Development of a generalized proportional integral control strategy for level control in a coupled tank system

F J Regino-Ubarnes¹², J A Gómez-Camperos¹², and A F Ruedas-Rodríguez¹²
¹ Grupo de Investigación en Ingeniería Aplicada, Universidad Francisco de Paula Santander Seccional Ocaña, Colombia
² Semillero de Investigación en Control y Automatización de Procesos, Universidad Francisco de Paula Santander Seccional Ocaña, Colombia

E-mail: fjreginou@ufpso.edu.co

Abstract. This work presents the analysis and design of a generalized proportional integral control strategy for a level control in a coupled tank system, such analysis is made under the active disturbance rejection. The principle objective is to show the advantages of the control strategy generalized proportional integral as to signals tracking and disturbance rejection, compared to conventional control techniques such as the proportional integral derivative control. To validate the process, the modeling of the system was carried out and with this model simulations were carried out under nominal conditions, applying the two control strategies. It was found that the generalized proportional integral control presented a better performance since it achieves a lower mean squared error percentage than the proportional integral derivative control, even in the presence of disturbances.

1. Introduction

Nowadays in the industry it is common to find liquid storage systems, which depending on the type of process it is possible to control the flow output or the level of that liquid. For the automatic control of these processes it is necessary to have a platform that allows to improve their performance through the variation of the control parameters, leading to more efficient processes [1].

Industrial chemical processes generally require liquids to be pumped, stored and transported from one tank to another. Many times, these liquids are processed by chemical treatments and mixed in the tanks, but it is always necessary to control the level of these. Often the tanks are so close together that the levels interact, a change in tank one affects the second tank [2].

In this document, generalized integral proportional control (GPI) is used to regulate the liquid level from a non-linear coupled tank system in MATLAB, Simulink environment. Initially, proportional–integral–derivative (PID) control is used, which is a restricted model approach. A different approach is then used using control techniques based on active rejection of disturbances [3-5].

2. System model

Figure 1 shows the schematic diagram of the coupled tank system used in this work. The analysis of this system depends on the type of flow, either laminar or turbulent. The system can be linearized if the changes in the variables are kept small around the values in a stable state. As the input flow (q₁) minus the output flow (q₀) during the small-time interval dt is equal to the additional quantity stored in the tank, as observed in Equation (1).
\[ Cdh = (q_i - q_o) \, dt \]  

**Figure 1.** Interaction coupled tank system [6].

In Equation (2) the resistance \( R \) is defined as the quotient of the deviation of the height in steady state over the deviation of the output flow in steady state:

\[ R = \frac{h}{q_0} \]  

Equation (3) shows the model of a tank with a constant \( R \), where \( h \) is the deviation around the height in stable state for tank, \( R \) is the resistance of valve 1 and \( q_1 \) is the flow through it.

\[ RC_1 \frac{dh}{dt} + h = R q_1 \]  

Equation (4), for the two-tank system with interaction:

\[ q_1 = \frac{h_1 - h_2}{R_1}, \]  

where \( h_1 \) and \( h_2 \) is the deviation around the height in stable state for tanks 1 and 2 respectively, \( R_1 \) is the resistance of valve 1 and \( q_1 \) is the flow through it.

Equations (5) represent the dynamic response for the tank 1 and Equations (6) represent the dynamic response for the tank 2. When applying the Laplace transform, and taking \( q \) as the system input and \( h_2 \) as the output, the system transfer function is shown in Equation (7).

\[ C_1 \frac{dh_1}{dt} = q - q_1 \]  

\[ C_2 \frac{dh_2}{dt} = q_1 - q_2 \]  

\[ \frac{H_2(s)}{Q(s)} = \frac{R_2}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_2) s + 1} \]  

The parameters for the case study are shown in Table 1. Considering the parameters shown in Table 1, the transfer function of the system is shown in Equation (8).

\[ \frac{H(s)}{Q(s)} = \frac{0.6}{4.8 s^2 + 6.8 s + 1} \]
Table 1. System values.

| Parameter | Value |
|-----------|-------|
| $R_1$     | $0.5 \frac{m}{(m^3/s)}$ |
| $R_2$     | $0.6 \frac{m}{(m^3/s)}$ |
| $C_1$     | $4 \text{ m}^3$ |
| $C_2$     | $4 \text{ m}^3$ |

3. Integral proportional control controller design

This type of control strategies, such as GPI techniques, are capable of rejecting different types of disturbances that adhere to the output of the system, being more robust in the face of unknown perturbations [7,8]. Considering the above it can be said that for a non-linear single input/single output (SISO) system of order $n$, disturbed and soft, as shown in Equation (9).

$$y^{(n)}(t) = \psi(t, y(t), y_t^{(n-1)}) + \varphi(t, y(t))u(t) + \xi(t) \quad (9)$$

It is said that the system is not disturbed for $\xi(t) \equiv 0$ and that it is differentially flat since it is possible to express all the variables of the system, including $u(t)$, in terms of differential functions of the flat output $y(t)$ i.e., functions of $y(t)$ and a finite number of its temporal derivatives [9, 10]. It is assumed that the exogenous perturbation $\Delta(t)$ is uniformly bounded.

Considering the above, it can be said that the system described in Equation (7) is differentially flat since it can be expressed as Equation (10):

$$y^{(n)}(t) = \kappa u(t) + \xi(t) \quad (10)$$

where $\xi(t)$ collects the external and internal perturbations and also the uncertainties of the system proper to the unmodeled dynamics thereof, is $m$-differentiable and uniformly bounded i.e. $\sup |\xi^{(m)}(t)| \leq \kappa$. Where $\kappa = \frac{R_2}{R_1C_1R_2C_2}$. It can be said that the system shown in Equation (7) is differentially flat since it can be expressed as Equation (11).

$$\frac{d^2h_2}{dt^2} = \frac{R_2}{R_1C_1R_2C_2} u(t) + \xi(t) \quad (11)$$

The GPI control is designed within the framework of "active disturbance rejection" [11,12], and it includes a polynomial model in time of the state-dependent disturbances and those that are of an exogenous nature without any special structure. In Equation (12), the structure of the GPI control is shown, where $\kappa = \frac{R_2}{R_1C_1R_2C_2}$ and the output $h_2$ of the system is the level in the second tank; $m$ is the order of the polynomial with which the disturbance approximates; $n$ is the order of the system; $K_{n+m}, \ldots, K_1, K_0$ correspond the gains of such polynomial; and $r$ is reference input.

$$u(t) = \frac{1}{\kappa} \left[ r^{(n)} - \left( \frac{K_{n+m}s^{n+m} + \ldots + K_1 s + K_0}{s^{m+1} + K_{n+m} s^{n+m-1} + \ldots + K_1 s + K_0} \right) (h_2 - r) \right] \quad (12)$$

For the case study, it is assumed that $\frac{d^{m+1} \xi(t)}{dt^{m+1}} = 0$, for $m = 2$, the GPI control structure can be written as Equation (13).
To test the control strategy applied to the system, a step reference signal was proposed (Figure 2). The tracking error was measured with the root-mean-square error (RMSE) and the results were compared with a PID controller.

Figure 2. Reference tracking of the PID.
Six poles were selected for the GPI controller with values of \([-400 - 500 - 600 - 700 - 800 - 900]\), Figure 3; these results are shown in Table 2.

![Graph](image1.png)

**Figure 3.** Reference tracking of the GPI.

| Table 2. Tracking error. |  |
|--------------------------|--|
| Est. ctrl | RMSP (Step) |
| PID | 0.4829% |
| GPI | 0.0025% |

For the disturbance rejection, a step type signal was added to the input of the plant at 50 s (Figure 4 and Figure 5), and the mean square error percentage was calculated, these results are shown in Table 3.

![Graph](image2.png)

**Figure 4.** Disturbance rejection of the PID

| Table 3. Step type disturbance. |  |
|---------------------------------|--|
| Est. ctrl | RMSP (Step) |
| PID | 0.5135% |
| GPI | 0.0025% |
5. Conclusions
The GPI control techniques present a very good response to different external disturbances, substantially improving the response time of the system with a rise time less than one second, surpassing the rise time of the PID control which is about four seconds. Additionally, this control technique presents a very good performance in terms of monitoring the reference signal with an RMSP of 0.0025%.

Compared to PID control, the proposed control performs better in terms of tracking references and rejection of disturbances. The results of the RMS percentage calculation show a clear advantage of GPI control over PID control.

References
[1] Nath U M, Datta S, Dey C 2015 Design and implementation of decentralized IMC-PI controllers for real time coupled tank process (India: Michael Faraday IET International Summit)
[2] Gomez-Camperos J A, Regino-Ubarnes F, Espinel-Blanco E E 2010 Experimental study for detection of leaks in horizontal pipelines Contemporary Engineering Sciences 11 5017
[3] Khalid M U, Kadri M B 2012 Liquid level control of nonlinear coupled tanks system using linear model predictive control (Pakistan: 8th IEEE International Conference on Emerging Technologies)
[4] Regino-Ubarnes F J, Espinel-Blanco E E, Ruedas-Rodriguez A F 2018 Generalized proportional integral control (GPI) design for a ball and beam system Contemporary Engineering Sciences 11(90) 4447
[5] Ramos-Fuentes G, Melo-Lagos I, Regino-Ubarnes F J 2016 Control repetitivo impar de alto orden de un rectificador monofásico: operación a frecuencia variable Técnico Lógicas 19(36) 63
[6] Ogata K 2010 Ingeniería de Control Moderna (Madrid: Pearson Educación, S.A.)
[7] Zurita-Bustamante E W, Linares-Flores J, Guzmán-Ramírez E, Sira-Ramírez H 2011 A comparison between the GPI and PID controllers for the stabilization of a DC–DC “buck” converter: A field programmable gate array implementation IEEE Transactions on Industrial Electronics 58(11) 5251
[8] Regino F J, Gómez J A and Espinel E E, 2018 Comparative study of three control techniques for the current loop of a boost bridgeless converter IEEE International Conference on Automation/XXIII Congress of the Chilean Association of Automatic Control (ICA-ACCA) (Concepción: IEEE)
[9] Sira-Ramírez H, Luviano-Juárez A, Cortés-Romero J 2011 Control lineal robusto de sistemas no lineales diferencialmente planos Rev. Iberoam. Automática e Informática Ind 8(1) 14
[10] Gao Z 2006 Active disturbance rejection control: A paradigm shift in feedback control system design (Minnesota: American Control Conference)
[11] Coral-enriquez H, Ramos G, Cortés J, 2015 Power factor correction and harmonic compensation in an active filter application through a discrete-time active disturbance rejection control approach (Chicago: American Control Conference)
[12] Liu N, Fei J, 2016 Active disturbance rejection control of active power filter 19th International Conference on Electrical Machines and Systems (ICEMS) (Chiba: IEEE)