Analytical and experimental investigation on cutting force in longitudinal-torsional coupled rotary ultrasonic machining zirconia ceramics

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Abstract
Ceramics and other hard-and-brittle materials are very effectively processed by longitudinal-torsional coupled rotary ultrasonic machining (LTC-RUM), which cutting force evolution and the effects of processing parameters on the material removal mechanism need to be optimized. This study introduced the LTC-RUM cutting force model of zirconia ceramics based on the brittle material removal mechanism. Firstly, the kinematic analysis of a single abrasive grain was performed, with further consideration of the material removal volume, the effective contact time, and the impact force per one ultrasonic vibration cycle. Then, the longitudinal-torsional coupled vibration of the core tool was analyzed from the wave energy conversion standpoint. The analytical model was finalized and experimentally verified by LTC-RUM tests, which results closely correlated with the ones predicted via the proposed model. With the spindle speed, feed rate, and ultrasonic power variation, the maximum discrepancies between the predicted and experimental cutting forces are 15.51, 13.24, and 8.3%, respectively. The results obtained are considered instrumental in predicting the effects of processing parameters on the cutting force during LTC-RUM of ceramics and their further optimization.

Keywords Zirconia ceramics · Cutting force · Rotary ultrasonic machining · Longitudinal-torsional coupled vibration · Analytical model

1 Introduction
Zirconia ceramics are metal oxide ceramic materials with excellent physical and chemical properties, including high mechanical strength, corrosion and wear resistance, and thermal stability, facilitating their wide application in aerospace, optical, biomedical, electronics, and other industries [1–4]. However, due to zirconia ceramics’ high hardness and brittleness, obtaining their high-quality surface with conventional machining technologies is problematic, due to high tool wear and processed workpieces’ damage risks. Numerous innovative technologies, such as grinding [5, 6], laser machining [7], electrolytic in-process dressing grinding [8], ultrasonic machining [9], ultrasonic vibration-assisted machining [10], and rotary ultrasonic machining (RUM) [11], have been recently implemented to mitigate the above problems.

The latter technique (RUM), which combines grinding and ultrasonic machining, became one of the most effective ceramics-processing methods [12, 13]. Many investigations have been conducted to explore the processing characteristics of RUM, which are schematically presented in Fig. 1.

The research on processing characteristics of RUM’s hole manufacturing of brittle materials primarily can be subdivided into two main categories. The first one involves comparative studies of processing characteristics of RUM and conventional grinding, including the material removal rate [14–17], processing performance [18–20] and modeling [21–24], damage formation mechanism [25, 26], and suppression methods [27–29]. Such studies proved that RUM outperformed conventional grinding due to the improved material removal rate (MRR), reduced cutting force, enhanced hole-processing accuracy and surface quality,
alleviated processing damage, and extended tool life. The second category’s research is focused on the effects of various processing factors, including tool parameters [30], cooling condition [31], spindle speed, feed rate [32], and ultrasonic amplitude [33], on the RUM performance. While the related studies provided more insight into the RUM perspectives for processing hard-brittle and other difficult-to-machining materials; most of them were related to the one-dimensional (1D) RUM derived from the ultrasonic-assisted machining (UAM). Although optimizing processing parameters via the 1D RUM scheme enhanced the processing performance, its capacity was restricted by the limit conditions.

With the development of USM technology and the effective performance of 2D vibration in ultrasonic vibration-assisted cutting, 2D vibration has been applied to rotary ultrasonic drilling (RUD). To date, there are three types of 2D vibration, including longitudinal and bending coupled (LBC) vibration [34], double bending coupled (DBC) vibration [35], longitudinal and torsional coupled (LTC) vibration [36]. Compared with the former two types, the LTC vibration has been more widely used in different types of UAM. For instance, Gao et al. studied the influence of longitudinal-torsional ultrasonic vibration (LTUV) on microhole drilling of Ti–6Al–4 V. They found that the cutting force was dramatically reduced compared to conventional drilling [37]. Liu et al. developed LTUV helical milling for hole-making of carbon-fiber-reinforced plastic (CFRP) composites. Compared to conventional helical milling, the LTUV-assisted one was more adapted to fracture fibers and provided better surface quality [38]. In particular, Cardoni et al. developed several ultrasonic drilling devices resonating in the LTUV mode, enhancing drilling performance [39]. The successful application of LTUV/LTC to the UAM case inspired the LTC-RUM processing of brittle materials. Thus, Wang et al. utilized the LTC-RUM to drill ceramic matrix composites and reported that it reduced the cutting force by over 50% compared to the conventional longitudinal RUM (Con-RUM) [40]. Ma et al. experimentally investigated the processing performance of zirconia ceramics with the LTC-RUM and revealed a significant improvement of the machined surface quality compared to Con-RUM [41]. However, the material removal mechanism of LTC-RUM in brittle material has received no adequate clarification or theoretically substantiated yet.

The cutting force is regarded as one of the most important output variables to assess the machining performance because it dramatically influences the surface damage, tool wear, cutting temperature, etc. [21]. Besides, an accurate cutting force model is required to describe the material removal mechanism during LTC-RUM. In the Con-RUM, the material removal and surface formation are related to the tool-end face and the periphery-surface abrasive grains, which control the respective cutting forces, particularly the normal cutting force acting along the feed direction. Despite a large number of available state-of-the-art publications, to the best of the authors’ knowledge, no investigations on the cutting force of LTC-RUM have been reported yet. This study attempts to identify the material removal mechanism and simulate the cutting force in the LTC-RUM as applied to zirconia ceramics. The available cutting force models based on the static mechanical properties for RUM [12, 13, 21, 22] were further refined in this paper by taking into account a single abrasive grain’s kinematics analysis of LTC-RUM, tool workpiece’s intermittent interaction, effective cutting time per one ultrasonic cycle, indentation depth, and the brittle fracture model of the material removal. Tests were also conducted to verify the proposed mode’s feasibility in predicting LTC-RUM cutting forces for various processing parameters.
2 Assumptions and approaches used in the cutting force model elaboration

2.1 Main assumptions

RUM is a complex process with many input variables: core drill diameter, concentration and size of abrasive particles, workpiece material’s properties, ultrasonic parameters (ultrasonic frequency and amplitude), and machining parameters (spindle speed and feed rate). The following five assumptions were used to facilitate the LTC vibration analysis in the RUM processing of zirconia ceramics.

1. The diamond abrasive particles were uniformly distributed along the end face of the core drill.
2. The effect of circumferential abrasive grains on the cutting force was ignored.
3. All abrasive particles had the same size and were octahedron-shaped.
4. The abrasive particles were perfectly rigid, had the same height, and all of them participated in the cutting process in each vibration cycle.
5. The ultrasonic frequency, amplitude, and torsional-to-longitudinal vibration ratio ($A_T/A_L$) remained unchanged during processing;
6. The material removal mainly occurred in the brittle fracture mode.

Some additional assumptions and simplifications were used, which will be provided in the following section.

2.2 The approach and main procedures used in the model elaboration

The main approach and procedures used in the model’s elaboration are schematically presented in Fig. 2. The model development started from a single abrasive grain, its trajectory analysis, the maximum impact force, and the effective cutting time per one vibration cycle. Based on the brittle fracture mode of the material removal, the material removal volume of a single abrasive grain was calculated by considering the scratching length of abrasive grains in the processed workpiece and the relevant properties of zirconia ceramics. Next, the material removal rate was assessed by superimposing the effective abrasive grains' effects, and the cutting force evolution during LTC-RUM was predicted. Then, the actual material removal volume of a single abrasive ($K_v$) was adjusted for the model realization based on the additional test results. Finally, drilling tests were conducted to verify the model’s feasibility and accuracy.

3 The LTC-RUM cutting force assessment

3.1 Kinematics analysis of a single abrasive particle

In LTC-RUM processing, an electroplated diamond core drill rotated with a spindle speed $n$ and moved toward the workpiece at a feed rate $f$, simultaneously vibrating with longitudinal amplitude of $A_L$ and torsional amplitude $A_T$. The LTC-RUM scheme is depicted in Fig. 3.
The abrasive particle’s axial and circumferential vibrations had the same frequency and phase during the LTC-RUM processing. The torsional amplitude could be approximated by the arc length of rotation angle caused by torsional vibration. Therefore, the kinematic trajectory of a single abrasive particle in LTC-RUM can be expressed as follows:

\[
\begin{align*}
  x &= r \cos\left(\frac{A_T}{r} \sin(2\pi f t) + \frac{\pi n t}{30}\right) \\
  y &= r \sin\left(\frac{A_T}{r} \sin(2\pi f t) + \frac{\pi n t}{30}\right) \\
  z &= A_\ell \sin(2\pi f t) + f_1 t
\end{align*}
\]  

where \( r \) is the tool’s average radius. At \( f_\omega = 0 \), Eq. (1) describes the moving trajectory of an abrasive grain during Con-RUM. The diamond abrasive grain trajectory in the above two cases is illustrated in Fig. 4.

As shown in Fig. 4, the LTC-RUM’s abrasive particle movement trajectory was a slanted sinusoidal curve, while that of Con-RUM was a sinusoidal curve. During the LTC-RUM processing, the motion direction of the abrasive particle coincided with the main motion direction or was opposite to it, leading to the intermittent cutting effect between the abrasive particles and workpiece. To appear the trajectory more legible in both cases, Eq. (1) can be expressed based on the cylindrical surface as follows:

\[
\begin{align*}
  l &= A_T \sin(2\pi f t) + 2\pi r n t \\
  z &= A_\ell \sin(2\pi f t) + f_1 t
\end{align*}
\]  

The grain’s trajectories in the LTC-RUM and Con-RUM processing were plotted in Figs. 5 and 6, respectively. Figure 5 presents the effect of ultrasonic parameters (ultrasonic frequency and amplitude) on the abrasive particle trajectory. As shown in Fig. 5a, the abrasive grain’s trajectory under both cases was reduced with an increase in frequency. The intermittent cutting effect during LTC-RUM can be enhanced, which is beneficial to reduce the cutting force. In addition, as shown in Fig. 5b, the ultrasonic vibration’s impact effect was strengthened with the ultrasonic amplitude, increasing the grinding depth of a single abrasive grain. This promoted the material removal rate (MRR).

Figure 6 presents the effect of processing parameters (spindle speed and feed rate) on the abrasive particle trajectory. With an increase in the spindle speed, as shown in Fig. 6a, the cycloidal trajectory in a vibration cycle during both cases became sparser, i.e., the rotary pattern of abrasive grains gradually weakened, indicating that the ultrasonic vibration superiority was suppressed to a certain extent. This reduced the number of reciprocating impact ironings at the same position of the workpiece surface by different
abrasive particles and deteriorated the machined surface quality. However, the rotary pattern of abrasive grains was not changed with the feed rate variation when other conditions remained constant, as shown in Fig. 6b. This can be attributed to the fact that the feed rate was much lower than the velocity of the longitudinal ultrasonic vibration, i.e., \( f_r < 2\pi f_A L \). Therefore, in LTC-RUM, a suitable ultrasonic parameter should be selected to match the spindle speed to enhance processing efficiency and quality.

### 3.2 Material removal mechanisms of ceramics

As shown in Fig. 3, the trajectory of the abrasive particle on the tool-end-face in LTC-RUM was not a simple superposition of Con-RUM and conventional grinding, while the abrasive grains contacted and separated with the workpiece material at an ultrasonic frequency during LTC-RUM. As depicted in Fig. 7, the abrasive grain can be regarded as multiple Vickers indenters, and the material firstly generated elastic and plastic deformations when the abrasive particle penetrated the workpiece. The median cracks were generated as the penetration depth continued to increase. The tensile residual stresses were generated when the abrasive particles were pulled out from the processed material, causing lateral cracking. The cracks initiated and propagated at the workpiece’s surface, promoting the material removal. Because the impact force of abrasive particles under the ultrasonic vibration on the workpiece exceeded that in conventional grinding, the material removal occurred via brittle fracture mode.

In the LTC-RUM, the abrasive grain-workpiece contact is depicted in Fig. 8, the effective contact time \( t_c \) can be calculated as follows:
where \( \lambda \) is a proportionality coefficient, \( \lambda \in (0.5,1) \) \([42]\), \( \delta \) is the indentation depth of diamond abrasive particles in the workpiece material induced by LTC-RUM.

According to the results of Liu et al. \([12]\), the relationship between the maximum impact force of a single diamond abrasive \( F_i \) and the penetration depth \( \delta \) can be expressed as follows:

\[
\tau_c = \frac{\lambda}{\pi f} \left[ \frac{\pi}{2} - \arcsin(1 - \frac{\delta}{A_L}) \right]
\]  

(3)

where \( \lambda \) is a proportionality coefficient, \( \lambda \in (0.5,1) \) \([42]\), \( \delta \) is the indentation depth of diamond abrasive particles in the workpiece material induced by LTC-RUM.

According to the results of Liu et al. \([12]\), the relationship between the maximum impact force of a single diamond abrasive \( F_i \) and the penetration depth \( \delta \) can be expressed as follows:

\[
F_i = \frac{2}{\pi} \sqrt{\tan^2 \psi + 2}
\]  

(4)

where \( \psi = 45^\circ \) is the abrasive grain tip’s semi-angle, and \( H_V \) is the microhardness of the workpiece.

It was assumed that all effective abrasive grains involved in the material removal with the same penetration depth \( \delta \), and the maximum impact force \( F_m \) of all abrasive grains can be derived as follows:

\[
F_m = N_i F_i
\]  

(5)
where \( N_a \) is the number of effective abrasive grains on the tool-end-face, which is assessed as follows:

\[
N_a = \frac{\pi C_1 C_a^{2/3} (d_o^2 - d_i^2)}{S_a^2} \tag{6}
\]

where \( C_a \) is the concentration of abrasive grains; \( S_a \) is the abrasive grain size; \( C_1 \) is a dimensionless constant \((C_1 = 0.02)\), while \( d_o \) and \( d_i \) are the outer and inner diameters of the core drill, respectively.

Since the ultrasonic frequency usually exceeds the dynamometer’s natural frequency and the processing system’s resonant frequency, only the average grinding force could be measured accurately. Therefore, during the LTC-RUM tests, the measured cutting force \( F_c \) failed to properly reflect the maximum impact force \( F_m \) under the action of ultrasonic vibration. Instead, based on the energy conservation theorem, the pulse \((I)\) of the maximum impact force \((F_m)\) during per vibration cycle can be expressed as

\[
I = \int_{cycle} F_m dt \approx F_m t_c \tag{7}
\]

The pulse value \( I \) for the cutting force \( F_c \) during per vibration cycle can be derived as

\[
I = \frac{F_c}{f} \tag{8}
\]

The relationship between \( F_c \) and \( F_m \) can be obtained by equating pulses in Eqs. (7) and (8).

\[
F_c = F_m t_c \tag{9}
\]

By substituting Eqs. (3)–(6) into Eq. (9), it could be obtained:

\[
F_c = \frac{\pi}{\lambda N_a} \left[ \frac{\pi}{2} - \arcsin(1 - \frac{\delta}{A_L}) \right]^{-1} F_c \tag{10}
\]

Substituting Eq. (10) into Eq. (4), the penetration depth \( \delta \) can be calculated by the following equation.

\[
\delta = \left( \frac{\pi}{24 N_a H \tan \psi \sqrt{\tan^2 \psi + 2(1/2)} - (1/\pi) \arcsin(1 - \delta/A_L)} \right)^{1/2} \tag{11}
\]

The brittle fracture mode controls the material removal mechanism during the LTC-RUM processing of brittle materials. As shown in Fig. 9, the single diamond grain impact generated the lateral and medial cracks in the workpiece. The material was removed by propagation and coalescence of lateral cracks.

According to Wang et al. [29], the lateral crack’s length \( (C_L) \) and depth \( (C_h) \) can be calculated as

\[
\begin{aligned}
C_L & = C_2 \left( \frac{1}{\tan \psi} \right)^{5/12} \left( \frac{E^{3/4}}{H \cdot K_{IC} (1 - v^2)^{1/2}} \right)^{1/2} (F_t)^{5/8} \\
C_h & = C_2 \left( \frac{1}{\tan \psi} \right)^{1/3} \left( \frac{E^{1/2}}{H} \right)^{1/2} (F_t)^{1/2}
\end{aligned}
\tag{12}
\]

where \( v, K_{IC}, \) and \( E \) are Poisson’s ratio, fracture toughness, and elastic modulus of the workpiece material, respectively; \( C_2 \) is the dimensionless constant \((C_2 = 0.226 [29])\).

As seen in Fig. 9, a single diamond grain’s fracture zone volume \( (V_0) \) corresponds to the \( A_0 A_1 A_2 A_3 A_4 \) pyramid. The effective scratching distance of a single grain per one vibration cycle during LTC-RUM \( (L) \) can be assessed as follows:

\[
L = A_7 \sin(2\pi f t_c) + \frac{2\pi nR}{60} t_c \tag{13}
\]

When a single grain length scratches the workpiece surface, the grain penetration depth increases from zero to \( \delta \) and then drops to zero. Consequently, the width and length of the lateral crack will increase from zero to the maximum value and then drop to zero again. As a result, the theoretical material removal volume \( V_0 \) of the fracture zone can be calculated as

\[
V_0 = V_{a_0 A_1 A_2 A_4} = \frac{1}{3} C_L C_h L \tag{14}
\]

During the LTC-RUM process, lateral cracks randomly propagate and interact as the grain impacts on the workpiece surface, increasing the differences between the actual \( (V) \) and the theoretical \( (V_0) \) values of the material removal volume per vibration cycle. To simplify the actual volume calculation, its value \( V \) can be derived by multiplying \( V_0 \) to a constant adjusting coefficient \( K_V \). Therefore, the material removal rate \( \text{MRR} \) of all grains in per vibration cycle can be expressed as
MRR = \frac{K}{\omega}V_0fN_a \quad (15)

In addition, the MRR can be obtained by the following equation:

\[ MRR = \frac{\pi(d_o^2 - d_i^2)}{4}f_r \quad (16) \]

By equating Eqs. (14) and (15), and substituting Eqs. (6), (10), (12), and (13) into Eq. (14), it can be obtained:

\[
F_c = \frac{3f_hH^{3/2}K^1/2C_1^{1/2}C_a^{1/2}(d_o^2 - d_i^2)^{9/8}(\frac{\pi}{2} - \arcsin(1 - \frac{\delta}{h}))^{9/8}}{4^{9/8}K_m^{17/8}C_2E^{7/8}S^{1/4}(\cot \psi)^{3/4} \left( A_T \sin(2\lambda(\frac{\pi}{2} - \arcsin(1 - \frac{\delta}{h})) + \frac{2\pi}{3\delta}f_\text{transverse} \right) \right]}^{8/9} \quad (17)
\]

According to Eq. (17), the cutting force \( F_c \) can be determined by such processing parameters as spindle speed, feed rate, ultrasonic frequency, ultrasonic amplitude, tool parameters, and material properties in the LTC-RUM of brittle materials. If the adjustment coefficient \( K_V \) is experimentally determined, only two parameters, \( F_c \) and \( \delta \), remain unknown. The relation between \( F_c \) and \( \delta \) has been obtained by Eq. (11) and combining Eq. (17), the cutting force \( F_c \) and penetration depth \( \delta \) can be acquired simultaneously.

**4 Obtaining the adjusting coefficient \( K_V \)**

**4.1 Experimental setup**

The rotary ultrasonic drilling tests were performed in this study using a modified CNC machine center (VMC850E) equipped with self-designed ultrasonic vibration devices. As presented in Fig. 10, the experimental setup comprised an ultrasonic vibration system, a cutting force measurement system, and a coolant system.

The ultrasonic vibration system included an ultrasonic generator, transducer, horn, and an electroplated diamond core tool. The ultrasonic generator with a maximum output power of 500 W converted the alternating current into high-frequency electric oscillation for the ultrasonic vibration system. Then, the piezoelectric transducer converted the electric oscillation into high-frequency mechanical vibration. However, the amplitude of mechanical vibration generated by the transducer was generally too low to be used for mechanical machining. Thus, the horn with helical slots was used as concentrators to amplify the ultrasonic vibration amplitude to applicable magnitudes. During the LTC-RUM, the longitudinal-torsional coupled vibration is applied to the core drill end, which is achieved by prefabricating four helical slots on the horn, as shown in Fig. 11.

When the ultrasonic wave generated by the transducer was transferred from the horn’s larger end to the smaller ones, the reflection of waves occurred in the helical slots. As shown in Fig. 11, under the action of helical slots, the incident longitudinal wave \( P_i \) was decomposed into the reflected longitudinal \( P_r \) and transverse \( P_t \) waves, respectively. Thus, the axial and circular components were produced at the horn’s smaller end, including longitudinal and torsional vibrations, respectively. The mechanism of longitudinal vibration conversion into torsional one via helical slots was described in more detail in our earlier study [43].

This study fabricated the horn with helical slots based on
the previous results. Then, a laser displacement sensor (LK-G10, KEYENCE, Japan) was used to measure the torsional ($A_T$) and longitudinal ($A_L$) vibration amplitudes of the output end face. The average amplitude ratio of torsional amplitude to longitudinal amplitude ($A_T/A_L$) was 0.52.

The diamond core tool was connected to the horn with an ER16 cone fit. The workpiece was fixed by a jaw vise grip and connected to a Kistler 9257B dynamometer through a fixture. As the forces in the $x$- and $y$-axes directions were smaller and substantially stable, the dynamometer was mounted on the worktable to measure the cutting force evolution along the $z$-axis direction during the test. A 4/8-channel laboratory charge amplifier (5070A) was used to amplify the electrical signal from the dynamometer, and then it was fed to a data recorder (2825A). Using Kistler’s Dyno Ware data acquisition software for cutting force measurement, the recorded data were stored and displayed via a PC. Because the dynamometer’s natural frequency was much lower than the ultrasonic frequency, the average grinding force components were used to assess the processing. The actual cutting force is shown in Fig. 12.

### 4.2 Design of experiments

The workpiece material was zirconia ceramics, which mechanical properties are listed in Table 1. All workpiece-simulating specimens were machined to the initial dimensions of $15 \times 10 \times 5$ mm.

In the experiment, tools 1 and 2 were utilized to obtain the adjusting coefficient $K_v$ and verify the cutting force model, respectively. The core tool parameters are listed in Table 2. The diamond core tool used in the experiment was manufactured by the Zhengzhou Research Institute for Abrasives & Grinding Co. Ltd. China. The workpiece mass values before and after machining (including the machined cylinder) were weighted by a precision balance (JCS-2000) during the experiment.

During the experiment, at the tool’s tuning length of 30 mm, the resonant frequencies of tools 1 and 2 were 35.16 and 35.12 kHz, respectively, which difference was negligible. Due to the experimental setup limitations, the ultrasonic vibration frequency was fixed for both tools in the tests, while the spindle speed, feed rate, and ultrasonic amplitude varied. The variation of ultrasonic amplitude for tools 1 and 2 was achieved by adjusting the ultrasonic generator power. As shown in Fig. 13, the ultrasonic amplitude dependence versus the ultrasonic power was nearly linear.

### Table 1 Mechanical properties of the zirconia ceramics material

| Properties               | Unit      | Value |
|--------------------------|-----------|-------|
| Elastic modulus $E$      | GPa       | 210   |
| Hardness $H_V$           | GPa       | 12    |
| Fracture toughness $K_{IC}$ | MPa m$^{1/2}$ | 6   |
| Density $\rho$           | g/cm$^3$  | 6.05  |
| Poisson’s ratio $v$      |           | 0.3   |
The detailed machining parameters were preset to obtain the adjusting coefficient $K_v$, as shown in Table 3. Each test was repeated three times, and a final value was obtained by averaging the measured results.

### 4.3 Analysis of experimental results

The actual volume ($V$) can be calculated as follows:

$$V = \frac{m_1 - m_2}{\rho N_a}$$

(18)

where $m_1$ and $m_2$ are the workpiece mass values before and after machining (including the machined cylinder), respectively.

According to Liu et al. [12], the value of $K_v$ varied only with the workpiece material, being independent of processing parameters. In this study, it was assessed via the following equation:

$$K_v = \frac{V}{V_0} = \frac{12H^3/2K_{ic}^{1/2}S_a^2(1 - v^2)^{1/4}(m_1 - m_2)}{\rho \pi f C_1 C_a^2 C_2 (\cot \psi)^{3/4} E^{7/8} \beta^9/8 (d_o^2 - d_i^2)A_T \sin(2\gamma^{7/8} - \text{arcsin}(1 - \frac{d_i}{A_T}) + \frac{4R_n}{30} \left(\frac{\pi}{2} - \text{arcsin}(1 - \frac{d_1}{A_T})\right))}$$

(19)

The values of $F_i$ in Eq. (18) can be derived via Eq. (4). According to Eq. (19), the relationship between the actual volume ($V$) and the theoretical volume ($V_0$) of material removal was obtained and plotted in Fig. 14. The slope of the regression line, namely 0.239, can be utilized to assess the $K_v$ value.

### 5 Cutting force prediction

Based on the developed mechanistic cutting force model, the cutting force was predicted for the particular processing conditions. Figure 15 presents the predicted cutting forces for different machining variables. It can be seen that the predicted cutting force decreased with spindle speed and ultrasonic amplitude but increased with the feed rate. According to Eq. (17), the cutting force $F_c$ can be reduced by increasing spindle speed and ultrasonic amplitude ($A_L$ and $A_T$) or

![Fig. 13 Experimental relationship between ultrasonic power and amplitude](image1)

![Fig. 14 Correlation between the actual and theoretical material removal volumes $V$ and $V_0$](image2)
decreasing the feed rate. In addition, according to Eq. (16), the \( MRR \) value remained unchanged unless the feed rate was changed. In other words, the indentation depth \( \delta \) needs to be decreased to keep the \( MRR \) unchanged with the spindle speed increasing. Besides, as shown in Fig. 6a, a sparser cycloidal trajectory of a single abrasive grain implies that the indentation depth \( \delta \) is reduced with the increase of spindle speed. Based on Eqs. (3), (4), and (10), the indentation depth reduction led to lower values of the effective contact time \( t_c \) and the maximum impact force \( F_i \), further reducing the cutting force \( F_c \). Therefore, the cutting force decreased with an increase in the spindle speed. Although the cycloidal trajectory of single abrasive grain slightly changed with the feed rate, the workpiece’s \( MRR \) was promoted, increasing the corresponding indentation depth \( \delta \). In contrast to the spindle speed effect, an increase in the feed rate raised the cutting force \( F_c \). The increased longitudinal amplitude led to larger indentation depth \( \delta \) and maximum impact force \( F_i \), but a smaller \( \delta/A_L \) ratio [44]. This, in turn, significantly reduced the effective contact time \( t_c \), compensating the effect of increased maximum impact force \( F_i \) and resulting in the cutting force \( F_c \) eventual reduction. Moreover, as shown in Fig. 8, under the action of the LTC vibration, the torsional ultrasonic amplitude \( A_T \) caused the diamond abrasive particles to be vertically pulled out from the workpiece, improving their intermittent cutting effect. Therefore, larger longitudinal and torsional amplitudes (\( A_L \) and \( A_T \)) are beneficial for cutting force reduction.

Fig. 15 Prediction relation between parameters and cutting force: a spindle speed, b feed rate, and c longitudinal and torsional amplitude
Pilot experimental verification

The pilot experiment was used to verify the cutting force model’s feasibility. The experimental setup was adopted as described in Sect. 4, and tool 2 was used in the test. The test parameters are listed in Table 4.

Based on the developed mechanistic cutting force model, the cutting force can be predicted for the processing conditions. Figure 16 compares the predicted and experimental cutting forces for different machining variables.

It can be seen that the predicted cutting force trends closely correlate with the experimental ones. The cutting force decreased with spindle speed and ultrasonic power but increased with the feed rate. Besides, the effects of spindle speed and feed rate on the cutting force were more pronounced than that of the ultrasonic power. With the variation of spindle speed, feed rate, and ultrasonic power, the maximum relative errors between the predicted and experimental results of the cutting force were 15.51, 13.24, and 8.3%, respectively. These errors can be attributed to the fact that the abrasive grain trajectory was enhanced by increasing spindle speed, reducing the penetration depth, and further reducing the cutting force, as shown in Fig. 6.

### Table 4 Test parameters

| Input variable       | Unit    | Values                        |
|----------------------|---------|-------------------------------|
| Spindle speed ($n$)  | r/min   | 1000, 2000, 3000, 4000, 5000  |
| Feed rate ($f_r$)    | mm/min  | 5, 10, 15, 20, 25             |
| Ultrasonic power     | %       | 20, 40, 60, 80                |

**Fig. 16** Predicted and experimental curves of cutting force versus various processing parameters: a spindle speed, b feed rate, and c ultrasonic power
The increased feed rate would increase the penetration depth, resulting in the cutting force increase. The longitudinal and torsional amplitude grow if the ultrasonic power is enhanced, strengthening the intermittent cutting effect and reducing the cutting force. Compared with the less significant effect of ultrasonic amplitude on the cutting force in Con-RUM [12], its effect during the LTC-RUM was relatively strong. In addition, to obtain the optimal processing effect, it is essential to comprehensively select the processing parameter, which will be envisaged in the follow-up study.

7 Conclusions

In this study, an analytical cutting force model was proposed for the LTC-RUM processing of zirconia ceramics. The actual LTC-RUM tests validated the presented model’s feasibility. The following conclusions can be drawn:

1. The kinematic motion of a single abrasive grain and its interaction with the processed workpiece were analyzed. The intermittent cutting effect was strengthened with increased ultrasonic frequency and ultrasonic amplitude. It was weakened with the spindle speed, being slightly affected by the feed rate.

2. It was found that LTC vibration improved the intermittent cutting effect of a single abrasive grain and could change the material removal mechanism. The cutting force variation under various processing parameters was analyzed based on the material removal mechanism of brittle fracture and experimentally validated. The cutting force negatively correlated with spindle speed and ultrasonic power (amplitude) and positively correlated with feed rate. With the variation of spindle speed, feed rate, and ultrasonic power, the maximum relative errors between the predicted and experimental results of the cutting force are 15.51, 13.24, and 8.3%, respectively.

The proposed model provided an accurate cutting force prediction in LTC-RUM of brittle material. It can be further refined for taking account of dynamic cutting forces and extended for predicting the cutting temperature, tool wear, and surface roughness in LTC-RUM.

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Author contribution Fan Chen and Wenbo Bie conceived the analysis and wrote the manuscript. Bo Zhao provided supervision on experimentation and manuscript preparation. Yingli Chang and Xiaobo Wang collected the data and revised the manuscript. Wenbo Bie, Yingli Chang, and Yuemin Zhang performed the experiment. The authors discussed each reference paper together and contributed useful ideas for this manuscript.

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Declarations

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