State-space modeling of the radio frequency inductively-coupled plasma generator

Rakesh Kumar Dewangan¹, Sangeeta B Punjabi¹, N K Joshi², D N Barve²
H A Mangalvedekar¹, B K Lande¹
¹Electrical Engineering Department, V J T I, Mumbai-400019
²Laser and Plasma Technology Division, BARC, Mumbai-400085

E-mail: ham.vjti@gmail.com

Abstract: Computational fluid dynamics models of RF-ICP are useful in understanding the basic transport phenomenon in an ICP torch under a wide variety of operating conditions. However, these models lack the ability to evaluate the effects of the plasma condition on the RF generator. In this paper, simulation of an induction plasma generator has been done using state space modelling by considering inductively coupled plasma as a part of RF network. The time dependent response of the RF-ICP generator circuit to given input excitation has been computed by extracting the circuit’s state-space variables and their constraint matrices. MATLAB 7.1 software has been used to solve the state equations. The values of RF coil current, frequency and plasma power has been measured experimentally also at different plate bias voltage. The simulated model is able to predict RF coil current, frequency, plasma power, overall efficiency of the generator. The simulated and measured values are in agreement with each other. This model can prove useful as a design tool for the Induction plasma generator.

1. Introduction:
Since the successful operation of a radio frequency induction-coupled plasma (RF-ICP) torch in 1961 by Reed [1], it has become the subject of considerable studies and induction plasma technology has found a number of industrial applications.

Over the last four decades, mathematical and numerical models of the RF-ICP have advanced to a considerable degree. A comprehensive review on 2-D modeling of inductively coupled plasma has been published by Mostaghimi [2]. Three dimensional modeling using commercial CFD code such as fluent [3-4] have also been reported. These models are very useful in understanding the basic transport phenomenon in an ICP torch under a wide variety of operating conditions and torch configuration. The mathematical models have been used to design torches [5], to simulate emission pattern in ICP atomic emission spectroscopy [6], to predict heat transfer to powder in plasma spraying [7], and to study non-equilibrium effects in argon [8-9] and helium ICP’s [10]. Typical two-dimensional (2-D) model [11] has been very successful in predicting the gas flow dynamics of the ICP.

However, in the above discussed mathematical models it was assumed that the plasma power and its frequency were constant. In reality the plasma power and frequency are not constant. They depend on plasma condition and plate bias voltage, so it is required to consider the induction plasma generator and radio frequency network (tank circuit) simultaneously. An integrated model [12] allows for the
computation of plasma impedance and the overall efficiency in addition to standard plasma flow and temperature fields. Recently, a number of publications have numerically simulated the induction plasma generators using state-space approach and transformer models for the plasma [13-15].

In this paper, an induction plasma generator in which the ICP torch is considered as part of the RF network has been simulated. The RF-ICP torch system designed and assembled at BARC, L&PT Division Mumbai has been considered in this model for numerical simulation. The system has a HVDC power supply (12 kV, 10A) which provides the required plate voltage to the oscillator. Oscillator used is basically a Colpitts oscillator with a common cathode connection and uses a single inductor which couples RF power to the plasma load. Argon is used as plasma gas.

Here the behavior of a dynamical system such as RF-ICP may be described by a set of n first order nonlinear ordinary differential equations. For solving any differential equation using available numerical techniques, the differential must be of first order [16-17]. The time dependent response of the circuit to given input excitation has been computed by extracting the circuit’s state-space variables and their constraint matrices. MATLAB 7.1 software has been used for numerical solutions of differential equation employing the Runge-Kutta method.

Voltage across the capacitors and current through inductors are taken as state-variables. The magnitude of the radio frequency oscillating coil current and its frequency is obtained from the solution of state equation. These can be used to calculate the dissipated plasma power and the overall efficiency of the RF-ICP model.

2. State space model of the RF-ICP
In the RF-ICP, the generator circuit of a typical Colpitts oscillator consists of a triode tube functioning as the oscillation maintaining device. The generator is a free-running type, in which the plasma torch is integrated into the tank circuit. The input power of the oscillator is supplied by a filtered feed-back controlled voltage supply that maintains a constant bias to the triode plate.

Since the operation of the power oscillator is in class C mode, it is necessary to model the triode as nonlinear active device. Its function is simulated by a pair of nonlinear voltage-controlled current sources—one replacing the plate and the other replacing the grid.

The circuit diagram of the RF-ICP generator developed indigenously at BARC is shown in figure 1. The plasma resistance $R_{\text{plasma}}$, coil resistance $R_{\text{coil}}$, and torch inductance $L_{\text{torch}}$ are nonlinear passive elements. Their experimentally measured values have been used for analysis.

![Circuit Diagram](image)

**Figure 1.** Circuit diagram for the RF-ICP Generator.
2.1 Selection of Tree and Co-tree Branches for RF-ICP Generator Circuit

Due to highly non-linear characteristics of the triode and plasma, the numerical solution of the ICP generator falls in the non-linear dynamic network category. The automated algorithm to extract governing equations for a non-linear dynamic circuit is described in [18]. To solve the above circuit diagram, we have to make its state space model. Any set of n linearly independent system variables may be used to describe the state of the system. These are referred to as the state variables; they form a minimal set of the dynamic variables that along with the inputs to the system provide a complete description of the system behavior. Any other system variables may be determined from knowledge of the state. A simplified circuit diagram for the preparation of state space model in general form is shown in figure 2.

One commonly used method is to first choose a tree with the following order of preference for tree branches: independent voltage sources, controlled voltage sources, capacitors, resistors, inductors, controlled current sources, and independent current sources. The branches that are not chosen as tree form the co-tree branches. After selecting the tree and co-tree branches, applying Kirchhoff’s voltage law to the tree branch loops and Kirchhoff’s current law to the co-tree nodes results in a set of network equilibrium equations. Such a tree is called a normal tree or proper tree for a linear active network. Under the assumptions that independent voltage sources contain no loops and independent current sources contain no cut-sets, any normal tree will contain all independent voltage sources and no independent current sources.

These trees can always be constructed under the two conditions just stated. The second – level subscript \( \ell \) has been used to represent the normal tree and \( \ell \) for the links (co-tree), then the branch voltage and current vectors (each independent source, voltage or current source is considered as individual branch, this allows more flexibility than the composite branches) may be partitioned as shown in figure 2.

![Figure 2. Simplified circuit diagram for the preparation of state space model in general form.](image)

In figure 2 solid lines represent tree branches and dotted lines represent co-tree branches. Those capacitors not in the normal tree are called excess capacitors, and those inductors in the normal tree are called excess inductors. We construct a normal tree by using all the independent voltage sources first and then supplementing with capacitors, resistors and inductors in the indicated order. We have to put a capacitor in the co-tree only when it forms a loop with some voltage sources and capacitors.
already chosen as tree branches. Thus, the fundamental loop corresponding to a capacitor in the co-tree will contain no resistors or inductors. Similarly, we have to put a resistor in the co-tree only when it forms a loop with some voltage sources, capacitors and resistors already chosen as tree branches. Therefore, fundamental loop corresponding to a resistor in the co-tree will not contain any inductors. Figure 3 shows the tree and co-tree diagram of RF-ICP Generator developed at BARC.

Figure 3. Tree and Co-tree diagram of the RF-ICP generator.

In RF-ICP model, input voltage sources are plate bias voltage (Epb) and grid bias voltage (Egb) and input current sources are plate current (Ip) and grid current (Ig). In this model there are total five state-variables. The state variables taken from the circuit for the preparation of the state-space model of RF-ICP generator are as follows:

\[ x(1) = \text{voltage across capacitor } C_1, \ x(2) = \text{voltage across capacitor } C_{c1}, \ x(3) = \text{voltage across capacitor } C_{c2}, \ x(4) = \text{current through choke coil}, \ x(5) = \text{Radio-Frequency coil current} \]

Hence the state-space model of the RF-ICP generator is represented as:

\[
\frac{d}{dt} \begin{bmatrix} v_{C_3} \\ i_{L_f} \end{bmatrix} = A \begin{bmatrix} v_{C_2} \\ i_{L_0} \end{bmatrix} + B \begin{bmatrix} V_3 \end{bmatrix} 
\]

Where,

\[
v_{C_3} = \begin{bmatrix} v_{C_1} \\ v_{C_{c1}} \\ v_{C_{c2}} \end{bmatrix}, \quad V_3 = \begin{bmatrix} E_{pb} \\ E_{gb} \end{bmatrix} 
\]

\[
i_{L_f} = \begin{bmatrix} i_{f} \\ i_{rad} \end{bmatrix}, \quad I_\ell = \begin{bmatrix} I_p \\ I_g \end{bmatrix} 
\]
Where $A$ and $B$ are the constraint matrices. $V_C$ represents the vector of voltage across the tree capacitors, $I_L$ represents the vector of current through the co-tree inductors, $V_E$ represents the vector of voltage sources in the tree branches, $I_J$ represents the vector of current sources in the co-tree branches.

3. Method of solution
At the beginning of the program execution as input, the topology of the circuit is entered along with the electrical parameters of each element. A MATLAB program organizes the topology into a state space matrix, and the transient signal output of the generator is computed for predetermined duration $\Delta t$. The overview of the plasma simulation using state-space modeling has been shown in figure 4.

![Diagram of the simulation process](image_url)

**Figure 4.** Overview of the plasma simulation using state-space modeling.

Once the circuit simulation is completed for a single interval the rms coil current, frequency, dissipated power of the plasma is computed. In this the following formulation is used to obtain the rms coil current value:

$$I_{coil}^{rms} = \frac{1}{\Delta t} \left( \int I^{2} \, dt \right)^{\frac{1}{2}}$$
where I_{coil} is the instantaneous current flowing in the torch coil. The frequency of the oscillation is obtained by computing the tank current signal intercepting the zero axis and dividing the number by the time interval. Dissipated power of the plasma can be obtained by considering the fact that the resistive dissipation due to the R_{plasma} in the generator circuit should be equal to the power dissipated by the plasma. Hence the dissipated power of the plasma in the circuit model is computed by the following formula:

\[ P_{\text{Plasma}} = R_{\text{Plasma}} \times \left( I_{rms}^{\text{coil}} \right)^2 \]

Here the experimental values of R_{plasma} have been taken from [19-20].

Also, the overall efficiency of the RF-ICP model is given by:

\[ \text{Overall efficiency} = \frac{P_{\text{Plasma}}}{P_{dc}} \]

4. Experimental results

For electrical measurements, DC plate power (P_{dc}) input to the oscillator tube is measured with a DC ammeter and voltmeter mounted on the power supply model. It is also required to measure the RF current by Rogowski coil with proper calibration factor, RF voltage by capacitive voltage divider, frequency of tank circuit, power factor angle by observing the wave shape of RF voltage and RF current on digital oscilloscope. It can be seen from experimental results that resonating frequency changes with the change of input power. This change is because of change in the reflected impedance of the plasma. The measured value of plasma inductance L_{plasma} and plasma resistance R_{plasma} are taken from [19]. All these electrical measurements are done at a particular gas flow rate of argon, for plasma it is 25 LPM and for sheath it is 50 LPM. Both measured and simulated values of the electrical parameters such as peak coil current, frequency and plasma power for RF-ICP torch system are shown in table 1 and table 2 respectively.

| Table 1. Experimental results for I_{cmax}, frequency, P_{plasma}. |
|---|
| DC plate bias Voltage(kV), V_{dc} | Input DC power (kW) | I_{cmax} (A) (peak coil current) | Frequency (MHz) | P_{plasma} (kW) |
| 2.6 | 5.2 | 52 | 3.1830 | 1.97 |
| 3.0 | 9.0 | 70 | 3.2258 | 4.21 |
| 3.4 | 13.6 | 95 | 3.3113 | 8.83 |
| 4.0 | 20.0 | 112 | 3.3186 | 13.58 |
| 4.4 | 26.4 | 122 | 3.3250 | 19.15 |

| Table 2. Simulated results for I_{cmax}, frequency, P_{plasma}. |
|---|
| DC plate bias Voltage(kV), V_{dc} | Input DC power (kW) | I_{cmax} (A) (peak coil current) | Frequency (MHz) | P_{plasma} (kW) |
| 2.6 | 5.2 | 60.95 | 3.4782 | 2.41 |
| 3.0 | 9.0 | 72.66 | 3.4722 | 4.46 |
| 3.4 | 13.6 | 85.13 | 3.5696 | 7.09 |
| 4.0 | 20.0 | 101.00 | 3.6023 | 11.04 |
| 4.4 | 26.4 | 112.00 | 3.6297 | 16.14 |
5. Results and discussions

Figure 5 shows the simulated wave form of the radio frequency coil current which is obtained from the MATLAB simulation of state space model of the RF-ICP generator circuit. The circuit condition under which the simulation has been done is mentioned in the box inside the figure 5. Peak coil current and its frequency have been obtained as $I_{c\max}=112$amps and $f=3.6297$MHz.

![Figure 5. Time response of the coil current.](image)

Fig.6 shows that predicted coil current by state space model and measured coil current from Rogowski coil are in good agreement. It can be seen that as plate bias voltage increases the coil current increases and thus the power delivered to plasma also increases.

![Figure 6. Comparison of simulated and experimental coil current versus plate bias voltage.](image)
One can control the power delivered to the plasma by varying the plate bias voltage. The frequency of oscillation is governed by the tank circuit in which plasma is an active element. It can be seen from Table 1 and Table 2 that frequency increase as the bias voltage increases and thus plasma impedance also varies. Fig. 7 shows that the reflected resistance of plasma also increases with plasma power. From figure 6 it can be seen that when the plate bias voltage increases, the magnitude of coil current also increases, which also increases power developed across the plasma. Therefore the plasma’s reflected resistance also increases as shown in figure 7.

![Figure 7. Reflected resistance of the plasma versus plasma power.](image)

![Figure 8. Plate bias voltage Vs Overall efficiency.](image)

Figure 8 shows both the experimental as well as simulated results for the overall efficiency versus plate bias voltage. The results indicate that the overall efficiency of the power transfer to the plasma increases with increasing plasma power as a function of plate bias voltage.

6. Conclusions
Using state space model the simulation of the radio-frequency ICP torch and RF power supply system has been made. Here the induction plasma is considered as a part of the RF network. The RF-ICP torch system has been indigenously developed at BARC, Mumbai. State space modeling approach estimates the steady state output electrical parameter of the generator. MATLAB 7.1 software is used for solving the differential equations which is quite handy and gives reliable and accurate results. By using both measured and simulated values of rms coil current and frequency, dissipated plasma power has been calculated. The experimental and simulation results for overall efficiency Vs plate bias voltage are in agreement with each other. By using the state space approach, various generator designs and component combinations can be tested successfully. Thus state space model can prove useful as a design tool for induction plasma generator.

Acknowledgement
The authors are thankful to Dr. L.M. Gantayet, Associate Director, Beam technology development group and Dr. A.K. Das, head L&PT Division for their support during the course of this work. The authors are thankful to Shri D. P. Chakravarthy for valuable discussions and suggestions during the experimental measurements. The fellowship given by BRNS during the course of this work is gratefully acknowledged by author Sangeeta B.P. Authors acknowledge the help provided by Mr. Mayank Prakash during the preparation of this manuscript.
References:
[1] Reed T B 1961 Induction plasma torch J. Appl. Phys. 32 821–24
[2] Mostaghimi J and Boulos M I 1994 Inductively Coupled Plasmas in Analytical Atomic Spectrometry Montaser A and Golightly D W Eds.2nd ed. VCH pp 949–84
[3] Bernadi D, Colombo V, Ghedini E, Mentrelli A and Trombetti T 2004 3-D numerical simulation and fully coupled heating in ICPT’s Eur. Physics J.D. 28 423-33
[4] Bernadi D, Colombo V, Ghendini E and Mentrelli A 2003 Three dimensional effects in the modelling and ICPT’s Eur. Phys.J.D. 25 271-77
[5] Yoshida T, Nakagawa K, Harada T, and Akashi K 1981 New design of a radio-frequency plasma torch Plasma Chem. Plasma Process. 1113–829
[6] Mostaghimi J, Proulx P, and Boulos M I 1985 Computer modeling of the emission attens for an ICP spectrochemical torch Spectrochim. Acta B, At. Spectros. 40 153–66
[7] Proulx P, Mostaghimi J, and Boulos M I 1985 Plasma-particle interaction effects in induction plasma modeling under dense loading conditions Int. J. Heat Mass Trans. 28 1327–36
[8] Mostaghimi J and Boulos M I 1990 Effect of frequency on local thermodynamic equilibrium in an inductively coupled plasma at atmospheric pressure J. Appl. Phys. 68 2643–48
[9] Suekane T, Taya T, Okuno Y and Kabashima S 1996 Numerical study of the non equilibrium inductively coupled plasma with metal vapor ionization IEEE Trans. Plasma Sci 24 1147
[10] Cai M, A. Montaser A, and Mostaghimi J 1995 A two-temperature model for the simulation of atmospheric-pressure helium ICP’s Appl. Spectrosc. Part B 49 1390–1402
[11] Mostaghimi J and Boulos M I 1989 Two-dimensional electromagnetic field effects in induction plasma modelling Plasma Chem. Plasma Proc. 9 25–44
[12] Merkhouf A and Boulos M I 1998 Integrated model for the radio frequency induction plasma torch and power supply system Plasma sources sci. Technology 7 599-606
[13] Chentouf A, Fouladgar J, and Daveley G 1995 A simplified method for calculation of the impedance of an induction plasma IEEE Trans. Magnetics 31 2100-03
[14] Roland E 1996 Analytical method for the analysis of the behaviour of a high frequency triode induction generator for a plasma inductive torch J de Physique III 3 1733-58
[15] Kim J, Mostaghimi J, Iravani R 1997 Performance analysis of a radio frequency induction plasma generator using nonlinear state-space approach IEEE Trans.on plasma science 25 1023-28
[16] Chapra S C, Canale R P 2007 Numerical methods for engineers NewDelhi Tata Mcgraw hill, fifth edition pp 563-98
[17] Hunt Brian R, Lipsman Ronald L Differential Equations with MATLAB(R) chap.5-11.
[18] Chua L O 1975 Computer aided analysis of electronic circuits Prentice hall publication pp 337-43
[19] Biswal Sushim 2007 Performance Analysis and Estimation of Electrical Parameter for 50kW, 4MHz Inductively coupled plasma Reactor Dept. Elect.Engg. M-Tech thesis V J T I Mumbai
[20] Dewangan Rakesh Kumar 2008 State space modeling of radio frequency inductively coupled plasma generator Dept. electrical engg. M-Tech thesis V J T I Mumbai