Computational technologies for the synthesis of decentralized control systems for multistage technological processes

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Abstract. This article discusses the development of two-level hierarchical control systems for complex technological processes. This area of research relates to the problems of creating robotic complexes as part of automation systems for managing continuous multi-stage processes according to economic criteria. For the synthesis of decentralized systems of optimal control of processes with a sequential structure, it is recommended to use the dynamic programming method, which allows providing optimal coordinating solutions and justify the setting of local tasks for managing the stages of the technological process, provided that the current states of the input products of these stages are accurately or approximately observed.

For an n-stage technological process in conditions of complete information, the problem of a coordinating subsystem is written, the solutions of which do not change for a set of values of input variables of the process and the time period of constancy of external and internal perturbations. It is shown that the implementation of algorithms for controlling individual stages of the process can be performed on the basis of microprocessor technology. Estimates of control error were obtained using a simulation model of a process control system with specified parameters. The article considers the possibility of applying the dynamic programming method in the management of multi-stage technological processes in conditions of incomplete information.

1. Introduction

Two-level decomposition algorithms for controlling a set of static technological processes with a sequential structure are considered. Synthesis of this class of control systems is performed by methods of global goal decomposition. As a result we obtain a coordinating control task and local process control tasks [1, 2]. Currently, decomposition methods for the synthesis of such control systems for static [3] and dynamic [4] complex objects, including under conditions of uncertainty, have been developed. The presented research area is relevant due to the development of microprocessor control systems for continuous technological processes in real time.

Decomposition methods in control theory are studied in the framework of the General theory of multilevel hierarchical systems, in which the upper control level (Center) changes the conditions for choosing solutions to problems of subsystems of the lower level, ensuring the optimality of the system as a whole. Currently, when designing computer control systems, the greatest attention is paid to control systems, the lower levels of which do not have their own goals. Then the algorithms of functioning (solving functions) of the Center and subsystems can be obtained by decomposing the global control problem. It should be noted that the possibility of using these methods depends on the structure of technological processes and the nature of the management task.
Multi-stage technological processes with a sequential structure of connecting of individual stages are widely used, for example, in the chemical industry [2]. Two-level management of such processes can be organized by dividing the set of perturbations into fast and slow processes. Fast processes include changes in the parameters of the initial and intermediate processing products, while relatively conservative parameters include economic indicators, equipment characteristics, and technology. Then the center's decision functions can depend only on fast perturbations and can be corrected when conservative parameters change. The decision functions of subsystems depend on fast perturbations, coordinating variables, and their design will be determined by the accepted method of decomposition of the global optimal control problem.

The described systems of computer control of technological processes are multifunctional. They include models for monitoring variables of controlled processes, computing systems of varying complexity, dispatching functions, and others. Below, we will limit ourselves to the study of computational technologies for synthesizing control algorithms for subsystems of a two-level control system and evaluating the accuracy of its functioning.

2. Computational technology for the synthesis of decentralized control systems using the dynamic programming method

In this paper, computational technologies for the synthesis of decentralized control systems are based on the use of the dynamic programming method (DP) in relation to the following global control problem: for each value of \( x_i \in X_i \), find the solving rules (control actions) of the \( \nu^* = (\nu^*_1, ..., \nu^*_n) \) from the condition:

\[
\Psi_0(x_i) = \max_{y_i \in Y_i} \left\{ \Psi_n(y_n) - \sum_{i=1}^{n} \varphi_i(y_i, x_i, \nu_i)/y_i = F_i(x_i, \nu_i); x_{i+1} = y_i; i = 1, ..., n-1 \right\}. \quad (1)
\]

According to the work (1) it is necessary to provide optimal control of an n-stage technological process \((n>1)\) with a sequential structure, each i-th stage of which is described by a triple of variables \((y_i, x_i, \nu_i)\), taking into account the conditions: \( y_i \in Y_i, x_i \in X_i, \nu_i \in V_i, i = 1, 2, ..., n \). Next, we assume that the introduced variables are vectors in spaces of corresponding dimensions. The relationship of variables for n stages is described by the functions \( \{ y_i = F_i(x_i, \nu_i); i = 1, 2, ..., n \} \), and the relationships of the process stages are set by the following conditions: \( \{x_{i+1} = y_i; i = 1, 2, ..., n-1 \} \). Below is an example of problem (1), which additionally highlights external perturbations.

The designations of variables in problem (1) correspond to the designations of the DP method: \( \Psi_n(y_n), \Psi_0(x_i) \) – the initial and final Bellman functions – the cost of material products depending on their parameters, and \( \{ \varphi_i(y_i, x_i, \nu_i); i = 1, 2, ..., n \} \) – scalar functions of financial costs of n stages of the process.

The process control problem (1) for fixed parameter values belongs to the class of mathematical programming problems, but in practice its solution is difficult due to its large dimension. In addition, in real processes, it is necessary to conduct operational coordination of management conditions at the stages of the entire process. The dynamic programming method turns out to be an effective way to solve problems of large dimensions and to substantiate the approval procedures [5-7].

Diagram of the decision problem by DP includes reverse, which is calculated of the Bellman function, and a straight stroke, which recovers the optimal solution of problem (1) and the optimal trajectory of the process input and output variables.

On the reverse course (we perform calculations starting from the number \( i = n \)), we find the optimal solving rules \( \nu_i = \tilde{\nu}_i(x_i) \) and Bellman functions \( \Psi_{i-1}(y_{i-1}) = \Psi_{i-1}(x_i) \):

\[
\tilde{\nu}_i(x_i) \in \text{Arg} \max_{\nu_i \in V_i} \left\{ \Psi_i(y_i) - \varphi_i(y_i, x_i, \nu_i)/y_i = F_i(x_i, \nu_i) \right\}, \quad x_i \in X_i; i = 1, ..., n; \quad (2)
\]

\[
\Psi_{i-1}(x_i) = \left\{ \Psi_i(y_i) - \varphi_i(y_i, x_i, \tilde{\nu}_i(x_i))/y_i = F_i(x_i, \tilde{\nu}_i(x_i)) \right\}, \quad x_i \in X_i; i = 1, ..., n. \quad (3)
\]
We start a straight course with the number \( i = 1 \) and make calculations according to the following scheme:

\[
x_i \Rightarrow v_i^* = \tilde{v}_i^*(x_i) \Rightarrow y_i^* = F_i(x_i, v_i^*) = x_2 \Rightarrow \ldots \Rightarrow v_n^* = \tilde{v}_n^*(x_n) \Rightarrow y_n^* = F_i(x_n^*, v_n^*). \tag{4}
\]

The computational technology of the DP method adequately reflects the process control schemes in real time: with known Bellman functions or with known decision rules, the lower level of individual stages is able to provide optimal technological modes, if the input values \( x_i \) are observed parameters. One of the options for organizing decentralized management is to use a coordinating subsystem (Center) and \( n \) lower-level management subsystems \( (LMS_i) \), where \( i = 1, \ldots, n \).

In this case, problem (1) is solved in real time by a two-level control system, in which the Center for all valid inputs \( x_i \), solves the problem of approximation of the Bellman family of functions once \( \Psi_i(y_i); i = 1, \ldots, n - 1 \). Subsystems \( LMS_i \) measure input variables in real time and calculate the current control effect for the decision rules found by the Center or by solving \( v_i = \tilde{v}_i^*(x_i) \) the problem (2).

The described computational technology of the control problem differs from the solution of the optimization problem, in which the Bellman functions are calculated only for a fixed value \( x_i \).

3. Analysis of the results of solving the optimal control problem on the example of a 4-stage technological process

Let us consider an example of problem (1). Let us assume that a process consisting of four stages is used for cleaning the product. The initial percentage of impurity \( x_i \) in the product, the price at which the product is sold after the fourth stage of purification, and the cost coefficients at each stage of the technological process are known \( a = (a_1, a_2, a_3, a_4) \). The cost functions and the cleaning effect are defined by expressions:

\[
\varphi_i(v_i) = \frac{a_i}{v_i}; \quad y_i = v_i \cdot x_i, \quad i = 1, \ldots, 4; \quad x_{i+1} = y_i, \quad i = 1, \ldots, 3, \tag{5}
\]

where \( v_i \in [0, 1] \) – the degree of purification is the control parameter of the \( i \)-th stage. The dependence of revenue on the final impurity content in the product \( y_4 \) is determined by the expression:

\[
\Psi_4(y_4) = C_0(1 - y_4). \tag{6}
\]

For each value we need to find the value \( x_i \in X_i \) of the vector \( v^* = (v_1^*, \ldots, v_4^*) \), that maximizes the profit from the sale of the final product:

\[
\Psi_0(x_i) = \max_{v_i \in (0, 1]} \left( \Psi_4(y_4) - \sum_{i=4}^4 \varphi_i(v_i) \right). \tag{7}
\]

The study of problem (6) was performed in an Excel environment for the source data of table 1. During calculations, the values of input variables of all stages were set in small increments at variable intervals of the segment \([0; 15]\), and functions (2) and (3) were represented as a series (example for Bellman functions):

\[
\Psi_i(y_i) = A_i^0 + A_i^1 y_i + A_i^2 y_i^2 + A_i^3 y_i^3 + A_i^4 y_i^4. \tag{8}
\]

The results of solving problem (6) are analyzed and their accuracy estimates are obtained using a numerical example with the parameters presented in table 1.

The accuracy of the results of control of cleaning processes was estimated by the average error in the functional solution of the problem (6) on the grid \( x_i \in [10; 15] \) by the DP method according to the solution scheme according to expressions (2), (3) and a two-level control system (scheme 2), in which the \( LMS_i \) subsystems found their control effects by solving local optimization problems at known values \( x_i \) for \( i = 1, 2, 3, 4 \).

The exact solution was obtained using the Excel "solution Search" tool. For the initial data of table...
1. estimates of errors of the studied computing technology were obtained, equal to 0.0038% and 0.0014%, respectively. The lower accuracy of the classical DP scheme is explained by the use of approximating expressions of solving functions in it instead of finding their optimal values in scheme 2.

Table 1. Source data for the management task (6).

| Name of parameter                  | Designation | Value  |
|-----------------------------------|-------------|--------|
| Number of stages                  | n           | 4      |
| The interval of initial values of the impurity, % | x_1         | [10; 15] |
| Price parameter                   | C_0         | 250    |
| Cost factor at the first stage    | a_1         | 6      |
| Cost factor for the second stage  | a_2         | 7      |
| Cost factor at the third stage    | a_3         | 8      |
| Cost factor at the fourth stage   | a_4         | 9      |

It should be noted that when changing the parameters specified in table 1, the Center needs to find new expressions for the family of Bellman functions, which will require the use of universal computers for managing real technological processes.

4. Computational technology for the synthesis of decentralized control systems in conditions of incomplete information

In practice, there is a problem of managing technological processes in conditions of incomplete information, which is studied in the literature as a problem of managing stochastic processes [7-10]. It is assumed below that the probabilistic characteristics of random parameters and measurements of process variables are known to the control system elements and do not change on the coordination interval (on the interval of Center decisions). Then the considered DP-based computing technology is applicable in management problems with incomplete information.

In this case, the statements of the problem of the Center for coordinating solutions of local subsystems and the problem LMS_i of synthesis of solving functions are written in the class of M-models of stochastic programming.

Let us consider a generalization of the problem (1) of managing discrete multistage processes, where the functions of technological costs and process models depend on the w_i perturbations at stage i: y_j = F_j(x_i, v_i, w_i); q_0 = q(x_i, v_i, w_i), i = 1, ..., n. We assume that the Center and subsystems in the control processes are not fully and variously informed about the values of the w = (w_1, ..., w_n) vector components.

We assume that w_i is a random vector whose components are independent of each other and from variables of other stages, and its probabilistic properties are known in the synthesis of the control system. The required properties of the considered stochastic optimal control system for the possibility of using DP are described, for example, in [6]. Further, without loss of generality, in order to simplify mathematical expressions, we assume that w_i ∈ W_i is a scalar parameter, i ∈ I_n = {1, ..., n}.

We will define, following [7, P. 243], the information structure of the decision rules in a decentralized management system. For a stage i ∈ I_n, we assume that LMS_i knows the current state of the managed process (a vector x_i) and some component composition of the vector w of subsequent stages, whose indexes are set by the set J_i ⊆ I = {i, ..., n}.

In accordance with the specified information structure the required type of decision rules for the LMS_i subsystem is written as:

\[ \bar{v}_i = \bar{v}_i(x_i; w_i, i \in J_i); \quad \tilde{v}_i \in V_i, i \in I_n, \]

where V_i is the set of implemented \( \bar{v}_i \) decision rules at stage \( i \in I_n \).

The information structure of the Center’s decisions is determined by the indices of the vector w
components. The Bellman function at step \( i \) of the DP reverse depends on them. Let's denote \( J_i^r, i \in I_n \) the set of specified indexes. Then the Bellman function \( \Psi_t \) for estimating the cost of technological products after stage \( i \) has the following form: \( \Psi_t = \Psi_t(y_i; w_i, i \in J_i^r) \).

To ensure the required composition of arguments for the decision rules (8), the families of sets \( J_i^r \) and \( J_i^r, i \in I_n \) must be agreed upon. For this purpose \( J_i^r \) should be defined as:

\[
J_i^r = \bigcap_{i+1} (J_{i+1} \setminus J_i^r), \quad I_n = \phi, \quad i \in I_n.
\]

In the present computational technology, there is a possibility to reduce the number of arguments of the Bellman function by phase-averaging for the components of \( w \), which is in the initial stages not taken into account in the final rules (8). Let's denote \( J_i^w \) the set of averaged parameters in step \( i \) of the reverse course. This set is defined as: \( J_i^w = J_i \cup \{i\} \setminus J_i^r; \quad J_0^w = \phi, i \in I_n \). If \( J_{i+1}^w = J_i \cup \{i\} \) is equal, the set \( J_i^w \) is empty and the Bellman function averaging operation is not performed at stage \( i \).

Similarly, we select a set of \( J_i^r \) indexes that are used for averaging the target function of the local subsystem: \( J_i^w = J_i \cup \{i\} \setminus J_i^r \). Vectors whose components are separated by sets \( J_i^r, J_i^r, J_i^w, J_i^w \) are denoted as \( w_i^r, w_i^r, w_i^w, w_i^w \), respectively.

Let in (8) the set \( V_i \) is bounded only by the set \( V_i \) of the values of the decision rules, and the set of indexes \( J_i, i \in I_n \) is given. Then, on the reverse course of the DP, we propose to find the decisive rules and Bellman functions from the following conditions (M-symbol of the averaging operator):

\[
\bar{v}_i(y_i, w_i^r) \in \arg \max_{v_i \in V_i} \left\{ \Psi_t(y_i, w_i^r) - \phi_t(y_i, x_i, v_i, w_i) / y_i = F_i(x_i, v_i, w_i) \right\} \quad x_i \in X_i, i \in I_n;
\]

\[
\Psi_{t-1}(x_i, w_{i-1}) = \left. \max_{v_i \in V_i} \left\{ \Psi_t(y_i, w_i^r) - \phi_t(y_i, x_i, \bar{v}_i^r, w_i^r) / y_i = F_i(x_i, \bar{v}_i^r, w_i^r) \right\} \right| x_i \in X_i, i \in I_n.
\]

A direct course in the control system is performed by the LMS according to the calculation scheme specified by formulas (4), successively solving problems (10) or using the optimal decision rules found by the Center.

Let's consider the problem of optimality of the proposed computational technology for the synthesis of solving functions (8) based on expressions (10) and (11). For this purpose, we use the variational extension (VE) approach in parametric programming [7]. We formulate the problem of finding optimal solving functions \( \bar{v}_i^r \in V_i \) with structure (8) as a problem of variational calculus:

\[
\Psi_0(x_i) = \max_{v_i \in V_i} \left\{ \Psi_t(y_i) - \sum_{i=1} \phi_t(y_i, x_i, v_i, w_i) / y_i = F_i(x_i, v_i, w_i); x_{i+1} = y_i, i = 1,...,n-1 \right\}.
\]

It should be noted that in General, the problem of optimal synthesis of controlled systems (12) is significantly more complex than traditional optimal control problems, in which the desired functions depend only on time at finite or infinite intervals. In (12), the required composition of the arguments of the desired functions can be a vector and is defined using sets \( V_i, i \in I_n \). The Complexity of the synthesis problem (12) does not limit the capabilities of the calculated technology, since it is used only in theoretical research.

We solve the synthesis problem (12) using the DP method and write the reverse-flow expressions similar to formulas (2) and (3):

\[
\bar{v}_i \in \arg \max_{v_i \in V_i} \left\{ \Psi_t(y_i, w_i^r) - \phi_t(y_i, x_i, \bar{v}_i, w_i) / y_i = F_i(x_i, \bar{v}_i, w_i) \right\} \quad x_i \in X_i, i \in I_n;
\]

\[
\bar{v}_i \in \arg \max_{v_i \in V_i} \left\{ \Psi_t(y_i, w_i^r) - \phi_t(y_i, x_i, \bar{v}_i, w_i) / y_i = F_i(x_i, \bar{v}_i, w_i) \right\} \quad x_i \in X_i, i \in I_n;
\]
In expressions (13) and (14), the arguments of Bellman functions are defined by the information structure of the elements of the synthesized system. Problem (13) is a problem of variational calculus. To prove the optimality of the proposed computational technology, it is sufficient to show that expressions (10) and (11) are equivalent to expressions (13) and (14). Let's consider a special case. Let's assume that in (8) the sets \( J_i \) contain indexes of all perturbations of subsequent stages, and the sets \( V_i, i \in I_n \) are limited only by the sets \( V_i, i \in I_n \) of the values of the desired functions. Then according to [7] BP problem (12) is equivalent to the following parametric programming problem:

\[
\Psi_0(x_i) = M_{w_i} \max_{y_i \in V_i} \left\{ \psi_n(y_n) - \sum_{i=1}^{n} \phi_i(y_i, x_i, v_i, w_i) \bigg/ y_i = F_i(x_i, v_i, w_i) \right\}. \tag{15}
\]

The reverse course of the DP for problem (15), taking into account the accepted information structure, coincides with expressions (10) and (11). Therefore, the proposed computational technology is optimal for the considered synthesis problem. The study of the optimality of decentralized control computing technology for other special cases can be performed using statistical tests.

5. The results of simulation of stochastic control systems

An example of process control under uncertainty is the problem of controlling the process of cleaning the mixture (5), (6), in which expression (5) has the form:

\[
\phi_i(v_i) = \frac{a_i}{v_i}; \quad y_i = w_i \cdot v_i \cdot x_i, \quad i = 1, \ldots, 4; \quad x_{i+1} = y_i, \quad i = 1, \ldots, 3, \tag{16}
\]

where the parameter \( w_i \) is an independent random variable that has a normal distribution with a unit expectation and a known variance \( D[w_i] \), \( i = 1, \ldots, 4 \).

In the special case of zero dispersion of the vector \( w \) (\( D[w_i] = 0, i \in I_4 \)), expression (16) coincides with expression (5) and the stochastic system under consideration becomes deterministic. Given this property, the stochastic control system is mathematically described by the formulas (6), (16), and the optimal decision rules are searched for in the form (8).

We will study the computational technology for the synthesis of decentralized control systems for two information structures. In the first, purely stochastic system (SS), the components of the vector \( w \) are unknown to subsystems, i.e. the set \( J_i \) is empty: \( J_i = \phi, i \in I_4 \).

For the SS system, we define index sets \( J^v_i, J^v_i, J^o_i \) of vectors \( w^v, w^f, w^o, i \in I_4 \), on which the Bellman function depends and on which averaging is performed in expressions (10), (11). According to (9), the set \( J^v_i \) is empty, and the sets \( J^v_i, J^o_i \) contain only the index \( i: J^v_i = J^o_i = \{i\}, i \in I_4 \). In this case, when synthesizing the SS system, the averaging of the target functions in (10), (11) is carried out by the \( w_i \) component of the vector \( w \).

The second variant of the information structure is a decentralized system (DS), in which local LMS subsystems know only their components of the \( w_i \) vector \( w \).

In this variant, the set \( J_i \) contains the index \( i: J_i = \{i\}, i \in I_4 \); \( J^v_i \) is an empty set; the set \( J^o_i \) contains only the index \( i: J^o_i = \{i\}, i \in I_4 \); the set \( J^l_i \) is empty: \( J^l_i = \phi, i \in I_4 \). Then there is no averaging operation in expression (10), and when calculating a family of Bellman functions according to (11), averaging is performed on the component \( w_i \) of the vector \( w \).

Let's describe the goals of computer simulation of SS and DS systems. The main goal is to show
for special cases that the solution of the problems of synthesis of decentralized control systems according to (10), (11) is optimal, i.e. coincides with the exact solution. Another goal is to evaluate the effect of using additional information in the DS system in comparison with the SS system.

The study of stochastic control systems of SS and DS was performed for numerical data in Table 1 with two values of variance \( D[w_i], i \in I_a \) that corresponded to the variation of Wi parameters in the expression (16) within ±10% and ±40%. The systems were compared based on the average values of efficiency functionals on the grid of the variable \( x_i \in [10\% ; 15\% ] \).

In all the numerous statistical tests conducted, the computational technology for the synthesis of decentralized systems provided optimal solutions according to (10) and (11). The next result of the simulation is the conclusion that the functioning of the SS system at different variances of random perturbations is statistically the same for average indicators. We assume that the principle of replacing random parameters with their mathematical expectations in this case allows ensuring the optimal functioning of the controlled system on average.

The study of DS system has shown a positive effect of the additional informational content of \( LMS_i \) subsystems. Average control efficiency for ±10% and ±40% variation component of the vector \( w \) significantly increased by 0.05% and 0.22%, respectively, compared with the performance of SS systems under similar conditions.

6. Conclusion

The article describes new computational technologies for the synthesis of optimal two-level discrete control systems with complete and incomplete information, in which a global problem similar to (1) is written in the class of M-models of stochastic programming.

In practice, solving the problems of technological control (1) and (6) with fixed values of parameters is difficult due to the large dimension. It should also be taken into account that in real processes there is an additional function of operational coordination of control conditions at the stages of the entire process, which requires significant computing resources.

The method of dynamic programming using Bellman functions allows implementing the decomposition of the optimal control problem of multi-stage processes and thereby reduce the complexity of its computational procedures (algorithms) and the corresponding computational costs, in other words, it is an effective way to solve problems of large dimensions and rationale the matching procedures.

In addition, the efficiency of solving problems of large dimensions can be increased by applying parallelization schemes in computational algorithms and using high-performance computing systems.

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