On direct measurement of the $W$ production charge asymmetry at the LHC

K. Lohwasser

Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH, United Kingdom
and Physikalisches Institut, Hermann-Herder-Str. 3, D-79104 Freiburg, Germany
E-mail: kristin.lohwasser@physik.uni-freiburg.de

J. Ferrando

Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH, United Kingdom
E-mail: j.ferrando1@physics.ox.ac.uk

Ç. İssever

Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH, United Kingdom
E-mail: c.issever1@physics.ox.ac.uk

Abstract: The prospects for making a direct measurement of the $W$ production charge asymmetry at the LHC are discussed. A modification to the method used at the Tevatron is proposed for measurements at the LHC. The expected sensitivity for such a measurement to parton distribution functions is compared to that for a measurement of the lepton charge asymmetry. The direct measurement approach is found to be less useful for placing constraints on parton distribution functions at the LHC than a measurement of the lepton charge asymmetry.

Keywords: Hadron-Hadron Scattering, Standard Model, Hadronic Colliders, Parton Mode
1. Introduction

The measurement of the $W$ production charge asymmetry at hadron-hadron colliders provides important information about parton distribution functions (PDFs). The simplest method is to measure the asymmetry in the pseudorapidity distribution of the charged leptons (the lepton asymmetry) arising from leptonic decay of the $W^{\pm}$ bosons. Such measurements have been performed at the Tevatron, for both $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ events, by both the CDF \cite{1, 2} and D0 \cite{3, 4} collaborations and the data have been included in global fits of parton distributions \cite{5, 6, 7}. The asymmetry is mainly sensitive to valence quark distributions \cite{8}, providing complementary information to that obtained from measurements of structure functions in deep inelastic scattering \cite{9, 10, 11, 12, 13}. Predictions for the asymmetry exist up to next-to-next-to-leading order in the strong coupling \cite{14}.

The CDF collaboration has recently published a direct measurement of the $W$ production charge asymmetry \cite{15}. This measurement used a technique first proposed by Bodek
et al. [16], in which the \( W \) boson rapidity is inferred event-by-event from the lepton four-momentum and the missing transverse momentum. This inference enables a measurement of the \( W \) asymmetry to be made at values of rapidity where the uncertainty on the prediction from theory is much larger than the uncertainty on the lepton asymmetry for any part of the pseudorapidity range.

The inclusion of the CDF \( W \) asymmetry [15] in PDF fits has not been straightforward: the data set gives the second largest contribution per data point to the \( \chi^2 \) of the NNPDF fit [7]. Attempts by the MSTW group [17] to include it in their fits have revealed tension with the electron asymmetry measured by D0 [3]. To investigate this the CDF collaboration have produced the electron asymmetry for the same dataset as used in the \( W \) asymmetry measurement with two interesting features revealed. Firstly this electron asymmetry agrees well with the D0 electron asymmetry and secondly, while the \( W \) asymmetry agrees very well with the predictions made using the CTEQ6.6 [18] PDF set, the CDF electron asymmetry disagrees with predictions made using the same PDF set. This apparent inconsistency may indicate biases which have not been accounted for in the method.

In this paper the feasibility of measuring directly the \( W \) production charge asymmetry at the LHC is investigated for the first time using simulated data. The expected sensitivity to the PDFs for such a measurement is compared to that for a measurement of the lepton asymmetry.

The paper is structured as follows: the simulated data samples used for this study are summarised in section 2; the procedure used to measure the \( W \) asymmetry is described in section 3. Problems specific to the LHC environment are described and modifications to the reconstruction procedure proposed in sections 4 and 5. The performance of the modified method is studied in greater detail and the expected statistical and systematic uncertainties on the \( W \) asymmetry are compared to the expected statistical uncertainties on the lepton asymmetry for different luminosities in section 6. Finally, the implications of this study for early measurements of \( W \) production charge asymmetries at the LHC are discussed in section 8.

2. Simulation of \( W \) production

The studies in this paper are based on samples generated using the Monte Carlo (MC) generators Pythia 8.120 [19] and Herwig++ 2.4.0 [20, 21]. The Pythia sample is a simulation of the process \( f \bar{f}' \rightarrow W \), where \( f \) and \( f' \) are fermions, at leading order (LO) in the strong coupling. The Herwig++ sample is a simulation of the \( W \) production process at next-to-leading order (NLO) in the strong coupling. As such, it represents the state-of-the-art in simulation of Drell-Yan vector boson production. The positive weight NLO matching scheme (POWHEG) [22, 23] was used in the generation of the Herwig++ sample. This approach consistently combines the NLO calculation and parton shower simulation and was implemented in Herwig++ by Hamilton et al. [24].

The PDFs used for the generation and the size of the samples used are specified in the appropriate sections of the paper. No detector simulation is applied. The following fiducial cuts are used in the event selection in order simulate the detector acceptance:
• a cut on the transverse momentum of the lepton, $P_T > 25$ GeV,

• a cut on the missing transverse momentum (the transverse momentum of the neutrino) in the event $\not{E}_T > 25$ GeV,

• a cut on the lepton pseudorapidity $|\eta^l| < 2.4$.

3. Analysis Technique

The procedure used to extract the $W$ production charge asymmetry as a function of the $W$ boson rapidity, $y_W$, follows closely that used by Bodek et al. [16]. It can be broken down into the following steps:

1. **Calculation of the $W$ rapidity solutions:** For each reconstructed event, the two possible $W$ rapidity values (referred to as “solutions”) are calculated from the missing transverse momentum in the event and the charged lepton momentum using a constraint on the $W$ boson mass.

2. **Weighting of the $W$ rapidity solutions:** Event-by-event, the solutions are filled into separate $y_W^+$ and $y_W^-$ histograms, using weights calculated from MC input as described in section [4]. If only one physical solution is found, the solution is filled with a weight of 1. Step 1 (calculation of the solutions) and step 2 (weighting of the solutions) are hereafter referred to as **full kinematic $W$ reconstruction**.

3. **Acceptance corrections:** The rapidities of the kinematically fully reconstructed $W$ bosons are corrected in each bin of $y_W$ for the detector acceptance and also for biases of the full kinematic $W$ reconstruction arising from the weighting procedure (as discussed in section [4]). The acceptance corrections are calculated bin-by-bin as the ratio between the generated $y_W^\pm$ distributions after cuts and full kinematic $W$ reconstruction and the MC generated $W$ distributions before cuts:

$$ Acc_{(\text{bin} \ i)} = \frac{\text{MC events after cuts and with full kinematic } W \text{ reconstruction}_{(\text{bin} \ i)}}{\text{MC events before cuts}_{(\text{bin} \ i)}} $$(3.1)

The acceptance correction is applied by multiplying $1/\text{Acc}_{(\text{bin} \ i)}$ with the content of bin $i$ of the $y_W$ distributions of the fully reconstructed $W$ bosons. Step 3 yields the **reconstructed $y_W$ distributions**, extrapolated to the whole phase space.

4. **Compare reconstructed and MC-input $y_W$ distributions:** If the reconstructed $y_W$ distributions and the MC input $y_W$ distributions are consistent with each other then the procedure stops (go to step 7). If they disagree, then the procedure must be iterated, with the MC input reweighted to describe the reconstructed distribution (go to step 5).
5. **Reweight input MC to reproduce the reconstructed $y_W$ distributions**: The input MC $y_W$ distributions without any cuts applied and the $y_W$ distributions reconstructed in step 1-3 are compared. A reweighting factor is extracted bin-by-bin:

$$r(\text{bin } i) = \frac{\text{experimentally determined } y_W^{(\text{bin } i)}}{\text{Input MC } y_W^{\text{true}(\text{bin } i)}}$$  \hspace{1cm} (3.2)

Each event in the input MC is re-weighted with the event weight $r$ ($y_W^{\text{true}}$ (MC input)).

6. **Recalculation of weights**: The weights used in step 2 and the acceptance corrections (step 3) are recalculated from the reweighted MC input and the steps 1-4 are repeated until convergence. This iteration reduces the dependence on the MC input.

7. **Measurement of the $W$ asymmetry**: When the reconstructed and modified input MC $y_W$ distributions are in agreement within their statistical uncertainty, the $W$ asymmetry is considered to be determined.

4. **Full kinematic $W$ reconstruction: weighting of the $W$ rapidity solutions**

In the weighting procedure [10] (step 2 from section 3) the twofold ambiguity for $p_T^\nu$ and thus for $y_W$ is resolved statistically with the help of MC predictions. The possible rapidity solutions $s_1$ and $s_2$ are weighted with their respective probabilities ($P_1$ and $P_2$) and the total probability is normalised to unity, $P = P_1 + P_2 = 1$. The probabilities of each solution occurring are derived, based on:

1. **Expected cross sections $d\sigma/dy_W$**: The cross sections $d\sigma/dy_W$ have a distinct behaviour as a function of $y_W$. The two different rapidity solutions can be compared as to which is more probable. The expected cross sections $d\sigma/dy_W$ predicted by Monte Carlo are used as probability density functions to determine this.

2. **Expected lepton decay angle and anti-quark/quark ratios**: The lepton decay angle in the $W$ rest frame, $\cos \theta^*$, follows a $(1 \pm \cos \theta^*)^2$ distribution, which can be used as another probability density function to weight the two rapidity solutions. The sign of the $(1 \pm \cos \theta^*)^2$ distribution depends on the helicity of the lepton and the helicity of the incoming parton with highest-x, hereafter referred to as “higher-x (anti-)quarks”. Therefore in the $\cos \theta^*$ weighting, the probability density function is built from the contributions of events with higher-x quarks $(1 \pm \cos \theta^*)^2$ and of events with higher-x anti-quarks $(1 \pm \cos \theta^*)^2$. The probability density function for the weighting is constructed as the sum of the two decay distributions. The contribution of the higher-x anti-quark events is adjusted using the parameter $\bar{q}/q$, which is defined as

$$\frac{\bar{q}}{q}(p_T^W, y_W) = \frac{\text{number of higher-x anti-quarks } (p_T^W, y_W)}{\text{number of higher-x quarks } (p_T^W, y_W)}$$  \hspace{1cm} (4.1)
The ratio $\frac{\bar{q}}{q}$ is a function of $p_T^W$ and $y_W$, it is determined from Monte Carlo simulations and therefore also depends on PDFs. Using $\frac{\bar{q}}{q}$, one can build the total probability density function from the sum of the two possible angular distributions for the LHC

$$P(\cos \theta^*) = (1 \pm \cos \theta^*)^2 + \frac{\bar{q}}{q}(p_T^W, y_W)(1 \pm \cos \theta^*)^2 \tag{4.2}$$

and similarly for the Tevatron \[16\].

The probability density function for the cross section and the decay angle can be combined to calculate weighting factors, $w_1$ and $w_2$, for $s_1$ and $s_2$, depending on the charge of the $W^\pm$:

$$w_{1,2}(W^\pm) = \frac{P(\cos \theta^*_{l^\pm, W^\pm}) \times (d\sigma/dy_{W^\pm})}{\sum_{i=1}^{2} \left(P(\cos \theta^*_{l^\pm, W^\pm}) \times (d\sigma/dy_{W^\pm})\right)} \tag{4.3}$$

In a reconstructed event each of the solutions 1 and 2 is weighted by $w_1$ and $w_2$ respectively and filled with that weight into histograms of $y_{W^\pm}$.

5. Performance of the kinematic $W$ reconstruction

The performance of the weighting and reconstruction of the $W$, steps 1 and 2 in the iterative procedure, was studied. This study was performed using $8 \times 10^6$ PYTHIA MC events generated with the MSTW08 NLO PDF set \[3\] for Tevatron ($p\bar{p}$, $\sqrt{s} = 1.96$ TeV) and LHC conditions ($pp$, $\sqrt{s} = 14$ TeV). The analysis technique was tested using the same MC sample as MC input and as ‘pseudo-data’. The transverse components of the neutrino’s momentum, $p_{\nu \perp x}$ and $p_{\nu \perp y}$ were taken directly from the MC. No acceptance corrections were applied for this part of the study.

5.1 Performance at the Tevatron

Figure \[3\] shows the weight assigned to the rapidity solution calculated in step 1 that was closest to the true rapidity. The figure shows the normalised distribution only for $W^-$ events but is very similar for $W^+$ events. The distribution exhibits a peak at 1 and lies mostly above 0.5, indicating that in the weighting procedure the solution closest to the true $y_W$ value is given the larger weight.

Figure \[3\] a) shows the asymmetry distributions for the Tevatron. Shown are the true $W$ asymmetry distributions from the MC with and without cuts. Also depicted is the $W$ asymmetry that arises when only the $W$ solution closest to the true $y_W$ value is used and the $W$ asymmetry based on the full kinematic reconstruction. All asymmetries are in agreement up to $y_W = 1.5$, beyond that there is some disagreement between the reconstructed and the true $W$ asymmetry. This can be better seen in figure \[3\] b), which shows the ratio of the fully reconstructed $W$ asymmetry (without acceptance corrections) and the true asymmetry. The difference between the reconstructed and the true asymmetry in the region defined by the acceptance cuts is only around 20%. The true asymmetry before cuts is at most 80% to 100% larger than the reconstructed asymmetry. Hence it can be
Figure 1: Normalised distributions of weights given to the rapidity solution that is closest to true $W^{-}$ rapidity in the Tevatron environment.

Figure 2: This figure shows the asymmetry as evaluated at various stages of the iterative weighting procedure (a). In b) the ratios of reconstructed asymmetry and true asymmetries (before and after applying cuts) are depicted.

seen that acceptance corrections correct not only for genuine detector acceptance effects but also for biases of the full kinematic reconstruction. However, the former are larger and more important than the latter.

5.2 Performance at the LHC

It is not immediately obvious that it is possible to transfer the full kinematic reconstruction technique to the LHC because of crucial differences between the Tevatron and LHC environments. The Tevatron is a $p\bar{p}$ collider operating at centre-of-mass energy $\sqrt{s} =1.96$ TeV, while the LHC is due to collide $pp$ beams at $\sqrt{s}$ up to 14 TeV. These differences pose a major challenge when applying the weighting method to the LHC environment. Colliding $pp$ beams, will break the mirror symmetry between $W^{+}$ and $W^{-}$. The asymmetry in $pp$ collisions is the same at positive and negative rapidities. At the LHC $W^{+}$ bosons are produced almost twice as often as $W^{-}$ and generally with a larger boost along the $Z$ axis. Hence the performance of the method will differ in the two environments.
The full kinematic reconstruction and the weights used were studied using a Pythia MC with the MSTW08 NLO PDF set. As already described above, the two solutions calculated in step 1, were weighted in step 2 using two basic considerations.

- **Shape of cross section as function of $y_W$:** At the LHC the cross section for $W$ production is much flatter and expands over a much larger range of $y_W$ than at the Tevatron. This is shown in figure 3. There is no peak in the $y_W$ distributions for central rapidities and the cross section begins to fall only above $y_W > 1.5 - 2.0$. Therefore, if both reconstructed solutions are within $-1.5 < y_W < 1.5$, both are equally probable. In particular, the reconstruction of $W^+$ rapidities suffers from this fact, since the longitudinal boost of $W^+$ bosons is generally larger than the boost of $W^-$, and therefore the rapidity distribution extends to higher $y_W$.

- **Ratios of leading $\bar{q}$ versus leading $q$:** A further problem is the contribution of $W$'s produced with a higher-$x$ anti-quark. If the higher-$x$ parton participating in the Drell-Yan $W$ production is an anti-quark, the $W$ is no longer produced with a boost parallel to the incoming quark, but antiparallel to the incoming quark. This introduces a change of sign in the expected $\cos \theta^*$ decay angle. In the weighting procedure this is accounted for by constructing the $\cos \theta^*$ weight such that it combines a weight for leading quark and leading anti-quark $W$ production according to their relative contributions by using the ratio $\bar{q}/q$ as a function of $p_T^W$ and $y_W$, as in equation (3).

At Tevatron centre of mass energies, leading quark production is indeed the most probable process, with $\bar{q}/q = 0.25$ at most. At the LHC however, the ratio $\bar{q}/q$ even extends to values as high as 1 as shown in figure 3. This introduces a strong ambiguity to the $\cos \theta^*$ weighting. This effect is particularly problematic for $W^-$ production because the ratio of anti-quark induced processes is higher than for $W^+$ production. The effect is also shown in figure 4, where the $\cos \theta^*$ distributions of $W^-$ (left) and $W^+$ (right) bosons are shown for $W$ production at the LHC. For the $W^-$ distributions, the contribution of higher-$x$ anti-quarks is especially large and two solutions with $\cos \theta^* \sim \cos \theta^*$ cannot be distinguished. Solutions with $\cos \theta^* \sim \cos \theta^*$ or $\cos \theta^* \sim \cos \theta^*$ are in fact most common and in these cases both solutions will be weighted with factors of about 0.5.

Figure 5 displays the weight given to the rapidity solution that was closest to the correct rapidity at the LHC. While at the Tevatron, this distribution peaks at 1 and is mostly larger than 0.5 (as shown in fig. 1), at the LHC most weights are around 0.5. For $W^-$ production the weights given to the right solution are on average at least marginally larger than 0.5, however for $W^+$ rapidities it is narrow and symmetric around 0.5, thus reducing the impact of the weighting procedure. The larger spread of $W^-$ weights comes from the different expected $d\sigma/dy_W$ rather than decay angle effects.

Again it is instructive to evaluate the asymmetry distribution at the various stages of the iterative weighting procedure. This is done in figure 6 a). Additionally, figure 6 b) depicts the ratio of the fully reconstructed asymmetry distribution to the true asymmetry distributions. The figure shows that in the central region, the corrections for intrinsic
Figure 3: Problems of the weighting procedure at the LHC: a) The normalised $y_{W\pm}$ distributions are flatter at the LHC (full markers) compared to the Tevatron (open markers). The ratios $\bar{q}/q$ used in the weighting procedure at the LHC are shown for $W^-$ in b) and for $W^+$ in c). For both charges, the ratio is considerably larger than 0.25, which is the maximal value at the Tevatron \cite{16}.

Figure 4: $\cos \theta^*$ distributions at the LHC. The total of the higher-$x$ quark (dotted line) and higher-$x$ anti-quark (dashed line) distributions is shown.

systematic biases of the full kinematic reconstruction dominates, while in the forward region genuine acceptance corrections dominate. Comparing this figure to the equivalent plots for the Tevatron (figure 2 b) reveals that the acceptance corrections are more important at the LHC than at the Tevatron. While at the Tevatron the ratio of the reconstructed asymmetry and the true asymmetry with cuts is between 0.8-1.1, the same ratio is 0.5-3.0 at the LHC. Equally the ratio of the reconstructed asymmetry and the true asymmetry without cuts is, at 1.5-3.0, larger at the LHC than at the the Tevatron (0.8-1.8).
Figure 5: The weight given to reconstructed rapidity solution closest to the true value of $y_W$ for $W^-$ and $W^+$ at the LHC.

Figure 6: a) the asymmetry as evaluated at various stages of the iterative weighting procedure. b) the ratios of reconstructed asymmetry and true asymmetries (with and without cuts).

6. Modification of the analysis technique for the LHC

At the LHC the full $W$ kinematic reconstruction suffers because in this kinematic region it is difficult to favour one rapidity solution over another. Even with the best possible weights, it would not possible to restore the true asymmetry perfectly. At both the Tevatron and the LHC, acceptance corrections compensate for intrinsic kinematic biases in the full kinematic reconstruction. As a result of these features a new simplified analysis technique for the LHC can be proposed: in this new simplified approach, the construction of weights for the full kinematic reconstruction is bypassed and instead each solution is filled with a weight of 0.5, equivalent to randomly choosing either solution. This approach relies then solely on the acceptance correction, which is then also the only step dependent on MC input. This simplifies the determination of the systematic errors of the method. The acceptance corrections should be applied in bins of $y_W$ and $p_T^W$. The iteration procedure with reweighting of the MC input is still necessary to remove the MC dependence. This approach is hereafter referred to as the “new scheme”.
7. Performance of the modified analysis technique at the LHC

The expected systematic uncertainties of a measurement in the new scheme were evaluated in comparison to the uncertainty on the predicted $W$-asymmetry and in comparison to the lepton asymmetry. Here, only the expected statistical errors and the inherent uncertainty of the modified $W$ measurement method are considered, other potential sources of experimental uncertainty are not taken into account.

7.1 Uncertainties on the acceptance corrections due to statistical fluctuations

It is not trivial to propagate the uncertainties from statistical fluctuations in the data sample and the acceptance corrections to a total statistical uncertainty on the $W$ asymmetry. The method reshuffles the number of events in the bins and might amplify statistical fluctuations as was also observed for the original implementation of the method [25].

In order to estimate the statistical uncertainties on the $W$ asymmetry, 450 toy experiments were carried out (375 in the case where the toy experiments contained $5 \times 10^6$ events each). Two Pythia samples generated with the CTEQ66 PDF, $\sqrt{s} = 14$ TeV were used. The MC input sample consisted of $18 \times 10^6$ events and was used to calculate the acceptance corrections. 450 sub samples of $1 \times 10^5$, $5 \times 10^5$, $1 \times 10^6$ and $5 \times 10^6$ events each were drawn randomly from the pseudo-data sample of $75 \times 10^6$ events, corresponding to integrated luminosities of 5.4, 27, 54 and 270 pb$^{-1}$ respectively. These sub-samples were not exclusive, no attempt was made to exclude events already used in one toy data sample from other toy samples of the same luminosity.

For each of the toy “data” samples, the $W$ rapidity solutions were calculated for each event and filled with a weight of 0.5. Next the acceptance corrections derived from the MC input sample were applied. The spread of the measured asymmetries in each bin of $y_W$ was then interpreted as the statistical uncertainty on the measured $W$ asymmetry. Figure 7 a) shows this spread for the toy ’data’ samples. The entries belonging to different bins in $y_W$ are shown in different grey tones. Figure 7 b) depicts the $W$ asymmetry with the uncertainty extracted in the toy experiments as bands for the different integrated luminosities. Figure c) shows the relative uncertainties on the $W$ asymmetry as a function of $y_W$.

7.2 Uncertainties on the acceptance corrections from PDFs

Uncertainties on the acceptance correction arising from the chosen MC $y_{W\pm}$ input distributions differing from those in data due to the underlying PDF distributions were also considered. In this study the CTEQ66 and MSTW08 PDF distributions were used. To estimate the uncertainty the PDF error sets based on the standard Hessian technique [6] were used.

Figure 8 shows the measured asymmetries and the uncertainty on them arising from PDF uncertainties for the CTEQ66 and the MSTW08 PDF sets, both shown as a ratio to the central value of CTEQ66. Around the PDF predictions and their uncertainties, a dot-dashed envelope is drawn, which is taken to give the PDF uncertainty on the asymmetry measurement due to the acceptance corrections. This error band is also centred around
Figure 7: a) the number of entries for each measured $W$ asymmetry. b) The statistical uncertainty corresponding to the spread as an absolute uncertainty on the asymmetry. c) The statistical uncertainty as a relative uncertainty on the asymmetry.

![Number of entries for each measured W asymmetry](image1)

![W asymmetry with statistical uncertainty](image2)

![Relative uncertainty on W asymmetry](image3)

**Figure 8:** The uncertainties on the measured $W$ asymmetry from statistics and from the PDF uncertainties in two different bins of $p_T^W$.

the CTEQ66 value and drawn as a grey band. Figure 8 shows on the left the relative PDF uncertainty on the asymmetry for $1.15 < p_T^W < 2.47$ GeV and for $11.3 < p_T^W < 24.1$ GeV on the right. For these and also the other low and mid-$p_T^W$ bins the uncertainties are between 5-12%, for larger values of $p_T^W$, the uncertainty tends to decrease.

7.3 Experimental uncertainty due to the detector

In order to estimate the effect of the reconstruction of the $W$ events in the detector, the
four-vectors of the generator level particles were smeared to approximate the effect of differing MC and data resolutions. This was done by applying a Gaussian resolution factor to the $p_x$ and $p_y$ variables of the neutrino and to the $p_x$, $p_y$ and $p_z$ variables of the lepton. These resolution factors were drawn randomly and independently for each of the variables from a Gaussian distribution centered around one with a width of 1% (lepton) and 5% (neutrino) which represent realistic uncertainties on the resolutions at the LHC \cite{26}. This was done 500 times for the same $\approx 16 \times 10^6$ Pythia events. The reconstructed asymmetry was corrected with the nominal acceptance corrections derived from the unsmeared sample. In a sizeable number of events, as a result of the smearing, no physical solution could be obtained for $y_W$. In these cases, the neutrino $p_T$ is scaled down in steps of 0.1 GeV until a valid solution is found, following the prescription of the CDF measurement, where the $E_T$ was assumed to be overestimated and scaled down \cite{15}.

Due to the fact that the $p_T$ spectrum of leptons is correlated with $\eta$, there is a slight bias in the mean of the smeared lepton asymmetry distributions with regard to the nominal (unsmearred) distribution. A similar but larger bias can also be observed in the measured $W$ asymmetry distribution. When calculating the experimental uncertainty, the bias of the mean of the 500 smeared distributions with regard to the nominal distribution is added in quadrature to the RMS of the 500 smeared distributions for both the lepton and the $W$ asymmetry.

7.4 Application of the modified scheme to pseudo-data

In the following the weighting procedure of the modified scheme is investigated for the LHC environment. Two cases were studied:

1. **Different PDFs**: A Pythia sample generated using the MSTW08 PDF ($8 \times 10^6$ events) was used as pseudo-data and a Pythia sample generated using the CTEQ66 PDF ($75 \times 10^6$ events) was used as MC input. 5 iteration steps were used.

2. **Higher orders**: An Herwig++ $W$ production sample generated using the MSTW08 PDF ($1 \times 10^6$ events) was used as pseudo-data and a Pythia sample generated using the MSTW08 PDF ($18 \times 10^6$ events) was used as MC input. 5 iteration steps were used.

Figure 3 a) compares the true $W$ asymmetries for the CTEQ66 MC input and the MSTW08 pseudo-data to the reconstructed $W$ asymmetries as obtained in the iterations steps 1-5. The reconstructed $W$ asymmetries from all of the iteration steps agree better with the MSTW08 pseudo-data than with the CTEQ66 MC input, showing that the method still works. This can also be seen in figure 3 b) which shows the ratio of the reconstructed $W$ asymmetries at each iteration step to the true $W$-asymmetry of the MSTW08 pseudo-data. The first iterations agree within 7%, for the other iterations larger deviations are visible. This is an effect of statistical fluctuations, the iteration procedure should be sensibly stopped after the first or second iteration for a measurement. This can be seen in figure 3 c), which shows the ratio of the reconstructed $W$ asymmetries of the iteration steps to the $W$ asymmetry of the specific previous iteration step or the input MC asymmetry in the case of
Figure 9: The true $W$ asymmetries for LO MC input (CTEQ66 PDF, triangles) and pseudo-data (MSTW08, circles) are compared a) to the reconstructed $W$ asymmetries as obtained in the iterations steps 1-5. b) depicts the ratio of the reconstructed $W$ asymmetries of the iteration steps to the true $W$ asymmetry of the pseudo-data. c) shows the ratio of the reconstructed $W$ asymmetries of the iteration steps to the $W$ asymmetry of the preceding iteration step.

a) $W$ asymmetry of MC input, pseudo-data and iteration steps

b) Ratio of reconstructed $W$ asymmetry with the true $W$ asymmetry of the pseudo-data.

c) Ratio of reconstructed $W$ asymmetry with the reconstructed $W$ asymmetry of the preceding iteration step.

the first iteration). Except for the first iteration step, all of the other iteration steps agree very well with the reconstructed asymmetries from the preceding iterations indicating the convergence of the iterative procedure. This demonstrates that the modified reweighting scheme is able to provide a good measurement of the $W$ asymmetry.

An indication for when the iteration should be stopped can also be taken from this last plot: As long as the ratio of the reconstructed $W$ asymmetry of iteration step $n$ to the reconstructed $W$ asymmetry of iteration step $n - 1$ grows significantly closer to one with every iteration $n$, the iteration still converges. If the ratio does not change significantly from some constant numbers (as e.g. in the first bin of $y_W$) or starts to deviate from one, the iteration starts to diverge and should be stopped well before this happens.

All MC codes only simulate the physics process up to a certain order in QCD. The impact of higher order QCD effects was investigated by studying the analysis technique using NLO pseudo-data generated with Herwig++ and LO MC input generated with PYTHIA each using the same PDF set. Figure II a) compares the true $W$ asymmetries for the LO MSTW08 MC input and the NLO MSTW08 pseudo-data to the reconstructed $W$ asymmetries. Again, the reconstructed $W$ asymmetries agree well with the true $W$ asymmetry of the pseudo-data, as also seen in figure II b). The agreement is of the level of 5% for the first iteration. Statistical effects are larger due to the size of the data sets (1 million events for the NLO MSTW08 pseudo-data and 18 million events for the LO MSTW08 MC input). This can also be seen in figure II c), where the deviations of the iteration step
from the previous iteration step are quite small for the first and the second iteration and then become systematically larger. Since this indicates a divergence, that grows with the number of iteration steps, the iteration should be stopped after the first or second iteration. Apart from the large statistical fluctuations, no significant systematic effects were encountered from higher order QCD.

8. Comparison of Uncertainties on the Lepton and $W$ asymmetry

In order to assess the possible advantage of the direct reconstruction of the $W$ asymmetry in comparison to a measurement of the lepton asymmetry, the expected statistical and systematic uncertainties on measurements of the two observables were compared to the uncertainty on the respective predictions arising from the PDFs. Here, for the $W$ asymmetry, the new scheme as described in section 6 was used and acceptance cuts were applied. For the lepton asymmetry, the same acceptance cuts were applied as would be the case for the usual experimental determination of this quantity.

Table 1 compares the relative uncertainties in detail for the $W$ and lepton asymmetries including experimental uncertainties estimated using Gaussian resolution functions of 1% for the lepton $P_T$ and 5% for the $\not{E}_T$.

Table 2 compares the uncertainties on the theoretical predictions from the PDFs ($\sigma_{PDF}$) to the total uncertainties on the measurement (both with and without the experimental uncertainty due to the detector resolution). Here, the measured $W$ asymmetry refers to
Table 1: Comparison of the relative uncertainties on the measured $W$ asymmetry and the lepton asymmetry for a sample corresponding to an integrated luminosity of 268 pb$^{-1}$. The statistical uncertainty in the bins is denoted $\sigma_{\text{stat}}$, uncertainties on acceptance corrections from PDFs are denoted $\sigma_{\text{acc}}$ (c.f section 3, step 3). Uncertainties from detector resolution are denoted as $\sigma_{\text{det}}$. The total uncertainty neglecting $\sigma_{\text{det}}$ is labeled $\sigma_{\text{meas}}^{\text{th}}$ and the total uncertainty including resolution effects $\sigma_{\text{meas}}^{\text{tot}}$.

| $y_W$ or $\eta$ | $\sigma_{\text{stat}}(W)$ | $\sigma_{\text{acc}}(W)$ | $\sigma_{\text{det}}(W)$ | $\sigma_{\text{meas}}^{\text{th}}(W)$ | $\sigma_{\text{meas}}^{\text{tot}}(W)$ | $\sigma_{\text{stat}}(l)$ | $\sigma_{\text{det}}(l)$ | $\sigma_{\text{meas}}^{\text{tot}}(l)$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 - 0.6        | 3.47           | 10.1           | 9.61           | 10.7           | 14.4           | 1.49           | 4.04           | 4.3            |
| 0.6 - 1.2      | 2.52           | 8.31           | 6.51           | 8.68           | 10.9           | 1.24           | 2.64           | 2.92           |
| 1.2 - 1.8      | 1.54           | 4.9            | 0.699          | 5.13           | 5.18           | 0.921          | 1.46           | 1.73           |
| 1.8 - 2.4      | 1.19           | 3.3            | 1.77           | 3.51           | 3.93           | 0.695          | 0.925          | 1.16           |
| 2.4 - 3        | 1.02           | 4.38           | 3.96           | 4.5            | 5.99           | -             | -             | -              |

The $W$ asymmetry extracted in the last step of the analysis technique (c.f section 3, step 7) with the modified scheme described in section 3. Table 2 also gives the ratios of the errors on the measurement, $\sigma_{\text{meas}}^{\text{tot}}$, to $\sigma_{\text{PDF}}$ for the $W$ asymmetry, $\mathcal{R}(W)$, and lepton asymmetry, $\mathcal{R}(l)$. These ratios give an estimate of how well a measurement could be used to constrain the PDFs. Also shown is the ratio of these ratios, $\mathcal{R}(W/l) = \mathcal{R}(W)/\mathcal{R}(l)$. This is an estimate of how the observables perform in comparison to each other, with a ratio of 0.3 predicting the $W$ asymmetry measurement to be only a third as well suited to constrain PDFs as the lepton asymmetry for all $y_W$ when the uncertainties without the experimental uncertainty are taken into account. The detailed numbers for different $y_W$ bins are given in table 2 and in which they are calculated for 268.1 pb$^{-1}$ integrated luminosity and are the relative uncertainties.

The numbers are crude estimates since only the generator level is studied. However, the message from table 2 is clear: the measurement of the lepton asymmetry will be much more appropriate to constrain the PDFs, regardless of whether only the inherent theoretical uncertainties from the method are considered or whether experimental uncertainties are also taken into account. Measuring the $W$ asymmetry has a power to constrain PDFs which is only 33% as good as a measurement of the lepton asymmetry, when only the inherent theoretical uncertainties on the method are taken into account (c.f. $\mathcal{R}(W/l)\sigma_{\text{PDF}}/\sigma_{\text{meas}}^{\text{th}}$). Taking into account experimental resolution effects, the lepton asymmetry still performs twice as well compared to the $W$ asymmetry measurement (c.f. $\mathcal{R}(W/l)\sigma_{\text{PDF}}/\sigma_{\text{exp}}^{\text{meas}}$). The ratio $\mathcal{R}_W/\mathcal{R}_l$ tends to decrease with higher luminosity.

It should be noted again that the situation at the Tevatron is more favourable for the measurement of the $W$ asymmetry. Since the lepton asymmetry at the Tevatron tends to be smaller than the $W$ asymmetry, at the Tevatron the lepton asymmetry is favoured if the systematic uncertainties are negligible or of the same absolute size on the $W$ and the lepton asymmetry.

9. Conclusions

In this paper the applicability of the analysis technique first developed for the Tevatron
Table 2: Comparison of the relative uncertainties on the measured W and lepton asymmetries and the uncertainties on the prediction arising from PDFs for a sample corresponding to an integrated luminosity of 268 pb$^{-1}$. The uncertainties on the theoretical predictions for the W and lepton asymmetry are denoted as $\sigma_{PDF}(W)$ and $\sigma_{PDF}(l)$ respectively. Ratios of uncertainties for W and lepton asymmetries are denoted as $R(W)$ or $R(l)$ respectively. Finally double ratios $R(W/l)$ are denoted as $R(W/l)$.

| $y_W$ or $\eta_l$ | $\sigma_{PDF}(W)$ | $\sigma_{PDF}(W)$ | $\sigma_{PDF}(W)$ | $\sigma_{PDF}(l)$ | $\sigma_{PDF}(l)$ | $\sigma_{PDF}(l)$ | $\sigma_{PDF}(l)$ | $\sigma_{PDF}(l)$ | $\sigma_{PDF}(l)$ | $\sigma_{PDF}(l)$ |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 0 - 0.6           | 24.5              | 2.29              | 1.7               | 11.9              | 8                 | 2.77              | 0.287             | 0.616             |                   |                   |
| 0.6 - 1.2         | 20.7              | 2.39              | 1.91              | 11.2              | 9.07              | 3.85              | 0.263             | 0.496             |                   |                   |
| 1.2 - 1.8         | 15.1              | 2.95              | 2.92              | 9.69              | 10.5              | 5.6               | 0.281             | 0.521             |                   |                   |
| 1.8 - 2.4         | 11.7              | 3.32              | 2.97              | 7.75              | 11.1              | 6.7               | 0.298             | 0.443             |                   |                   |
| 2.4 - 3           | 8.39              | 1.87              | 1.4               | -                 | -                 | -                 | -                 | -                 |                   |                   |

by Bodek et al. [10] to the LHC environment has been studied. The method is inherently less well suited for LHC conditions and a modification to the method has been proposed. This modification was found to be more robust and relies on MC input only for one step. The statistical and systematic uncertainties on the measurement of the W asymmetry were determined and compared to the PDF uncertainties on the prediction from theory. The expected experimental uncertainty on the measured W asymmetry is smaller than the theoretical uncertainty. However, the expected experimental uncertainty on the lepton asymmetry is even smaller in comparison to its theoretical uncertainties and thus better suited to constrain PDFs at the LHC. An estimate of the likely impact of experimental uncertainties due to the detector resolution were taken into account and the lepton asymmetry was still found to perform better than the W asymmetry by a factor of roughly 2. It is hence the opinion of the authors that early measurement at the LHC should focus on the lepton asymmetry rather than the direct W asymmetry to constrain the PDFs. The studies have been performed for a centre of mass energy of $\sqrt{s} = 14$ TeV. However, the main arguments hold also at $\sqrt{s} = 7$ TeV, so that these conclusions are not likely to change with the centre of mass energy.

Acknowledgements

This work was supported by the UK STFC. The authors would like to thank Yeon Chung, Bo-Young Han, Kevin McFarland and Arie Bodek for helpful clarifications of implementation of the method at the Tevatron. It is a pleasure to thank Claire Gwenlan for useful comments on the manuscript and to thank Eva Halkiadakis and Junjie Zhu for information about the comparison of the Tevatron W-asymmetry to the lepton asymmetry.

K. Lohwasser gratefully acknowledges support from Deutscher Akademischer Austauschdienst (DAAD).

References

[1] CDF Collaboration, F. Abe et al., Measurement of the lepton charge asymmetry in W boson
decays produced in pp collisions, Phys. Rev. Lett. 81 (1998) 5754–5759, [hep-ex/9809001].

[2] CDF Collaboration, D. E. Acosta et al., Measurement of the forward-backward charge asymmetry from \( W \rightarrow e\nu \) production in pp collisions at \( \sqrt{s} = 1.96 \) TeV, Phys. Rev. D71 (2005) 051104, [hep-ex/0501023].

[3] D0 Collaboration, V. M. Abazov et al., Measurement of the muon charge asymmetry from W boson decays, Phys. Rev. D77 (2008) 011106, [arXiv:0709.4254].

[4] D0 Collaboration, V. M. Abazov et al., Measurement of the electron charge asymmetry in pp → W + X → eν + X events at \( \sqrt{s} = 1.96 \)-TeV, Phys. Rev. Lett. 101 (2008) 211801, [arXiv:0807.3367].

[5] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Parton distributions for the LHC, arXiv:0901.0002.

[6] J. Pumplin et al., New generation of parton distributions with uncertainties from global QCD analysis, JHEP 07 (2002) 012, [hep-ph/0201195].

[7] R. D. Ball et al., A first unbiased global NLO determination of parton distributions and their uncertainties, arXiv:1002.4407.

[8] E. L. Berger, F. Halzen, C. S. Kim, and S. Willenbrock, Weak boson Production at Tevatron energies, Phys. Rev. D40 (1989) 83.

[9] ZEUS Collaboration, S. Chekanov et al., Measurement of charged current deep inelastic scattering cross sections with a longitudinally polarised electron beam at HERA, arXiv:0812.4620.

[10] ZEUS Collaboration, S. Chekanov et al., Measurement of high-Q^2 neutral current deep inelastic e^-p scattering cross sections with a longitudinally polarised electron beam at HERA, arXiv:0901.2385.

[11] H1 Collaboration, C. Adloff et al., Measurement and QCD analysis of neutral and charged current cross sections at HERA, Eur. Phys. J. C30 (2003) 1–32, [hep-ex/0304003].

[12] H1 Collaboration, F. D. Aaron et al., A Precision Measurement of the Inclusive ep Scattering Cross Section at HERA, Eur. Phys. J. C64 (2009) 561–587, arXiv:0904.3513.

[13] H1 and ZEUS Collaboration, F. D. Aaron et al., Combined Measurement and QCD Analysis of the Inclusive ep Scattering Cross Sections at HERA, JHEP 01 (2010) 109, arXiv:0911.0884.

[14] S. Catani, G. Ferrera, and M. Grazzini, W boson production at hadron colliders: the lepton charge asymmetry in NNLO QCD, arXiv:1002.3115.

[15] CDF Collaboration, T. Aaltonen et al., Direct Measurement of the W Production Charge Asymmetry in pp Collisions at \( \sqrt{s} = 1.96 \) TeV, Phys. Rev. Lett. 102 (2009) 181801, arXiv:0901.2169.

[16] A. Bodek, Y. Chung, B.-Y. Han, K. McFarland, and E. Halkiadakis, New analysis technique to measure the W production charge asymmetry at the Fermilab Tevatron, Phys. Rev. D77 (2008) 111301, arXiv:0711.2859.

[17] R. S. Thorne, A. D. Martin, W. J. Stirling, and G. Watt, The effects of combined HERA and recent Tevatron W→ lepton neutrino charge asymmetry data on the MSTW PDFs, arXiv:1006.2753. IPPP/10/44, Cavendish-HEP-10/12, CERN-PH-TH/2010-135.
[18] P. M. Nadolsky et al., Implications of CTEQ global analysis for collider observables, *Phys. Rev.* **D78** (2008) 013004, [arXiv:0802.0007](http://arxiv.org/abs/0802.0007).

[19] T. Sjostrand, S. Mrenna, and P. Skands, A Brief Introduction to PYTHIA 8.1, *Comput. Phys. Commun.* **178** (2008) 852–867, [arXiv:0710.3820](http://arxiv.org/abs/0710.3820).

[20] M. Bahr et al., Herwig++ Physics and Manual, *Eur. Phys. J.* **C58** (2008) 639–707, [arXiv:0803.0883](http://arxiv.org/abs/0803.0883).

[21] M. Bahr et al., Herwig++ 2.3 Release Note, [arXiv:0812.0529](http://arxiv.org/abs/0812.0529).

[22] P. Nason, A new method for combining NLO QCD with shower Monte Carlo algorithms, *JHEP* **11** (2004) 040, [hep-ph/0409146](http://arxiv.org/abs/hep-ph/0409146).

[23] S. Frixione, P. Nason, and C. Oleari, Matching NLO QCD computations with Parton Shower simulations: the POWHEG method, *JHEP* **11** (2007) 070, [arXiv:0709.2092](http://arxiv.org/abs/0709.2092).

[24] K. Hamilton, P. Richardson, and J. Tully, A Positive-Weight Next-to-Leading Order Monte Carlo Simulation of Drell-Yan Vector Boson Production, *JHEP* **10** (2008) 015, [arXiv:0806.0290](http://arxiv.org/abs/0806.0290).

[25] B.-Y. Han, Measurement of the W Boson Production Charge Asymmetry in pp Collisions. FERMILAB-THESIS-2008-15.

[26] K. Lohwasser, The W Charge Asymmetry: Measurement of the Proton Structure with the ATLAS Detector. CERN-THESIS-2010-069.