A CARBON-CYCLE BASED STOCHASTIC CELLULAR AUTOMATA CLIMATE MODEL

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Abstract
In this article a stochastic cellular automata model is examined, which has been developed to study a “small” world, where local changes may noticeably alter global characteristics. This is applied to a climate model, where global temperature is determined by an interplay between atmospheric carbon dioxide and carbon stored by plant life. The latter can be released by forest fires, giving rise to significant changes of global conditions within short time.

Keywords: Cellular Automata; Forest Fire; Climate Dynamics; Carbon Cycle

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1 Introduction

1.1 A Brief Note on Cellular Automata

Cellular Automata (CA), as studied first by von Neumann and Ulam [1], provide systems both interesting from a fundamental point of view and for practical purposes of model builders. Among the systems most frequently studied are CA versions of daisyworld [2, 3, 4] and forest fire models [5, 6, 7, 8, 9].

The main idea is to study a grid or network of identical units (cells), which each can take discrete states, and where the time evolution of each cell is governed by simple local rules: The state $s_i(t)$ of a cell labelled by $i$ at timestep $t$
is only determined by its previous state \( s_i(t-1) \) and the states \( s_{jk}(t-1) \) of a small number \( k \) of neighbours, labelled by \( j_k \).

There are different ways how to precisely define a neighbourhood, but typically the qualitative behaviour is typically unaffected by the details \([10, 11]\). This can be seen as a footprint of universality – the fact that microscopic details do not influence features of systems which are “critical” in the sense of Wilson \([12]\). The fact that all rules are local implies a finite maximum speed for the propagation of all effects.

Originally the rules governing cellular automata were strictly deterministic. One can extend this formalism (and make it both more flexible and more realistic) when including stochastic elements, i.e. give probabilities how the state of a cell depends on its own previous state and those of its neighbours. One can also replace the few discrete states by a (quasi)continuum to gain additional flexibility. The system studied in this article belongs to this class of Stochastic Cellular Automata (SCA) models, which the authors apply to a particular feature of climate-relevant dynamics.

1.2 A Brief Note on the Carbon Cycle

It is well-known that the global temperature depends strongly on the amount of greenhouse gases contained in the atmosphere. While \( \text{H}_2\text{O} \) vapor is the most important of them, its dynamic is extremely complicated and hard to treat reliably in models. Since the amount of \( \text{H}_2\text{O} \) vapor strongly depends on temperature, the water vapor cycle may largely act as a positive feedback cycle, enhancing the effects of other greenhouse gases.

For these reasons the authors focus on CO\(_2\), being the most important other greenhouse gas. Atmospheric CO\(_2\) is crucial for plant life and one typically expects enhanced growth for larger amount of CO\(_2\) available. This is indeed sometimes used as an argument against global warming \([13]\).

But in particular when accompanied by climate changes, enhanced plant growth can also mean increased vulnerability to large-scale forest fire, which can release considerable amounts of CO\(_2\) in very short time \([14]\) (though balancing aspects seem possible \([15]\)).

There exists a number of realistic climate models, which attempt to directly describe the terrestrial system \([16]\). These models, which are by now quite successful, are mostly based on the (simplified) treatment of atmosphere and ocean dynamics, but mostly neglect the plant life Carbon cycle. Therefore the present model, though significantly more simplistic, can be regarded as complementary to these approaches.
2 The Model

2.1 The Basic Idea

In this model one studies patches of land, which are, in the course of time, inhabited by plant life. During their growth process, these plants can store carbon from the atmosphere. The growth rate depends both on temperature (determined by geographical latitude and carbon dioxide in the atmosphere) and the amount of carbon available (i.e. again carbon dioxide in the atmosphere).

For each cell, optimum growth temperature and temperature tolerance are first set randomly and then subject to a simple evolutionary process. Uninhabited cells can become inhabited by spreading from neighbours. Their characteristic parameters are determined as a weighted average of those of all neighbours, affected by mutations.

In addition, there are spontaneous outbreaks of forest fires, and patches are more vulnerable to them, if they are more heavily overgrown. The fires can spread, and burning cells immediately release all carbon stored in them to the atmosphere in form of carbon dioxide.

Remark: Since the amount of oxygen in the atmosphere is not noticeably changed during all processes, in the following the distinction between “carbon” and “carbon dioxide” will be dismissed and always the word “carbon” is used, where it is implicitly assumed that a sufficient amount of oxygen is available for all processes and that carbon is deposited in the atmosphere in the form of carbon dioxide.

In the process of biological degradation and during fires, not only CO$_2$ but also considerable amounts of H$_2$O are released. While, for reasons discussed in section 1.2 we do not attempt to explicitly include water vapor effects, they may be – at least at some timescales – partially covered too.

2.2 Relation to Previous Work

The system studied in this approach is closely related to the Clar-Drossel-Schwabl forest-fire model \[7, 8\] which also combines slow growth with rapid burning of clusters. In contrast to the Clar-Drossel-Schwabl model, the present approach features stochastic propagation of the fire and – more important – the coupling to a global variable, to be interpreted as the atmospheric carbon dioxide content of the model world.

2.3 Implementation

The model is set up on a rectangular grid, described by several \([N_i \times N_j]\)-matrices, which are summarized and explained in table 1. Each cell represents a patch of land, possibly inhabited by a certain amount of plant life with individual characteristics. Boundary conditions are chosen periodic in the first \((i)\) and open in the second \((j)\) direction.
A simulation run is performed as follows:

**Initialization:** Cells are inhabited with starting density $\rho_{\text{start}}$, the parameters of inhabited cells are determined as $c_{ij} = U$, $\tau_{ij} = U$ and $\sigma_{ij} = 1 + \mu \cdot (2U - 1)$ with $U$ taken individually from a uniform $[0, 1)$-distribution. Now the following cycle is repeated $N_{\text{steps}}$ times:

1. **Spreading:** Each cell with at least one inhabited neighbour has a chance to become inhabited as well. The probability is given by

$$ p(i, j) = \frac{1}{4 + 4\pi_M} \sum_{i', j'} c_{i'j'} . $$

(1)

where the prime at the sum indicates summation over next neighbours, with diagonal elements $(i \pm 1, j \pm 1)$ weighted with the Moore factor $\pi_M = \frac{1}{2}$. In case the cell becomes inhabited, its parameters are determined as

$$ \tau_{ij} = \frac{1}{N} \sum_{i', j'} c_{i'j'} \tau_{i'j'} + \mu \cdot (2U - 1) , $$

(2)

$$ \sigma_{ij} = \frac{1}{N} \sum_{i', j'} c_{i'j'} \tau_{i'j'} (1 + \mu \cdot (2U - 1)) , $$

(3)

with $N := \sum_{i', j'} c_{i'j'}$ and $U$ individually taken from a uniform random distribution on $[0, 1)$. The row-averaged values

$$ \tau_j = \frac{1}{\sum_i \ell_{ij}} \sum_{i=1}^{N_i} \ell_{ij} \tau_{ij} \quad \text{and} \quad \sigma_j = \frac{1}{\sum_i \ell_{ij}} \sum_{i=1}^{N_i} \ell_{ij} \sigma_{ij} $$

(4)

are stored for later time series analysis.

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1In principle, one could introduce an additional multiplicative parameter $\alpha_{\text{spread}} \in (0, 1]$ in front of the sum, the same way it is done for the following processes. Our choice of $\alpha_{\text{spread}} = 1$ effectively fixes the timescale. Accordingly the rates of all other processes (except spreading of forest fires) are determined relative to this spreading rate.
2. **Determining global variables (1):** The carbon content \( C \) of the atmosphere is determined as

\[
C = 1 - \frac{1}{N_i N_j} \sum_{i,j} c_{ij}
\]

and the temperature is determined as

\[
T_j = \alpha_{\text{Tvar}} \frac{j - 1}{N_j - 1} + (1 - \alpha_{\text{Tvar}}) C.
\]

The value of \( C \) is stored for later time-series analysis.

3. **Growth:** Each cell grows with its individual rate,

\[
c_{ij}(t+1) = \text{med}\left\{0, c_{ij}(t) + \alpha_{\text{growth}} C \varphi(T_j, \tau_{ij}, p_{ij}), 1\right\}
\]

(7)

(\text{where “med” denotes the median value}) with the temperature adaptation model function

\[
\varphi(T_j, \tau_{ij}, \sigma_{ij}) = N \exp \left\{-\frac{(T_j - \tau_{ij})^2}{2\sigma_{ij}}\right\} - h_{\text{hostil}}.
\]

(8)

where the normalization \( N = N(\tau_{ij}, \sigma_{ij}) \) is chosen such that

\[
\int_0^1 \varphi(T_j, \tau_{ij}, \sigma_{ij}) = 1 - h_{\text{hostil}}.
\]

(9)

This simple Gaussian shape already shows the most important features one would expect of such a function. It allows both extreme specialization to a certain temperature and adaptation to a wider range. For \( h_{\text{hostil}} > 0 \) there may be temperature regions where growth is negative. If this causes \( c_{ij}(t+1) \) to be 0, the cell dies.

4. **Determining global variables (2):** Again Carbon content and temperature are determined according to equations (5) and (6).

5. **Forest fires:** Each inhabited cell \((i, j)\) has a chance to catch fire, with probability

\[
p = \alpha_{\text{fire}} T_j c_{ij}.
\]

(10)

If there any burning cells (cells that caught fire), a complementary simulation on a short timescale is started. The fire may spread from burning cells \( b_{ij} = 1 \) to their neighbours:

(a) In each step of the burning process, each inhabited cell can catch fire with probability

\[
p = T_j c_{ij} \frac{1}{4 + 4\pi M} \sum_{i'j'} b_{i'j'}.
\]

(11)
| param. | range | meaning |
|--------|-------|---------|
| $N_i$  | $\mathbb{N}_{\geq 2}$ | linear size of the grid in first direction (open b.c.) |
| $N_j$  | $\mathbb{N}_{\geq 2}$ | linear size of the grid in second direction (periodic b.c.) |
| $t_{\text{max}}$ | $2^N$ | simulation time |
| $\rho_{\text{start}}$ | $(0, 1]$ | initial density of inhabited cells |
| $\alpha_{\text{fire}}$ | $\mathbb{R}^+$ | probability of a cell spontaneously catching fire |
| $\alpha_{\text{growth}}$ | $\mathbb{R}^+$ | measure for growth rate |
| $h_{\text{hostil}}$ | $\mathbb{R}_{\geq 0}$ | "hostility" of environment |
| $\alpha_{\text{Tvar}}$ | $[0, 1]$ | Geographic variability of the temperature |
| $\mu$ | $[0, 1]$ | global mutation rate |

Figure 2: A summary of the system parameters, which are explained in detail in sec. 2.3.

(b) In each individual cell, the fire lasts for one timestep and is extinguished afterwards ($b_{ij} \rightarrow 0$). A burning cell which has been burning already in the last step is set back to uninhabited state.

The system parameters are summarized in figure 2.

3 Simulation and Results

3.1 Simulation and Data Analysis

The result of a typical simulation run is displayed in figure 3. After a simulation run, the following data is available:

- The time series of carbon $C$ in the atmosphere,
- the time series of average temperature adaptation

$$A(t) := 1 - \frac{1}{N_L(t)} \sum_{ij} \ell_{ij}(t) \max(|\tau_{ij}(t) - T_j(t)|, 0) ,$$

$$V(t) := \frac{1}{N_L(t)} \sum_{ij} \ell_{ij}(t) \sigma_{ij}(t) ,$$

where the definition $N_L(t) := \sum_{ij} \ell_{ij}(t)$ has been used. From these time series, the authors have extracted maximum, average and minimum values for $A$ and
Figure 3: Results of a simulation run for the parameter choice $N_i = 12$, $N_j = 24$, $t_{\text{max}} = 256$, $\rho_{\text{start}} = 0.9$, $\alpha_{\text{fire}} = 0.002$, $\alpha_{\text{growth}} = 0.1$, $h_{\text{hostil}} = 1$, $\alpha_{\text{Tvar}} = 0.5$, $\mu = 0.1$.

The graphical output contains the following elements:
In the first row the system parameters are summarized, the first plot displays the currently stored amount of carbon, the second one shows currently inhabited (red/light) and uninhabited (blue/dark) cells and the third one is a graphical representation of the last forest fires.
In the second row one has the Fourier spectrum (absolute values), including a power-law fit according to equation (14), an illustration of the total amount of carbon stored in the cells over the whole simulation run and the time series of carbon in the atmosphere.
The third row contains the current $\tau$-values, the current $\sigma$-values and a time series of average temperature adaptation (12).
The fourth row displays the time series of row-averaged $\tau$ and $\sigma$ and a time series of average versatility (13). (In case that row-averaged values were not well-defined for certain times [when no cell of that particular row was inhabited], this is reflected by white spots in the corresponding graphs, cf. figure 8.)
$V$ and certain characteristics of the Fourier spectrum of $C$ (examining only absolute values, i.e. the amplitude spectrum $X(n)$, which is closely related to the power spectrum).

These characteristics are the position and the value of the dominant maximum beyond $n = 1$ and the parameters of a power-law fit

$$P(n) \approx \frac{C}{n^\alpha} + o_{\text{offset}}. \quad (14)$$

In addition the authors have analyzed the number of exceptional increases of $C$ (usually caused by forest fires), which are defined as timesteps $\tau$, where $C(\tau) > C(\tau - 1)$, but $C(\tau - 1) < C(\tau - 2)$. and the average over the changes $\Delta C(\tau) := C(\tau) - C(\tau - 1)$.

### 3.2 Results

Comparison of simulation runs for various sets of parameters revealed that the most crucial parameters are $\alpha_{\text{fire}}$ and $h_{\text{hostil}}$. Thus the authors have performed simulations for values of these two parameters chosen on a grid. For a wide mesh size and $\alpha_{\text{Tvar}} = \frac{1}{2}$, the results are shown in figure 4.

Some of the most important features of the model are already clearly recognizable: For $\alpha_{\text{fire}} = 0$ there are no forest fires, so for small $h_{\text{hostil}}$ all carbon is stored in the cells. For large values of $h_{\text{hostil}}$, it is still possible that the whole population dies out due to lack of adaptation. This happens in general (for the chosen set of parameters) for $h_{\text{hostil}} \geq 1.75$, regardless of $\alpha_{\text{fire}}$.

For $h_{\text{hostil}} = 0$ and $\alpha_{\text{fire}}$ small, but nonzero, one finds a local maximum of atmospheric carbon. This peak is a key feature of the model and will be examined closely in the following analysis. The basic explanation is that very small $\alpha_{\text{fire}}$ implies long periods without fires, in which large amounts of carbon are stored, such that one forest fire can have devastating consequences and eliminate almost all cells at once, releasing large amounts of carbon.

The authors find adaptation maxima at $h_{\text{hostil}} \approx 1$ due to the fact that for smaller values of $h_{\text{hostil}}$ the punishment for bad adaptation is less severe, while for even larger values of $h_{\text{hostil}}$ the population more frequently dies out completely without having time to adapt.

To study these effects more closely, the authors have restricted the grid to a smaller region in parameter space. Performing again several simulation runs gave the results depicted in figure 5. In addition they changed the parameter $\alpha_{\text{Tvar}}$ to zero (figure 6) and one (figure 7) and find that the former case (where temperature is solely dominated by $C$) is significantly different, while the latter case gives results very similar to those obtained for $\alpha_{\text{Tvar}} = \frac{1}{2}$. 
Figure 4: Results of simulation runs (10 runs for each specific combination of parameters) for the parameter choice $N_i = 12, N_j = 24$, $t_{\text{max}} = 256$, $\rho_{\text{start}} = 0.9$, $\alpha_{\text{growth}} = 0.1$, $\alpha_{\text{Tvar}} = 0.5$, $\mu = 0.1$. The parameters $\alpha_f = \alpha_{\text{fire}}$ and $h_h = h_{\text{hostil}}$ have been varied as $\alpha_{\text{fire}} = 0.005 k$, $k = 0, \ldots, 20$ and $h_{\text{hostil}} = 0.25 \ell$, $\ell = 0, \ldots, 8$. On the left hand side, the main plot shows the average carbon content $C$ of the atmosphere. On the right hand side, the first row of graphs gives the amplitude of the dominant Fourier mode, $X_{\text{max}}$, its position $M_{\text{pos}}$ and the exponent $\alpha$ of the spectral power-law fit \cite{14}. The second row shows the average temperature adaptation ($A$), the total number $N_{\text{Cinc}}$ of timesteps for which $C$ increased (which corresponds to the number of significant forest fires) and the average change $\Delta C_{\text{inc}}$ of carbon content during such events. Plots of the same quantities are given one more time (second graph on the left hand side, third and forth row on the right hand side), but usually shown from a different viewing position. In the last row the standard deviations $\sigma$ of the seven quantities under consideration are given.
Figure 5: Results of a sequence of 10 simulation runs. Parameters are chosen as in figure 4 but only the most interesting region is examined by choosing $\alpha_{\text{fire}} = 0.001 \ k, \ k = 1, \ldots, 20$ and $h_{\text{hostil}} = 0.1 \ \ell, \ \ell = 0, \ldots, 15$. Again the graphs are displayed for two different viewing positions.

Note that for larger values of $\alpha_{\text{fire}}$ the carbon content $C$ is almost constant for small $h_{\text{hostil}}$ and increases monotonically as a function of $h_{\text{hostil}}$, starting at $h_{\text{hostil}} \approx 1$. For small values of $\alpha_{\text{fire}}$ and $h_{\text{hostil}}$, there are large fluctuations in $C$, to be noticed also in the graph for $\sigma(C)$. 
Figure 6: Results of a sequence of simulation runs. Parameters are chosen as in figure 5 except for $\alpha_{Tvar} = 0$. Again the main graphs are displayed for two different viewing positions.

For this choice of parameters, the temperature does not depend on geographical latitude, but only on $C$. Apparently in this situation, for small values of $\alpha_{fire}$ it is more likely that the atmospheric carbon level remains low, i.e. the cells inhabitance is stable. This is also reflected in the Fourier amplitude spectrum, where the maximum still has low intensity, indicating the absence of significant oscillations as they can be recognized in figure 3.
Figure 7: Results of a sequence of simulation runs. Parameters are chosen as in figure 5 except for $\alpha_{Tvar} = 1$. Again the graphs are displayed for two different viewing positions. In this case, where temperature is independent of the carbon content $C$, one finds results not so different from those displayed in figure 5. This, together with the significantly different results shown in figure 6 seems to indicate that the system characteristics change significantly for small values of $\alpha_{Tvar}$, i.e. the mostly $C$-dominated case.
3.3 Interplay of parameters

The model parameters are not completely indepent. For example it is clear that in figure 4 the population typically dies out for $h_{hostil} \geq 1.75$. One could expect, however, that a larger mutation rate $\mu$ may be beneficial in dealing with hostile environments and that one could find values of $\mu$ that allow the population to survive in most cases.

This is indeed the case, as illustrated in figure 8. For $h_{hostil} = 1.75$ and $\mu = 0.3$ the population typically survives. This is also true for $\mu = 0.5$, though in this case it repeatedly comes close to extinction. This is a hint that also in this model a mutation rate which is too large can threaten the survival of a species.

4 Possible Extensions of the Model

There are several possible extensions of this model, which might be interesting to study:

- One could modify the grid by of introduction of uninhabitable spots, distributed randomly or as clusters of some fractal shape. One may also consider the growth of such uninhabitable spots (which can be interpreted as spreading of cities and deserts).

- In a more complete model one could explicitly consider the influence of soil, which can store carbon from deceased plants. Another important factor is humidity, which can reduce the rate of forest fires and also has influence on temperature. This could help to make the model more realistic, but at the same time harder to analyze.

- One could dismiss the normalization condition $C \leq 1$ and increase the total amount of Carbon available in the system during the simulation run (to mimic the effect of burning fossil fuels).

- One could try to explicitly include the effects of water vapor by setting up a separate cycle for $H_2O$, coupled to the plant life grid, but endowed with a separate dynamics on a shorter timescale. It might even be worthwhile to think about coupling a Stochastic Cellular Automata Carbon climate model as proposed in this article to a more traditional model based on atmosphere and ocean dynamics.
Figure 8: Large values for $h_{hostil}$ can be partially compensated by increasing the mutation rate $\mu$. These plots show typical simulation runs for $h_{hostil} = 1.75$ and $\mu = 0.1$, $\mu = 0.3$ and $\mu = 0.5$. (Other parameters were chosen as in figure 3.)
• A fascinating extension may be the coupling of the system to an “ocean”, i.e. a carbon sink with limited absorption rate and temperature-dependent capacity.

This may be especially important, since the solubility of CO$_2$ in water decreases with increasing temperature$^2$. So while the oceans can store enormous amounts of CO$_2$, their capacity decreases with global warming. This could lead to a dramatic positive feedback chain:

In such a scenario, an increase of temperatures above a certain critical level could lead to a massive release of CO$_2$ from the oceans and thus a further rise of temperature. Confronted with this frightening perspective, even simple models which allow to study the dynamics of such catastrophic behaviour can be extremely valuable.

Apart from these extensions, there are also several aspects of the original model that should be studied in greater depth. The interplay of model parameters, as sketched in subsection 3.3, certainly deserves closer examination. In addition it may be interesting to study size-dependence, in particular the correspondence between “critical” $\alpha_{\text{fire}}$ and the extension of the grid (for example by considering the limit $N_i N_j \to \infty$, $\alpha_{\text{fire}} \to 0$ with the product $N_i N_j \alpha_{\text{fire}}$ held fixed).

5 Summary and Conclusions

The authors have studied a “small-world” model designed to describe the interplay between global climate, carbon storage and sudden outbreaks of forest fires.

They have seen that the least predictable behaviour is found for small, but nonzero values for the probability $\alpha_{\text{fire}}$ of a cell catching fire. While in most cases an increased hostility of the environment leads to a smaller average population and thus to a higher carbon content of the atmosphere, this is not necessarily true in the most complex (and probably most realistic) case of small, but nonzero $\alpha_{\text{fire}}$.

This “critical” behaviour is more dramatic in the case of temperature dominated by atmospheric carbon (instead of geographic parameters), in terms of this model for $\alpha_{T\text{var}} \approx 0$.

They have also seen that there exists an interesting interplay of parameters, as for example the fact that a larger hostility of the environment can be partially compensated by a higher mutation rate. In general, the model exhibits a rich structure worth further investigation, and offers the possibility of several interesting extensions.

$^2$This is in general true for the solubility of gases in liquids, while it is exactly the other way round for the solubility of solids in liquids. This is easy to understand, since enhanced thermal movement of the molecules makes it easier for gases to evaporate, while it makes it harder for solids to crystallize.
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