MATCHING THE LOW AND HIGH ENERGY DETERMINATIONS OF $\alpha_s(M_Z)$ IN THE MSSM

David GARCIA, Joan SOLÀ
Grup de Física Teòrica
and
Institut de Física d’Altes Energies
Universitat Autònoma de Barcelona
08193 Bellaterra (Barcelona), Catalonia, Spain

ABSTRACT

Recent calculations of supersymmetric corrections to the conflicting ratios $R_b$ and $R_c$ have shown that an alleged discrepancy between the SM predictions of these observables and the corresponding experimental values can be cured in the MSSM within a certain region of the parameter space. Here we show that, in this very same region, also a well-known discrepancy between the low and high energy determinations of $\alpha_s(M_Z)$ can be disposed of. Specifically, we find that the lineshape determination of the strong coupling constant, which in the SM points towards the large central value $\alpha_s(M_Z) \gtrsim 0.125$, can be matched up with the value suggested by the wealth of low-energy data, namely $\alpha_s(M_Z) \simeq 0.11$, which is smaller and more in line with the traditional QCD expectations at low energy. Our approach differs from previous analyses in that we argue that the desired matching could originate to a large extent from a purely electroweak supersymmetric quantum effect.
The world average of the various determinations of the strong coupling constant at the scale of the $Z$-boson mass within the Standard Model (SM) reads as follows:\(^1\):

$$\alpha_s(M_Z) = 0.118 \pm 0.007.$$  \hspace{1cm} (1)

However, one should not forget that this number follows from a gross averaging of several groups of data of very different nature whose respective central values show up statistically significant differences. Hence eq.\(^1\) is suspicious of hiding a potentially important problem within the SM. Even more: it could prompt us about physics beyond the SM, as has already been pointed out by several authors\(^2,3\). Indeed, one finds that the high-energy data, i.e. data taken directly at $q^2 = M_Z^2$ from lineshape and event shape analyses at LEP and SLD, turn out to cluster around a “large” value of $\alpha_s(M_Z)$. For example, the lineshape value is\(^1\)

$$\alpha_s(M_Z) = 0.126 \pm 0.007.$$ \hspace{1cm} (2)

In contrast, the low-energy data obtained from a great variety of sources (such as deep inelastic scattering, lepton scattering and quarkonium spectra) concentrates\(^4\) around a central value which is about 12\% smaller. For instance, the deep inelastic scattering result reads\(^1\)

$$\alpha_s(M_Z) = 0.112 \pm 0.005.$$ \hspace{1cm} (3)

We infer that the low-energy determination of $\alpha_s(M_Z)$ lies three standard deviations below the high-energy determination. Although the latter is consistent with the value obtained from the $\tau$-decay fraction measurement (a low-energy result), here what is in dispute is the theoretical method of calculation\(^5\) and the treatment of errors\(^3,6\).

The clash between the low and high energy determinations of $\alpha_s(M_Z)$ is a persisting result in the literature\(^1\) and it has become a controversial and disturbing problem (a sort of “$\alpha_s(M_Z)$ crisis”) within the context of the standard model picture of the strong interactions. Recently, Shifman has drawn much attention on this issue\(^3\). He strongly advocates in favour of $\alpha_s(M_Z) \simeq 0.11$—the deep inelastic result, eq.\(^3\), being a case in point—\(^4\) and argues that the large result\(^2\) is incompatible with crucial features of QCD; for example—following Shifman’s contention—, a strong coupling constant $\alpha_s(M_Z) > 0.11$ would entail $\Lambda_{QCD} > 200\text{ MeV}$, where $\Lambda_{QCD} \equiv \Lambda_{\text{MS}}(\frac{s}{1})$—suitable for deep inelastic scattering. In particular, for $\alpha_s(M_Z) \simeq 0.125$ one has to endure a corresponding value of $\Lambda_{QCD} \simeq 500\text{ MeV}$ which is considered far too large to explain the evolution of moments of the

\(^1\) Upon due account, of course, of the renormalization group running from the low-energy scale $q^2 << M_Z^2$ where the data have been taken up to $q^2 = M_Z^2$.

\(^2\) Lattice calculations also agree with the low-energy value\(^5\) and they produce $\alpha_s(M_Z) = 0.115$ or less\(^5\).
structure functions in deep inelastic scattering and the success of the operator product expansion in a wide range of QCD problems. On these grounds, solid and convincing as they may be, Shifman suggests that the large lineshape result (2) claims for new physics, in particular for unconventional strong interactions, such as e.g. supersymmetric interactions mediated by gluinos and squarks, which have not been taken into account in the theoretical calculation of the hadronic Z-width. Upon inclusion of these contributions, the high-energy value (2) should descend to the preferred low-energy value (3). On the basis of existing calculations of the SUSY-QCD corrections to the Z-width [8] one can sustain that this could be the case provided that gluinos have a mass of order of a few GeV and squarks are in the ballpark of 100 GeV. We remark that for $O(100)$ GeV gluinos, non-negligible corrections are also possible in certain regions of parameter space, but they invariably come out with the “wrong” sign, i.e. they would induce a lineshape value of $\alpha_s(M_Z)$ even larger than that of eq.(2) (see later on).

As for the light gluino scenario, it was already advocated long ago in the literature [9], and also more recently by several authors [10], though in all these cases from a different point of view: namely, as a means to slow down the renormalization group running of $\alpha_s(q^2)$ from the low-energy scale up to $q^2 = M_Z^2$ due to extra, negative, contributions to the strong coupling $\beta$-function. Unfortunately, the extremely narrow slit for the existence of these very light gluinos of $O(1)$ GeV is nearly–if not completely– closed experimentally.

It should be pointed out that alternative (non-SUSY) scenarios have also been proposed to cure the “$\alpha_s(M_Z)$ crisis”, such as e.g. the extended technicolor approaches of Ref.[11]. However, it remains to see whether they are, too, consistent with other observables like $R_b$ and $R_c$ (see below). Moreover, in contrast to the MSSM, technicolour models are not in very good shape to fit the plethora of available electroweak precision data at the moment [12].

On the other hand, from the point of view of grand desert supersymmetric Grand Unified Theories (SUSY-GUT’s), the typical range of large values (2) also obtains in all renormalization group analyses based on canonical assumptions [13]. However, as Ref.[15] shows, one can try to reconcile SUSY-GUT’s with the low-energy value (3) at the price of relaxing several common place assumptions on gaugino mass relations and in general by adopting a more phenomenological attitude towards the structure of the Minimal Supersymmetric Standard Model (MSSM) [16]. In a sense this is also the philosophy adopted in the rather different context of Refs.[17]-[20], where we systematically searched for the pattern of supersymmetrical particle masses preferred by phenomenology in order

\[ \alpha_s(M_Z) = 0.073 \pm 0.002 \quad [14]. \]
to solve an apparent discrepancy between theory and experiment in the ratio

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})},$$

(4)

where $\Gamma(Z \to \text{hadrons}) \equiv \sum_{q=u,d,c,s,b} \Gamma(Z \to q\bar{q})$ stands for the partial $Z$-width into hadrons resulting from a primarily produced quark-antiquark pair. Within the context of the SM, and for the present range of values of the top quark mass [21], $R_b$ is predicted to be below the experimental result[4]. In contrast, as it follows from our analysis [18] as well as from previous analyses in the literature [22], the theoretical prediction of $R_b$ in the MSSM could result in a net increase in the value of this observable. As a drawback, however, the theoretical improvement of $R_b$ can only be achieved within the framework of a general MSSM with a minimum number of assumptions on its spectrum of sparticle masses. In other words, the supersymmetric prediction of $R_b$ does not substantially improve in the context of “canonical” SUSY-GUT’s, such as e.g. in the so-called constrained MSSM [23] or in the simplest (and next to simplest) supergravity (SUGRA) scenarios [24]. Still, as already mentioned, one can try to amend this situation in non-minimal (e.g. string-like) SUSY-GUT’s [13]; after all, the minimal SUGRA models, although provide an economic and elegant framework, they might be based on oversimplified assumptions about physics at the unification scale, and they may ultimately prove to be incorrect.

Further studies of the authors [13] in the framework of the MSSM have shown that one can improve the theoretical prediction, not only of the ratio $R_b$, but also of the companion ratio

$$R_c = \frac{\Gamma(Z \to c\bar{c})}{\Gamma(Z \to \text{hadrons})}.$$ 

(5)

In contradistinction to $R_b$, this observable is predicted in the SM to be above the experimental measurement [4]. Quite intriguingly, it turns out that upon taking into account the supersymmetric electroweak quantum effects predicted by the general MSSM in a particular region of parameter space, the theoretical prediction can be substantially ameliorated [19]. And what is more, one can simultaneously alleviate the $R_b$ and $R_c$ “crises” in the MSSM. For this to be so it suffices that the following conditions are fulfilled:

i) $\tan \beta$ should be large enough, namely $\tan \beta \gtrsim \mathcal{O}(m_t/m_b)$;

ii) There should exist a light supersymmetric CP-odd (“pseudoscalar”) Higgs, $A^0$, of $\mathcal{O}(50)\,\text{GeV}$ ($m_{A^0} < 70\,\text{GeV}$, to avoid negative effects from charged higgses [18]);

iii) For a comfortable solution, there should also exist a light chargino, $\Psi^+_1$, in the $50 - 60\,\text{GeV}$ ballpark [18, 19].

We point out that similar results can also be obtained for very large $m_{A^0}$, i.e. for effectively decoupled non-standard higgses, and very low values of $\tan \beta$, typically $\tan \beta < 4$

4 See e.g. Ref.[4] for a complete report on the status of the Z-physics observables.
0.7, which insure a big contribution from the sparticles alone. In this case condition iii) above has to be supplemented with the requirement of a light stop whose mass is of the same order of magnitude as the chargino mass. However, the range \( \tan \beta < 1 \) is currently not supported by model building and we shall not consider this possibility as our favourite Ansatz.

In spite of not being strictly necessary for the \( Z \) observables under study, one might also like to have a light stop, \( \tilde{t}_1 \), even at high \( \tan \beta \). This could be necessary \([25]\) to make the MSSM compatible with the CLEO bound on \( B(b \to s\gamma) \) \([24]\). In this respect it should be recalled that light stops have been invoked in previous analyses of \( R_b \) in the MSSM which emphasized the region of moderate and small values of \( \tan \beta \) \([23]\). However, in a regime of large \( \tan \beta \), a light stop is not essential to account for the ratios \( R_b \) and \( R_c \) themselves, as explicitly shown in Refs. \([18, 19]\). Moreover, postulating a light stop on the sole basis of demanding consistency with the CLEO bound may not be too compelling; after all, a satisfactory treatment of conventional QCD corrections to \( B(b \to s\gamma) \), which are known to be very large, is still lacking \([27]\). Therefore, we should better be openminded at this point.

In the light of the above considerations, it would be desirable to explore whether the electroweak supersymmetric corrections may account at the same time for the disturbing “\( \alpha_s(M_Z) \) crisis” and to assess whether the purported solution space for the “\( R_b - R_c \) crisis” does overlap with that of the “\( \alpha_s(M_Z) \) crisis”, in which case the relevance of the supersymmetric solution would be significantly augmented.

In Figs.1-4 we deliver the final result of our analysis, which is based on the detailed calculations of all oblique and non-oblique one-loop supersymmetric electroweak quantum effects on the partial \( Z \)-widths into fermion-antifermion pairs in the MSSM, as presented in Refs. \([17, 18]\) whose notation and definitions we shall adopt henceforth. In the case under consideration it will suffice to consider the decay modes involved in the ratio of the hadronic and electronic partial \( Z \) widths:

\[
R \equiv \frac{\Gamma_h}{\Gamma_e} = \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)}.
\]  

This observable is the relevant quantity in our analysis, for it is directly involved in the hadronic peak cross-section

\[
\sigma^0_h \equiv \sigma(e^+e^- \to \text{hadrons})|_{s=M_Z^2} = \frac{12\pi}{M_Z^2} \left( \frac{\Gamma_e}{\Gamma_Z} \right)^2 R
\]

from which the lineshape determination of \( \alpha_s(M_Z) \) ensues. In the SM one computes \( \Gamma^{SM}_h = \Gamma^{SM}_h(\alpha_s(M_Z)) \) as a function of \( \alpha_s(M_Z) \), and by comparing it with the experimentally measured lineshape value of \( R \) (notice that \( R = \Gamma_h/\Gamma_e = \sigma^0_h/\sigma^0_e \)) one derives the result \([2],\)
where the error reflects uncertainties on the electroweak and QCD parts of the theoretical prediction, as well as on the lack of knowledge of the Higgs mass (assumed in the range 60 – 1000 GeV, with central value $m_H = 300$ GeV) and of the top quark mass whose determination is still rather poor [21].

In the context of the MSSM it is convenient to cast each partial width $\Gamma(Z \to f \bar{f})$ split up as follows:

$$\Gamma_{\text{MSSM}}^Z = \Gamma_{\text{RSM}}^Z + \delta \Gamma_{\text{MSSM}}^Z,$$

(8)

where $\Gamma_{\text{RSM}}^Z$ involves the contribution from a so-called “Reference Standard Model” (RSM): namely, the Standard Model with a Higgs mass set equal to the mass $m_{h^0}$ of the lightest $CP$-even Higgs scalar of the MSSM whereas $\delta \Gamma_{\text{MSSM}}^Z$ constitutes the total quantum departure of the MSSM prediction with respect to that RSM. Besides, $\delta \Gamma_{\text{MSSM}}^Z$ itself splits up naturally into two parts, viz. the extra two-doublet Higgs contribution $\delta \Gamma_H^Z$, in which the single Higgs part included in $\Gamma_{\text{RSM}}^Z$ has been subtracted out in order to avoid double-counting, and the SUSY contribution, $\delta \Gamma_{\text{SUSY}}^Z$, from the plethora of “genuine” ($R$-odd) supersymmetric particles:

$$\delta \Gamma_{\text{MSSM}}^Z = \delta \Gamma_H^Z + \delta \Gamma_{\text{SUSY}}^Z.$$

(9)

Similarly for the ratio (6), we may decompose the MSSM theoretical prediction as follows:

$$R_{\text{MSSM}}^Z = R_{\text{RSM}}^Z + \delta R_{\text{MSSM}}^Z,$$

(10)

where in an obvious meaning

$$\delta R_{\text{MSSM}}^Z = \delta R_H^Z + \delta R_{\text{SUSY}}^Z = R_{\text{RSM}}^Z \left( \frac{\delta \Gamma_{\text{MSSM}}^Z}{\Gamma_{\text{RSM}}^Z} - \frac{\delta \Gamma_{\text{MSSM}}^e}{\Gamma_{\text{RSM}}^e} \right).$$

(11)

Now, to compute the RSM contribution we shall adapt the convenient formula of Ref. [28] for $R_{\text{SM}}^Z$, in the following way:

$$R_{\text{RSM}}^Z = 19.968 \left( 1 - 2.4 \times 10^{-4} \ln \frac{m_{h^0}^2}{M_Z^2} \right) \left( 1 - 2.5 \times 10^{-4} \frac{m_t^2}{M_Z^2} \right) \left[ 1 + a_1 \frac{\alpha_s(M_Z)}{\pi} \right. \nonumber \\
+ a_2 \left( \frac{\alpha_s(M_Z)}{\pi} \right)^2 + a_3 \left( \frac{\alpha_s(M_Z)}{\pi} \right)^3 \left. \right],$$

(12)

with

$$a_1 = 1.060, \quad a_2 = 0.90 - 0.002 (m_t/\text{GeV} - 150), \quad a_3 = 15.$$

(13)

We have identified $R_{\text{SM}}^Z$ with $R_{\text{RSM}}^Z$ and straightforwardly incorporated the explicit dependence on the top quark mass and on the (CP-even) Higgs mass. For the latter we [5]For fully fledged analytical formulae on $\delta \Gamma_{\text{MSSM}}^Z$ and an enlarged numerical analysis, see the more comprehensive exposition of Ref. [21].
have set $m_H = m_{h^0}$ and consistently related it with the mass of the pseudoscalar, $m_{A^0}$, in the usual way predicted by the MSSM \[30\].

The very compact formula \[12\] is a fit to the exact theoretical calculation. It summarizes all the complicated quantum effects from conventional QCD up to 3-loop order and standard 1-loop electroweak physics up to leading 2-loop order, including the dominant 2-loop strong-electroweak mixed effects, and is accurate to within 0.0005 in $\alpha_s(M_Z)$ over a range $0.10 \lesssim \alpha_s(M_Z) \lesssim 0.15$ for Higgs and top quark masses in the ranges $50 \text{GeV} \lesssim m_{h^0} \lesssim 1000 \text{GeV}$ and $100 \text{GeV} \lesssim m_t \lesssim 200 \text{GeV}$ \[29\].

From eqs.\[10\]-\[13\] we numerically determine $\alpha_s(M_Z)$ in the MSSM upon equating $R_{\text{MSSM}}$ to the experimental value \[4\]

\[
R_{\exp} = 20.795 \pm 0.040. \tag{14}
\]

Throughout our numerical analysis we shall survey the general MSSM parameter space using the 8-tuple procedure devised in Ref.\[18\]:

\[
(\tan \beta, m_{A^0}, M, \mu, m_{\tilde{\nu}}, m_{\tilde{b}_L}, m_{\tilde{u}_L}, M_{LR}), \tag{15}
\]

where it is understood that the SUSY parameters in \[15\] will be picked from the typical intervals given in eq.(14) of Ref.\[18\]. For example, in Fig.1 we plot contour lines of constant $\alpha_s(M_Z)$ in a sufficiently large window of the $(\mu, M)$-parameter space, for fixed $\tan \beta = 50$ and $m_{A^0} = 50 \text{GeV}$, and for a given set of sfermion masses in the basic tuple \[15\]. We see that the electroweak SUSY effects are capable of significantly reducing the conventional lineshape result \[2\] down to the low-energy value \[3\]. Notice that there is a excluded region of parameter space (see the shaded area in Fig.1a) which comes about because we have subordinated our analysis of $\alpha_s(M_Z)$ to the MSSM prediction of the total $Z$-width, $\Gamma_Z$ \[17\]. The latter is experimentally bound to lie within the interval \[31\]

\[
\Gamma_{\exp}^Z = 2.4974 \pm 0.0038 \text{GeV}, \tag{16}
\]

whereas the SM prediction, which we identify with $\Gamma_{\text{SM}}^Z$, reads \[32\]

\[
\Gamma_{\text{SM}}^Z = 2.4922 \pm 0.0075 \pm 0.0033 \text{GeV}. \tag{17}
\]

Imposing the condition that $\delta \Gamma_{\text{MSSM}}^Z$ should not exceed to 1$\sigma$ the experimental value \[16\], with all errors (experimental and theoretical) added in quadrature, we find a forbidden area in the $(\mu, M)$-plane of Fig.1a. Similarly, in Fig.1b we consider the contour lines corresponding to a larger $\tan \beta$ and larger sfermion masses. In this case the shaded

\[6\] In the analytic formulae for $\delta R_{\text{MSSM}}$ \[20\], we do not incorporate the 2-loop non-standard Higgs effects on $\delta R^H$. They are expected to be very small, in part because of the tight SUSY constraints in the Higgs sector.
(excluded) area is much smaller, since we have selected a set of parameters yielding small universal contributions to $\Gamma_Y$, while the vertex contributions—specially from the neutral Higgs sector—remain fairly large.

Let us study the dependence on the various parameters. For instance, for a given set (15), the largest possible values of $\tan \beta$ render the smallest lineshape determinations of $\alpha_s(M_Z)$. However, one should not exceed the upper bound of $\tan \beta$, which roughly lies at $\tan \beta = 70$. A significant decrease of $\alpha_s(M_Z)$ can also be obtained for very large (decoupled) $m_{A^0}$ and very small $\tan \beta < 0.7$. In both (extreme) regimes of $\tan \beta$ one is bordering the limit of perturbation theory for the supersymmetric Yukawa couplings of charginos and neutralinos with the stop-sbottom sector:

$$h_t = \frac{g m_t}{\sqrt{2} M_W \sin \beta}, \quad h_b = \frac{g m_b}{\sqrt{2} M_W \cos \beta}.$$ (18)

In spite of the $\Gamma_Y$-constraint, we see from Figs.1a and 1b that a wide region of parameter space is phenomenologically accessible. Part of this region, however, is not so interesting. In fact, although large values of $|\mu|$ are permitted by the $\Gamma_Y$-constraint, the most relevant domain of the $(\mu, M)$-plane is the one characterized by the smallest possible values of $|\mu|$ compatible with the phenomenological bounds. The reason is that, in this region, charginos have a large higgsino component and are relatively light, viz. of $\mathcal{O}(50)$ GeV, and in these conditions one can simultaneously improve the MSSM prediction of $R_b$ and $R_c$ [17]-[19].

Another important feature of our analysis is the demand for a relatively light $\mathcal{O}(50)$ GeV CP-odd scalar. At high $\tan \beta$, it goes accompanied (in the MSSM) with a CP-even Higgs of about the same mass. Even though several lower mass limits on two-doublet Higgs sectors exist in the literature, the region $m_{A^0} \gtrsim 40$ GeV has not yet been convincingly excluded by LEP for $\tan \beta >> 1$ [34]. In fact, in contradistinction to the usual analyses of mass limits on the SM Higgs, the couplings of the MSSM Higgs sector [30] are such that the corresponding standard Bjorken decay $Z \to h^0 l^+l^-$ [35] for the lightest CP-even Higgs scalar in the MSSM cannot be used to set any stringent lower limit on its mass in the region of large $\tan \beta$. On the other hand, the complementary decay $Z \to h^0 A^0$, whose branching ratio increases dramatically at large $\tan \beta$, is rendered inefficient for $m_{A^0} \gtrsim (0.3-0.4) M_Z$ [32]. As a result the whole range $m_{A^0} \gtrsim 40$ GeV remains open at high $\tan \beta$ and therefore we are free to exploit the window around the lower band of permitted values of $m_{A^0}$ where the interesting effects take place [18].

As for the demand for light (higgsino-like) chargino-neutralinos of $50-60$ GeV [36], it is worthwhile remembering that they are imperative not only because they may contribute sizeable non-oblique corrections to some partial $Z$-widths, but also because they produce

7The detailed interaction Lagrangian is given e.g. in eqs.(18)-(19) of Ref.33.
large, and negative, oblique effects on all partial Z-widths. Since the oblique effects essentially cancel in the ratios \( \frac{3}{4}, \frac{3}{4} \), the latters are dominated by just the non-oblique corrections. The upshot of this game is that one may generate a pattern of quantum corrections in the MSSM characterized by non-negligible effects on \( R_b \) and \( R_c \) while at the same time leaving a small enough net correction on the total Z-width (Cf. Fig.1 of Ref.[17]) that respects the tight \( \Gamma_Z \)-constraint.

In Figs.2a-2b we plot the evolution of our corrections as a function of the pseudoscalar mass, \( m_{A^0} \), for three large values of \( \tan \beta \) in two antipodal points of the \((\mu, M)\)-plane. It becomes clear that a light pseudoscalar mass \( m_{A^0} \lesssim 70 \text{ GeV} \) is favoured by a possible solution to the “\( \alpha_s(M_Z) \) crisis” at high \( \tan \beta \). We see that, for \( \tan \beta > 50 \), we are able to approach the desired \( \alpha_s(M_Z) = 0.11 \) regime on the basis of pure electroweak corrections alone. Finally, in Figs.3 and 4 we test the sensitivity of \( \alpha_s(M_Z) \) on the sfermion spectrum and in particular on the mixing parameter of the stop mass matrix. Specifically, in Fig.3a we exhibit the (mild) sensitivity on \( m_{\tilde{u}} \) (assumed to be degenerate with \( m_{\tilde{c}} \)), and in Fig.3b we plot the (also light) dependence on the slepton masses through the sneutrino mass parameter \( m_{\tilde{\nu}} \). We deal separately in Fig.4 with the stop-sbottom case, due to the likely presence of mixing in the stop sector. From Fig.4a we verify that \( \alpha_s(M_Z) \) noticeably decreases with a decreasing sbottom mass. This was expected from the relatively large, and positive, supersymmetric contribution to the \( b\bar{b} \) decay mode which, at very high \( \tan \beta \), is dominated by the total yield from the pseudoscalar plus sbottom-neutralino vertex diagrams. Such a contribution overwhelms that of the stop-chargino vertex and renders the influence from the stop mass rather irrelevant. Indeed, for fixed sbottom mass, Fig.4b confirms that \( \alpha_s(M_Z) \) is insensitive to \( M_{LR} \), and so to the stop masses, thus proving our contention that a light stop is not essential in our analysis. This is in stark contrast to the low \( \tan \beta \) regime, where the stop-chargino vertex takes off and \( \alpha_s(M_Z) \) becomes very sensitive to \( M_{LR} \).

Some discussion is worth doing concerning the SUSY-QCD corrections as compared to the SUSY-electroweak corrections. For the formers to produce the desired quantum effect one needs very light values for the gluino masses, if squark masses are of the order of \( \mathcal{O}(100) \text{ GeV} \). This possibility, even though not fully excluded as long as gluinos of \( \mathcal{O}(1) \text{ GeV} \) remain phenomenologically viable, is almost ruled out. Alternatively, our calculation proves that the relevant effect could also drop from pure electroweak SUSY quantum physics. Remarkably enough, these electroweak supersymmetric corrections have two distinctive virtues, to wit: they are not only potentially larger than the strong supersymmetric corrections, but they even show the suitable alternation of signs so as to simultaneously adjust the \( R_b \) and \( R_c \) theoretical predictions. This feature is not possible for the SUSY-QCD corrections, since they are always positive for all light-
quark channels, in particular for the $c\bar{c}$ channel, and are either large and negative, or small and positive, for the $b\bar{b}$ channel. Thus they could contribute significantly both to $R_b$ and $R_c$ just in the wrong direction. This is specially so when $R_b$ receives large negative corrections—a circumstance which is tied to the hypothetical existence of light sbottoms. In this case, strong SUSY-QCD effects from sbottoms would dangerously interfere with the positive contributions from the SUSY-electroweak sector.

To prevent these negative corrections from occurring, we have assumed that the mixing parameter in the sbottom sector, $m_b(A_b - \mu \tan \beta)$, is small enough so that light sbottoms of order 45 GeV are forbidden. In contrast, in the SUSY-QCD calculation of Ref. [8], light sbottoms are allowed and go associated with large values of $\tan \beta$, the reason being that the trilinear soft SUSY-breaking parameters, $A_{b,t}$, have been arbitrarily set to zero. In general, however, the trilinear terms do not vanish and one may have large sbottom masses even for large $\tan \beta$. In this case the negative SUSY-QCD contributions on the $b\bar{b}$ mode are safely restrained and they do not spoil the positive SUSY-electroweak effects presented here.

In summary, we have demonstrated the existence of regions of the general MSSM parameter space where electroweak supersymmetric quantum physics could reveal itself as the “new physis” called for to simultaneously resolve the triple “$R_b - R_c - \alpha_s(M_Z)$ crisis”. These regions are characterized either by very large or—less likely—by very small values of $\tan \beta$ and also, respectively, by a light CP-even and a light CP-odd Higgs of the same mass, or by a very large (effectively decoupled) CP-odd Higgs. In both cases a light chargino-neutralino is mandatory. However, whereas in the low $\tan \beta$ region a light stop of $\mathcal{O}(50)$ GeV is also indispensable, in the high $\tan \beta$ region it is not needed at all; instead, an intermediately heavy sbottom of a few hundred GeV suffices to round off a comfortable solution to the triple crisis in the MSSM. As an extra bonus to be added up to our approach, we remark that the solution space just described does automatically preserve the successful SM prediction of the observables $M_W$, $\sin^2 \theta^l_{\text{eff}}$ and $A_{FB}^{bb}$. In other words, our RSM prediction for these observables is essentially the same as in the SM. The reason is simple: it stems from the fact that these observables are dominated by universal corrections and hence are weakly sensitive to the Higgs sector of the MSSM. Furthermore, $M_W$, $\sin^2 \theta^l_{\text{eff}}$ and $A_{FB}^{bb}$ are fairly blind, up to small L-R mixing effects, to the presence of RH stops and sbottoms which, due to the higgsino-like nature of the lightest chargino-neutralino, are the only ones involved in the relevant $Z$ vertices.

Despite it is not strictly necessary from the phenomenological point of view, it remains to see—if only as a standard theoretical prejudice—whether our favourite parameter region can be successfully implemented in some (non-minimal) SUSY-GUT scenario. For exam-

---

8Alternatively, gluinos could be very heavy and the SUSY-QCD contributions would totally decouple.
ple, in the string type unified scenario of Ref. \[15\] one finds that in order that the grand unified prediction of $\alpha_s(M_Z)$ be consistent as much as possible with the low-energy value (3), gluinos and squarks should be relatively light, viz. of $\mathcal{O}(100)\,\text{GeV}$, and the wino soft-mass parameter, $M$, should be much larger than the gluino mass. Notice that this feature is not incompatible with a chargino being mostly higgsino, as currently required in the MSSM literature on $R_b$. Furthermore, from the contour lines of $\alpha_s(M_Z)$ in the $(\mu, M)$-plane of Fig.1a we see that, for a given value of $\mu$, it is precisely in the region of large $M$ where $\alpha_s(M_Z)$ approaches the closest to the low-energy value, eq. (3), while at the same time preserves the experimental bound (16) on $\Gamma_Z^{\exp}$. As an optimized example, we take $\mu = -50\,\text{GeV}$, $M = 350\,\text{GeV}$ and the rest of the parameters as in Fig.1b, and we find $\alpha_s(M_Z) \simeq 0.111$, thus a value which is in excellent agreement with the deep inelastic scattering result (3) and being also compatible with the rest of the $Z$-observables. Still, it is true that our analysis favours the higgsino together with the lightest Higgs to be in the $\mathcal{O}(50)\,\text{GeV}$ range, a fact which could be at variance with the GUT spectrum derived in Ref. \[15\]. Clearly, more work is needed in the arena of supersymmetric grand unified theories before reaching a conclusive solution to the “$\alpha_s(M_Z)$ crisis” that is compatible with the experimental status of the various $Z$ observables. Be as it may, if this status remains unchanged in the next generation of experiments, a SUSY-GUT solution within the MSSM, if it exists at all, is bound to meet the general conditions reported in the present study.

**Added Note**

After completing our work, we became aware of a work by Chankowski and Pokorski \[37\] which projects the same preferred region of the MSSM parameter space as in our case, and a work by Kane, Stuart and Wells \[38\] where a fit analysis of the data is made on the basis of assuming (not computing!) the low energy value (3) of $\alpha_s(M_Z)$ in the MSSM. For comparison purposes, it may be useful to note that the latter work by Kane et al. does not incorporate the effect from the Higgs pseudoscalar contributions and as a consequence their approach is not adequately sensitive to the high $\tan \beta$ region. This leads them to overemphasize the– potentially problematic– existence of a too light stop, a possibility which, although sufficient, is not necessary to fit the electroweak data. In fact, as we have shown in the present and previous analyses \[17\]-\[19\], a complete treatment of the quantum effects from the MSSM with a Higgs sector containing a light pseudoscalar (with or without a light stop) may be crucial to account for the electroweak data on the $Z$, as also confirmed by the aforementioned work by Chankowski and Pokorski.
Acknowledgements

One of us (JS) is indebted to M. Martínez for providing details on the fit analysis of Ref.[28] which we have used in this paper. He is also obliged to M. Shifman for reading the manuscript and for sharing his insight on the QCD implications of a high $\alpha_s(M_Z)$. Conversations with P. Chankowski are also gratefully acknowledged. This work has been partially supported by CICYT under project No. AEN93-0474. DG has also been financed by a grant of the Comissionat per a Universitats i Recerca, Generalitat de Catalunya.

References

[1] S. Bethke, talk at the Tennessee International Symposium on Radiative Corrections, Gatlinburg, Tennessee, June 27-July 1, 1994 (to appear in the Proceedings); S. Bethke and S. Catani, A summary of $\alpha_s$ measurements, preprint CERN-TH.6484/92; G. Altarelli, QCD and experiment-status of $\alpha_s$ in QCD-20 years later Proc. of the 1992 Aachen Workshop, eds. P. Zerwas and H. Kastrup (World Scientific, Singapore, 1993), vol.1, pag. 172; I. Hinchliffe, Phys. Rev. D50 (1994) 1297.

[2] J. Erler and P. Langacker, Implications of high precision experiments and the CDF top quark candidates, preprint UPR-0632T, October 1994.

[3] M. Shifman, Mod. Phys. Lett. A10 (1995) 605; M. Shifman, Determining $\alpha_s(M_Z)$ from measurements at Z: how nature prompts us about new physics, talk at the International Symposium on Particle Theory and Phenomenology; Ames, Iowa, May 22-24 1995 (to appear in the Proceedings).

[4] M. Martínez, Results at LEP on electroweak parameters, preprint IFAE-UAB/94-02, September 1994 (to appear in: Proc. of the XXII International Meeting on Fundamental Physics: “The standard model and beyond”, Jaca, Spain, February 1994); and Precision tests of the standard model, preprint IFAE-UAB/95-01, January 1995 (Lectures given at ”Frontieres in Particle Physics”, Institute d’Etudes Scientifiques de Cargese, Corsica, France, August, 1994).

[5] For a recent review, see e.g. A. Pich, QCD predictions for the $\tau$ hadronic width: determination of $\alpha_s(M_{\tau}^2)$, preprint FTUV/94-71 (1994).

[6] G. Altarelli, $\alpha_s$ from $\tau$: a discussion of theoretical errors, preprint CERN-TH.7493/94 (1994).
[7] A.X. El-Khadra, G. Hockney, A. Kronfeld and P. Mackenzie, Phys. Rev. Lett 69 (1992) 729.

[8] A. Djouadi, M. Drees and H. König, Phys. Rev. D48 (1993) 3081; K. Hagiwar, H. Murayama, Phys. Lett. B246 (1990) 533.

[9] J. Ellis and H. Kowalski, Phys. Lett. B157 (1985) 437.

[10] L. Clavelli, Phys. Rev. D46 (1992) 2112; L. Clavelli, P. Coulter and K. Yuan, Phys. Rev. D47 (1993) 1973; M. Jezabek and J.H. Kühn, Phys. Lett. B301 (1993) 121; J. Ellis, D.V. Nanopoulos and D.A. Ross, Phys. Lett. B305 (1993) 375.

[11] R.S. Chivukula, E.H. Simmons and J. Terning, Phys. Lett. B331 (1994) 383; K. Hagiwara and N. Kitazawa, *Extended technicolor contribution to the Zbb vertex*, preprint KEK-TH-433 (1995); See also Ref.[2] and references therein.

[12] J. Ellis, G.L. Fogli and E. Lisi, Phys. Lett. B343 (1995) 282.

[13] For a recent analysis, see P. Langacker and N. Polonsky, *The strong coupling, unification, and recent data*, preprint UPR-0642T (1995).

[14] P. Langacker and N. Polonsky, Phys. Rev. D47 (1993) 4028.

[15] L. Roszkowski and M. Shifman, *Reconciling supersymmetric grand unification with $\alpha_s(M_Z) \simeq 0.11$*, preprint TPI-MINN-95/04-T (1995).

[16] H. Nilles, Phys. Rep. 110 (1984) 1; H. Haber and G. Kane, Phys. Rep. 117 (1985) 75; A. Lahanas and D. Nanopoulos, Phys. Rep. 145 (1987) 1; See also the exhaustive reprint collection *Supersymmetry* (2 vols.), ed. S. Ferrara (North Holland/World Scientific, Singapore, 1987).

[17] D. Garcia, R.A. Jiménez and J. Solà, Phys. Lett. B347 (1995) 309.

[18] D. Garcia, R.A. Jiménez and J. Solà, Phys. Lett. B347 (1995) 321.

[19] D. Garcia, J. Solà, *The quantum correlation $R_b - R_c$ in the MSSM: more hints of supersymmetry?*, preprint UAB-FT-358 (1995) [hep-ph/9502317] (Phys, Lett. B, in press).

[20] D. Garcia, R.A. Jiménez and J. Solà, *The width of the Z boson in the MSSM*, preprint UAB-FT in preparation; D. Garcia, PhD. Thesis, in preparation, Univ. Autònoma de Barcelona.
[21] F. Abe et al. (CDF Collab.), Phys. Rev. Lett. 73 (1994) 225; F. Abe et al (CDF Collab.), preprint FERMILAB-PUB-95/022-E (1995); S. Abachi et al (D0 Collab.), preprint FERMILAB-PUB-95/028-E (1995).

[22] M. Boulware and D. Finnell, Phys. Rev. D44 (1991) 2054; A. Djouadi, G. Girardi, C. Verzegnassi, W. Hollik and F.M. Renard, Nucl. Phys. B349 (1991) 48; G. Altarelli, R. Barbieri and F. Caravaglios, Phys. Lett. B314 (1993) 357; ibid Nucl. Phys. B405 (1993) 3.

[23] J.D. Wells, C. Kolda and G.L. Kane, Phys. Lett. B338 (1994) 219.

[24] J.E. Kim and G.T. Park, $\epsilon_b$ constraints on the minimal $SU(5)$ and $SU(5) \times U(1)$ supergravity models, preprint SNUTP 94-66, August 1994; M. Carena and C. Wagner, Higgs and supersymmetric particle signals at the infrared fixed point of the top quark mass, preprint CERN-TH.7393/94, August 1994; X. Wang, J.L. Lopez and D.V. Nanopoulos, $R_b$ in supergravity models, preprint CERN-TH.7553/95, January 1995.

[25] R. Garisto and J.N. Ng, Phys. Lett. B315 (1993) 372.

[26] B. Barish, et al., talk at the Int. Conf. on High Energy Physics, Glasgow, Scotland, July, 1994.

[27] A.J. Buras, M. Misiak, M. Münz and S. Pokorski, Nucl. Phys. B424 (1994) 374.

[28] T. Hebbeker, M. Martínez, G. Passarino, G. Quast, Phys. Lett. B331 (1994) 165.

[29] M. Martínez, private communication.

[30] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, The Higgs hunter’s guide (Addison-Wesley, New York, 1990).

[31] The LEP Collaborations ALEPH, DELPHI, L3, OPAL and the LEP Electroweak Working Group, CERN-PPE/93-157 and Contribution to the 27th International Conference on High Energy Physics, Glasgow, Scotland, July 1994 (to appear in the Proceedings).

[32] W. Hollik, Electroweak precision observables. An indirect access to the top quark, preprint KA-TP-2-1995 (1995).

[33] D. Garcia, R.A. Jiménez, J. Solà, W. Hollik, Nucl. Phys. B427 (1994) 53.
[34] Aleph Collab., Phys. Rep. 216 (1992) 254; Phys. Rev. D50 (1994) 1369; P. Abreu, et al. (DELPHI Collab.), Phys. Lett. B247 (1990) 148; O. Adriani, et. al. (L3 Collab.), Phys. Rep. 236 (1993) 1; P.J. Franzini and P. Taxil, in: Higgs search, Z Physics at LEP 1, vol.2, eds. G. Altarelli, R. Kleiss and C. Verzegnassi, CERN Yellow Report, CERN 89-08 (1989).

[35] J.D. Bjorken, in: Weak interactions at high energy and the production of new particles, Proc. of the SLAC Summer Institute on Particle Physics, 1976, ed. M. Zipf.

[36] Proc. of the workshop: Ten Years of SUSY Confronting Experiment, ed. J. Ellis, D.V. Nanopoulos and A. Savoy-Navarro, CERN, September 1992, CERN-TH.6707/92-PPE/92-180; H. Baer et al., Low energy supersymmetry phenomenology, preprint FSU-HEP-950401 (1995).

[37] P.H.Chankowski and S. Pokorski, Precision tests of the MSSM, preprint IFT-95/5.

[38] G.L. Kane, R.G. Stuart, J.D. Wells, A global fit of LEP/SLC data with light superpartners, preprint UM-TH-95-16.
Figure Captions

• **Fig.1** (a) Contour lines for the lineshape-determined $\alpha_s(M_Z)$ in the higgsino-gaugino $(\mu, M)$-plane of the MSSM. The sfermion spectrum is obtained from the basic parameter set ($[15]$) following the procedure of Ref. [$18$] with $\tan \beta = 50$, $m_{A^0} = 50 \text{ GeV}$, $m_{\tilde{\nu}_l} = m_{\tilde{\nu}_u} = m_{\tilde{b}} = 200 \text{ GeV}$ and $M_{LR} = 0$. The top quark mass is fixed at $m_t = 175 \text{ GeV}$. The blank region is excluded by the chargino-neutralino mass bounds [$17$], and the shaded area is excluded by the $\Gamma_Z$-constraint: $\delta \Gamma_Z^{\text{MSSM}} < 9 \text{ MeV}$; (b) As in case (a), but for $\tan \beta = 60$ and $m_{\tilde{\nu}_l} = m_{\tilde{\nu}_u} = m_{\tilde{b}} = 300 \text{ GeV}$.

• **Fig.2** (a) $\alpha_s(M_Z)$ as a function of the pseudoscalar mass $m_{A^0}$ for various $\tan \beta$, $(\mu, M) = (-60, 350) \text{ GeV}$ and the same sfermion spectrum as in Fig.1a. The curves are cut off from below by the $\Gamma_Z$-constraint; (b) As in case (a), but for $(\mu, M) = (-350, 60) \text{ GeV}$. The dashed curve corresponds to the RSM contribution alone, and the straight line to the SM with a fixed Higgs mass of $m_H = 300 \text{ GeV}$.

• **Fig.3** (a) $\alpha_s(M_Z)$ as a function of the squark masses $m_{\tilde{u}} = m_{\tilde{c}}$ for various $\tan \beta$. The remaining sfermion masses are as in Fig.1a, and $(\mu, M)$ fixed as in Fig.2a; (b) $\alpha_s(M_Z)$ as a function of the sneutrino masses $m_{\tilde{\nu}}$ and the rest of parameters as in case (a).

• **Fig.4** (a) Evolution of $\alpha_s(M_Z)$ in terms of $m_{\tilde{b}_L} = m_{\tilde{b}_R} \equiv m_{\tilde{b}}$, for $(\mu, M)$ as in Fig.2a and the rest of the parameters as in Fig.1a; (b) Dependence of $\alpha_s(M_Z)$ on the stop mixing parameter $M_{LR}$ for the same choice of the remaining parameters as in case (a).
Fig. 1 (a)
Fig. 1 (b)
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{\label{fig:2}
Fig. 2}
\end{figure}
Fig. 3
Fig. 4