Error analysis of mathematical problems on TIMSS: A case of Indonesian secondary students

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Abstract. Indonesian students’ competence in solving mathematical problems is still considered as weak. It was pointed out by the results of international assessment such as TIMSS. This might be caused by various types of errors made. Hence, this study aimed at identifying students’ errors in solving mathematical problems in TIMSS in the topic of numbers that considered as the fundamental concept in Mathematics. This study applied descriptive qualitative analysis. The subject was three students with most errors in the test indicators who were taken from 34 students of 8th graders. Data was obtained through paper and pencil test and student’s interview. The error analysis indicated that in solving Applying level problem, the type of error that students made was operational errors. In addition, for reasoning level problem, there are three types of errors made such as conceptual errors, operational errors and principal errors. Meanwhile, analysis of the causes of students’ errors showed that students did not comprehend the mathematical problems given.

1. Introduction
There is an extensive literature on error analysis in mathematics problem-solving. The main concern is to improve students’ learning and understanding of Mathematics. Several researches on error analysis on mathematics problem solving were done such as in the topic of the fraction. Abdullah et.al [1] shared several errors in solving HOTS problems such as comprehension error, transformation error, process skill error, and encoding error. Problem-solving is an activity that involves various actions in the mind of thought including accessing and using knowledge and experience [2]. Thus Polya in 1973 [3], teaching strategies that involve the use of non-routine problems in the classroom give students the opportunity to develop higher order thinking skills in the process of understanding, exploration, and application of mathematical concepts. Since problem-solving involves the activities of processing, the knowledge used in the process of solving the problems is different. As stated by Mayer [4], the knowledge covers such as knowledge of language and facts, knowledge of schemes, knowledge of algorithms, and strategic knowledge. In short, students need to equip themselves with various knowledge and high skills in problem-solving. Quoting Widiharto in 2008 [5], mathematical concepts and skills that are not fully mastered by students have led to difficulties and errors in solving mathematical problems. A study conducted by Susanti et al. [6] found out that students are difficult to solve problems that involve the use of HOTS and among the difficulties faced by them are reading and interpreting data, determining and delegating data, and making conclusions and arguments.
Herholdt and Sapire in 2014 [7] stated that error analysis, also referred to error pattern analysis, is the study of errors in learners’ work with a view to finding explanations for these reasoning errors. Not all errors can be attributed to reasoning faults; some are simply careless errors [8], identified as “slips” from Olivier [9], which can easily be corrected if the faulty process is pointed out to the learner. Slips are random errors in declarative or procedural knowledge, which do not indicate systematic misconceptions or conceptual problems [10]. Error analysis is concerned with the pervasive errors (or ‘bugs’) which learners make, based on their lack of conceptual or procedural understanding (Ketterlin-Geller & Yovanoff 2009).

As a result, the performance of Indonesian students in the international assessments that test students' thinking skills, namely Trends in International Mathematics and Science Studies (TIMSS) and Programme for International Student Assessment (PISA) was not at the satisfactory level. Based on the data of TIMSS 2011, Indonesian students’ mathematics achievement was in the ranked 38 out of 42 countries which followed the TIMSS with a score of 386 of the benchmark score of 500 [11]. Generally, one of the causes was that they were lack of knowledge in resolving TIMSS problems [12].

There are two domains that are being tested in TIMSS assessments, namely content and cognitive domains. For the tested cognitive domain, it covers applying, analyzing, and reasoning, which the components of higher-order thinking are in Bloom’s Taxonomy. Meanwhile, the tested content domain covers four areas of mathematics learning namely Numbers, Geometry, Algebra, Data, and Probability. The number is one of the fundamental subjects that is tested in TIMSS which percentages is 30% becomes the greatest percentages in the content domain on TIMSS (Mullis et al., 2013). The concept of number is connected and as the basis and prerequisite for the understanding of the next concept [13]. Therefore, it is important to know the errors of number problems on TIMSS.

This study was carried out to identify students’ errors in solving mathematics problem in TIMSS in the topic of numbers which considered as the fundamental concept in Mathematics.

2. Methodology

This study is a descriptive study that used a qualitative approach. The subject was three students with most errors in the test indicators who were taken from 34 students of 8th graders in a secondary school of the districts Sidoarjo, East Java. Data was obtained through paper and pencil test students’ interview. The instrument used for this study was a set of test questions to identify the types of students’ errors.

The items contained in the instrument have been adopted with the questions of international assessments of Trends in International Mathematics and Science Study (TIMSS). Four items were built in a subjective form, and the items contained the elements of Applying and Reasoning level problems as shown in Table 1.

The subjects of the study were required to answer the questions contained in the instruments that have been prepared under the supervision of individual mathematics teacher who taught the respective class. The time allocated to answer questions was 60 minutes. The errors were analyzed using three types of errors that are conceptual errors, operational errors, and principal errors.
| Item | Question                                                                                                                                                                                                                                     | Level   |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| 1    | Penny had a bag of marbles. She gave one-third of them to Rebecca, and then one-fourth of the remaining marbles to John. Penny then had 24 marbles left in the bag. How many marbles were in the bag to start with?                                      | Applying|
| 2    | P and Q represent two fractions on the number line above. P × Q = N. Shows the location of N on the number line?                                                                                                                               | Reasoning|
| 3    | Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.                                                                                                   | Reasoning|
| 4    | John and Cathy were told to divide a number by 100. By mistake, John multiplied the number by 100 and obtained an answer of 450. Cathy correctly divided the number by 100. What was her answer?                                                         | Applying|

3. Data analysis

3.1 Analysis of the types of errors

This section discusses the errors made by the students. There are three types of students’ errors; they are conceptual error, operational error, and principal error.

3.1.1. Conceptual error. Conceptual error occurs when the students are not able to apply the concept of number. This error recorded on Reasoning level problems only.

![Figure 1. Example of conceptual error](image-url)

Question: P and Q represent two fractions on the number line above. P × Q = N. Shows the location of N on the number line?

The student was not able to multiply two fractions and to determine the value of P and Q. He considered that the way to multiply two fractions is to multiply across that is multiplying the numerator of the first fraction with a denominator of the second fraction and the numerator of the
second fraction with the denominator of the first fraction, in which he failed to solve the following problem. He made a conceptual error such as multiply two fractions. From the interview, students were not able to understand the way of multiplying two fractions and to determine what value of the number line.

![Figure 2. Example of conceptual error](image)

The student was not able to identify the procedure and solve the problem. She considered that item 2 could be solved by algebra operation such as multiplication algebra. She thought whatever problem that contained alphabet must be solved by algebra solution. The interview showed that this error caused by the subject who did not comprehend the mathematical problem given.

3.1.2. Operational Error. Operational error occurs when the students are not able to calculate numbers. This error recorded on Applying and Reasoning level problems

- For applying level problem

  Question: John and Cathy were told to divide a number by 100. By mistake, John multiplied the number by 100 and obtained an answer of 450. Cathy correctly divided the number by 100. What was her answer?

![Figure 3. Example of operational error in applying level problem](image)

The student was not able to compute the operation. The subject considered that \(45 \times 100 = 450\), in which he failed to solve the following problem. He made an operational error such as on multiplying. The interview showed that this error caused because the subject was careless.

- For reasoning level problem

  Question: P and Q represent two fractions on the number line above. \(P \times Q = N\). Shows the location of \(N\) on the number line?

![Figure 4. Example of operational error in reasoning level problem](image)
The student was not able to compute the operation. The subject considered that \( \frac{1}{2} \times \frac{1}{3} = \frac{2}{2} \) in which he failed to solve the following problem. He made an operational error such as on multiplying two fractions. The interview showed that this error caused because the subject was not able to understand to compute two fractions.

Question: Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.

![Figure 5. Example of operational error in reasoning level problem](image)

Figure 5. Example of operational error in reasoning level problem

The student was not able to compute the operation. She made an operational error such as on multiplying whole numbers. The interview showed that this error caused because the subject was careless.

3.1.3. Principal Error. Principal error occurs when the students are not able to answer the final answer that is caused by the previous error in the problem-solving. This error recorded on Reasoning level problems only.

Question: Place the four digits 4, 6, 8, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.

![Figure 6. Example of principal error](image)

Figure 6. Example of principal error
The student was not able to determine the final answer. She made a major error. The interview showed that this error caused because the subject was not able to determine the suitable numbers that were produced greatest value if they were multiplied.

4. Conclusion
As conclusion, it was found that students tend to make three types of errors; they were conceptual error, operational error, and principal error. Some of conceptual errors were translating word problem to mathematics problem; using multiplication fraction, and determining fraction’s order in number line. The operational error that made by students was the subject could not use the operation correctly. For the principal error made by students was determining final solution due to the previous errors.

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