Sum Rate Fairness Trade-off-based Resource Allocation Technique for MISO NOMA Systems

Haitham Al-Obiedollah*, Kanopathipillai Cumanan*, Jeyarajan Thiagalingam†, Alister G. Burr*, Zhiguo Ding‡, and Octavia A. Dobre§

*Department of Electronic Engineering, University of York, York, YO10 5DD, UK
†STFC, Rutherford Appleton Laboratory, Oxford, OX11 0QX, UK
‡School of Electrical and Electronic Engineering, The University of Manchester, Manchester, UK
§Department of Electrical and Computer Engineering, Memorial University, St. John’s, Canada

Abstract—In this paper, we propose a beamforming design that jointly considers two conflicting performance metrics, namely the sum rate and fairness, for a multiple-input single-output non-orthogonal multiple access system. Unlike the conventional rate-aware beamforming designs, the proposed approach has the flexibility to assign different weights to the objectives (i.e., sum rate and fairness) according to the network requirements and the channel conditions. In particular, the proposed design is first formulated as a multi-objective optimization problem, and subsequently mapped to a single objective optimization (SOO) problem by exploiting the weighted sum approach combined with a prior articulation method. As the resulting SOO problem is non-convex, we use the sequential convex approximation technique, which introduces multiple slack variables, to solve the overall problem. Simulation results are provided to demonstrate the performance and the effectiveness of the proposed approach along with detailed comparisons with conventional rate-aware-based beamforming designs.

Index Terms—Beamforming design, multi-objective optimization, non-orthogonal multiple access, Pareto-optimal.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been proposed as a novel multiple access scheme to overcome the relatively poor spectral-efficiency of the conventional orthogonal multiple access (OMA) schemes [1], [2]. In the power-domain NOMA, superposition coding (SC) is employed to encode different signals with different power levels through power domain multiplexing [2]. In particular, the users with lower channel gains are assigned with higher power levels compared to those with higher channel gains [3]. At the receiver end, stronger users exploit successive interference cancellation (SIC) to subtract the interference from weaker users before detecting their own signals [2]. This multiple access technique, along with other disruptive technologies, such as massive multiple-input multiple-output (MIMO) and mmWave communication, has the potential to further improve the performance of the fifth generation (5G) and beyond wireless networks [4], [5]. Recently, different rate-aware beamforming designs have been proposed for multiple-input single-output (MISO) NOMA systems. For example, the sum rate maximization (SRM)-based design maximizes the sum rate of all users in the cell, however, without taking the individual users rates into account [6]. This approach significantly degrades the rates of the users with weaker channel conditions. To overcome this issue, in [7], a rate-fairness-based design has been developed through the weighted sum-rate maximization (WSRM). In WSRM, higher weights are assigned to weaker users’ rates to maintain the fairness between users in terms of their achievable rates. However, none of these conventional rate-aware-based designs consider either the instantaneous rate-requirements of the users, or the variations of the users’ channel strengths due to the mobility of the users. For example, SRM-based design is an appropriate beamforming design when the users have similar channel strengths. However, the SRM-based design is not capable of achieving a reasonable throughput for all users in a system where the channel strengths of the users vary significantly. Such cases, the weakest user will suffer from low quality of service. In particular, both performance metrics, the sum rate and the fairness among users are crucial performance metrics that have to be considered in 5G and beyond wireless networks [8], [9], [10]. Hence, the base station (BS) should have the flexibility to intelligently decide whether it needs to maximize the sum rate or the fairness among users, or to strike a good balance between them. The fairness index (FI) has been used to measure the fairness between users in terms of their achievable rates [11]. In particular, the FI of the system with K users is defined as follows [12], [13]:

\[
FI = \frac{(\sum_{i=1}^{K} R_i)^2}{K \sum_{i=1}^{K} R_i^2},
\]

where \( R_i \) denotes the achieved rate of the \( i^{th} \) user \( (u_i) \). The best fairness can be achieved when FI is one. Note that the FI and sum rate are conflicting performance metrics, which means that maximizing the sum rate will degrade the FI, and vice versa, especially with users with significantly different channel strengths.

Motivated by this discussion, we propose a novel beamforming design that jointly considers the conflicting performance metrics, i.e., the sum rate and fairness in a MISO NOMA system. In this joint design, the BS decides the importance
of each performance metric (i.e., sum rate and FI) through assigning a weight factor for each objective based on the service requirements and the channel conditions of the users. For instance, the BS will consider the fairness with higher weight when the channel strengths of the users are significantly different and the users expect to achieve the same quality of service in terms of their achievable throughput. On the other hand, more weight will be assigned for the sum rate when the users have similar channel conditions. Furthermore, this joint design has the capability to strike a balance between the sum rate and fairness through assigning appropriate weights to each performance metric. In particular, we formulate this trade-off-based design as a multi-objective optimization (MOO) problem, which is difficult to solve for an optimal solution. Therefore, we rewrite this MOO problem as a single objective optimization (SOO) problem by employing a prior articulation method combined with the weighted sum approach [14]–[15]. In the prior articulation scheme, the BS decides the weight of each objective in the MOO problem prior to designing the beamforming vectors. A weighted sum single objective function is used to represent the multi-objective functions [14]. However, the obtained SOO is non-convex and we employ sequential convex approximations (SCA) to solve it.

The remainder of the paper is organized as follows. Section II presents the system model and the problem formulation. Section III demonstrates the proposed technique to solve the developed optimization problem. Section IV provides simulation results to validate the effectiveness of the proposed beamforming design by comparing its performance with different beamforming techniques available in the literature. Finally, Section V concludes the paper.

Notations

We use lower case boldface letters for vectors and upper case boldface letters for matrices. $(\cdot)^H$ denotes complex conjugate transpose. $\Re(\cdot)$ and $\Im(\cdot)$ stand for real and imaginary parts of a complex number, respectively. The symbols $\mathbb{C}^N$ and $\mathbb{R}^N$ denote $N$-dimensional complex and real spaces, respectively. $|| \cdot ||_2$ and $| \cdot |$ represent the Euclidean norm of a vector and the absolute value of a complex number, respectively. $x \succ 0$ means that all the elements in the vector $x$ are greater than zero.

II. System Model And Problem Formulation

A. System Model

In this paper, we consider the downlink transmission of a MISO NOMA system, in which a BS equipped with $N$ antennas simultaneously transmits signals to $K$ single-antenna users. The transmitted signal from the BS can be written as

$$x = \sum_{i=1}^{K} w_i s_i,$$  (2)

where $s_i$ and $w_i \in \mathbb{C}^{N \times 1}$ denote the signal intended for the $i^{th}$ user $u_i$ and the corresponding beamforming vector, respectively. The received signal at $u_i$ can be written as

$$y_i = h_i^H w_i s_i + \sum_{j=1, j \neq i}^{K} h_i^H w_j s_j + n_i,$$  (3)

where $h_i \in \mathbb{C}^{N \times 1}$, $v_i \in \mathbb{K} = \{1, \cdots, K\}$ represents the channel coefficient vector between $u_i$ and the BS. These channel coefficients are modelled as $h_i = \sqrt{d_i^{-\kappa}} g_i$, where $\kappa, d_i$ and $g_i$ denote the path loss exponent, the distance between $u_i$ and the BS in meters, and the small scale fading, respectively. In addition, $n_i \sim CN(0, \sigma^2)$ represents the additive white Gaussian noise with zero-mean and variance $\sigma^2$. In power-domain NOMA, user ordering plays a crucial role for the performance of NOMA [16], which can only be determined through exhaustive search to achieve the optimal performance. However, for the sake of simplicity, we order the users based on their channel strengths as follows:

$$||h_1||_2^2 \leq ||h_2||_2^2 \leq \cdots \leq ||h_K||_2^2,$$ (4)

where $u_1$ and $u_K$ are refer to the weakest and strongest users in the system. Based on this user ordering in (4), $u_i$ has the capability to perform SIC through decoding and subtracting the signals intended for the $u_1, u_2, \cdots, u_{i-1}$ users prior to decoding its own signal [5]. Therefore, the received signal at $u_i$ after eliminating the first $i-1$ users’ signals using SIC can be written as [17]

$$\tilde{y}_i = h_i^H w_i s_i + \sum_{j=i+1}^{K} h_i^H w_j s_j + n_i. \forall i \in \mathbb{K}.$$ (5)

In particular, the signal intended for $u_i$ will be decoded at $u_k$ (i.e., $k \geq i$ ) with the following signal to interference plus noise ratio $SINR_i^{(k)}$:

$$SINR_i^{(k)} = \frac{|h_i^H w_i|^2}{\sum_{j=i+1}^{K} |h_k^H w_j|^2 + \sigma_d^2}, \quad \forall i \in \mathbb{K}, k \geq i. \quad (6)$$

In order to decode the $i^{th}$ user signal at different users, the $SINR$ of that signal should be more than a certain threshold. This imposes a condition $SINR_i^{(k)}(k \geq i)$ at all the strong users that they should satisfy the predefined SINR threshold to achieve a particular rate. Therefore, the SINR of the signal intended for $u_i$ can be defined as [6]

$$SINR_i = \min\{SINR_i^{(1)}, \cdots, SINR_i^{(K)}\}, \forall i \in \mathbb{K}, \quad (7)$$

and the rate of $u_i$ can be defined as

$$R_i = B_w \log(1 + SINR_i), \quad \forall i \in \mathbb{K}, \quad (8)$$

where $B_w$ is the available bandwidth for transmission, which is assumed to be one. Furthermore, the sum rate of this MISO
NOMA system is given by
\[ R = \sum_{i=1}^{K} R_i. \]  
(9)

In order to ensure that SIC is successfully implemented at all strong users, and to assign more power levels to the weaker users based on NOMA, the following SIC constraints should be included [18]:
\[ |h_i^H w_i|_2^2 \geq \cdots \geq |h_i^H w_K|_2^2, \quad \forall i \in K. \]  
(10)

In addition, the transmit power (\(P_{tr}\)) should not exceed the available power budget (\(P_{ava}\)) at the BS, which can be mathematically formulated as the following constraint:
\[ P_{tr} = \sum_{i=1}^{K} |w_i|_2^2 \leq P_{ava}. \]  
(11)

B. Problem Formulation

For the sake of notation simplicity, we denote the sum rate by \(f_1(\{w_i\}_{i=1}^{K})\) (i.e., \(f_1(\{w_i\}_{i=1}^{K}) = R\)), whereas FI is represented by \(f_2(\{w_i\}_{i=1}^{K})\) (i.e., \(f_2(\{w_i\}_{i=1}^{K}) = FI\)). In this work, we aim to jointly maximize the conflicting objectives (i.e., maximize \(f_1(\{w_i\}_{i=1}^{K})\) and \(f_2(\{w_i\}_{i=1}^{K})\)) subject to SIC and total transmit power constraints. This could be mathematically formulated as the following MOO problem:
\[ P_1: \max_{\{w_i\}_{i=1}^{K}} f(\{w_i\}_{i=1}^{K}) \]  
(12a)
subject to \([10], [11], \]  
(12b)
where \(f(\{w_i\}_{i=1}^{K})\) denotes the vector which consists of the both objective functions (i.e., \(f(\{w_i\}_{i=1}^{K}) = [f_1(\{w_i\}_{i=1}^{K}) \; f_2(\{w_i\}_{i=1}^{K})]^{T}\)). In fact, there exists no single global optimal solution that simultaneously maximizes \(f_1(\{w_i\}_{i=1}^{K})\) and \(f_2(\{w_i\}_{i=1}^{K})\) together. Therefore, to handle such a problem, the designers search for the best trade-off solutions according to the network conditions, which are named as the Pareto optimal solutions (non-dominated solutions) [15].

In particular, a feasible solution \(\{w_i^*\}_{i=1}^{K}\) is called a Pareto optimal solution if and only if there exists no other feasible solution \(\{w_i'\}_{i=1}^{K}\) such that \(f(\{w_i'\}_{i=1}^{K}) \succ f(\{w_i^*\}_{i=1}^{K})\). The set of all Pareto-optimal solutions is called the Pareto front [15]. Therefore, in the following section, we develop an effective approach to determine the Pareto-optimal solution of the MOO problem in [12].

III. PROPOSED METHODOLOGY

In this section, we provide an effective approach to solve the challenging MOO problem \(P_1\). In particular, this approach is developed by combining both the objective functions in \(P_1\) as a weighted single objective function, which we solve by using the SCA technique.

A. Single Objective Optimization

In order to reformulate the optimization problem in [12] into a tractable SOO problem, we employ the prior articulation scheme combining with the weighted sum approach. First, a weight factor \((\alpha_i)\) is assigned to each objective function such that \(\alpha_1 + \alpha_2 = 1\), where \(\alpha_i\) for \((i = 1, 2)\) reflects the importance of \(f_i(\{w_i\}_{i=1}^{K})\) in the overall MOO problem. Next, both objective functions in \(f(\{w_i\}_{i=1}^{K})\) are combined into a single objective function (utility function) [14]. A number of utility functions have been considered in the literature of MOO, however, we employ the weighted sum approach here as it achieves the Pareto-optimal solutions [15]. It is worth mentioning that we have to normalize each objective function by its maximum value (i.e., utopia point) prior to adding them in order to get an unit-less objective function with a maximum value of one for each normalized objective function. Based on this weighted sum-approach, the original MOO problem in [12] can be reformulated as a SOO as follows:

\[ \hat{P}_1: \max_{\{w_i\}_{i=1}^{K}} \alpha_1 f_1^*(\{w_i\}_{i=1}^{K}) + \alpha_2 f_2^*(\{w_i\}_{i=1}^{K}) \]  
(13a)
subject to \([10], [11], \]  
(13b)
where \(f_i^*(\{w_i\}_{i=1}^{K})\) is the normalized objective function of \(f_i(\{w_i\}_{i=1}^{K})\), which can be written as [19]
\[ f_i^*(\{w_i\}_{i=1}^{K}) = \frac{f_i(\{w_i\}_{i=1}^{K})}{f_i^{\max}}, \]  
(14)
where \(f_i^{\max}\) represents the maximum value of \(f_i(\{w_i\}_{i=1}^{K})\). In particular, the maximum value of the FI is one (i.e., \(f_2^{\max} = 1\)). On the other hand, the sum rate function (i.e., \(f_1(\{w_i\}_{i=1}^{K})\)) should be normalized by its maximum value prior to solving the SOO \(\hat{P}_1\), which can be determined by solving the following SRM problem:
\[ P_2: \max_{\{w_i\}_{i=1}^{K}} \sum_{i=1}^{K} R_i \]  
(15a)
subject to \([11], [10], \]  
(15b)

The SRM in [15] can be solved by exploiting the minorization maximization algorithm [6]. In the following subsection, we develop an effective approach to solve the SOO in [13]. It is worth to make two important observations regarding the SOO optimization problem \(\hat{P}_1\). Firstly, the weighted sum approach that is utilized to replace the original multi-objective functions by a single one produces the Pareto-optimal solutions of the original optimization problem \(P_1\) [15]. Secondly, the original MOO \(P_1\) turns out to be the conventional SRM problem when \(\alpha_1 = 1\). However, when \(\alpha_2 = 1\), \(\hat{P}_1\) becomes the max-min rate (MMR) optimization problem. In particular, the MMR solution achieves the same rate (i.e., a unity FI) for all the users in the system [20]. However, the WSRM and proportional fairness (PF) problems could be formulated from the original optimization problem \(P_1\) through appropriately scaling the weight factor between zero and one.

B. Sequential Convex Approximation

In this subsection, we solve the \(\hat{P}_1\) by approximating the non-convex functions in the objective and the constraints as convex ones through using the SCA approach [21]. To apply
this approach, we introduce multiple slack variables \((\xi_1, \xi_2, \xi)\) to represent the single objective function as follows:

\[
\begin{align*}
\alpha_1 f_1^i \left( \{w_i\}_{i=1}^K \right) & \geq \xi_1, \\
\alpha_2 f_2^i \left( \{w_i\}_{i=1}^K \right) & \geq \xi_2, \\
\xi_1 + \xi_2 & \geq \xi.
\end{align*}
\]

(16a) (16b) (16c)

Based on these new slack variables, the optimization problem in (13) can be rewritten as

\[
\begin{array}{ll}
\max_{\xi_1, \xi_2, \xi} & \xi \\
\text{s.t.} & \xi_1 + \xi_2 \geq \xi, \\
& (1 - \alpha) f_1^i \left( \{w_i\}_{i=1}^K \right) \geq \xi_1, \\
& \alpha f_2^i \left( \{w_i\}_{i=1}^K \right) \geq \xi_2, \\
& (\ref{10}), (\ref{11}).
\end{array}
\]

(17a) (17b) (17c) (17d) (17e)

where \(\alpha_2 = \alpha\) and \(\alpha_1 = 1 - \alpha\). It is obvious that \(P_1\) cannot be solved directly through existing convex optimization software due to the non-convex constraints. Hence, we approximate those constraints with convex ones and iteratively solve the SOO problem by updating the approximations in each iteration. Without loss of generality, the constraint in (17c) can be equivalently written as

\[
\sum_{i=1}^K R_i \geq \frac{f_{\text{max}}^\xi_i}{(1 - \alpha)}.
\]

(18)

By introducing a new set of slack variables, the constraint in (18) can be rewritten into the following set of constraints:

\[
\begin{align*}
(\ref{18}) & \Leftrightarrow \left\{ \begin{array}{l}
\sum_{i=1}^K r_i \geq f_{\text{max}}^\xi_i \left( 1 - \alpha \right), \\
R_i \geq r_i, \forall i \in \mathcal{K},
\end{array} \right. \\
\text{and this constraint is approximated by introducing the following new slack variables } & \gamma, \beta \text{ such that}
\end{align*}
\]

(19a) (19b)

Furthermore, the constraint in (19b) can be expressed as

\[
\log \left( 1 + \frac{|h_i^H w_i|^2}{\sum_{j=i+1}^K |h_k^H w_j|^2 + \sigma_k^2} \right) \geq r_i, \forall i \in \mathcal{K}, k \geq i.
\]

(20)

The non-convexity of (20) can be handled by introducing new slack variables, namely \(z_i\) and \(\rho_i\), such that the constraint in (20) is written into the following two constraints:

\[
\begin{align*}
1 + \frac{|h_i^H w_i|^2}{\sum_{j=i+1}^K |h_k^H w_j|^2 + \sigma_k^2} \geq z_i, \forall i \in \mathcal{K}, k \geq i, \\
z_i \geq 2^{r_i}, \forall i \in \mathcal{K}.
\end{align*}
\]

(21a) (21b)

In addition, the constraint in (21a) is similarly written with a new slack variable \(\eta_{i,k}\) as

\[
\begin{align*}
(21a) & \Leftrightarrow \left\{ \begin{array}{l}
|h_i^H w_i|^2 \geq (z_i - 1)\eta_{i,k}, \\
K \sum_{j=i+1}^K |h_k^H w_j|^2 + \sigma_k^2 \leq \eta_{i,k}^2.
\end{array} \right.
\end{align*}
\]

(22a) (22b)

To represent (22a) in a convex form, the square root of each term in the inequality is considered. Then, we approximate the right hand-side term of the inequality by convex-concave approximation \(\ref{22a}\) through the first-order Taylor series approximation \(\ref{6}\). Therefore, the constraint in (22a) can be approximated by the following linear inequality constraint:

\[
R(h_k^H w_i) \geq \sqrt{(z_i^{(n-1)} - 1)\eta_{i,k}^{(n-1)} + \sqrt{(z_i^{(n-1)} - 1)(\eta_{i,k} - \eta_{i,k}^{(n-1)})}} + 0.5 \sqrt{(z_i^{(n-1)} - 1)}(z_i - z_i^{(n-1)}), \forall i \in \mathcal{K}, k \geq i.
\]

(23)

where \(z_i^{(n-1)}\) and \(\eta_{i,k}^{(n-1)}\) represent the approximations of \(z_i\) and \(\eta_{i,k}\) at the \((n-1)\)th iteration, respectively. Furthermore, we can rewrite the constraint in (22a) as the following second order cone (SOC):

\[
\eta_{i,k} \geq ||h_i^H w_{i+1} h_i^H w_{i+2} \cdots h_i^H w_K \sigma_k^T||_2, \forall i \in \mathcal{K}, k \geq i.
\]

(24)

With multiple slack variables, the non-convex constraint in (17c) is now formulated as the following convex constraints:

\[
(\ref{17c}) \Leftrightarrow (\ref{19a}), (\ref{19b}), (\ref{23}), (\ref{24}).
\]

(25)

Next, we rewrite the constraint in (17d) in convex form by employing the same approximation techniques that have been already implemented to handle (17c). Firstly, we equivalently rewrite the constraint in (17d) as

\[
\frac{\sum_{i=1}^K r_i^2}{K \sum_{i=1}^K r_i^2} \geq \frac{\xi_2}{\alpha},
\]

(26)

and this constraint is approximated by introducing the following new slack variables \(\gamma, \beta\) such that

\[
\begin{align*}
(26) & \Leftrightarrow \left\{ \begin{array}{l}
\left( \sum_{i=1}^K r_i \right)^2 \geq \gamma \beta^2, \\
K \sum_{i=1}^K r_i^2 \leq \beta^2, \\
\gamma \geq \frac{\xi_2}{\alpha}.
\end{array} \right.
\end{align*}
\]

(27a) (27b) (27c)

To handle the non-convexity of the constraint in (27a), we use the same Taylor series approximation that was employed to derive the inequality in (23), as follows:

\[
\sum_{i=1}^K r_i \geq \sqrt{\gamma^{(n-1)}\beta^{(n-1)}} + 0.5 \frac{1}{\sqrt{\gamma^{(n-1)}}} \beta^{(n-1)}(\gamma - \gamma^{(n-1)}) + \sqrt{\gamma^{(n-1)}(\beta - \beta^{(n-1)})}.
\]

(28)

Similarly, the constraint in (27b) can be written as the following SOC constraint:

\[
\beta \geq \sqrt{K ||r_1 r_2 \cdots r_K||_2}.
\]

(29)
Hence, the non-convex constraint in (17d) can be now reformulated as the following convex constraints:

\[ 17d \Rightarrow (27e), (28), (29). \] (30)

Finally, we transform the non-convex SIC constraint in (10) by replacing each non-convex term of the inequality with a linear approximated term using the first-order Taylor series approximation, as shown below [23]

\[
|h_k^H w_j|^2 \geq \parallel [\Re(h_k^H w_{j(n-1)}) + \Im(h_k^H w_{j(n-1)})] ||^2 + 2\Re(h_k^H w_j)\Im(h_k^H w_j) + \Im(h_k^H w_j)\Re(h_k^H w_j) - \Re(h_k^H w_j)\Im(h_k^H w_j)(n-1))].
\] (31)

Through including these approximations, the original optimization problem \( P_1 \) can be reformulated as follows:

\[
\begin{align*}
\text{max} & \quad \xi \\
\text{s.t.} & \quad (10), (19b), (21b), (23), (24), (17b), (17e), (19a), (29), (27c), (28), (32c)
\end{align*}
\] (32a, 32b, 32c)

where \( \chi \) includes all the optimization parameters such that \( \chi = \{w_i, \xi, \xi_1, \xi_2, \beta, \gamma, r_i, \eta_i, k, z_i\}_k^{K+1} \). It is obvious that solving the optimization problem in (32) requires to initialize the parameters \( \chi^{(0)} \) and these parameters can be obtained by choosing a feasible beamforming vectors \( \{w_i^{(0)}\}_k^{K+1} \). Furthermore, the other slack variables can be determined through substituting \( \{w_i^{(0)}\}_k^{K+1} \) in the inequalities. The optimization problem in (32) is iteratively solved until the required accuracy is achieved such that \( |\xi^{(n)} - \xi^{(n-1)}| \) is less than a pre-defined threshold \( \varphi \).

IV. Simulation Results

In this section, we provide simulation results examining the effectiveness of the proposed sum rate-fairness trade-off based beamforming design. In particular, we consider a BS equipped with four transmit antennas (i.e., \( N = 4 \)) which transmits information to five single-antenna users. It is assumed that the users are located at distances of 50, 4, 3, 2 and 1 meters. Furthermore, we assume that all the channels are Rayleigh fading, with the path loss exponent and the noise variance of the channels are set to be two and one, respectively, whereas the available bandwidth for transmission is assumed to be \( B_w = 1 \) MHz. The threshold to terminate the iterative algorithm is chosen to be \( 0.001 \) (i.e., \( \varphi = 0.001 \)). We define the normalized transmit power (TX-SNR) in dB as \( \text{TX-SNR} (\text{dB}) = 10 \log_{10} \frac{P}{\sigma^2} \). Fig. 1 demonstrates the achieved sum rate and FI over the weight factor \( \alpha \). As expected, the problem \( \tilde{P}_1 \) becomes SRM at \( \alpha = 0 \), and the maximum sum rate is achieved at the cost of lower FI. Furthermore, the optimal fairness is achieved when the weight factor is set to be one (i.e., \( \alpha = 1 \)). In particular, the problem \( \tilde{P}_1 \) turns out to be MMR with \( \alpha = 1 \). However, the BS can appropriately choose a value for the weight factor \( \alpha \) so that a good balance between the sum rate and FI can be achieved. A good trade-off between these performance metrics can be achieved by choosing \( \alpha = 0.5 \), as shown in Fig. 1. Fig. 2 illustrates the rate variation of the weakest user for different transmit power for different weight factors \( \alpha \). For example, at TX-SNR = 30 dB, the rate of the weakest user achieves around 0.2 Mbps with \( \alpha = 0 \); however, this rate can be increased five times by setting the weight factor \( \alpha \) to 1. Hence, the BS has the flexibility to determine the achievable rate of the weakest user by appropriately choosing the weight factor \( \alpha \). Furthermore, we provide the Pareto-optimal solutions of the proposed joint sum rate-fairness-based beamforming design for different TX-SNR thresholds in Fig. 3. In particular, the Pareto-front is the set that consists of the best-trade off (Pareto-optimal) solutions for \( \alpha \). For instance, at TX-SNR = 25 dB, each point on the curve represents the best (sum rate, FI) solution that could be achieved with a particular weight of \( \alpha \). It is worth mentioning that for a given value of \( \alpha \), one of the performance metrics can be improved. However, this improvement is not without degrading the performance of the other metric.

Figure 1: Achieved sum rate and FI against the weight factor \( \alpha \), TX-SNR=30 dB.

Figure 2: The weakest user’s rate against the available transmit power for different weight factors \( \alpha \).
Table I: The impact of the weakest user distance (i.e., $d_1$) on the sum rate and FI with different weight factors, at TX-SNR=35 dB.

| Case 1, $d_1=10$ m | Case 2, $d_1=100$ m | Case 3 $d_1=1000$ m |
|---------------------|---------------------|---------------------|
| $\alpha = 0$        | $\beta_1$ (Mbps) | $\beta_2$ (Mbps) | $\beta_3$ (Mbps) | FI | $\beta_1$ (Mbps) | $\beta_2$ (Mbps) | $\beta_3$ (Mbps) | FI | $\beta_1$ (Mbps) | $\beta_2$ (Mbps) | $\beta_3$ (Mbps) | FI |
| 13.598              | 0.5357             | 8.9750              | 0.4224             |     | 13.4003             | 0.2033             | 9.2729              | 0.3845 | 12.3006             | 0.0042             | 9.2421              | 0.3750 |
| $\alpha = 0.25$     | 13.1764             | 2.0968              | 5.6811              | 0.9571 | 12.9283             | 0.5935             | 4.2581              | 0.7974 | 12.9127             | 0.0057             | 5.1300              | 0.6559 |
| $\alpha = 0.5$      | 12.9801             | 2.3456              | 3.0192              | 0.9923 | 12.6967             | 0.4279             | 3.4141              | 0.8487 | 12.3254             | 0.0081             | 3.5196              | 0.7930 |
| $\alpha = 0.75$     | 12.9111             | 2.3686              | 2.8195              | 0.9966 | 10.9314             | 0.9287             | 2.5853              | 0.9229 | 12.1129             | 0.0082             | 3.1948              | 0.8094 |
| $\alpha = 1$        | 12.7126             | 2.6919              | 2.4219              | 0.9999 | 7.6435              | 1.1216             | 1.3380              | 0.3484 | 0.1469              | 0.0340             | 0.0374              | 0.9951 |

Figure 3: Pareto-front for different TX-SNR thresholds.

Finally, Table I demonstrates the importance of the proposed sum rate-fairness trade-off-based design over the other fixed beamforming design. In particular, the BS decides the weights of each of the conflicting performance metrics (i.e., sum rate and FI) based on the instantaneous channel state information of the users. To explain this in a detailed manner, we present the rates of the weakest and strongest users, the achieved sum rate, and the FI through increasing the distance of the weakest user $d_1$ from 10-1000 meters, while the distances of the remaining four users in the system remain fixed. As seen in Table I, the SRM beamforming design (i.e., $\alpha = 0$) does not provide a better quality of service in terms of fairness and achievable rate of the weakest user as the distance between users increases. For example, the weakest user achieves only a rate of 0.0042 Mbps while the strongest user enjoys a rate of 9.2421 Mbps, with FI of 0.375. Hence, the BS can intelligently assign appropriate weights to maintain a good fairness among the users, such that the weakest user rate is reasonably increased.

V. CONCLUSIONS

In this paper, we have proposed a sum rate-fairness trade-off-based beamforming design for a MISO NOMA system. In this design, the BS has the flexibility to appropriately choose the weights of each objective according to the users' channel conditions. The beamforming design is formulated as a MOO problem which is hard to solve directly. To overcome this issue, a weighted sum approach combined with the prior articulation method is employed to reformulate the original problem as a SOO problem. Furthermore, the SCA technique is exploited to iteratively solve the weighted SOO problem. Simulation results indicate that the proposed approach is very effective when compared against the conventional rate-aware beamforming design.

ACKNOWLEDGEMENT

The work of K. Cumanan, A. Burr and Z. Ding was supported by H2020-MSCA-RISE-2015 under grant no: 690750.

REFERENCES

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, “Non-orthogonal multiple access (NOMA) for cellular future radio access,” in Proc. IEEE VTC Spring 2013, pp. 1–5.

[2] S. R. Islam, N. Avazov, O. A. Dobre, and K.-S. Kwak, “Power-domain non-orthogonal multiple access (NOMA) in 5G systems: potentials and challenges,” IEEE Commun. Surveys Tuts., vol. 19, no. 2, pp. 721 – 742, Oct. 2016.

[3] S. Islam, M. Zeng, O. A. Dobre, and K.-S. Kwak, “Resource allocation for downlink NOMA systems: Key techniques and open issues,” IEEE Wireless Commun., vol. 25, no. 2, pp. 40–47, Apr. 2018.

[4] Z. Ding, F. Adachi, and H. V. Poor, “The application of MIMO to non-orthogonal multiple access,” IEEE Trans. Wireless Commun., vol. 15, no. 1, pp. 537–552, Jan. 2016.

[5] Z. Chen, Z. Ding, P. Xu, and X. Dai, “Optimal precoding for a QoS optimization problem in two-user MISO-NOMA downlink,” IEEE Commun. Lett., vol. 20, no. 6, pp. 1263–1266, Jun. 2016.

[6] M. F. Hanif, Z. Ding, T. Ratnarajah, and G. K. Karagiannidis, “A minorization-maximization method for optimizing sum rate in the downlink of non-orthogonal multiple access systems,” IEEE Trans. Signal Process., vol. 64, no. 1, pp. 76–88, Jan. 2016.

[7] J. Wang, Q. Peng, Y. Huang, H.-M. Wang, and X. You, “Convexity of weighted sum rate maximization in NOMA systems,” IEEE Commun. Lett., vol. 24, no. 9, pp. 1323–1327, Sep. 2017.

[8] P. Xu, K. Cumanan, and Z. Yang, “Optimal power allocation scheme for noma with adaptive rates and alpha-fairness,” in Proc. IEEE GLOBECOM, 2017, pp. 1–6.

[9] K. Cumanan, R. Krishna, V. Sharma, and S. Lambbotharan, “Robust interference control techniques for multiuser cognitive radios using worst-case performance optimization,” in Proc. Asilomar Conf. Signal, Syst. Comput. IEEE, 2008, pp. 378–382.

[10] —, “A robust beamforming based interference control technique and its performance for cognitive radios,” in Communications and Information Technologies, 2008. ISCT 2008. International Symposium on., IEEE, 2008, pp. 9–13.

[11] R. Jain, D.-M. Chui, and W. R. Hawe, A quantitative measure of fairness and discrimination for resource allocation in shared computer system. Eastern Research Laboratory, Digital Equipment Corporation Hudson, MA, 1984, vol. 38.

[12] H. Shi, R. V. Prasad, E. Onur, and I. Niemegeers, “Fairness in wireless networks: Issues, measures and challenges,” IEEE Commun. Surveys Tuts., vol. 16, no. 1, pp. 5–23, 1st Quart. 2014.

[13] P. Xu and K. Cumanan, “Optimal power allocation scheme for non-orthogonal multiple access with $\alpha$-fairness,” IEEE J. Sel. Areas Commun., vol. 35, no. 10, pp. 2357–2369, 2017.

[14] R. T. Marler and J. S. Arora, “Survey of multi-objective optimization methods for engineering,” Struct. Multidiscip. Optim., vol. 26, no. 6, pp. 369–395, Apr. 2004.

[15] A. Zhou, B.-Y. Qu, H. Li, S.-Z. Zhao, P. N. Suganthan, and Q. Zhang, “Multi objective evolutionary algorithms: A survey of the state of the art,” Swarm and Evol. Comput., vol. 1, no. 1, pp. 32–49, 2011.

[16] F. Alavi, K. Cumanan, Z. Ding, and A. G. Burr, “Robust beamforming techniques for non-orthogonal multiple access systems with bounded channel uncertainties,” IEEE Commun. Lett., vol. 21, no. 9, pp. 2035–2036, 2017.
[17] H. Alobiedollah, K. Cumanan, J. Thiyagalingam, A. G. Burr, Z. Ding, and O. A. Dobre, “Energy efficiency fairness beamforming design for MISO NOMA systems,” in Proc. IEEE WCNC’19.

[18] F. Alavi, K. Cumanan, Z. Ding, and A. G. Burr, “Beamforming techniques for non-orthogonal multiple access in 5G cellular networks,” IEEE Trans. Veh. Technol., vol. 67, no. 10, pp. 9474–9487, Oct. 2018.

[19] K. Proos, G. Steven, O. Querin, and Y. Xie, “Multicriterion evolutionary structural optimization using the weighting and the global criterion methods,” AIAA journal, vol. 39, no. 10, pp. 2006–2012, 2001.

[20] K. Cumanan, L. Musavian, S. Lambotharan, and A. B. Gershman, “SINR balancing technique for downlink beamforming in cognitive radio networks,” IEEE Signal Process. Lett., vol. 17, no. 2, pp. 133–136, Feb. 2010.

[21] A. Beck, A. Ben-Tal, and L. Tetruashvili, “A sequential parametric convex approximation method with applications to nonconvex truss topology design problems,” J. Global Optimiz., vol. 47, no. 1, pp. 29–51, 2010.

[22] A. L. Yuille and A. Rangarajan, “The concave-convex procedure,” Neural computation, vol. 15, no. 4, pp. 915–936, Mar. 2003.

[23] H. Alobiedollah, K. Cumanan, J. Thiyagalingam, A. G. Burr, and Z. a. Ding, “Energy efficient beamforming design for MISO non-orthogonal multiple access systems,” Accepted in IEEE Trans. Wireless Commun., Feb. 2018.