Implications of Non-universal Soft Masses on Gauge Coupling Unification

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Abstract

We study the gauge coupling unification of the minimal supersymmetric standard model with non-universal soft scalar and gaugino masses. The unification scale of the gauge couplings is estimated for non-universal cases. It is sensitive to the non-universality. It turns out that these cases can be combined with the assumption of string unification, which leads to a prediction of $\sin^2 \theta_W(M_Z)$ and $k_1$, the normalisation of the $U(1)_Y$ generator. String unification predicts $k_1 = 1.3 - 1.4$. These values have non-trivial implications on string model building. Two-loop corrections are also calculated. Some of these cases exhibit a large discrepancy between experiment and string unification. We calculate string threshold corrections to explain the discrepancy.
1. Introduction

Supersymmetric models are some of the most promising extensions of the standard model (SM) [1]. For a phenomenologically viable scenario, supersymmetry (SUSY) is broken by soft SUSY breaking terms, scalar masses, gaugino masses and trilinear and bilinear couplings of scalar fields. These soft terms have implications on low energy physics.

Aspects of the soft terms can be derived from underlying supergravity theories or superstring theories. In most of the studies one assumes universality of soft scalar masses and gaugino masses at the unification scale \( M_X \). However breaking of supergravity in general leads to non-universal soft scalar masses and gaugino masses [2, 3, 4]. Further it was shown in ref. [5] how cases where soft scalar masses are fairly different from each other can be obtained. Therefore it is important to study SUSY models with non-universal soft masses and to show which phenomenological features are sensitive or insensitive to such non-universality.

Effects of the non-universality have been studied recently for some phenomenological aspects, e.g. gauge coupling unification, Yukawa coupling unification, radiative symmetry breaking and neutron electric dipole moment [6, 7]. Some types of non-universality are constrained by flavor changing neutral currents (FCNC) [8]. The gauge couplings of the SM groups are unified at \( M_X \approx 3 \times 10^{16}\text{GeV} \) within the framework of the minimal supersymmetric standard model (MSSM) with universal soft masses [4]. In ref. [6] it was shown that the gauge coupling unification of \( SU(3) \) and \( SU(2) \) is sensitive to the non-universality of the soft scalar masses. In most of the non-universal cases, the unification scale \( M_X \) rises higher than \( 3 \times 10^{16}\text{GeV} \). In some cases the unification scale \( M_X \) becomes higher than \( 10^{17}\text{GeV} \) and closer to a string scale \( M_{st} = 5.27 \times g_{st} \times 10^{17}\text{GeV} \) [10] with \( g_{st} \) being a universal string coupling of order one.
One of the strongest arguments in favour of supersymmetric grand unified theories (GUTs) is the unification of the three SM interactions at $M_X \approx 3 \times 10^{16}$GeV and the correct prediction of $\sin^2 \theta_W(M_Z)$ within the experimental bounds \[1\]. Unfortunately, not all features of GUTs are theoretically satisfactory, e.g. the doublet-triplet splitting problem. String theories realize another class of unified theories. One does not need a unified group like $SU(5)$ or $SO(10)$. Each gauge coupling $g_a$ is related to $g_{st}$ as $k_a g_a^2 = g_{st}^2$ at $M_{st}$ \[1\], where $k_a$ is a Kac-Moody level of each gauge group. Hence a direct string unification of the MSSM is free from the theoretical problems of SUSY-GUTs. For levels, we take $k_a = 1$ for $SU(3)$ and $SU(2)$ \[1\]. The normalization of the $U(1)_Y$ generator $k_1$ is treated as a free parameter \[14\].

The purpose of this paper is to study the evolution of the gauge couplings in the MSSM for non-universal cases and to consider their implications from the viewpoint of string theories. Here we discuss non-universal cases where gaugino masses as well as soft scalar masses are non-universal. The non-universality of the gaugino masses entails non-universality of the soft scalar masses by radiative corrections from $M_X$ or $M_{st}$ to the weak scale $M_Z$. To obtain a non-universal case at $M_X$ might require ‘fine tuning’ of parameters like goldstino angles. However, it is important to investigate all possibilities at present. We study the running of the gauge couplings in two ways. One is to run the renormalisation group equations (RGEs) of the gauge couplings bottom-up using experimental data and then to estimate the unification scale in each non-universal case. The other is to run the RGEs top-down assuming string unification and to compare with the experimental data. Results obtained in both ways are considered from the viewpoint of string theories, especially orbifold models \[15\].

This paper is organized as follows. In section 2 we review in brief soft masses derived from supergravity theories. In subsection 3.1 we estimate unification scales of $SU(3)$ and $SU(2)$ in non-universal cases using the exper-

\[1\] Non-abelian Gauge groups with $k_a \neq 1$ are discussed in refs. \[12\] \[13\].
imental values of $\alpha$, $\sin \theta_W$ and $\alpha_3$ and one-loop RGEs of the gauge couplings. Radiative corrections to soft masses are calculated. In subsection 3.2 we assume string unification and run one-loop RGEs from the string scale down to $M_Z$. Two-loop corrections are also computed. In section 4, we discuss results obtained in section 3 from the viewpoint of string theories. In subsection 4.1 we investigate threshold corrections due to massive string modes which are needed to explain the difference between experiment and the prediction by string unification. Implications of $k_1$ are discussed in subsection 4.2. Section 5 is devoted to conclusions and discussions.

2. Soft Masses

In this section we review soft scalar masses and gaugino masses derived from supergravity theories [3]. A supergravity Lagrangian is characterized by a Kähler potential $K$, a superpotential $W$ and a gauge kinetic function $f_a$, where the subscript $a$ represents a gauge group. Here we assume that fields $\Phi^m$ have a non-perturbative potential $\hat{W}(\Phi)$ leading to SUSY breaking. The Kähler potential and the total superpotential are expressed as follows

$$K = \kappa^{-2}\hat{K}(\Phi, \bar{\Phi}) + K(\Phi, \bar{\Phi})_{IJ}Q^I\bar{Q}^J + \left(\frac{1}{2}H(\Phi, \bar{\Phi})_{IJ}Q^I\bar{Q}^J + \text{h.c.}\right) + \cdots,$$

$$W = \hat{W}(\Phi) + \tilde{W}(\Phi, Q), \quad (2.1)$$

where $\kappa^2 = 8\pi/M_{pl}^2$ and $Q^I$ are chiral superfields. The dots stand for terms of higher orders in $Q^I$. We have a scalar potential $V$ as follows,

$$V = \kappa^{-2}e^G[G_\alpha(G^{-1})^{\alpha\beta}G_\beta - 3\kappa^{-2}], \quad (2.2)$$

where $G = K + \kappa^{-2}\log\kappa^6|W|^2$ and the indices $\alpha$ and $\beta$ represent $Q^I$ and $\Phi^m$. Here we do not consider the D-term contribution to $V$. The gravitino mass $m_{3/2}$ is obtained as

$$m_{3/2} = \kappa^2e^{K/2}|\hat{W}|. \quad (2.3)$$
In (2.2) we take the flat limit \((M_{pl} \to \infty)\) while keeping \(m_{3/2}\) fixed. Then we obtain soft scalar masses \(m_{I\bar{J}}\) for canonically normalized fields \(Q^I\) as follows,

\[
m^2_{I\bar{J}} = m^2_{3/2}K_{I\bar{J}} - F^m F^n \partial_m \partial_n K_{I\bar{J}} - \left( \partial_n K_{K\bar{J}} \right) K^{KL} \left( \partial_m K_{L\bar{J}} \right) + \kappa^2 V_0 K_{I\bar{J}},
\]

(2.4)

where \(F^m\) are F-terms of \(\Phi^m\), \(\partial_m\) represent derivatives with respect to \(\Phi^m\) and \(V_0\) is the cosmological constant, which is expressed as

\[
V_0 = \kappa^{-2} (F^m F^n \partial_m \partial_n \tilde{K} - 3m^2_{3/2}).
\]

(2.5)

Further we obtain gaugino masses \(M_a\) as

\[
M_a = F^m \partial_m \ln \text{Re} f_a.
\]

(2.6)

In general the form of the Kähler metric \(K_{I\bar{J}}\) depends on each field \(Q^I\) and the gauge kinetic term \(f_a\) is also dependent on the gauge group. Thus we have non-universal soft scalar masses and gaugino masses. Therefore it is important to study the effects of the non-universality on phenomenological aspects.

For example orbifold models have the following Kähler potential \([16]\):

\[
K = -\log(S + \bar{S}) - \sum_i \log(T^i + \bar{T}^i) + \prod_i \left( T^i + \bar{T}^i \right)^{n_i^I} Q^I \bar{Q}^I,
\]

(2.7)

where \(S\) is the dilaton field, \(T^i\) are moduli fields and \(n_i^I\) are modular weights of \(Q^I\) corresponding to \(T^i\). At tree level the gauge kinetic function is obtained as \(f_a = k_a S\) and it is independent of gauge groups. However one-loop corrections induce \(T\)-dependent threshold corrections, which depend on the gauge groups \([17]\). It is plausible that \(S\) and \(T^i\) contribute to the non-perturbative superpotential \(\hat{W}\) which breaks SUSY. If \(S\) contributes dominantly to the SUSY breaking, we obtain universal soft scalar and gaugino masses \([18]\). Otherwise the soft scalar masses depend on their modular weights and \(T\)-dependent threshold corrections lead to non-universal gaugino masses. Non-universality of soft scalar masses was discussed in refs.\([4, 5]\) under certain
assumptions about $\hat{W}$. In ref. [19] a concrete superpotential induced by gaugino condensation was used to derive soft scalar masses which are universal and heavier than a universal gaugino mass.

3. Running of gauge couplings
3.1 Unification scale

In this section we study effects of non-universal masses on the evolution of the gauge couplings within the framework of the MSSM. First of all we classify non-universalities of soft scalar masses and gaugino masses. For simplicity, we divide sfermions and gauginos into two groups, $\mathcal{A}$ and $\mathcal{B}$. We assume that the superpartners in Group $\mathcal{A}$ have a representative mass $M_S$ ($M_S \geq M_Z$) and the superpartners in Group $\mathcal{B}$ have a mass $M_Z$ at the weak scale. Note that soft scalar masses get radiative corrections of order of gaugino masses at low energies. For example squarks have at least a mass of order of a gluino mass at low energies. Thus we do not consider here the case where $\lambda_3$ belongs to $\mathcal{A}$ while squarks belong to $\mathcal{B}$. We consider the typical cases shown in Table 1, where $\lambda_a$ represent $SU(a)$ gauginos and $Q$, $U$, $D$ and $L$ denote squark doublets, up- and down-squark singlets and slepton doublets respectively. Case I corresponds to the ordinary MSSM with the SUSY breaking scale $M_S$. We assume slepton singlets $E$ and the $U(1)_Y$ gaugino $\lambda_1$ belonging to Group $\mathcal{B}$ in each case except in Case I where all superpartners belong to Group $\mathcal{A}$. Here the generation indices are abbreviated, because we assume degeneracies between the generations in order to avoid FCNC. Further, we assume that the Higgs sector below $M_S$ has the same structure as the SM. In Cases III – VI, soft scalar masses are non-universal. On top of that, gaugino masses are also non-universal in Cases A – D.

Now we study the unification scale of $SU(3)$ and $SU(2)$ gauge couplings in each non-universal case. Note that the gauge coupling of $U(1)_Y$ can always
be unified with the other couplings using a suitable value of $k_1$. We decouple matter fields of Group $\mathcal{A}$ at $M_S$ in the RGEs of the gauge couplings. Then the gauge coupling constant evolves at $\mu (\mu > M_S)$ as follows

$$
\alpha_a^{-1}(\mu) = \alpha_a^{-1}(M_Z) - \frac{\bar{b}_a}{2\pi} \ln \frac{M_S}{M_Z} - \frac{b_a}{2\pi} \ln \frac{\mu}{M_S},
$$

(3.1)

where $b_a$ are the one-loop $\beta$-function coefficients of the MSSM, i.e. $(b_1, b_2, b_3) = (11, 1, -3)$. Note that we take $\alpha_1 = g_1^2/4\pi$, because in our approach $k_1$ is a free parameter. In (3.1), $\bar{b}_a$ denote one-loop $\beta$-function coefficients between $M_S$ and $M_Z$. The values of $(\bar{b}_1, \bar{b}_2, \bar{b}_3)$ are obtained for each case as shown in the fourth column of Table 1.

Taking $\alpha_X = \alpha_3(M_X) = \alpha_2(M_X)$ in (3.1), we obtain the unification scale $M_X$ of the $SU(3)$ and $SU(2)$ gauge couplings as follows,

$$
\ln M_X = \frac{\bar{b}_2 - \bar{b}_3}{b_3 - b_2} \ln \frac{M_S}{M_Z} + \ln M_S + \frac{2\pi}{b_2 - b_3} (\alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z)).
$$

(3.2)

The smaller the value of $(\bar{b}_2 - \bar{b}_3)$ the higher is the unification scale $M_X$, because $b_3 - b_2 = -4$. Here we estimate the unification scales $M_X$ in non-universal cases using $M_Z = 91.173\,\text{GeV}$, $\alpha_2^{-1}(M_Z) = 127.9$, $\sin^2\theta_W(M_Z) = 0.2321$ and $\alpha_3(M_Z) = 0.118$. For Cases I and A – D, the unification scale $M_X$ is shown against $M_S$ in Figure 1, where $x$ and $y$ denote $\log_{10} M_S$ and $\log_{10} M_X$ (in GeV), respectively. In ref. [6], a similar figure on $M_X$ was shown for Case I – VI. For $M_S = 1\,\text{TeV}$, the unification scales $M_X$(in GeV) of all the cases are also found in the second column of Table 2, where the numbers in the parentheses represent the corresponding values of the case with $M_S = \sqrt{10}\,\text{GeV}$. In Case I, $M_X$ is stable against $M_S$. Because of the value $\bar{b}_2 - \bar{b}_3 = 23/6$ being very close to the value of $b_2 - b_3 = 4$ the $M_S$ dependence in (3.2) is suppressed. The unification scale is very sensitive to the non-universality of the gaugino masses. $SU(2)$ gauginos belonging to group $\mathcal{A}$ lead to a higher unification scale, while gluinos belonging to group $\mathcal{A}$ lead to a lower value. Most of the cases II – VI result in a higher unification.
scale than Case I. Note that the estimation of $M_X$ includes an uncertainty of order $10^{0.3}$GeV as usual [9]. Suppose that $\alpha_{st} = g_{st}^2/4\pi = 1/25$, then we have $M_{st} = 3.7 \times 10^{17}$GeV. If we take $M_S = 2.5$TeV for Case A, the unification scale $M_X$ becomes $3.7 \times 10^{17}$GeV. Case III with $M_S = 7.1$TeV and Case B with $M_S = 3.6$TeV lead to $M_X = 10^{17.3}$GeV, which coincides with $M_{st}$ within the uncertainty of $10^{0.3}$GeV.

Furthermore the third column of Table 2 shows values of the unified coupling $\alpha_X^{-1}$. In all cases the value of $\alpha_X^{-1}$ increases as $M_S$ becomes higher. Similarly we can run the $U(1)_Y$ gauge coupling $\alpha_1$ from $M_Z$ to $M_X$. The fourth column of Table 2 shows the ratios $\alpha_X/\alpha_1(M_X)$, which correspond to the values of $k_1$. The value $k_1$ is also sensitive to the non-universality. It takes values from 1.4 to 1.7 in the case with $M_S = 1$TeV. In some cases, e.g., Case V, VI, D, the value is stable around 1.6 against $M_S$. This value corresponds to the GUT prediction $k_1 = 5/3$. Even if $M_S = 10$TeV, Case V, VI and D lead to $\alpha_X/\alpha_1(M_X) = 1.58$, 1.64 and 1.67 respectively. It seems that a higher unification scale corresponds to a smaller value of $k_1$. In all cases with $M_S = M_Z$, we obtain $M_X = 10^{16.4}$GeV, $\alpha_X^{-1} = 24.4$ and $\alpha_X/\alpha_1(M_X) = 1.63$.

Our classification is based on the fact that by radiative corrections squarks are at least as heavy as gluinos and SU(2) doublet sfermions are at least as heavy as SU(2) gauginos, even if these sfermions are massless at $M_X$. Here we investigate this assumption by calculating these radiative corrections. We use the following RGEs,

$$\frac{dM_a}{dt} = \frac{\tilde{b}_a}{2\pi} \alpha_a M_a, \quad (3.3)$$

$$\frac{dm_I^2}{dt} = \frac{1}{2\pi} \left( -4 \sum_a C_a(R_I) M_a^2 \alpha_a + \text{(Yukawa terms)} \right), \quad (3.4)$$

where $t = \ln \mu$ and $C_a(R_I)$ is the quadratic Casimir of the representation $R_I$ corresponding to each scalar field. We neglect the contributions due to the Yukawa terms in (3.4). If SUSY is a good symmetry, then $\tilde{b}_a = b_a$. Thus
we have the relation $M_a(\mu)/\alpha_a(\mu) = \text{constant}$ for $\mu > M_S$. If sfermions decouple at $M_S$, $\tilde{b}_a$ is not always equal to $\bar{b}_a$ for energies below $M_S$. This is because one-loop corrections to the gaugino masses include graphs which have fermions and their superpartners simultaneously in internal lines. Therefore we obtain the following relation below $M_S$ for $\mu < M_S$,

$$
(M_a(\mu))^\tilde{b}_a(\alpha_a(\mu))^{-\bar{b}_a} = \text{constant}. \quad (3.5)
$$

We have $(\tilde{b}_1, \tilde{b}_3) = (6, -6)$ for Case A, $(\tilde{b}_1, \tilde{b}_3) = (3, -9)$ for Case B, $(\tilde{b}_1, \tilde{b}_2) = (9/2, -9/2)$ for Case C and $(\tilde{b}_1, \tilde{b}_2) = (3, -6)$ for Case D.

Using eqs. (3.3 – 3.5) we calculate radiative corrections to the gaugino masses $M_a$ and the soft scalar masses $m_I$ for Case A – D. Tables 3 and 4 show the results in the case with $M_S = 1\text{TeV}$. The second row of Table 3 lists ratios of $M_3(\mu)$ to $M_3(M_X)$, where $\mu = M_Z$ for Case A and B, and $\mu = M_S$ for Case C and D. Similarly corrections to $M_2$ and $M_1$ are found in the other rows. The second column shows these values for Case I for comparison. Table 4 lists corrections to the scalar masses $\Delta m^2_I \equiv m^2_I(m_I) - m^2(M_X)$. It is clear from (3.4) that $\Delta m^2_I$ is represented by a linear combination of $M_3^2(M_X)$, $M_2^2(M_X)$ and $M_1^2(M_X)$. Their coefficients of the linear combinations are found in Table 4. For example the third row for Case A represents $\Delta m^2_Q = m^2_Q(M_S) - m^2_Q(M_X)$ as

$$
\Delta m^2_Q = 5.0 \times M_3^2(M_X) + 0.46 \times M_2^2(M_X) + 4.3 \times 10^{-3} \times M_1^2(M_X). \quad (3.6)
$$

In Table 4 values of $\Delta m^2_U$ are omitted, because they are equal to the corresponding values of $\Delta m^2_U$ except small contributions due to $M_1$. The values in the parentheses of Table 3 and 4 represent the corresponding values of the case where $M_S = \sqrt{10}\text{TeV}$. For example the squark masses get the corrections $\Delta m^2_{Q,U,D} \approx 3.8 \times M_3^2(M_X)$ in Case C or D with $M_S = 1\text{TeV}$. Thus

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\(^2\)Some SUSY relations between several couplings are broken below the SUSY threshold. For example, gaugino-scalar-fermion couplings are independent of the corresponding gauge couplings below $M_S$. That leads to further corrections \[^{[20]}\]. We neglect here such corrections.
even though the squarks are massless at \( M_X \), they have \( m = 1.9 \times M_3(M_X) \) at 1TeV. On the other hand, the scalar fields \( L \) get the correction \( \Delta m_L^2 \approx 0.46 \times M^2_Z(M_X) \) in Case A or B with \( M_S = 1\text{TeV} \). Even though \( m_L \) vanishes at \( M_X \), the scalar field \( L \) acquires \( m_L = 0.68 \times M_3(M_X) = 0.82 \times M_2(1\text{TeV}) \) at 1TeV. To obtain the second relation, we use \( M_2(1\text{TeV})/M_2(M_X) = 0.83 \) for Case A and B shown in Table 3. Hence the scalar field \( L \) acquires a mass of order of \( M_2(M_X) \) or \( M_2(1\text{TeV}) \) through radiative corrections.

In the above, we have assumed that the masses of \( E \) and \( \lambda_1 \) are of order of \( M_Z \). Even if we alter this assumption, its effects on the results of Tables 3 and 4 can be neglected. For example we consider Case A where the masses of \( E \) and \( \lambda_1 \) are of order of \( M_S \). For \( M_S = 1\text{TeV} \), we have \( M_1(M_S)/M_1(M_X) = 0.39 \) and the coefficients of \( M_1^2(M_X) \) for \( \Delta m_U^2(= m_U^2(M_Z) - m_U^2(M_X)) \), \( \Delta m_L^2(= m_L^2(M_S) - m_L^2(M_X)) \) and \( \Delta m_E^2(= m_E^2(M_Z) - m_E^2(M_X)) \) become \( 6.8 \times 10^{-2}, 3.8 \times 10^{-2} \) and \( 1.5 \times 10^{-1} \) respectively. The other values of Tables 3 and 4 are not changed.

### 3.2 String unification

In SUSY-GUTs the normalisation \( k_1 \) of the \( U(1)_Y \) generator is determined by the requirement that the \( U(1)_Y \) generator belongs to the set of generators of the unification gauge group: \( k_1 = 5/3 \). A free parameter of the theory is \( M_X \), which can only be calculated with the knowledge (i.e. measurement) of the low energy gauge couplings, preferentially at \( M_Z \). In string unification scenarios, the situation is different. The gauge coupling constants \( g_a \) of the three SM interactions are related at the string scale \( M_X = M_{st} \) by [11]:

\[
k_3g_3^2 = k_2g_2^2 = k_1g_1^2, \tag{3.7}
\]

where we take \( k_2 = k_3 = 1 \). This boundary condition is independent of any GUT type unification of strong and weak interactions and is a proper consequence of string unification. On the other hand there is little model
independent information known about $k_1$, except that it should be a rational number with $k_1 \geq 1$.  

$$k_1 \geq 1. \quad (3.8)$$

The lepton singlet field $E$ is not allowed in models with $k_1 < 1$. As we will constrain ourselves to a model independent discussion, $k_1$ can therefore be regarded as a free parameter of the theory. Nevertheless, as shown in ref.[14], the predictivity of direct string unification is not smaller than in GUT unification, because in the string case the incorporation of gravity leads to another constraint yielding a prediction of the unification scale [10]: $M_{st} = 5.27 \times g_{st} \times 10^{17}$GeV. the experimental data At one-loop level, the running coupling constants with one intermediate breaking scale evolve according to the following equations:

$$\alpha_a^{-1}(M_Z) = \frac{k_a}{\alpha_{st}} + \frac{b_a}{2\pi} \ln \left( \frac{M_{st}}{M_S} \right) + \frac{\bar{b}_a}{2\pi} \ln \left( \frac{M_S}{M_Z} \right), \quad (3.9)$$

with the same notation adopted as before. After elimination of $k_1$ we obtain:

$$\sin^2 \theta_W(M_Z) = \frac{\alpha(M_Z)}{\alpha_2(M_Z)} \quad (3.10)$$

$$= \frac{\alpha_3(M_Z)}{g_{st}^2} \left( \frac{4\pi}{2\pi} \ln g_{st} + \frac{b_2}{2\pi} \ln \left( \frac{5.27 \times 10^{17}}{M_S} \right) + \frac{\bar{b}_2}{2\pi} \ln \left( \frac{M_S}{M_Z} \right) \right).$$

Using (3.3) for $\alpha_3(M_Z)$, we can compute $g_{st}$ in (3.10) and then obtain a constraint in the $\sin^2 \theta_W(M_Z) - \alpha_3(M_Z)$ plane. This constraint is plotted in Figure 2 for all of the cases with $M_S = 1$TeV. In this figure, $a$ and $\sin$ denote $\alpha_3(M_Z)$ and $\sin^2 \theta_W(M_Z)$ respectively and the experimental values for $\sin^2 \theta_W(M_Z) = 0.2321 \pm 0.004$ and $\alpha_3(M_Z) = 0.118 \pm 0.007$ are also displayed. Contrary to ref. [14] we allow for different values of $g_{st}$ in $M_{st}$. This gain of exactitude in $M_{st}$ causing changes of $\Delta M_{st} \approx -0.1 \times 10^{17}$GeV is however of the order of the uncertainty of the numerical factor 5.27 in $M_{st}$ [10].

Similarly we can get a relation between $k_1$ and $\sin^2 \theta_W(M_Z)$ or $k_1$ and $\alpha_3(M_Z)$, using (3.9) for $\alpha_1$. Further the second and third columns of Table 5
show the values $\sin^2 \theta_W(M_Z)$ and $\alpha_3(M_Z)$ of the nearest point of each curve to the center of the error cross. Corresponding values of $k_1$ are also found in the fourth column of Table 5. Case I – VI are fairly equal in their prediction of the Weinberg angle, although Case II, III and IV do a little bit better. Case I, V and VI are almost degenerate. Among the cases classified above Case A and B do exceptionally well, whereas Case C and D seem to be ruled out by this calculation. In addition all of the cases predict $k_1 = 1.3 - 1.4$, which is consistent with (3.8) and agrees with the result of ref. [14].

As an example, the qualitative features of the dependence on $M_S$ is shown in Figure 3 for Case I, II and A with some values of $M_S$. As expected from the discussion of the dependence of the unification scale on $M_S$, the curve for Case I is not very sensitive to a shift of $M_S$ from 1TeV to 10TeV, whereas the curve for Case A shows a stronger dependence on $M_S$. Case A can achieve exact accordance with experiment for $M_S = 2.5$TeV. That agrees with the result in section 3.1. For Case I we also considered $M_S = M_{st}$ - the non-supersymmetric string. The curve of Case I with $M_S = M_Z$ is almost degenerate with the curve of Case I with $M_S = 1$TeV. The non-supersymmetric string beats the supersymmetric string (Case I with $M_S = M_Z$) with respect to gauge coupling unification. In general, increase of $M_S$ results in a lowering of the curve for all cases except for Case C and D.

To get an idea of the effects of higher order corrections, we perform the calculation described above at two-loop level for some cases. Here, we considered Case I and II with $M_S = 1$TeV and Case I with $M_S = M_{st}$ (Figure 4). The RGEs of the gauge couplings at two-loop level are

$$\frac{d\alpha_a^{-1}}{dt} = -\frac{b_a}{2\pi} - \sum_{b=1}^{3} \frac{b_{ab}}{8\pi^2} \alpha_b.$$  

(3.11)

The two-loop $\beta$-coefficients $b_{ab}$ needed in this calculation read [21]:

$$\bar{b}_{ab}^I = \bar{b}_{ab}^{SM} = \begin{pmatrix} 199/18 & 9/2 & 44/3 \\ 3/2 & 35/6 & 12 \\ 11/6 & 9/2 & -26 \end{pmatrix}, \quad \bar{b}_{ab}^{II} = \begin{pmatrix} 307/18 & 9 & 68/3 \\ 3/2 & 163/6 & 12 \\ 11/6 & 9/2 & 22 \end{pmatrix}.$$
and \( b^{\text{MSSM}}_{ab} = \begin{pmatrix} 199/9 & 9 & 88/3 \\ 3 & 11 & 24 \\ 11/3 & 9 & 14 \end{pmatrix} \). \hspace{1cm} (3.12)

In all three cases the curve is lifted by two loop corrections, so that the accordance with the experimental data becomes worse. The non-supersymmetric string is most sensitive to the inclusion of higher order effects. Case II acquires a smaller correction. Similarly we can compute two-loop corrections for other cases, using corresponding two-loop \( \beta \)-coefficients [21]. In these cases, two-loop corrections are as small as in Case II.

The above results show that the theoretical predictions are not compatible with experiment for most cases. This discrepancy might be solved, if threshold corrections due to higher massive modes of string theories are taken into account. Assuming exact accordance at one-loop level with the measured values, we can estimate the threshold corrections from the value of the nearest point of each curve to the center of the error cross shown in Table 5. This will be done in the next section.

4. String theory

In this section we discuss the results obtained in the previous section from the viewpoint of orbifold models [15]. The orbifold construction is one of the simplest and most promising methods to construct four-dimensional string models.

4.1 String threshold corrections

String models have towers of higher massive modes, which bring about threshold corrections to the gauge couplings. Some parts of the string thresh-
old corrections depend on the vacuum expectation values of moduli fields $T$, which describe geometrical features of orbifolds [17]. In general orbifolds have three independent moduli fields $T^i$ ($i = 1, 2, 3$). Here we restrict ourselves to the overall moduli field $T = T^i$. The other corrections depend on the details of massive string spectra. Large values of $T$ could lead to large threshold corrections. The vacuum expectation value of the moduli field could be determined by a non-perturbative superpotential $\hat{W}$, which also breaks SUSY. In refs.[22] the values of $O(1)$ were obtained within the framework of the gaugino condensation scenario. On the other hand, the values of $O(10)$ were obtained in ref.[23], taking into account a one-loop effective potential. Hence the gauge coupling at $M_{st}$ is written as

$$\alpha^{-1} = \alpha_{st}^{-1} + \Delta_a(T) + \Delta'_a, \quad (4.1)$$

where the second term of the right hand side is the $T$-dependent threshold correction. The threshold correction reads [17],

$$\Delta_a(T) = \frac{b'_a - \delta_{GS}}{4\pi} \log[(T + T)|\eta(T)|^4], \quad (4.2)$$

where $b'_a$ represents a duality anomaly coefficient and $\delta_{GS}$ denotes a Green-Schwarz coefficient [24]. The former is determined by massless modes in string models. Further, the Dedekind function $\eta(T)$ is expressed as $\eta(T) = e^{-\pi T/12} \prod_{n=1}^{\infty} (1 - e^{-2\pi nT})$.

The investigation in the previous section shows some differences between experiment and direct string unification, except for Case A. Here we estimate string threshold corrections to explain the difference between experiment and the tree-level prediction by string unification. We represent the necessary threshold corrections by the values of $T$, neglecting $T$-independent threshold corrections $\Delta'_a$. In general, the bulk of the corrections comes from $T$-dependent parts. In refs.[4, 25] it was discussed to explain the discrepancy between the experiment and the string unification for the MSSM with $M_S = M_Z$ and $k_1 = 5/3$, using $\Delta_a(T)$. That analysis was extended to the cases with general values of $k_1$ in refs.[26].
At first we estimate threshold corrections necessary to explain discrepancy between the experiments and the results obtained in 3.2. Suppose that for $\alpha_3^{-1}(M_Z)$ and $\sin^2 \theta_W(M_Z)$ we denote deviations of the predicted values from the experiments as $\Delta \alpha_3^{-1}$ and $\Delta \sin^2 \theta_W$, then we need the following threshold corrections at $M_{st}$,

$$\Delta_3(T) = \Delta \alpha_3^{-1}, \quad \Delta_2(T) = \alpha^{-1}(M_Z) \Delta \sin^2 \theta_W. \quad (4.3)$$

We compare the values of $\alpha_3(M_Z)$ and $\sin^2 \theta_W(M_Z)$ of each case in Table 5 with experimental values $\alpha_3(M_Z) = 0.118$ and $\sin^2 \theta_W(M_Z) = 0.2321$. For example the deviations of these values for Case I require $\Delta_3(T) = 0.344$ and $\Delta_2(T) = 1.279$. These values of the threshold corrections are realized in (4.2) by $T = 6.6$ and 19, respectively, if we take $b_3' - \delta_{GS} = 1$ and $b_2' - \delta_{GS} = 1$. These values of $T$ are found in the fifth and the sixth columns of Table 5, where $B_a' \equiv b_a' - \delta_{GS}$. For the other cases we can estimate necessary values of $T$ similarly. Those values are listed in the fifth and sixth columns of Table 5. For all of the cases, the difference $\Delta \sin^2 \theta_W$ is more important than $\Delta \alpha_3^{-1}$. The seventh and eighth columns of Table 5 show values of $T$ deriving necessary threshold corrections for $\Delta \sin^2 \theta_W$ in the case with $b_2' - \delta_{GS} = 5$ and $b_2' - \delta_{GS} = 10$, respectively.

Next we estimate the threshold corrections to explain difference between the experiments and the results obtained in 3.1. Including the threshold corrections we have the running gauge couplings at $\mu (\mu > M_S)$ as,

$$\alpha_a^{-1}(\mu) = \alpha_{st} - \frac{b_a}{2\pi} \ln \frac{M_{st}}{\mu} + \frac{b_a' - \delta_{GS}}{4\pi} \log[(T + \bar{T})|\eta(T)|^4]. \quad (4.4)$$

Using (4.4), the string scale can be related with the unification scale $M_X$ where $\alpha_3(M_X) = \alpha_2(M_X)$ as follows [4],

$$\ln \frac{M_X}{M_{st}} = \Delta \frac{b'}{8} \ln[(T + \bar{T})|\eta(T)|^4], \quad (4.5)$$

where $\Delta b' \equiv b_3' - b_2'$. Note that $\ln[(T + \bar{T})|\eta(T)|^4]$ is always negative. If $M_X < M_{st}$, the duality anomaly coefficients should satisfy $b_3' > b_2'$. For
example the MSSM with $M_S = M_Z$ leads to $M_X = 10^{16.4}\text{GeV}$. This value of $M_X$ leads to $T = 24$ in the case with $\Delta b' = 1$. Similarly we can estimate the value of $T$ for each unification scale $M_X$ obtained in 3.1 using $M_S = 1\text{TeV}$. The results are found in the fifth, sixth and seventh columns of Table 2 for the cases with $\Delta b' = 1$, 5 and 10, respectively. The numbers in the parentheses of Table 2 correspond to $T$ in the case with $M_S = \sqrt{10}\text{TeV}$.

4.2 Level of $U(1)_Y$

Here we comment on the value of $k_1$. The discussion in 3.2 seems to show that the desirable value is $k_1 = 1.3 - 1.4$ for the string unification of the MSSM. Within the framework of orbifold models massless states satisfy the following condition \cite{15},

$$h + N_{OSC} + c - 1 = 0,$$

(4.6)

where $N_{OSC}$ is the oscillator number, $c$ is the ground state energy and $h$ is the conformal dimension due to the gauge parts. A state belonging to the representation $R$ of a non-abelian group $G$ contributes to the conformal dimension as follows,

$$h = \frac{C(R)}{C(G) + k}.$$

(4.7)

A state transforming under an abelian group with charge $Q$ gives $h = Q^2/k_1$. Under the condition that the MSSM matter fields are massless, the level of $U(1)_Y$ is restricted by a lower bound. These lower bounds are shown explicitly in refs.\cite{26}. For example the twisted sector of the $Z_3$ orbifold has $c = 1/3$. The level should satisfy the condition $k_1 \geq 4/3$ so that the chiral field $U$ appears in the twisted sector. Further the existence of the chiral field $E$ in the twisted sector requires $k \geq 3/2$, although the other MSSM matter

\footnote{In ref.\cite{26}, possible values of $\Delta b'$ were given explicitly for orbifold models. To obtain $\Delta b' = 5$ or 10 gives severe constraints on model building.}
fields in the twisted sector are allowed in the case with \( k_1 \geq 1 \). However, the condition on \( k_1 \) restricts not only \( U \) and \( E \) fields, but also the others, taking into account Yukawa couplings. In \( Z_3 \) orbifold models, the field \( E \) in the twisted sector is ruled out and the twisted sector cannot contain \( U \) for some cases. Orbifold models have selection rules for Yukawa couplings different from those obtained by gauge invariance \([27]\). These selection rules restrict the couplings of one sector to another ref.\([28]\). In \( Z_3 \) orbifold models, the untwisted matter fields are allowed to couple to the untwisted matter fields only. If the fields \( U \) and \( E \) are permitted in the untwisted sector only, the fields \( Q \) and \( L \) and the Higgs fields are allowed only in the untwisted sector to give the Yukawa couplings. Further this fact has a phenomenological implication that these Yukawa couplings give the same magnitude, i.e. Yukawa coupling unification, although twisted sectors could lead to a hierarchy structure of Yukawa couplings \([29]\). For the other orbifold models, we can discuss similar constraints. In general the field \( E \) is restricted to the twisted sector. Therefore the prediction of \( k_1 \) has sensitive effects on model building.

5. Conclusion

We have studied the gauge coupling unification in cases with non-universal soft scalar and gaugino masses. The unification scale of \( SU(3) \) and \( SU(2) \) gauge couplings is sensitive to the non-universality. Some cases lead to a unification scale \( M_X \) around \( M_{st} \). The ratio \( \alpha_x/\alpha_1 \) at \( M_X \) is also sensitive to the non-universality and varies from 1.4–1.7 for \( M_S = 1 \)TeV. We have also run the gauge couplings top-down assuming the string unification of the MSSM. That analysis seems to show that the preferred values of \( k_1 \) are in the range 1.3 – 1.4. These values of \( k_1 \) give some constraints on model building. Two-loop corrections depend on the non-universality, too. The non-SUSY case acquires larger two-loop corrections. The corrections are reduced by effects of some superpartners.
Several cases show discrepancies between experiment and the predictions by string unification of the MSSM. These discrepancies could be explained in terms of threshold corrections due to massive string modes. Threshold corrections are expressed in terms of the vacuum expectation values of the moduli fields. Some cases require $T$ of order $O(10)$.

It is interesting to analyze the other RGEs with decoupling as discussed here and to study the radiative symmetry. We have assumed that the SUSY breaking scale for the Higgs sector is $M_S$. One has to investigate the condition of the successful radiative symmetry breaking in the mass spectra of the non-universal cases. Further it is important to investigate which phenomenological aspects are sensitive or insensitive to the non-universality of other soft SUSY breaking parameters as well as soft scalar masses and gaugino masses.

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Table 1: Non-universal cases

| Case | \(A\) | \(B\) | \(b_1, b_2, b_3\) |
|------|-------|-------|----------------|
| I    | \(Q, U, D, L, E, \lambda, \lambda_3, \lambda_2, \lambda_1\) | \(E, \lambda_3, \lambda_2, \lambda_1\) | \(41/6, -19/6, -7\) |
| II   | \(Q, U, D, L\) | \(U, D, E, \lambda_3, \lambda_2, \lambda_1\) | \(47/6, -11/6, -5\) |
| III  | \(Q, L\) | \(U, D, E, \lambda_3, \lambda_2, \lambda_1\) | \(19/2, -11/6, -4\) |
| IV   | \(L\) | \(Q, U, D, E, \lambda_3, \lambda_2, \lambda_1\) | \(29/3, -1/3, -3\) |
| V    | \(Q, U, D\) | \(L, E, \lambda_3, \lambda_2, \lambda_1\) | \(25/3, -4/3, -5\) |
| VI   | \(U, D\) | \(Q, L, E, \lambda_3, \lambda_2, \lambda_1\) | \(17/2, 1/6, -4\) |
| A    | \(Q, L, \lambda_2\) | \(U, D, E, \lambda_3, \lambda_1\) | \(19/2, -19/6, -4\) |
| B    | \(Q, U, D, L, \lambda_2\) | \(E, \lambda_3, \lambda_1\) | \(47/6, -19/6, -5\) |
| C    | \(Q, U, D, \lambda_3\) | \(L, E, \lambda_2, \lambda_1\) | \(25/3, -4/3, -7\) |
| D    | \(Q, U, D, L, \lambda_3\) | \(E, \lambda_2, \lambda_1\) | \(47/6, -11/6, -7\) |

Table 2: Unification scale

| Case | \(\log_{10} M_X\) | \(\alpha_X^{-1}\) | \(\alpha_X/\alpha_1(M_X)\) | \(T(\Delta b' = 1)\) | \(T(\Delta b' = 5)\) | \(T(\Delta b' = 10)\) |
|------|-----------------|-----------------|-----------------|----------------|----------------|----------------|
| I    | 16.5            | 26.0            | 1.59            | 23             | 6.2             | 3.8             |
|      | (16.5)          | (26.7)          | (1.57)          | (23)           | (6.2)           | (3.8)           |
| II   | 16.7            | 25.6            | 1.58            | 19             | 5.4             | 3.4             |
|      | (16.8)          | (26.3)          | (1.55)          | (17)           | (4.9)           | (3.1)           |
| III  | 16.9            | 25.3            | 1.52            | 15             | 4.5             | 2.8             |
|      | (17.1)          | (25.7)          | (1.47)          | (11)           | (3.5)           | (2.3)           |
| IV   | 16.8            | 24.8            | 1.57            | 17             | 4.9             | 3.1             |
|      | (16.9)          | (25.0)          | (1.54)          | (15)           | (4.5)           | (2.8)           |
| V    | 16.5            | 25.2            | 1.60            | 23             | 6.2             | 3.8             |
|      | (16.6)          | (25.7)          | (1.59)          | (21)           | (5.8)           | (3.5)           |
| VI   | 16.4            | 24.8            | 1.63            | 24             | 6.6             | 4.1             |
|      | (16.4)          | (25.0)          | (1.64)          | (24)           | (6.6)           | (4.1)           |
| A    | 17.3            | 25.7            | 1.44            | 7.3            | 2.5             | 1.6             |
|      | (17.7)          | (26.3)          | (1.31)          | –              | –               | –               |
| B    | 17.0            | 25.8            | 1.50            | 13             | 4.0             | 2.6             |
|      | (17.3)          | (26.4)          | (1.44)          | (7.3)          | (2.5)           | (1.6)           |
| C    | 16.0            | 25.4            | 1.67            | 32             | 8.2             | 5.0             |
|      | (15.8)          | (25.9)          | (1.69)          | (35)           | (9.0)           | (5.4)           |
| D    | 16.1            | 25.6            | 1.65            | 30             | 7.8             | 4.7             |
|      | (16.0)          | (26.2)          | (1.66)          | (32)           | (8.2)           | (5.0)           |
Table 3: Radiative corrections to gaugino masses

| Case | I   | A     | B     | C     | D     |
|------|-----|-------|-------|-------|-------|
| $M_3/M_3(M_X)$ | 2.3 | 3.3   | 3.6   | 2.3   | 2.3   |
|       | (2.2) | (3.3) | (3.9) | (2.1) | (2.1) |
| $M_2/M_2(M_X)$ | 0.84 | 0.83  | 0.83  | 0.89  | 0.91  |
|       | (0.85) | (0.84) | (0.84) | (0.93) | (0.95) |
| $M_1/M_1(M_X)$ | 0.43 | 0.38  | 0.40  | 0.44  | 0.44  |
|       | (0.44) | (0.37) | (0.40) | (0.45) | (0.45) |

Table 4: Radiative corrections to soft scalar masses

| Case | A     | B     |
|------|-------|-------|
|      | $M_3^2$ | $M_2^2$ | $M_1^2$ | $M_3^2$ | $M_2^2$ | $M_1^2$ |
| $\Delta m_Q^2$ | 5.0 | 0.46 | $4.3 \times 10^{-3}$ | 4.6 | 0.46 | $4.2 \times 10^{-3}$ |
|       | (4.5) | (0.46) | (4.3 $\times 10^{-3}$) | (4.0) | (0.44) | (4.2 $\times 10^{-3}$) |
| $\Delta m_U^2$ | 6.7 | — | $7.0 \times 10^{-2}$ | 4.6 | — | $6.8 \times 10^{-2}$ |
|       | (6.7) | (—) | (7.1 $\times 10^{-2}$) | (4.0) | (—) | (6.8 $\times 10^{-2}$) |
| $\Delta m_L^2$ | — | 0.46 | $3.9 \times 10^{-2}$ | — | 0.46 | $3.8 \times 10^{-2}$ |
|       | (—) | (0.46) | (3.9 $\times 10^{-2}$) | (—) | (0.44) | (3.8 $\times 10^{-2}$) |
| $\Delta m_E^2$ | — | — | $1.6 \times 10^{-1}$ | — | — | $1.5 \times 10^{-1}$ |
|       | (—) | (—) | (1.6 $\times 10^{-1}$) | (—) | (—) | (1.6 $\times 10^{-1}$) |

| Case | C     | D     |
|------|-------|-------|
|      | $M_3^2$ | $M_2^2$ | $M_1^2$ | $M_3^2$ | $M_2^2$ | $M_1^2$ |
| $\Delta m_Q^2$ | 3.8 | 0.44 | $4.0 \times 10^{-3}$ | 3.8 | 0.44 | $4.0 \times 10^{-3}$ |
|       | (3.0) | (0.41) | (3.9 $\times 10^{-3}$) | (3.1) | (0.41) | (4.0 $\times 10^{-3}$) |
| $\Delta m_U^2$ | 3.8 | — | $6.5 \times 10^{-2}$ | 3.8 | — | $6.5 \times 10^{-2}$ |
|       | (3.0) | (—) | (6.3 $\times 10^{-2}$) | (3.1) | (—) | (6.4 $\times 10^{-2}$) |
| $\Delta m_L^2$ | — | 0.45 | $3.7 \times 10^{-2}$ | — | 0.44 | $3.7 \times 10^{-2}$ |
|       | (—) | (0.43) | (3.7 $\times 10^{-2}$) | (—) | (0.41) | (3.6 $\times 10^{-2}$) |
| $\Delta m_E^2$ | — | — | $1.5 \times 10^{-1}$ | — | — | $1.5 \times 10^{-1}$ |
|       | (—) | (—) | (1.5 $\times 10^{-1}$) | (—) | (—) | (1.5 $\times 10^{-1}$) |
Table 5: Nearest points to experimental value

| Case | $\alpha_3$ | $\sin^2 \theta_W$ | $k_1$ | $T(B'_3 = 1)$ | $T(B'_2 = 1)$ | $T(B'_2 = 5)$ | $T(B'_2 = 10)$ |
|------|-----------| ------------------|-------| --------------|--------------|--------------|---------------|
| I    | 0.123     | 0.242             | 1.34  | 6.6           | 19           | 5.3          | 3.4           |
| II   | 0.123     | 0.240             | 1.37  | 6.6           | 16           | 4.6          | 2.9           |
| III  | 0.121     | 0.238             | 1.37  | 4.7           | 12           | 3.8          | 2.4           |
| IV   | 0.122     | 0.239             | 1.39  | 5.7           | 14           | 4.2          | 2.7           |
| V    | 0.123     | 0.241             | 1.36  | 6.6           | 17           | 5.0          | 3.1           |
| VI   | 0.123     | 0.242             | 1.37  | 6.6           | 19           | 5.3          | 3.4           |
| A    | 0.120     | 0.235             | 1.38  | 3.6           | 7.2          | 2.4          | 1.5           |
| B    | 0.121     | 0.237             | 1.38  | 4.7           | 11           | 3.4          | 2.2           |
| C    | 0.126     | 0.246             | 1.31  | 9.2           | 25           | 6.8          | 4.2           |
| D    | 0.125     | 0.245             | 1.32  | 8.4           | 24           | 6.4          | 4.0           |

**Figure captions:**

1. Unification scale $M_X$ as a function of $M_S$ - the SUSY breaking scale - for Case I and Case A - D, where x and y denote $\log_{10} M_S$ and $\log_{10} M_X$ respectively.

2. One-loop calculation of $\sin^2 \theta_W(\alpha_3)$ at $M_Z$ with $M_S = 1$ TeV for all the cases under consideration, i.e., Case I - VI and Case A - D. In this and the following figures a stands for $\alpha_3(M_Z)$ and sin for $\sin^2 \theta_W(M_Z)$.

3. One-loop calculation of $\sin^2 \theta_W(M_Z)$ at $M_Z$ for Case I, II and A for different values of $M_S$.

4. One- and two-loop curves of $\sin^2 \theta_W(\alpha_3)$ at $M_Z$ with $M_S = 1$ TeV for Case I and II and with $M_S = M_{st}$ for Case I (non-supersymmetric string).
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