Research on Improved Triangular Mesh Surface Subdivision Algorithm

Hua Ma, Yanan Zhu, Xinya Niu and Wenshuai Wang*

School of Mathematical Statistics, Ningxia University, Yinchuan, China
Email: wws@nxu.edu.cn

Abstract. Subdivision surfaces have been widely used in computer graphics. They are also commonly used to subdivide surfaces in 3D geological modeling. The classic triangular mesh subdivision algorithms are Loop subdivision algorithm, Sqrt3 subdivision algorithm and Butterfly subdivision algorithm. Based on the realization of the three algorithms, the paper displays that the improved Butterfly subdivision algorithm, compared with the Sqrt3 subdivision algorithm and the Loop subdivision algorithm, can alleviate the contradiction between data volume and smoothness. It can also achieve a better fitness, and can deal with the smooth subdivision of the model well in 3D geological modeling and other applications.

1. Preface
Tessellation is a mesh subdivision of a three-dimensional model based on mesh discrete surfaces. Its basic idea is to define the limit of a mesh sequence and take the surface shape of the sequence limit. That is to say, the subdivision rule is repeatedly applied, and new nodes are continuously inserted. When the limit is reached, the mesh converges on a smooth curve or surface. Subdivision technology has the characteristics of arbitrary topology, scalability, consistency of representation [1], and has advantages in 3D model building, animation production, industrial modeling and so on.

In the process of establishing a three-dimensional model, in order to characterize the surface details of the object, a large number of geometric grids are required. Because triangle meshes have flexibility in representing surfaces, they are widely used. The three subdivision algorithms selected in this paper are based on the triangle mesh subdivision rule. Through comparative analysis, it is found that the butterfly subdivision algorithm and the Loop subdivision algorithm grow faster than the Sqrt3 subdivision algorithm. After the number of divisions, the subdivision effect of Sqrt3 is inferior to the Butterfly and Loop algorithms in terms of tightness and smoothness.

This article focuses on the algorithms of triangular mesh tessellation. As we all know, the subdivision algorithm is divided into approximation subdivision method and interpolation subdivision method according to whether the limit curve surface has passed the initial control vertex [2]. The Butterfly subdivision algorithm is an interpolation subdivision algorithm, and the Loop and sqrt3 subdivision algorithms are approximation subdivision algorithms.

2. Butterfly Subdivision Algorithm
Butterfly subdivision algorithm (also known as butterfly algorithm), named for its subdivision template similar to a butterfly [3], and first proposed by Dyn, Gregory and Levin in 1990 [4]. The
Butterfly subdivision algorithm is an interpolation subdivision algorithm that defines the \( C^1 \) continuous surface on an arbitrary triangular mesh. It is also a face splitting algorithm. Unlike the approximation subdivision algorithm, it cannot generate a limit surface with a slice polynomial representation [5].

The generation of the new vertex position of the Butterfly subdivision algorithm depends on 8 vertices, which is called an 8-point template, as shown in figure 1.

\[ a = \frac{1}{2}, \quad b = \frac{1}{8} + 2w, \quad c = \frac{-1}{16} - w \]

where \( w \) is the tension coefficient, which is used to control the smoothness of the limit surface.

However, the limit faces are not continuous at the singular points, but are continuous on a regular triangular mesh. This contradicts the 8-point template of the Butterfly subdivision algorithm, so later, in order to achieve a better smoothing effect in the singular point, Zorin et al. improved the Butterfly subdivision algorithm and obtained an improved Butterfly subdivision algorithm [7]. In the improved Butterfly subdivision algorithm, the range of vertex numbers is expanded to process the singular points based on the original 8-point template, as shown in figure 2. The improved Butterfly subdivision algorithm achieves \( C^0 \) continuity at the singular point, and the other points are \( C^1 \) continuous.

\[ \left( \begin{array}{c} s_0 \end{array} \right) \]

Figure 1. 8-point template of the Butterfly subdivision algorithm.

(a) (b) (c)

Figure 2. Improved butterfly subdivision algorithm subdivision rules.
In the improved Butterfly subdivision algorithm, the new vertices generated have the following conditions:

1. When the number of adjacent points of the vertex at both ends of the edge is six, as shown in Figure 2a, the weights of the vertices at different positions are set as follows:
   \[ a = \frac{1}{2} - w, \quad b = \frac{1}{8} + 2w, \quad c = -\frac{1}{16} - w, \quad d = w \]  
   where \( w \) is the tension coefficient, which is used to control the smoothness of the limit surface.

2. When the vertex of one end of the side is a singular point, as shown in Figure 2b, the position singularity generated by the new vertex is related to the number of its neighbors. The vertex weights of different positions are set as follows (\( N \) represents the number of adjacent points):
   \[ \begin{align*}
   q &= \frac{3}{4}, \quad S_0 = \frac{5}{12}, \quad S_1 = S_2 = -\frac{1}{12} \quad \text{for } N = 3 \\
   q &= \frac{3}{4}, \quad S_0 = \frac{3}{8}, \quad S_1 = S_2 = 0, \quad S_3 = -\frac{1}{8} \quad \text{for } N = 4 \\
   q &= \frac{3}{4}, \quad S_1 = \frac{1 + \cos \frac{2\pi m}{N} + \cos \frac{4\pi m}{N}}{N} \quad \text{for } N \geq 5
   \end{align*} \]

When the vertices at both ends of the edge are singular points, the operation in equation (2) must be performed for each singular point to obtain two vertices. The new vertices are generated at the average of the two vertices obtained in the previous step.

When the edge is a boundary, as shown in Figure 2c, the numbers in the figure are the weights of different positions.

3. Classification and Comparison of Algorithms

There are many kinds of surface subdivision algorithms now. However, the most of the schemes classified based on four criteria:

- The splitting type of grid topology (face split or vertex split);
- The differences in the elements made up the original mesh (triangular or quadrilateral);
- Whether the scheme is approximating or interpolating (approximation subdivision or interpolation subdivision);
- Continuity of the limit surfaces for meshes (\( C^1, C^2 \) etc.)

The three algorithms mentioned in this paper are classified and compared according to the above four criteria and the relationship between the number of grids in the splitting process, and the results are shown in Table 1.

| Classify criteria | Subdivision scheme | The type of grid topology | The elements of original mesh | Approximating or interpolating | Continuity | Grid quantity relation in splitting |
|-------------------|--------------------|--------------------------|-------------------------------|-------------------------------|------------|----------------------------------|
|                    | Loop subdivision   | face split               | triangular                    | Approximation                 | \( C^2 \)  | 1-4                              |
|                    | Butterfly subdivision | face split               | triangular                    | Interpolation                 | \( C^1 \)  | 1-3                              |
|                    | Sqrt3 subdivision  | face split               | triangular                    | Interpolation                 | \( C^2 \)  | 1-3                              |

The Loop subdivision algorithm is a spline-based approximation triangle face splitting subdivision algorithm that converts the original mesh into a triangular mesh, so the Loop subdivision can be applied to the subdivision of the original mesh of any polygon. The surface generated by this algorithm is \( C^1 \) continuous except for the singularity. The original butterfly subdivision algorithm is
defined on any triangle mesh. However, the limit surface \( \mathbf{C}^1 \) continuous on the regular grid except the singular point of the valence \( k=3 \) or \( k>7 \) \[8\]. Different from the approximation algorithm based on spline curve, this algorithm does not produce piecewise polynomial surface in the limit. The butterfly subdivision algorithm is modified in Ref. \[9\] which improved butterfly subdivision algorithm can generate continuous surfaces on any triangle mesh (see Ref. \[8\]). With the advantage of approaching the limit surface gradually \[10\], Sqrt3 subdivision algorithm is applied to the normal triangulation of plane. In each subdivision step, the number of triangles increases by three times, and the number of subdivision patches increases less than other algorithms. It can approach the subdivision surface accurately in the face splitting subdivision.

4. Performance Display

We take points from the surface of the polyhedron and the mushroom, get the corresponding vertex coordinate data of each triangle mesh, and run the programs of the three algorithms in Matlab respectively. The effect of the subdivision is shown as follows.

Form the subdivision results as shown in figures 3-8, we can find that both of the results of the Loop and the Butterfly subdivision algorithm are significantly better than the Sqrt3 subdivision algorithm, and the surfaces of them are also smoother. Butterfly’s subdivision results are not smooth as Loop, but retain the characteristics of the original image.

**Figure 3.** Results of the polyhedron of the Loop subdivision algorithm

**Figure 4.** Results of the polyhedron of the Sqrt3 subdivision algorithm

**Figure 5.** Results of the polyhedron of the Butterfly subdivision algorithm

**Figure 6.** Results of the mushroom of the Loop subdivision algorithm.
Figure 7. Results of the mushroom of the Sqrt3 subdivision algorithm.

Figure 8. Results of the mushroom of the butterfly subdivision algorithm.

The Butterfly subdivision algorithm closely reproduces the shape of the solid figure, but the surface quality is poor. Because the original mesh is interpolated, there is a trade-off between surface quality and interpolation. The closer the surface is to the interpolated surface, the lower the surface quality.

5. Conclusions
In practical problems, the rough model is processed by subdivision algorithm, which can transform the low-resolution model into a smooth surface, and the surface can be redrawn into a mesh composed of triangles subdivided by high-precision surface. The extensive application of triangular mesh surface subdivision promotes the continuous optimization of various algorithms. The Butterfly algorithm saves storage space by controlling the increase of mesh number and improves the surface quality after subdivision to make the model surface closer to the entity.

This paper mainly studies several triangle surface subdivision algorithms such as Sqrt3 Subdivision Algorithm, Loop Subdivision Algorithm and Butterfly Subdivision Algorithm. It is found that Butterfly Subdivision Algorithm, compared with Sqrt3 Subdivision Algorithm and Loop Subdivision Algorithm, alleviated the contradiction between the amount of data and the degree of smoothness, and to a certain extent could achieve a better fitness, and could handle the smooth segmentation of models well in applications such as 3d geological modeling. However, compared with the Loop Subdivision Algorithm, the smoothness of its surface still has some shortcomings.

Acknowledgments
This work was supported by the Natural Science Foundation of Ningxia (Grant No. 2018AAC03057) and the National Innovation and Entrepreneurship Training Program for College Students.

References
[1] Zhang X H 2006 Application of triangular mesh subdivision technology in the three-dimensional geological model Science Technology and Engineering 6 (01) 84-85 (in Chinese).
[2] Lu L M 2017 Application of Charles loop subdivision algorithm in computer graphics Journal of Puer University 33 (03) 32-34 (in Chinese).
[3] Liu Y 2013 Region Decomposition and Adaptive Subdivision in Electromagnetic Computing (Zhejiang: Zhejiang University) (in Chinese).
[4] Wu Y C 2013 Research on Related Algorithms of Subdivision Surface Theory (Hefei: Hefei University of Technology) (in Chinese).
[5] Gong N G 2017 *Construction of Adaptive Subdivision Surfaces Based on Triangular Domain* (Xi’an: Northwest Agriculture and Forestry University) (in Chinese).

[6] Dong P, Pan M and Wu Z X 2008 A kind of new adaptive refinement scheme based on modified butterfly subdivision method and its application on 3D geological models *Geography and Geo-Information Science* **24** (06) 34-38 (in Chinese).

[7] Lin J H, Liu X D, Liu G R and Chen M L 2011 Virtual crop DFFD form modeling based on butterfly subdivision scheme *Journal of Changchun University of Technology (Natural Science Edition)* **32** (01) 42-45 (in Chinese).

[8] Zorin D N 1998 *Stationary Subdivision and Multiresolution Surface Representations* (California Institute of Technology).

[9] Zorin D, Schroder P and Sweldens W 1996 Interpolating subdivision for meshes with arbitrary topology *Computer Graphics Proceedings* pp 43-53.