On the electrodynamics of a magnetized rotating accretor

Aldo Treves and Ewa Szuszkiewicz

International School for Advanced Studies, SISSA
via Beirut 2-4, 34013 Trieste, Italy

Summary

We consider the current flow in an accreting rotating neutron star (X-ray pulsator) and propose that the active magnetosphere is limited by the Alfvén surface. Formulae for the luminosity and torque due to currents circulating in the neutron star are given, and astrophysical consequences for systems like Her X-1 and LMXRB are shortly discussed.

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1 Introduction

The electrodynamics of an isolated rotating magnetic neutron star is the backbone of pulsar models. The fundamentals of the theory have been posed in the paper of Goldreich and Julian (1969). The complete problem has been recognized to be complicated requiring the use of rather advanced techniques of plasma physics.

The other astrophysical situation where one deals with magnetized neutron stars is that of X-ray pulsators, where the neutron star accretes matter from a close companion. The basis of the theory was formulated in the seventies, and its capacity to account for the spin up (down) of X-ray pulsators can be considered as a test of its validity (e.g. Lipunov 1992, and references therein).

An aspect which has not focused much attention is that of the study of the electrodynamic activity, powered by the rotation of the neutron star (à la pulsar), when the neutron star is surrounded by a plasma sphere due to the very accretion process. This is the argument which we consider here in a rather simplified way.

In section 2 we briefly summarize the basics of pulsar electrodynamics. In section 3 we recall some formulae relevant for describing accretion on a static magnetic neutron star. Our elementary formulation for the electrodynamics of a magnetic rotating accretor is proposed in section 4. Astrophysical applications with consideration to Her X-1 and of a representative case for low mass X-ray binaries are discussed in section 5.

2 Pulsar electrodynamics

The aligned pulsar - unipolar inductor analogue - was proposed in the seminal paper of Goldreich and Julian (1969). In particular it was recognized that the magnetosphere intrinsically divides into an equatorial inactive part, which corotates with the pulsar, and a polar part with current flowing along the field lines. The colatitude at the surface of the star (with the radius $a$) of the magnetic surface separating the two zones is given in the approximation of the dipole field by

$$\vartheta_c \simeq \left(\frac{a \omega}{c}\right)^{1/2},$$  

(1)

where $\omega$ is the angular velocity of a rotating pulsar and $c$ is the speed of light. The polar zone, which does not corotate with the pulsar, is active since it is subject to an electromotive force (emf). Similarly as for an unipolar inductor the value of the emf or the difference of an electrostatic potential, $\varphi$, between
the pole and a point at the latitude $\vartheta_c$ can be found as follows:

$$\Delta \varphi = E\ell \simeq \frac{a^3 B_0 \omega^2}{c^2},$$

(2)

where the electric field relates to the magnetic field at the surface of the star, $B_0$, through

$$E \simeq \frac{\ell \omega}{c} B_0$$

(3)

and the characteristic length is

$$\ell = \vartheta_c a.$$  

(4)

Goldreich and Julian (1969) assumed that the pulsar is isolated and showed that if the electric, $E$, and magnetic, $B$, fields fulfil the condition

$$E \cdot B = 0$$

(5)

at least approximately, a net charge density, to be expected, is given by

$$|\rho_c| \simeq \frac{\omega B_0}{2\pi c}.$$  

(6)

Assuming that the currents are due to a completely charge separated plasma, the current density is approximately given by

$$J = |\rho_c| c.$$  

(7)

One can then evaluate the total current

$$I \approx \pi \ell^2 J \simeq \frac{B_0 \vartheta_c a^3 \omega^2}{2c},$$

(8)

and the energy loss rate, which reads

$$\left( \frac{dW}{dt} \right)^P = I \Delta \varphi \simeq \frac{\omega^3 a^6 B_0^2}{2c^3}.$$  

(9)

This is the formula proposed by Goldreich and Julian (1969), which substantially coincides with the radiating dipole formula of Deutsch (1955).

A rather controversial and specific assumption of the Goldreich and Julian paper is that of complete charge separation. However it is noticeable that also in models where the currents are almost neutral, eq. (7) appears to be valid. In particular Beskin et al. (1993) has shown that if the plasma pressure is insignificant and the density of positive or negative charges, $|\rho|$, satisfies the following condition

$$|\rho| \gg |\rho_c|,$$  

(10)
the Grad-Shafranov equation combined with the boundary conditions for the active region can be explicitly solved. The important conclusion is that eqs (7)-(9) are recovered. In any case we assume in the following that (7) holds as order of magnitude.

3 Accretion onto a magnetized neutron star

Consider a non-rotating neutron star endowed of a magnetic field $B_0$ accreting a homogeneous medium at a rate $\dot{M}$. The associated luminosity is

$$\left( \frac{dW}{dt} \right)^A = \frac{G M \dot{M}}{a}. \quad (11)$$

The simplest picture asserts that the magnetic field can be neglected at radii larger than the Alfvén radius

$$r_A = \left( \frac{2 \mu^2}{\dot{M} \sqrt{2GM}} \right)^{2/7}, \quad (12)$$

where $\mu = \frac{1}{4} a^3 B_0$. This is the distance where the magnetic energy density approximately equals the energy density of the infalling medium. The critical magnetic surface of equatorial radius, $r_A$, intersects the neutron star surface at a colatitude

$$\vartheta_A = \left( \frac{a}{r_A} \right)^{1/2}. \quad (13)$$

Supposedly matter moves along the critical magnetic surface filling completely the magnetic funnels and hits the neutron star at two polar caps which have a radial extension

$$r = a \vartheta_A = \frac{a^{3/2}}{r_A^{1/2}}. \quad (14)$$

If centrifugal force at the Alfvén radius overcomes gravitation, accretion may be inhibited (propeller phase). However we consider here only the situation where accretion does occur, since we are interested in the short circuiting of the polar cap of the neutron star.
4 Electrodynamics of a rotating magnetized accretor

Suppose that the neutron star is rotating with the spin axis parallel to the magnetic axis, and that it is accreting at a rate $\dot{M}$. Further suppose that

$$\vartheta_A > \vartheta_c.$$  \hfill (15)

The region enveloped by the magnetic surface of colatitude $\vartheta_A$ will be electro-dynamically inactive. In fact it corotates with the neutron star and no emf is present. Moreover at least in the simplest picture, the accreting plasma cannot penetrate this region. The regions of electrodynamical activity will be those corresponding to the accretion funnels within colatitude $\vartheta_A$. The accreting material does not corotate with the star and its density dominates that of the plasma produced by the neutron star rotation. Therefore the case is closer to that treated by the Beskin, Gurevich and Istomin (1993) rather than to the original Goldreich and Julian one. The main difference is that the boundary of the active region is now $\vartheta_A$ and not $\vartheta_c$. It follows that the relevant equation for the electrodynamics will be again (10), since it is unaffected by the boundary condition for the inactive region. The equations for the emf, current and power are recovered substituting the characteristic length, $\ell$, by $r$ from equation (14), and therefore one has

$$\Delta^A \varphi \simeq \frac{\omega B_0}{c} r^2;$$  \hfill (16)

$$I^A \simeq \frac{\omega B_0 r^2}{2};$$  \hfill (17)

and

$$\left( \frac{dW}{dt} \right)^{AP} = I^A \Delta^A \varphi \simeq \frac{(\omega B_0)^2 r^4}{2 c} = \frac{2 \omega \mu^2}{r_A^2 r_L},$$  \hfill (18)

where

$$r_L = \frac{c}{\omega}.$$  \hfill (19)

The corresponding torque is given by

$$K^{AP} = \frac{2 \mu^2}{r_A^2 r_L}.$$  \hfill (20)

Eqs (18) and (20) are in our opinion reasonable expressions for the energy and angular momentum loss associated to currents reaching the neutron star.
Obviously they are not the result of a self consistent picture. The actual current structure remains an exceedingly complicated and still unsolved problem.

5 Some astrophysical consequences

In order to explore the astrophysical consequences of our scenario of the electrodynamics of accreting magnetized rotators we examine two cases which may be representative for X-ray binaries. In both the mass of a neutron star is taken equal to $1 \, M_\odot$ and its radius to $10^6$ cm. First we examine the case of Her X-1, a prototype of X-ray pulsators. We take $B_0 \simeq 10^{12}$ G, $P = 1.2$ s, and $\dot{M} = 10^{17}$ g/s. From the previous formulae one has

$$\left( \frac{dW}{dt} \right)^A \sim 10^{37} \text{erg/s} \quad (21)$$

$$\left( \frac{dW}{dt} \right)^{AP} \sim 10^{34} \text{erg/s} \quad (22)$$

$$K^{AP} \sim 10^{33} \text{erg} \quad (23)$$

The second case is that of luminous low mass X-ray binaries. Here we take $\dot{M} = 10^{18}$ g/s. The values of the magnetic field and period are unknown. We will consider $B_0 \simeq 10^9$ G and a period of 5 milliseconds, and scale accordingly. The result is

$$\left( \frac{dW}{dt} \right)^A \sim 10^{38} \text{erg/s} \quad (24)$$

$$\left( \frac{dW}{dt} \right)^{AP} \sim 10^{36} B_9^{6/7} P_5^{-2} \text{erg/s} \quad (25)$$

$$K^{AP} \sim 10^{33} \text{erg} \quad (26)$$

It is apparent that in both cases the energy released by the passage of currents is a small fraction ($10^{-2} - 10^{-3}$) of the accretion luminosity. It should be pointed out here that the interaction between the magnetosphere and infalling material, which is not included here, may affect this results. Our picture is obviously inadequate for spectral calculations. However one could argue that the current structure may build up a non thermal contribution. In particular it is tempting to apply this scheme to a system like Cyg X-3, where rotation of a rapid pulsar may coexist with a large accretion rate. The resulting model would then have close relation to the flywheel picture of the system (Treves and Bocci, 1987, Mineshinge, Fabian, Rees, 1991).
The other aspect where this picture may be relevant is that related on the torque acting on the neutron star (20). The torque is always a braking one and therefore tends to spin down the pulsar. Spin up and spin down on accreting neutron stars have been the focus of a diffuse interest, because they are directly related to the observed period derivative of X-ray pulsators. The theory is somewhat intricated and we refer for a recent presentation to Lipunov (1992). The spin up is supposedly dominated by the angular momentum transferred from the accretion disk at the last Keplerian orbit, and depends on the accretion rate. Here we focus on the spin down. In the usual picture the spin down torque is calculated through the emergence of a toroidal component of the magnetic field in the accretion disk (e.g. Lynden-Bell and Pringle 1974). Alternatively one can consider viscous friction between the plasma and the magnetosphere (e.g. Davies and Pringle 1981). Both approaches yield the formula (Lipunov 1992)

\[ K_L = \frac{\mu^2}{r_c^3} \]  

(27)

where \( r_c \) is the corotation radius

\[ r_c = \left( \frac{GM}{\omega^2} \right)^{1/3} \]  

(28)

Equation (27) is formally similar to our equation (20) where \( r_A r_c \) plays the role of \( r_c^3 \). Applying this expression to the two examined cases, we obtain for Her X-1

\[ K_L \sim 10^{34} \text{erg} \]  

(29)

and for the second representative case

\[ K_L \sim 10^{33} \text{erg} \]  

(30)

In the first case the torque considered by us is one order of magnitude lower than that given by (27). In the second case the torques are comparable. Since

\[ \frac{K^{AP}}{K_L} = \frac{r_c^3}{r_A^2 r_L} \sim \frac{M^{9/7} \dot{M}^{4/7}}{\omega \mu^{8/7}} \]  

(31)

it is clear that if the magnetic field of a considered LMXRB were smaller than \( 10^9 \) G the torque proposed here might even exceed the torque described by (27). This indicates that the braking corresponding to equation (20) should not be neglected.

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