Partially Strong $WW$ Scattering

Kingman Cheung\textsuperscript{1,2}, Cheng-Wei Chiang\textsuperscript{3,4}, Tzu-Chiang Yuan\textsuperscript{2}

\textsuperscript{1} Department of Physics, National Tsing Hua University, Hsinchu, Taiwan
\textsuperscript{2} Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan
\textsuperscript{3} Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chung-Li, Taiwan
\textsuperscript{4} Institute of Physics, Academia Sinica, Taipei, Taiwan

(Dated: April 14, 2008)

What if only a light Higgs boson is discovered at the CERN LHC? Conventional wisdom tells us that the scattering of longitudinal weak gauge bosons would not grow strong at high energies. We show that this is not always true. In some composite models, two-Higgs-doublet models, or even supersymmetric models, the presence of a light Higgs boson does not guarantee the complete unitarization of the $WW$ scattering. After the partial unitarization by the light Higgs boson, the $WW$ scattering becomes strongly interacting until it hits one or more heavier Higgs bosons or other strong dynamics. We analyze how the LHC experiments can reveal this interesting possibility of partially strong $WW$ scattering.

PACS numbers: 14.80.Bn, 14.80.Cp, 12.60.Fr, 12.15.Ji

Introduction – The CERN Large Hadron Collider (LHC) will commence soon to uncover the mystery of electroweak symmetry breaking (EWSB). The ultimate goal of the LHC is to search for the Higgs boson and hopefully any new physics beyond the standard model (SM). Physicists have been exciting about mapping new observations to the parameter spaces in various models, known as the “inverse LHC problem.” However, one may anticipate that only one light Higgs boson is found in the first few years of LHC run. This is perhaps one of the most pessimistic scenarios. A light Higgs boson $h$ of mass $m_h \lesssim 130$ GeV can be discovered through the $\gamma\gamma$ or $b\bar{b}$ modes. Since this mass is below the $WW$ or $ZZ$ threshold, it would be hard to probe how much this light Higgs boson is directly linked to EWSB. Several recent works have suggested precision measurements in the branching ratios of the light Higgs boson \textsuperscript{1,3} and $W_L W_L$ scattering \textsuperscript{1,3} to unravel the nature of EWSB.

In this Letter, we propose to use the scattering of longitudinal weak gauge bosons to probe whether the light Higgs boson completely or just partially unitarizes the scattering amplitudes. Longitudinal weak gauge boson scattering is an old idea \textsuperscript{4} and it has been used to impose a unitarity bound on the mass of a heavy Higgs boson. At high energies, the longitudinal components of the weak gauge bosons recall their identities as the Goldstone bosons of the EWSB sector \textsuperscript{3}. The scattering amplitudes of these Goldstone bosons with purely gauge contributions grow with energy as $s/m_W^2$, where $s$ is the squared center-of-mass (CM) energy of the $W_L W_L$ system. Here we used $W_L$ to generically denote either a $W$ or $Z$ boson, unless otherwise stated. In the SM with a light Higgs boson, the amplitude will be completely unitarized by the Higgs boson. Once $\sqrt{s}$ goes beyond the light Higgs boson mass, the scattering amplitude will no longer grow like $s/m_W^2$. If the SM with a light Higgs by itself were indeed an ultraviolet (UV) complete theory, that would be our final prediction albeit a boring one. However, many issues such as the fine-tuning problem in the Higgs boson mass, nonzero neutrino masses, dominant dark matter and dark energy contents in the Universe do not have easy solutions within the SM if not impossible. They all suggest that new physics must be involved in order to solve some or all of these puzzles.

In many extensions of the SM, e.g., two-Higgs-doublet model (2HDM), little Higgs model, etc, there is usually one light Higgs boson. However, the light Higgs boson may not be fully responsible for the symmetry breaking, so that longitudinal $W_L W_L$ scattering is only partially unitarized by the light Higgs boson. Such an idea was recently mentioned first in Ref. \textsuperscript{1} and then in Ref. \textsuperscript{2}. Terms effectively scaling like $s/m_W^2$ in the scattering amplitude comes back such that it becomes strong after hitting the light Higgs pole. At a sufficiently high energy, there will be the other part of EWSB sector, e.g., the heavier Higgs boson of the 2HDM or the UV completion of the little Higgs models, to eventually unitarize the $W_L W_L$ scattering. Nonetheless, if the scale of this UV part is far enough from the light Higgs boson, the onset of strong $W_L W_L$ scattering between the light Higgs mass and the UV scale should be discernible at the LHC. The main result of this work shows that longitudinal weak gauge boson scattering can indeed provide a useful means to probe the nature of EWSB associated with a light Higgs boson.

Methodology – In the SM, the $hWW$ coupling is $g_{hWW}^W = g m_W g^{\mu\nu}$, where $g$ is the SU(2) gauge coupling constant. For a concrete example consider the scattering of $W_L^+ W_L^- \to W_L^+ W_L^-$, which proceeds through the $t$- and $s$-channels of $\gamma$ and $Z$ exchanges, the 4-point vertex, and the $s$- and $t$-channels of Higgs exchanges. The longitudinal polarization of the $W$ boson can be expressed as $e^L_\mu(p) = p^\mu/m_W + v^\mu(p)$ with $v^\mu(p) \simeq -m_W/(2p^2)(p^+ - p^-) \sim O(m_W/E_W)$. In the CM system of $W_L^+(p_1) W_L^-(p_2) \to W_L^+(k_1) W_L^-(k_2)$, one
can choose $v^\mu(p_1) = -2(m_W/s)p_1^\mu$, and so on. The sum of the amplitudes of all gauge diagrams is, in the high energy limit,

$$i\mathcal{M}_{\text{gauge}} = -i\frac{g^2}{4m_W^2} u + O\left((E/m_W)^0\right), \tag{1}$$

where $E$ denotes the scattering energy. Note that the quartic term proportional to $E^4/m_W^4$, naively expected from the 4-point vertex is canceled by the $\gamma$- and $Z$-exchange diagrams. On the other hand, the sum of the two Higgs diagrams is

$$i\mathcal{M}_{\text{Higgs}} = -i\frac{g^2}{4m_W^2} \left[ \frac{(s - 2m_W^2)^2}{s - m_h^2} + \frac{(t - 2m_W^2)^2}{t - m_h^2} \right] \simeq -i\frac{g^2}{4m_W^2} u, \tag{2}$$

in the limit of $s \gg m_h^2, m_W^2$. Thus, the bad energy-growing term is delicately canceled between the gauge diagrams and the Higgs diagrams. This is a well-known fact in the SM. However, in some extended models that the light Higgs boson has only a fraction of the SM coupling strength with the gauge bosons, one expects the gauge amplitude to keep growing with $s$ after hitting the light Higgs pole.

Given our ignorance of what may lie beyond the SM, we follow the approach adopted by recent studies to parametrize the coupling $g_{hWW}$ as a fraction of its SM value. As a result, the Higgs amplitude in Eq. (2) becomes $\delta$ times the SM value. For small enough $\delta$, the total scattering amplitude will grow after the light Higgs pole due to incomplete cancellation of the bad high-energy behavior terms. This is true even for a rather large $\delta = 0.9$. We show in Fig. 1(a) the exact scattering cross sections for $W^+_LW^-_L \to W^+_LW^-_L$ versus $\sqrt{s_{WW}}$, where we have assumed $m_h = 200$ GeV. For the SM case the sum of amplitudes converges to $O((E/m_W)^0)$ terms, and the cross section thus drops like $1/\sqrt{s_{WW}}$. When the size of the Higgs amplitudes deviates from the SM value, even with a small amount (say $\delta = 0.9$), the cross section will cease falling but start climbing instead around $\sqrt{s_{WW}} \lesssim 1$ TeV. It turns around at lower $\sqrt{s_{WW}}$ for smaller $\delta$'s. A similar behavior happens in the $W^+_LW^-_L \to Z_LZ_L$ channel, as shown in Fig. 1(b), where the turn-around occurs at even lower energies. Not so dramatic feature can also be shown for the nonresonant channels, such as $W^+_LW^-_L \to W^+_LW^-_L$ and $W^+_LZ_L \to W^+_LZ_L$, where the cross sections only climb up gradually. Such behavior can be readily observed at the LHC. We will give some realistic event numbers later to support our claim.

We also analyze the partial-wave coefficients to determine when the unitarity is violated. Consider a clean isospin $I = 2$ channel, $W^+_LW^-_L \to W^+_LW^-_L$. The sum of the gauge amplitudes $i\mathcal{M}_{\text{gauge}} = \frac{i g^2}{4m_W^2} \left[ \frac{u_t}{m_W^4} + O\left((E/m_W)^0\right) \right]$, in which the quartic terms proportional to $E^4/m_W^4$ are again canceled. The SM Higgs boson with a full strength $g_{hWW}^\text{SM}$ contributes $i\mathcal{M}_{\text{Higgs}} = -i\frac{g^2}{4m_W^2} \left[ \frac{u_t}{m_W^4} + O\left((E/m_W)^0\right) \right]$, which is valid for $|f|, |u| \gg m_W^2, m_h^2$. It is clear that the bad energy-growing terms cancel each other such that their sum behaves well at high energies. Now as before, we assume that the coupling $g_{hWW}$ is a fraction $\sqrt{\delta}$ of its SM value so that the cancellation is only partial. In the high energy

\footnote{We note that in models with an extra $Z'$ boson, it is possible to have the $hZZ$ coupling modified due to $Z - Z'$ mixing while the $hWW$ coupling remains intact. The unitarization of the longitudinal weak gauge boson scattering in such models is somewhat different from what we discuss in this work and deserves a separate study.}
energies, of these amplitudes will be presented elsewhere. At high

the full expressions of the amplitudes, instead of the sim-
ple, unitarity limit can be read off when each curve reaches

behavior for various \( \delta \). One can then check the unitarity limit as a function of

in which light Higgs boson couples to the vector bo-

2. W e show in Fig. 2 the partial-wave coefficients

partial decay widths of a light Higgs boson, which will

LHC signals – W e show the invariant mass spectrum in

mass energy \( \sqrt{s_{WW}} \) for various \( \delta = 0 - 0.9 \).

tan \( \beta \) is the ratio of the VEVs of the two doublets and

\( \alpha \) is the mixing angle of the two CP even neutral Higgs

towards jet-tagging. The jet-tagging and central-jet vetoing

and various backgrounds were summarized in Refs. \[8\],

the LHC for various scattering channels in Table I, with

efficiencies under optimized cuts were listed there too.

$|\Re(a_0^i)| < 1/2$. Note that the matrix element of the

\( I = 1 \) channel at high energy is an odd function of \( \cos \theta \)

such that the partial wave \( a_0^1 \) does not show any growing

behavior for various \( \delta \). The unitarity limits that would

be obtained from \( a_1^2 \) are significantly weaker than those

from \( a_0^0, a_0^2 \) due to \( P \)-wave suppression. The most severe

violation of unitarity is in the \( a_0^0 \) channel. For example,

unitarity is violated at \( \sqrt{s_{WW}} \approx 1.7 \text{ (2.7)} \) TeV for

\( \delta = 0.5 \text{ (0.8)} \). The LHC may not be able to directly

probe such high CM energies. While the growing behav-

ior of the scattering amplitudes should be discernible at

much lower energies.

Various models – The simplest example of partially

strong weak gauge boson scattering is the 2HDM \[2\],

in which light Higgs boson couples to the vector bo-

son with a strength \( g_{hWW} = \sin(\beta - \alpha) g_{hWW}^{SM} \), where

\( g_{hWW} \) is the coupling constant between the light

Higgs boson and the vector bosons. If the other neutral

Higgs boson \( H \) is much heavier,

the weak gauge boson scattering amplitudes will enjoy

their growths as \( s/m_H^2 \), for the energy between the

two Higgs boson masses. This heavier neutral Higgs bo-

son couples to the weak gauge boson with a strength

\( g_{hWW} = \cos(\beta - \alpha) g_{hWW}^{SM} \) such that it can unitarize

the rest of the growing amplitudes when \( s_{WW} > m_H^2 \). A

general 2HDM has enough room in the parameter space

to allow \( \sin(\beta - \alpha) \) to be small while keeping the other

Higgs boson \( H \) heavy. However, in minimal supersym-

metric standard models (MSSM) the heavier the heavy

Higgs boson \( H \) is, the closer to 1 the factor \( \sin(\beta - \alpha) \)

will be. As shown in Ref. \[6\], it is possible to achieve a

light Higgs boson with a small \( \sin(\beta - \alpha) \) while keeping

the other neutral ones relatively light as well. Thus, no

appreciable strong weak gauge boson scattering can be

observed in the MSSM.

In the strongly-interacting light Higgs model \[1\], a

composite-like model for the light Higgs boson is as-

sumed with the size of the ratio \( g_{hWW} / g_{hWW}^{SM} \) smaller

than 1. All other heavier degrees of freedom are inte-

grated out and the effects are parameterized as an effec-

tive Lagrangian with an explicit UV cutoff. The partial

widths of the light Higgs boson will be affected. Also,

the weak gauge boson scattering amplitudes described

by some higher dimensional effective operators will also

grow with \( s \) until the cutoff is reached. Similarly, in a

model of multi-scalar doublets \[2\] all the heavy Higgs

bosons can be integrated out to give corrections to the

partial decay widths of a light Higgs boson, which will

affect significantly its discovery modes at the LHC.

$W W \rightarrow \ell^+ \ell^- \nu \bar{\nu}$ (EW A) \[7\] to estimate the event rates, which is good

enough to demonstrate the main idea here. The stud-

ies of strongly-interacting weak gauge boson scattering

and various backgrounds were summarized in Refs. \[8\],

based on the techniques of central-jet vetoing and for-

ward jet-tagging. The jet-tagging and central-jet vetoing

efficiencies under optimized cuts were listed there too.

The event rates predicted in this work are to be mul-

tiplied by those efficiencies. It is easy to see that with

\( a_0^2 = \frac{1}{64\pi} \int_{-1}^{1} d\cos \theta \mathcal{M}(W_L^- W_L^+ \rightarrow W_L^- W_L^+) \),

One can then check the unitarity limit as a function of

\( \delta \). The partial-wave coefficient for the dominant S-wave

scattering is

\( i \mathcal{M}^{\text{gauge}} + i \mathcal{M}^{\text{Higgs}} \approx \frac{g^2}{4m_W^2} (u + t) (1 - \delta) \). \( (3) \)

where the superscript 2 denotes the isospin of the \( W_L^- W_L^+ \)

system. Besides this \( I = 2 \) channel, one can also study

the \( I = 0, 1 \) partial waves. Unitarity requires \( |\Re(a_0^1)| \leq 1/2 \). We show in Fig. 2

the partial-wave coefficients \( a_0^i \) \( (I = 0, 1, 2) \) versus \( \sqrt{s_{WW}} \) for various \( \delta = 0 \) \(-0.9 \). We use

the full expressions of the amplitudes, instead of the sim-

plified expression like Eq. \( (3) \), in the evaluations. Details

of these amplitudes will be presented elsewhere. At high

energies, \( a_0^0 \)'s are positive while \( a_0^2 \)'s stay negative.

The unitarity limit can be read off when each curve reaches

\( \Re(a_0^1) = \pm 1/2 \). Note that the matrix element of the

\( I = 1 \) channel at high energy is an odd function of \( \cos \theta \)

such that the partial wave \( a_0^1 \) does not show any growing

behavior for various \( \delta \). The unitarity limits that would

be obtained from \( a_1^2 \) are significantly weaker than those

from \( a_0^0, a_0^2 \) due to \( P \)-wave suppression. The most severe

violation of unitarity is in the \( a_0^0 \) channel. For example,

unitarity is violated at \( \sqrt{s_{WW}} \approx 1.7 \text{ (2.7)} \) TeV for

\( \delta = 0.5 \text{ (0.8)} \). The LHC may not be able to directly

probe such high CM energies. While the growing behav-

ior of the scattering amplitudes should be discernible at

much lower energies.
TABLE I: Event rates for longitudinal weak gauge boson scattering at the LHC with a yearly luminosity of 100 fb⁻¹ using the EWA for \(\delta = 1\) (SM), 0.9, 0.5 and 0 (No Higgs). Branching ratios for the leptonic final states are summed for \(\ell = e, \mu\). We set \(m_h = 200\) GeV and \(M_W^{\text{EW}} = 300\) GeV.

| Subprocess | Number of Events | \(\delta = 1\) (SM) | 0.9 | 0.5 | 0 (No Higgs) |
|------------|------------------|---------------------|-----|-----|-------------|
| \(W^+_LW^-_L \rightarrow W^+_LW^-_L \rightarrow \ell^+\nu\ell^-\nu\) | | 21 | 26 | 57 | 118 |
| \(W^+_LW^-_L \rightarrow W^+_LW^-_L \rightarrow \ell^+\nu\ell^-\nu\) | | 8 | 7 | 17 | 67 |
| \(W^+_LZ_L \rightarrow W^+_LZ_L \rightarrow \ell^+\nu\ell^-\nu\) | | 4 | 5 | 13 | 33 |
| \(W^+_LW^-_L \rightarrow Z_LZ_L \rightarrow \ell^+\ell^-\ell^+\ell^-\) | | 0.04 | 0.12 | 2 | 9 |
| \(W^+_LW^-_L \rightarrow Z_LZ_L \rightarrow \ell^+\ell^-\nu\nu\) | | 0.25 | 0.74 | 12 | 50 |
| \(Z_LZ_L \rightarrow Z_LZ_L \rightarrow \ell^+\ell^-\ell^+\ell^-\) | | 0.4 | 0.32 | 0.08 | 0 |
| \(Z_LZ_L \rightarrow Z_LZ_L \rightarrow \ell^+\ell^-\nu\nu\) | | 2.4 | 2 | 0.5 | 0 |

\(\delta = 0.5\) significant enhancement to the event rates relative to the SM can be realized.

To conclude, detailed studies of longitudinal weak gauge boson scattering at the LHC can provide useful hints of new physics at a higher scale, despite only a light Higgs boson may be discovered during the first few years at the LHC. If unitarity is only partially fulfilled by the light Higgs, the scattering cross sections must be growing as energy increases before it reaches the other heavier Higgs bosons or other UV completions to achieve the full unitarization. These partial growths of the cross sections can be discernible at the LHC provided that the UV part is at a high scale. This can be realized in two- or multi-Higgs-doublet models with large \(\tan\beta\) as was studied recently in Refs. [2,3], which proposed using the precision measurements of light Higgs boson decays to explore effects from new physics. Our approach of using longitudinal weak gauge boson scattering is complementary to those works but more direct. Discovery of a light Higgs together with positive observations of partially strong \(WW\) scattering at the LHC will definitely indicate that the SM is just an effective theory of some more fundamental theories.

**Acknowledgments.** KC would like to thank the Institute of Theoretical Physics at the Chinese University of Hong Kong for hospitality. This work was supported in part by the NSC of Taiwan and by the NCTS.

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