On the chromoelectric permittivity and Debye screening in hot QCD

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Abstract. We study the response functions (chromo-electric susceptibilities) for an interacting quark-gluon plasma. The interaction effects have been encoded in the effective fugacities for quasi-partons which are extracted self-consistently from the two equations of state for hot QCD. The first one is the fully perturbative $O(g^5)$ EOS and, the second one which is $O(g^6 \ln(1/g))$, incorporates some non-perturbative effects. We find that response function shows large deviations from the ideal behavior. We further determine the temperature dependence of the Debye mass by fixing the effective coupling constant $Q^2$ which appears in the transport equation. We show that our formalism naturally yields the leading order HTL expression for the Debye mass if we employ the ideal EOS. Employing the Debye mass, we estimate the dissociation temperatures for various charmonium and bottomonium bound states. These results are consistent with the current theoretical studies.

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1 Introduction

It is expected that at high temperatures ($T \sim 150 – 200 \text{MeV}$) and high densities ($\rho \sim 10 \text{GeV}/\text{fm}^3$) nuclear matter undergoes a deconfinement transition to the quark-gluonic phase. This phase is under intense investigation in heavy ion collisions, and already, interesting results have been reported by Relativistic Heavy Ion Collider (RHIC) experiments [1]. As an important development, flow measurements [2] suggest that close to the transition temperature $T_c$, the quark-gluon plasma (QGP) phase is strongly interacting — showing an almost perfect liquid behavior, with very low viscosity to entropy ratio — rather than showing a behavior close to that of an ideal gas. See ref. [3] for a comprehensive review of experimental observations from RHIC, and ref. [4] for other recent experimental results. On the other hand, lattice computations [5, 6] also suggest that QGP is strongly interacting even at $T = 2T_c$. This finding has been reproduced by a number of other theoretical studies — by employing AdS/CFT correspondence in the strongly interacting regime of QCD [7, 8], by molecular dynamical simulations for classical strongly coupled systems [9], and by model calculations with Au-Au data from RHIC [10, 11]. In the backdrop of the above developments, a number of standard diagnostics, such as $J/\psi$ suppression and strangeness enhancement, which have been proposed to probe QGP also need to be re-examined. It is also of importance to address other transport properties, production and equilibration dynamics, and the physical manifestations of pre-equilibrium evolution.

If this is the case, as it indeed appears to be, then the plasma interactions would be largely in the non-perturbative regime; in this regime, few analytic techniques are available for a robust theoretical analysis. Effective interaction approaches are needed. In this direction, considerable work has already been done and we refer the reader to ref. [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] for some of the theoretical results.

The effective approaches emphasize the collective origin of the plasma properties which can be best understood within a semi-classical framework. Indeed, in a recent work [22], the successes of hydrodynamics in interpreting and understanding the experimental observations from RHIC has been reviewed. Since more exciting and discerning data are expected from LHC experiments soon, and given the above context, it is worthwhile exploring semi-classical techniques to understand the properties of QGP in heavy ion collisions. In this context, it is known by now [23, 24, 25, 26] that a classical behavior emerges naturally when one considers hard thermal loop (HTL) contributions. A local formulation of HTL effective action has...
been obtained by Blaizot and Iancu who have succeeded in rewriting the HTL effective theory as a kinetic theory with a Vlasov term [27, 28, 29, 30]. A significant development in this direction is the realization that the HTL effects are, in fact, essentially classical and that they are much easier to handle within the frame work of classical transport equations [31, 32, 33]. Thus, the semi-classical techniques appear to hold the promise of providing tools to understand the bulk properties of QGP.

The present paper continues the theme, and its central aim is to combine the kinetic equation approach which yields the transport properties, with the hot QCD equations of state to make predictions which can perhaps be tested in heavy ion collisions. Recently, Ranjan and Ravishankar have developed a systematic approach to determine fully the response functions of QGP, with a special emphasis on the color charge as a dynamical variable [18]. In parallel, Chandra, Kumar and Ravishankar have succeeded in adapting two hot QCD EOS to make predictions for heavy ion collisions [34, 35]. They have shown that the interaction effects which modify the equations of state can be expressed by absorbing them into effective fugacities ($\delta g,q$) and are evaluated up to $T(\bar{\mu}/g^5)_{36,37}$. The second EOS has a free parameter $\delta$, and is evaluated up to $O(g^6)\log(1/g)$ [38]. We denote it by EOS$\delta$, $\delta$ may be fine tuned to get a reasonably good agreement [38] with the lattice results [39], which we exploit here. Both the EOS are expected to be valid for $T \geq 2T_c$. [38].

We determine the Debye mass as a function of temperature first by combining the quasi-particle description of improved pQCD EOS with the semi-classical transport theory formulation of response functions for QGP. Employing the Debye mass so determined, we have estimated the dissociation temperatures for various quarkonia states. For our purpose, we consider two specific hot QCD equations of state: The first, which we call EOS1 is perturbative, with contributions up to $O(g^4)\log(1/g)$ [38]. The second EOS has a free parameter $\delta$, and is evaluated up to $O(g^6)\log(1/g)$ [38]. We denote it by EOS$\delta$, $\delta$ may be fine tuned to get a reasonably good agreement [38] with the lattice results [39], which we exploit here. Both the EOS are expected to be valid for $T \geq 2T_c$. [38].

The paper is organized as follows: In section 2, we introduce the two hot QCD equations of state and outline the recently developed method [34] to adapt them for making definite predictions for QGP at RHIC and the forthcoming experiments at LHC. In section 3, we obtain the expressions for the response functions of interacting QGP and in section 4, we study their temperature dependence in detail. Moreover, we study the Debye screening and the dissociation phenomenon of heavy quarkonia states in hot QCD medium. In doing so we also relate the phenomenological charge that occurs in the transport equation to lattice and experimental observables. We conclude the paper in section 5.

## 2 Hot QCD equations of state and their quasi-particle description

There are various equations of state proposed for QGP at RHIC. These include non-perturbative lattice EOS [39], hard thermal loop (HTL) resumed EOS [40] and perturbative hot QCD equations of state [34, 36, 38]. In the present paper, we seek to determine the chromo-electric response functions for QGP by employing two EOS: (i) the fully perturbative $O(g^4)$ hot QCD EOS proposed by Arnold and Zhai [34] and Zhai and Kastening [37], and (ii) the EOS of $O(g^6)(\ln(1/g) + \delta)$ determined by Kajantie et al [38], by incorporating contributions from non-perturbative scales, $gT$ and $g^2T$. We employ the method recently formulated by Ranjan and Ravishankar [18] to extract the chromo-electric permittivities of the medium. The EOS which we label EOS$\delta$ is given by

\[
P_{g^6}(\ln(1/g)) = \frac{8\pi^2}{45\beta^4} \left\{ \left( 1 + \frac{21N_f}{32} \right) - 15 \left( 1 + \frac{5N_f}{12} \right) \frac{\alpha_s}{\pi} \right. \\
+ 30 \left( 1 + \frac{N_f}{6} \right) \left( \frac{\alpha_s}{\pi} \right)^2 + \left[ 237.2 + 15.97N_f \right] \\
- 0.41N_f^2 + 135 \left( 1 + \frac{N_f}{6} \ln \frac{\alpha_s}{\pi} \right) \left( 1 + \frac{N_f}{6} \right) \\
- \frac{165}{8} \left( 1 + \frac{5N_f}{6} \ln \frac{\alpha_s}{\pi} \right) \left( 1 + \frac{5N_f}{6} \right) \\
+ \frac{1}{8} \left( 1 + \frac{N_f}{6} \right) \left[ -799.2 - 21.99N_f - 1.926N_f^2 \right] \\
+ \frac{495}{2} \left( 1 + \frac{N_f}{6} \right) \left( 1 + \frac{2N_f}{33} \right) \ln \frac{\alpha_s}{\pi} \left( 1 + \frac{N_f}{6} \right)^2 \\
+ \frac{8\pi^2}{45} T^4 \left[ 1134.8 + 65.89N_f + 7.653N_f^2 \right] \\
- \frac{1485}{2} \left( 1 + \frac{N_f}{6} \right) \left( 1 - \frac{2}{33} \right) \ln \frac{\alpha_s}{\pi} \left( 1 + \frac{N_f}{6} \right)^2 \\
\times \left( \frac{\alpha_s}{\pi} \right)^3 \left( \ln \frac{1}{\alpha_s} + \delta \right) \}. 
\]

EOS1 is obtained from this equation by dropping the last term which has contributions of $O(g^6)(\ln(1/g) + \delta)$. The phenomenological parameter $\delta$ is introduced in [38] to incorporate the undetermined contributions of $O(g^6)$. It also acts as a fitting parameter to get the best agreement with the lattice results.

EOS1 and EOS$\delta$ have several ambiguities, associated with the renormalization scale ($\mu_{\bar{M}_S}$), the scale parameter $A_T/A_{\bar{M}_S}$ which occurs in the expression for the running coupling constant $\alpha_s$, and the value of the phenomenological parameter $\delta$. The arbitrariness in fixing ($\mu_{\bar{M}_S}$) has been discussed well in literature and a popular way out is the BLM criterion due to Brodsky, Lepage and Mackenzie [41]. In this criterion, the value of ($\mu_{\bar{M}_S}$) is allowed to vary between $3\pi T$ and $4\pi T$ [42]. Here, we choose $\mu_{\bar{M}_S} = 2.15\pi T \approx 6.7527 \geq 2\pi T$. With this particular choice, all the contributions due to the logarithms containing $\mu_{\bar{M}_S}$ are very small.
The construction of the distribution functions that underlie the EOS, in terms of effective quarks and gluons which act as quasi-excitations, has been discussed by Chandra et al. [34] in the specific context of EOS1 and EOS8. To review the method briefly, all the terms that represent interactions are collected together by recasting them as effective fugacities (\(z_{q,g}\)) for the otherwise free quarks and gluons. Of course, the pure gauge theory case is simply obtained by putting the number of flavors, \(N_f = 0\) in the EOS. Thus, \(z_q\) represents the self interactions of the gluons, while \(z_g\) encapsulates the quark-quark and the quark gluon interaction terms. The quantities \(z_{q,g}\) can be determined from the two EOS self-consistently. In this procedure, all the temperature effects are contained in the effective fugacities \(z = z(\alpha_s(T/T_c))\), where we display the dependence on the temperature and coupling constant explicitly. It has been shown in ref. [34] (where the details can be found) that one can trade off the dependence of the effective fugacities on the renormalization scale (\(\mu_{\overline{MS}}\)) by their dependence on the critical temperature \(T_c\). For that purpose, one utilizes the one loop expression of \(\alpha_s(T)\) at finite temperature given by [44]. It should be borne in mind that the effective fugacity \(z_{q,g}\) introduced here has nothing to do with the usual fugacity which corresponds to the conservation of particle number. It is, therefore, unrelated to the baryon chemical potential in the case of quarks. \(z_{q,g}\) merely encodes the interaction effects present in hot QCD EOS. Its deviation from unity signifies the presence of non-ideal terms in the EOS.

In a recent work Chandra and Ravishankar [35] showed that the effective fugacities \(z_{q,g}\) modifies the dispersion relations. The modification captures the non-zero trace anomaly effects in hot QCD. They further employed this model to determine the shear viscosity and shear viscosity to entropy ratio as a function of temperature. This model has further been generalized to the pure lattice gauge theory EOS in [45] and the authors found that the model works remarkably well for lattice QCD. This quasi-particle description is analogous to Landau’s theory of Fermi liquids. The authors [50] have further determined the temperature dependence of gluon quenching parameter and studied the shear viscosity (\(\eta\)), and its ratio with entropy (\(\eta/S\)) employing the recent work by Asakawa, Müller and Bass [47] and Majumder, Müller and Wang [48].

In figs. 1 and 2, we display the behavior of EOS8 for various values of the parameter \(\delta\). The figures show the pure gauge theory contributions to the EOS and full QCD separately. We remark parenthetically that the studies in the earlier work [34] were confined to EOS1 and the special case \(\delta = 0\) in EOS8. For the details on EOS1 and EOS8 for \(\delta = 0\), we refer the reader to ref. [51] (see fig.1-7 of ref. [34]). First of all, we see that as \(\delta\) increases in magnitude, the EOS, for both pure gauge theory and full QCD, become softer, with \(P/P_t\) taking smaller values, we denote the the ratio \(P/P_t\) by \(R_1\). Kajantie [52] obtains the best fit with the lattice results of Boyd et al. [49] by choosing a value \(\delta = 0.7\). We find that to get agreement with the more recent results of Karsch [39], \(\delta \approx 1.0\) is preferred, when we consider \(T > 2T_c\). Over all, we find that the range of values \(0.8 \leq \delta \leq 1.2\) gives a reasonably good qualitative agreement with the lattice results for the screening lengths. Finally, we set \(A_{\overline{MS}} = T_c\), which is close to the value 0.87\(T_c\) found by Gupta [50]. Here, we wish to mention that there is an uncertainty in fixing the free parameter \(\delta\). This follows from the freedom in choosing the QCD renormalization scale at high temperature. This has been investigated in detail by Blaizot, Iancu and Rebbah [51]. The value of \(\delta\) in the present paper has been obtained by employing the one loop expression for the running coupling constant and the QCD renormalization scale determined in ref. [44].

The behavior of the corresponding fugacities, as a function of temperature, is shown in fig.3. It may be seen that \(0 < z_{q,g} < 1.0\) which ensures the convergence of the method to determine the effective fugacities from the hot
QCD EOS. We now proceed to determine the response of the plasma in the next section.

![Figure 3](image.png)

**Fig. 3.** (Color online) Effective parton fugacities \((z_{a,q})\) quarks determined from EOS6 as a function of temperature. Note that the behavior is shown for \(\delta = 1.0\).

### 3 Response functions for interacting qgp

Recently Ranjan and Ravishankar [18] have determined the form of chromo-electric response functions for collision less quark-gluon plasma within the framework of semi-classical transport theory. They have set up the transport equation in the extended phase space including the SU(3) group space corresponding to dynamical color degree of freedom. They have taken the distribution function in a coherent state basis defined over the extended single particle phase space \(R^6 \otimes C_G\), where \(C_G = G/H\) is the phase space corresponding to the color degree of freedom, obtained as a coset space by factoring the group space by the stabilizer group \(H\) of any reference state in the Hilbert space. Having been employed to study the ideal case, the formalism has not been applied to examine the behavior of the plasma with a realistic EOS. We employ the results of the previous section and rectify this drawback, by incorporating the interaction effects as represented by EOS1 and EOS6.

A brief comment on the response functions. In contrast to electrodynamic plasma, the chromo-electric response has a richer structure. Apart from the standard permittivity which we shall call Abelian and denote by \(\epsilon_A\), there are additional response functions, their number depending on the color carried by the partons. Thus, quarks have an additional response function which affects the non-Abelian coupling. The corresponding permittivity will be called non-Abelian, and denoted by \(\epsilon_N\). The two functions exhaust the response in the quark sector. The gluonic sector, arising from the adjoint representation of the gauge group admits yet another kind of response, corresponding to tensor excitations. These excitations are not allowed in the quark sector (which emerges from the fundamental representation of the gauge group). We consider each of these response functions for the interacting QGP. The response functions are obtained in the temporal gauge.

Consider first the familiar Abelian component of the response \(\epsilon_A\). For an isotropic plasma (in the absence of chromo-magnetic fields), its expression is given by [18]

\[
\tilde{\epsilon}_A(\omega, k) = 1 + Q^2 I_0(\omega, k)
\]

where \(Q^2 = Q^a Q^a\) is the color charge magnitude squared, and \(I_0\) is determined by the equilibrium distribution function thus:

\[
\int \frac{1}{\omega - \frac{k \cdot p}{\varepsilon} + i \tau} \frac{\partial f_{eq}}{\partial p_i} d^3p = k_i I_0(\omega, k),
\]

The non-Abelian response function, which has been evaluated in the long wavelength limit, is given by

\[
\tilde{\epsilon}_N(\omega, \omega') = \left(1 + \frac{Q^2 I_1(\omega', k') |_{k'=0}}{\omega}\right)
\]

where \(I_1\) is defined as

\[
I_1(\omega, k) = \frac{1}{3} Tr \left( \int \frac{p_i}{\omega - \frac{k \cdot p}{\varepsilon}} \frac{\partial f_{eq}}{\partial p_i} d^3p \right).
\]

We recall that the new constitutive Yang-Mills equations, in the presence of the medium, are given by

\[
\frac{Q^2 f^{alm}}{\omega} \int I_1(\omega', k') |_{k'=0} \tilde{A}^{i}_{l}(\omega - \omega', k - k') \times \tilde{E}^{m}_{l}(\omega', k') dw' d^3k' = 0.
\]

As pointed out in [18], the Abelian and non-Abelian responses are not independent of each other. Gauge invariance relates them, by virtue of which we can obtain both from a common generating function as follows:

\[
I_0 = \frac{1}{k^2} \frac{\partial}{\partial \omega} \int ln(\omega - \frac{k \cdot p}{\varepsilon}) k_i \partial p_i d^3p
\]

\[
I_1 = \frac{1}{3} Tr \left( \frac{\partial}{\partial k_j} \int ln(\omega - \frac{k \cdot p}{\varepsilon}) \partial p_i d^3p \right).
\]

We further recall that these expansions are determined when the system is displaced slightly from its equilibrium, in the collisionless limit.

#### 3.1 Ideal response

It is convenient to first write the expressions for the responses of ideal distributions for quarks and gluons. The
responses due to EOS1 and EOSδ get a simple modification over their ideal forms since we have mapped successfully the interaction effects into quasi free partons with effective fugacities. Thus, in the ideal case we have, for the quarks,

\[ \tilde{\epsilon}_A^{(q)} = \left[ 1 + \frac{2\pi^2 Q^2 T^2 N_f}{3k^2} \left\{ \frac{\omega}{k} \ln \left( \frac{\omega + k}{\omega - k} \right) + 2 \right\} \right] \]

and the non-Abelian response function is given by

\[ \tilde{\epsilon}_N^{(q)} = \left\{ 1 - \frac{4\pi^3 Q^2 T^2 N_f}{9} \frac{1}{\omega^2} \right\}. \]

The imaginary part of Abelian (\(\tilde{\epsilon}_A\)) and non-Abelian component (\(\tilde{\epsilon}_N\)) of the chromo-electric permittivity can be easily evaluated by the standard Landau \(i\epsilon\) prescription. These are needed to obtain Landau damping which we do not study here. The gluon effective charge \((Q_g^2)\) and the quark effective charges \((Q_q^2)\) are related to each other by the total number of color \(N_c\) for \(SU(N_c)\) as \(Q_g^2 = N_c Q_q^2\), where \(Q\) is the color charge of a single quark. It is easy to see that the gluonic susceptibility, \(\chi^{(q)} = \tilde{\epsilon}_A^{(N)} - 1\) is identical to the quark susceptibility, \(\chi^{(q)} = \tilde{\epsilon}_A^{(N)} - 1\); they are related as,

\[ \chi^{(q)} = \frac{N_f}{2N_c} \chi^{(g)} \]

3.2 Interaction effects

We now consider the modification that the above expressions undergo permittivities arising because of the new EOS. Recall that the corresponding equilibrium distribution functions differ from each other only in their form for the chemical potentials \(\mu_{q,g}\). The responses thus depend on the interactions implicitly through a formal dependence on \(z\).

Considering the gluonic case, i.e., pure gauge theory first, we get the expressions for the two permittivities as

\[ \tilde{\epsilon}_A = \left[ 1 + \frac{4\pi^3 N_c Q^2 T^2 g_2(z) g_2'(z)}{3k^2} \left\{ \frac{\omega}{k} \ln \left( \frac{\omega + k}{\omega - k} \right) + 2 \right\} \right] \]

and the non-Abelian response function as

\[ \tilde{\epsilon}_N = \left\{ 1 - \frac{8\pi^3 N_c Q^2 T^2 g_2(z) g_2'(z)}{9} \frac{1}{\omega^2} \right\}. \]

The function \(g_2(z)\) has the series expansion

\[ g_2(z) = \sum_{i=1}^{\infty} \frac{z^i}{i} \text{ for } z \ll 1. \]

Note that \(g_2'(1) = 1\) gives the ideal limit.

Similarly, the corresponding expressions in the quark sector are obtained as

\[ \tilde{\epsilon}_A = \left[ 1 + \frac{2\pi^3 Q^2 T^2 N_f f_2(z) f_2'(z)}{3k^2} \left\{ \frac{\omega}{k} \ln \left( \frac{\omega + k}{\omega - k} \right) + 2 \right\} \right] \]

and the non-Abelian response for effective quarks reads:

\[ \tilde{\epsilon}_N = \left\{ 1 - \frac{4\pi^3 Q^2 T^2 N_f f_2(z) f_2'(z)}{9} \frac{1}{\omega^2} \right\}. \]

The function \(f_2(z)\) defined via the integral below.

\[ \int_0^{\infty} \frac{x^{\nu-1}}{z^\nu e^{2\pi x} + 1} dx = \Gamma(\nu) f_\nu(z) \]

\[ f_\nu(z) = \sum_{l=1}^{\infty} (-1)^{l-1} \frac{\nu^l}{l^\nu} \frac{\Gamma(\nu)}{\Gamma(l+\nu)} \text{ for } z \ll 1 \]

and \(f_2'(1) = 1\).

4 Effective charges and relative susceptibilities

eq. (10) admit a simple physical interpretation, when compared with their counterparts eq. (7-8). Indeed, the sole effect of the interactions on the transport properties is to merely renormalize the quark and the gluon charges \(Q_{g,q}\) as shown below:

\[ Q^2_g \rightarrow Q^2_g g_2(z_g); \quad Q^2_q \rightarrow Q^2_q g_2(z_q). \]

The renormalization factors \(g_2(z_g), f_2(z_g)\) further possess the significance of chromo-electric susceptibilities, relative to the ideal values. To see that, we note that the Abelian and the non-Abelian strengths for gluons as well as quarks suffer the same renormalization reflecting the underlying gauge invariance. Furthermore, the expressions for the relative susceptibilities are given by,

\[ R = \frac{\chi(z)}{\chi(1)} = \frac{A(z)}{A(1)} = \frac{N(z)}{N(1)} = \left\{ \frac{f_2(z_g)}{g_2'(z_g)} \right\} \text{ for quarks, } \]

and

\[ R_{q,g} = \frac{\chi^{(q)}(z_g)}{\chi^{(q)}(z_g)} = \frac{A^{(q)}(z_g)}{A^{(q)}(z_g)} = \frac{N^{(q)}(z_g)}{N^{(q)}(z_g)} = \frac{f_2(z_g)}{g_2'(z_g)}. \]

Note that the relative susceptibilities are entirely functions of the single variable \(T/T_c\) and are independent of
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The dependence of the susceptibilities on \((\omega, k)\) has already been studied in detail in ref.\[18\]. We merely concentrate on the temperature dependence below.

Before we go on to discuss the susceptibilities and other bulk properties, we point out an essential care to be taken in using the above susceptibilities for determining the response of the plasma. For pure gauge theory, only the gluonic part contributes, while for the full QCD, we have to necessarily take the contribution from both the quark and the gluonic sector. We discuss both the cases below. The response functions for the full QCD is obtained by summing over the response function for quark as well as gluon plasma. The relative susceptibility for full QCD plasma is given by

\[
R' = \frac{N_f f_2'(z_q) + 2 N_c g_2'(z_g)}{N_f + 2 N_c}.
\]

### 4.1 Behavior of the susceptibilities

We now proceed to study the behavior of the relative susceptibilities displayed in eqs.\[14\], \[15\] and \[16\] as functions of temperature. As observed, relative susceptibilities for both quarks and gluons scale with \(T/T_c\). We have plotted the relative susceptibilities \(R, R_{gg}\) and \(R'\) as functions of \(T/T_c\) (See figs.4-6), for both EOS1 and EOS\(\delta\). Please note that we have chosen \(\delta = 1.0\) in EOS\(\delta\).

Fig.4 shows the relative susceptibility of a purely gluonic plasma as a function of temperature for EOS1 and EOS\(\delta\).

![Fig. 4. (Color online) Relative susceptibility, \(g_2'(z_g)\) (see eq. \[14\], for pure gauge theory plasma as a function of \(T/T_c\) for EOS1 and EOS\(\delta\) \((\delta = 1)\).](image)

We see From fig. 4 that the susceptibility of a purely gluonic plasma is weaker in the presence of interactions, approaching its ideal value asymptotically with increasing temperatures. Equivalently, there is a decrease in the value of the phenomenological coupling \(Q^2\), relative to its ideal value.

The behavior of quark gluon plasma is not qualitatively different from that of a purely gluonic plasma, as may be seen from fig.5. In other words, the quark contribution is of the same order as the purely gluonic contribution. However, the relative contribution from the quarks and the gluons does depend on the EOS considered. Indeed, with EOS1 (where interactions up to \(O(g^3)\) are included), fig.6 shows that the gluonic contribution dominates over the quark contribution for both \(N_f = 2\) and \(N_f = 3\) and same is true for EOS\(\delta\). The dominance is more pronounced for EOS1 as compare to EOS\(\delta\).

At this juncture, we clarify that by ideal we mean that we employ the ideal EOS. The expressions so obtained are the same as the one loop result, or equivalently, the leading order HTL result. Similarly, the phrase interacting QGP refers to the contribution of the non-ideal terms in the hot QCD EOS, which is equivalent to the inclusion of higher order corrections to the one loop result.
4.2 Debye mass from the chromoelectric susceptibility

The static or zero frequency limit of the response function leads to the well known relation with the Debye mass as follows, \( \epsilon(k, T) = 1 + M_D^2 k^2 \).

One can compute the Debye mass for full QCD by utilizing the additive nature of Abelian response functions of partons in QCD. The full permittivity for hot QCD plasma will be obtained,

\[
\epsilon(k, T) = 1 + \chi^{(g)}(z_g) + \chi^{(q)}(z_q)
\]

which naturally incorporates the additivity of Debye mass square in thermal QCD.

For an ideal QGP the Debye mass comes out to be,

\[
(M_D^2)^2 = 8\pi^3 Q^2 T^2 (N_c/3 + N_f/6).
\]

Interestingly, this matches with the leading order HTL expression for the Debye mass if one identifies the phenomenological charge \( Q \) as

\[
Q^2 = \frac{g^2(T)}{8\pi^3},
\]

where \( g(T) \) is the QCD running coupling constant at finite temperature. This fixes the phenomenological charge \( Q \). We employ this to study how the inclusion of interactions influence the Debye screening and the dissociation phenomenon of heavy quarkonia in a hot QCD medium.

The current approach to determine the Debye mass is rather different from that in ref.\[54\]. Here, we have determined the Debye mass from the chromo-electric permittivity recently obtained by Ranjan and Ravishankar\[18\]. On the other hand, the method adopted in ref.\[34\] is based on the earlier works of Kelly et. al\[31, 32\]. We remark that the two approaches are equivalent at the one loop level provided that the phenomenological charge is fixed appropriately.

For the interacting QGP the Debye mass can be written by,

\[
M_D^2 = 8\pi^3 Q^2 T^2 \frac{N_c}{3} \left( g'_2(z_g) + \frac{N_f}{2N_c} f'_2(z_g) \right).
\]

To see how the interactions influence the Debye screening, we consider the ratio, \( R_{M_D} = (M_D)^2/(M_D^0)^2 \), which is given as,

\[
R_{M_D} = \frac{2N_c g'_2(z_g) + N_f f'_2(z_g)}{2N_c + N_f}.
\]

The expression of \( R_{M_D} \) is the same as that for relative susceptibility \( R' \) which follows from the expression for the Debye mass of full QCD in terms of the response function (eq.17). It is clear from figs. 4 and 5 that the inclusion of the interactions significantly lower the Debye mass.

The Debye mass will approach its ideal value only asymptotically. We have shown the behavior of the screening length as a function of temperature for EOS1 and EOS\( \delta \) in fig. 7 and fig.8 respectively. To compare these results with lattice predictions of Zantow and Kazmarek\[52\], we choose \( T_c = 0.27 \) GeV for pure gauge theory and \( T_c = 0.203, 0.197 \) for 2- and 3-flavor QCD respectively. The screening length for both EOS1 and EOS\( \delta \) show qualitative agreement with these results in \[52\]. As expected, the agreement is more pronounced for EOS\( \delta \) as compare to EOS1. We shall now proceed to determine the dissociation temperatures for various quarkonia states in a hot QCD medium.

4.3 Dissociation temperatures

To determine the dissociation temperatures of heavy quarkonia, we follow a well known criterion–whenever the \( \text{rms} \) radius, \( r_{q\bar{q}} \geq 1/M_D \) for a particular heavy quarkonia bound state, the state will dissociate in the medium. The equality sign will yield the dissociation temperature. To this end, we shall employ the \( \text{rms} \) radius of various bound states of charmonium and bottomonium in ref.\[53\]. We have shown the dissociation temperatures for various quarkonia in Table I along with the flavor dependence. The dissociation temperatures decrease with increasing number of flavors. The \( J/\Psi \) dissociation temperature varies between 1.9\( T_c \) to 1.5\( T_c \) for EOS\( \delta \) and 2.28\( T_c \) to 1.58\( T_c \) for EOS1.
we have determined the dissociation temperatures by assuming the validity of EOS1 and EOS8 temperatures as small as $T \geq 1.5 T_c$.

We now turn our attention to compare hot QCD estimates for dissociation temperatures with other theoretical works. In a recent paper, Satz [25] has studied the dissociation of quarkonia states by studying their in-medium behavior. These estimates were based on the Schrödinger equation for the Cornell potential. In a more recent work, Alberico et al. [54] reported the dissociation temperatures for charmonium and bottomonium states for $N_f = 0$ and $N_f = 2$ QCD. They have solved the Schrödinger equation for the charmonium and bottomonium states at finite temperature in the presence of a temperature dependent potential—computed from the lattice QCD. The estimates for $T_D$ shown in Table I are consistent with these estimates [25, 54] and also consistent with other lattice results [55] for quenched QCD and predictions of dynamical $N_f = 2$ QCD by Aarts et al. [56]. Along these results, we wish to mention the very recent estimates on dissociation temperature reported by Mócsy and Pétreeczky [57] and Agotiya, Chandra and Patra [43]. Their estimates for $J/\Psi$ dissociation temperatures and for $T$ are significantly smaller than the earlier results and results quoted in Table 1.

5 Conclusions and Outlook

In conclusion, we have successfully extracted the quasi-free particle content of two hot QCD equations of states and used them to determine the chromo-electric permittivities within the standard Boltzmann-Vlasov kinetic approach. The Abelian and the non-Abelian components of the permittivity are obtained, for pure gauge theory and the full QCD. We have shown that the effect of the interactions is to merely renormalize the magnitude of the effective color charge, $Q$. We have determined the Debye mass from the chromo-electric susceptibility which yields the leading order HTL result for an ideal QGP. The viability of the two EOS, especially EOS8 is thus phenomenologically well supported. It would also be of interest to extend the analysis to other signatures like strangeness enhancement, and also for QGP with a finite baryonic chemical potential [58, 59] and the HTL and HDL equations of state [51, 60].

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