Is scale-invariance in gauge-Yukawa systems compatible with the graviton?

Nicolai Christiansen, Astrid Eichhorn, Aaron Held

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany
E-mail: n.christiansen@thphys.uni-heidelberg.de, a.eichhorn@thphys.uni-heidelberg.de, a.held@thphys.uni-heidelberg.de

Abstract: We explore whether perturbative interacting fixed points in matter systems can persist under the impact of quantum gravity. We first focus on semi-simple gauge theories and show that the leading order gravity contribution evaluated within the functional Renormalization Group framework preserves the perturbative fixed-point structure in these models discovered in [1]. We highlight that the quantum-gravity contribution alters the scaling dimension of the gauge coupling, such that the system exhibits an effective dimensional reduction. We secondly explore the effect of metric fluctuations on asymptotically safe gauge-Yukawa systems which feature an asymptotically safe fixed point [2]. The same effective dimensional reduction that takes effect in pure gauge theories also impacts gauge-Yukawa systems. There, it appears to lead to a split of the degenerate free fixed point into an interacting infrared attractive fixed point and a partially ultraviolet attractive free fixed point. The quantum-gravity induced infrared fixed point moves towards the asymptotically safe fixed point of the matter system, and annihilates it at a critical value of the gravity coupling. Even after that fixed-point annihilation, graviton effects leave behind new partially interacting fixed points for the matter sector.
1 Introduction

Asymptotic safety [3] generalizes the paradigm of asymptotic freedom [4, 5] and thereby provides ultraviolet (UV) completions for quantum field theories. It hinges on the existence of an interacting Renormalization Group (RG) fixed point. If the running couplings of a model approach such a fixed point with finitely many UV-attractive directions at high momentum scales, the model enters a scale-invariant regime, which allows one to “zoom in” on arbitrarily small length scales while keeping all observables finite at all scales and dependent on just a finite number of free parameters. In high-energy physics, the special case of asymptotic freedom, where the RG fixed point is a free one, plays a distinctive role, as it underlies the UV structure of non-Abelian gauge theories. Asymptotic safety, where the fixed point features residual interactions, is not simple to achieve in Standard-Model like theories in four dimensions, as all couplings in the Standard Model, with the exception of the Higgs mass parameter, are dimensionless. To achieve a scale-invariant regime at finite interaction strength, one must therefore balance quantum fluctuations of different fields against each other. On the other hand, in statistical physics, interacting RG fixed points are of course much more prevalent than free ones, showing that asymptotic safety is not just an exotic property of quantum field theories. Models away from their critical dimensionality, which can play a major role in statistical physics, feature dimensionful couplings. Then, interacting fixed points can arise from a balance between quantum and canonical scaling. Thus, manifold examples for interacting fixed points exist for models away from their critical dimensionality, e.g., in non-Abelian gauge theories in $d = 4 + \epsilon$ dimensions [6, 7], in O(N) models with a
“bound-state” field in $d = 6 - \epsilon$ dimensions [8–10], in the Gross-Neveu model in $d = 2 + \epsilon$ [11–13].

A breakthrough was reached in [2], where asymptotic safety in a particular class of gauge-Yukawa systems in four dimensions in the perturbative regime has been discovered. There, the interplay of quantum fluctuations of scalars, fermions and gauge fields allows to achieve a balance that leads to quantum scale-invariance in the UV [14, 15], interestingly without requiring supersymmetry [16]. The corresponding models feature similar building blocks to the Standard Model. Thus, the asymptotic-safety paradigm could potentially play a role in beyond-Standard-Model physics at scales below the Planck scale [17, 18], with possible consequences for phenomenological questions [19–22]. On the other hand, there are compelling hints for asymptotic safety of gravity without [23] and with [24] matter beyond the Planck scale, for reviews, see [25]. In this setting, it is an intriguing question whether asymptotic safety in pure matter systems can persist at intermediate scales, i.e., within the regime where gravity can be treated as an effective field theory.

In the following, we will explore leading-order quantum-gravity corrections on gauge-Yukawa systems. As a “warm-up”, we will first discuss quantum-gravity effects on the fixed-point structure of semi-simple gauge theories with fermions in Sec. 2, and then include scalars to explore the impact of quantum gravity on asymptotic safety in gauge-Yukawa systems in Sec. 3. In particular, we explore whether the fixed-point structure in gauge-Yukawa systems, which features a perturbative, asymptotically safe fixed point in the Veneziano limit, survives the effect of gravity fluctuations. Breaking with tradition, the answer to the question paraphrased in the title, will tentatively be positive.

2 Quantum-gravity corrections to the fixed-point structure in semi-simple gauge theories

In this work, we consider a regime where the running of the gravitational coupling is governed by classical behavior, i.e., the dimensionfull Newton coupling is constant. Its dimensionless counterpart must therefore run as a function of the momentum scale $k$ and is given in terms of the Planck scale $M_{\text{Planck}}$ according to

$$g = \frac{k^2}{M_{\text{Planck}}^2}. \quad (2.1)$$

Although the gravitational coupling $g$ is small below the Planck scale, its contributions to the $\beta$-functions in the gauge and matter sector can be important. The leading-order quantum-gravity effects are linear in the Newton coupling. In our setting, well below the Planck scale, we assume that it is sufficient to neglect all higher-order effects. In particular, this implies that the only relevant quantum-gravity effects are encoded in direct quantum-gravity contributions to the anomalous scaling of the matter couplings at their fixed points. Specifically, the preservation of global symmetries by quantum gravity within the effective field theory regime, that we observe in all approximations that have been explored within the framework of the functional RG, implies that the
quantum-gravity contribution to the running of a gauge coupling $\alpha$ is of the form
\[ \beta_\alpha |_{QG} = -D^{\text{grav}}_\alpha \alpha g. \] (2.2)

This contribution is universal in that it does not depend on the gauge group, i.e., quantum gravity is "blind" to non-spacetime indices. Thus, the gravity-correction to Abelian and non-Abelian gauge theories is the same. The critical piece of information is the sign of $D^{\text{grav}}_\alpha$. Within different schemes, this quantity has been calculated in the Abelian and non-Abelian case in \([26–30]\).

To evaluate the quantum-gravity contribution, we employ functional RG techniques \([31, 32]\). The functional RG allows us to probe the scale-dependence of a quantum field theory, introduced by a mass-like regularization scheme. Specifically, an infrared (IR) regulator function, that acts as a mass-like cutoff term, is added to the action in the generating functional. Beta functions are then obtained from the Wetterich equation \([31]\), that encodes the change in the effective dynamics under a change of the cutoff scale. The universal one-loop terms of canonically dimensionless couplings are straightforward to reproduce with the Wetterich equation. Two-loop contributions can also be obtained, see, e.g., \([33, 34]\), although the two-loop coefficient is not straightforward to obtain in the case of non-Abelian gauge theories \([35, 36]\). The Wetterich equation is an excellent method to explore interacting fixed points in systems with dimensionful couplings, such as the Wilson-Fisher fixed point in three dimensions. There, quantitatively precise values for the critical exponents can be obtained within this scheme \([37, 38]\). Thus, the Wetterich equation is a well-suited tool to evaluate quantum-gravity contributions to running couplings. For reviews of the technique, see \([39–45]\).

Let us note that due to the dimensionful nature of the Newton coupling, quantum-gravity effects should not be expected to have universal beta functions, even at the one-loop level. This is of course not a problem, as running couplings do not directly correspond to observables. On the other hand, we check explicitly that our choice of gauge does not impact our results for the signs of the gravity-contributions in App. A. For the metric fluctuations, we work in a covariant gauge with gauge parameters $\alpha = 0, \beta = 1$ in the main part. Our results are obtained with a Litim-type cutoff \([46]\) of the form
\[ R_k = Z_k (k^2 - p^2) \theta (k^2 - p^2). \]

The sign of the gravity contribution in Eq. (2.2) is critical to determine the impact of gravity on the gauge system. A negative sign, i.e. $D^{\text{grav}}_\alpha > 0$, preserves the full fixed-point structure of semi-simple gauge theories as analyzed in \([1]\). It is vital to note that the sign of the gravity contribution is gauge-independent, cf. App. A, see also \([28]\).

Here, we first explore the effect of the quantum-gravity correction on the fixed-point structure in semi-simple gauge theories. The model features an SU($N$) gauge group with gauge coupling $\alpha_N$ and $M$ fundamental Weyl fermions as well as $M$ antifundamental fermions. An SU(2)$_L$ subgroup of the global SU($M$)$_L \times$ SU($M$)$_R$ global symmetry is gauged, and features the gauge coupling $\alpha_2$. At particular values of ($N, M$), the system exhibits a fully IR-attractive interacting fixed point as well as two semi-interacting fixed points with one UV-attractive direction \([1]\). For the gravity-free beta functions, the corresponding results can be found in App. C1 of \([1]\). Here, we will repeat them for clarity; our new result consists in the addition of the gravity contribution. The full beta
functions read

\[
\beta_{\alpha_N} = \beta_{\alpha_N}^{\text{grav}} - \left( \frac{11}{2} - \frac{2}{3} \frac{M}{N} \right) \alpha_N^2 \left( \frac{17}{3} N^2 - \left( \frac{5}{3} N + \frac{N^2 - 1}{2 N} \right) M \right) \frac{\alpha_N^2}{(2\pi)^2} + \frac{3}{4} \alpha_N^2 \frac{\alpha_N^2}{(2\pi)^2},
\]

\[
\beta_{\alpha_2} = \beta_{\alpha_2}^{\text{grav}} - \left( \frac{22}{3} - \frac{2}{3} N \right) \alpha_2^2 \left( \frac{68}{3} - \frac{49}{12} N \right) \frac{\alpha_2^2}{(2\pi)^2} - \frac{1 - N^2}{4} \alpha_N \frac{\alpha_2^2}{(2\pi)^2},
\]

(2.3)

where the gravity contributions to the running of the gauge couplings is independent of the gauge group and takes the form

\[
\beta_{\alpha_{N/2}}^{\text{grav}} = -g \frac{\alpha_{N/2}}{2\pi}.
\]

(2.5)

Note that the gravity correction is linear in the gauge coupling, i.e., it acts like a correction to the scaling dimension of the system. In fact, if we take into account that the canonical dimension of the gauge couplings is given by

\[
d_i := [\alpha_i] = 4 - d,
\]

(2.6)

then in \( d < 4 \), the beta functions for the gauge couplings feature the leading-order terms

\[
\beta_{\alpha_i} = (d - 4) \alpha_i + ...
\]

(2.7)

In \( d = 4 + \epsilon \), those terms can balance against the terms from quantum fluctuations and induce asymptotic safety in pure Yang-Mills theories [6, 7]. Here, we discover that the quantum-gravity correction takes the same form, at least for positive Newton coupling, as a dimensional reduction. It is intriguing to observe that other forms of dimensional reduction have been observed in a variety of quantum-gravity approaches in the deep quantum-gravity regime, see, e.g., [47–52]. At this point, it is unclear whether a connection between these two observations can be established. We will now explore the consequences of the effective “dimensional reduction” encoded in Eqs. (2.3) and (2.4).

At the free, i.e., Gaussian fixed point, the gravity contribution becomes the leading one. Accordingly, it can have a severe impact on the matter system already at small values of \( g \). We observe that the effective dimensional reduction stabilizes the fixed-point structure of the system, cf. Fig. 1: For values of \( N, M \) where the free, semi-interacting and interacting fixed points of the system exist without gravity, they also persist at finite \( g \) and simply move a little further apart from each other. In particular, as quantum gravity acts in the direction of strengthening asymptotic freedom, the UV/IR-attractivity properties of the various fixed points remain intact: The free fixed point is fully UV attractive, and the interacting fixed point is fully IR attractive, with the semi-interacting fixed points featuring one UV attractive and one IR attractive direction. Note that under the inclusion of gravity, the fixed points become partial fixed points of the full system: As \( g \) does not approach a fixed point, but scales canonically, the fixed points in the matter system become scale-dependent. Thus fixed-point scaling in the matter system is no longer signalled by constant dimensionless couplings, but instead by couplings which
Figure 1. We show the flow towards the IR in the plane spanned by the two gauge couplings for $g = 0$ (upper left panel), $g = 0.2$ (upper right panel) and $g = -0.05$ (lower left panel) and a zoom in the vicinity of the free fixed point for $g = -0.05$ (lower right panel) for $N = 9$ and $M = 40$. IR attractive fixed points are shown in dark red (large dot), UV attractive ones in purple (small dot), and mixed (with one UV attractive and IR relevant one) in magenta (medium-size dots).

follow the gravitational scaling through the scale-dependence of the $g$-dependent fixed points. This “running scale-invariance” holds at values of $g$ which are not too small. Towards the IR, the standard fixed-point scaling of the pure matter system is restored quickly.

Crucially, the effect of gravity on the system would be completely different if the gravity contribution had the opposite sign – i.e., if gravity was repulsive instead of attractive – which we highlight in the lower two panels of Fig. 1: The opposite sign leads to a split of the free fixed point into four separate fixed points, one of which is now fully UV attractive. In particular, if the gravity correction came with a positive sign, it would push the four fixed points towards each other, and induce a fixed-point annihilation already at small $g$, leaving behind an IR-attractive Gaußian fixed point. It is intriguing that the sign of the gravity contribution that preserves the fixed-point structure in the system is also the one that could induce asymptotic freedom even in Abelian gauge theories [29, 30].
Figure 2. We contrast the existence of fixed points in the case without gravity (left panel) to the case with gravity (right panel, $g = 0.1$). The existence region of FP1 is shown with horizontal lines, that of FP2 with diagonal lines, and that of FP3 with vertical lines.

Figure 3. Left panel: We show the fixed-point value for $\alpha_2$ at FP1 as a function of $N$ for $g = 0$ (thick red line), $g = 0.1$ (magenta dotted line) and $g = 0.2$ (purple dashed line). Right panel: We show the fixed-point values at FP1 (blue, dotted lines, $\alpha_2 \neq 0$), FP2 (continuous red lines, $\alpha_{N,2} = 0$), and FP3 (green, dot-dashed lines, $\alpha_{N,3} < \alpha_{2,2}$) as a function of $g$ at $N = 12$, $M = 70$. For $g = 0$, all three fixed points lie on top of the Gaussian fixed point, whereas they continue to exist as (partially) interacting fixed points if gravity is included.

In particular, it seems that quantum gravity effects extend the regions where the various interacting fixed points exist, cf. Fig. 2 and Fig. 3. Following the terminology in [1], we denote the two fixed points which are UV attractive in one direction by FP1 and FP2, and the fully UV attractive one by FP3.

The mechanism is as follows: For instance in the case of FP1, the upper limit on $N$ for which the fixed point is viable at $g = 0$ follows from the condition $\alpha_{2,*} > 0$. The fixed-point value for $\alpha_2$ decreases as a function of $N$, until FP1 collides with the Gaussian fixed point, and tunnels through it to lie at negative values of $\alpha_2$. Intriguingly, the presence of metric fluctuations prevents this fixed-point collision. Hence, the fixed point continues to exist for large $N$ and only reaches the Gaussian fixed point in the limit $N \rightarrow \infty$. To understand the existence of fixed points in a larger region in $N, M$, let us focus on a fixed point which features $\alpha_{N,*} = 0, \alpha_{2,*} > 0$. In that case, it suffices
to analyze the effect of quantum gravity on $\beta_{N}$, which for $\alpha_N = 0$ has the following fixed-point solutions:

$$\alpha_{2+1} = 0, \quad \alpha_{2+2/3} = \frac{(88 - 8N) \pi \pm \sqrt{8\pi (3g(49N - 272) + 8\pi(N - 11)^2)}}{49N - 272}. \quad (2.8)$$

The additional term $\sim g$ lifts the degeneracy between $\alpha_{2+1}$ and $\alpha_{2+2}$, that exists for $g = 0$. Instead, there is one free fixed point and one interacting fixed point at $\alpha_{2+2} < 0$, which remains unphysical. Moreover, $\alpha_{2+3} > 0$ for all $N > 5.55$. Instead of crossing zero at $N = 11$, which happens for $g = 0$, the fixed-point value for $\alpha_{2+2}$ approaches zero asymptotically from the positive regime. The fixed point therefore exists for all values of $N > 5.55$, and is perturbative in the sense of $\alpha_{2+2} < 1$ for $N > 8.31 + 0.76g$. The attractivity properties of the fixed points remain unaltered, as critical exponents only change signs in fixed-point collision and these are avoided here.

To conclude this subsection, we summarize that quantum-gravity effects on the fixed-point structure in semi-simple gauge theories are similar to a (scale-dependent) effective dimensional reduction. They preserve asymptotic freedom, and therefore enlarge the regions in $N, M$ where one totally IR-attractive fixed point and two fixed points with one UV attractive direction each exist.

3 Quantum-gravity effects on gauge-Yukawa systems

3.1 Asymptotic safety for gauge-Yukawa systems with gravity

Following [2], we consider a gauge-Yukawa system with Yukawa coupling $y$ and gauge coupling $g$. We define

$$\alpha_y = \frac{y^2 N_C}{4\pi^2}, \quad \alpha_g = \frac{g^2 N_C}{4\pi^2}.$$  \quad (3.1)

The system contains an $SU(N_C)$ gauge field, $N_F$ flavors of fermions in the fundamental representation of the gauge group, and $N_F \times N_F$ complex scalar fields which are uncharged under the gauge group.

In [2], the $\beta$-functions of this system are explored in detail. Here we add the gravity contributions and arrive at

$$\beta_{\alpha_N} = \alpha_N^2 \left( \frac{4}{3} \epsilon + \left( 25 + \frac{26}{3} \epsilon \right) \alpha_N - 2 \left( \frac{11}{2} + \epsilon \right)^2 \alpha_y \right) + \beta_{\alpha_N}^{\text{grav}} \quad (3.2)$$

$$\beta_{\alpha_g} = \alpha_g \left( (13 + 2\epsilon) \alpha_y - 6\alpha_N \right) + \beta_{\alpha_g}^{\text{grav}}. \quad (3.3)$$

The parameter $\epsilon$ is defined as

$$\epsilon := \frac{N_F}{N_C} - \frac{11}{2}. \quad (3.4)$$

The Veneziano limit [53] is a particular case of a large $N$ limit, where the number of colors and flavors is taken to infinity, $N_F \to \infty, N_C \to \infty$, while their ratio is kept fixed. Hence, the parameter $\epsilon$ can be tuned continuously to any number, in particular it can be chosen to be arbitrarily small. This allowed the authors of [2] to use $\epsilon$ as a perturbative expansion parameter and opens up the possibility to investigate interacting fixed points which are under control in perturbation theory. The quantum-gravity
contribution $\beta_{\alpha_N}^{grav}$ to the running of the gauge coupling remains the same as in the previous section, see Eq. (2.5). The direct contribution to the running of a simple Yukawa coupling has been evaluated in [54–58], including anomalous dimensions in [59]. Here, we have also derived the gravity contributions to the $\beta$-functions in the gauge and matter sector for general gauges (see appendix A), thus generalizing the results of [30] and [59]. For definiteness, we now focus on the gauge $\alpha = 0, \beta = 1$, and highlight in App. A that none of the qualitative features of our analysis depends on the choice of gauge. The gravity contribution to the running of the Yukawa coupling takes the form

$$\beta_{\alpha_y}^{grav} = g \alpha_y \frac{17}{10\pi}. \tag{3.5}$$

It is essential to note that the gravity contributions to the running of the couplings in the gauge- and matter sector are linear in the gauge and matter couplings respectively. Within our assumption, where the running of $g$ is given by dimensional scaling only, this implies that the gravity contributions cannot be neglected in the large-$N$ limit, e.g., in the Veneziano limit. In that limit, a simple counting argument might suggest that quantum gravity effects should be negligible, as there are $N_F N_C$ fermions, $N_C^2 - 1$ gauge fields and $N_F^2$ scalars, whereas there is only one spin-2-mode. However, as the quantum-gravity correction is linear in the coupling, it acts like a change in the canonical scaling dimension. As the canonical scaling dimension of a coupling is of course not affected by the number of fields in the system, the quantum-gravity correction is not negligible in the Veneziano limit. One can also convince oneself of this property by starting with the system of couplings $\tilde{\alpha}_g = \alpha_g / N_C$, $\tilde{\alpha}_y = \alpha_y / N_C$ and performing the redefinition $\tilde{\alpha}_i \rightarrow \alpha_i$, thus making the transition to the beta functions in Eqs. (3.2), (3.3). This leads to a suppression of all terms in the beta function which do not scale with an appropriate positive power of $N_C$, except the terms linear in the couplings, as in this case the factors of $N_C$ on the left- and on the right-hand side of the $\beta$-functions cancel.

We observe that the gravity contributions to different matter couplings differ in their sign, thus the effective scaling dimensionality that combines the canonical dimension with the quantum-gravity induced one is different for each operator: While quantum-gravity corrections to the scale dependence of the Yukawa coupling act as if the effective dimension is increased, those to the scale dependence of the gauge coupling again act like a dimensional reduction [60]. Note that it is this effective dimensional reduction in the scaling dimension of the gauge coupling which fundamentally alters the phase structure of the gauge-gravity-system, even for small values of the Newton coupling: As gravity acts to strengthen asymptotic freedom, which stabilized the gauge system in the previous section, gravity is responsible for a destabilizing effect in the gauge-Yukawa system: In the presence of the linear gravity term, the degeneracy of the free fixed point is lifted and it splits into new interacting fixed points. These emerge in the regime $\alpha_N > 0, \alpha_y > 0$, cf. Fig. 4 and 5. Specifically, the coordinates of a new, fully interacting
fixed point are
\[
\alpha_{N*} = \frac{1}{80\pi(57 - 46\epsilon - 8\epsilon^2)} \left[ 51g(11 + 2\epsilon)^2 + 80\pi\epsilon(13 + 2\epsilon) \right. \\
- \sqrt{4800\pi g(13 + 2\epsilon)(-57 + 46\epsilon + 8\epsilon^2)} \left. + (51g(11 + 2\epsilon)^2 + 80\pi\epsilon(13 + 2\epsilon))^2 \right],
\]
\[
\alpha_{y*} = \frac{1}{40\pi(13 + 2\epsilon)(57 - 46\epsilon - 8\epsilon^2)} \left[ 240\pi\epsilon(13 + 2\epsilon) + 17g(861 + 580\epsilon + 68\epsilon^2) \right. \\
- 3\sqrt{4800\pi g(13 + 2\epsilon)(-57 + 46\epsilon + 8\epsilon^2)} + (51g(11 + 2\epsilon)^2 + 80\pi\epsilon(13 + 2\epsilon))^2 ]
\]
which clearly approaches the free fixed point in the limit \( g \to 0 \). We do not explicitly provide the expression for the critical exponents, which is rather lengthy, however they are both negative, when the fixed point is real. The critical exponents are defined via the stability matrix
\[
\theta_{1,2} = -\text{eig} \left( \frac{\partial^2 \beta_{\alpha_N}}{\partial \alpha_N^2} \frac{\partial \beta_{\alpha_N}}{\partial \alpha_y} \right) \bigg|_{\alpha_N = \alpha_{N*}, \alpha_y = \alpha_{y*}},
\]
such that a negative critical exponent corresponds to an IR attractive direction.

If the sign of the gravity-contribution to the gauge coupling was opposite, all the interacting fixed points which emerge from the free one under the impact of gravity would lie in the physically unacceptable regime \( \alpha_{N*} < 0 \), as they do in the case of semi-simple gauge theories in the previous section. This implies that the dynamics of the system closely resembles that of the system in \( d = 4 - \epsilon \) dimensions analyzed in [60].

The existence of the new fixed point can easily be understood from the structure of the beta function in the gauge sector, cf. Eq. (3.2). Schematically, it is given by
\[
\beta_{\alpha_N} = -D_{\alpha_N}^{\text{grav}} g \alpha_N - B \alpha_N^2 + C \alpha_N^3,
\]
where the coefficient \( D_{\alpha_N}^{\text{grav}} \) parameterizes the gravitational contributions to the running of the gauge coupling \( \alpha_N \). An asymptotically safe UV fixed point arises in the case of vanishing gravity, \( g = 0 \), if \( B < 0 \) and \( C < 0 \), which can be arranged with the help of the Yukawa coupling. With gravity, it is instructive to rewrite the beta function in the form
\[
\beta_{\alpha_N} = \alpha_N(-D_{\alpha_N}^{\text{grav}} g - \alpha_N (B - C \alpha_N)).
\]

Figure 4. We show the split of the Gaussian fixed point for \( \epsilon = 0.1 \) as a function of \( g \) (fixed-point values for couplings in the left panel and for critical exponents in the right panel). Dotted blue lines correspond to the IR-attractive interacting fixed point in Eq. (3.6) and Eq. (3.7).
The fixed point at $\alpha_N = 0$ does not change its location, but becomes UV attractive for $D_{\alpha_N}^{\text{grav}} g > 0$. The interacting UV fixed point at $\alpha_N = \frac{B}{C}$ is shifted and lies at

$$\alpha_{N*1} = \frac{B}{2C} + \sqrt{\frac{B^2}{4C^2} + \frac{D_{\alpha_N}^{\text{grav}} g}{C}}. \quad (3.11)$$

For small $g$, the shift is very mild, and the fixed point still lies in the vicinity of its previous value. Further, it remains UV-attractive, as gravity does not change the slope of the beta-function for large $\alpha_N$, since the leading large-$\alpha_N$ term is $g$ independent, and thus the outermost fixed point remains UV attractive. Since we now have two fixed points which are both UV attractive, it is obvious that there must be a third fixed-point solution which must lie inbetween those two. In fact, it is of course given by

$$\alpha_{N*2} = \frac{B}{2C} - \sqrt{\frac{B^2}{4C^2} + \frac{D_{\alpha_N}^{\text{grav}} g}{C}}, \quad (3.12)$$

which clearly emerges from the free fixed point as $g$ increases from zero. As $g$ increases, $\alpha_{N*2}$ approaches $\alpha_{N*1}$, until they annihilate and move into the complex plane at

$$g_{\text{crit}} = -\frac{B^2}{4C D_{\alpha_N}^{\text{grav}}}, \quad (3.13)$$

which is of course positive for $C < 0$ and $D_{\alpha_N}^{\text{grav}} > 0$. This analysis suggests that quantum gravity can have a severe impact on the system and destroy the UV completion of the pure matter system, substituting if for an asymptotically free one, as the asymptotically free fixed point is of course left behind when the two interacting fixed points annihilate.

As we are analyzing the EFT regime for gravity, none of the two UV fixed points can of course be a true UV fixed point of the full system. Instead, a UV completion for gravity is required beyond the Planck scale. The (partial) UV fixed points of the matter system could then arise at intermediate scales between the Planck scale and the electroweak scale and dominate the matter sector in that regime. In a particularly intriguing case, the UV completion for gravity might also be provided by the asymptotic-safety paradigm.

In the regime of small enough Newton coupling, we thus encounter a scenario where the interacting UV fixed point still exists in all cases. Towards the IR, the RG flow can move along the separatrix, which at finite $g$ connects the interacting UV to an interacting IR fixed point. As the scale decreases, the Newton coupling decreases further, such that the interacting IR fixed point moves back into the free fixed point. Thus, we have a setting where the flow approaches a partial fixed point of the system (it is of course not a fixed point in $g$), and that partial fixed point simultaneously moves away. In the deep IR, gravity fluctuations have of course switched off, and the remaining IR fixed point is a Gaussian one.

On the other hand, as $g$ grows, the interacting UV and IR fixed points can approach each other. Beyond a critical $g$, these two annihilate, as is signalled by a vanishing critical exponent, and they move into the complex plane, where they cease to be physically acceptable fixed points.

According to Eq. (2.1), the value of $g$ can be translated into a scale. In units of $M_{\text{Planck}}$, $g = 1$ exactly at the Planck scale and $g < 1$ for $k < M_{\text{Planck}}$. Hence, the critical
Figure 5. We show the flow in the plane of the two couplings for $\epsilon = 0.1$ and fixed $g = 7 \cdot 10^{-3}$ (left panel), where a fully interacting IR attractive fixed point exists in addition to the fully interacting UV attractive one; and the case with $g = 0.1$ (right panel), where the two fully interacting fixed points have annihilated, leaving behind a free fixed point and a partially interacting one, both of which have one UV attractive direction each.

Figure 6. We show the annihilation between the UV attractive and the IR attractive interacting fixed points at finite $g$ for $\epsilon = 0.1$ (left panel). The critical exponents (right panel) also show the same annihilation.

The critical value of $g_{\text{crit}}$, beyond which fixed-point annihilation occurs, can be translated into a scale $k_{\text{crit}}$: If the UV safe fixed point in the matter sector is to be phenomenologically relevant, it has to be reached at a scale below $k_{\text{crit}}$.

The critical value of $g$ at which the fixed-point annihilation occurs is a function of $\epsilon$,

$$ g_{\text{crit}} = \frac{80\pi}{867(11 + 2\epsilon)^3} \left(7410 - \sqrt{20(13 + 2\epsilon)^2(-57 + 46\epsilon + 8\epsilon^2)(-285 + \epsilon(2287 + 788\epsilon + 68\epsilon^2))} - \epsilon(31581 + 2\epsilon(7899 + 2\epsilon(635 + 34\epsilon))) \right). \quad (3.14) $$

In the limit $\epsilon \to 0$, where the fixed point in the pure matter system moves arbitrarily close to the free fixed point, an infinitesimal value of $g$ is sufficient to trigger a fixed-point annihilation,

$$ \lim_{\epsilon \to 0} g_{\text{crit}} = 0. \quad (3.15) $$
The fixed points reappear at a larger value of $g_{\text{reappear}}$. That value hardly depends on $\epsilon$ and lies at about $g_{\text{reappear}} \approx 0.29$, until it drops sharply at about $\epsilon \approx 0.1$. For all values of $\epsilon$, the value of $g_{\text{reappear}}$ lies dangerously close to the Planck scale, where it is unclear whether our leading-order treatment of quantum gravity fluctuations is still justified. Hence we emphasize that the reappearance of the fixed points might be an artifact of the approximation.

As gravitational contributions alter the UV behavior of the matter system, they can prevent the approach to the scale-invariant regime that would be reached in the pure matter system. Instead, a new scaling regime can be induced by quantum gravity fluctuations. If we assume a classical regime for gravity, $g$ runs with $k^2$. In that setting, the new fixed points inherit the scaling, exhibiting a form of “running scale-invariance”. For $g = \text{const}$, a standard form of scale invariance is recovered. Explicit trajectories in the different regimes of gravitational coupling strength ($g = 0$, $g < g_{\text{crit}}$ and $g > g_{\text{crit}}$) all share the same scale-invariant behavior in the IR, cf. Fig. 8 & 9. This follows as gravity is dynamically switched off towards the IR, which is encoded in the canonical running of the Newton coupling $g \sim k^2$, cf. Eq. (2.1). Here, we focus on that part of the phase diagram in which the matter system becomes weakly coupled in the IR. In turn, the different types of matter UV-completions that are realized for different strengths of gravitational fluctuations become apparent in signatures of UV fixed-point behavior at higher scales. While both, the pure-matter model ($g = 0$) and the weakly gravitating model ($g < g_{\text{crit}}$) exhibit similar fixed-point scaling towards the UV, the stronger gravitating system ($g > g_{\text{crit}}$) approaches a different UV fixed point (as the original one annihilated, cf. Figure 6). These different UV scale-invariant regimes are distinguishable. Therefore, UV matter-models like the gauge-Yukawa-model at hand in principle offer a setting in which tests of quantum-gravity effects below the Planck scale could appear feasible. Of course this would require that the corresponding matter models are realized beyond the Standard Model, and moreover this does not provide a way to test different UV completions for quantum gravity. Our analysis suggests that gravitational signatures, namely the modified UV-scaling in Fig. 8 and 9, are present at the scale $k_{\text{UV-scaling}}$ where the matter-model starts to flow towards its UV fixed point. This scale might well be significantly below the Planck scale, i.e., $k_{\text{UV-scaling}} < M_{\text{Planck}}$.

Finally, let us add the observation that in the original models discussed in [2],
Figure 8. We compare the running of $\alpha_N$ and $\alpha_y$ as a function of the RG “time” $t = \ln(k/k_0)$ for the case with and without gravity. We fix the scales by choosing $g_{|UV} = g(k_0)$. Without gravity (blue dashed lines) we initiate the flow close to the UV fixed point. Starting the flow with gravity (green continuous line) at the same initial condition, gravity fluctuations lead to modifications in the flow, before they switch off and the system is dominated by matter-induced running.

Figure 9. We compare the running of $\alpha_N$ and $\alpha_y$ as a function of the RG “time” $t = \ln(k/k_0)$ for the case with and without gravity. We fix the scales by choosing $g_{|UV} = g(k_0)$. We initiate the system at different UV starting points, such that the initial condition always corresponds to a UV fixed point. With gravity (red continuous line) and $g > g_{crit}$, the fixed point is the partially interacting one. As a function of the scale, the fixed point moves, since $g$ runs. In the IR, the same universal scaling regime is approached. Note that the flow towards the IR follows the $g$-dependent fixed point. This induces a much faster running in the case with gravity than the running along the separatrix in the gravity-free case, as $g$ runs quadratically with scale $k$.

an Abelian gauge group poses a challenge, as it is not possible to flip the sign of the one-loop coefficient of the beta function by increasing the number of charged fermions – a prerequisite that is necessary to obtain asymptotic safety in the non-Abelian gauge couplings. Here, we observe that we can generate a partial fixed point for a gauge-Yukawa-system with an additional Abelian gauge group by including quantum-gravity effects. The fixed point will be asymptotically safe in the non-Abelian gauge coupling
and the Yukawa coupling, but asymptotically free in the Abelian gauge coupling. As it becomes asymptotically free, the Abelian gauge coupling cannot alter the fixed-point structure in the other two couplings, as it vanishes at the fixed point. Moreover, the gravity-contribution to the beta function of the Abelian gauge coupling is the leading one at the free fixed point, i.e., the contributions from the other gauge coupling and the charged matter fields are completely subleading, and gravity induces asymptotic freedom for arbitrary matter content.

3.2 Connection to asymptotically safe quantum gravity with matter

The setting that we have explored so far treats quantum gravity within an effective-field theory regime. Such a treatment is expected to break down at the Planck scale. Beyond, the asymptotic-safety paradigm could provide a UV complete model of quantum gravity and matter. Here, we will briefly highlight how the fixed-point structure in gauge-Yukawa models could match onto a setting with asymptotically safe gravity in the UV. In such a case, the IR could be reached via a cascade of fixed points, starting with a fully asymptotically safe fixed point in the far UV and passing by a fixed point of the gauge-Yukawa system with running \( g \). The more RG “time” the flow spends in the vicinity of intermediate fixed points, the more one expects universality to set in, i.e., information on the underlying UV theory would be washed out and replaced by information on the universal scaling behavior in the vicinity of the intermediate fixed point. It is intriguing to observe that the fixed-point structure of the gauge-Yukawa system at \( g > g_{\text{crit}} \) appears to potentially be attainable at intermediate scales if one starts from an asymptotically safe matter-gravity model: There, studies indicate that the Yukawa coupling generically features a fixed point at \( \alpha_y = 0 \) [54–59]. Moreover, \( \alpha_N \) can feature a fixed point at which it is finite and IR-attractive [29], at least in the Abelian case. For the non-Abelian case, one would expect the possibility of an interacting, UV attractive fixed point within a certain part of the gravitational coupling space. Starting from such a fixed point, the RG flow might then be led towards the fixed point at \( \alpha_N > 0, \alpha_y = 0 \) in the right-hand panel of Fig. 5. That fixed point is of course only a partial fixed point of the full system, as the gravitational coupling no longer lives at a fixed point, but scales with \( k \) according to the effective-field-theory regime. Towards the deep IR, this fixed point then moves along a separatrix into the free fixed point at \( \alpha_N = 0, \alpha_y = 0 \), or alternatively is driven towards the strongly-coupled regime if \( \alpha_y \) is allowed to deviate from zero. In principle, it is of course also conceivable that the asymptotically safe matter-gravity fixed point lies at finite \( \alpha_y, \alpha_N \), and the flow towards the IR then passes by the interacting gauge-Yukawa fixed point and enters the strongly coupled regime at large \( \alpha_N \) in the deep IR.

3.3 Next-to-next-to leading order

At NNLO in the beta functions the scalar self-interactions come into play and contribute to the gauge-Yukawa sector, as discussed in [2]. For a \( N_F \times N_F \) complex matrix scalar, two quartic terms exist, namely

\[
L_U = u \text{Tr} \left( H^\dagger H \right)^2, \quad L_V = v \left( \text{Tr} H^\dagger H \right)^2, \quad \tag{3.16}
\]
where we have already made the transition to the Euclidean setting. Note that these Lagrange densities are independent of the metric, thus the coupling of these operators to metric fluctuations arises through the metric determinant and hence the gravity contributions to these scalar interactions are identical. In the gauge $\alpha = 0, \beta = 1$, we find the following result

$$\beta_u\big|_{\text{grav}} = u \frac{3}{\pi} g, \quad \beta_v\big|_{\text{grav}} = v \frac{3}{\pi} g.$$  

(3.17)

Note that the positive sign of the gravity contribution underlies a scenario in which the Higgs mass could become predictable starting from a fully asymptotically safe matter-gravity model [64, 65]. As the canonical dimensions of $u, v$ are $4 - d$, this result can again be read as an effective dimensional increase induced by metric fluctuations. For the couplings

$$\alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi^2)},$$  

(3.18)

this implies exactly the same gravity contribution. Stability of the potential is given if

$$\alpha_{h*} + \alpha_{v*} > 0.$$  

(3.19)

For matter theories without gravity, vacuum stability has been discussed in [63]. The UV fixed point from the next-to-leading-order analysis splits into two fixed points (at

Figure 10. Fixed-point values in NNLO at the UV fixed point (purple) and the IR fixed point (green) as a function of $g$ for $\epsilon = 0.1$.

Figure 11. We show the critical exponents in NNLO at the UV fixed point (purple) and the IR fixed point (green) as a function of $g$ for $\epsilon = 0.1$. The left panel shows that the fully IR attractive and the UV attractive fixed point approach each other and annihilate at the point where $\theta_1 = 0$. 
$g = 0$), of which only one satisfies the stability criterion. Similar to the behavior at next-to-leading order, under the effect of gravity the Gaussian fixed point again splits into several fixed points, one of which satisfies all criteria for viability and becomes a fully IR- attractive fixed point, cf. Fig. 11. At a critical value of $g$, it collides with the UV fixed point and they move off into the complex plane, cf. Fig. 10. In summary, the qualitative effects of graviton fluctuations on the gauge-Yukawa system do not appear to change when next-to-next-to-leading order terms in the $\beta$-functions of the matter sector are taken into account.

### 3.4 Impact of matter on the running of $g$

Until now, we have worked under the assumption that an effective-field-theory regime for quantum gravity exists in which the Planck scale does not run, i.e., the dimensionless Newton coupling runs quadratically with scale. We will now point out that this assumption might require further study in the Veneziano limit: Quantum fluctuations of matter impact the running of the Newton coupling [61, 66, 67] according to

$$\beta_g = 2g - \frac{g^2}{6\pi} (N_S + 2N_D - 4N_V), \quad (3.20)$$

where for the case of interest here the number of scalars $N_S = N_F^2$, the number of Dirac fermions $N_D = N_F N_C$ and the number of vectors is $N_V = N_C^2 - 1$. In Eq. (3.20), we have neglected the graviton loop contributions. As we take the Veneziano limit, the term quadratic in $g$ starts to feature a divergent coefficient. Matter fluctuations thus drive a fast running of the Newton coupling. Using Eq. (3.4), we write

$$\beta_g = 2g + \frac{g^2}{6\pi} N_C^2 \left( \left( \frac{11}{2} \right)^2 + 2 \left( \frac{11}{2} \right) - 4 + \frac{4}{N_C^2} \right). \quad (3.21)$$

Accordingly, we introduce

$$\tilde{g} = g N_C^2 \quad (3.22)$$

This provides us with a beta function for the rescaled Newton coupling

$$\beta_{\tilde{g}} = 2\tilde{g} + \frac{\tilde{g}^2}{6\pi} \left( \frac{149}{4} + 13\epsilon + \epsilon^2 \right). \quad (3.23)$$

We observe that matter fluctuations result in an accelerated running of the Newton coupling. On the other hand, the rescaling of the Newton coupling leads to a decoupling of graviton effects from the running of the matter couplings. In combination, we conclude that this regime requires further studies, and presumably a fully-fledged quantum gravity treatment.

### 4 Summary and conclusions

We explore the leading-order quantum-gravity effects on fixed points in semi-simple gauge theories and gauge-Yukawa theories within the framework of the functional RG. Our analysis pertains to the effective-field-theory regime for gravity, i.e., we make no assumptions about the UV completion for gravity, and explore the leading-order
quantum gravity effects only. We assume that for this study it is sufficient to work with a constant Planck mass, i.e., with a dimensionless Newton coupling that runs canonically. The range of validity of our analysis does therefore not extend beyond the Planck scale, where higher-order quantum gravity effects must be taken into account. Quantum-gravity contributions to the beta functions of the couplings in the system are linear in those couplings and in the dimensionless Newton coupling \( g \), i.e., they act like corrections to the dimensional scaling of the system. In particular, we observe that within the functional RG framework the quantum-gravity correction to the running of the gauge couplings can be re-interpreted as a running gauge coupling in a dimensionally reduced setting. This has several important consequences: Firstly, it implies that gauge theories which are asymptotically free remain so under the impact of quantum gravity. Therefore, the phase structure of semi-simple gauge theories as explored in [1], which feature IR as well as potential UV fixed points, remains unaffected by quantum gravity. The stabilizing effect of quantum gravity fluctuations even extends the region in parameter space in \((N,M)\), where fully and partially interacting fixed points exist. The values of all fixed points depend on the dimensionless Newton coupling and exhibit a corresponding scale-dependence, unless the dimensionless Newton coupling is held fixed.

Secondly, while the quantum-gravity effect on the Yukawa coupling and the scalar self-interaction takes the form of a lowered scaling dimension, i.e., of “dimensional increase”, the “dimensional reduction” of the gauge system through quantum gravity is decisive for the altered dynamics of the gauge-Yukawa system. It is intriguing that a dimensional reduction, typically exhibited by the spectral dimension has also been observed in the deep quantum regime in many quantum gravity approaches. Of course the scaling dimensions of particular matter couplings are conceptually different from the spectral dimension. Nevertheless one might speculate whether quantum gravity induces a dimensional reduction in some form in order to become UV complete, and the observed dimensional reduction in the gauge-Yukawa system is an imprint of this property that survives in the effective-field theory regime. The effective dimensional reduction induces a split of the IR- attractive free fixed point into several fixed points, one of which is interacting and totally IR- attractive. As a function of the dimensionless Newton coupling \( g \), the interacting IR fixed point moves towards the interacting UV fixed point which is already present in the gravity-free system. These two fixed points collide and annihilate. In other words, quantum-gravity effects can destroy the fixed-point structure in the system as they trigger a fixed-point collision. In the presence of \( g \), all but the free fixed point in the system depend on \( g \). If canonical scaling is assumed for \( g \), all fixed points therefore become parametrically scale dependent, realizing a particular form of universal behavior. For any value of \( \epsilon \), which controls the approach to the Veneziano limit for \( \epsilon \to 0 \), there is a critical value of the dimensionless Newton coupling \( g_{\text{crit}} \), such that this fixed-point collision occurs. Even as this occurs, another partially interacting fixed point is induced by graviton fluctuations and is left behind. We conclude that quantum gravity effects might have a significant impact on the fixed-point structure in these models, and can trigger fixed-point destructions.

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A Gauge dependence

We treat metric fluctuations around a flat background with a split according to

\[ g_{\mu \nu} = \delta_{\mu \nu} + \sqrt{\frac{32\pi}{M_{\text{Planck}}^2}} h_{\mu \nu}. \tag{A.1} \]

To obtain the propagator of metric fluctuations, we start from an Einstein-Hilbert action at vanishing cosmological constant, supplemented by a harmonic-type gauge condition,

\[ S = S_{\text{EH}} + S_{g}\]

\[ = -\frac{1}{16\pi G} \int d^4x \sqrt{g} R + \frac{1}{2\alpha} \int d^4x \left( \partial^\mu h_{\mu \nu} - \frac{1 + \beta}{4} \partial_\nu h \right) \delta^{\nu \lambda} \left( \partial^\kappa h_{\kappa \lambda} - \frac{1 + \beta}{4} \partial_\lambda h \right). \tag{A.2} \]

If we parameterize quantum-gravity corrections to the beta functions of the system in the form

\[ \beta_{\alpha_N} \bigg|_{\text{grav}} = -D_{\alpha_N}^{\text{grav}} g \alpha_N, \tag{A.3} \]

\[ \beta_{\alpha_y} \bigg|_{\text{grav}} = D_{\alpha_y}^{\text{grav}} g \alpha_y, \tag{A.4} \]

with \( D_{\alpha_N}^{\text{grav}} > 0, D_{\alpha_y}^{\text{grav}} > 0 \), then we observe that for relatively large \( \epsilon \), certain regions in the space of \( D_{\alpha_N}^{\text{grav}} \) and \( D_{\alpha_y}^{\text{grav}} \) do not lead to a fixed-point annihilation, cf. Fig. 12, and Fig. 13. As \( g \) increases, an intriguing dynamics is triggered in the matter sector: Depending on the relative size of the gravity-contribution to the flow of \( \alpha_N \) versus the flow of \( \alpha_y \), the interacting UV fixed point of the matter sector could run towards larger or smaller values as a function of \( g \). In the meantime, the newly emerged interacting IR fixed point runs towards larger values. Thus, these two fixed points either move towards each other while \( g \) increases, or both move to larger values. In the former case, there is a critical value \( g_{\text{crit}} \) at which fixed-point annihilation occurs, while there is no special value of \( g \) in the latter case. Thus it is crucial to explore whether the dynamics of the second case can be reached by a choice of gauge.

We obtain the following gauge-dependent gravity contributions to the beta functions, where we emphasize that \( \alpha \) is a gauge parameter here, not to be confused with the gauge coupling,

\[ -D_{\alpha_N}^{\text{grav}} = -\frac{(15 - 10\beta + \beta^2 + \alpha(11 - 6\beta + \beta^2))}{3\pi(\beta - 3)^2}, \tag{A.5} \]

\[ D_{\alpha_y}^{\text{grav}} = -\frac{(\alpha(3\beta^2 - 18\beta + 31) - 3(\beta - 1)^2)}{3\pi(\beta - 3)^2} - \frac{6(\alpha(5\beta^2 - 30\beta + 53) + 5\beta^2 - 50\beta + 81)}{16\pi(\beta - 3)^2} \]

\[ -\frac{(\alpha(\beta - 3) + 2(\beta + 3)\beta)}{40\pi(\beta - 3)} \frac{3(\alpha(\beta - 3) - 3(\beta + 13))}{280\pi(\beta - 3)} \]

\[ + \frac{(5\alpha(5\beta^2 - 30\beta + 53) + 35\beta^2 - 258\beta + 339)}{10\pi(\beta - 3)^2}. \tag{A.6} \]
Figure 12. We show fixed-point values and critical exponents for $D_{\alpha_N}^{\text{grav}} = 1$ and $D_{\alpha_y}^{\text{grav}} = 1$ (upper panels) and $D_{\alpha_y}^{\text{grav}} = 1.5$ (lower panels).

Figure 13. We show the region where the fixed-point collision is avoided for $\epsilon = 0.5$ in red, $\epsilon = 0.1$ in orange, $\epsilon = 0.01$ in yellow.

A.1 Next-to leading order

When the gauge parameter $\alpha$ is varied, we find the opposite behavior and larger values of $\alpha$ shift the critical value where the fixed point turns complex to smaller values. This behavior is shown in figure 15. We are now in position to analyze the gauge dependence of the fixed points. From the equations above one can infer the well-known pole of the gravitational contributions at $\beta = 3$. As a consequence, the results cannot be trusted in the vicinity of $\beta = 3$ due to the instability and we restrict ourselves to $\beta \lesssim 1$. The
Figure 14. We show the dependence of $g_{\text{crit}}$ on $\beta$ for $\epsilon = 1/10$ for different values of $\alpha$: $\alpha = 0$ (continuous blue line), $\alpha = 0.5$ (purple dashed line) and $\alpha = 1$ (green dotted line).

Figure 15. We show the dependence of $g_{\text{crit}}$ on $\alpha$ for $\epsilon = 1/10$ for different values of $\beta$: $\beta = 0$ (continuous blue line), $\beta = 1$ (purple dashed line) and $\beta = -1$ (green dotted line).

The main message of this analysis can be summarized as follows. The qualitative general result for the gauge dependence of $g_{\text{crit}}$ is given by

$$g_{\text{crit}} = \frac{160\pi}{(11 + 2\epsilon)^4 \left( -6(140 + \beta(\beta - 73)) + \alpha(263 + 27\beta(\beta - 6)) \right)^2} \cdot$$

$$\cdot \left[ -2(\beta - 3)^2(13 + 2\epsilon) \left( -1140(15 + (\beta - 10)\beta) + (64620 + \beta(1283\beta - 35699))\epsilon ight) 
+ 4(5220 + \beta(73\beta - 2809))\epsilon^2 + 12(140 + \beta(\beta - 73))\epsilon^3 \right]$$

$$+ \alpha(\beta - 3)^2(13 + 2\epsilon) \left( 2280(11 + \beta(\beta - 6)) + (11583 + 1427\beta(\beta - 6))\epsilon 
+ 4(2013 + 217\beta(\beta - 6))\epsilon^2 + 4(263 + 27\beta(\beta - 6))\epsilon^3 \right)$$

$$- 4\sqrt{5} \left\{ (15 + \beta(\beta - 10) + \alpha(11 + \beta(\beta - 6)))(13 + 2\epsilon)^2 \cdot$$

$$\cdot (-57 + 46\epsilon + 8\epsilon^2) \left( 1140(15 + \beta(\beta - 10) + \alpha(11 + \beta(\beta - 6))) 
+ (\alpha(21703 + 2347\beta(\beta - 6)) - 2(57720 + \beta(823\beta - 31099)))\epsilon 
+ 4(-9840 + 2\beta(2609 - 53\beta) + \alpha(2453 + 257\beta(\beta - 6)))\epsilon^2 
+ 4(-6(140 + \beta(\beta - 73)) + \alpha(263 + 27\beta(\beta - 6)))\epsilon^3 \right) \right\}^{\frac{1}{2}} \right\} \} (A.7)$$

The main message of this analysis can be summarized as follows. The qualitative
features of the fixed point structure are independent of $\beta \in (-\infty, 1]$ and $0 < \alpha < \infty$. For fixed $\alpha$, the fixed point annihilation is shifted towards smaller values of $g$ as $\beta$ is lowered and converges (for $\epsilon = 1/10$) towards

$$\lim_{\beta \to -\infty} g_{\text{crit}} = \frac{55\pi}{11586 + 40\sqrt{83603}} \approx 0.007,$$

(A.8)

with corresponding fixed point values

$$\lim_{\beta \to -\infty} \alpha_{N, *} = \frac{55(-545 + 2\sqrt{83603}}{74774} \approx 0.02,$$

(A.9)

and

$$\lim_{\beta \to -\infty} \alpha_{y, *} = \frac{5(-2573163 + 9580\sqrt{83603}}{87934224} \approx 0.01.$$

(A.10)

For fixed $\alpha$ the parametric dependence on $\beta$ is shown in figure 14. For any epsilon, there is value at which $g_{\text{crit}}$ disappears, i.e. where no fixed point annihilation takes place. However, this is typically in a region with large $\beta$. Moreover, one can see that there is a minimum at some $\alpha$-dependent value of $g_{\text{crit}}$ as a function of $\beta$, but there is no discontinuity or any further peculiar behavior of the curve all the way down to the smooth limit $\beta \to -\infty$.

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