Surface Topography Prediction Model for Free-form Surface Milling under a Dynamic System Response

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Abstract. At present, numerous studies on surface topography prediction models for plane workpieces have been performed at home and abroad. Given the complexity of curved surface models (especially free-form surfaces that cannot be expressed in analytic formulas), prediction models for the surface topography of free-form surfaces are rarely studied. This paper aims to establish and simulate a 3D surface topography model of ball-end milling for all types of curved surfaces, including simple surfaces and free-form surfaces. The dynamic factors influencing the surface topography, such as the spindle runout initial phase angle, runout amplitude, axial drift initial phase angle, and axial drift amplitude are considered in this model. Moreover, some processing parameters influencing the surface topography, such as cutter tooth number, machining inclination, radial cutting depth, feeding frequency and feed per tooth are also considered in this model. The simulation results can be used to optimize the milling process of the actual free-form surface to improve workpiece surface quality or to predict the surface topography of the given machining parameters.

1. Introduction
The surface topography of a workpiece refers to the residual height, texture and scar morphology that remains on the workpiece surface after machining [1]. Surface topography is generally characterized by roughness, waviness, shape error and frequency spectrum [2]. The surface topography of the workpiece directly affects its mechanical properties, such as friction, wear, contact stiffness, and assembly performance [3]. In the field of mechanical manufacturing, the virtual realization of machined surfaces occurs before actual processing, which can eliminate the test and error methods in advance to achieve ideal surface quality, reduce the finished cost and improve productivity [4, 5]. The size of the surface topography should be controlled in the effective area. Effectively predicting the surface topography of machined workpieces can optimize the actual machining process, improve the surface quality of machined workpieces or predict the surface topography of given machining parameters [6, 7].

Milling is widely used in the finishing of complex mold surfaces because these products have very high requirements for surface quality, so numerous studies have been performed in this regard. At present, scholars at home and abroad have performed some studies on the prediction of milling surface topography, and these studies can be divided into the following two categories.
1.1. Research on surface topography prediction based on end-mill milling

This research topic is represented by the following studies: based on the transformation matrix and vector algorithm in motion homogeneous coordinates, Dong and his co-author [8, 9] established the motion trajectory equation of cutter teeth in the process of end-mill peripheral milling and provided an algorithm to generate surface topography in peripheral milling. Wang et al. [10] presented a new model that utilized elliptical paths as cutting edge trajectories on 3D surface topography machined by peripheral milling. The cutter parallel axis offset and location angle were considered, which changed the location of the ellipse center and intersection point of the cutting edges. The effects of the cutter location position (CLP) geometric parameters, cutter parallel axis offset and curvature on the roughness were evaluated using numerical simulation. Both Arizmendi [11, 12] and Liu [13] considered the vibration of the cutting tool in the cutting process and proposed a model to predict the surface topography of peripheral milling.

1.2. Research on surface topography prediction based on ball-end milling

According to the definition of surface topography, surface topography was divided into two components of macroscopic shape error and microscopic surface roughness by Zhao et al. [14]. An integrated method that includes geometric modeling and neural networks was proposed to simulate and predict surface topography. Based on cutting dynamics, Yang [15] studied surface topography modeling and surface generation by constructing a geometric model and kinematic model of the cutting edge of a ball-end milling cutter. Based on the surface generation theory of generalized machining, the geometrical model and the milling motion model of ball-end milling cutters, Xu and his co-author [16] presented a surface topography model for a ball-end cutter, which took varying feedrate into full consideration. The affecting elements, including the surface's local geometry, the cutting edge shape, the path interval, and the relative motion of the tool-workpiece (especially the changes in the tool orientation and feedrate), were considered in this model. Lotfi et al. [17] developed a new milling profile prediction model for multi-axis ball-end milling based on the analysis of the contact area between the tool and workpiece based on the equation of the tool path and cutting edge relative to the real path of the workpiece. An analytical model for predicting the surface topography and roughness of five axis ball milling was first proposed by Ehsan Layegh K [18]. Irene et al. [19] established a model for predicting the shape and surface roughness of the ball milling process based on the geometric intersection between the tool and workpiece.

In these studies, the research objects were either horizontal planes or inclined planes. No research on curved surface topography has been performed to date. Due to the complexity of curved surface models (especially free-form surfaces that cannot be expressed by analysis formulas), limited research on prediction models of surface topography has been reported. Most of the surface topography prediction models ignored the dynamic effects of radial runout and axial drift of the machine spindle, which often resulted in a large deviation between the predicted surface topography and the actual situation.

In this paper, the milling of a free-form surface by a ball-end mill is taken as the research object. Dynamic factors influencing the spindle, such as the radial initial phase angle and amplitude of runout as well as the axial initial phase angle and amplitude of drift; geometric factors, such as the number of milling cutter teeth, machining inclination angle, radial cutting depth, feed times and feed rate per tooth; and machining parameters influencing surface topography are comprehensively considered. A 3D surface topography dynamic model suitable for all types of surfaces (including analytical surfaces and nonanalytical surfaces) is established.

2. Mathematical model of a 1 ball-end milling cutter

For workpiece processing, especially for curved surface parts, a ball-end milling cutter is the most ideal choice to ensure that the cutting edge of the milling cutter is tangent to the surface contour of the workpiece at the cutting point and to avoid accidental interference between the cutting edge and the working surface [20]. Ball-end milling cutters can be divided into two types according to the helix of the cutting edge: constant helix leads and constant helix angles. In this paper, the latter is used for the following research work.

When the cutting edge of the ball-end milling cutter is a plane edge, the tool coordinate $O_t –uvw$
is established, which is shown in Fig. 1, where $w$ represents the direction of the tool axis, and the rotation direction of the tool in the milling process is clockwise. Under the tool coordinate system, the coordinate expression of any point $P$ of the first cutting edge can be deduced according to formula (1) as follows:

\[
\begin{align*}
    u' &= R \cdot \sin(\alpha) \cdot \cos((\tan \gamma) \cdot \log \left(\cot \left(\frac{\epsilon_i}{2}\right)\right)) \\
    v' &= R \cdot \sin(\alpha) \cdot \sin((\tan \gamma) \cdot \log \left(\cot \left(\frac{\epsilon_i}{2}\right)\right)) \\
    w' &= R \cdot (1 - \cos(\alpha))
\end{align*}
\]

(1)

where $R$ is the radius of the ball-end milling cutter, $\alpha$ is the position angle of any point on the cutting edge of the tool, and $\gamma$ is the helix angle of the cutting edge.

The initial entrance angle of the $i$-th feed and the $j$-th cutting edge of the tool can be described by equation (2):

\[
\phi_{i,j} = \phi_{i,1} + 2 \cdot \pi \cdot (j - 1)/z_n
\]

(2)

where $z_n$ is the teeth number.

Under the tool coordinate system, the coordinates of any point $P$ of any cutting edge of the ball-end milling cutter can be deduced by substituting equation (2) into formula (1):

\[
\begin{align*}
    u &= R \cdot \sin(\alpha) \cdot \cos((\tan \gamma) \cdot \log \left(\cot \left(\frac{\epsilon_i}{2}\right)\right)) - 2 \cdot \pi \cdot (j - 1)/z_n \\
    v &= R \cdot \sin(\alpha) \cdot \sin((\tan \gamma) \cdot \log \left(\cot \left(\frac{\epsilon_i}{2}\right)\right)) - 2 \cdot \pi \cdot (j - 1)/z_n \\
    w &= R \cdot (1 - \cos(\alpha))
\end{align*}
\]

(3)

The rotation angle expression of tool rotation coordinate system $O'_t - u'v'w'$ relative to the tool coordinate $O_t - uvw$ can be expressed as follows:

\[
\theta_i = \phi_{i,1} - \omega \cdot t
\]

(4)

where $\omega$ is the angular speed of the milling cutter rotating with the spindle, and $t$ is the time from the $i$-th feed to the current time.

Therefore, at time $t$, the transformation matrix $T_1$ of any point $P$ of the cutting edge from the tool rotation coordinate system $O'_t - u'v'w'$ to the tool coordinate $O_t - uvw$ is denoted as follows:
In the milling process, the ball-end milling cutter performs interpolation motion. On the one hand, the ball-end milling cutter realizes linear or curved feeding motion along the feed direction; on the other hand, it realizes rotating motion around its own axis at the same time. Therefore, the coordinate expression of any point $P$ of any cutting edge in the coordinate system $O_t = uvw$ during tool motion is denoted equation (6):

$$\begin{bmatrix}
    u'' \\
    v'' \\
    w''
\end{bmatrix} = T_1 \begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix}$$

(6)

3. Influence of tool axis runout on workpiece surface topography

In the actual machining process, the tool will inevitably experience wear. In addition, due to the cumulative influencing factors, including the manufacturing accuracy error of the machine tool, the vibration coupling error of the machine tool, the installation accuracy error of the tool, and the wear of the spindle support-shaft of the machine tool for long-term work, deviation between the spindle axis of the machine tool and the tool axis, that is, rotation deviation, will appear, as shown in Fig. 2.

It is assumed that the radial motion deviation of the spindle is based on the rotation motion, and its diameter is runout $\Delta d_1$. In addition, the axial motion of the spindle is based on the cosine motion and its amplitude is $\Delta d_2$. To more accurately and completely describe the surface topography, the coordinate expression of tool center point $O$ in the tool coordinate system is determined as follows:

$$\begin{align*}
    u_0 &= \Delta d_1 \times \cos(\Delta \alpha_1 - \omega \times t) \\
    v_0 &= \Delta d_1 \times \sin(\Delta \alpha_1 - \omega \times t) \\
    w_0 &= \Delta d_2 \times \cos(\Delta \alpha_2 - \omega \times t)
\end{align*}$$

(7)

where $\Delta \alpha_1$ is the initial phase angle of spindle rotation runout, and $\Delta \alpha_2$ is the initial phase angle of axial drift.

Then, at time $t$, the coordinate transformation matrix expression of any point $P$ of the cutting edge transformed from the tool coordinate system to the spindle coordinate system is determined as follows:
4. Influence of tool inclination on workpiece surface topography

The proper inclination of the machining tool not only expands the range of the workpiece but also improves the tool processing environment, prolongs the tool life, reduces the residual height of the machined part surface, and finally improves the accuracy of the workpiece surface. In addition, given the small inclination between the tool axis and the machined workpiece surface, the effective diameter of the ball-end milling cutter actually participating in cutting will change, and the actual cutting speed will also change. As shown in Fig. 3, the transformation from the spindle coordinate system $O_t - UVW$ to the workpiece coordinate system $O_w - XYZ$ requires two rotations. First, rotating the $\beta_1$ angle around the $X$-axis, the rotation matrix is marked as $T_X$. Then, rotating the $\beta_2$ angle around the $Z$-axis, and the rotation matrix is marked as $T_Z$. The final transformation matrix from the coordinate system $O_t - UVW$ to the coordinate system $O_w - XYZ$ is determined as follows:

$$T_3 = T_X \ast T_Z$$

(9)

It should be noted that where $\beta_1$ and $\beta_2$ do not represent the machining inclination, their positive and negative symbols are determined according to the right-hand screw rule. Generally, assuming that the tool-axis tilts only around the $X$-axis and $Y$-axis or only around the $Y$-axis and $X$-axis in the workpiece coordinate system, these effects can be divided into four motion directions according to the combination mode: only around the $X$-axis ($\beta>0$), only around the $X$-axis ($\beta<0$), only around the $Y$-axis ($\beta>0$), and only around the $Y$-axis ($\beta<0$). The specific expression for $T_3$ can be written as:

$$T_3 = \begin{bmatrix}
\cos(\beta) & 0 & -\sin(\beta) & x_0 + (i-1) \ast a_e \\
0 & 1 & 0 & y_0 + \frac{n \ast z_0 \ast f_x \ast \pi}{60} \\
\sin(\beta) & 0 & \cos(\beta) & z_0 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad \text{(only around the $X$-axis)}$$

(10)

or

$$T_3 = \begin{bmatrix}
1 & 0 & 0 & x_0 + (i-1) \ast a_e \\
0 & \cos(\beta) & \sin(\beta) & y_0 + \frac{n \ast z_0 \ast f_x \ast \pi}{60} \\
0 & -\sin(\beta) & \cos(\beta) & z_0 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad \text{(only around the $Y$-axis)}$$

(11)

where $i$ refers to the $i$-th feed, $a_e$ is the radial cutting depth, $n$ is the spindle speed, $f_x$ refers to the feed rate per tooth, and $(x_0, y_0, z_0)$ is the coordinate value of the origin of the spindle coordinate system in the workpiece coordinate system.
5. Trajectory model of a ball-end milling cutter when milling a curved surface

There are many classification standards for curved surfaces. For example, according to the motion mode of the generatrix, these surfaces can be divided into rotating surfaces and nonrotating surfaces. According to the classification of mathematical analytical expressions, these surfaces can be divided into simple surfaces and complex surfaces (free-form surfaces). A simple surface can be expressed by a general mathematical analytical formula without design degrees of freedom. Common simple surfaces include cylindrical surfaces, elliptical paraboloids, and hyperboloids. Free-form surfaces refer to surfaces that cannot be expressed by general analytical formulas and have the characteristics of flexible design degrees of freedom. Common free-form surfaces include streamlined shells of automobiles, aircraft wings, human body sculptures, and steam turbine blades [21].

Surface fine machining and feature research are often used in the fields of precision machinery; molding; automobile, aerospace and other power equipment design and manufacturing. Due to the complexity and diversity of surfaces, surface machining and simulation modeling have always been technical bottlenecks for researchers to break through. Three-axis NC machine tools are mostly used for surface milling, and tools with multiple axes, such as four-axis or five-axis machine tools, are also used for milling more complex surfaces. During the machining process, the tool axis should always form a fixed inclination angle with the coordinate axis (Z-axis, as shown in Fig. 4) and realize intermittent feed machining along the feed direction (X-axis, as shown in Fig. 4). Finally, the surface topography of the machined surface is the envelope surface of the cutting tool motion path along each cutting rank (feed direction) [22, 23]. Ball-end milling cutters, drum cutters and end milling cutters are often used for surface milling. Because circular cutters and conical cutters have the advantages of machining 3D profiles and variable bevel contours, they are also commonly used to complete surface finishing.

In this paper, the 3D topography of various surfaces (including analytical surfaces and nonanalytical surfaces) machined by a ball-end milling cutter will be dynamically modeled and simulated. The first step of modeling is to deduce the new coordinate transformation of the origin of the spindle coordinate system under the workpiece coordinate system, as shown in formula (12).

The multiaxis machining mechanism of a curved surface approximates the contour of the machined surface through the linear interpolation motion of the tool, which inevitably produces machining errors. In view of this phenomenon, the motion process of the milling cutter is treated as multiple linear interpolation motions in this paper. Fig. 4 provides a schematic diagram of the tool path of the milling surface.
New coordinate expression of the origin of the machine tool spindle coordinate system in the workpiece coordinate system during surface machining is defined as follows:

\[
\begin{align*}
\bar{x}_{0WT} &= x_0 + (i-1) \cdot a_e \\
\bar{y}_{0WT} &= y_0 + (k-1) \cdot d_z + v_f \cdot \cos \theta \cdot (t - T(k,i)) \\
\bar{z}_{0WT} &= z_0 + d_k + v_f \cdot \sin \theta \cdot (t - T(k,i))
\end{align*}
\] (12)

where \((x_0,y_0,z_0)\) is the coordinate value of the origin of the machine tool spindle coordinate system under the workpiece coordinate system during the first linear feed of the milling cutter, \(i\) is the \(i\)-th intermittent feed, \(a_e\) is the radial cutting depth, \(t\) is the time from the \(i\)-th feed to the current time, \(k\) is the \(k\)-th point of the workpiece surface corresponding to the origin of the machine tool spindle coordinate system along the feed direction during the \(i\)-th feed, and \(\theta\) is the angle between the cutting speed direction and plane \(XY\) in the machining process, and \(T(k,i)\) is the time that the milling cutter passes from the beginning of the \(i\)-th feed to the \(k\)-th point on the workpiece surface along the current feed direction. Therefore, the general coordinate formula of any point \(P\) on the cutting edge of the ball-end milling cutter under the workpiece coordinate system can be deduced as follows:

\[
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = TT \ast \begin{bmatrix}
u \\
v \\
w \\
1
\end{bmatrix}
\] (13)

where \(TT\) is the transformation matrix from the tool coordinate system to the workpiece coordinate system during surface milling.

6. Steps of topography simulation when milling a curved surface with ball-end milling cutter

The simulation modeling of the curved surface of a workpiece adopts the idea of 3D space meshing. By comparing the machined surface with the original unmachined surface, it can be assumed that, in a sense, the machined surface is equivalent to the original unmachined surface, which is offset \(a_p\) unit distance downward along the normal vector of each grid point. Based on this notion, the topography simulation steps of ball-end milling cutters when milling curved surfaces can be summarized as follows:

**Step 1: Normalization of coordinate system origin.** For convenience, the origin of the workpiece surface (to be machined) coordinate system is set on a vertex of the initial surface, and it is taken as the initial surface so that it is easy to know that its projection on the plane of the workpiece coordinate system is an equal-bottom rectangle.
**Step 2: Surface offset.** The initial surface of the first step is offset upward by an \( a_p \) unit distance along the normal vector direction of each grid point, which is equivalent to the overall outward (upward) generation of a surface with the same shape and equal spacing from the initial surface.

**Step 3: Surface meshing.** The offset surface generated in the second step is divided into \( m \) and \( n \) equal parts along the \( X \)-axis direction and \( Y \)-axis direction, respectively, in the workpiece coordinate system, and the spacing of each equal part is \( d_x \) and \( d_y \), respectively. Then, according to the previous modeling method, the \( Z \) coordinate value expression of each grid point on the offset surface in the workpiece coordinate system is calculated, and the \( Z \) coordinate values of these grid points are stored in the matrix \( H(i, j) \).

**Step 4: Data update.** Compare the coordinate data of each grid point of the offset surface in the workpiece coordinate system calculated in the third step with the corresponding data of the previous cutting. If it is less than the previous cutting coordinate data, it is deemed to have cut into the surface of the workpiece surface. Then, the \( Z \) data of grid point \((i, j)\) replaces the previous one; otherwise, it will not be changed.

**Step 5: Topography generation.** The 3D surface topography is drawn and analyzed according to the final \( H(i, j) \).

7. Simulation example
In the mold industry, most workpieces are free-form surfaces. Nonuniform rational B-splines (NURBS) surfaces have the advantages of easy adjustment of control vertices, easy modification of weight factors, and generation of various required surface shapes. Therefore, these surfaces are widely used in the construction of free-form surfaces. In this example, taking the NURBS surface machined by a ball-end milling cutter as an example, the 3D shape simulation results of the surface are analyzed.

First, an NURBS surface is defined, and the degree, control vertices and node vectors of the surface are specified. Fig. 5 shows the original NURBS surface and its mesh-point normal vector. Figs. 6 (a) - (f) show the surface topography of NURBS surface milling after offset and reduction under the parameters of experiment nos. 1–6 in Table 1, respectively.

![Fig. 5 Construction of NURBS primitive surface and its mesh point normal vector](image-url)
Fig. 6 and Table 1 demonstrate that when each parameter changes, it has a great impact on the surface topography. For example, after calculation, the arithmetic mean deviation $S_{ba}$ of the 3D surface...
topography in Fig. 6(a) is 0.94 μm. However, in Fig. 6(b), \( S_{ba} = 1.25 \) μm. The simulation results can also be used to optimize the actual free-form surface milling process to improve the surface quality of the milling workpiece or predict the surface topography of given machining parameters, which is also the direction of further research.

8. Conclusions
This paper studies free-form surface machining with a ball-end milling cutter. The motion trajectory equation expression of any point of any cutting edge of the ball-end milling cutter is obtained using a tool mathematical model and transformation matrix of motion coordinates. Then, the tool motion envelope model is derived. Finally, according to the trajectory model and simulation steps of the ball-end milling cutter when the milling surface is milled, a 3D surface topography prediction model of the free-form surface is constructed. The model comprehensively considers dynamic factors, such as the initial phase angle and amplitude of spindle rotation runout and the initial phase angle and amplitude of axial drift, and geometric factors such as the number of milling cutter teeth, machining inclination angle, radial cutting depth, feed times and feed rate per tooth, as well as the influence of machining parameters on the surface topography. A dynamic model of 3D surface topography machining with a ball-end milling cutter suitable for all types of surfaces (including analytical and nonanalytical surfaces) is established, and several examples are simulated and compared.

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References
[1] Denkena, B., Böß, V., Nespor, D., et al. (2015) Prediction of the 3D surface topography after ball end milling and its influence on aerodynamics [J]. Procedia CIRP, 31:221–227.
[2] Li, S.J., Dong, Y., Li, Y., et al. (2019) Geometrical simulation and analysis of ball-end milling surface topography [J]. The International Journal of Advanced Manufacturing Technology, 102: 1885–1900.
[3] Wei, J., Hou, X., Sun, C. (2021) Modeling and simulation of surface topography in five-axis ball end milling [J]. Journal of Physics Conference Series, 1820:1–6.
[4] Wang, Z., Yuan, J., Yin, Z., et al. (2016) Surface topography and roughness of high-speed milled AlMn1Cu [J]. Chinese Journal of Mechanical Engineering, 29(6):1200–1207.
[5] Yu, T.B., Su, P.C., Zhang, J.Q., et al. (2010) A simulation system for grinding based on virtual reality [J]. Advanced Materials Research, 126−128:96–100.
[6] Wang, W., Li, Q.Z., Jiang, Y.F. (2020) A novel 3D surface topography prediction algorithm for complex ruled surface milling and partition process optimization [J]. The International Journal of Advanced Manufacturing Technology, 107:3817–3831.
[7] Yang, D., Liu, Z.Q. (2015) Surface topography analysis and cutting parameters optimization for peripheral milling titanium alloy Ti−6Al−4V [J]. Int. Journal of Refractory Metals and Hard Materials, 51:192–200.
[8] M Zheng, YH Dong. Research on simulation of peripheral milling surface topography machined by mills [J]. Manufacturing Automation, 2014, 36(8):62–64.
[9] Dong, Y.H., Li, Y., Zhang, Q. (2014) Research on simulation of surface topography machining by end milling using end-mills [J]. Manufacturing Automation, 36 (4):1-3.
[10] Wang, L.P., Ge, S.Y., Hao, S., et al. (2019) Elliptical model for surface topography prediction in five-axis flank milling [J]. Chinese Journal of Aeronautics, 33(4):1361−1374.
[11] Arizmendi, M., Campa, F.J., Fernández, J., et al. (2009) Model for surface topography prediction in peripheral milling considering tool vibration [J]. CIRP Annals- Manufacturing Technology, 58(1):93–96.
[12] Arizmendi, M., Fernández, J., Gil, A., et al. (2010) Model for the prediction of heterogeneity bands in the topography of surfaces machined by peripheral milling considering tool runout [J]. International Journal of Machine Tools & Manufacture, 50(1):51–64.

[13] Liu, J.B., Jia, J.J., Wang, R.Q., et al. (2021) State dependent regenerative stability and surface location error in peripheral milling of thin-walled parts [J]. International Journal of Mechanical Sciences, 196(2):229–238.

[14] Zhao, H.W., Zhang, S., Zhao, B., et al. (2014) Simulation and prediction of machined surface topography machined by ball-nose end mill [J]. Computer integrated manufacturing systems, 20(4):880–889.

[15] Yang, S.C., Han, P., Su, S., et al. (2021) Study on surface work hardening of titanium alloy milled by micro-textured ball milling cutter [J]. The International Journal of Advanced Manufacturing Technology, 112(7):2497–2508.

[16] Xu, J.T., Xu, L.K., Geng, Z., et al. (2020) 3D surface topography simulation and experiments for ball-end NC milling considering dynamic feedrate [J]. CIRP Journal of Manufacturing Science and Technology, 31(5):210–223.

[17] Lotfi, S., Wassila, B., Gilles, D. (2017) Cutter workpiece engagement region and surface topography prediction in five-axis ball-end milling [J]. Machining Science & Technology, (3):181–202.

[18] Ehsan Layegh K., S., Lazoglu, I. (2017) 3D surface topography analysis in 5-axis ball-end milling [J]. CIRP Annals-Manufacturing Technology, 66(1): 133–136.

[19] Buj-Corral, I., Vivancos-Calvet, J. (2012) A domingoz-fernández. surface topography in ball-end milling processes as a function of feed per tooth and radial depth of cut [J]. International Journal of Machine Tools & Manufacture, 53(1):151–159.

[20] Zhang, X., Zhang, J., Zheng, X.W., et al. (2017) Tool orientation optimization of 5-axis ball-end milling based on an accurate cutter/workpiece engagement model [J]. CIRP Journal of Manufacturing Science and Technology, 19:106–116.

[21] Takanashi, Y., Aoyama, H., Song, C.W. (2021) Generation method of cutting tool paths for high-speed and high-quality machining of free-form surfaces [J]. International Journal of Automation Technology, 15(4):521–528.

[22] Kazaal, R.S., Hamdan, K.H. (2019) Investigation the optimization of machining parameters to surface roughness in free form surface of composite material [J]. Al-Khwarizmi Engineering Journal, 15(2):60–69.

[23] Ozturk, B., Lazoglu, I. (2017) An analytical model for engagement regions in machining of 3-D free-form surfaces (analytical advancement of machining process) [J]. Proceedings of International Conference on Leading Edge Manufacturing in 21st century, 2005:1127–1132.