The Onset of Color Transparency in \((e, e'p)\) Reactions on Nuclei

N.N.Nikolaev\(^1,2\), A.Szczurek\(^1,3\), J.Speth\(^1\), J.Wambach\(^1,4\),
B.G.Zakharov\(^5\), V.R.Zoller\(^6\)

\(^1\)IKP(Theorie), Forschungszentrum Juelich GmbH.,
5170 Juelich, Germany
\(^2\)L.D.Landau Institute for Theoretical Physics,
GSP-1, 117940, ul.Kosygina 2, V-334 Moscow, Russia
\(^3\) Institute of Nuclear Physics, PL-31-342 Krakow, Poland
\(^4\)Department of Physics, University of Illinois at Urbana-Champaign,
Urbana, IL 61801, USA
\(^5\)Nuclear Research Institute, High Energy Laboratory,
Moscow State University, 177234 Moscow, Russia
\(^6\)Institute for Theoretical and Experimental Physics,
ul.B.Cheremushkinskaya 25, 117259 Moscow, Russia

Abstract

Quantum filtering of the ejectile wave packet from hard \(ep\) scattering on bound nucleons puts stringent constraints on the onset of color transparency in \((e, e'p)\) reactions in nuclei at moderate energies. Based on multiple-scattering theory, we derive a novel formula for nuclear transparency and discuss its energy dependence in terms of a color transparency sum rule.
1 Introduction.

There has been much discussion on the possibility of color transparency (weak final state interaction, weak nuclear attenuation) in \((e,e'p)\) reactions on nuclei [1-9]. It is expected that, at very large \(Q^2\) the nuclear attenuation of the ejected protons should vanish, as the hard \(ep\) scattering selects a small-size configuration of quarks [1,2]

\[
\rho \propto 1/Q ,
\]

and for the colourless \(q\bar{q}\) or 3q systems of small size \(\rho\) the interaction cross section is small [10,11]:

\[
\sigma(\rho) \propto \rho^2 .
\]

Nuclear attenuation will be weak if the small size (2) stays frozen while the ejectile propagates through the nucleus, i.e., if the so-called formation (coherence) length [12] \(l_f\) is larger than the radius of a nucleus \(R_A\):

\[
l_f = \gamma \frac{1}{\Delta m} \gtrsim R_A ,
\]

where \(\Delta m \sim (0.5-1.0)\text{GeV}\) is a typical level splitting of the nucleonic resonances and \(\gamma\) is the relativistic \(\gamma\)-factor of the ejectile state. One then needs for the ejectile proton energy (which is equal to the energy loss of the scattered electron) that \(\nu \gtrsim (3-5)\text{GeV} \cdot A^{1/3}\) and that

\[
Q^2 \approx 2m_N\nu \gtrsim (5-10)A^{1/3}(\text{GeV}/c)^2 .
\]

This shows that in the foreseeable future all the experimental data will correspond to \(l_f \sim R_A\).

In the practical terms, it is not clear which \(Q^2\) is large enough for the onset of the asymptotic shrinkage (1) [7,8]. Besides, even if the system of quarks emerging from the hard \(ep\) scattering vertex has a small size, much depends on how this wave packet evolves when propagating through the nucleus. Evidently, the small-size configurations, like any non-stationary state, should be expanded in terms of the hadronic mass-eigenstates. Each hadronic state has a large free-nucleon cross section, \(\gtrsim \sigma_{tot}(pN)\), and weak attenuation can only come from Gribov’s inelastic shadowing [13], i.e., from the quantum interference of the diagonal (elastic) and off-diagonal (inelastic) diffractive transitions between the nucleon and its excited states when the ejectile propagates in the nuclear medium. All these transitions should be properly phase-correlated. Small initial size of the ejectile requires many conspiring excited states and, to keep them all phase-correlated, may require very large incident energy.

The subject of this paper is the formulation of a quantitative criterion for the weak final-state interaction in terms of Gribov’s theory of inelastic shadowing [13]. We formulate a ”color transparency (CT) sum rule” which demonstrates clearly the close connection between the onset of CT and Gribov’s inelastic shadowing corrections. We derive a formula for nuclear transparency with complete treatment of the quantum interference effects in the propagation of the ejectile. This has not been done before. We find that the onset of CT vs. incident energy is very slow and is very sensitive to the mass spectrum of the diffractively produced states.
2 Color transparency sum rule

In order to set a frame of reference, let us consider a two-dimensional $q\bar{q}$ system with the vector $\vec{\rho}$ being its transverse size in the plane normal to ejectile's momentum. The ejectile can be described in terms of two complete sets of wave functions: the mass eigenstates $|i⟩$ and the fixed-$\vec{\rho}$ states $|\vec{\rho}⟩$. The mass-eigenstates $|i⟩$ can be expanded in the basis $|\vec{\rho}⟩$ yielding the stationary wave function

$$\Psi_i(\vec{\rho}) = \langle \vec{\rho}|i⟩ .$$

Vice versa, the $\vec{\rho}$-eigenstates can be expanded as

$$|\vec{\rho}⟩ = \sum_i \Psi_i(\vec{\rho})^* |i⟩ .$$

Let $\hat{\sigma}$ be the cross section or diffraction operator. In the $\vec{\rho}$-representation $\hat{\sigma}$ is a diagonal operator: $\hat{\sigma} = \sigma(\rho)$. In the mass-eigenstate basis the diagonal matrix elements give the total cross section: $\sigma_{\text{tot}}(iN) = \sigma_{ii} = \langle i|\hat{\sigma}|i⟩$ and the differential cross section for forward elastic scattering at $t = 0$, $d\sigma_{\text{el}}(iN)/dt|_{t=0} = \sigma_{ii}^2/16\pi$. The off-diagonal matrix elements $\sigma_{ik} = \langle i|\hat{\sigma}|k⟩$ describe the differential cross section of the forward diffraction excitation $kN \rightarrow iN$:

$$\frac{d\sigma_D(kN \rightarrow iN)}{dt}|_{t=0} = \frac{\sigma_{ik}^2}{16\pi} .$$

The mass-eigenstate expansion for $\sigma(\rho)$ reads as

$$\sigma(\rho) = \langle \vec{\rho}|\hat{\sigma}|\vec{\rho}⟩ = \sum_{i,k} \langle \vec{\rho}|k⟩ \langle k|\hat{\sigma}|i⟩ \langle i|\vec{\rho}⟩ = \sum_{i,k} \Psi_i^*(\vec{\rho})\Psi_k(\vec{\rho})\sigma_{ki} .$$

Experimentally, the diffraction excitation rate is much smaller than the elastic scattering rate, $\sigma_{ik} \ll \sigma_{ii}, \sigma_{kk}$ (see below), so that one might be tempted to neglect the off-diagonal terms $\propto \sigma_{ik}$ in (8). In doing so one runs into a contradiction with CT as formulated in eq. (2) since

$$\sigma(\rho) \approx \sum_i \Psi_i^*(\vec{\rho})\Psi_i(\vec{\rho})\sigma_{\text{tot}}(iN) \sim \sigma_{\text{tot}}(pN) \sum_i |\Psi(\vec{\rho})|^2 = \sigma_{\text{tot}}(pN) ,$$

with the l.h.s expected to vanish as $\rho \rightarrow 0$, whereas the r.h.s does not depend on $\rho$ at all.

This makes it obvious that, in the mass-eigenstate basis, CT has its origin in the strong cancellations between the diagonal and off-diagonal diffractive transitions [8,16]. QCD, as the theory of strong interactions, dictates a very special relationship between the wave functions at the origin, $\Psi_i(0)$, and the matrix of the diffraction transition amplitudes $\sigma_{ik}$, which may be called the "CT sum rule"

$$\sum_{i,k} \Psi_k(0)^*\Psi_i(0)\sigma_{ki} = 0 .$$
The time evolution of the wave packet \( |\vec{\rho}, t \rangle \) is given by
\[
|\vec{\rho}, t \rangle = \sum_i \Psi_i(\vec{\rho})^* |i \rangle \exp(-im_i t) \tag{11}
\]
and
\[
\langle \vec{\rho}, t | \hat{\sigma} | \vec{\rho}, t \rangle = \sum_{i,k} \langle \vec{\rho} | k \rangle \langle k | \hat{\sigma} | i \rangle \langle i | \vec{\rho} \rangle \exp[i(m_k - m_i)t]. \tag{12}
\]

The proper time \( t \) is related to the distance \( z \) from the \( ep \) scattering vertex as \( t = z/\gamma \). As soon as large phases \( (m_k - m_i)z/\gamma \sim z/l_f \geq 1 \) emerge in the phase factors of (12), they will destroy the delicate cancellations necessary for CT. Then, the crucial issue is how rapidly the CT sum rule (10) is saturated by the lowest-lying excitations of the nucleon. Saturation requires the excitation of high-lying nucleonic resonances and therefore impractically large \( Q^2 \) may be needed for an onset of CT.

3 Color transparency criterion and diffraction scattering

The CT sum rule (10) can be given a still more practical formulation. Since \( \sigma(\rho) \) is the eigenvalue of the cross section operator \( \hat{\sigma} \), the CT sum rule simply states that \( \hat{\sigma} \) has a vanishing eigenvalue [17].

The ejectile state \( |E \rangle \) emerging from the hard \( ep \) scattering is a coherent superposition of nucleonic states \( |i \rangle \) with the relative amplitudes given by the elastic and transition electromagnetic form factors \( G_{ip}(Q^2) = \langle i | J_{em} | p \rangle \). Thus
\[
|E \rangle = \sum G_{ip}(Q^2) |i \rangle. \tag{13}
\]

The proton, as the ground state of the mass operator, is expected to have the smallest size of all the nucleonic states. In order to form a wave packet \( |E \rangle \) of the size \( \rho \ll R_p \), where \( R_p \) is the proton radius, one has to mix in the higher excitations which have an ever growing size. According to eq. (12), however, the higher the excitation energy, the faster the corresponding admixture to \( |E \rangle \) becomes out of phase during the evolution, so that only finite number of excited states \( N_{\text{eff}}(Q) \) can interfere coherently at finite energy.

In order to set up a simple formalism for the \( Q \) (or \( \nu \)) dependence of \( N_{\text{eff}}(Q) \), let us start with the proton-nucleus total cross section. The conventional Glauber formula reads
\[
\sigma_{\text{tot}}(pA) = 2 \int d^2\vec{b} \left\{ 1 - \exp \left[ -\frac{1}{2} \sigma_{\text{tot}}(pN)T(b) \right] \right\} \tag{14}
\]
and corresponds to the sum of all multiple-scattering diagrams with the free-nucleon cross section \( \sigma_{\text{tot}}(pN) \) (Fig. 1a). Here \( b \) stands for the impact parameter, \( T(b) = \int_{-\infty}^{+\infty} dz n_A(b, z) \) is the familiar optical thickness of the nucleus and \( n_A(b, z) \) denotes the target matter density. The inclusion of off-diagonal diffractive transitions shown in Fig. 1b (Gribov’s inelastic shadowing [13]), corresponds to the coupled-channel generalization of (14) [14,15]
\[
\sigma_{\text{tot}}(pA) = 2 \int d^2\vec{b} \left\{ 1 - \langle p | \exp \left[ -\frac{1}{2} \hat{\sigma}T(b) \right] | p \rangle \right\}. \tag{15}
\]
In order to see how the high-energy formula (15) should be modified at moderate energy, consider the $\nu$-fold scattering contribution to (15)

$$\frac{1}{\nu!} T(b)^\nu (p|\hat{\sigma}^\nu |p) = \sum_{i,j,k} \sigma_{pi} \sigma_{ij} \ldots \sigma_{kp} \int_{-\infty}^{+\infty} dz_\nu n_A(b, z_\nu) \ldots \int_{-\infty}^{z_1} dz_1 n_A(b, z_1)$$  \quad (16)

Each off-diagonal transition $i N \rightarrow k N$ involves a longitudinal momentum transfer

$$\kappa_{ik} = \frac{m_k^2 - m_i^2}{2E} \approx \frac{1}{l_f}$$  \quad (17)

and a phase factor

$$\exp [i\kappa_{pi} z_\nu + i\kappa_{ij} z_{\nu-1} + \ldots + i\kappa_{kp} z_1]$$  \quad (18)

emerges in the integrand of the r.h.s. of (16). In time-ordered perturbation theory with conserved momentum $\vec{p}$ one finds the same phase factor when expanding the energy of the relativistic intermediate state as $\nu_i = \sqrt{p^2 + m_i^2} \approx p + m_i^2/2\nu$ and factoring out the common t-dependence in eq. (11).

Eq. (15) holds at high energies, when $l_f \gg R_A$, and the phase in (18) can be neglected. In this limit a convenient way to calculate the nuclear cross section is to diagonalize the diffraction matrix $\hat{\sigma}$ in order to find the corresponding eigenstates $|\alpha\rangle$ and eigenvalues $\sigma_{tot}(\alpha N) = \langle \alpha | \hat{\sigma} | \alpha \rangle$, then expand the proton wave function in these diffraction eigenstates

$$|p\rangle = \sum_{\alpha} a_{\alpha} |\alpha\rangle$$  \quad (19)

and calculate the cross section (15) as [14,15]

$$\sigma_{tot}(pA) = \sum_{\alpha} a_\alpha^* a_{\alpha} 2 \int d^2 \vec{b} \left\{ 1 - \exp \left[ -\frac{1}{2} \sigma_{tot}(\alpha N) T(b) \right] \right\} .$$  \quad (20)

In contrast, at lower energies, when $\kappa_{ik} R_A \sim R_A/l_f \gg 1$, contributions from the Gribov’s off-diagonal rescatterings will vanish upon integration over the nucleon’s positions $z_i$ and only the elastic rescattering, $p N \rightarrow p N$, contribution survives in the l.h.s. of (18). As a result, the nuclear cross section will be given by Glauber’s formula (14) with the free-nucleon cross section $\sigma_{tot}(pN)$.

At intermediate energies, the result of the $z$ integrations with the phase factor (18) in the r.h.s. of eq. (16) will be that the l.h.s of eq. (16) is multiplied by a certain form factor $F_\nu(b, \kappa_{ij})$. In a somewhat crude approximation, one can use for this form factor a factorized form

$$F_\nu(b, \kappa_{ij}) \approx \prod_{\mu} G_A(\kappa_{ij}) ,$$  \quad (21)

where $G_A(\kappa)$ is the charge form factor of the target nucleus (see, for instance, considerations in [18]). Then, at each rescattering vertex, the form factor $G_A(\kappa_{ik})$ can be absorbed into the corresponding rescattering amplitude

$$\sigma_{ik}^{eff} = \sigma_{ik} G_A(\kappa_{ik}) .$$  \quad (22)
Therefore, at moderate energy, one has to solve for the eigenvalues and eigenstates of the effective energy-dependent diffraction operator \( \hat{\sigma}^{\text{eff}} \) and calculate the nuclear cross section applying eqs. (19) and (20). The factorization (21) is an approximation, but it correctly describes the opening of new coherently interfering channels with

\[
m_i^2 - m_p^2 \sim \frac{\nu}{R_A}
\]

as the incident energy gradually increases. Notice, that for elastic rescatterings \( \kappa_{ii} = 0 \) and \( G_A(\kappa_{ii}) = 1 \). When the form factor becomes small and the Gaussian approximation becomes too crude, the corresponding state decouples anyway. Therefore, using the diffraction operator \( \hat{\sigma}^{\text{eff}} \) as defined by eq. (22) is justified.

In the \((e,e'p)\) reaction on nuclei the incident particle is the virtual photon emitted by the electron. In the corresponding multiple-scattering diagrams of Fig. 1 the rightmost vertex involves \( G_j(Q^2) \). We are interested in those quasielastic \( e + A \rightarrow e + p + A^* \) reactions, where one sums over all excitations of the target debris \( A^* \), excluding production of secondary particles (mesons). In the high-energy limit, the coupled-channel formula for the nuclear transparency or more precisely the transmission coefficient \( T_{RA} \) (which can easily be derived following the technique of Ref. 15; a detailed derivation will be presented elsewhere) takes the form

\[
T_{RA} = \frac{d\sigma_A}{Zd\sigma_p} = \frac{1}{|\sum_\alpha a_\alpha^* c_\alpha|^2} \sum_{\alpha,\beta} a_\alpha^* c_\alpha c_\beta a_\beta T(\Xi_{\alpha\beta}),
\]

where

\[
T(\sigma) = \frac{1}{A\sigma} \int d^2b \{1 - \exp [-\sigma T(b)]\},
\]

\[
\Xi_{\alpha\beta} = \frac{1}{2}(\sigma_{\text{tot}}(\alpha N) + \sigma_{\text{tot}}(\beta N) - \int d\Omega |f_\alpha(\theta)f_\beta(\theta)|),
\]

and \( c_\alpha \) is defined by expansion of the ejectile in the eigenvalues of the diffraction operator \( \hat{\sigma} \)

\[
|E\rangle = \sum_\alpha c_\alpha |\alpha\rangle.
\]

In (26) \( f_\alpha(\theta) \) is the corresponding elastic scattering amplitude \( (\sigma_{el} = \int d\Omega |f_\alpha(\theta)|^2) \). Notice, that the numerator of eq. (25) is precisely the inelastic nuclear cross section \( \sigma_{in}(hA) \) for the free-nucleon cross section \( \sigma_{\text{tot}}(hN) = \sigma \). The emergence of the non-trivial quantity \( \Xi_{\alpha\beta} \) demonstrates the importance of the quantal interference effects in the multiple-scattering problem [15].

At moderate energies, one has to calculate the nuclear transparency (24) using the diffraction operator \( \hat{\sigma}^{\text{eff}} \). Simultaneously, one has to apply the rule (22) to the electromagnetic vertex as well: \( G_{kp}(Q^2) \rightarrow G_{kp}(Q^2)G_A(\kappa_{kp}) \).

In the low-energy limit of \( l_f \ll R_A \) all inelastic excitations decouple and the coupled-channel problem reduces to a single-channel problem. In this case \( \sigma_{\text{tot}}(\alpha N) = \sigma_{\text{tot}}(pN) \), the last term in (24) is the elastic cross section \( \sigma_{el}(pN) \) and \( T_{RA} \) is given by eq. (23) with \( \sigma = \sigma_{in}(pN) \). This result is quite obvious, since the elastic rescatterings of the high-energy ejectile do not contribute to nuclear attenuation. Glauber and Matthiae have noticed this in 1970 in their multiple-scattering analysis [19] of the quasielastic proton-nucleus scattering (see also a recent paper by Kohama, Yazaki and Seki [9]).
4 Estimates for nuclear transparency

A possible signal of CT in \((e, e'p)\) reactions depends on how baryons are excited in both the hard electromagnetic and the soft diffractive scattering, i.e., on the internal structure of the nucleon. In high-energy QCD, the quark-spin changing diffractive transitions are weak. Hence the candidate states which could conspire to produce an ejectile of small transverse size ejectile are the \(N(938, \frac{1}{2}^+)\), \(N^*(1680, \frac{5}{2}^+)\), \(N^*(2220, \frac{3}{2}^+)\) as well as higher excitations. (In the two-dimensional case only the radial excitations contribute to the CT sum rule, in the three-dimensional case angular excitations to the same parity states contribute as well). Excitation of the \(N^*(1680, \frac{5}{2}^+)\) is one of the prominent channels of the diffraction-dissociation of nucleons with \(\sigma(NN \rightarrow N^*(1680, \frac{5}{2}^+)) \approx 0.17 \text{mb} \) as compared to \(\sigma_{el}(NN \rightarrow NN) \approx 7 \text{mb} \) [20, 21]. The total inclusive cross section of diffractive excitations sums up to \(\sim 2 \text{mb} \) [21].

As we have discussed above, the requirement of phase coherence, limits the number of interfering states \(N_{eff}(Q)\) from above. The case of \(N_{eff} = 1\) corresponds to \(\sigma_{min} = \sigma_{11} = \sigma_{tot}(pN)\) and represents the conventional nuclear attenuation. The case of \(M = 2\) gives

\[
\sigma_{min} = \frac{1}{2}(\sigma_{11} + \sigma_{22}) - \frac{1}{2}\sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}
\] (28)

so that \(\sigma_{min} \geq \sigma_{tot}(NN) - |\sigma_{12}|\). Judging from the magnitude of the diffraction-dissociation cross sections, one cannot gain much in transmission when the number of interfering states is small. Two states are simply not enough to form a wave packet of small size and small \(\sigma_{min}\) (excitation and coherent interference of many states is indeed important for CT).

There exists considerable evidence that the nucleon can be regarded as a quark core surrounded by a pionic cloud [22] which, in the simplest approximation, yields a wave function

\[
|N\rangle = \cos(\theta)|3q\rangle + \sin(\theta)|3q + \pi\rangle.
\] (29)

From the diffractive scattering point of view, one can estimate the core size from \(\sigma_{tot}(NN) \approx \sigma_{tot}((3q)N) + n_\pi \sigma_{tot}(\pi N)\), where \(n_\pi\) is the average number of pions in the nucleon. Here we have neglected the shadowing in the \(3q + \pi\) system. With \(\sigma_{tot}(\pi N) \approx 2/3\sigma_{tot}(NN)\) we then have

\[
\sigma_{tot}((3q)N) \approx (1 - \frac{2}{3}n_\pi)\sigma_{tot}(NN)
\] (30)

which gives a lower bound for \(\sigma_{tot}((3q)N)\). Evaluation of the number of pions in the nucleon with a realistic \(\pi N\) vertex function gives \(n_\pi \sim 0.4\) [22]. The same admixture of pions describes diffraction production of the continuum \(N\pi\) system by the so-called Drell-Hiida-Deck mechanism (for a review see [21]).

From the point of view of electromagnetic scattering, the \(3q + \pi\) admixture has a large radius and contributes significantly to the charge radius of the proton. However, because of the large size, the contribution of the \(3q + \pi\) component to the nucleon electromagnetic form factor vanishes rapidly at large \(Q^2\). Our estimates with realistic pion-nucleon vertex functions show that the 3-quark Fock state dominates already at \(Q^2 \sim 1\text{(GeV/c)}^2\). The electromagnetic form factor analysis suggests \(\langle R_{ch}^2 \rangle_{3q} \approx 0.8\langle R_{ch}^2 \rangle_{p}\), which is consistent with eq. (31).
Therefore, at \( Q^2 \sim 10(\text{GeV}/c)^2 \), the region of interest for CT, we can concentrate on excitations of the 3-quark core. In order to study the onset of color transparency, we calculate the diffraction matrix elements \( \sigma_{ik} \) in the quark-diquark model of the nucleon, using three-dimensional harmonic oscillator wave functions and choose the \( \rho \) dependence of the interaction cross section as

\[
\sigma(\rho) = \sigma_0 \left[ 1 - \exp \left( -\frac{\rho^2}{R_0^2} \right) \right].
\]  

(31)

This form reproduces the color transparency property (2) and saturates at large \( \rho \) in agreement with confinement [23]. Following the calculations in Ref. [23], we take \( \sigma_0 \approx 1.6\sigma_{\text{tot}}(pN) \) and \( R_0^2 \approx 0.55 \text{ fm}^2 \), such that the 3q-core interacts with the nucleon with the cross section \( \approx 0.8\sigma_{\text{tot}}(pN) \), cf. eq. (30). In a model of scalar diquarks and with the QCD suppression of the spin-flip transitions, the dominant diffraction transitions are to the even angular momentum and positive parity quark-diquark states with the angular momentum projection \( L_z = 0 \). Production of \( N^*(1680, \frac{5}{2}^+) \) is a prominent feature of the forward diffraction dissociation of nucleons, which suggests \( \Delta m = 2\hbar\omega \approx 700 \text{ MeV}/c^2 \). The radial excitations of nucleons in the non-relativistic quark model rather suggest \( \Delta m \approx 1200 \text{ MeV}/c^2 \). This may be a more realistic estimate if we ask for a large number of interfering states \( N_{\text{eff}}(Q) \approx 1, 3, 6, 10 \) with the number of interfering major shells \( K = 1, 2, 3, 4 \). For our purpose it is sufficient to take a Gaussian nuclear form factor

\[
G_A(\kappa) = \exp\left( -\frac{1}{6} \left( \frac{R_{ch}}{\kappa} \right)^2 \right)
\]

when evaluating the effective diffraction matrix \( \hat{\sigma}_{\text{eff}} \).

In Table 1 we show the structure of the diffraction matrix \( \hat{\sigma} \). Its off-diagonal matrix elements give realistic estimates for the diffraction dissociation cross sections \( \sigma_D(pN \rightarrow ip) \sim (\sigma_{\text{tot}}(iN)/\sigma_{\text{tot}}(pN))^2\sigma_{\text{el}}(pp) \). In Table 2 we show the eigenvalues of the truncated diffraction matrix \( \hat{\sigma} \). The departure of the minimal eigenvalue \( \sigma_{\text{min}} \) from zero measures the rate of saturation of the CT sum rule in the limited basis of \( N_{\text{eff}} \) states. The minimal eigenvalue \( \sigma_{\text{min}}(Q) \) decreases at large \( N_{\text{eff}}(Q) \), but this decrease is very slow (see below).

Fig. 2 displays the lowest eigenvalue of \( \hat{\sigma}_{\text{eff}} \) as a function of energy for few nuclei. The corresponding effective number of interfering shells \( K \) can easily be deduced from Table 2. The \( Q(\nu) \) dependence of \( \sigma_{\text{min}} \) can be understood from the condition (23). For instance, at \( \nu \sim 30 \text{ GeV} \) only the ground state (proton, \( K = 1 \)) and the two excited shells, \( K = 2, 3 \), can interfere coherently in the \((e, e'p)\) reaction on the carbon nucleus. Notice, that excitation of the same number of shells on different nuclei requires an energy \( \nu \propto A^{1/3} \), and we predict therefore slower onset of reduced nuclear attenuation for heavy nuclei as compared to higher nuclei.

We can optimize for the transparency by requiring that the ejectile state \(|E\rangle \) coincides with the eigenfunction corresponding to the smallest eigenvalue \( \sigma_{\text{min}} \) of the diffraction matrix. The resulting transparency will be given by formula (25) with

\[
\sigma \approx \sigma_{\text{min}} \left( 1 - \frac{\sigma_{\text{min}}}{\sigma_{\text{tot}}(pN)} \frac{\sigma_{\text{el}}(pN)}{\sigma_{\text{tot}}(pN)} \right).
\]

(32)

For instance, when \( K = 1, 2, 3 \) shells are included, the eigenstate with \( \sigma_{\text{min}} = 17.5 \text{ mb} \), appropriate for an energy \( \nu \sim 30 \text{ GeV} \), corresponds to the superposition of the harmonic
and cannot enhance the nuclear transparency above our estimates shown in Fig.3.

If \( \sigma \) then
\[
\langle T r \rangle \text{ is sensitive to the number of pions in nucleons, see also eq.}(30).
\]

We then find a somewhat stronger signal of transparency at large \( Q^2 \) will be roughly proportional to the product of wave functions at the origin, and the transition form factors will be of the same order as the elastic scattering form factor, in agreement with the expansion given by (33) and with the conclusions by Stoler from an analysis [24] of the SLAC electroproduction data. With two shells (\( \sigma_{min} = 22.1mb \)) and with four shells (\( \sigma_{min} = 14.0mb \)) the corresponding eigenstates are

\[
|1,0\rangle : |2,0\rangle : |1,2\rangle : |3,0\rangle : |2,2\rangle : |1,4\rangle = 0.77 : 0.44 : 0.31 : 0.26 : 0.22 : 0.11 .
\]

For the radial excitations, the asymptotics of the non-relativistic form factor will be roughly proportional to the product of wave functions at the origin, and the transition form factors will be of the same order as the elastic scattering form factor, in agreement with the expansion given by (33) and with the conclusions by Stoler from an analysis [24] of the SLAC electroproduction data. With two shells (\( \sigma_{min} = 22.1mb \)) and with four shells (\( \sigma_{min} = 14.0mb \)) the corresponding eigenstates are

\[
|1,0\rangle : |2,0\rangle : |1,2\rangle = 0.85 : 0.43 : 0.30 ,
\]

\[
|1,0\rangle : |2,0\rangle : |1,2\rangle : |3,0\rangle : |2,2\rangle : |1,4\rangle : |4,0\rangle : |3,2\rangle : |2,4\rangle : |1,6\rangle = 0.70 : 0.43 : 0.31 : 0.28 : 0.24 : 0.18 : 0.17 : 0.10 : 0.04
\]

In Fig.3 we show the energy dependence of the nuclear transparency factor \( T r_A \) for these least-attenuation states. As a reference point one should take the low-energy values of transparency evaluated from formula (25) with \( \sigma = \sigma_{in}(pN) \approx 30mb: \ T r_C = 0.61, T r_{Al} = 0.49, T r_{Cu} = 0.40, T r_{Pb} = 0.26, \) which are also shown in Fig.3. We remind that the curves shown correspond to the 3\( q \)-core stripped off the pionic cloud and having \( \sigma_{tot}(3q)N \approx 0.8\sigma_{tot}(pN) \). At \( \nu \sim 15 GeV \) we find \( \sim 35\% \) reduction of attenuation for carbon, and \( \sim 25\% \) effect for aluminum. The effect decreases significantly if one chooses \( \Delta m \approx 1.2 GeV/c^2 \).

Notice, that with the optimized ejectile state \( |E\rangle \), the nuclear transparency in \((e,e'N^*)\) reactions is the same as in \((e,e'p)\) reaction, although for resonances the free-nucleon cross section is larger than for protons, cf. the diagonal matrix elements in Table 1. This equality holds at high enough energies, when for excitation of the corresponding shell \( \kappa_{N^*p}R_A \ll 1 \).

As we have explained above, states of a large number of the oscillator shells must conspire to produce a wave packet of very small size. However, at energies of practical interest \( \nu \sim (10 - 30) GeV \) and \( Q^2 \sim (20 - 60)(GeV/c^2) \) the admixture of shells with \( K \gtrsim 4 \), even if present in the initial ejectile state \( |E\rangle \), will get very rapidly out of phase and cannot enhance the nuclear transparency above our estimates shown in Fig.3.

The principal parameter which controls the onset of CT is the size of the 3\( q \)-core, which is sensitive to the number of pions in nucleons, see also eq.(30). If \( \langle R_{ch/3q}^2 \rangle = 0.5\langle R_{ch/3q}^2 \rangle p \), then \( \sigma_{tot}(3q)N = 24mb \) and, compared to the previous case, \( \sigma_{min} \) will be \( \sim 25\% \) smaller. We then find a somewhat stronger signal of transparency at large \( Q^2 \).

The above conclusions hold for the electroproduction of pions as well, where the first excited shell corresponds to the \( \pi(1300, 0^-), A_1(1260, 1^+) \) and \( \pi_2(1670, 2^-) \) resonances. In this case the appropriate energy scale will be set by \( \Delta m \sim 1.2 GeV/c^2 \).
5 Discussion of the results

We have constructed a realistic model of CT effects in \((e,e'p)\) reactions on nuclei. Our formalism, embodied in the target-size and energy-dependent diffraction matrix \((21)\) and in the nuclear transparency formulae \((24)-(26)\), correctly describes the quantum interference effects in the ejectile propagation through the nucleus. Even after optimizing the ejectile state scattering for weak attenuation, we find rather slow onset of CT. This can be understood in terms of a large number of excited states needed to produce wave packets of small transverse size. At moderate energies the coherent interference condition constrains \(N_{eff}(Q)\) from above. Nevertheless, for light nuclei (carbon or aluminum) the predicted CT signal is strong enough for a decisive test of CT ideas at the highest energy at SLAC or at a new European Electron Facility currently under discussion. CT experiments in \((e,e'p)\) scattering are much discussed as a case for electronuclear facilities of next generation \([1-9,25-28]\). Our analysis strongly suggests that electron beams of \(\sim 30 \text{ GeV}\) are highly desirable in order to observe a CT signal for heavy nuclear targets.

Major differences from early papers on CT effects in \((e,e'p)\) scattering is as follows. Farrar, Frankfurt, Liu and Strikman \([3]\) (see also \([4]\)) use a classical expansion and attenuation model, which is too crude to describe the quantal interference effects. This has been discussed in Ref. \([29]\) for photoproduction of charmonium off nuclei. The approach of Jennings and Miller \([5]\) is similar to ours to the extent that they also start from the multiple scattering theory. Their QCD-motivated model for the diffraction operator, used in \([5]\), satisfies our CT sum rule. We differ, however, significantly in the treatment of the coherence constraint. While we treat it within the coupled-channel theory, Jennings and Miller reduce the coupled-channel problem to a single-channel problem with a complex cross section. After this paper was submitted for publication, we have learned of preliminary results of the NE18 experiment at SLAC, which confirm our predictions of a slow onset of CT in \((e,e'p)\) scattering \([30]\).

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Table 1: The diffraction matrix (matrix elements are in millibarns).

| \( n_r, L \) | (1, 0) | (2, 0) | (1, 2) | (3, 0) | (2, 2) | (1, 4) |
|-------------|-------|-------|-------|-------|-------|-------|
| (1, 0)      | 32.0  | -13.1 | -9.2  | -5.8  | -4.9  | -2.3  |
| (2, 0)      | -13.1 | 42.7  | 7.5   | -11.9 | -4.0  | 4.8   |
| (1, 2)      | -9.2  | 7.5   | 37.3  | 1.7   | -7.1  | -12.9 |
| (3, 0)      | -5.8  | -11.9 | 1.7   | 46.9  | 7.2   | -2.6  |
| (2, 2)      | -4.9  | -4.0  | -7.1  | 7.2   | 40.4  | 6.5   |
| (1, 4)      | -2.3  | 4.8   | -12.9 | -2.6  | 6.5   | 44.7  |

Table 2: The minimal eigenvalue of the diffraction matrix vs. the number of the coupled shells \( K \) (\( \Delta m = 0.7 \text{GeV}/c^2 \)).

| Number of the shells, \( K \) | Number of the coupled states, \( M \) | Excitation energy (GeV) | \( \sigma_{\text{min}} \) (mb) |
|------------------------------|-----------------------------------|------------------------|------------------|
| 1                            | 1                                 | 0.94                   | 32.0             |
| 2                            | 3                                 | 1.64                   | 22.1             |
| 3                            | 6                                 | 2.34                   | 17.1             |
| 4                            | 10                                | 3.04                   | 14.0             |
| 5                            | 15                                | 3.74                   | 11.8             |
| 6                            | 21                                | 4.34                   | 10.3             |
| 7                            | 28                                | 5.04                   | 9.1              |
Figure captions

Fig.1 - Elastic (a) and inelastic (b) shadowing in pA scattering.

Fig.2 - The minimal eigenvalue $\sigma_{\text{min}}(\nu)$ ($\nu = Q^2/2m_N$) of the diffraction matrix (22) for the 3$q$-core of the nucleon as a function of $\nu$ for different nuclei ($\langle R_{ch}^2 \rangle_{3q} = 0.8\langle R_{ch}^2 \rangle_p$ and $\Delta m = 0.7\text{GeV}/c^2$).

Fig.3 - The ejectile energy ($\nu = Q^2/2m_N$) dependence of the nuclear transparency $T_{rA}$ in the quasielastic ($e,e'p$) scattering on different nuclei at $\Delta m = 0.7\text{GeV}/c^2$ (solid curves) and $\Delta m = 1.2\text{GeV}/c^2$ (dotted curve for the carbon nucleus) for the 3$q$-core with $\langle R_{ch}^2 \rangle_{3q} = 0.8\langle R_{ch}^2 \rangle_p$. The dashed curves for the carbon and lead nuclei are for $\langle R_{ch}^2 \rangle_{3q} = 0.5\langle R_{ch}^2 \rangle_p$ and $\Delta m = 0.7\text{GeV}/c^2$. The low energy predictions are shown separately.