Chiral String-Soliton Model for the light chiral baryons

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Abstract

The Chiral String-Soliton Model is a joining of the two notions about the light chiral baryons: the chiral soliton models (like the Skyrme model) and the Quark-Gluon String models. The ChSS model is based on the Effective Chiral Lagrangian which was proposed in [1]. We have studied the physical properties of the light chiral baryon within the framework of this ChSS model.

Introduction

The problem of finding an appropriate theoretical model for the light baryons is one of the essential long-standing tasks in hadron physics. Many models of the light baryon were proposed so far. Ideologically all these models can be divided by two subclasses. The first group of models is based on the idea of the realistic chiral solution which firstly was proposed by Skyrme [2]. The baryon is treated within this approach as a topological soliton of the non-linear chiral meson field. The Chiral bag model [3] which was very popular during last time is a development of this idea. Despite of the fact that chiral bag model can describe the quark degrees of freedom in the baryon, there are no natural picture of the confinement in these models. The simulation on the lattice explicitly shows us that the confinement phenomenon in QCD is connected with the formation of the gluon string between the quarks. This phenomenon can be described phenomenologically by means of the effective linear interaction between the quarks. The models of such type form the second class of the models of the light baryons. A weak place of such models is that it is difficult to take into the account the chiral degrees of freedom of the meson cloud around baryon. It is very sad because the chiral effects play the essential role in the physics of light baryons. So the question is how to combine these two different points of view on the light baryons into the framework of a single model?

An important step has been made recently towards such model [1]. The effective chiral Lagrangian which contains the effects of both confinement and chiral symmetry breaking was proposed. It was shown that this Lagrangian is reproduced the Gell-Mann-Oakes-Renner relations for the light mesons and correctly describes the spectrum of radial excitations of $\pi$ and $K$ mesons. In our work we use this model for description of the light baryons.

1 Effective Chiral Lagrangian and soliton model

In this paper we explore bound states of valence quark using the chiral invariant effective $\sigma$-model lagrangian. Our model is a rather similar to the Soliton-Bag model [4]. We allow the additional string potential for quarks to realize the confinement phenomenon. The structure of our effective lagrangian is connected with the effective chiral theory which was derived from QCD in the framework of the field correlator method. Using the this method one can integrate
out gluon field $A_\mu$ and find the contributions from fermion determinant which generate the meson effective lagrangian. As a result one can get

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\pi + \mathcal{L}_{\text{int}},$$

where $\mathcal{L}_\psi = \overline{\psi}(i\gamma\partial - m_0)\psi$ and $\mathcal{L}_\pi$ is a self-interacting part of the meson field. The bullet point of the model is the interaction part of the lagrangian density $\mathcal{L}_{\text{int}}$:

$$\mathcal{L}_{\text{int}} = M_{\text{eff}}(x, y) \exp(i\gamma_5(\vec{\tau}\vec{\varphi})),$$

where $M_{\text{eff}}$ is the effective quark mass operator. It was shown in [1] that

$$M_{\text{eff}} \approx \sigma |\vec{x} - \vec{y}|$$

for large distance between quarks; $\sigma$ is the string tension. The physical interpretation of this result is quite clear. If the distance between the quarks becomes large, the effective quark-meson coupling becomes large too. This effect is not unexpected by reason of the string broken phenomenon with the production of the light meson.

Now we promote this idea of the effective quark-meson coupling for the baryon physics. In the first approximation let us consider the quarks which live in the self-consistent linear potential with the center in the point of the string junction. It means that in this approximation we neglect the contributions from quark-antiquark interaction in comparison with the interaction with the string junction. It is so-called $Y$-type picture of the baryon.

Let us consider the self-interacting, spin-0, isosinglet field $\phi(x)$, and the isotriplet pion field $\vec{\pi}(x)$ which interacts with the isodouplet, spin-$\frac{1}{2}$ quark field $\psi(x)$. The Lagrangian density $\mathcal{L}$ can be written as

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \vec{\pi} \partial^{\mu} \vec{\pi} + \overline{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi - M_{\text{eff}} \overline{\psi}[(\phi + i\gamma_5(\vec{\pi}\vec{\tau}))/f_\pi] \psi \frac{\lambda^2}{4}(\phi^2 + \vec{\pi}^2 - f_\pi^2)^2.$$

For the light quarks $m_0 \ll \sqrt{\sigma}$ so let us take $m_0 = 0$.

In the main-field approximation the $\phi$ and $\vec{\pi}$ field are taken as classical, time-independent $c$-number fields. The quark field is expanded in single-particle modes $\psi_n$ which satisfy the Dirac equation

$$-i\gamma_\mu \partial^\mu \psi_n - M_{\text{eff}}[(\phi + i\gamma_5(\vec{\pi}\vec{\tau}))/f_\pi] \psi_n = \mathcal{E}_n \psi_n. \quad (1)$$

If one put $N$ quarks into the lowest mode with energy $\mathcal{E}_0$, the total energy of the system is given by

$$E_{\text{tot}} = N\mathcal{E}_0 + \int d^3x [\frac{1}{2}(\partial_\phi)^2 + \frac{1}{2}(\vec{\partial}\vec{\pi})^2 + \frac{\lambda^2}{4}(\phi^2 + \vec{\pi}^2 - f_\pi^2)^2]. \quad (2)$$

The total energy $E_{\text{tot}}$ is a functional of the chiral fields $\phi$ and $\vec{\pi}$. At equilibrium point, the energy is stationary with respect to variations of the fields. One can get the equations of motion for meson fields by the minimizing of $E_{\text{tot}}$

$$-\vec{\partial}^2\phi + \lambda^2(\phi^2 + \vec{\pi}^2 - f_\pi^2)\phi = -NM_{\text{eff}}/f_\pi(\overline{\psi}_0\psi_0) \quad (3)$$

$$-\vec{\partial}^2\vec{\pi} + \lambda^2(\phi^2 + \vec{\pi}^2 - f_\pi^2)\vec{\pi} = -NM_{\text{eff}}/f_\pi(\overline{\psi}_0i\gamma_5\vec{\tau}\psi_0). \quad (4)$$
2 The hedgehog solution and the chiral angle parametrization

The equations (1), (3) and (4) are the self-consistent set of equations for Dirac orbitals \( \psi \) and chiral fields \( \phi \) and \( \pi \). In order to obtain the minimum energy solution, let us consider the spherically symmetrical hedgehog ansatz for the chiral fields

\[
\phi = \phi(r), \quad \pi = \pi(r)
\]

and the s-state solution for Dirac orbital

\[
\psi = \begin{pmatrix} u(r) \\ i(\bar{\sigma} \pi) v(r) \end{pmatrix} \chi
\]

where \( \chi \) is a state in which the spin and isospin of the fermion couple to zero. After substituting this ansatz into the equation of motion one can get:

\[
-\frac{1}{r} \frac{d^2}{dr^2}(r\phi) + \lambda^2(\phi^2 + \pi^2 - f^2) \phi = -N \frac{1}{f_\pi} \sigma r (u^2 - v^2)
\]

\[
-\frac{1}{r} \frac{d^2}{dr^2}(r\pi) + \frac{2}{r^2} \pi + \lambda^2(\phi^2 + \pi^2 - f^2) \pi = -2N \frac{1}{f_\pi} \sigma ru v
\]

\[
\frac{du}{dr} + \left( \mathcal{E} + \frac{1}{f_\pi} \sigma r \phi \right) v + \frac{1}{f_\pi} \sigma r \pi u = 0
\]

\[
\frac{dv}{dr} + \frac{2}{r} v + \left( -\mathcal{E} + \frac{1}{f_\pi} \sigma r \phi \right) u - \frac{1}{f_\pi} \sigma r \pi v = 0
\]

If the constant \( \lambda \) is large enough, the chiral fields restrict to the chiral circle \( \phi^2 + \pi^2 = f^2_\pi \). In this case it is possible to parameterize the chiral fields by means of a chiral angle \( \theta(r) \):

\[
\phi(r) = f_\pi \cos \theta(r), \quad \pi(r) = f_\pi \sin \theta(r).
\]

The equations of motion in terms of the chiral angle are:

\[
\theta'' + \frac{2}{r} \theta' - \frac{1}{r^2} \sin 2\theta = -\frac{N}{f^2_\pi} \sigma r [(u^2 - v^2) \sin \theta + 2uv \cos \theta]
\]

\[
\frac{du}{dr} + (\mathcal{E} + \sigma r \cos \theta) v + \sigma r \sin \theta u = 0
\]

\[
\frac{dv}{dr} + \frac{2}{r} v + ( -\mathcal{E} + \sigma r \cos \theta ) u - \sigma r \sin v = 0
\]

The fermion wave function is normalized to

\[
4\pi \int_0^\infty r^2 dr (u^2 + v^2) = 1
\]

The energy of the system in terms of the chiral angle is:

\[
E_{tot} = N\mathcal{E} + 2\pi f^2_\pi \int_0^\infty r^2 dr (\theta'^2 + \frac{2}{r^2} \sin^2 \theta)
\]

The chiral angle approximation is a very interesting due to numerical and theoretical reasons. We can consider this approximation as a first step of our consideration.
3  Chiral String-Soliton Model: numerical results

Before we start to discuss the numerical results of our simulation of the equations of motion (5), (6) and (7) let us consider the Dirac particle in the string potential without chiral soliton. One obtains the equations of motion in this case by substituting the vacuum value $\theta = 0$ into (6) and (7):

$$\frac{du}{dr} + (E + \sigma r) v = 0 \quad (9)$$

$$\frac{dv}{dr} + 2rv + (-E + \sigma r) u = 0 \quad (10)$$

The energy spectrum in this case can be found numerically and the lowest energy level is equal to $E_0 = 1.61\sqrt{\sigma}$. The value of $\sqrt{\sigma}$ was well studied in the lattice calculations and varies from 400 MeV to 420 MeV. Thereby the total energy of the system for $N = 3$ is

$$E_{str} = N E_0 \approx 2 \text{ GeV},$$

and this is obviously much more than the physical scale of the light baryon mass (about 1 GeV). Of course this fact is a sequence of the ignoring of the chiral effects. And the main motivation of our work is to show how such effects can be taken into account. Now we will demonstrate that the chiral soliton around baryon leads to diminishing of the total mass of the system down to the physical value about 1 GeV.

Now let us consider the set of equations (5), (6) and (7). As we point out the soliton sector for chiral field will be interesting for us. The topological chiral soliton corresponds to the following boundary condition:

$$\theta(r) \to \pi, \quad \text{as} \quad r \to 0;$$

$$\theta(r) \to 0, \quad \text{as} \quad r \to \infty.$$

The numerical results for the total energy of the self-consistent system of the $N = 3$ massless fermion and chiral topological soliton as the function of $\sqrt{\sigma}$ are presented in Fig. 1.

First of all, we see that for physical value of $\sqrt{\sigma}$ the total mass of the baryon is about 1230 MeV. It is close to the experimental data for the mass of $\Delta$-baryon: $M_{\Delta} = 1232$ MeV. Of course, is such simple model there is no possible to reproduce mass of the proton directly because the spin effects are not taken into account. It is possible to do by using of the standard technique [5] and it will be the object of our next studies.

Another interesting feature of this result is that for the physical values of the string tension $\sigma$ the energy of the Dirac orbital $E$ is very close to zero ($E = 0$ for $\sqrt{\sigma} = 394$ MeV). It means that the mass of the light baryons in this model almost forms due to the chiral soliton. The Dirac orbital is very essential for the stability of the system but it is clear that the meson cloud plays the dominant role in the dynamics. This fact is very interesting and intriguing, first of all due to the possible connection with the famous ”proton spin” paradox. We plan to discuss it in the future.

In Fig. 2 and Fig. 3 we show the radial functions and densities for the bound state of zero mode ($E = 0$ for $\sqrt{\sigma} = 394$ MeV). We see that the fermions are localized in a small region in the center and meson cloud surrounds this quark kernel.
In this paper we have studied the properties of the light baryon in the framework of the Chiral String-Soliton Model. In main-field approximation we interpret the baryon as the self-consistent bound state of the three valent quarks and meson soliton. We have shown that the generation of the meson cloud is energetically profitable in comparison with the bound state without chiral soliton. The Chiral String-Soliton Model leads to the good prediction for the mass of the light baryon.

Finally, we would like to emphasize that the ChSS model has no free parameter. Only the string tension one can vary in very short region. It means that the prediction power of this model is very strong. Another advantage of this approach is that the basis of this model is Effective Chiral Lagrangian which can derived from QCD \cite{1} and works very well in the meson sector. In our work we show that this Lagrangian is applicable for the physics of the light chiral baryon too.

References

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Figure 2: The radial functions $u(r)$, $v(r)$ and $\theta(r)$

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Figure 3: The radial density of the meson (1) and fermion (2) fields: 

1) \( (\theta'^2 + \frac{2}{r^2} \sin^2 \theta) \) and 
2) \( u^2 + v^2 \)