Optimised Traffic Flow at a Single Intersection: Traffic Responsive signalisation

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We propose a stochastic model for the intersection of two urban streets. The vehicular traffic at the intersection is controlled by a set of traffic lights which can be operated subject to fixed-time as well as traffic adaptive schemes. Vehicular dynamics is simulated within the framework of the probabilistic cellular automata and the delay experienced by the traffic at each individual street is evaluated for specified time intervals. Minimising the total delay of both streets gives rise to the optimum signalisation of traffic lights. We propose some traffic responsive signalisation algorithms which are based on the concept of cut-off queue length and cut-off density.

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I. INTRODUCTION

Modelled as a system of interacting particles driven far from equilibrium, vehicular traffic provides the possibility to study various aspects of truly non-equilibrium systems which are of current interest in statistical physics\(^1\text{-}^6\). For almost half century, physicists have been challenged to understand the fundamental principles governing the vehicular flow\(^4\text{-}^7\). Recently, discrete models such as cellular automata have provided a significant theoretical framework for the discipline of traffic flow modelling. The first cellular automata, the so-called BML model, was introduced by Biham, Middleton and Levine. This model described a simplified network of urban intersections\(^8\). Soon after, cellular automata, found their way in highway traffic through the pioneering work of Nagel and Schreckenberg\(^9\) which became the ancestor of many papers in the literature (for a review see ref.\(^1.2\) and the references therein). The BML model itself was later generalized to take into account several realistic features such as fragile traffic lights, independent turning of vehicles, and green-wave synchronization\(^10\text{-}^17\). The Nagel-Schreckenberg and BML model were recently combined to cast an upgraded version of city traffic models\(^18\). In a very recent paper, the model is now extended to account for different types of global signalisation\(^19\).

Despite the aforesaid efforts and those carried out by traffic engineers, the subject of optimal signalisation of realistic urban networks has not yet been comprehensively reviewed. In the above approaches, the main concern has been focused on the global strategies of the traffic network and frequently the role of isolated intersection have been suppressed. We believe that the optimisation of traffic flow at a single intersection is a substantial ingredient towards a global optimisation. Isolated intersections are fundamental operating units of the sophisticated and correlated urban network and thorough analysis of them would be advantageous toward the ultimate task of the global optimisation of the city network. In this regard, our objective in this paper is to analyze the traffic state of an isolated intersection in order to find a better insight into the problem. In addition to theoretical viewpoint, an investigation of isolated intersection could be of practical importance. To a very good approximation, marginal intersections in cities are unaffected by other intersections and can be regarded as isolated ones. Generally there are two basic types of control for traffic lights at intersections: fixed-cycle and traffic-responsive\(^20\text{-}^21\). Both of these methods can be implemented via centralized or decentralized strategies. The application of each method strongly depends on traffic condition and the topology of the city network. In this paper we study the basic features of traffic flow at a single intersection which is controlled both under fixed-cycle as well as traffic responsive schemes.

II. FORMULATION OF THE MODEL

An isolated intersection is formed at the intersection of two streets. The streets, in principle, can each carry two opposite flows of vehicles. Depending on the design of the intersection, different phases of movement can be defined (a phase of traffic is defined as the flow of vehicles that proceed an intersection without conflict). Here for simplicity we restrict ourselves to the simplest structure: a one-way to one-way intersection. With no loss of generality, we take the direction of the flow in the first street, hereafter referred to as the street A, northwards. The other street (hereafter referred to as street B) conducts a one-way eastward flow. Cars arrive at the south and west entrances of the intersection. Space and time are discretized in such a way that each chain is divided into cells which are the same size as a typical car length. Time is assumed to elapse in discrete steps. We take the number of cells to be \(L\) for both roads. Each cell can be either occupied by a car or being empty. Moreover, each car can take discrete-valued velocities \(1, 2, \ldots, v_{\text{max}}\). To be more specific, at each step of time, the system is characterized by the position and velocity configurations of cars and the traffic light state at each road.
The system evolves under a generalized discrete-time Nagel-Schreckenberg (NS) dynamics. The generalized model incorporates anticipation effects of driving habits. It modifies the standard NS model at its second step i.e., adjusting the velocities according to the space gap. Let us briefly explain the updating rules which are synchronously applied to all the vehicles. We denote the position, velocity and space gap (distance to its leading car) of a typical car at discrete time $t$ by $x(t)$, $v(t)$ and $g(t)$ and the same quantities for its leading car by $x(t)$, $v(t)$ and $g(t)$. Assuming that the expected velocity of the leading car, anticipated by its follower, in the next time step $t+1$ takes the form $v_{\text{anti}}^{(t)} = \min(g_{l}^{(t)}, v_{l}^{(t)})$, we define the effective gap as $g_{\text{eff}}^{(t)} = g^{(t)} + \max(v_{\text{anti}}^{(t)} - \text{gap}_{\text{secure}}, 0)$ in which $\text{gap}_{\text{secure}}$ is the minimal security gap. Concerning the above considerations, the following updating steps evolves the position and the velocity of each car.

1) Acceleration:
\[ v(t^{+1/3}) := \min(v(t) + 1, v_{\text{max}}) \]

2) Velocity adjustment:
\[ v(t^{+2/3}) := \min(g_{\text{eff}}^{(t^{+1/3})}, v(t^{+1/3})) \]

3) Random breaking with probability $p$:
\[
\text{if random } < p \text{ then } v(t^{+1}) := \max(v(t^{+2/3}) - 1, 0) \]

4) Movement:
\[ x(t^{+1}) := x(t) + v(t^{+1}) \]

Let us now specify the physical value of our time and space units. Ignoring the possibility of existence of long vehicles such as buses, trucks etc, the length of each cell is taken to be as 5.6 metres which is the typical bumper-to-bumper distance of cars in a waiting queue. Concerning the fact that in most of urban areas a speed-limit of 60 kilometre/hour should be kept by the drivers, we quantify the time step in such a way that $v_{\text{max}} = 6$ corresponds to the speed-limit value (taken as 60 km/h). In this regard, each time step equals two seconds and therefore each discrete increments of velocity signifies a value of 10km/h which is equivalent to a comfortable acceleration of $1.4 \text{ m/s}^2$. We have also set the horizon length $L = 70$ cells and $\text{gap}_{\text{sec}} = 1$. The state of the system at time $t+1$ is updated from that in time $t$ by applying the modified NS dynamical rules.

**Step 1 : signal determination.**

We first specify the signal states for all of the driving directions. In subsequent sections we will, in detail, explain the scheme at which the traffic lights change their colour.

**Step 2 : movement in the green road.**

At this stage, we update the position and velocities of cars on the green road according to the movement rules which are synchronously applied to each car.

**Step 3 : movement at the red road, delay evaluation.**

Here the updating is divided into two parts. In the first part, we evaluate the delay of cars waiting on the red period of the signalisation. In the second half, we update the position and velocities of the moving cars approaching the waiting queue. We should note that once the signal switches to red, the moving cars continue their movement until they come to a complete stop when reaching to the end of the waiting queue. As soon as a car comes to halt, it contributes to the total delay. In order to evaluate the delay, we measure the queue length (the number of stopped cars) at time step $t$ and denote it by the variable $Q$. We recall that $\text{pos}^{\text{arr}}[i, t] = 1$ for $i = 1 \cdot \cdot \cdot Q$ and zero at $t = Q + 1$. Delay at time step $t+1$ is obtained by adding the queue length $Q$ to the delay at time step $t$.

\[ delay(t+1) = delay(t) + Q(t) \quad (1) \]

This ensures that during the next time step, all the stopped cars contribute one step of time to the delay. The next part describes the update of positions and velocities of moving cars. Moving cars can potentially be found in the cells $Q + 2, Q + 3, \cdot \cdot \cdot, L$. We update their positions and velocities accordingly.

**step 4 : entrance of cars to the intersection.**

So far, we have dealt with those cars within the horizon of the intersection which goes up to the entry point located at site $L$. Here we discuss the entrance of cars into the intersection. Generally speaking, during each green period, a fraction of the queue will dissolve and
pass the intersection. If the average arrival rate of cars exceeds the maximum evacuation rate of the lane, then, on an average level, a fraction of a generic queue will not be able to go through the intersection and should wait until the next upcoming green period arrives. Correspondingly in the course of time, the remainders accumulate giving rise to a growing queue length. In reality we rarely observe this phenomena since an actual intersection is linked to other ones hence the possibility of such a rare event is restricted to very exceptional cases where there is an overwhelming large amount of incoming flux. Throughout the paper, we assume that the average inflow rate is sufficiently below this evacuation rate so that a generic queue will have enough green time to dissolve and that the intersection is able to support the incoming flux. In this case, there is a typical queue length which affects the motion of incoming cars. The motion of a car approaching the red light is affected via two factors: the distance to the end of the queue and the traffic light signal which acts as a hindrance to the incoming traffic flow. In the light traffic state under consideration, there exist an interaction distance to the queue beyond which one can, to a good extent, assume that the cars are moving without being affected by the traffic light signal. Away from this interaction zone, the cars move according to the movement rules without any hindrance. We take the position of the place at which the cars enters the horizon of the intersection to be 70 cell equivalent to 400 metres. The time head-ways between entering cars at this entry location vary in a random manner which consequently implies a random distance headway between successive entering cars. As a candidate for describing the statistical behaviour of random space gap of entering cars, we have chosen Poisson distribution. The Poisson distribution function has been used in a variety of phenomena incorporating the modelling of ”queue theories” and has proven to be a good estimation of reality. In addition, it has the merit of taking only discrete values which is desirable to us in the view of the fact that in our model the gap is a discrete variable. According to this distribution function the probability that the space gap between the car entering the intersection horizon and its predecessor be $n$ is: $p(n) = \frac{\lambda^n e^{-\lambda}}{n!}$ where the parameter $\lambda$ specifies the average as well as the variance of distribution function. The parameter $\lambda$ is a direct measurement of traffic volume. A large value of $\lambda$ describes a light traffic while on the other hand, a small-valued $\lambda$ corresponds to a heavy traffic state. Now we let the cars enter into the intersection's horizon. At the end of the movement rule, we evaluate the position of farthest cars on both streets. We denote them by last$A$ and last$B$. By definition, $posA[j,t] = 0$ for $j >$ last$A$. A similar statement applies to street $B$. In order to simulate the entrance of cars into the horizon of the intersection, we randomly choose an integer weighted by Poisson distribution function. This number represents the gap of the oncoming successor of the farthest car. Let us denote these numbers (headways) by $h_A$ and $h_B$ for street $A$ and $B$ respectively. Once the random gap is chosen, we create a car at the position last$A + h_A$ (last$B + h_B$) of the street $A$ ($B$) respectively. The created car survives provided the following constraint is satisfied: $last_A + h_A \leq L$ (last$B + h_B \leq L$). If the position of the created car exceeds the horizon length $L$, the creation procedure is rejected. The above ad hoc rules updates the configuration of the intersection in time. In the next section, we will explain our simulation results.

III. SIGNALISATION OF TRAFFIC LIGHTS: FIXED TIME SCHEME

In this controlling scheme, the traffic flow is controlled by a set of traffic lights which are operated in a fixed-cycle manner. The lights periodically go green with a fixed period (cycle length) $T$. This period is divided into two parts: in the first part, the traffic light is green for street $A$ (simultaneously red for street $B$). This part lasts for $T_g$ seconds $(T_g < T)$. In the second part, the lights change colour and the movement is allowed for the cars of road $B$. The second part lasts from $T_g$ to $T$. This behaviour is repeated periodically. Cars enter the intersection and a fraction of them experience the red light and consequently have to wait until they are allowed to go through intersection during the upcoming green periods. Now we raise the basic question:

how should one adjust the ratio of $\frac{T_{green}}{T}$ in order to optimise the throughput flow?.

There is now an almost well-established agreement on the quantitative definition of optimisation. Borrowing from the traffic engineering literature, we adopt optimised traffic as a state in which the total delay of vehicles is minimum. In one of our preceding works, we analytically evaluated the total delay in terms of arrival as well as that of the exit rates of vehicles. However, our approach was based on the simple assumption of the time-constancy of the arrival rates. In reality, we know that successive cars arrive with fluctuating time-headways which consequently induces time-varying arrival rates. In this paper we address the question of non-constant rates. In order to evaluate the delay, we have simulated the flow of vehicles. For the sake of simplicity, we have assumed that each street has a single lane. For streets with more than one lane, one simply should multiply the value of delay by the number of lanes.

A. Simulation Results: Symmetric inflow

We let the intersection evolve for 1800 time steps which is equal to a real time period of one hour. We let the green time of street $A$, $T_g$, vary from zero to $T$. For
each value of $T_g$, we evaluate the aggregate delay corresponding to both traffic lights during 1800 time steps. We have averaged the results of 50 independent runs of the programme. Let us first consider the symmetric traffic states in which the traffic conditions are equal for both roads. In this case, we equally load the intersection with entering cars spatially separated by random space gap (Poisson statistics) from each other. The following graphs depict the total delay curves as a function of $T_g$ allocated to road A for two values of cycle length.

![Total delay versus green time](image)

**Fig. 2 and 3:** The cycle length is 40 seconds (top graph) and 60 seconds (bottom graph). Total delay versus the green time of road A is sketched for various in-flow rates.

The general shape of the total delay curve resembles a "U-shaped" valley. The middle area, where the total waiting time is minimised, corresponds to a situation in which the evacuation rate of roads exceeds the in-flow and the queues can be dissolved during one green period and consequently all the waiting cars will be able to go through the intersection in the upcoming green period. The optimal traffic flow is obtained by keeping the lights at equal timing $T_g = \frac{T}{2}$. Before proceeding further, it would be useful to discuss, in detail, the conditions in which the intersection would not be able to support the influx and hence the queue length starts growing. Supposing the fraction $\frac{T_g}{T}$ of the cycle is given as green time to a street say A. Therefore during the one hour time interval, street A receives $3600 \frac{T_g}{T}$ seconds of green times. Using the fact that in green phase vehicles are going through the crossing with the approximate rate of 0.5/sec/lane, on an average level, the maximum out-flow capacity, denoted by $\langle C_{\text{max}} \rangle$ would be: $\langle C_{\text{max}} \rangle = 1800 \frac{T_g}{T}$. On the other hand, total number of in-flow can be estimated once the average space gap is given. To do so, we first obtain the average time headway of entering vehicle (in second), denoted by $\Delta \lambda_A^*$, as follows $\Delta \lambda_A^* = \frac{5.6}{v_{\text{max}}}$. In the above formula, we have assumed that entering cars have the maximum velocity and 5.6 denotes the cell length in metre. Concerning the above consideration, during one hour, the average number of entering cars are approximated by $\langle N_{in} \rangle = \frac{3600 \lambda_A^*}{\Delta \lambda_A^*}$. Avoiding the occurrence of growing queue, leads to the satisfaction of the following constraint $^{24}$:

$$\langle N_{in} \rangle \leq \langle C_{\text{max}} \rangle \quad (2)$$

Substituting maximum velocity by 16.7 m/s in the above constraint, gives rise to condition $T_g \geq \frac{6T}{\lambda_A}$. Taking $T = 60$ seconds, the minimum consistent value of $T_g$ for $\lambda = 13, 16$ and 19 cells would be 27, 22 and 19 seconds respectively. Alternatively, the lowest $\lambda_A$ is restricted to 12 cells or 67 metres. Similar arguments should be applied to street B. Concerning the fact that the green time of street B is $T - T_g$, one simply deduce that the condition $T - T_g \geq \frac{6T}{\lambda_B}$ should hold. Combining the two conditions on $T_g$, one arrives at the following inequality on $T_g$ for a stationary condition of the queues.

$$\frac{6T}{\lambda_A} \leq T_g \leq T(1 - \frac{6}{\lambda_B}) \quad (3)$$

Consequently, the allowed $\lambda_A, \lambda_B$ and $T_g$ should satisfy the above inequality, otherwise the queue will grow infinitely in time. From the above relation one concludes two necessary conditions on the rates: $\lambda_B \geq 6$ and $\frac{\lambda_A}{3} + \frac{1}{\lambda_B} \leq \frac{1}{6}$ which hold regardless of the value of green time. We note that the above arguments are based on simple mean-field approximation and no fluctuation has been taken into account. There are two origins of fluctuation, the first one concerns the stochastic space gap of cars which make the incoming flux deviated from a constant one, and second, the interaction among cars and randomness arising from the car movements. Therefore the region close to critical volume $\lambda_A = 12$ needs special treatment. Simulation results shows that away from the critical region, fluctuations do not violate our mean-field conclusions.
B. Asymmetric in-flow rates

In the asymmetric states, the entering rates into the roads differ from each other. An interesting asymmetric state is the intersection of a major to a minor street. A large fraction of urban intersections lies in the category of major-to-minor. The signalisation of these types of crossing is still a controversial subject. The main reason is that in most of intelligent real-time controller systems, the signalisation of these intersections are highly affected by the major intersections signalisation schemes which frequently overlook the local optimisation of minor intersections. In our model, a major-to-minor crossing is modelled by a light traffic in one road and a congested one in the other road. The following graph depicts the behaviour of the delay curve at a major-to-minor intersection. As observed, the minima of delay curves are shifted toward right which expresses the fact that the majority of green times should be given to the road with higher in-flow rate. Analytical considerations lead to distribution of green times in proportion to the in-flow rates of the roads\(^{24}\). To be more specific, let us denote the average arrival rate at street A and B by \(\alpha_A\) and \(\alpha_B\). Note that \(\alpha_A\) and \(\alpha_B\) are proportional to inverse of entering space gaps. We now evaluate the waiting time of street A during one complete cycle. The number of cars arriving the queue during the time interval \([t, t + dt]\) in the red period, which lasts \(T - T_g\) seconds, is \(\alpha_A dt\). These cars should wait \(T - T_g - t\) seconds until the upcoming green period starts. Therefore, their contribution to delay reads \(\alpha_A dt(T - T_g - t)\). The aggregate delay of street A during the cycle is obtained by summation of infinitesimal delays as follows:

\[
\int_0^{T-T_g} \alpha_A dt(T - T_g - t) = \frac{1}{2} \alpha_A (T - T_g)^2 \tag{4}
\]

Similarly the contribution of street B to total delay can simply be evaluated as \(\frac{1}{2} \alpha_B T_g^2\). Adding the street delays together and minimising it with respect to \(T_g\) gives rise to the optimal \(T_g^{opt}\):

\[
T_g^{opt} = \frac{\alpha_A}{\alpha_A + \alpha_B} T \tag{5}
\]

The above result corresponds to distribution of green times in the ratio of in-flow rates. Our simulations results are in good agreement to this conclusion. In fact the position of optimal green times, i.e., the minima of the delay curves, grow linearly with the linear decrease of in-flow rates which are in turn proportional to the inverse of average space gap.

Fig. 4: Total delay for asymmetric traffic volumes in a major-to-minor crossroads. The space gap of the major road is set to 13. The space gap of the minor road is varied. \(T=60\) seconds.

IV. SIGNALISATION OF TRAFFIC LIGHTS: TRAFFIC ADAPTIVE

We now discuss the traffic adaptive controlling scheme in which the light signalisation is adapted to the traffic at the intersection. Nowadays advanced traffic control systems anticipate the traffic approaching intersections. Traffic-responsive methods have shown a very good performance in controlling the city traffic and now a variety of schemes exists in the literature\(^{20,21,25,26}\). In these schemes, the data obtain via traffic detectors installed at the intersection is gathered for each movement direction and it is possible to count the queue lengths formed behind the red lights. One can also measure the time-headways between successive cars passing each lane detector. Thus it is possible to estimate the traffic volume existing at the intersection. There are various methods for distribution of green times. In what follows we try to explain some standard ones. In each scheme, the green time of a typical green street is terminated if some conditions are fulfilled. By green (red) street we mean the street for which the traffic light is green (red). We now state the termination conditions for each scheme.

Scheme (1): The queue length in the conflicting direction exceeds a cut-off value \(L_c\). This scheme only concerns the traffic states in the red street.

Scheme (2): The global car density in the green street falls below the cut-off value \(\rho_c\). Here the algorithm only considers the traffic state in the green street.
Scheme (3): Each direction is endowed with two control parameters \( L_c \) and \( \rho_c \). The green phase is terminated if the conditions: \( \rho^g \leq \rho_c \) and \( L^r \geq L_c \) are both satisfied.

Here the algorithm implements the traffic states in both streets. The superscripts ”r” and ”g” refer to words ”red” and ”green” respectively. We note that the first two schemes are special cases of the more general scheme (3). Schemes (1) and (2) are the limiting cases of schemes (3) by letting \( \rho_c \rightarrow 1 \) and \( L_c \rightarrow 0 \) respectively. In general, the numerical value of control parameters \( L_c \) and \( \rho_c \) could be taken different for each individual street. In what follows we present our simulation results for different types of signalisation schemes introduced above.

A. Simulation results: symmetric inflow

We let the intersection evolve for 1800 time steps which is equal to a real time period of one hour. We evaluate the aggregate delay for both streets as well the number of passed vehicle during 1800 time steps at each direction for different traffic situations. As the first situation, we have considered the symmetric traffic state in which the traffic conditions are equal for two streets. In this case, we equally load the streets with entering cars spatially separated by random space gap, obeying Poisson statistics, from each other. We first discuss the first scheme in which the lights change colour as soon as the queue length reaches the cut-off length. We set an equal cut-off length for both streets. The following graph depicts the total delay curve as a function of traffic volume for various cut-off lengths.

![Fig. 5: total delay in terms of average space gap of entering cars for some cut-off queue length \( L_c = 5, 6, 7, 8 \).](image)

We observe that for each cut-off length, total delay shows a slight decrease with respect to the in-flow rate. The graph demonstrate that the shorter the cut-off length, the less the total delay. The next graph shows the delay versus in-flow rate when the lights are signalised according to scheme (2). Here the green light is terminated below a certain occupancy in the corresponding moving flow.

![Fig. 6: total delay in terms of average space gap of entering cars for some cut-off densities.](image)

We observe that the first scheme gives rise to lower delay. The reason is that in relatively low traffic volume, more cars have to stop in the red light in order that the global density in the green direction reaches to its cut-off value. This raises the delay compared to scheme (1). Finally we discuss the third scheme in which two conditions should be satisfied for termination of green light. In this scheme both cut-off densities and cut-off lengths are taking into account. We have investigated the case where cut-off parameters are equal for both streets i.e., \( \rho_c^A = \rho_c^B = \rho_c \) and \( L_c^{(A)} = L_c^{(B)} = L_c \). The following table shows the delay table of one-hour performance for various values of \( \rho_c \) and \( L_c \). Traffic in-flow rates are \( \lambda_A = \lambda_B = 13 \).

| \( \rho_c \) | \( L_c \rightarrow 5 \) | \( 6 \) | \( 7 \) | \( 8 \) | \( 9 \) |
|---|---|---|---|---|---|
| 0.10 | 14000 | 15500 | 15900 | 17900 | 18600 |
| 0.20 | 13200 | 14900 | 14700 | 16500 | 17400 |
| 0.30 | 11500 | 12800 | 13200 | 15500 | 16400 |
| 0.40 | 11200 | 11900 | 12600 | 14000 | 15300 |
| 0.50 | 10900 | 11200 | 11700 | 12700 | 13900 |

The above result sets the optimal parameters at \( \rho_c = 0.5 \) and \( L_c = 5 \). We have tested a variety of other traffic volumes. The results demonstrate that in the control parameter space, the optimal region lies in high \( \rho_c \) and low \( L_c \). The above result shows that scheme (3) gives an improved delay with respect to scheme (2). Our simulation results shows that this conclusion can be made for other values of \( \lambda \) hence the third scheme is more optimal than the second one. However the results show that...
still the first scheme gives lower delays compared scheme (3). Specifically for $\lambda = 20$ cells, the predicted delays are 8500, 10100 and 9600 seconds respectively. The next graphs depict the behaviour of consecutive green times of streets. We have chosen the green times of street A. Fluctuating traffic volume at streets induces fluctuating green periods which are known as the green time plans in the traffic engineering literature\textsuperscript{21,26}. We note that a realistic optimising algorithm must satisfy the constraint that the duration of green phases should not be shorter that a minimal amount e.g. 10 sec or so. This minimal value reflects the actual time required for an immobile waiting queue to accelerate and make a considerable movement forward. Our simulation results shows that $L_c \geq 4$ corresponds to typical green periods greater than 12 seconds. It may be useful to analyze the statistics of green plans. Denoting the consecutive green times by $T_g^{(1)}, T_g^{(2)}, T_g^{(3)} \ldots$, We first obtain the basic statistical properties which are the moments of the distribution of the green times. The bottom figure shows the green plan histogram of street A for $\lambda_A = \lambda_B = 13$. For the mentioned traffic state, the average green time and the standard deviation of street A are 18 and 2.54 seconds respectively.

![Green times histogram of street A. $\lambda_A = \lambda_B = 13$ cells. The lights are controlled via scheme (1). The cut-off length is taken as 7 cars.](image)

B. asymmetric in-flow

scheme (1):

We now investigate the situations where the streets are not equally loaded. For this purpose, we fix the in-flow rate of street A at $\lambda_A = 13$ and vary the in-flow rate of street B. The following graph exhibits the total waiting time curves as well as those of each street for some cut-off lengths (taken equal for both streets). Here again the less delay is achieved for shorter cut-off lengths which in turn correspond to shorter green phases.

![Fig. 8: Total delay versus the inverse traffic volume of street B for various cut-off lengths. $\lambda_A$ is fixed at 13.](image)

We observe that decreasing the traffic volume in street B leads to a decrease in total delay as expected. In the following figure, the dependence of each street delay on the traffic volume of street B is exhibited in details. One observes that the delay of street B starts growing whereas that of street A decreases. This shows that the algorithm does not always act optimally for individual directions. In fact, the delay of street A, which is more congested than street B, behaves decreasingly while the delay of the less congested street, B, behaves increasingly. Nevertheless, the algorithm acts in an optimal manner when taking into account the whole intersection.

![Fig. 9: Delay of individual streets as a function of inverse traffic volume of street B. Traffic volume of street A is kept](image)
We have also examined the cases where cut-off lengths take different values for streets with non-equal incoming flux. Simulation results shows that optimal cut-off lengths should be taken equal and as short as possible. For instance, in the case $\lambda_A = 14$ and $\lambda_B = 24$ the minimised total delay when $L^c_A = L^c_B = 5$ is 8500 seconds.

scheme (2):

Let us now investigate the predictions of scheme (2). As explained, in this method, once the global density of the moving direction (in the vicinity of the intersection) falls below the cut-off density, green phase is terminated irrespective of the queue length in the red direction. The global density is obtained by measuring the occupancy of $L_\rho$ cells before the crossing point. The following graphs shows the one-hour delay behaviour of individual streets as well as that of the entire intersection for various controlling densities. Analogous to the predictions of first scheme, here the algorithm increases the delay in the minor road whereas the delay is decreased in the major road upon decreasing the traffic volume in the minor road.

![Graphs showing one-hour delay behaviour of individual streets and the entire intersection](image)

Fig. 10: Top: delay of individual streets as a function of inverse traffic volume of the minor street B. Traffic volume of street A (major street) is kept fix $\lambda_A = 13$ cells. Bottom: total delay versus the inverse traffic volume in the minor street for various controlling densities.

One observes that the delay predictions of the second scheme are higher than those of the first scheme. We have also examined the cases where cut-off densities of each street take non-equal values. The following table exhibits the values of one-hour total delay for various amounts of cut-off densities of each street. Traffic volumes correspond to $\lambda_A = 13$ and $\lambda_B = 18$ and $L_\rho$ is set to 10 cells.

| $\rho_B$ | $\rho_A$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|---------|---------|------|------|------|------|------|
| 0.10    | 14800   | 17100| 21000| 20900| 21100|
| 0.20    | 13700   | 15400| 15600| 16700| 16500|
| 0.30    | 12900   | 13300| 13700| 14400| 14600|
| 0.40    | 11400   | 12100| 13200| 14200| 14000|
| 0.50    | 10800   | 11300| 11800| 12400| 12900|
| 0.60    | 10600   | 10800| 10700| 10900| 11000|

We observe that optimal flow corresponds to set $\rho_A = 0.1$ and $\rho_B = 0.6$. In contrast to scheme (1) where optimal cut-off lengths are equal to each other, here we see that in asymmetric traffic volume, the optimal cut-off densities are non-equal. We note that other values of cut-off densities give rise to unrealistic green time plans and hence are excluded from the table. It would be useful to compare the minimal delay with the predicted one in the scheme (1). For the traffic volumes corresponding to $\lambda_A = 13$ and $\lambda_B = 18$, scheme (2) gives the minimal delay at 10600 seconds whereas the scheme (1) predicts the value 8700 seconds for $L_c = 5$. We have also obtained the total delay table for another asymmetric traffic volume where in-flow rates are $\lambda_A = 13$ and $\lambda_B = 25$. For this choice of traffic volumes, the optimal cut-off densities are $\rho_A = 0.10$ and $\rho_B = 0.40$ which are relatively close to the preceding optimal values. The values of minimal delays are 9300 and 8500 seconds in scheme (2) and (1) respectively. We have investigated a variety of traffic states corresponding to different in-flow rates. The results show that scheme (1) gives better optimal states than scheme (2). Before discussing scheme (3), we would like to discuss the role of parameter $L_\rho$. In fact, the measurement of global density depends on the length $L_\rho$. This length itself could be regarded as a control parameter. We have carried out simulations for various values of $L_\rho$. Our results shows that the $L_\rho = 10$ corresponds to the best choice. For instance the result obtained for the same traffic state investigated in the above table and $L_\rho = 12$ gives the optimal delay as 11000 seconds which is higher than the value predicted by $L_\rho = 10$.

scheme (3):

Finally we discuss the third scheme. In this scheme both cut-off densities and cut-off lengths are taking into account. The following graphs shows the one-hour delay behaviour of individual streets as well as that of the
entire intersection for various controlling densities. Analogous to the predictions of first and the second schemes, here the algorithm increases the delay in the minor road whereas the delay is decreased in the major road upon decreasing the traffic volume in the minor road. We have taken the cut-off parameters equal for both streets i.e., \( L_c^A = L_c^B \) and \( \rho_c^A = \rho_c^B \).

The table above shows that optimal traffic flow is obtained in the high-high region of \((\rho_c^A, \rho_c^B)\) sub-space of control parameters. The minimised delay is 9900 seconds. For the same in-flow rates, the values of minimal delays predicted by the schemes (2) and (1) are 10600 and 8700 seconds respectively. This comparison shows that the third scheme acts better than scheme (2) but is less efficient than the first scheme. Further simulation results carried out for other traffic states demonstrates that for \( \lambda_B \leq 19 \) (\( \lambda_A \) fixed) scheme (3) acts better than scheme (2) but is less efficient than scheme (2) for higher in-flow rate of the minor street. Nevertheless, scheme (1) gives the least delay for all asymmetric states.

V. SUMMARY AND CONCLUDING REMARKS

In this paper we have investigated the optimisation of vehicular traffic flow at an isolated signalised intersection in the framework of cellular automata. Marginal urban areas are places where single intersections are frequently designed and operated. Investigations on single junctions can be of practical relevance for various applications in city traffic. Basically there are two methods of signalisation of the traffic lights: fixed-time or adaptive. In traffic adaptive strategies, it has always been a subject of argument whether to control an intersection under a centralized or decentralized scheme. In special circumstances, decentralized local adaptive strategies operate more effectively than globally adaptive strategies and often show a very good performance. For this purpose, we have developed and analysed prescriptions for the traffic light signalisation at single intersections. Our simulation results confirm that in fixed-time scheme, green times should be distributed proportional to the traffic in-flow rates. We have simulated and analysed three traffic responsive signalisation schemes. The signalisation mechanisms are based on the concepts of cut-off queue lengths and densities. Two major conclusion can be made from our simulation results. First, the traffic adaptive scheme act more optimally than the fixed-time. Secondly, the best traffic adaptive scheme is the one in which the flow in the green direction is terminated as soon as the queue length in the opposite direction exceeds the cut-off value \( L_c \). In this method, the traffic states in the green direction are not taken into account. It should be noted that the results in this paper have been obtained under a modified version of Nagel-Schreckenberg model. Whether these result are robust against more realistic vehicle movement models needs more exploration and is the subject of our current investigation. Throughout the paper we have used the Poisson statistics for the headway of approaching cars. However the realistic in-flow traffic would certainly deviate from this statistics. Investigations on the impact of the entrance statistics on the above results will shed more lights onto the problem.

| \( \rho_c^A \) | \( \rho_c^B \) | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|-------------|-------------|-----|-----|-----|-----|-----|
| 0.10        | 12560       | 11600 | 11300 | 11000 | 10800 |
| 0.20        | 12240       | 11370 | 10950 | 10600 | 10500 |
| 0.30        | 12050       | 11200 | 10560 | 10300 | 9960  |
| 0.40        | 12160       | 11150 | 10470 | 10200 | 9980  |
| 0.50        | 11950       | 10760 | 10350 | 9970  | 9900  |

Fig. 11: Top: delay of individual streets as a function of traffic volume of the minor street B. Traffic volume of street A (major street) is kept fix \( \lambda_A = 13 \) cells. Bottom: total delay versus the traffic volume in the minor street for various controlling densities. Cut-off densities are taken 0.5 for both streets.

Another investigation considers the effect of cut-off densities for fixed values of cut-off lengths. The following table exhibits the delay when \( \lambda_A = 13 \) and \( \lambda_B = 18 \). \( L_c = 5 \) while \( \rho_c \) is varied.
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