Spin-charge separation and Kondo effect in an open quantum dot

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We study a quantum dot connected to the bulk by single-mode junctions at almost perfect conductance. Although the average charge $e\langle N \rangle$ of the dot is not discrete, its spin remains quantized: $s = 1/2$ or $s = 0$, depending (periodically) on the gate voltage. This drastic difference from the conventional mixed-valence regime stems from the existence of a broad-band, dense spectrum of discrete levels in the dot. In the doublet state, the Kondo effect develops at low temperatures. We find the Kondo temperature $T_K$, and the conductance at $T \lesssim T_K$.

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The Kondo effect is one of the most studied and best understood problems of many-body physics. Initially, the theory was developed to explain the increase of resistivity of a bulk metal with magnetic impurities at low temperatures [1]. Soon it was realized that Kondo’s mechanism works not only for electron scattering, but also for tunneling through barriers with magnetic impurities [2]. A non-perturbative theory of the Kondo effect has predicted that the cross-section of scattering off a magnetic impurity in the bulk reaches the unitary limit at zero temperature [3]. Similarly, the tunneling cross-section should approach the unitary limit at low temperature and bias [3] in the Kondo regime.

The Kondo problem can be discussed in the framework of Anderson’s impurity model [4]. The three parameters defining this model are: the on-site electron repulsion energy $U$, the one-electron on-site energy $\varepsilon_0$, and the level width $\Gamma$ formed by hybridization of the discrete level with the states in the bulk. The non-trivial behavior of the conductance occurs when the level is singly occupied and the temperature $T$ is below the Kondo temperature $T_K \simeq (U \Gamma)^{1/2} \exp\{\pi\varepsilon_0(\varepsilon_0 + U)/2U\}$, where $\varepsilon_0 < 0$ is measured from the Fermi level [1].

It is hard to vary these parameters for a magnetic impurity embedded in a host material. One has much more control over a quantum dot attached to leads by two adjustable junctions. Here, the role of the on-site repulsion $U$ is played by the charging energy $E_C = e^2/C$, where $C$ is the capacitance of the dot. The energy $\varepsilon_0$ can be tuned by varying the voltage on a gate which is capacitively coupled to the dot. In the interval $|N - (2n + 1)| < 1/2$ of the dimensionless gate voltage $N$, the energy $\varepsilon_0 = E_C[(2n + 1) - N - 1/2] < 0$, and the number of electrons $2n + 1$ on the dot is an odd integer. The level width is proportional to the sum of conductances $G = G_L + G_R$ of the left (L) and right (R) dot-lead junctions, and can be estimated as $\Gamma = (hG/8\pi^2e^2)\Delta$, where $\Delta$ is the discrete energy level spacing in the dot.

The experimental search for a tunable Kondo effect brought positive results [5] only recently. In retrospect it is clear, why such experiments were hard to perform. In the conventional Kondo regime, the number of electrons on the dot must be an odd integer. However, the number of electrons is quantized only if the conductance is small, $G \ll e^2/h$, and the gate voltage $N$ is away from half-integer values (see, e.g., [4]). Thus, in the case of a quantum dot, the magnitude of the negative exponent in the above formula for $T_K$ can be estimated as $|\pi\varepsilon_0(\varepsilon_0 + U)/2U| \sim (E_C/\Delta)(e^2/hG)$. Unlike an atom, a quantum dot has a non-degenerate, dense set of discrete levels, $\Delta \ll E_C$. Therefore, the negative exponent contains a product of two large parameters, $E_C/\Delta$ and $e^2/hG$. To bring $T_K$ within the reach of a modern low-temperature experiment, one may try smaller quantum dots in order to decrease $E_C/\Delta$: this route obviously has technological limitations. Another, complementary option is to increase the junction conductances, so that $G_{L,R}$ come close to $2e^2/h$. (This is the maximal conductance of a single-mode quantum point contact used [6] to couple the dot and the two-dimensional electron gas in a semiconductor heterostructure). However, at such values of $G_{L,R}$ the discreteness of the number of electrons on the dot is almost completely washed out [7]. Exercising this option, hence, raises a question about the nature of the Kondo effect in the absence of charge quantization. It is the main question we address in this Letter.

Below we will show that the spin of a quantum dot may remain quantized even if charge quantization is destroyed and the average charge $e\langle N \rangle$ is not integer. Spin-charge separation is possible because charge and spin excitations of the dot are controlled by two very different energies: $E_C$ and $\Delta$, respectively. The charge varies linearly with the gate voltage, $e\langle N \rangle \simeq eN$, if at least one of the junctions is almost in the reflectionless regime, $|r_{L,R}| \ll 1$, and its conductance $G_{L,R} \equiv (2e^2/h)(1 - |r_{L,R}|^2)$ is close to the conductance quantum. We will show that the spin quantization is preserved if the reflection amplitudes $r_{L,R}$ of the junctions satisfy the condition $|r_L|^2|r_R|^2 \gtrsim \Delta/E_C$. 


These two constraints on $r_{L,R}$ needed for spin-charge separation are clearly comparable at $\Delta/E_C \ll 1$.

Under the condition of spin-charge separation, the spin state of the dot remains singlet or doublet, depending on $eN$. If $\cos \pi N < 0$, the spin state is doublet, and the Kondo effect develops at low temperatures $T \lesssim T_K$. The Kondo temperature we find is

$$T_K \simeq \Delta \sqrt{\frac{\Delta}{T_0(N)}} \exp \left\{ -\frac{T_0(N)}{\Delta} \right\}; \quad (1a)$$

$$T_0(N) = \alpha E_C |r_L|^2 |r_R|^2 \cos^2 \pi N. \quad (1b)$$

Here $\alpha > 0$ is a numerical factor. Eqs. (1a) and (1b) demonstrate that in the case of weak backscattering in the junctions, the large parameter $E_C/\Delta$ in the Kondo temperature exponent may be compensated by a small factor $\propto |r_L|^2 |r_R|^2$. This compensation, resulting from quantum charge fluctuations in a dot with a dense spectrum of discrete states, leads to an enhancement of the Kondo temperature compared with the prediction for the conventional constant-interaction model to a one-mode channel connecting the dot with the bulk, and $D$ is the energy bandwidth for 1D fermions. The interaction term is $H_C = (E_C/2)[2\theta_{\rho}(0)/\sqrt{\pi} - N]^2$. The canonically conjugated Bose fields satisfy the commutation relations $[\phi_{\gamma'}(x'), \theta_{\gamma}(x)] = i(2/\sqrt{\pi})\delta_{\gamma,\gamma'}$, where $\gamma, \gamma' = \rho, s$. The operators $(2\sqrt{\pi} \nu \theta_{\rho}(x))/(2\sqrt{\pi})\nabla \theta_{\rho}(x)$ and $(2\sqrt{\pi} \nu \theta_{s}(x))/(2\sqrt{\pi})\nabla \theta_{s}(x)$ are the smooth parts of the electron charge ($\rho$) and spin ($s$) densities, respectively. The continuum of those electron states outside the dot, which are capable to pass through the junction, is mapped onto the Bose fields defined on the half-axis $[0, \infty)$. Similarly, states within a finite-size dot are mapped onto the fields defined on the interval $[-L, 0]$ with the boundary condition $\theta_{\rho,s}(-L) = 0$, which corresponds to $|r_L| = 1$. The length in this effective 1D problem is related to the average density of states $\nu_d \equiv 1/\Delta$ in the dot by $L \simeq \pi v_F \nu_d$, and scales proportionally to the area $A$ of the dot formed in a two-dimensional electron gas.

To the leading order in the reflection amplitude $|r_R| \ll 1$ and in the level spacing $\Delta/E_C \ll 1$, the average charge of the dot can be found by minimization of the energy $H_C$. This charge is not quantized, and, to this order, it varies linearly with the gate voltage, $(2\pi E_C)/(2\pi \nu \theta_{\rho}(0)) = eN$. Within the same approximation, the factor $\cos[2\sqrt{\pi} \nu \theta_{\rho}(0)]$ in Eq. (11) at low energies $E \ll E_C$ may be replaced by its average value. This procedure yields the effective Hamiltonian $\hat{H}_s = \hat{H}^s + \hat{H}^R$ for the spin mode,

$$\hat{H}^s = \frac{v_F}{2} \int_{-L}^{\infty} dx \left[ \frac{1}{2} (\nabla \phi_{\rho})^2 + 2(\nabla \theta_{\rho})^2 \right], \quad (5a)$$

$$\hat{H}^R = -\left[ \frac{4E_C}{\sqrt{\pi}} \nu D\right]^{1/2} |r_R| \cos(\pi N) \cos[2\sqrt{\pi} \nu \theta_{s}(0)]. \quad (5b)$$

This is a Hamiltonian of a one-mode, $g = 1/2$ Luttinger liquid with a barrier at $x = 0$. At $L \to \infty$ (i.e., at $E \gg \Delta$) the backscattering at the barrier, described by the Hamiltonian $\hat{H}^R$, is known to be a relevant perturbation $[12]$: even if $|r_R|$ is small, at low energy $E \to 0$ the amplitudes of transitions between the minima of the potential of (5a) scale to zero. These minima are $\theta_{\rho}(0) = \sqrt{\pi} n$ if $\cos \pi N > 0$, or $\theta_{x}(0) = \sqrt{\pi}(n + 1/2)$, if $\cos \pi N < 0$. The crossover from weak backscattering $|r_R(E)| \ll 1$ to weak tunneling $|r_R(E)| \ll 1$ occurs at $E \sim T_0(N)$, Eq. (3). To describe the low-energy $[E \lesssim T_0(N)]$ dynamics of the spin mode, it is convenient to project out all the states of the Luttinger liquid that are not pinned to the minima of the potential (5a). Transitions between various pinned states then are described by the tunnel Hamiltonian $\hat{H}_0 = \hat{H}_0 + \hat{H}_{xy} + \hat{H}_z$, where

$$\hat{H}_{xy} = -\frac{D^2}{2\pi T_0(N)} \cos \left\{ \sqrt{\pi} [\phi_{\rho}(+0) - \phi_{\rho}(-0)] \right\}, \quad (6a)$$

$$\hat{H}_z = \frac{v_F^2}{2 T_0(N)} \nabla \theta_{s}(-0) \nabla \theta_{s}(+0). \quad (6b)$$
Here a discontinuity of the variable $\phi_{\sigma}(x)$ at $x = 0$ is allowed, and the point $x = 0$ is excluded from the region of integration in Eq. (6a). The term $H_{xy}$, which is a sum of two operators of finite shifts for the field $\theta_{\sigma}(0)$, represents hops $\theta_{\sigma}(0) \rightarrow \theta_{\sigma}(0) \pm \sqrt{\pi}$ between pinned states. This term is familiar from the theory of DC transport in a Luttinger liquid [12]. However, the usual scaling argument [2] is insufficient for deriving the term (6b) and for establishing the exact coefficients in (6a) and (6b). We have accomplished these tasks by matching the current-current correlation function $\langle [I_x(t), I_x(0)] \rangle$ calculated from (6) with the proper asymptote of the exact result which we obtained starting with Eqs. (4) and proceeding along the lines of Ref. [4].

At $L \rightarrow \infty$ the ground state of the spin mode is infinitely degenerate, different states may be labeled by the discrete boundary values $\theta_{\sigma}(0)$. At finite $L$, however, this degeneracy is lifted due to the energy of spatial quantization, coming from the Hamiltonian (6a). If $\cos \pi \Delta < 0$, the spatial quantization entirely removes the degeneracy, and the lowest energy corresponds to $\theta_{\sigma}(0) = 0$ (spin state of the dot is $s = 0$). If $\cos \pi \Delta > 0$, the spatial quantization by itself, in the absence of tunneling, would leave the ground state doubly degenerate, $\theta_{\sigma}(0) = \pm \sqrt{\pi}/2$ (spin state of the dot is $s = 1/2$). Hamiltonian (6a) hybridizes the spin of the dot with the continuum of spin excitations in the lead. The Kondo effect consists essentially of this hybridization, which ultimately leads to the formation of a spin singlet in the entire system. The energy scale at which the hybridization occurs, is the Kondo temperature of the problem at hand.

To find $T_K$, it is convenient to return, following Haldane [4], from the boson fields at $x < 0$ and $x > 0$ to the fermion operators $\hat{\chi}_{\sigma}$, $\hat{\psi}_{\sigma}$, of the dot and lead respectively. The two parts of the Hamiltonian, (6a) and (6b), correspond, respectively, to the in-plane and Ising parts of the exchange interaction $\hat{H}_{ex}$, 

$$\hat{H}_{ex} = J_{RR} \hat{S}_R \hat{S}_d, \quad J_{RR} = 1/2T_0(N)\rho_d\rho_R. \quad (7)$$

Here $\hat{S}_R = \hat{\psi}_{\sigma_1}^\dagger (R_R) \sigma_{\sigma_1\sigma_2} \hat{\chi}_{\sigma_2}(R_R)$ and $\hat{S}_d = \hat{\chi}_{\sigma_1}^\dagger (R_R) \sigma_{\sigma_1\sigma_2} \hat{\psi}_{\sigma_2}(R_R)$ are the operators of spin density in the dot and in the lead, respectively, at the point $R_R$ of their contact; $\rho_d \equiv \nu_d/a$ and $\rho_R$ are the average densities of states in the dot and lead respectively. One can explicitly check that the initial $SU(2)$ symmetry of the problem is preserved, and the exchange interaction is isotropic. At low energies, $E \ll T_0(N)$, the dot can be considered as being completely detached from the lead, apart from the exchange interaction. Hence in this energy domain the spectrum of the dot is discrete with the smallest excitation energy $\sim \Delta$. At energy scales below $\Delta$ the system we consider is equivalent to the standard Kondo model with exchange constant $J_{RR} A$ and bandwidth $\Delta$. It allows us to use the known [12] result for the Kondo temperature, $T_K \simeq \Delta(2J_{RR} A)^{-1/2} \exp(-1/2J_{RR} A \rho_R)$, which leads to Eq. (4a), with energy $T_0(N)$ given by Eq. (3).

When deriving the Hamiltonian (6b), we have neglected the effects of spatial quantization coming from finite $L$. This is justified as long as $T_0(N) \gg \Delta$. The same condition ensures the smallness of $T_K$ compared to $\Delta$, and makes the singlet Kondo polaron at $\cos \pi \Delta < 0$ distinguishable from a trivial singlet state formed within the dot at $\cos \pi \Delta > 0$.

The most interesting manifestation of the Kondo effect is the enhanced conductance through a dot with two junctions. To consider the low-temperature conductance, we derive a Hamiltonian that generalizes Eq. (6) to the case of two junctions:

$$\hat{H}_{ex} = \left[ J_{LL} \hat{\psi}_{\sigma_1}^\dagger (R_L) \hat{\chi}_{\sigma_1}^\dagger (R_L) \hat{\chi}_{\sigma_1}(R_L) \hat{\psi}_{\sigma_2}(R_L) + J_{RR} \hat{\psi}_{\sigma_1}^\dagger (R_R) \hat{\chi}_{\sigma_1}^\dagger (R_R) \hat{\chi}_{\sigma_1}(R_R) \hat{\psi}_{\sigma_2}(R_R) + J_{LR} \hat{\psi}_{\sigma_1}^\dagger (R_L) \hat{\chi}_{\sigma_1}^\dagger (R_L) \hat{\chi}_{\sigma_2}(R_L) \hat{\psi}_{\sigma_2}(R_L) \right] \times \sigma_{\sigma_1\sigma_2} \sigma_{\sigma_3\sigma_4}. \quad (8)$$

The derivation of the low-energy theory goes through stages similar to Eqs. (4) and (5). We will explain first how to derive the relevant exchange constants in the least involved case of a strongly asymmetric set-up: $G_L \ll e^2/h$ and $|r_L| \ll 1$. In this case the largest constant $J_{RR} \propto G_L^2$ exists even in the limit $G_L = 0$, and is defined by Eq. (4b); the smallest constant, $J_{LL} \propto G_L^2$, is unimportant in the calculation of the conductance; the intermediate constant $J_{LR}$ is proportional to $G_L$. To find the proportionality coefficient, we calculate the conductance through the dot in the lowest-order perturbation theory in the Hamiltonian (6b), and obtain $G(T) = (\pi^4 e^2/3h) |J_{LR}|^2 |\rho_L \rho_R|^2 T^2$. When deriving this formula, we set also $T \gg \Delta$, which allows us now to compare $G(T)$ with the exact at $T = 0$ result [4] for the conductance of the same system. The comparison yields:

$$J_{LR}^2 = 4(h/e^2) G_L \left[ \pi e C T_0(N) \rho_L \rho_R \rho_R^2 \right]^{-1}. \quad (9)$$

At $T \lesssim \Delta$, only the lowest discrete level in the dot remains important. If the gate voltage is close to an odd integer, $\cos \pi \Delta < 0$, the level is spin-degenerate. This way, the initial problem of the dot, which has a dense spectrum of discrete levels, and is strongly coupled to the leads, is reduced to the problem of a single-level Kondo impurity in a tunnel junction [4]. Using the found values of the exchange constants, and the result (4) for a strongly asymmetric junction ($J_{LL} \ll J_{LR} \ll J_{RR}$), we obtain the conductance in the problem under consideration:

$$G_K(T/T_K, N) = (e^2/h)(J_{LR}/J_{RR})^2 T(T/T_K) \simeq (64/\pi^2) G_L |r_L|^2 (\cos \pi \Delta)^2 F(T/T_K). \quad (10)$$

Note that Kondo conductance (4) in the strongly asymmetric set-up is significantly smaller than the conductance quantum $e^2/h$ even at $T = 0$. The maximal
value of $G_K$ is substantially increased, if the asymmetry between the junctions is reduced, and the condition $G_L \ll e^2/h$ is lifted. To show this, we further generalize the above results to include the experimentally important case $|r_R| \ll |r_L| \ll 1$. Like in the case of a single strong junction considered above, the backscattering in the junctions becomes increasingly effective at low electron energies. Initially, at energies below $E_C$, the reflection amplitudes grow independently of each other [13] as $|r_{LR}(E)| \sim |r_{LR}|(E_C/E)^{1/4}$. Upon reducing the energy scale, the weaker junction reaches the cross-over region first: at $E \sim T_1 \equiv E_C|r_L|^4$ the backscattering in this junction becomes significant, $|r_L(E)| \sim 1$.

To consider conductance at temperatures $T \ll T_1$, we can formulate now an effective Hamiltonian, which acts within the narrow energy band $T_1$, and describes weak reflection in the right junction, $|r_R(T_1)| \sim |r_R/r_L|$, and strong reflection in the left junction, $|r_L(T_1)| \sim 1$. Both junctions eventually cross over into the weak tunneling regime at sufficiently low temperatures. Replacing $E_C$ by the bandwidth $T_1$ and $|r_R|$ by $|r_R/r_L|$ in Eq. (3), we find Eq. (13) for the new crossover temperature. Below it, the exchange Hamiltonian (3) is applicable. The largest exchange constant $J_{RR}$ is independent of $|r_L|$ in the leading approximation; it is still defined by Eq. (3) with $T_0(N)$ from Eq. (1). To find the new value of $J_{LR}$, we replace $E_C \to T_1$, $G_L \to (e^2/h)(1-|r_L(T_1)|^2) \sim e^2/h$, and use Eq. (14) for $T_0(N)$ in Eq. (3): the result is $J_{LR}^2 \sim |E_C| |r_L|^6 |r_R|^2 |\rho_L \rho_R|^2 |\Delta|^2$. Substituting the exchange constants $J_{RR}$ and $J_{LR}$ in Eq. (14), we arrive at Eq. (2).

FIG. 1. The overall temperature dependence of conductance. The estimates of the crossover temperatures and the two characteristic values of the conductance, $G_K \equiv G_K(0, N')$ and $G_d$, are given in the text.

We finally discuss the overall temperature dependence of the conductance, see Fig. 1. In this discussion, we use the above results for the Kondo regime, and the results of Refs. [13,14] for co-tunneling, generalized properly onto the case $|r_R| \ll |r_L| \ll 1$. The conductance decreases slowly [13], as the temperature is reduced from $E_C$ to $T_1$. At lower temperature, the leading mechanism of transport is inelastic co-tunneling, which yields $G \sim T/T_0$ and $G \sim T^2/T_0 T_0(N)$ at $T$ above and below $T_0(N)$, respectively. At yet lower temperature, the main contribution to the conductance $G(T)$ is provided by elastic co-tunneling, $G_d \sim (\Delta/T_1) \ln(T_1/\Delta)$. The crossover between the two co-tunneling mechanisms occurs at $T^* \sim \sqrt{T_0(N) \Delta \ln(T_1/\Delta)}$. It is instructive to compare $G_d$ with the zero-temperature Kondo conductance [6]. Taking into account the definition of $T_1$, we see that the Kondo mechanism dominates, if $T_0(N)/\Delta \gtrsim \ln(E_C|r_L|^4/\Delta)$. This condition simultaneously ensures the smallness of the Kondo temperature compared to the level spacing, so that the Kondo singlet state remains distinct.

In conclusion, we found that the spin of a quantum dot may remain quantized, even if the quantization of charge is destroyed by strong dot-dot tunneling. In the spin-doublet state, the Kondo effect develops at low temperature, yielding a non-monotonous temperature dependence of the conductance. We found that the Kondo temperature is significantly enhanced by charge fluctuations, compared to the standard case of weak dot-dot tunneling.

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