ONSET OF BENARD-MARANGONI INSTABILITIES IN A DOUBLE DIFFUSIVE BINARY FLUID LAYER WITH TEMPERATURE-DEPENDENT VISCOSITY

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Abstract. The effect of temperature-dependent viscosity in a horizontal double diffusive binary fluid layer is investigated. When the layer is heated from below, the convection of Benard-Marangoni will start to exists. Linear stability analysis is performed and the eigenvalues from few cases of boundary conditions were obtained. Galerkin method were used to solve the numerical calculation and marginal stability curve is obtained. Results shows that an increase of temperature-dependent viscosity will destabilized the system. The impact of double diffusive coefficients are also revealed. It is found that the effect of Soret parameter exhibits destabilizing reaction on the system while an opposite response is noted with an increase of Dufour parameter.

1. Introduction. Double diffusive or thermosolutal comes from the coupled temperature and salinity diffusion in a fluid. In a binary fluid, the competing gradient are Dufour (thermo-diffusion) and Soret (diffusion-thermo) effects. Nield and Kuznetsov [15] studied both effects in a nanofluid where stationary and oscillatory

2010 Mathematics Subject Classification. Primary: 76R10, 80A20; Secondary: 76A02.
Key words and phrases. temperature-dependent viscosity, binary fluid, double diffusive, Galerkin method, convection.

The present research was partially supported by MOHE for FRGS Vote no 5524808.
The reviewing process of the paper is handled by Gafurjan Ibragimov, Siti Hasana Sapar and Siti Nur Iqmal Ibrahim.

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modes were performed. Current research in binary fluid was done by Bergeon et al. [5] where they highlighted on the Soret effect and a year later Slavtchev et al. [20] extended the research by considering the nonlinear Soret effect. Saravanan and Sivakumar [19] examined the Soret and throughflow effects in a binary fluid.

Some researchers assume that fluid has a constant viscosity or may possess temperature-dependent viscosity (viscosity decreases exponentially with temperature) which may affect the onset of convection. Palm [16] initiated the study of variable viscosity on a steady convection. Other researchers also studied the temperature-dependent viscosity effects in different problems where Torrance and Turcotte [22] and Stengel et al. [21] studied in Benard instabilities and Slavtchev et al. [20], Cloot and Lebon [7] and Kozhoukharova and Rozé [11] in Marangoni instabilities. Cloot and Lebon [7] studied the steady Marangoni convection with undeformable surface and in microgravity. Abidin et al. [2] and Arifin and Abidin [3]- [4] studied the temperature-dependent viscosity effect together with others effects such as the feedback control, deformable surface and boundary effect in a fluid layer. White [23] did an overall study where he studied both theoretical and experimental in a Benard convection meanwhile Manga et al. [13] compared the temperature effect experimentally with boundary layer models.

Temperature dependent viscosity also has been integrated into other convection system where Franchi and Straughan [8] included the effect in a micropolar fluid and Ramirez and Saez [17] in a porous medium. It stated that a higher temperature will make the critical Rayleigh number decrease in both system. Ramirez and Saez [17] stated that temperature-dependent viscosity should be taken into account for every case studied since the effect has a huge impact on the instability of a convection. A similar result obtained by Lu and Chen [12] where the stability was enhanced by decreasing the temperature.

Here, we intend to investigate the effect of temperature-dependent viscosity in a double diffusive binary fluid layer. In future, we may consider the physical parameters such as the skin fraction, local Nusselt and Sherwood number [18].

2. Mathematical formulation. Two horizontal layers of quiescent double diffusive binary fluid with thickness $m$ is heated from below where the temperature difference is represented by $\Delta T$. We choose a Cartesian coordinate system where $(x, y)$ lies at the lower horizontal plane and $z$ pointing upward. For a Boussinesq approximation, we assumed the constant physical fluid properties except the density, $\rho$ and surface tension, $\sigma$ (for Marangoni convection) to vary upon temperature, $T$ and solute concentration, $S$ [6] and takes the form

$$\sigma = \sigma_0 - \sigma_t(T - T_0) + \sigma_s(S - S_0),$$

$$\rho = \rho_0[1 - \rho_t(T - T_0) + \rho_s(S - S_0)].$$

Here, $\sigma_0$ and $\rho_0$ are the values at the reference temperature, $T_0$ and at reference concentration, $S_0$. $\sigma_t$ and $\rho_t$ are the rate of change with temperature and $\sigma_s$ and $\rho_s$ are the rate of change of density with concentration. Let $S_0$ be at the lower boundary and $S_0 + \Delta S$ at the upper boundary. Due to the temperature-dependent viscosity in a binary mixture, the kinematic viscosity, $\mu$ follows [9] in the form

$$\mu = \mu_0 \exp[-\gamma(T - T_0)]$$
where $\mu_0$ is the reference kinematic viscosity value, $\gamma = \frac{\nu}{T^2_0}$ is a positive constant. The derivation will start from four governing equations 4 - 7 for the Benard-Marangoni convection following the analysis by Nield and Kuznetsov [15] and Nanjundappa et al. [14]

\[
\nabla \cdot \mathbf{h} = 0
\]

\[
\rho_0 \left[ \frac{\partial \mathbf{h}}{\partial t} + (\mathbf{h} \cdot \nabla) \mathbf{h} = -\nabla p + \nabla \cdot [\mu (\nabla^2 \mathbf{u} + \nabla \mathbf{h}^T)] + \rho g \varepsilon_z \right]
\]

\[
\frac{\partial T}{\partial t} + (\mathbf{h} \cdot \nabla) T = \kappa \nabla^2 T + D_T S \nabla^2 S
\]

\[
\frac{\partial S}{\partial s} + (\mathbf{h} \cdot \nabla) S = \kappa_s \nabla^2 S + D_{TS} \nabla^2 T
\]

where the variables are represented as follows; $\mathbf{h} = (u, v, w)$ is the velocity, $p$ (pressure), $g$ (gravity), $\varepsilon_z$ (unit vector in the $z$-direction), $\kappa$ (thermal diffusivity), $S$ (solute concentration), $\kappa_s$ (solutal diffusivity), $D_T$ (Soret diffusivity) and lastly, $D_{ST}$ is the Dufour diffusivity. The basic state of the fluid is

\[
(u, v, w) = (0, 0, 0), T = T_b(z) = T_c - \beta(z - \frac{m}{2}), \rho = \rho_0(z)
\]

\[
p = p_0(z) = p_0 - \rho_0 g z - \frac{1}{2} \rho_0 \alpha_4 g \beta z (z - m), S = S_0(z), \mu_0 = \mu_0 f(z)
\]

where $T_c = \frac{T_l + T_u}{2}$ is the average temperature between the lower boundary temperature, $T_l$ and the upper boundary, $T_u$ and $T_b = \frac{T_l + T_u}{2}$ is the temperature gradient. In this state, we perturb the system with the following form

\[
(u, v, w, T, p, \rho, S) = [0, 0, 0, T_b(z), p_0(z), \rho_0(z), S_0(z)] + [u', v', w', T', \rho', S']
\]

Subscript $b$ and primes quantities indicate the basic state and the perturbed variables. Using the following definitions, we non-dimensionalized equations 4 - 7

\[
(x', y', z') = \left(\frac{x, y, z}{m}, \frac{t}{\mu m^2}, \frac{u', v', w'}{\kappa}, \frac{p}{\mu \kappa \rho}, \frac{T'}{\Delta T}, \frac{S'}{\Delta S}, \frac{f}{\mu_0}\right)
\]

and by using equation 8 and equation 9, we obtain

\[
\nabla \cdot \mathbf{h}' = 0
\]

\[
\frac{1}{Pr} \frac{\partial h'}{\partial t} = -\nabla p' + f(z) \nabla \cdot (\nabla h' + \nabla h^T) + f(z) \nabla^2 h' + Ra T' c' + Rs Le S' c'_z
\]

\[
\frac{\partial T'}{\partial t} = w' + \nabla^2 T' + Df \nabla^2 S'
\]

\[
\frac{\partial S'}{\partial t} = w' + Le \nabla^2 S' + Sr \nabla^2 T'
\]

where $Ra = \frac{\rho g m^2 \Delta T}{\nu \kappa}$ (Rayleigh number), $Rs = \frac{\rho g m^2 \Delta S}{\nu \kappa \kappa_s}$ (Solutal Rayleigh number), $Le = \frac{\nu \kappa}{\kappa}$ (Lewis number), $Sr = \frac{D_S \Delta T}{\kappa \Delta S}$ (Soret number), $Df = \frac{D_{TS} \Delta S}{\kappa \Delta T}$ (Dufour number) and $Pr = \frac{\nu}{\kappa}$ (Prandtl number).

Operating equation 12 together with equation 11, it is rewrite as

\[
\frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 w' = f(z) \nabla^4 w' + 2 \frac{\partial f(z)}{\partial z} \nabla^2 \left( \frac{\partial w'}{\partial z} \right) + \frac{\partial^2 f(z)}{\partial z^2} \times (\nabla^2 w' - 2 \nabla^2 w')
\]

\[
+ Ra \nabla^2 T' + Rs Le \nabla^2 S'
\]
The equation of the normal form is given by
\[
(w', T', S') = [W(z), \Theta(z), \Upsilon(z)] e^{i[(ax + ay \beta_m)]} \tag{16}
\]
where this equation is being substituted into equations (13)-(15) to obtain the linearized form
\[
\mathcal{F}(D^2 - a^2)W + D^2 \mathcal{F}(D^2 - a^2)DW - a^2 Ra \Theta - \frac{1}{Le} a^2 Rs \Upsilon = 0 \tag{17}
\]
\[
W + (D^2 - a^2)\Theta + Df(D^2 - a^2)\Upsilon = 0 \tag{18}
\]
\[
W + Sr(D^2 - a^2)\Theta + Le(D^2 - a^2)\Upsilon = 0 \tag{19}
\]
\[a = \sqrt{a_x^2 + a_y^2} \] represents the wavenumber, \(D = \frac{d}{dz}\) represents the differential operator and \(\mathcal{F}(z) = \exp[B(z - \frac{1}{2}) + \frac{(T_0 - T_c)}{\beta_m}]\) where \(B = (\mu_t + \mu_s)\beta_m\) represents the dimensionless viscosity parameter.

For Rayleigh-Benard convection, the boundary conditions are as follows:
- Lower-upper boundaries are free-free
  \[W = D^2W = \Theta = \Upsilon = 0 \tag{20}\]
- Lower-upper boundaries are rigid-rigid
  \[W = DW = \Theta = \Upsilon = 0 \tag{21}\]
- Lower-upper boundaries are rigid-free
  \[W = DW = \Theta = \Upsilon = 0 \text{ at } z = 0 \tag{22}\]
  \[W = D^2W = \Theta = \Upsilon = 0 \text{ at } z = 1 \tag{23}\]

For Marangoni convection, the boundary conditions are as follows:
\[W = DW = \Theta = \Upsilon = 0 \text{ at } z = 1 \tag{24}\]
\[W = \mathcal{F}D^2W + Ma a^2 \Theta = D\Theta = \Upsilon = 0 \text{ at } z = 1 \tag{25}\]
where \(Ma\) is the Marangoni number. The method used to find an approximate solution is by Galerkin-type weighted residuals method where three variables, \(W_n, \Theta_n\) and \(\Upsilon_n\) are written in a series of trial function based on the boundary conditions. The Galerkin method were chose due to the computational efficiency and economy.

\[
W = \sum_{n=1}^{N} A_n W_n, \Theta = \sum_{n=1}^{N} B_n \Theta_n, \Upsilon = \sum_{n=1}^{N} C_n \Upsilon_n \tag{26}\]

The linearized equations 17 - 19 is multiply with the base functions 26 and later being integrate to obtain a 3 \times 3 linear algebraic equations in 3 unknowns \(A_n, B_n\) and \(C_n, p = 1, 2, 3, ..., N\) (natural number). The matrix determinant is calculated in order to obtain the eigenvalue, \(Ra\) or \(Ma\).

3. Results and discussion. Rayleigh-Benard convection (due to buoyancy) and Marangoni convection (due to surface tension) were studied in this research work. In the Rayleigh-Benard convection, the boundary conditions were set to be free-free, rigid-free and rigid-rigid representing the lower-upper boundaries. However, for Marangoni convection, only rigid-free condition is presented since rigid-free is the common slip condition considered in any convection. Further investigations need to be done for different slip condition in future. However, we expect the same results as Rayleigh-Benard convection.
3.1. **Rayleigh-Benard convection.** Temperature-dependent viscosity, $B$ effect is investigated numerically by using the Maple software. In each case, the minimum point of the marginal stability curves in the $(a, Ra)$ plane is identified. This minimum point act as the critical value of Rayleigh number represent by $Ra_c$ where this value represents the onset of convection. To validate the results, we neglect all the effects and compared the results with Nield and Kutzenotv [15] where we obtained the same critical values which are $Ra_c = 657.33$, $Ra_c = 1138.71$ and $Ra_c = 1750.15$ representing free-free, rigid-free and rigid-rigid boundaries respectively.

![Figure 1. Effect of $B$ to Rayleigh number](image1)

![Figure 2. Effect of $Sr$ to Rayleigh number](image2)

Figure 1 illustrate the effect of temperature-dependent viscosity, $B$ towards the Rayleigh number, $Ra$ with wave number, $a$ for values of $B = 1, 2$ and 3. It shows that...
marginal stability curves will shift downwards when the value of $B$ increases, which state that the temperature-dependent viscosity destabilized for all wavenumber, $a$. From figures 2 and 3, the trends of stability for two parameters which are the Soret, $Sr$ and Dufour number, $Df$ that exist in a double diffusive is investigated. The stability curves for the effects are shown in figure 2 and figure 3. As can be seen clearly in figure 2, $Ra$ decreases when $Sr$ increases and thus enhance the onset of convection due to the increase of temperature flux. Meanwhile, for Dufour parameter, it shows that an increase of Dufour parameter, $Df$ will increase the $Ra_c$ as illustrated in figure 3. Findings are similar with the previous results obtained in Abidin et al. [1] and in Hurle and Jakeman [10] where the coupled effect Dufour and Soret were considered in their research.
3.2. **Marangoni convection.** The results obtained in the Marangoni convection is to support the finding in Rayleigh-Benard convection. When a larger temperature-dependent viscosity, $B$ is tested, the Marangoni number, $Ma$ decreases as shown in figure 4. The trend of stability for the Soret effect, $Sr$ is also shown in the same figure. The variation of increasing the Soret effect decreases the Marangoni number. The effect of Dufour, $Df$ shows the opposite reaction where when $Df$ increases, $Ma$ also increases. This observation reveals that in Marangoni convection, $B$ and $Sr$ accelerate the arriving of convection and destabilize the system. Meanwhile, $Df$ delays the arriving of convection and stabilize the system.

3.3. **Lewis and Solutal Rayleigh Effect.** Lewis number, $Le$ and Solutal Rayleigh, $Sr$ effects exist due to the solutal balances in a binary fluid where $Le$ is the ratio between characteristic lengths for diffusion of heat and diffusion of mass and $Rs$ is the Rayleigh number representing the solutal balances. The effects to the system were shown in figure 5 and figure 6. Figure 5 represents the $Le$ effect on the critical Rayleigh number, $Ra_c$ against $B$. It is observed that $Le$ shows the same results as $B$ where an increase of the effects values make the system destabilized. The increasing values of $Rs$ and $B$ are shown together in figure 6. An increase of $Rs$ make the system more stable. From all the figures shown, the system is more stable when both boundaries are in a rigid slip condition as the value of $Ra_c$ is higher compared to the other two boundary conditions.

4. **Conclusion.** The temperature-dependent viscosity effect in a double diffusive binary is being examined in this research paper where results show that an increasing values of $B$, $Sr$ and $Le$ accelerates the onset of convection and $Rs$ and $Df$ will decelerates the onset of convection. In other words, $B$, $Sr$ and $Le$ destabilize the system and $Rs$ and $Df$ stabilize the system. Results obtained are similar to both Rayleigh-Benard and Marangoni convection. Results also show that rigid-rigid boundaries has higher stablity compared to rigid-free and free-free boundaries.

**Acknowledgments.** This research was supported by the Ministry of Higher Education Malaysia under FRGS (02-01-15-1703FR).
Figure 6. Effect of $B$ on $Ra_c$ for various values of $Rs$

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Received January 2018; 1st revision May 2018; final revision July 2018.

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