A First Taste of Dynamical Fermions with an $O(a)$ Improved Action.

M. Talevi\textsuperscript{a} *, for the UKQCD Collaboration

\textsuperscript{a} Department of Physics & Astronomy, University of Edinburgh, The King’s Buildings, EH9 3JZ (UK)

We present the first results obtained by the UKQCD Collaboration using a non-perturbatively $O(a)$ improved Wilson quark action with two degenerate dynamical flavours. The Symanzik improvement program is an attractive method which allows reduction of discretization errors order by order in $a$ in physical quantities. Recently, the improvement counterterm $c_{\text{SW}}(g_0^2)$ for the improved action \[ S = S_{\text{Wilson}} + ac_{\text{SW}}\int dx \bar{\psi} i 4 \sigma_{\mu\nu} F_{\mu\nu} \psi \] has been computed non-perturbatively, thus yielding full $O(a)$ improvement, both in the quenched approximation \cite{2}, and with $N_f = 2$ dynamical fermions \cite{3}.

The UKQCD Collaboration intends to exploit the action (1) with two degenerate sea quarks for its Dynamical Fermions Project. The calculations have been carried out on a Cray T3E recently installed in Edinburgh. The version on which these results were obtained consisted of 96 processor elements (PE), each of 900 MFlops peak speed, and our code sustained a speed of 25-30 GFlops.

The algorithm that has been implemented is a Generalized Hybrid Monte Carlo (GHMC) with even-odd preconditioning. A variant of the Sexton-Weingarten integration scheme has been employed. We refer the reader to \cite{4} for all details of the implementation, optimization and testing of the algorithm.

The simulation parameters are summarized in tab. 1. The value of $c_{\text{SW}}$ used was kindly communicated to us by the ALPHA Collaboration \cite{3}, albeit at a very preliminary stage. Hence the discrepancy between the number we chose with respect to the final result presented in \cite{3}. Since this is the first attempt to do hadronic physics with such an improved action in the dynamical case, even with slightly incomplete improvement we hope to gain much insight in terms of both the performance of the code and of the choice of the parameters for future runs. Thus, the results presented here should be regarded primarily as a testing ground to explore the feasibility of the new action and algorithm. The choice of a large coupling was also motivated on such grounds. Given the exploratory character of the simulations, we have limited ourselves to using local sources and have not exploited any form of smearing. Nevertheless, some light spectrum physics in the mass region around the strange quark mass can still be extracted.

For each of the four values of $\kappa_{\text{sea}}$ we have simulated valence quarks with the same set of four $\kappa$’s. One must keep in mind that in order to take a sensible chiral limit, we need to put $\kappa \equiv \kappa_{\text{sea}} = \kappa_{\text{val}}$, but the full ($\kappa_{\text{sea}}, \kappa_{\text{val}}$) plane can be exploited for “strange” physics \cite{5}. At $\beta = 5.2$ the $\pi/\rho$ mass ratios for the relevant $\kappa$’s are shown in tab. 2. These numbers show how relatively far we are from the chiral region.

1. Setting the scale

The critical point was determined from a linear extrapolation in $1/\kappa$ to vanishing $M_{\pi^+}^2$. The linear ansatz is well supported by the numerical

| $\beta$ | $c_{\text{SW}}$ | $L^3 \cdot T$ | $\kappa_{\text{sea}}$ | Conf |
|--------|----------------|---------------|----------------------|-------|
| 5.2    | 1.76           | $12^3 \cdot 24$ | 0.1370               | 50    |
|        | 0.1380         | 50            |
|        | 0.1390         | 50            |
|        | 0.1395         | 50            |

Table 1
Simulation parameters.
The result $\kappa_{\text{crit}} = 0.14033(3)$ was confirmed by the scaling behaviour of the GHMC algorithm as a function of $\kappa_{\text{sea}}$.

The lattice spacing was determined in two independent ways, cf. fig. 1a: from the scale $r_0$ and from $M_K$. We stress that neither method requires any extrapolation to $\kappa_{\text{crit}}$ and are completely consistent at all values of $\kappa_{\text{sea}}$. We view the standard approach of fixing the scale from the chiral extrapolation of $M_\rho$ as unreliable, given the strong non-linear behaviour in $1/\kappa$ shown by the data, cf. tab. 2. Moreover, in full QCD the $\rho$ can decay and it is therefore dubious to extrapolate to a region where it becomes unstable. In fig. 1b we show the static potential from which $r_0$ was calculated, together with the value of twice the static heavy-light meson mass, calculated for $\kappa = 0.1390$, which is the upper limit for the potential in the case of string breaking. Plotting the potential in units of $r_0$, we recover a universal curve, as shown in [5]. Our data does not exclude string breaking although the distances $r/r_0$ are too small and we need to explore larger distances to be able to draw any definite conclusion.

The strong dependence of $a$ on $\kappa_{\text{sea}}$ indicates the non-negligible effect of the improvement counterterm of the coupling, necessary for the full

\[
O(a) \text{ improvement of the action}
\]
\[
\tilde{g}_0^2 = g_0^2(1 + b_g(g_0^2) m_q a).
\]  

(2)

To one-loop in perturbation theory $b_g(g_0^2) = 0.012 N_f g_0^2 [8]$, which shows why this counterterm is not needed in the quenched approximation ($N_f = 0$). A non-perturbative method to evaluate $b_g$ has been proposed in [9], and its application is currently under investigation by us. Preliminary results at higher values of $\beta$ indicate a much less pronounced mass dependence of the lattice spacing, as expected.

### 2. Light Spectrum

We now give some preliminary results of the calculation of the light hadron spectrum. The standard way to address this point is to show the effective mass. In fig. 2 we report it for the pion and for the nucleon, for the different $\kappa$’s. The lines convey some of the information from the exponential fit: the central line is the value of the mass, the outer lines denote the error spread, and the extension of the lines the fit interval. The error has been obtained with the jackknife method. The information we can gather from them is that the plateau is quite evident for the pion, even with a relatively small volume, and the fit is quite reliable (a similar situation occurs for the $\rho$), taking into consideration the fact that this result is obtained without any type of smearing. On the other hand, and not surprisingly, the situation is worse for the nucleon. In both cases we expect a better isolation of the ground state from smearing. A full statistics study, including the effects of smearing, is underway [10]. Some of our results are summarized in tab. 2, in which we also present the variable $J = M_K \cdot d M_\rho/d M_\pi^2$.

| $\kappa$  | $M_\rho^2$ | $M_\pi/M_\rho$ | $M_N/M_\rho$ | $J$     |
|---------|----------|----------------|--------------|--------|
| 0.1370  | 1.21(1)  | 0.855(4)       | 1.56(2)      | 0.356(4)|
| 0.1380  | 0.87(1)  | 0.824(5)       | 1.51(2)      | 0.343(8)|
| 0.1390  | 0.48(1)  | 0.787(10)      | 1.56(2)      | 0.367(7)|
| 0.1395  | 0.31(1)  | 0.738(16)      | 1.59(4)      | 0.371(9)|

Table 2: Some light hadron results for different $\kappa$’s.
3. Improvement of the Axial Current

The improved on-shell local axial current
\[ A^I_\mu(x, c_A) = A_\mu(x) + ac_A \partial_\mu P(x) \quad (3) \]
can be determined by enforcing PCAC, as done in the Schrödinger Functional formalism \[ 2 \]. An equivalent approach, proposed in \[ 9 \], can be applied using standard hadronic correlators. The current quark mass (averaged over space)
\[ m_{PCAC}(t, c_A) = \frac{1}{2} \frac{\langle \partial_t A^I_0(t, c_A) P(0) \rangle}{\langle P(t) P(0) \rangle} \quad (4) \]
is required to be independent of \( t \) (up to \( O(a^2) \)) if the improvement coefficients \( c_{SW}, c_A \) and \( b_g \) are correctly fixed. We know that this is not completely the case for the present simulation, but we are here interested mainly in showing the viability of the method. Plotting \( \Delta m_{PCAC}(t^*, c_A) = m_{PCAC}(T/2, c_A) - m_{PCAC}(t^*, c_A) \), we fix \( c_A \) by looking at the zero intercept. In fig. 3 we report a typical example, which shows that, for a given value of \( t^* \), there is a distinct cross-over from negative to positive values at a fixed value of \( c_A \), which can be precisely determined by a linear interpolation. We have checked that other values of \( t^* \) do not yield contradictory information, as \( \Delta m_{PCAC} \) remains constant with \( c_A \) within statistical errors. This analysis suggests a value of \( c_A \)
much larger than the perturbative one, which is not surprising given the large coupling. Unfortunately, a comparison with the quenched case is not possible since from \[ 2 \] we have \( c_A \) only down to \( \beta = 6.0 \) and the \( (\beta, \kappa_{sea}) \) parameters we have used are best compared to a quenched value of \( \beta = 5.7 \), for which the lattice spacing is almost the same. A more detailed study, with higher statistics, is currently underway. Future runs with the correct \( c_{SW} \) and \( b_g \) are in progress.

I would like to thank G. Martinelli, G.C. Rossi, C.T. Sachrajda, S. Sharpe, M. Testa and all the members of the UKQCD Collaboration for fruitful discussions. We acknowledge the support of PPARC through grant GR/L22744 for the time allocation on the Cray T3E.

REFERENCES

1. B. Sheikholeslami and R. Wholert, Nucl.Phys.B259 (1985) 572.
2. M. Lüscher et al., Nucl.Phys.B491(1997)323.
3. K. Jansen, R. Sommer, these proceedings.
4. Z. Sroczynski et al., these proceedings.
5. S. Guesken, these proceedings.
6. R. Sommer, Nucl.Phys.B411 (1994) 839.
7. C.R. Allton et al., Nucl.Phys.B489(1997)427.
8. S. Sint and R. Sommer, Nucl.Phys.B465 (1996) 71.
9. G. Martinelli et al., hep-lat/9705018.
10. UKQCD Collaboration, in preparation.