The bases of the theory of light reflection and absorption by low-dimensional semiconductor objects (quantum wells, wires and dots) at both monochromatic and pulse irradiations and at any form of light pulses are developed. The semiconductor object may be placed in a stationary quantizing magnetic field. As an example the case of normal light incidence on a quantum well (QW) surface is considered. The width of the QW may be comparable to the light wave length and number of energy levels of electronic excitations is arbitrary. For Fourier-components of electric fields the integral equation (similar to the Dyson-equation) and solutions of this equation for some individual cases are obtained.

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The interest to optics time-dependent effects in semiconductor objects is great in last years [1-4]. This is connected with successful engineering of short light pulses what allows to investigate coherent phenomena in processes of light interactions with elementary excitations in various systems.

If a semiconductor object is irradiated with a short light pulse there appears a pulse of a secondary light radiation the form of which differs essentially from the form of primary light pulse and bears also the information on exited states in the object, for example about lifetimes of electron-hole pairs (EHP), about splittings of magneto-polaron energy spectrum and so on.

In general the secondary light radiation from material objects is powerful means of research of their internal structure. Both at monochromatic and at a pulse light irradiation there are two sorts secondary light radiation. For example, at irradiation of low-dimensional semiconductor objects the secondary light radiation of the first sort causes light reflection from these objects which may be resonant if the frequency $\omega_l$ of stimulating light coincides with the frequency $\omega_0$ of one of discrete energy levels of an electronic system. In bulk semiconductors the secondary radiation of the first sort causes difference of true electromagnetic fields from stimulating fields, i.e. causes a deviation of the dielectric susceptibility $\varepsilon$ from unit.

The second sort of secondary light radiation is light scattering, for example Raman scattering, which can not be described in terms of the dielectric susceptibility.

How does any secondary radiation from material objects appear? Stimulating light creates in systems of charged particles induced alternating electric currents and charge density fluctuations. Current and charge fluctuations cause secondary electromagnetic fields. This chem is applicable equally to bulk bodies and to low-dimensional systems, for instance, to quantum wells.

If to average densities of induced currents and charges (on the ground state of system, for example, in a case of zero temperature) and to calculate the induced electromagnetic fields, we obtain the secondary light radiation of the first sort. The secondary light radiation of the second sort, i.e. light scattering, is caused by fluctuations of induced current and charge densities. Below we investigate only the secondary light radiation of the first sort and light absorption.

Modern semiconductor technologies allow to make high quality quantum wells, when radiative broadening of an absorption line may be comparable to the contributions of non-radiative relaxation mechanisms or to exceed them. In such situation it is impossible to be limited by the lowest approximation on interaction of electrons with electromagnetic fields and it is necessary to take into account all orders of this interaction [5-13].

Below principals of the theory of the secondary radiation of the first sort from low-dimensional semiconductor objects are developed. The accent is made on situation when the object is placed in a quantizing stationary magnetic field. Results are applied to a case of light pulse irradiation with any pulse forms.

The statement is constructed as follows. In section I the expressions for averaged values of current and charge densities induced by a weak electromagnetic field in a system of limited in space charged particles are given. These expressions are applicable in case of any stationary potential, any interaction between particles and any constant magnetic fields. Contributions containing electric fields and derivative of electrical fields on coordinates
are separated. In further last contributions are considered small and are not taken into account.

Further in sections II-VII the averaged density of induced currents in low-dimensional semiconductor objects is calculated without taking into account the Coulomb interaction between electrons and holes and in section VII the result is generalized with taking into account of the excitonic effect.

In section VIII the concept of the conductivity tensor \( \sigma_{\alpha\beta}(k,\omega|\mathbf{r}) \) depending on spatial coordinates due to the spatial heterogeneity of low-dimensional semiconductor objects is introduced. The general formula for the tensor \( \sigma_{\alpha\beta}(k,\omega|\mathbf{r}) \) is applicable for quantum wells, wires and dots.

In section IX the conductivity tensor for a case of quantum wells in zero and quantizing magnetic fields is calculated. In section X the averaged density of induced currents is calculated for a specific case of normal light incidence on a quantum well surface.

In section XI the model appropriate to two degenerated valence bands and simplifying expressions for the averaged density of induced currents is described. It is shown that to this model the zero density of induced charges corresponds.

Using the formula for the retarded potential in section XII we express the vector potential through an integral containing the averaged density of induced currents. Knowing the vector potential we calculate induced electric fields. Since the induced current density depends on an electric field we obtain some integral equation for it.

In section XIII the integral equation is transformed with reference to an approximation of an infinitely deep quantum well.

In sections XIV-XVI the integral equation for electric fields is solved for special cases. In section XIV the case of many energy levels of electronic excitations is considered in a narrow quantum well, the width of which is much less than the length of a stimulating light wave. In section XV the equation is solved in a case of many energy levels in a wide quantum well in the lowest order on the interaction of electromagnetic fields with electrons. In section XVI electric fields are precisely determined in case of one energy level of an excitation in a wide quantum well.

At last, in section XVII it is shown how the expressions for induced fields are connected with the form of a stimulating light pulse.

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I. THE EXACT FORMULAS FOR THE AVERAGE INDUCED CURRENT AND CHARGE DENSITIES.

In [14] it is shown that averaged current and charge densities induced by external weak electromagnetic fields may be expressed through values of electric fields and their derivatives on coordinates as follows

\[
\langle 0| j_{i\alpha}(\mathbf{r},t)|0 \rangle = \langle 0| j_{i\alpha}(\mathbf{r},t)|0 \rangle_{I} + \langle 0| j_{i\alpha}(\mathbf{r},t)|0 \rangle_{II}, \tag{1}
\]

\[
\langle 0| \rho_{i}(\mathbf{r},t)|0 \rangle = \langle 0| \rho_{i}(\mathbf{r},t)|0 \rangle_{I} + \langle 0| \rho_{i}(\mathbf{r},t)|0 \rangle_{II}, \tag{2}
\]

where the subscript "I" means linear approximation on fields, indexes I and II - contributions containing fields and their derivatives on coordinates, respectively. The results are obtained

\[
\langle 0| j_{i\alpha}(\mathbf{r},t)|0 \rangle_{I} = \frac{i}{\hbar} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \times \langle 0| [j_{i\alpha}(\mathbf{r},t), d_{\beta}(\mathbf{r}',t')]|0 \rangle E_{\beta}(\mathbf{r}',t'), \tag{3}
\]

\[
\langle 0| j_{i\alpha}(\mathbf{r},t)|0 \rangle_{II} = \frac{e}{mc} \langle 0| \bar{d}_{\beta}(\mathbf{r})|0 \rangle \frac{\partial \alpha_{\beta}(\mathbf{r},t)}{\partial r_{\alpha}}, \tag{4}
\]

\[
\langle 0| \rho_{i}(\mathbf{r},t)|0 \rangle_{I} = \frac{i}{\hbar} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \times \langle 0| [\rho(\mathbf{r},t), d_{\beta}(\mathbf{r}',t')]|0 \rangle E_{\beta}(\mathbf{r}',t'), \tag{5}
\]

\[
\langle 0| \rho_{i}(\mathbf{r},t)|0 \rangle_{II} = -\frac{i}{\hbar} \int d\mathbf{r}' \int_{-\infty}^{t} dt' \times \langle 0| \rho(\mathbf{r},t), Y_{\beta\gamma}|0 \rangle \frac{\partial \alpha_{\beta}(\mathbf{r}',t')}{\partial r_{\gamma}}. \tag{6}
\]

The following designations are used: \( \langle 0| \ldots |0 \rangle \) is averaging on the ground state of system, \( j(\mathbf{r},t) \) and \( \rho(\mathbf{r},t) \) are the operators of current and charge densities in the interaction representation

\[
\mathbf{j}(\mathbf{r},t) = e^{i\mathcal{H}t/\hbar} \mathbf{j}(\mathbf{r}) e^{-i\mathcal{H}t/\hbar},
\]

\[
\rho(\mathbf{r},t) = e^{i\mathcal{H}t/\hbar} \rho(\mathbf{r}) e^{-i\mathcal{H}t/\hbar}, \tag{7}
\]

where \( \mathcal{H} \) is the Hamiltonian of the system

\[
\mathcal{H} = \frac{1}{2m} \sum_{i} \mathbf{p}_{i}^{2} + V(\mathbf{r}_{1}, \ldots \mathbf{r}_{N}),
\]

\[
\mathbf{p}_{i} = \mathbf{P}_{i} - \frac{e}{c} A(\mathbf{r}_{i}), \quad \mathbf{P}_{i} = -i\hbar \frac{\partial}{\partial r_{i}}; \tag{8}
\]

\( A(\mathbf{r}) \) is the vector potential appropriating to a constant quantizing magnetic field

\[
\mathbf{H}_{c} = \nabla \times A(\mathbf{r}),
\]

\( V(\mathbf{r}_{1}, \ldots \mathbf{r}_{N}) \) is the potential energy including an interaction between particles and external potential. The Hamiltonian (8) describes the system of \( N \) particles with a charge \( e \) and mass \( m \).
The operators \( j(r) \) and \( \rho(r) \) are determined as
\[
\mathbf{j}(r) = \sum_i j_i(r), \quad \rho(r) = \sum_i \rho_i(r),
\]
\[
j_i(r) = \frac{e}{2} \{ \delta(r - r_i) v_i + v_i \delta(r - r_i) \}, \quad v_i = \frac{p_i}{m},
\]
\[
\rho_i(r) = e \delta(r - r_i), \quad \text{(8a)}
\]
\( r_i \) is the coordinate of i-th particle. Let us introduce designations
\[
\mathbf{d}(r) = \sum_i \mathbf{r}_i \rho_i(r),
\]
\[
\bar{Y}_{\beta\gamma}(r) = \frac{1}{2} \sum_i (j_i \bar{r}_{i\beta} + \bar{r}_{i\beta} j_i)
\]
\[
\bar{r}_i = r_i - (0|r_i|0), \quad \text{(8b)}
\]
and also
\[
\mathbf{a}(r, t) = -c \int_0^\infty dt' \mathbf{E}(r, t').
\]
Fields are considered as classical, the temperature is equal zero. At a derivation of (1) - (6) we assumed that on the infinitely removed distances there are no charges and currents, and also that on times \( t \to -\infty \) the fields \( \mathbf{E}(r, t) \) and \( \mathbf{H}(r, t) \) are equal 0 what corresponds to adiabatic switching on of fields.

II. THE SECONDARY QUANTIZATION REPRESENTATION.

Below we do not take into account the contributions with the index II containing derivatives from electric fields on coordinates, considering these contributions as small in comparison to basic contributions with the index I. The discussion of this question see in [15].
Let us pass to the secondary quantization representation, using a set of ortho-normalized wave functions of particles \( \Psi \) satisfying to conditions
\[
\int dr \Psi_{m'}^*(r) \Psi_m(r) = \delta_{mm'}.
\]
Then operators of current and charge densities determined in (8a) look like
\[
j_\alpha(r) = \frac{e}{2m} \sum_{m,m'} \{ \Psi_{m'}^*(r) p_\alpha \Psi_m(r) \\
- \Psi_m(r) p_\alpha \Psi_{m'}^*(r) \} a^+_{m'm} a_m,
\]
\[
\rho(r) = e \sum_{m,m'} \Psi_{m'}^*(r) \Psi_m(r) a^+_{m'm} a_m.
\]
The operator \( \mathbf{d}(r) \) determined in (8b) in the secondary quantization representation looks like
\[
\bar{d}_\alpha(r) = e (r_\alpha - r_{0\alpha}) \sum_{m,m'} \Psi_{m'}^*(r) \Psi_m(r) a^+_{m'm} a_m,
\]
where
\[
r_{0\alpha} = \frac{1}{N} \sum_{m_0} \int d^3r \Psi_{m_0}^*(r) r_\alpha \Psi_m(r),
\]
\( m_0 \) is a set of \( N \) states occupied by particles in the ground state \( |0 \rangle \). Let us notice that \( r_\alpha - r_{0\alpha} \), included in the right hand part of (12), is invariant relatively a shift of origin of coordinates, i.e. a replacement \( r \to r + R \) where \( R \) is any vector.

III. CONSIDERATION OF SEMICONDUCTOR OBJECTS.

Let us consider a semiconductor quantum well, wire or dot, containing valence bands and a conductivity band. From group of indexes \( m \) we allocate indexes \( v \) and \( c \) of valence and conductivity bands, respectively. We neglect transitions in higher bands. We designate other indexes by \( \zeta \).
Let us calculate the averaged induced current density using initial expression (3). Into the RHS of (3) only non-diagonal matrix elements of the operators \( j_\alpha(r) \) and \( \bar{d}_\beta(r) \) introduce, because the operators stand inside of a commutator. Therefore in the RHS (3) we have to substitute
\[
j^{nd}_\alpha(r) = \frac{e}{2m} \sum_{v,\zeta,\zeta'} \{ \Psi_{\zeta'}^*(r) p_\alpha \Psi_{v\zeta}(r) \\
- \Psi_{v\zeta}(r) p_\alpha \Psi_{\zeta'}^*(r) \} a^+_{v\zeta} a_{v\zeta} + h.c.,
\]
\[
\bar{d}^{nd}_\alpha(r) = e \sum_{v,\zeta,\zeta'} \Psi_{\zeta'}^*(r) (r_\alpha - r_{0\alpha}) \Psi_{v\zeta}(r) \\
\times a^+_{v\zeta} a_{v\zeta} + h.c.,
\]
In (14) and below we consider electrons, therefore \( e = -|e| \), the mass \( m = m_0 \), where \( m_0 \) is the free electron mass. The superscript \( nd \) means a part of the operator having only non-diagonal matrix elements.

IV. THE EFFECTIVE MASS APPROXIMATION.

Let us consider that object sizes - width \( d \) of a QW or sizes of wires or dots - are much greater than the lattice constant \( a \) and the distances on which varies a slowly
varying part of the wave function, is much greater than $a$. Then the effective mass approximation is applicable according to which

$$
\Psi_{\mu\zeta}(r) = u_{0\mu}(r)\psi_{\mu\zeta}(r),
$$

where $u_{0\mu}(r)$ is the quickly varying dimensionless factor, $\psi_{\mu\zeta}(r)$ is the slowly varying factor, $\mu = c$ or $\mu = v$.

In (14) we neglect an action of the operators $P_\alpha$ on the slowly varying factor in wave functions (16). Then we have approximately

$$
j^\alpha_n(r) \simeq \frac{e}{2m_0} \sum_v \left\{ u^\alpha_{0v}(r) p_\alpha u_{0v}(r) - u_{0v}(r) p_\alpha u^\alpha_{0v}(r) \right\} \times \sum_{\zeta,\zeta'} \psi^\ast_{\zeta',\zeta}(r) \psi_{\zeta}(r) a^+_\zeta a_{\zeta'} + h.c.,
$$

(17)

$$
\tilde{d}^\alpha_n(r) \simeq e \sum_v u^\alpha_{0v}(r) u_{0v}(r) (r_\alpha - r_{\alpha 0}) \times \sum_{\zeta,\zeta'} \psi^\ast_{\zeta',\zeta}(r) \psi_{\zeta}(r) a^+_\zeta a_{\zeta'} + h.c.,
$$

(18)

We get rid of quickly varying factors in the RHS of (17) and (18). For this purpose we introduce the Fourier components

$$
j_\alpha(\kappa) = \int d^3r j_\alpha(r)e^{-i\kappa r},
$$

$$
\tilde{d}_\alpha(\kappa) = \int d^3r \tilde{d}_\alpha(r)e^{-i\kappa r}.
$$

(19)

If $\kappa a \ll 1$ we obtain approximately

$$
j^{nd}(\kappa) = \frac{e}{m_0} \sum_v \mathbf{p}_{cv} \sum_{\zeta,\zeta'} \left\{ \int d^3r \psi^\ast_{\zeta',\zeta}(r) \psi_{\zeta}(r) m(r)e^{-i\kappa r} \right\} \times a^+_\zeta a_{\zeta'} + h.c.,
$$

(20)

$$
\tilde{d}^{nd}(\kappa) = \sum_v \mathbf{d}_{cv} \sum_{\zeta,\zeta'} \left\{ \int d^3r \psi^\ast_{\zeta',\zeta}(r) \psi_{\zeta}(r) e^{-i\kappa r} \right\} \times a^+_\zeta a_{\zeta'} + h.c.,
$$

(21)

where

$$
\mathbf{p}_{cv} = \frac{1}{\Omega} \int d^3r u^\ast_{0v}(r) \mathbf{P} u_{0v}(r),
$$

$$
\mathbf{d}_{cv} = \frac{e}{\Omega} \int d^3r u^\ast_{0v}(r) \mathbf{P} u_{0v}(r),
$$

(22)

$\Omega$ is the volume of an elementary crystal cell on which the integration is made.

In the first equality of (22) we have replaced the operator $\mathbf{P}$ (see (8)) on $\mathbf{P}$. If the system is in a quantizing magnetic field $\mathbf{H}_c = rot \mathbf{A}(r)$ and in the operator $\mathbf{P}$ the term $-(e/c)\mathbf{A}(r)$ is present, it brings in the small contribution in $\mathbf{p}_{cv}$ and may be rejected in the effective mass approximation.

Assuming that in the future we interest only by long wave components of $j^{nd}(r)$ and $\mathbf{d}^{nd}(r)$, we pass back from $\kappa$ - representation to $r$ - representation and obtain

$$
j^{nd}(r) = \frac{e}{m_0} \sum_v \mathbf{p}_{cv} \sum_{\zeta,\zeta'} \psi^\ast_{\zeta',\zeta}(r) \psi_{\zeta}(r) \times a^+_\zeta a_{\zeta'} + h.c.,
$$

(23)

$$
\tilde{d}^{nd}(r) = \sum_v \mathbf{d}_{cv} \sum_{\zeta,\zeta'} \psi^\ast_{\zeta',\zeta}(r) \psi_{\zeta}(r) \times a^+_\zeta a_{\zeta'} + h.c.,
$$

(24)

V. ELECTRON WAVE FUNCTIONS IN QW.

Let us consider two concrete examples of electron long wave functions in the effective mass approximation in QWs. In the free electron case

$$
\psi_{k,\perp}(r) = \frac{1}{\sqrt{S_0}} e^{i\mathbf{k}_{\perp} \cdot \mathbf{r}} \varphi_l(z),
$$

(25)

where $S_0$ is the normalization area, the axis $z$ is directed perpendicularly to the QW plane, the real function $\varphi_l(z)$ corresponds to levels $l = 1, 2, ...$ of the size quantization of electrons. For QWs of a finite depth the function $\varphi_l(z)$ and energy levels, appropriate to them, are determined, for example, in [16].

The second example is the case of electrons in a QW in quantizing magnetic field $\mathbf{H}_c$ perpendicular to the QW plane. The axis $z$ is directed along a magnetic field. Let us choose the following gauge of the vector potential

$$
\mathbf{A}(r) = \mathbf{A}(0, xH, 0).
$$

(26)

Then electron wave functions look like

$$
\psi_{n, k_{||}}(r) = \Phi_n(x + a_H^2 k_{||} y) \frac{1}{\sqrt{L_y}} e^{i k_{||} y} \varphi_l(z),
$$

(27)

$$
\Phi_n(x) = \frac{1}{\sqrt{\pi}^{1/2} n! a_H} H_n(x/a_H) e^{-x^2/2a_H^2},
$$

(28)

$$
a_H = \sqrt{\frac{eH}{|e|}}
$$

(29)

$H_n(t)$ is an Hermitian polynomial, $L_y$ is the normalization length.
VI. THE CONCEPT OF A HOLE IN A VALENCE BAND.

Let us consider that quasi-momentum components of a hole \( k_{\parallel} = -k_{\parallel} \) and \( k_y \) (in a quantizing magnetic field \( H_c \)) and the operator \( a_{\eta}^+ \) of an electron annihilation in the valence band is equal to the operator of a hole creation. Let us introduce a set of indexes \( \eta \) describing quantum numbers of an electron-hole pair (EHP) and operator \( a_{\eta}^+ \) of a creation (annihilation) of an EHP. Then from (23) and (24) we obtain

\[
j^{nd}(r) = \frac{e}{m_0} \sum_{\eta} \{ p_{cv} F_\eta^*(r) a_{\eta}^+ + p_{cv}^* F_\eta(r) a_{\eta} \}, \tag{30}\]

\[d^{nd}(r) = \sum_{\eta} \{ d_{cv} F_\eta^*(r) a_{\eta}^+ + d_{cv}^* F_\eta(r) a_{\eta} \}, \tag{31}\]

where \( F_\eta(r) \) is the EHP wave function at \( r = r_h = r \), \( r_e(r_h) \) is the electron (hole) radius - vector.

In the case of free EHPs in QWs

\[
F_\eta(r) = \frac{1}{S_0} e^{i(k_{\parallel} r_{\parallel} + k_y r_y)} \varphi_\eta^c\varphi_\eta^v(z), \tag{32}\]

where the set \( \eta \) includes indexes \( v, k_{\parallel}, k_y, l_e, l_v \). The EHP energy counted from the ground state energy is equal

\[
E_\eta = \hbar \omega_\eta = \hbar \omega + \varepsilon_\eta^c + \varepsilon_\eta^v + \frac{\hbar^2 k_{\parallel}^2}{2m_e} + \frac{\hbar^2 k_y^2}{2m_v}, \tag{33}\]

where \( \hbar \omega_\eta \) is the band gap, \( m_e(m_v) \) is the electron (hole) effective mass.

In the case of EHPs in QWs in a quantizing magnetic field we have

\[
F_\eta(r) = \Phi_{n_e}(x + a_\eta^c k_{\parallel} v) \Phi_{n_v}(x - a_\eta^v k_y v) \times \frac{1}{L_y} e^{i(k_{\parallel} r_{\parallel} + k_y r_y)} \varphi_\eta^c\varphi_\eta^v(z), \quad \tag{34}\]

the set \( \eta \) includes indexes \( v, n_e, n_v, k_{\parallel}, k_y, l_e, l_v \). The appropriate energy is equal

\[
E_\eta = \hbar \omega_\eta + \varepsilon_\eta^c + \varepsilon_\eta^v + \hbar \Omega_{\eta}(n_e + 1/2) + \hbar \Omega_{\eta}(n_v + 1/2), \tag{35}\]

\[
\Omega_{\eta}(v) = \frac{|e| H_c}{m_{\eta}(v)c}, \tag{36}\]

is the cyclotron frequency of electron (hole in a band \( v \)). It is possible to show that the formulas (30) and (31) are applicable and in those cases when it is essential the Coulomb interaction between electrons and holes. For example, at \( H_c = 0 \) discrete energy levels correspond to excitonic states in QW. Then \( F_\eta(r) \) is the excitonic wave function at \( r_e = r_v = r \), and \( \eta \) is the set of indexes describing an exciton. In quantizing magnetic fields Coulomb forces can change a position of energy levels and affect on the function \( F_\eta(r) \). (Conditions of weak influence of Coulomb forces in quantizing magnetic fields see in [27].)

VII. AVERAGED INDUCED CURRENT DENSITIES IN SEMICONDUCTOR OBJECTS.

It is easy to find a connection between matrix elements \( p_{cv} \) and \( d_{cv} \) determined in (22), if to use a ratio

\[v_\alpha = \frac{i}{\hbar} [\mathcal{H}, r_\alpha],\]

from which follows

\[d_{cv} = - \frac{i e}{m_0 \omega_\eta} p_{cv}. \tag{37}\]

Substituting (37) in (31) and then (30) and (31) in (3), we obtain

\[0 \langle j_{1\alpha}(r,t) | 0 \rangle = \frac{e^2}{\hbar \omega_\eta m_0} \int d^3r' \int_{-\infty}^{\infty} dt' \Theta(t - t') \sum_{\eta,\eta'} (p_{cv}^+ p_{cv} F_\eta^*(r) F_\eta'(r') \langle 0 | a_{\eta'}(t') a_{\eta}^+(t) | 0 \rangle E_{\beta}(r',t'), \tag{38}\]

where \( \Theta(\tau) = 1 \) at \( \tau > 0 \) and \( \Theta(\tau) = 0 \) at \( \tau < 0 \).

Averaging on the ground state gives the result [17, page 48]

\[0 \langle j_{1\alpha}(r,t) | 0 \rangle = \delta_{\eta,\eta'} e^{i \omega_\eta(t' - t) - (\gamma_\eta / 2)(t' - t)}, \tag{39}\]

where \( \gamma_\eta \) is the non-radiative broadening of a state \( \eta \).

Substituting (39) in (38) and making replacement \( t' \rightarrow t + t' \), we obtain

\[0 \langle j_{1\alpha}(r,t) | 0 \rangle = \frac{e^2}{\hbar \omega_\eta m_0^2} \sum_{\eta} \times \left\{ p_{cv}^+ p_{cv} F_\eta^*(r) \int d^3r' F_\eta'(r') \int_{-\infty}^{\infty} dt' e^{i \omega_\eta t' - (\gamma_\eta / 2)t'} \times E_{\beta}(r', t + t'). \tag{40}\]

The result (40) is applicable in a wide area, for example, in case of exciton states at a zero magnetic field and in quantizing magnetic fields, i.e. at the account of Coulomb interaction of electrons and holes in those cases when it is essential. Certainly, the functions \( F_\eta(r) \) at the account of Coulomb forces will differ from (32) and (34). Besides the formula (40) is applicable in case of other low-dimensional semiconductor objects, for example, quantum wires or dots.
VIII. INTRODUCTION OF A CONDUCTIVITY TENSOR.

We write down the expression (40) as

$$\langle 0 | j_{1\alpha}(r,t) | 0 \rangle = \int d^3r \int_{-\infty}^{\infty} dt' \sigma_{\alpha\beta}(r',t'|r,t) \times E_\beta(r-r',t-t'),$$

(41)

where $\sigma_{\alpha\beta}(r',t'|r,t)$ is the conductivity tensor (the designation is borrowed from [18]). It follows from (40)

$$\sigma_{\alpha\beta}(r',t'|r,t) = \frac{\epsilon^2}{\hbar \omega_g m_0^2} \sum_\eta \left\{ \left( p_{cv\alpha}^* p_{cv\beta} F_\eta(r) F_\eta^*(r-r') \Theta(t') e^{-i\omega t'-(\gamma/2)t'} + p_{cv\alpha} p_{cv\beta}^* F_\eta^*(r) F_\eta(r-r') \Theta(t') e^{i\omega t'-(\gamma/2)t'} \right) \right\}.$$  

(42)

It follows from (42) that the tensor $\sigma_{\alpha\beta}(r',t'|r)$ does not depend on time $t$, if the potential energy $V(r_1 \ldots r_N)$ from (8) does not depend on $t$, what we mean. It is a consequence of time uniformity. Therefore the designation is used below

$$\sigma_{\alpha\beta}(r',t'|r,t) = \sigma_{\alpha\beta}(r',t'|r,t).$$

Let us make a Fourier transform. Let us write down the electric field as

$$E_\alpha(r,t) = E_\alpha^{(+)}(r,t) + E_\alpha^{(-)}(r,t),$$

(43)

where

$$E_\alpha^{(+)}(r,t) = \frac{1}{(2\pi)^3} \int d^3k \int_0^{\infty} d\omega E_\alpha(k,\omega) e^{i(kr-\omega t)},$$

(44)

$$E_\alpha^{(-)}(r,t) = (E_\alpha^{(+)}(r,t))^*,$$

(45)

$$E_\alpha(k,\omega) = \int d^3r \int_{-\infty}^{\infty} dt E_\alpha(r,t) e^{i(-kr+\omega t)}.$$  

(46)

The splitting (43) is usually made to not use negative frequencies $\omega$.

Let us introduce a Fourier-image $\sigma_{\alpha\beta}(r',t'|r)$ on variable $r', t'$

$$\sigma_{\alpha\beta}(r',t'|r) = \int d^3r' \int_{-\infty}^{\infty} dt' \sigma_{\alpha\beta}(r',t'|r) e^{i(-kr' + \omega t')}.$$  

(47)

Then with the help of (41), (44) and (45) we obtain

$$\langle 0 | j_{1\alpha}(r,t) | 0 \rangle = \frac{1}{(2\pi)^3} \int d^3k \int_0^{\infty} d\omega \times \sigma_{\alpha\beta}(k,\omega) e^{i(kr - i\omega t) + c.c.}.$$  

(48)

In a case of spatially - homogeneous systems, for example, bulk semiconductor crystals

$$\sigma_{\alpha\beta}(k,\omega) = \sigma_{\alpha\beta}(k,\omega).$$

(49)

From (42), making transformations (47) and executing integration on $t'$, we obtain

$$\sigma_{\alpha\beta}(k,\omega) = \frac{i\epsilon^2}{\hbar \omega_g m_0^2} \sum_\eta \left\{ \left( p_{cv\alpha}^* p_{cv\beta} F_\eta(r) F_\eta^*(r-r') \right) \right\}.$$  

(50)

Let us notice that the conductivity tensor has a property

$$\sigma_{\alpha\beta}^*(k,\omega) = \sigma_{\alpha\beta}(-k, -\omega).$$

(51)

Let us emphasize that the expressions (48) and (50) basically allow to calculate average induced current density of a particle at monochromatic and at any direction of incident light, for example, not only at normal, but also at slanting incidence of light on a QW’s plane.

IX. THE CONDUCTIVITY TENSOR IN QW.

It follows from (32) and (34) that for free EHPs in zero or quantizing magnetic field the function $F_\eta(r)$ may be represented as a product

$$F_\eta(r) = Q_{\pi}(r_\perp) \phi_\chi(z),$$

(52)

where $\pi$ is the set of indexes $v,k_{\perp},k_{\parallel}$ in $H_\epsilon = 0$ and $V,N_\epsilon,N_0,k_{\parallel},k_{\perp}$ in quantizing magnetic field, $\chi$ is the set of indexes $v,l_\parallel,l_\perp$,

$$\phi_\chi(z) = \varphi_0^\epsilon(z) \varphi_0^\epsilon(z).$$

(53)

The splitting (52) is applicable also when the Coulomb interaction of electrons and holes can essentially influence only on movement of particles along $z$ axis. It occurs under condition of [19]

$$a_{\text{exc}}^2 >> a_H^2,$$

where

$$a_{\text{exc}} = \frac{\hbar^2 \varepsilon_0}{\mu \bar{v}}$$

is the Wannier-Mott exciton radius in absence of a magnetic field, $\varepsilon_0$ is the static dielectric susceptibility, $\mu = m_e m_h/(m_e + m_h)$ is the effective mass, i.e. in a case of quantizing magnetic fields. At

$$a_{\text{exc}} >> d,$$
i.e. for narrow bands the Coulomb forces poorly influence on movement along an \( z \) axis and the functions \( \phi_\chi(z) \) look like (53). Otherwise

\[
a_{\text{exc}} \ll d
\]

the formula (53) is inapplicable. For GaAs

\[
a_{\text{exc}} = 146 A, \quad a_{\text{res}}^H = 57.2 A,
\]

where \( a_{\text{res}}^H \) corresponds to a magnetic field \( H_{\text{res}} \) at which has a place the magnetophonon resonance \( \Omega_{eH} = \omega_{LO} \). Using (52) we obtain from (50)

\[
\sigma_{\alpha\beta}(k, \omega | r) = \frac{ie^2}{\hbar \omega g m_0} e^{-ikz} \sum_\eta \phi_\chi(z) R^*_\eta(k)
\]

\[
\times \left\{ p_{\text{cv}} p_{\text{cv}}^* \delta \xi_{\perp, \parallel} \eta \quad \frac{e^{i\tau_r r'_z}}{\omega - \omega_\eta + i\gamma/2} \right\},
\]

(54)

For free EHPs at \( H_c = 0 \)

\[
Q_\pi(r_\perp) = \frac{1}{S_0} e^{i(k_{\perp} + k_{\parallel})r_\perp}.
\]

Substituting this expression in (54) and executing integration on \( r'_z \) we obtain

\[
\sigma_{\alpha\beta}(k, \omega | r) = \frac{ie^2}{\hbar \omega g m_0} e^{-ikz} \sum_\eta \phi_\chi(z) R^*_\eta(k)
\]

\[
\times \left\{ p_{\text{cv}} p_{\text{cv}}^* \delta \xi_{\perp, \parallel} \eta \quad \frac{e^{i\tau_r r'_z}}{\omega - \omega_\eta + i\gamma/2} \right\},
\]

(55)

where \( K_\perp = k_{\perp} + k_{\parallel} \)

\[
R^*_\eta(k) = \int_{-\infty}^{\infty}dz e^{-ikz} \phi_\chi(z),
\]

(56)

the energy \( \hbar \omega_\eta \) is determined in (33).

For EHP in a quantizing magnetic field

\[
Q_\pi(r_\perp) = \Phi_{\text{cv}}(x + a^2_H k_{xy}) \Phi_{\text{cv}}(x - a^2_H k_{xy})
\]

\[
\times \frac{1}{L_y} e^{i(k_{xu} + k_{yu})y}.
\]

(57)

Let us substitute (57) in (54), integrate on variable \( y' \) and make summation on indexes \( k_{xy} \) and \( k_{xy} \) from which the energy \( \hbar \omega_\eta \) (determined in (35)) does not depend. It results in

\[
\sigma_{\alpha\beta}(k, \omega | r) = \frac{ie^2}{2\pi \hbar \omega g m_0 a^2_H} e^{-ikz} \sum_\xi \phi_\chi(z) R^*_\chi(k)
\]

\[
\times \left\{ p_{\text{cv}} p_{\text{cv}}^* \sum_{n_x, n_y} (k_x, k_y) \quad \frac{e^{i\tau_r r'_z}}{\omega - \omega_\eta + i\gamma/2} \right\} +
\]

\[
\times \left\{ p_{\text{cv}} p_{\text{cv}}^* \sum_{n_x, n_y} (k_x, k_y) \quad \frac{e^{i\tau_r r'_z}}{\omega + \omega_\eta + i\gamma/2} \right\}.
\]

(58)

where

\[
\Xi_{n_x, n_y}(k_x, k_y) = \left| \int_{-\infty}^{\infty} dt \Phi_{n_x}(t) \Phi_{n_y}(t - a^2_H k_y e^{i k_x t}) \right|^2,
\]

(59)

\( \zeta \) is the set of indexes \( \chi, n_x, n_y \), the energy \( \hbar \omega_\zeta = \hbar \omega_\eta \).

Let us pay attention that \( \sigma_{\alpha\beta}(k, \omega | r) \) in QWs (at \( H_c = 0 \) or with \( H_c \) directed along \( z \) axis) is more convenient for concrete calculations, because an integration contour may be closed in the top or bottom half-plane.

In a quantizing magnetic field with the help of (48) and (58) we obtain

**X. NORMAL LIGHT INCIDENCE ON A QW SURFACE.**

At a normal light incidence the electric field \( E(r, t) \) depends only on variables \( z \) and \( t \). Let us introduce the Fourier-component of a field on variable \( t \)

\[
E_\beta(z, \omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} E_\beta(z, t).
\]

(61)

With the help of (48) and (55) it is possible to show that average induced density of a current at \( H_c = 0 \) is equal

\[
\langle 0 | j_{\text{cv}}(r, t) | 0 \rangle = \frac{1}{2\pi} \left( \frac{e}{m_0} \right)^2 \frac{1}{\hbar \omega g S_0} \sum_\chi \phi_\chi(z)
\]

\[
\times \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_{-\infty}^{\infty} dz' \phi_\chi(z') E_\beta(z', \omega)
\]

\[
\times \left\{ p_{\text{cv}} p_{\text{cv}}^* \sum_{k_\perp} (\omega - \omega_\kappa + i\gamma/2)^{-1}
\]

\[
+ p_{\text{cv}} p_{\text{cv}}^* \sum_{k_\perp} (\omega + \omega_\kappa + i\gamma/2)^{-1} \right\},
\]

(62)

where \( \kappa \) is the set of indexes \( \chi, k_\perp = k_{\perp} = -k_{\perp} \),

\[
\omega_\kappa = \omega_\eta + \varepsilon_\mu^2 / \hbar + \varepsilon_\nu^2 / \hbar + \frac{\hbar k^2}{2\mu}.
\]

(63)

In (62) we passed from integration on \( \omega \) in limits from 0 up to \( \infty \) to integration in limits from \( -\infty \) up to \( \infty \), what is more convenient for concrete calculations, because an integration contour may be closed in the top or bottom half-plane.

In a quantizing magnetic field with the help of (48) and (58) we obtain
\( \langle 0|j_{1\alpha}(r,t)|0 \rangle = \frac{1}{(2\pi)^2} \left( \frac{e}{m_0} \right)^2 \frac{1}{\hbar \omega_0 a_H^2} \sum_{\chi} \phi_{\chi}(z) \times \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_{-\infty}^{\infty} d\nu \phi_{\nu}(\nu) E_{\nu}(\nu, \omega) \times \left\{ p_{c\nu}^\alpha p_{c\nu}^\beta \sum_{n} (\omega - \omega_\nu + i\gamma_\nu/2)^{-1} \right\} + p_{c\nu}^\alpha p_{c\nu}^\beta \sum_{n} (\omega + \omega_\nu + i\gamma_\nu/2)^{-1} \right\}, \) (64)

where \( \lambda \) is the set of indexes \( \chi \) and \( n_\nu = n_\nu = n, \)

\( \omega_\lambda = \omega_g + \varepsilon_{l_\nu}^i / \hbar + \varepsilon_{l_\nu}^f / \hbar + \Omega_{\mu H}(n + 1/2), \)

\( \Omega_{\mu H} = \frac{|e| H_c}{\mu c}. \) (65)

At a derivation of (64) the ratio was used

\[ \Xi_{n_\nu, n_\nu}(k_x, k_y) = 0, k_y = 0) = \left| \int_{-\infty}^{\infty} dt \Phi_{n_\nu}(t) \Phi_{n_\nu}(t) \right|^2 = \delta_{n_\nu, n_\nu}, \] (66)

which corresponds to the following selection rule: at a normal incidence of light EHPs with identical Landau quantum numbers of electrons and holes are excited.

At \( H_c = 0 \) and a normal incidence of light EHPs with zero quasi-momentum in a QW plane are excited, therefore \( k_{\nu, \parallel} = -k_{\nu, \parallel}, \) that follows from the quasi-momentum conservation law in \( xy \) plane.

Let us notice that the expression (64) for quantizing magnetic fields differs from (62) for \( H_c = 0 \) only by replacement of the normalization area \( S_0 \) by \( 2\pi a_H^2 \) and the index \( k_{\parallel} \) by the index \( n. \)

**XI. THE MODEL SIMPLIFYING EXPRESSIONS FOR AN AVERAGE CURRENT DENSITY.**

Further a model is used which was applied in [20-28]. Vectors \( p_{c\nu} \) for two degenerated valence bands \( \nu = I \) and \( \nu = II \) look like

\[ p_{c\nu}^{I} = \frac{p_{c\nu}}{\sqrt{2}} (e_x - ie_y), \]

\[ p_{c\nu}^{II} = \frac{p_{c\nu}}{\sqrt{2}} (e_x + ie_y), \] (67)

where \( e_x \) and \( e_y \) are unite vectors along axes \( x \) and \( y, \)

\( p_{c\nu} \) is the real value. This model corresponds to heavy holes in a semiconductor with the zinc blend structure, if an axis is directed along an axis of symmetry of the 4-th order [29,30]. If to use vectors of circular polarization of stimulating light

\[ e_l = \frac{1}{\sqrt{2}}(e_x \pm ie_x), \] (68)

the property of preservation of a polarization vector is performed

\[ \sum_{\nu=I,II} p_{c\nu}^e (e_l p_{c\nu}^e) = \sum_{\nu=I,II} p_{c\nu}^e (e_l p_{c\nu}^{e*}) = e_l p_{c\nu}^e. \] (69)

Wave functions \( \varphi_{l_\nu}^i \) and energy levels \( \varepsilon_{l_\nu}^i \) do not depend on numbers of valence bands \( I \) and \( II. \)

Using the model (67) and results (62) and (64) the expression for average induced current density at \( H_c = 0 \) in a quantizing magnetic field we write down in an unified form

\[ \langle 0|j_{1\alpha}(r,t)|0 \rangle = \frac{i e \nu}{4 \pi^2} \sum_{\rho} \gamma_{\rho\nu} \Phi_{\rho}(z) \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \times \int_{-\infty}^{\infty} dz \phi_{\rho}(z') E_{\alpha}(z', \omega) \times (\omega - \omega_\rho + i\gamma_\rho/2)^{-1} + (\omega + \omega_\rho + i\gamma_\rho/2)^{-1}, \] (70)

where \( \nu \) is the refraction light factor and for \( H_c = 0 \)

\[ \gamma_{\rho\nu} = \gamma_{\nu} = 4\pi \frac{e^2 p_{c\nu}^2}{\hbar c v m_0} \frac{1}{\hbar c v m_0^2 \hbar \omega_g}. \] (71)

\( \rho \) is the set of parameters \( l_\nu, l_{\nu h}, k_{\parallel}, \) and for quantizing magnetic fields

\[ \gamma_{\rho\nu} = \gamma_{\nu} = 2 \frac{e^2 p_{c\nu}^2}{\hbar c v m_0^2} \frac{1}{\hbar \omega_g} = \frac{e^2 p_{c\nu}^2}{\hbar c v m_0^2} \frac{\Omega_{\mu H}}{\hbar \omega_g}, \] (72)

\( \Omega_{\mu H} = |e| H_c / m_0 c, \)

\( \rho \) is the set of indexes \( l_\nu, l_{\nu h}, n. \)

In the RHS \( \gamma_{\nu} \) is supplied with an index \( \rho \) though the RHSs of (71) and (72) do not depend on this index. It is made for the expression (70) would be applicable and to another situations, for example, to a case of the magnetopolaron resonance in a quantizing magnetic field. The physical sense of \( \gamma_{\rho\nu} \) will be opened below.

Let us notice that in the case of model (67) the important property is performed

\[ \text{div} \langle 0|j_{1}(r,t)|0 \rangle = 0, \]

therefore the average induced charge density is equal 0 what follows from the continuity equation.

**XII. CALCULATION OF A VECTOR POTENTIAL AND AN ELECTRIC FIELD.**

Knowing distribution of the average current density inside of QW, it is possible to determine a vector potential according to the known formula for retarded potentials (see, for example, [31, page 209])

\[ A(r, t) = \frac{1}{c} \int d^3r' \frac{j(r', t - \frac{r - r' \cdot \hat{r}}{c})}{|r - r'|} + A_0(r, t). \] (74)
It follows from (70) that the dependence of a current density on coordinates is determined only by \( \phi_{\rho}(z) \) in the sum on \( \rho \).

Let us calculate the integral
\[
I_\rho(\omega, z) = \int d^3r' \phi_{\rho}(z') e^{i\omega|z - z'|/c},
\]
which is equal to
\[
I_\rho(\omega, z) = \frac{2\pi i e}{\omega v} \left\{ \int_{-\infty}^{z} dz' \phi_{\rho}(z') e^{i\kappa(z - z')} + \int_{z}^{\infty} dz' \phi_{\rho}(z') e^{-i\kappa(z - z')} \right\},
\]
where \( \kappa = \omega v / c \). Using (70), (74) and (76) we obtain that the vector potential \( \vec{\phi} \) is equal 0 also, therefore
\[
E(z, t) = -\frac{1}{c} \frac{\partial A(z, t)}{\partial t},
\]
and accordingly we obtain
\[
E_\alpha(z, t) = -\frac{i}{4\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \sum_{\rho} \gamma_{\rho\rho} \int_{-\infty}^{z} dz' \phi_{\rho}(z') E_\alpha(z', \omega)
\times \left\{ \int_{-\infty}^{z} dz' \phi_{\rho}(z') e^{i\kappa(z - z')} + \int_{z}^{\infty} dz' \phi_{\rho}(z') e^{-i\kappa(z - z')} \right\}
\times \left\{ \frac{\omega - \omega_\rho + i\gamma_\rho / 2}{\omega - \omega_\rho + i\gamma_\rho / 2 - 1} \right\}
\times A_{o\alpha}(z, t),
\]
where \( E_{o\alpha}(z, t) \) is the exciting field. Let us make the Fourier transform of the left and right parts of (79) using (61). We have
\[
E_\alpha(z, \omega) = -\frac{i}{2} \sum_{\rho} \gamma_{\rho\rho} \int_{-\infty}^{z} dz' \phi_{\rho}(z') E_\alpha(z', \omega)
\times \left\{ \int_{-\infty}^{z} dz' \phi_{\rho}(z') e^{i\kappa(z - z')} + \int_{z}^{\infty} dz' \phi_{\rho}(z') e^{-i\kappa(z - z')} \right\}
\times \left\{ \frac{\omega - \omega_\rho + i\gamma_\rho / 2}{\omega - \omega_\rho + i\gamma_\rho / 2 - 1} \right\}
\times E_{o\alpha}(z, \omega),
\]
Thus, we have obtained the integral equation for Fourier-components of the electric field.

Let us write down the exciting field in the form
\[
E_0(z, t) = E_0 e^{i\omega_0 t} D_0(\omega) + c.c.,
\]
where \( p = t - \omega v z / c \).

For a monochromatic excitation with frequency \( \omega_1 \)
\[
D_0(\omega) = \delta(\omega - \omega_1).
\]
\( D_0(\omega) \) also may correspond to pulses of any durations and forms. It follows from (81)
\[
E_{o\alpha}(z, \omega) = 2\pi E_0 e^{i\omega_0 z / c} \left\{ e_\alpha D_0(\omega) + e_\alpha^* D_0(-\omega) \right\}.
\]
Let us write down the required solution as
\[
E(z, t) = \frac{E_l}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} E(z, \omega) + c.c.,
\]
Then for \( E(z, \omega) \) we obtain the equation
\[
E(z, \omega) = -\frac{i}{2} \sum_{\rho} \gamma_{\rho\rho} \int_{-\infty}^{\infty} dz' \phi_{\rho}(z') E(z', \omega)
\times \left\{ \int_{-\infty}^{z} dz' \phi_{\rho}(z') e^{i\kappa(z - z')} + \int_{z}^{\infty} dz' \phi_{\rho}(z') e^{-i\kappa(z - z')} \right\}
\times \left\{ \frac{\omega - \omega_\rho + i\gamma_\rho / 2}{\omega - \omega_\rho + i\gamma_\rho / 2 - 1} \right\}
\times 2\pi E_0 e^{i\kappa z} D_0(\omega).
\]

XIII. THE APPROXIMATION OF THE INFINITELY DEEP QW.

For the greater simplicity and demonstrativeness of solutions we consider a case of an infinitely deep QW, when wave functions \( \varphi_l(z) \) of electrons and holes are strictly limited by limits of QW and there is no any their penetration in a barrier, i.e. for free EHPs
\[
\varphi_l(z) = \frac{2}{\sqrt{d}} \sin \left( \frac{lz_\pi}{d} + \frac{l\pi}{2} \right), l = 1, 2, \ldots; -\frac{d}{2} \leq z \leq \frac{d}{2},
\]
\[
\varphi_l(z) = 0; z \leq -\frac{d}{2}, z \geq \frac{d}{2},
\]
\[
\varepsilon_i^e = \frac{\hbar^2 \pi^2 l^2}{2m_e d^2}, \varepsilon_i^h = \frac{\hbar^2 \pi^2 l^2}{2m_h d^2}.
\]
Then with the help of (84) we obtain
\[ \mathcal{E}(z, \omega) = -\frac{i}{2} \sum_{\rho} \gamma_{\rho} \int_{-\Delta/2}^{\Delta/2} dz' \phi_{\rho}(z') \mathcal{E}(z', \omega) \times \left\{ e^{i z} \int_{-\Delta/2}^{\Delta/2} dz' \phi_{\rho}(z') e^{-i z'} + e^{-i z} \int_{-\Delta/2}^{\Delta/2} dz' \phi_{\rho}(z') e^{i z'} \right\} \times \left\{ (\omega - \omega_{\rho} + i \gamma_{\rho}/2)^{-1} + (\omega + \omega_{\rho} + i \gamma_{\rho}/2)^{-1} \right\} + 2\pi E_0 e^{i \kappa d} D_0(\omega). \] (86)

**XIV. THE SOLUTION FOR A QW, WHICH WIDTH IS LESS THAN A LIGHT WAVE LENGTH.**

Let us consider the solution of (86) at

\[ \kappa d \ll 1. \]

For monochromatic irradiation \( \kappa l = \omega \nu / c \), for pulse irradiation frequencies \( \omega \) are essential, laying in an interval \( \pm \Delta \omega \) near the carrying pulse frequency \( \omega l \). \( \Delta \omega \) is of the order of \((\Delta t)^{-1}\), where \( \Delta t \) is the duration of a light pulse. In any case \( \omega \) is of the order of \( \omega_g \), where \( \hbar \omega_g \) is the band gap of the semiconductor. Let us search for the solution \( \mathcal{E}(z, \omega) \) at the left and to the right of QW, where only plane waves with frequencies \( \omega = c k / \nu \) may exist.

We search for solutions of type

\[ \mathcal{E}_{\text{left}}(z, \omega) = \mathcal{E}_0(z, \omega) + \Delta \mathcal{E}_{\text{left}}(z, \omega), \]
\[ \mathcal{E}_{\text{right}}(z, \omega) = \mathcal{E}_0(z, \omega) + \Delta \mathcal{E}_{\text{right}}(z, \omega), \] (87)

\[ \Delta \mathcal{E}_{\text{left}}(z, \omega) = 2\pi E_0 e^{-i \kappa d} D(\omega), \quad z \leq -d/2, \]
\[ \Delta \mathcal{E}_{\text{right}}(z, \omega) = 2\pi E_0 e^{i \kappa d} D(\omega), \quad z \geq d/2. \] (88)

We designate the field inside of QW by indexes QW

\[ \mathcal{E}_{QW}(z, \omega) = \mathcal{E}_0(z, \omega) + \Delta \mathcal{E}_{QW}(z, \omega). \] (89)

Let us determine the integral

\[ \int_{-d/2}^{d/2} dz' \mathcal{E}_{QW}(z, \omega) \] included in the RHS of (86), in an approximation \( \kappa d \ll 1 \). Substituting (89) in the integrand we obtain

\[ \int_{-d/2}^{d/2} dz' \mathcal{E}_{QW}(z, \omega) = \int_{-d/2}^{d/2} dz' \mathcal{E}_{QW}(z, \omega) + 2\pi E_0 D_0(\omega) C_{\rho}, \] (90)

where

\[ C_{\rho} = \int_{-d/2}^{d/2} dz \Phi_{\rho}(z). \]

The function \( \Delta \mathcal{E}_{QW}(z) \) is unknown, but on QW’s borders we have

\[ \Delta \mathcal{E}_{QW}(-d/2, \omega) = \Delta \mathcal{E}_{\text{left}}(-d/2, \omega) = \frac{2\pi E_0 e^{i \kappa d} D(\omega)}{\kappa \nu}, \]
\[ \Delta \mathcal{E}_{QW}(d/2, \omega) = \Delta \mathcal{E}_{\text{right}}(d/2, \omega) = \frac{2\pi E_0 e^{-i \kappa d} D(\omega)}{\kappa \nu}. \] (91)

From (91) it is clear that at \( \kappa d \ll 1 \)

\[ \Delta \mathcal{E}_{QW}(z, \omega) \approx 2\pi E_0 D(\omega). \] (92)

Substituting (92) in (90) we obtain

\[ \int_{-d/2}^{d/2} dz' \phi_{\rho}(z') \mathcal{E}_{QW}(z, \omega) = 2\pi E_0 D_0(\omega + D(\omega)) \rho. \] (93)

Using the first equality from (88) we write down the equation (86) for \( z < -d/2 \). In this area the integral

\[ \int_{-d/2}^{d/2} dz' e^{-i \kappa z' \phi_{\rho}(z')}, \]

included in the RHS of (86), is equal 0, and the integral

\[ \int_{z}^{d/2} dz' e^{i \kappa z' \phi_{\rho}(z')} = \int_{-d/2}^{d/2} dz' \phi_{\rho}(z') = C_{\rho}. \]

We obtain the equation for \( D(\omega) \) the solution of which is

\[ D(\omega) = \frac{-4\pi \chi(\omega) D_0(\omega)}{\kappa \nu(1 + 4\pi \chi(\omega))}, \] (94)

\[ \chi(\omega) = \frac{i}{8\pi} \sum_{\rho} \gamma_{\rho} C_{\rho}^2 \{ (\omega - \omega_{\rho} + i \gamma_{\rho}/2)^{-1} + (\omega + \omega_{\rho} + i \gamma_{\rho}/2)^{-1} \}. \] (95)

The equation (86) for \( z > d/2 \) also results in (95).

In a case of free movement of electrons and holes along \( z \) axis, when (53) is carried out,

\[ C_{\rho} = \delta_{l_{\rho}, l_{\rho}} \]

and

\[ \chi(\omega) = \frac{i}{8\pi} \sum_{\rho_0} \gamma_{\rho_0} \{ (\omega - \omega_{\rho_0} + i \gamma_{\rho_0}/2)^{-1} + (\omega + \omega_{\rho_0} + i \gamma_{\rho_0}/2)^{-1} \}. \]

\( \rho_0 \) is the set of indexes at \( l_{\rho} = l_{h} = l \), i.e. the set \( l, k_{\perp} \) for \( H_{\perp} = 0 \) and \( l, n \) for the case of a quantizing magnetic fields.

The energy levels are accordingly equal

\[ \omega_{\rho_0} = \bar{\omega}_{gl} + \frac{\hbar^2 k_{\perp}^2}{2\mu}, \quad \omega_{\rho_0} = \bar{\omega}_{gl} + \Omega_{\mu}(n + 1/2), \] (96)
where
\[ \bar{\omega}_{gl} = \omega_g + \frac{\hbar^2 \pi^2 l^2}{2 \mu d^2}. \]

For electric fields at the left and to the right of QW we obtain the expressions
\[
\mathbf{E}_{\text{left(right)}}(z, t) = \mathbf{E}_0(z, t) + \Delta \mathbf{E}_{\text{left(right)}}(z, t), \tag{97}
\]
\[
\Delta \mathbf{E}_{\text{left(right)}}(z, t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega(t + z/\nu)} D(\omega) + c.c., \tag{98}
\]
where the superscript concerns to an index \text{left}, the subscript - to an index \text{right}. With the help of expressions (97) and (98) it is possible to obtain formulas for reflected and absorbed by QW light fluxes in case of any number of energy levels in QW, any form of exciting pulse (including monochromatic irradiation) and any ratio of parameters \( \gamma_r \) and \( \gamma \) (radiative and non-radiative broadenings of excitations). It follows from (98) that the induced fields \( \Delta \mathbf{E}_{\text{left}}(z, t) \) and \( \Delta \mathbf{E}_{\text{right}}(z, t) \) differ only by direction of movement.

**XV. SOLUTIONS FOR QW, WHICH WIDTH IS COMPARABLE TO A LIGHT WAVE LENGTH. THE FIRST ORDER ON ELECTRON-LIGHT INTERACTION.**

The electric field \( \mathbf{E}(z, t) \) may be spread out in a series on electron-light interaction
\[
\mathbf{E}(z, t) = \mathbf{E}_0(z, t) + \mathbf{E}_1(z, t) + \mathbf{E}_2(z, t) + \ldots, \tag{99}
\]
where \( \mathbf{E}_0(z, t) \) is the exciting field. It is possible to obtain the following orders by iterations, using the equation (86).

In the first order we have
\[
\mathbf{E}_1(z, t) = \frac{\mathbf{E}_0}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \mathcal{E}_1(z, \omega) + c.c., \tag{100}
\]
\[
\mathcal{E}_1(z, \omega) = -\frac{i}{2} \sum_r \gamma_r \left[ \int_{-d/2}^{d/2} dz' e^{-i\kappa z'} \phi_{r}(z') \mathcal{E}_0(z', \omega) + \int_{d/2}^{\infty} dz' e^{-i\kappa z'} \phi_{r}(z') \right]
\]
\[ \times \left\{ e^{i\kappa z} \int_{-d/2}^{d/2} dz' e^{-i\kappa z'} \phi_{r}(z') + e^{-i\kappa z} \int_{d/2}^{\infty} dz' e^{i\kappa z'} \phi_{r}(z') \right\}
\]
\[ \times \{(\omega - \omega_p + i\gamma_p/2)^{-1} + (\omega + \omega_p + i\gamma_p/2)^{-1}\}. \tag{101}
\]
Using definition
\[
\mathcal{E}_0(z, \omega) = 2\pi \mathcal{E}_0 e^{i\kappa z} D_0(\omega), \tag{102}
\]
with the help of (100) we obtain
\[
E_1(z, \omega) = -iE_0 e^{i\kappa z} D_0(\omega)
\]
\[ \times D_0(\omega) \sum_r (\gamma_{r}\rho/2) R^*_{r}(\kappa)
\]
\[ \times \left\{ e^{i\kappa z} \int_{-d/2}^{d/2} dz' e^{-i\kappa z'} \phi_{r}(z') + e^{-i\kappa z} \int_{d/2}^{\infty} dz' e^{i\kappa z'} \phi_{r}(z') \right\}
\]
\[ \times \{(\omega - \omega_p + i\gamma_p/2)^{-1} + (\omega + \omega_p + i\gamma_p/2)^{-1}\} + c.c., \tag{103}
\]
where the designation is introduced
\[
R_{r}(\kappa) = \int_{-d/2}^{d/2} dz \phi_{r}(z) e^{-i\kappa z}. \tag{104}
\]
For free electrons and holes in an infinitely deep QW
\[
R_{\rho}(\kappa) = \int_{-d/2}^{d/2} dz \varphi_{l_\rho}(z) \varphi_{l_\rho}(z) e^{-i\kappa z}. \tag{105}
\]
The functions \( \varphi_{l}(z) \) are determined in (85). From (103) we obtain that the fields at the left and to the right of QW are accordingly equal
\[
\mathbf{E}_{1\text{left}}(z, t) = -iE_0 e^{i\kappa z} D_0(\omega)
\]
\[ \times D_0(\omega) \sum_r (\gamma_{r}\rho/2) (R^*_{r}(\kappa))^2
\]
\[ \times \{(\omega - \omega_p + i\gamma_p/2)^{-1} + (\omega + \omega_p + i\gamma_p/2)^{-1}\} + c.c., \tag{106}
\]
\[
\mathbf{E}_{1\text{right}}(z, t) = -iE_0 e^{i\kappa z} D_0(\omega)
\]
\[ \times D_0(\omega) \sum_r (\gamma_{r}\rho/2) |R_{r}(\kappa)|^2
\]
\[ \times \{(\omega - \omega_p + i\gamma_p/2)^{-1} + (\omega + \omega_p + i\gamma_p/2)^{-1}\} + c.c.. \tag{107}
\]
And for a field \( \mathbf{E}_{1QW}(z, t) \) inside of QW it is necessary to use (103). From (103) - (107) it follows that in a case of wide QW there is allowable a creation of EHPs with quantum numbers
\[
l_e \neq l_h, \tag{108}
\]
and also it appears there a dependence of fields on a QW’s width \( d \) contained in factors \( R_{r}(\kappa) \). It is possible to show that, if to use functions (85), the factor \( R^*_{r}(\kappa)/R_{r}(\kappa) \), included in the relation \( \mathbf{E}_{1\text{left}}(z, t)/\mathbf{E}_{1\text{right}}(z, t) \), depends
on indexes \( l_e \) and \( l_h \) as follows: if \( l_e \) and \( l_h \) are of identical parity, \( R^*_l(k)/R_p(k) = 1 \), if \( l_e \) and \( l_h \) are of different parity, \( R^*_l(k)/R_p(k) = -1 \).

Substituting the first order result (101) in the RHS of (86) one obtains the second order result, etc. So it is possible to calculate all series (99). But we apply another method for calculations of fields in case of wide QWs, i.e. under condition of \( \kappa d \geq 1 \).

**XVI. SOLUTIONS FOR WIDE QW IN A CASE OF ONE ENERGY LEVEL.**

In a case when one energy level is essential the equation (86) may be solved exactly. Introducing designations

\[
\omega_p = \omega_0, \quad \gamma_p = \gamma, \quad \phi_p(z) = \phi(z), \quad \gamma_{\tau p} = \gamma_{\tau} \tag{99}
\]

we re-write (86) as

\[
\mathcal{E}(z, \omega) = \frac{\Gamma}{2} \gamma_M \left\{ e^{i\kappa z} \int_{d/2}^{d/2} dz'e^{-i\kappa z'} \phi(z') + e^{-i\kappa z} \int_{d/2}^{-d/2} dz'e^{i\kappa z'} \phi(z') \right\} \times \left\{ (\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1} \right\} + 2\pi \mathcal{E}_0 e^{i\kappa z} D_0(\omega), \tag{110}
\]

where the designation is introduced

\[
M(\omega) = \int_{-d/2}^{d/2} dz' \phi(z') \mathcal{E}(z', \omega). \tag{111}
\]

Let us multiply both parts of (110) on \( \phi(z) \) and integrate on \( z \) in limits from \(-d/2\) up to \( d/2\). It results in an equation for \( M(\omega) \), which solution is

\[
M(\omega) = 2\pi \mathcal{E}_0 D_0(\omega) R^*(\kappa) \left\{ 1 + \frac{i}{2} \gamma_{\tau} J(\kappa) \right\} \times \left\{ (\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1} \right\}^{-1} \tag{112}
\]

where

\[
J(\kappa) = \int_{-d/2}^{d/2} dz \phi(z) \left\{ e^{i\kappa z} \int_{d/2}^{d/2} dz'e^{-i\kappa z'} \phi(z') + e^{-i\kappa z} \int_{d/2}^{-d/2} dz'e^{i\kappa z'} \phi(z') \right\}. \tag{113}
\]

It is possible to show that

\[
J(\kappa) = |R(\kappa)|^2 + iQ(\kappa). \tag{114}
\]

Substituting (111) in (109) we obtain the solution of our task. Using (83) we find induced electric fields at the left and to the right of QW

\[
\Delta \mathcal{E}_{left}(z, t) = -ie_0 G_0(\gamma_{\tau}/2) \int_{-\infty}^{\infty} d\omega e^{-i\kappa z - i\omega t} D_0(\omega) \times (R^*(\kappa))^2 \left[ (\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1} \right] \times \left\{ 1 + i(\gamma/2)(|R(\kappa)|^2 + iQ(\kappa)) \right\} \times \left\{ [(\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1}]^{-1} \right\}^{-1}. \tag{115}
\]

\[
\Delta \mathcal{E}_{right}(z, t) = -ie_0 G_0(\gamma_{\tau}/2) \int_{-\infty}^{\infty} d\omega e^{i\kappa z - i\omega t} D_0(\omega) \times (R^*(\kappa))^2 \left[ (\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1} \right] \times \left\{ 1 + i(\gamma/2)(|R(\kappa)|^2 + iQ(\kappa)) \right\} \times \left\{ [(\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1}]^{-1} \right\}^{-1}. \tag{116}
\]

The value

\[
\gamma_{\tau}(\omega) = \gamma_{\tau}|R(\kappa)|^2 \tag{117}
\]

at \( \omega = \omega_0 \) and at the account of (72) coincides with calculated in [27] radiative broadening of EHP in a quantizing magnetic field at \( N_e = n_h = n, K_\perp = 0 \) for any values \( \omega_0 d/c \).

Neglecting the non-resonant contribution \( (\omega + \omega_0 + i\gamma/2)^{-1} \), we obtain from (114) and (115) the results [26]

\[
\Delta\mathcal{E}_{left}(z, t) = -ie_0 G_0(\gamma_{\tau}/2) \int_{-\infty}^{\infty} d\omega \times e^{-i\kappa z - i\omega t}(\gamma_{\tau}(\omega)/2) D_0(\omega) e^{i\alpha} \int_\omega^{\omega_0 + \Delta + i(\gamma + \gamma_{\tau}(\omega))/2}, \tag{118}
\]

\[
\Delta\mathcal{E}_{right}(z, t) = -ie_0 G_0(\gamma_{\tau}/2) \int_{-\infty}^{\infty} d\omega \times e^{i\kappa z - i\omega t}(\gamma_{\tau}(\omega)/2) D_0(\omega) \int_\omega^{\omega_0 + \Delta + i(\gamma + \gamma_{\tau}(\omega))/2}, \tag{119}
\]

where

\[
e^{i\alpha} = \frac{R^*(\kappa)}{R(\kappa)}, \quad \Delta = (\gamma/2)Q(\kappa). \tag{120}
\]

Let us notice that above this section we did not use the formula (53), applicable only in case of free movement of electrons and holes along \( z \) axis and assumed only performance of (52).

In case of use (53) with substitution of functions (85) it is possible to transform the expression for \( R(\kappa) \) and \( R^*(\kappa) \) to

\[
R(\kappa) = \frac{1}{2} \int_{-d/2}^{d/2} dz e^{-i\kappa z}. \tag{121}
\]

*In [26] in the formulas (47) and (48) the typing errors are admitted: instead of \( \gamma_{\tau} \) it is necessary to read \( \gamma_{\tau} e^{-i\kappa d/2} \).
\begin{equation}
\times \{ \cos[(\pi/d)(l_e - l_h)z + (\pi/2)(l_e - l_h)z] \\
- \cos[(\pi/d)(l_e + l_h)z + (\pi/2)(l_e + l_h)] \} \}
\end{equation}

\begin{equation}
R^*(\kappa) = \frac{1}{2} \int_{-d/2}^{d/2} e^{-inz} \\
\times \{ \cos[(\pi/d)(l_e - l_h)z - (\pi/2)(l_e - l_h)] \\
- \cos[(\pi/d)(l_e + l_h)z - (\pi/2)(l_e + l_h)] \} 
\end{equation}

In case of narrow QWs at \( \kappa d << 1 \) we have
\begin{equation}
R(\kappa) = R^*(\kappa) = \delta_{l_e, l_h} 
\end{equation}

It means that light creates only EHPs with identical numbers of the size quantization of electrons and holes (in a limit of infinite deep QWs). In a case \( \kappa d \geq 1 \) EHPs are born with various \( l_e \) and \( l_h \), i.e. there may be an excitation of much more energy levels.

**XVII. MONOCHROMATIC AND PULSE EXCITATION.**

In [7-15] the following expression is used for a light pulse
\begin{equation}
E(z, t) = E_0 [e^{-i\omega_1 p} + e^{i\omega_1 p} \Theta(p) e^{-\gamma_1 p/2} + 1 - \Theta(p) e^{\gamma_1 p/2}], 
\end{equation}

where \( \omega_1 \) is the carrying frequency, \( p = t - z\nu/c, \Theta(p) \) is the Haeviside function. Decomposing a pulse on frequencies, we have
\begin{equation}
E(z, t) = E_0 e^{i\omega} \int_{-\infty}^{\infty} d\omega e^{-i\omega p} D_0(\omega) + c.c. 
\end{equation}

where
\begin{equation}
D_0(\omega) = \frac{i}{2\pi} [(\omega - \omega_1 + i\gamma_1/2)^{-1} - (\omega - \omega_1 - i\gamma_1/2)^{-1}].
\end{equation}

Under condition of \( \gamma_1 = \gamma_2 = \gamma \) the pulse is symmetric, its duration is of order \( \gamma^{-1} \). At \( \gamma_1 \to 0 \) we obtain
\begin{equation}
D_0(\omega) = \delta(\omega - \omega_1),
\end{equation}
what corresponds to monochromatic irradiation. At \( \gamma_2 \to 0 \) the pulse is asymmetrical and has a very much abrupt front.

The case of an monochromatic irradiation is considered in [26,27], only asymmetric pulse - in [20-22], only symmetric pulse - in [23,24,28], symmetric and asymmetric pulses - in [25].

**XVIII. CONCLUSIONS.**

It is possible to allocate two most important results. First, the expressions (48) and (50) for the averaged density of the induced current is applicable to any semiconductor objects in case of any number of energy levels of electronic excitations and at any form of a stimulating light pulse and also at anyone direction of light concerning crystal axes.

Second, (84) is the integral equation for Fourier-components of the electric field in a case of normal incidence of light on a QW, which width can be comparable to the light wave length, and number of energy levels of excitations is anyone, what in particular corresponds to a QW in a quantizing magnetic field. The equation is applicable both for monochromatic and for pulse light excitation. With the help of these results it is possible to solve a plenty of tasks on optics of low-dimensional semiconductor objects.
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