Revisiting $\gamma\gamma$ absorption for UHE photons with $l\bar{l}$ and $q\bar{q}$ production

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We revisit the estimate of cosmic opacity to ultra-high energy (UHE) $\gamma$ rays ($E > 10^{18}$ eV) considering, besides the $\gamma\gamma \rightarrow e^+e^-$ and $\gamma\gamma \rightarrow e^+e^-e^+e^-$ processes, the production of other leptons ($l$) and quarks ($q$). The most important channels are those leading to the production of light $q\bar{q}$ ($q = u, d$) followed by hadronization and $\mu^\pm$, while the production of $\tau^\pm$ and $s\bar{s}, c\bar{c}$ followed by hadronization are subdominant. We consider also the single neutral meson production but it does not play a significant role. We recalculate the optical depth by considering all these processes, and we find that at UHE absorption is currently underestimated. For a distance $\sim 130$ Mpc (approximately equal to the GZK radius) the photon survival probability is smaller by about one order of magnitude with respect to the current estimates.

Keywords: ultra-high energy photons; photon-photon interaction; gamma rays; leptons; quarks

I. INTRODUCTION

Propagation of $\gamma$ rays in the Universe is severely limited by absorption through the interaction with soft radiation backgrounds, leading to the production of particle/antiparticle pairs [1]. The most probable reaction is the $e^\pm$ pair production $\gamma\gamma \rightarrow e^+e^-$, which starts to become important above few tens of GeV for sources at cosmological distances. Here the main targets are the IR-optical-UV photons belonging to the so called extragalactic background light (EBL), the fossil record of light produced by stars along the whole cosmic history (e.g. [2]). At energies above $\sim 10^{14}$ eV the major source of opacity is instead the cosmic microwave background (CMB), which restrics the mean free path of $\gamma$-rays of energy $\sim 10^{15}$ eV to few tens of Megaparsecs (e.g. [3]). Ultra-high energy $\gamma$ rays (UHE; energies above $10^{18}$ eV) interact with the Rayleigh-Jeans tail of the CMB spectrum and the cosmic radio background (RB). The potential detection of UHE photons is quite
relevant, since they provide a natural probe for several fundamental processes [4, 5]. In particular, UHE photons are the by-product of the photo-meson reactions which restrict the propagation of UHE cosmic rays to \( \simeq 100 \) Mpc (the so-called GZK radius) [6]. Currently, only upper bounds on the UHE flux have been derived [4]. Because photons produced at cosmic distances suffer from absorption during their propagation to the Earth, any comparison of these upper limits with theoretical expectations must rely on the accurate estimate of the cosmic opacity at UHE [6, 7].

So far, the optical depth \( \tau_\gamma \) at UHE has been modeled by using specific codes [7] which, however, include only pure QED effects (and often consider only the \( \gamma \gamma \rightarrow e^+e^- \) process). The aim of this Letter is to investigate the potential relevance of quark-producing and lepton-producing reactions, which are possible for photons in such an energy range because of the large energies available in the center of mass (c.m.) frame \( S_{\text{CM}} \). These processes have been discussed several years ago in the framework of effective treatments (e.g. [8] and references therein). Here, we reconsider them in the light of the state-of-the-art knowledge.

The Letter is organized as follows. In Sect. II we review the pair production process in general, in Sect. III and Sect. IV we sketch the electron double pair production and the single neutral meson production, respectively, in Sect. V we compute the optical depth for \( \gamma \) rays, in Sect. VI we present our results, while in Sect. VII we summarize our conclusions.

II. PAIR PRODUCTION CROSS SECTION

In order to get a feeling of what happens at different energy regimes, we recalculate the cross section \( \sigma \) of the process \( \gamma \gamma \rightarrow f\bar{f} \), where \( \gamma \) is a photon and \( f \) is a generic charged fermion of mass \( m \). The process \( \gamma \gamma \rightarrow f\bar{f} \) is related to the Compton scattering by crossing symmetry [9]. We compute \( \sigma_{\gamma\gamma\rightarrow f\bar{f}} \) in \( S_{\text{CM}} \) so that the two incident photons have four-momenta \( k_1 \equiv (\omega, \vec{\omega}) \) and \( k_2 \equiv (\omega, \vec{\omega}) \) and the outgoing fermions have four-momenta \( p_1 \equiv (\omega, \vec{p}) \) and \( p_2 \equiv (\omega, -\vec{p}) \), where \( \omega \) is the photon energy and \( \vec{p} \) is the fermion three-momentum (see Fig. 1). Integrating over the total solid angle \( \Omega \) we get the total cross section [10]

\[
\sigma_{\gamma\gamma\rightarrow f\bar{f}}(\omega, p) = C \frac{4\pi\alpha^2 Q^4}{4\omega^2} \left\{ \left( 1 + \frac{m^2}{\omega^2} - \frac{1}{2} \frac{m^4}{\omega^4} \right) \ln \left[ \frac{(\omega + p)^2}{m^2} \right] - \frac{p}{\omega} \left( 1 + \frac{m^2}{\omega^2} \right) \right\}, \tag{1}
\]
Fig. 2: Cross section $\sigma_{\gamma\gamma \rightarrow ff}$ for the production of $e^+e^-$ and of $u\bar{u}$. Reported are also the thresholds above which the processes are allowed. The real threshold for the process $\gamma\gamma \rightarrow u\bar{u}$ is that to produce $2\pi^0$ since the quarks cannot exist free and must be confined, $\pi^0$ being the lowest massive particle wherein the up quark can be confined. $\sigma_{\gamma\gamma \rightarrow u\bar{u}}^{\text{max}}$ is the maximal cross section for the process $\gamma\gamma \rightarrow u\bar{u}$ taking the minimal realistic mass for the up quark propagator, $m = m_q \simeq 50$ MeV, while $\sigma_{\gamma\gamma \rightarrow u\bar{u}}^{\text{min}}$ is the minimal cross section for the process $\gamma\gamma \rightarrow u\bar{u}$ taking the mass for the up quark propagator, $m = m_q = m_{\pi^0} \simeq 135$ MeV (more about this in the text).

where $\alpha$ is the fine-structure constant, $Q$ is the relative charge of the fermion with respect to the electron elementary charge, and $C$ is the number of particle species for single fermions e.g. $C = 1$ for leptons while $C = 3$ for quarks. The fermion dispersion relation $\omega^2 = p^2 + m^2$ allows as to express $\sigma_{\gamma\gamma \rightarrow ff}$ of Eq. (1) in terms of $\omega$ only, and employing the Lorentz-invariant Mandelstam variable $s = (k_1 + k_2)^2 = 4\omega^2$ we can rewrite $\sigma_{\gamma\gamma \rightarrow ff}$ in terms of $s$ alone. As an example, the behavior of $\sigma_{\gamma\gamma \rightarrow ff}$ for the production of $e^+e^-$ and of $u\bar{u}$ is reported in Fig. 2 while its analytical expression at high energies reads

$$\sigma_{\gamma\gamma \rightarrow ff}(s)\bigg|_{s \gg m^2} \simeq C \frac{4\pi\alpha^2Q^4}{s}\left[\ln\left(\frac{s}{m^2}\right) - 1\right],$$

which is consistent with the one reported e.g. in [11] with $C = 1$ and $Q = 1$.

Actually, as far as quarks are concerned we focus our attention on light quarks, here to be specific only $u\bar{u}$ since – owing to Eq. (1) – they give a contribution larger than $s\bar{s}$ and $c\bar{c}$ (more
about the last two processes in Sect. VI), but everything that follows regarding $u\bar{u}$ applies equally well to the considered quark-antiquark pairs and the other lepton-antilepton pairs.

In order to evaluate the optical depth of UHE photons interacting with soft background photons, we work in a generic inertial reference frame $S$, wherein $E$ denotes the energy of the hard photon and $\epsilon$ that of the soft background photon. As a result, $s$ reads

$$s = 4\omega^2 = 2E\epsilon(1 - \cos \varphi),$$

(3)

where $\varphi$ is the angle between the two photon three-momenta. In order to translate Eq. (1) from $S_{CM}$ to $S$, we insert the fermion dispersion relation and Eq. (3) into Eq. (1), thereby obtaining Eq. (1) as written in $S$ and denoted by $\sigma_{\gamma\gamma\rightarrow f\bar{f}}(E,\epsilon,\varphi)$, which will be used to compute the corresponding contribution $\tau_{\gamma\gamma\rightarrow f\bar{f}}$ to $\tau_{\gamma}$.

Let us proceed to specialize the cross section $\sigma_{\gamma\gamma\rightarrow f\bar{f}}(E,\epsilon,\varphi)$ to leptons and quarks.

• For leptons $Q = 1$, $C = 1$ and $m = m_l$ is the lepton mass. As it is clear from Fig. 2 – where $\sigma_{\gamma\gamma\rightarrow e^+e^-}(E,\epsilon,\varphi)$ is plotted – such a cross section is meaningful only above the c.m. energy threshold $s_{\text{min}} = 4m_l^2$, in order for the production of two leptons in the final state (in this case $e^+e^-$) to be kinematically allowed.

• For quarks the situation is more complex. In this case $C = 3$ (since quarks exist in three colors), while $Q = 2/3$ for $u, c$ quarks and $Q = 1/3$ for $d, s$ quarks. A critical issue concerns their mass. In fact, what is the real quark mass to be used is not a trivial question: non-perturbative QCD effects are expected to increase the bare quark mass so that the quark propagator become ‘dressed’ [12]. The actual value of the mass in question is still rather uncertain. In principle, the actual quark mass may span a range from its bare value up to the mass of the lightest meson containing the considered quark, since two of them are produced. However, values for the quark mass close to its bare mass appear as unrealistic because of the non-perturbative QCD effects. So, the realistic mass for the considered quark is believed to be not so far from the mass of the lightest meson containing the quark in question. In order to take into account such an uncertainty we consider a variation of the order of $\Lambda_{\text{QCD}}$. For instance, for light quarks we contemplate a mass range $m_q = (50 - 135)\text{ MeV}$, since the lightest meson containing the considered quarks is the neutral pion $\pi^0$ whose mass is $m_{\pi^0} \simeq 135\text{ MeV}$ [13]. This is the reason why in Fig. 2 we have plotted $\sigma_{\gamma\gamma\rightarrow u\bar{u}}^{\text{max},\text{min}}$ which takes into account the whole range of possible quark masses: $\sigma_{\gamma\gamma\rightarrow u\bar{u}}^{\text{max},\text{min}}$ possesses a similar analytical behavior as compared to $\sigma_{\gamma\gamma\rightarrow e^+e^-}$.

Another difference with respect to the case of leptons in the final state concerns the threshold for a reaction to be kinematically allowed. Indeed, for quarks the threshold $4m_q^2$ is unphysical, since free quarks do not exist. Thus, it is necessary to have a c.m. energy large enough to produce a pair of the lightest mesons containing the considered quark: namely, for the up quark we have to produce two $\pi^0$ so that the physical threshold becomes $4m_{\pi^0}^2$ (see Fig. 2) – which transforms the process $\gamma\gamma \rightarrow u\bar{u}$ into $\gamma\gamma \rightarrow \pi^0\pi^0$ from a phenomenological point of view [14].

III. DOUBLE PAIR PRODUCTION

At very high energies the double pair production cross section can be well approximated by [8]

$$\sigma_{\gamma\gamma\rightarrow e^+e^-} (s) = 6.45 \times 10^{-30} \left(1 - \frac{16m_l^2}{s}\right)^6 \text{ cm}^2,$$

(4)
where \( m_e \) is the electron mass. At lower energies a more accurate description of \( \sigma_{\gamma\gamma\rightarrow e^+e^-+e^-} \) is unnecessary since the single electron-positron pair production cross section dominates over \( \sigma_{\gamma\gamma\rightarrow e^+e^-+e^-} \).

IV. SINGLE NEUTRAL MESON PRODUCTION

We consider here the production of the neutral mesons \( \pi^0, \eta, \eta' \) and \( \eta_c \) via the \( \gamma\gamma \) channel.

For \( \pi^0 \) the dominant decay mode is into two photons following the process \( \pi^0 \rightarrow \gamma\gamma \). Here, we consider the inverse process of single \( \pi^0 \) production \( \gamma\gamma \rightarrow \pi^0 \) whose cross section is

\[
\sigma_{\gamma\gamma\rightarrow \pi^0}(s) = \frac{8\pi^2}{m_{\pi^0}^2} \Gamma_{\pi^0\rightarrow \gamma\gamma} \delta\left(s - m_{\pi^0}^2\right),
\]

where \( \Gamma_{\pi^0\rightarrow \gamma\gamma} = 7.82 \text{ eV} \) is the experimental \( \pi^0 \) decay rate \[13\]. Obviously, the \( \pi^0 \) production cross section includes a Dirac delta function because of the four-momentum conservation: in the center of mass frame the total energy of the two incident photons must be exactly equal to \( m_{\pi^0} \).

For \( \eta, \eta' \) and \( \eta_c \) the situation is similar apart from the fact that not always the \( \gamma\gamma \) channel represents the dominant decay mode. In Eq. [5] we must replace \( m_{\pi^0} \) and \( \Gamma_{\pi^0\rightarrow \gamma\gamma} \) with the corresponding quantities for \( \eta, \eta' \) and \( \eta_c \). For the masses we take \( m_\eta = 548 \text{ MeV}, m_{\eta'} = 958 \text{ MeV} \) and \( m_{\eta_c} = 2984 \text{ MeV} \) and for the \( \gamma\gamma \) decay rates we have \( \Gamma_{\eta\rightarrow \gamma\gamma} = 0.51 \text{ keV}, \Gamma_{\eta'\rightarrow \gamma\gamma} = 4.3 \text{ keV} \) and \( \Gamma_{\eta_c\rightarrow \gamma\gamma} = 5 \text{ keV} \[13\].

V. OPTICAL DEPTH

The optical depth \( \tau_{\gamma}(E_0, z_s) \) at redshift \( z_s \) of a hard photon of energy \( E = (1 + z_s)E_0 - E_0 \) being the observed energy in \( S \) – interacting with background soft photons of energy \( \epsilon \) is computed by multiplying the background spectral number density \( n_\gamma(\epsilon, z) \) with the cross section of two interacting photons \( \sigma_{\gamma\gamma}(E(\epsilon), \epsilon(z), \varphi) \) and next integrating over \( z, \varphi \) and \( \epsilon(z) \) \[1, 15\]. Thus, we have

\[
\tau_{\gamma}(E_0, z_s) = \int_0^{z_s} \frac{dz}{dz} \int_{-1}^1 d(\cos \varphi) \frac{1 - \cos \varphi}{2} \int_{\epsilon_{\min}(E(\epsilon), \varphi)}^\infty \epsilon n_\gamma(\epsilon, z) \sigma_{\gamma\gamma}(E(\epsilon), \epsilon, \varphi) \, d\epsilon,
\]

where

\[
\frac{dz}{dz} = \frac{1}{H_0 (1 + z) \left[ \Omega_\Lambda + \Omega_M (1 + z)^3 \right]^{1/2}}.
\]

In the standard \( \Lambda \text{CDM} \) cosmological model we take for definiteness \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), while \( \Omega_\Lambda = 0.7 \) and \( \Omega_M = 0.3 \). In addition, we have

\[
\epsilon_{\min}(E, \varphi) \equiv \frac{2 m^2}{E (1 - \cos \varphi)},
\]

where for leptons \( m \) is the mass of the produced leptons, while, following the discussion of Sect. II, for quarks \( m \) is the mass of the lightest produced meson mass containing the considered quarks.

Once \( n_\gamma(\epsilon(z), z) \) is known, \( \tau_{\gamma}(E_0, z) \) can be computed exactly; generally the integration over \( \epsilon(z) \) in Eq. (6) must be performed numerically. Concerning \( n_\gamma(\epsilon(z), z) \) of the soft photon background we consider the EBL, the CMB and the RB. Many methods exist in the literature to estimate.
the EBL background but quite remarkably they substantially agree in the redshift range where they overlap, so that nowadays the EBL is known to a very good accuracy. We adopt here the model by Gilmore et al. [16] mainly because the values of \( n_\gamma(z, z) \) are tabulated. Similar results can be obtained e.g. by employing the model of Franceschini & Rodighiero [17]. For the CMB we consider the standard temperature value \( T = 2.73 \text{K} \) and concerning the RB we use the most recent available data with a low-frequency cutoff placed at 2 MHz [18].

VI. RESULTS

We are now in position to evaluate the contribution to the optical depth \( \tau_\gamma \) arising from all considered processes. For leptons the calculation of the optical depth \( \tau_{\gamma,\gamma \rightarrow \ell^\pm} (l = e, \mu, \tau) \) directly follows from Sect. II and Sect. V. We can get \( \tau_{\gamma,\gamma \rightarrow e^+ e^-} \) by combining Sect. III and Sect. V, and \( \tau_{\gamma,\gamma \rightarrow \pi^0, \eta, \eta', \eta_c} \) by combining Sect. IV and Sect. V. For quarks the question is not trivial: as briefly discussed in Sect. II non-perturbative QCD effects modify the effective quark mass in such a way that it is assumed to be close to the mass of the lightest meson containing it with an uncertainty estimated of the order of \( \Lambda_{\text{QCD}} \) (as we shall see, this uncertainty mostly affects the first family, moderately the second family and nothing at all the third family). Hence, we schematically consider: 50 MeV < \( m_q \) < \( m_{\pi^0} \simeq 135 \text{MeV} \) for light quarks \( (q = u, d) \), 250 MeV < \( m_q \) < \( m_K \simeq 495 \text{MeV} \) and 1500 MeV < \( m_q \) < \( m_D \simeq 1865 \text{MeV} \). We discard the bottom and top quarks, since they are totally irrelevant for our considerations.

In Fig. 3 we plot the optical depth at \( z_s = 0.03 \) (corresponding to a distance \( \simeq 130 \text{ Mpc} \)) of all processes considered so far, along with the total optical depth \( \tau_{\gamma,\text{tot}} \). The shadowed areas represent the uncertainty of \( \tau_\gamma \) due to the uncertainty on the quark masses.

At energies \( E_0 \gtrsim 10^{18} \text{eV} \) \( \tau_{\gamma,\text{tot}} \) starts to differ in a sizable way from the optical depth arising from the process \( \gamma \gamma \rightarrow e^+ e^- \) alone, which represents the dominant contribution at almost all energies. As we said, in the literature also \( \gamma \gamma \rightarrow e^+ e^- e^+ e^- \) has been considered (e.g. [7]): this process starts to become important at energies above \( E_0 \gtrsim 10^{18} \text{eV} \).

In order to figure out the importance of the considered processes besides \( \gamma \gamma \rightarrow e^+ e^- \), it is compelling to compute the photon survival probability \( P_{\gamma} \) – which is related to \( \tau_\gamma \) by \( P_{\gamma} = e^{-\tau_\gamma} \) – since this is the actual observable quantity. In the upper panel of Fig. 4 we plot the total photon survival probability \( P_{\gamma,\text{tot}} \) that takes into account all considered processes and the photon survival probability \( P_{\gamma,\gamma \rightarrow e^+ e^-} \) due to the process \( \gamma \gamma \rightarrow e^+ e^- \) only. For \( P_{\gamma,\text{tot}} \) we plot also the shadowed area with a similar meaning of Fig. 3. The lower and upper limits \( P_{\gamma,\text{tot}}^{\text{UL}} \) and \( P_{\gamma,\text{tot}}^{\text{LL}} \) are also shown: they are associated to the lowest and highest quark masses under consideration, respectively. The lower panel of Fig. 4 uses the same conventions of the upper panel and quantifies the importance of the new considered processes with respect to the process \( \gamma \gamma \rightarrow e^+ e^- \) alone. Thus, in this panel we plot the ratio between \( P_{\gamma,\gamma \rightarrow e^+ e^-} \) and \( P_{\gamma,\text{tot}} \).

We see that the new processes considered in this Letter are not negligible in the UHE band since at redshift \( z_s = 0.03 \) (\( \simeq 130 \text{ Mpc} \)) they give a correction of \( \mathcal{O}(10\%) \) to the total optical depth \( \tau_{\gamma,\text{tot}} \) and more manifestly a decrease of the total survival probability \( P_{\gamma,\text{tot}} \) by a factor of \( \mathcal{O}(10) \).

In particular, as we can see from Fig. 3 the most important channels in addition to \( \gamma \gamma \rightarrow e^+ e^- \) and \( \gamma \gamma \rightarrow e^+ e^- e^+ e^- \) are those leading to the production of light \( q\bar{q} \) \((q = u, d)\) followed by hadronization and \( \mu^+ \mu^- \), while the production of \( \tau^\pm \) and \( ss, c\bar{c} \) followed by hadronization are less important. The single neutral meson production that takes into account \( \pi^0, \eta, \eta' \) and \( \eta_c \) does not play a significant role as compared to the above-mentioned processes: only around \( E_0 \sim 5 \times 10^{18} \text{eV} \) it represents the leading contribute of those that must be added to the processes \( \gamma \gamma \rightarrow e^+ e^- \) and \( \gamma \gamma \rightarrow e^+ e^- e^+ e^- \).
FIG. 3: Optical depth for all considered processes for a source at $z_s = 0.03$ ($\approx 130$ Mpc). The colored areas represent the uncertainty on the optical depth related to the channels $\gamma\gamma \rightarrow q\bar{q}$ caused by the uncertainty affecting the quark masses. See the text for more details and for the effective quark masses adopted for the curves.

VII. DISCUSSION AND CONCLUSION

In this Letter we have investigated the relevance of quark- and lepton-producing reactions and the single neutral meson production process – beyond the processes $\gamma\gamma \rightarrow e^+e^-$ and $\gamma\gamma \rightarrow e^+e^-e^+e^-$ – for the transparency of UHE photons. As far as the quark sector is concerned, non-perturbative QCD effects complicate the calculation and give rise to some uncertainties in the final $\tau_{\gamma,\text{tot}}$ or equivalently in $P_{\gamma,\text{tot}}$. Yet, our results are sizable even in the most conservative case (assuming the largest value for the quark masses). Specifically, $\tau_{\gamma,\text{tot}}$ changes by $\mathcal{O}(10\%)$ and $P_{\gamma,\text{tot}}$ is dramatically modified of a factor of $\mathcal{O}(10)$ at $z_s = 0.03$. This latter fact is extremely important for the calculation of upper limits of UHE photon flux (e.g. [4]), which should be modified according to our findings.
FIG. 4: Survival probability $P_\gamma$ for UHE photons at $z_s = 0.03$ ($\approx 130$ Mpc). In the upper panel the total photon survival probability $P_{\gamma,\text{tot}}$ (with area of uncertainty caused by uncertainty about quark masses and lower and upper limits $P_{\gamma,\text{tot}}^{\text{LL}}$ and $P_{\gamma,\text{tot}}^{\text{UL}}$) and the photon survival probability $P_{\gamma,\gamma\gamma\rightarrow e^+e^-}$ due to the process $\gamma\gamma \rightarrow e^+e^-$ are plotted. In the lower panel the ratio between $P_{\gamma,\gamma\gamma\rightarrow e^+e^-}$ and $P_{\gamma,\text{tot}}$ is drawn by using the same conventions of the upper panel.

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Since the mass of the charged pions $\pi^+, \pi^-$ is similar to the one of $\pi^0$ also the process $\gamma\gamma \rightarrow \pi^+\pi^-$ may occur at comparable energies. However, it is not important for us what happens after the $u\bar{u}$ production. What matters for us is only the energy threshold which is similar for the two processes.

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