Nuclear spintronics

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Abstract

The electron spin transport in condensed matter, Spintronics, is a subject of rapidly growing interest both scientifically and from the point of view of applications to modern and future electronics. In many cases the electron spin transport cannot be described adequately without accounting for the hyperfine interaction between electron and nuclear spins. Here, the progress in physics and applications of these phenomena will be reviewed.

1 Introduction

Hyperfine interactions between the nuclear and electron spins play a crucial role in a large variety of physical phenomena in normal and superconducting metals [1, 2], bulk semiconductors [3] and in quantum Hall and mesoscopic systems [4]. Very recently the indirect hyperfine interaction between nuclear spin qubits in semiconductor based quantum computer proposals attracted growing attention [5].

Application of high magnetic fields is a very powerful tool for studying the electronic properties of a large variety of metals, semiconductors and superconductors. Due to the Landau quantization of electron motion in sufficiently strong magnetic field, most of the transport properties, such as magnetization, conductivity etc. experience magnetic quantum oscillations (QO) [4]. The Landau quantization is most spectacularly manifest in the electronic magneto-transport in low dimensional conductors. A striking example are the celebrated quantum Hall effects [4].

Apart from the anomalous enhancement of the well known QO in 2DES one expects also strong QO in physical properties which are not sensitive to the
magnetic field in isotropic three dimensional metals. It was suggested in [8] that in quasi-two-dimensional metals under strong magnetic fields the nuclear spin lattice relaxation rate \( T_1^{-1} \) should exhibit strong magnetic oscillations.

This should be compared with the Korringa relaxation law [1, 2] usually observed in three-dimensional normal metals, which results in a magnetic field independent nuclear spin-relaxation rate. This line of research seems to be useful in dense quasi-two-dimensional electronic systems, as is the case, for example in synthetic metals (GIC’s etc.) and low-dimensional organic compounds.

A completely new line of research, the hyperfine interaction between nuclear and electron spins in low dimensional semiconductors and nanostructures, has been developed during the last decade both theoretically [9]-[34] and experimentally [35]-[50].

Very recently the interplay between the nuclear spin ordering at ultra-low temperatures and superconductivity have attracted rapidly increasing theoretical [51]-[53] and experimental [54]-[56] interest.

Here I will outline the theoretical concepts and experimental achievements in the new and quickly developing field of nuclear spintronics.

2 Quantized Nuclear Spin Relaxation Effect: QN-SRE.

2.1 Activation law for \( 1/T_1 \) in QHE systems.

In metals and doped semiconductors, usually, the leading contribution to the spin-lattice relaxation process is due to the hyperfine Fermi contact interaction between the nuclear spins and the conduction electron spins [1]. This interaction is represented by the Hamiltonian: \( \hat{H}_{int} = -\gamma_n \hbar \hat{I}_i \cdot \hat{H}_e \) where \( \gamma_n \) is the nuclear gyromagnetic ratio, \( I_i \) is the nuclear spin and \( H_e \) is the magnetic field on the nuclear site, produced by electron orbital and spin magnetic moments: \( \hat{H}_e = -g\beta \sum \frac{4\pi\delta_e}{3} \hat{s}_e \delta \left( \hat{r}_e - \hat{R}_i \right) \). Here \( \hat{r}_e \) is the electron radius-vector, \( \hat{s}_e \) is the electron spin operator, \( \beta = \frac{e}{m \hbar \gamma_n} \) is the Bohr magneton and \( g \) is the electronic \( g \)-factor.

The nuclear spin-lattice relaxation rate \( T_1^{-1} \), caused by the hyperfine Fermi contact interaction between the nuclear spins and the conduction electron spins, is related to the local spin-spin correlation function through the equation:

\[
T_1^{-1} = \frac{32\pi^2}{9} \gamma_n^2 g^2 \beta^2 \int_{-\infty}^{\infty} e^{-i\omega_n t} \left\{ < S^+ (\mathbf{R}, t) S^- (\mathbf{R}, 0) > \right\} dt
\]

where \( S^+ (\mathbf{R}) \), \( S^- (\mathbf{R}) \) are the transverse components of the electron spin density operator at the nuclear position \( \mathbf{R} \), and \( \omega_n \) is the nuclear magnetic resonance frequency.
The rate of the nuclear spin-relaxation in metals is, usually, proportional to the temperature and to the square of the electronic density of states at the Fermi energy (the Korringa law [1]). This follows from:

\[
\frac{1}{T_1} \propto \int_0^\infty |< i | V | f >|^2 \rho(E_i)\rho(E_f) f(E_i) \\
\times [1 - f(E_f)]\delta(E_f - E_i + \gamma_N H_0) 
\]

(2)

At low temperatures: 
\[ f(E)[1 - f(E)] \propto k_B T \frac{\partial f}{\partial E} \]where \( k_B T \) is the temperature in the energy units and we arrive at the linear in temperature (Korringa) law

\[
T_1^{-1} \sim k_B T \rho^2 (E_F) 
\]

(3)

and \( \rho (E_F) \) is the electron density of states at the Fermi level.

In high magnetic fields and in systems with reduced dimensionality this simple argumentation does not hold, since the electron spectrum acquires field induced [9] or size quantized [19, 33] energy gaps.

It was conjectured in [9] that in quantum Hall effect systems the nuclear spin relaxation rate should have an activation behavior

\[
T_1^{-1} \sim \exp \left\{ -\frac{\Delta (B)}{k_B T} \right\} 
\]

(4)

where \( \Delta (B) \) is either \( g\mu_B B \), the electron Zeeman gap (odd filling factors) or \( \hbar \omega_c \), the Landau levels gap (even filling factors), instead of the usual Korringa law.

This unusual magnetic field dependence of the nuclear spin relaxation reflects the fact that the energy gaps in the spectrum of two-dimensional electrons in strong magnetic fields (either Zeeman splitting or the Landau levels gap) are orders of magnitude larger than the nuclear Zeeman energy. Indeed, the energy needed to reverse the spin of an electron in the external magnetic field \( H_0 \) is \( \Delta E_{el} = 2g\mu_B H_0 \), which is much larger (by a factor of \( \frac{M_B}{m_e} \approx 10^3 \), \( M_B \) and \( m_e \))
being the nuclear and free electron masses) than the energy $\gamma_n H_0$ provided by reversing the nuclear spin. Therefore the delta function in Eq. (2) can not be realized and the simultaneous spin flip of the nuclear and the electron spins (flip-flop) Fig. 1, is severely restricted by the energy conservation. The discreteness of the electron spectrum will manifest, at finite temperatures, in an activation type of the magnetic field dependence of the nuclear spin relaxation rate, $T_1^{-1}$, Eq. (4), similar to that of the magnetoresistance $\rho_{xx}$ in the QHE, [9].

In isotropic 3D electron systems, in a strong magnetic field : $\hbar \omega_c > k_B T$, where the kinetic energy of the electron motion perpendicular to the field is quantized (the Landau levels), the kinetic energy of the electron motion parallel to the field should change in order to ensure the energy conservation of the process. Thus, in the "isotropic" model, the electron spin - flip will be accompanied by a simultaneous change of the Landau level and of the kinetic energy parallel to the field, $E_z$: $\Delta E_z = \hbar \omega_c (n' - n) + \gamma_n H_0 - \hbar \omega_z$. While this is impossible for an ideal 2D system in a strong magnetic field, it may take place in quasi-two-dimensional conductors, as is the case in superlattices, for certain regions of parameters.

Because of the existence of energy gaps in the electron spectrum of a 2DES under strong magnetic fields (the QHE systems), finite nuclear spin relaxation times $T_1$ could be expected only if 2DES is subjected to different kinds of external potentials, such as short range impurities, [9, 10, 11], long range potential fluctuations in a heterojunction [12, 13] and edge states [14].

In sufficiently clean heterojunctions, however, where the FQHE and Wigner crystallization could be observed, the mechanisms mentioned above are extremely inefficient. At relatively high temperatures a phonon assisted mechanism for relaxing the polarized nuclear spins can be operative [15].

Since the electron Zeeman energy gap reduces the effectiveness of the contact interaction between the nuclear and electron spins, in very clean limit the alternative relaxation channels, like the magnetic electron-nuclei interaction (dipolar), may start to be operative [16]. The dipole-dipole interaction does not conserve the total spin, and is not sensitive, therefore, to the existence of the Zeeman gap in the electron spectrum. In this process the spin angular momentum of nuclei is converted, as a result of the interaction, to the orbital momentum of the electron gas.

### 2.2 Spin-excitons

Much of the recent attention paid to hyperfine interactions under conditions of the quantum Hall effect is connected with correlation effects in 2DES. This is based on the notion of a spin-exciton: the elementary excitation over the Zeeman gap dressed by the Coulomb interaction [57]. This results in a strong enhancement (up to a factor of 100, as is the case in GaAs) of the effective $g(k)$ -factor.

Due to the Coulomb interaction the spin-excitions are bound states of electron-hole pairs which, unlike the individual electrons or holes, can propagate freely under the influence of a magnetic field due to their zero electric charge. These el-
elementary excitations are, therefore, chargeless particles with a nearly parabolic dispersion in the low-k limit. At \( k = 0 \) the gap is equal to the "bare" Zeeman splitting.

The energy spectrum of spin-excitons on the ground Landau level: \( n=0 \) is \[ E_{\text{sp}}(k) = |g| \mu_B H_0 + \sqrt{\pi} \left[ 1 - I_0(k^2/4) \exp \left( -k^2/4 \right) \right] \]. In the parabolic approximation (small exciton momenta), the dispersion relation reads: \[ E_{\text{sp}}(k) \approx |g| \mu_B H_0 + k^2/2m_{se} \]

where \( m_{se} \equiv 1/4 \sqrt{\pi/e^2} \) is the definition of the spin-exciton mass.

The invariance of the energy gap with respect to the electron-electron interaction is associated with the fact that in creating a quasielectron-quasihole pair excitation at the same point in space (i.e. with center of mass momentum \( k = 0 \)) the energy decrease due to the Coulomb attraction is exactly cancelled by the increase in the exchange energy. Thus the energy gap for the creation of a widely separated (i.e. with \( k \rightarrow \infty \)) quasielectron-quasihole pair (large spin-exciton) is equal to the exchange energy associated with the hole.

2.2.1 Long-range random potential

Iordanskii et al. [12] have studied nuclear spin relaxation taking into account the creation of spin-excitons in the flip-flop process. The energy for the creation of a spin-exciton can be provided by the long range impurity potential in a process, where the electron turns its spin while its center of orbit is displaced to a region with lower potential energy.

As shown in Fig. 2, the overlap of the initial and final location of the electron wave functions, centered at \( x_1 \) and \( x_2 \) respectively, is: \[ \exp\left[ -\frac{(x_1-x_0)^2}{\sigma^2_{np}} - \frac{(x_2-x_0)^2}{\sigma^2_{np}} \right] \]. Here \( x_0 \) is the nuclear position. Nuclear spin relaxation by the conduction electron spin in the vicinity of a potential fluctuation is effective when the nuclear spin is positioned in the region of the overlapping initial and final states of the

Figure 2: Electron-nuclear spin flip flop in the vicinity of the impurity potential.
electron wave function.

The energy conservation in the spin-exciton creation process can be written in the form: \( \mu_B g H_0 + E(p) = x \nabla U \). This defines the gradient of electric potential caused by the impurity, sufficient to create a spin-exciton during a flip-flop process. The probability of finding such a fluctuation is exponentially small:

\[
\exp\left[-\frac{\left(\nabla U\right)^2}{2\langle \nabla U^2 \rangle}\right]
\]

The electronic DOS in high quality heterojunctions is defined by charged impurities at a distance of the order of the spacer dimension. This results, usually, in a gaussian random potential, which is smooth on the scale of the magnetic length. Because the free spin-exciton energy is large compared to the average potential value, it can be achieved only due to rather large fluctuations of the random potential. It is possible, therefore, to use the standard method of optimal fluctuation, to get the expression for DOS \[12\].

### 2.2.2 Nuclear spin diffusion

Apart from the direct nuclear spin relaxation, important information about the electron system can be obtained from nuclear spin diffusion processes. This is the case when the nuclear spins are polarized in a small part of a sample as it was experimentally observed in \[36, 37\].

To explain these experimental observations, Bychkov et al. \[17\] have suggested a new mechanism for indirect nuclear spin coupling via the exchange of spin excitons. The spin diffusion rate from a given nuclear site \( \vec{R}_a \) within the polarized region is proportional to the rate of transition probability \( P(\vec{R}_a) \) for the polarization of the nuclear spin \( \downarrow \), located at \( \vec{R}_a \), to be transferred to a nuclear spin \( \uparrow \), positioned at \( \vec{R}_b \), outside the polarized region, via the exchange of virtual spin excitons, Fig. 3. The virtual character of the spin-excitons, transferring the nuclear spin polarization, removes the problem of the energy conservation, typical for a single flip-flop process. Furthermore, the virtual spin-excitons are neutral entities, which can propagate freely in the presence of a magnetic field. In this model the electron interactions play a crucial role: the kinetic energy of a spin exciton is due to the Coulomb attraction between the electron and the hole. Thus the proposed mechanism yields the possibility of transferring nuclear spin polarization over a distance much longer than the magnetic length \( \ell_B \). The long range nature of this mechanism is of considerable importance when the size of the region of excited nuclear spins, \( L_{ex} \), is much larger than the magnetic length \( \ell_B \).

As it is shown in \[17\], the potential of the nuclear spin - spin interaction, mediated by the exchange of spin-excitons, is a monotonic function of the distance between the two nuclei with the asymptotics: \( U(R_{ab}) \propto -\sqrt{\frac{d}{\epsilon_p}} e^{-\frac{R_{ab}}{\sqrt{\epsilon_p}}} \). This is typical for the interaction, mediated by the exchange of quasiparticles with an energy gap at \( q \rightarrow 0 \), as is the case for the spin-exciton dispersion. The range \( \Delta R \) of this potential is determined by the critical wave number \( k_0 = \frac{\lambda_B}{\ell_B} \sqrt{\frac{\epsilon_p}{\epsilon_c}} \) as it follows from the uncertainty principle: \( \Delta R \cdot k_0 \approx 1 \).
Figure 3: Indirect interaction between two nuclear spins via conduction electron spin.

The negative sign of this interaction corresponds to attraction between the nuclear spins and may cause, at sufficiently low temperatures, a ferromagnetically ordered nuclear state in QHE systems.

3 NONEQUILIBRIUM NUCLEAR SPIN POLARIZATION

3.1 Meso-Nucleo-Spinics

Once the nuclear spins are completely polarized, they produce a hyperfine field of the order of several Teslas, strongly influencing all the electronic transport [35]-[43]. This may result in a series of new nonequilibrium mesoscopic phenomena (meso-nucleo-spinics). Among them are a) HABE-the hyperfine field induced Aharonov-Bohm effect [19], b) HAHE- the hyperfine field induced Anomalous Hall effect [18], c) the nuclear spin polarization induced quantum dots (NSPI QD) [30] and nanowires (NSPI NW) [31].

The nonequilibrium nuclear spin polarization may strongly influence the correlated electron states, resulting in creation of SkyrNuons i.e. skyrmions, localized by the nonuniform hyperfine field of polarized nuclei Fig. 3.
3.1.1 Hyperfine Aharonov-Bohm Effect

The physics of the HABE: the Aharonov-Bohm effect driven by the hyperfine field of polarized nuclei, developed in [19], can be understood along the following lines.

Persistent currents (PC) in mesoscopic rings reflect the broken clockwise-anticlockwise symmetry of charge carriers momenta caused, usually, by an external vector potential. Experimentally PC are observed when an adiabatically slow time dependent external magnetic field is applied along the ring axis. The magnetic field variation results in an oscillatory behavior of the diamagnetic moment, with the magnetic flux quantum \( \Phi_0 = \frac{hc}{e} \), which is a manifestation of the Aharonov-Bohm effect (ABE).

In a quantum ring with a nonequilibrium nuclear spin population the ABE-like oscillations of PC with time may exist, during the time interval of the order of nuclear spin relaxation time \( T_1 \), [19], since the hyperfine field breaks the spin symmetry of charged carriers. Combined with a strong spin-orbital interaction (SOI), in systems without center of inversion [58] this results in the breaking of the rotational symmetry of diamagnetic currents in a ring. Under the topologically nontrivial spatial nuclear spin distribution the hyperfine field produces an adiabatically slow time variation of the Berry phase of the electron wave function. The time variation of this topological phase may result in observable oscillations of a diamagnetic moment (the PC).

Once the nuclear spins are polarized, i.e. if \( \langle \sum_i I_i \rangle \neq 0 \), the charge carrier spins feel the effective hyperfine field \( B_{hypf} = B_{hypf}^0 \exp(-t/T_1) \) which lifts the spin degeneracy even in the absence of an external magnetic field. In GaAs/AlGaAs one may achieve a spin splitting due to hyperfine field of the order of one tenth of the Fermi energy [37].

A typical term for a heterojunction SOI is the Bychkov-Rashba term [58]:

\[
\hat{H}_{so} = \frac{\alpha}{\hbar} [\sigma \times p] \nu,
\]

where \( \alpha = 0.6 \cdot 10^{-3} eV cm \) for holes, and \( \alpha = 0.25 \cdot 10^{-3} eV cm \) for electrons, \( \sigma \), \( p \) are the charge carrier spin and momentum and \( \nu \) is the normal to the surface. It can be rewritten in the form \( \hat{H}_{so} = \frac{e}{\hbar c} p A_{eff} \), where \( A_{eff} = \frac{e}{\hbar c} [\nu \times \sigma] \). Under the conditions of a topologically nontrivial orientation of \( A_{eff} \) the wave function of a charge carrier encircling the ring gains a phase shift similar to the one in an external magnetic field as in the ordinary ABE. This phase shift can be estimated as follows:

\[
2\pi \Theta = \frac{e}{\hbar c} \oint \mathbf{A}_{eff} \cdot d\mathbf{l} = \frac{m^*}{\hbar^2} \langle \sigma(t)|\alpha \rangle \sim \frac{m^*}{\hbar^2} \alpha L, \]

where \( L \) is the ring perimeter. To observe the oscillatory PC connected with the adiabatically slow time-dependent \( \langle \sigma(t) \rangle \), \( L \) is supposed to be less than the phase breaking length. Taking the realistic values for \( L \approx 3 \mu m \) and \( \langle \sigma \rangle \approx 0.05 \div 0.1 \), the estimate: \( 2\pi \Theta \sim 5 \div 10 \) shows the experimental feasibility of this effect.

There is a marked difference between the periodic time dependence of standard ABE oscillations, which are usually observed under the condition of linear time variation of the applied magnetic field, and the hyperfine field driven oscillations which die out due to the exponential time dependence of the nuclear polarization.
It is worth pointing out that the PC depends not only on $B_{hypf}$, it also has an oscillating dependence on $SOI$ coupling parameter $\alpha$ \[33\].

### 3.1.2 Hyper-anomalous Hall effect

The anomalous Hall effect, (AHE), is caused by the spin-orbit interaction, $SOI$, combined with carrier magnetization. Due to $SOI$, electrons with their spin polarization parallel to the magnetization axis will be deflected at right angles to the directions of the electric current and of the magnetization while electrons with antiparallel spin polarization will be deflected in the opposite direction. Thus, if the two spin populations are not equal there appears a net current in the transverse direction. Until now, studies of the anomalous Hall effect have been limited to the case where the carrier magnetization is induced either by the external magnetic field or by the ordering of the magnetic impurities. The magnetic field, however, produces a much larger normal Hall effect which makes experimental studies quite difficult.

Bednarski et al. \[18\] have suggested that the hyperfine field induced anomalous Hall effect, $HAHE$, can be obtained under conditions of strong nuclear spin polarization even in the absence of an external magnetic field. In this model the role of the hyperfine field is to split the conduction band, thus creating the magnetization of the carriers, and to introduce a long time scale dependence into electronic spin polarization.

It is outlined in \[18\], that the magnetic field enters the results due to non-commutativity of the $k$-vector components, while the contact hyperfine field is a fictitious magnetic field, acting only on electronic spins, and is not connected with any real vector potential. Thus the presence of polarized nuclei can not be reduced to a trivial and formal replacement of $B$ by $B + B_{HF}(t)$. For example, the hyperfine field does not appear explicitly in the cyclotron frequency $\omega_c$ while it can influence $\omega_c$ via a spin dependent effective mass.

### 3.1.3 Skyrmions and Skyr-Nuons

Skyrmions, in QHE systems, are the topologically nontrivial spin excitations around filling factor $\nu = 1$ \[59\] which arise as a condensate of interacting spin excitons \[60\]. The Coulomb interaction acts to enlarge the Skyrmion size while the Zeeman splitting tends to collapse Skyrmions. The interplay between these factors determines the final distribution of spins within a Skyrmion, and its characteristic length scales. The resulting radius $R$ corresponds to the region where both these energies are of the same value, and grows weakly to infinity as the g-factor goes to zero \[61\], thus reflecting the importance of the long range Coulomb repulsion associated with the Skyrmion charge in the zero g-factor limit.

In NMR experiments on Skyrmions \[38, 39, 44, 45, 40\], the nuclear spins are strongly polarized. The sample inhomogeneity will result in a strong inhomogeneity of the hyperfine field and therefore spatially varying electron Zeeman splitting. This may result in a strong localization of skyrmions \[52, 41\], result-
Figure 4: Skyr-Nuon is a Skyrmion localized by the interaction with the nuclear spins.

3.2 Universal Residual Resistance

The possibility that the hyperfine interaction between the conduction electron spins and nuclear spins may result in hyperfine universal residual resistivity (HURR) in clean conductors at very low temperatures was studied theoretically by Dyugaev et al. [51]. Apart from the fundamental nature of this problem, the natural limitations on the mean free path are decisive in semiconductor based high speed electronic devices, like heterojunctions and quantum wells. The space periodicity of nuclei plays no specific role, as long as the nuclear spins are disordered and act as magnetic impurities with the concentration $C_n \approx 1$. This scattering is not operative at extremely low temperatures, in the $\mu K$ region when the nuclear spins are ferro- or antiferromagnetically ordered.

The residual "nuclear" resistivity is due to the Fermi (contact) hyperfine interaction between the nuclear and the conduction electron spins:

$$V_{en} = -\frac{8\pi}{3}\mu_e\mu_h \Psi_e^2(0) \equiv \mu_n H_e$$

(5)

here $\mu_e$ and $\mu_h$ are the operators of the electron and nuclear magnetic moments, $\Psi_e^2(0) \propto Z$ is the value of the conduction electron wave function on the nuclei with the nuclear charge $Z$ and $H_e$ is the magnetic field induced on nuclei by the electrons.

The estimate of $V_{en}$ in atomic units: $\hbar = m_e = e = 1$ is: $V_{en} \approx Z\alpha^2 \frac{m_n}{M_n} Ry$ where $m_e, M_n$ are the electron and the nucleon masses, respectively; $Ry = 27$ eV and $\alpha = \frac{1}{137}$ is the fine structure constant.

In metals the effective electron-nuclear interaction constant is $g_n \equiv \frac{V_{en}}{\epsilon_F} \approx 10^{-7}Z \frac{Ry}{\epsilon_F}$ where the Fermi energy $\epsilon_F$ varies in a wide interval $0.01 \div 1$. $g_n$ is $10^{-6}$ for Li and $10^{-3}$ for the rare earth metals.
The total residual resistivity is therefore a sum of the impurity $\rho_o(T \to 0) \sim C_o$ and the nuclear spin $\rho_n(T \to 0) \sim g_n^2$ contributions: $\rho_n(T \to 0) \approx \rho_o(C_o + g_n^2)$. The nuclear contribution to resistivity starts to be operative when the impurity concentration is $C_o \sim g_n^2$.

In the limit of an ideally pure ($C_o = 0$) metal the universal residual resistivity $\rho_{URR}$ is, therefore $\rho_{URR} \geq \rho_o g_n^2$ and the mean free path is limited by $10^{-8}$ cm. This yields $10^4$ cm in light atom conductors, as $Li$, for example and $10^{-2}$ cm for the rare earth metals. It is interesting to note that in zero-nuclear spin materials the $HURR$ should be absent.

In materials with the nuclear magnetic moments $I \neq \frac{1}{2}$, even without external magnetic field, their $2I + 1$ degeneracy is partially lifted by quadrupole effects (in the case of cubic crystal symmetry the quadrupole splitting of the nuclear levels may happen due to the defects [1, 2, 63] and dislocations [64]. The hyperfine nuclear contribution to $\rho_n(T)$ in this case will have a different temperature dependence since $\mu_n H$ should be replaced by the characteristic quadrupole splitting of energy levels.

While the normal metals have a quite similar electronic structure, the experimentally observed temperature dependence of the dephasing time $\tau_\phi$ is quite different. This was shown in [65, 66], where the value of $\tau_\phi$ was defined by the magnetoresistance measurements of long metallic wires $Cu, Au, Ag$ in a wide temperature interval $10^{-2} < T < 10^4$ K. In $Cu$ and $Au$ wires $\tau_\phi$ saturates at low temperatures which contradicts the standard theory [67]. Strangely enough the $Ag$ wires do not show saturation [66] at the lowest temperatures, in accordance with [67].

It is conjectured in [22] that the influence of the quadrupole nuclear spin splitting on the phase coherence time $\tau_\phi$ can be a clue to this puzzle. Indeed, the nuclear spins of both $Cu$ and $Au$ have a strong quadrupole moment ($s = 3/2$) and may act as inelastic two-level scatterers once their degeneracy is lifted by the static impurities [63], dislocations [64] and other imperfections. It is known that (see [68] and references therein) the nondegenerate two level scatterers may introduce inelastic phase breaking scattering of conduction electrons.

This may be not the case for $Ag$ nuclei since their spin is $s = \frac{1}{2}$. In this case the quadrupole splitting of nuclei spins by imperfections is negligible. In the absence of magnetic Zeeman splitting therefore the nuclear spins in $Ag$ samples will act just as a set of elastic scatterers, and the temperature dependence of $\tau_\phi$ should obey the standard theory [67].

### 3.3 Nuclear spins and superconducting order

The problem of coexistence of the superconducting and magnetic ordering, in spite of its long history [69], is still among the enigmas of modern condensed matter physics. Most of the theoretical and experimental efforts were devoted to studies of the coexistence of electron ferromagnetism and superconductivity. The possibility of a reduction of $H_c(T)$ by the nuclear ferromagnetism was outlined by Dyugaev et al. in [51]. Later on it was theoretically considered in
In [51] it was suggested that apart from the influence of the "electromagnetic" part of the polarized nuclear spins on the superconducting order, the hyperfine coupling between the nuclear spins and conduction electron spins may play a crucial role on the coexistence between superconducting state and nuclear ferromagnetism [52]. Moreover, it was conjectured in [70] that the hyperfine part of the nuclear-spin-electron interaction may result in the appearance of a nonuniform superconducting order parameter, the so called Fulde-Ferrel-Larkin-Ovchinnikov state (FFLO) [71].

The FFLO state was thought originally to take place in superconductors with magnetically ordered magnetic impurities [71]. The main difficulty, however, in the observation of the FFLO caused by magnetic impurity ordering is in the simultaneous action of the "electromagnetic" and "exchange" parts of the magnetic impurities on the superconducting order. In most of the known superconductors the "electromagnetic" part destroys the superconducting order before the "exchange" part modifies the BCS condensate to a nonuniform FFLO state.

In the case of nuclear spin ferromagnetic ordering, the nuclear magnetic moment $\mu_n = \hbar g_i M_i$, is at least three orders of magnitude smaller than the electron Bohr magneton $\mu_e = \hbar g_e m_e$, so that the "electromagnetic" part of the nuclear spin fields is quite low, compared to that of the magnetic impurities. On the other hand the "exchange" part is strongly dependent on the nuclear charge $Z$. As it was shown in [70], in some materials the interplay between these two contributions can be in favor of the "exchange" part, thus providing the necessary conditions for the appearance of the FFLO.

4 NUCLEAR SPINS AS QUBITS

4.1 Quantum Hall Quantum Computation

The hyperfine interactions are believed to play a central role in solid state electronics based realizations of future quantum computation devices [5].

Privman et al. in [20] have proposed a quantum computer realization based on hyperfine interactions between the conduction electrons and nuclear spins embedded in a two-dimensional electron system in the quantum-Hall effect. For modifications and improvements of this model see a recent review [5].

The general idea is as follows: consider a chain of spin-1/2 nuclei in an effectively two-dimensional heterojunction or quantum well subjected to a strong magnetic field. The typical separation should be comparable to the magnetic length $\ell_H = \sqrt{\hbar c/eH}$, where $H$ is the applied magnetic field, perpendicular to the 2D layer. This length is of the order of 100 Å. The control of individual nuclear spins is by electromagnetic-radiation pulses in the nuclear magnetic resonance (NMR) frequency range. Differentiation between individual nuclear spins can be achieved by a combination of several methods. First, one can use
different nuclei. Secondly, they can be positioned in different crystalline environments. The latter can be controlled by implanting atoms and complexes into the host material. Use of a magnetic field gradient could be also contemplated, but there are severe limitations on the field variation owing to the need to maintain the QHE electronic state. There is no apparent limit on how many different spins can be arranged in a chain. It may also be appropriate to utilize small clusters of nuclear spins, rather than individual spins. These can be made coherent by lowering the temperature to the order of several $\mu$K [43].

Maniv et al. [27] have suggested a physical process of preparing a coherent state in QH ferromagnets. Let us assume that at time $t = -t_0 < 0$ the filling factor was tuned to a fixed value $\nu = \nu_0 \neq 1$ and then kept constant until $t = 0$. If $t_0 \gg T_2 (\nu_0)$ then at $t = 0$ the nuclear spin system is in the ground state corresponding to the 2D electron system at $\nu = \nu_0$. Suppose that at time $t = 0$ the filling factor is quickly switched (i.e., on a time scale much shorter than $T_2 (\nu_0)$) back to $\nu = 1$ so that the nuclear spin system is suddenly trapped in its instantaneous configuration corresponding to $\nu = \nu_0 \neq 1$. Thus the nuclear spins for a long time $t (\gg T_2 (\nu_0))$ will find themselves almost frozen in the ground state corresponding to the 2D electron system at $\nu = \nu_0$, since $T_2 (\nu = 1) \gg T_2 (\nu_0)$.

4.2 Decoherence and Dephasing ($T_2$).

Maniv et al. [27], have considered the effect of vacuum quantum fluctuations in the QH ferromagnetic state on the decoherence of nuclear spins. It was shown there that the virtual excitations of spin excitons [57], which have a large energy gap (on the scale of the nuclear Zeeman energy) above the ferromagnetic ground state energy, lead to fast incomplete decoherence in the nuclear spin system. It is found that a system of many nuclear spins, coupled to the electronic spins in the 2D electron gas through the Fermi contact hyperfine interaction, partially loses its phase coherence during the short (electronic) time $\hbar/\varepsilon_{sp}$, even under the ideal conditions of the QHE, where both $T_1$, and $T_2$ are infinitely long. The effect arises as a result of vacuum quantum fluctuations associated with virtual excitations of spin waves (or spin excitons) by the nuclear spins. The incompleteness of the resulting decoherence is due to the large energy gap of these excitations whereas the extreme weakness of the hyperfine interaction with the 2D electron gas guarantees that the loss of coherence of a single nuclear spin is extremely small.

5 EXPERIMENTS

The measurement of the nuclear spin-relaxation in heterojunctions is a challenging experimental problem, since the direct detection of the NMR signals in solids requires usually $10^{17} - 10^{20}$ nuclei. The number of nuclear spins interacting with the two-dimensional electrons is however, much smaller: $10^{12} - 10^{15}$.

The first successful measurements of the magnetic field dependence of $T_1^{-1}$
under QHE conditions were performed in a series of elegant experiments by the K. von Klitzing group, [35]. Combining ESR and resistivity measurement techniques they have observed the shifting of the ESR resonance frequency by the hyperfine field of nonequilibrium nuclear spin population, which is the well known Overhauser shift [1, 2]. In this experiment the 2D electron Zeeman splitting is tuned to the pumping frequency. The angular momentum gained by a 2DEG electron, excited to the upper Zeeman branch, is then transferred to the nuclear spins, thus creating a nonequilibrium nuclear spin population.

These measurements show a close similarity between the magnetic field dependence of the nuclear spin-relaxation rate and the magnetoresistance in quantum Hall effect, as it was suggested theoretically in [9], thus demonstrating clearly the importance of the coupling of nuclear spins to the conduction electron spins in the nuclear relaxation processes in these systems.

Various experimental techniques were used since and in what follows we will describe shortly the main developments and achievements in experimental studies of the hyperfine coupling between the nuclear spins and the electrons in QHE, mesoscopic and superconducting systems.

Another way of measuring the nuclear spin relaxation and diffusion in a heterojunction under strong magnetic field by transport techniques (spin-diode) was demonstrated by Kane et al., [36]. They have reported measurements performed on "spin diodes" : junctions between two coplanar 2DEG’s in which \( \nu < 1 \) on one side and \( \nu > 1 \) on the other. The Fermi level \( E_F \) crosses between spin levels at the junction. In such a device the 2DEG is highly conducting except in a narrow region (with a width of the order of several hundred angstroms) where \( \nu = 1 \).

Wald et al. [37] presented the experimental evidence for the effects of nuclear spin diffusion and the electron-nuclear Zeeman interaction on interedge state scattering. Polarization of nuclear spins by dc current has proven to be a rich source for new, not always yet understood, phenomena. This is the case, for example of anomalous spikes in resistivity around certain fractional filling factors, observed by Kronmuller et al. [41] and in resistively detected NMR in QHE regime reported by Desrat et al. [42]. Influence of nonequilibrium nuclear spin polarization on Hall conductivity and magnetoresistance was observed and studied in detail by Gauss et al. [43].

In 1994 [38] Barrett et al. observed, for the first time, a sharp NMR signal in multi-quantum wells, using the Lampel [72] technique of polarizing the nuclear spins by optical pumping of interband transitions with near-infrared laser light (OPNMR). Polarization of nuclei results in a significant enhancement of the NMR sensitivity, since the resonance in a two-level system results in equalizing their population. The difference in the population is, obviously, maximal when the spins are completely polarized.

Detailed studies of the Knight shift data suggested [38] that the usually accepted picture of electron spins, aligned parallel to the external field, should be modified to include the possibility of topologically nontrivial nuclear spin orientations, the Skyrmions. Optical polarization of nuclear spins was used also as a tool for reducing the Zeeman splitting of 2D electrons by Kukushkin et al.
This resulted in a noticeable enhancement of the skyrmionic excitations. Similar results are reported by Vitkalov et al. 45.

Very recently, a modern ultra-sensitive "standard" NMR spin-echo technique was employed to study the physics of quantum Hall effect 46 in GaAs/AlGaAS multi-quantum well heterostructures. The spin polarisation of 2DES in the quantum limit was investigated and the experimental data support the non-interacting Composite Fermion model in the vicinity of the filling factor $\nu = \frac{1}{2}$. Using the same technique the polarization of 2DES near $\nu = \frac{2}{3}$ was investigated 47. It was shown there that a quantum phase transition from a partially polarized to a fully polarized state can be driven by increasing the ratio between the Zeeman and Coulomb energies.

An amazing phenomenon, following from the hyperfine coupling between the electron and the nuclear spins, is the giant enhancement of the low temperature heat capacity of GaAs quantum wells near the filling factor $\nu = 1$, discovered in 1996 by Bayot et al. 48. As other thermodynamic properties, it experiences quantum oscillations, following from the oscillatory density of states $D(E_F)$ at the Fermi level.

At about $T = 25 mK$, and in clean samples, Bayot et al. 48 have observed anomalous deviations from the free electron model, in the specific heat value (up to four orders of magnitude) for the filling factor in the range $0.5 \leq \nu \leq 1.5$. Their explanation is that in this interval of parameters, the electron system couples strongly to nuclear spin system with a concentration of several orders of magnitude larger than the electron one.

This raises a question about the origin of the strong coupling between the electron and the nuclear spins in the interval $0.5 \leq \nu \leq 1.5$. The guess is the skyrmions, since they are predicted to appear just in the same interval of the filling factor. Additional support for this mechanism is in the results of 49, where the disappearance of the nuclear spin contribution to the heat capacity was reported, as the ratio between the Zeeman and Coulomb energies exceeds a certain critical value. The Zeeman splitting of electrons was modified in these experiments by tilting the magnetic field.

A new very promising technique for measuring spatially varying nuclear spin polarization within a GaAs sample is reported in 50. In the force detected NMR (FDNMR) the sample is mounted on a microcantilever in an applied magnetic field. A nearby magnetic particle creates a gradient of magnetic field which exerts a force on the magnetized sample and triggers the cantilever oscillations. FDNMR is capable to perform the magnetic resonance imaging of the sample with a very high accuracy. This method can be useful in defining the spatial distribution of nuclear spin polarization in non homogeneous samples. This information may be crucial for understanding different peculiarities of data obtained by previously described methods.

A new world of the low-temperature physics of the hyperfine interactions in superconducting metals opens in the $\mu K$ region, where the nuclear spins start ordering 54-56. Reduction of the critical magnetic field of superconductors by the ferromagnetic ordering of the nuclear spins has been recently discovered by the Pobell group 54. They have studied the magnetic critical field $H_c(T)$
of a metallic compound \( \text{AuIn}_2 \) where the superconductivity sets up at \( T_{ce} = 0.207K \). They have observed, in \( \text{AuIn}_2 \), the nuclear spin ferromagnetic ordering at \( T_{cn} = 35\mu K \). It was observed in these experiments that the magnetic critical field \( H_{c0} = 14.5 \) G is lowered by almost a factor of two at \( T < T_{cn} \).

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