A geomechanics classification for the rating of railroad subgrade performance

António Gomes Correia1 • Ana Ramos1

Abstract The type of subgrade of a railroad foundation is vital to the overall performance of the track structure. With the train speed and tonnage increase, as well as environmental changes, the evaluation and influence of subgrade are even more paramount in the railroad track structure performance. A geomechanics classification for subgrade is proposed coupling the stiffness (resilient modulus) and permanent deformation behaviour evaluated by means of repeated triaxial loading tests. This classification covers from fine- to coarse-grained soils, grouped by UIC and ASTM. For this achievement, we first summarize the main models for estimating resilient modulus and permanent deformation, including the evaluation of their robustness and their sensitivity to mechanical and environmental parameters. This is followed by the procedure required to arrive at the geomechanical classification and rating, as well as a discussion of the influence of environmental factors. This work is the first attempt to obtain a new geomechanical classification that can be a useful tool in the evaluation and modelling of the foundation of railway structures.

Keywords Subgrade • Resilient modulus • Permanent deformation • Geomechanical classification

Nomenclature

| Symbol | Description |
|--------|-------------|
| $\gamma$ | Dry unit weight |
| $S$ | Degree of saturation |
| $a$ | Initial tangent modulus |
| $q_u$ | Unconfined compressive strength |
| $\%$ finer #200 | Percent passing #200 sieve |
| LL | Liquid limit |
| $\%$ clay | Percent finer than 0.002 mm |
| $W_L$ | Liquid limit |
| $W_p$ | Plastic limit |
| IP | Index plasticity |
| $W$ | Moisture content |
| $W_o$ | Optimum moisture content |
| $C_u$ | Coefficient of uniformity |
| $C_c$ | Coefficient of gradation |
| $D_n$ | Diameter corresponding to $n\%$ of passed material in a grain size distribution curve |
| CBR | California bearing ratio |

1 Introduction

The understanding and knowledge about the deformation and failure mechanisms of the geomaterials under repeated loadings are extremely important for the proper design and maintenance planning in the railway structures [1–4]. Typically, the subgrade layer of railway track is composed of geomaterials (mainly coarse-grained soils due to its better performance when compared to the fine-grained soils) and presents two types of deformation when submitted to cyclic loads: the recoverable (quasi-elastic, resilient or reversible) and permanent (or plastic)
deformations, which have significant importance in the long-term performance of the substructure [5–7]. Regarding the resilient deformation, the concepts were introduced by AASHTO in 1986 [8]. This property is often used to characterize the materials in several railway layers [9–11]. About the permanent deformation, an accurate estimation of the amount of cumulative settlement will benefit railway structures to avoid a mediocre performance and consequently higher annual maintenance costs [12–14]. Overall, resilient and permanent deformations are both important parameters for the design of railway structures [15–18]. However, the complexity and time-consuming of laboratory tests, mainly for permanent deformation evaluation, conduct to the development of predictive models based on the available data from the existing laboratory and/or field results.

In this line, one of the main objectives of this work is to review the available resilient and permanent deformation models in the scope of the geomaterials published in the specialized bibliography considering the information about the conditions for its development (properties of the materials tested, compaction tests, degree of compaction, moisture content, etc.) and the main variables/factors that can influence the response of the material. This means that this analysis also includes external factors as the physical state of the soil, which is often difficult to control because it depends on other environmental aspects. A first attempt was already performed for permanent deformation models in the work developed by [19]. Consequently, this work intends to complement it with the resilient deformation models, including comparisons among some of the selected models that are based on different materials with different classifications (UIC and ASTM), properties, granulometry and physical states. By coupling both resilient- and permanent-based deformations, we rate the geomaterial classifications according to the predicted resilient and permanent deformation performances, which is believed to be a helpful tool in the design of the railway structures. It is also noted that this rating should be interpreted under normalized conditions because it depends on several properties and soil state conditions. Based on these results, a novel geomechanical classification for track subgrade/formation that couples the resilient modulus and the permanent deformation is purposed using a similar approach developed by [20].

2 Influence of subgrade in design and performance of railroad track structures

The quality and the support given by the subgrade are important aspects in the performance of the railroad track structure. The subgrade is an integrant component of the track structures (ballasted and ballastless tracks), and its properties are important in the performance of track and track quality. Indeed, this is why subgrade performance indicators are important and should be implemented into the track designs and assessments. Thus, there are numerous means of measuring and predicting subgrade performance, which include the laboratory and in situ tests. The in situ tests can be advantageous since the results are representative of the real conditions and some can be obtained quickly and more cost-effective. However, laboratory tests could be also very interesting and necessary to carry out parametric studies considering variations occurring during the service life of the structure. Indeed, recently, new methods have been developed to determine the resilient modulus, such as the performance of the cyclic lightweight deflectometer test [21].

Thus, despite the importance of the subgrade and its significant impact on the track quality and required maintenance, the evaluation of the subgrade is not straightforward since there is an extensive list of factors that can affect its short- and long-term performance: the moisture content and/or, suction, and/or degree of saturation, temperature (freezing and thawing), shear strength, stiffness parameters, consolidation, etc. Indeed, recent studies show the influence of soil suction on the deformation characteristics of railway formation materials [22]. Furthermore, other variables can affect the performance of the subgrade related to the train type (passenger or freight), axle load, train speed, train configuration, drainage, rail condition, tie spacing, and wheel condition [23–25]. Thus, these factors added to the environmental factors can affect the performance of the subgrade and also the global performance of the track structure. Indeed, the poor performance of the subgrade can lead to some issues in the railroad track structure: ballast fouling, ballast pockets, pumping of soil fines through the ballast, and slope stability failure. Thus, there are two main types of subgrade failure (mostly related to the fine subgrade soils): subgrade progressive shear failure and excessive subgrade plastic deformation which are well documented [15, 17, 26]. More recent works in Refs. [27] and [28] emphasize that low- and medium-plasticity soils can also be subjected to different types of failure such as fluidization and mud pumping due to excessive increase in axle loads of freight trains in recent years. Nevertheless, these phenomena and many others like liquefaction (large displacement caused by repeated loading, saturated silt, and fine sand), massive shear failure (slope stability—this failure is associated with the weight of the train, inadequate soil strain and increase of water content), consolidation settlement (the static soil stress increased due to the embankment weight and saturated fine-grained soils), frost action (the track become rough caused by periodic freezing), swelling/shrinkage,
slope erosion and soil collapse caused by ground settlement are out of scope of this paper.

The design of the subgrade (design period corresponds to 50–100 years) is defined according to the type of traffic, bearing capacity of the subgrade, configuration of the track, climatic and hydro-geological conditions [15, 29, 30]. Therefore, the subgrade should be designed to show a good short- and long-term performance, resisting to failure and excessive deformation, respectively, induced by repeated loads. The short performance is influenced directly by the resilient modulus (that has influence on the stress levels on the subgrade) and by the strength parameters, among other factors. The stress paths and the strength parameters (such as cohesion and friction angle) have a significant effect on the development of the permanent deformation. (This topic will be analysed in more detail further in this work.) Usually, the design of the ballasted track involves the determination of the deformation and stresses at critical locations in the track (sleeper-ballast or ballast-sub-ballast contacts). Posteriorly, the magnitude of the settlements and stresses (induced by the passage of the trains) is compared with allowable values in an iterative process where the dimensions of the sleepers and/or thickness of the granular layers are adjusted. These methodologies are applied by several authors and standards:

- Reference [15] developed a methodology that prevents progressive shear failure and also excessive plastic deformation where the plastic strain ($e_p$) and the cumulative deformation should be limited to 2% and 25 mm, respectively.
- Reference [31] presented a set of references for the design and maintenance of track substructure. According to [31], there are four soil quality classes: QS0 (unsuitable soils), QS1 (poor soils), QS2 (average soils) and QS3 (good soils). This concept will be explored further in this work. From the soils’ classification and thickness of the granular layers, the track is evaluated in three categories: P1, P2 and P3 (mediocre, average and good, respectively).
- British Rail Method and Network Rail Code defined recommendations related to the thickness of the trackbed layers [17].
- West Japan Railways defined a limit value for bearing capacity (288 kPa). Below this value, the improvement of the foundation is required. Thus, the bearing capacity of 288 kPa equates to a compressive strength $\sigma_s$ of approximately 112 kPa assuming a cohesion model plastic solution to a simple strip footing where $\sigma_s = 2.57\sigma_c$.

These methods allow preventing the development of an overstressed subgrade that can present cumulative permanent deformation (formation of ballast pocket) or progressive shear failure (subgrade squeezing) under repeated loads. Indeed, these are common failure modes of typical subgrades composed of fine-grained cohesive soils [15, 26, 30, 32, 33]. These types of problems can lead to an increase in dynamic loads and accelerate the deterioration of the track.

Regarding the ballastless track, the design theories of the ballastless track vary across the world according to the experience and background of each country [34]. Thus, two important parameters should be included in the design of railway tracks related to the short- and long-term performance: the resilient modulus ($M_r$) and the permanent deformation ($e_p$). These two parameters should be analysed together in an integrated approach since, from a simplistic point of view, the stress levels at the rail track subgrade vary with the value of its $M_r$, which have impact on the permanent deformation. These parameters will be analysed together with the UIC classification in this work to purposed a novel geomechanical classification.

### 3 Resilient modulus

The resilient modulus is used to characterize the recoverable, reversible or quasi-elastic behaviour. In the traditional theory of elasticity, the elastic properties (which are material parameters) are defined, mostly, by the Poisson’s ratio ($\nu$) and Young’s modulus ($E$). In the analysis of the response of the material when submitted to cyclic loads, the modulus of the elasticity is replaced by the resilient modulus to consider the nonlinearity (i.e. dependence on stress level) on the performance of the material, as well as the inelastic properties of the materials, i.e. the loading and unloading of the stress–strain curve are not totally overlapped due to the dissipation of the energy, as depicted in Fig. 1.

![Fig. 1 Definition of the recoverable and permanent deformation](https://example.com/fig1.png)
Indeed, in the case of repeated load tests with constant confining pressure, the resilient modulus and the Poisson’s ratio are defined, respectively, by the following expressions:

\[ M_r = \frac{\sigma_d}{\varepsilon_{1r}}, \quad \nu = -\frac{\varepsilon_{1t}}{\varepsilon_{1r}}, \]  

(1)

where \( \sigma_d \) is the deviatoric stress, \( \varepsilon_{1r} \) is the recoverable axial deformation and \( \varepsilon_{1t} \) is the recoverable horizontal deformation.

The test method used to determine the experimental value of the resilient modulus is described in [8] and is based on the application of deviatoric stress under constant confining stress. The response of the materials varies according to its own nature. The cohesive materials are more sensitive to the deviator stress than the confining pressure since the experimental results show that there is a decrease of the resilient modulus with the increase in the deviatoric stress. On the other hand, the granular materials show a different tendency since the resilient modulus increases with confining stress. The more recent models are dependent on important factors regarding environmental aspects such as moisture content, suction and degree of saturation [9, 35–40]. Additionally, recent work by [41] also proposes a methodology to obtain the damping from these type of tests using the standard AASHTOT-307–93 [42].

The computational modelling of resilient behaviour is a very difficult task, due to the necessity of decoding complex behaviours into simple mathematical expressions and procedures for routine analysis.

### 3.1 Mechanistic-empirical modelling approach

The empirical approaches can be used in the design procedures of railway structures. This means that a relationship is established between the design inputs (materials, loads, environment, geometry) and, for example, rail track failures through experience, experimentation or a combination of both. The more complex empirical approaches are based on empirical equations derived from experiments. The relationship between rail track failure and the physical phenomena is described by empirical equations.

The mechanistic-empirical modelling approaches used to determine the resilient modulus are divided according to the type of material tested (granular or cohesive). Furthermore, the models more complex depending on the physical state of the material are also presented. Moreover, in appendix, the state of the art of resilient models developed based on laboratory tests (mostly cyclic triaxial tests) considering all types of materials (such as clays, silts, sands and gravels) is summarized.

#### 3.1.1 Cohesive materials

In the cohesive materials, the fines content and its mineralogy are important factors in the analysis, which means that the resilient modulus should be dependent on the soil physical state. Indeed, in cohesive materials, certain characteristics may have more importance in the value of the resilient modulus, besides the stress state, such as the moisture content, suction and saturation degree. In the work developed by [43], the authors state that the new generation models not only provide a better fit than the older models, but they also provide a reasonable fit to the data that can capture the effects of stress state, soil type, soil structure and the soil physical state quite effectively. Furthermore, according to [44], there are prediction models that combine the effect of the stress state and matric suction on the resilient modulus. These models not only reflect the relationship between the resilient modulus of subgrade soils and the stresses but also the effects of seasonal variation of moisture content on the resilient modulus. The authors also conclude that the degrees of stress and moisture content have a significant impact on the resilient modulus of compacted cohesive soils. Moreover, the results show that the resilient modulus of the tested cohesive soils increases with the increase in effective confining pressure, matric suction, and degree of compaction and decreases with the increase in deviator stress and moisture content. Therefore, lower moisture content concomitantly with a higher degree of compaction is beneficial to the stiffness of the subgrade, mainly in those not sensitive to water content changes.

Considering the extensive bibliography about this topic, several models were developed to describe the resilient modulus and are summarized in Table 1.

Furthermore, it is also important to take into account the type of approach in terms of total and/or effective stresses. In the recent work developed by [52], the developed model can describe the phenomena of modulus development during the cyclic undrained condition and takes into account the actual values of effective stress (\( p' \)), excess pore water pressure, the loading characteristic and the position of the effective stress path to the failure line.

#### 3.1.2 Granular materials

The modelling of the resilient behaviour in the case of granular materials can be performed through two different approaches: simulation of the resilient modulus considering a constant value of the Poisson’s ratio and by separation of the deformation response of the material into the volumetric and shear components, which is more complex [53].
The models expressed by the resilient modulus and constant Poisson’s ratio are very easy to understand, very simple and easy to implement numerically. In fact, initially, in terms of variables, the models were only dependent on the confining stress and material parameters and/or the sum of the principal stresses. Due to the necessity to include the influence of the deviator stress, modified the well-known $k$–$\theta$ model, introducing the influence of the mean and deviatory stress ($p$ and $q$, respectively):

$$E = k_1 \theta^{k_2} q^{k_3},$$

where $\theta$ is the sum of principal stresses, and $k_1$, $k_2$ and $k_3$ are parameters dependent on the state of the soil and its characteristics.

Posteriorly, some models were improved considering, for example, the porosity of the material and variable confining stress. However, some of the previous models show drawbacks concerning the assumption of a constant value of the Poisson’s ratio: despite the good results regarding the axial deformations, it was difficult to simulate correctly the volumetric and radial deformations. Indeed, some authors also suggested that the use of these models could lead to Poisson’s ratios superior to 0.5. However, some of these models are still used and were also improved because of their simplicity.

Nevertheless, in order to solve Poisson’s ratio problem, other models were developed, expressing the resilient behaviour through the shear modulus ($G$) and bulk modulus ($K$). This formulation (in terms of $G$ and $K$) is more comprehensive than the models only characterized by the resilient modulus and Poisson’s ratio since it is dependent on more complex laboratory tests and with a more generic interpretation than a cyclic axial loading.

Indeed, Ref. [48] cited by [60] identifies three important conditions in the application and formulation of these models:

- In each increment of calculus, a linear elastic behaviour is adopted.
- The shear and volumetric components of stress and strain are analysed independently.
- Better adjust to the experimental results, mostly when the solicitation presents a 3D character.

Reference [61] developed a well-known model dependent on bulk modulus and shear modulus:

$$K = \frac{P}{\varepsilon_v},$$

$$G = \frac{3Q}{\varepsilon_q},$$

where $\varepsilon_v$ and $\varepsilon_q$ are respectively the volumetric and shear resilient strains:

$$\varepsilon_v = \frac{1}{K_1}P^n \left[1 - \beta \left(\frac{Q}{P}\right)^\alpha\right],$$

$$\varepsilon_q = \frac{1}{3G_1}P^n \left(\frac{Q}{P}\right),$$

where $K_1$, $\beta$ and $G_1$ are parameters of the model, with $\beta = (1 - n)K_1/6G_1$. In this model, there is a clear division between the volumetric and shear deformation and the values of $K$ and $G$ are stress-dependent ($P$ and $Q$, respectively).

Posteriorly, this model was updated by other authors [53, 62–64] in

| Models          | Expressions                                      | Parameters                                      | Authors                        |
|-----------------|--------------------------------------------------|-------------------------------------------------|--------------------------------|
| Bilinear        | $M = K_1 + K_2 \sigma_d$ when $\sigma_d < \sigma_{di}$ | $K_1, K_2, K_3$ and $K_4$                      | Thompson and Robnett [46]     |
|                 | $M = K_1 + K_2 \sigma_d$ when $\sigma_d > \sigma_{di}$ | $k$ and $n$ are parameters dependent on the type of soil and physical state; the parameter $n$ usually has a negative value | Moossazadeh and Witczak [47] |
| Power           | $M = k \sigma_d^n$                              | $\sigma_d$ is the confining stress              | Brown, Lashine [48]           |
| Power (with the influence of the confining stress for overconsolidated saturated soils) | $M = k \left(\frac{\sigma_d}{\sigma_d^0}\right)^n$ | $\sigma_d^0$ is the confining stress, $\sigma_d$ is the confining stress Brown, Lashine [48] |
| Semi-log        | $M = 10^{k-n \sigma_d}$                         | $k$ and $n$ are empirical parameters            | Fredlund, Bergan [49]         |
| Hyperbolic      | $M = \frac{k + n \sigma_d}{\sigma_d}$           | $M$, in ksi and $\sigma_d$ in psi                | Drumm and Pierce [50]         |
| Octahedral      | $M = k \frac{\sigma_{oct}^n}{\tau_{oct}^n}$     | $\sigma_{oct}$ and $\tau_{oct}$ are the shear and normal octahedral stresses | Shackel [51]                  |

Table 1 Resilient moduli for cohesive materials (adapted from [45])

Rail. Eng. Science (2022) 30(3):323–359 © Springer
order to consider the anisotropy of the material, which is an important characteristic regarding the deposition and compaction conditions. Indeed, [63] and [53] purpose and apply an orthotropic version of Boyce’s model introducing an anisotropic coefficient ($\gamma$).

### 3.2 Models dependent on the physical state—suction and degree of saturation

Recently, some studies were developed in order to include the physical state into the resilient models [9, 35, 36, 65].

In this section, a brief reference is made regarding the models dependent on the suction and degree of saturation. These models were developed based on experimental results and include suction as one of the main parameters. In the case of non-saturated soils, three important variables influence recoverable/reversible behaviour [66]:

- net confining stress: $\sigma_3 - u_a$
- deviatoric stress: $\sigma_1 - \sigma_3$
- suction matrix: $\psi_m = u_a - u_w$,

where $\sigma_3$ is the confining stress, $\sigma_1$ is the axial stress, $u_a$ is the air pressure and $u_w$ is the water pressure.

Based on Uzan’s model, Ref. [67] developed a model that includes the suction in the determination of the resilient modulus:

$$M_t = k_p \left( \frac{\theta_{oat} - 3\Delta u_{w-sat}}{p_a} \right)^{k_2} \left( \frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} \left( \frac{\psi_m - \Delta \psi_m}{p_a} + 1 \right)^{k_4},$$

(7)

where $p_a$ is atmospheric pressure; $\tau_{oct}$ is octahedral shear stress; $\theta_{oat}$ is net bulk stress, $\theta_{oat} = \theta - 3u_a$, and $\theta_{oat}$ is bulk stress; $\Delta \psi_m$ is the variation of the suction matrix regarding the initial suction ($\Delta u_{w-sat} = 0$); and $\Delta u_{w-sat}$ represents the increase of the pore pressure in saturated conditions ($\psi_m = 0$).

Despite the simplicity of the model formulation (which is based on the work developed by [56]), the characterization of the suction increases the complexity and application of these types of models since this is a parameter that is difficult to characterize.

Recent studies show the influence of the water content (with special attention on the role of the suction in the interpretation of the results) and consider the possibility to apply an effective stress approach to characterize the changes in normalized resilient modulus as a function of a single parameter that includes the total stresses and suction [37]. Moreover, some studies are focused on the importance of the degree of saturation (and the controlling in the soil compaction) and its relationship with the soil structure design. In fact, the degree of saturation and also the CBR (California bearing ratio) are parameters that are easy to obtain and easy to use when compared to the suction. The authors conclude that CBR (soaked and unsoaked) and also the elastic shear modulus (representative of the stress–strain behaviour at small strains of the subgrade), unconfined compression strength, cyclic undrained shear strength, among other parameters, of unsaturated soil are controlled by $\rho_a$ and $S_r$ at the end of compaction [39]. More specific studies about compaction conditions and percentage of fines can be found in [68] and [69].

### 3.3 Main differences of the resilient modulus formulation—analysis and comparison

As previously described, the resilient behaviour is dependent on several factors such as the stress level, fines contents, particle shape, grading, density, maximum grain size, aggregate type, moisture content, stress history and number of load cycles. From all of them, two important factors stand out: the applied stress level and moisture content. The stress (in terms of confining stress, sum of principal stresses and deviator stress) is the most important factor in the analysis of the resilient behaviour since its impact on the response of the material is very significant. Indeed, most of the geomaterials (especially the granular materials) show a stress dependency under repeated loads. To replicate this nonlinearity and time dependency, the resilient response of the materials is not solved by the traditional elasticity theory. However, as mentioned previously, there are other formulations to characterize the response of the material such as the replacement of the resilient modulus and Poisson’s ratio by the shear and bulk modulus.

This formulation is simple and based on curve-fitting. Indeed, this approach deals better with the nonlinear response of the geomaterials (mostly in the case of granular materials) from a theoretical point of view, and it is more realistic regarding the physical meaning in a 3D stress regime. However, as mentioned previously, these models have a more complex formulation and the parameterization is also difficult to obtain through the available test data [70].

Another relevant aspect is related to the fact that some of the resilient models only fit the laboratory data used for their own development, which means that the mathematical formulation can only be used in a particular situation.

The models that include the influence of the moisture content and/or degree of saturation describe better the material conditions. In the case of the granular materials, the granulometry and the fines content (in a compacted well-graded material) have a significant influence on the response of the material since the water cannot drain freely since it is kept in the pores of the materials [71]. Indeed, when the saturation is close to the maximum, the resilient behaviour is affected [72]. Nevertheless, these models are more complex since it is difficult to characterize these parameters related to the suction and degree of saturation.
More recently, some resilient models have been developed using machine learning and statistical techniques and optimization tools as described in the work developed by [73] and [74].

Despite the several advantages of the empirical models, the main limitation is related to the confidence in the extrapolation of the analysis beyond the conditions in which they were defined. The tables presented in appendix (Tables 17–21) contain information regarding the type of soils (UIC and ASTM classification), source (authors), the mathematical models and respective variables and empirical constants. This information is quite significant in the modelling of the subgrade of the railway tracks since supports information about the materials used to define the model, as well as its physical conditions such as moisture content and dry density.

4 Permanent deformation

The permanent deformation is induced by repeated traffic loading and may occur in railway structures. The development of permanent deformation is the result of the accumulation of deformation throughout millions of cycles, which is a complex process and depends on several factors such as the number of load cycles ($N$), stress levels, loading history, the effect of the principal stress rotation, moisture content, density, aggregate type and particle size distribution. This development of permanent deformation during $N$ loading cycles can stabilize or, in the worst situation, may lead to the ultimate collapse of the structure (excessive rutting). Regarding the moisture content, despite the extensive research about the influence of this parameter on the reversible deformation, a recent study was developed to study the effects of the compaction moisture content on the plastic behaviour of fine soils [75].

To predict permanent deformation, several approaches were proposed: numerical simulations using elastoplastic models, shakedown theory or mechanistic-empirical permanent deformation models based on laboratory tests such as cyclic triaxial tests or the hollow cylinder tests [19]. Throughout the analyses presented by these authors, more emphasis is given to the mechanistic-empirical permanent deformation models from the laboratory testing until the modelling. In fact, the work done in this paper for the resilient modulus is a mirror of the one for permanent deformation.

5 Geomechanics classification and the rating

5.1 Resilient modulus: parametric study

Considering the extended information presented in Sect. 3 and also the tables presented in appendix, a first attempt was done to compare the resilient modulus of different materials under a certain stress level. This analysis is important to comprehend how the models differ from each other in terms of formulation and the importance of certain variables.

The models were calibrated considering the same reference stress path for all materials during the calibration process. The stress path described by [76] was used. This stress path is characterized by a cyclic deviator stress of 24 kPa and a constant confining stress of 60 kPa. During the cyclic tests, the stress ratio ($\sigma_d/\sigma_c$) was kept constant at 0.4, which is a representative ratio in the subgrade of a rail track full-scale model test. It is noted that other confining stresses from 60 to 210 kPa were also tested.

In this analysis, four models were applied in order to compare the resilient modulus for the coarse-grained soils, as depicted in Table 2. In order to facilitate the interpretation of Table 2, $C$ and $m$ are material constants and $\sigma_v$ is the effective vertical stress (more details in Tables 19 and 20).

The first part of this work is based on the experimental results presented in [80]. The authors studied six granular materials representative of the base and sub-base materials used on flexible pavements. [80] performed rapid shear tests and repeated load tests in order to determine the shear strength parameters (cohesion and friction angle), resilient modulus, rutting potential and moisture susceptibility. The properties of the materials are presented in Table 3.

The constants of each model were determined by regression analyses. The values of these constants reflect the material properties as well as the loading cycle applied in each test that may vary from author to author. For example, in the work developed by [80], the specimens were conditioned for 1000 load repetitions while in the

| Table 2 | Models considered in the analysis of the coarse-grained soils |
|---------|---------------------------------------------------------------|
| Refs.   | Models                                                       |
| [55, 77, 78] | $M_t = k_1 \left( \frac{\sigma_v}{\sigma_0} \right)^{k_1}$ |
| [56]   | $M_t = k_1 P_0 \left( \frac{\theta}{P_0} \right)^{k_2} \left( \frac{\sigma_v}{P_0} \right)^{k_3}$ |
| [79]   | $E_v = C \sigma_v^m$                                         |
| [59]   | $M_t = k_1 P_0 \left( \frac{\theta}{P_0} \right)^{k_2} \left( \frac{\sigma_v}{P_0} \right)^{k_3}$ |
work developed by [9], the authors follow the procedures described in AASHTO T307-99. However, for practical purposes, the conditioning adopted for resilient modulus determination following the standard procedures (AASHTO T307-99, [40]) intends to assure that the number of cycles during resilient modulus determination will not affect the results. The results of the four models and the two materials are presented in Table 4. It is important to refer that for material 2, due to the lack of data, it was not possible to present the regression results for the model developed by [79] and MEPDG model. Analysing Table 4, in general, the models present similar r-squared values. Therefore, considering the obtained constants and the stress path selected, the models were compared in terms of the resilient modulus, as depicted in Table 5. It can be observed that the models present similar results. Material 2 presents an inferior resilient modulus when compared to the well-graded gravel, as expected. Regarding the material classified as QS3-GW, it is evident the close values between the Uzan’s model, $k_1 \theta^2$ and MEPDG model.

After the analysis of the coarse-grained soils, the fine-grained soils were evaluated. In the case of the fine-grained soils, four models (Table 6) and two materials (Table 7) were compared. The materials were tested through the triaxial test and both materials were compacted at optimum water content and 2% above the optimum. The measured resilient modulus is based on the work developed by [9].

### Table 3 Properties of the materials (coarse-grained soils)

| No. | UIC and ASTM classification | Materials | Properties | Observations |
|-----|------------------------------|-----------|------------|--------------|
| 1   | QS3 GW                       | Well-graded gravel | $C_u = 27$; $C_c = 2$ (PI$_{min} = NP$; PI$_{max} = 6$) $W_L,\max = 25$ $W = 6.3\%$ $W_{OMC} = 6.8\%$ | Granular material used as based and sub-base in the flexible pavement sections; |
| 2   | QS2/QS3 SW/SM-ML             | Sand      | $C_u = 20$; $C_c = 0.8–1.1$ (PI$_{min} = NP$; PI$_{max} = 6$) $W_L,\max = 25$ $W = 7.7\%$ $W_{OMC} = 7.7\%$ | The target moisture content and densities were selected based on AASHTO T99 test results and field-measured values [80] |

### Table 4 Summary of the values of the regression analysis based on experimental results

| Models | Material classification | Parameters | Regression analysis $R^2$ |
|--------|-------------------------|------------|---------------------------|
| $M_t = k_1 \theta^2$ | Well-graded gravel | $k_1 = 5176$; $k_2 = 0.64$ | 0.997 |
|        | QS3-GW                  |            |                           |
|        | Sand                    | $k_1 = 36,942$; $k_2 = 0.32$ | 0.933 |
| $M_t = k_1 \frac{p_0}{p_0^0} \theta^2 \left( \frac{q}{p_0^0} \right)^{k_3}$ | QS3-GW | $k_1 = 5306$; $k_2 = 0.59$; $k_3 = 0.05$ | 0.998 |
|        | Sand                    | $k_1 = 34,336$; $k_2 = 0.45$; $k_3 = -0.15$ | 0.942 |
| $M_t = k_1 \frac{p_0}{p_0^0} \left( \frac{\tau_{oc1}}{p_0^0} + 1 \right)^{k_3}$ | QS3-GW | $k_1 = 1.02$; $k_2 = 0.62$; $k_3 = -0.01$ | 0.997 |
| $E_v = C \sigma_v^m$ | Well-graded gravel | $C = 6768$; $m = 0.65$ | 0.988 |
|        | QS3-GW                  |            |                           |

[330] A. Gomes Correia, A. Ramos
Considering the results obtained by [9] regarding the measured resilient modulus, the constants of the parameters were determined through regression analyses (Table 8). However, it is important to refer that the MEPDG model is defined in terms of total stresses, which means that the regression analysis is performed for each material on both moisture contents (opt and opt + 2%). The remaining models are dependent on effective stresses, which means that the regression analysis is performed considering the whole set of moisture content (opt and opt + 2%). This type of information allows analyzing the sensibility of the resilient modulus regarding the moisture content. The models developed by [9, 35] and [81] are dependent on the $\psi_m$ and $\chi_m$ parameters, which means that the results are presented considering $M_r^{\text{opt}}$ and $M_r^{\text{opt+2\%}}$, although the regression is performed with the whole set (opt and opt + 2%).

The model developed by [81] is simpler than the remaining models, and the influence of the moisture content is indirectly included in the suction matrix in the initial mean normal stress ($p_0^\prime$). From multiple linear regression analyses, the empirical constants were determined, and the values are presented in Table 8.

From the constant parameters and the selected stress path, the resilient moduli of the materials were found, as depicted in Table 9. Considering the obtained results for the fine- and coarse-grained materials, the MEPDG model [59] will be adopted in the following analysis, despite its complexity when compared to the remaining models. The main reason is related to its universal character since the model can be

### Table 5 Determination of the resilient modulus for the coarse-grained materials

| Models | Material classification | Resilient modulus (MPa) |
|--------|-------------------------|-------------------------|
| $M_i = k_i \theta_i^{\psi_i}$ | Well-graded gravel QS3-GW | 156 |
| $M_i = k_i p_0 \left( \frac{\theta}{p_0} \right)^{\psi_i} \left( \frac{q}{p_0} \right)^{\chi_i}$ | Sand QS2/QS3-SW/SM-ML | 109 |
| $M_i = k_i P_0 \left( \frac{\theta}{p_0} \right)^{\psi_i} \left( \frac{q}{p_0} \right)^{\chi_i}$ | Well-graded gravel QS3-GW | 143 |
| $M_i = k_i P_0 \left( \frac{\theta}{p_0} \right)^{\psi_i} \left( \frac{q}{p_0} \right)^{\chi_i}$ | Sand QS2/QS3-SW/SM-ML | 131 |
| $M_i = k_i P_a \left( \frac{\theta}{p_0} + \psi_m \right)^{\psi_i} \left( \frac{q}{p_0} + 1 \right)^{\chi_i}$ | Well-graded gravel QS3-GW | 159 |
| $E_v = C \sigma_v^{\psi_i}$ | Well-graded gravel QS3-GW | 118 |

### Table 6 Models selected for the fine-grained soils

| Refs. | Models | Materials |
|-------|--------|-----------|
| [59]  | $M_i = k_i p_0 \left( \frac{\theta}{p_0} \right)^{\psi_i} \left( \frac{q}{p_0} \right)^{\chi_i}$ | Lean clay QS1-CL |
| [35]  | $M_i = k_i (\sigma_v + \psi_m)^{\psi_i}$ | Lean clay QS1-CL-ML |
| [9]   | $M_i = k_i P_a \left( \frac{\theta + \chi_m}{P_a} \right)^{\psi_i} \left( \frac{q}{P_a} + 1 \right)^{\chi_i}$ | Lean clay QS1-CL-ML |
| [81]  | $E_i = A + B p_0 + C \sigma_v^{\psi_i}$ | Silt-lean clay QS1-CL-ML |

### Table 7 Properties of the fine-grained materials

| No. | UIC and ASTM classification | Materials | Properties | Observations |
|-----|------------------------------|-----------|------------|-------------|
| 1   | QS1-CL                       | Lean clay | $W_L = 30.8\%$; $W_p = 18.4\%$; $IP = 12.3\%$; | The effect of the matric suction will be included in the effective stress |
|     | Low plasticity               |           |            |             |
| 2   | QS1-CL-ML                    | Silt-lean clay | $W_L = 27.8\%$; $W_p = 19.8\%$; $IP = 8\%$; | The triaxial tests following the AASHTO T307-99 procedure were conducted on both materials |
|     | Low plasticity               |           |            |             |
Table 8 Regression results for the fine-grained materials

| Models | Material classification | Parameters | Regression analysis $R^2$ |
|--------|-------------------------|------------|---------------------------|
| $M_r = k_1 p_0 \left( \frac{\theta}{p_0} \right)^{k_2} \left( \frac{\tau_{ct1}}{p_0} + 1 \right)^{k_3}$ | QS1-CL | $k_1 = 0.55$ | 0.98 |
| Low plasticity | | $k_2 = 0.15$ | |
| Opt | | $k_3 = -1.82$ | |
| QS1-CL | | $k_1 = 0.35$ | 0.90 |
| Low plasticity | | $k_2 = 0.14$ | |
| opt + 2% | | $k_3 = -1.61$ | |
| QS1-CL-ML | | $k_1 = 0.97$ | 0.96 |
| Low plasticity | | $k_2 = -0.17$ | |
| Opt | | $k_3 = -1.97$ | |
| QS1-CL-ML | | $k_1 = 0.68$ | 0.87 |
| Low plasticity | | $k_2 = -0.10$ | |
| opt + 2% | | $k_3 = -1.13$ | |
| $M_r = k_1 (\sigma_a + \chi \psi_m)^{k_2}$ | QS1-CL | $k_1 = 5.45$ | Difficult adjustment |
| Low plasticity | | $k_2 = 0.36$ | |
| QS1-CL-ML | | $k_1 = 72.11$ | Difficult adjustment |
| Low plasticity | | $k_2 = -0.04$ | |
| $M_r = k_1 p \left( \frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left( \frac{\tau_{ct1}}{p_a} + 1 \right)^{k_3}$ | QS1-CL | $k_1 = 0.24$ | 0.62 |
| Low plasticity | | $k_2 = 0.85$ | |
| | | $k_3 = -2.29$ | |
| QS1-CL-ML | | $k_1 = 0.78$ | 0.61 |
| Low plasticity | | $k_2 = 0.13$ | |
| | | $k_3 = -2.01$ | |
| $E_r = A + B p_0^2 - C q_t$ | QS1-CL | $A = 3499.51$ | 0.91 |
| Low plasticity | | $B = 261.25$ | |
| | | $C = -190.78$ | |
| QS1-CL-ML | | $A = 60,443.76$ | 0.69 |
| Low plasticity | | $B = -167.96$ | |
| | | $C = -474.39$ | |

Table 9 Resilient modulus of the fine-grained materials

| Models | Material classification | Resilient modulus (MPa) |
|--------|-------------------------|------------------------|
| $M_r = k_1 p_0 \left( \frac{\theta}{p_0} \right)^{k_2} \left( \frac{\tau_{ct1}}{p_0} + 1 \right)^{k_3}$ | QS1-CL | 50 (opt) |
| Low plasticity | | 33 (opt + 2%) |
| QS1-CL-ML | | 70 (opt) |
| Low plasticity | | 56 (opt + 2%) |
| $M_r = k_1 (\sigma_a + \chi \psi_m)^{k_2}$ | QS1-CL | Difficult adjustment |
| Low plasticity | | Difficult adjustment |
| QS1-CL-ML | | Difficult adjustment |
| $M_r = k_1 p \left( \frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left( \frac{\tau_{ct1}}{p_a} + 1 \right)^{k_3}$ | QS1-CL | 53 (opt) |
| Low plasticity | | 47 (opt + 2%) |
| QS1-CL-ML | | 73 (opt) |
| Low plasticity | | 72 (opt + 2%) |
| $E_r = A + B p_0^2 - C q_t$ | QS1-CL | 83 (opt) |
| Low plasticity | | 76 (opt + 2%) |
| QS1-CL-ML | | 80 (opt) |
| Low plasticity | | 71 (opt + 2%) |
applied in the prediction of the resilient modulus of all types of geomaterials.

This analysis shows a possible way to rank materials according to the resilient modulus, as depicted in Fig. 2. In the case of the material classified as QS2/QS3-SW/SM-ML, instead of the MEPDG model, the Uzan’s model was used due to the lack of information. (It was not possible to perform a regression analysis.) However, as the previous analysis shows, the MEPDG and Uzan’s models present similar results. All the materials present in Fig. 2 are at optimum moisture content, except the material classified as QS2/QS3 SW/SM-ML. In this case, the moisture content is close to the optimum conditions.

5.2 Permanent deformation: parametric study

The comparison regarding the permanent deformation models was already presented in the work developed by [19]. As in the previous exercise, in this parametric study, the authors attempted to compare different permanent deformation models available for different types of soils considering the empirical permanent deformation model developed by [76]. Thus, the results showed that two soils can be integrated into the same classification (UIC or ASTM), but the laboratory conditions may differ greatly and therefore lead to different results in terms of permanent deformation. The calibration was performed to find the best-fit for the experimental data through the parameters $e_{\pi,0}$, $\alpha$ and $B$ (which correspond to the model constants associated with the material properties). The models were calibrated considering the same stress paths (described by [76]). As in the resilient modulus analysis, the materials selected are representative of different types of materials.

In order to understand better the ranking, the materials selected for the preliminary analysis of the permanent deformation are depicted in Table 10. This table also includes information about the ASTM and UIC classifications. To complete the information about the materials, the physical properties, state conditions and strength properties of the materials are depicted in Tables 11, 12 and 13, which would be extremely important to understand the novel geomechanical classification purposed.

In the work developed by [19], the ranking process is described. The final ranking is depicted in Fig. 3.

6 Geomechanical classification

Considering the results presented in the previous sections, two models were selected to characterize the resilient modulus and permanent deformation of the geomaterials. Indeed, this is an attempt to relate the resilient modulus with the permanent deformation under a certain stress level. As in the previous analysis, the selected stress path is characterized by a cyclic deviator stress equal to 24 kPa and a confining constant stress equal to 60 kPa. The permanent deformation was determined considering a number of load cycles equal to 30,000. The selected materials are

| Refs | Soils | ASTM classification | UIC classification | Observations |
|------|-------|---------------------|--------------------|--------------|
| [76] | Sand  | SP                  | QS2                | Compacted at field conditions and saturated |
|      | Silt  | CL-ML               | QS1                | Compacted at field conditions and saturated |
| [82] | Silty sand (42.2% fines) | SM                 | QS1                | Compacted at optimum compaction—standard proctor |
| [66] | Silty sand (27.4% fines) | SM                 | QS1                | Compacted at optimum compaction—standard proctor |
| [83] | Well-graded Sand | SP                 | QS2                | Compacted at optimum compaction—standard proctor |
|      | Poor-graded Sand | SW                 | QS3                | Compacted at optimum compaction—standard Proctor |
presented in Table 14. Some of the materials are the same used in Sects. 5.1 and 5.2, and others were added to increase the robustness of the analysis, the ranking and the impact of the final result of the geomechanical classification. These “auxiliary” data only have information about the resilient modulus or permanent deformation (they were used to define the “limits” of the classification). The work developed by [84] was also included but due to the significant information about both types of deformation (resilient and permanent) under different water content conditions. It is important to refer that, for these particular materials, are presented, whenever it is possible, the values of both deformations: resilient and permanent deformation.

This table also includes information about compaction conditions.

The resilient modulus and permanent deformation of the selected materials (Table 14) are presented in Table 15. The materials with both values of $M_r$ and $e_p$ allow validating the mechanical classification.

From the results presented in Table 15, it is possible to define guiding/limit values associated with the UIC classification: QS1, QS2 and QS3 [31]. This work only presents a preliminary geomechanical classification. The lack of information about the models’ parameters found in the bibliography (namely the models that include the suction and other complex model coefficients) and the necessity and difficulty to performed back analysis to find important

---

**Table 11** Physical properties of the materials—granulometry [19]

| Material Properties | C. Bruynweg [83] | Crusher [83] | Silty clay (42.2% fines content) [82] | Silty clay (27.4% fines content) [66] | Silt [76] | Coarse sand [76] |
|---------------------|------------------|-------------|-----------------------------------|-----------------------------------|---------|-----------------|
| $C_u$               | 2.100            | 10.50       | 28.0                              | 33.0                              | 2.51    | 4.68            |
| $C_c$               | 1.050            | 1.250       | 0.54                              | 0.75                              | 1.32    | 0.62            |
| $D_{10}$            | 0.148            | 0.217       | –                                 | –                                 | –       | –               |
| $D_{50}$            | 0.219            | 0.784       | –                                 | –                                 | –       | –               |
| $D_{90}$            | 0.280            | 1.722       | –                                 | –                                 | –       | –               |
| $D_{60}$            | 0.310            | 2.280       | –                                 | –                                 | –       | –               |
| CBR (%)             | 22.00            | 15.70       | –                                 | –                                 | –       | –               |

**Table 12** State conditions of the materials [19]

| Material properties | C. Bruynweg [83] | Crusher [83] | Silty clay (42.2% fines content) [82] | Silty clay (27.4% fines content) [66] | Silt [76] | Coarse sand [76] |
|---------------------|------------------|-------------|-----------------------------------|-----------------------------------|---------|-----------------|
| $\gamma_{dry,max}$ (kg/m$^3$) | 1723            | 1755        | 1998                              | 2070                              | 1620    | 2110            |
| $W_{opt}$ (%)       | 12.5             | 10.5        | 10.1                              | 7.6                               | –       | –               |
| $\gamma_{wet}$ (kg/m$^3$) | 1942            | 1937        | –                                 | –                                 | 35      | Non-plastic     |
| Liquid limit (%)    | Non-plastic      | Non-plastic | Non-plastic                       | Non-plastic                       | –       | –               |
| Plastic limit       | Non-plastic      | Non-plastic | Non-plastic                       | Non-plastic                       | 24      | Non-plastic     |

**Table 13** Strength properties of the materials [19]

| Material properties | C. Bruynweg [83] | Crusher [83] | Silty clay (42.2% fines content) [82] | Silty clay (27.4% fines content) [66] | Silt [76] | Coarse sand [76] |
|---------------------|------------------|-------------|-----------------------------------|-----------------------------------|---------|-----------------|
| $\phi$ (°)          | 48.2             | 50.20       | 36.18                             | 45.66                             | 11.7*   | 0.00*           |
| $c$ (kPa)           | 5.60             | 8.680       | 15.82                             | 10.43                             | 16.4*   | 33.0*           |
| $s$ (kPa)**         | 9.90             | 14.90       | 31.80                             | 19.10                             | 24.8    | 0.00            |
| $m$**               | 1.98             | 2.070       | 1.470                             | 1.880                             | 0.62    | 1.33            |

* In this case, the author described the saturated samples (silt and coarse sand) in $\phi'$ and $c'$

**s** and $m$ are the parameters used to define the failure envelope: $q = s + mp$
parameters (as the cohesion and friction angle used to define the failure criterion) reduced the number of models and materials that could be used in this geomechanical classification. Besides, some of the models found in the bibliography tested materials classified as QS0, which implies its elimination in the geomechanical classification, since these materials require mechanical improvement.

The geomechanical classification is an attempt to define a novel helpful guide to be used as a support for the modelling and design of the substructure. Indeed, this novel geomechanical classification tries to rank soils in a very similar way developed by [20] that was inspired by [40]. This work allows understating which are the acceptable values of the permanent deformation and resilient modulus according to the type of material and its classification. In this particular case, [31] classification was used as depicted in Fig. 4.

Analysing Fig. 4, a significant range of values was obtained for each classification. This was expected due to the mineralogical nature and physical properties of each material, which can have an influence on several parameters used in the determination of the resilient modulus and permanent deformation. Indeed, in the case of the materials classified as QS1 (fine soils), the type of the material (CL, CH, ML, or MH) associated with its plasticity’s characteristics and consistency index (as well as the percentage of fines) can have a significant impact in the permanent deformation and resilient behaviour, as depicted in Fig. 5. For the materials classified as QS2 and QS3, the type of materials (sand or gravel) as well as its granulometry (well- or poor-graded) and the percentage of fines (in the well-graded materials) can also influence the response of the material in terms of resilient and permanent deformations.

With this work, it was possible to define subsets associated with the properties of the soils, despite the need for more information. For example, as Fig. 5 shows, a silty soil (classified as QS1-ML) presents a superior resilient modulus when compared to clayed soils (QS1-CL). Regarding the granular materials, the subsets can be defined by the granulometry of the materials (well- or poor-graded). However, it is important to refer that the percentage of fines can also influence the obtained results. Indeed, despite the importance of the granulometry and plasticity properties, the fines content and the moisture content have also significant importance, mostly in the fine soils. As mentioned previously, the work developed by [84] shows the influence of the moisture content in the soils. Thus, [84] selected and studied the behaviour of the clay considering three compaction moisture content and dry unit weight conditions representing dry of optimum moisture content level, optimum moisture content level (corresponds to 95% of maximum dry unit weight), and wet of optimum moisture content level. From the results obtained by [84], an estimation was performed (in terms of resilient modulus and permanent deformation), and the results are presented in Figs. 6 and 7. These results are based on the properties of the clay described in Table 16. Analysing Fig. 7, the moisture content can influence both deformations, which means that can affect severely the track performance. In a more detailed analysis, it is possible to identify a reduction of around 200% in the resilient modulus from dry to wet conditions. This difference is significant, which means that the moisture content is an important parameter since influences the performance of the subgrade and corroborates the findings of [20, 86] in the case of non-standard unbound granular materials for pavements.
Table 14 Properties of the materials

| Information | UIC and ASTM classification | Materials | Properties | Observations |
|-------------|-----------------------------|-----------|------------|--------------|
| [50]        | QS1-CL                      | –         | $W_L = 28.5\%$; $W_P = 19.2\%$; $\text{IP} = 9.3\%$; $G_s = 2.73$; Clay content 20.0\%; Passing #200 73.0\%; Maximum dry density 17.36 kN/m$^3$ | – |
| From the resilient modulus parametric study | QS1 Low plasticity CL Lean clay | $W_L = 30.8\%$; $W_P = 18.4\%$; $\text{IP} = 12.3\%$ | The effect of the matric suction will be included in the effective stress; the triaxial tests following the AASHTO T307-99 procedure were conducted on both materials |
| [84]        | QS1 Lean clay (CL) Silty clay | $W_L = 28.19\%$; $G_s = 2.63$; $\text{IP} = 12.55\%$ (low plasticity); Passing #200 80\%; Maximum dry density 17.10 kN/m$^3$; $W_{	ext{opt}} = 17.11\%$; $c = 60$ kPa; $\phi = 18^\circ$ | Compacted at optimum compaction—standard proctor |
| From the permanent deformation parametric study | QS1 CL-ML Silt | $W_L = 35\%$; $W_P = 24\%$; $\text{IP} = 11\%$ (low to medium plasticity); $G_s = 2.67$; $C_u = 2.51$; $C_c = 1.32$; Maximum dry density 15.89 kN/m$^3$; Hydraulic permeability $5.3 \times 10^{-7}$ m/s; $c' = 11.7$ kPa; $\phi' = 16.4^\circ$ | Compacted at field conditions and saturated |
| From the resilient modulus parametric study | QS1 Low plasticity CL-ML Silt-lean clay | $W_L = 27.8\%$; $W_P = 19.8\%$; $\text{IP} = 8\%$ | The effect of the matric suction will be included in the effective stress; The triaxial tests following the AASHTO T307-99 procedure were conducted on both materials |
| Information | UIC and ASTM classification | Materials                  | Properties                                                                 | Observations                                      |
|-------------|----------------------------|----------------------------|---------------------------------------------------------------------------|---------------------------------------------------|
| From the permanent deformation parametric study | QS1 SM                     | Silty sand (42.2% fines)   | $C_u \approx 28; C_c \approx 0.54; G_s = 2.68; W_{opt} = 10.1\%; $         | Compacted at optimum compaction—standard proctor |
|             |                            |                            | Fines content = 42.2%; Maximum dry density 19.6 kN/m$^3$                 |                                                   |
| From the permanent deformation parametric study | QS1 SM                     | Silty sand (27.4% fines)   | $C_u \approx 33; C_c \approx 0.75; G_s = 2.67; W_{opt} = 7.6\%; $         | Compacted at optimum compaction—standard proctor |
|             |                            |                            | Fines content = 27.4%; Maximum dry density 20.3 kN/m$^3$                 |                                                   |
| [84]        | QS1 Silt (SM)              | Silty clayed sand          | $W_{L} = 16.70\%; G_s = 2.70; IP = 7.50\% (low plasticity);             | Compacted at optimum compaction—standard proctor |
|             |                            |                            | Passing #200 38\%; Maximum dry unit weight 16.9 kN/m$^3$                 |                                                   |
|             |                            |                            | $W_{opt} = 19.3\%; c = 103$ kPa; $                                      |                                                   |
|             |                            |                            | $\phi = 35^\circ$                                                        |                                                   |
| From the permanent deformation parametric study | QS2 SP                     | Well-graded sand           | $C_u = 2.10; C_c = 1.05; $                                                  | Compacted at optimum compaction—standard proctor |
|             |                            |                            | Maximum dry density 16.90 kN/m$^3$                                       |                                                   |
|             |                            |                            | $W_{opt} = 12.5\%; $                                                     |                                                   |
|             |                            |                            | $c = 5.60$ kPa; $\phi = 48.2^\circ$                                      |                                                   |
| [58]        | QS2 SP                     | Poorly graded Sand         | $G_s = 2.74; C_u \approx 2.0; C_c \approx 1.04$                          |                                                   |
| [83]        | QS2 SP                     | Poorly graded Sand         | $C_u \approx 1.88; C_c \approx 1.04; $                                   |                                                   |
|             |                            |                            | Maximum dry density 16.56 kN/m$^3$                                       | Standard proctor                                  |
|             |                            |                            | $W_{opt} = 14.0\%;$                                                      |                                                   |
|             |                            |                            | $c = 6.34$ kPa; $\phi = 41.8^\circ$                                      |                                                   |
| From the permanent deformation parametric study | QS2 SP                     | Sand                       | $G_s = 2.66; C_u = 4.8$                                                   | Compacted at field conditions and saturated       |
|             |                            |                            | $C_c = 0.62; $                                                          |                                                   |
|             |                            |                            | Maximum dry density 20.69 kN/m$^3$                                       |                                                   |
|             |                            |                            | Minimum dry density 15.89 kN/m$^3$                                       |                                                   |
|             |                            |                            | Hydraulic permeability $3.2 \times 10^{-5}$ m/s; $c^* = 0$ kPa; $\phi^* = 33^\circ$ |                                                   |
| Information                                      | UIC and ASTM classification | Materials          | Properties | Observations                                                                                       |
|-------------------------------------------------|-----------------------------|---------------------|------------|---------------------------------------------------------------------------------------------------|
| [84]                                            | QS2 SP                      | Poorly graded sand  | $W_L = 26.40\%$; $G_s = 2.71\%$; $C_u = 1.79$; $C_c = 0.89$; $W_{opt} = 13.70\%$; $W_{o} = 15.70$ kN/m$^3$; $W_{opt} = 13.70\%$; $c = 20$ kPa; $\phi = 42^\circ$ | Compacted at optimum compaction—standard proctor |
| [85]                                            | QS2-QS3 – LA = 22; MED = 15; Fines content = 10% | –                   | $C_u = 20$; $C_c = 0.8–1.1$; $W_L;_{max} = 25$; $W = 7.7\%$; $W_{OMC} = 7.7\%$ | Modified proctor |
| From the resilient modulus parametric study      | QS2/QS3 SW/SM-ML             | Sand                | $C_u = 10.5$; $C_c = 1.25$; $W_{opt} = 10.5\%$; $c = 8.68$ kPa; $\phi = 50.2^\circ$ | Granular material used as base and sub-base in the flexible pavement sections; The target moisture content and densities were selected based on AASHTO T99 test results and field-measured values [80] |
| From the permanent deformation parametric study  | QS3 SW                     | Poor-graded sand    | $C_u = 27$; $C_c = 2$; $(P_l_{max} = NP; P_l_{max} = 6)$ $W_L;_{max} = 25$; $W = 6.3\%$; $W_{OMC} = 6.8\%$ | Compacted at optimum compaction—standard proctor |
| From the resilient modulus parametric study      | QS3 GW                     | Well-graded gravel  | $G_s = 2.69$; $C_u \approx 61$; $C_c \approx 2.11$ | Granular material used as base and sub-base in the flexible pavement sections; The target moisture content and densities were selected based on AASHTO T99 test results and field-measured values [80] |
| [58]                                            | QS3 GW                     | Well-graded gravel  | –          | –                                                                                                 |
Table 15 Resilient modulus and permanent deformation of the selected materials

| Information | Materials classification | Resilient modulus (Mpa) | Permanent deformation $\varepsilon_p$ (%) |
|-------------|--------------------------|--------------------------|------------------------------------------|
| [50] From the resilient modulus parametric study | QS1-CL                   | 37                       | –                                        |
| [84] From the permanent deformation parametric study | QS1-CL                   | –                        | 0.01864                                  |
| From the resilient modulus parametric study | QS1-CL/ML                | 50                       | –                                        |
| Silt       |                          |                           |                                          |
| From the permanent deformation parametric study | QS1-CL-ML                | 70                       | –                                        |
| Low plasticity |                             |                           |                                          |
| From the resilient modulus parametric study | Silty sand (42.2% fines) | 258                      | 0.01385                                  |
| QS1-SM     |                          |                           |                                          |
| From the permanent deformation parametric study | Silty sand (27.4% fines) | 294                      | 0.01251                                  |
| QS1-SM     |                          |                           |                                          |
| [84] From the permanent deformation parametric study | Well-graded sand         | 511                      | 0.00658                                  |
| QS2-SP     |                          |                           |                                          |
| [58] From the permanent deformation parametric study | Sand                     | 673                      | –                                        |
| QS2-SP     |                          |                           |                                          |
| [83] From the permanent deformation parametric study | Sand                     | –                        | $9.85 \times 10^{-10}$                   |
| QS2-QS3    |                          | 172                      | 0.01873                                  |
| [84] From the resilient modulus parametric study | Poor-graded sand         | 178                      | 0.00084                                  |
| QS2/QS-SW/SW-ML |                       |                           |                                          |
| From the resilient modulus parametric study | Well-graded sand         | 131                      | –                                        |
| QS3-SW     |                          |                           |                                          |
| From the permanent deformation parametric study | Poor-graded sand         | 178                      | 0.00084                                  |
| QS3-GW     |                          | 159                      | 0.00320                                  |
| [58]       | QS3-GW                   | 862                      | –                                        |

Fig. 4 Mechanical classification based on the resilient modulus and permanent deformation
7 Conclusions

The capability to determine the resilient modulus and also the permanent deformation of geomaterials is imperative in the modelling, design and evaluation of the performance of railway structures. This work attempts to identify and summarize the main models to estimate the resilient modulus and also permanent deformation. At the beginning of the analysis, the most simplistic models are presented. In the case of the resilient modulus, the models should include the influence of the deviatoric and confining stresses and the constant parameters should reflect the influence of other factors as the moisture content and physical state of the material. The mechanistic-empirical models summarized in Annex were divided according to the type of material (clay, silt, sand and gravel) to better understand the model’s formulation and the conditions of the material when tested in terms of its granulometry ($C_u$ and $C_c$) and plasticity properties ($W_L$, $W_P$, and IP). Tables also include some observations regarding the laboratory tests and classification of the material (UIC and ASTM).

From the available mechanistic-empirical approach, it was important to evaluate the robustness of the models and their sensitivity. Thus, some models and materials were selected to perform a parametric study in terms of resilient modulus. A similar analysis was already performed for the permanent deformation in the work developed by [19]. The selection depended on the data available for each material and the variability of the geomaterials (type of soils) in terms of granulometry, percentage of fines, moisture content and plasticity properties. The analysis shows better results in the case of the coarse-grained soils. (The $R$-
squared value is higher and the obtained values for resilient modulus are similar.) This fact can be explained by the complexity of the nature of fine-grained soils, namely the variation of its properties in terms of plasticity and fines content. At the end of this analysis, one model was selected to rating the geomechanical performance. The MEPDG model shows better results for both materials. Indeed, this choice is also justified by its universal character, which means that can be applied to all types of materials. As expected, the comparison of the results of the resilient modulus shows that the values increase with the UIC classification: higher resilient modulus is related to well-graded materials (QS2 and QS3).

The geomechanical classification is based on the parametric studies of the resilient modulus (applying the MEPDG model) and the permanent deformation (applying Chen’s model). Moreover, other materials were added to this analysis because of the available information about the moisture content. The classification presents a wide range of results in terms of permanent deformation and resilient modulus. This range is a reflection of the variation of the properties of the materials with the same classification (QS1, QS2 and QS3) that influence both types of deformations. Indeed, this work is the first attempt to obtain a novel geomechanical classification that can be a helpful tool in the modelling of the foundation of the railway structures since gather information about the UIC classification (which is a reference in the railway works) and permanent and resilient deformations. Nevertheless, several limitations are recognized in this proposal, which were discussed throughout this paper. These limitations should be overcome in future work updating this proposal with a wide range of geomaterials properly characterized (physical and mechanical characteristics) and tested under different state conditions (deviation to the optimum compaction conditions), which should include variations of water content/suction/degree of saturation and density.

Appendix

An extensive list of resilient models based on laboratory tests is presented in this appendix. Although these tables were constructed uniformly (especially regarding the nomenclature of the variables and the parameters and characteristics of the materials), the authors maintained the original symbols described in the original paper. For example, the symbol $E_r$ is used to describe the resilient modulus, which is usually represented by the symbol $M_r$.

Some models were developed using certain SI units, and where possible, these units are described in the column “variables and empirical constants”.

Regarding the resilient modulus tables, [87] defined $w_c$ as optimum moisture content (which is equal to $W_{opt}$) and $w_{cr}$ as the relation between the actual moisture content and the optimum moisture content. In this analysis, the fines are measured through the passing sieve #200 or through the percentage of fines. Furthermore, the maximum dry unit weight is also defined as maximum dry density. The constants such as $A$, $B$, $C$ or $k_i$ are defined as fitting, regression, material and model parameters according to the original formulation defined by the author. The symbols $p_a$ and $p_0$ are defined as atmospheric stress/pressure and reference stress and it is different from $p_u$ which means unit pressure.

Regarding the stress variables, the symbol $\theta$ is defined as bulk stress or the sum of principal stresses, and the deviator stress can be represented by the symbols $q$, $q_t$ and $\sigma_d$. The symbol $q$ is different from $q_w$ which represents the mean value of the deviator stress (Tables 17–21).
### Table 17 Models that describe resilient modulus of fine-grained soils—clays

| UIC  | ASTM classification | Ref. | Equation model                                                                 | Variables and empirical constants                                                                 | Parameters and characteristics | Observations                                                                                                                                 |
|------|---------------------|------|--------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| QS0* | CH                  | [88] | $M_r = k_ip_a \left( \frac{\theta_{net}}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \left( \psi_m \right)^{k_4}$ | $\theta_{net}$ is the net bulk stress ($\theta_{net} = 0 - 3\mu_s$); $\tau_{oct}$ is the octahedral shear stress; $\psi_m$ is the matric suction; $p_a$ is the atmospheric stress; $k_i$ are the material parameters | $W_L = 61\%$; $W_p = 25\%$; $IP = 36\%$; $G_s = 2.69$ | The model is similar to Cary and Zapata [67] model. The last term related to the suction parameter is normalized with the net bulk stress instead of the atmospheric pressure. The model does not consider the effect of the pore-water pressure under saturated conditions. |
|      |                     |      |                                                                              |                                                                                               |                               | The model is developed based on four tested fine-grained soils. This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests. The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa. All the test soils were prepared at OPT water content and at a dry density corresponding to 98\% or 103\% relative compaction for a standard proctor. |
| QS0  | CL                  | [36] | $M_r = k_ip_a \left( \frac{0 - 3k_4}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + k_5 \right)^{k_3}$ | $M_{rs}$ is the modulus at certain suction (or moisture); $M_{rs}$ is the modulus at optimum moisture content; $M_{opt}$ is the modulus at optimum moisture content; $c$ and $d$ are model parameters; $p_a$ is the atmospheric pressure (101 kPa); $k_i$ are fitting parameters | $W_L = 85\%$; $IP = 52\%$; $G_s = 2.75$; Sand content = 3.1\%; Silt content = 21.2\%; Clay content = 75.2\%; Fines content = 96.4\%; $W_{opt}$ = 27.5\%; Maximum dry unit weight = 14.4 kN/m$^3$ | The model is developed based on four tested fine-grained soils. This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests. The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa. All the test soils were prepared at OPT water content and at a dry density corresponding to 98\% or 103\% relative compaction for a standard proctor. |
Table 17 continued

| UIC   | ASTM classification | Ref. | Equation model | Variables and empirical constants | Parameters and characteristics | Observations |
|-------|---------------------|------|----------------|-----------------------------------|--------------------------------|--------------|
| QS0-QS1 CL | Medium plasticity | [50] | \( E_r = \frac{a' + b' \sigma_{av}}{\sigma_{av}} \) for \( \sigma_{av} > 0 \) | \( a' \) and \( b' \) are material parameters: \( a' = 318.2 + 0.337(q_u) + 0.73(\% \text{clay}) + 2.26(\Pi) - 0.915(c) - 2.19(S) - 0.304(\% \text{finer} \#200) \) | \( W_L = 42.1\%; \ W_P = 22.0\%; \ IP = 20.1\%; \ G_s = 2.76; \) Clay content = 36\%; Passing sieve \#200 = 93\% | The hyperbola predicts excessive values of \( E_r \) when \( \sigma_{av} \) is close to zero |
| QS1 CL | Low plasticity | \( b' = 2.10 + 0.00039 \left( \frac{1}{\sigma_{av}} \right) + 0.104(q_u) + 0.09(\Pi) \) | \( 0.10(\% \text{finer} \#200) \) | \( W_L = 38.8\%; \ W_P = 23.3\%; \ IP = 15.5\%; \ G_s = 2.65; \) Clay content = 18.0\%; Passing sieve \#200 = 100.0\%; Maximum dry density = 15.98 kN/m³ | |
| QS1 CL | Low plasticity | \( W_L = 29.5\%; \ W_P = 20.1\%; \ IP = 9.4\%; \ G_s = 2.67; \) Clay content = 16.0\%; Passing sieve \#200 = 78.0\%; Maximum dry density = 17.16 kN/m³ | | |
| QS1 CL | Low plasticity | \( W_L = 28.5\%; \ W_P = 19.2\%; \ IP = 9.3\%; \ G_s = 2.73; \) Clay content = 20.0\%; Passing sieve \#200 = 73.0\%; Maximum dry density = 17.36 kN/m³ | | |
| UIC | ASTM classification | Ref. | Equation model | Variables and empirical constants | Parameters and characteristics | Observations |
|-----|---------------------|------|----------------|-----------------------------------|--------------------------------|--------------|
| QS1 | CL | [36] | $M_r = k_1 \rho_a \left( \frac{\theta - 3k_3}{\rho_a} \right)^{k_2} \left( \frac{\tau_{oct}}{\rho_a} + k_5 \right)^{k_3}$ | MR$_{opt}$ is the modulus ratio; $M_r$ is the modulus at certain suction (or moisture); $M_{opt}$ is the modulus at optimum moisture content; $\rho_a$ is the atmospheric pressure (101 kPa); $k_i$ are fitting parameters | $W_L = 26.0\%$; $IP = 9.0\%$; $G_s = 2.66$; Sand content = 36.3%; Silt content = 45.3%; Clay content = 14.5%; Per cent fines = 59.7%; $W_{opt} = 16.0\%$; Maximum dry unit weight = 17.70 kN/m$^3$ | Model developed based on four tested fine-grained soils. This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests. The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa. All the test soils were prepared at OPT water content and at a dry density corresponding to 98% or 103% relative compaction for a standard proctor The model takes into account, explicitly, the effect of the confining and octahedral shear stress. It is used for a large range of unbound materials. The effect of the matric suction is included in the effective stress. The model is similar to the model developed by Uzan [56] |
| QS1 | CL | [9] | $M_r = k_1 \rho_a \left( \frac{\theta + 2\psi_m}{\rho_a} \right)^{k_2} \left( \frac{\tau_{oct}}{\rho_a} + \frac{1}{\psi_m} \right)^{k_3}$ | $\theta$ is the bulk stress; $\tau_{oct}$ is the octahedral shear stress; $\psi_m$ is the matric suction; $k_i$ are the material parameters | $W_L = 30.8\%$; $W_P = 18.4\%$; $IP = 12.3\%$; Passing sieve #200 = 68.8%; $W_{opt} = 16.5\%$; Maximum dry density = 17.70 kN/m$^3$ | |
Table 17 continued

| UIC | ASTM classification | Ref. | Equation model | Variables and empirical constants | Parameters and characteristics | Observations |
|-----|---------------------|------|----------------|-------------------------------------|-------------------------------|-------------|
| ≤ QS1 | – | [87] | \( M_r = k_1 P_a \left( 1 + \frac{\sigma_d}{1 + \sigma_c} \right)^{k_2} \) | \( M_r \) is the resilient modulus (MPa); | \( \gamma_d = 16.92 \text{ kN/m}^3; \gamma_{dp} = 0.97; \) | \( k_1 \) is dependent on \( \gamma_{dp}, LL \) and \( w_c \). |
| (based on AASHTO classification) | | | | \( w_c = 1.48; w_c = 20.7\% \); | Optimun \( w_c = 14.0\% \); Maximum dry density = 17.4 \text{ kN/m}^3; | \( k_2 \) is dependent on \( \gamma_{dp}, LL, w_c \), and #200, where \( \gamma_{dp} \) and \( w_{ce} \) are the density and moisture ratio, respectively. |
| QS1 | CL-ML | [9] | \( M_r = k_1 P_a \left( \frac{\theta + \psi_m}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \) | \( \theta \) is the bulk stress; | \( \gamma_d = 19.87 \text{ kN/m}^3; \gamma_{dp} = 1.10; \) | The model takes into account, explicitly, the effect of the confining and octahedral shear stress. |
| Low plasticity | | | \( \tau_{oct} \) is the octahedral shear stress; | \( w_{ce} = 0.82; w_c = 11.5\% \); | Maximum dry density = 18.4 \text{ kN/m}^3; | It is used for a large range of unbound materials. |
| | | | \( \psi_m \) is the Bishop’s effective stress parameter; | Optimun \( w_c = 14.0\% \); | Passing sieve #200 = 44%; | The effect of the matric suction is included in the effective stress. |
| | | | \( \psi_m \) is the matric suction; | Maximum dry density = 17.75 \text{ kN/m}^3; | Passing sieve #200 = 56.3%; | The model is similar to the model developed by Uzan [56]. |
| | | | \( k_i \) are the material parameters | W_L = 27.8\%; W_P = 19.8\%; IP = 8\%; | W_{opt} = 14.2\%; | The last term related to the suction parameter is normalized with the net bulk stress instead of the atmospheric pressure. |
| QS1** | CL | [88] | \( M_r = k_1 P_a \left( \frac{\theta_{net}}{\theta_{net}} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \) | \( \theta_{net} \) is the net bulk stress \( (\theta_{net} = \theta - 3 \mu_a)\); | W_L = 37\%; W_P = 18\%; IP = 19\%; G_s = 2.69 | The model does not consider the effect of the pore-water pressure under saturated conditions. |
| (no information about the particle size curve) | | | \( \tau_{oct} \) is the octahedral shear stress; | | | |
| Low to medium plasticity | | | \( \psi_m \) is the matric suction; | | | |
| | | | \( P_a \) is the atmospheric stress; | | | |
| | | | \( k_i \) are the material parameters | | | |

*Classification based on the plasticity index and AASHTO classification (A-7-5: > 35% passing the 0.075 mm sieve)

**Classification based on the plasticity index and AASHTO classification (A-7-6: > 35% passing the 0.075 mm sieve)
Table 18 Models that describe resilient modulus of fine-grained soils—sils

| UIC | ASTM classification | Ref. | Equation model | Variables and Empirical Constants | Parameters and characteristics | Observations |
|-----|---------------------|------|----------------|------------------------------------|--------------------------------|--------------|
| QS0 | MH                  | [35] | $M_t = k_1 p_a (\sigma_d + \chi \psi_m)^{k_2}$ | $\chi$ is the Bishop’s effective stress parameters (it is function of the degree of saturation of the soil); $\psi_m$ is the matric suction; $k_i$ are the materials' parameters; $W_L = 54.0\%$; $IP = 20\%$; $G_s = 2.71$; $W_{opt} = 18\%$; Maximum dry density = 17.26 kN/m$^3$ | This model does not take into account the effect of confining stress. The model can be applied to a large range of moisture content. The model is dependent on only two material parameters. |
| QS0 | MH                  | [50] | $E_t = \frac{a' + b' \sigma_d}{\sigma_d}$ for $\sigma_d > 0$ | $a'$ and $b'$ are material parameters: $a' = 318.2 + 0.337(q_u) + 0.73(\%clay) + 2.26(PI) - 0.915(\gamma) - 2.19(S) - 0.304(\%finer #200)$; $b' = 2.10 + 0.00039 \left( \frac{1}{a} \right) + 0.104(q_u) + 0.09(\%finer #200)$ | $W_L = 68.5\%$; $W_p = 39.2\%$; $IP = 29.3\%$; $G_s = 2.71$; Clay content = 28.7%; Passing sieve #200 = 80.0%; Maximum dry density = 12.55 kN/m$^3$ | The hyperbola predicts excessive values of $E_t$ when $\sigma_d$ is close to zero. |
| QS0-QS1 | MH | [35] | $M_t = k_1 p_a (\sigma_d + \chi \psi_m)^{k_2}$ | $\chi$ is the Bishop’s effective stress parameters (it is function of the degree of saturation of the soil); $\psi_m$ is the matric suction; $k_i$ are the materials' parameters; $G_s = 2.67$; $W_L = 50.0\%$; $IP = 23\%$; $W_{opt} = 17\%$; Maximum dry density = 17.65 kN/m$^3$ | This model does not take into account the effect of confining stress. The model can be applied to a large range of moisture content. The model is dependent on only two material parameters. |
### Table 18 continued

| UIC | ASTM classification | Ref. | Equation model | Variables and Empirical Constants | Parameters and characteristics | Observations |
|-----|----------------------|------|----------------|-----------------------------------|---------------------------------|--------------|
| QS1 | ML                   | [36] | $M_t = k_1 p_a \left( \frac{ \theta - 3 k_4 }{ p_a } \right)^k_2 \left( \frac{ \tau_{oct} }{ p_a } + k_3 \right)^k_5$ | $M_{R;opt}$ is the modulus ratio; $M_{eq}$ is the modulus at certain suction (or moisture); $M_{R;opt}$ is the modulus at optimum moisture content; $c$ and $d$ are model parameters; $p_a$ is the atmospheric pressure (101 kPa); $k_i$ are fitting parameters | $W_L = 28.0\%$; $IP = 11.0\%$; $G_s = 2.69$; Sand content = 11.9%; Silt content = 82.4%; Clay content = 5.7%; Fines content = 88.1%; $W_{opt} = 13.5\%$; Maximum dry unit weight = 17.9 kN/m$^3$ | The model is developed based on four tested fine-grained soils. This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests. The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa. All the test soils were prepared at OPT water content and at a dry density corresponding to 98% or 103% relative compaction for a standard proctor |
| QS1 | ML                   | [88] | $M_t = k_1 p_a \left( \frac{ \theta_{net} }{ p_a } \right)^k_2 \left( \frac{ \tau_{oct} }{ p_a } + 1 \right)^k_3 \left( \frac{ \psi_m }{ \theta_{net} + 1 } \right)^k_4$ | $\theta_{net}$ is the net bulk stress ($\theta_{net} = \theta - 3 u_a$); $\tau_{oct}$ is the octahedral shear stress; $\psi_m$ is the matric suction; $p_a$ is the atmospheric stress; $k_i$ are the material parameters | $W_L = 43.2\%$; $W_P = 29.2\%$; $IP = 14.0\%$; $G_s = 2.73$; Sand content = 72%; Silt content = 24%; Clay content = 4%; $C_u = 4.55$; $C_c = 0.61$; $W_{opt} = 16.3\%$; Maximum dry density = 17.26 kN/m$^3$ | The model is similar to Cary and Zapata [67] model. The last term related to the suction parameter is normalized with the net bulk stress instead of the atmospheric pressure. The model does not consider the effect of the pore-water pressure under saturated conditions |
Table 18 continued

| UIC classification | ASTM classification | Ref. | Equation model | Variables and Empirical Constants | Parameters and characteristics | Observations |
|-------------------|----------------------|------|----------------|----------------------------------|--------------------------------|--------------|
| QS1 ML Low plasticity | [50] | $E_r = \frac{a' + b'\sigma_d}{\sigma_d}$ for $\sigma_d > 0$ | $a'$ and $b'$ are material parameters: $a' = 318.2 + 0.337(q_u) + 0.73(\% \text{clay}) + 2.26(\text{PI}) - 0.915(G_o) - 2.19(S)$ - 0.304(\% finer #200) $b' = 2.10 + 0.00039 \left(\frac{1}{\sigma_d}\right) + 0.104(q_u) + 0.09(LL) - 0.10(\% \text{finer} #200)$ | $W_L = 36.2\%; \ W_P = 34.1\%; \ IP = 2.1\%; \ G_s = 2.66\%; \ Clay \ content = 18.0\%; \ Passing \ sieve \ #200 = 80.0\%; \ Maximum \ dry \ density = 13.14 \ kN/m^3$ | W_L = 36.2%; W_P = 34.1%; IP = 2.1%; G_s = 2.66; Clay content = 18.0%; Passing sieve #200 = 80.0%; Maximum dry density = 13.14 kN/m^3 |
| QS1 ML Medium plasticity | [89] | $M_t = k_i p_o \sigma_d^{\psi_m} \psi_m^k$ | $\sigma_d$ is the deviator stress; $\psi_m$ is the matric suction; $k_i$ are the regression parameters | $W_L = 46.4\%; \ W_P = 28.8\%; \ IP = 17.6\%; \ G_s = 2.66\%; \ Clay \ content = 38\%; \ Passing \ sieve \ #200 = 52\%; \ W_{opt} = 19.5\%; \ Maximum \ dry \ density = 16.28 \ kN/m^3$ | The hyperbola predicts excessive values of $E_r$ when $\sigma_d$ is close to zero | The model does not take into account the effect of confining stress. It is only valid for a limited range of moisture content. When the soils get saturated and matric suction is equal to zero (high moisture content) the resilient modulus is close to zero. Low moisture content values lead to high (unrealistic) resilient modulus. The suction was determined through the paper filter method |
| UIC | ASTM classification | Ref. | Equation model | Variables and empirical constants | Parameters and characteristics | Observations |
|-----|---------------------|------|----------------|---------------------------------|-------------------------------|--------------|
| QS1 | SM-CL               | [50] | \( E_r = \frac{a' + b\sigma_d}{\sigma_d} \) for \( \sigma_d > 0 \) | \( a' \) and \( b' \) are material parameters: \( a' = 318.2 + 0.337(q_u) + 0.73(\%\text{clay}) + 2.26(\text{PI}) - 0.915(\gamma) - 2.19(S) - 0.304(\%\text{finer #200}) \) \( b' = 2.10 + 0.00039\left(\frac{1}{\sigma_m}\right) + 0.104(q_u) + 0.09(\text{LL}) - 0.10(\%\text{finer #200}) \) | \( W_L = 21.0\% ; W_P = 14.1\% ; IP = 6.9\% ; G_s = 2.65\% ; \text{Clay content} = 16.0\% ; \text{Passing sieve #200} = 36.0\% ; \) | The hyperbola predicts excessive values of \( E_r \) when \( \sigma_d \) is close to zero |
| QS1 | SM                 |      | \( M_r = k_1 \rho_a \left( \frac{\theta_{\text{net}} - 3\Delta\psi_{\text{m-sat}}}{\psi_{\text{m}} - \Delta\psi_{\text{m}} + 1} \right)^{k_2} \) \( k_1 \) | \( \theta_{\text{net}} \) is the net bulk stress (\( \theta_{\text{net}} = \theta - 3\mu_d \)) ; \( \Delta\mu_{\text{m-sat}} \) is the pore-water pressure build-up in a saturated condition (\( \psi_{\text{m}} = 0 \)) ; \( \Delta\psi_{\text{m}} \) is the relative change in matric suction with respect to the initial matric suction in an unsaturated condition (\( \Delta\mu_{\text{m-sat}} = 0 \)) | \( W_L = 20.7\% ; W_P = 19.0\% ; IP = 1.7\% ; G_s = 2.60\% ; \text{Clay content} = 17.0\% ; \text{Passing sieve #200} = 20.0\% ; \) | Maximum dry density = 18.04 kN/m³ |
| QS1 | SC                 | [67] | \( M_r = k \frac{\sigma_{\text{oct}}^2}{\sigma_{\text{oct}}} \) \( k \) | \( \sigma_{\text{oct}} \) and \( \tau_{\text{oct}} \) are the octahedral normal and shear stresses, respectively | \( W_L = 22\% ; W_P = 18\% ; IP = 5\% ; G_s = 2.71\% ; \text{Passing sieve #200} = 47\% \) | The model is based on the universal model with an additional term that includes the matric suction effects into the resilient modulus |
| QS1 | SC                 | [51] | \( M_r = k \frac{\sigma_{\text{oct}}^2}{\sigma_{\text{oct}}} \) \( k \) | \( \sigma_{\text{oct}} \) and \( \tau_{\text{oct}} \) are the octahedral normal and shear stresses, respectively | \( W_L = 26\% ; W_P = 17.6\% ; IP = 8.4\% ; 60\% \text{ sand and 40\% kaolinite by weight} \) | This model is difficult to apply [86] |
| UIC ASTM classification | Ref. | Equation model | Variables and empirical constants | Parameters and characteristics | Observations |
|-------------------------|------|----------------|-----------------------------------|-----------------------------|--------------|
| QS2 SP Poor-graded sand | [58] | $M_r = A \left( \frac{p_m}{p_0} \right)^B \left( \frac{p_u}{p_0} \right)^C$ | $q_m$ is the mean value of deviator stress; $p_m$ is the mean value of mean normal stress; $p$ is the mean normal stress; $p_u$ is unit pressure (1 kPa); $A$, $B$ and $C$ are constants | $G_s = 2.64$; $C_u = 2.5$; $C_c = 0.54$ | This model is based on triaxial tests carried out in six aggregates. |
| QS2                    |      |                |                                   |                             | The model is based on laboratory triaxial testing (constant and variable confining pressure) [70]. |
| QS3 Well-graded sand   | [57] | $M_r = A(n_{max} - n)p_0 \left( \frac{\theta}{p_0} \right)^{0.5}$ | $\theta$ is the sum of principal stresses; $p_0$ is the reference stress, 100 kPa; $n$ is the porosity of the material; $n_{max}$ is the theoretical maximum value of porosity when $M_r = 0$; $A$ is a material parameter | $C_u = 5.9$; $C_c = 1.2$ | This model is based on laboratory triaxial testing (constant confining pressure) [70]. |
|                        |      | $M_r = B(n_{max} - n)p_0 \left( \frac{\theta}{p_0} \right)^{0.7} \left( \frac{q}{p_0} \right)^{-0.2}$ | $n_{max}$ is the theoretical maximum value of porosity when $M_r = 0$; $B$ is a material parameter | $C_u = 1.6$; $C_c = 1.0$ | The model is based on tests carried out in several types of coarse-grained granular materials typically used in unbound layers of road and railroad pavements. |
|                        |      |                |                                   |                             | The density was incorporated in the model by the porosity parameter. |
|                        |      |                |                                   |                             | Materials classified as natural gravels. |
|                        |      |                |                                   |                             | The study also includes the crushed gravel and crushed rock. |
| QS3 SW Well-graded sand| [79] | $E_v = C \sigma_v^n$ | $C$ is a parameter that depends on the mineralogy of the particles, shape of the grains and particle size curve (in the case of granular materials. In the case of the cohesive materials; it also depends on the percentage of fines and Atterberg limits | $D_{50} = 3.5$ mm; $G_s = 2.71$; $C_u = 28$; $C_c = 1.0$ | The vertical Young’s modulus is independent of the lateral stress. |
Table 20  Models that describe resilient modulus of granular soils—gravels

| UIC    | ASTM classification | Ref. | Equation model | Variables and empirical constants | Parameters and characteristics | Observations |
|--------|---------------------|------|----------------|-----------------------------------|--------------------------------|--------------|
| QS2    | GP-GM               | [67] | $M_t = k_U p_a \left( \frac{\theta_{net} - 3 \Delta \mu_{sat}}{\theta_{net}} \right) \frac{1}{k_1} \frac{\psi_m - \Delta \psi_m}{p_a} \frac{1}{k_2} \frac{1}{C_0} \frac{h_{net}}{C_{18}} \frac{D_u w_{sat}}{C_{19}} \left( \psi_m \right) \frac{1}{k_3} \frac{2}{C_0} \frac{D_w m_{sat}}{C_{18}} \frac{p_a}{C_3} \frac{1}{C_{19}}$ | $\theta_{net}$ is the net bulk stress $(\theta_{net} = \theta - 3 \mu)$; $\Delta \mu_{sat}$ is the pore-water pressure build-up in a saturated condition $(\psi_m = 0)$; $\Delta \psi_m$ is the relative change in matric suction with respect to the initial matric suction in an unsaturated condition $(\Delta \mu_{sat} = 0)$; $IP = NP; G_s = 2.71$; Passing sieve #200 = 7%; $W_{opt} = 7.2%$; Maximum dry density $= 22.16 \text{kN/m}^3$ | The model is based on the universal model with an additional term that includes the matric suction effects into the resilient modulus. |
| QS2    | GP-GM Poor-graded gravel | [58] | $M_t = A \left( \frac{P_M}{P_n} \right)^B \left( \frac{P_u}{p} \right)^C$ | $q_m$ is the mean value of deviator stress; $p_m$ is the mean value of mean normal stress; $p_n$ is unit pressure $(1 \text{kPa})$; $A, B$ and $C$ are constants | $G_s = 2.69; C_u \approx 5.3; C_c \approx 4.7$ | This model is based on the triaxial tests carried out in six aggregates. The model is based on laboratory triaxial testing (constant and variable confining pressure) [70] |
| QS3    | GW Well-graded gravel | | | | |
| QS2-QS3 | GP Poor-graded gravel | [79] | $E_v = C \sigma_v^{m}$ | $C$ is a parameter that depends on the mineralogy of the particles, shape of the grains, particle size curve (in the case of the granular materials) | $D_{so} = 8.5 \text{ mm}; G_s = 2.71; C_u = 53; C_c \approx 4.5$ | The vertical Young’s modulus is independent of the lateral stress. The parameter $C$, in the case of the cohesive materials, also depends on the percentage of fines and Atterberg limits |
| $\geq$ QS2 (based on AASHTO classification) | | [87] | $M_t = k_1 P_a \left( 1 + \frac{\sigma_d}{1 + \sigma_c} \right)^k_1$ | $M_t$ is the resilient modulus (MPa); $\sigma_d$ is the deviator stress (kPa); $\sigma_c$ is the confining pressure (kPa); $P_a$ is the atmospheric pressure (kPa); $k_1$ and $k_2$ are model parameters | $\gamma_d = 16.07 \text{kN/m}^3; \gamma_d = 1.02; w_c = 16.3\%; \psi_{cr} = 1.05\%$; Optimum $w_c = 15.5\%$; Maximum dry density $= 15.7 \text{kN/m}^3$; Passing sieve #200 = 11% $\gamma_d = 18.04 \text{kN/m}^3; \gamma_d = 1.10; w_c = 1.15\%; \psi_{cr} = 17.32\%$; Optimum $w_c = 15.0\%$; Maximum dry density $= 16.7 \text{kN/m}^3$; Passing sieve #200 = 34% | $k_1$ is dependent on $\gamma_{cr}, w_{cr}, #200$ and $C_u$ where $\gamma_d$ and $w_c$ are the density and moisture ratio, respectively. $k_2$ is dependent on $\gamma_{cr}, w_{cr}, #200$ and $C_u$. |
| Ref. | Equation model | Variables and empirical constants | Observations |
|------|----------------|-----------------------------------|--------------|
| [90] | $M_r = k_1 (\sigma_3)^{k_2}$ | $\sigma_3$ is the confining pressure; $k_1$ and $k_2$ are material constants | This model is based on the resilient modulus and Poisson’s ratio; Model based on laboratory triaxial testing (constant confining pressure) |
| [77] | $M_r = k_1 (\theta)^{k_2}$ | $\theta$ is the bulk stress (sum of the principal stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$); $k_i$ are the regression parameters | Commonly known as the $k$-$\theta$ model; hyperbolic relationship. |
| [91] | $M_r = k_1 (\sigma_3)^{k_2}$ | $\sigma_3$ is the confining pressure; $k_1$ and $k_2$ are material constants | This model assumes a constant Poisson’s ratio. However, several studies show that this parameter varies with the applied stress. The effect of stress on the resilient modulus is only considered by the bulk stress (sum of the principal stresses). It is a simplified version of the universal model. This model is based on laboratory triaxial testing (constant confining pressure) [70]. The model fits well in the case of tests with constant confining pressure and shows poor results in the case of variable confining pressure [53]. |
| [92] | $M_r = K_1 + K_2 \sigma_d$, when $\sigma_d < \sigma_{dh}$ $M_r = K_3 + K_4 \sigma_d$, when $\sigma_d > \sigma_{dh}$ | $K_i, K_2, K_3, \text{ and } K_4$ are model parameters dependent upon soil type and its physical state (normally, $K_2$ and $K_4$ are negative); $\sigma_{dh}$ is the deviator stress at which the slope of $M_r$ versus $\sigma_d$ changes | This is a bilinear model |
| [93] | $\log M_r = c_{ld} - m_{ld}(\sigma_1 - \sigma_3)$ | $c_{ld}$ and $m_{ld}$ are functions of matrix suction; $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses | The model was not fully validated with laboratory data. This model is characterized as a semi-log model |
| [61] | $\varepsilon_v = \frac{1}{K_1} \rho^p \left[ 1 - \beta \left( \frac{q}{p} \right)^2 \right]$ $\varepsilon_s = \frac{1}{3G_1} \rho^p \left( \frac{q}{p} \right)$ $K = \frac{p}{\varepsilon_v}$ $G = 3 \frac{q}{\varepsilon_q}$ | $p$ is the mean normal stress; $q$ is the deviator stress; $\varepsilon_v$ is the volumetric strain and $\varepsilon_s$ is the shear strain; $K$ is the bulk modulus; $G$ is the shear modulus | This model uses shear-volumetric approach. This is a nonlinear elastic model that takes into account the effect of the stress path and it is expressed in terms of the bulk and shear modulus, that are stress-dependent. The model is elastic and follows the application of the theorem of reciprocity. The shear and volumetric components of stress and strain are treated separately. The granular materials show an inelastic response and this elastic model can lead to inaccurate predictions. The model is based on variable confining pressure. Sweere [94] used this model but keep the relationship between shear and volumetric strains with stresses independent from each other |
### Table 21 continued

| Ref. | Equation model | Variables and empirical constants | Observations |
|------|----------------|-----------------------------------|--------------|
| [47] | $M_r = k_1 (\sigma_d)^{k_2}$ | $k_1$ and $k_2$ are constants dependent on the material; $\sigma_d$ is the deviator stress; | The model is characterized by a power model. A modified elastic layered computer program was made to determine equivalent subgrade modulus values. Multiple regression techniques were used to determine predictive equations for the equivalent subgrade modulus. This model was widely used by other authors and good results were obtained (in agreement with test results) |
| [56] | $M_r = k_1 p_0 \left( \frac{\partial}{p_0} \right)^{k_2} \left( \frac{q}{p_0} \right)^{k_3}$ or $M_r = k_1 p_0 \left( \frac{\tau_{oct}}{p_0} \right)^{k_2}$ | $\theta$ is the bulk stress (sum of the principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$); $\tau_{oct}$ is the octahedral shear stress; $p_0$ is the atmospheric pressure; $k_i$ are the material parameters | The model shows good results in the case of pre-failure stresses. However, in the case of stresses that go beyond the static failure, the predictions are poor. This model is suitable for a large range of materials: from fine-grained soils to coarse-granular materials. This model is based on laboratory triaxial testing (constant confining pressure) [70] |
| [95] | $e_v = \delta \left[ \frac{B}{A} \right] \ln \left( \frac{q}{p} \right) \left[ 1 - C \left( \frac{q}{p} \right)^2 \right]$ | $p$ is the mean normal stress; $q$ is the deviator stress; $A$, $B$, $C$, $D$ and $E$ are material constants | This model uses shear-volumetric approach. Two materials were considered: well-graded crushed limestone and uniformly graded limestone. This model is also known as the contour model and takes into account effective mean and deviatoric stresses and stress path dependency. This model predicts the shear strain (considering the length of the stress path corresponding to the load cycle). The material parameters in the two parts of the contour model are independent of each other. In fact, independent material parameters mean that the model is inelastic (unlike the Boyce model) |
| [96] | $M_r = k_1 \left( J_2 \right)^{k_2}$ | $k_1$ and $k_2$ are constants; $J_2$ is the second stress invariant; $\tau_{oct}$ is the octahedral shear stress | Model based on laboratory triaxial testing (constant confining pressure) [70] |
| [97] | $M_r = k_1 \left( \frac{p}{q} \right)^{k_2}$ | $k_1$ is a material constant; $p$ is the mean normal stress; $q$ is the deviator stress; | Model based on laboratory triaxial testing (constant confining pressure) [70] |
| [98] | $e_v = A (\delta np)^B (\delta p)^C - D \left[ \delta \left( \ln \frac{\sigma_1}{\sigma_3} \right) \right]^E$ | $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses $T$ and $S$ are 2D stress invariants | Model developed for prediction of the results of triaxial and hollow cylinder tests. The model separates the volumetric and shear components as the Boyce's and contour models. Karasahin [58] verified the model but found it very difficult to obtain the material parameters by nonlinear regression analysis |
| Ref. | Equation model | Variables and empirical constants | Observations |
|------|----------------|-----------------------------------|--------------|
| [99] | $M_r = k_1 \theta^{2.3}$ | $\theta$ is the bulk stress (sum of the principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$); $k_1$ is a constant parameter; | $k$-$\theta$ model that includes a failure term (the impact is insignificant until the failure is approached). The failure deviator stress is determined based on the confining pressure and the static triaxial shear stress. This model is based on laboratory triaxial testing (constant confining pressure) [70]. |
| [100] | Developed by Mayhew | $R_m = A_1 h^{k_2}$, $A_1$ is a regression coefficient determined from laboratory data and $R$ is the stress/strength ratio (deviator stress divided by the failure deviator stress). |
| [101] | and re-written by Brown and Selig | $e_v = \frac{1}{K} \frac{p}{q} \left[ 1 - C \left( \frac{q}{p} \right)^2 \right]$ | This model is similar to the contour model. The author plots the shear and volumetric strains in the $p$-$q$ space. The author concludes that the stress path length (included in the contour model) had little impact on the shear strain response. The model removes the stress path dependence and the equations of the contour model turn into the inelastic modified version of the Boyce model. |
| [102] | $\varepsilon_v = \frac{1}{K_1} \frac{p}{q} \left[ 1 - C \left( \frac{q}{p} \right)^2 \right]$ | $p$ is the mean normal stress; $q$ is the deviator stress; $\varepsilon_{v,r}$ is the recoverable volumetric strain and $\varepsilon_{x,r}$ is the shear strain; $K$ is the bulk modulus; $G$ is the shear stress. |
| [103] | $M_r = N_1 q N_2 (\sigma_3)^{N_3}$ | $\sigma_3$ is the confining pressure; $q$ is the deviator stress. |
| [80] | $R_{m1} = 0.98 - 0.28(w - w_{opt}) + 0.029(w - w_{opt})^2$ | $R_{m1} = M_r/M_{r,opt}$ for the case of constant dry density; $M_r$ is the resilient modulus at moisture content $w$ (%) and the same dry density as $M_{r,opt}$; $M_{r,opt}$ is the resilient modulus at the maximum dry density and optimum moisture content $w_{opt}$ (%) for any compaction effort. |
| [86] | $R_{m1} = M_r/M_{r,opt}$ for the case of constant dry density; $M_r$ is the resilient modulus at moisture content $w$ (%) and the same dry density as $M_{r,opt}$; $M_{r,opt}$ is the resilient modulus at the maximum dry density and optimum moisture content $w_{opt}$ (%) for any compaction effort. | Model based on laboratory triaxial testing (constant confining pressure) [70]. This model includes the soil’s physical state by two quantities: moisture content and dry density that are related to the soil-compaction curve. The results are based on repeated load triaxial test results on fine-grained soils from literature (Seed, Chan [104]; Sauer and Montsmith [105]; Culley [106]; Robnett and Thompson [107]; Fredlund, Began [93]; Edil and Moran [108]; Kirwan, Farrell [109]; Ellino and Davidson [110]). The power-law model was selected for the representation of the relationship between resilient modulus and deviator stress. |
Table 21 continued

| Ref. | Equation model | Variables and empirical constants | Observations |
|------|----------------|-----------------------------------|--------------|
| [86] | $R_m = 0.96 - 0.18\{w - w_{opt}\} + 0.0067(w - w_{opt})^2$ | $R_{m2} = M/M_{r,opt}$ for constant compaction effort; $M_r$ is the resilient modulus at moisture (%) and the same compaction effort as $M_{r,opt}$ | This model includes the soil’s physical state by two quantities: moisture content and dry density that are related to the soil-compaction curve. |
| [116] | $E_v = \frac{p^*}{\rho_0} \left[ \gamma + 2 \frac{A - 1}{18G_1} (\frac{q}{p^*})^2 + \frac{\gamma - 1}{3G_1} \left( \frac{q}{p^*} \right) \right]$ | $p^* = \frac{5G}{3\gamma_1^2} + (\gamma_1 - 3) \quad \gamma$ is the coefficient of anisotropy; The parameter $\gamma$ means that the material is isotropic (when $\gamma = 1$) and the material behaves more anisotropically as $\gamma$ deviates further from $\gamma = 1$; when $\gamma < 1$, the material is stiffer vertically than horizontally; | The power-law model was selected for the representation of the relationship between resilient modulus and deviator stress This model is based on the isotropic Boyce’s model and is also known as the anisotropic Boyce model. |
| [117] | $E_v = \frac{\partial \sigma_v}{\partial e_v} = E_v, \sigma_v = 0, v = E_v / \sigma_v \quad F(e) / F(0)$ | $\sigma_v = \frac{5G}{3\gamma_1^2} + (\gamma_1 - 3) \quad \gamma$ is the coefficient of anisotropy; The parameter $\gamma$ means that the material is isotropic (when $\gamma = 1$) and the material behaves more anisotropically as $\gamma$ deviates further from $\gamma = 1$; when $\gamma < 1$, the material is stiffer vertically than horizontally; | The power-law model was selected for the representation of the relationship between resilient modulus and deviator stress This model is based on the isotropic Boyce’s model and is also known as the anisotropic Boyce model. |
| [118] | $M_\tau = 10^4 k_1 p_0 \left( \frac{\theta}{p_0} \right)^{k_1} \left( \frac{\tau_{oct}}{p_0} + 1 \right)^{k_2}$ | $A = a + \frac{b-a}{1+exp(-\beta(\delta-S_{opt}))}$ | The cross-anisotropic-hypo-quasi-elasticity model This model depends on four material constants ($E_v, v, n$ and $a$). The model can predict nonlinear and anistropic resilient stress–strain properties for general stress paths under general stress conditions | The cross-anisotropic-hypo-quasi-elasticity model This model depends on four material constants ($E_v, v, n$ and $a$). The model can predict nonlinear and anistropic resilient stress–strain properties for general stress paths under general stress conditions. |

Cross-anisotropic-hypo-quasi-elasticity model

This model depends on four material constants ($E_v, v, n$ and $a$). The model can predict nonlinear and anisotropic resilient stress–strain properties for general stress paths under general stress conditions.
This model can be used in the case of fine-grained and coarse-grained materials (the difference is expressed in terms of the parameters $\alpha$, $\beta$, and $k_m$). The model does not combine the effects of the stress state and the moisture content.

The model estimated the change of the resilient modulus at a certain moisture content and a certain known stress state. Based on an intensive literature review study, it was developed to characterize the effect of moisture changes in moduli due to changes in moisture. This is an analysis based on repeated load triaxial testing carried out on recompacted soils.

### References

1. Li D, Selig ET (1996) Cumulative plastic deformation for fine-grained subgrade soils. J Geotech Geoenviron Eng 122(12):1006–1013
2. Luo X et al (2017) Mechanistic-empirical models for better consideration of subgrade and unbound layers influence on pavement performance. Transp Geotech 13:52–68
3. Mones Ruiz HA, Grabe PJ, Maina JW (2019) A mechanistic-empirical method for the characterisation of railway track formation. Transp Geotech 18:10–24
4. Alabassi Y, Hussein M (2021) Geomechanical modelling of railroad ballast: a review. Archiv Comput Methods Eng 28(3):815–839
5. Ling X et al (2017) Permanent deformation characteristics of coarse grained subgrade soils under train-induced repeated load. Adv Mater Sci Eng 7:15
6. Mostaqr Rahman Md, Gassman SL (2019) Permanent deformation characteristics of coarse grained subgrade soils using repeated load triaxial tests. In: Eighth international conference on case histories in geotechnical engineering, Philadelphia. pp 599–609
7. Lu C et al (2021) Resilient and permanent deformation behaviors of construction and demolition wastes in unbound pavement base and subbase applications. Transp Geotech 28
8. AASHTO (1994) Standard test method for determining the resilient modulus of soils and aggregate materials
9. Liang RY, Rababah S, Khasawneh M (2008) Predicting moisture-dependent resilient modulus of cohesive soils using soil suction concept. J Transp Eng 134(1):34–40
10. Nguyen BT, Mohajerani A (2016) Resilient modulus of fine-grained soil and a simple testing and calculation method for determining an average resilient modulus value for pavement design. Transp Geotech 7:59–70
11. Titi HH, Matar MG (2018) Estimating resilient modulus of base aggregates for mechanistic-empirical pavement design and performance evaluation. Transp Geotech 17:141–153
12. Barksdale RD (1972) Laboratory evaluation of rutting in base course materials. In: Proc. 3rd int. conf. on the structural design of asphalt pavements, London, pp 161–174

**Acknowledgements** This work was partially carried out under the framework of In2Track, a research project of Shift2Rail. This work was partly financed by FCT/MCTES through national funds (PID-DAC) under the R&D Unit Institute for Sustainability and Innovation in Structural Engineering (ISISE), under reference UIDB/04029/2020. It has been also financially supported by national funds through FCT—Foundation for Science and Technology, under grant agreement [PD/BD/127814/2016] attributed to Ana Ramos.

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.
13. Monisinth CL, Ogawa N, Freeme CR (1975) Permanent deformation characteristics of subgrade soils due to repeated loading. Transp Res Rec 537:1–17
14. Puppala AJ, Mohammad LN, Allen A (1999) Permanent deformation characterization of subgrade soils from RLT test. J Mater Civ Eng 11(4):274–282
15. Li D, Selig ET (1998) Method for railroad track foundation design. I: development. J Geotech Geoenviron Eng 124(4):316–322
16. Li D, Selig ET (1998) Method for railroad track foundation design. II: applications. Techn Pap 124(4):323–329
17. Burrow M, Bowness D, Ghataoa G (2007) A comparison of railway track foundation design methods. Proc Inst Mech Eng Part F J Rail Rapid Transit 221(1):1–12
18. Sun QD, Indraratna B, Nimbalkar S (2016) Deformation and degradation mechanisms of railway ballast at high frequency cyclic loading. J Geotech Geoenviron Eng 142(1):04015056
19. Ramos A, Correia AG, Indraratna B et al (2020) Mechanistic-empirical permanent deformation models: Laboratory testing, modelling and ranking. Transp Geotech 23:100326
20. Coronado O, Caicedo B, Taibi S et al (2011) A macro geomechanical approach to rank non-standard unbound granular materials for pavements. Eng Geol 119(1–2):64–73
21. Kuttah D (2021) Determining the resilient modulus of sandy subgrade using cyclic light weight deflectometer test. Transp Geotech 27:100482
22. Schulz-Poblete MV, Gräbe PJ, Jacobsz SW (2019) The influence of soil suction on the deformation characteristics of railway formation materials. Transp Geotech 18:111–123
23. Neidhart T, Shultz G (2011) Dynamic stability of railway tracks—DyStatIFIT, an innovation in testing. In: Géotechnique ferroviaire. Symposium international, Paris
24. Powrie W, Pen LL, Milne D, Thompson D (2019) Train loading effects in railway geotechnical engineering: ground response, analysis, measurement and interpretation. Transp Geotech 21:100261
25. Grossoni I, Powrie W, Zervos A (2021) Modelling railway ballasted track settlement in vehicle-track interaction analysis. Transp Geotech 26:100433
26. Sayeed MA, Shahin MA (2017) Design of ballasted railway track foundations using numerical modelling. Part I Dev Can Geotech J 55(3):353–368
27. Indraratna B et al (2020) Laboratory study on subgrade fluidization under undrained cyclic triaxial loading. Can Geotech J 57(11):1767–1779
28. Nguyen TT, Indraratna B (2021) Rail track degradation under mud pumping evaluated through site and laboratory investigations. Int J Rail Transp. https://doi.org/10.1080/23248378.2021.1878947
29. Burrow Mpn, Bowness D, Ghataoa GS (2006) A comparison of railway track foundation design methods. U.o.B. School of Engineering, Edgbaston, Birmingham
30. Li D et al (2016) Railway geotechnics. CRC Press - Taylor & Francis Group, Boca Raton
31. UIC (2008) UIC Code 719 R. Earthworks and track bed for railway lines
32. Boler H, Mishra D, Hou W et al (2018) Understanding track substructure behavior: Field instrumentation data analysis and development of numerical models. Transp Geotech 17(Part A):109–121
33. Li X, Vanapalli SK (2021) Simulation of progressive shear failure in railway foundation. Transp Geotech 29:100550
34. Liu X, Zhao P, Dai F (2011) Advances in design theories of high-speed railway ballastless tracks. J Modern Transp 19(3):154–162
35. Yang S, Huang W, Tai Y (2005) Variation of resilient modulus with soil suction for compacted subgrade soils. Transp Res Rec J Transp Res Board 1913(1):99–106
36. Sawangsuriya A, Edil TB, Benson CH (2009) Effect of suction on resilient modulus of compacted fine-grained subgrade soils. Transp Res Rec J Transp Res Board 2101(1):82–87
37. Coronado O, Caicedo B, Taibi S et al (2016) Effect of water content on the resilient behavior of non standard unbound granular materials. Transp Geotech 7:29–39
38. Maadani O, El Halim AOA (2017) Environmental considerations in the AASHTO mechanistic-empirical pavement design guide: impacts on performance. J Cold Reg Eng 31(3):04017008
39. Tatsuoka F, GomesCorreia A (2018) Importance of controlling the degree of saturation in soil compaction linked to soil structure design. Transp Geotech 17:3–23
40. EN13286–7, Unbound and hydraulically bound mixtures. In Part 7: Cyclic load triaxial test for unbound mixtures 2004, CEN-European Committee for Standardization, Brussels
41. Pereira C, Nazarian S, Correia AG (2017) Extracting damping information from resilient modulus tests. J Mater Civ Eng 29(12):04017233–1–6
42. AASHTO (2007) Standard specifications for transportation materials and methods of sampling and testing. 2: Tests. In: T-307–93. 2007: Washington, DC
43. Ooi PSK, Archilla AR, Sandefur KG (2004) Resilient modulus models for compacted cohesive soils. Transp Res Rec J Transp Res Board 1874(1):115–124
44. Zhang J, et al (2019) Prediction of resilient modulus of compacted cohesive soils in South China. Int J Geomech 19(7):04019068
45. Nimbalkar S, Punetha P, Kaeunruen S (2020) Performance improvement of ballasted railway tracks using geocells: present state of the art, in geocells: advances and applications. Springer, Singapore
46. Thompson M, Robnett Q (1976) Resilient properties of subgrade soils. Report No. FHWA-IL-UI-160, Federal Highway Administration, Washington DC
47. Moosazadeh JM, Witzczak MW (1981) Prediction of subgrade moduli for soil that exhibits nonlinear behavior. Transp Res Rec J Transp Res Board 810:9–17
48. Brown SF, Lashine AKF, Hyde AFl (1975) Repeated load triaxial testing of a silty clay. Geotechnique 25(1):95–114
49. Fredlund DG, Bergan AT, Sauer EK (1975) Deformation characterization of subgrade soils for highways and runways in northern environments. Can Geotech J 12(2):213–223
50. Drumm EC, Pierce TJ (1990) Estimation of subgrade resilient modulus from standard tests. J Geotech Eng 116(5):774–789
51. Shackel, B., The derivation of complex stress-strain relations. In: Proc. 8th Int. Conf. on Soil Mech. and Found. Engrg., Moscow, 1973:353–359
52. Sas W, Głuchowski A, Gabryś K et al (2017) Resilient modulus characterization of compacted cohesive subgrade soil. Appl Sci 7(370):1–20
53. Gomes Correia A, Hornych P, Akou Y (1999) Review of models and modelling of unbound granular materials In: Proceedings of an international workshop on modelling and advanced testing for unbound granular materials. Lisbon
54. Dunlap WS (1963) A report on a mathematical model describing the deformation characteristics of granular materials. Technical report 1, Project 2–8–62–27. 1963, Texas A &M University: TTI
55. Brown SF, Pell PS (1967) An experimental investigation of the stresses, strains and deflections in layered pavement structure subjected to dynamic loads. In Proceedings of the 2nd international conference on structural design of asphalt pavements. Ann Arbor, pp 384–403
56. Uzan J (1985) Characterization of granular material. Transp Res Rec 1022:52–59
57. Koliosja P (1997) Factors affecting deformation properties of coarse grained granular materials. In: Proceedings of the fourteenth international conference on soil mechanics and foundation engineering, Hamburg.
58. Karasahin M (1993) Resilient behaviour of granular materials for analysis of highway pavements. University of Nottingham, Nottingham
59. ARA Inc (2004) ERES Division, Guide for mechanistic-empirical design of new and rehabilitated pavement structures, National Research Council
60. Lekarp F, Isacsson U, Dawson A (2000) Permanent strain response of unbound aggregates. J Transp Eng 126(1):76–83
61. Boyce JR (1980) A non-linear model for the elastic behaviour of granular materials under repeated loading. In: Proc. Int. symp. soils under cyclic and transient loading. Swansea, pp 285–294
62. Jouve P, Elhannani M (1994) Application de modèles non linéaires au calcul des chaussées souples. Bulletin des liaison Laboratoires des Pont et Chaussées 190:39–55
63. Hornych P, Kazai A, Piau JM (1998) Study of the resilient behaviour of unbound granular materials. In: Proceedings 5th int. conf. on the bearing capacity of roads and airfields
64. Fortunato E (2005) Renovação de plataformas ferroviárias Estudos relativos à capacidade de carga. Faculty of Engineering of the University of Porto, Porto
65. Salour F, Erlingssson S (2015) Resilient modulus modelling of unsaturated subgrade soils: laboratory investigation of silty sand subgrade. Road Mater Pavement Des 16(3):553–568
66. Erlingssson S, Rahman S, Salour F (2017) Characteristic of unbound granular materials and subgrades based on multi stage RLT testing. Transp Geotech 13:28–42
67. Cary CE, Zapata CE (2011) Resilient modulus for unsaturated unbound materials. Road Mater Pavement Des 12(3):615–638
68. Gomes Correia A (2000) Influence of compaction conditions on resilient and permanent deformations of aggregates mixtures of granite. In: Gomes Correia A, Quibel A (eds) Compaction of soils and granular materials. Presse Nationale des Ports et des Chaussées, Paris, pp 27–39
69. Duong TV et al (2016) Effects of water and fines contents on the resilient modulus of the interlayer soil of railway substructure. Acta Geotech 11(1):51–59
70. Lekarp F, Isacsson U, Dawson A (2000) State of the art. J resilient response of unbound aggregates. J Transp Eng 126(1):66–75
71. Raad L, Minassian G, Gartin S (1992) Characterization of saturated granular bases under repeated loads. Transp Res Rec 1369:73–82
72. Vuong B (1992) Influence of density and moisture content on dynamic stress-strain behaviour of a low plasticity crushed rock. Road Transp Res 1(2):88–100
73. Khasawneh MA, Al-jamal NF (2019) Modeling resilient modulus of fine-grained materials using different statistical techniques. Transp Geotech 21:100263
74. Hanandehe S, Ardah A, Abu-Farsakh M (2020) Using artificial neural network and genetics algorithm to estimate the resilient modulus for stabilized subgrade and propose new empirical formula. Transp Geotech 24:100358
75. Lima CDA, Motta LMG, Aragão FTS (2021) A permanent deformation predictive model for fine tropical soils considering the effects of the compaction moisture content on material selection. Transp Geotech 28:100534
76. Chen R, Chen J, Zhao X et al (2014) Cumulative settlement of track subgrade in high-speed railway under varying water levels. Int J Rail Transp 2(4):205–220
77. Seed HB, Mitry FG, Monismith CL et al (1967) Prediction of flexible pavement deflections from laboratory repeated load tests, No 35, Transportation Research Board, National Cooperative Highway Research Program
78. Hicks RG, Monismith CL (1971) Factors influencing the resilient properties of granular materials. Highway Res Rec 345:15–31
79. Gomes Correia A., et al (2001) Small strain stiffness under different isotropic and anisotropic stress conditions of two granular granite materials. Advanced laboratory stress-strain testing of geomaterials, Balkema, Swets & Zeitlinger
80. Garg N, Thompson MR (1997) Triaxial characterization of Minnesota road research project granular materials. Transp Res Rec 1577(1):27–36
81. Dawson AR, Gomes Correia A (1993) The effects of subgrade clay condition on the structural behaviour of road pavements, Flexible Pavements, Balkema
82. Salour F, Erlingssson S (2016) Characterization of permanent deformation of silty sand subgrades from multistage RLT tests. Procedia Eng 143:300–307
83. Huurman M (1997) Permanent deformation in concrete block pavement. Delft University of Technology, Delft
84. Puppala AJ, Saride S, Chomtid S (2009) Experimental and modeling studies of permanent strains of subgrade soils. J Geotech Geoenviron Eng 135(10):1379–1389
85. Gidel G, Denys B, Hornych P et al (2001) A new approach for investigating the permanent deformation behaviour of unbound granular material using the repeated load triaxial apparatus. Bulletin des Laboratoires des Pont et Chaussées 233:5–21
86. Li D, Selig ET (1994) Resilient modulus for fine-grained subgrade soils. J Geotech Eng 120(6):939–957
87. Rahim AM, George KP (2005) Models to estimate subgrade resilient modulus for pavement design. Int J Pavement Eng 6(2):89–96
88. Ng CWW, Zhou C, Yuan Q et al (2013) Resilient modulus of unsaturated subgrade soil: experimental and theoretical investigations. Can Geotech J 50(2):223–232
89. Parreira AB, Goncalves RF (2000) The influence of moisture content and soil suction on the resilient modulus of a lateritic subgrade soil. In: GeoEng—an international conference on geotechnical and geological engineering. Melbourne
90. Monismith CL, Seed HB, Mitry, FG et al. (1967) Prediction of pavement deflections from laboratory tests. In: Proc., 2nd int. conf. struct. des of Asphalt Pavements, Ann Arbo, pp 53–88
91. Brown SF, Pell PS (1967) An experimental investigation of the stresses, strains and deflections in a layered pavement structure subjected to dynamic loads. In: Proc., 2nd int. conf. struct. des. of Asphalt Pavements. 1967, pp. 487–504
92. Thompson MR, Robnett QL (1976) Final report, resilient properties of subgrade soils. FHWA-IL-U1–160, Univ. of Illinois, Urbana, 111
93. Fredlund DG, Began AT, Wong PK (1977) Relation between resilient modulus and stress conditions for cohesive subgrade soils. Transp Res Rec 62:73–81
94. Sweere GTH (1990) Unbound granular bases for roads. Dissertation, Delft University of Technology, Netherlands
95. Brown SF, Pappin JW (1985) Modeling of granular materials in pavements. Trans Res Rec 1022:45–51
96. Johnson TC, Berg RL, DMillio A (1986) Frost action predictive techniques: an overview of research results. Transp Res Rec 1089:147–161
97. Tam WA, Brown SF (1988) Use of the falling weight deflectometer for in situ evaluation of granular materials in pavements. In: Proc., 14th ARRB conf., Canberra, pp 155–63
98. Thom N (1988) Design of road foundations. Dissertation, University of Nottingham
99. Elliot RP, Lourdesnathan D (1989) Improved characterization model for granular bases. Transp Res Rec 1277:128–133
100. Mayhew HC (1983) Resilient properties of unbound road base under repeated triaxial loading. TRRL Laboratory Report, Crowthorne, UK
101. Brown SF, Selig ET (1991) The design of pavement and rail track foundations. In: O’Reilly MP, Brown SF (eds) Cyclic loading of soils: from theory to design. Blackie and Son Ltd., Glasgow, Scotland, pp 249–305
102. Elhannani M (1991) Modélisation et simulation numérique des Chaussées Souplées. Dissertation, University of Nantes, Nantes
103. Pezo RF (1993) A general method of reporting resilient modulus tests of soils—a pavement engineer’s point of view. In: 72nd Annu. Meeting of the TRB. Washington, DC
104. Seed HB, Chan CK, Lee CE (1962). Resilience characteristics of subgrade soils and their relation to fatigue failures in asphalt pavement. In: Proc. first int. conf. on struct. design of asphalt Pavements. Ann Arbor, pp 77–113
105. Sauer EK, Monismith CL (1968) Influence of soil suction on behavior of a glacial till subjected to repeated loading. Highway Res Rec 215:8–23
106. Culley RW (1971) Effect of freeze-thaw cycling on stress-strain characteristics and volume change of a till subjected to repetitive loading. Can Geotech J 8(3):359–371
107. Robnett QL, Thompson MR (1976) Effect of lime treatment on the resilient behavior of fine-grained soils. Transp Res Rec 560:11–20
108. Edil TB, Moran SE (1979) Soil-water potential and resilient behavior of subgrade soils. Transp Res Rec 705:54–63
109. Kirwan RW, Farrell ER, Maher MLJ (1979) Repeated load parameters of a glacial till related to moisture content and density. In: 7th European conference on soil mechanics and foundation engineering, England, pp 69–74
110. Eltino MK, Davidson JL (1989) Modeling field moisture in resilient moduli testing. In: Resilient moduli of soils: laboratory conditions, ASCE Geotech. Special Publication, No. 24, ASCE. New York, pp 31–51
111. Kallas BF, Riley J (1967) Mechanical properties of asphalt pavement materials. In: 2nd int. conf. on struct. design of asphalt pavements. Ann Arbor, pp 931–952
112. Shifley LH Jr, Monismith CL (1968) Test road to determine the influence of subgrade characteristics on the transient deflections of asphalt concrete pavements. In: Report No. TE 68–5, office of res. services. Univ. of California, Berkeley
113. Tanimoto K, Nishi M (1970) On resilience characteristics of some soils under repeated loading. Soils Found 10(1):75–92
114. Edris EVJ, Lytton RL (1976) Dynamic properties of subgrade soils, including environmental effects. TTI-2–18–74–164–3, Texas Transp. Inst., Texas A&M Univ., College Station, Tex.
115. Pezo RF et al (1991) Aspects of a reliable resilient modulus testing system. In: Transp. res. Board preprint, 70th annual meeting. Washington, D.C
116. Hornych P, Kazai A, Piau JM (1998) Study of the resilient behaviour of unbound granular materials. In: Proc. 5th conference on bearing capacity of roads and airfields. Trondheim.
117. Tatsuoka F et al. (1999) Non-linear resilient behaviour of unbound granular materials by the cross-anisotropic hypo-quasi-elasticity model. In: Unbound granular materials - laboratory testing, in-situ testing and modelling, Lisbon
118. Witzczak MW, Andrei D, Houston WN (2000) Resilient modulus as function of soil moisture - summary of predictive models. In Development of the 2002 guide for the development of new and rehabilitated pavement structures, NCHRP 1–37 A, Inter team technical report (seasonal 1). Arizona State University, Tempe, Arizona