Stellar Kinematics of the Spiral Structure in the Solar Neighborhood

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ABSTRACT

We present a method, based on the kinematic analysis of the Galactic disk stars, to clarify whether the internal motions of the stellar system in spiral arms follow those expected in the density wave theory. The method relies on the comparison with the linear relation between the phases of spatial positions and epicyclic motions of stars, as drawn from the theory. The application of the method to the 78 Galactic Cepheids near the Sun, for which accurate proper motions are available from the Hipparcos Catalogue, has revealed that these Cepheids hold no correlation between both phases, thereby implying that their motions are in contradiction with the theoretical predictions. Possible reasons for this discrepancy are discussed and future prospects are outlined.

Subject headings: Galaxy: kinematics - spiral arm - density wave - Hipparcos

1. Introduction

Spiral structures of galaxies have been studied for a long time in order to understand how these structures are formed (e.g., Roberts, Roberts & Shu 1975; Rohlfs 1977; Binney & Tremaine 1987). One of the proposed models to explain spiral arms is that they are just material arms, where the stars originally making up a spiral arm remains in the arm even at the later time. However this simple model holds a wellknown problem which is called “winding problem”: the differential rotation in galactic disks winds up the arm in

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a short time compared with the age of galaxies, so that the spiral pattern would be too tightly wound compared with the observed spiral structures. In contrast, the currently most popular model, which is free from the winding problem, is the density wave theory (Lin & Shu 1964), where a spiral arm is regarded as a wave and wavelike oscillation of stellar motions propagates through galactic disks. In this picture, the global spiral pattern is sustained independently of individual stars moving at different angular velocities. For a comprehensive review of the density wave theory, see, e.g., Rohlfs (1977) and Binney & Tremaine (1987).

The density wave theory has been suggested by various observational aspects in spiral galaxies, including the relative distributions of dust lane, interstellar gas, and H II regions across the arms (Fujimoto 1968; Roberts 1969; Rohlfs 1977), intensity distribution of radio continuum radiation (Mathewson, van der Kruit, & Brouw 1972), and systematic variation of gaseous velocity fields near the arms (e.g., Visser 1980). In particular, recent high-resolution observations using CO emission have revealed detailed streaming motions of molecular gas, which are generally in agreement with predictions of the density wave models (e.g., Kuno & Nakai 1997; Aalto et al. 1999). However, we note that these observational results provide only an outcome of nonlinear interaction between interstellar matter and background stellar arms, and it is yet unknown whether the motions of stars themselves, which make up spiral pattern, actually follow those predicted by the density wave theory. In this regard, the direct access to detailed stellar kinematics in disks is possible only in our Galaxy.

Here, we present a method to clarify this issue, based on the analysis of local kinematics of disk stars. We then apply the method to 78 Cepheids in the solar neighborhood, for which the precise data of proper motions are available from the Hipparcos Catalogue (ESA 1997). Also, the distances to these sample stars can be accurately estimated from the period-luminosity relation, so that combined with the radial velocity data, the full three-dimensional velocities are available. We note here that although we focus on the local kinematics of spiral arms in this work, the method we develop here can be applied to the motions of more remote stars distributed over a whole disk, for which precise astrometric data will be provided by the next-generation satellites such as FAME and GAIA.

Our paper is organized as follows. In §2, we describe the method to determine whether or not the motions of stars agree with those expected in the density wave theory. In §3, we show the detail of the sample stars and the fundamental parameters of our Galaxy adopted in this work. The application of our method to the sample stars is shown in §4. Finally, §5 is devoted to discussion and conclusions.
2. METHOD

All orbits of the disk stars in our Galaxy are not perfectly circular. The discrepancy between the motion of a star and its reference circular orbit, called orbit of the guiding center, can be represented by an epicyclic motion of the star (Binney & Tremaine 1987). We describe here a method to compare the epicyclic motions of the stars observed near the Sun with those expected in the density wave theory.

First, we define the “position phase”, \( \chi \), as a function of the position of a star in the Galactic plane, by assuming that the shape of the spiral arm is logarithmic as follows:

\[
\chi = \ln \left( \frac{r}{r_g} \right) \tan i - \theta + \Omega_p t ,
\]

where \( r \) is the distance between the star and the Galactic Center (GC), \( r_g \) is the distance between the Sun and GC, \( i \) is the pitch angle of the spiral arm, \( \theta \) is the angle between \( r \) and \( r_g \), and \( \Omega_p \) is the angular velocity of the spiral pattern in the azimuthal direction. We note that all the stars on the same arm have the same value of \( \chi \) (Figure 1).

Second, we define the phase of the epicyclic motion of a star, \( \phi \), which is the angle between the velocity vector of the epicyclic motion and the direction perpendicular to a spiral arm (Figure 1). Then, if the stellar motions in the arm obey the density wave theory, all the stars on the same arm have the same value of \( \phi \) as well as the same value of \( \chi \). Also, when the number of arms is \( m \), the epicyclic frequency \( \kappa \) of the stellar motions is equal to \( m \) times as large as the angular frequency \( \Omega - \Omega_p \) \([\kappa = m(\Omega - \Omega_p)]\). If so, since \( d\phi/dt = -\kappa \), and \( d\chi/dt = \Omega_p - d\theta/dt = \Omega_p - \Omega \) from equation (1), we obtain the following linear relation between \( \phi \) and \( \chi \),

\[
\phi \equiv m\chi \pmod{2\pi},
\]

if the stellar motions obey the density wave theory.

Thus, by assessing this linear relation between observed \( \phi \) and \( \chi \), it is possible to investigate whether or not the observed stellar motions follow those predicted by the density wave theory. We can also derive the number of the arms, \( m \), from the slope in the \( \phi - \chi \) relation. In Figure 3, we show the case \( m = 4 \) (solid lines) as an example.

3. DATA

We adopt the Galactic Cepheids as the tracers of stellar motions in spiral arms, because these bright stars can be seen from a long distance and so we can investigate a wide region around the Sun. Also, these young populations show only a small deviation
from circular rotation, thereby allowing us to analyze the internal kinematics of spiral arms alone; relatively old stars, such as dwarf stars, have too large velocity dispersions, possibly due to repeated gravitational interactions with massive clouds (Spitzer & Schwarzschild 1953) in addition to the effect of spiral arms. Furthermore we can obtain accurate distances for Cepheids, based on the relation between pulsation period and absolute magnitude. We do not use the Hipparcos parallaxes, which have generally large errors for many Cepheids beyond \( \sim 100 \) pc from the Sun. We adopt the Cepheid catalog compiled by Mishurov et al. (1997) for distances and radial velocities, and the Hipparcos Catalogue for proper motions, to calculate the individual motions of the Cepheids in the Galactic plane.

In the Mishurov et al. catalog, Cepheids in the region of \( r > 4 \) kpc and those in a binary system are excluded. Also, nearby Cepheids in the region of \( r < 0.5 \) kpc are excluded in order to reduce the local effects like Gould’s Belt. Furthermore, Cepheids whose pulsation periods exceed 9 days are also excluded, because they are supposed to be extremely young objects. In addition, we further exclude the Cepheids within 1 kpc for the following reason. When we compare the observed distribution of Cepheids in the \( \phi - \chi \) plane with the theoretical prediction in a quantitative manner, as will be described later, we assign larger statistical weights to the Cepheids having a smaller observational error [eq. (3)]. This leads to larger weights to the Cepheids located close to the Sun, say \( r < 1 \) kpc, so that the result of the analysis will be largely determined by only small number of Cepheids in a small region near the Sun. We also exclude two Cepheids whose peculiar velocity is exceptionally large, over 50 km s\(^{-1}\), compared to other ones. As a consequence, we adopt 78 Cepheids in this work, and their spatial distribution is shown in Figure 2.

We adopt \( r_g = 8.3 \) kpc in our analysis, which is approximately an average of observed values ranging from 8.1 kpc to 8.5 kpc (Kerr & Lynden-Bell 1986; Hanson 1987; Pont, Mayor & Burk 1994; Feast & Whitelock 1997). As for the pitch angle of spiral arms, \( i \), in our Galaxy, several authors investigated one of the conspicuous spiral arms near the Sun, the Sagittarius arm, based on the spatial distributions of open clusters, CO emissions, or O-B2 clusters, and arrived at several to about 20° (Pavlovskaya & Suchkov 1984; Dame et al. 1986; Grabelsky et al. 1988; Alfaro, Cabrera-Cano & Delgado 1992). In this work, we adopt \( i = -8.0^\circ \) in the solar neighborhood, where the negative value for \( i \) denotes a trailing arm. We have found that even if we change the values of these parameters over a likely range, the result shown below remains essentially unchanged.

In order to analyze the epicyclic motion of the Cepheids, we require to subtract both the local solar motion with respect to the local standard of rest and the effect of the Galactic differential rotation from the observed motions. As the local solar motion, we adopt 15.5 km s\(^{-1}\) in the direction to \( l_\odot=45^\circ \) and \( b_\odot = 23.6^\circ \) (Kulikovskij 1985). To
estimate the effect of the Galactic differential rotation, we assume that in the concerned region within about 4 kpc from the Sun, the Galactic rotation is monotonously changing with distance from the Sun, where the Oort constant $A$ is assumed to be $13.0 \text{ km s}^{-1}\text{kpc}^{-1}$. We again note that the result shown in §4 is essentially independent of the value of $A$ or the assumption for differential rotation.

4. RESULTS

We present the relation between the phases of the epicyclic motions of the Cepheids $\phi$ and their position phases $\chi$ in Figure 3. We also show the case for the 4-armed galaxy which obeys the density wave theory (solid lines). If the motions of the Cepheids in the solar neighborhood are in accordance with those expected in the density wave theory, we expect the similar linear relation between $\phi$ and $\chi$. However, such a linear relation is not apparent for the Cepheids, which may imply that these objects in the region we investigate do not obey the density wave theory.

To be more quantitative, we analyze the motions of the Cepheids by using the sum of squares of the deviations for the observed $\phi$’s from the theoretically expected ones. We define this deviation, $\Delta^2$, as follows:

$$\Delta^2 \equiv \sum_i \frac{\delta\phi_i^2}{\sigma_i^2},$$

where $\delta\phi_i$ is defined as,

$$\delta\phi_i \equiv \min(|\phi_i - \phi_{DW}(\chi_i)|, 2\pi - |\phi_i - \phi_{DW}(\chi_i)|),$$

where $\sigma_i$ is the error in the phase $\phi_i$ of the star $i$, and $\phi_{DW}(\chi_i)$ is the phase when the motion of the star $i$ obeys the density wave theory. Here we note that the error $\sigma_i$ includes both the observational error and the velocity dispersion of the Cepheids. We adopt $13 \text{ km s}^{-1}$ for the velocity dispersion. The quantity $\delta\phi_i$ denotes the discrepancy between $\phi_i$ and $\phi_{DW}(\chi_i)$, for which we take the smaller value of $|\phi_i - \phi_{DW}(\chi_i)|$ and $2\pi - |\phi_i - \phi_{DW}(\chi_i)|$, because $\phi_i$ has a period of $2\pi$. The phase $\phi_{DW}(\chi_i)$ is determined by minimizing the deviation $\Delta^2$ for each value of the arm number $m$.

If the motions of stars are in accordance with those expected in the density wave theory, the expected value of the deviation $\Delta^2$ is about the same as the number of the sample stars (78 in this work), because the distribution of $\delta\phi_i^2$ is normalized by its standard dispersion $\sigma_i^2$. Thus, in this case, we will obtain a minimum of $\Delta^2$ with $\Delta^2_{\text{min}} \approx 80$ at a specific arm number $m = m_q$, whereas at other $m$’s, the value of $\Delta^2$ will be systematically
larger than 80; the larger ratio of $\Delta^2/\Delta^2_{\text{min}}$ implies more likely the observed stellar motions match the density wave theory. On the other hand, if $\Delta^2$ is always larger than 80 at all $m$’s, we may conclude that the stellar motions are totally inconsistent with those expected in the theory.

We plot the deviation $\Delta^2$ as a function of $m$ in Figure 4. Solid line shows the result for our Cepheid sample. It is apparent that $\Delta^2$ is around 250 for all values of the parameter $m$, without showing a noticeable minimum with $\Delta^2 \sim 80$.

In order to examine the significance of this result, we investigate three hypothetical models for comparison. As the first model, we randomly select $\phi$, independently of $\chi$, so that there is no correlation between $\phi$ and $\chi$. We then assign the same errors $\sigma_i$ as those for the Cepheids to each $\phi$. Thick dashed line in Figure 4 shows $\Delta^2$ vs. $m$ derived from this “random model”. It follows that the properties of $\Delta^2$ as a function of $m$ are basically the same as for the Cepheids, thereby implying that the phase $\phi$ of the Cepheids is random in the solar neighborhood. The second model, as shown by thin dashed line, follows the density wave theory with $m = 4$. We assign the same errors $\sigma_i$ as those for the Cepheids. The value of $\Delta^2$ is around 250, except for the case of $m = 4$ at the minimum of $\Delta^2$ ($\Delta^2_{\text{min}} \sim 80$). This behavior of $\Delta^2$ is in sharp contrast to the case of the Cepheids. The last model (dotted line) is similar to the second one, except for the assignment of larger errors $\sigma_i$: 1.4 times as much as those of the Cepheids, in order to see the effect of $\sigma_i$ on the result. In this case, the value $\Delta^2$ is smaller than other cases: $\Delta \sim 150$ except for the case of $m = 4$ at the minimum of $\Delta^2$ ($\Delta^2_{\text{min}} \sim 80$). Thus, even if the errors $\sigma_i$ are large, which gives rise to small $\Delta^2/\Delta^2_{\text{min}}$, $\Delta^2$ has a noticeable minimum at a specific value of $m$, in sharp contrast to the case of the Cepheids.

Therefore, these results suggest us that the motions of the Cepheids in the solar neighborhood are inconsistent with those expected in the density wave theory, whereas the random model, in which the phases $\phi$ and $\chi$ are randomly selected, reproduces the observation in a reasonable manner.

5. DISCUSSION AND CONCLUSIONS

We have presented a method, based on the analysis of local kinematics of disk stars, to clarify whether the motions of the stars, which make up the spiral arms, follow those expected in the density wave theory. The method utilizes the comparison with the expected linear relation between the “position phases” of the stars $\chi$ and those of their epicyclic motions $\phi$, as given in equation (2). The application of the method to the 78 Galactic
Cepheids within 4 kpc from the Sun, for which accurate proper motions are available from \textit{Hipparcos}, has revealed that the relation between $\chi$ and $\phi$ for the Cepheids does not show the expected linear relation. Based on the quantitative analysis using the deviation $\Delta^2$, we conclude that the observed motions of the Cepheids are well reproduced by the random model having no correlation between $\chi$ and $\phi$.

There are a couple of possibilities to explain the current results, even if the spiral arms follow the density wave theory. First, the spiral structure around the Sun may not be simple as given in equation \cite{equation}. In fact, many of our sample Cepheids belong to the local spiral structure called the Orion arm, for which the definite conclusion on its spatial structure is yet to be reached. It is frequently expressed as “Orion spur”, having a rather irregular pattern compared to other large-scale arms, Sagittarius and Perseus arm (see, e.g., Gilmore, King, & van der Kruit 1989). The existence of the Orion arm may give disturbances on the density wave motions of stars induced by these large-scale arms. Second, the Cepheids we have adopted here may still convey systematic velocities of dense gas clouds from which these stars were formed, in the form of the streaming motions. If there still exist some individual streaming motions among the sample stars, such motions may violate the ideal linear relation between $\chi$ and $\phi$ expected for the density wave motions. Third, the Cepheids we have adopted here may have already experienced some scattering by dense gas clouds, thus having large velocity dispersions \cite{Spitzer & Schwarzschild 1953}. However, our experiment in §4 (dotted line in Figure 4) implies that the effect of velocity dispersions of our sample on the result appears to be minor.

In order to settle the last issue described above more clearly, we have repeated our analysis using younger populations with smaller velocity dispersions than the Cepheids. As such young stars, we have adopted the O-B5 stars, although due to their fainter luminosities than Cepheids, the sample with available proper motions is confined to the narrower region near the Sun. These sample stars are taken from the NASA SKY2000 Master Star Catalog Ver. 2 (Sande et al. 1998) which provides almost 300,000 stars brighter than 8 mag. The catalog contains many basic quantities, such as MK classification, luminosity class, apparent magnitude, color, radial velocity, and so on. We have then calibrated distances using \textit{Hipparcos} parallaxes or spectroscopic distances using the program kindly supplied by Drs. M. Sôma and M. Yoshizawa, and also obtained accurate proper motions by the cross-identification with the \textit{Hipparcos} and ACT Reference Catalogs \cite{Urban, Corbin, & Wycoff 1998}. After removing binaries and multiples, we have selected 773 O-B5 stars for which full three-dimensional velocities are available. Then, the application of the method we have developed here has revealed that the deviation $\Delta^2$ as a function of $m$ remains essentially constant of the order of 3000, without showing any noticeable minimum at a specific value of $m$. Thus, even the motions of such young populations with small velocity
dispersions are in contradiction with the density wave theory. We note here that most of these O-B5 stars are located within $\sim 1$ kpc from the Sun, so the effect of the local irregular spiral on the result cannot be negligible.

More definite conclusions on the issue we have addressed here require the assembly and analysis of much larger numbers of stars with accurate distances and proper motions, so that the statistical fluctuation in the result can be significantly reduced. Also, it is necessary to assemble the data of more remote stars over a large fraction of the disk, thereby diminishing effects of local irregular spiral structures on the kinematic analysis. Indeed, next-generation satellites such as FAME and GAIA will provide very precise astrometric data for huge numbers of the Galactic stars, and will thus offer us an opportunity to assess detailed motions of disk stars in conjunction with the density wave theory.

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Fig. 1.— Schematic diagram showing the definition of phases, \( \chi \) and \( \phi \), in the Galactic Plane. Each vector corresponds to a velocity vector of each star in epicyclic motion.
Fig. 2.— Positions of the Cepheids in the Galactic plane. The position of the Sun is the origin of the coordinate axes. The Galactic Center is located in the negative direction of the y-axis with $x = 0$. 
Fig. 3.— The phase $\phi/2\pi$ is plotted against $\chi/2\pi$ for our sample of Cepheids. The solid lines show the relation expected for a 4-armed galaxy that obeys the density wave theory.
Fig. 4.— The sum of square of the deviations, $\Delta^2$, as a function of arm-number $m$. The solid, dashed, thin dashed, and dotted lines denote Cepheid data, random model, 4-arm model ($\sigma = \sigma_{cep}$), and 4-arm model ($\sigma = 1.4\sigma_{cep}$), respectively. Here $\sigma_{cep}$ stands for the velocity error in the original Cepheid sample.