Nuclear electromagnetic currents to fourth order in chiral effective field theory

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Abstract Recently, we have shown that the continuity equation for the nuclear vector and axial current operators acquires additional terms if the latter depend on the energy transfer. We analyze in detail the electromagnetic single-nucleon four-current operators and verify the validity of the modified continuity equation for all one- and two-nucleon contributions up to fourth order in the chiral expansion. We also derive, for the first time, the leading contribution to the three-nucleon charge operator which appears at this order. Our study completes the derivation of the electroweak nuclear currents to fourth order in the chiral expansion.

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Chiral effective field theory has been extensively used in the past decades to derive nuclear forces and the corresponding vector and axial charge and current operators. Presently, the two-nucleon force contributions have been worked out completely up through fifth order $Q^5$ of the chiral expansion, i.e. up to $N^4\text{LO}$ [1,2,3,4,5]. Here and in what follows, the expansion parameter is defined as $Q \in \{M_\pi/A_\eta, \ p/A_\eta\}$, where $p$ refers to four- (three-)momenta of external pions (nucleons), while $A_\eta$ is the breakdown scale of the chiral expansion. In the Goldstone boson and single-baryon sectors one usually assumes $A_\eta = A_\rho \sim 4\pi F_\pi \sim 1 \text{ GeV}$ [6]. In the few-nucleon sector, the breakdown scale is estimated to be of the order of $A_\rho \sim 600 \text{ MeV}$ [7,8]. The expressions for the three- and four-nucleon forces up to fourth order, i.e. $Q^4 \text{ or } N^3\text{LO}$, calculated using dimensional regularization can be found in Refs. [9,10,11,12,13,14]. Most of the $N^4\text{LO}$ contributions to the three-nucleon force have also been worked out [15,16,17,18].

Nuclear electroweak current operators have also attracted considerable attention. Starting from the pioneering (but incomplete and kinematically restricted) studies by Park et al. [19,20], the exchange electromagnetic currents have been worked out to the leading one-loop order $Q$, i.e. $N^3\text{LO}$ using the method of unitary transformation (UT) [25,26,27] in Refs. [28,29] and time-ordered perturbation theory in Refs. [22,23,24], see also Ref. [30] for a related discussion. In fact, the UT method was first used in this context in Ref. [31]. The main idea behind the method of UT is to find a unitary operator in the pion-nucleon Fock space which decouples its purely nucleonic subspace, called $\eta$-space, from the rest of the Fock space. Nuclear forces are then identified with the resulting decoupled Hamiltonian which acts on the nucleonic subspace. Notice that it is always possible to perform additional UTs on the $\eta$-space which affect the off-shell behavior of the nuclear potentials but do not change observable quantities. This unitary ambiguity was found to be considerably reduced if one demands renormalizability of the nuclear potentials [11,26]. While the on-shell scattering amplitude calculated in chiral effective field theory is defined unambiguously and guaranteed to be renormalizable, this does not hold anymore for irreducible (i.e. non-iterative) contributions alone, which are associated with nuclear forces and current operators. Renormalizability of nuclear potentials can, however, be enforced by performing suitable UTs on the $\eta$-space. For a broad class of UTs considered in Refs. [11,26,13], the resulting renormalized static contributions to the nuclear forces up to $N^4\text{LO}$ were found

\[ Q^{-3} \text{ and } Q^{-1}, \]

respectively. When the nucleon mass $m$ is counted according to $m \sim A_\rho^2/Q$ as done here and commonly employed in few-nucleon studies, see e.g. Refs. [2,3,13,14,15,16], no corrections to the charge and current operators appear at order $Q^{-2}$. Thus, the terms LO, NLO, $N^2\text{LO}$ and $N^3\text{LO}$ refer to the contributions at orders $Q^{-3}$, $Q^{-1}$, $Q^0$ and $Q$, respectively. Notice that the authors of Refs. [22,23,24] employ the counting scheme with $m \sim A_\rho$ leading to a different expansion pattern.
to be determined unambiguously, while the leading relativistic corrections depend on two arbitrary phases $\beta_3$ and $\beta_9$ \cite{14,7}, see also \cite{32} and references therein for a related earlier discussion of this kind of unitary ambiguity.

In the presence of external classical sources, the UTs determined in the strong sector as outlined above allow one to derive the corresponding charge and current operators. To enforce renormalizability also for the four-current operators one, however, needs to employ a more general class of $\eta$-space UTs, whose generators involve a single insertion of an external classical sources. At the order considered they are parametrized via \cite{29}²

$$S = - \sum_{i=1}^{7} \bar{\beta}_i S_i ,$$ (1.1)

where $\bar{\beta}_i \in \mathbb{R}$ are the transformation “angles” while the corresponding anti-hermitian generators $S_i$ have the form

$$S_1 = \eta \left[ J_0^{(0)} \frac{\lambda^2}{E_\pi^2} H^{(2)}_{22} - H^{(2)}_{22} \frac{\lambda^2}{E_\pi^2} J_0^{(-1)} \right] \eta ,$$

$$S_2 = \eta \left[ H^{(1)}_{21} \frac{\lambda^1}{E_\pi^2} J_0^{(-1)} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} - H^{(1)}_{21} \frac{\lambda^1}{E_\pi} J_0^{(-1)} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} \right] \eta ,$$

$$S_3 = \eta \left[ J_0^{(1)} \frac{\lambda^1}{E_\pi^2} H^{(1)}_{21} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} - H^{(1)}_{21} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} \eta J_0^{(-1)} \right] \eta ,$$

$$S_4 = \eta \left[ J_0^{(1)} \frac{\lambda^2}{E_\pi^2} H^{(1)}_{21} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} - H^{(1)}_{21} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} \lambda^2 J_0^{(-1)} \right] \eta ,$$

$$S_5 = \eta \left[ J_0^{(-1)} \frac{\lambda^2}{E_\pi^2} H^{(1)}_{21} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} - H^{(1)}_{21} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} \lambda^2 J_0^{(-1)} \right] \eta ,$$

$$S_6 = \eta \left[ H^{(1)}_{21} \frac{\lambda^1}{E_\pi^2} J_0^{(0)} - J_0^{(0)} \frac{\lambda^1}{E_\pi} H^{(1)}_{21} \right] \eta .$$ (1.2)

Here, $\eta$ and $\lambda^i$ denote the projection operators onto the purely nucleonic part of the Fock space and Fock-space components with $i$ pions, respectively. Further, $E_\pi = \sum_i \sqrt{p_i^2 + M_\pi^2}$ is the total energy of pions with three-momenta $p_i$ in the intermediate $\lambda^i$-state while $H^{(1)}_{ab}$ ($J^{(1)}_{ab}$) refer to the vertices in the effective chiral Hamiltonian with a nucleon and $b$ pion fields without (with a single insertion of) external electromagnetic sources. The superscript $\kappa$ is related to the canonical field dimension of the fields via $\kappa = d + (3/2)n + p - 4$, where $d$, $n$ and $p$ denote the number of derivatives or $M_\pi$-insertions and the nucleon

\footnote{The minus sign is missing in Eq. (4.3) of Ref. \cite{29} and for the quantities $\delta J_{a2,c5,e6,c7}$ in Eq. (4.6) of that paper.}
and pion field operators, respectively. In the method of UT, all contributions to the nuclear forces and currents are written as time-ordered sequences of vertices and energy denominators. A contribution involving \( N \) vertices of type \( \kappa_i, i = 1, \ldots, N \), appears at order \( Q^\nu \) with \( \nu = \sum_i \kappa_i - 2 \) \( (\nu = \sum_i \kappa_i - 3) \) for the forces (currents). More details on the notation and the method of UT can be found in Ref. [11,26], while the explicit form of the relevant terms in the effective Lagrangian and Hamiltonian is given in Ref. [29]. For the sake of completeness, we also give the expressions for the generators of the already mentioned \( \eta \)-space UTs which parametrize the unitary ambiguity of the leading relativistic corrections to the nuclear forces and currents:

\[
S' = \bar{\beta}_8 S_8 + \bar{\beta}_9 S_9,
\]

with the operators \( S_{8,9} \) given by

\[
S_8 = \eta \left[ \tilde{H}_2^{(2)} \eta H_2^{(1)} \frac{\lambda_1}{E_\pi^2} H_2^{(1)} - H_2^{(1)} \frac{\lambda_1}{E_\pi^2} H_2^{(1)} \eta \tilde{H}_2^{(2)} \right] \eta,
\]

\[
S_9 = \eta \left[ \tilde{H}_2^{(3)} \frac{\lambda_1}{E_\pi^2} H_2^{(1)} - H_2^{(1)} \frac{\lambda_1}{E_\pi^2} \tilde{H}_2^{(3)} \right] \eta.
\]

Here, \( \tilde{H}_i^{(\kappa)} \) denote the \( 1/m \)-corrections to the corresponding vertices.

The UTs listed in Eqs. (1.1,1.2) induce contributions to the electromagnetic charge and current operators at N\(^3\)LO and higher chiral orders which restore their renormalizability upon a suitable choice of \( \bar{\beta}_i \). More precisely, it was found in Ref. [29] that the expressions for the two-nucleon charge and current operators do not depend on the parameters \( \bar{\beta}_2, \bar{\beta}_7 \) and \( \bar{\beta}_5 - \bar{\beta}_6 \). Further, renormalizability of the one-loop contributions to the one-pion exchange current operator, calculated in dimensional regularization, was found to require the choice

\[
\bar{\beta}_1 = 1, \quad \bar{\beta}_4 - 3\bar{\beta}_5 - 3\bar{\beta}_6 = 2,
\]

while the parameter \( \bar{\beta}_3 \) was set to \( \bar{\beta}_3 = 0 \). With this choice, the one-loop contributions to the electromagnetic charge and current operators still depend on an arbitrary phase \( \bar{\beta}_4 \), for which the value of \( \bar{\beta}_4 = -1 \) was adopted in Ref. [29].

In Ref. [33], the framework outlined above was extended to derive the nuclear axial charge and current operators. Similarly to the vector currents, renormalized expressions could only be obtained by taking into account additional \( \eta \)-space UTs whose generators depend on the external axial sources and have been parametrized in terms of the real phases \( \alpha_i^{ax}, i = 1,2,\ldots,33 \). While renormalizability alone does not completely eliminate the resulting unitary

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\(^3\) Requiring, in addition to renormalizability of the single- and two-nucleon current operators, also factorizability of the exchanged pions in the three-nucleon charge operator, see [33] for more details, leads to the stronger constraints on the phases, namely \( \bar{\beta}_1 = 1, \bar{\beta}_4 = -1 \) and \( \bar{\beta}_5 = \bar{\beta}_6 = -1/2 \).
ambiguous, the requirement of the pion-pole contributions to the axial current to match the corresponding terms in the nuclear potentials was shown to result in unique expressions. Ref. [33] also provides a detailed discussion of the subtleties associated with the explicit time dependence of the additional UTs due to the dependence of their generators on the external classical sources. It is shown in that paper that the explicit time dependence of the UTs is, in general, expected to yield contributions which depend on the energy transfer \( k_0 \) of the external source. Also the continuity equations, which are manifestations of gauge invariance and the chiral symmetry, get modified and take a more general form that involves the energy-dependent contributions to the charge and current operators, see Eq. (2.42) of Ref. [33]. The validity of the continuity equation for the axial current was explicitly verified in that paper. For a related recent work on the nuclear axial currents in the framework of time-ordered perturbation theory see Refs. [34].

The purpose of this study is to complete the derivation of the electromagnetic currents at N^3LO initiated in [28,29] and to investigate the implications of the findings of Ref. [33]. In particular, we discuss in detail the expressions for the single-nucleon charge and current operators including the relevant relativistic corrections and provide their parametrization in terms of the corresponding nucleon form factors. We also work out the three-nucleon contributions to the charge operator, which completes the derivation of the nuclear electromagnetic currents at N^3LO. Last but not least, we explicitly verify the validity of the continuity equation for all considered classes of diagrams. Notice that since single-nucleon contributions were not considered in our earlier papers [28,29], the continuity equation could so far only be verified explicitly for the static two-pion exchange terms.

Our paper is organized as follows. In Sec. 2 we discuss the single-nucleon charge and current operators. Special attention is paid to the energy-transfer-dependent contributions, which lead to the already mentioned modified form of the continuity equation. In Sec. 3 we use the derived single-nucleon current to explicitly verify the validity of the continuity equation for the one-pion exchange (OPE) and short-range two-nucleon operators. Next, in Sec. 4 we derive the leading three-nucleon contributions to the electromagnetic charge operator. We observe that there are no three-nucleon current contributions up to the considered order. The main results of our paper are summarized in Sec. 5.

2 Single nucleon electromagnetic current

Here and in what follows, we stay as close as possible to the notation employed in Ref. [33]. In particular, the one-, two- and three-nucleon four-current oper-
ators are defined according to
\[ \langle p' | \hat{V}_1^\mu | p \rangle =: \delta^{(3)}(p' - p - k) V_1^\mu, \]
\[ \langle p'_1 p'_2 | \hat{V}_2^\mu | p_1 p_2 \rangle =: (2\pi)^{-3} \delta^{(3)}(p'_1 + p'_2 - p_1 - p_2) V_2^\mu, \]
\[ \langle p'_1 p'_2 p'_3 | \hat{V}_3^\mu | p_1 p_2 p_3 \rangle =: (2\pi)^{-6} \delta^{(3)}(p'_1 + p'_2 + p'_3 - p_1 - p_2 - p_3 - k) V_3^\mu, \]
where \( p_i (p'_i) \) denotes the incoming (outgoing) momentum of nucleon \( i \) while \( k \) is the momentum of the external electromagnetic source. The electromagnetic current operators we are interested in here are linear combinations of isoscalar and isovector quantities. For the sake of compactness, we refrain from explicitly showing isospin indices except for the isospin Pauli matrices \( \tau \). Further, \( \hat{X} \) means that the quantity \( X \) is to be regarded as an operator rather than matrix element with respect to the nucleon momenta. Finally, we employ throughout our work the same choice of the unitary phases \( \bar{\beta}_i \) as adopted in Ref. \[29\] and explained in the previous section.

2.1 Parametrization of the single nucleon current in terms of the form factors

Single-nucleon current operators can be conveniently parametrized in terms of the Dirac and Pauli electromagnetic form factors \( F_1(Q^2) \) and \( F_2(Q^2) \), respectively. In the relativistic kinematics, the on-shell single-nucleon current is given by
\[ V_1^\mu = \frac{e}{2m} \bar{u}(p') \left[ \gamma^\mu F_1(Q^2) + i \frac{1}{2m} \sigma^{\mu\nu} k_\nu F_2(Q^2) \right] u(p), \]
where the \( \gamma^\mu \) are Dirac matrices, \( \sigma^{\mu\nu} = i [\gamma^\mu, \gamma^\nu] / 2 \), \( k = p' - p \) is the four-momentum transfer while \( Q^2 = -k_\mu k^\mu \). The Dirac spinors are normalized according to
\[ \bar{u}(p) u(p) = 2m. \]

As already pointed out above, we suppress the isospin indices to simplify the notation. In the following, we will express all our results in terms of the Sachs electric and magnetic form factors
\[ G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m} F_2(Q^2), \]
\[ G_M(Q^2) = F_1(Q^2) + F_2(Q^2). \]

The \( 1/m \)-expansion of the non-relativistically normalized electromagnetic current is defined according to
\[ \sqrt{\frac{m}{E_p'}} \sqrt{\frac{m}{E_p}} V_1^\mu = \chi' \left( V_1^\mu_{1N: \text{static}} + V_1^\mu_{1N: 1/m} + V_1^\mu_{1N: 1/m^2} + O\left( \frac{1}{m^3} \right) \right) \chi, \]

\[ (2.5) \]
where \( E(p) = \sqrt{m^2 + p^2} \), while \( \chi' \) and \( \chi \) refer to Pauli spinors. In terms of the Sachs form factors, the charge operator is parametrized via

\[
V_{1N: \text{static}}^0 = e G_E(Q^2),
\]

\[
V_{1N: 1/m}^0 = \frac{i e}{2m^2} k \cdot (k_1 \times \sigma) G_M(Q^2),
\]

\[
V_{1N: 1/m^2}^0 = -\frac{e}{8m^2} [Q^2 + 2i k \cdot (k_1 \times \sigma)] G_E(Q^2),
\]

while the current operator is given by

\[
V_{1N: \text{static}}^1 = -\frac{i e}{2m} k \times \sigma G_M(Q^2),
\]

\[
V_{1N: 1/m}^1 = e \frac{k_1}{m} G_E(Q^2),
\]

\[
V_{1N: 1/m^2}^1 = \frac{e}{16m^3} \left[ i k \times (2k_1^2 + Q^2) + 2i k \times k_1 k_1 \cdot \sigma \right. \\
\left. + 2k_1 (i k \cdot (k_1 \times \sigma) + Q^2) - 2k k \cdot k_1 + 6i k \cdot \sigma k \cdot k_1 \right] G_M(Q^2),
\]

where \( k_1 = (p' + p)/2 \) and \( \sigma \) refers to the Pauli matrices in spin space. The powers of the nucleon mass in Eqs. (2.6,2.7) can be traced back to the convention, according to which the nonrelativistic expansion of the electric and magnetic Sachs form factors starts from terms of orders \( m^0 \) and \( m^1 \), respectively. Notice further that replacing \( Q^2 \) by \( k^2 \) does not affect Eqs. (2.6,2.7) since \( Q^2 = k^2 + O(1/m^2) \).

Before discussing the results for the charge and current operators in the method of UT, which may generally be expected to differ from the on-shell expressions given in Eqs. (2.6,2.7) by off-shell terms, it is instructive to address the validity of the continuity equation for the on-shell contributions alone. The general form of the continuity equation for the vector current at the considered order in the chiral expansion can be written as

\[
k \cdot \hat{V}(k,0) - \left[ \hat{H}, \hat{V}_0(k,0) \right] - \frac{\partial}{\partial k_0} \left( k \cdot \sigma \left[ \hat{H}, \hat{V}_0(k,k_0) \right] \right) = 0, \quad (2.8)
\]

where \( H \) denotes the strong part of the nuclear Hamiltonian. The longitudinal part of the on-shell single nucleon current operator is given by

\[
k \cdot V_{1N: \text{static}} = 0,
\]

\[
k \cdot V_{1N: 1/m} = \frac{k \cdot k_1}{m} e G_E(Q^2),
\]

\[
k \cdot V_{1N: 1/m^2} = i \frac{k \cdot k_1}{2m^2} k \cdot (k_1 \times \sigma) e G_M(Q^2),
\]

while the commutator of the kinetic energy with the on-shell charge operator reads

\[
\left[ \hat{p}^2, V_{1N: \text{static}}^0 \right] = \frac{k \cdot k_1}{m} e G_E(Q^2),
\]
\[ \left[ \frac{\hat{p}^2}{2m}, \hat{V}_{1N:1/m}^0 \right] = i \frac{k \cdot k_1}{2m^3} k \cdot (k_1 \times \sigma) e G_M(Q^2). \] (2.10)

Thus, if there is no dependence on the energy transfer, the parametrized on-shell current satisfies the continuity equation in the single-nucleon sector. This is, however, not the only possibility. If, for example, the current operator has an off-shell longitudinal component given by

\[ V_{1N: \text{off-shell}}^1 = k \left( k_0 - \frac{k \cdot k_1}{m} \right) e X, \] (2.11)

where \( X \) is an arbitrary function of momenta, the single-nucleon electromagnetic current

\[ V_{1N}^0 = V_{1N: \text{static}}^0 + V_{1N:1/m}^0 + V_{1N:1/m^2}^0, \]

\[ V_{1N}^1 = V_{1N: \text{static}}^1 + V_{1N:1/m}^1 + V_{1N:1/m^2}^1 + V_{1N: \text{off-shell}}^1 \] (2.12)

still satisfies the continuity equation (2.8) since

\[ k \cdot V_{1N: \text{off-shell}}^1(k, k_0) = -\frac{k \cdot k_1}{m} e X, \] (2.13)

and

\[ \left[ \frac{\hat{p}^2}{2m}, \frac{\partial}{\partial k_0} k \cdot V_{1N: \text{off-shell}}^1(k, k_0) \right] = \frac{k \cdot k_1}{m} e X. \] (2.14)

We now turn to the method of UT. Performing explicit calculations, we find that the on-shell expressions for the nuclear charge and current operators to have the same form as given in Eqs. (2.6) and (2.7). Using the same choice of the additional \( \eta \)-space UTs as adopted in Ref. [29] and described in Sec. 1, we, however, find an additional off-shell contribution to the single-nucleon current operator which depends on the phase \( \propto \bar{\beta}_6 \), see Eq. (1.2). Moreover, using dimensional regularization to calculate the loop integrals at order \( Q \), we obtain a divergent contribution to the current of the form

\[ V_{1N: \text{singular}}(Q) = -k \left( k_0 - \frac{k \cdot k_1}{m} \right) \frac{\tau_3}{d-4} \frac{2\bar{\beta}_6 + 1}{6(4\pi F^2)^2} \] (2.15)

where \( d \) denotes the number of space-time dimensions. Thus, renormalizability of the current operators requires taking \( \bar{\beta}_6 = -1/2 \), and the finite pieces of single-nucleon current operator does explicitly depend on the energy transfer \( k_0 \). The corresponding off-shell contribution to the current operator can be expressed in terms of the form factors as

\[ V_{1N: \text{off-shell}} = k \left( k_0 - \frac{k \cdot k_1}{m} \right) e \frac{Q^2}{Q^2} \] (2.16)

\[ \times \left[ (G_E(Q^2) - G_E(0)) + i \frac{k \cdot (k_1 \times \sigma)}{2m^2} (G_M(Q^2) - G_M(0)) \right]. \]
Clearly, the unitary transformation $\propto \tilde{\beta}_0$ in Eq. (1.2) only induces terms in $V_{1N; \text{off-shell}}$ up to order $Q$. In Appendix A we, however, provide an explicit form of the unitary transformation in the Hilbert space, which exactly generates the off-shell terms given in Eq. (2.16).

2.2 Chiral expansion of the Sachs form factors

In order to verify the validity of the continuity equation we provide in this section explicit expressions for the single-nucleon form factors at lowest orders in the chiral expansion, as already employed in Ref. [31]. We refer the reader to Ref. [35,36] and references therein for a detailed analysis of the electromagnetic form factors of the nucleon in relativistic formulations of baryon chiral perturbation theory. We employ in this section the standard power counting used in the single-nucleon heavy baryon approach, where the nucleon mass is treated on the same footing as the chiral symmetry breaking scale $m \sim \Lambda_{\chi}$.

The corresponding powers of the soft scale relative to the dominant contribution will be written in square brackets to avoid a possible confusion with the counting scheme employed throughout the rest of this paper as explained in Sec. 1. For what concerns the relativistic corrections, we only list below the contributions which are relevant to the derivation of the current operators at the considered order. For the sake of simplicity, we do not differentiate between the various quantities in the chiral limit and their physical values here.

- The leading contributions to the form factors $G_E(Q^2)$ and $G_M(Q^2)$ have the form

$$G_E^{[Q^0]}(Q^2) = \frac{1}{2} (1 + \tau_3),$$
$$G_M^{[Q^0]}(Q^2) = \frac{1}{2} (1 + \kappa_s + (1 + \kappa_v) \tau_3), \quad (2.17)$$

where $\kappa_s$ and $\kappa_v$ refer to the isoscalar and isovector anomalous magnetic moments of the nucleon, respectively.

- At order $Q^1$, there are only loop contributions to the magnetic form factor

$$G_E^{[Q^1]}(Q^2) = 0,$$
$$G_M^{[Q^1]}(Q^2) = \frac{mg_A^2 \tau_3}{16 F_{\pi}^2 \pi} \left[ M_{\pi} - (4M_{\pi}^2 + Q^2) A(|k|) \right], \quad (2.18)$$

where $g_A$, $F_\pi$ and $M_{\pi}$ refer to the nucleon axial-vector coupling, pion decay constant and pion mass, respectively. Further, the loop function $A(|k|)$ is given by

$$A(k) = \frac{1}{2|k|} \arctan \frac{|k|}{2M_{\pi}}. \quad (2.19)$$
At order $Q^2$ one has further contributions to both electromagnetic form factors. These read

\[
G_E^{[Q^2]}(Q^2) = \frac{1}{6(4\pi F)^2} \left[ 4(1 + 2\alpha_A^2)M^2 + (1 + 5\alpha_A^2)Q^2 \right] L(|k|) \tau_3
\]

\[+ \frac{\tau_3}{36(4\pi F)^2} \left[ -24(1 + 2\alpha_A^2)M^2 - Q^2(5 + 13\alpha_A^2) \right]
\]

\[+ Q^2(2d_7 + \bar{d}_6 \tau_3),
\]

\[
G_M^{[Q^2]}(Q^2) = m \left[ \frac{c_4 \tau_3}{9(4\pi F)^2} \right] \left[ 24M^2(L(|k|) - 1) + Q^2(6L(|k|) - 5) \right]
\]

\[\quad - 2Q^2(2\epsilon_{54} + \bar{\epsilon}_{44} \tau_3) \right] + \frac{\tau_3}{36(4\pi F)^2} \left[ 24(4\alpha_A^2 - 1)M^2(1 - L(|k|)) \right]
\]

\[\quad - Q^2(5 - 23\alpha_A^2 + 6(7\alpha_A^2 - 1)L(|k|)) \right],
\]

where

\[
L(|k|) = \sqrt{k^2 + 4M^2} \log \frac{\sqrt{k^2 + 4M^2} + |k|}{2M}\]

with the loop function $L(|k|)$ given by

At order $Q^3$ one has to take into account the leading two-loop contributions, whose calculation goes beyond the scope of our study, as well as the additional relativistic corrections at the one-loop level given by

\[
G_E^{[Q^3]}(Q^2) = \frac{g^2 A \tau_3}{32\pi F^2 m} \left[ M_x (M^2_x - 2Q^2) - (2M^2_x + Q^2)^2 A(|k|) \right],
\]

\[
G_M^{[Q^3]}(Q^2) = \frac{g^2 Q^2}{512\pi F^2 m} \left[ 2(16M^2_x + 5Q^2)A(|k|)\tau_3 \right]
\]

\[\quad + \left( 3(1 + \kappa_x) + (11 - \kappa_x)\tau_3 \right)M_x \right],
\]

Notice that the chiral expansion of the electromagnetic form factors of the nucleon is known to converge rather slowly. The form factors are known to be largely driven by vector mesons \[35,39\], which are not included as explicit degrees of freedom in the approach we are using. In this sense, it is certainly more advantageous from the phenomenological point of view to employ empirical parametrizations of the single-nucleon form factors in the single-nucleon charge and current operators when performing few-nucleon calculations, rather than to strictly rely on their chiral expansion. Such an approach is, in fact, adopted in most of the studies in the few-nucleon sector, see e.g. the early work in Ref. \[38\].
3 Current conservation

Having derived the single-nucleon contributions we are now in the position to explicitly demonstrate current conservation by verifying the validity of the continuity equation (2.8).

The single-nucleon contributions to the current operator have already been shown to fulfill the continuity equation in Sec. 2.1. The leading two-nucleon contributions to the current operator at order $Q^{-1}$, stemming from the OPE at tree level, are also well known to fulfill the continuity equation. Further, as already pointed out in the introduction, the two-pion exchange contributions to the current operator at order $Q$ are shown to be conserved in Ref. [28]. Thus, it remains to verify the validity of the continuity equation for the order-$Q$ short-range pieces and corrections to the OPE.

The OPE contributions at the leading one-loop level have been calculated in [29] without considering possible contributions induced by the explicit time dependence of the imposed UTs in Eqs. (1.1,1.2). We have verified that using the choice of unitary phases $\bar{\beta}_i$ from Ref. [29] described in Sec. 1, the OPE contributions to the charge and current operators up to order $Q^1$ do not depend on the energy transfer $k_0$. Thus, the complete expressions for the OPE contributions to the charge and current operators at order $Q$ are given in Eqs. (4.28)-(4.31) of Ref. [29]. We now verify the validity of the continuity equation at order $Q^2$ for the OPE contributions and begin with the longitudinal part of the current operator

$$ k \cdot V^{(Q)}_{2N:1\pi} = -\frac{\bar{\alpha}}{g_\pi M^2} \frac{M^2}{F^2_\pi} \left[ \tau^{(1)} \times \tau^{(2)} \right] \frac{q_1 \cdot \sigma^{(1)} q_1 \cdot \sigma^{(2)}}{q_1^2 + M^2} + (1 \leftrightarrow 2) $$

$$ = \left[ \hat{H}^{(Q^2)}_{2N:1\pi}, \hat{V}^{0(Q^{-1})}_{1N} \right], \quad (3.1) $$

where the order-$Q^2$ correction to OPE potential is given by

$$ H^{(Q^2)}_{2N:1\pi} = \frac{\bar{d}_{18} g_{A M^2}}{F^2_\pi} \frac{q \cdot \sigma^{(1)} q \cdot \sigma^{(2)}}{q^2 + M^2} \tau^{(1)} \cdot \tau^{(2)}, \quad (3.2) $$

with $q$ denoting the three-momentum transfer of the nucleons, and the LEC $\bar{d}_{18}$ parameterizes the deviation from the Goldberger-Treiman relation [39].

The leading-order single-nucleon charge operator has the form

$$ V^{0(Q^{-3})}_{1N} = \frac{e}{2} (1 + \tau_3). \quad (3.3) $$

Here and in what follows, $\sigma^{(i)}$ and $\tau^{(i)}$ refer to the Pauli spin and isospin matrices of nucleon $i$. On the other hand, the longitudinal component of the energy-transfer-dependent current is given by

$$ \frac{\partial}{\partial k_0} k \cdot V^{(Q)}_{1N: off-shell} = e G^{(Q^2)}_{1N, static} = V^{0(Q^{-1})}_{1N, static}, \quad (3.4) $$
so that
\[
\left[ \hat{H}_{2N:1\pi}^{(Q^0)}, \hat{V}_{1N:static}^{(Q^{-1})} - \frac{\partial}{\partial k_0} k \cdot \hat{V}_{1N:off-shell}^{(Q)} \right] = 0. \tag{3.5}
\]

The double commutator in the continuity equation (2.8) contributes at orders higher than \(Q^2\). Thus, we conclude that the continuity equation for the contributions of the OPE-range is fulfilled at order \(Q^2\) in the static limit of \(m \to \infty\). Notice that without the \(\frac{\partial}{\partial k_0} k \cdot \hat{V}_{1N:off-shell}^{(Q)}\) contribution, the traditional form of the continuity equation \(k \cdot \hat{V} = [\hat{H}, \hat{V}^0]\) would not be satisfied. The validity of the continuity equation for the leading relativistic corrections at the same chiral order follows trivially from \(\hat{V}_{2N:1\pi,1/m}^{(Q)} = 0\) along with the observation
\[
\frac{\partial}{\partial k_0} \hat{V}_{2N:cont}^{(Q)} 1N:1/m = \frac{\partial}{\partial k_0} \hat{V}_{1N:off-shell}^{(Q)} = 0, \tag{3.6}
\]
which implies that
\[
\left[ \hat{H}_{2N:1\pi}^{(Q^0)}, \hat{V}_{1N:1/m}^{(Q^{-1})} \right] = \frac{\partial^2}{\partial k_0^2} \frac{k}{2m} \cdot \hat{V}_{2N:1\pi}^{(Q^0)}
= \left[ \hat{H}_{2N:1\pi}^{(Q^0)}, \frac{\partial}{\partial k_0} k \cdot \hat{V}_{1N:off-shell}^{(Q)} \right] = 0. \tag{3.7}
\]

We now turn to the short-range terms. The leading static contribution to the current operator is generated by tree-level diagrams at order \(Q^1\) and given in Eq. (5.3) of [29]. Clearly, the terms in \(k \cdot \hat{V}_{2N:cont}^{(Q)}\) proportional to the LECs \(C_i\) coincide with the corresponding contributions in \([\hat{H}_{2N:cont}, \hat{V}_{1N:static}^{(Q^{-3})}]\), while the ones proportional to the LECs \(L_i\) are purely transversal. Further, due to Eq. (3.4), we obtain
\[
\left[ \hat{H}_{2N:cont}^{(Q^0)}, \hat{V}_{1N:static}^{(Q^{-1})} - \frac{\partial}{\partial k_0} k \cdot \hat{V}_{1N:off-shell}^{(Q)} \right] = 0. \tag{3.8}
\]
Thus, the continuity equation for the short-range current is fulfilled at order \(Q^2\) in the static limit. Since
\[
\hat{V}_{2N:cont,1/m}^{(Q)} 1N:1/m = \frac{\partial}{\partial k_0} \hat{V}_{1N:cont}^{(Q^0)} 2N:1/m = \hat{V}_{1N:off-shell}^{(Q)} = 0, \tag{3.9}
\]
the continuity equation is trivially fulfilled for the short-range relativistic corrections at order \(Q^2\) as well.
Fig. 1 Diagrams generating the $g_A^4$-contribution to the long-range three-nucleon electromagnetic charge operator at $N^3$LO. Diagrams resulting from the application of the time reversal and permutation operations are not shown. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order.

Fig. 2 Diagrams generating the $g_A^2$-contribution to the long-range three-nucleon electromagnetic charge operator at $N^3$LO. Diagrams resulting from the application of the time reversal and permutation operations are not shown. For notations, see Fig. 1.

Fig. 3 Diagrams generating the $g_A^2$-contribution to the three-nucleon electromagnetic charge operator at $N^3$LO which involve a single insertion of the LO NN contact interactions. Diagrams resulting from the application of the time reversal and permutation operations are not shown. For notations, see Fig. 1.

4 Three-nucleon charge operator

While the leading contributions to the three-nucleon current appear at order $Q^2$ which is beyond the accuracy of our study, the three-nucleon charge operator receives the leading contributions already at $N^3$LO. Using the method of UT, we obtain the following result for the diagrams $\sim g_A^4$ shown in Fig. 1.

$$V_{3N;\pi}^{0(Q)} = \frac{-\epsilon g_A^4}{8F_\pi^2(q_1^2 + M_\pi^2)(q_1^2 + q_2^2 + M_\pi^2)} \left( \frac{(q_1 + q_2) \cdot \sigma^{(3)}}{(q_1 + q_2)^2 + M_\pi^2} + \frac{q_3 \cdot \sigma^{(3)}}{q_3^2 + M_\pi^2} \right)$$

$$\times \left[ \left[ \tau^{(1)} \times \tau^{(3)} \right]_{\beta} + \tau^{(1)} \cdot \tau^{(3)} \tau^{(2)}_{\beta} - \tau^{(2)} \cdot \tau^{(3)} \tau^{(1)}_{\beta} \right] (q_1^2 + q_1 \cdot q_2)$$

$$+ 5 \text{ permutations},$$

while the $\sim g_A^2$ contributions visualized in Fig. 2 have the form
Table 1 Chiral expansion of the nuclear electromagnetic current operator up to $N^3\text{LO}$. LO, NLO, $N^2\text{LO}$ and $N^3\text{LO}$ refer to chiral orders $Q^{-3}$, $Q^{-1}$, $Q^0$ and $Q$, respectively. The single-nucleon contributions are given in Eqs. (2.7) and (2.16).

| order | single-nucleon | two-nucleon | three-nucleon |
|-------|----------------|-------------|---------------|
| LO    | $V_{1\text{N}}:\text{static}$ | $V_{2\text{N}}:1\pi$, Eq. (4.16) of [29] | — |
| NLO   | $V_{1\text{N}}:1/m$ | $V_{2\text{N}}:1\pi$, Eq. (4.28) of [29] | — |
| $N^2\text{LO}$ | $V_{1\text{N}}:\text{static}$ | — | — |
| $N^3\text{LO}$ | $V_{1\text{N}}:1/m$ + $V_{1\text{N}}:\text{off-shell}$ | $V_{2\text{N}}:2\pi$, Eq. (2.18) of [28] + $V_{1\text{N}}:\text{cont}$, Eq. (5.3) of [29] | — |

$$
V^{0(Q)}_{3\text{N}:\pi} = \frac{\epsilon g_A^2}{16 F_2^2} \left( \frac{\mathbf{q}_1 \cdot \mathbf{\sigma}^{(1)}}{\mathbf{q}_1^2 + M_\pi^2} \left( \frac{(\mathbf{q}_1 + \mathbf{q}_2)^2 + M_\pi^2}{\mathbf{q}_3^2 + M_\pi^2} \right) \right) + \frac{\mathbf{q}_3 \cdot \mathbf{\sigma}^{(3)}}{\mathbf{q}_3^2 + M_\pi^2} + 5 \text{ permutations.} \quad (4.2)
$$

There are also diagrams involving a single insertion of the leading-order contact interactions shown in Fig. 3, which lead to the following result:

$$
V^{0(Q)}_{3\text{N}:\text{cont}} = -\left[ \tau^{(1)} \times \mathbf{\tau}^{(3)} \right]_3 \frac{\epsilon g_A^2}{2 F_2^2} \left( \frac{\mathbf{q}_2 + \mathbf{q}_3 \cdot \mathbf{\sigma}^{(2)} \times \mathbf{\sigma}^{(3)}}{\mathbf{q}_2 + \mathbf{q}_3 \cdot \mathbf{\sigma}^{(2)} + M_\pi^2} \right) + \frac{\mathbf{q}_1 \cdot \mathbf{\sigma}^{(1)}}{\mathbf{q}_1^2 + M_\pi^2} + \frac{(\mathbf{q}_2 + \mathbf{q}_3) \cdot \mathbf{\sigma}^{(1)}}{(\mathbf{q}_2 + \mathbf{q}_3)^2 + M_\pi^2} + 5 \text{ permutations.} \quad (4.3)
$$

To the best of our knowledge, the three-nucleon contributions to the charge operator have not been considered before in the framework of chiral effective field theory.

5 Summary and conclusions

In this paper, we have completed the derivation of the nuclear electromagnetic current and charge operators to fourth order ($N^3\text{LO}$) in the chiral expansion. The corresponding contributions, obtained using dimensional regularization, are summarized in Tables 1 and 2.

The main results of our study can be summarized as follows:

– We have analyzed the single-nucleon contributions to the electromagnetic four-current at the one-loop level using the method of UT and identified the choice of the unitary transformations leading to a renormalized result. We also provide a parametrization of the single-nucleon operators including the leading and subleading relativistic corrections in terms of the
Table 2 Chiral expansion of the nuclear electromagnetic charge operator up to $N^3\!LO$. LO, NLO, $N^2\!LO$ and $N^3\!LO$ refer to chiral orders $Q^{-3}$, $Q^{-1}$, $Q^0$ and $Q$, respectively. The single-nucleon contributions are given in Eq. (2.6).

| order   | single-nucleon | two-nucleon | three-nucleon |
|---------|----------------|-------------|---------------|
| LO      | $V^0_{1N:\text{static}}$ | —           | —             |
| NLO     | $V^0_{1N:\text{static}}$ | —           | —             |
| $N^2\!LO$ | $V^0_{1N:\text{static}}$ | —           | —             |
| $N^3\!LO$ | $V^0_{1N:\text{static}}$ | $V^0_{2N,1\pi}$, Eq. (4.30) of [29] | $V^0_{3N,1\pi}$, Eq. (4.1) |
|         | $+ V^0_{1N,1/m}$ | $+ V^0_{2N,2\pi}$, Eq. (2.19) of [28] | $+ V^0_{3N,2\pi}$, Eq. (4.2) |
|         | $+ V^0_{1N,1/m^2}$ | $+ V^0_{2N,\text{cont}}$, Eq. (5.6) of [29] | $+ V^0_{3N,\text{cont}}$, Eq. (7.3) |
|         | $+ V^0_{2N,1\pi,1/m}$, Eq. (4.30) of [29] | —           | —             |

Different conventions are being used in the literature for the leading-order two-nucleon contact interactions $\propto C_{S,T}$. To match the convention of Refs. [2,4,7], the factors of $32F_2^2$ in Eq. (5.7) of [29] should be replaced by $16F_2^2$.

Electromagnetic form factors of the nucleon. Apart from the well known on-shell pieces, see e.g. [40], we found an additional energy-dependent off-shell contribution to the current operator whose form is dictated by the renormalizability constraint.

- We have verified the completeness of the expressions for the two-nucleon contributions derived in Refs. [28,29] without considering possible energy-dependent pieces due to the explicit time-dependence of the employed unitary transformations, see Ref. [33] for more details. For the choice of unitary phases $\bar{\beta}_i$ adopted in Refs. [29], the two-nucleon current operator $V^\mu_{2N}$ is found to be $k_0$-independent.

- We have worked out the leading three-nucleon contributions to the charge operator which appears at $N^3\!LO$.

- With the complete $N^3\!LO$ result for the electromagnetic current $V^\mu$ at hand, we were able to explicitly verify the validity of the modified form of the continuity equation derived in Ref. [39] at order $Q^2$ for all one- and two-nucleon contributions.

The obtained results can be used to study electromagnetic properties of nuclei such as the form factors, photo- and electrodisintegration reactions and the radiative capture processes with light nuclei in the framework of chiral effective field theory with fully consistent nuclear forces and currents. This, however, would require the development and implementation of the regularization procedure, which is consistent with the one employed in the high-precision semilocal chiral nuclear potentials of Ref. [4]. Work along these lines is in progress.
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A Unitary transformation to exactly generate the off-shell single-nucleon current in Eq. (2.16)

In this appendix we construct a unitary transformation, which generates the off-shell single-nucleon longitudinal current in Eq. (2.16).

In order to see how the nuclear operators are affected by time-dependent unitary transformations, we perform an additional unitary transformation of the kind

$$ U(t) = \exp \left( J(t) \right), \quad J(t) = \int d^3 x \, v_\mu(x,t) \, X^\mu(x). $$

where $v_\mu(x)$ is an external vector source and $X_\mu(x)$ is some antihermitean operator. The unitary transformation changes the Hamilton operator in the presence of the vector source to

$$ W[v] \rightarrow U(t) W[v] U(t) + \left( i \frac{\partial}{\partial t} U(t)^\dagger \right) U(t) = W[v] + [\hat{H}, \hat{J}(t)] - i \frac{\partial}{\partial t} J(t) + O(v^2), $$

with $H := W[0]$. For the vector current operator one obtains

$$ V_\mu(k) = - \left. \frac{\delta W[v]}{\delta \tilde{v}_\mu(k,k_0)} \right|_{v=0}, \quad \tilde{v}_\mu(k,k_0) := \int \frac{d^4 x}{(2\pi)^4} e^{ik \cdot x} \tilde{v}_\mu(x), $$

see [33] for more details, where the operator $W[v]$ is taken at $t = 0$:

$$ W[v] = H + \int d^3 x \, v_\mu(x,0) \tilde{V}_\mu(x) = H + \int d^4 k \, \tilde{v}_\mu(k,k_0) V^\mu(k). $$

Using

$$ J(0) = \int d^3 x \, v^\mu(x,0) X_\mu(x) = \int d^4 k \, \tilde{v}^\mu(k,k_0) \tilde{X}_\mu(k), $$

$$ \left. \frac{\partial}{\partial t} J(t) \right|_{t=0} = \int d^3 x \, i \, v^\mu(x,0) X_\mu(x) = \int d^4 k \, k_0 \tilde{v}^\mu(k,k_0) \tilde{X}_\mu(k), $$

where

$$ \tilde{X}^\mu(k) := \int d^3 x \, e^{ik \cdot x} X^\mu(x), $$

we obtain

$$ V^\mu(k) \rightarrow V^\mu(k) - \left( k_0 \tilde{X}^\mu(k) - [\hat{H}, \tilde{X}^\mu(k)] \right). $$

We adopt here the commonly accepted sign convention for the electromagnetic current, which differs from the one used in our paper [33].
Thus, choosing
\[
\langle p' | \tilde{X}^0(k) | p \rangle = 0,
\]
\[
\langle p' | \tilde{X}(k) | p \rangle = \chi' \left( -\frac{e}{k^2} \left( G_E(k^2) - G_E(0) \right) + \frac{i}{2m^2} k \cdot (k_1 \times \sigma) (G_M(k^2)
- G_M(0)) \right) \chi,
\]
we obtain the off-shell term given in Eq. (2.16). It is important to emphasize that the derivation of Eq. (1.7) does not rely on chiral perturbation theory. No approximations were made to derive this equation (except for neglecting the contributions involving more than a single insertion of the external vector source).

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