A global fit to determine the pseudoscalar mixing angle and the gluonium content of the $\eta'$ meson

The KLOE collaboration:

F. Ambrosino, A. Antonelli, M. Antonelli, F. Archilli, P. Beltrame, G. Bencivenni, S. Bertolucci, C. Bini, C. Bloise, S. Bocchetta, F. Bossi, P. Branchini, G. Capon, T. Capussela, F. Ceradini, P. Ciambrone, E. De Lucia, A. De Santis, P. De Simone, G. De Zorzi, A. Denig, A. Di Domenico, C. Di Donato, B. Di Micco, M. Dreucci, G. Felici, S. Fiore, P. Franzini, C. Gatti, P. Gauzzi, S. Giovannella, E. Graziani, M. Jacewicz, G. Lanfranchi, J. Lee-Franzini, M. Martini, P. Massarotti, S. Meola, S. Miscetti, M. Moulson, S. Müller, M. Napolitano, F. Nguyen, M. Palutan, E. Pasqualucci, A. Passeri, V. Patera, P. Santangelo, B. Sciascia, M. Testa, L. Tortora, P. Valente, G. Venanzoni, R. Versaci, G. Xu, dimicco@fis.uniroma3.it

Abstract: We update the values of the $\eta$-$\eta'$ mixing angle and of the $\eta'$ gluonium content by fitting our measurement $R_\phi = \text{BR}(\phi \to \eta'\gamma)/\text{BR}(\phi \to \eta\gamma)$ together with several vector meson radiative decays to pseudoscalars ($V \to P\gamma$), pseudoscalar mesons radiative decays to vectors ($P \to V\gamma$) and the $\eta' \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$ widths. From the fit we extract a gluonium fraction of $Z^2_G = 0.12 \pm 0.04$, the pseudoscalar mixing angle $\psi_P = (40.4 \pm 0.6)^\circ$ and the $\phi-\omega$ mixing angle $\psi_V = (3.32 \pm 0.09)^\circ$. $Z^2_G$ and $\psi_P$ are fairly consistent with those previously published. We also evaluate the impact on the $\eta'$ gluonium content determination of future experimental improvements of the $\eta'$ branching ratios and decay width.

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1Corresponding authors: B. Di Micco dimicco@fis.uniroma3.it
1. Introduction

The \( \eta' \) meson, being almost a pure SU(3)_{flavour} singlet, is considered a good candidate to host a gluon condensate. The question of a gluonium component in the \( \eta' \) meson has been extensively investigated in the past but it is still without a definitive conclusion [1]. We extract the \( \eta' \) gluonium content and the \( \eta-\eta' \) mixing angle in the constituent quark model according to the Rosner [2] approach with the modifications introduced in ref. [3] as described in the following. We use the same method of ref. [4]; in addition, we also introduce in the fit the \( \pi^0 \rightarrow \gamma\gamma \) and \( \eta' \rightarrow \gamma\gamma \) branching fractions according to the prescription of ref. [5]. This method relates our measurement of the ratio \( R_{\phi} = \frac{BR(\phi \rightarrow \eta'\gamma)}{BR(\phi \rightarrow \eta\gamma)} \) [6], to the \( \eta' \) gluonium content and to the \( \eta,\eta' \) mixing angle. The same quantities were extracted in our previous analysis [6] with some assumptions. This has given rise to some objections from refs. [4] and [7]. Here we give an answer to these objections and we repeat the fit taking into account their comments. Then we repeat the fit with recently updated experimental results.

The \( \eta \) and \( \eta' \) states can be represented in the base \(|N\rangle = (|u\bar{u}\rangle + |d\bar{d}\rangle)/\sqrt{2}, |S\rangle = |s\bar{s}\rangle\) and \(|G\rangle = |\text{gluonium}\rangle\) as:

\[
|\eta'\rangle = \cos \psi_G \sin \psi_P |N\rangle + \cos \psi_G \cos \psi_P |S\rangle + \sin \psi_G |G\rangle \\
|\eta\rangle = \cos \psi_P |N\rangle - \sin \psi_P |S\rangle
\]  

(1.1)  
(1.2)

where \( \psi_P \) is the \( \eta-\eta' \) mixing angle and \( Z_G^2 = \sin^2 \psi_G \) is the gluonium fraction in the \( \eta' \) meson. According to ref. [8] the state \(|G\rangle\) could be the \( \eta(1405) \): a pseudoscalar glue ball candidate.
The ratio \( R_\phi = \frac{\text{BR}(\phi \to \eta'\gamma)/\text{BR}(\phi \to \eta\gamma)}{} \) is related to the \( \psi_P \) and \( \psi_G \) parameters by the formula [3]:

\[
R_\phi = \cot^2 \frac{\psi_P}{\cos^2 \psi_G} \left(1 - \frac{m_s}{\bar{m}} \frac{Z_\phi}{Z_s} \tan \psi_V \right)^2 \left(\frac{p_{\eta'}}{p_\eta}\right)^3 \tag{1.3}
\]

where \( p_{\eta'} \) and \( p_\eta \) are the momenta of the \( \eta' \) and \( \eta \) meson respectively in the \( \phi \) reference frame, \( m_s/\bar{m} = 2m_s/(m_u + m_d) \) is the constituent quark masses ratio and \( \psi_V \) is the \( \phi-\omega \) mixing angle. Following ref. [3] we define the constant \( C_q = \langle q\bar{q}_\rho|q\bar{q}_\eta \rangle \) as the overlap between the spatial wave functions of the quark-antiquark pair in the \( \rho \) and the \( \eta \) meson. Isospin symmetry is assumed exact, so that \( m_u = m_d = \bar{m} \) and the following further relations follow:

\[
C_q = \langle q\bar{q}_\eta\rangle = \langle q\bar{q}_\rho\rangle, \quad C_s = \langle s\bar{s}_\eta\rangle \quad \text{and} \quad C_\pi = \langle q\bar{q}_\pi\rangle = \langle q\bar{q}_\eta\rangle \frac{p_{\eta'}}{p_\eta}
\]

where we indicate with \( \langle q\bar{q}_\eta\rangle \) and \( \langle q\bar{q}_\rho\rangle \) the \( q\bar{q} \) spatial wave function in the \( \eta \) and \( \omega \) mesons, and with \( \langle s\bar{s}_\eta\rangle \) the \( s\bar{s} \) spatial wave function in the \( \eta \) and \( \phi \) mesons. The parameters \( Z_q \) and \( Z_s \) are the ratios: \( Z_q = C_q/C_\pi \) and \( Z_s = C_s/C_\pi \). In this model SU(3)\_flavour breaking effects are accounted for by the different values of the effective quark masses, \( m_s > m_u = m_d = \bar{m} \), and by \( Z_q \neq Z_s \).

In our previous analysis [4] the parameters \( Z_s, Z_q, \psi_V \) and \( m_s/\bar{m} \) were taken from ref. [3] where \( \text{BR}(\phi \to \eta'\gamma) \) and \( \text{BR}(\phi \to \eta\gamma) \) were fitted together with other \( V \to P\gamma \) decay rates (\( V \) indicates the vector mesons \( \rho, \omega, \phi \) and \( P \) the pseudoscalars \( \pi^0, \eta, \eta' \)) assuming no \( \eta' \) gluonium content. We fitted [3] our measurement

\[
R_\phi = \frac{\text{BR}(\phi \to \eta'\gamma)/\text{BR}(\phi \to \eta\gamma)}{} = (4.77 \pm 0.09\text{stat.} \pm 0.19\text{syst.}) \times 10^{-3}
\]

together with the available data [3] on \( \Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma), \Gamma(\eta' \to \rho\gamma)/\Gamma(\omega \to \pi^0\gamma) \) and \( \Gamma(\eta' \to \omega\gamma)/\Gamma(\omega \to \pi^0\gamma) \). The dependence of these ratios from the mixing angle \( \psi_P \) and the gluonium content \( \psi_G \) is given by the following equations:

\[
X_{\eta'} = \sin \psi_P \cos \psi_G, \quad Y_{\eta'} = \cos \psi_P \cos \psi_G
\]

\[
\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{9} \left( \frac{m_{\eta'}}{m_{\pi^0}} \right)^3 \frac{5f_\pi}{f_q} \frac{\cos \psi_G}{\sin \psi_P} + \frac{\sqrt{2}f_\pi}{f_s} \cos \psi_G \sin \psi_P \right)^2 \tag{1.4}
\]

\[
\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{Z_q^2}{\cos^2 \psi_V} \left( \frac{m_{\eta'}^2 - m_\rho^2}{m_\omega^2 - m_\pi^2} \cdot \frac{m_\omega}{m_{\eta'}} \right)^3 X_{\eta'}^2 \tag{1.5}
\]

\[
\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{3} \left( \frac{m_{\eta'}^2 - m_\omega^2}{m_\omega^2 - m_\pi^2} \cdot \frac{m_\omega}{m_{\eta'}} \right)^3 \left[ Z_q X_{\eta'} + 2 \frac{m_\bar{m}}{m_s} Z_s \frac{\tan \psi_V Y_{\eta'}}{m_{\eta'}} \right]^2 \tag{1.6}
\]

where \( f_\pi \) is the pion decay constant and \( f_q \) and \( f_s \) are the decay constants of the isospin singlet states (mainly \( \eta, \eta' \) mesons) at no anomaly limit [3]. The fit result was \( \psi_P = (39.7 \pm 0.7)^\circ \) and \( Z_G^2 = \sin^2 \psi_G = 0.14 \pm 0.04, \) \( P(\chi^2) = 49\% \). Imposing \( \psi_G = 0 \) the \( \chi^2 \) probability of the fit decreased to 1%.
In ref. [3], a procedure similar to [3] has been adopted, but they fitted also the
gluonium component in the \( \eta' \) wave function that was previously fixed at zero. The result
\( Z_G^2 = 0.04 \pm 0.09 \) deviates 1σ from our value but with a larger error. In refs. [4] and [7] this
difference was attributed to the use in our fit of the parameters \( Z_s \) and \( Z_q \) obtained in ref. [8]
assuming no gluonium content. However further tests of the fit procedure showed that
\( Z_G^2 \) and \( \psi_P \) are marginally sensitive to large variations of \( Z_q \) and \( Z_s \) [10]. Here we repeat
the fit with a larger number of free parameters, including \( Z_s \) and \( Z_q \).

2. Fit description

In order to enlarge the set of parameters used in the fit we add to the equations (1.4-1.6)
the following further relations that can be derived directly from [4]:

\[
\frac{\Gamma(\omega \to \eta \gamma)}{\Gamma(\omega \to \pi^0 \gamma)} = \frac{1}{9} \left[ Z_q \cos \psi_P - 2 \frac{\bar{m}}{m_s} Z_s \tan \psi_V \sin \psi_P \right]^2 \left( \frac{m_\omega^2 - m_\eta^2}{m_\omega^2 - m_\pi^0_s} \right)^3 \tag{2.1}
\]

\[
\frac{\Gamma(\rho \to \eta \gamma)}{\Gamma(\omega \to \pi^0 \gamma)} = Z_Q^2 \cos^2 \psi_P \left( \frac{m_\rho^2 - m_\eta^2}{m_\rho^2 - m_\pi^0_s} \right)^3 \tag{2.2}
\]

\[
\frac{\Gamma(\phi \to \eta \gamma)}{\Gamma(\omega \to \pi^0 \gamma)} = \frac{1}{9} \left[ Z_q \tan \psi_V \cos \psi_P + 2 \frac{\bar{m}}{m_s} Z_s \sin \psi_P \right]^2 \left( \frac{m_\phi^2 - m_\eta^2}{m_\phi^2 - m_\pi^0 \phi} \right)^3 \tag{2.3}
\]

\[
\frac{\Gamma(\phi \to \pi^0 \gamma)}{\Gamma(\omega \to \pi^0 \gamma)} = \tan^2 \psi_V \cdot \left( \frac{m_\phi^2 - m_\pi^0_s}{m_\phi^2 - m_\pi^0 \phi} \right)^3 \tag{2.4}
\]

\[
\frac{\Gamma(K^{*+} \to K^{+} \gamma)}{\Gamma(K^{*0} \to K^{0} \gamma)} = \left( \frac{2 m_s}{m} - 1 \right)^2 \left( \frac{m_{K^{*+}}^2 - m_{K^{+}}^2}{m_{K^{*0}}^2 - m_{K^{0}}^2} \right)^3 \tag{2.5}
\]

Notice that, differently from [4] where the VP\( \gamma \) couplings are fitted, we fit directly ratios
of partial decay widths. This allows both to reduce the parameters involved in the fit
(two of them cancel out in the ratios), and to simplify the error treatment using quantities
proportional (directly or inversely) to the experimental measurements. As an example, a
ratio of \( \Gamma' \)s is written as:

\[
\frac{\Gamma(\eta' \to \rho \gamma)}{\Gamma(\omega \to \pi^0 \gamma)} = \frac{\text{BR}(\eta' \to \rho \gamma)}{\text{BR}(\omega \to \pi^0 \gamma)} \frac{\Gamma_{\eta'}}{\Gamma_{\omega}}
\]

In this way the correlation matrix among the \( \eta' \) branching ratios and the decay widths can
be used directly. The fit is performed minimising the \( \chi^2 \) function:

\[
\chi^2 = \sum_{i,j} (y_i - y_i^{\text{th}}) (V^{-1})_{ij} (y_j - y_j^{\text{th}})
\]

where \( y_i \) are the experimental measurements of the ratios on the left side of equations (1.4-
1.6) and (2.1-2.5), \( y_i^{\text{th}} \) are the theoretical predictions on the right side of the same equations
and \( V^{-1} \) is the inverse of the covariance matrix. This last quantity is obtained summing
the contribution from the experimental error on branching ratios, decay widths and their
Table 1: Correlation matrix of the \( \eta' \) branching ratios from PDG-2006 fit [9].

| \( \rho \gamma \) | \( \pi^0 \pi^0 \eta \) | \( \omega \gamma \) | \( \gamma \gamma \) | \( 3\pi^0 \) | \( \Gamma_{\eta'} \) |
|-------------------|------------------|-----------------|-----------------|--------|------------------|
| \(-0.34\)         | \(-0.78\)         | \(-0.35\)       | \(-0.26\)       | \(-0.28\) | \(0.32\)         |
| \(-0.29\)         | \(0.32\)          | \(0.24\)        | \(0.26\)        | \(0.35\) | \(-0.02\)        |
|                   |                   |                 |                 | \(0.11\) | \(-0.24\)        |
|                   |                   |                 |                 | \(0.11\) | \(-0.05\)        |
|                   |                   |                 |                 | \(0.09\) | \(-0.88\)        |
|                   |                   |                 |                 |         | \(-0.88\)        |
|                   |                   |                 |                 |         | \(-0.24\)        |
|                   |                   |                 |                 |         | \(-0.05\)        |
|                   |                   |                 |                 |         | \(-0.24\)        |
|                   |                   |                 |                 |         | \(-0.88\)        |
|                   |                   |                 |                 |         | \(-0.05\)        |
|                   |                   |                 |                 |         | \(-0.88\)        |
|                   |                   |                 |                 |         | \(-0.24\)        |
|                   |                   |                 |                 |         | \(-0.05\)        |

correlations and the uncertainty from theoretical inputs. The method is the same of our previous fit [6], but only the parameters \( f_q/f_\pi = 1.00 \pm 0.01 \) and \( f_s/f_\pi = 1.4 \pm 0.014 \) [11] are taken as input, while in the past also \( Z_s, Z_q, \psi_V \) and \( m_s/m_\pi \) were fixed. The parameters \( f_q/f_\pi \) and \( f_s/f_\pi \) involve only the ratio \( \Gamma(\eta' \rightarrow \gamma \gamma)/\Gamma(\pi^0 \rightarrow \gamma \gamma) \). The contribution from theoretical error is evaluated by standard error propagation:

\[
V^{th} = A C A^T
\]

where \( C \) is the covariance matrix of the uncorrelated parameters \( x_1 = f_q/f_\pi \) and \( x_2 = f_s/f_\pi \) and \( A \) is:

\[
(A)_{ij} = \frac{\partial y_{ij}^{th}}{\partial x_j}
\]

The covariance matrix \( V \) is indeed:

\[
V = V^{exp} + V^{th}
\]

where \( V^{exp} \) is the covariance matrix of the data used in the fit. Particularly relevant is the correlation between the \( \eta' \) branching fractions and the decay widths shown in tab. 1.

The \( \eta' \) width and the \( \eta' \rightarrow \gamma \gamma \) branching ratio are 88\% correlated. This is because the \( \eta' \) width is evaluated dividing the \( \Gamma(\eta' \rightarrow \gamma \gamma) \), obtained by measuring the cross section \( \sigma(e^+e^- \rightarrow \eta' e^+e^-) \), by the \( \eta' \rightarrow \gamma \gamma \) branching ratio. The fit results and the correlation matrix are shown in tab. 2. The gluonium fraction \( Z_G^2 = 0.105 \pm 0.037 \) is 2.8\( \sigma \) from zero. In order to identify the measurements which require the presence of the gluonium in the \( \eta' \) we have repeated the fit fixing \( Z_G \) at zero. The results of the fit are shown in tab. 3. The \( \chi^2 \) probability is now quite low, reflecting the 2.8\( \sigma \) effect seen in the previous fit, while the pseudoscalar mixing angle is quite stable.

In fig. 1 we show for each measurement the pulls of the two fits, defined as \( p_i = (y_i - y_i^{th})/\sigma y_i \). The measurement which does not fit in the no-gluonium hypothesis is the ratio \( \Gamma(\eta' \rightarrow \gamma \gamma)/\Gamma(\pi^0 \rightarrow \gamma \gamma) \), whose pull is less than \(-3\), bringing the \( \chi^2 \) probability to 1.1\%. It becomes \(-1\) when gluonium is allowed. We have then repeated the fit without using the \( \Gamma(\eta' \rightarrow \gamma \gamma)/\Gamma(\pi^0 \rightarrow \gamma \gamma) \) information. The result is compared with ref. 4 in tab. 3. The results of the two fits are in agreement, making it evident that the origin of the discrepancy with ref. 4 is due to their neglecting the \( \Gamma(\eta' \rightarrow \gamma \gamma)/\Gamma(\pi^0 \rightarrow \gamma \gamma) \) datum. In ref. 4 the couplings among the vectors and the pseudoscalar mesons are used in place...
of the width ratio. The couplings are related to the partial decay width by the following
formulae:

$$\Gamma(V \rightarrow P\gamma) = \frac{1}{3} \frac{g_{VPP}^2}{4\pi} |\vec{p}_\gamma|^3,$$

$$\Gamma(P \rightarrow V\gamma) = \frac{g_{VPP}^2}{4\pi} |\vec{p}_\gamma|^3$$

In order to make a full comparison between the two methods we have performed the fit
also using the couplings and we have obtain the same results \[12\].
|                | Fit with PDG-2006 | Fit of ref. [4] |
|----------------|-------------------|----------------|
| $\chi^2$/ndf (CL) | 1.8/2 (41%)       | 4.2/4 (38%)    |
| $Z_G^2$         | 0.03 ± 0.06       | 0.04 ± 0.09    |
| $\psi_G$        | $(10 \pm 10)\degree$ | $(12 \pm 13)\degree$ |
| $\psi_P$        | $(41.6 \pm 0.8)\degree$ | $(41.4 \pm 1.3)\degree$ |
| $Z_q$           | 0.85 ± 0.03       | 0.86 ± 0.03    |
| $Z_s$           | 0.78 ± 0.05       | 0.79 ± 0.05    |
| $\psi_V$        | $(3.16 \pm 0.10)\degree$ | $(3.2 \pm 0.1)\degree$ |
| $m_s/\bar{m}$   | 1.24 ± 0.07       | 1.24 ± 0.07    |

Table 3: Comparison among the fit results without the $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ measurements and the results of ref. [4]. PDG-2006 data [9] have been used in both fits.

|                | $Z_G$ free | $Z_G = 0$ fixed |
|----------------|------------|-----------------|
| $\chi^2$/ndf (CL) | 7.9/3 (5%) | 15/4 ($5 \times 10^{-3}$) |
| $Z_G^2$         | 0.097 ± 0.037 | 0 fixed      |
| $\psi_P$        | $(41.0 \pm 0.7)\degree$ | $(41.7 \pm 0.5)\degree$ |
| $Z_q$           | 0.86 ± 0.02   | 0.86 ± 0.02    |
| $Z_s$           | 0.79 ± 0.05   | 0.78 ± 0.05    |
| $\psi_V$        | $(3.17 \pm 0.09)\degree$ | $(3.19 \pm 0.09)\degree$ |
| $m_s/\bar{m}$   | 1.24 ± 0.07   | 1.24 ± 0.07    |

Table 4: Fit results using the PDG-2008 data.

3. Update with the recent PDG results

In the Review of Particle Physics [13] new measurements of the $\rho$, $\omega$, $\eta$ and $\eta'$ mesons have been included, which change slightly the partial decay widths used in the fit. Therefore we repeat the fit using these updated values together with our $R_{\phi}$ measurement. All the correlation coefficients among the measurements are taken into account in the fit. The results of the fit are shown in tab. 4 and the correlation matrix in tab. 5; the pulls of the fit are shown in fig. 2.

The results in tab. 4 show that the gluonium hypothesis is still highly favoured with respect to the null gluonium hypothesis. Nevertheless the fit probability is quite low also in the gluonium hypothesis: it goes from 17% using PDG06 data to 5% using PDG08 data. The reason of the worsening of the fit is found comparing the pulls of the new fit (fig. 2) with the previous one (fig. 1). In particular, the pull of the ratio $\Gamma(\omega \to \eta\gamma)/\Gamma(\omega \to \pi^0\gamma)$ goes from -0.93 using PDG-2006 data
measurements from the best fit is still ∼e the η′(ω to +0.39. This happens because the PDG estimate of the BR(ω to ηγ) to 0.39. This happens because the PDG estimate of the BR(ω to ηγ) has changed from (4.9 ± 0.5) × 10^{-4} to (4.6 ± 0.4) × 10^{-4}, lowering the Γ's ratio and worsening the pull\(^1\).

The BR(ω to ηγ) and BR(ρ to ηγ) PDG values are dominated by the measurement of the e^+e^- → ηγ cross section by SND\(^{[14]}\) as a function of \(\sqrt{s}\) in the \(\rho, \omega, \phi\) mass range. From the measured cross section they extract the \(\rho, \omega\) partial decay widths assuming the Vector Meson Dominance model and a parametrisation for the \(\rho'\) resonance. Some correlation is therefore expected between the \(\rho\) and the \(\omega\) partial decay widths which are not discussed in ref.\(^{[14]}\), moreover the decay widths are model dependent. The average value reported by PDG-2008, BR(ω to ηγ) = (6.3 ± 1.3) × 10^{-4}, is dominated by a model independent measurement\(^{[15]}\) and is 1.2σ away from the PDG fit. Using this value for BR(ω to ηγ) we obtain a much better \(\chi^2\) probability: \(P(\chi^2) = 28\%\) in the gluonium hypothesis and 1.1% fixing the gluonium at zero\(^{[12]}\). Both gluonium content and pseudoscalar mixing angle are unchanged (\(Z_G^2 = 0.11 ± 0.04\), \(\psi_P = (40.6 ± 0.7)°\) in the gluonium hypothesis). Therefore we will use the average value for BR(ω to ηγ) in the following.

4. Update with the new KLOE measurement of BR(ω to π^0γ)

The relations (1.3\(^{[3]}\) and (2.1\(^{[24]}\) are dependent from the \(ω → π^0γ\) decay rate. Recently we have improved the measurement of this branching fraction BR(ω to π^0γ) = (8.09 ± 0.14)\%\(^{[16]}\). This value is about 3σ different from the PDG 2008 value: BR(ω to π^0γ) = (8.92 ± 0.24)\%. We then performed the fit using our measurement of BR(ω to π^0γ).

Moreover \(f_q/f_{\pi}\) and \(f_s/f_{\pi}\) have been fixed according to ref.\(^{[4]}\). In the exact isospin

\(^1\)A fit without the \(Γ(η' → γγ)/Γ(π^1 → γγ)\) ratio has been performed in order to check its effect on the \(ω → ηγ\) and \(ρ → ηγ\) pulls. In this case the \(χ^2/ndf\) of the fit is high also in the null gluonium hypothesis, nevertheless the \(Γ(ω → ηγ)/Γ(ω → π^0γ)\) pull is -0.68 while the \(Γ(ρ → ηγ)/Γ(ω → π^0γ)\) pull is +1.1. In other words the BR(ω to ηγ) fits better while the BR(ρ → ηγ) fits worse. The difference of the two measurements from the best fit is still ∼ 2σ, therefore the poor \(χ^2\) with PDG-2008 data is not due to \(η' → γγ\) but to the inconsistency between Γ(ω to ηγ) and Γ(ρ to ηγ) measurements.
symmetry approximation the relations

\[ f_q = f_\pi; \quad f_s = \sqrt{2f_K^2 - f_\pi^2} \]

hold, where \( f_\pi \) and \( f_K \) are the \( \pi \) and \( K \) decay constants. Therefore \( f_s/f_\pi = \sqrt{2f_K^2/f_\pi^2 - 1} \).

Using \( f_K/f_\pi \) from lattice calculation \[19\] we get:

\[ \frac{f_q}{f_\pi} = 1 \quad \frac{f_s}{f_\pi} = 1.352 \pm 0.007. \]  (4.1)

The results of the fit are shown in tab. 6 and the correlation matrix in tab. 7.

### Table 6: Fit results using PDG-2008 inputs, BR(\( \omega \to \eta \gamma \)) from PDG direct measurement average and the KLOE BR(\( \omega \to \pi^0 \gamma \)) and \( R_\phi \). The equations (4.1) have been used for the \( f_q/f_\pi \) and \( f_s/f_\pi \) parameters.

| \( \chi^2/\text{ndf (CL)} \) | \( Z_G \) free | \( Z_G = 0 \) fixed |
|-----------------------------|---------------|-------------------|
| \( Z_G^2 \)                 | 0.115 ± 0.036 | 0                 |
| \( \psi_P \)                | (40.4 ± 0.6)^0 | (41.4 ± 0.5)^0    |
| \( Z_q \)                   | 0.936 ± 0.025 | 0.927 ± 0.023     |
| \( Z_s \)                   | 0.83 ± 0.05   | 0.82 ± 0.05       |
| \( \psi_V \)                | (3.32 ± 0.09)^0 | (3.34 ± 0.09)^0  |
| \( m_s/\bar{m} \)           | 1.24 ± 0.07   | 1.24 ± 0.07       |

### Table 7: Correlation matrix of the fit shown in tab. 6.

| \( \psi_P \) | -0.507 |
| \( Z_q \)   | 0.063  -0.018 |
| \( Z_s \)   | 0.092  -0.189  0.013 |
| \( \psi_V \) | -0.059  -0.012  0.045  0.028 |
| \( m_s/\bar{m} \) | -0.002  0.003  0.001  0.949  0.000 |
| \( Z_G^2 \) | \( \psi_P \) \( Z_q \) \( Z_s \) \( \psi_V \) |

5. Prospects with improved measurements

We have shown that within the precision of the available data the crucial measurement sensitive to the \( \eta' \) gluonium content is \( \Gamma(\eta' \to \gamma \gamma)/\Gamma(\pi^0 \to \gamma \gamma) \). The theoretical framework
Figure 3: 68% confidence level regions of the shown measurements in the $Z_2^2, \psi_P$ plane.

Table 8: Fitted values of the $\Gamma$ ratios.

| width ratio         | fitted value | error | width ratio         | fitted value | error |
|---------------------|--------------|-------|---------------------|--------------|-------|
| $\Gamma(\eta'\rightarrow\gamma\gamma)$ | 570          | 35    | $\Gamma(\rho\rightarrow\eta\gamma)$ | 0.061        | 0.004 |
| $\Gamma(\phi'\rightarrow\rho\gamma)$ | 0.0735       | 0.007 | $\Gamma(\omega'\rightarrow\pi^0\gamma)$ | 0.072        | 0.003 |
| $\Gamma(\phi\rightarrow\eta'\gamma)$ | 0.0047       | 0.0002| $\Gamma(\phi\rightarrow\eta\gamma)$ | 0.0079       | 0.0004|
| $\Gamma(\phi\rightarrow\omega\gamma)$ | 0.0087       | 0.0010| $\Gamma(K^+\rightarrow K^+\gamma)$ | 0.43         | 0.06  |
| $\Gamma(\omega\rightarrow\pi^0\gamma)$ | 0.0064       | 0.0015| $\Gamma(\phi\rightarrow\pi^0\gamma)$ | 0.0079       | 0.0004|

Table 9: Correlation coefficients of the best-fitted values.

| $\eta'\rho\gamma$ | $\phi\eta'\gamma$ | $\eta'\omega\gamma$ | $\omega\eta\gamma$ | $\rho\eta\gamma$ | $\phi\eta\gamma$ | $\phi\pi^0\gamma$ | $K^+\rightarrow K^+\gamma$ |
|--------------------|--------------------|----------------------|--------------------|------------------|------------------|-------------------|--------------------------|
| 0.28               | 0                  | 0.24 0.48 0          | 0                  | 0.13 0.08 0.05   | 0.28 0.18 0.10 0 | 0.17 0.11 0.06 0.36   | 0 0 0 0 0 0 0 0           |

used to describe $\eta' \rightarrow \gamma\gamma$ and $V \rightarrow P\gamma$ transition is different. In fact while in the first case we have a quark-antiquark annihilation into two photons, in the second case we have a transition among two meson with a photon emission via the spin flip of one of the two
Table 10: Fit values assuming 1% error on the $\eta'$ branching fractions: left: using $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$, right: without using $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$.

|        | with $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ | without $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ |
|--------|------------------------------------------------------|------------------------------------------------------|
| $Z_G^2$ | $0.12 \pm 0.03$                                      | $0.11 \pm 0.04$                                      |
| $\psi_P$ | $(40.5 \pm 0.6)\degree$                              | $(40.5 \pm 0.6)\degree$                              |
| $Z_{NS}$ | $0.93 \pm 0.02$                                       | $0.93 \pm 0.03$                                       |
| $Z_S$    | $0.83 \pm 0.05$                                       | $0.83 \pm 0.05$                                       |
| $\psi_V$ | $(3.32 \pm 0.08)\degree$                              | $(3.32 \pm 0.09)\degree$                              |
| $m_{\rho}/\bar{m}$ | $1.24 \pm 0.07$                                      | $1.24 \pm 0.07$                                      |

quarks. Therefore it is important both to reach a sensitivity to the gluonium independently from the $\eta' \to \gamma\gamma$ decay and to measure again the $\eta' \to \gamma\gamma$ branching ratio.

The $\eta' \to \gamma\gamma$ branching ratio affects directly the $\eta' \to \gamma\gamma$ decay width while the $\eta' \to \pi^+\pi^-\eta$ and the $\eta' \to \pi^0\pi^0\eta$ branching ratios affect the systematic errors on the $R_\phi$ measurement [3]. The increase of the precision on all main $\eta'$ branching fractions at 1% level could lead the gluonium contribution to $\sim 4\sigma$ (see tab. 8). The results shown in the table are obtained assuming as central value of all measurements the width ratios shown in tab. 8 and assigning to all $\eta'$ branching ratios a 1% error (the error on $\Gamma_{\eta'}$ and $R_\phi$ has been recomputed accordingly); the correlation matrix is taken equal to the present measurements. The fit gives a large $\eta'$ gluonium component even without using the $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ measurement (see tab. 10, right column).

In fig. 4 the expected 68% CL contour in the plane $\psi_P, Z_G^2$ is shown. Notice that the $\eta' \to \gamma\gamma$ measurement would be not needed anymore to have a significative gluonium component.

A big improvement in the gluonium content determination can be obtained by measuring the $\eta'$ decay width directly through the measurement of $\sigma(e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta')$. The measurement of this cross section at 1% level and the $\eta' \to \gamma\gamma$ branching ratio at the same level of accuracy would allow to determine the $\eta'$ width:

$$
\Gamma_{\eta'} = \frac{\Gamma(\eta' \to \gamma\gamma)}{BR(\eta' \to \gamma\gamma)}
$$

with a fractional error of $\sim 1.4\%$. In fig. 4, right the 68% CL contours in the $(\psi_P, Z_G^2)$ parameter plane are shown for all $\eta'$ related measurements. The improvement in the fit accuracy is evident. The gluonium contribution would be measured at $\sim 5\sigma$'s.

6. Conclusions

The origin of the difference between the KLOE result [3] on the $\eta'$ gluonium content and ref. [4] is the use of the $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$ measurement. A global fit to all measured $V \to P\gamma$ and $P \to V\gamma$ transitions of light mesons has been performed extracting all the relevant parameters. The new results are slightly different from our previous results but confirm the presence of a significative gluonium contribution in the $\eta'$.
Figure 4: Future prospects: one sigma confidence regions in the $(\psi P, Z^2_G)$ plane assuming present fitted measurements but with reduced errors: left: with 1% error on $\eta'$ branching ratios, right: as before plus 1% precision on $\Gamma(\eta' \rightarrow \gamma\gamma)$.

The gluonium content and the mixing angle are again marginally affected by this measurement while a different $\phi - \omega$ mixing angle is obtained.

Finally, we have estimated the impact of future improvements in the measurements of the $\eta'$ branching ratios and $\eta'$ width in the gluonium content determination. A $5\sigma$ evidence could be obtained with a 1% precision on each measurement.

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