Drag and Diffusion of Heavy Quarks in a hot and anisotropic QCD medium

P. K. Srivastava* and Binoy Krishna Patra†

Department of Physics, Indian Institute of Technology Roorkee, Roorkee 247667, INDIA

The propagation of heavy quarks (HQs) in a medium was quite often modeled by the Fokker-Plank (FP) equation. Since the transport coefficients, related to drag and diffusion processes are the main ingredients in the FP equation, the evolution of HQs is thus effectively controlled by them. At the initial stage of the relativistic heavy ion collisions, asymptotic weak-coupling causes the free-streaming motions of partons in the beam direction and the expansion in transverse directions are almost frozen, hence an anisotropy in the momentum space sets in. Since HQs are too produced in the same time therefore the study of the effect of momentum anisotropy on the drag and diffusion coefficients becomes adversely desirable. In this article we have thus studied the drag and diffusion of HQs in the anisotropic medium and found that the presence of the anisotropy reduces both drag and diffusion coefficients. In addition, the anisotropy introduces an angular dependence to both the drag and diffusion coefficients, as a result both coefficients get inflated when the partons are moving transverse to the direction of anisotropy than parallel to the direction of anisotropy.

PACS numbers: 12.38.Mh, 12.38.Gc, 25.75.Nq, 24.10.Pa

I. INTRODUCTION

The main outcome of the relativistic heavy ion collision (rHIC) experiments is the creation of a deconfined medium of strongly interacting quarks and gluons, known as quark gluon plasma (QGP). Since the HQs are mainly produced at the initial stage of heavy-ion collisions and the thermalization time of HQs are of the order of the lifetime of QGP therefore HQs are the suitable candidate to probe the QGP. Due to the large mass of HQ, it was expected from perturbative QCD, the nuclear suppression factor, $R_{AA}$ would have been large, viz. $R_{AA} \sim 0.6$ and 0.8 - 0.9 for charm and bottom quarks, respectively [1, 2] and the elliptic flow (measured as $v_2$) of heavy flavoured hadrons were expected to be smaller than the light hadrons [2] but the experimental data reveals the opposite trend, i.e. smaller $R_{AA}$ and large $v_2$ [3–6]. Therefore, to circumvent the contradictory observation, investigation for the evolution of HQs with the proper input and the accompanied energy-loss mechanism in the hot and dense QGP becomes essential.

For the evolution of HQs in the medium, understanding about the energy-loss mechanisms of HQs becomes vital. There are mainly two mechanisms for the energy loss of HQs: the first one is the medium induced gluon radiation (radiation energy loss) and the other one is the quasielastic scattering with the background medium partons (collisional energy loss). Earlier it was thought that the medium induced gluon radiation is the dominant one but recent studies suggest that this process is suppressed by the large mass of heavy quark, dubbed as “dead-cone effect” [7, 8], thus the collisional energy loss is then considered to be responsible especially at lower energies eg., RHIC energy [9]. However, at LHC energy the “dead-cone effect” is not so pronounced thus the radiational energy loss become commensurate again. In brief, the issue is not yet settled and one can only say that both mechanisms for the energy loss are equally important to understand the experimental data for both $R_{AA}$ and $v_2$ [11].

Since the evolution of HQs in phase space can be envisaged as the motion of a nonequilibrated particle in an equilibrium medium therefore the motion of HQs can be thought as the random motion or the Brownian motion in the heat bath of an equilibrated plasma because the mass of HQ is much smaller than the temperature of the medium. Thus the trajectories of HQs due to its random motion can be quantified by the Langevin dynamics [12], which, however, can also be studied by the Fokker-Plank equation [13–18] in the limit of soft scattering (Landau) approximation. Other approaches have also been employed to study the HQs dynamics at RHIC and LHC energies, viz. relativistic Boltzmann transport approach [19–22], where the Boltzmann equation is solved numerically by discretizing the space into a 3-dimensional lattice and the collisional integral is modeled by the stochastic sampling of the collision probability. Instead of a constant coupling, the running coupling and the improved scattering matrix [18, 23–26] within perturbative QCD framework supplemented by hard thermal loop (HTL) scheme has been employed to improve upon the collision integral and hence the drag and diffusion coefficients can be refined further. Since the emergence of hadronic phase is inevitable in rHIC therefore some efforts have also been made to understand the evolution of heavy flavours in hadronic medium [27, 28], which decipher to subtract the hadronic contribution from the data to separate the effect of QGP alone. Recently [28], authors have shown that even a weak coupling of heavy flavour hadrons to the hadronic medium can lead to a noticeable contribution to the total elliptic flow. The afore-

---

*prasu111@gmail.com
†binoyfph@iitr.ac.in
said discussions are limited to the weak coupling limit, thus some groups used the complementary setup of the gauge-gravity duality \cite{29-31} to understand the heavy flavour dynamics at strong coupling limit in heavy ion collisions. In summary, after so many efforts, all models face some difficulties to describe both $R_{AA}$ and $v_2$ of heavy mesons simultaneously.

The ultimate aim of the studies on the the drag and diffusion coefficients is to determine the transverse and the azimuthal momentum distribution of HQs. The fluctuation of the momentum encoded in the diffusion coefficient can be understood in terms of the random forces acting on the heavy quarks, which is defined by the auto-correlation of the random forces \cite{27}. On the other hand, the drag coefficient, in relaxation-time approximation, is related to the kinetic equilibration rate of HQs in a thermal medium \cite{30}. Asymptotically the drag and diffusion coefficients are related by the fluctuation-dissipation theorem (FDT): $D/\gamma = \text{Energy of HQ} \times \text{temperature}$ (Non-relativistically the FDT relation is $D/\gamma = \text{Mass} \times T$), where $D$ and $\gamma$ are the momentum and drag coefficients, respectively. One of our aim in this article is to check the FDT theorem.

Nowadays it is expected that the rHIC collisions at the initial stage may induce an anisotropy in the momentum space due to the asymptotic free expansion of the fireball in the beam direction compared to its transverse direction. Thus the strongly interacting fluid created in rHIC possesses momentum-space anisotropies in the local rest frame \cite{32, 33} for a short duration of time in the initial stage. Since the heavy quarks too are produced in the initial stage of the collision therefore the momentum-space anisotropies may have important implications on heavy quark dynamics. This anisotropy subsequently induces the Chromo-Weibal instability \cite{34} in the medium, which may significantly affect the HQ drag and diffusion coefficients \cite{34} and facilitates early equilibration of the medium. Thus it is desirable to study the effect of momentum-space anisotropy on the drag and diffusion coefficients of HQs, which, in turn may have significant impact on the experimental observables, e.g. $R_{AA}$ and $v_2$ \cite{35}.

In this article we have thus explored the effect of momentum anisotropy on the drag and diffusion coefficients due to collisional energy loss only when a test charm quark evolves in a hot anisotropic QCD medium. In our calculation we employ the one-loop running coupling constant and the Debye mass in the leading and next-to-leading order to see the effect of the regulator on the $t$-channel matrix element, which appears in the collision integral. Our work is thus organized as follows: First, in subsection II A we revisited the drag and diffusion coefficients arising due to collisional energy loss alone in an isotropic medium. Here we closely follow the kinematics used by Svetitsky \cite{36} with the corrected matrix elements made in Ref. \cite{37}. We then move on to an anisotropic medium in subsection II B, where it is found that for weak anisotropic limit (anisotropy parameter, $\xi \ll 1$), both coefficients can be decomposed into the dominant isotropic and the sub-leading anisotropic contributions. Later we demonstrate our results for an isotropic medium in subsection III A and understand the salient features of both coefficients as a function of momentum, temperature etc. and its connection with the microscopic properties of HQs evolution from the point of view of statistical mechanics. With these understanding in isotropic medium, we then explain the numerical results for anisotropic medium in subsection III B. We have noticed that how the momentum anisotropy affects the coefficients and finally transpires to the equilibration rate. Finally we conclude in Section IV.

II. MODEL FORMALISM

A. Isotropic Case

Since the thermalization of HQs is very slow compared to the light quarks and gluons therefore a description of the motion of non-equilibrated degrees of freedom in the background of equilibrated degrees of freedom is required. The appropriate framework is provided by the Fokker-Planck equation. Therefore we start with the Boltzmann transport equation describing a nonequilibrium statistical system as follows:

\[ \left( \frac{\partial}{\partial t} + \frac{p \cdot \partial}{E} + F \frac{\partial}{\partial p} \right) f(x, p, t) = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}}. \]  

(1)

For $2 \leftrightarrow 2$ interaction the collisional integral appearing in the right hand side of above transport equation can be written as:

\[ \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} = \int d^3k [w(p + k, k)f(p + k) - w(p, k)f(p)], \]  

(2)

where $w(p, k)$ is the rate of collision which encodes the change of HQ momentum from $p$ to $p - k$ and can be expressed as \cite{36}:

\[ w(p, k) = g \int \frac{d^3q}{(2\pi)^3} f(q) v_{\text{rel}} \sigma_{p,q \to p-k,q+k}, \]  

(3)

Here $f$ is the phase space distribution of the bulk constituents, $v_{\text{rel}}$ is the relative velocity between the two collision partners, $\sigma$ represents the cross-section and $g$ is the statistical degeneracy of the particles in QCD medium.

Using the soft-scattering Landau approximation in the collision integral, the resulting Fokker-Planck equation (1) is cast in the form

\[ \frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} \left[ B_{ij}(p)f \right] \right], \]  

(4)

where the kernels are defined as:

\[ A_i = \int d^3k \, w(p, k)k_i, \]  

(5)
and
\[ B_{ij} = \frac{1}{2} \int d^3k \; w(p, k)kJ_iJ_j. \] (6)

In low momentum transfer limit (\(|p| \to 0\)), kernels are reduced into
\[ A_i = \gamma_i p, \] (7)
and
\[ B_{ij} = D \delta_{ij}, \] (8)
where \( \gamma \) and \( D \) are the drag and diffusion coefficients, respectively. The generic integral appeared for both the drag and diffusion coefficients for HQ in a hot and isotropic medium of massless quarks and gluons is given by [38]:
\[
\langle F_{iso}(p) \rangle = \frac{1}{512\pi E_p} \int_0^\infty dq \frac{q^2}{E_q} dq \\
\times \int_{-1}^1 d(cos\chi) \frac{s-M^2}{s} f^0(q) \\
\times \int_{-1}^1 d(cos\theta) \frac{1}{2q} \sum |M|^2 \\
\times \int_0^{2\pi} F(p') d\phi,
\] (9)
where \( M \) and \( g_Q \) are the masses and degeneracy factors of heavy quarks, respectively (here we use \( M = 1.5 \text{ GeV} \) for charm quark) and \( f^0(q) \), the equilibrium distribution function of the massless quarks and gluons is given by
\[
f^0(q) = \frac{1}{\exp (E_q/T) \pm 1}\] (10)
where \( \pm \) sign is for quarks and gluons, respectively.

Depending on the drag or diffusion, the function, \( F(p') \) in the integral (9) is given by
\[
F(p') = \langle \langle 1 \rangle \rangle - \frac{\langle p \cdot p' \rangle}{p^2},
\] (11)
for the drag coefficient whereas for the diffusion coefficient
\[
F(p') = \frac{1}{4} \left[ \langle p^2 \rangle \right] - \frac{\langle p \cdot p' \rangle^2}{p^2},
\] (12)
where the dot product, \( p \cdot p' \) is calculated from the expression [36]:
\[
p \cdot p' = E_p E_{p'} - \bar{E}_p^2 + p^2 \cos \theta
\] (13)
where \( E_p, E_{p'} \) are the energies of incident and scattered heavy quarks in the lab frame, \( \bar{E}_p \) and \( \bar{p} \) are the energy and momentum of heavy quarks in the CM frame, respectively, and \( \theta \) is the CM scattering angle (hereafter the cap represents the variables in the CM frame). The energy, \( E_p \) in the laboratory frame is related to the CM frame by inverse Lorentz transformation:
\[
E_p = \gamma \left( \bar{E}_p + \bar{v} \cdot \bar{p} \right)
\] (14)
where the Lorentz factor and velocity in the CM frame are given by,
\[
\gamma = \frac{E_p + E_q}{\sqrt{s}}
\] (15)
\[
\bar{v} = \frac{p + q}{E_p + E_q}
\] (16)
respectively. The energy, \( \bar{E}_p \) and the magnitude of the momentum \( |\bar{p}| \) in the CM frame can be written as:
\[
|\bar{p}| = \frac{s-M^2}{2\sqrt{s}},
\] (17)
\[
\bar{E}_p = \sqrt{|\bar{p}|^2 + M^2},
\] (18)
respectively. Now the dot product \( \bar{v} \cdot \bar{p} \), in (14) can be calculated by the using Lorentz transformation of \( p \) from lab frame to CM frame
\[
\bar{p} = \gamma (p - \bar{v} E_p).
\] (19)
as
\[
\bar{v} \cdot \bar{p} = \gamma (p \cdot \bar{v} - |\bar{v}|^2 E_p),
\] (20)
which can be further simplified as:
\[
\bar{p} \cdot \bar{v} = \gamma \left( \frac{|\bar{p}|^2 + p \cdot q - |\bar{v}|^2 E_p}{E_p + E_q} \right).
\] (21)
The factor, \( N \) in Eq. (14) can be obtained as [36]:
\[
N^2 = |\bar{v}|^2 - \frac{(p \cdot \bar{v})^2}{|\bar{p}|^2}
\] (22)
and the Mandelstam variable, \( s \) in CM frame is given by
\[
s = (E_p + E_q)^2 - |p|^2 - |q|^2 - 2 |p||q| \cos \chi.
\] (23)
where \( \chi \) is the angle between \( p \) and \( q \).

In the present work, we consider the collisional energy loss of HQs, where the HQs are scattered quasi-elastically with the partons in QGP medium: \( Q(p) + q, q, g(q) \rightarrow Q(p') + q, q, g(q') \) (The quantities inside the bracket denotes the four momentum of the particle). If the HQ is scattered by the quark (anti-quark) then the matrix element for the corresponding process is [36, 37]:
\[
|M|^2_{q \rightarrow q} = 256\pi^2 \alpha_s^2 \left[ \frac{(M^2 - u)^2 + (s - M^2)^2 + 2M^2t}{(t - \mu^2)^2} \right],
\] (24)
whereas the matrix element for the gluon scattering is given by [36, 37]:

\[
|M|_{g_{c} ightarrow g_{c}}^{2} = \pi^{2} \alpha_{s}^{2} \left[ \frac{3072(s - M^{2})(M^{2} - u)}{(t - \mu^{2})^{2}} + \frac{2048}{3} \frac{(s - M^{2})(M^{2} - u)}{(s - M^{2})^{2}} + \frac{8}{3} \frac{2M^{2}(M^{2} + u)}{(M^{2} - u)} \right] \tag{25}
\]

In the above Eqs. (24, 25), the \(t\) and \(u\) variables are given by

\[
t = 2\bar{p}^{2}(\cos \theta - 1) \tag{26}
\]
\[
u = 2M^{2} - s - t \tag{27}
\]

and \(\mu^{2}\) is the regulator, which is needed to shield the infra-red divergences arising in the \(t\)-channel scattering amplitude. In our calculation we take it as the leading-order Debye mass \(m_{D}^{2}\) as [39]:

\[
m_{D}^{2} = T^{2} \left[ g^{2} \left( \frac{N_{c}}{3} + \frac{N_{f}}{6} \right) \right] \tag{28}
\]

where \(g\) is the strong QCD coupling in one-loop.

### B. Anisotropic Case

Recently it is envisaged that the partonic system generated in ultra-relativistic heavy-ion collisions at the nascent stage may not be necessarily isotropic in the momentum space rather the medium exhibits a momentum anisotropy due to the rapid expansion in the longitudinal direction compared to the transverse directions [40–44]. This motivates us to study the transport coefficients related to drag and diffusion processes in such anisotropic medium.

If the anisotropy is small then the anisotropic distribution is obtained by either stretching or squeezing the isotropic distribution along a certain direction, thereby preserving a cylindrical symmetry in momentum space. In particular, the anisotropic distribution relevant for relativistic heavy ion collision can be approximated by removing particles with the large momentum component along the direction of anisotropy, \(\mathbf{n}\) as [45, 46]:

\[
f_{\text{aniso}}(q) = f_{\text{iso}} \left( \sqrt{q^{2} + \xi(q \cdot n)^2} \right), \tag{29}
\]

where \(f_{\text{iso}}\) is an arbitrary isotropic distribution function and \(\xi\) is the anisotropic parameter and is generally defined as:

\[
\xi = \frac{\langle q_{T}^{2} \rangle}{2 \langle q_{L}^{2} \rangle} - 1, \tag{30}
\]

where \(q_{L} = q \cdot n\) and \(q_{T} = q - n(q \cdot n)\) are the components of momentum parallel and perpendicular to \(n\) respectively. There have been significant advancements in the dynamical models used to simulate plasma evolution having momentum-space anisotropies [47–51]. One of us have studied the effects of momentum anisotropy on the quarkonia states by the leading-anisotropic correction to the resummed gluon propagator [52, 53] which subsequently affects the suppression of quarkonium production at RHIC and LHC. Recently we have investigated the effect of momentum anisotropy on one the transport coefficients, namely the electrical conductivity [54].

If the distribution function is nearly an ideal gas distribution and the anisotropy, \(\xi\) is small then \(\xi\) can be related to the shear viscosity of the medium via the one-dimensional Bjorken expansion in the Navier-Stokes limit [56]:

\[
\xi = \frac{10 \eta}{T \tau s}, \tag{31}
\]

For an expanding system, non-vanishing viscosity implies the finite relaxation time in the momentum space, hence an anisotropy of the particle momenta does appear inherently, for example relaxation time in the momentum space, therefore the quark distribution function in the anisotropic medium can be approximated for a baryonless medium (\(\mu_{B} = 0\)):

\[
f_{\text{aniso}}(q; T) = \frac{1}{e^{ \langle \sqrt{q^{2} + \xi(q \cdot n)^2} + m^{2} \rangle / T} + 1}. \tag{32}
\]

For weakly anisotropic systems (\(\xi \ll 1\)), one can expand the distribution function and keep the leading term in \(\xi\) only:

\[
f_{\text{aniso}}(q; T) = \frac{1}{e^{E_{q}/T} + 1} + \frac{\xi}{2E_{q}/T} \langle q \cdot n \rangle^{2} e^{E_{q}/T} \left( e^{E_{q}/T} + 1 \right)^{2},
\]

\[
= f^{0}(q) - \frac{\xi}{2E_{q}/T} \langle q \cdot n \rangle^{2} f^{\theta^{2}} e^{E_{q}/T}, \tag{33}
\]

where \(q \equiv (q \sin \chi \cos \Phi, q \sin \chi \sin \Phi, q \cos \chi)\) and \(n \equiv (\sin \beta, 0, \cos \beta)\) or (0, \(\sin \beta, \cos \beta\)) is the angle between \(q\) and \(n\).

Therefore the drag and diffusion coefficients in a weakly anisotropic medium may be obtained by replacing the phase-space distribution in the anisotropic medium...
\[ \langle F(p) \rangle = \frac{1}{1024\pi^5 \gamma_p} \int_0^\infty \int_0^\pi \int_0^{2\pi} q^2 \sin\chi \, dq \, d\chi \, d\Phi \frac{\xi}{2E_qT} f_{\text{aniso}}(q) \times \int_0^1 \frac{d(\cos\hat{\phi})}{gQ} \sum |M|^2 \times \int_0^{2\pi} F(p') \, d\phi. \] 

Since \( f_{\text{aniso}} \) has two part: isotropic and correction due to momentum anisotropy. The resulting expression for drag and diffusion has two parts. The isotropic part is similar to Eq.(9). The expression for anisotropic part is as follows:

\[ \langle F_{\text{aniso}}(p) \rangle = -\frac{1}{1024\pi^5 \gamma_p} \int_0^\infty \int_0^\pi \int_0^{2\pi} q^2 \sin\chi \, dq \, d\chi \, d\Phi \frac{\xi}{2E_qT} \times f^{\text{iso}}(q) e^{E_q/T \omega^{1/2}} \times \int_0^1 \frac{d(\cos\hat{\phi})}{gQ} \sum |M|^2 \times \int_0^{2\pi} F(p') \, d\phi. \] 

Now using the definition of \( Q \) and \( n \), one can get the expression for \( (Q \cdot n)^2 \) as follows:

\[ (Q \cdot n)^2 = q^2 \sin^2\chi \cos^2\Phi \sin^2\beta + q^2 \cos^2\chi \cos^2\beta + 2q^2 \sin\chi \cos\chi \sin\beta \cos\beta \cos\Phi. \] 

Putting this value in Eq. (36) and integrating over \( \Phi \), we can get the modified expression as follows for the anisotropic correction part:

\[ \langle F_{\text{aniso}}(p) \rangle = -\frac{\xi}{1024\pi^5 \gamma_p} \frac{1}{2E_qT} \int_0^\infty \int_0^\pi q^4 \, dq \, d\chi \left[ \pi(1 - \cos^2\chi) \sin^2\beta + 2\pi \cos^2\chi \cos^2\beta \right] \times f^{\text{iso}}(q) e^{E_q/T \omega^{1/2}} \times \int_0^1 \frac{d(\cos\hat{\phi})}{gQ} \sum |M|^2 \times \int_0^{2\pi} F(p') \, d\phi. \]

Thus the total drag and/or diffusion coefficient for an anisotropic QGP is:

\[ \langle F(p) \rangle = \langle F_{\text{iso}}(p) \rangle + \langle F_{\text{aniso}}(p) \rangle. \]
During the evolution of heavy quarks in medium, the charm quarks are dragged by both quarks and gluons, thus we calculate separately the contribution by quarks, gluons and their sum total, which are shown by the dash-dotted, dotted and solid curves, respectively. In our calculation of the matrix element in $t$-channel, we take the regulator, $\mu$ as the leading-order Debye mass, $m_D(T)$, unlike a constant value used in other calculations. For the temperature dependence of the Debye mass, we take the strong coupling ($g_s$) from one-loop expression. We found that the drag coefficient decreases when the momentum of the charm quark increases. This observation agrees with common sense because the relative speed of the charm quark with respect to the medium increases with the increase its momentum and hence the drag coefficient decreases. The above observation can be understood from the point of view of statistical mechanics in the following way: The equilibration rate of HQs in phase space decreases with the increase in its momentum, hence the drag coefficient for HQs should decrease with its momentum because the drag coefficient is related linearly to the kinetic equilibration rate. This understanding will later be useful to understand the variation of diffusion coefficient with the momentum (in Fig. 1(b)). Another observation of Fig. 1 (a) is that the momentum dependence of the drag coefficient are mostly emanated from the gluon scattering due to their abundance and the large contribution to the cross-section whereas the contribution by light quarks is meagre.

On the other hand the momentum dependence of the diffusion coefficient is opposite, i.e. $D_T$ increases with the momentum (shown in Figure 1b) because it is easy for HQs having larger momentum to diffuse in the system compared to HQs of lower momentum. From the point of view of statistical physics, the diffusion coefficient is a measure of the equilibration time (inverse of the equilibration rate) thus the coefficient should be greater when the HQ momentum becomes larger. Like the drag coefficient, the gluons contribute substantially to the momentum dependence of diffusion coefficient compared to the meagre contribution by quarks.

In Fig. 2(a), we have studied how a relativistic charm quark is diffused or dragged while evolving in a static isotropic QCD medium when the temperature of the medium changes from lower to higher values. We found that the drag coefficient becomes small and increases with the temperature slowly whereas the diffusion coefficient increases with the temperature rapidly, so their separation ($\gamma_T - D_T$) increases with the temperature. This can be understood qualitatively: since the momentum is much higher than the temperature of the medium therefore the physical scale set here is the momentum of HQ only. If the temperature of a thermal medium is increased then the constituents of the medium exert more and more random force on the test particle, thus the increase of temperature causes the motion of the test particle more random. Since the diffusion is asymptotically related to the temperature of the medium, therefore the diffusion coefficient increase with the increase of the temperature. Similar to Fig. 2a, in Figure 2 b, we have explored how do the drag and diffusion coefficients depend on the temperature for a non-relativistic charm quark ($p = 0.001$ GeV). Since the relative speed of HQ becomes small therefore the drag coefficient becomes large. By the same reasoning as in Figure 2a, the relevant scale set here is the temperature of the medium, not the momentum of the test HQ. Thus both the equilibration rate and the random force exerted by the partons increase with the temperature for HQs having low momentum. Hence both drag and diffusion coefficients increase with the temperature. The increase of diffusion coefficient with the temperature in both figures (Figure 2 a & b) is understandable because the number of constituents faced by the test particle increases with temperature (for massless case, $n \propto T^3$) and thus the random force exerted on HQ by the constituents increases.

As we mentioned earlier in Figs 1 and 2, we have taken the regulator in the $t$ channel matrix element by the Debye mass in leading-order ($m_D^{LO}$). The Debye mass in the leading-order is correct in the weak coupling regime when the coupling constant is very small $g << T$. However when $g \sim T$ then higher order corrections also arise in the Debye mass [57]. Kajantie et al. [58] computed these contributions of $O(g^2T)$ and $O(g^3T)$ from a three-dimensional effective field theory. Here we wish to see the effect of the regulator on the drag and diffusion coefficients due to the corrections in the Debye mass. Thus, we have used two regulators : $\mu = m_D^{LO}$ and $1.4m_D^{LO}$, where the factor 1.4 takes into account the next-to-leading order corrections, for relativistic ($p=5$ GeV) and non-relativistic ($p=0.001$ GeV) in Figures 3 a and b, respectively. We observed that the inclusion of higher order effects in the Debye mass as the regulator decreases both drag and diffusion coefficients. To be specific, the
changes in drag coefficient of non-relativistic HQs due to increase in the regulator are about 35%–40% while going from T=0.2 GeV to T=0.6 GeV whereas in relativistic regime the change in drag coefficient (≈ 35%) is almost independent to the change in temperature. Further the change in the value of diffusion is about 30%–35% as one goes from lower to higher temperature at non-relativistic momentum but again the percentage of change in diffusion (≈ 35%) remains independent to the temperature at relativistic momentum p = 5 GeV.

To check the validity of the fluctuation dissipation theorem (FDT) for a picture where the non-equilibrated degrees of freedom (in this case it is heavy quarks) evolves in the background of equilibrated degrees of freedom, we have studied the ratio of the diffusion to drag coefficient (D<sub>T</sub>/γ<sub>T</sub>) for dynamical HQs in isotropic medium at a temperature, T=0.2 GeV (in Figure 4). As we know that the ratio is asymptotically related to the temperature of the medium so we have also plotted the quantity Energy(E) × T in the same figure. We observed that the FDT is almost satisfied in the non-relativistic limit of HQ momentum but is violated as the HQ momentum becomes more and more relativistic. This observation seems more plausible because FDT is satisfied only if a non-equilibrated degrees of freedom evolves in an ideal heat bath and undergoes through linear damping [61]. Our result is consistent with other calculations [59], where the KLN factorization is employed to model the pre-equilibrium momentum space gluon distribution [60].

**B. For anisotropic QGP**

As we discussed in the preamble that the system produced at the early stage of ultra-relativistic heavy ion collisions exhibits an anisotropy in the momentum space, thus we aim to explore the effect of anisotropy on the heavy quark evolution because heavy quarks are also produced at early stages of the collision. Since the anisotropy introduces an angular dependence in the drag coefficient so we have calculated the coefficient for different values of anisotropy parameter for two cases: a) when the partons move transverse to the direction of anisotropy (β = π/2) in Figure 5 a and b) when the partons moves along the direction of anisotropy (β = 0) in Figure 5 b. The immediate observation is that the drag coefficient always decreases with the anisotropy (ξ ≠ 0) for both parallel and perpendicular alignment, which can be understood qualitatively: In the small anisotropic limit, the anisotropic distribution function for partons may be approximated as an isotropic distribution function by removing particles with a large mo-

---

**FIG. 4:** Ratio of diffusion to drag (D<sub>T</sub>/γ<sub>T</sub>) with respect to HQ momentum p at fixed value of QGP temperature T = 0.2 GeV. We have also plotted the product of HQ energy (E) and temperature (T) by dash-dotted curve for comparison.

**FIG. 5:** Variation of drag coefficient with respect to HQ momentum for three different values of anisotropy parameter i.e., ξ = 0, 0.3, and 0.6) at a medium temperature T = 0.2 GeV for (a) perpendicular (β = π/2), and (b) parallel (β = 0) case.

**FIG. 6:** Variation of diffusion coefficient with respect to HQ momentum for three different values of anisotropy parameter i.e., ξ = 0, 0.3, and 0.6 at a medium temperature T = 0.2 GeV for (a) perpendicular (β = π/2), and (b) parallel (β = 0) case.
momentum component along the direction of anisotropy, \(\hat{n}\), which causes a reduction of the number of partons around a test heavy quark in a given phase-space point \((n_{\text{aniso}} \approx n_{\text{iso}}/\sqrt{1 + \xi})\). As a result, while propagating in the medium, HQ encountered less number of scatterings and hence the equilibration rate becomes smaller in anisotropic plasma, resulting a decrease in the drag coefficient. It can also be seen that if the temperature of the medium is increased then the (negative) correction due to anisotropy increases, as a result the coefficient decreases sharply for all values of HQ momentum. Another important observation is that the drag coefficient of HQ in parallel alignment \((\vec{q} \parallel \vec{n})\) is always less than the perpendicular alignment \((\vec{q} \perp \vec{n})\). This is due to the fact that for parallel alignment the momentum of partons got effectively shifted towards higher momentum side \(q^2 \rightarrow q^2 + \xi (\vec{q} \cdot \hat{n})^2\), \(\xi > 0\) thus a large chunk of higher momentum particles do not contribute to the scattering of partons with HQ whereas for perpendicular alignment, the shift of partons to higher momentum side does not arise that much. Similarly we can see from Figs. 6 (a) and (b) that the diffusion coefficient also decreases due to the momentum space anisotropy. Further, like drag coefficient, diffusion coefficient of HQ also becomes smaller in parallel alignment than the perpendicular alignment.

In conclusion, we have first revisited the propagation of charm quarks in a hot isotropic medium of quarks and gluons by its transport coefficients - drag and diffusion coefficients and then explore the dependencies of the coefficients on the charm quark momentum, the temperature of the medium and the regulator in the \(t\)-channel matrix element in the form of Debye mass. We have understood the results through the connection of the coefficients with the equilibration rate from the statistical mechanics point of view. With these understanding from the isotropic medium we move on to a medium which exhibits a momentum anisotropy and calculated both drag and diffusion coefficients in an anisotropic medium. The important finding of our calculation is that both drag and diffusion coefficients of charm quarks get waned due to the momentum anisotropy, which may be created at the early stages of the relativistic collisions. Moreover the anisotropy causes a direction dependent sizable modifications to both drag and diffusion, as a result the (negative) correction due to anisotropy, when the partons are moving parallel to the direction of anisotropy, is larger than when the partons are moving perpendicular to the direction of anisotropy. Thus the study of drag and diffusion coefficients in anisotropic medium will open up further applications to the phenomenology of relativistic heavy ion collisions.

IV. ACKNOWLEDGMENTS

PKS and BK is thankful for financial assistance from Council of Scientific and Industrial Research (No. CSR-656-PHY), Government of India. Authors also acknowledge the fruitful discussion with S. Das, R. Rapp and H. van Hees during the course of this work.

[1] M. Djordjevic, M. Gyulasssly, R. Vogt, and S. Wicks, Phys. Lett. B 632, 81 (2006).
[2] N. Armesto, M. Cacciari, A. Dainese, C. A. Salgado, and U. A. Wiedemann, Phys. Lett. B 637, 362 (2006).
[3] V. Greco, C. M. Ko, and R. Rapp, Phys. Lett. B 595, 202 (2004).
[4] A. Adare et al. (PHENIX Collaboration) Phys. Rev. Lett. 98, 172301 (2007).
[5] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 96, 032301 (2006).
[6] B. Abelev et al. (ALICE Collaboration), J. High Energy Phys. 09, 112 (2012).
[7] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B 519, 199 (2001).
[8] R. A. Lacey et al., Phys. Rev. Lett. 103, 142302 (2009).
[9] M. Mishra, V. J. Menon, B. K. Patra, arXiv:0708.0494v2[hep-ph].
[10] B. G. Zakharov, JETP Lett. 86, 444 (2007); P. A.urenche and B. G. Zakharov, ibid. 90, 237 (2009).
[11] S. Wicks, W. Horowitz, M. Djordjevic, M. Gyulassly, Nucl. Phys. A 784, 426 (2007).
[12] B. K. Patra, V. J. Menon, Nucl. Phys. A 708, 353 (2002).
[13] M. G. Mustafa, D. Pal, D. K. Srivastava, Phys. Rev. C 57, 889 (1998).
[14] G. D. Moore, D. Teaney, Phys. Rev. C 71, 064904 (2005).
[15] H. van Hees, V. Greco, R. Rapp, Phys. Rev. C 73, 034913 (2006).
[16] H. van Hees, M. Mannarelli, V. Greco, R. Rapp, Phys. Rev. Lett. 100, 192301 (2008).
[17] M. He, R. J. Fries, R. Rapp, Phys. Rev. Lett. 110, 112301 (2013).
[18] W. M. Alberico, et al., Eur. Phys. J. C 71, 1666 (2011); 73, 2481 (2013).
[19] B. Zhang, L.-W. Chen, C.-M. Ko, Phys. Rev. C 72, 024906 (2005).
[20] D. Molnar, Eur. Phys. J. C 49, 181 (2007).
[21] S. K. Das, F. Scardina, S. Plumari, V. Greco, Phys. Rev. C 90, 044901 (2014).
[22] J. Uphoff, O. Fochler, Z. Xu, and C. Greiner, Phys. Rev. C 84, 024908 (2011).
[23] P. B. Gossiaux, J. Aichelin, Phys. Rev. C 78, 014904 (2008).
[24] S. Cao, S. A. Bass, Phys. Rev. C 84, 064902 (2011).
[25] C. M. Ko, W. Liu, Nucl. Phys. A 783, 23c (2007).
[26] S. Mazumder, T. Bhattacharyya and J. Alam, arXiv:1209.1917v2[hep-ph].
[27] M. Laine, J. High. Energy Phys. 04, 124 (2011).
[28] M. He, R. J. Fries, R. Rapp, Phys. Lett. B 701, 445 (2011).
[29] S. S. Gubser, Phys. Rev. D 74, 126005 (2006).
[30] J. Casalderrey-Solana, D. Teaney, Phys. Rev. D 74, 085012 (2006).
[31] W. A. Horowitz, M. Gyulassy, J. Phys. G 35, 104152 (2014).
[32] M. Strickland, arXiv:1401.1188v1[nucl-th] (2014).
[33] M. Strickland, Nucl. Phys. A 926, 92 (2014); arXiv: 1312.2285[hep-ph] (2013).
[34] V. Chandra, and S. K. Das, arXiv:1506.07805v3 [nucl-th].
[35] S. K. Das, F. Scardina, S. Plumari, V. Greco, Phys. Lett. B 747, 260 (2015).
[36] B. Svetitsky, Phys. Rev. D 37, 2484 (1988).
[37] B. L. Combridge, Nucl. Phys. B 151, 429 (1979).
[38] S. K. Das, V. Chandra, J. Alam, J. Phys. G. 41, 015102 (2013).
[39] E. V. Shuryak, Sov. Phys. JETP 47, 212 (1978).
[40] A. Dumitru, Y. Guo and M. Strickland, Phys. Lett. B 662, 37 (2008).
[41] A. Dumitru, Y. Guo, A. Mocsy and M. Strickland, Phys. Rev. D 79, 054019 (2009).
[42] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. 100, 1 (1983).
[43] A. H. Mueller and J.-W. Qiu, Nucl. Phys. B 268, 427 (1986).
[44] J. P. Blaizot and A. H. Mueller, Nucl. Phys. B 289, 847 (1987).
[45] P. Romatschke and M. Strickland, Phys. Rev. D 68, 036004 (2003).
[46] P. Romatschke and M. Strickland, Phys. Rev. D 70, 116006 (2004).
[47] M. Martinez and M. Strickland, Nucl. Phys. A 856, 68 (2011); Nucl. Phys. A 848, 183 (2010)
[48] M. Martinez, R. Ryblewski, and M. Strickland, Phys. Rev. C 85, 064913 (2012).
[49] R. Ryblewski, and W. Florkowski, J. Phys. G 38, 015104 (2011); Euro. Phys. J. C 71, 1761 (2011).
[50] R. Ryblewski, W. Florkowski, Phys. Rev. C 85, 064901 (2012);W. Florkowski, R. Ryblewski, and M. Strickland, Phys. Rev. D 86, 085023 (2012).
[51] W. Florkowski and R. Ryblewski, Phys. Rev. C 83, 034907 (2011).
[52] L. Thakur, N. Haque, U. Kakade, and Binoy Krishna Patra, Phys. Rev. D 88, 054022 (2013).
[53] L. Thakur, U. Kakade, and Binoy Krishna Patra, Phys. Rev. D 89, 094020 (2014).
[54] P. K. Srivastava, Lata Thakur, B. K. Patra, Phys. Rev. C 91, 044903 (2015).
[55] A. Peshier, B. Kampfer, O. P. Pavlenko and G. Soff, Phys. Lett. B 337, 235 (1994).
[56] S. K. Das, M. Ruggieri, S. Mazumder, V. Greco, J. Alam, J. Phys. G 42, 095108 (2015).
[57] H. J. Drescher and Y. Nara, Phys. Rev. C 75, 034905 (2007).
[58] K. Kajantie, M. Laine, J. Peisa, A. Rajantie, K. Rummukainen, and M. E. Shaposhnikov, Phys. Rev. Lett. 79, 3130 (1997).
[59] S. K. Das, M. Ruggieri, S. Mazumder, V. Greco, J. Alam, J. Phys. G 42, 095108 (2015).
[60] T. Hirano and Y. Nara, Phys. Rev. C 75, 034905 (2007); T. Hirano and Y. Nara, Phys. Rev. C 79, 064904 (2009).
[61] D. B. Walton, J. Rafelski, Phys. Rev. Lett. 84, 31 (1999).