The study on the magnetic separation efficiency of the reverse water technological liquids from scales in industrial production. Part 3. improving the magnetic separation technology

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Abstract. The magnetic coagulation process mechanism features occurring in magnetic separators are disclosed. Magnetic coagulation occurs only at the threshold concentrations of ferro-particles in aqueous ferromagnetic suspensions. The basis of convergence (aggregation) mechanism of ferro-particles in a gradient external field is the difference in the ferro-particles’ speed with different masses. Based on mathematical modeling, it was found that during coagulation, the residual concentrations of ferro-impurities cannot exceed certain limit values, and mathematical expressions are obtained for calculating the distributions of the dispersed composition of aggregated particles. The combination of optimization procedures for the magnetic separators’ geometric parameters and the cleaning characteristics taking into account magnetic coagulation makes it possible to ensure the high levels of cleaning efficiency of finely dispersed ferromagnetic suspensions while reducing the cost of magnetic separators up to two times at the design stage.

Introduction
Magnetic coagulation is an integral process in the purification of water process liquids (WPL) from ferromagnetic impurities under the influence of an external magnetic field. In the wastewater treatment practice at metallurgical enterprises by magnetic separators, the little-studied phenomenon of magnetic coagulation, affecting the magnetic cleaning efficiency, is of great practical interest. The magnetic coagulation mathematical model construction for technological processes of magnetic treatment of reversed water will make it possible to control the magnetic coagulation process.

In the mathematical modeling of this phenomenon, it is necessary to take into account the physical mechanisms of particle approaching and sticking together, which include the free molecular (Brownian) motion, diffusion, motion due to electric, magnetic and centrifugal forces or gravitational deposition, etc. The fundamental task of process modeling of solid particles’ consolidation in dispersed systems is to describe the distribution of suspended particles by their size (or mass) as a function of time, which is implemented by the general stochastic theory [1]. This theory is based on discrete and continuous nonlinear integral differential equations (IDE) of M. Smolukhovsky,
expressing the concentration of \( c_i(t) \) \( i \) -dimensional particles or the function \( f(t,m) \) of the solid phase dispersed composition distribution density:

\[
\frac{dc_i}{dt} = \frac{1}{2} \sum_{j=1}^{i-1} K_{i-j} c_i c_j - c_i \sum_{j=1}^{\infty} K_{i,j} c_j ;
\]

\[
\frac{\partial f(t,m)}{\partial t} = \frac{1}{2} \int_0^m K(m', m - m') f(m') f(m - m') dm' - f(m) \int_0^\infty K(m, m') f(m') dm',
\]

(1)

Moreover, the coagulation mechanisms determine the symmetric function \( K(m_i, m_j) \), which is the coagulation core. The core characterizes the probability (frequency) of the particles’ collision with masses \( m_i \) and \( m_j \), which is inversely proportional to the time they were in a segregate state.

Due to the non-linearity of the integral differential equations (IDE) of M. Smolukhovsky, the problem in the general case has no solution; for each particular case of the initial particles’ distribution, the special mathematical studies, individual algorithms, and limiting machine resources are required.

For magnetic coagulation, the problem in this formulation has not been solved yet [2].

However, the performance purification characteristics’ calculation of the aqueous liquids from the solid impurities, taking into account the coagulation using the Smolukhovsky’s equations, becomes an insurmountably difficult task in the practice of creating and operating the technical means for purifying water technological liquids. Therefore, to build a mathematical model of magnetic coagulation, a different approach that differs from the approach implemented by M. Smolukhovsky is proposed.

The coagulation process’s kinetics will be based on the approximation of a binary union of particles of mass \( m_i \) and mass \( m_j \), creating the aggregates of mass \( m_k = m_i + m_j \).

If we take an ordered set consisting of \( N \) discrete particles with the ordered masses \( m_i = M \cdot i \), where \( i = 1, 2, ..., N ; \) \( M \) is the sampling interval of particle masses, then the number of binary aggregates with a fixed mass \( m_k = i_1 + j_1 = (m_i + m_j) \) will be determined by a random law of arrangement of particles’ pairs with the numbers \( i_1 \) and \( j_1 \) relative to each other and their physical mechanism approach. Note that a fixed value \( m_k = i_1 + j_1 \) can be implemented by the several sets of numbers \( i_1 \) and \( j_1 \) (for example, for \( k = 7, i_1 + j_1 = (1 + 6); (2 + 5); (3 + 4) \)). The set of binary aggregates \( x_k \) with mass \( m_k \) form a discrete random variable \( X \). We denote the probability of a random quantity \( X \) taking a mass value \( m_k \) as \( g(m_k) \).

If the set consists of arbitrary particles with the distribution \( p_i[M] \), then the disordered dispersed composition of the particles leads to the formation of an additional number of binary aggregates \( y_k \) of the same mass \( m_k = i_2 + j_2 = (m_i + m_j) \). In this case, the aggregate of the mass \( m_k \) binary aggregates’ additional number forms a discrete random variable \( Y \).

The combination of the total number of aggregates \( z_k = x_k + y_k \) with the mass \( m_k \) forms a discrete random variable \( Z = X + Y \) with the distribution probability \( p_k[M] \).

The distribution of the starting particles is independent of coagulation processes. In the approximation of weakly concentrated suspensions, the initial distribution of particles will not affect the coagulation mechanism (in mechanical engineering and in the metallurgical industry, spent WPLs contain no more than 0.01 - 1% solid and ferromagnetic impurities). Therefore, the random variables \( X \) and \( Y \) are independent. Then, the distribution probability of the dispersed composition during coagulation in accordance with the provisions of probability theory [3] can be expressed by the compression of the independent random variables’ discrete distribution functions.

\[
\sum_{k=-\infty}^{\infty} p_i[kM] g^*(n-kM)
\]
Taking into account, that there are no negative and zero particle masses, the compression formula takes the following form:

\[
p_h[nM] = \sum_{k=1}^{\infty} p_{kM}g^*[(n - k)M],
\]

where \( \delta[n] \) is the unit reference [4].

Similarly, instead of the continuous Smolukhovsky equation, we use the convolution formula of continuous random variables as a mathematical model of coagulation processes:

\[
f_k(m) = \int_{0}^{m} f_i(\eta)g(m\eta)d\eta,
\]

where \( f_k(m) \) is the probability density function of the dispersed phase distribution by the mass during coagulation; \( f_i(m) \) is the probability density function of the initial dispersed phase distribution by mass (before coagulation); \( g(m) \) is the probability density function that characterizes the coagulation process, which is called the weight function.

In a gradient magnetic field of magnetic separators, a larger ferro-particle catches up with a smaller ferro-particle. For such a mechanism of the ferro-particles’ convergence, the continuous weight function was determined in [5].

\[
g(m) = N(MG_N)^{-1} \left\{ \sum_{n=1}^{N/2} \sin(2n - mM^{-1})(\pi(2n - mM^{-1}))^{-1} + \sum_{n=1}^{N/2-1} (q_{2n+1,1} - q_{n+1,n})\sin(2n + 1 - mM^{-1})(\pi(2n - mM^{-1}))^{-1} \right\},
\]

where \( G_N; q_{2k,1}, q_{2k-1,1}, q_{k,k-1} \) – are the calculated coefficients, \( N \) – plays the role of a sampling parameter (the number of necessary samples), \( M \) – is a sampling interval of the continuous mass that can be related with the mathematical expectations of the masses over the probability density distribution functions of the dispersed composition of the initial ferro-particles and aggregates, respectively:

\[
\bar{m}_i = \int_{0}^{\infty} m f_i(m)dm;
\]

\[
\bar{m}_k = \int_{0}^{\infty} m dm \int_{0}^{m} f_i(\xi)g(m - \xi)d\xi.
\]

Substituting (4) into (6) and integrating the latter, it was obtained:

\[
\bar{m}_k = 3.7\pi^{-1}N(N - 1)^{-1}[\bar{m}_i(1 - 2N^{-1}) + M(0.5N - 2)].
\]

In [2], the law of mass conservation was confirmed \( \bar{m}N_\nu = C \equiv inv \) during coagulation of polydisperse ferro-impurities in a dispersion (water) medium, where \( \bar{m} \) is the average mass of aggregates; \( N_\nu \) – average concentration of impurity particles’ number, \( m^{-3} \); \( C \) is the average mass concentration of impurities, \( kg / m^3 \). Therefore, to characterize the coagulation depth, we introduce the coefficient \( \gamma_k \) of the coagulation depth, which we set as a criterion for the magnetic coagulation process:

\[
\gamma_k = \bar{m}_k \cdot \bar{m}_i^{-1} = N_\nu^{-1} \cdot N_{\nu k}^{-1}.
\]
Substituting (7) into (8), the expression for determining the mass discretization interval was obtained:

$$M = 2\bar{m}(N - 4)^{-1}\left[\pi\gamma_k(N - 1)(3,7N)^{-1} + 2N^{-1} - 1\right].$$

(9)

Substituting (9) into (4), we can get the most important dependence of the weight function on the coagulation depth’s coefficient:

$$g(m) \equiv g(m, \gamma_k).$$

(10)

Figure 1 shows in graphical form the results of a numerical experiment on the mathematical model of magnetic coagulation (2), (4), (9), which demonstrates a significant dependence of the distribution nature of the ferro-impurities’ dispersed composition during magnetic coagulation on the coagulation depth $\gamma_k$

**Figure 1.** Theoretical dependences of the probability density distribution of the dispersed phase by mass: 1- initial; 2- during coagulation for $a - \bar{\gamma}_k = 1.5$; $b - \bar{\gamma}_k = 3.5$

From the dependencies in Figure 1 it follows that due to the ferro-particles’ aggregation, their ferro-impurities dispersed composition distribution probability density functions are deformed towards an increase in the average mass of the aggregates $\bar{m}_k$ with an increase in the coagulation depth coefficient.

We will carry out mathematical modeling of the coagulation depth coefficient $\gamma_{KV}$ on the basis of the physical approach. If the combination of ferro-particles into pairs takes place over the average time $\bar{t}_{KV}$, then the average concentration of particles will be decreased by 2 times. If $T_m$ is the time interval for the dispersed system to remain in the coagulation zone, then the concentration of ferromagnetic particles during this time will decrease by $2T_m/\bar{t}_{KV}$ times. According to (8), the coefficient of magnetic coagulation depth $\gamma_{KV}$ in the $v$- cycle of the binary association will take the following form:

$$\gamma_{KV} = \langle m_v \rangle \langle m_n \rangle^{-1} = N_v(N_{\nu v})^{-1} = 2T_m\langle v_{KV-1} \rangle [C_0 \langle m_{\nu-1} \rangle^{-1}]^\frac{1}{3},$$

(11)

where

$$\tau_{KV} = \langle l_{v-1} \rangle \langle v_{KV-1} \rangle^{-1} = \left[\langle v_{KV-1} \rangle (N_{\nu v-1})^{-1}\right]^{\frac{1}{3}};$$

(12)

$\bar{t}_{v-1} = (N_{\nu v-1})^{\frac{1}{3}}$ is the average distance between ferro-units in the $v - 1$- cycle; $C_0$ is the initial concentration, kg / m$^3$; $\bar{C}_v$ is the average mass concentration of ferro-particles in suspension, kg / m$^3$; $\bar{\varepsilon}_0(\bar{d}_i)$ is the average value of the magnetic separator purification degree without taking into account the coagulation; $\langle m_{\nu-1} \rangle$ and $N_{\nu v-1}$ are the average mass and the average concentration of ferro-particles in the interval between $v - 1$ and $v$ cycles of the binary association of particles, kg and m$^{-1}$.

The rate of binary coagulation (the speed at which the $i$-particle catches up with the $j$-particle)
corresponds to the individual velocities’ difference \((v_{mi} - v_{mj})\), where \(v_{mi}, v_{mj}\) are the velocities of the corresponding particles relative to the dispersion medium (WPL), m/s. Then the average coagulation rate for all possible interacting \(i\) and \(j\) particles is determined by the sum of the differences in their velocities:

\[
\bar{v}_{kv-1} = \sum_{i=2}^{N} p_{ij} \sum_{j=1}^{i-1} (v_{mi} - v_{mj}) = \sum_{i=2}^{N} (i - 1) p_{ij} (v_{mi} - \bar{v}_{mi-1}),
\]  

where \(p_{ij}\) is the probability of pairing \(i,j\) ferro-particles [5]; \(\bar{v}_{mi-1}\) the average speed for ferromagnetic particles with masses from \(M\) to \(M(i - 1)\) relative to the WPL flow.

Magnetic coagulation for separation processes is valuable for particles with small sizes \(d_f < 5\) microns. For such ferro-particles, according to the methodology of mathematical trajectory modeling described in the second article of this cycle, the particle velocity is estimated by the following expression:

\[
\bar{v}_m = C_{1p} m(f_k(\alpha_{0k})^{-1} - \bar{r}); \quad C_{1p} = K_{0k} \alpha_1 \cdot \mu_0 \cdot \chi_{m0} \cdot \bar{H}_m^2(d_{sl}) \left(3\pi \rho_f d_{o5} H_{os}\right)^{-1},
\]  

where the ferro-particle velocity \(\bar{v}_m\) is related to the maximum magnetic field intensity \(\bar{H}_m^2(d_{sl})\) with the magnetic characteristics \(\mu_0, \chi_{m0}, d_{o5}, H_{os}\) of the ferroparticles (respectively, the magnetic constant, the maximum value of the magnetic susceptibility of the ferromagnetic material ferroparticles, approximation parameters of the dependence \(\chi_m(d, H)\), with its mass density \(\rho_f\), and also with the geometry of the magnetic separator \(K_{0k}\), \((f_k(\alpha_{0k})^{-1} - \bar{r})\) and the hydrodynamic properties of WPL \(\eta\) (dynamic viscosity).

Representing the mass of the \(i^{th}\) particle as \(m_{i_{kv-1}} = i M_{v-1}\) and substituting it into the formula (14) and then in (13), we obtain the average coagulation rate averaged over the particles’ masses:

\[
\bar{v}_{kv-1} = 0.5 M_{v-1} C_{1p} (f_k(\alpha_{0k})^{-1} - \bar{r}) \sum_{i=2}^{N} i (i - 1) p_{ij},
\]  

Substituting (15) in (11), as well as the maximum particle residence time in the active separation region \(T_n = t_{01}(q_n)\), we obtain the dependence \(y_k(q_n)\), which is averaged over all trajectories, we obtain the average value of the coagulation depth coefficient for the characteristics of the magnetic process coagulation during the separation of WPL in each individual magnetic separator:

\[
\bar{y}_k = \left[8 \bar{m}_{ik} \frac{2}{3} \left(1 - \bar{e}_{0k}(d_1)\right) \bar{C}_{ik} \frac{1}{3} C_{1p} (f_k(\alpha_{0k})^{-1} - \bar{r}) \cdot \bar{r} \cdot (\pi \frac{2}{5} \bar{V}_{liqc})^{-1}\right]^3 \times
\]

\[
\times (1 + 3 \sigma_{mi k}(\bar{m}_{ik})^{-1}) (\sum_{i=2}^{N} i (i - 1) p_{ij}) N^{-1} \left(\int_{0.25 \pi}^{0.5 \pi} \ln t g(0.5 q_n) d q_n\right).
\]  

Substituting (16) into (7) and then into (4) and (2), we get the opportunity to predict the change in the probability density of the distribution for the dispersed composition \(f_k(m)\) of the aggregated ferromagnetic impurities. In addition, each \(k^{th}\) step of the magnetic separator will provide the following residual concentration of ferrous impurities in WPL:

\[
C_{0kk} = C_{ik} \left[1 - \int_{0}^{\infty} \varepsilon_k(d_f) f_{k-1,k}(d_f) d(d_f)\right].
\]  

The obtained expressions reveal the dependence of the coagulation process on the magnetic separators’ operating mode.

In particular, the coefficient of coagulation depth (16) is proportional to the average mass of ferroparticles \(\bar{m}_{ik}\) (and, accordingly, average particle size), their average mass concentration at the inlet of the separation stage \(C_{ik}\) and inversely proportional to the average velocity \(\bar{V}_{liqc}\) of the WPL flow. The nature of the magnetic coagulation influence on the quality of cleaning a single-row magnetic
separator is shown in Figure 2.

It is important to emphasize that magnetic coagulation begins to appear from a certain threshold value of the initial impurities’ concentration $C_{\text{imp}}$ (extremum point): the curves 1, 2, 3, which correspond to the WPL speeds $v_{\text{iqn}} = 0.001; 0.025; 0.05$ m/s at the inlet of the separator stage, belong to the experimental dependences. Theoretical curves are adequate to experimental curves according to the Fisher criterion at a significance level of 5%.

![Figure 2](image)

**Figure 2.** Experimental (solid lines) and theoretical (shown with the lines $\bullet\bullet\bullet$; $\bigcirc\bigcirc\bigcirc$; $\bullet\bullet\bullet\bullet$) the dependences of the residual concentration $C_0$ of ferro-impurities in purified WPL on the initial concentration of impurities $C_i$ for the separator from barium ferrite magnets: 1, 2, 3 — respectively, with $v_{\text{iqn}} = 0.001; 0.025; 0.05$ m/s;

$$\bar{d} = 10 \mu m$$

The experimental and theoretical dependences $C_0(C_i)$ presented in Fig. 2 are the cleaning quality characteristics for magnetic separators. Without taking coagulation into account, the residual concentration $C_0$ during magnetic separation proportionally increases with increasing concentration of ferro-particles in the initial WPL. In the presence of magnetic coagulation, such an increase in the residual concentration of ferro-impurities in purified WPL is limited by the limiting values of $C_{\text{opt}}$ at the corresponding threshold concentrations of ferro-impurities $C_{\text{imp}}$ in the initial WPL.

Summarizing the theoretical and experimental dependences $C_0(C_i)$, the threshold concentration $C_{\text{imp}}$ can be estimated by substituting the following condition in (16):

$$\bar{y}_k(C_{\text{imp}}) = 1.1$$  \hspace{1cm} (18)

Magnetic separators made of neodymium-boron permanent magnets are more suitable for fine cleaning. Figure 3 shows the theoretical classes of cleaning quality characteristics for a single-row magnetic separator of neodymium-boron permanent magnets at different values of the water flow rate at the inlet of the cleaner.

Using the dependences obtained on the basis of a computational experiment on the developed mathematical models (Figure 3), the possibilities of single-row magnetic separators for fine purification of ferromagnetic suspensions are revealed. The obtained characteristics clearly demonstrate that the maximum values of the residual concentration of ferro-particles are set by the value of the water flow rate.
Theoretical dependences of the residual concentration $C_0$ of ferromagnetic impurities in purified WPL, depending on the concentration of impurities $Q_i$ in the initial WPL for a single-row separator of neodymium-boron magnets: $a$ - for $d_i = 1$ μm, where 1, 2, 3, for $v_{liqn} = 0.001; 0.01; 0.025$ m/s respectively; $b$ - with $d_i = 4.5$ μm, where 1, 2, 3 - for $v_{liqn} = 0.05; 0.1; 0.2$ m/s respectively.

Thus, the developed mathematical model of magnetic coagulation allows predicting an increase in the magnetic separators’ efficiency for cleaning WPL from ferromagnetic impurities.

Permanent magnets determine the basic cost of the magnetic separators. Therefore, it is advisable to carry out the parametric optimization of the magnetic separators according to the criterion of the minimum price corresponding to the optimal geometric parameters of the magnetic system.

A mathematical problem is formulated as follows.

The magnetic separator, which is a cell of $q$ permanent magnets organized in $N$ rows ($N$ separation stages) (Figure 4) is given.

Objective function: $q = q(a, N)$, where $a$ is the distance between the magnets’ axis, m.

It is necessary to determine the element $(a_0, N_0)$, on the set $(a, N)$, on which $q(a, N) \rightarrow \min$, while satisfying the required level of cleaning quality $C_0 \leq C_d$ and the required level of productivity $Q = \text{const}$, where $Q$ in m$^3$/s, where $C_0$ is the concentration of ferro-particles at the separator outlet, kg / m$^3$; $C_d$ – is the permissible concentration of ferrous impurities in the purified suspension kg / m$^3$. We also assume that $v_{liqn} = \text{const}$; $l_m = \text{const}$, where $v_{liqn}$ is the WPL flow velocity at the magnetic separator input, m/s; $l_m$ – is the magnetic element length, m.

Figure 4. Permanent magnets’ (PM) grid

Noting that the residual concentration of $C_0$ in the magnetic separator (Figure 3) cannot exceed the limiting values of $C_{olim}$ corresponding to the magnetic coagulation onset; therefore, we perform the parametric optimization in the absence of magnetic coagulation mode.

The objective function is represented as follows:
Composing a mathematical expression of optimization condition guarantees the quality of cleaning, taking into account the accumulation of a ferro-sludge layer on the magnets’ surface of each separation stage. The latest model is based on a simulated class of suspension separation quality characteristics in [6], (Fig. 5), which is approximated by a second-order exponential polynomial: For this, we use the mathematical model developed in the second work of this cycle of the suspension separation quality characteristics’ class (20):

\[ e(d, d_{sl}, a) = 1 - e^{-h(d_{sl}, a)d}; \quad h(d_{sl}, a) = B_0(a)e^{-\gamma(a)d_{sl}} + B_1(a)d_{sl}, \quad (20) \]

where \( d_{sl} \) is the ferro-particles sediment layer thickness on the surface of the magnetic system.

Applying the concepts of transmission coefficient and an average degree of purification (as well as expressions (12) - (20) from the first article of this cycle) taking into account (20), we obtain for the first stage of the separator:

\[ \bar{L}_1 = \int_0^\infty e^{-h_1 d_l} f_{inp}(d_f) d(d_f); \quad f_1(d_f) = (\bar{L}_1)^{-1}e^{-h_1 d_f}f_{inp}(d_f) \quad (21) \]

Similarly, the second stage of the expression to determine was obtained:

\[ \bar{L}_2 = \bar{L}_1^{-1} \int_0^\infty e^{-(h_1+h_2)d_f} f_{inp}(d_f) d(d_f); \quad f_2(d_f) = (\bar{L}_1 \bar{L}_2)^{-1}e^{-(h_1+h_2)d_f}f_{inp}(d_f), \quad (22) \]

where \( h_2 = h(d_{sl2}, a) \), \( f_{inp}(d_f) \) is the probability density of the ferro-particles distribution by size at the input of the separator.

For the last stage of the separator, we have respectively:

\[ \bar{L}_N = (\prod_{k=1}^{N_0-1} \bar{L}_k)^{-1} \int_0^\infty e^{-(2_{\nu=1}^{N_0} h_\nu)d_f} f_{inp}(d_f) d(d_f); \quad f_N(d_f) = (\prod_{\nu=1}^{N_0} \bar{L}_\nu)^{-1}e^{-(2_{\nu=1}^{N_0} h_\nu)d_f}f_{inp}(d_f), \quad (23) \]

where \( h_N = h(d_{slN}, a) \).

Having adopted the normal-logarithmic distribution law of the ferro-particles \( f_i(d) \) dispersed composition in the initial suspension with the parameters: \( \bar{d}_l \) is the average size of the ferro-particles; \( s = \ln(1 + \sigma^2/\bar{d}_l^2) \equiv \sigma^2/\bar{d}_l^2 \), for \( \sigma^2/\bar{d}_l^2 \ll 1 \), where \( \sigma \) – is the root-mean-square deviation of the ferro-particles’ size, then based on (23 ), we get:

\[ \bar{L}_N \equiv (\prod_{k=1}^{N_0-1} \bar{L}_k)^{-1} e^{-\bar{d}_l(2_{\nu=1}^{N_0} h_\nu)}; \quad f_N(d_f) = (\prod_{\nu=1}^{N_0} \bar{L}_\nu)^{-1} e^{-(2_{\nu=1}^{N_0} h_\nu)d_f}f_1(d), \quad (24) \]

Considering that the separation process proceeds with approximately the same ferro-sludge deposition rate, which is close to the deposition rate \( m_l(0) \) in the operating mode at the initial moment of this process at \( d_{sl} = 0 \), therefore substituting \( d_{sl} = 0 \) for all the separation series in (20) taking into account (24), we obtain the expression for the separation condition:

\[ C_0 = C_l \bar{L}_l(\bar{d}_l) \equiv C_l e^{-\bar{d}_l N_0 B_0(a)} \leq C_d. \quad (25) \]

Based on the objective function (19) and the condition (25), a system of differential equations is formed by the Lagrange multiplier method to determine the conditional extremum, the solution of which gave the optimal size of the distance between the location axis of the permanent magnets in the separator:

\[ a_0 = -B_0(a) \frac{\partial a}{\partial B_0(a)}; \quad \alpha = a_0. \quad (26) \]
The value of $a_0$ is determined by the approximation dependence $B_0(a)$, which is associated with the permanent magnet’s size.

The number of steps $N_0$ is expressed from the condition (25) taking into account (26) and (20) with the limiting value of the ferro-sludge layer thickness $d_{sl} = d_{sl\text{lim}}$:

$$N_0 \equiv (B_0(a))^{-1} \left[ \frac{1}{d_i} \ln C_i(C_0) - B_1(a) d_{sl\text{lim}} \right].$$  \hspace{1cm} (27)

Based on the constructed techniques, a magnetic separator with magnetic rods of neodymium-boron magnets was calculated according to the initial parameters presented in Table 1. According to the calculations, a two-row magnetic separator No. 1 provides a residual concentration of $C_0 = 6 \text{ mg/l}$ at the outlet, while a single-row separator No. 2 provides a residual concentration of $C_0 = 0,2 \text{ mg/l}$ at the outlet.

The specific cost price of permanent magnets $C_{Qm}$ decreased by half compared with the cost price of separator No. 1 is calculated without taking into account magnetic coagulation in the single-row separator ($N_0 = 1$). No. 2, calculated taking into account magnetic coagulation with a guaranteed supply of the purification degree ($C_0 \ll C_d$).

### Table 1. The Neodymium-Boron Magnetic Separators’ Parameters, calculated without and with consideration of magnetic coagulation

| Type | $N_0$ | $\psi_{iQn}$, m/s | $d_i$, $\mu$m | $\sigma$, $\mu$m | $Q$, m$^3$/h | $C_i$, mg/l | $C_d$, mg/l | $C_o$, mg/l | $a$, m | Number of magn. $q$ | Unit cost $C_{Qm}$ |
|------|------|----------------|-------------|----------------|-----------|-------------|-------------|-------------|------|----------------|----------------|
| 1    | 2    | 0.03          | 5           | 1.7            | 100       | 80          | 5           | 6           | 0.048| 1640           | 100[\%]        |
| 2    | 1    | 0.03          | 5           | 1.7            | 100       | 80          | 5           | 0.2         | 0.048| 820            | 50[\%]          |

### Summary

Theoretical and experimental studies have made it possible to establish a number of the magnetic coagulation process mechanism features occurring in magnetic separators:

1. Magnetic coagulation occurs only at threshold concentrations of ferro-particles in water ferromagnetic suspensions.
2. The basis of the ferro-particles convergence (aggregation) mechanism in a gradient external field is the difference in the speeds of ferro-particles with different masses.
3. During coagulation, the residual concentration of ferro-impurities in water suspensions cannot exceed certain limit values.
4. Mathematical expressions are determined for calculating the aggregated particles dispersed composition distributions.
5. The procedures’ combination for optimizing the geometric parameters of the magnetic separators and their cleaning characteristics, taking into account magnetic coagulation, allows us to provide the specified high levels of purification of finely dispersed ferromagnetic suspensions while reducing the magnetic separators’ cost up to two times.

Thus, the designed mathematical model of the magnetic coagulation process occurring in a magnetic separator allows optimizing the operating mode of the technological process for magnetic cleaning of the reverse water from scale and other ferro-magnetic impurities and significantly reduce the magnetic separators’ cost with a guaranteed level of cleaning quality.

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