Lessons of Quantum 2D Dilaton Gravity

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ABSTRACT

Based on the analysis of two dimensional dilaton gravity we argue that the semiclassical equations of black hole formation and evaporation should not be interpreted in terms of expectation values of operators in the exact quantum theory, but rather as WKB trajectories. Thus at the semiclassical level it does not seem possible to formulate a notion of quantum mechanical information loss.

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The development of two-dimensional dilaton gravity models (CGHS) \[1 - 5\] raises the possibility of understanding some conceptual issues in quantum gravity. In particular it may be hoped that the question of information loss in the evaporation of black holes \[6\] might be resolved within this simple model. Another issue that may be addressed is the problem of time in a quantum theory of gravity. Actually, as discussed in \[7\], in order to resolve the first issue it is necessary to have some understanding of the second.

In the literature on quantum gravity the semiclassical approximation is often confused with the Born-Oppenheimer approximation. As shown in \[8\] the latter approximation (which for 4D gravity consists of expanding in powers of $M^{-1}_P$), when applied to the Wheeler-DeWitt (WDW) equation, leads to a Schrödinger equation for the matter wave function in a background geometry that is a solution of the vacuum Einstein equation. However, in much of the work on black hole evaporation that attempts to include the effect of backreaction, it is assumed that the gravitational field may be treated classically according to the equation $G_{\mu\nu} = 8\pi G < T_{\mu\nu}>$, where the expectation value is to be taken in a quantum state of matter that satisfies the Schrödinger equation. Although many authors have attempted to obtain this from the WDW equation, there is no real derivation of this approximation. In fact the source of the problem in our opinion is the appearance of an expectation value, i.e. a quantity that violates the superposition principle. Clearly, starting from a linear equation like the WDW equation, it is unlikely that there can be a satisfactory derivation of a non-linear “approximation”.§

How then are we to understand the equations that have been discussed in

‡ The well known argument for information loss assumes that there is a notion of time evolution of quantum states defined on a series of space-like surfaces in an evaporating black hole space time that is a solution of the semiclassical equation. It is then implicitly assumed (in the case of the argument for no information doubling the assumption is explicit) that the superposition principle is valid for the matter quantum state. This is however not compatible with the assumption of a definite classical solution determined by the expectation value of the stress tensor. For a concrete calculation illustrating the problems involved see \[9\].

§ For a recent review of the problems involved see \[7\].
two dimensional dilaton gravity? What is the interpretation of these so-called
semiclassical equations? How can they be obtained from an exact formulation
of the theory? In [10, 7] the problems associated with what is usually called
the semiclassical approximation were discussed⁷. In this note we discuss other
possibilities for deriving a semiclassical interpretation, including the use of coherent
states that satisfy the constraints. We argue that the only consistent interpretation
of the semiclassical equations found in the literature is to regard these equations
as defining the WKB trajectories of the Wheeler-DeWitt wave function. We also
show that the quantum analog of the ADM Hamiltonian (that may be taken to
be conjugate to the time measured by clocks at infinity) in these dilaton gravity
theories plays no role in the local physics. Finally we discuss the implications for
the problem of information loss.

In conformal gauge, the Liouville-like version of the CGHS action, which in-
corporates corrections coming from the functional integral measure, is given by [2,
3]

\[
S = \frac{1}{4\pi} \int d^2\sigma \left[ \mp \partial_+ X \partial_- X \pm \partial_+ Y \partial_- Y + \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i + 2\lambda^2 e^{\mp \sqrt{2} \kappa (X \mp Y)} \right].
\]  (1)

Here \(\kappa = \frac{N-24}{6}\), \(N\) being the number of matter fields*. We have omitted in the
above the ghost action that comes from gauge fixing to the conformal gauge since
we are going to discuss only semiclassical, i.e. large \(N\), effects. Thus for the
purposes of this paper \(\kappa\) may be replaced by \(N/6\). Also in the above we will choose
the lower sign corresponding to \(\kappa > 0\). The field variables in the above are related
to the original variables \(\phi\) and \(\rho\) that occur in the CGHS action, gauge fixed to the

⁷ Actually in that paper the discussion at the end was couched in terms of deBroglie-Bohm
trajectories, which of course are supposed to describe the complete quantum mechanical
situation, and therefore involved potentially controversial interpretational issues. However,
for the purpose of just discussing the emergence of the semiclassical physics, it is suffi-
cient to consider the leading order approximation, which just gives the well-known WKB
trajectories.

* Note that this definition has the opposite sign to the \(\kappa\) defined in [2].
conformal gauge \((g_{\alpha\beta} = e^{2\phi}\eta_{\alpha\beta})\), through the following relations:

\[
Y = \sqrt{2\kappa} \left[ \rho + \kappa^{-1} e^{-2\phi} - \frac{2}{\kappa} \int d\phi e^{-2\phi} h(\phi) \right], \quad X = 2\sqrt{\frac{2}{\kappa}} \int d\phi P(\phi), \tag{2}
\]

where \(P(\phi) = e^{-2\phi}[(1 + \overline{T})^2 - \kappa e^{2\phi}(1 + h)]^{\frac{1}{2}}\). In (2), the functions \(h(\phi), \overline{T}(\phi)\) parametrize quantum (measure) corrections that may come in when defining the theory with respect to a translationally invariant measure (see [4] for details).

The discussion of Hawking evaporation has been based on the solution of the classical equations of motion coming from (1) and imposing the classical constraint equations: \(T_{\pm}^{X,Y} + T_{\pm}^{f} = 0\). What interpretation do these equations have from the standpoint of the exact quantum theory of (1)? It has already been pointed out [10, 7] that these equations cannot be interpreted as the expectation values of the field operators in physical states.\(^\ast\)\(^\ast\) The reason is the well-known problem of time in quantum gravity. How then are these equations to be interpreted.

The first issue is whether it is possible to derive the semiclassical equations starting from (say) coherent states of the DDF operators constructed in [11, 10]. The DDF operators \((\hat{A})\) are gravitationally dressed creation and annihilation operators for the \(f\) fields that commute with the total stress-tensor operator. Thus states constructed from them (by operating on the Fock vacuum) will satisfy the physical state condition (at least in the form \(T_{\text{total}}^+|\Psi> = 0\) or the BRST form, if not in the Wheeler-DeWitt form) and one may ask whether the semiclassical equations have an interpretation as expectation values in these states. Now as pointed out in [10], the expectation values of the field operators \(\hat{f}, \hat{X}, \hat{Y}\) in these states are independent of time if the time translation generator is the usual one, namely the spatial integral of the time-time component of the total stress tensor. However one might use a different Hamiltonian as the time translation generator. In fact, one might think that the natural Hamiltonian to use is \(\hat{H}_d = \int d\omega \omega \hat{A}^\dagger(\omega) \hat{A}(\omega)\).

\(^\ast\)\(^\ast\) The physical states are normally defined as solutions of the Wheeler-DeWitt equation or the BRST condition \(Q_{\text{BRST}}|\Psi> = 0\).
Clearly the field operator

\[ f_d(\sigma^+, \sigma^-) = \int_{\omega > 0} d\omega \left[ A_+(\omega)e^{-i\omega\sigma^+} + A_+^\dagger(\omega)e^{i\omega\sigma^+} + (+ \rightarrow -) \right], \]  

(3)

which satisfies the free field equation \( \partial_+ \partial_- f_d = 0 \), evolves in time according to the Heisenberg equation \( \dot{f}_d = i [H_d, f_d] \). Coherent states may then be defined as (using a somewhat schematic notation)

\[ |f_c> = e^{\int d\sigma f_c \partial f_d} : |0>, \]  

(4)

and the classical field \( f_c \) is the expectation value of the dressed field operator in this state. However, it is not possible to obtain the rest of the semiclassical system of equations. In particular the time evolution of the geometry (i.e. the fields \( X \) and \( Y \)) as defined by this new Hamiltonian is now given by a non-local equation since the Hamiltonian density is non-local in those fields**. Also, even though the constraint equation \( T^{(+)}_{\text{tot}}|\Psi> = 0 \) implies that \( <f_c|T^{X,Y} + T^f|f_c> = 0 \), the usual semiclassical equations cannot be obtained from this since that would mean again taking the expectation values of \( f, X, Y, \) etc. in the physical states, and then we are back to the old problem of the time independence of the expectation values.

In some recent papers (for instance [12, 13]) the equations of the theory are interpreted in the following fashion. Consider the constraint equation in the Kruskal gauge (which in our notation means the choice of coordinates such that \( X + Y = 0 \)):

\[ \sqrt{\kappa} \frac{\partial^2}{2} X(x^\pm) = \partial_{\pm} f_c \partial_{\pm} f_c. \]

It is then proposed that the left hand side of this equation (i.e. the geometrical fields) be treated as classical fields and the right hand side as the expectation value

** This is easily seen by examining the definitions for the DDF operators, see Refs. [11, 10].
of the matter field operator in a coherent state of the $f$-field\textsuperscript{****}. But as discussed in detail in [7] this is precisely the interpretation that cannot be obtained if one starts from the Wheeler-DeWitt equation (or any other implementation of the constraint as a linear condition on the states). An alternative that one might consider is the imposition of a non-linear condition $\langle T^{total} \rangle = 0$ as was proposed in [10]. Here one has a Schrödinger evolution of the states, but the linear superposition principle has been abandoned. It is not clear whether this is a viable theory of quantum gravity, but it is perhaps the only option available if one wants to interpret the semiclassical theory in terms of expectation values of field operators. However there are still ambiguities in the treatment of the constraints that we wish to highlight now.

The states we wish to consider are coherent states of the matter and dilaton gravity fields. These may be built either on the vacuum that is annihilated by operators defined with respect to the Kruskal coordinates $x^\pm$, or by that defined with respect to the sigma coordinates $\sigma^\pm = \pm \lambda^{-1} \ln (\pm \lambda x^\pm)$. Thus we have two possible states to consider: $|\, , \, K >$ or $|\, , \, \sigma >$, where the commas denote the values of the classical field configurations for the $X$, $Y$, and $f$ fields. However there are two possible operators to consider as well. Namely

\begin{equation}
\hat{T}^{X,Y} + : \hat{T}^f ;_K,
\end{equation}

and

\begin{equation}
\hat{T}^{X,Y} + : \hat{T}^f ;_\sigma.
\end{equation}

Note that these operators are just the expressions for the total stress tensor operator, apart from the ghost stress tensor that we ignore since we are just interested in the large $N$ limit. In this same limit we may ignore the normal ordering of the $X, Y$ stress tensor. Now if we are to consider the physical state conditions as the equation that the expectation values of these operators is zero, then clearly there

\textsuperscript{****} I.e. (4) with $\hat{f}_d$ replaced by $\hat{f}$
are four possibilities. Equating the expectation value in the Kruskal states of the Kruskal normal-ordered operator (5) to zero we have,

\[-\sqrt{\frac{\kappa}{2}} \partial_{\pm}^2 X(x) + \partial_{\pm} \bar{f} \partial_{\pm} f(x) = 0.\]  

(7)

The fields in the above equation are the classical fields out of which the coherent states are constructed. The fields in the Kruskal gauge are barred to distinguish their functional form from that of the fields in the sigma gauge. Of course, since all the fields in the above equation are scalars, we have \(\overline{X}(x) = X(\sigma)\) and \(\overline{f}(x) = f(\sigma)\). Similarly, equating the expectation value in the sigma states of the sigma normal ordered operator (6), we have

\[- \left[ \sqrt{\frac{\kappa}{2}} (\partial_{\pm}^2 X \mp \partial_{\pm} X)(\sigma) + \frac{\kappa \lambda^2}{4} \right] + \partial_{\pm} f \partial_{\pm} f(\sigma) = 0.\]  

(8)

If one now uses also the equations of motion for these fields then it is possible to show that the relation (8) is the one corresponding to Hawking radiation [1-5] whilst (7) seems to describe the static situation of a black hole in equilibrium with a radiation bath [14].

On the other hand one may also evaluate the Kruskal normal-ordered operator (5) in the sigma states using

\[\hat{T} f : \sigma (\sigma) = \left( \frac{dx}{d\sigma} \right)^2 \hat{T}_f : K (x) - \frac{\kappa \lambda^2}{4}\]

and get

\[- \sqrt{\frac{\kappa}{2}} (\partial_{\pm}^2 X \mp \partial_{\pm} X)(\sigma) + \partial_{\pm} f \partial_{\pm} f(\sigma) = 0.\]  

(9)

This is nothing but the coordinate transformed version of (7) and describes the static situation. Similarly, evaluating the sigma normal ordered operator (6) in the Kruskal states gives us

\[-\sqrt{\frac{\kappa}{2}} \partial_{\pm}^2 \overline{X}(x) + \partial_{\pm} \bar{f} \partial_{\pm} f(x) = 0,\]  

(10)

which is of course just (8) evaluated in Kruskal coordinates.
The moral of the story is that it is irrelevant which state the expectation value is taken, only the operator appears to be relevant. This should be compared with the usual notion, obtained from intuition derived from working in a fixed background, that the two vacua (the Hartle-Hawking one, i.e. the one that is annihilated by the positive frequency modes defined in the Kruskal gauge, and the Unruh vacuum defined relative to the sigma coordinates) are what determine the two different physical situations. In any case, the fact that one has to abandon the superposition principle in order to justify this expectation value interpretation makes it unattractive. Also one cannot within this interpretation formulate the usual arguments for information loss which depend on being able to deal with (a linear subspace of) the Hilbert space at least for the matter sector.

This brings us to the interpretation of the semiclassical equations as WKB trajectories. To facilitate the (formal) justification of our approximations it is convenient to explicitly introduce $\hbar$ into the discussion. Thus in all of the above equations $\kappa$ should actually be replaced by $\kappa\hbar$: these terms come from measure corrections and thus have an explicit factor of $\hbar$. Now the $\frac{1}{\kappa}$ expansion is done keeping $\kappa\hbar$ fixed at $O(1)$, and therefore it is nothing but the usual (WKB) semiclassical expansion in powers of $\hbar$. In fact it is convenient to choose $\kappa\hbar = 1$. Introducing the fields $\zeta_+ = \ln 2 + \sqrt{2} (X + Y)$ and $\zeta_- = \sqrt{2} (X - Y)$, the (total) stress tensors then take the form (ignoring the ghosts)

\[
T_{\pm\pm} = \frac{1}{4} \left[ \partial_\pm \zeta_+ \partial_\pm \zeta_- + \partial_\pm^2 (\zeta_+ - \zeta_-) \right] + \frac{1}{2} \sum_{i=1}^{N} \partial_\pm f_i \partial_\pm f_i \\
T_{+-} = -\frac{1}{4} \partial_+ \partial_- (\zeta_+ - \zeta_-) - \frac{1}{2} \lambda^2 e^{\zeta_+}.
\]

Using the canonical momenta $\Pi_\pm = \frac{1}{16\pi} \dot{\zeta}_\mp$, $\Pi_{f_i} = \frac{1}{8\pi} \dot{f}_i$, the Hamiltonian and momentum densities are:
\[ T_{00} = 32\pi^2 \Pi_+ \Pi_- + \frac{1}{8} \left[ \zeta'_+ \zeta'_- + 2(\zeta''_+ - \zeta''_-) - 8\lambda^2 e^{\zeta^+} \right] + \frac{1}{4} \sum_{i=1}^{N} \left[ f_i^2 + 16\pi^2 \Pi_i^2 \right] \]

\[ T_{01} = 2\pi \left[ \zeta'_- \Pi_- + \zeta'_+ \Pi_+ + 2 \frac{\partial}{\partial \sigma} (\Pi_- - \Pi_+) + \sum_{i=1}^{N} f_i^i \Pi_i \right]. \]

We now quantize using \( \Pi_u \rightarrow \frac{\hbar}{i} \frac{\delta}{\delta u} \) to obtain the corresponding operators \( \hat{T}_{00} \) and \( \hat{T}_{01} \). The quantum mechanical constraint (Wheeler-DeWitt) equations become:

\[ \hat{T}_{00} \Psi [\zeta_+, \zeta_-, f_i] = 0, \quad \text{and} \quad \hat{T}_{01} \Psi [\zeta_+, \zeta_-, f_i] = 0. \] (11)

Writing \( \Psi \) in the form

\[ \Psi = R [\zeta_+, \zeta_-, f_i] \exp \left\{ \frac{i}{\hbar} S [\zeta_+, \zeta_-, f_i] \right\} \]

we obtain for the real part of \( \hat{T}_{00} \Psi = 0 \),

\[ 32\pi^2 \frac{\delta S}{\delta \zeta_+} \frac{\delta S}{\delta \zeta_-} + V[\zeta_+, \zeta_-] + V_m[f'] + Q + 4\pi^2 \sum_{i=1}^{N} \left( \frac{\delta S}{\delta f_i} \right)^2 = 0 \] (12)

with

\[ V[\zeta_+, \zeta_-] = \frac{1}{8} \left[ \zeta'_+ \zeta'_- + 2(\zeta''_+ - \zeta''_-) - 8\lambda^2 e^{\zeta^+} \right], \quad V_m[f'] = \frac{1}{4} \sum_{i=1}^{N} f_i^{i2}, \]

\[ Q = -\hbar^2 \left[ \frac{32\pi^2}{\kappa} \frac{1}{R} \frac{\delta^2 R}{\delta \zeta_+ \delta \zeta_-} + \frac{4\pi^2}{R} \sum_{i=1}^{N} \frac{\delta^2 R}{\delta f_i^2} \right]. \]

The imaginary part of \( \hat{T}_{00} \Psi = 0 \) gives

\[ \sum_{i=1}^{N} \frac{\delta}{\delta f_i} \left( R^2 \frac{\delta S}{\delta f_i} \right) + 4 \left[ \frac{\delta}{\delta \zeta_+} \left( R^2 \frac{\delta S}{\delta \zeta_-} \right) + \frac{\delta}{\delta \zeta_-} \left( R^2 \frac{\delta S}{\delta \zeta_+} \right) \right] = 0. \] (13)
Now let us write out the WKB (semiclassical) expansion as,

\[
S[\zeta_+, \zeta_-, f_i] = S_{-1}[\zeta_+, \zeta_-, f_i] + \hbar S_0[\zeta_+, \zeta_-, f_i] + \hbar^2 S_1[\zeta_+, \zeta_-, f_i] + \ldots
\]

\[
R[\zeta_+, \zeta_-, f_i] = R_0[\zeta_+, \zeta_-, f_i] + \hbar R_1[\zeta_+, \zeta_-, f_i] + \ldots.
\]

Then the leading order WKB approximation to the Wheeler-DeWitt wave function is

\[
\Psi \simeq \Psi_{WKB} = R_0 e^{\frac{i}{\hbar} S_{-1}},
\]

where

\[
32\pi^2 \frac{\delta S_{-1}}{\delta \zeta_+} \frac{\delta S_{-1}}{\delta \zeta_-} + V[\zeta_+, \zeta_-] + V_m[f'] + 4\pi^2 \sum_{i=1}^{N} \left( \frac{\delta S_{-1}}{\delta f_i} \right)^2 = 0, \tag{14}
\]

and \(R_0\) satisfies (13) with \(R\) replaced by \(R_0\) and \(S\) by \(S_0\).

The equation (14) is just the classical Hamilton-Jacobi equation and is equivalent to the classical equations of motion once we introduce the classical trajectory through the equations,

\[
\frac{1}{16\pi} \dot{Z}_\pm(\tau, \sigma) = \Pi_\pm = \frac{\delta S_{-1}}{\delta \zeta_-} \bigg|_{\zeta_\pm = \zeta_\pm_{f_i = f_i}} \Rightarrow \frac{1}{4\pi} \dot{F}_i(\tau, \sigma) = \Pi_f = \frac{\delta S_{-1}}{\delta f_i} \bigg|_{\zeta_\pm = \zeta_\pm_{f_i = f_i}}. \tag{15}
\]

Differentiating these equations with respect to time, and the spatial integral of (14) with respect to \(\zeta_\pm\) and \(f_i\), we obtain the classical equations of motion for the Liouville-like theory:

\[
\ddot{Z}_- - Z'' - 8\epsilon Z_+ = 0, \quad \ddot{Z}_+ - Z''_+ = 0, \quad \ddot{F} - F'' = 0.
\]

Substituting (15) into (14), and similarly deriving the corresponding equation for

\[\text{I.e. the classical CGHS theory modified by the measure corrections that are of order } \kappa \hbar \text{ and have in our large } N \text{ theory been promoted to the status of classical effects.}\]
the $T_{01}$ constraint, one gets the classical constraint equations

\[ 0 = \frac{1}{8} \left( \dot{Z}_+ \dot{Z}_- + Z'_+ Z'_- + 2 \left( Z''_+ - Z''_- \right) - 8 \lambda^2 e^{Z_+} \right) + \frac{1}{4} \sum_{i=1}^{N} \left( F'^2_i + F_i'^2 \right), \]

\[ 0 = \frac{1}{2} \left( \left( Z'_- + 2 \frac{\partial}{\partial \sigma} \right) \dot{Z}_+ + \left( Z'_+ - 2 \frac{\partial}{\partial \sigma} \right) \dot{Z}_- \right) + \sum_{i=1}^{N} F'_i \dot{F}_i. \]

These equations are precisely those which have been extensively discussed recently in connection with Hawking radiation. In our view these have to be viewed not as equations for expectation values of field operators but as the equations for WKB trajectories. A notion of time evolution emerges only along a trajectory, and it should be stressed that at the level of the WKB wave function itself there is no Schrödinger time evolution since it is still just an (approximate) solution to the Wheeler-DeWitt equation. This should be contrasted with what emerges from the Born-Oppenheimer approximation to the WdW equation. In (four-dimensional) minisuperspace for instance this approximation corresponds to holding $\bar{\hbar}$ fixed and doing a large $M_P$ (i.e. compared to matter energy density) expansion. In this case (as first shown by Rubakov and Lapchinsky [8]) one obtains a Schrödinger equation for the matter wave function with time evolution determined by a classical trajectory for the geometry (scale factor). In the 2D dilaton-gravity case the corresponding approximation consists of taking the large $N$ (or $\kappa$) limit while holding $\bar{\hbar}$ fixed. Again one obtains [7] a Schrödinger equation for the matter wave function in a classical geometrical background. However it should be stressed that in this approximation there is no back reaction. In fact, as explained in detail in [7], there is no systematic way of getting a Schrödinger equation for matter in a geometrical background that is backreacting to the evolution of the matter.

By contrast, in the semiclassical WKB approach there is both time evolution and backreaction of the geometry to matter, but in a purely (semi-) classical manner, i.e. along a WKB trajectory. Here there is no Schrödinger evolution of the matter wave function. In order to get time evolution and backreaction one has to
pick a (classical) trajectory in the WKB sense. As far as we can see this is the only interpretation (starting from an exact formulation of the corresponding quantum gravity) that is available for the semiclassical pictures of blackhole formation and evaporation that have been discussed in connection with CGHS models.

Finally let us consider the possibility of introducing time “as measured by” clocks at infinity. Since we are dealing with a asymptotically flat space time classically we can define a boundary Hamiltonian, i.e. the ADM energy. The question at hand is whether there is a quantum analogue of this which enables us to define Schrödinger evolution and avoid the problems discussed above. The Hamiltonian of the theory may be written (going back to the $X, Y$ field basis) as (slightly changing the normalization from that used before)

$$H_0 = \frac{1}{2} \int d\sigma \left[ 2(\Pi^2_X - \Pi^2_Y) + \frac{1}{2}(X'^2 - Y'^2) + 2Y'' - 4\lambda^2 e^{(X+Y)} \right] + H^\partial$$

As Regge and Teitelboim[15] have pointed out, in order to get Hamilton’s equations from this and therefore to interpret this object as the time translation generator, it is necessary to add a boundary term $H^\partial$ to compensate for the boundary contribution that arises in the course of deriving the equations of motion. The total classical Hamiltonian (time translation generator) is thus $H = H_0 + H^\partial \approx H^\partial$. Let us now consider (16) as an operator Hamiltonian in the quantum theory ($X, Y, f$, are now operators) and look at the derivation of the Heisenberg equations. Using the commutation relations $[X(\sigma), \Pi_X(\sigma')] = i\delta(\sigma - \sigma')$ etc. we get $i\dot{X} = 2i\Pi_X$ and

$$i\dot{\Pi}_X = [\Pi_X(\sigma), H_0]$$

$$= \frac{1}{2} \int d\sigma' \left[ X'(\sigma)(-i\partial_\sigma \delta(\sigma - \sigma')) - 4\lambda^2(-i\delta(\sigma - \sigma')e^{(X+Y)(\sigma')}) \right]$$

$$= \frac{i}{2} \left( X''(\sigma) + 4\lambda^2 e^{(X+Y)(\sigma)} \right)$$

Combining these two equations we get the equation of motion for $X$, and similarly for $Y$. The point of the above elementary exercise is to highlight the fact that in the quantum theory, unlike in the classical theory, there is no need for a boundary
term since we are dealing with distributions. Alternatively one may define the theory rigorously as a CFT in a box (see for instance [16]) and then take the size of the box to infinity. Again the irrelevance of the boundary term is apparent.

Even though such a term is not required by the Regge-Teitelboim argument, one may still add an operator valued surface term to the Hamiltonian and then argue that there is a Schrödinger time evolution with respect to that. In other words one may write,

\[ i\hbar \frac{\partial \Psi}{\partial t} = (H_0 + H_\partial)\Psi = H_\partial \Psi \]

(for physical states \( \Psi \)). However, because local operators will commute with a boundary Hamiltonian defined at spatial infinity, their expectation values in physical states will be independent of the time defined above.

What lessons can we draw from the discussion in this paper for the question of information loss? It is our contention that this can be posed only if one can derive a picture of Schrödinger evolution in a background geometry that behaves classically and backreacts to the matter evolution. But this is precisely what is not available to us. As far as we can see there are but two options. One is to use the Born-Oppenheimer approximation. In this case one has a notion of quantum mechanical states for the matter sector that evolve according to Schrödinger’s equation. The background however is a solution of the vacuum Einstein equation and does not backreact. On the other hand one can take the WKB semiclassical approximation. Here back reaction is included and the well-known Penrose diagram of black hole formation and evaporation can be justified. However both the geometry and the matter have to be treated as the classical trajectories that one obtains in the WKB approximation. There is no notion of Schrödinger evolution of the matter wave function. Within this context the question of loss of quantum mechanical information cannot even be posed.

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