Universal Scaling in Mixing Correlated Growth with Randomness

A. Kolakowska, M. A. Novotny, and P. S. Verma

Department of Physics and Astronomy, and the ERC Center for Computational Sciences,
P.O. Box 5167, Mississippi State, MS 39762-5167

(Dated: March 23, 2022)

We study two-component growth that mixes random deposition (RD) with a correlated growth process that occurs with probability \( p \). We find that these composite systems are in the universality class of the correlated growth process. For RD blends with either Edwards-Wilkinson or Kardar-Parisi-Zhang processes, we identify a nonuniversal parameter in the universal scaling in \( p \).

PACS numbers: 68.90.+g, 89.75.Da, 05.10.-a, 02.50.Fz

Many properties of complex systems can be uncovered by statistical analysis of some representative nonequilibrium interfaces. Mainstream studies of surface growth and interfacial roughening focus on one-component growth, or homoepitaxy, and large-scale properties. On the microscopic level, nonequilibrium interfaces have been studied in a variety of discrete simulation models such as ballistic deposition (BD), Eden or solid-on-solid models. While one-component growths are well understood \([1]\), the same cannot be claimed about composite systems, even as simple as binary growth in one spatial dimension. Several mixed-growth models studied in the recent decade \([2, 3]\) reveal new and nontrivial properties. Questions that we address here concern the universality of such composite systems. As we shall show, the presence of randomness slows down the dynamics of the correlation processes. Nevertheless, the universality class of the combined processes is the same as the universality class of a correlation process. This is an outcome of scaling in \( p \). One consequence of this observation is a magnifying-glass effect that RD-blending has on the time-evolution of the surface roughness. This effect can be useful in revealing hidden features of a correlated growth when designing simulation models. Intuitively, since RD carries no correlations of its own, it may be expected that its admixtures should not lead to a new universality class. Yet, demonstration of this is not so trivial since, as we shall make evident by the results of several simulations, some of the parameters involved in the universal scaling may be nonuniversal. Results presented here for \((1 + 1)\) dimensions can be easily extended to multidimensions.

Consider aggregation models where particles fall onto a one-dimensional substrate of \( L \) sites, where they may be accepted in accordance to a rule that generates correlations among the sites. This pure-correlated growth occurs with probability \( p \) and competes with RD growth that occurs with probability \( q = 1 - p \). When a particle is accepted at a site, the site increases its height by \( \Delta h \). Roughness of the growing surface is measured by the interface width \( w(t) \) at time \( t \): \( w^2(t) = \langle (\sum_{k=1}^{\lambda t} h_k(t) − \bar{h}(t))^2 \rangle \), where \( h_k(t) \) is the height at site \( k \) and \( \bar{h}(t) \) is its mean over \( L \) sites (angular brackets denote the mean over \( N \) configurations).

In a pure-correlated growth \((p = 1)\), assuming elementary linear and nonlinear models, the self-affined roughness obeys the Family-Vicsek (FV) scaling \([4]\),

\[
w^2(t) = L^{2\alpha} F(t/L^z), \tag{1}\]

where \( F(y) \) gives two evolution limits: \( F(y) \sim y^{2\alpha/z} \) if \( y \ll 1 \) (growth); and, \( F(y) \sim \text{const} \) if \( y \gg 1 \) (saturation). The cross-over time \( t_x \) from growth to saturation is given by the dynamic exponent \( z \), \( t_x \sim L^z \) (Fig. 1). At saturation the width does not depend on time, \( w^2 \sim L^{2\alpha} \), where \( \alpha \) is the roughness exponent. During the growth \( w^2(t) \sim t^{2\beta} \), where \( \beta = \alpha/z \). Exponents \( z \), \( \alpha \) and \( \beta \) are universal. This means, two different simulation models of two different correlation mechanisms will generate the same type of scaling, with consistent values of exponents, provided these mechanisms represent the same type of correlation process, i.e., belong to one universality class. Dynamics of the buildup of correlations and dynamical scaling are described within a continuum model by a stochastic growth equation. One example is the Kardar-Parisi-Zhang (KPZ) equation \([5]\)

\[
h_t = v(t) + \nu_0 h_{xx} + (\lambda_0/2) h_x^2 + \eta(x,t), \tag{2}\]

where \( h = h(x,t) \) is the height field (subscripts denote partial derivatives; \( x \) is the coordinate along the substrate), \( v \) is the mean interface velocity, and \( \eta \) is the white noise \((\nu_0 \text{ and } \lambda_0 \text{ are coefficients})\). In the KPZ universality class, governed by Eq. \((2)\), \( \alpha + z = 2 \) and \( \alpha = 1/2 \). When \( \lambda_0 = 0 \), Eq. \((2)\) becomes the Edwards-Wilkinson (EW) equation \([6]\), defining the EW universality with \( 2\alpha + 1 = z \) and \( \alpha = 1/2 \). When \( \nu_0 = \lambda_0 = 0 \), Eq. \((2)\) describes uncorrelated processes of RD universality, characterized by \( \beta = 1/2 \), \( t_x = \infty \), and the absence of scaling in \( L \). For EW processes, Eq. \((1)\) expresses the invariance
of the EW equation under the scaling

\[ x \rightarrow Lx, \ h \rightarrow L^\alpha h, \ t \rightarrow L^\beta t. \]  

Similarly, for KPZ processes it expresses the invariance of the convective derivative in the Burger’s equation.

In simulations, \( t \) is the number of deposited monolayers. The first step is RD to a flat substrate. The system retains the memory of this initial condition for \( t_0 \) steps, where \( t_0 \) depends on the particulars of the model, i.e., \( t_0 \) is a nonuniversal parameter. In this start-up regime \( w(t) \) does not scale; scaling occurs only for \( t > t_0 \) (Fig. 1).

In deriving the scaling hypotheses, we are guided by the following four models. **Model A**: for \( p = 1 \) is RD with surface relaxation where \( \Delta h = 1 \) [1,8], known to be in the EW universality class (for \( p < 1 \), studied in [2,8]).

**Model B**, being introduced here, \( \Delta h \) is sampled from a uniform distribution of unit mean and the substrate is sampled sequentially at each time. When \( p = 1 \): particles that fall on the local interface minima are always accepted; particles that fall on local maxima slide down to either of the neighboring sites with probability 1/2; and, particles that fall on local slopes slide down to nearest-neighbor sites. **Model B** for \( p = 1 \) is in the EW class (Fig. 1). It simulates, e.g., deposition of a sticky non-granular material of variable droplet size. **Model C**: for \( p = 1 \) is BD with \( \Delta h = 1 \), known to be of KPZ universality [1] (for \( p < 1 \), studied in [2,10]). In **Model D**, \( \Delta h \) is sampled from a Poisson distribution of unit mean, and each monolayer is obtained by sequential sampling. When \( p = 1 \) in **Model D**, particles are deposited only to local surface minima. This case is in the KPZ universality class [9]. **Model D** simulates, e.g., conservative updates in a system of asynchronous processors [11,12]. We stress that, although in one universality class for \( p = 1 \), **Models C and D** are essentially different simulations (as is the pair \( A \) and \( B \)).

In all models, evolutions \( w^2(t) \) form two-parameter families of curves \((L \text{ and } p \text{ being parameters})\) that for any \( p \in (0; 1] \) look like those in Fig. 1 but with \( t_0 \equiv t_0(p) \geq t_0(1) \), \( t_x \equiv t_x(p) \geq t_x(1) \), and at saturation \( w^2(p) \geq w^2(1) \). The curves saturate due to only one component, the pure-correlated deposition, since the other component, RD, introduces no correlations. At saturation, the observed lateral correlation length is \( \xi(p) \sim L \) and \( t_x(p) \sim L^p \); thus, \( \xi(p) \sim t_x^{1/p}(p) \); and, the widths scale in \( L \) as \( w^2(p) \sim L^{2\alpha(p)} \). Plots of the scaled widths \( \langle w^2(p)/L^{2\alpha} \rangle \) (Fig. 2) show that they generally scale in \( p \) as \( w^2 \sim L^{2\alpha}/p^{2\delta} \), where \( \delta \) is some parameter. Is \( \delta \) a universal exponent? **Models A and B** (Figs. 2a-b) may suggest a universal value \( \delta = 1 \) for the RD-EW mix. But **Models C and D** (Figs. 2c-d) show that \( \delta p = 2\delta c \). Accordingly, \( \delta \) is not universal because for the RD-KPZ mix its value is clearly related to the technicalities of these models. In the RD-EW case there is no reason to believe that \( \delta = 1 \) is not accidental. Scale invariance of the EW equation is not sufficient to furnish \( \delta \).

Since \( t_x(p) \sim L^p \) and \( w^2 \sim L^{2\alpha(p)}/p^{2\delta} \) for any \( p \neq 0 \), the roughness must scale as \( w^2(t)/w^2 \sim F(t(p)/L) \). This scaling in \( L \) collapses all curves \( w^2(t) \) to one-parameter families \((p \text{ being the only parameter now})\) presented in Figs. 3a, 4a, 5a and 6a. As RD components do not build correlations, this collapse is obtained with the scaling

![FIG. 1: Model B for pure-correlated growth: (a) Time evolutions of the interface width \((t_0 \text{ marks the end of the initial nonscaling regime})\); (b) Scaling function for \( t > t_0 \), \( z \), \( \alpha \) and \( \beta \) are consistent with the EW universality. Here, \( N \approx 100 \).](image1.png)

![FIG. 2: Scaled widths at saturation vs parameter \(1/p^{2\delta} \): (a) and (b) are for Models A and B, respectively (mix of RD with EW processes); (c) and (d) are for Models C and D, respectively (mix of RD with KPZ). Reference lines have slope 1. Data are scaled with the exponent values shown here.](image2.png)
laws from the corresponding universality classes of processes that build up correlations. Explicitly, \( z = 2\alpha + 1 \) and \( z = 2 - \alpha \) for blending RD with EW and KPZ processes, respectively. To further collapse the data in \( p \), i.e., to find \( t(p) \) in the argument of function \( F \), we analyze the invariance of the corresponding continuum equations under simultaneous affine transformations:

\[
x \rightarrow Lx, \ h \rightarrow hL^{\alpha} / g, \ t \rightarrow tL^{\beta} / f,
\]

assuming \( g \) and \( f \) being arbitrary suitable functions of \( p \).

Scaling (4) is the superposition of scaling (3) with

\[
x \rightarrow x' = x, \ h \rightarrow h' = h / g(p), \ t \rightarrow t' = t / f(p).
\]

Invariance analysis under the component scaling (3) leads to Eq. (11) and signature-scaling laws of KPZ and EW processes. This justifies the data collapse in \( L \). The component scaling (3) transforms Eq. (4) to:

\[
h'_{t'} = v' + \nu'(p)h'_{xx}, \quad \lambda'(p) = \lambda_0 g(p) f(p).
\]

From scaling at saturation we obtained \( g(p) = p^\delta \). Thus, the continuum equation for the RD-KPZ mix is

\[
h_t = v(t) + \nu_0 p^{\delta} h_{xx} + (\lambda_0/2) p^{3\delta} h_x^2 + \eta(x, t).
\]

In the limits \( p \rightarrow 1 \) and \( p \rightarrow 0 \) Eq. (11) describes the dynamics of pure processes, i.e., the KPZ-type and RD, respectively. Similarly, the invariance of the EW equation under scaling (4) gives Eqs. (6)–(7). This leads to the continuum equation for the RD-EW mix:

\[
w^2(t) = \frac{L^{2\alpha}}{p^{2\delta}} F\left(\frac{p^{2\delta}}{L^\alpha} t\right), \tag{11}
\]

where \( \alpha \) and \( z \) are universal exponents of the component process that builds up correlations, and \( \delta \) is nonuniversal. This result is illustrated in Figs. 3b, 4b, 5b and 6b.

Our results, Eqs. (9)–(11), show that mixing RD with a correlated growth preserves the universality of the correlated growth. Physical justification is in the uncorrelated nature of RD. As can be seen from Eqs. (6)–(11), RD blending reduces the values of coefficients \( \nu \) and \( \lambda \) relative to the original noise strength. In other words, the net outcome is a noisier dynamics. The analysis presented here by the examples of EW and KPZ processes in \( (1+1) \) dimensions is easily extended to other growth processes in \( (1+n) \) dimensions. It is enough to notice that Eq. (6) is generally valid when scaling (3) applies to growth equations of the type \( h_t(\vec{x}, t) = (\text{operator})h + \eta(\vec{x}, t) \), where \( \vec{x} \) is \( n \) dimensional. Hence the conclusion: if a correlated growth belongs to a given universality class, its mix with
RD will remain in the same class. The only effects of the RD admixture are the simultaneous dilatations of the fundamental time and height scales in accordance with scaling (5) (and Eq. (6)). The net consequence of these is a slowdown in the dynamics of building up the correlations, reflected in the change of the lateral correlation length $\xi_0(p) \sim t(p)^{1/z} = \xi_1(1)/\sqrt{f(p)}$. In a sense, RD blending is like applying a magnifying glass to the evolution curves $w(t)$: the smaller the $p$ the better the magnification. In particular, in a two-component growth that mixes RD with either EW or KPZ processes, these dilatations explicitly are $h \to h/p^\delta$ and $t \to t/p^{2\beta}$, where $\delta$ is nonuniversal and reflects the particulars of the deposition. The stretching in time causes the initial nonscaling regime $t_0(1)$ in curves $w(t)$ to be amplified as $t_0(p) = t_0(1)/p^{2\delta}$. One consequence of this amplification is a clear observation of the RD growth (with $\beta = 1/2$) for initial times $t < t_0$ when the growth starts from a flat substrate (e.g., observed in Figs. 5 and 6). Note, if $p \to 0$ this initial phase becomes infinitely long as this is the limit of RD growth. In simulations, when $p$ is known, by a prudent design of a model, magnifying effects of RD blending may prove advantageous in revealing hidden features of a correlated growth. However, in the laboratory, the presence of randomness in the growth process will obscure a clear-cut observation of the expected scaling.

This work is supported by NSF grant DMR-0426488, and by the ERC CCS at MSU. It used resources of the NERSC Center, supported by the Office of Science of the US DoE under contract No. DE-AC03-76SF00098.

References

[1] A.-L. Barabasi and H. E. Stanley, Fractal Concepts in Surface Growth (Cambridge University Press, Cambridge, 1995).
[2] C. M. Horowitz and E. V. Albano, Eur. Phys. J. B 31, 563 (2003).
[3] F. D. A. Aarao Reis, Phys. Rev. E 66, 027101 (2002); A. Chame and F. D. A. Aarao Reis, Phys. Rev. E 66, 051104 (2002); T. J. Da Silva and J. G. Moreira, Phys. Rev. E 63, 041601 (2001); B. Drossel and M. Kardar, Phys. Rev. Lett. 85, 614 (2000); H. F. El-Nashar and H. A. Cerdeira, Phys. Rev. E 61, 6149 (2000); M. Kotrla, F. Slanina, and M. Predota, Phys. Rev. B 58, 100003 (1998); W. Wang and H. A. Cerdeira, Phys. Rev. E 47, 3357 (1993); M. Ausloos, N. Vandewalle, and R. Cloots, Europhys. Lett. 24, 629 (1993); P. Pelligrini and R. Jullien, Phys. Rev. Lett. 64, 1745 (1990).
[4] F. Family and T. Vicsek, J. Phys. A 18, L75 (1985).
[5] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. 56, 889 (1986).
[6] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A 381, 17 (1982).
[7] A. Kolakowska, M. A. Novotny, and P. S. Verma, Phys. Rev. E 70, 051602 (2004).
[8] F. Family, J. Phys. A 19, L441 (1986).
[9] C. M. Horowitz, R. A. Monetti, and E. V. Albano, Phys. Rev. E 63, 066132 (2001).
[10] C. M. Horowitz and E. V. Albano, J. Phys. A: Math. Gen. 34, 357 (2001).
[11] A. Kolakowska and M. A. Novotny, in Artificial Intelligence and Computer Science, edited by S. Shannon (Nova Science Publishers, Inc., New York, 2005), p.151.