Student’s representation of fraction through ELPSA framework

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Abstract. Representation has an important role in developing students’ mathematical ability. Therefore, learning materials need to be developed. One of them is by implementing experience, language, pictorial, symbol, and application (ELPSA) framework. The aim of this research is to analyze students’ mathematical representation on fraction material after implementing ELPSA framework. There were 20 students of Year 7 from one of the junior high schools in West Lombok, Indonesia who joined the lessons. However, only six students were involved in the interview section to investigate their answers about the representation of fraction. This research shows that the students still have difficulties in the representation of fraction. Errors in representation on fractional material also occur due to students’ mistakes in understanding fraction unit. Students’ representation of fraction should be conducted through exercise continually.

1. Introduction

Representation can be interpreted as a process of modeling concrete matter in the real problem into abstract concept or symbol [1]. Mathematical representation is used to plan, monitor, and control the mathematical problem-solving process [2]. While Lowrie and Patahuddin state that concrete representations supported students’ learning as they move towards more abstract concepts and ideas [3]. Interpreting the given image is called encoding. The decoding process in which learners are given problems in the form of words and learners need to interpret the problem and present it in the form of sketches, diagrams or drawings.

In relation to the fractional concept, there are seven types of representations according to [4] as follows: (1) part group, congruent part; (2) part-whole, congruent parts; (3) part group, non-congruent parts; (4) part group, comparison (comparing the number of members or objects of two sets); (5) Number line, a line of numbers containing a line segment with a length of one unit; the segment of the line is divided into \(b\) equal parts of length; (6) part whole, comparison; and (7) part whole, non-congruent parts.

Representation of fraction using number line is more difficult for students to understand than others [5]. However, according to Wong and Evans, most elementary school students can use symbolic representation but find difficulties in representing images in the topic of fractional [6]. Therefore, students’ ability in representing fraction is necessary to be developed. Several studies have been conducted by [4,7,8]. However, there is no research conducted in developing learning material through the ELPSA Framework to improve students’ representation ability on fraction.
ELPSA framework is a learning approach consisting of elements experience, language, pictorial, symbol, and application. This framework is based on constructivism learning theories and its social nature. The ELPSA framework views learning as an active process in which students construct their own way of understanding something through individual thought processes and social interactions with others [3]. Pictorial components in ELPSA are components that are closely related to mathematical representation. This paper discusses the students' representation ability after the implementation of the learning materials of ELPSA framework.

2. Method
This research is design research to obtain learning materials namely lesson plan, students’ worksheet and mathematical representation test through ELPSA framework. Process of lesson materials development is started by conducting (1) preliminary investigation by organizing an assessment of curriculum and fraction topic, (2) prorotyping phase by designing and validation process of lesson plans, worksheets, and mathematical representation test, (3) assessment phase by implementing the learning materials and test the representation skill of the students [9]. This paper only discusses the phase of the learning materials especially related to assessing the ability of students' mathematical representation after implementation of ELPSA framework. Before implementation, authors have designed the teaching materials. Two experts were involved in this study in assessing the validity of teaching materials. According to them, the teaching materials were valid. There are seven lessons of fraction that have been implemented. Each lesson takes 80 minutes. This study only discusses the students’ mathematical representation after implement the first lesson about basic concepts of fraction.

One of the authors has discussed and performed (as simulation) some activities of the first lesson to the teacher before implementation of the teaching materials. Authors and teacher have joined several Professional Development workshop about ELPSA framework at Mataram, Indonesia. These workshops for mathematics teachers were conducted by Government Partnership for Development (GPFD) Project between the University of Canberra, Australia and IKIP Mataram, Indonesia (see www.elpsa.org).

The participants of this research were 20 students (one female and nine males) of Year 7 of one of junior high school in West Lombok, Indonesia. The student's mathematical representation is measured by three indicators: (1) solve problems by involving mathematical expressions; (2) create an image to clarify the problem and facilitate its completion; and (3) create a problem situation based on the data or representation given and write it down [10]. The test for 40 minutes was performed after the students learned the fraction through ELPSA Framework. After doing the test and reviewing the answers of the students, the authors conducted interviews with six students to trace their responses to three questions of mathematical representation. Table 1 shows the indicator and the item test.

| Mathematical Representation Indicators | Item Test |
|----------------------------------------|-----------|
| Solve problems by involving mathematical expressions | 1         |
| Create an image to clarify the problem and facilitate its completion | 2         |
| Create a problem situation based on the data or representation given and write it down | 3         |

3. Result and discussion
Student's answer to the test of mathematical representation is described as follows.
Question 1: Please divide the following square into four equal parts in different ways.
The students were expected to be able to answer six pictures on the provided answer sheet. Generally, the students drew such fraction as follows.

From 20 students only one student who answered five correct answers, 15 students answered four correct answers, and three students answered three correct answers. Students’ wrong answer was described as follows.

Therefore, the researchers conducted the interview with students to confirm their response. Excerpts of interviews between the researchers and Student 1 (S1) are as follows.

Researchers : Please explain how did you obtain this answer?
(Researchers pointed drawing created by S1)

Student 1 (S1) : For this one, (appointed drawing 1) I just created line from top to bottom, Mam!, for this one (appointed drawing 2) I created line sidelong, for this one (appointed drawing 3) I created line from the top to the bottom and sidelong, for this one (appointed drawing 4), I created line from the side corner to this side edge, for this one (appointed drawing 5), I created cross line, and for this one (appointed drawing 6) I just did the same as Question 5 however I rotated it, Mam!

Researchers : Please read the question carefully.
Student 1 (S1) : (Reading the question)
Researchers : What about this one? (Appointed drawing 1, 2, 3, and 4).
             : Do they have the same size?
Student 1 (S1) : Yes, Mam! Yes, they do.
Researchers : How about this one? (Appointed drawing 5 and 6).
             : What do you think?
Student 1 (S1) : All of them are not the same, however, two parts which face to face has the same size.
Researchers : Are you sure that is right?
Student 1 (S1) : I am pretty sure Mam.

It should be six different representations to state fraction $\frac{1}{4}$ as follows.

![Image of representations]

Figure 4. Answer key Question 1.

Question 2: Please represent fraction $\frac{1}{3}$ in various ways!
There was 90% of the students answered it by representation as follows.

![Image of representations]

Figure 5. Students’ representation in answering Question 2.

The researchers conducted interview with the students to confirm their answer. Here are excerpts of the interview between the researchers and Student 2.

Researchers : Please take a look at your answer for question no.2
             : Could you explain how did you obtain this answer?
Student 2 (S2) : Rechecking his answer. Yes, Mam, this is a form of fraction $\frac{1}{3}$
Researchers : Is there any other representation?
Student 2 (S2) : Yes, there is. However, I can’t remember it. This is all I can remember.

This matter provides information that 90% of students have limited ability in declaring fraction $\frac{1}{3}$ in various representations. There are only 10% of students who answered by using three representations to declare fraction $\frac{1}{3}$ as described in Figure 4 as follows.
Figure 6. Students’ representation in answering Question 2.

Here are excerpts of the interview between the researcher and student 3 (S3):

Researchers: Could you explain how did you obtain this answer? (Pointed S3’s answer sheet).
Student 3 (S3): Firstly, I created a square divided into 3 areas, but I shaded one of the areas which equals to $\frac{1}{3}$, secondly, I used number line. Third, I drew 3 flowers, if $\frac{1}{3}$ of the 3 flowers is one flower Mam.

Researchers: Please take a look at the number line! Pay attention to the position of the $\frac{1}{3}$.
Student 3 (S3): I was wrong Mam, it should be here (writing the position which was deemed correct by S3.

However, during the teaching and learning process, the teacher has explained in various ways to represent fraction $\frac{1}{2}$. In this research, the students still have difficulties in mathematical representation. This is supported by opinion [6] who stated that most of the Elementary students able to use representation symbolically and have difficulties to present it in the form of drawing or fraction materials.

Question 3: Please answer this question!

If this hexagon equals to $\frac{1}{3}$, could you please make drawing represent the fraction as below?

Figure 7. Question 3.

For question no. 3 point a, there were 80% of the students answered with figure 8(a). This is the correct answer to find out fraction equals to $\frac{1}{2}$ and $\frac{1}{3}$.

Fraction equals to $\frac{1}{2}$ is: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} \ldots$

Fraction equals to $\frac{1}{3}$ is: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} \ldots$

$\frac{1}{3} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$

There were 10% of the students provided the wrong answer as stated in Figure 8(b) and 5% of the students provided the wrong answer as stated in Figure 8(c), also 5% of them provided the wrong answer as stated in Figure 8(d).
Figure 8. Students’ representation in answering Question 3.

For question no.3 point b, there were 50% students provided correct answer as shown in figure 9(a) and 5% was appropriate to the expected answer because this based on answer 3a. Due to, 1 hexagon equals $\frac{1}{2}$ therefore for 2 hexagon equals to 1 and $\frac{1}{6} = \frac{2}{12}$ therefore it was obtained $\frac{7}{6}$. There were 5% of the students provided the wrong answer as shown in Figure 9(b) and 45% of them provided the wrong answer as shown in Figure 9(c).

Figure 9. Students’ representation in answering Question 3b.

The researchers conducted the interview to find out the students’ reason and answer as shown in Figure 9 (b) and (c). Here is the excerpt from interviews between researchers and S2 and S3.

Researchers : Please pay attention to your answer for question no.3b. Could you please explain how did you obtain this?
Student 2 (S2) : Question 3b. The question is $\frac{7}{6}$. It has been known that 1 hexagon equals to $\frac{1}{2}$ if its value is 1, therefore, there are 2 hexagons which are shaded. The rest is $\frac{1}{6}$. Then I divided 1 hexagon become 6 then I shaded into 1 area, Mam.

Researchers : Could you explain how did you get the answer for Question 3b?
Student 3 (S3) : It was from 3a Mam, I divided hexagon 3a becomes 6, so every part equals to $\frac{1}{12}$ therefore $\frac{12}{12} = 1$. I drew another which equals to 1, so there were 4 hexagons. The rest should be $\frac{1}{6}$ I shaded another 1 part, Mam, therefore, it equals to $\frac{7}{6} = \frac{12}{6}$

The problem occurred on question no. 3 because the students forgot the information on that question stated that the hexagon equals to $\frac{1}{2}$. This occurred as well on [7] concerning the students’ mistakes in understanding fraction unit on the number line. He stated that mistakes based on the understanding $\frac{7}{6}$ part of a fraction unit.

The ELPSA framework is a cyclical learning design approach, through daily life experiences, mathematical conversations, visual stimuli, symbol notation, and applications [3]. The teacher has implemented the learning materials of ELPSA framework to support the students’ representation skill
of fraction, especially through the pictorial component of ELPSA framework. Nevertheless, the students still have difficulties in solving mathematical representation about fraction.

During the lesson, students have learned about dividing the squares into two equal parts in different ways. Based on observation, most of the students in the classroom found the correct different ways. On the other hand, when students were asked to divide the square into four equal parts in different ways, only one of 20 students who answered five correct answers, 15 students answered four correct answers, and 3 students answered three correct answers. It means, dividing the square into four equal parts is more difficult for them than into two equal parts in different ways for students.

Most of the students’ representations about fraction $\frac{1}{2}$ are only used part of a whole representation. This representation is familiar to Indonesian students [11]. The most difficult problem of students is the question no. 3b. There were only 50% of students provided the correct answer to represents fraction $\frac{7}{8}$ when the representation of fraction $\frac{1}{2}$ is given. All three questions never presented in Indonesian mathematics textbook so that these questions are non-routine for students. Students have some errors and difficulties in solving non-routine problems [12] and they need some informal strategies to solve them [13].

One meeting of the lesson is not enough to develop the students’ representation. It is necessary for the students to obtain exercise any longer. In conducting mathematical representation, the students are required to have such a high level of creativity. Creative thinking is a process used when we produce a new invention [14]. Teachers have the important role to support students’ mathematical representation.

4. Conclusions

Three indicators measure the student's mathematical representation. The most difficult indicator is creating a problem situation based on the data or representation given. Student mathematical representation on fraction should be support continuously and required more exercises. One meeting is not enough to develop representation. In conducting mathematical representation, students are also required to have high-level creativity. Errors in the representation of fraction also occur due to students' mistakes in understanding fraction units.

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