Constraints on a seesaw model leading to Quasi-Degenerate neutrinos and signatures at the LHC

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Abstract

We consider a variant of TeV scale seesaw model in which three additional heavy right handed neutrinos are added to the standard model to generate the quasi-degenerate light neutrinos. This model is theoretically interesting since it can be fully rebuilt from the experimental data of neutrino oscillations except for an unknown factor in the Dirac Yukawa coupling. We study the constrains on this coupling coming from meta-stability of electro-weak vacuum. Even stronger bound comes from the lepton flavor violating decays on this model, especially in a heavy neutrino mass scenario which is within the collider reach. Bestowed with these constrained parameters, we explore the production and discovery potential coming from these heavy neutrinos at the 14 TeV run of Large Hadron Collider. Signatures with tri-lepton final state together with backgrounds are considered in a realistic simulation.

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1. INTRODUCTION

Recent discovery of neutral scalar [1, 2] with a mass around 126 GeV and gradual confirmation of its Standard Model (SM) Higgs like nature settled the most convincing and self-consisting model of particle physics. However, several experimental observations along with theoretical questions keep high energy physics community unconvinced that we have yet found our ultimate theory and complete periodic table of particles. So the quest for a new physics beyond the standard model (BSM) is underway both theoretically and experimentally especially with Large Hadron Collider exploring the new horizon of energy and luminosity.

Breakthrough with the Higgs boson also opens up the possibility of exploring new physics by studying the stability of the electroweak vacuum [3, 4]. For the SM to be the only valid theory, vacuum should be stable up to Planck scale $M_P \approx (1.2 \times 10^{19})$ GeV which indicates that the Higgs self-coupling must remain positive through Renormalization Group (RG) running up to the Planck scale [5–8]. However, it has been shown [9] that achieving absolute stability within the SM is severely restricted. Yet the self coupling is not largely negative near Planck scale which implies that the SM vacuum might be metastable [10, 11]. This hypothesis can act as a window to explore new physics considering that the SM vacuum should not go to unstable region [9, 12, 13]. At least it should remain in the metastable region after inclusion of the effect of new physics.

Seesaw models those lead to light neutrino masses are studied in the context of (meta) stability of the electroweak vacuum [3, 4, 14–17], lepton flavor violating (LFV) decay [18, 19], neutrino less double beta decay ($0\nu\beta\beta$) (for a recent review, see [20, 21]) and new physics signatures of such models at present colliders [22–40]. Seesaw models which consist of extra heavy fields added to the SM predict hierarchical light neutrino mass spectrum (such as, normal hierarchy and inverted hierarchy) as well as degenerate light neutrino mass spectrum [41–46]. With recent results from Planck data [47], degenerate mass spectrum becomes severely restricted, although quasi-degenerate (QD) mass spectrum [41–46] is not fully ruled out. It is worthwhile to study QD models in the light of new constraints coming from vacuum (meta)stability and lepton flavor violation (LFV), also to investigate the possibility of observing signatures of this model at the upcoming 14 TeV LHC.

In this paper we consider a variant of TeV scale seesaw model consists of three heavy neutrinos along with the SM, which leads to quasi-degenerate light neutrino mass spectrum. We explore the constraints on the parameters (neutrino Yukawa matrix) coming from the metastability bound. The neutrino Yukawa matrix is constrained significantly from the metastability condition while
having weak dependence on right handed heavy neutrino mass \[17\]. The allowed parameter space has been restricted further by combining it with the bound coming from lepton flavor violating (LFV) decay process such as $\mu \rightarrow e \gamma$. We found that the LFV constraint is more restrictive in the lighter mass range (up to $\sim 400$ GeV) of heavy neutrino. For higher masses the LFV constraint becomes weaker and at some point goes beyond the perturbativity limit effectively leaving no constraints on neutrino Yukawa matrix. The experimental uncertainties from top quark mass, strong coupling constant, and particularly those from the neutrino data permits a notable window in the constrained value of neutrino Yukawa coupling.

Once we found the constrained parameters in this model where neutrino Yukawa matrix is fully reconstructible with the present oscillation data, we study the collider signatures of the heavy neutrinos at 14 TeV LHC. Heavy neutrinos can be produced dominantly through $s$-channel production process associated with lepton which subsequently produce tri-lepton signal along with missing transverse energy coming from non-detection of light neutrino. We have considered the leading order production and performed the particle level realistic simulation to estimate this signal using MadGraph and PYTHIA. Besides $s$-channel process, heavy neutrino can also be produced through vector boson fusion (VBF) process, where weak gauge bosons originating from two oppositely moving partons ‘fuse’ to produce these heavy neutrinos. In the VBF production channel, the final new physics signal is accompanied by two forward tagged jets. Since there is no color connection between the two forward tagged jets, the central region is devoid of any color activity. This significantly lowers the background making weak signals more prominent. These features were exploited not only in the Higgs search (see, \[48\] and references therein), but also proposed as an avenue to explore new physics \[49–52\] at the hadron collider. However, in our case we found that the VBF production cross section of heavy neutrino production is too low to provide any conclusive signature in the proposed luminosity.

Organization of the paper goes as follows: Sec. 2 contains a brief description of the model leading to the quasi-degenerate light neutrinos. Vacuum metastability and LFV bounds are discussed in Sec. 3 and Sec. 4 respectively. We also briefly discuss neutrino less double beta decay in this model in Sec. 5. Thereafter we proceed for collider search strategy by discussing the heavy neutrino production channels at the LHC and its decay in Sec. 6. Detailed simulation, event selection criteria together with expected signal and background results are presented in sec. 7 followed by discovery potential in Sec. 8. Finally we summarize and conclude in Sec. 9.
2. THE MODEL

We extend the Standard Model (SM) particle spectra by adding three heavy right handed neutrinos having mass at TeV scale. The additional part of the Lagrangian is given by

$$L_{\text{ext}} = -\bar{\nu}_L^c N_R Y_{\nu} l_L - \frac{1}{2} N_R M N_R^c + \text{h.c.},$$

where $l_L$ is the left handed lepton doublet, $\phi$ is the SM Higgs doublet and $\tilde{\phi}$ is given by $\tilde{\phi} = i \sigma^2 \phi^*$. The right handed singlet heavy neutrino field is denoted by $N_R$. $(Y_{\nu})_{ji}$ are the elements of the Dirac Yukawa coupling matrix of dimension $(3 \times 3)$ in the present model with first(second) index is assigned for heavy(light) neutrinos. After spontaneous symmetry breaking the Higgs field acquire vacuum expectation value $v$, consequently the light neutrino mass matrix is given by

$$m_{\nu} = m_D^T M^{-1} m_D.$$  

Where Dirac mass term is given by $m_D = Y_{\nu} v / \sqrt{2}$. Using the parameterization based on Casas and Ibarra [53], texture of the Yukawa coupling matrix $Y_{\nu}$ can be expressed as

$$Y_{\nu} = \frac{\sqrt{2}}{v} \sqrt{M^d} R \sqrt{m_D^d U_{\text{PMNS}}^T},$$

where $M^d$ and $m_D^d$ are the heavy and light neutrino mass matrices respectively in their diagonal basis. $U_{\text{PMNS}}$ is the light neutrino mixing matrix, given by

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -c_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix} P,$$

with $c_{mn} = \cos \theta_{mn}$, $s_{mn} = \sin \theta_{mn}$ and $\delta$ is the Dirac CP phase. $P$ is the Majorana phase matrix, expressed as $P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$. For this parameterization of $Y_{\nu}$, clearly measurable parameters from the low energy neutrino experiments enters through $m_D^d$ and $U_{\text{PMNS}}$. Whereas all unknown parameters are originated from $M^d$ as well as from complex matrix $R$. For simplicity $M^d$ has been approximated with a single parameter of heavy neutrino mass. $R$ is a complex orthogonal matrix which satisfies the relation $RR^T = I$. Hence, the matrix $R$ can be written as,

$$R = O e^{iA},$$

$^\dagger$ In the present work we have taken $M$ to be diagonal which implies $M$ and $M^d$ are equivalent.
where $O$ and $A$ are real orthogonal and real antisymmetric matrices respectively. For nearly degenerate light neutrinos one can absorb $O$ in the $U_{\text{PMNS}}$ [54]. General form of the antisymmetric matrix $A$ can be expressed in terms of three unknown parameters

$$A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix},$$

with $a, b, c \in \mathbb{R}$. Expanding and rewriting in terms of a new parameter $\omega = \sqrt{a^2 + b^2 + c^2}$ one would obtain

$$e^{iA} = 1 - \frac{\cosh \omega - 1}{\omega^2} A^2 + i \frac{\sinh \omega}{\omega} A.$$  \hspace{1cm} (7)

In order to reduce the number of free parameters in our analysis, we choose $a = b = c = \omega/\sqrt{3}$. Now, we are left with a single unknown parameter $\omega$ (together with single unknown heavy neutrino mass scale $M_R$ as diagonal entries of matrix $M^d$) that will be constrained by imposing the bound of metastability of the electroweak vacuum and non observation of LFV decay process. These constraints would in turn be reflected in terms of norm for Yukawa coupling matrix which is extremely crucial in production of the heavy neutrinos and essentially determine the discovery potential at the collider. Since $Y_\nu$ is a complex square matrix of dimension three, magnitude of which can be best represented in terms of the norm of $Y_\nu$,

$$\text{Tr}[Y_\nu^\dagger Y_\nu] = \frac{2 M_R}{v^2} \text{Tr} \left[ \sqrt{m_\nu^d} R^\dagger R \sqrt{m_\nu^d} \right],$$

$$\simeq \frac{2 M_R}{v^2} m_0 (1 + 2 \cosh(2 \omega)).$$ \hspace{1cm} (8)

(9)

For demonstration, contours of constant values of $\text{Tr}[Y_\nu^\dagger Y_\nu]$ is shown in Fig. 1 with the parameters $\omega$ and common light neutrino mass scale $m_0$. For our analysis, the common mass scale for light neutrinos is chosen to be $m_0 \simeq 0.07$ eV whereas heavy neutrino mass is fixed at 100 GeV. As evident from the figure, for a fixed value of $m_0$, different values of $\text{Tr}[Y_\nu^\dagger Y_\nu]$ can be obtained by varying $\omega$ accordingly. To present one example, for this particular choice of degenerate light(heavy) neutrino mass of 0.07 eV(100 GeV), the norm $\text{Tr}[Y_\nu^\dagger Y_\nu] \simeq 0.2$ can be considered for choice of the parameter $\omega = 13.7$ [14, 54, 55].

\[^4\text{Satisfying } \det(O) = \det(R).\]
FIG. 1: Parametric plot of $\text{Tr}[Y_u^\dagger Y_u]$ with $\omega$ and common light neutrino mass scale $m_0$. Heavy neutrino mass fixed at 100 GeV. The numbers in the plot indicates the values of $\text{Tr}[Y_u^\dagger Y_u]$ for the different set of parameters $\omega$ and $m_0$.

3. METASTABILITY BOUND

The SM potential at tree level is given as

$$V(\phi) = \lambda \left( \phi^\dagger \phi \right)^2 - m^2 \phi^\dagger \phi.$$  \hfill (10)

The physical Higgs mass, in the above convention, is defined as $m_h^2 = 2\lambda v^2$. The Renormalization Group equation (RGE) of $\lambda$ can be expressed up to $i^{th}$ loop as

$$\frac{d\lambda}{d \ln \mu} = \sum_i \frac{\beta_\lambda^{(i)}}{(16\pi^2)^i},$$  \hfill (11)

where $\mu$ is the renormalization scale. The $\beta$ function for one loop is given as,

$$\beta_\lambda^{(1)} = 24 \lambda^2 - \left( \frac{9}{5} g_1^2 + 9 g_2^2 \right) \lambda + \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 + 4 T \lambda - 2 Y,$$  \hfill (12)

where,

$$T = \text{Tr} \left[ 3 Y_u^\dagger Y_u + 3 Y_d^\dagger Y_d + Y_l^\dagger Y_l + Y_\nu^\dagger Y_\nu \right],$$ \hfill (13)

$$Y = \text{Tr} \left[ 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2 + (Y_l^\dagger Y_l)^2 + (Y_\nu^\dagger Y_\nu)^2 \right].$$ \hfill (14)

and $g_i$'s are the gauge coupling constants. Grand Unified Theory (GUT) modification for the $U(1)$ gauge coupling has been incorporated. $Y_u$, $Y_d$ and $Y_l$ denote the Yukawa coupling matrices for the up type quark, down type quark and charged lepton respectively. Expectedly, dominant contribution comes from the top Yukawa (up type quark) running and one loop $\beta$ function is
governed by the following equation:

$$\beta_{Y_u}^{(1)} = Y_u \left[ \frac{3}{2} Y_u Y_u^\dagger + \frac{3}{2} Y_d Y_d^\dagger + T - \left( \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right) \right].$$

(15)

Three loop RGE for Higgs self coupling ($\lambda$), the top Yukawa and the gauge couplings has been used in the numerical analysis \[56–63\]. Matching corrections for top Yukawa has been taken up to three loop QCD \[64\], one loop electroweak \[65, 66\] and $O(\alpha_s)$ \[7, 67\] while for Higgs self coupling, it has been taken up to two loop \[9, 68\]. The Higgs self coupling also receives additional contribution from the higher order corrections of the effective potential. The loop corrected effective self coupling denoted by $\tilde{\lambda}$, is given by \[17, 69, 70\],

$$\tilde{\lambda} = \lambda - \frac{1}{32 \pi^2} \left[ \frac{3}{8} (g_1^2 + g_2^2)^2 \left( \frac{1}{3} - \ln \left( \frac{g_1^2 + g_2^2}{4} \right) \right) + 6 g_t^4 \left( \ln \frac{g_t^2}{2} - 1 \right) + \frac{3}{4} g_2^2 \left( \frac{1}{3} - \ln g_2^2/4 \right) \right] + \left( Y_{\nu} Y_{\nu}^\dagger \right)_{ii} \left( \ln \left( \frac{Y_{\nu} Y_{\nu}^\dagger}{2} \right)_{ii} - 1 \right) + \left( Y_{\nu} Y_{\nu}^\dagger \right)_{jj} \left( \ln \left( \frac{Y_{\nu} Y_{\nu}^\dagger}{2} \right)_{jj} - 1 \right) + \frac{Y_t^4}{(16 \pi^2)^2} \times $$

$$\left[ g_2^2 \left\{ 24 \left( \ln \frac{Y_{\nu} Y_{\nu}^\dagger}{2} \right)^2 - 64 \ln \frac{Y_{\nu} Y_{\nu}^\dagger}{2} + 72 \right\} - \frac{3}{2} Y_t^2 \left\{ 3 \left( \ln \frac{Y_{\nu} Y_{\nu}^\dagger}{2} \right)^2 - 16 \ln \frac{Y_{\nu} Y_{\nu}^\dagger}{2} + 23 + \frac{\pi^2}{3} \right\} \right],$$

(16)

where $i, j$ denote the number of generation of light and heavy neutrinos respectively. The absolute stability of the electro weak vacuum implies $\tilde{\lambda} \geq 0$ up to Planck scale. However as shown in \[9\], the absolute stability is highly restrictive. In this light we shall consider metastability i.e. transition time from a metastable vacuum towards instability should be greater than the age of the universe. In other words the transition probability through quantum tunneling should be less than unity.

The tunneling probability within the semi-classical approximation is given by (at zero temperature) \[10, 11, 71, 72\]

$$p = \max_{\mu < \Lambda} V_U \mu^4 \exp \left( -\frac{8\pi^2}{3 \lambda(\mu)} \right),$$

(17)

where $\Lambda$ is the cutoff scale and $V_U$ is volume of the past light-cone, taken as $\tau^4$. Here $\tau$ is the age of the universe taken from Planck data as $\tau = 4.35 \times 10^{17}$ sec \[73\]. For the vacuum to be metastable, one should have $p < 1$ which can be recast in terms of a lower bound on $\lambda$, as given below

$$|\lambda| < \lambda_{\text{meta}}^{\max} = \frac{8\pi^2}{3} \frac{1}{4 \ln (\tau \mu)}. \quad (18)$$

The above equation can be utilized to put an upper bound on $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$ from the running of $\lambda$ as a function of the heavy neutrino mass $M_R$. This has been displayed in Fig. 2 as horizontal slanting lines corresponding to different choices of the top mass and strong coupling. Now, the region below this line is consistent with the metastability bound.

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\* We incorporated two loop correction due to the SM and one loop correction due to neutrino Yukawa couplings.
FIG. 2: Allowed region of the Yukawa norm $\text{Tr}[Y_\nu^+ Y_\nu]$ as a function of the heavy neutrino mass $M_R$ by imposing combined constraints coming from metastability of the electroweak vacuum as well as lepton flavor violating decay ($\mu \to e\gamma$). Choice of Higgs mass fixed at $m_h = 126$ GeV. The horizontal slanting lines represent the upper bound on $\text{Tr}[Y_\nu^+ Y_\nu]$ consistent with the metastability bound, as in Eq. 18. Three lines are due to three different set of values for top mass and strong coupling \[74, 75\]. The shaded area to the right of the curved line are allowed after putting experimental upper limit from the lepton flavor violating constraint as used in Eq. 23. The green shaded region (region B) is allowed absolutely for experimentally measured values of oscillation parameters together with unknown phases. Additionally, yellow shaded region (region A) originated from the 3$\sigma$ uncertainty in the data and full range of unknown phases \[76\]. Any value of oscillation parameters within the available range will reside in region A. These contributions are evident from Eq. 22 and 8. Hence, the region marked “Disallowed” is strictly ruled out from LFV.

4. LEPTON FLAVOR VIOLATION BOUND

Lepton flavor violating decay processes get significant contribution from the heavy neutrino due to its relatively low mass scale compared to the canonical seesaw mechanism. The experimental upper limit on $\mu \to e\gamma$ processes can be translated to an upper bound on $\text{Tr}[Y_\nu^+ Y_\nu]$ as a function of $M_R$. Branching ratio of $\mu \to e\gamma$ \[77\] is given by

$$\text{Br} (\mu \to e\gamma) = \frac{3\alpha}{8\pi} \left| \sum_j V_{e j} V_{j \mu}^\dagger f(x_j) \right|^2,$$

where dependence of heavy neutrino mass is expressed in terms of dimensionless parameter $x_j = (M_{R_j}^2/m_W^2)$ in a slowly varying function,

$$f(x) = \frac{x \left( 1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x \right)}{2(1-x)^4},$$

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$$f(x) = \frac{x \left( 1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x \right)}{2(1-x)^4}.$$
In our present case, right handed neutrinos are degenerate, i.e., $M_{R_j} = M_R$. The light-heavy mixing matrix $V$ is obtained through the diagonalization of the full neutral lepton mass matrix \[ V = m_D^1 (M^{-1})^* U_R, \] (21)

where $U_R^{†\dagger}$ is a unitary matrix that diagonalizes $M$. Using Eq. [3] and 21 with Eq. [19] one gets,

\[
\text{Br} (\mu \to e\gamma) = \frac{3 \alpha}{8 \pi M_R^2} \left[ f\left(\frac{M_R^2}{m_W^2}\right)\right]^2 \left| U_{\text{PMNS}} \sqrt{m_{\nu}^d} R^\dagger \sqrt{m_{\nu}^d} U_{\text{PMNS}}^\dagger \right|^2
\] (22)

and $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$ is given by Eq. [9]. From Eq. [22], [8] and [9] one can see the angular and phase dependence of the branching ratio comes from the $U_{\text{PMNS}}$, whereas the magnitude of the branching ratio is encoded in $\sqrt{m_{\nu}^d} R^\dagger \sqrt{m_{\nu}^d}$ whose modulus is proportional to $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$. The analytical expression of $\text{Br}(\mu \to e\gamma)$ is somewhat lengthy and hence omitted here. Subjected to the present experimental upper bound on the $\mu \to e\gamma$ process \[ \text{Br} (\mu \to e\gamma) < 5.7 \times 10^{-13}, \] (23)

one would obtain, numerically, an upper bound on $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$ by inverting Eq. [22]. However due to uncertainty in the measurement of the oscillation parameters, the upper bound on this norm would lead to a band which is depicted in Fig. [2]. The yellow shaded region (region A) indicates the uncertainty in the upper bound of $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$ coming from the $3\sigma$ uncertainty of the oscillation parameters together with all the phases being varied in the full range \[ \text{76}. \] The right hand side of the region is always allowed from LFV bound irrespective of $3\sigma$ variation of oscillation data which is indicated by green shaded region (region B). However taking the $3\sigma$ uncertainty under consideration, only the part marked “Disallowed” is ruled out after imposing the LFV bound. From the figure we can see that the LFV bound is stronger than metastability bound for $M_R \lesssim 400$ GeV, beyond which the metastability bound becomes stronger. Consideration of both metastability and LFV makes the full shaded region (both region A & B) below the slanting line to be the allowed region of parameter space of $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$. For our analysis, we have used the maximum value of $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$ allowed from these constraints which reflects as the optimistic parameter. To get some notion of related neutrino oscillation parameters, we list them in Table [I] which lead to the upper and lower edges (curves) of the yellow shaded region ‘A’ as described in Fig. [2]. These two curves correspond to the maximum values of Yukawa norms $\text{Tr}[Y_{\nu}^\dagger Y_{\nu}]$ correspond to most optimistic and conservative scenario. Note that any other value of angles and phases within the available range will reside in region A.

‡‡ $U_R$ is identity matrix in the present scenario as $M$ is diagonal.
| Bounds                      | Parameters |
|-----------------------------|------------|
| Optimistic (upper)          | $\theta_{12}$ | $0.21\pi$ |
|                             | $\theta_{23}$ | $0.27\pi$ |
|                             | $\theta_{13}$ | $0.06\pi$ |
|                             | $\delta$     | $0.47\pi$ |
|                             | $\alpha_1$   | $0.18\pi$ |
|                             | $\alpha_2$   | $1.48\pi$ |
| Conservative (lower)        | $\theta_{12}$ | $0.19\pi$ |
|                             | $\theta_{23}$ | $0.24\pi$ |
|                             | $\theta_{13}$ | $0.05\pi$ |
|                             | $\delta$     | $1.65\pi$ |
|                             | $\alpha_1$   | $0.09\pi$ |
|                             | $\alpha_2$   | $0.29\pi$ |

TABLE I: Values of oscillation parameters leading to the upper and lower edges (curves) of the yellow shaded region ‘A’ in Fig. 2. These two curves represent maximum values of Yukawa norms $\text{Tr}[Y_\nu^\dagger Y_\nu]$ correspond to most optimistic and conservative scenario. As described in Fig. 2, variation of these parameters were done from the $3\sigma$ uncertainty in the data and full range of all the phases. 

5. NEUTRINO LESS DOUBLE BETA DECAY

In this section we briefly discuss the contribution of this particular model towards neutrino less double beta decay ($0\nu\beta\beta$). The general expression of half-life for $0\nu\beta\beta$ in the context of Type-I seesaw is given by [80, 81]

$$T_{1/2}^{-1} = G \frac{|M_{\nu}|^2}{m_e^2} \sum_i (U_{\text{PMNS}}^2)_{ei}^2 \left( m_{d_i}^d \right)^2 + \sum_j \langle p^2 \rangle \frac{V_{ej}^2}{M_{R_j}},$$

(24)

where $G = 7.93 \times 10^{-15}$ yr$^{-1}$, $M_{\nu}$ is the nuclear matrix element due to light neutrino exchange and $m_e$ being the electron mass. $\langle p^2 \rangle$ in the second term, which is due to the contributions from heavy singlet neutrinos, is given by [82]

$$\langle p^2 \rangle = -m_e m_p \frac{M_N}{M_{\nu}},$$

(25)

which is taken to be $\langle p^2 \rangle = -(182 \text{ MeV})^2$ [80]. Here $m_p$ is the proton mass and $M_N$ is the nuclear matrix element due to heavy neutrino exchange.

The first and the second term in Eq. (24) represent contributions from light and heavy neutrinos respectively and thus summed over corresponding number of light (heavy) neutrinos. Accordingly with the help of Eq. (3) and (21) the second term can be expressed as,

$$\frac{\langle p^2 \rangle}{M_{R}^2} \left( U_{\text{PMNS}} \sqrt{m_{d_i}^d R_i^d} \right)^2 \left( U_{\text{PMNS}}^T \right)_{ei} = \frac{\langle p^2 \rangle}{M_{R}} \left( U_{\text{PMNS}}^T \right)_{ei} \left( m_{d_i}^d \right)^2.$$

(26)

Consequently Eq. (24) becomes

$$T_{1/2}^{-1} = G \frac{|M_{\nu}|^2}{m_e^2} \left( 1 + \frac{\langle p^2 \rangle}{M_{R}} \right)^2 \left( U_{\text{PMNS}}^T \right)_{ei} \left( m_{d_i}^d \right)^2.$$

(27)

One can notice that the contribution on $0\nu\beta\beta$ from heavy neutrinos is extremely tiny, e.g. only 0.001% of the light neutrino contribution can come towards the half-life of $0\nu\beta\beta$ even for a heavy
neutrino mass of 100 GeV. This contribution is even suppressed as the mass increased. Although light neutrino contribution to the neutrino less double beta decay can be sizable and can possibly be explored in the future experiments, the heavy neutrino contribution in this scenario can be neglected. In the next section, we would explore the production of these heavy neutrinos at the collider and discuss the discovery potential for 14 TeV large hadron Collider.

6. SIGNATURES AT THE LHC

Heavy neutrinos can be produced dominantly in \( s \)-channel W-boson exchange at the hadron collider. We also explored the corresponding VBF production associated with two forward jets. At the leading order calculation, parton level processes producing heavy neutrinos (\( N \)) at the mass basis are as follows:

\[
q\bar{q}' \rightarrow W^\pm \rightarrow l^\pm N \quad (s\text{-channel}),
\]

\[
q\bar{q}' \rightarrow l^\pm N q q'' \quad (VBF),
\]

where \( q \) represents suitable parton and associated leptons are \( l \equiv (e, \mu, \tau) \). In Fig. 3 (left panel) the total cross section for these processes are shown as a function of heavy neutrino mass after applying the pre-selection cuts \( p_T^l > 20 \) GeV and \( |\eta_l| < 2.5 \). The solid (dashed) line is showing leading order production cross section through \( s \)-channel (VBF) process. From the figure it is evident that the VBF cross section is insufficient hence we shall not discuss this production mechanism afterwards and concentrate only on \( s \)-channel process for phenomenological analysis.

For our simulation we consider the maximum allowed value coming from \( \text{Tr}[Y^\dagger \nu Y \nu] \) satisfying combined LFV and meta stability bounds as depicted in Fig. 2 together with neutrino oscillation data within their uncertainties. One can notice that the higher values of Yukawa coupling is permitted from these constraints once we move towards higher mass of heavy neutrinos. This improved Yukawa coupling up to 400 GeV can enhance the cross section. But this effect is somewhat offset (and not clearly visible) by the much stronger drop due to heavier mass production.\footnote{Interestingly the effect is visible for VBF case where cross section drop due to heavy mass production is smaller.}

We have used MadGraph5 \footnote{MadGraph5} to simulate the production and decay of heavy neutrinos. Parton distribution function CTEQ6L1 \footnote{CTEQ6L1} has been used and the factorization scale is set at heavy neutrino mass.

The heavy neutrino can decay into weak gauge bosons (\( W^\pm, Z \)) or the Higgs boson (\( H \)) in...
FIG. 3: (Left panel) Total cross section is plotted for leading order s-channel heavy neutrino production (solid line) associated with charged lepton at the 14 TeV LHC. Basic pre-selection cuts $p_{T\ell} \geq 20$ GeV and $|\eta_{\ell}| \leq 2.5$ are applied and choice of parameters are compatible with the neutrino oscillation data constrained with vacuum metastability and LFV. The dotted line shows the corresponding VBF production cross section, where basic VBF cuts were used in addition to the pre-selection cuts. (Right panel) Demonstration of the decay branching ratios of the heavy neutrino in different channels as a function of mass. Total decay width is also shown with red-solid line.

association with leptons because of mixing between light and heavy neutrinos:

$$ N \rightarrow W^{\pm} \ell^{\mp}/Z\nu_l/H\nu_l. \quad (29) $$

Branching ratio of $N$ in these channels are shown in Fig. 3 (right panel) with varying heavy neutrino mass $M_R$. In this plot the red-solid line is showing the total decay width ($\Gamma_N$) of heavy neutrino. The figure manifests that $W\mu$ channel is the dominant decay mode for low mass region and saturates at $\sim 42\%$ for $M_R \gtrsim 300$ GeV. Both $H\nu$ and $Z\nu$ channels saturate at $\sim 25\%$ in the high mass region leaving approximately 7% for the $W\tau$ channel whereas $We$ channel is negligible.

Identifying dominant decay modes for the heavy neutrino, one can notice that the decay into charged lepton of muon type associated with onshell W boson can finally produce tri-lepton signal with missing transverse momentum. This can be vital channel searching for QD heavy neutrinos at the hadron collider. Since it was shown earlier [40] that the separation of these tri-lepton signals into separate flavor states can carry useful informations on the hierarchical structures of light neutrinos associated with the model. Hence we would also consider flavor allocated cross sections for signal and the backgrounds.
### Selection Criteria

| Criteria                                      | Requirement                                      |
|----------------------------------------------|--------------------------------------------------|
| Lepton identification criteria               | $|\eta\ell| < 2.5$ and $p_{T\ell} > 20$ GeV          |
| Detector efficiency for leptons              | Electron efficiency (for $e^- \& e^+$): 0.7 (70%) |
|                                              | Muon efficiency (for $\mu^- \& \mu^+$): 0.9 (90%) |
| Smearing                                     | Gaussian smearing of electron energy and muon $p_T$ |
| Jet reconstruction                           | `PYCELL` cone algorithm in `PYTHIA`               |
| Lepton-jet separation                        | $\Delta R_{lj} \geq 0.4$ (for all jets)         |
| Lepton-lepton separation                     | $\Delta R_{ll} \geq 0.2$                        |
| Lepton-photon separation                     | $\Delta R_{l\gamma} \geq 0.2$ for all $p_{T\gamma} > 10$ GeV |
| Hadronic activity                            | Hadronic activity for each lepton: $\sum_{i} p_{T_{had}}/p_{Ti} < 0.2$ (radius of the cone around the lepton) |
| Final $p_T$ cuts for leptons                 | $p_{T\ell_1} > 30$ GeV, $p_{T\ell_2} > 30$ GeV and $p_{T\ell_3} > 20$ GeV |
| Missing $p_T$ cut                            | $p_T > 30$ GeV                                   |
| $Z$-veto $^a$                                 | $|m_{\ell_1\ell_2} - M_Z| \geq 6\Gamma_Z$       |
| VBF Cuts                                     |                                                  |
| Central jet veto                             | On any additional jet with $p_{T3} > 20$ GeV, and $|\eta_0| < 2$ events are discarded. |
|                                              | Pseudorapidity difference between the average of the two forward jets and the third jet: $\eta_0 = \eta_3 - (\eta_1 + \eta_2)/2$. |
| Pseudorapidity [85] of charged leptons       | $\eta_{j,\text{min}} < \eta\ell < \eta_{j,\text{max}}$ |
| Cut applied to jets                          | $p_{Tj_1,j_2} > 20$ GeV                         |
|                                              | $M_{j_1,j_2} > 600$ GeV                         |
|                                              | $\eta_{j_1}, \eta_{j_2} < 0$ and $|\eta_{j_1} - \eta_{j_2}| > 4$ |

$^a$Invariant mass for the same flavored and opposite sign lepton pair, $m_{\ell_1\ell_2}$, must be sufficiently away from $Z$ pole.

| TABLE II: Selection criteria used in simulation. |

7. SIMULATION AND RESULTS

To analyze signals for heavy neutrino, we have implemented this model in `FeynRules` [86] to generate the Feynman rules compatible for `MadGraph`. Parton level cross sections were generated using `MadGraph5` and for showering and hadronization of the lhe [87] event file, `PYTHIA6` [88] has been used.

To enhance the signal over background, the selection criteria, tabulated in Table. III has been
TABLE III: Final tri-lepton with $E_T$ signal cross section in fb produced through $s$-channel heavy neutrino for the benchmark mass $M_R = 100$ GeV at the 14 TeV LHC. All event selection cuts were applied (Table II) except the VBF cuts as described in the text. We have also classified total tri-lepton signals into four different flavor combination of leptons and presented expected cross section in each category.

| Total signal cross section (fb) | Flavor allocated cross section (fb) |
|--------------------------------|-----------------------------------|
| 6.595                         | eee 8.17×10^{-4} 0.118 3.200 3.276 |

implemented. In top portion of this table, all selection parameters and efficiencies were listed. VBF cut part is additional criteria only for VBF part of the analysis. For detail see references [37, 40].

Following from our earlier discussion on heavy neutrino production and decay, we are looking for tri-lepton production at the LHC,

$$pp \rightarrow \ell^+ N \rightarrow \ell^+ (W^\pm \ell^\mp /Z\nu) \rightarrow e^\pm e^\pm e^\mp /\mu^\pm \mu^\mp /\mu^\pm \mu^\mp + E_T.$$

Cross section of final tri-lepton signal through $s$-channel heavy neutrino production at 14 TeV LHC for a benchmark point of $M_R = 100$ GeV is listed in Table III. Here we have incorporated all event selection criteria except the VBF cuts. Total contribution from all the light leptons ($e, \mu$) as well as the differential contributions from the four flavor combinations are also presented.

All the standard model channels those can mimic this tri-lepton signal with missing $E_T$ are considered for the estimation of SM background. For such simulation events are generated using ALPGEN [89] at the parton level and then passed into PYTHIA for hadronization and showering. We have used the same selection criteria as tabulated in Table II. Inclusive cross section for $\ell^+ \ell^\pm \ell^\mp \nu_\ell$ final state from the SM is 32.722 fb. Details of individual channel’s contribution towards the SM background can be found in [37, 40, 90].

8. DISCOVERY POTENTIAL

With our understanding on signal strength of producing tri-leptons from heavy neutrino and possible sources of leading background, it is convenient to present our result in terms of significance which we express as $S/\sqrt{S + B}$, where $S (B)$ = $\mathcal{L} \sigma_S (B)$. $\mathcal{L}$ is the integrated luminosity of available data from the experiment and $\sigma_S (B)$ is the final cross section of the signal (background) after all event selection cut and with model parameters of the model satisfying metastability and LFV bound. Fig. 4 demonstrates the expected significance of the $s$-channel production of heavy neutrino with mass $M_R = 100$ GeV as a function of integrated luminosity at 14 TeV LHC. $3\sigma (5\sigma)$
significance can be achieved with integrated luminosity $\sim 8(23)\ fb^{-1}$.

9. CONCLUSION

In this work we have considered a TeV scale seesaw model that leads to quasi degenerate light neutrino mass spectrum. The model is fully reconstructible from oscillation parameters apart from an unknown factor parameterized by a constant $\omega$. We have demonstrated that the norm of Yukawa $\text{Tr}[Y^\dagger_{\nu}Y_{\nu}]$ can choose arbitrary magnitude with different choices of $\omega$ and the common light neutrino mass scale $m_0$. Consequently we have obtained bounds on $\text{Tr}[Y^\dagger_{\nu}Y_{\nu}]$ from both the consideration of the metastability of the electroweak vacuum as well as lepton flavor violation. The LFV bound turns out to be stronger than the metastability bound in the low $M_R$ region which is $\text{Tr}[Y^\dagger_{\nu}Y_{\nu}] \simeq (0.18)^2$ at $M_R = 100$ GeV and gradually increases to become $\simeq (0.48)^2$ at around $M_R = 400$ GeV. Beyond that mass range, LFV bound becomes weaker than the metastability bound. The later remains slowly varying with $M_R$. However, contribution of the heavy neutrino towards the neutrino less double beta decay is insignificant in this model compared to the light neutrino contribution.

The constrained model parameters were then used to study the production and decay modes of the heavy neutrino at the LHC. We have studied tri-lepton associated with missing $E_T$ signal coming from the $s$-channel production of the heavy neutrino with realistic selection criteria as well as detailed simulation. However, the similar signal along with two forward tagged jets, coming
through the production of heavy neutrino perceived in vector boson fusion comes with much smaller cross section at the present scenario. With a benchmark point of heavy neutrino mass $M_R = 100$ GeV, we have presented the discovery potential of heavy neutrino, fitted to the model, with $3\sigma (5\sigma)$ significance for integrated luminosity $8(23) \text{ fb}^{-1}$ at the LHC.

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