Probing the critical exponent of superfluid fraction in a strongly interacting Fermi gas

Hui Hu\(^1\) and Xia-Ji Liu\(^\ddagger\)

\(^1\)Centre for Atom Optics and Ultrafast Spectroscopy, Swinburne University of Technology, Melbourne 3122, Australia

(Dated: October 15, 2013)

We theoretically investigate the critical behavior of second sound mode in a harmonically trapped ultracold atomic Fermi gas with resonant interactions. Near the superfluid phase transition with critical temperature \(T_c\), the frequency or the sound velocity of second sound mode depends crucially on the critical exponent \(\beta\) of superfluid fraction. In an isotropic harmonic trap, we predict that the mode frequency diverges like \((1 - T/T_c)^{\beta - 1/2}\) when \(\beta < 1/2\). In a highly elongated trap, the speed of second sound reduces by a factor \(1/\sqrt{2\beta + 1}\) from that in a homogeneous three-dimensional superfluid. Our prediction could be readily tested by measurements of second sound wave propagation in a setup such as that exploited by Sidorenkov et al. [Nature 498, 78 (2013)] for resonantly interacting lithium-6 atoms, once the experimental precision is improved.

PACS numbers: 03.75.Kk, 03.75.Ss, 67.25.D-

I. INTRODUCTION

Superfluidity, a remarkable state of matter in which particles flows with zero resistance, is a ubiquitous quantum phenomenon occurring in diverse systems ranging from liquid helium, high-temperature superconductors, to neutron stars \([1, 2]\). While at zero temperature all the particles in the system participate into the superfluid motion, at finite temperatures because of thermal excitations only a portion of particles - named as superfluid fraction - behaves in such a way. The remaining particles comprise a normal fluid component that behaves like an ordinary fluid \([3, 4]\). To characterize superfluidity, it is therefore crucial to understand the superfluid fraction, which, unfortunately is notoriously difficult to calculate microscopically for strongly interacting quantum systems, especially near the superfluid phase transition. In this respect, the recently realized ultracold atomic Fermi gases with controllable interatomic interactions and external harmonic trapping potentials \([3, 6]\), known as a new type of strongly interacting superfluid, provide unique opportunities to explore superfluidity and understand superfluid fraction in the strongly interacting regime. In this paper, we propose that the critical behavior of superfluid fraction of a resonantly interacting atomic Fermi gas at unitarity (where atoms occupying unlike spin states interact with an infinitely large scattering length) could be well characterized through the measurement of second sound propagation.

Second sound, as well as first sound, is a coupled oscillation of the superfluid and normal fluid components at finite temperatures \([3, 4]\). In contrast to first sound, which is an in-phase oscillation of the two components (i.e., density oscillation), second sound is an out-of-phase oscillation (i.e., temperature or entropy wave) and depends very sensitively on the superfluid fraction. Therefore, it presents arguably the most dramatic manifestation of superfluidity. Indeed, in superfluid helium the accurate determination of superfluid fraction below the lambda point is provided by the measurement of second sound \([1, 2]\). Very recently, for a resonantly interacting Fermi gas of lithium-6 atoms confined in highly elongated harmonic traps, the superfluid fraction is qualitatively extracted from the measurement of second sound velocity along the weakly confined axial direction, as reported by Sidorenkov et al. \([8]\). This milestone experiment already imposes a grand challenge, since the theoretical predictions for the temperature dependence of the superfluid fraction in the unitary limit are rather incomplete \([3, 12]\). Our proposal, together with future second sound measurement with better precision in such ultracold atomic systems, allows an accurate determination of the critical behavior of the superfluid fraction just below the superfluid phase transition.

Our main results are briefly summarized as follows. We consider both isotropic and highly elongated harmonic traps. The latter situation is exactly the setup exploited in the current experiment \([8]\). For isotropic traps, we find that slightly below the superfluid transition temperature \(T_c\), the mode frequency of the second sound diverges as \((1 - T/T_c)^{\beta - 1/2}\) if the critical exponent of the superfluid fraction \(\beta < 1/2\). While for highly elongated traps, the speed of second sound along the weakly confined direction reduces by a factor \(1/\sqrt{2\beta + 1}\) from that in a three-dimensional free space. In both cases, the sensitive dependence of the second sound mode on the critical exponent leads to an accurate calibration of \(\beta\).

II. TWO-FLOW HYDRODYNAMICS

First and second sound are well described by the equations of two-fluid hydrodynamics first derived by Landau \([1]\). As discussed in the previous works \([3, 13, 14]\), in the dissipationless regime the solutions of these hydrodynamic equations with frequency \(\omega\) at temperature \(T\) can
be derived by minimizing a variational action, which, in terms of displacement fields \( \mathbf{u}_s(\mathbf{r}) \) and \( \mathbf{u}_n(\mathbf{r}) \), is given by,

\[
S = \frac{1}{2} \int \text{d}t \left[ \omega_s^2 (\rho_s u_s^2 + \rho_n u_n^2) - \frac{1}{\rho_0} \left( \frac{\partial P}{\partial \rho} \right)_s (\delta \rho)^2 - 2\rho_0 \left( \frac{\partial T}{\partial \rho} \right)_s (\delta \rho)(\delta s) + \left( \frac{\partial T}{\partial \rho} \right)_s (\delta s)^2 \right]. \tag{1}
\]

Here, \( \rho_s(\mathbf{r}) \) and \( \rho_n(\mathbf{r}) \) are the superfluid and normal fluid densities for a gas with total mass density \( \rho(\mathbf{r}) \equiv \rho_s + \rho_n \). \( P(\mathbf{r}) \) is the local pressure of the gas and \( s(\mathbf{r}) = s/\rho \) is the entropy per unit mass. \( \delta \rho(\mathbf{r}) = -\nabla \cdot [\rho_s \mathbf{u}_s + \rho_n \mathbf{u}_n] \) and \( \delta s(\mathbf{r}) = -\mathbf{u}_s \cdot \nabla s_0 + (\delta s_0/\rho_0) \nabla \cdot [\rho_0(\mathbf{u}_s - \mathbf{u}_n)] \) are the density and entropy fluctuations, respectively. The displacement fields are related to the superfluid and normal velocity fields by \( d\mathbf{u}_s/\text{d}t = \mathbf{v}_s \) and \( d\mathbf{u}_n/\text{d}t = \mathbf{v}_n \). The effect of the external harmonic trapping potential \( V_T(\mathbf{r}) = m\omega_T^2 z^2_0/2 + m\omega_s^2 z^2/2 \) enters Eq. (1) through the position dependent equilibrium thermodynamic functions, which we have indicated by the subscript “0”. For a resonantly interacting Fermi gas, all these thermodynamic functions - except the superfluid density - are known to certain precision, owing to the recent experimental analysis of the homogeneous equation of state performed by the MIT team [15] by using the universality relations satisfied by the unitary Fermi gas [16, 17]. Throughout the work, we calculate the trapped density profile and thermodynamic functions based on the smoothed experimental MIT data [13] and the local density approximation (LDA) which amount to setting a local chemical potential \( \mu(\mathbf{r}) = \mu - V_T(\mathbf{r}) \), where \( \mu \) is the chemical potential at the trap center.

In the absence of the coupling term between density and entropy fluctuations (i.e., \( \rho_0(\partial T/\partial \rho)_s = 0 \)), Eq. (1) admits two decoupled solutions: a pure in-phase mode with the Ansatz \( \mathbf{u}_s(\mathbf{r}) = \mathbf{u}_n(\mathbf{r}) = \mathbf{u}^{(1)}(\mathbf{r}) \) and a pure out-of-phase mode with \( \rho_0 \mathbf{u}_s^{(2)}(\mathbf{r}) + \rho_0 \mathbf{u}_n^{(2)}(\mathbf{r}) = 0 \), which may be referred to as first and second sound, respectively. These first and second sound modes are the exact variational solutions for pure density \( |\delta \rho(\mathbf{r})| = 0 \) and pure temperature \( |\delta s(\mathbf{r})| = 0 \) oscillations. When the coefficient \( \rho_0(\partial T/\partial \rho)_s \) is nonzero, the first and second sound are necessarily coupled. This coupling can be conveniently characterized by the dimensionless Landau-Placzek (LP) parameter \( \epsilon_{\text{LP}} \equiv \gamma - 1 \) [18], where \( \gamma \equiv \tilde{c}_v/\tilde{c}_s \) is the ratio between the equilibrium specific heats per unit mass at constant pressure \( [\tilde{c}_P = T(\partial s/\partial T)|_P] \) and density \( [\tilde{c}_s = T(\partial s/\partial T)|_s] \). In superfluid helium, \( \tilde{c}_v \approx \tilde{c}_s \) or \( \epsilon_{\text{LP}} \approx 0 \), implying \( \rho_0(\partial T/\partial \rho)_s \approx 0 \). Thus, the solutions of the two-fluid hydrodynamic equations for superfluid helium are perfectly described by decoupled first and second sound modes. For resonantly interacting atomic Fermi gases, the universality relations give rise to \( \rho_0(\partial T/\partial \rho)_s = 2T/3 \neq 0 \) [9]. Close to the superfluid transition temperature \( T_c \approx 0.167T_F \), where \( T_F \) is the Fermi temperature, we estimate from the MIT data that the LP parameter is about \( \epsilon_{\text{LP}} \approx 0.4 \). Therefore, similarly to superfluid liquid helium, the solutions of two-fluid equations for a unitary Fermi gas are well approximated by weakly coupled first and second sound modes. We note that the smallness of the LP parameter in superfluid helium and unitary Fermi gas and hence the weak coupling between their first and second modes is a general consequence of strong interactions [18]. Note also that the existence of harmonic traps will significantly reduce the sound mode coupling, as we shall see later. Hereafter, we focus on the second sound mode by neglecting its coupling to the first sound mode. In the past, the first sound mode of a unitary Fermi gas has been studied in greater detail, both at zero temperature [10, 22] and finite temperatures [23].

### III. SECOND SOUND NEAR SUPERFLUID TRANSITION

Inserting the Ansatz \( \mathbf{u}_s^{(2)}(\mathbf{r}) = -\rho_0/\rho_0 \mathbf{u}_n^{(2)}(\mathbf{r}) \) for second sound into Eq. (1), taking the variation with respect to \( \mathbf{u}_n^{(2)}(\mathbf{r}) \) and making use of standard thermodynamic relations, we obtain the following equation for the superfluid displacement field [14]:

\[
\omega_s^2 \mathbf{u}_n^{(2)} = -\tilde{s}_0 \nabla \left[ \frac{1}{\rho_0} \left( \frac{\partial T}{\partial s} \right)_s \nabla \cdot \left( \frac{s_0 \rho_0}{\rho_0} \mathbf{u}_n^{(2)} \right) \right]. \tag{2}
\]

As \( \delta \rho(\mathbf{r}) = 0 \) for second sound, we may also rewrite the above equation into a closed form for the temperature fluctuation \( \delta T(\mathbf{r}) = (\partial T/\partial \rho)_s \delta \rho(\mathbf{r}) + (\partial T/\partial s)_s \delta s(\mathbf{r}) = \rho_0^{-1}(\partial T/\partial s)_s \nabla \cdot [s_0 \rho_0 \mathbf{u}_n^{(2)}/\rho_0] \), where \( \delta s(\mathbf{r}) \) is the entropy fluctuation described below Eq. (1). This gives rise to,

\[
\omega_s^2 \delta T(\mathbf{r}) = -\frac{1}{\rho_0} \left( \frac{\partial T}{\partial s} \right)_s \nabla \cdot \left[ \frac{s_0 \rho_0}{\rho_0} \nabla \delta T(\mathbf{r}) \right]. \tag{3}
\]

In homogeneous space, where \( \delta T(\mathbf{r}) \propto e^{i\mathbf{k}\cdot\mathbf{r}} \), we recover the well-known result for the second sound velocity:

\[
\epsilon_{2,\text{hom}}^2 = T(s_0 \rho_0)/(c_v \rho_0). \tag{4}
\]

In the presence of harmonic traps, it seems to be cumbersome to directly solve Eq. (3). Fortunately, near the superfluid transition this equation could be greatly simplified, as the temperature oscillation has to be restricted in a small superfluid area around the trap center and therefore we may safely neglect the position dependence of all thermodynamic functions - except the superfluid density. Furthermore, close to transition it is reasonable to assume the following critical behavior for superfluid fraction,

\[
\frac{\rho_0}{\rho_0} = \eta \left( 1 - \frac{T}{T_c} \right)^{2\beta},
\]

where the constant \( \eta \) and the critical exponent \( \beta \) are to be determined for a unitary Fermi gas. It is known from the Leggett model of pairing at unitarity that the mean-field BCS wave function gives rise to \( \eta = 2 \) and \( \beta = 1/2 \) [24].
However, a superfluid with a two-component order parameter (and a bosonic fluctuation spectrum) generally undergoes a second order phase transition with a superfluid density that varies as \( \rho_s \propto (T_c - T)^{2/3} \) close to the transition, independent of the interaction strength. Indeed, in superfluid liquid helium, the second sound measurement suggests that \( \eta \approx 3.2 \) and \( \beta \approx 1/3 \) [7]. With these considerations, we find that within LDA:

\[
\omega^2 \delta T = -\frac{\eta k_B T_F f_s^2}{m f_s} \nabla \cdot \left\{ \left( 1 - \frac{T}{T_c} \right)^{2\beta} \left[ 1 - \frac{T V_T (r)}{T (T_F - T)} \left( T_F f_\mu - T f'_\mu \right) \right]^{2\beta} \nabla \delta T \right\}
\]  

(5)

where we have expressed the entropy and chemical potential of a uniform unitary Fermi gas in terms of dimensionless functions of the reduced temperature \( T/T_F \): \( s = n k_B f_s(T/T_F) \) and \( \mu = k_B T_F f_\mu(T/T_F) \); \( T_F = h^2 (3\pi^2 n)^{2/3} / (2mk_B) \) is the Fermi temperature at the trap center with density \( n \), and \( f_{s,\mu} = df_{s,\mu}/dT(T/T_F) \).

It is readily seen that the superfluid area is restricted to \( r_\perp \leq R_\perp s \) and \( z \leq R_z s \), where \( m \omega^2 R^2 / 2 = m \omega^2 R^2 / 2 = (T_c/T - 1) k_B (T_F f_\mu - T f'_\mu) \). By introducing the scaled coordinates \( \tilde{r}_\perp = r_\perp / R_\perp s \) and \( \tilde{z} = z / R_z s \), we may rewrite Eq. (5) into the following dimensionless form,

\[
\tilde{\omega}^2 \delta \tilde{T} + \nabla \cdot \left[ \left( 1 - \tilde{r}^2 - \tilde{z}^2 / \lambda^2 \right)^{2\beta} \nabla \delta \tilde{T} \right] = 0,
\]  

(6)

where \( \lambda \equiv \omega_T / \omega_\perp \) is the aspect ratio of the harmonic trap and a reduced mode frequency \( \tilde{\omega} \) is defined by

\[
\tilde{\omega} \equiv \sqrt{D} (1 - T/T_c)^{\beta-1/2} \omega_T
\]  

(7)

with \( D = \eta (T T_F f_s^2)/(2 T_0 f_s(T_F f_\mu - T f'_\mu)) \). In Fig. 1, we show the temperature dependence of the parameter \( D \) near the superfluid transition, calculated using the MIT data for the equation of state of a unitary Fermi gas [13]. It is typically at about 0.2. From Eq. (7), one may realize immediately that for any non-zero discrete mode frequency, it would become divergent when temperature approaches to the superfluid transition temperature if the critical exponent \( \beta < 1/2 \).

### A. Isotropic traps

For an isotropic harmonic trap, we may recast Eq. (6) into a one-dimensional differential equation, for example, in the sector of zero angular momentum \( l = 0 \) (i.e., breathing modes),

\[
\left[ R(x) \frac{d^2}{dx^2} + P(x) \frac{d}{dx} + Q(x) \delta T(x) \right] = 0,
\]  

(8)

where \( x \equiv \tilde{r}^2 \leq 1 \) and the coefficients \( R(x) = x(1 - x)^{2\beta+1} \), \( P(x) = [3/2 - (3/2 + 2\beta)x] (1 - x)^{2\beta} \), and \( Q(x) = \tilde{\omega}^2 (1 - x)/4 \). It can be solved numerically by using a
cies decrease quickly with increasing the critical exponent $\beta$. When $\beta$ is an integer or half-integer, our numerical results could be examined analytically, as the solutions for the temperature fluctuation are simply polynomials and therefore the mode frequency are known precisely. Indeed, for $\beta = 1/2$ (i.e., mean-field superfluid fraction), Eq. (6) has exactly the same structure as the hydrodynamic equation that describes oscillations of a zero-temperature Bose condensate [27, 28]. It admits analytical solutions for harmonic traps with arbitrary aspect ratio. In the case of isotropic traps, the reduced mode frequency is given by

$$\omega_{nl} = 2\sqrt{n^2 + n/2 + 1/2}.$$  

Now, we are able to calculate the mode frequency, by using Eq. (7). The two lowest mode frequencies are shown in Fig. 3 for superfluid-helium-like superfluid fraction (black solid lines) or mean-field-like superfluid fraction (red dashed lines). In the former case, the divergence of the mode frequency near superfluid transition is evident, as we may anticipate.

For isotropic traps, it is worth noting that the equations of two-fluid hydrodynamics can be fully solved by using a variational approach [14]. We have performed such a calculation with a mean-field superfluid fraction. The results for the lowest second-sound mode frequency are reported in Fig. 3 by blue solid circles. The good agreement between the full variational calculation and the prediction of the simplified second-sound Eq. (9) gives a reasonable justification for all the assumptions that we have made to derive Eq. (6), including neglecting the coupling between first and second sound modes and the ignorance of the position dependence of thermodynamic functions used in Eq. (8).

**B. Highly elongated traps**

Experimentally, a resonantly interacting atomic Fermi gas is trapped in highly elongated harmonic trapping potentials. The second-sound is excited and propagated one-dimensionally along the long trap direction [8]. For such a configuration, we may assume that the temperature fluctuation has the form: $\delta T(\tilde{r}_\perp, \tilde{z}) = \delta T(\tilde{r}_{\perp}) e^{ikz}$, where $\tilde{k} = kR_{1+s}$ is the reduced wave vector. The fluctuation field $\delta T(\tilde{r}_{\perp})$ then satisfies,

$$\left[\omega^2 - \tilde{k}^2 (1 - \tilde{r}^2_\perp)^{2\beta}\right]\delta T + \tilde{\nabla} \cdot \left[(1 - \tilde{r}^2_\perp)^{2\beta} \tilde{\nabla}\delta T\right] = 0.$$  

For any reduced wave vector $\tilde{k}$, similar to the case of isotropic traps, the above equation in the sector of $l = 0$ can be rewritten in a one-dimensional (1D) differential form Eq. (11) by setting $x = \tilde{r}^2_\perp$, but with new coefficients $P(x) = [1 - (1 + 2\beta)x](1 - x)^{2\beta}$ and $Q(x) = [\omega^2 - (1 - x)^{2\beta}(1 - x)/4$. It can be solved numerically following Ref. [20]. Fig. 4 shows the results for $\beta = 1/3$. 

![Fig. 3: (Color online) Temperature dependence of the two (lowest breathing) second-sound mode frequency of an isotropically trapped unitary Fermi gas near the superfluid transition temperature. For a critical exponent $\beta < 1/2$, the mode frequency diverges at the transition. The blue solid circles are the full variational results of the two-fluid hydrodynamic equations for the lowest breathing second-sound mode.](image1)

![Fig. 4: (Color online) Reduced second-sound mode frequency for a unitary Fermi gas confined in highly elongated harmonic traps for the critical exponent $\beta = 1/3$ (a) and $\beta = 1/2$ (b). We assume that the second sound can propagate freely along the long trap axis. In (c), we show the reduced second-sound velocity $\tilde{c} = \partial \tilde{\omega} / \partial \tilde{k}$ for the lowest phonon modes.](image2)
and $\beta = 1/2$. In the latter case, our result in Fig. 4(b) agrees exactly with earlier prediction on first-sound propagation of a zero-temperature Bose condensate in highly elongated harmonic traps \cite{29, 31}, as it should be.

In Fig. 4, we observe multi-branches in the spectrum, each of which corresponds to a discrete radial excitation \cite{29, 31}. The lowest branch is of particular interest, as it resembles the phonon mode in free space and is the easiest mode to excite experimentally. The associated second-sound velocity is given by $c_{2,1D} = \partial \omega / \partial k = (\partial \omega / \partial k)_{c_{2,\text{hom}}}$. Thus, the velocity of second-sound propagated in quasi-1D geometry is reduced by a factor of $\tilde{c} = \partial \omega / \partial k$, with respect to the bulk value $c_{2,\text{hom}}$. The similar quenching of first-sound velocity due to confinement was pointed out earlier for a unitary Fermi gas \cite{30} or a Bose condensate \cite{29}. The value of $\tilde{c}$ may be calculated by integrating out the transverse coordinate $\tilde{r}_\perp$ in Eq. (9):

$$\tilde{c}^2 = \int d\tilde{r}_\perp (1 - \tilde{r}_\perp^2)^2 \delta T(\tilde{r}_\perp) / \int d\tilde{r}_\perp \delta T(\tilde{r}_\perp).$$

In the limit of long wave length ($\tilde{k} \rightarrow 0$), where the temperature fluctuation $\delta T(\tilde{r}_\perp)$ is radially independent, we find that $\tilde{c} = 1/\sqrt{2\beta + 1}$, in agreement with our numerical result shown in Fig. 4(c).

In Fig. 5 we report the 1D second-sound velocity $c_{2,\text{hom}}/\sqrt{2\beta + 1}$ by using the assumed superfluid-helium or mean-field like superfluid fraction Eq. (4). For comparison, we show also the experimental data extracted from Fig. 3(a) of Ref. \cite{8}. Close to the superfluid transition (i.e., $T > 0.95T_c$), our prediction of the 1D second-sound velocity with superfluid-helium-like superfluid fraction agrees reasonably well with the measurement. By noting that the superfluid fraction of a unitary Fermi gas resembles that of superfluid helium \cite{8}, this agreement somehow is an indication of the quenched second sound velocity. However, a quantitative experimental determination of the critical exponent $\beta$ requires a much better precision of data.

C. Conclusions

In conclusion, we have investigated theoretically how the second sound of a harmonically trapped unitary Fermi gas is affected by the critical exponent $\beta$ of superfluid-helium near the superfluid phase transition when temperature $T$ approaches to the critical temperature $T_c$. In an isotropic trap, the sound frequency goes like $(1 - T/T_c)^{\beta-1/2}$ and therefore exhibit clearly a divergence when $\beta < 1/2$. In an experimentally exploited highly elongated trap, the second sound velocity along the long trap axis reduces by a factor $1/\sqrt{2\beta + 1}$ with respect to its bulk value. Our prediction could be used to measure directly the critical exponent $\beta$ in future experiments, if the experimental accuracy gets improved.

Acknowledgments

The present work is dedicated to in memory of Professor Allan Griffin, who was always enthusiastic on observing the second sound in resonantly interacting atomic Fermi gases and was the driving force of our recent works \cite{8, 14, 18}. We thank Edward Taylor, Allan Griffin, Lev Pitaevskii and Sandro Stringari for their stimulating discussions during the early stage of this work in 2009. This work is supported by the ARC Discovery Projects (Grant No. DP0984522 and DP0984637) and NFRP-China (Grant No. 2011CB921502).

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