Effect of exchange interaction on fidelity of quantum state transfer from a photon qubit to an electron-spin qubit

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We analyzed the fidelity of the quantum state transfer (QST) from a photon-polarization qubit to an electron-spin-polarization qubit in a semiconductor quantum dot, with special attention to the exchange interaction between the electron and the simultaneously created hole. In order to realize a high-fidelity QST we had to separate the electron and hole as soon as possible, since the electron-hole exchange interaction modifies the orientation of the electron spin. Thus, we propose a double-dot structure to separate the electron and hole quickly, and show that the fidelity of the QST can reach as high as 0.996 if the resonant tunneling condition is satisfied.

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Quantum state transfer (QST) has attracted enormous attention as one of the key concepts in quantum information science. Quantum information can take several different forms such as photons, nuclear spin of atoms, and electron spin of quantum dots. All of these physical realizations of quantum information are called “qubits”. Since each qubit has its own merits and demerits, we have to choose the right qubit for each process. The photon-polarization qubit is the most convenient medium for sharing quantum information between distant locations. Presently, we can distribute quantum keys over 122 km of standard telecom fiber. On the other hand, the electron-spin qubit is the most convenient medium for quantum gate and quantum memory in a semiconductor quantum dot since coupling among the qubits can easily be controlled by gate voltage. Electron-spin qubits are a promising candidate for the realization of a scalable quantum computer. It is then a logical next step to study the QST from a photon qubit to an electron-spin qubit in order to construct efficient quantum information processing devices.

In 2001, Vrijen and Yablonovitch proposed a spin-coherent semiconductor photo-detector which transfers the quantum information from a photon-polarization qubit to an electron-spin qubit. Such a quantum-state-coherent photo-detector is a basic element of a quantum repeater, which enables us to drastically expand the distance of quantum key distribution. They showed that the well-known optical orientation in semiconductor heterostructure can be used for the QST.

The spin-coherent semiconductor photo-detector has an optically active quantum well where the quantum information is transferred from photon polarization to electron spin. The $k$-vector of the incident photon is parallel to the growth direction of the well. The energy levels of the well are shown in Fig. 1(a). In order to carry out the photon-spin QST, the spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ of an electron should be degenerate in the presence of a magnetic field. Therefore, we have to tune the electron spin $g$-factor to be zero, $g_e = 0$, with the help of $g$-factor engineering. The $g$-factor engineering can be realized by using the proximity of the electron wave function into the barrier layer. In the quantum well system, the $g_e$ can be estimated as $g_e = gw_W + (1 - w)g_B$, where $g_W$ and $g_B$ are the $g$-factor of the well and that of the barrier, respectively, and $w$ is the occupation probability of the electron in the well. Appropriate choice of the structure and the material enables us to obtain $g_e = 0$. The degeneracy between the heavy-hole states and light-hole states is lifted if the material is placed under tensile strain. The uniform magnetic field $B$ is applied along the $z$-direction to lift the degeneracy of the light-hole states $|\psi^\uparrow\rangle_{lh} = (|J = 3/2, m_J = 1/2\rangle + |J = 3/2, m_J = -1/2\rangle)/\sqrt{2}$ and $|\psi^-\rangle_{lh} = (|J = 3/2, m_J = 1/2\rangle - |J = 3/2, m_J = -1/2\rangle)/\sqrt{2}$. The Zeeman splitting between these two states is given by $g_h\mu_B B$, where $\mu_B$ is the Bohr magneton. The Zeeman splitting of the electron spin states is assumed to be zero. According to the selection rule, the electron with the spin up state along the $z$-direction $|\uparrow\rangle$ is excited in the quantum dot by a right-handed circularly polarized photon $|\sigma^+\rangle$. Similarly, a left-handed circularly polarized photon $|\sigma^-\rangle$ excites the electron in the spin-down state $|\downarrow\rangle$. In these two cases, a hole in the $|\psi^\uparrow\rangle_{lh}$ state is created in the dot simultaneously. After elimination of the hole, the superposition of the polarized photon $\alpha_+ |\sigma^+\rangle + \alpha_- |\sigma^-\rangle$ is transferred to the superposition of the electron spin $\alpha_+ |\uparrow\rangle + \alpha_- |\downarrow\rangle$.

One of the main obstacles to high-fidelity QST in a spin-coherent semiconductor photo detector is the exchange interaction between the electron and the simultaneously created hole. In this paper, we analyze the effect of the exchange interaction on the fidelity of the QST from a photon-polarization qubit to an electron-spin qubit. For high-fidelity QST we have to extract the hole as soon as possible. We propose a double-well structure to separate the carrier using resonant tunneling. Quick extraction of the carrier using resonant tunneling in the double-well structure was extensively studied by Gurvitz, and experimentally demonstrated by Cohen. Using the double-well structure, quick extraction of the hole can be realized without...
thinning the barrier width, and then we can minimize the deviation of $g_0$ from zero. We solved the Schrödinger equations of the photo-detector using realistic parameters of semiconductor heterostructure and showed that the coherence time of the QST can reach as high as 0.996 under the resonant tunneling condition.

The system we consider is the semiconductor heterostructure shown in Fig. 1(b), which consists of two dots, dot1 and dot2. Dot2 is connected with the continuum through the tunneling barrier. The strong confinement of the dot structure prolongs the electron-spin coherence time $T_2$. The gates are attached to the dots to control the energy levels. Since our interest is in the effect of the electron-hole exchange interaction on the fidelity of the QST, we restricted our study to the dynamics of the system after the electron-hole pair is created in dot1 by an incident photon. We assume that the dipolar interaction is so small that we can neglect the recombination process. We also assume that the mismatch of the electron energy levels between two quantum dots is much larger than the inter-dot tunneling coupling, and therefore that the electron is localized in dot1.

The wave function of the system can be written as

$$ \Psi(t) = \sum_{s=\uparrow,\downarrow} \phi_{1s}(t) |s\rangle_{1h} + \sum_{s=\uparrow,\downarrow} \phi_{2s}(t) |s\rangle_{2h} + \sum_{s=\uparrow,\downarrow} \sum_l \psi_{ls}(t) |sl\rangle, $$

(1)

where $s=\uparrow,\downarrow$ denotes the electron state with spin $s$, $h_{1(2)}$ the hole state in dot1(2), and $l$ the hole state in the continuum. Note that the state of the hole in the dot1 $h_1$ is assumed to be restricted into the top-most light-hole state $|\psi^+\rangle$. Here, $\phi_{1s}(t), \phi_{2s}(t),$ and $\psi_{ls}(t)$ are coefficients to be determined by solving the Schrödinger equation.

The Hamiltonian of the system is expressed as

$$ H = \sum_{s=\uparrow,\downarrow} (\omega_e + \omega_1) |s\rangle_{1h}\langle s|_{1h} + \sum_{s=\uparrow,\downarrow} \omega_J |s\rangle_{1h}\langle s|_{1h} $$

$$ + \sum_{s=\uparrow,\downarrow} \{ \delta |s\rangle_{1h}\langle s|_{1h} + \text{h.c.} \} + \sum_{s=\uparrow,\downarrow} (\omega_e + \omega_2) |s\rangle_{2h}\langle s|_{2h} $$

$$ + \sum_{s=\uparrow,\downarrow} \sum_l \{ W_l |s\rangle_{2h}\langle s|_{2l} + \text{h.c.} \} $$

$$ + \sum_{s=\uparrow,\downarrow} \sum_l (\omega_e + \omega_l) |s\rangle_{2l}\langle s|_{2l}, $$

(2)

where $\omega_e$ is the energy level of the electron in dot1, $\omega_{1(2)}$ the hole energy level in dot1(2), $\omega_J$ the electron-hole exchange interaction, $\omega_l$ the hole energy level in the continuum, $\delta$ the coupling between dot1 and dot2, $W_l$ the coupling between dot2 and continuum state $l$, and $\tilde{s}$ the electron spin opposite to $s$. Here, we set $\hbar = 1$. In a zincblende crystal, the electron-hole exchange interaction is given by $a s \cdot J + b \sum_{x,y,z} s_x J_{x,y,z}^2$, where $a, b$ are coefficients, $s$ and $J$ represent the electron and hole spin, respectively. In Eq. (2), we consider the coupling term between two degenerated states $|\uparrow\rangle_{h1}$ and $|\downarrow\rangle_{h1}$, $\omega_J = (2a + 5b)/4$. We can neglect the coupling with the other states since the typical energy scale of the exchange interaction ($\sim 10\mu eV$) is much smaller than the Zeeman splitting between the light-hole states ($\sim 1 \text{meV}$ at $B = 5T$).

The dynamics of the system are obtained by solving the following Schrödinger equation:

$$ \dot{\phi}_{1s}(t) = -i(\omega_e + \omega_1)\phi_{1s}(t) - i\omega_J\phi_{1s}(t) - i\delta\phi_{2s}(t), $$

(3)

$$ \dot{\phi}_{2s}(t) = -i(\omega_e + \omega_2)\phi_{2s}(t) - i\delta\phi_{1s}(t) $$

$$ - i\sum_l W_l\psi_{ls}(t), $$

(4)

$$ \dot{\psi}_{ls}(t) = -i(\omega_e + \omega_l)\psi_{ls}(t) - iW_l^*\phi_{2s}. $$

(5)

These equations can be simplified by changing the electron spin basis from the eigenstates ($|\uparrow\rangle,|\downarrow\rangle$) of $\sigma_z$ to the eigenstates ($|+\rangle,|-\rangle$) of $\sigma_x$, introducing

$$ \phi_{1\pm}(t) = (\phi_{1\uparrow}(t) \pm \phi_{1\downarrow}(t))/\sqrt{2}, $$

(6)

$$ \phi_{2\pm}(t) = (\phi_{2\uparrow}(t) \pm \phi_{2\downarrow}(t))/\sqrt{2}, $$

(7)

$$ \psi_{\pm}(t) = (\psi_{\uparrow}(t) \pm \psi_{\downarrow}(t))/\sqrt{2}. $$

(8)

Eqs (3)-(5) are rewritten as

$$ \dot{\phi}_{1\sigma}(t) = -i(\omega_e + \omega_1 + \omega_{J\sigma})\phi_{1\sigma}(t) - i\delta\phi_{2\sigma}(t), $$

(9)

$$ \dot{\phi}_{2\sigma}(t) = -i(\omega_e + \omega_2)\phi_{2\sigma}(t) - i\delta\phi_{1\sigma}(t) $$

$$ - i\sum_l W_l\psi_{ls}(t), $$

(10)

$$ \dot{\psi}_{\sigma}(t) = -i(\omega_e + \omega_l)\psi_{\sigma}(t) - iW_l^*\phi_{2\sigma}(t), $$

(11)

where $\sigma = \pm$, $\omega_{J\pm} = \pm\omega_J$. One can easily see that Eqs (9)-(11) are separable with respect to the index $\sigma = \pm$. 
The solution of Eq. (11) is obtained as

$$\psi_\sigma(t) = -iW_i^* \int_0^t dt' e^{-i(\omega_\sigma + \omega_f)(t-t')} \phi_{2\sigma}(t').$$  \hspace{1cm} (12)$$

The tunneling process of the hole from the dot2 to the continuum is characterized by the spectral density function $\gamma_\omega(\omega) \equiv \pi \sum |W_i|^2 \delta(\omega - \omega_f)$. Given the density of the state of the hole in the continuum is dense around $\omega \sim \omega_f$, we can treat $\gamma_\omega(\omega)$ as a constant, which corresponds to the Markov approximation. Substituting Eq. (12) into Eq. (10), and applying the Markov approximation, we have

$$\dot{\phi}_{2\sigma}(t) = -i(\omega_\sigma + \omega_f - i\gamma_\omega)\phi_{2\sigma}(t) - i\delta^* \psi_\sigma(t).$$  \hspace{1cm} (13)$$

Here, $\gamma_\omega$ represents the tunneling rate of the hole from the dot2 to the continuum.

Applying the Laplace transformation $\hat{\psi}_\sigma(p) = \int_0^\infty dt e^{-pt} \psi_\sigma(t)$, Eqs. (12) and (13) can be expressed as

$$p\hat{\phi}_{1\sigma}(p) - \beta_\sigma = -i(\omega_\sigma + \omega_f - i\gamma_\omega)\hat{\phi}_{1\sigma}(p) - i\delta^* \hat{\psi}_\sigma(p),$$  \hspace{1cm} (14)$$

$$p\hat{\phi}_{2\sigma}(p) = -i(\omega_\sigma + \omega_f - i\gamma_\omega)\hat{\phi}_{2\sigma}(p) - i\delta^* \hat{\psi}_\sigma(p),$$  \hspace{1cm} (15)$$

where $\beta_\omega$ are the coefficients for the linear combination of electron spin states $|\pm\rangle$ at $t = 0$. Then we obtain

$$\hat{\phi}_{1\sigma}(p) = \beta_\sigma \frac{p+i(\omega_\sigma + \omega_f - i\gamma_\omega)}{p+i(\omega_\sigma + \omega_f - i\gamma_\omega)} \hat{\phi}_{1\sigma}(p) + \frac{|\delta|^2}{p+i(\omega_\sigma + \omega_f - i\gamma_\omega)} \hat{\phi}_{1\sigma}(p),$$  \hspace{1cm} (16)$$

$$\hat{\phi}_{2\sigma}(p) = \frac{-i\delta^*}{p+i(\omega_\sigma + \omega_f - i\gamma_\omega)} \hat{\phi}_{1\sigma}(p).$$  \hspace{1cm} (17)$$

The fidelity of the QST is defined as $F = \langle \Psi(0)|\rho(\infty)|\Psi(0) \rangle$, where $\rho(t)$ is the reduced density matrix, and $|\Psi(0)\rangle = \beta_+ |\uparrow\rangle + \beta_- |\downarrow\rangle$ is the initial state of the spin. Each component of $\rho(t)$ is defined as

$$\rho_{\sigma\sigma'}(t) = \phi_{1\sigma}(t)\phi_{2\sigma'}(t) + \phi_{2\sigma}(t)\phi_{2\sigma'}(t) + \sum_l \psi_{\sigma}(t)\tilde{\psi}_{\sigma'}(t).$$  \hspace{1cm} (18)$$

In the limit of $t \to \infty$, the hole is in the continuum and $\phi_{1\sigma}(\infty) = \phi_{2\sigma}(\infty) = 0$. Hence, we have $\rho_{\sigma\sigma'}(\infty) = \sum_l \psi_{\sigma}(\infty)\tilde{\psi}_{\sigma'}(\infty)$. The reduced density matrix $\rho_{\sigma\sigma'}(\infty)$ can be easily evaluated by moving to the interaction picture. In the interaction picture, the reduced density matrix is expressed as $\rho_{\sigma\sigma'}(\infty) = \sum_l \tilde{\psi}_{\sigma}(\infty)\tilde{\psi}_{\sigma'}(\infty)$, where $\tilde{\psi}_{\sigma}(t) = e^{i(\omega_{\sigma}+\omega_f)t}\psi_{\sigma}(t)$. From Eqs. (12) and (17), $\tilde{\psi}_{\sigma}(t)$ is given by

$$\tilde{\psi}_{\sigma}(\infty) = -iW_i^* \hat{\psi}_{2\sigma}(t) = \beta_\sigma \frac{1}{f_{\sigma}(\omega_f)} \int d\omega f_{\sigma}(\omega) f_{\sigma'}(\omega)|\delta|^2, \hspace{1cm} (19)$$

where $f_{\sigma}(\omega) = (\omega_\sigma - \omega + i\gamma_\omega)(\omega_f - \omega - i\gamma_\omega) - |\delta|^2$. Thus we have $\rho_{\sigma\sigma'}(\infty) = \beta_\sigma \beta_{\sigma'}^* I_{\sigma\sigma'}$, where

$$I_{\sigma\sigma'} = \frac{|\delta|^2 |\gamma_\omega|^2}{\pi} \int d\omega \frac{1}{f_{\sigma}(\omega)f_{\sigma'}(\omega)}. \hspace{1cm} (20)$$

Finally, the fidelity of the QST is obtained as

$$F = 1 - 2|\beta_+|^2 |\beta_-|^2 (1 - |\rho_{++}|). \hspace{1cm} (21)$$

Equation (21) shows that the fidelity depends strongly on the initial state of the electron spin, $\beta_\omega$. If the initial state of the electron spin is $|\uparrow\rangle$ or $|\downarrow\rangle$, the electron-hole exchange interaction does not affect the fidelity since the initial state is the eigenstate of the electron-hole exchange interaction. For the general initial states with $|\beta_+|^2 |\beta_-|^2 \neq 0$, the fidelity is reduced from unity by the electron-hole exchange interaction. In Fig. 2, we plot the fidelity $F$ as a function of the electron-hole exchange interaction, $\omega_f$, and the tunneling rate of the hole from dot2 to the continuum, $\gamma_\omega$. We assume that the hole energy levels in dot1 and dot2 are the same, i.e., $\omega_1 = \omega_2$. The initial state of the electron spin is taken to be $|\uparrow\rangle$. We now proceed to the estimation of the fidelity.
of the electron spin using realistic parameters of GaAs/Al$_{0.8}$In$_{0.2}$As heterostructure [30]. We can set the electron $g$-factor in the quantum dot to be zero by adjusting the thickness of the GaAs layer. The energy levels shown in Fig. 1 (a) can be realized in this heterostructure since tensile strain is applied to the GaAs layer. The inter-dot coupling, $\delta$, can be calculated by considering the boundary conditions for the wave function for the hole $g$-factor. We estimate $|\delta| = 0.8$ meV for GaAs/Al$_{0.8}$In$_{0.2}$As heterostructure with $g_z = 0$. The typical value of the electron-hole exchange interaction in the quantum dot is $\omega_J = 40$ meV [32]. In Fig. 4 we plot the fidelity $F$ as a function of $\gamma_h$. As mentioned before, the fidelity takes its maximum value $F = 0.996$ under the resonant condition at $\gamma_h = 0.8$ meV.

In conclusion, we analyzed the effect of the electron-hole exchange interaction on the QST in a spin-coherent semiconductor photo-detector. We have shown that the fidelity decreases as the strength of the exchange interaction increases, and that it depends on the initial state of the electron spin. We have also shown that a high-fidelity ($F \sim 0.996$) QST is possible using the double-dot structure under the resonant tunneling condition of a hole.

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