Mass and structure of the nucleon: Gluon trace anomaly versus spontaneous symmetry breaking

Martin Schumacher

II. Physikalisches Institut der Universität Göttingen, Friedrich-Hund-Platz 1
D-37077 Göttingen, Germany

Abstract

Two different approaches to mass and structure of the nucleon are discussed in recent works, viz. (case i) the QCD lagrangian evaluated via lattice calculations and (case ii) spontaneous symmetry breaking mediated by the $\sigma$ field. These approaches are complementary in the sense that the QCD lagrangian makes use of the gluon content of the nucleon entering in terms of the gluon trace-anomaly and ignores the effects of $q\bar{q}$ vacuum polarization, whereas in spontaneous symmetry breaking masses are formed by attaching $q\bar{q}$ pairs to the valence quarks, thus giving them a definite mass which is named the constituent mass. By the same process the $q\bar{q}$ pairs of the vacuum polarization acquire mass and in this form are the elements of the quark condensate, having an up-quark and a down-quark component. A linear combination of these two components in the form $\sigma = 1/\sqrt{2}(u\bar{u} + d\bar{d})$ shows up as the $\sigma$ field. It is shown that (case i) corresponds to an unstable nucleon configuration whereas (case ii) corresponds to a stable nucleon configuration as observed in low-energy photo-nuclear and pion-nuclear reactions.

1 Introduction

In the last decades great effort has been devoted to the question how far hadrons can be described in terms of first-principle QCD and what effective forms of QCD are valid in the low-energy limit. First-principle QCD makes use of the QCD lagrangian containing the gluon trace-anomaly and the masses and wave-functions of the current quarks. Constructing a nucleon in terms of these two components leads to a nucleon mass in the form $M_N = M_0 + \sigma_{\pi N}$, where $M_0$ is the gluonic part remaining nonzero in the chiral limit and $\sigma_{\pi N}$ the $\sigma$ term which vanishes in the chiral limit. The term “chiral limit” denotes the hypothetical case where the effects of the Higgs boson are disregarded. The $\sigma$ term is known from experimental investigations and has a widely adopted value of $\sigma_{\pi N} = 45$ MeV. Then, from a nucleon mass of $M_N = 939$ MeV we arrive at $M_0 = 894$ MeV. Lattice QCD is used to confirm that this latter quantity can be traced back to the gluonic trace anomaly. The present status of these activities will be reviewed in the following sections.

The linear $\sigma$ model ($L\sigma M$) and the Nambu–Jona-Lasinio (NJL) model both are effective field theories for the mass of constituent quarks and of the $\sigma$ meson. These two effective field theories had the characteristics of a toy model as long as the $\sigma$ meson had not been discovered. This has changed completely after the $\sigma$ meson has been definitely observed as part of the constituent-quark structure in a Compton scattering experiment.
where a mass of this particle of \( m_\sigma = 600 \pm 70 \) MeV has been determined. Arguments based on the NJL model led to a mass of \( m_\sigma = 666 \) MeV \[1,2\]. This latter mass proved to be compatible with three fundamental structure constants of the nucleon, \( \text{viz.} \) the mass, the magnetic moment and the polarizability. This means that as far as the constituent-quark mass and structure and the \( \sigma \)-meson mass and structure are concerned, the NJL model is the low-energy effective field theory of first choice.

2 Status of lattice calculations based on the gluon trace anomaly

For the nucleon electroweak (EW) spontaneous symmetry breaking mediated by the Higgs boson generates only 2\% of the observed mass. The missing 98\% are explained in two different ways, \( \text{viz.} \)

- via perturbative QCD evaluated through calculation on the lattice and
- via low-energy QCD as provided by the NJL model.

In QCD the trace of the energy momentum tensor becomes\[3,4\]

\[
\theta_\mu^\mu = \frac{\beta(g)}{2g} Tr(G_{\mu\nu}G^{\mu\nu}) + \sum_{\text{flavors}} m_t \bar{\psi}_t \psi_t,
\]

where \( \beta(g) \) is the QCD \( \beta \) function. Apart from the quark mass term the energy momentum tensor has an extra term proportional to the squared gluon field tensor, which is referred to as the trace anomaly. We ignore the heavy quarks (using \( N_f = 3 \)) and work to leading order in \( \alpha_s \). Then, with the normalization \( \bar{u}u = 2M_N \) the nucleon mass becomes

\[
M_N = (2M_N)^{-1} \langle N(p) | - \frac{9\alpha_s}{4\pi} Tr(G_{\mu\nu}G^{\mu\nu}) + m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d + m_s \bar{\psi}_s \psi_s | N(P) \rangle,
\]

(2)

where the trace anomaly term survives in the chiral limit. The trace anomaly including the strange quark leads to

\[
M_0 = (2M_N)^{-1} \langle N(p) | - \frac{9\alpha_s}{4\pi} Tr(G_{\mu\nu}G^{\mu\nu}) + m_s \bar{\psi}_s \psi_s | N(P) \rangle = 894 \mp 8 \text{ MeV},
\]

(3)

and the term including the up and down quark

\[
\sigma_{\pi N} = (2M_N)^{-1} \langle N(p) | + m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d | N(P) \rangle = 45 \pm 8 \text{ MeV}.
\]

(4)

Eqs. (2) and (3) contain the effects of the strange quark which, however, may be disregarded in case of the nucleon.

The mass value given in Eq. (4) has been obtained by experiments. The mass term given in Eq. (3) is the difference between the nucleon mass and the mass given in Eq. (4). Therefore, the two errors are anticorrelated. QCD on the lattice is used to verify the mass value of Eq. (3). A problem entering into these lattice calculations is that sufficiently small lattice constants lead to too large computer times. Therefore, the lattice calculations have to be carried out at unphysically large lattice constants. Thereafter, extrapolations
to the physics point have to be made. The tool for these extrapolations is borrowed from baryon chiral perturbation theory \((B\chi PT)\). The status of these lattice calculations is described in \([5]\). In \([5]\) the nucleon mass \(M_N\) and the \(\sigma\) term are studied in the covariant baryon perturbation theory \((B\chi PT)\) up to chiral order \(p^4\). Fits have been made using \(B\chi PT\) with and without explicit \(\Delta\) degrees of freedom to combined lattice QCD (lQCD) data from various collaborations. In total 10 low-energy constants (LECs) were needed, some of which have been fitted to the lQCD data. The masses obtained through these fits are typically in the range \(M_N = 862 - 904\) and the \(\sigma\) terms in the range \(\sigma_{\pi N} = 64 - 36\) MeV. Further information is given in \([6,7]\).

The disadvantage of representing the nucleon mass in terms of the gluon trace anomaly is that this representation does not lead to a reasonable model of the nucleon. In \([3]\) it has been pointed out that a nucleon with properties predicted by the QCD lagrangian does not go well with the widely accepted constituent-quark model of the nucleon. The explanation is that constituent quarks are a property of the nucleon in the low-energy limit which may be investigated in low-energy photo-nuclear and pion-nuclear reactions. These reactions definitely show that the low-energy structure of the nucleon does not indicate any sign of a gluon content, but rather constituent quarks and mesons are observed. In case of Compton scattering the process consists of two parts, named the \(s\)-channel part and the \(t\)-channel part. The \(s\)-channel part proceeds through resonant and nonresonant excitation of the nucleon, the \(t\)-channel part through a \(\sigma\) meson located on the constituent quarks.

### 3 Illustration of the topic in terms of the mexican-hat potential

The structure of the nucleon appears different depending on the type of investigation. The parameter discriminating between the different types of observed structures is the momentum transfer \(Q\) introduced in connection with deep inelastic electron, muon and neutrino scattering experiments. Using Heisenberg’s uncertainty principle in the form \(Q\Delta x = \hbar\) we may interpret \(\Delta x = \hbar/Q\) as the spacial resolution of the experiment. The range of structures extends from asymptotic freedom \((Q \to \infty)\) to confinement \((Q = 0)\). In these two limiting cases the structures of the nucleon and the degrees of freedom are different, i.e. quarks and gluons in the \((Q \to \infty)\) case and constituent quarks and mesons in the \((Q = 0)\) case. In the \((Q \to \infty)\) case the interquark coupling constant is small \(\alpha_s(Q \to \infty) \to 0\). This has the consequence that the \(q\bar{q}\) current-quark Dirac sea is completely decoupled from the current-quarks and gluons of the valence sector. In Figure 1 this case corresponds to the center of the mexican-hat potential. In terms of the mexican-hat potential this is an unstable equilibrium corresponding to an unstable form of the nucleon structure. The appropriate theoretical approach to this modification is perturbative QCD. There is a widespread belief that the mass of the nucleon may be given by Eq. \([2]\), i.e. by a current-quark component and a gluon component. The latter component requires some explanation. We know that in case of an electron the related electric field does not contribute to the mass of the electron whereas in case of gluons it is believed that the gluon component is responsible for \(\sim 95\%\) of the nucleon.
mass. This difference between the electromagnetic and the gluon case is related to the fact that gluons carry color charges leading to a gluon-gluon interaction. This gluon-gluon interaction makes resonant gluonic states named glue-balls possible. This leads to the mass of the nucleon but not to a reasonable model of the nucleon as observed in the low-energy domain.

Up to this point we have described the structure of the nucleon as corresponding to the maximum of the mexican hat potential. Here the relevant degrees of freedom are current quarks and gluons which cannot provide a reasonable model of the nucleon as observed experimentally in the low-energy domain. This is different when we use the tools of low-energy QCD, i.e. the tools as provided by the NJL model. The NJL model provides us with a quantitative description of the physics in the minimum of the mexican hat potential (see Figure 1) and to realistic degrees of freedom in the low-energy domain. These are constituent quarks and mesons.

The minimum of the mexican hat potential is located at the vacuum expectation value of the \( \sigma \) field. This vacuum expectation value is given by the pion decay constant in the chiral limit (cl)

\[
v \equiv \langle \sigma \rangle \equiv f_\pi^{cl} = 89.8 \text{ MeV}. \tag{5}
\]

Then the mass of the \( \sigma \) particle in the chiral limit is given by

\[
m_\sigma^{cl} = 2M = \frac{4\pi}{\sqrt{3}} f_\pi^{cl} = 652 \text{ MeV}. \tag{6}
\]

The mass of the constituent quark in the chiral limit \( M \) entering into Eq. (6) is related

Figure 1: Mexican hat potential for the \( \sigma \) meson. In the center of the potential we find the current-quark gluon phase. At the expectation value of the \( \sigma \) field we find the constituent-quark meson phase.
to the pion decay constant $f^{cl}_\pi$ in the chiral limit via the following two relations [2]

\begin{align*}
M &= -\frac{8iN_c g^2}{(m^{cl}_\pi)^2} \int \frac{d^4p}{(2\pi)^4} \frac{M}{p^2 - M^2}, \\
\left. f^{cl}_\pi \right| &= -4iN_c g M \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - M^2)^2}
\end{align*}

leading to

\[ M = \frac{2\pi}{\sqrt{3}} f^{cl}_\pi. \]

From the mass of the sigma meson in the chiral limit as given Eq. (6) the mass of the $\sigma$ meson with the effects of the Higgs boson included is obtained via

\[ m_\sigma = m^{cl}_\sigma + m^0_u + m^0_d = 666 \text{ MeV}, \]

where $m^0_u = 5 \text{ MeV}$ and $m^0_d = 9 \text{ MeV}$ are the current-quark masses of the up quark and the down quark, respectively. The same result is obtained from the relation

\[ m_\sigma = \sqrt{(m^{cl}_\sigma)^2 + (m^\pi)^2} = 666 \text{ MeV}, \]

where $m^\pi$ is the average pion mass. For details see [1][2]. The constituent-quark masses with the effects of the Higgs boson included are given by

\begin{align*}
\left. m_u \right| &= \frac{1}{2} m^{cl}_\sigma + m^0_u = 331 \text{ MeV}, \\
\left. m_d \right| &= \frac{1}{2} m^{cl}_\sigma + m^0_d = 335 \text{ MeV}.
\end{align*}

The relations (12) and (13) are valid because the binding of the two constituent quarks in the $\sigma$ meson is small so that the mass of the $\sigma$ may be regarded as the sum of the masses of the two constituent quarks.

### 4 The mass and structure of the nucleon

In the foregoing section we have shown that the constituent-quark masses $m_u = 331 \text{ MeV}$ and $m_d = 335 \text{ MeV}$ are well founded values in the framework of chiral symmetry breaking. Differing from the case of the $\sigma$ meson, in case of the nucleon we have to take into account the effects of a binding energy which reduces the mass. Without this mass reduction the mass of the nucleon would be

\[ m^0_p = 997 \text{ MeV}, \quad m^0_n = 1001 \text{ MeV}. \]

This leads to a binding energy $B/A$ per constituent quark ($A=3$) in the two isospin partners $p$ and $n$ as given in the following table. Constituent quarks are the building blocks of the nucleon in a way closely resembling that of nucleons in a nucleus. A difference between the two cases is that constituent quarks in a nucleon cannot be extracted from the nucleon because of the color charges. The binding energy $B/A$ of constituent quarks
in a nucleon is a consequence of the interquark forces which are mediated by mesons as in case of nucleons in a nucleus. We, therefore expect binding energies $B/A$ per constituent quark to be of the same order of magnitude as the binding energies $B/A$ per nucleon in the light nuclei $^3\text{H}$ and $^3\text{He}$. These are are 2.83 MeV and 2.57 MeV, respectively. This means that the binding energies of constituent quarks in the nucleons are about a factor 7.5 larger than the corresponding numbers for nucleons in the light nuclei $^3\text{H}$ and $^3\text{He}$. This result is very plausible because of the smaller distances in the nucleon and because of possible residual gluonic components in the inter-quark forces. These residual gluonic components are not expected in case of nucleons in nuclei.

In a tentative attempt to understand the difference of the numbers $B/A$ in the proton and the neutron we write down the interquark Coulomb energy in the form

$$ U = \sum_{i,j,i<j} \frac{e_i e_j}{r_{ij}} \alpha_{em} \hbar. \quad (15) $$

Then with $\langle r_{ij} \rangle \approx 0.3$ fm we arrive at $U_p \approx 0$ MeV and $U_n \approx -1.6$ MeV. This tentative consideration explains the difference of the $B/A$ values for the proton and the neutron as being mainly due to a Coulomb attraction in the neutron.

## 5 Summary and conclusion

As a conclusion we may state that the NJL model together with a small contribution due to Coulomb forces quantitatively explains the mass of the nucleon at a percent level of precision. It has been shown previously [1] that this approach also leads to a quantitative explanation of the fundamental structure constants of the nucleon, viz. the magnetic moment and the electromagnetic polarizability.

This success contrasts with the current-quark-gluon approach where a model of the nucleon is applied which does not contain the experimentally observed structure of the nucleon at low energies. Furthermore, the computational tools entering into the approach provided by lattice calculations are only applicable for comparatively large lattice constants, so that extrapolations to the physical point are required. The method making this extrapolation possible is provided by $\chi$PT.

## References

[1] Martin Schumacher, Ann.Phys. (Berlin) 526, 215 (2014); arXiv: 1403.7804 [hep-ph].
[2] Martin Schumacher, Pramana, - J. Phys. 87 : 44 (2016) ; arXiv:1506.00410 [hep-ph].

[3] John F. Donoghue, Eugene Golowich, Barry R. Holstein “Dynamics of the Standard Model” Cambridge Monographs (1992).

[4] Anthony W. Thomas, Wolfram Weise, “The Structure of the Nucleon” Wiley-VCH (2000).

[5] L. Alvares-Ruso, T. Ledwig, J. Martin Camalich, and M.J. Vicente-Vacas, Phys. Rev. D 88, 054507 (2013), arXiv:1304.0483 [hep-ph].

[6] G.S. Bali, et al., Nucl. Phys. B 866 (2013) 1- 25, arXiv:1206.7034 [hep-lat].

[7] M. Procura, T.R. Hemmert, W. Weise, Phys. Rev. D 69 (2004) 034505, arXiv:hep-lat/0309020.