Anderson-Mott Transition in a Magnetic Field: Corrections to Scaling

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It is shown that the Anderson-Mott metal-insulator transition of paramagnetic, interacting disordered electrons in an external magnetic field is in the same universality class as the transition from a ferromagnetic metal to a ferromagnetic insulator discussed recently. As a consequence, large corrections to scaling exist in the magnetic-field universality class, which have been neglected in previous theoretical descriptions. The nature and consequences of these corrections to scaling are discussed.

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I. INTRODUCTION

The metal-insulator transition of interacting, disordered electrons at zero temperature ($T = 0$), or Anderson-Mott transition (AMT), has been studied theoretically near two-dimensions ($d = 2 + \epsilon$) as well as in high dimensions ($d > 6$) and $d \lesssim 6$). In the latter case one finds the complete universality that is characteristic of Landau theories, while in $d = 2 + \epsilon$ there is a large number of different universality classes. This is because the AMT in low dimensions is driven by soft modes, and the soft-mode spectrum of electrons at $T = 0$ is determined by the symmetries of the underlying field theory. Accordingly, systems in external magnetic fields, in the presence of magnetic impurities, in the presence of spin-orbit scattering, etc., all constitute different universality classes. A priori it is not clear whether the high-dimensional or the low-dimensional theories provide a better description of the situation in $d = 3$. Experimentally, while different universality classes seem to exist, the situation is also far from clear. A serious problem is posed by the fact that the minimum distance from the critical point, and the ease and accuracy with which the distance from the critical point can be controlled, are much inferior to what can be routinely achieved near thermal phase transitions. Consequently, one should expect corrections to scaling to play an important role in the experimentally accessible region of parameter space, and their incorporation into any interpretation of data to be essential. This expectation notwithstanding, however, corrections to scaling near an AMT have been largely ignored by both experimentalists and theorists, with the exception of a suggestion that strong (logarithmic) corrections to scaling may be responsible for the ill-understood observations in phosphorus-doped silicon.

Recently, another example of an AMT has been considered near two-dimensions, namely, the quantum phase transition from a ferromagnetic metal to a ferromagnetic insulator. Since both an intrinsic magnetization and an external magnetic field break the rotational symmetry in spin space and give a mass to the transverse spin-triplet particle-hole excitations, one expects this transition to be related to the AMT in an external field. An important question in this context relates to the Goldstone modes or spin waves that are present in a ferromagnet, but not in a system in an external field. Since the Goldstone modes contribute to the soft-mode spectrum, one might expect them to influence the critical behavior. However, it has been shown in Ref. 8 that this is not the case. This makes it likely that the two transitions indeed belong to the same universality class. The effective model for the ferromagnetic MIT that was derived and studied in Ref. 8, however, is different from the one for the magnetic-field universality class considered before and the former contains strong corrections to scaling that were absent in the latter. This suggests that the existing description of the magnetic-field universality class is incomplete.

In this note we show that the two transitions belong indeed to the same universality class, as has been suggested in Ref. 9 and that the previously studied model for the magnetic-field AMT missed terms that contribute important corrections to scaling. As a result of these corrections to scaling, a reliable experimental determination of the critical exponents is not possible from the currently available experimental data.

II. EFFECTIVE ACTION

We consider the usual microscopic model for an AMT in an external magnetic field. That is, we assume that the orbital effects of the magnetic field serve only to suppress the soft modes in the particle-particle or Cooper channel and consider only the Zeeman term explicitly.
The latter’s contribution to the action is

$$S_B = \frac{1}{2} g_L \mu_B B \int dx \, T \sum_n \sum_{\alpha} \left[ \bar{\psi}^{\alpha}_{n,\uparrow}(x) \psi^{\alpha}_{n,\uparrow}(x) - \bar{\psi}^{\alpha}_{n,\downarrow}(x) \psi^{\alpha}_{n,\downarrow}(x) \right].$$  \hspace{1cm} (2.1)

Here $g_L$ is the g-factor, $\mu_B$ is the Bohr magneton, $B$ is the magnetic field, and $T$ is the temperature. $\psi_{n,\uparrow}$ and $\psi_{n,\downarrow}$ are fermionic fields for up- and down-spin electrons, with $n$ the index of a fermionic Matsubara frequency, $\omega_n = 2\pi T (n + 1/2)$, and $\alpha$ a replica index to deal with the quenched disorder.

Within the formalism of Ref. [8], bilinear products of fermion fields are expressed in terms of a classical matrix field $Q$, whose elements are related to the fermion fields by means of the isomorphism

$$Q_{12} \cong \frac{i}{2} \begin{pmatrix} -\psi^{\uparrow}_1 \bar{\psi}^{\uparrow}_2 & -\psi^{\uparrow}_1 \bar{\psi}^{\uparrow}_3 & -\psi^{\uparrow}_1 \bar{\psi}^{\uparrow}_4 & \psi^{\uparrow}_1 \psi^{\uparrow}_2 & \psi^{\uparrow}_1 \psi^{\uparrow}_3 & \psi^{\uparrow}_1 \psi^{\uparrow}_4 \\ -\psi^{\downarrow}_1 \bar{\psi}^{\downarrow}_2 & -\psi^{\downarrow}_1 \bar{\psi}^{\downarrow}_3 & -\psi^{\downarrow}_1 \bar{\psi}^{\downarrow}_4 & \psi^{\downarrow}_1 \psi^{\downarrow}_2 & \psi^{\downarrow}_1 \psi^{\downarrow}_3 & \psi^{\downarrow}_1 \psi^{\downarrow}_4 \\ -\psi^{\downarrow}_2 \bar{\psi}^{\downarrow}_3 & -\psi^{\downarrow}_2 \bar{\psi}^{\downarrow}_4 & -\psi^{\downarrow}_2 \bar{\psi}^{\downarrow}_1 & \psi^{\downarrow}_2 \psi^{\downarrow}_3 & \psi^{\downarrow}_2 \psi^{\downarrow}_4 & \psi^{\downarrow}_2 \psi^{\downarrow}_1 \\ \psi^{\downarrow}_3 \bar{\psi}^{\downarrow}_4 & \psi^{\downarrow}_3 \bar{\psi}^{\downarrow}_1 & \psi^{\downarrow}_3 \bar{\psi}^{\downarrow}_2 & \psi^{\downarrow}_3 \psi^{\downarrow}_4 & \psi^{\downarrow}_3 \psi^{\downarrow}_1 & \psi^{\downarrow}_3 \psi^{\downarrow}_2 \\ -\psi^{\uparrow}_3 \bar{\psi}^{\uparrow}_4 & -\psi^{\uparrow}_3 \bar{\psi}^{\uparrow}_1 & -\psi^{\uparrow}_3 \bar{\psi}^{\uparrow}_2 & \psi^{\uparrow}_3 \psi^{\uparrow}_4 & \psi^{\uparrow}_3 \psi^{\uparrow}_1 & \psi^{\uparrow}_3 \psi^{\uparrow}_2 \\ -\psi^{\uparrow}_4 \bar{\psi}^{\uparrow}_1 & -\psi^{\uparrow}_4 \bar{\psi}^{\uparrow}_2 & -\psi^{\uparrow}_4 \bar{\psi}^{\uparrow}_3 & \psi^{\uparrow}_4 \psi^{\uparrow}_1 & \psi^{\uparrow}_4 \psi^{\uparrow}_2 & \psi^{\uparrow}_4 \psi^{\uparrow}_3 \end{pmatrix}.$$ \hspace{1cm} (2.2)

Here all fields are understood to be taken at position $x$, and $1 \equiv (n_1, \alpha_1)$, etc. In terms of these matrix fields, the Zeeman term can be written

$$\mathcal{A}_B = -i h \int dx \, \text{tr} \left[ (\tau_3 \otimes s_3) Q(x) \right].$$ \hspace{1cm} (2.3)

where $h = g_L \mu_B B$, $\text{tr}$ is a trace over all discrete indices of the matrix $Q$, and $\tau_i = -s_3 = -i \sigma_z$, with $\sigma_z$ a Pauli matrix. More generally, it is useful to expand $Q$ into a spin-quantum basis,

$$Q_{12}(x) = \sum_{r=0,3} \sum_{i=0,3} \frac{i}{2} Q_{12}(x) \left( \tau_r \otimes s_i \right),$$ \hspace{1cm} (2.4)

with $\tau_0 = s_0 = 1$ the unit $2 \times 2$ matrix. The partition function can be written in terms of a functional integration over the field $Q$ and an auxiliary field $\tilde{A}$.

$$Z = \int D[Q] D[\tilde{A}] \, e^{\mathcal{A}[Q, \tilde{A}]},$$ \hspace{1cm} (2.5a)

with an action

$$\mathcal{A}[Q, \tilde{A}] = \mathcal{A}_{B=0}[Q, \tilde{A}] + \mathcal{A}_B[Q].$$ \hspace{1cm} (2.5b)

The action in the absence of a magnetic field, $\mathcal{A}_{B=0}$, is the same as in Ref. [8].

The further formal development proceeds in exact analogy to Ref. [8], and we will therefore be very brief. The action allows for a saddle-point solution that can be written in terms of two Green functions, $G$ and $F$, which are related to the saddle-point values of the matrix elements $\delta Q$ and $\tilde{Q}$, respectively. They obey the equations

\begin{align}
(i \omega_n - \xi_k - \Sigma_n) \, G_n(k) + \Delta_n \, F_n(k) &= 1, \quad (2.6a) \\
(i \omega_n - \xi_k - \Sigma_n) \, F_n(k) + \Delta_n \, G_n(k) &= 0. \quad (2.6b)
\end{align}

Here $\xi_k = k^2/2m - \epsilon_F$ with $m$ the electron mass and $\epsilon_F$ the Fermi energy. The two self-energies $\Sigma$ and $\Delta$ are given by

\begin{align}
\Sigma_n &= \frac{1}{\pi N_F \tau} \frac{1}{V} \sum_k G_n(k), \quad (2.7a) \\
\Delta_n &= \Delta - \frac{1}{\pi N_F \tau} \frac{1}{V} \sum_k F_n(k), \quad (2.7b)
\end{align}

with

$$\Delta = h + 2\Gamma^{(i)} T \sum_n \frac{1}{V} \sum_k F_n(k).$$ \hspace{1cm} (2.7c)

Here $\Gamma^{(i)}$ is the spin-triplet interaction amplitude defined in Ref. [8]. $\tau$ is the transport relaxation time due to the quenched disorder, and $N_F$ is the density of states at the Fermi level. The appearance of $h$ in Eq. (2.7c) is the only difference between the saddle-point equations here and those in Ref. [8] and it results from the Zeeman term in the action. For later reference we note that the field configuration obtained by putting $\Delta = 0$ is not a saddle point, in contrast to the situation in the absence of an external field.

The discussion of the saddle-point solution proceeds as in Ref. [8]. In particular, $\Delta$ obeys the equation

$$\Delta = h - T \sum_n \frac{1}{V} \sum_k \frac{2\Gamma^{(i)} \Delta}{(i \omega_n - \xi_k + \text{sgn} \omega_n/2\tau)^2 - \Delta^2}. \quad (2.8a)$$

For $h = 0$ there is a nonzero physical solution only if the Stoner criterion $N_F \Gamma^{(i)} > 1$ is fulfilled [8]. In paramagnetic systems, $N_F \Gamma^{(i)}$ is smaller than the critical value, and $\Delta = 0$ in the absence of a magnetic field. However, for nonzero $h$ we have

$$\Delta = h \left[ 1 + N_F \Gamma^{(i)} \right] + O(h^3). \quad (2.8b)$$

Since $\Delta$ is proportional to the magnetization, this reflects the fact that a magnetic field polarizes the spins.

We see that, at the saddle-point level, the only difference between the current situation and the one considered in Ref. [8] is the origin of the nonzero value of $\Delta$. More generally, as far as an effective theory that concentrates on soft-mode effects is concerned, the only difference lies in the existence of Goldstone modes in a magnetic system. Since the latter turn out to be irrelevant for describing the metal-insulator transition [8], the procedure of expanding about the saddle-point, separating soft and massive modes, and deriving an effective theory for the metal-insulator transition is therefore the same in the magnetic field and the ferromagnetic cases,
respectively, and we can simply take over the result for the latter. The final effective action thus reads

\[
\mathcal{A} = \frac{-1}{2G} \int dx \text{tr} [\nabla \bar{Q}(x)]^2 + 2H \int dx \text{tr} [\Omega \bar{Q}(x)] - \frac{1}{2G_3} \int dx \text{tr} \left( (\tau_3 \otimes s_3) [\nabla \bar{Q}(x)]^2 \right) + 2H_3 \int dx \text{tr} \left[ (\tau_3 \otimes s_3) \Omega \bar{Q}(x) \right] + A_{\text{int}}(\bar{Q}). \tag{2.9a}
\]

Here

\[
\Omega_{12} = (\tau_0 \otimes s_0) \delta_{12} 2\pi T n , \tag{2.9b}
\]

is a bosonic Matsubara frequency matrix. The bare values of the coupling constants \( G, G_3, H, \) and \( H_3 \) are

\[
\begin{align*}
1/G &= \frac{\pi}{4} m (\sigma_0^+ + \sigma_0^-) , \tag{2.9c} \\
1/G_3 &= \frac{\pi}{4} m (\sigma_0^+ - \sigma_0^-) , \tag{2.9d} \\
H &= \frac{\pi}{16} (N_F^+ + N_F^-) , \tag{2.9e} \\
H_3 &= \frac{\pi}{16} (N_F^+ - N_F^-) , \tag{2.9f}
\end{align*}
\]

where \( \sigma_0^\pm \) and \( N_F^\pm \) are the Boltzmann conductivity and the bare density of states at the Fermi level, respectively, of systems whose Fermi energy has been shifted by \( \pm \Delta \) from its level in the absence of a magnetic field. Notice that \( 1/G_3 \) and \( H_3 \) vanish for a vanishing magnetic field, and for small fields are linear in the field, see Eqs. (2.9d, 2.9f). The interaction piece of the action consists of three terms,

\[
A_{\text{int}}(\bar{Q}) = A^{(s)}_{\text{int}}[\bar{Q}] + A^{(t)}_{\text{int}}[\bar{Q}] + A^{(3)}_{\text{int}}[\bar{Q}] , \tag{2.9g}
\]

\[
A^{(s)}_{\text{int}}[\bar{Q}] = -\frac{\pi T}{4} K_s \int dx \sum_{1234} \delta_{\alpha_1 \alpha_2} \delta_{\alpha_3 \alpha_4} \delta_{1-2-4-3} \\
\times \sum_r (-)^\tau \text{tr} \left[ (\tau_r \otimes s_0) Q_{12}(x) \right] \\
\times \text{tr} \left[ (\tau_r \otimes s_0) Q_{34}(x) \right] , \tag{2.9h}
\]

\[
A^{(t)}_{\text{int}}[\bar{Q}] = -\frac{\pi T}{4} K_t \int dx \sum_{1234} \delta_{\alpha_1 \alpha_2} \delta_{\alpha_3 \alpha_4} \delta_{1-2-4-3} \\
\times \sum_r (-)^\tau \text{tr} \left[ (\tau_r \otimes s_3) Q_{12}(x) \right] \\
\times \text{tr} \left[ (\tau_r \otimes s_3) Q_{34}(x) \right] , \tag{2.9i}
\]

\[
A^{(3)}_{\text{int}}[\bar{Q}] = -4\pi T K_3 \int dx \sum_{1234} \delta_{\alpha_1 \alpha_2} \delta_{\alpha_3 \alpha_4} \delta_{1-2-4-3} \\
\sum_{rs} \sum_{ij} m_{rs,ij} \tau_r^{\dagger} Q_{12}(x) \tau_s Q_{34}(x) , \tag{2.9j}
\]

where

\[
m_{rs,ij} = \frac{1}{4} \text{tr} \left( \tau_3 \tau_r \tau_s \right) \text{tr} \left( s_3 s_i s_j \right) , \tag{2.9k}
\]

and \( K_t = 2\pi \Gamma^{(t)} \) and \( K_s \) are the usual spin-triplet and spin-singlet interaction amplitudes that are also present in the absence of a magnetic field. \( K_3 \), like \( 1/G_3 \) and \( H_3 \), is magnetic field dependent, and vanishes linearly with the field for small fields. Since it is absent from the bare action, but generated by the renormalization group at one-loop order, it is also proportional to the disorder. Finally,

\[
\bar{Q}_{12} = \hat{Q}_{12} - \delta_{12} (\tau_0 \otimes s_0) \omega_n , \tag{2.9l}
\]

and \( \bar{Q} \) is subject to the constraints

\[
\bar{Q}^2(x) \equiv 1 , \quad \bar{Q}^\dagger = \bar{Q} , \quad \text{tr} \bar{Q}(x) \equiv 0 . \tag{2.9m}
\]

III. METAL-INSULATOR TRANSITION

We recognize Eqs. (2.9) as the generalized nonlinear \( \sigma \) model proposed by Finkelstein for the magnetic field universality class, except that the latter was missing the terms proportional to \( 1/G_3, H_3, \) and \( K_3 \). As was shown in Ref. 8, these terms do not change the asymptotic critical behavior. Choosing the correlation length exponent \( \nu \), the critical exponent for the density of states \( \beta \), and the dynamical critical exponent \( z \) as the three independent exponents, we thus have, to lowest order in an expansion in \( \epsilon = d - 2 \),

\[
\nu = 1/\epsilon + O(1) , \tag{3.1a}
\]

\[
\beta = 1/2\epsilon(1 - \ln 2) , \tag{3.1b}
\]

\[
z = d , \tag{3.1c}
\]

and the critical exponent for the conductivity, \( s = \nu(d - 2) \), is

\[
s = 1 + O(\epsilon) . \tag{3.1d}
\]

While the additional coupling constants \( 1/G_3, H_3, \) and \( K_3 \) do not contribute to the leading critical behavior, they lead to corrections to scaling. As was shown in Ref. 8, the least irrelevant operator related to these coupling constants has a scale dimension

\[
\lambda_3 = -\frac{3 \ln 2 - 2}{1 - \ln 2} \epsilon + O(\epsilon^2) . \tag{3.2}
\]

Consequently, in the vicinity of the metal-insulator transition any observable \( \omega \) obeys the homogeneity law

\[
\omega(t, T, u, \ldots) = b^{-\omega/\omega (tb^{1/\nu}, T b^\beta, \omega b^{\lambda_3}, \ldots)} , \tag{3.3}
\]

where \( t \) is the dimensionless distance from the critical point. Apart from (or instead of) the temperature \( T \), other generalized external fields may appear as arguments of \( \omega \), e.g., a frequency, or an electric field, depending on the nature of the observable under consideration.
u denotes the least irrelevant variable (which is a linear combination of $1/G_3$, $H_3$, and $K_3$), $x_\omega$ is the scale dimension of $\omega$, $b$ is an arbitrary scale factor, and the ellipses denote the dependence of $\omega$ on variables that are more irrelevant than $u$. These corrections to scaling were absent from the original model of Ref. 2.

IV. DISCUSSION

To put our results in context, we give a brief discussion of corrections to scaling in general, the additional terms in Eq. (2.9a) in particular, and why they were missed in previous treatments. Asymptotically close to a critical point, observables obey the homogeneity law, Eq. (6.3), and of the various coupling constants only those with positive scale dimensions need to be kept. Apart from the dimensionless distance from the critical point, whose scale dimension is by definition the inverse correlation length exponent $1/\nu > 0$, the only coupling constants with positive scale dimensions are generalized external fields like the temperature in our example. Technically, the homogeneity law appears as the solution of a renormalization group equation, which takes the form of a Callan-Symanzik equation or a similar partial differential equation. If one is not asymptotically close to the transition, corrections to the asymptotic scaling behavior appear from two sources: (1) The solution of the renormalization group equation becomes more complicated than a generalized homogeneous function, and needs to be expanded in a power series. (2) The “irrelevant operators”, i.e. coupling constants whose scale dimensions are negative, can no longer be ignored. Keeping them, and ordering them with respect to their irrelevancy, leads to the Wegner expansion. In our case, the leading term in the Wegner expansion, which we have kept in Eq. (1.1), is more important than the corrections to scaling of type (1). In fact, our value $-\nu \lambda_3 \approx 0.26$ is rather small for a leading Wegner exponent, which typically is on the order of 0.5. Since these large corrections to scaling result from the coupling constants $1/G_3$, $H_3$, and $K_3$ in Eqs. (2.9), it is important to keep these terms.

This brings us to our next discussion topic, viz. the origin of these additional terms in the action. As we have mentioned in Sec. 1 above, a field configuration that has $\Delta = 0$, and thus describes non-spin polarized electrons, is not a saddle-point solution of the field theory, Eqs. (6.3), underlying the effective action. Reference 2 used the implicit assumption that the only effect of the magnetic field, as far as the metal-insulator transition is concerned, is to give a mass to certain modes that are massless in the absence of an external field. If this were true, then one could use a field configuration with $\Delta = 0$ as the starting point for deriving an effective action. The result of this procedure is Eqs. (2.9) with $1/G_3 = H_3 = K_3 = 0$. By the nature of the renormalization group flow equations, zero bare values for these coupling constants imply that they are not generated under renormalization either. The effective theory that was originally proposed for this problem is therefore incomplete. As we have seen, this omission has an influence, not on the asymptotic scaling properties, but on the corrections to scaling, within the resulting effective field theory. Since the additional terms discussed above describe a physical effect, namely the spin polarization due to the magnetic field that was neglected in the original treatment, we conclude that the leading corrections to scaling discussed in this paper are an immediate, if a priori not entirely obvious, consequence of this spin polarization. We also note that there is no obvious reason underlying our result that the additional terms do not change the asymptotic critical behavior; this appears to be accidental. In particular, it needs to be stressed that this conclusion holds only in an $\epsilon$-expansion near two-dimensions, and in $d = 3$ the new terms might well influence the asymptotic scaling behavior.

We finally discuss the experimental implications of our results. For definiteness, let us consider the conductivity $\sigma$, whose scale dimension is $x_\sigma = s/\nu$. A common way to analyze experiments is to extrapolate the data to $T = 0$ and consider $\sigma(t, T = 0)$. From Eq. (6.3), we have

$$\sigma(t, T = 0) = \sigma_0 t^{\nu} \left[ 1 + \text{const.} \times e^{-t^{\nu \lambda_3}} \right].$$

Here $\sigma_0$ is a microscopic conductivity scale on the order of the Boltzmann conductivity, and the constant is nonuniversal. Assuming that the constant is of $O(1)$, in order to achieve, e.g., a 10% accuracy in a determination of the value of $s$ from a log-log plot, the $t$-range needs to be restricted to $t < (0.1/\nu |\lambda_3|)^{1/\nu |\lambda_3|}$. Extrapolating the results of our one-loop approximation to $d = 3$, we have $\nu |\lambda_3| \approx 0.26$, or $t < 0.026$. If the constant is larger, as is the case at many critical points, the asymptotic critical region is even smaller. For instance, for const. $= 1/|\nu \lambda_3| \approx 4$, we need $t \lesssim 10^{-4}$ to measure $s$ with a 10% accuracy. Such small values of $t$ are not currently achievable for Anderson-Mott transitions. The technique that yields the best control over $t$, viz. stress tuning, allows to probe the region $10^{-2} < t < 10^{-3}$, while other methods are restricted to $t \lesssim 10^{-2}$ or larger. We conclude that it is currently not possible to determine the asymptotic critical exponents for the magnetic-field universality class of the Anderson-Mott transition with any meaningful accuracy.

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This approximation is justified in disordered systems subject to weak fields, when the disorder induced broadening of the single-particle spectrum is large compared to the magnetic cyclotron energy.

Notice that a factor of $-i$ is missing in Eq. (3.143) of Ref. 3.

Here we consider the case of a long-range Coulomb interaction in the underlying microscopic model. Models with short-range interactions form a different universality class.

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If one or more of the irrelevant operators are so-called dangerous irrelevant variables for the observable under consideration, they can not be ignored even in the asymptotic regime, see, e.g., Ref. 12.

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