Curved branes from string dualities

G Papadopoulos†, J G Russo‡ and A A Tseytlin§∥

† Department of Mathematics, King’s College London, Strand, London WC2R 2LS, UK
‡ Departamento de Física, Universidad de Buenos Aires, Ciudad Universitaria, Buenos Aires, Argentina
§ Department of Physics, Ohio State University, 174 West 18th Avenue, Columbus, OH 43210, USA
E-mail: gpapas@mth.kcl.ac.uk, russo@df.uba.ar and tseytlin@mps.ohio-state.edu

Received 14 December 1999

Abstract. We describe a simple method for generating new string solutions for which the brane worldvolume is a curved space. As a starting point we use solutions with NS-NS charges combined with two-dimensional CFTs representing different parts of spacetime. We illustrate our method with many examples, some of which are associated with conformally invariant sigma models. Using U-duality, we also obtain supergravity solutions with RR charges which can be interpreted as D-branes with non-trivial worldvolume geometry. In particular, we discuss the case of a D5-brane wrapped on $AdS_3 \times S^3$, a solution interpolating between $AdS_3 \times S^3 \times \mathbb{R}^5$ and $AdS_3 \times S^3 \times S^3 \times \mathbb{R}$, and a D3-brane wrapped over $S^3 \times \mathbb{R}$ or $AdS_2 \times S^2$. Another class of solutions we discuss involves NS5-branes intersecting over a 3-space and NS5-branes intersecting over a line. These solutions are similar to D7-brane or cosmic string backgrounds.

PACS number: 1125

1. Introduction

Branes whose worldvolumes are curved spaces have widespread applications in string theory, M-theory and in studies of four- and five-dimensional black holes. In particular, they have been used to identify the non-perturbative states of strings in lower dimensions in various compactifications. However, in most cases, the branes wrapped over curved spaces were treated as probes and their back-reaction on spacetime geometry was ignored. This is a consistent procedure for D-branes which are wrapped on various homology cycles at small string coupling but in general this is not the case. Another application of branes with curved worldvolume, or ‘curved branes’ for short, is in scenarios where the $(3+1)$-dimensional universe is regarded as a brane moving in a higher dimensional spacetime. In particular, in some models one can introduce gravity localized on the brane and study gravitational collapse. Cosmological models have also been investigated in this context.

Our aim is to provide a systematic method to construct such solutions. For this we begin with configurations that have NS–NS charges only. The advantage of starting with NS–NS configurations is that the conformal invariance of the associated sigma models can be used to check the consistency of the backgrounds in string theory. In this way we construct a large number of configurations that can be interpreted as curved branes. We find that many
of these curved brane configurations are limits of intersecting brane configurations possibly superposed with plane waves and KK-monopoles. The curved brane configurations can thus be constructed from the ‘elementary’ solutions of the NS–NS sector which are associated with conformally invariant sigma models. These include the following:

(a) The plane wave [1].
(b) The fundamental string (T-dual to plane wave) [2, 3].
(c) The NS5-brane [4] associated with a generic harmonic function.
(d) The KK-monopole [5] (or ‘KK 5-brane’ T-dual to the NS5-brane) and many other hyper-Kähler metrics in four and eight dimensions.

The exactness of most of the above solutions in string theory is based on the ultraviolet finiteness of (4, q)-supersymmetric two-dimensional sigma models [6, 7]. There is an associated ‘smeared’ solution for many of the above configurations. This mostly applies to plane wave, fundamental string, NS5-brane and KK-monopole all preserving 1/2 of the maximal spacetime supersymmetry. The ‘smearing’ is achieved by choosing the harmonic functions for these solutions to be invariant under some translational isometries, i.e., to be harmonic in a subspace of the transverse space of the original solution. The smeared solutions share the conformal invariant properties of the generic solutions.

The above ‘elementary’ solutions can be directly superposed to construct other solutions which preserve various fractions of spacetime supersymmetry. All these solutions are associated to intersecting M-brane configurations [8, 9]. This can be achieved in various ways using, for example, the harmonic function rule of [9] or the calibration methods of [10]. The simplest solutions of that type are branes localized in all common transverse dimensions, but, in general, one can localize the brane of a lower dimension in that of a higher dimension (the lower-dimensional brane satisfies the Laplace equation in curved geometry of the higher-dimensional brane [11]). Some of the combinations that are known to be conformal to all orders in \(\alpha'\) are the following:

(a) The fundamental string superposed with a plane wave [3].
(b) The fundamental string/NS5-brane system, the NS5-brane superposed with a plane wave [11, 12] and various T-dual configurations which include the fundamental string/KK-monopole and the KK-monopole superposed with a plane wave.
(c) The direct sum of two NS5-branes [13] and the intersection of two NS5-branes on a line.
(d) Intersecting NS5-branes at \(Sp(2)\)-angles on a line and toric hyper-Kähler metrics in eight dimensions [14, 15].

These are mostly the solutions that preserve 1/4 of the spacetime supersymmetries, apart from the last case which consists of solutions that preserve at least a fraction 3/32.

An alternative way of constructing solutions, which leads to a broader class of solutions, can be obtained by ‘curving’ the transverse space of a plane wave or a fundamental string by putting there any CFT with correct central charge and solving the corresponding curved space Laplace equations for harmonic functions. This general point of view and examples have already been discussed in [3, 11]. In the case of the NS5-brane, one can also curve the six worldvolume directions of the NS5-brane or those of the KK monopole by putting there any CFT with the required central charge. For example, this can be achieved using a WZW model (possibly with a linear dilaton to preserve the value 6 of the central charge of the internal space), or a plane wave or a fundamental string localized on the NS5-brane, or, to leading order in \(\alpha'\), any Ricci-flat space of dimension 1 + 5.

From the above solutions of the NS–NS sector one can easily find, using S- and T-dualities, many others which have RR charges. This is in parallel to the construction of
the D5 + D1 solution which was found [16] by applying S-duality to the previously known NS5 + NS1 configuration [11]. In particular, this will lead to new curved D-brane solutions in 10 dimensions. The 11-dimensional counterpart of such solutions will be generalizations of M2 and M5 solutions having curved internal and/or external parts.

To summarize, the key advantage of the approach based on starting with an NS–NS solution is that one does not need to solve the supergravity equations directly. In fact, one can use simple CFT composition rules in NS–NS sector and then apply T- and S-dualities. In addition, the supersymmetry of some of the resulting solutions is easy to show in the sigma-model framework.

Applying this method, we will find new solutions that include D-branes wrapped over a group space or a coset space. For example, we will show that there is a curved D5-brane solution with worldvolume \( AdS_3 \times S^3 \) (with an extra RR 2-form background). Another example is a curved D3-brane with the worldvolume being the Nappi–Witten [17] space or the four-dimensional null coset of [18]. In addition, we will find a Euclidean curved D3-brane with worldvolume \( R \times S^3 \) and with linear dilaton.

We shall also consider smeared NS5-branes, NS5-brane intersections on three-dimensional space or on a line and a similar intersection with one NS5-brane replaced by a KK-monopole. Many of these solutions are determined by harmonic functions on a two-dimensional transverse space. Apart from the smeared NS5-branes, such solutions and their \( U \)-duals are solutions to the leading-order in \( \alpha' \) and they can be obtained from known M5-brane intersecting solutions [8, 9, 19] by dimensionally reducing to type IIA \( D = 10 \) theory along an overall transverse direction. Such solutions will be shown to admit a certain complex structure, and they are similar to the cosmic string solution of [21]. Consequently, their overall transverse directions can be curved using the freedom of choosing the harmonic functions in two dimensions. The D7 + D3 bound state is \( U \)-dual to the intersecting NS5-brane configuration on a three-dimensional space. A question that remains open is whether the D7 + D3 bound state is an exact string solution.

2. Brane solutions with curved internal and transverse space

2.1. The ansatz

From the NS–NS sector branes, the fundamental string and plane wave are the natural starting points for constructing solutions with curved transverse space, while the NS5-brane and the KK monopole are more appropriate for finding brane solutions with curved worldvolume. To describe the configuration that describes both possibilities in the NS–NS sector, we consider an eight-dimensional manifold \( M \) with metric \( d\hat{s}^2 \), dilaton \( \hat{\phi} \) and closed 3-form \( \hat{H} \). Then we write the ansatz [11]

\[
\begin{align*}
    ds^2 &= 2g_1^{-1}du(dv + A + g_2du) + d\hat{s}^2, \\
    \hat{H} &= du \wedge dv \wedge dg_1^{-1} + du \wedge d(g_1^{-1}A) + \hat{H}, \\
    e^{2\phi} &= g_1^{-1}e^{2\hat{\phi}},
\end{align*}
\]

(2.1)

where \( g_1 \) and \( g_2 \) are functions on \( M \) associated with the fundamental string and the plane wave, respectively, \( A \) is a (locally) defined 1-form on \( M \) associated with a rotation of the string. In this ansatz, the NS5-brane solution and possible NS5-brane intersections or any other NS–NS background that allows a fundamental string superposition are described by the geometry of

\[\text{† The multiple NS5-brane configuration intersecting over a three-dimensional space can also be obtained, by smearing and \( U \)-duality, from a combination of a fundamental string and a plane wave [20]. However, although the latter defines a conformal sigma model, the \( U \)-dual configurations may receive higher order \( \alpha' \) corrections.}\]
To find the string backgrounds described by this ansatz, we either substitute (2.1) into the supergravity field equations or use the conformal invariance of sigma models to investigate the consistency of the propagation of a string probe. The former coincides with the leading order approximation in $\alpha'$ of the latter. The supergravity Killing spinor equations for (2.1) were investigated in [22] and many new supersymmetric solutions were given.

The dynamics of a string probe in the NS–NS background (2.1) is described by a two-dimensional sigma model with Lagrangian

$$L_{10} = g_1^{-1}[\partial u \bar{\partial} v + A_\mu \partial u \bar{\partial} x^\mu + g_2 \partial u \bar{\partial} u] + \gamma_{ab} \partial x^a \bar{\partial} x^b + \hat{B}_{ab} \partial x^a \bar{\partial} x^b + \mathcal{R}(\frac{1}{\alpha'} \log g_1 + \hat{\phi}),$$

where $\mathcal{R}=\alpha' R(2)$. The vanishing of the beta-functions in the leading order in $\alpha'$ gives

$$\partial_a(\sqrt{\gamma} e^{-2\hat{\phi}} \gamma_{ab} \partial b) = 0, \quad \partial_a(\sqrt{\gamma} e^{-2\hat{\phi}} \hat{H}_{abc} \partial c) = 0,$$

$$\hat{B}_{ab} - \frac{1}{2} \hat{H}_{ace} \hat{H}_{b}^{\ce} + 2 \nabla_a \hat{\phi} = 0, \quad \hat{a} (\sqrt{\gamma} e^{-2\hat{\phi}} \hat{H}^{ab}c) = 0,$$

where

$$F_{ab} = 2 \hat{\delta}_{ab} A_0,$$

These are precisely the field equations of the common sector of supergravity theories adapted to the ansatz.

2.2. Conformal curved branes

Some of the solutions described by the ansatz above are exact to all orders in $\alpha'$. To give some examples, we begin with the NS5-brane background for which the associated sigma model is [4] ($\mu = 0, 1, \ldots, 5; m, n = 1, \ldots, 4$)

$$L_{10} = L_{0}^{(0)}(x) + L_4(y) = \partial x^\mu \bar{\partial} x^\mu + (H_5 \delta_{mn} + B_{mn})(y) \partial y^m \bar{\partial} y^n + \mathcal{R}(y),$$

$$e^{2\hat{\phi}} = H_5(y) = 1 + \frac{P}{y^2}, \quad dB = *dH_5.$$  

This is a conformal model because the interacting part $L_4$ admits (4,4) two-dimensional supersymmetry [6]. The same applies to many parallel NS5-branes for which $H_5$ is a generic harmonic function on $\mathbb{R}^4$.

Next, we can replace the six-dimensional flat Minkowski worldvolume directions of the NS5-brane with any conformal sigma model with the same central charge. It is clear that the new sigma model will be again conformal. Some examples are:

1. A plane wave propagating in the NS5-brane worldvolume ($i = 1, 2, 3, 4$)

$$L_6 = \partial u \bar{\partial} u + g_2(u, x) \partial u \bar{\partial} x^i + \partial x^i \bar{\partial} x^i,$$

where $g_2$ is a harmonic function, e.g., $g_2 = K = 1 + \frac{Q}{x^2}$. With this choice of $g_2$ the plane wave is localized on the worldvolume of NS5-brane but it is not localized along its transverse directions.

2. A WZW model with central charge $c = 6$. In particular, one can take the Lagrangian $L_6$ to be that of $SL(2, \mathbb{R}) \times SU(2)$ WZW model which in supersymmetric cases has the same $c$ as the free six-dimensional theory. Other examples include the $\mathbb{R}^{1,2} \times SU(2)$ WZW model with linear dilaton in a spatial direction or some (super)cosets.

3. The Nappi–Witten [17] WZW model supplemented with a free CFT of two scalars.
(4) In general, \( L_6 \) is allowed to depend on the transverse coordinates of NS5-brane. For example, the function \( g_1 \) in (2.1) associated with the fundamental string is allowed to depend on worldvolume \( x \) and transverse \( y \) coordinates \([11]\). In particular, we get

\[
L_{I0} = g_1^{-1}(x, y)\partial u \hat{\partial} v + \partial x^i \hat{\partial} x^i + (H_5 \delta_{mn} + B_{mn})(y)\partial y^m \hat{\partial} y^n + \mathcal{R}\phi, \tag{2.6}
\]

where \( H_1 = g_1 \) satisfies

\[
[\partial_y^2 + H_5(y)\partial_x^2]H_1(x, y) = 0. \tag{2.7}
\]

There are many solutions to this equation. One is that of a string smeared over the NS5-brane but localized in transverse directions,

\[
g_1 = H_1 = 1 + \frac{Q}{y^2},
\]

which is the standard case, and another is of a string localized on NS5-brane but smeared in all of the transverse directions,

\[
g_1 = H_1 = 1 + \frac{Q}{x^2}.
\]

This latter case is \( T \)-dual to the ‘wave on the NS5-brane’ example mentioned above.

(5) Another choice is a magnetic field on the brane, based on the CFT investigated in \([23]\). It is a solvable two-dimensional theory defined by the Lagrangian

\[
L_6 = \partial \rho \hat{\partial} \rho + F(\rho)\rho^2 \left[\partial \phi + (b_1 + b_2)\partial y\right] \left[\hat{\partial} \phi + (b_1 - b_2)\hat{\partial} y\right] + \partial y \hat{\partial} y + \partial x_i \hat{\partial} x_i + \frac{1}{2}\mathcal{R}\log F, \tag{2.8}
\]

where

\[
F = \frac{1}{1 + b_2^2 \rho^2}.
\]

Here \( b_1, b_2 \) are constants, \( y \) is a periodic coordinate of radius \( R \), and \( \rho, \phi \) are polar coordinates. The model describes a magnetic flux tube in a Melvin-type geometry. The curvature of the geometry is caused by the magnetic field energy density. There are two magnetic fields, corresponding to the \( U(1) \times U(1) \) gauge fields associated to the components \( g_{xy} \) and \( B_{xy} \). The flux lines are confined to a radius \( \rho \sim 1/b_2 \) and \( \rho \sim 1/b_1 \), respectively. Although there is no residual supersymmetry, the sigma model is conformal to all orders in \( \alpha' \).

(6) An NS5-brane placed on the worldvolume of the original NS5-brane. The most general solution of this type is the one constructed in \([10,15]\) with the interpretation of NS5-branes intersecting on a line at \( Sp(2) \) angles. The corresponding two-dimensional sigma model has at least \( (4,0) \) supersymmetry, depending on the choice of rotational parameters, and therefore is finite to all orders in \( \alpha' \). However, it may still receive \( \alpha' \) corrections produced by finite counterterms which one needs to add to ensure cancellation of sigma model and supersymmetry anomalies \([7]\). In the special case of real rotational parameters, the sigma model supersymmetry is \( (4,4) \) and the background does not receive \( \alpha' \) corrections.

The above list can be enlarged by solutions which solve the supergravity equations only to leading \( \alpha' \) order. In particular, one can take any Ricci-flat \( (1 + 5) \)-dimensional space, i.e.

\[
L_6 = g_{\mu\nu}(x)\partial x^\mu \hat{\partial} x^\nu, \quad R^g_{\mu\nu} = 0.
\]

Solutions of that type were discussed in \([24–27]\). A particular class of them contains various four-dimensional hyper-Kähler metrics supplemented with the two-dimensional Minkowski space. Such backgrounds are exact CFTs.
2.3. Conformal branes with curved transverse space

In order to find brane solutions with curved transverse space, it is convenient to begin with the fundamental string solution

\[ L_{10} = g_1^{-1}(x) \partial u \partial v - \frac{1}{2} R \ln g_1 + L_{10}^{(0)}(x), \quad L_{8}^{(0)} = \partial y^a \partial y^a. \]

(2.9)

Then we can substitute the flat transverse directions with some CFT. For example, the SU(2) WZW model with linear dilaton, or NS5-brane and others. The explicit expression for \( g_1 \) is then found to the leading order in \( \alpha' \) by solving the field equations (2.3) which in this case reduce to

\[ \partial_u (\sqrt{\gamma} y^{ab} e^{-2 \phi} \partial_\theta) g_1 = 0. \]

Some explicit examples involving WZW-type backgrounds have been discussed [3]. To the leading order in \( \alpha' \), the transverse part can be replaced by any Ricci flat metric, leading to the following background

\[ dx^2 = g_1^{-1}(y)(-dt^2 + dx^2) + \gamma_{ab}(y) dy^a dy^b, \]
\[ B_{tt} = g_1^{-1}, \quad e^{-2 \phi} = g_1, \]
\[ R^\gamma_{ab} = 0, \quad \partial_u (y^{ab} \sqrt{\gamma} \partial_\theta) g_1(y) = 0. \]

(2.10)

Another class of solutions can be obtained to the leading order in \( \alpha' \) by taking the transverse space to be the configuration of NS5-branes intersecting at \( Sp(2) \) angles on a line [15]. To the same order, such solutions can be superposed with a plane wave and rotation giving an even larger class of backgrounds [22].

2.4. Brane solutions generated by U-dualities

Using \( T \)- and \( S \)-dualities we can construct configurations with both NS–NS and R–R charges from the solutions of the previous sections which have only NS–NS charges. This method is well known and we shall not present an exhaustive catalogue of all possibilities. Instead, we shall present a few examples. Applying \( S \)-duality transformation to the model (2.4) with the longitudinal part given by any of the models \( L_6 \) in section 2.1, one obtains a curved D5-brane solution. For example, if we choose the wave model (2.5), we find a solution describing a D5-brane with a wave localized on it. Other similar D-brane solutions can be constructed by applying \( T \)-duality. For example, if we assume that \( g_2 = K \) depends only on \( u \) and \( x^a = (x^1, x^2) \) but not on \( z^I = (x^3, x^4) \) (i.e. it describes a wave in four dimensions smeared in the other two), then \( T \)-duality along \( x^3, x^4 \) gives a D3-brane solution with a wave localized on it (with the whole configuration smeared in the two \( z^I \) directions). The metric is given by

\[ ds^2_{10} = H_5^{-1/2}(y)[du dv + K(u, x)[du^2 + dx^a dx^b] + H_3^{1/2}(y)[dz^k dz^l + dy^a dy^b]. \]

Similarly, \( S \)-duality applied to the solution (2.6) gives a D5-brane with D1 localized on it, i.e.

\[ ds^2_{10} = H_5^{-1/2}(y)[H_1^{-1}(x)du dv + dx^a dx^b] + H_3^{1/2}(y)[dz^k dz^l + dy^a dy^b]. \]

A \( p_1 \)-brane within a \( p \)-brane, with \( p_1 < p \), can also be constructed by \( U \)-dualities. The solutions for these composite branes have the following generic form

\[ ds^2 = H_p^{-1/2}(\tilde{r})[H_{p_1}^{-1}(\tilde{r})(-dt^2 + dx_1^2 + \ldots + dx_{p_1}^2) + H_{p_1}^{1/2}(\tilde{r})(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2_{8-p})] + H_{p_1}^{1/2}(\tilde{r}) H_{p}^{1/2}(\tilde{r})(dr^2 + r^2 d\Omega^2_{8-p}), \]

(2.11)

\[ e^{-2(\phi - \phi_0)} = H_p^{(p-3)/2}(r) H_{p_1}^{(p_1-3)/2}(\tilde{r}), \quad H_p(r) = 1 + \frac{e_p}{r_{p-1}}, \]

(2.12)

\[ F_{8-p} = Q \Omega_{8-p}, \quad F_{p-p_1} = \dot{\Omega}_{p-p_1}. \]

(2.13)
This solution is valid for $p - p_1 = 4$, $p_1 < 3$.

One can also construct curved Dp-brane solutions of the form
\[ ds^2 = H_p^{-1/2}(y)g_{\mu\nu}(x)dx^\mu dx^\nu + H_1^{1/2}(y)dy^m dy^n, \]
where $H_p(y)$ is a harmonic function in the curved space $ds^2_{p-p} = h_{mn}(y)dy^m dy^n$, which is transverse to the brane, and $g_{\mu\nu}(x)$, $h_{mn}(y)$ are Ricci-flat metrics.

Analogous solutions can be constructed in 11-dimensional supergravity. In particular, the following background represents a curved M5-brane:
\[ ds^2_{11} = H^{-1/3}(r)g_{\mu\nu}(x)dx^\mu dx^\nu + H^{2/3}(r)(dr^2 + r^2 d\Omega_4^2), \]
\[ dB = 6Q\Omega_4, \quad H(r) = 1 + \frac{Q}{r^3}, \quad R_{\mu\nu} = 0. \]
(2.16)

When the metric (2.16) has translational isometry, this background can be connected to the previous backgrounds upon dimensional reduction and suitable $U$-duality transformations.

We would like to emphasize, however, that the cases with Ricci-flat longitudinal and/or transverse spaces are just a small subclass of more general backgrounds. These more general configurations describe various interesting cases which include branes on group spaces, brane black holes and cosmological models with non-constant dilaton.

3. Some particular cases

3.1. D5-brane wrapped on $AdS_3 \times S^3$

One example is D5-brane wrapped on $AdS_3 \times S_3$. This solution can be obtained by an $S$-duality transformation applied to the NS5-brane model described in the previous section, with worldvolume $L_6$ given by the $SL(2, \mathbb{R}) \times SU(2)$ WZW model.

An alternative derivation, which is also useful for further generalizations, can be given by starting with the system $1_{\text{NS}} + 5_{\text{NS}} + 5_{\text{NS}}'$, with metric, 2-form and dilaton given by [28]
\[ ds^2 = g_1^{-1}(x, y)(-dr^2 + dz^2) + H_2(x)dx^m dx^n + H_3(y)dy^m dy^n, \]
\[ dB = dg_1^{-1} \wedge dt \wedge dz + *dH_5 + *dH_5', \quad e^{2\phi} = \frac{H_5(x)H_5(y)}{g_1(x, y)}, \]
(3.1)
(3.2)

where
\[ [H_5'(y)\partial_x^2 + H_5(x)\partial_y^2]g_1(x, y) = 0. \]
(3.3)

A particular solution is
\[ g_1(x, y) = H_1(x)H_1'(y), \]
(3.4)

with
\[ H_1 = 1 + \frac{R_1^2}{x^2}, \quad H_5 = 1 + \frac{R_5^2}{x^2}, \quad H_5' = 1 + \frac{R_1^2}{y^2}, \quad H_5'' = 1 + \frac{R_5^2}{y^2}, \]
(3.5)

which corresponds to the ‘dyonic string’ generalization [29] of the $5_{\text{NS}} + 5_{\text{NS}}$ solution of [13]. Different choices of the harmonic functions give a number of interesting models; related solutions have been discussed in [14, 22, 30–32].

To continue, let us further choose $H_1' = 1$, in which case the solution is just a direct product of NS1 + NS5 configuration and another NS5-brane. Then we use the fact that the near-horizon
limit of the NS1+NS5 configuration is simply the $SL(2) \times SU(2)$ WZW theory [11]. Explicitly, as $x \to 0$, the metric (3.1) becomes

$$ds^2 = \frac{x^2}{R_1^2}(-dt^2 + dz^2) + R_1^2 \frac{dx^2}{x^2} + R_1^2 d\Omega_3^2 + H_2(y)(dy^2 + y^2 d\Omega_5^2), \quad (3.6)$$

where we have set for simplicity $H_1 = H_5$. The resulting sigma model describes a curved NS5-brane with worldvolume $AdS_3 \times S^3$ and defines an exact CFT. Applying $S$-duality, we obtain the type IIB solution representing a curved D5-brane with worldvolume $AdS_3 \times S^3$ space. The associated metric is

$$ds^2 = H_s^{-1/2} \left[ \frac{x^2}{R_1^2}(-dt^2 + dz^2) + R_1^2 \frac{dx^2}{x^2} + R_1^2 d\Omega_3^2 \right] + H_s^{1/2}(dy^2 + y^2 d\Omega_5^2), \quad (3.7)$$

which is supplemented with the obvious dilaton and RR 2-form backgrounds.

The above solution may have applications to the study of the decoupling limit of branes wrapped on curved spaces. Other solutions can be constructed by applying $T$-duality along isometric directions of $AdS_3 \times S^3$.

### 3.2. D1–D5–D5 system with $AdS_3 \times S^3 \times S^3 \times \mathbb{R}$ horizon geometry

Construction of another example requires us to begin again with (3.1), (3.4) but now choose $H_1 = H_5$ and $H_2 = H'_5$ in which case $R_1 = R_5 \equiv R$ and $R'_1 = R'_5 \equiv R'$; see also [30]. Now we take the near-horizon limit $y \to 0$. Rescaling the coordinates, the metric becomes

$$ds^2 = H_1^{-1} \frac{u^2}{R^2}(-dt^2 + dz^2) + R^2 \frac{du^2}{u^2} + H_1(dx^2 + x^2 d\Omega_3^2) + R^2 d\Omega_3^2. \quad (3.8)$$

Since the dilaton is constant, the $S$-dual metric describing a D1–D5–D5 system is the same as (3.8). The corresponding RR 2-form field strength is given by (3.2).

The solution (3.8) interpolates between the two exact conformal models, as it is easy to show. Indeed, as $x \to \infty$, the metric approaches that of $AdS_3 \times S^3 \times \mathbb{R}^4$. For $x \to 0$, the metric takes the form

$$ds^2 = u^2 R^2 \frac{du^2}{u^2} + R^2 \frac{dX^2}{X^2} + R^2 d\Omega_3^2 + R^2 d\Omega_5^2, \quad (3.9)$$

where we have rescaled the coordinates. Introducing new variables $Y$ and $X$ by

$$u = R \exp \left[ \frac{R_0}{R^2} Y + \frac{R_0}{RR'} X \right], \quad u' = R' \exp \left[ \frac{R_0}{R'^2} Y - \frac{R_0}{RR'} X \right],$$

$$\frac{1}{R_0} = \frac{1}{R^2} + \frac{1}{R'^2},$$

we get

$$ds^2 = e^{\frac{2}{R_0}} (-dt^2 + dz^2) + dY^2 + dX^2 + R^2 d\Omega_3^2 + R^2 d\Omega_5^2, \quad (3.10)$$

This is a direct product of the three-dimensional anti-de Sitter space of radius $R_0$, the two 3-spheres with radii $R$ and $R'$, respectively, and a real line (parametrized by $X$). String theory on such space and its duality to two-dimensional superconformal theories was investigated in [42]. Similar models to those of $AdS_3 \times S^3 \times S^3 \times S^1$ have also recently been studied in [33].
3.3. D3-brane wrapped over non-trivial four-dimensional spaces

It is straightforward to find the solution representing a D3-brane ‘wrapped’ over a Ricci-flat space. In the ‘decoupling limit of [34], we obtain \( (R^2 = \sqrt{4\pi g_s N}, \alpha' = 1) \)

\[
\text{d}s^2 = \frac{r^2}{R^2} g_{\mu\nu}(x) \text{d}x^\mu \text{d}x^\nu + R^2 \frac{\text{d}r^2}{r^2} + r^2 \Omega_5^2,
\]

with constant dilaton \( e^\phi = g_s \) and \( R_{\mu\nu} = 0 \). The metric is of the form \( X_5 \times S^5 \), where \( X_5 \) is an Einstein space having a Ricci-flat 4-manifold with metric \( g_{\mu\nu} \) as a boundary. Type-IIB superstring theory on this background (3.11) should be dual to an \( \mathcal{N} = 4 \) SU(N) Yang–Mills theory on a four-dimensional manifold described by \( g_{\mu\nu} \) [35].

Like the D5-brane, the D3-brane can be also wrapped over non-Ricci-flat spaces with non-trivial 2-form field and/or dilaton. One particular example is obtained by taking the parallel sigma model in the horizon limit and dilaton are given by

\[
\phi = R_0^{-1} z, \quad H = 1 + \frac{R^4}{r^4}.
\]

Let us now construct a D3-brane wrapped over \( AdS_3 \times S^2 \). This is related to the D5-brane wrapped on \( AdS_3 \times S^3 \) by T-duality along isometric directions of \( AdS_3 \times S^3 \), but it can also be constructed directly from an exact conformal model as follows. We start with the conformal \( \sigma \)-model representing the intersection of an NS5-brane, a Kaluza–Klein monopole (with Kaluza–Klein coordinate \( x^5 \)), a fundamental string, and a wave along the string direction \( x^1 \) [12]. The NS and KK 5-branes lie on the directions \( x^1, x^3, x^4, x^5, x^6 \). T-duality in \( x^5 \) exchanges their charges. In four dimensions, this background leads to the dyonic BPS black hole (which contains the Reissner–Nordstrom solution as a particular case). Let us consider the case when all four charges are equal. The near-horizon geometry is \( AdS_3 \times S^2 \times \mathbb{R}^6 \), the \( AdS_3 \times S^2 \) part of the space being described by the coordinates \( x^0, x^1, x^4, x^6 \).

In order to incorporate a D3-brane wrapped on \( AdS_2 \times S^2 \), we start with this conformal sigma model in the horizon limit \( AdS_2 \times S^2 \times \mathbb{R}^6 \). We add an NS5-brane lying along the space \( (x^1, x^2, x^7, x^8, x^9) \). The transverse part of this extra brane is thus the flat space \( (x^3, x^4, x^5, x^6) \),

† If one is interested in calculating the gauge theory partition function, one can take the longitudinal space to be some hyper-Kähler manifold, for which various properties of the instanton moduli space are known. In particular, for certain ALE spaces, or K3, exact information is available for any SU(N) [36]. The partition function \( Z \) could then be compared with the string theory partition function on this background.
so that the resulting metric is given by

\[ ds_{10B}^2 = \frac{1}{x_7^2} \left( -dx_0^2 + R^2 dx_5^2 \right) + R^2 d\Omega_3^2 + dx_1^2 + dx_2^2 + H(r) \left[ dr^2 + r^2 d\Omega_3^2 \right], \]

\[ e^{2\phi} = H = 1 + \frac{Q}{r^2}, \quad r^2 = x_7^2 + x_5^2 + x_6^2. \]

Then, we perform S-duality and T-duality transformations in the directions \( x^1, x^2 \), which transform the extra NS5-brane into a D3-brane smeared in the directions \( (x^1, x^2) \), and the original brane configuration of [12] into a (near-horizon) configuration representing a D5-brane (lying on \( x^2, x^3, x^4, x^5, x^6 \) ) and an NS5-brane (on \( x^1, x^3, x^4, x^5, x^6 \) ), a D1-brane (along \( x^3 \)) and a fundamental string (along \( x^1 \)). The resulting background is given by

\[ ds_{10B}^2 = H^{-1/2}(r) \left[ \frac{1}{x_7^2} (-dx_0^2 + R^2 dx_5^2) + R^2 d\Omega_3^2 \right] + H^{1/2}(r) \left[ dx_1^2 + dx_2^2 + dr^2 + r^2 d\Omega_3^2 \right], \]

(3.14)

\[ H = 1 + \frac{Q}{r^2}, \quad F_5 = Q(1 + \phi)(\Omega_3 \wedge dx_1 \wedge dx_2), \quad e^{\phi} = g_s , \]

(3.15)

\[ H_{3}^{NS} = \frac{1}{R} \, dx_0 \wedge dx_1 \wedge dx_7 + R \, \Omega_2 \wedge dx_7, \]

\[ H_{3}^{R} = \frac{1}{R} \, dx_0 \wedge dx_5 \wedge dx_7 + R \, \Omega_2 \wedge dx_7. \]

(3.16)

### 3.4. D2-brane on \( S^2 \)

Many of the solutions that we have presented above contain the metric of an odd-dimensional sphere \( S^{2n+1} \). In all such cases, one can use the Hopf fibration \( S^3 \to S^{2n+1} \to CP^n \) and perform T-duality along \( S^3 \) in a way similar to that of [44]. It turns out that for D-brane backgrounds in the dual picture, the Hopf fibration untwists and the sphere is replaced by \( CP^n \).

For an application, let us consider the solution (3.12), (3.13). This is a Euclidean D3-brane wrapped on \( S^3 \). Next we use the Hopf fibration \( S^3 \to S^3 \to S^2, CP^1 = S^2 \), to obtain a solution with the interpretation of a Euclidean D2-brane wrapped on \( S^2 \). The metric and dilaton of this background is

\[ ds_{10B}^2 = H^{-1/2}(r) \left[ dz^2 + R_0^2 d\Omega_2^2 \right] + H^{1/2}(r) \left[ d\theta^2 + dr^2 + r^2 d\Omega_2^2 \right], \]

\[ e^{2\phi} = e^{2z/R_0} H^{1/2}. \]

(3.17)

The 11-dimensional supergravity solution representing an M2-brane wrapped on \( S^2 \) is then constructed by applying the usual formula of dimensional reduction.

### 4. Solutions with two-dimensional transverse space

Now we are going to consider a different class of NS–NS solutions which are determined by functions which depend on two variables. Many examples of such solutions have the interpretation of intersecting branes with a two-dimensional overall transverse space. We can either curve the worldvolume directions of the branes involved in the intersection or the overall transverse space. The investigation of the former case is similar to that presented in the previous section and we shall not repeat the analysis. In the latter case we first remark that any two-dimensional metric is conformal to the flat one and that in two dimensions there is a freedom of defining harmonic function by adding the real part of a holomorphic one. A product
of these harmonic functions is the conformal coefficient of the overall transverse space. As we shall see, making different choices for the harmonic functions, we effectively curve the overall transverse space of the configuration. This phenomenon is similar to what happens in the case of the cosmic string background [21]. In fact, we shall find that one of our solutions reduces to that of the cosmic string. As a byproduct of our analysis, we shall give two examples of Calabi–Yau metrics with holonomy SU(3) and SU(4), respectively.

4.1. Single ‘smeared’ NS5-brane

Let us start with an NS5-brane solution smeared in two transverse dimensions. As we have already mentioned, this is an exact conformal model just like the standard NS5-brane. In particular, we smear the directions \( \{ \xi_a; a = 1, 2 \} = (x_3, x_4) \) and keep the dependence of the solution on the coordinates \( \{ x_i; i = 1, 2 \} \). In such a case, the NS–NS 3-form field strength is

\[
(\mathbb{d}B)_{123} = \epsilon_{ij} \partial_j H, \quad \partial_i \partial_i H = 0. \tag{4.1}
\]

Next, we consider a holomorphic function \( \mathcal{H} = H + iB \) of \( z = x_1 + ix_2 \) that solves

\[
\partial \bar{\partial} H = 0, \quad \partial = \partial / \partial z.
\]

Then

\[
\partial_i H = \epsilon_{ij} \partial_j B,
\]

and so the non-zero 2 \times 2 components of \( B_{mn} \) can be chosen as

\[
B_{ab} = \epsilon_{ab} B. \tag{4.2}
\]

The resulting NS5-brane sigma model can be written as

\[
L = -\partial t \bar{\partial} t + \partial y_a \bar{\partial} y_a + (H \delta_{ab} + B \epsilon_{ab})(x) \partial \bar{\partial} \xi_a \bar{\partial} \xi_b + H(x) \partial x_i \bar{\partial} x_i + \frac{1}{2} R \ln H(x). \tag{4.3}
\]

Note that

\[
(H \delta_{ab} + B \epsilon_{ab})(x) \partial \bar{\partial} \xi_a \bar{\partial} \xi_b \equiv \frac{1}{2} [\mathcal{H}(z) \partial \bar{\partial} \xi + \text{c.c.}],
\]

where \( \xi = \xi_1 + i\xi_2 \). Since the torsion in two dimensions is trivial, i.e. adding \( B_{ij} = \epsilon_{ij} B(x) \) gives a total derivative which we can drop, we can write this in a complex form

\[
L = -\partial t \bar{\partial} t + \partial y_a \bar{\partial} y_a + \frac{1}{2} [\mathcal{H}(z) (\partial \bar{\partial} \xi + \partial \bar{\partial} \bar{\xi}) + \bar{\mathcal{H}}(z) (\partial \bar{\partial} \bar{\xi} + \partial \bar{\partial} \xi)] + \frac{1}{2} R \ln H(x). \tag{4.4}
\]

This background is related, via \( T \)-duality, to that of the D7-brane [38], and, via compactification, to the cosmic string geometry of [21]. More specifically, \( T \)-duality along a spatial 5-brane direction gives the IIB NS5-brane; \( S \)-duality and \( T \)-duality along \( \xi_a \) gives the D7-brane. \( \xi_a \) are directions of 2-torus with trivial complex structure, and the background modulus field is \( E_{ab} = H \delta_{ab} + B \epsilon_{ab} \), or

\[
\rho = \rho_1 + i\rho_2 = B + i(\det G_{ab})^{1/2} = -i\mathcal{H}, \quad e^{2\phi} = H = \rho_2.
\]

\( T \)-duality along one of the \( \xi_a \) directions gives the corresponding smeared version of the KK monopole:

\[
L = -\partial t \bar{\partial} t + \partial y_a \bar{\partial} y_a + H^{-1}(x) [\partial \xi_1 + B(x) \partial \xi_2] [\bar{\partial} \bar{\xi}_1 + B(x) \bar{\partial} \bar{\xi}_2] + H(x) \partial \bar{\partial} \xi_2 + H(x) \partial x_i \bar{\partial} x_i \tag{4.5}
\]

or

\[
L = -\partial t \bar{\partial} t + \partial y_a \bar{\partial} y_a + H^{-1}(x) |\partial \xi_1 + \tau(x) \partial \xi_2|^2 + H(x) \partial x_i \bar{\partial} x_i, \tag{4.6}
\]

where the modulus of the 2-torus is \( \tau = B + iH = \rho \). Compactifying to four dimensions, this reduces indeed to the model of [21], i.e. \( T^2 \times \mathbb{R}^2 \).
4.2. NS5-brane and KK monopole

One can also start with the conformal model describing NS5-brane + KK monopole which is smeared in one transverse direction, say $y_3$. Then we find that

$$L = - \partial t \bar{\partial} t + \partial y_n \bar{\partial} y_n + H_1(x)H_{\perp}^{-1}(x) \left[ \partial z + a_1(x) \partial x_1 \right] \left[ \partial \bar{z} + a_1(x) \partial \bar{x}_1 \right] + H_2(x) \left[ \partial x_3 + a_3(x) \partial x_3 \right] \right] \right]$$

$$+ \partial z \partial \bar{z} + \partial x_3 \partial \bar{x}_3 + \partial \eta \partial H_1 + \partial \eta \partial H_2, \quad \partial \eta a_1 = \epsilon_{ij} \partial j H_1, \quad \partial \partial H_2,$

where $i = 1, 2$ and we have made a natural special choice of the 2-form gauge potential. We can further identify $b_1$ and $a_3$ with the imaginary parts of the holomorphic functions $H_1$ and $H_2$ that have $H_1$ and $H_2$ as their real parts.

This background is $U$-dual, in particular, to $D6 + NS5$ configuration with one isometric direction, or to $D7 +$ KK-monopole.

4.3. Intersecting NS5-branes

Let us now consider the solution of two NS5-branes intersecting on a three-dimensional space and localized only in the overall two transverse directions. The corresponding M-theory solution is that of $M5 \perp M5$ smeared in the 11-th transverse direction. The NS5-brane system is related, by an $S$-duality and a chain of two $T$-dualities, to a “D3-brane on a D7-brane” configuration. This is 1/4 BPS state and the solution is given by the harmonic function rule. The solution is smeared in all eight common directions and depends only on the two overall transverse directions. The sigma-model action for this NS5-brane configuration is

$$L = - \partial t \bar{\partial} t + \partial y_n \bar{\partial} y_n + (H_1(x)H_{\perp}^{-1}(x) \left[ \partial z + a_1(x) \partial x_1 \right] \left[ \partial \bar{z} + a_1(x) \partial \bar{x}_1 \right] \right] \right]$$

where $x = y_3$ and $\eta$ is a scalar field that solves the two-dimensional Laplace equation, while $B_{\mu \nu}$ is determined by

$$(d B)_{\xi, \xi} = \epsilon_{ij} \partial_i H, \quad (d B')_{\eta, \eta} = \epsilon_{ij} \partial_j H'. \quad (4.9)$$

To put the action in the above simple local form we made the assumption that $H$ and $H'$ are real parts of the two holomorphic functions $\mathcal{H}$ and $\mathcal{H}'$, and used the same considerations as above for a single NS5-brane smeared in two directions (which is an obvious special case $H' = 1$) to determine the antisymmetric tensor in terms of $Im \mathcal{H} = B$ and $Im \mathcal{H}' = B'$. We remark that the $5_{NS} \perp 5_{NS}$ solution with a different special choice of $H$ and $H'$ (such that $H + iH'$ is a holomorphic function) was considered in [39].

The resulting action has a remarkably simple structure

$$L = - \partial t \bar{\partial} t + \partial y_n \bar{\partial} y_n + \frac{1}{2} \left[ \mathcal{H}(z) \partial \xi \partial \bar{\xi} + \mathcal{H}'(z) \partial \eta \partial \bar{\eta} \right] + c.c.$$
that $\nabla^{(s)} I = 0$ and $\nabla^{(-)} J = 0$ which implies that the holonomy of $\nabla^{(s)}$ is a subgroup of $U(3)$. In fact, it turns out that the holonomy of both connections is $SU(3)$. Using this, one can conclude that the solution indeed preserves 1/4 of the supersymmetry. As a consequence, the sigma model (4.10) admits a (2,2)-supersymmetric extension. This amount of supersymmetry by itself is not, however, sufficient to prove that this sigma model is finite to all orders in perturbation theory since the argument of [6] does not apply. It may nevertheless be an exact CFT due to the simple form of the action.

Another consequence of the $SU(3)$ holonomy is that these metrics are associated with Calabi–Yau ones. In particular, let us $T$-dualize twice, once along the $\xi$ and once along the $\eta$ coordinates. Using the results in section (4.1), we find that

$$L = -\partial \bar{t} \bar{t} + \partial y_n \bar{y}_n + H^{-1}(x) |\partial \xi_1 + \tau(x) \partial \xi_2|^2$$
$$+ H^{-1}(x) |\partial \eta_1 + \tau'(x) \partial \eta_2|^2 + H(x) H'(x) \partial x_i \bar{\partial} x_i ,$$  

(4.11)

where $\tau = -i\mathcal{H}$ and $\tau' = -i\mathcal{H}'$. The non-trivial part of the sigma-model action is associated with a six-dimensional non-compact Calabi–Yau metric.

4.4. Three NS5-branes intersecting on a line

The metric of the supergravity solution of three NS5-branes pairwise intersecting on three-dimensional spaces, and all – on a line, is

$$ds^2 = -dt^2 + dy^2 + H_1(x) d\xi_1 + H_2(x) d\eta_1 + H_3(x) d\zeta + H_1 H_2 H_3 dz d\bar{z} .$$  

(4.12)

Here $\xi, \eta, \zeta$ and $z$ are complex coordinates and $H_1, H_2, H_3$ are harmonic functions in the overall transverse space spanned by $(z, \bar{z})$. This solution may be viewed as a reduction of $M5 \perp M5 \perp M5$ background in 11 dimensions [8, 9]. Performing similar calculations to those in the previous cases, the sigma-model action is found to be

$$L = -\partial t \bar{\partial} t + \partial y \bar{\partial} y + \frac{1}{2} \left[ H_1(z) \partial \xi_1 \bar{\partial} \bar{\xi}_1 + H_2(z) \partial \eta_1 \bar{\partial} \bar{\eta}_1 + H_3(z) \partial \zeta \bar{\partial} \bar{\zeta} \right] + c.c.$$  

$$+ \frac{1}{2} H_1(x) H_2(x) H_3(x) (\partial z \bar{\partial} \bar{z} + c.c.) + \frac{1}{2} R \ln[H_1(x) H_2(x) H_3(x)] ,$$  

(4.13)

where $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ are holomorphic functions of $z$. The above solution preserves 1/8 of the spacetime supersymmetry, as can be seen by studying the holonomy of the connections $\nabla^{(s)}$. In particular, the holonomy of both connections is $SU(4)$. The corresponding bosonic sigma model admits a (2,2)-supersymmetric extension which again is not enough to establish that the model is conformal using the argument of [6].

As in the previous case, the above NS5-brane geometry is associated with an eight-dimensional Calabi–Yau one. Indeed, using $T$-duality in three directions chosen in a way similar to that of the previous example, we find

$$L = -\partial t \bar{\partial} t + \partial y \bar{\partial} y + H_1^{-1}(x) |\partial \xi_1 + \tau_1(x) \partial \xi_2|^2 + H_2^{-1}(x) |\partial \eta_1 + \tau_2(x) \partial \eta_2|^2$$
$$+ H_3^{-1}(x) |\partial \zeta_1 + \tau_3(x) \partial \zeta_2|^2 + H_1(x) H_2(x) H_3(x) \partial x_i \bar{\partial} x_i ,$$  

(4.14)

where $\tau = -i\mathcal{H}_1, \tau_2 = -i\mathcal{H}_2$ and $\tau_3 = -i\mathcal{H}_3$. The Calabi–Yau metric is given by the non-trivial part of the sigma-model action.

5. Conclusions

We have obtained brane solutions with a curved worldvolume by starting with conformal two-dimensional sigma models describing the propagation of strings in NS–NS backgrounds. The advantage of this method is the simplicity and transparency of the construction of brane solutions. Many of the solutions that have already appeared in the literature [24–27] can then
be easily derived by dualities, without solving any differential equations. One of the main points of the present discussion is that branes with a Ricci-flat worldvolume constitute only a special case of a more general class of configurations. The worldvolume of a brane can be any suitable CFT, which may describe non-Ricci flat spaces with non-constant dilaton and non-vanishing 2-form field strengths. In particular, we have presented examples of the solutions representing branes wrapped over group spaces.

Further progress in constructing interesting curved brane solutions is related to the understanding of the back-reaction of branes wrapped on homology cycles of compact spaces in string and M-theory compactifications. It is clear from our results that apart from the topological information about the cycles one also needs some geometrical data. In some situations the geometry of the normal bundle of the cycle will also be relevant†.

Acknowledgments

GP is supported by the Royal Society. JR is supported by UBA, Conicet and Fundación Antorchas. The work of AT is supported in part by the DOE grant no DOE/ER/01545-777, by the EC TMR programme ERBFMRX-CT96-0045, INTAS grant no 96-538, and NATO grant PST.CLG 974965.

References

[1] Amati D and Klimcik C 1989 Nonperturbative computation of the Weyl anomaly for a class of nontrivial backgrounds Phys. Lett. B 219 443
Horowitz G T and Steif A R 1990 Space-time singularities in string theory Phys. Rev. Lett. 64 260
Horowitz G T and Steif A R 1990 Strings in strong gravitational fields Phys. Rev. D 42 1950
Tseytlin A A 1993 String vacuum backgrounds with covariantly constant null Killing vector and 2-d quantum gravity Nucl. Phys. B 390 153
(Many references are available in Tseytlin A A 1992 Preprint hep-th/9209023)
[2] Dabholkar A, Gibbons G, Harvey J A and Ruiz Ruiz F 1990 Superstrings and solitons Nucl. Phys. B 340 33
Horowitz G T and Tseytlin A A 1994 On exact solutions and singularities in string theory Phys. Rev. D 50 5204
(Horowitz G T and Tseytlin A A 1994 Preprint hep-th/9406067)
[3] Callan C G, Harvey J A and Strominger A 1991 World sheet approach to heterotic instantons and solitons Nucl. Phys. B 359 611
[5] Sorkin R D 1983 Kaluza-Klein monopole Phys. Rev. Lett. 51 87
Gross D J and Perry M J 1983 Magnetic monopoles in Kaluza-Klein theories Nucl. Phys. B 226 29
[6] Howe P S and Papadopoulos G 1987 Ultraviolet behaviour of two-dimensional nonlinear sigma models Nucl. Phys. B 289 264
Hove P S and Papadopoulos G 1988 Further remarks on the geometry of two-dimensional nonlinear sigma models Class. Quantum Grav. 5 1677
[7] Howe P S and Papadopoulos G 1992 Finiteness and anomalies in (4,0)-supersymmetric sigma models Nucl. Phys. B 381 360
(Howe P S and Papadopoulos G 1992 Preprint hep-th/9203070)
[8] Papadopoulos G and Townsend P K 1996 Intersecting M-branes Phys. Lett. B 380 273
(Papadopoulos G and Townsend P K 1996 Preprint hep-th/9603087)
[9] Tseytlin A A 1996 Harmonic superpositions of M-branes Nucl. Phys. B 475 149
(Tseytlin A A 1996 Preprint hep-th/9604035)
[10] Papadopoulos G and Teschendorff A Grassmanians calibrations and five-brane intersections Preprint hep-th/9811034
[11] Tseytlin A A 1996 Extreme dyonic black holes in string theory Mod. Phys. Lett. A 11 689
(Tseytlin A A 1996 Preprint hep-th/9601177)

† In this paper we have assumed an orthogonal decomposition between the worldvolume and the transverse spaces. In general, if one wraps a brane over a cycle one may not have an orthogonal decomposition in the metric because the normal bundle of the cycle can be twisted. Examples of such solutions were discussed in [41].
Curved branes from string dualities

[12] Cvetić M and Tseytlin A A 1996 General class of BPS saturated dyonic black holes as exact superstring solutions Phys. Lett. B 366 95
(Cvetić M and Tseytlin A A 1995 Preprint hep-th/9510097)

Cvetić M and Tseytlin A A 1996 Solitonic strings and BPS saturated dyonic black holes Phys. Rev. D 53 5619
(Cvetić M and Tseytlin A A 1995 Preprint hep-th/9512031)

[13] Khuri R R 1993 Remark on string solitons Phys. Rev. D 48 2947

[14] Gauntlett J P, Gibbons G W, Papadopoulos G and Townsend P K 1997 Hyper-Kaehler manifolds and multiply intersecting branes Phys. Lett. B 366 95
(Gauntlett J P, Gibbons G W, Papadopoulos G and Townsend P K 1997 Preprint hep-th/9702202)

Papadopoulos G and Teschendorff A 1998 Multigluee five-brane intersections Phys. Lett. 443 159
(Papadopoulos G and Teschendorff A 1998 Preprint hep-th/9806191)

[16] Callan C G and Maldacena J M 1996 D-brane approach to black hole quantum mechanics Nucl. Phys. B 472 591

[17] Nappi C R and Witten E 1993 A WZW model based on a nonsemisimple group Phys. Rev. D 48 2947

[18] Ivanščuk V D and Melnikov V N 1996 Intersecting p-brane solutions in multidimensional gravity and M-theory Preprint hep-th/9612089

Ivanščuk V D and Melnikov V N 1998 Multidimensional sigma-models with composite electric p-branes Grav. Cosmol. 4 145
(Ivanščuk V D and Melnikov V N 1997 Preprint gr-qc/9705005)

Grebeniuk M A, Ivanščuk V D and Melnikov V N 1998 Multidimensional cosmology for intersecting p-branes with static internal spaces Grav. Cosmol. 4 145
(Grebeniuk M A, Ivanščuk V D and Melnikov V N 1998 Preprint gr-qc/9804042)

[25] Brecher D and Perry M J 2000 Ricci-flat branes Nucl. Phys. B 566 151
(Brecher D and Perry M J 1999 Preprint hep-th/9908018)

[26] Janssen B 2000 Curved branes and cosmological (a,b)-models J. High Energy Phys. JHEP01(2000)044

[27] Figueroa-O’Farrill J M 1999 More Ricci-flat branes Phys. Lett. B 471 128
(Figueroa-O’Farrill J M 1999 Preprint hep-th/9810086)

[28] Tseytlin A A 1997 Composite BPS configurations of p-branes in 10 and 11 dimensions Class. Quantum Grav. 14 2085
(Tseytlin A A 1997 Preprint hep-th/9702163)

[29] Duff M J, Ferrara S, Khuri R R and Rahmfeld J 1995 Supersymmetry and dual string solitons Phys. Lett. B 356 479
(Duff M J, Ferrara S, Khuri R R and Rahmfeld J 1995 Preprint hep-th/9506057)

[30] Cowdall P M and Townsend P K 1998 Gauged supergravity vacua from intersecting branes Phys. Lett. B 429 281
(Cowdall P M and Townsend P K 1998 Preprint hep-th/9801165)

[31] Boonstra H J, Peeters B and Skenderis K 1998 Brane intersections, anti-de Sitter spacetimes and dual superconformal theories Nucl. Phys. B 533 127
(Boonstra H J, Peeters B and Skenderis K 1998 Preprint hep-th/9803231)
[32] Gauntlett J P, Meyer R C and Townsend P K 1999 Supersymmetry of rotating branes Phys. Rev. D 59 025001
(Gauntlett J P, Meyer R C and Townsend P K 1998 Preprint hep-th/9809065)
[33] Elitzur S, Feinerman O, Giveon A and Tsabar D 1999 String Theory on $AdS_5 \times S^3 \times S^3 \times S^1$ Phys. Lett. B 449 180
(Elitzur S, Feinerman O, Giveon A and Tsabar D 1998 Preprint hep-th/9811245)
[34] Maldacena J M 1998 The large N limit of superconformal field theories and supergravity Adv. Theor. Math. Phys. 2 231
(Maldacena J M 1997 Preprint hep-th/9711200)
[35] Witten E 1998 Anti-de Sitter space and holography Adv. Theor. Math. Phys. 2 253
(Witten E 1998 Preprint hep-th/9802150)
[36] Vafa C and Witten E 1994 A strong coupling test of S-duality Nucl. Phys. B 431 3
(Vafa C and Witten E 1994 Preprint hep-th/9408074)
[37] Duff M J, Lu H and Pope C N 1998 $AdS(5) \times S(5)$ untwisted Nucl. Phys. B 532 181
(Duff M J, Lu H and Pope C N 1998 Preprint hep-th/9803061)
[38] Gibbons G W, Green M B and Perry M J 1996 Instantons and seven-branes in type IIB superstring theory Phys. Lett. B 370 37
(Gibbons G W, Green M B and Perry M J 1995 Preprint hep-th/9511080)
[39] Papadopoulos G 1998 $T$-duality and the worldvolume solitons of five-branes and KK monopoles Phys. Lett. B 434 277
(Papadopoulos G 1997 Preprint hep-th/9712162)
[40] Bourdeau M and Lopes Cardoso G 1998 Finite energy solutions in three-dimensional heterotic string theory
Nucl. Phys. B 522 137
(Bourdeau M and Lopes Cardoso G 1997 Preprint hep-th/9709174)
[41] Minasian R and Tsimpis D 1999 On the geometry of non-trivially embedded branes
Preprint hep-th/9911042
[42] de Boer J, Pasquucci A and Skenderis K 1999 $AdS/CFT$ dualities involving large $2d$ $N = 4$ superconformal
symmetry
Preprint hep-th/9904073
(Cvetič M and Tseytlin A 1995 Preprint hep-th/9510097)
[43] Cvetić M and Tseytlin A 1996 Solitonic strings and BPS saturated dyonic black holes Phys. Rev. D 53 5619
(Cvetić M and Tseytlin A 1995 Preprint hep-th/9512031)
[44] Duff M J, Lu H and Pope C N 1997 Supersymmetry without supersymmetry
Phys. Lett. B 409 136
(Duff M J, Lu H and Pope C N 1997 Preprint hep-th/9704189)