Abstract

It was recently shown by the authors that the Fritzsch ansatz for the quark mass matrices prescribed at the supersymmetric grand unified scale is compatible with a moderately heavy top quark ($m_t \simeq 120-150 \text{ GeV}$). Here we extend the ansatz to incorporate the charged leptons and the neutrinos. It is found that the $\nu_e - \nu_\mu$ mixing angle is small and consistent with the MSW solution of the solar neutrino puzzle. Furthermore, the model predicts observable $\nu_\mu - \nu_\tau$ oscillations with $\sin^2 2\theta_{\mu\tau} \simeq 0.1$ and $\nu_\tau$ mass in the $(1-3) \text{ eV}$ range.

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Employing symmetries to constrain the form of quark and lepton mass matrices is an attractive concept, since it often leads to relations involving the otherwise arbitrary fermionic observables (masses, mixing angles and CP-violating phases). A particularly simple and elegant texture for the quark mass matrices was proposed some time ago for three families of fermions by Fritzsch.\textsuperscript{1} Only the heaviest (top) family has a direct mass in this scheme, while the lighter family masses are generated via nearest neighbor mixing. Generalizing to include the charged leptons, the mass matrices $M_{u,d,\ell}$ for the up–quarks, down–quarks and charged–leptons are given by

$$ M_{u,d,\ell} = P_{u,d,\ell} \begin{pmatrix} 0 & a_{u,d,\ell} & 0 \\ a_{u,d,\ell} & 0 & b_{u,d,\ell} \\ 0 & b_{u,d,\ell} & c_{u,d,\ell} \end{pmatrix} Q_{u,d,\ell}. $$ (1)

Here $P_{u,d,\ell}$ and $Q_{u,d,\ell}$ are diagonal phase matrices and $a, b, c$ are real (positive) quantities. The zero’s of these matrices are enforced by certain symmetries (either discrete or continuous). Parity invariance is used to ensure the symmetrical nature of the magnitudes of the various elements. Models which generate such textures include those with left–right symmetry,\textsuperscript{1,2} as well as $SO(10)$ where parity is a spontaneously broken symmetry.\textsuperscript{3}

In the quark sector, the Fritzsch ansatz leads to predictions for the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing angles in terms of the quark mass ratios and two phase parameters (denoted by $\sigma$ and $\tau$). Even though the original mass matrices have several more phase degrees of freedom, only the two are observable. All others can be rotated away by redefining the fermion fields. Of special concern is the prediction of the Fritzsch
ansatz for $|V_{cb}|$, which takes the form

$$|V_{cb}^0| = \left| \sqrt{\frac{m_0^b}{m_b^0}} - e^{i(\sigma - \tau)} \sqrt{\frac{m_0^c}{m_t^0}} \right|.$$  \hspace{1cm} (2)

Here the superscript on the quantities is used to emphasize the fact that the relation holds at whatever scale the Fritzsch texture holds. A natural scale at which the symmetries of the Fritzsch ansatz may be broken is the supersymmetric grand unification (SUSY GUT) scale $M_G \simeq 10^{16}$ GeV. Implications of eq. (2) at low energies are then to be evaluated by evolving the quark mass ratios and the mixing angles from $M_G$ to the weak scale, using the renormalization group equations.

If eq. (2) holds at the weak scale, it can be verified that the experimental value of $|V_{cb}| = (0.043 \pm 0.009)$ sets an upper limit of about 90 GeV on the top quark mass.\(^4\) This comes about since the first term on the right-hand side in eq. (2) is at least 0.15, and so needs a large cancellation from the second term to agree with the observed value of $|V_{cb}|$. Such a low value of $m_t$ is on the verge of being excluded by the CDF search for the top quark, which sets a lower limit on $m_t$ of 91 GeV.\(^5\) It also is in conflict with estimates from the one-loop radiative corrections within the standard model, which prefers a moderately heavy top, say in the range of $(120 - 160)$ GeV.\(^6\) Could one therefore conclude that the ansatz of eq. (1) has been excluded by experiments?

In a recent paper\(^7\) we have shown that if the Fritzsch ansatz is prescribed at a supersymmetric grand unified scale $M_G \simeq 10^{16}$ GeV, relation (2) can in fact lead to a low energy prediction for $|V_{cb}|$ which is consistent with observations even for a moderately heavy top quark ($m_t \lesssim 150$ GeV). (Throughout
this paper we shall denote by $m_t$ the running mass $m_t(m_t)$, which is related to the pole mass by $m_t^{\text{pole}} = m_t(m_t)[1 + 4\alpha_s(m_t)/(3\pi)]$. For large values of $\tan\beta$ (the ratio of the vacuum expectation values of the two higgs doublets which SUSY mandates) relation (2) renormalizes in a desirable way making it consistent with observations. For example, the renormalized relation for $|V_{cb}|$ at the weak scale reads (for $m_t = 140$ GeV and $\tan\beta = 60$)

$$|V_{cb}| = 0.89 \sqrt{\frac{m_s}{m_b}} - 1.12 e^{i(\sigma-\tau)} \sqrt{\frac{m_c}{m_t}}.$$  \hspace{1cm} (3)

(Quantities without the superscripts refer to their weak scale values.) With an optimal choice of $m_s(1 \text{ GeV}) = 120 \text{ MeV}$, $m_b(m_b) = 4.35 \text{ GeV}$, $m_c(m_c) = 1.32 \text{ GeV}$ and $\sigma = \tau$, we find that $|V_{cb}| = 0.046$, in agreement with observations. (Without the renormalization factors, the value for $|V_{cb}|$ corresponding to the same input numbers would have been $\simeq 0.073$!)

Another testable prediction of the Fritzsch ansatz for quarks is for the ratio\(^7\) $|V_{ub}|/|V_{cb}| \simeq \sqrt{m_u/m_c} \simeq 0.06$ which is in good agreement with the recent CLEO-II data on charmless $B$–meson decay. The relation for the Cabibbo angle, $|V_{us}| \simeq |\sqrt{m_d/m_s} - e^{i\sigma} \sqrt{m_u/m_c}|$, fixes the phase $\sigma$ to be $\simeq \pi/2$.

The purpose of this paper is to extend the Fritzsch ansatz to the charged lepton and neutrino sectors. We are motivated by the success of the ansatz in the quark sector when prescribed within a SUSY GUT framework: not only does it predict an acceptable value for $|V_{cb}|$, in the process it also fixes the important parameter $\tan\beta$ to be large, close to 60. (This value corresponds to the infra–red fixed point solution for the $b$-quark Yukawa coupling $h_b$.\(^7\)) An extension of the ansatz to the lepton sector can lead to predictions for the down–quark masses in terms of the charged leptons as well as determine
some of the neutrino oscillation parameters. The $\nu_\mu - \nu_\tau$ system turns out to be a particularly interesting case.

We envision the matrices of eq. (1) as arising from an underlying SUSY $SO(10)$ type grand unification. It is well known that the quark–lepton symmetry embedded in $SO(10)$ enables one to relate the quark masses with those of leptons. For the charged leptons, we will assume that the mass matrix has the Fritzsch form as in eq. (1), with the (33), (12) and (21) elements arising from a 10–plet of Higgs. Thus we identify

$$a_\ell = a_d, \quad c_\ell = c_d. \quad (4)$$

The (23) and (32) elements, on the other hand, should receive contributions from a 10 as well as a 126 of Higgs. If only a 10 contributed to the (23) and (32) entries, one would have the asymptotic relation $m_\mu^0 = m_\tau^0$, which, after the renormalization group corrections, is phenomenologically unacceptable. Similarly, if only a 126 contributed to the (23) and (32) elements, $m_\mu^0 = 9m_s^0$ will follow, which again is unacceptable.

Relations (4) lead to two successful asymptotic predictions given by

$$m_b^0 - m_s^0 + m_d^0 = m_\tau^0 - m_\mu^0 + m_e^0, \quad m_d^0 m_s^0 m_b^0 = m_e^0 m_\mu^0 m_\tau^0. \quad (5)$$

For $m_t = 130 \text{ GeV}$ and $\tan\beta = 60$, the first of these relations predicts $m_b(m_b) \simeq 4.2 \text{ GeV}$ which is in good agreement with the spectroscopic determinations. The second relation leads to $m_d(1 \text{ GeV}) \simeq 7 \text{ MeV}$ (if $m_s(1 \text{ GeV}) = 140 \text{ MeV}$), also in good agreement with observations. Note that the relations in eq. (5) are identical to two of the predictions of the Georgi–Jarlskog ansatz.10
Before discussing the neutrino sector, one remark is in order regarding
the phase matrices $P_\ell$, $Q_\ell$ and their relationship with the matrices $P_d$ and
$Q_d$ of eq. (1). Although these phases were not relevant for the determination
of the mass eigenvalues, they do play a role in the mixing angles. Without
loss of generality, we can choose $P_\ell = P_d$ and $Q_\ell = Q_d$, provided that we
allow the parameter $b_\ell$ to be complex, $b_\ell = |b_\ell|e^{i\alpha}$, which is what we do in
the following.

Turning next to the neutrino sector, we assume the Dirac neutrino matrix
$M_\nu^D$ to have the Fritzsch form. We shall further assume that the elements of
$M_\nu^D$ arise from a Higgs 10–plet, resulting in the identity

$$M_\nu^D = M_u . \tag{6}$$

The light neutrino masses depend on $M_\nu^D$ as well as on the form of the heavy
Majorana matrix $M_\nu^M$ for the right–handed neutrinos ($\nu_R$’s). To arrive at a
simple and predictive spectrum, a judicious choice of $M_\nu^M$ is needed. (See
refs. 11–13 for specific examples.) We first note that a 126–plet of Higgs
which generates such mass entries, was already used in the (23) and (32)
elements of $M_d$ and $M_\ell$. The simplest possibility is then to use the same 126
to generate (23) and (32) entries in the $\nu_R$ Majorana matrix. Now $M_\nu^M$ should
have rank three in order to make the see–saw mechanism effective for all three
neutrino species. Keeping the number of parameters at a minimum, this is
best done by allowing a non–zero (11) element in $M_\nu^M$. So our extension of
the Fritzsch ansatz to the heavy Majorana matrix is given by

$$M_\nu^M = \begin{pmatrix} M' & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix} . \tag{7}$$
The elements $M, M'$ are in general complex.

We are now in a position to discuss neutrino oscillations within the model. Let us first absorb the phase matrices $P_{u,d}, Q_{u,d}$ from $M_u, M_d, M_\ell$ and $M^D_\nu$. Since $P_u \neq P_d$, this would alter the charged current matrix from an identity to the matrix
\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & e^{i\tau}
\end{pmatrix}
\]  
both in the quark and in the leptonic charged currents. Note that this phase rotation would alter the phases of $M$ and $M'$ in eq. (7), but they were anyway arbitrary to begin with. The advantage of this way of proceeding is that some of the phase parameters in the leptonic CKM matrix will be the same as those in the quark sector. If we now make a further redefinition of lepton fields so that $M_\ell$ becomes real (i.e., remove the phase $\alpha$ from $|b_\ell|e^{i\alpha}$), the leptonic charged current will have the form
\[
K = \begin{pmatrix}
e^{i\alpha} & 0 & 0 \\
0 & e^{i(\sigma-\alpha)} & 0 \\
0 & 0 & e^{i\tau}
\end{pmatrix}.
\]

The light neutrino matrix $M^\text{light}_\nu$ is obtained from the see–saw formula as
\[
M^\text{light}_\nu = M^D_\nu (M^M_\nu)^{-1} (M^D_\nu)^T.
\]  
Making use of the relations
\[
c_u \simeq m^0_t, \quad b_u \simeq (m^0_\ell m^0_\nu)^{1/2}, \quad a_u \simeq (m^0_\nu m^0_\ell)^{1/2}
\]
(similar relations hold for other sectors), $M^\text{light}_\nu$ can be expressed as
\[
M^\text{light}_\nu \simeq \frac{(m^0_\nu)^2}{M} \begin{pmatrix}
0 & \sqrt{m^0_\nu m^0_\ell} & \sqrt{m^0_\nu m^0_\nu} \\
\sqrt{m^0_\nu m^0_\ell} & m^0_\nu & m^0_\ell \\
\sqrt{m^0_\nu m^0_\nu} & m^0_\ell & 2 m^0_\nu
\end{pmatrix}.
\]
Here \( r = |M'/M| \) and \( \gamma \) is their relative phase.

The eigenvalues of the light neutrino mass matrix are readily obtained:

\[
\begin{align*}
    m_{\nu_1} &\simeq \frac{(m^0_t)^2}{M} \left( \frac{m^0_u}{m^0_t} \right)^2 r \\
    m_{\nu_2} &\simeq \frac{1}{2} \frac{(m^0_t)^2}{M} \left( \frac{m^0_c}{m^0_t} \right)^{\frac{3}{2}} \\
    m_{\nu_3} &\simeq \frac{2}{M} \frac{(m^0_t)^2}{m^0_t} \frac{m^0_c}{m^0_t} . \tag{13}
\end{align*}
\]

Note that in eq. (12) the parameter \( r \) is accompanied with a very small co-efficient \( \left( \frac{m^0_u}{m^0_t} \right) \left( \frac{m^0_c}{m^0_t} \right) \). It becomes relevant only in determining the mass of \( \nu_1 \) which, given the hierarchy in the masses, is unimportant for neutrino oscillations. Similarly, the phase \( \gamma \) is an irrelevant variable, disappearing from all physical observables.

The leptonic CKM matrix is obtained from

\[
V_{\text{lepton}}^{KM} = O^T_\nu K O_\ell \tag{14}
\]

where \( K \) is the phase matrix of eq. (9), and \( O_\nu \) and \( O_\ell \) are the orthogonal matrices that diagonalize \( M^\text{light}_\nu \) and \( M_\ell \): \( O^T_\ell M_\ell O_\ell = M_\ell(\text{diagonal}) \) and \( O^T_\nu M^\text{light}_\nu O_\nu = M^\text{light}_\nu(\text{diagonal}) \). They are given by

\[
O_\nu \simeq \begin{pmatrix}
1 & -\sqrt{\frac{m^0_t}{m^0_\mu}} & \frac{1}{2} \sqrt{\frac{m^0_t}{m^0_\tau}} \\
\sqrt{\frac{m^0_\mu}{m^0_t}} & 1 & \frac{1}{2} \sqrt{\frac{m^0_\mu}{m^0_\tau}} \\
-\frac{1}{2} \sqrt{\frac{m^0_\mu}{m^0_\tau}} & \frac{1}{2} \frac{m^0_\mu}{m^0_\tau} & 1
\end{pmatrix} \tag{15}
\]

\[
O_\ell \simeq \begin{pmatrix}
1 & -\sqrt{\frac{m^0_\mu}{m^0_\mu}} & \sqrt{\frac{m^0_\mu}{m^0_\tau}} \\
\sqrt{\frac{m^0_\mu}{m^0_\mu}} & 1 & \sqrt{\frac{m^0_\mu}{m^0_\tau}} \\
-\sqrt{\frac{m^0_\mu}{m^0_\tau}} & \sqrt{\frac{m^0_\mu}{m^0_\mu}} & 1
\end{pmatrix} . \tag{16}
\]
The resulting leptonic CKM elements are

$$|V_{1\mu}| \approx |V_{2e}| \approx \sqrt{|m_0^e - e^{i(\sigma - 2\alpha)} \sqrt{|m_0^0|}}$$
$$|V_{2\tau}| \approx |V_{3\mu}| \approx \sqrt{|m_0^\mu - \frac{1}{2}e^{i(\tau - \sigma + \alpha)} \sqrt{|m_0^0|}}$$
$$|V_{1\tau}| \approx \sqrt{|m_0^e m_0^\mu m_0^\tau + e^{i(\sigma - 2\alpha)} \sqrt{|m_0^0| m_0^0 m_0^\tau} - e^{i(\tau - \alpha)} \sqrt{|m_0^0| m_0^\mu}}$$
$$|V_{3e}| \approx \frac{1}{2} \sqrt{|m_0^u m_0^0 m_0^t + \frac{1}{2}e^{i(\sigma - 2\alpha)} \sqrt{|m_0^0| m_0^0 m_0^t} - e^{i(\tau - \alpha)} \sqrt{|m_0^0| m_0^\mu}|}. \quad (17)$$

The leptonic mixing angles do not run below the scale of $B - L$ breaking, since the right–handed neutrinos decouple at that scale. However, the relations (13) and (17) are in terms of the asymptotic masses of quarks and charged leptons. It is therefore necessary to extrapolate the low energy masses to the SUSY GUT scale using the (one loop) renormalization group equation for the mass ratios and mixing angles including the effect of a heavy third family. In ref. (7) a detailed analysis of this type was carried out. We extend it here to include the charged lepton mass ratios. The variations of these quantities as functions of the top quark mass corresponding to $\tan \beta = 60$ are shown in Fig. (1). As input values we chose $\alpha_1(M_Z) = 0.01013$, $\alpha_2(M_Z) = 0.03322$, $\alpha_3(M_Z) = 0.115$ and $M_G = 10^{16} \text{ GeV}$. Our numerical results are in agreement with the analytic results presented in ref. (15). For $m_t = 130 \text{ GeV}$ the relevant renormalization factors are found to be

$$\left( \frac{m_0^e}{m_t^e} \right) = 0.64 \left( \frac{m_c}{m_t} \right); \quad \left( \frac{m_0^\mu}{m_t^\mu} \right) = 0.64 \left( \frac{m_u}{m_t} \right)$$
\[
\left( \frac{m_\mu^0}{m_\tau^0} \right) = 0.53 \left( \frac{m_\mu}{m_\tau} \right);
\left( \frac{m_e^0}{m_\mu^0} \right) = 0.53 \left( \frac{m_e}{m_\mu} \right). \quad (18)
\]

From eq. (13), it follows that
\[
\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{m_e^0}{4m_t^0} . \quad (19)
\]

Choosing \( m_c(m_c) = 1.27 \text{GeV} \) and noting that the QCD running factor from \( m_c \) to \( m_t \) is \( \simeq 0.6 \), we find that
\[
\frac{m_{\nu_2}}{m_{\nu_3}} \simeq 9 \times 10^{-4} . \quad (20)
\]

From eq. (17) one sees that the \( \nu_\mu - \nu_\tau \) oscillation angle is rather large, in the range of \((15 - 25)\%\). For such a large mixing, there is an upper limit of about 2.5 eV on the \( \nu_\tau \) mass arising from oscillation experiments.\(^{16}\) Eq. (20) translates this into an upper limit on \( m_{\nu_2} \leq 2.3 \times 10^{-3} \text{eV} \). Remarkably, this is in the right range for \( \nu_e - \nu_\mu \) MSW oscillations, corresponding to \( \Delta m^2_{e\mu} \simeq 5.4 \times 10^{-6} \text{eV}^2 \).\(^{17}\) We also see that the \( \nu_\tau \) mass \( (m_{\nu_3}) \) cannot be less than about 1 eV, otherwise \( \nu_\mu \) would be outside of the MSW range. A neutrino mass in the range of \((1 - 3) \text{eV}\) is cosmologically significant and plays the role of the hot component of dark matter, which is suggested\(^{18}\) by the recent COBE data.

As for the \( \nu_e - \nu_\mu \) mixing angle, first note that the ratios of masses involving the first two families do not run. The two terms \( \sqrt{m_e/m_\mu} \) and \( \sqrt{m_\mu/m_e} \) are numerically about equal. This means that the mixing angle can be small and quite different from the naive expectation that \( |V_{1\mu}| \simeq |V_{us}| \simeq 0.22 \). The combined SAGE/GALLEX experiments in fact prefer a mixing angle in the \( \nu_e - \nu_\mu \) sector which is significantly smaller than \( |V_{us}| \). For a ‘central’ value of \( |V_{1\mu}| = 0.05 \) implied by SAGE/GALLEX, we can fix all the other oscillation parameters of the model. Using \( m_\alpha(1 \text{GeV}) = 5.1 \text{MeV}, \ m_c(m_c) = \)
1.27 GeV, we determine $\sqrt{m_u/m_c} = 0.062$, to be compared with $\sqrt{m_c/m_\mu} = 0.07$. Then, from the first relation of eq. (17), we find $\cos(\sigma - \alpha) = 0.72$, or $\sigma - \alpha \simeq \pi/4$. From the quark sector, we also know that $\sigma \simeq \pi/2$ (to get the Cabibbo angle right), so $\alpha \simeq \pi/4$ is preferred. Using this value of $\alpha$ and $\tau \simeq \sigma$ (needed for acceptable $|V_{cb}|$), we can calculate

$$|V_{2\tau}| \simeq |V_{3\mu}| \simeq 0.158; \quad |V_{3e}| \simeq 0.011; \quad |V_{1\tau}| \simeq 0.0098.$$  \hspace{1cm} (21)

With such a significant mixing in the (2-3) sector along with the constraint $m_{\nu_\tau} \geq 1 \, eV$, it follows that $\nu_\mu - \nu_\tau$ oscillations should be observable in the planned CHORUS/NOMAD experiments at CERN and the proposed Fermilab experiment P803.\textsuperscript{19} The $\nu_e - \nu_\tau$ mixing may play a significant role in astrophysical settings, for example in blowing up the supernova core.

At this stage it is worthwhile to assess the predictive power of the Fritzsch ansatz including the leptonic generalization proposed here and compare its predictions with those of other popular ansatzes.\textsuperscript{20–22} In the charged fermion sector there are altogether 9 parameters ($a_{u,d}$, $b_{u,d}$, $c_{u,d}$, $\sigma$, $\tau$ and $|b_\ell|$) and 13 observables, thereby leading to 4 predictions. In addition, the parameter $\tan\beta$ is determined. The inclusion of neutrinos adds three additional parameters, $|M|$, $|M'|$ (as noted earlier, their relative phase became irrelevant) and the phase $\alpha$ associated with $b_\ell$. There are now 9 more observables (3 neutrino masses, 3 mixing angles and 3 CP violating phases). The 23 ($= 13 + 1 + 9$) observables are determined in terms of 12 parameters, thereby resulting in 11 predictions. In the charged fermion sector, the predictions include the two mass relations of eq. (5), $|V_{ub}|/|V_{cb}| \simeq \sqrt{m_u/m_c} \simeq 0.06$, $\tan\beta \simeq 60$ and the prediction for the CP violating parameter in the CKM matrix.\textsuperscript{7} In the neutrino sector, they include the mass ratio $m_{\nu_2}/m_{\nu_3}$ of eq. (19), the two
mixing angles in eq. (21), and the three CP violating phases. The three distinguishing features of the Fritzsch ansatz are (i) $\tan\beta$ is large, (ii) the top quark is moderately heavy, $m_t \simeq 120 - 150$ GeV, and (iii) $|V_{cb}|$ can be close to its central value. At least two of the predictions should distinguish it from the ansatz of ref. (22), which predicts $m_t \gtrsim 170$ GeV and $|V_{cb}| \gtrsim 0.052$.

In conclusion, motivated by the success the Fritzsch ansatz enjoys in the quark sector, we have proposed a generalization to include the leptons.\textsuperscript{23} We found that the minimal scheme which incorporates the charged leptons and neutrinos predicts a small $\nu_e - \nu_\mu$ mixing angle which is consistent with the MSW resolution of the solar neutrino problem. The $\nu_\tau$ mass is predicted to be in the range $(1 - 3)$ eV, making it a suitable candidate for the hot component of dark matter. The planned $\nu_\mu - \nu_\tau$ oscillation experiments and the discovery of the top quark in the mass range $m_t \simeq (120 - 150)$ GeV will provide crucial tests of the idea.

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Figure Caption

Fig. 1: The running factors $f(M_X)/f(M_Z)$ for $f = |V_{cb}|$ (solid) $|m_c/m_t|$ (dot–dash), $|m_\mu/m_\tau|$ (solid) and $|m_s/m_b|$ (dashed) versus $m_t$. The running factors for $|V_{ub}|$, $|V_{td}|$, $|V_{ts}|$ are identical to that of $|V_{cb}|$. Similarly, $|m_e/m_\tau|$ runs as $|m_\mu/m_\tau|$, $|m_d/m_b|$ runs as $|m_s/m_b|$ and $|m_u/m_t|$ as $|m_c/m_t|$.