Ensuring Progress for Multiple Mobile Robots via Space Partitioning, Motion Rules, and Adaptively Centralized Conflict Resolution

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Abstract—In environments where multiple robots must coordinate in a shared space, decentralized approaches allow for decoupled planning at the cost of global guarantees, while centralized approaches make the opposite trade-off. These solutions make a range of assumptions — commonly, that all the robots share the same planning strategies. In this work, we present a framework that ensures progress for all robots without assumptions on any robot’s planning strategy by (1) generating a partition of the environment into “flow”, “open”, and “passage” regions and (2) imposing a set of rules for robot motion in these regions. These rules for robot motion prevent deadlock through an adaptively centralized protocol for resolving spatial conflicts between robots. Our proposed framework ensures progress for all robots without a grid-like discretization of the environment or strong requirements on robot communication, coordination, or cooperation. Each robot can freely choose how to plan and coordinate for itself, without being vulnerable to other robots or groups of robots blocking them from their goals, as long as they follow the rules when necessary. We describe our space partition and motion rules, prove that the motion rules suffice to guarantee progress in partitioned environments, and demonstrate several cases in simulated polygonal environments. This work strikes a balance between each robot’s planning independence and a guarantee that each robot can always reach any goal in finite time.

I. INTRODUCTION

Some of the most successful real-world applications of robotics, including deployments in warehouses and industrial settings, require coordinating multiple mobile robots in a shared space to avoid conflicts. Most approaches to multi-robot coordination in these settings assume that all robots share the same planning strategies. While this assumption may hold in real-world spaces today, in the future we expect that mobile robots controlled by different entities and equipped with different planning strategies will have to coexist safely in spaces such as shared warehouses, docks, and offices such that each robot is able to access shared resources and make progress.

In this paper, we introduce the concept of a motion pact, which lays out a set of rules for motion in a shared space independent of any particular planning strategy and provides certain guarantees if all robots in the space follow those rules. We propose one such motion pact which guarantees progress for all robots while imposing relatively lightweight restrictions on each robot’s space of planning choices, ensuring that each robot can largely plan, coordinate, and move independently from other robots in a decentralized fashion as long as it obeys the rules when necessary. The motion pact achieves this by (1) dividing up the shared environment into regions (Section IV), and (2) installing a set of motion rules (Section V) that consist of guided motion in certain regions and an online adaptive centralization mechanism for resolving spatial conflicts (Section V-C). We prove that progress is guaranteed for all robots if they all honor the motion rules (Section VI) — in particular, that at any instant, any robot is able to access any location in finite time — and demonstrate this practically via a proof-of-concept simulation implementation (Section VII).

II. RELATED WORK

Multi-robot path planning and coordination has been studied extensively \cite{2–4, 6, 8–13, 15, 18, 20, 23–29, 31, 32}. Much of this work can be characterized by its degree of centralization: fully centralized methods \cite{1, 2, 20, 26} tend to offer guarantees around progress (i.e., ensuring that all robots reach their goals) and optimality with regards to path length, but are typically computationally expensive and limited in their scalability. At the other end of this spectrum, fully decentralized methods \cite{11, 17, 25, 28} most often take the opposite trade-off: high scalability, but weaker guarantees of progress and optimal performance. Many approaches, including ours, lie between these extremes; we call these semi-centralized \cite{6, 16, 22, 29}.

Our approach permits robots to remain fully decentralized and decoupled until a conflict arises during execution, at which time a subset of the robots must coordinate. It combines and builds on work in semi-centralized planning and space partitioning to offer guaranteed progress without making strong assumptions on robot planning methods. Our space partition prevents deadlock in narrow passageways, ensuring that every point in the workspace will always be safely reachable in finite time. We resolve conflicts in tightly-packed space with a protocol that builds on priority-based planning \cite{28} and local conflict resolution \cite{6, 13, 14, 21}. We also guarantee progress and are complete; we do not provide any guarantee on path length optimality.

Van den Berg et al. \cite{26} is similar to our approach in that it performs maximally decoupled centralized planning for a set of robots by computing optimal partitions of robots to centrally coordinate. Their algorithm runs offline and creates sequentially executed plans among the partitioned subsets of robots. In contrast, our approach is designed to work online: we do not explicitly partition robots, and instead induce an
We impose a global density cap: can become stuck when the space is too densely occupied, \( E \) refers to a flow space regions according to the algorithm described in Appendix II. These to their goals in finite time.

Finally, let each robot be independently controlled, with its own navigation strategy and goals, and initialize the \( r_i \in \mathbb{R} \), and that each \( r_i \in \mathbb{R} \) has a maximum speed bounded by \( V \in \mathbb{R}^+ \) and (2) uses a local safety controller to never overlap with any obstacle in \( O \) or other robot \( r_j \in \mathbb{R} \), \( i \neq j \).

Finally, let each robot be independently controlled, with its own navigation strategy and goals, and initialize the \( r_i \in \mathbb{R} \) to unique, non-overlapping positions in \( \mathcal{E} \) at timestep \( t = 0 \).

We wish to guarantee that all \( r_i \in \mathbb{R} \) can safely navigate to their goals in finite time.

### III. Problem Setup

Let \( \mathcal{R} \) be a finite set of omnidirectional robots in a two-dimensional polygonal environment \( \mathcal{E} \) consisting of free space \( \mathcal{F} \) and polygonal obstacles \( O \). Assume each robot \( r_i \in \mathcal{R} \) is modeled as a disc with the same fixed radius \( \rho \), and that each \( r_i \in \mathcal{R} \) (1) has a maximum speed bounded by \( V \in \mathbb{R}^+ \) and (2) uses a local safety controller to never overlap with any obstacle in \( O \) or other robot \( r_j \in \mathcal{R} \), \( i \neq j \).

We partition the free space of an environment into three classes of space: flow space, open space, and passage space according to the algorithm described in Appendix II. These classes are composed of, respectively, flow regions, open regions, and passage regions. In the following, a “spot” refers to a \( B(\rho) \subseteq \mathcal{F} \), an exactly robot-shaped disc in the free space of \( \mathcal{E} \). As disc-modeled robots in arbitrary environments can become stuck when the space is too densely occupied, we impose a global density cap:

**Assumption: Global Density Cap**

Let \( N \) be the total number of open, flow, and passage regions in \( \mathcal{F} \) for some environment \( \mathcal{E} \). Let the maximal sphere packing\(^a\) of a space be \( M \). Then the global cap on the number of robots allowed in \( \mathcal{E} \) is \( M - N \). In other words, there is always \( N \) spots' worth of unoccupied space.

\(^a\)Sphere packing is difficult and well-studied [19]. Similar sphere-modeled multi-agent robotics work assumes conservative estimates of density, often in the range 50 – 60% [5] or lower. Identifying \( M \), the upper bound, in practice is non-trivial, but determining a number below \( M \) that generously upholds the density assumption while outperforming the density of 50% is achievable.

In open space, robots can move freely; in flow space, there is an associated directional constraint on robot motion; passage regions help robots move between flow regions.

We define the following: given a bounded environment \( \mathcal{E} \) with free space \( \mathcal{F} \) and robots of radius \( \rho \), we partition \( \mathcal{F} \) into sets \( \Omega \) (the open space), \( \Phi \) (the flow space), and \( \Psi \) (the passage regions) such that:

1. \( \Phi \) contains a minimally one-robot-wide corridor around every obstacle in \( \mathcal{E} \) and the interior of \( \mathcal{E} \)'s perimeter.
2. Every point in \( \mathcal{F} \) that can be reached by a robot without following any rules can be reached by a robot following our proposed motion rules in Section V in \( \Omega \), \( \Phi \), or \( \Psi \).
3. Every point in \( \Phi \) constrains robot motion to move in a particular direction (clockwise or counter-clockwise) around the perimeter and each obstacle in \( \mathcal{E} \), and that these directions are compatible with each other — that is, that there are no two adjacent points directing a robot to move in opposing directions.
4. \( \Psi \) is composed of passage regions \( \mathcal{P} \), where each passage region is a spot (a robot-shaped disc) that connects adjacent flow regions.

### IV. Space Partitioning: Regions

We partition the free space of an environment into three classes of space: flow space, open space, and passage space according to the algorithm described in Appendix II. These classes are composed of, respectively, flow regions, open regions, and passage regions. In the following, a “spot” refers to a \( B(\rho) \subseteq \mathcal{F} \), an exactly robot-shaped disc in the free space of \( \mathcal{E} \). As disc-modeled robots in arbitrary environments can become stuck when the space is too densely occupied, we impose a global density cap:

In open space, robots can move freely; in flow space, there is an associated directional constraint on robot motion; passage regions help robots move between flow regions.

At any time, a robot is exclusive either in an open, flow, or passage region, or is transitioning between two regions. These conditions are disjoint — a robot may not be in two types of region at the same time without transitioning.

To transition between regions, a robot must use the “space request” mechanism defined in Section V-F. In the following, let \( r_i \in \mathcal{R} \) be a robot, \( r_i(t) \) its position at time \( t \), and \( \vec{v}_i(t) \) its velocity vector at time \( t \). We identify \( r_i \) with its geometry, i.e. saying “all of \( r_i \) is contained by a region” means that the \( \rho \)-radius disc representing \( r_i \) is contained by the region.

#### A. Determining a robot’s region and transition state

We determine whether \( r_i \) is transitioning between regions or in a particular region type at time \( t \) by the following sequence of checks:

1. If \( r_i(t) \cap P \neq \emptyset \) for a passage region \( P \in \Psi \), \( r_i \) is in a passage region.
2. Else, if \( r_i(t) \) is completely contained in a flow region \( F \subset \Phi \), \( r_i \) is in a flow region.
3. Else, if \( r_i(t) \) is completely contained by an open region \( O \subset \Omega \) (i.e. \( B_\rho(r_i(t)) \subset O \)), \( r_i \) is in an open region.
4. Otherwise, if none of the other conditions hold, \( r_i \) is transitioning between regions.

#### B. Constraints on robot motion

If \( r_i(t) \) is in a flow region, its velocity \( \vec{v}_i(t) \) must satisfy \( \vec{v}_i(t) \cdot \vec{d}_c \geq 0 \), for \( c_i \) the centroid of \( r_i(t) \) and \( d_c \) the flow direction at \( c_i \). In any region type, the pushing rules defined in Section V-F apply and may constrain the motion of \( r_i \). If \( r_i \) is transitioning between regions, it must abide by all of the motion constraints for the region types it is transitioning between (e.g. a robot moving into a flow region cannot move against the direction of flow at its transition point).
C. Space Requests

Space requests are the core of our adaptive centralization mechanism. Robots use space requests to (1) transition between regions and (2) resolve spatial conflicts (i.e., conflicting attempts to occupy a space). Space requests are tuples \((S,d,T)\) comprising:

**The primary spot** \(S\): The primary spot represents the goal space, if any, that the robot wishes to access. \(S\) is a radius \(\rho\) ball\(^1\) lying entirely in one region if the robot has a goal, or null otherwise.

**The duration** \(d\): A real number representing the amount of time the robot would like to spend in the primary spot.

**The transition spot** \(T\): The transition spot represents the space, if any, that the robot may need to access to move between two regions. \(T\) is a radius \(\rho\) ball\(^1\) in a region adjacent to the robot’s current location if the robot is moving between regions, or null otherwise.

To use space requests, each robot \(r_i\) maintains the following:

**Priority:** A tuple \(L_i = (\tau_i, \ell_i)\) where \(\tau_i\) is a timestamp and \(\ell_i \in \mathbb{N}\) is a unique label assigned to \(r_i\). “Timestamp priority” refers to \(\tau_i\) alone. If \(r_i\) does not have an active space request at some time \(t\), then \(\tau_i = \infty\).

**Personal space request:** \(SR_{pi} = (S_{pi}, d_{pi}, T_{pi})\), a space request used for the robot’s unforced goals and transitions, as in Section V-D.

**Forced space request:** \(SR_{fi} = (S_{fi}, d_{fi}, T_{fi})\), a space request used for the robot’s forced goals and transitions, as in Section V-E. \(d_{fi}\) is always zero.

**Pusher identity:** \(I_p\), a record of the robot (if any) which originally initiated the pushing “network” containing \(r_i\). If \(r_i\) is not being pushed, \(I_p = \ell_i\).

D. Moving with personal space requests

Space requests for voluntary movement are a way of reserving access to a desired space. A robot \(r_i\) communicates its intent to move into a space by submitting a space request, the request is checked to ensure that it is valid and won’t cause a collision, and then, once granted access to the requested space, \(r_i\) can safely move in. There are three stages to using a personal space request \((S_{pi}, d_{pi}, T_{pi})\): (1) making the request, (2) moving into the request once it is granted, and (3) completing the request.

**Making a space request:** A robot \(r_i\) can make a new space request if \(S_{pi}\), the primary spot of its personal space request, is currently null. To do so, \(r_i\) sets \(S_{pi}\) to the spot it wishes to move into, sets \(d_{pi}\) to the duration it wishes to spend in \(S_{pi}\), and communicates \(SR_{pi}\) to the space request arbiter \(\chi\) (Section V-F). \(r_i\) then waits for the arbiter to notify it that its request has been granted and set its timestamp priority to the time of approval. While it waits, \(r_i\) cannot move unless forced (per Section V-E). \(r_i\) can cancel its personal space request at any time by setting \(S_{pi}\) to null and communicating \(SR_{pi}\) to \(\chi\).

**Moving into a granted space request:** Once \(SR_{pi}\) is granted, \(r_i\) must move into \(S_{pi}\) as quickly as possible, following the shortest collision-free path to the requested space (abiding by motion rules). If this path crosses between regions, \(r_i\) must use the transition spot \(T_{pi}\) from \(SR_{pi}\), by setting \(T_{pi}\) to a spot (1) tangent to \(r_i\) and (2) completely in the region \(r_i\) wishes to enter, and communicating the modified \(SR_{pi}\) to \(\chi\) for approval of the transition per the above. Once \(r_i\) changes regions, the transition spot is reset to null.

**Completing a space request:** Once \(r_i\) has moved into \(S_{pi}\) (i.e., \(r_i\) and \(S_{pi}\) align) for \(d_{pi}\) seconds, the space request is completed and \(SR_{pi}\) is reset to \((null, 0.0, null)\).

E. Forced space requests and pushing

To guarantee progress for all robots, we sometimes need to make one or more robots move out of the space another robot wants to occupy. We do so by introducing a notion of “pushing”: a mechanism by which a high priority robot can clear its path. At a high level, pushing is triggered when a higher priority robot encounters a lower priority robot in its way. If that lower priority robot can move out of the higher priority robot’s path, it must; otherwise, if its own path is blocked, it transitively pushes out, borrowing the higher priority to do so. This iterative pushing, combined with a total ordering of robot priorities, ensures that the original higher priority robot will be able to proceed toward its goal in finite time (proving this is the primary focus of Section VI).

In more detail: a robot \(r_i\) can (in different circumstances) **push** and **be pushed**. Each of these processes uses each robot’s priority \(L_i\) and forced space request \(SR_{fi}\).

**Comparing priorities:** We impose a total order on robot priorities by saying that, for \(i \neq j\), \(L_i < L_j\) iff \(\tau_i\) is later...
than \( r_j \) or \( r_i = r_j \) and \( \ell_i < \ell_j \). This order ensures that earlier requests have higher priority, and thus that a space request’s priority monotonically increases until it is completed.

**Pushing:** When a robot \( r_i \) is moving to complete its space request (personal or forced) and comes tangent to a robot \( r_j \) with lower timestamp priority, then \( r_i \) pushes \( r_j \). \( r_j \) first sets \( I_{p_j} = I_{p_i} \) and adopts the pusher’s \( (r_j) \) timestamp priority for its forced space request \( SR_{f_j} \). We then define a “pushing vector” recursively to derive how \( r_j \) has to move from \( r_i \’s \) pushing vector. If \( r_i \) is not being pushed (i.e., it’s the pusher), then its pushing vector is the vector from \( r_i(t) \) to \( r_j(t) \). If \( r_i \) is in a flow region, then its pushing vector is the flow direction. Otherwise, \( r_i \’s \) pushing vector also abides by the following recursive definition. When \( r_i \) pushes robot \( r_j \), \( r_j \) selects a unit vector \( p_j \) with an angle \( \theta \) within \( \pi/2 \) of \( r_i \’s \) pushing vector \( (p_i) \) and averages \( p_0 \) and \( p_i \) to generate \( p_j \).

**Being pushed:** A robot \( r_j \) is pushed when it comes tangent to \(^2\) a robot \( r_i \) with equal or higher priority. There is one notable exception: to ensure that no robot can transitively push itself, if the timestamp priorities of \( r_j \) and \( r_i \) are equal, but \( I_{p_j} = \ell_j \) (i.e., \( r_j \) originated the pushing network containing \( r_i \)), then \( r_j \) is not considered pushed. When \( r_j \) is pushed, it sets \( T_{p_j} = \text{null} \) (cancelling any pending requests to transition between regions for voluntary movement), and adopts \( r_i \’s \) timestamp priority. \( r_j \) must immediately attempt to move away from \( r_i \) into a spot adjacent to its current location within \( \pm \frac{\pi}{2} \) radians of \( r_i \’s \) pushing vector. If there exists an unoccupied adjacent spot within the allowed angle range, \( r_j \) must move into it. If \( r_j \) needs to place a space request to move (i.e., there are no unoccupied adjacent spots within the allowed angle range), then \( r_j \) makes and completes a forced space request using \( SR_{f_j} \) in the angle range, following the same process as for personal space requests with \( r_i \’s \) timestamp priority for the request. A forced space request is immediately canceled if the robot is no longer being pushed or transitioning to satisfy the pushed request. \( r_j \) updates its pushing vector as described above.

When equal priority robots push each other and neither originated the pushing network, their pushing vectors are averaged, ensuring a consistent overall pushing direction. Pushing is re-evaluated every timestep, so pre-established redundant forced space requests are automatically canceled.

Pushing is covered in more detail in Appendix I.

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Note that this does not imply physical contact between robots; \( \rho \) can be chosen to provide a buffer of space around each robot.

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Fig. 3: An example of how pushing occurs when TR (gray) is moving to granted space request (gray circle), the desired motion is shown as a red arrow. Each pushed robot’s direction is indicated by an arrow (in particular, observe that the purple robot overlaps the requested spot, so it must move forward in the flow to get out of the way).

### The Passage Region Exception

Space requests work differently when a robot \( r_i \) wants to occupy a passage region \( P \).

1. The primary or transition spot (depending on why \( r_i \) wants to occupy \( P \)) becomes a capsule region defined by \( P \cup B_{EXIT} \), where \( B_{EXIT} \) is a radius \( \rho \) ball adjacent to \( P \) in the flow region \( r_i \) wishes to exit into.
2. A robot pushed by \( r_i \) can pass through the capsule to fulfill pushing, though it must clear the capsule as quickly as possible.

This allows robots to “escape” packed flow regions to allow \( r_i \) in and prevents deadlock caused by \( r_i \) blocking other robots from moving out of its way (see Fig. 4).

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**F. Global Space Request Arbiter**

The global space request arbiter \( \chi \) is in charge of space requests. We refer to \( \chi \) as an adaptive centralization mechanism because it coordinates precisely the set of robots which must be moved for a given robot to make progress and determines this set online. In practice, the arbiter could be implemented as a centralized entity or through local communication directly between robots. The arbiter has an initial role of randomly assigning a unique ID \( \ell_i \) to each robot \( r_i \). It may reassign \( \ell_i \) for all robots at fixed intervals to provide better overall fairness in access to resources (as \( \ell_i \) breaks ties when comparing priorities). However, the arbiter’s main job is to keep track of space requests and their priorities. Every time a robot makes a space request, it is submitted to the global arbiter with the timestamp \( \tau \) when it was placed. The arbiter keeps track of the priorities \( (\tau_i, \ell_i) \) of all robots that have made a space request, removes cancelled requests, and uses this information to send pushing information to the robots and grant requests.

\( \chi \) must approve all primary and transition spots for all space requests. The approval process first checks that the requested spot is valid (i.e. that it is contained in the free space). The arbiter immediately grants space requests that have valid spots and do not require transitioning into a flow or passage region. Requests to transition into a flow or passage region are approved once the requested spot is clear...
(i.e. does not contain any robots). When a request is made to transition into a flow or passage region, the arbiter informs nearby robots; robots with lower timestamp priorities must stay clear of the spot or exit it if they are currently occupying it. If two conflicting requests (i.e. the requested spots overlap) with the same timestamp priority are ready to be granted, \( \chi \) grants the request with the higher-priority value of \( \ell \).

VI. Analysis

We will now show that following the rules defined in Section V in an environment partitioned according to Section IV suffices to guarantee progress for every robot in the shared space.

**Theorem 1** (Progress). For any valid environment \( E \) with robots \( R \), pick (without loss of generality) some \( r_i \in R \) with a valid space request \( SR \). Then \( r_i \) will reach satisfy \( SR \) within finite time.

We build our proof of Theorem 1 by first showing that our motion rules result in progress in a series of simplified environment types, then compose these results together to guarantee progress for all valid environments. For space, we provide sketches of the full proofs.

**Lemma 1.** Assume any space request is satisfied in finite time. Then every space request will eventually have the highest priority.

**Proof Sketch.** Each robot can have at most one personal space request and one forced space request a time. The global space request arbiter processes space requests in order of submission. Thus, there are a finite number of requests, processed in order of arrival, and (by assumption) each is completed in finite time. Further, by the ordering defined on request priorities, a request’s priority (1) is monotonically increasing and (2) increases whenever a previously submitted request is completed. Therefore, every space request will eventually have the highest priority.

We introduce the following symbols for proof sketch brevity. Let \( \gamma \) be a finite set of transitively adjacent open, passage, and flow regions in an environment \( E \), and let \( \Gamma \subseteq \mathcal{E} \) be the set of all possible \( \gamma \) given environment \( E \). Finally, let \( TR \) signify the robot \( r_i \) with current global highest priority.

**Lemma 2.** Let \( \Gamma' \) be a single infinite obstacle-free open region, and note that \(| R |\) is finite. Let the current TR be robot \( r_i \), and let \( r_i \) be a finite distance away from its granted space request \( SR \). Both \( r_i \) and \( SR \) lie in \( \Gamma' \). Then \( r_i \) will complete \( SR \) in finite time.

**Proof Sketch.** Assume \( \Gamma' \) and \( r_i \) as given. We will show that \( r_i \) will satisfy its request in finite time by induction on \(| R |\).

**Base case:** Let \(| R | = 2\), containing TR \( r_i \) and one other robot \( r_j \). There can be at most one robot blocking the straight line path from \( r_i(t) \) to its goal. By the rules for being pushed, and since \( r_i \) is TR, the obstructing robot \( r_j \) will move away from \( r_i \) immediately if there is an adjacent open spot. By construction of \( \Gamma' \), there must be an empty spot adjacent to \( r_j \). Therefore, \( r_j \) will always clear \( r_i \)'s path in finite time and thus \( r_i \) will always complete \( SR \) in finite time.

**Inductive Hypothesis:** Let \(| R | \leq k \) for \( k \in \mathbb{N} \). Then TR \( r_i \) will complete \( SR \) in finite time.

**Inductive Step:** Let \(| R | = k+1\). There are at most \( k \) robots that lie within the straight line path from \( r_i \) to \( SR \). Robots on this path may have their movement blocked by at most \( k-1 \) robots. Since \( \Gamma' \) is infinite, there always exists an empty spot transitively adjacent to every \( r \in R \). By the pushing rules, the blocking robots directly adjacent to said empty spot must immediately move into it. Thus, combining this motion with the inductive hypothesis, each robot obstructing \( r_i \)'s path is able to and will in finite time move out of the way.

**NOTE:** As described in the pushing rules the pushing vector of the \( n \)th robot in a pushing chain is defined as \( p_n = \frac{p_{n-1} + p_n}{2} \), where \( p_{n-1} \) is the pushing vector of the robot that pushed \( p_n \). This sequence of vectors’ angles in the global frame is bounded above by the original top robot’s pushing vector angle \( \pm \pi/2 \). This ensures that no chain of pushed robots will ever circle back towards the original pusher (TR) and close a cycle with the original TR (which would preclude progress). This process is also described in Appendix I.

**Lemma 3.** Let \( \Gamma' \) be the subset of \( \Gamma \) such that each \( \gamma \in \Gamma' \) consists of only a single flow network — a maximal subset of \( \Phi \cup \Psi \) for which all regions are adjacent to at least one other region in the subset. Let \( r_i \) be TR with space request \( SR \) through a passage region \( P \) to exit into a flow region \( F \). \( r_i \) will complete \( SR \) in finite time.

**Proof Sketch.** Note that (per the passage region exception in note 1) space requests in passage regions reserve a capsule region \( C \) instead of a radius \( \rho \) ball. The capsule overlaps both the passage region \( P \) and an open spot in the exit flow region adjacent to the capsule. We will call this open spot \( B_{exit} \) and the flow region it belongs to \( F_{exit} \). \( F_{exit} \) flows in one direction by construction, moving away from \( P \). The proof proceeds by cases:

**Case 1:** \( C \) is already clear. \( r_i \) can move into its requested passage region and exit region immediately.

**Case 2:** There are a finite set of robots ordered in proximity to \( P \{r_1, r_2, ..., r_j, ..., r_k\} \) in such that \( \{r_1, ..., r_j\} \) lie in \( B_{exit} \cup C \) where \( i \leq k \). A capsule can intersect at most 3 robots at once, so there must be at least \( j \) robots worth of unoccupied space in \( \gamma \in \Gamma' \) because of assumption 1.

If the flow region has \( j \) unoccupied robot’s worth of space, robots \( \{r_{j+1}, ..., r_k\} \) moving forward in the flow will cause the \( j \) unoccupied robot’s worth of space to move behind \( r_{i+1} \) and thus provide enough room for \( \{r_1, ..., r_j\} \) to empty \( C \). If the flow region does not have \( j \) unoccupied robot’s worth of space, then \( j \) robots need to exit \( F_{exit} \) to another flow region through a passage region. If the adjacent flow regions have \( j \) unoccupied robot’s worth of space, each robot leaving \( F_{exit} \) will use TR’s priority to reserve the passage region and transition into the adjacent flow regions and like the prior argument there will be enough room made for the \( j \) robots to clear \( C \). In the worst case, pushing loops back to \( r_i \), meaning that a robot \( \not\in \{r_1, ..., r_j\} \) must pass through \( C \) to
Claim: If finite time.

Proof Sketch. Sub-Lemma 1: Region transitions take finite to the flow region network. Then TR is a flow region network and a single open region adjacent to Open: moving between flow regions via a passage region. There are three cases to consider, by transition type:

(Flow → Flow): Lemma 3 makes the claim hold for \( r_j \) moving between flow regions via a passage region.

(Flow → Open): If there is not enough room for \( r_j \in \Phi \) to move into an adjacent spot in \( \Omega \), it will push outward into \( \Omega \). If there is sufficient unoccupied space in \( \Omega \), then this pushing will cause robots to occupy as much of this space as possible — gathering together into a more densely packed group — to try to clear a spot for \( r_j \) in finite time. Otherwise, the pushing network will propagate until a robot is pushed into \( \Phi \), clearing a spot for \( r_j \) in finite time. These cases are exhaustive by assumption 1.

(Open → Flow): If the flow network is not full, Lemma 4 means that \( r_j \) can use a space request to transition in finite time. If the network is full, assumption 1 implies that there is a free spot in the open region. Thus a robot can exit the full flow network to make room for \( r_j \) to enter; by the rules for pushing (which causes robots in the flow network to push out into open space), this will happen in finite time.

Case 1: \( SR \) is in the open region \( O \).

\((r_i \in O)\): We use the same inductive structure as Lemma 2 with a different base case. An unoccupied spot somewhere in \( \gamma \) is guaranteed by assumption 1. Either this unoccupied spot is in \( O \) and pushing ensures the space request will be completed in finite time (by condensing the robots in \( O \), or the spot is in the adjacent flow network and pushing ensures an obstructing robot will be pushed into the flow network allowing the space request to be completed in finite time.

\((r_i \in F)\): If \( r_i \) is in the flow network and its space request is in \( O \), \( r_i \) can reach a spot adjacent to \( O \) by Lemma 4. Then, by sub-lemma 1, it can enter \( O \) in finite time.

Case 2: \( SR \) is in a flow region \( F \).

\((r_i \in F)\): By Lemma 4, \( r_i \) can reach its request. \( (r_i \in O)\): \( r_i \) can make it to a spot adjacent to the flow network via Case 1, and then transition across the boundary by sub-lemma 1. \( r_i \) can then follow the flow network motion as per Lemma 4 to make it to its space request.

Corollary 1. Let \( \Gamma' \) be the subset of \( \Gamma \) such that each \( \gamma \in \Gamma' \) is a finite number of transitively adjacent flow region networks and transitively adjacent open regions. The TR \( r_i \) will reach its requested spot in finite time.

Proof Sketch. Observation 1: Full Flow Region In a packed flow network, the pushing rules imply that all robots will join \( r_i \)'s pushing network until one can exit into an adjacent region.

Observation 2: Full Open Region In a packed open region, a robot originating a pushing network will push all robots on one side of the plane orthogonal to the robot’s pushing vector until a robot is able to exit into an adjacent flow region. At this stage, it is not known if the whole open region is packed. If the pushing network pushes back into the same open region, all robots in the region will be included in the pushing network and the region is known to be full. This follows because the initial pushing vector and the pushing vectors re-entering the open region will have an angle of less than or equal to \( \frac{\pi}{2} \) radians between them, creating “pushing planes” that cover the open region.

Lemma 5 ensures that if there is an unoccupied spot in a region adjacent to the TR, pushing can and will use this spot for \( r_i \) to achieve its goal. assumption 1 ensures there are \( N \) unoccupied spots in the space but does not guarantee they are — at any given time — in a region adjacent to the TR. However, in a \( \gamma \) with more than one flow network adjacent to an open space, we need to prove that as pushing propagates, the unoccupied space guaranteed by assumption 1 will not move in cycles between regions without ever being moved to the region where \( SR \) lies. By construction of \( \gamma \), all regions are transitively adjacent to all other regions in \( \gamma \). A pushing network will push into any adjacent unoccupied spots, or expand the pushing network to encompass any adjacent robots. Therefore, observation 1 and observation 2 mean that pushing will always grow to cover larger subsets of the environment and find any possible unoccupied spots.
Proof Sketch. Proving Theorem 1 (Progress) From Corollary 1 we know that any TR will reach its space request in finite time. From Lemma 1 we know if all space requests are satisfied in finite time, all robots with a space request become TR in finite time. Thus, space requests will be satisfied in finite time, and progress for all robots is guaranteed.

VII. EVALUATION

We have implemented a proof-of-concept simulation of our proposed approach, as shown in Figs. 1 to 3. In our supplementary material, we include a video demonstrating how robots move, resolve contention for space, and push in packed environments when abiding by our motion rules. Each robot in our simulation creates and follows their own plan without knowledge of other robots’ planning details. We show examples of several possible polygonal environments partitioned as per Section IV.

VIII. CONCLUSION

In this work we present a method for ensuring progress for every robot in a shared multi-robot space via a motion pact. Our motion pact is comprised of a partitioning of the environment freespace along with a set of motion rules that allow robots to plan independently and flexibly, with protection from being blocked (either intentionally or inadvertently) by other robots’ behavior. We prove that the rules guarantee progress for every robot, and demonstrate a proof-of-concept implementation of our system.

This work assumes that all robots are holonomic and capable of instantaneous bounded acceleration. Beyond relaxing this assumption, this work leaves several exciting open questions. For example, one could consider re-imaging the form of goals to encompass team-based tasks, where multiple robots must collaborate to satisfy their objectives rather than individually reach a location. With such an expansion, robots can work in teams, or even switch between teams, to more flexibly move and work in a shared space.

APPENDIX A: PUSHING LOGIC

We now describe, in Algorithm 1, the logic for computing the pushing angles and velocity vectors for each robot in the set R at a timestep t. Algorithm 1 depends on the following subroutines:

- activeSR (rᵢ): Returns the first non-null space request for rᵢ, in the order Tⱼ, Sⱼ, Tₚ, Sₚ.
- adoptpriority (rᵢ, rⱼ): Copies the timestamp τ from rᵢ’s priority and the identity rₚ of the original pushing robot for rᵢ to rⱼ.
- arc (pushⱼ): Returns the set of angles within ±τ radians of pushⱼ.
- overlaps (d₁, d₂): Returns true iff the discs d₁ and d₂ intersect. If called with robots rᵢ, rⱼ as arguments, d₁ and d₂ are the Bₚ-sized discs centered at the positions of rᵢ and rⱼ, respectively.

At a high level, Algorithm 1 iterates through each pair of robots rᵢ, rⱼ ∈ R checking for each rⱼ that is close enough to a higher-priority rᵢ, or the active space request of a higher-priority rᵢ, for pushing to engage. We compute the angle between the velocity vector pushⱼ of rᵢ and the vector pointing from rᵢ (or its active space request) to rⱼ (Line 10), and, if this angle is within the acceptable ±τ range of the original pushing vector, propagate the pushing priority of rᵢ to rⱼ and set rⱼ’s velocity vector to move out of the way. This process repeats until no robot’s velocity vector changes, at which time the robots in R move accordingly.

APPENDIX B: REGION GENERATION

As described in Section IV, our approach requires partitioning the free space F of a bounded environment E into three sets: the open space Ω, the flow space Φ, and the passage region space Ψ. In this section, we describe constructing such a partition. We will refer to the set of all obstacles and the internal perimeter of the environment as flow generating objects. Our algorithm proceeds through the following high-level steps (with further detail in corresponding subsections):

1) Construct initial flow regions around each flow generating object to capture a space wide enough for a line of robots around each object’s perimeter (Appendix II-A).
2) Separately construct inflated flow regions around each flow generating object (Appendix II-A). These regions capture subsets of the free space too narrow to hold three robots side by side.
   a) In areas where two inflated flow regions overlap, split the region lengthwise between corresponding flow generating objects.
3) Construct single-lane regions from areas where two initial flow regions overlap (Appendix II-C) to capture flow space areas too narrow for two robots side by side.
4) Place passage regions (Appendix II-D) to connect different flow regions in Φ.
5) Smooth the perimeter of open space (Appendix II-E).
6) Assign directions to each flow region (Appendix II-F).
7) Return the flow regions as $\Phi$, the passage regions as $\Psi$, and the remainder of the free space as $\Omega$.

A. Initial Flow Regions: The initial flow regions are constructed by taking the Minkowski sum of each object $O_i$ and a $B_\rho$-sized disc. Each flow region is $f_i = (O_i \oplus B_\rho) \setminus O_i$. If any $f_i$ intersects itself or a flow generating object, the environment is invalid and we return an error.

B. Inflated Flow Regions: The inflated flow regions are constructed the same as the initial flow regions, but using a $1.5 * B_\rho$-sized disc. If two inflated flow regions overlap, generating initial flow regions without modification creates a subset of $\Omega$ too narrow to hold a robot. Instead, we split the inflated region overlaps along their central axis and add each divided-up area onto their corresponding initial flow regions.

C. Single-lane Regions: Anywhere that two initial flow regions overlap is a subset of the flow space too narrow to hold two robots: both initial flow regions cannot exist in the same space. We construct single-lane regions by joining the two generating objects at the ends of each overlap and creating a new flow region between these bounds.

D. Passage Regions: We place a passage region (per Section IV) on both ends of each single-lane region and at each point where three or more flow regions meet. This (1) ensures that transitioning for single-lane regions is controlled by space requests and (2) covers “sharp corners” in flow regions.

E. Smoothing: Convolve a $B_\rho$-sized disc with the inner perimeter of the open space (i.e. $\mathcal{F} \setminus \Phi$). Anywhere that this disc is in contact with the inner perimeter at two points, we expand the corresponding flow regions out to meet the disc. Smoothing ensures that no reachable point in $\mathcal{F}$ is made unreachable by the flow/open space partition.

F. Direction Assignment: We assign directions to each flow region by winding either clockwise or counterclockwise around the centroid of its generating object (for single-lane regions, we arbitrarily choose one of the generating objects). We then return the flow, single-lane, and passage regions.

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