Simulation Studies on the Stability of the Vortex-Glass Order

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The stability of the three-dimensional vortex-glass order in random type-II superconductors with point disorder is investigated by equilibrium Monte Carlo simulations based on a lattice XY model with a uniform field threading the system. It is found that the vortex-glass order, which stably exists in the absence of screening, is destroyed by the screening effect, corroborating the previous finding based on the spatially isotropic gauge-glass model. Estimated critical exponents, however, deviate considerably from the values reported for the gauge-glass model.

Due to the enhanced effect of fluctuations, the problem of the phase diagram of high-$T_c$ superconductors in applied magnetic fields is highly nontrivial and has attracted much interest recently. For random type-II superconductors with point disorder, possible existence of an equilibrium thermodynamic phase called the vortex-glass (VG) phase, where the vortex was pinned on a long length scale by randomly distributed point-pinning centers, was proposed [1]. In such a VG state, the phase of the condensate wavefunction is frozen in time but randomly in space, with a vanishing linear resistivity $\rho_L$. It is a truly superconducting state separated from the vortex-liquid phase with a nonzero $\rho_L$ via a continuous VG transition.

This proposal was supported by subsequent experiments. In particular, transport measurements on films [2] and twinned single crystals [3] gave evidence for the occurrence of a continuous transition into the glassy superconducting state. It should be noticed, however, that these samples often contain extended defects with correlated disorder, such as grain boundaries, twins and dislocations. Since these extended defects could pin the vortex more efficiently than point defects, the possibility still remains that extended defects play a crucial role in the experimentally observed "VG transitions", and the sample only with point defects behaves differently [4-7].

Stability of the hypothetical VG state was also studied by numerical simulations. Here, it is essential to obtain the data of appropriate thermodynamic quantities in true equilibrium and to carefully analyze its size dependence. So far, such calculations have been limited almost exclusively to a highly simplified model called the gauge-glass model. Previous simulations on the three-dimensional (3D) gauge-glass model have indicated that, while the stable VG phase exists in the absence of screening [4-7], the finite screening effect inherent to real superconductors eventually destabilizes it [10,11].

Meanwhile, it has been recognized that the gauge-glass model has some obvious drawbacks [8]. First, it is a spatially isotropic model without a net field threading the system, in contrast to the reality. Second, the source of quenched randomness is artificial. The gauge-glass model is a random flux model where the quenched randomness appears in the phase factor associated with the flux. In reality, the quenched component of the flux is uniform, nothing but the external field, and the quenched randomness occurs in the superconducting coupling. It remains unclear whether these simplifications underlying the gauge-glass model really affect the basic physics of the VG ordering in 3D.

The purpose of the present letter is to introduce a model in which the above limitations of the gauge-glass model are cured, and to examine by extensive Monte Carlo (MC) simulations the nature of the 3D VG ordering with and without screening beyond the gauge-glass model. It is found that, as in the gauge-glass model, the VG phase is stable in the absence of screening but the finite screening effect destabilizes it.

We consider the dimensionless Hamiltonian[12,13],

$$\mathcal{H}/J = - \sum_{<ij>} J_{ij} \cos(\theta_i - \theta_j - \lambda_{ij}) + \frac{\lambda_0^2}{2} \sum_p (\nabla \times \vec{A} - \Phi_{ext})^2, \quad (1)$$

where $J$ is the typical coupling strength, $\theta_i$ is the phase of the condensate at the $i$-th site of a simple cubic lattice, $\vec{A}$ is the fluctuating gauge potential at each link of the lattice, the lattice curl $\nabla \times \vec{A}$ is the directed sum of $A_{ij}$’s around a plaquette with $A_{ij} = -A_{ji}$, and $\lambda_0$ is the bare penetration depth in units of lattice spacing. $\Phi_{ext}$ is an external flux threading the elementary plaquette $p$, which is equal to $\theta$ if the plaquette is on the $xy$-plane and zero otherwise, i.e., a uniform field is applied along the $z$-direction. The first sum in (1) is taken over all nearest-neighbor pairs, while the second sum over all elementary plaquettes. Fluctuating variables to be summed over are the phase variables, $\theta_i$, at each site and the gauge variables, $A_{ij}$, at each link. In order to allow for the flux penetration into the system, we impose free boundary conditions in all directions for both $\theta_i$ and $A_{ij}$ [12,13]. Quenched randomness occurs only in the superconducting coupling $J_{ij}$ which is assumed to be an independent random variable uniformly distributed between $[0,2]$. We stress that the aforementioned drawbacks of the gauge-glass model have been cured now: The present model has a uniform field threading the system and the quenched randomness occurs in the superconducting coupling, not
in the gauge field.

In addition to the global U(1) gauge symmetry, the Hamiltonian (1) has a local gauge symmetry, i.e., the invariance under the local transformation \( \theta_i \to \theta_i + \Delta \) and \( A_{i+\delta} \to A_{i+\delta} + \Delta \) for an arbitrary site \( i \) (\( A_{i+\delta}'s \) are all link variables emanating from the site \( i \)). We adopt the Coulomb gauge, imposing the condition, \( \text{div} \mathbf{A} = \sum_\delta A_{i+\delta} = 0 \), at every site \( i \). In the limit of vanishing screening \( \lambda_0 \to \infty \), the link variable \( A_{ij} \) is quenched to the external-field value, and the fluctuating variable becomes the phase variable \( \theta_i \) only.

Simulation is performed based on the exchange MC method where the systems at neighboring temperatures are occasionally exchanged \[14\]. We run two independent sequences of systems (replica 1 and 2) in parallel, and compute a complex overlap \( q \) between the local superconducting order parameters of the two replicas \( \psi_i^{(1,2)} \equiv \exp(i \theta_i^{(1,2)}) \),

\[
q = \frac{1}{N} \sum_i \psi_i^{(1)} \ast \bar{\psi}_i^{(2)}, \tag{2}
\]

where the summation is taken over all \( N = L^3 \) sites. In terms of the overlap \( q \), the Binder ratio is calculated by

\[
g(L) = 2 - \frac{<|q|^4>}{<|q|^2>^2}, \tag{3}
\]

where \( \langle \cdots \rangle \) represents the thermal average and \( [\cdots] \) represents the average over bond disorder. Note that, thanks to the Coulomb-gauge condition, the superconducting order parameter, which is originally not local-gauge invariant, becomes a nontrivial quantity.

We deal mainly with two cases; [I] no screening corresponding to \( \lambda_0 = \infty \), and [II] finite screening corresponding to \( \lambda_0 = 2 \), whereas some data are taken for the case of stronger screening corresponding to \( \lambda_0 = 1 \). In either case, we fix the field intensity to \( h = 1 \) which corresponds to \( f = 1/(2\pi) \approx 0.159 \) flux quanta per plaquette. We have chosen the fractional value of \( f \) to avoid the commensurability effect associated with the vortex-lattice formation. The lattice sizes studied are \( L = 6, 8, 10, 12 \) and \( 16 (\lambda_0 = \infty) \), and \( L = 6, 8, 10, 12 (\lambda_0 = 2) \). Equilibration is checked by monitoring the stability of the results against at least three-times longer runs for a subset of samples. Sample average is taken over \( 300 (L = 6) \), 200-300 \( (L = 8) \), 120 \( (L = 10) \), 75-150 \( (L = 12) \) and 136 \( (L = 16) \) independent bond realizations.

We begin with the case of no screening \( (\lambda_0 = \infty) \). The size and temperature dependence of the calculated Binder ratio is shown in Fig.1(a). As can be seen from Fig.1(a), \( g(L) \) for different \( L \ge 8 \) tends to merge, or weakly cross, at \( T/J = 0.68 \pm 0.02 \), indicating that the VG transition occurs at a finite temperature in the absence of screening. Observed near marginal behavior suggests that \( D = 3 \) is close to the lower critical dimension.

In the present model, as well as in reality, the nature of fluctuations along the field (longitudinal direction) and perpendicular to the field (transverse direction) could differ. An extreme possibility here may be that the VG order occurs only in some spatial component, say, in the transverse component, keeping the other (longitudinal) component disordered \[14\]. Indeed, the possibility of such “two-dimensional” or purely “transverse” vortex order has been discussed in the literature as a “decoupling” transition \[14\]. In order to probe such exotic possibility, we introduce a transverse Binder ratio in terms of the layer-overlap \( q_k' \) defined for the \( k \)-th \( xy \)-layer of the lattice by

\[
q_k' = \frac{1}{L^2} \sum_{i \in k} \psi_i^{(1)} \ast \bar{\psi}_i^{(2)}, \tag{4}
\]

\[
g_{\text{trans}}(L) = 2 - \frac{(1/L) \sum_k [|q_k'|^4]}{(1/L) \sum_k [|q_k'|^2]^2}, \tag{5}
\]

When the VG order occurs in each layer, \( g_{\text{trans}}(L \to \infty) \) should be nonzero. In particular, if the purely transverse VG order is to occur as a consequence of the
layer-decoupling, $g_{\text{trans}}(L \to \infty)$ should stay finite while $g(L \to \infty)$ vanishes.

The calculated $g_{\text{trans}}(L)$ is shown in Fig.1(b). As can be seen from Fig.1(b), $g_{\text{trans}}(L)$ exhibits behavior quite similar to $g(L)$, revealing a merging or a weak crossing at $T/J = 0.67 \pm 0.02$. This indicates that the present model exhibits only a single bulk VG transition where both the transverse and longitudinal components order model exhibits only a single bulk VG transition where both the transverse and longitudinal components order simultaneously.

We found no evidence of anisotropic scaling, although the errors with $
u \simeq 4.0$ and $
u \simeq 3.5$ from $g_{\text{trans}}$ and $q_{\text{trans}}^{(2)}$, which again agree within the errors. (Note that $\eta$ should be equal to -1 for a $T = 0$ transition with nondegenerate ground state as expected in the present model.) Hence, the scaling also appears to be isotropic with $\nu = 3.5 \pm 1.0$.

The obtained exponents are summarized in Table 1 and are compared with the values reported for the spatially isotropic gauge-glass model with and without screening. Apparently, there exists a significant deviation between the two results. In particular, $\nu$ of the present model appears to be significantly larger than $\nu$ of the gauge-glass model, which might suggest that the present model lies in a universality class different from that of the gauge-glass model.

Finally, we wish to discuss the experimental implication of our results. The present study suggests, in accord
with the previous studies based on the gauge-glass model, that in random type-II superconductors with point disorder there should be no VG phase at finite temperature in the strict sense. As discussed, experimental data on films and twinned crystals supporting the occurrence of a thermodynamic transition into the truly superconducting glassy state might well reflect the properties associated with extended defects. In this connection, the properties of a sample exclusively containing point defects is of great interest [4-6]. Recently, Petrean et al measured the transport properties of such sample, untwinned, proton irradiated YBCO [10]. These authors observed an Ohmic behavior at all temperature studied, but found that the linear resistivity appeared to vanish at a finite $T_g^*$ as $\rho_L \sim (T-T_g^*)^s$ with a universal exponent $s \approx 5.3\pm 0.7$. Since the non-Ohmic region could not directly be reached in these measurements, it is not clear at the present stage whether the non-Ohmic behavior really sets in below the apparent $T_g^*$ deduced from the high-temperature Ohmic regime. An another possibility, which is entirely consistent with the present result, may be that the power-law decrease of $\rho_L$ eventually breaks down at some temperature close to $T_g^*$ yielding a small but nonzero $\rho_L$ even at $T < T_g^*$. If this is the case, the universal critical behavior of $\rho_L$ observed above $T_g^*$ should be governed by the $\lambda_0 = \infty$ VG fixed point [1,20], which, however, should eventually be unstable against the screening effect. Indeed, our present estimate of $s = (z-1)\nu \approx 5.1$ for the $\lambda_0 = \infty$ transition is close to the experimental value of Ref. [19]. It might be interesting to experimentally determine the exponents $\nu$, $z$ and $\eta$ separately, as well as to go further down to lower temperatures in order to examine whether the non-Ohmic behavior really sets in below $T_g^*$.

In summary, we have introduced a VG model which possesses a uniform field and cures the limitations of the gauge-glass model. Extensive simulations show that, while the stable vortex-glass phase occurs in the absence of screening, it is eventually destroyed by the screening effect. Critical exponents associated the VG transitions appear to differ from those reported for the gauge-glass model, suggesting that real VG transitions may lie in a different universality class.

The numerical calculation has been performed on the FACOM VPP500 at the supercomputer center, ISSP, University of Tokyo. The author is thankful to R. Ikeda and S. Okuma for useful discussion.

### Table 1: Critical exponents of the present model compared with the values reported for the gauge-glass model for both cases of $\lambda_0 = \infty$ (no screening) and $\lambda_0 < \infty$ (finite screening).

| $\lambda_0$ | $\nu$ | $\eta$ | $z$ |
|-------------|-------|-------|-----|
| $\lambda_0 = \infty$ present | 2.2(4) | $-0.5(2)$ | 3.3(5) |
| $(T_g > 0)$ gauge glass | 1.3(4) | $-0.3(7)$ | 4.7(7) |
| $\lambda_0 < \infty$ present | 3.5(10) | $-1$ | |
| $(T_g = 0)$ gauge glass | 1.05(10) | $1.1$ | 1.05(3) |