Abstract—In this paper, an event-triggered control protocol is developed to investigate flocking control of Lagrangian systems, where event-triggering conditions are proposed to determine when the velocities of the agents are transmitted to their neighbours. In particular, the proposed controller is distributed, since it only depends on the available information of each agent on their own reference frame. In addition, we derive sufficient conditions to avoid Zeno behaviour. Numerical simulations are provided to show the effectiveness of the proposed control law.

Index Terms—Event-triggered control, flocking, Lagrangian systems.

I. INTRODUCTION

FLocking, swarming, and schooling are common emergent collective motion behaviors exhibited in nature. These natural collective behaviors can be leveraged in multirobot systems to safely transport large cohesive groups of robots within a workspace [1]. To capture these effects, Reynolds introduced three heuristic rules in [2]: cohesion; alignment; and separation, to reproduce flocking motions in computer graphics. Later, these rules were used to construct flocking control algorithms. In [3], the authors designed a control law that captures the following three Reynolds rules by using a collective potential function and a velocity consensus term:

- **Cohesion**: Each agent should stay close to its neighbours.
- **Alignment**: Each agent should synchronize its velocity with its neighbours.
- **Separation**: Agents cannot collide with their neighbours.

E. Aranda-Escolástico and M. Guinaldo have been supported by the Spanish Ministry of Science and Innovation under Projects CI-CYT RTI2018-094655-B-I00, RTI2018-096590-B-I00 and by Agencia Estatal de Investigación (AEI) under the project PID2020-112658RB-I00/AEI/10.13039/501100011033. L. Colombo have been founded by ‘la Caixa’ Foundation under the project “Decentralized strategies for cooperative robotic swarms” with project code LCF/BQ/PI19/11690016, and by the Spanish Ministry of Science and Innovation under Project PID2019-106715GB-C21.

E. Aranda-Escolástico is with the Department of Software and Systems Engineering, Universidad Nacional de Educación a Distancia (UNED), 28040 Madrid, Spain, earanda@issi.uned.es

L. J. Colombo is with the Centre for Automation and Robotics (CSIC-UPM), Ctra. M300 Campo Real, Km 0.200, Arganda del Rey - 28500 Madrid, Spain, leonardo.colombo@car.upm-csic.es

M. Guinaldo is with the Computer Science and Automatic Control Department, UNED, Juan del Rosal 16, 28040, Madrid, Spain, mguinaldo@dia.uned.es

©2021 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.
partial linearization to implement adaptive control laws [27]–[29]. We further provide sufficient conditions to avoid Zeno behavior in the proposed setting with a trigger function that only depends on the local error and broadcasted states, and does not require additional variables to exclude the Zeno behavior [27].

We begin by reviewing the background about graph theory and forced Euler-Lagrange equations for flocking control in Section II. Section III addresses the problem of event-triggered control for flocking of Euler-Lagrange systems and provide sufficient conditions to avoid Zeno behavior. Finally, a numerical example with a swarm of 50 underwater vehicles is presented in Section IV.

II. BACKGROUND AND PROBLEM FORMULATION

A. Graph theory

Consider an undirected graph denoted by $G = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, 2, \ldots, s\}$ denotes a finite and nonempty set of nodes and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ a set of unordered pairs of nodes. Denote by $|\mathcal{N}|$ the cardinality of the set $\mathcal{N}$. Neighbor’s relationships of an agent $i \in \mathcal{N}$ are described by the set $\mathcal{N}_i := \{j \in \mathcal{N} : \{i, j\} \in \mathcal{E}\}$.

An arc $\{j, i\} \in \mathcal{E}$ describes that nodes $i, j$ receive each other’s information reciprocally. A path between $i_1$ and $i_k$, $k \leq |\mathcal{N}|$, is a sequence of arcs of the form $\{i_1, i_2\}, \{i_2, i_3\}, \ldots, \{i_{k-1}, i_k\}$. If each node of an undirected graph $\mathcal{G}$ has an undirected arc graph $\mathcal{G}$ and an adjacency matrix, a matrix $A = [a_{ij}]_{|\mathcal{N}| \times |\mathcal{N}|}$ defined by $a_{ij} > 0$ if $\{i, j\} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Since $\mathcal{G}$ is undirected, $A$ is a symmetric matrix, i.e. $a_{ij} = a_{ji}$, for all $i, j \in \mathcal{N}$.

B. Agents Dynamics: Forced Euler-Lagrange equations

Consider $s \geq 2$ autonomous agents whose positions are denoted by $q_i \in \mathbb{R}^d$, and denote by $q \in \mathbb{R}^{d|\mathcal{N}|}$ the stacked vector of agents’ positions.

The neighbor relationships between agents are described by the undirected graph $\mathcal{G}$ which is assumed to be time-invariant and connected. The stacked vector of relative positions between neighboring agents, denoted by $z \in \mathbb{R}^{d|\mathcal{E}|}$, is given by $z = B^{-1}q$, where $B := B \otimes I_d \in \mathbb{R}^{d|\mathcal{N}| \times d|\mathcal{E}|}$, with $B$ being the incidence matrix for $\mathcal{G}$. Note that $zk_r \in \mathbb{R}^d$ and $zk_{k+|\mathcal{E}|} \in \mathbb{R}^d$ in $z$ correspond to $q_i - q_j$ and $q_j - q_i$ for the edge $E_k$. Define $zk_r := q_i - q_j$ and consider the desired distance between neighboring agents over the edge $E_k$ as $dk_r$.

Next, assume the motion of the agent $i \in \mathcal{N}$ is determined by a Lagrangian function $L_i : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$, that is, the Euler-Lagrange equations for $L_i$ describe the dynamics for the system. The Lagrangian function for agent $i \in \mathcal{N}$, in generalized coordinates, is given by $L_i(q_i, \dot{q}_i) = K_i(q_i, \dot{q}_i) - U_i(q_i)$, where $K_i$ and $U_i$ are the kinetic and potential energies, respectively.

While conservative forces are included into the potential energy, a non-conservative force between agents on an edge can be defined by a smooth map $F_{ij} : (\mathbb{R}^d \times \mathbb{R}^d) \times (\mathbb{R}^d \times \mathbb{R}^d) \rightarrow (\mathbb{R}^d \times \mathbb{R}^d)$.

We can expand the forced Euler-Lagrange equations, by computing the time derivative. Expanding the previous expression, equations (I) takes the form

$$\frac{\partial^2 L_i}{\partial q_i \partial \dot{q}_j} \dot{q}_i + \frac{\partial^2 L_i}{\partial q_j \partial \dot{q}_i} \dot{q}_j = F_{ij}(q_i, \dot{q}_i, \dot{q}_j, \dot{q}_j) + \frac{\partial L_i}{\partial \dot{q}_i}$$

Equations (I) determine a system of implicit second-order differential equations. The Lagrangian $L_i$ is said to be regular (see for instance [31]), if for each $i \in N$, the $(d|\mathcal{N}| \times d|\mathcal{N}|)$ block matrix $M_i(q_i, \dot{q}_i)$ with blocks $M_i(q_i, \dot{q}_i) := \left(\frac{\partial^2 L_i}{\partial q_j \partial \dot{q}_i} \right)_{d \times d}$ is non-singular. In such a case, the local existence and uniqueness of solutions is guaranteed for any given initial condition.

Remark 1: Note that the flocking stabilization systems (i.e., flocking control for double-integrator agents [8], [10]) can be seen as forced Euler-Lagrange equations (I) by considering the Lagrangian function $L : \mathbb{R}^{d|\mathcal{N}|} \times \mathbb{R}^{d|\mathcal{N}|} \rightarrow \mathbb{R}$ given by

$$L(q, \dot{q}) = \frac{1}{2} \sum_{i=1}^{|\mathcal{N}|} \left(\|\dot{q}_i\|^2 - 2 \sum_{j \in \mathcal{N}_i} V_{ij}(q_i, q_j)\right),$$

together with $F_{ij} = \sum k_{j \in \mathcal{N}_i} (\dot{q}_j - \dot{q}_i)$ and $\dot{q}_i = v_i$.

C. Problem formulation

Consider a network given by $s \geq 2$ agents, each one with a dynamics evolving according to the Euler-Lagrange equations associated with the Lagrangian $L_i : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$. The network is modelled by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ that is assumed to be connected and time-invariant.

Form equations (II) we can identify the Coriolis and Mass matrices associated with the Lagrangian $L_i$. These matrices must satisfy (see for instance [31]):

(P1) $M_i(q_i, \dot{q}_i)$ is positive definite and bounded for any $q_i \in \mathbb{R}^d$ and $i \in \mathcal{N}$. That is, there exists $\omega_i, \theta_i \in \mathbb{R}_{>0}$ such that $\omega_i I_{d \times d} \preceq M_i(q_i, \dot{q}_i) \preceq \theta_i I_{d \times d}$.

(P2) $M_i(q_i, \dot{q}_i) = 2C_i(q_i, \dot{q}_i)$ is skew-symmetric, for each $i \in \mathcal{N}$.

(P3) $C_i(q_i, \dot{q}_i)$ is bounded w.r.t. $\dot{q}_i$ for each $i \in \mathcal{N}$ and linearly bounded w.r.t. $\dot{q}_i$. That is, there exists $c_i \in \mathbb{R}_{>0}$ such that for all $i \in \mathcal{N}$, $\|C_i(q_i, \dot{q}_i)\| \leq c_i \|\dot{q}_i\|$.

(P4) If $\dot{q}_i, \dot{q}_i \in L^\infty$, then $\frac{d}{dt} C_i(q_i, \dot{q}_i)$ is a bounded operator.
The goal is to show that under these conditions agents can achieve flocking motion based on Raynolds rules of alignment, cohesion and separation under an event-triggered framework. It is demonstrated that each agent is equipped with the following sensing and communication capabilities:

(A1) Agent \( i \in \mathcal{N} \) is able to measure the distance to its nearest neighbours continuously.

(A2) Agent \( i \in \mathcal{N} \) transmits its velocity to its neighbours at fixed instants of time, which should be determined.

Note that these assumptions agree with the reality, in the sense that many mobile robots are equipped with a set of sensors that allow them to measure its velocity and relative positions respect to other robots or obstacles. Nevertheless, getting a measurement or an estimation of other agents’ velocities is not easy, and flocking control requires agents to exchange this information to synchronize themselves with their nearest neighbors. Otherwise, additional assumptions over the system model and a significant increase on the computation onboard are needed [28], [29], [32]. Though recent works have addressed the problem of achieving cooperative control objectives in multi-agent systems with position measurements only (such as in [33] for the consensus problem of double integrator agents), the design of observers in cooperative Euler-Lagrange systems is generally high dimensional and complex [34]–[36]. Hence, we propose to employ an event-based strategy for the transmission of the velocity measurements in order to reduce the communication exchange. In an event-triggered policy, the agent \( j \in \mathcal{N}_i \) sends the information at instants \( t_k^j \) with \( k \in \mathbb{N} \) and these measurements are obtained by the agent \( i \in \mathcal{N}_j \).

III. CONTROLLER DESIGN

In this section we provide an event-triggered control law for flocking of Euler-Lagrange systems. In particular, we consider \( s \geq 2 \) agents with dynamics described by the Euler-Lagrange equations as in Section II-B. Note that we can write equations (2) as

\[
\begin{align*}
M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i &= F_i, \\
V_i &= \sum_{j \in \mathcal{N}_i} V_{ij}(\|z_{jk}\|).
\end{align*}
\]  

(4)

where \( M_i \) and \( C_i \) satisfies properties (P1)-(P4) and \( F_i := F_{ij}(q_i, q_j, \dot{q}_i, \dot{q}_j) \) is the control law to be designed for each agent to satisfy Raynolds rules of flocking. According to the Reynolds model [2], the motion of each agent in the flock is defined by the three rules of alignment, cohesion and separation, weighted by positive constant coefficients \( \alpha_i \) and \( \beta_i \). We will refer to them as Reynolds gains.

To satisfy these rules, we consider for each agent a term in the control law based on the gradient of the potential function

\[
V_i = \sum_{j \in \mathcal{N}_i} V_{ij}(\|z_{jk}\|).
\]

For any \( t \in [t_k^i, t_{k+1}^i) \) such that

\[
t_{k+1}^i = \inf\{t > t_k : f_i(e_i(t), \dot{q}_i(t)) > 0\},
\]

where \( f_i : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R} \) is the triggering function for the \( i \)-th agent given by

\[
f_i(t, e_i(t)) = \|e_i(t)\|^2 - \frac{\sigma_i}{2} \sum_{j \in \mathcal{N}_i} \beta_i \|\dot{q}_i(t_k^i) - \dot{q}_j(t_k^i)\|^2
\]

(10)

where \( \sigma_i \in \mathbb{R}_{>0} \) determines the number of triggered events.

Hence, the proposed control law for the coordination of multiple mechanical systems satisfying Raynolds rules of flocking, based on an event-triggered protocol, is given by

\[
F_{ij}(q_i, q_j, \dot{q}_i, \dot{q}_j) = -\alpha_i \sum_{j \in \mathcal{N}_i} \nabla_{q_i} V_{ij} - \frac{\partial L}{\partial q_i} - \beta_i \sum_{j \in \mathcal{N}_i} (\dot{q}_i(t_k^i) - \dot{q}_j(t_k^i)),
\]

(11)
so, the control law $F_i$ for agent $i \in N$ is given by

$$F_i(q_i, q_j, \dot{q}_i, \dot{q}_j) = -\alpha_i \sum_{j \in N_i} \nabla_{q_i} V_{ij} - \beta_i \sum_{j \in N_i} (\dot{q}_i(t_k^j) - \dot{q}_j(t_k^j)).$$

(12)

The next result shows convergence to flocking motion with the designed event triggered control law.

**Theorem 1:** Under the sensing and communication assumptions (A1)-(A2), a network of $s \geq 2$ agents with dynamics (4) and control law (12) achieves stable flocking if the control gain and the parameter of the event-triggering function (10) fulfill $\beta_i > 0$ and $0 < \sigma_i < 1$ for all $i \in N$, respectively.

**Proof:** Consider a candidate Lyapunov function

$$V = \sum_{i=1}^{\left| N \right|} \sum_{j \in N_i} \alpha_i V_{ij} + \frac{1}{2} \sum_{i=1}^{\left| N \right|} \dot{q}_i^T M_i \dot{q}_i.$$  
(13)

Its time derivative is

$$\dot{V} = \sum_{i=1}^{\left| N \right|} \sum_{j \in N_i} \alpha_i \dot{q}_i^T \nabla_{q_i} V_{ij} + \frac{1}{2} \sum_{i=1}^{\left| N \right|} \dot{q}_i^T M_i \dot{q}_i.$$

(14)

By using equation (3), equation (14) can be written as

$$\dot{V} = \sum_{i=1}^{\left| N \right|} \sum_{j \in N_i} \alpha_i \dot{q}_i^T \nabla_{q_i} V_{ij} + \frac{1}{2} \sum_{i=1}^{\left| N \right|} \dot{q}_i^T M_i \dot{q}_i.$$

The first term reduces to

$$\sum_{i=1}^{\left| N \right|} \sum_{j \in N_i} \alpha_i \dot{q}_i^T \nabla_{q_i} V_{ij} + \frac{1}{2} \sum_{i=1}^{\left| N \right|} \dot{q}_i^T M_i \dot{q}_i.$$  

(15)

On the one hand, since the graph is undirected and connected, the first term in (15) can be rewritten such as

$$\sum_{i=1}^{\left| N \right|} \sum_{j \in N_i} \alpha_i \dot{q}_i^T \nabla_{q_i} V_{ij} + \frac{1}{2} \sum_{i=1}^{\left| N \right|} \dot{q}_i^T M_i \dot{q}_i$$

$$\leq \sum_{i=1}^{\left| N \right|} \sum_{j \in N_i} \alpha_i \dot{q}_i^T \nabla_{q_i} V_{ij} + \frac{1}{2} \sum_{i=1}^{\left| N \right|} \dot{q}_i^T M_i \dot{q}_i.$$

(16)

This implies that $V$ is bounded and has a limit. Therefore, $\lim_{t \to \infty} \|\dot{q}_i(t_k^j) - \dot{q}_j(t_k^j)\| = 0$. Observing (3) and (16), this implies that $e_i \to 0$ when $t \to \infty$ and, consequently, $\|\dot{q}_i - \dot{q}_j\| \to 0$ when $t \to \infty$. Moreover, since $V$ is bounded, then $\dot{V}_{ij}$ is also bounded. Thus, collisions between the interconnected agents are avoided, because of (V1), i.e., since $V_{ij} \to \pm \infty$ when $\|z_k\| \to 0$.

In any event-triggered control framework it is important to show that Zeno behaviour is avoided. Next we provide sufficient conditions to avoid Zeno behaviour.

**Corollary 1:** The event-triggered control law (12) avoids Zeno behaviour whenever (V3) holds.

**Proof:** We next show that a minimum interevent time $0 \leq t_m \leq t_{k+1} - t_k$ exists for all $i \in N$. To do that, we consider the time derivative of $e_i$ for $t \in [t_k^i, t_{k+1}^i]$

$$\frac{d}{dt} e_i(t) = -\dot{q}_i = M_i^{-1}(q_i) \left( \alpha \sum_{j \in N_i} \nabla_{q_i} V_{ij}(t) + \beta \sum_{j \in N_i} (\dot{q}_i(t_k^j) - \dot{q}_j(t_k^j)) + C_i(q_i, \dot{q}_i) \right).$$

By properties (P1)-(P3), $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$ are bounded. $\dot{q}_i$ is bounded because we have shown that the Lyapunov function is bounded in Theorem 1. Therefore, since $\nabla_{q_i} V_{ij}(t)$ is also bounded, then

$$\frac{d}{dt} e_i(t) \leq \|e_i(t)\| \leq l,$$

(17)

where $l$ is a positive constant. Taking $t \in [t_k^i, t_{k+1}^i]$, we obtain $\|e_i(t)\| \leq l(t-t_k^i) \leq l(t_{k+1}^i - t_k^i)$. Thus, $t_{k+1}^i - t_k^i \geq \|e_i(t)\|/l$ which is larger than 0 if $e_i(t) \neq 0$. Since $e_i(t) = 0$ is maintained only if the control objectives are achieved, the Zeno effect is avoided.

**IV. NUMERICAL EXAMPLE**

In this section, we test the distributed event-based control law (12) with a network of underwater vehicles. A detailed model of the vehicles is described in [22]. We consider a network of $s = 50$ fully actuated vehicles (rigid bodies) evolving on the special Euclidean group $SE(3)$ of rigid motions in the space. Any element of $SE(3)$ is given by

$$g_i = \begin{bmatrix} R_i & b_i \\ 0 & 1 \end{bmatrix}$$

with $R_i \in SO(3)$ describing the orientation for the $i^{th}$-body as a rotation matrix and $b_i = (b_i^x, b_i^y, b_i^z) \in \mathbb{R}^3$ is the position of the center of mass for the $i^{th}$-body in inertial frame, with $b_i = (\bar{b}_i^x, \bar{b}_i^y, \bar{b}_i^z)$ representing the velocity vector for agent $i$ in the directions $x, y, z$, respectively. The graph defining the neighbor’s relations is randomly generated (ensuring that it is connected and undirected).

Besides, in some rigid body applications, the mass matrix is usually given by $M_i = m_i \textbf{I}_3$, where $m_i$ is the mass of the body and $\textbf{I}_3$ its matrix of inertia moments. We will consider models for underwater vehicles where the elements of $M_i$ may be different due to the fact that added masses have to be taken into account.

For simplicity in the model, assume that possible dissipating forces acting on the body under the water are negligible. The potential energy for the $i^{th}$ underwater vehicle is given by

$$U_i(R_i, b_i) = \rho g_i (\bar{r}_i, R_i^T \bar{v}_3) + (\rho \bar{r}_i - m_i) g \bar{b}_i^2,$$

where $g$ is the
The control equations are given by

\[ \dot{\mathbf{r}}_i = \mathbf{v}_i, \quad \mathbf{v}_i = M_i \mathbf{v}_i - R_i^T (m_i - \rho_i \bar{g}) e_z + u_i, \]

\[ \mathbf{J}_i \mathbf{\Omega}_i = \mathbf{J}_i \mathbf{\Omega}_i + M_i \mathbf{v}_i \times \mathbf{v}_i - \rho_i \bar{g} \times (R_i^T e_z) + \bar{a}_i, \]

for \( i = 1, 2, 3, \) with \( \mathbf{\Omega}_i = (\Omega_{i1}^x, \Omega_{i2}^y, \Omega_{i3}^z) \) \( \in \mathbb{R}^3 \) orientation of agent \( i \) and \( \mathbf{\Omega}_i \) its associated skew-symmetric matrix under the \( \text{hat} \) isomorphism \( \hat{} : \mathbb{R}^3 \to \mathfrak{so}(3) \) [31]. For numerical simulations we consider all rigid bodies have mass (including added masses) \( m_i = 123.8 \text{ kg} \), and inertia matrices \( M_i = m_i \mathbf{I}_i + \text{diag}(65, 70, 75) \text{ kg} \), \( \mathbf{J}_i = \text{diag}(5.46, 5.29, 5.72) \text{ kg m}^2 \text{ and } \mathbf{I}_i = \mathbf{I}_{3 \times 3} \text{ kg m}^2 \) with \( \mathbf{I}_{3 \times 3} \) the \( (3 \times 3) \)-identity matrix. Also assume that \( \rho_i \bar{g} = 1215.8 \text{ N} \) and \( \bar{r}_i = (0, 0, -0.007)^T \text{ m} \).

We set the parameters of the control law and the event-triggering condition as \( \beta_i = 10 \text{ and } \sigma_i = 0.01 \) for \( i = 1, ..., 50 \), respectively. We choose the potential function described in [37], which satisfies (V1)-(V3) and whose gradient is

\[ \nabla q_j V_{ij} = \begin{cases} (0, 0, 0) & \|z_k\| > R \\
\frac{2\pi z_k \sin(2\pi (z_k - \|d_k\|))}{\|z_k\|} & z_k < \|z_k\| \leq R \\
0 & \|z_k\| \leq d_k \end{cases}, \]

where \( d_k = 0.5 \text{ m} \) and \( R > \max_k d_k \) is a positive constant set to 1. Initial conditions are also randomly chosen.

In Figure 1 we can observe the trajectory followed by the agents. First, agents are distanced by the repulsive potential while consensus in linear velocities is achieved and the agents maintain the formation (Figure 2 the velocities of the agents converge to three values corresponding to \( x, y, z \) directions). From the communication point of view, the average number of generated events is 99, i.e., each agent transmitted its velocity 99 times (in average) to their neighbours, so the average inter-event time is 2.02 s. The agent 44 generated the minimum number of transmissions (62) to its neighbours, while the agent 6 generated the maximum number (144). To illustrate the distribution of events over time, three agents have been selected (agents 1, 25 and 50), and the instances of event times are depicted as example in Figure 3. At the beginning, the agents need to exchange information of their velocities very frequently (but not continuously according to Corollary 1). However, once they are close to the consensus in velocities, the transmission of information is clearly reduced and communication resources are optimized.

We evaluate flocking behavior through the metrics described in [38]. The average minimum distance to a neighbor measures the cohesion between the agents. In this case, due to the nature of the potential, the distance between the agents is stabilized far enough to avoid collisions. Since the graph is fixed, we can use the average velocity difference to measure the consensus of velocities in 3D, which is clearly achieved in the example. These results are depicted in Figure 4.

V. CONCLUSIONS

We have designed an event-triggered control protocol for flocking control of multi-agent Lagrangian systems. In particular, the proposed controller in this work is distributed: it only depends on the available information of each agent on their own reference frame, the velocity of its neighbors at event times, and to the desired distance to the neighbors. We have also provided sufficient conditions to avoid Zeno behavior. The results show that the control objective is achieved while the amount of communication is reduced thanks to the event-triggering communication. Future work includes extension to
system with delays and underactuated systems with partially unknown dynamic.

**References**

[1] M. Rubenstein, A. Cornejo, and R. Nagpal, “Programmable self-assembly in a thousand-robot swarm,” *Science*, vol. 345, no. 6198, pp. 795–799, 2014.

[2] C. W. Reynolds, “Flocks, herds and schools: A distributed behavioral model,” in *Proceedings of the 14th Annual Conference on Computer Graphics and Interactive Techniques*, 1987, pp. 25–34.

[3] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, “Stable flocking of mobile agents, part I: Fixed topology,” in *42nd Conference on Decision and Control*, vol. 2. IEEE, 2003, pp. 2010–2015.

[4] R. Olfati-Saber and R. M. Murray, “Distributed cooperative control of multiple vehicle formations using structural potential functions,” *IFAC Proceedings Volumes*, vol. 35, no. 1, pp. 495–500, 2002.

[5] R. Olfati-Saber, “Flocking for multi-agent dynamic systems: Algorithms and theory,” *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[6] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, “Flocking in fixed and switching networks,” *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[7] F. Cucker and S. Smale, “Emergent behavior in flocks,” *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 852–862, 2007.

[8] M. Deghat, B. D. O. Anderson, and Z. Lin, “Combined flocking and distance-based shape control of multi-agent formations,” *IEEE Transactions on Automatic Control*, vol. 61, no. 7, pp. 1824–1837, 2016.

[9] Z. Sun, S. Mou, M. Deghat, B. D. O. Anderson, and A. S. Morse, “Finite time distance-based rigid formation stabilization and flocking,” *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 9183–9189, 2014.

[10] Z. Sun and B. D. O. Anderson, “Rigid formation control systems modelled by double integrators: System dynamics and convergence analysis,” in *Proceedings of the 5th Australian Control Conference*, 2015, pp. 241–246.

[11] A. Tavasoli, S. Taghvaei, and M. Eghtesad, “Flocking of a team of Lagrangian agents,” in *Proceedings of the IEEE International Conference on Robotics and Biomimetics*, 2009, pp. 1433–1438.

[12] H. Wang, “Flocking of networked uncertain Euler-Lagrange systems on directed graphs,” *Automatica*, vol. 49, no. 9, pp. 2774–2779, 2013.

[13] S. Ghapani, J. Mei, and W. Ren, “Flocking with a moving leader for multiple uncertain Lagrange systems,” in *Proceedings of the American Control Conference*, 2014, pp. 3189–3194.

[14] S. Ghapani, F. Mei, W. Ren, and Y. Song, “Fully distributed flocking with a moving leader for Lagrange networks with parametric uncertainties,” *Automatica*, vol. 67, pp. 67–76, 2016.

[15] X. Li, H. Su, and M. Z. Chen, “Flocking of networked Euler–Lagrange systems with uncertain parameters and time-delays under directed graphs,” *Nonlinear Dynamics*, vol. 85, no. 1, pp. 415–424, 2016.

[16] Y. Dong and J. Huang, “Consensus and flocking with connectivity preservation of uncertain Euler–Lagrange multi-agent systems,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 140, no. 9, p. 091011, 2018.

[17] Y. Mao, L. Dou, H. Fang, and J. Chen, “Distributed flocking of Lagrangian systems with global connectivity maintenance,” in *Proceedings of the IEEE International Conference on Cyber Technology in Automation, Control and Intelligent Systems*, 2013, pp. 69–74.

[18] S. Yazdani and H. Su, “Adaptive leader-follower flocking for uncertain Lagrange systems with input saturation and external disturbances,” in *Proceedings 37th Chinese Control Conference*, 2018, pp. 7210–7214.

[19] Z. Feng and G. Hu, “Connectivity-preserving flocking for networked Lagrange systems with time-varying actuator faults,” *Automatica*, vol. 109, p. 108509, 2019.

[20] M. Guinaldo, D. V. Dimarogonas, K. H. Johansson, J. Sánchez, and S. Dormido, “Distributed event-based control strategies for interconnected linear systems,” *IEEE Control Theory & Applications*, vol. 7, no. 6, pp. 877–886, 2013.

[21] E. Aranda-Escólástico, M. Guinaldo, R. Heradio, J. Chacon, H. Vergas, J. Sánchez, and S. Dormido, “Event-based control: A bibliometric analysis of twenty years of research,” *IEEE Access*, vol. 8, pp. 47188–47208, 2020.

[22] E. Aranda-Escólástico, L. Colombo, and M. Guinaldo, “Periodic event-triggered target leader control for Lagrangian systems with discrete-time delays,” *ISA Transactions*, vol. 117, pp. 139–149, 2021.

[23] P. Yu, L. Ding, Z.-W. Liu, and Z.-H. Guan, “Leader–follower flocking based on distributed event-triggered hybrid control,” *International Journal of Robust and Nonlinear Control*, vol. 26, pp. 143–153, 2016.

[24] Y. Shen, Z. Kong, and L. Ding, “Flocking of multi-agent system with nonlinear dynamics via distributed event-triggered control,” *Applied Sciences*, vol. 9, no. 7, p. 1336, 2019.

[25] F. Sun, R. Wang, W. Zhu, and Y. Li, “Flocking in nonlinear multi-agent systems with time-varying delay via event-triggered control,” *Applied Mathematics and Computation*, vol. 350, pp. 66–77, 2019.

[26] X. Liu, C. Du, P. Lu, and D. Yang, “Decentralised consensus for multiple Lagrangian systems based on event-triggered strategy,” *International Journal of Control*, vol. 89, no. 6, pp. 1111–1124, 2016.

[27] X. Jin, W. Du, W. He, L. Kocarev, Y. Tang, and J. Kurths, “Twisting-based finite-time consensus for Euler-Lagrange systems with an event-triggered strategy,” *IEEE Transactions on Network Science and Engineering*, vol. 7, no. 3, pp. 1007–1018, 2019.

[28] L. Chen, C. Li, B. Xiao, and Y. Guo, “Formation-containment control of networked Euler-Lagrange systems: An event-triggered framework,” *ISA Transactions*, vol. 86, pp. 87–97, 2019.

[29] X.-Y. Yao, H.-F. Ding, M.-F. Ge, and J. H. Park, “Event-triggered synchronization control of networked Euler-Lagrange systems without requiring relative velocity information,” *Information Sciences*, vol. 508, pp. 183–199, 2020.

[30] L. Colombo, P. Moreno, M. Ye, H. G. de Marina, and M. Cao, “Forced variational integrator for distance-based shape control with flocking behavior of multi-agent systems,” *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 3348–3353, 2020.

[31] R. M. Murray, Z. Li, S. S. Sastry, and S. S. Sastry, *A mathematical introduction to robotic manipulation*, 1994.

[32] L.-M. Chen, C.-J. Li, J. Mei, and G.-F. Ma, “Adaptive cooperative formation-containment control for networked euler–lagrange systems without using relative velocity information,” *IEEE Control Theory & Applications*, vol. 11, no. 9, pp. 1450–1458, 2017.

[33] S. A. Ajwad, T. Menard, E. Moulay, M. Defoort, and P. Coirault, “Observer based leader-following consensus of second-order multi-agent systems with nonuniform sampled position data,” *Journal of the Franklin Institute*, vol. 356, no. 16, pp. 10 031–10 057, 2019.

[34] E. Nino, “Consensus of Euler-Lagrange systems using only position measurements,” *IEEE Transactions on Control of Network Systems*, vol. 5, no. 1, pp. 489–498, 2016.

[35] Q. Yang, H. Fang, J. Chen, Z.-P. Jiang, and M. Cao, “Distributed global output-feedback control for a class of Euler-Lagrange systems,” *IEEE Transactions on Automatic Control*, vol. 62, no. 9, pp. 4855–4861, 2017.

[36] Z. Peng, L. Liu, and J. Wang, “Output-feedback flocking control of multiple autonomous surface vehicles based on data-driven adaptive extended state observers,” *IEEE Transactions on Cybernetics*, 2020.

[37] Y. Cao and W. Ren, “Distributed coordinated tracking with reduced communication via a variable structure approach,” *IEEE Transactions on Automatic Control*, vol. 57, no. 1, pp. 33–48, 2011.

[38] Z. T. Jiahao, L. Pan, and M. A. Hsieh, “Learning to swarm with knowledge-based neural ordinary differential equations,” *arXiv preprint arXiv:2109.04927*, 2021.
