PHENOMENOLOGY OF PARTICLE PRODUCTION AND PROPAGATION IN STRING-MOTIVATED CANONICAL NONCOMMUTATIVE SPACETIME

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We outline a phenomenological programme for the search of effects induced by (string-motivated) canonical noncommutative spacetime. The tests we propose are based, in analogy with a corresponding programme developed over the last few years for the study of Lie-algebra noncommutative spacetimes, on the role of the noncommutativity parameters in the $E(p)$ dispersion relation. We focus on the role of deformed dispersion relations in particle-production collision processes, where the noncommutativity parameters would affect the threshold equation, and in the dispersion of gamma rays observed from distant astrophysical sources. We emphasize that the studies here proposed have the advantage of involving particles of relatively high energies, and may therefore be less sensitive to “contamination” (through IR/UV mixing) from the UV sector of the theory. We also explore the possibility that the relevant deformation of the dispersion relations could be responsible for the experimentally-observed violations of the GZK cutoff for cosmic rays and could have a role in the observation of hard photons from distant astrophysical sources.

I. INTRODUCTION AND SUMMARY

Most approaches to the unification of general relativity and quantum mechanics lead to the emergence, in one or another way, of noncommutative geometry. While one can in principle consider a relatively wide class of flat noncommutative spacetimes (“quantum Minkowski”), most studies focus on the two simplest examples; “canonical noncommutative spacetimes” $(\mu, \nu, \beta = 0, 1, 2, 3)$

$$[x_\mu, x_\nu] = i\theta_{\mu,\nu}$$  \hspace{1cm} (1)

and “Lie-algebra noncommutative spacetimes”

$$[x_\mu, x_\nu] = iC_{\mu,\nu}^{\beta} x_\beta .$$  \hspace{1cm} (2)

The canonical type (1) was originally proposed in the context of attempts to develop a new fundamental picture of spacetime. More recently, (2) is proving useful in the description of string theory in presence of certain backgrounds (see, e.g., Refs. [1,2,3,4]). String theory in these backgrounds admits description (in the sense of effective theories) in terms of a field theory in the noncommutative spacetimes (1), with the tensor $\theta_{\mu,\nu}$ reflecting the properties of the specific background. Among Lie-algebra, type (2), noncommutative versions of flat (Minkowski) spacetime, research has mostly focused on $\kappa$-Minkowski spacetime $(l, m = 1, 2, 3)$

$$[x_m, t] = i\lambda x_m , \hspace{1cm} [x_m, x_l] = 0 .$$  \hspace{1cm} (3)

This form of noncommutativity can provide, for fixed value of the scale $\lambda$, a new fundamental description of (flat) spacetime, which in particular hosts a maximum momentum $\frac{1}{\lambda}$. No effective-theory applications (e.g., again for the description of physics in some background fields) has yet been found for (3).

1But in order to play a role in a fundamental picture of spacetime the $\theta_{\mu,\nu}$ cannot be constants (e.g. they should themselves be elements of an enlarged algebra, together with the coordinates (3)).
Several authors have proposed techniques that allow to place experimental bounds on the (dimensionful) parameters \( \theta_{\mu,\nu} \) of canonical spacetime and on the \( \lambda \) of \( \kappa \)-Minkowski Lie-algebra spacetime, have been rather different and this is partly understandable in light of the differences between the two scenarios, particularly

- The fact that (3) could provide a fundamental picture of spacetime encourages the assumption that \( \lambda \) be of the order of the Planck length \( L_p \approx 1.6 \times 10^{-33} \text{cm} \), so the search of experimental tests is naturally aiming for corresponding sensitivities. Instead, in its popular string-theory application, the \( \theta_{\mu,\nu} \) parameters of (3) reflect the properties of a background field of the corresponding string-theory context, and therefore there is no natural estimate for the \( \theta_{\mu,\nu} \) (they will depend on the strength of the background field).

- The new effects predicted by field theories in canonical spacetime (1) could be observably large also in the low-energy regime. Instead field theories in the Lie-algebra spacetime (3) predict noncommutativity effects that are vanishingly small for low-energy particles, but become significant in the high-energy regime (where the particle/probes have wavelength short enough to be affected by the small, \( \lambda \)-suppressed, noncommutative effects).

- As one easily realizes based on the fact that the \( \theta_{\mu,\nu} \) of (3) reflect the properties of a background in the corresponding string-theory context, the \( \theta_{\mu,\nu} \) parameters cannot be treated as observer-independent (the string-theory background takes different form/value in different inertial frames and the \( \theta_{\mu,\nu} \) transform accordingly). This must be taken into account in the analysis of the experimental contexts that could set bounds on the \( \theta_{\mu,\nu} \), and in particular it has important implications for the task of combining the limits obtained by different experiments. Instead, as clarified in Refs. [14,15], noncommutativity of type (3) does not identify a preferred inertial frame (\( \lambda \) is observer-independent), but rather reflects a deformed action of the boost generators. All limits on \( \lambda \) can therefore be combined straightforwardly.

In spite of the fact that indeed these differences between the two noncommutativity scenarios are rather significant, in this paper we show that some of the phenomenological contexts that are being analyzed from a \( \kappa \)-Minkowski, (3), perspective can also deserve interest from the perspective of canonical noncommutativity (1). The key ingredient for the experimental studies we consider is a deformed dispersion relation. Deformed dispersion relations arise naturally in noncommutative spacetimes. The conventional special-relativistic dispersion relation \( E^2 = p^2 + m^2 \) reflects the classical Lorentz symmetries of classical Minkowski spacetime. Noncommutative versions of Minkowski spacetime do not enjoy the same classical symmetries, and they therefore naturally lead to deformed dispersion relations. The \( \kappa \)-Minkowski spacetime (3) is known to be invariant under a group of deformed Lorentz transformations; there is no loss of symmetries (infinitesimal Lorentz transformations still correspond to 6 generators, which were already well understood in the mathematical-physics studies of deformed Poincaré algebras (12,29)), but the nature of these symmetries is different from the classical case and there is a corresponding deformation of the dispersion relation. Canonical noncommutative spacetimes (1) are clearly less symmetric than classical Minkowski spacetime (again, this is easily understood considering the presence of a background field in the corresponding string-theory picture). The loss of symmetries is reflected in a deformation of the dispersion relation.

Our proposal of investigating the implications of the deformed dispersion relations of the canonical case (1) in the same high-energy contexts previously considered for the deformed dispersion relations of \( \kappa \)-Minkowski might at first appear surprising. Whereas the significance of the dispersion-relation deformations encountered in \( \kappa \)-Minkowski increases with energy, the most significant dispersion-relation-deformation effects in canonical noncommutative spacetimes are formally found at low energies. But, as we show here, the relevant observational constraints are significant for canonical-noncommutativity in spite of the relatively high energies involved. We therefore argue that these observations may be relevant for several scenarios based on canonical noncommutativity. In particular, these observations appear to be relevant in attempts to test the idea of canonical noncommutativity in regimes which are immune from

\[ \text{Limits on the } \theta_{\mu,\nu} \text{ obtained using data analyses that assume different inertial frames cannot be directly combined/compared; one must first transform all analyses into a single inertial frame (e.g. the one naturally identified by the cosmic microwave background). This is unfortunately not taken into account by some authors.} \]

\[ \text{Besides the fact that in } \kappa \text{-Minkowski Lie-algebra spacetime there is no symmetry loss (only symmetry deformation), while in canonical noncommutative spacetimes there is a net loss of symmetries, another important difference is the fact that in field theories on } \kappa \text{-Minkowski the deformation of the dispersion relation is already encountered at tree level, while field theories on canonical noncommutative spacetimes have an unmodified (i.e. special-relativistic) tree-level propagator and the dispersion-relation deformation emerges only through loop corrections.} \]
the “contamination” of unknown features of the UV sector. If unknown new structures come in at a UV scale \( \Lambda \) they can also affect (through IR/UV mixing) the predictions of the theory at low-energy scales.

We adopt in this paper a strictly phenomenological viewpoint, but we are concerned with the rather preliminary status of the understanding of the infrared problems of field theories in canonical noncommutative spacetimes. Limits on \( \theta_{\mu,\nu} \) are being set in some studies based on low-energy considerations, but the IR/UV mixing introduces in these low-energy predictions a strong sensitivity (see, e.g., Refs. \([22,30]\)) to the structure of the unknown UV sector.

Some of the anomalous effects encountered in naïve low-energy studies could be made “even worse” (more anomalous with respect to conventional theories) as a result of effects induced through the IR/UV mixing \([22]\). It is also conceivable that the opposite might happen: the unknown UV sector might have such special properties that it “cures” (through the IR/UV mixing) the anomalous features encountered in naïve low-energy analysis. For example, it is sufficient to have SUSY (supersymmetry) in the UV sector to remove several troublesome low-energy predictions of canonical noncommutativity, and on the basis of the SUSY example it appears plausible \([30]\) that other features of the UV sector might positively affect the nature of the low-energy predictions.

High-energy data are already important, in theories in commutative spacetime, as a way to test theories in different regimes, providing information complementary to that obtained in low-energy studies, but in the case of canonical noncommutative spacetimes high-energy observations might be even more significant, since low-energy data provide indications which are subject to possible modification as a result of IR/UV mixing. In order to give a more explicit description of the type of scenarios we are considering, let us consider an example in which

(i) The canonical-noncommutativity scale \( \sqrt{\theta} \) is of order \( \sqrt{\theta} \sim 10^{-25}cm \sim (10^{11}GeV)^{-1} \).

(ii) SUSY (supersymmetry) is eventually “restored”, at least at some UV scale \( M_{susy} \).

(iii) The description of spacetime based exclusively on canonical noncommutativity is only applicable up to a UV cutoff \( \Lambda \sim 10^{19}GeV \). For energies above the UV cutoff spacetime acquires additional quantum features, even beyond noncommutativity, that are perhaps described by string theory or some other candidate “quantum gravity” theory. At the scale \( \Lambda \) we could also expect additional particle degrees of freedom.

In such a scenario the low-energy regime (say, \( E < 10^{3}GeV \sim 1/(\Lambda \theta) \)) could be largely unaffected by the spacetime noncommutativity, if the properties of the UV sector (through the IR/UV mixing) conspired to largely cancel out the characteristic low-energy predictions of canonical noncommutativity (in the same way and beyond what is understood to be the effect of UV SUSY on the low-energy sector of canonical noncommutative theories). Since the UV (\( E > \Lambda \)) sector is unknown and its features affect low-energy phenomena it is indeed plausible that the net result of noncommutativity combined with new UV physics would be giving us back physical predictions that are very close to the ones of an ordinary low-energy theory in commutative spacetime. This may require a large fine-tuning of the UV sector, but from a strictly phenomenological perspective it cannot be excluded, and it cannot be excluded if one is aiming for unconditional high-confidence bounds on the noncommutativity parameters.

So in the scenario characterized by hypotheses (i),(ii),(iii) the theory in canonical noncommutative spacetime might be completely indistinguishable from a corresponding theory in commutative spacetime at low-energy scales, scales such that \( E < 1/(\theta \Lambda) \). But above the scale \( 1/(\theta \Lambda) \) the theory in canonical noncommutative spacetime is “protected” from the UV: the predictions of a theory in canonical noncommutative spacetime in the energy range \( 1/(\theta \Lambda) < E < 1/(\sqrt{\theta}) \) are largely insensitive to the UV sector. We are not claiming that this scenario in which low-energy predictions are unaffected (as a result of mixing with a fine-tuned UV sector) by canonical noncommutativity is the only possibility (perhaps indeed it only occurs upon severe fine-tuning of the UV sector), but we are interested here in this particular possibility, especially since it would represent a huge challenge from a phenomenological perspective.

In the next Section we discuss (relying in part on results which had already appeared in the literature) the emergence of deformed dispersion relations in field theories (mostly QED) on canonical noncommutative spacetimes. We emphasize that this effect is particularly significant for uncharged particles; in fact, in canonical noncommutative spacetimes it is possible to describe the particles that we observe as neutral with respect to the electromagnetic

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4Because of the nature of the study we are reporting, we find useful sometimes to consider a single length scale \( \sqrt{\theta} \) to characterize the matrix \( \theta_{\mu,\nu} \). Of course, in principle there can be large differences between the non-zero entries of the \( \theta_{\mu,\nu} \) matrix, and the line of argument we adopt is applicable also to this more general situation (in some cases it would be sufficient to take \( \sqrt{\theta} \) as the largest length scale in \( \theta_{\mu,\nu} \)).

5Actually, the necessary fine-tuning of the UV sector might not even have to be very large \([30]\); as mentioned, simply assuming that SUSY is indeed restored at some UV scale already leads to significant reduction of the “anomalies” that would otherwise affect theories in canonical noncommutative spacetime in the low-energy regime.
interactions as particles that do interact with the photon (as a result of the structure of the “star product”) in contexts in which the $\theta_{\mu,\nu}$ parameters cannot be neglected. Loop corrections, and in particular photon dressing, of neutral-particle propagators in canonical noncommutative spacetimes generally induce corrections to the dispersion relation.

In Section III, in preparation for the later discussion of experimental tests, we discuss the type of contexts in which our phenomenological strategy appears to have greater potential for insight.

In Section IV we outline a phenomenological programme which can lead to bounds on (or estimates of) the $\theta_{\mu,\nu}$ parameters by studying the implications of the deformed dispersion relations discussed in Section II. We focus on the implications of a deformed dispersion relation $E^2 = f(p) \neq p^2 + m^2$ for: (E1) the threshold condition for particle production in certain collision processes, and (E2) the dispersion of signals observed from distant astrophysical sources. Since our objective here is the one of outlining a relatively wide phenomenological programme, postponing to future studies a detailed analysis of each of the proposals, we just list a few experimental contexts and provide estimates of the sensitivity levels that appear to be within reach of these experimental contexts. Concerning the implications of the deformed dispersion relations for the dispersion of signals observed from distant astrophysical sources, we focus on the case of gamma-ray bursts [31], whose observation provides powerful constraints on dispersion for photons, and on the case of 1987a-type supernovae, which can be used to constrain possible deformations of the dispersion relation for neutrinos. Concerning threshold conditions, we argue that high sensitivity to nonvanishing values of the $\theta_{\mu,\nu}$ parameters can be achieved by analyzing the threshold for electron-positron pair production from photon-photon collisions at energy scales relevant for the expected cutoff on the energies of hard photons observed from distant astrophysical sources. Similarly, deformed thresholds associated with dispersion-relation deformations can significantly affect the GZK cutoff for the observation of cosmic rays. Ultra-high-energy cosmic-ray protons, with energies in excess of the GZK cutoff, should not be detected by our observatories because they should lose energy (thereby complying with the GZK cutoff) through photo-pion production off cosmic-microwave-background photons. If the present (commutative spacetime) estimates of the GZK hard-proton cutoff and of the analogous hard-photon cutoff were confirmed experimentally, we could obtain very stringent limits on the $\theta_{\mu,\nu}$. Interestingly, observations of cosmic rays presently appear [32,27] to be in conflict with the cutoff estimates that assume the conventional dispersion relation (and the associated classical picture of spacetime).

Finally, Section V is devoted to some closing remarks, particularly concerning the outlook of the phenomenological programme here outlined.

## II. DEFORMED DISPERSION RELATIONS IN CANONICAL NONCOMMUTATIVE SPACETIME

The subject of field theory in canonical noncommutative spacetimes has been extensively studied, and there is a wide literature where the reader can find rather pedagogical introductions (see, e.g., Refs. [8,43-53,54,55,56,57,58,59,60]). We here just mention some well-established features of these field theories, and focus on a few loop corrections that are relevant for the analysis of deformed dispersion relations. The tree-level propagators are unmodified by the noncommutativity. Loop corrections to the two-point functions introduce terms that correspond to deformations of the dispersion relations and reflect the loss of symmetry associated with nonvanishing $\theta_{\mu,\nu}$ parameters. Particularly rich are the structures of the dressed photon propagator [60] and of the photon corrections to the propagators of particles that are neutral in the commutative-spacetime limit. Certain gauge-invariant actions in canonical noncommutative spacetime describe particles that are coupled to the gauge field only for nonvanishing $\theta_{\mu,\nu}$, i.e., these are particles that in the $\theta_{\mu,\nu} \rightarrow 0$ limit no longer interact with the gauge field. These particles are natural candidates to describe the neutral particles that we observe. In the remainder of this section we discuss the $\theta_{\mu,\nu}$ deformation of the dispersion relations for photons, neutral spin-1/2 fermions, and neutral spin-0 bosons. Here and in the following we are describing as (QED-)“neutral” the type of particles described above (no interactions with the photon in the $\theta_{\mu,\nu} \rightarrow 0$ limit, but

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6The loop corrections we are interested in concern the photon self-energy at one loop, and the one-loop contributions involving the photon to the self-energies of “neutral” spin-1/2 fermions and “neutral” spin-0 bosons. Our results on the photon self-energy reproduce the ones of the previous analysis reported in Ref. [60]. We did not find in the literature any analogous analysis of the self-energy of neutral spin-1/2 fermions. For the self-energy of neutral spin-0 bosons a related analysis has been reported in Ref. [61]; however, that study adopted a different gauge choice and it is not clear to us (it was not explicitly stated) whether it concerned “neutral” or “charged” spin-0 bosons (if it was meant to consider neutral particles, it would investigate the same loop corrections which are here of interest and our results would be in disagreement with some of the results of Ref. [61], even taking into account the different choice of gauge).
some $\theta_{\mu,\nu}$-dependent interactions with the photon in the noncommutative spacetime). We work in Feynman gauge, we adopt the standard notation

$$\tilde{p}_\mu \equiv p^\alpha \theta_{\mu,\alpha},$$

and we also assume throughout that $\theta_{\mu,0} = 0 = \theta_{0,\mu},$ i.e. we consider the case in which only (some of) the space components $\theta_{i,j}$ are nonvanishing. Although we are planning (as discussed in Section I) to develop a phenomenology for field theory in canonical noncommutative spacetime at certain relatively high energies, we will not explicitly focus on these energy scales in this section. However, as discussed in Sections III and IV, our phenomenological programme is primarily sensitive to the leading $\theta$-dependent deformations of the dispersion relations, and the fact that we intend to analyze contexts involving relatively high energy scales allows us to analyze self-energies with the implicit assumption that terms of order $\Lambda^{-2}$ can be neglected with respect to terms of order $p^2 \theta^2$, when $p$ is an external momentum ($p > 1/(\theta \Lambda)$). To the cutoff scale $\Lambda$ we attribute consistently the interpretation discussed in Section I: we assume that at some scale $\Lambda$ our field theories in canonical noncommutative spacetime fail to apply in a rather significant way, perhaps as a result of the presence of new stringy or quantum-gravity effects, and that, because of the IR/UV mixing [8,30], this unknown UV physics could affect profoundly the predictions of the theory at scales below $1/(\theta \Lambda)$.

### A. Deformed dispersion relation for photons

In order to see the emergence of a deformed dispersion relation for photons in canonical noncommutative spacetimes it is sufficient to consider one-loop contributions to the photon propagator from diagrams involving as virtual particles either photons themselves or other “neutral” particles. We shall often use the notation $\gamma$ for photons, $\nu$ for neutral spin-1/2 fermions, and $\Phi$ for neutral spin-0 bosons. Some relevant interaction vertices of the field theory in canonical noncommutative spacetime are:

- the four-$\gamma$ vertex

$$-4ig^2 \left( (g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}) \sin \frac{p_1 p_2}{2} \sin \frac{p_3 p_4}{2} + \right.$$

$$\left. + (g^{\alpha\delta} g^{\gamma\beta} - g^{\alpha\beta} g^{\gamma\delta}) \sin \frac{p_3 p_1}{2} \sin \frac{p_2 p_4}{2} + \right.$$

$$\left. + (g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta}) \sin \frac{p_3 p_4}{2} \sin \frac{p_2 p_3}{2} \right)$$

with every $p_i$ exiting the vertex;

- the three-$\gamma$ vertex

$$-2g \sin \frac{p_1 p_2}{2} \left( g^{\alpha\beta} (p_1 - p_2)^\gamma + g^{\alpha\gamma} (p_3 - p_1)^\beta + g^{\beta\gamma} (p_2 - p_3)^\alpha \right)$$

with every $p_i$ exiting the vertex;

- the $\gamma$-$\nu$-$\nu$ vertex

$$2g^\mu \sin \frac{p_1 p_2}{2}$$

where $p_1$ ($p_2$) is the momentum of the incoming (outgoing) $\nu$;

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7. The case of space/time noncommutativity ($\theta_{0i} \neq 0$) is not necessarily void of interest [41], but it is more delicate, especially in light of possible concerns for unitarity [42]. Since our analysis is not focusing on this point we will simply assume that $\theta_{0i} = 0$. 

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• the $\gamma-\Phi-\Phi$ vertex

$$4ig^2g^{\mu\nu} \left( \sin \frac{\tilde{p}_2 p_4}{2} \sin \frac{\tilde{p}_1 p_3}{2} + \sin \frac{\tilde{p}_2 p_3}{2} \sin \frac{\tilde{p}_1 p_4}{2} \right)$$  \hfill (8)\

where $p_1, p_2$ are the photons’ momenta and $p_3, p_4$ the scalars’ momenta, and they all exit the vertex;

• and the $\gamma-\Phi-\Phi$ vertex

$$2g (p_1 + p_2)^\mu \sin \frac{\tilde{p}_1 p_2}{2}$$  \hfill (9)\

where $p_1$ ($p_2$) is the momentum of the incoming (outgoing) scalar.

• In addition we will also need the $\gamma$-ghost-ghost vertex

$$2gp_2^\mu \sin \frac{\tilde{p}_1 p_2}{2}$$  \hfill (10)\

where $p_1$ ($p_2$) is the momentum of the incoming (outgoing) ghost.

There are three nontrivial pure-gauge one-loop contributions to the photon self-energy. For external photons with momentum $p$ and polarizations $\mu$ and $\nu$ one finds

• a tadpole-type diagram, using the vertex (5),

$$i\Sigma_{\gamma;1}^{\mu\nu} = -2ig^2 \int \frac{d^4k}{(2\pi)^4} -\frac{i\delta_{\rho\sigma}}{k^2} (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho} - g^{\mu\rho} g^{\nu\sigma}) \sin^2 \frac{\tilde{p}k}{2}$$  \hfill (11)\

• a photon-loop diagram, using the vertex (6),

$$i\Sigma_{\gamma;2}^{\mu\nu} = 2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i\delta_{\rho\sigma}}{k^2} (g^{\mu\sigma} (p-k)^\rho + g^{\mu\rho} (-2p-k)^\sigma + g^{\rho\sigma} (2k+p)^\mu)$$

$$\frac{-i\delta_{\rho\sigma}}{(p-k)^2} \left( g^{\mu\beta} (-p+k)^\alpha + g^{\nu\alpha} (2p+k)^\beta + g^{\alpha\beta} (-p-2k)^\nu \right) \sin^2 \frac{\tilde{p}k}{2}$$  \hfill (12)\

• and a ghost-loop diagram, using the vertex (10),

$$i\Sigma_{\gamma;ghost}^{\mu\nu} = 4g^2 \int \frac{d^4k}{(2\pi)^4} \frac{\delta_{\rho\sigma}}{k^2} (p+k)^\mu \frac{i}{(p-k)^2} k^\nu \sin^2 \frac{\tilde{p}k}{2}$$  \hfill (13)\

With straightforward calculations one finds [8] that the dominant $\theta$-dependent contribution from these diagrams are

$$i\Sigma_{\gamma;1}^{\mu\nu} = \frac{3ig^2g^{\mu\nu}}{2\pi^2 |p|^2}$$  \hfill (14)
\[ i\Sigma_{\gamma;\gamma;2} = \frac{-ig^2}{4\pi^2} \left( \gamma^\mu \gamma_\nu \gamma^\nu - \frac{10}{3} \frac{\bar{p}^\mu \bar{p}^\nu}{|\bar{p}|^4} \right), \quad (15) \]

and

\[ i\Sigma^{\mu\nu}_{\gamma;\text{ghost}} = \frac{ig^2}{4\pi^2} \left( -2 \frac{\bar{p}^\mu \bar{p}^\nu}{|\bar{p}|^4} + \frac{g^{\mu\nu}}{|\bar{p}|^2} \right). \quad (16) \]

Next let us consider the one-loop contribution to the photon self-energy that involves virtual neutral fermions, using the vertex (7):

\[ i\Sigma_{\mu\nu} = 4g^2 \int \frac{d^4k}{(2\pi)^4} \sin^2 \frac{\bar{p}k}{2} \left( \gamma^\mu \frac{i}{\gamma^\rho (p + k)^\rho - m} \gamma^\nu \frac{i}{\gamma^\sigma k_\sigma - m} \right). \quad (17) \]

from which we extract again the dominant \( \theta \)-dependent contribution

\[ i\Sigma_{\mu\nu} = -\frac{4ig^2}{\pi^2} \frac{\bar{p}^\mu \bar{p}^\nu}{|\bar{p}|^4}. \quad (18) \]

Finally we consider the two nontrivial one-loop contributions to the photon self-energy that involve virtual neutral scalars: a tadpole-type diagram, using the vertex (8),

\[ i\Sigma_{\gamma;\Phi;1} = 4ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \sin^2 \frac{\bar{p}k}{2}, \quad (19) \]

and a scalar-loop diagram, using the vertex (9),

\[ i\Sigma_{\gamma;\Phi;2} = -2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \left( (p + k)^\mu (p + 2k)^\nu \sin^2 \frac{\bar{p}k}{2} \right), \quad (20) \]

whose dominant \( \theta \)-dependent contributions are

\[ i\Sigma_{\gamma;\Phi;1} = \frac{ig^2}{2\pi^2 \bar{p}^2}, \quad (21) \]

and

\[ i\Sigma_{\gamma;\Phi;2} = -\frac{ig^2}{2\pi^2} \left( -2 \frac{\bar{p}^\mu \bar{p}^\nu}{|\bar{p}|^4} + \frac{g^{\mu\nu}}{|\bar{p}|^2} \right). \quad (22) \]

Combining these results one obtains [8] the total contribution to the photon self-energy:

\[ i\Sigma_{\gamma;\gamma} = \frac{ig^2}{\pi^2} \left( N_s + 2 - 2N_f \right) \frac{\bar{p}^\mu \bar{p}^\nu}{|\bar{p}|^4}. \quad (23) \]

where \( N_s \) and \( N_f \) denote the number of neutral scalar fields and the number of neutral fermion fields in the theory. It is important to notice [8] that in a supersymmetric field theory in canonical noncommutative spacetime, which would accommodate an equal number of bosonic and fermionic degrees of freedom, this correction term vanishes. However, while we do want to emphasize the implications that apply in particular to supersymmetric theories, we are here interested in general in the predictions of field theories in canonical noncommutative spacetime. There is at present no direct experimental evidence of supersymmetry, and accordingly in our phenomenological analysis we will assume that either there is no supersymmetry at all or that supersymmetry is broken at low energies (processes below the supersymmetry-restoration scale \( M_{\text{susy}} \), here treated as a phenomenological parameter). It is therefore important from our perspective to explore the physical content of (23) when supersymmetry is absent (or not yet restored). It is convenient to consider the simple case in which \( \theta_{i,j} \) is only nontrivial in the (1,2)-plane: \( \theta_{1,2} = -\theta_{2,1} \equiv \theta 
eq 0, \theta_{1,3} = \theta_{2,3} = 0 \). Relevant observations have already been reported in Ref. [8]: according to (23) the two, transversely
polarized, physical degrees of freedom of the photon satisfy different dispersion relations, reflecting the loss of Lorentz invariance introduced by $\theta$. If, for example, $\tilde{p}$ is in the 1-direction, in the case we are considering, $\theta_{1,3} = \theta_{2,3} = \theta_{\mu,0} = 0$, one finds that the degree of freedom polarized in the direction orthogonal to $\tilde{p}$ satisfies the ordinary special-relativistic dispersion relation

$$p_0^2 = \tilde{p}_0^2,$$

while the degree of freedom polarized in the direction parallel to $\tilde{p}$ satisfies a deformed dispersion relation of the type

$$p_0^2 = \tilde{p}_0^2 + \frac{\zeta_\gamma}{\tilde{p}_0^2},$$

where $\zeta_\gamma$ is a number that depends, according to (23), on the coupling constant and on the number of bosonic and fermionic degrees of freedom present in the theory. Besides the polarization dependence, these dispersion relations are strongly characterized by the term $\zeta_\gamma/\tilde{p}_0^2$, which reflects the mentioned IR/UV mixing.

### B. Deformed dispersion relation for neutral spin-0 bosons

We now analyze in the same way the self-energy of scalars (neutral spin-0 bosons) in canonical noncommutative spacetime. We focus on the loop contributions involving virtual photons and on the loop contributions involving virtual scalars (self-interactions). There are two nontrivial one-loop contributions to the scalar self-energy that involve virtual photons (for a scalar of external momentum $p$):

- a tadpole-type diagram, using the vertex (8),

$$i\Sigma_{\Phi;\gamma;1} = 4ig^2g^{\mu\nu}\int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\mu\nu}}{k^2} \frac{\sin^2 \frac{\tilde{p}k}{2}}{2},$$

- and a sunset-type diagram, using the vertex (9),

$$i\Sigma_{\Phi;\gamma;2} = -4g^2\int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\mu\nu}}{(p+k)^\mu (p+k)^\nu} \frac{i}{k^2 - m^2} \frac{\sin^2 \frac{\tilde{p}k}{2}}{2}.$$

Again, the dominant $\theta$-dependent contribution from these diagrams can be established with straightforward calculations, finding

$$i\Sigma_{\Phi;\gamma;1} = -\frac{2ig^2}{\pi^2 |\tilde{p}|^2},$$

and

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10 On this issue of the breaking of Lorentz invariance there is sometimes some confusion. At the fundamental level these theories in canonical noncommutative spacetimes are still special-relativistic in the ordinary sense. However, $\theta$ has the role of a background (as well understood in the corresponding string-theory picture) and of course effective field theories in presence of some background do not enjoy Lorentz symmetry. The Lorentz invariance of the theory becomes manifest only in formalisms that take into account both the transformations of the quantum fields and of the background. The effective field theory does have a preferred frame (which one can identify by selecting, e.g., a frame in which the form of the background is particularly simple) but this is of course fully consistent with special relativity. The situation is completely different in Lie-algebra noncommutative spacetimes. For example, it was recently shown [14] that $\kappa$-Minkowski does not reflect the properties of a background and does not have a preferred frame, both $\lambda$ and $c$ have in $\kappa$-Minkowski roles which are completely analogous to the role that only $c$ enjoys in classical Minkowski: the role of an observer-independent kinematic scale [14][15], left invariant under (deformed) Lorentz transformations (and in particular the observer-independent scale $\lambda$ acquires the role of the inverse of the maximum momentum of the theory [15][16][17]). Instead in canonical noncommutative spacetimes the $\theta_{\mu,\nu}$ is not invariant under Lorentz transformations; it transforms of course like a tensor. More on this type of considerations can be found in Ref. [14].

11 In the contexts analyzed in Section IV one finds that terms of the type $1/\tilde{p}^2$ are more significant than the ones of the type $\ln(\tilde{p}^2)$. In light of this observation we consistently neglect logarithmic $\theta$ dependence when terms of the type $1/\tilde{p}^2$ are present.
The one-loop (tadpole) contribution to the scalar self-energy due to its self-interactions is a well-known prototype result of field theory in canonical noncommutative spacetime. One finds that the most important effect of the noncommutativity in this self-interaction tadpole contribution is again of the type $1/\tilde{p}^2$. Therefore combining photon dressing and self dressing of the propagator one finds a deformed dispersion relation for neutral spin-0 bosons of the type

$$p_0^2 = \bar{p}^2 + m_\Phi^2 + \frac{\zeta_\Phi}{\bar{p}^2},$$ (30)

where again $\zeta_\Phi$ is a number that depends on the coupling constants of the theory and on the number and type of fields involved.

### C. Deformed dispersion relation for neutral spin-1/2 fermions

We close this Section by performing an analogous dispersion-relation analysis for neutral spin-1/2 fermions. We only consider the self-energy contribution that is due to interactions with the photon. The relevant sunset-type diagram, using the vertex $\tilde{p}$, corresponds to the integral

$$i\Sigma_{\nu;\gamma} = -4g^2 \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \gamma^\rho \frac{i}{\gamma^\tau k^\sigma - m} \gamma^\nu - ig_{\mu\nu} \frac{-(p - k)^2}{2} \sin^2 \frac{\tilde{p}k}{2};$$ (31)

which can be evaluated exactly, obtaining:

$$i\Sigma_{\nu;\gamma} = \frac{2g^2}{(4\pi)^2} (-\gamma^\mu p_\mu + 4m) \ln \frac{\Lambda^2}{\Lambda_{eff}^2},$$ (32)

where $\Lambda$ is a conventional high-momentum cutoff and $\Lambda_{eff}$

$$\frac{1}{\Lambda_{eff}^2} = \frac{1}{\Lambda^2} + \frac{\tilde{p}^2}{4}.$$ (33)

The physical implications are analogous to the ones found for photons and neutral spin-0 bosons. For neutral spin-1/2 fermions one has again a deformation of the dispersion relation which (if the fermion has mass) is singular in the infrared; however, the singularity is softer, only logarithmic, and accordingly the magnitude of the effects to be expected (whether or not the fermion has mass) at relatively small momenta is not as significant as, e.g., for the photon. We will therefore not express high expectations for the bounds on $\theta$ that can be placed using observations of neutral spin-1/2 fermions, but still we will comment on some types of experiments/observations that are sensitive to the dispersion-relation deformation experienced by these particles in canonical noncommutative spacetime.

### III. OBJECTIVES OF HIGH-ENERGY $\theta_{\mu,\nu}$ PHENOMENOLOGY

In the next Section we discuss certain classes of observations in astrophysics in which the deformed dispersion relations that are characteristic of field theory in canonical noncommutative spacetime can have significant implications. These observations in astrophysics involve particles of relatively high energy (all relevant processes involve at least one particle with $E > 1$ TeV) and the nature of the observations is such that they can be used to constrain (or search for) even rather small deviations from the standard $E^2 = m^2 + p^2$ dispersion relation.
We are bringing these observations to the attention of the community interest in canonical noncommutativity since deformed dispersion relations are a characteristic feature of field theories in canonical-noncommutative spacetime. These observations may therefore be relevant for several scenarios based on canonical noncommutativity. For example, as mentioned in Section I, these observations appear to be relevant in attempts to test the idea of canonical noncommutativity in regimes which are immune from the “contamination” of unknown features of the UV sector. If unknown new structures come in at a UV scale Λ they can also affect (through IR/UV mixing) the predictions of the theory at energy scales below 1/(θΛ). By looking for ultra-high-energy tests of canonical noncommutativity one is essentially hoping to gain access to energy regimes that are high enough to be above the scale 1/(θΛ).

As mentioned in Section I, it is even plausible to contemplate canonical-noncommutativity theories in which, as a result of certain corresponding hierarchies between the relevant energy scales and of corresponding properties of the UV sector, noncommutativity leads to negligible effects below the scale 1/(θΛ), but leads to the characteristic dispersion-relation-deformation effects above the scale 1/(θΛ). Because of the IR/UV mixing it cannot be excluded that some properties of the (unknown) UV sector might lead to exact (or nearly-exact) cancellations of the anomalous effects that canonical noncommutativity would otherwise predict for processes below the scale 1/(θΛ). The highest-energy processes which we witness in astrophysics provide us an opportunity to test canonical noncommutativity in a way that would be protected from the (however unlikely) possibility of such a “conspiracy” (UV structures that conspire to cancel out the characteristic effects of canonical noncommutativity in the infrared).

Of course one of the unknowns of our study is the “infrared scale” 1/(θΛ). As a result, the analysis of relevant astrophysics contexts at relatively high energies will lead us to conclusions of the “either/or” type. If a certain deformation of the dispersion relation is not seen in observations conducted at an energy/momentum scale E this can be interpreted in two ways: one should either constrain the noncommutativity parameters accordingly (to suppress the unwanted effect) or assume that even at the relatively high energy scale E some unknown properties of the UV sector have managed to conspire to cancel out the unwanted dispersive effects (i.e. even the energy scale E is below 1/(θΛ)).

In some of the astrophysical processes that we consider, in addition to the high-energy particle there is also a very soft particle, typically a soft photon. We will assume that these very soft particles are unaffected by noncommutativity, since this is consistent with the conservative approach we are adopting: we want to test the idea of canonical noncommutativity relying exclusively on its characteristic departures from commutative-spacetime predictions at energy scales larger than 1/(θΛ).

For our programme of experimental searches of the effects predicted by field theories in canonical noncommutative spacetime another key issue is the one of the identification of the SUSY-restoration scale $M_{\text{susy}}$. While the phenomenology we discuss is of even wider applicability, our key focus will be on scenarios in which SUSY is present in the UV sector (in order to contribute to the “cure” of the infrared problems), but the restoration of SUSY occurs at ultrahigh energies, possibly above the noncommutativity scale 1/(√θ). In such a situation SUSY can still effectively contribute to the “cure” of the infrared problems of canonical noncommutativity, although it would occur at energy scales typically higher than the ones usually considered for SUSY restoration in commutative spacetimes affected by the “hierarchy problem”. We feel that the role of SUSY in commutative spacetime and in canonical noncommutative spacetime should be distinguished more carefully than usually done in the literature. In commutative spacetime Wilson decoupling between the UV and the IR sectors is at work, and SUSY is only needed in order to provide an elegant explanation for the hierarchy of mass scales present in the Standard Model of particle physics, which would otherwise be puzzling in light of the corresponding renormalization-group results. In canonical noncommutative spacetime the renormalization group analysis is alarming already before considering problems of mass-scale hierarchy: much more troubling problems are present as a result of the lack of Wilson IR/UV decoupling. SUSY is desperately needed in order to cure some unacceptable IR problems, whereas in commutative spacetime SUSY improves the compellingness of our description of particle physics, by eliminating the (phenomenologically acceptable but “unnatural”) fine-tuning otherwise required by the mass-scale hierarchy. In the study of theories in commutative spacetime, because of the reasons just mentioned, we have become accustomed to assuming a SUSY-restoration scale of the order of 17TeV or 10TeV. Theories in commutative spacetime are consistent even without SUSY restoration, but assuming SUSY restoration at 17TeV or 10TeV a more compelling picture emerges. In the new subject of theories in canonical noncommutative spacetime we probably must assume SUSY restoration, at least in the UV sector. Whether or not theories in canonical noncommutative spacetime become more compelling with a lower scale of SUSY restoration remains to be established (it requires more studies in which the renormalization group is applied to “realistic” models based on canonical noncommutativity).
IV. HIGH-ENERGY ASTROPHYSICS OBSERVATIONS AND CANONICAL NONCOMMUTATIVITY

As announced in Section I, we are proposing to test the predictions of canonical noncommutative spacetime using the same techniques previously developed to search for the effects possibly induced by the parameter $\lambda$ of the Lie-algebra noncommutative spacetime $\mathfrak{g}$. In fact, both classes of noncommutative spacetimes are characterized by deformations of the special-relativistic dispersion relation, and can therefore both be tested using experiments with good sensitivity to such deformations. We focus here on the role of deformed dispersion relations for the evaluation of the threshold condition for particle production in certain collision processes, and for the dispersion of signals observed from distant astrophysical sources. Since our objective here is the one of outlining a relatively wide phenomenological programme, postponing to future studies a detailed analysis of each of the proposals, we just list a few experimental tests and provide estimates of the sensitivity levels that appear to be within reach of these tests. Our sensitivity estimates are also rather tentative: the relevant astrophysics studies are experiencing a fast rate of improvement and one can expect these sensitivities to increase significantly over the next few years.

A. Gamma-ray astrophysics

It is a rather general feature \cite{25} of “quantized” (discretized, noncommutative...) spacetimes to induce anomalous particle-propagation properties. In some pictures \cite{13,14,15} of “spacetime foam” (not involving noncommutative geometry) one describes foam as a sort of spacetime medium that, like other media, induces dispersion. As discussed in the preceding Sections, in quantum pictures of spacetime based on canonical or Lie-algebra noncommutative geometry one also automatically finds deformations of the dispersion relation. The observation that gamma-ray astrophysics could be used to search for the effects of such deformed dispersion relations was put forward in Ref. \cite{13}, focusing on foam-induced dispersion. The use of the same gamma-ray astrophysics for tests of the predictions of $\kappa$-Minkowski Lie-algebra noncommutative spacetimes was then discussed in Refs. \cite{23,13,14}. Here we observe that gamma-ray astrophysics can also be used to set bounds on the $\theta_{\mu\nu}$ parameters of canonical noncommutative spacetimes. Bounds on deformations of the dispersion relation can be set by analyzing time-of-arrival versus energy correlations of gamma-ray bursts that reach our detectors coming from far away galaxies. In presence of a deformed dispersion relation (which implies that photons of different energies travel at different speeds) photons emitted in a relatively short time (a burst) should reach our detectors with a larger time spread, and the spreading should depend on energy difference.

Gamma-ray bursts \cite{31} travel over large distances, $\sim 10^{10}$ light years, and are observed to maintain time-of-arrival correlation over very short time scales, $\sim 10^{-3}$s. For photons in the bursts that have energies in the 100KeV-1MeV range, as the ones observed by the BATSE detector \cite{31}, this has of course allowed to establish \cite{13,14,15} that in vacuo dispersion (if at all present) is small enough to induce relative time-of-arrival delays between photons with energy differences of a few hundred KeV that are below the $10^{-3}$s level. This turns out to set a stringent limit on the $\lambda$ parameter of the $\kappa$-Minkowski Lie-algebra spacetime, $\lambda < 10^{-30} cm$, and experiments now in preparation will allow to probe even smaller, subPlanckian, deformation scales \cite{43,13,17,18}. Limits of comparable significance can be obtained analyzing the bursts of photons emitted by blazars \cite{19}. An analysis of data on the Markarian 421 blazar allowed to establish \cite{43} that photons with energies of a few TeV (and comparable energy differences within the burst) acquire relative time-of-arrival delays that are below the $10^{3}$s level for travel over distances of order 100Mpc. This again sets a limit on $\lambda$ that is of order $\lambda < 10^{-30}cm$.

With respect to the analysis of these astrophysics contexts in $\kappa$-Minkowski Lie-algebra spacetime, the analysis in canonical noncommutative spacetimes is complicated by the polarization dependence of the deformation of the dispersion relation for photons. However, it appears safe to assume that the emissions by gamma-ray-bursts and blazars are largely unpolarized, and therefore canonical noncommutative spacetimes would imply that at least a portion of the bursts (also depending on the position of the source, which of course fixes the direction of propagation of the photons that reach us from the source) would manifest a dispersion-induced effect. A short-duration light burst travelling in a birefringent medium increases its time spread over time.

As announced we just want to estimate this type of sensitivities and we want to focus on data that are obtained as far from the infrared as possible. Considering the TeV photons of the mentioned blazar and the nominal \cite{19} $\delta T/T \sim 10^{-12}$ accuracy (with $\delta T$ the observed level of time-of-arrival simultaneity and $T$ the overall time of flight), the $(\rho\theta)^{-2}$ behaviour of the deformation of the dispersion relation\cite{43} leads to expected (polarization-dependent and)

\textsuperscript{13}In our estimates we take the numerical factors $\zeta$ introduced in Section II to be of order 1. They are probably smaller than 1 (they involve the QED coupling constant $\alpha$) but at this preliminary stage we are only trying to establish a rough picture of
energy-dependent time delays of \((\Delta \vartheta)^{-2} \sim (TeV^2 \vartheta)^{-2} T\). Therefore, if terms of the type \((p\vartheta)^{-2}\) are present in the dispersion relation that applies to photons at energies in the TeV range, then necessarily one must impose \(\sqrt{\vartheta} > 10^{-14} cm\), i.e. \(1/\sqrt{\vartheta} < 1 GeV\) (this assures that \((TeV^2 \vartheta)^{-2} \ll 10^{-12}\), so that the observed level of time-of-arrival simultaneity is not in conflict with canonical noncommutativity).

One can confidently exclude spacetime noncommutativity at the GeV scale, and therefore this analysis imposes that even for TeV photons the \((p\vartheta)^{-2}\) behaviour of the deformation of the dispersion relation is not acceptable phenomenologically. One must therefore either reject canonical noncommutativity altogether or assume that the UV sector is effective in eliminating the \((p\vartheta)^{-2}\) corrections all the way up to the TeV scale (which can be accomplished by introducing suitable SUSY in the UV).

In a certain sense our result is negative, since it does not leave us with any hope of finding one day the effects of canonical noncommutativity through this gamma-ray-dispersion studies. A positive result would have been to find present limits on TeV gamma-rays dispersion to be consistent with some still relatively high noncommutativity energy scale \(1/\sqrt{\vartheta}\). This would have left us with the hope that future more sensitive searches of TeV gamma-rays dispersion might find some evidence of dispersion and that this evidence could be interpreted in terms of canonical noncommutativity.

### B. Hard-photon FIRB-absorption threshold

Both in the study of spacetime-foam models \([43,44,45]\) and in the study of \(\kappa\)-Minkowski Lie-algebra noncommutative spacetimes the emerging deformed dispersion relations have also been analyzed \([46,14,27,32,50,51]\) for what concerns their implications for the determination of the threshold conditions for particle-production in collision processes. The interested reader can find these previous results in Refs. \([43,44,45,27,32,50,51]\), and references therein. Here we apply the same line of analysis to the deformed dispersion relations that emerge in canonical noncommutative spacetimes. An interested reader can find these previous results in Refs. \([43,44,45,27,32,50,51]\), and references therein.

In the case of canonical noncommutative spacetimes it is important to take into account the polarization dependence of the deformation of the photon dispersion relation. This would lead to different threshold conditions for different polarizations. If the threshold is increased only for photons with a certain polarization, then one would expect that at the conventional special-relativistic threshold there would be a suppression of the flux, but only partial. Another delicate issue is the one of the very soft (FIRB) photon involved in the process. As discussed in the previous sections, we will neglect possible canonical-noncommutativity deformations of the dispersion relation for such low-energy photons.

Observational evidence with respect to this absorption prediction is still preliminary but a cut off on the hard-photon energy \(E\) and the soft FIRB photon energy \(\epsilon\) should satisfy the kinematical condition for the production of an electron-positron pair: \(E > m^2/\epsilon\). The end result is that (with some uncertainty due to our partial knowledge of the FIRB) absorption by the FIBR should become significant for hard photons of energies of 10-15 TeV and higher.

This hard-photon absorption phenomenon is analyzed using a simple kinematical requirement. The balance of the hard photon energy \(E\) and the soft FIRB photon energy \(\epsilon\) should satisfy the kinematical condition for the production of an electron-positron pair: \(E > m^2/\epsilon\). The end result is that (with some uncertainty due to our partial knowledge of the FIRB) absorption by the FIBR should become significant for hard photons of energies of 10-15 TeV and higher.  

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the \(\vartheta\)-sensitivities of some relevant experiments, and we would not be too concerned even with inaccuracies of 1 or 2 orders of magnitude.

\(^{14}\)We are making the (apparently robust) assumption that is not possible to make a working phenomenological proposal based on a noncommutativity energy scale \(1/\sqrt{\vartheta}\) with value that falls within the energy scales to which we do have access in laboratory experiments, such a particle accelerators.

\(^{15}\)In principle, canonical noncommutativity could lead to large deformations for infrared photons, but the corresponding predictions are subject to severe modification by the UV sector, through IR/UV mixing. Moreover, at infrared energies we have a huge amount of data confirming the special-relativistic behaviour of photons, and therefore (if canonical noncommutativity must at all be studied) an a priori assumption of phenomenology in canonical noncommutative spacetimes must be that the unknown UV sector should cure the potential problems of the far infrared.
photons.

We shall therefore assume that only the hard photon is affected by the deformed dispersion relation, while the FIBR photon involved in the process obeys the ordinary special-relativistic dispersion relation. Again, postponing a more detailed analysis to future studies, we estimate here the range of the $\theta_{\mu, \nu}$ parameters that could significantly affect the absorption threshold for multi-TeV photons from blazars. Since the deformation of the dispersion relation is governed by correction terms of the type $(p\theta)^{-2}$, it is easy to verify that the absorption threshold could be significantly affected only if $(p\theta)^{-2} \gtrsim m^2\theta$. In the case of interest $p \sim 10^{20}\text{TeV}$, and therefore the threshold can be significantly affected only if the value of $\sqrt{\theta}$ is at least as large as $10^{-15}\text{cm} \sim (10^{-2}\text{TeV})^{-1}$. Again we are finding a limit on the noncommutativity energy scale, $1/\sqrt{\theta} < 10\text{GeV}$, which on other grounds (see previous Subsection) one can confidently exclude. Therefore, if indeed more refined data confirm that the threshold for multi-TeV photons from blazars is correctly predicted by the ordinary special-relativistic dispersion relation, this analysis imposes that even for photons of energies of order $10\text{TeV}$ the $(p\theta)^{-2}$ behaviour of the deformation of the dispersion relation is not acceptable phenomenologically. One must therefore either reject canonical noncommutativity altogether or assume that the UV sector is effective in eliminating the $(p\theta)^{-2}$ corrections all the way up to the $10\text{TeV}$ scale (which can be accomplished by introducing suitable SUSY in the UV).

C. Cosmic-ray threshold

Ultra-high-energy cosmic rays can interact with the Cosmic Microwave Background Radiation (CMBR), producing pions ($p + \gamma \rightarrow p + \pi$). This phenomenon can be analyzed just like the pair-production process relevant for observations of multi-TeV photons, which we discussed in the previous Subsection. Taking into account the typical energy of CMBR photons, and assuming the validity of the kinematic rules for the production of particles in our present, classical and continuous, description of spacetime (ordinary relativistic kinematics and dispersion relation), one finds that these interactions should lead to an upper limit $E < 5 \cdot 10^{19}\text{eV}$, the GZK limit $[54]$, on the energy of observed cosmic rays. Essentially, cosmic rays emitted with energies in excess of the GZK limit should lose energy on the way to Earth by producing pions, and, as a result, should still satisfy the GZK limit when detected by our observatories.

As for the case of the multi-TeV photons, a deformed dispersion relation can affect the prediction for the $p + \gamma \rightarrow p + \pi$ threshold. Again we refer the reader interested in the analysis of the cosmic-ray paradox within other quantum-spacetime frameworks to previous results in the literature $[16, 14, 12, 17, 58]$; here we just intend to focus on canonical noncommutative spacetimes and estimate values of the $\theta_{\mu, \nu}$ parameters that could significantly affect the GZK cosmic-ray threshold.

The process $p + \gamma \rightarrow p + \pi$ does not involve any hard photons, and, for the reasons discussed above, we will assume that the soft CMBR photon should be analyzed according to the conventional special-relativistic dispersion relation. For the proton, which is electrically charged, strong anomalies in the dispersion relation are not expected. The most significant dispersion-relation deformation relevant for this process could be attributed to the neutral/uncharged pion $\pi^0$. Assuming that $(p\theta)^{-2}$ corrections to dispersion relations do indeed characterize the kinematics of this process, we observe that these deformations would be significant at the GZK scale if $(p\theta)^{-2} \gtrsim m_{\pi}m_p$, with $p \sim 10^{20}\text{eV}$. Therefore, for $\sqrt{\theta} < 10^{-20}\text{cm} \sim (10^2\text{TeV})^{-1}$ there could be significant implications for this type of experimental studies.

This possible implication of canonical noncommutativity certainly deserves interest. In this cosmic-ray context the prediction of the standard special-relativistic dispersion relation can be questioned. As mentioned this standard dispersion relation leads to the GZK limit, and instead, observations of several cosmic-rays above the GZK limit (with energies as high as $3 \cdot 10^{20}\text{eV}$) have been reported $[33]$. The fact that canonical noncommutativity at or above the $1/\sqrt{\theta} \sim (10^2\text{TeV})$ scale could significantly affect the GZK prediction could be used in attempts to explain these surprising violations of the standard cosmic-ray paradox. Canonical noncommutativity with $1/\sqrt{\theta} \sim (10^2\text{TeV})$ also appears to be plausible, since we do not yet have access to the $10^2\text{TeV}$ scale in laboratory experiments.

In order for the dispersion relation for particle of energies $\sim 10^{20}\text{eV}$ to be affected by $(p\theta)^{-2}$ corrections it is necessary $[30]$ to assume that SUSY is not yet restored at $10^{20}\text{eV} = 10^8\text{TeV}$. This is consistent with the experimental information so far available, but it would be in conflict with the expectations that are formulated in the most popular

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$^{16}$Our wording here is rather prudent as a result of the fact that the pion is a composite particle. In Lie-algebra noncommutative spacetimes it appears $[14]$ that composite particles are subject to a deformed dispersion relation that is different from the one for fundamental particles. We are not aware of analogous results in canonical noncommutative spacetimes, but of course this issue would be important for the kinematics of $p + \gamma \rightarrow p + \pi$. 
theoretical scenarios for particle physics in commutative spacetime. As emphasized in Section III, the role of SUSY in commutative spacetime and in canonical noncommutative spacetime should be distinguished more carefully than usually done in the literature. In commutative spacetime Wilson decoupling between the UV and the IR sectors is at work, and this has a nontrivial role in the expectation that SUSY should be restored well below $10^8 TeV$ (probably already at a scale of a few $TeVs$). In canonical noncommutativity there is no Wilson IR/UV decoupling and it appears plausible to contemplate a much higher SUSY-restoration scale, possibly above $10^8 TeV$.

D. 1987a-type supernovae

Our final observation concerning possible experimental investigations of the dispersion-relation deformations that emerge in canonical noncommutative spacetimes involves neutrinos. We have seen in Section II that also neutral spin-$1/2$ particles could acquire a dispersion-relation deformation, but with a somewhat softer (logarithmic, rather than inverse-square power) dependence on “$p\theta$". This might mean that, if any of these experimental searches ends up being successful, the first positive results are unlikely to come from data on neutrino kinematics. Still, one should keep in mind that neutrinos are a particularly clean spacetime probe. The identification of quantum-spacetime effects that can be qualitatively described as “spacetime-medium effects”, such as the ones induced by canonical noncommutativity, can sometime be obstructed by the fact that most particles also interact with more conventional (electro-magnetic) media, as indeed is the case for the particles that reach our detectors from far-away galaxies.

Neutrinos are only endowed with weak-interaction charges. They are therefore mostly insensitive to this conventional dispersion-inducing effects, and as a result could provide clean signatures of spacetime-induced dispersion. In this respect supernovae of the type of 1987a might provide a interesting laboratory because of the relatively high energy of the observed neutrinos ($\sim 100 MeV$), the relatively large distances travelled ($\sim 10^3$, $10^4$ light years), and the short (below-second) duration of the bursts.

V. CLOSING REMARKS

We have argued that certain types of experimental tests which were previously considered in the literature on Lie-algebra noncommutative spacetimes, can also be of interest for investigations of canonical noncommutative spacetimes. The relevance of these experimental contexts comes from the fact that they rely on deformed dispersion relations, a feature that is present in both types of noncommutative spacetimes.

The observations involve particles of relatively high energies. This is welcome in Lie-algebra noncommutative spacetimes because the effects are fully confined to the high-energy regime, and we argued that it should also be welcome in canonical noncommutative spacetimes in light of the peculiar dependence of the infrared sector on the unknown structure of the UV sector. With low-energy tests of canonical noncommutativity we probe predictions of the theory which are highly sensitive to the structure of the unknown UV sector. If one manages to gain access to observations that pertain the regime $E > 1/(\Lambda \theta)$ the unwanted sensitivity to the UV sector would be avoided.

In two types of observations involving TeV and multi-TeV photons we were led to the conclusion that (if not rejected altogether) requires a UV sector which (through IR/UV mixing) eliminates the $(p\theta)^{-2}$ corrections to the dispersion relation all the way up to the $10 TeV$ scale (which can be accomplished by introducing suitable SUSY in the UV).

Potentially more exciting is our preliminary analysis of ultra-high-energy cosmic rays from the canonical-noncommutativity perspective. We found that the type of deformed dispersion relations predicted by canonical noncommutativity could explain the puzzling observations of cosmic rays above the GZK cutoff ($5-10^7 TeV$). This would require a noncommutativity scale in the neighborhood of $1/\sqrt{\theta} \sim 10^2 TeV$. We clarified that in order to obtain a working canonical noncommutativity scenario with this attractive prediction it appears necessary to obtain a satisfactory description of pions as composed by fundamental quarks (a description which is turning out to troublesome in other noncommutative spacetimes) and a satisfactory canonical-noncommutativity physical picture in which SUSY is restored only at scales higher than $10^8 TeV$.

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17 As mentioned, canonical noncommutative spacetimes require the existence of a preferred frame, and most of their features, especially concerning the associated dispersion-relation deformations, can be described in close analogy with ordinary (commutative-spacetime) physics in presence of a medium (think, for example, of the laws that govern propagation of light in certain crystals).

18 Interestingly, if spacetime was indeed described by a canonical noncommutative geometry, neutrinos would acquire at once both a deformed dispersion relation and the ability to interact, however softly, with electromagnetic media.
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