Perturbative QCD Applied to Baryons

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Abstract.
We review standard applications of perturbative QCD to baryon production, and argue by examining data that it is generally relevant at high but experimentally feasible momentum transfers. Then we consider some new initiatives, particularly meson photoproduction off baryons and the seeming quagmire of $\Delta(1232)$ electroproduction.

1 Introduction
This talk is a special one at a workshop dedicated to nonperturbative methods in baryon physics. It discusses the other side of things, namely perturbative QCD (pQCD) applied to baryons, with particular emphasis on applications to exclusive and semi-exclusive reactions.

We will start out in the next section discussing what I will call “standard old stuff,” reviewing methods of calculation and scaling and normalization predictions that are well known to many, and seeing in what kinematic regime pQCD seems to work and how well it works there. I might say now that I am an optimist, thinking that pQCD results can be valid when momentum transfers are only a few GeV. The “standard old stuff” will come in three headings, namely the scaling behavior expected for amplitudes at high momentum transfer, with comparison to data, the polarization behavior expected for amplitudes at high momentum transfer, with comparison to data, and some review of results that have been gotten in the few cases where normalized calculations are possible.

To balance the old, section 3 will present a selection of new initiatives using pQCD, focusing on semi-exclusive reactions and connections between low and high momentum transfer behavior of $\Delta(1232)$ electroproduction.

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2 Standard Old Stuff

2.1 Scaling—expectations and data

Perturbative QCD for exclusive reactions begins by drawing all the relevant lowest order Feynman diagrams. There can be many for a given process and calculating all of them can be time consuming. However, the scaling behavior is generally the same for all the diagrams, and can be ferreted out relatively easily. The general categories of processes are form factors at high momentum transfer, or quasi-elastic reactions at high $s$ at fixed large $\theta_{CM}$. An example of the latter, specifically for $\gamma p \rightarrow \pi^+ n$, is given in the Figure below. The momentum transfer dependence comes from the internal propagators—a $1/Q^2$ for each gluon propagator (where $Q$ is some momentum scale) and a $1/Q$ quark propagators—and a factor $Q$ for each quark line. 

Figure 1. One lowest order diagram for $\gamma p \rightarrow \pi^+ n$.

The amplitude represented by this diagram has four quark lines and three each of internal quark and gluon propagators. Hence

$$\mathcal{M} \propto Q^4 Q^{-3} (Q^2)^{-3} = Q^{-5} \propto s^{-5/2},$$

and the differential cross section is

$$\frac{d\sigma}{dt} = \frac{1}{16 \pi s^2} |\mathcal{M}|^2 \propto s^7.$$

Does it work? Here is a plot of $s^7 d\sigma/dt$ vs. $s$ for $\theta_{CM} = 90^\circ$.

Figure 2. Scaled cross section for $\gamma p \rightarrow \pi^+ n$.

The bumps at low $s$ are resonance excitations, and the pQCD expectation appears to succeed just above resonance region.
Form factors for electron elastic or quasi-elastic scattering from a hadron with $N$ constituents generally go like,

$$F(Q^2) \propto 1/(Q^2)^{N-1}. \tag{3}$$

For baryon elastic or transition form factors this means $F \propto 1/Q^4$. (At least the leading form factor falls like this; there may be form factors that are zero to leading order, which then fall faster.)

Paul Stoler \cite{4} has produced the following plots:

![Figure 3: Form factors for two transition form factors, divided by $F_{\text{dipole}}$.](image)

Figure 3. Form factors for two transition form factors, divided by $F_{\text{dipole}}$.

For reasons of space, we have shown only the nucleon to $N(1535)$ and to $\Delta(1232)$ transition form factors. The dipole form is $(1 + Q^2/0.71\text{GeV}^2)^{-2}$, so a flat curve is what pQCD predicts. There are also plots for the elastic case and the $N(1688)$ region, which look rather like the $N(1535)$. Hence the pQCD results are successful, except for the $\Delta(1232)$.

The $\Delta(1232)$ falls faster than the others. There is a reason within the pQCD framework for this and a discussion will come in section 2.3. Also, there has been a suggestion that the $N(1535)$ is a $\Lambda K$ bound state. This makes the minimum Fock component a 5 constituent state, with a faster form factor falloff according to Eqn (3). This is not supported by the data.

### 2.2 Polarization—expectations and data

The scaling rules tell us the leading scaling behavior, assuming nothing else suppresses the amplitude farther. In particular, there can be farther suppression if the helicity conservation rules are violated. The basic rule is that, neglecting quark mass and binding, the quark helicity is conserved in interactions with either gluons or photons. If all interactions are at close range, the orbital angular momentum of the quarks can be neglected, and then the helicity of the hadrons overall must be conserved. Each unit violation of the helicity conservation rule costs a factor of $O(m/Q)$ where $m$ is some mass scale and $Q$ is some momentum transfer scale \cite{2, 3}.

The nucleon electromagnetic form factors give a simple example. Thinking in the Breit frame, a transverse photon with helicity $+1$ hitting a nucleon with helicity $+1/2$ gives a final state nucleon also of helicity $+1/2$. Hadron helicity
is conserved; The previous rules apply. The result in terms of $G_M$ comes from

$$G_+ = \frac{1}{2 m_N} \langle R, \lambda' = \frac{1}{2}, \epsilon_\mu(+) \cdot j^\mu(0) | N, \lambda = \frac{1}{2} \rangle = \frac{Q}{m_N \sqrt{2}} G_M \propto \frac{1}{Q^3} \quad (4)$$

and so one gets $G_M \propto 1/Q^4$, which is well known to be true. However, bringing in a longitudinal photon leads to a final helicity of $-1/2$, and so the amplitude should be suppressed by a power of $Q$, and

$$G_0 = \frac{1}{2 m_N} \langle R, \lambda' = \frac{1}{2} | \epsilon^{(0)} \cdot j^{\mu}(0) | N, \lambda = \frac{1}{2} \rangle = G_E \propto \frac{1}{Q^4}. \quad (5)$$

Thus for the Pauli form factor $F_2$ (using $\tau \equiv Q^2/4m_N^2$),

$$F_2 = \frac{G_M + G_E}{1 + \tau} \propto \frac{1}{Q^6}. \quad (6)$$

Comparing to $F_1$ in the figure ($F_1 = G_E + \tau G_M/(1 + \tau) \propto 1/Q^4$), one sees that this prediction from hadron helicity conservation proves to be true in nature [5].

![Figure 4](image-url)  

**Figure 4.** Checking the $F_2$ scaling behavior vs. data.

### 2.3 Normalized calculations

When normalized calculations can be done, they become the heart of the perturbative predictions for exclusive reactions. For example, for some typical form factor the whole high momentum transfer calculation is

$$F(Q^2) = \int [dx][dy] \phi(x, Q^2) T(x, y, Q^2) \phi(y, Q^2) \quad (7)$$

Here $\phi(x)$ is the distribution amplitude for the final baryon, simply related to its wave function, and describes finding three quarks with substantially parallel momenta, with a tolerance related to the scale $Q$, and with momentum fractions $x_i$; $\phi(y)$ is the same for the initial state. The distribution amplitudes are only
weakly dependent on $Q$. The main, power law, $Q$ dependence comes from the amplitude $T$, which describes one quark absorbing a large momentum transfer $Q$ and sharing it with the other quarks so they are all parallel moving in the final state. It is calculated in perturbation theory.

The wave function or distribution amplitudes cannot be calculated in perturbation theory. One gets them using QCD sum rules to get moments of wave functions, which become constraints on model wave functions, and model wave functions have been offered by, for the nucleon, CZ and COZ (Chernyak, Oglublin, Zhitnitsky) and KS (King-Sachrajda) and GS (Gari-Stefanis).

These all lead to good results for proton $G_M$ (of course),

$$Q^3 G_+(p \to p) \approx 0.75 \text{ GeV}^3,$$

with

$$Q^3 G_+(N \to \Delta) \approx 0.08 \text{ GeV}^3$$

and

$$Q^3 G_+(p \to N^*(1535)) \approx 0.46 \text{ GeV}^3.$$  

For definiteness, these use KS for the nucleon and CP (Carlson-Poor) for the $\Delta$ and $S_{11}$ (with apologies to FOZZ (Farrar, Oglublin, Zhang, Zhitnitsky) and BP (Bonekamp-Pfeil)) [6].

The asymptotic $\Delta$ transition amplitude is small. Hence what we see in the data shown earlier is still the subleading part of the transition. A deep reason not known. Still, we can claim that the DDR (Disappearing Delta Resonance) is understood within pQCD.

A quick summary of this quick review is that pQCD has a decent record in explaining data at high but feasible momentum transfers, for single baryons.

### 3 New Initiatives

#### 3.1 Semi-exclusive reactions

A semi-exclusive reaction is one where one or a few, but not all, of the hadrons in a final are observed. We will focus on pion photoproduction $[^3][8], \gamma p \to \pi X$.

We will also suppose that the transverse momentum of the pion is high, and that the recoil mass $m_X$ is high. These provisos ensure that perturbation theory can be used in the calculations.

We hope to learn or supplement what we know about:

- the polarized and unpolarized gluon distributions of the target,
- the quark distributions for high $x$, and
- the pion wave function at short range.

To proceed, let the transverse momenta be high enough (say $k_\perp > 2 \text{ GeV}$) so that vector meson dominance is a small contributor. The pion in $\gamma p \to \pi X$ comes either from a parton emerging in some direction and fragmenting (so
that the pion is part of a jet) or—at the very highest transverse momenta—directly as part of the short range process (whence the pion is kinematically isolated).

Where, fragmentation dominates, about 1/3 to 1/2 of rate comes from gluon targets in the proton. Note the importance of the high pion transverse momentum, and not just for allowing perturbative calculations. There has to be a recoiling particle, hence the process must be higher order. Then it is possible for the gluon target process to be of the same order of magnitude as a quark target process.

One quantity to consider is

$$E \equiv A_{LL} \equiv \frac{d\sigma_{R+} - d\sigma_{R-}}{d\sigma_{R+} + d\sigma_{R-}}$$

as a function of $k_\perp$. The $R$ refers to the right handed polarization of the photon, and the “±” gives the helicity of the target proton. The corresponding quantity for the subprocess $\gamma g \rightarrow q\bar{q}$ is $(-100\%)$, so that there is a possibility of great sensitivity to the gluon polarization. This is borne out by actual calculations using a variety of proposed gluon in the proton distributions [8].

We will close this section with one more comment. As lower energies it is harder to find a fragmentation region between the direct pion production and VMD regions. Help may be available in fishing out gluon target events by looking two jets or two hadrons $180^\circ$ apart in azimuth angle. Think of the two parton level diagrams.

Figure 5. ‘Gluon fusion’ and ‘quark Compton’ subgraphs for pion photoproduction.

Fragmenting $q$’s give faster hadrons than fragmenting glue. Perhaps observing two pions with some cut like each $k_\perp$ above 1.5 GeV suffices to ensure that gluon fusion dominates quark Compton [9] even at CEBAF with 12 GeV.

3.2 Approach to pQCD in $\Delta(1232)$ electroproduction

Electroproduction of the $\Delta(1232)$, $\gamma^* + N \rightarrow \Delta$, is a tough place to see pQCD at work for two reasons. One is that the low $Q^2$ starting point is so different from the asymptotic ending point. In terms of the multipole amplitudes, the quark model expectation, born out by data, is that the so-called electromagnetic ratio (EMR) or $E_{1+}/M_{1+}$ is essentially zero at low $Q^2$, whereas the high $Q^2$ pQCD prediction is that same ratio is unity. The other is that the leading term asymptotically is unusually small, as we have already noted in section 2.3.

Since pQCD seems to work at a few GeV$^2$ in more normal cases, we [11] thought we should examine how the probably delayed approach to the pQCD
result might go as a function of $Q^2$. We did so by choosing simple forms that would give the correct results at low and high $Q^2$ and that obeyed a few principles. We worked using the language of helicity amplitudes, say the $G_+$ and $G_-$ defined in section 2.2. The principles were basically three: the falloffs of $G_+$ and $G_-$ should be $1/Q^3$ and $1/Q^5$ asymptotically; another is that there should be a kinematic zero in the amplitude at a (timelike) $Q^2$ where the $\Delta$ does not recoil when produced off a standing nucleon; and another is the high $Q^2$ normalization (with due regard for the uncertainties of the calculation) of $G_+$ that was quoted in section 2.3.

At the photon point, $Q^2 = 0$, the overall normalization of the two helicity amplitudes were fixed by comparing to existing data. The size of $G_-$, essentially given by the mass parameter governing its falloff in $Q^2$, was also determined from unseparated in helicity data on $\Delta$ electroproduction. Some tweaking of the $G_+$ mass parameter was also needed: there was some information about $E_{1+}/M_{1+}$ at 3 GeV$^2$ even before the recent CEBAF data was released. Results of our fits are shown in the Figure below.

![Graph](image)

**Figure 6.** The electromagnetic ratio for $\Delta$ electroproduction

The solid curve is our preferred fit; the dashed curve is a naive fit that did not fit the unseparated data well, and the not so different dotted curve has an asymptotic $G_+$ that was in our opinion too large even given generous uncertainties in the calculated value. It appears that even in this tough situation there will be some push toward the pQCD result by 10 GeV$^2$ momentum transfer.

We have only mentioned two new initiatives because of space and time limitations. Others exist, notably the idea of off-forward parton distributions and applications to deeply virtual Compton scattering and meson electroproduction and also including new work on inclusive/exclusive connections.

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