QCD CORRECTIONS TO THE $t \rightarrow H^{+}b$ DECAY WITHIN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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ABSTRACT

I present the contribution of gluinos and scalar quarks to the decay rate of the top quark into a charged Higgs boson and a bottom quark within the minimal supersymmetric standard model, including the mixing of the scalar partners of the left- and right-handed top quark. I show that for certain values of the supersymmetric parameters the standard QCD loop corrections to this decay mode are diminished or enhanced by several 10 per cent. I show that not only a small value of 3 GeV for the gluino mass (small mass window) but also much larger values of several hundreds of GeV’s have a non-negligible effect on this decay rate, against general belief. Last but not least, if the ratio of the vacuum expectation values of the Higgs bosons are taken in the limit of $v_1 \ll v_2$ I obtain a drastic enhancement due to a $\tan \beta$ dependence in the couplings.

March 1994

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I. INTRODUCTION

Recently there have been a lot of interest in the electroweak loop corrections \cite{1,2} as well as to the QCD loop corrections \cite{3,4,5,6} to the top quark decay into a charged Higgs boson and a bottom quark.

In the standard model we have no charged Higgs particle and therefore this decay can be used as a test for models beyond the standard model; such as a two Higgs doublet model \cite[see eg. 7 and references therein]{7} and the minimal supersymmetric extensions of the standard model (MSSM) \cite{8,9}, which is the favorite model beyond the standard model.

In this paper I take the last one as the underlying model to consider the QCD corrections to the $t \rightarrow H^+ b$ decay mode. In \cite{6} it was shown that the effect of the mass of the bottom quark to this decay rate is negligible. It was also shown that the ratio of the first order to the zeroth order is constant about $-9\%$ for a wide range of the Higgs mass $(0 \leq m_{H^+} \leq 90$ GeV$)$; the top quark mass was taken to be 150 GeV.

In this paper I present the QCD loop corrections to the $t \rightarrow H^+ b$ decay if gluinos and scalar quarks are taken within the loop as shown in Fig.1. Throughout the calculations I neglect the mass of the bottom quark, but I do not neglect the mixing of the scalar partners of the left- and right-handed top quark, which is proportional to the top quark mass.

In the next section I present the calculation of the Feynman diagram given in Fig.1 and give the results. In the last section I discuss the results and end with the conclusions.

II. SUSY QCD CORRECTIONS

In the MSSM the interaction Lagrangian relevant to the decay $t \rightarrow H^+ b$ is given by:

\[ \mathcal{L} = \frac{g_2}{\sqrt{2}} \frac{m_{\text{top}}}{m_W} V_{tb} \cot \beta (H^+ \bar{t} P_L b + H^{++} \bar{b} P_R t) \]  

where $H^+ = \cos \beta H_2^+ - \sin \beta H_1^+$, $\cot \beta = v_1 / v_2$ is the ratio of the vacuum expectation values (vev) of the two Higgs doublets and $P_{L,R}$ are the chiral projection operators.

The Lagrangian of eq.(1) leads to the following decay rate for $m_b \ll m_{\text{top}}$:

\[ \Gamma^0 (t \rightarrow H^+ b) = \frac{G_F}{\sqrt{2}} |V_{tb}|^2 \cot^2 \beta \frac{1}{8\pi} m_{\text{top}}^3 (1 - \frac{m_{H^+}^2}{m_{\text{top}}^2})^2 \]  

To calculate the 1 loop diagram given in Fig.1 we need the couplings of the scalar quarks to the charged Higgs boson and the scalar-quark-gluino-quark coupling. The first coupling is given in Fig.115* in \cite{7} and the latter one in eq.(C89) in \cite{9}.

When neglecting the bottom quark mass only the scalar partner of the left handed bottom quark $\tilde{b}_L$ occurs within the loop whereas for the top quark we have to

* $\mu$ has to be replaced by $-\mu$
take both left- and right-handed superpartner $\tilde{t}_L$ and $\tilde{t}_R$ into account. Furthermore since the mixing of $\tilde{t}_L$ and $\tilde{t}_R$ is proportional to the top quark mass we have to include the full scalar top quark matrix which is given by [10]:

$$M^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m^2_{\text{top}} + 0.35D^2_Z & -m_{\text{top}}(A_{\text{top}} + \mu \cot \beta) \\ -m_{\text{top}}(A_{\text{top}} + \mu \cot \beta) & m^2_{\tilde{t}_R} + m^2_{\text{top}} + 0.16D^2_Z \end{pmatrix}$$

(3)

where $D^2_Z = m^2_Z \cos 2\beta$.

$m^2_{\tilde{t}_{L,R}}$ are soft breaking masses. $A_{\text{top}}$ is a trilinear scalar interaction parameter and $\mu$ is the supersymmetric mass mixing term of the Higgs bosons. The mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ are related to the current eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ by:

$$\tilde{t}_1 = \cos \Theta \tilde{t}_L + \sin \Theta \tilde{t}_R \quad \tilde{t}_2 = -\sin \Theta \tilde{t}_L + \cos \Theta \tilde{t}_R$$

(4)

In the following we take $m_{\tilde{t}_L} = m_{\tilde{t}_R} = m_S = A_{\text{top}}$ (global SUSY), $m^2_{\tilde{b}_1} = m^2_S - 0.42D^2_Z$ and $m^2_{\tilde{b}_2} = m^2_S - 0.08D^2_Z$. With neglecting bottom quark mass the scalar partners of the left and right handed bottom quarks do not mix and therefore $m_{\tilde{b}_1} = m_{\tilde{b}_1}$. The gluino mass $m_\tilde{g}$ is a free parameter, which in general is supposed to be larger than 100 GeV, although there is still the possibility of a small gluino mass window in the order of 1 GeV [11,12].

The results of the calculation of the loop diagram in Fig.1 are finite, there are no dimensional divergencies. As a result I get for the first order in $\alpha_s$:

$$\Gamma_{\text{MS}}(t \to H^+ b) = \Gamma^0(t \to H^+ b) \left[ 1 - \frac{2\alpha_s}{3\pi}(S + A) \right]$$

(5)

$$S = S_t + \frac{m_\tilde{g}}{m_{\text{top}}} S_{\tilde{g}}$$

$$A = A_t + \frac{m_\tilde{g}}{m_{\text{top}}} A_{\tilde{g}}$$

$$S_t = K_{11}[c_\Theta^2 C^\tilde{b}_1 \tilde{t}_1 + s_\Theta^2 C^\tilde{b}_1 \tilde{t}_2 + K_{21}[s_\Theta c_\Theta (C^\tilde{b}_1 \tilde{t}_1 - C^\tilde{b}_1 \tilde{t}_2)]$$

$$A_t = S_t$$

$$S_{\tilde{g}} = K_{11}[c_\Theta s_\Theta (C^0 \tilde{b}_1 \tilde{t}_2 - C^0 \tilde{b}_1 \tilde{t}_1)] - K_{21}[c_\Theta s_\Theta (C^\tilde{b}_1 \tilde{t}_2 + s_\Theta C^\tilde{b}_1 \tilde{t}_1)]$$

$$A_{\tilde{g}} = S_{\tilde{g}}$$

$$K_{11} = 1 - \frac{m^2_W}{m^2_{\text{top}}} \tan \beta \sin 2\beta$$

$$K_{21} = \frac{1}{m_{\text{top}}}(A_{\text{top}} + \mu \tan \beta)$$

$$C^\tilde{b}_0 \tilde{t}_i = -\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m^2_{\text{top}}}{f^\tilde{g} \tilde{b}_i}$$

$$C^\tilde{b}_1 \tilde{t}_i = -\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m^2_{\text{top}} \alpha_1}{f^\tilde{g} \tilde{b}_i}$$

$$f^\tilde{b}_i \tilde{t}_i = m^2_\tilde{g} - (m^2_\tilde{g} - m^2_{\tilde{t}_i}) \alpha_1 - (m^2_\tilde{g} - m^2_{\tilde{b}_i}) \alpha_2 - m^2_{\text{top}} \alpha_1 (1 - \alpha_1 - \alpha_2) - m^2_{\tilde{H}+} \alpha_1 \alpha_2$$
where \(c_\Theta = \cos \Theta\) and \(s_\Theta = \sin \Theta\). The S and A in eq.(5) indicate that the contribution comes from the scalar - and axial scalar coupling of the matrix element. The Feynman integration can be done numerically.

In eq.(10) in [6] the authors present the results of the standard QCD 1 loop corrections within the two Higgs doublet model and the MSSM, which I will include in my calculation. As a final result I obtain:

\[
\Gamma^1(t \rightarrow H^+b) - \Gamma^0(t \rightarrow H^+b) = \left[ 1 + \frac{4\alpha_s}{3\pi} \tilde{G}'_+ - \frac{2\alpha_s}{3\pi}(S + A) \right]
\]

\[
\tilde{G}'_+ = 2\text{Li}_2(1 - \chi^2) - \frac{\chi^2}{1 - \chi^2} \log \chi^2 + \log \chi^2 \log (1 - \chi^2) + \frac{1}{\chi^2} (1 - \frac{5}{2}\chi^2)
\]

\[
\log (1 - \chi^2) - \frac{2\pi^2}{3} + \frac{9}{4}
\]

\[
\chi^2 = \frac{m^2_{H^+}}{m^2_{\text{top}}}
\]

Eq.(6) gives the full \(O(\alpha_s)\) QCD correction within the MSSM. In the next section I will discuss the result.

III. DISCUSSIONS

To compare the standard QCD correction given in [6] with the gluino contribution via eq.(5) I present in Fig.2–4 the results for different masses of the gluinos and different values of the \(\mu\), \(A_{\text{top}}\) parameters and \(\tan \beta\). In the MSSM we have \(m^2_{H^+} = m^2_W + m^2_{H^0}\) where \(H^0\) is the pseudo Higgs particle. That is the mass of the charged Higgs particle has to be larger than the mass of the W boson. I have set the top quark mass to be the recently released CDF value of 174 GeV [13], the charged Higgs mass to be equal \(m_W\) and the vevs to be equal \(v_1 = v_2\). In Fig.2 I set \(\mu = 0 = A_{\text{top}}\). It should be kept in mind that \(\mu = 0\) is unrealistic, because it leads in general to chargino masses below the experimental limit of 45 GeV. With \(\sin \Theta = 0 = K_{21}\) we see from eq.(5) that only \(S_t\) and \(A_t\) lead to a contribution to the first order in \(\alpha_s\). The scalar top quark masses vary from 181 GeV to 482 GeV and \(m_{b_i} = m_S\). In Fig.2 I present the results for three different values of the gluino mass that is 3 GeV (solid line), 100 GeV (dotted line) and 500 GeV (dash-dotted line). The standard contribution of [6] is presented by the solid straight line and lies at \(-9.5\%\).

As a result we have that for small values of the scalar mass \(m_S\) and a small gluino mass the standard result is diminished by a non-negligible amount down to \(-7\%\). If the Higgs mass is enhanced all curves are pushed up closer to 0, but the shape of the curves remain the same. For \(m_{H^+} = 120\) GeV the standard QCD correction is about \(-8.1\%\). The effect of \(v_1 \ll v_2\) is that the curves for the different gluino masses are pushed closer to the standard QCD correction.

In Fig.3 we consider the case with \(\mu = 500\) GeV and \(A_{\text{top}} = m_S\) again with \(v_1 = v_2\) and the same three different gluino masses. In this case the lighter scalar top quark mass is about 250 GeV for \(m_S\) smaller than 100 GeV, decreases constantly to
about 70 GeV for $m_S = 350$ GeV and increases again to 260 GeV in the range considered here. The heavier one varies from 358 GeV to 631 GeV. Here $\cos\Theta = 1/\sqrt{2}$ and $K_{21} > K_{11}$. Therefore in this case $S_{\tilde{g}}$ and $A_{\tilde{g}}$ contribute more to the decay rate than $S_t$ and $A_t$.

As a result we have in this case that the standard QCD corrections are diminished for small gluino masses whereas we get an enhancement up to $-18\%$ for a gluino mass of 500 GeV. Changing the $\mu$- parameter hardly effects the results. Enhancing the Higgs mass leads to the same changings as mentioned above.

We obtain a totally different result in this case when we consider $v_1 \ll v_2$. From eq.(5) we see that the coupling $K_{21}$ is dominated by the $\tan\beta$ term and therefore we have $K_{21} \gg K_{11}$. The gluino mass becomes more important. For very large gluino masses ($m_{\tilde{g}} \gg 100$ GeV) the 1 loop contribution $\Gamma^1(t \to H^+b)$ is decreasing again. In Fig.4 we have taken e.g. $v_2 = 10 \cdot v_1$ with $\mu = 500$ GeV and $A_{\text{top}} = m_S$ as before. Here $\cos\Theta \approx 1/\sqrt{2}$, the lighter scalar top quark mass is about 115 to 110 GeV for $m_S$ smaller than 100 GeV and increases constantly to 379 GeV for $m_S = 450$ GeV. The heavier one varies from 219 GeV to 564 GeV. The heavy scalar bottom quark mass varies from 78 GeV to 454 GeV and the lighter one from 56 GeV to 451 GeV.

As a result we see that the gluino mass contribution enhances the standard QCD correction drastically. This decay mode therefore can be used to put constraints on the ratio of the vevs $v_1$ and $v_2$. Smaller values for $\mu$ diminishes the results whereas higher values for $\mu$ enlarges them.

$v_1 \ll v_2$ has to be taken with care, because we neglected the mass of the bottom quark. If I take $v_2 = 2 \cdot v_1$ in the last case, that is still in the limit of $m_t \tan\beta \ll m_{\text{top}} \cot\beta$ I do get the same shape as in Fig.4, but with the values pushed a bit closer to the standard model.

III. CONCLUSIONS

In this paper I presented the results of the calculation of the 1 loop correction to the decay rate $t \to H^+b$ when gluinos, the scalar partner of the left- handed bottom quark and the scalar partners of the left- and right- handed top quark are taken within the relevant loop diagram. We presented two cases with the vevs of the Higgs bosons to be taken equal where $\mu = 0 = A_{\text{top}}$ and the other one with $\mu = 500$ GeV and $A_{\text{top}} = m_S$. I have shown that the standard QCD corrections are changed from $-9.5\%$ to $-7\%$ for a small gluino mass and to $-18\%$ for a large gluino mass if we set the top quark mass to be 174 GeV and the charged Higgs boson mass to be equal to $m_W$. Enhancing the Higgs mass lead to values closer to 0.

Finally I considered the case $v_2 = 10 \cdot v_1$ which lead to a large enhancement of the standard QCD corrections due to a $\tan\beta$ dependence of the couplings. This decay rate therefore may be a good decay mode to constrain the ratio of the vevs of the Higgs bosons once the MSSM is proven to be the correct theory to describe nature.

Finally as it is well known I want to mention that the most competitive decay mode to $t \to H^+b$ is the equivalent decay mode of $t \to W^+b$. Within the MSSM the electroweak corrections to this decay mode was recently considered in [14]. The
QCD corrections with a gluino within the relevant loop diagram lead to divergencies, which have to be renormalised. A full analysis of the gluino contribution to this decay rate has not been done yet and will be presented elsewhere [15].

IV. ACKNOWLEDGMENT

I would like to thank the physics department of Carleton university for the use of their computer facilities. The figures were done with the very user friendly program PLOTDATA from TRIUMF.

This work was partially funded by funds from the N.S.E.R.C. of Canada and les Fonds F.C.A.R. du Québec.

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FIGURE CAPTIONS

Fig.1 The diagram with scalar quarks and gluino within the loop, which contribute to the top quark decay into a charged Higgs boson and bottom quark.
Fig.2 The ratio of $\Gamma^1/\Gamma^0$ as a function of the scalar mass $m_S$ for 3 different values of the gluino mass: 3 GeV (solid line), 100 GeV (dotted line) and 500 GeV (dash-dotted line) with $\mu = 0 = A_t$ and $v_1 = v_2$. The top mass has been taken to be 174 GeV and the Higgs mass to be $m_W$. The straight solid line is the standard QCD corrections as given in [6].
Fig.3 The same as in Fig.2 with $\mu = 500$ GeV and $A_{\text{top}} = m_S$.
Fig.4 The same as in Fig.3 with $v_2 = 10 \cdot v_1$. 
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