Comparing Wilson and Clover quenched $SU(3)$ spectroscopy with an improved gauge action

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We present results of quenched $SU(3)$ hadron spectroscopy comparing $O(a)$ improved Wilson (Clover) fermions with conventional Wilson fermions. The configurations were generated using an $O(a^2)$ improved 6-link $SU(3)$ pure gauge action at $\beta$’s corresponding to lattice spacings of 0.15, 0.18, 0.20, 0.33, and 0.43 fm. We find evidence that fermionic scaling violations are consistent with $O(a^2)$ for Clover and $O(a)$ with a nonnegligible $O(a^2)$ term for standard Wilson fermions. This latter mixed ansatz makes a reliable continuum extrapolation problematic for Wilson fermions. We also find that the slope of the scaling violations is roughly 250 MeV for both Wilson and Clover fermions.

1. INTRODUCTION

The Symanzik action improvement program has been proposed as a way to reduce scaling violations in the approach to the continuum limit from a lattice action. In this contribution, we report on our investigations into the nature of scaling violations inherent in improved actions. We perform $SU(3)$ quenched spectroscopy using a one-loop tadpole-improved Symanzik pure gauge action, and compare the tree-level tadpole-improved Clover fermion action with the standard Wilson fermionic action. We measure the hadron spectrum and the string tension at lattice spacings 0.15, 0.18, 0.20, 0.25, 0.33, and 0.43 fm. The goal of this work is to measure the lattice spacing dependence of the hadron spectrum. We find significant scaling violations in $am_{\rho}/\sqrt{a^2\sigma}$ consistent with $O(a^2)$ for Clover fermions and $O(a)$ for Wilson fermions. However, Wilson fermions show a nonnegligible $O(a^2)$ term. Without this correction, continuum extrapolations of $am_{\rho}/\sqrt{a^2\sigma}$ decrease with increasing $\sqrt{a^2\sigma}$; hence, reliable continuum extrapolations are problematic.

2. ACTIONS

In this work, we used the tadpole improved 1-loop correct Symanzik pure gauge action described in Ref. Classically, this action has $O(a^4)$ errors but quantum effects induce $O(a^2a^2)$ errors. However, it was shown that the gauge action is insensitive to nonperturbative tuning of the coefficients suggesting that the $O(a^2a^2)$ errors are small.

We compute hadron spectroscopy using the Wilson action, and the tadpole-improved version of the tree-level Clover fermionic action proposed in. With this tree-level action, one expects quantum errors of $O(aa)$. However, Naik computed the one loop and the dominant two loop contributions to $c(g_0^a)$ and finds that after tadpole improvement the coefficient of $g_0^a$ is 0.016. Therefore, we expect to see only the dominant $O(a^2)$ fermionic errors.

Recently, the Clover coefficient with the Wilson gauge action was computed nonperturbatively and was found to deviate significantly from the tadpole value. Since our gauge actions differ from at $O(a^2)$, we cannot use this result in the present work.
3. OBSERVABLES

To set the scale, \( a \), we used the string tension. We computed finite \( T \) approximations to the static quark potential using time-like Wilson loops \( W(\vec{R}, T) \) which were constructed using ‘APE’-smeared spatial links \( [7,4] \). We extracted the string tension from the fitted “effective” potentials.

For hadron measurements, we used correlated multi-state fits to multiple correlation functions as discussed in \( [4] \). We computed quark propagators in Coulomb gauge at several \( \kappa \) values for each \( \beta \). We used two gaussian source smearing functions with smeared and local sinks. Fitting ranges were chosen by an automated procedure \( [4] \).

4. SIMULATIONS AND RESULTS

We generated 100 quenched configurations using the Symanzik gauge action \( [1,3] \) on a \( 16^3 \times 32 \) lattice at \( \beta = 7.90, 7.75, 7.60, 7.40, 7.10, \) and 6.80. We expect finite volume effects to be small since the box size is 2.40fm at \( \beta = 7.90 \). We originally had problems with exceptional configurations on \( 8^3 \times 16 \) at \( \beta = 6.80 \). With the larger physical volume, we reduced this problem. The cost of inversions increased; however, we compensated for this by using the method of ‘\( Z(3) \)’ fermion sources \( [8] \) to increase the statistics per inversion.

The results of the chiral extrapolations of our best fits are listed in Table \( [3] \) along with the string tension. The lattice spacings we quote used \( \sqrt{a^2\sigma} = 440\text{MeV} \).

Our main result for the determination of scaling violations is in Figure \( [1] \). We extrapolate the vector mass to the physical \( \rho/\pi \) ratio and plot the result of \( \rho_m/\sqrt{a^2\sigma} \) for Wilson and Clover. Fitting the functional form of the scaling violations, we find that the confidence level \( Q \) is 0.37 for Wilson assuming \( \mathcal{O}(a) \) errors and \( 10^{-6} \) assuming \( \mathcal{O}(a^2) \). We are confident then that the Wilson scaling violations are not consistent with only \( \mathcal{O}(a^2) \). However, as we drop the lower \( \beta \) points in the \( \mathcal{O}(a) \) ansatz, we \( Q \) and the intercept rise indicating that quadratic scaling corrections are not small. This is not too surprising since the
Clover result varies by 23% over the range of the fit while the Wilson result varies 48%. Fitting to both $O(a)$ and $O(a^2)$ we have $Q = 0.62$ and an intercept of $1.79(7)$ with an $O(a^2)$ term that is two standard deviations away from 0. This mixed ansatz is plotted in Figure 1.

For Clover, unfortunately, $Q$ is large for both $O(a)$ and $O(a^2)$ ansätze; hence, we can not discern the scaling violation order solely by $Q$. However, the extrapolated value of $am_\rho/\sqrt{a^2\sigma}$ is $1.98(2)$ compared to $1.77(1)$ for $O(a)$ and $O(a^2)$ ansätze, resp. Hence, only the $O(a^2)$ ansatz is compatible with the Wilson continuum prediction. We find that the $O(a^2)$ fit is stable after dropping low $\beta$ points. Furthermore, the GF11 and MILC collaborations find a consistent continuum extrapolation using Wilson and Staggered fermions, resp. Using the mixed $O(a)$ and $O(a^2)$ ansatz for Clover, we find the fit has a linear term that is zero within errors; hence, we will ignore this fit. To adequately resolve the problem of $O(a)$ and $O(a^2)$ scaling, we would need a simulation at $a < 0.08$ fm.

Another main feature of the extrapolation is the slope. For Wilson fermions, we find the coefficients of the $O(a)$ and $O(a^2)$ terms (the slopes) are $280$ MeV and $(160$ MeV$)^2$. For Clover, we find the coefficient of the $O(a^2)$ term is $(230$ MeV$)^2$. These slopes are consistent with the characteristic soft scale of light spectroscopy $\Lambda_{QCD}$. We find smaller slopes for the nucleon and delta.

In Figure 2, we plot the ratio of $am_N/am_\rho$ assuming $O(a^2)$ scaling violations for Clover. We find a continuum value of $1.28(2)$ that is consistent with the GF11 result 8; however, it is different than the experimental value of 1.22. For the delta, we obtain a continuum value of $1.67(4)$ for Clover compared to $1.61(8)$ for GF11. Our Wilson continuum extrapolations using only an $O(a)$ ansatz are consistently below the Clover and GF11 extrapolations.

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