Performance analysis of multiuser MIMO OFDM systems incorporating feedback delay and feedback error

S Surya, M Kanthimathi and B Rajalakshmi
Department of Electronics and Communication Engineering, Sri Sairam Engineering College, Kancheepuram Dt. – 600044, Tamilnadu, India.

E-mail: surya.ece@sairam.edu.in  kanthimathiece@sairam.edu.in, rajaralakshmib.ece@sairam.edu.in

Abstract: In multiuser Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems the performance can be improved when full Channel State Information (CSI) is available at the transmitter. The characterization of downlink channel is estimated at the receiver and feedback to the transmitter. But the transmitter may acquire false information due to delay or errors in the feedback channel. The resulting channel capacity loss and Bit Error Rate (BER) performance are evaluated. The BER and the capacity of MIMO-OFDM systems are analyzed by varying the parameters of the system.

Keywords: MIMO-OFDM, Feedback delay, Feedback error, Inter Carrier Interference (ICI), capacity.

1. INTRODUCTION

The best performance in Broadcast multiuser MIMO is achieved when CSI is available at the transmitter by the use of feedback [1, 2]. The known channel properties of a channel is referred as CSI. For downlink broadcast systems, CSI is required to cancel or reduce interference during simultaneous transmission to multiple users [2]. In [3] Zero-Forcing (ZF) beamforming is used to form a beam in the desired direction. The effect of channel-dependent scheduling and a common Base Station (BS) that has knowledge of the instantaneous channel quality is considered in [4]. The notion of mean feedback has been introduced in [5] for flat-fading multiantenna channels. Adapting the transmitted signal across multiple users is an additional degree of freedom that can be leveraged in most communication systems. Clearly, as the number of users increases the level of channel knowledge required also increases. when the transmitter does not have priori channel information practical system implementations creates challenging problems [6]. It is assumed that the BS broadcasts downlink pilot symbols so that each Mobile Station (MS) estimates its own channel state vector. The MSs feedback this prediction through some CSIT feedback channel (uplink).

Most prior work assumed that the feedback channel has zero-delay on limited feedback beamforming. The effects of CSI feedback delay on BER or capacity in MIMO communication system have been discussed in [7]–[9]. In [10] the CSI source bit rate and an upper limit on the feedback throughput gain as a function of the CSI feedback delay have been derived for limited feedback beamforming systems over temporally-correlated channels. In [11] adaptive OFDM subcarrier allocation approach to minimize the transmit power is discussed. In [12], ICI due to feedback error is assumed and various performance measures have been derived.
The remaining paper is organized as follows: The system model is given in section 2. In section 3 the performance measures of a multiuser MIMO OFDM system with feedback delay and error is derived. Numerical results and the conclusions are given in section 4 and 5.

2. SYSTEM DESCRIPTION

A multiuser MIMO OFDM system consists of a base station with $N_t$ transmit antennas, which simultaneously transmit to $U$ users each equipped with $N_r$ receive antennas as in Fig. 1. Let $X_u[n]$ be the complex symbol in which $b_u[n]$ bits are transmitted on the $n^{th}$ subcarrier. It is assumed that each receiver has knowledge about its own channel only. A vector, $c[n] = [c_1[n], c_2[n], \ldots, c_{N_r}[n]]^T$, $d[n] = [d_1[n], d_2[n], \ldots, d_{N_r}[n]]^T$ denotes the $N_r \times 1$ beamforming vector and $N_t \times 1$ weighted combining vector for the $n^{th}$ subcarrier, respectively. Then the transmitted signal vector for the $n^{th}$ subcarrier is given by

$$ r_u[n] = \sum_{u=1}^{U} H_u[n] c[n] X_u[n], $$

where $H_u[n] \in \mathbb{C}^{N_r \times N_t}$ is the $u^{th}$ users’ channel matrix, $r_u[n] \in \mathbb{C}^{N_t\times1}$ is the $u^{th}$ user transmitted signal vector. Beamforming technique sends information to multiple users. Let $X_u[n]$ denote the scalar symbol for the $u^{th}$ user in the $n^{th}$ subcarrier, normalized so that $E|X_u[n]|^2 = 1$. The channel matrix of the $u^{th}$ user for $n^{th}$ subcarrier is given by

$$ H_u[n] = \begin{bmatrix} H_{u11}[n] & \cdots & H_{u1N_r}[n] \\ \vdots & \ddots & \vdots \\ H_{uN_r1}[n] & \cdots & H_{uN_rN_t}[n] \end{bmatrix}, $$

where

$$ H_u^{ij} = h_u^{ij}(1) + h_u^{ij}(2) + \ldots + h_u^{ij}(L) $$

Here, $i$ is the receive antenna index ($i = 1, 2, \ldots, N_r$), $j$ is the transmit antenna index, ($j = 1, 2, \ldots, N_t$), $u$ denotes user index ($u = 1, 2, \ldots, U$) and $L$ denotes the number of multipaths. The maximization of the received SNR is optimum by using Beamforming technique. Based on this, beamforming vector, $c[n]$,
and the weighted combining vector, \( d[n] \), for the \( n^{th} \) subcarrier is selected. The optimum beamforming vector is given in [11] as

\[ \begin{align*}
\mathbf{d}[n] &= (\mathbf{H}_u[n] \mathbf{v}_{u,\text{max}}[n])^H \\
\end{align*} \]  

(4)

where \( \mathbf{v}_{u,\text{max}} \) is the eigenvector of the maximum eigenvalue, \( \lambda_{u,\text{max}}[n] \). The received signal vector at the \( n^{th} \) subcarrier is a \( N_r \times 1 \) vector given by

\[ \mathbf{y}_u[n] = \mathbf{H}_u[n]\mathbf{c}[n]\mathbf{X}_u[n] + \mathbf{w}_u[n] \]  

(5)

where \( \mathbf{w}_u[n] \) is the complex AWGN vector. The corresponding optimum weighted combining vector is given by

\[ \mathbf{d}[n] = (\mathbf{H}_u[n] \mathbf{v}_{u,\text{max}}[n])^H \]  

(6)

Hence the combined received symbol is given by

\[ \mathbf{z}_u[n] = (\mathbf{H}_u[n]\mathbf{v}_{u,\text{max}}[n])^H (\mathbf{H}_u[n]\mathbf{c}[n]\mathbf{X}_u[n] + \mathbf{w}_u[n]). \]  

(7)

The SNR of the \( n^{th} \) subcarrier is given by

\[ \text{SNR}[n] = \frac{E_s}{N_0} \mathbf{c}[n]^H \mathbf{H}_u[n] \mathbf{c}[n] \]  

(8)

where \( E_s \) stands for expectation. Here, \( E_s/N_0 \) is equal to the transmit SNR. Therefore, the received SNR is maximized by maximizing the quantity, \( \mathbf{c}[n]^H \mathbf{H}_u[n] \mathbf{c}[n] \). Let the eigen-decomposition of the be

\[ \mathbf{H}_u[n]^H \mathbf{H}_u[n] = \mathbf{V}_u[n] \Lambda_u[n] \mathbf{V}_u[n]^H \]  

(9)

where \( \Lambda_u[n] \) is a diagonal matrix whose diagonal elements are eigenvalues of \( \mathbf{H}_u[n]^H \mathbf{H}_u[n] \), and \( \mathbf{V}_u[n] \) is a unitary matrix whose columns are the eigenvectors of \( \mathbf{H}_u[n]^H \mathbf{H}_u[n] \). Then the optimal beamforming vector is chosen from (4). The maximum received SNR of the \( n^{th} \) subcarrier is given by [11]

\[ \text{SNR}[n] = \frac{E_s \lambda_{u,\text{max}}[n]}{N_0} \]  

(10)

where \( N_0 \) is the single sided noise spectral density, and \( E_s \) is the energy per symbol.

3. PERFORMANCE EVALUATION

**BER Analysis of MU MIMO OFDM with feedback error:** The receiver estimates the downlink channel and feedback one of the predefined channel characterization code words indices to the transmitter. In practice, the feedback channel will introduce errors in the transmission of the indices and the resulting channel capacity loss and error probability are evaluated. The effect of feedback channel errors is the increased level of ICI in OFDM systems. OFDM is sensitive to Carrier Frequency Offset (CFO) and phase noise. The CFO is caused by misalignment in carrier frequencies or Doppler shift [13].

The analysis of the MU MIMO-OFDM system presented in (7) is now extended for the nonzero FO case which may vary from symbol to symbol but it is same for all subcarriers of an OFDM symbol. In the MU MIMO OFDM system, the received symbol at the \( n^{th} \) subcarrier is:

\[ \mathbf{z}_u[n] = \mathbf{H}_u[n]\mathbf{v}_{u,\text{max}}[n]^H \mathbf{H}_u[n]\mathbf{c}[n]\mathbf{X}_u[n] + \sum_{p=0, p \neq n}^{N_s-1} S_{t,r}(p-n)\mathbf{H}_u[p]\mathbf{c}[n]\mathbf{X}_u[p]\mathbf{v}_{u,\text{max}}[n]^H + \mathbf{v}_{u,\text{max}}[n]^H \mathbf{w}_u[n] \]  

(11)

Let \( I_d(n) \) denote ICI from the other subcarriers to the received \( n^{th} \) subcarrier. Let the NFO be \( \epsilon_{t,r} \) from Tx antenna \( t \) and Rx antenna \( r \). The ICI term, \( I_d(n) \), at the \( n^{th} \) subcarrier be \( I_{t,r}(n) \) caused by the \( t^{th} \) transmit antenna be
\[ I_u[n] = \sum_{t=1}^{N_t} I_{t,r}(n), \]  
\[ \text{where} \]
\[ I_{t,r}(n) = \sum_{p=0, p \neq n}^{N_c-1} S_{t,r}(p-n) H_{u}[p] c[n] X_u[p] v_{u,\text{max}}[n] H^H \]  
and the coefficients of \( S(n) \) in (13) are expressed in [13] as
\[ S(n) = \frac{\sin[\pi(n + \varepsilon_{t,r})]}{N_c \sin[\pi n / N_c(n + \varepsilon_{t,r})]} \exp\left[j\pi \left(1 - \frac{1}{N_c}\right)(n + \varepsilon_{t,r})\right]. \]

Equation (11) can now be simplified as [12]
\[ Z_u[n] = (H_u[n] v_{u,\text{max}}[n])^H (H_u[n] c[n] X_u[n] S(0) + I_u[n] + \tilde{W}_u[n]). \]

The equivalent noise at each received subcarrier is given as
\[ \tilde{Z}[n] = I[n] + \tilde{W}[n] \]  
where \( I_u[n] \) is ICI and \( W_u[n] \) is thermal noise.

The variance, of \( I_u[n] \) is given in [12] as
\[ \sigma^2_{t,r} = 1 = S(0) \]  
where
\[ S(0) = [\text{sinc}(\epsilon_0)]^2, \]  
where \( \epsilon_0 \) is the normalized CFO.

The maximum received SNR of \( n^{th} \) subcarrier is given as
\[ \text{SNR}[n] = \frac{E_{ul\text{max}} n S(0)}{N_0 + (1 - S(0))}. \]

The important criterion to evaluate the performance of communication systems is the Bit Error Rate (BER). For OFDM communication systems, the main sources affecting its BER performance are AWGN and ICI. A theoretical BER derivation needs the distribution function of the ICI. But it is normally unknown. In the following discussion, \( \varepsilon \) in the BER expression represents the CFO. The CFO can be considered to be uniformly distributed [14]. Then, with the assumption that \( \varepsilon \) is uniformly distributed over the region \([a, b]\), the BER can be written as
\[ p_e(\varepsilon|M) = \int_a^b \frac{\sqrt{M - 1}}{\sqrt{M \log_2 M}} \text{erfc} \left( \frac{3 \log_2 M y S(0)}{2(M - 1)} \right) \]  
\[ + \frac{\sqrt{M - 2}}{\sqrt{M \log_2 M}} \text{erfc} \left( \frac{3 \log_2 M y S(0)}{2(M - 1)} \right) \]  
f(\varepsilon) \, d\varepsilon, \]  
where \( y \) is the average SNR given in (19).

**Capacity with Feedback Delay:** When the CSI is not available at transmit side then the power is equally distributed to all channels to attain a capacity of MIMO system. When the CSI is available, the eigenvalues correspond to the gain of the channels is obtained by Singular Value Decomposition (SVD) technique. Ergodic capacity is calculated by averaging over the PDF of instantaneous SNR of the channel. The capacity is given by
\[ C = \int_0^\infty \log (1 + \rho \lambda) f(\lambda) \, d\lambda. \]  
Consider the subchannel fading, i.e., \( H_u[n] \) as Rayleigh flat-fading and \( |H_u[n]|^2 \) for each user in each subchannel is a Chi-square distributed random variable with \( N_t N_r \) degrees of freedom, and the capacity with feedback delay is given by
where \( \lambda_{a,f,max} [n] = \left| \rho_k \right|^2 + N_t (1 - \left| \rho_k \right|^2) \) and \( \Gamma(z) = \int_0^\infty e^{-t} dt \) is the gamma function.

The closed form expression may not exist for the integral in (22). Hence, numerical integration has been performed. The weight vectors at both sides are not updated when there is a delay in the feedback, so that the channels orthogonality may get affected as the channel changes in time when the weight vectors are applied.

**Capacity with Feedback error:** The effect of feedback error is the increased level of ICI. OFDM systems are more sensitive to frequency synchronization errors [12]. The capacity of MU MIMO OFDM with feedback error is given as

\[
C = \frac{1}{U} \sum_{u=1}^{U} \log_2 \left( 1 + \frac{\gamma_u}{U N_t N_c} \right) \sum_{k=0}^{m-1} \frac{k!}{(k+n-m)!} \left[ L_n^m (\lambda) \right]^2 \lambda^{n-m} e^{-\lambda} d\lambda, \tag{23}
\]

where \( \gamma_u = \frac{E_s A_{u,max} [n] S(O)}{N_s + (1 - S(O))} \) and \( L_k^n (x) = \frac{1}{k!} e^x \frac{d^k}{dx^k} (e^{-x} x^n) \) is the associated Laguerre polynomial of order \( k \).

The impact of ICI is negligible when the FO is smaller than thermal noise, when the FO is large, with increased signal power, ICI causes performance loss.

4. **NUMERICAL RESULTS**

Figure 2(a) shows the BER of a closed loop MU MIMO OFDM system with error in feedback for different numbers of transmit and receive antennas. The graphs are obtained using the expression (20). As the number of transmit and receive antennas increases, the BER decreases. Figure 2(b) shows BER graphs for different numbers of subcarriers with feedback error. Performance can be improved by using a large number of subcarriers.
Figure 2. BER with feedback error for a) different number of antenna configurations, and b) different number of subcarriers.

Figure 3(a) shows BER graphs for different noise variances with feedback error. When the noise power increases, the BER increases. Figure 3(b) depicts the BER graphs for different modulation orders in a closed loop MU MIMO OFDM with feedback error. As shown in Fig. 3(a), an additional 4–5 dB of SNR is required to transmit an extra bit per dimension to maintain an average BER.

Figure 4(a) shows the capacity versus gain curves for a Rayleigh fading channel with feedback delay. The capacity decreases when the number of users increases. Figure 4(b) shows the capacity with feedback delay for different number of antennas. It is concluded that the increased number of antennas, the capacity increases and finally the system performance.
Figure 3. BER with feedback error for a) different modulation order, and b) different noise variances.

Figure 4. Capacity with feedback delay for (a) different number of users and (b) different antenna configurations.

Figure 5(a) and 5(b) shows the capacity of a MU MIMO OFDM system with ICI due to error in the feedback channel for different users and antennas. From Figure 5, it is observed that the transmit power and the number of users has significant effect on the capacity. Figure 6 shows the effect of feedback error on the capacity of MIMO OFDM system. As the FO increases, ICI also increases, and capacity decreases.
5. CONCLUSIONS

In this paper, it is discussed that how the performance of MU MIMO OFDM degrades with feedback delay and feedback error. The effect of feedback delay has been analyzed and the numerical results for various system specifications have been plotted. It is analyzed that the system capacity degrades when

Figure 5. Capacity with ICI due to feedback error for (a) different number of users and (b) different antenna configurations.

Figure 6. Capacity with feedback error for various FOs of a MU MIMO OFDM system.
the feedback is not updated properly. The ICI due to error in the feedback has been assumed. The BER performance and capacity, with feedback error has been analyzed and plotted for various system parameters. Analytical results show that the performance of a system degrades with feedback delay and feedback error.

REFERENCES

[1] Rappaport, T. S., *Wireless Communications, Principles and Practice*, 2nd edition Prentice Hall Inc., NJ, 2001.

[2] Duman, T. M., and Ghrayeb, A., *Coding for MIMO Communication Systems*, John Wiley & Sons, Ltd., The Atrium, Southern Gate, Chichester, England, 2007.

[3] Volker Kuhn, V., *Wireless Communications over MIMO Channels Applications to CDMA and Multiple antenna Systems*, John Wiley & Sons, Ltd., The Atrium, Southern Gate, Chichester, England, 2006.

[4] Prasad, R., *OFDM for Wireless Communications Systems*, Artech House, Inc., Boston, London, 2004.

[5] Xia, P., Zhou, S., and Giannakis, G. B., “Adaptive MIMO OFDM based on Partial Channel State Information,” *IEEE Transactions on Signal Processing*, vol. 52, no. 1, pp. 202-213, Jan. 2004.

[6] Love, D. J., Heath, R.W., Lau, V. K. N., Gesbert, D., Rao, B. D., Andrews, M., “An overview of limited feedback in wireless communication systems,” *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1341 – 1365, Oct. 2008.

[7] Au, E., Jin, S., McKay, M. R., Mow, M. R., Gao, X., and Collings, I. B., “Analytical performance of MIMO-SVD systems in Ricean fading channels with channel estimation error and feedback delay,” *IEEE Transactions on Communications*, vol. 7, no. 4, pp. 1315 – 1325, April 2008.

[8] Du, J., Li, Y., Gu, D., Molisch, A. F., and Zhang, J., “Estimation of performance loss due to delay in channel feedback in MIMO systems,” *Proceedings of IEEE Vehicular Technology Conference*, Atlanta, USA, vol. 3, pp. 1619–1622, Sep. 2004.

[9] Kobayashi, K., Ohtsuki, T., and Kaneko, T., “MIMO systems in the presence of feedback delay,” *Proceedings of IEEE International Conference on Communications*, Istanbul, Turkey, vol. 9, pp. 4102–4106, June 2006.

[10] Huang, K., Heath, R.W., Andrews, J.G., “Limited Feedback Beamforming over Temporally Correlated Channels,” *IEEE Transactions on Signal Processing*, vol. 57, no. 5, pp. 1959 – 1975, May 2009.

[11] Hu, Z., Zhu, G., Xia, Y., Liu, G., “Multiuser subcarrier and bit allocation for MIMO OFDM systems with perfect and partial channel information,” *IEEE Wireless Communications and Networking Conference*, Atlanta, NY, vol. 2, pp. 1188-1193, March 2004.

[12] Dao, N. D., Tellambura, C., “Intercarrier Interference Self-Cancellation Space-Frequency Codes for MIMO-OFDM,” *IEEE Transactions on Vehicular Technology*, vol. 54, no. 5, pp. 1729-1738, Sep. 2005.

[13] Sathananthan, K., Tellambura, C., “Probability of Error Calculation of OFDM Systems with Frequency Offset,” *IEEE Transactions on Communications*, vol. 49, no. 11, pp. 1884-1888, Nov. 2001.

[14] Zhao, Y., Haggman, S. G., “BER Analysis of OFDM Communication Systems with Intercarrier Interference,” *International Conference on Communication Technology*, Beijing, China, vol. 2, pp. 1-5, Oct. 1998.