Simplified sigmoidal curve fitting for 6 MV FFF photon beam of Halcyon to determine field size for beam commissioning and quality assurance

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Abstract

Background

Delivery of a single energy 6 MV flattening filter free (FFF) photon beam is a main characteristic of an O-ring gantry type linear accelerator (linac) Halcyon. The purpose of this study is to determine the field size of the beam through an application of the simplified sigmoidal curve fitting (SCF) to the beam profiles obtained from the preconfigured reference data of Halcyon, and then to compare its parametrization with the measured beam data from Halcyon.

Methods

After a mathematical definition of the SCF using four coefficients, the defined curves were fitted to both the reference and measured data. When a high agreement between the fitting curve and the profiles in each data, the field sizes were determined by identifying the maximum point along the third derivative of the fitting curve. The curve fitting included the field sizes for beam profiles as 2 × 2, 4 × 4, 6 × 6, 8 × 8, 10 × 10, 20 × 20 and 28 × 28 cm² as a function of depths (at 1.3, 5, 10, 20 cm). The results of the field size from the reference data were compared with the results in the measured data using same condition.

Results

All fitting curves show an average agreement ratio higher than 97% and the values of goodness of fit, R², as better than 0.99. The differences of the field size between the reference data and the measured data were within the range of 0 to 0.2 cm. The least difference of the field sizes at depth 10 cm which is a surface to axis distance was reported.

Conclusion

The application of the SCF has been proved to accurately obtain the field size of the preconfigured reference and of measured FFF photon beam data for Halcyon. The current work can be useful to the beam commissioning as a countercheck methodology to the field size from the reference data in the treatment planning system of a newly installed Halcyon and to the routine quality assurance to ascertain the correctness of field sizes clinically used with the Halcyon.
The field size of a radiation beam in radiotherapy with the use of a linear accelerator (linac) means an area for the radiation delivery. For this reason, the determination of an accurate field sizes is one of significant parameters for the delivery of the radiation and of an important process in a quality assurance (QA)[1, 2]. Normally, this field size can be determined during a commissioning procedure which requires a considerable time due to a repeatable radiation delivery to a water phantom for scanning profiles as well as a validation[3, 4]. While a conventional linac systems have been equipped with a flattening filter (FF) type to deliver the radiation beam of an uniform dose distribution, a method of a full width at half maximum (FWHM) is the conventionally representative methodology for the determination of the field size for the FF beam[5]. Correctly, the definition of the field size for the FF beam is based on a point of off-axis at the dose of 50% after the dose normalization of a central axis (CAX) as 100%. This FWHM methodology is suitable for the determination of the field size of FF beam since it has the uniform region around the CAX. According to Task Group Report #142 of AAPM, several parameters such as flatness, symmetry and penumbra should be considered to specify the FF beam[6]. Recently, the use of the flattening filter-free (FFF) beam has been more preferably used in radiotherapy than the use of the FF beam due to the advanced radiotherapy technologies such as intensity modulated radiotherapy (IMRT), volumetric modulated arc therapy (VMAT) and stereotactic radiosurgery [7–12]. Because the high precision radiotherapy like above techniques does not require the use of the flat homogeneous profile, the method of FWHM is not suitable for determining the field size of the FFF beam which shows a specific shape of the dose profile with relatively higher peak at the CAX.

Halcyon (Varian Medical Systems Inc., Palo Alto, USA), which is a linac with an O-ring gantry type, is one of radiotherapy machine using the 6 MV FFF beam, like Tomotherapy and Cyberknife. There have been several studies to define the field sizes of a FFF beam[13, 14]. The most representative method to determine the field size of the FFF beam is the use of the inflection point (IP) on the penumbra region of the beam’s profile[15, 16]. Nevertheless, there are some uncertainties to obtain correct IP from beam data measurement. In order to consider this uncertainty, Pönisch et al. proposed a method to identify the IP at the field edge of the FFF beam with the same level of FF beam[17]. Because the
position of the IP can be different according to the positional stepping size error to obtain the beam profile. Fogliata et al. suggested the formulation of re-normalization to overcome the uncertainty in IP due to the stepping size [9]. Basically, although these two methods to define the field size of FFF beam are originally based on the profiles of the FF beam, with the large amount of values, the position error by the measurement still have existed from both methods. From the advanced studies, the parameterized gradient-based method (PGM) that is complementing these two methods was proposed to determine the field size of FFF beam using a mathematical model [18, 19]. However, the parameters of the profile were partially reported [20]. In addition, since the PGM method also applied the mathematical model originated from the measured data, the uncertainty by the measurement still have existed as initial two methods. Therefore, the purpose of this study is to determine the field size of the Halcyon using only 6 MV FFF beam through the application of the simplified sigmoidal curve fitting (SCF). To compare the uncertainty by the measurement, firstly we created fitting curves using a simplified mathematical sigmoidal equation based on the preconfigured reference data in the Halcyon treatment panning system. The SCF have been applied to the seven field sizes as function of depths for both the preconfigured reference data and measured data using four coefficients to specify a SCF. The field size as obtained by SCF fitting to measured data was compared with the field size from the reference beam to ensure reliability of the SCF in this study. This methodology with simplified process can be applied to determine the field size during repeatedly periodic QA and commissioning process for Halcyon [13, 14, 21].

Methods
Preparation of data
A preconfigured reference beam dataset (reference data) generated by the vendor has been stored in the treatment planning system when a new Halcyon is installed. The reference data included the lateral dose profiles for field size of 2 × 2, 4 × 4, 6 × 6, 8 × 8, 10 × 10, 20 × 20 and 28 × 28 cm² as function of depth at 1.3, 5, 10, and 20 cm into the water phantom. In order to compare the field size from the reference data with that from the measured data, the measurement was performed under the same conditions in the reference data. A source to surface distance (SSD) was set at 90 cm. The
CC13 ionization chamber and the Blue Phantom water tank (IBA Dosimetry, Schwarzenbruck, Germany) were used to measure the relative dose profiles for field size > 4 × 4 cm². For field sizes ≤ 4 × 4 cm², an edge diode detector (Sun Nuclear, Melbourne, FL, USA) was used. The scanning step for the acquisition of the profile on the measurement line along the off axis position was 0.1 cm with increment per step. All measurement values were processed by using OminiPro Accept7 (version 7.4.24.0) software (IBA dosimetry, Schwarzenbruck, Germany).

Definition of fitting using sigmoidal curve
The sigmoidal curve is originated from the sigmoid function which has been used at the field of the signal process. The shape of the sigmoidal curve is given by Eq, (1),

\[ f(x) = \gamma \left( \frac{1}{1 + e^{x \beta + \delta}} \right) + \delta \]  

(1)

The coefficients \( \alpha, \beta, \gamma, \) and \( \delta \) are used to determine shape of the curve \( f(x) \). The coefficient \( \alpha \) controls the gradient of the sigmoidal curve. The higher value of \( \alpha \) makes the curve gradient steeper. The coefficient \( \beta \) is related with the horizontal movement of the whole sigmoidal curve. The higher value of \( \beta \) let the sigmoidal curve move further to the right hand side of the curve. The coefficient \( \gamma \) determines the location of the only upper end of the sigmoidal curve. The higher value of \( \gamma \) let the upper end of the sigmoidal curve locate at a higher position side. The coefficient \( \delta \) determines the vertical movement of the whole sigmoidal curve. The higher value of \( \delta \) let the sigmoidal curve move to more upward direction. Thus, the coefficients \( \alpha \) and \( \gamma \) contribute to transform the shape of the curve. The coefficients \( \beta \) and \( \delta \) change the location of the sigmoidal curve.

After the upload of the profile to MATLAB (2019 version, MathWorks Inc, Sherborn, MA, USA), the SCF has been performed by the change of each coefficient until the sigmoidal curve overlapped on the profile. However, because the physical range of the sigmoidal curve cannot cover the whole range of the profile due to the original shape of the curve, the physical range of the sigmoidal curve was limited until the profile can cover the maximum range using the only sigmoidal curve. Through this methodology, all SCFs have been done to all profiles.

Verification of agreement for fitting curves
To verify the accuracy of the fitting curve based on the sigmoidal curve with the profiles, the average agreement ratio (AAR) between the values in the fitting curve \( f_i \) and the values in profiles \( x_i \) at the same step position were calculated using Eq. (2), which shows the agreement between values from profile and fitting.

\[
\text{AAR} (\%) = 100 - \frac{1}{n} \sum_i \left( \frac{|x_i - f_i|}{x_i} \right) \times 100
\]  

(2)

In this study, if the AAR is higher than 97%, the optimization for the fitting terminates because the sufficient accuracy for the fitting has been obtained and the four coefficients \( \alpha, \beta, \gamma, \text{ and } \delta \) are used to define shape of the final fitting curve. Moreover, an additional verification was performed through the evaluation of goodness of fit \( R^2 \) (Eq. (3))

\[
R^2 = 1 - \frac{\sum (f_i - y_i)^2}{\sum (y_i - \bar{y})^2}
\]  

(3)

where \( \bar{y} \) is the mean of all \( f_i \) values on fitting curve, and the \( y_i \) is a value on profile. The same validation procedures were applied to the measured data and to the reference data.

Identification of specific regions & points

In this study, in order to describe the sigmoidal curve, three regions and two points have been assigned for the definition to the half-side of SCF (Fig. 1(a)). The three regions included the introductory region (IR), the growing region (GR), and plateau region (PR). The IR is the region starting to increase on the sigmoidal curve. The GR is continuously increasing region on the sigmoidal curve. The PR is the region slowing down to increase. These regions can be identified through the second derivative of the sigmoidal curve as shown Fig. 1(a). The range between the rightmost point and the maximum point on the second derivative curve was defined as the IR. The range between the maximum point and the minimum point on the second derivative curve was defined as the GR. The region between the minimum point and the leftmost point on the second derivative curve was defined as the PR. Because of the specification of the sigmoidal shape, there are two specific points as singular point (SP) and inflection point (IP), both of which these couple of points could be identified from the third derivative curve of the sigmoidal curve. The SP is the minimum point between the
range of IR and the GR (Eq. (4)). The IP is another minimum point at the range between the GR and the PR as below (Eq. (5)). When there is no point either the IP or the SP, the re-fitting process was performed from 2.2 part in this session.

\[
\text{SP (IR}< x < \text{GR}) = \min \left( \frac{\partial}{\partial x} \frac{\partial}{\partial x} \frac{\partial}{\partial x} y \left( \frac{1}{1+e^{ax+by}} + \delta \right) \right) \tag{4}
\]

\[
\text{IP (GR}< x < \text{PR}) = \min \left( \frac{\partial}{\partial x} \frac{\partial}{\partial x} \frac{\partial}{\partial x} y \left( \frac{1}{1+e^{ax+by}} + \delta \right) \right) \tag{5}
\]

**Determination of the Field Size**

After the SP and the IP have been obtained, the determined field size (DFS) can be identified as the maximum point on the third derivative curve between the SP and the IP as below Eq. (6). Figure 1(b) shows the conceptual DFS on the third derivative curve and the actual example of the DFS has been demonstrated with fitting to the profile at Fig. 1(b)

\[
\text{DFS} = 2 \times \max \left( \frac{\partial}{\partial x} \frac{\partial}{\partial x} \frac{\partial}{\partial x} y \left( \frac{1}{1+e^{ax+by}} + \delta \right) \right) \tag{6}
\]

The factor 2 in the equation (Eq. (6)) was due to that fact which only right hand half side of the symmetric open beam profile was used for the curve fitting.

**Results**

**The Accuracy of the fitting curve**

Figure 2 shows all the final fitting curves using SCF with half of the profiles in the reference data. The profiles for all field sizes (2 × 2, 4 × 4, 6 × 6, 8 × 8, 10 × 10, 20 × 20, and 28 × 28 cm²) were demonstrated as a function of depth (1.3, 5, 10, and 20 cm) as (a), (b), (c), and (d), respectively. The fitting curve is up on each profile as indicated by the red circles. On the other hand, Fig. 3 shows all the final fitting curves using same SCF with the half of profiles in the measured data. It also includes the profiles for all field sizes with variable depths: 1.3, 5, 10, and 20 cm as (a), (b), (c), and (d), respectively. The profiles in Figs. 2 and 3 along the field size were distinguished by their color, and have been normalized by relative dose at the CAX. The X-axis shows the off-axis position from the CAX. All of the fitting curves show a good agreement with each profile. The accuracy for all fitting curves were evaluated using method of AAR and R². The performance were reported as over 97% for
AAR and over 0.99 for $R^2$.

Table 1 tabulates all of coefficients to form the final sigmoidal curves for all field sizes with all depths using SCF before determination of the field size. The differences of the coefficient values at all depths were reported as $\alpha$ maximum difference 1.3, $\beta$ maximum difference 4.4, $\gamma$ maximum difference 4.0, $\delta$ maximum difference 1.7 and all of coefficients minimum difference 0. All values for IP and SP are also tabulated. The maximum and minimum difference for results in IP were 0.15 cm and 0 cm, respectively. The maximum and minimum difference for results in SP were 0.18 cm and 0 cm, respectively.

Table 2 tabulates all values for DFS from the fitting curve to all profiles at different depth. For the field sizes, the maximum difference was 0.2 cm and the minimum difference shows 0 cm. The results for depth 10 cm shows the least difference between the reference data and the measured data.

Discussion
Although there have been previous studies for the determination of the field size of the conventional FF beam, such as the FWHM, with which is not suitable for the Halcyon linac’s profile due to the difference in CAX of the dose. The limitation of the current measurement is time consuming to apply a fine scanning step of 0.1 cm in the QA and commissioning process. MSM, techniques by Pönisch et al. and the method of the re-normalization, which is a typical definition methods for the FFF beam field size, could contain uncertainty arising the scanning step [9, 15–17]. The uncertainty at the location of the inflection point may occur from the measured data for using less fine scanning step. On the other hand, although the PGM method shows the advantage in that it does not affect to the size of scanning step, its analytical fitting also obtained the parameters from the measured data[18]. Thus, if the measured data is not accurate due to the several conditions, the fitting also would be unreliable to apply to the profiles. In this study, the use of the preconfigured reference data and mathematical fitting curve to determine the field allows us to compare the high accuracy and the good reliability of the parametrization method through direct comparison between the reference data and measured data.
From the study of Lebron et al., because the parameter values for the small field of the profile (≤ 4 × 4 cm²) were not shown, the information for small field size were unclear[18-20]. However, the current study presented the determined field size for even small field (e.g. 2 × 2, 4 × 4 cm²). Most small fields from linacs, including Halcyon, may frequently be used for IMRT or VMAT. In particular, a significant dose difference can be deviated from the true dose for off axis position within the steep dose fall off region.

Therefore the current parametrization to small field size is useful not only as a countercheck to the preconfigured reference data to the Halcyon treatment planning system but also as a routine commissioning and QA tool to ascertain the correctness of field size used in clinical application of the Halcyon system. It is meaningful in that the criteria can be established itself through the use of the reference data. This work with the adequate parameters’ information and simple methodology could be useful for the especially new Halcyon users who must perform the validation of the preconfigured reference for the beam commissioning process as well as the QA because beam data can vary from machine to machine of the same model and same vendor as described by AAPM TG,51,100,106[13, 14, 21].

Conclusions
The determination of the field size using the simple SCF method has been established for radiation beam of 6 MV FFF from Halcyon. This method covers all field sizes including small field sizes. The coefficients for the fitting and field sizes between the reference data and the measured data were in good accord and can be used as the repeated countercheck for other users using the same model of linac.

Abbreviations
QA: Quality assurance; IMRT: Intensity modulated radiation therapy; VMAT: Volumetric modulated arc therapy, SCF: Sigmoidal fitting curve; RD: Reference data; MD: Measured data; AAR: Average agreement ratio; IR: Introductory region; GR: Growing region; PR: Plateau region; SP: Singular point; IP: Inflection point; DFS: Determined field size

Declarations

Ethics approval and consent to participate
Not applicable

Consent for publication
Not applicable

Availability of data and materials
The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Competing interests
The authors declare that they have no competing interests.

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Authors’ contribution
Study concept and design: MC, DY, MT, HM, TS. Data acquisition: MT, KM, MO, HM. Data analysis and interpretation: MC, DY, ML, MT, KM, MO, MK, SKD, HM, TS. Computer program modeling: MC, DY, MK. Manuscript preparation and editing: MC, ML, DY, MT, HM, TS. Manuscripts review: All authors read and approved the final manuscript.

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Tables
Table 1. Comparison between reference data and measured data for all coefficients and IP,SP

| Field Size | RD | MD | RD | MD | RD | MD |
|------------|----|----|----|----|----|----|
| (cm^2)     | α  | α  | β  | β  | γ  | γ  |
| 2 x 2      | 11.20 | 9.90 | -10.30 | -9.20 | 97.50 | 97.50 |
| 4 x 4      | 10.00 | 8.80 | -18.30 | -16.20 | 97.50 | 93.50 |
| 6 x 6      | 6.06 | 5.20 | -16.50 | -14.50 | 94.50 | 93.00 |
| 8 x 8      | 6.03 | 5.70 | -22.00 | -20.60 | 92.00 | 93.00 |
| 10 x 10    | 6.01 | 6.30 | -27.50 | -27.80 | 89.50 | 89.60 |
| 20 x 20    | 5.85 | 5.90 | -53.50 | -53.80 | 73.00 | 73.00 |
| 28 x 28    | 5.70 | 5.90 | -72.30 | -74.90 | 62.50 | 62.50 |
| 2 x 2      | 10.20 | 9.80 | -9.80 | -9.20 | 95.00 | 95.00 |
| 4 x 4      | 10.00 | 9.40 | -19.00 | -17.40 | 90.50 | 90.70 |
| 6 x 6      | 5.50 | 5.30 | -15.60 | -14.80 | 91.00 | 91.00 |
| 8 x 8      | 5.30 | 5.30 | -20.10 | -19.90 | 88.00 | 88.00 |
| 10 x 10    | 5.30 | 4.80 | -25.20 | -24.60 | 84.30 | 85.80 |
| 20 x 20    | 5.10 | 5.10 | -48.30 | -48.40 | 70.00 | 69.60 |
| 28 x 28    | 5.00 | 5.00 | -66.50 | -66.30 | 60.00 | 59.50 |
| 2 x 2      | 8.70 | 8.80 | -8.70 | -8.80 | 96.00 | 96.40 |
| 4 x 4      | 9.00 | 8.80 | -18.10 | -17.50 | 90.00 | 88.50 |
| 6 x 6      | 5.50 | 5.10 | -16.40 | -15.10 | 88.00 | 88.00 |
| 8 x 8      | 5.10 | 4.90 | -20.40 | -19.40 | 86.50 | 84.80 |
| 10 x 10    | 5.10 | 4.60 | -25.50 | -22.80 | 81.80 | 91.00 |
| 20 x 20    | 4.90 | 4.90 | -48.80 | -48.90 | 64.70 | 63.60 |
| 28 x 28    | 4.80 | 4.80 | -67.30 | -66.90 | 53.50 | 53.50 |
| 2 x 2      | 7.80 | 7.70 | -8.65 | -8.50 | 94.50 | 96.20 |
| 4 x 4      | 7.50 | 7.10 | -16.50 | -15.50 | 89.70 | 89.40 |
| 6 x 6      | 4.80 | 4.80 | -15.80 | -15.30 | 97.00 | 86.30 |
| 8 x 8      | 4.80 | 4.80 | -21.10 | -21.20 | 82.00 | 79.20 |
| 10 x 10    | 4.90 | 4.10 | -26.90 | -22.50 | 75.50 | 76.00 |
| 20 x 20    | 4.75 | 5.00 | -54.30 | -55.00 | 56.00 | 54.30 |
| 28 x 28    | 5.00 | 5.00 | -77.00 | -77.00 | 45.50 | 45.00 |

Table 2. Determination of field size for reference data and measured data
| Depth   | Field Size (cm²) | RD |
|---------|------------------|----|
| 1.30 cm |                  |    |
| 2 × 2   |                  |    |
| 4 × 4   |                  |    |
| 6 × 6   |                  |    |
| 8 × 8   |                  |    |
| 10 × 10 |                  |    |
| 20 × 20 |                  |    |
| 28 × 28 |                  |    |
| 5.00 cm |                  |    |
| 2 × 2   |                  |    |
| 4 × 4   |                  |    |
| 6 × 6   |                  |    |
| 8 × 8   |                  |    |
| 10 × 10 |                  |    |
| 20 × 20 |                  |    |
| 28 × 28 |                  |    |
| 10.00 cm|                  |    |
| 2 × 2   |                  |    |
| 4 × 4   |                  |    |
| 6 × 6   |                  |    |
| 8 × 8   |                  |    |
| 10 × 10 |                  |    |
| 20 × 20 |                  |    |
| 28 × 28 |                  |    |
| 20.00 cm|                  |    |
| 2 × 2   |                  |    |
| 4 × 4   |                  |    |
| 6 × 6   |                  |    |
| 8 × 8   |                  |    |
| 10 × 10 |                  |    |
| 20 × 20 |                  |    |
| 28 × 28 |                  |    |

Figures
(a) An example for explaining the method for the identification of three regions and two specific points. The three regions were defined by the second derivative and two specific point was set at the minimum points of the third derivative curve: Identification of the position for introductory region (IR; Yellow color region), the growing region (GR; Green color region), and plateau region (PR; Red color region). (b) The singular point (SP), inflection point (IP). The red dot shows the fitting curve up on the profile (Blue line). The black dotted line shows the shape of the third derivative curve. A comparison of the whole fitting curve with the third derivative: the positions for the SP, IP, and DFS/2 on the black dotted line were shown.
The fitting curves and the profiles from the reference data (RD). The profiles according to the field size $2 \times 2$, $4 \times 4$, $6 \times 6$, $8 \times 8$, $10 \times 10$, $20 \times 20$, and $28 \times 28$ cm$^2$ along with the fitting curves for each beam profile (red circles). The depth for the profile were (a) 1.3 cm, (b) 5 cm, (c) 10 cm, and (d) 20 cm.
Figure 3

The fitting curves and the profiles from the measured data (MD). The profiles according to the field size 2 × 2, 4 × 4, 6 × 6, 8 × 8, 10 × 10, 20 × 20, and 28 × 28 cm² along with the fitting curves for each beam profile (red circles). The depth for the profile were (a) 1.3 cm, (b) 5 cm, (c) 10 cm, and (d) 20 cm.