Effect of Interfacial Energy on the Drain Sink Formation Height

Niklas KOJOLA, Shinichiro YOKOYA and Pär JÖNSSON

Division of Applied Process Metallurgy, Royal Institute of Technology, SE-100 44, Stockholm, Sweden.
(Received on October 17, 2008; accepted on January 20, 2009)

The effect of the liquid/gas interfacial energy on the drain sink formation height has been studied using both mathematical and physical modeling. Initially, the mathematical model predictions of the drain sink formation height were compared with data from physical modeling, representing a situation with a low interfacial energy. The agreement was found to be good. Thereafter, mathematical modeling was done to evaluate the drain sink formation height at higher interfacial energies. In addition, an analytical expression was derived for the prediction of the drain sink formation height. The calculations using this equation were found to align well with both the experimental data as well as the numerical predictions. This analytical equation is suitable to use for determination of the drain sink formation height as function of the liquid/gas interfacial energy, liquid density, outlet radius and outlet length. In order to demonstrate its industrial usefulness the equation was used to predict drain sink formation heights for steel in a system with geometrical dimensions relevant for steel production. It was found that an increased interfacial energy lowered the drain sink formation height severely for small outlets, low-density fluids and short outlet lengths. In real plant practice, the predictions with the analytical equation yield that the effect of steel/argon interfacial energy decreases the predicted drain sink formation height by approximately 10% compared to if the interfacial energy was neglected.

KEY WORDS: drain sink; mathematical model; interfacial energy; height; physical model.

1. Introduction

Reoxidation of the steel and dissolution of macro inclusions into the liquid bath are phenomena that are most likely to occur during metallurgical teeming operations.1) Possible outcomes of reoxidation are nozzle clogging, surface defects in steel sheets and loss of easily oxidized alloy elements. Although the teeming techniques, mostly due to technical progress like sliding gates, stopper rods, electrical slag detection systems etc., have improved over the years, there are still technical problems to deal with. More specifically, slag carry-over from ladle to tundish due to late stage teeming phenomena like vortex and drain sink still poses a great threat to the final quality of the steel and efficiency of the steel works production process.2) Although product quality and safety urges for an early termination of the ladle teeming, increased productivity demands the opposite. Thus, in production, an optimization of these two conflicting aspects needs to be done.

The vortex is a non-rotational swirling flow caused by conservation of angular momentum in combination with some initial bulk rotation during the teeming operation. Under some circumstances, the vortex can form an open funnel shaped core through the liquid, which can transport slag to the outlet. The vortex has been analyzed in previous works.1–3) The current report focuses on the drain sink phenomena. This is a non-rotational, non-swirling flow caused by higher volumetric flow capacity through the outlet than the largest possible flow capacity to the outlet. An analogy can be seen for the flow situation in a waterfall. Visually, the drain sink, similar to vortex, appears as a funnel shaped core, extending from the surface of the liquid, through the bulk and to the outlet. Also similar to the vortex, the drain sink can transport a second phase, i.e. slag, through the core and to the outlet.

The drain sink phenomenon has been investigated thoroughly in the open literature.1,3–10) More specifically, it has been suggested to be the main problem causing slag carry-over during bottom teeming of vessels.1,4) Important properties of the drain sink formation height are independence of nozzle position,1,3) initial filling height, vessel diameter and the shape of the vessel bottom (flat, hemispherical, sloped or inclined).1,3,5) The drain sink formation height is also reported to be a linear function of the outlet radius.1,3,5) Thus, it is clear that to minimize the negative effect of drain sink on steel plant production, the nozzle can be partially closed during late stages of teeming in order to decrease the drain sink formation height.3) Furthermore, a tilted bottom of the vessel decreases the amount of steel left when the drain sink occurs.4,9) In order to demonstrate its industrial usefulness the equation was used to predict drain sink formation heights for steel in a system with geometrical dimensions relevant for steel production. It was found that an increased interfacial energy lowered the drain sink formation height severely for small outlets, low-density fluids and short outlet lengths. In real plant practice, the predictions with the analytical equation yield that the effect of steel/argon interfacial energy decreases the predicted drain sink formation height by approximately 10% compared to if the interfacial energy was neglected.
bers for the drain sink flows usually are very high.\textsuperscript{3)}

So far, the experimental works on the drain sink formation relevant for ladle teeming has focused on physical modeling by using water-systems. Although the Reynold’s numbers for free-flowing water and liquid steel systems are similar, the Weber number, shown in Eq. (1) where $\rho$ is density, $v$ is velocity, $L$ is characteristic length and $\sigma$ is interfacial energy, indicating the impact of interfacial energy on the flow, differs severely. More specifically, the Weber number ratio during drain sink for a water/air system compared to a steel/argon system with similar geometries is approximately 4.

$$ We = \frac{\rho v^2 L}{\sigma} \quad \text{(1)} $$

In order to obtain an increased understanding of the drain sink formation in real steel production, the present paper has investigated the effect of free surface interfacial energy on drain sink formation height. Since it is difficult to control the interfacial energy of the liquid during cold physical modeling, a mathematical modeling approach was chosen in the present paper. A numerical model of a teeming process of water was created. The model simulated a water/air system and was verified by data from water/air model experiments. Then, the numerical model was used to calculate the drain sink formation height for higher interfacial energies, while all other variables were kept constant. Due to the lack of exactness in the turbulence models as well as in the numerical discretization of the physical problem, the results should be treated as qualitative indications rather than a perfect model of reality. Moreover, a simple analytical expression was derived and compared to the results obtained from the numerical simulation as well as the physical model experiments.

2. Analytical Theory

The hollow drain sink is formed because the outflow capacity through the outlet exceeds the maximum bulk flow capacity towards the outlet. Previous models predicting the drain sink formation height have made the predictions based on the hydrodynamics with liquid velocities determined only from the hydrostatic pressure and vertical position. However, if the surface tension (or the resistance to create new surface) of the liquid is considered, there will be a pressure drop over the liquid surface if the surface becomes deformed. If this is applied to the drain sink problem, the velocities towards the outlet can increase compared to when the surface tension is not considered.

A simple analytical calculation based on a cylindrical drain sink control volume situated just above the ladle outlet, between the ladle bottom and the liquid surface, will be done below in order to derive a simple analytical model to predict the drain sink formation height. The geometry used in the derivation together with the drain sink formation process are shown in Fig. 1, where the outlet radius ($R$), the bath surface height ($H$) and vertical distance ($h$) are defined.

The vertically directed force, $F_\sigma$, acting on a circle border surrounding a circular outlet of radius $R$ and created by the interfacial energy $\sigma$ of the interface between the primary liquid and the surrounding gas phase, can be expressed as shown in Eq. (2):

$$ F_\sigma = 2 \pi R \sigma \quad \text{(2)} $$

Thus, a pressure difference $\Delta P$ between opposite sides of the interface can be resisted over the outlet. The resulting vertical force created by the pressure difference, $F_p$, can be expressed as:

$$ F_p = -\pi R^2 \Delta P \quad \text{(3)} $$

At mechanical equilibrium, the forces balance each other as shown in Eq. (4):

$$ F_p + F_\sigma = 0 \quad \text{(4)} $$

Thus, combining Eq. (2), Eq. (3) and Eq. (4), the pressure difference between the two sides of the interface can be expressed as a function of the outlet radius and the interfacial energy:

$$ \Delta P = \frac{2 \sigma}{R} \quad \text{(5)} $$

The Bernoulli equation can be applied to compare i) a point at rest, and at atmospheric pressure somewhere at the gas–liquid surface at height $H$, and ii) a point at the surface of the drain sink control volume, where the liquid is flowing at maximum speed $v$ at height $h$. Furthermore, a pressure fall, $\Delta P$, over the interface can be resisted by the surface tension, $\sigma$. Hence, the maximum velocity of the liquid can increase. The resulting equation is the following:

$$ \rho g H + \Delta P = \frac{\rho v^2}{2} + \rho g h \quad \text{(6)} $$

Fig. 1. Schematic sketch of critical steps in the drain sink formation process defining the variables used in the model derivation, 1: normal teeming, 2: drain sink formation 3: fully developed drain sink, 4: present model of drain sink formation.
where \( \rho \) is the liquid density and \( g \) is the acceleration due to gravity. Now, if Eq. (5) is inserted into Eq. (6), it is possible to derive an expression for the speed, \( v \), at a point on the drain sink control volume at height \( h \) above the vessel bottom:

\[
v = \sqrt{\frac{2}{3\rho} \left( g(H - h) + \frac{2\sigma}{\rho R} \right)} \quad \text{..................(7)}
\]

Thereafter, all flow velocities directed towards the outlet can be assumed to be horizontal, due to the low bath level. Therefore, integration over the control volume surface can be done to obtain the total volume flow towards the outlet, \( Q_{TO} \):

\[
Q_{TO} = 2\pi R \int_0^H vdh \quad \text{..................(8)}
\]

If the expression for the speed at the point on the drain sink control volume from Eq. (7) is inserted in Eq. (8), the following equation can be obtained:

\[
Q_{TO} = 2\pi R \int_0^H \sqrt{\frac{2}{3\rho} \left( g(H - h) + \frac{2\sigma}{\rho R} \right)} dh \quad \text{..................(9)}
\]

After integration of Eq. (9) and inserting the limits, a simple expression for the flow towards the outlet appears:

\[
Q_{TO} = \frac{4\pi R^2}{3\rho} \left( gH + \frac{2\sigma}{\rho R} \right)^{3/2} - \left( \frac{2\sigma}{\rho R} \right)^{3/2} \quad \text{..................(10)}
\]

The flow through the outlet can be calculated if the point at rest at the interface is compared to a point at the end of the outlet. This is done assuming atmospheric pressure and a distance \( l \) (outlet length) beneath the gravitational axis origin. Thus, the volume flow through the outlet, \( Q_{TH} \), can be expressed as:

\[
Q_{TH} = \pi R^2 \sqrt{2gh(H + l)} \quad \text{..................(11)}
\]

Since no accumulation is likely to occur in the outlet pipe, the maximum flows to and through the outlet are equal, as shown in Eq. (12):

\[
Q_{TH} = Q_{TO} \quad \text{..................(12)}
\]

Thus, given that Eq. (10) and Eq. (11) are equal, the critical condition expression shown in Eq. (13) is valid at the drain sink formation height:

\[
\frac{4}{3} \left( H + \frac{2\sigma}{\rho gR} \right)^{3/2} - \left( \frac{2\sigma}{\rho gR} \right)^{3/2} = R_e \sqrt{H + l} \quad \text{..................(13)}
\]

The simple assessment shown above resulted in a new analytical drain sink model defined by Eq. (13). Here, it is clear that it is possible to calculate the drain sink height as a function of the bath surface height, interfacial energy, density, outlet radius and outlet length. However, it is important to note that, during the derivation, no attention has been taken to include the effect of the supernatant phase density on the drain sink height.

3. Numerical Model

A numerical model was developed to simulate the emptying of the liquid through the outlet geometry shown in Fig. 1.

- **Assumptions**
  - Newtonian fluids
  - Incompressible fluids
  - Constant molecular viscosity of the fluids
  - Constant density of the fluids
  - Axis-symmetry
  - Only axial and radial velocity components
  - No mass sources
  - Isothermal system

- **Equations**

  The general transportation equation of some property, \( \Phi \), is shown in Eq. (14) where \( \rho \) is density, \( u \) is the mean velocity vector, \( \Gamma \) is the diffusion coefficient and \( S_\Phi \) is a source term (accounting for external impact on the property, i.e. if \( \Phi \) is a velocity, \( S_\Phi \) also contains the pressure influence).

\[
\frac{\partial}{\partial t} \left( \rho \Phi \right) + \nabla \cdot (\rho \Phi u) = \nabla \cdot (\Gamma \nabla \Phi) + S_\Phi \quad \text{..................(14)}
\]

In Table 1, the parameters of different conserved properties in a cylindrical coordinate system are shown.

- **Turbulence Model**

  The Realizable \( k-\varepsilon \) turbulence model was used in the calculations. The difference between the standard semi-empirical \( k-\varepsilon \) model and the Realizable \( k-\varepsilon \) model is that the treatment of the dissipation rate, \( \varepsilon \), in the latter is derived from an exact equation for the transport of the mean-square vorticity fluctuation, while the former one relies on a physical reasoning rather than mathematically exact equations.

| Table 1. Property parameters for Eq. (2). |
|-------------------------------|--------|--------|
| **Conserved property:** \( \Phi \) | **\( \Gamma \)** | **\( S_\Phi \)** |
| Mass | 1 | 0 | 0 |
| Axial momentum | \( u_x \) | \( \mu \) | \( -\frac{\rho \varepsilon}{\eta} + \rho g_x F_{3x} \) |
| Radial momentum | \( u_r \) | \( \mu \) | \( -\frac{\rho \varepsilon}{\eta} + \rho g_r F_{3r} \) |
| \( K \) | \( k \) | \( \frac{\mu + \frac{\mu_t}{\sigma_t}}{\sigma_t} \) | \( G_k - p\varepsilon \) |
| (realizable model) | \( \varepsilon \) | \( \frac{\mu + \frac{\mu_t}{\sigma_t}}{\sigma_t} \) | \( pC_{\varepsilon} \sqrt{\frac{G_k}{\mu}} - C_{\varepsilon} \rho g R_{\varepsilon} k^2 \) |

Notes:

- \( \mu_\varepsilon = C_{\mu_\varepsilon} \rho R_{\varepsilon} k^2 \)
- \( G_k = 2\mu_\varepsilon \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right)^2 + \frac{\partial u_z}{\partial z} \)
- \( C_1 = \max \left( 0.43, \frac{a_3}{\eta + k} \right) \)
- \( \eta = \frac{C_\eta}{\rho R_{\eta}} \)
- \( \nu = \frac{\eta}{\rho R_{\eta}} \)
- \( C_{\varepsilon} = 0.09 \quad C_{\varepsilon} = 1.44 \quad C_{\varepsilon} = 1.99 \quad a_3 = 1 \quad a_4 = 1.2 \)
• Computational Domain, Boundary Conditions and Solver

The grid was uniform and structured consisting of quadrilateral cells of constant size. For the pressure, discretization of the computational domain was done with PRESTO! and for the transport equations, a second-order upwind-scheme was used. The pressure and velocity was coupled using the SIMPLEC scheme. To track the interface between the two fluids, the volume of fluid (VOF) scheme was used. The calculations were performed on Intel-based computers with Windows OS (2000) and the solver Fluent®. The grids were created using Gambit®.

Computational simulations were done on two different quadrilateral cell grids, one with cell size 0.1 mm² and another with cell size 0.2 mm². The geometries of the model are shown in Fig. 2 and the time step of the numerical solution was $10^{-3}$ s. The domain initially contained water as indicated in Fig. 2. The rest of the domain was air at 1 atm pressure. The effect of surface tension was modeled by the continuum-surface force (CSF) model in which the surface force appears as a body force in the grid cells in which the two-phase interface is present, as shown in Eq. (15):

$$F_V = \frac{2\sigma V}{(\rho_1 + \rho_2)}$$

where $F_V$ is the volume force caused by the interfacial energy, $\sigma$ is the interfacial energy, $\kappa$ is the curvature of the surface, $\rho_c$ is the actual density of the grid cell, $\rho_1$ and $\rho_2$ are the densities of liquid phase and gas phase respectively and $\alpha$ is the volume fraction of the primary phase.

In Table 2, physical data used in the model is shown. To elucidate the effect of the interfacial energy on the drain sink formation, the interfacial energy between the water and the gas was varied between the different simulations, from 0.072 to 2.0 N/m. On the lower end this interval covers the interfacial energy of water and air at 25°C and as well as the approximate interfacial energies between liquid iron and argon gas and liquid iron and slag on the upper end.

All initial velocities were zero and the flow boundary condition for walls was no slip. The liquid/wall interaction was handled by a dynamic boundary condition, i.e. the contact angle between the wall and the liquid, $\sigma$, was defined as 90°. This boundary condition only affects the cells close to the wall and gas liquid interface (why it is called dynamic).

Usually, some small symmetrical surface waves were induced during opening of the outlet, which was why a slight oscillation of the bath surface and the surface dimple (preceeding the drain sink formation) occurred. Therefore, the drain sink formation height was defined as the bath height, $H$, at which the lowest point of the outlet-penetrating surface dimple reached 0.5$H$, as shown in Fig. 3.

4. Experimental Work

A circular and transparent vessel with an inner diameter of 320 mm was made from acrylic resin. It was equipped with a centrally placed circular outlet of a 40 mm diameter. In the physical model, water was used as the primary liquid to simulate steel. The water’s kinematic viscosity at 25°C is $1 \times 10^{-6}$ m²/s. This is very similar to the kinematic viscosity of liquid steel which is $0.8 \times 10^{-6}$ m²/s at 1600°C. The experiments were recorded using a digital video camera placed at the outlet level, 4 meters from the experimental vessel. The drain sink formation height was defined as the bath height when the visible surface dimple began to grow towards the outlet. The growth procedure from dimple formation to outlet penetration took less than one second. The experimental set-up gave a geometrical reading error of less than 1% of the measured drain sink formation height value. Several initial water filling heights were tested and all gave the same result with respect to the drain sink

Table 2. Physical data of the model and model phases.

| Phase        | Density ($\rho$) | Kinematic Viscosity ($\nu$) |
|--------------|------------------|-----------------------------|
| Gas (air)    | $1.225 \text{ kg/m}^3$ | $1.789 \times 10^{-5} \text{ kg/m/s}$ |
| Liquid       | $1.460 \times 10^{-5} \text{ kg/m/s}$ | $1.003 \text{ kg/m/s}$ |
| Interfacial energy gas/liquid | $0.072-2.0 \text{ N/m}$ |

Fig. 2. Image of the numerical domain, thick black lines are constant pressure boundaries, $P=1$ atm. The dashed grey line defines the position of axis-symmetric boundary condition and the thin black lines are walls. All measurements are given in mm, the gravity, $g$, was 9.82 N/kg.

Fig. 3. Definition of drain sink formation height, $H$.

Fig. 4. Sketch over the physical modeling experimental set-up. All measurements are given in mm.
formation height. A sketch over the experimental set-up is shown in Fig. 4.

5. Results and Discussion

Drain sink formation height data simulated using the numerical model and calculated using the analytical model given in Eq. (13) are all shown in Fig. 5 as a function of the interfacial energy. In addition, experimental data from the physical modeling experiments are presented. The drain sink formation height is found to decrease approximately exponentially with an increased interfacial energy. Also, as clearly can be seen, the calculations using the analytical model, Eq. (13), align fairly well with the result from the physical modeling experiments and the predictions by the numerical model. Furthermore, it is seen that the results from the numerical simulations scatter slightly for different grid sizes, but are clearly in the same order of magnitude as the experiments.

Of course, the numerical solution to a time dependent free surface flow is very complex. The existing turbulence models are not in any way an exact description of reality. In addition, the numerical convergence of time-dependent 2-dimensional axis-symmetric flow equations is complicated. Thus, the nature of the problem both physically and mathematically causes errors in the solution, so that a perfect match with reality is not likely to occur. In addition, the analyzing of the results also contains an error because the definition of the variable of interest. More specifically, the drain sink formation height requires an inexact approximation of the time when it appears. However, since two different grids with different number of cells were used in the numerical simulations, it is possible to grasp a brief idea of the error related to the numerical solution. Although both grids give results close to the physical model, the larger cell size was used for the majority of the simulations since it was less time consuming.

It should be stressed that the analytical model derived in the present paper, Eq. (13), is a very simple assessment mainly supposed to give qualitative predictions of the drain sink formation height as function of interfacial energy between the phases. The cylindrical control volume is fit for low bath levels (when the bath level is lower than the outlet radius), since the flow under such conditions usually is directed horizontally between the bath surface and the vessel bottom. The small surface dimple shown in Fig. 1 has purely a cylindrical shape. This is probably one of the most severe assumptions in the model, since such a condition is physically impossible due to the required sharp edges of the liquid. Empirical modification of the model can easily be done by for example multiplying some average shape factor with the surface energy term. However, this did not seem necessary in the current study. In addition, the effect of the assumed shape of the control volume decreases with an increased interfacial energy. This can be seen if comparing Fig. 6, a captured image of a numerical calculation of water with an interfacial energy of 1.0 N/m during late stage teeming, to the control volume shown in Fig. 1. Figure 6 clearly shows that the surface dimple, compared to the dimple seen Fig. 3, where the interfacial energy was 0.072 N/m, tends to form to a more cylindrical shape.

Another issue necessary to point out is the handling of the maximum under-pressure due to the interfacial energy. In the present calculations, the lower limit of the under-pressure has been determined by the interfacial energy alone. However, in reality, the potential under-pressure is always limited by the suction effect of the system, i.e. the under-pressure cannot be lower than the gravity-induced pressure difference between the point of interest and the lowest point of the outlet because That would lead to an upstream back flow. Thus, the under-pressure cannot be lower than ($\rho g l$), where $l$ is the vertical distance between the lowest point of the outlet and the present position. The effect of this issue is that when the interfacial energy can withstand a larger pressure difference than the system can create, no drain sink will form.

As can be seen in the results, the drain sink formation height is severely lowered if the interfacial energy between water and air (0.072 N/m at 298 K)$^{21}$ is altered to be the interfacial energy between liquid iron and argon 10% H$_2$ gas (1.85 N/m at 1 873 K).$^{17}$ Thus, it can be expected that interfacial energy has a strong effect on the drain sink height when it becomes large. Nevertheless, it is important to note that the analytical model, as seen in Eq. (13), predicts that, unlike when drain sink is analyzed without consideration to interfacial energy, the density of the fluid has a damping effect on the interfacial energy term. In Eq. (13) it can also be seen that an increased outlet radius will decrease the lowering effect of the interfacial energy on the drain sink formation height. Thus, the presented results cannot be directly converted to steel. Furthermore, bulk steel production seldom exposes the steel bath surface to Ar10%H$_2$-gas. However, the interfacial energy between a steel (ShKh15, a ball-bearing steel containing 15% Cr) and different slags

Fig. 5. Drain sink formation height as a function of interfacial energy. Data are shown from all numerical predictions as well as from the physical experiment. In addition, the predictions by the model shown in Eq. (13) are also given. The outlet length was set to 0.01 m.

Fig. 6. Predicted shape of liquid/gas interface at a late stage of teeming of water, for an interfacial energy of 1.0 N/m.
have been measured to be of similar magnitude. More specifically, it was found to be in the range of 0.6 N/m for 2FeO–SiO$_2$ to 1.3 N/m for CaO–Al$_2$O$_3$ and 1.5 N/m for MgO–SiO$_2$. Furthermore, the interfacial energy between liquid steel and pure alumina has a value of 2.5 N/m.

In order to obtain an increased understanding of the effect of the interfacial energy on the drain sink formation height under varying outlet radius and liquid density conditions, calculations using Eq. (13) were done. In Fig. 7, predictions of the drain sink formation height as a function of the outlet radius and the interfacial energy are shown. As clearly can be seen, the calculations show that the drain sink formation height, at low interfacial energies, is a linear function of the outlet radius. This is in agreement with previous studies. However, this relationship is only true for low interfacial-tension values, as for water in contact with air. At high interfacial energies, the drain sink formation height increases over the outlet radius with a higher power than 1. It is also notable that the relative decrease of the drain-sink formation height, caused by an increased interfacial energy, decreases with an increased outlet size. This can be understood, since the surface forces carrying the under-pressure can only act along the borders of the outlet (you cannot lift yourself by pulling your own hair), which is proportional to $R$ while the pressure acts on the area of the outlet which increases with $R^2$.

In Fig. 8, the effect of the liquid density and the interfacial energy on the drain sink formation height is illustrated. As seen, the effect of density becomes fairly large when the interfacial energy is high. Since the kinetic energy at constant liquid velocity increases with the liquid density, surface forces must withstand a higher pressure when the density is high compared to when it is low. Thus, the drain sink will form earlier when the density is high.

Since the drain sink formation is directly dependent on the rate of discharge of the liquid, it might also be interesting to observe the effect of outlet length together with interfacial energy on the formation height. In Fig. 9, this is shown. As clearly can be seen, an increased outlet length results in an increased drain sink formation height. However, an increased outlet length also leads to a decreased effect of an increased interfacial energy on the drain sink formation height. This is because the longer outlet pipe can increase the discharge rate by the sucking effect to unlimited values, while the increase in fluid velocity through the outlet that can be obtained due to the effect of the interfacial energy is limited.

Finally, the analytical Eq. (13), has been applied for an industrial case. More specifically, the drain sink formation height as a function of outlet radius for two different outlet lengths, 0.0 m and 0.2 m respectively. Graphs are also shown for the corresponding drain sink formation height for a similar liquid, but with an interfacial energy that equals 0.

![Fig. 7. Analytical model predictions of the drain sink formation height as a function of the interfacial energy and the outlet radius. The density of the liquid and the outlet length are set to 1 000 kg/m$^3$ and 0 m, respectively.](image1)

![Fig. 8. Analytical model predictions of the drain sink formation height as a function of the interfacial energy and the density of the liquid. The outlet radius and the outlet length are set to 0.02 m and 7 000 kg/m$^3$, respectively.](image2)

![Fig. 9. Analytical model predictions of the drain sink formation height as a function of the interfacial energy and the outlet length. The outlet radius and the liquid density are set to 0.02 m and 7 000 kg/m$^3$, respectively.](image3)

![Fig. 10. Analytical model predictions of the drain sink formation height as a function of the outlet radius for two different outlet lengths, 0.0 m and 0.2 m respectively. Graphs are also shown for the corresponding drain sink formation height for a similar liquid, but with an interfacial energy that equals 0.](image4)
istic from a steel production point-of-view. In order to understand the impact of the interfacial energy on the drain sink formation height predictions for the same system, but for a liquid with zero interfacial energy, are also shown. As can be seen, the effect of the interfacial energy decreases with an increasing outlet size, but does still have an impact on the drain sink formation height of approximately −10% at realistic steel production dimensions. This result seems to be reasonable considering the dimensionless Weber number shown in Eq. (1). For flows with Weber numbers ≪1, the interfacial energy is of dominating importance. Some Weber numbers for the systems analyzed in the present paper are shown in Table 3. As can be seen, the Weber number was low (1.6) for a low interfacial energy in the system shown in Fig. 5 while it was higher (109.6) for a low value of the interfacial energy in the same system. For a steel production system, the Weber number was computed to be approximately 40, which is why some weak effect of the interfacial energy could be expected.

### 6. Conclusions

The drain sink formation height has been studied using physical and numerical modeling. In the same domain, the analytical and numerical model predictions were in good agreement with experimentally determined drain sink formation heights. Thereafter, the numerical model was used to predict the drain sink formation height at various interfacial energies. Moreover, a simple analytical model was derived which predicts the drain sink formation height as function of outlet radius, liquid density, outlet length and interfacial energy. The newly developed analytical model and the numerical simulations showed good agreement regarding the drain sink predictions. According to the simulations, the interfacial energy has a strong effect on the drain sink formation height. The analytical model yields that the effect of interfacial energy is largest for liquids with a low density and liquids teemed through narrow outlets. The effect decreases with an increasing liquid density and an increased outlet radius, but it does not disappear within the domain of reasonable values of the both. For steel and steel production outlet geometries, it was predicted that the effect of interfacial energy decreases the drain sink formation height by approximately 10%.

### Nomenclature

| Symbol | Definition |
|--------|------------|
| \( F_s \) | Surface force (N) |
| \( F_p \) | Pressure force (N) |
| \( R \) | Outlet radius (m) |
| \( \sigma \) | Interfacial energy (N m\(^{-1}\)) |
| \( \Delta P \) | Pressure difference (Pa) |
| \( g \) | Acceleration due to gravity (N kg\(^{-1}\)) |
| \( \rho \) | Density (kg m\(^{-3}\)) |
| \( H \) | Bath surface height (m) |
| \( h \) | Vertical distance (m) |
| \( v \) | Velocity (m s\(^{-1}\)) |
| \( \Phi \) | Transferred quantity (−) |
| \( \mu \) | Molecular viscosity (Pa s) |
| \( p \) | Pressure (N m\(^{-2}\)) |
| \( \beta \) | Axial comp. of \( g \) (N kg\(^{-1}\)) |
| \( \gamma \) | Radial comp. of \( g \) (N kg\(^{-1}\)) |
| \( \varepsilon \) | Turbulent dissipation (m\(^2\) s\(^{-1}\)) |
| \( \nu \) | Kinematic viscosity (m\(^2\) s\(^{-1}\)) |
| \( We \) | Weber number (−) |

### Table 3

Weber number calculated for various systems, the characteristic length is defined as the outlet radius.

| \( p \) [kg/m\(^3\)] | \( \Delta P \) [Pa] | \( H \) [m] | \( \sigma \) [N/m] | \( We \) [-] |
|-----------------|-----------------|----------|------------------|-------------|
| 1000            | 0.02            | 0.02     | 0.03             | 0.072       | 109.1      |
| 1000            | 0.02            | 0.008    | 0.40             | 2.000       | 1.6        |
| 7000            | 0.03            | 0.020    | 0.03             | 1.850       | 44.6       |

### REFERENCES

1. P. Andrzejewski, A. Diener and W. Plueckhll: Steel Res., 58 (1987), 547.
2. A. V. Kuklev, V. V. Tinyakov, Yu. M. Aizin, V. N. Gushchin and V. A. Ul’yanov: Metallurgist, 48 (2004), 207.
3. R. Sankaranarayanan and R. I. L. Guthrie: Ironmaking Steelmaking, 29 (2002), 147.
4. G. Mazzaferro, M. Piva, S. Ferro, P. Bissio, M. Iglesias, A. Calvo and M. Goldsmit: Ironmaking Steelmaking, 31 (2004), 503.
5. S. C. Koria and U. Kanth: Steel Res., 5 (1994), 8.
6. M. Dubke and K. Schweidtferger: Ironmaking Steelmaking, 17 (1990), 184.
7. P. Hammerschmid, K. H. Tacke, H. Popper, L. Weber, M. Dubke and K. Schweidtferger: Ironmaking Steelmaking, 11 (1984), 332.
8. R. Sankaranarayanan and R. Guthrie: Proc. to Int. Symp. on Developments in Ladle Steelmaking and Continuous Casting, CIM-MET Soc., Hamilton, Ontario, (1990), 66.
9. M. Nadif, J. Lehman, M. Burty and J. F. Domgin: Proc. of the 7th Int. Conf. on Clean Steel, OMBKE, Balatonfured, Hungary, (2007), 38.
10. B. Lubin and G. Springer: J. Fluid Mech., 29 (1967), 385.
11. L. Brinkmeyer and S. D. Melville: Ironmaking Steelmaking, 22 (1995), 45.
12. A. J. Fay: Introduction to Fluid Mechanics, MIT Press, Cambridge, Massachusetts, (1994), 17.
13. D. R. Poirier and G. H. Geiger: Transport Phenomena in Materials Processing, The Minerals, Metals & Materials Society, Warrendale, PA, (1994), 21.
14. T.-H. Shih, W. W. Liou, A. Shabir, Z. Yang and J. Zhu: Comput. Fluids, 24 (1995), 227.
15. C. W. Hirt and B. D. Nichols: J. Comput. Phys., 39 (1981), 201.
16. J. U. Brackbill, D. B. Kothe and C. Zemach: J. Comput. Phys., 100 (1992), 335.
17. D. R. Lide and H. V. Kehiaian: CRC Handbook of Thermophysical and Thermochemical Data, 1st ed., CRC Press, Boca Raton, FL, (1994), 203.
18. J. Lee, A. Kiyose, M. Tanaka and T. Tanaka: ISIJ Int., 46 (2006), 1810.