A more comprehensive formulation of evolutionary equations

Ingrid Hartmann

Zusammenfassung

As mathematical model for the evolutionary equations of species the master equation is chosen. Two formulations will be demonstrated to include the changes of parameters into the master equation - that is, on the one hand, the formation of a second master equation for the development of parameters, and, on the other hand, the use of the Wigner distribution to describe the development of parameters. Moreover, the Wigner distribution is used to describe morphic fields and involved in the theory of selforganization.

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1 Introduction

In the course of the past few years, in the frame of the theory of selforganization, attempts have been made by a great number of authors [1a, 1b, 1c, 1d] to describe the development (evolution) of biological systems. In this way,

1Ingrid Hartmann, Querstrasse 4a, 18442 Buschenhagen, Germany
the competition of species, which, in each case, have a certain amount of characteristic features, could be simulated and the dynamic behaviour for certain attempts in the growing and decay rates could be determined. However, these attempts lead to stagnation as soon as a stationary behaviour is reached. The reason is that - in the author’s opinion - the change of the parameters of species (for instance of the growing rates), which could change the behaviour once achieved, was not included in the equations in an adequate way. Attempts have been made [1b, 1d, 1g, 1h] to change these features of species with the help of mutation terms via chance, so that it turns out to be possible basically (in principle) to reach another section of the parameter room that shows another dynamic behaviour. In this way, a more or less diffuse walk through the parameter room is possible. In some papers we have tried to solve these problems [1e, 1f] assuming that the development of the parameters during the evolution process happens in a way that the function, which is characteristic for the particular species, becomes optimal. However, these attempts have had the disadvantage that the discovering of this efficiency function - provided that it actually exists - is subject to a certain arbitrariness. On the other hand, this attempt contains the additional assumption that the development of the parameters is not inherently contained in the evolutionary equations. In the present paper, the author is going to demonstrate another way that allows to include changes of the parameters directly into the evolutionary equations of species.

2 Behaviour under constant parameters

The behaviour under constant parameters will be demonstrated at the following example (a detailed description is given in [1b]). The total number of sorts amounts to $N = N_1 + N_2$. The sort $N_1$ increases with growing rate $E_1$ and the sort $N_2$ with the growing rate $E_2$. The elementary stochastic process is the growing of one sort at the expense of the other sort.

$$
N_1 \rightarrow N_1 - 1 \\
N_2 \rightarrow N_2 + 1
$$

(1)

It is:

$$
N_1 + N_2 = \text{const.}
$$

(2)
be the rate of the spontaneous transition from j to i. For the corresponding
transition probabilities valids:

\[
W^+(N_2) = E_2 N_1 (N_2 / N) + a_{21} \\
W^-(N_2) = E_1 N_2 (N_1 / N) + a_{12}
\]

(3)

The master equation can then be written for the probability
\[P(N_2, t)\] :

\[
\frac{\partial P(N_2, t)}{\partial t} = [a_{21} + \frac{E_2}{N} (N_2 - 1)(N - N_2 + 1)]P(N_2 - 1, t) \\
+ [a_{12} + \frac{E_1}{N} (N_2 + 1)(N - N_2 - 1)]P(N_2 + 1, t) \\
- [a_{12} + a_{21} + \frac{1}{N}(E_1 + E_2)N_2(N - N_2)]P(N_2, t)
\]

(4)

The stationary solution is shown in [1b].
In the stationary case with stochastic description a probability distribution
\[P^{eq}(N_2) \neq 0\] for \[N_2 \neq 0\] is achieved, that means that the two sorts can coexist
with each other in a certain manner, but in the deterministic case the sort
with the greatest growing rate wins. (However, this field of problems shall
not be dealt with in the present connection.) Now, how can the change of
the features \(a_{ij}, E_i\) be included in order to meet the conditions of the gradual
changes of the parameters? In the next sections two proposals will be made
to that matter.

3 Behaviour under time-varying parameters

On the basis of equation 4 the following attempt shall be made:

\[
\frac{\partial P(N_2, t)}{\partial t} = [\langle a_{21} \rangle + \frac{\langle E_2 \rangle}{N} (N_2 - 1)(N - N_2 + 1)]P(N_2 - 1, t) \\
+ [\langle a_{12} \rangle + \frac{\langle E_1 \rangle}{N} (N_2 + 1)(N - N_2 - 1)]P(N_2 + 1, t) \\
- [\langle a_{12} \rangle + \langle a_{21} \rangle + \frac{1}{N}(\langle E_1 \rangle + \langle E_2 \rangle)N_2(N - N_2)]P(N_2, t)
\]

(5)

The parameters \(a_{ij}, E_i\) are time-dependent and have a specific eigentime
\(\tau_{ij}, \tau_i\), according to which they are developing.
As simplified consideration the following attempt shall be made:
$a_{12}$ develops in the course of time to $a_{12}'$,
$a_{21}$ develops in the course of time to $a_{21}'$,
$E_1$ develops in the course of time to $E_1'$,
$E_2$ develops in the course of time to $E_2'$.

It valids:

\[
a_{12} + a_{12}' = a_m^{(1)} = \text{const.}
\]
\[
a_{21} + a_{21}' = a_m^{(2)} = \text{const.}
\]
\[
E_1 + E_1' = E^{(1)} = \text{const.}
\]
\[
E_2 + E_2' = E^{(2)} = \text{const.}
\]

(6)

The description of the temporal development be demonstrated by the example of $E_1(t)$. $E_1(t)$ obey the following equation:

\[
W^+(E_1) = d_{11'}E_1' = d_{11'}(E^{(1)} - E_1)
\]
\[
W^-(E_1) = d_{11'}E_1.
\]

(7)

The master equation

\[
\frac{\partial P_{E_1}(E_1, t)}{\partial t} = W^+(E_1 - 1, t)P(E_1 - 1, t) + W^-(E_1 + 1, t)P(E_1 + 1, t)
\]
\[
- [W^+(E_1, t) + W^-(E_1, t)]P(E_1, t).
\]

(8)

then reads as follows:

\[
\frac{\partial P_{E_1}(E_1, t)}{\partial t} = d_{11'}[E^{(1)} - (E_1 - 1)]P(E_1 - 1, t) + d_{11'}(E_1 + 1)P(E_1 + 1, t)
\]
\[
- [d_{11'}(E^{(1)} - E_1) + d_{11'}E_1]P(E_1, t).
\]

(9)

The stationary solution says:

\[
P_{E_1}^{eq}(E_1) = \left( \frac{E^{(1)}}{E_1} \right) \left[ \frac{d_{11'}}{d_{11'} + d_{11'}} \right] E_1 \left[ \frac{d_{11'}}{d_{11'} + d_{11'}} \right] E^{(1)} - E_1
\]
\[
\langle E_1(t \to \infty) \rangle = \left( \frac{d_{11'}}{d_{11'} + d_{11'}} \right) E^{(1)} = pE^{(1)}
\]
\[ \langle E_1(t \to \infty) \rangle = \left( \frac{d_{11'}}{d_{1'1} + d_{11'}} \right) E^{(1)} = (1 - p)E^{(1)} \]

mit

\[ p = \frac{d_{1'1}}{d_{1'1} + d_{11'}}. \]

Consequently, the time-dependent solution is:

\[ \langle E_1(t) \rangle = pE^{(1)} + (E_0 - pE^{(1)}) \exp(-(d_{1'1} + d_{11'})t) \]

\[ \langle E_{1'}(t) \rangle = E^{(1)} - \langle E_1(t) \rangle = E^{(1)}(1 - p) \]

\[ + (pE^{(1)} - E_0) \exp(-(d_{1'1} + d_{11'})t) \]

mit

\[ E_0 - \text{starting value.} \] (11)

\( P(t) \) is given in [3a].

Analogous to this the equations for \( E_2(t) \) can be formulated. In the case of strong temporal separation of the development of the species from the development of the features follows equation 5 with the stationary solutions:

\[ \langle E_1(\infty) \rangle = pE^{(1)} \]

\[ \langle E_2(\infty) \rangle = qE^{(2)} \]

with

\[ q = \frac{d_{2'2}}{d_{2'2} + d_{22'}}. \] (12)

By analogy the considerations for \( a_{12} \) and \( a_{21} \) can be continued. In the case of a not so strong temporal separation, that is if the time-development of \( E_j \) has to be considered as well, follows from equation 5 and 11:

\[ \frac{\partial P(N_2, t)}{\partial t} = \langle a_{12} \rangle P(N_2 - 1, t) \]

\[ + (E^{(2)}q + (E_{02} - qE^{(2)}) \exp(-(d_{2'2} + d_{22'})t)) \frac{(N_2 - 1)}{N}(N - N_2 + 1)P(N_2 - 1, t) \]

\[ + \langle a_{21} \rangle P(N_2 + 1, t) \]

\[ + (E^{(1)}p + (E_{01} - pE^{(1)}) \exp(-(d_{1'1} + d_{11'})t)) \frac{(N_2 + 1)}{N}(N - N_2 - 1)P(N_2 + 1, t) \]
Thereby, the solution of this equation will become extremely complicated. For the present, it may remain here as an attempt proposal. In an analogous way, the other terms can be dealt with as well. For space reasons, this question will not be considered in the present paper.

4 Behaviour under time-varying parameters with the help of the description by the Wigner-distribution

A further possibility to include the development of the parameters in the master equation offers the following attempt with the help of the Wigner-distribution.

$E_i$ be the feature of a species which in the course of the time $t$ can be changed. It has turned out to be necessary to introduce - in addition to the ”normal” time $t$ - characteristic times for the particular species. So let $E_i$ be reverse proportional to a characteristic eigentime $\tau_i$ of the species:

$$E_i \sim \frac{1}{\tau_i} \quad (14)$$

and so it valids:

$$E_i \sim \omega_i \quad (15)$$

with $\omega_i$ the characteristic frequency of a species. (The principle of the method, for simplification, shall be demonstrated at one frequency only. It can be extended to a characteristic frequency spectrum. (Considerations about different times can be found in [4a, 1e, 1f].)
Now the attempt shall be made:

\[ s^{(i)} = \sum_{j=1}^{n} \cos(E_j^{(i)} t) \exp(-\alpha_j (t - t_j)^2), \]  

(16)

by which it shall be described that the species have the feature \( E_j^{(i)} \) at a certain time \( t_j \) hat. Furthermore, the attempt shows that the sort can potentially have all frequencies \( \omega_j^{(i)} \) however actually it happens at certain times \( t_j \). In this way, each sort is characterised by a tuple \( (E_j^{(i)}, t_j, \alpha_j) \) or \( (\omega_j^{(i)}, t_j, \alpha_j) \) respectively, and so it represents an energy-time-unit. From quantum mechanics the Wigner-distribution is known:

\[ W(q, p) = \int_{-\infty}^{\infty} \exp(-j2\pi x^2/h) \phi(q + \frac{x}{2}) \phi^*(q - \frac{x}{2}) \, dx \]  

(17)

(\( \phi \) - wave function, \( q \) - position, \( p \) - momentum, \( h \) - Planck’s constant).

In electrical engineering it is applied as follows:

\[ W(\omega, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-j\tau \omega) f(t + \frac{\tau}{2}) f^*(t - \frac{\tau}{2}) \, d\tau \]  

(18)

(\( f(t) \) - signal, \( \omega \) - frequency).

In this case, it serves for the analysis of time-varying spectrums and a distribution over \( \omega \) and \( t \).

In our case, if we consider \( s_i(t) \) according to equation 16 as "wave-function" of the species, it means that

\[ W(\omega, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-j\tau \omega) s(t + \frac{\tau}{2}) s^*(t - \frac{\tau}{2}) \, d\omega. \]  

(19)

demonstrates the probability of the features of the species. The mean property \( \langle E_j^{(i)} \rangle \) can be obtained by formation of the moments of the Wigner-distribution [4b], and it can be included in this form in the master-equation 5.

In the following, these considerations will be demonstrated in detail for the case \( n = 2 \). From equation 16 and equation 19 follows then:

\[ W^{(1)} = W_{11}^{(1)} + W_{12}^{(1)} + W_{21}^{(1)} + W_{22}^{(1)}. \]  

(20)

Consequently, the Wigner-distribution, according to equation 19, is composed of two autocorrelation terms \( W_{ii}^{(1)} \) and two crosscorrelation terms \( W_{ij}^{(1)} \) which
result from terms with equal or different indices respectively.
The calculation gives the following terms:

\[ W_{11}^{(1)} = \frac{1}{4} \sqrt{\frac{\pi^4}{\alpha}} \exp[-\alpha(t - t_1)^2] \]
\[ \left[ 2 \cos(2E_1 t) \exp\left(-\frac{E^2}{\alpha}\right) + \exp\left(-\frac{(E + E_1)^2}{\alpha}\right) + \exp\left(-\frac{(E - E_1)^2}{\alpha}\right) \right] \]

\[ W_{22}^{(1)} = \frac{1}{4} \sqrt{\frac{\pi^4}{\alpha}} \exp[-\alpha(t - t_2)^2] \]
\[ \left[ 2 \cos(2E_2 t) \exp\left(-\frac{E^2}{\alpha}\right) + \exp\left(-\frac{(E + E_2)^2}{\alpha}\right) + \exp\left(-\frac{(E - E_2)^2}{\alpha}\right) \right] \]

\[ W_{Kreuz} = W_{12}^{(1)} + W_{21}^{(1)} = \]
\[ \frac{1}{4} \sqrt{\frac{\pi^2}{\alpha}} \exp[-\alpha(t - t_1)^2] \exp[-\alpha(t - t_2)^2] \]
\[ \left( 2 \cos[(E_1 + E_2)t] \left[ \exp\left(-\frac{[E + \frac{(E_2 - E_1)^2}{2\alpha}]^2}{2\alpha}\right) + \exp\left(-\frac{[E - \frac{(E_2 - E_1)^2}{2\alpha}]^2}{2\alpha}\right) \right] \]
\[ + 2 \cos[(E_2 - E_1)t] \left[ \exp\left(-\frac{[E + \frac{(E_2 + E_1)^2}{2\alpha}]^2}{2\alpha}\right) + \exp\left(-\frac{[E - \frac{(E_2 + E_1)^2}{2\alpha}]^2}{2\alpha}\right) \right] \right) \]

(For simplification it was calculated with \( \alpha_1 = \alpha_2 = \alpha \).) From the calculation of the local moment of the Wigner-distribution

\[ \langle E(t) \rangle = \frac{1}{2\pi} \int_0^\infty EW(t, E) dE/p_s \]
with
\[ p_s = \frac{1}{2\pi} \int_0^{\infty} W(t, E) dE \] (22)
results in special, with \( p_s \) being the momentary energy of \( s \):

\[ \langle E_{11}^{(1)}(t) \rangle = \frac{1}{4\sqrt{\pi}} \exp(-\alpha(t - t_1)^2) \]
\[ \left[ 2 \cos(2E_1 t) \frac{\alpha}{2} + \frac{\alpha}{2} + E_1 \sqrt{\frac{\pi \alpha}{2}} \right] / p_s \]
\begin{align*}
\langle E_{11}^{(1)}(t) \rangle &= \frac{1}{4\pi\sqrt{\alpha}} \exp(-\alpha(t-t_2)^2) \\
&\quad \left[ 2\cos(2E_2t)\frac{\alpha}{2} + \frac{\alpha}{2} + E_2\sqrt{\pi\alpha} \right] / p_s \\
\langle E_{12}^{(1)}(t) \rangle &= \frac{1}{8\pi\sqrt{\alpha}} \exp(-\alpha(t-t_1)^2) \exp(-\alpha(t-t_2)^2) \\
&\quad \left( 2\cos((E_1 + E_2)t) \left[ \alpha + \left( \frac{E_2 - E_1}{2} \right) \sqrt{\pi\alpha} \right] \right. \\
&\quad \left. + 2\cos((E_2 - E_1)t) \left[ \alpha + \left( \frac{E_2 + E_1}{2} \right) \sqrt{\pi\alpha} \right] \right) / p_s \\
\langle E^{(1)}(t) \rangle &= \langle E_{11}^{(1)}(t) \rangle + \langle E_{22}^{(1)}(t) \rangle + \langle E_{12}^{(1)}(t) \rangle 
\end{align*}

These expressions for the mean values can be included into the master equation. In this way, we can get a more extensive description. (The other parameters can be dealt with in a analogous way.) So the development of the parameters can directly be included into the master equation and a walk from pik to pik in the probability distribution \( P(N_2, t) \) can happen because of the fact that at certain times certain feature values are adopted so that again new relations with other sorts are in another way formed.

\section{Conclusions}

The author considers the Wigner-distribution is an adequate description for species since it can describe the time-development and, at the same time, the frequency-development of the species (or the development of the characteristic times of species respectively). The author, in a further publication, will come back to these problems. In the present paper, there is only given a short outline. The Wigner-distribution is considered as probability distribution on the time behaviour of species and, therefore, represents an "information field".

To inform means to bring in shape. According to the present attempt, certain values \( E_i \) are taken at certain times \( t_i \). Potential features \( E_i \) are made real features \( E'_i \), which means they are brought in shape. In this respect, the Wigner-distribution can be considered as information distribution for species. This description, in the author’s opinion, corresponds with the description
given by R. Sheldrake [5a, 5b], who describes the development of features with the help of morphic fields. The morphic field as probability - and information field [5b] is described in the present attempt of the Wigner-distribution. Furthermore, it can be demonstrated [5c] that the Wigner-distribution included as probability distribution in the relation

\[ S = k \ln(W) \]  

produces additional terms which in the theory of selforganization contribute to structure formation.

In this way, the term **information** takes up a central place in the theory of selforganization [5c]

- as mental potential that, at a certain time, is brought in shape - .

The author thinks that the present considerations could be one step forward on the way to integrate the term **consciousness** in the ”world of physics” - an idea which recently has repeatedly been considered by a number of famous physicists [5d, 5e, 5f, 5g].

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