Parity violation in ferromagnet-superconductor heterostructures with strong spin-orbit coupling

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We study spectroscopic properties of ferromagnetic-superconductor heterostructures with strong spin-orbit coupling of the Rashba type and in the presence of exchange fields. The superconducting layer (film) experiences both an intrinsic spin-orbit field and an exchange field due to the proximity to ferromagnetic layers (films). We analyse the temperature dependence of the order parameter for superconductivity at various values of exchange field and spin-orbit coupling, and describe momentum-dependent properties that exhibit parity violation. Furthermore, we show that parity violation can be probed in tunneling experiments of the single-particle density of states and in photoemission experiments of the momentum distribution.

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The interplay of magnetism and superconductivity has played a very important role in several materials including Cuprate [1] and Pnictides [2], while the interplay between spin-orbit effects and superconductivity has been important in the case of non-centro-symmetric [3] and topological [4] superconductors. In the case of Cuprate and Pnictides the interplay of magnetism and superconductivity leads to a very rich phase diagram and to unconventional behavior such as d-wave and multi-s-wave order parameters, but there is no evidence that the superconducting ground states of these systems violate parity. Similarly in the case of non-centro-symmetric or topological superconductors, where spin-orbit coupling (SOC) terms lead to parity-odd matrix elements, ground state properties do not exhibit parity violation.

In this paper, we study a simple case of the interplay of magnetism, superconductivity and spin-orbit coupling to a superconducting ground state that violates parity. In Fig. 1 we show possible geometries for the realization of such ground state. We focus on the simpler bilayer and trilayer cases shown in Figs. 1(a), and 1(b), respectively. However, the effect also can exist in the multilayered systems illustrated in Fig. 1(c). In the bilayer and trilayer cases, we show that when the spin-orbit-coupled superconducting layer experiences a strong in-plane exchange field due to the proximity to a ferromagnetic layer, it may no longer exhibit properties with well defined parity. Parity is violated in the superconducting layer when its critical temperature is lower than the ferromagnetic ordering temperature. This parity violation manifests itself in spectral properties of the superconducting layers such as the quasi-particle excitation spectrum, single-particle density of states, and spin-dependent momentum distribution. The latter two properties may be measured via tunnelling and photoemission experiments. Our theoretical findings point into a new experimental direction for ferromagnet-superconductor multilayers, beyond the traditional proximity effects [5], magnetic couplings across the superconducting layer [6], and Josephson coupling across a ferromagnetic layer [7]. Our results are particularly relevant to recent experimental results that show the emergence of exchange interactions in ferromagnet-superconductor multilayers consisting of Manganites and Cuprates [8, 9], where if SOC exists, then parity violation will also emerge.

In the heterostructures (multilayered systems) shown in Fig. 1, the layers that become superconducting (SC) across a ferromagnetic layer [8]. Our results are particularly relevant to recent experimental results that show the emergence of exchange interactions in ferromagnet-superconductor multilayers consisting of Manganites and Cuprates [8, 9], where if SOC exists, then parity violation will also emerge.

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Figure 1. (color online) Layered heterostructures consisting of ferromagnets (FM) and superconductors (SC) with spin-orbit coupling in (a) bilayer, (b) trilayer, and (c) multilayer configurations.

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our specific calculations to this case. For the bilayer and trilayer cases the Hamiltonian density of the combined ferromagnet-superconductor (FMSC) system has three contributions $\mathcal{H}(\mathbf{r}) = \mathcal{H}_S(\mathbf{r}) + \mathcal{H}_{FS}(\mathbf{r}) + \mathcal{H}_F(\mathbf{r})$. The first term of $\mathcal{H}(\mathbf{r})$ is

$$\mathcal{H}_S(\mathbf{r}) = \sum_{s,s'} \psi_1^s(\mathbf{r}) [\hat{\mathcal{K}}_1 + \mathbf{H}_{SO}(-i\nabla)]_{ss'} \psi_{s'}(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})$$

(1)

describing the superconducting layer, with $\hat{\mathcal{K}}_1 = -\nabla^2/2m - \mu$ being the kinetic energy, $\mathbf{H}_{SO}(-i\nabla)_{ss'}$ being the SOC, and $\mathcal{H}_I = -g \psi_1^s(\mathbf{r}) \psi_1^s(\mathbf{r}) \psi_1^s(\mathbf{r})$ being the local interaction term. Here, $\psi_1^s(\mathbf{r})$ is a creation operator of an electron with spin $s$ located at the point $\mathbf{r}$. The second term $\mathcal{H}_{FS}(\mathbf{r}) = -J_{FS,\nu} \psi_1^s(\mathbf{r}) \sigma_\nu \psi_{s'}(\mathbf{r}) \cdot \mathbf{S}_{F,\nu}(\mathbf{r} + \mathbf{a})$, where $\mathbf{S}_{F,\nu}$ is a spin in the ferromagnetic layer located at distance $\mathbf{a}$ away from the superconducting layer along the $\nu$ direction, $\sigma_\nu$ is the Pauli matrix, and $J_{FS,\nu}$ is the exchange coupling along the $\nu$ direction. The third term $\mathcal{H}_F(\mathbf{r}) = K_F(\mathbf{r}) - J_{\mu\nu} \sum_{i<j} \mathbf{S}_{F,\mu}(\mathbf{r}_i) \cdot \mathbf{S}_{F,\nu}(\mathbf{r}_j)$ describes the ferromagnetic layers, which can be itinerant (localized) if the magnetic state is metallic (insulating) with non-zero (zero) kinetic energy density $K_F(\mathbf{r})$. In either case, when in-plane ferromagnetism sets in at $T_M$, the SC layers experience a strong parallel exchange field. For definiteness, we assume that the FM layer is insulating and governed by a magnetic Hamiltonian density corresponding to the XYZ model: $\mathcal{H}_F(\mathbf{r}) = -J_F \sum_{i<j} \mathbf{S}_{F,\nu}(\mathbf{r}_i) \cdot \mathbf{S}_{F,\nu}(\mathbf{r}_j)$, where $\nu = x, y, z$. For simplicity, we take the case of $J_y \gg \{J_x, J_z\}$, such that the magnetization points essentially along the $y$-direction of the superconductor [10].

In this case, the effective Hamiltonian matrix for the superconducting layer acquires the simple form

$$\mathbf{H}_0(\mathbf{k}) = \begin{pmatrix} \hat{K}_1(k) & -h_+^r(k) & 0 & -\Delta \\ -h_-^r(k) & \hat{K}_1(k) & \Delta & 0 \\ 0 & \Delta^* & -\hat{K}_1(-k) & h_+^r(-k) \\ -\Delta^* & 0 & h_+^r(-k) & -\hat{K}_1(-k) \end{pmatrix}$$

(2)

in the four-dimensional Nambu basis $\Psi^\dagger(\mathbf{k}) = (\psi_1^r(\mathbf{k}), \psi_1^d(\mathbf{k}), \psi_1(-\mathbf{k}), \psi_1(-\mathbf{k}))$. Here, the kinetic energy for the $\uparrow$ spin is $\hat{K}_1 = k^2/2m - \mu - h_z$ and for the $\downarrow$ spin is $\hat{K}_1 = k^2/2m - \mu + h_z$, while the order parameter for superconductivity $\Delta = |\Delta| \text{e}^{i\phi}$ with $|\Delta|$ being its magnitude, and with $\phi$ being its phase. The spin-flip field $h_\perp(k) = h_x(k) + \text{i} h_y(k)$ is the complex representation of the sum of the components of the exchange and spin-orbit fields felt by the electrons in the superconducting layer. The exchange fields are $h_\nu = J_\nu \langle \mathbf{S}_{F,\nu} \rangle$, where $\nu = x, y, z$, while the spin-orbit fields are assumed to be of the Rashba type [11] $h_R(k) = -\text{v}_R k_y + i\text{v}_R k_z$.

Since $J_y \gg \{J_x, J_z\}$ the magnetization in the FM layer points along the $y$ direction, then $h_z = h_y = 0$, but $h_y \neq 0$, which leads to the total spin-flip field $h_\perp(k) = -v_R k_y + \text{i}(h_y + v_R k_z)$.

For simplicity, we consider only case II, in the limit of $T_M \gg T_c$, which is sufficient to produce a parity violating superconducting state. In this case, the thermodynamic potential corresponding to $\mathbf{H}_0(k)$ defined in Eq. (2) is

$$\Omega_0 = V \frac{|\Delta|^2}{g} - \frac{1}{2} \sum_{k,j} \ln \left[ 1 + \exp \left( \frac{-E_j(k)}{T} \right) \right] + \sum_{k} K_+(k)$$

(3)

where $K_+(k) = \frac{1}{2} \left( \hat{K}_1(k) + \hat{K}_1(-k) \right)$ is a reference kinetic energy, and $E_j(k)$ are the eigenvalues of $\mathbf{H}_0(k)$. The saddle-point order parameter equation

$$\frac{V}{g} |\Delta|^2 = -\frac{1}{4} \sum_{k,j} n_F(E_j(k)) \frac{\partial E_j(k)}{\partial |\Delta|}$$

(4)

is obtained from the condition $\delta \Omega_0 / \delta |\Delta| = 0$. While the number equation

$$N_0 = \sum_k \left( 1 - \frac{1}{2} \sum_j n_F(E_j(k)) \frac{\partial E_j(k)}{\partial \mu} \right)$$

(5)

fixes the chemical potential $\mu$ and is obtained from the relation $N_0 = -\partial \Omega_0 / \partial \mu$. In the expressions above $n_F$ is the Fermi function. These two equations need to be solved numerically and self-consistently [12].

In calculations we assume that the constants have values $g/V = 4.49 \times 10^{-6} e_F$ and $N_0/V = k_F^2/(2\pi)$. The solution obtained is checked for the minimum condition $\partial^2 \Omega_0 / \partial |\Delta|^2 > 0$ to guarantee the thermodynamic stability of the system.

In order to solve for $|\Delta|$ and $\mu$, we need to calculate explicitly the eigenvalues $E_j(k)$. Notice that when there is no magnetization in the ferromagnet, the effective exchange fields are zero and all the matrix elements of $\mathbf{H}_0(k)$ have well-defined parity, i.e., $h_\perp(k)$ is odd, while all the other elements are even in momentum space. However, when the in-plane exchange field $(h_x, h_y, 0)$ is non-zero, the spin-flip matrix element $h_\perp(k)$ does not have a

Figure 2. (color online) Sketches of normalized dependencies of the magnetization $M(T)/M(0)$ (red-dashed line) and the order parameter $|\Delta(T)|/|\Delta(0)|$ (black-solid line) as functions of temperature $T$. Two cases are possible: (a) $T_M < T_c$, (b) $T_M > T_c$. 
well defined parity, while all the other elements remain parity even. This fact alone, leads to the emergence of a parity violating quasi-particle (quasi-hole) energy spectrum when there is an in-plane component of the total exchange field. If the exchange field is zero (no ferromagnetism), or if the magnetization (exchange field) is only along the z-direction, then there is no parity violation in the excitation spectrum and in the superconducting state. The solution for the excitation spectra can be obtained analytically, but the expressions are extremely cumbersome in the parity violating case, thus we prefer to obtain the eigenvalues of $H_0(k)$ numerically, by making a base transformation $\Phi(k) = U(k)\Psi(k)$ where $\Phi(k)$ is the four-dimensional spinor describing quasi-particles and quasi-holes, and $U(k)$ is the unitary matrix of coherence factors that diagonalizes $H_0(k)$.

Numerical results for $|\Delta(T)|/\varepsilon_F$ are shown in Fig. 3 for various values of $h_y/\varepsilon_F$ and $v_F/\varepsilon_F$, where $\varepsilon_F$ ($v_F$) is the Fermi energy (velocity) of the non-interacting Fermi gas without SOC. Notice that increasing $h_y$ tends to suppress superconductivity with zero center-of-mass (CM) momentum pairing due to pair breaking. However, finite SOC tends to stabilize superconductivity since its momentum-dependent spin-flip field induces a triplet component in the order parameter which counteracts the pair breaking effect. In order to see the violation of parity in the SC layers, it is essential to measure momentum dependent quantities via spectroscopic techniques. Thus, next, we investigate three spectroscopic quantities that contain valuable information about parity violation in the superconducting state.

The first property is the quasi-particle excitation spectrum consisting of the two upper branches of eigenvalues $E_j(k)$, shown in Fig. 1 where a clearly parity violating excitation spectrum is present when the magnetization (exchange field $h_y$) is non-zero. The direction that affects the overall parity of the excitation spectrum is $k_x$ because the magnetization of the ferromagnet is assumed to point out along the $y$-direction alone. This leads to the spin-flip field $h_x(k) = -v_R k_y + i(h_y + v_R k_x)$, that depends on the combination $h_y + v_R k_x$, and, therefore, does not have well defined parity. More generally, when the magnetization of the ferromagnet has components along the $x$ and $y$ directions then $h_k(k) = (h_y - v_R k_y) + i(h_y + v_R k_x)$, and the excitation spectrum violates parity both along the $x$ and $y$ directions.

The second property is the electronic density of states (DOS), which can be obtained from the resolvent operator matrix $G(\omega, k) = [\omega - H_0(k)]^{-1}$ in terms of the imaginary part of the diagonal elements of $G$ as $\rho_\parallel(\omega) = -(1/\pi) \text{Im} G_{ii}(\omega + i\delta, k)$, where $i = (\uparrow, \downarrow)$ labels the spins in the original basis, with the spin quantization axis chosen to be along the perpendicular direction ($z$) to the films. The resulting expression in terms of the matrix of coherence factors is simply

$$\rho_{\parallel,\downarrow}(\omega) = \sum_{j,k} \frac{1}{2} |U_{1j}(k)|^2 \delta(\omega - E_j(k)),$$

where $\uparrow (\downarrow)$ corresponds to the up (down) spin of the particle with respect to the direction of the exchange field $h_y$. The spin-dependent DOS is illustrated in Figs. 1(c) and 1(g). Notice that Eq. (6) differs from the quasi-particle DOS $\rho_{qp,j}(\omega) = \sum_k \delta(\omega - E_j(k))$ due to the presence of the coherence factors $U_{ij}$. The main effects of $h_y \neq 0$ is to create a parity-violating asymmetry in the low-energy quasi-particle bands and produce the split-peak structure in the single-particle DOS seen in Fig. 1(g) in comparison to the $h_y = 0$ case shown in Fig. 1(c). Furthermore, the spin-up and spin-down electronic DOS are very different from each other as the coherence factors are highly sensitive to the presence of the exchange field $h_y$. We show only the $\omega > 0$ region as the $\omega < 0$ region can be obtained by the transformation: $\rho_\parallel(\omega) = \rho_\parallel(-\omega)$ and $\rho_\downarrow(\omega) = \rho_\uparrow(-\omega)$. In the presence of $h_y$ the system remains gapped, but the induced triplet component of the order parameter in the generalized helicity basis acquires a $k_x$ component in addition to the $k_x + ik_y$ contribution when $h_y = 0$. The former contribution is responsible for the linear density of states right above the quasiparticle gap. Either the spin-dependent or total DOS $\rho_T(\omega) = \rho_\uparrow(\omega) + \rho_\downarrow(\omega)$ may be measured via tunneling experiments [13–16].

The third property that we analyse is the spin-dependent momentum distribution $n_s(k) = \langle \psi_s^\dagger(k) \psi_s(k) \rangle$, where $s = (\uparrow, \downarrow)$. The resulting expression in terms of the coherence factors $U_{ij}$ and the quasi-particle operators $(\phi^\dagger, \phi)$ is $n_{\parallel,\downarrow}(k) = \sum_j \frac{1}{2} |U_{1j}(k)|^2 + iU_{2j}(k)|^2 (\phi^\dagger k \phi)|_{\parallel,\downarrow}(k)$,
which can be further expressed in terms of the Fermi function $n_F$ and the eigenvalues $E_j(k)$ as

$$n_{\uparrow,\downarrow}(k) = \sum_j \frac{1}{2} [U_{1j}(k) \pm iU_{2j}(k)]^2 n_F(E_j(k)). \quad (7)$$

We show the spin-dependent momentum distributions in Fig. 4 along with their asymmetric parts $n_{s, asym}(k_x, k_y) = \frac{1}{2} [n_s(k_x, k_y) - n_s(-k_x, k_y)]$. The asymmetry of the distribution, which also represents parity violation, arises from the presence of both the in-plane exchange field and the in-plane Rashba SOC. In the absence of either one of these terms, the momentum distribution would be parity even in both $k_x$ and $k_y$ directions. The total momentum distribution $n(k) = n_\uparrow(k) + n_\downarrow(k)$ is also parity violating, but the effect is smaller. Spin-dependent momentum distributions may be measured via the recently developed spin- and angle-resolved photoemission spectroscopy (Spin-ARPES) [17, 18], while total momentum distributions may be measured via standard ARPES [20, 21].

In summary, we studied spectroscopic properties in ferromagnet-superconductor heterostructures and showed that a parity violating superconducting state can exist, when the superconducting layers possess strong spin-orbit coupling. We focused on the bilayered and trilayered heterostructures, where the Curie temperature $T_M$ of the ferromagnetic layers was larger than the critical temperature $T_c$ of the superconducting layers. However, a similar effect also occurs when $T_M < T_c$. Where

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Value \tabularnewline
\hline
$T/\varepsilon_F$ & 0.007 \tabularnewline
$eR/\varepsilon_F$ & 0.30 \tabularnewline
$\mu/\varepsilon_F$ & 1.02 \tabularnewline
$|\Delta|/\varepsilon_F$ & 0.015 \tabularnewline
\hline
\end{tabular}
\caption{Parameters used for spin-dependent momentum distributions.}
\label{tab:parameters}
\end{table}
the superconducting state is first parity preserving, and then below $T_M$ becomes parity violating. We found that if the in-plane Rashba spin-orbit coupling is zero, then the superconducting state is always parity preserving even if the ferromagnet is in its ordered state. Thus, we concluded that it is necessary to have both an in-plane magnetization (exchange field) and in-plane spin-orbit coupling for the emergence of a parity violating superconducting state. Finally, we showed such parity violation can be detected through the measurement of spectroscopic properties such as the quasi-particle excitation spectrum, single-particle density of states and spin-dependent momentum distributions.

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[1] E. Dagotto, Rev. Mod. Phys. 66, 763 (1994).
[2] D. C. Johnston, Adv. Phys. 59, 803 (2010).
[3] E. Bauer and M. Sigrist (Eds.), Non-centrosymmetric superconductors, Lecture Notes in Physics vol. 847, Springer-Verlag, Berlin-Heidelberg (2012).
[4] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010); X. L. Qi and S. C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[5] A. I. Buzdin, Rev. Mod. Phys. 77, 935 (2005).
[6] C. A. R. Sá de Melo, Phys. Rev. Lett. 79, 1933 (1997).
[7] T. Kontos, M. Aprili, J. Lesueur, F. Genêt, B. Stephanidis, and R. Boursier, Phys. Rev Lett. 89, 137007 (2002).
[8] Y. Liu, C. Visani, N. M. Nemes, M. R. Fitzsimmons, L. Y. Zhu, J. Tornos, M. Garcia-Hernandez, M. Zhernenkov, A. Hoffmann, C. Leon, J. Santamaria, and S. G. E. te Velthuis, Phys. Rev. Lett. 108, 207205 (2012).
[9] S. R. Giblin, J. W. Taylor, J. A. Duffy, M. W. Butchers, C. Utfeld, S. B. Dugdale, T. Nakamura, C. Visani, and J. Santamaria, Phys. Rev. Lett. 109, 137005 (2012).
[10] This assumption is not necessary for the emergence of parity violation. Any in-plane magnetization will lead to parity violation in the superconducting state.
[11] E. I. Rashba, Sov. Phys. Solid State 2, 1109 (1960).
[12] Since $T_M \gg T_c$, the magnetization $M_\nu = \langle S_{F,\nu} \rangle$ in the FM layer is well developed and its major effect in the SC layer is to introduce a proximity induced exchange field with components $(h_x, h_y, h_z)$. The value of $M_y$ in the ferromagnet can be obtained separately. For instance, when $J_y \gg \{J_x, J_z\}$, only $h_y \neq 0$, and the self-consistency equation for $h_y(T) = h_y(0) \tanh [h_y(T)/T]$.
[13] I. Giaever, Rev. Mod. Phys. 46, 245 (1974); B. D. Josephson, Rev. Mod. Phys. 46, 251 (1974); G. Binnig and H. Rohrer, Rev. Mod. Phys. 59, 615 (1987).
[14] Niv Levy, Tong Zhang, Jeonghoon Ha, Fred Sharifi, Alec Talin, Young Kuk, and Joseph A. Stroscio, Phys. Rev. Lett. 110, 117001 (2013).
[15] R. Wiesendanger, Rev. Mod. Phys. 81, 1495 (2009).
[16] P. M. Tedrow and R. Meservey, Phys. Rev. Lett. 26, 192 (1971); R. Wiesendanger, H.-J. Güntherodt, G. Güntherodt, R. J. Gambino, and R. Ruf, Phys. Rev. Lett. 65, 247 (1990).
[17] D. Hsieh, Y. Xia, D. Qian, L. Wray, J. H. Dil, F. Meier, J. Osterwalder, L. Patthey, J. G. Checkelsky, N. P. Ong, A. V. Fedorov, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, M. Z. Hasan Nature 460, 1101 (2009).
[18] Chris Jozwiak, Cheol-Hwan Park, Kenneth Gotlieb, Choongyu Hwang, Dung-Hai Lee, Steven G. Louie, Jonathan D. Denlinger, Costel R. Rotundu, Robert J. Birgeneau, Zahid Hussain, Alessandra Lanzara. Nature Physics 9, 293 (2013).
[19] C. Jozwiak, Y. L. Chen, A. V. Fedorov, J. G. Analytis, C. R. Rotundu, A. K. Schmid, J. D. Denlinger, Y.-D. Chuang, D.-H. Lee, I. R. Fisher, R. J. Birgeneau, Z.-X. Shen, Z. Hussain, and A. Lanzara Phys. Rev. B 84, 165113 (2011).
[20] Andrea Damasceli, Zahid Hussain, and Zhi-Xun Shen, Rev. Mod. Phys. 75, 473 (2003).
[21] D. H. Lu, I. M. Vishik, M. Yi, Y. L. Chen, R. G. Moore, and Z.-X. Shen, Ann. Rev. of Cond. Mat. Phys. 3, 129 (2012).