THE LIMITS ON COSMOLOGICAL ANISOTROPIES AND INHOMOGENEITIES FROM COBE DATA

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ABSTRACT

Assuming that the cosmological principle holds, Maartens, Ellis, & Stoeger (MES) recently constructed a detailed scheme linking anisotropies in the cosmic background radiation (CBR) with anisotropies and inhomogeneities in the large-scale structure of the universe and showed how to place limits on those anisotropies and inhomogeneities simply by using CMB quadrupole and octupole limits. First, we indicate and discuss the connection between the covariant multiple moments of the temperature anisotropy used in the MES scheme and the quadrupole and octupole results from COBE. Then we introduce those results into the MES limit equations to obtain definite quantitative limits on the complete set of cosmological measures of anisotropy and inhomogeneity. We find that all the anisotropy measures are less than $10^{-3}$ in the case of those not affected by the expansion rate $H$ and less than $10^{-6}$ Mpc$^{-1}$ in the case of those which are. These results demonstrate quantitatively that the observable universe is indeed close to Friedmann-Lemaître-Robertson-Walker (FLRW) on the largest scales and can be modeled adequately by an almost-FLRW model—that is, the anisotropies and inhomogeneities characterizing the observable universe on the largest scales are not too large to be considered perturbations to FLRW.

Subject headings: cosmic microwave background — cosmology: theory — large-scale structure of universe

1. INTRODUCTION

People have long considered that the near-isotropy of the cosmic microwave background (CMB) radiation indicates that the universe on very large scales is isotropic and homogeneous, or nearly so. The first rigorous result supporting this conjecture was the theorem proved by Ehlers, Geren, & Sachs (1968, hereafter EGS): “If a family of freely falling observers measure self-gravitating background radiation to be everywhere exactly isotropic, then the universe is exactly Friedmann-Lemaître-Robertson-Walker (FLRW).” Later, Grishchuk & Zeldovich (1978), analyzing the FLRW perturbations, argued that, if CBR anisotropies are very small, all anisotropies and inhomogeneities on scales larger than the horizon should also be very small. Much more recently, Stoeger, Maartens, & Ellis (1995, hereafter SME) proved a significant generalization of this theorem: “If the Einstein-Liouville equations are satisfied in an expanding universe, where there is present pressure-free matter with 4-velocity vector field $u^a$ ($u_au^a = -1$) such that (freely propagating) background radiation is everywhere almost-isotropic relative to $u^a$, then spacetime is almost FLRW.” These results obviously provide a fundamental link between observational and theoretical cosmology, one that promises to reveal key aspects of the very large scale structure of the universe without relying on the often uncertain observational measurements of local and intermediate-scale structures. It should be noted that both the EGS and the SME results depend on assuming that the cosmological principle holds—expressed here in terms of the near isotropy of the background radiation relative to every fundamental observer in the spacetime. Goodman (1995) and Maartens, Ellis, & Stoeger (1995a), however, have pointed out recently the important fact that the cosmological principle itself is partially testable via the Sunyaev-Zeldovich effect.

Employing this connection between CBR anisotropies and the full range of cosmological isotropies and inhomogeneities, Maartens et al. (1995a) and Maartens, Ellis, & Stoeger (1995b) (hereafter MESa, MESb) have developed a detailed scheme demonstrating how limits on the temperature anisotropies of the CMB imply rigorous limits on the anisotropy and inhomogeneity of the universe. In this paper we show how CMB anisotropy data are inserted into the theoretically derived limits of this scheme to constrain strongly the vorticity, shear, spatial gradients, Weyl tensor components, and other measures of deviation from FLRW for the observable universe. Then we introduce the limits COBE places on the dipole, quadrupole, and octupole of the CBR to determine those limits quantitatively.

In the next section we shall give briefly the key MESa-MESb equations that constitute the CMB limits on the anisotropy and inhomogeneity of the universe. Then, in the third section, we shall discuss the relationship of the usual CMB multipole results with the temperature anisotropy multipoles in the MESa-MESb equations. Finally, in § 4, we shall present the COBE quadrupole and octupole observational results and use these to arrive at limits on all the cosmological anisotropy and inhomogeneity measures mentioned above.

Maartens, Ellis, & Stoeger (1996) recently provided such limits by introducing order-of-magnitude COBE results of $\varepsilon_2 \approx \varepsilon_3 \approx 10^{-5}$, but they did not discuss the relationship between temperature anisotropy multipoles used in their scheme and those in terms of which the COBE results are given. Here we fill that gap, and then we use the up-to-date COBE results for the rms quadrupole and octupole to obtain improved limits.

2. THE MES EQUATIONS

MES have written the covariant temperature isotropy multipoles as $\tau_{\ell_1 \ldots \ell_n}$, where $L$ is the multipole number.
Thus, for instance, \( \tau_{ab}, \tau_{abc} \) are, respectively, the components of the dipole, quadrupole, and octupole of the CMB temperature anisotropy. This form of the harmonic decomposition, which is given in detail in Maartens et al. (1995a), is equivalent to that formulated in terms of spherical harmonics (Ellis, Matravers, & Treciokas 1983; Ellis, Treciokas, & Matravers 1983).

MES have assumed that there are observed bounds on these temperature anisotropy multipoles, so that there exist \( O(1) \) constants \( \varepsilon_k \) such that

\[
| \tau_{\alpha_1...\alpha_k} | < \varepsilon_k .
\]

The absolute value brackets have been defined to be the square root of the sum of the squares of the components of a given vector or tensor (Maartens et al. 1995a). Thus, \( \varepsilon_1 \) gives the limits on the dipole components, \( \varepsilon_2 \) gives the limits on the quadrupole components, \( \varepsilon_3 \) gives the limits on the octupole moments, and so on.

Then, using the strong observational assumptions on the spatial gradients and time derivatives of the temperature harmonics they adopted in Maartens et al. (1995b), they have the observational limit equations on various kinematic, dynamic, and geometric indicators of anisotropy and inhomogeneity (Maartens et al. 1995a, 1995b):

\[
\frac{|\hat{\psi}_a|}{H} = 4 \frac{|\hat{\psi}_a T|}{T} < H \left( 12 \varepsilon_1 + 24 \frac{1}{5} \varepsilon_2 \right),
\]

\[
|\sigma_{ab}| \theta < 5 \varepsilon_1 + 3 \varepsilon_2 + \frac{3}{7} \varepsilon_3,
\]

\[
|\omega_{ab}| \theta < 10 \varepsilon_1 + \frac{2}{15} \varepsilon_2,
\]

\[
\frac{|\hat{\psi}_a \rho|}{H} < \frac{9}{2} H \varepsilon_2 + \left( \frac{H}{\Omega_M} \right) (60 \varepsilon_1 + 134 \varepsilon_2 + 6 \varepsilon_3)
\]

\[
+ \left( \frac{\Omega_K}{\Omega_M} \right) H \left( 16 \varepsilon_1 + \frac{61}{3} \varepsilon_2 \right),
\]

\[
\frac{|\hat{\psi}_a \Theta|}{H} < H \left( \frac{205}{3} \varepsilon_1 + 8 \varepsilon_2 \right) + 4(2 \Omega_K + \Omega_M) H \varepsilon_1,
\]

\[
\frac{|E_{ab}|}{\Theta} < H \left( \frac{55}{3} \varepsilon_1 + \frac{103}{3} \varepsilon_2 + \frac{23}{7} \varepsilon_3 \right)
\]

\[
+ \frac{4}{45} (11 \Omega_K + 15 \Omega_M) H \varepsilon_2,
\]

\[
\frac{|H_{ab}|}{\Theta} < H \left( \frac{16}{3} \varepsilon_1 + \frac{52}{15} \varepsilon_2 + \frac{1}{21} \varepsilon_3 \right).
\]

Here \( \mu \) and \( \rho \) are the radiation and matter densities, respectively, and \( \Omega_K \) and \( \Omega_M \) are the ratios of the radiation energy density and the matter densities, respectively, to the critical density of the universe. \( H \), of course, is the Hubble parameter, and \( T \) is the CMB temperature; \( \sigma_{ab} \) is the shear of the congruence of timelike geodesics in the universe, \( \omega_{ab} \) is its vorticity, \( \Theta = u^a a_a \) is its expansion scalar, and \( E_{ab} \) and \( H_{ab} \) are the electric and magnetic components, respectively, of the Weyl tensor. These quantities, along with the spatial gradients of \( \rho, \mu, \) and \( \Theta \), describe the anisotropy and inhomogeneity of the spacetime. The \( \hat{\psi}_a \) operator expresses the covariant spatial gradient in the spacetime. It is defined by

\[
\nabla_a Q_{ab} = h_{ab} E_a E_b V_a Q_{ab},
\]

where \( h_{ab} = g_{ab} + u_a u_b \) is the projection tensor in the rest spaces of the geodesically moving observers and \( V_a \) is the covariant derivative defined by the metric \( g_{ab} \). Finally, it should be noted that equations (2)–(8) hold independently of the statistics of the underlying matter-density fluctuations—they do not assume that the fluctuations are Gaussian.

If we can determine \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) from CMB measurements and have observational estimates of \( H, \Omega_K, \) and \( \Omega_M \) from other observations, then we can determine limits on all these anisotropy and inhomogeneity indicators. We shall proceed to do this in § 4. But first, in the next section, we need to discuss the relationship between the \( \tau_{\alpha_1...\alpha_k} \) given in the equations and the multipole moment results determined by the COBE Differential Microwave Radiometers (DRMs) and other CMB anisotropy detectors.

3. CMB MULTIPOLe ANISOTROPY DATA

We have mentioned above that the harmonic decomposition represented by the \( \tau_{\alpha_1...\alpha_k} \) is equivalent to that in terms of spherical harmonics. But the question then is whether or not the multipole results recovered from, say, COBE DRM data can be simply substituted into our limit equations. Are the multipole results presented in the COBE papers measurements of the \( \tau_{\alpha_1...\alpha_k} \) in our limit equations? The answer to this question is “yes,” as long as we use the real rms dipole, quadrupole, and octupole moments they obtain and not those associated with obtaining the power spectrum, which are often the focus of their reported results; as long as we determine the numerical factors relating these moments, defined in terms of Legendre polynomials, to the \( \tau_{\alpha_1...\alpha_k} \) (see § 4 below); and as long as we realize that they are usually given as the square root of the sum of the squares of the \((2L + 1)\) components of the \( L \)-pole, the rms \( L \)-pole—not as values of each separate component of the multipole in question. Furthermore, in the COBE data the multipoles are quoted for \( L \)th order, instead of \( \delta T/T \) for which our \( \tau_{\alpha_1...\alpha_k} \) values are the multipoles. Therefore, the COBE rms values published all have units of \( \mu K \). To translate these values into what we need, we must thus divide them by \( T \), the average CMB background temperature over the sky.

As a relevant example, Bennett et al. (1994) give results for the square of the rms quadrupole amplitude \( Q_{rms}^2 \):

\[
Q_{rms}^2 = \tau^2 (\frac{1}{2} Q_1^2 + Q_2^2 + Q_3^2 + Q_4^2 + Q_5^2),
\]

where the five quadrupole components \( Q_i \) are given in terms of Galactic coordinates \( l \) and \( b \) (Galactic longitude and latitude, respectively) by the expansion

\[
Q(l, b) = Q_1 (3 \sin^2 b - 1)/2 + Q_2 \sin 2b \cos l + Q_3
\]

\[
\times \sin 2b \sin l + Q_4 \cos^2 b \cos 2l + Q_5 \cos^2 b \sin 2l.
\]

The “strange” coefficients in equation (9) are due to the fact that the basis vectors are orthogonal but not orthonormal (see Partridge 1995, p. 189); \( Q_{rms}^2 \) does not contain a factor of \((2L + 1)^{-1} = \frac{1}{2L+1} \). From 4 years of data, the COBE workers give us a best-fit value of \((C. F. Smoot, private communication; Kogut et al. 1996)

\[
Q_{rms} = 10.7 \pm 7 \mu K ,
\]

with a 95% confidence limit. We can modify this result (see below) for use in equations (2)–(8). This quantity is to be
distinguished carefully in the COBE results from $Q_{\text{rms-ps}}$, which is often referred to and which is not the true best-fit value of the quadrupole, but rather the value of the quadrupole derived from a power spectrum fit (when a power law is assumed) of the other higher order multipole moments (see Bennett et al. 1994; Smoot et al. 1992).

The dipole moment can be neglected according to the fairly well justified assumption that it is all due to our peculiar motion with respect to the rest frame of the microwave background. In fact, this is what is done in the COBE anisotropy analysis (Bennett et al. 1994). However, we should be aware that there could in principle be a small non-Doppler contribution to the CMB dipole (see Maartens et al. 1995a, 1996 concerning this). Thus, in our calculations below we shall set

$$\epsilon_1 = 0 .$$

The COBE workers have not yet published the rms octupole from their data, but G. Smoot (private communication) has kindly informed us that the rms octupole results from COBE are

$$O_{\text{rms}} = 16 \pm 8 \, \mu\text{K} .$$

This is consistent with the results given by Wright et al. (1994) in their Figure 1.

There are several important observational and data-reduction issues and one theoretical issue that we should mention briefly here, to provide the background against which we can understand and appreciate these COBE multipole results.

The first is that the COBE DMR experiment does not measure directly the dipole, quadrupole, and octupole moments of the temperature anisotropy, but rather the two-point correlation function of the temperature anisotropy (Smoot et al. 1992; Padmanabhan 1993, p. 220; Partridge 1995),

$$C(\alpha) = \langle S(n) S(m) \rangle = \sum_{l=1}^{\infty} \Delta T_l^2 \, W(l)^2 \, P_{l}(\cos \alpha) ,$$

where $\alpha$ is the angle between the two points, $n$ and $m$ are unit vectors denoting the two different directions, so that $\cos \alpha = n \cdot m$, and the angle brackets signify the average over all pairs of points on the sky with separation angle $\alpha$. Furthermore, $S(n) \equiv \delta T(n)$, the temperature anisotropy in a given direction $n$ on the plane of the sky. The $\Delta T_l^2$ (with $l \equiv L$) are the squares of the rotationally invariant rms multipole moments (thus, $\Delta T_2^2 = Q_{\text{rms}}^2$; see Bennett et al. 1994, for instance),

$$\Delta T_l^2 = \frac{1}{4\pi} \sum_{m} |a_{lm}|^2 ,$$

where the $a_{lm}$ values are the coefficients of the expansion of the temperature anisotropy in spherical harmonics, i.e.,

$$S(n) \equiv S(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi) ,$$

$\theta$ and $\phi$ being, of course, the angular coordinates at which the temperature anisotropy is being measured. In equation (14), finally, $P_l(\cos \alpha)$ is just the Legendre polynomial of degree $l$ given as a function of $\cos \alpha$, and $W(l)$ is the window function, which describes the smoothing properties of the instrument’s beam. For detectors measuring large-angle CMB anisotropies, it essentially weights the multipole moments in such a way that the higher multipoles are smoothed over; that is, the instrument is insensitive to anisotropies on angular scales less than a certain $l$-pole, and $W(l)$ describes the insensitivity and resulting transfer of power in the measurement from higher multipoles to lower ones. Thus, when the actual quadrupole or octupole is determined from the data, $\Delta T_l$ must be deconvolved from $W(l)$. When the rms dipole, quadrupole, and octupole results are given by the COBE researchers, this deconvolution has already been performed. This is one of the procedures that must be effected to give us the real rms multipole moments.

The second major observational and data reduction problem is that in the COBE measurements there is a great deal of contamination by experimental systematic errors, including Galactic emission (see Bennett et al. 1994; Smoot et al. 1992; and references therein). The lower multipole moments are the most susceptible to these distortions. Furthermore, when the data are processed, the entire region containing the Galaxy is removed from the data set. This “Galactic cut” destroys the orthogonality of the spherical harmonics and leads to further aliasing of higher order multipole power onto the lower multipoles (dipole, quadrupole, octupole, etc.). Corrections for this are estimated on the basis of Monte Carlo simulations (Bennett et al. 1994) and included in the published values for the rms multipoles. There are a number of other complex issues that it has been necessary to resolve in arriving at these values (see Bennett et al. 1994; Smoot et al. 1992; Wright et al. 1994 for further discussion).

Finally, there is the theoretical-observational issue of “cosmic variance.” Actually, cosmic variance does not affect what we are concerned with here, as we shall see. But it is important to realize why it does not. It may explain why the multipoles we measure have the values they have relative to theoretical models of the perturbation spectrum, but it does not lead to observational errors, which would have to be corrected for. It does affect the comparison of the measured multipole power spectrum with the theoretical power spectrum predicted from, say, inflationary models (Abbott & Wise 1984). If the primordial perturbation spectrum originated due to fluctuations in the inflaton field, as we think it did, then the values of the temperature anisotropy multipoles they induce will be random variables with a certain distribution, probably Gaussian with zero mean (Liddle & Lyth 1993), and thus with a certain variance. Our observable universe is only one realization of that ensemble of universes represented by the probability distribution. Therefore, the value for each multipole we obtain from our observations will give us just one point of the distribution, which, in general, will not reflect the ensemble averaged value. It will deviate from it by a certain amount, which can be estimated theoretically by the variance (Liddle & Lyth 1993). This variance goes as $2/(2l + 1)$ and so will be more significant for the lower multipoles (Abbott & Wise 1984; Smoot et al. 1992). Our concern here, however, is not to compare the observed power spectrum of CMB anisotropies with the theoretical spectrum of density perturbations generated by an inflationary scenario. It is merely to use the best values of the CMB multipole moments we have available, however they are generated and whatever their spectrum, to set definite limits on the large-scale anisotropy and inhomogeneity of the observable universe itself. Thus, cosmic variance falls outside those issues that we need to consider in arriving at those limits.
4. CALCULATING THE ANISOTROPY AND INHOMOGENEITY LIMITS

We are now ready to use the values for the rms dipole, quadrupole, and octupole of the Cobe team, which have obtained so far to determine the anisotropy and inhomogeneity of the universe on very large scales. As indicated above, we set the dipole equal to zero (eq. [12]). Thus, we shall set \( \epsilon_i = 0 \) in our equations (2)–(8). In order to transform equations (11) and (13) into values of \( |\tau_{ab}| \) and \( |\tau_{abc}| \), respectively, we need to divide the results of equations (11) and (13) by \( T = 2.73 \) K, since our multipoles are for \( \delta T / T \). We do not need to divide them by \( (2l + 1)^{1/2} \), since our multipole quantities are given as the absolute values, which we defined as the square root of the sum of the squares of the components (see above). Finally, we also need to relate the \( \Delta T^2 \), the coefficients in the usual Legendre polynomial expansion, equation (14), to the \( |\tau_{a1...a3}| \). The numerical relationship is (Gebbie 1996)

\[
\langle \tau_{a1} \tau^{a1} \rangle = \frac{3}{2} \frac{(2l)!}{2(l!)^2} \Delta T^2_l, \tag{17}
\]

assuming Guassian fluctuations. This gives

\[
|\tau_{ab}|^2 = 13.5Q_{\text{rms}}^2, \tag{18}
\]

and

\[
|\tau_{abc}|^2 = 67.5Q_{\text{rms}}^2. \tag{19}
\]

We obtain, therefore,

\[
\langle \epsilon_2 \rangle = 1.4 \pm 0.9 \times 10^{-5} \tag{20}
\]

and

\[
\langle \epsilon_3 \rangle = 4.8 \pm 2.4 \times 10^{-5}. \tag{21}
\]

In examining equations (2)–(8), we notice that they are not expressed in terms of average values of the components. Let us now consider them to be equations for the rms averages of the indicators they represent, which we can do, since our \( \epsilon \) values are all positive. Now writing the Hubble parameter as \( H = 100 \) km s\(^{-1}\) Mpc\(^{-1}\), \( 0.4 < h < 1.0 \), and neglecting terms containing the factor \( \Omega_K \), since this is presently so small \( (\Omega_K = 4.11 h^{-2} \times 10^{-5}; \text{Roos 1994, p. 99}) \), we obtain

\[
\left\langle \frac{\nabla_a \mu}{\mu} \right\rangle < 3.8h \times 10^{-8} \text{ Mpc}^{-1}, \tag{22}
\]

\[
\left\langle \frac{\sigma_{ab}}{\Theta} \right\rangle < 1.0 \times 10^{-4}, \tag{23}
\]

\[
\left\langle \frac{\psi_{ab}}{\Theta} \right\rangle < 3.2 \times 10^{-6}, \tag{24}
\]

\[
\left\langle \frac{\|V_a\rho\|}{\Theta} \right\rangle < (1.22\Omega_M^{-1} + 0.04)h \times 10^{-6} \text{ Mpc}^{-1}, \tag{25}
\]

\[
\left\langle \frac{\|V_a\Theta\|}{\Theta} \right\rangle < 6.4h \times 10^{-8} \text{ Mpc}^{-1}, \tag{26}
\]

\[
\left\langle \frac{|H_{ab}|}{\Theta} \right\rangle < (3.5 + 0.3\Omega_M)h \times 10^{-7} \text{ Mpc}^{-1}, \tag{27}
\]

In equations with \( H \) we have put a factor of \( c^{-1} \) back into the equations to give the correct order of magnitude and the correct dimensions. This factor is hidden in the normalization of the 4-velocity \( \nu \), which figures implicitly in the equations.

These are the limits we desire on the anisotropies and the inhomogeneities of the universe. In the past, other groups have put limits on the shear and the vorticety using limits on CBR isotropies (Bajtlik et al. 1986; Martinez-Gonzalez & Saz 1995; Barrow, Juszkiewicz, & Sonoda 1985). But in doing so they assumed exact spatial homogeneity, which yields a limit on shear that is too strong (Maartens et al. 1996). Our limiting scheme, as we have mentioned, does not assume exact homogeneity, or even that the inhomogeneities and anisotropies are small. The result that they are small is due to the fact that the CBR anisotropies are small. The primary assumption we have made is a weak form of theCopernican principle: that all fundamental observers in the relevant spacetime domain measure at most the same level of CBR anisotropy. This, as we have pointed out, is at least partially testable and fully falsifiable. A single observation demonstrating that CBR anisotropies relative to some other cluster of galaxies are large compared to those we observe would banish that assumption. Finally, our approach provides limits on the full range of possible isotropies and inhomogeneities, including spatial gradients of the radiation and matter densities, and of the expansion parameter, and the electric and magnetic components of the Weyl tensor. It is worth pointing out that the Weyl tensor components measure those parts of the gravitational curvature that are not determined locally by the mass-energy distribution (that is, via the Einstein field equations). Instead, they are determined by the Bianchi identities, and so, in a sense, they are due to the mass-energy distribution at other, more distant points (Hawking & Ellis 1973, pp. 85–88; Maartens et al. 1996). \( H_{ab} \), the magnetic Weyl tensor, measures the amount of gravitational radiation in the spacetime. It vanishes in the case in which exact spatial homogeneity is assumed. \( E_{ab} \), the electric Weyl tensor, which also vanishes in that case, measures the tidal, shear-inducing force of the global gravitational field. Thus, it manifests its presence by the shear it introduces in timelike and null congruences of geodesics—worldlines of particles. In fact, the measurement of null shear in bundles of light rays would be the clearest signature of the presence of nonzero \( E_{ab} \) (see Hawking & Ellis 1973).

We see clearly from our results, equations (19)–(25), that in every case the anisotropy and inhomogeneity measures for the universe on the largest scales are very small, despite the significant inhomogeneities that have been detected on intermediate scales. This provides very strong justification for considering the observable universe to be almost FLRW on large scales; that is, the deviations from FLRW are small enough to be considered perturbations.

Finally, we might wonder what the relevance of these limits is to “the averaging problem” (see Zotov & Stoeger 1995, and references therein) in cosmology. Do these results demonstrate that the universe is almost FLRW whatever the resolution of those issues happen to be? Unfortunately, that is not the case. The analysis we have implemented here assumes that the effective theory of gravity on cosmological
scales is general relativity. The averaging problem calls that assumption into question.

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