Random Matrix Crossovers and Quantum Critical Crossovers for Interacting Electrons in Quantum Dots

Ganpathy Murthy

Department of Physics and Astronomy, University of Kentucky, Lexington KY 40506-0055

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Quantum dots with large Thouless number $g$ embody a regime where both disorder and interactions can be treated nonperturbatively using large-$N$ techniques (with $N = g$) and quantum phase transitions can be studied. Here we focus on dots where the noninteracting Hamiltonian is drawn from a crossover ensemble between two symmetry classes, where the crossover parameter introduces a new, tunable energy scale independent of and much smaller than the Thouless energy. We show that the quantum critical regime, dominated by collective critical fluctuations, can be accessed at the new energy scale. The nonperturbative physics of this regime can only be described by the large-$N$ approach, as we illustrate with two experimentally relevant examples.

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Mesoscopic systems show many novel properties which are not present in bulk systems\[1], such as the Coulomb Blockade in the zero-bias conductance through a quantum dot, or persistent currents in mesoscopic rings penetrated by a magnetic flux. Disorder renders the single-particle states chaotic\[2], with the statistics of the energies and wavefunctions controlled by Random Matrix Theory (RMT)\[3], which also controls the correlations between different states separated by less than the Thouless energy $E_T$ (related to the ergodicization time for a particle $\tau_{\text{erg}}$ by the Uncertainty Principle $E_T = \hbar/\tau_{\text{erg}}$). For mean single-particle level spacing $\delta$, $g = E_T/\delta$.

Since disorder breaks all the spatial symmetries, only time-reversal $T$ and possibly Kramers degeneracy remain. There are three classical symmetry classes\[3], the gaussian orthogonal ensemble or GOE ($T$ intact, no spin-orbit coupling), the unitary or GUE ($T$ broken), and the symplectic or GSE ($T$ intact, with spin-orbit coupling). More recently, other classes have been identified for disordered superconductors\[2] and quantum dots constructed from two-dimensional semiconductor heterostructures with spin-orbit coupling\[3].

The question of how to incorporate electron-electron interactions in mesoscopic systems has enjoyed much attention. A recent proposal has been (somewhat inaccurately) dubbed the “Universal Hamiltonian”\[4, 5]. Its central idea is to start with a general Hamiltonian

$$H = \sum_{\alpha,s} \epsilon_{\alpha,s} c_{\alpha,s}^{\dagger} c_{\alpha,s} + \frac{1}{2} \sum_{\alpha \beta \gamma \delta, s s'} V_{\alpha \beta \gamma \delta}^{s s'} c_{\alpha,s}^{\dagger} c_{\beta,s'}^{\dagger} c_{\gamma,s'} c_{\delta,s}$$ (1)

where $\alpha, s$ are single-particle eigenstates of the kinetic energy (including spin or Kramers degeneracy), and $c^{\dagger}$ are canonical fermion operators. For a given symmetry of the kinetic term, only special matrix elements of the interaction have a nonzero ensemble average\[6]. In the GOE, only $V_{\alpha \beta \gamma \delta}^{s s'}$, $V_{\alpha \beta \gamma \delta}^{s s'}$, and $V_{\alpha \beta \gamma \delta}^{s s'}$ survive ensemble averaging. Matrix elements which do not survive ensemble-averaging are small (of typical size $\delta/g$) as are the sample-to-sample fluctuations of the terms which do survive\[6]. Finally, in the large-$g$ limit all small terms are dropped\[4], leading to the Universal Hamiltonian

$$H_U = \sum_{\alpha,s} \epsilon_{\alpha,s} c_{\alpha,s}^{\dagger} c_{\alpha,s} + \frac{U_0}{2} N^2 - JS^2 + \lambda T \gamma T$$ (2)

where $N$ is the total particle number, $S$ is the total spin, and $T = \sum c_{\alpha, s} c_{\alpha, s}^{\dagger}$ in addition to the charging energy, $H_U$ has an exchange energy $J$ and a superconducting coupling $\lambda$. This last term is absent in the GUE, while the exchange term disappears in the GSE. In the large-$g$ limit only interaction terms which are invariant under the symmetries of the kinetic term appear in $H_U$\[4, 5].

However, the correct way to determine if a coupling is relevant is to carry out a renormalization group (RG) analysis\[7]. If the coupling grows under RG, it must be kept, no matter how small it was initially, and irrespective of whether it had an ensemble average in a particular symmetry class. Using RG, the author and Harsh Mathur have shown that in ballistic quantum dots $H_U$ is unstable to Landau Fermi liquid couplings for sufficiently strong coupling\[8]. Using a variant of the large-$N$ approach (which subsumes the RG\[10]), the author and R. Shankar have constructed\[10] a controlled theory of the quantum phase transition in the large-$g$ limit. For finite $g$ we identified\[10, 11] a weak-coupling regime which is controlled by $H_U$, a strong-coupling regime with a distorted Fermi surface, and a fan-shaped many-body quantum critical regime\[12] (QCR) in which the physics is dominated by collective critical fluctuations. For weak coupling, the QCR can be accessed at nonzero frequency or temperature $(\omega, T) > E_{\text{QCR}}$, the characteristic energy scale for the crossover in our case being\[10, 11] $E_{\text{QCR}} = r E_T$, where $r$ is a dimensionless distance from the criticality. For weak coupling, perturbation theory around $H_U$ is valid only for $(\omega, T) \ll E_{\text{QCR}}$, whereas the large-$N$ approach is valid for all $(\omega, T) \ll E_T$\[11].

Close to the transition $(r \ll 1)$ the large-$N$ approach is superior, but is superfluous deep in the weak-coupling regime $(r \approx 1)$.

The focus of this paper is a class of problems where the QCR can be accessed at a tunable energy independent...
of, and much smaller than $E_T$, even at weak-coupling, and where the large-$N$ approach is essential to capturing the physics. These problems occur in systems where the single-particle Hamiltonian is undergoing a crossover from one symmetry class to another, say from the GOE to the GUE\textsuperscript{[2]}, achieved by turning on the orbital effects of a magnetic field. For Thouless number $g \gg 1$ of the original GOE, the crossover Hamiltonian is a $g \times g$ matrix

$$H_X(\alpha) = H_{GOE} + \frac{\alpha}{\sqrt{g}} H_{GUE}$$

(3)

where $\alpha$ is the crossover parameter. Properties of single eigenvector\textsuperscript{[3]}, and more recently multiple eigenvector\textsuperscript{[4]} correlations, have been computed in the crossover. For $2\alpha^2 = gx \gg 1$, the following ensemble-averaged correlations hold for the eigenstates $\psi_\mu(i)$, where $\mu \neq \nu$ label the states and $i, j, k, l$ the original orthogonal labels:

$$\langle \psi_\mu^*(i) \psi_\nu(j) \rangle = \frac{1}{g^2} \delta_{\mu \nu} \delta_{ij}$$

$$\langle \psi_\mu^*(i) \psi_\nu^*(j) \psi_\nu(k) \psi_\mu(l) \rangle = \frac{\delta_{\mu \nu} \delta_{ij}}{g^2} + \frac{\delta_{\mu \nu} \delta_{ik}}{g^2} E_X \frac{1}{\pi} \left( \frac{E_X}{\pi} - \epsilon_{\mu} + \epsilon_{\nu} \right)$$

(4)

The last term on the second line shows the extra correlations induced in the crossover\textsuperscript{[4]}. The energy scale $E_X = gx \delta/\pi$ represents a window within which GUE correlations have spread. When $E_X \approx E_T$, the crossover is complete. Similar expressions hold for the GOE to GSE crossover as well\textsuperscript{[4]}. It was pointed out\textsuperscript{[4]} that interaction matrix elements have enhanced sample-to-sample fluctuations in crossover ensembles, controlled by $1/g_X$ and not $1/g$. Thus a naive application of the symmetry principle leading to the Universal Hamiltonian is problematic\textsuperscript{[4] [13].}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Different regimes and crossover energy scales}
\end{figure}

We are now ready to state our central results. Consider phase transitions driven by interactions which commute with the symmetries of the initial ensemble but not with those of the crossover ensemble. Then, (i) The energy scale $E_{QCX}$ characterizing the many-body weak-coupling to quantum-critical crossover is proportional to $E_X$. (ii) The large-$N$ approach gives a nonperturbative description of the physics valid for energies $\omega, T \ll E_T$ (regions I and II in Fig\textsuperscript{[1]}, while perturbative treatments based on $H_U$ are valid\textsuperscript{[11]} only for $\omega, T \ll E_{QCX} \approx E_X$ (region I). (iii) Tuning the single-particle crossover offers a powerful way to control and access the quantum-critical crossover. These results apply equally to diffusive and ballistic dots. Finally, RMT is much better controlled in the crossover regime $E_X \ll E_T$\textsuperscript{[13]} than when applied to the entire Thouless shell\textsuperscript{[10]}. The tunability of the QCR is potentially very important for quantum dots fabricated from two-dimensional semiconductor heterostructures, since current samples appear to be on the weak-coupling side of the Stoner and other transitions (see references in refs.\textsuperscript{[7, 11]}). An illustration relevant for these samples is provided by the crossover from the GOE to the new “intermediate” symplectic ensemble\textsuperscript{[3]} with small spin-orbit coupling in two-dimensional heterostructures. Here the physical crossover parameter $\alpha$ is a function of $L/L_{SO}$, where $L_{SO}$ is the spin-orbit scattering length. Keeping the lowest leading order in this ratio ($\alpha = C_1 L^2/L_{SO}^2$, where $C_1$ is a factor of order unity) leads\textsuperscript{[3]} to a Hamiltonian which conserves $S_z$, but not the total spin. The Universal Hamiltonian for this case has recently been worked out by Alhassid and Rupp\textsuperscript{[14]} (AR). For the sake of simplicity, let us ignore all interactions except exchange. The Hamiltonian can be written as

$$H = \sum \epsilon_{\mu \nu} c_{\mu \uparrow}^\dagger c_{\nu \downarrow} - J_z S_z^2 - J (S_x^2 + S_y^2)$$

(5)

The last term would be absent in the Universal Hamiltonian\textsuperscript{[17]}, since it does not commute with the kinetic term, but physically it should be present because the high-energy processes which led to it are still present. AR\textsuperscript{[17]} show that the spin operators can be written as

$$S_+ = \sum_{\mu \nu} M_{\mu \nu} c_{\mu \uparrow}^\dagger c_{\nu \downarrow}$$

(6)

with $S_- = (S_+)^\dagger$. The quantities $M_{\mu \nu}$ satisfy

$$\langle |M_{\mu \nu}|^2 \rangle = \frac{E_X \delta/\pi}{E_X^2 + (\epsilon_{\mu \uparrow} - \epsilon_{\nu \downarrow})^2}$$

(7)

where $E_X = C_2 \alpha^2 \pi E_T$ (similar to Eq.\textsuperscript{[14]}). It is straightforward to carry out the Hubbard-Stratanovich transformation, integrate out fermions, and obtain an effective action for the collective variables, here $\sigma_z, \sigma_y$ coupling to $S_z, S_y$. Deep in the crossover $E_X \gg \delta$ the effective action is self-averaging\textsuperscript{[10] [14]}. Calling $J = J \delta$, and taking $g \to \infty$ and setting $J_z = 0$ for simplicity, to quadratic order we have the Euclidean action

$$S_{eff} = \delta \int \frac{d\omega}{2\pi} |\tilde{\sigma}(\omega)|^2 \left( \frac{1}{f} - \frac{1}{1 + |\omega|/E_X} \right)$$

(8)

The critical coupling is $J^* = 1$, which is also the bulk critical value, exactly as in ref.\textsuperscript{[16]} (in fact, the quadratic effective action for the model of ref.\textsuperscript{[16]}, which the authors did not compute, would be identical to Eq.\textsuperscript{[14]}). The retarded $S_+ S_- $ correlation function in weak-coupling
\( \tilde{J} < 1 \) at \( T = 0 \) is read off by making the replacement \( i\omega \rightarrow \omega + i0^+ \) in Eq. (9). The imaginary part of this correlator has the scaling form valid for \( \omega \ll E_T \)

\[
\frac{1}{\pi} \text{Im} \langle 0 | S_+ (\omega) S_- (-\omega) | 0 \rangle = \frac{j^2}{\pi r} \frac{x}{1 + x^2} \tag{9}
\]

where \( r = 1 - \tilde{J} \) and \( x = \omega / E_T \) is the scaling variable. For \( \omega \gg E_T \approx E_{QCSX} \) the system crosses over to the QCR. Eq. (10) illustrates all our central results, the fact that \( E_{QCSX} \propto E_X \), the fact that the large-\( N \) approach\([10, 11]\) captures the nonperturbative physics of collective critical fluctuations, and the tunability of the crossover to the QCR.

To illustrate the generality of these results, consider the GOE to GUE crossover in three-dimensional ballistic/chaotic superconducting nanoparticles of linear size \( L \). The single-particle spacing is \( \delta \approx h^2/\text{mk}_F L^3 \) and the Thouless energy is \( E_T \approx h^2 k_F / m L \), leading to a Thouless number \( g \approx (k_F L)^2 \). A commonly-used model to understand superconductivity in such particles\([12]\) in the absence of an orbital magnetic field is the reduced BCS Hamiltonian

\[
H_{BCS} = \sum_{i,s} \epsilon_i c^{\dagger}_{i,s} c_{i,s} - \delta \sum \lambda T^{\dagger} T \tag{10}
\]

where \( i, s \) are orthogonal state and spin labels, \( \tilde{\lambda} > 0 \) is the attractive dimensionless BCS coupling valid in an energy shell of width \( 2\hbar \omega_D \) around the Fermi energy, and \( T = \sum c_{i,s} c_{i,s}^{\dagger} \). Much recent work has concentrated on ultrasmall grains\([13]\) of size a few nanometers, where \( \delta \) is comparable to the bulk BCS gap \( \Delta = 2\hbar \omega_D \exp (-1/\tilde{\lambda}) \). The orbital effects of an external magnetic field can be captured by the crossover Hamiltonian of Eq. (10), with \( \alpha/\sqrt{\delta} = C (\phi / \phi_0) \), where \( \phi = BL^2 \) is the flux through the sample, \( \phi_0 = h/e \) is the flux quantum, and \( C \) is a constant of order unity. This leads to \( E_X = 2C^2 (\phi / \phi_0)^2 E_T \propto B^2 L^3 \). For grains of Pb or Al smaller than 10nm and fields in the sub-Tesla range this crossover scale is smaller than \( \delta \), and thus orbital magnetic effects are of no importance for their physics. However, for grains in the size range 10-30nm in moderate fields (see, for example, ref. \([14]\)), the orbital effects will be relevant, while the Zeeman coupling will be small. We will assume that the critical temperature of the superconductor is smaller than the Thouless energy of the particle, true of Pb or Al particles in this size regime. Also, \( L \) should be smaller than the London penetration depth. In the presence of an orbital field, the kinetic energy of Eq. (11) will be replaced by a random matrix drawn from the ensemble of Eq. (10). Since the high-energy processes which led to an attractive BCS interaction are not modified by the small orbital field, the BCS interaction must be kept, though it no longer commutes with the symmetries of the kinetic energy. This leads to

\[
H_{BCS \times} = \sum_{\mu \nu} \epsilon_\mu c^{\dagger}_{\mu, \sigma} c_{\mu, \sigma} - \delta \sum \lambda T^{\dagger} T \tag{11}
\]

\[
T = \sum_{\mu \nu} M_{\mu \nu} c_{\nu, \uparrow} c_{\mu, \uparrow} : \quad M_{\mu \nu} = \sum_i \psi_\mu (i) \psi_\nu (i) \tag{12}
\]

To study the magnetization of the particle in the crossover we start with the partition function \( Z = \text{Tr} (\exp -\beta H) \) where \( \beta = 1/T \) is the inverse temperature. We convert the partition function into an imaginary time path integral and use the Hubbard-Stratanovich identity to decompose the interaction, leading to the imaginary time Lagrangian

\[
\mathcal{L} = \frac{\sigma^2}{\delta \lambda} \sum_{\mu, \nu} \tilde{c}_{\mu, \sigma} (\partial_\tau - \epsilon_\mu) c_{\mu, \sigma} + \sigma \tilde{T} + \sigma T \tag{13}
\]

where \( \sigma, \tilde{\sigma} \) are the bosonic Hubbard-Stratanovich fields representing the BCS order parameter and \( \tilde{c}, c \) are Grassman fields representing fermions. The fermions are integrated out, and the resulting action for \( \sigma, \tilde{\sigma} \) is expanded to second order to obtain

\[
S_{\text{eff}} \approx \frac{\delta}{\beta} \sum_n |\sigma (i \omega_n)|^2 (\frac{1}{2} - f_n (\beta, E_X, i \omega_n)) \tag{14}
\]

\[
f_n (\beta, E_X, i \omega_n) = \frac{\delta}{\beta} \sum_{\mu \nu} |M_{\mu \nu}|^2 \frac{1 - \delta_n (\epsilon_\mu - \epsilon_\nu)}{\epsilon_\mu + \epsilon_\nu - |\omega_n|} \tag{15}
\]

where \( \omega_n = 2\pi n / \beta \), and the sums are restricted to \( |\epsilon_\mu|, |\epsilon_\nu| < h \omega_D \). We see that the correlations between different states \( \mu, \nu \) play an important role. Deep in the crossover (for \( E_X \gg \delta \)) we can replace \( |M_{\mu \nu}|^2 \) by its ensemble average (dominated by the last term of Eq. (14)), just as in our previous work\([14, 15]\). We will also henceforth replace the summations over energy eigenstates by energy integrations with the appropriate cutoffs. The sums can be approximately carried out to give

\[
f_n \approx \frac{1}{2} \log \left( \frac{4 \hbar \omega_D^2 + \omega_n^2}{C' \beta^2 + (E_X + |\omega_n|)^2} \right) \tag{16}
\]

where \( C' \approx 3.08 \) is chosen to obtain the correct transition temperature in the absence of a magnetic field. The coefficient of the quadratic term for the \( \omega_n = 0 \) term as \( T \rightarrow 0, \beta \rightarrow \infty \) is \( 1 / 4 \log 2 \), leading to a continuous phase transition at \( \lambda_0 = 1 / (2 \hbar \omega_D / E_X) \).

On the normal side \( \lambda < \lambda_0 \) \( \sigma, \tilde{\sigma} \) fluctuate, and the quadratic action is sufficient to explore their fluctuations. One can now integrate out \( \sigma, \tilde{\sigma} \) to obtain the contribution of their thermal fluctuations to the free energy. One then obtains for the magnetization

\[
M = - \frac{\partial F}{\partial B} = M_{\text{nonint}} + \lambda L^2 \frac{dE_X}{d\phi} \sum_n \frac{\partial \mu}{\partial E_X} \frac{1}{1 - \lambda f_n} \tag{17}
\]

where \( M_{\text{nonint}} \) is the contribution to the magnetization from noninteracting electrons (intimately connected to the noninteracting persistent current\([14]\)).

To see the physics of the crossover into the QCR as \( T \) increases\([12]\) in the most transparent way, we take the weak-coupling limit \( \lambda \rightarrow 0, \hbar \omega_D \rightarrow \infty \) such that \( T_c \) remains invariant. Defining \( r = \lambda^{-1} - (\lambda_0)^{-1} \), we obtain for the fluctuation magnetization in the scaling region

\[
-M_0 \sum_{n=-\infty}^{\infty} \frac{x (1 + 2 \pi |n| x) / (C' x^2 + (1 + 2 \pi |n| x)^2)}{r + \frac{1}{2} \log (C' x^2 + (1 + 2 \pi |n| x)^2)} \tag{18}
\]
where $x = T/E_X$ is the scaling variable, and $M_0 = L^2 \sqrt{8C^2 E_X/E_T/\pi \phi_0^2}$ sets the $T$-independent scale for the magnetization. At quantum criticality $r = 0$. For $r > 0$ and $T \gg E_X$ the second log in the denominator dominates, and the system crosses over into the QCR. In this case, unlike the previous one, the critical coupling $\lambda^c$ depends on $E_X$, making the critical point tunable as well. Note that this result is nonperturbative in $E_X$, $T$, and $\lambda$, and offers a perspective complementary to previous results which are perturbative in one or more of these parameters [20, 21, 22]. The regime of validity of Eq. (18) is $E_T \gg (T, E_X) \gg \delta$, and $r > 0$.

In summary, we have identified a class of mesoscopic problems in which access to the quantum critical regime (where the physics is dominated by collective critical fluctuations) can be obtained at energies $E_{QCX}$ much smaller than $E_T$ by tuning the crossover scale $E_X$ between ensembles with different single-particle symmetry. Perturbative approaches in these systems are limited to $(\omega, T) \ll E_{QCX} \approx E_X$, whereas the large-$N$ approach [10, 11] is successful in capturing their nonperturbative physics in the entire range $(\omega, T) \ll E_T$. Symmetries crossovers in weak-coupling large-$g$ dots offer theoretical control over and experimental access to strongly correlated physics. It would be interesting (and directly relevant to superconducting particles) to see the crossover from the strong-coupling regime to the quantum critical regime, as well as to determine the signatures of the weak-coupling to quantum critical crossover in the transport properties of semiconductor quantum dots.

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