MAGNETIC FIELDS IN SUPERNOVAE

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Abstract  
A relatively modest value of the initial rotation of the iron core, a period of $\sim 6 - 31$ s, will give a very rapidly rotating protoneutron star and hence strong differential rotation with respect to the infalling matter. Under these conditions, a seed field is expected to be amplified by the MRI and to grow exponentially. Exponential growth of the field on the time scale $\Omega^{-1}$ by the magnetorotational instability (MRI) will dominate the linear growth process of field line “wrapping” with the same characteristic time. The shear is strongest at the boundary of the newly formed protoneutron star. Modest initial rotation velocities of the iron core result in sub–Keplerian rotation and a sub–equipartition magnetic field that nevertheless produce substantial MHD luminosity and hoop stresses: saturation fields of order $10^{15} - 10^{16}$ G develop $\sim 300$ msec after bounce with an associated MHD luminosity of $\sim 10^{49} - 10^{53}$ erg s$^{-1}$. Bi-polar flows driven by this MHD power can affect or even cause the explosions associated with core-collapse supernovae. If the initial rotation is too slow, then there will not be enough rotational energy to power the supernova despite the high luminosities. The MRI should be active and may qualitatively alter the flow if a black hole forms directly or after a fall-back delay.
Keywords: supernova, jets, MHD

Introduction

Accumulating evidence shows that core collapse supernovae are distinctly and significantly asymmetric. A number of supernova remnants show intrinsic “bilateral” structure (Dubner et al. 2002). Jet and counter jet structures have been mapped for Cas A in the optical (Fesen & Gunderson 1996; Fesen 2001; and references therein), and the intermediate mass elements are ejected in a roughly toroidal configuration (Hughes et al. 2000; Hwang et al. 2000; Willingale et al. 2002). The debris of SN 1987A has an axis that roughly aligns with the small axis of the rings (Pun et al. 2001; Wang et al. 2002). Spectropolarimetry shows that substantial asymmetry is ubiquitous in core-collapse supernovae, and that a significant fraction of core-collapse supernovae have a bi-polar structure (Wang et al. 1996, 2001). The strength of the asymmetry observed with polarimetry is higher (several %) in supernovae of Type Ib and Ic that represent exploding bare non-degenerate cores (Wang et al. 2001). The degree of asymmetry also rises as a function of time for Type II supernovae (from $\lesssim 1\%$ to $\gtrsim 1\%$) as the ejecta expand and the photosphere recedes (Wang et al. 2001; Leonard et al. 2000, 2001). Both of these trends suggest that it is the core collapse mechanism itself that is responsible for the asymmetry.

Two possibilities are being actively explored to account for the observed asymmetries. One is associated with the rotational effect on convection (Fryer & Heger 2000), and another is due to the effect of jets (Khokhlov et al. 1999; Wheeler et al. 2000; Wheeler, Meier & Wilson 2002). Jet calculations have established that non-relativistic axial jets of energy of order $10^{51}$ erg originating within the collapsed core can initiate a bi-polar asymmetric supernova explosion that is consistent with the spectropolarimetry (Khokhlov et al. 1999; Khokhlov & Höflich 2001; Höflich et al. 2001). The result is that heavy elements (e.g. O, Ca) are characteristically ejected in tori along the equator. Iron, silicon and other heavy elements in Cas A are distributed in this way (Hwang et al. 2000), and there is some evidence for this distribution in SN 1987A (Wang et al. 2002). Radioactive matter ejected in the jets can alter the ionization structure and hence the shape of the photosphere of the envelope even if the density structure is spherically symmetric (Höflich et al. 2001). This will generate a finite polarization, even though the density distribution is spherical and the jets are stopped deep within the star and may account for the early polarization observed in Type II supernovae (Leonard et al. 2000; Wang et al. 2001). If one of the pair of axial jets is somewhat stronger than the other, jets can, in principle,
also account for pulsar runaway velocities that are parallel to the spin axis (Helfand et al. 2001, and references therein). While a combination of neutrino–induced and jet–induced explosion may prove necessary for complete understanding of core-collapse explosions, jets of the strength computed by Khokhlov et al. (1999) are sufficient for supernova explosions.

Immediately after the discovery of pulsars there were suggestions that rotation and magnetic fields could be a significant factor in the explosion mechanism (Ostriker & Gunn 1971; Bisnovatyi-Kogan 1971; Bisnovatyi-Kogan & Ruzmaikin 1976; Kundt 1976). Typical dipole fields of $10^{12}$ G and rotation periods of several to several tens of milliseconds yield electrodynamic power of $\sim 10^{44-45} \text{erg s}^{-1}$ that is insufficient to produce a strong explosion. The evidence for asymmetries and the possibility that bi-polar flows or jets can account for the observations suggest that this issue must be revisited. The fact that pulsars like those in the Crab and Vela remnants have jet-like protrusions (Weisskopf et al. 2000; Helfand et al. 2001) also encourages this line of thought. The present-day jets in young pulsars may be vestiges of much more powerful MHD jets that occurred when the pulsar was born. The transient values of the magnetic field and rotation could have greatly exceeded those observed today. Tapping that energy to power the explosion could be the very mechanism that results in the modest values of rotation and field the pulsars display after the ejecta have dispersed.

Possible physical mechanisms for inducing axial flows, asymmetric supernovae, and related phenomena driven by magnetorotational effects were considered by Wheeler et al. (2000), who focused on the effect of the resulting net dipole field, and Wheeler, Meier, & Wilson (2002) explored the capacity of the toroidal field to generate axial jets by analogy with magneto-centrifugal models of jets in AGN (Koide et al. 2000; and references therein). Wheeler et al. (2000), Wheeler, Meier, & Wilson (2002), and, indeed, all previous work considered only amplification of the field by “wrapping,” a process that increases the field linearly, and hence rather slowly in time. In addition, reconnection might limit the field before it can be wrapped the thousands of times necessary to be interesting. Akiyama et al. (2002) considered the effects of magnetic shearing, the magnetorotational instability (MRI; Balbus & Hawley 1991, 1998), on the strongly shearing environment that must exist in a nascent neutron star. This instability is expected to lead to the rapid exponential growth of the magnetic field with characteristic time scale of order the rotational period. While this instability has been widely explored in the context of accretion disks, this was the first time it has been applied to core collapse. This instability must inevitably occur in core collapse.
and is likely to be the dominant mechanism for the production of magnetic flux in the context of core collapse. This process has the capacity to produce fields that are sufficiently strong to affect, if not cause, the explosion.

1. The Magneto-Rotational Instability

Akiyama et al. (2002) simulated the collapse of a model iron core of a 15 M⊙ progenitor with a one-dimensional flux-limited diffusion code (Myra et al. 1987). The evolution of the angular velocity profile Ω(𝑟) was computed using the radial density profiles produced by the core collapse code assuming that the specific angular momentum of a given shell is constant. The magnetic field was obtained using the resulting ρ(𝑟), Ω(𝑟), and dΩ/dr profiles according to the theory of the MRI. The MHD luminosity and hoop stress were estimated from the resulting magnetic field.

1.1 Angular Velocity Profile

The 15 M⊙ model of Heger et al. (2000) attains an angular velocity of 10 rad s⁻¹ (see their Fig. 8) in the center of the iron core at the precollapse stage. Their simulations did not include the effects of a magnetic field. It is possible that the iron core rotates slower if the effect of magnetic braking is included (Spruit & Phinney 1998; Heger & Woosley 2002). Fryer & Heger (2000) studied the rotational effects on pure hydrodynamic core collapse explosions with initial velocity profiles obtained by Heger et al. (2000) with a central rotational velocity of 4 rad s⁻¹. Akiyama et al. (2002) adopted the initial angular velocity profile, Ω₀(𝑟), of Fryer & Heger (2000) (hereafter called the FH profile) as one case to study in addition to an analytic (MM profile) form (Möchmeyer & Müller 1989; Yamada & Sato 1994; Fryer & Heger 2000) and solid body profiles. The adopted profiles, characterized by the initial central value of the rotational frequency, Ω₀,c had small enough angular momentum that little departure from spherical geometry will occur.

It is inevitable that the collapsing core spins up and generates strong differential rotation for very general choices of the Ω₀(𝑟) profile, since the inner regions collapse larger relative distances than the outer regions. A strong shear must form at the boundary of the protoneutron star (PNS). At bounce, the original homologous core has a positive gradient in Ω(𝑟), and about 50 ms after bounce, the density profile is nearly identical to that of the initial iron core, giving a nearly flat rotation profile. After that, the density profile becomes somewhat more centrally
condensed than the original iron core and the rotation profile decreases monotonically outward even deep within the PNS (Fig. 1, 2).

Figure 1. Rotational profiles and $\Omega/\Omega_{\text{KEP}}$ for the initial differential rotation cases (MM and FH) with $\Omega_{0,c} = 1.0$ rad s$^{-1}$. The collapse generates strong differential rotation at the boundary of the initial homologous core. The rotation is always sub-Keplerian.

Ruderman et al. (2000) noted that the collapse of a white dwarf to a PNS gives a positive $\Omega(r)$ gradient since the relativistic degenerate core of the white dwarf has a steeper density profile than the PNS. The PNS will thus be relatively more compact for a given central density. There are two important differences in the calculations of Akiyama et al. (2002). The most critical is that the core collapsing is not in isolation as for the accretion-induced collapse scenario. Rather, the PNS forms within the massive star collapse ambience, and the PNS must be strongly differentially rotating with respect to the still-infalling matter. This will generate a strong shear at the boundary of the PNS that would
not pertain to a collapsing isolated white dwarf. Another, more subtle, difference is the equation of state. The equation of state of a partially degenerate iron core is not as different as for the highly relativistic white dwarf collapsing to a non-relativistic neutron star.

### 1.2 Magnetic Field

The MRI generates turbulence in a magnetized rotating fluid body that amplifies the magnetic field and transfers angular momentum. The MRI should pertain in this environment and amplify the magnetic field exponentially and perhaps, in turn, power MHD bi-polar flow or jets. Key questions are the amplitude of the magnetic field and the effect on the dynamics.
Ignoring entropy gradients, the condition for the instability of the slow magnetosonic waves in a magnetized, differentially rotating plasma is (Balbus & Hawley 1991, 1998):

\[
\frac{d\Omega^2}{d\ln r} + (k \cdot v_A)^2 < 0,
\]

where

\[
v_A = \frac{B}{\sqrt{4\pi \rho}}
\]

is the Alfvén velocity. When the magnetic field is very small, and/or the wavelength is very long, \((k \cdot v_A)^2\) is negligible, and the instability criterion for the MRI is simply that the angular velocity gradient be negative (Balbus & Hawley 1991, 1998), i.e.:

\[
\frac{d\Omega^2}{d\ln r} < 0.
\]

The growth of the magnetic field associated with the MRI is exponential with characteristic time scale of order \(\Omega^{-1}\). The time scale for the maximum growing mode is given by (Balbus & Hawley 1998):

\[
\tau_{\text{max}} = 4\pi \left| \frac{d\Omega}{d\ln r} \right|^{-1}.
\]

We thus expect the MRI to dominate any process such as “wrapping of field lines” (cf. Wheeler et al. 2000 and references therein) that only grows linearly in time, even if on about the same time scale. The MRI will also operate under conditions of moderate rotation that are not sufficient to compete with the PNS convective time scales to drive the sort of \(\alpha - \Omega\) dynamo invoked by, e.g., Duncan & Thompson (1992). The resulting unstable flow is expected to become non-linear, develop turbulence, and drive a dynamo that amplifies and sustains the field.

An order of magnitude estimate for the saturation field can be obtained by equating the shearing length scale \(\ell_{\text{shear}} \sim dr/d\ln \Omega\) to the characteristic mode scale \(\ell_{\text{mode}} \sim v_A \cdot (d\Omega/d\ln r)^{-1}\). The resulting saturation magnetic field is given by:

\[
B_{\text{sat}}^2 \sim 4\pi \rho r^2 \Omega^2.
\]

This is the same result as obtained by setting the Alfvén velocity equal to the local rotational velocity, \(v_A = r\Omega\).

The empirical value of the saturation field obtained by the numerical simulation of Hawley et al. (1996) is:

\[
B_{\text{sim}} = \sqrt{\frac{\rho}{\pi r}} r\Omega
\]
This saturation field is achieved after turbulence is fully established, which takes about 20 rotations following the initial exponential growth (Hawley et al. 1996). For conditions of rotation at much less than Keplerian, these saturation fields are much less than the equipartition field for which \( B^2/8\pi \) is comparable to the ambient pressure, i.e. for the current calculations, \( c_s \gg r\Omega \sim v_A \).

When a vertical seed field exists, the maximum unstable growing mode (Balbus & Hawley 1998) implies a saturation field of:

\[
B_{\text{max}}^2 = -4\pi\rho\lambda_{\text{max}}^2 \Omega^2 \cdot \left[ \frac{1}{8\pi^2} \left( 1 + \frac{\ln(\Omega^2)}{8 \ln r} \right) \frac{d\ln(\Omega^2)}{\ln r} \right],
\]

where \( \lambda_{\text{max}} \) is the wavelength of the maximum growing mode which is not allowed to exceed the local radius \( r \). With \( \lambda_{\text{max}} = r \), eq. (7) becomes:

\[
B_{\text{max}}^2 = -B_{\text{sat}}^2 \cdot \left[ \frac{1}{8\pi^2} \left( 1 + \frac{\ln(\Omega^2)}{8 \ln r} \right) \frac{d\ln(\Omega^2)}{\ln r} \right].
\]

This expression for the saturation field depends on the shear explicitly as well as indirectly through the stability criterion. Note that for the maximum growing mode the expression for \( B_{\text{max}}^2 \) acquires a negative value when

\[
\frac{d\Omega^2}{\ln r} < -8\Omega^2 \text{ or }, \quad \kappa^2 < -4\Omega^2 < 0,
\]

where \( \kappa \) is the epicyclic frequency:

\[
\kappa^2 = \frac{1}{r^3} \frac{d\left(r^4\Omega^2\right)}{dr} = 4\Omega^2 + \frac{d\Omega^2}{\ln r}. \quad (10)
\]

When eq. (9) is true, the epicyclic motion dominates over the MRI and prevents growth of the perturbation. Akiyama et al. (2002) turned off field amplification when this condition arose. In practice, the gradient of \( \Omega \) may be reduced by mixing due to the epicyclic motion, and the MRI may eventually be active in a region in which it was at first suppressed by a strong negative gradient of \( \Omega(r) \). Akiyama et al. (2002) also discuss the situation when the protoneutron star is convectively unstable.

1.3 Results

The shear is the strongest at the boundary of the initial homologous core. At bounce the shear is positive inside of the initial homologous
core, and the region is stable against the MRI though convective instability can destabilize the structure. The solid body profile possesses a similar shear profile to the FH and MM profiles.

Even a relatively modest value of $\Omega_0$ gives a very rapidly rotating PNS and hence strong differential rotation with respect to the infalling matter. At bounce, the peak of $\Omega/\Omega_{\text{kep}}$ is at the boundary of the initial homologous core. At later times, however, the peak moves to the second hump which is located inside the stalled shock. This hump is at the same location as a maximum in entropy which is caused by shocked material with higher density.

Figure 3. Magnetic field that of $B_{\text{sat}}$ (eq. 5) and the ratio $\beta^{-1} = P_{\text{mag}}/P_{\text{gas}}$ for MM, FH, and solid body profiles.

For the given initial rotational profiles, the magnetic fields of eq. (5) and eq. (8) are amplified exponentially with the time scale of eq. (4). The resulting magnetic field for $B_{\text{sat}}$ (eq. 5) and the ratio
\[ \beta^{-1} \equiv \frac{P_{\text{mag}}}{P_{\text{gas}}} \] (where \( \beta \) is the conventional \( \beta \) in plasma physics) are presented in Fig. 3. Given the limitation of the current calculations, we can only argue that these fields are roughly representative of what one would expect during core collapse.

For the cases with initial differential rotation, the peak values of the magnetic field at the end of our calculation at 387 ms after bounce are:

- For the FH profile: \( B_{\text{sat}} = 2.7 \times 10^{16} \) G and \( B_{\text{max}} = 2.5 \times 10^{15} \) G.
- For the MM profile: \( B_{\text{sat}} = 2.5 \times 10^{16} \) G and \( B_{\text{max}} = 2.5 \times 10^{15} \) G.
- For the solid body profile: \( B_{\text{sat}} = 3.7 \times 10^{15} \) G and \( B_{\text{max}} = 4.0 \times 10^{14} \) G.

The amplitude of the magnetic field is remarkably high and above the QED limit (\( B_{\text{Q}} = 4.4 \times 10^{13} \) G), but remains less than equipartition. For the case of \( B_{\text{sat}} \), \( P_{\text{mag}} \) is above 10% of \( P_{\text{gas}} \), and magnetic buoyancy may limit growth of the magnetic field (Wheeler, Meier, & Wilson 2002).

We expect that the magnetic field generated by the MRI will power MHD bi-polar outflow. The characteristic power of non-relativistic MHD outflow is given by Blandford & Payne (1982; see also Meier 1999, Wheeler, Meier & Wilson 2002):

\[ L_{\text{MHD}} = \frac{B^2 r^3 \Omega}{2}. \] (11)

The outflow carries energy, angular momentum and mass. Employing this characteristic power of a Blandford & Payne type MHD outflow, the outflow luminosity \( L_{\text{MHD}} \) calculated for the three initial rotational profiles, MM, FH, and solid body. The profiles of this MHD luminosity mimic those of the magnetic field. For the cases with initial differential profiles, the peaks of the MHD luminosity are at the boundary of the PNS.

Our calculations are limited to sub-Keplerian rotation and sub-equipartition fields, and yet they potentially produce significant MHD luminosity (Fig. 4): for the saturation field \( B_{\text{sat}} \), the maximum values 387 ms after bounce are \( 3.8 \times 10^{53} \) erg s\(^{-1} \) for FH, \( 2.5 \times 10^{53} \) erg s\(^{-1} \) for MM, and \( 1.9 \times 10^{51} \) erg s\(^{-1} \) for the initial solid rotation. For the saturation field \( B_{\text{max}} \), the peak values of MHD luminosity 387 ms after bounce are \( 5.5 \times 10^{51} \) erg s\(^{-1} \) for FH, \( 4.2 \times 10^{51} \) erg s\(^{-1} \) for MM, and \( 8.2 \times 10^{49} \) erg s\(^{-1} \) for the initial solid rotation. The investigation of how the MHD luminosity can be turned into a bi-polar flow is left for future work, although we outline some possibilities in the discussion below.

2. Discussion and Conclusions

No one would doubt that the progenitors of core collapse supernovae rotate and possess some magnetic field. The question has always been whether rotation and magnetic fields would be incidental perturbations
Figure 4. MHD jet luminosity in units of 1 foe (= $10^{51}$ erg) corresponding to $B_{\text{sat}}$. Sub-Keplerian rotation with MM, FH, and solid body initial rotation profiles results in high luminosity.

or a critical factor in understanding the explosion. Akiyama et al. (2002) have shown that with plausible rotation from contemporary stellar evolution calculations and any finite seed field with a component parallel to the rotation axis, the magnetorotational instability can lead to the rapid exponential growth of the magnetic field to substantial values on times of a fraction of a second, comparable to the core collapse time. Even a relatively modest value of initial rotation gives a very rapidly rotating PNS and hence strong differential rotation with respect to the infalling matter.

This result promises to be robust because the instability condition for the MRI is basically only that the gradient in angular velocity be negative. This condition is broadly satisfied in core collapse environments. Rotation can weaken supernova explosions without magnetic field (Fryer & Heger 2000); on the other hand, rotational energy can be converted to magnetic energy that can power MHD bi-polar flow that may promote supernova explosions. The implication is that rotation and magnetic fields cannot be ignored in the core collapse context.

As expected, the shear and hence the saturation fields are often highest at the boundary of the PNS where strong MHD activity is anticipated. Even artificially limiting the post-collapse rotation to sub-
Keplerian values as done by Akiyama et al. (2002), we find fields in excess of $10^{15}$ G near the boundary of the neutron star are produced. While this field strength is sub–equipartition, the implied MHD luminosities are of order $10^{52}$ erg s$^{-1}$. This is a substantial luminosity and could, alone, power a supernova explosion if sustained for a sufficiently long time, a fraction of a second. As pointed out by Wheeler, Meier & Wilson (2002), the fields do not have to be comparable to equipartition to be important because they can catalyze the conversion of the large reservoir of rotational energy into buoyant, bi-polar MHD flow. Higher rates of initial rotation that are within the bounds of the evolutionary calculations could lead to even larger post-collapse rotation and even larger magnetic fields. If the initial rotation of the iron core proves to be substantially lower than we have explored here, then the MRI would be of little consequence to the explosion. The MHD luminosities derived here are comparable to the typical neutrino luminosities derived from core collapse, $\sim 10^{52}$ erg s$^{-1}$. One important difference is that the matter beyond the PNS is increasingly transparent to this neutrino luminosity, whereas the MHD power is deposited locally in the plasma. Another difference is that the neutrino luminosity is basically radial so it resists the inward fall of the collapse, the very source of the neutrino luminosity itself. In contrast, hoop stresses associated with the magnetic field (see below) will tend to pull inward and force matter selectively up the rotation axis.

We note that for complete self-consistency, one should apply the MRI to the evolution of rotating stars where even a weak field renders the Høiland dynamical stability criterion “all but useless” in the words of Balbus & Hawley (1998). Recent calculations by Heger & Woosley (2002) based on a prescription for magnetic viscosity by Spruit (2002) yield rapidly rotating iron cores. Heger & Woosley (2002) find PNS rotation rates of 4 to 8 ms, consistent with the values we have explored here. Clearly, much more must be done to understand the magnetorotational evolution of supernova progenitors.

The configuration of the magnetic field in a precollapse iron core is not well understood. In this calculation we have assumed there exists a seed vertical field to calculate the growth of the field due to the MRI; however, the MRI can amplify other components of the magnetic field. The final configuration of the magnetic field after collapse may be less uncertain since the system has a strongly preferred direction due to rotation. Most of the shear is in the radial direction, so the radial component is greatly amplified by the MRI and turned into toroidal field due to differential rotation (Balbus & Hawley 1998). The dominant component is most likely to be the toroidal field.
Another uncertainty is the rotational profile. It is not clear what profile to use in the PNS, since, we note, even the rotational profile of the Sun is not well understood. A full understanding of the rotational state of a PNS remains a large challenge.

We have assumed various prescriptions for the saturation field. All are variations on the theme that, within factors of order 2π, the saturation field will be given by the condition \( v_A \sim r\Omega \). In the numerical disk simulations, about 20 rotations are required to reach saturation. The region of maximum shear in these calculations, around 15 km, typically has an angular velocity of 500 rad s\(^{-1}\) or a period of about 0.013 s. That means that by the end of the current calculations at 0.387 s, there have been about 30 rotations. Although the prescriptions for the growth and saturation fields we use here are heuristic, this aspect of our results is certainly commensurate with the numerical simulations of the MRI.

The issues of the saturation field and the nature of astrophysical dynamos are still vigorously explored. Vishniac & Cho (2001) conclude that the MRI has the required properties for a dynamo, anisotropic turbulence in a shearing flow, to generate both disordered and ordered fields of large strength. The saturation limits we have adopted here are consistent with those found in numerical calculations of the MRI saturation, but this topic clearly deserves more study.

Both the magnetic pressure and the magnetic viscosity are small for the sub–Keplerian conditions explored here. For most cases \( \beta^{-1} \) is less than 0.1 for the conditions we have assumed, (the \( B_{\text{sat}} \) case for FH with \( \Omega_{0,c} = 1.0 \) pushes this limit), so the direct dynamical effect of the magnetic field is expected to be small. The viscous time scale is \( \tau_{\text{vis}} \sim (\alpha\Omega)^{-1}(r/h)^2 \), where \( \alpha \) is the viscosity parameter and \( h \) is the vertical scale height, with \( h \sim r \) for our case. For a magnetically-dominated viscosity,

\[
\alpha \sim \frac{B_t B_\phi}{4\pi P} = \left( \frac{B_t}{B_\phi} \right) \frac{B_\phi^2}{4\pi P} \sim 2 \left( \frac{B_t}{B_\phi} \right) \beta^{-1}. \tag{12}
\]

With this expression for \( \alpha \), the viscous time becomes:

\[
\tau_{\text{vis}} \sim \frac{1}{2} \left( \frac{B_\phi}{B_t} \right) \left( \frac{1}{\beta^{-1}\Omega} \right) \gg \Omega^{-1}, \tag{13}
\]

where the final inequality follows from \( B_\phi > B_t \) and \( \beta^{-1} < 1 \). This prescription for viscosity is reasonable in the absence of convection. In the portions of the structure that are convective, the viscosity could be enhanced significantly.

We have not discussed the role of neutrinos here, although the processes of neutrino loss and de-leptonization are included in our calculation of the cooling PNS. It is possible that the neutrino flux affects
the magnetic buoyancy (Thompson & Murray 2002) and that the magnetic fields affect the neutrino emissivity (Thompson & Duncan 1996) and interactions with the plasma (Laming 1999). The time scale for shear viscosity due to neutrino diffusion is much longer than the times of interest here, although magnetic fields and turbulence can make it shorter (Goussard et al. 1998). The MRI provides magnetic field and turbulence, so this issue deserves further study. In addition to affecting the shear, the neutrino viscosity might also affect the turbulence needed to make the MRI work.

An obvious imperative is to now understand the behavior of the strong magnetic fields we believe are likely to be attendant to any core collapse situation. The fields will generate strong pressure anisotropies that can lead to dynamic response even when the magnetic pressure is small compared to the isotropic ambient gas pressure. As argued in Wheeler, Meier & Wilson (2002), a dominant toroidal component is a natural condition to form a collimated magneto-centrifugal wind, and hence polar flow. A first example of driving a polar flow with the MRI is given by Hawley & Balbus (2002).

The MRI is expected to yield a combination of large scale and small scale magnetic fields. A key ingredient to force flow up the axis and to collimate it is the hoop stress from the resulting field. Hoop stresses and other aspects of the strongly anisotropic Maxwell stress tensor will tend to lead to enhanced flow inward on the equator and up the axis, thus promoting a jet. These hoop stresses will occur for a field with a large scale toroidal component, but also in cases with only a small scale, turbulent field, i.e. when $<B_0^2>=0$ but $<B_\phi^2> \neq 0$ (Ogilvie 2001, Williams 2002).

Akiyama et al. (2002) found that the acceleration implied by the hoop stresses of the saturation fields, $a_{\text{hoop}} = B_\phi^2/4\pi \rho r$, was competitive with, and could even exceed, the net acceleration of the pressure gradient and gravity. The large scale toroidal field is thus likely to affect the dynamics by accelerating matter inward along cylindrical radii. The flow, thus compressed, is likely to be channeled up the rotation axes to begin the bi-polar flow that will be further accelerated by hoop and torsional stresses from the field, the “spring and fling” outlined in Wheeler, Meier & Wilson (2002).

While there has been some excellent work on the generation and propagation of jets through stars in the context of the “collapsar” model (Aloy et al. 1999, 2000; MacFadyen & Woosley 1999; MacFadyen, Woosley & Heger 2000; Zhang, Woosley & MacFadyen 2002) and for supernovae (Khokhlov et al. 1999; Khokhlov & Höflich 2001; Höflich, Wang, and Khokhlov 2001), none of this numerical work has taken ex-
Explicit account of rotation and magnetic fields in the origin and propagation of the jet. The same is true for the associated analytic work on jet propagation (Tan, Matzner, & McKee 2001; Matzner 2002; Mészáros & Rees 2001; Ramírez-Ruiz, Celotti & Rees 2002).

The dynamics of MHD jets may depart substantially from pure hydrodynamical jets, since they will tend to preserve the flux in the Poynting flow and be subject to hoop stresses and other magnetic phenomena. In addition, reconnection can accelerate the matter (Spruit, Daigne & Drenkhahn 2001). The magnetic forces in the jet may affect the collimation of the jet and the efficiency with which the surrounding cocoon is heated and expands in a transverse manner. A sufficiently strong magnetic field could thus alter the efficiency with which the propagating jet deposits energy into the stellar envelope. The magnetic field can also affect the stability of the jet. Large scale helical fields tend to be unstable to the kink instability. Li (2002) has recently argued that the effective hoop stresses of small scale turbulent fields could collimate magnetic jets and stabilize the flow against kinking.

Understanding of the role of magnetic fields in supernovae may also shed light on the production of collimated jets and magnetic field in the more extreme case of \(\gamma\)-ray bursts. One of the outstanding questions associated with \(\gamma\)-ray bursts is the origin of the magnetic field that is implicit in all the modeling of synchrotron emission. The fields deduced from the modeling are comparable to, but substantially less than, equipartition. Such fields cannot arise simply from shock compression of the ambient field of the ISM. While some schemes for generating this field in the \(\gamma\)-ray burst shock have been proposed (Medvedev & Loeb 1999), there is no generally-accepted understanding of the origin of this strong field.

Jets arising from the rotation and magnetic fields of neutron stars are likely to be important in asymmetric core-collapse supernova explosions (Wheeler et al. 2000; Wheeler, Meier, & Wilson 2002; Akiyama et al. 2002). Rotation and magnetic fields are critical in current models for the origin of jets in everything from protostars to AGN (Meier et al. 2001) and are very likely to be involved in the rapidly rotating environment that must occur if core collapse to a black hole is to produce anything like a \(\gamma\)-ray burst in the collapsar scenario. If the magnetic field plays a significant role in launching a relativistic \(\gamma\)-ray burst jet from within a collapsing star, then the magnetic field may also play a role in the propagation, collimation, and stability of that jet within and beyond the star. These factors have not been considered quantitatively. If magnetic flux is carried out of the star in the jet, then the magnetic field required
to explain the observed synchrotron radiation may already be present and will not have to be generated in situ.

The MRI can operate under conditions of moderate rotation. This means that the MRI will be at work even as the disk of material described by MacFadyen & Woosley (1999) begins to form and makes a transition from a non-Keplerian to quasi-Keplerian flow. The resulting unstable flow is expected to become non-linear, develop turbulence, and drive a dynamo that amplifies and sustains the field.

For a complete understanding of the physics in a core collapse supernova explosion, a combination of neutrino–induced and jet–induced explosion may be required. Understanding the myriad implications of this statement will be a rich exploration.

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