The Fubini-Furlan-Rossetti sum rule reexamined

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Abstract

We review the status of a Fubini-Furlan-Rossetti sum rule for the photopro-duction of pions from nucleons, indicating how well this sum rule is satisfied using a recent analysis of pion photoproduction data. Our results are dis-cussed in light of a recent paper comparing the Gerasimov-Drell-Hearn and spin-dependent polarizability sum rules.
The development of Chiral Perturbation theory (ChPT) and extended current algebra has led to a renewed interest in a number of sum rules derived in the 1960’s. Examples include the Gerasimov-Drell-Hearn\[1\] (GDH) and Weinberg\[2\] sum rules, as well as sum rules for the nucleon electric, magnetic and spin-dependent polarizabilities\[3\]. In the following, we will consider another sum rule\[4\], due to Fubini, Furlan and Rossetti (FFR), which has not attracted as much attention. We will also briefly comment on a recent paper\[5\] which compares the GDH sum rule with a related one for the nucleon spin-polarizability.

Our study of the FFR sum rule was preceded by a reexamination of the GDH sum rule in the context of extended current algebras. While the GDH sum rule was first derived from a dispersion relation (unsubtracted) and the low-energy theorem (LET) for Compton scattering, it was later obtained from the commutation relations of vector current densities. In Ref.\[5\] the extended current algebra of Chang and Liang\[6\] was shown to imply a modified GDH sum rule. (It was observed\[8\] that modified currents would lead to modified sum rules soon after the original GDH sum rule appeared.) An estimation of this modification was shown to account for an apparent discrepancy\[7\] in the original sum rule.

In their discussion of modified sum rules, the authors of Ref.\[8\] mentioned in passing that a similar procedure could be used to determine modifications to the
FFR sum rule. This sum rule relates nucleon magnetic moments to an integral over the invariant amplitude \((A_1)\) for single-pion photoproduction. The FFR sum rule has the form\[9\]

\[
g_A \left( \frac{e \kappa_{V,S}}{2M} \right) = \frac{2f_\pi}{\pi} \int \text{Im} A_1^{(+,0)}(\nu) \frac{d\nu}{\nu}
\]

where \(\kappa_{V,S}\) is the isovector (isoscalar) anomalous magnetic of the nucleon, given by \((\kappa_p \mp \kappa_n)/2\). The invariant amplitude \(A_1\) corresponds to the amplitude associated with \(\gamma_5\gamma \cdot \epsilon \gamma \cdot k\) in the paper of Chew, Goldberger, Low, and Nambu\[10\]. The required isospin combinations are given\[10\], in terms of charge-channel information, by

\[
A_1^{(+,0)} = \left( A_1 (\gamma p \rightarrow \pi^0 p) \pm A_1 (\gamma n \rightarrow \pi^0 n) \right) / 2.
\]

Here the amplitude for photoproduction of \(\pi^0 n\) states is inferred from measurements in the three other charge channels.

Empirical evaluation of the integral in Eq.(1) is (in principle) much simpler than the integral in the GDH sum rule – which involves contributions from multi-pion final states. Unfortunately, there are two problems which make a precise check more difficult. Unlike the GDH sum rule, the FFR sum rule is not exact. It requires use of the Goldberger-Treiman relation\[11\]. In addition, convergence of the associated integral is expected to be less rapid than was found in the GDH sum rule.

Regardless of the above qualifications, early attempts to evaluate the integral in Eq.(1) were encouraging. An analysis\[4\] using the \(P_{33}(1232)\) and \(D_{13}(1520)\) resonances
found good agreement for both $\kappa^V$ and $\kappa^S$. A subsequent study\[12\], using an early multipole analysis\[13\], found 85% of the prediction for $\kappa^V$ but did not present results for the isoscalar combination. In Ref.\[12\] the threshold behavior of the multipoles was modified by a factor to account for a non-zero pion mass\[14\].

This brings us to the reason for re-examining the FFR sum rule. If this sum rule is in fact valid, as the early studies suggest, it puts a constraint on the single-pion photoproduction multipoles. The integral for $\kappa^S$, in particular, involves a delicate cancellation between amplitudes, and thus provides a useful test of the single-pion production input to the GDH sum rule. Other tests of the GDH integral (including the $\pi\pi N$ contributions) have been made recently by Sandorfi et al.\[3\]. In Ref.\[3\], the multipole input to the GDH and spin-dependent polarizability sum rules was compared to predictions from ChPT\[15\]. The integrals in these sum rules involve the difference of helicity 3/2 and 1/2 total cross sections weighted by different powers of the photon energy. Though the difference of proton and neutron GDH sum rules has a serious problem\[7\], the difference of proton and neutron spin-dependent polarizabilities was found to agree with ChPT.

While such comparisons are interesting, our poor knowledge of the $\pi\pi N$ contribution is an impediment. Early estimates of the $\pi\pi N$ contributions were based upon the resonance spectrum found in analyses of $\pi N$ elastic scattering data. This ne-
glects contributions from possible ‘missing states’ which couple very weakly to the \( \pi N \) channel. (Though the FFR sum rule is not exact, we at least understand the approximation (PCAC) we are making.)

We first evaluated the integrals given in Eq. (1) using the results of a multipole analysis\[^1\] from 1993. The integral giving \( \kappa^V \) is heavily dominated by the \( P_{33}(1232) \) contribution, while the integral corresponding to \( \kappa^S \) shows sensitivity to cancellations between the low and high energy parts. The result for \( \kappa^V \) was 1.96, remarkably close to the predicted value. (Exact agreement is not expected.) The integral corresponding to \( \kappa^S \), however, gave only about 25\% of its predicted value. This discrepancy is beyond what one would expect from the use of PCAC.

The FFR and GDH integrals were subsequently re-evaluated using our most recent results from the analysis of single-pion photoproduction data. This more recent analysis was the result of a critical examination of the entire database, and resulted in a significantly improved fit\[^2\]. The analysis was also extended to 2 GeV in order to better regularize the results at our end-point energy of 1.8 GeV. The isovector components of the FFR and GDH integrals were not changed significantly by the more recent analysis. The GDH isovector-isoscalar (VS) component, however, was qualitatively different in the region just below the \( D_{13}(1520) \) resonance and also near the high energy limit (1.8 GeV) of the multipole solution. While these changes did
not alter the character of the VS result (the disagreement of this component with
the sum rule prediction remains), the isoscalar FFR integral is very sensitive to the
$D_{13}(1520)$ resonance region and was substantially different ($\kappa^S$ changed from $-0.015$
to $-0.069$). The energy dependence of the FFR integrand is displayed in Fig. 1.

In summary, the FFR sum rule for $\kappa^V$ appears to be reasonably well satisfied,
as was the case for the isovector GDH sum rule. We also see that the FFR integral
does not converge as quickly as the analogous GDH integral. The isoscalar result
is less certain. The existence of significant structure apart from the $D_{13}$ resonance
suggests that early success$^4$ with this component of the FFR sum rule was fortuitous.
While the isoscalar integral gives a result of the correct magnitude (compare the above
results with $(\kappa_p+\kappa_n)/2 \approx -0.06$), we cannot claim quantitative agreement. However,
we should note that the isoscalar component of the FFR sum rule appears to have
less problems than the VS component of the GDH sum rule. (Estimates of the VS
component disagree with the sum rule in both magnitude and sign.) This tends to
weaken arguments that require a large discrepancy in the single-pion photoproduction
multipoles in order to explain the GDH discrepancy. It would be helpful if high-quality
photoproduction measurements could be extended a further 1 GeV in order to test
the convergence of both the FFR and GDH sum rules.

If extended current algebra does indeed contribute to the FFR sum rule (as sug-
gested in Ref.[8]), the results presented here should provide a useful test for the form proposed by Chang and Liang[6]. While the isoscalar FFR sum rule would likely provide the most sensitive check on any such contribution, the phenomenological evaluation of the associated integral is not yet sufficiently stable for more than an order-of-magnitude test.

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[14] As the extrapolation $m_\pi \to 0$ is not well defined, we have used the physical photoproduction amplitudes.

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[16] These results were obtained from VPI solution FA93. This solution was also used in the study of Ref.[3].

[17] The more recent multipole solution is called SP95. The global fit to data below 1.8 GeV is substantially better (the $\chi^2$/datum is near 2.5 as compared to 3.4 for solution FA93). The current VPI solutions can be obtained from the WWW server at: http://clsaid.phys.vt.edu

[18] Both the VS component of the GDH sum rule and the FFR sum rule for $\kappa^S$ are sensitive to neutron target data. It should be noted that our multipole solutions do not agree well with the existing neutron target data above 800 MeV.
Figure captions

Figure 1. Integrand for Eq.(1) using multipole solution SP95 (see text). (a) Contribution to $\kappa^V$, (b) Contribution to $\kappa^S$. 
This figure "fig1-1.png" is available in "png" format from:

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This figure "fig1-2.png" is available in "png" format from:

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