Band structure and reflectance for a nonlinear one-dimensional photonic crystal

S. Gutiérrez-López, A. Castellanos-Moreno, A. Corella-Madueño, and R. A. Rosas
Departamento de Física, Universidad de Sonora, Apartado Postal 1626, Hermosillo, Sonora, México

J. A. Reyes∗
Departamento de Física
Universidad Autónoma Metropolitana, Ixtapalapa, Apartado Postal 55 534 09340, México D. F., México

Abstract

We consider a model for a one-dimensional photonic crystal formed by a succession of Kerr-type equidistant spaceless interfaces immersed in a linear medium. We calculate the band structure and reflectance of this structure as a function of the incident wave intensity, and find two main behaviors: the appearance of prohibited bands, and the separation and narrowing of these bands. A system with these features is obtained by alternating very thin slabs of a soft matter material with thicker solid films, which can be used to design a device to control light propagation for specific wavelength intervals and light intensities.

PACS. 42.70.Df; 42.65.Tg; 77.84.Nh

∗On leave from Instituto de Física, Universidad Nacional Autónoma de México
Photonic crystals PCs —spatially periodic, composite materials that can exhibit photonic band PB gaps for light propagation—have reached a mature state of development, with their optical properties well understood and with many realized and potential applications [1]. Most of this research deals with PCs whose characteristics are fixed, that is, once they have been fabricated there is no possibility to alter their optical response. A recent trend, however, concerns tunable or active PCs; by this we imply that, by means of some external agent, it becomes feasible to change the optical properties of the PC continuously and reversibly. This could lead to tunable optical waveguides, switches, limiters, and polarizers; to reconfigurable optical networks; and to electrooptic interconnects in microelectronics. We can classify tunable PCs according to two broad categories. For one of these, an external agent causes structural changes with no alteration of the dielectric constants of the constituent materials. In the other category the configuration of the PC remains the same, and it is some material property of the PC that is affected by the external agent. Structural tuning has been proposed or accomplished by means of mechanical stress applied to a polymer opal of nanoscale spheres, by applying an electric field to a PC on a piezoelectric substrate; by incorporating diodes in a 2D structure of wires; and by applying a magnetic field to periodically distributed magnetic particles [2]. On the other hand, tuning through the alteration of some material property involves the incorporation of a ferromagnetic or ferroelectric material in the PC—to be tuned by an external magnetic or electric field, respectively [3]. In particular, LCs are well-established electro-optic materials that can be tuned by means of pressure, heat, and applied electric or magnetic field. Their incorporation within a PC is of particular interest because of the possibility of selectively tuning PB gaps, as reviewed recently [4].

In this letter we consider a system whose schematic plot is shown in fig.1. This PC consists of a succession of spaceless nonlinear interfaces immersed in a linear medium. The nonlinear interfaces are made of a soft matter medium whose nonlinear response is that of a Kerr material for which the nonlinear response is proportional to the square of the magnitude of the intensity of the light [5]. A system with these features can be managed by alternating very thin slabs of a soft matter material like a nematic LC, with thicker solid films. Hence, for a low power laser beam (for instance He-Ne) the nonlinear index refraction of the solid stacks is to be negligible in comparison with that of the nematic films whose usual value is more than eight orders of magnitude larger than that of a solid [6]. In this model we shall ignore the thickness of the nonlinear slabs located in \( x = na \) and approximate their optical response by using the Kerr model. This allow us to write the following propagation equation

\[
\frac{d^2 \psi}{dx^2} + \left( k^2 - \frac{n_2}{a} \sum_{n=-\infty}^{\infty} \delta(x - na) |\psi|^2 \right) \psi = 0 \quad (1)
\]

where \( n_2 \) is the nonlinear refraction index and \( \psi \) is the amplitude of the propagating wave. For points \( x \neq na \), the general solution of this equation is of the form: \( \psi = Ae^{ikx} + Be^{-ikx} \). However, after a standard integration of Eq. (1)
around the points $x = na$, we can establish the boundary conditions

$$\psi(0) = e^{\chi a} \psi(a), e^{\chi a} \frac{d\psi(a)}{dx} - e^{\chi a} \frac{d\psi(0)}{dx} = e^{\chi a} g\psi(0) |\psi(0)|^2,$$

(2)

where we have already introduced the Bloch parameter $\chi a$ to characterize the momentum in each unit cell. Upon applying these conditions to the general solution, permit us to write the following transcendental equation

$$f(z, p, \chi a) = p \cos(z - \chi a) \sin^{3^3} z + \frac{1}{2} \cos(z - \chi a) [1 - \cos(z + \chi a)] = 0,$$

(3)

where $z = ka$, $p = B^2 gka$ and $g = \frac{2a}{\chi a}$. In Fig. 2 we have illustrated the forbidden and permitted bands that we have found by solving Eq. (3) numerically, where $f$ is plotted in the vertical axis and $z$ is shown in the horizontal one.

Each plot is the result of the superposition of the different curves obtained by varying the parameter $\chi a$ in the function $f(z, p, \chi a)$. Hence, the roots of Eq. (3) correspond to the points where curves cross the horizontal axis and thus each crossing give us a point of the permitted band. Conversely, there is a prohibited band whenever there is a white interval (no crossing point).

Fig. 3 depicts prohibited bands in the interval $0 \leq z \leq 10$ and show how they appear and disappear versus the parameter $p$.

Some interesting remarks of these plots are the following. Fig. 3a for the interval $0 \leq p \leq 1$ shows that there are no prohibited bands for $p = 0$ whereas a prohibited band appears at $p = 0.03$ that widens, until a very narrow prohibited band appears in $p = 0.3$. For the interval $0.3 \leq p < 0.7$, the first band is narrowed and the second one is widened. Simultaneously, the first band is displaced to the left until the adjacent allowed band located in $x = 0$ disappears. For $0.7 \leq p \leq 0.9$, a third forbidden band is added and it also widens against $p$. Fig. 3b for the interval $1 \leq p \leq 1000$, a fourth prohibited band is added, and all of them become narrower. For $p = 100$ the left prohibited band is split into two narrow bands, and for $p \geq 200$ there are no more bands. In general terms we can address two characteristic behaviors of these bands, one for the appearance of the prohibited bands, and another for the separation and narrowing of the mentioned bands.

Instead of an infinite array, for experimental purposes it is wealth to consider a finite array of nonlinear interfaces $n = 1, 2, \ldots$ immersed also in a linear medium. It is convenient to write the general solution for a nonlinear material in the following form

$$\psi(x) = c_n e^{ikx} + d_n e^{-ikx},$$

(4)

where the subscript $n$ corresponds the stack number $n$. The boundary conditions given by Eq. (2) can be expressed in the $n$-th interface as

$$c_n e^{-ikx} + d_n e^{ikx} = c_{n-1} e^{-ikx} + d_{n-1} e^{ikx}$$

(5)

and

$$-ikc_n e^{-ikx} + ikd_n e^{ikx} - (-ikc_{n-1} e^{-ikx} + ikd_{n-1} e^{ikx})
= g |c_{n-1} e^{-ikx} + d_{n-1} e^{ikx}|^2 (c_{n-1} e^{-ikx} + d_{n-1} e^{ikx}).$$

(6)
An additional condition stems from the fact that we shall assume that a wave travels from the left, and no wave is coming from the right

\[ \begin{align*}
    d_0 &= d_n(0) = A, \\
    c_0 &= c_n(0) = B, \\
    c_n(N) &= 0 = C_N.
\end{align*} \tag{7} \]

We solve the system of recurrence equations defined by Eqs. (5) and (6) with the initial conditions Eq. (7) by straightforward substitution, and calculate directly the reflectance defined by

\[ R = |B|^2 / |A|^2. \]

Here we denote by \( n \) the number of nonlinear interfaces and \( N - 1 \) the total number of them contained in the finite array of stack. In this way the system will have \( N \) linear regions.

Fig. 4a shows the reflectance, \( R(A, k) \) as function of the independent variables \( A \) and \( k \). Fig. 4b exhibits the same information but depicted in a contour plot. We present first the simplest case for which \( n = 1 \ (N = 2) \), and \( g = 0.2 \) to get some useful insight. It can be observed that \( R(A, k) \) goes to 1, provided that \( A \) tends to 4, for any value of each \( k \), except for the interval \( 2.1 < k < 2.3 \), where the material becomes transparent.

This region disappears when \( g \) tends to 1, as one can see in Fig. 5, to reach finally the opacity \( (R(A, k) \simeq 1) \) for \( g = 6 \), with \( N = 2 \).

Fig. 6 displays a panel plots where \( g \) and \( N \) are varying. notice that by keeping \( g \) constant and increasing \( N \), the trough in the middle of the surface is maintained.

Further calculations allow us to address the following interesting remarks. For the case of a only one nonlinear interface \( (N = 2) \), \( R(A, k) \) increases by enlarging \( A \) and keeping constant the value of \( k \). Nevertheless, there exists a transparency stripe situated approximately in the \( k \)-interval \([2, 3]\) which narrows when \( g \) grows until it disappears for \( g = 6 \). Similarly, for the case of three nonlinear interfaces \( (N = 4) \): \( R(A, k) \) grows for \( k \) fixed by enlarging \( A \). Also, there is a transparency stripe which is narrowing but it does not disappear even for \( g = 6 \). The behavior is similar even when the system consists of five nonlinear interface \( (N = 6) \). A general feature observed in all these calculations is that by increasing both \( g \) and \( N \), the border of the transparency band is no longer smooth.

We have elaborated a model for a simple nonlinear one-dimensional PC for which the nonlinearity is concentrated in certain spaceless barriers periodically located. This system can be constructed by piling up slabs of a nonlinear soft matter material like a LC inserted between thicker solid films. We have calculated analytically the band structure of this periodic structure. We have shown that by increasing the intensity of the normal incident wave, there appear more prohibited bands which separate each other and get narrow. The same qualitative behavior is shown by the reflectance of a finite array of alternating stacks for which forbidden bands displace and narrow versus the signal intensity.

Our results suggest to design devices based on this optical structure in order to prevent the propagation of light whose intensity is either larger or lower than certain threshold, for a specific given interval of the wavelength spectrum.
References

[1] C. M. Soukoulis, ed., Photonic Crystals and Light Localization in the 21st Century Kluwer Academic, Dordrecht, 2001; P. Lodahl, A. F. van Driel, I. S. Nikolaev, A. Irman, K. Overgaag, D. Vanmaekelbergh, and W. L. Vos, Nature London 430, 654 2004; K. Bush, S. Lölkes, R. B. Wehrspohn, and H. Föll, eds., Photonic Crystals: Advances in Design, Fabrication, and Characterization Wiley-VCH, 2004.

[2] S. Kim and V. Gopalan, Appl. Phys. Lett. 78, 3015(2001); J. D. Joannopoulos R. D. Meade, and J. N. Winn, Photonic Crystals (Princeton University Press 1995); K. Sakoda, Optical Properties of Photonic Crystals (Springer 2001); M. Golosovsky, Y. Saado, and D. Davidov, Appl. Phys. Lett. 75, 4168 (1999); Y. Saado, M. Golosovsky, D. Davidov, and A. Frenkel, Phys. Rev. B 66, 195108 (2002).

[3] A. Figotin, Y. A. Godin, and I. Vitebski, Phys. Rev. B 57, 2841 (1998); Chul-Sik Kee, Jae-Eun Kim, Hae Yong Park, Ikmo Park, and H. Lim, Phys. Rev. B 61, 15523 (2000); C. –S. Kee, J. –E. Kim, and H. Y. Park, Phys. Rev. E 57, 2327 (1998); J. Zhou, C. Q. Sun, K. Pita, Y. L. Lam, Y. Zhou, S. L. Ng, C. H. Kam, L. T. Li, and Z. L. Gui, Appl. Phys. Lett. 78, 661 (2001).

[4] H. S. Kitzerow and J. P. Reithaimer Ref. 1 c , Chap. 9.

[5] R.W. Boyd, Nonlinear Optics, (Academic Press, London, UK, 1992).

[6] N. V. Tabiryan, A. V. Sukhov and B. Ya. Zeldovich, Mol. Cryst. Liq. Cryst., 136 1 (1986); Khoo I. C. Prog. Optics, 26 108 (1988); I.C. Khoo, H. Li, Appl. Phys. B, 1, 573 (1993).

[7] S. Gasiorowicz, Quantum Physics, 2nd Edition, Chapter 5, (Wiley, New York, 1996).

[8] N. W. Ashcroft and N. D. Mermin, Solid State Physics, Chapter 8, (Saunder, Philadelphia, 1976).
Fig. 1 Schematic plot of the nonlinear photonic crystal. The interfaces exhibit Kerr-type optical response whereas the rest of the material displays a linear response.

Fig. 2 Band structure of the nonlinear PC. $f(z,p,\chi_a)$ versus $z$ parametrized by $\chi_a$. The roots of Eq. (3) correspond to the points where curves cut the $z$–axis and thus each crossing give us a point of the permitted band. a) $p = 0.03$, b) $p = 0.3$, c) $p = 0.7$ and d) $p = 0.9$.

Fig. 3 The clearer intervals denote the forbidden bands versus light intensity $p$ (vertical axis) in the interval $0 \leq z \leq 10$. a) $0 \leq p \leq 1$ and b) $0 \leq p \leq 1000$.

Fig 4 a) Reflectance $R$ against $A$ and $k$ for $n = 1$ ($N = 2$), and $g = 0.2$. b) Contour plot of $R$ versus $A$ and $k$ and the same parameters. The clearer regions correspond to larger values of $R$.

Fig 5 The same as Fig. 4 but for $g = 1$ and $n = 1$ ($N = 2$).

Fig. 6 Table of contour plots of $R$ against $A$ and $k$ for the shown values of $g$ and $N$. 
This figure "Fig_1.png" is available in "png" format from:

http://arxiv.org/ps/1104.2311v1
This figure "Fig_2.png" is available in "png" format from:

http://arxiv.org/ps/1104.2311v1
This figure "Fig_3.png" is available in "png" format from:

http://arxiv.org/ps/1104.2311v1
This figure "Fig_4.png" is available in "png" format from:

http://arxiv.org/ps/1104.2311v1
This figure "Fig_5.png" is available in "png" format from:

http://arxiv.org/ps/1104.2311v1
This figure "Fig_6.png" is available in "png" format from:

http://arxiv.org/ps/1104.2311v1