An analysis of the effects of territory properties on population behaviors and evacuation management during disasters using coupled dynamical systems

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Abstract
One of the major current challenges in the field of security and safety of populations is to advance further in the understanding and the ability to anticipate their behaviors when faced with threats or disasters. Several factors such as the hazard properties or the culture of risk can influence people behavior during a disaster. In this paper, we assume that the spatial configuration of the site where the disaster takes place has also a significant impact on collective behaviors. For this, we use a mathematical model based on meta-population networks, in order to design realistic evacuation scenarios. This model, called the Alert-Panic-Control model, allows, on the one hand, to take into account the temporal dynamics of collective behaviors in front of disaster and on the other hand, the spatial context considered in terms of site maximum capacity. From the results of a in situ experiment carried out in 2019 with the population of Le Havre (France) concerning an industrial accident hypothesis, and in particular from the different evacuation paths chosen by the respondents, three scenarios of evacuation of the place located below the street level and in front of the Niemeyer Cultural Centre were built. The results are able to quantify how the contagion of panic has a major impact or not on an evacuation forced by specific territorial properties. Too narrow paths can cause panic phenomena due to bottleneck. They also highlight that the function of the refuge places, recreational function in this case, must be taken into account insofar as they can gather many people, impeding the evacuation of the population exposed to the danger.

Keywords: Coupled dynamical systems, Human behaviors, Evacuation, Territorial risk management

Introduction
Risk and disaster prevention has become a major societal issue over the past years. During the last decades, the number of disasters has increased and major crises have revealed the complexity of these events. Besides being a physical, natural or technological event, a catastrophe also has human, economic, financial, social, environmental and cultural effects. Modern societies, whatever their level of development, are still
unprepared to face the complexity of disasters and in general the population does not know how to behave in such situations. This partial ignorance of human behavior in front of catastrophic events is not only peculiar to population and/or decision-makers. It also depends on the difficulties of researchers in the understanding the range of behaviors adopted during a disaster (Crocq 2013), their sequence, dynamics and interdependence (Provitolo et al. 2015).

Research in geography shows that the influence of territorial and spatial contexts must be taken into account in order to better understand the collective behavioral dynamics during a catastrophic event (Dubos-Paillard et al. 2021; Provitolo et al. 2020). Geographers consider that space is not a neutral support. Produced by societies, it is heterogeneous and anisotropic. As a result, it constrains the adopted behaviors, if only by the topography and the organisation of the buildings and the infrastructures in an urban area. In urban areas where the spatial extent of the disaster is large, the variety of observed behaviors may be wide. This variety also depends on the location, the intensity of the event, the social group, etc (Provitolo et al. 2015; Fischer 1998). Thus, for a same kind of disaster people's responses can be different. Moreover, during a disaster, individuals scarcely adopt the same behavior during the whole event. Indeed, we often observe a sequence of several behaviors.

Nevertheless, researchers have few empirical information about the real evolution of population response, because of the difficulties to collect real-time observation and to analyse human reactions during disasters. Therefore, the main sources of information about the behaviors adopted during a disaster are interviews and surveys carried out with operational actors, residents and victims after (or before) a disaster and in a specific territory.

In order to advance further in this research field, an interdisciplinary approach has been undertaken under the Com2SiCa research program. It combines an innovative experiment methodology with a mathematical modeling based on the APC (Alert-Panic-Control) model applied to a network (Lanza et al. 2021).

The experiment consisted in the implementation of an in situ simulation based on a sound immersion in a real place where the occurrence of a disaster was plausible (Lago et al. 2022). During the sound immersion, interviewees were asked to react to what they heard in the soundtrack. This permitted to observe and record real-time participants’ behaviors, to monitor the stress level before and after the sound immersion, and to get information about the escape routes chosen during the experience.

In literature, a mathematical model describing the temporal dynamics of collective behaviors when faced with a disaster has been proposed (Verdière et al. 2014; Cantin et al. 2016). In order to consider the influence of the territorial properties, in previous works we modeled the geographical area under study as a network (Lanza et al. 2021; Cantin 2017). The territory is therefore subdivided in different areas that correspond to the nodes of the network. By exploiting a meta-population approach, each node hosts a sub-population whose dynamics is governed by a APC model. Moreover, this sub-population can move to the adjacent nodes of the network and influence their dynamics.

The aim of this work is to exploit the information collected during the surveys as an input to the mathematical model. Our objective is not to identify parameters from data in order to reproduce the evolution of a complex situation subject to many hazards. Our
purpose here is to exploit the information on the trajectories adopted in the stressful survey situation to design several realistic evacuation networks and scenarios. In particular, we are interested to investigate to what extent the spatial context and the urban functions influence the individual and collective behavioral dynamics. We show that our approach and our mathematical model permit to evaluate how behavioral dynamics impact evacuation paths in terms of hazard potential. This paper is structured as follows. First, the protocol of the in situ experiments and their results are briefly presented. Then, the Alert-Panic-Control Behavior (APC) mathematical model and its implementation on a network are explained. Finally numerical simulations of realistic evacuation scenarios designed by exploiting the data from the surveys are shown and discussed. We show that the maximal number of individuals that a place can host and the presence of people in a shelter at the onset of the event have an influence on the behavioral dynamics and can lead to a crisis in the crisis phenomena.

**Field surveys to identify behavioral responses and escape trajectories in stressful situations**

**Surveys protocol**

In 2018 and 2019, two in situ experiments (Lago et al. 2022) have been conducted: the first one on the seafront of Nice (France) where a tsunami hazard scenario has been investigated, and the second one in Le Havre (France) where the occurrence of a major technological catastrophic event has been considered. The surveys carried out aimed to immerse the participants in two distinct disaster scenarios, one of natural origin, the other of technological one. Their common factor is to simulate a sudden, rapid occurrence with little or no warning signs of a major event.

In this paper we mainly focus on the survey in Le Havre. The in situ experiment in Le Havre took place in two different part of the city: at the seaside and in the city center, near the Niemeyer Cultural Centre. This place is of particular importance since it is a semi-closed esplanade with buildings open to the public (a city library, Le Volcan theater and a restaurant). Moreover, it is below the street level and surrounded by residential buildings, see Fig. 1. Three different exits can be taken in order to go up to the street level: via a narrow spiral footbridge it is possible to arrive to a small terrace with some restaurants and a porch (denoted by B in Fig. 1), while a larger staircase and an access ramp lead to Louis Brindeau Street (denoted by C in Fig. 1) and Place Perret square. Due to its location in the basement, behavioral reactions take place without being aware of the events that could occur in the city. The difficulty of identifying the origin of a loud blast is likely to increase the stress because the interpretations can be different from one person to another (terrorist attacks, industrial explosion, collapse of a building, etc). When people cannot clearly identify the source of the danger, imitation processes are often important and can lead to collective panic (Helbing et al. 2002; Moussaid 2010).

The purpose of these experiments was to analyze people reactions facing a simulated danger, and to observe and record the different behavioral sequences of each interviewee immersed into a scenario of a sudden, unforeseen (without pre-warning signs) disaster. The investigation seeks to provide the keys to understanding the
questionings, the reasonings and the behaviors that one might expect in such situations. Between in situ simulation and interview, the investigation was structured in three steps (see Fig. 2):

- visual immersion
- sound immersion
- debriefing of the experience.

In particular, the second step consisted in a role-play scenario, where the interviewee was asked to listen to an audio track and react in the more naturally way. The sound
immersion aims to project the interviewee into a dynamic and fictitious (but credible) situation of a sudden and unforeseen disaster. During this immersion, the person, depending on his/her emotional response, which may be more or less strong, reacts to the situation by adopting different behaviors. During this phase, no specific instructions were given to the volunteer except to listen carefully to the audio track. He/she reacts as he/she wishes, the researcher does not ask questions or suggest to adopt a particular behavior.

Categorization of human behaviors during a catastrophic event

The analysis of data from individual interviews with residents has enabled us to identify nearly twenty reactions (panic or reflective flight, mutual aid, seeking shelter, or simply moving away from the danger zone, to give just a few examples) (Dubos-Paillard et al. 2021; Tricot et al. 2021). As this diversity of behavioral reactions cannot be integrated into mathematical models, we have found a consensus between thematichians and modelers while keeping the detail of the information obtained from the surveys. We have thus categorized this behavioral diversity on the basis of two criteria used in emotional psychology to qualify reactions: the variables of emotional load and emotional regulation. The first takes into account the level of stress and nervousness, while the second refers to the ability to control this excess of emotions (Russell 1980). These two key variables allowed us to categorize the diversity of behavioral responses observed during the survey into three meta-behaviors: alert, controlled and panic behaviors, presented below.

- Alert behaviors (A) are micro-behaviors that can be observed from a motor point of view (e.g., a start or a rapid eye movement to scan what is happening in the near visual environment). This behavior marks a transition from the behavior that was appropriate for the current activity (e.g., jogging or driving to work). In an alert state, the person has an emotional charge which is weak, due to the state of uncertainty of the situation. This means that there has not yet been a significant rise in stress.

- Control behaviors (C) have the common feature of being reflective behaviors where the emotional load, more or less strong, is regulated. By regulating their emotions, the person seeks to adapt their reactions to the context of the disaster. These reactions can be very varied. It is possible to observe behaviors such as controlled flight, taking shelter, and mutual aid, as well as less virtuous behaviors, such as theft or looting. In addition, it should be noted that not everyone behaves in the same way when faced with an identical situation. During an explosion, for example, people may seek shelter in the nearest building, others may instead prefer to flee and return home.

- Panic behaviors (P) are uncontrolled behaviors where fear-related emotions have taken over. Panic behavior involves a strong emotional charge and weak regulation, ineffective in regaining a controlled state. Different behaviors can signify a state of panic: panic flight or, on the contrary, stupor (Crocq 2013). The first is characterized by movement while the second, in contrast, is marked by immobility.
Trajectories adopted in stressful situations
Among the control and panic behaviors, some produce movement while others are marked by immobility. Flight (reasoned or under the influence of panic), evacuation and taking shelter behaviors observed during the survey campaign carried out in Le Havre, are associated with movement trajectories. In the Niemeyer Cultural Centre area in Le Havre, 13 respondents out of 16 adopted movement trajectories, i.e., 81% of respondents. One of the objectives of the surveys carried out was precisely to capture these behaviors and the specific trajectories taken by the respondents. By immersing people in a simulated disaster context, filming their reactions and equipping them with a smartwatch, we were able to map all of the paths taken to leave the danger zone (Ranarimahefa 2020). The extracted information from the interviews has been gathered and treated so as to render a geographical visualization of the different trajectories chosen by people. This map (Fig. 3) shows two different rationales for movement: some of the respondents tried to leave the danger zone (identified by them as being the basement area of the Niemeyer Cultural Centre) so as not to be trapped, while the others instead stayed in this zone in order to reach another building in a few seconds, in this case a library, in order to take shelter there. Among the people leaving the area, we can identify three major trajectories: some took the wide stairs or the narrower footbridge to leave the premises on foot, others preferred to go via the access ramp, especially those on a bicycle.

These collected, spatialized data allow us to discretize the space under study, to build the corresponding network and to feed the mathematical model presented below, in order to carry out realistic simulations and anticipate the spatio-temporal behavioral dynamics.

Mathematical model
The APC model
The Alert-Panic-Control (APC) model, proposed for the first time in Lanza et al. (2021), as the previous model developed in Verdière et al. (2014), Provitolo et al. (2015), Cantin et al. (2016), is a nonlinear ODE compartmental model inspired by the classical epidemic compartmental models such as the SIR (Susceptible-Infected-Recovered) one (Hethcote 2000). It is based on the behavioral categorization proposed in the previous section and people is assumed to evolve among five main categories of behaviors during a catastrophe: the daily behaviors before the catastrophic event, the states of alert, panic and control, the behaviors of everyday life after the disaster. They can also die or be severely injured (victims). We consider two types of transitions from one behavior to another: the ones due to intrinsic motivations, that are peculiar to each individual and depend on the past, the individual characteristics, etc, or the ones due to imitation processes. Indeed, human behavior is often ruled by imitation and social comparison with others (Drury et al. 2009).

We briefly present the variables and the equations of the model, firstly introduced in Lanza et al. (2021). We note $t_0$ the initial moment of the catastrophic event and for $t \geq t_0$ and:
• $a(t)$ the number of individuals in a state of alert,
• $p(t)$ the number of individuals in a state of panic,
• $c(t)$ the number of individuals in a state of control,
• $q(t)$ the number of individuals in the everyday behaviors,
• $b(t)$ all the people in a behavior of everyday life after the disaster,
• $v(t)$ the number of individuals who lose their lives during the disaster.
The APC model equations are the following:

\[
\begin{align*}
\frac{da}{dt} &= \gamma(t)q - (B_1 + B_2 + D_a)a - F(a,c)\frac{ac}{N} - G(a,p)\frac{ap}{N} + B_3c + B_4p, \\
\frac{dp}{dt} &= B_2a + C_2c - (B_4 + C_1 + D_p)p + G(a,p)\frac{ap}{N} - H(c,p)\frac{cp}{N}, \\
\frac{dc}{dt} &= B_1a + C_1p - (B_3 + C_2 + D_c)c + F(a,c)\frac{ac}{N} + H(c,p)\frac{cp}{N} - \varphi(t)c, \\
\frac{dq}{dt} &= -\gamma(t)q, \\
\frac{db}{dt} &= \varphi(t)c, \\
\frac{dv}{dt} &= D_aa + D_cc + D_pp.
\end{align*}
\]  

(1)

According to the flow diagram of Fig. 4, we assumed that, before the event, all the people adopt an everyday behavior \( q \). The occurrence of a sudden catastrophic event is modeled by function \( \gamma \). Once the event is triggered, people firstly pass through a state of alert, hence the term \( \gamma(t)q \) in the equation for \( a \) in system (1). For an unexpected and without warning signs event, function \( \gamma \) is defined as

\[
\gamma(t) = \zeta(t, \tau_0, \tau_1) = \begin{cases} 
0 & \text{si } t < \tau_0 \\
\frac{1}{2} - \frac{1}{2}\cos\left(\frac{t - \tau_0}{\tau_1 - \tau_0}\pi\right) & \text{si } \tau_0 \leq t \leq \tau_1 \\
1 & \text{si } t > \tau_1
\end{cases}
\]  

(2)

At \( t = \tau_0 \) the event takes place and at \( t = \tau_1 \) the majority of the population in the daily behavior becomes alerted.

Then, alerted people adopt a panic or a control behavior, according to an intrinsic behavioral transition or an imitation process. All the linear terms in system (1) represent intrinsic transitions, while the nonlinear functions \( F, G \) and \( H \) model the transitions due to imitation. In the following, we call them imitation functions. Function \( F \) represents the imitation of the alert people toward the control one, and is defined as:
\[
F(a, c) \frac{ac}{N} = a_1 \xi \left( \frac{c}{a + \varepsilon} \right) \cdot \frac{a}{N} \cdot c, \tag{3}
\]

where \( \xi(s) = \frac{s^2}{1 + s^2} \), \( N = N(t) = q(t) + a(t) + p(t) + c(t) + b(t) \), and \( \varepsilon << 1 \).

Since we are interested in a significant population size, a classical proportional incidence rate is considered (Arino and Van den Driessche 2006; Blackwood and Childs 2018). Function \( \xi \), represented in Fig. 5, takes into account the assumption that the minority tends to adopt by imitation the behavior of the majority.

In the same way we can define the two other imitation functions:

\[
\begin{cases}
G(a, p) = \beta \xi \left( \frac{p}{a + \varepsilon} \right) \\
2em \quad H(c, p) = \gamma_{p \to c} \xi \left( \frac{c}{p + \varepsilon} \right) - \gamma_{c \to p} \xi \left( \frac{p}{c + \varepsilon} \right).
\end{cases}
\tag{4}
\]

Finally, we supposed that everyday behaviors can be adopted once again after a certain time and only starting from a controlled behavior. This transition is taken into account by the term \(-\varphi(t)c\) in the second equation of system (1). Function \( \varphi \), as function \( \gamma \), has to be chosen according to the nature of the disaster under study. In analogy with function \( \gamma \), it is defined as:

\[
\varphi(t) = \xi(t, \tau_2, \tau_3) \begin{cases}
0 & \text{si } t < \tau_2 \\
\frac{1}{2} - \frac{1}{2} \cos \left( \frac{t - \tau_2}{\tau_3 - \tau_2} \pi \right) & \text{si } \tau_2 \leq t \leq \tau_3 \\
1 & \text{si } t > \tau_3
\end{cases}
\tag{5}
\]

The pseudo-daily behavior cannot be adopted before \( t = \tau_2 \) and since \( t = \tau_3 \) this transition is at its maximum.

It is worth remarking that, by adding up all the equations, we obtain

\[
\frac{da}{dt} + \frac{dc}{dt} + \frac{dp}{dt} + \frac{dq}{dt} + \frac{db}{dt} + \frac{dv}{dt} = 0,
\]

thus the population is constant in time.

The variables, the functions and the parameters of the APC model are summarized in Tables 1, 2 and 3, respectively.
The APC model on networks

The APC model presented before takes into account only the temporal dynamics of the different behavioral categories. However, the territory and its properties play an important role on the dynamics of the human behaviors. To include this essential aspect, we exploit a mixed approach, based on complex networks and nonlinear differential equations. First of all, we show how the spatial environment is modeled by a complex network adapted to the terrain under study. Then the equations of the APC model on network are presented.

Modeling a territory with networks

Our purpose is to study the spatio-temporal dynamics of the different behaviors during a catastrophic event in a geographical area. In order to exploit a complex network
approach, the first step is to divide the territory under study in different sub-areas, that will be the nodes of our network. An example of a subdivision of the urban area around the Niemeyer Cultural Centre can be found in Fig. 6. The nodes represent a region endowed with some specific properties (for instance a surface) and containing populations subjected to the panel of behaviors of the APC model (Lanza et al. 2021).

![Example of a subdivision of the geographical area around the Niemeyer Cultural Centre in Le Havre (France). The information on the escape routes chosen by the interviewees (Fig. 3) have guided the selection of the sub-regions. In particular, five sub-areas, being the nodes of our network, have been identified.](image_url)
Here physical displacements of people are considered. Therefore, one directed edge between two nodes models the fact that people can move from one node to the other. It is worth remarking that the close tie between the network structure and the geographical characteristics of the territory under study makes our APC model on network very different to the meta-population epidemic models in literature (Arino 2009).

In particular, our network has some properties that directly stem from the fact that our nodes represent geographical sub-areas:

- each node $i$ has a surface that we denote $S_i$;
- we suppose that each node $i$ has a maximum capacity, denoted by $N_i^{\text{max}}$, i.e. a maximum number of individuals that the node can host. We assume that $\rho = 3$ people per $m^2$ is the density beyond which motion begins to stop flowing and population slows down due to its mass (Hermant 2012). Thus, the maximal capacity of node $i$ is calculated as
  $$N_i^{\text{max}} = \rho S_i \approx 3 S_i.$$
- Each directed edge has a weight, that is the coupling coefficient. It takes into account the fact that an edge represents people displacements between nodes. It depends on the properties of the node and on the population that is located within, according to the following formula:
  $$\eta_{jk}^j = \frac{L_{ik} \langle v_j^j \rangle}{S_i}, \quad 1 \leq i, k \leq n, i \neq k, \quad j \in \{a, p, c\}$$
  where:
  - $L_{ik}$ is the width of the exit from node $i$ to node $k$
  - $\langle v_j^j \rangle$ is the average speed of the population $j$ leaving node $i$ (it is noteworthy that this quantity is computed here only with the horizontal components of speed vectors)
  - $S_i$ is the global surface of node $i$ that people leave.

Figure 7 shows an example of two nodes. the esplanade between the two volcano buildings who has a surface of $S_1 \approx 1500 \, m^2$, and the large staircase that leads to the Louis Brindeau Street, whose surface is about $S_2 = 300 \, m^2$. People can move from the first node to the second one through a passage whose width is $L_{12} = 30 \, m^2$. People in each category of behavior have a different average speed. We suppose that people in the alert behavior barely move, since people in an alert state are usually in search of information. Moreover, we suppose that the speed of individuals in panic is less than the one of individuals in a control behavior. This choice has two reasons: firstly among the people in a panic behavior some of them could have a freeze response, and secondly others do not always take the right route to escape. Therefore, for all $1 \leq i, k \leq n, i \neq k$ we have
  $$\eta_{ik}^a \ll \eta_{ik}^p < \eta_{ik}^c$$
The mathematical equations

Here are our main assumptions for the APC model on network:

A1: the network consists of \( n \) non-identical nodes, that is the parameters of the APC model on each node can be different,

A2: only linear couplings are considered, \( i.e. \) only physical displacements are taken into account. In addition, no behavioral transitions are allowed during the displacement,

A3: the network population is constant and equal to \( N \). No population can enter or leave the network. On the other hand, the population on each node \( N_i \) depends on time, since individuals move from one node to another,

A4: each node \( k \) has a maximum capacity, \( i.e. \) a maximum number of individuals \( N_k^{\text{max}} \) that the node can receive,

A5: the transitions towards panic can also depend on the density on each node. Indeed, we assume that, the more population is in the node, the more it has a tendency to lose control,

A6: the individuals speed depends on the density in the node. The greater is the density, the lower is the speed,

A7: the victims and the population in a daily behavior do not move from one node to another one,

A8: depending on the disaster, we can have nodes that are not directly impacted by the hazard and therefore where the population continue to show a daily behavior
even after the occurrence of the disaster. In this case, only the arrival of population from the other nodes triggers the APC model on those nodes.

A9: population can go back to behaviors close to everyday life only on previously defined refuge nodes. The presence of people who exhibit again a pseudo-daily behavior can help individuals in a panic state to become controlled.

In the following, let us denote by

- \( N_i = N_i(t) = q_i(t) + a_i(t) + p_i(t) + c_i(t) + b_i(t) \) the number of individuals in the \( i \)-th node at time \( t \).
- \( N^{\text{out}}(i) \) the out-neighbors set of node \( i \) (that is the nodes that are adjacent to node \( i \) and whose edge starts from \( i \)) (West 1996),
- \( N^{\text{in}}(i) \) the in-neighbors set of node \( i \) (that is the nodes that are adjacent to node \( i \) and whose edge comes into \( i \))

In Fig. 8, an example of network where the out-neighbors and the in-neighbors of a node are highlighted is presented.

Thus, the equation for the individuals in the alert behavior of node \( i (i = 1, \ldots, n) \) reads as:

\[
\frac{da_i}{dt} = -\left( b_1' + b_2' + D_a' \right) a_i - F^a(a_i, c_i) \frac{a_i c_i}{N_i} - G(a_i, p_i) \frac{a_i p_i}{N_i} + b_3 c_i + b_4 p_i \\
+ \sum_{k \in N^{\text{in}}(i)} \left( 1 - \frac{N_i}{N_i^{\text{max}}} \right) \theta_k^a a_k - \sum_{k \in N^{\text{out}}(i)} \left( 1 - \frac{N_k}{N_k^{\text{max}}} \right) \theta_k^a a_i \\
+ \delta' \gamma'(t) q_i + (1 - \delta') \left( \sum_{k \in N^{\text{in}}(i)} \left( 1 - \frac{N_i}{N_i^{\text{max}}} \right) \left( \theta_k^a a_k + \theta_k^c c_k + \theta_k^p p_k \right) \right) \frac{q_i}{N_i} \\
+ (a_i + c_i + p_i) \frac{q_i}{N_i}. \tag{7}
\]

The terms in the first line of Eq. (7) represent the intrinsic and imitation transitions, that are already present in the simple APC model (1). Moreover, we have the coupling terms that model the fact that individuals in alert behavior can move from one node to another. In particular,
represents the out-coming flux of node $i$. The term $1 - \frac{N_k}{N_{\text{max}}^k}$ in Eq. (8) takes into account that population in node $i$ can move to node $k$ only if node $k$ has not reached its maximal capacity (assumption A4). Furthermore, we assume that the average speed is an affine function of the density, that is

$$\theta_{ik}^a = \eta_{ik}^a \left( w_i + 1 - \frac{N_i}{N_{\text{max}}^i} \right) = \frac{L_{ik}}{S_i} \left( \nu_i^a \right) \left( w_i + 1 - \frac{N_i}{N_{\text{max}}^i} \right),$$

with $w_i \in [0, 1]$. Indeed, the speed of the alert population $(\nu_i^a) \left( w_i + 1 - \frac{N_i}{N_{\text{max}}^i} \right)$ is supposed to decrease as the density increases (assumption A6). Moreover, we suppose that, when the $i$-th node is completely full, that is when $N_i = N_{\text{max}}^i$, people inside the node move with a speed equal to $(\nu_i^a)w_i$.

In the same way,

$$\sum_{k \in N_{\text{out}}(i)} \left( 1 - \frac{N_i}{N_{\text{max}}^i} \right) \theta_{ki}^a a_k$$

models the incoming flux and the fact that node $i$ can take in new individuals only if it has not yet reached its maximal capacity (assumption A4). The terms

$$\delta^i \gamma^i(t)q_i + (1 - \delta^i) \left( \sum_{k \in N_{\text{in}}(i)} 1 - \frac{N_i}{N_{\text{max}}^i} \right) \left( \theta_{ki}^a d_k + \theta_{ki}^c c_k + \theta_{ki}^p p_k \right) \frac{q_i}{N_i}$$

model assumption A8. The dynamics on each node can be triggered both by the hazard and by all the population arriving from the adjacent nodes, represented by

$$\sum_{k \in N_{\text{in}}(i)} \left( 1 - \frac{N_i}{N_{\text{max}}^i} \right) \left( \theta_{ki}^a d_k + \theta_{ki}^c c_k + \theta_{ki}^p p_k \right),$$

since only people in alert, panic and control behaviors can move. Parameter $\delta^i \in [0, 1]$ is a sort of weight that takes into account which one of these two sources is the one that contributes the most to the onset of the APC dynamics on the node. Moreover, the term

$$(a_i + c_i + p_i) \frac{q_i}{N_i}$$

takes into account that people in a daily behavior interact with the individuals in the alert, panic or control behavior that are already present in the node, and become alerted.

The equations for the individuals in panic and control behaviors have a similar structure, that is the terms specific to the transitions already present in the APC model (1) plus the coupling terms:
where in the refuge nodes people in panic behavior interact also with the individuals that have gone back to a pseudo-normal behavior and become controlled (assumption A9). In particular, in order to model the assumption A5, that is the fact that the increase of the density in one node amplifies the panic behaviors, the intrinsic transition from control to panic is modified with respect to the same term in the a-spatial APC model (1), as follows:

\[
\frac{dp_i}{dt} = B^i_d a_i + \tilde{C}^i_2 \left(1 + \frac{N_i}{N_{i_{\text{max}}}}\right) c_i - \left(B^i_k + C^i_1 + D^i_p\right)p_i + G^i(a_i, p_i) \frac{a_i p_i}{N_i} - H^i(c_i + b_i, p_i) \frac{(c_i + b_i)p_i}{N_i} + \sum_{k \in N^{in}(i)} \left(1 - \frac{N_k}{N_{k_{\text{max}}}}\right) \theta^p_{ik} p_k - \sum_{k \in N^{out}(i)} \left(1 - \frac{N_k}{N_{k_{\text{max}}}}\right) \theta^p_{ik} p_k,
\]

\[
\frac{dc_i}{dt} = B^i_1 a_i + C^i_1 p_i - \left(B^i_3 + \tilde{C}^i_2 \left(1 + \frac{N_i}{N_{i_{\text{max}}}}\right) \right) c_i - D^i_c c_i + F^i(a_i, c_i) \frac{a_i c_i}{N_i} - F^i(a_i, c_i) \frac{a_i c_i}{N_i} + \sum_{k \in N^{in}(i)} \left(1 - \frac{N_k}{N_{k_{\text{max}}}}\right) \theta^c_{ik} c_k - \sum_{k \in N^{out}(i)} \left(1 - \frac{N_k}{N_{k_{\text{max}}}}\right) \theta^c_{ik} c_k,
\]

where

\[
H^i(c_i + b_i, p_i) = \gamma^i_{p \rightarrow c} \xi \left(\frac{c_i + b_i}{p_i + \epsilon}\right) - \gamma^i_{c \rightarrow p} \xi \left(\frac{p_i}{c_i + b_i + \epsilon}\right)
\]

since in the refuge nodes people in panic behavior interact also with the individuals that have gone back to a pseudo-normal behavior and become controlled (assumption A9).

In particular, in order to model the assumption A5, that is the fact that the increase of the density in one node amplifies the panic behaviors, the intrinsic transition from control to panic is modified with respect to the same term in the a-spatial APC model (1), as follows:

\[
-\tilde{C}^i_2 \left(1 + \frac{N_i}{N_{i_{\text{max}}}}\right) c
\]

where \(\tilde{C}^i_2 = \frac{C^i_2}{2}\), and \(C^i_2\) is the parameter associated to the intrinsic transition from control to panic in the a-spatial APC model. This definition permits to link the parameter of the a-spatial APC model (1), where the notion of density is not present, and the one of the APC model on network. Indeed, \(C^i_2\) can be interpreted as the maximum value that we can consider for this transition and we reach it when the node is saturated (that is when \(N_i = N_{i_{\text{max}}})\).

Writing down in the same way the equations for people in a daily behavior, for the individuals that in the shelters go back to a pseudo-daily behaviors and for victims, the APC model on a network of \(n\) nodes consists in the following \(6n\) equations (\(i = 1, \ldots, n\):
\[
\frac{da_i}{dt} = -\left(B_i^t + B_i^e + D_i^\theta\right)a_i - F^i(a_i, c_i)\frac{a_i c_i}{N_i} - G(a_i, p_i)\frac{a_i p_i}{N_i} + B_i^c c_i + B_i^p p_i + \delta^i \gamma^i(t) q_i \\
\quad + (1 - \delta^i) \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_i}{N_i^{\max}}\right) \theta^p_k a_k + \theta^c_k c_k + \theta^p_k p_k \frac{q_i}{N_i} \\
\quad + (a_i + c_i + p_i) \frac{q_i}{N_i} + \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_i}{N_i^{\max}}\right) \theta^p_k a_k - \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_k}{N_k^{\max}}\right) \theta^p_k a_k,
\]

\[
\frac{dp_i}{dt} = B_i^e a_i + \tilde{C}_2^i \left(1 + \frac{N_i}{N_i^{\max}}\right)c_i - \left(B_i^t + C_i^t + D_i^\theta\right)p_i + G^i(a_i, p_i)\frac{a_i p_i}{N_i} \\
\quad - H^i(c_i + b_i, p_i)\frac{(c_i + b_i)p_i}{N_i} + \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_i}{N_i^{\max}}\right) \theta^p_k p_k \\
\quad - \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_k}{N_k^{\max}}\right) \theta^p_k c_k,
\]

\[
\frac{dc_i}{dt} = B_i^e a_i + \tilde{C}_2^i\left(1 + \frac{N_i}{N_i^{\max}}\right)c_i - \left(B_i^t + C_i^t + D_i^\theta\right)c_i + F^i(a_i, c_i)\frac{a_i c_i}{N_i} \\
\quad + H^i(c_i + b_i, p_i)\frac{(c_i + b_i)p_i}{N_i} - \gamma^i(t)c_i + \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_i}{N_i^{\max}}\right) \theta^p_k c_k \\
\quad - \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_k}{N_k^{\max}}\right) \theta^p_k c_k,
\]

\[
\frac{dq_i}{dt} = -\delta^i \gamma^i(t) q_i - (1 - \delta^i) \sum_{k \in \mathcal{N}^{out}(i)} \left(1 - \frac{N_i}{N_i^{\max}}\right) \theta^p_k a_k + \theta^c_k c_k + \theta^p_k p_k \frac{q_i}{N_i} \\
\quad - (a_i + c_i + p_i) \frac{q_i}{N_i},
\]

\[
\frac{d\xi_i}{dt} = \psi^i(t)c_i,
\]

\[
\frac{dv_i}{dt} = D_i^e a_i + D_i^c c_i + D^p_i p_i,
\]

with

\[
\begin{align*}
F^i(a_i, c_i) &= \alpha^i \xi \left(\frac{c_i}{a_i + \varepsilon}\right) \\
[2em] G^i(a_i, p_i) &= \beta^i \xi \left(\frac{p_i}{a_i + \varepsilon}\right) \\
[2em] H^i(c_i + b_i, p_i) &= \gamma^i \xi \left(\frac{c_i + b_i}{p_i + \varepsilon}\right) - \gamma^i_{p \to c} \xi \left(\frac{p_i}{c_i + b_i + \varepsilon}\right).
\end{align*}
\]

(11)

It is worth noting that the sum of all the equations of system (10) is equal to zero. This means that the total population in the network is constant, as stated in A3.

As in the simple APC model (1), we suppose that at \(t = t_0\) all the people is in a daily behavior. It means that we consider the following initial condition \((i = 1, \ldots, n)\):

\[
a_i(t_0) = p_i(t_0) = c_i(t_0) = b_i(t_0) = v_i(t_0) = 0, \quad q_i(t_0) = q_i^0,
\]
with $\sum_{i=1}^{n} q_i^0 = N$.

Assumption A9 is taken into account by considering $\varphi_i = 0$ for all the nodes that are not sufficiently far from the impact zone of the disaster and are not labeled as refuge nodes. Therefore, in these nodes, since $b_i(0) = 0$, we have that $b_i(t) = 0$ for all $t \geq t_0$, that is no one adopts again a pseudo-daily behavior after the beginning of the APC dynamics within the node.

### Numerical simulations

#### Description of the scenarios

The extracted information from the interviews fed a set of scenarios based on a population of about 1500 individuals. Le Volcan theater is designed to host up to 800 members in the audience, without counting the employees. The doors of the theater and the library has just closed as well as the restaurant which in the basement. That is why we consider a total of 1500 people in the esplanade at the beginning of the simulations.

We assume then that sudden several violent explosions resound. Due to their location in the basement, people cannot identify the origin of the blast (industrial accident, collapse of a building or terrorist attacks). This situation leads to high levels of stress among the population. The objective of the population is then to leave the esplanade whose spatial configuration is relatively closed and rapidly considered as a potential source of danger. According to our in situ survey, two paths are preferred by respondents, as shown in Fig. 3.

The first one consists of taking the spiral footbridge, whose width is about 2 meters. Even if people are aware that this escape path is narrow, it is the closest to the esplanade and gives access to a relatively small terrace (see Fig. 1). According to the survey results, this alternative was chosen by around 20% of the interviewees (two people over eleven have chosen this path), which represents 300 persons that we have located in an area of $300 \text{ m}^2$ next to the spiral footbridge. These elements enable us to design the first scenario of the esplanade evacuation.

In a second scenario, which is an alternative to the first one, many people are located on the patios of the bars and restaurants located on the terrace. This situation differs from the first scenario where there were a very small number of individuals on the terrace. Our objective is to analyze if the presence of many people could have a significant impact on the evacuation process.

Finally, the third scenario relies on the second path of Fig. 6, which is longer, but whose staircase is wider for leaving the basement (30 meters). It gives access to Louis Brindeau Street and to the large Place Perret square (see the green area on the top of Fig. 6) that offers a broad perspective on the city and enables to more easily identify the origin of the explosions. We suppose that the remaining 80% of people, i.e. 1200 individuals located on an area of $1200 \text{ m}^2$, have chosen this alternative.

In all cases, we considered that the access to the terrace or the square has a reassuring effect on the population (satisfaction at having left the basement, better visibility and contact with people able to give information about the situation) while a feeling of stress prevails in the other areas (basement, footbridge and staircase). Therefore, the terrace and Place Perret square are considered as refuge areas.
Design of the network

The novelty of our approach is that the choice of the topology of the network takes into account the real configuration of the place where the surveys were carried out. Moreover, some of the involved geographical areas exhibit limited capacities to host people, therefore some nodes of the network under study—including the refuge nodes—may have a small maximum capacity $N_k^{\text{max}}$.

The main trajectories chosen by people for evacuating the esplanade have guided us to design the networks under study in the different scenarios. In both cases (the two main trajectories), a network of three nodes has been designed:

- the first node represents the esplanade in front of Le Volcan theater and the city library. It is the same for both trajectories. Indeed, this location is relatively large with a high maximal capacity. This capacity has been estimated considering the available surface of the area and an averaged maximal density of people. As mentioned before, the whole initial population can be divided into two groups according to their escaping path choices. One group accounts of 300 people in the first two scenarios occupying a surface of 300 m², that corresponds to a maximal capacity of 900 people, when considering an average maximal density of 3 people per m². In the third scenario, we consider an initial population of 1200 people occupying 1200 m², that gives a maximal capacity of 3600 people.

- The second node corresponds either to the narrow spiral footbridge in path 1, or to the large staircase in path 2 (see Figs. 1 and 6). The two maximal capacities are very different in these two cases: in the spiral footbridge, which enables to reach the ground level, the surface of the area has been estimated approximately to 50 m², therefore it can hold $N_k^{\text{max}} = 150$ persons maximum if we consider $3 \text{ p/m}^2$ as the average density in an evacuation process. In the second path, the staircase is very large with a surface evaluated to 300 m², so we obtain a maximal capacity of $N_k^{\text{max}} = 900$ people at the same time.

- Finally the third node can be considered in each case as a refuge: in the path 1, the refuge is on the road level and is a relatively small terrace with many restaurants and bars. This is the reason why we consider two scenarios with different initial conditions on this node: one with a few people already on the place (5 people) and another one with a crowdy place (295 people). Moreover, the place is full of obstacles (plant pots, seats,...), this is why we may consider this refuge not so large in terms of carrying capacity and we have chosen a maximal capacity of 500 people maximum in this zone/node. For the path 2 towards Louis Brindeau Street, the situation is more convenient because this path ends on a big square whose surface is around $50 \text{ m} \times 50 \text{ m} = 2500 \text{ m}^2$. Thus, this square may carry up to 7500 people.

Finally, for the first path we have $L_{1,2} = L_{2,3} = 2 \text{ m}$, since the footbridge width is about 2 m wide. For the second one, the staircase is 30 m wide, so we take $L_{1,2} = L_{2,3} = 30 \text{ m}$.

Several authors assume that the free flow walking speed on a flat surface is on average about 1.3 m/s (Hermant 2012; Fruin 1971; Muccini et al. 2019). This velocity evolves according to several factors as the social composition of the crowd, the culture, the density or the presence of stairs. Indeed, the speed of individuals decreases as the age or the
density of the crowd increases, or when people have to go upstairs or to go downstairs. According to Fruin (1971) the free flow speed on stairs can almost be divided by two if we consider upward walking speed on a short stairway. However, this approximation does not take into account the level of stress of the population due to an exposition to a danger. For instance, during the world trade center attack, people went down the stairs at an estimated speed of 0.3 m/s (Blake et al. 2004). Therefore people in high stress (or in panic) have a lower speed in average. Furthermore, we assume that the population in alert states do not move. Finally, we suppose \( w_1 = w_2 = 0.2 \), that is, when a node is completely full, the average speed of the population is equal to 20\% of their free flow speed, according to the empirical data from the survey paper (Hermant 2012).

For all these reasons, we will consider different speeds and derived coupling coefficients, which are summarized in Table 4.

Table 4 Coupling setup. Here two networks are considered, the ones labelled Path 1 and Path 2 in Fig. 6

| Network   | Coupling \( \eta_{1,2} \) | Coupling \( \eta_{2,3} \) |
|-----------|-----------------|-----------------|
| Path 1    |                  |                  |
|           | \( S_1 = 300 \text{ m}^2; L_{1,2} = 2 \text{ m}; w_1 = 0.2 \) | \( S_2 = 50 \text{ m}^2; L_{2,3} = 2 \text{ m}; w_2 = 0.2 \) |
|           | \( <\nu_{p}> = 1 \text{ m s}^{-1}; <\nu_{c}> = 1.3 \text{ m s}^{-1} \) | \( <\nu_{p}> = 0.5 \text{ m s}^{-1}; <\nu_{c}> = 0.65 \text{ m s}^{-1} \) |
|           | \( \eta_{1,2}^p = 0.4; \eta_{1,2}^c = 0.52 \) | \( \eta_{2,3}^p = 1.2; \eta_{2,3}^c = 1.62 \) |
| Path 2    |                  |                  |
|           | \( S_1 = 1200 \text{ m}^2; L_{1,2} = 30 \text{ m}; w_1 = 0.2 \) | \( S_2 = 300 \text{ m}^2; L_{2,3} = 30 \text{ m}; w_2 = 0.2 \) |
|           | \( <\nu_{p}> = 1 \text{ m s}^{-1}; <\nu_{c}> = 1.3 \text{ m s}^{-1} \) | \( <\nu_{p}> = 0.5 \text{ m s}^{-1}; <\nu_{c}> = 0.65 \text{ m s}^{-1} \) |
|           | \( \eta_{1,2}^p = 1.5; \eta_{1,2}^c = 1.95 \) | \( \eta_{2,3}^p = 3; \eta_{2,3}^c = 3.9 \) |

Table 5 Initial conditions and node maximal capacities for the three scenarios under study

| Scenario   | Node 1 (\( N_1(0) \)) | Node 1 (\( N_{\text{max}} \)) | Node 2 (\( N_2(0) \)) | Node 2 (\( N_{\text{max}} \)) | Node 3 (\( N_3(0) \)) | Node 3 (\( N_{\text{max}} \)) |
|------------|-----------------|-----------------|----------------|-----------------|----------------|-----------------|
| Scenario 1 | 300             | 300             | 5              | 50              | 50              | 500             |
| Scenario 2 | 300             | 300             | 295            | 250             | 250             | 2500            |
| Scenario 3 | 1200            | 1200            | 5              | 50              | 50              | 7500            |

According to Fruin (1971) the free flow speed on stairs can almost be divided by two if we consider upward walking speed on a short stairway. However, this approximation does not take into account the level of stress of the population due to an exposition to a danger. For instance, during the world trade center attack, people went down the stairs at an estimated speed of 0.3 m/s (Blake et al. 2004). Therefore people in high stress (or in panic) have a lower speed in average. Furthermore, we assume that the population in alert states do not move. Finally, we suppose \( w_1 = w_2 = 0.2 \), that is, when a node is completely full, the average speed of the population is equal to 20\% of their free flow speed, according to the empirical data from the survey paper (Hermant 2012).

For all these reasons, we will consider different speeds and derived coupling coefficients, which are summarized in Table 4.

Simulation setup

According to the three scenarios explained before and the two paths/networks under study, we have chosen the initial conditions summarized in Table 5.

Moreover, for all the scenarios, we suppose that

- the explosion is almost instantaneous and the different nodes experience the event at the same time. Thus, \( t_0^i = 0 \) and \( t_1^i = 0.5 \), for all \( i = 1, 2, 3 \).
- the explosion triggers the behavioral dynamics in node 1, that is \( \delta^1 = 1 \), while in node 2 and 3 the arrival of people from the other nodes plays a role as important as the event itself. Thus \( \delta^2 = \delta^3 = 0.5 \).
- the involved population has a low level of risk culture. The intrinsic transitions towards panic are more likely than the ones towards control.
• in the refuge zones people tend to imitate the individuals in control behavior.
• the third node of the networks under study is the only refuge node. Moreover, in the
refuge nodes people can attain a pseudo-daily behavior only after 50 min. Therefore
\( \phi_1 = \phi_2 = 0 \) and \( \tau_3^2 = 50 \) and \( \tau_3^3 = 80 \) min.

These assumptions lead to the parameter choices in Table 6.

Numerical results and discussion
The numerical results for the three scenarios are represented in Fig. 9. Each column cor-
responds to a scenario. For each scenario, the total population on each node, and the
number of individuals in daily, alert, panic, control and pseudo-daily behaviors for each
node of the network are plotted.

Scenarios 1 and 3
The first scenario shows that the 300 individuals who escaped via the spiral staircase
(node 2) were all able to take refuge on the terrace (node 3) within 22 min. This evacu-
ation is faster in the third scenario for the 1200 people who fled by the wide staircase
(30 metres wide), since it lasted about 11 min. In both cases, it takes from 0 to 5 min for
people of node 1 to understand that they are exposed to a possible danger. Even if the
evacuation via the spiral staircase is slower, no bottleneck is observed. At most, there are
45 people, whereas the capacity of the staircase is 150 persons.

Panic behaviors (panic flight, agitation, freezing response, automatic movements) take
over in both node 1 and node 2. The arrival of individuals in panic on node 3 is visible at
the beginning of the simulation. Indeed, for about 15 min for scenario 1 and for between
8–10 min for scenario 3, the arrival of panicked people justifies the fact that there are as
many controlled people as panicked ones. However, the reassuring information provided
by the five people present at the beginning on the node 3 and/or a better view of what is
happening are likely to gradually reassure the population.

Scenario 2
The issue is more problematic in the second scenario. Effectively the terrace is almost
at its full capacity due to the many people in restaurants and bars. Moreover numerous
tables, chairs and various obstacles restrict the space for people who try to escape the
esplanade via the small staircase. In this case, the time to understand that something is

| Parameters Values | Parameters Values | Parameters Values |
|-------------------|-------------------|-------------------|
| \( B_1^1 = B_2^1 = B_3^1 \) 0.2 | \( C_1^1 = C_2^1 = C_3^1 \) 0.1 | \( D_0^1 = D_0^2 = D_0^3 \) 0.001 |
| \( B_2^2 = B_3^2 \) 0.25 | \( C_1^2 = C_2^2 = C_3^2 \) 0.15 | \( D_0^2 = D_0^3 \) 0.001 |
| \( B_3^3 \) 0.005 | \( B_4^1 = B_4^2 = B_4^3 \) 0.005 | \( D_0^1 = D_0^2 = D_0^3 \) 0.001 |
| \( \alpha_1^1 = \alpha_2^1 \) 0.3 | \( \alpha_1^3 = \alpha_2^3 \) 0.5 | \( \beta_1^1 = \beta_2^1 \) 0.5 |
| \( \beta_2^1 \) 0.3 | \( \gamma_1^{c \rightarrow p} = \gamma_2^{c \rightarrow p} \) 0.5 | \( \gamma_3^{c \rightarrow p} \) 0.3 |
| \( \gamma_0^{c \rightarrow c} \) 0.5 | \( \gamma_0^{p \rightarrow c} = \gamma_0^{p \rightarrow c} \) 0.3 | \( \delta_1 \) 1 |
| \( \delta^2 = \delta^3 \) 0.5 | \( \tau_0^1 = \tau_0^2 = \tau_0^3 \) 0 | \( \tau_1^1 = \tau_1^2 = \tau_1^3 \) 0.5 |
| \( \phi_1 = \phi_2 \) 0 | \( \tau_2^1 \) 50 | \( \tau_3^1 \) 80 |
going wrong (state of alert) persists less than 5 min, and then a bottleneck appears in the footbridge node. This situation is due to the footbridge capacity which is nearly reached after about 10 min (about 110 people are in node 2), and even more to the progressive saturation of the terrace. The situation in nodes 2 and 3 gives rise to panic phenomena classically observed in densely crowded areas during disaster. The risk is high to observe trampling, compression, crushes. The ripple effect is that people on the esplanade are also blocked and it takes more than 40 min to evacuate the area next to the narrow footbridge instead of the 22 min observed in the first scenario.

This second scenario is particularly interesting because it highlights a crisis within a crisis phenomenon. Indeed, for a part of the population on the esplanade, the terrace...
seems to be the best location to take refuge in. We can assume that people arriving from the other nodes do not know that the terrace is already crowded and that the capacity of this place is nearly reached. This could explain the observed situation of panic predominance. If these people at the beginning of the simulation had this information, they could have chosen the other path.

**Discussion**

The specificity of our case study is that the density increases sharply when people enter the stairs where speed is divided by two. The high density combined with the slackening of the flow leads to low average speeds for panicked and controlled people.

Moreover, by comparing scenarios 1 and 2, we remark that the number of individuals present at $t = 0$ in the refuge node 3 has an important role on the dynamics of the whole network. If node 3 is quite full of people from the beginning, on the one hand, it can rapidly reach its maximum capacity so bottlenecks can occur; on the other hand, due to its high density, panic reactions can easily take place. In order to clear up this last phenomenon, let us consider what happens in node 3 at $t = 40$ min, when the APC dynamics on the node is well established and none has already returned in a pseudo-daily behavior. In particular, let us focus on the proportion of individuals in panic in node 3 at $t = 40$ min, that is

$$p^{40} = \frac{p_3(t = 40)}{N_3(t = 40)}$$

Figure 10 shows $p^{40}$ as a function of $N_3(0)$, that is the population at $t = 0$ in node 3. All the parameters are here settled as in Scenarios 1 and 2, only $N_3(0)$ varies. It is possible to see that for small values of $N_3(0)$, the proportion of individuals in panic is low. Thus, in this case, node 3 is a proper refuge node and panic is managed. If, at the beginning of the catastrophe, node 3 hosts more than 240 individuals, then a crisis in the crisis phenomena can occur and panic gets the upper hand, since the node becomes rapidly full and cannot welcome other people who tried to get there.

Furthermore, we remark that in our simulations the two networks are not linked whereas the population leaves the same place. This connection between different simple networks will be the next step of our work. However, several empirical studies about evacuation have shown that rational behaviors are not always observed. Individuals may collectively take the same path, while other exits are possible. Firstly because herding or imitation behavior occur frequently in exit choices. People behave as a group by putting aside their ability to act as individuals, leading some to choose the most congested exit, rather than an exit with less people (Saloma and Perez 2007; Lovreglio et al. 2014). Secondly, because some experiments have shown that people often use familiar exits. For Nilsson et al. (2009), a familiar exit can be for example the entrance of a building or the ordinary exit.

Finally, we note that, whatever the scenario 1 or 2, the choice of path 1 seems to be a wrong decision in both cases, leading eventually to highly hazardous situations because of the emergence of panic behaviors on the narrow footbridge. This type of analysis and conclusion may be considered as a significant contribution of our simulation model to operational staff: it could be efficient to block the way to such hazardous paths and to
drive people towards the safest exits so as to improve the global evacuation process and the risk management.

Conclusions
The lack of knowledge about the spatio-temporal dynamics of human behavior faced with catastrophic events is still significant. In order to advance further in this research field, an interdisciplinary approach, combining in situ experiments and mathematical modeling, has been recently adopted. In this paper, we show how the survey results can feed our network mathematical model in order to design numerical simulations of realistic scenarios. Here, we focused on the in situ experiment that took place in the urban area around the Niemeyer Cultural Centre in Le Havre (France) two years ago. We have shown how the territorial configuration and the interviewees’ responses, through their specific evacuation paths, guided us in the construction of the network under study. We have therefore brought to consider two separated networks of three nodes, with different properties. Based on these networks some simulations have been achieved. Results confirm the idea that the maximal capacity of each node and the initial conditions (that is the number of individuals on the node at the onset of the catastrophe) play an important role on the sequence of events and on the level of panic behaviors in some particular configurations. The main contribution of this approach lies in the predictive...
ability of our simulation tool for evaluating one evacuation path or another in terms of hazard potential.

The next steps of this research is to consider bigger networks or specific cases in order to evaluate their different potential evacuation paths by simulation. Our perspective work is also to pursue the mathematical analysis of the APC model on networks and study how parameters influence the dynamics of the whole network.

Abbreviations
APC: Alert-Panic-Control; ODE: Ordinary differential equations; SIR: Susceptible-infected-recovered.

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Authors’ contributions
VL, EDP and RC were responsible for conception and analysis of the numerical simulations, and wrote the manuscript. DP analyzed the survey results, provided the maps of the escape trajectories and contributed to writing the manuscript. AB provided guidance towards the research and provided comments on the manuscript. All authors read and approved the final manuscript.

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Declarations
Competing interests
The authors declare that they have no competing interests.

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