ANTIBOUND AND RESONANT STATES IN HALO NUCLEI

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Abstract

An unified shell model scheme to evaluate simultaneously the contribution of antibound states and Gamow resonances to the nuclear halos is presented. The calculations, performed in the complex energy plane, are applied to the case of \textsuperscript{11}Li. It is found that \textsuperscript{11}Li may develop a resonant state excitation near the breakup threshold.

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It is by now well established that halos in nuclei are produced by particles moving in single-particle states which extend far in space. The setting for this to occur requires neutron configurations consisting mainly of unbound s- and p-waves since in this case no barrier large enough to trap the nucleons inside the nuclear core will be present. It is the pairing interaction that holds the escaping neutrons together, granting them their Borromean character [1].

Stated in this simplified fashion the explanation of halos seems perhaps obvious. However it took many years of intense work, experimental as well as theoretical, to reach the understanding of halos. It is not our intention here to quote the vast amount of papers written on this subject. A recent review, including abundant references, can be found in Ref. [2]. For our purpose it is enough to point out that for the description of $^{11}\text{Li}$, which is the typical halo nucleus, the two relevant single-particle states, as specified by the experimental spectrum of $^{10}\text{Li}$ [2,3], consist of a low-lying $s_{1/2}$ state, which probably is an antibound (or virtual) state at about -50 keV, and a $p_{1/2}$-resonance at about 540 keV. A second $p$-resonance at around 0.250 MeV may also exist [4]. The two-body correlations induce a bound ground state in $^{11}\text{Li}$ at about -0.295 MeV. Notice that we are here using the shell-model language, where the core ($^{9}\text{Li}$) is considered as inert, the single-particle states are given by $^{10}\text{Li}$ and the two-body nucleus is $^{11}\text{Li}$ [5].

The existence of a very low-lying virtual $s$-state in $^{10}\text{Li}$ has important consequences for the correlations developed in $^{11}\text{Li}$ [6]. As discussed below, an antibound state close to the continuum threshold enhances the localisation of the low-lying scattering states. Therefore the $s$-wave content of the ground state of $^{11}\text{Li}$ is also increased, reaching the corresponding (large) experimental value. Moreover, the antibound state in $^{10}\text{Li}$ can affect the excited spectrum of $^{11}\text{Li}$ as well as the ground state. Since the dineutron system has also a virtual state close to threshold, in each of the two-body subsystems of $^{11}\text{Li}$ there is an antibound state near zero energy. Thus $^{11}\text{Li}$ fulfils the Efimov conditions [7] and consequently it may form low-lying excited states close to the breakup threshold. The possibility of such excitations was suggested in Ref. [8] in connection to the momentum distribution of the fragments of $^{11}\text{Li}$.

In all the studies done until now the role of the antibound $s$-state in $^{11}\text{Li}$ was discussed only indirectly, i.e. through the associated scattering length. One of the aims of this letter is to present for the first time a formalism in which antibound states are treated as ordinary single-particle states. That is, the antibound states will form part of the single-particle representation in the same fashion as bound states do in standard shell model calculations. For this we will use an extended version of the shell model on the complex energy plane (CXSM) [9,10]. Within this formalism we will investigate the existence of low-lying excited resonant states of Efimov type, for which there are no accurate calculations yet.

The CXSM is a powerful method to analyse the influence of the single-particle resonances upon the two-particle correlated states as well as to understand the developing of bound states by the two-body force acting upon states embedded in the continuum. In its initial form [9,10] the representation corresponding to the CXSM consists of Gamow resonances, bound states and complex scattering states. These, together with the antibound states, are poles of the one-particle Green function and, therefore, we will refer to them as ”poles”.

The space spanned by this basis (Berggren space) [11] is determined by a metric (Berggren metric) which is non-Hermitian. Since on the real energy axis the Berggren
space coincides with the Hilbert space, any physical quantity evaluated within the CXSM coincides with the corresponding one evaluated within the standard shell model. In this sense one can assert that the CXSM is a generalization of the shell-model and, as in this model, the many-body energies and corresponding wavefunctions can readily be evaluated and analysed.

The CXSM can be easily extended to take into account the antibound states. The corresponding single-particle representation contains the antibound states on the same footing as the other discrete elements, i.e. bound states and Gamow resonances. We will show how to construct this representation for the case of $^{11}\text{Li}$.

In the first step of the calculation one evaluates the single-particle states of the unbound nucleus $^{10}\text{Li}$. As in Ref. [12], for the central field we choose a Woods-Saxon potential with different depths for even and odd orbital angular momenta $l$. The single-particle states as well as the scattering waves will be evaluated by using the high precision piecewise perturbation method [13]. Since for the resonant state $0p_{1/2}$ the experimental data are uncertain we will perform two calculations corresponding to the energies 200 keV and 500 keV. At the same time, this will allow us to assess the influence of the $p_{1/2}$ resonance upon the halo formation. For clarity of presentation we will start studying the 200 keV case and afterwards the 500 keV case will be presented. At the end we will compare the results and discuss the similarities and differences between the two cases.

For the 200 keV case we use the Woods-Saxon potential given by $a = 0.67$ fm, $r_0 = 1.27$ fm, $V_0 = 50$ (36.9) MeV and $V_{so} = 16.5$ (12.624) MeV for $l$ even (odd). With these parameters we found the single-particle bound states $0s_{1/2}$ at $-23.278$ MeV and $0p_{3/2}$ at $-2.589$ MeV forming the $^9\text{Li}$ core. The valence poles are the low lying resonances $0p_{1/2}$ at $(0.195,-0.047)$ MeV and $0d_{5/2}$ at $(2.731,-0.545)$ MeV and the wide resonance $0d_{3/2}$ at $(6.458,-5.003)$ MeV. Besides, the state $1s_{1/2}$ appears as an antibound state$^1$ at $-0.050$ MeV. We thus reproduce the experimental single-particle energies giving from the very beginning unequivocal endorsement to the low lying $s$-state as due to an antibound state.

We also found other resonances at high energies. However we include in the basis single-particle states lying up to 10 MeV of excitation energy only. We found that expanding the basis from this limit does not produce any effect upon the calculation up to the six digits of precision that we require.

The non-resonant continuum in the CXSM representation is given by the points in the contour embracing the resonances [11]. One can use different contours for different partial waves, but it is important to keep in mind that the corresponding Berggren space and therefore the calculated quantities on the complex energy plane will depend on the contours. Only on the real energy axis the calculated quantities do not depend upon the shapes of the contours. This property will allow us to check the precision of our calculation. That is, the calculation of physical quantities by using any contour should coincide with those evaluated on the real energy axis, i.e. by means of the continuum shell-model [12]. Let us stress again that differences between the CXSM and the standard shell-model appear only when

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$^1$The principal quantum number $n$ labelling the single-particle states indicates that the corresponding wave functions are localized in a region inside the nucleus and that its real part has in that region $n$ nodes, excluding the origin.
evaluating resonances \[10\].

In the calculations presented below we will use the contours shown in Fig. 1. One sees in this figure that for the partial waves containing a Gamow resonance we take the conventional CXSM contour defined by the vertices B \([9,10]\) (see also Ref. \[14\]). Instead, for the antibound state we introduce the new type of countour defined by the vertices A. The general requirements which such contours should fulfil are discussed in Ref. \[15\]. With proper values for the energies corresponding to the vertices of these contours one can readily include in the basis the three resonances and the antibound state mentioned above. Notice that in our basis no single-particle bound state is present.

The next step in the calculation is to adopt a residual interaction. We will use a separable force and, therefore, the Hamiltonian matrix reduces to a dispersion relation (for details see Ref. \[10\]). To evaluate the ground state of \(^{11}\)Li we adjust the strength \(G\) of the separable interaction to reproduce the corresponding energy, i. e. -295 keV. We thus obtained \(G = 0.00194\) MeV.

With the mean field and the two-body interaction thus established we evaluated the ground state wave function. First we performed the calculations by choosing the real energy as a contour. In this case the wave function is spread over many components. The largest of these components correspond to configurations \(p_{1/2} \otimes p_{1/2}\) lying close to 400 keV (i. e. about twice the energy of the \(0p_{1/2}\) resonance) and \(s_{1/2} \otimes s_{1/2}\) lying close to threshold (i. e. close to twice the energy of the antibound state). The wave function consists of 44 \(\%\) \(s\)-states, 48 \(\%\) \(p\)-states and 8 \(\%\) \(d\)-states, as expected \[2,3\].

A remarkable feature of the calculation is that the antibound state exerts such a strong effect upon the two-particle wavefunction. From a CXSM point of view this is because the energy corresponding to the configuration \((1s_{1/2})^2\) is very close (in the complex energy plane) to the two-particle energy. But it can also be understood from a continuum shell-model point of view. That is, the radial scattering wave function with energies \(E = \hbar^2k^2/2\mu\) \((k\) real and positive\) close to a bound or antibound state with energy \(E_0\) \((k_0 = \pm ik_0|,\) respectively\) lying near threshold can be written as \[16\],

\[
\mathcal{R}_l(kr) \approx \sqrt{2k|k_0|} \mathcal{R}_l(|k_0|r) \tag{1}
\]

which shows that close to threshold the radial dependence of the scattering wave functions is independent of the energy, except the square root factor. This factor is maximum for \(k = |k_0|\). Therefore, the matrix elements of the two-body interaction are large within a relative large interval close to threshold. This induces large components of the two-body wave function in that energy region. Notice that it does not matter whether the S-matrix pole \(E_0\) corresponds to a bound or to an antibound state. The effect is exactly the same.

This feature can be seen in Fig. 2, where we give the localization of the scattering states, \(L(E)\), defined as

\[
L(E) = \int_0^{1.2R_N} \mathcal{R}_l^2(kr)r^2dr \tag{2}
\]

where \(R_N = 2.6\) fm is the core nuclear radius.

One sees in Fig. 2 a strong increase of the localization as the energies of the poles approaches threshold. This properry is also responsible for a similar increase in the elastic
cross section, as seen in Figs. 12 and 13 of Ref. [17]. Therefore one expects large elastic cross sections at low energies in halo nuclei, particularly in $^{11}$Li.

So far we have shown the advantages of the CXSM to evaluate the effect of the antibound state on already known properties of the ground state of the halo nucleus $^{11}$Li. However, the transparency of the method becomes essential in the search for other physically meaningful two-particle states in the continuum. In what follows we discuss the problem of low-lying $0^+$ excitations and their influence upon the neutron halo.

The possibility of low-lying excitations in $^{11}$Li, with energies below the first experimentally measured excitation at 1.2 MeV, was considered in Ref. [8]. In this reference it is argued that a low-lying narrow resonance at around 0.2 – 0.4 MeV would be consistent with the momentum distribution of the $^{11}$Li fragments. To our knowledge the existence of such a low-lying resonance in $^{11}$Li was not investigated in microscopical calculations so far.

Within the CXSM two-particle resonances are easy to calculate since they appear as a result of the diagonalization of the Hamiltonian (which in our case reduces to the solution of the dispersion relation) in the complex energy plane. Thus we found that the first excited state (i. e. the state $0^+_2$) appears at the complex energy $(0.202,-0.137)$ MeV. The corresponding wave function consists of nearly 100% $p$-states, with a small admixture of $s$-states.

It is interesting to analyse how this state is built up by the two-body interaction starting from the zeroth-order configuration $(0p_{1/2})^2$. For this we increased the interaction gradually starting from $G=0$, as seen in Fig. 3. As the attractive interaction increases the resonance becomes narrower and approaches threshold, as expected from perturbation theory. However, a point is reached where continuum configurations become important and the resonance widens. This happens at $G=0.0005$ MeV in the figure. Up to this point the resonance is a purely $(0p_{1/2})^2$ state and, therefore, it is localized inside the nucleus. That is, it is a physically meaningful resonance. But from here on other configurations become important. These configurations are overwhelmingly those where one of the particles moves in the continuum and the other in the resonance $0p_{1/2}$. More specifically, in the two-particle resonance that we are studying the most important states in the continuum are those corresponding to $p_{1/2}$ waves with energies corresponding to the points on the segment $B_1 - B_2$ in Fig. 1. At $G = G_0=0.00194$ MeV, corresponding to the $G$-value fitting the ground state, there is a strong mixing with the continuum configurations. Increasing $G$ farther the configuration $(0p_{1/2})^2$ looses its importance, the resonance is split in a number of pieces and eventually dissolves into the continuum. However, since these configurations virtually include only $p$-waves the wavefunctions still consist of only $p$-states.

The analysis that we have done so far is based upon the assumption that the $p$ resonance in $^{10}$Li is located at 200 keV. To see the influence of this resonance on the structure of the halo we will now analyse the ground and the excited states of $^{11}$Li by using the 500 keV case. For this we adopted the Woods-Saxon depth $V_0 = 35.366$ MeV for odd $l$-values, keeping all other parameters as before. We thus obtained the energy $(0.470,-0.197)$ MeV for the state $0p_{1/2}$. The state $0p_{3/2}$, belonging to the core, is found now at -2.016 MeV. The other odd $l$-value poles lie beyond the range of energies included here. But, nevertheless, we have checked that they do not affect the results.

We kept the two-particle interaction used in the previous case, except that the strength necessary to adjust the energy of $^{11}$Li(gs) is now $G = 0.00694$ MeV.
As before, we found that on the real energy axis the ground state wave function is spread in many components. The largest of them lie close to threshold for the configurations $s_{1/2} \otimes s_{1/2}$ and around 1 MeV for the $p_{1/2} \otimes p_{1/2}$ configurations. The wave function consists of 49 % $s$-states, 39 % $p$-states and 12 % $d$-states, which is also within the range of accepted values [2,3].

Since the position of the $p_{1/2}$ pole seems likely to correspond to the present 500 keV case [3], we will analyse here the effects of the antibound and the Gamow poles upon the ground state of $^{11}$Li by using the contours of Fig. 1. We therefore present in Table I the contribution of different configurations to that ground state. The corresponding complex amplitudes depend on the chosen contours and have no direct physical meaning. But the total content of a given partial wave in the bound ground state wave function, which is a physical quantity, does not depend upon the chosen contour. From Table I we can see that for the $p$ and $d$ waves the configurations are built mainly on the corresponding Gamow resonances. The situation is different for the $s$-wave since apart from the configurations built upon the antibound state there is also an important contribution coming from the complex scattering states. This contribution is given mainly by those $s$ scattering states located on the segments $(0, 0) - A_1$ and $A_1 - A_2$ of Fig. 1, which are the closest to the antibound state.

Up to this point there is not much difference between the 200 and the 500 keV cases, which may explain why various studies of the halo structure of $^{11}$Li(gs) with the common feature of having low-lying $s$- and $p$-states, provide similar results [3]. This is because the wave function of $^{11}$Li(gs) is mainly controlled by low-spin single-particle states lying close to the continuum threshold. The exact positions of the resonances do not influence the wave function very much. Even the two-particle interaction (if reasonable) is not so important in the evaluation of the properties of $^{11}$Li(gs) since this state is a Cooper pair [18] and as such is mainly induced by the Pauli principle acting upon the valence particles and those in the core. However, the two-body interaction as well as the position of the single-particle poles may have a fundamental importance to determine the physically meaningful excited states. The states arising from the particles moving in the continuum are not localized inside the nucleus and, therefore, will be weakly affected or not affected at all by the interaction. This can be seen in Fig. 4, where we present the evolution of the state $0^+_2$ as a function of the strength $G$. The $0p_{1/2}$ resonance is now wider and higher in energy than before. As a result, the point corresponding to $G=0$ in the figure is closer to points coming from the continuum contour. Yet, these continuum states do not seem to affect the resonance as $G$ increases. That is, the behaviour of the resonance as $G$ is varied is very similar to that in Fig. 3 as well as to resonances in non-halo nuclei [10]. The reason for this is that those points, label "s-states" in the figure, correspond to configurations of the type $cs_{1/2}ls_{1/2}$, where "c" labels points in the segments $(0, 0) - A_1$ and $A_1 - A_2$ of Fig. 1. The overlap between these configurations and the mainly $(0p_{1/2})^2$ configuration of the resonance is small. Only at large values of $G$ (above $G=0.004$ MeV in the figure), the resonance starts to feel the presence of the continuum states. The remarkable feature of the figure is the sudden turning down of the curve corresponding to the physical resonance at $G=0.005$ MeV. As $G$ increases in this region the continuum plays a mounting role. As in the 200 keV case above, the most important of the continuum configurations are those in which one particle moves in the continuum and the other in the $0p_{1/2}$ resonance. There could be many comparatively large configurations of these type and it would not be useful to give all of them. More instructive
is to show their contribution to the normalization of the wave function in this case where, in contrast to the ground state case of Table I, the zeroth order energies are not very close to the energy of the state $0^+_2$. We thus define $S(l)$ as the sum of the squares of the amplitudes corresponding to configurations where at least one of the two particles moves in continuum states. On the real energy axis the sum of $S(l = 1)$ and $X^2((0p_{1/2})^2)$, where $X$ is the wave function amplitude, is the probability of the p-content of the wave function. This is a quantity that we evaluated above for the ground state. The dependence of these quantities upon $G$ corresponding to the state $0^+_2$ is shown in Table II. Since we are studying states with complex energies the numbers $S$ as well as $X^2$ are complex in this Table. Moreover, their absolute values could be larger than 1 although the sum of all possible l-contributions is normalized to (1,0). This is a good example of the non-Hermitian character of the Berggren metric.

One sees in this Table that the two-particle resonance starts to mix with the continuum at $G= 0.005$ MeV and at $G=G_0$ it is composed mainly of continuum configurations. Therefore, at this point it has already lost its localization features. It has become a part of the continuum background.

We are now in a position to recognize other systems where halos may be present. We thus looked for nuclei which may be considered shell-model cores lying on the neutron drip line with low-lying single-particle resonances carrying low-spin. Following the trend of single-particle states in the relativistic mean field calculations we found that $Z=20$, $N=50$ may be such a core. In order to simulate the order of the single particle states given by the relativistic calculations we used a Wood-Saxon potential defined by $a = 0.67$ fm, $r_0 = 1.27$ fm, $V_0 = 39$ MeV and $V_{so} = 22$ MeV. With this potential the antibound $2s_{1/2}$ state (note that $n=2$) appears again at -0.050 MeV. But now the next valence shells are $1d_{5/2}$ at $(0.469,-0.048)$ MeV, $1d_{3/2}$ at $(2.080,-1.525)$ MeV, $0g_{7/2}$ at $(6.739,-0.738)$ MeV and $0h_{11/2}$ at $(5.344,-0.102)$ MeV. The states in the core are ordered as usual. As expected, the highest of these is the state $0g_{9/2}$, lying at -2.276 MeV. Using the same separable interaction as before and assuming again that the ground state of $^{72}$Ca lies at -295 keV, we obtained for the strength of the interaction the value $G_0=0.00174$ MeV. Close to this $G_0$ value we found also a low-lying two-particle resonance with the energy of about $(0.550,-0.350)$ MeV. The behaviour of this $0^+_2$ resonance as a function of $G$ is very similar to the 500 keV case of Fig. 4. The discussion performed there is also valid here and, therefore, we will not analysed this rather academic case farther. But it is important to point out that in this and the other cases presented here, we have been careful to choose contours that leave the region around the two-particle resonances in the complex energy plane free of continuum configurations. We thus established an “allowed” region [10]. Otherwise the two-particle resonance would be embedded in a see of continuum states, making the calculations difficult and the evaluated quantities unreliable.

In conclusion, we have presented in this letter a formalism that treats the antibound states exactly in the same fashion as bound states and Gamow resonances in the framework of a shell model scheme in the complex energy plane. We found, as expected, that antibound states lying close to the continuum threshold are of a fundamental importance to build up the halo. But we also found that an excited low-lying two-particle resonance may exist in halo nuclei. For the case of $^{11}$Li this low-lying resonant appears in the energy range of $0.2 - 0.5$ MeV. This energy range is the same as the one suggested in Ref. [8] in connection
to the momentum distribution of $^{11}$Li fragments. However, the calculations exhibit a very drastic change in the structure of the resonant excited state when the strength of the force is approaching the value used for the determination of the ground state. Do to this it is rather difficult to conclude whether this state is a physical resonance or not. Further efforts directed to the measuring of the eventual low-lying resonance excitations in $^{11}$Li would be essential in order to settle this issue.

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FIGURES

FIG. 1. In the CXSM the contour is replaced by integration points [11]. The triangles indicate the points belonging to the $l = 0$ contour used to incorporate the antibound state in the Berggren basis. The circles indicate the points corresponding to the contour embracing the resonances with $l \neq 0$.

FIG. 2. Localization $L(E)$, Eq. 2, in the presence of low-lying antibound (a) and bound (b) $s$-states. The numbers labeling the curves are the energies of the poles in MeV.

FIG. 3. Evolution of the energies of the two-particle resonance $^{11}\text{Li}(0^+_2)$ as a function of $G$ (in MeV) for the 200 keV case. The numbers labeling the open circles are the values of $G \times 10^4$ MeV. The strength adjusted to obtain the ground state is $G_0 = 19.4 \times 10^{-4}$ MeV.

FIG. 4. As Fig. 3 for the 500 keV case except that the numbers are the values of $G \times 10^3$ MeV and $G_0 = 6.94 \times 10^{-3}$ MeV.
TABLES

**TABLE I.** The contribution of partial wave configurations \((s_{1/2})^2\), \((p_{1/2})^2\) and \((d_{5/2})^2\) to the ground state wave function calculated in the complex energy plane. For each partial wave are given the square amplitude of the pole-pole term and the sum of the square amplitudes corresponding to the pole-scattering and scattering-scattering terms. The total contribution of each partial wave is given in the last line.

|             | \((s_{1/2})^2\)       | \((p_{1/2})^2\)       | \((d_{5/2})^2\)       |
|-------------|-----------------------|-----------------------|-----------------------|
| pole-pole   | (12.936, -0.039)      | (0.642, -0.204)       | (0.127, 0.011)        |
| pole-scat.  | (-29.365, 0.079)      | (-0.279, 0.221)       | (-0.031, -0.018)      |
| scat.-scat. | (16.921, -0.040)      | (0.022, -0.017)       | (-0.002, 0.007)       |
| total       | (0.492, 0.0)          | (0.385, 0.0)          | (0.094, 0.0)          |

**TABLE II.** Square of the wave function amplitude X\(((nlj)^{2})\), where \((nlj)\) indicates the quantum number of the single-particle resonance, and the sum \(S(l)\) corresponding to the physical resonance of Fig. 4. The values of \(G\) are in MeV and \(G_0 = 0.00694\).

| \(G\)   | \(X^2((ls_{1/2})^{2})\) | \(S(0)\)                  | \(X^2((0p_{1/2})^{2})\) | \(S(1)\)                  |
|---------|--------------------------|---------------------------|--------------------------|---------------------------|
| 0.001   | (0.00, 0.00)              | (-0.00, 0.00)             | (1.00, 0.00)             | (0.00, -0.00)             |
| 0.003   | (0.05, 0.02)              | (-0.10, 0.10)             | (1.06, 0.06)             | (0.00, -0.05)             |
| 0.005   | (-0.01, 0.11)             | (-0.25, -0.12)            | (0.59, 0.30)             | (0.47, -0.19)             |
| \(G_0\) | (-0.04, 0.02)             | (-0.00, -0.11)            | (0.23, 0.18)             | (0.74, -0.12)             |
| 0.008   | (-0.03, 0.01)             | (0.01, -0.08)             | (0.18, 0.14)             | (0.79, -0.10)             |
