Stability of 3D black hole with torsion

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Abstract

Using $N = 1 + 1$ supersymmetric extension of the three-dimensional gravity with torsion, we show that a generic black hole has no exact supersymmetries, the extremal black hole has only one, while the zero-energy black hole has two. Combining these results with the asymptotic supersymmetry algebra, we are naturally led to interpret the zero-energy black hole as the ground state of the Ramond sector, and analogously, the anti-de Sitter solution as the ground state of the Neveu-Schwartz sector.

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1 Introduction

In the previous two and a half decades, three-dimensional (3D) gravity has been successfully used for exploring basic features of the gravitational dynamics. Within the traditional approach based on general relativity (GR) and the underlying Riemannian geometry of spacetime, a number of remarkable results has been achieved [1, 2, 3, 4, 5, 6]. However, 3D gravity also represents an arena for testing a more general, gauge-theoretic conception of gravity based on Riemann-Cartan geometry—the geometry that is characterized by both the curvature and the torsion [7, 8]. Today, fifteen years after Mielke and Baekler proposed a general topological model for 3D gravity with torsion [9, 10], this approach represents a respectable framework for studying the gravitational dynamics [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Recently, using $N = 1 + 1$ supersymmetric extension of 3D gravity with torsion, it was shown that there exists a suitable supersymmetric generalization of the anti-de Sitter (AdS) asymptotic conditions, which leads to the asymptotic superconformal symmetry [21]. In the present paper, we explore exact supersymmetries of the black hole with torsion and use the asymptotic supersymmetry algebra to study its stability properties. Our results are a natural generalization of those obtained for the BTZ black hole in [23, 24].

The paper is organized as follows. In section 2 we recall some basic aspects of the topological 3D gravity with torsion and its supersymmetric extension with $N = 1 + 1$ gravitini. Sections 3 and 4 contain basic results of the paper. In section 3, we solve the Killing spinor equation for the general black hole configuration and show that: (1) a generic

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black hole has no exact supersymmetries, (2) the extremal black hole has only one, while (3) the black hole with zero energy has two exact supersymmetries. In section 4, we combine these results with the asymptotic supersymmetry algebra to show that the zero-energy black hole can be interpreted as the ground state of the Ramond sector, with periodic boundary conditions for gravitini. Similarly, the AdS solution is identified as the ground state of the Neveu-Schwartz sector, with anti-periodic boundary conditions. Finally, section 5 is devoted to concluding remarks.

Our conventions are the same as in [21]: the Latin indices \((i,j,k,\ldots)\) refer to the local orthonormal frame, the Greek indices \((\mu,\nu,\rho,\ldots)\) refer to the coordinate frame, and both run over \(0,1,2\); the metric components in the local Lorentz frame are \(\eta_{ij} = (+,−,−)\); totally antisymmetric object \(\varepsilon^{ijk}\) is normalized by \(\varepsilon^{012} = +1\); gamma matrices are pure imaginary, \((\gamma_0,\gamma_1,\gamma_2) = (−\sigma^2,i\sigma^3,i\sigma^1)\) with \(\sigma^k\) the Pauli matrices, and Majorana spinors are real.

2 Supersymmetric 3D gravity with torsion

Riemann-Cartan geometry. Theory of gravity with torsion can be naturally described as Poincaré gauge theory (PGT), with an underlying spacetime structure corresponding to Riemann-Cartan geometry [7, 8]. Basic gravitational variables in PGT are the triad field \(b^i\) and the Lorentz connection \(A^{ij} = −A^{ji}\) (1-forms), and the corresponding field strengths are the torsion \(T^i\) and the curvature \(R^{ij}\) (2-forms). In 3D, we can simplify the notation by introducing \(A^{ij} = −\varepsilon^{ijk}\omega^k\) and \(R^{ij} = −\varepsilon^{ijk}R^k\), which yields:

\[
T^i = db^i + \varepsilon^i_{jk}\omega^j \wedge b^k, \quad R^i = d\omega^i + \frac{1}{2} \varepsilon^{ijk}\omega^j \wedge \omega^k. \tag{2.1}
\]

Gauge symmetries of the theory are local translations and local Lorentz rotations. The covariant derivative \(\nabla \equiv \nabla(\omega)\) acts on a general tangent-frame spinor/tensor in accordance with its spinorial/tensorial structure; when \(X\) is a form, \(\nabla X := \nabla \wedge X\).

The metric structure of PGT is defined by \(g = \eta_{ij}b^i \otimes b^j\). Metric and connection are related to each other by the metricity condition, \(\nabla g = 0\), which corresponds to Riemann-Cartan geometry of spacetime. In PGT, we have a useful identity

\[
\omega^i \equiv \bar{\omega}^i + K^i, \tag{2.2}
\]

where \(\bar{\omega}^i\) is the Levi-Civita (Riemannian) connection, and \(K^i\) is the contortion 1-form, defined implicitly by \(T^i = \varepsilon^i_{mn}K^m \wedge b^n\).

Generalized dynamics. General gravitational dynamics in Riemann-Cartan spacetime is determined by Lagrangians which are at most quadratic in field strengths. Omitting the quadratic terms, we arrive at the topological Mielke-Baekler model for 3D gravity [9, 10]:

\[
I_0 = 2a \int b^i R_i - \frac{A}{3} \int \varepsilon_{ijk}b^i b^j b^k + \alpha_3 I_{CS}[\omega] + \alpha_4 \int b^i T_i, \tag{2.3}
\]

where the wedge product sign \(\wedge\) is omitted for simplicity. The first term with \(a = 1/16\pi G\) is the usual Einstein-Cartan action, the second term is a cosmological term, \(I_{CS}[\omega]\) is the Chern-Simons action for the Lorentz connection, \(I_{CS}[\omega] = \int \left( \omega^i d\omega_i + \frac{1}{3} \varepsilon_{ijk} \omega^i \omega^j \omega^k \right)\), and the
last term is a torsion counterpart of the first one. The Mielke-Baekler model is a natural
generalization of GR with a cosmological constant.

In the sector $\alpha_3 \alpha_4 - a^2 \neq 0$, the vacuum field equations take the simple form

$$2T^i = p\varepsilon^i_{jk} b^j b^k, \quad 2R^i = q\varepsilon^i_{jk} b^j b^k, \quad (2.4)$$

where

$$p = \frac{\alpha_3 \Lambda + \alpha_4 a}{\alpha_3 \alpha_4 - a^2}, \quad q = -\frac{(\alpha_4)^2 + a\Lambda}{\alpha_3 \alpha_4 - a^2}.$$

Thus, vacuum solutions are characterized by constant torsion and constant curvature.

In Riemann-Cartan spacetime, one can use the identity (2.2) to express the curvature $R^i$ in terms of its Riemannian piece $\tilde{R}^i = R^i(\tilde{\omega})$ and the contortion. The resulting identity, combined with the on-shell relation $K^i = p b^i/2$, leads to

$$\tilde{R}^{ij} = -\Lambda_{\text{eff}} b^i \wedge b^j, \quad \Lambda_{\text{eff}} := q - \frac{1}{4} p^2, \quad (2.5)$$

where $\Lambda_{\text{eff}}$ is the effective cosmological constant. The form of $\tilde{R}^{ij}$ implies that our spacetime has maximally symmetric metric.

**Black hole with torsion.** For negative $\Lambda_{\text{eff}}$, the Mielke-Baekler model has an exact vacuum solution, the black hole with torsion [11, 12, 13, 14, 15, 16], which is a natural generalization of the well-known BTZ black hole [4, 5]. In static coordinates $x^\mu = (t, r, \varphi)$ (with $0 \leq \varphi < 2\pi$), the BTZ metric is given as

$$ds^2 = N^2 dt^2 - N^{-2} dr^2 - r^2 (d\varphi + N_{\varphi} dt)^2, \quad N^2 = \left(-8Gm + \frac{r^2}{\ell^2} + \frac{16G^2 J^2}{r^2}\right), \quad N_{\varphi} = \frac{4GJ}{r^2}. \quad (2.6)$$

For the black hole with torsion, the triad field is taken as

$$b^0 = N dt, \quad b^1 = N^{-1} dr, \quad b^2 = r (d\varphi + N_{\varphi} dt), \quad (2.7a)$$

while the connection, in accordance with (2.2), has the form

$$\omega^i = \tilde{\omega}^i + \frac{p}{2} b^i, \quad (2.7b)$$

where the Riemannian connection $\tilde{\omega}^i$ reads:

$$\tilde{\omega}^0 = -N d\varphi, \quad \tilde{\omega}^1 = N^{-1} N_{\varphi} dr, \quad \tilde{\omega}^2 = -\frac{r dt}{\ell} - r N_{\varphi} d\varphi. \quad (2.7c)$$

Energy and angular momentum of the black hole with torsion differ from the corresponding GR expressions:

$$E = m + \frac{\alpha_3}{a} \left(\frac{pm}{2} - \frac{J}{\ell^2}\right), \quad M = J + \frac{\alpha_3}{a} \left(\frac{pJ}{2} - m\right). \quad (2.8)$$

The black hole manifold is topologically $R^2 \times S^1$. The AdS solution (AdS$_3$) is locally isometric to the black hole and can be formally obtained from (2.7) by taking $J = 0$ and $8Gm = -1$. However, AdS$_3$ has a different topology in which $\varphi$ is not periodic.
Supersymmetric extension. There exists a simple locally supersymmetric extension of 3D gravity with torsion, based on the action (2.4). The extension includes two gravitini fields and is usually referred to as $N = 1 + 1$ AdS supergravity [2, 20, 21]. Consider the action

$$I = I_0 - g \int \left( \bar{\psi} \nabla \psi - i \mu \bar{\psi} b^i \gamma_i \psi \right) - g' \int \left( \bar{\psi}' \nabla \psi' - i \mu' \bar{\psi}' b^i \gamma_i \psi' \right),$$

(2.9)

where $\psi^\Pi$ are the gravitini fields (1-forms), $\nabla \psi^\Pi = (d - i \frac{2}{\ell} \omega^m \gamma_m) \psi^\Pi$ are their covariant derivatives, and Dirac matrices are pure imaginary. The action is invariant under the local supersymmetry transformations with spinorial parameters $\varepsilon^\Pi = (\varepsilon, \varepsilon')$:

$$\delta S_{b^i} = i \bar{\varepsilon} \gamma^i \psi + i \bar{\varepsilon}' \gamma^i \psi',$$
$$\delta S_{\omega^i} = -2i \mu' \bar{\varepsilon} \gamma^i \psi - 2i \mu \bar{\varepsilon}' \gamma^i \psi' ,$$
$$\delta S_{\psi^\Pi} = 4a \left( \nabla \varepsilon^\Pi - i \mu^\Pi b^k \gamma_k \varepsilon^\Pi \right).$$

(2.10)

provided the coupling constants $g, g'$ and $\mu^\Pi = (\mu, \mu')$ satisfy the relations

$$2ag = a - 2 \mu' \alpha_3 , \quad 2ag' = a - 2 \mu \alpha_3 ,$$
$$2\mu + \frac{p}{2} = \frac{1}{\ell} , \quad 2\mu' + \frac{p}{2} = -\frac{1}{\ell} .$$

Here, $\ell$ is the AdS radius, $\ell^{-1} = \mu - \mu'$, and for $\mu$ and $\mu'$ real and different from each other, the effective cosmological constant is negative: $\Lambda_{\text{eff}} = - (\mu - \mu')^2 \equiv -1/\ell^2 < 0$.

The commutator algebra of the local super-Poincaré transformations closes on shell, which is sufficient for exploring the asymptotic symmetries.

The variation of the action with respect to $b^i$ and $\omega^i$ yields the supersymmetric extension of the field equations (2.4), the variations with respect to $\bar{\psi}$ and $\bar{\psi}'$ yields the gravitini field equations. The black hole and AdS$_3$ can be regarded as exact solutions of the supersymmetric field equations with zero gravitini, $\psi = \psi' = 0$.

3 Killing spinors

Since the black hole and AdS$_3$ are maximally symmetric solutions of 3D gravity, their symmetries are locally identical. However, the existence of different global structures implies completely different symmetries in the large.

Exact supersymmetries. In the supersymmetric theory (2.9), the supersymmetry transformations that leave the black hole or AdS$_3$ configuration with $\psi = \psi' = 0$ invariant, are called exact supersymmetries. Since the conditions $\delta_S b^i = \delta_S \omega^i = 0$ are automatically satisfied for $\psi = \psi' = 0$, the spinor parameters $(\varepsilon, \varepsilon')$ of exact supersymmetries are defined solely by the requirements $\delta_S \psi = \delta_S \psi' = 0$. Expressing $\omega^i$ in terms of the Levi-Civita connection $\bar{\omega}^i$ as in (2.7b), these requirements take the form

$$\bar{\nabla} \varepsilon = \frac{i}{2\ell} b^k \gamma_k \varepsilon , \quad \bar{\nabla} \varepsilon' = -\frac{i}{2\ell} b^k \gamma_k \varepsilon' ,$$

(3.1)

where $\bar{\nabla}$ is Riemannian covariant derivative. Equations (3.1) are called the Killing spinor equations; they define the spinor parameters $(\varepsilon, \varepsilon')$ of the exact supersymmetries. For the
configuration \((2.7)\), the Killing spinor equations take the form:

\[
\partial_+ \varepsilon = 0, \quad \partial_+^2 \varepsilon - 2G \left( m - \frac{J}{\ell} \right) \varepsilon = 0, \\
\partial_\ell \varepsilon + \frac{N^{-1}}{2} \left( \frac{4GJ}{r^2} + \frac{1}{\ell} \right) \sigma^3 \varepsilon = 0,
\]

(3.2a)

and similarly:

\[
\partial_- \varepsilon' = 0, \quad \partial_-^2 \varepsilon' - 2G \left( m + \frac{J}{\ell} \right) \varepsilon' = 0, \\
\partial_\ell \varepsilon' + \frac{N^{-1}}{2} \left( \frac{4GJ}{r^2} - \frac{1}{\ell} \right) \sigma^3 \varepsilon' = 0,
\]

(3.2b)

where \(x^\pm = t/\ell \pm \varphi\). These equations depend on the black hole parameters \(m\) and \(J\) through the combination \(\lambda^2_\pm = 2G \left( m \pm \frac{J}{\ell} \right)\). Among their solutions, only those that are in agreement with the global properties of the black hole/AdS are acceptable.

**Killing spinors for AdS\(_3\).** The AdS solution is characterized by \(\lambda^2_\pm = -\frac{1}{4}\), and it possesses four Killing spinors, two for \(\varepsilon\) and two for \(\varepsilon'\):

\[
\varepsilon = \left( \sqrt{\frac{N_* + 1}{2}} \sigma^3 - \sqrt{\frac{N_* - 1}{2}} \right) \left( \cos \frac{x^-}{2} + i \sigma^2 \sin \frac{x^-}{2} \right) \zeta, \\
\varepsilon' = \left( \sqrt{\frac{N_* + 1}{2}} \sigma^3 + \sqrt{\frac{N_* - 1}{2}} \right) \left( \cos \frac{x^+}{2} + i \sigma^2 \sin \frac{x^+}{2} \right) \zeta'.
\]

(3.3)

Here, \(N_* = \sqrt{1 + r^2/\ell^2}\) is the value of \(N\) at AdS\(_3\) and \(\zeta, \zeta'\) are constant spinors. The Killing spinors are anti-periodic, which is in agreement with the global structure of AdS\(_3\).

**Killing spinors for the black hole.** Since the black hole is obtained from AdS\(_3\) by a process of identification, it possesses locally the same number of Killing spinors as AdS\(_3\). Globally, however, spinors on the black hole manifold can be either periodic or anti-periodic \([4, 5, 23, 24]\), which is a serious restriction on the solutions of the Killing spinor equations (3.2a). The form of this restriction depends on the values of the black hole parameters \(m\) and \(J\).

The reality of the black hole horizons, ensured by the condition \(|J| \leq m\ell\), implies \(\lambda^2_\pm \geq 0\).

1. For a generic black hole with \(|J| < m\ell\), we have \(\lambda^2_\pm > 0\) and \(\lambda^2_\pm > 0\). Consequently, there are no periodic/anti-periodic solutions of (3.2a), and no Killing spinors.

2. Consider now the extreme black hole with \(|J| = m\ell\) and \(m \neq 0\).

   (a) For \(J = m\ell\), we have \(\lambda^2_\pm = 0\) and \(\lambda^2_\pm > 0\). The periodic/anti-periodic solution for \(\varepsilon'\) does not exist, while the solution for \(\varepsilon\) has the form

   \[
   \varepsilon = \left( |u|^{1/2} + |u|^{-1/2} \right) - \text{sgn} \left( |u|^{1/2} - |u|^{-1/2} \right) \sigma_3 \frac{1 - x^- \sigma^-}{2} \zeta, \quad (3.4a)
   \]

   where \(u = r/\ell - 4Gm\ell/r\). The Killing spinor \(\varepsilon\) is compatible with the periodicity in \(\varphi\) if the term linear in \(x^-\) dissapears, i.e. if

   \[
   \zeta \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
   \]
Thus, we have here only one Killing spinor.
(b) Similarly, in the case $J = -m\ell$, i.e. $\lambda_+^2 > 0$ and $\lambda_+^2 = 0$, there are no periodic/anti-periodic solutions for $\varepsilon$, while globally acceptable solution for $\varepsilon'$ takes the form

$$\varepsilon' = \left[ (|u|^{1/2} + |u|^{-1/2}) + \text{sgn}(u) (|u|^{1/2} - |u|^{-1/2}) \sigma_3 \right] \frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{3.4b}$$

3. The case $m = J = 0$, with $\lambda_+^2 = 0 = \lambda_+^2 = 0$, corresponds to the zero-energy black hole, for which both $E$ and $M$ vanish. The related Killing spinors can be obtained from the extreme black hole solutions (3.4) in the limit $m \to 0$. The periodicity requirement implies the existence of two Killing spinors:

$$\varepsilon \sim \sqrt{\frac{r}{\ell}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \varepsilon' \sim \sqrt{\frac{r}{\ell}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{3.5}$$

Thus, a generic black hole has no Killing spinors, the extreme black hole has only one, while the zero-energy black hole has two.

4 Stability

The existence of Killing spinors for the black hole/AdS$_3$, combined with the asymptotic supersymmetry algebra, has serious implications for the stability of these solutions.

Asymptotic symmetries. For $\Lambda_{\text{eff}} < 0$, the asymptotic structure of 3D gravity with torsion is well understood [15, 16]. It can be generalized to the supersymmetric theory (2.9) by a suitable completion of the asymptotic conditions in the fermionic sector. The asymptotic symmetry is defined as a subset of local super-Poincaré transformations that leaves the adopted asymptotic configuration invariant. It is characterized by four chiral parameters: two of them are bosonic, $T^\pm(x^\pm)$, while the other two are fermionic, $\epsilon^\pm(x^\pm)$. The asymptotic Poisson bracket algebra of the canonical generators $\tilde{G}(T^\pm, \epsilon^\pm)$ is given (in the quantum-mechanical notation) by two independent copies of the super-Virasoro algebra [21]:

$$[L^\pm_n, L^\pm_m] = (n - m)L^\pm_{n+m} + \frac{c^\pm}{12} n^3 \delta_{m+n,0},$$

$$[L^\pm_n, Q^\pm_m] = \left( \frac{1}{2} n - m \right) Q^\pm_{m+n},$$

$$\{Q^\pm_n, Q^\pm_m\} = 2L^\pm_{n+m} + \frac{c^\pm}{3} n^2 \delta_{m+n,0}, \tag{4.1}$$

where $L^\pm_n$ and $Q^\pm_n$ are Fourier modes of $\tilde{G}(T^\pm, \epsilon^\pm)$, and $c^\pm$ are classical central charges:

$$c^- = 12 \cdot 2\pi\ell ag, \quad c^+ = 12 \cdot 2\pi\ell ag'.$$

In the sectors with periodic (Ramond) or anti-periodic (Neveu-Schwartz) boundary conditions for fermions [4, 5], index of $Q^\pm_n$ takes on integer or half-integer values, respectively. The reality properties of the modes are: $(L^\pm)^\dagger_n = L^\pm_{-n}$, $(Q^\pm)^\dagger_n = Q^\pm_{-n}$.
The eigenvalues of the bosonic operators $L_0^\pm$ can be expressed in terms of the energy $E$ and angular momentum $M$ of the system. For the black hole/AdS$_3$ solution (2.7), we have:

$$L_0^\pm = \frac{\ell E \pm M}{2} = (\ell m \pm J) \frac{c^+}{48\pi a\ell}.$$  \hspace{1cm} (4.2)

One should stress that the asymptotic supersymmetry algebra (4.1) is defined by ignoring (factoring out) pure gauge transformations [1, 15, 16, 23, 24]. The algebra has interesting implications on the stability properties of the black hole/AdS$_3$.

**Ramond sector.** In the Ramond sector (with periodic boundary conditions), the sub-algebra of (4.1) with vanishing central charge, generated by $(L_0^+ \pm Q_0^-)$, yields:

$$L_0^\pm = Q_0^\mp Q_0^\pm = Q_0^\mp Q_0^\pm \geq 0.$$ \hspace{1cm} (4.3)

These relations are equivalent to $|M| \leq \ell E$, which implies $E \geq 0$.

We have seen that the zero-energy black hole configuration has two Killing spinors, which are the parameters of two exact supersymmetries, generated by the action of $Q_0^\pm$. On the other hand, the zero-energy black hole saturates the bounds (4.3), which means that it has the lowest values of the conserved charges $L_0^\pm$, compared to any other black hole configuration. Consequently, the zero-energy black hole can be naturally interpreted as the black hole ground state (the ground state of the Ramond sector).

One should note that for an extremal black hole, only one of the bounds in (4.3) is saturated, while for a generic black hole, both $L_0^-$ and $L_0^+$ are strictly positive.

**Neveu-Schwarz sector.** The AdS solution belongs to the Neveu-Schwarz sector (anti-periodic boundary conditions), with (the value of $L_0^\pm$) $=-c^+/24$. If one makes the shift $L_0^\pm \rightarrow L_0^\pm - c^+/24$, the value of the new $L_0^\mp$ on AdS$_3$ vanish. The $osp(1|2) \otimes osp(1|2)$ subalgebra with vanishing charge is generated by $(L_{\mp 1}, L_0, Q_{\mp 1/2})^\pm$, and it implies

$$2L_0^\mp = Q_{1/2}^\mp Q_{-1/2}^\mp + Q_{-1/2}^\mp Q_{1/2}^\mp = Q_{1/2}^\mp (Q_{1/2}^\mp)^\dagger + Q_{-1/2}^\mp (Q_{-1/2}^\mp)^\dagger \geq 0.$$ \hspace{1cm} (4.4)

The AdS solution has four Killing spinors, which are the parameters of four exact supersymmetries, generated by the action of $(Q_{1/2}^\mp, Q_{-1/2}^\mp)$, and it saturates the bounds (4.4). Thus, we are led to interpret AdS$_3$ as the ground state of the Neveu-Schwarz sector.

### 5 Concluding remarks

In this paper, we used the $N = 1 + 1$ supersymmetric extension of 3D gravity with torsion to investigate exact supersymmetries and stability of the black hole with torsion.

(1) We showed that a generic black hole has no exact supersymmetries, the extremal black hole has only one, while the zero-energy black hole has two exact supersymmetries.

(2) Using the asymptotic superconformal symmetry of the theory, we found that the zero-energy black hole and AdS$_3$ can be naturally interpreted as the ground states of the sectors with periodic and anti-periodic boundary conditions, respectively.

These results are a natural generalization of those found earlier in Riemannian GR.
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