Connected Vehicle Data-Driven Fixed-Time Traffic Signal Control Considering Cyclic Time-Dependent Vehicle Arrivals Based on Cumulative Flow Diagram

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Abstract—Fixed-time control is a widely adopted and cost-effective method for signalized intersections. However, existing studies utilizing connected vehicle (CV) data have not effectively addressed fixed-time control due to their reliance on specific vehicle arrival assumptions. To overcome this limitation, this study presents a novel traffic control approach for fixed-time signalized intersections based on a cumulative flow diagram (CFD) framework. The proposed method comprises a CFD model and a multi-objective optimization model. The CFD model establishes analytical relationships between traffic flow operations and varying signal timing parameters, with intersection demand estimated using a novel weighted maximum likelihood estimation method. A multi-objective optimization model based on CFD is formulated to minimize exceeded queue dissipation time as the primary objective and average delay as the secondary objective, which is applicable under both undersaturated and oversaturated traffic conditions. Leveraging the data-driven nature of the CFD model, a specially designed bi-level particle swarm optimization-based algorithm is employed to determine optimal cycle length and offset if applicable) and green ratios separately. Evaluation results demonstrate that the proposed method outperforms Synchro, a conventional approach, in terms of average delay and queue under various traffic conditions. Moreover, the proposed method exhibits the capability to handle specialized scenarios involving spillbacks.

Index Terms—Connected vehicles, cumulative flow diagram model, signal optimization, time-dependent vehicle arrivals, arrival rate estimation, spillback.

I. INTRODUCTION

Signalized intersections are one of the most common types of bottlenecks in urban road networks; thus, traffic signal control plays an important role in improving urban traffic flow efficiency. Traffic signal control is commonly classified into three categories: fixed-time control, actuated control, and adaptive control. The latter two can adjust traffic signal timings based on real-time traffic flow information collected via infrastructure-based sensors [1]. However, the real-world applications of the two control methods are often constrained by the limited coverage, malfunction, and high maintenance cost of the sensors [2], [3]. In contrast, fixed-time signal control has proven to be a cost-effective alternative and is widely implemented at signalized intersections in many Asian countries, such as Japan, Korea, and China [4]. Therefore, it is important to improve the performance of fixed-time signal control using the available technologies [5], [6].

Existing fixed-time signal control optimization methods commonly consider traffic volume as basic input and rely on traditional delay models, such as the Webster delay model and the Highway Capacity Manual delay model, as well as specific assumptions on vehicle arrivals, such as uniform and random arrivals [7]. In real-world applications, the collection of traffic volumes incurs a significant cost, traffic volume data are not always available [8], vehicle arrival assumptions often deviate from reality, and the traditional delay models require careful validation and generate unreasonable results, particularly for near-saturated and oversaturated conditions [9]. The above limitations significantly hinder the practical application of existing fixed-time signal control optimization methods.

In the past decade, there has been a significant advancement in intelligent mobility and connected vehicle technologies, enabling access to massive amounts of high-resolution vehicle trajectory data from mobile sensors (smartphones, GPS-equipped devices). In general, vehicles equipped with mobile sensors can transmit their instantaneous positions and speeds with a high frequency (i.e., every 1–5 s), referred to as connected vehicles (CVs). Currently, many mobility service and data service companies can provide CV data without the need to deploy field infrastructures, e.g., Amap, Uber, and Wejo. In comparison with traditional fixed-location sensors, CV trajectory data can serve as a cost-effective data source for monitoring network-wide traffic flows, thus demonstrating the significant potential for facilitating the dynamic evaluation and optimization of traffic signals [10], [11].

Numerous studies have been devoted to applying CV trajectory data for the estimation of traffic flow parameters and
performance measures at signalized intersections, including queue length [12], [13], [14], [15], traffic volume [16], [17], [18], [19], and delay [20], [21]. These studies have reported promising results, demonstrating the considerable potential of CV trajectory data. Nevertheless, the methods proposed in these studies cannot be applied directly to signal control optimization because they do not provide analytical relationships between performance measures and varying signal timing parameters.

Some studies have focused on the optimization of fixed-time signal control based on CV trajectory data. Early studies mainly utilized the travel information provided by the CV trajectory data to optimize the offset for arterial coordination signal control [22], [23]. Yao et al. [24] proposed a STREAM model to optimize all signal timing parameters of arterial control, namely cycle length, green splits, and offset. This method first estimated the arrival rate between CVs, then achieved the evolution of the delay of CVs with signal timing parameters, and finally achieved arterial coordination control by minimizing the average delay of CVs. Aiming at network signal timing coordination, Yan et al. [25] proposed a network-level multiband signal coordination model. This study first used historical CV trajectories to identify major traffic streams in urban road networks and then optimized the progression bands for the identified major traffic streams.

Ma et al. [5] proposed a signal timing optimization model for isolated intersections. This study assumed that the vehicle arrival distribution is uniform, and CV trajectories during the cycle follow the same-ratio principles, which is the basis for the evolution of CV trajectories under varying signal timing parameters. However, vehicle arrivals are largely affected by signal control at the upstream intersections; thus, the assumption of uniform vehicle arrivals is ideal for practical applications. In addition, when varying the cycle length and green splits, the same-ratio principles of CVs can barely be satisfied because of varying vehicle arrivals during the cycle. In summary, the existing studies on fixed-time signal control at isolated intersections, either based on fixed-location detectors or CV trajectories, require specific assumptions on vehicle arrival or the penetration rate of CVs, which significantly constrains their real-world applications.

Meanwhile, a few studies have developed real-time signal control methods based on CV trajectory data [26], [27], [28], [29], [30], targeting future scenarios when CVs become popular. Nevertheless, because the information of regular vehicles needs to be estimated based on the information provided by CVs, these real-time control methods typically require a high penetration rate of CVs. Acceptable performance can only be achieved when the penetration rate of CVs exceeds 30% [5], [31]. Furthermore, the CV trajectory data are usually owned by companies such as Amap, Uber, and Wejo, whereas traffic signals are operated by traffic management sectors, and real-time collaboration between them still involves issues such as data security. Besides, it is expected to take years to be able to apply vehicle-to-infrastructure technology in a large-scale network [32]. Therefore, the implementation of CV-based real-time signal control is likely to be difficult even in the near future. In contrast, fixed-time signal control does not require real-time CV trajectory data and has great demand in the real world; therefore, it is worthwhile to develop fixed-time signal control optimization methods by utilizing offline CV trajectory data.

Therefore, this study proposes a CV trajectory-driven cumulative flow diagram (CFD) model for fixed-time signal timing optimization at isolated intersections. CFD represents a traffic flow profile that illustrates all vehicle accumulation and dissipation processes of a signalized intersection. In our previous research, we performed CFD estimation and prediction for a single phase based on CV trajectories for the initial and candidate signal timing plans, which are nonlinear and non-parametric [33]. In this study, a general linearized CFD model formulating CFD evolution under varying signal timing plans is first proposed, where the traffic demand of the intersection is estimated based on a novel statistical method, namely weighted maximum likelihood estimation (WMLE). This CFD model is CV trajectory data-driven and requires no prior assumptions regarding vehicle arrival patterns and the CV penetration rate. Then, based on the proposed CFD model, a multi-objective optimization model is proposed to optimize the signal timing plans for both undersaturated and oversaturated conditions. In the multi-objective optimization model, the primary objective is to ensure that the queues are clear before the yellow start time, and the secondary objective is to minimize the average delay of the intersection. To solve this multi-objective optimization problem, a bi-level particle swarm optimization (PSO)-based algorithm is designed to determine the optimal cycle length and its corresponding green splits.

The major contributions of this study are summarized as follows:

1) This study introduces a novel statistical method for estimating arrival rates at signalized intersections. Unlike traditional approaches that assume first-in-first-out vehicle arrivals, our method relaxes this assumption. Besides, by considering the number of connected vehicles (CVs), we demonstrate that estimation errors of individual phases can be mitigated, leading to a significant improvement in the overall accuracy of intersection estimation.

2) Building upon the framework proposed by [33], this study further advances the field by presenting a generalized, parameterized, and linearized CFD model. This model establishes analytical relationships between traffic flow operations and varying signal timing parameters, while accounting for the time-dependent nature of vehicle arrivals throughout the signal cycle. Notably, our model does not rely on any prior assumptions regarding vehicle arrival patterns, enhancing its versatility and applicability.

3) Leveraging the capabilities of the CFD model, we develop a hierarchical multi-objective signal timing optimization model applicable to both undersaturated and oversaturated traffic conditions. By considering the time-dependent vehicle arrivals during the cycle, our proposed CFD-based optimization model achieves additional signal control benefits by effectively optimizing the signal timing parameters, particularly the offset of the signal timing plan. Furthermore, we demonstrate the extensibility of our model by showcasing its ability to
handle specialized scenarios, such as accounting for the spillback of specific traffic flows.

4) Collectively, this study, in conjunction with the work conducted in [33], presents a comprehensive CFD-based framework for integrated traffic performance evaluation and traffic signal optimization based on CV data. This framework offers a more cost-effective approach to traffic operation management at signalized intersections compared to conventional approaches that rely on fixed-position detectors, such as Synchro. By leveraging CV data, our framework provides enhanced accuracy, flexibility, and efficiency in managing traffic flow at signalized intersections, thereby contributing to the advancement of intelligent transportation systems.

II. CUMULATIVE FLOW DIAGRAM MODEL

In our previous study [33], we presented a CV trajectory-driven CFD estimation and prediction framework specifically designed for a single phase at signalized intersections. This framework utilized non-parametric and non-linear methods to profile the CFD under undersaturated traffic conditions, showcasing its potential for signal control optimization through CFD prediction. However, the application of this framework for signal control optimization still faces several challenges:

1) Model parameterization and linearization: The previous framework employed a nonparametric and nonlinear approach, resulting in a high computational complexity that poses difficulties in solving the optimization problem. To overcome this challenge, it is necessary to introduce parameterization and linearization techniques to make the model more computationally tractable.

2) Generalized CFD model with a unified origin: The previous framework considered a single stream and used the corresponding red start time as the origin. However, this formulation is not directly applicable for signal optimization purposes. Thus, it becomes crucial to establish a unified origin for each stream and derive a generalized CFD model that encompasses all streams.

3) Consideration of oversaturated traffic conditions: The previous framework focused exclusively on unsaturated scenarios, neglecting the impact of initial queues and oversaturated traffic conditions. To make the framework more robust and applicable in real-world settings, it is essential to extend its capabilities to handle oversaturated traffic conditions and consider the influence of initial queues on signal control optimization.

In this section, we present an extension of the previous CFD framework by introducing a generalized, linearized, and parameterized CFD model capable of accommodating both undersaturated and oversaturated traffic conditions. It is important to note that, in this study, we adopt a common CFD profile for the entire analysis period, representing the traffic flow pattern. This approach aligns with conventional fixed-time signal timing optimization models, assuming that the vehicle arrivals remain consistent across all cycles. While this assumption may not fully reflect the traffic arrival variability observed in practice, it serves as a rigorous model formulation. Moreover, the significance of this study lies in the consideration of detailed time-dependent vehicle arrivals throughout the signal cycle, enhancing the accuracy and realism of the proposed framework.

The basic idea of the CFD model is as follows: First, by approximating the CV arrivals based on a piecewise constant function, the cumulative arrival curve $S_a$ is modeled as a piecewise linear function associated with the cycle length $C$ and changes in the offset $\Delta \phi$ of the analysis period; then, the queue discharging wave speed is fitted by Huber regression and the cumulative departure curve $S_d$ is modeled as a linear function of the green phase parameters, that is, effective green start time $t_{egs}$ and end time $t_{egc}$; eventually, the CFD model is obtained based on $S_a$ and $S_d$, and several performance measures, such as volume, queue length, total delay, and average delay, can be further derived. The flowchart of the proposed CFD model is shown in Fig. 1.

The following assumptions are required: 1) Vehicle arrivals during the cycle are time-dependent while vehicle arrivals at different cycles within the same TOD can be represented by a common cycle. This is a commonly used assumption for fixed-time signal timing optimization. 2) The upstream signal timing as well as vehicle inputs are constant before and after signal timing optimization of the study intersection. This assumption defines this study as targeting isolated intersections. 3) The departure process of queued vehicles at the stopline is homogeneous, which suggests that we can fit the cumulative departure curve with a straight line.

A. Cumulative Arrival Curve

In this section, the cumulative arrival curve is modeled as a function associated with two variables to be optimized: cycle length $C$ and changes in the offset $\Delta \phi$. Note that, the offset here indicates the start time of the first cycle of the studied isolated intersection relative to the system reference point.
Step 3: approximating \( \alpha (t) \) with a piecewise function

Nevertheless, the statistical distribution is discontinuous and nonlinear. Therefore, we use a piecewise constant function to approximate it, as shown in Fig. 2(c). Note that when the penetration rates are low (e.g., no more than 10%), very few CVs would be observed during the analysis period for those phases with low volume. This makes \( \alpha_h (t) \) sparser and more challenging to fit directly. Thus, under such conditions, the application of a kernel density estimator is first recommended to smooth and fit the data [33], [34], and then the use of a piecewise constant function is recommended to approximate the KDE-fitted distribution. The piecewise constant function \( \alpha (t) \) can be estimated by solving the following optimization problem to minimize the difference between \( \alpha (t) \) and \( \alpha_h (t) \),

\[
P1 : \min_{\alpha_m, \tau_m} \sum_{m=1}^{M} \left[ \sum_{t=\tau_{m-1}}^{\tau_m} (\alpha_h (t) - \alpha_m)^2 \right]
\]

where \( \tau_0 = 0 \) and \( \tau_M \) is equal to the cycle length \( C \); \( M \) is the total number of segments of the piecewise constant function, whose optimal value is discussed in evaluation; and \( \alpha_m \) is the value of \( \alpha (t) \) during the period \( [\tau_{m-1}, \tau_m) \).

The key to solving P1 is to determine the best segmentation points, that is, \( \tau_m, m = 1, 2, \ldots, M-1 \). Then, the corresponding value \( \alpha_m \) for periods \( [\tau_{m-1}, \tau_m) \) \( (m = 1, 2, \ldots, M) \) can be easily determined by minimizing the sum of squared errors (SSE). However, it is computationally infeasible to consider every possible combination of the segmentation points. Therefore, a tree-based method, the regression tree, is adopted to solve the optimal segmentation points [35]. The segmentation process is achieved by recursive binary segmentation, which is a top-down greedy approach. In our case, this approach proceeds as follows:

1. First, given \( \hat{\alpha}_h (t) \), we search for an optimal segmentation time point \( st_1 \) that minimizes the overall SSE as

\[
st_1 = \arg \min_{t_s} \sum_{t=0}^{t_1} (\hat{\alpha}_h (t) - \bar{\alpha}_1)^2 + \sum_{t=t+1}^{C} (\hat{\alpha}_h (t) - \bar{\alpha}_2)^2
\]

where \( \bar{\alpha}_1 \) and \( \bar{\alpha}_2 \) are the average of two parts, respectively.

2. Then, we repeat this process on the part with the largest SSE until the stopping criterion associated with the overall SSE differences is reached or \( M - 1 \) points are obtained.

3. Eventually, after we partition \( \hat{\alpha}_h (t) \) into \( M \) parts in terms of time, the estimator of each part is determined by minimizing the SSE.

Therefore, a piecewise constant function of \( \alpha (t) \) in the general form can be obtained as follows,

\[
\alpha (t) = \begin{cases} 
\alpha_1, & 0 \leq t < \tau_1 \\
\alpha_2, & \tau_1 \leq t < \tau_2 \\
\vdots & \\
\alpha_M, & \tau_{M-1} \leq t < \tau_M 
\end{cases}
\]

Since the integral of \( \lambda (t) \) over the time of the cycle should be equal to the traffic volume of the cycle, i.e., \( \lambda_0 C \), we have

\[
\int_0^C \lambda (t) dt = \int_0^C \lambda_0 \alpha (t) dt = \lambda_0 C,
\]

then we can obtain

\[
\int_0^C \alpha (t) dt = C, \quad \text{i.e.,} \quad \sum_{m=1}^{M} [\alpha_m (\tau_m - \tau_{m-1})] = C.
\]
2) Mapping Relation Between Arrival Rate and Wave Speed: In Tan et al. [33], the time mapping relation between the instantaneous wave speed and arrival rate was derived, as presented in Eq. (6). For a specific moment \( t \) in the cycle, whose instantaneous arrival rate is \( \dot{\lambda} (t) \), its corresponding wave speed should be \( w (t') \) at moment \( t' \). The detailed derivation process can be found in Tan et al. [33].

\[
w (t') = \frac{v^2}{v - d_0 \dot{\lambda} (t)} - v, \quad 0 \leq t' \leq C \tag{6}
\]

where \( d_0 \) denotes the vehicle jam spacing; \( v \) denotes the free-flow speed; \( t' = t (1 - d_0 \dot{\lambda} (t) / v) \); \( t' \in [0, C (1 - d_0 \dot{\lambda}_0 / v)] \); and \( \dot{\lambda}_i = \dot{\lambda}_0 \int_0^1 \dot{\alpha} (s) \, ds / t \).

Thus, given the piecewise constant \( \alpha (t) \), the wave speed is also a piecewise constant function,

\[
w (t) = \begin{cases} \frac{v d_0 \dot{\lambda}_0 \alpha_1}{v - d_0 \dot{\lambda}_0 \alpha_1}, & \tau_0' \leq t < \tau_1' \\
\frac{v d_0 \dot{\lambda}_0 \alpha_2}{v - d_0 \dot{\lambda}_0 \alpha_2}, & \tau_1' \leq t < \tau_2' \\
\vdots & \\
\frac{v d_0 \dot{\lambda}_0 \alpha_M}{v - d_0 \dot{\lambda}_0 \alpha_M}, & \tau_{M-1}' \leq t < \tau_M' \end{cases}
\]

where \( \tau'_m = \tau_m - d_0 \dot{\lambda}_0 \sum_{e=1}^m [\alpha_e (\tau_e - \tau_{e-1})] / v, m = 1, 2, \ldots, M \).

3) Piecewise Linear Cumulative Arrival Curve: Finally, as shown in Fig. 3, by integrating the wave speed in terms of time, a piecewise linear cumulative arrival curve \( S_a \) of vehicles can be acquired as

\[
S_a (t) = w_m t - w_m \tau_{m-1}' + \sum_{e=1}^{m-1} w_e (\tau_e' - \tau_{e-1}'), \quad \tau_{m-1}' \leq t < \tau_m', \quad m = 1, 2, \ldots, M \tag{8}
\]

In particular, \( m - 1 \) should be no less than \( 1 \) for \( \sum_{e=1}^{m-1} X \); however, here we assume that when \( m - 1 = 0 \), we have \( \sum_{e=1}^0 X = 0 \) for any \( X \). Thus, when \( m = 1 \), Eq. (8) still holds.

**B. Cumulative Departure Curve**

To model the cumulative departure curve under candidate signal plans, we must first estimate the queue discharging wave speed, that is, the slope of the cumulative departure curve \( w^m \), under the initial signal timing plan. Tan et al. [33] proposed a robust regression method, that is, Huber regression [36], for queue discharging wave speed estimation. The evaluation results indicate that Huber regression is robust against outliers (such as abnormal discharging points deviating significantly from the majority and those caused by other traffic events near an intersection) and can significantly improve the accuracy of CFD estimation. Similar to Tan et al. [33], the Huber regression is also used to estimate \( w^m \) under the initial signal timing plan, and we can obtain the following:

\[
S_d^0 (t) = w^m t + \xi^0 \tag{9}
\]

where \( S_d^0 \) represents the cumulative departure curve under the initial signal timing plan; \( w^m \) denotes the queue discharging wave speed; and \( \xi^0 \) is a constant value.

Notably, for a specific cycle, the green start time of the subsequent phase is determined by the green time of the previous phase. In other words, if we consider the red start time of each phase as the origin of the corresponding cumulative arrival curve, the cycle origin of each phase differs. Then, for each feasible solution of cycle length and green splits, \( \alpha (t) \) must be regenerated for all phases, which significantly increases the computational burden and makes the signal timing optimization problem challenging to solve. Therefore, we set the green start time of the first phase in the cycle as the common origin for all phases. In this manner, given a feasible \( C \), the \( \alpha (t) \) for all phases only needs to be generated once. Thus, the cumulative departure curve \( S_d \) for any studied phase can be modeled as a function of the corresponding effective green start time \( t_{egs} \) and end time \( t_{edge} \).

As depicted in Fig. 4, considering the general case in which the green start time of the studied phase is not the origin of the cycle, the red time is divided into two parts by the green time, that is, \( R_a \) (or \( C - t_{egs} \)) and \( R_b \) (or \( t_{edge} \)). Notably, the cumulative departure curve is not expected to start from \( t_{egs} \) because the queue \( Q_a \) accumulated during \( R_a \) dissipates early in the green time.

\[
Q_a = S_a (C) - S_a (t_{egs})
\]

\[
= d_0 \dot{\lambda}_0 C - d_0 \left[ \lambda_0 \sum_{e=1}^s \alpha_e (\tau_e - \tau_{e-1}) + \lambda_0 \alpha_{s+1} (t_{edge} - \tau_s) \right],
\]

\[
\tau_s \leq t_{edge} < \tau_{s+1}, s = 0, 1, \ldots, M - 1 \tag{10}
\]

where \( \lambda_0 C \) denotes the total volume of the cycle, and the terms within square brackets represent the number of vehicles arriving before \( t_{edge} \).
The corresponding dissipation time of \( Q_a \) can be calculated as follows:

\[
T_a = Q_a \left( \frac{1}{\hat{w}_m} + \frac{1}{v_f} \right)
\]

where \( v_f \) denotes the average queue departure speed of queued vehicles and \( v_f \leq v \).

Thus, the cumulative departure curve can be determined as follows:

\[
S_d(t) = \hat{w}_m (t - t_{egs} - T_a)
\]

C. CFD Model

As both the cumulative arrival and departure curves were modeled as functions of the signal timing parameters, the CFD model was formulated. Based on this model, several critical performance measures for signal timing optimization can be derived as functions of the signal timing parameters.

As shown in Fig. 5, the queue length consists of two parts, that is, \( Q_a \) and \( Q_b \), where \( Q_b \) is determined by the point at which \( S_q(t) \) and \( S_d(t) \) intersect, as follows:

\[
t'_{boq} = \frac{\hat{w}_m (t_{egs} + T_a) - w_q (t'_{q-1} - 1) + S_q (t'_{q-1})}{\hat{w}_m - w_q}
\]

\[
Q_b = \frac{w_q (t_{egs} + T_a) - w_q (t'_{q-1} - 1) + S_q (t'_{q-1})}{1 - w_q / \hat{w}_m}
\]

where \( t'_{boq} \) is assumed to be located in \( [t'_{q-1}, t'_{q}] \), \( 1 \leq q \leq M \); \( w_q \) is the wave speed between \( [t'_{q-1}, t'_{q}] \); and \( S_q (t'_{q-1}) = \sum_{q=1}^{q-1} w_e (t'_{\epsilon} - t'_{\epsilon-1}) = d_0 \lambda_0 \sum_{\epsilon=1}^{q-1} a_\epsilon (t_e - t_{\epsilon-1}) \) as indicated by Eq. (8).

Note that the delay study in this refers to the stopped delay, which is the amount of time vehicles are actually stopped owing to the signal. The total delay consists of three parts, that is, \( A_1 \), \( A_2 \), and \( A_3 \), where \( A_1 + A_2 \) denotes the delay produced by vehicles in \( Q_a \), and \( A_3 \) denotes the delay produced by vehicles in \( Q_b \).

\[
A_1 = 0.5 \sum_{\epsilon=1+1}^{M} \left( \left( t_\epsilon - t_{\epsilon-1} \right) \left[ S_a (t_\epsilon) + S_a (t'_{\epsilon-1}) \right] - 2S_a (t_\epsilon) \right)
\]

\[
+ 0.5 (t'_{boq} - t'_{q-1}) \left[ S_a (t'_{q-1}) + Q_a \right]
\]

\[
A_2 = 0.5 Q_a \times \left( 2t_{egs} + T_a + \frac{Q_a}{v_f} - Q_a \right)
\]

\[
A_3 = 0.5 \sum_{\epsilon=1}^{q-1} \left( t'_\epsilon - t'_{\epsilon-1} \right) \left[ S_a (t'_\epsilon) + S_a (t'_{\epsilon-1}) \right]
\]

D. Parameter Estimation for CFD

In the CFD model, \( \lambda_0 \) is a critical parameter to be estimated because it reflects the traffic demands and, to some extent, determines the cycle length and green splits. Several studies have attempted to estimate the traffic volume based on CV trajectories [16], [17], [18], [19], [33]. However, these studies still have limitations: 1) Existing studies can only estimate the traffic volume for undersaturated conditions and are not applicable for oversaturated conditions. 2) Existing statistical approaches, such as those proposed by Zheng and Liu [19] and Yao et al. [18], commonly adopt the relative arrival time and queuing position of consecutive CVs for model construction and assume that the queue process of vehicles conforms to the first-in-first-out (FIFO) principle, which is not always true in the case of multiple lanes. Data-driven methods, such as the one proposed by Tang et al. [16], are typically computationally burdensome. 3) Existing studies estimate the traffic volume for each phase separately. Some phases may be overestimated, whereas others may be underestimated. This may lead to a mismatch between the green ratios and the traffic demands of these phases during optimization, reducing the benefits of the optimized signal plan.

To overcome the aforementioned limitations, in this study, we introduced a novel statistical method, namely WMLE along with redistribution, to estimate the intersection arrival rates under both undersaturated and oversaturated conditions. First, the WMLE is adopted to initially estimate the arrival rates for each phase. When constructing the weighted maximum likelihood function, all CVs are treated as independent observations of the arrival rate based on their arrival time and queuing positions during the cycle, which relaxes the assumption of FIFO. Then, the number of CVs is adopted to redistribute the estimates of all phases, which can neutralize the estimation errors and improve the accuracy of the estimates of all phases.

As indicated by Eq. (1), the vehicle arrivals during the cycle are modeled using \( \lambda_0 \alpha (t) \). Given the initial signal timing plan, \( \alpha (t) \) can be obtained using Eq. (5). Then, each queued CV can be treated as an independent observation of the arrival rate, as shown in Fig. 6. Thus, we have the probability density function of the \( i \)-th queued CV observed during the analysis period based on the time-dependent Poisson process [12], [15], [18], [19]:

\[
f \left( n_i, t'^{ec}_i | \lambda_0 \right) = \left[ \frac{\lambda_0 t'^{ec}_i \alpha (t) dt}{n_i!} \right] e^{-\lambda_0 t'^{ec}_i \alpha (t) dt}
\]

where \( n_i \) denotes the number of vehicles joining the queue before the \( i \)-th queued CV during the same cycle. For vehicles
queued more than once, only the earliest queuing positions of these CVs were considered.

Under constantly oversaturated conditions, the queue infinitely increases, making the proposed estimation method inapplicable. Under slightly or temporally oversaturated conditions, the overflow queue is relatively stable, although it still exists during the analysis period, and the proposed method is applicable. Under undersaturated conditions, \( n_i \) denotes the exact queuing position of the CVs, while for temporally oversaturated conditions, the average overflow queue should be excluded:

\[
n_i = \begin{cases} \frac{n^q_i}{n^q_i - n^{ov}_i}, & \text{undersaturated} \\ \frac{n^q_i}{n^q_i - n^{ov}_i}, & \text{oversaturated} \end{cases} \tag{22}
\]

where \( n^q_i \) denotes the queuing position of the i-th queued CV and \( n^{ov}_i \) denotes the average overflow queue of the analysis period. In this study, a queuing position is identified as an overflow queue position if more than 50% of the queued CVs observed at this position queue more than once, and \( n^{ov}_i \) represents the maximum overflow queue position.

Then, the WMLE [37] is adopted to formulate the weighted likelihood of \( N_q \) queued CVs as

\[
L (\lambda_0) = \prod_{i=1}^{N_q} f (n_i, t^{ec}_i | \lambda_0) {w_i}^k , \tag{23}
\]

where \( w_i \) denotes the weight of the i-th queued CV.

WMLE is proposed because, with outliers, the maximum likelihood estimation method may produce large errors. Besides, the confidence level of the samples may be different. Therefore, the WMLE method assigns weights to each sample, so as to achieve a robust estimation of parameters [37].

In our cases, a greater arrival time \( t^{ec}_i \) covers a longer cycle length, indicating a more reliable observation of the arrival rate. In addition, the measurement error of \( n_i \) has a low impact on the result with a greater \( t^{ec}_i \). Therefore, in our case, considering the time-dependent characteristics of vehicle arrivals during the cycle, we use \( \frac{\lambda_0}{t^{ec}_i} \alpha (t) dt \) as the weight of observations, and the log-likelihood can be written as

\[
\log L (\lambda_0) = \sum_{i=1}^{N_q} \int_0^{t^{ec}_i} \frac{\lambda_0}{t^{ec}_i} \alpha (t) dt \log f (n_i, t^{ec}_i | \lambda_0). \tag{24}
\]

By computing the derivative of this log-likelihood, \( \lambda_0 \) was initially estimated as

\[
\hat{\lambda}_0 = \frac{\sum_{i=1}^{N_q} n_i \int_0^{t^{ec}_i} \alpha (t) dt}{\sum_{i=1}^{N_q} \left( \frac{\lambda_0}{t^{ec}_i} \alpha (t) \right)^2} \tag{25}
\]

Existing studies have shown that the lower the penetration rate, the more random the distribution of CVs in the population, which may result in more errors and variances in the estimation of \( \lambda_0 \). Similarly, the proposed \( \lambda_0 \) estimation method also faces the same problem. To optimize the signal timing plan, we must estimate \( \lambda_0 \) for each flow. Among the estimated arrival rates for different flows, some may be overestimated, while others may be underestimated. This may lead to a mismatch between the optimized green ratios and true traffic demands, reducing the benefits of signal control. To reduce the impact of this mismatch problem, we use the number of CVs to redistribute the estimated \( \lambda_0 \) for each phase as follows:

\[
\tilde{\lambda}_0^k = \left( \frac{0.5 \lambda_0}{\frac{K}{\sum_{j=1}^{K} z^j \lambda_0} + 0.5 N_k} \right) \times \sum_{j=1}^{K} z^j \lambda_0^j \tag{26}
\]

where \( \lambda_0^k \) denotes the redistributed arrival rate used for later signal timing optimization; \( \hat{\lambda}_0^k \) is the arrival rate initially estimated by WMLE; \( z^k \) is the lane number of phase \( k \); and \( N_k \) denotes the observed number of CVs of phase \( k \); \( K \) is the total number of phases in which the queued CVs were observed during the analysis period; \( z^j \lambda_0^j \) indicates the total arrivals of \( K \) phases; \( \sum_{j=1}^{K} z^j \lambda_0^j \) indicates the proportion of the estimates of phase \( k \) to the total; \( \sum_{j=1}^{K} z^j \lambda_0^j \) indicates the proportion of the number of observed CVs of phase \( k \) to the total; items in brackets indicate that we give the estimates the same weight as the number of trajectories to modify the initial estimates.

In corner cases, some phases may only have non-queued CVs due to well coordination between adjacent intersections, then the proposed WMLE method is not applicable. Their average arrival rates can be estimated by the number of CVs as follows:

\[
\tilde{\lambda}_0^{k'} = \frac{1}{z^{k'}} \left( \frac{N^{k'}}{\sum_{j=1}^{K} N_j} \right) \times \sum_{j=1}^{K} z^j \lambda_0^j \tag{27}
\]

where \( \tilde{\lambda}_0^{k'} \) is the average arrival rate of phase \( k' \), which did not observe any queued CVs during the analysis period; \( N^{k'} \) is the number of (non-queued) CVs of phase \( k' \).

To sum up, as long as at least one phase has queue CVs, whose average arrival rate can be estimated by the WMLE method, we can still use Eq. (27) to obtain the estimates of the other phases without queued CVs. That is to say, our method fails only if all phases of the intersection have no queued CVs. Note that, in future real-world applications, the arrival rates estimated by other state-of-the-art methods or detected by other sensors can also be used as alternatives.

In addition to the average arrival rate, other basic parameters must be calibrated based on the observed CV trajectories. For each phase, the free-flow speed \( v_l \) for each phase is adopted as the 95-th percentage speed value of all trajectory points of the corresponding phases; the average queue departure speed \( v_l \) is adopted as the average speed value of the queued CVs when they travel through the stop line; and the saturated headway
Identify whether the phase is oversaturated, as follows:

\[ Q_{\text{over}}^k = \max \{ 0, Q_{\text{total}}^k - \left( \frac{t_{\text{egs}}^k - t_{\text{egs}}^0}{h_s^0} \right) d_0 \} \]  

If \( Q_{\text{over}}^k > 0 \), the phase \( k \) is oversaturated; otherwise, it is undersaturated.

The proposed CFD model indicates the average traffic flow profile during the analysis period. The case of \( Q_{\text{over}}^k > 0 \) implies that the average demand exceeds the capacity, and theoretically, the queue increases infinitely. Under oversaturated conditions, the present CFD model cannot adequately represent the periodic traffic flow profile. However, although \( Q_{\text{over}}^k \) does not represent a real overflow queue, it can still, to some extent, reflect the severity of the oversaturated condition of the phase. In contrast, when \( Q_{\text{over}}^k = 0 \), for the cases wherein the average demand is smaller than but close to the capacity, owing to the fluctuation of traffic demand, overflow queues may also exist for some cycles. Therefore, in this study, a more conservative criterion similar to \( Q_{\text{over}}^k \) was adopted to determine the saturation condition of the studied phase, as follows:

\[ G_{\text{exceed}}^k = \max \{ 0, \frac{Q_{\text{total}}^k h_s^k}{d_0} + t_{\text{egs}}^k - g_{\text{exceed}}^k \} \]  

where \( G_{\text{exceed}}^k \) is defined as the exceeded queue dissipation time representing the difference between the queue dissipation time \( Q_{\text{total}}^k h_s^k/d_0 + t_{\text{egs}}^k \) and the green end time \( g_{\text{exceed}}^k \). If \( G_{\text{exceed}}^k \) is greater than \( 0 \), the phase can dissipate before the green end time, and the phase is identified as an undersaturated condition; otherwise, it is oversaturated.

For oversaturated traffic conditions, we expect the optimized intersection to be in an undersaturated traffic condition with as small an average delay as possible. While for undersaturated traffic conditions, we expect to minimize the average delay at the intersection. Thus, a hierarchical multi-objective structure is proposed for the signal timing optimization of both undersaturated and oversaturated traffic conditions:

\[
\min_{C, g_e, g_f} f_{\text{green}} = \sum_{k=1}^{8} z_k G_{\text{exceed}}^k \tag{31a}
\]

\[
\min_{C, g_e, g_f} f_{\text{delay}} = \sum_{k=1}^{8} z_k D_{\text{total}}^k \tag{31b}
\]

The primary objective is to ensure that the queues are clear before the yellow start time, which, to some extent, buffers the traffic fluctuation; this also ensures that the intersection is undersaturated by satisfying Eq. (28). The secondary objective is to minimize the theoretical average delay derived by the CFD model, which is commonly used for undersaturated conditions. The total delay and total volume derived by the CFD model are both values per lane per cycle; thus, they are weighted by the number of lanes. When the intersection is oversaturated, the primary objective would drive the model to optimize the plan in the direction of undersaturated traffic conditions, and then minimize the average delay with the secondary objective. If the intersection is undersaturated, only the secondary objective would come into play.

Furthermore, we can scalarize the hierarchical multi-objective optimization problem into a single-objective optimization problem by weighting \( f_{\text{green}} \) with a sufficiently large constant \( \delta \), as follows:

\[
P_2 : \min_{C, g_e, g_f} f_{\text{hybrid}} = f_{\text{delay}} + \delta f_{\text{green}} \tag{32}
\]

In particular, for intersections with short link or short left-turn bay, they may suffer link spillback or left-turn spillback due to inappropriate traffic signal control. At this point we
need to specifically design our objective to prevent spillback, i.e., ensure that the theoretical queue length will not exceed a queue threshold.

Then we have the following variable to measure the difference between the queue threshold and the theoretical queue length by the CFD model,

$$ Q_{spill}^k = \max \{0, Q_{total}^k - \zeta L_{link}^k \} \quad (33) $$

where $L_{link}$ indicates the link length or left-turn bay length. Note that $\zeta$ is an empirical value related to the variance of the traffic arrivals. Then, our objective becomes,

$$ P3 : \min_{C, g_s, g_e} f'_\text{hybrid} = f_{delay} + \delta(f_{green} + f_{spill}) \quad (34) $$

where $f_{spill} = \sum_{k=1}^{8} Q_{spill}^k$.

B. Constraints

Based on the example phase structure in Fig. 7, the following structural constraints can be formulated as follows:

$$ C = \sum_{k=1}^{4} \left( g_e^k - g_s^k + y^k + r_a^k \right) \quad (35a) $$

$$ C = \sum_{k=5}^{8} \left( g_e^k - g_s^k + y^k + r_a^k \right) \quad (35b) $$

$$ g_s^1 - g_s^5 + g_s^7 = g_s^2 - g_s^6 - g_e^6 \quad (35c) $$

$$ y^k + r_a^k = g_s^{k+1} \quad k = 1, 2, 3, 5, 6, 7 \quad (35d) $$

$$ g_s^k - g_s^1 \geq C_{min} \quad k = 1, 2, \ldots, 8 \quad (35e) $$

$$ C_{max} \geq C \geq C_{min} \quad (35f) $$

Eq. (35a-d) confirms that the green phases should satisfy the structure of the dual ring, as shown in Fig. 7. In addition, the green time should be limited by a lower bound, which is determined by the time at which the pedestrian passes through the intersection when the pedestrian is considered, as shown in Eq. (35e). In general, the cycle length should be bounded within an appropriate range, as shown in Eq. (35f).

C. Solution

As mentioned earlier, for each feasible cycle length, $\alpha (t)$ needs to be regenerated based on CV trajectories by solving P1; thus, the cumulative arrival curve varies for different cycle lengths owing to its data-driven property. This implies that it is impossible to calculate the gradient of the objective function, and it is quite difficult to solve P2 using exact algorithms. Therefore, an advanced meta-heuristic, PSO algorithm, is tailored into a bi-level form to solve P2, which solves the optimal cycle length and green split separately, as shown in Fig. 9. The PSO algorithm can solve complex non-differentiable optimization problems, which makes no assumptions regarding the targeted problem and can efficiently search a large feasible region [24], [38].

In the bi-level PSO-based algorithm, the cycle length and green splits are optimized separately, which allows us to less repeat the process of fitting $\alpha (t)$ by solving P1, thereby effectively improving the computational efficiency of the solution algorithm.

IV. Evaluation

A. Test Scenarios

The performance of the proposed method was evaluated based on VISSIM simulation. To verify the effectiveness of the proposed method under both random arrival and platoon arrival patterns, a simulation model was built based on three consecutive intersections on Jinling Road, Changzhou City, China, as shown in Fig. 10(a). The subject intersection is in the middle, that is, the Jinling-Taigu intersection. The lane allocation and the initial plan of the subject intersection are also shown in Fig. 10(a), which are provided by the local traffic management center. In this case, the right-turning flow was not controlled by signals. To pursue more realistic scenarios, the truck ratio and turning ratio were collected by the license plate recognition data deployed at the stop lines of three intersections and processed to calibrate the simulation model. The simulation model was run for 9000 s, where the first 900 s was to warm up and the last 900 s was to ensure the integrity of vehicle trajectories. Thus, two hours of data from 900 to 8100 s in the simulation were used for evaluation.
To better demonstrate the advantages of the proposed method, signal timing plans generated by Synchro were also applied in VISSIM and used for comparison. The lost time for each phase was 6 s, consisting of 3 s of start-up lost time, 1 s of an unused portion of the yellow time, and 2 s of all red time. Four typical traffic conditions indicated by the demand to capacity (d/c) under the initial plan were tested in the simulation, as shown in Fig. 10(b) and (c), including medium (d/c = 0.6), high (d/c = 0.8), near-saturated (d/c = 1.0), and oversaturated (d/c = 1.2) conditions. The CVs were sampled from the population under different penetration rates, that is, 5–50%. Each penetration rate was randomly sampled 10 times with different sampling seeds to ensure result reliability. To better reflect the adaptability of the model to traffic flows, the time required for pedestrians to pass through the intersection was not considered in our cases; thus, the minimum green time was set as 5 s for all phases [7].

Fig. 10. Basic information of the simulation model.

To better demonstrate the advantages of the proposed method, signal timing plans generated by Synchro were also applied in VISSIM and used for comparison. The lost time for each phase was 6 s, consisting of 3 s of start-up lost time, 1 s of an unused portion of the yellow time, and 2 s of all red time. Four typical traffic conditions indicated by the demand to capacity (d/c) under the initial plan were tested in the simulation, as shown in Fig. 10(b) and (c), including medium (d/c = 0.6), high (d/c = 0.8), near-saturated (d/c = 1.0), and oversaturated (d/c = 1.2) conditions. The CVs were sampled from the population under different penetration rates, that is, 5–50%. Each penetration rate was randomly sampled 10 times with different sampling seeds to ensure result reliability. To better reflect the adaptability of the model to traffic flows, the time required for pedestrians to pass through the intersection was not considered in our cases; thus, the minimum green time was set as 5 s for all phases [7]. Notably, the input of Synchro is the ground truth volume output by VISSIM, whereas the input of the proposed method is the CV trajectories. All optimized plans are applied in VISSIM again, with all other settings maintained the same as those in the initial plan. The jam spacing \( d_0 \) is empirically set to 7 m in the evaluation.

B. Estimation of \( \lambda(t) \)

Before we apply the optimization model, the estimation of \( \lambda(t) \) must be evaluated, which includes the approximation of \( \alpha(t) \) and estimation of the average arrival rate \( \lambda_0 \).

1) Approximation of \( \alpha(t) \): \( \alpha(t) \) must be approximated based on CV trajectories by the regression tree method for each feasible cycle length, in which two critical parameters need to be predetermined, that is, the maximum number of segments and the minimum length of segments. This section aims to determine the most suitable combination of the two critical parameters.

We tested the approximation of \( \alpha(t) \) based on a CV dataset collected from the real world at the Fuzhong-Huanggang intersection in Shenzhen, China. This dataset was used for the CFD estimation by Tan et al. [33]. The approximation results of \( \alpha(t) \) for different combinations of parameters are shown in Fig. 11(a). By fixing the minimum lengths of segments, the SSE descends faster when the maximum number of segments is smaller. With an increase in the maximum number of segments, the SSE decreases while the descending rate becomes smaller until it remains unchanged. Thus, for different minimum lengths of segments, we can obtain the optimal maximum numbers of segments, which are 6, 9, and 18 when the minimum lengths of segments are 5 s, 10 s, and 15 s, respectively.

The approximation results of the three optimal combinations of the two parameters are shown in Fig. 11(b). As can be seen, with a smaller minimum length of segments, approximated \( \alpha(t) \) can better accommodate the distribution of CV arrivals. However, notably, because the CV trajectories are randomly sampled from the population, the arrival distribution of CVs may not perfectly represent the arrival distribution of the population, especially when the penetration rates are low. In addition, considering the impact of the upstream intersection, vehicle arrivals at downstream intersections typically exhibit bimodal characteristics. Therefore, adopting a small
minimum length of segments may introduce errors caused by the random sampling of CVs. In summary, the recommended and adopted combination of parameters for $\alpha(t)$ approximation in the proposed method is 9 for the maximum number of segments and 10 s for the minimum length of segments, which not only captures the characteristics of the CV arrival distribution well but is also less sensitive to fluctuations in the distribution.

2) Estimation of $\lambda_0$: The average arrival rate $\lambda_0$ is a critical input of the CFD; Tan et al. [33] found that its estimation accuracy determines the accuracy of the CFD estimation and prediction. In other words, it can determine the reliability of the proposed CFD model. In this section, the estimation of $\lambda_0$ for all phases at the studied intersection is evaluated under different d/c and penetration rates.

The mean absolute percentage error (MAPE) was adopted to evaluate the accuracy of the estimates:

$$MAPE = \frac{1}{8} \sum_{k=1}^{8} \left| \frac{x_k - \hat{x}_k}{x_k} \right|$$

where $x_k$ is the ground truth value of phase $k$ and $\hat{x}_k$ is the estimate of phase $k$.

The arrival rate estimation results are shown in Fig. 12. The line that indicates the performance after the distribution is obtained from Eq. (26), which uses the number of CVs to correct the initial estimates, that is, the line before distribution. Note that the redistribution process did not change the sum of the estimates, that is, the total volume of the intersection. As shown, for all values of d/c, the MAPEs of the two estimates decrease as the penetration rates increase, particularly when the penetration rate is less than 15%. The accuracy was significantly improved after redistribution based on the number of CVs. In addition, the error bar, that is, the standard deviation of 10 estimation errors, shows that the estimates become more stable after redistribution. When the penetration rates are as low as 5%, the average MAPE for the 10 experiments is no more than 12% for 0.6 d/c and no more than 8% for a higher d/c. When the penetration rates are 10%, which is close to the real-world condition, the average MAPE is no more than 8% for 0.6 d/c and approximately 6% for a higher d/c.

In Fig. 13, an example showing the estimation error of each phase is provided when d/c is 1.2, and the penetration rate is 10%. The percentage error of $\lambda_0$ estimation of the eight phases shows that before redistribution, the maximum overestimation reaches 22.5% and the minimum underestimation reaches $-12.5%$. After redistribution, the range from underestimation to overestimation is reduced by 10.6%, from $-7.9\%$ to $16.5\%$. The results show that the estimation errors of most phases (6/8) are reduced after redistribution, leading to a smaller $MAPE$ of the intersection on average. Thus, we can conclude that considering the number of CVs of phases for traffic volume estimation can significantly neutralize the estimation errors and improve overall accuracy.

C. Optimization Results

After obtaining the average arrival rates for all phases, the CFD model can be applied to optimize the signal timing plan. As mentioned previously, the proposed optimization model was solved using a bi-level PSO-based algorithm. The upper level compared the optimal solution of the lower-level optimization for all feasible cycle lengths, while the lower level adopted the PSO method to attain optimal green splits. All experiments were conducted on a laptop with a 1.8 GHz 8-core i7-8550U CPU and 16 GB RAM. The model required approximately 30 s to attain the optimal green splits for the PSO method. The calculation speed of PSO was determined by the swarm size and stopping criterion. In our cases, the swarm size was set to 120, and the stopping criterion was that the optimal result remained unchanged for more than 50 iterations. Normally, the algorithm stops after approximately 70 iterations.

1) Different d/c: First, the proposed optimization model was tested under different degrees of saturation. The penetration rate of the CVs, in this case, was 10%, which is close to real-world conditions. The improvement was used to evaluate the performance of the optimized plans compared with the initial plan.

$$IMP = \frac{x_{ini} - x_{opt}}{x_{ini}}$$

where $x_{ini}$ is the value of the performance measures under the initial plan, and $x_{opt}$ is the value under the optimized plans.

Fig. 14 shows the average delay of the initial plan, Synchro plan, and plan using the proposed model (denoted as the CFD plan). The results of the CFD plan were an average of 10 experiments. The error bars indicate that the proposed model can produce stable results for different CV samples. As shown, for all d/c, both the Synchro plan and the CFD plan can significantly improve the average delay compared with
the initial plan, and the CFD plan performs better than the Synchro plan. When the d/c is 0.6, the average delays of the Synchro plan and that of the CFD plan are almost the same; the improvement of the CFD plan reaches 59.50%, which is only 0.03% higher than that of the Synchro plan. When d/c is 1.0, the delay improvement of the CFD plan is 35.15%, which is the smallest among all d/c. However, the delay improvement of the Synchro plan is even 5.8% less than that of the CFD plan, which is the largest difference among all d/c. When the d/c is 0.8, the delay improvements of the CFD plan and the Synchro plan are 49.62% and 46.04%, respectively. While for oversaturated conditions, i.e., when d/c is equal to 1.2, the delay improvements of the CFD plan and the Synchro plan are 41.25% and 39.30%, respectively.

Some other commonly used performance measures output by VISSIM, including the average queue, stops, and throughput of the CFD plan and the Synchro plan are shown in Table I. Similar to the average delay, the improvement of the average queue of the CFD plan is also significantly greater than the Synchro plan for all d/c. As for stops, the Synchro plan performs better than the CFD plan for all d/c. Except for d/c = 0.8, the average number of stops of the CFD plan under other d/c increases compared with the initial plan. Note that the stops outputted by VISSIM also include stops caused by lane-changing behaviors, which are actually greater than stops caused by signal control. When the intersection is oversaturated with a d/c of 1.2, the throughputs of both the Synchro plan and CFD plan are increased.

The Synchro plan and the average CFD plan for the 10 experiments are shown in Fig. 15. The cycle length of the CFD plan is always smaller than that of the Synchro plan, and this difference is particularly significant at higher d/c. The green ratio shows that the CFD plan tends to allocate a greater green ratio to the second barrier in this case compared to the Synchro plan.

Fig. 16 shows a typical case of aggregated trajectories of a CFD plan and the Synchro plan when the d/c is 0.8 and the penetration rate is 10%. Trajectories in blue are vehicles that are not queued, in gray are vehicles queued once, and in red are vehicles queued more than once. The red and green points indicate the critical trajectory points when the vehicles joined the queue and left the queue, respectively. Vehicle arrivals of phase 1 are platoon arrivals affected by the upstream intersection, whereas vehicle arrivals in phase 8 are random arrivals. The cycle length of the CFD plan is 80 s, whereas that of the Synchro plan is 90 s.

As shown, owing to traffic demand fluctuations, the CFD plan and the Synchro plan both have a small proportion of vehicles queued twice. Besides, the cycle length of the CFD plan is smaller than that of the Synchro plan, and although the proportions of vehicles queued once and queued twice of phases 6 and 8 of the CFD plan are slightly greater than in the Synchro plan, the values of the average delay are still smaller. Notably, phase 1 of the CFD plan shows specific arrival patterns during the cycle because the 80 s of the
cycle length is exactly half of the upstream intersection. The proportions of vehicles queued once and queued twice of the CFD plan are smaller than those of vehicles queued once and queued twice of the Synchro plan, and so is the average delay. This means that phase 1 received extra benefits from its fixed arrival patterns during the cycle. This also indicates that we can further improve the performance of the signal control at urban intersections by optimizing the offset of the targeted intersection, which determines the arrival patterns during the cycle along with the cycle length.

2) **Different Penetration Rates:** In this section, we evaluate the performance of the proposed optimization model under different penetration rates, and d/c was set to 0.8. As shown in Fig. 17, the overall trends of the average delay, average queue, and stops of the CFD plan decrease with increasing penetration rates, despite some fluctuations due to the randomness of CVs among the 10 experiments. One exception is the throughput, which peaked at a 20% penetration rate and reached troughs at 5% and 40% penetration rates. Compared with the Synchro plan, the average delay and average queue of the CFD plan are significantly smaller. Nevertheless, the CFD plan performs slightly worse in terms of the stops and throughput. Notably, the variance of the average performance measures under different penetration rates was small. For instance, the average delay was 25.9 s/veh at the 50% penetration rate, while it only increased by 1.2 s/veh, i.e., 27.1 s/veh, at the 5% penetration rate.

The average CFD plans for the different penetration rates are shown in Fig. 18. The cycle lengths of the CFD plan under different penetration rates remained almost unchanged, at about 80 s, which is smaller than that of the Synchro, whose cycle length is 90 s. As is shown, the green splits of the CFD plan are also stable under different penetration rates.

3) **Considering the Offset:** A noteworthy advantage of the proposed CFD-based optimization model is that the dynamic vehicle arrivals during the cycle are modeled by the CFD model based on CV trajectories. Although for fixed-time signal timing optimization at isolated intersections, we cannot optimize the offset to make that the majority of vehicles in the major stream do not experience stops, as existing studies done in arterial or network coordination with common cycle length, we can still let the majority of vehicles at the studied isolated intersection experience less delay because we have reconstructed detail arrival distribution of vehicles during the cycle based on CFD model.

As indicated by Eq. (2), the evolution of vehicle arrivals during the cycle is associated with the cycle length and the offset of the signal timing plan of the analysis period. In the previous evaluation, the offset was fixed at 0, that is, \( \Delta \phi = 0 \), and only the cycle length \( C \) was optimized in the upper level of the solution algorithm. In this section, both the cycle length \( C \) and the offset \( \Delta \phi \) were optimized in the upper level of the solution algorithm.

An example experiment with 0.8 d/c and 10% penetration rate was conducted. Note that when d/c was 0.8, the cycle length of the Synchro plan was 90 s. The Synchro software can only optimize the “offset” if the cycle length is half or the same as the cycle length of upstream and downstream intersections (160 s). Thus, the Synchro plan, in this case, is generated with the cycle length fixed at 80 s, which was proved to be better than the case with a cycle length of 90 s by our previous results.

The performance measures of the optimal CFD plan without considering \( \Delta \phi \) (denoted as CFD0), CFD plan considering \( \Delta \phi \) (denoted as CFD75), Synchro plan without considering \( \Delta \phi \) (denoted as Syn0), and Synchro plan considering \( \Delta \phi \) (denoted as Syn60) are shown in Table II. After we considered \( \Delta \phi \) in the optimization, the average delay, average queue, and stops of both CFD75 and Synchro60 decreased. However, the optimal \( \Delta \phi \) values of the CFD plan and the Synchro plan were different, which were 75 s and 60 s, respectively. The average delay and queue of CFD75 were significantly smaller than those of Synchro60, while the stops were slightly greater, and the throughput was slightly smaller.

To further evaluate the effect of considering \( \Delta \phi \) during the optimization, the detailed plans of CFD0, CFD75, and Synchro60 with the same cycle length and their average delay in terms of phases are shown in Fig. 19; thus, the effect of the cycle length on the results can be excluded. As shown, the green splits of CFD0 and CFD75 were the same, and...
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Fig. 19. Comparison of average delay in terms of phases.

In our NEMA phase structure, vehicle arrivals of phases in barrier 1 (i.e., phases 1, 2, 5, and 6) are affected by the signal control of upstream intersections and thereby are platoon arrivals, while the other phases (phases 3, 4, 7, and 8) in barrier 2 are random arrivals. Therefore, the effect of considering $\Delta \varphi$ must be discussed separately.

For phases with random arrivals in barrier 2, changing $\Delta \varphi$ has a limited effect on their performance, as indicated by the average delay of CFD0 and CFD75 in Fig. 19(b). The average delays of the second barrier of CFD0 and CFD75 are 33.16 s/veh and 33.02 s/veh, respectively. However, the average delay of the second barrier of Synchro60 is 35.31 s/veh, which is much greater. This is mainly caused by the smaller green lengths of phases 4 and 7 of Synchro60. For phases with platoon arrivals in barrier 1, we can see that the average delay of CFD75 is 31.54 s/veh, which is obviously smaller than that of CFD0, whose average delay of barrier 1 is 32.78 s/veh. Although Syn60 allocates a longer green length to barrier 1 than CFD75, its average delay of barrier 1 is still slightly greater than that of CFD75.

Synchro60 was also similar. The green lengths of the four phases of Synchro60 are the same as those of CFD75, which are phases 1, 3, 6, and 8. The differences between the other phases were just 1 s.

In summary, we can conclude that, for urban intersections where vehicle arrivals are affected by the signal control of upstream intersections, the proposed CFD-based optimization model can obtain an almost optimal $\Delta \varphi$, which is much better than that obtained by Synchro plans.

4) Considering the Spillback: In this section, we aim to validate the effectiveness of the CFD-based method in addressing the issue of left-turn spillback at an intersection. We consider a scenario where there is a short left-turn bay spanning 70 meters on the west link. Consequently, we anticipate that the queue length for phase 3, which corresponds to the target left-turn movement, will not exceed 70 meters during the evaluation period. Thus, our objective focuses on optimizing P3.

Table III presents the performance of the CFD-based method with and without considering the spillback of phase 3.

To validate whether the optimized $\Delta \varphi$ by the CFD-based model is optimal, the average delay of the CFD plan under different $\Delta \varphi$ is output by VISSIM, as shown in Fig. 20, where the cycle length and green splits are fixed. The triangle indicates the corresponding optimal $\Delta \varphi$ output by VISSIM, while the square indicates the $\Delta \varphi$ optimized by the proposed CFD-based model or Synchro. As shown, the optimal $\Delta \varphi$ for the CFD plan is 70 s or 72 s, whereas the optimized $\Delta \varphi$ is 75 s, which is the second-best value. However, $\Delta \varphi$ optimized by Synchro (60 s) is far from the corresponding optimal value (71 s). In summary, we can conclude that, for urban intersections where vehicle arrivals are affected by the signal control of upstream intersections, the proposed CFD-based optimization model can obtain an almost optimal $\Delta \varphi$, which is much better than that obtained by Synchro plans.

In this particular case, we adopt a d/c of 0.8 and a penetration rate of 0.2. Given that we employ the mean arrival rate as an input, we set the value of $\zeta$ in Equation (33) as 0.5. Table III presents the performance of the CFD-based method with and without considering the spillback of phase 3.

The results demonstrate that the spillback issue of the target left-turn flow can be effectively mitigated when it is properly incorporated into the CFD-based optimization model. However, this improvement comes at the cost of reduced overall intersection performance. This trade-off arises because, in order to prevent the target flow from experiencing spillback, the model assigns it a higher proportion of green time or reduces the cycle length. Consequently, the performance of other traffic flows is compromised, resulting in increased overall delay at the intersection.

V. CONCLUSION AND FUTURE STUDIES

In this study, we proposed a CFD-based signal timing optimization method for fixed-time signal control at isolated intersections, which is driven by the CV trajectory and does not require prior vehicle arrival pattern assumptions. The CFD model profiles the vehicle arrival and departure processes under varying signal timing plans, in which the traffic demand is estimated based on the WMLE method. For signal timing optimization, a multi-objective optimization
model was proposed for both undersaturated and oversaturated traffic conditions. The primary objective was to minimize the exceeded queue dissipation time of the intersection, aiming to dissipate the queue before the yellow start time as much as possible, while the secondary objective was to minimize the average delay of the intersection. Considering the data-driven property of the CFD model, a bi-level PSO-based algorithm was proposed for the solution. Notably, a significant advantage of the proposed method is that the dynamic vehicle arrivals during the cycle are considered and generated based on CV trajectories, which enables our method to not only attain better cycle length and green splits than those of Synchro but also further optimize the offset of the signal timing plan, thereby achieving better performance.

The proposed method was evaluated based on simulation data and compared with Synchro, which uses the ground truth traffic volume as input. The results showed that the proposed method performed better than Synchro under varying traffic conditions in terms of the average delay and average queue length. As the penetration rates increased, the average delay, average queue length, and stops of the proposed method decreased. Even under a low penetration rate, i.e., 5%, the proposed method still outperformed Synchro in terms of the average delay and average queue length. When considering the offset in optimization, both the average delay, average queue length, and stops of the CFD plan were further decreased, which also outperformed the Synchro plan.

In particular, we found that: 1) for arrival rate estimation, considering the number of CVs of phases can significantly neutralize the estimation errors and improve the overall accuracy; 2) the proposed CFD-based optimization model can obtain a smaller and better cycle length than those of Synchro under varying traffic conditions in terms of the average delay and average queue length; 3) under different penetration rates, the optimal plan by the proposed method is quite stable; 4) because the dynamic vehicle arrivals during the cycle are considered in the CFD model, some phases of the CFD plan can receive additional benefits from the fixed arrival patterns; 5) by optimizing the offset, the performance of the proposed CFD-based optimization model can be further improved; 6) the proposed CFD-based optimization model can obtain an almost optimal offset, which is also better than that of the Synchro plan; and 7) the CFD model has the ability to further take into account the spillback of some of the flows at the cost of increased overall delays.

There are several possible directions for future research: 1) Currently, the proposed arrival rate estimation method simply uses the number of CVs to redistribute the estimates and achieve a rough estimation of those phases without queued CVs, which is straightforward but rough. In the future, we will extend the current WMLE method to multiple phases and integrate the quantity information of CVs as prior information, and thus, develop a joint estimation method for multi-phase traffic demands based on Bayesian theory. 2) Although the CFD model can identify the oversaturated condition, it cannot accurately represent the periodic traffic flow profile of this traffic condition. Thus, future works can focus on the CFD model for oversaturated conditions. 3) Presently, the CFD model can only represent the average traffic flow profile while the traffic demand fluctuations are not considered. Therefore, future studies may improve the CFD model and consider cycle-to-cycle traffic demand fluctuations. 4) Although the proposed method achieves offset optimization, it is still essentially a fixed-time optimization model for isolated intersections. If we can combine the signal timing parameters of adjacent intersections in the CFD model, it is promising to achieve arterial or network coordination based on CV data. However, regarding the data-driven nature of the CFD model, how simplifying the optimization model and solving it efficiently will be challenging.

**Credit Authorship Contribution Statement**

Chaopeng Tan: conceptualization, methodology, validation, writing-original draft, writing-review and editing. Yumin Cao: validation, writing-review and editing. Xuegang Ban: conceptualization, writing-review and editing, supervision. Keshuang Tang: conceptualization, funding acquisition, writing-review and editing, supervision.

**Appendix**

**Notation List**

**Notations of the CFD Model for the Studied Phase**

- $S_0$: Cumulative arrival curve;  
- $S_d$: Cumulative departure curve;  
- $t_{gs}$: The effective green start time, decision variable;  
- $t_{ge}$: The effective green end time, decision variable;  
- $C$: The cycle length, decision variable;  
- $Δφ$: The change of offset, decision variable;  
- $λ(t)$: Time-dependent arrival rate function during the cycle;  
- $λ̄_0$: Average arrival flow rate;  
- $α(t)$: Time-dependent scaling parameter of the arrival rate during the cycle;  
- $α_h(t)$: Statistical distribution of the expected cycle arrival time of CVs;  
- $M$: The total number of segments of $α(t)$;  
- $α_m$: The value of $α(t)$ during the period $[τ_m−1, τ_m]$;  
- $τ_m$: The $m$-th split point of $α(t)$;  
- $w(τ')$: Time dependent wave speed, $τ' = τ \left(1 - d_0 τ' / v \right)$;  
- $v$: The free-flow speed;  
- $d_0$: The vehicle jam spacing;  
- $λ̄_v$: The average arrival rate of the period $[0, t)$;  
- $τ'_m$: The $m$-th Turning point of the time dependent wave speed, $τ'_m = τ_m - d_0 λ̄_0 \sum_{ε=1}^{m} [α_ε (τ_ε - τ_{ε-1})] / v$;  
- $w_m$: The wave speed during the period $[τ_m−1, τ'_m]$;  
- $S_0^d$: The cumulative departure curve under the initial signal timing plan;
Notations of the Optimization Model

\( \hat{w}^n \) The queue discharging wave speed;
\( \xi^0 \) A constant value in \( s^0(t) \);
\( R_a \) The red time before the green time;
\( R_b \) The red time after the green;
\( Q_a \) The queue accumulated during \( R_a \);
\( T_d \) The dissipation time of \( Q_a \);
\( v_l \) The average queue leaving speed of queued vehicles;
\( Q_b \) The queue length accumulated during \( R_b \), which is the point where \( S_a(t) \) and \( S_d(t) \) intersect;
\( t_{beg} \) The time when \( S_a(t) \) and \( S_d(t) \) intersect;
\( A_1, A_2, A_3 \) Intermediate parameters for total delay calculation;
\( V_{total} \) Traffic volume;
\( Q_{total} \) Queue length;
\( D_{total} \) Total delay;

\( C \) Cycle length, decision variable;
\( g_s^k \) Green start time phase of \( k \), decision variable;
\( g_{egs}^k \) Effective green start time of phase \( k \), \( t_{egs}^k = g_s^k + t_{eg}^k \);
\( g_{ege}^k \) Effective green end time of phase \( k \), \( t_{ege}^k = g_{egs}^k + y^k - t_{e}^k \);
\( t_l^k \) Start-up lost time of phase \( k \);
\( t_c^k \) Clearance lost time of phase \( k \);
\( t_y^k \) Yellow time of phase \( k \);
\( t_r^k \) All red time of phase \( k \);
\( h^k \) Saturated headway of phase \( k \);
\( l^k \) Lane number of phase \( k \);
\( t_{min} \) Minimum green time of phase \( k \);
\( Q_{total}^k \) Queue length of phase \( k \), derived from CFD model;
\( Q_{over}^k \) Theoretical overflow of phase \( k \), derived from CFD model;
\( D_{total}^k \) The total delay of phase \( k \), derived from CFD model;
\( V_{total}^k \) The total volume of phase \( k \), derived from CFD model;
\( G_{exceed}^k \) Exceeded queue dissipation time of phase \( k \), which indicates whether the queue can dissipate before the green end time;
\( f_{green} \) Primary objective value, the sum of differences between queue dissipation time and green time minus start-up lost time of all phases;
\( f_{delay} \) Secondary objective value, the average delay of the intersection;
\( \delta \) Sufficiently large constant;
\( f_{hybrid} \) The single-objective value that combines primary and secondary objective;
\( f_{opt}^C \) The optimal objective value of the lower-level optimization problem;
\( C_{min} \) Minimum cycle length;
\( C_{max} \) Maximum cycle length;
\( g_s \) Vector of green phase start time;
\( g_e \) Vector of green phase end time;

References

[1] S. Lee, S. C. Wong, and P. Varaiya, “Group-based hierarchical adaptive traffic-signal control—Part I: Formulation,” Transp. Res. B, Methodol., vol. 105, pp. 1–18, Nov. 2017.
[2] X. (Jeff) Ban, P. Hao, and Z. Sun, “Real time queue length estimation for signalized intersections using travel times from mobile sensors,” Transp. Res. C, Emerg. Technol., vol. 19, no. 6, pp. 1133–1156, Dec. 2011.
[3] X. Zhan, R. Li, and S. V. Ukkusuri, “Lane-based real-time queue length estimation using license plate recognition data,” Transp. Res. C, Emerg. Technol., vol. 57, pp. 85–102, Aug. 2015.
[4] K. Tang, K. Li, M. Li, and D. Liu, “11–China,” Global Practices Road Traffic Signal Control, K. Tang et al., Eds. Amsterdam, The Netherlands: Elsevier, 2019, pp. 185–215.
[5] W. Ma, L. Wan, C. Yu, L. Zou, and J. Zheng, “Multi-objective optimization of traffic signals based on vehicle trajectory data at isolated intersections,” Transp. Res. C, Emerg. Technol., vol. 120, Nov. 2020, Art. no. 102821.
[6] Y. Yin, “Robust optimal traffic signal timing,” Transp. Res. B, Methodol., vol. 42, no. 10, pp. 911–924, Dec. 2008.
[7] Transportation Research Board, Highway Capacity Manual 2010, 5th ed., Transportation Research Board, National Research Council, Washington, DC, USA, 2010.
[8] X. Chen, Z. He, and L. Sun, “A Bayesian tensor decomposition approach for spatiotemporal traffic data imputation,” Transp. Res. C, Emerg. Technol., vol. 98, pp. 73–84, Jan. 2019.
[9] C. Cheng, Y. Du, L. Sun, and Y. Ji, “Review on theoretical delay estimation model for signalized intersections,” Transp. Res., vol. 36, no. 4, pp. 479–499, Jul. 2016.
[10] Q. Guo, L. Li, and X. J. Ban, “Urban traffic signal control with connected and automated vehicles: A survey,” Transp. Res. C, Emerg. Technol., vol. 101, pp. 313–334, Apr. 2019.
[11] K. Yang, S. I. Guler, and M. Menendez, “Isolated intersection control for various levels of vehicle technology: Conventional, connected, and automated vehicles,” Transp. Res. C, Emerg. Technol., vol. 72, pp. 109–129, Nov. 2016.
[12] C. Tan, J. Yao, K. Tang, and J. Sun, “Cycle-based queue length estimation for signalized intersections using sparse vehicle trajectory data,” IEEE Trans. Intell. Transp. Syst., vol. 22, no. 1, pp. 91–106, Jan. 2021.
[13] H. Zhang, H. X. Liu, P. Chen, G. Yu, and Y. Wang, “Cycle-based end of queue estimation at signalized intersections using low-penetration-rate vehicle trajectories,” IEEE Trans. Intell. Transp. Syst., vol. 21, no. 8, pp. 3257–3272, Aug. 2020.
[14] M. Ramezani and N. Geroliminis, “Queue profile estimation in congested urban networks with probe data,” Comput.-Aided Civil Infrastruct. Eng., vol. 30, no. 6, pp. 414–432, Jun. 2015.
[15] P. Hao, X. Ban, D. Guo, and Q. Ji, “Cycle-by-cycle intersection queue length distribution estimation using sample travel times,” Transp. Res. B, Methodol., vol. 68, pp. 185–204, Oct. 2014.
[16] K. Tang, C. Tan, Y. Cao, J. Yao, and J. Sun, “A tensor decomposition method for cycle-based traffic volume estimation using sampled vehicle trajectories,” Transp. Res. C, Emerg. Technol., vol. 118, Sep. 2020, Art. no. 102739.
[17] Y. Zhao, J. Zheng, W. Wang, X. Wang, Y. Meng, and H. X. Liu, “Estimation of queue lengths, probe vehicle penetration rates, and traffic volumes at signalized intersections using probe vehicle trajectories,” Transp. Res. Rec., J. Transp. Res. Board, vol. 2673, no. 1, pp. 660–670, Nov. 2019.
[18] J. Yao, F. Li, K. Tang, and S. Jian, “Sampled trajectory data-driven method of cycle-based volume estimation for signalized intersections by hybridizing shockwave theory and probability distribution,” IEEE Trans. Intell. Transp. Syst., vol. 21, no. 6, pp. 2615–2627, Jun. 2020.
[19] J. Zheng and H. X. Liu, “Estimating traffic volumes for signalized intersections using connected vehicle data,” Transp. Res. C, Emerg. Technol., vol. 79, pp. 347–362, Jun. 2017.
[20] X. Ban, R. Herring, P. Hao, and A. M. Bayen, “Delay pattern estimation for signalized intersections using sampled travel times,” Transp. Res. Rec., J. Transp. Res. Board, vol. 2130, no. 1, pp. 109–119, Jan. 2009.
[21] X. Liu, F. Lu, H. Zhang, and P. Qiu, “Intersection delay estimation from floating car data via principal curves: A case study on Beijing’s road network,” *Frontiers Earth Sci.*, vol. 7, no. 2, pp. 206–216, Jun. 2013.

[22] J. Zhang, Y. Cheng, S. He, and B. Ran, “Improving method of real-time offset tuning for arterial signal coordination using probe trajectory data,” *Adv. Mech. Eng.*, vol. 9, no. 1, Jan. 2017, Art. no. 168781401668335.

[23] C. M. Day et al., “Detector-free optimization of traffic signal offsets with connected vehicle data,” *Transp. Res. Rec.: J. Transp. Res. Board*, vol. 2620, no. 1, pp. 54–68, Jan. 2017.

[24] J. Yao, C. Tan, and K. Tang, “An optimization model for arterial coordination control based on sampled vehicle trajectories: The STREAM model,” *Transp. Res. C, Emerg. Technol.*, vol. 109, pp. 211–232, Dec. 2019.

[25] H. Yan, F. He, X. Lin, J. Yu, M. Li, and Y. Wang, “Network-level multiband signal coordination scheme based on vehicle trajectory data,” *Transp. Res. C, Emerg. Technol.*, vol. 107, pp. 266–286, Oct. 2019.

[26] J. Lee, B. Park, and I. Yun, “Cumulative travel-time responsive real-time intersection control algorithm in the connected vehicle environment,” *J. Transp. Eng.*, vol. 139, no. 10, pp. 1020–1029, Oct. 2013.

[27] W. Li and X. Ban, “Connected vehicles based traffic signal timing optimization,” *IEE Trans. Intell. Transp. Syst.*, vol. 20, no. 12, pp. 4354–4366, Dec. 2019.

[28] Y. Feng, K. L. Head, S. Khoshmaghan, and M. Zamanpour, “A real-time adaptive signal control in a connected vehicle environment,” *Transp. Res. C, Emerg. Technol.*, vol. 55, pp. 460–473, Jun. 2015.

[29] Z. Yao, Y. Jiang, B. Zhao, X. Luo, and B. Peng, “A dynamic optimization method for adaptive signal control in a connected vehicle environment,” *J. Intell. Transp. Syst.*, vol. 24, no. 2, pp. 184–200, Mar. 2020.

[30] X. Liang, S. I. Gurle, and Y. V. Gayah, “An equitable traffic signal control scheme at isolated signalized intersections using connected vehicle technology,” *Transp. Res. C, Emerg. Technol.*, vol. 110, pp. 81–97, Jan. 2020.

[31] S. M. A. B. A. Islam, A. Hajbabaie, and H. M. A. Aziz, “A real-time network-level traffic signal control methodology with partial connected vehicle information,” *Transp. Res. C, Emerg. Technol.*, vol. 121, Dec. 2020, Art. no. 102830.

[32] P. Bansal and K. M. Kockelman, “Forecasting Americans’ long-term adoption of connected and autonomous vehicle technologies,” *Transp. Res. A, Policy Pract.*, vol. 95, pp. 49–63, Jan. 2017.

[33] C. Tan, J. Yao, X. Ban, and K. Tang, “Cumulative flow diagram estimation and prediction based on sampled vehicle trajectories at signalized intersections,” *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 8, pp. 11325–11337, Aug. 2022.

[34] E. Parzen, “On Estimation of a Probability Density Function and Mode,” *Ann. Math. Statist.*, vol. 33, no. 3, pp. 1065–1076, 1962.

[35] L. Breiman, J. Friedman, C. J. Stone, and R. A. Olshen, *Classification and Regression Trees*. Boca Raton, FL, USA: CRC Press, 1984.

[36] P. J. Huber, “Robust estimation of a location parameter,” *Ann. Math. Stat.*, vol. 35, no. 1, pp. 73–101, 1964.

[37] E. S. Ahmed, A. I. Volodin, and A. A. Hussein, “Robust weighted likelihood estimation of exponential parameters,” *IEEE Trans. Rel.*, vol. 54, no. 3, pp. 389–395, Sep. 2005.

[38] J. Kennedy and R. Eberhart, “Particle swarm optimization,” in *Proc. Int. Conf. Neural Netw. (ICNN)*, vol. 4, Perth, WA, Australia, 1995, pp. 1942–1948, doi: 10.1109/ICNN.1995.488968.