Re-examination of long distance effects in $b \rightarrow s l^+ l^-$

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Abstract

We re-analyse the long distance contributions to the process $b \rightarrow s l^+ l^-$. Full $q^2$-behavior of the vector meson dominance amplitude is used together with the effect of Terasaki suppression, and comparisons with the previous results are given. We show that the interference between short- and long distance contributions makes it difficult to extract the short distance information from the dominant long distance background, either in the dilepton invariant mass distribution or in the single lepton energy spectrum.

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Rare decays through the flavor changing $b \to s$ transitions provide good test of the standard model (SM), and are expected to give signals of new physics [1]. The branching ratio of the process $b \to s\gamma$, which has already been measured by the CLEO collaboration [2], is within the SM predictions. Unlike the decay $b \to s\gamma$, the process $b \to s l^+ l^-(l = e \text{ or } \mu)$ is expected to be dominated by long distance contributions through the mechanism of vector meson dominance (VMD) [3]. However, it was usually believed that the long distance (resonance) contributions arise only in some particular region of the invariant mass spectrum of the dilepton pair [4], since the involved resonance $\psi(\psi')$ peak is very sharp. Detailed calculation [3] shows that there exists significant interference between the short and long distance contributions, which leaves only a small portion of kinematic region at low dilepton invariant mass where the interference effect by the resonances is small. The energy spectrum of single lepton has also been given in [5] where a window of nearly pure short distance information is found.

In the present work, we will re-examine both the dilepton invariant mass and the single lepton energy spectrums using an alternative treatment of the VMD amplitude. In the previous analysis of the cascade decays $b \to s \psi(\psi')$ and $\psi(\psi') \to l^+ l^-$, an effective description is made for the later electromagnetic transition $\psi(\psi') \to l^+ l^-$ [3, 5]. The dependence of the VMD amplitude on the square of the dilepton invariant mass, $q^2$, is approximated by that of the resonance mass $m_\psi^2$ or $m_{\psi'}^2$ in the denominator of the photon propagator. This approximation is only valid near the resonance region, and consequently, the previous analysis are not complete in the whole phase space [4].

Let us start with the short distance contributions to $b \to s l^+ l^-$ with $l = e$ or $\mu$. The short distance contributions come from box, $Z$ and photon penguin diagrams. The QCD corrected effective Hamiltonian in SM is [4]:

$$
\mathcal{H}_{\text{eff}} = \frac{\alpha G_F}{\sqrt{2\pi}} V_{tb} V_{ts} \left[ \left( C_9^{\text{eff}} \bar{s} \gamma^\mu P_L b + \frac{2C_7^{\text{eff}} m_b}{q^2} \bar{s} \gamma^\mu P_R b \right) \bar{l} \gamma_\mu l + C_{10} (\bar{s} \gamma^\mu P_L b) \bar{l} \gamma_{\mu 5} l \right],
$$

1This has also been noted in [6] in the discussion of exclusive decays of B meson, and in [3] in the charm quark sector.
with $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, and $q = p_{l+} + p_{l-}$ is the invariant mass of the dilepton.

The analytic Wilson coefficients $C_{eff}^7(\mu)$, $C_{eff}^9(\mu)$, and $C_{10}(\mu)$ are given in Ref. [8]. Under the leading logarithmic approximation, we get the numerical results at $\mu = m_b$ as:

$$C_{eff}^7 = -0.315, \quad C_{10} = -4.642, \quad (2)$$

and to the next-to-leading order,

$$C_{eff}^9 = 4.227 + 0.124 \omega(\hat{s}) + 0.359 g(m_c/m_b, \hat{s}) + 0.034 g(1, \hat{s}) + 0.033 g(0, \hat{s}), \quad (3)$$

where $\hat{s} = q^2/m_b^2$. The function $\omega(\hat{s})$ and $g(z, \hat{s})$ can be found in ref. [8]. Here for numerical evaluation, we use $m_{top} = 176$GeV [9], $m_b = 4.8$GeV, $m_c = 1.4$GeV, $\Lambda_{QCD} = 225$MeV [10].

By normalizing to the semileptonic rate, the strong dependence on the b-quark mass cancels out. The differential decay rate $d\Gamma(B \to X_s l^+ l^-)/d\hat{s}$, where $\hat{s} = (p_{l+} + p_{l-})^2/m_B^2$, is given by

$$\frac{1}{\Gamma(B \to X_c e \bar{\nu})} \frac{d\Gamma(B \to X_s l^+ l^-)}{d\hat{s}} = \frac{\alpha^2_{QED}}{4\pi^2 f(m_c/m_b)} (1 - \hat{s})^2 \left[ (1 + 2\hat{s}) \left( |C_{eff}^9|^2 + C_{10}^2 \right) \right.
\left. + \left( 1 + \frac{2}{\hat{s}} \right) |C_{eff}^7|^2 + 12 C_{eff}^7 \text{Re}(C_{eff}^9) \right], \quad (4)$$

where $f(m_c/m_b)$ is the phase space factor:

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x.$$

If we take the experimental result $Br(B \to X_c e \bar{\nu}) = 10.8\%$ [10], the differential decay rate of $\mathcal{B} \to X_s \mu^+ \mu^-$ is found, which is depicted in Fig.1 as the dash-dotted line.

In addition, there are also long distance resonance contributions from $c \bar{c}$ state. There are six known resonances in the $c \bar{c}$ system that can contribute to this decay mode [11]. The lowest two, $\psi$ and $\psi'$, were considered in the previous analyses [3, 5]. Here we also consider the same two resonances. The higher resonances will also contribute, but they are less important in our case of discussing the uncertainties, as will be shown later.
Applying the VMD mechanism, the long distance contribution is through $b \to s \psi$, and $\psi \to \gamma \to l^+ l^-$, where the resonance can also be $\psi'$. These give the effective Lagrangian

$$L_{\text{res}} = \frac{16\pi^2 a_2 g^2_\psi(q^2)}{3q^2(q^2 - m^2_\psi + i m_\psi \Gamma_\psi)} \frac{\alpha G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^* (\bar{s} \gamma^\mu P_L b) \bar{l} \gamma^\mu l + (\psi \to \psi'),$$  \hspace{1cm} (5)$$

where $a_2 = C_1 + C_2/3$ is a QCD corrected coefficient of the four quark operators. Below we will use the phenomenological value $a_2 = 0.24$ which comes from fitting the data of $B$ meson decays\cite{14}. Note that the expression (5) differs from the previous ones\cite{3, 5} by keeping the photon propagator as $-i g_{\mu\nu}/q^2$ instead of $-i g_{\mu\nu}/m^2_\psi$ or $-i g_{\mu\nu}/m^2_{\psi'}$. Thus it holds in the whole kinematic region.

The effective coupling of a vector meson $g_V(q^2)(V = \psi, \psi')$ is defined by

$$<0|\bar{c} \gamma_\mu c|V(q)> = g_V(q^2) \epsilon^V_\mu,$$  \hspace{1cm} (6)$$

where $\epsilon^V_\mu$ is the polarization vector of the vector meson $V$. On the mass-shell of the vector meson, $g_V(q^2)$ is replaced by the decay constant $g_V(m^2_V)$ which can be obtained from the leptonic width of the vector meson:

$$\Gamma(V \to \ell^+ \ell^-) = \frac{16\pi \alpha^2}{27 m^3_V} g^2_V(m^2_V).$$  \hspace{1cm} (7)$$

The structure of eqn.(5) is the same as that of the operator $O_9$. It is convenient to include the resonance contribution in eqn.(3) by simply making the replacement

$$C^\text{eff}_9 \to C^\text{eff}_9 = C^\text{eff}_9 + \frac{16\pi^2 a_2 g^2_\psi(q^2)}{3q^2(q^2 - m^2_\psi + i m_\psi \Gamma_\psi)} + (\psi \to \psi').$$  \hspace{1cm} (8)$$

Assuming a constant coupling $g^2_\psi(q^2) \equiv g^2_\psi(m^2_\psi)$ as done in \cite{3, 5}, the numerical result is given in Fig.1 as the dashed line. It is easy to see that, this spectrum is enhanced in the low $q^2$ region due to the explicit inclusion of the photon propagator. From Fig.1, we can also expect that higher resonances other than $\psi$ or $\psi'$ contribute mainly in the region $0.6 < \hat{s} < 1$ where we are not interested, since near this tail of the spectrum no useful short distance information is expected to emerge.
The assumption made above on the constant coupling of $g^2_\psi(q^2)$ can be improved by accounting for the mechanism of Terasaki suppression for the $\psi-\gamma$ conversion\cite{13}. In the framework of VMD, the data on photoproduction of $\psi$ indicates a large suppression of $g_\psi(0)$ compared to $g_\psi(m^2_\psi)$\cite{14}. This has been confirmed in \cite{13} by constraining the dominant long distance contribution to $s \to d\gamma$ using the present upper bound on the $\Omega^- \to \Xi^-\gamma$ decay rate. As a result, it can be concluded that this suppression results in a much smaller long distance contribution to $b \to s\gamma$ transition\cite{14}. Now we use a momentum dependent $g_V(q^2)(V = \psi, \psi')$ in $\mathcal{L}_{\text{res}}$, which is used in \cite{16} to obtain a reduced resonance to non-resonance interference where a broader region of invariant mass spectrum sensitive to short distance physics is claimed.

The momentum dependence of $g_V(q^2)(V = \psi, \psi')$ derived using a dispersion relation\cite{13} is

$$g_V(q^2) = g_V(0) \left( 1 + \frac{q^2}{c_V} \left[ d_V - h(q^2) \right] \right),$$

(9)

where $c_\psi = 0.54$, $c_{\psi'} = 0.77$ and $d_\psi = d_{\psi'} = 0.043$. $h(q^2)$ is defined by

$$h(q^2) = \frac{1}{16\pi^2 r} \left\{ -4 - \frac{20r}{3} + 4(1+2r)\sqrt{1 - \frac{1}{r}} - 1 \tan^{-1} \frac{1}{\sqrt{1 - \frac{1}{r}}} \right\}$$

(10)

with $r = q^2/m^2_V$ for $0 \leq q^2 \leq m^2_V$. As a result, eqn.(9), which is valid for $0 \leq q^2 \leq m^2_V$, is an interpolation of $g_V$ from the photoproduction experimental data on $g_V(0)$ to $g_V(m^2_V)$ from the leptonic width based on quark-loop diagram. We assume $g_V(q^2) = g_V(m^2_V)$ for $q^2 > m^2_V$, mainly due to the fact that the behavior of the $\psi-\gamma$ conversion strength is not clear in this region, and is not important in our case (see below).

Applying Terasaki’s formula \cite{9} for the $q^2$ dependence of $g_V(q^2)$, the differential decay rate of $b \to sl^+l^-$ receives suppression in low $q^2$ region. However, there is still significant interference between the resonance and the short distance contributions, due to the factor $1/q^2$ coming from the propagator of the virtual photon. This is also shown in Fig.1.

Now we turn to the energy spectrum of single lepton. The integration over $q^2$ is complicated, since many functions here involve $\hat{s}$. We simply give the numerical results in Fig.2 for
$l = \mu$ (see also [3]). One can see that, if the $1/q^2$ behavior is replaced by $1/m^2_\psi$ (or $1/m^2_\psi'$) everywhere, there is almost no contribution from the $\psi, \psi'$ resonance when $\beta < 0.2$. This result is what has been arrived in Ref.[3]. If, however, this $1/q^2$ is retained, there are also contributions from the resonances even in the low $\beta$ region and consequently, the resonance background is still serious. Including the effect of Terasaki suppression of $g_N(q^2)$ in the low $q^2$ region, the result is also shown in Fig.2 where the resonance background is only half reduced.

From both Fig.1 and Fig.2, we can observe that the long distance VMD contributions to the process $b \rightarrow s l^+ l^-$ are large, if an alternative treatment of the electromagnetic subprocess is performed. It can also be seen that the single lepton energy spectrum is almost useless in extraction of any short distance information from the resonance background. The total branching ratio of $b \rightarrow s l^+ l^-$ turn out to be $3.6 \times 10^{-4}$ or $4.9 \times 10^{-4}$ with the effect of Terasaki suppression included or not.

We have treated the resonance contribution from $\psi, \psi'$ to $b \rightarrow s l^+ l^-$ alternatively without using the effective description of the electromagnetic sub-process $\psi(\psi') \rightarrow \gamma l^+ l^-$. The long distance contributions are found to be significant, especially in the single lepton energy spectrum. Concerning other higher resonances, and also other uncertainties existed in this decay mode[8], we conclude that it is difficult to extract short distance information, which is sensitive to new physics, from the dominant long distance contributions.

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References

[1] For review see A.J. Buras, M.K. Harlander, “Heavy flavors”, p58-201, Eds. A.J. Buras, M. Lindner, World Scientific, Singapore; A. Ali, Nucl. Phys. B, Proc. Suppl. 39BC, 408-425 (1995); S. Playfer and S. Stone, HEPSY 95-01.

[2] R. Ammar, et al., CLEO Collaboration, Phys. Rev. Lett. 71, 674 (1993); M.S.Alam et al., CLEO Collaboration, Phys. Rev. Lett. 74, 2885 (1995).

[3] N.G. Deshpande, J. Trampetic and K. Panose, Phys. Rev. D39, 1461 (1989); C.S. Lim, T. Morozumi and A.I. Sanda, Phys. Lett. B218, 343 (1989); P.J. O’Donnell, and H.K.K. Tung, Phys. Rev. D43, 2067 (1991); N. Paver and Riazuddin, Phys. Rev. D45, 978 (1992).

[4] W.S Hou and R.S. Willey, Phys. Lett. B202 (1988)591; R. Grigjanis et al. Phys. Lett. B213 (1988) 247; Phys. Lett. 223 (1989) 239; Phys. Rev. D42 (1990) 245; B. Grinstein, M.J. Savage and M.B. Wise, Nucl. Phys. B319 (1989) 271; G. Cella, G. Ricciardi, A. Vicere, Phys. Lett. B258 (1991) 212; M. Misiak, Nucl. Phys. B393 (1993) 23; (E) ibid, B439 (1995) 461.

[5] P.J. O’Donnell, M. Sutherland, and H.K.K. Tung, Phys. Rev. D46, 4091 (1992).

[6] Z. Ligeti and M. B. Wise, Phys. Rev. D53, 4937 (1996).

[7] P. Singer and D.-X. Zhang, preprint TECHNION-PH-96-10, to be published in Phys. Rev. D55 (1997).

[8] A.J. Buras and M. Münz, Phys. Rev. D52 (1995) 186.

[9] F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 74, 2626 (1995).

[10] Particle Data Group, Phys. Rev. D50, 3-I (1994).

[11] F. Krüger and L.M. Sehgal, Phys. Lett. B380 (1996) 199.
[12] H.-Y. Cheng and B. Tseng, Phys. Rev. D51, 6259 (1995); M. Gourdin, A.N. Kamal, Y.Y. Keum and X.Y. Pham, Phys. Lett. B333, 507 (1994).

[13] K. Terasaki, Nuov. Cim. 66A, 475 (1981).

[14] N.G. Deshpande, X.G. He and J. Trampetic, Phys. Lett. B367, 362 (1996).

[15] G. Eilam, A. Ioannissian, R.R. Mendel and P. Singer, TECHNION-PH-95-18.

[16] M.R. Ahmady, Phys. Rev. D53, 2843 (1996).

**Figure Captions**

Fig.1 The differential decay rate via \( \hat{s} = m_{\mu\mu}^2/m_b^2 \). The dash-dotted and the dotted lines correspond to the results without resonance contribution and with resonance contribution included as [3, 5], respectively. The dashed and solid lines are the results with resonance included with the treatment of the electromagnetic sub-processes using eqn. (5), while constant coupling \( g_V(q^2) = g_V(m_{V}^2) \) and (9) are used, respectively.

Fig.2 Same as Fig.1 for the differential decay rate via \( \beta = E^-/m_b \).
