Evidence for Collective Multiparticle Correlations in \(p\)-Pb Collisions

V. Khachatryan et al.

(CMS Collaboration)

(Received 18 February 2015; revised manuscript received 19 April 2015; published 29 June 2015)

The second-order azimuthal anisotropy Fourier harmonics, \(v_2\), are obtained in \(p\)-Pb and PbPb collisions over a wide pseudorapidity (\(\eta\)) range based on correlations among six or more charged particles. The \(p\)-Pb data, corresponding to an integrated luminosity of 35 nb\(^{-1}\), were collected during the 2013 LHC \(p\)-Pb run at a nucleon-nucleon center-of-mass energy of 5.02 TeV by the CMS experiment. A sample of semiperipheral PbPb collision data at \(\sqrt{s_{NN}} = 2.76\) TeV, corresponding to an integrated luminosity of 2.5 \(\mu\)b\(^{-1}\) and covering a similar range of particle multiplicities as the \(p\)-Pb data, is also analyzed for comparison. The six- and eight-particle cumulant and the Lee-Yang zeros methods are used to extract the \(v_2\) coefficients, extending previous studies of two- and four-particle correlations. For both the \(p\)-Pb and PbPb systems, the \(v_2\) values obtained with correlations among more than four particles are consistent with previously published four-particle results. These data support the interpretation of a collective origin for the previously observed long-range (large \(\Delta\eta\)) correlations in collision systems with a very small overlapping region.

DOI: 10.1103/PhysRevLett.115.012301

PACS numbers: 25.75.Gz

Measurements at the CERN LHC have led to the discovery of two-particle azimuthal correlation structures at large relative pseudorapidity (long range) in proton-proton (\(pp\)) [1] and proton-lead (\(p\)-Pb) [2–5] collisions. Similar long-range structure has also been observed for \(\sqrt{s_{NN}} = 200\) GeV deuteron-gold (\(d + Au\)) collisions at RHIC [6,7]. The results extend previous studies of relativistic heavy-ion collisions, such as for the copper-copper [8], gold-gold [8–12], and lead-lead (PbPb) [13–18] systems, where similar long-range, two-particle correlations at small relative azimuthal angle \(|\Delta\phi| \approx 0\) were first observed. A fundamental question is whether the observed behavior results from correlations exclusively between particle pairs, or if it is a multiparticle, collective effect. It has been suggested that the hydrodynamic collective flow of a strongly interacting and expanding medium [19–21] is responsible for these long-range correlations in central and midcentral heavy-ion collisions. The origin of the observed long-range correlations in collision systems with a small overlapping region, such as for \(pp\) and \(p\)-Pb collisions, is not clear since for these systems the formation of an extended hot medium is not necessarily expected. Various theoretical models have been proposed to interpret the \(pp\) [22,23] and \(p\)-Pb results, including initial-state gluon saturation without any final state interactions [24,25] and, similar to what is thought to occur in heavier systems, hydrodynamic behavior that develops in a conjectured high-density medium [26–28]. These models have been successful in describing different aspects of the previous experimental results.

To further investigate the multiparticle nature of the observed long-range correlation phenomena, in this Letter we present measurements of correlations among six or more charged particles for \(p\)-Pb collisions at a center-of-mass energy per nucleon pair of \(\sqrt{s_{NN}} = 5.02\) TeV. The azimuthal dependence of particle production is typically characterized by an expansion in Fourier harmonics \((v_n)\) [29]. In hydrodynamic models, the second \((v_2)\) and third \((v_3)\) harmonics, called “elliptic” and “triangular” flow [30], respectively, directly reflect the response to the initial collision geometry and fluctuations [31–33], providing insight into the fundamental transport properties of the medium. First attempts to establish the multiparticle nature of the correlations observed in \(p\)-Pb collisions were presented in Refs. [34,35] by directly measuring four-particle azimuthal correlations, where the elliptic flow signal was obtained using the four-particle cumulant method [36]. However, four-particle correlations can still be affected by contributions from noncollective effects such as fragmentation of back-to-back jets. By extending the studies to six- and eight-particle cumulants [36] and by also obtaining results using the Lee-Yang zeros (LYZ) method, which involves correlations among all detected particles...
[37,38], it is possible to further explore the collective nature of the correlations. High-statistics data obtained by the CMS experiment during the 2013 p-Pb run at the LHC are used. With a sample of very high final state multiplicity p-Pb collisions, the correlation data have been studied in a regime that is comparable to the charged particle multiplicity of the 50% most peripheral (semiperipheral) PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

The CMS detector comprises a number of subsystems [39]. The results in this Letter are mainly based on the silicon tracker information. The silicon tracker, located in the 3.8 T field of a superconducting solenoid, consists of 1440 silicon pixel and 15148 silicon strip detector modules. The silicon tracker measures charged particles within the pseudorapidity range $|\eta| < 2.5$, and it provides an impact parameter resolution of $\approx 15$ $\mu m$ and a transverse momentum ($p_T$) resolution better than 1.5% at $p_T \approx 100$ GeV/$c$. The electromagnetic (ECAL) and hadron (HCAL) calorimeters are also located inside the solenoid and cover the pseudorapidity range $|\eta| < 3.0$. The HCAL barrel and end caps are sampling calorimeters composed of brass and scintillator plates. The ECAL consists of lead tungstate crystals arranged in a quasiprojective geometry. Iron and quartz-fiber Čerenkov hadron forward (HF) calorimeters cover the range $2.9 < |\eta| < 5.2$ on either side of the interaction region. These HF calorimeters are azimuthally subdivided into 20° modular wedges and further segmented to form $0.175 \times 0.175$ rad ($\Delta \eta \times \Delta \phi$) “towers.” The detailed Monte Carlo (MC) simulation of the CMS detector response is based on GEANT4 [40].

The analysis is performed using data recorded by CMS during the LHC p-Pb run in 2013. The data set corresponds to an integrated luminosity of 35 nb$^{-1}$. The beam energies were 4 TeV for protons and 1.58 TeV per nucleon for lead nuclei, resulting in $\sqrt{s_{NN}} = 5.02$ TeV. The beam directions were reversed during the run, allowing a check of one potential source of systematic uncertainties. As a result of the energy difference between the colliding beams, the nucleon-nucleon center of mass in the p-Pb collisions is not at rest with respect to the laboratory frame. Massless particles emitted at $\eta_{cm} = 0$ in the nucleon-nucleon center-of-mass frame will be detected at $\eta = -0.465$ (clockwise proton beam) or 0.465 (counterclockwise proton beam) in the laboratory frame. A sample of $\sqrt{s_{NN}} = 2.76$ TeV PbPb data collected during the 2011 LHC heavy-ion run, corresponding to an integrated luminosity of 2.3 $\mu$b$^{-1}$, is also analyzed for comparison purposes. The triggers and event selection, as well as track reconstruction and selection, are summarized below and are identical to those used in Ref. [35].

Minimum bias (MB) p-Pb events were triggered by requiring at least one track with $p_T > 0.4$ GeV/$c$ to be found in the pixel tracker for a p-Pb bunch crossing. Only a small fraction ($\sim 10^{-2}$) of all MB triggered events were recorded (i.e., the trigger was “prescaled”) because of hardware limits on the data acquisition rate. In order to select high-multiplicity p-Pb collisions, a dedicated high-multiplicity trigger was implemented using the CMS level-1 (L1) and high-level trigger (HLT) systems. At L1, three triggers requiring the total transverse energy summed over ECAL and HCAL to be greater than 20, 40, and 60 GeV were used since these cuts selected roughly the same events as the three HLT multiplicity selections discussed below. On-line track reconstruction for the HLT was based on the three layers of pixel detectors, and it required a track origin within a cylindrical region of length 30 cm along the beam and a radius 0.2 cm perpendicular to the beam around the nominal interaction point. For each event, the vertex reconstructed with the highest number of pixel tracks was selected. The number of pixel tracks ($N_{\text{tracks}}^\text{on-line}$) with $|\eta| < 2.4$, $p_T > 0.4$ GeV/$c$, and a distance of closest approach to this vertex of 0.4 cm or less, was determined for each event. Several high-multiplicity ranges were defined with prescale factors that were progressively reduced until, for the highest multiplicity events, no prescaling was applied.

In the off-line analysis, hadronic collisions are selected by requiring a coincidence of at least one HF calorimeter tower containing more than 3 GeV of total energy in each of the HF detectors. Only towers within $3 < |\eta| < 5$ are used to avoid the edges of the HF acceptance. Events are also required to contain at least one reconstructed primary vertex within 15 cm of the nominal interaction point along the beam axis and within 0.15 cm transverse to the beam trajectory. At least two reconstructed tracks are required to be associated with the primary vertex. The beam related background is suppressed by rejecting events for which less than 25% of all reconstructed tracks pass the track selection criteria of this analysis. The p-Pb instantaneous luminosity provided by the LHC in the 2013 run resulted in an approximately 3% probability of at least one additional interaction occurring in the same bunch crossing. Following the procedure developed in Ref. [35] for rejecting such “pileup” events, a 99.8% purity of single-interaction events is achieved for the p-Pb collisions belonging to the highest multiplicity class studied in this Letter. In p-Pb interactions simulated with the EPOS [41] and HIJING [42] event generators, requiring at least one primary particle with total energy $E > 3$ GeV in each of the $\eta$ ranges $-5 < \eta < -3$ and $3 < \eta < 5$ is found to select 97%–98% of the total inelastic hadronic cross section.

The CMS “high-quality” tracks described in Ref. [43] are used in this analysis. Additionally, a reconstructed track is only considered as a candidate track from the primary vertex if the significance of the separation along the beam axis $(z)$ between the track and the best vertex, $d_z/\sigma(d_z)$, and the significance of the track-vertex impact parameter measured transverse to the beam, $d_T/\sigma(d_T)$, are each less than 3. The relative uncertainty in the transverse momentum measurement, $\sigma(p_T)/p_T$, is required to be less than...
10%. To ensure high tracking efficiency and to reduce the rate of incorrectly reconstructed tracks, only tracks within $|\eta| < 2.4$ and with $0.3 < p_T < 3.0$ GeV/c are used in the analysis. A different $p_T$ cutoff of 0.4 GeV/c is used in the multiplicity determination because of constraints on the online processing time for the HLT.

The entire $p$-Pb data set is divided into classes of reconstructed track multiplicity, $N_{\text{trk}}^\text{off-line}$. The multiplicity classification in this analysis is identical to that used in Ref. [35], where more details are provided, including a table relating $N_{\text{trk}}^\text{off-line}$ to the fraction of the MB triggered events. A subset of semiperipheral PbPb data collected during the 2011 LHC heavy-ion run with a MB trigger is reprocessed using the same event selection and track reconstruction as for the present $p$-Pb analysis. A description of the 2011 PbPb data can be found in Ref. [44].

Extending the previous two- and four-particle azimuthal correlation measurements of Ref. [35], six- and eight-particle azimuthal correlations [36] are evaluated in this analysis as

$$
\langle 6 \rangle \equiv \langle e^{i(n\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \rangle,
\langle 8 \rangle \equiv \langle e^{i(n\phi_1+\phi_2+\phi_3+\phi_4-\phi_5-\phi_6)} \rangle.
$$

(1)

Here $\phi_i$ ($i = 1, \ldots, 8$) are the azimuthal angles of one unique combination of multiple particles in an event, $n$ is the harmonic number, and $\langle \cdots \rangle$ represents the average over all combinations from all events within a given multiplicity range. The corresponding cumulants, $c_n\{6\}$ and $c_n\{8\}$, are calculated as follows:

$$
c_n\{6\} = \langle 6 \rangle - 9 \times \langle 4 \rangle \langle 2 \rangle + 12 \times \langle 2 \rangle^3,
$$

$$
c_n\{8\} = \langle 8 \rangle - 16 \times \langle 6 \rangle \langle 2 \rangle - 18 \times \langle 4 \rangle^2
+ 144 \times \langle 4 \rangle^2 \langle 2 \rangle - 144 \langle 2 \rangle^4,
$$

(2)

using the $Q$-cumulant method as formulated in Ref. [36], where $\langle 2 \rangle$ and $\langle 4 \rangle$ are defined similarly as in Eq. (1). The Fourier harmonics $v_n$ that characterize the global azimuthal behavior are related to the multiparticle correlations [45] using

$$
v_n\{6\} = \sqrt{\frac{1}{4} c_n\{6\}},
v_n\{8\} = \sqrt{\frac{1}{33} c_n\{8\}}.
$$

(3)

To account for detector effects, such as the tracking efficiency, the $Q$-cumulant method was extended in Ref. [45] to allow for particles having different weights. Each reconstructed track is weighted by a correction factor to account for the reconstruction efficiency, detector acceptance, and fraction of misreconstructed tracks. This factor is derived as a function of $p_T$ and $\eta$, as described in Refs. [13,14], based on MC simulations. The combined geometrical acceptance and efficiency for track reconstruction exceeds 60% for $p_T \approx 0.3$ GeV/c and $|\eta| < 2.4$. The efficiency is greater than 90% in the $|\eta| < 1$ region for $p_T > 0.6$ GeV/c. For the entire multiplicity range (up to $N_{\text{trk}}^\text{off-line} \sim 350$) studied in this Letter, no dependence of the tracking efficiency on multiplicity is found and the rate of misreconstructed tracks remains at the 1%–2% level. The software package provided by Ref. [45] is used to implement the weights of the individual tracks in the cumulant calculations.

The LYZ method [37,38] allows a direct study of the large-order behavior by using the asymptotic form of the cumulant expansion to relate locations of the zeros of a generating function to the azimuthal correlations. This method has been employed in previous CMS PbPb analyses [17,46]. For each multiplicity bin, the $v_2$ harmonic averaged over $0.3 < p_T < 3.0$ GeV/c is found using an integral generating function [17]. Similar to the cumulant methods, a weight for each track is implemented to account for detector-related effects. In both methods, the statistical uncertainties are evaluated from data by dividing the data set into 20 subsets with roughly equal numbers of events and evaluating the standard deviation of the resulting distributions of the cumulant or $v_2$ ($\text{LYZ}$) values. In the case of a low multiplicity or small flow signal, the LYZ method may overestimate the true collective flow. This effect was studied using MC pseudoexperiments for the event multiplicities covered in this analysis, and a small correction is applied to the data. The correction is less than 3% in the lowest multiplicity bin and becomes much smaller in higher-multiplicity bins. This correction is also included in the quoted LYZ systematic uncertainties.

Systematic uncertainties are estimated by varying the track quality requirements, by comparing the results using efficiency correction tables from different MC event generators, and by exploring the sensitivity of the results to the vertex position and to the $N_{\text{trk}}^\text{off-line}$ bin width. For the $p$-Pb data, potential HLT bias and pileup effects are also studied by requiring the presence of only a single reconstructed vertex. No evident $N_{\text{trk}}^\text{off-line}$ or beam direction dependent systematic effects are observed. For $p$-Pb collisions, a 5% systematic uncertainty is obtained for $v_2\{6\}$ and a 6% uncertainty is found for both $v_2\{8\}$ and $v_2\{\text{LYZ}\}$. The corresponding uncertainties for PbPb collisions are 2% for $v_2\{6\}$ and $v_2\{8\}$, and 4% for $v_2\{\text{LYZ}\}$.

In Fig. 1, the six- and eight-particle cumulants, $c_2\{6\}$ and $c_2\{8\}$, for particle $p_T$ of 0.3–3.0 GeV/c in 2.76 TeV PbPb and 5.02 TeV $p$-Pb collisions are shown as a function of event multiplicity. The cumulants shown are required to be at least 2 standard deviations away from their physics boundaries ($c_2\{6\}/\sigma_{c_2\{6\}} > 2$, $c_2\{8\}/\sigma_{c_2\{8\}} < -2$) so that the statistical uncertainties can be propagated as Gaussian.
fluctuations [47]. Nonzero multiparticle correlation signals are observed in both PbPb and p-Pb collisions. The p-Pb data exhibit larger statistical uncertainties than the PbPb results, mainly because of the smaller magnitudes of the correlation signals. Because of the limited sample size, the $c_2\{6\}$ and $c_2\{8\}$ values in p-Pb collisions are derived for a smaller range in $N_{\text{off-line}}$.

The second-order anisotropy Fourier harmonics, $v_2$, averaged over the $p_T$ range of 0.3–3.0 GeV/c, are shown in Fig. 2 based on six- and eight-particle cumulants [Eq. (3)] for 2.76 TeV PbPb (left panel) and 5.02 TeV p-Pb (right panel) collisions, as a function of event multiplicity. The open symbols are $v_2$ results extracted by CMS using two- and four-particle correlations [35]. The $v_2$ values derived using the LYZ method involving correlations among all particles are also shown. For each multiplicity bin, the values of $v_2\{4\}$, $v_2\{6\}$, $v_2\{8\}$, and $v_2\{\text{LYZ}\}$ for p-Pb collisions are found to be in agreement within 10%. For part of the multiplicity range, the values for $v_2\{4\}$ are larger than the others by a statistically significant amount, although still within 10%. The corresponding PbPb values are consistently higher than for p-Pb collisions, but within the PbPb system are found to be in agreement within 2% for most multiplicity ranges and within 10% for all multiplicities. This supports the collective nature of the observed correlations, i.e., involving all particles from each system, and is inconsistent with a jet-related origin involving correlations among only a few particles. The $v_2$ data from two-particle correlations are consistently above the multiparticle correlation data. This behavior can be understood in hydrodynamic models, where event-by-event participant geometry fluctuations of the $v_2$ coefficient are expected to affect the two- and multiparticle cumulants differently [48,49]. Note that, to minimize jet-related nonflow effects, the $v_2\{2\}$ values are obtained with an $\eta$ gap of 2 units between the two particles. Possible residual nonflow effects resulting from back-to-back jet correlations are estimated using very low multiplicity events in Ref. [35]. Based on this analysis, such nonflow effects are expected to make a negligible contribution to $v_2\{2\}$ in very high multiplicity events. In PbPb collisions, the $v_2$ values from all methods show an increase with multiplicity, while little multiplicity dependence is seen for the p-Pb data. This difference might reflect the presence of a lenticular overlap geometry in PbPb collisions—which is not expected in p-Pb collisions—that gives rise to a large (and varying) initial elliptic asymmetry in the PbPb system.

The effect of fluctuation-driven initial-state eccentricities on multiparticle cumulants has recently been explored in the context of hydrodynamic behavior of the resulting medium [50,51]. For fluctuation-driven initial-state conditions, ratios of $v_2$ values derived from various orders of multiparticle cumulants are predicted to follow a universal behavior [50]. In Fig. 3, ratios of $v_2\{6\}/v_2\{4\}$ (top panel) and $v_2\{8\}/v_2\{6\}$ (bottom panel) are calculated and plotted against $v_2\{4\}/v_2\{2\}$ in p-Pb collisions at $\sqrt{s_{\text{NN}}} = 5.02$ TeV. The $v_2\{2\}$ and $v_2\{4\}$ data are taken from previously published CMS results [35]. The solid curves correspond to theoretical predictions for both large and small systems based on hydrodynamics and the assumption that the initial-state geometry is purely driven by

FIG. 1 (color online). The cumulant $c_2\{6\}$ and $-c_2\{8\}$ results as a function of $N_{\text{off-line}}$ for PbPb and p-Pb reactions. Error bars and shaded areas denote the statistical and systematic uncertainties, respectively.

FIG. 2 (color online). The $v_2$ values as a function of $N_{\text{off-line}}$. Open data points are the published two- and four-particle $v_2$ results [35]. Solid data points are $v_2$ results obtained from six- and eight-particle cumulants, and LYZ methods, averaged over the particle $p_T$ range of 0.3–3.0 GeV/c, in PbPb at $\sqrt{s_{\text{NN}}} = 2.76$ TeV (left panel) and p-Pb at $\sqrt{s_{\text{NN}}} = 5.02$ TeV (right panel). Statistical and systematic uncertainties are indicated by the error bars and the shaded regions, respectively.
Recently, other theoretical models based on quantum chromodynamics, and not involving hydrodynamics, have also been suggested to explain the observed multiparticle correlations in p-Pb collisions [52,53]. Unlike the descriptions based on hydrodynamic behavior, these models do not require significant final state interactions among quarks and gluons. They suggest similar values for \( v_2\{4\}, v_2\{6\}, v_2\{8\}, \) and \( v_2\{\text{LYZ}\} \)—without yet, however, providing quantitative predictions.

In summary, multiparticle azimuthal correlations among six, eight, and all particles have been measured in p-Pb collisions at \( \sqrt{s_{\text{NN}}} = 5.02 \) TeV by the CMS experiment. The new measurements extend previous CMS two- and four-particle correlation analyses of p-Pb collisions and strongly constrain possible explanations for the observed correlations. A direct comparison of the correlation data for p-Pb and PbPb collisions is presented as a function of particle multiplicity. Averaging over the particle \( p_T \) range of \( 0.3-3.0 \) GeV/c, multiparticle correlation signals are observed in both p-Pb and PbPb collisions. The second-order azimuthal anisotropy Fourier harmonic, \( v_2 \), is extracted using six- and eight-particle cumulants and using the LYZ method which involves all particles. The \( v_2 \) values obtained using correlation methods including four or more particles are consistent within \( \pm 2\% \) for the PbPb system, and within \( \pm 10\% \) for the p-Pb system. This measurement supports the collective nature of the observed correlations. The ratios of \( v_2 \) values obtained using different numbers of particles are found to be consistent with hydrodynamic model calculations for p-Pb collisions.

We congratulate our colleagues in the CERN accelerator departments for the excellent performance of the LHC and thank the technical and administrative staffs at CERN and at other CMS institutes for their contributions to the success of the CMS effort. In addition, we gratefully acknowledge the computing centers and personnel of the Worldwide LHC Computing Grid for delivering so effectively the computing infrastructure essential to our analyses. Finally, we acknowledge the enduring support for the construction and operation of the LHC and the CMS detector provided by the following funding agencies: BMWFW and FWF (Austria); FNRS and FWO (Belgium); CNPq, CAPES, FAPERJ, and FAPESP (Brazil); MES (Bulgaria); CERN; CAS, MoST, and NSFC (China); COLCIENCIAS (Colombia); MSES and CSF (Croatia); RPF (Cyprus); MoER, ERC IUT, and ERDF (Estonia); Academy of Finland, MEC, and HIP (Finland); CEA and CNRS/IN2P3 (France); BMBF, DFG, and HGF (Germany); GSRT (Greece); OTKA and NIH (Hungary); DAE and DST (India); IPM (Iran); SFI (Ireland); INFN (Italy); MSIP and NRF (Republic of Korea); LAS (Lithuania); MOE and UM (Malaysia); CINVESTAV, CONACYT, SEP, and UASLP-FAI (Mexico); MBIE (New Zealand); PAEC (Pakistan); MSHE and NSC (Poland); FCT (Portugal); JINR (Dubna); MON, RosAtom, RAS, and RFBR (Russia); MESTD (Serbia); SEIDI and CPAN (Spain);
Swiss Funding Agencies (Switzerland); MST (Taipei); TheEPCenter, IPST, STAR, and NSTDA (Thailand); TUBITAK and TAEK (Turkey); NASU and SFFR (Ukraine); STFC (United Kingdom); DOE and NSF (U.S.).

[1] CMS Collaboration, Observation of long-range near-side angular correlations in proton-proton collisions at the LHC, J. High Energy Phys. 09 (2010) 091.
[2] CMS Collaboration, Observation of long-range near-side angular correlations in proton-lead collisions at the LHC, Phys. Lett. B 718, 795 (2013).
[3] ALICE Collaboration, Long-range angular correlations on the near and away side in p-Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV, Phys. Lett. B 719, 29 (2013).
[4] ATLAS Collaboration, Observation of Associated Near-Side and Away-Side Long-Range Correlations in \( \sqrt{s_{NN}} = 5.02 \) TeV Proton-Lead Collisions with the ATLAS Detector, Phys. Rev. Lett. 110, 182302 (2013).
[5] ALICE Collaboration, Multiparticle azimuthal correlations in p-Pb and Pb-Pb collisions at the CERN Large Hadron Collider, Phys. Rev. C 90, 054901 (2014).
[6] A. Adare et al. (PHENIX Collaboration), Quadrupole Anisotropy in Dihadron Azimuthal Correlations in Central \( d + Au \) Collisions at \( \sqrt{s_{NN}} = 200 \) GeV, Phys. Rev. Lett. 111, 212301 (2013).
[7] A. Adare et al. (PHENIX Collaboration), Measurment of Long-Range Angular Correlation and Quadrupole Anisotropy of Pions and (Anti)Protons in Central \( d + Au \) Collisions at \( \sqrt{s_{NN}} = 200 \) GeV, Phys. Rev. Lett. 114, 192301 (2015).
[8] B. Alver et al. (PHOBOS Collaboration), System size dependence of cluster properties from two-particle angular correlations in Cu + Cu and Au + Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV, Phys. Rev. C 81, 024904 (2010).
[9] J. Adams et al. (STAR Collaboration), Distributions of Charged Hadrons Associated with High Transverse Momentum Particles in \( pp \) and \( Au + Au \) Collisions at \( \sqrt{s_{NN}} = 200 \) GeV, Phys. Rev. Lett. 95, 152301 (2005).
[10] B.I. Abelev et al. (STAR Collaboration), Long range rapidity correlations and jet production in high energy nuclear collisions, Phys. Rev. C 80, 064912 (2009).
[11] B. Alver et al. (PHOBOS Collaboration), High Transverse Momentum Triggered Correlations over a Large Pseudorapidity Acceptance in \( Au + Au \) Collisions at \( \sqrt{s_{NN}} = 200 \) GeV, Phys. Rev. Lett. 104, 062301 (2010).
[12] B.I. Abelev et al. (STAR Collaboration), Three-Particle Coincidence of the Long Range Pseudorapidity Correlation in High Energy Nucleus-Nucleus Collisions, Phys. Rev. Lett. 105, 022301 (2010).
[13] CMS Collaboration, Long-range and short-range dihadron angular correlations in central PbPb collisions at a nucleon-nucleon center of mass energy of 2.76 TeV, J. High Energy Phys. 07 (2011) 076.
[14] CMS Collaboration, Centrality dependence of dihadron correlations and azimuthal anisotropy harmonics in PbPb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, Eur. Phys. J. C 72, 2012 (2012).
[15] ALICE Collaboration, Harmonic decomposition of two-particle angular correlations in Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, Phys. Lett. B 708, 249 (2012).
[16] ATLAS Collaboration, Measurement of the azimuthal anisotropy for charged particle production in \( \sqrt{s_{NN}} = 2.76 \) TeV lead-lead collisions with the ATLAS detector, Phys. Rev. C 86, 014907 (2012).
[17] CMS Collaboration, Measurement of the elliptic anisotropy of charged particles produced in PbPb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, Phys. Rev. C 87, 014902 (2013).
[18] CMS Collaboration, Studies of azimuthal dihadron correlations in ultra-central PbPb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, J. High Energy Phys. 02 (2014) 088.
[19] J-Y. Ollitrault, Anisotropy as a signature of transverse collective flow, Phys. Rev. D 46, 229 (1992).
[20] U. Heinz and R. Snellings, Collective flow and viscosity in relativistic heavy-ion collisions, Annu. Rev. Nucl. Part. Sci. 63, 123 (2013).
[21] C. Gale, S. Jeon, and B. Schenke, Hydrodynamic modeling of heavy-ion collisions, Int. J. Mod. Phys. A 28, 1340011 (2013).
[22] W. Li, Observation of a “Ridge” correlation structure in high multiplicity proton-proton collisions: A brief review, Mod. Phys. Lett. A 27, 1230018 (2012).
[23] J.D. Bjorken, S.J. Brodsky, and A. Scharff Goldhaber, Possible multiparticle ridge-like correlations in very high multiplicity proton-proton collisions, Phys. Lett. B 726, 344 (2013).
[24] K. Dusling and R. Venugopalan, Explanation of systematics of CMS \( p + Pb \) high multiplicity di-hadron data at \( \sqrt{s_{NN}} = 5.02 \) TeV, Phys. Rev. D 87, 054014 (2013).
[25] K. Dusling and R. Venugopalan, Evidence for BFKL and saturation dynamics from dihadron spectra at the LHC, Phys. Rev. D 87, 051502 (2013).
[26] B. Schenke and R Venugopalan, Eccentric Protons? Sensitivity of Flow to System Size and Shape in \( p + p \) and \( Pb + Pb \) Collisions, Phys. Rev. Lett. 113, 102301 (2014).
[27] P. Bozek, Collective flow in \( p-Pb \) and \( d-Pb \) collisions at \( \sqrt{s_{NN}} \) energies, Phys. Rev. C 85, 014911 (2012).
[28] P. Bozek and W. Broniowski, Correlations from hydrodynamic flow in \( p-Pb \) collisions, Phys. Lett. B 718, 1557 (2013).
[29] S. Voloshin and Y. Zhang, Flow study in relativistic nuclear collisions by Fourier expansion of azimuthal particle distributions, Z. Phys. C 70, 665 (1996).
[30] B. Alver and G. Roland, Collision geometry fluctuations and triangular flow in heavy-ion collisions, Phys. Rev. C 81, 054905 (2010); 82, 039903(E) (2010).
[31] B.H. Alver, C. Gombeaud, M. Luzum, and J.-Y. Ollitrault, Triangular flow in hydrodynamics and transport theory, Phys. Rev. C 82, 034913 (2010).
[32] B. Schenke, S. Jeon, and C. Gale, Elliptic and Triangular Flow in Event-by-Event \( D = 3 + 1 \) Viscous Hydrodynamics, Phys. Rev. Lett. 106, 042301 (2011).
[33] Z. Qiu, C. Shen, and U. Heinz, Hydrodynamic elliptic and triangular flow in Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, Phys. Lett. B 707, 151 (2012).
[34] ATLAS Collaboration, Measurement with the ATLAS detector of multi-particle azimuthal correlations in \( p + Pb \)
collisions at $\sqrt{s_{NN}} = 5.02$ TeV, Phys. Lett. B 725, 60 (2013).

[35] CMS Collaboration, Multiplicity and transverse momentum dependence of two- and four-particle correlations in $p$-$p$ and $p$-$p$ collisions, Phys. Lett. B 724, 213 (2013).

[36] A. Bilandzic, R. Snellings, and S. Voloshin, Flow analysis with cumulants: Direct calculations, Phys. Rev. C 83, 044913 (2011).

[37] R. S. Bhalerao, N. Borghini, and J. Y. Ollitrault, Analysis of anisotropic flow with Lee-Yang zeroes, Nucl. Phys. A727, 373 (2003).

[38] N. Borghini, R. S. Bhalerao, and J. Y. Ollitrault, Anisotropic flow from Lee-Yang zeroes: A practical guide, J. Phys. G 30, S1213 (2004).

[39] CMS Collaboration, The CMS experiment at the CERN LHC, JINST 3, S08004 (2008).

[40] S. Agostinelli et al. (GEANT4 Collaboration), GEANT4: A simulation toolkit, Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).

[41] S. Porteboeuf, T. Pierog, and K. Werner, Producing hard processes regarding the complete event: The EPOS event generator, arXiv:1006.2967.

[42] M. Gyulassy and X.-N. Wang, Hijing 1.0: A Monte Carlo program for parton and particle production in high-energy hadronic and nuclear collisions, Comput. Phys. Commun. 83, 307 (1994).

[43] CMS Collaboration, Description and performance of track and primary-vertex reconstruction with the CMS tracker, JINST 9, P10009 (2014).

[44] CMS Collaboration, Measurement of higher-order harmonic azimuthal anisotropy in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, Phys. Rev. Lett. 109, 022301 (2012).

[45] A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen, and Y. Zhou, Generic framework for anisotropic flow analyses with multi-particle azimuthal correlations, Phys. Rev. C 89, 064904 (2014).

[46] CMS Collaboration, Azimuthal Anisotropy of Charged Particles at High Transverse Momenta in PbPb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV, Phys. Lett. B 725, 60 (2013).

[47] V. Khachatryan, A. M. Sirunyan, A. Tumasyan, W. Adam, T. Bergauer, M. Dragicevic, J. Erö, M. Friedi, R. Frühwirth, V. M. Ghete, C. Hartl, N. Hörmann, J. Hrubec, M. Jeitler, W. Kiesenhofer, V. Knünz, M. Krammer, I. Krätschmer, D. Liko, I. Mikulec, D. Rabady, C.-E. Wulz, V. Mossolov, N. Shumeiko, J. Suarez Gonzalez, A. Olbrechts, Q. Python, D. Strom, S. Tavernier, W. Van Doninck, P. Van Mulders, G. P. Van Onsem, I. Villella, A. Van Spilbeeck, F. Blekman, S. Blyweert, J. D. Aldá Júnior, G. A. Alves, L. Brito, G. C. Mora Herrera, M. E. Pol, P. Rebello Teles, W. Carvalho, J. Chinellato, A. Cimmino, S. Costantini, S. Crucy, A. Fagot, G. Garcia, C. Caillol, B. Clerbaux, G. De Lentdecker, D. Dobur, L. Favart, A. P. R. Gay, A. Grebenyuk, A. Léonard, S. Alderweireldt, T. Cornelis, E. A. De Wolf, X. Janssen, A. Knutsson, J. Lauwers, S. Luyckx, S. Ochesanu, R. Rougny, M. Van De Klundert, H. Van Haevermaet, P. Van Mechelen, N. Van Remortel, A. Van Spilbeeck, F. Blekman, S. Blyweert, J. D. Aldá Júnior, G. A. Alves, L. Brito, G. C. Mora Herrera, M. E. Pol, P. Rebello Teles, W. Carvalho, J. Chinellato, A. Cimmino, S. Costantini, S. Crucy, A. Fagot, G. Garcia, C. Caillol, B. Clerbaux, G. De Lentdecker, D. Dobur, L. Favart, A. P. R. Gay, A. Grebenyuk, A. Léonard, S. Alderweireldt, T. Cornelis, E. A. De Wolf, X. Janssen, A. Knutsson, J. Lauwers, S. Luyckx, S. Ochesanu, R. Rougny, M. Van De Klundert, H. Van Haevermaet, P. Van Mechelen, N. Van Remortel.
130 Cornell University, Ithaca, USA
131 Fairfield University, Fairfield, USA
132 Fermi National Accelerator Laboratory, Batavia, USA
133 University of Florida, Gainesville, USA
134 Florida International University, Miami, USA
135 Florida State University, Tallahassee, USA
136 Florida Institute of Technology, Melbourne, USA
137 University of Illinois at Chicago (UIC), Chicago, USA
138 The University of Iowa, Iowa City, USA
139 Johns Hopkins University, Baltimore, USA
140 The University of Kansas, Lawrence, USA
141 Kansas State University, Manhattan, USA
142 Lawrence Livermore National Laboratory, Livermore, USA
143 University of Maryland, College Park, USA
144 Massachusetts Institute of Technology, Cambridge, USA
145 University of Minnesota, Minneapolis, USA
146 University of Mississippi, Oxford, USA
147 University of Nebraska-Lincoln, Lincoln, USA
148 State University of New York at Buffalo, Buffalo, USA
149 Northeastern University, Boston, USA
150 Northwestern University, Evanston, USA
151 University of Notre Dame, Notre Dame, USA
152 The Ohio State University, Columbus, USA
153 Princeton University, Princeton, USA
154 University of Puerto Rico, Mayaguez, USA
155 Purdue University, West Lafayette, USA
156 Purdue University Calumet, Hammond, USA
157 Rice University, Houston, USA
158 University of Rochester, Rochester, USA
159 The Rockefeller University, New York, USA
160 Rutgers, The State University of New Jersey, Piscataway, USA
161 University of Tennessee, Knoxville, USA
162 Texas A&M University, College Station, USA
163 Texas Tech University, Lubbock, USA
164 Vanderbilt University, Nashville, USA
165 University of Virginia, Charlottesville, USA
166 Wayne State University, Detroit, USA
167 University of Wisconsin, Madison, USA

aDeceased.
bAlso at Vienna University of Technology, Vienna, Austria.
cAlso at CERN, European Organization for Nuclear Research, Geneva, Switzerland.
dAlso at Institut Pluridisciplinaire Hubert Curien, Université de Strasbourg, Université de Haute Alsace Mulhouse, CNRS/IN2P3, Strasbourg, France.
eAlso at National Institute of Chemical Physics and Biophysics, Tallinn, Estonia.
fAlso at Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia.
gAlso at Universidade Estadual de Campinas, Campinas, Brazil.
hAlso at Laboratoire Leprince-Ringuet, Ecole Polytechnique, IN2P3-CNRS, Palaiseau, France.
iAlso at Université Libre de Bruxelles, Bruxelles, Belgium.
jAlso at Joint Institute for Nuclear Research, Dubna, Russia.
kAlso at Suez University, Suez, Egypt.
lAlso at Cairo University, Cairo, Egypt.
mAlso at Fayoum University, El-Fayoum, Egypt.
nAlso at British University in Egypt, Cairo, Egypt.
oAlso at Ain Shams University, Cairo, Egypt.
pAlso at Université de Haute Alsace, Mulhouse, France.
qAlso at Brandenburg University of Technology, Cottbus, Germany.
rAlso at Institute of Nuclear Research ATOMKI, Debrecen, Hungary.
sAlso at Eötvös Loránd University, Budapest, Hungary.
tAlso at University of Debrecen, Debrecen, Hungary.
