A partial $\mu$-$\tau$ symmetry and its prediction for leptonic CP violation

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Abstract

We find that the lepton flavor mixing matrix $U$ should possess a partial $\mu$-$\tau$ permutation symmetry $|U_{\mu 1}| = |U_{\tau 1}|$, and the latter predicts a novel correlation between the Dirac CP-violating phase $\delta$ and three flavor mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ in the standard parametrization. Inputting the best-fit values of these angles reported by Capozzi et al, we obtain the prediction $\delta \simeq 255^\circ$ in the normal neutrino mass ordering, which is in good agreement with the best-fit result $\delta \simeq 250^\circ$. In this connection the inverted neutrino mass ordering is slightly disfavored. If this partial $\mu$-$\tau$ symmetry is specified to be $|U_{\mu 1}| = |U_{\tau 1}| = 1/\sqrt{6}$, one can reproduce the phenomenologically-favored relation $\sin^2\theta_{12} = (1 - 2 \tan^2\theta_{13})/3$ and a viable two-parameter description of $U$ which were first uncovered in 2006. Moreover, we point out that the octant of $\theta_{23}$ and the quadrant of $\delta$ can be resolved thanks to the slight violation of $|U_{\mu 2}| = |U_{\tau 2}|$ and $|U_{\mu 3}| = |U_{\tau 3}|$ either at the tree level or from radiative corrections.

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Thanks to a number of successful experiments which have convincingly revealed the solar, atmospheric, reactor and accelerator neutrino (or antineutrino) oscillations \cite{1}, the fact that the three known neutrinos have finite masses and lepton flavors are significantly mixed has been established on solid ground. While the origin of neutrino masses requires the existence of new physics which remains unknown or unsure to us, the leptonic weak charged-current interactions can be described in the standard way as follows:

\[- \mathcal{L}_{ee} = \frac{g}{\sqrt{2}} (e^\mu \tau_\nu) L \gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^- + \text{h.c.}, \tag{1}\]

where the charged-lepton and neutrino fields are all the mass eigenstates, and $U$ denotes the $3 \times 3$ lepton flavor mixing matrix \cite{2}. If $U$ is unitary, it can be parametrized in terms of three mixing angles and three CP-violating phases:

\[U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i \delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i \delta} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i \delta} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i \delta} & c_{13} c_{23} \end{pmatrix} P_\nu, \tag{2}\]

where $c_{ij} \equiv \cos \theta_{ij}, s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13, 23$), $\delta$ is often referred to as the Dirac CP-violating phase, and $P_\nu = \text{Diag} \{e^{i \theta}, e^{i \theta}, 1\}$ is a diagonal Majorana phase matrix which has nothing to do with neutrino oscillations. So far $\theta_{12}, \theta_{13}$ and $\theta_{23}$ have all been measured to a good degree of accuracy, and a preliminary hint for a nontrivial value of $\delta$ has also been obtained from a global analysis of current neutrino oscillation data \cite{3}. Here let us quote the results obtained recently by Capozzi et al. \cite{4} and list them in Table 1, from which the best-fit values of $\theta_{12}, \theta_{13}, \theta_{23}$ and $\delta$ are found to be

\[\theta_{12} \simeq 33.7^\circ, \quad \theta_{13} \simeq 8.8^\circ, \quad \theta_{23} \simeq 41.4^\circ, \quad \delta \simeq 250^\circ, \tag{3}\]

for the normal neutrino mass ordering (i.e., $m_1 < m_2 < m_3$); or

\[\theta_{12} \simeq 33.7^\circ, \quad \theta_{13} \simeq 8.9^\circ, \quad \theta_{23} \simeq 42.4^\circ, \quad \delta \simeq 236^\circ, \tag{4}\]

for the inverted neutrino mass ordering (i.e., $m_3 < m_1 < m_2$). How to understand the observed pattern of lepton flavor mixing is one of the fundamental issues in particle physics. Before $\theta_{13}$ was measured, a lot of phenomenological interest had been paid to the $\mu$-$\tau$ permutation symmetry for model building. The $\mu$-$\tau$ symmetry is powerful in the sense that it can constrain the neutrino mass texture and predict $\theta_{23} = 45^\circ$ together with $\theta_{13} = 0^\circ$ or $\delta = 90^\circ$ (or $270^\circ$), but its validity has been being challenged since a relatively large value of $\theta_{13}$ was uncovered by the Daya Bay experiment \cite{5} in 2012. Once the octant of $\theta_{23}$ is fixed and the value of $\delta$ is measured in the near or foreseeable future, it will be much easier to probe the possible flavor symmetry behind $U$ and even pin down the relevant symmetry breaking effects.

In this Letter we stress that even a partial $\mu$-$\tau$ symmetry, defined as $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, \text{or } 3$), is powerful enough for us to establish some testable relations among the three neutrino mixing angles and the Dirac CP-violating phase. Note that $|U_{\mu 3}| = |U_{\tau 3}|$ is trivial because it predicts $\theta_{23} = 45^\circ$, which is apparently in conflict with the best-fit value of $\theta_{23}$. But we find that the partial $\mu$-$\tau$ symmetry $|U_{\mu 1}| = |U_{\tau 1}|$ is quite nontrivial and can predict a novel correlation between $\delta$ and $(\theta_{12}, \theta_{13}, \theta_{23})$, from which we obtain the prediction $\delta \simeq 255^\circ$ in the normal neutrino mass ordering. This result is certainly in good agreement with the best-fit result $\delta \simeq 250^\circ$ given in Eq. (3). In this connection the inverted neutrino mass ordering is slightly disfavored. Moreover, $|U_{\mu 2}| = |U_{\tau 2}|$ is phenomenologically disfavored. If the partial

\[1\]
\begin{table}
\centering
\begin{tabular}{|l|l|l|l|}
\hline
Parameter & Best fit & 1σ range & 2σ range & 3σ range \\
\hline
\multicolumn{5}{|c|}{Normal neutrino mass ordering (m_1 < m_2 < m_3)} \\
\hline
\sin^2 \theta_{12}/10^{-1} & 3.08 & 2.91 — 3.25 & 2.75 — 3.42 & 2.59 — 3.59 \\
\sin^2 \theta_{13}/10^{-2} & 2.34 & 2.15 — 2.54 & 1.95 — 2.74 & 1.76 — 2.95 \\
\sin^2 \theta_{23}/10^{-1} & 4.37 & 4.14 — 4.70 & 3.93 — 5.52 & 3.74 — 6.26 \\
\delta/180° & 1.39 & 1.12 — 1.77 & 0.00 — 0.16 ± 0.86 — 2.00 & 0.00 — 2.00 \\
\hline
\multicolumn{5}{|c|}{Inverted neutrino mass ordering (m_3 < m_1 < m_2)} \\
\hline
\sin^2 \theta_{12}/10^{-1} & 3.08 & 2.91 — 3.25 & 2.75 — 3.42 & 2.59 — 3.59 \\
\sin^2 \theta_{13}/10^{-2} & 2.40 & 2.18 — 2.59 & 1.98 — 2.79 & 1.78 — 2.98 \\
\sin^2 \theta_{23}/10^{-1} & 4.55 & 4.24 — 5.94 & 4.00 — 6.20 & 3.80 — 6.41 \\
\delta/180° & 1.31 & 0.98 — 1.60 & 0.00 — 0.02 ± 0.70 — 2.00 & 0.00 — 2.00 \\
\hline
\end{tabular}
\caption{The best-fit values, together with the 1σ, 2σ and 3σ intervals, for the three neutrino mixing angles and the Dirac CP-violating phase from a global analysis of current experimental data \cite{4}.}
\end{table}

\(\mu-\tau\) symmetry is specified to be \(|U_{\mu 1}| = |U_{\tau 1}| = 1/\sqrt{6}\), one can easily reproduce the successful relation \(\sin^2 \theta_{12} = (1 - 2\tan^2 \theta_{13})/3\) and a viable two-parameter description of \(U\) which were first proposed and discussed in 2006. We thus come to an important conclusion from looking at the global fit of the present neutrino oscillation data: behind the observed pattern of lepton flavor mixing should be the partial \(\mu-\tau\) symmetry \(|U_{\mu 1}| = |U_{\tau 1}|\), and leptonic CP violation must be quite significant. This observation may serve as one of the guiding principles for further model building in the lepton sector. We also point out that the octant of \(\theta_{23}\) and the quadrant of \(\delta\) can be resolved thanks to the slight violation of \(|U_{\mu 2}| = |U_{\tau 2}|\) and \(|U_{\mu 3}| = |U_{\tau 3}|\) either at the tree level or from radiative corrections.

Without involving any model-dependent uncertainties, let us focus on the \(\mu-\tau\) permutation symmetry of the lepton flavor mixing matrix \(U\) itself. As already pointed out in Ref. \cite{4}, the equalities \(|U_{\mu i}| = |U_{\tau i}|\) (for \(i = 1, 2, 3\)) simultaneously hold if the following conditions are satisfied:

- Case A: \(\theta_{23} = 45°\) and \(\theta_{13} = 0°\). This possibility has been excluded by the Daya Bay experiment \cite{5};

- Case B: \(\theta_{23} = 45°\) and \(\delta = 90°\) or \(270°\). This possibility is disfavored but still allowed by the present experimental data at the 2σ level, as one can see from Table 1.

Case B is clearly illustrated by the \(\delta\)-versus-\(\theta_{23}\) plot shown in Fig. 1, in which the crosspoints \(P\) and \(Q\) correspond to \(\delta = 90°\) and \(270°\), respectively. The line dictated by \(|U_{\mu 3}| = |U_{\tau 3}|\) is actually independent of \(\delta\) in the given parametrization of \(U\), while the curves dictated by \(|U_{\mu 1}| = |U_{\tau 1}|\) and \(|U_{\mu 2}| = |U_{\tau 2}|\) involve some uncertainties coming from the 1σ ranges of \(\theta_{12}\) and \(\theta_{13}\).

When the best-fit values of \(\delta\) and \(\theta_{23}\) are taken into account, Fig. 1 shows that the partial \(\mu-\tau\) symmetry \(|U_{\mu 1}| = |U_{\tau 1}|\) is amazingly favored if the neutrino mass ordering is normal. This fact is easily understood, since \(|U_{\mu 1}| = |U_{\tau 1}|\) predicts a simple but nontrivial correlation among \(\theta_{12}\), \(\theta_{13}\), \(\theta_{23}\) and \(\delta\):

\[
\cos \delta = \frac{1}{2} (\sin^2 \theta_{13} - \tan^2 \theta_{12}) \cot \theta_{12} \csc \theta_{13} \cot 2\theta_{23}.
\]  

Given \(\tan^2 \theta_{12} > \sin^2 \theta_{13}\), the CP-violating phase \(\delta\) must be in the second or third quadrant if \(\theta_{23} < 45°\) holds. This observation is qualitatively consistent with the best-fit results of \(\theta_{23}\) and \(\delta\) in Eq. (3) or (4).
Figure 1: The numerical correlation between $\delta$ and $\theta_{23}$ as dictated by the partial $\mu$-$\tau$ permutation symmetry $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) in the normal or inverted neutrino mass ordering, where the dotted lines and shaded areas come respectively from inputting the best-fit values and $1\sigma$ ranges of $\theta_{12}$ and $\theta_{13}$, while the hashed regions denote the $1\sigma$ ranges of $\delta$ and $\theta_{23}$ [4]. Moreover, the horizontal dotted-dashed line stands for the best-fit value of $\theta_{23}$. Three dotted lines cross at points $P$ and $Q$, corresponding to $(\theta_{23}, \delta) = (45^\circ, 90^\circ)$ and $(45^\circ, 270^\circ)$, respectively. Note that the location of the best-fit values $(\theta_{23}, \delta) \simeq (41.4^\circ, 250^\circ)$ shown in Fig. 1(a) is marked by the point in the blue circle and very close to the location of $(\theta_{23}, \delta) \simeq (41.4^\circ, 255^\circ)$ as predicted by Eq. (5). But in the inverted neutrino mass ordering case shown in Fig. 1(b), there is an obvious discrepancy between the best-fit value $\delta \simeq 236^\circ$ and the prediction $\delta \simeq 259^\circ$.

Using the best-fit values of $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ as the inputs, we find that Eq. (5) quantitatively predicts

$$\delta \simeq \begin{cases} 105^\circ \text{ or } 255^\circ & \text{(normal mass ordering)} , \\ 101^\circ \text{ or } 259^\circ & \text{(inverted mass ordering)} . \end{cases}$$

One can see that the prediction $\delta \simeq 255^\circ$ is in good agreement with the preliminary best-fit result $\delta \simeq 250^\circ$. 


in the normal neutrino mass ordering, while the prediction $\delta \approx 259^\circ$ in the inverted neutrino mass ordering is slightly disfavored as compared with the corresponding best-fit result $\delta \approx 236^\circ$. Fig. 1 shows the location of the best-fit points of $\theta_{23}$ and $\delta$. It coincides with the curve dictated by $|U_{\mu_1}| = |U_{\tau_1}|$ to a good degree of accuracy when the neutrino mass ordering is normal.

For the sake of comparison, we have also examined the possible partial $\mu-\tau$ symmetries by taking account of a recent and independent global analysis of the neutrino oscillation data done by Forero et al in Ref. [7]. The numerical results are shown in Fig. 2. Since the global-fit results obtained in Ref. [7] slightly favor the second octant of $\theta_{23}$, the best-fit value of $\delta$ is closer to the prediction based on the partial $\mu-\tau$ symmetry $|U_{\mu_2}| = |U_{\tau_2}|$ in the normal neutrino mass ordering. In the inverted neutrino mass ordering, however, the best-fit value of $\delta$ is lying between the predictions from $|U_{\mu_1}| = |U_{\tau_1}|$ and $|U_{\mu_2}| = |U_{\tau_2}|$, with a slight preference for the former. In this work we concentrate our attention only on the the partial $\mu-\tau$ symmetry $|U_{\mu_1}| = |U_{\tau_1}|$ and its phenomenological implications.

Note also that the partial $\mu-\tau$ permutation symmetry $|U_{\mu_1}| = |U_{\tau_1}|$ immediately leads us to the relation

$$
|U_{\mu_2}|^2 - |U_{\tau_2}|^2 = |U_{\tau_3}|^2 - |U_{\mu_3}|^2 = \cos^2 \theta_{13} \cos 2\theta_{23} = \cos^2 \theta_{13} \sin 2\epsilon ,
$$

where $\epsilon \equiv 45^\circ - \theta_{23}$ characterizes the octant of $\theta_{23}$. This result indicates that the deviation of $\theta_{23}$ from $45^\circ$ actually originates from a slight violation of the partial $\mu-\tau$ symmetry $|U_{\mu_2}| = |U_{\tau_2}|$ or $|U_{\mu_3}| = |U_{\tau_3}|$. In other words, $|U_{\tau_3}| > |U_{\mu_3}|$ and $|U_{\mu_2}| > |U_{\tau_2}|$ hold if $\theta_{23}$ lies in the first octant (i.e., $\epsilon > 0$); and the inverse inequalities hold if $\theta_{23}$ lies in the second octant (i.e., $\epsilon < 0$). Because of $\cos \delta \propto \tan 2\epsilon$, as one can see from Eq. (5), the quadrant of $\delta$ is also associated with the sign of $\epsilon$. In view of the best-fit value of $\theta_{23}$ given in Eq. (3) or (4), one obtains $\epsilon \simeq 3.6^\circ$ (or $2.6^\circ$) in the normal (or inverted) neutrino mass ordering. The upcoming neutrino oscillation experiments will verify or falsify such a preliminary result and then allow us to resolve the octant of $\theta_{23}$. On the theoretical side, the smallness of $\epsilon$ can naturally be ascribed to either a tree-level perturbation or a radiative correction to the exact $\mu-\tau$ symmetry.

One might argue that the best-fit result of $\delta$ remains too preliminary (i.e., $\sin \delta < 0$ at the 1.6$\sigma$ or 90% confidence level in the normal neutrino mass ordering [4]) to be believable. This is certainly true, but the history of particle physics tells us that a preliminary result coming from a reliable global analysis of relevant experimental data often turns out to be the truth. The closest example of this kind should be the global-fit “prediction” for a nonzero and unsuppressed $\theta_{13}$ [8]: $\sin^2 \theta_{13} = 0.016 \pm 0.010$ (1$\sigma$), which proved to be essentially correct after the first direct measurement of $\theta_{13}$ was reported by the Daya Bay experiment about three and a half years later. In fact, the T2K measurement of a relatively strong $\nu_\mu \rightarrow \nu_\tau$ appearance signal [9] plays an important role in the global fit to make $\theta_{13}$ consistent with the Daya Bay result and drive a slight but intriguing preference for $\delta \approx 1.5\pi$ [4]. Taking all these things seriously, we expect that the global fit of current neutrino oscillation data can provide useful guidance again in connection with the CP-violating phase $\delta$, especially before a direct measurement of $\delta$ itself is available.

It is therefore meaningful to stress that the fairly good consistency of Eq. (5) with the best-fit results of $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and $\delta$ in the normal neutrino mass ordering case might not just be a numerical accident. Provided Eq. (5) is correct or essentially correct, one may draw at least two important phenomenological conclusions: 1) leptonic CP violation in neutrino oscillations must be quite significant, because its strength, which is measured by the Jarlskog invariant $J_\nu = c_{12}s_{12}c_{13}^2s_{13}c_{23}s_{23}\sin \delta$ [10], must be at the percent level for $\delta \approx 250^\circ$; 2) behind the observed pattern of lepton flavor mixing should be the partial $\mu-\tau$ permutation symmetry $|U_{\mu_1}| = |U_{\tau_1}|$ or its approximation. Hence a viable model for neutrino mass generation and flavor mixing ought to accommodate this partial $\mu-\tau$ symmetry at low energies, no matter how complicated its original flavor symmetry groups are [11].
In a specific neutrino mass model, of course, either the full $\mu$-$\tau$ symmetry or a partial one should impose strong constraints on the elements $\langle m \rangle_{\alpha\beta}$ (for $\alpha, \beta = e, \mu, \tau$) of the effective Majorana neutrino mass matrix $M_{\nu} = U \tilde{M}_{\nu} U^T$ with $\tilde{M}_{\nu} = \text{Diag}\{m_1, m_2, m_3\}$. For instance,

$$\langle m \rangle_{\mu\mu} + \langle m \rangle_{\tau\tau} - 2\langle m \rangle_{\mu\tau} = \sum_{i=1}^{3} \left[ m_i (U_{\mu i} - U_{\tau i}) \right]^2$$

(8)

holds in general, but the $\mu$-$\tau$ permutation symmetry $|U_{\mu i}| = |U_{\tau i}|$ may simplify it to some extent, leading
to one or two correlative relations among the three matrix elements \(\langle m \rangle_{\mu e}, \langle m \rangle_{\tau\tau}\) and \(\langle m \rangle_{\mu\tau}\). The partial \(\mu-\tau\) symmetry \(|U_{\mu 1}| = |U_{\tau 1}| = 1/\sqrt{6}\) implies that such correlative relations might not be very simple, but it is still possible to obtain very impressive predictions for lepton flavor mixing and CP violation. This point has already been seen in the above discussions, and it will become more transparent and convincing in a much more specific case to be discussed later on.

To be more specific, let us focus on the partial \(\mu-\tau\) symmetry \(|U_{\mu 1}| = |U_{\tau 1}| = 1/\sqrt{6}\). Then \(|U_{e1}| = 2/\sqrt{6}\) can be derived from the unitarity of \(U\). Taking \(U_{e1} = \cos \alpha, U_{\mu 1} = -\sin \alpha \cos \beta\) and \(U_{\tau 1} = \sin \alpha \sin \beta\) in a way analogous to the Kobayashi-Maskawa (KM) parametrization \([12]\), we obtain \(\alpha = \arctan (1/\sqrt{2}) \simeq 35.3^\circ\) and \(\beta = 45^\circ\). So we are left with a KM-like parametrization of the lepton flavor mixing matrix \(U\), in which another flavor mixing angle \(\theta\) and the Dirac CP-violating phase \(\phi\) are free parameters:

\[
U = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} & \frac{\sin \theta}{\sqrt{3}} e^{-i\phi} \\
\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} & \frac{\sin \theta}{\sqrt{3}} e^{-i\phi} \\
\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} & -\frac{\sin \theta}{\sqrt{3}} e^{i\phi}
\end{pmatrix} P_\nu.
\]

This interesting neutrino mixing pattern was first proposed by us \([13]\) and Lam \([14]\) in 2006, and it was later referred to as the TM1 mixing pattern \([15]\). It can actually be obtained from the tri-bimaximal neutrino mixing pattern \([16]\) multiplied by a complex \((2,3)\) rotation matrix from its right-hand side. Some of the phenomenological implications of Eq. (8) have already been explored in the literature (see, e.g., Refs. \([13]\) and \([17]\)), and it can easily be derived from a given neutrino mass model by implementing an \(A_4\) or \(S_4\) flavor symmetry \([14]\) \([18]\). Comparing this KM-like representation of \(U\) with the standard one in Eq. (2) allows us to reproduce the very striking prediction \([13]\):

\[
\sin^2 \theta_{12} = \frac{1}{3} \left(1 - 2 \tan^2 \theta_{13}\right).
\]

Given \(\theta_{13} \simeq 8.8^\circ\), for example, Eq. (10) leads us to \(\theta_{12} \simeq 34.3^\circ\), a result which is very close to the best-fit value of \(\theta_{12}\) given in Eq. (3) — the deviation is only about 0.6°! This good agreement is more intuitive in the \(\theta_{12}\)-versus-\(\theta_{13}\) plot as shown by Fig. 3, in which the best-fit values and uncertainties of these two flavor mixing parameters have been taken into account \(^4\). Moreover, the deviation of \(\theta_{23}\) from 45° is related to \(\delta\) in a much simpler way in this specific flavor mixing scenario:

\[
\tan 2\epsilon = -\frac{2 \sin \theta_{13} \sqrt{2 \left(1 - 3 \sin^2 \theta_{13}\right)}}{1 - 5 \sin^2 \theta_{13}} \cos \delta.
\]

This relation allows us to obtain \(\delta \simeq 254^\circ\) from \(\theta_{13} \simeq 8.8^\circ\) and \(\epsilon \simeq 3.6^\circ\) in the normal neutrino mass ordering, compatible with the best-fit result \(\delta \simeq 250^\circ\) in Eq. (3) to a good degree of accuracy.

\(^4\)It is worth mentioning that the best-fit result \(\theta_{12} = 34.6^\circ\) obtained from \([17]\) coincides with the prediction of Eq. (10) to a better degree of accuracy. Nevertheless, the best-fit values of \(\theta_{23}\) achieved from this global analysis and the one done by Gonzalez-Garcia \textit{et al} in Ref. \([3]\) prefer the second octant (i.e., \(\theta_{23} > 45^\circ\)), and hence they are in conflict with the partial \(\mu-\tau\) symmetry \(|U_{\mu 1}| = |U_{\tau 1}|\) and its consequences [e.g., Eq. (5)]. While the octant of \(\theta_{23}\) remains an open issue today, we believe that it must be associated with a partial or approximate \(\mu-\tau\) symmetry. In this sense we find that \(\theta_{23} < 45^\circ\) as obtained by Capozzi \textit{et al} \([4]\) is phenomenologically more favored.
Figure 3: A comparison between the relation $\sin^2 \theta_{12} = (1 - 2 \tan^2 \theta_{13})/3$ (red line) and the best-fit values of $\theta_{12}$ and $\theta_{13}$. The allowed $1\sigma$, $2\sigma$ and $3\sigma$ regions of both parameters are constructed from their global-fit results in the normal neutrino mass ordering \cite{4}, and a Gaussian distribution of the uncertainties is assumed for illustration. If $\theta_{13} \simeq 8.8^\circ$ is input, one will arrive at the prediction $\theta_{12} \simeq 34.3^\circ$ (black dot), which is consistent with the best-fit result $\theta_{12} \simeq 33.7^\circ$ (red dot) within $1\sigma$.

We conclude that Eq. (9) should nowadays be the simplest and most favored neutrino mixing pattern, which respects the partial $\mu$-$\tau$ permutation symmetry $|U_{\mu 1}| = |U_{\tau 1}|$ and may originate from a violation of the full $\mu$-$\tau$ symmetry or discrete flavor symmetries. Some comments in this connection are in order.

- **Soft breaking of $\mu$-$\tau$ symmetry.** As indicated by Eq. (9), the lepton flavor mixing matrix $U$ with the partial $\mu$-$\tau$ symmetry $|U_{\mu 1}| = |U_{\tau 1}|$ can be regarded as the tri-bimaximal mixing pattern corrected by a $(2, 3)$ rotation, which can be induced by a $\mu$-$\tau$ symmetry breaking term in the neutrino mass matrix $M_\nu$. In the canonical type-I seesaw model, this soft term may come from the heavy Majorana neutrino mass matrix; while in the type-II seesaw model, it may arise from the breaking of $\mu$-$\tau$ symmetry in the potential of triplet and doublet Higgs fields.

- **Discrete flavor symmetries.** As a general guiding principle, one may search for a proper flavor symmetry by investigating the residual symmetry group $G_\ell$ in the charged-lepton sector and $G_\nu$ in the neutrino sector \cite{14}. In the basis where the charged-lepton mass matrix is diagonal, for instance, the residual symmetry in the charged-lepton sector is $G_\ell = Z_3$, while that in the neutrino sector is $G_\nu = Z_2$ as implied by the first column of $U$ in Eq. (9). Therefore, a finite discrete symmetry group which contains $G_\ell$ and $G_\nu$ as the subgroups can be a good starting point for model building to achieve the partial $\mu$-$\tau$ permutation symmetry under consideration \cite{18}.

- **Radiative breaking of $\mu$-$\tau$ symmetry.** A specific flavor symmetry model, which can simultaneously predict $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ or $270^\circ$ \cite{19}, is very likely to work at a superhigh-energy scale (e.g., close to the grand unification scale $\Lambda \sim 10^{16}$ GeV or around the conventional seesaw scale
Λ \sim 10^{14} \text{ GeV). In this case it is possible to preserve the partial \( \mu-\tau \) symmetry \(|U_{\mu 1}| = |U_{\tau 1}|\) but break the equalities \(|U_{\mu 2}| = |U_{\tau 2}|\) and \(|U_{\mu 3}| = |U_{\tau 3}|\) at the electroweak scale by taking into account the renormalization-group running effects on the lepton flavor mixing matrix \( U \). In other words, the octant of \( \theta_{23} \) and the quadrant of \( \delta \) can be resolved with the help of radiative corrections to the full \( \mu-\tau \) symmetry.

So there is a lot of room for model building to understand why the observed pattern of lepton flavor mixing exhibits an approximate \( \mu-\tau \) permutation symmetry.

To summarize, we have taken a close and serious look at the global-fit results of current neutrino oscillation data and found that the partial \( \mu-\tau \) permutation symmetry \(|U_{\mu 1}| = |U_{\tau 1}|\) should be behind the observed pattern of lepton flavor mixing. This simple but interesting symmetry predicts a novel correlation between the Dirac CP-violating phase \( \delta \) and three flavor mixing angles \( \theta_{12}, \theta_{13} \) and \( \theta_{23} \) in the standard parametrization, as shown in Eq. (5). Inputting the best-fit values of those mixing angles as reported by Capozzi et al [4], we have obtained the prediction \( \delta \simeq 255^\circ \) in the normal neutrino mass ordering, which is in good agreement with the best-fit result \( \delta \simeq 250^\circ \). In comparison, the inverted neutrino mass order case is slightly disfavored in this connection. Specifying this partial \( \mu-\tau \) symmetry to be \(|U_{\mu 1}| = |U_{\tau 1}| = 1/\sqrt{6}\), we have reproduced (Fig. 3) the remarkable relation \( \sin^2 \theta_{12} = (1 - 2 \tan^2 \theta_{13})/3 \) which was first uncovered in 2006 and is consistent very well with the present experimental data. In this case we are also left with a viable two-parameter description of the lepton flavor mixing matrix \( U \), and it can easily be derived from some neutrino mass models with concrete flavor symmetries. Moreover, we have pointed out that the octant of \( \theta_{23} \) and the quadrant of \( \delta \) can be resolved thanks to the slight violation of \(|U_{\mu 2}| = |U_{\tau 2}|\) and \(|U_{\mu 3}| = |U_{\tau 3}|\) either at the tree level or from radiative corrections.

Even though the normal neutrino mass ordering is favored in our discussions, it does not necessarily mean that it would be hopeless to observe a signal of the neutrinoless double-beta decay. The reason is simply that a normal but weakly hierarchical (or even nearly degenerate) neutrino mass spectrum can also result in a promising value of the effective Majorana neutrino mass term \(|\langle m \rangle_{ee}|\), which is actually comparable with the expected value in the inverted neutrino mass ordering case [20].

On the other hand, \( \delta \simeq 250^\circ \) implies a very significant effect of leptonic CP violation, which will be observed in the next-generation long-baseline neutrino oscillation experiments. One probably questions the preliminary hint of \( \delta \simeq 250^\circ \) as obtained from a global analysis of the available neutrino oscillation data, but a fairly good lesson learnt recently from the global-fit determination of \( \theta_{13} \) is so encouraging that the situation for \( \delta \) deserves very serious attention. A similar argument works for the octant of \( \theta_{23} \), which must be associated with a partial or approximate \( \mu-\tau \) symmetry. That is why we are strongly motivated to highlight the partial \( \mu-\tau \) symmetry \(|U_{\mu 1}| = |U_{\tau 1}|\) and its striking prediction for CP violation in the lepton sector. They will soon be tested by the upcoming experimental data.

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