Non-diagonal contributions to $Z\gamma V^*$ vertex, polarizations and bounds on $Z\bar{t}q$ couplings

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The non-diagonal contributions to the trilinear gauge bosons couplings $Z\gamma V^*$ ($V = Z, \gamma$) at the one-loop level are calculated using complex flavor-changing neutral currents mediated by the $Z$ boson. We find that it is not possible to generate non-diagonal contributions to the vertex $Z\gamma \gamma^*$ using these couplings, whereas for the vertex $Z\gamma Z^*$ only the $CP$-conserving form factor $h_Z^X$ is induced. Constraints on the $Z\bar{t}c$ couplings are obtained from current LHC data. Our bounds are $|g_{tZ}|, |g_{Z\gamma}| \leq 0.007$ and $|g_{tZ}^V|, |g_{Z\gamma}^V| \leq 0.0095$ in terms of the vector and axial couplings, while in terms of the coefficients of the effective field theory framework these bounds are $|C_{g^{\gamma(1+3)}}|^2, |C_{g^{\gamma(1+3)}_r}| \leq 0.12$ and $|C_{g^{Z(2+3)}}^\gamma|, |C_{g^{Z(2+3)}_r}| \leq 0.16$. The new contributions to $h_Z^X$ are of order $10^{-6} - 10^{-7}$. We also study the polarized partial widths of the $V^* \to Z\gamma$ process, as they are useful for expressing LHC cross-sections. Furthermore, two types of left-right asymmetries are also discussed. The polarized $\Gamma(V^* \to Z\gamma)$ show significant deviations from SM, whereas the asymmetries are only non-vanishing in theories beyond the SM. Thus, the polarized observables are highly sensitive to new physics effects, especially those related to $CP$-violation.

I. INTRODUCTION

The current limits on the trilinear neutral gauge bosons couplings (TNGBCs) reported by the CMS and ATLAS collaborations at $\sqrt{s} = 13$ TeV [1–2] are close to the Standard Model (SM) prediction [3, 4]. These bounds have been improved one order of magnitude compared with previous LHC analysis [5–6]. Therefore, with the LHC’s current Run 3 and its luminosity upgrade, the TNGBCs can be measured in the following years. Regarding the detection of TNGBCs, corrections from beyond SM (BSM) theories and the search for new observables will play an important role.

In the SM, a vertex with three neutral gauge bosons can only exist if at least one of them is off-shell and it is induced through dimension-six and dimension-eight operators [3–7]. These operators give rise to $CP$-conserving and $CP$-violating form factors, where the former can be generated at the one-loop level in the SM, whereas the $CP$ violation can be induced at the same level in models of new physics [3, 10], and hence, new sources of $CP$ violation are possible. According to Sakharov’s conditions [11], the violation of $CP$ symmetry is necessary to explain the invariance between matter and antimatter in the universe. The TNGBCs have been studied within a lot of extensions of the SM, such as minimal supersymmetric standard model (MSSM) [3, 12], the $CP$-violating two-Higgs doublet model (2HDM) [13, 14], models with axial and vector fermion couplings [13], models with extended scalar sectors [16, 17], the effective Lagrangian approach [7], models with $CP$ violation [18] and models with flavor changing neutral currents (FCNCs) [10]. The effects of the TNGBCs in BSM theories at future colliders have been also revisited very recently [19–30], whereas the first experimental results were obtained at the LEP [27–29] and the Tevatron [30–32] colliders. On the other hand, the phenomenology of these couplings has been analyzed long ago [12, 33–39] and also in the last years [40–55] as a renew interest has emerged.

New contributions to TNGBCs could arise from $t\bar{q}Z$ ($q = c, u$) FCNCs couplings [10]. Such a process can not be generated at tree level in the SM and is very suppressed as the $B(t \to Zc)$ branching ratio is of order $10^{-14}$, whereas $B(t \to Zu)$ can be three orders of magnitude smaller [46]. At the LHC the relevant top quark FCNCs processes are studied by the ATLAS and CMS collaborations and include decays such as $t \to j\ell^+\ell^-$ [47], $t \to Hq$ [48–51], $t \to gg$ [52, 54], $t \to \gamma q$ [53, 56] and $t \to Zq$ [57–59]. The implications of these process have been also studied at future colliders such as the FCC-he [60–62], CEPC [63] and lepton-hadron colliders [64, 65]. The process $t\bar{q}Z$ interaction can be induced through the effective field theory (EFT) method [66–68]. To describe these new interactions, the EFT framework requires in principle a complete basis of operators, which have been constrained at the LHC [1, 2, 57, 58]. Under the context of the TNGBCs, the polarizations effects have been
In this work, we study the non-diagonal contributions to the TNGBCs $Z\gamma V^*$ ($V = Z, \gamma$) arising from a general model where FCNCs mediated by the $Z$ gauge boson are allowed. The rest of the presentation is organized as follows. In Sec. II we present a review of the TNGBCs of type $Z\gamma V^*$ ($V = Z, \gamma$) and the basics of the EFT used to introduce the FCNCs couplings. We discuss in Sec. III the main steps of our calculations and show our analytic results in terms of the Passarino-Veltman scalar functions. In Sec. IV we obtain the bounds on FCNCs mediated by the $Z$ boson, which are used in Sec. V to study numerical analysis. Finally, in Sec. VI the conclusions and outlook are presented.

II. THEORETICAL FRAMEWORK

In this section we introduce the theoretical description of the TNGBCs and the FCNCs of the $Z$ gauge boson. The notation in Ref. [3] is adopted.

A. Trilinear gauge boson couplings

In this work, we are interested in TNGBCs of the form $Z\gamma V^*$ ($V = Z, \gamma$), which arise from dimension-six and dimension-eight operators [8]. Following the kinematics in Fig. 1, the $Z\gamma V^*$ coupling can be parametrized as follows:

$$\Gamma_{Z\gamma V^*}^{\alpha\beta\mu}(p_1, p_2, q) = \frac{i(q^2 - m_Z^2)}{m_Z^2} \left[ h_1^V \left( p_1^\mu g^{\alpha\beta} + p_2^\mu g^{\beta\alpha} \right) + \frac{h_2^V}{m_Z^2} q^\alpha \left( q \cdot p_2 g^{\mu\beta} - p_2^\mu q^\beta \right) 
- h_3^V \epsilon^{\mu\alpha\beta\rho} p_{2\rho} - \frac{h_4^V}{m_Z^2} q^{\alpha} \epsilon^{\mu\beta\rho\sigma} q_{\rho} p_{2\sigma} \right],$$

(1)

where $h_1^V$ and $h_2^V$ correspond to the $CP$-conserving form factors, whereas $h_3^V$ and $h_4^V$ are $CP$-violating. In the SM, the form factor $h_3^V$ is the only one induced in both cases ($V = \gamma, Z$) via a fermion loop and are of order $10^{-4}$ [3]. On the other hand, the $CP$ violation can be induced in the vertex $Z\gamma Z^*$ through charged scalars at the one-loop level [17]. The general form of Eq. (1) for three off-shell bosons can be found in Refs. [7, 8]. It is observed in Eq. (1) that the coupling $Z\gamma V^*$ is only non-vanishing if the $V$ boson is off-shell, while for the on-shell case ($q^2 = m_V^2$) the coupling can not exist because of Bose statistics and angular momentum conservation.

The current bounds on the $CP$-conserving form factors $h_3^V$ and $h_4^V$ ($V = \gamma, Z$) at the 95% C.L. were obtained by the ATLAS collaboration analyzing the rate and kinematics properties of the $Z\gamma$ production at $\sqrt{s} = 13$ TeV [2]:

$$- 3.7 \times 10^{-4} < h_3^{\gamma} < 3.7 \times 10^{-4},$$

(2)

$$- 4.4 \times 10^{-7} < h_4^{\gamma} < 4.3 \times 10^{-7},$$

(3)

$$- 3.2 \times 10^{-4} < h_3^{Z} < 3.3 \times 10^{-4},$$

(4)

$$- 4.5 \times 10^{-7} < h_4^{Z} < 4.4 \times 10^{-7}.$$  

(5)

these results are in terms of the $CP$-conserving parameters $h_{3,4}^V$ as they do not interfere with the $CP$-violating form factors $h_{1,2}^V$, and their sensitivities to the TNGBC are almost identical [34]. In general, the $h_1^V$ form factors are complex and functions of the off-shell boson four momentum ($q^2$). The absorptive parts are a consequence of the optical theorem and could be also extracted from the Cutkosky rules [75], which yield identical results through the usual Feynman diagram calculation [76]. It has been pointed out that the imaginary part can be larger than the real one [3, 10]. Recently, off-shell couplings with complex form factors have also been revisited, for instance, the coupling of the gluon with a quark-antiquark pair [77, 79] and Higgs boson couplings [69].
FIG. 1. Kinematics for the TNGBCs $Z\gamma V^*$ ($V = \gamma, Z$)

B. The effective field theory framework

The top quark FCNCs can be described by an effective Lagrangian approach, which is based on dimension-four and -five operators that satisfy Lorentz and $SU(3)_C \times U(1)_{EM}$ gauge symmetries. For the case of FCNCs mediated by the $Z$ boson, this effective Lagrangian is

$$L_{eff} = -\frac{e^2}{2SW_{EW}} \tilde{T}_{\gamma\mu} \left( g_{VZ}^{tq} - \gamma^{5} g_{AZ}^{tq} \right) q^{\mu} Z$$

where the factors $g_{VZ}^{tq}$ and $g_{AZ}^{tq}$ are complex in general and satisfy: $g_{VZ}^{tq} = g_{VZ}^{tq \dagger}$ by hermiticity and $|g_{VZ}^{tq}| + |g_{AZ}^{tq}| = 1$. On the other hand, the effective Lagrangian under the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry that induces FCNCs couplings of the top quark can be written as

$$L_{EFT} = L_{SM} + \sum_i \frac{c_i}{\Lambda^2} O_i + H.c,$$

where the $O_i$ operators are in general dimensions-six. The parameter $\Lambda$ can be seen as the scale of new physics and the dimensionless coefficients $c_i$ parametrize the strength of the new physics couplings. It is well known that an EFT is only appropriate up to energies of order $\Lambda$. We note in Eq. (7) that the SM is recovered in the limit $\Lambda \to \infty$.

Assuming that at least at the LHC energies range the new physics still preserves the SM gauge invariance, Therefore, the Lagrangian (6) could be used as an equivalent parametrization of the $SU(3)_C \times SU(2)_L \times U(1)_Y$-invariant operators. In such a case, the couplings in Eq. (6) can be written in terms of the gauge-invariant operators coefficients, the scale $\Lambda$ and the SM parameters as [67, 80]

$$g_{VZ}^{tq} = \frac{m_t^2}{\Lambda^2} \left[ C_{\phi_u}^{(a+3)*} + C_{\phi_q}^{-(a+3)*} \right], \quad g_{AZ}^{tq} = -\frac{m_t^2}{\Lambda^2} \left[ C_{\phi_u}^{(a+3)*} - C_{\phi_q}^{-(a+3)*} \right],$$

where $a = 1, 2$ indicates the family of the light quark.

III. ANALYTICAL RESULTS

In this section, we discuss the contributions to the TNGBCs $Z\gamma V^*$ ($V = \gamma, Z$) arising from complex FCNCs couplings of the form of Eq. (6). It is clear that for $V = \gamma$ is not possible induce none type of new contributions, whereas for the $V = Z$ case we find that the $h_{Z1}^Z$ and $h_{Z2}^Z$ CP-violation form factors are not induced. On the other hand, a new contribution to the form factor $h_{Z3}^Z$ can be obtained from the generic diagram shown in Fig. 2. For our calculation, we consider conserved vector currents and Bose symmetry. Additionally, the Passarino-Veltman reduction scheme with the help of the FeynCalc [81] and Package-X [82] packages are used.

A. $Z\gamma Z^*$ coupling

For this case, there are 4 contributing Feynman diagrams but because of gauge invariance we only need to calculate the amplitude $M^{\alpha\beta\mu}$ of the diagram depicted in Fig. 2 the remaining diagrams can be obtained in a straightforward form as follows. We have to consider an extra diagram from the exchange of fermions running into the loop. Therefore, the second amplitude is $M^{\alpha\beta\mu}(f_i \leftrightarrow f_j)$, and due to Bose statistics we also consider the permutation of the two $Z$ bosons in the previous diagrams, which in terms of the amplitude $M^{\alpha\beta\mu}$ reads as $M^{\alpha\beta\mu}(p_1 \leftrightarrow -q)$. 
As we pointed out, it is not possible to induce the $CP$-violating form factors \( h_3^{Z}\) through complex FCNCs couplings mediated by the $Z$ boson. The same occurs in vertex $ZZ\gamma^*$ \[10\]. On the other hand, a non-diagonal contribution to the $CP$-conserving form factor $h_3^{Z}$ is obtained and reads as

\[
h_3^{Z} = - \sum_{i,j} N_f Q_i e^2 m_Z^2 \text{Re}(g_A^{ij} g_V^{ij}) \left\{ \frac{2 m_Z^2 q^2}{4 \pi^2 s_W^2 c_W^2} \right\}
\]

where $N_f$, $m_i$, and $Q_i$ are the color number, mass and electric charge of fermion $f_i$. As we are considering neutral currents we note that $Q_j = Q_i$. The Eq. \[9\] agrees with the case of real FCNC couplings of the $Z$ boson reported in Ref. \[3\].

### 1. Asymptotic behavior

We also study the high-energy limit $q^2 \gg m_i^2, m_j^2, m_Z^2$, in such case Eq. \[9\] reduces to

\[
h_3^{Z} \approx \sum_{i,j} N_f Q_i e^2 m_Z^2 \text{Re}(g_A^{ij} g_V^{ij}) \left\{ \frac{2 m_Z^2 q^2}{4 \pi^2 s_W^2 c_W^2} \right\}
\]

The Eq. \[10\] agrees with the Flavor-conserving case ($m_i = m_j$). Nevertheless, we must take into account that our result is twice as big as the result reported in Ref. \[3\] since we are considering the double number of Feynman diagrams, which arise from the fermion exchange ($f_i \leftrightarrow f_j$). We note from Eq. \[10\] that $h_3^{Z} \to 0$ in the high-energy limit as $q^2$ increases.

### B. Polarizations

We now present the analytical expressions for the polarized and unpolarized $\Gamma(V^* \to Z\gamma)$ partial widths. These results are useful as the cross-section of the observed process at colliders: $q\bar{q} \to V^* \to Z\gamma \to \ell\ell\gamma$ can be written in terms of the $V^* \to Z\gamma$ ($V = Z, \gamma$) partial widths as follows:

\[
\sigma(q\bar{q} \to V^* \to Z\gamma \to \ell\ell\gamma) \sim \sigma(q\bar{q} \to V^*) \Gamma(V^* \to Z\gamma) \Gamma(Z \to \ell\ell),
\]

where the unpolarized partial width of the $V^* \to Z\gamma$ process can be replaced by the polarized partial width $\Gamma^{\lambda_1,\lambda_2}(V^* \to Z\gamma)$, with $\lambda_1$ and $\lambda_2$ the polarizations of on shell $Z$ boson and the photon, respectively. The polarizations in diboson final states can lead to interesting new physics effects \[69\], which can also be observed in different process \[70\]. The TNGBC have been studied in the context of polarized beams \[19\] \[40\] \[42\] \[71\] \[74\] and considering the polarizations of the boson in the final state \[20\] \[22\] \[37\].
Following the vertex function in Eq. [1] for the $V^*Z\gamma$ ($V = Z, \gamma$) interactions, the unpolarized and polarized partial widths for the $V^* \rightarrow Z\gamma$ process can be calculated. We consider the $h_i^V$ form factor as complex:

$$h_i^V = \text{Re}[h_i^V] + i \text{Im}[h_i^V], \quad (V = \gamma, Z) \quad \text{and} \quad (i = 1, 2, 3, 4),$$

which lead to the amplitude of the production of a $Z\gamma$ pair through an off-shell $V$ ($V = Z, \gamma$) boson:

$$\mathcal{M} = -\epsilon \left(\frac{q^2 - m_Z^2}{m_Z^2}\right) \left(\text{Re}[h_1^V] + i \text{Im}[h_1^V]\right) \left(p_1^\mu g_\mu^\alpha + p_2^\mu g_\mu^\beta\right) + \frac{\text{Re}[h_2^V] + i \text{Im}[h_2^V]}{m_Z^2} q^\alpha \left(q \cdot p_2 g_\mu^\alpha - p_2^\mu g_\mu^\beta\right)
- \left(\text{Re}[h_3^V] + i \text{Im}[h_3^V]\right) \epsilon^{\mu\alpha\beta\sigma} q_{p_2^\mu} \text{Re}[h_4^V] + i \text{Im}[h_4^V] q^\beta q_{p_2^\beta} \epsilon^\sigma(p_1, \lambda_1) \epsilon^\mu(p_2, \lambda_2) \epsilon(q, \lambda_3),$$

where $\lambda$ and $\epsilon(r, \lambda_i)$ ($i = 1, 2, 3$) correspond to the polarizations and vector polarizations of the neutral gauge bosons, respectively. In the rest of this section, we calculate the $\Gamma(V^* \rightarrow Z\gamma)$ unpolarized and polarized partial widths.

### 1. Unpolarized partial widths

From the amplitude [13], we can obtain the unpolarized partial widths of the $V^* \rightarrow Z\gamma$ ($V = Z, \gamma$) process in terms of the real and absorptive parts of the $h_i^V$ ($i = 1, 2, 3, 4$) form factors. For an off-shell $Z$ boson, the partial width is given as

$$\Gamma_{Z^* \rightarrow Z\gamma} = \frac{e^2 (m_Z - Q)^4 (m_Z + Q)^4 (Q^2 - m_Z^2)}{128\pi Q^4 m_V^{10}} \left\{ -2Q^2 m_Z^2 \left(\text{Re}[h_2^Z]^2 + \text{Re}[h_4^Z]^2\right) + 2\left(\text{Re}[h_1^Z] \text{Re}[h_2^Z] \right. \right.$$

$$+ \text{Re}[h_3^Z] \text{Re}[h_4^Z] + \text{Im}[h_2^Z] \text{Im}[h_4^Z] + \text{Im}[h_3^Z] \text{Im}[h_4^Z]\left\} + \text{Im}[h_2^Z]^2 + \text{Im}[h_4^Z]^2 + Q^4 \left(\text{Re}[h_2^Z]^2 + \text{Re}[h_4^Z]^2 + \text{Im}[h_2^Z]^2 + \text{Im}[h_4^Z]^2\right)\right\}$$

whereas for an off-shell photon we obtain

$$\Gamma_{\gamma^* \rightarrow Z\gamma} = \frac{e^2 Q (Q^2 - m_Z^2)^3}{128\pi m_V^{10}} \left\{ -2Q^2 m_Z^2 \left(\text{Re}[h_2^\gamma]^2 + \text{Re}[h_4^\gamma]^2\right) + 2\left(\text{Re}[h_1^\gamma] \text{Re}[h_2^\gamma] \right. \right.$$

$$+ \text{Im}[h_1^\gamma] \text{Im}[h_2^\gamma] + \text{Im}[h_3^\gamma] \text{Im}[h_4^\gamma]\left\} + \text{Im}[h_2^\gamma]^2 + \text{Im}[h_4^\gamma]^2 + Q^4 \left(\text{Re}[h_2^\gamma]^2 + \text{Re}[h_4^\gamma]^2 + \text{Im}[h_2^\gamma]^2 + \text{Im}[h_4^\gamma]^2\right)\right\},$$

where we have introduced the notation $Q \equiv |q|$ for the norm of the four momentum of the off-shell particle $V$. It is noted that in both cases there are not interference terms between the $CP$-violating and $CP$-conserving form factors. For our results in Eqs [14]-[15], we have not averaged over the initial polarizations as the $V^*$ boson is off-shell.

### 2. Polarized partial widths

From amplitude [13], we can also obtain the polarized partial widths considering only polarizations of the on-shell gauge bosons:

$$\Gamma^{\lambda_1\lambda_2}(V^* \rightarrow Z\gamma), \quad (V = Z, \gamma),$$

(16)
with $\lambda_1$ and $\lambda_2$ the polarizations of the $Z$ and $\gamma$ bosons, respectively. In a frame where the off-shell $V$ boson is in rest and the $Z(\gamma)$ boson moves along the positive(negative) $x$ axis, the polarization vectors are

$$\epsilon(p_1, 0) = \frac{1}{2m_Z}Q (m_Z^2 - Q^2, m_Z^2 + Q^2, 0, 0), \quad (17)$$

$$\epsilon(p_1, L/R) = \frac{1}{\sqrt{2}}(0, 0, -i, \mp 1), \quad (18)$$

$$\epsilon(p_2, L/R) = \frac{1}{\sqrt{2}}(0, 0, i, \pm 1), \quad (19)$$

with $L$ and $R$ the left and right polarizations, respectively.

The different polarized partial widths for the $Z^* \rightarrow Z\gamma$ process are

$$\Gamma^{LL}(Z^* \rightarrow Z\gamma) = \frac{e^2 (Q^2 - m_Z^2)}{64\pi^3 m_Z^6} \left\{ \frac{1}{2} \left( \text{Re}[h_{1Z}^2] - \text{Im}[h_{1Z}^2] \right) \text{Re}[h_{3Z}^2] + 2\left( \text{Im}[h_{1Z}^2] \text{Re}[h_{3Z}^2] - \text{Im}[h_{3Z}^2] \text{Re}[h_{1Z}^2] \right) \right\}, \quad (20)$$

$$\Gamma^{RR}(Z^* \rightarrow Z\gamma) = \frac{e^2 (Q^2 - m_Z^2)}{64\pi^3 m_Z^6} \left\{ \frac{1}{2} \left( \text{Re}[h_{1Z}^2] - \text{Im}[h_{1Z}^2] \right) \text{Re}[h_{3Z}^2] - 2\left( \text{Im}[h_{1Z}^2] \text{Re}[h_{3Z}^2] - \text{Im}[h_{3Z}^2] \text{Re}[h_{1Z}^2] \right) \right\}, \quad (21)$$

$$\Gamma^{0L}(Z^* \rightarrow Z\gamma) = \frac{e^2 (m_Z - Q)^2}{256\pi^3 m_Z^{10/3}} \left\{ -2Q^2m_Z^2 \left( \text{Re}[h_{2Z}^2] - \text{Im}[h_{2Z}^2] \right)^2 + \text{Im}[h_{2Z}^2] \left( \text{Re}[h_{4Z}^2] + \text{Im}[h_{4Z}^2] \right)^2 \right\}, \quad (22)$$

$$\Gamma^{0R}(Z^* \rightarrow Z\gamma) = \frac{e^2 (m_Z - Q)^2}{256\pi^3 m_Z^{10/3}} \left\{ -2Q^2m_Z^2 \left( \text{Re}[h_{2Z}^2] + \text{Im}[h_{2Z}^2] \right)^2 + \text{Im}[h_{2Z}^2] \left( \text{Re}[h_{4Z}^2] - \text{Im}[h_{4Z}^2] \right)^2 \right\}. \quad (23)$$

At the one-loop in the SM, the $h_{3Z}^2$ form factor is the only one induced, and the Eqs. (20)-(23) reduce to the same expression:

$$\Gamma_{SM}^{\kappa}(Z^* \rightarrow Z\gamma) = \frac{e^2 (Q^2 - m_Z^2)}{64\pi m_Z^2} \left\{ \text{Re}[h_{1Z}^2]^2 + \text{Im}[h_{1Z}^2]^2 \right\}, \quad (\kappa = LL, RR, 0L, 0R). \quad (24)$$

On the other hand, for the $\gamma^* \rightarrow Z\gamma$ process we only find two different polarized partial widths, which can be written in terms of the previous results as follows

$$\Gamma^{0\lambda}(\gamma^* \rightarrow Z\gamma) = \frac{Q^4 (Q^2 - m_Z^2)^2}{(m_Z - Q)^4 (m_Z + Q)^4} \Gamma^{0\lambda}(Z^* \rightarrow Z\gamma), \quad (25)$$
for \( \lambda = L, R \). For the SM at the one loop level, both result reduce to the same expression according to Eq. \( \text{(24)} \). Summing over the different polarizations, we can obtain Eq. \( \text{(14)-15} \).

We observe that the \( \Gamma_{LL/RR}^{Z\gamma}(Z^* \rightarrow Z\gamma) \) partial widths are only in terms of the \( CP \)-violating and \( CP \)-conserving form factors \( h_1^Z \) and \( h_3^Z \), respectively. This is clearly different from the unpolarized case, where the partial width involve all the four form factors, making it challenging to discern each contribution. Therefore, studying the polarizations can make it easier to distinguish the \( CP \)-violating contributions, specially those related to the \( h_1^Z \) form factor. We also note that the partial widths are in terms of the mixing between the \( CP \)-conserving and \( CP \)-violating form factors. This can also be useful for differentiating the individual contributions of the various form factors. In fact, when comparing the left and right polarizations, we note that the interference terms differ by a sign. Hence new observables can be defined to determine each \( h_1^Z \) contributions. Of special interest are those related to the \( CP \)-violating form factors, as they are not studied directly at the LHC \( \text{(2)} \) \( \text{(34)} \). Finally, it is observed that in both unpolarized and polarized cases, the partial widths increase dramatically at high values of \( Q \). Thus, the validity of the numerical results should be carefully addressed.

\[ A_{LR} = \frac{\Gamma_{LL}^{Z\gamma}(Z^* \rightarrow Z\gamma) - \Gamma_{RR}^{Z\gamma}(Z^* \rightarrow Z\gamma)}{\Gamma_{LL}^{Z\gamma}(Z^* \rightarrow Z\gamma) + \Gamma_{RR}^{Z\gamma}(Z^* \rightarrow Z\gamma)} \]  \hspace{1cm} \text{(26)}

where using Eq. \( \text{(20)} \) and \( \text{(21)} \) the left-right asymmetry in terms of the \( h_1^Z \) and \( h_3^Z \) form factors is

\[ A_{LR} = \frac{2 \left( \text{Im}[h_1^Z]\text{Re}[h_3^Z] - \text{Im}[h_3^Z]\text{Re}[h_1^Z] \right)}{\text{Im}[h_1^Z]^2 + \text{Im}[h_3^Z]^2 + \text{Re}[h_1^Z]^2 + \text{Re}[h_3^Z]^2}. \]  \hspace{1cm} \text{(27)}

The \( A_{LR} \) asymmetry provides a sensitive probe for new physics beyond the SM. Since in the SM predicts a vanishing \( A_{LR} \) at the one-loop level due to the absence of \( CP \)-violating contributions, any observed non-zero asymmetry would be a clear signal of new physics. Moreover, experimental measurements of \( A_{LR} \) at the LHC can lead to set stringent limits on the magnitude of the \( h_1^Z \) form factor. This is because the \( A_{LR} \) asymmetry is directly proportional to the interference between \( CP \)-conserving \( h_3^Z \) and \( CP \)-violating \( h_1^Z \) from factors. To obtain a non-zero \( A_{LR} \) asymmetry complex and \( CP \)-violating form factors are required. These complex form factors introduce phase differences that lead to observable \( CP \)-violating effects in the final state. Thus, studying the \( A_{LR} \) asymmetry not only helps to constrain the \( h_1^Z \) form factor but also enhances our understanding of the underlying dynamics of \( CP \) violation in electroweak interactions.

The second asymmetry includes both an off-shell \( Z \) boson or photon scenarios and can be defined as

\[ A_{0LR}^V = \frac{\Gamma_{LL}(V^* \rightarrow Z\gamma) - \Gamma_{RR}(V^* \rightarrow Z\gamma)}{\Gamma_{LL}(V^* \rightarrow Z\gamma) + \Gamma_{RR}(V^* \rightarrow Z\gamma)} \quad (V = Z, \gamma), \]  \hspace{1cm} \text{(28)}

from Eq. \( \text{(22)-25} \) we find that \( A_{0LR}^V = A_{0LR}^\gamma \) and the new asymmetry can be written as

\[ A_{0LR}^V = \frac{g(Q)}{h(Q)}. \]  \hspace{1cm} \text{(29)}

with the \( g(Q) \) and \( h(Q) \) functions given as

\[ g(Q) = 4Q^2m_Z^2 \left\{ \text{Im}[h_1^V]\text{Re}[h_1^V] + \text{Im}[h_3^V] + \text{Im}[h_4^V]\right\} \text{Re}[h_2^V] - \text{Im}[h_3^V]\text{Re}[h_1^V] - \text{Im}[h_4^V]\left(\text{Re}[h_3^V] + \text{Re}[h_4^V]\right) \]
\[ + 2m_Z^4 \left\{ (2\text{Im}[h_1^V] + \text{Im}[h_3^V]) (2\text{Re}[h_2^V] + \text{Re}[h_4^V]) - (2\text{Im}[h_3^V] + \text{Im}[h_4^V]) (2\text{Re}[h_1^V] + \text{Re}[h_2^V]) \right\} \]
\[ + 2Q^4 \left\{ \text{Im}[h_2^V]\text{Re}[h_2^V] - \text{Im}[h_3^V]\text{Re}[h_3^V] \right\}. \]  \hspace{1cm} \text{(30)}
\[ h(Q) = -2Q^2m_Z^2 \left\{ \text{Im}[h_2^T]^2 + 2\text{Im}[h_1^T]\text{Im}[h_2^T] + \text{Im}[h_Y^T]^2 + 2\text{Im}[h_Y^T]\text{Re}[h_Y^T]^2 + \text{Re}[h_Y^T]^2 \right\} \\
+ 2\text{Re}[h_1^T]\text{Re}[h_Y^T] + 2\text{Re}[h_Y^T]\text{Re}[h_Y^T]^2 + m_Z^2 \left\{ (2\text{Im}[h_1^T] + \text{Im}[h_Y^T])^2 + (2\text{Im}[h_Y^T] + \text{Im}[h_Y^T])^2 \right\} \\
+ (2\text{Re}[h_Y^T] + \text{Re}[h_Y^T])^2 + (2\text{Re}[h_Y^T] + \text{Re}[h_Y^T])^2 \right\} + Q^4 \left\{ \text{Im}[h_Y^T]^2 + \text{Im}[h_Y^T]^2 + \text{Re}[h_Y^T]^2 \right\}. \] (31)

Similar to the \( A_{LR} \) asymmetry, we find that only interference terms between \( CP \)-violating and \( CP \)-conserving form factors appear in the denominator of \( A_{VLR}^V \) \((V = Z, \gamma)\). In this case, to induce a nonzero asymmetry, complex couplings and at least one non-vanishing \( CP \)-violating form factor are required. In contrast with the former asymmetry, the \( A_{VLR}^V \) \((V = Z, \gamma)\) allows to study both \( CP \)-violating form factors \( h_1^T \) and \( h_Y^T \). Therefore, the \( A_{VLR}^V \) asymmetry is also a good opportunity to observe new physics. The angular asymmetries in TNGBC are also sensitive to new physics \([19, 20, 22, 42, 71, 73]\). It has been pointed out that comparable bounds with the current LHC limits are possible through these type of asymmetries \([22]\). Furthermore, they provide an independent check on the results obtained from the \( A_{LR} \) and \( A_{VLR}^V \). By comparing the constraints derived from different measurements, we can reach a more comprehensive understanding of physics beyond the SM.

**IV. BOUNDS ON FCNC Z COUPLINGS THROUGH THE EFT FRAMEWORK**

As we are interested in numerical values of the new contributions to the form factor \( h_2^T \), it is necessary to obtain constraints on the couplings \( g_A^{tq} \) and \( g_V^{tq} \). We expect that the dominant results arise from FCNCs where the top quark is involved \([10]\). To achieve these bounds we use the width of the \( t \rightarrow qZ \) decay, which in terms of the vector and axial couplings and for negligible \( m_q \) can be written as

\[
\Gamma_{t \rightarrow qZ} = \frac{N_f e^2 m_t^3}{64 \pi c_W m_Z^2 s_W^2} \left| g_A^{tq} \right|^2 + \left| g_V^{tq} \right|^2 \left( 1 - m_Z^2/m_t^2 \right)^2 \left( 1 + 2 m_Z^2/m_t^2 \right). \] (32)

The most recent limits on the branching ratios \( B(t \rightarrow qZ) \) obtained by ATLAS collaboration are: \( B(t \rightarrow uZ) < 1.7 \times 10^{-4} \) and \( B(t \rightarrow cZ) < 2.4 \times 10^{-4} \) at 95\% confidence-level while the analysis was not sensitive to handedness of the couplings \([58]\). On the other hand, for a left-handed coupling the bounds are \( B(t \rightarrow uZ) < 6.2 \times 10^{-5} \) and \( B(t \rightarrow cZ) < 13 \times 10^{-5} \) with 95\% confidence-level, whereas for a right-handed coupling they are \( B(t \rightarrow uZ) < 6.6 \times 10^{-5} \) and \( B(t \rightarrow cZ) < 12 \times 10^{-5} \) \([59]\). The SM prediction for \( B(t \rightarrow cZ) \) is of order \( 10^{-14} \) \([58]\).

From Eq. \([32]\) we obtain

\[
B(t \rightarrow qZ) = 2.7471 \left( \left| g_A^{tq} \right|^2 + \left| g_V^{tq} \right|^2 \right). \] (33)

Moreover, using Eq. \([8]\) it is also possible to set bounds on the coefficients \( C_{\phi \mu}^{(a+3)} \) and \( C_{\phi \mu}^{-(a+3)} \). Since we look for the most stringent limits, we use the results where the helicity of the particles has been considered. The branching ratio for left(right)-handed particles is

\[
B(t \rightarrow qZ) = 1.37355 \left( \left| g_A^{tq} \right|^2 \pm 2 \left| g_A^{tq} \right| \left| g_V^{tq} \right| \cos \theta + \left| g_V^{tq} \right|^2 \right), \] (34)

with \( \theta \) the sum of the \( g_A^{tq} \) and \( g_V^{tq} \) phases. Then, from Eq. \([34]\) we obtain the allowed areas on the \( \left| g_A^{tq} \right| \) vs \( \left| g_A^{tq} \right| \) and \( \left| C_{\phi \mu}^{(a+3)} \right| \) vs \( \left| C_{\phi \mu}^{-(a+3)} \right| \) planes, which are shown in Fig. \(3\) considering \( \theta = 0 \). The blue-solid (magenta-dashed) line corresponds to \( Ztc \) (\( Ztu \)) coupling. We note that in general the vector \( (g_A^{tq}) \) and axial \( (g_A^{tq}) \) couplings are of order \( 10^{-2} \) \((10^{-3})\), which is one order of magnitude tighter than our previous estimation \([10]\), whereas the coefficients \( C_{\phi \mu}^{(a+3)} \) and \( C_{\phi \mu}^{-(a+3)} \) can be as large as \( 10^{-1} \) for \( \Lambda = 1000 \text{ GeV} \). Our limits can be summarized as

\[
\left| g_A^{tq} \right|, \left| g_A^{tq} \right| \leq 0.007, \quad \left| g_V^{tq} \right|, \left| g_V^{tq} \right| \leq 0.0095, \quad \left| C_{\phi \mu}^{(a+3)} \right|, \left| C_{\phi \mu}^{-(a+3)} \right| \leq 0.12, \quad \left| C_{\phi \mu}^{(a+3)} \right|, \left| C_{\phi \mu}^{-(a+3)} \right| \leq 0.16. \] (35)

(36)

Bounds on EFT coefficients related to top new physics have been recently obtained at the LHC and compiled in Ref. \([83]\). The values presented in Eq. \([36]\) are lower than those reported in Refs. \([67, 68]\). Other limits can be obtained through \( B \) physics \([84]\).
FIG. 3. Allowed area with 95% confidence-level in the $|g_V^{tq}|$ vs $|g_A^{tq}|$ (left) and $|C^{(a+3)}_{\phi q}|$ vs $|C^{-(a+3)}_{\phi q}|$ (right) planes from the most recent bounds on the branching ratio $t \to Zq$ \cite{59}. The solid-line (dashed-line) boundaries correspond to the $Zt\bar{c}$ ($Zt\bar{u}$) couplings. We have considered $\theta = 0$ and $\Lambda = 1000$ GeV.

On the other hand, bounds on FCNCs couplings between down quarks the $Z$ boson have been achieved from Kaon and $B$ mesons decays \cite{85–89}, they are of order $10^{-3}$ for both the vector and axial couplings. Furthermore, the best bounds on lepton flavor violating couplings mediated by the $Z$ boson arise from the decays $\mu \to eee$, $\tau^- \to e^- \mu^+ \mu^-$, and $\tau^- \to \mu^- \mu^+ \mu^-$, which yield to values of order $10^{-3}$ $- 10^{-6}$ \cite{90}. Moreover, limits on the branching ratios $B(Z \to \ell_1 \ell_2)$ obtained by the ATLAS and CMS \cite{91, 92} collaborations lead to values of order $10^{-3}$ $- 10^{-4}$, which agree with the results obtained through muon and tau decays.

V. NUMERICAL ANALYSIS

We present in this section the numerical evaluation of the non-diagonal contribution to the $CP$-conserving form factor $h_Z^2$ and the polarized partial widths $\Gamma_{\lambda_1\lambda_2}(V^* \to Z\gamma)$. For our analysis, we have used the LoopTools \cite{93} package to evaluate the Passarino-Veltman scalar functions.

A. Non-diagonal contribution to $h_Z^2$

We summarize in Table I the values of the FCNC couplings used in our numerical analysis. The vector and axial couplings of the $Zt\bar{c}$ and $Zt\bar{u}$ interactions agree with the bounds obtained in Fig. 3. For the remaining FCNC couplings, we consider values of order $10^{-3}$. The fermion contributions arising from flavor violating interactions, as described in Lagrangian \cite{6}, are not considered since they are three or more orders of magnitude smaller than the quarks contributions. The phase between the vector an axial couplings $\theta$ is set to zero.

| TABLE I. Values for the FCNC couplings of the quarks considered to study the non-diagonal contributions to the $CP$-conserving form factor $h_Z^2$. For the couplings with the top quark we use the results in Fig. 3 |
|---------------------------------------------------------------|
| $\ell_c$ | $\ell_u$ | $\ell_d$ | $d_i d_j$ |
| $g_{V}^{t_{1/2}}$ | 0.009 | 0.002 | 0.001 | 0.001 |
| $g_{A}^{t_{1/2}}$ | 0.001 | 0.005 | 0.002 | 0.006 |
In Fig. 4, we show the non-diagonal contributions to \( h_{Z}^{3} \) as function of the norm of the off-shell Z boson momentum \( Q \). The dominant contributions for the real and imaginary parts of \( h_{Z}^{3} \) arise from the up quarks contribution, whereas the real down quark contribution is one order of magnitude lower. The absorptive part is neglected as it is of order \( 10^{-11} \). It is noted that the imaginary part is larger than the real one in the energy region \( 200 \text{ GeV} \leq Q \leq 400 \text{ GeV} \). The up quark contribution to the absorptive part of \( h_{Z}^{3} \) is non-zero around \( Q = m_{t} \), where the particles attached to \( Z^{*} \) can be on-shell. In the SM, this behavior occurs at \( Q = 2m_{t} \). [3]

**FIG. 4.** Behavior of the \( CP \)- conserving form factor \( h_{Z}^{3} \) as a function of the momentum of the virtual Z boson. We use the values depicted in Tab. [1] Only the relevant contributions are shown.

As the highest contributions are expected to arise from the \( Z\bar{t}c \) coupling, we show in Fig. 5 the contour lines of the real part of \( h_{Z}^{3} \) in the plane \( |g_{Z}^{\bar{t}}| \) vs \( |g_{Z}^{c}| \), where the values in Tab. [1] for the remaining couplings were considered. The norm of the off-shell Z gauge boson is set to \( Q = 300 \text{ GeV} \). In general, we find that \( h_{Z}^{3} \) can be of order \( 10^{-6} \). The same behavior is observed at different energies and for the imaginary part. Our results are two orders of magnitude smaller than the SM prediction. In the past, the non-diagonal contributions to \( h_{Z}^{3} \) has been calculated under the MSMM context, which are estimated to be of order \( 10^{-4} \) [3]. Nevertheless, as no clear sign of superpartners has been observed at the LHC [94], we focus on a model-independent framework presented in Sec. [1] to obtain a more realistic study of the non-diagonal contributions to the \( CP \)-conserving form factor \( h_{Z}^{3} \).
FIG. 5. Contour lines of the real part of the CP-conserving form factor $h_Z^3$ in the plane $|g_{tc}^V|$ vs $|g_{tc}^A|$. We set $Q = 300$ and use the values in Tab. I for the remaining couplings. The imaginary part shows a similar behavior.

### B. Unpolarized and polarized partial widths

We now study the behavior of the polarized partial widths in new physics scenarios. Since the new contributions to $h_Z^3$ are not relevant, we use the SM contributions to $h_V^1$ ($V = Z, \gamma$) reported in Ref. [3], which are of order $10^{-4}$. For the remaining form factors, their values are shown in Table II, where four different scenarios are considered. They agree with the current bounds on the CP-conserving form factors [2], whereas for the CP-violating ones similar values are used. To analyze the effects of different signs in the interference terms between the CP-violating and CP-conserving form factors, negative values are also considered. Furthermore, we also contemplate the case where the imaginary contribution is larger than the real one. In scenario IV, the dependence on $Q$ is examined for the $h_Y^1$ CP-violating form factor.

**TABLE II.** Numerical values for the $h_Y^i$ ($V = Z, \gamma$ and $i = 1, 2, 4$) form factors considered in our numerical analysis. For $h_Y^3$ we use the SM model result reported in Ref. [3].

| Scenario | Re[$h_Y^1$] | Im[$h_Y^1$] | Re[$h_Y^2$] | Im[$h_Y^2$] | Re[$h_Y^4$] | Im[$h_Y^4$] |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|
| I        | 1 x 10^{-4} | -3 x 10^{-4} | -1 x 10^{-4} | 2 x 10^{-4} | 4 x 10^{-7} | 3 x 10^{-7} |
| II       | -1 x 10^{-4} | 3 x 10^{-4} | -1 x 10^{-4} | 2 x 10^{-4} | -4 x 10^{-7} | 3 x 10^{-7} |
| III      | 3 x 10^{-4} | 3 x 10^{-4} | -1 x 10^{-7} | 4 x 10^{-7} | 4 x 10^{-7} | -3 x 10^{-7} |
| IV       | 0.1 Re[$h_Y^3$] | -0.4 Im[$h_Y^3$] | -1 x 10^{-7} | 4 x 10^{-7} | -3 x 10^{-7} | 1 x 10^{-7} |

The numerical evaluation must be addressed carefully, as many authors have been pointed out that the TNGBC can lead to unphysical results [33, 34]. Such behavior can be avoided if the form factors are defined as

$$h_Y^i(Q) = \frac{h_{i0}^V}{(1 + Q^2/\Lambda^2)^n}, \quad (37)$$

where $h_{i0}^V$ is a constant and $\Lambda$ is an the energy scale introduced to prevent that the the observables from growing rapidly as $Q$ increases. In the literature, it is common to use $n = 3$ for $h_Y^1$ and $n = 4$ for $h_Y^2$, whereas $\Lambda$ is of order of TeVs or $\infty$ [95]. For $Q \ll \Lambda$, the $h_Y^i(Q)$ form factors are almost a constant. Therefore, for the study of the polarized
partial widths we consider low energies in a range $300 \text{ GeV} \leq Q \leq 400 \text{ GeV}$. In such an energy level, the form factors can be taken as constants, whereas the SM contributions to $h_V^Z(Q)$ ($V = Z, \gamma$) are of the same order [3]. At smaller values of $Q$ we obtain similar results to the presented in the rest of this section. On the other hand, at high energies the partial widths grow rapidly and our results are not longer valid.

![Graphs showing polarized partial widths for $Z^* \rightarrow Z\gamma$](image)

**FIG. 6.** Polarized partial widths for the $Z^* \rightarrow Z\gamma$ process as a function of $Q$ the norm of the four momentum of the virtual $Z$ boson. We study the new physics scenarios in Table. [II] and the SM prediction at the one-loop level.

In Fig. [6] we show the behavior of the polarized partial widths for the $Z^* \rightarrow Z\gamma$ process. The four scenarios in Table. [II] are considered. Moreover, the polarized cases in the SM at the one-loop level in Eq. [24] are also plotted to compare with the new physics scenarios. It is found that the larger values are reached in scenario I, and they are of order $10^{-3}$ for the $0L$ and $0R$ polarizations. For the remaining scenarios, the polarized partial widths are of order $10^{-4} - 10^{-6}$. In scenario III, it is noted that for small $CP$-violating contributions, the polarized partial widths related to the left or right polarizations are similar. Furthermore, relevant deviations from the SM case are observed in scenarios I-III, which is a consequence of the new physics contributions. Therefore, the study of the polarizations in the $Z \rightarrow Z\gamma$ process is a good channel to detect physics beyond of the SM. On the other hand, at low energies in scenario IV, we do not find distinguishable fluctuations from the SM case. Nevertheless, for $Q > 2m_t$, where the $h_Z^Z$ and $h_T^Z$ develop and imaginary part, the new physics scenarios differ considerably from the SM. This shows that the polarized partial widths are sensitive to the absorptive parts, especially in regions where the imaginary contributions become significant. This behavior underscore the importance of precise measurements on polarizations of gauge bosons to explore new physics phenomena.
FIG. 7. Polarized partial widths for the $\gamma^* \rightarrow Z\gamma$ process as a function of $Q$ the norm of the four momentum of the virtual $Z$ boson. We study the new scenarios I and IV in Table III whereas for the remaining cases similar results are obtained. The SM prediction at the one-loop level is also shown.

In Fig. 9 the polarized partial widths for $V^* = \gamma$ are shown. We only plot the scenarios I and IV in Table III as similar results are obtained for the remaining cases. The polarized SM case is also considered. We observe that an identical behavior to the $Z^* \rightarrow Z\gamma$ process, specially in the scenario IV, the contribution from the absorptive part of the $CP$-conserving form factor $h_3^{\gamma}$ becomes relevant and notorious at $Q = 2m_t$. Therefore, the polarized partial widths for the $\gamma^* \rightarrow Z\gamma$ process are also sensitive to the imaginary and beyond SM contributions. In general, the values of $\Gamma^{0L,0R}(\gamma^* \rightarrow Z\gamma)$ are of orders $10^{-3} - 10^{-5}$.

C. Asymmetries

FIG. 8. Allowed
We now study the left-right asymmetries defined in Sec. III B 3. They are sensitive to the absorptive and CP-violating contributions. In Fig. 8, the $A_{\text{LR}}$ asymmetry is plotted as a function of $Q$ for the new physics scenarios in Table I. It is observed that the larger values are obtained for scenarios I and II, where a difference of sign between the real and absorptive part of the CP-violating form factor $h^Z_{ij}$ is imposed. The $A_{\text{LR}}$ asymmetry can reach values of the unity in such scenarios. We also note that from $Q = 2m_t$, the energy where the $h^Z_{ij}$ form factor becomes complex, the asymmetries show a change in their behavior. This indicates that the absorptive part have distinguishable effects on the polarized asymmetries. Such a behavior is also observed in Fig. 8, where the $A_{\text{LR}}^Z$ (left plot) and $A_{\text{LR}}^Z$ (right plot) asymmetries are shown. It is found that in both cases the asymmetry behaves similar and the can also reach values close to the unity. The scenario IV is particularly interesting. For the $A_{\text{LR}}$ case, it is zero for $Q < 2m_t$ because the $h^Z_{ij}$ and $h^Z_{ij}$ form factors are real in this energy region. However, such a behavior is not true for the $A_{\text{LR}}^Z$ asymmetries since they depend on the four $h^Z_{ij}$ ($i=1,2,3,4$) form factors, and non-zero values are obtained for energies lower than $2m_t$.

In brief, as the asymmetries are zero in the SM, they can be excellent channels to look for new physics effects, specially those related to CP-violation. In summary, since the asymmetries are zero in the SM, they serve as excellent channels for probing new physics effects, especially those related to CP-violation. They provide a clear signal of potential deviations from the SM and are valuable tools in the search for new physics.

![Figure 9](https://example.com/figure9.png)

**FIG. 9.** Allowed

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**VI. CONCLUSIONS AND OUTLOOK**

In this work, we have presented a calculation of non-diagonal contributions to the vertex $Z\gamma V^*$ ($V = Z, \gamma$) in a generic model, where complex FCNCs couplings mediated by the $Z$ boson are considered. We find that only new contributions $Z\gamma Z^*$ is induced, whereas no signs of CP violation are observed in the $Z\gamma V^*$ ($V = Z, \gamma$) vertex. Our result is presented in terms of the Passarino-Veltman scalar functions and can be reduced to the case of real FCNCs couplings reported in the literature. To study the behavior of $h^Z_{ij}$ we obtain constraints on the FCNCs couplings $Ztq$ using the current limits on the branching ratios $B(t \rightarrow Zq)$ reported by ATLAS collaboration. Our bounds are $|g_{\nu\ell}^\nu|, |g_{\alpha\ell}^\alpha| \leq 0.007$ and $|g_{\nu\ell}^\nu|, |g_{\alpha\ell}^\alpha| \leq 0.0095$. These results can also be expressed in terms of the EFT coefficients, in such framework our constrains read as $|C_{\phi\mu}^{(1+3)}|, |C_{\phiq}^{(1+3)}| \leq 0.12$ and $|C_{\phiq}^{(2+3)}|, |C_{\phiq}^{(2+3)}| \leq 0.16$. We find that the dominant contributions to $h^Z_{ij}$ arise from up quarks FCNC couplings and can be up order $10^{-6}$. This is two order of magnitude smaller than in the MSSM. Nevertheless, our result is more realistic as supersymmetric models are ruled out by experimental data. Moreover, since the current experimental limits on TNGBCs are close to the SM prediction they could be tested at the LHC shortly. Thus, non-diagonal contributions will play an important role in the incoming runs of the LHC and more reasonable estimations for TNGBCs will be necessary.

The unpolarized and polarized partial widths of the $V^* \rightarrow Z\gamma$ process are also calculated. These results are useful to derive cross-sections that are studied at the LHC, since they can be written in terms of $\Gamma(V^* \rightarrow Z\gamma)$. We noted that the unpolarized case is more sensitive to new physics effects, as these results depend on the CP-violating form factors. Numerically, it is found that significant deviations from the SM can be observed in polarized amplitudes...
that they are highly sensitive to CP-violating effects and the absorptive parts of the $h_V^i$ form factors, as they are required to induce non-vanishing asymmetries. In summary, the study of polarized $Z$ and $\gamma$ bosons at colliders offer a valuable opportunity to measure physics beyond the SM. This thorough approach enhances our ability to search and understand new physics in high energy experiments.

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