Associated production of the doubly-charged scalar pair with the Higgs boson in the Georgi–Machacek model at the ILC

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ABSTRACT

Besides the SM-like Higgs boson $h$, the Georgi–Machacek (GM) model predicts the existence of doubly-charged Higgs bosons $H_5^{±±}$ in the 5-plet representation, which can be seen the typical particles in this model. We first used the latest Higgs boson diphoton signal strength data to find the allowed region at $2\sigma$ confidence level on the plane of the scalar mass values $m_{H_+}$ and the triple scalar coupling parameter $b_{hHH}$, and then focus on the study of the triple Higgs production process $e^+e^-\rightarrow hH_5^{±±}$ at the future International Linear collider (ILC). Our numerical results show that, the values of the production cross section are very sensitive to the triple Higgs coupling strength $b_{hHH}$ and can reach the level several fb in the reasonable parameter space. Considering the same-sign diboson decay $H_5^{±±}\rightarrow W^±W^±$, the expected discovery reach at the future ILC experiments are also studied.

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1. Introduction

Although the observed properties of the 125 GeV Higgs boson [1] are in excellent agreement with the prediction of the Standard Model (SM) [2], it does not unquestionably mean that the Higgs sector in the SM is the unique choice. Such Higgs-like resonance can also be well explained in various new physic (NP) scenarios beyond the SM. One such scenario is the Georgi–Machacek (GM) model [3,4], which is an extension of the SM with a complex triplet of hypercharge $Y=1$ and a real triplet of $Y=0$ under the SM $SU(2)_L\times U(1)_Y$ gauge symmetry. With the triplet vacuum expectation values (VEVs), the neutrino masses can be generated through the type-II seesaw mechanism [5].

Besides the SM-like Higgs boson $h$, the GM model also predicts the existence of several Higgs multiplets under the custodial symmetry: one singlet ($H_1$), one triplet ($H_3^0$, $H_3^±$, and $H_3^{±±}$), and one quintet ($H_5^{±±}$, $H_5^0$, $H_5^±$). Recently, there have been many phenomenological studies about searching for these exotic scalars at colliders [6–9]. An important feature of the GM model is that the doubly-charged Higgs boson ($H_5^{±±}$) can lead to phenomenologically prominent and interesting signatures at colliders: decays into a pair of like-sign leptons or $W$ bosons, depending on the magnitude of $v_Δ$. On the other hand, the couplings between the SM-like Higgs boson $h$ and the weak gauge bosons can be stronger than their SM values as a result of mixing between the Higgs doublet and triplet fields [10–16] and lead to discriminative phenomena [17–19]. Very recently, the authors of Ref. [20] have studied whether the singlet Higgs boson in the custodial Higgs triplet model can serve as the candidate for the 750 GeV resonance reported by ATLAS and CMS experiments [21].

As mentioned above, the doubly-charged Higgs boson can be seen as the typical particles in the GM model. If a doubly charged Higgs boson is discovered at LHC, it will be crucial to determine its relevant couplings at the future high-energy linear colliders due to clean environment and high luminosity, such as the International Linear Collider (ILC) with a center-of-mass (c.m.) energy in the range of 500 to 1600 GeV [22]. In the GM model, the SM-like Higgs boson $h$ can couples with other Higgs bosons pairs via the triple scalar couplings $hSS$ and their values can be determined by other model parameters, as shown in Ref. [10]. In this paper, we only consider the triple scalar coupling $hH_5^{±±}H_5^{±±}$ and take its coupling strength as a free parameter. We concentrate on the study of the constraints between the coupling parameter and the doubly-charged Higgs boson mass by using the latest LHC Higgs diphoton signal strength and how one can test the GM model at the ILC.

The structure of this paper is as follows. In Sec. 2, we briefly review the GM model and review the current experimental constraints on the model parameters. In Sec. 3, we study the production cross section for the process $e^+e^-\rightarrow hH_5^{±±}H_5^{±±}$.

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Some phenomenological analyses and the detectable probability are also given. Finally, we give our conclusion in Sec. 4.

2. The GM model and the experimental constraints

2.1. Brief review on the model

We first give a concise introduction to the model. In the GM model [3,4], the scalar sector consists of is composed of an isospin doublet Higgs field $\phi$ with the hypercharge $Y = 1/2$, a real triplet field $\xi$ with $Y = 0$, and a complex triplet field, $\chi$, with $Y = 1$. These scalar fields can be expressed in the $SU(2)_L \times SU(2)_R$ covariant form as:

$$\Phi = \begin{pmatrix} \phi^0 \xi^0 \chi^0 \\ (\phi^+)^* \xi^+ \chi^+ \end{pmatrix}, \Delta = \begin{pmatrix} \chi^0 \xi^0 \\ (\chi^+)^* \xi^+ \end{pmatrix}.$$  \tag{1}

The neutral components can be parameterized as

$$\phi^0 = \frac{1}{\sqrt{2}} (\phi_0 + \phi_i i), \chi^0 = \frac{1}{\sqrt{2}} (\chi_i + i \xi_i), \xi^0 = \xi_i + \xi_r,$$  \tag{2}

where $\phi_0$, $\phi_i$, $\chi^0$, $\xi_i$, $\xi_r$, and $\xi_r$ are the vacuum expectation values (VEVs) of $\phi$, $\chi$ and $\xi$, respectively. In order to preserve the global $SU(2)_L \times SU(2)_R$ symmetry, one can take $\phi^0 \equiv v_\phi \equiv \nu_\Delta$. In this case, the electroweak $\rho$ parameter defined by $\rho_{\text{tree}} = m_W^2 (m_\gamma^2 \cos^2 \theta_W) = 1.0$, unity at tree level. The Fermi constant $G_F$ fixes the combination of VEVs, $v_H = v_\phi^2 + v_\chi^2 = 1/2 (2G_F) = (246 \text{ GeV})^2$.

After symmetry breaking, the physical scalar fields can be organized by their transformation properties under the custodial $SU(2)_V$ symmetry into a 5-plet, i.e., $H_5 = (H_{5+}, H_{5-}, H_3^0)$, a 3-plet, i.e., $H_3 = (H_{3+}, H_{3-}, H_3^0)$, and two singlets ($h$ and $H_1$), where $h$ can be seen the discovered 125 GeV Higgs boson. Because of the $SU(2)_V$ invariance, different charged Higgs boson states belonging to the same $SU(2)_V$ multiplet are degenerate in mass. Furthermore, the latter two singlets can mix with each other with mixing angles $\theta_{hi}$ and $\alpha$. The mixing angle $\theta_{hi}$ is defined by

$$c_{hi} = \cos \theta_{hi} = \frac{\phi_{hi}}{v}, \quad s_{hi} = \sin \theta_{hi} = \frac{2 \sqrt{2} v_A}{v},$$  \tag{3}

while $\alpha$ is determined by the quartic coupling constants in the Higgs potential. For simplicity, we here take $\sin \alpha = 0$ and think that the SM-like Higgs boson $h$ composed entirely of the $SU(2)_L$ doublet.

The explicit forms of the Higgs potential and the masses of these scalars can be found, for example, in Refs. [6,10]. The Higgs potential contains four dimensionful ($\mu_1$, $\mu_2$, $M_1$ and $M_2$) and five dimensionless parameters ($\lambda_1-\lambda_5$) and these parameters are independent of the VEV, but they can be constrained by the perturbative unitarity and vacuum stability [6,10,23], the oblique parameters ($S$, $T$, $U$, $R_0$, and $B$-meson observables [6,14–16,24,25]). For the triplet VEV, the strongest of the indirect experimental bounds arises from measurements of $b \to s \gamma$ [25], which constrain the triplet VEV $v_\Delta \leq 65 \text{ GeV}$ ($s_H \leq 0.75$).

The double-charged Higgs boson can be searched using the like-sign dilepton and diboson modes. At the LHC 7 TeV, $\mathcal{L}_{\text{int}} = 4.7 \text{ fb}^{-1}$ and assuming $\text{Br}(H^{\pm \pm} \rightarrow \ell^+ \ell^-) \approx 100\%$, the lower mass limit on $H^{\pm \pm}$ is about 409 GeV for ATLAS, and 459 GeV for CMS, respectively [26]. At the LHC 8 TeV and $\mathcal{L}_{\text{int}} = 20.3 \text{ fb}^{-1}$, ATLAS has recently pushed the most stringent lower limit up to 550 GeV in the $\ell^+ \ell^-$ channel [27]. The search for $H^{\pm \pm}$ based on the diboson channel has been studied in Ref. [28], where they concluded that the lower bound on mass of $H^{\pm \pm}$ can be derived from the like-sign dilepton limit, which is about 60 GeV for LHC 7 TeV and extends to about 84 GeV for LHC 8 TeV. Ref. [29] has studied the possibility for searching the doubly charged Higgs bosons via the signal of dilepton + missing energy + jets and find that the GM model can be ultimately verified or ruled out at the LHC with 14 TeV in a clean weak boson fusion (WBF) selection. Very recently, the ATLAS like-sign WW/ZZ cross-section measurement, excludes a doubly-charged Higgs $H_\pm^\pm$ with masses in the range $140 \leq m_\pm \leq 400 \text{ GeV}$ at $s_{\text{ATL}} = 0.5$, and $100 \leq m_\pm \leq 700 \text{ GeV}$ at $s_{\text{ATL}} = 1$, under the assumption of $\text{Br}(H_\pm^\pm \rightarrow W^+ W^-) = 100\%$ [30]. However, when we take the typical value for the triplet VEV as $v_\Delta = 1 \text{ GeV}$ ($s_H = 0.011$), these can be relaxed and the masses of these scalars can be safely in the range of several hundreds GeV.

2.2. Constraints from the LHC Higgs diphoton data

As we know, the decay $h \rightarrow \gamma \gamma$ played a central role in the discovery of the SM higgs boson. While the tree-level couplings of $h h V V$ ($V = W, Z$) and $h f f$ in the GM model are deviated from those of the SM Higgs boson. In the limit of $\sin \alpha = 0$, the ratios of the $h h V V$ and $h f f$ couplings to their respective SM values are given by

$$c_{hVV} = c_H, \quad c_{hff} = \frac{1}{c_H^2}.\tag{4}$$

On the other hand, the charged scalars $H^{\pm}$, $H_{3}^{\pm}$ and $H_{5}^{\pm}$ propagating in the loop can also give sizable contributions on the diphoton decay mode. The triple-scalar coupling involving the SM-like Higgs $h$ are given by

$$\lambda_{hH^{\pm}H_{3}^{\pm}} = -\lambda_{hH_{3}^{\pm}H_{5}^{\pm}} = g_{hHHV},\tag{5}$$

$$\lambda_{hH_{3}^{\pm}H_{5}^{\pm}} = -\frac{1}{c_H^2} (2c_H^2 m_h^2 + s_H^2 m_h).\tag{6}$$

Note that in Eq. (5), the coupling parameter $g_{hHHV}$ can also be parameterized as other forms including the different model parameters: $\lambda_3-\lambda_5$ and $M_2$, as shown in the appendix (A1) of [10]. For $\sin \alpha = 0$, the coupling strength of $hH^{\pm}H_{3}^{\pm}$ can be determined by $(4\lambda_2 + \lambda_5) v_\phi$. Considering the constraints of the perturbative unitarity and vacuum stability, the following maximum range can be obtained, as shown in Ref. [10]:

$$\lambda_2 \in (-\frac{2}{3}, \frac{2}{3}), \quad \lambda_5 \in (-\frac{8}{3}, \frac{8}{3}).\tag{7}$$

Here we mainly consider the three-scalar coupling for $hH_{3}H_{5}$ and thus only take $g_{hHHH}$ as one free parameter. For $v_\phi \approx v$, the maximum range from unitarity can be obtained as

$$g_{hHHH} \in (-\frac{16}{3}, \frac{16}{3}) \approx (-16.7, 16.7).\tag{8}$$

The LHC diphoton rate of Higgs boson in the GM model normalized to the SM prediction can be written as

$$R_{\gamma\gamma} = \frac{\sigma_{GM}(pp \rightarrow h)}{\sigma_{SM}(pp \rightarrow h)} \times \frac{\text{Br}_{GM}(h \rightarrow \gamma \gamma)}{\text{Br}_{SM}(h \rightarrow \gamma \gamma)} \approx \frac{\Gamma_{GM}(h \rightarrow \gamma \gamma)}{\Gamma_{SM}(h \rightarrow \gamma \gamma)}.\tag{9}$$

The corresponding expressions are given in the Appendix A.

The current signal strength of the diphoton decay channel $R_{\gamma\gamma}$ is $1.17 \pm 0.27$ at ATLAS [31] and $1.12^{+0.37}_{-0.32}$ at CMS [32]. Both of them are consistent with SM at 1σ level, but still have a relatively large uncertainty. In Fig. 1, we show the allowed region on
doubly-charged Higgs bosons mass $m_H$ and the coupling parameter $g_{hHH}$ with $v_A = 1$ GeV at 2σ level for the ATLAS and CMS results, respectively. In our numerical estimation, we find that the contribution from the singly-charged scalars $H_5^{\pm}$ are much smaller than that for the charged scalars in the 5-plet, and thus we safely take $m_{H_5} = m_{H_5}$. On the other hand, the constraint is stronger than that for the perturbative unitarity and vacuum stability for the light doubly charged Higgs bosons. For example, when we take $m_{H_5} = 200$ GeV, $250$ GeV and $300$ GeV, the absolute value of $g_{hHH}$ should be approximately less than 3, 4 and 6, respectively.

3. Numerical results and discussions

3.1. The production cross section

From above discussions, we can see that the doubly-charged scalars can be associated produced with the SM-like Higgs $h$ via the processes $e^+e^- \rightarrow hH_5^{\pm}H_5^{\mp}$ at the ILC. The relevant Feynman diagrams are shown in Fig. 1.

![Fig. 1](Image)

Fig. 1. (Colour on-line.) The allowed region of $R_{r\gamma}$ on $(m_H, g_{hHH})$ with $v_A = 1$ GeV and for (a) ATLAS, and (b) CMS at 2σ level.

![Feynman diagrams](Image)

Fig. 2. Feynman diagrams of the process $e^+e^- \rightarrow hH_5^{\pm}H_5^{\mp}$ in the GM model.

The SM input parameters relevant in our calculation are taken as [33]:

$$
M_W = 80.385 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}, \quad \Gamma_Z = 2.495 \text{ GeV}.
$$

The SM-like Higgs boson mass is taken as $m_h = 125.09$ GeV [34]. The free parameters involved in our calculations are the masses of doubly-charged scalars $m_H$ and the triple scalar coupling parameter $g_{hHH}$. The calculation of the cross sections is performed using the CalcHEP [35] and checked with MadGraph5-aMC@NLO [36] at the leading order (LO), where the model files are combined with the FeynRules package [37].

In Fig. 3, we plot the production cross sections $\sigma$ as a function of $m_H$ for $m_h = 125$ GeV and various of $g_{hHH}$ as indicated for two proposed center-of-mass (c.m.) energy: $\sqrt{s} = 1.0$ TeV and 1.5 TeV, respectively. One can see from Fig. 3 that: (i) The values of the production cross section are very sensitive to the coupling parameter $g_{hHH}$ and increase with the increasing $g_{hHH}$. This is because in the Fig. 2 (a–b), the cross section is mainly proportional to
3.2. Observability of the process

For the degenerate case for the Higgs masses, the possible decays of the doubly charged Higgs boson \( H^{\pm\pm} \) are the same-sign dilepton channel and the same-sign diboson channel. As shown in [6], the dominant decay channel is mainly dependent on the magnitude of \( v_\Delta \). In Fig. 4, we plot the branching ratio of \( Br(H_5^{\pm\pm} \to W^{\pm}W^{\pm}) \) versus the value of \( v_\Delta \) for three typical values of \( m_H = 200, 400 \) and 600 GeV, respectively. We can see that the value of \( Br(H_5^{\pm\pm} \to W^{\pm}W^{\pm}) \) increases with \( m_H \) increasing in the moderate region for \( v_\Delta \). However, for \( v_\Delta \geq 0.01 \) GeV, the decay branching ratio of \( H_5^{\pm\pm} \) is \( Br(H_5^{\pm\pm} \to W^{\pm}W^{\pm}) \simeq 100\% \) and our result is consistent with that in Refs. [6,42]. Thus in the following analysis, it is safely to take \( Br(H_5^{\pm\pm} \to W^{\pm}W^{\pm}) = 1 \) for \( v_\Delta = 1 \) GeV.

![Graph](image)

**Fig. 4.** The branching ratios of \( Br(H_5^{\pm\pm} \to W^{\pm}W^{\pm}) \) versus \( v_\Delta \) for three typical values of \( m_H = 200 \) (solid), 400 GeV (dashed) and 600 GeV (dotted), respectively.

Next, we turn to event generation and simulation of the process

\[ e^+e^- \to h(\to b\bar{b})H^{++}(\to W^+W^+) \]

\[ \to (\ell^+\ell^-\nu\bar{\nu})H^{--}(\to W^-W^- \to jjjj) \]

where \( \ell = e, \mu \). The signal and background events are generated at leading order using MadGraph5-aMC@NLO. PYTHIA [38] is utilized for parton shower and hadronization.Delphes 3 [39] is then employed to account for the detector simulations where the (mis-)tagging efficiencies and fake rates are assumed to be their default values. The anti-k_{t} algorithm [40] with the jet radius of 0.4 is used to reconstruct jets. Finally we use MadAnalysis5 [41] for analysis. Here we take the input parameters as \( g_{Hhh} = 1 \) and \( \sqrt{s} = 1 \) TeV. The above numbers are obtained after taking the following basic kinematic cuts.
where $p_T$ and $\eta$ are the transverse momentum and the pseudorapidity of final particles. $\Delta R_{ij} = \sqrt{(\Delta \phi_{ij})^2 + (\Delta \eta_{ij})^2}$ (i and j running over all particles in the final state) is the particle separation with $\Delta \phi_{ij}$ and $\Delta \eta_{ij}$ being the separation in the azimuth angle and rapidity respectively.

In Fig. 5, we present the normalized invariant mass distribution of $M_{bb}$ for signal and backgrounds. As can be seen the invariant mass distribution peaks at the Higgs boson mass for signal events while backgrounds have wide distributions. As a result, applying a mass window cut can reduce the backgrounds contributions. We require the reconstructed invariant mass of the Higgs boson to satisfy

$$90 \text{ GeV} < M_{bb} < 150 \text{ GeV}. \quad (13)$$

Fig. 6 shows the invariant mass $M_{\ell^+\ell^+}$ for the $\ell^+\ell^+$ system (left panel) and the transverse mass $M_T$ (right panel) distributions for $\ell^+\ell^+E_T^{\text{miss}}$ system, where the transverse mass $M_T$ is defined as

$$M_T^2 = (\sqrt{(p_{T,i} + p_{T,j})^2 + (\mathbf{p}_{T,i} + \mathbf{p}_{T,j})^2 + E_T^{\text{miss}}})^2$$

$$- |\mathbf{p}_{T,i} + \mathbf{p}_{T,j} + E_T^{\text{miss}}|^2,$$ 

where $\mathbf{p}_{T,i}$ are the transverse momentums of the charged leptons and $E_T^{\text{miss}}$ is the missing transverse momentum determined by the negative sum of visible momenta in the transverse direction. One can see that the suitable cuts on the invariant mass $M_{\ell^+\ell^+}$ and the transverse mass $M_T$ are also useful to measure the doubly charged Higgs bosons mass $m_H$.

Table 1 summarizes the cross sections for the signal and backgrounds after applying the basic cuts and the reconstructed Higgs boson mass cut, respectively. One can see that the SM backgrounds can be reduced efficiently and at the level of $10^{-4}$ fb. Furthermore, the cross section of signal is directly proportional to the coupling strength $g_{HH}$. For $g_{HH} = 1$ and $m_H = 200$ GeV, there will be only two signal events survive at the ILC with an integrated luminosity of 1000 fb$^{-1}$ after the basic cuts. Thus, we here do not apply other cuts on the signal and backgrounds and the high integrated luminosity is needed to detect more signals at the future ILC experiments.

In order to discuss the observation of doubly charged Higgs bosons, we need to calculate the statistical significance (SS) as $S/\sqrt{S+B}$ where $S$ and $B$ denote the number of the signal and background events, respectively. In Fig. 7, we plot the lowest necessary luminosity with $3\sigma$ observation limits depending on the absolute values of $g_{HH}$ at $\sqrt{s} = 1$ TeV for three typical values of $m_H$, where the absolute values of $g_{HH}$ are according with the constraints of the perturbative unitarity and the current LHC Higgs dates. One can see that the high integrated luminosity is needed for probing these signals due to the small production rates. In the reasonable parameter spaces, the signal can be detected via this process at the future ILC experiments, even under the basic cuts for the detector. Otherwise, if no signal is observed, it means that triple scalar coupling strength $g_{HH}$ might be very small.

4. Conclusion

The Georgi–Machacek model is one of the attractive new physics models, which predicts the existence of the doubly-charged Higgs bosons $H^{\pm\pm}$ and their masses can be in the range of several hundreds GeV. In this letter, we first used the latest Higgs boson diphoton signal strength data to find the allowed region at $2\sigma$ confidence level on the plane of the scalar mass values $m_H$ and the triple scalar coupling parameter $g_{HH}$, and then examined the possibility of detecting the triple Higgs bosons ($hH^+_5 + H^-_5$) signal of two like-sign lepton ($e$ or $\mu$) + two $b$-jets + four jets + missing $E_T^{\text{miss}}$ at the ILC. The numerical results show that:

1. For the light doubly charged Higgs bosons, the constraints from the current LHC Higgs diphoton datas are stronger than those from the perturbative unitarity and vacuum stability. For example, when we take $m_H = 200$ GeV and 300 GeV, the
upper limits for the absolute values of coupling strength are about 3 and 6, respectively.

2. The production cross section of the process \( e^+e^- \rightarrow hH^+_5 \rightarrow W^\pm W^\pm \rightarrow \ell^\pm \nu \ell^\pm v \) is very sensitive to the triple coupling parameter \( g_{HHH} \) and increases with the increasing \( g_{HHH} \). Considering the subsequent decay of \( H^+_5 \rightarrow W^\pm W^\pm, W^\pm \rightarrow \ell^\pm \nu, \ell^\pm v \) and \( h \rightarrow bb \), the characteristic signal of \( hH^+_5 \rightarrow H^-_5 \) might be two like-sign lepton (\( e \) or \( \mu \)) plus two \( b \)-jets and four jets with missing \( E_T \).

In the allowed parameter space, the production rates can exceed the 3\( \sigma \) sensitivity of at the future ILC experiments with high integrated luminosity.

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Appendix A. The expressions for \( \Gamma(h \rightarrow \gamma \gamma) \) in the GM model

In the GM model, the charged fermion (\( f \)), gauge boson (\( W \)) and scalar (\( s \)) can contribute to the decay width \( h \rightarrow \gamma \gamma \), which are given by [42]

\[
\Gamma(h \rightarrow \gamma \gamma) = \frac{\alpha^2 m_h^3}{256 \pi^5 v^2} \left| \sum_f N_f^2 \frac{V_{hff}^2}{2 m_f^2} A_1(\tau_f) \right|^2 \tag{A.1}
\]

where \( \tau_f = m_h^2/4m_f^2 \), \( \lambda_f = m_f^2/4m_f^2 \), \( Q_W = 1 \), \( Q_f^s = c_s^2 \). \( Q_f^s \) are the electric charges of fermion and scalar. \( N_f^2 \) is the color factor for fermion \( f \). \( Q_f^L = I^L_f = I^L_f (Q_f, 3) - Q_f (Q_f, 3) \) with \( I^L_f (Q_f, 3) \) being the third isospin components of chiral fermions (scalar). \( \lambda_{hss} \) is the coupling constant of \( hss \). The loop functions \( A_1(\tau_f) \) in Eq. (A.1) are defined as

\[
A_0(\tau) = -[\tau - f(\tau)] \tau^{-2}, \quad A_{1/2}(\tau) = 2[\tau + (\tau - 1)f(\tau)] \tau^{-2}.
\]

with the functions \( f(\tau) \) given by

\[
f(\tau) = \begin{cases} 
(\sin^{-1} \sqrt{\tau})^2, & \tau \leq 1 \\
\frac{1}{4} \log \frac{1 + \sqrt{\tau - 1}}{1 - \sqrt{\tau - 1}} - \pi^2, & \tau > 1.
\end{cases}
\]
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