Wilson loops from supergravity and string theory

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Abstract. We present a theorem that determines the value of the Wilson loop associated with a Nambu-Goto action which generalizes the action of the $AdS_5 \times S_5$ model. In particular we derive sufficient conditions for confining behavior. We then apply this theorem to various string models. We go beyond the classical string picture by incorporating quadratic quantum fluctuations. We show that the bosonic determinant of $D_p$ branes with 16 supersymmetries yields a Luscher term. We confirm that the free energy associated with a BPS configuration of a single quark is free from divergences. We show that unlike for a string in flat space time in the case of $AdS_5 \times S_5$ the fermionic determinant does not cancel the bosonic one. For a setup that corresponds to a confining gauge theory the correction to the potential is attractive. We determine the form of the Wilson loop for actions that include non trivial $B_{\mu\nu}$ field. The issue of an exact determination of the value of the stringy Wilson loop is discussed.

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1. Classical Wilson loops - general results

Wilson loops were derived from the Nambu-Goto action associated with the $AdS_5 \times S^5$ supergravity\textsuperscript{[1]}\textsuperscript{[2]} and from several other related actions\textsuperscript{[3]}\textsuperscript{[4]}\textsuperscript{[5]}\textsuperscript{[6]}\textsuperscript{[7]}\textsuperscript{[8]}\textsuperscript{[9]}\textsuperscript{[10]} We introduce here a space-time metric that unifies all these models and determine the corresponding static potential\textsuperscript{[10]}.

Consider a 10d space-time metric

$$ds^2 = -G_{00}(s)dt^2 + G_{x||x||}(s)dx_{||}^2 + G_{ss}(s)ds^2 + G_{xTxT}(s)dx_T^2$$

(1)

where $x_{||}$ are $p$ space coordinates on a $D_p$ brane and $s$ and $x_T$ are the transverse coordinates. The corresponding Nambu-Goto action is

$$S_{NG} = \int d\sigma d\tau \sqrt{det[\partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu}]}$$
Upon using $\tau = t$ and $\sigma = x$, where $x$ is one of the $x_i$ coordinates, the action for a static configuration reduces to

$$S_{NG} = T \cdot \int dx \sqrt{f^2(x) + g^2(x)(\partial_x s)^2}$$

where

$$f^2(x) \equiv G_{00}(s(x))G_{x|x}(s(x)) \quad g^2(x) \equiv G_{00}(s(x))G_{ss}(s(x)) \quad (2)$$

and $T$ is the time interval. The equation of motion (geodesic line)

$$\frac{ds}{dx} = \pm \frac{f(s)}{g(s)} \cdot \sqrt{\frac{f^2(s) - f^2(s_0)}{f(s_0)}}$$

For a static string configuration connecting “Quarks” separated by a distance

$$L = \int dx = 2 \int_{s_0}^{s_1} g(s) \frac{f(s_0)}{f(s)} \sqrt{\frac{f^2(s) - f^2(s_0)}{f(s_0)}} ds$$

The NG action and the corresponding energy $E = \frac{S_{NG}}{T}$ are divergent. The action is renormalized by subtracting the quark masses\[^1\]. For the $AdS_5 \times S^5$ case it is equivalent to the procedure suggested by \[^1\].

$$m_q = \int_0^{s_1} g(s) ds \quad (3)$$

So that the renormalized quark anti-quark potential is

$$E = f(s_0) \cdot L + 2 \int_{s_0}^{s_1} \frac{g(s)}{f(s)}(\sqrt{f^2(s) - f^2(s_0)} - f(s)) ds - 2 \int_0^{s_0} g(s) ds \quad (4)$$

The behavior of the potential is determined by the following theorem\[^{10}\].

**Theorem 1.** Let $S_{NG}$ be the NG action defined above, with functions $f(s), g(s)$ such that:

(i) $f(s)$ is analytic for $0 < s < \infty$. At $s = 0$, ( we take here that the minimum of $f$ is at $s = 0$ ) its expansion is:

$$f(s) = f(0) + a_k s^k + O(s^{k+1})$$

with $k > 0$ , $a_k > 0$.

(ii) $g(s)$ is smooth for $0 < s < \infty$. At $s = 0$, its expansion is:

$$g(s) = b_j s^j + O(s^{j+1})$$

with $j > -1$ , $b_j > 0$.
(iii) \( f(s), g(s) \geq 0 \) for \( 0 \leq s < \infty \).

(iv) \( f'(s) > 0 \) for \( 0 < s < \infty \).

(v) \( \int_{-\infty}^{\infty} g(s)/f^2(s) \, ds < \infty \).

Then for (large enough) \( L \) there will be an even geodesic line asymptoting from both sides to \( s = \infty \), and \( x = \pm L/2 \). The associated potential is

(i) if \( f(0) > 0 \), then

(a) if \( k = 2(j+1) \),

\[
E = f(0) \cdot L - 2 \kappa + O((\log L)^\beta e^{-\alpha L})
\]

(b) if \( k > 2(j+1) \),

\[
E = f(0) \cdot L - 2 \kappa - d \cdot L^{\frac{k+2(j+1)}{k-2(j+1)}} + O(L^\gamma).
\]

where \( \gamma = -\frac{k+2(j+1)}{k-2(j+1)} - \frac{1}{k/2-j} \) and \( \beta \) and \( \kappa \) and \( d \) and \( C_{n,m} \) are positive constants determined by the string configuration.

In particular, there is **linear confinement**

(ii) if \( f(0) = 0 \), then if \( k > j+1 \),

\[
E = -d' \cdot L^{-\frac{j+1}{k-j-1}} + O(L^{\gamma'})
\]

where \( \gamma' = -\frac{j+1}{k-j-1} - \frac{2k-j-1}{(2k-j)(k-j-1)} \) and \( d' \) is a coefficient determined by the classical configuration.

In particular, there is **no confinement**

A detailed proof of this theorem is given in [10]. As a consequence of this theorem it is straightforward that a sufficient condition for confinement is if either of the two conditions is obeyed:

(i) \( f \) has a minimum at \( s_{\text{min}} \) and \( f(s_{\text{min}}) \neq 0 \) or

(ii) \( g \) diverges at \( s_{\text{div}} \) and \( f(s_{\text{div}}) \neq 0 \).
## 2. Applications to various models

| Model | Nambu-Goto Lagrangian | Energy |
|-------|-----------------------|--------|
| $AdS_5 \times S^5$ | $\sqrt{U^4/R^4} + (U')^2$ | $-\frac{2\sqrt{2}e^{3/2R^2}}{\Gamma(\frac{3}{4})^4} \cdot L^{-1}$ |
| non-conformal $D_p$ brane (16 supersymmetries) | $\sqrt{(U/R)^{7-p} + (U')^2}$ | $-d' \cdot L^{-2/(5-p)} + O(l^{-2/(5-p)-2(6-p)/(5-p)(7-p)})$ |
| Pure YM in 4d at finite temperature | $\sqrt{(U/R)^4(1 - (U_T/U)^4) + (U')^2}$ | $\sim L^{-1}(1 - c(LT)^4)$ for $L << L_c$ full screening $L > L_c$ |
| Dual model of Pure YM in 3d | $\sqrt{(U/R)^4 + (U')^2(1 - (U_T/U)^4)}^{-1}$ | $\frac{U^2}{2\pi R^2} \cdot L - 2\kappa + O(\log L e^{-\alpha L})$ |
| Dual model of Pure YM in 4d | $\sqrt{(U/R)^3 + (U')^2(1 - (U_T/U)^3)}^{-1}$ | $\frac{U^3}{2\pi R^{3/2}} \cdot L - 2\kappa + O(\log L e^{-\alpha L})$ |
| Rotating $D_3$ brane | $\sqrt{C \sqrt{\frac{U^6}{U_0^6} \Delta + (U')^2 - \frac{U^2 \Delta}{1 - a^4 U^4/U_0^4 - U^6/U_0^6}}}$ | $4/3\frac{U^2}{R^2} CL + ...$ |
| $D_3 + D_{-1}$ system | $\sqrt{(U^4/R^4 + q) + (U')^2(1 + qR^4/U^4)}$ | $qL + ...$ |
| MQCD system | $2\sqrt{2\zeta} \sqrt{\cosh(s/R_{11})} \sqrt{1 + s^2}$ | $E = 2\sqrt{2\zeta} \cdot L - 2\kappa + O(\log L e^{-1/\sqrt{2}R_{11} L})$ |
| 't Hooft loop | $\frac{1}{g_Y^2 M} \sqrt{(U/R)^4(1 - (U_T/U)^3) + (U')^2}$ | full screening of monopole pair |
3. Quantum fluctuations

So far we have discussed Wilson loops from their correspondence to certain classical string configuration. Now we write down the machinery to incorporate quantum fluctuations and present some preliminary results about the QM determinant of some of the classical setups discussed above [19].

We start with introducing quantum fluctuations around the classical bosonic configuration

\[ x^\mu(\sigma, \tau) = x^\mu_{cl}(\sigma, \tau) + \xi^\mu(\sigma, \tau) \]

The quantum corrections of the Wilson line is

\[ \langle W \rangle = e^{-E_{cl}(L)T} \int \prod_a d\xi_a \exp \left( - \int d^2\sigma \sum_a \xi^a O^a \xi^a \right) \]

where \( \xi^a \) are the fluctuations left after gauge fixing. The corresponding free energy is

\[ F_B = - \log Z(2) = - \sum_a \frac{1}{2} \log \det O_a \]

3.1. Gauge fixing

In the classical treatment it is convenient to choose for the worldsheet coordinates \( \tau = x^0 \) and \( \sigma = x \). In computing the quantum corrections it seems that there are several equivalent gauge fixings. One can still use the gauge of above, namely set \( \xi_x = 0 \), or fix \( \sigma = u, \xi_u = 0 \) (we denote here \( s \) by \( u \)) so that there are no fluctuations in the space-time metric. However, it turns out that those gauges suffer from singularities at the minimum of the configuration \( u_0 \). A gauge that is free from those singularities is the “normal coordinate gauge” \( \sigma = u_{cl} \) and the fluctuation in \( x, u \) plane is in the coordinate normal to \( u_{cl} \).

3.2. General form of the bosonic determinant

In the \( \sigma = u \) gauge and after a change of variables the free energy is given by

\[ F_B = - \frac{1}{2} \log \det O_x - \frac{(p - 1)}{2} \log \det O_{x_{II}} - \frac{(8 - p)}{2} \log \det O_{x_T} \]

where

\[ \dot{O}_x = \left[ \partial_x \left( \frac{1 - \frac{f^2(u_0)}{f^2(u_{cl})}}{f^2(u_{cl})} \right) \partial_x + \frac{G_{xx}(u_{cl})}{G_{tt}(u_{cl})} \left( \frac{f^2(u_{cl})}{f^2(u_0)} - 1 \right) \partial_t^2 \right] \]

\[ \dot{O}_{x_{II}} = \left[ \partial_x \left( \frac{G_{yy}(u_{cl})}{G_{xx}(u_{cl})} \right) \partial_x + \frac{G_{yy}(u_{cl})}{G_{tt}(u_{cl})} \left( \frac{f^2(u_{cl})}{f^2(u_0)} \right) \partial_t^2 \right] \]

(6)
with $\hat{O} = \frac{2}{f(u_0)}O$ and a similar expression for $\hat{O}_{x_T}$. The boundary conditions are $\hat{\xi}(-L/2, t) = \hat{\xi}(L/2, t) = 0$

3.3. Bosonic fluctuations in flat space-time

Let us recall first the determinant in flat space-time. The fluctuations in this case are determined by the following action

$$S_{(2)} = \frac{1}{2} \int d\sigma d\tau \sum_{i=1}^{D-2} \left[ (\partial_\sigma \xi_i)^2 + (\partial_\tau \xi_i)^2 \right]$$

The corresponding eigenvalues are

$$E_{n,m} = \left( \frac{n\pi}{L} \right)^2 + \left( \frac{m\pi}{T} \right)^2$$

and the free energy is given by

$$-\frac{2}{D-2}F_B = \log \prod_{nm} \frac{1}{E_{n,m}} = \frac{T}{2L} \sum_n n + O(L)$$

Regulating this result using Riemann $\zeta$ function we find that the quantum correction to the linear quark anti-quark potential is

$$\Delta V(L) = -\frac{1}{T}F_B = -(D-2)\frac{\pi^2}{24} \cdot \frac{1}{L}$$

which is the so-called Lüscher term [15].

3.4. General scaling relation, and the $L$ dependence of $\Delta V$

Consider an operator of the form

$$\mathcal{O}[A, B] = A^2 F_t(v) \partial_t^2 + B^2 \partial_v (F_v(v) \partial_v)$$

The correction to the potential $V[A, B]$ due to fluctuations determined by such an operator is

$$V[A, B] = (B/A) \cdot V[1, 1]$$

For the operators that describe the fluctuations associated with metrics such that

$$f(u) = au^k \quad g(u) = bu^j$$

like for instance for the $D_p$ brane solution in the near horizon limit we find that $A^2 = bu_0^j$, $B^2 = \frac{a^2}{b} u_0^{2k-j-2} \rightarrow B/A = \frac{a}{b} u_0^{k-j-1}$. Therefore, the potential is proportional to

$$B/A = \frac{a}{b} u_0^{k-j-1} \rightarrow \Delta V \propto L^{-1}$$

Thus, the quantum correction of the quark anti-quark potential is of Lüscher type [14] for models of $D_p$ branes with 16 supersymmetries, in particular also the $AdS_5 \times S^5$ model.
3.5. The fermionic fluctuations in flat space-time

The NSR action of the type II superstring with a RR fields like on $AdS_5 \times S^5$ is not known. On the other hand the manifestly space-time supersymmetric Green Schwarz action was written down for the $AdS_5 \times S^5$ case\cite{20} To demonstrate the use of the GS action we start with the fermionic determinant in flat space-time

The fermionic part of the $\kappa$ gauged fixed GS-action is

$$S_{F}^{\text{flat}} = 2i \int d\sigma d\tau \bar{\psi} \Gamma^i \partial_i \psi$$

where $\psi$ is a Weyl-Majorana spinor, $\Gamma^i$ are the SO(1,9) gamma matrices, $i,j = 1,2$ and we explicitly considered a flat classical string. Thus the fermionic operator is

$$\hat{O}_F = D_F = \Gamma^i \partial_i \quad (\hat{O}_F)^2 = \Delta = \partial_x^2 - \partial_t^2$$

The total free energy is

$$F = 8 \times \left( -\frac{1}{2} \log \det \Delta + \log \det D_F \right) = 0$$

since for $D=10$, we have 8 transverse coordinates and 8 components of the unfixed Weyl-Majorana spinor. Thus in flat space-time the energy associated with the supersymmetric string is not corrected by quantum fluctuations.

3.6. The determinant for a free BPS quark of $AdS_5 \times S^5$

The $\kappa$ fixed GS action\cite{21} is based on treating the target space as the coset $SU(2,2|4)/(SO(1,4) \times SO(5))$. The action incorporates the coupling to the RR field. The square of the operator associated with the fermionic fluctuations is

$$8 \times \left( \mathcal{O}_\psi^2 = \partial_\sigma^2 + \partial_\tau^2 - \frac{3}{4\sigma^2} \right)$$

The bosonic operators are of the form

$$3 \times \left[ \mathcal{O}_x = \partial_\sigma^2 + \partial_\tau^2 - \frac{2}{\sigma^2} \right]$$

$$5 \times \left[ \mathcal{O}_\theta = \partial_\sigma^2 + \partial_\tau^2 \right]$$

where $\{x^0, x, u, \theta\} \equiv \{\tau, \sigma, \xi \cdot \xi , \xi \theta \}$ and $\theta$ is the coordinate on the $S^5$. According to a theorem of McKean and Singer\cite{22} the divergences of a Laplacian type operator of the form

$$\Delta = \nabla^2 + X = -\frac{1}{\sqrt{g}} D_a (g^{ab} \sqrt{g} D_b) + X$$
vanish if there is a match between the fermionic and bosonic coefficients of $\nabla^2$ and $X$. In the present case there are 8 bosonic and 8 fermionic $\nabla^2$ terms and hence there is no quadratic divergences, and the coefficients of $X$ are $8 \times 3/4$ from the fermions and $3 \times 2$ from the bosons so there are also no logarithmic divergences. It is thus clear that the divergent parts of the determinant associated with the supersymmetric fluctuation of a BPS string “quark” vanish. This problem is related to issues associated with certain BPS soliton solutions[23].

3.7. The determinant for a Wilson line of $AdS_5 \times S^5$

The GS action was further simplified by using a particular $\kappa$ fixing[21]

$$S_{GS} = \int d^2 \sigma \sqrt{g} g^{\alpha \beta} (y^2 \partial_\alpha x^\mu - 2i \bar{\psi} \Gamma^\mu \partial_\alpha \psi) [\partial_\beta x^\mu - 2i \bar{\psi} \Gamma^\mu \partial_\beta \psi] + \frac{1}{y^2} \partial_\alpha y^t \partial_\beta y^t) + 4 \epsilon^{\alpha \beta} \partial_\alpha y^t \bar{\psi} \Gamma^t \partial_\beta \psi]$$

where $\psi$ is a Majorana-Weyl spinor and the $AdS_5 \times S^5$ metric is written in terms of the $4 + 6$ coordinates $ds^2 = y^2 dx_{11}^2 + \frac{1}{y^2} dy_{xT}^2$. The bosonic operators in the normal gauge now read

$$2 \times \mathcal{O}_{x_{11}} = \partial_x^2 - \frac{u^4}{u_0^4} \partial_t^2$$

$$5 \times \mathcal{O}_{\theta} = \partial_x^2 - \frac{u^4}{u_0^4} \partial_t^2 + 2 \frac{u^6}{u_0^6}$$

$$1 \times \mathcal{O}_{\text{normal}} = \partial_x^2 - \frac{u^4}{u_0^4} \partial_t^2 + 5 u^2 - 3 \frac{u^4}{u_0^2}$$

The fermionic part of the action for the classical solution leads to the operator

$$\hat{O}_\psi = \frac{u_0^2}{R^2} \Gamma^1 \partial_x + \left( \frac{u_{cl}^4}{u_0^2 R^2} \Gamma^0 + \frac{u_{cl}^4}{R^4} \cdot \sqrt{u_{cl}^4 - u_0^4} \Gamma^2 \right) \partial_t$$

where we use $\Gamma$ matrices of $SO(1,4)$, the $AdS_5$ tangent space. Squaring this operator, we find

$$\left( \frac{R^2}{u_0^2} \hat{O}_F \right)^2 = \partial_x^2 - \frac{u_{cl}^4}{u_0^4} \partial_t^2 = \frac{R^2}{u_0^2} \hat{O}_y$$

Thus the transverse fluctuations $\mathcal{O}_{x_{11}}$ are cancelled by the fermionic fluctuations. We are left with 6 fermionic degrees of freedom, the normal bosonic fluctuation and 5 additional bosonic fluctuations associated with $\mathcal{O}_{\theta}$. Using our general result we know that the quantum correction of the potential is of a Luscher type. The universal coefficient and
in particular its sign has not yet been determined. In [24] the bosonic and fermionic determinants were analyzed in a different gauge fixing procedure. It was found there that the answer is not free from logarithmic divergences.

3.8. The determinant for “confining scenarios”

Let us consider first the the setup which is dual to the pure YM theory in 3d. For that case

\[ f(u) = u^2 / R^2 \quad \text{and} \quad g(u) = (1 - (u_T / u)^4)^{-1/2} \]  

(11)

In the large \( L \) limit

\[ \hat{O}_{x_I} \longrightarrow \frac{u_T^2}{2} \left[ \partial_x^2 + \partial_t^2 \right] \]  

(12)

\[ \hat{O}_t \longrightarrow 2u_T^2 e^{-2u_T L} \left[ \partial_x^2 + \partial_t^2 \right] \]  

(13)

\[ \hat{O}_n \longrightarrow \left[ \frac{4u_T^2}{2R^4} + \frac{1}{2} \partial_x^2 + \frac{1}{2} \partial_t^2 \right] \]  

(14)

We see that the operators for transverse fluctuations, \( \hat{O}_{x_I}, \hat{O}_t \), turn out to be simply the Laplacian in flat spacetime, multiplied by overall factors, which are irrelevant. Therefore, the transverse fluctuations yield the standard Lüscher term proportional to \( 1/L \). The longitudinal normal fluctuations give rise to an operator \( \hat{O}_n \) corresponding to a scalar field with mass \( 2u_T^2 / R^2 = \alpha \). Such a field contributes a Yukawa like term

\[ \approx -\sqrt{\alpha} e^{-\alpha L} \]  

\[ \sqrt{L} \]

to the potential. Thus, in the metric that corresponds to the “pure YM case” there are 7 Luscher type modes and one massive mode. It can be shown that a similar behavior occurs in the general confining setup [19]. Had the fermionic modes been those of flat space-time then the total coefficient in front of the Luscher term would have been a repulsive Culomb like potential [7] which contradicts gauge dynamics [25]. However the point is that due to the RR flux the corresponding GS action cannot be that of a flat space-time. Indeed the fermionic fluctuations also become massive so that the total interaction is attractive after all which is in accordance with [8].

4. On the exact determination of Wilson loops

[26] So far we have discussed the determination of Wilson loops from the classical string description and the way quantum fluctuations modify the classical result. An interesting question to address is whether in certain circumstances one can find an exact expression for the Wilson loop. We start with a string in flat space-time.
Consider the bosonic string in flat space-time with the boundary conditions

\[ X^i(\sigma = 0) = 0 \quad X^i(\sigma = \pi) = L^i \quad \text{with} \quad L^i L_i = L^2 \]

The energy of the lowest \textbf{tachyonic} state is given by

\[ E^2 = P^i P_i + m_{\text{tach}}^2 = \left( \frac{L^i}{2\pi \alpha'} \right)^2 - \left( \frac{D - 2}{24} \right) \frac{1}{\alpha'} \]

so that

\[ E = T_{st} L \sqrt{1 - \frac{(D-2)}{24} \frac{1}{T_{st} L^2}} \]

For \( L >> (T_s)^{-1/2} \) this can be expanded to yield

\[ \sim T_{st} L - \frac{\pi}{24} \frac{(D - 2)}{L} + \ldots \]

where the string tension \( T_{st}^{-1} = 2\pi \alpha' \). Thus this expansion yields the Luscher quadratic fluctuation term. Moreover, this result is identical to the expression for the NG action derived for a bosonic string in flat space-time in the large \( D \) limit

\[ D \to \infty \quad \frac{\pi}{24 T_{st} L^2} \to 0 \quad \frac{D \pi}{24 T_{st} L^2} \to \text{finite} \]

It is straightforward to realize that for a static classical configurations \( E_{\text{Poly}} = S_{NG} \).

A more challenging question is whether one can find such “exact” solution for a non-flat space-time. A naive conjecture is that for the \( AdS_5 \times S^5 \) the result is

\[ \sim -\sqrt{g^2 N} L \sqrt{1 + \frac{c}{\sqrt{g^2 N}}} \]

However, whereas its large \( g^2 N \) expansion includes the lresult of \( \text{[1]} \) and a non trivial Luscher term, it does not permit a smooth extrapolation to the weak coupling region where the potential behaves like \( \sim -\frac{g^2 N}{L} \).

### 4.1. Wilson loops for string actions with WZ term

In general exact results are known for non trivial backgrounds of group manifolds and coset spaces. The sigma model associated with such target spaces is equipped with a WZ term. The bosonic action is therefore

\[ S_B = S_{NG} + \int d^2 \sigma e^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu \nu} \]

For the case that the only non-trivial component of \( B_{\mu \nu} \) is \( B_{01} = B(u) \) one finds that for \( B \neq f \) ( \( f \) was defined in \( \text{(2)} \))

\[ S_{NG+WZ} = \int_{u_0}^{\infty} du \frac{a}{f} \frac{f^2 - B(f_0 + B - B_0)}{\sqrt{f^2 - (f_0 + B - B_0)^2}} \]
\[ S_{NG+WZ} = 0 \quad \text{for} \quad B = f \]

For the former case

\[ S = (f_0 + B_0)L + 2 \int_{u_0}^{\infty} du \frac{g}{f} \sqrt{f^2 - (f_0 + B - B_0)^2} \]

For string theories where \( f_0 + B_0 \), which is the value of \( B + f \) at the minimum of the string configuration, does not vanish the Wilson loop admits an area law behavior with a string tension equal to \( f_0 + B_0 \).

In the string model associated with a three dimensional \( SL(2, R) \) group manifold, and in the “near extremal” corresponding models of \( SL(2, R) \times R \). The \( B \) term matches the \( f \), namely \( f = B \) so that the Wilson line is a straight line and the energy \( E = 0 \).

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