Simple scaling laws for astrophysical jets

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ABSTRACT

The idea of a unified model for all astrophysical jets has been considered for quite some time. We present here a few scaling laws relevant to all type of astrophysical jets, analogous to those of Sams et al. (1996) which are widely used for astrophysical black holes. We use Buckingham’s theorem of dimensional analysis to obtain a family of dimensional relations among the physical quantities associated to astrophysical jets.

Subject headings: hydrodynamics – galaxies: jets – ISM: jets and outflows – quasars: general – gamma rays: bursts

1. Introduction

Although the first report of an astrophysical jet was made by Curtis (1918), these objects were extensively studied much later with radio astronomy techniques (Reber 1940). Quasars, and radiogalaxies were discovered and later gathered in a unified model which proposed a dusty torus around the nucleus of the source (Antonucci & Miller 1985). Years later, some galactic sources showed similar features to the ones presented by quasars and radiogalaxies, i.e. relativistic fluxes, a central engine, symmetrical collimated jets, radiating lobes, and apparent superluminal motions (cf. Sunyaev et al. 1991). These objects are usually identified as µ–quasars. Optical and X-ray observations showed other similar non–relativistic sources in the galaxy associated to H–H objects (cf. Gouveia Dal Pino 2004). Lately, the strong explosions found in long Gamma Ray Bursts (GRB) have been modelled as collapsars, in which a jet of a very short lifetime is associated to the observed phenomena (cf. Kulkarni et al. 1999; Castro-Tirado et al. 1999).

The similarities between all astrophysical jets, mainly those between quasars and micro–quasars, and the scaling laws for black holes proposed by Sams et al. (1996) and Rees (1998)
made us search for the possible existence of some scaling laws that may occur to astrophysical jets in terms of very simple physical parameters, such as the magnetic field associated to the accretion disc, accretion rate and mass of the central object. These sort of relations have been studied in a very different way by Heinz & Sunyaev (2003); Heinz et al. (2005) giving scalings between the flux $F_\nu$ at a frequency $\nu$ and the mass of the central object.

The present work presents a few mathematical relations that appear naturally as a consequence of dimensional analysis and Buckingham’s Π theorem. We begin by considering some of the most natural physical dimensional quantities that have to be included to describe some of the physical phenomena related to all classes of jets. We then calculate the dimensional relations associated to these quantities. Finally, we briefly discuss these relations and their physical relevance to astrophysical jets.

2. Analysis

A complete set of quantities that must appear in the physical description of a particular astrophysical jet is complicated. However, there are some essential physical ingredients that must enter into the description of the problem. To begin with, the mass $M$ of the central object must accrete material from its surroundings at an accretion rate $\dot{M}$. Now, because gravity and magnetic fields $B$ are necessary in order to generate jets, Newton’s constant of gravity $G$ and the velocity of light $c$ must also be taken into account. If in addition there are certain characteristic quantities such as length $l$, density $\rho$ and velocity $v$ (e.g. these can be associated to the jet’s length, the density of the surrounding medium and the jet’s ejection velocity respectively), then the jet’s kinetic luminosity or kinetic power $L$ is an important parameter that must be related in a general way to all these quantities in the following manner:

$$L = L(\dot{M}, M, c, G, B, l, v, \rho).$$

Using Buckingham’s Π theorem of dimensional analysis (Buckingham 1914; Sedov 1993) the following non–trivial dimensionless parameters are found

$$\Pi_1 = \frac{L}{Mc^2}, \quad \Pi_2 = \frac{G\dot{M}}{c^3}, \quad \Pi_3 = \frac{Bc^{1/2}M}{M^{3/2}}, \quad \Pi_4 = \frac{l\dot{M}}{Mc^3}, \quad \Pi_5 = \frac{\rho c^3M^2}{M^3}.\quad (2)$$
The parameter $\Pi_2$ can be rewritten as

$$\Pi_2 = \left(\frac{GM}{c^2}\right) \left(\frac{\dot{M}}{M}\right) \frac{1}{c}. \quad (2)$$

Since the quantity

$$\tau := \frac{M}{\dot{M}} \quad (3)$$

has dimensions of time, according to Buckingham’s $\Pi$ theorem it represents a characteristic time for our problem. For the case we are analysing, it represents the time for which the central object doubles its mass. Using equation (3) we can write $\Pi_2$ as

$$\Pi_2 = \frac{r_s}{2\tau c}, \quad (4)$$

where $r_s$ is the Schwarzschild radius. This relation naturally defines a length

$$\lambda \sim c\tau, \quad (5)$$

which can be thought of as the maximum possible length a jet could have, since $\tau$ is roughly an upper limit to the lifetime of the source.

On the other hand, from equation (2) it is found that

$$\Pi_6 := \Pi_2^{2/3} \Pi_3 = \left(\frac{GM}{c^2}\right)^{3/2} \sqrt{\frac{M_c^2}{B}}. \quad (6)$$

The quantity $GM/c^2$ is of the order of the gravitational radius, therefore the denominator of equation (6) defines a length $r_j$ given by

$$r_j \propto \frac{M_1^{1/3} c_{2/3}^2}{B_{2/3}} \approx 10^2 \left(\frac{M}{M_\odot}\right)^{1/3} \left(\frac{B}{1G}\right)^{-2/3} \text{ pc}. \quad (7)$$

From now on, we use typical values for different physical quantities related to astrophysical jets as presented in Table 1. With these numerical values, it follows that extragalactic radio sources and $\mu$-quasars are such that $r_j \propto 10^4\text{ pc}$ and $r_j \propto 10\text{ pc}$ respectively. These lengths are fairly similar to the associated length of their corresponding jets. Therefore, if
Table 1: Typical values for different physical quantities involved in the accretion–ejection phenomena of astrophysical jets. All values of luminosities, central masses and velocities in terms of the Lorentz factor $\Gamma$ were taken from Robson (1996); Reipurth et al. (1997); Ferrari (1998); Ford & Tsvetanov (1999); Meier (2002); Vilhu (2002); Cherepashchuk et al. (2003); Calvet et al. (2004); Mendoza et al. (2005); Mirabel (2004); Gouveia Dal Pino (2004). Magnetic field values were taken from Blandford (1990); Carilli et al. (1996); Lovelace & Romanova (1996); Koide et al. (1998); Camenzind (1999); Meier (2002); Wu et al. (2002); Smirnov et al. (2003); Uzdensky & MacFadyen (2006).

| Object | AGN | $\mu$-quasars | H–H | GRB |
|--------|-----|----------------|------|-----|
| Size [pc] | $\sim 10^5$ | $\lesssim 10$ | < a few | $\sim 10^{-5} - 10^{-1}$ |
| Luminosity [$L_\odot$] | $10^7 - 10^{19}$ | $< 10^5$ | $10^1 - 10^4$ | $10^{21}$ |
| Central mass [$M_\odot$] | $10^6 - 10^9$ | $1 - 10$ | $< 10$ | $1 - 10$ |
| Lorentz factor [$\Gamma$] | $10 - 10^3$ | $> 10$ | $\lesssim 1.0000005$ | $100 - 300$ |
| Magnetic field [G] | $\sim 100$ | $\sim 100$ | < 200 | $\sim 10^{16}$ |

we identify the length $r_j$ as the length of the jet, a constant of proportionality $\sim 1$ is needed in equation (7), and so

$$r_j \approx 100 \left( \frac{M}{M_\odot} \right)^{1/3} \left( \frac{B}{1 \text{G}} \right)^{-2/3} \text{ pc.} \quad (8)$$

Since equation (6) is roughly the the Schwarzschild radius $r_S$ divided by jet’s length $r_j$ to the power $3/2$, then

$$\Pi_6 = \frac{(Bl^{3/2}) (GM^2/l)^{3/2}}{(Mc^2)^2} \ll 1, \quad (9)$$

which in turn implies that

$$B \ll \frac{c^4}{G^{3/2}M} \approx 10^{23} (M/M_\odot)^{-1} \text{ G.} \quad (10)$$

The right hand side of this inequality is the maximum upper limit for the magnetic field associated to the accretion disc about the central object. For “extreme” micro–quasars like SS 433 and GRB’s the magnetic field $B$ reaches values $\gtrsim 10^{16} \text{ G}$ (cf. Meier 2002; Uzdensky & MacFadyen 2006), so that this upper limit works better for those objects.

From equation (2) it follows that
\[ \Pi_7 := \frac{\Pi_1}{\Pi_2 \Pi_3} = \frac{L \dot{M}}{B^2 M^2 G}, \]

and so

\[ L \propto 10^{-7} \left( \frac{B}{1 \text{ G}} \right)^2 \left( \frac{M}{M_\odot} \right)^2 \left( \frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right)^{-1} L_\odot. \] (11)

For the case of quasars and \( \mu \)-quasars, using typical values from Table 1, it follows that the kinetic power \( L \propto 10^{15} L_\odot \) and \( L \propto 10^8 L_\odot \) respectively. In order to normalise it to the observed values, we can set a constant of proportionality \( \sim 10^{-6} \) in equation (11), and thus the jet power relation as a function of the magnetic field, accretion rate and mass of the central object is given by

\[ L \approx 10^{-13} \left( \frac{B}{1 \text{ G}} \right)^2 \left( \frac{M}{M_\odot} \right)^2 \left( \frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right)^{-1} L_\odot. \] (12)

3. Discussion

According to the previous analysis astrophysical jets exist due to a precise combination of electromagnetic, mechanical and gravitational processes independently of the physical mechanisms behind the central engine.

A trivial dimensionless parameter that is obtained using Buckingham’s \( \Pi \) theorem of dimensional analysis applied to equation (1) is the ratio \( v/c \). With this, it is possible to form another dimensionless parameter given by \( \Pi_2 \Pi_4^2 \Pi_5/(v/c)^2 \), which leads to the dimensionless quantity \( \Lambda := \rho G l^2 / v^2 \) used by Mendoza et al. (2005) in order to obtain a maximum length for an astrophysical jet. As explained by Mendoza et al., this maximum size is most probably determined by the interaction of the jet and its cocoon with their surrounding environment, leading to the generation of Kelvin–Helmholtz instabilities.

Applying the results of equations (7) and (12) to GRB jets with a canonical magnetic field \( B \sim 10^{16} \text{ G} \), leads to wrong output kinetic luminosities and typical sizes of GRB jets. To correct these, proportionality factors of \( \sim 10^9 \) in the jet’s length \( r_j \) and of \( 10^{-5} \) in the kinetic power \( L \) in equations (7) and (12) have to be used respectively. The reason for this might be due to the fact that these jets have very short life times and so, they hardly resemble a traditional steady jet. Also, there might be some particular physical mechanisms that make
a dimensionless combination changing the proportionality factors in equations (7) and (12) in such a way that they give the correct value needed for these class of jets.

There are well known kinetic luminosities that appear in the literature related to the ejection of jets from different sources. As an example, in the Blandford & Payne (1982) model the luminosity takes the following form

\[ L = B_p^2 R^3 \Omega, \]

where \( \Omega \) is the angular velocity of the poloidal component of the magnetic field \( B_p \) and \( R \) is the size of the rotating region. It is possible to obtain equation (13) with the model presented in this article if we proceed as follows. Let us include in the functional relation (1) an important parameter of the problem, namely an angular velocity \( \Omega \), so that

\[ L = L(\dot{M}, M, c, G, B, l, v, \rho, \Omega). \]

It is then possible to build another dimensionless parameter \( \Pi_7 \) given by

\[ \Pi_7 := \frac{\Omega M}{\dot{M}}. \]

The Blandford & Payne (1982) kinetic luminosity is obtained using equations (2) and (15) with the introduction of a new dimensionless parameter \( \Pi_8 := \Pi_3^2 \Pi_4^2 \Pi_7 / \Pi_1 \).

In the same manner, let us define a new dimensionless parameter \( \Pi_9 := \Pi_3^2 \Pi_4^2 / \Pi_1 \) which reproduces the general dimensional shape of the kinetic luminosity of the Blandford & Znajek (1977) model, giving (Meier 2002)

\[ L \propto \frac{1}{c} B^2 R^4 \Omega^2. \]

The constant of proportionality in this equation has a value of \( \sim 0.1-0.03 \) depending on the geometry of the problem.

Of all our results, it is striking the fact that the jet power is inversely proportional to the accretion rate associated with it. This is probably due to the following. For a fixed value of the mass of the central object (in any case, for the time that accretion takes place, the mass of the central object does not increase too much) when the accretion mass rate increases, the magnetic field lines anchored to the plasma tend to pack up and thus, the field’s intensity increases in such a way as to get the correct result given by equation (12).
All the results presented in this article are in agreement with powerful extragalactic jets and jets associated with $\mu$–quasars and Herbig–Haro objects. They are also in agreement with jets associated to long gamma–ray bursts if proportionality factors are adjusted for the corresponding physical quantities.

Our main result, that the kinetic luminosity of the jet as a function of the magnetic field, the accretion rate and the mass of the central object is of general validity, independently of whether the same physical mechanism produces jets from galactic sources (i.e. Herbig–Haro, $\mu$-quasar and possibly jets associated to Gamma Ray Bursts) or extragalactic ones. Indeed, if the physics behind the accretion–ejection mechanism that occurs in jets works differently for each class of jets, then the luminosity output will be given by different mathematical expressions. For example, the Blandford & Payne (1982) and Blandford & Znajek (1977) models, are given by equations (13) and (16) respectively. However, for these two particular cases, we have proven above that if the Luminosity is a general function given by equation (14), then both cases lead to the luminosity relation (11). On the other hand, if the physical mechanism that generates jets at all scales is the same, relation (11) is also of general validity if the luminosity is a function described by (1), or equivalently by (14).

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