A smart scheme for relay selection in cooperative wireless communication systems

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Abstract
The performance of multiple-input multiple-output systems using spatial multiplexing can be degraded when the spatial information channels are correlated. This work proposes a solution to this problem, which is based on a cooperative wireless communication system. Within the cooperative system, the relay is selected either randomly or using the smart selection scheme, a simple and distributed approach proposed herein. These relay selection schemes are evaluated for several situations within a small cell environment, using a simulator that generates frequency-selective channel realizations. The simulation results show that the smart selection scheme yields high capacity gains close to the theoretical maximum gain.

Keywords: Channel capacity, Cooperative techniques, MIMO transmission, Wireless communications

1 Introduction
In wireless communication systems, the wavelength and the distance between transmitter and receiver terminals are not the only factors that characterize the radio wave propagation phenomenon. All the interacting objects present in the surrounding environment where waves bounce, as well as their dimension and composition, also contribute to the propagation phenomenon. These interacting objects are usually grouped into clusters and induce scattering, where one or more non-uniformities in the medium force radio waves to deviate from a straight trajectory. Scattering leads to a propagation phenomenon known as multipath propagation, where the transmitted data reach the receiver multiple times by two or more paths and/or at different time instants. These different copies of the transmitted signal, having each one a different attenuation, delay, and phase shift, create an amplified or attenuated received signal power, depending on whether the interference is constructive or destructive, respectively. This is a random process designated as multipath fading, which may vary according to time, space, and/or frequency. Another type of fading is the shadow fading, also a random process due to shadowing from obstacles affecting the wave propagation.

Fading can be very harmful for any wireless communication system as it can cause a strong destructive interference resulting in a deep loss of signal, which in turn can lead to data transmission failure. One way to cope with this issue is to use multiple-input multiple-output (MIMO) systems. These systems can be defined, in a simple way, as wireless communication systems equipped with multiple antenna elements at the transmitter and at the receiver. MIMO systems exploit multipath and fading propagation phenomena so as to achieve high spectral efficiencies without requiring extra frequency spectrum and transmission power [1,2]. However, the real benefit of MIMO systems does not come from multiple antennas by themselves, but from the way these systems process the antennas’ signals using, e.g., spatial diversity and spatial multiplexing [3]:

- Spatial diversity is a powerful technique to mitigate fading and increase link reliability; it combines, in the receiver, different signals from the radio channel, originated by multipath propagation, in order to obtain the source’s stream in better conditions. With this MIMO technique, receiver antennas can provide power gain and, if space-time codes are used, spatial diversity transmission gain can also be achieved. All of this can be reached without requiring channel knowledge at the transmitter, i.e., without channel state information (CSI) prior to any data transmission.
• **Spatial multiplexing** is a technique that exploits differences in the spatial signatures (e.g., caused by rich scattering) of multiplexed data streams onto the wireless channel so as to separate the different signals, i.e., orthogonal information channels are created when there is significant spatial decorrelation. This can be seen as an additional spatial dimension for communication that yields a degree-of-freedom gain without additional power, time, or bandwidth. Hence, the system capacity can be increased linearly by a factor $n$, where $n$ is the minimum number of transmit and receive antennas. This MIMO technique can be used with or without CSI at the transmitter (a system with full CSI can lead to higher spectral efficiencies than a system where CSI is only available at the receiver).

The use of MIMO systems in the spatial multiplexing mode may bring improvements in terms of spectral efficiency. However, since the spectral efficiency gain lies on the fact that the user is in the presence of rich multipath, the MIMO spectral efficiency gain will decrease for spatially correlated channels. One possible way to circumvent this problem is to increase the separation among the antennas at a communication end, resulting in a higher antenna decorrelation. For the base station (BS) side, increasing the antenna array size might not be a problem, but for the mobile station (MS) side, a large array could lead to an oversized handset, which is not attractive for a user. A question then arises: if a virtual MS antenna array is considered, i.e., a distributed array, would this solve the correlated channels’ problem?

The virtual MS antenna array could be implemented using cooperation among MSs. The basic idea of a cooperative wireless communication system is that each user transmits his/her data, acting also as a cooperative agent for another user, i.e., each terminal works both as a user and as a relay. Using this technique, independent copies of the transmitted signal are traditionally used to combat fading effects through spatial diversity generated in a distributed manner [4,5]. Since the MSs share their antennas and other resources, the virtual MS antenna array is obtained through distributed transmission and signal processing, which can also be seen as a virtual MIMO system, even for single-antenna terminals.

Considering single-antenna systems, relay networks can resemble the spatial diversity of MIMO systems. This has been studied before, and effective protocols like *amplify-and-forward* (AF) and *decode-and-forward* (DF) have been defined [6,7]; in the former, a relay simply amplifies what it receives, while in the latter, a relay decodes, re-encodes, and retransmits the signal. Capacity bounds for single-relay MIMO channels, where every terminal in the network has multiple antennas, were presented in [8]. For a multiple MIMO relay network, where spatial diversity is achieved through cooperation among all the relays available in the network, quantitative capacity results were presented in [9]. In [9], it was also shown that using relays as active scatterers allows to recover the spatial multiplexing gain in poor scattering environments. A discussion and comparison of different relaying schemes for MIMO terminals were presented in [10]. Based on the AF and DF protocols, the authors proposed another hybrid method, namely *filter, amplify, and forward*, where the relays apply a spatial filter to the received signals, this filter being based on the CSI obtained only by decoding a training sequence.

Some issues arise when using cooperative communication, such as how to select which relay(s) will be used to cooperate and how often they are reassigned. Compared to cooperative diversity schemes where all relays transmit their message to the destination concurrently, several recent results have demonstrated the performance and implementation advantages of one or a subset of relays assisting transmissions in a wireless network [10-21]. With relay selection, the complexity, synchronization, and overhead of the cooperative system are relaxed, leading to an increase in the spectral efficiency.

The simplest routing scheme for selecting a cooperation partner corresponds to MSs choosing their relay(s) randomly. The major advantage of this *random selection* scheme is a very low scheme complexity, resulting in low selection time and signaling overhead. However, choosing the relay(s) randomly might not be very efficient in terms of gain. For frequency-flat fading channels, it was proven in [7] that in some situations no diversity gain can be obtained if the random selection scheme is used, whereas for frequency-selective channels, it was shown in [21] that only in some cases the random relay selection can achieve the same diversity as the best relay selection.

Considering single-antenna systems, more efficient relay selection algorithms have been proposed. The authors of [11] presented a centralized method for grouping and partner selection to achieve full network-wide cooperative diversity, in which enough partners are assigned to each transmitter in order to improve the probability of decoding messages successfully. Other works followed a game theory approach to find a solution to the relay selection problem [12-14]; this problem is formulated as a market competition in which multiple relays compete with each other to offer resources (e.g., power, bandwidth) to a single user. In [15], a power allocation scheme along with a relay subset selection scheme considering generic noise and interference were proposed in order to reduce the signaling overhead required for CSI acquisition.

For MIMO systems, a routing scheme based only on the pathloss of the channel was proposed in [10], which approaches the performance of an optimal routing scheme if the number of transmit and receive antennas...
or relay antennas is high. In [16], antenna selection was addressed (i.e., antenna pair selection at the relays) so as to obtain increased performance and low complexity, where the criteria are based on the instantaneous mean square error (MMSE). A set of joint antenna selection and relay selection algorithms for a variety of MMSE-based reception techniques was proposed in [17].

The previous cited works [10-17], which present relay selection algorithms, only consider frequency-flat fading channels. Nevertheless, broadband communication wireless systems have a transmission bandwidth larger than the coherence bandwidth of the channel, yielding a frequency-selective channel. Taking this fact into account, a power control, bandwidth allocation, relay selection, and relay strategy choice in an orthogonal frequency-division multiple-access (OFDMA) based relay network for single-antenna systems was proposed in [18]. Also, for single-antenna systems, the authors of [19] proposed two relay selection strategies for systems employing orthogonal frequency-division multiplexing (OFDM), one on a per-subcarrier basis and the other on an all-subcarrier basis. Other works, as far as single-antenna systems are concerned, either provided close-to-optimal relay assignment and power allocation across users supported by a single relay [20] or explored the use of relay selection when all channels have inter-symbolic interference [21].

None of the previous studies dealt with relay selection algorithms for MIMO systems in realistic frequency-selective environments (even considering single-antenna systems, only [20,21] used realistic channel models). Moreover, the research referred above was aimed at obtaining good relay(s) in terms of cooperative spatial diversity. The works either considered algorithms related to the best relay selection (BRS), the simplest selection method which is based on the instantaneous end-to-end signal-to-noise Ratio (SNR), or partial relay selection (PRS) algorithms, i.e., methods in which the relay choice only takes into account the source-relay or the relay-destination SNR. However, when addressing the spatial multiplexing case, both the BRS and PRS approaches are ineffective, and the resulting loss in capacity can be significant, as can be seen in [17], in which capacity-based selection was also considered. This behavior can be interpreted physically: the goal of a cooperative spatial multiplexing system is to help the receiver to separate the different data streams; thus, it is not good to use the signals from two relays that are highly correlated, even if both have high SNR.

In this work, we propose a solution to enhance the capacity of wireless communication systems in small cells through the use of MIMO systems and cooperation between terminals. Some new ideas are explored:

- The virtual MS antenna array is based on MIMO spatial multiplexing mode.
- A simple and smart distributed way of selecting a good cooperative agent is proposed.
- A realistic approach is adopted by using a simulator that generates realistic frequency-selective channel realizations.

The rest of the article is organized as follows. Section 2 describes the proposed cooperative wireless communication system, along with the smart selection scheme for selecting the cooperative agent. Simulation results of the capacity gain obtained by using the cooperative system in a small cell scenario are presented in Section 3. Finally, conclusions are given in Section 4.

2 Cooperative wireless communication system

In this section, the suggested cooperative wireless communication system based on MIMO spatial multiplexing mode is first described. Next, the MIMO channel capacity and its bounds are discussed. Afterwards, the smart selection scheme is presented, which is a simple and distributed scheme for selecting the cooperative agent proposed in this work. Some application examples of the suggested cooperative system are given at the end of the section.

2.1 System description

From an analytical point of view, a MIMO system can be expressed as

\[ \tilde{s}_g(f) = H(f)s_T(f) + \tilde{n}(f), \]  

where \( \tilde{s}_T(f) \) corresponds to the transmitted signal vector from \( n_T \) antenna elements at the transmitter, \( \tilde{s}_g(f) \) stands for the received signal vector at \( n_R \) antenna elements at the receiver, \( H(f) \in C^{nR \times nT} \) denotes the MIMO channel matrix associated with a radio propagation channel snapshot, \( \tilde{n}(f) \) represents the additive white Gaussian noise vector, and \( f \) corresponds to the carrier frequency. The \((r, t)\)th component of \( H(f) \), \( h_{r,t}(f) \), represents the link between the \( t \)th transmitter antenna and the \( r \)th receiver antenna, which depends on the radio environment.

A MS in non-cooperative mode (with a real antenna array) experiences the following MIMO channel matrix for the downlink (an example is depicted in Figure 1a for a MIMO system with \( n_T = n_R = 2 \))

\[
H_R = \begin{bmatrix}
H_{R}^{(1)} & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots \\
& \cdots & \cdots & \cdots \\
H_{R,n_R} & \cdots & \cdots & H_{R,n_R,n_T}
\end{bmatrix}
\]  

(2)
For instance, if the signal to/from the antenna (hence, $\vec{H}_R^{(i)}$ where $HR$ respectively, for a MIMO system with downlink and the uplink are presented in Figure 2a,b, the antennas of a cooperative MS (examples for the one or more antennas of this MS are redirected through BS antenna (thus, $\vec{H}_R^{(i)}$ refers to the signal at the $i$th MS antenna).

For the uplink (an example is shown in Figure 1b for a MIMO system with $n_T = n_R = 2$), a MS without cooperation experiences the following MIMO channel matrix:

$$H_T = \begin{bmatrix}
\vec{H}_T^{(1)} & \vec{H}_T^{(2)} & \cdots & \vec{H}_T^{(n_T)} \\
H_{T,1,1} & H_{T,1,2} & \cdots & H_{T,1,n_T} \\
H_{T,2,1} & H_{T,2,2} & \cdots & H_{T,2,n_T} \\
\vdots & \vdots & & \vdots \\
H_{T,n_T,1} & H_{T,n_T,2} & \cdots & H_{T,n_T,n_T}
\end{bmatrix}$$

(3)

where $H_{T,k,i}$ stands for a complex value corresponding to the link between the $k$th BS antenna and the $i$th MS antenna (hence, $\vec{H}_R^{(i)}$ denotes the signal at the $i$th MS antenna).

When a MS is in cooperative mode, the signals to/from one or more antennas of this MS are redirected through the antennas of a cooperative MS (examples for the downlink and the uplink are presented in Figure 2a,b, respectively, for a MIMO system with $n_T = n_R = 2$). For instance, if the signal to/from the $m$th antenna of the assisted MS is replaced by a cooperative signal, the corresponding MIMO virtual channel matrix for the downlink is given by

$$H_{R,virtual} = \begin{bmatrix}
\vec{H}_R^{(1)} \\
\vec{H}_R^{(m-1)} \\
\vec{H}_R^{(m)} \\
\vec{H}_R^{(n_T)} \\
H_{R,1,1} & H_{R,1,2} & \cdots & H_{R,1,n_T} \\
\vdots & \vdots & & \vdots \\
H_{R,m-1,1} & H_{R,m-1,2} & \cdots & H_{R,m-1,n_T} \\
H_{R,m+1,1} & H_{R,m+1,2} & \cdots & H_{R,m+1,n_T} \\
\vdots & \vdots & & \vdots \\
H_{R,n_T,1} & H_{R,n_T,2} & \cdots & H_{R,n_T,n_T}
\end{bmatrix}$$

(4)

where $H_{R_{coop},l,k}$ denotes a complex value corresponding to the link between the $k$th BS antenna and the $i$th cooperative MS antenna; for the uplink, the corresponding MIMO virtual channel matrix is given by

$$H_{T,virtual} = \begin{bmatrix}
\vec{H}_T^{(1)} & \vec{H}_T^{(m-1)} & \vec{H}_T^{(m)} & \cdots & \vec{H}_T^{(n_T)} \\
H_{T,1,1} & H_{T,1,m} & H_{T,1,2} & \cdots & H_{T,1,n_T} \\
H_{T,2,1} & H_{T,2,m-1} & H_{T,2,2} & \cdots & H_{T,2,n_T} \\
\vdots & \vdots & \vdots & & \vdots \\
H_{T,n_T,1} & H_{T,n_T,m-1} & H_{T,n_T,2} & \cdots & H_{T,n_T,n_T}
\end{bmatrix}$$

(5)
where $H_{\text{coop}}^{T}$ denotes a complex value corresponding to the link between the $i$th cooperative MS antenna and the $k$th BS antenna.

Within this wireless system, a cooperative MS receives a signal from the BS or from the assisted MS (whether in the downlink or the uplink, respectively) and then forwards this signal to the destination (assisted MS or BS, respectively) without decoding. Besides simplifying the cooperative MS’s work, it is useful from a security point of view since the transmitted data can be encrypted, allowing only the assisted MS, the BS or both to be able to decode the message successfully.

We assume that the signals exchanged by a cooperative MS with the assisted MS (related to $\tilde{H}_{\text{coop}}^{R}/\tilde{H}_{\text{coop}}^{T}$) can be transmitted using another technology (secondary wireless interface), such as Bluetooth [22] or Wi-Fi [23], in order to avoid the interference between cooperating terminals and the communication with the BS (main wireless interface). Also, we assume that the MS can only select relays which are able to decode the messages exchanged in the secondary wireless interface successfully. This can be achieved if each message is followed by a cyclic redundancy check (CRC) code and if the relays that receive corrupted messages report that they are unable to help the MS. As an example, consider the downlink scenario of the suggested cooperative system, in which the selected relay receives a signal from the BS in the main wireless interface. This signal is then encapsulated using the technology of the secondary wireless interface and, finally, it is forwarded to the assisted MS. The same idea is already put into practice, allowing smartphones to work as Wi-Fi hotspots (secondary wireless interface), while the Internet connection is established through a cellular operator (main wireless interface).

In a perfect exchange of information between a cooperative MS and the assisted MS, when a MS is assisted by using the proposed cooperative system, one or more antennas of the assisted MS are a few meters away from the MS position, yielding a virtual MS antenna array.

### 2.2 MIMO channel capacity

As stated in the introduction, systems with full CSI can yield higher spectral efficiencies, i.e., higher channel capacities. Unfortunately, obtaining channel knowledge at the transmitter is not an easy task [24]. Hence, in this work, only systems without CSI at the transmitter will be considered.

The MIMO channel capacity expressed in bits per second per hertz (bit/s/Hz) is given by [1,2]

$$
C = \log_2 \det \left( \mathbf{I}_{n_{R}} + \frac{\text{SNR}}{n_{T}} \mathbf{H}_{F} \mathbf{H}_{F}^{H} \right)
= \log_2 \det \left( \mathbf{I}_{n_{T}} + \frac{\text{SNR}}{n_{T}} \mathbf{H}_{F}^{H} \mathbf{H}_{F} \right),
$$

where $\mathbf{I}_{n_{R}}$ (or $\mathbf{I}_{n_{T}}$) stands for the identity matrix of size $n_{R}$ ($n_{T}$), $\cdot^{H}$ represents the conjugate transpose operation and $\mathbf{H}_{F}$ denotes the normalized matrix of a given channel realization $\mathbf{H}$ using the Frobenius norm $\| \cdot \|_{F}$, i.e.,

$$
\mathbf{H}_{F} = \frac{1}{\rho} \mathbf{H}, \quad \rho = \sqrt{\frac{1}{n_{T}n_{R}} \| \mathbf{H} \|_{F}^{2}}, \quad \| \mathbf{H} \|_{F} = \sqrt{\text{tr}(\mathbf{H}^{H}\mathbf{H})},
$$

where $\text{tr}(\cdot)$ corresponds to the trace function. With this matrix normalization, the channel attenuation is reflected in the SNR, allowing the influence of the channel correlation properties on the channel capacity to become visible.

Another way to compute the MIMO channel capacity is through the use of the eigenvalues $\gamma_{1}, \ldots, \gamma_{n}$ of the channel correlation matrix $\mathbf{W}$, defined by

$$
\mathbf{W} = \begin{cases} 
\mathbf{H}_{F} \mathbf{H}_{F}^{H} & n_{R} \leq n_{T} \\
\mathbf{H}_{F}^{H} \mathbf{H}_{F} & n_{R} > n_{T}
\end{cases},
$$

which is a Hermitian non-negative definite matrix and hence has $n$ real non-negative eigenvalues, where $n$ is the minimum number of transmit and receive antennas, i.e.,

$$
n = \min(n_{T}, n_{R}).
$$

Therefore, the capacity formula (6) can be rewritten as

$$
C = \sum_{i=1}^{n} \log_2 \left( 1 + \frac{\text{SNR}}{n_{T}} \gamma_{i} \right).
$$

Expression (10) has as underlying assumption that a wireless narrowband channel is being used, so the channel realization $\mathbf{H}$ is constant for the whole bandwidth. In the case of a wideband channel, the channel realizations have a frequency-selective feature, i.e., $\mathbf{H}$ is frequency-dependent. To obtain the capacity in this situation, the channel bandwidth can be split into $Q$ flat-fading subchannels represented by the corresponding channel correlation matrix, $\mathbf{W}_{q}$, given by

$$
\mathbf{W}_{q} = \begin{cases} 
\mathbf{H}_{F}(f_{q}) \mathbf{H}_{F}(f_{q})^{H} & n_{R} \leq n_{T} \\
\mathbf{H}_{F}(f_{q})^{H} \mathbf{H}_{F}(f_{q}) & n_{R} > n_{T}
\end{cases},
$$

where $f_{q}$ stands for the $q$th subcarrier central frequency. Denoting $\gamma_{i,q}$ as the $i$th eigenvalue of $\mathbf{W}_{q}$ and looking at the different frequency subchannels as parallel subchannels, the channel capacity of a frequency-selective MIMO channel can be expressed as [3]

$$
C = \frac{1}{Q} \sum_{q=1}^{Q} C_{q}
= \frac{1}{Q} \sum_{q=1}^{Q} \sum_{i=1}^{n} \log_2 \left( 1 + \frac{\text{SNR}}{n_{T}} \gamma_{i,q} \right),
$$

where $C_{q}$ stands for the $q$th subchannel capacity.
The MIMO channel capacity achieves its minimum value when the radio channel is completely correlated among the different MIMO links for all $Q$ flat-fading subchannels, value which is given by

$$C_{\text{min}} = \log_2 (1 + n_R \text{SNR}).$$

(13)

On the other hand, if the radio channel is completely decorrelated among the different MIMO links, then the MIMO channel capacity achieves its maximum value, which is given by

$$C_{\text{max}} = n \log_2 \left(1 + \text{SNR} \frac{n_R}{n}\right).$$

(14)

The derivation of these bounds is presented in the Appendix.

### 2.3 Smart selection scheme

The optimal selection scheme would be the one that always chooses the relay that yields the highest cooperative capacity gain. This could be achieved if a MS compared all the cooperative capacities provided by the cooperative MSs before the best relay is selected. However, this scheme might conduct to high signaling overhead, since capacity tests have to be performed for every flat-fading subchannel. Also, the time required for this procedure might lead to a non-viable solution because the channel matrices corresponding to the tested channel realizations might change in the meantime.

In order to reduce the required overhead, the selection time, and the overall scheme complexity, a simpler approach is proposed in this subsection, which will be designated from now on as smart selection. To introduce this scheme, note that from the eigenvalues properties, the matrix $W_q$ verifies the condition

$$\det(W_q) = \prod_{i=1}^n |\gamma_{i,q}|.$$

(15)

Taking into account that all eigenvalues of $W_q$ are non-negative and also that their sum is equal to $n_T n_R$ (c.f. (26) in the Appendix), if the goal is to determine which eigenvalues maximize the determinant of $W_q$, i.e.,

$$\arg \max_{\gamma_{i,q} \geq 0} \det(W_q) = \arg \max_{\gamma_{i,q} \geq 0} \prod_{i=1}^n |\gamma_{i,q}|,$$

(16)

the solution is given by the inequality of arithmetic and geometric means. This inequality states that the arithmetic mean (average) of a list of non-negative real numbers is greater than or equal to the geometric mean of the same list; it further states that the two means are equal if and only if every number in the list is the same. Hence, the solution of (16) is that all eigenvalues have the same value, i.e.,

$$\arg \max_{\gamma_{i,q} \geq 0} \det(W_q) \Rightarrow \gamma_{i,q} = \frac{n_T n_R}{n}, \enspace \forall i.$$

(17)

Noticing that the maximum channel capacity value is also obtained when all eigenvalues have the same value (c.f. (32) in the Appendix), from (17), we conclude that a higher determinant value of the channel correlation matrix leads to a higher capacity value.

The smart selection scheme is based on this conclusion: the MS receives, from a cooperative MS, the coefficients corresponding to $H^R/T_{\text{coop}}$ of only one flat-fading subchannel and then computes the determinant of the corresponding virtual channel correlation matrix:

$$W_{q,\text{virtual}} = \begin{cases} H_{R/T,\text{virtual}}(f_q) H^T_{R/T,\text{virtual}}(f_q) & n_R \leq n_T \\ H^T_{R/T,\text{virtual}}(f_q) H_{R/T,\text{virtual}}(f_q) & n_R > n_T \end{cases}.$$  

(18)

This procedure is repeated for all the cooperating MSs and, finally, the chosen relay is the one that yields the highest determinant value of $W_{q,\text{virtual}}$. For the particular case where $H_{R/T,\text{virtual}}(f_q)$ is a square matrix ($n_T = n_R$), it is possible to write

$$\det(W_{q,\text{virtual}}) = \det(H_{R/T,\text{virtual}}(f_q) H^T_{R/T,\text{virtual}}(f_q)) = \left|\det(H_{R/T,\text{virtual}}(f_q))\right|^2,$$

(19)

so that the MS can have an easier procedure and compute only $|\det(H_{\text{virtual}})|$ for each cooperating terminal, choosing, in the end, the relay that yields the highest determinant absolute value of $H_{R/T,\text{virtual}}(f_q)$.

Since the choice is based only on one flat-fading subchannel (e.g., the signal corresponding to an OFDM pilot channel), the proposed scheme is not optimal; hence, bad choices will still be made if, for cooperating terminals, this specific subchannel is not much correlated and if the other subchannels are highly correlated. On the other hand, less overhead and time are required for the selection procedure, not only because less tests are performed (for only one instead of all subchannels), but also because it is computationally simpler to test the determinant than the capacity expression (12), which even for a MIMO square matrix always requires matrix multiplications.

The smart selection scheme requires the MSs to have the subchannel CSI, which could be hard to implement in the uplink (where MSs are transmitters); however, in the downlink this CSI is already available at the MSs (acting as receivers) since it is required for data transmission using...
MIMO spatial multiplexing. Therefore, the smart selection scheme can be considered as being distributed for the downlink transmission, as no processing is needed at the BS (for the uplink, the BS would have to feedback the CSI that it is experiencing as a receiver).

From a security point of view, the proposed scheme requires only the cooperative agents to decode the training sequence of only one frequency subchannel, and so these are the only data that must be available to all users.

2.4 Application examples

Broadcast situations are cases for which the suggested cooperative system is well suited. For instance, consider a BS, on a football stadium, broadcasting TV replays for the spectators to watch on their handsets. The signal received at one of the MS’s antennas could be used not only for receiving data in this MS, but also as the cooperating signal for a partner, so all users would easily enhance their channel capacity by using this cooperative system instead of using their real antenna arrays. Additionally, the smart selection scheme is straightforward to implement: as the receiver CSI needed for MIMO transmission is always computed by all the MSs, the CSI required by this scheme is already available.

Single-antenna MSs can also use the proposed cooperative system to enhance their channel capacity, but there is a drawback: since the BS transmission is envisioned for a MIMO system using spatial multiplexing (each communication end requires a minimum of n antennas for the n × n MIMO), cooperation would be essential for data transmission (the option to fallback to co-located antennas would not be available). However, this can also be seen as a way to encourage cooperation.

3 Simulation results

This section provides simulation results for the capacity gain obtained using the proposed cooperative communication system in a microcell scenario, which is one of the environments encompassed by small cells. First, the MIMO channel model and test environment are presented, followed by the capacity gain results using the random selection and the smart selection schemes for selecting the cooperative agents. A comparison between these relay selection schemes, the optimal exhaustive search scheme, and a greedy selection scheme is provided at the end of the section.

3.1 MIMO channel model and test environment

Given a channel model that generates channel impulse responses (CIRs), a Fourier transform is applied to the CIRs in order to compute $H(f)$. In this work, the CIRs are generated using a validated implementation of the COST 273 channel model for microcell scenarios [25]. This channel model uses time-delayed multipath replicas of a transmitted signal to simulate the signal that a MS receives. These replicas have their origin in $M$ clusters of scatterers; these scatterers are stochastically generated within the channel model. Each cluster is characterized by $L$ multipath components (MPCs), with each MPC having an associated angle of departure (AoD), angle of arrival (AoA), power, and delay.

Considering that both BS and MSs are equipped with uniform linear arrays, with omnidirectional unitary gain pattern for each antenna element, $h_{l,t}(f)$ is then given by

$$h_{l,t}(f) = \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{l=1}^{L} a_{m,l} e^{-j\frac{2\pi}{\lambda} d_l \sin(\phi_{m,l}, \text{AoD})} \times e^{-j2\pi f \tau_n} g(\tau_{m,l}, \tau_n),$$

where $a_{m,l}$ represents the complex amplitude of the $l$th MPC of the $m$th cluster, $d_l$ ($d_r$) stands for the distance from the transmitter (receiver) antenna element $t$ ($r$) to the reference antenna (for the reference antenna $d_1 = 0$), $\phi_{m,l}, \text{AoD}$ ($\phi_{m,l}, \text{AoA}$) denotes the AoD (AoA) for the $(m, l)$th MPC at the transmitter (receiver) with respect to its broadside, and $\lambda$ is the wavelength; to account for the effect introduced by the system bandwidth, $B$, $h_{l,t}(f)$ is described by $N$ delay bins, having each bin $\tau_n$ a width of $1/B$ [26]. The function $g(\tau_{m,l}, \tau_n)$ is given by

$$g(\tau_{m,l}, \tau_n) = \begin{cases} 1 & \tau_{m,l} \in [\tau_{n-1}, \tau_n] \\ 0 & \text{otherwise} \end{cases},$$

where $\tau_{m,l}$ corresponds to the $(m, l)$th MPC’s excess delay regarding the line-of-sight (LoS) radio path. With this formulation, several MPCs can be contained within a delay bin and, depending on their phase relationship, small-scale fading occurs due to either constructive or destructive interference.

The small cell test environment is characterized by a BS located at the center, $(x, y) = (0, 0)$, of a 100 m × 100 m microcell area. Following the conclusions of [27,28], where it was shown that the capacity values are higher for regions which are transverse to the plane defined by the BS antenna array, reference MSs are placed in positions $(x, y) = (\pm 25, 0)$ m, which are in a high capacity region (HCR), and in positions $(x, y) = (0, \pm 25)$ m, which are in a low capacity region (LCR). Possible cooperative MSs are positioned around the reference MSs up to a maximum range of 10 m, being evenly separated from each other by 1 m. The BS and the MSs antennas are at a 10 and 1.5 m height, respectively.

The MIMO configuration used for the simulation is characterized by a $2 \times 2$ antenna array setting with 20 MHz bandwidth, which is one of the long-term evolution configurations [29]. The operating frequency is set to 2 GHz and two spacings between the antenna elements are assumed as reference scenarios: one with a spacing
of 0.5λ and 1.0λ for the MS arrays and the BS array (2(0.5λ) × 2(1.0λ) MIMO), respectively, and another with a spacing of 0.5λ and 2.5λ (2(0.5λ) × 2(2.5λ) MIMO), respectively.

Taking into account the frequency-selective feature of the channel, the simulated MIMO channel matrices are split into Q = 512 subchannels, i.e., each subchannel has a bandwidth of 39.0625 kHz, which corresponds to the coherence bandwidth of channels that have a delay spread of 12.8 μs [3]. In order to simulate a bad channel in terms of attenuation experienced by the MSs, a SNR of 5 dB is used in (12), in all experiments.

To simulate inter-link correlation among different MSs properly, the simulator simultaneously generates, in a given simulation run, the radio channel snapshots for all the MSs. All the following results are based on 200 simulations runs, where each run corresponds to a different propagation scenario.

Considering the LoS and the non-LoS (NLoS) situations, Table 1 presents the average channel capacity, \( \bar{C}_{\text{real}} \) (\( \bar{C} \) represents the average operation), obtained by the reference MSs using the real antenna array (non-cooperative mode, i.e., using \( H_{R/T} \)).

Noticing that from (14), the theoretical maximum channel capacity for a 2 × 2 MIMO is \( C_{\text{max}} = 4.11 \text{ bit/s/Hz} \), Table 2 presents the theoretical maximum capacity gain that the cooperative system can achieve, taking into account the values presented in Table 1. This gain is defined as

\[
\text{Gain}_{\text{theo}} = \left[ \frac{C_{\text{max}}}{C_{\text{real}}} - 1 \right]. \tag{22}
\]

The values presented in Table 2 will be used to evaluate the performance of the random selection and of the smart selection schemes.

### 3.2 Random selection scheme performance

Figure 3 shows the cumulative distribution of the cooperative capacity gain obtained using the random selection scheme for the 2(0.5λ) × 2(1.0λ) MIMO, with the cooperating MSs positioned around the reference MSs up to a maximum range of 5 m. This gain is given by

\[
\text{Gain}_{\text{random}} = \frac{C_{\text{virtual},\text{random}}}{C_{\text{real}}} - 1, \tag{23}
\]

where \( C_{\text{virtual},\text{random}} \) stands for the capacity obtained using a virtual MS antenna array (cooperative mode, i.e., using \( H_{R/T,\text{virtual}} \)) with the random selection scheme.

From Figure 3 we can conclude that for about 20% of the cases (a reference MS either in a HCR or in a LCR, with or without LoS), it is a bad choice to use cooperation. However, it can be argued that if the average cooperative capacity gain (\( \text{Gain}_{\text{random}} \), which considers both positive and negative gains) is high, then the cooperation should be used. Having this in mind, Figure 4 presents, for several maximum cooperation ranges, a comparison of the bad choices’ percentage, along with the average cooperative capacity gain, \( \text{Gain}_{\text{random}} \), and also the difference between this gain and the theoretical maximum cooperative capacity gain presented in Table 2, \( \text{Gain}_{\text{theo}} - \text{Gain}_{\text{random}} \) (since Figure 4a,b,c share the same legend, this is only shown in the first figure for the sake of readability).

These results show that when the maximum cooperation range increases, not only the percentage of bad choices decreases but also the average capacity gain generally increases. This was expected, as more cooperative MSs with a higher degree of channel decorrelation regarding the reference MS are being considered; yet,

![Table 2 Theoretical maximum cooperative capacity gain](image)

| MIMO          | Gain\(_{\text{theo}}\) (HCR) [%] | Gain\(_{\text{theo}}\) (LCR) [%] |
|---------------|---------------------------------|---------------------------------|
|               | LoS    | NLoS   | LoS    | NLoS   |
| 2(0.5λ) × 2(1.0λ) | 39.1   | 32.4   | 41.8   | 36.5   |
| 2(0.5λ) × 2(2.5λ) | 38.9   | 30.6   | 41.4   | 34.7   |
also more cooperative MSs with a negative effect become available and can, in some cases, start lowering the gain (e.g., $2(0.5\lambda) \times 2(2.5\lambda)$ MIMO, with the MS in a HCR). From Figure 4c, we can conclude that although this cooperative system yields a capacity gain with the random selection scheme, this gain is not enough to solve the correlated channels’ problem since there is at least a 15% margin of improvement for all the situations under consideration.

### 3.3 Smart selection scheme performance

Figure 5 presents the resulting bad choices’ percentage for the smart selection scheme, along with the average cooperative capacity gain, $Gain_{\text{smart}}$, and also the difference between this gain and the theoretical maximum cooperative capacity gain presented in Table 2, $Gain_{\text{theo}} - Gain_{\text{random}}$.

These results show that the smart selection scheme is much better than the random scheme, yielding a lower percentage of bad choices and a higher average cooperative capacity gain; additionally, this gain never decreases as the cooperation range increases. In spite of the fact that this selection scheme is not optimal, since bad choices are always made (see Figure 5a), we can conclude from Figure 5c that the smart selection scheme yields near-optimal results, because for some of the considered situations the difference between the theoretical maximum gain and the achieved gain is less than 5%. As an example, for the reference situation $2(0.5\lambda) \times 2(2.5\lambda)$ MIMO, with the MS in a HCR with LoS, the gain difference
is less than 2% if at least a 5 m cooperation range is assumed.

3.4 Relay selection scheme comparison

Considering the reference situation $2(0.5\lambda) \times 2(1.0\lambda)$ MIMO, with the MS in a HCR with LoS, Figure 6 presents a comparison of the average cooperative capacity gain obtained using four different relay selection schemes: random selection (Gain_{random}), smart selection (Gain_{smart}), exhaustive search (Gain_{max}), and greedy selection (Gain_{greedy}); the theoretical maximum cooperative capacity gain (Gain_{theo}) is also shown. The exhaustive search corresponds to the optimal relay selection scheme, where the reference MS compares all the cooperative capacities, i.e., capacities for all flat-fading subchannels provided by all the cooperative MSs, and then chooses the relay that yields the highest global cooperative capacity. The greedy selection scheme corresponds to an adaptation of the method presented in [16] for the suggested cooperative system. First, the MS computes the cooperative capacity value (base value) using one relay, for all flat-fading subchannels. Next, it chooses another relay and, afterwards, it computes the new cooperative capacity value. If the new value is higher than the base value, then the MS adopts this new capacity value as the base one and repeats the previous procedure; otherwise, if the new value is lower than the base value, the MS stops the search for a better cooperative capacity value and selects the relay corresponding to the current cooperative capacity base value.
smart selection, is also proposed. Simulation results show that the cooperative system using the smart selection scheme can improve the channel capacity and mitigate the correlated channels’ effects, yielding capacity gains close to the theoretical maximum gain.

**Appendix**

**MIMO channel capacity bounds**

To derive the bounds of the MIMO channel capacity, let us first recall that from the eigenvalues properties, the trace of a channel correlation matrix, \( W_q \), can be given by

\[
\text{tr}(W_q) = \sum_{i=1}^{n} \gamma_i q.
\]  

If the normalization expressions (7) are applied to an already normalized matrix \( H_F(f_q) \), resulting in \( \rho = 1 \) and

\[
n_T n_R = \| H_F(f_q) \|^2_F
\]

\[
= \text{tr}(H_F(f_q) H_F^T(f_q))
\]

\[
= \text{tr}(H_F(f_q) H_F^T(f_q))
\]

\[
= \text{tr}(W_q).
\]  

combining (24) with (25) allows to write

\[
\sum_{i=1}^{n} \gamma_i q = n_T n_R.
\]  

Consider now the case in which the MIMO channel capacity achieves its minimum value. From a physical point of view, this corresponds to a radio channel which is completely correlated among the different MIMO links for all \( Q \) flat-fading subchannels. From an analytical point of view, all channel correlation matrices \( W_q (q = 1, \ldots, Q) \) will have a rank equal to one, i.e., all \( n \) eigenvalues but one will be zero; taking into account (26),

\[
\begin{cases}
\gamma_i q = n_T n_R & i = 1 \\
\gamma_i q = 0 & i \neq 1 , \forall q.
\end{cases}
\]  

Applying this result to the channel capacity expression (12), it yields

\[
C_{\min} = \frac{1}{Q} \sum_{q=1}^{Q} \log_2 \left( 1 + \frac{\text{SNR}}{n_T n_R} \right)
\]

\[
= \log_2 (1 + n_q \text{SNR}).
\]  

Let us use a proof by contradiction to show that the channel capacity presented in (28) is indeed the minimum value. Assume that there exists a \( C_{\min}' < C_{\min} \) such that the eigenvalues for obtaining \( C_{\min}' \) are given by (taking also into account (26))

\[
\begin{cases}
\gamma_i q = n_T n_R - \epsilon , i = 1 \\
\gamma_i q = \epsilon , \quad i = 2 , \quad \epsilon > 0 , \forall q, \\
\gamma_i q = 0 , \quad i \neq [1, 2]
\end{cases}
\]  

Figure 6 Average cooperative capacity gain comparison between different relay selection schemes. 2(0.5\(m\)) \times 2(1.0\(m\)) MIMO, with the MS in a HCR with LoS.

By observing Figure 6, it can be seen that the performance of the smart selection scheme is only overtaken by the exhaustive search scheme, but still the cooperative gain difference between these two schemes is much lower than the resulting gain difference using the other relay selection schemes. Similar results were obtained for other reference situations. These results are even more interesting if we analyze the complexity required by the selection schemes: considering a MIMO square matrix (\( n_T = n_R \)) and a certain number \( N_{\text{relay}} \) of available relays, the exhaustive search requires \( N_{\text{relay}} \times Q \) matrix multiplications plus \( N_{\text{relay}} \times Q \) determinant computations, besides \( N_{\text{relay}} - 1 \) comparisons; the greedy selection requires at least \( 2 \times Q \) matrix multiplications plus \( 2 \times Q \) determinant computations and one comparison, and at most the same as the exhaustive search; the smart selection only requires \( N_{\text{relay}} \) determinant computations, besides \( N_{\text{relay}} - 1 \) comparisons (c.f. Section 2.3). Since the number of flat-fading sub-channels (\( Q \)) is usually much larger than the number of available relays (\( N_{\text{relay}} \)), we can conclude that the smart selection yields a good tradeoff between complexity and performance.

4 Conclusions

In the presence of spatially correlated channels, MIMO systems that use spatial multiplexing have their performance affected and experience a lower channel capacity than expected. In this work, two relay selection schemes within a cooperative wireless communication system were evaluated so as to deal with the correlated channels’ problem. Besides considering the trivial random selection scheme, a simple and distributed relay selection scheme,
allowing to write

\[
C_{\text{min}}' = \frac{1}{Q} \sum_{q=1}^{Q} \left[ \log_2 \left( 1 + \frac{SNR}{nT} \left( n^{-1} \Delta_T - \epsilon \right) \right) + \log_2 \left( 1 + \frac{SNR}{nT} \epsilon \right) \right] = \log_2 \left( 1 + nR \text{SNR} + \frac{SNR^2}{nT^2} \epsilon (nT nR - \epsilon) \right).
\]

(30)

From (28) and (30), and as it is assumed that \( C_{\text{min}}' < C_{\text{min}} \), the following result holds:

\[
C_{\text{min}}' < C_{\text{min}} \Rightarrow \left[ 1 + nR \text{SNR} + \frac{SNR^2}{nT^2} \epsilon (nT nR - \epsilon) \right] < [1 + nR \text{SNR}] \\
\Rightarrow \epsilon (nT nR - \epsilon) < 0 \\
\Rightarrow \epsilon < 0 \text{ } \lor \text{ } nT nR < .\]

(31)

Since \( \epsilon \) must be positive (c.f. (29)) and recalling that the eigenvalues are always non-negative (so \( \epsilon \) can never be greater than \( nT nR \), c.f. (29)), it follows that (31) is false. Hence, \( C_{\text{min}} \) is indeed the minimum channel capacity value.

The case in which the MIMO channel capacity achieves its maximum value corresponds to a radio channel which is completely decorrelated among the different MIMO links for all \( Q \) flat-fading subchannels. From an analytical point of view, all channel correlation matrices \( W_q \) (\( q = 1, \ldots, Q \)) will have full rank, with all \( n \) eigenvalues having the same value. Thus, and taking into account (26), the eigenvalues that allow to achieve the maximum channel capacity are given by

\[
y_{i,q} = \frac{nT nR}{n}, \text{ } \forall i, q.
\]

(32)

Applying this result to the channel capacity expression (12), it yields

\[
C_{\text{max}} = \frac{1}{Q} \sum_{q=1}^{Q} \sum_{i=1}^{n} \log_2 \left( 1 + \frac{SNR}{nT} \frac{nR}{n} \right) = \log_2 \left( \left( 1 + SNR \frac{nR}{n} \right)^n \right) = n \log_2 \left( 1 + SNR \frac{nR}{n} \right).
\]

(33)

To show that the channel capacity presented in (33) is indeed the maximum value, assume now that there exists a \( C_{\text{max}}' > C_{\text{max}} \) such that the eigenvalues for obtaining \( C_{\text{max}}' \) are given by (taking also into account (26))

\[
\begin{align*}
y_{i,q} &= \frac{nT nR}{n} - \delta, \text{ } i = 1 \\
y_{i,q} &= \frac{nT nR}{n} + \delta, \text{ } i = 2 \\
y_{i,q} &= \frac{nT nR}{n}, \text{ } i \neq \{1,2\}
\end{align*}
\]

allowing to write

\[
C_{\text{max}}' = \frac{1}{Q} \sum_{q=1}^{Q} \left[ \log_2 \left( 1 + \frac{SNR}{nT} \frac{nT nR}{n} - \delta \right) + \log_2 \left( 1 + \frac{SNR}{nT} \frac{nT nR}{n} + \delta \right) \right] + \sum_{i=3}^{n} \log_2 \left( 1 + \frac{SNR}{nT} \frac{nR}{n} \right)
\]

\[
= \log_2 \left[ \left( 1 + \frac{SNR}{nT} \frac{nR}{n} \right)^{n-2} \left( \frac{\delta}{nT} \right)^{2} \right].
\]

(35)

From (33) and (35), and as it is assumed that \( C_{\text{max}}' > C_{\text{max}} \), the following expression must hold:

\[
C_{\text{max}}' > C_{\text{max}} \Rightarrow \left[ 1 + SNR \frac{nR}{n} \right]^{n-2} \left( \frac{\delta}{nT} \right)^{2} > \left( 1 + SNR \frac{nR}{n} \right)^{n}. \]

(36)

Since all the variables in (36) are greater than zero, this expression is false. Hence, \( C_{\text{max}} \) is indeed the maximum channel capacity value.

Competing interests
The authors declare that they have no competing interests.

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