Modelling of mechanical and filtration processes near the well with regard to anisotropy

To cite this article: V I Karev et al 2018 J. Phys.: Conf. Ser. 991 012039

View the article online for updates and enhancements.
Modelling of mechanical and filtration processes near the well with regard to anisotropy

V I Karev*, D M Klimov**, Yu F Kovalenko***, and K B Ustinov****
Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow, Russia
E-mail: *klimov@ipmnet.ru, **wikarev@ipmnet.ru, ***perfolinkgeo@yandex.ru, ****ustinov@ipmnet.ru

Abstract. A geomechanical approach to modeling deformation and seepage is presented. Three stages of modeling are described: choice of an appropriate mechanical model and its adaptation to the case in question, experimental determination of parameters of the model, simulation of processes of seepage for particular configurations of the well. The applied model allows describing the main specific characteristics of mechanical behavior of the collector: the influence of the pore pressure on deformation; the influence of not only shear but also comprehensive stresses and pore pressure on the transition to inelastic behavior; the appearance of inelastic volumetric deformation and its nontrivial dependence on the stress state; the anisotropy of elastic, strength and seepage properties; non-obvious dependence of permeability on the stress strain state. The model unites essential characteristics of Hill’s plastic flow theory for anisotropic materials and the Drucker–Prager theory for inelastic deformation of soils. The results of experimental determination of the involved parameters obtained using true triaxial loading system for the collector of Vladimir Filanovsky field in the Caspian Sea are presented.

1. Introduction
The process of hydrocarbon fluid seepage into productive wells is affected by many factors, one of which, and not the least significant, is the changes in the pore structure, and hence in permeability, due to deformation processes in collectors governed by the stress state. The situation is aggravated due to the back influence of seepage in rearrangement of the stress strain state, and hence it is necessary to solve the coupled elastic-plastic and filtration problem.

The approach of modeling used throughout the current research consists in the following key stages:

- choosing and, if necessary, developing an appropriate model of deformation and filtration;
- experimental determination of the parameters involved;
- solving the coupled mechanical (determining the stress strain state) and seepage (determining the fluid flow and the production rate) problem on the basis of the chosen model and experimentally obtained parameters.

The results of the research on these three stages are described below in the particular case of Filanovsky field collector.
2. Model of deformation and filtration

2.1. Generalities

One of the main goals of the current research, consists in choosing and developing an appropriate mechanical model of deformation and fracture of the material in the collector and the seepage processes in it. An adequate model should be able to describe the key characteristics of the process, among which we emphasize the following ones:

1. the influence of pore pressure on deformation;
2. the nonlinear stress-strain behavior with essential role of inelastic ("plastic") strains, and the influence of the level of not only shear but also comprehensive stress (and pore pressure) on the transition to inelastic behavior;
3. the possible inelastic volumetric deformation and its nontrivial dependence on the stress state: at least the absence of proportionality between inelastic volumetric strain and volumetric stress (violation of the associative law);
4. the possible anisotropy of elastic, strength and seepage properties;
5. the non-obvious dependence of permeability on the stress strain state.

All of these characteristics have been well described by a number of suggested theories. Thus, the influence of pore pressure on deformation (p. 1) is described by the theory of poroelasticity [1–3]. The influence of comprehensive stress on the level of elastic/plastic transition (p. 2) is described by the Moor–Coulomb and Drucker–Prager criteria [4–6]. To describe the inelastic volumetric deformation nonproportional to the volumetric stress (p. 3), the concept of dilatancy was proposed [7–10], according to which the former is supposed to depend on the intensity of shear strain, with possible violation of the associative law. The anisotropy in the theory of plastic flow was taken into account by Hill [11, 12], and further the Hill theory was modified to account for the difference in the behavior in compression and in tension [13, 14]. The dependence of permeability on the stress state may vary in a wide range depending on the particular collector and is usually determined from experiments.

The above-mentioned models, which describe the listed characteristics of deformation and filtration, form a basis for solving geomechanical problems. Moreover, some of them have already been implemented to commercial codes. Meanwhile, describing some characteristics each of these models does not describe the others, which does not allow their direct application. Therefore, one of the main goals of the current study consists in adoption and further development of these models aiming at taking the whole set of the above-listed specific features into account.

2.2. Basic equation of the model

The fluid flow is described by Darcy’s law

\[(\kappa p_i)_{,i} = 0,\]  

(1)

where \(p\) is the pore pressure; \(\kappa\) is the permeability which may generally depend on the coordinate, pore pressure, the stress state and other factors; the indexes after comma stand for the derivative with respect to the corresponding coordinate.

The stress strain state is described by the laws of poroelasticity [2, 3]. Their generalization to the case of plastic deformations may be written as

\[\sigma_{ij, i} = 0,\]  

(2)

\[s_{ij} = \sigma_{ij} + \alpha_p p \delta_{ij},\]  

(3)

\[s_{ij} = \Lambda_{ijkl} \varepsilon^{E}_{kl},\]  

(4)

\[\varepsilon^{E}_{ij} = \varepsilon^{T}_{ij} + \varepsilon^{P}_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}).\]  

(5)
Here $\sigma_{ij}$ and $s_{ij}$ are components of the total and effective (acting on solid skeleton) stress tensors; $\varepsilon^T_{ij}$, $\varepsilon^E_{ij}$, and $\varepsilon^P_{ij}$ are components of the total, elastic and plastic strain tensors; $u_i$ are components of the displacement vector; $\Lambda_{ijkl}$ are components of elasticity tensor (in the case of elastic isotropy, they are expressed in terms of two constants); $0 \leq \alpha_P \leq 1$ is Bio’s coefficient characterizing the structure of porosity; for well permeable collectors, $\alpha_P$ approaches the unity from below [5]. For practical reasons, one can set $\alpha_P = 1$. Before reaching the criterion of elastic-plastic transition $\varepsilon^P_{ij} = 0$, equations (1)–(5) together with the boundary conditions for stresses (or displacements) and the pore pressure form a closed system.

To describe the plastic deformation, one needs to specify the plasticity criterion and the plasticity potential. Consider a criterion of the type of [13] modified for taking account of the effects of poroelasticity

$$F = \left[ G^0_{(23)}(s_{22} - s_{33})^2 + G^0_{(13)}(s_{11} - s_{33})^2 + G^0_{(12)}(s_{11} - s_{22})^2 + 2L^0_{(23)}s_{23}^2 + 2L^0_{(13)}s_{31}^2 + 2L^0_{(12)}s_{12}^2 \right]^{1/2} + (B^0_{(1)}s_{11} + B^0_{(2)}s_{22} + B^0_{(3)}s_{33}) - 1 > 0.$$  \hspace{0.5cm} (6)

Here $s_{ij}$ are components of the effective stress tensor in the coordinate frame related to the material axes of isotropy; $G^0_{(ij)}, L^0_{(ij)}, B^0_{(ij)}$ are material constants [11–14].

For a transverse isotropic medium with the isotropy plane with the normal $n_3$, the number of constants in (6) reduces to

$$G^0_{(13)} = G^0_{(23)}, \quad L^0_{(13)} = L^0_{(23)}, \quad L^0_{(12)} = G^0_{(13)} + 2G^0_{(12)}, \quad B^0_{(1)} = B^0_{(2)}.$$  \hspace{0.5cm} (7)

In the isotropic case,

$$G^0_{(13)} = G^0_{(23)} = G^0_{(12)} = G, \quad L^0_{(13)} = L^0_{(23)} = L^0_{(12)} = 3G, \quad B^0_{(1)} = B^0_{(2)} = B^0_{(3)} = \frac{1}{3}B, \quad (8)$$

neglecting the effects related to the pore pressure (replacing effective stresses with the total ones) results in reducing (6) to the Drucker–Prager criterion [5].

On the other hand, for the vanishing influence of normal stresses on the elastic-plastic transition and effects related to pore pressure, equation (6) reduces to the Hill criterion [11, 12] of anisotropy plasticity.

According to the plastic flow theory (although modified for taking account of hydrostatic stresses), the actual state in the space of stresses during the active loading remains belonging to the yield surface (generalization of plasticity criterion (6))

$$F = \left[ G^0_{(23)}(s_{22} - s_{33})^2 + G^0_{(13)}(s_{11} - s_{33})^2 + G^0_{(12)}(s_{11} - s_{22})^2 ight. \left. + 2L^0_{(23)}s_{23}^2 + 2L^0_{(13)}s_{31}^2 + 2L^0_{(12)}s_{12}^2 \right]^{1/2} + (B^0_{(1)}s_{11} + B^0_{(2)}s_{22} + B^0_{(3)}s_{33}) - A(k), \quad (9)$$

where the dimensionless function $A(k)$ ($A(0) = 1$) serves as a characteristic of hardening, supposed to be isotropic for the model under consideration; $k$ is the parameter of hardening and, for a widely used variant of the theory, $k$ is associated with the work of plastic strains

$$dk = s_{ij} \varepsilon^P_{ij}. \quad (10)$$

During active loading, the stress growth is accompanied by plastic strain growth. In the frame of the plastic flow theory, the increment (“rate”) of plastic strain $\varepsilon^P_{ij}$ is expressed in terms of the plastic potential $Q$ as

$$d\varepsilon^P_{ij} = d\lambda \frac{\partial Q}{\partial s_{ij}}. \quad (11)$$

Here $d\lambda$ is a still unknown coefficient. In the classical theory of plasticity, equating the plastic potential with the yield surface (such a relation is referred to as the associative law) allows elegant
deducing to constructing constitutive laws capable of adequate description of the observed stress strain relations. However, using the associative law to describe the deformation of rocks and soils leads to a rather strong discrepancy from the phenomenon, namely, the predicted volumetric inelastic deformations are usually much greater than the actual ones. Therefore, the so-called non-associative laws with $Q \neq F$ are used to describe the deformation of rocks and soils [9, 10, 15].

Hereafter, the form of the plastic potential $Q$ is chosen similarly to the form of the yield function $F$ in equation (9) with dissimilar factors at the linear terms of stresses

$$Q = G_{(23)}^0(s_{22} - s_{33})^2 + G_{(13)}^0(s_{11} - s_{33})^2 + G_{(12)}^0(s_{11} - s_{22})^2 + 2L_{(23)}^0 s_{13}^2 + 2L_{(13)}^0 s_{31}^2 + 2L_{(12)}^0 s_{12}^2 + (B_{(1)}^1 s_{11} + B_{(2)}^1 s_{22} + B_{(3)}^1 s_{33}) - A(k). \quad (12)$$

In the case of vanishing or negligible inelastic volumetric strains (dilatancy), the following restriction takes place

$$B_{(1)}^1 + B_{(2)}^1 + B_{(2)}^1 = 0. \quad (13)$$

For a number of rocks and soils, this equation, although not exact, serves as a rather accurate approximation.

To provide the deviator associativity, it is sufficient to set

$$B_{(i)}^1 = B_{(i)}^0 - B_0, \quad B_0 = \frac{1}{3} \sum_{j=1}^{3} B_{(j)}^0. \quad (14)$$

The value of $d\lambda$ is determined from equation (9). The differential of $F$ should vanish

$$\frac{\partial F}{\partial s_{ij}} ds_{ij} + \frac{\partial F}{\partial k} dk = 0. \quad (15)$$

The substitution of $dk$ from equation (10) into this relation yields

$$\frac{\partial F}{\partial s_{ij}} ds_{ij} + \frac{\partial F}{\partial k} s_{ij} d\varepsilon_{ij}^p = 0. \quad (16)$$

Expressing the value of plastic strain in terms of the plastic potential (11), one finds

$$d\lambda = \frac{\partial F}{\partial s_{ij}} ds_{ij} \left( \frac{\partial Q}{\partial s_{ij}} s_{ij} \right)^{-1}. \quad (17)$$

Here $H \equiv -\partial F/\partial k$ is the material function determined from experiments. In the first approximation, it may be considered as the constant $H = E_p^{-1}$, where $E_p$ is the plasticity modulus.

Thus, for given $F$, $Q$, and $H$, equations (11), (10), and (17) form a closed system.

3. Experimental determination of the model parameters

The experimental part of the study consists in determining the model parameters. Experiments were carried using the experimental unit IPMech RAS Triaxial Independent Loading Test System (TILTS) [16] for cubic specimens (40 x 40 x 40 mm) for one of lithotypes of the Filanovsky field collector. Two types of the loading program were used: uniaxial loading with lateral pressure and generalized shear [17–19].

According to the loading program of the first type, the specimens were subjected to comprehensive pressure up to a certain value (2, 10, or 20 MPA) and then loaded along one of the axis up to the level of elastic/plastic transition in the displacement controlled mode.
Figure 1. Stress-strain curves for the loading program of the second type; the maximal stress corresponding to the vertical (a) and horizontal (b) directions.

Table 1. Results of tests for the loading program of the first type (Specimen Pr-1).

| Cycle | Lateral pressure, MPa | $E \times 10^{-3}$, MPa | $E_p \times 10^{-3}$, MPa | $S^*_2$, MPa |
|-------|-----------------------|--------------------------|---------------------------|--------------|
| 1     | 2                     | 1.80                     | 0.18                      | 27           |
| 2     | 10                    | 5.42                     | 0.54                      | 51           |
| 3     | 20                    | 7.78                     | 0.78                      | 75           |

During the second loading program, the specimens experience volumetric compression up to the level corresponding to the in situ stress state, and then the loading continues along one of the axes accompanied with unloading along another axis so that the stress along the third axis remains constant; the average stress is conserved at this stage. The stress state constructed in this program corresponds to the state near the well.

The permeability was measured along the direction of the minimal stress during the whole period. The tests were carried out for two orientations of the minimal stresses: along the layers and normally to the layers, which corresponds to two different points at the contour of the horizontal well. As a result, the dependence of permeability on the stress and the yield stresses corresponding to these loading conditions was obtained.

The results of tests according to the loading programs of the first type are presented in table 1. Figure 1 presents the stress strain curves in the cases of the maximal stress aligned in vertical (figure 1a) and one of the horizontal (figure 1b) directions. Figure 2 presents the dependence of permeability on the stress intensity. Figure 3 presents experimental and calculated data about the critical stress according to criterion (9).

The parameters of yield criteria (6) were determined from the whole set of experiments of both types (the property corresponds to transverse isotropy (7)) by means of the least square method $G_{(12)}^0 = 2.7 \times 10^{-3}$, $G_{(13)}^0 = G_{(23)}^0 = 1.27 \times 10^{-3}$, $B_{(1)}^0 = B_{(2)}^0 = 1.95 \times 10^{-2}$, $B_{(3)}^0 = 3.28 \times 10^{-2}$, $L_{(13)}^0 = 1.0 \times 10^{-2}$. It was impossible to determine the last parameter from the available experiments, its value was chosen by analogy with samples of similar lithotype [18]. The parameters of plastic potential (12) deviating from the corresponding parameters of the yield criteria are determined by formula (14): $B_{(1)}^1 = -4.44 \times 10^{-3}$, $B_{(3)}^1 = 8.88 \times 10^{-3}$. 

---

**Figure 1.** Stress-strain curves for the loading program of the second type; the maximal stress corresponding to the vertical (a) and horizontal (b) directions.

**Table 1.** Results of tests for the loading program of the first type (Specimen Pr-1).

| Cycle | Lateral pressure, MPa | $E \times 10^{-3}$, MPa | $E_p \times 10^{-3}$, MPa | $S^*_2$, MPa |
|-------|-----------------------|--------------------------|---------------------------|--------------|
| 1     | 2                     | 1.80                     | 0.18                      | 27           |
| 2     | 10                    | 5.42                     | 0.54                      | 51           |
| 3     | 20                    | 7.78                     | 0.78                      | 75           |

During the second loading program, the specimens experience volumetric compression up to the level corresponding to the in situ stress state, and then the loading continues along one of the axes accompanied with unloading along another axis so that the stress along the third axis remains constant; the average stress is conserved at this stage. The stress state constructed in this program corresponds to the state near the well.

The permeability was measured along the direction of the minimal stress during the whole period. The tests were carried out for two orientations of the minimal stresses: along the layers and normally to the layers, which corresponds to two different points at the contour of the horizontal well. As a result, the dependence of permeability on the stress and the yield stresses corresponding to these loading conditions was obtained.

The results of tests according to the loading programs of the first type are presented in table 1. Figure 1 presents the stress strain curves in the cases of the maximal stress aligned in vertical (figure 1a) and one of the horizontal (figure 1b) directions. Figure 2 presents the dependence of permeability on the stress intensity. Figure 3 presents experimental and calculated data about the critical stress according to criterion (9).

The parameters of yield criteria (6) were determined from the whole set of experiments of both types (the property corresponds to transverse isotropy (7)) by means of the least square method $G_{(12)}^0 = 2.7 \times 10^{-3}$, $G_{(13)}^0 = G_{(23)}^0 = 1.27 \times 10^{-3}$, $B_{(1)}^0 = B_{(2)}^0 = 1.95 \times 10^{-2}$, $B_{(3)}^0 = 3.28 \times 10^{-2}$, $L_{(13)}^0 = 1.0 \times 10^{-2}$. It was impossible to determine the last parameter from the available experiments, its value was chosen by analogy with samples of similar lithotype [18]. The parameters of plastic potential (12) deviating from the corresponding parameters of the yield criteria are determined by formula (14): $B_{(1)}^1 = -4.44 \times 10^{-3}$, $B_{(3)}^1 = 8.88 \times 10^{-3}$. 

---

**Figure 1.** Stress-strain curves for the loading program of the second type; the maximal stress corresponding to the vertical (a) and horizontal (b) directions.

**Table 1.** Results of tests for the loading program of the first type (Specimen Pr-1).

| Cycle | Lateral pressure, MPa | $E \times 10^{-3}$, MPa | $E_p \times 10^{-3}$, MPa | $S^*_2$, MPa |
|-------|-----------------------|--------------------------|---------------------------|--------------|
| 1     | 2                     | 1.80                     | 0.18                      | 27           |
| 2     | 10                    | 5.42                     | 0.54                      | 51           |
| 3     | 20                    | 7.78                     | 0.78                      | 75           |
4. Simulation

Two variants of well faces are considered: simple unsupported well and unsupported well with perforation. The former case corresponds to a rather simple geometry, which allows comparing the obtained results of simulation with the analytical results. The latter case corresponds to a more complex situation. In both cases, the normal stresses and pressure at the well contour were assumed to be vanishing. The stress state at a far distance from the well was supposed to be the hydrostatic compression of magnitude corresponding to the weight of the overlying rocks; the pore pressure at a far distance from the well was set to be corresponding to hydrostatic.

For each configuration, the finite element (FEM) simulation was carried out as follows.

1. The problem of fluid seepage was solved to obtain the first iteration of the pore pressure.
2. The problem of poroelastoplasticity was solved to obtain the distribution of effective stresses, elastic and plastic strains. This stage consisted in the following substages.
   2.1. For the calculated field of pore pressure, the uncoupled problem of poroelasticity was solved.
   2.2. The parameters of plasticity for each element were modified according to the calculated fields of the attained stresses and pore pressure levels.
   2.3. The problem of poroelastoplasticity was solved for the modified properties of the elements.
3. In accordance with the experimentally obtained law of the permeability dependence on the stress strain state, the permeability of each element was modified in accordance with the calculated stress field.
4. The problem of fluid seepage was solved, the flows distribution and the production rate were obtained.

The calculations were performed in 3-D using meshes with 22356 nodes and 44001 elements corresponding (due to symmetry) to quarters of the initial areas. The geometrical properties were: the well radius, $R = 0.1$ m; the length of the cut $L = 0.46$ m; the thickness of the cut $h = 0.02$ m (figure 4). The mechanical properties were listed in the preceding section.

The boundary conditions were the following: normal stress and pore pressure on the outer boundary were 31 MPa and 13 MPa, respectively; normal stress and pore pressure at the well

---

**Figure 2.** Dependence of permeability on the stress intensity.

**Figure 3.** Value of critical stress $S_{cr}$ depending on the lateral pressure for tests of the second type. The dots correspond to experimental points, the line corresponds to modelling.
and cut surfaces were zero. For the normal to the well surfaces, the zero displacements and zero fluid flow were set.

The production rates relative to the “ideal” well (for which the permeability is constant and equal to the initial permeability) are presented in table 2. The distributions of the stress intensity and plastic strains for both configurations are presented in figure 5.

**Conclusion**

The geomechanical approach to modeling the deformation and seepage is presented with regard to the anisotropy of elastic and plastic properties and dependence of the yield transition on
volumetric stresses. Three stages of modeling are described, namely: choosing an appropriate mechanical model and its adaptation to the case in question, obtaining parameters of the model from direct experiments, computing the stress state and the parameters of the fluid flow for particular configurations of the well.

The used model allows describing the main specific characteristics of the mechanical behavior of the collector: the influence of pore pressure on deformation; the influence of not only shear but also comprehensive stress and pore pressure on the transition to inelastic behavior; the appearance of inelastic volumetric deformation and its nontrivial dependence on the stress state; the anisotropy of elastic, strength and seepage properties; the non-obvious dependence of the permeability on stress strain state. The model unites essential characteristics of Hill’s plastic flow theory for anisotropic materials and the Drucker–Prager theory for inelastic deformation of soils.

The procedure of experimental determination of the involved parameters for the Filanovsky field using the experimental unit IPMech RAS Triaxial Independent Loading Test System (TILTS) is presented, and the experimental results are given.

The results of numerical simulation for the described model and the experimentally obtained parameters are presented. The cases of unsupported and perforated well ends are considered. The stress concentrations and the production rates were calculated.

The results of the study demonstrate that the proposed approach can be used to address geomechanical problems for optimizing technological processes.

Acknowledgments
This work is supported by the Russian Foundation for Basic Research, grant No. 16-11-10325.

References
[1] Terzaghi K 1925 Erdbaumechanik auf Bodenphysikalischer Grundlage (Leipzig & Wien: Franz Deuticke)
[2] Biot M A 1935 Le problème de la consolidation des matières argileuses sous une charge Ann. Soc. Sc. de Brux. Ser. B 55 110–3
[3] Hristianovich S A and Jeltov Yu P 1955 On hydraulic break of a petroliferous layer Izv. AN SSSR. OTN. 3 3–41
[4] Coulomb C A 1776 Essai sur une application des règles des maximis et minimis à quelques problèmes de statique relatifs, à l’architecture Mem. Acad. Roy. Div. Sav. 7 343–87
[5] Drucker D C and Prager W 1952 Soil mechanics and plastic analysis for limit design Quart. of Appl. Math. 10 (2) 157–65
[6] Goodman R E 1980 Introduction to rocks mechanics New York: John Wiley and Sons
[7] Reynolds O 1885 On the dilatancy of media composed of rigid particles in contact, with experimental illustrations Phil. Mag. Ser. 5 20 (127) 469–81
[8] Mead W J 1925 The geologic rôle of dilatancy J. Geology 33 (5) 685–98
[9] Nikolaevskii V N 1967 On interconnection of volume and shear strains and on shock waves in soft soils Dokl. Akad. Nauk SSSR 177 542–3
[10] Nikolaevskii V N 1996 Geomechanics and Fluid Dynamics (Moscow: Nedra) p 448 [in Russian]
[11] Hill R 1948 A theory of the yielding and plastic flow of anisotropic metals Proc. Roy. Soc. London A 193 281–97
[12] Hill R 1983 The Mathematical Theory of Plasticity New York: Oxford University Press
[13] Caddell R M, Raghava E S, and Atkins A G 1973 A yield criterion for anisotropic and pressure dependent solids such as oriented polymers J. Mater. sci. 8 1641–6
[14] Deshpande V S, Fleck N A, and Ashby M F 2001 Effective properties of the octet-truss lattice material J. Mech. Phys. Solids 49 1747–69
[15] Ustinov K B 2016 On application of models of plastic flow to description of inelastic behavior of anisotropic rocks Processes in geomecha 3 (7) 278–87
[16] Karev V I and Kovalenko Yu F 2013 Triaxial loading system as a tool for solving geotechnical problems of oil and gas production In True Triaxial Testing of Rocks (Leiden: CRC Press/Balkema) pp 301–10
[17] Karev V I, Klimov D M, Kovalenko Yu F, and Ustinov K B 2016 Fracture model of anisotropic rocks under complex loading Phys. Mesomech. 19 (6) 34–40
[18] Karev V I, Klimov D M, Kovalenko Yu F, and Ustinov K B 2016 Fracture of Sedimentary Rocks under a Complex Triaxial Stress State Mech. Solids 51 (5) 522–6