Fermion and scalar phenomenology of a 2-Higgs doublet model with $S_3$

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We propose a 2-Higgs doublet model where the symmetry is extended by $S_3 \otimes Z_3 \otimes Z'_3 \otimes Z_{14}$ and the field content is enlarged by extra $SU(2)_L$ singlet scalar fields. $S_3$ makes the model predictive and leads to viable fermion masses and mixing. The observed hierarchy of the quark masses arises from the $Z'_3$ and $Z_{14}$ symmetries. The light neutrino masses are generated through a type I seesaw mechanism with two heavy Majorana neutrinos. In the lepton sector we obtain mixing angles that are nearly tri-bi-maximal, in an excellent agreement with the observed lepton parameters. The vacuum expectation values required for the model are naturally obtained from the scalar potential, and we analyze the scalar sector properties further constraining the model through rare top decays (like $t \rightarrow ch$), the $h \rightarrow \gamma\gamma$ decay channel and the $T$ and $S$ parameters.

I. INTRODUCTION

The flavor puzzle is not understood in the context of the Standard Model (SM), which does not specify the Yukawa structures and has no justification for the number of generations. As such, extensions addressing the fermion masses and mixing are particularly appealing. With neutrino experiments increasingly constraining the mixing angles in the leptonic sector many models focus only on this sector, aiming to explain the near tri-bi-maximal structure of the PMNS matrix through some non-Abelian symmetry.

Discrete flavor symmetries have shown a lot of promise and $S_3$, as the smallest non-Abelian group has been considerably studied in the literature since [1], with interesting results for quarks, leptons or both, and remains a popular group [2–15]. Other popular groups are the smallest groups with triplet representations, particularly $A_4$ which has only a triplet and three distinct singlets. $A_4$ was used in [16–20] and more recently in [21–35]. With just triplets and singlet representations the groups $T_7$ [36–43] and $\Delta(27)$ [44–52] are also promising as flavor symmetries. For recent reviews on the use of discrete flavor groups, see Refs. [53, 54].

In this work we make use of the $S_3$ group to formulate a 2-Higgs doublet model (2HDM) with an extra $S_3 \otimes Z_3 \otimes Z'_3 \otimes Z_{14}$ symmetry. Assigning the SM fermions under this symmetry and using scalars transforming under the different irreducible representations of $S_3$, we provide an existence proof of models leading to the viable mixing inspired quark textures presented in [55], by building a minimal realization. We then consider the model in the lepton sector where we obtain viable masses and mixing angles by using assignments that lead to a charged lepton texture similar to that of the down-type quarks, with the neutrino sector being completed through a type I seesaw. We discuss the scalar potential in some detail, showing it leads to the Vacuum Expectation Values (VEVs) used to obtain the fermion masses, and analyzing phenomenological processes that constrain the parameters of the model such as $t \rightarrow ch$ and $h \rightarrow \gamma\gamma$.

The paper is outlined as follows. In Section II we describe the field and symmetry content of the model, including a brief revision of the quark mass and mixing angles presented in [55] (Section IIIA) and the equivalent analysis for the lepton sector (Section IIIB). Section III contains the analysis of the phenomenology associated with the extended scalar sector, presenting the Yukawa couplings, an analysis of rare top decays, then considering the $h \rightarrow \gamma\gamma$ rate (Section IIIC), and the $T$ and $S$ parameters (Section IIID). We present our conclusions in Section IV. We relegate some technical discussions that are relevant for the paper to the Appendix.
We consider an extension of the SM with extra scalar fields and discrete symmetries, which reproduces the predictive mixing inspired textures proposed in Ref. [55], i.e. the Cabbibo mixing arises from the down-type quark sector whereas the up-type quark sector contributes to the remaining mixing angles. These textures describe the charged fermion masses and quark mixing pattern in terms of different powers of the Wolfenstein parameter \( \lambda = 0.225 \) and order one parameters. Because of the required mismatch between the down-type quark and up-type quark textures, to obtain these textures in a model we use two Higgs doublets distinguished by a symmetry (in our model, a \( Z_3 \)). In the following, we describe our 2HDM with the inclusion of the \( S_3 \) \( \otimes \) \( Z_3 \) \( \otimes \) \( Z'_3 \) \( \otimes \) \( Z_{14} \) discrete symmetry and four singlet scalar fields, assigned in a \( S_3 \) doublet, one \( S_3 \) trivial singlet and one \( S_3 \) non trivial singlet. We use the \( S_3 \) discrete group since it is the smallest non-Abelian group, having a doublet and two singlets as irreducible representations.

The full symmetry \( G \) of the model is broken spontaneously in two steps:

\[
G = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes S_3 \otimes Z_3 \otimes Z'_3 \otimes Z_{14} \\
\Downarrow \Lambda
\]

\[
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_3 \\
\Downarrow \Lambda_{EW}
\]

\[
SU(3)_C \otimes U(1)_{em},
\]

where the different symmetry breaking scales satisfy the following hierarchy \( \Lambda \gg \Lambda_{EW} \), where \( \Lambda_{EW} = 246 \) GeV is the electroweak symmetry breaking scale.

The content of the model, which includes the particle assignments under the different symmetries, is shown in Tables I and II. The \( S_3 \) symmetry reduces the number of parameters in the Yukawa sector of this 2HDM making it more predictive. The \( Z_3 \) symmetry allows to completely decouple the bottom quark from the remaining down and strange quarks. As can be seen from the scalar field assignments, the two scalar \( SU(2)_L \) doublets have different \( Z_3 \) charges (\( \phi_1 \) being neutral). The \( Z'_3 \) and \( Z_{14} \) symmetries shape the hierarchical structure of the quark mass matrices necessary to get a realistic pattern of quark masses and mixing.

The Higgs doublets \( \phi_l \) \((l = 1, 2)\) acquire VEVs that break \( SU(2)_L \)

\[
\phi_l = \begin{pmatrix} 0 \\ \frac{v_l}{\sqrt{2}} \end{pmatrix}, \quad l = 1, 2.
\]

We decompose the Higgs fields around this minimum as

\[
\phi_l = \begin{pmatrix} \varphi^+_l \\ \frac{1}{\sqrt{2}} (v_l + \rho_l + i\eta_l) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} (\omega_l + i\tau_l) \\ \frac{1}{\sqrt{2}} (v_l + \rho_l + i\eta_l) \end{pmatrix},
\]

Table I: Assignments of the SM fermions under the flavor symmetries.

| Field | \( q_{1L} \) | \( q_{2L} \) | \( q_{3L} \) | \( U_R \) | \( u_{3R} \) | \( d_{1R} \) | \( d_{2R} \) | \( d_{3R} \) | \( l_{1L} \) | \( l_{2L} \) | \( l_{3L} \) | \( l_{1R} \) | \( l_{2R} \) | \( l_{3R} \) | \( u_{1R} \) | \( u_{2R} \) |
|-------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( S_3 \) | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| \( Z_3 \) | 0 | 0 | 1 | 0 | 1 | 2 | 2 | 1 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| \( Z'_3 \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( Z_{14} \) | -3 | -2 | 0 | 1 | 0 | 4 | 3 | 3 | -3 | 0 | 0 | 4 | 5 | 3 | 0 | 0 |

Table II: Assignments of the scalars under \( SU(2)_L \) and the flavor symmetries.
where
\[ \langle \rho_l \rangle = \langle \eta_l \rangle = \langle \omega_l \rangle = \langle \tau_l \rangle = 0, \quad \text{for } l = 1, 2. \tag{4} \]

From an analysis of the scalar potential (see Appendix B), we obtain the following VEVs for the SM singlet scalars:
\[ \langle \xi \rangle = v_\zeta (1, 0) \quad \langle \chi \rangle = v_\chi \quad \langle \zeta \rangle = v_\zeta, \tag{5} \]

i.e., the VEV of \( \xi \) is aligned as \((1, 0)\) in the \( S_3 \) direction.

For the up and down-type quarks, the Yukawa terms invariant under the symmetries are
\[ L^U_Y = \varepsilon^{(u)}_{33} \bar{q}_{3L} \phi_1 u_{3R} + \varepsilon^{(u)}_{22} \bar{q}_{2L} \phi_2 u_{3R} \frac{\chi^2}{\Lambda^2} + \varepsilon^{(u)}_{13} \bar{q}_{1L} \phi_2 u_{3R} \frac{\chi^3}{\Lambda^3} + \frac{\xi^3}{\Lambda^4} + \varepsilon^{(u)}_{11} \bar{q}_{1L} \phi_1 U_R \frac{\xi^3 \zeta^3}{\Lambda^8} + h.c. \tag{6} \]
\[ L^D_Y = \varepsilon^{(d)}_{33} \bar{q}_{3L} \phi_1 d_{3R} \frac{\chi^3}{\Lambda^3} + \varepsilon^{(d)}_{22} \bar{q}_{2L} \phi_2 d_{2R} \frac{\chi^5}{\Lambda^5} + \varepsilon^{(d)}_{12} \bar{q}_{1L} \phi_2 d_{2R} \frac{\chi^6}{\Lambda^6} + \varepsilon^{(d)}_{21} \bar{q}_{2L} \phi_2 d_{1R} \frac{\chi^7}{\Lambda^7} + h.c. \tag{7} \]

The invariant Yukawa terms for charged leptons and neutrinos are
\[ L^L_Y = \varepsilon^{(l)}_{33} \bar{l}_{3L} \phi_1 \bar{l}_{3R} \frac{\chi^3}{\Lambda^3} + \varepsilon^{(l)}_{22} \bar{l}_{2L} \phi_2 \bar{l}_{2R} \frac{\chi^5}{\Lambda^5} + \varepsilon^{(l)}_{12} \bar{l}_{1L} \phi_2 \bar{l}_{2R} \frac{\chi^6}{\Lambda^6} + \varepsilon^{(l)}_{21} \bar{l}_{2L} \phi_2 \bar{l}_{1R} \frac{\chi^7}{\Lambda^7} + h.c. \tag{8} \]
\[ L^\nu_Y = \varepsilon^{(\nu)}_{11} \bar{\nu}_{1L} \phi_1 \nu_{1R} \frac{\chi^3}{\Lambda^3} + \varepsilon^{(\nu)}_{12} \bar{\nu}_{1L} \phi_2 \nu_{1R} \frac{\chi^5}{\Lambda^5} + \varepsilon^{(\nu)}_{21} \bar{\nu}_{2L} \phi_1 \nu_{1R} + \varepsilon^{(\nu)}_{22} \bar{\nu}_{2L} \phi_1 \nu_{1R} + \varepsilon^{(\nu)}_{13} \bar{\nu}_{1L} \phi_3 \nu_{1R} + \varepsilon^{(\nu)}_{31} \bar{\nu}_{3L} \phi_1 \nu_{1R} + M_1 \bar{\nu}_{1R} \nu_{1R} + M_2 \bar{\nu}_{2R} \nu_{2R} + M_3 \bar{\nu}_{3R} \nu_{3R} + h.c. \tag{9} \]

The \( Z_{14} \) symmetry is the smallest cyclic symmetry that allows \( \chi \) in the Yukawa terms responsible for the down quark and electron masses, which we want to suppress by \( \lambda^7 \) (\( \lambda = 0.225 \) is one of the Wolfenstein parameters) without requiring small dimensionless Yukawa couplings. Furthermore, the \( Z'_3 \) symmetry is responsible for coupling the scalar \( \zeta \) with \( U_R \) as well as with \( l_{1R} \), which helps to explain the smallness of the up quark and electron mass in this model. The hierarchy of charged fermion masses and quark mixing matrix elements is therefore explained by both the \( Z'_3 \) and \( Z_{14} \) symmetries. Given that in this scenario the quark masses are related with the quark mixing parameters, we set the VEVs of the \( SU(2)_L \) singlet scalars with respect to the Wolfenstein parameter \( \lambda \) and the new physics scale \( \Lambda \):
\[ v_\zeta \sim v_\chi \sim v_\zeta = \lambda \Lambda. \tag{10} \]

These scalars therefore acquire VEVs at a scale unrelated with \( \Lambda_{EW} \). We have checked numerically that this regime is a valid minimum of the global potential for a suitable region of the parameter space (see Appendix B). As we will see in the following sections, in order to obtain realistic fermion masses and mixing without requiring a strong hierarchy among the Yukawa couplings, the VEVs of the \( SU(2)_L \) doublets \((v_1 \text{ and } v_2)\) should be of the same order of magnitude.

### A. Quark masses and mixing

Using Eqs. (6) and (7) we find the mass matrices for up and down-type quarks in the form:
\[ M_U = \frac{\nu}{\sqrt{2}} \begin{pmatrix} c_1 \lambda^8 & 0 & a_1 \lambda^3 \\ 0 & b_1 \lambda^4 & a_2 \lambda^3 \\ 0 & 0 & a_3 \end{pmatrix}, \quad M_D = \frac{\nu}{\sqrt{2}} \begin{pmatrix} e_1 \lambda^7 & f_1 \lambda^6 & 0 \\ e_2 \lambda^6 & f_2 \lambda^5 & 0 \\ 0 & 0 & g_1 \lambda^3 \end{pmatrix}, \tag{11} \]

where \( a_k, b_k, c_k, 1, 2, 3 \), \( b_1, c_1, f_1, f_2, e_1 \) and \( e_2 \) are \( O(1) \) parameters. Here we assume that all dimensionless parameters given in Eq. (11) are real excepting \( a_3 \), which we assume to be complex. These are the viable quark textures presented in [55], which we briefly review here.
The hermitian combinations $M_U M_U^\dagger$ and $M_D M_D^T$ are

\[
M_U M_U^\dagger = \frac{v^2}{2} \begin{pmatrix}
|a_1|^2 \lambda^6 + c_1^2 \lambda^{16} & a_1 a_2 \lambda^5 & a_1 a_3 \lambda^3 \\
a_1^* a_2 \lambda^5 & a_2^2 \lambda^4 + b_1^2 \lambda^8 & a_2 a_3 \lambda^2 \\
a_1^* a_3 \lambda^3 & a_2 a_3 \lambda^2 & a_3^2
\end{pmatrix},
\]

and

\[
M_D M_D^T = \frac{v^2}{2} \begin{pmatrix}
\lambda^{14} c_1^2 + \lambda^{12} f_1^2 & e_1 e_2 \lambda^{13} + f_1 f_2 \lambda^{11} & 0 \\
e_1 e_2 \lambda^{13} + f_1 f_2 \lambda^{11} & \lambda^{12} c_2^2 + \lambda^{10} f_2^2 & 0 \\
0 & 0 & \lambda^6 g_1^2
\end{pmatrix},
\]

and are approximately diagonalized by unitary rotation matrices $R_U$ and $R_D$:

\[
R_U^\dagger M_U M_U^\dagger R_U = \begin{pmatrix}
m_u^2 & 0 & 0 \\
0 & m_e^2 & 0 \\
0 & 0 & m_\mu^2
\end{pmatrix}, \quad R_U \simeq \begin{pmatrix}
c_{13} & s_{13} s_{23} e^{i \delta} & -c_{23} s_{13} e^{i \delta} \\
0 & c_{23} & s_{23} \\
s_{13} e^{-i \delta} & -c_{13} s_{23} & c_{13} e^{i \delta}
\end{pmatrix},
\]

\[
R_D^T M_D M_D^T R_D = \begin{pmatrix}
m_d^2 & 0 & 0 \\
0 & m_s^2 & 0 \\
0 & 0 & m_b^2
\end{pmatrix}, \quad R_D = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ (with $i \neq j$ and $i, j = 1, 2, 3$). $\theta_{ij}$ and $\delta$ are the quark mixing angles and the CP violating phase, respectively, in the usual parametrization. They are given by

\[
\begin{align*}
\tan \theta_{12} & \simeq \frac{f_1}{f_2}, \\
\tan \theta_{23} & \simeq \frac{a_2}{a_3} \lambda^2, \\
\tan \theta_{13} & \simeq \frac{|a_1|}{a_3} \lambda^3, \quad \delta = -\arg(a_1).
\end{align*}
\]

Therefore, the up and down-type quark masses are approximately given by

\[
\begin{align*}
m_u & \simeq c_1 \lambda^8 \frac{v}{\sqrt{2}}, \\
m_e & \simeq b_1 \lambda^4 \frac{v}{\sqrt{2}}, \\
m_\mu & \simeq a_3 \frac{v}{\sqrt{2}}, \\
m_d & \simeq |e_1 f_2 - e_2 f_1| \frac{\lambda^7 v}{\sqrt{2}}, \\
m_s & \simeq f_2 \lambda^5 \frac{v}{\sqrt{2}}, \\
m_b & \simeq g_1 \lambda^3 \frac{v}{\sqrt{2}}.
\end{align*}
\]

We also find that the CKM quark mixing matrix is approximately

\[
V_{CKM} = R_U^\dagger R_D \simeq \begin{pmatrix}
c_{12} c_{13} & c_{13} s_{12} & e^{i \delta} s_{13} \\
e^{-i \delta} c_{12} s_{13} s_{23} - c_{23} s_{12} & c_{12} c_{23} + e^{-i \delta} s_{12} s_{13} s_{23} & -c_{13} s_{23} \\
-s_{12} s_{23} - e^{-i \delta} c_{12} c_{23} s_{13} & c_{12} s_{23} - e^{-i \delta} c_{23} s_{13} s_{23} & c_{13} c_{23}
\end{pmatrix}.
\]

It is noteworthy that Eq. (11) provides an elegant understanding of all SM fermion masses and mixing angles through their scalings by powers of the Wolfenstein parameter $\lambda = 0.225$ with $O(1)$ coefficients.

The Wolfenstein parametrization \[56\] of the CKM matrix is:

\[
V_W \simeq \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix},
\]

with

\[
\begin{align*}
\lambda &= 0.22537 \pm 0.00061, \quad A = 0.814^{+0.023}_{-0.024}, \\
\bar{\rho} &= 0.117 \pm 0.021, \quad \bar{\eta} = 0.353 \pm 0.013, \\
\bar{\rho} &\simeq \rho \left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} \simeq \eta \left(1 - \frac{\lambda^2}{2}\right).
\end{align*}
\]
From the comparison with (20), we find:

\[ a_3 \simeq 1, \quad a_2 \simeq A \simeq 0.81, \quad a_1 \simeq -A \sqrt{\rho^2 + \eta^2} e^{i\delta} \simeq -0.3e^{i\delta}, \quad \delta = 67^\circ, \quad b_1 \simeq \frac{m_c}{\lambda^4 m_t} \simeq 1.43, \quad c_1 \simeq \frac{m_u}{\lambda^8 m_t} \simeq 1.27. \]  

\[ (24) \]

Note that \( a_1 \) is required to be complex, as previously assumed, and its magnitude is a bit smaller than the remaining \( \mathcal{O}(1) \) coefficients.

Since the charged fermion masses and quark mixing hierarchy arises from the \( Z'_3 \otimes Z_{14} \) symmetry breaking, and in order to have the right value of the Cabbibo mixing, we need \( e_2 \approx f_2 \). We fit the parameters \( e_1, f_1, f_2 \) and \( g_1 \) in Eq. \( (11) \) to reproduce the down-type quark masses and quark mixing parameters. As can be seen from the above formulas, the quark sector of our model contains ten effective free parameters, i.e., \( |a_1|, a_2, a_3, b_1, c_1, e_1, f_1, f_2, g_1 \) and the phase \( \text{arg}(a_1) \), to describe the quark mass and mixing pattern, which is characterized by ten physical observables, i.e., the six quark masses, the three mixing angles and the CP violating phase. Furthermore, in our model these parameters are of the same order of magnitude. The results for the down-type quark masses, the three quark mixing angles and the CP violating phase \( \delta \) in Tables III and IV correspond to the best fit values:

\[ e_1 \simeq 0.84, \quad f_1 \simeq 0.4, \quad f_2 \simeq 0.57, \quad g_1 \simeq 1.42. \]  

\[ (26) \]

As pointed out in \cite{55}, the CKM matrix in our model is consistent with the experimental data. The agreement of our model with the experimental data is as good as in the models of Refs. \cite{9, 11, 29, 33, 47, 57, 58} and better than, for example, those in Refs. \cite{59–66}. The obtained and experimental values of the magnitudes of the CKM parameters, i.e., three quark mixing parameters and the CP violating phase \( \delta \) are shown in Table III. The experimental values of the CKM magnitudes and the Jarlskog invariant are taken from Ref. \cite{67}, whereas the experimental values of the quark masses, which are given at the \( M_Z \) scale, have been taken from Ref. \cite{68}.

| Observable | Model value | Experimental value |
|------------|-------------|---------------------|
| \( m_u(\text{MeV}) \) | 1.47 | 1.45^{+0.56}_{-0.45} |
| \( m_c(\text{MeV}) \) | 641 | 635 ± 86 |
| \( m_t(\text{GeV}) \) | 172.2 | 172.1 ± 0.6 ± 0.9 |
| \( m_d(\text{MeV}) \) | 3.00 | 2.95^{+0.5}_{-0.4} |
| \( m_s(\text{MeV}) \) | 59.2 | 57.7^{+16.4}_{-15.7} |
| \( m_b(\text{GeV}) \) | 2.82 | 2.82^{+0.09}_{-0.04} |

Table III: Model and experimental values of the quark masses.

| Observable | Model value | Experimental value |
|------------|-------------|---------------------|
| \( \sin \theta_{12} \) | 0.2257 | 0.2254 |
| \( \sin \theta_{23} \) | 0.0412 | 0.0413 |
| \( \sin \theta_{13} \) | 0.00352 | 0.00350 |
| \( \delta \) | 68° | 68° |

Table IV: Model and experimental values of CKM parameters.

**B. Lepton masses and mixing**

This \( S_3 \) flavor model obtains the viable quark textures proposed in \cite{55} as shown in section II A. We now proceed to analyze the lepton sector of the model. From the charged lepton Yukawa terms of Eq. \( (8) \) it follows that the charged lepton mass matrix takes the following form:

\[
M_l = \frac{v}{\sqrt{2}} \begin{pmatrix} x_1 \lambda^8 & 0 & 0 \\
0 & y_1 \lambda^5 & z_1 \lambda^3 \\
0 & y_2 \lambda^5 & z_2 \lambda^3 \end{pmatrix}.
\]

\[ (27) \]
where $x_1, y_1, y_2, z_1, z_2$, are $\mathcal{O}(1)$ parameters, assumed to be real, for simplicity.

Then, the charged lepton mass matrix satisfies the following relations:

$$M_l M_l^T = \frac{v^2}{2} \begin{pmatrix} x_1^2 \lambda^6 & 0 & 0 \\ 0 & z_1^2 \lambda^6 + y_1^2 \lambda^{10} & 0 \\ 0 & 0 & z_2^2 \lambda^6 + y_2^2 \lambda^{10} \end{pmatrix},$$  \hspace{1cm} (28)

$$M_l^T M_l = \frac{v^2}{2} \begin{pmatrix} x_1^2 \lambda^6 & 0 & 0 \\ 0 & (y_1^2 + y_2^2) \lambda^{10} & (y_1 z_1 + y_2 z_2) \lambda^8 \\ 0 & (y_1 z_1 + y_2 z_2) \lambda^8 & (z_1^2 + z_2^2) \lambda^6 \end{pmatrix}. $$  \hspace{1cm} (29)

Therefore, the matrix $M_l M_l^T$ can be diagonalized by rotation matrix $R_l$ according to:

$$R_l^T M_l M_l^T R_l = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}, \hspace{1cm} R_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_l & -\sin \theta_l \\ 0 & \sin \theta_l & \cos \theta_l \end{pmatrix}, \hspace{1cm} \tan \theta_l \simeq -\frac{z_1}{z_2}. $$  \hspace{1cm} (30)

The charged lepton masses are approximately given by

$$m_e \simeq x_1 \lambda^8 \frac{v}{\sqrt{2}}, \hspace{1cm} m_\mu \simeq \left| y_1 z_2 - y_2 z_1 \right| \lambda^5 \frac{v}{\sqrt{2}}, \hspace{1cm} m_\tau \simeq \sqrt{z_1^2 + z_2^2} \lambda^3 \frac{v}{\sqrt{2}}. $$  \hspace{1cm} (31)

From the neutrino Yukawa terms it follows that the full $5 \times 5$ neutrino mass matrix is

$$M_\nu = \begin{pmatrix} 0_{3 \times 3} & M_\nu^D \\ (M_\nu^D)^T & M_R \end{pmatrix}, $$  \hspace{1cm} (32)

where:

$$M_\nu^D = \begin{pmatrix} \lambda^3 \epsilon_{11} \frac{v}{\sqrt{2}} & \lambda^3 \epsilon_{12} \frac{v}{\sqrt{2}} \\ \epsilon_{21} \frac{v}{\sqrt{2}} & \epsilon_{22} \frac{v}{\sqrt{2}} \\ \epsilon_{31} \frac{v}{\sqrt{2}} & \epsilon_{32} \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} A & F \\ B & E \end{pmatrix}^T, \hspace{1cm} M_R = \begin{pmatrix} M_1 & \frac{1}{2} M_{12} \\ \frac{1}{2} M_{12} & M_2 \end{pmatrix}. $$  \hspace{1cm} (33)

Since $(M_R)_{ii} >> v$, the light neutrino mass matrix is generated through a type I seesaw mechanism and is given by

$$M_L = M_\nu^D M_R^{-1} (M_\nu^D)^T = \begin{pmatrix} A & F \\ B & E \end{pmatrix} \begin{pmatrix} \frac{4 M_2 M_1}{M_{12} - 4 M_1 M_2} & \frac{2 M_{12}}{M_{12} - 4 M_1 M_2} \\ \frac{2 M_{12}}{M_{12} - 4 M_1 M_2} & \frac{2 M_2}{M_{12} - 4 M_1 M_2} \end{pmatrix} \begin{pmatrix} A & B & C \\ F & E & D \end{pmatrix}. $$  \hspace{1cm} (34)

In order to demonstrate these structures can be fit to the data, we set $\varphi = \varphi$ for simplicity, to obtain

$$M_L = \begin{pmatrix} W^2 & \kappa W X W Y & W Y \cos \varphi & W Y \cos (\varphi - \varphi) \\ W X \cos \varphi & X^2 & \kappa X Y \cos \varphi & \kappa X Y \cos (\varphi - \varphi) \\ W Y \cos (\varphi - \varphi) & \kappa X Y \cos \varphi & Y^2 \end{pmatrix}, \hspace{1cm} \kappa = \cos \varphi.  $$  \hspace{1cm} (35)
Assuming that the neutrino Yukawa couplings are real, we find that for the normal (NH) and inverted (IH) mass hierarchies, the light neutrino mass matrix is diagonalized by a rotation matrix $R_{\nu}$, according to

$$R_{\nu}^{T} M_{L} R_{\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix}, \quad R_{\nu} = \begin{pmatrix} \frac{-Y}{\sqrt{W^{2} + Y^{2}}} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} \\ 0 & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} & -\frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{\nu} \\ \frac{Y}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{\nu} & \frac{Y}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} \end{pmatrix}, \quad \text{for NH} \quad (36)$$

$$\tan \theta_{\nu} = -\sqrt{\frac{m_{3} - X^{2}}{X^{2} - m_{2}}}, \quad m_{\nu 1} = 0, \quad m_{\nu 2,3} = \frac{W^{2} + X^{2} + Y^{2}}{2} \mp \sqrt{(W^{2} - X^{2} + Y^{2})^{2} - 4k^{2}X^{2}(W^{2} + Y^{2})}$$

$$R_{\nu}^{T} M_{L} R_{\nu} = \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix}, \quad R_{\nu} = \begin{pmatrix} \frac{-Y}{\sqrt{W^{2} + Y^{2}}} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} \\ 0 & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} & -\frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{\nu} \\ \frac{Y}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{\nu} & \frac{Y}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{\nu} \end{pmatrix}, \quad \text{for IH} \quad (37)$$

$$\tan \theta_{\nu} = -\sqrt{\frac{m_{2} - X^{2}}{X^{2} - m_{1}}}, \quad m_{\nu 1,2} = \frac{W^{2} + X^{2} + Y^{2}}{2} \mp \frac{1}{2} \sqrt{(W^{2} - X^{2} + Y^{2})^{2} - 4k^{2}X^{2}(W^{2} + Y^{2})}, \quad m_{\nu 3} = 0.$$

The smallness of the active neutrinos masses is a consequence of their scaling with the inverse of the large Majorana neutrino masses, as expected from the type I seesaw mechanism implemented in our model.

With the rotation matrices in the charged lepton sector $R_{l}$, Eq. (30), and the neutrino sector $R_{\nu}$, Eqs. (36) and (37) for NH and IH, respectively, we obtain the PMNS mixing matrix

$$U = R_{l}^{T} R_{\nu} = \begin{pmatrix} \frac{-Y}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{l} \cos \theta_{\nu} + \frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{l} \sin \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{l} \cos \theta_{\nu} - \cos \theta_{l} \sin \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{l} \cos \theta_{\nu} - \sin \theta_{l} \sin \theta_{\nu} \\ \frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{l} \cos \theta_{\nu} & \frac{Y}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{l} \cos \theta_{\nu} - \cos \theta_{l} \sin \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{l} \cos \theta_{\nu} + \sin \theta_{l} \sin \theta_{\nu} \\ \frac{W}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{l} \sin \theta_{\nu} & \frac{Y}{\sqrt{W^{2} + Y^{2}}} \sin \theta_{l} \cos \theta_{\nu} & \frac{W}{\sqrt{W^{2} + Y^{2}}} \cos \theta_{l} \cos \theta_{\nu} + \sin \theta_{l} \sin \theta_{\nu} \end{pmatrix}$$

By comparing with the standard parametrization we derive the mixing angles for NH and IH

$$\sin^{2} \theta_{12} = \frac{W^{2} \sin^{2} \theta_{\nu}}{W^{2} + (1 - \cos^{2} \theta_{\nu}) W^{2}}; \quad \sin^{2} \theta_{13} = \frac{W^{2} \cos^{2} \theta_{\nu}}{W^{2} + Y^{2}},$$

$$\sin^{2} \theta_{23} = \frac{(\sqrt{W^{2} + Y^{2}} \sin \theta_{l} \cos \theta_{\nu} - Y \cos \theta_{\nu} \sin \theta_{l})^{2}}{(1 - \cos^{2} \theta_{\nu}) W^{2} + Y^{2}}, \quad \text{for NH} \quad (39)$$

$$\sin^{2} \theta_{12} = \frac{Y^{2} \sin^{2} \theta_{\nu}}{W^{2} + (1 - \cos^{2} \theta_{\nu}) Y^{2}}; \quad \sin^{2} \theta_{13} = \frac{Y^{2} \cos^{2} \theta_{\nu}}{W^{2} + Y^{2}},$$

$$\sin^{2} \theta_{23} = \frac{(\sqrt{W^{2} + Y^{2}} \sin \theta_{l} \cos \theta_{\nu} - W \cos \theta_{\nu} \sin \theta_{l})^{2}}{(1 - \cos^{2} \theta_{\nu}) Y^{2} + W^{2}}, \quad \text{for IH}. \quad (40)$$

We further simplify the analysis by considering

$$x_{1} = y_{2} = z_{1},$$

so that the charged lepton masses will be determined by three dimensionless effective parameters, i.e., $x_{1}$, $y_{1}$ and $z_{2}$, whereas the neutrino mass squared splittings and neutrino mixing parameters will be controlled by four dimensionless
effective parameters, i.e., $\kappa$, $W$, $X$ and $Y$. Varying the parameters $x_1$, $y_1$, $z_2$, $\kappa$, $W$, $X$ and $Y$, we fit the charged lepton masses, the neutrino mass squared splittings $\Delta m^2_{31}$, $\Delta m^2_{32}$ (defined as $\Delta m^2_{ij} = m^2_i - m^2_j$) and the leptonic mixing angles $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ to their experimental values for NH and IH. Therefore the lepton sector of our model contains seven effective free parameters, i.e., $x_1$, $y_1$, $z_2$, $\kappa$, $W$, $X$ and $Y$, and describes the lepton masses and mixing pattern, characterized by eight physical observables, i.e., the three charged lepton masses, the two neutrino mass squared splittings and leptonic mixing angles. The results shown in Table V correspond to the following best-fit values:

$$\kappa \simeq 0.45, \quad W \simeq 0.13 \, eV^{1/2}, \quad X \simeq 0.11 \, eV^{1/2}, \quad Y \simeq 0.18 \, eV^{1/2},$$

$$x_1 \simeq 0.42, \quad y_1 \simeq 1.39, \quad z_2 \simeq 0.77, \quad \text{for NH},$$

$$\kappa \simeq 4.03 \times 10^{-3}, \quad W \simeq 0.18 \, eV^{1/2}, \quad X \simeq 0.22 \, eV^{1/2}, \quad Y \simeq 0.13 \, eV^{1/2},$$

$$x_1 \simeq 0.42, \quad y_1 \simeq 1.38, \quad z_2 \simeq 0.78, \quad \text{for IH}. \quad (42)$$

Using the best-fit values given above, we obtain the following neutrino masses for NH and IH

$$m_1 = 0, \quad m_2 \approx 9 \text{meV}, \quad m_3 \approx 50 \text{meV}, \quad \text{for NH}, \quad (44)$$

$$m_1 \approx 49 \text{meV}, \quad m_2 \approx 50 \text{meV}, \quad m_3 = 0, \quad \text{for IH}. \quad (45)$$

The obtained and experimental values of the observables in the lepton sector are shown in Table V. Given that the lightest neutrino is predicted to be massless in our model, the neutrino masses are hierarchical, which puts the overall neutrino mass scale below the current experimental reach (the same applies to the cosmological bound $\sum^3_{k=1} m_{\nu_k} < 0.23 \, eV$ on the sum of the neutrino masses [69, 70]). Therefore, our model fulfills the cosmological contraints on neutrino masses for both normal and inverted hierarchies.

The experimental values of the charged lepton masses, which are given at the $M_Z$ scale, have been taken from Ref. [68], whereas the experimental values of the neutrino mass squared splittings and leptonic mixing angles for both NH and IH, are taken from Ref. [71]. The obtained charged lepton masses, neutrino mass squared splittings and lepton mixing angles are in excellent agreement with the experimental data, showing that the model can perfectly account for all the observables in the lepton sector. We recall that for the sake of simplicity, we assumed all leptonic parameters to be real and further restricted the set of parameters, but a non-vanishing CP violating phase in the PMNS mixing matrix can be generated by allowing one or several parameters in the neutrino mass matrix of Eq. (32) to be complex.

| Observable       | Model value | Experimental value |
|------------------|-------------|--------------------|
| $m_e (MeV)$      | 0.487       | 0.487              |
| $m_\mu (MeV)$    | 102.8       | 102.8 ± 0.0003     |
| $m_\tau (GeV)$   | 1.75        | 1.75 ± 0.0003      |
| $\Delta m^2_{31} (10^{-5}eV^2)$ (NH) | 7.60 | 7.60 ± 0.19 ± 0.18 |
| $\Delta m^2_{31} (10^{-3}eV^2)$ (NH) | 2.48 | 2.48 ± 0.05 ± 0.07 |
| $\sin^2 \theta_{12} (NH)$ | 0.323 | 0.323 ± 0.016     |
| $\sin^2 \theta_{23} (NH)$ | 0.567 | 0.567 ± 0.032 ± 0.128 |
| $\sin^2 \theta_{13} (NH)$ | 0.0234 | 0.0234 ± 0.0020 |
| $\Delta m^2_{31} (10^{-5}eV^2)$ (IH) | 7.60 | 7.60 ± 0.19 ± 0.18 |
| $\Delta m^2_{31} (10^{-3}eV^2)$ (IH) | 2.48 | 2.48 ± 0.05 ± 0.06 |
| $\sin^2 \theta_{12} (IH)$ | 0.323 | 0.323 ± 0.016     |
| $\sin^2 \theta_{23} (IH)$ | 0.573 | 0.573 ± 0.025 ± 0.043 |
| $\sin^2 \theta_{13} (IH)$ | 0.0240 | 0.0240 ± 0.0019 |

Table V: Model and experimental values of the lepton sector observables, for normal (NH) and inverted (IH) hierarchies.

We can now predict the amplitude for neutrinoless double beta ($0\nu\beta\beta$) decay in our model, which is proportional to the effective Majorana neutrino mass

$$m_{\beta\beta} = \left| \sum_k U^2_{ek} m_{\nu_k} \right|,$$  

(46)
where $U_{k}^{2}$ and $m_{\nu_{k}}$ are the PMNS mixing matrix elements and the Majorana neutrino masses, respectively. Then, from Eqs. (38) and (42)-(45), we predict the following effective neutrino mass for both hierarchies:

$$m_{\beta\beta} = \begin{cases} 4 \text{ meV} & \text{for NH} \\ 50 \text{ meV} & \text{for IH} \end{cases}$$

(47)

This is beyond the reach of the present and forthcoming $0\nu\beta\beta$ decay experiments. The present best upper limit on this parameter $m_{\beta\beta} \leq 160$ meV comes from the recently quoted EXO-200 experiment [72, 73] $T_{1/2}^{0\nu\beta\beta}(^{136}\text{Xe}) \geq 1.6 \times 10^{25}$ yr at the 90\% CL. This limit will be improved within the not too distant future. The GERDA experiment [74, 75] is currently moving to "phase-II", at the end of which it is expected to reach $T_{1/2}^{0\nu\beta\beta}(^{76}\text{Ge}) \geq 2 \times 10^{26}$ yr, corresponding to $m_{\beta\beta} \leq 100$ MeV. A bolometric CUORE experiment, using $^{130}\text{Te}$ [76], is currently under construction. Its estimated sensitivity is around $T_{1/2}^{0\nu\beta\beta}(^{130}\text{Te}) \sim 10^{26}$ yr corresponding to $m_{\beta\beta} \leq 50$ meV. There are also proposals for ton-scale next-to-next generation $0\nu\beta\beta$ experiments with $^{136}\text{Xe}$ [77, 78] and $^{76}\text{Ge}$ [74, 79] claiming sensitivities over $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr, corresponding to $m_{\beta\beta} \sim 12 - 30$ meV. For recent experimental reviews, see for example Ref. [80] and references therein. Thus, according to Eq. (47), our model predicts $T_{1/2}^{0\nu\beta\beta}$ at the level of sensitivities of the next generation or next-to-next generation $0\nu\beta\beta$ experiments.

### III. SCALAR PHENOMENOLOGY

The renormalizable scalar potential involving only the $SU(2)$ doublets $\phi_{i}$ is

$$V(\phi_{i}) = -\sum_{i=1}^{2} \mu_{i}^{2}(\phi_{i}^{\dagger}\phi_{i}) + \sum_{i=1}^{2} \kappa_{i}(\phi_{i}^{\dagger}\phi_{i})^{2},$$

$$V(\phi_{1}, \phi_{2}) = \gamma_{12}(\phi_{1}^{\dagger}\phi_{1})(\phi_{2}^{\dagger}\phi_{2}) + \kappa_{12}(\phi_{1}^{\dagger}\phi_{1})(\phi_{2}^{\dagger}\phi_{2}),$$

$$V(\xi, \chi, \zeta, \phi_{i}) = \left(\lambda_{\xi}(\xi\xi) + \lambda_{\chi}(\chi^{\dagger}\chi) + \lambda_{\zeta}(\zeta^{\dagger}\zeta)\right)\sum_{i=1}^{2} \lambda_{i,1}(\phi_{i}^{\dagger}\phi_{i}),$$

whereas the remaining terms are

$$V(\xi) = -\mu_{\xi}^{2}(\xi\xi) + \gamma_{\xi,3}(\xi\xi)_{2}\xi + \kappa_{\xi,1}(\xi\xi)_{1}(\xi\xi)_{1} + \kappa_{\xi,2}(\xi\xi)_{2}(\xi\xi)_{2},$$

$$V(\chi) = -\mu_{\chi}^{2}(\chi^{\dagger}\chi) + \kappa_{\chi}(\chi^{\dagger}\chi)^{2},$$

$$V(\zeta) = -\mu_{\zeta}^{2}(\zeta^{\dagger}\zeta) + \kappa_{\zeta}(\zeta^{\dagger}\zeta)^{2},$$

$$V(\xi, \chi, \zeta) = \lambda_{2}(\xi\xi)(\chi^{\dagger}\chi) + \lambda_{3}(\xi\xi)(\zeta^{\dagger}\zeta) + \lambda_{4}(\zeta^{\dagger}\zeta)(\chi^{\dagger}\chi).$$

To obtain a viable low-energy model with one CP-odd and one charged Goldstone boson, we consider the following soft breaking terms:

$$V_{\text{soft}}(\zeta, \chi) = -\mu_{\chi}^{2}(\zeta\chi + \chi^{\dagger}\chi),$$

(48)

$$V_{\text{soft}}(\phi_{1}, \phi_{2}) = -\mu_{12}^{2}\left[(\phi_{1}^{\dagger}\phi_{1}) + (\phi_{2}^{\dagger}\phi_{2})\right].$$

(49)

The mass matrices of the low-energy CP-even neutral scalars $\rho_{1,2}$ , CP-odd neutral scalars $\eta_{1,2}$ and charged scalars $\varphi_{1,2}^{\pm}$ can be written as

$$M_{1} = \frac{1}{2} \left(\begin{array}{cc} 2\kappa_{1}v_{1}^{2} + \frac{\mu_{2}}{v_{1}}\mu_{12}^{2} & \gamma v_{1}v_{2} - \mu_{12}^{2} \\ \gamma v_{1}v_{2} - \mu_{12}^{2} & 2\kappa_{2}v_{2}^{2} + \frac{\mu_{2}}{v_{2}}\mu_{12}^{2} \end{array}\right),$$

$$M_{2} = \frac{\mu_{12}^{2}}{2} \left(\begin{array}{cc} v_{1} & -1 \\ -1 & v_{2} \end{array}\right),$$

$$M_{3} = \frac{\mu_{12}^{2} + \kappa_{12}v_{1}v_{2}}{2} \left(\begin{array}{cc} v_{1} & -1 \\ -1 & v_{2} \end{array}\right).$$

(50)
The physical low-energy scalar mass eigenstates are connected with the weak scalar states by the following relations

\[
\begin{pmatrix}
  h \\
  H
\end{pmatrix} = \begin{pmatrix}
  \sin \alpha & -\cos \alpha \\
  -\cos \alpha & -\sin \alpha
\end{pmatrix} \begin{pmatrix}
  \rho_1 \\
  \rho_2
\end{pmatrix},
\]
\[
\tan 2\alpha = \frac{2(\gamma v_1 v_2 - \mu_{12}^2)}{2(\kappa_1 v_1^2 - \kappa_2 v_2^2) + \mu_{12}^2 \left( \frac{v_1}{v_2} - \frac{v_2}{v_1} \right)},
\]
(51)

\[
\begin{pmatrix}
  \pi^0 \\
  A^0
\end{pmatrix} = \begin{pmatrix}
  \cos \beta & \sin \beta \\
  \sin \beta & -\cos \beta
\end{pmatrix} \begin{pmatrix}
  \eta_1 \\
  \eta_2
\end{pmatrix},
\]
\[
\pi^\pm = \begin{pmatrix}
  \cos \beta & \sin \beta \\
  \sin \beta & -\cos \beta
\end{pmatrix} \begin{pmatrix}
  \varphi_1^\pm \\
  \varphi_2^\pm
\end{pmatrix},
\]
\[
\tan \beta = \frac{v_2}{v_1}
\]

with the low-energy physical scalar masses given by

\[
m^2_h = \frac{1}{2v_1} \left( \kappa_1 v_1^3 + \kappa_2 v_1 v_2^2 + \mu_{12}^2 v_2 - v_1 \sqrt{\gamma^2 v_1^2 v_2^2 - 2\gamma \mu_{12} v_1 v_2 + \kappa_1^2 v_1^4 - 2\kappa_1 \kappa_2 v_1^2 v_2^2 + \kappa_2^2 v_2^4 + \mu_{12}^4} \right),
\]
(52)

\[
m^2_H = \frac{1}{2v_1} \left( \kappa_1 v_1^3 + \kappa_2 v_1 v_2^2 + \mu_{12}^2 v_2 + v_1 \sqrt{\gamma^2 v_1^2 v_2^2 - 2\gamma \mu_{12} v_1 v_2 + \kappa_1^2 v_1^4 - 2\kappa_1 \kappa_2 v_1^2 v_2^2 + \kappa_2^2 v_2^4 + \mu_{12}^4} \right),
\]
(53)

\[
m^2_{A^0} = \frac{\mu_{12}^2}{2} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right),
\]
\[
m^2_{H^\pm} = \frac{\mu_{12}^2 + \kappa_1 v_1 v_2}{2} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right).
\]
(54)

The physical low-energy scalar spectrum of our model includes two massive charged Higgses ($H^\pm$), one CP-odd Higgs ($A^0$) and two neutral CP-even Higgs ($h, H^0$) bosons. The scalar $h$ is identified as the SM-like 126 GeV Higgs boson found at the LHC. It is noteworthy that the neutral $\pi^0$ and charged $\pi^\pm$ Goldstone bosons are associated with the longitudinal components of the $Z$ and $W^\pm$ gauge bosons, respectively.

Thanks to the specific shape of the Yukawa couplings dictated by the discrete symmetries, the present model is flavor conserving in the down-type and charged lepton sectors because for those sectors we have a special case of Yukawa alignment [33]~[35]. $\phi_2$ generates the masses of the first two down-type quark generations, whereas $\phi_1$ is responsible only for the bottom Yukawa, conversely, $\phi_2$ is associated only with the electron Yukawa, while $\phi_1$ generates the masses of the remaining charged leptons. The Yukawa couplings of both doublets are therefore aligned in these sectors. Due to the lack of Flavor Changing Neutral Currents (FCNCs) in the down-type sector, tightly constrained Kaon and B-meson mixings are protected against neutral scalar contributions. Mixing occurs exclusively in the up-type sector, where both $\phi_1$ and $\phi_2$ couple to the third generation of up-type quarks. Consequently, top quark FCNCs arise that can be exploited as a probe of new physics since associated processes are strongly suppressed in the SM. Explicitly, we obtain the following structures for the up and down-type Yukawas in the scalar and fermion mass bases using the rotation matrices [14], [20], [31] and the corresponding transformations of the right handed fields.

\[
Y^d_h = \begin{pmatrix}
  y^d_{hh} \\
  y^d_{Hh} \\
  y^d_{H_h}
\end{pmatrix} = \sqrt{2} \begin{pmatrix}
  -\frac{c_w m_h}{v_{\gamma\beta}} & 0 & 0 \\
  0 & \frac{c_w m_h}{v_{\gamma\beta}} & 0 \\
  0 & 0 & \frac{m_h s_h}{v_{\gamma\beta}}
\end{pmatrix},
\]
(55)

\[
Y^d_H = \begin{pmatrix}
  y^d_{dd} \\
  y^d_{Hd} \\
  y^d_{H_H}
\end{pmatrix} = \sqrt{2} \begin{pmatrix}
  -\frac{m_h s_h}{v_{\gamma\beta}} & 0 & 0 \\
  0 & \frac{m_h s_h}{v_{\gamma\beta}} & 0 \\
  0 & 0 & \frac{v v_{\gamma\beta}}{s_h c_h}
\end{pmatrix},
\]
(56)

\[
Y^u_h = \begin{pmatrix}
  y^u_{uu} \\
  y^u_{uc} \\
  y^u_{uH}
\end{pmatrix} \simeq \sqrt{2} \begin{pmatrix}
  \frac{m_u s_h}{v_{c\beta}} & 0 & \frac{m_u}{v} V_{ub} V_{ub} \left( c_{s_h} + s_{c_h} \right) \\
  0 & \frac{m_u s_h}{v_{c\beta}} & \frac{m_u}{v} V_{ub} V_{ub} \left( c_{s_h} + s_{c_h} \right) \\
  0 & 0 & \frac{m_u}{v} \left( t^2 c_{s\beta} - s_{c\beta} \right)
\end{pmatrix},
\]
(57)

\[
Y^u_H = \begin{pmatrix}
  y^u_{uu} \\
  y^u_{uc} \\
  y^u_{uH}
\end{pmatrix} \simeq \sqrt{2} \begin{pmatrix}
  -\frac{c_w m_h}{v_{\gamma\beta}} & 0 & \frac{m_u}{v} V_{ub} V_{ub} \left( c_{s_h} - s_{c_h} \right) \\
  0 & -\frac{c_w m_h}{v_{\gamma\beta}} & \frac{m_u}{v} V_{ub} V_{ub} \left( c_{s_h} - s_{c_h} \right) \\
  0 & 0 & -\frac{m_u}{v} \left( V_{ub}^2 c_{c\beta} + s_{c\beta} \right)
\end{pmatrix},
\]
(58)

with the notations $\sin(x) \equiv s_x$, $\cos(x) \equiv c_x$ and $\tan(x) \equiv t_x$ and $V_{ij}$ denote the CKM matrix elements. Furthermore, the mixing angles $\alpha$ and $\beta$ are defined in Eq. [51]. As in other 2HDMs the couplings depend crucially on the parameters $\alpha$ and $\beta$, but should comply with the current bounds if $\tan \beta$ is neither unnaturally large or small, in
which cases deviations from the bottom and top Yukawa couplings with respect to the SM will become very large. This agrees with our previous statement that the fermion mass hierarchies and mixing are best explained by tan$\beta$ values of $O(1)$. As explained above, FCNCs are absent in the down-type quark sector since the matrices $Y_{d,ct}^{h, H}$ do not have off-diagonal entries. The up-type Yukawa couplings $Y_{ut,ct}^{h, H}$, however, allow for the tree-level decays $t \rightarrow q h$ ($q = u, c$), whose branching ratios are currently limited by ATLAS to $\text{Br}(t \rightarrow q h) < 0.79\%$ @ 95% C.L. [86] and by CMS to $\text{Br}(t \rightarrow q c) < 0.56\%$ @ 95% C.L (observed limit) and $\text{Br}(t \rightarrow q c) < 0.65_{-0.19}^{+0.10}\%$ (expected limit) [87]. Since $y_{ut}$ is negligibly small compared to $y_{ct}$, we consider only the stronger CMS constraint that can be interpreted as an upper bound on the off-diagonal top Yukawas to

$$\sqrt{|y_{ct}^h|^2 + |y_{ct}^H|^2} = \sqrt{\frac{2m_t}{v}} \sqrt{|V_{tb} V_{cb} \left( \frac{s_\alpha}{c_\beta} + \frac{c_\alpha}{s_\beta} \right)|^2} < 0.14,$$

which translates to

$$\frac{|c_\alpha - s_\beta|}{c_\beta s_\beta} \lesssim 3.40.$$

The $t \rightarrow ch$ channel is particularly interesting since its branching ratio $\text{Br}(t \rightarrow hc)_{\text{SM}} \approx 10^{-15}$ [86] is extremely suppressed in the SM, but can be potentially large in our model allowing it to be probed at future collider experiments. As shown in Fig. 1, our model predictions can reach branching ratios of $O(0.01\%)$ in some regions of the $\alpha - \beta$ plane, allowing to further constrain our model parameter space with experimental searches for rare top decays.

Recently an analysis of up-type FCNCs in the 2HDM type III has been performed [88] parametrizing the flavor violating $y_{ct}^h$ coupling as $y_{ct}^h = \frac{1}{2} \lambda_{ct} \sqrt{2m_t m_c}$ according to the Cheng–Sher Ansatz [89] (this type of FCNC was shown to be remarkably stable under radiative corrections [90]). Focusing on the $cc \rightarrow tt$ as well as the $t \rightarrow cg$ channels, they find that $\lambda_{ct}$ can still take values of up to 10–20 depending on the neutral heavy Higgs mass. With $y_{ct}^h \propto \frac{1}{\sqrt{v}} V_{cb} V_{tb} \sqrt{2m_t}$ our model corresponds to $\lambda_{ct} \approx \frac{1}{2}$ and is therefore well below the critical region. Indeed, following the analysis of [91] we find numerically that the loop induced decays $t \rightarrow cg$, $t \rightarrow c\gamma$ and $t \rightarrow cZ$ are several orders of magnitude below the current LHC sensitivity. Explicitly, varying the free model parameters $\alpha, \beta$ and the scalar masses $m_H, m_A$ and $m_{H^\pm}$, we expect the branching ratios to be approximately

$$\text{Br}(t \rightarrow cg) \sim O(10^{-9}), \quad \text{Br}(t \rightarrow c\gamma) \sim O(10^{-12}), \quad \text{Br}(t \rightarrow cZ) \sim O(10^{-13}),$$

as opposed to the current upper limits from ATLAS and CMS [92, 93]

$$\text{Br}(t \rightarrow cg) < 1.6 \times 10^{-4}, \quad \text{Br}(t \rightarrow c\gamma, cZ) < 5 \times 10^{-4}.$$

![Figure 1](image-url)
Figure 2: (a) $\text{Br}(t \to h g)$ in the $\alpha - \beta$ plane with $m_{H} = m_{A} = 500 \text{ GeV}$. (b) $\text{Br}(t \to h g)$ as a function of $m_{H}$ and $m_{A}$ for $\alpha = \pi/3$ and $\beta = \pi/4$. The decay rate is to a large extent independent of the charged Higgs mass $m_{H^\pm}$.

The largest branching ratio of the three channels, $\text{Br}(t \to c g)$, is shown in Fig. 2 as a function of $\alpha$ and $\beta$ for fixed $m_{H}$ and $m_{A}$ (a), as well as for variable $m_{H}$ and $m_{A}$ with fixed $\alpha$ and $\beta$ (b). As it turns out, the charged Higgs contribution is tiny and does not affect the prediction for any values of $m_{H^\pm}$.

In the charged lepton sector we obtain

$$ Y_{h} = \sqrt{2} \begin{pmatrix} y_{e h}^l & y_{\mu h}^l & y_{\tau h}^l \\ y_{e h}^\prime & y_{\mu h}^\prime & y_{\tau h}^\prime \\ y_{e h}^\prime & y_{\mu h}^\prime & y_{\tau h}^\prime \end{pmatrix} = \sqrt{2} \begin{pmatrix} -\frac{m_{\tau}}{\sqrt{v}} & 0 & 0 \\ 0 & m_{\mu} \tan \beta & 0 \\ 0 & 0 & m_{\tau} \cot \beta \end{pmatrix}, \quad (63) $$

$$ Y_{H} = \sqrt{2} \begin{pmatrix} y_{e H}^l & y_{\mu H}^l & y_{\tau H}^l \\ y_{e H}^\prime & y_{\mu H}^\prime & y_{\tau H}^\prime \\ y_{e H}^\prime & y_{\mu H}^\prime & y_{\tau H}^\prime \end{pmatrix} = \sqrt{2} \begin{pmatrix} -\frac{m_{\tau}}{\sqrt{v}} & 0 & 0 \\ 0 & -\frac{m_{\mu}}{v \cot \beta} & 0 \\ 0 & 0 & -\frac{m_{\tau}}{v \cot \beta} \end{pmatrix}. \quad (64) $$

The charged leptons are also free of FCNCs due to the lack of off-diagonal Yukawa couplings. Consequently, the recently reported anomaly in $h \to \mu \tau$ decays cannot be explained in our present model, even though it was possible to account for this in other multi-Higgs models with $S_3$ or other discrete symmetries $[94,97]$. The charged Higgs couplings that are relevant, e.g., for $B_{s,d}^0 - \bar{B}_{s,d}^0$ mixing and the radiative decays $b \to q \gamma$ ($q = s,d$), are given by

$$ Y_{H^\pm}^L = \sqrt{2} \begin{pmatrix} y_{ud} & y_{ds} & y_{dt} \\ y_{su} & y_{sc} & y_{st} \\ y_{bu} & y_{bc} & y_{bt} \end{pmatrix} = \sqrt{2} \begin{pmatrix} V_{td} \frac{m_{d}}{v} & \frac{m_{u}}{v} & -V_{td} \frac{m_{u}}{v} \\ \frac{m_{d}}{v} & V_{td} \frac{m_{u}}{v} & -V_{td} \frac{m_{u}}{v} \\ 0 & 0 & V_{td} \frac{m_{u}}{v} \end{pmatrix}, \quad (65) $$

$$ Y_{H^\pm}^R = \sqrt{2} \begin{pmatrix} y_{ud} & y_{us} & y_{ub} \\ y_{cd} & y_{cs} & y_{cb} \\ y_{bd} & y_{bs} & y_{bb} \end{pmatrix} = \sqrt{2} \begin{pmatrix} V_{td} \frac{m_{d}}{v} & V_{td} \frac{m_{u}}{v} & V_{ub} \frac{m_{d}}{v} \\ V_{td} \frac{m_{d}}{v} & V_{td} \frac{m_{u}}{v} & V_{ub} \frac{m_{d}}{v} \\ 0 & 0 & V_{ub} \frac{m_{d}}{v} \end{pmatrix}, \quad (66) $$

$$ Y_{H^\pm}^{\mu} = \sqrt{2} \frac{m_{e}}{v t_{\beta}}, \quad Y_{H^\pm}^{\nu} = \sqrt{2} \frac{m_{\mu}}{v} t_{\beta} (c_{\beta} - s_{\beta}), \quad Y_{H^\pm}^{\tau} = \sqrt{2} \frac{m_{\tau}}{v} t_{\beta} (c_{\beta} + s_{\beta}). \quad (67) $$
In our 2HDM the $h \to \gamma\gamma$ decay receives additional contributions from loops with charged scalars $H^{\pm}$, as shown in Fig. 3 and therefore sets bounds on the masses of these scalars as well as on the angles $\alpha$ and $\beta$. The explicit form of the $h \to \gamma\gamma$ decay rate is

$$\Gamma (h \to \gamma\gamma) = \frac{\alpha_{em}^2 m_h^3}{256 \pi^3 v^2} \sum_f a_{hff} N_C Q_f^2 F_{1/2} (q_f) + a_{hWW} F_1 (q_W) + \frac{\lambda_{hH^{\pm}H^{\mp}} v}{2 m_{H^{\pm}}} F_0 (q_{H^{\pm}}) \bigg| ^2. \quad (68)$$

Here $q_i$ are the mass ratios $q_i = \frac{m_i^2}{M_i^2}$, with $M_i = M_f, M_W$, and $m_{H^{\pm}}, \alpha_{em}$ is the fine structure constant, $N_C$ is the color factor ($N_C = 1$ for leptons, $N_C = 3$ for quarks), and $Q_f$ is the electric charge of the fermion in the loop. From the fermion-loop contributions we consider only the dominant top quark term. Furthermore, $\lambda_{hH^{\pm}H^{\mp}}$ is the trilinear coupling between the SM-like Higgs and a pair of charged Higgses, which is given by

$$\lambda_{hH^{\pm}H^{\mp}} = -\frac{\gamma_{12}^2 + \kappa_{12}^2}{2} v \sin 2\beta \cos (\alpha + \beta). \quad (69)$$

Besides that $a_{htt}$ and $a_{hWW}$ are the deviation factors from the SM Higgs-top quark coupling and the SM Higgs-$W$
gauge boson coupling, respectively (in the SM these factors are unity). These deviation factors are given by

\[ a_{htt} \simeq \frac{\sin \alpha}{\cos \beta}, \]
\[ a_{hWW} = \sin (\alpha - \beta), \]

where in \( a_{htt} \) we neglected the contribution suppressed by small CKM entries.

The dimensionless loop factors \( F_{1/2} (g) \) and \( F_1 (g) \) (for spin-1/2 and spin-1 particles in the loop, respectively) are \[104, 106]\n
\[
F_{1/2} (g) = 2 \left[ g + (g-1) f (g) \right] g^{-2}, \\
F_1 (g) = - \left[ 2g^2 + 3g + 3 (2g-1) f (g) \right] g^{-2}, \\
F_0 (g) = - \left[ g - f (g) \right] g^{-2},
\]

with

\[
f (g) = \begin{cases} 
\arcsin^2 \sqrt{g}, & \text{for } g \leq 1 \\
-\frac{1}{2} \left[ \ln \left( \frac{1+\sqrt{1-g^-3}}{1-\sqrt{1-g^-3}} \right) - i\pi \right]^2, & \text{for } g > 1.
\end{cases}
\]

In what follows we determine the constraints that the Higgs diphoton signal strength imposes on our model. To this end, we introduce the ratio \( R_{\gamma\gamma} \), which normalizes the \( \gamma \gamma \) signal predicted by our model relative to that of the SM:

\[
R_{\gamma\gamma} = \frac{\sigma (pp \to h) \Gamma (h \to \gamma\gamma)}{\sigma (pp \to h)_{SM} \Gamma (h \to \gamma\gamma)_{SM}} \simeq a_{htt}^2 \frac{\Gamma (h \to \gamma\gamma)}{\Gamma (h \to \gamma\gamma)_{SM}}.
\]

The normalization given by Eq. (70) for \( h \to \gamma\gamma \) was also used in Refs. \[94, 108–113\]. The ratio \( R_{\gamma\gamma} \) has been measured by CMS and ATLAS with the best-fit signals \[113, 115\].

\[
R^{CMS}_{\gamma\gamma} = 1.14^{+0.26}_{-0.23} \quad \text{and} \quad R^{ATLAS}_{\gamma\gamma} = 1.17 \pm 0.27.
\]

Figure (4(a)) shows the sensitivity of the ratio \( R_{\gamma\gamma} \) under variations of the mixing angle \( \alpha \) for \( m_{H^\pm} = 500 \text{ GeV}, \gamma_{12} + \kappa_{12} = 1 \) and different values of the mixing angle \( \beta \). It follows that as the mixing angle \( \beta \) is increased, the range of \( \alpha \) consistent with LHC observations of \( h \to \gamma\gamma \) moves away from \( \pi/2 \). On the other hand, the decay rate is largely independent of the charged Higgs mass or the sum of the couplings \( \gamma_{12} + \kappa_{12} \), which is consistent with the contribution mediated by charged scalars to the \( h \to \gamma\gamma \) process being a small correction. In fact we checked numerically it stays almost constant when \( m_{H^\pm} \) is varied from 500 GeV to 1 TeV for fixed values of \( \alpha, \beta \), and the quartic couplings of the scalar potential. For the same values of the charged Higgs mass and quartic couplings, we show in Figure (4(b)) the Z-shaped allowed region in the \( \alpha-\beta \) plane that is consistent with the Higgs diphoton decay rate constraints at the LHC, and overlay it with the relatively weak bound in Eq. (60) that arises from top quark FCNCs.

### B. \( T \) and \( S \) parameters

The extra scalars affect the oblique corrections of the SM, and these values are measured in high precision experiments. Consequently, they act as a further constraint on the validity of our model. The oblique corrections are parametrized in terms of the two well-known quantities \( T \) and \( S \). In this section we calculate one-loop contributions to the oblique parameters \( T \) and \( S \) defined as \[116, 118]\n
\[
T = \frac{\Pi_{33} (q^2) - \Pi_{11} (q^2)}{\alpha_{EM} (M_Z^2) M_W^2} \bigg|_{q^2 = 0}, \quad S = \frac{2 \sin 2\beta_W}{\alpha_{EM} (M_Z)} \frac{d \Pi_{30} (q^2)}{dq^2} \bigg|_{q^2 = 0}.
\]

\( \Pi_{11} (0), \Pi_{33} (0), \) and \( \Pi_{30} (q^2) \) are the vacuum polarization amplitudes with \( \{W^\mu_1, W^\dagger_1\}, \{W^3_2, W^3_3\} \) and \( \{W^3_2, B_\mu\} \) external gauge bosons, respectively, where \( q \) is their momentum. We note that in the definitions of the \( T \) and \( S \) parameters, the new physics is assumed to be heavy when compared to \( M_W \) and \( M_Z \).

The Feynman diagrams contributing to the \( T \) and \( S \) parameters are shown in Figs. 5 and 6.
We split the $T$ and $S$ emphasizing the contributions arising from new physics as $T = T_{SM} + \Delta T$ and $S = S_{SM} + \Delta S$, where $T_{SM}$ and $S_{SM}$ are the SM contributions given by

$$T_{SM} = \frac{3}{16\pi\cos^2\theta_W} \ln\left(\frac{m_h^2}{m_W^2}\right),$$

$$S_{SM} = \frac{1}{12\pi} \ln\left(\frac{m_h^2}{m_W^2}\right),$$

while $\Delta T$ and $\Delta S$ contain all the contributions involving in our model the heavy scalars

$$\Delta T \simeq -\frac{3\cos^2(\alpha - \beta)}{16\pi\cos^2\theta_W} \ln\left(\frac{m_{H^0}^2}{m_h^2}\right) + \frac{1}{16\pi^2 v^2 \alpha_{EM}(M_Z)} \left[m_{H^\pm}^2 - F\left(m_{A^0}^2, m_{H^\pm}^2\right)\right]$$

$$+ \frac{\sin^2(\alpha - \beta)}{16\pi^2 v^2 \alpha_{EM}(M_Z)} \left[F\left(m_{H^0}^2, m_{A^0}^2\right) - F\left(m_{H^\pm}^2, m_{H^\pm}^2\right)\right]$$

$$+ \frac{\cos^2(\alpha - \beta)}{16\pi^2 v^2 \alpha_{EM}(M_Z)} \left[F\left(m_{A^0}^2, m_{A^0}^2\right) - F\left(m_{H^0}^2, m_{H^\pm}^2\right)\right],$$

$$\Delta S \simeq \frac{1}{12\pi} \left[\cos^2(\alpha - \beta) \ln\left(\frac{m_{H^0}^2}{m_h^2}\right) + \sin^2(\alpha - \beta) K\left(m_{H^0}^2, m_{A^0}^2, m_{H^\pm}^2\right) + \cos^2(\alpha - \beta) K\left(m_{A^0}^2, m_{A^0}^2, m_{H^\pm}^2\right)\right],$$

where we introduced the functions $[104, 119, 125]$

$$F\left(m_1^2, m_2^2\right) = \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln\left(\frac{m_1^2}{m_2^2}\right),$$

$$\lim_{m_2 \to m_1} F\left(m_1^2, m_2^2\right) = m_1^2,$$

$$K\left(m_1^2, m_2^2, m_3^2\right) = \frac{1}{(m_2^2 - m_1^2)^3} \left\{m_1^4 (3m_2^2 - m_1^2) \ln\left(\frac{m_1^2}{m_2^2}\right) - m_2^4 (3m_1^2 - m_2^2) \ln\left(\frac{m_2^2}{m_3^2}\right) - \frac{1}{6} \left[27m_1^2 m_3^2 (m_1^2 - m_2^2) + 5 (m_2^6 - m_3^6)\right]\right\},$$

$$\lim_{m_3 \to m_1} K\left(m_1^2, m_2^2, m_3^2\right) = m_1^2.$$
Figure 5: One-loop Feynman diagrams contributing to the $T$ parameter. The fields $H^1$ and $H^2$ are linear combinations of the charged Higgses $H^\pm$, similarly to how $W^\pm$ gauge bosons are defined in terms of $W^1$ and $W^2$. Likewise, the fields $\pi^1$ and $\pi^2$ are linear combinations of the charged Goldstone bosons $\pi^\pm$.

with the properties

$$
\lim_{m_1 \to m_2} K(m_1^2, m_2^2, m_3^2) = K_1(m_2^2, m_3^2) = \ln \left( \frac{m_2^2}{m_3^2} \right),
$$

$$
\lim_{m_2 \to m_3} K(m_1^2, m_2^2, m_3^2) = K_2(m_1^2, m_3^2) = \frac{-5m_1^6 + 27m_1^4m_3^2 - 27m_1^2m_3^4 + 6 (m_1^6 - 3m_1^4m_3^2) \ln \left( \frac{m_2^2}{m_3^2} \right) + 5m_3^6}{6 (m_1^2 - m_3^2)^3},
$$

$$
\lim_{m_1 \to m_3} K(m_1^2, m_2^2, m_3^2) = K_2(m_2^2, m_3^2).
$$

The experimental results on $T$ and $S$ restrict $\Delta T$ and $\Delta S$ to lie inside a region in the $\Delta S - \Delta T$ plane. At the 95% confidence level, these are the elliptic contours shown in Fig. 7. The origin $\Delta S = \Delta T = 0$ is the SM value with $m_h = 125.5$ GeV and $m_t = 176$ GeV. We analyze the $T$ and $S$ parameter constraints on our model by considering two benchmark scenarios, in both keeping $\alpha - \beta = \frac{\pi}{2}$. In the first scenario we assume that the CP-even and CP-odd neutral Higgs bosons have degenerate masses of 500 GeV, below which the LHC has not detected any scalars beyond the SM-like state. In this first scenario, we find that the $T$ and $S$ parameters constrain the charged Higgs masses to the range $550$ GeV $\leq m_{H^\pm} \leq 580$ GeV, which is consistent with the lower bound $m_{H^\pm} \geq 500$ GeV obtained from $b \to s\gamma$ constraints [99]. In the second scenario, we assume that the charged Higgses and CP-even neutral Higgses have degenerate masses of 500 GeV. In this second scenario, the $T$ and $S$ parameter constraints are fulfilled if the CP-odd neutral Higgs boson mass is in the range $375$ GeV $\leq m_{A^0} \leq 495$ GeV.
We have constructed a viable 2-Higgs doublet extension of the Standard Model which features additionally an $S_3$ flavor symmetry and extra scalars that break $S_3$. This leads to textures for fermion masses, and consists in an existence proof of models leading to the quark texture in [55]. Overall, the model can fit the observed masses, CKM and PMNS mixing angles very well. The model has in total seventeen effective free parameters, which are fitted to reproduce the experimental values of eighteen observables in the quark and lepton sectors, i.e., nine charged fermion masses, two neutrino mass squared splittings, three lepton mixing parameters, three quark mixing angles and one CP violating phase of the CKM quark mixing matrix. The model predicts one massless neutrino for both normal and inverted hierarchies in the active neutrino mass spectrum as well as an effective Majorana neutrino mass, relevant for neutrinoless double beta decay, with values $m_{\beta\beta} = 4$ meV and 50 meV, for the normal and the inverted neutrino spectrum, respectively. In the latter case our prediction is within the declared reach of the next generation bolometric CUORE experiment [76] or, more realistically, of the next-to-next generation tonne-scale $0\nu\beta\beta$-decay experiments. The sum of the light active neutrino masses in our model is 59 meV and 0.1 eV for the normal and the inverted neutrino spectrum, respectively, which is consistent with the cosmological bound $\sum_{k=1}^{3} m_{\nu_k} < 0.23$ eV. The additional scalars mediate flavor changing neutral current processes, but due to the specific shape of the Yukawa couplings dictated by the flavor symmetry these processes occur only in the up-type quark sector. In the scalar sector the enlarged field content of the model leads to constraints from both rare top decays and from a $h \rightarrow \gamma\gamma$ rate that can be distinguished from the SM prediction. Among rare top decays, $t \rightarrow ch$ is particularly promising as its branching ratio can reach $O(0.01\%)$ in our model. With respect to the $h \rightarrow \gamma\gamma$, we find that it depends only slightly on the mass of the charged Higgs and the dependence on the quartic scalar couplings is negligible, but the dominant top quark and vector boson contributions are modified in our model and allow us to place constraints on the hierarchy of the $SU(2)$ doublet VEVs ($\beta$) and the mixing of their CP-even mass eigenstates ($\alpha$) that are much stronger than those obtained from the up-type quark flavor changing processes. We also showed for a few benchmark scenarios that our model is compatible with the present bounds for the oblique parameters $T$ and $S$.

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Figure 7: The $\Delta S - \Delta T$ plane, where the ellipses contain the experimentally allowed region at 95% confidence level taken from [126–128]. We set $\alpha - \beta = \frac{\pi}{2}$. Figures (a) and (b) correspond to $m_{A^0} = m_{H^0} = 500$ GeV and $m_{H^0} = m_{H^\pm} = 500$ GeV, respectively. The charged Higgs and CP-odd neutral Higgs boson masses vary between $550$ GeV $\leq m_{H^\pm} \leq 580$ GeV (Fig. 7(a)), $375$ GeV $\leq m_{A^0} \leq 495$ GeV (Fig. 7(b)). The nearly vertical lines going up towards the ellipses correspond to $\Delta T$ and $\Delta S$ parameters in our model as masses are varied in the aforementioned ranges.

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Appendix A: The product rules for $S_3$.

The $S_3$ group has three irreducible representations: $1$, $1'$ and $2$. Denoting the basis vectors for two $S_3$ doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and $y'$ a non trivial $S_3$ singlet, the $S_3$ multiplication rules are [129]:

\[
\begin{align}
(x_1 \quad x_2) \otimes (y_1 \quad y_2) & = (x_1 y_1 + x_2 y_2)_1 + (x_1 y_2 - x_2 y_1)_1' + \left( \frac{x_2 y_2 - x_1 y_1}{x_1 y_2 + x_2 y_1} \right)_2, \\
(x_1 \quad x_2) \otimes (y') & = \left( \frac{-x_2 y'}{x_1 y'} \right), \quad (x')_1 \otimes (y')_1 = (x' y')_1. 
\end{align}
\]

Appendix B: Decoupling and $S_3$ VEVs

We assume that all SM singlet scalars acquire VEVs much larger than the electroweak symmetry breaking scale. This implies that the mixing angle between the scalar singlets and the $SU(2)$ doublet scalars is strongly suppressed since it is of the order of $v_{1,2}^2/\Lambda^2$, as follows from the method of recursive expansion of Refs. [130, 132]. Consequently, the mixing between these scalar singlets and the SM Higgs doublets can be neglected. We also checked numerically that the masses of the low-energy scalars are nearly unaffected by SM singlet VEVs of $O(500$ GeV) and higher.

For simplicity we assume a CP invariant scalar potential with only real couplings as done in Refs. [10, 11, 42, 94]. In the regime where the VEVs decouple, and also because the $1'$ scalar $\zeta$ is charged under $Z'_{3}$, the relevant terms for determining the direction of the $\xi$ VEV in $S_3$ are

\[
V(\xi) = -\mu_\xi^2 (\xi \xi)_1 + \gamma_{\xi,3} (\xi \xi)_2 \xi + \kappa_{\xi,1} (\xi \xi)_1 (\xi \xi)_1 + \kappa_{\xi,2} (\xi \xi)_2 (\xi \xi)_2 + \kappa_{\xi,3} [ (\xi \xi)_2 \xi ]_2 \xi, \quad (B1)
\]
From the minimization conditions of the high-energy scalar potential, we find the following relations:

\[
\begin{align*}
\frac{\partial \langle V \rangle}{\partial v_{\xi_1}} &= 2v_{\xi_1} \left[ \mu_\xi^2 + 2 \left( \kappa_{\xi,1} + \kappa_{\xi,2} + \kappa_{\xi,3} \right) \left( v_{\xi_1}^2 + v_{\xi_2}^2 \right) \right] + 3\gamma_{\xi,3} \left( v_{\xi_2}^2 - v_{\xi_1}^2 \right) = 0, \\
\frac{\partial \langle V \rangle}{\partial v_{\xi_2}} &= 2v_{\xi_2} \left[ \left( \mu_\xi^2 + 2 \left( \kappa_{\xi,1} + \kappa_{\xi,2} + \kappa_{\xi,3} \right) \right) \left( v_{\xi_1}^2 + v_{\xi_2}^2 \right) \right] + 3\gamma_{\xi,3} v_{\xi_1} = 0,
\end{align*}
\]

(B2)

Then, from an analysis of the minimization equations given by Eq. (B2), we obtain for a large range of the parameter space the following VEV direction for \( \xi \):

\[
\langle \xi \rangle = v_\xi (1, 0).
\]

(B3)

From the expressions given in Eq. (B2), and using the vacuum configuration for the \( S_3 \) scalar doublets given in Eq. (5), we find the relation between the parameters and the magnitude of the VEV:

\[
\mu_\xi^2 = -2v_\xi^2 \left[ 3\gamma_{\xi,3} + 4 \left( \kappa_{\xi,1} + \kappa_{\xi,2} + \kappa_{\xi,3} \right) v_\xi \right],
\]

(B4)

These results show that the VEV direction for the \( S_3 \) doublet \( \xi \) in Eq. (5) is consistent with a global minimum of the scalar potential of our model.

[1] S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978). doi:10.1016/0370-2693(78)90172-7
[2] H. Cardenas, A. C. B. Machado, V. Pleitez and J.-A. Rodriguez, Phys. Rev. D 87, no. 3, 035028 (2013) doi:10.1103/PhysRevD.87.035028 [arXiv:1212.1665 [hep-ph]].
[3] A. G. Dias, A. C. B. Machado and C. C. Nishi, Phys. Rev. D 86, 093005 (2012) doi:10.1103/PhysRevD.86.093005 [arXiv:1206.6362 [hep-ph]].
[4] S. Dev, R. R. Gautam and L. Singh, Phys. Lett. B 708, 284 (2012) doi:10.1016/j.physletb.2012.01.051 [arXiv:1201.3755 [hep-ph]].
[5] D. Meloni, JHEP 1205, 124 (2012) doi:10.1007/JHEP05(2012)124 [arXiv:1203.3126 [hep-ph]].
[6] F. González Canales, A. Mondragón, M. Mondragón, U. J. Saldaña Salazar and L. Velasco-Sevilla, Phys. Rev. D 88, 096004 (2013) doi:10.1103/PhysRevD.88.096004 [arXiv:1304.6644 [hep-ph]].
[7] E. Ma and B. Melic, Phys. Lett. B 725, 402 (2013) doi:10.1016/j.physletb.2013.07.015 [arXiv:1303.6928 [hep-ph]].
[8] Y. Kajiyama, H. Okada and K. Yagyu, Nucl. Phys. B 887, 358 (2014) doi:10.1016/j.nuclphysb.2014.08.009 [arXiv:1309.6234 [hep-ph]].
[9] A. E. Cárcamo Hernández, R. Martinez and F. Ochoa, Eur. Phys. J. C 76, no. 11, 634 (2016) doi:10.1140/epjc/s10052-016-4480-3 [arXiv:1309.6567 [hep-ph]].
[10] A. E. Cárcamo Hernández, E. Cataño Mur and R. Martinez, Phys. Rev. D 90, no. 7, 073001 (2014) doi:10.1103/PhysRevD.90.073001 [arXiv:1407.5217 [hep-ph]].
[11] A. E. Cárcamo Hernández, R. Martinez and J. Nisperuza, Eur. Phys. J. C 75, no. 2, 72 (2015) doi:10.1140/epjc/s10052-015-3278-z [arXiv:1401.0937 [hep-ph]].
[12] V. V. Vien and H. N. Long, Zh. Eksp. Teor. Fiz. 145, 991 (2014) [J. Exp. Theor. Phys. 118, no. 6, 869 (2014)] doi:10.7868/S0044451014060044, 10.1134/S1063776114050173 [arXiv:1404.6119 [hep-ph]].
[13] E. Ma and R. Srivastava, Phys. Lett. B 741, 217 (2015) doi:10.1016/j.physletb.2014.12.049 [arXiv:1411.5042 [hep-ph]].
[14] D. Das and U. K. Dey, Phys. Rev. D 89, no. 9, 095025 (2014) Erratum: [Phys. Rev. D 91, no. 3, 039905 (2015) doi:10.1103/PhysRevD.91.039905, 10.1103/PhysRevD.89.095025 [arXiv:1404.2491 [hep-ph]].
[15] D. Das, U. K. Dey and P. B. Pal, Phys. Lett. B 753, 315 (2016) doi:10.1016/j.physletb.2015.12.038 [arXiv:1507.06509 [hep-ph]].
