An optimization tool to design the field of a solar power tower plant allowing heliostats of different sizes

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SUMMARY

The design of a solar power tower plant involves the optimization of the heliostat field layout. Fields are usually designed to have all heliostats of identical size. Although the use of a single heliostat size has been questioned in the literature, there are no tools to design fields with heliostats of several sizes at the same time. In this paper, the problem of optimizing the heliostat field layout of a system with heliostats of different sizes is addressed. We present an optimization tool to design solar plants allowing two heliostat sizes. The methodology is illustrated with a particular example considering different heliostat costs. Copyright © 2017 John Wiley & Sons, Ltd.

KEY WORDS
solar thermal power tower; field layout; multi-size-heliostat field; heuristic algorithm; greedy algorithm

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1. INTRODUCTION

Solar power tower (SPT) system is known as one of the most promising technologies for producing solar electricity because of the high temperatures reached that result in high thermodynamic performances; some reviews on solar thermal electricity technology are discussed in [1–3]. In an SPT system, direct solar radiation is reflected and concentrated by the heliostat field in a receiver placed at the top of the tower. At the receiver, the solar energy is converted into thermal energy by heating a fluid, which can be used to generate electricity through a conventional thermodynamic cycle. The heliostat field is composed of a group of mirrors having usually two-axis tracking system to reflect the direct light from the sun to the receiver aperture.

The heliostat optical design influences the overall performance of the system (ratio of ground coverage, number of heliostats, receiver size and tower height). At the same time, the optical design is influenced by the cost (manufacturing and assembly processes, canting, installation, calibration, etc.) and wind loads among others; see [4–8].

The optimization of the field layout to minimize the leverized cost of energy (LCOE), even when a single heliostat size (single-size-heliostat field) is considered, is a challenging problem. It involves non-convex constraints; the objective function is non-smooth, has no closed form, is hard to compute and multimodal and, moreover, involves a high number of variables (hundreds or thousands of heliostats); see [9,10]. Because of these characteristics, the optimization problem is actually complex.

For the single-size-heliostat field design problem to be solved, there exist several algorithms in the literature [11]. Efficient single-size-heliostat fields have been designed with large or big heliostats (148 m\textsuperscript{2} ATS [4], 121 m\textsuperscript{2} Sanlucar [12], 116 m\textsuperscript{2} Sener [13]) and also using small or micro heliostats (16 m\textsuperscript{2} AORA Solar and Heliko-DLR [4], 7.5 m\textsuperscript{2} [14], 4.3 m\textsuperscript{2} SHP-CSIRO [7]).

With the purpose of reducing the complexity of the problem, a geometrical pattern is frequently imposed; that is, a pattern-based field is designed. Usually, a radially staggered [15–19], spiral [20] or grid [21] distribution is used. Thus, the heliostat locations are calculated through the optimization of a low number of parameters describing the geometry of the selected pattern. More recently, algorithms yielding pattern-free layouts also appeared in the literature; see [22,23].
All these optimization algorithms consider a unique and fixed heliostat size. However, as captured in the literature, an extensive catalogue of heliostats sizes is available in the market. Since 1970s, different research programmes have been financed to study the heliostat design aiming to reduce the heliostat costs while maintaining the collected energy.

Original ideas have been developed in recent studies (hexagonal [6], bubble [4], minimirror array [24] and other geometries [8,25,26]); see also [13,27–29]. Some promising prototypes are the following: the autonomous heliostat (CIEMAT PCHA project [30,31]), the EASY heliostat (hEliostat for eAsy and Smart deploYment [32]) and the SCS5 eSolar next generation heliostat [33].

Therefore, the heliostat size, instead of being fixed, becomes a new design variable in the optimization problem. A possible approach to address this new problem is to solve a single-size-heliostat field optimization problem for each heliostat size available and then select as final solution the best field obtained. However, as pointed out in several studies [4,13,32,34], the use of single-size-heliostat fields, with identical heliostat sizes, may not lead to optimal fields.

More specifically, there is a patent where the design of ‘mixed fields’ appears in the literature (a very exploratory approach, see [35]). In this approach, fixed zones are considered (with a simple geometric pattern) to locate the different heliostat sizes. However, the shapes and positions of these zones are dependent on the heliostats technologies and the project characteristics, and there appears the need for ad hoc analysis.

The study of the usefulness of multi-size-heliostat fields, that is, the combination of different sizes in the same field, appears as a natural question. For this purpose, an optimization tool, which allows the design of a field layout with multiple heliostat sizes, is needed.

In this paper, we address the design problem of mixing two heliostat sizes in the same field. We will focus on the optimization of the field developing a pattern-free heuristic method without fixing any zone for each heliostat size location. We attempt to develop a flexible location strategy, valid under any constraint over the field region available.

Two rectangular-shaped heliostat sizes are considered with the same pedestal height and safe distance. For simplicity, we will assume that the tower and receiver are fixed (to address the whole optimization problem, see [23]), and, as usual, all heliostats will be assumed to focus onto the same target point: the aperture centre. A minimal power input \( P_0 \) in the receiver is set, which will determine the minimal number of heliostats to be located. Finally, the feasible available region has a fixed annulus shape around the tower, but it is big enough to not interfere in the heliostat location.

Using two different technologies (heliostat sizes) in the same solar installation instead of only one increases the complexity because new heliostat aiming, control, cleaning and maintenance strategies are to be determined. These issues would need to be studied in detail for each project; see, for instance, [36].

The purpose of this paper is to give a methodological tool to design fields with different heliostat sizes, so that planners can analyse if the loss in economies of scale and increase in design in maintenance complexity are compensated by a higher efficiency of the so-obtained field. An additional motivation of our study is to show that not only the (classical) variables can be important for the design of an efficient field; contrarily, additional characteristics of the heliostats can play an important role.

The rest of the paper is organized as follows. In Section 2, we describe the main ingredients affecting the behaviour and performance of the SPT system. Locating heliostats of different sizes amounts to solving a nontrivial optimization problem. How to address such problem is explained in Section 3. In Section 4, an illustrative example is presented where we apply the proposed optimization tool to a typical plant design. Finally, in Section 5, our main results are summarized, and some perspectives for further work are presented.

2. PROBLEM STATEMENT

In this section, we explain the meaning of the variables involved in the optimization process. We also present the constraints that have to be satisfied, the cost and energy functions (which are the elements to be considered for the computation of the objective function LCOE) and the optimization problem itself.

2.1. Variables

The heliostats locations, given by the coordinates \((x, y)\) of their centres, and the heliostat sizes \(d\) are the variables to be used. From now on, we will denote by \(\Omega\) the collections of coordinates of the centres and sizes of the heliostats, namely, \((x, y, d)\). The set \(\Omega\) is described as follows:

\[
\Omega = \{(x_i, y_i, d_i)\} \text{ for } i \in [1, N] \text{ with } (x_i, y_i) \in S \text{ and } d_i \in D,\]

where \(N\) denotes the total number of heliostats, \(S\) is the set of heliostats coordinates and \(D\) is the set of heliostat sizes. We assume that the set \(D\) is finite.

For simplicity, all heliostats are assumed to be rectangular and have the same pedestal height, although they can have different dimensions. Note that these assumptions help to reduce the computation of the shading and blocking effects caused by large-size heliostats on the smaller ones.

2.2. Constraints

Usually, when designing an SPT system, a fixed time is used to size the plant; see [37,38]. In order to size the field, we impose at the receiver aperture for this fixed time a minimal power input requirement. This time is known in
the literature as the design point, denoted here by $T_d$. Let $\Pi_{rd}(\Omega)$ be the power input obtained at the design point. Then, the following constraint has to be satisfied:

$$\Pi_{rd}(\Omega) \geq \Pi_0. \tag{2}$$

The minimal number of heliostats required to build a feasible solution (field) will be affected by this constraint.

The heliostats must be located within a given region $S_0 \subset \mathbb{R}^2$:

$$S \subset S_0. \tag{3}$$

In this paper, $S_0$ is an annulus comprising a minimal radius (because of technical reason, e.g. to allow the access of a crane to the tower) and a maximal radius (because of low efficiency after this distance). This region is fully described in Table II.

The heliostats located in the field have to rotate freely avoiding collisions with other heliostats. Consequently, we have to include constraints forcing the heliostats not to overlap:

$$|| (x,y) - (x',y') || \geq \delta(d) + \delta(d')$$

$$\forall (x,y,d), (x',y', d') \in \Omega \text{ with } (x,y) \neq (x',y'), \tag{4}$$

where the security distance (radius of the clear-out circle) for heliostat size $d$ is $\delta(d) = 0.5 \text{ diag}(d) + 0.5 \ d_c$. Here, diag$(d)$ denotes the heliostat diagonal, and $d_c$ is a positive constant called safe distance, related to installation errors and heliostat accessibility. In this paper, the safe distance value $d_c$ remains equal for all the heliostat sizes.

### 2.3. Functions

The cost and the annual energy are the functions involved in the objective function. The cost function $C = C(\Omega)$ takes into account the investment in power plant equipment (tower, receiver and heliostat field), purchasing of land and civil engineering costs:

$$C(\Omega) := K + \Psi(\Omega), \text{ with } \Psi(\Omega) := \sum_{d \in D} c(d) N_d. \tag{5}$$

where $K$ is a constant including all fixed costs (independent of the configuration of the heliostat field) and $\Psi(\Omega)$ represents the heliostat field cost function. The number of heliostats of each size is denoted by $N_d$, and $c(d)$ denotes the cost per heliostat of size $d$. All costs associated with the heliostats (mirror modules, support structure, drives, pedestal, foundation, field wiring, etc.) are included in $c(d)$, and for simplicity, they are supposed to be independent of the heliostat position.

The annual energy input function $E = E(\Omega)$ takes the form:

$$E(\Omega) := \int_0^T \sum_{i=1}^N \varphi(t, x_i, y_i, d_i, \Omega) dt, \tag{6}$$

where $\varphi(t)$ is the so-called instantaneous direct solar radiation and $\varphi$ represents the product of the heliostat efficiencies (usual in this framework), that is, $\varphi = f_{ref} f_{at} f_{cos} f_{sh} f_{in} f_{int}$.

Specifically, $f_{ref}$ is the heliostat reflectance factor, $f_{at}$ is the atmospheric efficiency [39], $f_{cos}$ is the cosine efficiency [39], $f_{sh}$ is the shading and blocking efficiency [16,40]; finally, $f_{in}$ is the interception efficiency or spillage factor [41].

The annual energy of the plant is computed with a procedure similar to Nevada Solar Power Optimization Code [42]. We refer the reader to [39,41] for further details. We have developed a Matlab prototype to adapt the energy calculation when having different heliostat sizes and, in particular, address the shading and blocking effects.

### 2.4. Optimization problem

The optimization problem we are addressing can be written as follows:

$$\begin{aligned}
\text{Minimize} & \quad F(\Omega) = C(\Omega)/E(\Omega) \\
\text{Subject to} & \quad \Pi_{rd}(\Omega) \geq \Pi_0 \\
& \quad \Omega \subset S_0 \times D \\
& \quad || (x, y) - (x',y') || \geq \delta(d) + \delta(d') \\
& \quad \text{for } (x,y,d), (x',y', d') \in \Omega \text{ with } (x,y) \neq (x',y').
\end{aligned} \tag{7}$$

In this problem, the number of heliostats is not fixed in advance. Note that, even fixing this number, the huge amount of heliostats in recent commercial plants makes this problem very difficult to solve, as pointed out in [11].

Some of the heliostat efficiency functions depend on the heliostat area and/or the position in the field (interception efficiency [10,43], atmospheric efficiency [11,20], astigmatism effects, etc.). Hence, the heliostats’ annual energy per unit area values is different depending on their positions; see Figure 1. These values are similar in general for both sizes although they have a different behaviour in regions below the two quadrant diagonals (because of the interception efficiency together with the astigmatism effects; Figure 2).

### 3. FIELD OPTIMIZATION: EXPANSION–CONTRACTION ALGORITHM

The aim of this paper is the design of multi-size-heliostat fields. The Expansion–contraction algorithm is proposed as a strategy that allows the pattern-free location of two different heliostat sizes. It works under any selected size having arbitrary aspect ratio, cost and so on (see pseudo-code in Algorithm 1).

The algorithm starts with a large-size-heliostat field (computed using the Greedy algorithm to reach the power input constraint (2), detailed in Section 3.1) and complements it by inserting small-size heliostats. Two consecutive phases, called expansion and contraction, are applied...
and repeated until a stopping condition is fulfilled. At the expansion phase, small-size heliostats are inserted. At the contraction phase, the worst heliostats are sequentially deleted according to their LCOE per unit area values. Both phases are explained in detail in Section 3.2.

In order to allow the possibility of mixed-fields when having large and small heliostats, the algorithm starts locating large heliostats first. Therefore, large-size heliostats will be located at the best positions taking advantage of the most favourable region near the tower. If smaller heliostats were considered as first candidates, only infeasible positions would remain for the location of large heliostats if required. This way, the idea presented in [44] is followed: ‘The best strategy to fill a case with stone, pebble and sand is as follows. First filling the case with the stones and then filling the gap left from the stones with pebbles and in the same way, filling the gap left from pebbles with sand. Since filling in opposite direction may leave the stones or pebbles outside’. One possible disadvantage of our method is the increase of the computational time if smaller heliostats are preferable.

We describe our approach in the case of two sizes (large and small heliostats). The resolution of the problem when having three or more sizes, although not direct, could be used as an underlying idea in our methodology.

3.1. Greedy algorithm

The procedure presented in this paper makes use of the Greedy algorithm to locate the heliostats once the heliostat size is selected. This algorithm was presented in [23] to design an optimal field layout with a single heliostat size.

The proposed Greedy algorithm sequentially locates the heliostats one by one in the field at the best feasible position. The annual energy values are modified at each step because of the shading and blocking effects that the new heliostat produces in the field.

For simplicity, it is assumed that the heliostat cost is independent of the heliostat location, so that the annual energy function $E$ can be viewed as the objective function. The heliostats are located freely, without any pre-arranged distribution. Only two geometrical constraints have to be taken into account: the field shape constraint (3) and the constraints to avoid heliostat collisions (4). Obviously, the first problem involves locating the first heliostat centre when only the field shape constraint is considered. This problem has an easy-to-handle objective function, because of the absence of shading and blocking effects. In return, when we have already located $k-1$ heliostats and we have obtained a field $\Omega^{k-1} = \{(x_1,y_1,d_1),\ldots,(x_{k-1},y_{k-1},d_{k-1})\}$ that fulfils (3) and (4), the problem (P$^k$) described in the succeeding texts is difficult to solve, because non-convex constraints are involved and the energy function has a complex shape because of the shading and blocking effects.

Let us introduce the notation $\Omega^k = \Omega^{k-1} \cup \{(x,y,d)\}$, where $(x,y)$ denotes the variables with respect to which we maximize in problem (P$^k$). Now, we focus on the problem of finding the optimal location of a new heliostat:

\[
\begin{align*}
\text{Maximize} & \quad E(\Omega^{k-1} \cup \{(x,y,d)\}) \\
\text{Subject to} & \quad (x,y) \in S_0 \\
& \quad ||(x,y) - (x',y')|| \geq \delta(d) + \delta(d') \quad \forall (x',y',d') \in \Omega^{k-1}.
\end{align*}
\]

However, when $k > 0$, this problem becomes multimodal because of the shading and blocking effects. A multi-start procedure is used to avoid local minima starting from several randomly selected feasible positions.

The final solution is chosen according to the annual energy given by each configuration. The final number of heliostats is given by the algorithm. It stops when the power requirement is reached.
Optimization based on a pure greedy heuristic may not necessarily be optimal, for example, [45]; it is frequently used in combinatorial optimization theory and practice. Its common use may be due to its simplicity and, as stated in such paper, because it is widely assumed that it often provides solutions that are significantly better than the worst ones.

### 3.2. Expansion and contraction phases

The Expansion–contraction algorithm starts with a feasible large-size-heliostat field that reaches the power input constraint (2) and then makes a series of expansion–contraction steps.

The expansion phase consists of over-sizing the large-size field using small-size heliostats until a prescribed power input value $\Pi^+$, greater than $\Pi_0$, is reached. The small-size heliostats are located one by one following the greedy algorithm, recalculating the shading and blocking effects at each step. Small-size heliostats are expected to fill-in possible holes between the large-size heliostats already located due to their smaller area. Moreover, as can be seen in the contour lines shown in Figure 2, they reach higher energy per unit area values in lateral regions.

Once the oversized multi-size-heliostat field is obtained, the heliostats are arranged according to their LCOE per unit area values. At the contraction phase, the heliostats reaching lowest values are (sequentially) deleted, and the number of selected heliostats is determined by (2). This phase has to follow a sequential procedure because once a heliostat is deleted, the shading and blocking effects over its neighbours change, and therefore, their values have to be recalculated. This process can be carried out selecting carefully the active neighbours in order to avoid the recalculation of the annual energy of the whole field and, consequently, reducing the computational time. Oversizing and selection are well known in the field layout problem, as they are usually used in combination with some fixed-pattern strategies; see [16,19,20].

### 4. ILLUSTRATIVE EXAMPLE

The aim of this section is to illustrate the previously described expansion–contraction algorithm. In our example, we consider the large-size heliostat of the kind HLarge, whose area is 121.34 m$^2$ (similar to the usual heliostat used with the selected tower-receiver configuration, see Table II, the heliostat Sanlucar120 [12,46]). The small-size heliostats are of the HSmall kind, with area 4.35 m$^2$ (similar to the SCS5 eSolar heliostat [33]). In Table I and Figure 3, both are fully described.

In the Section 1, we discussed some possible advantages and disadvantages of mixing different heliostat sizes from the cost and performance point of view. However, this technology is not mature enough to clearly determine the associated costs. Several studies [4,28] support a reduction on the heliostat cost per unit area for small heliostats compared with large heliostats. Despite this, it is also known that small heliostats need more wiring, controllers and so on.

Our methodology allows the use of any combination of heliostat sizes and costs and tests their performances. Therefore, an individual study should be performed for each specific project. We attempt to present this example to be used as starting point.

In this example, the heliostat cost per unit area is set to 158.61 $/m^2$, and two different cost scenarios are studied:

- Scenario-100: The costs per unit area are identical for both sizes. The LCOE function is denoted by $F_{100}$.
- Scenario-80: Only for small-size heliostats, the cost per unit area is set to the 80%. $F_{80}$ denotes the LCOE function.

In this paper, the pedestal height and safe distance remain the same for all heliostat sizes. These assumptions may certainly be viewed strong and help to reduce the shading and blocking effects caused by large-size heliostats over the smaller ones. However, note that the selected sizes have different aspect ratio, which implies a different clear-out ratio. Therefore, these effects and the computed solutions will depend strongly on the selected heliostat sizes.

### Table I. Parameter values (heliostat sizes).

| Heliostat parameter | Large-size | Small-size |
|---------------------|------------|------------|
| Name                | HLarge     | HSmall     |
| Width (m)           | 12.84      | 3.21       |
| Height (m)          | 9.45       | 1.36       |
| Optical height $z_0$ (m) | 5.17 | 5.17 |
| Diagonal (m)        | 15.94      | 3.49       |
| Safe distance $d_s$ (m) | 1.70 | 1.70 |
| Security distance $d_l$ (m) | 17.64 | 5.19 |
| Aspect ratio (width/height) | 1.36 | 2.36 |
| Total area $A_2$ (m$^2$) | 121.34 | 4.35 |
| Heliostat cost Scenario-100 ($/m^2$) | 158.61 | 158.61 |
| Heliostat cost Scenario-80 ($/m^2$) | 158.61 | 126.89 |

Algorithm 1 Expansion-Contraction

Require: $\Pi_0$ and $\Pi^+$

\begin{algorithm}
$F_0, F(0)$ initial values while $F(\ell)$ increases and $k \leq k_{max}$ do
- Expansion Phase:
  - $\ell$ = 0. Stop when $\Pi_\ell$ is reached.
- $F_{\ell}(F(\ell))$ initial values while $F(\ell)$ increases and $k \leq k_{max}$ do
  - $k = k + 1$ initial step
  - $\ell = \ell$ initial step
  - $F(\ell) = F(\ell)$ initial step
  - $\ell = \ell$ initial step
- Stop when $\Pi_\ell$ is reached.

- Contraction Phase:
  - Delete the worst heliostats until $\Pi_\ell$ is reached
  - According to their LCOE per unit area.
- Update:
  - $k = k + 1$ initial step
  - $\ell = \ell$ initial step
  - $F(\ell) = F(\ell)$ initial step
- end while

return $[\Pi_\ell, F(\Pi_\ell)]$ best configuration.

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In view of the problem parameters detailed in Table II, the point of the field where a single heliostat collects its maximum thermal energy value can be calculated. With our settings, this value is obtained at coordinates (74.41, 0), that is, 0.74 tower heights to the north. Note that this value belongs to the interval [0.5, 1], given in [17]. Therefore, as can also be appreciated in Figure 1, this region (near the tower) is, in principle, the most favourable to locate heliostats, and a higher density of heliostats is expected there, as pointed out in [7,20].

The annual energy per unit area generated by one single heliostat is very similar for both sizes in general. However, the interception efficiency values differ, especially in regions below the two quadrant diagonals, and furnish better results with the small-size heliostat (dashed lines in Figure 2).

The expansion–contraction algorithm described in Section 3.2 has been implemented in Matlab®, using the fmincon routine to solve the involved optimization subproblems. The specific values for the receiver-field parameters are shown in Table II.

In order to compare our results, we use a reference system similar to the PS10 configuration called PS10-592, similar to a solar commercial plant located in Seville; see Figure 4(a).

The power input required at the design point $\Pi_0$ is selected by the authors to be 45.03 MWth in order to have a field of similar size to the reference field selected. The value for the upper limit $\Pi^*$ is set to 49.51 MWth (an increase of 10% on $\Pi_0$). The initial field $\Omega_0$, see Figure 4(b), is obtained with the greedy algorithm considering the power requirements $\Pi_0$. Note that any heliostat field could be used instead, multi-size or single-size.

Let us also mention that our approach does not impose a priori any prescribed or preferred location for heliostats of a given (small or large) size. Contrarily, we try to leave this completely free. As detailed in Table II and Figure 1, the feasible region has an annulus shape. However, note that in the following examples, the heliostats are located by the algorithm automatically at the north area, where higher energy values are reached (Figures 5 and 6).

Two different cost scenarios are studied, called Scenario-100 and Scenario-80. The results for both scenarios are shown in Tables III and IV, where $N_{\text{del}}$ denotes the number of large-size heliostats deleted by the algorithm at each iteration. At each scenario, the algorithm stops when no improvement in the LCOE value is found. The final fields obtained are $\Omega_5$ (Figure 7(a)) and $\Omega_{12}$.
As it can be seen in the resulting fields, there exist some holes and visual irregularities (because of heliostat(s) deleted at the last iterate and/or the nature of the problem: many local optima and non-convex constraints). In order to address these irregularities, the small-size heliostats can be directly relocated again, obtaining the regularized fields $\Omega_5^R$ and $\Omega_12^R$. Figures 7(b) and 8(b), respectively. However, the shading and blocking effects could increase with this simple compactification, and the annual energy value could be reduced. This is the case with the field $\Omega_5$ where the resulting field achieve a worst result than the previous field. In order to further improve the objective function, a pattern-free refinement procedure called ‘heliostat field improvement’ can be applied; see [48].

In Figures 5 and 6, a detail of the expansion–contraction phase for both cost scenarios is presented. The initial, middle and final steps are shown for both cost scenarios. The heliostats selected to be removed are marked with black filled squares (large-size heliostats) and small circles (small-size heliostats). These heliostats are sequentially selected by the algorithm to be eliminated because of their low LCOE per unit area values. As expected, the number of large-size heliostats deleted increases as the heliostat cost per unit area of small size decreases, and different solutions are obtained depending on the fixed scenario.

Table V summarizes the results obtained with the best multi-size fields calculated with the expansion–contraction algorithm ($\Omega_5$ and $\Omega_{12}-R$), the single-size fields used as starting point $\Omega_0$ and reference field $\text{PS10-592}$. In order to analyse the results, we also incorporate a small-size-heliostat field achieving $\Pi_0$. This field is denoted by $\Omega_0^\pi$; see Figure 4(c).

The LCOE result obtained at the worst multi-size scenario (Table III, Scenario-100) is similar to the reference plant $\text{PS10-592}$ and shows an improvement over the field obtained with the greedy algorithm $\Omega_0$. In this scenario, the best field obtained with the expansion–contraction algorithm is $\Omega_5$. 

(Figure 8(a)) with Scenario-100 and Scenario-80, respectively.
For Scenario-80, $\Omega_{12}$-R improves the LCOE value of the reference field PS10-592 and achieves a similar value than the best single-size-heliostat field $\Omega_0$.

Note that, with the same heliostats sizes, if we reduce the heliostat cost per unit area of small-size (for instance...
applying Scenario-60), multi-size-heliostat fields would not be advantageous, as it is optimal to use heliostats of just one size.

It can be seen in the results that the obtained multi-size-heliostat fields reach similar results but are not able to improve the single-size-heliostat fields obtained so far.
5. CONCLUDING REMARKS AND EXTENSIONS

In this paper, we present an optimization strategy that allows one to use various sizes of heliostats in the very same solar field. The proposed expansion–contraction algorithm is flexible enough to combine in an appropriate manner heliostats of different sizes and simultaneously considers both the location and the size of the heliostats. The algorithm tends to locate large-size heliostats in the most efficient regions of the field, and small-size heliostats (of the same cost/m²), near the borders and to fill-in the holes between large sizes heliostats when advantageous.

Using the expansion–contraction algorithm, a detailed comparative study can be performed, taking into account the different heliostats sizes available at the time of building an SPT system (having different aspect ratio, cost per unit area, etc.). With the illustrative example presented, multi-size-heliostat fields with similar LCOE values to the one of the single-size-heliostat fields are obtained. Hence, under the selected sizes and costs scenario, there are no advantages in mixing heliostats sizes in the same plant. In the following, we comment some of the assumptions made during the paper and possible extensions of this work.

In this paper, the algorithm has been applied with two rectangular heliostat sizes. The problem of considering more than two rectangular heliostats sizes or even sizes of different shape is a more complex problem that need to be studied. We have no evidence that the methodology here presented based on the stone-pebble-sand idea will work for this problem.

For simplicity, we have considered that all the heliostats have the same height of elevation axis and, also, that the aiming point is unique. Considering different pedestal heights for each size (or even different heights for the same size) and also including an aiming strategy are very interesting and more complex problems that need to be studied in detail. This will be the subject of future work.

The pattern-free location strategy used in this paper can be extended to successfully cover many other situations, as for instance ground irregularities, the effect of tower shading, variable (stochastic) meteorological data and multi-tower plants [49]. Note that the fields obtained with pattern-free strategies are less regular than the traditional pattern-based fields. However, new cleaning and maintenance strategies can be used for this kind of fields [50].

Table V. Results summary.

| Field  | $N$  | $N_{\text{small}}$ | $N_{\text{large}}$ | $\Pi_{12}$ (Ω) | $E(\Omega)$ | $F_{100}(\Omega)$ | $F_{80}(\Omega)$ |
|--------|-----|---------------------|---------------------|----------------|-------------|-----------------|----------------|
| PS10-592 | 592 | 0                   | 592                 | 45.03         | 127.4       | 0.01615         | 0.01615        |
| Ω2 HLarge | 617 | 0                   | 617                 | 45.06         | 126.0       | 0.01822         | 0.01822        |
| Ω3 HSmall | 18848 | 0               | 18848               | 45.03         | 130.1       | 0.01878         | 0.01615        |
| Ω1 | 4138 | 3647               | 491                 | 45.04         | 127.0       | 0.01816         | 0.01764        |
| Ω12-R | 13493 | 13322              | 171                 | 45.51         | 130.2       | 0.01824         | 0.01638        |
and if necessary, road access can be directly included without modifying the algorithm. In our approach, it is assumed that the same cleaning technology is going to be used with both sizes, and therefore, the same safe distance is considered for both sizes. If different technologies are used, different safe distances must be used, and this would lead to considerable complications in the formulation of the problem.

In practice, not only the heliostat field but also the tower-receiver subsystem must be optimized. This can be carried out following an Alternating algorithm, as suggested in [23]. This algorithm consists to sequentially optimize the field layout for a given tower-receiver design and, then, optimize the tower-receiver subsystem for the previously obtained field. This process is repeated until no improvement in the objective function is found. Other interesting situations appear when we consider a field with several towers and/or receivers; see [51] for some results in this direction.

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