One-dimensional constrained inversion of electrical source transient electromagnetic method

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Abstract. The damped least squares inversion principle is applied to the transient electromagnetic one-dimensional inversion of electrical sources, and a new model is obtained by continuously iterating the initial model, thereby fitting the observed transient electromagnetic response, and performing one-dimensional inversion through induced electromotive force play. In the damped least squares inversion, constraints are added to the Jacobian matrix, and simultaneous constraint equations and conventional inversion equations are solved. By weighting the constraint parameters, the difference between adjacent resistivities and layer thicknesses is minimized. Finally, K-type and H-type theoretical models were used to verify the reliability of the algorithm, and compared with the conventional transient electromagnetic damping least squares inversion.

1. Introduction

Transient electromagnetic method (TEM) is a widely used time-domain electromagnetic detection method based on the principle of electromagnetic induction. This method has the advantages of observing pure secondary field, high resolution, high sensitivity, simple structure and so on. It is widely used in the fields of engineering geological survey, advanced tunnel prediction, seawater intrusion survey, metal mineral exploration, coal mine geological hazard investigation, etc. [1-3]. Transient electromagnetic method can be divided into magnetic source transient electromagnetic method and power source transient electromagnetic method according to the nature of the emission source. The method of transmission using an ungrounded loop is the magnetic source transient electromagnetic method, and the method of transmitting through a long ground wire is power source Variable electromagnetic method.

With the rapid development of the computer level, the forward and inversion of the transient electromagnetic method have been rapidly developed. Due to the complexity of the two-dimensional and three-dimensional transient electromagnetic theory, the current method of interpretation of the two-dimensional and three-dimensional transient electromagnetic inversion is still Staying on theoretical research, it has not been used in actual production. At present, there are more research and applications of one-dimensional inversion. Among them, the more widely used one-dimensional linear inversion methods are: Gauss-Newton method, OCCAM inversion, conjugate gradient method, damped least square method, etc. It has the advantages of faster convergence speed and shorter calculation time, but there are still some problems, such as poor inversion stability, easy to fall into local minima, and the initial model has a large influence on the inversion results. In view of the existing problems, some scholars introduced Occam inversion method in the study of tem of central loop observation device. Although it reduced the dependence of inversion results on the initial model, there was a large amount of calculation of Lagrangian operator[4]; Some scholars have used the...
damped least square algorithm in the calculation of one-dimensional transient electromagnetic inversion of large fixed source loop, which has realized the automatic adjustment of damping factor and accelerated the inversion speed, but for iterative control, there are many human interventions [5]. Some scholars use the Gauss Newton method to calculate the one-dimensional inversion of large fixed source transient electromagnetic, which has a good application effect, but the inversion results depend too much on the initial model [6]. In this paper, the Jacobian matrix constraint processing, joint constraint equation and conventional inversion equation are used to optimize the resistivity and layer thickness. Finally, the calculation of the theoretical model shows the effectiveness of the algorithm.

2. Algorithm theory

2.1. One-dimensional forward

In the actual survey work, the construction method of electrical source transient electromagnetic method is mainly based on surface emission and surface reception. The vertical magnetic field generated by the long-line source transient electromagnetic can be obtained by integrating the dipole field expression along the long line:

\[ H_z = \frac{Ids}{4\pi} \frac{y}{r} \int_0^{r_{\text{TE}}} (1 + r_{\text{TE}}) e^{i\omega \lambda} \frac{\lambda^2}{\mu_0} J_1(\lambda \rho) d\lambda \]  

(1)

Where \((x, y, z)\) is the coordinate of the receiving point, \(ds\) is the length of the dipole, \(r\) is the distance from the receiving point to the dipole, \(r_{\text{TE}}\) is the reflection coefficient in TE mode, and \(J_1(\lambda \rho)\) is the first-order shell Searle function, among them, \(r_{\text{TE}} = \frac{Y_0 - \hat{Y}_1}{Y_0 + \hat{Y}_1}\), \(Y_0 = \frac{u_0}{z_0}\), \(z_0 = i\omega \mu_0\), assuming \(N\) layers, the following recursive formula:

\[ \hat{Y}_n = Y_n Y_{n+1} + Y_n \tanh(u_n h_n) \]

\[ Y_n + Y_{n+1} \tanh(u_n h_n) \]  

(2)

In the above formula, \(Y_n = \frac{u_n}{z_n}\), \(u_n = (k_x^2 + k_y^2 - k_n^2)^{1/2}\), and \(k_x^2 + k_y^2 = \lambda^2\), \(k_n^2 = -\frac{\varepsilon_n}{\varepsilon_n} \hat{Y}_n = \omega^2 \mu_n \varepsilon_n - i\omega \mu_n \sigma_n\), recursing layer by layer starting from the \(N\)th layer can get \(\hat{Y}_1\).

When calculating the vertical magnetic field \(H_z\), the integral along the long wire can be replaced by the cumulative approximation of the field generated by each small segment of length \(\Delta x\) (the subunit located at \((x_n,0,0)\)). Therefore, we get:

\[ H_z = \frac{Iy}{4\pi} \sum_{n=1}^{N} \frac{\Delta x}{r_n} \int_0^{r_{\text{TE}}} (1 + r_{\text{TE}}) e^{i\omega \lambda} \frac{\lambda^2}{\mu_0} J_1(\lambda r_n) d\lambda \]  

(3)

Finally, the conversion from frequency domain to time domain is realized by G-S transformation, and the time domain response of the transient electromagnetic field of the long wire source can be obtained.

2.2. One-dimensional constraint inversion

The damped least squares principle is applied to the one-dimensional inversion of electrical source transient electromagnetics, and its objective function is:

\[ P(m, d) = \| \ln A(m) - \ln d \|^2 + \alpha \sum_{j=1}^{2N-1} \left( \frac{\Delta m_j}{m_j} \right)^2 \]  

(4)
In the formula, \( \| \ln A(m) - \ln d \|^2 \) is a data fitting function, also known as a poor fit, and 
\[
\alpha \sum_{j=1}^{2N-1} \left( \frac{\Delta m_j}{m_j} \right)^2
\]
is a stable function; \( A(m) \) is a long-line source transient electromagnetic one-dimensional forward operator, that is, the induction obtained by each forward iteration. Electromotive force response, \( d \) is the measured data of a long lead source transient electromagnetic arbitrary measurement point, that is, the transient electromagnetic induced electromotive force response, \( \alpha \) is a damping factor, and \( \Delta m \) is a model parameter correction amount, \( m = [\rho_1, \rho_2, \ldots, \rho_N, h_1, h_2, \ldots, h_{N-1}] \), where \( N \) represents the number of inversion layers, \( \rho_1, \rho_2, \ldots, \rho_N \) represent the resistivity from the first layer to the \( N \)th layer, and \( h_1, h_2, \ldots, h_{N-1} \) respectively represent layer thickness from layer 1 to layer \( N-1 \). Differentiate the above formula and take the derivative to zero to obtain the iterative equation:

\[
(U_m^T U_m + \alpha D) \Delta m = U_m^T [\ln d - \ln A(m)]
\]

In the formula, \( U_m = U_{yj} = \frac{1}{A} \frac{\partial A}{\partial \rho_j} \), \( D = \text{diag} \left( \frac{1}{m_j^2} \right) \), where \( m \) is the current iterative model parameter, \( j \) is the loop amount of the model parameter, and \( i \) is the loop amount of the number of frequency points.

The inversion step is to set up the initial model and obtain a new model \( m(i) \) \((i = 1, 2, \ldots, 2n-1)\) through continuous iteration, that is, the resistivity of the \( n \)-layer formation and the formation of the \( n-1 \) layer formation. Thickness to fit the observed transient electromagnetic response \( b(j) \) \((j = 1, 2, \ldots, a)\), where \( j \) is the number of time points. In the inversion, the linear conjugate gradient method is used to solve the linear equations. In order to avoid the instability of the inversion due to the large difference in the magnitude of the resistivity as a model parameter, the logarithm of the resistivity and the layer thickness is used as the model parameter in the inversion, that is: \( m = [\ln(\rho_1), \ln(\rho_2), \ldots, \ln(\rho_N)] \), \( \ln(h_1), \ln(h_2), \ldots, \ln(h_{N-1}) \), then the Jacobian matrix

\[
\frac{\partial A}{\partial (\ln \rho_j)} = \rho_j^{-1} \frac{\partial A}{\partial \rho_j}, \quad \frac{\partial A}{\partial (\ln h_j)} = \rho_j \frac{\partial A}{\partial h_j}.
\]

In this paper, the weighted constraints are mainly added to the Jacobi matrix, and then the constraint equation and the conventional inversion equation are solved simultaneously. The central idea is to make the difference between adjacent resistivity parameters and layer thickness parameters as small as possible to achieve smoothness. The weighting matrix \( R_p \) is a difference operator. According to Auken (2009)[7], \( R_p m - c_r = 0 \), this transformation can be obtained

\[
\Delta r_p = R_p \Delta m
\]

In the formula, \( r_p = -R_p m_0 \) and \( \Delta m = m - m_0 \). Where \( R_p \) is a sparse matrix composed of \( W_r \) and \( W_h \), which can be expressed as \( R_p = [R_{p1}, R_{p2}] \), and

\[
R_{p1} = \begin{bmatrix}
    w_{r1} & -w_{r1} & 0 & 0 \\
    0 & w_{r2} & -w_{r2} & 0 \\
    0 & 0 & M & O & M \\
    0 & 0 & 0 & L & w_{rL}
\end{bmatrix}, \quad R_{p2} = \begin{bmatrix}
    w_{h1} & -w_{h1} & L & 0 & 0 & 0 \\
    0 & w_{h2} & -w_{h2} & 0 & 0 \\
    0 & 0 & M & O & M \\
    0 & 0 & 0 & w_{mL-1} & w_{mL-1} \\
    0 & 0 & 0 & L & 0 & 0 & 0
\end{bmatrix}
\]

The specific values of \( W_r \) and \( W_h \) are based on the requirements of the inversion. The larger the value, the stronger the constraint and the smoother the curve.

From the relationship between the data residual and the model correction, \( F \Delta m = \Delta d_{\text{obs}} \), where \( F \) is the Jacobian matrix and \( \Delta d_{\text{obs}} \) is the residual of the observation data. Combined (5), the iterative equation under the constraints is obtained:
\[
\begin{bmatrix}
F \\
R_p
\end{bmatrix}
\Delta m = \begin{bmatrix}
\Delta d^{obs} \\
\Delta r_p
\end{bmatrix}
\]  \hspace{1cm} (6)

And \( \Delta r_p \ll \Delta d^{obs} \), which can be recorded as 0, then there is \( F_r = \begin{bmatrix} F \\ R_p \end{bmatrix} \). Adding the damping factor \( \alpha \), the iterative equation becomes

\[
(F_r^T F_r + \alpha) \Delta m = F_r^T [d - A(m_0)]
\]  \hspace{1cm} (7)

The other steps are the same as the conventional damped least squares inversion.

3. Model study

In order to verify the validity of one-dimensional constrained inversion, K-type and H-type models are constructed. The resistivity of K-type model is 100 \( \Omega \cdot \text{m} \), 1000 \( \Omega \cdot \text{m} \), 100 \( \Omega \cdot \text{m} \) from top to bottom, and the layer thickness is 0.2 km, 0.3 km respectively; The resistivity of H-type model is 1000 \( \Omega \cdot \text{m} \), 100 \( \Omega \cdot \text{m} \), 1000 \( \Omega \cdot \text{m} \) from top to bottom, and the layer thickness is 0.2 km, 0.3 km respectively; In inversion, \( \alpha = 10 \), using the 15-layer model as the initial model, the transmission and reception distance \( r = 1000 \text{ m} \), the emission current \( I = 10 \text{ A} \), the length of the electrical source \( L = 100 \text{ m} \), and the weighted constraint parameters \( w_{r1} = 0.1 \) and \( w_{r2} = 0.1 \). The results of the inversion are shown in Figure 1.

From the inversion results in Figure 1, it can be seen that the inversion results of K-type and H-type models have good fitting effect, and the inversion results basically conform to the changes of geoelectric parameters of the real model. The inversion results of H-type model are better than those of K-type model, and the attenuation voltage curves of the two models have good fitting and high precision, which shows the rationality of the inversion results.
Fig. 1 Inversion results of theoretical model. (a) K-model inversion results; (b) K-model attenuation voltage fitting curve; (c) H-model inversion results; (d) H-model attenuation voltage fitting curve.

3.1. Constraint results comparison

A five layer theoretical model is constructed, with resistivity of 500 Ω·m, 50 Ω·m, 800 Ω·m, 50 Ω·m, 400 Ω·m from top to bottom, layer thickness of 0.5 km, 0.1 km, 0.5 km, 0.4 km, and other parameters consistent with the inversion parameters of K and H models.

Fig. 2 Inversion results of the five-layer theoretical model. (a) Unconstrained inversion results; (b) Weighted constraint inversion results.

Compared with Fig. a and b in Fig. 2, the inversion result in Fig. b reduces the surface instability and makes the shallow layer result closer to the real model, and Fig. B reflects the real resistivity value in the third layer with higher accuracy. Compared with the two methods, the weighted constrained inversion results reduce the redundant structure, reduce the change rate of inversion results, enhance the vertical continuity of the stratum, and more in line with the real stratum situation.

4. Model study

In this paper, the inversion results of K-type, H-type and five layers theoretical models can reflect the relative changes of strata, which shows the effectiveness of the algorithm for different geoelectric structures, and proves the effectiveness of one-dimensional inversion of transient electromagnetic of
power supply. In this paper, the damping factor is selected reasonably to ensure the inversion stability and improve the inversion resolution to the greatest extent. By selecting the appropriate initial model resistivity, transceiver distance, emission current, electrical source length, and more reasonable weighted constraint parameters, the redundant structure in inversion can be properly reduced and the false anomaly can be suppressed. The inversion results of weighted constraints are closer to the real model, which is mainly due to the sensitive response of transient electromagnetic field to low resistivity body and insensitive to high resistivity body.

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