Finite Pressure Corrections the to Parton Structure of Baryon Inside a Nuclear Medium.

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Our model calculations performed in the frame of the Relativistic Mean Field (RMF) approach show how important are the modifications of baryon Structure Function (SF) in Nuclear Matter (NM) above the saturation point. They originated from the conservation of a parton longitudinal momenta - essential in the explanation of the EMC effect at the saturation point of NM. For higher density the finite pressure corrections emerge from the Hungholt-van Hove theorem valid for NM. The density evolution of the nuclear SF seems to be stronger for higher densities. Here we show that the course of Equation o State (EoS) in our modified Walecka model is very close to that obtained from extensive DBHF calculations with a Bonn A potential. The nuclear compressibility decreases. Our model - a nonlinear extension of nuclear RMF, has no additional parameters. Modelling deep inelastic scattering on nuclear, neutron or (strange) matter with finite pressure, we attempt to predict also the change of baryon masses in a strange nuclear medium. These changes are derived from the Momentum Sum Rule (MSR) of quark longitudinal momenta for different constituents. The increasing pressure between baryons starts to increase baryon Fermi energies $e_F$ in comparison to average baryon energies $e_A$, and consequently the MSR is broken by the factor $e_F/e_A$ from the hadron level in the convolution model. To compensate this factor which increases the longitudinal momentum for nuclear partons, the baryon SF in the nuclear medium and their masses have to be adjusted. Here we assume that, independently from density, quarks and gluons carry the same amount of longitudinal momenta - the similar assumption is used in the most nuclear models with parton degrees of freedom.

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I. THE NUCLEAR DEEPLY INELASTIC LIMIT - NUCLEAR EQUILIBRIUM

In the nuclear deep inelastic scattering on nuclei our time-distance resolution is given by variable $z$:

$$
z = 1/(x M_N)$$

which measures the propagation time of the hit quark caring the $x$ fraction of the longitudinal momentum of the nucleon of mass $M_B$. Start with the scenario where the partonic mean free paths $z$ are much shorter then the average distances between nucleons. This means that partons (inside mesons) remain inside the "volume" of a given nucleon and therefore we can treat nucleons as noninteracting objects remaining on the energy shell not affected by neighboring nucleons.

In the light cone formulation, $x_A$ corresponds to the nuclear fraction of quark longitudinal momentum $p_A^+ = p_q^0 + p_A^3$ and is equal (in the nuclear rest frame) to the ratio $x_A = p_A^+/M_A$. But the composite nucleus is made of hadrons which are distributed with longitudinal momenta $p_h^+$, where $h = N, \pi, ...$ stands for nucleons, virtual pions, ...

In the convolution model, a fraction of parton longitudinal momenta $x_A$ in the nucleus is given as the product $x_A = x_h \ast y_h$ of fractions: parton momenta in hadrons $x_h \equiv Q^2/(2 M_h \nu) = p_q^0/p_h^+$ and longitudinal momenta of hadrons in the nucleus $y_h = p_h^+ / M_A$. The nuclear dynamics of given hadrons in the nucleus is described by the distribution function $\rho^h(y \equiv y_h)$ and SF $F^h_2(x \equiv x_h)$ describes its parton structure. In the convolution model restricted to nucleons and pions (lightest virtual mesons) the nuclear SF $F^A_2$ is described by the formula:

$$
F^A_2(x_A) = \int dy \int dx \delta(x_A - xy)(\rho^N(y) F^B_2(x) + \rho^\pi(y) F^\pi_2(x))
$$

where $F^\pi_2$ and $F^B_2$ are the parton distributions in the virtual pion and in the bound nucleon. The nucleon distribution

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\[ \rho^A(y) = \frac{4}{\rho} \int_{|p| > p_F} \frac{S_N(p)d^3p}{(2\pi)^3} (1 + \frac{p_F}{E_p}) \delta(y - p^*/\varepsilon_N) \]

\[ = \frac{3}{4} \frac{\varepsilon_N}{k_F} \left[ \left( \frac{p_F}{\varepsilon_N} \right)^2 - \left( y - \frac{E_F}{\varepsilon_N} \right)^2 \right], \tag{3} \]

Here the nucleon spectral function was taken in the impulse approximation: \( S_N = n(p)\delta(p^o - (E^*(p) + U_V)) \) and \( E^*(p) = \sqrt{M_N^2 + p^2} \). \( E_F \) is the nucleon Fermi energy and \( y \) takes the values given by the inequality \((E_F - p_F)/\varepsilon_N < y < (E_F + p_F)/\varepsilon_N\).

The MSR for the nucleonic part is sensitive to the Fermi energy as can be seen from the integral:

\[ \int dy y\rho^A(y) = \frac{E_F}{\varepsilon_N} \tag{4} \]

Thus the nucleonic part of MSR gives a factor \( E_F/\varepsilon_N \) which is equal to 1 at the saturation point. If the nucleon SF is not changed in the medium (no EMC effect except Fermi motion) the total (5) MSR is satisfied without nuclear pions:

\[ \frac{1}{A} \int F^A_2(x)dx = \int dy y\rho^A(y) \int F^N_2(x)dx = \int F^N_2(x)dx \tag{5} \]

Summarizing, good description \([6, 7]\) of these deeply inelastic processes without gluon degrees of freedom allows us to assume that fraction of momentum carried by quarks does not change from nucleon to nucleus (~one half, the rest is carried by gluons). We will assume that balance also above the saturation point of NM. Now for the non zero pressure the Fermi energy in NM is no longer equal to the average binding energy, and corrections to MSR \([11]\) proportional to the pressure which will now be investigated.

**II. NON-EQUILIBRIUM CORRECTION TO NUCLEAR DISTRIBUTION.**

For finite pressure very important is well known Hugenholtz van Hove relation between \( E_F, \varepsilon_N \) and pressure \( p \) (see for example) \([10]\). The Fermi energy is defined as density derivative of the total nuclear energy \( E = A\varepsilon_N \):

\[ E_F = \frac{d}{d\rho} \left( \frac{E}{\Omega} \right) \]

\[ E_F = \varepsilon_N + \rho \frac{d\varepsilon_N}{d\rho} \tag{6} \]

where \( A/\rho = \Omega \) gives the volume. At the saturation point \( E_F = \varepsilon_A \). But for negative pressure \( p \)

\[ \int dy y\rho^A(y) = \frac{E_F}{\varepsilon_N} < 1. \tag{7} \]

and we have room for additional pion inside NM (even with unmodified nucleons) in the Björgen limit (parton picture).

**A. Positive pressure.**

Consider the additional pion contributions. The amount of 1% of the total nuclear momentum \([3, 4, 5]\) was estimated from (8) for \( x > x_L \) due to the smaller, \( x \) dependent nucleon mass \( M_B(x) \). For higher density, the average distances between nucleons are smaller, therefore parameter \( x_L \) \([??]\) will increase with density. It means that the room for nuclear pions given by (8) from \( x \) dependent nucleon mass \( M_B \) will be reduced for higher densities and a dependence of \( M_B \) from Björgen \( x \) will vanish gradually. Also the pion effective cross section is strongly reduced at high nuclear densities above the threshold in \( N + N = N + N + \pi \) reaction calculated in Dirac-Brueckner approach \([11]\) (also with RPA insertions to self energy of \( N \) and \( \Delta \) \([12]\) included). Therefore for positive pressure the nuclear pions carry much less than 1% of the nuclear longitudinal momentum and dealing with a non-equilibrium correction to the nuclear distribution \([2]\) we will restrict considerations to the nucleon part without additional virtual pions between them

\[ \frac{1}{A} F^A_2(x)dx = \frac{1}{A} \int dy \rho^A(y) \int F^N_2(x/y). \tag{8} \]
FIG. 1: Constant nucleon mass $M_N$ (Walecka) and density dependent mass $M_{\text{med}}$ from our "Mass Mod." model. Also effective mass $M^*$ in these approaches. Both models are calculated for $S_1$ and $S_2$ parametrization.

The Equation of State (EOS) for NM has to match the saturation point with compressibility $K^{-1} = 9\rho^2 \frac{d^2 E}{d\rho^2}$ but then the behavior for higher densities is different for different RMF models. We compare here two extreme examples: stiff model of Walecka [13] and nonrelativistic expansion in powers of Fermi momentum [?]. For linear coupling in the standard Walecka model at saturation density of NM compressibility is too large ($K^{-1} \approx 560$ MeV). The energy $E_{\text{press}} = p/\rho$ shown in Fig. ?? influences the nuclear EOS. In both models, two coupling constants of the theory are fixed by the empirical saturation density. Our approach is different. In this work we consider the change of the nucleon mass with the change of the parton distribution (nucleon SF) above the saturation point. The increasing pressure between nucleons starts to increase the $E_F(4)$ and consequently the sum rule (5) is broken by the factor $E_F/\varepsilon_N > 1$.

To compensate this factor which increases the longitudinal momentum (5) of nuclear partons, the nucleon SF in the nuclear medium has to be changed. For good estimate, in order to proceed without new parameters, assume (similarly to (??)) that the changes of SF will be included through the changes of Björken $x$ in the medium. Multiplying the argument of the SF by a factor $E_F/\varepsilon_N$ the SF will be squeezed towards smaller $x$ and the total fraction of longitudinal momentum will be smaller by a factor $\varepsilon_N/E_F$:

$$\int_0^1 F_N^2 \left( \frac{E_F}{\varepsilon_N} x \right) dx_N = \frac{\varepsilon_N}{E_F} \int_0^1 F_N^2 \left( x \right) dx \approx \frac{\varepsilon_N}{E_F} \int_0^1 F_N^2 \left( x \right) dx \ (9)$$

Here in the integral we neglect the small contributions from $x > 1$ region originated from NN correlations.

Now, with the help of Eq.(4) and Eq.(9), the nuclear MSR is satisfied:

$$\frac{1}{A} \int F_A^A \left( x_A \right) dx_A = \int dyy \rho^A \left( y \right) \int \frac{E_F}{\varepsilon_N} \int F_N^2 \left( \frac{M_N}{M_{\text{med}}} x \right) dx \approx \int F_N^2 \left( x \right) dx \ (10)$$

This means that quarks in the nucleus carry the same fraction of longitudinal momentum as in bare nucleons. On the other hand the integral (9) corresponds (??) to the total sum of the quark longitudinal momenta $p_q^+ = p_q^0 + p_q^3$ inside a nucleon, which is proportional to a total nucleon rest energy or the nucleon mass. Consequently, the nucleon mass $M_N$ will be changed for $\varrho \geq \varrho_0$ to the mass $M_{\text{med}}$ by the gradually decreasing factor $\varepsilon_N/E_F$:

$$M_{\text{med}} = \frac{\varepsilon_N}{E_F} M_N = M_N \left( 1 - \frac{\varrho}{\varrho_0} \frac{\varepsilon_N}{\varepsilon_N} \right) \approx M_N \left( 1 - \frac{p}{\varepsilon_N} \right) \ (11)$$

which decreases as the pressure increases. This explicit mass dependence from energy $\varepsilon_N$ and energy derivative (11) is plugged into the following standard Walecka RMF equations [13] for energy per nucleon $\varepsilon_N$ and effective mass $M^*$:
III. CONCLUSION

Here, our model applied to the linear Walecka model for nuclear and neutron matter make the EoS softer, close to semi-empirical analysis\cite{18} and close to DBHF calculation with a realistic Bonn A potential\cite{19}. Other features of the Walecka model, including a good value of the spin-orbit force remain in our model unchanged. Our results suggest corrections above the saturation density to any RMF model, with constant nucleon mass and unmodified parton SF. The stiffness of EoS recently discussed\cite{20} in application to compact and neutron stars is important when studying star properties (mass-radius constraint). Partial support of the Ministry of Science and Higher Education under the Research Project No. N N202046237 for is acknowledged.

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