Simple design to estimate time-lapse microgravity response due to shallow subsurface density redistribution caused by land subsidence

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Abstract. A simple design for modeling shallow subsurface density redistribution due to land subsidence is designed to obtain the time-lapse microgravity response. The subsurface model at each point of gravity observation is represented by a rectangular prism. A numerical example of computational modeling is performed to estimate the effect of land subsidence to the data of a time-lapse microgravity. Simple numerical simulations with an initial model that have flat topography, homogeneous density, and homogeneous compaction thickness are carried out in variations of geological and hydrological information that are often found in a study area. Additional algorithms to accommodate information on topographic variations, density variations, and compaction thickness variations in the horizontal direction also shown with illustration. Field data application for this study utilize rough estimation of the geology and the land subsidence rate in Bandung Basin. The estimation results with numerical simulations give time-lapse microgravity anomaly 0.78 to 28.61 μGal/m and field data application give an anomaly up to 10 μGal.

1. Introduction
The change in the value of gravity that occurs at the time-lapse microgravity (TLM) observation point can be caused by several factors. In the research of Tampone et al. [1], changes in gravity values for simulation studies in hydrocarbon production areas consist of: changes in the position of the observation point, deformation below the earth's surface, changes in groundwater level, and changes in the reservoir layer (fluid substitution). For case studies in the research area where there is no hydrocarbon production activity, in general, the target depth of the TLM anomaly is relatively shallower (changes in the water table and changes in the soil surface). Groundwater level changes involve fluid substitution in the rock pore space below the surface (not discussed in this study), while subsidence estimates (which are often monitored by repeated GPS surveys and/or InSAR) provide information on changes in the position of the TLM observation points and redistribution estimates, density of rock near the surface.

In this study, the main focus of research is related to TLM modeling due to changes in the soil surface followed by a redistribution of rock density near the surface. The modeling approach is carried out with a simple volume element in the form of a rectangular prism. The subsidence estimation results provide a change in the dimension of volume elements in the vertical column (z-axis of the Cartesian coordinates) below the surface. The modeling computation is carried out by taking into account two main changes, namely: changes in the position of the observation point and redistribution of density below the surface (illustration in Figure 1). Several researchers have conducted several studies ([1]-[3]) for subsurface deformation modeling. Several approaches and study results for the effect of gravity on changes in the position of the observation point have also been carried out by several researchers ([4]-[
using theoretical approach of the free-air gradient, using a theoretical approach of free-air and Bouguer slab [6], vertical correction of gravity gradient ([7]-[8]), and observational studies [9].

Figure 1. An illustration of a comparison of observation points based on changes in the subsurface from the initial and final conditions in the TLM data acquisition scheme which explains the correction due to subsidence factors using the following approach: a) vertical gradient value or free-air correction, b) free-air coarse and correction slab Bouguer, c) modeling changes in density and observation points due to subsidence.

The illustration in Figure 1 shows a sketch of changes in the position of the observation point and conditions below the surface. The theoretical approach of the free-air gradient or vertical gradient correction as a correction for changes in the earth's surface vertically is shown in Figure 1a. The approach to correction like this is still very rough and can still be improved because based on the illustration (Figure 1a) the position of the observation in the initial conditions is considered to be a certain height above the surface (although at the time of the data acquisition activity, the gravimeters were both placed on the surface of the earth and very much endeavored to do. at the same position in the initial and final conditions). Figure 1b is an illustration for the relatively same observation position from the earth's surface between the initial and final conditions, but also accompanied by subsidence information. In the illustration in Figure 1b, it is shown that the density distribution is still the same in the initial and final conditions but has different volume sizes (one of the near-surface activities that allows this illustration is the activity of taking material from the surface by a hoe or heavy equipment). In general, the free-air and Bouguer slab correction in Figure 1b does not involve redistribution of subsurface density due to subsidence. Figure 1c describes an illustration that is almost the same as Figure 1b, but the mass below the surface does not change so that there is a density redistribution due to subsidence which is shown in gray.
2. Modeling Computation (Simple Design)

The discretization below the surface is approached by a simple set of elements (Figure 2). For practical purposes, discretization of observational data corresponds to discretization of bodies under the surface. Each observational data on a particular topography below is modeled with a prism that is parallel to the x-axis, y-axis, and z-axis. Each volume element has a uniform density change value ($\Delta \rho$) with the prism volume being $dV = dx dy dz$. Each volume element can then add up its effect cumulatively at each point of gravity observation on the surface.

Based on Figure 2, a simple design illustration of the subsurface model for computation of the TLM anomaly is shown by the initial surface and the final surface at a certain time interval with the change in subsurface density from the initial state (white color) to the final state (gray color). The change in density is a consequence of the mass conservation formulation as shown by Tempone et al. [1] as follows:

$$\rho_2 = \frac{\rho_1 V_1}{V_2}. \quad (1)$$

At a certain time interval, a prism volume element is simply modeled where only the $dz$ value changes, while the $dx$ and $dy$ values are assumed to be constant. Based on Figure 2, the $dz$ value is limited to the lower limit estimate of the compacting. This approach simplifies the lower bound ($z_b$) which is the same at a certain time interval, so that the calculation formulation of density change is shown in equation (2) as follows:

$$\rho_2 = \frac{\rho_1 dx dy dz_1}{dx dy dz_2} = \frac{\rho_1 dz_1}{dz_2} = \frac{\rho_1 (z_1 - z_b)}{(z_2 - z_b)}. \quad (2)$$

The $z_1$ and $z_2$ values are input data from the topographic differences in the initial and final conditions. The change in density ($\Delta \rho$) is then shown in equation (3) as:

$$\Delta \rho = \rho_2 - \rho_1 = \frac{\rho_1 (z_1 - z_b) - \rho_1 (z_2 - z_b)}{(z_2 - z_b)} = \frac{\rho_1 (z_1 - z_2)}{(z_2 - z_b)}. \quad (3)$$
Synthetically, the TLM anomaly value ($\Delta g$) from two gravitational observations ($g_1$ and $g_2$) at different topographic coordinates ($z_1$ and $z_2$) from a certain time interval ($t_1$ to $t_2$) is shown by the mathematical expression in equations (4) and (5):

$$
\Delta g = g_2(x, y, z_2, t_2) - g_1(x, y, z_1, t_1), \tag{4}
$$

$$
\Delta g = \gamma \rho_2 \int \frac{z_2}{(r_2)^2} dV_2 - \gamma \rho_1 \int \frac{z_1}{(r_1)^2} dV_1. \tag{5}
$$

In equation (5) the values of $r_1$ and $r_2$ are the distance between the observation point and the prism body in the initial and final conditions. Equation (5) for TLM involves two integrals, each of which can be solved numerically. Numerical formulations can be used as has been shown by Wahyudi et al [10].

3. Program Testing with Numerical Examples

This section shows program testing with numerical examples based on the simple modeling design descriptions described in the previous section. The simplified visualization for this explanation will be used for the vertical section sections of a simple numerical subsidence model example. Figure 3 shows an illustration of the model change due to subsidence at certain time intervals (zoom in on a depth scale of 0 to 3 meters). At each point of observation (in red) there is an initial model under the surface represented by a discretization of a simple object in the form of a rectangular prism with a density value of 2.5 g/cc (Figure 3a). The illustration for surface change due to subsidence is shown with a vector arrow in the vertical direction (Figure 3b). The final model after the subsidence information is accounted for for the density change in each rectangular prism model is shown in Figure 3c. Equation (3) which is used to obtain the density value estimate for each discretization of the model elements gives density values in the range of 2.5 g/cc to 2.55 g/cc.

![Figure 3](image.png)

Figure 3. An illustration of model change due to subsidence involves a simple design: (a) initial model with $dy = 500$ m, (b) vector change in vertical direction (subsidence direction) at certain time intervals, and (c) final model with near surface density change.

Synthetically, the models in Figures 3a and 3c can be used in calculating the gravity anomaly response, where each anomaly is gravitational observations (synthetic data) $g_1$ and $g_2$, respectively. An illustration of the synthetic data calculation is shown in Figure 4. If we look at Figures 4a and 4c, it can...
be seen that the synthetic data on the resulting edge shows a very drastic change as the edge effect of the limited computation model. Regarding the edge effects of the model computation, the main focus regarding the TLM anomaly will be on the center of the model. Figures 4b and 4d illustrate the zoom out of the models in Figures 3a and 3c, to provide more complete information about the models involved in computation. The model in figure 4 shows the thickness of the compacting zone for calculations determined to be 50 meters below ground level.

As a simplification of the illustrations in Figures 3 and 4, the computation of the TLM anomaly and its change model is shown in Figure 5. The TLM anomaly in Figure 5a is generated based on calculations using equation (5). The subsurface model (Figure 5b) is represented by the density change values from the initial and final models. The change in density in Figure 5b is shown in the range of 0 to 0.05 g/cc with an illustration of subsidence at certain intervals represented by a vector in the vertical direction.

![Figure 4. Illustration of synthetic data calculation (g1 and g2): (a) initial model gravity response, (b) zoom out initial model, (c) final model gravity response, and (d) zoom out final model.](image)

In the illustration in Figure 5, it is shown that the maximum TLM anomaly value correlates with the position of the maximum subsidence as well, this is related to the maximum compacting process for the compacting zone scenario and the uniform initial model density (2.5 g/cc and 50 meters respectively). As shown in the subsidence arrow vector (Figures 3b and 5b) the calculated value of vertical change is in the range of 0 to 1 meter, this will help provide a rough estimate of the maximum effect of subsidence on a flat topography in units of μGal/m. Figure 6 shows some of the estimation results (in μGal/m) for the effect of subsidence in the initial model which has a flat topography and uniform rock density. The estimation results in Figure 6 are consecutive simulations for several densities of 1.5 to 3.0 g/cc and successive simulations for several rock layer thicknesses in the computed zone calculation (10 to 300 meters). The curve in Figure 6 provides a larger estimated value for the initial large-density model (3.0 g/cc) and occurs in a thicker compacted zone (300 meters). Estimates of the effect of subsidence and change in density were simple designs with the computational input parameters described in the range 0.78 to 28.61 μGal/m.
Figure 5. Computational illustration: (a) TLM anomaly and (b) density change model due to subsidence at a certain time interval (subsidence is represented by a vector in the vertical direction).

Figure 6. Estimation of the effect of subsidence and near surface density change (in $\mu$Gal/m) for the successive simulations of the initial model densities as well as the successive simulations for several thickness packed layers.

The estimation results of the model in Figure 6 need to be considered for: the initial model with a flat topography, the initial model with a homogeneous density value throughout the discretization of the model under its surface, and the thickness of the homogeneous compacted zone. To carry out an application in an area, the simple computational design that has been described needs to be added with an algorithm to accommodate some of the more real information found in the research area. The topography in the study area in the horizontal direction will tend to vary in topography, as well as the density of the initial model also tends to be not homogeneous (varies according to the geological
information in the study area). The compacted zone in a study area also tends to vary in thickness in the horizontal direction. Information on compacted zone thickness can be used based on groundwater utilization well data or conceptual information on hydrological models in a study area. If this information is not available, calculations for a more rough estimate can use a specific, homogeneous value of the thickness of the hydrological layer of a study area.

Figure 7a shows an illustrative example for the initial model with variations in topography, variations in density, and variations in the thickness of the compacted zone in the horizontal direction. Then an illustrative example for subsidence information (which can often be obtained from the results of monitoring with repeated GPS surveys and/or InSAR) is shown with a vector arrow in the vertical direction in Figure 7b. The information in Figure 7 is used for input computation of TLM anomalies. The topographic variation of the initial model is in the elevation range of 700 to 1000 meters above sea level (depth = 0) or has a negative value for the slope of the positive vertical axis downward or depth. The density variation of the initial models is in the range 2.0 to 2.5 g/cc, while the variation for the thickness of the compacted zone is also shown in the range of 100 to 200 meters. At certain time intervals, subsidence information is also synthetically varied in the range of 0 to 1 meter. Figure 8a shows the TLM anomaly calculated by equation (5) in the range 2 to 80 μGal. The density change in Figure 8b is shown in the range of 0 to 0.015 g/cc with an illustration of subsidence at a certain time interval shown by a vector in the vertical direction.

**Figure 7.** Illustration of the initial model and vector changes in the vertical direction. (a) Initial model with varying topography, density, and thickness of the compacted zone in the horizontal direction. (b) Vector change in the vertical direction (subsidence direction) occurs at certain intervals.

**Figure 8.** An illustration of computation using the information in Figure 7: (a) TLM anomaly and (b) density change model due to subsidence at a certain time interval (subsidence is shown as a vector in the vertical direction).
4. TLM Anomaly Due to Changes of Topography and Near Surface Density in Bandung

The hydrological information of the research area in Bandung as the initial model to be used in this study is based on the discretization of the conceptual geological model in the publication of Nurliana and Widodo [11]. The discretization of the geological model as shown in Figure 9 provides information on rock formation in the study area. This information will be used as an estimate of the average rock density value of the rectangular vertical prism volume element below the surface. In the discretization of the model, information on topographic variations and variations in the thickness of the zone layers can be extracted in the horizontal direction of the study area. Figure 9 also shows the subsidence estimate in the study area based on the results of the discretization of the average research results of Abidin et al [12]. The subsidence estimate is shown according to the position of the North-South line according to the model trajectory in Figure 9.

Figure 10a shows the initial model with variations in topography, variations in density, and variations in the thickness of the compacted zone in the horizontal direction. Subsidence information is shown with a vector arrow in the vertical direction in Figure 10b. The information in Figure 10 is used for the computational input of the TLM anomaly as shown in Figure 11a. The density change over time as shown in Figure 11b is $15 \times 10^{-4} \text{g/cc}$. The computational results by accommodating the input data in the study area gave TLM anomalies that reached $10 \mu\text{Gal}$. The descriptive statistics and histograms of the computational results of the TLM anomaly along the North-South path are shown in Figure 12.

![Figure 9](image9.png)

**Figure 9.** Illustration of discretization of subsurface geological models based on the publication of Nurliana and Widodo [11] and also the estimated average subsidence based on the publication of Abidin et al [12].

![Figure 10](image10.png)

**Figure 10.** Illustration of the initial model and vector changes in the vertical direction in the study area. (a) Initial model with varying topography, density, and thickness of the compacted zone in the horizontal direction. (b) Vector change in the vertical direction (subsidence direction) occurs at certain intervals.
Figure 11. Computational illustration using information in the study area: (a) TLM anomaly and (b) density change model due to subsidence at a certain time interval (subsidence is shown as a vector in the vertical direction).

Figure 12. Histogram and descriptive statistics of the TLM anomaly calculated from input data in the study area along the North-South section.

5. Conclusion
In this study, a simple design for TLM modeling of near-surface density redistribution with subsidence estimation results has been shown. A simple numerical simulation on an initial model that has a flat topography, homogeneous density, and homogeneous compressed zone thickness is carried out consecutively for variations in geological and hydrological information that are often found in a study area. The estimation results with numerical simulations of the change in the position of the measurement point and the change in density to the initial model at certain time intervals give TLM anomaly 0.78 to 28.61 μGal/m. Additional algorithms to accommodate topographic variations, initial model density variations, and compacted zone layer thickness variations can be used to obtain computational estimates of the results in the study area. The estimation results in the study area by accommodating the annual trend of subsidence rate gave a time-lapse microgravity anomaly response reaching 10 μGal.

References
[1] Tempone P, Landrø M and Fjær E 2012 4D gravity response of compacting reservoirs: Analytical approach Geophysics 77 G45-G54
[2] Bonafede M and Mazzanti M 1998 Modelling gravity variations consistent with ground deformation in the Campi Flegrei caldera (Italy) Journal of Volcanology and Geothermal Research 81 137-157
[3] Battaglia M and Segall P 2004 The Interpretation of Gravity Changes and Crustal Deformation in Active Volcanic Areas *Pure appl. geophys* **161** 1453-1467

[4] Battaglia M, Gottsmann J, Carbone D and Fernández J 2008 4D volcano gravimetry *Geophysics* **73** WA3-WA18

[5] Kobe M, Gabriel G, Weise A and Vogel D 2019 Time-lapse gravity and levelling surveys reveal mass loss and ongoing subsidence in the urban subrosion-prone area of Bad Frankenhausen, Germany *Solid Earth* **10** 599–619

[6] Wahyudi E J, Dahrin D and Alawiyah S 2010 Interpretasi Data Anomali Gayaberat Mikro Antar Waktu untuk Menentukan Pola Pergerakan Air Tanah di Semarang *Buletin Meteorologi Klimatologi dan Geofisika* **6** 287-294

[7] Hunt T M and Sugihara M 2000 Correcting for Effects of Ground Subsidence Microgravity Monitoring *New Zealand Geothermal Workshop* 109-114

[8] Jacob T, Bayer R, Chery J and Le Moigne N 2010 Time-lapse microgravity surveys reveal water storage heterogeneity of a karst aquifer *Journal of Geophysical Research* **115** 1-18

[9] Olsson P–A, Milne G, Scherneck H–G and Ågren J 2015 The relation between gravity rate of change and vertical displacement in previously glaciated areas *Journal of Geodynamics* **83** 76-84

[10] Wahyudi E J, Santoso D, Kadir W G A and Alawiyah S 2014 Designing a Genetic Algorithm for Efficient Calculation in Time-Lapse Gravity Inversion *J. Eng. Technol. Sci.* **46** 59-79

[11] Nurliana L and Widodo L E 2009 Potensi Imbuhan dan Imbuhan Airtanah Cekungan Airtanah Bandung *JTM XVI* 261-268

[12] Abidin H Z, Gumilar I, Andreas H, Sidiq T P and Fukuda Y 2011 Study on Causes and Impacts of Land Subsidence in Bandung Basin, Indonesia *FIG Working Week Bridging the Gap between Cultures* 1-18