A Free-field Representation of the Screening Currents of $U_q(\widehat{sl}(3))$

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Abstract

We construct five independent screening currents associated with the $U_q(\widehat{sl}(3))$ quantum current algebra. The screening currents are expressed as exponentials of the eight basic deformed bosonic fields that are required in the quantum analogue of the Wakimoto realization of the current algebra. Four of the screening currents are ‘simple’, in that each one is given as a single exponential field. The fifth is expressed as an infinite sum of exponential fields. For reasons we discuss, we expect that the structure of the screening currents for a general quantum affine algebra will be similar to the $U_q(\widehat{sl}(3))$ case.

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1 Introduction

It has recently been realized that quantum affine algebras play the role of dynamical symmetries in many two-dimensional quantum integrable systems. This is the case in both continuum theories such as the Sine-Gordon model [1, 2], and in lattice models such as the $XXZ$ quantum spin chain [3, 4]. For the spin-$\frac{1}{2}$ $XXZ$ chain (the anisotropic quantum Heisenberg model), the dynamical symmetry is that of $U_q(\widehat{sl}(2))$ at level 1. The calculation of equal-time correlation functions of local operators in this model turns into the problem of calculating traces of the vertex operator interwiners defined in [3, 4] as

$$\tilde{\Phi}_V^\mu\lambda(z) : V(\lambda) \rightarrow V(\mu) \otimes V_z.$$ (1.1)

Here $V(\lambda)$ and $V(\mu)$ are the irreducible highest weight representations of $U_q(\widehat{sl}(2))$ associated with the dominant integral weights $\lambda$ and $\mu$, and $V_z \simeq V \otimes \mathbb{C}[z, z^{-1}]$ is the evaluation representation. This analysis of the $XXZ$ model has been generalized, both to higher spin [6, 7], in which case the dynamical symmetry is $U_q(\widehat{sl}(2))$ at level $k = 2s$, and to $sl(n)$ spin chains [8], where the symmetry is $U_q(\widehat{sl}(n))$.

The desired traces may be obtained via two different approaches. Firstly, it is possible to derive quantum difference equation obeyed by these objects [9, 10]. However, generically, these equations will have an infinite number of solutions, and some additional information about the expected analytic structure of the ‘true’ solution is required in order to isolate it. The second approach is to use the free field representation of the algebra and vertex operators - where it exists. This approach yields an integral expression [4, 7, 6, 8], which in some simple cases can be integrated explicitly, and in other cases yield enough analytic information in order to select the correct solution of the difference equation for the same quantity [8].

The main problem associated with the free field realization is that in general the Fock space constructed from the free fields is completely reducible, i.e., it contains infinitely many null states. The unique irreducible subspace, which is embedded in this large Fock space, and over which one should calculate the traces, can be obtained by modding out the null states and their descendants [11, 12]. This problem has been resolved in the context of conformal field theory through the use of screening current operators [13, 14, 15]. The main property of
a screening current is that its operator product expansions (OPEs) with the current algebra under consideration are either regular or total derivatives.

In the context of conformal field theories with Kac-Moody symmetry, it is known that the screening currents split into two different sets [16]. The screening currents in the first set are each expressed as a finite sum of products of two terms. The first term consists of a first derivative of the basic free bosonic fields which arise in the Wakimoto realization of the current algebra [17]. The second term is expressed as an exponential of these same basic fields. The screening currents in this first set may be constructed in an algorithmic way from the bosonized expressions for the raising step currents [18]. The screening currents in the second set are expressed only as exponentials of the basic fields; they do not depend on their derivatives [16]. These screening currents are now related to both raising and lowering step currents, and there is the additional complication that some of them involve infinite sums of such exponentials.

In this paper we are interested in the case of quantum affine algebras. It has recently been shown in Ref. [19] that for \( U_q(\widehat{sl}(n)) \) there exists an analogue of the first set of screening currents described above. The screening currents now depend on both free deformed bosonic fields and their first quantum derivatives, and again can be read off from the bosonization of the raising step currents of the quantum current algebra. \( U_q(\widehat{sl}(n)) \) is presently the only algebra for which a Wakimoto realization in terms of free deformed bosons is available [19]. The purpose of this paper is to show that the second set of screening currents also has a quantum analogue. We choose the special case of \( U_q(\widehat{sl}(3)) \) because in the classical case \( \widehat{sl}(3) \) is the simplest example which embodies all the features of \( \widehat{g} \): the screening currents which involve infinite sums appear here but not in \( \widehat{sl}(2) \) (for \( \widehat{sl}(2) \) only the screening currents which involve single exponentials appear). For the ‘classical case’ of \( \widehat{sl}(n) \), no more new features, other than computational complexity, arise in explicit computation of the screening currents than are present for \( \widehat{sl}(3) \) [16]. It is legitimate to expect that the \( U_q(\widehat{sl}(3)) \) current algebra will also maintain its special status in the quantum case. Nevertheless, we will briefly discuss the screening currents in the general case of \( U_q(\widehat{sl}(n)) \).

The paper is organized as follows. In Section 2, we review the bosonization of the \( U_q(\widehat{sl}(3)) \) quantum current algebra at general level \( k \). In section 3, we present in four succes-
sive steps our method for constructing the second set of screening currents associated with this algebra. In section 4, by way of comparison, we discuss the simpler case of \( U_q(\hat{sl}(2)) \).

In addition we comment on the extension of our method to \( U_q(\hat{sl}(n)) \) and make a conjecture concerning the single exponential screening currents. Section 5 is devoted to our conclusions. In appendix A, we give the definition of a general quantum current algebra \( U_q(\hat{g}) \). In appendix B, we present four tables of OPEs corresponding to the four steps of section 3.

## 2 Bosonization of the \( U_q(\hat{sl}(3)) \) Current Algebra

The definition of a quantum affine algebra was given in references \[20, 21, 22\]. For a brief summary of this definition, and that of the associated quantum current algebra, see Appendix A (more details are given in reference \[19\]). Here we review the bosonization (the Wakimoto realization) of the quantum current algebra of \( U_q(\hat{sl}(3)) \).

The number of deformed bosonic fields required for the free-field representation of the \( U_q(\hat{sl}(n)) \) current algebra is equal to the dimension of \( sl(n) \), i.e., \( n^2 - 1 \). These fields may all be expressed in terms of the following fundamental \( d^i \) fields (\( i = 1, \ldots, n^2 - 1 \)) \[19\]:

\[
\begin{align*}
d^i(M, N|z, \alpha) &\equiv \frac{M}{N}d^i + \frac{M}{N}d_0^i \ln(z) - \sum_{n \neq 0} \frac{q^{-\alpha|n||Mn|}}{|n|} d_n^i z^{-n}, \\
d^i(M|z, \alpha) &\equiv d^i(M, 1|z, \alpha) = Md^i + M d_0^i \ln(z) - \sum_{n \neq 0} \frac{q^{-\alpha|n|}}{|n|^2} d_n^i z^{-n}, \\
d^i(z, \alpha) &\equiv d^i(1, 1|z, \alpha) = d^i + d_0^i \ln(z) - \sum_{n \neq 0} \frac{q^{-\alpha|n|}}{|n|} d_n^i z^{-n}, \\
d^i_{\pm}(z) &\equiv d^i(zq^{\pm(\alpha-1)}, \alpha) - d^i(zq^{\mp\alpha}, \alpha - 1) = \pm d_0^i \ln q \pm (q - q^{-1}) \sum_{n>0} d_n^i z^{-n}.
\end{align*}
\]

These fields satisfy the following useful identities:

\[
d^i(M, N|zq^\delta, \alpha) + d^i(M', N|zq^\gamma, \alpha) = d^i(M + M', N|zq^\delta, \alpha),
\]

with \( \delta = \begin{cases} 
\gamma + M, & \text{if } M + M' = \beta - \gamma, \\
\gamma - M, & \text{if } M + M' = \gamma - \beta,
\end{cases} \]

\[
d^i(wq^\beta, \alpha) + d^i(\beta|wq^{-1}, \alpha) = d^i(wq^{-\beta}, \alpha) + d^i(\beta|wq, \alpha).
\]

Let us recall that the OPE of the exponentials of two fields \( d^i(M|z, \alpha) \) and \( d^j(M'|w, \beta) \) is given by

\[
e^{d^i(M|z, \alpha)} e^{d^j(M'|w, \beta)} = e^{<d^i(M|z, \alpha)d^j(M'|w, \beta)>}; e^{d^i(M|z, \alpha)+d^j(M'|w, \beta)} ;.
\]

(2.4)
where as usual the normal ordering symbol \(\ldots\) means that the annihilation modes \(\{d^i_n, n \geq 0\}\) are moved to the right of the creation modes \(\{d^i_n, n < 0\}\) and of the shift mode \(d^i\). For later purposes, we list the three generic vacuum expectation values that will be required:

\[
< d^i(M|z, \alpha) d^j(M'|w, \beta) > = MM'[d^i_0, d^j] - \sum_{n>0} q^{-(\alpha+\beta)n} |Mn[Mn'][M'\prime]| n!^2 z^{-n} w^n, \\
< d^i_+(z) d^j(M|w, \alpha) > = M[d^i_0, d^j] \ln q + (q - q^{-1}) \sum_{n>0} q^{-\alpha n} |Mn[Mn'][M'\prime]| n!^2 z^{-n} w^n, \\
< d^i_-(z) d^j(M|w, \alpha) > = -M[d^i_0, d^j] \ln q. \\
\tag{2.5}
\]

In what follows, all simple operators or products of operators defined at the points \(zq^n\) with the same point \(z\), for some integers \(n\), are understood to be normal ordered. The q-integer \([n]\) is as usual defined by

\[
[n] = \frac{q^n - q^{-n}}{q - q^{-1}}. \tag{2.6}
\]

For \(U_q(sl(3))\), there are thus 8 bosonic fields labeled as \(a^i \equiv d^i, i = 1, 2; b^i \equiv d^{i+2}, i = 1, 2, 3; c^i \equiv d^{i+5}, i = 1, 2, 3\). The corresponding deformed Heisenberg algebras are \(2.9\)

\[
[a^i_n, a^j_m] = \frac{1}{n} [n(k+3)] [a^{ij} n] \delta^{n+m,0}, \quad [a^i_0, a^j] = (k+3) a^{ij}, \quad i, j = 1, 2; \\
[b^i_n, b^j_m] = -\frac{1}{n} [n] [a^{ij} n] \delta^{n+m,0} \delta^{ij}, \quad [b^i_0, b^j] = -\delta^{ij}, \quad i, j = 1, 2, 3; \\
[c^i_n, c^j_m] = \frac{1}{n} [n] [a^{ij} n] \delta^{n+m,0} \delta^{ij}, \quad [c^i_0, c^j] = \delta^{ij}, \quad i, j = 1, 2, 3;
\tag{2.7}
\]

where \(a^{ij}\) is the Cartan matrix of \(sl(3)\), i.e.,

\[
(a^{ij}) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}. \tag{2.8}
\]

All commutators of different letters vanish, and \(k \in \mathbb{Z}\) is called the level of the algebra. We use the identities \(2.9\) to re-express the bosonization of the current generators of \(U_q(sl(3))\) given in ref. \(19\) in a suitable form for our purposes:

\[
E^{-1}(z) = \ J^1(z) + J^2(z), \\
E^{-2}(z) = \ J^3(z) - J^1(z), \\
E^{+1}(z) = -J^3(z), \\
E^{+2}(z) = -J^9(z) - J^7(z), \\
\psi^i_\pm(zq^{\pm k/2}) = e^{\varphi^i_\pm(z)},
\]

where,

\[
J^i(z) = e^{X^i(z)} D_z e^{Y^i(z)}, \quad i = 1, \ldots, 7; \tag{2.10}
\]
and

\[
\begin{align*}
X^1(z) &= -a^1(z, (k + 1)/2) - b^1(k + 1|z, 1) + b^2(z, k + 1) - b^3(z, k + 2) - c^1(k|z, 0), \\
X^2(z) &= a^1(zq^{(k+3)/2}, 1) - a^1(zq^{(k+1)/2}, 0) - b^2(zq^{k+3}, 0) + b^3(zq^{k+3}, 1) + c^3(zq^{k+2}, 0), \\
X^3(z) &= -a^2(z, (k + 1)/2) - b^2(k + 2|z, 1) - c^2(k + 1|z, 0), \\
X^4(z) &= a^2(zq^{-(k+3)/2}, 1) - a^2(zq^{-(k+1)/2}, 0) - b^1(zq^{-(k+1)}, 1) + b^2(z^{-(k+3)}, 1) \\
&\quad - b^2(zq^{-(k+1)}, -1) + b^3(zq^{-(k+1)}, 0) + c^3(zq^{-(k+1)}, 0), \\
X^5(z) &= -b^1(z, -1), \\
X^6(z) &= -b^2(qz, -1) + b^3(qz, 0) - b^3(z, -1) - b^1(q^2z, 0) + b^1(qz, -1), \\
X^7(z) &= b^1(z, 0) + c^1(z, 0) - b^3(z, -1), \\
Y^1(z) &= a^1(z, (k + 3)/2, 0) + b^1(k + 2|z, 1) - b^2(z, k + 2) + b^3(z, k + 3) + c^1(k + 1|z, 0), \\
Y^2(z) &= -c^2(zq^{k+2}, 0), \\
Y^3(z) &= a^2(z, (k + 3)/2) + b^2(k + 3|z, 1) + c^2(k + 2|z, 0), \\
Y^4(z) &= -c^1(z^{-(k+1)}, 0), \\
Y^5(z) &= -c^1(z, 0), \\
Y^6(z) &= -c^2(qz, 0), \\
Y^7(z) &= -c^3(z, 0), \\
\varphi^1_\pm(z) &= a^1_\pm(zq^{\pm(k+3)/2}) + b^1_\pm(zq^{\pm(k+2)}) + b^1_\pm(zq^{\pm(k+2)}) - b^2_\pm(zq^{\pm(k+2)}) + b^3_\pm(zq^{\pm(k+3)}), \\
\varphi^2_\pm(z) &= a^2_\pm(zq^{\pm(k+3)/2}) - b^1_\pm(zq^{\pm(k+1)}) + b^2_\pm(zq^{\pm(k+1)}) + b^2_\pm(zq^{\pm(k+3)}) + b^3_\pm(zq^{\pm(k+3)}).
\end{align*}
\]

(2.11)

Here the quantum derivative \( _1D_w \) appears. It is defined more generally by

\[
_pD_w f(z) = \frac{f(zq^p) - f(zq^{-p})}{z(q - q^{-1})}, \quad p \in \mathbb{Z}/\{0\}. \tag{2.12}
\]

For latter use, we also introduce the notation,

\[
J^i_\pm(z) = \frac{J^i_\pm(z) - J^i_\pm(z)}{z(q - q^{-1})}, \quad \text{with} \quad J^i_\pm(z) = e^{X_i(z)}e^{Y_i(zq^{\pm1})}. \tag{2.13}
\]
3  The Screening Currents of $U_q(sl(3))$

The screening currents are constructed by carrying out the following sequence of steps.

Step 1

Let $J(z) = e^{X(z)} \mathcal{D}_z e^{Y(z)}$ be any of the seven basic currents defining the step currents $E^{±,i}(z), i = 1, 2$ of equation (2.9), and let $G(w) = e^{g(w)}$ be a candidate screening current constructed such that its OPE with $J(z)$ is the following total quantum derivative:

$$J(z)G(w) = \frac{q^\alpha}{z(q - q^{-1})} \left( \frac{q^\beta J_+(z)G(w)}{z - wq^\beta} - \frac{q^{-\beta} J_-(z)G(w)}{z - wq^{-\beta}} \right) \sim q^\alpha \mathcal{D}_w \left( \frac{f(w)}{z - w} \right), \quad (3.14)$$

where $\sim$ means ‘equal up to regular terms’, $\alpha$ and $\beta$ are integers to be determined, and $f(w)$ satisfies the condition

$$f(w) =: J_+(w)G(wq^{-\beta}) := J_-(w)G(wq^\beta), \quad (3.15)$$

This condition translates into the following relation:

$$Y(wq) + g(wq^{-\beta}) = Y(wq^{-1}) + g(wq^\beta). \quad (3.16)$$

There are two obvious solutions to this latter equation:

$$\beta = 1 \quad \text{and} \quad g(w) = Y(w), \quad (3.17)$$
$$\beta = -1 \quad \text{and} \quad g(w) = -Y(w).$$

However, for the OPE of $G(w) = e^{\beta Y(w)}$ with $J(z)$ to then be a total derivative, we also require the following:

$$J_{\pm}(z)e^{Y(w)} = \begin{cases} \frac{q^{\alpha\pm1}}{z-wq^{\mp1}} : J_{\pm}(z)e^{Y(w)} : , & \text{for } \beta = 1, \\ q^{-\alpha\pm1}(z-wq^{\mp1}) : J_{\pm}(z)e^{Y(w)} : , & \text{for } \beta = -1. \end{cases} \quad (3.18)$$

In Table 1 we compute all the OPEs of the form $J^i(z)e^{Y^i(w)}, i = 1, \ldots, 7$. Examining this table and relation (3.18), one can see that it is possible to satisfy condition (3.14) if we choose $G^i(z) = e^{\beta^i Y^i(z)}$ for all $i = 1, \ldots, 7$, with the $\beta^i$ given by

$$\beta^i = \begin{cases} -1, & i = 2, 4, 5, 6, 7 \\ +1, & i = 1, 3 \end{cases}. \quad (3.19)$$
Note that one can also read off the value $\alpha^i$ appearing in each OPE $J^i(z)G^i(w)$ from Table 1. We find

$$\alpha^i = \begin{cases} 
0, & i = 1, 3 \\
-k - 2, & i = 2 \\
k + 1, & i = 4 \\
0, & i = 5, 7 \\
-1 & i = 6
\end{cases}$$

(3.20)

**Step 2**

For any of the $G^j(w), j = 1, \ldots, 7$, to be a genuine screening current, its OPEs with all the remaining currents, i.e., $J^i(z)G^j(w), i \neq j$ and $\psi^\pm_i(zq^{\pm k/2})G^j(w)$, must be either regular or total quantum derivatives. We give these OPEs in Table 2. They are indeed all regular except for the OPE $J^2(z)G^1(w)$, which is neither regular nor a total derivative. Therefore $G^1(w) = e^{Y_1(w)}$ fails to be a genuine screening current. It might at this stage appear that we have already constructed six screening currents. However, as $G^2(z) = G^6(zq^{k+1})$ and $G^4(z) = G^5(zq^{-k-1})$, there are only four independent screening currents (it is trivial to see that if $G(w)$ is a screening current, then so is $G(wq^n), n \in \mathbb{Z}$). We denote these currents by

$$S^2(w) = G^2(w),$$
$$S^3(w) = G^3(w),$$
$$S^4(w) = G^4(w),$$
$$S^7(w) = G^7(w).$$

(3.21)

We have shown that from each basic current $J^i(z), i = 2, \ldots, 7$, one can construct a screening current $S^i(w)$. Putting it another way - the number of screening currents one can construct from each current $E^\pm, i(z), i \neq 1$, is equal to the number of $J^i(z)$ it contains. This rule fails for $E^{-1}(z) = J^1(z) + J^2(z)$: it has so far led to only one screening current $S^2(w)$ (the one associated with $J^2(z)$) instead of two. This is because $J^1(z)$ fails to yield a screening current on its own. However, we show in the next step that all is not lost; this failure can still be corrected in a nontrivial way with the help of $J^2(z)$.
Step 3

Our aim in this section is to extract a second candidate screening current $G^{(1,2)}(w)$ from $E^{-1}(z)$ by using both $J^1(z)$ and $J^2(z)$ simultaneously. To this end, we compute the OPEs involving $J^i_+(z)e^{Y^j(w)}; i, j = 1, 2$ and $J^i_-(z)e^{X^1(u) - X^2(w)}; i = 1, 2$. These OPEs are presented in Table 3. As discussed above, the OPE $J^1_+(z)e^{Y^1(w)}$ is a total quantum derivative, i.e., $e^{Y^1(w)}$ is a screening current with respect to $J^1(z)$; the problem is that the OPE $J^2_+(z)e^{Y^1(w)}$ is singular with poles at $z = wq^{\pm 1}$. The idea is to correct this problem using a recursive technique. This is achieved as follows. First note that according to Table 3 the OPEs of the operator

$$G^{(1,2)}_0(w) \equiv e^{Y^1(w) + Y^2(w)}$$

(3.22)

with $J^i(z), i = 1, 2$, though neither regular nor total quantum derivative, have the following ‘nice’ forms:

$$J^1(z)G^{(1,2)}_0(w) = e^{Y^2(w)}D_w \left( e^{X^1(w) + Y^1(wq) + Y^1(wq^{-1})} \right),$$

$$J^2(z)G^{(1,2)}_0(w) = q^{k+2}e^{Y^1(w)}D_w \left( e^{X^2(w) + Y^2(wq) + Y^2(wq^{-1})} \right).$$

(3.23)

The relevance of these nice forms stems from the two possible quantum analogues of the chain rule:

$$1D_w(f(w)h(w)) = h(wq)1D_wf(w) + f(wq^{-1})1D_wh(w),$$

(3.24)

$$1D_w(f(w)h(w)) = h(wq^{-1})1D_wh(w) + f(wq)1D_wf(w).$$

(3.25)

To appreciate the role of these two different rules, let us consider the OPE $J^1(z)G^{(1,2)}_0(w)$ of equation 3.23. If we construct two operators, which we denote by $q^{m_{-1}}G^{(1,2),m_{-1}}_{-1}(w)$, where $m_{-1} = 0, 1$ and $m_{-1} \in \mathbb{Z}$, such that

$$J^2(z)q^{m_{-1}}G^{(1,2),m_{-1}}_{-1}(w) = \frac{e^{X^1(wq^{2m_{-1}-1}) + Y^1(wq^{2m_{-1}-1}) + Y^1(wq^{2m_{-1}-2})}}{z - wq^{2m_{-1}-1}}1D_we^{Y^2(wq^{2m_{-1}-1})},$$

(3.26)

then the following OPEs add up to a total quantum derivative:

$$J^1(z)G^{(1,2)}_0(w) + J^2(z)q^{m_{-1}}G^{(1,2),m_{-1}}_{-1}(w) = 1D_w \left( e^{X^1(w) + Y^1(wq) + Y^1(wq^{-1}) + Y^2(wq^{2m_{-1}-1})} \right).$$

(3.27)

The two values, $m_{-1} = 0, 1$, correspond to using 3.24 and 3.25 respectively for the rhs. Here we use the convention that $G^{(1,2),m_{n}}(w)$ (that will be introduced shortly for general $n \in \mathbb{Z}$) are
pure exponential operators with no overall multiplicative factors. In this notation, \( G^{(1,2)}_0(w) \) is identified with \( G^{(1,2),m_0=0}_0(w) \), with \( \alpha_0^{m_0=0} = 0 \). From the requirement and Table 3 we find that the operator \( q^{m_{-1}} G^{(1,2),m_{-1}}_{-1}(w) \) must be given by

\[
G^{(1,2),m_{-1}}_{-1}(w) = e^{X^1(wq^{2m_{-1}-1})-X^2(wq^{m_{-1}-1})+Y^1(wq^{2m_{-1}})+Y^1(wq^{2m_{-1}-2})}, \quad m_{-1} = 0, 1; \quad \text{and,} \\
\alpha_{m_{-1}} = 2m_{-1} - k - 5.
\]

(3.28)

From the preceding discussion, a natural candidate for our screening is therefore \( G^{(1,2)}(w) = q^{m_0=0} G^{(1,2),m_0}(w) + q^{m_{-1}} G^{(1,2),m_{-1}}_{-1}(w) \). However, using Table 3 one finds that the crossing terms \( J^2(z)q^{m_0=0} G^{(1,2),m_0}(w) \) and \( J^1(z)q^{m_{-1}} G^{(1,2),m_{-1}}_{-1}(w) \) in the OPE \( E^{1,-}(z) G^{(1,2)}(w) \) do not add up to be either regular or a total quantum derivative. They are given respectively by and

\[
J^1(z)q^{m_{-1}} G^{(1,2),m_{-1}}_{-1}(w) = q^{m_{-1}-2m_{-1}+1} e^{-X^2(q^{2m_{-1}-1})} \left[ D_w \left( \frac{e^{h(w)}}{z-wq^{2m_{-1}}} \right) \right] + q^{m_{-1}-1} \left( \frac{e^{h(w)-X^2(wq^{2m_{-1}-1})}}{(z-wq^{2m_{-1}})(z-wq^{2m_{-1}-1})} \right),
\]

(3.29)

with \( h(w) = X^1(wq^{2m_{-1}})+X^1(wq^{2m_{-1}-2})+Y^1(wq^{2m_{-1}})+Y^1(wq^{2m_{-1}-2})+Y^1(wq^{2m_{-1}-3}) \).

(3.30)

This means that one has to correct the above operator \( G^{(1,2)}(w) \) by adding in two more terms, denoted by \( q^{m_1} G^{(1,2),m_1}_1(w) \) and \( q^{m_{-2}} G^{(1,2),m_{-2}}_{-2}(w) \), such that the OPEs \( J^1(z)q^{m_1} G^{(1,2),m_1}_1(w) + J^2(z)q^{m_0} G^{(1,2),m_0}(w) \) and \( J^1(z)q^{m_{-1}} G^{(1,2),m_{-1}}_{-1}(w) + J^2(z)q^{m_{-2}} G^{(1,2),m_{-2}}_{-2}(w) \) are total quantum derivatives.

This process continues infinitely many times. Table 3 allows us to perform the appropriate corrections recursively to end up with the following candidate screening current, which is expressed as a series,

\[
G^{(1,2)}(w) = \sum_{n \in \mathbb{Z}} q^{m_n} G^{(1,2),m_n}_n(w),
\]

(3.31)

where

\[
G^{(1,2),m_n}_n(w) = \exp \left( -\sum_{i=1}^{n-1} (X^1(wq^{2m_n-2i+1}) - X^2(wq^{2m_n-2i+1})) \right) - \sum_{i=0}^{n-1} Y^1(wq^{2m_n-2i}) + \sum_{i=0}^{n-1} Y^2(wq^{2m_n-2i}), \quad n > 0,
\]

(3.32)

\[
G^{(1,2),m_{-n}}_{-n}(w) = \exp \left( \sum_{i=1}^{n} (X^1(wq^{2m_{-n}-2i+1}) - X^1(wq^{2m_{-n}-2i+1})) \right) + \sum_{i=0}^{n-1} Y^1(wq^{2m_{-n}-2i}) - \sum_{i=1}^{n-1} Y^2(wq^{2m_{-n}-2i}), \quad n > 0,
\]

\[
G^{(1,2),m_0}_0(w) = \exp \left( (Y^1(w) + Y^2(w)) \right),
\]

9
and
\[ \alpha_n^{m_n} = -n^2 + n(k + 2) + 2m_n, \quad n > 0, \]
\[ \alpha_{m-n}^{m-n} = -n^2 - n(k + 4) + 2m_{n-1}, \quad n > 0, \]
\[ \alpha_0^{m_0} = 0. \]

For \( n > 0, \) \( m_{\pm n} \) is such that \( 0 \leq m_{\pm n} \leq n, \) and is defined recursively from \( m_0 = 0 \) by either \( m_{\pm n} = m_{\pm(n-1)} \) or \( m_{\pm n} = m_{\pm(n-1)} + 1, \) depending on whether we use \( 3.24 \) or \( 3.25 \) respectively to construct \( G^{(1,2),m_{\pm n}}(w) \) from \( G^{(1,2),m_{\pm(n-1)}}(w) \). In the former case, i.e., \( m_{\pm n} = m_{\pm(n-1)}, \) \( n > 0, \) one can easily check that
\[ J^1(z) q^{\alpha_n^{m_n}} G^{(1,2),m_n}(w) + J^2(z) q^{\alpha_{m-n}^{m-n}} G^{(1,2),m_{n-1}}(w) = q^{m_n-n-2m_{n-1}} D_w \left( \frac{F(w)}{z-wq^{2m_n}} \right), \quad n \geq 1, \]
\[ J^1(z) q^{\alpha_n^{m_n}} G^{(1,2),m_n}(w) + J^2(z) q^{\alpha_{m-n}^{m-n}} G^{(1,2),m_{n-1}}(w) = q^{m_n-n-2m_{n-1}} D_w \left( \frac{H(w)}{z-wq^{2m_n}} \right), \quad n \leq 0, \]
where,
\[ F(w) = : J^1(wq^{2m_n}) G^{(1,2),m_n}(wq) : =: J^2(wq^{2m_n}) G^{(1,2),m_{n-1}}(wq), \]
\[ H(w) = : J^1_+(wq^{2m_n}) G^{(1,2),m_n}(wq^{-1}) := J^2_+(wq^{2m_n}) G^{(1,2),m_{n-1}}(wq^{-1}) :. \]
\[ \alpha_n^{m_n} = \alpha_{n-1}^{m_{n-1}} - 2n + k + 3, \quad n \geq 1, \]
\[ \alpha_n^{m_n} = \alpha_{n-1}^{m_{n-1}} - 2n + k + 5, \quad n \leq 0. \]

In the latter case, i.e., \( m_{\pm n} = m_{\pm(n-1)} + 1, \) \( n > 0, \) we find
\[ J^1(z) q^{\alpha_n^{m_n}} G^{(1,2),m_n}(w) + J^2(z) q^{\alpha_{m-n}^{m-n}} G^{(1,2),m_{n-1}}(w) = q^{m_n-n-2m_{n-1}} D_w \left( \frac{F'(w)}{z-wq^{2m_n-2}} \right), \quad n \geq 1, \]
\[ J^1(z) q^{\alpha_n^{m_n}} G^{(1,2),m_n}(w) + J^2(z) q^{\alpha_{m-n}^{m-n}} G^{(1,2),m_{n-1}}(w) = q^{m_n-n-2m_{n-1}} D_w \left( \frac{H'(w)}{z-wq^{2m_n-2}} \right), \quad n \leq 0, \]
where,
\[ F'(w) = : J^1_+(wq^{2m_n-2n}) G^{(1,2),m_n}(wq^{-1}) := J^2_+(wq^{2m_n-2n}) G^{(1,2),m_{n-1}}(wq), \]
\[ H'(w) = : J^1_+(wq^{2m_n+2n}) G^{(1,2),m_n}(wq) :=: J^2_+(wq^{2m_n+2n}) G^{(1,2),m_{n-1}}(wq^{-1}) :. \]
\[ \alpha_n^{m_n} = \alpha_{n-1}^{m_{n-1}} - 2n + k + 5, \quad n \geq 1, \]
\[ \alpha_n^{m_n} = \alpha_{n-1}^{m_{n-1}} - 2n + k + 3, \quad n \leq 0. \]

From the above relations it is therefore clear that the OPE \( E^{1,-}(z) G^{(1,2)}(w) \) (with \( G^{(1,2)}(w) \) given by \( 3.31 \)) is a sum of an infinite number of quantum total derivative terms. The sum is also a total quantum derivative since the quantum derivative is linear.
There are an infinite number of ways of choosing the \( \{m_n\} \) and hence an infinite number of screening current candidates. However \( G^{(1,2)}(w) \) does not yet qualify as a genuine screening current - we must also make sure that its OPEs with the remaining currents \( E^{2-}(z), E^{1+}(z), \psi_{\pm}^{i}(zq^{\pm k/2}); i = 1, 2 \), are regular or quantum total derivatives. This is the subject of the fourth and final step.

**Step 4**

In order to check the consistency of \( G^{(12)}(z) \) with the remaining currents \( E^{2-}(z), E^{1+}(z), \psi_{\pm}^{i}(zq^{\pm k/2}); i = 1, 2 \), we construct Table 4. This table consists of all elementary OPEs \( J^{i}(z)e^{X(w)} - X^{2}(w), J^{i}(z)e^{Y(w)}, J^{i}(z)e^{Y^{2}(w)}; i = 3, \ldots, 7, \psi_{\pm}^{i}(zq^{\pm k/2})e^{X(w)} - X^{2}(w), \psi_{\pm}^{i}(zq^{\pm k/2})e^{Y(w)} \) and \( \psi_{\pm}^{i}(zq^{\pm k/2})e^{Y^{2}(w)}; i = 1, 2 \). From this table one can easily show that the relation

\[
J^{3}(z)q_{\alpha_{n+1}}^{m_{n+1}+1}G^{(1,2),m_{n+1}}_{n+1}(w) - J^{4}(z)q_{\alpha_{n}}^{m_{n}}G^{(1,2),m_{n}}_{n}(w) \sim \text{regular}, \quad n \in \mathbb{Z},
\]

is true if the following conditions are satisfied:

\[
m_{n} = m_{n+1}, \quad n \geq 0,
\]
\[
m_{n} = m_{n+1} + 1, \quad n < 0,
\]

Since \( m_{0} = 0 \) this means that

\[
m_{n} = 0, \quad n \geq 0,
\]
\[
m_{n} = -n, \quad n < 0.
\]

These two conditions uniquely select a single candidate screening current from the infinite set of candidates. Now that all the parameters are fixed, all that remains is to check that the OPEs of this unique screening current candidate \( G^{(12)}(z) \) with the currents \( E^{1+}(z) = -J^{5}(z), E^{2+}(z) = -J^{6}(z) - J^{7}(z) \) and \( \psi_{\pm}^{i}(zq^{\pm k/2}) \) are regular. This is confirmed by the following relations:

\[
\psi_{\pm}^{i}(zq^{\pm k/2})q_{\alpha_{n}}^{m_{n}}G^{(1,2),m_{n}}_{n}(w) = \text{regular}, \quad n \in \mathbb{Z},
\]
\[
E^{1+}(z)q_{\alpha_{n}}^{m_{n}}G^{(1,2),m_{n}}_{n}(w) = -J^{5}(z)q_{\alpha_{n}}^{m_{n}}G^{(1,2),m_{n}}_{n}(w) = \text{regular}, \quad n \in \mathbb{Z},
\]
\[
-J^{6}(z)q_{\alpha_{n}}^{m_{n}}G^{(1,2),m_{n}}_{n}(w) - J^{7}(z)q_{\alpha_{n+1}}^{m_{n+1}}G^{(1,2),m_{n+1}}_{n+1}(w) = \text{regular}, \quad n \in \mathbb{Z}.
\]
The last relation obviously means that the OPE of $G^{(12)}(z)$ with $E^{2+}(z) = -J^6(z) - J^7(z)$ is also regular. Therefore the status of $G^{(12)}(z)$ can be elevated to that of a genuine screening current which we denote by $S^{1,2}(w)$.

4 Generalization to $U_q(\widehat{sl}(n))$

As far as the application of our method to other quantum affine algebras is concerned, the $U_q(\widehat{sl}(2))$ case is simplest because, unlike $U_q(\widehat{sl}(3))$, its currents $E^\pm(z)$ and $\psi^\pm(z)$ do not involve sums over basic fields $J(z)$. They are given simply by

\[
E^-(z) = e^{X^1(z)}D_w e^{Y^1(w)},
E^+(z) = e^{X^2(z)}D_w e^{Y^2(w)},
\psi^\pm(zq^{\pm k/2}) = e^{a^1_{\pm}(zq^{\pm (k+2)/2}) + b^1_{\pm}(zq^{\pm k}) + b^1_{\pm}(zq^{\pm (k+2)})},
\]

where

\[
X^1(z) = -a^1(z, k/2) - b^1(k + 1|z, 1) - c^1(k|z, 0),
X^2(z) = -b^1(z, -1),
Y^1(z) = a^1(z, (k + 2)/2) + b^1(k + 2|z, 1) + c^1(k + 1|z, 0),
Y^2(z) = -c^1(z, 0).
\]

Here the corresponding Heisenberg algebras are defined by

\[
[a^0_n, a^1_m] = \frac{1}{n} [n(k + 2)][2n] \delta^{n+m,0}, \quad [a^1_0, a^1] = 2(k + 2),
[b^1_n, b^1_m] = -\frac{1}{n} [n]^2 \delta^{n+m,0}, \quad [b^1_0, b^1] = -1,
[c^1_n, c^1_m] = \frac{1}{n} [n]^2 \delta^{n+m,0}, \quad [c^1_0, c^1] = 1.
\]

All the other commutation relations are trivial. Since each of the currents $E^\pm(z)$ contains only one term of the form $e^{X(z)}D_w e^{Y(w)}$, it is possible to extract the two single exponential screening currents $S^1(z) = e^{\beta^1 Y^1(w)}$ and $S^2(z) = e^{\beta^2 Y^2(w)}$ from them. No screening current with an infinite number of exponential terms is present. Furthermore, for $U_q(\widehat{sl}(2))$, the analogue of our Table 1 leads to $\beta^1 = 1$ and $\beta^2 = -1$.

Recall that $U_q(\widehat{sl}(3))$ has the property that one of its step currents is expressed as a single term of the form $J(z)$ defined by 2.9 and the others are the sum of two such terms. Thus our method can in principle be extended to other quantum affine algebras who’s step currents
are finite linear combinations of terms of the form $J^i(z)$. This structure of currents is present for $U_q(\widehat{sl(n)})$ (the only quantum affine algebra for which a Wakimoto realization in terms of deformed bosonic fields currently exists [19]), and very plausibly for general quantum affine algebras. Most probably, the only problem with extending our technique will be the complexity of the Wakimoto realizations of the other quantum affine algebras. For example the Wakimoto realization of $U_q(\widehat{sl(n)})$, $n > 3$, requires $n^2 - 1$ free deformed bosonic fields $a^i, i = 1, \ldots, n - 1$; $b^i, i = 1, \ldots, n(n - 1)/2$ and $c^i, i = 1, \ldots, n(n - 1)/2$. In addition some of its step currents are sums of several basic currents $J^i(z), i = 1, \ldots, m$. This means that corresponding screening currents might be expressed as $m - 1$ infinite sums of exponential terms. The maximum value of $m$ is $n - 1$ and the corresponding algebras of the above bosonic fields are still given by 2.8 with 3 being replaced by $n$ (see Ref. [19] for more details). Based on our results for both $U_q(\widehat{sl(2)})$ and $U_q(\widehat{sl(3)})$, we conjecture that those screening currents of $U_q(\widehat{sl(n)})$ expressible as single exponentials are given by $S^i(z) = e^{c^i(z,0)}, i = 1, \ldots, n(n - 1)/2$ and $S(z) = e^{a^{n-1}(z,(k+n)/2)+b^{n-1}(k+n|z,1)+c^{n-1}(k+n-1|z,0)}$. Here the bosonic fields $a^{n-1}, b^{n-1}, c^{n-1}$ and $c^i, i = 1, \ldots, n(n - 1)/2$ are identified respectively with $a^{n-1}, b^{n-1,n}, c^{n-1,n}$ and $c^i,j, 1 \leq i < j \leq n$ of Ref. [19].

5 Conclusions

To summarise, we have found five independent screening currents for $U_q(\widehat{sl(3)})$. Four of these are expressed as single exponential operators in terms of free deformed bosonic fields. They are given by the relations 3.21. The fifth is more complicated and is written as an infinite sum of exponential operators, each of which is given in terms of free deformed bosonic fields. As in the classical case, these are all the screening currents that it is possible to construct with our method. It is still an open problem to show their uniqueness. One application of the above screening currents in the case of both $U_q(\widehat{sl(2)})$ and $U_q(\widehat{sl(3)})$ is to the computation of the correlation functions of the higher spin versions of XXZ model and their $sl(3)$ generalisations.
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A The $U_q(\hat{g})$ Current Algebra

A.1 $U_q(\hat{g})$

The quantum affine algebra $U_q(\hat{g})$ [22, 20, 23], associated with a rank $r$ Lie Algebra $g$, is generated by the elements, $\{e_i^\pm, t_i, i = 0, \cdots, r\}$ such that

\[
[t_i, t_j] = 0, \\
t_i e_j^\pm t_i^{-1} = q^{±a_{ij}} e_j^\pm, \\
[e_i, f_j] = \delta_{ij} t_i^{-1} t_i - \frac{1-a_{ij}}{q-q^{-1}} \\
\sum_{r=0}^{1-a_{ij}} (-1)^r \frac{1-a_{ij}}{r} (e_i^\pm)^{1-a_{ij}} (e_j^\pm)^r = 0,
\]

where $a_{ij}$ is the extended Cartan matrix of the affine algebra $\hat{g}$, and $[\frac{n}{m}] = [n]/([m]![n-m]!)$, $[n]! = [n][n-1] \cdots [1]$ with $[n] = (q^n - q^{-n})/(q - q^{-1})$.

A.2 The Drinfeld Realization

Drinfeld has shown [21] that $U_q(\hat{g})$ is isomorphic to the algebra generated by $\{E_{n}^{±,i}, H_{n\neq 0}^{i}, K_i, \gamma; i = 0, \cdots, r; n \in \mathbb{Z}\}$ [23]:

\[
[K_i, H_n^i] = 0, \\
K_i E_{n}^{±,i} K_i^{-1} = q^{±a_{ij}} E_{n}^{±,j}, \\
[H_n^i, H_m^j] = \frac{1}{n} [q^{ij} n]_{q-q^{-1}} \delta_{n+m,0} \\
[H_n^i, E_{m}^{±,j}] = \pm \frac{1}{n} [q^{ij} n]_{q^{-1/2}} E_{n+m}^{±,j} \\
[E_{n}^{±,i}, E_{m}^{±,j}] = \frac{\delta_{ij}}{q-q^{-1}} (\gamma^{(n-m)/2} \psi_{+1,n+m} - \gamma^{-(n-m)/2} \psi_{-1,n+m}) \\
[E_{n+1}^{±,i}, E_{m}^{±,j}]_{q^{±a_{ij}}} + [E_{m+1}^{±,j}, E_{n}^{±,i}]_{q^{±a_{ij}}} = 0 \\
[E_{n}^{±,i}, E_{n}^{±,j}] = 0, \text{ for } a_{ij} = 0 \\
[E_{n}^{±,i}, [E_{m}^{±,i}, E_{m}^{±,j}]_{q^{±1}}]_{q^{±1}} + [E_{m}^{±,i}, [E_{n}^{±,i}, E_{m}^{±,j}]_{q^{±1}}]_{q^{±1}} = 0, \text{ for } a_{ij} = -1,
\]

where

\[
\sum_{\pm n \geq 0} \psi_{±,n} z^{-n} = K_i^{±1} \exp \left( \pm (q-q^{-1}) \sum_{\pm n > 0} H_n^i z^{-n} \right), \quad \text{(A.47)}
\]

and the q-commutator is defined by

\[
[X, Y]_{q^a} \equiv XY - q^a YX. \quad \text{(A.48)}
\]
A.3 The Quantum Current Algebra

For the purpose of bosonization, it is convenient to re-express \( A.47 \) as a current algebra \[19\] that is generated by the currents

\[
E^{\pm,i}(z) = \sum_{n \in \mathbb{Z}} E^{\pm,i}_n z^{-n-1}, \quad \psi^i_\pm(z) = \sum_{n \geq 0} \psi^i_{\pm,n} z^{-n},
\]

such that

\[
\left[ \psi^i_\pm(z), \psi^j_\pm(w) \right] = 0,
\]

\[
(z - q^{a_{ij}} \gamma^{-1} w)(z - q^{-a_{ij}} \gamma w) \psi^i_+(z) \psi^j_+(w) = (z - q^{a_{ij}} \gamma w)(z - q^{-a_{ij}} \gamma^{-1} w) \psi^j_+(w) \psi^i_+(z),
\]

\[
(z - q^{a_{ij}} \gamma^{\pm} w) \psi^i_+(z) E^{\pm,j}(w) = (q^{\pm a_{ij}} z - \gamma^{\pm \frac{1}{2}} w) E^{\pm,j}(w) \psi^i_+(z),
\]

\[
(z - q^{a_{ij}} \gamma^{\pm} w) E^{\pm,j}(z) \psi^i_+(w) = (q^{\pm a_{ij}} z - \gamma^{\pm \frac{1}{2}} w) \psi^i_+(w) E^{\pm,j}(z),
\]

\[
\left[ E^{+,i}(z), E^{-,j}(w) \right] = \frac{\delta^{ij}}{(q - q^{-1})z w} (\delta(w/\gamma z) \psi^i_+(\gamma^{\frac{1}{2}} w) - \delta(z w/\gamma) \psi^i_+(\gamma^{-\frac{1}{2}} w)),
\]

\[
(z - q^{a_{ij}} w) E^{+,i}(z) E^{\pm,j}(w) = (q^{a_{ij}} z - w) E^{\pm,j}(w) E^{+,i}(z),
\]

\[
E^{+,i}(z) E^{\pm,j}(w) = E^{\pm,j}(w) E^{+,i}(z), \quad a_{ij} = 0,
\]

\[
E^{+,i}(z_1) E^{+,i}(z_2) E^{\pm,j}(w) = (q + q^{-1}) E^{+,i}(z_1) E^{\pm,j}(w) E^{+,i}(z_2) + E^{+,i}(w) E^{+,i}(z_1) E^{+,i}(z_2) + (z_1 \leftrightarrow z_2) = 0, \quad \text{for } a_{ij} = -1,
\]

where \( \delta(z) = \sum_{n \in \mathbb{Z}} z^n \). For \( U_q(\widehat{sl}(3)) \) there are six currents,

\[
E^{+,i}(z), E^{-,i}(z), \psi^i_\pm(z), \quad i = 1, 2.
\]

The bosonization of these currents for arbitrary level and \( n \) is constructed in \[19\]. The version given in Section 2 for \( U_q(\widehat{sl}(3)) \) is in a slightly different form which is suitable for the purpose of constructing the screening currents.
B OPEs

In this section, we list the OPEs relevant to the discussion of section 3. The four tables refer to the OPEs required in the four stages of the argument presented in that section.

Table 1

\[
\begin{align*}
J^i_+(z)e^{Y^i(w)} & = \frac{q^{\pm 1}}{z-wq^{\mp 1}} : J^i_+(z)e^{Y^i(w)} : , \quad i = 1, 3; \\
J^2_+(z)e^{Y^2(w)} & = q^{k+2\pm 1}(z - wq^{\mp 1}) : J^2_+(z)e^{Y^2(w)} :, \\
J^4_+(z)e^{Y^4(w)} & = q^{-k-1\pm 1}(z - wq^{\mp 1}) : J^4_+(z)e^{Y^4(w)} :, \\
J^i_+(z)e^{Y^i(w)} & = q^{\pm 1}(z - wq^{\mp 1}) : J^i_+(z)e^{Y^i(w)} :, \quad i = 5, 7; \\
J^6_+(z)e^{Y^6(w)} & = q^{1\pm 1}(z - wq^{\mp 1}) : J^6_+(z)e^{Y^6(w)} :.
\end{align*}
\]

Table 2

\[
\begin{align*}
J^i_+(z)e^{\beta Y^j(w)} & = \text{regular, where } (i, j) \in \{(i, j); (i, j) \neq (2, 1); i = 1, \cdots, 7; j = 2, 3, 4, 7\}, \\
J^2_+(z)e^{Y^1(w)} & = \frac{1}{(z-wq)(z-wq^{-1})} : J^2_+(z)e^{Y^1(w)} :, \\
\psi^i_+(zq^{\pm k/2})e^{\beta Y^j(w)} & = \text{regular, } i = 1, 2 \quad j = 1, \ldots, 7.
\end{align*}
\]

Table 3

\[
\begin{align*}
J^1_+(z)e^{X^1(w)-X^2(w)} & = (z - w) : J^1_+(z)e^{X^1(w)-X^2(w)} :, \\
J^1_+(z)e^{Y^1(w)} & = \frac{q^{\pm 1}}{z-wq^{\mp 1}} : J^1_+(z)e^{Y^1(w)} :, \\
J^1_+(z)e^{Y^2(w)} & = : J^1_+(z)e^{Y^2(w)} :, \\
J^2_+(z)e^{X^1(w)-X^2(w)} & = q^{k+4}(z - wq^{2})(z - w)(z - wq^{-2}) : J^2_+(z)e^{X^1(w)-X^2(w)} :, \\
J^2_+(z)e^{Y^1(w)} & = \frac{1}{(z-wq)(z-wq^{-1})} : J^2_+(z)e^{Y^1(w)} :, \\
J^2_+(z)e^{Y^2(w)} & = q^{k+2\pm 1}(z - wq^{\mp 1}) : J^2_+(z)e^{Y^2(w)} :.
\end{align*}
\]
\[
J_3^3(z)e^{X^1(w)} = \frac{1}{z-w} : J_3^3(z)e^{X^1(w)} :
\]
\[
J_3^3(z)e^{X^2(w)} = q^{\pm(k+3)}(z - wq^{k+2\mp(k+3)}) : J_3^3(z)e^{X^2(w)} :
\]
\[
J_3^3(z)e^{Y^1(w)} = (z - w) : J_3^3(z)e^{Y^1(w)} :
\]
\[
J_3^3(z)e^{Y^2(w)} = \frac{q^{\pm(k+2)}}{z-wq^{k+2\mp(k+2)}} : J_3^3(z)e^{Y^2(w)} :
\]
\[
J_4^5(z)e^{X^1(w)} = q^{-1-k+1}(z-w^{1+k}) : J_4^5(z)e^{X^1(w)} :
\]
\[
J_4^5(z)e^{X^2(w)} = q^{-2} : J_4^5(z)e^{X^2(w)} :
\]
\[
J_4^5(z)e^{Y^1(w)} = q^{k+1}(z-w^{k+1}) : J_4^5(z)e^{Y^1(w)} :
\]
\[
J_4^5(z)e^{Y^2(w)} = : J_4^5(z)e^{Y^2(w)} :
\]
\[
J_5^5(z)e^{X^1(w)} = \frac{q^{\pm k}}{(z-wq^{\mp k})} : J_5^5(z)e^{X^1(w)} :
\]
\[
J_5^5(z)e^{X^2(w)} = : J_5^5(z)e^{X^2(w)} :
\]
\[
J_5^5(z)e^{Y^1(w)} = q^{\mp(k+1)}(z - w^{(k+1)}) : J_5^5(z)e^{Y^1(w)} :
\]
\[
J_5^5(z)e^{Y^2(w)} = : J_5^5(z)e^{Y^2(w)} :
\]
\[
J_6^6(z)e^{X^1(w)} = q^{1-k}(z - wq^{-k-1}) : J_6^6(z)e^{X^1(w)} :
\]
\[
J_6^6(z)e^{X^2(w)} = \frac{q^{-2}}{(z-wq^{-1})} : J_6^6(z)e^{X^2(w)} :
\]
\[
J_6^6(z)e^{Y^1(w)} = \frac{q^k}{z-wq^k} : J_6^6(z)e^{Y^1(w)} :
\]
\[
J_6^6(z)e^{Y^2(w)} = q^{1\pm 1}(z - wq^{k+1\mp 1}) : J_6^6(z)e^{Y^2(w)} :
\]
\[
J_7^7(z)e^{X^1(w)} = : J_7^7(z)e^{X^1(w)} :
\]
\[
J_7^7(z)e^{X^2(w)} = q^{\mp 1}(z - wq^{k+3}) : J_7^7(z)e^{X^2(w)} :
\]
\[
J_7^7(z)e^{Y^1(w)} = : J_7^7(z)e^{Y^1(w)} :
\]
\[
J_7^7(z)e^{Y^2(w)} = : J_7^7(z)e^{Y^2(w)} :
\]
\[
\psi_i^i(zq^{1+k/2})e^{\pm(X^1(w)-X^2(w))} = : \psi_i^i(zq^{1+k/2})e^{\pm(X^1(w)-X^2(w))} :: \ i = 1, 2;
\]
\[
\psi_i^i(zq^{1+k/2})e^{\pm Y^1(w)} = : \psi_i^i(zq^{1+k/2})e^{\pm Y^1(w)} :: \ i = 1, 2;
\]
\[
\psi_i^i(zq^{1+k/2})e^{\pm Y^2(w)} = : \psi_i^i(zq^{1+k/2})e^{\pm Y^2(w)} :: \ i = 1, 2.
\]
References

[1] D. Bernard and A. Leclair. *Comm. Math. Phys.*, 142:99, 1992.

[2] D. Bernard and A. Leclair. *Nucl. Phys.*, B399:709, 1993.

[3] B. Davies, O. Foda, M. Jimbo, T. Miwa, and A. Nakayashiki. *Comm. Math. Phys.*, 151:89, 1993.

[4] M. Jimbo, K. Miki, T. Miwa, and A. Nakayashiki. Correlation functions of the XXZ model for $\Delta < -1$, 1992. RIMS preprint.

[5] I. B. Frenkel and N. Yu Reshetikhin. *Comm. Math. Phys.*, 146:1, 1992.

[6] A. H. Bougourzi and Robert A. Weston. N-point correlation functions of the spin-1 XXZ model, 1993. Preprint CRM-1896, In Press Nucl. Phys. B.

[7] M. Idzumi. Correlation functions of the spin-1 analog of the XXZ model, 1993. RIMS preprint, RIMS-926; PhD Thesis.

[8] Y. Koyama. Staggered polarization, May 1993. RIMS preprint.

[9] M. Jimbo, T. Miwa, and A. Nakayashiki. *J. Phys. A: Math. Gen.*, 26:2199, 1993.

[10] O. Foda, M. Jimbo, T. Miwa, K. Miki, and A. Nakayashiki. Vertex operators of solvable lattice models, 1993. RIMS preprint.

[11] B. L. Feigin and E. V. Frenkel. *Comm. Math. Phys.*, 128:61, 1990.

[12] F. G. Malikov, B. L. Feigin, and D. B. Fuks. *Funct. Anal. Appl.*, 20:103, 1986.

[13] Vl. S. Dotsenko and V. A. Fateev. *Nucl. Phys.*, B240:312, 1984.

[14] Vl.S. Dotsenko. *Nucl. Phys.*, B358:547, 1991.

[15] D. Bernard and G. Felder. *Comm. Math. Phys.*, 127:145, 1990.

[16] A. H. Bougourzi, Q. Ho-Kim, Y. Kikuchi, and C. S. Lam. *Int. J. Mod. Phys.*, A6:4181, 1991.
[17] M. Wakimoto. *Comm. Math. Phys.*, 104:605, 1986.

[18] P. Bouwknegt, J. McCarthy, and K. Pilch. *Phys. Lett.*, 234B:297, 1990.

[19] H. Awata, S. Odake, and J. Shiraishi. Free boson realization of $u_q(sl_N)$, 1993. RIMS-924, YITP/K-1018 preprint.

[20] V. G. Drinfeld. *Soviet Math. Doklady*, 32:254, 1985.

[21] V. G. Drinfeld, 1986. Proc. ICM, Am. Math. Soc., Berkeley, CA.

[22] M. Jimbo. *Lett. Math. Phys.*, 10:63, 1985.

[23] V. G. Drinfeld. *Soviet Math. Doklady*, 36:212, 1988.