Black holes in full quantum gravity

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Received 3 October 2009, in final form 2 November 2009
Published 25 November 2009
Online at stacks.iop.org/CQG/26/245009

Abstract
Quantum black holes have been studied extensively in quantum gravity and string theory, using various semiclassical or background-dependent approaches. We explore the possibility of studying black holes in the full non-perturbative quantum theory, without recurring to semiclassical considerations, and in the context of loop quantum gravity. We propose a definition of a quantum black hole as the collection of the quantum degrees of freedom that do not influence observables at infinity. From this definition, it follows that for an observer at infinity a black hole is described by an SU(2) intertwining operator. The dimension of the Hilbert space of such intertwiners grows exponentially with the horizon area. These considerations shed some light on the physical nature of the microstates contributing to the black hole entropy. In particular, it can be seen that the microstates being counted for the entropy have the interpretation of describing different horizon shapes. The space of black hole microstates described here is related to the one arrived at recently by Engle \textit{et al} (2009, arXiv:0905.3168) and sometime ago by Smolin (1995, \textit{J. Math. Phys.} \textbf{36} 6417), but obtained here directly within the full quantum theory.

PACS number: 04.60.Pp

1. Introduction

Considerable progress has been obtained in understanding the microphysics of black hole entropy using loop quantum gravity [1], following the first pioneering works started in the late nineties [2, 3]. So far, however, the description of black holes has relied on some mixture of quantum theory and classical analysis of black hole geometry: for instance, one can characterize a black hole classically [4], and then quantize part of the classical-theory phase space that contains the black hole. Is it possible, instead, to describe black holes entirely within the non-perturbative quantum theory of spacetime [5–7]?

In this paper we suggest a direction for answering this question. We propose a simple definition of a quantum black hole within the full quantum loop theory, as a region of a spin

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network which is not ‘visible’ from infinity. This is in the same spirit of the global analysis that is possible in classical general relativity, where properties of horizons and black holes can be obtained by studying their implicit definition, even without being able to solve the equations of motion and writing the metric explicitly [8].

We use this definition to study how black holes are characterized quantum mechanically and find that they can be described by $SU(2)$ intertwining operators. The Hilbert space of such operators is intimately related with the space of states of $SU(2)$ Chern–Simon (CS) theory on a punctured surface: the two spaces have the same dimension if the CS level is high enough, and the former arises from the latter in the limit when the level is taken to infinity. The space of $SU(2)$ Chern–Simon states as describing the black hole quantum microstates was obtained recently by Engle et al [9] developing the work of Ashtekar et al [1], that is, using a semiclassical approach where a boundary condition is imposed on the classical theory before quantization. It is also the space obtained a time ago by Smolin [3], using related semiclassical considerations. This paper shows that a Hilbert space closely related to that of $SU(2)$ CS theory emerges from a very natural characterization of quantum black holes in the full theory.

Furthermore, we argue that, contrary to what is often assumed, these states are distinguishable, in an appropriate sense, by measurements outside the hole. They are related to the fluctuations of the extrinsic geometry of the black horizon.

2. Preliminaries

We refer to [5] for an introduction to loop quantum gravity and for notation. We briefly recall here only the elements of the theory that are needed below. The theory is defined over a kinematical Hilbert space $\mathcal{K}_{aux}$ and an algebra $A$ of operators on $\mathcal{K}_{aux}$. The space $\mathcal{K}_{aux}$ admits a linear subspace $\mathcal{K}_\Gamma$ for each graph $\Gamma$ embedded in a three-dimensional manifold $\Sigma$. The space $\mathcal{K}_\Gamma$ is the Hilbert space of an $SU(2)$ lattice gauge theory on $\Gamma$. That is $\mathcal{K}_\Gamma = L^2[SU(2)^{\Sigma}]$, where $L$ is the number of links in $\Gamma$. The theory is invariant under local $SU(2)$ transformations. The $SU(2)$ gauge invariant states live in the subspace $H_\Gamma = L^2[SU(2)^{\Sigma}/SU(2)^{\Sigma}'] \subset \mathcal{K}_\Gamma$, where $N$ is the number of nodes in $\Gamma$ and the action of the gauge transformations is $\psi(U) \rightarrow \psi(V^{-1}UfVf^{-1})$ where $i(l)$ and $f(l)$ are the initial and final nodes of the link $l$ and $U, V \in SU(2)$. Peter–Weyl theorem implies that $\mathcal{K}_\Gamma$ decomposes as

$$\mathcal{K}_\Gamma = \bigoplus_i \otimes_n A(H_j),$$

where $l$ are the links in $\Gamma$, $n$ are the nodes, $a$ are the links adjacent to the node $n$ and $H_j$ is the Hilbert space of the spin $j$ irreducible representation of $SU(2)$. Then

$$H_\Gamma = \bigoplus_i \otimes_n \{\text{Inv} \left[ \otimes_a H_j \right]\},$$

where the space $H_n = \text{Inv} \left[ \otimes_a H_j \right]$ is that of $SU(2)$-invariant tensors—intertwiners—of the node $n$. The $SU(2)$-gauge invariant state space of the theory $H_{aux}$ is composed by (is the projective limit of) all the $H_\Gamma$ spaces. Choosing a basis $i_n$ in each $H_n$ of each $H_\Gamma$, we obtain the spin network basis $|S\rangle \equiv |\Gamma,j_i,i_n\rangle$ for the $SU(2)$-gauge invariant states of the theory.

$H_{aux}$ carries a unitary representation of the group $\text{Diff}_S$ of the diffeomorphisms of $\Sigma$. This allows us to define and solve the 3D diffeomorphism gauge of the theory. The Hilbert space of diffe-invariant states $H_{diff}$ admits an orthonormal basis $|s\rangle$, where $s$ (referred to as $s$-knot or simply a spin network) is an equivalence class of embedded spin networks $S$ under diffeomorphisms. There exists a projection operator $\pi : H_{aux} \rightarrow H_{diff}$, which can be

\[4\] Technically, $K_{aux}$ is a projective limit built from the spaces $K_\Gamma$ [5].
intuitively viewed as the exponentiation of the quantum diffeomorphism constraint operator. An operator \( O \in A \) on \( \mathcal{H}_{\text{aux}} \) is diffeomorphism invariant if there exists an operator \( O_{\text{diff}} \) on \( \mathcal{H}_{\text{diff}} \) such that \( \pi O = O_{\text{diff}} \pi \).

The first assumption on which our result relies is that there exists a projection operator \( P : \mathcal{H}_{\text{diff}} \to \mathcal{H}_{\text{phys}} \) implementing the dynamics. Here \( \mathcal{H}_{\text{phys}} \) is the space of the physical states (‘the solutions of the Wheeler–DeWitt equation’). An operator \( O \) on \( \mathcal{H}_{\text{diff}} \) is gauge invariant (i.e. is a ‘physical operator’) if there exists an operator \( O_{\text{phys}} \) on \( \mathcal{H}_{\text{phys}} \) such that \( PO_{\text{diff}} = O_{\text{phys}} P \). The set of operators \( O \in A \) on \( \mathcal{H}_{\text{aux}} \) such that \( P \pi O = O_{\text{phys}} P \) form the gauge invariant observable algebra \( A_{\text{ph}} \). There are various attempts to define the quantum Hamiltonian constraint [5] or directly the operator \( P \) [10], but the argument we present here depends only on the existence of \( P \), and not on its specific form.

Our second assumption concerns asymptotic flatness. Most of the work in loop quantum gravity has so far assumed 3D physical space to be compact. Here, we assume that a suitable extension of the theory to the asymptotically flat case exists. In particular, in what follows we make use of the notion of an asymptotic observer.

Consider classical general relativity in the asymptotically flat case. Let \( \mathcal{C} \) be the space of the initial (Cauchy) data of the theory on a spacelike surface \( \Sigma \). Using the evolution equations, we can compute the value of the gravitational field at any spacetime point in the future of \( \Sigma \). In particular, given the initial data we can compute the value of the gravitational field at future null infinity. Therefore, observables at null infinity are functions on the initial-data phase space. They are non-local and very complicated functions, since writing them explicitly requires solving the equations of motion, but they are nevertheless implicitly well defined. Let \( O \) be one such observable quantity at future null infinity.

In the quantum theory, the space \( \mathcal{C} \) of the initial data is promoted to a state space \( \mathcal{H}_{\text{aux}} \), and functions on \( \mathcal{C} \) are promoted to the operators on \( \mathcal{H}_{\text{aux}} \). Then to every observable \( O \) at future null infinity there is a corresponding (Heisenberg) operator \( \hat{O} \) on \( \mathcal{H}_{\text{aux}} \). These operators define the algebra \( A_{\infty} \), which is a subalgebra of the algebra of physical operators. Observables at null infinity are gauge invariant because they do not depend on arbitrary lapses or shifts of \( \Sigma \). Therefore, the operators \( \hat{O} \) must commute with the projection operators \( \pi \) and \( P \), and be well defined on \( \mathcal{H}_{\text{phys}} \). Our assumptions above are thus equivalent to an assumption that the algebra \( A_{\text{ph}} \) of the gauge invariant operators contains a subalgebra \( A_{\infty} \) of the operators \( \hat{O} \), that corresponds to all possible observations of the gravitational field that can be made at future null infinity. What follows depends on the existence, not on the explicit form of this algebra.

3. Definition of a quantum black hole

Since we are working in the canonical formalism, we need a notion of a black hole at a spacelike surface. In classical general relativity, a black hole is a region of space which is outside the past of future null infinity. This can be formulated as follows. Consider a spacelike surface \( \Sigma \), and initial data \( c \) on \( \Sigma \). Then a region \( \mathcal{R} \) of \( \Sigma \) is inside a black hole if all observables at null infinity have the same value on \( c \) and on any other initial data \( c' \) that are the same as \( c \) outside \( \mathcal{R} \). Of course, to determine explicitly if a certain region is or is not inside a black hole is a nontrivial task (as numerical relativity people know well), since one must in principle evolve the data to infinity in order to find out; nevertheless (again, as numerical relativity people know well), the notion is well defined.

It is useful to define the external and internal geometry of a (open) spacial region \( \mathcal{R} \) as follows: the external geometry of the region is an ensemble of the properties of the geometry that can be measured by local observables which are not in the region. Examples include the intrinsic (e.g. area) as well as extrinsic (e.g. extrinsic curvature) geometry of the boundary of
the region. The internal geometry of the region is the ensemble of the properties of the region that can be measured only by local observables in the interior of the region. An example is the volume of the region. Thus, a region $\mathcal{R}$ is inside a black hole if all observables at null infinity are independent of the internal geometry of $\mathcal{R}$, namely if they have the same value on any other initial data that have the same external geometry. Let us try to capture the same idea in the quantum theory.

Consider a spin network state $|S\rangle$. It defines a state $S : A \to \mathbb{C}$ over the algebra $A$ of the observables by $S(O) = \langle S | O | S \rangle$. Consider a region $\mathcal{R}$ in $\Sigma$. Denote $A_{\mathcal{R}}$ the subalgebra of the observable algebra $A$ formed by all local observables with support in $\mathcal{R}$. (Recall that $A$ is formed by the operators on the kinematical Hilbert space $\mathcal{K}_{\text{aux}}$, where the diffeomorphism constraint has not yet been imposed. Hence, $A$ contains local observables, with support on finite spacial regions.) Call $S_{\mathcal{R}}$ the restriction of $S$ to $A_{\mathcal{R}}$. Similarly, let $S_{\infty}$ be the restriction of $S$ to $A_{\text{in}}$. Given a spin network $S$, let us say that an open region $\mathcal{R}$ is a ‘hidden region’ in the quantum state $|S\rangle$ iff

$$S_{\overline{\mathcal{R}}} = S_{\mathcal{R}} \Rightarrow S_{\infty} = S_{\infty},$$

where $\overline{\mathcal{R}}$ is the complement of $\mathcal{R}$. The algebra $A_{\overline{\mathcal{R}}}$ is formed by all local observables that have no support on $\mathcal{R}$.

These are the observables that read the ‘outside geometry’ of $\mathcal{R}$. Therefore, this definition captures precisely the notion of a region that does not affect infinity: any other state $|S'\rangle$ which is equal to $|S\rangle$ outside the hidden region (meaning: that is indistinguishable from $|S\rangle$ by means of local measurements outside the hidden region) is also indistinguishable from $|S\rangle$ when observed at infinity. That is part of $|S\rangle$ inside the hidden region does not affect the future infinity.

Let us now call the maximal hidden region of a state $|S\rangle$ a ‘black hole’ region and denote it as $\mathcal{BH}(S)$. A spin network with a black hole splits into two parts: we call a ‘quantum black hole’ the portion of $S$ inside the hole, that is, the open graph $\Gamma_{\text{BH}} := \Gamma \cap \mathcal{BH}(S)$ with its colourings, and denote it $S_{\text{BH}} := (\Gamma_{\text{BH}}, j_{\text{BH}}, i_{\text{BH}})$. For simplicity, we consider here only the situation when the quantum black hole $S_{\text{BH}}$ is connected. We call $S_{\text{ext}} := (\Gamma_{\text{ext}}, j_{\text{ex}}, i_{\text{ex}})$, the rest of the spin network, that is, the open graph $\Gamma \cap \overline{\mathcal{BH}}(S)$, with its colourings. Let us call ‘internal black hole geometry’ the set of quantum numbers $S_{\text{BH}} := (\Gamma_{\text{BH}}, j_{\text{BH}}, i_{\text{BH}})$ and ‘external geometry’ the set of quantum numbers $S_{\text{ext}} := (\Gamma_{\text{ext}}, j_{\text{ex}}, i_{\text{ex}})$.

Since knowledge of the data at a node includes the knowledge about the links that arrive to this node, all links that are bounded by nodes in $S_{\text{ext}}$ are also in $S_{\text{ext}}$. Then the links of $S_{\text{ext}}$ split into two groups: those that are bounded by two external nodes and those that are bounded by a node in $S_{\text{ext}}$ and a node in $S_{\text{BH}}$. It is natural to call these second kind ‘horizon links’. They form the open legs of the graph of $S_{\text{ext}}$. Pictorially, they are the links that puncture the horizon of the black hole.

It is important to observe that all definitions above are given in terms of a spin network $S$. The black hole region is only defined as part of the graph of $S$, and in particular the horizon is only defined as a collection of links that separate $S$ into an external and an internal component.

In other words, a quantum black hole defined in this way is not a sharp surface in the manifold $\Sigma$; it is only a split of a spin network. A consequence is that the notion is immediately three-dimensionally diffeomorphism invariant, and thus comes down to $\mathcal{H}_{\text{diff}}$.

In the next section we study the properties of a quantum black hole just defined.

### 4. Observability and entropy

The split $S \to (S_{\text{BH}}, S_{\text{ext}})$ between the internal and external part of the black hole determines a split in the Hilbert space $\mathcal{H}_\Gamma$, where $\Gamma$ is the graph of $S$. Indeed we can write $\mathcal{H} = \mathcal{H}_{\text{BH}} \oplus \mathcal{H}_{\text{ext}}$,
where
\[ \mathcal{H}_{\text{ext}} = \bigoplus_{j_{\text{ext}}} \bigotimes_{a_{\text{ext}}} \left( \otimes_a H_{j_a} \right). \]
and
\[ \mathcal{H}_{\text{BH}} = \bigoplus_{j_{\text{BH}}} \bigotimes_{a_{\text{BH}}} \left( \otimes_a H_{j_a} \right). \]

Since states in both spaces live on open graphs, they transform nontrivially under local \( SU(2) \) gauge transformation. If we label with an integer \( p = 1, \ldots, P \) the horizon links, the states in \( \mathcal{H}_{\text{BH}} \) and \( \mathcal{H}_{\text{ext}} \) live in a representation of \( SU(2)^P \) with spin \( \{j_p\} \), namely in \( \mathcal{H}_{\text{horizon}} = \bigotimes_p \mathcal{H}_{j_p} \).

In other words, the states of both spaces have free magnetic indices where the graph \( \Gamma \) has been cut.

Consider two states \(|S\rangle\) and \(|S'\rangle\). Let us say that they are ‘equivalent’ if \( S_{\text{ext}} = S'_{\text{ext}} \), that is, if they are indistinguishable by measurements outside the black hole region. Denote the corresponding equivalence classes by \([|S\rangle]\). Because of the very nature of the horizon, for all observers that do not enter the horizon, a state containing a black hole is effectively described by the class \([|S\rangle]\).

A crucial observation is now the following. One may be tempted to deduce from the above considerations that the states \([|S\rangle]\) are fully determined by the external geometry of the hole, namely by the quantum numbers \( S_{\text{ext}} = (\Gamma_{\text{BH}}, j_{\text{ext}}, l_{\text{ext}}) \). But this is not the case. A state \([|S\rangle]\) is determined by more degrees of freedom than those characterizing its outside geometry.

To see this, consider an operator defined as follows. Let \( T_{\text{pp}'}^{\alpha}(x, y) \) be the ‘two-hand’ grasping operator in terms of which loop quantum gravity was initially defined [6]. This is the operator \( T^{\alpha \beta}[\alpha](x, y) = \text{tr}[E^\alpha(x)U_{\alpha_1}(y)E^\beta(y)] \), where \( E^\alpha(x) \) is the Ashtekar electric field, \( U_{\alpha_1} \) is the holonomy of the Ashtekar connection, and \( \alpha \) and \( \alpha_1 \) are two lines connecting \( x \) and \( y \). Let \( T_{\text{pp}} = \int_{\Sigma_p} \frac{d^2x}{2} \int_{\Sigma_p} \frac{d^2y}{2} n_a(x) n_b(y) T_{\text{pp}'}^{\alpha}(x, y) \), where \( \Sigma_p \) is a small surface punctured by the link \( p \) and \( n_a \) is its normal. A moment of reflection shows that this operator has support outside the black hole. However, it reads the properties of \(|S\rangle\) that depend on features of the spin network \( S \) inside the black hole. This can be seen easily by acting with this operator on two links \( p \) and \( p' \) bounded by a node \( n \) that is inside a BH; the action of the operator depends on the intertwiner at \( n \).

This shows that observables in the outside region can read some features of \(|S\rangle\) which are not captured by the quantum numbers \( S_{\text{ext}} = (\Gamma_{\text{ext}}, j_{\text{ext}}, l_{\text{ext}}) \). In other words, the ‘external geometry’, defined as what can be observed by observers with support outside the hole, is more rich than the ‘outside geometry’, defined by \((\Gamma_{\text{ext}}, j_{\text{ext}}, l_{\text{ext}})\). Indeed, to help intuition, note that even a change of a intertwiner ‘deep inside’ \( \Gamma_{\text{BH}} \) can be detected by the observable \( T_{\text{pp}'} \). What are thus these additional degrees of freedom?

A moment of reflection shows that the additional degrees of freedom that can be observed by external observers are completely captured as follows. We have seen that a state in \( \mathcal{H}_{\text{BH}} \) transforms as a vector in \( \mathcal{H}_{\text{horizon}} = \bigotimes_p \mathcal{H}_{j_p} \). The operators \( E^\alpha(x) \) act as \( SU(2) \) generators on each \( \mathcal{H}_{j_p} \). The \( SU(2) \) invariance implies that only the globally \( SU(2) \) gauge invariant subspace of this space is physically relevant. Therefore, the degrees of freedom that can be read out by observables outside the hole, and are not captured by \( S_{\text{ext}} := (\Gamma_{\text{BH}}, j_{\text{ext}}, l_{\text{ext}}) \), are entirely determined by the state space
\[ \mathcal{H}_{\text{horizon}} = \text{Inv}[\bigotimes_p \mathcal{H}_{j_p}], \]
where the operator \( T_{\text{pp}'} \) acts as \( T_{\text{pp}'} \sim J_p^i J_{p'}^i \), where \( J_p^i \), \( i = 1, 2, 3 \), are the \( SU(2) \) generators in \( H_{j_p} \). Thus, we conclude that
\[ [[|S\rangle]] = [\Gamma_{\text{BH}}, j_{\text{ext}}, l_{\text{ext}}]. \]
where \( I_{BH} \in \mathcal{H}_{\text{horizon}} \) is a single intertwiner. In other words, from the point of view of the outside observer, a black hole behaves as a (possibly gigantic) single intertwiner, which intertwines all the links puncturing its horizon.

Suppose now that we are in a statistical mechanical context and want to count the number of states subject to given conditions. Suppose that we know the outside geometry (area) of the black hole horizon. Then we must associate with the black hole an entropy equal to the (logarithm of the) number of states compatible with this outside geometry. This number is given by the dimension of the Hilbert space \( \mathcal{H}_{\text{horizon}} \):

\[
N = \dim \mathcal{H}_{\text{horizon}} = \dim \text{Inv} \left[ \otimes_p \mathcal{H}_{j_p} \right].
\]

This dimension is given by the classical formula

\[
N = \frac{2}{\pi} \int_0^\pi d\theta \sin^2(\theta/2) \prod_p \chi^{h_r}(\theta),
\]

where \( \chi^j(\theta) = \sin((j+1/2)\theta)/\sin(\theta/2) \) are the \( SU(2) \) characters. For a large number of punctures this goes as \( N \sim \prod_p (2j_p + 1) \), which grows exponentially as a function of the BH horizon area.

The space \( \mathcal{H}_{\text{horizon}} \) is related to the state space of a Chern–Simon theory with punctures \( j_p \) [9, 11], with the former arising from the later in the limit of the CS level \( k \to \infty \). Furthermore, the two spaces have precisely the same dimension for any \( k \) larger than a given value. The space of states of CS theory on a sphere with punctures as describing quantum states of black holes has been arrived at in [3, 9] using semiclassical considerations based on quantizations of theories with boundaries.

Finally, note that the operator \( T_{pp'} \) essentially reads out the ‘angle’ between the normal to the horizon at the punctures \( l_p \) and \( l_{p'} \). It can therefore be interpreted as an operator reading the extrinsic curvature of the horizon. Thus, the states that are being counted in (2) are those corresponding to different horizon shapes.

5. Conclusion

We have given a purely quantum mechanical definition of a black hole as part of a spin network state that is not accessible to observables based at infinity. Semiclassical considerations, such as the analysis of boundary conditions at a classical horizon, play no role in this definition. We have observed that the graph outside the hole, with its intertwiners and spins, is not sufficient to describe all degrees of freedom that can be measured from the exterior of the hole. Additional degrees of freedom are needed. We have shown that these additional degrees of freedom are described by a Hilbert space \( \mathcal{H}_{\text{horizon}} \), whose elements are intertwiners between all links puncturing the horizon. This space is related to the Hilbert space of \( SU(2) \) Chern–Simon theory with punctures.

Surface states of a black hole are described by Chern–Simon theory also in the analysis of [1]. The proposal for using \( SU(2) \) Chern–Simon theory for this recently resurfaced in [9], but in fact has a longer history. It was discussed in the context of loop quantum gravity by Smolin in [3], following earlier suggestions by Crane. Here, a related version of this proposal is recovered directly within the full loop quantum gravity.

We have observed that the operators that read the information in \( \mathcal{H}_{\text{horizon}} \) are the angle operators between the punctures, and have an intuitive interpretation as measuring the extrinsic curvature of the horizon. If the outside geometry is fixed, a black hole is still characterized by a number of states. These can be seen as describing the ‘shape’ of the horizon. (For a more detailed discussion, see [12], and the book [5].) It is important to emphasize that these degrees
of freedom are observable from the exterior of the black hole; if they were not observable they would not contribute to the black hole entropy. Indeed, if they had no effect on the external world, and in particular, had no effect on the heat exchanges between the hole and the rest of the world, they would not affect the entropy.

The notion of horizon used here is based on the traditional one (the boundary of the past of future null-infinity), and it has the same limitations. It would be interesting to find an extension of our construction that could capture also the notion of isolated horizon \[4\]. In this way, in particular, one could extend the result presented here also to the scenario where information is recovered during, or at the end of, the Hawking evaporation, and where, according to the traditional definition, there is no horizon \[13\].

We close with a simple comment. If a black hole, seen from its exterior, is described by an intertwiner, then an intertwiner can be viewed as a sort of black hole. This means that a semiclassical (‘weave’) spin network state with Planck-scale intertwiners can be viewed as made up with a large number of Planck-scale black holes. This intuitive image brings loop quantum gravity closer to Wheeler’s initial intuition of a Planck-scale foam. At trans-Planckian scale, the quantum energy fluctuations are such that spacetime disappears into micro-black-holes. The intertwiners of the states of loop quantum gravity can be seen as those ‘elementary’ Wheeler’s micro-black-holes.

Acknowledgment

This paper is an edited version of notes taken by one of us (CR) in March 98, following a long discussion with the other author (KK). At the time, we ended up discarding this idea because the precise state counting in \[1\] is based on a \(U(1)\) CS theory and is not given by \(2\). The recent arguments presented in \[9\] renew the relevance in this idea, in our opinion. CR thanks Alejandro Perez for discussing his work before publication and for numerous inputs on the present paper. KK was supported by an EPSRC Advanced Fellowship.

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