Is there a bound $^{3}\Lambda n$?

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Abstract

The HypHI Collaboration at GSI argued recently for a $^{3}\Lambda n$ ($\Lambda nn$) bound state from the observation of its two-body $t+\pi^-$ weak-decay mode. We derive constraints from several hypernuclear systems, in particular from the $A=4$ hypernuclei with full consideration of $\Lambda N \leftrightarrow \Sigma N$ coupling, to rule out a bound $^{3}\Lambda n$.

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1. Introduction

The lightest established $\Lambda$ hypernucleus is known since the early days of hypernuclear physics to be $^{3}\Lambda H^{T=0}$, in which the $\Lambda$ hyperon is weakly bound to the $T=0$ deuteron core, with ground-state (g.s.) separation energy $B_{\Lambda}(^{3}\Lambda H)=0.13\pm0.05$ MeV and spin-parity $J^P=\frac{1}{2}^+$. There is no evidence for a bound spin-flip partner with $J^P=\frac{3}{2}^+$. For a brief review on related results deduced from past emulsion studies of light hypernuclei, see Ref. [1].

As for $^{3}\Lambda H^{T=1}$, given the very weak binding of the $\Lambda$ hyperon in the $T=0$ g.s., and that the $T=1\,NN$ system is unbound, it is unlikely to be particle stable against decay to $\Lambda+p+n$. Similarly, assuming charge independence, $\Lambda nn$ is not expected to be particle stable. As early as 1959 just six years following the discovery of the first $\Lambda$ hypernucleus, it was concluded by Downs and Dalitz upon performing variational calculations of both $T=0,1\,\Lambda NN$
systems that the isotriplet \((^3\Lambda, ^3\Lambda H_T=1, ^3\Lambda He)\) hypernuclei do not form bound states \([2]\). This issue was revisited in Refs. \([3, 4, 5]\) using various versions of Nijmegen hyperon-nucleon \((YN)\) potentials within \(\Lambda NN\) Faddeev equations for states with total orbital angular momentum \(L = 0\) and all possible values of total angular momentum \(J\) and isospin \(T\). Again, no \(\Lambda nn\) bound state was found in any of these studies as long as \(^3\Lambda H_T=0(J^P = \frac{1}{2}^+)\) was only slightly bound. Similar conclusions were reached in Refs. \([6, 7, 8]\) based on chiral constituent quark model \(YN\) interactions, and in Ref. \([9]\) based on recently constructed NLO chiral EFT \(YN\) interactions \([10]\). Note that \(\Lambda N \leftrightarrow \Sigma N\) coupling was fully implemented in the more recent \(^3\Lambda n\) studies \([4, 5, 6, 7, 8, 9]\). A more general discussion of stability vs. instability for \(^3\Lambda n\) in the context of neutral hypernuclei with strangeness \(-1\) and \(-2\) has been given very recently in Ref. \([11]\).

A claim for particle stability of \(^3\Lambda n\) has been made recently by the HypHI Collaboration \([12]\) observing a signal in the \(t + \pi^-\) invariant mass distribution following the bombardment of a fixed graphite target by \(^6\text{Li}\) projectiles at 2\(A\) GeV in the GSI laboratory. The binding energy of the conjectured weakly decaying \(^3\Lambda n\) is \(0.5\pm1.1\pm2.2\) MeV, with a large standard deviation \(\sigma=5.4\pm1.4\) MeV. As noted above, there is unanimous theoretical consensus based on \(\Lambda NN\) bound-state calculations that \(^3\Lambda n\) cannot be particle stable. However, possible connections to other hypernuclear systems, in particular the \(A=4\) bound isodoublet hypernuclei \((^4\Lambda H, ^4\Lambda He)\), need to be explored. The present work addresses this issue by establishing connections that make it clear why a bound \(^3\Lambda n\) cannot be accommodated into hypernuclear physics.

Assuming charge-symmetric \(\Lambda N\) interactions, \(V_{\Lambda p} = V_{\Lambda n}\), we demonstrate some unacceptable implications of a bound \(^3\Lambda n\) to \(\Lambda p\) scattering in Sect. \([2]\) and to \(^3\Lambda H_T=0\) in Sect. \([3]\). Consequences of \(A = 4\) hypernuclear spectroscopy with full consideration of charge-symmetric \(\Lambda N \leftrightarrow \Sigma N\) couplings are derived for \(^3\Lambda n\) in Sect. \([4]\) by applying methods that differ from those used in the combined analysis of \(A = 3\) and \(A = 4\) hypernuclei by Hiyama et al. \([5]\), reaffirming that \(^3\Lambda n\) is unbound. Our results are discussed and summarized in Sect. \([5]\) with additional remarks made on the possible role of charge-symmetry breaking (CSB) and \(\Lambda NN\) interaction three-body effects, concluding that a bound \(^3\Lambda n\) interpretation of the \(t + \pi^-\) signal in the HypHI experiment is outside the scope of present-day hypernuclear physics.
2. $^3\Lambda n$ vs. $\Lambda p$ scattering

To make a straightforward connection between the low-energy $\Lambda N$ scattering parameters and the three-body $\Lambda NN$ system we follow the method of Ref. [3] in solving $YNN$ Faddeev equations with two-body $YN$ input pairwise separable interactions constructed directly from given low-energy $YN$ scattering parameters. For simplicity we neglect in this section the spin dependence of the low-energy $\Lambda N$ scattering parameters, setting $a_s = a_t$ for the scattering length and $r_s = r_t$ with values $r=2.5$ or 3.5 fm for the effective range, spanning thereby a range of values commensurate with most theoretical models and also with the analysis of measured $\Lambda p$ cross sections at low energies [13]. By using Yamaguchi form factors within rank-one separable interactions, we then compute critical values of scattering length $a$ required to bind successively the $T = 0$ and $T = 1 \Lambda NN$ systems, with results shown in Table 1.

Table 1: Values of the spin-independent $\Lambda N$ scattering length $a$ required to bind $T = 0$ and $T = 1 \Lambda NN$ states as indicated, for two representative values of the spin-independent effective range $r$, and calculated values of the $\Lambda p$ total cross section at $p_\Lambda = 145$ MeV/c. The measured value at the lowest momentum bin available is $\sigma_{\Lambda p}^{tot}(p_\Lambda = 145\pm 25$ MeV/c$)=180\pm 22$ mb [13]. Calculated values of $B_\Lambda(^3\Lambda H T=0)$ are listed in the last column for $\Lambda N$ interactions that just bind $^3\Lambda n$, in contrast to $B_\Lambda^{exp}(^3\Lambda H)$=0.13$\pm 0.05$ MeV.

| $r$ (fm) | $a$ (fm) | $\sigma_{\Lambda p}^{tot}$ (mb) | $a$ (fm) | $\sigma_{\Lambda p}^{tot}$ (mb) | $\sigma_{\Lambda p}$ (mb) | $B_\Lambda^{T=0}$ (MeV) |
|---------|---------|-----------------|---------|-----------------|-----------------|-----------------|
| 2.5     | -1.185  | 129.7           | -1.498  | 192.5           | -4.491          | 953.8           | 2.59            |
| 3.5     | -1.405  | 152.4           | -1.895  | 239.7           | -5.930          | 943.1           | 1.74            |

Exceptionally large values of $\Lambda N$ scattering lengths are seen to be required to bind $^3\Lambda n$, and the low-energy $\Lambda p$ cross sections thereby implied exceed substantially the measured cross sections as shown by the $\Lambda N$ cross sections evaluated at the lowest momentum bin reported in Ref. [13]. Of the three $B_\Lambda$ values tested in the table, only $B_\Lambda^{T=0}=0.13$ MeV is consistent with the reported $\Lambda p$ cross sections, including their uncertainties. In the last column of the table we also listed the $\Lambda$ separation energies in $^3\Lambda H$ that result once $^3\Lambda n$ has just been brought to bind. These calculated values are much too big to be reconciled with $B_\Lambda^{exp}(^3\Lambda H)$=0.13$\pm 0.05$ MeV.
3. $^3\Lambda n$ vs. $^3\Lambda H$

The $^3\Lambda n$ vs. $^3\Lambda H$ discussion in this section is limited to using $s$-wave $\Lambda N$ effective interactions, providing a straightforward extension of earlier studies [2, 3]. Effects of possibly substantial $\Lambda N \leftrightarrow \Sigma N$ coupling, as generated by strong one-pion exchange in Nijmegen meson-exchange potentials [14] and in recent chirally based potentials [10], are discussed in Sect. 4.

Table 2: $^3\Lambda n$ vs. $^3\Lambda H$ discussion in this section is limited to using $s$-wave $\Lambda N$ effective interactions, providing a straightforward extension of earlier studies [2, 3]. Effects of possibly substantial $\Lambda N \leftrightarrow \Sigma N$ coupling, as generated by strong one-pion exchange in Nijmegen meson-exchange potentials [14] and in recent chirally based potentials [10], are discussed in Sect. 4.

| $x$ | $^3\Lambda n$ FD($E = 0$) | $B_{\Lambda}(3\Lambda H^T=0;\frac{1}{2}^+)$ | $B_{\Lambda}(3\Lambda H^T=0;\frac{3}{2}^+)$ |
|-----|-----------------|-----------------|-----------------|
| 1.00 | 0.55 | 0.096 | unbound |
| 1.10 | 0.47 | 0.147 | 0.124 |
| 1.20 | 0.39 | 0.211 | 0.448 |
| 1.30 | 0.31 | 0.288 | 0.986 |
| 1.40 | 0.21 | 0.381 | 1.704 |
| 1.50 | 0.12 | 0.488 | 2.598 |
| 1.60 | +0.015 | 0.612 | 3.659 |
| 1.61 | +0.004 | 0.625 | 3.772 |
| 1.62 | −0.006 | 0.638 | 3.890 |

Following Ref. [3] we solve Faddeev equations for $^3\Lambda n$ and $^3\Lambda H$ using simple Yamaguchi separable $s$-wave interactions fitted to prescribed input values of singlet and triplet scattering lengths $a$ and effective ranges $r$, thereby relaxing the spin-independence assumption of the preceding section. Of the four Nijmegen interaction models A,B,C,D studied there, only C reproduces the observed binding energy of $^3\Lambda H$, binding also the $\frac{3}{2}^+$ spin-flip excited state just 11 keV above the $\frac{1}{2}^+$ g.s. To get rid of this excited state, we have slightly changed the input parameters of model C. In this model, denoted C', the input $\Lambda N$ low-energy parameters are (in fm):

$$a_s = -2.03, \quad r_s = 3.66, \quad a_t = -1.39, \quad r_t = 3.32.$$  \hspace{1cm} (1)

The $^3\Lambda H^T=0(J^P=\frac{1}{2}^+, \frac{3}{2}^+)$ separation energies obtained by solving the appropriate $\Lambda NN$ Faddeev equations are listed in Table 2. The row marked $x = 1$
corresponds to using $\Lambda N$ interaction based on the low-energy parameters Eq. (1), and subsequent rows correspond to multiplying the $\Lambda N$ triplet interaction $V_t$ by $x > 1$ in order to bind $^3\Lambda n$ ($^3\Lambda H^{T=1}$).

Inspection of Table 2 shows that while the $\Lambda$ separation energies increase upon varying $x$, a by-product of this increase is that $^3\Lambda H^{T=0}(\frac{3}{2}^+)$ quickly overtakes $^3\Lambda H^{T=0}(\frac{1}{2}^+)$ becoming $^3\Lambda H$ g.s. This is understood by observing that the weights with which $V_t$ and the singlet interaction $V_s$ enter a simple folding expression for the $\Lambda$–core interaction are given by

$$J^P = \frac{1}{2}^+ : (T + \frac{1}{2}) V_t + (\frac{3}{2} - T) V_s, \quad J^P = \frac{3}{2}^+ : 2V_t,$$

so that $V_t$ is the only $\Lambda N$-interaction component affecting $^3\Lambda H^{T=0}(\frac{3}{2}^+)$ besides being more effective in binding $^3\Lambda n$ than binding $^3\Lambda H^{T=0}(\frac{1}{2}^+)$. Subsequently, beginning with $x = 1.614$, $^3\Lambda n$ becomes bound as indicated by the corresponding Fredholm determinant at $E = 0$ going through zero. Note that the $(2J+1)$-averaged $B^{\Lambda=0}(^3\Lambda H)$ is then $\approx 2.76$ MeV, in rough agreement with the spin-independent analysis of the previous section (cf. first row in Table 1). Similar results are obtained when replacing the parameters (1) of model C’ by those of model C, used in Ref. [3], and repeating the procedure summarized in Table 2. A bound $^3\Lambda n$ is therefore in strong disagreement with the binding energy $B^{\exp}(^3\Lambda H) = 0.13 \pm 0.05$ MeV determined for $^3\Lambda H$ g.s. and with its spin-parity $J^P = \frac{1}{2}^+$.

4. $^3\Lambda n$ vs. $^4\Lambda H$

$\Lambda N \leftrightarrow \Sigma N$ coupling cannot be ignored in quantitative calculations of $\Lambda$ hypernuclear binding energies. One-pion exchange induces a strong coupling in the $YN^3S_1 - ^3D_1$ channel which dominates the effective $V_t$ contribution in $^3\Lambda H$ three-body calculations, independently of whether using NSC97-related $YN$ interactions as in Refs. [4, 5] or NLO chiral $YN$ interactions in Ref. [15]. In the $YN^1S_0$ channel, in contrast, $\Lambda N \leftrightarrow \Sigma N$ coupling is weak. Here we employ $G$-matrix $0s_Y0s_Y$ effective interactions devised by Akaishi et al. [16] from the Nijmegen soft-core interaction model NSC97 and used in binding energy calculations of the $A = 4, 5 \Lambda$ hypernuclei. Of particular significance in the present context is the $\approx 1.1$ MeV splitting of the $0^+_{g.s.}, 1^+_{\text{exc}}$ spin-doublet levels in the isodoublet hypernuclei $^4\Lambda H - ^4\Lambda He$ which cannot be reconciled with theory without substantial $\Lambda N \leftrightarrow \Sigma N$ contribution. These $0s_Y0s_Y$ effective interactions were extended by Millener to the $p$ shell and tested there.
successfully in a comprehensive analysis of hypernuclear $\gamma$-ray measurements \cite{17}. For a recent application to neutron-rich hypernuclei, see Ref. \cite{18}. The $0s_N0s_Y$ $\Lambda N \leftrightarrow \Sigma N$ effective interaction $V_{\Lambda \Sigma}$ assumes a spin-dependent central interaction form

$$V_{\Lambda \Sigma} = (\bar{V}_{\Lambda \Sigma} + \Delta_{\Lambda \Sigma} \bar{s}_N \cdot \bar{s}_Y) \sqrt{4/3} \bar{t}_N \cdot \bar{t}_{\Lambda \Sigma},$$

where $\bar{t}_{\Lambda \Sigma}$ converts a $\Lambda$ to $\Sigma$ in isospace, with matrix elements

$$\bar{V}_{\Lambda \Sigma} = 2.96(3.35) \text{ MeV}, \quad \Delta_{\Lambda \Sigma} = 5.09(5.76) \text{ MeV}$$

derived from the Nijmegen model version NSC97e (NSC97f) as given in Ref. \cite{18} (Ref. \cite{19}). As for the diagonal $0s_N0s_Y$ interactions, we will constrain the spin-dependent $\Lambda N$ interaction $\Delta_{\Lambda \Lambda}$ matrix elements by fitting, together with $\bar{V}_{\Lambda \Sigma}$ and $\Delta_{\Lambda \Sigma}$, to the excitation spectrum of the $A = 4$ hypernuclei. Finally, the detailed properties of the $\Sigma N$ interaction hardly matter in view of the large energy denominators of order $M_{\Sigma} - M_{\Lambda} \approx 80 \text{ MeV}$ with which they appear. The binding-energy contribution arising from $V_{\Lambda \Sigma}$ is then given to a good approximation schematically by $|\langle V_{\Lambda \Sigma} \rangle|^2/80$ (in MeV).

The nonvanishing matrix elements of the spin-independent term in Eq. (3) are given in closed form by

$$\langle (J_N T, s_{\Sigma t_{\Sigma}})JT | V_{\Lambda \Sigma} (\Delta_{\Lambda \Sigma} = 0) | (J_N T, s_{\Lambda t_{\Lambda}})JT \rangle = \sqrt{4T(T + 1)/3} \bar{V}_{\Lambda \Sigma},$$

where $s_{\Sigma} = s_{\Lambda} = \frac{1}{2}$, $t_{\Sigma} = 1$, $t_{\Lambda} = 0$. This term is diagonal in the nuclear core, specified here by its total angular momentum $J_N$ and isospin $T$, with matrix elements that resemble the Fermi matrix elements in $\beta$ decay of the core nucleus. Similarly, matrix elements of the spin-spin term in Eq. (3) involve the SU(4) generator $\sum_j \bar{s}_{Nj} \bar{t}_{Nj}$ for the core, connecting core states with large Gamow-Teller transition matrix elements. A complete listing of these $\Lambda N \leftrightarrow \Sigma N$ Fermi and Gamow-Teller matrix elements together with corresponding $\Lambda N$ spin-spin matrix elements for the $A = 3, 4$ $\Lambda$ hypernuclei is given in the first three rows of Table \ref{table:3} and the resulting binding-energy contributions arising from $V_{\Lambda \Sigma}$ are listed in the last two rows, including two-body as well as three-body terms.

The last two columns of the table list matrix elements and binding-energy contributions for the $A = 4$ states, marked here by $^{4}_\Lambda \text{H}$. Fermi and Gamow-Teller contributions are added coherently because both $\bar{V}_{\Lambda \Sigma}$ and $\Delta_{\Lambda \Sigma}$ connect to the same and only spin-isospin SU(4) $0s_N0s_Y$ intermediate state available.
Table 3: Nonvanishing $\Lambda N$ spin-spin matrix elements as well as Fermi (F) and Gamow-Teller (GT) nonvanishing matrix elements of $V_{\Lambda\Sigma}$, Eq. (3), are listed in the first three rows for $^3\Lambda H(T, J^P)$ and $^4\Lambda H(T, J^P)$ $0\Lambda$ states. Estimates of the total $\Lambda\Sigma$ contributions to binding energies, using the NSC97e parameter values [4], are given in MeV in the last two rows. Note: $\Delta_{\Lambda\Lambda}$ is positive for binding-energy contributions.

\[
\begin{array}{cccccc}
\Lambda N \times \Delta_{\Lambda\Lambda} & ^3\Lambda H(0, {1\over 2}^+) & ^3\Lambda H(0, {3\over 2}^+) & ^3\Lambda H(1, {1\over 2}^+) & ^4\Lambda H({1\over 2}, 0^+) & ^4\Lambda H({1\over 2}, {1\over 2}^+)
\hline
\Lambda N \times \Delta_{\Lambda\Lambda} & 1 & -1/2 & - & 3/4 & -1/4 \\
F \times \bar{V}_{\Lambda\Sigma} & - & - & 2\sqrt{2/3} & 1 & 1 \\
GT \times \Delta_{\Lambda\Sigma} & \sqrt{3}/2 & - & -1/2 & 3/4 & -1/4 \\
{1\over s_0}(|F|^2 + |GT|^2) & 0.243 & - & 0.373 & - & - \\
{s_0}(|F + GT|^2) & - & - & - & 0.574 & 0.036 \\
\end{array}
\]

The $\Lambda N \leftrightarrow \Sigma N$ transition matrix elements are seen to provide about half of the observed 1.1 MeV $0^+_{g.s.}$ - $1^+_{exc}$ splitting in the $A = 4$ hypernuclei, the rest must then be assigned to the $\Lambda N$ spin-spin matrix element $\Delta_{\Lambda\Lambda}$. For the $A = 3$ states, marked here by $^3\Lambda H$, Fermi and Gamow-Teller contributions are added incoherently owing to different intermediate states involved in these transitions, with binding-energy contributions obtained upon assuming implicitly same-size nucleon and hyperon wavefunctions as for $A = 4$. Since $^3\Lambda H(0, {3\over 2}^+)$ is weakly bound, the actual $A = 3$ contributions are expected to be somewhat suppressed, with matrix-element suppression factor $\eta$ estimated to be about $\eta \approx 0.7$ – 0.8. Even so, given the size of both $\Lambda N$ spin-spin and $\Lambda\Sigma$ transition binding-energy negative contributions to $^3\Lambda H(0, {3\over 2}^+)$ with respect to $^3\Lambda H(0, {1\over 2}^+)$ g.s., it is safe to conclude that $^3\Lambda H(0, {3\over 2}^+)$ is unbound.

Focusing on discussion of $^3\Lambda H(1, {1\over 2}^+)$, particularly relative to $^3\Lambda H(0, {1\over 2}^+)$ g.s., we first go to the SU(4) limit of nuclear-core dynamics in which the dineutron becomes bound and degenerate with the deuteron, and where the difference in $\Lambda$ separation energies of $^3\Lambda H(1, {1\over 2}^+)$ and $^3\Lambda H(0, {1\over 2}^+)$ according to Table 3 is given (in MeV) by

\[
\delta B_\Lambda \equiv B^{T=1}_\Lambda( {1\over 2}^+) - B^{T=0}_\Lambda( {1\over 2}^+) = \eta^2 (0.373 - 0.243) - \eta \Delta_{\Lambda\Lambda}.
\]

To maximize this energy difference we neglect the $\Lambda N$ spin-spin contribution, thereby letting $\Delta_{\Lambda\Lambda} \to 0$, and compensate by doubling the $\Lambda\Sigma$ contribution in order to keep $E(1^+) - E(0^+) \approx 1.1$ MeV in $^4\Lambda H$ intact. For $\eta = 1$, expected
to be a fair approximation in this SU(4) limit, we obtain $\delta B_{\Lambda}^{\text{max}} = 0.26$ MeV, and so by charge independence the $\Lambda$ separation energy in this hypothetically bound $^{3}_{\Lambda}n$ with respect to the bound dineutron core is $0.39 \pm 0.05$ MeV. Precisely the same result is obtained if Nijmegen model NSC97f $\Lambda\Sigma$ matrix elements from [4], in parentheses there, are used instead. Next, by solving $\Lambda nn$ Faddeev equations we fit a $\Lambda N$ spin-independent Yamaguchi separable interaction that reproduces $B_{\Lambda}(^{3}_{\Lambda}n) = 0.39$ MeV, with $B(^{2}n) = 2.23$ MeV as in the deuteron. For a chosen value of 2.5 fm for the $\Lambda N$ effective range, this requires a $\Lambda N$ scattering length of $-1.804$ fm. For $nn$ interaction we used Yamaguchi separable potential determined by the $NN$ $T = 0$ low-energy parameters $a_s = 5.4$ fm, $r_s = 1.75$ fm, resulting in $B(^{2}n) = 2.23$ MeV which equals the deuteron binding energy in this SU(4) limit. We then perform a series of $\Lambda nn$ Faddeev calculations keeping the $\Lambda N$ interaction as is, but breaking SU(4) progressively by varying the $nn$ interaction to reach $a_s = -17.6$ fm and $r_s = 2.88$ fm as appropriate in the real world to the unbound dineutron. This is documented in Table 4.

Table 4: Binding energy $B(^{2}n)$ (in MeV) of two neutrons in a separable Yamaguchi potential specified by scattering length $a_s$ and effective range $r_s$ (both in fm) in the $^{1}S_{0}$ channel, and $\Lambda$ separation energy $B_{\Lambda}(^{3}_{\Lambda}n)$ (in MeV) obtained by solving $\Lambda nn$ Faddeev equations with a separable Yamaguchi $\Lambda N$ spin-independent interaction specified by scattering length $a = -1.804$ fm and effective range $r = 2.5$ fm. The $B(^{2}n)_{\text{approx}}$ values are obtained using Eq. 7.

| $a_s$ | $r_s$ | $B(^{2}n)$ | $B(^{2}n)_{\text{approx}}$ | $B_{\Lambda}(^{3}_{\Lambda}n)$ |
|------|------|------------|-----------------------------|-----------------------------|
| 5.4  | 1.75 | 2.23       | 2.24                        | 0.39                        |
| 5.4  | 2.25 | 2.79       | 2.87                        | 0.27                        |
| 5.4  | 2.881| 4.98       | -                           | 0.16                        |
| 6.0  | 2.881| 2.86       | 3.20                        | 0.11                        |
| 7.0  | 2.881| 1.64       | 1.68                        | 0.06                        |
| 9.0  | 2.881| 0.80       | 0.80                        | 0.01                        |
| 13.0 | 2.881| 0.32       | 0.32                        | 0.003                       |
| 17.612| 2.881| 0.16       | 0.16                        | –                           |
| -17.612| 2.881| –          | –                           | –                           |

The table demonstrates the behavior of the dineutron binding energy $B(^{2}n)$ and the $^{3}_{\Lambda}n$ binding energy $B(^{3}_{\Lambda}n) = B(^{2}n) + B_{\Lambda}(^{3}_{\Lambda}n)$ upon varying the $NN$ low-energy scattering parameters from values given by the $T = 0$ $pn$...
interaction down to the empirical values for the $T = 1$ $nn$ interaction. This is done in two stages. First, increasing the effective range while keeping the scattering length fixed, $B(2n)$ increases whereas $B\Lambda(3\Lambda n)$ steadily decreases. In the second stage, while keeping the effective range fixed at its final empirical $nn$ value, the scattering length is varied by increasing it and then crossing from a large positive value associated with a loosely bound dineutron to the empirical large negative value of $a_{nn}$ associated with a virtual dineutron. During this stage, $B(2n)$ too decreases steadily until $3\Lambda n$ is no longer bound.

With $B\Lambda(3\Lambda n) \ll B(2n)$ holding over the full range of variation exhibited in Table 4, it is clear that the behavior of $B(3\Lambda n)$ follows closely that of $B(2n)$. For fairly small values of $B(2n)$, say $B(2n) \lesssim 3$ MeV, $B(3\Lambda n)$ is quite accurately reproduced by the effective-range expansion approximation

$$B(2n)_{\text{approx}} = \frac{\hbar^2}{M_n r_s^2} \left(1 - \sqrt{1 - \frac{2r_s}{a_s}}\right)^2,$$  

as shown by comparing the exact and approximate values of $B(2n)$ listed in the table.

It is worth noting in Table 4 that the dissociation of $3\Lambda n$ occurs while the dineutron is still bound, although quite weakly. The final result of no $3\Lambda n$ bound state, for a virtual dineutron and $\Lambda N$ low-energy scattering parameters listed in the caption to Table 4, should come at no surprise given that a considerably larger-size $\Lambda N$ scattering length was found to be required in the Faddeev calculations listed in Table 1 to bind $3\Lambda n$. Although a specific value of 2.5 fm for the $\Lambda N$ effective range was used in our actual demonstration, similar results are obtained for other reasonable choices of the $\Lambda N$ effective range.

5. Discussion and conclusion

We have shown in this work that the $\Lambda N$ interactions required to bind $3\Lambda n$ are inconsistent with the measured $\Lambda p$ scattering cross sections at low energies, with $3\Lambda H_{g.s.}$ binding energy, and with the $0^+_{g.s.} - 1^+_{exc}$ excitation energy of the $A = 4 \Lambda$ hypernuclei. Although simple $\Lambda N$ interactions were used to simulate the more realistic NSC97 interactions, the consequences of

\footnote{A decrease of $B\Lambda$ upon increasing one of the effective ranges in a few-body calculation was noted and discussed by Gibson and Lehman [20].}
accepting a bound $^3\Lambda n$ for $\Lambda$ hypernuclear data are sufficiently strong that
the use of more refined interactions is unlikely to modify any of the conclusions reached here. Of the three hypernuclear systems related here to $^3\Lambda n$, we attach special significance to the $A = 4 \Lambda$ hypernuclei where only the 1.1 MeV $0^+_{\text{g.s.}}-1^+_{\text{exc}}$ excitation energy is involved in our model building. This excitation energy is intimately connected to $\Lambda N \leftrightarrow \Sigma N$ coupling effects in the $A = 4$ hypernuclei [16] which have been further incorporated and tested successfully to reproduce excitation spectra in $p$-shell hypernuclei [17]. We judiciously avoided relying on the absolute binding energy of the $0^+_{\text{g.s.}}$ of the $A = 4 \Lambda$ hypernuclei because it has not been yet reproduced satisfactorily in few-body calculations that use theoretically derived $YN$ potentials, as stressed recently by Nogga [15]. This difficulty might be associated with missing three-body $\Lambda NN$ interaction terms, other than those incorporated here by including $\Lambda N \leftrightarrow \Sigma N$ coupling.

Of the $\Lambda NN$ interactions considered in past hypernuclear calculations, those arising from an intermediate $\Sigma(1385)$ hyperon resonance [21] are independent of the spin of the $\Lambda$ and thus would not affect the $0^+_{\text{g.s.}}-1^+_{\text{exc}}$ spin-flip excitation upon which our considerations rest. The spin-isospin dependence of the central component of this interaction is given by $-\vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$ which assumes the same value $+3$ for both $J^P = \frac{1}{2}^+$ states in the $A = 3$ hypernuclei. A dispersive $\Lambda NN$ repulsive contribution with $\Lambda$ spin dependence given by $(1 + \frac{1}{3} \vec{\sigma}_\Lambda \cdot \vec{S}_{12})$, where $\vec{S}_{12} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$, was considered in VMC calculations of light hypernuclei [22]. This gives $1(\frac{1}{3})$ for the $T = 1(0), J^P = \frac{1}{2}^+$ $A = 3$ states, namely more repulsion for $^3\Lambda n$ than for $^3\Lambda H_{\text{g.s.}}$. Another form of dispersive $\Lambda NN$ contribution suggested in Ref. [23] depends on spin and isospin through the factor $-\vec{\tau}_1 \cdot \vec{\tau}_2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{\sigma}_\Lambda \cdot \vec{S}_{12})$ which assumes values $+3(-3)$ for the $T = 1(0), J^P = \frac{1}{2}^+$ states, repulsive for $^3\Lambda n$ while attractive for $^3\Lambda H_{\text{g.s.}}$. The latter two dispersive $\Lambda NN$ interaction forms were found in Ref. [24] capable of accounting for a substantial fraction of the $0^+_{\text{g.s.}}-1^+_{\text{exc}}$ excitation in the $A = 4$ hypernuclei, but obviously neither of them would add attraction to $^3\Lambda n$ relative to $^3\Lambda H_{\text{g.s.}}$. This brief survey of three-body $\Lambda NN$ phenomenology offers, therefore, no plausible solution of the $^3\Lambda n$ puzzle.

A comment on CSB effects in light $\Lambda$ hypernuclei and whether or not CSB might resolve the $^3\Lambda n$ puzzle is in order before concluding the present study. For the known $T = \frac{1}{2}$ isodoublet of $A = 4$ hypernuclear $0^+_{\text{g.s.}}$ levels $\Delta B^\text{exp}_\Lambda(A = 4) \equiv B\Lambda(\Lambda\text{He}) - B\Lambda(\Lambda\text{H}) = 0.35 \pm 0.04$ MeV [1] is exceptionally large and defies explanation in modern $YN$ interaction models, see Table 9 in
Ref. [15] where the recently constructed NLO chiral \(YN\) interactions [10] are shown to yield only \(\Delta B_{\Lambda}^{\text{calc}}(A = 4) \approx 50\) keV. This \(\Delta B_{\Lambda}(A = 4)\) arises largely from kinetic energies depending on which charged \(\Sigma\) hyperon mass is used. The same CSB effect will result in smaller \(B_{\Lambda}(A\Lambda n)\) values relative to those calculated, as done here, using a charge symmetric calculation. Therefore, CSB contributions are also unlikely to resolve the \(A\Lambda n\) puzzle.

How does one then explain the HypHI \(t + \pi^-\) signal which is naturally assigned to the two-body weak decay \(\Lambda n \rightarrow t + \pi^-\)? This problem is aggravated by a similar one addressing a \(d + \pi^-\) signal, also observed in the HypHI experiment, the most straightforward assignment of which would be due to the two-body weak decay of a bound \(\Lambda n\) system: \(\Lambda n \rightarrow d + \pi^-\). No plausible solution has been offered to these puzzles and more work on other possible origins of \(d + \pi^-\) and \(t + \pi^-\) signals is called for.

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