DETERMINATION OF $V_{us}$: RECENT PROGRESSES FROM THEORY

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Abstract

Recent experimental and theoretical results on kaon semileptonic decays have significantly improved the determination of the CKM matrix element $V_{us}$. After briefly summarizing the impact of the new experimental determinations, I will concentrate in this talk on the theoretical progresses, coming in particular from lattice QCD calculations. These results lead to the estimate $|V_{us}| = 0.2250 \pm 0.0021$, in good agreement with the expectation based on the determination of $|V_{ud}|$ and the unitarity of the CKM matrix.
1 Introduction

The determination of the Cabibbo angle is of particular phenomenological and theoretical interest since it provides at present the most stringent unitarity test of the CKM matrix. This is expressed by the “first row” unitarity condition:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.$$  \hspace{1cm} (1)

Since $|V_{ub}| \sim 10^{-3}$, its contribution to Eq. (1) can be safely neglected.

The value of $|V_{ud}|$ is accurately determined from nuclear superallowed $0^+ \rightarrow 0^+$ beta decays. An analysis of the results, based on nine different nuclear transitions, leads to the very precise estimate $^1$

$$|V_{ud}| = 0.9740 \pm 0.0005.$$  \hspace{1cm} (2)

This determination of $|V_{ud}|$ is more accurate, by approximately a factor three, than the one obtained from the analysis of neutron beta decay. In the neutron case, the error is dominated by the uncertainty on the contribution of the weak axial current, which is determined experimentally, $g_A/g_V = 1.2720 \pm 0.0022$. It is also worth to mention that a new measurement of the neutron lifetime has been recently presented $^2$ whose new value, $\tau_n = (885.7 \pm 0.8)$ sec., differs by more than six standard deviations with respect to the previous average quoted by the PDG, $\tau_n = (885.7 \pm 0.8)$ sec. $^3$. Combined together, the neutron beta decay results lead to the determination $|V_{ud}| = 0.9750 \pm 0.0017$, in agreement with Eq. (2) but with a much larger error. In the following, I will take the estimate in Eq. (2) as the final average of $|V_{ud}|$ and concentrate the discussion on the remaining entry of Eq. (1), the matrix element $|V_{us}|$.

The most accurate determination of $|V_{us}|$ is obtained from semileptonic kaon decays ($K_{\ell3}$). The analysis of the experimental data gives access to the quantity $|V_{us}| \cdot f_+(0)$, where $f_+(0)$ is the vector form factor at zero four-momentum transfer square. In the SU(3) limit, vector current conservation implies $f_+(0) = 1$. The deviation of $f_+(0)$ from unity represents the main source of theoretical uncertainty. This deviation has been estimated many years ago by Leutwyler and Roos (LR) $^6$, who combined a leading order analysis in chiral perturbation theory (ChPT) with a quark model calculation. They obtained $f_+^{K^0\pi^+}(0) = 0.961 \pm 0.008$, and this value still represents the referential estimate $^6$.

$^1$After this talk, the estimate of $|V_{ud}|$ from nuclear superallowed decays has been updated by Marciano at the CKM 2005 Workshop on the Unitarity Triangle. The new estimate, whose uncertainty is further reduced, reads $|V_{ud}| = 0.9739 \pm 0.0003$. $^2$
Figure 1: Experimental results for $|V_{us}| \cdot f_+(0)$. The “EXP” and “THEORY” bands indicate respectively the average of the new experimental results and the unitarity prediction combined with the LR and lattice (see Sect. 4) determination of the vector form factor.

By averaging old experimental results for $K^\ell_3$ decays with the recent measurement by E865 at BNL \(^7\), and using the LR determination of the vector form factor, the PDG quotes $|V_{us}| = 0.2200 \pm 0.0026$ \(^8\). This value, once combined with the determination of $|V_{ud}|$ given in Eq. (2), implies about 2$\sigma$ deviation from the CKM unitarity condition, i.e. $|V_{us}|^\text{unit.} \simeq \sqrt{1 - |V_{ud}|^2} = 0.2265 \pm 0.0022$.

2 $K^\ell_3$ decays: the new experimental results

With respect to the PDG 2004 analysis, however, a significant novelty is represented by several new experimental results, for both charged and neutral $K^\ell_3$ decays, which have been recently presented by KTeV \(^9\), NA48 \(^10\), and KLOE \(^11\). Expressed in terms of $|V_{us}| \cdot f_+(0)$, these determinations are shown in Fig. 1 together with the BNL result and the averages of the old $K^\ell_3$ results quoted by the PDG. Remarkably, the average of the new results, \(^11\), represented by the darker band in the plot (“EXP”), is in very good agreement with the unitarity prediction, once the LR determination of the vector form factor is taken into account. The unitarity prediction is shown in Fig. 1 by the lighter band (“THEORY”).
3 $K\ell^3$ decays: theory status

An important theoretical progress in the determination of $|V_{us}|$ is represented by the first (quenched) lattice determination, with significant accuracy, of the vector form factor at zero momentum transfer $f_+(0)$ [12]. The lattice result turns out to be in very good agreement with the quark model estimate obtained by LR, thus putting the evaluation of this form factor on a firmer theoretical basis. Before outlining the strategy of the lattice calculation, I would like to summarize the theoretical status of the $f_+(0)$ evaluations.

A good theoretical control on $K\ell^3$ transitions is assured by the Ademollo-Gatto (AG) theorem [13], which states that $f_+(0)$ is renormalized only by terms of at least second order in the breaking of SU(3)-flavor symmetry. Nevertheless, the error on the shift of $f_+(0)$ from unity represents not only the main source of theoretical uncertainty but it also dominates the overall error in the determination of $|V_{us}|$.

The amount of SU(3) breaking due to light quark masses can be investigated within ChPT, by performing an expansion of the form $f_+(0) = 1 + f_2 + f_4 + \ldots$, where $f_n = \mathcal{O}(p^n) = \mathcal{O}[M_K^2/(4\pi f_\pi)^n]$. Thanks to the AG theorem, the first non-trivial term in the chiral expansion, $f_2$, does not receive contributions of local operators appearing in the effective theory and can be computed unambiguously in terms of $M_K$, $M_\pi$ and $f_\pi$ ($f_2 = -0.023$, in the $K^0 \to \pi^-$ case [13]). The higher-order terms of the chiral expansion involve instead the coefficients of local chiral operators, that are difficult to estimate. The quark model calculation by LR provides an estimate of the next-to-leading correction $f_4$, and it is based on a general parameterization of the SU(3) breaking structure of the pseudoscalar meson wave functions.

An important progress in this study is represented by the complete two-loop ChPT calculation of $f_4$, performed in Refs. [14, 15]. In Ref. [15], the result has been written in the form

$$f_4 = \Delta(\mu) - \frac{8}{F_\pi^2} [C_{12}(\mu) + C_{34}(\mu)] (M_K^2 - M_\pi^2)^2,$$

where $\Delta(\mu)$ is expressed in terms of chiral logs and the $\mathcal{O}(p^4)$ low-energy constants, while the second term is the analytic one coming from the $\mathcal{O}(p^6)$ chiral Lagrangian. As can be seen from Eq. (3), this local contribution involves a single combination of two (unknown) chiral coefficients entering the effective Lagrangian at $\mathcal{O}(p^6)$. In addition, the separation between non-local and local contribution quantitatively depends on the choice of the renormalization scale $\mu$, only the whole result for $f_4$ being scale independent. This dependence is found to be large [16]: for instance, at three typical values of the scale one finds

$$\Delta(\mu) = \begin{cases} 0.031, & \mu = M_\eta \\ 0.015, & \mu = M_\rho \\ 0.004, & \mu = 1 \text{ GeV} \end{cases}.$$
An important observation by Bijnens and Talavera is that the combination of low-energy constants entering $f_4$ could be in principle constrained by experimental data on the slope and curvature of the scalar form factor. The required level of experimental precision, however, is far from what is currently achieved. Thus, one is left with either the LR result or other model dependent estimates of the local term in Eq. (3). Recent attempts in this direction include the estimate by resonance saturation obtained in Ref. 17 and the dispersive analysis of Ref. 18. The model results, however, are in disagreement within each other. In addition, the large scale dependence of the $O(p^6)$ loop calculation shown in Eq. (4) seems to indicate that the error $\pm 0.010$ quoted in Refs. 15, 16, 18 might be underestimated.

For all these reasons, a first principle lattice determination of the vector form factor is of great phenomenological relevance.

4 Strategy of the lattice calculation

The first lattice calculation of the vector form factor at zero momentum transfer has been recently presented in Ref. 12. In order to reach the challenging goal of about 1% error on the lattice determination of $f_4(0)$, a new strategy has been proposed and applied in the quenched approximation. This strategy is based on three steps.

1) Precise evaluation of the scalar form factor $f_0(q^2)$ at $q^2 = q_{\text{max}}^2$

This evaluation follows a procedure originally proposed by the FNAL group to study heavy-light form factors. For $K^{\ell+3}$ decays, the scalar form factor $f_0(q^2)$ can be calculated very efficiently at $q^2 = q_{\text{max}}^2 = (M_K - M_\pi)^2$ by studying the following double ratio of matrix elements,

$$\frac{\langle K | \bar{s} \gamma_0 u | K \rangle \langle K | \bar{u} \gamma_0 s | \pi \rangle}{\langle K | \bar{s} \gamma_0 s | K \rangle \langle \pi | \bar{u} \gamma_0 u | \pi \rangle} = \frac{(M_K + M_\pi)^2}{4M_K M_\pi} \left| f_0(q_{\text{max}}^2) \right|^2,$$

where all the external particles are taken at rest. There are several crucial advantages in using the double ratio which are described in details in Ref. 12. The most important point is that this ratio gives values of $f_0(q_{\text{max}}^2)$ with a statistical uncertainty smaller than 0.1%, as it is illustrated in Fig. 2 (left).

2 A different factorization between local and non-local contributions has been considered in Ref. 17, which partly reduces the dependence on the factorization scale.
2) Extrapolation of $f_0(q_{\text{max}}^2)$ to $f_0(0) = f_+(0)$

This extrapolation is performed by studying the $q^2$-dependence of $f_0(q^2)$. New suitable double ratios are also introduced in this step, that improve the statistical accuracy of $f_0(q^2)$. The quality of the extrapolation is shown in Fig. 2 (right). Three different functional forms in $q^2$ have been considered, namely a polar, a linear and a quadratic one. The lattice result for the slope $\lambda_0$ of the scalar form factor is in very good agreement with the recent accurate determination from KTeV $^{20}$. In the case of the polar fit, for instance, the lattice result is $\lambda_0 = 0.0122(22)$ (in units of $M_\pi^2$) to be compared with the experimental determination $\lambda_0 = 0.0141(1)$.

3) Extrapolation to the physical masses

The physical value of $f_+(0)$ is finally reached after extrapolating the lattice results to the physical kaon and pion masses. The problem of the chiral extrapolation is substantially simplified if the AG theorem (which holds also in the quenched approximation $^{21}$) is taken into account and if the leading (quenched) chiral logs are subtracted. This is achieved by introducing the following ratio

$$R = \frac{\Delta f}{(M_K^2 - M_\pi^2)^2} = \frac{1 + f_0^q - f_+(0)}{(M_K^2 - M_\pi^2)^2},$$

(6)
where \( f_2^q \) represents the leading chiral contribution calculated in quenched ChPT \(^{12}\) and the quadratic dependence on \((M_K^2 - M_\pi^2)\), driven by the AG theorem, is factorized out. It should be emphasized that the subtraction of \( f_2^q \) in Eq. (6) does not imply necessarily a good convergence of (quenched) ChPT at \( \mathcal{O}(p^4) \) for the meson masses used in the lattice simulation. The aim of the subtraction is to access directly on the lattice the quantity \( \Delta f \), defined in Eq. (6) in such a way that its chiral expansion starts at \( \mathcal{O}(p^6) \) independently of the values of the meson masses. After the subtraction of \( f_2^q \), the ratio \( R \) of Eq. (6) is smoothly extrapolated in the meson masses as illustrated in Fig. 3.

In order to check the stability of the results, linear, quadratic and logarithmic fits have been considered. The chiral extrapolation leads to the final result

\[
\Delta f_{f_{+\pi^0}}(0) = 0.960 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}},
\]

where the systematic error does not include quenching effects beyond \( \mathcal{O}(p^4) \). Removing this error represents one of the major goals of future lattice studies of \( K_{\ell 3} \) decays. Remarkably, two preliminary unquenched calculations have been already presented. The results read

\[
\Delta f_{f_{+\pi^0}}(0) = 0.962 \pm 0.006 \pm 0.007, \tag{8}
\]

\[
\Delta f_{f_{+\pi^0}}(0) = 0.954 \pm 0.009, \tag{9}
\]

in very good agreement with the quenched estimate of Eq. (7).
The value $7$ compares well with the LR result $f_{K^0\pi^-}(0) = 0.961 \pm 0.008$ quoted by the PDG and, once combined with the average of the more recent experimental results, implies

$$|V_{us}| = 0.2256 \pm 0.0022,$$

in good agreement with the unitarity prediction.

A strategy similar to the one discussed above has been also applied to study hyperon semileptonic decays on the lattice, and preliminary results have been presented in 24).

5 Conclusions

We have discussed the most recent experimental and theoretical progresses achieved in the determination of $V_{us}$ from semileptonic kaon decays. On the theoretical side, the main novelty is represented by the first lattice QCD calculation of the $K^\ell_3$ vector form factor at zero-momentum transfer, $f_+(0)$. This calculation is the first one obtained by using a non-perturbative method based only on QCD, except for the quenched approximation. Once combined with the new measurements of kaon semileptonic decays, the lattice result leads to a determination of $V_{us}$ in very good agreement with the expectation based on the determination of $V_{ud}$ and the unitarity of the CKM matrix.

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