**R-band host galaxy contamination of TeV γ-ray blazar Mrk 501: effects of aperture size and seeing**

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**Abstract** We simulated the *R*-band contribution of the host galaxy of TeV γ-ray BL Lac object Mrk 501 in different aperture sizes and seeing conditions. An intensive set of observations was acquired with the 1.02 m optical telescope, managed by Yunnan Observatories, from 2010 May 15 to 18. Based on the host subtraction data usually used in the literature, the subtraction of host galaxy contamination results in significant seeing-brightness correlations. These correlations would lead to illusive large amplitude variations at short timescales, which will mask the intrinsic microvariability, thus giving rise to difficulty in detecting the intrinsic microvariability. Both aperture size and seeing condition influence the flux measurements, but the aperture size impacts the result more significantly. Based on the parameters of an elliptical galaxy provided in the literature, we simulated the host contributions of Mrk 501 in different aperture sizes and seeing conditions. Our simulation data of the host galaxy obviously weaken these significant seeing-brightness correlations for the host-subtracted brightness of Mrk 501, and can help us discover the intrinsic short timescale microvariability. The pure nuclear flux is \(\sim 8.0\) mJy in the *R* band, i.e., the AGN has a magnitude of *R* \(\sim 13.96\) mag.

**Key words:** galaxies: active — BL Lacertae objects: individual (Mrk 501) — techniques: photometric — methods: data analysis

**1 INTRODUCTION**

Blazars are an extreme subclass of active galactic nuclei (AGNs), including BL Lacertae (BL Lac) objects and flat spectrum radio quasars (FSRQs) (e.g., Angel & Stockman 1980; Urry & Padovani 1995; Fossati et al. 1998; Böttcher & Dermer 2002; Maraschi & Tavecchio 2003). They are characterized by rapid and strong variability over the whole electromagnetic spectrum, high and variable polarization from the optical to radio bands, and prominent non-thermal emission at all wavelengths. In general, these extreme properties can be generated from a relativistic jet with a viewing angle less than \(10^\circ\) (e.g., Blandford & Königl 1979; Urry & Padovani 1995). The broadband spectral energy distributions (SEDs) of blazars usually exhibit a double peak profile. The first component extends from infrared to ultraviolet or soft X-ray, and the second is located in the GeV/TeV gamma-ray bands (e.g., Ghisellini et al. 1998; Abdo et al. 2010). The first peak is generally believed to be the synchrotron radiation of relativistic electrons in the jet. The second peak
is attributed to inverse-Compton scattering of the same electron population that produces the synchrotron radiation (e.g., Dermer & Schlickeiser 1993; Böttcher 2007; Neronov et al. 2012).

Due to the property of strong variability in BL Lac objects, the photometric technique is widely used to investigate the structures, radiation mechanisms, dynamics and masses of the central supermassive black holes (e.g., Ciprini et al. 2003, 2007; Gupta et al. 2008b; Liu & Bai 2015; Dai et al. 2015; Guo et al. 2017). However, the host galaxies often exhibit strong radiation in the optical to near-infrared (NIR) bands. Thus, contamination from the host galaxies may influence the photometric results, especially for nearby extended sources. The photometric aperture is either a dynamic aperture or a fixed aperture. A dynamic aperture could be a few times the seeing, and the case of an extended source will result in a significant dependence of the photometric magnitudes on the seeing. There is not a dependence on the seeing for a point source at high redshift. A fixed aperture and a dynamic aperture could result in similar influences on the photometric results for an extended source due to the seeing (see Feng et al. 2017). For point sources, strong host galaxies could dilute the variability amplitudes of AGNs. Besides, the color indices and the SEDs of AGNs will be influenced. Since an extended source is resolved, different aperture sizes and seeing conditions would introduce large uncertainties in photometry at different epochs.

However, the host galaxies of nearby BL Lac objects are elliptical galaxies, which are huge (with effective radius $R_e \sim 10$ kpc) and luminous ($M_R \sim -24.0$ mag) (e.g., Falomo & Kotilainen 1999; Urry et al. 1999, 2000; Scarpa et al. 2000; Kotilainen & Falomo 2004; Nilsson et al. 2003, 2007; Hyvönen et al. 2007). Even though some BL Lac objects may show signs of interaction with companions (e.g., Stickel et al. 1993; Falomo 1996; Heidt et al. 1999; Falomo & Ulrich 2000), there is no clear evidence in most cases that the nuclear activity is triggered by interaction (Nilsson et al. 1999, 2007). For most BL Lac objects, the morphologies of host galaxies are indistinguishable from similar normal elliptical galaxies (Scarpa et al. 2000). Thus, the host galaxies of BL Lac objects can be simulated based on normal elliptical galaxies.

Mrk 501 is a prototype nearby BL Lac object (redshift $z = 0.034$), which has been widely studied over the past two decades in the entire electromagnetic spectrum (e.g., Stickel et al. 1993; Quinn et al. 1996; Catanese et al. 1997; Samuelson et al. 1998; Xie et al. 1999, 2001; Konopelko et al. 2003; Gupta et al. 2008a, 2012; Albert et al. 2007; Abdo et al. 2011; Shukla et al. 2015; Ahnen et al. 2017). In the high energy regime from X-ray to TeV, Mrk 501 is one of the brightest extragalactic sources (Abdo et al. 2011). Many studies attempted to investigate its properties in the optical bands (e.g., Xie et al. 1999, 2001; Gupta et al. 2008a, 2012; Xiong et al. 2016). Based on the host subtraction data presented in Nilsson et al. (2007), widely used in previous photometric studies, the subtraction of host galaxy contamination results in a significant seeing-magnitude correlation for Mrk 501 (Feng et al. 2017). Researches related to its variability will need a reasonable subtraction of the host galaxy, which should (partly) eliminate this significant seeing-brightness correlation.

In this paper, we present observations of Mrk 501 in the $R$ band from 2010 May 15 to 18. In order to obtain the host components in different aperture radii and seeing conditions, we use a two-dimensional simulation method to model the host galaxy. The structure of this paper is as follows: Section 2 presents the observations and data reduction; Section 3 gives the details on simulations; Section 4 draws conclusions, and discussion is presented in Section 5.

2 OBSERVATIONS AND DATA REDUCTION

The observations of Mrk 501 were carried out with the 1.02 m optical telescope administered by Yunnan Observatories. This telescope is a classical Cassegrain telescope located in Kunming, China. An Andor AW436 CCD ($2048 \times 2048$ pixels) camera was mounted at the f/13.3 Cassegrain focus of the 1.02 m telescope. The entire field of view of the CCD is $\sim 7.3 \times 7.3$ arcmin², and each pixel corresponds to 0.21″ in both dimensions. The CCD readout noise and gain are 6.33 electrons and 2.0 electrons/ADU, respectively (e.g., Dai et al. 2015; Xiong et al. 2016). We selected the standard Johnson broadband filters to carry out observations in the $R$ band, and 326 valid exposures were obtained in four nights from 2010 May 15 to 2010 May 18. The exposure time is 150 seconds for each frame. Table 1 presents the complete observation log. For each image, the standard stars are always in the same field as the object.

Because the magnitudes of the standard stars are considered constant, the brightness of the object could be calibrated using these standard stars (e.g., Bai et al. 1998; Fan et al. 2014; Zhang et al. 2004, 2008). There are six
Table 1  Observation Log and Results of IDV Observations of Mrk 501

| Date       | N  | Exposure (s) | σ (star 1−star 6) |
|------------|----|--------------|------------------|
| 2010 May 15| 88 | 150          | 0.005            |
| 2010 May 16| 88 | 150          | 0.007            |
| 2010 May 17| 80 | 150          | 0.005            |
| 2010 May 18| 70 | 150          | 0.005            |

Notes: Column (1): observation date; Col. (2): observation number; Col. (3): exposure time; Col. (4): standard deviation of the (star 1−star 6).

Figure 1 shows the relationship between the FWHMs and magnitudes in different apertures for each night, and Figure 2 shows the corresponding relationship of the FWHMs and fluxes. Figures 1 and 2 indicate that both the FWHM and aperture affect the photometric results. The brightness increases as the aperture increases, and decreases as the seeing increases. An increasing aperture will collect more light, and increasing seeing will scatter more light out of the aperture.

3 SIMULATIONS OF HOST GALAXY

The host galaxy of Mrk 501 is an elliptical galaxy (e.g., Nilsson et al. 1999, 2003; Hyvönen et al. 2007). Thus, we simulated the host galaxy using a two-dimensional model, which assumes the surface brightness $I(r)$ follows the Sersic law $\sim r^{\beta}$ (Sersic 1968; Caon et al. 1993; Nilsson et al. 1999). The formula for $I(r)$ is

$$I(r) = I(r_e)\text{dex} \left\{ -b_\beta \left( \frac{r}{r_e} \right)^{-\beta} - 1 \right\},$$

where $\beta$ is the shape parameter and $r_e$ is the effective radius (containing half of the total luminosity). A $\beta$-dependent constant $b_\beta$ is defined as

$$b_\beta = \frac{0.868}{\beta} - 0.142,$$

and

$$I(r_e) = \frac{f_R}{K_\beta r_e^2 (1 - \epsilon)},$$

where $f_R$ is the total flux of the galaxy, $\epsilon$ is the ellipticity and $K_\beta$ can be derived from

$$K_\beta = \text{dex} \left( 0.030 \log^2 \beta - 0.441 \log \beta + 1.079 \right),$$

where $\text{dex} \left( x \right) = 10^x$. Equations (1) to (4) indicate that if we obtain the parameters of $\beta$, $\epsilon$, $r_e$ and $f_R$, we could confirm the surface brightness ($I(r)$) distribution of the host galaxy. Combining with the position angle $\theta$, we can simulate the host of Mrk 501 in the observed images. However, the lower resolution and relatively poor S/N restrict how accurately we can measure values of the above parameters. Fortunately, Nilsson et al. (1999) have obtained all the above parameters from high-resolution images in the $R$ band. The free $\beta +$ core model was adopted in our simulations (based on properties of BL Lac objects and the de Vaucouleurs model (e.g., Makino et al. 1990)). We simulated the host component of Mrk 501 and convolved the simulation results.
### Table 2 Data Observed on 2010 May 15

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5331.69936 | 4.121 | 6.985         |
| 5331.70187 | 4.335 | 7.214         |
| 5331.70372 | 4.307 | 7.174         |
| 5331.70559 | 4.256 | 7.147         |
| 5331.71116 | 4.056 | 6.921         |

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| 5331.70559 | 4.256 | 7.147         |
| 5331.71116 | 4.056 | 6.921         |

### Table 3 Data Observed on 2010 May 16

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5332.64491 | 3.160 | 5.719         |
| 5332.64729 | 3.349 | 5.994         |
| 5332.64929 | 3.383 | 6.050         |
| 5332.65115 | 3.446 | 6.123         |
| 5332.65300 | 3.501 | 6.202         |
| 5332.65486 | 3.282 | 5.907         |

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5332.64491 | 3.160 | 5.719         |
| 5332.64729 | 3.349 | 5.994         |
| 5332.64929 | 3.383 | 6.050         |
| 5332.65115 | 3.446 | 6.123         |
| 5332.65300 | 3.501 | 6.202         |
| 5332.65486 | 3.282 | 5.907         |

### Table 4 Data Observed on 2010 May 17

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5333.68653 | 4.079 | 6.921         |
| 5333.68796 | 4.053 | 6.902         |
| 5333.69064 | 3.803 | 6.591         |
| 5333.69250 | 3.931 | 6.745         |
| 5333.69432 | 3.761 | 6.543         |
| 5333.69621 | 4.193 | 7.069         |

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5333.68653 | 4.079 | 6.921         |
| 5333.68796 | 4.053 | 6.902         |
| 5333.69064 | 3.803 | 6.591         |
| 5333.69250 | 3.931 | 6.745         |
| 5333.69432 | 3.761 | 6.543         |
| 5333.69621 | 4.193 | 7.069         |

### Table 5 Data Observed on 2010 May 18

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5334.70033 | 3.717 | 6.489         |
| 5334.70280 | 3.530 | 6.225         |
| 5334.70466 | 4.019 | 6.838         |
| 5334.70651 | 3.782 | 6.561         |
| 5334.70836 | 3.772 | 6.543         |
| 5334.71023 | 3.619 | 6.323         |

| MJD       | Apert | FWHM (arcsec) |
|-----------|-------|---------------|
| 5334.70033 | 3.717 | 6.489         |
| 5334.70280 | 3.530 | 6.225         |
| 5334.70466 | 4.019 | 6.838         |
| 5334.70651 | 3.782 | 6.561         |
| 5334.70836 | 3.772 | 6.543         |
| 5334.71023 | 3.619 | 6.323         |

Notes: This table is available in its entirety at [http://www.raa-journal.org/docs/Supp/3443fengTable2.txt](http://www.raa-journal.org/docs/Supp/3443fengTable2.txt). The other notes are the same as those in Table 2.
into 28 different FWHMs with the point spread function (PSF) of a Gaussian profile. The FWHMs of the convolved images are from 0.5″ to 5.9″ with a bin size of 0.2″. We performed the photometry using 111 fixed apertures from 1.0″ to 12.0″ with a bin size of 0.1″. Table 6 shows flux simulations for the host galaxy under different FWHMs and apertures.

Figure 3 shows the relationships between brightness, FWHM and aperture. Our simulation results are very different from those in Nilsson et al. (2007). The host subtraction based on the subtraction data in Nilsson et al. (2007) led to a significant seeing-brightness correlation for Mrk 501 (see an example presented in fig. 2 in Feng et al. 2017). Thus, a reasonable host subtraction is needed for the optical photometry of Mrk 501.

We used two methods to compare the simulation results with our observations. First, we checked the observed images to determine photometric regions where the S/N ratios are high enough (i.e., > 5). This is normally achieved with an aperture radius of 5″. We measured the brightness of the images within annular apertures with radii of 3.5″ − 4.5″ and 4.5″ − 5.0″.

Figure 4 shows comparisons between the simulated and observed results in the same annular apertures. The simulations and observations are (marginally) consistent with each other in the case of 3.5″ − 4.5″ except for 2010 May 17 (see Fig. 4). In general, the observed results are lower than the simulation results in the case of 4.5″ − 5.0″. This may arise from low S/N ratios at those annular apertures. The host galaxy of Mrk 501 is a low surface brightness galaxy, and this will result in lower S/N ratios at larger annular apertures. The deviations of simulations from observations in the case of 3.5″ − 4.5″ are less than those in the case of 4.5″ − 5.0″. Combining four panels in Figure 4 into one panel (see Fig. 5), we find that simulations are marginally consistent with observations in the case of 3.5″ − 4.5″, and the deviations of observations from simulations in the case of 3.5″ − 4.5″ are less than those in the case of 4.5″ − 5.0″. Observations need an exposure time to obtain a certain S/N ratio. A low S/N ratio may result in a lower flux measurement...
compared to the flux simulation based on a high S/N ratio image presented in Nilsson et al. (1999). Another method is based on the fact that the brightness difference between simulation and observation is the contribution of AGN, i.e., the observed flux is a combination of AGN and its host galaxy flux, while the simulation result only contains the host component. For a relatively large photometric aperture (nearly including all the AGN flux, e.g., an aperture radius of 4.0″ including 99% of the AGN flux), the differences between simulations and observations should be a constant for the different seeing conditions. The observed results are very consistent with the vertically shifted simulation results for aperture radii from 3.0″ to 6.0″ in the flux versus FWHM diagram (see

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**Fig. 2** The relationships between FWHM and flux for different photometric aperture radii in our observations. The solid lines are the simulation results moved vertically by the average differences between the original simulations and the corresponding observational data (the solid circles).

**Table 6** Simulation Data for the Host Galaxy of Mrk 501

| Apert FWHM | 0.5 | 0.7 | 0.9 | 1.1 | 1.3 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 | 2.7 | 2.9 | ... | 5.9 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.0        | 2.615 | 2.138 | 1.732 | 1.425 | 1.190 | 1.012 | 0.872 | 0.762 | 0.672 | 0.599 | 0.537 | 0.486 | 0.441 | ... | 0.164 |
| 1.1        | 2.928 | 2.456 | 2.023 | 1.681 | 1.413 | 1.207 | 1.043 | 0.913 | 0.806 | 0.719 | 0.646 | 0.585 | 0.532 | ... | 0.199 |
| 1.2        | 3.231 | 2.775 | 2.322 | 1.949 | 1.649 | 1.415 | 1.226 | 1.076 | 0.951 | 0.850 | 0.764 | 0.692 | 0.630 | ... | 0.236 |
| 1.3        | 3.512 | 3.082 | 2.620 | 2.222 | 1.893 | 1.632 | 1.419 | 1.249 | 1.096 | 0.990 | 0.891 | 0.808 | 0.736 | ... | 0.277 |
| 1.4        | 3.782 | 3.383 | 2.920 | 2.503 | 2.147 | 1.860 | 1.623 | 1.431 | 1.271 | 1.139 | 1.026 | 0.931 | 0.849 | ... | 0.321 |
| 1.5        | 4.034 | 3.668 | 3.213 | 2.783 | 2.405 | 2.095 | 1.834 | 1.622 | 1.443 | 1.296 | 1.168 | 1.062 | 0.969 | ... | 0.368 |
| ...        | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  |
| 12.0       | 15.554 | 15.542 | 15.527 | 15.509 | 15.486 | 15.460 | 15.429 | 15.394 | 15.354 | 15.311 | 15.259 | 15.206 | 15.143 | ... | 12.889 |

Notes: This table is available in its entirety in a machine-readable form in the online version of the journal at [http://www.raajournal.org/docs/Supp/3443fengTable6.txt](http://www.raajournal.org/docs/Supp/3443fengTable6.txt). A portion is shown here for guidance regarding its form and content. Apert: aperture radius in units of arcsec, presented in Col. (1). FWHM is in units of arcsec, presented inCols. (2)–(16). The fluxes are in units of mJy.
Fig. 3 The relationships between FWHM and brightness for different apertures with the simulated host galaxy of Mrk 501 (solid lines). The dot-dashed lines represent the results of Nilsson et al. (2007). The numbers associated with lines in the plots are the photometric aperture radii.

Fig. 4 Fluxes in annular apertures for different seeing (FWHM). In each panel, the solid line represents the simulation results from $3.5''$ to $4.5''$, and the circles represent the observed results in the same annular apertures. Also in each panel, the dotted line indicates the simulation results from $4.5''$ to $5.0''$, and the triangles indicate the observed results in the same annular apertures.

Fig. 2). There are very similar trends between the vertically shifted simulations and the observational results for the other aperture radii in Figure 2. These slight differences between simulations and observations may be from the fact that the corresponding aperture radii are either too small or too large ($< 3.0''$ or $> 6.0''$).

We calculated the average difference between the simulations and the observational results in Figure 2.
The shifted simulation results are very consistent with the observational data, and the average difference can be regarded as the AGN flux. The mean flux of the AGN is $\sim 8.0 \text{ mJy}$, which corresponds to $R = 13.96 \text{ mag}$ ($F = 3.08 \times 10^{-0.4 \times R + 3} \text{ Jy}$ (Nilsson et al. 2007)). Thus, the AGN’s contribution to the total flux of the source is $\sim 13.3\%$. Compared with the brightness obtained in Nilsson et al. (1999), $R = 14.45 \text{ mag}$, AGN Mrk 501 brightened by $\sim 57\%$ in our observations. According to our simulations, we subtracted the host contribu-
tion and investigated whether there are still significant seeing-brightness correlations for AGN Mrk 501. The host-subtracted fluxes versus FWHMs are presented in Figure 6. There is no correlation on 2010 May 18. Though the host-subtraction based on our simulations can (obviously) weaken the significant correlations found in Feng et al. (2017), there are still correlations for 2010 May 15 and 17, and an obvious correlation on 2010 May 16.

Figure 7 shows the host-subtracted flux versus seeing distribution in the case of a 5.0′′ aperture. The host-subtracted flux versus seeing distribution shows that the larger photometric aperture radii can further weaken the host-subtracted brightness-seeing correlation. Thus,
our simulations can basically give a reasonable host-subtraction. The obvious correlation on 2010 May 16 might be from the smaller photometric aperture relative to the average seeing. The host-subtracted flux light curves show that the darkening variations found in Feng et al. (2017) still exist in the light curve on 2010 May 15 even though the host contribution has been subtracted (see Fig. 8). There is a flare with a duration of \(~\sim\)1 hour on 2010 May 18 around MJD 5334.75 (see Fig. 8), which was not found in Feng et al. (2017). This confirms that the fake large amplitude fast variability due to the seeing effect can mask intrinsic microvariability in Mrk 501. This kind of fake rapid and strong variability due to the seeing effect will mask the intrinsic microvariability in Mrk 501, and will lead to difficulty in detecting intrinsic microvariability in similar sources with brighter host galaxies, e.g., Mrk 421.

4 CONCLUSIONS

Based on the intensive set of observations acquired with the 1.02 m optical telescope, administered by Yunnan Observatories, from 2010 May 15 to 18, and a two-dimensional model of an elliptical galaxy, we simulated the \(R\)-band contribution of the host galaxy of TeV \(\gamma\)-ray BL Lac object Mrk 501. The simulated brightness in the different aperture radii and seeing conditions shows correlations between the seeing and brightness for the host galaxy, and these correlations are confirmed well by the observational data. The differences between the simulation fluxes and the observational data are due to contribution from AGN Mrk 501, and the host-subtracted brightness of Mrk 501 can obviously weaken these significant correlations found in Feng et al. (2017). There is no correlation between the seeing and the host-subtracted brightness on 2010 May 18. However, there are correlations on 2010 May 15 and 17, and an obvious correlation on 2010 May 16. The larger photometric aperture radii with respect to the average seeing can further weaken the correlation on 2010 May 16 (see Figs. 6 and 2). These correlations led to illusive large amplitude variations on short timescales, which can mask the intrinsic microvariability and then lead to difficulty in detecting the intrinsic microvariability. The host-subtracted brightness light curves confirm the darkening variations on 2010 May 15 found in Feng et al. (2017), and revealed a flare with a duration of \(~\sim\)1 hour on 2010 May 18. Both the aperture size and the seeing condition influence the photometric results, but the aperture size can have a stronger influence. The pure nuclear flux is \(~\sim\)8.0 mJy. Compared with the result observed in July 1996 (Nilsson et al. 1999), the AGN Mrk 501 brightened by a factor of \(~\sim\)57%. Simulation data of the host galaxy of Mrk 501 are given for the different aperture radii and seeing conditions (online Table 6).

5 DISCUSSION

The correlation between seeing FWHM and brightness within a certain aperture is obvious for the intensive set of observations acquired from 2010 May 15 to 18. At the same time, the flux of the target is higher as the aperture radius is larger. The larger aperture radius will cover more area of an extended source, and thus will collect more light in the aperture. Thus, the total brightness will be monotonically increasing with aperture radius. This indicates that a fixed aperture is better than a dynamic aperture in performing photometry for Mrk 501. This point was tested in Feng et al. (2017), where a fixed aperture was used to measure photometry. Brightness monotonically decreases with increasing FWHM of seeing in the fixed aperture. This can be explained in that the larger PSF due to the worse seeing will scatter more light out of a fixed aperture. Another feature is that the PSF effect is more significant for a smaller aperture (less than 3.0\(')\). This is due to the fact that the amount of scattered light from an AGN changes with different PSFs. Therefore, the photometry of Mrk 501 should use a large fixed aperture, which can collect almost all the light of an AGN. In addition, it is necessary to correct the influence of seeing.

Figure 3 shows the simulation results for the host galaxy of Mrk 501, and the two panels in Figure 3 have similar relationships as those in Figures 1 and 2. The brightness curve shapes from the simulation results are very similar to those of observations for the same aperture and the same range of FWHM. However, the results in Figure 3 are somewhat different from the results in Figures 1 and 2, especially for the small apertures, and this difference is mainly due to the AGN component. We tested the reliability of the simulations via two methods (see Sect. 3), and both tests indicate that the simulations are robust (see Fig. 2). The results in Figure 3 can be used to correct the host contamination of Mrk 501, and the corresponding values are given in Table 6. Nilsson et al. (2007) provided a similar table (table B.1). Comparing our simulation results to theirs (see Fig. 3), we found some differences. Though these two results indicate that the host fluxes strongly depend on photometric apertures,
the values from the same aperture and PSF are inconsistent. Especially within a small aperture radius ($\lesssim 3.0''$), the difference is significant. For a fixed aperture, the relationships between brightness and FWHM are significantly different for these two results. The brightness of the host galaxy decreases as the FWHM increases (see Fig. 3). These trends are opposite to the results in Nilsson et al. (2007). The influence of the seeing on the variability amplitude is significant in our results. After we subtracted the contamination of the host galaxy using the results of Nilsson et al. (2007), the relationships are still significant for the brightness and seeing FWHM. Otherwise, if the brightness of the host galaxy monotonically increases with the FWHM, the outer part of the host galaxy would be brighter than the central part. This is inconsistent with the reversal of surface brightness distribution for an elliptical galaxy.

The simulations and observations indicate that the AGN contribution of Mrk 501 is $\sim 13.3\%$. This means that even if the variability of an AGN is up to 10%, we can only detect a magnitude change of $\sim 0.01$ mag for the whole galaxy. This variability amplitude is approximately the limiting accuracy of photometry for some telescopes. Therefore, it is not easy to detect this variability in Mrk 501. The effects of the photometric aperture and observational seeing are significant for the photometric results, and most previous works did not take into account the effects of the two factors. This might lead to some fake variability in some previous works for Mrk 501, and the relevant results should be reconsidered. Our studies suggest that a fixed aperture, which depends on the seeing condition, is better than a dynamic aperture, and host galaxy subtraction is necessary. Our simulations give a reasonable host-subtraction. Strong host contamination also impacts the color, polarization and SED of an AGN. Thus, it is meaningful to subtract the host component before investigating properties of Mrk 501.

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