Kinetic Theory of the Quantum Field Systems With Unstable Vacuum

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Abstract — The description of quantum field systems with meta-stable vacuum is motivated by studies of many physical problems (the decay of disoriented chiral condensate [1], the resonant decay of CP-odd meta-stable states [2,3], self-consistent model of QGP pre-equilibrium evolution [4], the phase transition problem in the systems with broken symmetry [5] etc). A non-perturbative approach based on the kinetic description within the framework of the quasi-particle representation was proposed here. We restrict ourselves to scalar field theory. As an example, the models with potentials of polynomial type are considered.

1 Introduction

Separating of the classical background field is a well known method in solving QFT problems in non-pertrubative approach [6,7,8]. In these methods the quantum fluctuations can be described by perturbation theory. We consider a particular class of scalar field theories with the background field, which is strong enough to generate quasiparticles and propose here the kinetic approach for the description of this process. The back reaction mechanism, i.e. the particle production influence on background field is also discussed. Using the oscillator representation, we derive the generalized kinetic equation with non-perturbative source term for description of particle-antiparticle creation under action of background field and equation of motion for it. The efficiency of this method was demonstrate previously in the framework of scalar QED [9]. As an illustrative example we consider one-component scalar theory with double-well potential. In this example, we study some features of proposed approach, in particular, the selection problem of stable vacuum state, what allows to
avoid appearance of tachyonic regimes. The similar analysis is possible for some other models of such kind: the Friedberg-Lee model \[10\], the non-linear $\eta$ meson model of Witten–Di Vecchia–Veneziano \[2,3\], and so on (e.g., \[1,11\]).

We use the metric $g^{\mu\nu} = \text{diag}(1,-1,-1,-1)$ and assume $\hbar = c = 1$.

2 System of Basic Equations

Let us consider the scalar particle Lagrangian with a self-interaction

$$\mathcal{L}[\Phi] = \frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{2} m_{0}^{2} \Phi^{2} - V[\Phi].$$ \hspace{1cm} (1)

It is assumed that the field variable $\Phi$ can be decomposed into the classical space homogeneous background field $\phi_{0}(t)$ and the fluctuation part $\phi(x)$,

$$\Phi(x) = \phi_{0}(t) + \phi(x), \quad \langle \phi(x) \rangle = 0, \quad \langle \Phi(x) \rangle = \phi_{0}(t),$$ \hspace{1cm} (2)

where symbol $\langle \ldots \rangle$ denotes some averaging procedure.

The potential energy expansion in powers of $\phi(x)$ can be performed:

$$V[\Phi] = V[\phi_{0}] + R_{1}[\phi_{0}] \phi + \frac{1}{2} R_{2}[\phi_{0}] \phi^{2} + Q[\phi_{0}, \phi],$$ \hspace{1cm} (3)

where $R_{1}[\phi_{0}]$ and $R_{2}[\phi_{0}]$ are the first and the second coefficients of the expansion, $Q[\phi_{0}, \phi]$ contains high order terms and will be neglected in the current article (non-dissipative approximation). The equation of motion after field decomposition \[2\] is (dots denote derivatives with respect to time):

$$\left[ \partial_{\mu} \partial^{\mu} + m^{2}(t) \right] \phi + \ddot{\phi}_{0} + m_{0}^{2} \phi_{0} + R_{1}[\phi_{0}] + \frac{1}{2} \frac{dR_{2}[\phi_{0}]}{d\phi_{0}} \phi^{2} = 0,$$ \hspace{1cm} (4)

where $m(t)$ is the time dependent mass,

$$m^{2}(t) = m_{0}^{2} + R_{2}[\phi_{0}].$$ \hspace{1cm} (5)

After averaging of the Eq.\[4\] we get the equation of motion for the background field:

$$\ddot{\phi}_{0} + m_{0}^{2} \phi_{0} + R_{1}[\phi_{0}] + \frac{1}{2} \frac{dR_{2}[\phi_{0}]}{d\phi_{0}} \langle \phi^{2} \rangle = 0.$$ \hspace{1cm} (6)

The kinetic equation(KE) for the fluctuations can be obtained either by the Bogoliubov transformation \[4,6\] or by the oscillator representation approach \[9\]. We found the last one more efficient in application to the current model.
The derivation of KE is fulfilled in the quasiparticle representation, in which the corresponding quadratic Hamiltonian has the diagonal form. One can find the detailed formalism in the works \[4, 9\]. The resulting KE has the following form:

\[
\frac{df(\vec{p}, t)}{dt} = \frac{1}{2} \Delta(\vec{p}, t) \int_{-\infty}^{t} dt' \Delta(\vec{p}, t') [1 + 2f(\vec{p}, t)] \cos \left[ 2 \int_{t'}^{t} d\tau \omega(\vec{p}, \tau) \right],
\]

(7)

where \( f(\vec{p}, t) \) is the distribution function of particles and the transition amplitude is given by \( \Delta(\vec{p}, t) = \dot{\omega}(\vec{p}, t)/\omega(\vec{p}, t) \), where \( \omega(\vec{p}, t) = \sqrt{m^2(t) + \vec{p}^2} \).

The KE (7) can be transformed to a system of ordinary differential equations, which is convenient for numerical analysis \[4\]

\[
2 \dot{f} = \Delta v_1, \quad \dot{v}_1 = \Delta(1 + 2f) - 2\omega v_2, \quad \dot{v}_2 = 2\omega v_1.
\]

(8)

Finally, the average value \( \langle in|\phi^2(x)|in \rangle \) from Eq. (6) is equal to

\[
\langle in|\phi^2(x)|in \rangle = \int \frac{d^3p}{2\omega(\vec{p}, t)} \left[ 1 + 2f(\vec{p}, t) + v_1(\vec{p}, t) \right].
\]

(9)

As a result, the equation of motion for background field Eq.(6) will have the form (the vacuum unit is omitted here)

\[
\ddot{\phi}_0 + m_0^2 \phi_0 + R_1[\phi_0] + \frac{1}{2} \frac{dR_2[\phi_0]}{d\phi_0} \int \frac{d^3p}{\omega(\vec{p}, t)} \left[ f(\vec{p}, t) + \frac{1}{2} v_1(\vec{p}, t) \right] = 0.
\]

(10)

The KE (7) (or equivalent equation system (8)) and Eq.(10) form the closed system of equations of back reaction problem. For a description of vacuum particle creation we use the total particle density

\[
n(t) = \int \frac{d\vec{p}}{(2\pi)^3} f(\vec{p}, t),
\]

(11)

as well as condensate and created particle energies defined as

\[
E_c = \frac{1}{2} \dot{\phi}_0^2 + \frac{1}{2} m_0^2 \phi_0^2 + \frac{1}{4} \lambda \phi_0^4, \quad E_q = \int \frac{d^3p}{(2\pi)^3} \omega(\vec{p}, t) f(\vec{p}, t).
\]

(12)
Figure 1: Time evolution for the symmetric $\Phi^4$ potential. Parameters: $m_0 = 1$, $\phi_0(0) = 1.2$, $\lambda = 1$; (a) evolution of the mean field; (b) evolution of the particle density; (c) evolution of the energy; (d) momentum spectrum of particles at time $t = 10$ and $t = 50$.

3 Examples: some polynomial potentials

3.1. As the first example here we consider the $\Phi^4$ theory $V[\Phi] = \lambda \Phi^4 / 4$, $\lambda > 0$. In this case we have $R_1 = \lambda \phi_0^3$, $R_2 = 3\lambda \phi_0^2$ and the time dependent mass $m^2(t) = m_0^2 + 3\lambda \phi_0^2$. The Eq. (10) is

$$\ddot{\phi}_0 + \left( m_0^2 + 3\lambda \int \frac{d^3p}{\omega(t)} \left[ f + \frac{1}{2} v_1 \right] \right) \phi_0 + \lambda \phi_0^3 = 0. \quad (13)$$

In numerical calculations we apply the zero initial conditions for the distribution function and nonzero one for the background field $\phi_0(t_0) = 1.2$. The choice of parameters ($\lambda = 1$ and $m_0 = 1$) as well as initial conditions is motivated by the desire to make a comparison between our work and [5], where
authors offered the alternative method for description of quantum systems under strong background field action.

As it can be seen on Fig.1 at the early evolution stage all energy is mainly concentrated in the field oscillation. For \( t < 50 \) we have a slow growing of number density. However it drastically increases at \( t \sim 50 \) and after this time the quantum energy dominates over classical one. Note that full energy of the system is conserved precisely. Fig.1 also shows that the spectra of distribution function is close to quasi-equilibrium one. Thus fluctuation temperature can be calculated, but it is beyond the scope of this paper.

3.2. The double-well potential \( \lambda \Phi^4/4 - \mu^2 \Phi^2/2 \) results in the time dependent masses of quasiparticles and background field excitations, which are defined by Eq.5 with substitution \( m_0^2 \rightarrow -\mu^2 \). The stable (non-tachyonic) solutions are absent in the neighbourhood of the point \( \Phi = 0 \). The transition of the system with broken symmetry to the stable state is realized by the shift at one of two stable points, \( \Phi = \Psi_\pm + \Psi \), with the following selection of the background field, \( \Psi = \phi_0 + \phi \), where \( \Psi_\pm = \pm \mu/\sqrt{\lambda} \). After these transformations we have:

\[
\ddot{\phi} + 2\mu^2 \phi_0 + 3\lambda \Psi_\pm \phi_0^2 + \lambda \phi_0^3 + 3\lambda (\Psi_\pm + \phi_0) \int \frac{d^3p}{\omega(\vec{p}, t)} \left[ f + \frac{1}{2} v_1 \right] = 0 \tag{14}
\]

and the transition amplitude in the KE

\[
\Delta(\vec{p}, t) = \frac{3}{2} \lambda \phi_0 (\phi_0 + \Psi_\pm) \omega_\pm^2(\vec{p}, t), \tag{15}
\]

where \( \omega_\pm \) are defined by the time dependent masses

\[
m_\pm^2(t) = m_0^2 + 2\mu^2 + 3\lambda \phi_0 (\phi_0 + 2\Psi_\pm). \tag{16}
\]

It is seen from Eqs.14 and 16 that this redefinition of vacuum state leads to the stable (non-tachyonic) evolution of the system. Detailed research of these basic equations will be done in the future.

This work was partly supported by a Russian Federations State Committee for Higher Education grant(E02-3.3-210) and RFBR grant(03-02-16877).

We are grateful to J. Baacke and D. Blaschke for interest to this work.

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