Weak Decays of $b$ and $c$ Quarks

Report of Topical Group RF1

Rare Processes and Precision Measurements Frontier

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Precision measurements in weak decays of heavy flavored hadrons can test in unique ways our understanding of the fundamental interactions and of the observed baryon asymmetry in the Universe. The high sensitivity of such decays to beyond-Standard-Model physics, combined with the lack of major discoveries in direct production of new particles, motivates the continuation of a strong heavy-flavor program in the next decades. The observation of several anomalies by the BaBar, Belle and LHCb experiments in such decays, including evidence for violation of lepton universality, provides particular motivation to vigorously pursue this program. While the mass scales probed by direct searches for non-Standard-Model phenomena at the energy frontier will only marginally increase in the near future, a substantial advancement is expected in the study of weak decays of $b$ and $c$ quarks. The next 10 to 20 years will see the development of a highly synergistic program of experiments at both $pp$ and $e^+e^-$ colliders. This program will be complemented by advancements in theory, including both lattice and continuum calculations. Experimental measurements and theory predictions of several key observables will reach unprecedented precision and will allow to test the Standard Model in ways that have not been possible thus far. With a strong participation in this program, the US high-energy-physics community will remain at the forefront of indirect searches for new physics and retain its leading role in expanding humankind’s understanding of fundamental interactions.
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Introduction

This report describes the physics case for precision studies of weak decays of $b$ and $c$ quarks, and it discusses the experimental and theory programs needed to exploit these physics opportunities in the next decades. It is based on the several white papers submitted by the community and, in particular, on Refs. [1–4] – which provide an overview of the contributions discussing weak decays of $b$ and $c$ quarks. This report is not a review of heavy-quark physics, and no attempt has been made to provide complete references to prior work. Section 1 discusses the motivation for heavy-quark physics, giving an overview of its unique potential for discoveries of new dynamics up to very high energy scales. Section 2 presents the experimental efforts planned/proposed over the 2020-2030 decades, followed in Sec. 3 by a discussion of the opportunities for a continued heavy-quark-physics program at facilities further into the future. The expected theory progress is then outlined in Sec. 4. The report concludes in Sec. 5 with our recommendation to ensure a strong involvement of the U.S. high-energy-physics (HEP) community in this field of research.

1 The path to discovery in heavy-quark physics

The power of indirect searches for new fundamental physics in rare processes and precision measurements is rooted in the basic principles of quantum field theory. The probability amplitude for the transition from a certain initial state to a certain final state is the sum of all possible Feynman diagrams with these initial and final states. Crucially, the internal (virtual) particles in Feynman diagrams are not required to be on the mass shell, which means that arbitrarily heavy particles will contribute to the amplitude as long as there is no symmetry forbidding a coupling to them. The effects of the heavy $W^-$ bosons, for example, are seen in the $\beta$ decay of a neutron at much lower energy. Similarly, the charm, bottom, and top quarks were all predicted theoretically through their contributions as virtual particles to explain the observed phenomena of flavor-changing processes at lower energy, well before these particles could be produced directly.

The study of weak decays of $b$ and $c$ quarks – or more generally of quark-flavor physics – has been essential in constructing the Standard Model (SM), and may very well also point us to what lies beyond. Many questions left unanswered by the SM, such as those about the origin of the large matter-antimatter asymmetry in the Universe, the mechanism giving neutrinos their masses, and the observed patterns and hierarchies in the many “fundamental” parameters, are directly related to flavor physics. More generally, proposed extensions of the SM, for example supersymmetry (which can alleviate the electroweak hierarchy problem and provides a dark-matter candidate), typically introduce new sources of flavor-changing interactions and new sources of charge-parity symmetry ($CP$) violation. Flavor-physics measurements may reveal such effects, and can tightly constrain the parameter spaces of new theories (see, e.g., Refs. [5–7]). Because the dependence on new particle masses and (flavor-violating) couplings is different than in the on-shell production, the new physics (NP) searches performed in weak decays of $b$ and $c$ quarks are also complementary to the direct searches at the energy frontier.

The unique properties and the richness of possible final states of weak decays of $b$ and $c$ quarks result in a particularly high potential for discovering new fundamental physics. As an example, rare $b$ and $c$ decays – being strongly suppressed in the SM – are potentially sensitive to very high
The Wilson coefficients $C_{bs\mu\mu}^9$ and $C_{bs\mu\mu}^{10}$ are the NP contributions to the couplings of the operators $O_9 = (\bar{s}\gamma_\mu b_L)(\bar{\mu}\gamma_\mu\mu)$ and $O_{10} = (\bar{s}\gamma_\mu b_L)(\bar{\mu}\gamma_\mu\gamma_5\mu)$, respectively. The global fit result is inconsistent with the SM point (the origin) by $\sim 5\sigma$.

NP scales of several 10’s to 100 TeV [2]. Intriguingly, current experimental results for $b \to s\mu^+\mu^-$ branching fractions and angular observables, as well as ratios of $b \to s\mu^+\mu^-$ and $b \to se^+e^-$ branching fractions, already show a coherent pattern of deviations from SM predictions [1]. The muon to electron ratios are predicted to be close to 1 in the SM with essentially no hadronic uncertainties, but are observed to be closer to 0.75 on average, suggesting violation of lepton-flavor universality. According to some analyses [8–11], fits to the experimental data and theory inputs yield pulls of $\gtrsim 5\sigma$ with respect to the SM, as shown for example in Fig. 1. Moreover, restricting the fits to only the ratios of $b \to s\mu^+\mu^-$ and $b \to se^+e^-$ branching fractions along with $B(B_s^0 \to \mu^+\mu^-)$ still yields a significance for NP of $\gtrsim 4\sigma$ [8–11].

The tree-level $b \to c\tau^+\nu$ decays are not rare but are nevertheless expected to be quite sensitive to physics beyond the SM as a result of the large $\tau$ lepton mass (for example, a charged Higgs boson would couple much more strongly to the $\tau$ than to the other leptons) [1]. Experimental results are available for ratios of $b \to c\tau^-\bar{\nu}$ to $b \to c\ell^-\bar{\nu}$ branching fractions, typically denoted as $R(X_c)$ where $X_c$ is the charmed hadron in the final state. The world averages of experimental results for $R(D)$ and $R(D^*)$ exceed the SM predictions with a combined significance of $\sim 3\sigma$, again pointing to violation of lepton-flavor universality (Fig. 2).

The $b \to s\ell^+\ell^-$ and $b \to c\tau^+\bar{\nu}$ anomalies have led to significant efforts by the HEP theory...
Figure 2: The ratios of branching fractions $R(D) = \mathcal{B}(B \to D\tau^+\nu)/\mathcal{B}(B \to D\ell^+\nu)$ and $R(D^*) = \mathcal{B}(B \to D^*\tau^+\nu)/\mathcal{B}(B \to D^*\ell^+\nu)$, where $\ell$ denotes muons or electrons, are predicted precisely in the SM to be $R(D) = 0.299 \pm 0.003$, $R(D^*) = 0.254 \pm 0.005$ (the black point in this figure). The averages of experimental measurements of these ratios correspond to the red ellipse, which exceed the SM predictions with a combined significance of about $3.3\sigma$ [12].

...
Figure 3: Schematic representation of the (top) $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay and (bottom) $B_s^0 - \bar{B}_s^0$ mixing amplitudes as sums over all possible Feynman diagrams. The diagrams on the left are examples of SM contributions, while the diagram on the right is an example of an NP contribution in theories with a flavor-changing neutral gauge boson $Z'$.

Fig. 4) by requiring NP to have minimal flavor violation, i.e., the absence of new sources of flavor violation beyond the SM Yukawa couplings (e.g., as in composite Higgs models). In the minimal-flavor-violation case, the sensitivity becomes comparable to, and complements, other indirect probes of NP such as electroweak precision observables or Higgs couplings (see, e.g., Refs. [23–25]).

The coming two decades will not bring a substantial increase in the energy scales directly probed at colliders [25]. However, as discussed in the following, a remarkable increase in precision is expected for many heavy-quark observables. Weak decays of $b$ and $c$ quarks then offer a unique opportunity to reveal new phenomena, and/or strongly shape our expectations for beyond-SM dynamics, before the next energy-frontier machine will become available (see, e.g., the expected impact on the bounds on the NP scale from $\Delta F = 2$ transitions in Fig. 4).

2 Experimental efforts in the next two decades

Several experiments with beauty- and charm-quark physics in their programs are in operation or planned for the 2020s (Fig. 5). The Belle II experiment at the SuperKEKB asymmetric $e^+ e^-$ collider is a major improvement over its predecessors Belle and BaBar. The experiment has been running since 2019 and, until SuperKEKB Long Shutdown 1 in Summer 2022, has collected $\sim 430 \text{fb}^{-1}$ of integrated luminosity – corresponding to roughly the sample size collected by BaBar. During the same period SuperKEKB has achieved a record peak luminosity of $4.7 \times 10^{34} \text{cm}^{-2} \text{s}^{-1}$. The experiment will produce a variety of world-leading results continuously as it proceeds towards the goal of collecting an integrated luminosity of $50 \text{ab}^{-1}$ by the mid-2030s [28, 29]. To achieve this
Figure 4: Present (lighter) and future (darker) lower bounds at 95% confidence level on the NP scale $\Lambda$ from $\Delta F = 2$ transitions [26, 27]. The Wilson coefficients $C_i = F_i L_i / \Lambda^2$ ($i = 1, \ldots, 5$) are the coupling of the NP dimension-six operators governing the $\Delta F = 2$ transition:

$Q_{1i} = (t_{jL}^\alpha \gamma_\mu q_{iL}^\alpha)(t_{jL}^\gamma \gamma_\mu q_{iL}^\gamma)$, $Q_{2i} = (t_{jR}^\alpha q_{iL}^\alpha)(q_{jR}^\beta q_{iL}^\beta)$, $Q_{3i} = (q_{jR}^\alpha q_{iL}^\beta)(q_{jR}^\beta q_{iL}^\alpha)$, $Q_{4i} = (q_{jR}^\alpha q_{iL}^\beta)(t_{jL}^\beta q_{iR}^\beta)$, and $Q_{5i} = (q_{jR}^\alpha q_{iL}^\beta)(t_{jL}^\beta q_{iR}^\beta)$ (with $\alpha$ and $\beta$ being color indices). On the left, NP is assumed to have arbitrary flavor structure ($F_i = 1$) and to be strongly coupled with no loop suppression ($L_i = 1$); on the right, NP is assumed to have minimal-flavor-violation couplings ($F_i = V_{CKM}$) and to enter at one loop with weak coupling ($L_i = \alpha_2^2$, with $\alpha_2^2$ being the weak structure constant). The future bounds are based on expected sensitivities at Belle II (50 ab$^{-1}$), BESIII, LHCb Upgrade II (300 fb$^{-1}$), and ATLAS/CMS (3 ab$^{-1}$), and on improved theory inputs.

The LHCb experiment, after having collected about 9 fb$^{-1}$ of $pp$ collisions during Run 1 and 2 of the LHC, has just started the operation of its first upgrade during Run 3. By 2032, LHCb Upgrade I expects to collect a sample corresponding to an integrated luminosity of 50 fb$^{-1}$ [36]. Considering the increased collision energy and the enhanced online-selection efficiency, the yield of beauty and charm hadrons available for analyses will increase by factors of 5 to 10, depending on the final states, compared to the currently available data from Run 1 and Run 2. On the same timescale, ATLAS and CMS will continue to contribute significantly in some selected areas, such as...
Figure 5: Timeline of planned/proposed experiments in the next two decades [29, 33–35].

as in decays with dimuons in the final state [37]. For the HL-LHC both experiments are planning
significant modification of the detectors (Phase-2 upgrades scheduled during Long Shutdown 3 in
2026-2028) to maintain effective data taking and event reconstruction at increased luminosity and
pileup. Particularly relevant for heavy-flavor physics are upgrades to the tracking systems, which
would result in improved mass and decay-time resolutions, and to the trigger systems, to maintain
the online selection efficient at the relatively low transverse momenta typical of the final state
muons from beauty decays.

Finally, the BESIII experiment at BEPCII uses $e^+e^-$ collisions with center-of-mass energies
ranging from 2 to 5 GeV to study the broad spectrum of physics accessible in the $\tau$-charm energy
region [38]. Since the start of operations in 2009, BESIII has collected more than 35 fb$^{-1}$ of data,
comprising several data samples that are particularly useful for studying weak decays of charm
hadrons, such as 5 fb$^{-1}$ of $\psi(3770) \rightarrow D^0\bar{D}^0$ data, 3 fb$^{-1}$ at $\sqrt{s} = 4.178$ (near the $D^+_c\bar{D}^-_c$ threshold),
and more than 3 fb$^{-1}$ at $\sqrt{s} = 4.64$ GeV (above the $\Lambda^+_c\bar{\Lambda}_c^-$ threshold). The experiment will run
at least for the next 5-10 years, during which new upgrades for both the detector and accelerator
are being considered. In particular, BEPCII upgrades aim to first increase the maximum collision
energy to 5.6 GeV and then to increase the peak luminosity by a factor of three (for collision energies
above 4 GeV). The goal is for BESIII to integrate 20 fb$^{-1}$ at $\sqrt{s} = 3.773$ GeV (before the scheduled
BEPCII upgrade in June 2024), 6 fb$^{-1}$ at $\sqrt{s} = 4.178$, and 5 fb$^{-1}$ at $\sqrt{s} = 4.64$ GeV [34].

These experimental efforts will complement one another, making possible a wide range of pre-
cision measurements that would be unfeasible at a single facility. Since most of the current heavy-
flavor results are severely limited by their statistical precision, the ability to access much larger
samples of $b$- and $c$-hadron decays is crucial. In this respect, the production rate of heavy flavored
hadrons in $pp$ collisions gives LHC experiments a clear advantage with respect to the beauty and
charm “factories” operating at $e^+e^-$ colliders. However, such advantage is mostly exploited in final
states made of only charged particles thanks to the excellent tracking and vertexing detectors that,
by precisely measuring their properties, allow to discriminate signal particles from backgrounds. In
addition, all species of $b$ hadrons are produced at the LHC, including bottom-strange mesons and bottom baryons, which are kinematically forbidden at Belle II. Belle II and BESIII have unique capabilities that give them advantages over hadron-collider experiments despite the lower production rates. Reconstruction of neutral particles (photons and neutral pions) is nearly as efficient and precise as that of charged particles. Because the initial state is known and the detectors are nearly hermetic, fully-inclusive final states and reconstruction of particles with no direct signature in the detector (such as neutrinos and $K_L^0$ mesons) becomes accessible. Reconstruction efficiencies at beauty and charm factories are largely uniform as a function of the decay kinematics, which offers an advantage in measurements that involve multibody decays. Coherent $B^0\bar{B}^0$ pair production at Belle II makes possible efficient determination of the flavor of the neutral $B$ meson at production (flavor tagging), which is key for several time-dependent $CP$-violation measurements. Coherent $D^0\bar{D}^0$ pair production through the $\psi(3770) \rightarrow D^0\bar{D}^0$ process at BESIII enables the measurement of quantum-correlated observables that cannot be accessed elsewhere. Examples are strong-phase differences between the $D^0$ and $\bar{D}^0$ amplitudes, which are important inputs for the precise determination of the CKM angle $\gamma$ and of the charm-mixing parameters in a model-independent fashion at Belle II and LHCb [39–44].

The HL-LHC will extend the LHC program through the 2030s and the first half of the 2040s. While no major upgrades are yet planned for ATLAS and CMS during this period, LHCb proposes to upgrade the entire detector to be able to run at an instantaneous luminosity of around $1.5 \times 10^{34} \text{cm}^{-2} \text{s}^{-1}$, and collect a total of 300 fb$^{-1}$ by the end of Run 6 (in the early 2040s) [36,45]. The proposal is to install the new detector (Upgrade II) during LHC Long Shutdown 4 (2033-2034), with some preparatory work (Upgrade Ib) to be performed already during Long Shutdown 3 (2026-2028). The Upgrade Ib will also have benefits for the physics performance during Run 4, beyond what has been projected for LHCb Upgrade I. The challenge for the LHCb Upgrade II resides mostly in maintaining and extending the strengths of the LHCb Upgrade I detector, including its flexible software trigger, in the much harsher environment resulting from $\sim 40$ interactions per bunch crossings. The detector must sustain radiation doses of up to 400 MRad per year, be highly segmented to cope with large occupancy, and integrate timing information (with tens of ps resolution) in the readout to be able to associate hits with the right primary interaction. These challenges require the development of novel technologies, some of which will likely be deployed in future HEP experiments [46].

At Belle II, studies have started to explore upgrades beyond the currently planned program, such as beam polarization and ultra-high luminosity. The beam-polarization upgrade offers unique and powerful sensitivities to NP via precision measurements of neutral-current couplings at 10 GeV, and via studies of $\tau$-lepton properties and decays [47]. Accelerator upgrades to reach a peak luminosity in excess of $1 \times 10^{36} \text{cm}^{-2} \text{s}^{-1}$ and collect $\sim 250 \text{ab}^{-1}$ of integrated luminosity have recently been discussed [30]. If timely, such an upgrade may effectively complement the heavy-flavor program of the HL-LHC experiments. However, the feasibility from the accelerator perspective is still unclear, and so is the upgrade timeline.

With BESIII expected to end by around 2030, a Super $\tau$-Charm factory (STCF) [35,48] has been proposed in China to continue and extend the physics program with $e^+e^-$ collisions at energies between 2 and 7 GeV and with peak luminosity of at least $5 \times 10^{34} \text{cm}^{-2} \text{s}^{-1}$. The current schedule foresees the construction to occur between 2024 and 2030, and at least 10 years of operations. Upgrades to further increase the luminosity and for the implementation of a polarized $e^-$ beam are
also proposed for 2041-2042, followed by an additional 5 years of data taking.

2.1 Expected experimental progress on key observables

In a short summary such as this report, it is impractical to discuss all interesting observables in $b$ and $c$ physics. Thus we focus on a small subset of observables that are currently of high interest, either because their measurements show possible deviations from SM predictions, or because they are limited by experimental uncertainties and therefore offer an opportunity for significant improvement in precision over the next 10-20 years. A summary of the experimental prospects for many of these key measurements is given in Tab. 1.

2.1.1 Lepton-flavor-universality tests

The next decade should clarify the hints of lepton-flavor-universality violation observed in recent years thanks to the large data samples expected at LHCb Upgrade I and Belle II. Measurements of LFU observables in $b \rightarrow s \ell^+\ell^-$ decays will reach 1%-level uncertainties, a precision sufficient to establish or reject the level of LFU violation seen in the current measurements. The LHCb Upgrade II data set will then open new avenues with sensitivity to even cleaner observables, such as the difference between the values of $C_9$ and $C_{10}$ for $b \rightarrow s e^+e^-$ and $b \rightarrow s \mu^+\mu^-$ transitions, through the measurements of angular distributions [76,77]. The achievable precision will be crucial to distinguish between different NP models (Fig. 6). Additionally, the LHCb Upgrade II sample will allow lepton-flavor-universality tests in the related, and further suppressed, $b \rightarrow d \ell^+\ell^-$ transitions, which would further constrain the dynamics. For example, the statistical precision on the ratio $B(B^+ \rightarrow \pi^+ \mu^+ \mu^-)/B(B^+ \rightarrow \pi^+ e^+ e^-)$ is expected to reach a few percent.

Tests of lepton-flavor universality in $B^0 \rightarrow D^{*-}\tau^+\nu$ decays are expected to be dominated by Belle II, thanks to the ability to constrain the kinematics of the undetected neutrinos by utilizing the precise knowledge of the $B^0\overline{B}^0$-pair production mechanism. LHCb will also contribute, in particular, by performing measurements of semitauonic rates of other $b$ hadrons not accessible at Belle II, such as $B_s^0 \rightarrow D_s^{*-}\tau^+\nu$, $B_c^+ \rightarrow J/\psi \tau^+\nu$ and $A_b^0 \rightarrow A_c^+ \tau^-\overline{\nu}$. Measurements of observables related to angular distributions, such as the $\tau^+$ and $D^{*-}$ polarization fractions, will provide supplementary sensitivity to non-SM physics and key information to decipher the dynamics (see, e.g., Ref. [79–83]). Furthermore, Belle II has the unique ability to measure the inclusive ratio $R(X) = B(B \rightarrow X \tau^+\nu)/B(B \rightarrow X \ell^+\ell^-)$ – where $X$ is any system made of one or more hadrons – whose phenomenological interpretation is based on different theory inputs compared to the exclusive observables, and will perform, for the first time, measurements of $b \rightarrow u \tau^-\overline{\nu}$ decays. As shown in Fig. 6, Belle II will achieve $\mathcal{O}(1\%)$ sensitivities on most quantities using 50 ab$^{-1}$ of integrated luminosity.

Other possibilities to test electron vs. muon universality in semileptonic charm and beauty decays have also been recently proposed, with many having good prospects at BESIII, STCF and Belle II [38,48,84–86]. One example is the difference between the lepton forward-backward asymmetries for $B^0 \rightarrow D^{*-}\mu^+\nu$ and $B^0 \rightarrow D^{*-}e^+\nu$ decays [84–86]. This difference can be precisely measured at Belle II.
| Observable                                                                 | Current best | Belle II 50 ab⁻¹ | Belle II 250 ab⁻¹ | LHCb 50 fb⁻¹ | LHCb 300 fb⁻¹ | ATLAS 3 ab⁻¹ | CMS 3 ab⁻¹ | BESIII 20 fb⁻¹ (*) | STCF 1 ab⁻¹ (*) |
|----------------------------------------------------------------------------|--------------|-------------------|-------------------|--------------|--------------|-------------|-------------|-------------------|-----------------|
| **Lepton-flavor-universality tests**                                       |              |                   |                   |              |              |             |             |                   |                 |
| $R_K(1 < q^2 < 6 \text{ GeV}^2/c^4)$                                      | 0.044 [49]   | 0.036             | 0.016             | 0.017        | 0.007        |             |             |                   |                 |
| $R_K(1 < q^2 < 6 \text{ GeV}^2/c^4)$                                      | 0.12 [50]    | 0.032             | 0.014             | 0.022        | 0.009        |             |             |                   |                 |
| $R(D)$                                                                    | 0.037 [51]   | 0.008             | < 0.003           | na           | na           |             |             |                   |                 |
| $R(D^*)$                                                                  | 0.018 [51]   | 0.0045            | < 0.003           | 0.005        | 0.002        |             |             |                   |                 |
| **Rare decays**                                                           |              |                   |                   |              |              |             |             |                   |                 |
| $B^0 \rightarrow \mu^+ \mu^-$ [10⁻⁹]                                      | 0.46 [52,53] | na                | 0.16              | 0.46–0.55    | 0.39         |             |             |                   |                 |
| $B^0 \rightarrow \mu^+ \mu^- / B^0 \rightarrow \mu^+ \mu^-$              | 0.69 [52,53] | 0.27              | 0.11              | na           | 0.21         |             |             |                   |                 |
| $B(B^0 \rightarrow K^{*0} \tau^+ \tau^-)$ UL [10⁻³]                      | 2.0 [54,55]  | 0.5               | na                |             |             |             |             |                   |                 |
| $B/B_{SM}(B^+ \rightarrow K^+ \nu \overline{\nu})$                       | 1.4 [56,57]  | 0.08–0.11         | na                |             |             |             |             |                   |                 |
| $B(B \rightarrow X_s \gamma)$                                            | 10% [58,59]  | 2–4%              | na                |             |             |             |             |                   |                 |
| **CKM tests and CP violation**                                            |              |                   |                   |              |              |             |             |                   |                 |
| $\alpha$                                                                  | 5° [60]      | 0.6°              | 0.3°              |             |             |             |             |                   |                 |
| $\sin 2\beta(B^0 \rightarrow J/\psi K^0_S)$                              | 0.029 [61]   | 0.005             | 0.002             | 0.006        | 0.003        |             |             |                   |                 |
| $\gamma$                                                                  | 4° [62]      | 1.5°              | 0.8°              | 1°           | 0.35°        |             |             |                   |                 |
| $\phi_A(B^0 \rightarrow J/\psi \phi)$                                    | 32 mrad [63] | 10 mrad           | 4 mrad            | 4–9 mrad     | 5–6 mrad     |             |             |                   |                 |
| $|V_{ub}| / |V_{cb}|(A_0^+ \rightarrow p \mu^- \nu)$                         | 5% [64,65]   | 2%                | < 1%             | na           | na           |             |             |                   |                 |
| $|J_D+| / |V_{cd}|(D^+ \rightarrow \mu^+ \nu)$                               | 6% [66]      | 2%                | 1%               |             |             |             |             |                   |                 |
| $S_{CP}(B^0 \rightarrow \eta K^0_S)$                                      | 0.08 [68,69] | 0.015             | 0.007             | na           | na           |             |             |                   |                 |
| $A_{CP}(B^0 \rightarrow K^0_S \pi^0)$                                    | 0.15 [68,70] | 0.025             | 0.018             | na           | na           |             |             |                   |                 |
| $A_{CP}(D^0 \rightarrow \pi^+ \pi^-)$                                    | 11 × 10⁻³ [71] | 1.7 × 10⁻³ | na                | na           | na           |             |             |                   |                 |
| $\Delta A(D^0 \rightarrow K^0_S \pi^+ \pi^-)$                           | 18 × 10⁻⁵ [72] | na                | na                | 4.1 × 10⁻⁵   | 1.6 × 10⁻⁵   |             |             |                   |                 |
| $A_{CP}(D^0 \rightarrow K^+ K^- , \pi^+ \pi^-)$                          | 11 × 10⁻⁵ [73] | na                | na                | 3.2 × 10⁻⁵   | 1.2 × 10⁻⁵   |             |             |                   |                 |

Table 1: Projected uncertainties (or 90% CL upper limits) in several key heavy-flavor observables over the next two decades. A missing entry means that the observable cannot be measured, the abbreviation na means that, although the observable can be measured, the projected uncertainty is not available. Projections are taken from Refs. [28,30,74] (Belle II), Refs. [45, 75] (LHCb), Ref. [37] (ATLAS and CMS), Refs. [34, 48] (BESIII and STCF). (∗) Integrated luminosity at $\sqrt{s} = 3.773$. (∗) Projected uncertainties on $\gamma$ resulting from BESIII/STCF measurements of the $D$ strong-phase differences, which will contribute as external inputs to the Belle II and LHCb measurements.
2.1.2 Rare decays

The purely leptonic rare decay $B^0_s \to \mu^+\mu^-$ has very small branching fraction in the SM and, as such, it has been historically considered one of the “golden” channels for flavor-changing neutral-current $b$-hadron decays. The average of the experimental measurements of $\mathcal{B}(B^0_s \to \mu^+\mu^-)$, $(3.01 \pm 0.35) \times 10^{-9}$, is dominated by statistical uncertainties that are larger than the theory uncertainties on the predicted SM value, $(3.66 \pm 0.14) \times 10^{-9}$ [87]. The experimental precision is expected to closely approach the SM uncertainty with the LHCb Upgrade II data set and to match it when combining with ATLAS and CMS 3 ab$^{-1}$ results (Tab. 1). A related probe for beyond-SM dynamics, which is particularly powerful in constraining supersymmetry or models with minimal flavor violation, is the ratio between $B^0 \to \mu^+\mu^-$ and $B^0_s \to \mu^+\mu^-$ branching fractions. This will be measured with better than 10% relative precision by the combination of the HL-LHC measurements. Thanks to its superior vertexing and flavor-tagging capabilities as compared to ATLAS and CMS, LHCb Upgrade II will furthermore have the unique ability to measure the $CP$-violation parameters $A_{\Delta \Gamma}$ and $S_{\Delta \Gamma}$ with a time-dependent analysis of $B^0_s \to \mu^+\mu^-$ decays [45]. These are important observables that, if different from their SM expectations of unity and zero respectively, would provide unambiguous evidence for new dynamics [88].

Besides from lepton-flavor-violation tests, differential decay rates of semileptonic $b \to s\ell^+\ell^-$ decays as a function of the squared dilepton mass, $q^2$, and of angular observables provide a wealth of information to constrain all Wilson coefficients in the effective Hamiltonian. Hints of NP in $C^9$ firstly arose from measurements of the $q^2$-dependent $B^0 \to K^{*0}\mu^+\mu^-$ and $B_s^0 \to \phi\mu^+\mu^-$ angular distributions and branching ratios at LHCb [89–95]. Experimental progress in this area is expected to be dominated by LHCb, with contributions from Belle II and the Phase-2 upgrades of ATLAS and
The absence of charged leptons in the final state – which removes theory uncertainties due to charm-loop effects [96] – makes $b \to s\ell^+\ell^-$ decays particularly interesting complementary probes of the non-SM physics scenarios proposed to explain the $b \to s\ell^+\ell^-$ anomalies [97]. Belle II will be the only experiment capable of exploring these key channels in the next decades. As an example, it has the potential to observe $B^+ \to K^+\nu\bar{\nu}$ decays at the SM rate with only $5 \text{ab}^{-1}$ of integrated luminosity and severely constrain various non-SM extensions such as models with leptoquarks, axions, feebly interacting, or dark-matter particles [28]. Given that many NP models predict the largest effects for the 3rd generation, additional complementary information will arise from improved searches at LHCb and Belle II of $b \to s\tau^+\tau^-$ decays. For example, the limits on $B(B^0_s \to \tau^+\tau^-)$ and $B(B^+ \to K^+\tau^+\tau^-)$ are both expected to improve by an order of magnitude or more at the end of LHCb Upgrade II [2,98]; and Belle II is expected to improve current limits on the $B^0 \to \tau^+\tau^-$ and $B^0 \to K^{*0}\tau^+\tau^-$ branching fractions by factors of 20 and 4, respectively, with $50\text{ab}^{-1}$ of integrated luminosity [28,74].

LHCb and Belle II will access a wide range of $b \to s(d)\gamma$ decays. Belle II is expected to have the best sensitivity in the next decade to observables based on exclusive $B$ decays, reaching on many percent or sub-percent precision with $50 \text{ab}^{-1}$ of integrated luminosity [28,74]. LHCb has unique access to time-dependent $CP$-violation observables in $B^0_s \to \phi\gamma$ decays, for which an improved calorimeter during Upgrade II would be critical to keep systematic uncertainties comparable or below the expected statistical uncertainties [36,45]. Belle II will also study these transitions inclusively. The precise and reliable SM prediction of the inclusive $B \to X_s\gamma$ rate, where $X_s$ identifies a particle or system of particles with strangeness, makes it an extremely sensitive probe for beyond-SM dynamics [99,100]. In addition, the inclusive analysis enables the determination of observables like the $b$-quark mass and can provide input to determinations of $|V_{ub}|$ [74]. Depending on the assumed detector performance in rejecting neutral hadrons faking photons, Belle II will reach a relative precision on $B(B \to X_s\gamma)$ between 2% and 4% with the $50 \text{ab}^{-1}$ data set, which is comparable to the theory prediction. Moreover, it will have the ability to explore the photon-energy spectrum at much lower energies than before [28,74].

Rare and forbidden decays of charm hadrons probe beyond-SM contributions in $c \to u$ transitions and are therefore complementary to searches done in the $b$ sector. Despite the SM rate being dominated by long-distance dynamics, the effective GIM cancellation and (approximate) symmetries of the charm system allow to define various null-test observables related to angular distributions and $CP$ violation, which have high discovery potential in the near future [101]. First measurements of such observables in $D^0 \to K^+K^-\mu^+\mu^-$ and $D^0 \to \pi^+\pi^-\mu^+\mu^-$ decays have been recently performed at LHCb [102]. The experimental precision has already reached $0.1-0.01$ and, for the $D^0 \to K^+K^-\mu^+\mu^-$ results, the overall agreement with the SM is at the level of $2.7\sigma$. These measurements are expected to reach sub-percent precision during Upgrade II, and will be complemented by studies of other rare charm decays and by tests of lepton-flavor universality [75]. Lepton-flavor universality tests and unique studies of $c \to u\nu\bar{\nu}$ decays are expected to be possible also at $e^+e^-$ colliders [38,48,74].

### 2.1.3 Precise CKM-unitarity tests and new sources of $CP$ violation

Precise tests of CKM unitarity remain crucial in constraining dynamics beyond the SM, particularly if new sources of $CP$ violation are present. Figure 7 shows the current constraints on the
The angle $\gamma$ is the only $CP$-violation parameter of the SM that can be measured exclusively from tree-level processes, such as from the interference between $B^-(\to D^f\to f)K^-$ and $B^-\to D^0(\to f)K^-$ decay amplitudes where $f$ is any final state directly accessible to both $D^0$ and $\bar{D}^0$ mesons. The theoretical uncertainties enter only at the level of one-loop electroweak corrections and are below $O(10^{-6})$, because all the required hadronic matrix elements can be experimentally measured when enough final states $f$ are taken into account. New $CP$ violating effects in non-leptonic tree-level decays can modify the SM relation between $\gamma$ and the CKM elements by several degrees, making precise ($\simeq 1^\circ$) measurements of $\gamma$ powerful probes of NP [104, 105]. The present experimental uncertainty on $\gamma$ is about 4$^\circ$ [12], dominated by LHCb measurements [62] with important inputs from CLEO and BESIII [106,107], and it is limited by statistics. The experimental progress in the next decades should bring the uncertainty down to 0.35$^\circ$, provided that strong-phase differences between $D^0$ and $\bar{D}^0$ amplitudes are measured with sufficient precision. The sample of coherent $D^0\bar{D}^0$ pairs expected to be collected at BESIII will contribute 0.4$^\circ$ to the $\gamma$ uncertainty. Hence, either larger samples of $\psi(3770)\to D^0\bar{D}^0$ data will need to be collected (e.g., at the STCF), or new methods to constrain these hadronic parameters will need to be developed.

The smallest and least well-known CKM matrix element magnitude is $|V_{ub}|$. In the standard unitarity triangle, $|V_{ub}|$ constrains the length of the side opposite to the precisely measured angle $\beta$ (Fig. 7). On the other hand, $|V_{cb}|$ normalizes the triangle and indirectly enters in SM predictions for many processes in flavor physics. For example, in the $b$ sector, $B(B_s^0\to \mu^+\mu^-)$ is approximately proportional to $|V_{ub}|^2$; in the kaon sector, $B(K_L^0\to \pi^0\nu\bar{\nu})$ and $\epsilon_K$ behave like $|V_{ub}|^4$ and $|V_{cb}|^{3.4}$, respectively [109]. For both $|V_{ub}|$ and $|V_{cb}|$, there are persistent tensions between exclusive and inclusive determinations from semileptonic $b$-hadron decays, as shown in Fig. 8. The reason for this discrepancy is unknown. Most indications point to possibly inconsistent experimental or theory...
The precision with semileptonic decays is limited by the lattice-QCD computation of the decay rate. Unique contributions from LHCb will come from measurements of rates of a variety of decays performed by CLEO, BaBar, Belle, and BESIII, together with lattice calculations of the corresponding form factors or decay constants [12, 108, 113–115]. The most precise determinations yield 1%-level uncertainties, with sub-percent precision recently achieved for $|V_{cb}|$ from $D \to K \ell^+\nu$ [116]. The precision with semileptonic decays is limited by the lattice-QCD computation of the decay rate.

Figure 8: (Left) Constraints in the $|V_{cb}|$-$|V_{ub}|$ plane from exclusive measurements and lattice QCD (colored bands, and their combination yielding the black point in the center) as well as inclusive measurements and operator-product expansions (blue point in the upper right) [108]. (Right) Projections of uncertainties in exclusive measurements of $|V_{ub}|$ from $B^0 \to \pi^- \ell^+\nu$ decays as functions of integrated luminosity at Belle II [74]. Projections are separately made for analysis in which the partner $B$ meson is reconstructed (tagged) or not (untagged), and for current or future (expected) lattice QCD inputs.

inputs, but interpretations in terms of non-SM physics cannot be excluded [110]. Improving both the exclusive and inclusive determinations of $|V_{ub}|$ and $|V_{cb}|$ is a high priority that will require a combined experiment-theory effort. Experimentally, Belle II will drive the global progress throughout the next decades. With the 50 ab$^{-1}$ of integrated luminosity, it will achieve $\mathcal{O}(1\%)$ precision on inclusive $|V_{ub}|$ and will double the global precision in exclusive $|V_{ub}|$ results, independently of any improvement in theoretical inputs. With the same sample, Belle II will reach $\mathcal{O}(1\%)$ precision on both inclusive and exclusive determinations of $|V_{cb}|$. Expected progress in lattice QCD will then offer further significant improvement as, e.g., shown in Fig. 8 for $|V_{ub}|$, and as discussed in Sec. 4. Unique contributions from LHCb will come from measurements of rates of a variety of $b$-hadron decays. LHCb has already performed measurements of $|V_{cb}|$ using $B_s^0 \to D_s^{*+} \mu^+\nu$ decays and of the ratio $|V_{ub}|/|V_{cb}|$ using semileptonic $A_0^b$ and $B_s^0$ decays [66, 111, 112]. These offer complementary sensitivity, particularly because they are subject to different theory inputs. The planned detector improvements in Upgrade II will significantly enhance the opportunities for more studies based on other $B_s^0$ decays modes and on $B^+_c$ mesons.

Determinations of $|V_{cs}|$ and $|V_{cd}|$ rely on rate measurements of leptonic and semileptonic charm decays performed by CLEO, BaBar, Belle, and BESIII, together with lattice calculations of the corresponding form factors or decay constants [12, 108, 113–115]. The most precise determinations yield 1%-level uncertainties, with sub-percent precision recently achieved for $|V_{cs}|$ from $D \to K \ell^+\nu$ [116].
form-factors. Measurements with leptonic decays are, instead, limited by experimental uncertainties. The relative uncertainties in $|V_{cs}|$ and $|V_{cd}|$ from leptonic decays are expected to be reduced at BESIII from 2.6% and 1.2% to approximately 1.1% and 0.9%, respectively [38]. Further improvement will be possible when combining with measurements at Belle II [74] and at the STCF, provided that systematic uncertainties can be reduced well below the 1% level [48].

New sources of $CP$ violation departing from the CKM picture can be effectively searched for in neutral meson mixing, where the knowledge of the $CP$-violating mixing phases are limited by the experimental statistical uncertainties. This is particularly true for the $B^0_s$-$\bar{B}^0_s$ mixing phase $\phi_s \approx -2\beta_s$, which is suppressed by a factor $0.05$ compared to the $B^0$-$\bar{B}^0$ phase $\beta$. The world average value, based on measurements from the LHC and Tevatron experiments, has a precision of 19 mrad [12]. The SM prediction, taken as the indirect determination of $-2\beta_s$ via the global CKM fit to experimental data, has an uncertainty of 0.8 mrad [103]. Such estimation, however, neglects contribution to $\phi_s$ from subleading penguin amplitudes that are estimated to be smaller than 21 mrad [117]. Improved measurements of $\phi_s$ will be performed at LHCb, ATLAS and CMS, with a combined projected uncertainty based exclusively on the $B^0_s \to J/\psi \phi$ channel estimated to go below 4 mrad. To possibly expose NP, the expected progress in experimental precision will require improved constraints on the SM penguin contributions, which can be achieved by measuring $SU(3)$-related decays [75, 118, 119]. Other avenues for beyond-SM sources of $CP$ violation are penguin-dominated $b \to q\bar{q}s$ decays such as $B^0_s \to \phi\phi$ or $B^0_s \to K^{*0}\bar{K}^{*0}$. The sensitivities expected at LHCb on these channels are shown in Fig. 9. Similarly, in the $B^0$ system, time-dependent $CP$ asymmetries in $B^0 \to \phi K^0_S$ and $B^0 \to \eta' K^0_S$ can be compared to determinations of $\sin 2\beta$ based on tree-dominated $b \to c\bar{c}s$ transitions, such as $B^0 \to J/\psi K^0_S$ [4].

Charmless $B$ decays give access to $\alpha$, the least known angle of the CKM unitarity triangle, which also suffers from much larger theory uncertainties compared to other CKM angles. Appropriate combinations of measurements from decays related by isospin symmetries, such as $B^0 \to (\pi\pi)^0$, $(\rho\pi)^0$, $(\rho\rho)^0$ and $a_1^+\pi^\mp$, reduce the impact of hadronic uncertainties and yield a robust direct determinations of $\alpha$ with a 4° uncertainty [12]. The determination of $\alpha$ is expected to be dominated

Figure 9: Projected sensitivity at LHCb for $\phi_s$ using various decay modes [45]. The SM prediction and its uncertainty [103] is also shown as the grey band.
Figure 10: Projected uncertainty as a function of the expected Belle II sample size in (left) the decay-time-integrated $CP$ asymmetry of $B^0 \to \pi^0\pi^0$ decays and (right panel) in the $B \to K\pi$ isospin sum rule $I_{K\pi}$. The solid red curve shows the projection assuming updates on the complete set of $B \to K\pi$ measurements. The dashed grey curve represents the projection assuming no Belle II inputs [28].

by Belle II with a projected uncertainty of $0.6^\circ$ with $50\, \text{ab}^{-1}$, provided that there is also an improved understanding of the size of isospin breaking (e.g., using $B \to \pi\eta'$ decays). Isospin symmetry also provides so-called sum rules, which are linear combinations of branching fraction and $CP$ asymmetries that offer null tests of the SM. An interesting case is provided by the so-called $K\pi$ puzzle, a long-standing $3\sigma$-deviation anomaly associated with the difference between direct $CP$ asymmetries in $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$ decays, and more generally from the isospin sum rule $I_{K\pi}$ relating $B^0 \to K^+\pi^-$, $B^+ \to K^0\pi^+$, $B^+ \to K^+\pi^0$ and $B^0 \to K^0\pi^0$ decays [120]. Since $I_{K\pi}$ is predicted to be zero within $\mathcal{O}(1\%)$ in the SM, a precise determination of all inputs offers a reliable and precise null test of the SM. The current experimental sensitivity of $\sim 13\%$, limited by the BaBar and Belle measurements of the $B^0 \to K^0\pi^0$ mode, is expected to be improved by Belle II in the next decade as shown in Fig. 10.

Unique opportunities to search for new sources of $CP$ violation in the up-type-quark sector are provided by the study of charm hadrons. LHCb made the first observation of $CP$ violation in charm decays in 2019 by measuring a nonzero difference in the $CP$ asymmetries of $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays, $\Delta A_{CP}$ [121]. In contrast to $b$ decays, loop amplitudes in charm are severely suppressed by the GIM mechanism and SM $CP$ violation arises mostly from the interference of tree-level amplitudes, possibly associated with rescattering [122, 123]. Rescattering amplitudes are challenging to compute and make the interpretation of the observed $CP$ violation ambiguous. Precise measurements of $CP$ asymmetries in other decay channels help constrain the rescattering effect and are therefore crucial to understand the underlying dynamics [4]. The Cabibbo-suppressed decay $D^+ \to \pi^+\pi^0$ is a particularly interesting mode as it proceeds essentially via a single tree amplitude in the SM [124, 125]. Thus, observing direct $CP$ violation in $D^+ \to \pi^+\pi^0$ would be a robust indication of new dynamics. While LHCb will continue to dominate the precision on $CP$ asymmetries in decay modes with charged particles in the final state [75], in the next decade Belle II
Figure 11: Projected sensitivity at LHCb to the parameters of CP violation in charm mixing, $|q/p|$ and $\phi_D$, assuming the current central values of experimental observables [45]. Contours shaded with different levels of darkness indicate 68.3% and 95.4% confidence-level regions.

is expected to dominate the precision on $A_{CP}(D^+ \rightarrow \pi^+\pi^0)$ and will be the only experiment able to precisely measure modes with only neutrals [28], such as $D^0 \rightarrow \pi^0\pi^0$ in which CP violation is expected to be of a measureable size [126].

LHCb’s recent observation of a nonzero mass difference between neutral charm eigenstates [72] paves the way for future precision measurements of mixing parameters and searches of CP-violation effects in $D^0, \bar{D}^0$ mixing. The current constraints on CP violation in charm mixing are at least an order of magnitude above the expected SM contribution and are limited by statistical experimental uncertainties [12,127,128]. The data sets expected from the operation of the Upgrade I and Upgrade II detectors make LHCb the only planned experiment with a realistic possibility of observing CP violation in charm mixing, since it has the best sensitivity to decay modes such as $D^0 \rightarrow K^-\pi^+$, $D^0 \rightarrow K^+K^-$, $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^0_S\pi^+\pi^-$ [75]. As examples, projected sensitivities for the mixing-induced CP-violation observables $A_\Gamma$ in $D^0 \rightarrow K^+K^-, \pi^+\pi^-$ and $\Delta x$ in $D^0 \rightarrow K^0_S\pi^+\pi^-$ are reported in Tab. 1; the projected sensitivity for the parameters $|q/p|$ and $\phi_D$, resulting from the combination of these and other measurements at LHCb, is shown in Fig. 11.

3 Farther into the future

The emergence of high-energy $e^+e^-$ circular collider projects to accurately study the properties of the Higgs boson opens a new appealing perspective for a continued heavy-quark-physics program past the HL-LHC era [25,129–132]. The proposed machines will operate at all the relevant electroweak thresholds ($Z^0$, $H^0$, $W^+W^-$, $tt\bar{t}$) and will give access to abundant samples of $b$- and $c$-hadron decays. As an example, the expected number of $Z^0$ decays to be collected with FCC-ee
will provide about 20 times more $B$ mesons, and about 9 times more charm hadrons, than expected at Belle II with 50 ab$^{-1}$ [131]. Moreover, similarly to LHCb, all $b$-flavored particles will be produced and with a significant boost to allow precise measurements of decay-time-dependent observables.

FCC-ee and/or CEPC will be able to complement many of the studies performed at Belle II and LHCb, and significantly extend the program in several critical areas. A particular strength will be the ability to make sensitive studies of modes containing neutral hadrons, photons and neutrinos, with much larger sample sizes than will be available at Belle II. This possibility will enable FCC-ee/CEPC to harness a wide range of charm-meson decay modes in measurements of $\gamma$ from $B^- \rightarrow DK^-$ and $B^0_s \rightarrow D^{*+}K^-$ decays. It is expected that the flavour-tagging efficiency will be significantly higher than at the LHC, bringing corresponding gains for time-dependent measurements of $B^0_s$ decays [133, 134]. Other interesting possibilities include modes relevant for the angle $\alpha$; e.g., precise measurements of the time-dependent CP asymmetries in $B^0 \rightarrow \pi^0\pi^0$ can be performed making use of both the $\gamma\gamma$ and $e^+e^-\gamma$ decays of the $\pi^0$ meson. At a high-energy $e^+e^-$ collider, another approach will open up for precise measurements of CKM matrix elements that has no systematic limitation due to the knowledge of hadronic inputs, e.g. from lattice QCD: direct determination from hadronic decays of $W^+$ bosons. Several $10^8 W^+$ boson decays will be collected when operating FCC-ee and/or CEPC at the $W^+W^-$ threshold and above, making possible, e.g., measurements of $|V_{cb}|$ with up to an order of magnitude improved precision with respect to present results. In addition to the measurement of CKM-related observables, FCC-ee/CEPC will perform studies of a wide range of suppressed flavor-changing neutral-current processes, such as $b \rightarrow s(d)\ell^+\ell^-$, $b \rightarrow s(d)\tau^+\tau^-$ and $b \rightarrow s(d)\nu\bar{\nu}$ (see, e.g., Refs. [135–137]). This program will be extended to the analysis of favored, but experimentally challenging, modes, where the SM predictions are reliable and beyond-SM effects could be pronounced (e.g., the decays $B^+_c \rightarrow \mu^+\nu$ and $B^+_c \rightarrow \tau^+\nu$ [138, 139]). The analysis of these channels, together with that of radiative flavor-changing neutral currents in both the beauty and charm sectors, will provide stringent tests of the SM and have high discovery potential for NP. Moreover, baryons with $b$ and $c$ quarks from $Z^0$ decays are strongly longitudinally polarized, giving access to new types of angular observables [140].

Measurements of weak decays of $b$ and $c$ quarks place specific demands on the detector design, particularly in the areas of vertexing, calorimetry and particle identification. These requirements are not necessarily the same as those required for electroweak and Higgs physics, and motivate a machine with four interaction points with one experiment devoted to heavy-flavor physics.

With a Higgs factory likely to represent the medium-term future of particle physics, the long-term future of the field crucially depends on the next generation of high-energy colliders to push forward the reach for direct production of NP particles [25]. Precise measurements of flavor observables in the next decades are likely to provide unique inputs that can have a major impact on the motivation and planning of such facilities. As an example, NP in $b \rightarrow s\mu^+\mu^-$ transitions would suggest that a muon collider of 10 TeV (or higher) has greater potential for a direct discovery than a 100 TeV $pp$ collider [13, 14].

4 Expected theory progress

The expected experimental advances in weak decays of $b$ and $c$ hadrons must be complemented by commensurate advances in theoretical calculations. In many cases, measurements involving
hadrons cannot be related to the underlying short-distance physics of interest (such as CKM matrix elements) without calculations of hadronic matrix elements, using lattice QCD or other approaches, such as the operator product expansion (OPE) for inclusive observables. In addition, the Wilson coefficients of weak effective operators need to be calculated in the SM and in NP models, and efforts in model building are required to find ultraviolet-complete explanations of observed deviations from the SM that ideally also address some of the other fundamental questions about the Universe outside of flavor physics.

While for some observables, such as purely leptonic $B^{0(s)}_s$-meson decay rates, theoretical predictions are currently more precise than experiment, the opposite is true for other observables, such as the CP asymmetries in hadronic charm decays where reliable SM predictions are still unavailable. For the determinations of $|V_{ub}|$ and $|V_{cb}|$ in semileptonic $b$-hadron decays, the uncertainties are currently more evenly split between experiment and theory, and both need to improve at a similar pace.

The following subsections summarize the status and prospects of theoretical calculations for selected important quantities or processes. Many of these topics are also discussed in the Theory-Frontier topical-group-6 report [141].

4.1 Exclusive leptonic and semileptonic decays

At leading order in weak effective theory and at leading order in QED, the nonperturbative QCD contributions to exclusive leptonic or semileptonic decays are given by matrix elements of the form $\langle 0 | \bar{q} \Gamma q | h \rangle$ or $\langle h' | \bar{q} \Gamma q | h \rangle$, respectively, where $h$ and $h'$ denote the hadrons in the initial and final states. These matrix elements can be expressed in terms of Lorentz-scalar decay constants or form factors, respectively, and are required inputs in the determination of $|V_{ub}|$, $|V_{cb}|$, $|V_{cd}|$, $|V_{cs}|$, in the theory predictions for lepton-flavor universality ratios, and in the theory of rare $b$ and $c$ decays. In general, studying the quark-level transitions of interest in multiple different decay channels involving hadrons with different spin helps in disentangling different operator structures of NP couplings. For example, the baryonic decay $\Lambda^0_b \to p\mu^-\bar{\nu}$ provides constraints on right-handed couplings in the $b \to u$ weak effective Hamiltonian which are complementary to those from $B \to \pi\mu^-\bar{\nu}$ [66,142,143]. Furthermore, theoretical predictions may also be needed for transitions to higher-lying states that contribute to the backgrounds in measurements, such as $B \to D^{**}\ell^+\nu$ [144–146].

Numerical lattice-gauge-theory computations now provide the most precise results for decay constants and for many form factors, and the precision can be improved even more in the future [108,113–115]. Light-cone sum rules and QCD factorization/soft-collinear effective theory (see, e.g., Refs. [147–156]) can be used for final states or kinematic regions that are challenging for lattice QCD. In addition, heavy-quark effective theory (HQET) can provide useful relations, especially for $b \to c$ transitions (see, e.g., Refs. [154,157]); here it is anticipated that long-standing questions about higher-order corrections can be answered with new experimental and lattice results in the coming years [110]. Another important theoretical tool is dispersion relations based on analyticity and unitarity, which yield bounds on form factors that can stabilize kinematic extrapolations [158].

For the charm-meson decay constants $f_D$ and $f_{D_s}$, the current averages of lattice results in pure QCD and in the isospin limit have total uncertainties of 0.3% and 0.2%, respectively, while the uncertainties for $f_B$ and $f_{B_s}$ are 0.7% and 0.6%, respectively. Key in achieving this precision was the use of the same type of relativistic lattice action for the heavy quarks as used for the light quarks,
which eliminates the systematic uncertainty associated with the matching of the vector and axial-vector currents from the lattice to the continuum. For bottom quarks, this approach still requires extrapolations in the heavy-quark mass, as most lattices used in the analyses have a spacing $a$ that is not small enough to satisfy $am_b < 1$. For semileptonic form factors, first calculations using the fully relativistic approach have been performed for $D \to K$ \cite{116, 159}, $B \to K$ \cite{159}, $B \to D^*$ \cite{160}, $B^0_s \to D_s^*$ \cite{161, 162}, $B^+_c \to J/\psi$ \cite{163}, and $B \to \pi$ \cite{164}. Reaching sub-percent precision for the $b$-hadron semileptonic form factors, will require large numbers of samples (gauge configurations/source locations) to reduce the statistical uncertainties (especially for $B \to \pi$, which is intrinsically noisier than, e.g., $B^0_s \to D_s^-$) and ultrafine lattices, demanding leadership-class computing resources \cite{165}.

If the hadron in the final state is a resonance with non-negligible decay width, a rigorous theoretical treatment requires computing transition matrix elements to the multi-hadron asymptotic final states to which the resonance couples, followed (if desired) by analytic continuation to the resonance pole. The mathematical formalism (known as the Lellouch-L"uisher formalism) relating the infinite-volume transition matrix elements of interest with the finite-volume transition matrix elements accessible on the lattice is well-established for $1 \to 2$ transitions \cite{166–175} and of the electromagnetic $\pi\gamma^* \to \rho(\to \pi\pi)$ transition \cite{181–183}. The formalism is also applicable to semileptonic decays with two-body resonances in the final state, such as $B \to \rho(\to \pi\pi)\ell^+\nu$, $B \to K^*(892)(\to K\pi)\ell^+\ell^-$, $B \to K^*_0(700)(\to K\pi)\ell^+\ell^-$. Here, it is possible to compute the $B \to \pi\pi$ or $B \to K\pi$ transition form factors for the relevant partial waves (e.g., $P$ wave and $S$ wave) as functions of both the dilepton invariant mass squared, $q^2$, and the $\pi\pi$ or $K\pi$ center-of-momentum energy squared, $s$. The analysis must take into account rescattering into other multi-hadron channels if $s$ is above their thresholds, and neglecting such channels thus places an upper limit on the accessible range of $s$. It is known how to take into account multiple coupled two-body channels (e.g., $\pi\pi + K\bar{K}$), and work is underway to extend the formalism to the three-body and higher sectors \cite{184, 185}. Dispersion theory can also be used to describe the $s$ dependence, see e.g. \cite{186}.

4.2 QED corrections to leptonic and semileptonic decays

With QCD uncertainties reduced to the sub-percent level, further theoretical improvements are needed in the treatment of QED corrections. This applies in particular to soft or collinear photons that can lead to large logarithms, and to hadron-structure-dependent effects, neither of which are accounted for by the commonly included Sirlin factor $\eta_{EW}$ \cite{187}. Soft photon radiation in experiments is modelled using PHOTOS \cite{188}, which however neglects radiation from charged initial-state particles and other important effects \cite{189}.

Significant progress in the treatment of QED corrections has been made recently in effective-field-theory-based approaches and factorization \cite{187, 189–194}. Sizeable hard-collinear logarithms were identified and were shown to be absent at the structure-dependent level using gauge invariance \cite{191}. Factorization in QCD×QED is still a rather new subject under active development. It must be noted that, as soon as non-perturbative soft matrix elements are evaluated in factorization including QED, light-cone distribution amplitudes need to be generalized accordingly \cite{195}.

First lattice-QCD calculations of structure-dependent QED corrections to leptonic decays have been performed for pion and kaon leptonic decays \cite{196, 197}, and this approach is in principle also
applicable to $B_{(s)}$ and $D_{(s)}$ leptonic decays. In Refs. [196, 197], real-photon emission was treated in the point-like approximation, which is sufficient for $\pi$ and $K$ decays to muons, but not for decays to electrons. Since then, full structure-dependent lattice calculations of real-photon emission in leptonic decays have also been performed [198–201]. These calculations are interesting not only in the context of QED corrections to leptonic decays but can also describe radiative leptonic decays with hard photons. The hard photon lifts the helicity suppression and, in the case of $B_{(s)}^0 \to \ell^+\ell^-\gamma$, provides sensitivity to a larger set of operators in the weak effective Hamiltonian. Furthermore, radiative leptonic $B$ decays at high photon energy are well suited to constrain the first inverse moment of the $B$-meson light-cone distribution amplitude, an important parameter in the theory of nonleptonic $B$ decays [202–217].

Recently, lattice calculations have also been demonstrated for $P \to \ell\nu\ell'^+\ell'^-$, where the $\ell'^+\ell'^-$ pair is produced through a virtual photon [218, 219]. Work is also underway to compute structure-dependent QED corrections to semileptonic decays on the lattice, which is substantially more challenging compared to leptonic decays [220]. The electroweak box diagrams have also been computed on the lattice for kaon semileptonic decays [221]. Further progress with QED corrections to semileptonic decays, and extensions of the calculations from light/strange mesons to charm and bottom mesons, would be desirable.

4.3 Exclusive rare $b$ and $c$ hadron decays

The theory uncertainties for rare $b$ and $c$ decays vary widely among different types of processes and observables [1, 2]. Muon-to-electron lepton-flavor-universality ratios such as $R_K$ are predicted to be very close to unity in the SM, and one of the main sources of uncertainty is QED, as discussed in the previous Section and in Ref. [1]. The branching fractions of the purely leptonic $B_{(s)}^0 \to \mu^+\mu^-$ are also predicted quite precisely, thanks to sub-percent-precision lattice-QCD results for the decay constants [222] and recent progress with perturbative QCD and QED corrections [87]; the dominant source of uncertainty in these branching fractions is now $|V_{cb}|$.

The theory of semileptonic $b \to s\ell^+\ell^-$ branching fractions and angular observables depends on both local hadronic matrix elements of quark-bilinear currents, described by local form factors, and nonlocal hadronic matrix elements involving four-quark or quark-gluon operators together with the quark electromagnetic current at a different spacetime point. Both are important sources of uncertainty. Higher-precision lattice calculations of the local form factors are expected in the future as discussed in Sec. 4.1. For the nonlocal matrix elements, the charm contributions are the most significant and problematic. At high $q^2$, the nonlocal matrix elements may be approximated using a local OPE [223, 224], but the OPE is unable to predict the detailed $q^2$-dependence associated with broad charmonium resonances in this region. At low $q^2$, the available approaches include QCD factorization [225] and the light-cone OPE [96, 226]. The latter calculation was recently further improved and combined with dispersive OPE bounds [227, 228]. The theory of the nonlocal matrix elements at low $q^2$ is more challenging for $A_0^0$ decays, where new types of nonfactorizable spectator-scattering contributions arise [229, 230]. In contrast to semileptonic modes with charged leptons, dineutrino modes ($b \to s\nu\nu\pi$) do not receive contributions from these nonlocal matrix elements and can be predicted precisely using just the local form factors. In $b \to d\ell^+\ell^-$ decays, non-local effects are qualitatively different as contributions from $\rho$ and $\omega$ resonances are not suppressed [2, 231, 232]. Controlling those effects will be crucial to establish exclusive $b \to d\ell^+\ell^-$ decays as important probes.
of NP [233].

Rare charm decays involving the transitions $c \to u \ell^+ \ell^-$ or $c \to u \gamma$ are subject to large long-distance contributions that dominate the short-distance contributions by orders of magnitude [234–237]. Nevertheless, as already mentioned, one can construct observables that are strongly suppressed in the SM and serve as clean null tests, such as $CP$ asymmetries and angular observables that vanish in the SM [238–243]. Additional probes for NP are lepton-flavor-violating transitions and dineutrino modes [241–244].

4.4 Inclusive semileptonic and radiative decays

The standard approach for calculating the inclusive $B \to X_c \ell^\mp \nu$ decay rate is the heavy-quark expansion (HQE). Using the optical theorem, the inclusive rate is expressed in terms of a forward matrix element of a product of two weak currents at different spacetime points. This is then treated with the heavy-quark/operator-product expansion that yields a series of local operators, organized by powers of $1/m_b$ [110], where the leading term corresponds to the “partonic rate” and terms up to order $1/m_b^5$ are known at tree level. The hadronic matrix elements are fitted to experimental data [245, 246] (in principle, they can also be calculated using lattice QCD [247–249]), and the matching coefficients are calculated perturbatively. For the partonic rate, the full kinematic distribution is known at order $\alpha_s^2$, and the total rate at $\alpha_s^3$ [250]. The full kinematic distribution of the $1/m_b^2$ contribution and the total rate at order $1/m_b^3$ have been calculated at order $\alpha_s$ [251–253]. The theory uncertainty in inclusive $|V_{cb}|$ determinations has reached the 1% level [245]. Looking ahead, the proliferation of HQE parameters at higher orders can be reduced by considering reparametrization-invariant observables, in particular $q^2$ moments [254], as was already implemented in Ref. [246]. Higher-order $\alpha_s$ corrections can be calculated, and improvements in the heavy-quark mass determination as well as novel heavy-quark mass schemes may be beneficial. At sub-percent level, possible duality violations and problems of HQE convergence [255] as well as QED corrections should be investigated further. The $b \to u \ell^\mp \nu$ and $b \to c \tau^- (\to \ell^- \nu \nu \nu)$ backgrounds also need to be treated carefully [256].

The determination of $|V_{ub}|$ from inclusive $B \to X_u \ell^\mp \nu$ measurements is substantially more complicated due to the large charm background. Cutting away the $b \to c \ell^- \nu$ contribution with a requirement on the lepton energy leaves only the endpoint region with $2E_\ell/m_b \sim 1$, where the local HQE breaks down. In this region, one needs to use a light-cone OPE, such that the HQE parameters are replaced by nonlocal matrix elements, the so-called shape functions [257,258]. The shape functions can be evaluated through a combination of fits to the differential data and QCD-based modeling; see Ref. [110] for details and prospects for improvements.

Also very important are the inclusive rare decays $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$ [2], which can provide tight constraints on NP with different systematic uncertainties compared to the exclusive rare $b$ decays. The inclusive $B \to X_s \gamma$ branching fraction currently has a 5% theory uncertainty [99], while Belle II may reach 2% as shown in Tab. 1. The theory uncertainty can be reduced further by completing the NNLO QCD corrections without interpolation in the charm mass and by controlling nonperturbative effects that are expected to give few-% contributions. For $B \to X_s \ell^+ \ell^-$ decays at low $q^2$ the situation is similar [100]. At high $q^2$ the uncertainties are larger due to the breakdown of the HQE, as with $B \to X_u \ell^\mp \nu$ decays. At high $q^2$ it is advantageous to normalize the $B \to X_s \ell^+ \ell^-$ rate to the $B \to X_u \ell^\mp \nu$ rate with the same kinematic cut [259].
Finally, significant progress in lattice QCD has been made in direct computations of the forward matrix elements of the two weak currents that are needed to describe inclusive decay rates. Extracting the hadronic tensor from a Euclidean four-point function requires solving an inverse Laplace transform. This is an ill-posed problem in principle, but the severity of the problem can be reduced by limiting the energy resolution, which at the same time controls finite-volume effects [260–264] (see also Ref. [265] for earlier work). Exploratory computations of inclusive semileptonic decay rates, along with a comparison to OPE predictions, have been performed successfully at a lower-than physical $b$-quark mass [262,266,267]. This paves the way for further calculations with physical parameters and controlled systematic uncertainties.

4.5 Neutral-meson mixing

The mass differences $\Delta m_d$ and $\Delta m_s$ in $B^0$-$\bar{B}^0$ and $B^0_s$-$\bar{B}^0_s$ mixing are known from experimental measurements with an extraordinary precision of 0.4% and 0.03%, respectively [12]. The theoretical uncertainties are at the 10% level [268,269]. Reducing them will therefore have a significant impact on CKM constraints and on many NP models, including models proposed as explanations of the $b \to s\ell^+\ell^-$ anomalies.

The theoretical description of $\Delta m_d$ and $\Delta m_s$ at leading-order in weak effective theory involves the hadronic matrix elements $\langle \bar{B}^0_{(s)} | Q_i | B^0_{(s)} \rangle$ of five local four-fermion $\Delta B = 2$ operators $Q_i$, of which only $Q_1$ contributes in the SM (see the caption of Fig. 4). These matrix elements can be calculated using lattice QCD [268, 270–274] or sum rules [275–279]. Lattice results for the $SU(3)_{\text{breaking}}$ ratio $\xi$, formed using the ratio of $B^0_{(s)}$ and $B^0$-meson mixing parameters, have reached percent-level precision [268, 271–274, 280–282]. For the mixing parameters themselves, there are currently some tensions among the lattice results from different groups employing different renormalization schemes, lattice discretizations, and numbers of dynamical quark flavors that need to be understood and resolved. It is likely possible to achieve sub-percent uncertainties within the next five years, at which point QED effects become important [114]. The sum rules estimates can also be further improved by determining $1/m_b$ corrections to the strict HQET limit, by determining NNLO-QCD corrections to the QCD-HQET matching or by considering the sum rule in full QCD [3].

For the width difference $\Delta \Gamma_s$, a large contribution to the theory uncertainty is due to the hadronic matrix elements of dimension-7 operators. A first lattice-QCD calculation of these operators has been reported in Ref. [283], leading to a SM prediction of $\Delta \Gamma_s = 0.092 \pm 0.014 \text{ ps}^{-1}$, which can be compared to the current experimental average of $0.084 \pm 0.005 \text{ ps}^{-1}$ [12]. Hence, further substantial improvements in the theory uncertainties would have a big impact, particularly given that the experimental uncertainties at the LHC are projected to decrease even further [37,75].

In $D^0$-$\bar{D}^0$ mixing, the $CP$-violating contributions can be described reasonably well by local hadronic matrix elements of $\Delta C = 2$ operators. These matrix elements have already been determined via sum rules [277] and lattice QCD [284–286] with uncertainties of order 5-10%, and further improvements are possible using standard methods in the coming years. However, the overall mixing process is dominated by long-distance effects, corresponding to hadronic matrix elements of two $\Delta C = 1$ operators at different spacetime points. Analogous nonlocal matrix elements have been computed on the lattice for the kinematically simpler case of kaon mixing [287–289], but substantial further theoretical and algorithmic developments are needed to extend this work to the case of charm mixing. It may be possible to employ novel methods similar to those developed for lattice
calculations of inclusive decay rates (cf. Sec. 4.4) [113].

4.6 Heavy-hadron lifetimes

Comparisons between theory and experiment for heavy-hadron lifetimes, or lifetime ratios, can also provide useful constraints on physics beyond the SM. Examples are the constraints on beyond-SM explanations of the $R(D^{*})$ anomalies from $\tau_{B^{\pm}}$ [290], and constraints from $\tau_{B^{0}}/\tau_{B^{0}}$ on NP in $b \to s\tau^{+}\tau^{-}$, on $B$-meson-based baryogenesis, and on NP in nonleptonic decays such as $b \to c\bar{c}s$ [291]. A recent experimental surprise is the LHCb measurement of the $\Omega_{c}^{0}$ lifetime, which rearranged the hierarchy of the charm-baryon lifetimes [292–296].

Similar to inclusive semileptonic decays, the standard theoretical tool is the HQE (and some of the Wilson coefficients and hadronic matrix elements are shared between these applications). Significant sources of theoretical uncertainty for $b$-hadron lifetimes are spectator effects described by four-quark operators, and the Darwin term [291, 295, 297–300]. There have been exploratory lattice calculations of spectator effects more than 20 years ago [301–305]. New state-of-the-art lattice calculations would be desirable to complement sum-rule calculations.

4.7 Nonleptonic bottom-hadron decays

Nonleptonic bottom-hadron decays are widely used to study $CP$ violation, both in the SM and beyond. As discussed in Sec. 2.1.3, the CKM angles $\alpha$, $\beta$, and $\gamma$ can be determined from combinations of multiple nonleptonic $B_{(s)}$ decay modes that allow the dominant hadronic matrix elements to be obtained from the experimental data. However, except for the case of $\gamma$ [306], subleading effects will no longer be negligible at the increased precision expected from experiments in the coming years, and need to be understood theoretically. An example is the $V_{ub}$ (“penguin”) contamination in $B^{0} \to J/\psi K_{S}^{0}$ and $B_{s}^{0} \to J/\psi \phi$, which affects determination of the CKM angles $\beta$ and $\beta_{s}$.

Flavor-symmetry relations are widely used in theoretical studies of nonleptonic $b$ decays [307–311]. Calculating hadronic matrix elements for nonleptonic $b$ decays directly is very challenging. Commonly used approaches are perturbative QCD, QCD factorization/soft-collinear effective theory, and light-cone sum rules, often combined with an expansion in $1/m_{b}$ [117,203,312–330]. Recently, factorization was also extended to include QED [192,331]. Naturally, one may ask whether lattice-QCD calculations of nonleptonic $b$-hadron decays are possible. The lattice methods developed for exclusive decays with two or three hadrons in the final state [166–175,184,185] are insufficient for processes like $B \to \pi\pi$ due to the high center-of-mass energy of the mesons in the final state, at which rescattering to many different states with more than three hadrons would affect the finite-volume matrix elements. However, lattice calculations can contribute in other ways to the theory of nonleptonic $b$ decays, for example by providing first-principles predictions of the $B$-meson light-cone distribution amplitude [332–335].

An interesting puzzle has emerged concerning the branching fractions of $B_{(s)}^{0} \to D_{(s)}^{\ast \pm}\{\pi^{+}, K^{+}\}$ decays, where improved QCD-factorization predictions are several standard deviations higher than experimental measurements [323,336]. The estimated theoretical uncertainties include, for the first time, the $O(\Lambda_{QCD}/m_{b})$ corrections. The decays considered include some of the simplest, color-allowed processes that are unaffected by penguin or annihilation contributions, and it appears unlikely that large nonfactorizable effects can explain the deviations [337]. Quasi-elastic rescattering
effects were studied in Ref. [338], where it was found that such effects cannot simultaneously account for the measured branching ratios of the color-allowed and color-suppressed decays. Beyond-the-SM explanations of the puzzle have been explored, for example, in Refs. [339, 340], and may lead to large enhancements to $CP$ asymmetries that could be observed experimentally [341]. Significant deviations from the SM predictions using QCD factorization were recently also found in dedicated measurements of the ratios between $\Gamma(B^0 \to D^{(*)}h)$ and $d\Gamma(B^0 \to D^{(*)} \ell^- \nu)/dq^2|_{q^2=m_b^2}$ [342].

While much of the work to date has focused on processes with $B(s)$ mesons, the increased statistics from the continued running of the LHC will likely allow to observe $CP$ violation also in $b$-baryon decays. A non-vanishing $CP$ asymmetry is a measure of direct $CP$ violation, as baryons and antibaryons do not undergo mixing because of baryon number conservation. Theoretical studies of $CP$ violation in $b$-baryon decays can be found, e.g., in Refs. [343–351].

4.8 Nonleptonic charm-hadron decays

In charm decays, the size of diagrams from penguin operators is determined by the ratio $m_b/m_W \ll 1$, which is much smaller than in the case of bottom decays, where the relevant ratio is $m_t/m_W > 1$. It follows that penguin operators are usually not relevant for charm decays, and the GIM mechanism is extremely effective. In singly Cabibbo-suppressed decays, $CP$ violation in the SM is proportional to the small non-unitary contribution of the relevant $2 \times 2$ submatrix of the CKM matrix.

Direct charm $CP$ violation has been observed by LHCb in a difference of direct $CP$ asymmetries, $\Delta a_{CP}^{dir} \equiv a_{CP}^{dir}(D^0 \to K^+K^-) - a_{CP}^{dir}(D^0 \to \pi^+\pi^-)$, with the result $(-0.161 \pm 0.028)\%$ [12]. The SM prediction is of order $\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{QCD}$, with $r_{QCD}$ being a ratio of pure low-energy QCD amplitudes. Calculating the QCD amplitudes from first principles is even more challenging than in the case of nonleptonic $b$ decays, due to the stronger QCD coupling at the lower energy and the lower heavy-quark mass, meaning that many of the theoretical tools discussed in the previous Section are less suitable. Depending on the methods used to estimate $r_{QCD}$, one arrives at very different interpretations of the result [122, 123, 352–357]. Further work is clearly needed. The theoretical understanding can also be improved by combining measurements of the $CP$ asymmetries in all singly-Cabibbo-suppressed charm-meson decays, taking advantage of flavor-$SU(3)$ sum rules. In addition, the long-term prospects for direct lattice-QCD calculations of the relevant QCD amplitudes using the Lellouch-Lüscher approach [166–175, 184, 185] are better than for nonleptonic $B$ decays, due to the smaller number of open multi-hadron channels at $\sqrt{s} \sim m_D$.

Like in the bottom sector, there are also interesting opportunities to study charm $CP$ violation in decays of baryons. Theoretical aspects are discussed in Refs. [358–363].

4.9 Model building

The ultimate goal of the efforts discussed in this report is to find clear manifestations of physics beyond the SM, and to narrow down the structure and parameters of what theory will replace the SM. The efforts to construct new candidate theories, i.e., beyond-SM model building, are discussed from a general point of view in the Theory-Frontier topical-group-8 report [364] and, in the context of flavor physics, in Refs. [1, 365].

Beyond-SM model building may be approached from different directions. Many models are primarily designed to address questions relating to naturalness problems, dark matter, or baryoge-
nescis. Nevertheless, such models often lead to new sources of flavor-changing interactions that may be observed in weak decays of \(b\) or \(c\) quarks, leading to tight constraints. On the other hand, the observation of deviations from the SM in weak decays of \(b\) or \(c\) quarks motivates a directed effort to build models that can explain these deviations while remaining consistent with other measurements. Typically, the measurements are initially interpreted in a model-independent way at the level of Wilson coefficients in a low-energy effective theory. If the scale of NP is far above the weak scale, the low-energy effective NP operators must be invariant under the SM gauge group. This condition imposes interesting new relations among different processes [16,366–369]. The next level could be simplified models with new particles capable of producing the observed deviations, but not subject to the requirement of providing a renormalizable, self-consistent theory. Finding an ultraviolet completion would then be the next step.

Many of the models constructed to explain the \(b \rightarrow s\ell^+\ell^-\) anomalies contain \(Z'\) bosons [370–389], typically with masses of a few TeV, and therefore within reach of the LHC or future colliders [390–395]. Another possibility are leptoquarks [368,369,396–414], which can arise in many different beyond-SM scenarios, including in composite models [397,402,410], in the MSSM with \(R\)-parity violation (in the form of the sbottom squark) [415–419], and as gauge bosons models in models with enlarged gauge groups [420–429]. Several models can simultaneously explain the \(b \rightarrow s\ell^+\ell^-\) and \(b \rightarrow c\tau^-\bar{\nu}\) anomalies, and/or simultaneously address other questions, for example by providing a dark-matter candidate. Interestingly, if requiring that a single mediator is responsible for the effects in both \(b \rightarrow s\ell^+\ell^-\) and \(b \rightarrow c\tau^-\bar{\nu}\) at tree level, there is a unique choice of leptoquark: the \(SU(3)\)-triplet, \(SU(2)\)-singlet, hypercharge-\(2/3\) vector leptoquark, usually denoted as \(U_1\). This leptoquark may be one of the gauge bosons in a generalized version [420,421,430–432] of the Pati-Salam grand unified theory [433].

5 Current and future U.S. involvement

The U.S. has been a leader in heavy-quark physics, involving a vigorous community and a series of extremely successful domestic experiments. The CLEO experiment at Cornell and the BaBar experiment at SLAC have been fundamental in today’s understanding of \(B\) and charm physics, and the CDF and D0 experiments at the Tevatron have pioneered the study of \(B^0\) mesons and \(b\)-baryons in addition to set the bases for precise heavy-quark physics at hadron colliders. Such a strong domestic program did not limit participation in offshore experiments, such as Belle at KEK in Japan. Since the shutdown of PEP-II and of the Tevatron, the U.S. heavy-flavor community has exclusively relied on offshore experiments, namely LHCb, Belle II and BESIII. About 60 physicists from six U.S. institutions participate to the LHCb collaboration. U.S. groups have lead the design, construction and commissioning of the upstream silicon tracker for Upgrade I; the development of trigger algorithms for real-time analysis; and several key physics measurements, including searches for lepton-flavor-universality violation, \(CP\)-violation in the \(B^0\) system, and discovery of tetra- and penta-quark states. The Belle II collaboration includes about 80 physicists from 17 U.S. institutions. BNL hosts a Tier 1 GRID computing facility for Belle II. U.S. groups have played major roles in the design, construction, commissioning, and operations of the Belle II hadron- and muon-identification detectors; in critical machine-detector interface studies of beam backgrounds; in management and leadership positions, and in data analysis. The BESIII collaboration currently
includes six physicists from three US institutions. The BESIII U.S. groups contribute to data analysis and have senior members holding significant leadership positions, such as in the publication committee. The combined physics output of LHCb, Belle II and BESIII is in the order of 220 publications per year\(^1\) – comparable to experiments at the energy frontier, which typically rely on much larger funding and U.S. participation.

International recognition of the importance of a continued heavy-flavor-physics program in the next decades is evident from the commitments in Europe and Asia. An example is the strong support given by the 2020 European Strategy Update to the study of heavy-flavor physics at the HL-LHC. For the U.S. HEP program to have the breadth to assure meaningful role in future discoveries, support for a significant U.S. participation in future heavy-flavor experiments needs to be assured. U.S. groups can have leading roles in the design and construction of detector and data-acquisition-system upgrades planned over the next ten years. Hence, U.S. contributions to LHCb Upgrade II and future upgrades of Belle II must be encouraged.

The successful experience of the LHC has demonstrated that experiments at energy-frontier facilities can be effectively complemented by experiments focusing on indirect searches for beyond-SM physics through the study of weak decays of \(b\) and \(c\) quarks. While the identification of the next energy-frontier facility will be mostly motivated by the need to understand the mechanism behind the electroweak-symmetry breaking and/or by the need to increase the reach of direct searches, the important role of heavy-quark physics should still be considered as key for a broad and rich HEP program. Strong U.S. participation in these efforts should also be encouraged.

The experimental progress in heavy-flavor physics has often benefited from a close collaboration with the theory community. The U.S. has strong theory groups working on quark-flavor physics, which are internationally recognized and influential. Scientists in the U.S. have pioneered heavy-quark effective theory, nonrelativistic QCD, heavy-hadron chiral perturbation theory, and lattice formulations for heavy quarks. Lattice gauge theory plays a crucial role for the physics program discussed in this report, and the U.S. has a very active lattice community. In flavor physics, the Fermilab Lattice, HPQCD, MILC, and RBC-UKQCD Collaborations are particularly influential.

The majority of lattice-field theory researchers in the U.S. are also members of the USQCD collaboration, which was founded in 1999 for the purpose of creating and utilizing software and dedicated hardware resources for lattice gauge theory calculations. As of April 2022, USQCD has 169 members. USQCD software is open-source and used widely by the worldwide community. In order for the strong theory and lattice efforts in the U.S. to continue, stable support for researchers and for computing resources is essential.

In the next decades, weak decays of \(b\) and \(c\) quarks will offer a unique opportunity to reveal NP that is not directly accessible at the energy frontier. A healthy U.S. HEP program will endeavor to be among the leaders in this research.

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\(^1\)With Belle II accumulating more and more data this number is expected to increase by at least 20% over the next decade.
References

[1] D. Guadagnoli and P. Koppenburg, *Lepton-flavor violation and lepton-flavor-universality violation in b and c decays*, in 2022 Snowmass Summer Study, 2022 [2207.01851].

[2] F. Archilli and W. Altmannshofer, *Rare decays of b and c hadrons*, in 2022 Snowmass Summer Study, 2022 [2206.11331].

[3] A. Lenz and S. Monteil, *High precision in CKM unitarity tests in b and c decays*, in 2022 Snowmass Summer Study, 2022 [2207.11055].

[4] A. Dery, Y. Grossman, S. Schacht and D. Tonelli, *CP violation in b and c quark decays*, in 2022 Snowmass Summer Study, 2022 [2209.07429].

[5] F. Mahmoudi, *New constraints on supersymmetric models from b → sγ*, *JHEP* 12 (2007) 026 [0710.3791].

[6] A.G. Akeroyd, F. Mahmoudi and D. Martinez Santos, *The decay B → μ⁺μ⁻: updated SUSY constraints and prospects*, *JHEP* 12 (2011) 088 [1108.3018].

[7] O. Atkinson, M. Black, C. Englert, A. Lenz, A. Rusov and J. Wynne, *The Flavourful Present and Future of 2HDMs at the Collider Energy Frontier*, 2202.08807.

[8] L.-S. Geng, B. Grinstein, S. Jäger, S.-Y. Li, J. Martin Camalich and R.-X. Shi, *Implications of new evidence for lepton-universality violation in b → sℓ⁺ℓ⁻ decays*, *Phys. Rev. D* 104 (2021) 035029 [2103.12738].

[9] W. Altmannshofer and P. Stangl, *New physics in rare B decays after Moriond 2021*, *Eur. Phys. J. C* 81 (2021) 952 [2103.13370].

[10] T. Hurth, F. Mahmoudi, D.M. Santos and S. Neshatpour, *More indications for lepton nonuniversality in b → sℓ⁺ℓ⁻*, *Phys. Lett. B* 824 (2022) 136838 [2104.10058].

[11] M. Algueró, B. Capdevila, S. Descotes-Genon, J. Matias and M. Novoa-Brunet, *b → sℓ⁺ℓ⁻ global fits after R_{K^0} and R_{K^{*+}}*, *Eur. Phys. J. C* 82 (2022) 326 [2104.08921].

[12] Y. Amhis et al., *Averages of b-hadron, c-hadron, and τ-lepton properties as of 2021*, 2206.07501, and online updates at https://hflav.web.cern.ch.

[13] W. Altmannshofer, S.A. Gadam and S. Profumo, *Probing New Physics with μ⁺μ⁻ → bs at a Muon Collider*, in 2022 Snowmass Summer Study, 2022 [2203.07496].
[14] A. Azatov, F. Garosi, A. Greljo, D. Marzocca, J. Salko and S. Trifinopoulos, *New Physics in $b\to s\mu\mu$: FCC-hh or a Muon Collider?*, 2205.13552.

[15] T. Bose et al., *Physics Beyond the Standard Model at Energy Frontier*, in *2022 Snowmass Summer Study*, 2022.

[16] M. Bordone, G. Isidori and S. Trifinopoulos, *Semileptonic B-physics anomalies: A general EFT analysis within \(U(2)^n\) flavor symmetry*, Phys. Rev. D96 (2017) 015038 [1702.07238].

[17] M. Bordone, D. Buttazzo, G. Isidori and J. Monnard, *Probing Lepton Flavour Universality with \(K\to\pi\nu\bar{\nu}\) decays*, Eur. Phys. J. C77 (2017) 618 [1705.10729].

[18] S. Fajfer, N. Koˇ snik and L. Vale Silva, *Footprints of leptoquarks: from \(R_{K^{(*)}}\) to \(K\to\pi\nu\bar{\nu}\)*, Eur. Phys. J. C78 (2018) 275 [1802.00786].

[19] M. Borsato, V.V. Gligorov, D. Guadagnoli, D. Martinez Santos and O. Sumensari, *Effective-field-theory arguments for pursuing lepton-flavor-violating \(K\) decays at LHCb*, Phys. Rev. D 99 (2019) 055017 [1808.02006].

[20] S. Descotes-Genon, S. Fajfer, J.F. Kamenik and M. Novoa-Brunet, *Implications of \(b\to s\mu\mu\) anomalies for future measurements of \(B\to K^{(*)}\nu\bar{\nu}\) and \(K\to\pi\nu\bar{\nu}\)*, Phys. Lett. B 809 (2020) 135769 [2005.03734].

[21] L. Di Luzio, M. Kirk and A. Lenz, *\(B_s^0-\bar{B}_s^0\) mixing interplay with \(B\) anomalies*, in *10th International Workshop on the CKM Unitarity Triangle*, 11, 2018 [1811.12884].

[22] J. Charles, S. Descotes-Genon, Z. Ligeti, S. Monteil, M. Papucci, K. Trabelsi et al., *New physics in \(B\) meson mixing: future sensitivity and limitations*, Phys. Rev. D 102 (2020) 056023 [2006.04824].

[23] J. de Blas, M. Ciuchini, E. Franco, S. Mishima, M. Pierini, L. Reina et al., *Electroweak precision observables and Higgs-boson signal strengths in the Standard Model and beyond: present and future*, JHEP 12 (2016) 135 [1608.01509].

[24] J. Ellis, C.W. Murphy, V. Sanz and T. You, *Updated Global SMEFT Fit to Higgs, Diboson and Electroweak Data*, JHEP 06 (2018) 146 [1803.03252].

[25] M. Narain, L. Reina, A. Tricoli et al., *Energy Frontier Report*, in *2022 Snowmass Summer Study*, 2022.

[26] UTfit collaboration, *Model-independent constraints on \(\Delta F = 2\) operators and the scale of new physics*, JHEP 03 (2008) 049 [0707.0636].

[27] A. Cerri et al., *Opportunities in Flavour Physics at the HL-LHC and HE-LHC*, CERN Yellow Rep. Monogr. 7 (2019) 867 [1812.07638].

[28] Belle II collaboration, *Belle II physics reach and plans for the next decade and beyond*, in *2022 Snowmass Summer Study*, 2022 [2207.06307].
[29] Projection of integrated luminosity delivered by SuperKEKB to Belle II, https://confluence.desy.de/display/BII/Belle+II+Luminosity, accessed on 2022-06-20.

[30] Belle II collaboration, The Belle II Detector Upgrade Program, in 2022 Snowmass Summer Study, 2022 [2203.11349].

[31] M. Endo et al., Japan’s Strategy for Future Projects in High Energy Physics, in 2022 Snowmass Summer Study, 2022 [2203.13979].

[32] A. Natochii et al., Beam background expectations for Belle II at SuperKEKB, in 2022 Snowmass Summer Study, 2022 [2203.05731].

[33] Longer term LHC schedule, https://lhc-commissioning.web.cern.ch/schedule/LHC-long-term.htm, accessed on 2022-06-02.

[34] BESIII collaboration, Future Physics Programme of BESIII, Chin. Phys. C 44 (2020) 040001 [1912.05983].

[35] STCF Working Group collaboration, Physics Program of the Super Tau-Charm Factory, PoS BEAUTY2020 (2021) 060.

[36] LHCb collaboration, Future physics potential of LHCb, in 2022 Snowmass Summer Study, 2022 [LHCb-PUB-2022-012].

[37] ATLAS and CMS collaboration, Physics with the Phase-2 ATLAS and CMS Detectors, in 2022 Snowmass Summer Study, 2022 [ATL-PHYS-PUB-2022-018, CMS PAS FTR-22-001].

[38] BESIII collaboration, Physics in the \( \tau \)-charm Region at BESIII, in 2022 Snowmass Summer Study, 2022 [2204.08943].

[39] A. Giri, Y. Grossman, A. Soffer and J. Zupan, Determining gamma using \( B^\pm \to D K^\pm \) with multibody \( D \) decays, Phys. Rev. D68 (2003) 054018 [hep-ph/0303187].

[40] A. Bondar and A. Poluektov, Feasibility study of model-independent approach to \( \phi_3 \) measurement using Dalitz plot analysis, Eur. Phys. J. C47 (2006) 347 [hep-ph/0510246].

[41] A. Bondar and A. Poluektov, The use of quantum-correlated \( D^0 \) decays for \( \phi_3 \) measurement, Eur. Phys. J. C55 (2008) 51 [0801.0840].

[42] A. Bondar, A. Poluektov and V. Vorobiev, Charm mixing in the model-independent analysis of correlated \( D^0 \bar{D}^0 \) decays, Phys. Rev. D82 (2010) 034033 [1004.2350].

[43] A. Di Canto, J. Garra Ticó, T. Gershon, N. Jurik, M. Martinelli, T. Pilař et al., Novel method for measuring charm-mixing parameters using multibody decays, Phys. Rev. D 99 (2019) 012007 [1811.01032].

[44] A. Poluektov, Unbinned model-independent measurements with coherent admixtures of multibody neutral \( D \) meson decays, Eur. Phys. J. C 78 (2018) 121 [1712.08326].
[45] LHCb collaboration, Framework TDR for the LHCb Upgrade II – Opportunities in flavour physics, and beyond, in the HL-LHC era, Tech. Rep. CERN-LHCC-2021-012, LHCb-TDR-023 (2021).

[46] P. Barbeau, P. Merkel, J. Zhang et al., Instrumentation Frontier Report, in 2022 Snowmass Summer Study, 2022.

[47] D.M. Asner et al., Upgrading SuperKEKB with a Polarized Electron Beam: Discovery Potential and Proposed Implementation, in 2022 Snowmass Summer Study, 2022 [2205.12847].

[48] H.-Y. Cheng, X.-R. Lyu and Z.-Z. Xing, Charm Physics in the High-Luminosity Super \( \tau \)-Charm Factory, in 2022 Snowmass Summer Study, 2022 [2203.03211].

[49] LHCb collaboration, Test of lepton universality in beauty-quark decays, Nature Phys. 18 (2022) 277 [2103.11769].

[50] LHCb collaboration, Test of lepton universality with \( B^0 \to K^{*0}\ell^+\ell^- \) decays, JHEP 08 (2017) 055 [1705.05802].

[51] Belle collaboration, Measurement of \( R(D) \) and \( R(D^*) \) with a semileptonic tagging method, Phys. Rev. Lett. 124 (2020) 161803 [1910.05864].

[52] LHCb collaboration, Analysis of neutral \( B \)-meson decays into two muons, Phys. Rev. Lett. 128 (2022) 041801 [2108.09284].

[53] LHCb collaboration, Measurement of the \( B_s^0 \to \mu^+\mu^- \) decay properties and search for the \( B^0 \to \mu^+\mu^- \) and \( B_s^0 \to \mu^+\mu^-\gamma \) decays, Phys. Rev. D 105 (2022) 012010 [2108.09283].

[54] BaBar collaboration, Search for \( B^+ \to K^+\tau^+\tau^- \) at the BaBar experiment, Phys. Rev. Lett. 118 (2017) 031802 [1605.09637].

[55] Belle collaboration, Search for the decay \( B^0 \to K^{*0}\tau^+\tau^- \) at the Belle experiment, 2110.03871.

[56] BaBar collaboration, Precision Measurement of the \( B \to X_s\gamma \) Photon Energy Spectrum, Branching Fraction, and Direct CP asymmetry \( A_{CP}(B \to X_{s+d}\gamma) \), Phys. Rev. Lett. 109 (2012) 191801 [1207.2690].

[57] Belle collaboration, Measurement of the \( \overline{B} \to X_s\gamma \) Branching Fraction with a Sum of Exclusive Decays, Phys. Rev. D 91 (2015) 052004 [1411.7198].

[58] BaBar, Belle collaboration, The Physics of the \( B \) Factories, Eur. Phys. J. C 74 (2014) 3026 [1406.6311].
Belle collaboration, *Precise measurement of the CP violation parameter sin2φ₁ in B⁰ \to (c\bar{c})K⁰ decays*, *Phys. Rev. Lett.* **108** (2012) 171802 [1201.4643].

LHCb collaboration, *Simultaneous determination of CKM angle γ and charm mixing parameters*, *JHEP* **12** (2021) 141 [2110.02350].

LHCb collaboration, *Updated measurement of time-dependent CP-violating observables in B_{s}^{0} \to J/ψK^{+}K^{-} decays*, *Eur. Phys. J. C* **79** (2019) 706 [1906.08356], Erratum *Eur. Phys. J. C* **80** (2020) 601.

LHCb collaboration, *Study of B \to ωK_{S}^{0}, η′K_{S}^{0}, and π^{0}K_{S}^{0} decays*, *Phys. Rev. D* **83** (2011) 052003 [0809.1174].

LHCb collaboration, *Measurement of CP asymmetries in B\to K^{*}π^{0} decays*, *Phys. Rev. D* **81** (2010) 011101 [0809.4366].

LHCb collaboration, *Search for CP violation in D_{(s)}^{+} \to h^{+}π^{0} and D_{(s)}^{+} \to h^{+}η decays*, *JHEP* **06** (2021) 019 [2103.11058].

LHCb collaboration, *Observation of the Mass Difference Between Neutral Charm-Meson Eigenstates*, *Phys. Rev. Lett.* **127** (2021) 111801 [2106.03744].

LHCb collaboration, *Search for time-dependent CP violation in D^{0} \to K^{+}K^{-} and D^{0} \to π^{+}π^{-} decays*, *Phys. Rev. D* **104** (2021) 072010 [2105.09889].

BELLE II collaboration, *The Belle II Physics Book*, *PTEP* **2019** (2019) 123C01 [1808.10567], Erratum *PTEP* **2020** (2020) 029201.

LHCb collaboration, *Physics case for an LHCb Upgrade II – Opportunities in flavour physics, and beyond, in the HL-LHC era*, 1808.08865.

A. Mauri, N. Serra and R. Silva Coutinho, *Towards establishing lepton flavor universality violation in B \to K^{*}ℓ^{+}ℓ^{-} decays*, *Phys. Rev. D* **99** (2019) 013007 [1805.06401].
[77] A. Sibidanov, T.E. Browder, S. Dubey, S. Kohani, R. Mandal, S. Sandilya et al., *A New Tool for Detecting BSM Physics in B → K^±\ell^- Decays*, in 2022 Snowmass Summer Study, 2022 [2203.06847].

[78] T. Keck et al., *The Full Event Interpretation: An Exclusive Tagging Algorithm for the Belle II Experiment*, *Comput. Softw. Big Sci.* **3** (2019) 6 [1807.08680].

[79] M. Duraisamy and A. Datta, *The full B → D^{(*)}\tau^-\nu_\tau angular distribution and CP violating triple products*, *JHEP* **09** (2013) 059 [1302.7031].

[80] Z. Ligeti, M. Papucci and D.J. Robinson, *New Physics in the Visible Final States of B → D^{(*)}\tau^+\nu_\tau*, *JHEP* **01** (2017) 083 [1610.02045].

[81] D. Hill, M. John, W. Ke and A. Poluektov, *Model-independent method for measuring the angular coefficients of B^0 → D^{**}\tau^-\nu_\tau decays*, *JHEP* **11** (2019) 133 [1908.04643].

[82] B. Bhattacharya, A. Datta, S. Kamali and D. London, *A measurable angular distribution for B^0 → D^{*}\tau^-\nu_\tau decays*, *JHEP* **07** (2020) 194 [2005.03032].

[83] F.U. Bernlochner, S. Duell, Z. Ligeti, M. Papucci and D.J. Robinson, *Das ist der HAMMER: Consistent new physics interpretations of semileptonic decays*, *Eur. Phys. J. C* **80** (2020) 883 [2002.00020].

[84] C. Bobeth, M. Bordone, N. Gubernari, M. Jung and D. van Dyk, *Lepton-flavour non-universality of B_q → \ell^+\ell^- angular distributions in and beyond the Standard Model*, *Eur. Phys. J. C* **81** (2021) 984 [2104.02094].

[85] B. Bhattacharya, T. Browder, Q. Campagna, A. Datta, S. Dubey, L. Mukherjee et al., *A new tool to search for physics beyond the Standard Model in B^0 → D^{*}\ell^-\nu_\ell*, in 2022 Snowmass Summer Study, 2022 [2203.07189].

[86] B. Bhattacharya, T.E. Browder, Q. Campagna, A. Datta, S. Dubey, L. Mukherjee et al., *Implications for the \Delta A_{FB} anomaly in B^0 → D^{*}\ell^-\nu_\ell using a new Monte Carlo Event Generator*, 2206.11283.

[87] M. Beneke, C. Bobeth and R. Szafron, *Power-enhanced leading-logarithmic QED corrections to B_q → \mu^+\mu^-, JHEP* **10** (2019) 232 [1908.07011].

[88] K. De Bruyn, R. Fleischer, R. Knegjens, P. Koppenburg, M. Merk, A. Pellegrino et al., *Probing new physics via the B_s^0 → \mu^+\mu^- effective lifetime*, *Phys. Rev. Lett.* **109** (2012) 041801 [1204.1737].

[89] LHCb collaboration, *Measurement of Form-Factor-Independent Observables in the Decay B^0 → K^{*0}\mu^+\mu^-, Phys. Rev. Lett.* **111** (2013) 191801 [1308.1707].

[90] LHCb collaboration, *Differential branching fraction and angular analysis of the decay B^0 → K^{*0}\mu^+\mu^-, JHEP* **08** (2013) 131 [1304.6325].
S. Descotes-Genon, J. Matias and J. Virto, *Understanding the B → K∗μ+μ− Anomaly*, Phys. Rev. D 88 (2013) 074002 [1307.5683].

R.R. Horgan, Z. Liu, S. Meinel and M. Wingate, *Calculation of B0 → K∗0μ+μ− and B0 → f′(1525)μ+μ− observables using form factors from lattice QCD*, Phys. Rev. Lett. 112 (2014) 212003 [1310.3887].

LHCb collaboration, *Branching Fraction Measurements of the Rare B0s → φμ+μ− and B0s → f′(1525)μ+μ− Decays*, Phys. Rev. Lett. 127 (2021) 151801 [2105.14007].

A. Khodjamirian, T. Mannel, A.A. Pivovarov and Y.M. Wang, *Charm-loop effect in B → K(l+)+l− and B → K∗γ*, JHEP 09 (2010) 089 [1006.4945].

T.E. Browder, N.G. Deshpande, R. Mandal and R. Sinha, *Impact of B → Kνν measurements on beyond the Standard Model theories*, Phys. Rev. D 104 (2021) 053007 [2107.01080].

C. Cornella, G. Isidori, M. König, S. Liechti, P. Owen and N. Serra, *Hunting for B+ → K+τ+τ− imprints on the B+ → K+μ+μ− dimuon spectrum*, Eur. Phys. J. C 80 (2020) 1095 [2001.04470].

M. Misiak, A. Rehman and M. Steinhauser, *Towards B → Xsγ at the NNLO in QCD without interpolation in mc*, JHEP 06 (2020) 175 [2002.01548].

T. Huber, T. Hurth, J. Jenkins, E. Lunghi, Q. Qin and K.K. Vos, *Phenomenology of inclusive B → Xsγ for the Belle II era*, JHEP 10 (2020) 088 [2007.04191].

H. Gisbert, M. Golz and D.S. Mitzel, *Theoretical and experimental status of rare charm decays*, Mod. Phys. Lett. A 36 (2021) 213002 [2011.09478].

LHCb collaboration, *Angular Analysis of D0 → π+π−μ+μ− and D0 → K+K−μ+μ− Decays and Search for CP Violation*, Phys. Rev. Lett. 128 (2022) 221801 [2111.03327].

CKMfitter group collaboration, *Current status of the Standard Model CKM fit and constraints on ∆F = 2 new physics*, Phys. Rev. D91 (2015) 073007 [1501.05013], updated results and plots available at http://ckmfitter.in2p3.fr/.

J. Brod, A. Lenz, G. Tetlalmatzi-Xolocotzi and M. Wiebusch, *New physics effects in tree-level decays and the precision in the determination of the quark mixing angle γ*, Phys. Rev. D 92 (2015) 033002 [1412.1446].

A. Lenz and G. Tetlalmatzi-Xolocotzi, *Model-independent bounds on new physics effects in non-leptonic tree-level decays of B mesons*, JHEP 07 (2020) 177 [1912.07621].
CLEO collaboration, Model-independent determination of the strong-phase difference between $D^0$ and $\bar{D}^0 \to K_{S,L}^0 h^+ h^-$ ($h = \pi, K$) and its impact on the measurement of the CKM angle $\gamma/\phi_3$, Phys. Rev. D 82 (2010) 112006 [1010.2817].

BESIII collaboration, Model-independent determination of the relative strong-phase difference between $D^0$ and $\bar{D}^0 \to K_{S,L}^0 \pi^+ \pi^-$ and its impact on the measurement of the CKM angle $\gamma/\phi_3$, Phys. Rev. D 101 (2020) 112002 [2003.00091].

Y. Aoki et al., FLAG Review 2021, 2111.09849.

A.J. Buras and E. Venturini, Searching for New Physics in Rare K and B Decays without $|V_{cb}|$ and $|V_{ub}|$ Uncertainties, Acta Phys. Polon. B 53 (2021) A1 [2109.11032].

P. Gambino et al., Challenges in semileptonic B decays, Eur. Phys. J. C 80 (2020) 966 [2006.07287].

LHCb collaboration, Measurement of $|V_{cb}|$ with $B_s^0 \to D_s^{*-} \mu^+ \nu_\mu$ decays, Phys. Rev. D 101 (2020) 072004 [2001.03225].

LHCb collaboration, First observation of the decay $B_s^0 \to K^- \mu^+ \nu_\mu$ and Measurement of $|V_{ub}|/|V_{cb}|$, Phys. Rev. Lett. 126 (2021) 081804 [2012.05143].

C. Lehner, S. Meinel et al., Opportunities for Lattice QCD in Quark and Lepton Flavor Physics, Eur. Phys. J. A 55 (2019) 195 [1904.09479].

P.A. Boyle et al., A lattice QCD perspective on weak decays of $b$ and $c$ quarks, in 2022 Snowmass Summer Study, 5, 2022 [2205.15373].

USQCD collaboration, Lattice QCD and Particle Physics, in 2022 Snowmass Summer Study, 7, 2022 [2207.07641].

HPQCD collaboration, Improved $|V_{cs}|$ determination using precise lattice QCD form factors for $D \to K\ell\nu$, Phys. Rev. D 104 (2021) 034505 [2104.09883].

P. Frings, U. Nierste and M. Wiebusch, Penguin contributions to CP phases in $B_{d,s}$ decays to charmonium, Phys. Rev. Lett. 115 (2015) 061802 [1503.00859].

LHCb collaboration, Measurement of the CP-violating phase $\beta$ in $B^0 \to J/\psi \pi^+ \pi^-$ decays and limits on penguin effects, Phys. Lett. B 742 (2015) 38 [1411.1634].

LHCb collaboration, Measurement of CP violation parameters and polarisation fractions in $B_s^0 \to J/\psi \bar{K}^{*0}$ decays, JHEP 11 (2015) 082 [1509.00400].

M. Gronau, A Precise sum rule among four $B \to K\pi$ CP asymmetries, Phys. Lett. B 627 (2005) 82 [hep-ph/0508047].

LHCb collaboration, Observation of CP Violation in Charm Decays, Phys. Rev. Lett. 122 (2019) 211803 [1903.08726].

36
[122] Y. Grossman and S. Schacht, *The emergence of the $\Delta U = 0$ rule in charm physics*, JHEP 07 (2019) 020 [1903.10952].

[123] I. Bediaga, T. Frederico and P. Magalhaes, *Enhanced charm CP asymmetries from final state interactions*, 2203.04056.

[124] F. Buccella, M. Lusignoli, G. Mangano, G. Miele, A. Pugliese and P. Santorelli, *CP Violating asymmetries in charged D meson decays*, Phys. Lett. B 302 (1993) 319 [hep-ph/9212253].

[125] Y. Grossman, A.L. Kagan and J. Zupan, *Testing for new physics in singly Cabibbo suppressed D decays*, Phys. Rev. D 85 (2012) 114036 [1204.3557].

[126] D. Wang, *Evidence of $A_{CP}(D^0 \rightarrow \pi^+\pi^-)$ implies observable CP violation in the $D^0 \rightarrow \pi^0\pi^0$ decay*, 2207.11053.

[127] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, *How large can the SM contribution to CP violation in $D^0$-$\bar{D}^0$ mixing be?*, JHEP 03 (2010) 009 [1002.4794].

[128] A.L. Kagan and L. Silvestrini, *Dispersive and absorptive CP violation in $D^0$-$\bar{D}^0$ mixing*, Phys. Rev. D 103 (2021) 053008 [2001.07207].

[129] P.C. Bhat et al., *Future Collider Options for the US*, in 2022 Snowmass Summer Study, 2022 [2203.08088].

[130] Z. Liu and L.-T. Wang, *Physics at Future Colliders: the Interplay Between Energy and Luminosity*, in 2022 Snowmass Summer Study, 2022 [2205.00031].

[131] G. Bernardi et al., *The Future Circular Collider: a Summary for the US 2021 Snowmass Process*, in 2022 Snowmass Summer Study, 3, 2022 [2203.06520].

[132] CEPC Physics Study Group collaboration, *The Physics potential of the CEPC. Prepared for the US Snowmass Community Planning Exercise (Snowmass 2021)*, in 2022 Snowmass Summer Study, 5, 2022 [2205.08553].

[133] R. Aleksan, L. Oliver and E. Perez, *CP violation and determination of the bs flat unitarity triangle at an FCC-ee*, Phys. Rev. D 105 (2022) 053008 [2107.02002].

[134] X. Li, M. Ruan and M. Zhao, *Prospect for measurement of CP-violation phase $\phi_s$ study in the $B^0_s \rightarrow J/\psi\phi$ channel at future Z factory*, 2205.10565.

[135] J.F. Kamenik, S. Monteil, A. Semkiv and L.V. Silva, *Lepton polarization asymmetries in rare semi-tauonic $b \rightarrow s$ exclusive decays at FCC-ee*, Eur. Phys. J. C 77 (2017) 701 [1705.11106].

[136] L. Li and T. Liu, *$b \rightarrow s\tau^+\tau^-$ physics at future Z factories*, JHEP 06 (2021) 064 [2012.00665].

[137] L. Li, M. Ruan, Y. Wang and Y. Wang, *Analysis of $B_s \rightarrow \phi\nu\bar{\nu}$ at CEPC*, Phys. Rev. D 105 (2022) 114036 [2201.07374].
[138] T. Zheng, J. Xu, L. Cao, D. Yu, W. Wang, S. Prell et al., Analysis of $B_c \to \tau \nu$ at CEPC, *Chin. Phys. C* **45** (2021) 023001 [2007.08234].

[139] Y. Amhis, M. Hartmann, C. Helsens, D. Hill and O. Sumensari, Prospects for $B_c^+ \to \tau^+ \nu$ at FCC-ee, *JHEP* **12** (2021) 133 [2105.13330].

[140] G. Hiller and A. Kagan, Probing for new physics in polarized $\Lambda_b$ decays at the $Z$, *Phys. Rev. D* **65** (2002) 074038 [hep-ph/0108074].

[141] R. Boughezal and Z. Ligeti, Theory Techniques for Precision Physics, in 2022 Snowmass Summer Study, 2022.

[142] W. Detmold, C. Lehner and S. Meinel, $\Lambda_b \to p \ell^- \bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks, *Phys. Rev. D* **92** (2015) 034503 [1503.01421].

[143] J. Albrecht, F. Bernlochner, M. Kenzie, S. Reichert, D. Straub and A. Tully, Future prospects for exploring present day anomalies in flavour physics measurements with Belle II and LHCb, 1709.10308.

[144] F.U. Bernlochner and Z. Ligeti, Semileptonic $B_{(s)}$ decays to excited charmed mesons with $e, \mu, \tau$ and searching for new physics with $R(D^{**})$, *Phys. Rev. D* **95** (2017) 014022 [1606.09300].

[145] F.U. Bernlochner, Z. Ligeti and D.J. Robinson, Model independent analysis of semileptonic $B$ decays to $D^{**}$ for arbitrary new physics, *Phys. Rev. D* **97** (2018) 075011 [1711.03110].

[146] F.U. Bernlochner, M.F. Sevilla, D.J. Robinson and G. Wormser, Semitauonic $b$-hadron decays: A lepton flavor universality laboratory, *Rev. Mod. Phys.* **94** (2022) 015003 [2101.08326].

[147] P. Ball and R. Zwicky, $B_{d,s} \to \rho, \omega, K^*, \phi$ decay form-factors from light-cone sum rules revisited, *Phys. Rev. D* **71** (2005) 014029 [hep-ph/0412079].

[148] Y.-M. Wang and Y.-L. Shen, QCD corrections to $B \to \pi$ form factors from light-cone sum rules, *Nucl. Phys. B* **898** (2015) 563 [1506.00667].

[149] Y.-M. Wang and Y.-L. Shen, Perturbative Corrections to $\Lambda_b \to A$ Form Factors from QCD Light-Cone Sum Rules, *JHEP* **02** (2016) 179 [1511.09036].

[150] S. Cheng, A. Khodjamirian and J. Virto, $B \to \pi \pi$ Form Factors from Light-Cone Sum Rules with $B$-meson Distribution Amplitudes, *JHEP* **05** (2017) 157 [1701.01633].

[151] N. Gubernari, A. Kokulu and D. van Dyk, $B \to P$ and $B \to V$ Form Factors from $B$-Meson Light-Cone Sum Rules beyond Leading Twist, *JHEP* **01** (2019) 150 [1811.00983].

[152] T. Feldmann, D. Van Dyk and K.K. Vos, Revisiting $B \to \pi \ell \nu$ at large dipion masses, *JHEP* **10** (2018) 030 [1807.01924].

[153] S. Descotes-Genon, A. Khodjamirian and J. Virto, Light-cone sum rules for $B \to K \pi$ form factors and applications to rare decays, *JHEP* **12** (2019) 083 [1908.02267].
M. Bordone, M. Jung and D. van Dyk, *Theory determination of $B \to D^{(*)} \ell^- \bar{\nu}$ form factors at $O(1/m_c^2)$*, *Eur. Phys. J. C* **80** (2020) 74 [1908.09398].

J. Gao, T. Huber, Y. Ji, C. Wang, Y.-M. Wang and Y.-B. Wei, *$B \to D\ell\nu$ form factors beyond leading power and extraction of $|V_{cb}|$ and $R(D)$*, *JHEP* **05** (2022) 024 [2112.12674].

A. Khodjamirian and M. Wald, *$B$-meson decay into a proton and dark antibaryon from QCD light-cone sum rules*, *Phys. Lett. B* **834** (2022) 137434 [2206.11601].

F.U. Bernlochner, Z. Ligeti, D.J. Robinson and W.L. Sutcliffe, *Precise predictions for $\Lambda_b \to \Lambda_c$ semileptonic decays*, *Phys. Rev. D* **99** (2019) 055008 [1812.07593].

I. Caprini, *Functional Analysis and Optimization Methods in Hadron Physics*, SpringerBriefs in Physics, Springer (2019), 10.1007/978-3-030-18948-8.

W.G. Parrott, C. Bouchard and C.T.H. Davies, *$B \to K$ and $D \to K$ form factors from fully relativistic lattice QCD*, 2207.12468.

T. Kaneko, Y. Aoki, B. Colquhoun, M. Faur, H. Fukaya, S. Hashimoto et al., *$B \to D^{(*)}\ell\nu$ semileptonic decays in lattice QCD with domain-wall heavy quarks*, in 38th International Symposium on Lattice Field Theory, 12, 2021 [2112.13775].

E. McLean, C.T.H. Davies, J. Koponen and A.T. Lytle, *$B_s \to D_s\ell\nu$ Form Factors for the full $q^2$ range from Lattice QCD with non-perturbatively normalized currents*, *Phys. Rev. D* **101** (2020) 074513 [1906.00701].

HPQCD collaboration, *$B_s \to D^*_s$ form factors for the full $q^2$ range from lattice QCD*, *Phys. Rev. D* **105** (2022) 094506 [2105.11433].

HPQCD collaboration, *$B_c \to J/\psi$ form factors for the full $q^2$ range from lattice QCD*, *Phys. Rev. D* **102** (2020) 094518 [2007.06957].

JLQCD collaboration, *Form factors of $B \to \pi\ell\nu$ and a determination of $|V_{ub}|$ with Möbius domain-wall-fermions*, *Phys. Rev. D* **106** (2022) 054502 [2203.04938].

P. Boyle et al., *Lattice QCD and the Computational Frontier*, in 2022 Snowmass Summer Study, 2022 [2204.00039].

M. Lüscher, *Volume Dependence of the Energy Spectrum in Massive Quantum Field Theories*, *Commun. Math. Phys.* **105** (1986) 153.

M. Lüscher, *Signatures of unstable particles in finite volume*, *Nucl. Phys. B* **364** (1991) 237.

L. Lellouch and M. Lüscher, *Weak transition matrix elements from finite volume correlation functions*, *Commun. Math. Phys.* **219** (2001) 31 [hep-lat/0003023].

C.J.D. Lin, G. Martinelli, C.T. Sachrajda and M. Testa, *$K \to \pi\pi$ decays in a finite volume*, *Nucl. Phys. B* **619** (2001) 467 [hep-lat/0104006].
[170] N.H. Christ, C. Kim and T. Yamazaki, *Finite volume corrections to the two-particle decay of states with non-zero momentum*, Phys. Rev. D 72 (2005) 114506 [hep-lat/0507009].

[171] M.T. Hansen and S.R. Sharpe, *Multiple-channel generalization of Lellouch-Luscher formula*, Phys. Rev. D 86 (2012) 016007 [1204.0826].

[172] R.A. Briceño, M.T. Hansen and A. Walker-Loud, *Multichannel 1 → 2 transition amplitudes in a finite volume*, Phys. Rev. D 91 (2015) 034501 [1406.5965].

[173] R.A. Briceño and M.T. Hansen, *Multichannel 0 → 2 and 1 → 2 transition amplitudes for arbitrary spin particles in a finite volume*, Phys. Rev. D 92 (2015) 074509 [1502.04314].

[174] A. Agadjanov, V. Bernard, U.-G. Meißner and A. Rusetsky, *The B → K*\(^*\) form factors on the lattice*, Nucl. Phys. B 910 (2016) 387 [1605.03386].

[175] R.A. Briceño, J.J. Dudek and L. Leskovec, *Constraining 1 + J→2 coupled-channel amplitudes in finite-volume*, Phys. Rev. D 104 (2021) 054509 [2105.02017].

[176] T. Blum et al., *The K → (ππ)\(_{I=2}\) Decay Amplitude from Lattice QCD*, Phys. Rev. Lett. 108 (2012) 141601 [1111.1699].

[177] RBC, UKQCD collaboration, *Standard Model Prediction for Direct CP Violation in K → ππ Decay*, Phys. Rev. Lett. 115 (2015) 212001 [1505.07863].

[178] N. Ishizuka, K.I. Ishikawa, A. Ukawa and T. Yoshié, *Calculation of K → ππ decay amplitudes with improved Wilson fermion action in non-zero momentum frame in lattice QCD*, Phys. Rev. D 98 (2018) 114512 [1809.03893].

[179] RBC, UKQCD collaboration, *Direct CP violation and the ΔI = 1/2 rule in K → ππ decay from the standard model*, Phys. Rev. D 102 (2020) 054509 [2004.09440].

[180] RBC, UKQCD collaboration, *Lattice determination of I=0 and 2 ππ scattering phase shifts with a physical pion mass*, Phys. Rev. D 104 (2021) 114506 [2103.15131].

[181] R.A. Briceno, J.J. Dudek, R.G. Edwards, C.J. Shultz, C.E. Thomas and D.J. Wilson, *The resonant π⁺γ → π⁺π⁰ amplitude from Quantum Chromodynamics*, Phys. Rev. Lett. 115 (2015) 242001 [1507.06622].

[182] R.A. Briceño, J.J. Dudek, R.G. Edwards, C.J. Shultz, C.E. Thomas and D.J. Wilson, *The ππ → πγ\(^*\) amplitude and the resonant ρ → πγ\(^*\) transition from lattice QCD*, Phys. Rev. D 93 (2016) 114508 [1604.03530].

[183] C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies et al., *πγ → ππ transition and the ρ radiative decay width from lattice QCD*, Phys. Rev. D 98 (2018) 074502 [1807.08357].

[184] F. Müller and A. Rusetsky, *On the three-particle analog of the Lellouch-Lüscher formula*, JHEP 03 (2021) 152 [2012.13957].
M.T. Hansen, F. Romero-López and S.R. Sharpe, Decay amplitudes to three hadrons from finite-volume matrix elements, *JHEP* 04 (2021) 113 [2101.10246].

X.-W. Kang, B. Kubis, C. Hanhart and U.-G. Meißner, $B_d$ decays and the extraction of $|V_{ub}|$, *Phys. Rev. D* 89 (2014) 053015 [1312.1193].

A. Sirlin, Large $m_W$, $m_Z$ Behavior of the $\mathcal{O}(\alpha)$ Corrections to Semileptonic Processes Mediated by $W$, *Nucl. Phys. B* 196 (1982) 83.

P. Golonka and Z. Was, *PHOTOS Monte Carlo: A Precision tool for QED corrections in $Z$ and $W$ decays*, *Eur. Phys. J. C* 45 (2006) 97 [hep-ph/0506026].

R. Szafron, *QED Corrections: Open Challenges*, in CKM 2021, 2021.

M. Beneke, C. Bobeth and R. Szafron, Enhanced electromagnetic correction to the rare $B$-meson decay $B_{s,d} \to \mu^+\mu^-$, *Phys. Rev. Lett.* 120 (2018) 011801 [1708.09152].

G. Isidori, S. Nabebaccus and R. Zwicky, *QED corrections in $\overline{B} \to K \ell^+\ell^-$ at the double-differential level*, *JHEP* 12 (2020) 104 [2009.09029].

M. Beneke, P. Böer, G. Finauri and K.K. Vos, *QED factorization of two-body non-leptonic and semi-leptonic $B$ to charm decays*, *JHEP* 10 (2021) 223 [2107.03819].

G. Isidori, D. Lancierini, S. Nabebaccus and R. Zwicky, *QED in $B \to K \ell^+\ell^-$ LFU ratios: Theory versus Experiment, a Monte Carlo Study*, 2205.08635.

R. Zwicky, *Notes on QED Corrections in Weak Decays*, *Symmetry* 13 (2021) 2036 [2205.06194].

M. Beneke, P. Böer, J.-N. Toelstede and K.K. Vos, *Light-cone distribution amplitudes of light mesons with QED effects*, *JHEP* 11 (2021) 059 [2108.05589].

D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula et al., *First lattice calculation of the QED corrections to leptonic decay rates*, *Phys. Rev. Lett.* 120 (2018) 072001 [1711.06537].

M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo et al., *Light-meson leptonic decay rates in lattice QCD+QED*, *Phys. Rev. D* 100 (2019) 034514 [1904.08731].

C. Kane, C. Lehner, S. Meinel and A. Soni, *Radiative leptonic decays on the lattice*, *PoS LATTICE2019* (2019) 134 [1907.00279].

A. Desiderio et al., *First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons*, *Phys. Rev. D* 103 (2021) 014502 [2006.05358].

R. Frezzotti, M. Garofalo, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo et al., *Comparison of lattice QCD+QED predictions for radiative leptonic decays of light mesons with experimental data*, *Phys. Rev. D* 103 (2021) 053005 [2012.02120].
[201] C. Kane, D. Giusti, C. Lehner, S. Meinel and A. Soni, Controlling unwanted exponentials in lattice calculations of radiative leptonic decays, PoS LATTICE2021 (2022) 162 [2110.13196].

[202] G.P. Korchemsky, D. Pirjol and T.-M. Yan, Radiative leptonic decays of B mesons in QCD, Phys. Rev. D61 (2000) 114510 [hep-ph/9911427].

[203] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, QCD factorization for $B \to \pi \pi$ decays: Strong phases and CP violation in the heavy quark limit, Phys. Rev. Lett. 83 (1999) 1914 [hep-ph/9905312].

[204] S. Descotes-Genon and C.T. Sachrajda, Factorization, the light cone distribution amplitude of the $B$ meson and the radiative decay $B \to \gamma \ell \nu_\ell$, Nucl. Phys. B650 (2003) 356 [hep-ph/0209216].

[205] E. Lunghi, D. Pirjol and D. Wyler, Factorization in leptonic radiative $B \to \gamma e \nu$ decays, Nucl. Phys. B649 (2003) 349 [hep-ph/0210091].

[206] V.M. Braun and A. Khodjamirian, Soft contribution to $B \to \gamma \ell \nu_\ell$ and the $B$-meson distribution amplitude, Phys. Lett. B718 (2013) 1014 [1210.4453].

[207] Y.-M. Wang, Factorization and dispersion relations for radiative leptonic $B$ decay, JHEP 09 (2016) 159 [1606.03080].

[208] M. Beneke, V.M. Braun, Y. Ji and Y.-B. Wei, Radiative leptonic decay $B \to \gamma \ell \nu_\ell$ with subleading power corrections, JHEP 07 (2018) 154 [1804.04962].

[209] Y.-M. Wang and Y.-L. Shen, Subleading-power corrections to the radiative leptonic $B \to \gamma \ell \nu$ decay in QCD, JHEP 05 (2018) 184 [1803.06667].

[210] Y.-L. Shen, Z.-T. Zou and Y.-B. Wei, Subleading power corrections to $B \to \gamma \ell \nu$ decay in PQCD approach, Phys. Rev. D 99 (2019) 016004 [1811.08250].

[211] A. Khodjamirian, R. Mandal and T. Mannel, Inverse moment of the $B_s$-meson distribution amplitude from QCD sum rule, JHEP 10 (2020) 043 [2008.03935].

[212] M. Beneke, C. Bobeth and Y.-M. Wang, $B_{d,s} \to \gamma \ell \bar{\ell}$ decay with an energetic photon, JHEP 12 (2020) 148 [2008.12494].

[213] Y.-L. Shen, Y.-B. Wei, X.-C. Zhao and S.-H. Zhou, Revisiting radiative leptonic $B$ decay, Chin. Phys. C 44 (2020) 123106 [2009.03480].

[214] A. Carvunis, F. Dettori, S. Gangal, D. Guadagnoli and C. Normand, On the effective lifetime of $B_s \to \mu \mu \gamma$, JHEP 12 (2021) 078 [2102.13390].

[215] L.-S. Lu, Factorization of radiative leptonic $D$-meson decay with sub-leading power corrections, Chin. Phys. C 45 (2021) 073101 [2104.01562].

[216] T. Janowski, B. Pullin and R. Zwicky, Charged and neutral $B_{u,d,s} \to \gamma$ form factors from light cone sum rules at NLO, JHEP 12 (2021) 008 [2106.13616].
[217] B. Pullin and R. Zwicky, *Radiative decays of heavy-light mesons and the $f_{H,H^*}^{(T)}$ decay constants*, JHEP 09 (2021) 023 [2106.13617].

[218] X.-Y. Tuo, X. Feng, L.-C. Jin and T. Wang, *Lattice QCD calculation of $K \rightarrow \ell\nu\ell'^{+}\ell'^{-}$ decay width*, Phys. Rev. D 105 (2022) 054518 [2103.11331].

[219] G. Gagliardi, V. Lubicz, G. Martinelli, F. Mazzetti, C.T. Sachrajda, F. Sanfilippo et al., *Virtual photon emission in leptonic decays of charged pseudoscalar mesons*, Phys. Rev. D 105 (2022) 114507 [2202.03833].

[220] C.T. Sachrajda, M. Di Carlo, G. Martinelli, D. Giusti, V. Lubicz, F. Sanfilippo et al., *Radiative corrections to semileptonic decay rates*, PoS LATTICE2019 (2019) 162 [1910.07342].

[221] P.-X. Ma, X. Feng, M. Gorchtein, L.-C. Jin and C.-Y. Seng, *Lattice QCD calculation of the electroweak box diagrams for the kaon semileptonic decays*, Phys. Rev. D 103 (2021) 114503 [2102.12048].

[222] A. Bazavov et al., *B- and D-meson leptonic decay constants from four-flavor lattice QCD*, Phys. Rev. D 98 (2018) 074512 [1712.09262].

[223] B. Grinstein and D. Pirjol, *Exclusive rare $B \rightarrow K^{*}\ell^{+}\ell^{-}$ decays at low recoil: Controlling the long-distance effects*, Phys. Rev. D 70 (2004) 114005 [hep-ph/0404250].

[224] M. Beylich, G. Buchalla and T. Feldmann, *Theory of $B \rightarrow K^{(*)}\ell^{+}\ell^{-}$ decays at high $q^2$: OPE and quark-hadron duality*, Eur. Phys. J. C 71 (2011) 1635 [1101.5118].

[225] M. Beneke, T. Feldmann and D. Seidel, *Systematic approach to exclusive $B \rightarrow V\ell^{+}\ell^{-}, V\gamma$ decays*, Nucl. Phys. B 612 (2001) 25 [hep-ph/0106067].

[226] A. Khodjamirian, T. Mannel and Y.M. Wang, *B$ \rightarrow K\ell^{+}\ell^{-}$ decay at large hadronic recoil*, JHEP 02 (2013) 010 [1211.0234].

[227] N. Gubernari, D. van Dyk and J. Virto, *Non-local matrix elements in $B_{(s)} \rightarrow \{K^{(*)}, \phi\}\ell^{+}\ell^{-}$, JHEP 02 (2021) 088 [2011.09813].

[228] N. Gubernari, M. Reboud, D. van Dyk and J. Virto, *Improved Theory Predictions and Global Analysis of Exclusive $b \rightarrow s\mu^{+}\mu^{-}$ Processes*, 2206.03797.

[229] Y.-M. Wang, *Beauty baryon decays: a theoretical overview*, J. Phys. Conf. Ser. 556 (2014) 012050.

[230] T. Feldmann, *Theory: Angular Distributions in Rare B Decays*, PoS BEAUTY2020 (2021) 018 [2101.04314].

[231] C. Hambrock, A. Khodjamirian and A. Rusov, *Hadronic effects and observables in $B \rightarrow \pi\ell^{+}\ell^{-}$ decay at large recoil*, Phys. Rev. D 92 (2015) 074020 [1506.07760].

43
A. Khodjamirian and A.V. Rusov, $B_s \to K\ell\nu_\ell$ and $B_{(s)} \to \pi(K)\ell^+\ell^-$ decays at large recoil and CKM matrix elements, *JHEP* **08** (2017) 112 [1703.04765].

A.V. Rusov, *Probing New Physics in $b \to d$ transitions*, *JHEP* **07** (2020) 158 [1911.12819].

G. Burdman, E. Golowich, J.L. Hewett and S. Pakvasa, *Rare charm decays in the standard model and beyond*, *Phys. Rev. D* **66** (2002) 014009 [hep-ph/0112235].

S. Fajfer and N. Košnik, *Prospects of discovering new physics in rare charm decays*, *Eur. Phys. J. C* **75** (2015) 567 [1510.00965].

S. de Boer and G. Hiller, *Flavor and new physics opportunities with rare charm decays into leptons*, *Phys. Rev. D* **93** (2016) 074001 [1510.00311].

A. Bharucha, D. Boito and C. Méaux, *Disentangling QCD and new physics in $D^+ \to \pi^+\ell^+\ell^-$*, *JHEP* **04** (2021) 158 [2011.12856].

S. de Boer and G. Hiller, *Rare radiative charm decays within the standard model and beyond*, *JHEP* **08** (2017) 091 [1701.06392].

S. Meinel, $\Lambda_c \to N$ form factors from lattice QCD and phenomenology of $\Lambda_c \to n\ell^+\nu_\ell$ and $\Lambda_c \to p\mu^+\mu^-$ decays, *Phys. Rev. D* **97** (2018) 034511 [1712.05783].

S. de Boer and G. Hiller, *Null tests from angular distributions in $D \to P_1P_2l^+l^-$, $l = e, \mu$ decays on and off peak*, *Phys. Rev. D* **98** (2018) 035041 [1805.08516].

R. Bause, M. Golz, G. Hiller and A. Tayduganov, *The new physics reach of null tests with $D \to \pi\ell\ell$ and $D_s \to K\ell\ell$ decays*, *Eur. Phys. J. C* **80** (2021) 208 [2107.13010].

M. Golz, G. Hiller and T. Magorsch, *Probing for new physics with rare charm baryon ($\Lambda_c$, $\Xi_c$, $\Omega_c$) decays*, *JHEP* **09** (2021) 208 [2107.13010].

M. Golz, G. Hiller and T. Magorsch, *Pinning down $|\Delta c| = |\Delta u| = 1$ couplings with rare charm baryon decays*, *Eur. Phys. J. C* **82** (2022) 357 [2202.02331].

R. Bause, H. Gisbert, M. Golz and G. Hiller, *Rare charm $c \to w\bar{w}\nu$ dineutrino null tests for $e^+e^-$ machines*, *Phys. Rev. D* **103** (2021) 015033 [2010.02225].

M. Bordone, B. Capdevila and P. Gambino, *Three loop calculations and inclusive $V_{cb}$*, *Phys. Lett. B* **822** (2021) 136679 [2107.00604].

F. Bernlochner, M. Fael, K. Olschewsky, E. Persson, R. van Tonder, K.K. Vos et al., *First extraction of inclusive $V_{cb}$ from $q^2$ moments*, [2205.10274].

A.S. Kronfeld and J.N. Simone, *Computation of $\Lambda_1$ and $\lambda_1$ with lattice QCD*, *Phys. Lett. B* **490** (2000) 228 [hep-ph/0006345], Erratum Phys. Lett. B **495** (2000) 441.
[248] P. Gambino, A. Melis and S. Simula, *Extraction of heavy-quark-expansion parameters from unquenched lattice data on pseudoscalar and vector heavy-light meson masses*, Phys. Rev. D 96 (2017) 014511 [1704.06105].

[249] Fermilab Lattice, MILC, TUMQCD collaboration, *Up-, down-, strange-, charm-, and bottom-quark masses from four-flavor lattice QCD*, Phys. Rev. D 98 (2018) 054517 [1802.04248].

[250] M. Fael, K. Schönhwald and M. Steinhauser, *Third order corrections to the semileptonic b → c and the muon decays*, Phys. Rev. D 104 (2021) 016003 [2011.13654].

[251] A. Alberti, T. Ewerth, P. Gambino and S. Nandi, *Kinetic operator effects in B → Xcℓν at O(αs)*, Nucl. Phys. B 870 (2013) 16 [1212.5082].

[252] A. Alberti, P. Gambino and S. Nandi, *Perturbative corrections to power suppressed effects in semileptonic B decays*, JHEP 01 (2014) 147 [1311.7381].

[253] T. Mannel, D. Moreno and A.A. Pivovarov, *NLO QCD corrections to inclusive b → cℓν decay spectra up to 1/m_b^3*, Phys. Rev. D 105 (2022) 054033 [2112.03875].

[254] M. Fael, T. Mannel and K. Keri Vos, *V_{cb} determination from inclusive b → c decays: an alternative method*, JHEP 02 (2019) 177 [1812.07472].

[255] T. Mannel, *V_{cb} and V_{ub} Continuum QCD Theory Overview*, talk at Snowmass workshop Theory meets experiment on |V_{ub}| and |V_{cb}|, 2021.

[256] T. Mannel, M. Rahimi and K.K. Vos, *Impact of background effects on the inclusive V_{cb} determination*, JHEP 09 (2021) 051 [2105.02163].

[257] C.W. Bauer, M.E. Luke and T. Mannel, *Light cone distribution functions for B decays at subleading order in 1/m_b*, Phys. Rev. D 68 (2003) 094001 [hep-ph/0102089].

[258] C.W. Bauer, M. Luke and T. Mannel, *Subleading shape functions in B → X_uℓν and the determination of |V_{ub}|*, Phys. Lett. B 543 (2002) 261 [hep-ph/0205150].

[259] Z. Ligeti and F.J. Tackmann, *Precise predictions for B → X_sℓ^+ℓ^− in the large q^2 region*, Phys. Lett. B 653 (2007) 404 [0707.1694].

[260] M.T. Hansen, H.B. Meyer and D. Robaina, *From deep inelastic scattering to heavy-flavor semileptonic decays: Total rates into multihadron final states from lattice QCD*, Phys. Rev. D 96 (2017) 094513 [1704.08993].

[261] M. Hansen, A. Lupo and N. Tantalo, *Extraction of spectral densities from lattice correlators*, Phys. Rev. D 99 (2019) 094508 [1903.06476].

[262] P. Gambino and S. Hashimoto, *Inclusive Semileptonic Decays from Lattice QCD*, Phys. Rev. Lett. 125 (2020) 032001 [2005.13730].

[263] T. DeGrand, *Remarks about weighted energy integrals over Minkowski spectral functions from Euclidean lattice data*, Phys. Rev. D 106 (2022) 014504 [2203.04393].
[264] W. Jay, *Inclusive Semileptonic Decays from LQCD (overview)*, talk at Snowmass workshop Theory meets experiment on $|V_{ub}|$ and $|V_{cb}|$, 2021.

[265] S. Hashimoto, *Inclusive semi-leptonic $B$ meson decay structure functions from lattice QCD*, *PTEP* 2017 (2017) 053B03 [1703.01881].

[266] S. Maechler, P. Gambino and S. Hashimoto, *Comparison of lattice QCD results for inclusive semi-leptonic decays $B$ mesons with the OPE*, *PoS LATTICE2021* (2022) 512 [2111.02833].

[267] P. Gambino, S. Hashimoto, S. Mächler, M. Panero, F. Sanfilippo, S. Simula et al., *Lattice QCD study of inclusive semileptonic decays of heavy mesons*, *JHEP* 07 (2022) 083 [2203.11762].

[268] R.J. Dowdall, C.T.H. Davies, R.R. Horgan, G.P. Lepage, C.J. Monahan, J. Shigemitsu et al., *Neutral $B$-meson mixing from full lattice QCD at the physical point*, *Phys. Rev. D* 100 (2019) 094508 [1907.01025].

[269] L. Di Luzio, M. Kirk, A. Lenz and T. Rauh, ∆$m_s$ theory precision confronts flavour anomalies, *JHEP* 12 (2019) 009 [1909.11087].

[270] E. Dalgic, A. Gray, E. Gamiz, C.T.H. Davies, G.P. Lepage, J. Shigemitsu et al., $B^0_s$-$\bar{B}^0_s$ mixing parameters from unquenched lattice QCD, *Phys. Rev. D* 76 (2007) 011501 [hep-lat/0610104].

[271] HPQCD collaboration, *Neutral $B$ Meson Mixing in Unquenched Lattice QCD*, *Phys. Rev. D* 80 (2009) 014503 [0902.1815].

[272] ETM collaboration, *$B$-physics from $N_f = 2$ tmQCD: the Standard Model and beyond*, *JHEP* 03 (2014) 016 [1308.1851].

[273] Y. Aoki, T. Ishikawa, T. Izubuchi, C. Lehner and A. Soni, *Neutral $B$ meson mixings and $B$ meson decay constants with static heavy and domain-wall light quarks*, *Phys. Rev. D* 91 (2015) 114505 [1406.6192].

[274] Fermilab Lattice, MILC collaboration, $B^0(s)$-$\bar{B}^0(s)$-mixing matrix elements from lattice QCD for the Standard Model and beyond, *Phys. Rev. D* 93 (2016) 113016 [1602.03560].

[275] A.G. Grozin, R. Klein, T. Mannel and A.A. Pivovarov, $B^0$-$\bar{B}^0$ mixing at next-to-leading order, *Phys. Rev. D* 94 (2016) 034024 [1606.06054].

[276] A.G. Grozin, T. Mannel and A.A. Pivovarov, Towards a Next-to-Next-to-Leading Order analysis of matching in $B^0$-$\bar{B}^0$ mixing, *Phys. Rev. D* 96 (2017) 074032 [1706.05910].

[277] M. Kirk, A. Lenz and T. Rauh, Dimension-six matrix elements for meson mixing and lifetimes from sum rules, *JHEP* 12 (2017) 068 [1711.02100], Erratum JHEP 06 (2020) 162.

[278] A.G. Grozin, T. Mannel and A.A. Pivovarov, $B^0$-$\bar{B}^0$ mixing: Matching to HQET at NNLO, *Phys. Rev. D* 98 (2018) 054020 [1806.00253].
D. King, A. Lenz and T. Rauh, $B^0_s$ mixing observables and $|V_{td}/V_{ts}|$ from sum rules, JHEP 05 (2019) 034 [1904.00940].

C. Albertus et al., Neutral $B$-meson mixing from unquenched lattice QCD with domain-wall light quarks and static $b$-quarks, Phys. Rev. D 82 (2010) 014505 [1001.2023].

A. Bazavov et al., Neutral $B$-meson mixing from three-flavor lattice QCD: Determination of the $SU(3)$-breaking ratio $\xi$, Phys. Rev. D 86 (2012) 034503 [1205.7013].

RBC/UKQCD collaboration, $SU(3)$-breaking ratios for $D(s)$ and $B(s)$ mesons, 1812.08791.

A. Bazavov et al., Short-distance matrix elements for $D^{0}$-meson mixing for $N_f=2$ lattice QCD, Phys. Rev. D 97 (2018) 034513 [1706.04622].

[289] Z. Bai, N.H. Christ, T. Izubuchi, C.T. Sachrajda, A. Soni and J. Yu, $K_0^0-L_0^0$ mass difference from Lattice QCD, Phys. Rev. Lett. 113 (2014) 112003 [1406.0916].

N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, Effects of finite volume on the $K_{L}^{0}$-$K_{S}^{0}$ mass difference, Phys. Rev. D 91 (2015) 114510 [1504.01170].

R. Alonso, B. Grinstein and J. Martin Camalich, Lifetime of $B^-_c$ Constrains Explanations for Anomalies in $B \rightarrow D^{(*)}\tau\nu$, Phys. Rev. Lett. 118 (2017) 081802 [1611.06676].

J. Gratrex, B. Melić and I. Nišandžić, Lifetimes of singly charmed hadrons, JHEP 07 (2022) 058 [2204.11935].
Belle II collaboration, Measurement of the $\Omega_c^0$ lifetime at Belle II, 2208.08573.

A. Lenz, M.L. Piscopo and A.V. Rusov, Contribution of the Darwin operator to non-leptonic decays of heavy quarks, JHEP 12 (2020) 199 [2004.09527].

T. Mannel, D. Moreno and A. Pivovarov, Heavy quark expansion for heavy hadron lifetimes: completing the $1/m_3^3$ corrections, JHEP 08 (2020) 089 [2004.09485].

D. King, A. Lenz and T. Rauh, SU(3) breaking effects in $B$ and $D$ meson lifetimes, JHEP 06 (2022) 134 [2112.03691].

A. Lenz, M.L. Piscopo and A.V. Rusov, Disintegration of beauty: a precision study, 2208.02643.

D. Becirevic, Theoretical progress in describing the $B$ meson lifetimes, PoS HEP2001 (2001) 098 [hep-ph/0110124].

D. Becirevic, D. Meloni, A. Retico, V. Gimenez, V. Lubicz and G. Martinelli, A Theoretical prediction of the $B_{(s)}$-meson lifetime difference, Eur. Phys. J. C 18 (2000) 157 [hep-ph/0006135].

J. Flynn and C.J.D. Lin, $B_{(s)}^0-\bar{B}_{(s)}^0$ mixing and b hadron lifetimes from lattice QCD, J. Phys. G 27 (2001) 1245 [hep-ph/0012154].

UKQCD collaboration, An Exploratory lattice study of spectator effects in inclusive decays of the $\Lambda_0$ baryon, Phys. Lett. B 468 (1999) 143 [hep-lat/9906031], Erratum Phys. Lett. B 525 (2002) 360.

UKQCD collaboration, A Lattice study of spectator effects in inclusive decays of $B$ mesons, Nucl. Phys. B 534 (1998) 373 [hep-lat/9805028].

J. Brod and J. Zupan, The ultimate theoretical error on $\gamma$ from $B \rightarrow DK$ decays, JHEP 01 (2014) 051 [1308.5663].

D. Zeppenfeld, SU(3) Relations for B Meson Decays, Z. Phys. C 8 (1981) 77.

M. Gronau and D. London, Isospin analysis of CP asymmetries in $B$ decays, Phys. Rev. Lett. 65 (1990) 3381.

M. Gronau, O.F. Hernandez, D. London and J.L. Rosner, Decays of $B$ mesons to two light pseudoscalars, Phys. Rev. D 50 (1994) 4529 [hep-ph/9404283].

M. Gronau, O.F. Hernandez, D. London and J.L. Rosner, Broken SU(3) symmetry in two-body $B$ decays, Phys. Rev. D 52 (1995) 6356 [hep-ph/9504326].

M. Ciuchini, M. Pierini and L. Silvestrini, Theoretical uncertainty in sin 2$\beta$: An Update, in 6th International Workshop on the CKM Unitarity Triangle, 2, 2011 [1102.0392].
[312] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, QCD factorization for exclusive, nonleptonic $B$ meson decays: General arguments and the case of heavy light final states, *Nucl. Phys. B* **591** (2000) 313 [hep-ph/0006124].

[313] Y.Y. Keum, H.-N. Li and A.I. Sanda, Penguin enhancement and $B \to K\pi$ decays in perturbative QCD, *Phys. Rev. D* **63** (2001) 054008 [hep-ph/0004173].

[314] Y.-Y. Keum, H.-n. Li and A.I. Sanda, Fat penguins and imaginary penguins in perturbative QCD, *Phys. Lett. B* **504** (2001) 6 [hep-ph/0004004].

[315] C.-D. Lu, K. Ukai and M.-Z. Yang, Branching ratio and CP violation of $B \to \pi\pi$ decays in perturbative QCD approach, *Phys. Rev. D* **63** (2001) 074009 [hep-ph/0004213].

[316] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters, *Nucl. Phys. B* **606** (2001) 245 [hep-ph/0104110].

[317] C.W. Bauer, D. Pirjol and I.W. Stewart, Soft collinear factorization in effective field theory, *Phys. Rev. D* **65** (2002) 054022 [hep-ph/0109045].

[318] M. Beneke and M. Neubert, QCD factorization for $B \to PP$ and $B \to PV$ decays, *Nucl. Phys. B* **675** (2003) 333 [hep-ph/0308039].

[319] C.W. Bauer, D. Pirjol, I.Z. Rothstein and I.W. Stewart, $B \to M_1 M_2$: Factorization, charming penguins, strong phases, and polarization, *Phys. Rev. D* **70** (2004) 054015 [hep-ph/0401188].

[320] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Comment on “$B \to M_1 M_2$: Factorization, charming penguins, strong phases, and polarization”, *Phys. Rev. D* **72** (2005) 098501 [hep-ph/0411171].

[321] T. Feldmann and T. Hurth, Non-factorizable contributions to $B \to \pi\pi$ decays, *JHEP* **11** (2004) 037 [hep-ph/0408188].

[322] A. Ali, G. Kramer, Y. Li, C.-D. Lu, Y.-L. Shen, W. Wang et al., Charmless non-leptonic $B_s$ decays to $PP$, $PV$ and $VV$ final states in the pQCD approach, *Phys. Rev. D* **76** (2007) 074018 [hep-ph/0703162].

[323] T. Huber, S. Kränkl and X.-Q. Li, Two-body non-leptonic heavy-to-heavy decays at NNLO in QCD factorization, *JHEP* **09** (2016) 112 [1606.02888].

[324] S.-H. Zhou, Q.-A. Zhang, W.-R. Lyu and C.-D. Lü, Analysis of Charmless Two-body $B$ decays in Factorization Assisted Topological Amplitude Approach, *Eur. Phys. J. C* **77** (2017) 125 [1608.02819].

[325] C. Wang, Q.-A. Zhang, Y. Li and C.-D. Lu, Charmless $B(s) \to VV$ Decays in Factorization-Assisted Topological-Amplitude Approach, *Eur. Phys. J. C* **77** (2017) 333 [1701.01300].
[326] T. Huber, J. Virto and K.K. Vos, Three-Body Non-Leptonic Heavy-to-heavy B Decays at NNLO in QCD, *JHEP* **11** (2020) 103 [2007.08881].

[327] G. Bell, M. Beneke, T. Huber and X.-Q. Li, Two-loop non-leptonic penguin amplitude in QCD factorization, *JHEP* **04** (2020) 055 [2002.03262].

[328] J. Hua, H.-n. Li, C.-D. Lu, W. Wang and Z.-P. Xing, Global analysis of hadronic two-body B decays in the perturbative QCD approach, *Phys. Rev. D* **104** (2021) 016025 [2012.15074].

[329] T. Huber, Some recent developments in nonleptonic B decays, in 11th International Workshop on the CKM Unitarity Triangle, 4, 2022 [2204.06224].

[330] C.-D. Lü, Y.-L. Shen, C. Wang and Y.-M. Wang, Enhanced Next-to-Leading-Order Corrections to Weak Annihilation B-Meson Decays, 2022.08073.

[331] M. Beneke, P. Böer, J.-N. Toelstede and K.K. Vos, QED factorization of non-leptonic B decays, *JHEP* **11** (2020) 081 [2008.10615].

[332] H. Kawamura and K. Tanaka, Coordinate-space calculation of radiative corrections to the B-meson distribution amplitudes: light-cone vs. static distributions, *PoS RADCOR2017* (2018) 076.

[333] W. Wang, Y.-M. Wang, J. Xu and S. Zhao, B-meson light-cone distribution amplitude from Euclidean quantities, *Phys. Rev. D* **102** (2020) 011502 [1908.09933].

[334] S. Zhao and A.V. Radyushkin, B-meson Ioffe-time distribution amplitude at short distances, *Phys. Rev. D* **103** (2021) 054022 [2006.05663].

[335] J. Xu, X.-R. Zhang and S. Zhao, Inverse moment of the B-meson quasidistribution amplitude, *Phys. Rev. D* **106** (2022) L011503 [2202.13648].

[336] M. Bordone, N. Gubernari, T. Huber, M. Jung and D. van Dyk, A puzzle in $B^0(s) \to D^{(*)+}(s) \{\pi-, K-\}$ decays and extraction of the $f_s/f_d$ fragmentation fraction, *Eur. Phys. J. C* **80** (2020) 951 [2007.10338].

[337] N. Gubernari, A puzzle in $\bar{B}^0(s) \to D^{(*)+}(s) \{\pi-, K-\}$ decays, talk at the workshop Status and prospects of Non-leptonic B meson decays, 2022.

[338] M. Endo, S. Iguro and S. Mishima, Revisiting rescattering contributions to $\bar{B}(s) \to D^{(*)} M$ decays, *JHEP* **01** (2022) 147 [2109.10811].

[339] S. Iguro and T. Kitahara, Implications for new physics from a novel puzzle in $\bar{B}^0(s) \to D^{(*)+}(s) \{\pi-, K-\}$ decays, *Phys. Rev. D* **102** (2020) 071701 [2008.01086].

[340] F.-M. Cai, W.-J. Deng, X.-Q. Li and Y.-D. Yang, Probing new physics in class-I B-meson decays into heavy-light final states, *JHEP* **10** (2021) 235 [2103.04138].
[341] T. Gershon, A. Lenz, A.V. Rusov and N. Skidmore, Testing the Standard Model with CP asymmetries in flavor-specific nonleptonic decays, *Phys. Rev. D* **105** (2022) 115023 [2111.04478].

[342] Belle collaboration, Measurements of the branching fractions $\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$ and tests of QCD factorization, 2207.00134.

[343] C.-D. Lu, Y.-M. Wang, H. Zou, A. Ali and G. Kramer, Anatomy of the pQCD Approach to the Baryonic Decays $\Lambda_b \to p\pi, pK$, *Phys. Rev. D* **80** (2009) 034011 [0906.1479].

[344] Y.K. Hsiao and C.Q. Geng, Direct CP violation in $\Lambda_b$ decays, *Phys. Rev. D* **91** (2015) 116007 [1412.1899].

[345] M. Gronau and J.L. Rosner, Flavor SU(3) and $\Lambda_b$ decays, *Phys. Rev. D* **89** (2014) 037501 [1312.5730], Erratum *Phys. Rev. D* **91** (2015) 119902.

[346] X.-G. He and G.-N. Li, Predictive CP violating relations for charmless two-body decays of beauty baryons $\Xi_b^{-0}$ and $\Lambda_b^0$ with flavor SU(3) symmetry, *Phys. Lett. B* **750** (2015) 82 [1501.00646].

[347] S. Roy, R. Sinha and N.G. Deshpande, Nonleptonic beauty baryon decays and CP asymmetries based on an SU(3)-flavor analysis, *Phys. Rev. D* **101** (2020) 036018 [1911.01121].

[348] A. Dery, M. Ghosh, Y. Grossman and S. Schacht, SU(3)$_F$ analysis for beauty baryon decays, *JHEP* **03** (2020) 165 [2001.05397].

[349] S. Roy, R. Sinha and N.G. Deshpande, Beauty baryon nonleptonic decays into decuplet baryons and CP-asymmetries based on an SU(3)-flavor analysis, *Phys. Rev. D* **102** (2020) 053007 [2006.16813].

[350] W. Bensalem, A. Datta and D. London, T violating triple product correlations in charmless $\Lambda_b$ decays, *Phys. Lett. B* **538** (2002) 309 [hep-ph/0205009].

[351] G. Durieux, CP violation in multibody decays of beauty baryons, *JHEP* **10** (2016) 005 [1608.03288].

[352] J. Brod, A.L. Kagan and J. Zupan, Size of direct CP violation in singly Cabibbo-suppressed D decays, *Phys. Rev. D* **86** (2012) 014023 [1111.5000].

[353] A. Soni, Resonance enhancement of Charm CP, 1905.00907.

[354] S. Schacht and A. Soni, Enhancement of charm CP violation due to nearby resonances, *Phys. Lett. B* **825** (2022) 136855 [2110.07619].

[355] A. Khodjamirian and A.A. Petrov, Direct CP asymmetry in $D \to \pi^-\pi^+$ and $D \to K^-K^+$ in QCD-based approach, *Phys. Lett. B* **774** (2017) 235 [1706.07780].

[356] M. Chala, A. Lenz, A.V. Rusov and J. Scholtz, $\Delta A_{CP}$ within the Standard Model and beyond, *JHEP* **07** (2019) 161 [1903.10490].
[357] H.-N. Li, C.-D. Lü and F.-S. Yu, *Implications on the first observation of charm CPV at LHCb*, 1903.10638.

[358] X.-W. Kang, H.-B. Li, G.-R. Lu and A. Datta, *Study of CP violation in Λc+ decay*, Int. J. Mod. Phys. A 26 (2011) 2523 [1003.5494].

[359] I.I. Bigi, *Probing CP Asymmetries in Charm Baryons Decays*, 1206.4554.

[360] H.-Y. Cheng, X.-W. Kang and F. Xu, *Singly Cabibbo-suppressed hadronic decays of Λc+*, Phys. Rev. D 97 (2018) 074028 [1801.08625].

[361] Y. Grossman and S. Schacht, *U- Spin Sum Rules for CP Asymmetries of Three-Body Charmed Baryon Decays*, Phys. Rev. D 99 (2019) 033005 [1811.11188].

[362] D. Wang, *Sum rules for CP asymmetries of charmed baryon decays in the SU(3)F limit*, Eur. Phys. J. C 79 (2019) 429 [1901.01776].

[363] X.-D. Shi, X.-W. Kang, I. Bigi, W.-P. Wang and H.-P. Peng, *Prospects for CP and P violation in Λc decays at Super Tau Charm Facility*, Phys. Rev. D 100 (2019) 113002 [1904.12415].

[364] P.J. Fox, G.D. Kribs and H. Murayama, *BSM Model Building*, in 2022 Snowmass Summer Study, 2022.

[365] W. Altmannshofer and J. Zupan, *Flavor Model Building*, in 2022 Snowmass Summer Study, 2022 [2203.07726].

[366] B. Bhattacharya, A. Datta, D. London and S. Shivashankara, *Simultaneous Explanation of the R_K and R(D^{(*)}) Puzzles*, Phys. Lett. B742 (2015) 370 [1412.7164].

[367] R. Alonso, B. Grinstein and J. Martin Camalich, *SU(2) × U(1) gauge invariance and the shape of new physics in rare B decays*, Phys. Rev. Lett. 113 (2014) 241802 [1407.7044].

[368] R. Barbieri, G. Isidori, A. Pattori and F. Senia, *Anomalies in B-decays and U(2) flavour symmetry*, Eur. Phys. J. C 76 (2016) 67 [1512.01560].

[369] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B-physics anomalies: a guide to combined explanations*, JHEP 11 (2017) 044 [1706.07808].

[370] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Quark flavor transitions in L_μ – L_τ models*, Phys. Rev. D 89 (2014) 095033 [1403.1269].

[371] A. Crivellin, G. D’Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, Phys. Rev. D 91 (2015) 075006 [1503.03477].

[372] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, *Family nonuniversal Z’ models with protected flavor-changing interactions*, Phys. Rev. D 92 (2015) 015007 [1505.03079].

[373] K. Fuyuto, W.-S. Hou and M. Kohda, *Z’-induced FCNC decays of top, beauty, and strange quarks*, Phys. Rev. D 93 (2016) 054021 [1512.09026].
[374] C. Bonilla, T. Modak, R. Srivastava and J.W.F. Valle, \( U(1)_{B_3 - 3L_\mu} \) gauge symmetry as a simple description of \( b \to s \) anomalies, \textit{Phys. Rev. D} \textbf{98} (2018) 095002 [1705.00915].

[375] D. Bhatia, S. Chakraborty and A. Dighe, Neutrino mixing and \( R_K \) anomaly in \( U(1)_X \) models: a bottom-up approach, \textit{JHEP} \textbf{03} (2017) 117 [1701.05825].

[376] R. Alonso, P. Cox, C. Han and T.T. Yanagida, Flavoured \( B - \) \( L \) local symmetry and anomalous rare \( B \) decays, \textit{Phys. Lett. B} \textbf{774} (2017) 643 [1705.03858].

[377] S.F. King, Flavourful \( Z' \) models for \( R_K^{(*)} \), \textit{JHEP} \textbf{08} (2017) 019 [1706.06100].

[378] B.C. Allanach and J. Davighi, Third family hypercharge model for \( R_K^{(*)} \) and aspects of the fermion mass problem, \textit{JHEP} \textbf{12} (2018) 075 [1809.01158].

[379] D. Bečirević, I. Doršner, S. Fajfer, N. Košnik, D.A. Faroughy and O. Sumensari, Scalar leptoquarks from grand unified theories to accommodate the \( B \)-physics anomalies, \textit{Phys. Rev. D} \textbf{98} (2018) 055003 [1806.05689].

[380] W. Altmannshofer, J. Davighi and M. Nardecchia, Gauging the accidental symmetries of the standard model, and implications for the flavor anomalies, \textit{Phys. Rev. D} \textbf{101} (2020) 015004 [1909.02021].

[381] D. Bhatia, N. Desai and A. Dighe, Frugal \( U(1)_X \) models with non-minimal flavor violation for \( b \to s \ell \ell \) anomalies and neutrino mixing, \textit{JHEP} \textbf{04} (2022) 163 [2109.07093].

[382] A. Greljo, P. Stangl and A.E. Thomsen, A model of muon anomalies, \textit{Phys. Lett. B} \textbf{820} (2021) 136554 [2103.13991].

[383] A. Greljo, Y. Soreq, P. Stangl, A.E. Thomsen and J. Zupan, Muonic Force Behind Flavor Anomalies, \textbf{2107.07518}.

[384] C. Niehoff, P. Stangl and D.M. Straub, Violation of lepton flavour universality in composite Higgs models, \textit{Phys. Lett. B} \textbf{747} (2015) 182 [1503.03865].

[385] F. Sannino, P. Stangl, D.M. Straub and A.E. Thomsen, Flavor Physics and Flavor Anomalies in Minimal Fundamental Partial Compositeness, \textit{Phys. Rev. D} \textbf{97} (2018) 115046 [1712.07646].

[386] A. Carmona and F. Goertz, Recent \( B \) physics anomalies: a first hint for compositeness?, \textit{Eur. Phys. J. C} \textbf{78} (2018) 979 [1712.02536].

[387] Y. Chung, Flavorful composite Higgs model: Connecting the \( B \) anomalies with the hierarchy problem, \textit{Phys. Rev. D} \textbf{104} (2021) 115027 [2108.08511].

[388] D. Marzocca and S. Trifinopoulos, Minimal Explanation of Flavor Anomalies: \( B \)-Meson Decays, Muon Magnetic Moment, and the Cabibbo Angle, \textit{Phys. Rev. Lett.} \textbf{127} (2021) 061803 [2104.05730].

[389] D. Bečirević, I. Doršner, S. Fajfer, D.A. Faroughy, F. Jaffredo, N. Košnik et al., On a model with two scalar leptoquarks – \( R_2 \) and \( S_3 \), \textbf{2206.09717}.
[390] A. Greljo and D. Marzocca, High-$p_T$ dilepton tails and flavor physics, Eur. Phys. J. C 77 (2017) 548 [1704.09015].

[391] B.C. Allanach, B. Gripaios and T. You, The case for future hadron colliders from $B \to K^{(*)} \mu^+ \mu^-$ decays, JHEP 03 (2018) 021 [1710.06363].

[392] M. Abdullah, M. Dalchenko, B. Dutta, R. Eusebi, P. Huang, T. Kamon et al., Bottom-quark fusion processes at the LHC for probing $Z'$ models and $B$-meson decay anomalies, Phys. Rev. D 97 (2018) 075035 [1707.07016].

[393] M. Kohda, T. Modak and A. Soffer, Identifying a $Z'$ behind $b \to s\ell\ell$ anomalies at the LHC, Phys. Rev. D 97 (2018) 115019 [1803.07492].

[394] B.C. Allanach, J.M. Butterworth and T. Corbett, Collider constraints on $Z'$ models for neutral current $B$-anomalies, JHEP 08 (2019) 106 [1904.10954].

[395] G.-y. Huang, F.S. Queiroz and W. Rodejohann, Gauged $L_\mu-L_\tau$ at a muon collider, Phys. Rev. D 103 (2021) 095005 [2101.04956].

[396] G. Hiller and M. Schmaltz, $R_K$ and future $b \to s\ell\ell$ physics beyond the standard model opportunities, Phys. Rev. D 90 (2014) 054014 [1408.1627].

[397] B. Gripaios, M. Nardecchia and S.A. Renner, Composite leptoquarks and anomalies in $B$-meson decays, JHEP 05 (2015) 006 [1412.1791].

[398] R. Alonso, B. Grinstein and J. Martin Camalich, Lepton universality violation and lepton flavor conservation in $B$-meson decays, JHEP 10 (2015) 184 [1505.05164].

[399] M. Bauer and M. Neubert, Minimal Leptoquark Explanation for the $R_{D^{(*)}}$, $R_K$, and $(g - 2)_\mu$ Anomalies, Phys. Rev. Lett. 116 (2016) 141802 [1511.01900].

[400] L. Calibbi, A. Crivellin and T. Ota, Effective Field Theory Approach to $b \to s\ell\ell^{(*)}$, $B \to K^{(*)}\nu\bar{\nu}$ and $B \to D^{(*)}\tau\bar{\nu}$ with Third Generation Couplings, Phys. Rev. Lett. 115 (2015) 181801 [1506.02661].

[401] S. Fajfer and N. Košnik, Vector leptoquark resolution of $R_K$ and $R_{D^{(*)}}$ puzzles, Phys. Lett. B 755 (2016) 270 [1511.06024].

[402] R. Barbieri, C.W. Murphy and F. Senia, $B$-decay Anomalies in a Composite Leptoquark Model, Eur. Phys. J. C 77 (2017) 8 [1611.04930].

[403] B. Bhattacharya, A. Datta, J.-P. Guévin, D. London and R. Watanabe, Simultaneous Explanation of the $R_K$ and $R_{D^{(*)}}$ Puzzles: a Model Analysis, JHEP 01 (2017) 015 [1609.09078].

[404] G. Hiller, D. Loose and K. Schönwald, Leptoquark Flavor Patterns & $B$ Decay Anomalies, JHEP 12 (2016) 027 [1609.08896].
[405] D. Bećirević, N. Košnik, O. Sumensari and R. Zukanovich Funchal, Palatable Leptoquark Scenarios for Lepton Flavor Violation in Exclusive $b \to s \ell_1 \ell_2$ modes, JHEP 11 (2016) 035 [1608.07583].

[406] Y. Cai, J. Gargalionis, M.A. Schmidt and R.R. Volkas, Reconsidering the One Leptoquark solution: flavor anomalies and neutrino mass, JHEP 10 (2017) 047 [1704.05849].

[407] A. Crivellin, D. Müller and T. Ota, Simultaneous explanation of $R_{D^{(*)}}$ and $b \to s \mu^+\mu^-$: the last scalar leptoquarks standing, JHEP 09 (2017) 040 [1703.09226].

[408] N. Assad, B. Fornal and B. Grinstein, Baryon Number and Lepton Universality Violation in Leptoquark and Diquark Models, Phys. Lett. B777 (2018) 324 [1708.06350].

[409] A. Angelescu, D. Bećirević, D.A. Faroughy and O. Sumensari, Closing the window on single leptoquark solutions to the $B$-physics anomalies, JHEP 10 (2018) 183 [1808.08179].

[410] D. Marzocca, Addressing the $B$-physics anomalies in a fundamental Composite Higgs Model, JHEP 07 (2018) 121 [1803.10972].

[411] O. Popov, M.A. Schmidt and G. White, $R_2$ as a single leptoquark solution to $R_{D^{(*)}}$ and $R_{K^{(*)}}$, Phys. Rev. D 100 (2019) 035028 [1905.06339].

[412] C. Cornella, J. Fuentes-Martin and G. Isidori, Revisiting the vector leptoquark explanation of the $B$-physics anomalies, JHEP 07 (2018) 169 [1903.11517].

[413] J. Fuentes-Martín, G. Isidori, M. König and N. Selimović, Vector Leptoquarks Beyond Tree Level III: Vector-like Fermions and Flavor-Changing Transitions, Phys. Rev. D102 (2020) 115015 [2009.11296].

[414] P. Fileviez Perez and C. Murgui, Flavor anomalies and quark-lepton unification, Phys. Rev. D 106 (2022) 035033 [2203.07381].

[415] N.G. Deshpande and X.-G. He, Consequences of $R$-parity violating interactions for anomalies in $\bar{B} \to D^{(*)}\tau\nu$ and $b \to s\mu^+\mu^-$, Eur. Phys. J. C 77 (2017) 134 [1608.04817].

[416] D. Das, C. Hati, G. Kumar and N. Mahajan, Scrutinizing $R$-parity violating interactions in light of $R_{K^{(*)}}$ data, Phys. Rev. D 96 (2017) 095033 [1705.09188].

[417] W. Altmannshofer, P.S. Bhupal Dev and A. Soni, $R_{D^{(*)}}$ anomaly: A possible hint for natural supersymmetry with $R$-parity violation, Phys. Rev. D 96 (2017) 095010 [1704.06659].

[418] K. Earl and T. Grégoire, Contributions to $b \to s\ell\ell$ Anomalies from $R$-Parity Violating Interactions, JHEP 08 (2018) 201 [1806.01343].

[419] S. Trifinopoulos, Revisiting $R$-parity violating interactions as an explanation of the $B$-physics anomalies, Eur. Phys. J. C 78 (2018) 803 [1807.01638].

[420] L. Di Luzio, A. Greljo and M. Nardecchia, Gauge leptoquark as the origin of $B$-physics anomalies, Phys. Rev. D 96 (2017) 115011 [1708.08450].
[421] M. Bordone, C. Cornella, J. Fuentes-Martín and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, Phys. Lett. B **779** (2018) 317 [1712.01368].

[422] R. Barbieri and A. Tesi, *B-decay anomalies in Pati-Salam SU(4)*, Eur. Phys. J. C **78** (2018) 193 [1712.06844].

[423] L. Calibbi, A. Crivellin and T. Li, *Model of vector leptoquarks in view of the B-physics anomalies*, Phys. Rev. D **98** (2018) 115002 [1709.00692].

[424] A. Greljo and B.A. Stefanek, *Third family quark-lepton unification at the TeV scale*, Phys. Lett. B **782** (2018) 131 [1802.04274].

[425] M. Blanke and A. Crivellin, *B Meson Anomalies in a Pati-Salam Model within the Randall-Sundrum Background*, Phys. Rev. Lett. **121** (2018) 011801 [1801.07256].

[426] S. Balaji, R. Foot and M.A. Schmidt, *Chiral SU(4) explanation of the b → s anomalies*, Phys. Rev. D **99** (2019) 015029 [1809.07562].

[427] B. Fornal, S.A. Gadam and B. Grinstein, *Left-Right SU(4) Vector Leptoquark Model for Flavor Anomalies*, Phys. Rev. D **99** (2019) 055025 [1812.01603].

[428] J. Heeck and D. Teresi, *Pati-Salam explanations of the B-meson anomalies*, JHEP **12** (2018) 103 [1808.07492].

[429] P. Fileviez Perez and M.B. Wise, *Low Scale Quark-Lepton Unification*, Phys. Rev. D **88** (2013) 057703 [1307.6213].

[430] M. Bordone, C. Cornella, J. Fuentes-Martín and G. Isidori, *Low-energy signatures of the PS$^3$ model: from B-physics anomalies to LFV*, JHEP **10** (2018) 148 [1805.09328].

[431] J. Fuentes-Martín, G. Isidori, J.M. Lizana, N. Selimovic and B.A. Stefanek, *Flavor hierarchies, flavor anomalies, and Higgs mass from a warped extra dimension*, Phys. Lett. B **834** (2022) 137382 [2203.01952].

[432] R. Barbieri and R. Ziegler, *Quark masses, CKM angles and Lepton Flavour Universality violation*, JHEP **07** (2019) 023 [1904.04121].

[433] J.C. Pati and A. Salam, *Lepton Number as the Fourth Color*, Phys. Rev. D**10** (1974) 275

Erratum Phys. Rev. D **11** (1975) 703.