Design and analysis of a novel yokeless mover permanent magnet linear generator for free piston engine converter

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Abstract
Free piston engine generator (FPEG) is one of the recent solutions by researchers used in hybrid vehicles, which eventually can contribute to the reduction of environmental pollutants from public transports. The purpose of the this work is to introduce a novel dual-stator permanent magnet linear generator with yokeless mover (DSPMLG) to improve the key parameters of the linear generator used in FPEG systems. In this work, an analytical model of the DSPMLG magnetic field is developed to investigate magnetic field flux density distribution and for back-EMF prediction. Then, a two-dimensional (2D) model of the DSPMLG is put forth using the finite element method (FEM) to validate the analytical results. In the next section, 1D modelling and simulation of a two-stroke free-piston engine (FPE) with dual-piston structure is done using Ricardo’s WAVE software. The obtained results are used in the FEM simulation and prototype development in order to accurately evaluate the performance of the introduced generator in the FPEG system. Since the FPE velocity profile distorts the back-EMF, the effect of the velocity profile on back-EMF distortion is investigated using fast Fourier transform. Also, the DSPMLG is optimised for minimum back-EMF distortion, considering the maximum output power. To validate the simulation results, the DSPMLG is developed and the obtained experimental results have proper conformity with the simulation results. Finally, in order to weigh up the effectiveness of the proposed generator, its structure is compared with other reported structures in previous research studies based on the key indicators required for the linear generator used in FPEGs.

1 | INTRODUCTION

With reference to the latest report from the UNFCCC, the Earth experiences temperature changes above 1.5°C annually, which, if ignored, could bring about a catastrophe in the years to come. Analyses indicate that public transport causes 16% of this temperature increase [1]. Over the past years, considerable efforts have been made to replace conventional public transport with new vehicles capable of working with clean fuels, which can reduce the emission of environmental pollutants caused by public transport by up to 43% [2]. The utilisation of the new FPEG structure is one of the solutions proposed by researchers to be implemented in series hybrid vehicles [3]. However, efforts to improve the performance of this system still continue. The FPEG consists of a free-piston engine (FPE) coupled with a linear generator. The three-dimensional (3D) structure of the FPEG is illustrated in Figure 1. An FPE is a linear crankless combustion engine, which operates without any rotating mechanism [4]. The engine operates directly via dynamic balancing of the longitudinal forces acting on a single moving translator, which produces linear reciprocating motion [5]. The FPE shown in Figure 1 is a basic configuration of the dual-piston type of this structure. In dual-piston type configuration, there are two combustion chambers and a linear generator is placed in the middle without any rebound device. Each combustion chamber will act as the rebound device alternately during cyclic operation. The simplicity and the small number of FPE components make it a reliable structure to be utilised in a variety of systems. The FPE structure is compact and efficient and can work with renewable energy and clean fuels [6–8].
Furthermore, it makes it possible to use advanced combustion strategies, such as homogenous charge compression ignition (HCCI), where combustion is possible when the temperature and pressure are at their optimum values [9].

Another component of the FPEG structure is the linear generator that must be selected based on the specific operating conditions of the system. Among all the types of linear generators, the tubular PM type has received more attention due to its ability to generate a strong magnetic field, its acceptable efficiency, and elimination of fringing and stator-slotting effects [10–12]. In the following paragraph, a summary of previous research on linear generators used in FPEGs is provided.

In [13], a tubular PM linear machine with quasi-Halbach magnetised armature is introduced for use in the FPEG. This structure is optimised through a heuristic approach using leading design parameters for efficiency, power factor, force density, and converter Volt-Amps rating. In this work, power density is not explicitly reported but system efficiency is reported to be 90% at its most optimal state. On the other hand, the thrust coefficient is equal to 13.3 N/A, which is a relatively low value. In [14], the transverse-flux PM linear generator is introduced for use in the FPEG. This structure, compared to other PM linear generators presented in this work, has lower thrust fluctuation and higher efficiency that is approximately 86.5%. Nevertheless, the introduced structure has low power density and power factor. On the other hand, the mover mass is relatively high in this structure. In [15], a tubular PM linear generator with an axially magnetised PM is introduced. In this structure, an axially magnetized PM along with ferromagnetic rings is used to remove the mover back iron and consequently reduce the mover mass. The power density in this structure is approximately equal to 0.528 kW/kg. However, in this structure, thrust fluctuation is increased and efforts have been made to reduce it by changing the size of the end teeth. In [16], a linear induction generator with a tubular structure is used. In the introduced structure, an aluminium tube is used without back iron as the mover, which causes a drastic decrease in the mover mass. However, this structure has very low power density and efficiency. In [17], a linear generator with two internal and external PM movers is introduced in which the power density is higher compared to the other two structures in [14, 15], but due to the use of two moving magnets, the mover mass is higher leading to reduced generator acceleration. In [18], a quasi-Halbach PM linear tubular generator is presented with ferrite magnets for the FPEG. This study focusses on field analysis and does not provide results on mover mass, power density, and thrust. The use of a ferrite magnet to provide 0.25 T flux density, with 9 mm thickness, has increased the mover mass and reduced the power density. In [19], a PM linear generator with a coreless moving-coil and four-stroke FPE is investigated. In this structure, PM cooling is improved due to the placement of the PMs on the stator; however, the mover mass increases significantly (4.3 kg). In [20], a plate moving-magnet linear generator is used. In this structure, the mover mass is reduced, but on the other hand, the power density (0.18 kW/kg) and thrust fluctuation (15.8 N/A) have low values.

A review of previous research reveals that three parameters, high power density, high thrust coefficient, and low mover mass, are the key indicators in selecting the linear generator used in the FPEG. In this work, the new dual-stator PM linear generator with a yokeless mover (DSPMLG) is introduced aiming at improving these indicators. Besides, normal force and back-EMF THD, which are highly important for improving the performance of the FPEG system, have been evaluated. This work fulfils the following objectives:

- The reduction of mover mass in the linear generator used in the FPEG has some advantages such as fast response, high operating frequency, and high acceleration. Due to the complete removal of the back iron, the mover mass is reduced in the introduced structure. Moreover, the use of an internal stator in the useless space of the DSPMLG increases the power density. In addition, due to the type of winding used in the internal and external stators, the produced thrust coefficient increases. The performance comparison of the proposed structure with the previous ones based on the key indicators of the linear generator used in the FPEG system is given in the comparative study section.
- The mover velocity profile of the linear generator used in the FPEG system is highly effective in its performance. In previous research, a sinusoidal velocity profile has been used.
to actuate the generator mover and evaluate the performance of the linear generator. Herein, the FPE is simulated using Ricardo’s WAVE software and its velocity profile is obtained (Section 4.1). To accurately evaluate the performance of the linear generator, the obtained velocity profile is used for simulation and experimental testing. The generator mover’s actuation with the obtained velocity profile causes distortion in the back-EMF produced by the generator. Then, the back-EMF THD is investigated and the introduced structure is optimised to reduce it (Section 4.3).

- Elimination of vibration and the reduction of the normal force on the mover in a linear generator can guarantee its smooth movement, which is essential for the generator used in the FPEG system with high speed and frequency. The presence of high normal force in the previous structures is one of the issues that received less attention, and no solution has been offered to reduce it. The dual-stator structure introduced in this work is a remarkably effective solution to reduce the normal force and vibrations caused by significant reduction of the axial component of the field flux density. This parameter is considered in Section 4.2.

2 | DSPMLG STRUCTURE

Figure 2 depicts a 3D structure of the DSPMLG. Different parts of the generator including coil, internal and external stators, PM, retaining rod, and aluminium shell are made clear in the figure. As shown in the figure, the PM mover in this structure is sandwiched between the cylindrical internal and external stator. The PMs used in this structure are made of NdFeB with \( H_C = 905 \text{ kA/m} \) and \( B_r = 1.2 \text{ T} \). The ring PMs are radially magnetised and placed side by side in reverse polarity using three retaining rods. The retaining rod is made of non-magnetic material with high mechanical strength, which is coupled with the FPE shaft by a ring of the same material. The internal and external stators are made of silicon steel using radial lamination technique. The external stator is kept fixed by an aluminium shell and the internal stator by a shaft. The machine is equipped with 12 modular windings, 6 of which are embedded on the internal stator and the other 6 are embedded on the external stator and are connected together in series to increase the output voltage. In this structure, since an internal stator is used, a mover back iron is not required to close the flux path. The path of the flux closure is shown in Figure 3.

3 | ELECTROMAGNETIC ANALYSIS

3.1 | Analytical model

There are generally two methods of the finite element method (FEM) and magnetic equivalent circuit to formulate the magnetic field. Being accurate and practical, the FEM is

![DSPMLG flux path](image1)

**FIGURE 3** DSPMLG flux path

![3D structure of the proposed DSPMLG](image2)

**FIGURE 2** (a) 3D structure of the proposed DSPMLG (b) longitudinal section of slotless DSPMLG
a tool used for calculating the magnetic field by considering the non-linear properties of ferromagnetic materials [21, 22]. However, this method is very time consuming and does not provide an insight into the design parameters. The magnetic equivalent circuit method is a simple yet practical method for modelling the magnetic field and creating a mathematical relationship between the design and output parameters. Nevertheless, this method is not accurate enough for complex flux paths [23]. To acquire exact knowledge of the magnetic field that is directly related to the geometry of the machine, an analytical model that can balance the calculation and accuracy is required [24]. As a result, in this work, an analytical model is presented using mathematical modelling of the generator’s magnetic field by series expansion of the solution in Laplace and Poisson equations. Finally, the results obtained from the analytical model are validated by the FEM results.

3.1.1 Magnetic field formulation

The analytical model of the magnetic field can be used to predict machine performance such as its flux linkage, field energy, and produced back-EMF [25, 26]. In the present work, the magnetic field distribution for a DSPMLG shown in Figure 2b is formulated using the Laplace and Poisson equations with these assumptions in mind: (i) the length of the linear machine is assumed to be infinite, (ii) the generator is assumed to be slotless, (iii) the magnetic permeability of the stator is assumed to be infinite.

The DSPMLG is divided into three regions based on the magnetic properties of the material: internal air gap and internal stator winding (Region 1), PMs (Region 2) and external air gap and external stator winding (Region 3). The Laplace equation for the magnetic vector potential \( A \) in Regions 1 and 3 and the Poisson equation for the magnetic vector potential in Region 2 are identified by the following equations in a cylindrical coordinate system:

\[
\frac{\partial^2 A_{1,3,\theta}}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_{1,3,\theta}}{\partial r} \right) = 0
\]  

(1)

\[
\frac{\partial^2 A_{2,\theta}}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_{2,\theta}}{\partial r} \right) = -\mu_0 \nabla \times M
\]  

(2)

where \( \mu_0 \) is the permeability of free space with a value of 4\( \pi \times 10^{-7} \) H/m, \( M = B_{\text{rem}}/\mu_0 \) the residual magnetisation vector in A/m, and \( B_{\text{rem}} \) the remanence. With the series expansions of the solution and the fact that \( A_{\theta} \) must be a periodic function in the \( z \)-direction, the general solution of the Laplace and Poisson equations would be as follows:

\[
A_{\theta} = \sum_{n=1,2,\ldots} \left[ a_n I_1(m_n r) + b_n K_1(m_n r) \right] \cos(m_n z)
\]  

(3)

where \( m \) is a real number, \( I_1(.) \) is the modified Bessel function of the first kind of order one and \( K_1(.) \) is also the modified Bessel function of the second kind of order one.

To calculate a particular solution of the Poisson equation, it seems necessary to write the \( M \) function based on harmonic distribution. As PMs are magnetised only in the \( r \) direction, \( M = M_r \hat{r} \). Besides, PMs are positioned in a way that \( M_r \) is a periodic function with \( 2\tau_p \) period, where \( \tau_p \) is the magnet pole pitch. Thus, the \( M_r \) function can be written as follows:

\[
M_r = \sum_{n=1,2,\ldots} c_n \cos(m_n z)
\]  

(4)

where, \( P_n = \frac{4\mu_0 \sin \left( \frac{(2n-1)\pi}{2} \right)}{\tau_p} \) and \( m_n = \frac{(2n-1)\pi}{\tau_p} \). By substituting (4) in (2) and assuming that the particular solution of the Poisson equation is just a function of \( r \) and \( z \), the particular solution of the Poisson equation can be considered as follows:

\[
A_{\theta} = \sum_{n=1,2,\ldots} \frac{\pi L_1(m_n r)}{2m_n^2} P_n \sin(m_n z)
\]  

(5)

where \( L_1(.) \) is the modified Struve function of order one. Since \( \nabla \times B_i = -\nabla^2 A_i \) the flux density in Regions 1 and 2 can be obtained with the following equations:

\[
B_{1r} = \sum_{n=1,2,\ldots} -m_n \left[ a_n I_1(m_n r) + b_n K_1(m_n r) \right] \cos(m_n z)
\]  

(6)

\[
B_{2r} = \sum_{n=1,2,\ldots} -m_n \left[ a_n I_1(m_n r) + b_n K_1(m_n r) \right] \cos(m_n z) + \frac{\pi L_1(m_n r)}{2m_n^2} P_n \cos(m_n z)
\]  

(7)

\[
B_{3r} = \sum_{n=1,2,\ldots} -m_n \left[ a_n I_1(m_n r) + b_n K_1(m_n r) \right] \cos(m_n z)
\]  

(8)

\[
B_{1z} = \sum_{n=1,2,\ldots} m_n \left[ a_n I_0(m_n r) - b_n K_0(m_n r) \right] \sin(m_n z)
\]  

(9)
\[ B_{2z} = \sum_{n=1}^{\infty} m_n \left\{ a_{2n} I_0(m_n r) - b_{2n} K_0(m_n r) \right\} \sin(m_n z) + \frac{\pi L_0(m_n r)}{2m_n^2} P_n \sin(m_n z) \]  

\[ B_{2\ell} = \sum_{n=1,\ldots} m_n [a_{2\ell} I_0(m_n r) + b_{2\ell} K_0(m_n r)] \sin(m_n z) \]  

where \( I_0() \) is the modified Bessel function of the first kind of order zero; \( K_0() \) is also the modified Bessel function of the second kind of order zero; \( L_0() \) is the modified Struve function of order zero.

The coefficients of the magnetic flux density are calculated using boundary conditions. According to the introduced structure and the two principles (1, the component perpendicular to the surface of the flux density changes continuously in two adjacent regions; 2, the tangential component of the magnetic field intensity changes continuously in two adjacent regions as the surface current is considered zero in this structure), the boundary conditions are taken as follows:

\[ B_{1z}|_{r=R_1} = 0, B_{2z}|_{r=R_4} = 0 \]

\[ B_{2z}|_{r=R_3} = B_{1z}|_{r=R_1}, H_{2z}|_{r=R_3} = H_{1z}|_{r=R_3} \]

\[ B_{2z}|_{r=R_2} = B_{1z}|_{r=R_2}, H_{2z}|_{r=R_2} = H_{1z}|_{r=R_2} \]  

The constants of the equations are calculated as follows:

\[ a_{1n} = a_{2n} + \alpha, b_{2n} = b_{1n} + \beta, a_{3n} = a_{2n} + \gamma, b_{2n} = b_{3n} + \xi \]

\[ a_{1n} = \frac{c_2}{c_1} b_{1n}, a_{3n} = \frac{c_0}{c_2} b_{3n} \]  

where,

\[ \alpha = \frac{c_5 c_7 + c_4 c_6}{c_5 c_7 + c_4 c_6} \pi P_p \frac{\pi}{2m_n^2} \]

\[ \beta = \frac{c_1 c_8 + c_5 c_6}{c_1 c_4 + c_1 c_7} \pi P_p \frac{\pi}{2m_n^2} \]

\[ \gamma = \frac{c_1 c_{15} + c_2 c_{16}}{c_1 c_{15} + c_2 c_{14}} \pi P_p \frac{\pi}{2m_n^2} \]

\[ \xi = \frac{c_1 c_{16} + c_1 c_{14}}{c_1 c_{16} + c_1 c_{15}} \pi P_p \frac{\pi}{2m_n^2} \]  

where,

\[ I_0(m_n R_1) = c_1, K_0(m_n R_1) = c_2 \]

\[ I_1(m_n R_2) = c_3, K_1(m_n R_2) = c_4, L_1(m_n R_2) = c_5 \]

\[ I_0(m_n R_3) = c_6, K_0(m_n R_3) = c_7, L_0(m_n R_3) = c_8 \]

\[ I_0(m_n R_4) = c_9, K_0(m_n R_4) = c_{10} \]

\[ I_1(m_n R_5) = c_{11}, K_1(m_n R_5) = c_{12}, L_1(m_n R_5) = c_{13} \]

\[ I_0(m_n R_5) = c_{14}, K_0(m_n R_5) = c_{15}, L_0(m_n R_5) = c_{16} \]

The stator slotting effect of the slotted DSPMLG has been considered using Carter’s coefficient.

### 3.1.2 Back-EMF formulation

Since the back-EMF is derived from the flux linkage \( \lambda \), to calculate the back-EMF, the flux linkage must be calculated first. A flux linkage is obtained from the sum of the fluxes passing through the windings \( \phi \). So, the flux linkage in internal and external windings are as follows:

\[ \lambda_E = N_w \phi_E, \lambda_I = N_w \phi_I \]  

in which \( N_w \) is the winding turns. The flux passing through a winding in \( \phi_w \) position is calculated as per the equation below for internal and external stators:

\[ d\phi_E = B_{3\gamma}(z)2\pi R_w dz \]

\[ d\phi_I = B_{1\gamma}(z)2\pi R_w dz \]

Note that \( z = z_0 + \nu t \), where \( z_0 \) is the initial position of the winding and \( v \) is the mover velocity. Henceforth, the back-EMF for each phase \( e_{ph} \) is calculated as follows:

\[ e_E = -\sum_{j=1}^{m} N_w \frac{d\phi_{Ej}}{dt} \]

\[ e_I = -\sum_{j=1}^{m} N_w \frac{d\phi_{Ij}}{dt} \]

\[ e_{ph} = e_E + e_I \]  

### 3.2 Validation with FEM model

This section validates the formulation obtained previously in Section 3.1 by FEM. The main design parameters of the DSPMLG are given in Table 1. \( R_{f-R_2}, R_{ws}, \) and \( R_{as} \) are defined in Figure 2 and \( \tau_{PM} \) is the PM pitch. Analytical solutions and FEM calculations will be obtained for both the slotless and slotted DSPMLG.
Figure 4 illustrates the radial component of the magnetic field ($B_r$) at the centre of the outer air gap ($r = 42.5$ mm for the slotless model and $r = 35.6$ mm for the slotted model) and the axial component of the magnetic field ($B_z$) at the centre of the PM ($r = 32.5$ mm) obtained using FEM and analytical calculations in the slotless and slotted DSPMLG. As predicted, the $B_r$ around the $z = 0$ axis is even symmetric. Since $B_r$ distribution is sinusoidal in the air gap, it causes the produced back-EMF to be sinusoidal and to lack harmony. Unlike $B_r$, $B_z$ around the $z = 0$ axis is an odd symmetric function. Depending on the type of dual stator structure, $B_z$ has a very small value, which in turn will reduce the normal force and vibration of the linear machine. As can be deduced from the results, the analytical results are properly aligned with the FEM results. Figure 4 indicates that in the slotted DSPMLG, $B_r$ has been increased and its distribution becomes quasi-rectangular rather than sinusoidal in pattern.

Figure 5 represents the back EMF of all three phases when the generator mover is actuated with a constant velocity of 4 m/s obtained using the analytical calculations and FEM calculations, which indicates strict conformity between the analytical and FEM results. Commensurate with predictions, the back-EMF waveform is sinusoidal with respect to the radial flux density waveform in Region 1, yet fast Fourier transform (FFT) is used to investigate the harmonics in the back EMF more accurately. As illustrated in Figure 6, in addition to the fundamental frequency, only harmonics of order 3, 5, and 7 are actuated and the waveform is noise free. The total harmonic distortion (THD) values for phases a, b, and c are 10.8%, 12.4%, and 9.9%, respectively.

4 | PERFORMANCE ANALYSIS

To evaluate the performance of the generator used in the FPEG, the generator mover was actuated with a sinusoidal velocity profile in previous research. However, in this work, in order to accurately study the performance of the proposed structure, a FPE is simulated using Ricardo’s WAVE software to achieve the mover position and velocity profile in Subsection 4.1. Then in Subsection 4.2, the DSPMLG is simulated and investigated with the obtained velocity profile. In Subsection 4.3, the distortion in the back EMF is minimised by optimising some performance and dimensional parameters. Finally, the DSPMLG is evaluated magnetically with the final dimensional specifications.

4.1 | FPE simulation

From a combustion cycle point of view, FPEs can function as two-stroke and four-stroke engines. In the four-stroke structure, the linear electric generator must operate as the motor in the intake, compression, and exhaust cycles and as the generator in the power cycle, making the piston move...
erratically and causing complexity in the linear machine commutation [27]. However, in a two-stroke structure, a linear machine always operates as a generator. Moreover, in a two-stroke structure, the heat release process is close to the constant volume process, which lowers the maximum cylinder pressure and compression ratio. Thus, using a two-stroke cycle reduces noise, increases efficiency, and decreases residual gases in the cylinder. There are several combinations of FPEs’ structure and linear generators, but the dual-piston structure (Figure 1) is more suitable for generating electrical energy with an FPEG due to the elimination of the rebound device [28]. According to the explanations given in this section, a two-stroke dual-piston engine has been selected in this work and the performance of the engine has been simulated and evaluated.

Modelling and simulation of a 1D two-stroke FPE with dual-piston structure has been done with Ricardo’s WAVE 2019.1/IGNITE 2018.1 software. To simulate the FPE, a two-stroke crankshaft commercial engine, called the 2-MIX Stihl, is initially selected as the base model. The engine specifications are given in Table 2. The selected crankshaft engine is transformed into a two-stroke free-piston motor using the imposed piston submodel. The simulation in Ricardo’s WAVE software is undertaken using three sub-models as shown in Figure 7. Also, WAVE build is used as a preprocessor for the simulations’ initial setup. The geometric properties of the model and the boundary conditions in this submodel are converted to a suitable input format for the solver. WAVE solver is used for solving 1D dynamic and thermodynamic time-dependent equations.

Ultimately, using WAVE post, which is a postprocessor, the results are prepared in the required forms. The simulation is run for frequencies from 10 to 80 Hz. The variations in the FPE output power for different frequencies are shown in Figure 8. As the figure shows, the output power is maximised for the frequencies between 35 and 75 Hz, while at the frequency 55 Hz, the maximum output power is obtained from the simulated FPE. Since the scavenging process for the two-stroke FPE depends on the exhaust valve opening (EVO) and intake valve opening (IVO), in Figure 9 the intake and exhaust valves timing are shown for the 55 Hz frequency. Depending on how the intake and exhaust valves are actuated, the position and velocity profiles of the piston during the steady state are shown in Figure 10.

### 4.2 DSPMLG performance

The specific generator parameters are listed in Table 3. Based on sizing equations of the linear electrical machine and the design consideration of the linear generator used in the FPEG systems, dimensional parameters of the DSPMLG are calculated and presented in Table 4. The DSPMLG is simulated in 3D by FEM in the Maxwell software. Figure 11 demonstrates

| Table 2 | The specification of Stihl 2MIX engine |
|---------|---------------------------------------|
| Parameter | Value |
| Capacity (cc) | 65 |
| Bore (mm) | 50 |
| Stroke (mm) | 45 |
| Geometric compression | 9.5:1 |
| Valve lift (mm) | 4 |
| Intake valve diameter (mm) | 20 |
| Exhaust valve diameter (mm) | 18 |
| Maximum power (kW) | 2.6 |

![Three primary subprograms in Ricardo WAVE software](image)

**Figure 7** Three primary subprograms in Ricardo WAVE software

![FPE output power obtained from Ricardo’s WAVE simulation](image)

**Figure 8** FPE output power obtained from Ricardo’s WAVE simulation

![Intake and exhaust valves timing for FPE (at 55 Hz)](image)

**Figure 9** Intake and exhaust valves timing for FPE (at 55 Hz)
the FEM 3D model along with longitudinal cross-section of the DSPMLG after meshing. As can be seen in this figure, triangular selective meshing is applied to the model, which improves the accuracy of the results. For a detailed study of the DSPMLG results, the mover is actuated with the velocity and position profile shown in Figure 10.

As discussed earlier in Section 3.2, one of the major benefits of this structure is the reduction of the axial flux density. Since normal force is generated by the axial magnetic field, it is anticipated that the normal force will decrease as the axial flux density reduces. Figure 12 shows the normal force on the mover. Consistent with the figure, the normal force is 710 N. As can be seen from this figure, the normal force generated by the internal and external stators are in opposite directions and most of the normal force generated by the external stator winding is neutralised by the normal force generated by the internal stator winding. As a result, it reduces the normal force on the mover.

Figure 13 depicts the FFT associated with no-load back-EMF when the mover is actuated with the velocity profile in Figure 10. As shown in Figure 13, the back-EMF fundamental frequency generated by the DSPMLG is 165 Hz. the back-EMF of all three phases has harmonics of order 2, 3, 4, and 5 in addition to the fundamental frequency. The FFT results indicate that the generated back-EMF, in addition to the mentioned harmonics, has a significant amount of noise, and consequently, to measure the harmonic distortion of the back-EMF waveform, THD + N (total harmonic distortion plus noise) has been applied. THD + N is the residual signal with only the fundamental component removed. It is important to note that the THD measurement does not include noise terms, while THD + N does. Often when a THD specification is quoted, it is really a THD + N specification since most measurement systems do not differentiate harmonically related signals from other signals [29]. The THD + N values for phases a, b, and c are 52.6%, 66.8% and 28.2%, respectively. The THD value is very different for various phases. This difference suggests that the position of the winding affects the generated harmonics. Thus, since the value of the harmonics in each phase changes according to the position of the winding, the following index has been used for comparison:
\[ THD_{\text{total}} = \text{Average}(THD_{\text{phase a}}, THD_{\text{phase b}}, THD_{\text{phase c}}) \] (19)

Therefore, the total back-EMF THD when the mover is actuated with the velocity profile in Figure 10 is 49.2% compared with the THD obtained from Section 3.2 (constant velocity), it is evident that the change in the mover velocity profile is the reason for the harmonics created in the back-EMF.

### 4.3 Back-EMF THD Reduction

In this section, parameters affecting the back-EMF THD have been optimised to reduce the back-EMF THD to its minimum value, taking into account the output power of the DSPMLG within an acceptable limit. The method used for optimisation is simple, innovative, and multi-stage. At the end of each stage, the obtained outputs are analysed and the optimisation process’ continuation depends on the design requirements. The optimisation process includes five steps as follows:

a. Determining the parameters affecting the back-EMF.

b. Determining the selected parameters’ range to reduce back-EMF THD and keep the output power within the possible range.

c. Simultaneous optimisation of selected parameters in order to find the common optimal range for minimum back-EMF THD and maximum output power.

d. Choosing the most optimal result.

e. Comparing the optimised design with the initial design.

The relationships obtained from the analytical model in Section 3.1 show that the mover translating frequency is effective on the back EMF. On the other hand, it is clear from the structure that the level of alignment of PMs and stator teeth is impressive on the distribution of air gap flux density and also the back EMF. Therefore, the PM pitch \( \tau_{PM} \) to slot pitch \( \tau \), ratio, which is called \( K_{PM} \), is considered as a parameter affecting the back EMF and consequently, its THD. Hence, the two parameters of the mover translating frequency and \( K_{PM} \) are considered to be evaluated in the back-EMF THD reduction process. It is noteworthy that the PM’s thickness and the air gap length on both sides have been kept constant to prevent saturation in the yoke and stator teeth.

In the next stage of optimisation, the \( K_{PM} \) value is considered constant and so is the frequency change in the range of 35–75 Hz, which is the range to achieve the maximum output power of FPE. Figure 14 shows the back-

![Figure 12](image12.png) **Figure 12** Normal force on the mover (at load = 40Ω and velocity = 4m/s)

![Figure 13](image13.png) **Figure 13** FFT associated with no-load back EMF (mover is actuated with FPE velocity profile)
EMF THD variations and the output power of the DSPMLG with frequency. As can be seen from the figure, the range of 45–65 Hz can be taken as the optimal range for the minimum back-EMF THD considering the maximum output power of the DSPMLG. Then, the frequency value to receive the maximum power from FPE (55 Hz) is considered as the mover translating frequency and $K_{lpm}$ is changed. Figure 15 illustrates the changes in the back-EMF THD and the output power of the DSPMLG with $K_{lpm}$. As evident in the figure, the range of 0.78–0.9 can be considered as the preferred range for achieving minimum back-EMF THD considering the maximum output power from the DSPMLG.

In the next stage, the frequency and $K_{lpm}$ are simultaneously optimised in the range obtained from the previous stage to achieve the minimum back-EMF THD and the maximum output power. Figure 16 shows the output power and back-EMF THD variations with frequency and $K_{lpm}$. As shown in the figure, in the range of 47.5–53.7 Hz and 0.81–0.88 of $K_{lpm}$, the back-EMF THD rate does not exceed 40% while this range provides output power between 1085.6 and 1267.5 W. Thus, frequency 49 Hz and $K_{lpm}$ 0.88 are considered as optimal values for these two parameters.

Finally, the back-EMF THD for the optimal values obtained is equal to 29.5%. It decreased 40.04% compared to its value before optimisation. On the other hand, the output

**FIGURE 14** DSPMLG output power and back-EMF THD variations against frequency.

**FIGURE 15** DSPMLG output power and back-EMF THD variations against $K_{lpm}$.

**FIGURE 16** (a) DSPMLG output power variations against frequency and $K_{lpm}$. (b) Back-EMF THD variations against frequency and $K_{lpm}$.
power of the DSPMLG after optimisation is 1245.7 W. It increased 11.2% compared with its value before optimisation.

### 4.4 DSPMLG Electromagnetic Model

Figure 17 illustrates the flux density for the maximum and minimum alignment between the magnet and the stator teeth, respectively. In the maximum alignment, the DSPMLG consists of five magnetic circuits and the maximum flux density in the stator teeth is equal to 1.65 T while in the minimum alignment, the structure has two main magnetic circuits and the maximum flux density in the stator yoke is 1.5 T. It is evident that the flux density in both cases does not exceed 1.8 T and consequently, no saturation is indicated. As shown, in this structure the flux lines rotate in short paths that reduce iron losses. On the other hand, due to the removal of the back iron and the use of two stators on both sides of the PM, the leakage flux is negligible, which is one of the advantages of this structure. In both the cases (maximum and minimum alignment), the field flux density at the air gap is equal to 0.8 T.

### 5 EXPERIMENTAL RESULTS

To study the mechanical and movement constraints and to prevent any failure in the prototype manufacturing process, all the DSPMLG components and the motion transmission system are designed and simulated in SOLIDWORKS 2017 (Figure 18). In the following section, after studying all the montage and movement constraints, the construction drawing of all designed parts are provided in the DRAWING environment of the software. Finally, to validate the simulation results the DSPMLG is manufactured and tested.

The measurement system and the test table are shown in Figure 19. The internal and external stators are made of silicon steel using the radial lamination technique. In both stators, 2205 TSA epoxy VPI resin with a thermal index of up to 220°C is used to increase the packing coil factor. In order to clarify the mover structure details, the assembled mover along with the mover’s disassembled parts are shown in Figure 20. As shown clearly, the mover is comprised of nine PM rings, four retaining rods to hold the magnet rings next to each other, six steel rods, and two end earrings to transfer the movement from the prime mover to the mover. It should be noted that detailed properties of the mover material are presented in Table 5. A linear variable differential transformer is used to measure mover position and speed.

Mainly in previous research works that focused on the design and analysis of the performance of linear generators in FPEGs, a high-speed and high-frequency linear actuator or a rotating electric motor and a crankshaft were used to actuate the generator mover to reduce costs and more importantly, to focus on the linear generator’s performance and eliminate possible FPE-related errors. The deficit in those systems is that the prime mover (linear actuator or rotating motor and crankshaft) does not fully implement the FPE performance, but in this research, the position and velocity profile of the simulated FPE obtained in Subsection 4.1, using a rotating motor and crankshaft with variable frequency drive (VFD), are applied to actuate the prototype linear generator mover. Considering the 49 Hz frequency and the maximum speed of 4 m/s, a 3600 rpm motor with 3.7 kW output power has been used. Considering the maximum mover stroke of 35 mm, the
crankshaft radius is 17.5 mm. A commercial variable frequency drive ABB ACS880-01-032A-E202 (shown in Figure 19) has been used to control the speed of the rotating motor and, accordingly, the velocity of the prototype DSPMLG mover.

The rotating motor speed is controlled using VFD with the open-loop PID method, which is a macro on ABB drives. Figure 21a demonstrates the velocity and position of the DSPMLG mover. As shown, by controlling the rotating motor speed using VFD, the generator mover velocity is very close to that of the simulated FPE. The back-EMF produced by the generator in the no-load condition for the obtained velocity profile is shown in Figure 21b. Clearly, in time 10.25 ms, due to zero velocity, the back EMF of all three phases is zero and the maximum back EMF of all three phases is obtained in the interval of 1.9–7.2 ms when the velocity is almost constant and has the highest value.

Table 6 shows the comparison between the simulation and the measurement results of the resistance and inductance of each phase along with the effective value of the back EMF. A tuning capacitor with an appropriate value (105 µF) is determined to compensate the winding inductance effect. To analyse the performance of the DSPMLG in load condition, output power variations with Ohmic load (shown in Figure 19) for frequencies 47–53 Hz have been evaluated. As illustrated in Figure 22, considering the use of the tuning capacitor in order to eliminate the generator winding inductance effect, irrespective of frequency values, the maximum power is obtained at load 17.5 Ω. As the results show, the output power is increased as the frequency decreases.
Table 5 Material properties of the mover and maximum equivalent stress on mover parts

| Part          | Material                               | Density (kg/m³) | Young's modulus (GPa) | Yield strength (MPa) | Maximum stress in simulation (MPa) |
|---------------|----------------------------------------|----------------|-----------------------|----------------------|-----------------------------------|
| PM            | NdFe₂₃B, sintered                       | 7500           | 160                   | 285                  | 23                                |
| Retaining rod | Polyamide-imide, PA1 Rod, RS Stock No. 257-7085 | 1400           | 12                    | 150                  | 3.6                               |
| End earring   | Carbon steel (Grade CK45)              | 7850           | 200                   | 340                  | 54                                |
| Steel rod     | Carbon steel (Grade CK45)              | 7850           | 200                   | 340                  | 75                                |

Figure 21 (a) Velocity and position profile of the DSPMLG mover (b) produced back EMF of the DSPMLG in no-load condition (mover is actuated with FPE speed profile)

Table 6 Measured and simulation results of electrical parameters

| Parameter       | Unit      | Measured | FEM       |
|-----------------|-----------|----------|-----------|
| Back-EMF (Eₛ)   | Vₑms     | 176      | 180       |
| Inductance (Lₙ) | mH        | 11.2     | 10.7      |
| Resistance (Rₛ)| Ω         | 1.8      | 1.6       |

Figure 23 illustrates the input and output power of the DSPMLG for 17.5 Ω load at 4 m/s and 49 Hz, in one reciprocating cycles of the actuator. The maximum power obtained from the prototype DSPMLG equals 3660W and the average power equals 1473.6 W. Considering the maximum and average simulated power values, there is 6.4% difference. This difference, indeed, can be mainly due to mechanical losses, magnetic properties of the materials, or measurement differences.

Considering the average values of the input and output power, the prototype DSPMLG’s efficiency is equal to 90.2%. Finally, since the weight and volume of the generator are equal to 3.05 kg and 7.7 × 10⁻⁴ m³, the power density relative to the weight and volume of the generator is 0.482 kW/kg and 1912 kW/m³, respectively.

Figure 22 DSPMLG output power variations against load

Figure 24 depicts the thrust coefficient. For this purpose, the mover is kept at zero position, the one phase current is increased to 10 A, and the thrust is measured using a load cell. Since the mover is not moved, friction has not affected the results. As clarified in the figure, for currents above 7 A the relationship between the thrust and the current is non-linear, and this is due to the saturation of the used iron core. For currents less than 6 A, the thrust coefficient is equal to 51 N/A.
Since mechanical stress analysis of the mover (whole moving body as illustrated in Figure 20) is more important compared to other components of the DSPMLG, mechanical stress is only analysed for the mover. Transient dynamic analysis for the velocity profile in Figure 21 and the normal force in Figure 12 is performed to assess stress intensity in the moving body. In this regard, a finite element model using the ANSYS Workbench 2020. R1 software is prepared. To reach good accuracy, the maximum time step is selected as $10^{-6}$ s. In addition, a 0.01 proportional damping ratio is considered and the simulation is performed long enough to reach the steady-state solution. Table 5 presents the material properties considered for the analysis.

Figure 25a depicts the maximum equivalent stress on the mover. Moreover, as the mechanical stress on PMs and retaining rods is of immense importance, the stress on them is shown separately in Figure 25b,c. The values of the maximum stress on the all parts during the analysis together with the yield strength of the materials based on the datasheet provided by the manufacturer are given in Table 5. Considering the high difference between the maximum stress that the parts experience and the yield strength of the materials, it is evident that none of the parts would fail.

Figure 26 illustrates the maximum stress on the mover during three reciprocations and shows that the stress intensity is acceptable.

### 7 | COMPARATIVE STUDY

In this section, the structure proposed in this work (DSPMLG) and several linear generator structures with yokeless mover, which have been previously introduced by researchers to decrease mover mass for the FPEGs’ use, are compared based...
Structure A is a yokeless tubular induction linear generator that has been introduced and optimised to increase power density. The structure of this generator is comprehensively explained in [13]. Structure B is a tubular coreless moving-coil linear generator that has been introduced to decrease mover mass and facilitate PMs' cooling. The structure and functional specifications of this generator are given in detail in [16]. Structure C is a plate moving-magnet linear generator that has been introduced to reduce the mover mass, boost the efficiency, and increase the response receiving speed as explained in [17]. Structure D is a DSPMLG which is introduced in the present work.

With regards to the fact that the size of the introduced machines and consequently, their structural and functional characteristics are different from one another in the previous works, the selected comparative indicators are defined in such a way that they are not affected by the size and nominal characteristics of linear generators. Table 7 shows the quantitative indicators of the four structures mentioned. As evident in the results, generator D (DSPMLG) is a more suitable structure to be used in FPEGs based on the introduced parameters. The DSPMLG has advantages such as high power density, high thrust coefficient, very low normal force, and low mover mass, which are key indicators in the selection of linear machines used in FPEGs.

For qualitative indicators, three indicators have been investigated, including structure complexity and building setbacks, use of special materials, and performance problems.

- **Structure complexity and building setbacks:**

**Structure C:** The use of the generator in FPEGs (high speed and frequency) requires an accurate mechanical structure provision and special building conditions due to the use of two shafts on both sides of the plate, and as stated in the article, the structure will face serious problems for high power generation.

- **Use of special materials:**

**Structure B:** The coil skeleton used to carry the coil must be made of a specific material with special characteristics like being light, tenacious, and non-magnetic, which has not even been mentioned in the work. Besides, in order to prevent the air gap and mover mass from increasing, there exists a space limitation for the coil, and as a result, low turn coils have been used. To compensate for this and increase the produced back-EMF, large magnets have been provided in the stator. Due to the improper use of large magnets, the flux density in the ferromagnetic materials (stator back iron) has reached 2.4 T. That is why an iron with a special B-H curve has been used to prevent back-iron saturation.

- **Performance problems:**

**Structure A:** This generator is not self-actuated, which means that it needs a power source to generate a magnetic flux, at least at the start. Hence, the induction generator must be

![Figure 25](image1.png)

**Figure 25** The maximum equivalent stress distribution contour, (a) whole body, (b) PMs, (c) retaining rods

![Figure 26](image2.png)

**Figure 26** The maximum equivalent stress versus time on quantitative and qualitative comparative indicators. The introduced indicators are defined considering the essential requirements of the linear generator used in the FPEG systems.
connected to a power source to supply the initial power. Considering the independency of the FPEG system for hybrid vehicles and emergency power units from the network, the use of this generator is limited to the FPEG system.

**Structure B:** Due to the use of the coil skeleton and its connection to the shaft with the bolt, the connection point of the coil skeleton with the shaft may lead to a performance deficit considering the high speed and frequency of FPEGs. Moreover, this can also reduce fault tolerance (eccentricity).

As discussed, the introduced structure D does not have an acute problem from the defined qualitative indicators’ standpoint and its only problem, which is common in all PM-moving generators, is about the PMs’ cooling that will be addressed in further studies. To summarise, the DSPMLG structure is a better choice for an FPEG in terms of quantitative and qualitative indicators.

This is to highlight that all presented information and results are extracted and presented from previous research without any change or deviation from the source.

### 8 | CONCLUSION

In this work, the DSPMLG structure is introduced with the aim of improving the key parameters of linear generators used in the FPEG system. The mover mass, which is an important parameter in linear generators used in high frequency and high speed, is reduced in this structure. On the other hand, the power density of the DSPMLG is equal to 0.52 kW/kg, which is higher compared to other previously proposed structures.

| Parameter                              | A   | B   | C   | D   |
|----------------------------------------|-----|-----|-----|-----|
| Total mass power density (kW/kg)       | 0.08| 0.05| 0.18| 0.48|
| Mover mass power density (kW/kg)       | 1.68| 0.78| 1.42| 2.26|
| Volume power density (x10^3 kW/m^3)    | 0.23| 0.17| 0.45| 1.9 |
| Thrust coefficient (N/A)               | \   | 31  | 15.8| 51  |
| EMF coefficient (V/(m/s))              | 34  | 32  | 16.4| 44.1|
| Normal force (N)                       | High| Low | Very low | Very low |

### CONFLICT OF INTEREST

The authors declare no conflict of interest.

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