Radiative corrections to the leptonic Dirac CP-violating phase

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Abstract

Since the smallest leptonic mixing angle $\theta_{13}$ has been measured to be relatively large, it is quite promising to constrain or determine the leptonic Dirac CP-violating phase $\delta$ in future neutrino oscillation experiments. Given some typical values of $\delta = \pi/2$, $\pi$, and $3\pi/2$ at the low-energy scale, as well as current experimental results of the other neutrino parameters, we perform a systematic study of the radiative corrections to $\delta$ by using the one-loop renormalization group equations in the minimal supersymmetric standard model and the universal extra-dimensional model. It turns out that $\delta$ is rather stable against radiative corrections in both models, except for the minimal supersymmetric standard model with a very large value of $\tan \beta$. Both cases of Majorana and Dirac neutrinos are discussed. In addition, we use the preliminary indication of $\delta = (1.08^{+0.28}_{-0.31}) \pi$ or $\delta = (1.67^{+0.37}_{-0.77}) \pi$ from the latest global-fit analyses of data from neutrino oscillation experiments to illustrate how it will be modified by radiative corrections.

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I. INTRODUCTION

In the last two decades, our knowledge on neutrinos has been greatly improved by a number of elegant neutrino oscillation experiments [1]. Now, we are convinced that neutrinos are massive, and they can transform from one flavor to another when propagating in vacuum or in matter. The lepton flavor mixing phenomenon can be described by a $3 \times 3$ unitary matrix $V$, namely the leptonic mixing matrix, which is conventionally parametrized through three mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$, as well as three CP-violating phases $\delta$, $\rho$ and $\sigma$, viz.,

$$V = U \cdot P \equiv \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix} \cdot P,$$  

(1)

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$ for $ij = 12, 13, 23$. Note that $P = \text{diag}(e^{i \rho}, e^{i \sigma}, 1)$ is a diagonal matrix with $\rho$ and $\sigma$ being two Majorana-type CP-violating phases if neutrinos are Majorana particles, while $P = 1$ if neutrinos are Dirac particles. Current experimental data indicate that the three leptonic mixing angles are $\theta_{12} \approx 34^\circ$, $\theta_{13} \approx 9^\circ$ and $\theta_{23} \approx 40^\circ$. Two independent neutrino mass-squared differences are found to be $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \approx 7.5 \times 10^{-5} \text{ eV}^2$ and $|\Delta m_{31}^2| \equiv |m_3^2 - m_1^2| \approx 2.5 \times 10^{-3} \text{ eV}^2$. The latest global-fit results of neutrino parameters are shown in Table I. However, we are still unclear whether the neutrino mass ordering is normal (i.e., $\Delta m_{31}^2 > 0$) or inverted (i.e., $\Delta m_{31}^2 < 0$), and the leptonic Dirac CP-violating phase $\delta$ remains experimentally unconstrained.

The recent results from Daya Bay [2] and RENO [3] reactor neutrino experiments have established that $\theta_{13} \approx 9^\circ$, which is rather large. Hence, it is quite promising to determine the leptonic Dirac CP-violating phase $\delta$ by comparing the oscillation probabilities of neutrinos and antineutrinos in future long-baseline neutrino oscillation experiments [4]. In addition, the km$^3$-scale neutrino telescopes (e.g., IceCube and KM3NeT) could provide us with useful and complementary information about leptonic CP violation by precisely measuring the flavor composition of ultrahigh-energy astrophysical neutrinos [5]. If the Deep Core of the IceCube detector is made denser to lower the energy threshold down to a few GeV, such as the proposal PINGU [6], a large amount of atmospheric neutrino events can be collected and used to determine the neutrino mass hierarchy and perhaps the leptonic CP-violating phase $\delta$. On the other hand, a lot of neutrino mass models based on discrete flavor symmetries or
TABLE I: The best-fit values and 1σ ranges of the neutrino parameters from the latest global-fit analyses of neutrino oscillation experiments, where the normal neutrino mass hierarchy is assumed.

| parameter          | Ref. [11]   | Ref. [12]   | Ref. [13]   |
|--------------------|-------------|-------------|-------------|
| \(\sin^2 \theta_{12}\) | 0.307       | 0.300       | 0.320       |
|                    | 0.291 – 0.325 | 0.287 – 0.313 | 0.303 – 0.336 |
| \(\sin^2 \theta_{13}\) | 0.0241      | 0.0230      | 0.0246      |
|                    | 0.0216 – 0.0266 | 0.0207 – 0.0253 | 0.0218 – 0.0275 |
| \(\sin^2 \theta_{23}\) | 0.386       | 0.410       | 0.427       |
|                    | 0.365 – 0.410 | 0.385 – 0.447 | 0.400 – 0.461 |
| \(\Delta m_{21}^2 / 10^{-5} \text{eV}^2\) | 7.54        | 7.50        | 7.62        |
|                    | 7.32 – 7.80 | 7.32 – 7.69 | 7.43 – 7.81 |
| \(\Delta m_{31}^2 / 10^{-3} \text{eV}^2\) | 2.51        | 2.47        | 2.55        |
|                    | 2.41 – 2.57 | 2.40 – 2.54 | 2.46 – 2.61 |
| \(\delta / \pi\) | 1.08        | 1.67        | 0.8         |
|                    | 0.77 – 1.36 | 0.90 – 2.03 | 0 – 2.0     |

phenomenological assumptions have recently been proposed to describe the observed leptonic mixing pattern, in particular a relatively large \(\theta_{13}\). Interestingly, the leptonic CP-violating phase \(\delta\) has been predicted in some models to be rather large (e.g., \(\delta > \pi/3\)) or even maximal (i.e., \(\delta = \pi/2\)) [8, 9]. In other models, leptonic CP violation is shown to be absent, namely \(\delta = 0\) or \(\pi\) [10]. It is worthwhile to mention that the latest global-fit analyses of neutrino oscillation experiments yield \(\delta = (1.08^{+0.28}_{-0.31}) \pi\) [11] and \(\delta = (1.67^{+0.37}_{-0.77}) \pi\) [12], although the 1σ errors are still quite large.\(^1\) Therefore, we have already obtained some preliminary information on the leptonic CP-violating phase \(\delta\) from the global-fit analyses.

In this work, we are concerned with how the theoretical predictions or the observed value of \(\delta\) will be modified by the radiative corrections when running from a low-energy scale to a superhigh-energy scale. This question does make sense if we believe that there exists at some superhigh-energy scale a unified theory for flavor mixing and CP violation in both quark and lepton sectors. Once the leptonic CP-violating phase \(\delta\) is measured in future neutrino oscillation experiments, the renormalization group (RG) evolution of \(\delta\) will tell us how large or small it will be at a given superhigh-energy scale. As a matter of fact,

\(^1\) The best-fit value is found to be \(\delta = 0.8 \pi\) for the normal mass hierarchy and \(\delta = -0.03 \pi\) for the inverted mass hierarchy by another global-fit group [13]. However, there is no constraint on \(\delta\) within the 1σ range.
the running of leptonic mixing parameters has been extensively discussed in the literature [14, 15], and more recently in Ref. [16], where the authors concentrate on the newly measured $\theta_{13}$. Different from the previous works, we focus on $\delta$ and perform a systematic study of its running behavior in the minimal supersymmetric standard model (MSSM) and in the universal extra-dimensional model (UEDM). The motivation for such a study is two-fold: (1) The leptonic CP-violating phase $\delta$ is the last fundamental parameter (except for the neutrino mass hierarchy) to be measured in the future neutrino oscillation experiments, and now both the theoretical models and the global-fit analysis can provide us with preferred values of $\delta$ at the low-energy scale. (2) The models with supersymmetry or extra spatial dimensions are the most natural extensions of the SM, which can solve the gauge hierarchy problem and offer good candidates for the dark matter.

In lack of a complete theory for neutrino mass generation, we implement the dimension-five Weinberg operator to account for tiny Majorana neutrino masses [17]. The RG running of $\delta$ in the case of Dirac neutrinos will be considered as well for comparison and completeness.

The remaining part of the present paper is organized as follows. In Sec. II, we set up the basic framework for the RG running of leptonic mixing parameters in the case of Majorana neutrinos. The renormalization group equation (RGE) of $\delta$ is derived analytically, and solved numerically. Section III is devoted to the RG running of $\delta$ in the case of Dirac neutrinos in the MSSM. We summarize our conclusions in Sec. IV. The complete set of RGE’s in the SM, MSSM, and UEDM for Majorana neutrinos are collected in Appendix A, while those in the SM and MSSM for Dirac neutrinos in Appendix B.

II. RUNNING OF CP-VIOLATING PHASE: MAJORANA NEUTRINOS

First of all, we derive the RGE for the leptonic CP-violating phase $\delta$, assuming that neutrinos are Majorana particles. Without loss of generality, we introduce the dimension-five Weinberg operator responsible for neutrino masses [17]:

$$- \mathcal{L}_{\nu} = \frac{1}{2} (\bar{\ell} H) \cdot \kappa \cdot (H^T \ell^C) + h.c.,$$

(2)

where $\ell$ and $H$ stand for the lepton and Higgs doublet fields, respectively, and $\kappa$ is a symmetric and complex matrix of the inverse mass dimension. After electroweak symmetry breaking,
TABLE II: Explicit expressions of $\text{Re} \left[ (U^\dagger U)_{ij} \right]$ and $\text{Im} \left[ (U^\dagger U)_{ij} \right]$ for $i \leq j$ in the standard parametrization of leptonic mixing matrix.

| $ij$ | Re $\left[ (U^\dagger U)_{ij} \right]$ | Im $\left[ (U^\dagger U)_{ij} \right]$ |
|------|--------------------------------|----------------------------------|
| 11   | 0                               | $+2 s_{12} c_{12} s_{13} s_\delta \dot{\theta}_{23} + c_{12}^2 s_{13}^2 \dot{\delta}$ |
| 22   | 0                               | $-2 s_{12} c_{12} s_{13} s_\delta \dot{\theta}_{23} + s_{12}^2 s_{13}^2 \dot{\delta}$ |
| 33   | 0                               | $-s_{13}^2 \dot{\delta}$ |
| 12   | $\dot{\theta}_{12} + s_{13} s_\delta \dot{\theta}_{23}$ | $-(c_{12}^2 - s_{12}^2) s_{13} s_\delta \dot{\theta}_{23} + c_{12} s_{13} s_{12} s_{13}^2 \dot{\delta}$ |
| 13   | $-s_{12} c_{13} \dot{\theta}_{23} + c_{12} c_\delta \dot{\theta}_{13} - c_{12} s_{13} c_{13} s_\delta \dot{\delta}$ | $-c_{12} s_\delta \dot{\theta}_{13} - c_{12} s_{13} c_{13} c_\delta \dot{\delta}$ |
| 23   | $+c_{12} c_{13} \dot{\theta}_{23} + s_{12} c_\delta \dot{\theta}_{13} - s_{12} s_{13} c_{13} s_\delta \dot{\delta}$ | $-s_{12} s_\delta \dot{\theta}_{13} - s_{12} s_{13} c_{13} c_\delta \dot{\delta}$ |

The mass matrix of three light Majorana neutrinos is given by $M_\nu = \kappa v^2$ with $v \approx 174$ GeV being the vacuum expectation value (vev) of the SM Higgs field, or by $M_\nu = \kappa (v \sin \beta)^2$ with $\tan \beta$ being the ratio of the vev’s of two Higgs doublets in the MSSM. Note that we are working within an effective theory, and consider the running of neutrino mixing parameters below the cutoff scale $\Lambda$ where new physics takes effects.

At one-loop level, the evolution of $\kappa$ is governed by

$$16\pi^2 \frac{d\kappa}{dt} = \alpha_\kappa + C_\kappa \left[ \left( Y_l Y_l^\dagger \right) \kappa + \kappa \left( Y_l Y_l^\dagger \right)^T \right],$$

(3)

where $t \equiv \ln(\mu/\Lambda_{\text{EW}})$ with $\mu$ being an arbitrary renormalization scale between the electroweak scale $\Lambda_{\text{EW}} \approx 100$ GeV and a cutoff scale where new physics comes into play, and $Y_l$ is the Yukawa coupling matrix of the charged leptons. The coefficients $\alpha_\kappa$ and $C_\kappa$ are flavor universal, and have been explicitly given in Appendix A for the SM, the MSSM, and the UEDM. It is worth stressing that Eq. (3) takes on the same form in all the models under consideration. However, the coefficients in the RGE’s may differ. We will distinguish them by adding the corresponding superscripts to these coefficients, as shown in Appendix A.

### A. Analytical Results

Since the RGE’s of neutrino mass matrix $M_\nu = \kappa v^2$ in the SM and UEDM, or $M_\nu = \kappa (v \sin \beta)^2$ in the MSSM, are given by the same formula in Eq. (3), the evolution of neutrino
TABLE III: The coefficients $R_{\alpha ij}$ and $T_{\alpha ij}$ for $\alpha = e, \mu, \tau$ and $ij = 12, 13, 23$ in the standard parametrization of leptonic mixing matrix.

| $R_{\alpha ij}$ | 12 | 13 | 23 |
|-----------------|----|----|----|
| $e$             | $s_{12}c_{12}c_{13}^2$ | $c_{12}s_{13}c_{13}$ | $s_{12}s_{13}c_{13}$ |
| $\mu$           | $s_{12}c_{12}(s_{23}^2s_{13}^2 - c_{23}^2)$ | $- (s_{12}c_{23} + c_{12}s_{23}s_{13}c_{\delta})s_{23}c_{13}$ | $+(c_{12}c_{23} - s_{12}s_{23}s_{13}c_{\delta})s_{23}c_{13}$ |
| $\tau$          | $s_{12}c_{12}(c_{23}^2s_{13}^2 - s_{23}^2)$ | $+(s_{12}s_{23} - c_{12}c_{23}s_{13}c_{\delta})c_{23}c_{13}$ | $-(c_{12}s_{23} + s_{12}c_{23}s_{13}c_{\delta})c_{23}c_{13}$ |

| $T_{\alpha ij}$ | 12 | 13 | 23 |
|-----------------|----|----|----|
| $e$             | 0  | 0  | 0  |
| $\mu$           | $s_{23}c_{23}s_{13}s_{\delta}$ | $c_{12}s_{23}^2s_{13}c_{13}s_{\delta}$ | $s_{12}s_{23}^2s_{13}c_{13}s_{\delta}$ |
| $\tau$          | $- s_{23}c_{23}s_{13}s_{\delta}$ | $c_{12}s_{23}^2s_{13}c_{13}s_{\delta}$ | $s_{12}s_{23}^2s_{13}c_{13}s_{\delta}$ |

mass eigenvalues and leptonic mixing parameters can be figured out in the same way. In flavor basis, where the Yukawa coupling matrix of the charged leptons is diagonal, namely $Y_l = D_l \equiv \text{diag}(y_e, y_\mu, y_\tau)$, $\kappa$ can be diagonalized by the leptonic mixing matrix $V$, namely $V^\dagger \kappa V^* = \tilde{\kappa} \equiv \text{diag}(\kappa_1, \kappa_2, \kappa_3)$. Generally speaking, an arbitrary $3 \times 3$ unitary matrix $V'$ can be factorized as $V' = QUP$, where $Q = \text{diag}(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})$ and $P = \text{diag}(e^{i\rho}, e^{i\sigma}, 1)$ are pure phase matrices, while the unitary matrix $U$ consists of three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and the Dirac CP-violating phase $\delta$ [cf. Eq. (1)]. Although the phases $\phi_\alpha$ (for $\alpha = e, \mu, \tau$) are unphysical and can be removed by rephasing the charged-lepton fields, we will keep them in the derivation of the RGE’s for neutrino masses and leptonic mixing parameters.

Since $y_e^2 \ll y_\mu^2 \ll y_\tau^2$, we take into account the dominant contribution from the tau-lepton Yukawa coupling to the RGE of $\kappa$. Following Ref. [18], one obtains

$$16\pi^2 \frac{d\kappa_i}{dt} = \kappa_i \left( \alpha_\kappa + 2C_\kappa y_\tau^2 |U_{\tau i}|^2 \right),$$

where $\alpha_\kappa$ and $C_\kappa$ should bear the corresponding superscripts when Eq. (4) is applied to a specific model. Given $m_i = \kappa_i v^2$ (for $i = 1, 2, 3$), we observe that Eq. (4) determines the evolution of absolute neutrino masses. Moreover, it is straightforward to find that $U_{\alpha i}$, $\rho$, $\sigma$,
and \( \phi_\alpha \) (for \( \alpha = e, \mu, \tau \) and \( i = 1, 2, 3 \)) have to fulfill the following equations:

\[
\begin{align*}
\text{Im} \left[ (U^\dagger \dot{U})_{11} \right] + \sum_\alpha |U_{\alpha 1}|^2 \dot{\phi}_\alpha + \dot{\rho} &= 0, \\
\text{Im} \left[ (U^\dagger \dot{U})_{22} \right] + \sum_\alpha |U_{\alpha 2}|^2 \dot{\phi}_\alpha + \dot{\sigma} &= 0, \\
\text{Im} \left[ (U^\dagger \dot{U})_{33} \right] + \sum_\alpha |U_{\alpha 3}|^2 \dot{\phi}_\alpha &= 0, \\
\end{align*}
\]

(5)

and

\[
\begin{align*}
\text{Re} \left[ (U^\dagger \dot{U})_{12} \right] - \sum_\alpha T^\alpha_{12} \dot{\phi}_\alpha &= -\frac{C_s y_t^2}{32\pi^2} \left\{ \frac{\tilde{c}_{12}}{s_{13}} \left[ s_{2(\rho-\sigma)}T^r_{12} + c_{2(\rho-\sigma)}R^r_{12} \right] + \tilde{c}_{12} T^r_{12} \right\}, \\
\text{Im} \left[ (U^\dagger \dot{U})_{12} \right] + \sum_\alpha R^\alpha_{12} \dot{\phi}_\alpha &= \frac{C_s y_t^2}{32\pi^2} \left\{ \frac{\tilde{c}_{12}}{s_{13}} \left[ s_{2(\rho-\sigma)}R^r_{12} - c_{2(\rho-\sigma)}T^r_{12} \right] + \tilde{c}_{12} T^r_{12} \right\}, \\
\text{Re} \left[ (U^\dagger \dot{U})_{13} \right] - \sum_\alpha T^\alpha_{13} \dot{\phi}_\alpha &= -\frac{C_s y_t^2}{32\pi^2} \left\{ \frac{\tilde{c}_{12}}{s_{13}} \left[ s_{2(\rho-\sigma)}T^r_{13} + c_{2(\rho-\sigma)}R^r_{13} \right] + \tilde{c}_{12} T^r_{13} \right\}, \\
\text{Im} \left[ (U^\dagger \dot{U})_{13} \right] + \sum_\alpha R^\alpha_{13} \dot{\phi}_\alpha &= \frac{C_s y_t^2}{32\pi^2} \left\{ \frac{\tilde{c}_{12}}{s_{13}} \left[ s_{2(\rho-\sigma)}R^r_{13} - c_{2(\rho-\sigma)}T^r_{13} \right] + \tilde{c}_{12} T^r_{13} \right\}, \\
\text{Re} \left[ (U^\dagger \dot{U})_{23} \right] - \sum_\alpha T^\alpha_{23} \dot{\phi}_\alpha &= -\frac{C_s y_t^2}{32\pi^2} \left\{ \frac{\tilde{c}_{23}}{s_{13}} \left[ s_{2(\sigma-\delta)}T^r_{23} + c_{2(\sigma-\delta)}R^r_{23} \right] + \tilde{c}_{23} T^r_{23} \right\}, \\
\text{Im} \left[ (U^\dagger \dot{U})_{23} \right] + \sum_\alpha R^\alpha_{23} \dot{\phi}_\alpha &= \frac{C_s y_t^2}{32\pi^2} \left\{ \frac{\tilde{c}_{23}}{s_{13}} \left[ s_{2(\sigma-\delta)}R^r_{23} - c_{2(\sigma-\delta)}T^r_{23} \right] + \tilde{c}_{23} T^r_{23} \right\},
\end{align*}
\]

(6)

where \( \tilde{c}_{ij} \equiv 4k_i k_j/(\kappa_i^2 - \kappa_j^2) \) and \( \tilde{c}_{ij} \equiv 2(\kappa_i^2 + \kappa_j^2)/(\kappa_i^2 - \kappa_j^2) \) have been defined, and the overdot refers to the derivative with respect to the running parameter \( t \). In addition, \( R^\alpha_{ij} \equiv \text{Re} \,(U^*_{ai}U_{aj}) \) and \( T^\alpha_{ij} \equiv \text{Im} \,(U^*_{ai}U_{aj}) \). Given the standard parametrization of \( U \) in Eq. (1), the matrix elements of \( U^\dagger \dot{U} \) are shown in Table II, while the coefficients \( R^\alpha_{ij} \) and \( T^\alpha_{ij} \) are given in Table III. Note that Eqs. (5) and (6) form an array of differential equations linear in \( \{ \hat{\theta}_{12}, \hat{\theta}_{13}, \hat{\theta}_{23}, \hat{\delta}, \hat{\rho}, \hat{\sigma}, \hat{\phi}_{\alpha i}, \hat{\phi}_{\mu}, \hat{\phi}_{\tau} \} \), which can be explicitly solved. As a result, the RGE of \( \delta \) can be approximately written as

\[
\dot{\delta} \approx \frac{C_s y_t^2}{32\pi^2} \left\{ \frac{s_{12}c_{12}s_{23}c_{23}}{s_{13}} \left[ s_\delta (\tilde{c}_{32} - \tilde{c}_{31}) + (s_{(\delta+2\sigma)} \tilde{c}_{32} - s_{(\delta+2\rho)} \tilde{c}_{31}) \right] \\
- \tilde{c}_{21} s_{23} s_{2(\rho-\sigma)} - (c^2_{23} - s^2_{23}) s_{2(\rho-\sigma)} s_{23} + s_{2(\delta+2\rho)} s_{23} + s_{2(\delta+\rho)} s_{12} s_{23} \right\} .
\]

(7)
Since the last two terms in the third line of Eq. (7) are proportional to $s_{13}$ and $s_{13}^2$, we have neglected the terms further suppressed by $O(\Delta m_{31}^2/|\Delta m_{31}^2|)$. If neutrino masses are nearly degenerate $m_i^2 \gg |\Delta m_{31}^2| \gg m_i^2 - m_j^2$, and $\hat{\xi}_{ij} \approx \hat{s}_{ij} \approx 4m_i^2/(m_i^2 - m_j^2)$) and $\hat{\xi}_{21} \gg |\hat{\xi}_{32}|, |\hat{\xi}_{31}| \gg 1$, and thus, the RG evolution of $\delta$ could be significant. To next-to-leading order, Eq. (7) approximates to

$$\dot{\delta} \approx -\frac{C_\kappa}{8\pi^2} \frac{m_1}{m_{21}^2} \left\{ \frac{s_{23}^2 s_{2(\rho - \sigma)}}{s_{12} c_{12} s_{13}} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2} s_{12}^2 c_{12}^2 c_2^2 (\delta + \rho + \sigma)(\rho - \sigma) \right\}, \quad (8)$$

where we have taken $m_1$ as the absolute neutrino mass and ignored the difference between $\Delta m_{21}^2$ and $\Delta m_{32}^2$. Some comments are in order:

- In general, the evolution of $\delta$ is dominated by the leading-order term $-\hat{\xi}_{21} s_{23} s_{2(\rho - \sigma)}$ on the right-hand side of Eq. (7). At higher order, if the terms suppressed by $|\hat{\xi}_{31}|/\hat{\xi}_{21} = \Delta m_{21}^2/|\Delta m_{31}^2| \approx 1/30$ are taken into account, then those by $s_{13}^2 \approx 1/40$ should also be kept for consistency, since they are of the same order of magnitude, as we have done in Eq. (8). The relative error in Eq. (7) is at the level of $s_{13} |\hat{\xi}_{31}|/\hat{\xi}_{21} \approx 0.5 \%$, given the best-fit values of $\theta_{13}$ and neutrino mass-squared differences.

- It is evident from Eq. (7) that the evolution of $\delta$ is entangled with that of three mixing angles and two Majorana CP-violating phases. In particular, it depends crucially on the Majorana phases $\rho$ and $\sigma$. It has been found that the Dirac CP-violating phase $\delta$ can be radiatively generated from $\rho$ and $\sigma$, even if the initial value of $\delta$ is vanishing $[19]$. On the other hand, the RG evolution of $\delta$ becomes negligible when $\rho \approx \sigma$, while the mixing angle $\theta_{12}$ is quite sensitive to the RG effect in this case.

- The RGE’s of $\delta$ in the SM, the MSSM, and the UEDM are given by the same formula in Eq. (7), but with different values of the coefficient $C_\kappa$. We have $C_\kappa^{\text{SM}} = -3/2$ in the SM, while $C_\kappa^{\text{MSSM}} = 1$ in the MSSM and $C_\kappa^{\text{UEDM}} = -3(1 + s)/2$ in the UEDM, respectively. Therefore, given the same Majorana CP-violating phases and leptonic mixing angles, the evolution of $\delta$ in the MSSM will be in the direction opposite to that in the SM and the UEDM.

Finally, we observe from Eq. (5) that the identity $\dot{\phi}_e + \dot{\phi}_\mu + \dot{\phi}_\tau + \dot{\rho} + \dot{\sigma} = 0$ holds in the standard parametrization of $U$. The proof is as follows. Given a general non-singular matrix $X$, whose elements are functions of the running parameter $t$, one can prove that
\[ \frac{d[\det(X)]}{dt} = \det(X) \cdot \text{tr}[X^{-1}(dX/dt)]. \] If we take \( X \) to be a unitary matrix \( U \) with \( \det(U) = 1 \) and \( U^{-1} = U^\dagger \), then \( \text{tr}(U^\dagger U) = 0 \) can be obtained. This observation together with Eq. (5) leads to the identity \( \dot{\phi}_e + \dot{\phi}_\mu + \dot{\phi}_\tau + \dot{\rho} + \dot{\sigma} = 0 \). However, this identity depends on the specific parametrization of \( U \). For instance, if \( \det(U) = e^{-i\phi} \) with \( \phi \) being the Dirac CP-violating phase, then we have \( \dot{\phi} = \dot{\phi}_e + \dot{\phi}_\mu + \dot{\phi}_\tau + \dot{\rho} + \dot{\sigma} \), as shown in Ref. [18].

B. Numerical Results

We proceed in this subsection with the numerical solution to the RGE of the leptonic Dirac CP-violating phase \( \delta \). Since the evolution of \( \delta \) in the SM is negligible even in the case of a nearly-degenerate neutrino mass spectrum, we consider only the MSSM and the UEDM. Note that no approximations to the RGE of \( \delta \) will be made in our numerical calculations. Our numerical results are shown in Fig. 1, and the main points are summarized as follows.

In the MSSM, we have taken two typical values of \( \tan \beta = 10 \) and \( \tan \beta = 30 \) for illustration. In both cases, the absolute neutrino mass \( m_1 = 0.1 \) eV is assumed, which is consistent with the cosmological bound \( m_1 + m_2 + m_3 < 1.3 \) eV (95 \% C.L.) from the WMAP Collaboration [20]. For the initial values of \( \delta \) at the electroweak scale, we have chosen \( \delta = \pi/2, \pi, \) and \( 3\pi/2 \) as typical examples. Since the tau-lepton Yukawa coupling is given by \( y_\tau^2 = m_\tau^2(1 + \tan^2 \beta)/v^2 \) in the MSSM, the evolution of \( \delta \) should be significantly enhanced for a large value of \( \tan \beta \), as shown in the upper plots of Fig. 1. For \( \tan \beta = 30 \), the RG running of \( \delta \) is quite significant. In particular, even if \( \delta = \pi \) is found at the low-energy scale, namely, there is no CP-violating effect in neutrino oscillation experiments, the maximal CP-violating phase \( \delta = \pi/2 \) or \( 3\pi/2 \) can be achieved at the cutoff scale \( \Lambda = 10^{14} \) GeV. In other words, one can change from the scenario with a zero CP-violating phase to that with a maximal CP-violating phase, or vice versa. For \( \tan \beta = 10 \), the radiative correction to \( \delta \) is at most 10 \% even at \( \Lambda = 10^{14} \) GeV.

In the UEDM, we have input two different values of the absolute neutrino mass \( m_1 = 0.1 \) eV and \( m_1 = 0.5 \) eV. As shown in the lower plots of Fig. 1, \( \delta \) is rather stable against radiative corrections for \( m_1 = 0.1 \) eV. Even for \( m_1 = 0.5 \) eV, which is marginally in tension with the cosmological bound, the relative change of \( \delta \) at the cutoff scale \( \Lambda = 3 \times 10^4 \) GeV is not larger than 10 \%. The cutoff scale \( \Lambda = 3 \times 10^4 \) GeV in the UEDM has been chosen to avoid the Landau pole, where the Higgs mass is \( M_H = 125 \) GeV and \( R^{-1} = 10 \) TeV with
FIG. 1: Evolution of $\delta$ for Majorana neutrinos in the MSSM (upper plots) and in the UEDM (lower plots). The initial values $\delta = \pi/2$, $\delta = \pi$, and $\delta = 3\pi/2$ are assumed, while the Majorana CP-violating phases $\rho$ and $\sigma$ are marginalized. The values of $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and $\Delta m_{21}^{2}$, $\Delta m_{31}^{2}$ in the 1$\sigma$ ranges from the global-fit analysis (for $\Delta m_{31}^{2} > 0$) have been used as input [12].

$R$ being the radius of the compactified extra dimension. Since the valid energy range in the UEDM is much smaller than that in the MSSM, the RG running does not develop as much. However, it should be noted that the RG running in UEDM is actually in the form of a power law, and thus can be more significant than in the SM and in the MSSM.

It should also be noted that the Majorana CP-violating phases $\rho$ and $\sigma$ have been marginalized over the range $[0, \pi)$ in our numerical results. If the specific values of $\rho$ and $\sigma$ are chosen, the variation of $\delta$ will be even smaller. Therefore, we conclude that the leptonic Dirac CP-violating phase $\delta$ is stable against radiative corrections in all the models under consideration, except for the MSSM with a large value of $\tan \beta$. In comparison, the Dirac CP-violating phase in the quark sector is stable even in the MSSM with a large value of $\tan \beta$, since the quark mass spectrum is strongly hierarchical.

Now, we turn to the RG running behavior of $\delta$ by taking the global-fit results $\delta = (1.08^{+0.28}_{-0.31}) \pi$ [11] and $\delta = (1.67^{+0.37}_{-0.77}) \pi$ [12] as input. Since the present uncertainty is large,
we will choose the $1\sigma$ range for illustration. In the upper plots of Fig. 2, the allowed regions of $\delta$ at the superhigh-energy scale have been given in the MSSM. In the case of $\tan \beta = 30$, one can observe that $\delta$ is almost arbitrary within $[0, 2\pi)$ due to the large uncertainty of the input, so any predictions for $\delta$ from a high-energy flavor model could be made consistent with the low-energy observations by the RG running. This is true for the global-fit results from both groups [11, 12]. In reality, any observable effects of CP violation should be related to the Jarlskog invariant $J \equiv s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2s_\delta$. Therefore, we also show the RG running of $J$ in the MSSM for $\tan \beta = 10, 30, 50$, in the lower plots of Fig. 2. It can be observed that $J$ at a superhigh-energy scale could be quite different from that at the low-energy scale, in particular for $\tan \beta = 30$ and $\tan \beta = 50$. 

FIG. 2: Allowed values of the leptonic CP-violating phase $\delta$ (upper plots) and the Jarlskog invariant $J$ (lower plots) for Majorana neutrinos at $1\sigma$ C.L. with $\tan \beta = 10$ (dark red or dark gray) and $\tan \beta = 30$ (light red or gray) in the MSSM. The result of $J$ in the MSSM with $\tan \beta = 50$ is also given in the lower plots (yellow or light gray). The global-fit data from Ref. [11] are adopted for the left column, while that from Ref. [12] for the right column.
III. RUNNING OF CP-VIOLATING PHASE: DIRAC NEUTRINOS

The possibility for neutrinos to be Dirac particles has never been experimentally excluded. Moreover, it has been shown that the leptogenesis mechanism responsible for the matter-antimatter asymmetry in our Universe also works well in a different way for Dirac neutrinos \[21\]. Hence, we assume neutrinos to be Dirac particles, and give them masses through the coupling to the Higgs doublet \(- \ell_L Y \nu_R H + \text{h.c.}\) with \(Y\) being the neutrino Yukawa coupling matrix. It is convenient to write the RGE’s of Dirac neutrino parameters as \[22\]

\[
16\pi^2 \frac{d\omega}{dt} = 2\alpha_{\nu} \omega + C_{\nu,l} \left[ \left( Y_l Y_l^\dagger \right) \omega + \omega (Y_l Y_l^\dagger) \right],
\]

where \(\omega \equiv Y_{\nu} Y_{\nu}^\dagger\) has been defined. The RGE’s of \(\kappa\) in the SM and the MSSM take the same form in Eq. (8), but with different coefficients \(\alpha_{\nu}\) and \(C_{\nu,l}\), as given in Appendix B. Since the beta function for Dirac neutrino Yukawa couplings is currently not available in the UEDM, we consider only the SM and the MSSM. Similarly, as in the Majorana neutrino case, we find the RGE for the leptonic Dirac CP-violating phase \(\delta\) in the case of Dirac neutrinos

\[
\dot{\delta} \approx - \frac{C_{\nu,l} y_2^2 s_{23} c_{23} s_{13} s_\delta}{16\pi^2} \frac{s_{12} c_{12}}{s_{13}^2} \left[ \xi_{21} + (c_{12}^2 \xi_{32} - s_{12}^2 \xi_{31}) + \frac{s_{12}^2 c_{12}^2}{s_{13}^2} (\xi_{32} - \xi_{31}) \right],
\]

where \(\xi_{ij} \equiv (m_i^2 + m_j^2)/(m_i^2 - m_j^2)\) has been defined. The relative error in the above equation is at the level of \(s_{13} (\Delta m^2_{21}/|\Delta m^2_{31}|)^2 \sim 10^{-4}\). It is worth mentioning that the last term in Eq. (10) is comparable in magnitude to the second term, since the suppression by a factor of \(\Delta m^2_{21}/|\Delta m^2_{31}|\) is compensated by the enhancement from \(1/s_{13}^2\). Some general comments are in order:

- The evolution of \(\delta\) is proportional to \(s_\delta\) at all orders, so \(\delta\) will be kept unchanged by the RG running if \(s_\delta = 0\), namely, \(\delta = 0\) or \(\delta = \pi\). In other words, if leptonic CP violation is absent at low energies, it will never be generated by RG running. This is quite different from the Majorana case, where \(\delta\) can be radiatively generated via the non-vanishing Majorana CP-violating phases even if \(\delta = 0\) or \(\delta = \pi\) has been used as an initial condition.

- Two qualitative differences between the SM and the MSSM should be noted. First, the tau-Yukawa coupling \(y_\tau^2 = m_\tau^2 (1 + \tan^2 \beta)/v^2\) in the MSSM is significantly enhanced
for a large value of \( \tan \beta \). Hence, the RG effect is more remarkable than that in the SM. Second, the coefficient \( C_{\nu,l} \) takes opposite signs in the SM and in the MSSM, indicating the evolution of \( \delta \) in opposite directions in these two models.

To illustrate the RG running behavior of \( \delta \) in the Dirac neutrino case, we have shown in Fig. 3 two typical examples in the MSSM. In both examples, the initial values of \( \delta \) have been taken to be \( \pi/2 \) and \( 3\pi/2 \), and the absolute neutrino mass is \( m_1 = 0.1 \text{ eV} \). The left plot is for \( \tan \beta = 10 \), while the right for \( \tan \beta = 30 \). Note that the beta function of \( \delta \) is proportional to \( -s_\delta \) in Eq. (9), where \( C_{\nu,l} = 1 \) in the MSSM. Therefore, \( \delta \) increases for \( \delta = 3\pi/2 \), while it decreases for \( \delta = \pi/2 \), as the energy scale evolves towards higher energies. This feature can be clearly observed in Fig. 3. Furthermore, the variation of \( \delta \) at any energy scale is quite small, compared to that in the case of Majorana neutrinos, where the arbitrary Majorana CP-violating phases play an important role in the evolution of \( \delta \). As we have already mentioned, \( \delta \) will be kept unchanged if the initial values lead to \( s_\delta = 0 \), so the trivial cases of \( \delta = 0 \) and \( \delta = \pi \) have not been considered.

Now, we continue with the global-fit results of \( \delta \) in Refs. [11, 12] as initial values. The RG running of \( \delta \) in the MSSM for \( \tan \beta = 10, 30 \) and \( \tan \beta = 50 \) have been shown in the upper and middle plots of Fig. 4, respectively. As before, the absolute neutrino mass \( m_1 = 0.1 \text{ eV} \) is assumed. In the former case, the RG running effects are insignificant, which is in accordance with the results in Fig. 3. In the latter case, however, it is interesting to note that a wide range of values \( \delta \in [0.2\pi, 1.8\pi] \) cannot be reached at the superhigh-energy
FIG. 4: Allowed values of the leptonic Dirac CP-violating phase $\delta$ (upper and middle plots) and the Jarlskog invariant $\mathcal{J}$ (lower plots) for Dirac neutrinos at $1\sigma$ C.L. with $\tan \beta = 10$ (dark red or dark gray), $\tan \beta = 30$ (light red or gray) and $\tan \beta = 50$ (yellow or light gray) in the MSSM. The absolute neutrino mass $m_1 = 0.1$ eV has been assumed. The global-fit data from Ref. [11] are adopted for the left column, while that from Ref. [12] for the right column.

scale $\Lambda = 10^{14}$ GeV, no matter what initial value of $\delta$ is chosen. The reason for this behavior is that the mixing angle $\theta_{13}$ is approaching zero around $\Lambda' = 10^8$ GeV. In the limit of an extremely small value of $\theta_{13}$, Eq. (8) can be written as

$$\dot{\delta} \approx - \frac{y_\tau^2}{16\pi^2} s_{12} c_{12} s_{23} c_{23} s_\delta s_{13}^{-1} (\xi_{32} - \xi_{31}) ,$$

(11)
where \( C_{\nu,l} = 1 \) has been chosen for the MSSM. Therefore, the RG running of \( \delta \) will be rapidly accelerated around \( \Lambda' = 10^8 \) GeV to the large-value region for \( s_\delta < 0 \) (i.e., \( \delta > \pi \)), while to the small-value region for \( s_\delta > 0 \) (i.e., \( \delta < \pi \)). This observation applies also to any initial value of \( \delta \). In fact, we have numerically checked the whole parameter region of \( \delta \in [0,2\pi) \) at low energies, and found that only \([0,0.2\pi]\) and \([1.8\pi,2\pi]\) can be reached at high energies. However, the exact allowed range of \( \delta \) at high-energy scales really depends on the initial values of \( \delta \) and three mixing angles. For \( \delta = \pi \), the RG running of \( \delta \) will be absent, but \( \theta_{13} \) becomes negative above \( \Lambda' = 10^8 \) GeV, so we have to redefine \( \delta \to \delta \pm \pi \) to make \( \theta_{13} \) positive, leading to \( \delta = 0 \) or \( 2\pi \) at high-energy scales. In the lower plots of Fig. 4, the evolution of the Jarlskog invariant \( J \) is shown. Unlike the Dirac CP-violating phase \( \delta \) itself, the physical observable \( J \) evolves smoothly over the whole range of energy scales, as it should. For \( \tan \beta = 50 \), the value of \( |J| \) can initially be as large as 2 \%, it becomes vanishingly small at \( \Lambda = 10^{14} \) GeV. One reason for this is that \( \delta \) shrinks into a small region around \( 0 \) or \( 2\pi \) at the high-energy scale, as indicated in the middle plots of Fig. 4. Obviously, the evolution of the three mixing angles is also relevant here.

**IV. FURTHER DISCUSSIONS**

In Secs. II and III, we have examined the RG running behaviors of the leptonic Dirac CP-violating phase \( \delta \) in the cases of Majorana neutrinos and Dirac neutrinos, respectively. Now, we compare these two cases and summarize the main differences:

- In the Majorana case, the two Majorana CP-violating phases are playing a crucial role in the RG running of \( \delta \). One can start from a CP-conserving scenario with \( \delta = 0 \) or \( \pi \) at the low-energy scale, and end up with a CP-violating scenario even with \( \delta = \pi/2 \) or \( 3\pi/2 \). In the Dirac case, the evolution of \( \delta \) is proportional to \( s_\delta \), so the CP conservation at the low-energy scale definitely implies that CP violation is absent at a superhigh-energy scale.

- The mixing angle \( \theta_{13} \) could approach zero at some high-energy scale \( \Lambda' \) in both cases if a large value of \( \tan \beta \) is assumed in the MSSM. On the other hand, there exist in the RGE's of \( \delta \) some terms inversely proportional to \( s_{13} \). Therefore, the RG running behavior of \( \delta \) will be dramatically changed around \( \Lambda' \). Given the global-fit values of
$\delta$ within the $1\sigma$ range, it turns out that $\delta$ could be arbitrary at the high-energy scale in the Majorana case due to the marginalization over $\rho$ and $\sigma$. In the Dirac case, $\delta$ is found to be in two narrow ranges $[0, 0.2\pi]$ or $[1.8\pi, 2\pi]$ in the MSSM with $\tan \beta = 50$.

However, if a concrete mass model for Majorana neutrinos or Dirac neutrinos is assumed, the RG running of $\delta$ may depend on the model details. In particular, when new particles or interactions come into play at some intermediate energy scale, the RGE’s of the neutrino parameters are completely changed \cite{23}. Hence, we have assumed that this is not the case in the previous discussions, at least below the cutoff scale.

As we have mentioned before, many flavor symmetry models, which are intended for describing the observed leptonic mixing angles, predict the leptonic Dirac CP-violating phase $\delta$. For instance, it has been shown in Ref. \cite{9} that $\delta \approx 2\pi/3$ (or $4\pi/3$) and $\delta \approx \pi/3$ (or $5\pi/3$) for different breaking patterns of the $A_4$ flavor symmetry in the type-I seesaw model, where three heavy right-handed neutrino singlets are introduced to realize the dimension-five Weinberg operator. If the vacuum alignment problem is further solved in the framework of supersymmetry, significant radiative corrections to these theoretical predictions of $\delta$ could be possible. Thus, the leptonic Dirac CP-violating phase to be measured in neutrino oscillation experiments is related by the RG running to the theoretical prediction at the seesaw scale. On the other hand, the CP-violating and out-of-equilibrium decays of the heavy right-handed neutrinos can generate the lepton number asymmetry in the early Universe, which will be converted into the baryon number asymmetry via the SM sphaleron processes. In this case, the leptonic CP violation in neutrino oscillations can be associated with the matter-antimatter asymmetry in our Universe.

V. SUMMARY

Thanks to the recent measurements of $\theta_{13}$ in the Daya Bay and RENO experiments, the discovery of CP violation in neutrino oscillation experiments seems to be promising if the leptonic CP violation really exists and the leptonic Dirac CP-violating phase $\delta$ happens to be far away from 0 or $\pi$. On the other hand, we have already had a preliminary result for the leptonic CP-violating phase $\delta$ from the global-fit analysis of all kinds of neutrino oscillation experiments, namely $\delta = (1.08^{+0.28}_{-0.31}) \pi$ \cite{11} and $\delta = (1.67^{+0.37}_{-0.77}) \pi$ \cite{12}. Therefore, we are well motivated to study the RG running of $\delta$ from the low-energy scale to a superhigh-energy
In the case of Majorana neutrinos, we have introduced the dimension-five Weinberg operator to account for neutrino masses. The RGE of $\delta$ has been derived analytically in great detail for the SM, the MSSM, and the UEDM, and a self-consistent approximation to it has been given as well. By a self-consistent approximation, we mean that the RGE of $\delta$ has been expanded in terms of $s_{13}^2$ and $\Delta m^2_{21}/|\Delta m^2_{31}|$, and all the terms of the same order of magnitude should be preserved. It turns out that $\delta$ is rather stable against radiative corrections in all these models, except for the case of a large $\tan \beta$ in the MSSM (e.g., $\tan \beta = 30$ together with a nearly degenerate neutrino mass spectrum). In this case, the Majorana CP-violating phases play an important role in the evolution of $\delta$ such that a maximal phase $\delta = \pi/2$ or $3\pi/2$ can be radiatively generated at a superhigh-energy scale even if $\delta = \pi$ (i.e., no CP-violating effects in neutrino oscillation experiments) at the low-energy scale. The evolution of $\delta$ and the Jarlskog invariant $J$ have been illustrated by taking the $1\sigma$ global-fit results of $\delta$ as input.

In the case of Dirac neutrinos, we have derived the RGE of $\delta$ in the SM and MSSM, and the self-consistent approximation to it has been made. Note that a nearly degenerate neutrino mass spectrum and the absolute neutrino mass $m_1 = 0.1$ eV are assumed in our analysis. The RG running effect of $\delta$ can be neglected in the SM and in the MSSM with a small $\tan \beta$ (e.g., $\tan \beta \leq 10$). However, $\delta$ can be modified by more than 30% for $\tan \beta = 30$. The evolution of $\delta$ and the Jarlskog invariant $J$ have been examined by inputting the $1\sigma$ global-fit results of $\delta$. In the case of $\tan \beta = 50$, $\delta$ in the range of $[0.2\pi, 1.8\pi]$ is found to be unreachable at $\Lambda = 10^{14}$ GeV, since the mixing angle $\theta_{13}$ approaches zero at some intermediate scale (e.g., $\Lambda' = 10^8$ GeV), which forces $\delta$ to be in a large-value region for $\delta > \pi$ or a small-value region for $\delta < \pi$. At the same time, the Jarlskog invariant $J$ becomes vanishingly small at a superhigh-energy scale.

As we already know some information and will soon learn more about the leptonic Dirac CP-violating phase $\delta$, it is thus meaningful to see how large it will be at a superhigh-energy scale. At such an energy scale, the leptonic Dirac CP-violating phase might be related to the quark Dirac CP-violating phase in a unified flavor model, or to the generation of matter-antimatter asymmetry in our Universe via the leptogenesis mechanism. In any case, the precise determination of $\delta$ in the ongoing and upcoming neutrino oscillation experiments or at a future neutrino factory will shed light on the flavor dynamics at a high-energy scale.
Acknowledgments

H.Z. would like to thank for financial support the Göran Gustafsson Foundation, and for the hospitality the KTH Royal Institute of Technology, where part of this work was performed. This work was supported by the Swedish Research Council (Vetenskapsrådet), contract no. 621-2011-3985 (T.O.), the Max Planck Society through the Strategic Innovation Fund in the project MANITOP (H.Z.), and the Göran Gustafsson Foundation (S.Z.).

Appendix A: RGE’s for Majorana neutrinos

1. The SM

In the SM extended with the dimension-five Weinberg operator, the RGE for \( \kappa \) has already been given in Eq. (3), while those for the Yukawa coupling matrices \( Y_f \) of charged fermions (i.e., \( f = l \) for charged leptons, \( f = u \) for up-type quarks and \( f = d \) for down-type quarks) can be written as

\[
16\pi^2 \frac{dY_f}{dt} = \left[ \alpha_{f,SM}^{SM} + C_{f,SM}^{SM} \left( Y_f Y_f^\dagger \right) \right] Y_f, \\
16\pi^2 \frac{dY_u}{dt} = \left[ \alpha_u^{SM} + C_{u,u,SM}^{SM} \left( Y_u Y_u^\dagger \right) + C_{u,d,SM}^{SM} \left( Y_d Y_d^\dagger \right) \right] Y_u, \\
16\pi^2 \frac{dY_d}{dt} = \left[ \alpha_d^{SM} + C_{d,u,SM}^{SM} \left( Y_u Y_u^\dagger \right) + C_{d,d,SM}^{SM} \left( Y_d Y_d^\dagger \right) \right] Y_d. 
\]  
(A1)

The relevant coefficients in Eqs. (3) and (A1) are \( C_{\kappa}^{SM} = C_{u,d}^{SM} = C_{d,u}^{SM} = -3/2, C_{l,l}^{SM} = C_{u,u}^{SM} = C_{d,d}^{SM} = +3/2, \) and

\[
\begin{align*}
\alpha_{\kappa}^{SM} &= -3g_2^2 + \lambda + 2T_{M}^{SM}, \\
\alpha_{l}^{SM} &= -\frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + T_{M}^{SM}, \\
\alpha_{u}^{SM} &= -\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 + 8g_3^2 + T_{M}^{SM}, \\
\alpha_{d}^{SM} &= -\frac{1}{3}g_1 - \frac{9}{4}g_2^2 + 8g_3^2 + T_{M}^{SM}. 
\end{align*}
\]  
(A2)
with $T_{SM}^M \equiv \operatorname{tr} \left[ 3 \left( Y_u^T Y_u' \right) + 3 \left( Y_d^T Y_d' \right) + \left( Y_l^T Y_l' \right) \right]$. The RGE’s for the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings $g_3$, $g_2$, and $g_1$ are given by

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 \quad (A3)$$

with $(b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7)$. The quartic coupling $\lambda$ of the Higgs field appears in the RGE of $\kappa$, which affects the evolution of absolute neutrino masses. It should satisfy the following RGE

$$16\pi^2 \frac{d\lambda}{dt} = 6\lambda^2 - 3\lambda \left( \frac{3}{5} g_1^2 + 3g_2^2 \right) + \frac{3}{2} \left( \frac{9}{25} g_1^2 + \frac{6}{5} g_1^2 g_2^2 + 3g_2^2 \right)$$

$$+ 4\lambda T_{SM}^M - 8 \operatorname{tr} \left[ 3 \left( Y_u^T Y_u' \right)^2 + 3 \left( Y_d^T Y_d' \right)^2 + \left( Y_l^T Y_l' \right)^2 \right] . \quad (A4)$$

It is worth mentioning that if the experimental uncertainties of the top quark mass $M_t$ and the strong coupling $\alpha_s$ are taken into account, the SM vacuum could be stable up to the Planck scale $\Lambda_{Pl} = 1.2 \times 10^{19}$ GeV [24], even for a Higgs mass $M_H = 125$ GeV indicated by the recent results of the ATLAS and CMS experiments.

2. The MSSM

In the MSSM, the RGE’s in Eqs. (3) and (A1) are still applicable, but the relevant flavor-universal coefficients are as follows: $C_{\kappa}^{\text{MSSM}} = C_{u,d}^{\text{MSSM}} = C_{d,u}^{\text{MSSM}} = 1$, $C_{l,d}^{\text{MSSM}} = C_{u,u}^{\text{MSSM}} = C_{d,d}^{\text{MSSM}} = 3$, and

$$\alpha_{\kappa}^{\text{MSSM}} = -\frac{6}{5} g_1^2 - 6g_2^2 + 6 \operatorname{tr} \left( Y_u^T Y_u' \right) ,$$

$$\alpha_l^{\text{MSSM}} = -\frac{9}{5} g_1^2 - 3g_2^2 + \operatorname{tr} \left[ 3 \left( Y_d^T Y_d' \right) + \left( Y_l^T Y_l' \right) \right] ,$$

$$\alpha_u^{\text{MSSM}} = \frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 + 36 \operatorname{tr} \left( Y_u^T Y_u' \right) ,$$

$$\alpha_d^{\text{MSSM}} = -\frac{7}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 + \operatorname{tr} \left[ 3 \left( Y_d^T Y_d' \right) + \left( Y_l^T Y_l' \right) \right] . \quad (A5)$$

The RGE’s for the gauge couplings are given in Eq. (A3), but with $(b_1^{\text{MSSM}}, b_2^{\text{MSSM}}, b_3^{\text{MSSM}}) = (33/5, 1, -3)$ in the beta functions. As we can see from the RGE of $\kappa$, the running neutrino parameters are determined by the charged-lepton Yukawa coupling matrix $Y_l$, especially the
tau-lepton Yukawa coupling $y_\tau^2 = m_\tau^2 (1 + \tan^2 \beta)/v^2$, which could significantly be enhanced for a large value of $\tan \beta$. Such a unique feature can make the RG running of leptonic mixing parameters remarkable in the MSSM.

3. The UEDM

In the UEDM, all the SM fields are promoted to a higher-dimensional spacetime, so every SM particle is accompanied by a tower of Kaluza–Klein (KK) modes. In the simplest UEDM with only one extra spatial dimension, which is compactified on an $S^1/Z_2$ orbifold with radius $R$, the KK parity defined as $(-1)^n$ for the $n$-th KK mode is conserved after compactification. The mass scale of the first excited KK mode, i.e., $\mu_0 \equiv R^{-1}$, has been constrained to be larger than about 300 GeV.

If we extend the UEDM by an effective operator $(\bar{\ell} H) \cdot \hat{\kappa} \cdot (H^T \ell^C)/2$ to accommodate Majorana neutrino masses, just as in Eq. (1), then the effective Majorana neutrino mass matrix after electroweak symmetry breaking is $M_\nu = \kappa v^2$ with $\kappa = \hat{\kappa}/(\pi R)$. The RGE of $\kappa$ now receives contributions from the KK modes, which are excited at the energy scale of interest. More explicitly, the RGE’s for $\kappa$ and the Yukawa coupling matrices of the charged fermions are also given by Eqs. (3) and (A1), but with the following coefficients

$$
\alpha_{\kappa}^{\text{UEDM}} = \alpha_{\kappa}^{\text{SM}} + s \left( -\frac{1}{4} g_1^2 - \frac{11}{4} g_2^2 + \lambda + 4 T_{\text{M}}^{\text{SM}} \right),
$$

$$
\alpha_{l}^{\text{UEDM}} = \alpha_{l}^{\text{SM}} + s \left( -\frac{33}{8} g_1^2 - \frac{15}{8} g_2^2 + 2 T_{\text{M}}^{\text{SM}} \right),
$$

$$
\alpha_{u}^{\text{UEDM}} = \alpha_{u}^{\text{SM}} + s \left( -\frac{101}{72} g_1^2 - \frac{15}{8} g_2^2 - \frac{28}{3} g_3^2 + 2 T_{\text{M}}^{\text{SM}} \right),
$$

$$
\alpha_{d}^{\text{UEDM}} = \alpha_{d}^{\text{SM}} + s \left( -\frac{17}{72} g_1^2 - \frac{15}{8} g_2^2 - \frac{28}{3} g_3^2 + 2 T_{\text{M}}^{\text{SM}} \right),
$$

(A6)

and $C_x^{\text{UEDM}} = C_x^{\text{SM}} (1+s)$ with “x” being any relevant subscript. Note that $s \equiv [\mu/\mu_0]$ counts the number of excited KK modes for a given energy scale $\mu$. In addition, the coefficients in the beta functions of gauge couplings turn out to be

$$
b_{1}^{\text{UEDM}} = b_{1}^{\text{SM}} + \frac{27}{2} s , \quad b_{2}^{\text{UEDM}} = b_{2}^{\text{SM}} + \frac{7}{6} s , \quad b_{3}^{\text{UEDM}} = b_{3}^{\text{SM}} - \frac{5}{2} s .
$$

(A7)

Finally, the RGE for the quartic Higgs coupling $\lambda$ is quite relevant in the UEDM, as in the
SM case. It has been found to be $25$

$$16\pi^2 \frac{d\lambda}{dt} = 6(1 + s)\lambda^2 - 3(1 + s)\lambda \left( \frac{3}{5} g_1^2 + 3 g_2^2 \right) + 3 \left(1 + \frac{4}{3} s\right) \left( \frac{9}{25} g_1^4 + \frac{6}{5} g_1 g_2^2 + 3 g_4^2 \right)$$

$$+ 4(1 + 2s)\lambda T_{SM}^D - 8(1 + 2s) \text{tr} \left[ 3 \left( Y_u Y_u^\dagger \right)^2 + 3 \left( Y_d Y_d^\dagger \right)^2 + \left( Y_l Y_l^\dagger \right)^2 \right].$$  \hspace{1cm} (A8)

**Appendix B: RGE’s for Dirac neutrinos**

If the SM is extended with three right-handed neutrino singlets, then neutrinos acquire Dirac masses in the same way as the charged leptons and quarks do. At one-loop level, the RGE’s of the fermion Yukawa coupling matrices read $22$:

$$16\pi^2 \frac{dY_\nu}{dt} = \left[ \alpha_{\nu}^{SM} + C_{\nu,\nu}^{SM} \left( Y_\nu Y_\nu^\dagger \right) + C_{\nu,l}^{SM} \left( Y_l Y_l^\dagger \right) \right] Y_\nu,$$

$$16\pi^2 \frac{dY_l}{dt} = \left[ \alpha_{l}^{SM} + C_{l,\nu}^{SM} \left( Y_\nu Y_\nu^\dagger \right) + C_{l,l}^{SM} \left( Y_l Y_l^\dagger \right) \right] Y_l,$$

$$16\pi^2 \frac{dY_u}{dt} = \left[ \alpha_{u}^{SM} + C_{u,u}^{SM} \left( Y_u Y_u^\dagger \right) + C_{u,d}^{SM} \left( Y_d Y_d^\dagger \right) \right] Y_u,$$

$$16\pi^2 \frac{dY_d}{dt} = \left[ \alpha_{d}^{SM} + C_{d,u}^{SM} \left( Y_u Y_u^\dagger \right) + C_{d,d}^{SM} \left( Y_d Y_d^\dagger \right) \right] Y_d,$$ \hspace{1cm} (B1)

where $C_{f,g}^{SM} = +3/2$ (for $f = g$) and $-3/2$ (for $f \neq g$), and

$$\alpha_{\nu}^{SM} = -\frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 + T_D^{SM},$$

$$\alpha_{l}^{SM} = -\frac{9}{4} g_1^2 - \frac{9}{4} g_2^2 + T_D^{SM},$$

$$\alpha_{u}^{SM} = -\frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 + T_D^{SM},$$

$$\alpha_{d}^{SM} = -\frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 + T_D^{SM}$$ \hspace{1cm} (B2)

with $T_D^{SM} \equiv \text{tr} \left[ 3 \left( Y_u Y_u^\dagger \right) + 3 \left( Y_d Y_d^\dagger \right) + \left( Y_l Y_l^\dagger \right) \right]$. The RGE’s of fermion Yukawa coupling matrices are the same as in Eq. (B1) for the MSSM, but with different coefficients,
namely $C_{f,g}^{\text{MSSM}} = +3$ (for $f = g$) and $+1$ (for $f \neq g$), and

$$
\begin{align*}
\alpha_\nu^{\text{MSSM}} &= - \frac{3}{5} g_1^2 - 3 g_2^2 + \text{tr} \left[ 3 \left( Y_u Y_u^\dagger + (Y_e Y_e^\dagger) \right) \right], \\
\alpha_l^{\text{MSSM}} &= - \frac{9}{5} g_1^2 - 3 g_2^2 + \text{tr} \left[ 3 \left( Y_l Y_l^\dagger + (Y_e Y_e^\dagger) \right) \right], \\
\alpha_u^{\text{MSSM}} &= - \frac{13}{15} g_1^2 - 3 g_2^2 - \frac{16}{3} g_3^2 + \text{tr} \left[ 3 \left( Y_u Y_u^\dagger + (Y_e Y_e^\dagger) \right) \right], \\
\alpha_d^{\text{MSSM}} &= - \frac{7}{15} g_1^2 - 3 g_2^2 - \frac{16}{3} g_3^2 + \text{tr} \left[ 3 \left( Y_d Y_d^\dagger + (Y_l Y_l^\dagger) \right) \right].
\end{align*}
$$

(B3)

The RGE’s of three gauge couplings $g_1$, $g_2$, and $g_3$ are the same as those in the case of Majorana neutrinos [see Eq. (A3)].

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