Design of Distributed Fractional Order PID Type Dynamic Matrix Controller for Large-Scale Process Systems

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ABSTRACT As a typical representative of distributed model predictive control, distributed dynamic matrix control (DDMC) is able to satisfy the basic control requirements for large-scale systems. However, the constraints and disturbances in actual industrial process usually lead to the slow set-point target tracking, large overshoot and weak anti-interference ability of the system. Therefore, the relevant requirements may not be met for some complex industrial processes. The existing distributed PID type dynamic matrix control (PID-DDMC) method can improve the control performance, but it maybe not accurate enough in some cases. Based on this background, this article introduces fractional order PID (FOPID) into distributed dynamic matrix control, and proposes a distributed fractional order PID type dynamic matrix control (FOPID-DDMC) algorithm. To compare with the conventional PID control, it expands the control and parameter setting range of the controller, and makes the control effect of the controller more accurate. Furthermore, the coupling effect among subsystems is dispelled by adopting the Nash optimal theory, and information interaction between the subsystems through network communication is realized, thereby, completing the optimization of the whole large-scale system. Finally, through a numerical simulation example and a level-temperature control process, the feasibility of the proposed algorithm is demonstrated by comparing with the traditional DDMC and PID-DDMC.

INDEX TERMS Distributed dynamic matrix control, fractional order PID, Nash optimal, large-scale process systems.

I. INTRODUCTION

With the development of modern society, the progress of science and technology and the growth of communication network, the industrial system is developing towards to large-scale and complex. Generally, an industrial system consists of many subsystems, among which there is the circulation of material, energy and information. In research of the large-scale system, people pay more attention to use distributed structure to reduce the complexity of computing [1]–[3].

Due to the mature application of model predictive control (MPC) in the actual industrial field [4]–[7], the distributed MPC proposed by the combination of MPC and distributed control is also developing and attracting research [8], [9]. Based on the idea of control with local information, several small-scale subsystems are decomposed from complex large-scale system for solution. This can reduce the calculation burden, as well as improve the overall system performance. Morosan et al. [10] put forward a model predictive control method for building temperature regulation, which can reduce energy consumption without reducing human comfort. It extends the predictive control strategy of single area building to multi area building examples, adopts a distributed predictive control algorithm which only exchanges information once in each time step, and has good control performance and low calculation requirements. Richard et al. put forward a novel distributed model predictive control (DMPC) for coupled constraint system in [11], which also transforms a single large-scale optimization problem into several smaller optimization problems, among which all decisions meet coupling constraints by transferring relevant plan data. A new distributed model predictive control method presented in [12] coordinates each decomposed
subsystem controller through the global performance indicators, also considers the interaction between subsystems, which improves the system global performance. In [13], an amine gas sweetening plant under the DMPC is studied. For linear discrete systems, a novel distributed predictive control algorithm [14] is presented, where each subsystem only needs its neighbor state variable reference trajectory, which reduces the transmission of information. Paper [15] presented a DMPC method based on Nash optimization and took the shell heavy oil fractionator benchmark as an example.

Traditional PID control method is widespread utilized in complex process industry due to simple structure, good stability and convenient regulation. Tang et al. put forward an optimal fuzzy PID controller in [16], which endows the controller with certain adaptive control ability and makes it have better control effect. According to several representative process models, an improved internal model control filter structure has been put forward, and a PID controller with better anti-interference response is designed in [17]. However, due to the deviation in building the mathematical model for the actual industry, the traditional PID control sometimes cannot meet the accuracy requirement of indexes [18], so various improved PID control algorithms are gradually studied and produced. Based on the state space form of system model, an improved PID control method proposed by Zhang et al. [19] is obtained by combining the extended nonminimal state space model predictive functional control strategy with PID control strategy. Hu et al. introduced the temperature control system of electromechanical chamber rely on incomplete differential PID method in [20]. Ge et al. [21] explicitly included the system uncertainty in the problem description, and designed a robust PID controller. To deal with time-varying nonlinearity and complexity in the process of chemical industry, an improved PID control scheme is proposed in [22], which uses neural network to linearize the nonlinear process. However, the above improved algorithm only has P, I and D parameters, which does not change the control method greatly. In paper [23], the fractional order model of a nonlinear system and its controller design and Implementation on FPGA are studied. Zhang et al. [24] proposed an improved extended non-minimal state space fractional order MPC method, which makes the industrial heating furnace’s temperature control more accurate. In literature [25], Zou et al. proposed a new method to realize PFC using fractional order system description, which improved the control system’s performance. The appearance of fractional order PID, which is a combination of fractional order control and PID control, improves the overall control performance of the system. Different from conventional PID, the FOPID has two more parameters λ and μ. It not only has the merits of former, such as good stability and convenient adjustment, but also expands the parameter setting range and control range of the controller, providing more superior control performance for production unit. In [26], for power system, the authors combined with FOPID controller and gas Brownian motion optimization to solve the load frequency control problem Li et al. designed a FOPID controller [27] to solve the problem of frequent switching control of pumped storage units. Hamamci et al. proposed a method to stabilize a given time delay fractional order system by using fractional order PID controller in [28]. Researchers presented and studied the application of FOPID controller in automatic voltage regulator [29].

Distributed dynamic matrix control, as a typical representative of DMPC uses the theory and thought of distributed control for reference and decomposes complicated large-scale system into multiple subsystems. Each subsystem is controlled through an independent dynamic matrix controller, and each controller transmits information through network interconnection, and based on the idea of Nash optimization, control optimization of the whole large-scale system can be realized [30], [31]. In [32], for fractional order system, Wang et al. designed a distributed PID type dynamic matrix control (PID-DDMC) algorithm. Wei et al. proposed a distributed energy-saving dynamic matrix control method in [33] to ensure the safety and stability of heavy-duty trains and energy-saving operation. Jin designed a distributed dynamic matrix control method in [34], which combines association estimation with multi-agent technology to achieve global optimal control by coordinating agents. For a large-scale nonlinear uncertain system that consists of many subsystems, a DMPC method based on multi rate sampling of network is studied [35]. Li et al. studied the distributed predictive control of a series of continuous time decoupled nonlinear systems with communication delay in [36], and presented a delay dependent DMPC algorithm. The paper [37] proposed a new distributed economic model predictive control method to solve the problem of fuel efficiency-oriented networked vehicle platooning control under nonlinear dynamics and safety constraints. Due to the inevitable disturbance and unknown factors in the actual industrial process, the traditional DDMC method may not satisfy some production index with high control accuracy requirements [38]. Therefore, developing a new DDMC scheme is necessary for improving system global performance.

In order to control large-scale system more accurately, a distributed FOPID dynamic matrix control algorithm is proposed by combining FOPID and DDMC, which guarantees the good control performance of large-scale system, as well as further improves the freedom of control parameter design. At the same time, in order to make the system achieve better control effect, researchers also put forward a lot of methods to set parameters. Bingul and Karahan [39] studied and designed the integer order PID and fractional order PID controller tuned by particle swarm optimization (PSO) and artificial bee colony (ABC) algorithms, and verified the effectiveness of the controller tuning algorithm, as well as gave the conclusion that the controller adjusted by ABC has better dynamic performance and robustness than PSO in time-domain performance index. Bingul and Karahan [40] also introduced and studied the effectiveness of swarm intelligence algorithm for the controller tuning and control of fractional systems.
In [41], two fractional order PI controllers are proposed and designed for a class of fractional order systems based on gain and phase margin (GPM) tuning methods. These tuning methods make the controller obtain good control parameters in the tuning process, so that the system has good control performance.

Because the proposed control method combines fractional order PID and dynamic matrix control, it inherits the advantages of both. Fractional order PID not only inherits the advantages of traditional PID control, such as simple structure and strong robustness, but also expands the control range and parameter setting range of the controller, making the control effect more superior. After introducing fractional order PID, a controller based on distributed dynamic matrix control is designed, it divides the complicated large-scale system into subsystems, with each subsystem being controlled through a corresponding FOPID dynamic matrix controller, and uses Nash optimal theory to eliminate the coupling effect among subsystems, which can not only ensure the overall performance of large-scale system, but also make up for the shortcomings of traditional distributed dynamic matrix control. Compared with the traditional distributed integer order PID type dynamic matrix control optimization method, the integral order and differential order are added to expand the control range and parameter setting range of the controller, so that the system control effect is more superior, and the system flexibility, robustness and overall control performance are improved. Because of its good control performance, it can be used in complex large-scale process industry to obtain better benefits.

The paper is organized as follows. In part 2, the basic concept of FOPID and the tuning rules of related parameters are introduced. The design of distributed FOPID type dynamic matrix controller is introduced in part 3. In section 4, the effectiveness of the distributed fractional order PID type dynamic matrix control is verified by two simulations and comparison is done with other two control methods Finally, section 5 makes the conclusion.

II. PRELIMINARIES OF FRACTIONAL ORDER PID

In this chapter, the preliminaries of FOPID are introduced from two aspects: the calculation of control increment of fractional order PID and the tuning rules for its controller.

A. CALCULATION OF CONTROL INCREMENT \(\Delta u(k)\) OF INCREMENTAL FOPID

According to the existing research, the transfer function of integer order PID is

\[
    u(s) = \left[ K_p + \frac{K_i}{s} + K_d s \right] e(s)
\]

(1)

the transfer function of fractional order PID is

\[
    u(s) = \left[ K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \right] e(s)
\]

(2)

Obviously, we can see that the fractional order PID has two more parameters \(\lambda\) and \(\mu\) than the integer order PID, which makes the tuning range of the controller parameters wider, the controller can control the controlled object more flexibly and get better control effect.

In the time domain, the FOPID is explained as follows.

\[
    u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^\mu e(t)
\]

(3)

where \(u(t)\) is the output value of the controller at time instant \(t\), \(e(t)\) is the deviation value of the controller input at time instant \(t\). \(K_p, K_i, K_d\) symbolize proportional gain, integral constant and differential constant respectively, \(\lambda\) and \(\mu\) respectively expressed as integral order and differential orders, \(D\) denotes the basic operators of fractional calculus.

For numerical calculation of FOPID, the transfer function needs to be discretized as follows:

\[
    u(k) = K_p e(k) + K_i T_s^{\lambda} \sum_{j=0}^{k} q_j e(k-j) + K_d T_s^{-\mu} \sum_{j=0}^{k} d_j e(k-j)
\]

(4)

where, \(u(k)\) is the output value of the controller, \(e(k)\) is the deviation value of the controller input. \(T_s\) is sampling time. \(q_j, d_j\) represent binomial coefficient, \(q_0 = 1, q_j = (1 - \frac{1+\lambda}{J}) q_{j-1}, d_0 = 1, d_j = (1 - \frac{1-\mu}{J}) d_{j-1}\).

Combined with equation (3) and (4), the incremental FOPID control \(\Delta u(k)\) can be obtained as follows

\[
    \Delta u(k) = u(k) - u(k-1)
\]

\[
    = K_p \Delta e(k) + K_i T_s^{\lambda} \left[ e(k) - \sum_{j=1}^{k} \frac{1+\lambda}{J} q_{j-1} e(k-j) \right] + K_d T_s^{-\mu} \left[ e(k) - \sum_{j=1}^{k} \frac{1-\mu}{J} d_{j-1} e(k-j) \right]
\]

\[
    = K_p \Delta e(k) + (K_i T_s^{\lambda} + K_d T_s^{-\mu}) e(k) - \sum_{j=1}^{k} \left( \frac{1+\lambda}{J} K_i T_s^{\lambda} q_{j-1} + \frac{1-\mu}{J} K_d T_s^{-\mu} d_{j-1} \right) \times e(k-j)
\]

(5)

In order to simplify equation (5), we define

\[
    K_a = K_i T_s^{\lambda} + K_d T_s^{-\mu},
\]

\[
    K_j = - \left( \frac{1+\lambda}{J} K_i T_s^{\lambda} q_{j-1} + \frac{1-\mu}{J} K_d T_s^{-\mu} d_{j-1} \right)
\]

Then the incremental FOPID control increment \(\Delta u(k)\) can be simplified as

\[
    \Delta u(k) = K_p \Delta e(k) + K_a e(k) + \sum_{j=1}^{k} K_j e(k-j)
\]

(6)
B. THE FUNCTION AND TUNING RULES OF λ AND µ IN FRACTIONAL ORDER PID CONTROLLER

In the different equations of FOPID controller, when λ and µ are both 1, the traditional integer order PID is formed. When λ is 1 and µ is 0, PI controller is formed. On the contrary, when λ is 0 and µ is 1, it is the PD controller. Therefore, PI, PD and PID are all special forms of fractional order PID.

Owing to the utilization of integral order λ and differential order µ, the fractional order PID has two more adjustable parameters than the traditional integer order PID controller, and its parameter setting range is also greatly increased. It can be known from Figure 1 that PI, PD and PID controllers are only a few special points in the control plane. The control range of FOPID controller can be extended to the whole plane by the arbitrary choice of integral order and differential order parameters, and as the range of parameter setting becomes larger, the control range of FOPID controller will be larger, which can make the control system track the reference trajectory more accurately and meet the requirements of control accuracy and stability.

To sum up, for the study of λ and µ, the main function of λ is to reduce the steady-state error, and µ is to adjust the system’s overshoot. Only when these two parameters are adjusted to a proper matching degree, the control effect of the system will be better. The five parameters of FOPID controller are selected reasonably, and the designed fractional order PID can control the system more accurately than the integer order PID.

C. TUNING RULES OF FRACTIONAL ORDER PID PARAMETERS

At present, there are three main methods to adjust parameter of FOPID controller, which are the dominant pole method, the optimization method, and the gain and phase margin (GPM). The gain and phase margin are important indexes to measure the quality of control system. Here, the referenced GPM method [41] is introduced as follows:

Given the phase margin $\phi_m$ and cut-off frequency $\omega_c$ of the system

1) The phase characteristic of the control system at the cut-off frequency is as follow.

$$\text{Arg}[H(j\omega_c)] = \text{Arg}[C(j\omega_c)G(j\omega_c)] = -\pi + \phi_m$$

(7)

2) The gain characteristic of the control system at the cut-off frequency is as follow.

$$|H(j\omega_c)|_{dB} = |C(j\omega_c)G(j\omega_c)|_{dB} = 0$$

(8)

3) The gain robust condition of the system is as follow.

$$\left(\frac{d}{d\omega} \text{Arg}[C(j\omega_c)G(j\omega_c)]\right)_{\omega=\omega_c} = 0$$

(9)

Combining the above three setting rules and by using simultaneous equation and equation calculation, the parameters of IOPID controller and FOPID controller can be calculated respectively.

1) PARAMETER TUNING OF INTEGRAL ORDER PID BASED ON GPM

It is assumed that the transfer function of the controlled object is

$$G(s) = \frac{k}{Ts + 1}e^{-\tau s}$$

(10)

The transfer function of IOPID is:

$$C(s) = k_p + \frac{k_i}{s} + k_ds$$

(11)

Then the frequency response, phase and gain of the controlled object are expressed as

$$G(j\omega) = \frac{k}{jT\omega + 1}e^{-j\tau \omega}$$

(12)

$$\text{Arg}[G(j\omega)] = -\arctan(\omega T) - \tau \omega$$

(13)

$$|G(j\omega)| = \frac{k}{\sqrt{1 + (T\omega)^2}}$$

(14)
The frequency response, phase and gain of the IOPID controller are expressed as

\[ C(j\omega) = k_p + j\left(k_d\omega - \frac{k_i}{\omega}\right) \]  
(15)

\[ \text{Arg}[C(j\omega)] = \text{arctan}\left(\frac{(k_d\omega^2 - k_i)}{(\omega k_p)}\right) \]  
(16)

\[ |C(j\omega)| = \sqrt{k_p^2 + (k_d\omega - k_i/\omega)^2} \]  
(17)

Thus, the transfer function and frequency characteristic of the system based on IOPID controller are described as follows.

\[ H(s) = G(s)C(s) \]  
(18)

\[ H(j\omega) = G(j\omega)C(j\omega) \]  
(19)

Then, the phase and gain of the open-loop frequency response are

\[ \text{Arg}[H(j\omega)] = \text{arctan}\left(\frac{(k_d\omega^2 - k_i)}{(\omega k_p)}\right) \]  
\[ -\text{arctan}(\omega T) - \tau \omega \]  
(20)

\[ |H(j\omega)| = |C(j\omega)||G(j\omega)| \]  
(21)

According to formula (7) - (9), the following parameter relations are obtained

1) The phase characteristic of the control system at the cut-off frequency is as follows

\[ \text{Arg}[H(j\omega_c)] = \text{arctan}\left(\frac{(k_d\omega_c^2 - k_i)}{(\omega_c k_p)}\right) \]  
\[ -\text{arctan}(\omega_c T) - \tau \omega_c = -\pi + \phi_m \]  
(22)

2) The gain characteristic of the control system at the cut-off frequency is as follows

\[ |H(j\omega_c)| = |C(j\omega_c)||G(j\omega_c)| \]  
\[ = \sqrt{k_p^2 + (k_d\omega_c - k_i/\omega_c)^2} \]  
(23)

3) The gain robust condition of the control system is as follows

\[ \frac{d}{d\omega}\left[\frac{\text{Arg}[C(j\omega_c)G(j\omega_c)]}{\omega=\omega_c}\right] = \begin{cases} \text{arctan}\left(\frac{(k_d\omega_c^2 - k_i)}{(\omega_c k_p)}\right) \\ -\text{arctan}(\omega_c T) - \tau \omega_c \end{cases} = 0 \]  
(24)

Sort out the above three equations and simplify them, we have

\[ \frac{k_d\omega^2 - k_i}{\omega k_p} = \Omega_1 \]  
(25)

\[ \frac{k_d k_p \omega^2 + k_i k_p}{(\omega k_p)^2 + (k_d \omega - k_i)^2} = \frac{T}{\Omega_2} + \tau \]  
(26)

where,

\[ \Omega_1 = \tan[\arctan(\omega T) + \tau \omega + \phi_m], \quad \Omega_2 = 1 + (\omega T)^2. \]

After simplified calculation, we can obtain

\[ k_p = \frac{\sqrt{\Omega_2}}{k \sqrt{1 + \Omega_1^2}} \]  
(28)

\[ k_i = \frac{1}{2k} \left[ \frac{\sqrt{1 + \Omega_1^2}}{\sqrt{\Omega_2}} \left( T \omega_c^2 + \Omega_1 \omega_c \sqrt{\Omega_2} \right) \right] \]  
(29)

\[ k_d = \frac{1}{2k} \left[ \frac{\sqrt{1 + \Omega_1^2}}{\sqrt{\Omega_2}} \left( T + \Omega_2 \tau \right) + \frac{\Omega_1 \omega_c \sqrt{\Omega_2}}{1 + \Omega_1^2} \right] \]  
(30)

Finally, the feasibility of IOPID controller can be verified by Bode diagram.

2) PARAMETER TUNING OF FRACTIONAL ORDER PID BASED ON GPM

Same as before, the phase and gain of the FOPID controller are

\[ \text{Arg}[C(j\omega)] = \arctan(\psi_1/\psi_2) \]  
(31)

\[ |C(j\omega)| = \sqrt{\psi_1^2 + \psi_2^2} \]  
(32)

where

\[ \psi_1 = k_d \omega^{\mu} \sin(\mu \pi / 2) - k_i \omega^{-\lambda} \sin(\mu \pi / 2) \]

\[ \psi_2 = k_p + k_i \omega^{-\lambda} \cos(\lambda \pi / 2) + k_d \omega^{\mu} \cos(\mu \pi / 2) \]

Take the same controlled object as in equation (10), the transfer function and frequency characteristic of the system based on FOPID controller are

\[ H(s) = G(s)C(s) \]  
(33)

\[ H(j\omega) = G(j\omega)C(j\omega) \]  
(34)

Its corresponding phase and gain of the open-loop frequency response are

\[ \text{Arg}[H(j\omega)] = \arctan(\psi_1/\psi_2) - \arctan(\omega T) - \tau \omega \]  
(35)

\[ |H(j\omega)| = |C(j\omega)||G(j\omega)| = \frac{k \sqrt{\psi_1^2 + \psi_2^2}}{1 + (\omega T)^2} \]  
(36)

1) The phase characteristic of the control system at the cut-off frequency is as follows

\[ \text{Arg}[H(j\omega_c)] = \arctan(\psi_1/\psi_2) - \arctan(\omega_c T) - \tau \omega_c = -\pi + \phi_m \]  
(37)
(2) The gain characteristic of the control system at the cut-off frequency is as follows
\[
|H(j\omega_c)| = |C(j\omega_c)| |G(j\omega_c)| = \frac{k \sqrt{\psi_1^2 + \psi_2^2}}{1 + (T\omega_c)^2} = 1
\]  
(38)

The gain robust condition of the system is
\[
\left( \frac{d}{d\omega} \text{Arg} (H(j\omega_c)) \right)_{\omega=\omega_c} = 0
\]
where,
\[
\psi_1 = k_1\omega_c^\mu \sin (\mu \pi / 2) - k_1\omega_c^{-\mu} \sin (\mu \pi / 2)
\]
\[
\psi_2 = k_2 + k_1\omega_c^{-\lambda} \cos (\lambda \pi / 2) + k_2\omega_c^{\lambda} \cos (\mu \pi / 2)
\]
\[
\psi_3 = k_3 \left( k_3\mu \omega_c^{\mu-\lambda} \sin (\mu \pi / 2) + k_3\lambda \omega_c^{\lambda-\mu} \sin (\lambda \pi / 2) \right)
\]
\[
\psi_4 = k_4 \left( k_4\mu \omega_c^{\mu-\lambda} \sin ((\lambda + \mu) \pi / 2) (\lambda + \mu) \right)
\]

With equation (38) as the main function, the nonlinear optimization function is used to solve the equations, and five parameters can be obtained respectively. Then the feasibility of the controller can be verified by Bode diagram.

III. DISTRIBUTED FRACTIONAL ORDER PID TYPE DYNAMIC MATRIX CONTROL

In this part, a distributed fractional order PID type dynamic matrix control algorithm is proposed through introducing fractional order PID operator into the performance indicator of distributed dynamic matrix control.

A. PREDICTION MODEL IN DISTRIBUTED DYNAMIC MATRIX CONTROL

Large-scale systems exist widely in the actual industrial production process with complex structure and strong coupling effect. Generally, the output of one subsystem will be affected by the input of the other subsystems. Thus, the study assumes that a large-scale system with \( N \) input and \( N \) output is as follows:
\[
G(s) = \begin{bmatrix}
K_{11}e^{-\tau_{11}s} & K_{12}e^{-\tau_{12}s} & \cdots & K_{1N}e^{-\tau_{1N}s} \\
\frac{T_{11}s + 1}{T_{12}s + 1} & \frac{T_{12}s + 1}{T_{13}s + 1} & \cdots & \frac{T_{1N}s + 1}{T_{1N}s + 1} \\
\frac{T_{21}s + 1}{T_{22}s + 1} & \frac{T_{22}s + 1}{T_{23}s + 1} & \cdots & \frac{T_{2N}s + 1}{T_{2N}s + 1} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{T_{N1}s + 1}{T_{N2}s + 1} & \frac{T_{N2}s + 1}{T_{N3}s + 1} & \cdots & \frac{T_{NN}s + 1}{T_{NN}s + 1}
\end{bmatrix}
\]  
(41)

where, \( K_{ij}, T_{ij}, \tau_{ij} \) respectively mean the steady-state gain, the time constant and lag time of the multivariable process and \( i, j = 1, 2, \cdots, N \).

According to the theory of distributed control, the problem of solving large-scale complex system with \( N \) input and \( N \) output can be decentralized to \( N \) subsystems. In dynamic matrix control, the sampling value of unit step response needs to be measured by carrying out step response experiment. A step signal is put into the process model, then a step response curve can be obtained. Further, filter the step response curve and record the corresponding step response data of the curve at each sampling time \( T_l \). When the error between \( a_{ij}(k') \) \( (k' > L_{ij}) \) and \( a_{ij}(L_{ij}) \) tends to 0, \( a_{ij}(L_{ij}) \) can be regarded as the steady-state value of the step response. Thus, a step response model vector \( a_{ij} \) between the \( i \) input and the \( j \) output is:
\[
a_{ij} = [a_{ij}(1), a_{ij}(2), \ldots, a_{ij}(L_{ij})]^T
\]  
(42)

where, \( a_{ij}(k') \) represents step response sample value when \( t_L = k'T_l, k' \) is the current number of samples, and \( L_{ij} \) denotes the modeling horizon of the \( j \) input to the \( i \) output.

Then establish the dynamic matrix of the process through the step response model vector \( a_{ij} \) is
\[
A_{ij} = \begin{bmatrix}
a_{ij}(1) & a_{ij}(2) & \cdots & a_{ij}(1) \\
\vdots & \vdots & \ddots & \vdots \\
a_{ij}(M-1) & a_{ij}(M-2) & \cdots & a_{ij}(1) \\
\vdots & \vdots & \ddots & \vdots \\
a_{ij}(P-1) & a_{ij}(P-2) & \cdots & a_{ij}(P+M-1)
\end{bmatrix}
\]  
(43)

where, \( A_{ij} \) is the \( P \times M \) order dynamic matrix, and \( a_{ij}(k) \) denotes the step response data of the \( i \) input to the \( j \) output. \( P, M \) are the optimized horizon and control horizon of the distributed dynamic matrix control algorithm respectively.

Add \( \Delta u_i(k-1), \Delta u_2(k-1), \cdots, \Delta u_N(k-1) \) of each subsystem at time instant \( k-1 \), we can obtain the model predicted value \( y_{i,p}(k-1) \) of the \( i \) subsystem.
\[
y_{i,p}(k-1) = y_{i,0}(k-1) + A_{ii,0}\Delta u_i(k-1) + \sum_{j=1,j\neq i}^{n} A_{ii,j}\Delta u_j(k-1)
\]  
(44)

where,
\[
y_{i,p}(k-1) = \begin{bmatrix}
y_{i,1}(k-1) \\
y_{i,2}(k-1) \\
\vdots \\
y_{i,N}(k-1) \\
y_{i,1}(k+L-1|k-1) \\
y_{i,2}(k+L-1|k-1) \\
\vdots \\
y_{i,N}(k+L-1|k-1)
\end{bmatrix}
\]
\[ y_{i,0}(k) = S y_{i,0}(k) \]

\[
S = \begin{bmatrix}
0 & 1 & 0 & \cdots & \cdots \\
0 & 0 & 1 & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix}
\]

The predicted output value of the \( i \) subsystem under \( M \) continuous control increments is

\[
y_{i,PM}(k) = y_{i,P0}(k) + A_{ii}\Delta u_{i,M}(k) + \sum_{j=1, j\neq i}^{n} A_{ij}\Delta u_{j,M}(k)
\]  \((48)\)

where

\[
y_{i,P0}(k) = \begin{bmatrix}
y_{i,M}(k+1|k) \\
y_{i,M}(k+2|k) \\
\vdots \\
y_{i,M}(k+P|k)
\end{bmatrix},
\]

\[
\Delta u_{i,M}(k) = \begin{bmatrix}
\Delta u_{i}(k) \\
\Delta u_{i}(k+1) \\
\vdots \\
\Delta u_{i}(k+M-1)
\end{bmatrix},
\]

\[
\Delta u_{j,M}(k) = \begin{bmatrix}
\Delta u_{j}(k) \\
\Delta u_{j}(k+1) \\
\vdots \\
\Delta u_{j}(k+M-1)
\end{bmatrix}
\]

\[ y_{i,0}(k+1|k), y_{i,0}(k+2|k), \ldots, y_{i,0}(k+P|k) \] are the initial predicted output of the \( i \) subsystem from time instant \( k \) to time instant \( k+1, k+2, \ldots, k+P \).

**B. SELECTION OF PERFORMANCE INDEX FOR DISTRIBUTED FRACTIONAL ORDER PID TYPE DYNAMIC MATRIX CONTROL**

Introducing FOPID into the objective function of DDMC and for the \( i \) subsystem, we have:

\[
\min J_i(k) = \Delta E_0^i(k)^T K_p^i \Delta E_0^i(k) + E_0^i(k)^T K_i^i E_0^i(k) + \sum_{j=1}^{k} E_0^i(k-j)^T K_j^i E_0^i(k-j)
\]

\[ + \Delta u_{i,M}(k)^T R_i \Delta u_{i,M}(k) \]  \((49)\)

where

\[
E_0^i(k) = w_i(k) - y_{i,PM}(k)
\]

\[
\Delta E_0^i(k) = \Delta w_i(k) - \Delta y_{i,PM}(k)
\]

\[
E_0^i(k-j) = w_i(k-j) - y_{i,PM}(k-j)
\]

\[
\Delta E_0^i(k) = \left[ \Delta e_0^i(k+1), \Delta e_0^i(k+2), \ldots, \Delta e_0^i(k+P) \right]^T
\]

\[
E_0^i(k-j) = \left[ \Delta e_0^i(k-j+1), \Delta e_0^i(k-j+2), \ldots, \Delta e_0^i(k-j+P) \right]^T
\]

\[
w_i(k) = \left[ w_i(k+1), w_i(k+2), \ldots, w_i(k+P) \right]^T
\]

\[
w_i(\varepsilon) = \beta^\varepsilon y_i(k) + (1-\beta^\varepsilon)c(k) (\varepsilon = 1, \ldots, P)
\]
Here, \( E_0^i (k) \) is the output error of the \( i \) subsystem, \( \Delta \) is difference operator, \( R_i = \text{diag}(r_{i1}, r_{i2}, \ldots, r_{ip}) \) is control weight coefficient matrix, \( w_i(k+\epsilon) \) is the expected output trajectory given by the \( i \) subsystem, \( y_i(k) \) and \( e(k) \) denote the actual and expected output of the \( i \) subsystem at time instant \( k \), and \( \beta \) is reference trajectory softening factor.

According to the above formula, \( \Delta e_0^i(k + \epsilon) \) can be concluded as

\[
\Delta e_0^i(k + \epsilon) = \Delta w_i(k + \epsilon) - \Delta y_{i,M}(k + \epsilon|k) \\
= w_i(k + \epsilon) - w_i(k + \epsilon - 1) \\
- [y_{i,M}(k + \epsilon|k) - y_{i,M}(k + \epsilon - 1|k)] \\
= [w_i(k + \epsilon - 1) - y_{i,M}(k + \epsilon - 1|k)] \\
= e_0^i(k + \epsilon) - e_0^i(k + \epsilon - 1) \quad (50)
\]

Through introducing matrix

\[
S_1 = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
-1 & 1 & \cdots & 0 \\
0 & -1 & 1 & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & -1 & 1
\end{bmatrix}
\]

We can obtain

\[
\Delta E_0^i(k) = S_1 E_0^i(k) \quad (51)
\]

To sum up, the \( i \) subsystem performance indicator can be obtained:

\[
\begin{align*}
\min J_i(k) &= E_0^i(k)^T S_1^T K_p^i S_1 E_0^i(k) + E_0^i(k)^T K_i^i E_0^i(k) \\
&+ \sum_{j=1}^{k} E_0^i(k - j)^T K_j^i E_0^i(k - j) \\
&+ \Delta u_{i,M}(k)^T R_i \Delta u_{i,M}(k) \\
\end{align*} \quad (52)
\]

making \( Q_i = S_1^T K_p^i S_1 + K_i^i \).

Then it can be rewritten as follows

\[
\begin{align*}
\min J_i(k) &= E_0^i(k)^T Q_i E_0^i(k) + \sum_{j=1}^{k} E_0^i(k - j)^T K_j^i E_0^i(k - j) \\
&+ \Delta u_{i,M}(k)^T R_i \Delta u_{i,M}(k) \\
\end{align*} \quad (53)
\]

### C. DESIGN OF DISTRIBUTED FRACTIONAL ORDER PID TYPE DYNAMIC MATRIX CONTROLLER

According to the Nash optimization strategy, the \( i \) subsystem takes \( \Delta u_{i,M}(k) \) as the control variable to minimize the performance index in Eq.(53).

Calculate \( \frac{\partial J_i(k)}{\partial \Delta u_{i,M}(k)} = 0 \), the Nash optimal result of the \( i \) subsystem is obtained

\[
\Delta u_{i,M}^*(k) = (A_{ii}^T Q_i A_{ii} + R_i)^{-1} (A_{ii}^T Q_i [w_i(k) - y_{i,P0}(k)] - \sum_{j=1, j \neq i}^{n} A_{ij} \Delta u_{j,M}^*(k)) \\
+ \sum_{j=1}^{n} A_{ij} \Delta u_{j,M}(k - j)) \quad (54)
\]

Repeat the above steps, the new iterative Nash optimal result of the \( i \) subsystem at time instant \( k \) can be obtained as follows

\[
\Delta u_{i,M}^+(k) = (A_{ii}^T Q_i A_{ii} + R_i)^{-1} (A_{ii}^T Q_i [w_i(k) - y_{i,P0}(k)] - \sum_{j=1}^{n} A_{ij} \Delta u_{j,M}(k - j)) \\
+ \sum_{j=1}^{n} A_{ij} \Delta u_{j,M}(k - j)) \quad (55)
\]

Consequently, the Nash optimal control law of the whole system at time instant \( k \) is derived.

\[
\Delta u_{M}^+(k) = [\Delta u_{1,M}^+(k), \Delta u_{2,M}^+(k), \ldots, \Delta u_{N,M}^+(k)]^T \\
(56)
\]

Taking the first term of the Nash optimal result of the \( i \) subsystem at time instant \( k \) as the real-time control law \( \Delta u_i(k) \), we have

\[
\Delta u_i(k) = [1, 0, \ldots, 0] \Delta u_{i,M}^*(k) \quad (57)
\]

Then we can obtain \( u_i(k) = u_i(k - 1) + \Delta u_i(k) \) of the \( i \) subsystem for the subsystem.

At the next moment, repeat the above steps and continue to solve the new round of real-time control law.

### IV. SIMULATION STUDY

In this part, the presented method is compared with conventional DDMC and PID-DDMC methods to verify the effectiveness.

#### A. CASE 1

Considering a complex three inputs and three outputs large scale system \([31]\)

\[
G(s) = \begin{bmatrix}
e^{-4s} & e^{-6s} & e^{-2s} \\
79s + 1 & 87s + 1 & 120s + 1 \\
-1.25e^{-2s} & 3.75e^{-6s} & e^{-3s} \\
33s + 1 & 56s + 1 & 38s + 1 \\
-2e^{-4s} & 2e^{-4s} & 3.5e^{-2s} \\
120s + 1 & 145s + 1 & 85s + 1
\end{bmatrix}
\quad (58)
\]

For the proposed complex system, it is necessary to divide it on the basis of the actual situation and the three subsystems
are obtained as follows

Subsystem 1: \( G_1(s) = \frac{e^{-4s}}{79s + 1} \) \hspace{1cm} (59)

Subsystem 2: \( G_2(s) = \frac{3.75e^{-6s}}{36s + 1} \) \hspace{1cm} (60)

Subsystem 3: \( G_3(s) = \frac{3.5e^{-2s}}{85s + 1} \) \hspace{1cm} (61)

1) IOPID CONTROLLER DESIGN OF SUBSYSTEM 1

According to section 2.3.1, based on parameters \( T = 79, \tau = 4, k = 1, \omega_c = 0.1, \phi_m = 50^\circ \), we can obtain \( k_p = 7.26, k_i = 0.39, k_d = 6.56 \). Then the PID controller is obtained as follows

\( C(s) = 7.26 + \frac{0.39}{s} + 6.56s \) \hspace{1cm} (62)

The Bode plot of the system based on PID is presented in Figure 3. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

2) IOPID CONTROLLER DESIGN OF SUBSYSTEM 2

Same as above, based on parameters \( T = 36, \tau = 6, k = 3.75, \omega_c = 0.1, \phi_m = 50^\circ \), we can obtain \( k_p = 0.928, k_i = 0.064, k_d = 2.777 \). The PID controller is as follows

\( C(s) = 0.928 + \frac{0.064}{s} + 2.777s \) \hspace{1cm} (63)

The Bode plot of the system based on PID is demonstrated in Figure 4. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

3) IOPID CONTROLLER DESIGN OF SUBSYSTEM 3

Based on parameters \( T = 85, \tau = 2, k = 3.5, \omega_c = 0.3, \phi_m = 50^\circ \), we can obtain \( k_p = 7.22, k_i = 0.857, k_d = 6 \). The PID controller is as follows

\( C(s) = 7.22 + \frac{0.857}{s} + 6s \) \hspace{1cm} (64)

The Bode plot of the system based on PID is presented in Figure 5. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

4) FOPID CONTROLLER DESIGN OF SUBSYSTEM 1

According to section 2.3.2 and parameters \( T = 79, \tau = 4, k = 1, \omega_c = 0.1, \phi_m = 50^\circ, k_p = 1.9249, k_i = 0.4223, k_d = 6 \), we can obtain the FOPID controller as follows

\( C(s) = 1.9249 + \frac{0.4223}{s} + 6s \) \hspace{1cm} (65)

The Bode plot of the system based on FOPID is presented in Figure 6. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.
\( k_d = 8.6483, \lambda = 1.0554, \mu = 0.1629 \) can be obtained. The FOPID controller is described as follows

\[
C(s) = 1.9249 + \frac{0.4223}{s^{1.0554}} + 8.6483^{0.1629}
\]

The Bode plot of the subsystem based on FOPID is shown in Figure 6. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

5) FOPID CONTROLLER DESIGN OF SUBSYSTEM 2

For subsystem 2 and parameters \( T = 36, \tau = 6, k = 3.75, \omega_c = 0.1, \phi_m = 50^\circ \), we can obtain \( k_p = 0.1436, k_i = 0.03576, k_d = 1.566, \lambda = 1.2816, \mu = 0.1499 \). The FOPID controller is

\[
C(s) = 0.1436 + \frac{0.03576}{s^{1.2816}} + 1.566^{0.1499}
\]

The Bode plot of the subsystem based on FOPID is presented in Figure 7. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

6) FOPID CONTROLLER DESIGN OF SUBSYSTEM 3

For subsystem 3 with parameters \( T = 85, \tau = 2, k = 3.5, \omega_c = 0.3, \phi_m = 50^\circ \), we can obtain \( k_p = 5.1898, k_i = 1.3744, k_d = 10.36, \lambda = 1.0909, \mu = 0.6201 \). The FOPID controller is as follows

\[
C(s) = 5.1898 + \frac{1.3744}{s^{1.0909}} + 10.36^{0.6201}
\]

The Bode plot of the subsystem based on FOPID is shown in Figure 8. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.
TABLE 1. Control parameters of three DDMC approaches.

| Parameters | DDMC | PID-DDMC | FOPID-DDMC |
|------------|------|----------|------------|
| $P$        | 6    |          |            |
| $M$        | 3    |          |            |
| $N$        | 300  |          |            |
| $T_s$      | 2s   |          |            |
| $\alpha$  | 1    |          |            |
| $\beta$   | 0.95 |          |            |
| $r_1, r_2, r_3$ | 0.1, 0.2, 0.8 |      |            |
| $q_1, q_2, q_3$ | 1, 1, 1 |      |            |

$\kappa^1_p, \kappa^1_\alpha, \kappa^1_\mu$ | 7.26, 0.39, 6.56, \etc | 1.9249, 0.4223, 8.6483, \etc |
| $\kappa^2_p, \kappa^2_\alpha, \kappa^2_\mu$ | 0.928, 0.064, 2.777, \etc | 0.1436, 0.03576, 1.566, 1.2816, \etc |
| $\kappa^3_p, \kappa^3_\alpha, \kappa^3_\mu$ | 7.2, 0.857, 6.0, \etc | 5.1898, 1.3744, 10.36, 1.0909, \etc |

The control time domain $M$ indicates the number of future control changes to be determined in the performance index, generally $M \leq P$.

The sampling time $T_s$ is based on the real-time nature of the computer’s memory and control, and a reasonable value is selected, which is directly related to the dynamic response information and type of the control system.

The model length $N$ affects the integrity of model parameters including dynamic information of objects.

Error correction coefficient $\alpha$, when the uncertainty and environmental disturbance in the process make the output predicted value deviate from the actual value, it plays the role of error correction. The softening coefficient $\beta$ in the reference trajectory is mainly set to make the output tracking curve of the process object softer.

The main function of the component elements $r_i$ of the control weight matrix $R$ is to suppress the drastic changes caused by the control increment, but its addition has no direct relationship with the improvement of system stability. The improvement of system stability is mainly realized by parameters $P$ and $M$.

In the conventional distributed dynamic matrix control, the component elements $q_i$ of the error weight matrix $Q$ represent the weight of the output trajectory approaching the reference trajectory at different times. Here, we often take its value as 1.

The three parameters of PID and the five parameters of FOPID are all adjusted by GPM algorithm.

Considering the model mismatch in the actual industrial production process, we introduce the cases of model/plant match and mismatch to verify the effectiveness of FOPID-DDMC algorithm.

a: MODEL/PLANT MATCH

For the case of model/plant match, the responses of each subsystem are as shown.

The specific analysis data obtained through measurement are as follows.

Under the circumstance of model/plant match, the following conclusions can be drawn from table 2 and figures 9 ∼ 11:

TABLE 2. Performance indexes for three DDMC approaches in model matching.

| Time index | DDMC | PID-DDMC | FOPID-DDMC |
|------------|------|----------|------------|
| Rise time: | $t_r$ | Subsystem 1: 1.270s | Subsystem 1: 1.248s | Subsystem 1: 1.246s |
| $t_s$ | Subsystem 1: 3.126s | Subsystem 1: 3.116s | Subsystem 1: 3.114s |
| Peak time: | $t_P$ | Subsystem 1: 1.696s | Subsystem 1: 1.696s | Subsystem 1: 1.664s |
| $t_s$ | Subsystem 1: 3.235s | Subsystem 1: 3.213s | Subsystem 1: 3.212s |
| Settling time: | $t_s$ | Subsystem 1: 1.384s | Subsystem 1: 1.366s | Subsystem 1: 1.334s |
| $t_s$ | Subsystem 1: 3.323s | Subsystem 1: 3.454s | Subsystem 1: 3.352s |
| Overshoot: | $\sigma$ | Subsystem 1: 1.0.34% | Subsystem 1: 1.0.23% | Subsystem 1: 1.0.13% |
| $t_s$ | Subsystem 1: 2.115% | Subsystem 1: 2.105% | Subsystem 1: 2.105% |
| $t_s$ | Subsystem 1: 1.142% | Subsystem 1: 1.124% | Subsystem 1: 1.124% |
| Recovery time: | $t_h$ | Subsystem 1: 2.248s | Subsystem 1: 2.222s | Subsystem 1: 1.922s |
| $t_s$ | Subsystem 1: 2.968s | Subsystem 1: 2.968s | Subsystem 1: 2.968s |
| $t_s$ | Subsystem 1: 3.128s | Subsystem 1: 3.114s | Subsystem 1: 3.922s |

FIGURE 9. Response curves of subsystem 1.

FIGURE 10. Response curves of subsystem 2.
1) As for the rise, peak and settling time of each subsystem, FOPID-DDMC algorithm takes less time than the other two algorithms. Obviously, adding fractional order PID control makes the algorithm control more accurate, which improves the speed of the system.

2) In terms of the overshoot, it is obvious that the value of distributed FOPID dynamic matrix control is reduced compared with the other two algorithms, which demonstrates that the presented control method has better dynamic stability.

3) Concerning the recovery time after suffering from the interference, the proposed method is obviously faster than the other two methods to recover near the tracking value, which shows the algorithm has stronger robustness.

To sum up, the performance of distributed FOPID type dynamic matrix control algorithm is better than traditional distributed dynamic matrix control algorithm and distributed PID type dynamic matrix control algorithm in terms of rapidity, stability and robustness.

b: MODEL/PLANT MISMATCH

Since there are various unpredictable factors in practical processes, which will lead to deviations in model establishment, so the performance of the proposed method also should be considered under the circumstance of model/plant mismatch. On basis of control model considered, the Monte Carlo method is utilized to generate three groups of model/plant mismatches (the maximum scope of mismatch is ±30%).

1) The first group

Subsystem 1: \[ G_1(s) = \frac{1.213e^{-4.343s}}{84.23s + 1} \] \[ \frac{3.425e^{-6.123s}}{41.21s + 1} \] \[ \frac{2.731e^{-2.12s}}{90.12s + 1} \] (68)

Subsystem 2: \[ G_2(s) = \frac{1.111e^{-3.531s}}{71.2s + 1} \] \[ \frac{4.175e^{-7.62s}}{44.3s + 1} \] (71)

Subsystem 3: \[ G_3(s) = \frac{2.871e^{-2.28s}}{102.12s + 1} \] (73)

In the first group of model mismatch, the output and control variables of each subsystem are exhibited in Fig. 12~14.

2) The second group

Subsystem 1: \[ G_1(s) = \frac{1.111e^{-3.531s}}{71.2s + 1} \] \[ \frac{4.175e^{-7.62s}}{44.3s + 1} \] (72)

In the second group of model mismatch, the output and control variables of each subsystem are exhibited in Fig. 15~17.
3) The third group

Subsystem 1: \[ G_1(s) = \frac{0.838e^{-4.878s}}{82s + 1} \] (74)

Subsystem 2: \[ G_2(s) = \frac{3.75e^{-6.56s}}{42.3s + 1} \] (75)

Subsystem 3: \[ G_3(s) = \frac{4.23e^{-2.6s}}{87.3s + 1} \] (76)

In the third group of model mismatch, the output and control variables of each subsystem are shown in Fig. 18 to Fig. 20.

1) Concerning the rise and settling time of each subsystem, FOPID-DDMC is better than the other two methods, indicating that its performance in terms of rapidity is the best.

2) For the overshoot, it is obvious that the value of distributed FOPID dynamic matrix control is reduced compared with the other two algorithms, which shows the proposed algorithm has better dynamic stability.

3) As for the recovery time after suffering the disturbance, the proposed method is obviously faster than the other two methods to recover near the tracking value, which demonstrates that the robustness of the control system is improved.

Similarly, in the case of model/plant mismatch, the performance of distributed fractional order PID type dynamic matrix control algorithm is better than traditional distributed dynamic matrix control algorithm and distributed PID type dynamic matrix control algorithm in terms of rapidity, stability and robustness.

In a word, the distributed fractional order PID dynamic matrix control method can not only approach the set-point value and deal with the disturbance, but also has better control performance in stability and robustness compared with the conventional distributed model predictive control algorithm.

B. CASE 2

To further testify the effectiveness of the control algorithm, a level-temperature reactor process proposed in [42] is simulated in this article. The physical figure of the process device is also given in the literature as shown in Figure 21. The experimental device consists of water tank with heater. The data transfer is realized by personal computer by data acquisition cards, and the controller is implemented through Emerson Delta-V DCS software. The first input is the water inflow with the help of the solenoid valve, and the power control unit of the heater is the second input. The first output is the liquid level, and the second output is the liquid temperature in the reaction tank. The working principle of the level-temperature reactor process is as follows. The cold water in the tank is injected into the reaction tank through the control of the solenoid valve, and the liquid level in the reaction tank is controlled according to the pressure sensor, personal computer, solenoid valve and other devices. Then the water in the reaction tank is heated by the heater. According to the temperature sensor, personal computer, heater power control unit and other devices, the temperature and liquid level in the reaction tank are to be controlled.

Concerning the level-temperature reactor, the pseudo-random-binary-sequence (PRBS) is used to input the excitation system dynamics, and the parametric model is obtained. The empirical model of the reactor is

\[
G_1(s) = \begin{bmatrix}
1.6838e^{-28.8168s} & -0.0186e^{-6.2360s} \\
63.0715s + 1 & 29.499s + 1 \\
-0.0289e^{-30.0s} & 0.0492e^{-16.9862s} \\
276.8893s + 1 & 62.3681s + 1
\end{bmatrix}
\] (77)
As for the level-temperature reactor, the system is also divided into the following two subsystems:

Subsystem 1: \[ G_1(s) = \frac{1.6838e^{-28.8168s}}{63.0715s + 1} \] (78)
Subsystem 2: \[ G_2(s) = \frac{0.0492e^{-16.9862s}}{62.3681s + 1} \] (79)

1) IOPID CONTROLLER DESIGN OF SUBSYSTEM 1 AND SUBSYSTEM 2

According to section 2.3.1 and parameters \( T = 63.0715 \), \( \tau = 28.8168 \), \( k = 1.6838 \), \( \omega_c = 0.02 \), \( \phi_m = 50^\circ \), we can obtain \( k_p = 0.6694 \), \( k_i = 0.02133 \), \( k_d = 19.223 \). Then the PID controller can be obtained as follows

\[ C(s) = 0.6694 + \frac{0.02133}{s} + 19.223s \] (80)

The Bode plot of the subsystem based on PID is shown in Figure 22. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

2) IOPID CONTROLLER DESIGN OF SUBSYSTEM 2

For parameters \( T = 62.3681 \), \( \tau = 16.9862 \), \( k = 0.0492 \), \( \omega_c = 0.02 \), \( \phi_m = 50^\circ \), we can obtain \( k_p = 16.563 \), \( k_i = 0.807 \), \( k_d = 620.284 \). Then the PID controller is obtained as follows

\[ C(s) = 16.563 + \frac{0.807}{s} + 620.284s \] (81)

3) FOPID CONTROLLER DESIGN OF SUBSYSTEM 1

According to section 2.3.2 and parameters \( T = 63.0715 \), \( \tau = 28.8168 \), \( k = 1.6838 \), \( \omega_c = 0.02 \), \( \phi_m = 50^\circ \), we can obtain \( k_p = 0.1363 \), \( k_i = 0.0098 \), \( k_d = 1.9071 \), \( \lambda = 1.1793 \), \( \mu = 0.2052 \). Then the FOPID controller is as follows

\[ C(s) = 0.1363 + \frac{0.0098}{s^{1.1793}} + 1.9071s^{0.2052} \] (82)

The Bode plot of the subsystem based on FOPID is shown in Figure 23. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

4) FOPID CONTROLLER DESIGN OF SUBSYSTEM 2

For subsystem 2 and parameters \( T = 62.3681 \), \( \tau = 16.9862 \), \( k = 0.0492 \), \( \omega_c = 0.02 \), \( \phi_m = 50^\circ \), we can obtain \( k_p = 5.3821 \), \( k_i = 0.43089 \), \( k_d = 55.4288 \), \( \lambda = 1.1364 \), \( \mu = 0.2524 \). The FOPID controller is as follows

\[ C(s) = 5.3821 + \frac{0.43089}{s^{1.1364}} + 55.4288s^{0.2524} \] (83)
The Bode plot of the subsystem based on FOPID is shown in Figure 25. According to the figure, the conclusion can be obtained that in the case of satisfying the phase margin, the phase near the cut-off frequency is flat to meet the design requirements, which can ensure that the system has strong robustness when the open-loop gain changes.

The set-point of the three methods are chosen as 1. Meanwhile, so as to test the anti-disturbance performance of each method, an output disturbance of \(-0.2\) is added at time instant 500. The error accuracy of Nash optimization strategy is specified as 0.01. According to the setting method in section 2.3, Table 3 lists the control parameters of the three algorithms.

Here we introduce the cases of model/plant match and mismatch to further verify the effectiveness of FOPID-DDMC algorithm.

\(\alpha: \textit{MODEL/PLANT MATCH}\)

In the case of model/plant match, the responses of each subsystem are exhibited in Fig.26∼27.

| Parameters | DDMC | PID-DDMC | FOPID-DDMC |
|------------|------|----------|------------|
| \(P\)      | 20   | 1        | 300        |
| \(M\)      | 5    | 1        | 5s         |
| \(N\)      | 1    | 1        | 1.095      |
| \(\alpha\) | 1    | 1        | 0.696      |
| \(\beta\)  | 0.95 | 1        | 1.363      |
| \(r_1, r_2\)| 0.1, 0.2 | \ | \ |
| \(q_1, q_2\)| 1, 1 | \ | \ |
| \(k_0^1, k_1^1, k_2^1\) | 0.6694, 0.0213, 19.223 | 0.1363, 1.9071, 1.1793 | 0.0098, 1.0971, 0.2052 |
| \(|k_0^2, k_1^2, k_2^2| \) | 16.563, 0.807, 620.284 | 5.3821, 5.5428, 1.1364 | 0.4309, 0.2524 |
| \(|k_0^3, k_1^3, k_2^3| \) | \ | \ | \ |

The specific analysis data obtained through measurement are as follows.

Under the situation of model matching, the following conclusions can be drawn from Table 4 and Figs. 26 ∼ 27

1) As for the rise, peak and settling time of each subsystem, FOPID-DDMC algorithm takes less time than the other two algorithms. Obviously, the proposed FOPID controller provide more accurate control effect, which improves the speed of the system.

2) In terms of the overshoot, it is clear that the value of distributed FOPID type dynamic matrix control is almost the same as the other two algorithms.

3) Concerning the recovery time after suffering the disturbance, the proposed method is obviously faster than the other two methods to recover near the tracking value, which means that the algorithm has stronger robustness.
TABLE 4. Performance indexes for three DDMC approaches in model matching.

| Time index | DDMC | PID-DDMC | FOPID-DDMC |
|------------|------|----------|-------------|
| Rise time: $t_r$ | Subsystem 1: 2.76s | Subsystem 1: 2.40s | Subsystem 1: 2.10s |
| Subsystem 2: 2.45s | Subsystem 2: 2.60s | Subsystem 2: 2.65s |
| Peak time: $t_p$ | Subsystem 1: 1.69s | Subsystem 1: 1.60s | Subsystem 1: 1.54s |
| Subsystem 2: 1.80s | Subsystem 2: 1.40s | Subsystem 2: 2.00s |
| Settling time: $t_s$ | Subsystem 1: 1.40s | Subsystem 1: 1.36s | Subsystem 1: 1.31s |
| Subsystem 2: 2.66s | Subsystem 2: 2.59s | Subsystem 2: 2.73s |
| Overshoot: $\sigma$ | Subsystem 1: 10.4% | Subsystem 1: 9.3% | Subsystem 1: 9.1% |
| Subsystem 2: 2.0% | Subsystem 2: 2.0% | Subsystem 2: 2.0% |
| Recovery time: $t_r$ | Subsystem 1: 1.29s | Subsystem 1: 1.22s | Subsystem 1: 1.21s |
| Subsystem 2: 2.34s | Subsystem 2: 2.30s | Subsystem 2: 2.23s |

To sum up, the distributed FOPID type dynamic matrix control algorithm is better than traditional distributed dynamic matrix control algorithm and distributed PID type dynamic matrix control algorithm in terms of overall performance.

b: MODEL/PLANT MISMATCH

The Monte Carlo method is utilized to generate three groups of model/plant mismatches (the maximum scope of mismatch is ±30%).

1) The first group

Subsystem 1: \[ G_1(s) = \frac{1.8521e^{-30.216s}}{66.225s + 1} \] (84)

Subsystem 2: \[ G_2(s) = \frac{0.0447e^{-15.48s}}{74.841s + 1} \] (85)

In the first group of model mismatch, the output and control variables of each subsystem are exhibited in Fig. 28~29.

2) The second group

Subsystem 1: \[ G_1(s) = \frac{1.53e^{-27.43s}}{52.55s + 1} \] (86)

Subsystem 2: \[ G_2(s) = \frac{0.054e^{-16.17s}}{51.97s + 1} \] (87)

In the second group of model mismatch, the output and control variables of each subsystem are exhibited in Fig. 30~31.

3) The third group

Subsystem 1: \[ G_1(s) = \frac{1.32e^{-75.68s}}{26.55s + 1} \] (88)

Subsystem 2: \[ G_2(s) = \frac{0.041e^{-14.23s}}{60.32s + 1} \] (89)

In the third group of model mismatch, the output and control variables of each subsystem are exhibited in Fig. 32~33.

In the case of model mismatch, the following conclusions can be drawn from figures 28 ~ 33:

1) Concerning the rise time and settling time of each subsystem, FOPID-DDMC algorithm is better than the other two, indicating that its performance of rapidity is the best.
For the overshoot, it is clear that the three algorithms have almost the same performance.

3) As to the recovery time after suffering the disturbance, the proposed method is obviously faster than the other two methods to recover near the tracking value.

In a word, the distributed fractional order PID dynamic matrix control method can not only approach the set-point value and deal with the disturbance quickly in the actual production process, but also has better control performance in stability and robustness compared with the traditional distributed model predictive control algorithm.

V. CONCLUSION

This article proposed a distributed fractional order PID type dynamic matrix control algorithm. It divides the complicated large-scale system into subsystems, with each subsystem being controlled through a corresponding FOPID dynamic matrix controller, and uses Nash optimal theory to eliminate the coupling effect among subsystems. Because fractional order PID is introduced into the objective function, two additional control parameters $\lambda$ and $\mu$ are added compared with traditional PID operator which makes the tuning rules of the controller more flexible. Regrettably, due to the complexity of fractional PID and distributed control, the proposed controller so far can only deal with unconstrained complex large-scale systems. But, the proposed controller has a wider control range, superior control accuracy, and a certain degree of improvement in the system’s ensemble performance. Compared with the distributed PID type dynamic matrix control, it further optimizes the system’s rapidity, stability and robustness. Finally, for a three-input three-output coupling system and a level-temperature control process, the feasibility of the proposed controller is verified by comparing with traditional distributed model predictive control algorithm.

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