TILTED BIANCHI TYPE I COSMOLOGICAL MODELS FILLED WITH DISORDERED RADIATION IN GENERAL RELATIVITY REVISITED

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Abstract. Tilted Bianchi type I cosmological models filled with disordered radiation in presence of a bulk viscous fluid and heat flow are investigated. The coefficient of bulk viscosity is assumed to be a power function of mass density. Some physical and geometric properties of the models are also discussed.

Keywords: Cosmology; Bianchi type I Universe; Tilted Models.

1. Introduction

General Relativity describes the state in which radiation concentrates around a star. Klein (1948) worked on it and obtained an approximate solution to Einsteinian field equations in spherical symmetry for a distribution of diffused radiation. Many other researchers (Singh and Abdussattar, 1973; Roy and Bali, 1977) have worked on this topic and obtained exact static spherically and cylindrically symmetric solutions of Einstein’s field equations with exception as well. Roy and Singh (1977) have obtained a non-static plane symmetric spacetime filled with disordered radiation. Teixeira, Wolk and Som (1977) investigated a model filled with source free disordered distribution of electromagnetic radiation in Einstein’s general relativity.

The general dynamics of tilted models have been studied by King and Ellis (1973) and Ellis and King (1974). The cosmological models with heat flow have been also studied by Coley and Tupper (1983, 1984); Roy and Banerjee (1988). Ellis and Baldwin (1985) have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. Beesham (1986) derived tilted Bianchi type V cosmological models in the scale-covariant theory. A tilted cold dark matter cosmological scenario has been discussed by Cen, Nickolay,
Kofman and Ostriker (1992).

The majority of the studies in cosmology involve a perfect fluid. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation suggests analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. The model studied by Murphy (1973) possessed an interesting feature in that the big bang type of singularity of infinite spacetime curvature does not occur to be a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity (Padmanabhan and Chitre, 1987; Johri and Sudarshan, 1988; Maartens, 1995; Zimdahl, 1996; Pradhan, Saraykar and Beesham, 1997; Kalyani and Singh (1997; Singh, Beesham and Mbokazi, 1998; Pradhan et al., 2001, 2002). This motivates to study cosmological bulk viscous fluid model.

Recently Bali and Meena (2002) have investigated two tilted cosmological models filled with disordered radiation of perfect fluid and heat flow. Meena and Bali (2002) have obtained two conformally flat tilted Bianchi type V cosmological models. Very recently tilted Bianchi type I cosmological model for perfect fluid distribution in presence of magnetic field is investigated by Bali and Sharma (2003). In this paper, we propose to find tilted Bianchi type I cosmological models filled with disordered radiation in presence of a bulk viscous fluid and heat flow.

2. Field Equations

We consider the Bianchi type I metric in the form

\[ ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \]

(1)

where A, B and C are function of t only.

The Einstein’s field equations are given by

\[ R_{ij}^i - \frac{1}{2} R g_{ij} = -8\pi T_{ij} \quad (c = 1, \ G = 1 \ \text{in gravitational unit}) \]

(2)
where $R^j_i$ is the Ricci tensor; $R = g^{ij} R_{ij}$ is the Ricci scalar; and $T^j_i$ is the stress energy-tensor in the presence of bulk stress given by

$$T^j_i = (\rho + \bar{p})v_i v^j + \bar{p}g^j_i + q_i v^j + v_i q^j,$$

and

$$\bar{p} = p - \xi v^i_i.$$  

Here $\rho$, $p$, $\bar{p}$ and $\xi$ are the energy density, isotropic pressure, effective pressure, flow vector, bulk viscous coefficient respectively and $v_i$ is the flow vector satisfying the relations

$$g_{ij} v^i v^j = -1$$

$$q_i q^j > 0,$$

$$q_i v^i = 0,$$

where $q_i$ is the heat conduction vector orthogonal to $v_i$. The fluid flow vector has the component $(\sinh \lambda A, 0, 0, \cosh \lambda)$ satisfying Eq. (5) and $\lambda$ is the tilt angle.

The Einstein’s field equations (2) for the line element (1) has been set up as

$$-8\pi[\rho + \bar{p}] \sinh^2 \lambda + \bar{p} + 2q_1 \frac{\sinh \lambda}{A} = \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_{4}C_{4}}{BC},$$

$$-8\pi\bar{p} = \frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_{4}C_{4}}{AC},$$

$$-8\pi\bar{p} = \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_{4}B_{4}}{AB},$$

$$-8\pi[-(\rho + \bar{p}) \cosh^2 \lambda + \bar{p} - 2q_1 \frac{\sinh \lambda}{A}] = \frac{A_{4}B_{4}}{AB} + \frac{A_{4}C_{4}}{AC} + \frac{B_{4}C_{4}}{BC},$$

$$(\rho + \bar{p})A \sinh \lambda \cosh \lambda + q_1 \cosh \lambda + q_1 \frac{\sinh^2 \lambda}{\cosh \lambda} = 0,$$

where the suffix 4 at the symbols $A$, $B$, $C$ denotes ordinary differentiation with respect to $t$.

### 3. Solutions of the field equations

Equations (8) - (12) with Eq. (4) are five independent equations in eight unknowns $A$, $B$, $C$, $\rho$, $p$, $\xi$, $q$ and $\lambda$. For the complete determinacy of the system, we need three extra conditions.

We assume the model is filled with disordered radiation which leads to

$$\rho = 3p$$
and
\[ A = (B C)^n \] (14)
where \( n \) is any positive real number. Eqs. (8) and (11) lead to
\[ \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = 8\pi(\rho - p + 2\xi\theta) \] (15)
Using Eq. (13) in Eq. (15) reduces to
\[ \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = 16\pi(\rho - \xi\theta) \] (16)
Equations (9) and (10) lead to
\[ \frac{C_{44}}{C} - \frac{B_{44}}{B} + \frac{A_4(B_4 - C_4)}{B} = 0 \] (17)
which leads to
\[ \frac{\mu_4}{\mu} = \frac{k}{\epsilon^{n+1}} \] (18)
where \( BC = \epsilon, \frac{B}{C} = \nu \) and \( k \) is a constant of integration. Eqs. (9) and (10) also give
\[ \frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = -16\pi(p - 2\xi\theta) \] (19)
From Eqs. (16) and (19), we obtain
\[ \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = 0 \] (20)
which can be rewritten as
\[ (n + 1)\frac{\epsilon_{44}}{\epsilon} + n^2\frac{\epsilon_4^2}{\epsilon^2} - \frac{1}{4}\frac{\epsilon_4^2}{\epsilon^2} + \frac{1}{4}\frac{\nu_4^2}{\nu^2} = 0 \] (21)
where \( A = \epsilon^n \). From Eqs. (18) and (21), we get
\[ \frac{f f^1 + (4n^2 - 1)f^2}{4(n + 1)\epsilon} = \frac{k^2}{4(n + 1)\epsilon^{2n+1}} \] (22)
where \( \epsilon_4 = f(\epsilon) \). Eq. (22) leads to
\[ f^2 = \frac{k^2 + (4n + 1)k_1\epsilon^{\frac{4n+1}{2}}} {(4n + 1)\epsilon^{2n}} \] (23)
where \( k_1 \) ia a constant of integration. On integrating Eq. (18) we obtain
\[ \log \nu = k\sqrt{(4n + 1)} \int \frac{d\epsilon}{\epsilon \sqrt{k^2 + k_1(4n + 1)\epsilon^{\frac{4n+1}{2}}}} \] (24)
Hence the metric (1) reduces to the form

$$ds^2 = -\frac{dr^2}{f^2} + \epsilon^{2n} dx^2 + \epsilon \nu dy^2 + \frac{\epsilon}{\nu} dz^2 \tag{25}$$

where $\nu$ is determined by Eq. (24).

By using the following transformation

$$\epsilon = T, \quad x = X, \quad y = Y, \quad z = Z$$

the metric (25) takes the form

$$ds^2 = -\left[ \frac{(4n + 1)T^{2n}}{k^2 + (4n + 1)k_1 T^{2(n+1)}} \right] dT^2 + T^{2n} dX^2 + T \nu dY^2 + \frac{T}{\nu} dZ^2 \tag{26}$$

where

$$\log \nu = k\sqrt{(4n + 1)} \int \frac{dT}{T\sqrt{k^2 + k_1(4n + 1)T^{2(n+1)}}} \tag{27}$$

The pressure and density of the model (26) are given by

$$8\pi(p - \xi \theta) = \frac{(4n + 1)k_1}{8(n + 1)T^{\frac{2n^2 + 4n + 3}{2(n+1)}}} \tag{28}$$

$$8\pi(\rho - 3\xi \theta) = \frac{3(4n + 1)k_1}{8(n + 1)T^{\frac{4n^2 + 4n + 3}{2(n+1)}}} \tag{29}$$

The tilt angle $\lambda$ is given by

$$\cosh^2 \lambda = \frac{2n + 3}{2(2n + 1)} \tag{30}$$

Hence

$$\sinh^2 \lambda = \frac{1 - 2n}{2(2n + 1)} \tag{31}$$

The scalar of the expansion $\theta$, calculated for the flow vector $v^i$, is given by

$$\theta = \frac{(n + 1)}{T^{n+1}} \sqrt{\frac{(2n + 3)[k^2 + (4n + 1)k_1 T^{2(n+1)}]}{2(2n + 1)(4n + 1)}} \tag{32}$$

If we put $\xi = 0$ in Eqs. (28)-(29), we get the solutions as obtained by Bali and Meena (2002).

Thus, given $\xi(t)$ we can solve the system for the physical quantities.
Therefore to apply the third condition, let us assume the following \textit{adhoc} law (Maartens, 1995; Zimdahl, 1996)

\[\xi(t) = \xi_0 \rho^m\]  \hfill (33)

where \(\xi_0\) and \(m\) are real constants. If \(m = 1\), Eq. (33) may correspond to a radiative fluid (Weinberg, 1972), whereas \(m = \frac{2}{3}\) may correspond to a string-dominated universe. However, more realistic models (Santos, 1985) are based on lying the regime \(0 \leq m \leq \frac{1}{2}\).

3.1. Model I: \((\xi = \xi_0)\)

When \(m = 0\), Eq. (33) reduces to \(\xi = \xi_0\) and hence Eqs. (28) and (29) with the use of Eq. (32) lead to

\[
p = \frac{(n + 1)\xi_0}{T^{n + 1}} \sqrt{\frac{2(n + 3)k^2 + (4n + 1)k_1 T^{\frac{2n + 1}{2(n + 1)}}}{2(2n + 1)(4n + 1)}} + \frac{(4n + 1)k_1}{64(n + 1)\pi T^{\frac{4n^2 + 4n + 3}{2(n + 1)}}} \hfill (34)
\]

\[
\rho = \frac{(n + 1)\xi_0}{3T^{n + 1}} \sqrt{\frac{2(n + 3)k^2 + (4n + 1)k_1 T^{\frac{2n + 1}{2(n + 1)}}}{2(2n + 1)(4n + 1)}} + \frac{(4n + 1)k_1}{192(n + 1)\pi T^{\frac{4n^2 + 4n + 3}{2(n + 1)}}} \hfill (35)
\]

3.2. Model II: \((\xi = \xi_0\rho)\)

When \(m = 0\), Eq. (33) reduces to \(\xi = \xi_0\rho\) and hence Eqs. (28) and (29) with the use of Eq. (32) lead to

\[
p = \frac{(4n + 1)k_1}{64\pi(n + 1)T^{\frac{2n^2 + 1}{2(n + 1)}}} \left[ T^{(n + 1)} - 3(n + 1)\xi_0 \sqrt{(2n + 3)} \left( k^2 + (4n + 1)k_1 T^{\frac{2n + 1}{2(n + 1)}} \right) \right] \hfill (36)
\]

\[
\rho = \frac{(4n + 1)k_1}{192\pi(n + 1)T^{\frac{2n^2 + 1}{2(n + 1)}}} \left[ T^{(n + 1)} - 3(n + 1)\xi_0 \sqrt{(2n + 3)} \left( k^2 + (4n + 1)k_1 T^{\frac{4n + 1}{2(n + 1)}} \right) \right] \hfill (37)
\]
4. Some Physical and Geometric Properties of the Models

The flow vector $v^i$ and heat conduction vector $q^i$ for the models (26) are obtained by Bali and Meena (2002)

$$v^1 = \frac{1}{T^n} \sqrt{\frac{1 - 2n}{2(2n + 1)}},$$

$$v^4 = \sqrt{\frac{2n + 3}{2(2n + 1)}},$$

$$q^1 = - \frac{b(4n + 1)(2n + 3)}{64\pi (n + 1)T^{\frac{4n^2 + 4n + 2}{2(2n + 1)}}} \sqrt{\frac{1 - 2n}{2(2n + 1)}},$$

$$q^4 = \frac{b(4n + 1)^2(1 - 2n)}{64\pi (n + 1)} \sqrt{\frac{2n + 3}{2(2n + 1)}} \left[ \frac{T^{\frac{2n^2 + 2n - 3}{2(n + 1)}}}{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}} \right].$$

The non-vanishing components of shear tensor ($\sigma_{ij}$) and rotation tensor ($\omega_{ij}$) are obtained as

$$\sigma_{11} = \frac{(2n - 1)}{6} \left[ \frac{2n + 3}{2n + 1} \right]^\frac{3}{2} T^{n-1} \sqrt{\frac{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}}{2(4n + 1)}},$$

$$\sigma_{22} = \nu \left[ \frac{2n + 3}{2(2n + 1)(4n + 1)} \right] \left[ \frac{3a\sqrt{4n + 1} + (1 - 2n)\sqrt{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}}}{6T^n} \right],$$

where $\nu$ is already given by (27).

$$\sigma_{33} = \frac{1}{\nu} \left[ \frac{2n + 3}{2(2n + 1)(4n + 1)} \right] \left[ (1 - 2n)\sqrt{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}} - 3a\sqrt{4n + 1} \right],$$

$$\sigma_{44} = - \frac{(1 - 2n)^2}{6(2n + 1)} \left[ \frac{2n + 3}{2(2n + 1)(4n + 1)} \right] \left[ \frac{\sqrt{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}}}{T^{n+1}} \right],$$

$$\sigma_{14} = \frac{(6 - 2n - 32n^2)}{12(2n + 1)^\frac{3}{2}} \left[ \frac{2n + 3}{2(4n + 1)} \right] \left[ \frac{\sqrt{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}}}{T} \right],$$

$$\omega_{14} = n \left[ \frac{(1 - 2n)}{2(2n + 1)(4n + 1)} \right] \left[ \frac{\sqrt{a^2 + b(4n + 1)T^{\frac{4n+1}{2(n+1)}}}}{T} \right].$$
The rates of expansion $H_i$ in the direction of $X$, $Y$, $Z$-axes are given by

$$H_1 = \frac{n}{\sqrt{4n+1}} \left[ \sqrt{a^2+b(4n+1)T^{2n+1}} \right],$$

(48)

$$H_2 = \frac{\sqrt{a^2+b(4n+1)T^{2n+1}} + a\sqrt{4n+1}}{2\sqrt{4n+1}T^{n+1}},$$

(49)

$$H_2 = \frac{\sqrt{a^2+b(4n+1)T^{2n+1}} - a\sqrt{4n+1}}{2\sqrt{4n+1}T^{n+1}}.,$$

(50)

The models in general represent shearing and rotating universes. The expansion in the models decreases as time increases and the expansion in the models stops at $T = \infty$. There is a big bang in the models at $T = 0$ if $n + 1 > 0$. There is no rotation in the models for $n = 0$ but it is shearing and goes on decreasing as time increases. For $n = 0$, the Hubble constant $H_1 = 0, H_2 \neq 0, H_3 \neq 0$. The fluid velocity vectors $v^1$ and $v^4$ for $n = 0$, are given by $v^1 = \frac{1}{\sqrt{2}}, v^4 = \sqrt{\frac{3}{2}}$. Both density and pressure in the models become zero at $T = \infty$. Since $\lim_{t \to \infty} \frac{\epsilon}{\rho} \neq 0$, the models do not approach isotropy for large values of $T$ for general value of $n$ and $n = 0$ also.

5. Particular Models

We consider the metric

$$ds^2 = -dt^2 + dx^2 + B^2 dy^2 + C^2 dz^2,$$

(51)

where $B, C$ are functions of $t$ alone.

Here we only assume that the model is filled with disordered radiation which leads Eq. (13). Following Bali and Meena (2002), we obtain, from Eqs. (51), (2) and (3)

$$B^2 = \frac{k_1(f - k)^4}{k_2^4},$$

(52)

$$C^2 = \frac{f + k}{k_1 k_2},$$

(53)

where $k, k_1$ and $k_2$ are constants of integration. Here $BC = \epsilon, \frac{B}{C} = \nu, \epsilon_4 = f(\epsilon)$ and

$$\epsilon = \left(\frac{f^2 k^2}{k_2}\right)^2$$

(54)
\[ \nu = k_1 \left( \frac{f - k}{f + k} \right)^2 \] (55)

Thus, after the suitable transformation of coordinates, the metric (51) reduces to the form

\[ ds^2 = \frac{16}{k_2^2} (T^2 - k^2)^2 dT^2 + dX^2 + (T - k)^2 dY^2 + (T + k)^4 dZ^2 \] (56)

The pressure and density for the model (56) are given by

\[ \bar{p} = (p - \xi \theta) = \frac{k_2^4}{64\pi(T^2 - k^2)^3} \] (57)

\[ (\rho - 3\xi \theta) = \frac{3k_2^4}{64\pi(T^2 - k^2)^3} \] (58)

The tilt angle \( \lambda \) is given by

\[ \cosh \lambda = \sqrt{3} \] (59)

The scalar expansion \( \theta \) calculated for the flow vector \( v^i \), is given by

\[ \theta = \sqrt{\frac{3}{2}} \left[ \frac{k_2 T}{(T^2 - k^2)^3} \right] \] (60)

Thus, given \( \xi(t) \) one can solve the system for the physical quantities.

5.1. Model I: (\( \xi = \xi_0 \))

When \( m = 0 \), Eq. (33) reduces to \( \xi = \xi_0 \) and hence Eqs. (57) and (58) with the use of Eq. (60) lead to

\[ p = \frac{k_2^4}{(T^2 - k^2)^2} \left[ \sqrt{\frac{3}{2}} \xi_0 T + \frac{k_2^2}{64\pi(T^2 - k^2)} \right] \] (61)

\[ \rho = \frac{3k_2^4}{(T^2 - k^2)^2} \left[ \sqrt{\frac{3}{2}} \xi_0 T + \frac{k_2^2}{64\pi(T^2 - k^2)} \right] \] (62)

5.2. Model I: (\( \xi = \xi_0 \rho \))

When \( m = 1 \), Eq. (33) reduces to \( \xi = \xi_0 \rho \) and hence Eqs. (57) and (58) with the help of Eq. (60) lead to

\[ p = \frac{k_2^4}{32\pi(T^2 - k^2)} \left[ \frac{2(T^2 - k^2)^2 - 3\sqrt{6} k_2^2 T}{k_2^2} \right] \] (63)
\[ p = \frac{3k_4^4}{32\pi(T^2 - k^2)\left[2(T^2 - k^2)^2 - 3\sqrt{6}k^2T\right]} \quad (64) \]

6. Some Physical and Geometric Properties of Particular Models

The non-vanishing components of shear tensor \( \sigma_{ij} \) and rotation tensor \( \omega_{ij} \) for the models (56) are given by (Bali and Meena, 2002)

\[
\sigma_{11} = -\frac{1}{2} \sqrt{\frac{3}{2}} \left[ \frac{k_4^2 T}{(T^2 - k^2)^2} \right], \quad (65)
\]

\[
\sigma_{22} = \frac{k_1}{6} \sqrt{\frac{3}{2}} \left[ \frac{(T + 3k)(T - k)^2}{(T + k)^2} \right], \quad (66)
\]

\[
\sigma_{33} = \frac{1}{6k} \sqrt{\frac{3}{2}} \left[ \frac{(T - 3k)(T + k)^2}{(T - k)^2} \right], \quad (67)
\]

\[
\sigma_{44} = -\frac{1}{6} \sqrt{\frac{3}{2}} \left[ \frac{k_4^2 T}{(T^2 - k^2)^2} \right], \quad (68)
\]

\[
\sigma_{14} = \frac{1}{2\sqrt{2}} \left[ \frac{k_4^2 T}{(T^2 - k^2)^2} \right], \quad (69)
\]

\[
\omega_{14} = 0 \quad (70)
\]

The expressions for \( v_1, v_4, q_1 \) and \( q_4 \) for the metric (56) are obtained as

\[
v_1 = \frac{1}{\sqrt{2}}, \quad (71)
\]

\[
v_4 = -\frac{16}{k_4^2} (T^2 - k^2)^2 \sqrt{\frac{3}{2}}, \quad (72)
\]

\[
q_1 = -\frac{3k_4^4}{64\sqrt{2}\pi(T^2 - a^2)^3}, \quad (73)
\]

\[
q_4 = \frac{3k_4^4}{64\sqrt{6}\pi(T^2 - a^2)^3}. \quad (74)
\]

The rate of expansion \( H_i \) in the directions of \( X, Y, Z \)-axes is given by

\[
H_1 = 0, \quad (75)
\]

\[
H_2 = \frac{k_4^2}{2(T + k)(T - k)^2}, \quad (76)
\]
\[ H_3 = \frac{k_2^2}{2(T-k)(T+k)^2}. \]  

(77)

The models start with a big bang at \( T = \pm k \) and the expansion in the models decreases as time increases and the expansion in the models stops at \( T = 0 \). The x-component of the Hubble parameter is zero due to the assumption of metric. However, the y and z components become infinite at \( T = k \). The models, in general, represent shearing and non-rotating universe.

Since

\[ \lim_{t \to \infty} \frac{\sigma}{\theta} \neq 0. \]

Hence the models do not approach isotropy for large values of \( T \). When \( T \to k \), \( q_1 \to -\infty \) and \( q_4 \) is constant. There is a singularity in the models at \( T = k \). This singularity is pan cake type (MacCallum, 1971) as \( g_{11} \) and \( g_{22} \) is zero at the singularity \( T = k \).

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