Structural strength of iso-polyhedral beryllium alloy rotating mirror for ultra-high-speed camera

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Abstract: This study aims to examine the influence law of polyhedron structure on the spatial mechanical properties of ultra-high-speed rotating mirrors. To this end, polyhedral beryllium alloy rotating mirrors are investigated on the basis of elastoplastic theory and finite element method. The maximum stress is located at the end position of the contact between the shaft and the mirror body. Stress increases with the number of mirror faces. The different structures have a negative stress gradient. The structural strength of rotating mirror is affected by the strength of the mirror body material in high-speed rotation of the tensile force of centrifugal force. The lateral deformation of the mirror surface is caused by the combined effect of compression of centrifugal force generated by the material of sharp-corner and the stretching of tensile force caused by the material at the centre of the mirror at high-speed rotation. The amount of mirror surface deformation is not proportional to the number of faces. The rotating mirror with iso-quadrangular structure has the best lateral deformation effect. This research provides a theoretical basis for the research and design of rotating mirrors with high potential value.

1. Introduction

Polyhedron rotating scanning mirrors (RM) have a wide range of applications in various industries, such as ultra-high-speed photography, laser micro-nano processing, laser scanning and laser Q-switching [1-8]. In ultra-high-speed photography, RMs are the core component of rotating-mirror ultra-high-speed cameras; meanwhile, the structure and mechanical properties of these mirrors directly determine the time resolution, spatial resolution, imaging quality and operational reliability of these cameras [7-11]. The laser Q-switching of RMs is reflective Q-switching; this Q-switching is characterised by the absence of insertion loss, a simple structure, insensitivity to polarisation and dual refraction effect; furthermore, no light damage is observed, and a high repetition frequency laser modulation is achieved because 100% modulation depth can be reached [12-16]. In comparison with galvanometer scanning, the rotating-mirror raster scanning used in the laser processing industry has fixed scanning angle, fast rotating-mirror scanning speed, high repeatability of laser beam scanning and high damage threshold characteristics; as such, it can achieve large-angle, large-scale, ultra-high-speed and high-precision processing [17-18].

The strength of the RM is the basis and guarantee of the mirror’s function, and the demand for RMs varies in different applications. The strength of the same materials with different cross-sectional shapes varies at the same operating speed. In other words, RMs with different section shapes have varied
destruction speed. Brixner and Shaw obtained the Brixner diagram of the relationship between the number of RMs and the relative speed\(^{[19-20]}\). Jingzhen studied the dynamic characteristics of aluminium alloy RM\(^{[21]}\). On the basis of 3D elastoplastic theory, the finite element method was introduced; a numerical simulation method of the strength of the aluminium alloy rotating-mirror and the beryllium alloy rotating-mirror with equilateral trilateral structure was also established\(^{[22]}\). Erez and Partom\(^{[23]}\) reported that the largest impact on rotating-mirror-type ultra-high-speed camera is the lateral deformation of the mirror; they also reported that the horizontal deformation of the mirror surface in the optical system introduced an uncorrected optical element to change the conjugate relationship of the image. In the case of RMs with the same shape, size and speed, lateral deformation is related to the specific stiffness of the mirrors, and the deformation amount of the rectangular section is proportional to the Poisson ratio. Hongbin\(^{[11]}\) proposed a geometric compensation method for the amount of mirror surface deformation and designed aluminium RMs with honeycomb structure. However, as we know, there is no theoretical or numerical analysis reports on structural strength of polyhedral beryllium alloy RM with different structure.

In this paper, we theoretically and numerically investigate the structural strength of polyhedral beryllium alloy RM with the same circumscribed circle radius of cross section, rotation speed and other constraint condition. The maximum stress is located at the end position of the contact between the shaft and the mirror body. The maximum stress increases with the number of mirror faces. The different RMs have a negative stress gradient. The structural strength of RM is affected by the strength of the mirror body material in high-speed rotation of the tensile force of centrifugal force. The lateral deformation of the mirror surface is caused by the combined effect of compression of centrifugal force generated by the material of sharp-corner and the stretching of tensile force caused by the material at the centre of the mirror at high-speed rotation. The lateral deformation of the isosceles triangular prism and the iso-quadrangular RMs is inward concave. The lateral deformation of iso-quadrangular prism is better than that of iso-triangular structure. This research provides a theoretical basis for the research, selection and design of RMs with high potential value.

2. Theoretical analysis
RMs are continuous dense objects. No gap exists amongst particles that make up RMs. Hence, the material is uniform. The physical properties of each part do not change with the coordinate position, and elastic constants do not change with coordinates. The constants of each material do not change due to stress or strain. The RM generates smaller displacement under the effect of the load than the dimension of the mirror itself. To treat any point on the RM as a hexahedral infinitesimal structure, the stress–strain state of any node of the RM can be expressed with six stress components and six strain components. The six stress components are as follows:

\[
\sigma_i = \begin{bmatrix} \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx} \end{bmatrix}^T
\]

where \(\sigma_x\) and \(\sigma_y\) are normal stresses in the direction of axis \(x, y, z\); \(\tau_{xy}, \tau_{yz}\) and \(\tau_{zx}\) are shear stresses.

The six strain components are as follows:

\[
\varepsilon_i = \begin{bmatrix} \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \end{bmatrix}^T
\]

where \(\gamma_{xy}, \gamma_{yz}\) and \(\gamma_{zx}\) are shearing strains; \(\varepsilon_x, \varepsilon_y\) and \(\varepsilon_z\) are normal strains.

Under the action of the external load, the stress state at any point on the RM satisfies the following equilibrium differential equation\(^{[24]}\):

\[
\sigma_{ij, \cdot} + F_j = \rho \frac{\partial^2 u_j}{\partial t^2}
\]

Following elastoplastic theory, the relationship between strain and displacement satisfies the following geometric equation\(^{[10]}\):
The stress–strain relationship between the RM material in the elastic region satisfies Hooke’s law\cite{24}, as follows:

\[
\varepsilon_{ij} = \frac{1}{2} \left( u_{i,j} + u_{j,i} \right) \quad (4)
\]

where \( \mu \) is Poisson’s ratio, and \( E \) is elastic modulus.

The working speed of the RM and displacement boundary at the constraint are known. In other words, during the solution of the RM, the part of surface force and part of the displacement boundary are identified. At the same time, the stress boundary conditions are as follows\cite{24}:

On the boundary of \( S_x \), the mirror surface

\[
\sigma_{ij} n_j = X_i \quad (6)
\]

On the boundary of \( S_u \), the displacement

\[
u_i = \ddot{u}_i. \quad (7)
\]

When the RM is running at a high speed, its density, mass and structure and the constraint conditions are known. Following elastoplastic theory, the stress component of the RM can be obtained initially. The stress component can then be replaced into the physical equation. Finally, the corresponding RM strain component can be obtained. However, the six stress components, six strain components and three displacement components of the RM are completely independent from time to time. In other words, even if the strain components are six, only three displacement components are solved. Hence, the stress and strain components of the RM must satisfy the Beltrami–Michell strain coordination equation at the same time\cite{24}.

\[
\sigma_{ij,kk} + \frac{1}{1+\mu} \varepsilon_{kk,ij} = 0 \quad (8)
\]

The differential equation for any point of the RM has six stress components, six strain components and three displacement components, thereby totalling to 15 equations. The boundary conditions are given in this paper. These 15 components are dependent of each other. If the motion of a rigid body is disregarded, then a unique solution to stress, strain and displacement at any point in the mirror is available in theory following uniqueness theorem of elastic mechanics solution. However, finding a real solution is complex. Therefore, the solution must be provided via the finite element method.

3. **Numerical Analysis**

The RM is unaffected by axial force softened during operation. The axial motion force produced by the gyro effect during operation is small and insignificant. Figure 1 shows the finite element model of a RM. The mirror has two parts, namely, a spinning axis and a mirror body, which are glued together. The bearing inner ring and spinning axis are face contact. The spinning axis and bearing inner ring cylindrical face are not fully in contact. For safety reasons, this part of the constraint is reduced to the line constraint in the middle part of the bearing inner ring. Correspondingly, at one end of the coupling, the RM constraint is reduced to fixed hinge support, i.e. \( U_X = U_Y = U_Z = 0 \). The other end of the RM model is the active hinge support, and the corresponding constraint mode is \( U_X = U_Y = 0 \), as shown in Figure 1. In the finite element model of the RM, the operating speed of the mirror is defined as \( 30 \times 10^4 \) rpm. The constraint mode defined in the model does not affect the deflection of the spinning axis end because of the special support mode of the RM. Thus, it can simulate the actual situation of the RM. Figure 2 shows an iso-polyhedron RM with different sections. Each mirror has the same circumscribed circle radius of \( R_1 \), mirror length of \( L_1 \) and spinning axis size. The rotating-mirror material is S-200 high-strength beryllium alloy extrusion profile, with a density of 1,880 kg/m\(^3\), a limit strength of 310 MPa, an elastic module of 700 GPa, a specific stiffness of 16.5, a Poisson’s ratio of 0.03 and a specific strength of 389. The rotating shaft material is AISI 4340 high-strength steel alloy, with a density of 7,800 kg/m\(^3\), a limit
strength of 200 MPa, an elastic module of 1,600 GPa, a specific stiffness of 25.6, a Poisson's ratio of 0.26 and a specific strength of 205.

Figure 1 The three-dimensional structure and cross-sectional view of RM prism model. (a) and (b) are iso-triangular prism. (c) and (d) are iso-quadrangular prism. L1 is the length of mirror body. L2 is the length of shaft segment 1. L3 is the length of shaft segment 2; R1 is the circumradius of mirror body. R2 is the radius of shaft segment 1. R3 is the radius of shaft segment 2.

4. Results and Discussion
Figure 2 shows a stress contour map and deformation curve graph of the RM with iso-triangular prism structure. When the speed is 30×10^4 rpm, the maximum node stress of the iso-triangular prism beryllium RM is 58.7 MPa. It is located in the end-face position of the contact between the mirror and the mirror body. The maximum stress is less than the limit strength of the beryllium alloy, and the safety factor is 11.9. In theory, no strength failure, such as an iso-triangular prism beryllium RM, will occur. The stress contours near the spinning axis are approximately circular. The stress on the mirror body gradually diminishes along the radial direction. Within the inscribed cylinder of the mirror body, the stress contour line is convex outwards along the diameter. Outside the inscribed cylinder, the stress contour lines at the three sharp-corner sections are concaved along the radial direction. In the direction away from the axis, although the stress value of the mirror is also decreased along the radial, the contours change near the position of the inscribed cylinder in the mirror, from outward convex to inward concave. The part where the contour is outward convex, and inward concave is subjected to a combination of tension and compression.

At the centre of the mirror, the letter delineated by the contour near the centre of the mirror is H, whereas the letter delineated by the contour within the mirror body is G along the radial direction. On this basis, the stress inside the mirror body is less than the stress near the surface of the mirror body. Furthermore, for the material in the central area of the mirror, in addition to the tensile force due to the centrifugal force generated by the material itself, the material is subjected to the centrifugal force generated by part of the material with outer sharp corners of the inscribed cylinder of the mirror body
during high speed rotation. As a result, the material on both sides of the middle area of the mirror surface is subjected to the tensile force along the radial direction and pointing to the two edges. Under the action of centrifugal force, the mirror centre material drifts away from the two sharp corners nearby. As a result, the compression effect is produced on the centre of the mirror. At the same time, the deformation of the concave is created at the centre of the mirror. Hence, the stress of this part of the area is greater than that in the nearby area. In general, the RM stress is gradually decreasing along the radial direction, especially in the inscribed cylinder of the mirror body. Hence, a negative stress gradient exists along the radial direction. In other words, during operation, the strength of the iso-triangular prism beryllium RM is primarily affected by the tensile stress.

In the mirror finite element modelling, one face of the RM is perpendicular to the X-Z plane. In this manner, the amount of mirror surface deformation can be identified. The node displacement on the intersection of the mirror surface where the plane intersects vertically with the X-Z plane is extracted as the axial displacement of the mirror surface. Thus, on the plane of X-Z, the strain of any point in this line can be regarded as the main strain. The other strain component is zero. The node to the axis along the X axis displacement is the amount of mirror surface axis displacement generated by the strain. The displacement of the X-axis direction of the node on the connection of the two edges of the mirror can be extracted as the horizontal deformation of the node. The node displacement of all nodes on this straight line can be regarded as the horizontal deformation of the mirror surface. At the same time, the mirror surface lateral size of the RM is different at the radius of the outer circle. Therefore, the mirror surface lateral deformation volume with small mirror size will be larger than the mirror surface horizontal size, directly with the mirror's relative lateral amount to evaluate the mirror deformation. However, the mirror surface deformation features of the RM cannot be qualitatively described. The amount of mirror surface deformation (μm) on the unit length of each 1 m is defined as the mirror surface deformation coefficient and as a parameter to measure and evaluate the distortion of RM surface.

![Image](Figure.2 The von Mises stress and deformation of RM with iso-triangular prism structure. (a) the isometric view of the stress, (b) a cross-sectional view of the stress, (c) lateral deformation; (d) axial deformation)
The horizontal deformation curve of the iso-triangular prism beryllium RM is shown in Figure 2 (c). The mirror surface lateral deformation and axial deformation curve of the iso-triangular prism beryllium RM are all symmetrical about its centre. The lateral relative deformation of the beryllium RM is 0.1453 μm. The axial relative deformation amount is 0.056 μm. The unit horizontal deformation coefficient is 8.389, whereas the unit longitudinal deformation coefficient is 1.748. On the basis of the amount of lateral deformation of the RM, the mirror is concaved along the radial direction. This result is consistent with the previous analysis. The axial deformation of the RM is less than the lateral deformation. Hence, the mirror surface deformation is due to the three sharp corners of the mirror inscribed cylinder outside the mirror to produce a large centrifugal force to the centre of the mirror. As a result, the material at the centre of the mirror is compressed.

Figure 3 shows the von Mises stress and deformation of RM with quadrangular prism structure. The maximum stress of the RM with an iso-quadrangular is 78.8 MPa. It is located in the end position of the spinning axis and mirror surface contact. The stress decreases in turn along the radial direction, and a negative stress gradient exists. The stress contour of the RM is symmetrical about X-axis and Y-axis. The inscribed cylinder of iso-quadrangular RM is larger than that of the iso-triangular prism RM. It needs additional materials for inscribed cylinder and fewer materials for sharp corner, which has a small tensile force on the mirror surface. On the contour map of the RM, the stress value of the mirror surface is greater than that near the part of the mirror structure. Moreover, the centrifugal force generated by the four sharp-corner materials in the RM during high-speed rotation has a compression effect on the material in the centre of the mirror surface. The centrifugal force generated by this part of the material when the mirror rotates has a compression effect on the tensile effect of the sharp-corner material. Meanwhile, the lateral deformation of the mirror surface is concaved inwards (Figure 3 (c)). However, in comparison with the RM with iso-triangular prism structure, the number of the RMs with iso-quadrangular increases. The radius of the inscribed cylinder in the mirror is relatively large. The material of the sharp corner part of the mirror is relatively reduced. Its compression effect on the centre of the mirror is also reduced. On the basis of these results, the compression effect of the sharp corner of the material on the centre of the mirror and the tensile force of the material to the centre of the mirror offset against each other, thereby greatly reducing the lateral deformation of the mirror surface.
Figures 3 (c) and (d) show the deformation curve of iso-quadrangular mirror surface. The mirror surface lateral deformation and axial deformation curve are symmetrical about its centre. The lateral relative deformation volume is 0.0039 μm, and the axial relative deformation is 0.0617 μm. In addition, the unit lateral deformation coefficient is 0.276, and the unit longitudinal deformation coefficient is 1.898. The amount of mirror surface lateral deformation is much less than the amount of mirror surface deformation of the iso-triangular prism. The mirror lateral relative deformation curve of the iso-quadrangular RM is 4.23 to 9.87 mm, with good deformation effect. The flatness is good in the RM operation. The compression effect of the sharp angle part of the iso-quadrangular RM is much less than that of the iso-triangular prism structure. The tensile effect of the material at the centre of the mirror under the action of centrifugal force cancels the compression effect of the sharp part of the material on the mirror surface and reduces the lateral deformation of the mirror surface.

5. Conclusions
Under the same constraints and loads, the negative stress gradient exists in the RM with different structures for the equivalent outer radius mirror. On this basis, the major effect on the strength of the mirror is the tensile effect of the centrifugal force generated by the mirror body material during high-speed rotation. The lateral deformation of the mirror surface is caused by the combined effect of compression of centrifugal force generated by the material of sharp corner and the tretching of tensile force caused by the material at the centre of the mirror at high-speed rotation. The amount of mirror surface deformation of the RM is not proportional to the number of mirror faces. The iso-quadrangular RM has the smallest lateral deformation, the smallest unit lateral deformation amount and the best lateral deformation effect.

Acknowledgments
This research was financially supported by the Natural National Science Foundation of China (NSFC) (grant numbers 61575129); and the Project of Characteristic Innovation in Higher Education of Guangdong, China (grant number 2018KTSCX347); and the Stable Support Project of Shenzhen Higher Education Institutions (SZWD2021008); and the Teaching Reform Research Project of Shenzhen technology University (grant number 60119).

References
[1] Owen,J., Davies. R. (1949) High-speed recording by a rotating-mirror method, Nature 164752.
[2] Ancona.A., Roser.F., Rademaker K., Limpert J., Nolte S., Tunnerrmm A. (2008) High speed laser drilling of metals using a high repetition rate, high average power ultrafast fiber CPA system, Opt Express 16:8958-8968.
[3] Mao Y., Flueraru C., Sherif S., Chang S. (2009) High performance wavelength-swept laser with mode-locking technique for optical coherence tomography, Opt Commun. 282:88-92.
[4] Cheng J., Liu C., Shang S., Dearden G., Watkins K., (2013)A review of ultrafast laser materials micromachining, Optics & Laser Technology 46:88-102.
[5] Courvoisier F., Stoian R., Couairon A.,(2016)Ultrafast laser micro- and nano-processing with nondiffracting and curved beams, Optics & Laser Technology 80:125-137.
[6] Winkler M.A. (1964) Rotating Mirror Vibration, Rev Sci Instrum. 35:790-792.
[7] Gelderblom E.C., Vos H.J., Mastik F., Faez T., Luan Y., Kokhuis T.J.A., Jong N. D., Versluis M., (2012)Brandaris 128 ultra-high-speed imaging facility: 10 years of operation, updates, and enhanced features, Rev Sci Instrum. 83:103706.
[8] Li C.B., Liu M.Q., Ruan S.Z., Chen S.J., Ren X.K., Du C.L., Huang H.B.(2018) Strength reliability of rotating mirrors for ultra-high-speed cameras, Optik 174: 363-371.
[9] Li C.B., Liu M.Q., Ren X.K., Du C.L., Huang H.B., Ruan S.C. (2018) Mechanical analysis on magnesium alloy rotating mirror for ultra-high-speed camera, Young Scientists Forum 2017, SPIE, pp. 107103L.
[10] Li C.B., Huang H.B., Zhao J.Q., Ruan S.C. (2018) A high strength magnesium alloy-based rotating
mirror for an ultra-high speed camera, Optik 157:85-92.
[11] Li J.Z., Sun F.S., Huang H.B., Gong X.D., Ca Y. (2014) Upgrading optical information of rotating mirror cameras, Rev Sci Instrum. 85:113701.
[12] Ionin A.A., Selezniev L.V., Sinitsyn D.V., Sunchugasheva E.S., Zemtsov D.S. (2017) Q-switched slab RF discharge CO laser, Laser Phys Lett. 14:5500.
[13] Bizjak A., Nemeš K., Možina J. (2011) Rotating-Mirror Q-Switched Er:YAG Laser for Optodynamic Studies, Strojniški vestnik – Journal of Mechanical Engineering. 57: 3-10.
[14] Skorczakowski M., Pichola W., Śviderski J., Nyga P., Galeck L., Maciejewska M., Zajac A., Gross S., Heinrich A., Bragan T., Kasprzak J. (2011) 30mJ, TEM00, high repetition rate, mechanically Q-switched Er:YAG laser operating at 2940 nm, Opto- Electron Rev. 19: 206-210.
[15] Scheps R., Myers J.F. (1994) Performance of a diode-pumped laser repetitively Q-switched with a mechanical shutter, Applied Optics 33:969-978.
[16] Marinček M., Lukac M. (1993) Development of EM Field in Lasers with Rotating Mirror Q-Switch, IEEE J Quantum Elect. 29:2405-2412.
[17] Jones C., Hann D.B., Voisey K.T., Aitken S. (2017) Application of high speed filming techniques to the study of rearwards melt ejection in laser drilling, J Laser Appl. 29:22205.
[18] Loor R. D. (2013) Polygon Scanner System for Ultra Short Pulsed Laser Micro-Machining Applications, Physics Procedia 41:544-551.
[19] B. Brixner. (1965) Rotating Steel Mirrors—Failure and Success, Rev Sci Instrum. 36:1297.
[20] B. Brixner. (1967) Rotating-Mirror Sweeping-Image Spectrograph, Rev Sci Instrum. 38 :287.
[21] Li J.Z., Sun F.S., Gong X.D. (2005) Studies on dynamic behavior of rotating mirrors, Proc. of SPIE. 5638:118-123.
[22] Huang H.B., Li J.Z., Gong X.D., Sun F.S., Hui B. (2007) Numerical simulation on surface deformation for rotating mirrors of ultra-high-speed photography, Pro.SPIE:62797M.
[23] Erez A., Partom Y., (1966) Calculation of Surface Distortions of Rotating Mirrors and their Effect on Streak Camera Resolution, Appl Optics. 5: 723-727.
[24] Atanackovic T. M. (2000) Theory of Elasticity for Scientists and Engineers, Springer Science Business Media, LLC, Boston, pp. 16-168.