New integrable version of the degenerate supersymmetric t-J model

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Abstract

A new integrable version of the degenerate supersymmetric t-J model is proposed. In this formulation instead of restricting single occupancy of electrons at each lattice site we may have up to two electrons at each site. As a requirement of exact integrability the hopping interaction turns out to be correlated with the density of electrons in the neighboring sites. The exact solution of the model is obtained through the coordinate Bethe ansatz.
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Recently there has been considerable interest in studying low-dimensional electronic models with strong correlation due to the possibility that the normal state of the two-dimensional novel superconductivity may share some interesting features of a one-dimensional interacting electron system. Exactly solvable models of strongly correlated electrons which are generalizations of the Hubbard and t-J models have been formulated and investigated [1 - 14]. It is noteworthy that some of these models [7,8,10] have superconducting behavior with dominant superconducting correlation functions.

The t-J model is a lattice model on the restricted electronic Hilbert space, where the occurrence of two electrons on the same lattice site is forbidden. The Hamiltonian of the extended version of the t-J model with the generalized spin $S = (N - 1)/2$ has the form [15,16,10]

$$H = -t \sum_{j=1}^{L} P_1 \left\{ \sum_{\alpha=1}^{N} \left( c_{j,\alpha}^+ c_{j+1,\alpha} + c_{j+1,\alpha}^+ c_{j,\alpha} \right) + J \sum_{\alpha \neq \beta}^{N} \left( c_{j,\alpha}^+ c_{j+1,\beta} c_{j+1,\alpha} - n_{j,\alpha} n_{j+1,\beta} \right) \right\} P_1$$

(1)

where $c_{j,\alpha}$ and $n_{j,\alpha} = c_{j,\alpha}^+ c_{j,\alpha}$ are the standard fermionic and density operators and $P_1$ projects out from the Hilbert space the states with more than single occupancy at a given site. The model (1) is not integrable for all the values of the parameter $J/t$ and arbitrary band-filling, but only at the supersymmetric point $J = t$. By construction the spin degrees of freedom of model (1) form a $SU(N)$-invariant subspace.

In this communication we introduce a new version of the supersymmetric t-J model. In this version instead of the operator $P_1$ we introduce the operator $P_2$ which is a projection operator into the subspace where at most double occupancy is allowed at the sites. Differently from (1) instead of $N$ allowed states we have in this case $(N^2 + N + 2)/2$ possible states at a given site $j$, namely

$$|0 >; \quad c_{j,\alpha}^+ |0 > (\alpha = 1, 2, ..., N);$$

$$c_{j,\alpha}^+ c_{j,\beta}^+ |0 > (\alpha \neq \beta = 1, 2, ..., N).$$

(2)

In a periodic lattice of length $L$ the Hamiltonian of our model reads
\[ H = - \sum_{j=1}^{L} P_2 H_{j,j+1} P_2, \]
\[ H_{j,k} = \sum_{\alpha} (c_{j,\alpha}^+ c_{k,\alpha} + h.c.) \]
\[ \times \{ \exp[\frac{1}{2} \eta \sum_{\beta (\neq \alpha)} (n_{j,\beta} + n_{k,\beta})] - \sum_{\beta \neq \gamma (\neq \alpha)} n_{j,\beta} n_{k,\gamma} \} \]
\[ + \sum_{\alpha < \beta} [U(n_{j,\alpha}n_{j,\beta} + n_{k,\alpha}n_{k,\beta}) + t_p(c_{j,\alpha}^+ c_{j,\beta}^+ c_{k,\alpha} c_{k,\beta} + h.c.)] \]
\[ - J \sum_{\alpha, \beta} c_{j,\alpha}^+ c_{j,\beta}^+ c_{j,\gamma} c_{k,\alpha} c_{k,\beta} \sum_{\gamma \neq (\alpha, \beta)} (n_{j,\gamma} + n_{k,\gamma}) - \sum_{\gamma \neq \delta \neq (\alpha, \beta)} n_{j,\gamma} n_{k,\delta} \]
\[ - \sum_{\alpha < \beta \neq \gamma} (c_{j,\alpha}^+ c_{j,\beta}^+ c_{j,\gamma} c_{k,\alpha} c_{k,\beta} c_{k,\gamma} + h.c.) \]
\[ + J \sum_{\alpha < \beta \neq \gamma < \delta} c_{j,\alpha}^+ c_{j,\beta}^+ c_{j,\gamma} c_{j,\delta}^+ c_{j,\gamma} c_{k,\alpha} c_{k,\beta} c_{k,\gamma} \]
\[ - J \sum_{\alpha < \beta \neq \gamma} (n_{j,\alpha} n_{j,\beta} n_{k,\gamma} + n_{k,\alpha} n_{k,\beta} n_{j,\gamma}) \]
\[ + J \sum_{\alpha < \beta \neq \gamma < \delta} n_{j,\alpha} n_{j,\beta} n_{j,\gamma} n_{k,\delta}. \]

Similarly as in the standard t-J model we have established the integrability of (1) only at a special point

\[ t_p = J = U = \varepsilon (e^{\eta} - 1), \text{ at } \eta = \ln 2, \varepsilon = \pm 1 \]

(4)

It is interesting to point that in the case \( N = 2 \), differently from (1), the model (3) is integrable at an arbitrary parameter \( \eta \) [12 - 14].

The model (1) has been constructed in the following way. We consider all possible interactions that conserve the number of particles of each species separately and satisfy the constraint imposed by \( P_2 \) that not more than double occupancy is allowed at each site. Moreover we impose for the amplitude of the eigenfunctions of the Hamiltonian (1) in the sector with \( n_\alpha \) particles of species \( \alpha (\alpha = 1, 2, ..., N) \) the following ansatz

\[ \Psi(x_1, \alpha_1; ..., x_n, \alpha_n) = \sum_P A_{P_1 P_2 ... P_n}^{Q_1 ... Q_n} \prod_{j=1}^{n} \exp(ikP_j x_{Q_j}) \]

(5)

where \( Q = (Q_1, ..., Q_n) \) is the permutation of the \( N \) particles such that their coordinates satisfy \( 1 \leq x_{Q_1} \leq x_{Q_2} \leq ... \leq x_{Q_n} \leq L \). The sum is over all permutations \( P = (P_1, ..., P_n) \) of integers 1, 2, ..., \( n \). In the case we have two particles at the same site \( x_{Q_1} = x_{Q_2} \) the ansatz (5) is replaced by
\[ \Psi(x_1, \alpha_1; \ldots; x_n, \alpha_n) = \sum_P A_{\alpha_1 \ldots \alpha_Q}^{\alpha_{Q+1} \ldots \alpha_n} \prod_{j=1}^{n} \exp(ikP_j x_{Q_j}), \]  

where the bar at the \( l \)th and \((l + 1)\)th position of the superscript indicates the pair position. Secondly we consider the model (3) in the sector where we have only two electrons. In this case the projector \( P_2 \) does not produce any restrictions and our problem is exactly equivalent to that of a model with only two species. The coefficients \( A_{\alpha_1 \ldots \alpha_n}^{\alpha_{Q+1} \ldots \alpha_n} \) arising from the different permutation \( Q \) are connected with each other by the elements of the two-particle S-matrix

\[ A_{\ldots \alpha_1 \alpha_2 \ldots} = \sum_{\delta, \gamma = 1}^{N} S_{\alpha_\delta}^{\alpha_\gamma}(k_{p_1}, k_{p_2}) A_{\ldots \delta \gamma \ldots}. \]  

A necessary condition of the compatibility of equations (7) is the fulfillment of the Yang-Baxter equations [17,18]. There are two integrable models with \( N = 2 \) and their S-matrices have such factorizable form. These models are the Hubbard model [1] and the correlated hopping model [6,7,12-14]. We may try to use as the fundamental building block of a general S-matrix of our model (3) the two-particle scattering matrix of these models. Here we use the second possibility and choose the S-matrix as in the correlated hopping model[12 - 14]

\[ S_{1,2}(k_1, k_2) = \frac{\lambda_1 - \lambda_2 - iP_{12}}{\lambda_1 - \lambda_2 - i}, \]  

where the operator \( P_{12} \) interchanges the species variables \( \alpha_1 \) and \( \alpha_2 \) and

\[ \lambda_j = \lambda(k_j) = \begin{cases} \cot \frac{1}{2}k; & \varepsilon = -1 \\ -\tan \frac{1}{2}k; & \varepsilon = +1 \end{cases}. \]  

To complete the proof of the Bethe ansatz (5-6) we must consider the eigenvalue equations in the sector where the total number of particles \( n = 3, 4 \). This gives a complicated system of equations for the coupling constants of the Hamiltonian (1). The solution of this system is presented in the Hamiltonian (3). The periodic boundary condition for the system on the finite interval (0,L) gives us the Bethe ansatz equation. In order to obtain these equations we must diagonalize the transfer matrix of a related inhomogeneous vertex model with non-intersecting strings [19 - 20]. The Bethe ansatz equations are written in terms of the charge rapidities \( \lambda^{(0)} \) and additional \( N - 1 \)-spin rapidities \( \lambda^{(l)} (l = 1, \ldots, N - 1) \).
\[
\left( \frac{\lambda_j^{(0)} - i}{\lambda_j^{(0)} + i} \right)^L = \frac{M_1}{\prod_{\alpha=1}^{M_1} \frac{\lambda_j^{(0)} - \lambda_{0}^{(1)} - i/2}{\lambda_j^{(0)} - \lambda_{0}^{(1)} + i/2}}
\]

\[
= \frac{M_l}{\prod_{\beta=1}^{M_l} \frac{\lambda_{0}^{(l)} - \lambda_{\beta}^{(l)} - i}{\lambda_{0}^{(l)} - \lambda_{\beta}^{(l)} + i}} = -\prod_{\beta=1}^{M_l} \frac{\lambda_{0}^{(l)} - \lambda_{\beta}^{(l-1)} - i/2}{\lambda_{0}^{(l)} - \lambda_{\beta}^{(l-1)} + i/2} \prod_{\beta=1}^{M_{l+1}} \frac{\lambda_{0}^{(l)} - \lambda_{\beta}^{(l+1)} - i/2}{\lambda_{0}^{(l)} - \lambda_{\beta}^{(l+1)} + i/2}
\]

(10)

\[l = 1, ..., N - 1, M_0 = n, M_N = 0, \alpha = 1, ..., M_l, \lambda_j^{(0)} = \lambda_j,\]

where \(n_j = M_{j-1} - M_j\) is the number of particles of species \(j\). The eigenenergies of the system are given in terms of the solutions \(\{\lambda^{(k)}_{\alpha}\}\) and are given by

\[E = -2 \sum_{j=1}^{n} \cos k_j = 2 \varepsilon (n - \sum_{j=1}^{n} \frac{2}{(\lambda_j^{(0)})^2 + 1}).\]

(11)

The corresponding magnetization of the eigenstate is given by a pure Zeeman splitting of the \(SU(N)\) spin multiplet

\[S^z = \frac{1}{2} (N - 1) n - \sum_{l=1}^{N-1} M_l.\]

(12)

The ground state and excitations of the system are given by inserting in (11) the solutions of equations (10). The rapidities have, in general, complex values and their classification is analogous to that of the degenerate electron gas with an attractive \(\delta\)-function potential[21]:

(i) real charge rapidities, belonging to the set \(\lambda_{0}^{(0)}\), and correspond to unpaired propagating electrons; (ii) complex spin and charge rapidities, which correspond to bound states of electrons with different spin components; and (iii) strings of complex spin rapidities, representing spin states. A complex of \(m\) electrons \((m \leq 0)\) is characterized [22] by one real \(\zeta^{(m-1)}\) rapidity and the remaining ones given by

\[\lambda_{p_l}^{(l)} = \zeta^{(m-1)} + i\frac{p_l}{2},\]

(13)

\[l \leq m - 1, \quad p_l = -(m - l - 1), -(m - l - 3), ..., (m - l - 1).\]

These spin and charge strings form the classes (i) and (ii) above mentioned. In the class (iii) there is a set of rapidities \(\{\lambda_{\alpha}^{(l)}\}\) \((l = 1, 2, ..., N - 1)\), forming asymptotically strings of maximum size \(n\),

\[n = \sum_{j=1}^{n} n_j = M_{j-1} - M_j,\]

where \(n_j = M_{j-1} - M_j\) is the number of particles of species \(j\). The eigenenergies of the system are given in terms of the solutions \(\{\lambda^{(k)}_{\alpha}\}\) and are given by

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\[n = \sum_{j=1}^{n} n_j = M_{j-1} - M_j,\]
\[\lambda_n^{(l)\mu} = \Lambda_n^{(l)} + i\mu/2 + \delta^{(l)\mu}, \quad \mu = -(n-1), -(n-3), ..., (n-1).\]

Here \(\delta^{(l)\mu} = O(e^{-aL}), a > 0\), vanishes in the thermodynamic limit \(L \rightarrow \infty\) and \(\Lambda_n^{(l)}\) are real numbers. The above rapidities (13) and (14) are substituted into (10) and the resulting coupled equations for the real \(\zeta^{(l)}\) and \(\Lambda_n^{(l)}\) are logarithmized. This procedure generated a set of integer quantum numbers for each set of rapidities: \(\rho^{(l)}(\zeta)\) for the real \(\zeta^{(l)}\), \(l = 0, ..., N-1\) and similarly for the corresponding "hole" functions. In the thermodynamic limit the Bethe ansatz equations reduce to sets of coupled linear integral equations for the density functions.

After Fourier transforming these equations we obtain

\[
\hat{\rho}_n^{(l)}(\omega) + \hat{\rho}^{(l)}(\omega) + \sum_{q=0}^{N-1} \hat{\rho}^{(q)}(\omega) \exp\left[-(l + q - p_{l,q})|\omega|/2\right] \\
\times \frac{\sinh[(p_{l,q} + 1)\omega/2]}{\sinh(\omega/2)} + \sum_{n=1}^{\infty} \hat{\sigma}_n^{(l+1)}(\omega) \exp(-n|\omega|/2) \\
= 2e^{-(l+1)|\omega|/2} \cosh \omega/2, \quad l = 0, ..., N-1, \quad (15)
\]

\[
\hat{\sigma}_n^{(l)}(\omega) = \hat{\rho}_n^{(l-1)}(\omega) \exp(-m|\omega|/2) \\
+ \sum_{n=1}^{\infty} [\hat{\sigma}_n^{(l-1)}(\omega) + \hat{\sigma}_n^{(l+1)}(\omega) - \cosh(\omega/2)\hat{\sigma}_n^{(l)}(\omega)] \\
\times \exp(-\max(m, n)|\omega|/2) \frac{\sinh[\min(m, n)\omega/2]}{\sinh(\omega/2)}, \quad l = 1, ..., N-1. \quad (16)
\]

The last set of equations holds for \(m = 1, ..., \infty\) with \(\sigma_m^{(0)}, \sigma_{mh}^{(0)}, \sigma_m^{(N)},\) and \(\sigma_{mh}^{(N)}\) being identically zero, and \(p_{l,q} = \min(l, q) - \delta_{l,q}\) in eq. (15). These equations are identical to those appearing in the degenerate electron gas with an attractive \(\delta\)-function potential \([21]\) and, apart from the driving terms, are also identical to those of the degenerate supersymmetrical t-J model \([14, 22]\). The expression for the energy however is different and is given by

\[
\frac{1}{L} E = 2\varepsilon\{\rho - \sum_{l=0}^{N-1} \int d\zeta \rho^{(l)}(\zeta)\left[\frac{(l+2)}{(l+2)^2/4 + \zeta^2} + \frac{l}{l^2/4 + \zeta^2}\right]\}. \quad (17)
\]

Next we discuss the ground state properties. First consider \(\varepsilon = -1\). As in the ordinary t-J model the ground state is described by the bound complexes (13) with the maximum length \(N\). In order to accommodate such complexes in the ground state we assume that the number
of electrons $n$ is a multiple of $N$. In the thermodynamical limit, $L, n \to \infty$ we obtain, from (15-16),
\[
2\pi \rho(\Lambda) + \left[ \int_{-v_0}^{-v_0} + \int_{v_0}^{N} \right] \left[ \sum_{l=1}^{N-1} \theta'(\frac{1}{l}(\Lambda - \Lambda')) \rho(\Lambda')d\Lambda' \right] = \theta\left(\frac{2\Lambda}{N+1}\right) + \theta\left(\frac{2\Lambda}{N-1}\right)
\]
\[
\left[ \int_{-v_0}^{-v_0} + \int_{v_0}^{N} \right] \rho(\Lambda)d\Lambda = \frac{1}{N} \rho, \quad \rho = \frac{n}{L}, \quad (18)
\]
where
\[
\theta'(\frac{1}{l}\Lambda) = \frac{2l}{l^2 + \Lambda^2}.
\]
The solution $\rho(\Lambda)$ of these equations yields the ground-state energy per site
\[
\frac{1}{L}E = -2\{\rho - \left[ \int_{-v_0}^{-v_0} + \int_{v_0}^{N} \right] \left[ \theta'(\frac{2\Lambda}{N+1}) + \theta'(\frac{2\Lambda}{N-1}) \right] \rho(\Lambda)d\Lambda \}. \quad (19)
\]
In order to solve numerically the integral equations (18) it is convenient to rewrite them in the following form
\[
\rho(\Lambda) - \frac{1}{2\pi} \int_{-v_0}^{v_0} K_i(\Lambda - \Lambda')\rho(\Lambda')d\Lambda' = \frac{1}{2\pi} K_2(\Lambda),
\]
\[
\int_{-v_0}^{v_0} \rho(\Lambda)d\Lambda = 2 - \rho, \quad (20)
\]
where
\[
K_i(\Lambda) = \int_{-\infty}^{\infty} \tilde{K}_i(p)e^{-ip\Lambda}dp \quad i = 1, 2,
\]
\[
\tilde{K}_1(p) = \frac{e^{-|p|/2}\sinh\frac{1}{2}(N-1)p}{\sinh\frac{1}{2}Np}; \quad \tilde{K}_2(p) = \frac{e^{-|p|/2}\sinh p}{\sinh\frac{1}{2}Np}.
\]
For the ground-state energy we now have
\[
\frac{1}{L}E = -2\{\rho - \Phi_2(0) + \int_{-v_0}^{v_0} K_2(-\Lambda)\rho(\Lambda)d\Lambda \}, \quad (21)
\]
where
\[
\Phi_2(\Lambda) = \int_{-\infty}^{\infty} (e^{-\frac{N+1}{2}|p|} + e^{-\frac{N-1}{2}|p|}) K_2(p)e^{-ip\Lambda}dp. \quad (22)
\]
The energy density $E/L$ (19) as a function of the density $\rho = n/L$ is shown in Fig. 1 for several values of $N$. For the sake of comparison we show in Fig. 2 the ground-state
The energy of the ordinary degenerated t-J model, for some values of $N$. These figures show that for small values of the density the ground-state energy of both models are the same. The same similarity is also expected for all densities $\rho \leq 1$ but for large values of $N$. On the other hand, for small values of $N$ and large densities, where the site-exclusion effect is more important, these figures show quite distinct behavior for both models.

Secondly consider the case of ferromagnetic coupling $\varepsilon = +1$. In contrast with the ordinary t-J model the ground state is described by bound complexes where the rapidities $\lambda^{(0)}$ have length 2. We have in this case an equal number of electrons with two different spin components. The situation is analogous to that of the correlated hopping model [14]. The energy density,

$$\frac{1}{N}E = 2\{\rho - \int_{-v_0}^{v_0} [\theta'(\frac{2}{3}\Lambda) + \theta'(2\Lambda)]\rho^{(1)}(\Lambda)d\Lambda\}$$

is now given in terms of the solution of the integral equations

$$2\pi \rho^{(1)}(\Lambda) + \int_{-v_0}^{v_0} \theta'(\Lambda - \Lambda')\rho^{(1)}(\Lambda')d\Lambda' = \theta'(\frac{2}{3}\Lambda) + \theta'(2\Lambda),$$

$$\int_{-v_0}^{v_0} \rho^{(1)}(\Lambda)d\Lambda = \frac{1}{2}\rho.$$  \hspace{1cm} (24)

The energy density $E/L$, as a function of $\rho$, obtained from the numerical solution of (23-24), is shown in Fig. 3.

To summarise, we have presented an integrable $(N^2 + N + 2)/2$-state version of the degenerate supersymmetric $t - J$ model where at most two electrons are allowed in a given site. We have solved the model by the coordinate Bethe ansatz method and derived the Bethe ansatz equations. Our results naturally raise interesting problems to be solved: the construction of the anisotropic version of the Hamiltonian (1) as well as its graduated version. In the latter case one should have an integrable model of interacting chains with spin $S = 1/2$ and 1. The isotropic version for $N = 2$ of this model has been solved recently by Frahm et al. [23]. Finally it is possible to construct new generalized integrable models by the introduction of the operator $P_l (l > 2)$ which is a projection operator into the subspace where at most $l$-times occupancy is allowed at the sites. As a fundamental model we may use in this case
the generalization of the correlated hopping model where we have \( l \) distinct species with single particle as well as multi-particle hopping. For \( l = 3 \) we may use as the fundamental building block of the general scattering matrix the S-matrix of a model which was constructed recently in refs. [24,25]. For an arbitrary \( l \), as a result of this construction, we may obtain the integrable model with the symmetry of quantum superalgebra \( U_q[gl(m + 1|N - m)]_{k = l} \). Certainly it will be interesting to study the physical properties of these quantum chains particularly their phase diagram and critical exponents.

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Figure Captions

Figure 1 - Ground-state energy per site $E/L$ as a function of the density $\rho = n/L$ for the exact integrable model (3) with $t_p = J = U = \epsilon = 1$ and some values of $N$.

Figure 2 - Ground-state energy per site $E/L$ as a function of the density $\rho = n/L$ for the degenerated t-J model (1) at $J = t = 1$, for some values of $N$.

Figure 3 - Ground-state energy per site $E/L$ as a function of the density $\rho = n/L$ for the exact integrable model (3) with $t_p = J = U = \epsilon = -1$. 
