Glimpses of Flow Development and Degradation During Type B Drag Reduction by Aqueous Solutions of Polyacrylamide B1120

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Abstract Flow development and degradation during Type B turbulent drag reduction by 0.10 to 10 wppm solutions of a partially-hydrolysed polyacrylamide B1120 of MW = 18x10⁶ was studied in a smooth pipe of ID = 4.60 mm and L/D = 210 at Reynolds numbers from 10000 to 80000 and wall shear stresses Tw from 8 to 600 Pa. B1120 solutions exhibited facets of a Type B ladder, including segments roughly parallel to, but displaced upward from, the P-K line; those that attained asymptotic maximum drag reduction at low Re√fb but departed downwards into the polymeric regime at a higher retro-onset Re√f; and segments at MDR for all Re√f. Axial flow enhancement profiles of S′ vs L/D reflected a superposition of flow development and polymer degradation effects, the former increasing and the latter diminishing S′ with increasing distance downstream. Solutions that induced normalized flow enhancements S′/S′m < 0.4 developed akin to solvent, with L_e,p/D = L_e,n/D < 42.3, while those at maximum drag reduction showed entrance lengths L_e,m/D ~ 117, roughly 3 times the solvent L_e,n/D. Degradation kinetics were inferred by first detecting a falloff point (Re√f∧, S′∧), of maximum observed flow enhancement, for each polymer solution. A plot of S′∧ vs C revealed S′∧ linear in C at low C, with lower bound [S′] = 5.0 wppm⁻¹, and S′∧ independent of C at high C, with upper bound S′₀ = 15.9. The ratio S′/S′∧ in any pipe section was interpreted to be the undegraded fraction of original polymer therein. Semi-log plots of (S′/S′∧) at a section vs transit time from pipe entrance thereto revealed first order kinetics, from which apparent degradation rate constants k_deg s⁻¹ and entrance severities −ln(S′/S′∧)₀ were extracted. At constant C, k_deg increased linearly with increasing wall shear stress Tw, and at constant Tw, k_deg was independent of C, providing a B1120 degradation modulus (k_deg/Tw) = (0.012 ± 0.001) (Pa s)⁻¹ for 8 < Tw Pa < 600, 0.30 < C wppm < 10. Entrance severities were negligible below a threshold Tw<sub>e</sub> ~ 30 Pa and increased linearly with increasing Tw for Tw > Tw<sub>e</sub>. The foregoing methods were applied to Type A
drag reduction by 0.10 to 10 wppm solutions of a polyethyleneoxide PEO P309, MW = 11x10^6, in a smooth pipe of ID = 7.77 mm and L/D = 220 at Re from 4000 to 115000. P309 solutions that induced S'/S'_m < 0.4 developed akin to solvent, with L_e,p/D = L_e,n/D < 23, while those at MDR had entrance lengths L_e,m/D ~ 93, roughly 4 times the solvent L_e,n/D. P309 solutions described a Type A fan distorted by polymer degradation. A typical trajectory departed the P-K line at an onset point Re√f* followed by ascending and descending polymeric regime segments separated by a falloff point Re√f^∧, of maximum flow enhancement; for all P309 solutions, onset Re√f* = 550 ± 100 and falloff Re√f^∧ = 2550 ± 250, the interval between them delineating Type A drag reduction unaffected by degradation. A plot of falloff S'^∧ vs C for PEO P309 solutions bore a striking resemblance to the analogous S'^∧ vs C plot for solutions of PAMH B1120, indicating that the initial Type A drag reduction by P309 after onset at Re√f* had evolved to Type B drag reduction by falloff at Re√f^∧. Presuming that Type B behaviour persisted past falloff permitted inference of P309 degradation kinetics; k_deg was found to increase linearly with increasing Tw at constant C and was independent of C at constant Tw, providing a P309 degradation modulus (k_deg/Tw) = (0.011 ± 0.002) (Pa s)^{-1} for 4 < Tw Pa < 400, 0.10 < C wppm < 5.0. Comparisons between the present degradation kinetics and previous literature showed (k_deg/Tw) data from laboratory pipes of D ~ 0.01 m to lie on a simple extension of (k_deg/Tw) data from pipelines of D ~ 0.1 m and 1.0 m, along a power-law relation (k_deg/Tw) = 10^{-5.4}.D^{-1.6}. Intrinsic slips derived from PAMH B1120 and PEO P309-at-falloff experiments were compared with previous examples from Type B drag reduction by polymers with vinylic and glycosidic backbones, showing: (i) For a given polymer, [S'] was independent of Re√f and pipe ID, implying insensitivity to both micro- and macro-scales of turbulence; and (ii) [S'] increased linearly with increasing polymer chain contour length Lc, the proportionality constant β = 0.053 ± 0.036 enabling estimation of flow enhancement S' = C.Lc.β for all Type B drag reduction by polymers.

**Keywords** Drag reduction · Turbulent flow · Polyelectrolyte solutions · Polymer degradation

1 Introduction

Glimpses of flow development and degradation were obtained during Type B turbulent drag reduction by dilute deionized water solutions of a partially-hydrolysed high molecular weight polyacrylamide B1120 in a smooth pipe of inside diameter D = 4.60 mm and normalized length L/D = 210. The experiments provided new estimates of the entrance length associated with turbulent flow at the asymptotic maximum drag reduction and of the apparent kinetic rate constants for B1120 degradation during less than maximum, but nevertheless considerable, turbulent drag reduction in the polymeric regime.

By way of background, turbulent drag reduction by polymer solutions comprizes two additive-insensitive flow regimes [1], respectively the Prandtl-Karman (P-K) law:

\[
\frac{1}{\sqrt{f}} = 4.0 \log \text{Re}\sqrt{f} - 0.4
\]  

(1)

And the maximum drag reduction (MDR) asymptote:

\[
\frac{1}{\sqrt{f}} = 19.0 \log \text{Re}\sqrt{f} - 32.4
\]  

(2)
These envelop a third, “polymeric”, regime wherein additive properties matter. Within the polymeric regime there occur two extremes of flow behaviour, termed Types A and B [2], respectively exhibited by initially randomly-coiled macromolecules, such as polyethylene-oxides (PEO) in deionized water (DW), and by extended polyelectrolytes, such as the present partially hydrolysed polyacrylamide (PAMH), B1120 in DW. On P-K coordinates, using subscripts p and n to respectively denote polymer solution and solvent, Type A behaviour yields a “fan” of friction factor segments radiating outwards from a common “onset” point on Eq. 1, their slopes increasing with increasing concentration C so that flow enhancement (informally called “slip”), \( S' = (1/\sqrt{f_p} - 1/\sqrt{f_n})Re\sqrt{f} \), increases with increasing C and also with increasing \( Re\sqrt{f} \). Type B behavior yields a “ladder” of segments, a typical rung of given C departing from Eq. 2 at a “retro-onset” point, then lying roughly parallel to but displaced upwards from Eq. 1, the more so with increasing C, so flow enhancement increases with increasing C but is independent of \( Re\sqrt{f} \).

Early experiments [3–6] had revealed that drag reduction in a pipe decreased with distance downstream, on account of polymer degradation, by midpoint-scission [7]; this was aptly termed “falloff” by Hoyt [3]. These studies also revealed that the pipe entrance influenced drag reduction, for example: (i) inserting an entrance orifice of half the pipe diameter, to trigger transition [4], did not affect the onset of drag reduction by several aqueous PEO solutions in a 2.92 mm ID \( \times \) 250 L/D pipe but diminished drag reduction by the solutions at flowrates 10 times those at onset and (ii) in the flow of a 50 wppm solution of a PEO of \( MW = 1.0 \times 10^6 \) [5, Fig. 8], the smooth nozzle and square cut-off pipe inlets on a 6.30 mm ID \( \times \) 1700 L/D pipe exhibited identical fractional drag reductions increasing from 0.46 to 0.70 as \( Re \) increased from 4000 to 20000, but by \( Re = 50000 \) these were 0.75 and 0.67, respectively, the latter inlet causing a bit more degradation than the former. These early works established that polymer drag reduction is always accompanied by polymer degradation, and vitiated by it, to an extent governed by degradation kinetics. All experimental studies of drag reduction thus implicitly incorporate the degradation severities induced by the flow apparatus, with severity meant in the chemical reactor sense of \( -\ln(C/C_0) \), where \( (C/C_0) \) is the fraction of original polymer remaining.

In the present work, flow enhancement by B1120 was observed to both decrease and increase with increasing distance downstream. The former was attributed to polymer degradation and used to infer its severity and apparent kinetics while the latter was attributed to flow development and used to estimate entrance lengths \( L_{e,m}/D \) at MDR, as previously attempted in [8] for drag reduction by PEOs. No previous experimental data could be found for flow development at MDR. However, DNS of a channel flow at MDR, performed by Min, Choi and Yoo 2003 [9], predicted \( L_{e,m}/D = 400 \) for \( Re = 3000 \). This simulation, using a fully-developed turbulent flow field of Newtonian fluid as an initial condition and an Oldroyd-B model to express polymer solution stress, evinces two reservations. First, that there is yet no experimental method to discern, let alone verify, how or even if stress is “carried” by polymers in a drag-reduced turbulent flow. And second that the numerical scheme implicitly requires macromolecules to alternate between their coiled and stretched conformations, in order to exchange strain energy with the turbulent flow, but this cannot occur during Type B drag reduction by polyelectrolytic macromolecules that are exclusively in their stretched conformations, and locked therein by electrostatic forces unrelated to the flow. Reservations notwithstanding, it is noteworthy that these authors were able to estimate \( L_{e,m}/D \) by DNS before experimental data were available; and further that their DNS at MDR [9], and at lower DR but higher \( Re = 15000 \) [10], also provided mean velocity and rms axial turbulent intensity profiles in broad accord with long-known, experimentally-established, features of these profiles during drag reduction [11].
The sections that follow concern experimental details, results and analysis of friction factor data for solvent and polymer solutions, discussion of the present work with respect to previous literature, and an itemized summary.

2 Experimental

The test pipe was assembled from 7 identical segments of electro-polished stainless steel (EPSS) tubing, each of $D = 4.60$ mm and $L/D = 30$, with a radial pressure tap of 0.40 mm ID at $L/D = 27.5$, and bore average roughness $k = 0.00018$ mm (relative roughness $D/2k = 12800$). The assembled pipe had a square cut-off entrance, set as axial origin $X = 0$, and six pressure tap pairs j&k (1&2, 2&3, 3&4, 4&5, 5&6, 6&7) at average normalized downstream distances $L/D$ (42.3, 72.1, 101.9, 131.7, 161.5, 191.3). Flow entered the test pipe through a “trigger” ball valve, orifice ID 7.1 mm, kept fully-open (Trigger = 0°), that provided enough disturbance to ensure fully turbulent flow at $Re \sim 4000$.

The test pipe with trigger was placed in a single-pass progressive cavity pump-driven flow system, described in [12], fed from a 220 liter run tank that held either the deionized water (DW) solvent or polymer solution, maintained at temperature $T = 22.0 \pm 1.0$ C. Pipe Reynolds numbers $Re = DU/\nu_n$, formed with solvent kinematic viscosity, were varied from 10000 to 80000 and wall shear stresses $Tw$ from 8 to 600 Pa. The polymer, PAMH B1120, $MW = 18.3 \times 10^6$, was dissolved in DW to provide dilute solutions of concentrations $C$ from 0.10 to 10 weight parts per million (wppm) by a two-stage process. As-received polymer powder was first converted to a (typically) 5000 wppm “master batch” in a baffled 10 liter stirred tank following the manufacturers’ recommendations. An appropriate volume of master batch was then added to the 220 liter run tank with momentarily vigorous stirring, for dispersal, then gentle stirring, for dissolution, for (typically) 15 min, followed by an unstirred holding period, for full solvation, of (typically) 60 min prior to use. Selected polymer concentrations were run in duplicate and these results commonly agreed to within system experimental errors, 1% in flowrate and 2% in pressure gradient, attesting to the overall reproducibility of polymer makeup procedures. Flow enhancements observed with the polymer solutions in the test pipe segments varied over the entire possible range $0 < S' < S'_m$, from $S' \sim 0$, near the P-K law Eq. 1, for the lowest $C = 0.10$ wppm to $S' = 15.9$, near the MDR asymptote Eq. 2, attained by $C = 10$ wppm.

3 Results and Analysis

3.1 Solvent

Figure 1 shows friction factors from two DW solvent runs that bracketed all the polymer solution runs.

At any fixed turbulent flowrate, DW friction factors (symbols) were constant to within 1.1% for all tap pairs, that is, solvent flows were fully developed, with “entrance length” $Le/D < 42.3$. This accords with entrance lengths $Le,n/D \sim 25$ to 40 cited for turbulent Newtonian pipe flow [13]. Too, friction factors for DW in all pipe segments adhered to the Prandtl-Karman law within $\pm 0.2\ 1/\sqrt{f}$ units, that is, all pipe segments were hydrodynamically smooth. Finally, the DW data were regressed to provide P-K relations for each tap pair, shown by light lines in Fig. 1, that served as “baselines” for accurate extraction
Fig. 1 Solvent DW friction factors for individual tap pairs (symbols) with regressions (light lines). Pipe ID = 4.60 mm. P-K, MDR (heavy lines) denote Eqs. 1 and 2 of polymer-induced flow enhancement in each pipe segment at any flowrate and polymer concentration.

3.2 Polymer solutions

3.2.1 Flow development

Figure 2 depicts axial flow enhancement profiles, $S'$ vs $L/D$ for B1120 solutions at $Re = 42300$, which exceeded the fall off $Re^* \sim 10000$ beyond which degradation became significant.

The lowest $C = 0.10$ wppm exhibited $S' = 0.1 \pm 0.1$, below detection limits. The next two $C = (0.30, 1.04)$ wppm provide distinct $S' = (0.4 \pm 0.1, 1.9 \pm 0.1)$; $S'$ increased with increasing $C$ but decreased slightly with increasing $L/D$ in each case, the latter decreases likely reflecting polymer degradation. The two $C = (3.23, 3.03)$ wppm show $S' = (7.5, 6.8)$ at $L/D = 42$ increasing to $S' = (8.0, 7.4)$ at $L/D = 72$ and then decreasing monotonically to $S' = (7.2, 6.5)$ at $L/D = 191$; this pattern likely reflects flow development for $42 < L/D < 72$ and polymer degradation for $72 < L/D < 191$. Finally, for $C = 9.97$ wppm, $S' = 10.8$ at $L/D = 42$ increased monotonically to $S' = (15.2, 15.9, 16.0)$ at $L/D = (132, 162, 191)$, attaining the maximum $S'_{m} \sim 15.9$ at entrance length $L_{e,m}/D \sim 132$.

Flow enhancement profiles for all $C = (0.10, 0.30, 1.10, 3.03, 3.23, 9.97)$ wppm at all $Re = (9900, 21000, 42300, 82700)$ revealed that:

(i) In the polymeric regime there is a superposition of flow development and polymer degradation effects, the former increasing and the latter diminishing $S'$ with increasing distance downstream. Solutions that induced normalized flow enhancements
Fig. 2  Axial flow enhancement profiles at Re = 42300. Pipe ID = 4.60 mm; Polymer = B1120, C = 0.10 to 10 wppm; $S'_m = 15.9$ is the asymptotic maximum flow enhancement

$S'/S'_m < 0.4$ developed akin to solvent, with $L_{c,p}/D = L_{c,n}/D < 42.3$, while those that induced $S'/S'_m > 0.4$ took longer to develop, with $L_{c,p}/D > L_{c,n}/D$.

(ii) In the maximum drag reduction regime, four instances of axial flow development, at (C wppm, Re) = (3.03, 10300), (9.97, 10200, 21800, 44200) are shown in Fig. 3, using Cartesian coordinates of normalized flow enhancement $S'/S'_m$ vs. L/D. These data provide average $S'/S'_m = (0.73, 0.91, 0.92, 0.98, 1.00, 1.02)$ at L/D = (42, 72, 102, 132, 162, 191). If $S'/S'_m \sim 0.95$ is taken as a criterion for development, then the entrance length at MDR was $L_{c,m}/D \sim 117$.

3.2.2 Polymer degradation

Figure 4 shows further facets of B1120 flow development at MDR and degradation in the polymeric regime.

For DW (black), at any fixed Re$/\sqrt{f}$, the individual friction factors from all tap pairs were virtually identical. Too, all DW friction factors can be seen to adhere closely to the P-K law for $800 < Re$/\sqrt{f} < 6000$. Data for C = 1.04 wppm (red) provide a glimpse of Type B behavior at the two lowest Re$/\sqrt{f} \sim 630$ and 1260 which lie roughly parallel to the P-K line but displaced upwards by $S' \sim 4.2$ 1/$\sqrt{f}$ units; at the two highest Re$/\sqrt{f} \sim 2640$ and 5120, $S'$ falls off, to 2.0 then 0.9. For C = 3.03 wppm (green) at the lowest Re$/\sqrt{f} \sim 540$, downstream tap pairs 3&4 to 6&7 (L/D 102 to 191) cluster on the MDR line while upstream tap pairs 2&3 and 1&2 lie respectively $\sim 0.5$ and $\sim 2.0$ 1/$\sqrt{f}$ units below. The next highest Re$/\sqrt{f} \sim 1000$ illustrates retro-onset; downstream tap pairs 2&3 to 6&7 remain clustered with $S' \sim 10$, but depart from MDR into the polymeric regime, with the upstream pair 1&2 far below. At the two highest Re$/\sqrt{f} \sim 2190$ and 5050, $S'$ falls off precipitously for all tap pairs, to $\sim 7$ and then $\sim 2$. 
Fig. 3  Normalized axial flow enhancement profiles at MDR. Pipe ID = 4.60 mm; Polymer = B1120, C wppm = 3.03, 9.97

Fig. 4  Polymer solution friction factors for individual tap pairs (symbols). Pipe ID = 4.60 mm; Polymer = B1120, C wppm = 0.0 (DW), 1.04 (red), 3.03 (green). P-K, MDR denote Eqs. 1 and 2
Define a “falloff point” as the one with the highest flow enhancement $S'/$ experimentally observed for a given polymer solution in any section of the test pipe. Figure 4 then provides a pair of falloff points, $(Re, f^\wedge, S^\wedge) = (620, 4.8)$ for $C = 1.04$ wppm and $(Re, f^\wedge, S^\wedge) = (980, 10.5)$ for $C = 3.03$ wppm. Falloff flow enhancements found in this manner for all B1120 solutions are plotted in Fig. 5.

At low concentrations, $C = 0.3$ and 1.0 wppm, $S'/$ is seen to increase linearly with increasing $C$, with lower bound, called the intrinsic slip, $[S''] = \lim_{C \to 0} (S'/C) = 5.0$ wppm$^{-1}$, whereas at high concentration, $C = 10$ wppm, $S'/$ becomes essentially independent of $C$, attaining a maximum asymptotic upper bound of $S'_m = 15.9$. The foregoing $S'/$ versus $C$ data for B1120 in a 4.60 mm pipe accord with previous results [14] for Type B drag reduction by B1120 in a 10.9 mm pipe which showed $S'$ to increase linearly with increasing $C$ at low concentrations from 0.3 to 6.6 wppm, with intrinsic slip $[S'] = 1.80$ wppm$^{-1}$. If $S'$ varies linearly with $C$, and falloff slip $S'/$ is associated with the original concentration $C$ of a polymer solution, then, at any flowrate in any pipe section, it follows that the ratio of observed to falloff slips, $S'/S'/$, must physically represent the fraction of original polymer yet undegraded. Too, the transit time from pipe entrance to any section $j&k$ is $t_{res} = D(L/D)_{j&k}/U$. A semi-log plot of $(S'/S'\wedge)$ vs $t_{res}$ for a single concentration $C = 1.04$ wppm at Re from 9600 to 78500 is presented in Fig. 6.

At each Re, degradation severity $\ln(S'/S'\wedge)$ is seen to be linear in $t_{res}$, implying first order kinetics according to the relation:

$$-\ln(S'/S'\wedge) = -\ln(S'/S'\wedge)_0 + k_{deg} \cdot t_{res}$$

Where $-\ln(S'/S'\wedge)_0$ is an “entrance severity”, reflecting initial degradation at the pipe entrance, and the best-fit slope $k_{deg}$ s$^{-1}$ is an apparent first order degradation rate constant, that increases visibly with increasing Re. Degradation kinetics seen in Fig. 6 are summarized in Table 1 at each Re, showing the average wall shear stress $T_w$ Pa as a measure of

![Fig. 5 Falloff Flow Enhancements for PAMH B1120. Bounding relations are: Intrinsic Slip $[S'] = \lim_{C \to 0} (S'/C) = 5.0$ and $S'_m = 15.9$](image-url)
turbulent flow strength, the apparent rate constant $k_{\text{deg}}$ s$^{-1}$, the fraction of polymer already degraded on arrival at the pipe entrance ($S'/S'^\wedge$)$_0$, and the fraction finally degraded at the pipe exit, ($S'/S'^\wedge$)$_L$. The difference between the latter two being the fraction degraded in the pipe itself. It is noteworthy that at the lowest Re = 9600 the polymer entered the pipe essentially undegraded and suffered $\sim$ 0.05 fractional degradation in the pipe; however, at all higher Re, the fractional degradation prior to entry far exceeded that in the pipe, being respectively 0.15 vs 0.07 at Re = 20000, 0.56 vs 0.11 at Re = 40000 and 0.84 vs 0.04 at Re = 78500.

Table 1 suggested that the rate constant $k_{\text{deg}}$ increased roughly linearly with increasing wall shear stress $T_w$, and with this guidance, Fig. 7, a log-log plot of $k_{\text{deg}}$ vs $T_w$ from all experiments, revealed that $k_{\text{deg}}$ increased linearly with increasing $T_w$ at any fixed concentration C from 0.30 to 10 wppm, and was essentially independent of C at any fixed $T_w$ from 8 to 600 Pa, the latter invariance a further indication of first order kinetics.

An approximate dimensional expression for B1120 degradation kinetics, the dashed line in Fig. 7, is:

$$k_{\text{deg}} = (0.012 \pm 0.001) T_w; \ 8 < T_w \text{ Pa} < 600; \ 0.30 < C \text{ wppm} < 10$$

Table 1  Apparent degradation kinetics of C = 1.04 wppm B1120 in a 4.60 mm ID $\times$ 210 L/D smooth pipe

| Re  | 9600 | 20000 | 40000 | 78500 |
|-----|------|-------|-------|-------|
| $T_w$ [Pa] | 9.0  | 36    | 160   | 590   |
| $k_{\text{deg}}$ [s$^{-1}$] | 0.15 ± 0.13 | 0.38 ± 0.17 | 2.6 ± 0.1 | 5.4 ± 1.2 |
| ($S'/S'^\wedge$)$_0$ | 1.01 ± 0.04 | 0.85 ± 0.02 | 0.44 ± 0.01 | 0.16 ± 0.01 |
| ($S'/S'^\wedge$)$_L$ | 0.95  | 0.78  | 0.33  | 0.12  |
Also, as seen in Fig. 6 and Table 1, the entrance severity \(-\ln(S'/S'^{\wedge})_0\) broadly increased with increasing wall shear stress \(T_w\). More detailed scrutiny showed that the entrance severity was essentially negligible below a threshold \(T_{we} = 30\) Pa and increased roughly linearly with increasing \(T_w\) above it:

\[
-\ln(S'/S'^{\wedge})_0 = 0; \quad T_w < T_{we} = 30\; Pa
\]  

\[
-\ln(S'/S'^{\wedge})_0 = (0.0031 \pm 0.0003) \cdot (T_w - T_{we}); \quad T_{we} < T_w < 600\; Pa
\]  

Equations 4 and 5a and b summarize the present B1120 degradation data in a 4.60 mm ID x 210 L/D smooth pipe at \(Re\sqrt{f}\) from 600 to 6000.

4 Discussion

Discussion is divided into two sections, comparisons with previous work and aspects of Type B drag reduction. The first section attempts to compare present results with previous literature, specifically studies of flow development and degradation during Type A drag reduction by solutions of PEO in DW [8]; Paterson’s [5] study of PEO degradation; and the works of Horn [6] and of Motier [15] on degradation of a poly-alpha-olefin co-polymer (POAC) in Alaska and North Sea crude oils. The second section considers Type B physics, specifically the disposition of macromolecular chains within the pipe and the observed insensitivity of Type B drag reduction to turbulence macro- and micro-scales; correlation of Type B intrinsic slips to polymeric parameters; and facets of the drag reduction mechanism revealed by the present experiments.
4.1 Comparisons with previous work

4.1.1 Flow development

Figures 8 and 9 depict axial flow enhancement profiles [8] for Type A drag reduction by DW solutions of a PEO, P309, MW = 11.4 × 10^6, C = 0.1 to 10.0 wppm, in a smooth pipe of 7.77 mm ID × 220 L/D × 21900 D/2k (relative roughness) at each of two fixed flowrates, Re = 23000 and 91000, respectively below and above the falloff point, Re^∧∼43000, beyond which degradation became significant for these polymer solutions.

In Fig. 8, Re = 23000 < Re^∧, the three lowest C = 0.1, 0.2 and 0.5 wppm each exhibit S′ independent of L/D but increasing with increasing C, and this pattern persists, approximately, for the middle C = 1 and 2 wppm too. However, for the two highest C = 5 and 10 wppm, S′ increased with increasing L/D at low L/D < 100 and then became roughly constant for L/D > 100. For C = 10 wppm, specifically, S′ = 8 at L/D = 23, increased monotonically to S′ = 12 at L/D = 93 and then remained constant at S′ = 12 ~ S′_m for 93 < L/D < 199. Flow development was thus dependent upon drag reduction; solutions that induced S′ < 5 required L/D < 22, akin to solvent, while those inducing S′ > 5 took longer to develop, with one attaining S′_m ~ 12 requiring L/D ~ 93.

In Fig. 9, Re = 91000 > Re^∧, the three lowest C = 0.1, 0.2, 0.5 wppm and the two middle C = 1, 2 wppm each exhibit local S′ decreasing perceptibly and monotonically with increasing L/D, while the average S′ increases with increasing C. As examples, C = 0.5 wppm shows S′ = 1.9 at L/D = 23 decreasing to S′ = 1.3 at L/D = 199, while at the middling L/D = 93, S′ = 0.2 for C = 0.1 wppm increases monotonically to S′ = 5.8 for C = 2 wppm. For the two highest C = 5 and 10 wppm, S′ increased with increasing L/D past the lowest L/D = 23, attained a broad maximum, and then decreased with increasing L/D towards the highest L/D = 199. For C = 10 wppm, specifically, S′ = 11.4 at L/D =

![Figure 8](image-url) Axial flow enhancement profiles at Re = 22800. Pipe ID = 7.77 mm; Polymer = P309, C = 0.10 to 10 wppm; S′_m = 12.8 is the asymptotic maximum flow enhancement

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Fig. 9 Axial flow enhancement profiles at Re = 91300. Pipe ID = 7.77 mm; Polymer = P309, C = 0.10 to 10 wppm; S'_m = 19.6 is the asymptotic maximum flow enhancement

23, attained its highest S' = 17.0 at L/D = 164, and then decreased slightly to S' = 16.6 at L/D = 199, with all S' < S'_m = 19.6. Flow development was still dependent upon drag reduction, with solutions that induced S' < 8 developing akin to solvent, and those that induced S' > 8 taking longer, but in all cases polymer degradation appreciably diminished S', the more so further downstream.

Evident similarities between Figs. 8 and 9, for Type A drag reduction by PEO solutions, and Fig. 2, for Type B drag reduction by PAMH solutions, show that, irrespective of the polymer additive and the gross flow behavior it induces, axial flow enhancement profiles always reflect a superposition of flow development and polymer degradation effects that respectively increase and diminish S' with increasing distance downstream. Joint consideration of all PAMH B1120 and PEO P309 data provide the following summary:

(i) In the polymeric regime, solutions that induced normalized flow enhancements S'/S'_m < 0.4 developed akin to solvent, with L_e,p/D = L_e,n/D, while those that induced S'/S'_m > 0.4 took longer to develop, with further approximate L_e,p/D ~ 75 for S'/S'_m ~ 0.6.

(ii) In the maximum drag reduction regime, six instances of axial flow development for C = 10 wppm P309 at Re√f = 800 to 2500 (Re = 14000 to 80000) are shown in Fig. 10, on Cartesian coordinates of normalized flow enhancement S'/S'_m vs. L/D; these provide an entrance length at MDR of L_e,m/D ~ 93.

Additional MDR results from [8] (not shown) for 200 and 500 wppm DW solutions of another PEO, N60K, MW = 3.2 × 10^6, in an EPSS pipe of ID = 4.58 mm, L/D = 180, D/2k = 12900, 6 segments, at Re from 10000 to 80000 provided L_e,m/D ~ 102. As summarized in Table 2, MDR asymptotes attained experimentally using 2 pipe diameters, 2 species and 3 grades of polymers, with MW from 3.2 × 10^6 to 18.3 × 10^6 and C from 3 to 500 wppm, all yielded the same entrance lengths L_e,m/D = 105 ± 15, about 3 times the Newtonian L_e,n/D ~ 25 to 40. The last row of Table 2 shows that Min, Choi and Yoo’s [9] DNS of a
channel flow at $Re = 3000$ predicted $L_{e,m}/D = 400$, roughly four-fold higher than observed in the present pipe flows at $Re$ from 10000 to 80000. The DNS study provided no “reference” Newtonian entrance length $L_{e,n}/D$, precluding comparisons with the experimentally observed ratio $(L_{e,m}/D)/(L_{e,n}/D) \sim 3.5$.

### 4.1.2 Polymer degradation

Figure 11 shows further facets of PEO P309 flow development at MDR and degradation in the polymeric regime.

For DW (black), at any fixed $Re \sqrt{f}$, the individual friction factors from all tap pairs were virtually identical and all friction factors can be seen to adhere closely to the P-K law for $300 < Re \sqrt{f} < 10000$. Friction factors for $C = 10$ wppm P309 (green) show a well-defined onset of drag reduction at $Re \sqrt{f^*} \sim 400$, a steep polymeric regime Type A segment.

### Table 2 Entrance lengths at the asymptotic maximum drag reduction in pipes

| Pipe ID | Polymer/Grade | MW | C  | Re     | $L_{e}/D$ | Source              |
|---------|---------------|----|----|--------|----------|---------------------|
| 0.460   | ×10^{-6}      | 0  | 10 | 10000 - 80000 | < 42 | This work          |
| 0.460   | PAMH/B1120    | 18.3 | 3  | 10000 - 44000 | 117  | Virk & Shields [8] |
| 0.777   | 0 (DW)        | 4000 - 115000 | < 23 | Virk & Shields [8] |
| 0.777   | PEO/P309      | 11.4 | 10 | 14000 - 80000 | 93   | Min, Choi & Yoo [9] |
| 0.458   | PEO/N60K      | 3.2 | 200, 500 | 10000 - 80000 | 102  | Min, Choi & Yoo [9] |
| DNS     | “at MDR”      | 3000 | 400 | 30000 | 400 | Min, Choi & Yoo [9] |
400 < Re/ν < 800 that ends on the MDR asymptote, Eq. 2, which is then followed for 800 < Re/ν < 2500 as described in the preceding section. Data for C = 1 wppm P309 (red), show onset at Re/ν* ∼ 600 off the P-K line and into the polymeric regime along a trajectory that resembles a fountain, with an initial “jet” segment 1250 < Re/ν < 2500, wherein tap pairs cluster closely together while flow enhancement increases with increasing Re/ν, followed by a “spray” segment 2500 < Re/ν < 7000, wherein tap pairs separate, upstream higher than downstream, their separation increasing with increasing Re/ν while flow enhancement decreases with increasing Re/ν. The preceding define a characteristic “falloff point”, (Re/ν∧, S′∧) = (2500, 4.3), of maximum flow enhancement, beyond which degradation becomes significant. Results from all solutions provided onset Re/ν* = 550 ± 100 and falloff Re/ν∧ = 2550 ± 250; the interval between these represent a roughly 20-fold range of wall shear stress within which P309 solutions induce “pure” Type A drag reduction, little affected by polymer degradation.

Falloff flow enhancements found for all P309 solutions are plotted in Fig. 12.

At low concentrations, 0.1 < C < 1.0, S′∧ increased linearly with increasing C, with lower bound, intrinsic slip, [S′] = LimC→0(S′∧/C) = 5.1 wppm⁻¹, whereas at high concentration, C = 10 wppm, S′∧ became essentially independent of C, approaching the maximum asymptotic upper bound of S′ = 19.1. The S′∧ versus C data for PEO P309 in a 7.77 mm pipe bear a striking resemblance to the falloff S′∧ vs C results in Fig. 5 for Type B drag reduction by PAMH B1120 in a 4.60 mm pipe as well as to earlier S′ vs C data [14] for Type B drag reduction by B1120 in a 10.9 mm pipe. Further noteworthy is that the intrinsic slip [S′] = 5.1 wppm⁻¹ for P309 in Fig. 12 is of roughly the magnitude of [S′] = 5.0 wppm⁻¹ found for B1120, that possesses a similar carbon-to-carbon bonded backbone with roughly the same number of links N ∼ 0.5 × 10⁶. Falloff flow enhancements thus reveal that the initial Type A drag reduction by PEO P309 after onset at Re/ν* evolved
Fig. 12 Falloff Flow Enhancements for PEO P309. Bounding relations are: Intrinsic Slip $[S'] = \lim_{C \to 0} (S'/C) = 5.1 \text{ wppm}^{-1}$ and $S'_m = 19.1$

Presuming that Type B behaviour persists past falloff permits the inference of P309 degradation kinetics as earlier done for B1120, invoking the linear variation of $S'$ vs $C$ at low $C$, and associating falloff slip $S'^\wedge$ with the original polymer concentration $C$, such that, at any flowrate in any pipe section, the ratio of observed to falloff slips, $S'/S'^\wedge$, physically represents the fraction of original polymer yet undegraded.

Figure 13 is a semi-log plot of $S'/S'^\wedge$ vs $t_{res}$, transit time, for a single concentration $C = 1.0 \text{ wppm}$ of P309 at $Re$ from 45900 to 115000. At each $Re$, degradation severity $-\ln(S'/S'^\wedge)$ is seen to be linear in $t_{res}$, implying first order kinetics according to Eq. 3, with best-fit slope $k_{deg} \text{ s}^{-1}$, the apparent degradation rate constant, increasing with increasing $Re$, and entrance severity $-\ln(S'/S'^\wedge)_0 \sim 0$ for $Re \leq 56900$ but increasing with increasing $Re$ for $Re \geq 73300$. Degradation kinetics for 1.0 wppm P309 seen in Fig. 13 are summarized in Table 3 in a manner analogous to that whereby the degradation of 1.04 wppm B1120 shown in Fig. 6 was summarized in Table 1. Comparisons between Table 3 and Table 1 show that for both 1.0 wppm P309 in a 7.77 mm pipe and 1.04 wppm B1120 in a 4.60 mm pipe, the rate constant $k_{deg}$ increased roughly linearly with increasing wall shear stress $Tw$, with similar magnitudes $k_{deg} \sim 1 \text{ s}^{-1}$ at fixed flow strength $Tw \sim 100 \text{ Pa}$. However, at roughly equal flow strengths, $Tw \sim 160 – 190 \text{ Pa}$, the fractional polymer degradation prior to entry versus that in the pipe itself differed greatly, being respectively 0.18 vs 0.22 for P309 at $Re = 91500$ and 0.56 vs 0.11 for B1120 $Re = 40000$; that is, entrance degradation for B1120 in the 4.60 mm pipe far exceeded that for P309 in the 7.77 mm pipe, even as fractional degradations in the pipes themselves were of similar magnitudes.

Figure 14 is a log-log plot of $k_{deg}$ vs $Tw$ from all P309 experiments, showing that $k_{deg}$ increased linearly with increasing $Tw$ at any fixed concentration $C$ from 0.10 to 5.0 wppm,
and was essentially independent of C at any fixed Tw from 4 to 400 Pa, the latter invariance a further indication of first order kinetics.

An approximate dimensional expression for P309 degradation kinetics, denoted by the dashed line in Fig. 14, is:

$$k_{deg} = (0.011 \pm 0.002) \cdot T_w; 4 < T_w \text{ Pa} < 400; 0.10 < C \text{ wppm} < 5.0$$

(6)

Also, from Fig. 13 and Table 3, the entrance severity \(-\ln(S'/S'^\wedge_0)\) broadly increased with increasing wall shear stress Tw, with more detailed scrutiny showing entrance severity

| Re   | 45900 | 56900 | 73300 | 91500 | 115000 |
|------|-------|-------|-------|-------|--------|
| Tw [Pa] | 48.2  | 71.9  | 120   | 191   | 314    |
| $k_{deg}$ [s$^{-1}$] | 0.48 ± 0.08 | 0.80 ± 0.06 | 1.39 ± 0.11 | 2.15 ± 0.23 | 3.11 ± 0.33 |
| $(S'/S'^\wedge)_0$ | 1.03 ± 0.02 | 1.04 ± 0.01 | 0.96 ± 0.01 | 0.82 ± 0.02 | 0.63 ± 0.02 |
| $(S'/S'^\wedge)_L$ | 0.90  | 0.86  | 0.74  | 0.60  | 0.44   |

Note: A physical $(S'/S'^\wedge)_0 > 1.00$ (slightly) at $Re = 45900$ and 56900 in Table 3 and in Fig. 13 is not experimental error.

It reflects an imperfect first approximation of falloff slip $S'^\wedge = 4.32$ from raw data in Fig. 11.

A second approximation (not used) of $S'^\wedge = 4.47$ would drive $(S'/S'^\wedge)_0 \to 1.00$ at both $Re = 45900$ and 56900.
Fig. 14  Apparent degradation kinetics of PEO P309. Pipe ID = 7.77 mm, Re 18000 – 115000; dashed line represents Eq. 6

essentially negligible below a threshold $T_{we} = 80$ Pa and increasing roughly linearly with increasing $T_w$ above it:

$$-\ln\left(\frac{S'}{S'^\wedge}\right)_0 = 0; \ T_w < T_{we} = 80 \ Pa$$ (7a)

$$-\ln\left(\frac{S'}{S'^\wedge}\right)_0 = (0.0022 \pm 0.0002) \ (T_w - T_{we}); \ T_{we} < T_w < 400 \ Pa$$ (7b)

Equations 6 and 7a and b summarize PEO P309 degradation data in a 7.77 mm ID $\times$ 220 L/D smooth pipe at $Re_{\sqrt{f}}$ from 800 to 8000 in a manner analogous to Eqs. 4 and 5a and b for B1120 degradation in a 4.60 mm ID $\times$ 210 L/D smooth pipe at $Re_{\sqrt{f}}$ from 600 to 6000. Degradation rate constants $k_{deg}$ for both polymers increase linearly with increasing wall shear stress $T_w$ and exhibit similar magnitudes of $(k_{deg}/T_w) = (0.012 \pm 0.001) \ (Pa \ s)^{-1}$ for B1120 and $(0.011 \pm 0.002) \ (Pa \ s)^{-1}$ for P309. Entrance severities for both polymers are negligible below a threshold $T_{we}$ and increase roughly linearly with increasing $T_w$ above $T_{we}$; however, the respective magnitudes of $T_{we} = 30$ and 80 Pa and the slope $[-\ln\left(\frac{S'}{S'^\wedge}\right)_0]/(T_w - T_{we}) = (0.0031 \pm 0.0003)$ and $(0.0022 \pm 0.0002)$ differ between B1120 and P309 to reflect the considerably lower entrance degradation observed with the latter.

If we accept an inherently linear relationship between $k_{deg}$ and $T_w$, as observed in Figs. 7 and 14 and represented by Eqs. 4 and 6, then the ratio $(k_{deg}/T_w) \ (Pa \ s)^{-1}$, a “degradation modulus”, can be used to compare the present results with previous degradation studies at widely different experimental conditions [5, 6, 15], summarized in Table 4.

Table 4 compares the present (entries 1, 2) with previous degradation kinetics derived from the works of Paterson and Abernathy [5] (entries 3, 4), Horn, Motier and Munk [6] (entries 5, 6), and Motier, Chou and Kommareddi [15] (entry 7). Rows, from the top
| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|---|---|---|---|---|---|
| Source | Present | Present, [8] | [5] | [5] | [6] | [6] | [15] |
| Solvent | Water | Water | Water | Water | Crude, AK | Crude, AK | Crude, NS |
| T K | 295 | 297 | 296 | 296 | 317 | 324 | 308 |
| $\rho$ kg/m$^3$ | 998 | 998 | 998 | 998 | 870 | 870 | 820 |
| $\eta$ Pa s | 0.00096 | 0.00092 | 0.00095 | 0.00095 | 0.011 | 0.010 | 0.0074 |
| Polymer Grade | B1120 | P309 | W301 | W301 | FLO-8B | FLO-8B | DRA |
| MW $\times 10^{-6}$ | 18.3 | 11.4 | 8.0 | 8.0 | $\sim$ 35 | $\sim$ 35 | $\sim$ 35 |
| N $\times 10^{-6}$ | 0.52 | 0.78 | 0.54 | 0.54 | $\sim$ 0.6 | $\sim$ 0.6 | $\sim$ 0.6 |
| Lc $\mu$m | 65 | 98 | 69 | 69 | $\sim$ 70 | $\sim$ 70 | $\sim$ 70 |
| C wppm | 0.3 - 10.0 | 0.1 - 5.0 | 1.0 | 1.0 | 0.14 - 1.08 | 2.7, 3.8, 5.4 | 0.7, 2.1 |
| D m | 0.00460 | 0.00777 | 0.0063 | 0.0174 | 0.052 | 1.194 | 0.049 |
| L/D | 210 | 220 | 1700 | 670 | 1200 | 250000 | 5000 |
| Re | 9600-88000 | 18000-115000 | 12000-25000 | 56000 | 7300 | 320000 | 13000 |
| Tw Pa | 10-600 | 40-400 | 7-28 | 15 ± 3 | 8.0 ± 2.0 | 9.6 ± 1.0 | 13 ± 2 |
| DR | 0.02 – 0.77 | 0.03 – 0.67 | 0.33 – 0.43 | 0.41 | 0.20 – 0.57 | 0.31, 0.38, 0.43 | 0.41, 0.67 |
Table 4  (continued)

| Entry | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|-------|-------|-------|-------|-------|-------|-------|-------|
| \( \text{Re}\sqrt{f} \) | 800-6000 | 800-8000 | 750-1500 | 3100 | 440 – 600 | 15000 | 850, 630 |
| \( S' \) | 1/\( \sqrt{f} \) | 0.1 – 15.9 | 0.1 – 17 | 4.0 – 5.2 | 4.6 | 1.5 – 6.5 | 3.9, 5.2, 6.3 | 4.0, 9.9 |
| Method | Falloff | Falloff | Falloff | Falloff | Falloff | Falloff | Falloff |
| \( k_{\text{deg}} \) | s\(^{-1} \) | 0.15-6.5 | 0.4-4.0 | 0.04 – 0.16 | 0.065± 0.015 | (1.6± 0.5) \( \times 10^{-2} \) | (1.4± 0.1) \( \times 10^{-5} \) | (6.2± 0.5) \( \times 10^{-3} \) |
| \( (k_{\text{deg}}/Tw) \) | (Pa s\(^{-1} \) | 0.012± 0.001 | 0.011± 0.002 | 0.006± 0.001 | 0.0045± 0.001 | (1.9± 0.2) \( \times 10^{-3} \) | (1.45± 0.05) \( \times 10^{-6} \) | (4.9± 0.3) \( \times 10^{-4} \) |

Notes: Solvents: Water and Crude Oils from AK – Alaska North Slope, NS – North Sea.
\( \rho \) = solvent density.
\( \eta \) = solvent viscosity.
Polymer: PAOC – Poly-alpha-olefin-copolymer, e.g. poly-(hex-1-ene + dodec-1-ene).
PAMH – Partially-hydrolysed Polyacrylamide (-CH2-C(CONH2)H-)n; PEO – Polyethyleneoxide (-CH2-CH2-O-)n;
MW = Molecular weight; Present, [5, 8] from intrinsic viscosity measurements; others are (rough) estimates.
N = Number of backbone chain links = MW/m where m (weight per link) = 35.5 (PAMH), 14.7 (PEO), 63 (PAOC).
Lc = Polymer chain contour length.
Re = Reynolds number formed with solvent kinematic viscosity.
DR = Fractional drag reduction = \( (1 – (f_p/f_n))_{\text{Re}} \)
down, list the solvent and its physical properties; the polymer, its molecular weight, backbone chain links, contour length and the concentration(s) employed; the pipe diameter and non-dimensional length; flow conditions, namely, Reynolds number, wall shear stress and fractional drag reduction as well as $Re \sqrt{f}$ and flow enhancement $S'$; and the apparent degradation rate constant(s) and modulus ($k_{\text{deg}}/Tw$). Row 19 lists the methods used to derive $k_{\text{deg}}$: The present entries 1 and 2 arose from the falloff-based calculations described above; entries 3 and 4 were also derived from the falloff data in [5], to somewhat improve on those authors' own estimates of degradation kinetics from the intrinsic viscosities of polymer solutions extracted from upstream and downstream sampling ports in their pipe; and entries 5, 6 and 7 [6, 15] were derived from linear “double reciprocal plots” of $1/\text{DR}$ vs $1/C$ [4] established for undegraded polymer solutions upstream and applied to the (lesser) drag reduction measured downstream.

Figure 15, a log-log plot of the modulus ($k_{\text{deg}}/Tw$) vs pipe diameter $D$ for all entries in Table 4, illustrates the profound decrease in degradation kinetics with increasing pipe diameter, first reported by Horn, Motier and Munk 1989 [6].

Also noteworthy, the present ($k_{\text{deg}}/Tw$) data from laboratory pipes of $D \sim 0.01$ m seem to lie on a simple extension of the ($k_{\text{deg}}/Tw$) data from oil pipelines of $D \sim 0.1$ m and 1.0 m. An approximate fit of all the data from Table 4 is shown by the dashed line in Fig. 15 which represents the dimensional relation:

$$\log(k_{\text{deg}}/Tw) = -5.4 - 1.6 \log(D); \ 0.004 < Dm < 1.2$$

Fig. 15 The effect of pipe diameter upon polymer degradation kinetics during drag reduction. Sources, polymer abbreviations from Table 4
The observed -1.6 power law dependence of $k_{\text{deg}}$ upon pipe diameter has thus far defied explanation in the literature. In this regard note that $k_{\text{deg}}$ is an “apparent” rate constant, averaged over the entire pipe cross section, whereas, during drag reduction, polymer may actually degrade in a thin “elastic sublayer” [16] near, but not at, the pipe wall, wherein the polymer-turbulence interaction responsible for drag reduction is believed to occur. As an example, recall from Section 3.2.2 Fig. 4 that 1.04 wppm B1120 had a falloff point (Re$\sqrt{f^*}$, $S'\wedge$) = (620, 4.8) from which, by [16], non-dimensional viscous sublayer, elastic sublayer and centerline locations are ($y_+^v$, $y_+^e$, $R^+$) = (11.6, 24.3, 219), that is, the elastic sublayer occupies area fraction $A_e/A = 0.10$, about a tenth of the pipe cross section, and therefore the real degradation rate constant may well be of order 10 times the apparent $k_{\text{deg}}$ obtained in experiments. Further, by approximate “wall-law” algebra, $A_e/A \sim 10.1*S'/Re\sqrt{f} \propto D^{-1}$ so that, at fixed drag reduction and wall shear stress, a substantial portion, -1, of the observed -1.6 power-law dependence of $k_{\text{deg}}$ upon D might simply reflect the elastic sublayer occupying a smaller fraction of the pipe cross section in larger pipes. Finally, the non-dimensional counterpart of Eq. 8, with ordinate and abscissa respectively $k^* = \eta k_{\text{deg}}/T_w$ and $D^* = D/L_c$, is:

$$\log k^* = -2.8 - 1.0 \log D^*$$  \hspace{1cm} (9)

Equation 9 embodies a -1 power-law dependence of $k_{\text{deg}}$ upon D but provides a poorer correlation, $R^2 = 0.70$, of the data in Table 4 than the original Eq. 8, $R^2 = 0.94$.

Polymer degradation kinetics are of interest in the possible extension of pipeline drag reduction (hundreds of current examples) to maritime applications (no examples). Consider a hypothetical containership of length and velocity $(L, U) = (347 \text{ m}, 12.9 \text{ m/s} \text{ (25 knots)})$ moving on a quiescent ocean with the flat plate boundary layer on its hull described by Reynolds number, friction factor, boundary layer thickness and mean wall shear stress $(\text{Re}_L, C_F, \delta, T_w m) = (4.35E+9, 1.32E-3, 1.52 \text{ m}, 110 \text{ Pa})$. Suppose the ship is drag-reduced with the present 1.04 wppm of PAMH B1120 in seawater (physical properties assumed same as DW), that provides an undegraded initial flow enhancement $S' = 4.8$ 1/$\sqrt{f}$ units, corresponding to fractional hull drag reduction $\text{DR} = 0.22$. To estimate the severity of B1120 degradation in the ship boundary layer from the present degradation results, assume that the dependence of degradation upon pipe diameter applies also to a 2-D boundary layer, and treat the boundary layer as a pipe of equivalent diameter $D = 2\delta = 3.04 \text{ m}$; then extrapolate Eq. 8 a bit past the bounds of data in Fig. 15 to estimate $(k_{\text{deg}}/T_w) = 0.67E-6 \text{ (Pa s)}^{-1}$ and thence $k_{\text{deg}} = 0.67E-6*110 = 0.74E-4 \text{ s}^{-1}$. Average residence time in the boundary layer is $t_{\text{res}} = L/U = 27 \text{ s}$, so degradation severity will be $k_{\text{deg}} t_{\text{res}} = 0.0020$, which, being $<< 1$, also represents the (negligible) fraction of polymer degraded. Maritime applications of drag reduction, at least those represented by vessels resembling containerships, thus appear unlikely to be impeded by polymer degradation.

4.2 Aspects of type B drag reduction

4.2.1 Type B physics

Table 5 summarizes the present and some previous intrinsic slip data for Type B drag reduction by a variety of polymers with vinylic and glycosidic backbones. Columns encompass: Polymer molecular weight, backbone chain links, chain contour length, chain diameter and concentrations employed; pipe ID and L/D; Re$\sqrt{f}$ and the associated turbulence macroscales.
Table 5  Intrinsic slips in Type B drag reduction by polymer solutions

| Entry | Source | Polymer | Grade | MW \( \times 10^{-6} \) | N \( \times 10^{-3} \) | Lc \( \mu m \) | d \( nm \) | C \( \text{wppm} \) | D \( mm \) | L/D | Re \( \Lambda \) | \( \lambda \) | \([S']\) \( \text{wppm}^{-1} \) | \( \beta \) |
|-------|--------|---------|-------|-----------------|----------------|--------|-----|--------|--------|-----|---------|--------|-----------------|-----|
| 1     | Present | PAMH    | B1120 | 18.3            | 515            | 64.8   | 0.76 | 0.1 - 10 | 4.60   | 210 | 1000    | 920    | 6.5             | 5.00 ± 0.55 | 0.077          |
| 2     | Present | PEO     | P309  | 11.4            | 776            | 97.6   | 0.48 | 0.1 - 10 | 7.77   | 220 | 2500    | 1554   | 4.4             | 5.07 ± 0.84 | 0.052          |
| 3     | [8]    | PEO     | N60K  | 3.2             | 218            | 27.4   | 0.48 | 1 - 500  | 4.58   | 180 | 2500    | 916    | 2.6             | 2.10 ± 0.15 | 0.077          |
| 4     | [2]    | PAMH    | B1110 | 15              | 423            | 53.2   | 0.76 | 3 - 100  | 9.45   | 200 | 1600    | 1892   | 8.4             | 0.72 ± 0.10 | 0.014          |
| 5     | [18]   | PAMH    | C832  | 15              | 423            | 53.2   | 0.76 | 1 - 20   | 14.6   | 208 | 1000    | 2920   | 20.6            | 0.74 ± 0.15 | 0.014          |
| 6     | [14]   | PAMH    | B1120 | 20              | 563            | 71.0   | 0.76 | 0.3 - 30 | 10.9   | 240 | 1600    | 2180   | 9.6             | 1.8 ± 0.2   | 0.025          |
| 7     | [17]   | DIU     | GV-XT | 4               | 8.06           | 4.53   | 1.76 | 2 - 2100 | 4.57   | 110 | 4000    | 914    | 1.6             | 0.12 ± 0.01 | 0.027          |
| 8     | [12]   | DIU     | GV-XT | 4               | 8.06           | 4.53   | 1.76 | 10 - 500 | 7.82   | 150 | 5000    | 1564   | 2.2             | 0.122 ± 0.035 | 0.027        |
| 9     | [17]   | DIU     | GV-XT | 4               | 8.06           | 4.53   | 1.76 | 10 - 2400 | 15.85  | 340 | 5000    | 3170   | 4.5             | 0.12 ± 0.02 | 0.026          |
| 10    | [17]   | XAN     | XV-L  | 2               | 2.11           | 1.18   | 1.90 | 10 - 200 | 4.57   | 110 | 4000    | 914    | 1.6             | 0.12 ± 0.02 | 0.101         |
| 11    | [12]   | XAN     | XV-L  | 2               | 2.11           | 1.18   | 1.90 | 10 - 500 | 7.82   | 150 | 5000    | 1564   | 2.2             | 0.126 ± 0.023 | 0.107        |
| 12    | [17]   | XAN     | XV-L  | 2               | 2.11           | 1.18   | 1.90 | 20 - 2600 | 15.85  | 340 | 5000    | 3170   | 4.5             | 0.11 ± 0.03 | 0.095          |

Notes: Solvent was deionized water DW in all cases.
Polymer: PAMH – Partially-hydrolysed Polyacrylamide; PEO – Polyethyleneoxide; DIU – Diutan gum; XAN – Xanthan gum.
MW = Molecular weight; Entries 1, 2, 3 from intrinsic viscosity measurements; others are (rough) estimates.
N = Number of backbone chain links = MW/m, where m = 35.5 (PAMH), 14.7 (PEO), 248 (DIU), 475 (XAN).
Lc = Polymer chain contour length = p.N, where p (length per link) = 0.126 nm (PAMH, PEO), 0.562 nm (DIU, XAN).
d = Polymer chain diameter, of cylinder enveloping PAMH, PEO at contour lengths and DIU, XAN double helices.
\( \Lambda \) = Macroscale \( \sim 0.2D \).
\( \lambda \) = Microscale \( \sim \nu/\tau \), where \( \nu \) = kinematic viscosity, \( \tau \) = friction velocity.
\([S']\) = Intrinsic slip, \( \text{Lim}_{C \rightarrow 0}(S' \gamma/C) \).
\( \beta \) = \([S']/Lc\), “Type B constant”, dimensional, with \([S']\) in \text{wppm}^{-1} and Lc in \( \mu m \).
and microscales; the intrinsic slip and a “Type B constant” $\beta = [S']/Lc$. Macromolecular parameters in Table 5 were improved relative to previous [12] by incorporating the tactility of PAMH [19] and X-Ray structure data for the double-helix conformations of both the biopolymers Diutan [20, 21] and Xanthan [22–24].

Consider the disposition of macromolecular chains during Type B drag reduction using results for PAMH B1120 in row 1 of Table 5 as a representative example. Form a characteristic concentration that provides unit flow enhancement $S' = 1$ from the reciprocal of the intrinsic slip: $C_1 = 1/[S'] = 0.20$ wpmm. The corresponding molecular concentration is $n_1 = N_{avv,v}/(C_1/MW) = 6.6E+15$ chains/m$^3$ with each chain a rod of length and diameter $(Lc, d) = (65, 0.00076) \mu m$, with aspect ratio $a = Lc/d = 86000$ and physical volume $v_{ch} = Lc.(\pi d^2/4) = 2.9E-23$ m$^3$. If each chain solely occupied a sphere of diameter $Lc$, with spherical “excluded” volume $v_{sp} = (\pi Lc^3/6) = 1.4E-13$ m$^3$, then the chain volume fraction $\phi = n_1.v_{sp} = 940 >> 1$ would be impossibly high. Individual chains must therefore be aligned to some extent, to maintain physical $\phi = 1$. Next, suppose chains are imperfectly aligned in the flow direction, with chain axis at (small) angle $\theta$ with respect to the pipe axis. Chain excluded volume is now essentially cylindrical $v_{cy} = Lc.Cos(\theta).(\pi d^2/4).(1 + a.Sin(\theta))^2$, with chain volume fraction $\phi = n_1.v_{cy} = n_1.v_{ch}.Cos(\theta).(1 + a.Sin(\theta))^2$. Uniform alignment in the flow direction, $\theta = 0$, provides $\phi = n_1.v_{ch} = 1.9E-7 << 1$ that is easily accommodated. However, on account of the chain aspect ratio $a >> 1$, a (miniscule) misalignment $\theta = 0.027$ rad (1.5 deg) increases chain volume fraction to its upper bound $\phi = 1$. The foregoing illustrates the delicate nature of chain alignment during drag reduction, with the entire available volume fraction range $1.9E-7 < \phi < 1$ traversed over a narrow band of alignment angles $0 < \theta < 0.027$.

Continuing with Row 1 of Table 5 for B1120 as an example, the chain contour length $Lc = 65 \mu m$ is an order of magnitude smaller than turbulence macroscale $\Lambda = 920 \mu m$ but an order of magnitude larger than the turbulence microscale $\lambda = 6.5 \mu m$. The ordering $\lambda << Lc << \Lambda$ holds for all vinylic polymers, entries 1 – 6 in Table 5 but changes to $\lambda \sim Lc << \Lambda$ for the glycosidic polymers, entries 7 – 12, the latter with contour lengths of the same order of magnitude as the microscale. Also, at characteristic concentration $C_1 = 0.20$ wpmm, where B1120 exhibits unit flow enhancement $S' = 1$, the elastic sublayer model [16] provides non-dimensional viscous sublayer, elastic sublayer and centerline locations $(y_1^*, y_2^*, R^*, \Gamma^*) = (11.6, 13.5, 346)$, that yield nondimensional elastic sublayer thickness $\delta^* = 1.9$ and physical thickness $\delta = 13 \mu m < Lc = 65 \mu m$ but still >> macromolecular diameter $d = 0.76$ nm, and therefore capable of accommodating numerous chains aligned in the flow direction.

Finally, Table 5 illustrates the remarkable insensitivity of Type B drag reduction to the micro- and macro-scales of turbulence. Insensitivity to microscale has always been implicit in the “ladder” structure of Type B trajectories on P-K coordinates, with each rung exhibiting constant flow enhancement $S'$ independent of $Re_s \sqrt{f} = \sqrt{2DuT/\nu}$ and, thence, microscale $\lambda \sim \nu/\sqrt{f}$. However, entries 7 – 12 of Table 5, for the Diutan and Xanthan gum biopolymers, also reveal macroscale insensitivity, in that intrinsic slips $[S'] = 0.12$ remain invariant in each of three pipes of $D = 4.6, 7.8$, and $16$ mm at high $Re_s \sqrt{f} = 4000$ - 5000. Another example of macroscale insensitivity, quoted in [12], can be seen in Fig. 1 of [25], wherein a single 150 wpmm Xanthan/DW solution yielded slips $S' \sim 9.5 \pm 1.5$ in each of $3$ pipes of $D = 5, 10$, and $20$ mm at $Re_s \sqrt{f} = 1000$ – 5000. The observed insensitivity to turbulence scales nurtures the hope that Type B drag reduction might eventually be described by macromolecular parameters alone, independent of flow conditions.
4.2.2 Intrinsic slip correlation

Intrinsic slip \([S']\) is derived at infinite dilution, reflecting the efficacy of an individual macromolecule, so we seek its relation to molecular parameters. Perusal of Table 5 revealed intrinsic slip \([S']\) to be roughly linear in polymer contour length \(L_c\) and Fig. 16 is the corresponding log-log plot with dashed line representing a dimensional correlation of currently available data for Type B drag reduction by polymeric additives:

\[
[S'] = \beta L_c; \quad \beta = 0.053 \pm 0.036 \text{ (wppm } \mu \text{m})^{-1}
\]  \(10a\)

At low polymer concentrations, where \(S'\) is linear in \(C\), the preceding reduces to:

\[
S' = C \beta L_c; \quad C < S'_m/[S]
\]  \(10b\)

Equations 10a and b crudely describe Type B drag reduction by polymeric additives with a single molecular parameter, the fully-extended contour length \(L_c\), and a dimensional Type B constant, \(\beta\).

A non-dimensional form of Eq. 10a can be inferred from polymer chain parameters by (i), on the RHS, converting \(L_c\) to chain aspect ratio \(a = L_c/d\), and (ii), on the LHS, recognizing that the reciprocal of intrinsic slip \(1/[S'] = C_1\) is a characteristic concentration required to induce unit flow enhancement, with associated molecular concentration \(n_1 = N_{avo}(C_1/MW)\), that, with chain physical volume \(v_{ch} = L_c.(\pi d^2/4)\), defines a non-dimensional chain volume fraction \(\varphi_1 = n_1.v_{ch}\), whence (iii) the non-dimensional Type B constant \(\beta^* = \beta.[(4x10^{12}/\pi.N_{avo}).(m.p/d).(1/\rho)]\). \(\beta^*\) is related to \(\beta\) by three bracketed entities within the square bracket that respectively comprise relevant numerical constants, polymer chain parameters, and solvent density. Polymer chain parameters listed in Table 4 and under Table 5 provide the ratio \(\beta^*/\beta = 700 \pm 210\) (wppm \(\mu\)m) and thus, from Eq. 10a,

![Fig. 16 Intrinsic slips in Type B drag reduction. Polymer abbreviations from Table 5](image-url)
β* = 37 ± 25; too, separate linear regression of (1/φ1) versus a, from data in Table 5, yielded β* = 27 ± 5; an approximate non-dimensional relation that melds both results is:

\[
\frac{1}{\phi_1} = \beta^* \cdot a; \quad \beta^* = 35 \pm 15
\]

Equation 11 evinces an appealing simplicity: \(\phi_1 \sim 1/(\beta^* \cdot a)\), that is, in Type B drag reduction, the volume fraction of additive required to induce unit slip decreases in direct proportion to the increase in its aspect ratio.

Although Eq. 11 derives from the present experiments with polymeric additives, its physical interpretation implies a more universal, additive-independent, inverse relationship between \(\phi_1\) and a. Literature for Type B drag reduction by aqueous suspensions of non-polymeric additives reveals that:

(i) Nylon fibres [26] adhere to Eq. 11 for 25 < a < 400 with \(\beta^* = 5.3 \pm 0.9\);
(ii) Paper fibres [27, 28] follow Eq. 11 for 48 < a < 90 with \(\beta^* = 13 \pm 1\);
(iii) Asbestos fibres [29, 30], both of the same \(a = 37500\), yield \(\beta^* = 4.4 \pm 0.9\); and
(iv) Milling yellow crystals [31], with \(a = 5.7\), exhibit \(\beta^* = 260\).

Thus suspensions of fibres and crystals seem also to follow Eq. 11, but with proportionality constants \(\beta^*\) that can differ from the polymer-derived \(\beta^* = 35 \pm 15\) by an order of magnitude, either lower (asbestos fibres) or higher (milling yellow crystals).

4.2.3 Facets of the drag reduction mechanism

The close correspondences between degradation results in Section 3.2.2, during Type B drag reduction by PAMH B1120, and in Section 4.1.2, during Type A drag reduction by PEO P309, have mechanistic significance. Near-identical falloff slip plots, \(S' \wedge \) vs C, in Figs. 5 and 12 imply that drag reduction by PEO P309 evolves from Type A at onset \(Re_{\sqrt{f^*}} = 550\) to Type B at falloff \(Re_{\sqrt{f^*}} = 2550\). That P309, a Polyox and bastion of Type A behavior, eventually evolves to Type B behavior at falloff is further evidence that all drag reduction is fundamentally of Type B, with Type A merely reflecting the dynamics of initially-coiled, ineffective, macromolecules being stretched to their extended, effective, conformations by the turbulent flow. The foregoing also resolves a mechanistic question of long standing [18, 32]: Is drag reduction induced by the act of macromolecular extension or by the extended chains alone? The latter evidently prevail.

Previous thoughts on the mechanism of drag reduction [11, 16, 32] are briefly noted. Overall, the P-K law Eq. 1 and the MDR asymptote Eq. 2 can be viewed as the limiting affine states of turbulent flow in a pipe, respectively the N-state and the M-state, with mean and turbulent flow structures characterized by their turbulent transport diffusivities, that is, their mixing length constants, \(X_n = 0.40\) and \(X_m = 0.085\). In greater detail, during drag reduction in the polymeric regime, the mean velocity profile underlying Type B friction factor behavior comprises three zones [16] outward from the wall, namely (i) \(0 < y^+ < y_v^+\), a viscous sublayer, (ii) \(y_v^+ < y^+ < y_e^+\), an elastic sublayer, reflecting M-state polymer-turbulence interaction with \(X_m = 0.085\), and (iii) \(y_e^+ < y^+ < R^+\), a Newtonian plug with N-state \(X_n = 0.40\). At the highest level of detail, cross-sectional turbulent energy balances show that the energy deficit of the the N-state core, region (iii), wherein dissipation \(>\) production and turbulent intensities (hence kinetic energy levels) are low, is satisfied by diffusion of kinetic energy from the M-state wall-proximate region (ii), wherein production \(>\) dissipation and turbulent intensities and kinetic energy levels are high. Maintenance of the requisite wall to core energy flux, despite diminished diffusivity, relative to Newtonian, in
the M-state elastic sublayer requires a higher turbulent kinetic energy level therein, which is
achieved by thickening, with associated non-dimensional “slip” $S^+$, that is, drag reduction.
A modest elaboration of the preceding mechanism can be gained by attributing the dif-
ferences in diffusivity between the M-state elastic sublayer, region (ii), and the N-state
Newtonian plug, region (iii), to differences in macromolecular chain alignment. During drag
reduction the entire pipe cross section is pervaded by chains dissolved in solvent and, from
Section 4.2.1, they must all be aligned in the axial direction to some extent. Imagine that the
ensemble-average (mis-)alignment angles of the chains with respect to the pipe axis are $\theta_e$
in the elastic sublayer and $\theta_n$ in the Newtonian plug, where $\theta = 0$ represents perfect align-
ment, and $\theta_e < \theta_n$ because $X_m < X_n$, that is, better axial alignment should diminish radial
transport. For the earlier PAMH B1120 example we can then estimate a range of alignments
$0 < \theta_e < \theta_n < 0.027$ rad; there is yet no indication of what actual alignment angles might
be and how such subtle differences between them might induce a switch from M-state to
N-state turbulent flow structures.

5 Summary

1. Flow development and degradation during Type B turbulent drag reduction by 0.10 to
10 wppm DW solutions of a partially-hydrolysed polyacrylamide B1120 of MW = 18.3
$\times 10^6$ was studied in an EPSS test pipe of ID = 4.60 mm, L/D = 210, relative roughness
D/2k = 12800, assembled from 7 identical segments. Pipe Reynolds numbers Re were
varied from 10000 to 80000 and wall shear stresses $T_w$ from 8 to 600 Pa. Solvent DW
friction factors in all pipe segments at all Re adhered to the Prandtl-Karman law within
$\pm 0.2 \sqrt{f}$ units, indicative of fully-developed flow in a hydrodynamically smooth
pipe.
2. B1120 solutions exhibited many facets of Type B drag reduction. Friction factors for
$C = 1.04$ wppm lay roughly parallel to the P-K line but displaced upwards by $S' \sim
4.2 \sqrt{f}$ units at the two lowest Re/$\sqrt{f} \sim 630$ and 1260; those for $C = 3.03$ wppm
attained MDR at the lowest Re/$\sqrt{f} \sim 540$ and exhibited retro-onset at Re/$\sqrt{f} \sim 1000$,
departing from MDR into the polymeric regime; and friction factors for $C = 9.97$ wppm
adhered to MDR for Re/$\sqrt{f} \sim 520$, 900, and 1650. Flow enhancements observed with
the polymer solutions varied over the entire possible range $0 < S' < S'_m$, from $S' \sim 0$, near the P-K law, for the lowest $C = 0.10$ wppm to $S' \sim 16$, near the MDR asymptote, attained by $C = 9.97$ wppm.
3. Axial flow enhancement profiles of $S'$ vs L/D for B1120 solutions reflected a superpo-
sition of flow development and polymer degradation effects, the former increasing and
the latter diminishing $S'$ with increasing distance downstream. In the polymeric regime,
solutions that induced normalized flow enhancements $S'/S'_m < 0.4$ developed akin to
solvent, with $L_{e,p}/D = L_{e,n}/D < 42.3$. In the maximum drag reduction regime, for four
instances of axial flow development, at $C = 3.03$ wppm, Re = 10300, and $C = 9.97$
wppm, Re = 10200, 21800, 44200, the entrance length $L_{e,m}/D \sim 117$, roughly 3 times
the solvent $L_{e,n}/D$.
4. B1120 degradation kinetics were inferred by first detecting a “falloff point”, of maxi-
mum observed flow enhancement, for each polymer solution, such as $(Re\sqrt{f}, S'^\wedge) =
(620, 4.8)$ for $C = 1.04$ wppm. A plot of $S'^\wedge$ vs $C$ then revealed $S'^\wedge$ linear in $C$ at low
$C$, with lower bound $[S'] = \lim_{C \to 0}(S'/C) = 5.0$ wppm$^{-1}$, and $S'^\wedge$ independent of $C$
at high $C$, with upper bound the asymptotic maximum $S'_m = 15.9$. The ratio $S'/S'^\wedge$ in any
pipe section was therefore interpreted to be the undegraded fraction of original polymer
7. P309 falloff flow enhancements $S'/S''$ at a section vs transit time from pipe entrance to that section showed first order kinetics, from which apparent degradation rate constants $k_{\text{deg}}$ s$^{-1}$ and entrance severities $-\ln(S'/S'')_0$ were extracted. Data for all solutions showed that $k_{\text{deg}}$ increased with increasing wall shear stress $T_w$ at constant $C$, and that $k_{\text{deg}}$ was independent of $C$ at constant $T_w$, providing a final B1120 degradation modulus $(k_{\text{deg}}/T_w) = (0.012 \pm 0.001)$ (Pa s)$^{-1}$ for $8 < T_w$ Pa $< 600$, $0.30 < C$ wppm $< 10$. Entrance severities, that reflect fractional degradation at the pipe entrance, were also related to flow strength, being negligible below a threshold $T_w \sim 30$ Pa and increasing roughly linearly with increasing $T_w$ for $T_w > T_w^e$.

5. Methods for analysis of Type B drag reduction by PAMH B1120 were applied to a study of flow development and degradation during Type A drag reduction by 0.1 to 10.0 wppm DW solutions of PEO P309, MW $= 11.4 \times 10^6$, in an EPSS pipe of ID $= 7.77$ mm, $L/D = 220$, $D/2k = 21900$, 7 segments, at $Re$ from 4000 to 115000. Axial flow enhancement profiles for P309 solutions in the polymeric regime showed those that induced $S'/S'_m < 0.4$ to develop akin to solvent, with $L_{c,p}/D = L_{c,n}/D < 23$, while those that induced $S'/S'_m \sim 0.6$ developed by $L_{c,p}/D \sim 75$; in the maximum drag reduction regime there were six instances of $S'/S'_m \sim 1$, with $C = 10$ wppm and $Re$ from 14000 to 80000, that provided entrance lengths at MDR of $L_{c,m}/D \sim 93$. Additional MDR results for 200 and 500 wppm DW solutions of another PEO, N60K, MW $= 3.2 \times 10^6$, in an EPSS pipe of ID $= 4.58$ mm, $L/D = 180$, $D/2k = 12900$, 6 segments, at $Re$ from 10000 to 80000 provided $L_{c,m}/D \sim 102$. Overall, MDR asymptotes attained experimentally using 2 pipe diameters, 2 species and 3 grades of polymers, with MW from $3.2 \times 10^6$ to $18.3 \times 10^6$ and $C$ from 3 to 500 wppm, all yielded the same entrance lengths $L_{c,m}/D = 105 \pm 15$, about 3 times the Newtonian $L_{c,n}/D \sim 25$ to 40.

6. Friction factors for solutions of PEO P309 described a Type A fan distorted by polymer degradation. The highest $C = 10$ wppm showed a well-defined onset off the P-K line at $Re\sqrt{f^e} \sim 400$, followed by steep ascent into the polymeric regime to attain the MDR asymptote, which was adhered to for $800 < Re\sqrt{f} < 2500$. Lower $C = 1$ wppm also showed onset, at $Re\sqrt{f^e} \sim 600$, but the polymeric regime trajectory resembled a fountain, with an initial ascending “jet” segment $1250 < Re\sqrt{f} < 2500$, wherein tap pairs clustered closely together while flow enhancement increased, followed by a descending “spray” segment $2500 < Re\sqrt{f} < 7000$, wherein tap pairs separated; upstream higher than downstream, while flow enhancement decreased; the preceding defined a characteristic “falloff point”, $(Re\sqrt{f^e}, S''/S'') = (2500, 4.3)$, of maximum flow enhancement. Data from all P309 solutions provided onset $Re\sqrt{f^e} = 550 \pm 100$ and falloff $Re\sqrt{f^e} = 2550 \pm 250$, the interval between them delineating “pure” Type A drag reduction, little affected by polymer degradation.

7. P309 falloff flow enhancements $S''/S'$ increased linearly with increasing $C$ at low concentrations, $0.1 < C < 1.0$, with lower bound $[S'] = \lim_{C \to 0}(S'/C) = 5.1$ wppm$^{-1}$, whereas at high concentration, $C = 10$ wppm, $S''/S'$ became essentially independent of $C$, approaching an asymptotic upper bound $S'_m = 19.1$. A plot of falloff $S''/S'$ vs $C$ for PEO P309 solutions bore a striking resemblance, in both shape and magnitudes, to the analogous $S''/S'$ vs $C$ plot for solutions of PAMH B1120, indicating that the initial Type A drag reduction by P309 after onset at $Re\sqrt{f^e}$ had evolved to Type B drag reduction by falloff at $Re\sqrt{f^e}$. Presuming that Type B behaviour persisted past falloff permitted the inference of P309 degradation kinetics as earlier done for B1120, invoking the linear variation of $S'$ vs $C$ at low $C$, and associating falloff slip $S''/S'$ with the original polymer concentration $C$, such that the ratio of observed to falloff slips $S'/S''$ in any pipe section could be considered the undegraded fraction of original polymer therein.
Kinetics results for P309 closely mirrored those for B1120, with $k_{\text{deg}}$ that increased linearly with increasing $T_w$ at constant $C$ and were independent of $C$ at constant $T_w$, providing a final P309 degradation modulus $(k_{\text{deg}}/T_w) = (0.011 \pm 0.002) \, (\text{Pa s})^{-1}$ for $4 < T_w \, \text{Pa} < 400, 0.10 < C \, \text{wppm} < 5.0$. The associated entrance severities $-\ln(S'/S'^\wedge 0)$ were negligible below a threshold $T_w \sim 80 \, \text{Pa}$ and increased roughly linearly with increasing $T_w$ for $T_w > T_w$.

8. Comparisons between the present PAMH B1120 and PEO P309 degradation kinetics and previous literature at widely different experimental conditions showed $(k_{\text{deg}}/T_w)$ data from laboratory pipes of $D \sim 0.01 \, \text{m}$ to lie on a simple extension of $(k_{\text{deg}}/T_w)$ data from oil pipelines of $D \sim 0.1 \, \text{m}$ and $1.0 \, \text{m}$, along a power-law relation $(k_{\text{deg}}/T_w) = 10^{-5.4} \, D^{-1.6}$ (dimensional, $(k_{\text{deg}}/T_w)$ (Pa s)$^{-1}$, D m) that reflects the profound decrease in apparent first order degradation rate constant with increasing pipe diameter.

9. Intrinsic slips, $[S'] = \lim_{C \to 0} (S'/C)$, derived from PAMH B1120 and PEO P309-at-falloff experiments were compared with previous data for Type B drag reduction by a variety of polymers with vinlylic and glycosidic backbones, to discern the influence of turbulent flow scales and molecular parameters. For a given polymer, $[S']$ was essentially independent of $Re\sqrt{f}$ and pipe ID, implying insensitivity to both micro- and macro-scales of turbulence; also, $[S']$ increased roughly linearly with increasing polymer chain contour length $L_c$, the proportionality constant $\beta = 0.053 \pm 0.036$ (wppm $\mu m$)$^{-1}$ enabling estimation of flow enhancement $S' = C.L_c.\beta$ for all Type B drag reduction by polymers.

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Compliance with Ethical Standards

Conflict of interests The author declares he has no conflict of interest.

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