Abstract— This paper investigates the DC-link voltage control of an active rectifier that is supplied by a variable speed permanent magnet synchronous generator. This configuration is commonly encountered in gearless wind energy conversion systems as well as in variable speed generating units. The proposed control strategy uses an optimal voltage vector based modulated model predictive control to achieve direct power control. The studied scheme combines the advantages of finite control set Model Predictive Control (MPC) and control techniques that use pulse width modulator. The fast dynamics of the former are obtained during large transients and constant switching frequency operation, of the latter, is ensured in steady state. At each sampling instant, all switching states are evaluated and the two adjacent states that give minimum error in the controlled variables are selected. The duty-cycle of each of these vectors is computed through linear combination and appropriately limited for over-modulation. Simulations and Co-simulation results presented in the paper show interesting results. The control strategy has been developed on an FPGA control platform and experimental results at steady state are shown, which guarantee the computational feasibility of the control strategy.

Keywords— permanent magnet synchronous generator, direct power control, modulated model predictive control, optimal voltage vectors

I. INTRODUCTION AND OBJECTIVE OF THE WORK

Permanent magnet (PM) machines are generally employed for servo and traction applications [1]. In variable speed drives, they are a viable solution in applications that demand high power density, such as integrated starter generators in aerospace sector [2]. These machines are also very effective in gearless wind energy conversion systems where bulky mechanical gearboxes are avoided for minimizing load on wind turbine tower [3]–[5]. With a direct-drive (gearless) permanent magnet synchronous generator (PMSG), a varying wind speed means variable frequency ac voltages being induced at the generator terminals. To be able to couple this variable frequency source with a constant frequency grid, a power electronic interface is necessary.

Most often, the power electronic interface consists of a diode rectifier feeding a dc-link at the output of which another power electronic converter delivers power to the grid [6]. However, a discontinuous conduction pattern in diode bridge rectifiers means greater current ripple and, hence, a substantial torque ripple at the shaft [6]. An active front end (AFE) converter that replaces diodes with bidirectional power electronic switches alleviates this discontinuous conduction problem. The control of this AFE converter has been widely reported in the literature.

An increasing research effort is directed towards MPC applications for AFE control [7], [8]. A ripple-reduced model predictive direct power control of an AFE is presented in [7]. The active and reactive power slopes are determined based on system state equations. Using these power slopes, the active and reactive powers for the next sampling instant are predicted for seven voltage vectors corresponding to available states of a three-phase inverter. A cost function is evaluated that compares the reference and predicted powers. Unlike finite control states FCS-MPC in which only one voltage vector is applied, [7] uses two voltages whose duties are computed through analytical solution. It should be noted that this technique either applies two active vectors or one active and one zero vector in a control period. Use of two active and a zero vector in the same control period is not analysed. Duty-cycles are optimized in [8] for model predictive current control of an AFE. The control is implemented in dq rotating reference frame. In a given sector, errors in controlled currents are evaluated for the two voltage vectors at the sector boundary. An optimization problem is solved to compute duty-cycles for each active vector in the current sector, which is then transformed to phase duty cycles. Performance of this predictive duty-cycle algorithm is compared with deadbeat, FCS-MPC and linear controllers.

A predictive hybrid pulse width modulation strategy is introduced in [9] for AFEs. The method focuses on minimization of current distortions and switching losses at each sampling instant. Predictive control is used to estimate current distortions and switching losses for every PWM sequence and the sequence that minimizes the cost function is chosen. As such, the control strategy does not deal with the dynamics of control. The application of predictive direct power control to minimize ripple in active power is presented in [10]. Starting from a conventional MPC that identifies the voltage vector with minimum power error, instead of applying this vector for a complete control period its duty-cycle is computed that results in reduced active power ripple. For the remaining time of a control period a zero vector is applied. Thus, only one active vector is used in a given control period.

In this paper, the control of an active front end power converter is implemented through optimal voltage vector selection starting from the basic model predictive control algorithm. The control strategy is adapted from [11], however, the implementation takes place in the rotating dq reference frame. The system under consideration is a variable speed PMSG supplying a DC load. Starting from the discrete equations of the considered system, the power errors at the next sampling instant are predicted for all the inverter states. Then, the two adjacent inverter states that minimize the error in the active and reactive powers are selected. The duty-cycle
for the application of each state is decided by whether it is possible to reach the reference power values in one sampling instant or not. In the first case, when the reference power can be achieved in one sample time, the solution lies in the linear regulation range and the duty-cycles are computed through linear combination of the two inverter states. In case the system constraints do not allow reaching the target in one sample time, the over-modulation condition is encountered. To preserve optimality condition also in over-modulation, the duty-cycles for each state are linearly scaled. In this way, the algorithm preserves the fast dynamic characteristics of FCS-MPC as well as maintains constant switching frequency of modulator based control schemes.

II. SYSTEM MODELING

With respect to the dq0 rotating frame, the equations representing the continuous time model of the PMSG (1) are considered together with the equation describing the DC-link capacitor dynamics, where \( i_{cp} \) stands for the capacitor current, as represented in Fig. 1.

\[
\begin{align*}
L_d \frac{di_d}{dt} &= v_{sd} - R_l i_d - \omega L_q i_q \\
L_q \frac{di_q}{dt} &= v_{sq} - R_l i_q - \omega L_d i_d - \omega \lambda_{pm} \\
C \frac{dv_{dc}}{dt} &= i_{cp}
\end{align*}
\]

(1)

Applying the Kirchhoff law to the node “A” in Fig. 1, the capacitor current is expressed as the sum of the load current \( i_l \) and the rectified current \( i_{dc} \).

From the continuous time model, forward Euler method is applied to obtain the discrete time model of the system (2).

\[
\begin{align*}
i_d^{(k+1)} &= \left(1 - T_s \frac{R_l}{L_d}\right) i_d^{(k)} + v_{sd}^{(k)} \frac{T_s}{L_d} + i_q^{(k)} \omega T_s \frac{L_q}{L_d} \\
i_q^{(k+1)} &= \left(1 - T_s \frac{R_l}{L_q}\right) i_q^{(k)} + v_{sq}^{(k)} \frac{T_s}{L_q} - i_d^{(k)} \omega T_s \frac{L_d}{L_q} \\
v_{dc}^{(k+1)} &= v_{dc}^{(k)} + \frac{T_s}{C} (i_{dc}^{(k)} - i_l)
\end{align*}
\]

(2)

Where \( T_s \) is the sampling period, \( R_l \) is the generator phase resistance, \( L_d \) and \( L_q \) are the machine inductances, \( \lambda_{pm} \) is the magnetic flux, \( \omega \) is the frequency of the electrical quantities and \( v_{sd} \) \( v_{sq} \) are the control actions applied by the inverter to the system. For a given DC-link voltage, the voltage vector resulting from each inverter state is given by (3).

\[
\begin{pmatrix}
0.66667 & -0.3333 & -0.3333 \\
-0.66667 & 0.3333 & 0.3333 \\
-0.3333 & 0.6667 & -0.3333 \\
0.3333 & -0.6667 & 0.3333 \\
-0.3333 & 0.3333 & 0.6667 \\
0.3333 & 0.3333 & -0.6667 \\
0 & 0 & 0
\end{pmatrix}
\]

(3)

III. MODULATED MPC STRATEGY

The purpose of the proposed control strategy is to regulate the DC-link voltage while keeping the reactive power to zero. The desired value of the DC voltage is reached considering the link between the active power flow produced by the PMSM and the change in the capacitor voltage. As already known, the only possible control actions in FCS-MPC are those generated by the eight different converter states. The control action is thus discrete, resulting in a low power quality at steady state. In this paper, an inversion of the electrical model is performed as in [11]. As already known, finding an analytical inversion would require extremely high computational effort. The electrical model inversion is thus obtained through linear approximation. Fig. 2 shows the hexagon created by the finite control actions on the q0 plane. It is possible to consider the plane of axes \( e_{id} \) and \( e_{iq} \) where \( e_{id} \) stands for the error between the predicted values of \( i_d \) and its reference and \( e_{iq} \) represents the error on the q-axis current. On this plane, the predictions associated to the non-zero voltage vectors form an irregular hexagon.
Once the couple of adjacent voltage vectors is chosen, the duty-cycle associated to each of them is calculated, according to the resolution of the linear system expressed by (5).

In the over-modulation region, that is when, $d_i + d_j > 1$, duty-cycles are opportunely scaled according to (6).

\[
\begin{align*}
  d_i' &= \frac{d_i}{d_i + d_j} \\
  d_j' &= \frac{d_j}{d_i + d_j} \\
  \bar{V}_s &= \bar{V}_x \cdot d_x + \bar{V}_y \cdot d_y
\end{align*}
\]  

The resulting voltage vector applied to the system can be expressed as in (7). The proposed control strategy does not consider an explicit formulation of a cost function, being the control action chosen on the basis of the explained mathematical procedure. The latter aims to identify the two voltage vectors that, linearly combined, lead to the smallest error on the controlled quantities. In order to keep the reactive power equal to zero, $i_q$ reference value is set to zero, while to opportunely regulate the capacitor voltage, a reference value of the $q$-axes current is generated ($i_{q,R}$), based on the estimation of the active power flow needed to both feed the load and follow the voltage reference [12]. The control algorithm starts computing the first step of prediction on the basis of the previous control action applied to the system. The values of $i_{dc,[k+1]}$, $v_{dc,[k+1]}$, and $v_{dc,[k-1]}$ are then calculated. The predicted value of the capacitor voltage $v_{dc,[k-1]}$ is obtained estimating the DC current $i_{dc}$ as a function of the inverter states applied to the system and the phase currents. This solution is considered because of the specific nature of the proposed MPC strategy, which returns as a result different converter states to be applied in a single control period. The adopted solution allows improving the $i_{dc}$ estimation and consequently the whole control algorithm.

\[
\begin{align*}
  i_{dc,M} &= \bar{s} \cdot \left[ \begin{array}{c}
  i_a \\
  i_b \\
  i_c \\
  i_q
\end{array} \right] + d_x \cdot \left[ \begin{array}{c}
  i_a \\
  i_b \\
  i_c \\
  i_q
\end{array} \right] d_x + d_y \cdot \left[ \begin{array}{c}
  i_a \\
  i_b \\
  i_c \\
  i_q
\end{array} \right] d_y
\end{align*}
\]  

Second step of prediction starts with the calculation of the $i_{dc}$ reference value. To do that, a proper power balance is defined considering that the active power flow produced by the PMSG ($P_{ac}$) has to account for both the power required by the load ($P_l$), the power needed to change the DC-link voltage ($P_v$) and the power losses. Equations (9) describe the existing relation between the change in the stored energy of a capacitor ($\Delta E_c$) and its voltage variation, as well as the resulting power flow.

\[
\Delta E_c = \frac{C}{2} (V_{ac,d,k}^2 - V_{ac,d,[k+1]}^2)
\]  

\[
P_v = \frac{\Delta E_c}{T_t}
\]

The power required by the load is determined as the product of the load current, which is here assumed measurable, and the desired DC-link voltage. Power losses are considered as Joule losses due to the stator resistance. It means that the required active power flow at instant $[k+2]$ is:

\[
P_{ac,[k+2]} = \frac{C}{2T_t} (V_{ac,d,k}^2 - V_{ac,d,[k+1]}^2) + (V_{ac,q,k}^2) + R_s (i_{q,R,k}^2 + i_{q,R,[k+1]}^2)
\]

The active power produced by the PMSG is expressed in terms of the q-axes current by (11) and it is then possible to evaluate the $i_q$ reference through (12).

\[
P_{ac} = \frac{3}{2} \rho_{dc} \omega_m i_q^2
\]

\[
i_q^2 = \frac{1}{\rho_{dc} \omega_m} \left( \frac{C}{T_t} (V_{ac,d,k}^2 - V_{ac,d,[k+1]}^2) + 2V_{ac,d,[k+1]} i_q + 2R_s (i_{q,R,k}^2 + i_{q,R,[k+1]}^2) \right)
\]

The parameter $T_t$ represents the period of time in which the reference value has to be reached. Consequently, it takes into account the dynamics of the control action. It has to be underlined that this parameter acts only on the power that in the proposed control strategy is required by the capacitor to increase the voltage at its terminals. When the measured voltage enters a close range of the reference value, the control dynamics is reduced increasing the value of $T_t$, in order to significantly decrease the voltage ripple around the voltage reference value and thus improve the phase currents waveform. An appropriate value of $T_t$ should be selected on the basis of the desired trade-off between current waveform quality, steady state error on the DC voltage reference and DC voltage ripple. A simplified scheme of the proposed control strategy is presented in Fig. 3.

IV. RESULTS

A. Simulation Results

Preliminary results have been achieved through a simulation model whose parameters are shown in Table 1. Typical issues of real implementation have been considered in the simulation, such as delay time on the application of the control action (finite time of the calculation) and parasitic elements such as switches conduction losses of the three phase inverter. A resistive load has been considered. Control performances are shown in Fig. 4 and 5. Fig. 4 shows the phase currents and the measured voltage when the load resistance is decreased from 132 $\Omega$ to 72 $\Omega$, while Fig. 5 shows the same quantities when a step is applied to the DC voltage reference and the load is kept constant and equal to 132 $\Omega$. 
Co-simulation Results

Between the simulation and the experimental stage, the NI LabVIEW Multisim Co-simulation tool has been used to verify the correct FPGA implementation of the control strategy. This tool allows simulating the physical system through NI Multisim and simultaneously implement the control strategy writing the FPGA code on LabVIEW FPGA. Proceeding like that, it has been possible to significantly reduce the time needed to reach a correct FPGA implementation of the code and thus the experimental stage. The tool, thanks to the use of a Control and Simulation Loop (black loop in Fig. 6), guarantees the right timing between the simulation of the physical system and that of the simulated control platform. Only few minor changes to the simulated FPGA code have to be made before loading it on the real control platform. Fig. 6 shows the block diagram of the Co-simulation code. For each iteration of the “Control & Simulation Loop” the Multisim model of the system is executed, while the output values of the FPGA simulated code are returned each $T_s$.

Table 1 Simulation, Co-simulation and experimental set-up main parameters

| Parameter     | Value       |
|---------------|-------------|
| L_d           | 0.450 mH    |
| $\lambda_p_m$| 0.04386 Wb  |
| L_q           | 0.450 mH    |
| R_s           | 0.1 Ω       |
| $T_s$         | 50 µs       |
| C             | 1 mF        |
| PMSG $p_p$    | 12          |
| $T_t$         | 2.5 ms      |

Co-simulation has been carried out considering the same parameters of the simulation stage. A dead-time of 1 µs was added to the switches behavior. The PMSG is rotating at a constant speed, about 2000 rpm which leads to a frequency of the electrical quantities around 400 Hz. For the behavior of the control algorithm during a step in the voltage reference, as shown in Fig. 5 and in Fig. 7, analogous performances have been put in evidence by both simulation and Co-simulation. Indeed, in both cases, it takes a time equal to about 10 ms for the control algorithm to reach the new reference value. In order to avoid too high transient currents, a limitation in the value of the q-axis current ($i_q$) intervenes as it is possible to notice in both figures. In this case, the current limitation was set to 50 A. Again, when the load resistance was decreased from 132 Ω to 72 Ω, simulation and Co-simulation returned comparable results as shown in Fig. 4 and Fig. 8. Once again, the fast dynamics of the control allows to keep the DC voltage really close to the reference value, even during the fast variation of the load conditions.

As already mentioned, this dynamics is regulated by the parameter $T_t$, which allows to increase or decrease the period of time in which the reference voltage has to be reached. By the way, this parameter acts only on the power flow that is delivered to the capacitor and not on the total power produced by the PMSG. This means that the best value that one can choose for $T_t$ in terms of currents ripple, control
Fig. 6. NI LabVIEW FPGA and Multisim Co-Simulation code.

Fig. 7. Co-Simulation. Phase currents and DC voltage when the load resistance is decreased from 132 to 72 Ω.

Fig. 8. Co-Simulation. Phase currents and DC voltage during a reference voltage step.
of the controlled system have been evaluated. In addition, the phase currents has been calculated to be equal to 4.2%. The chosen value of $T_t$ is set to 2.5 ms. The Total Harmonic Distortion of the capacitor ($C_p$) while the load resistance is equal to about 132 Ω and the value of the $T_t$ parameter is set to 2.5 ms. The Total Harmonic Distortion of the DC voltage fixed, the weight of the power delivered to the load ($P_L$), decreases with the decrease of the load resistance. The previous considerations have been taken into account to select an appropriate value of $T_t$; once that the characteristics of the controlled system have been evaluated. In addition, $T_t$ is expressed as an integer multiple of the sampling period. The chosen value of $T_t$ for both simulation and Co-simulation is reported in Table 1.

### B. Experimental Results

The developed control strategy has been implemented on the National Instruments System-on-Module (formally sbRIO9651), with a dedicated control board (PED-Board®). The whole control algorithm is developed on the FPGA while the Real-Time target takes into account the external communication and the system management actions. The active rectifier is a SiC power converter which allows to achieve high switching frequencies, thus improving power quality of the phase currents. The entire electrical drive is showed in Fig. 9.

Fig. 10 shows the steady state behavior of the control algorithm when the voltage reference is 700 V, the load resistance is equal to about 132 Ω and the value of the $T_t$ parameter is set to 2.5 ms. The Total Harmonic Distortion of the phase currents has been calculated to be equal to 4.2%.

![Experimental setup](image)

**Fig. 9.** Experimental setup.

![A-phase current and DC voltage at steady state](image)

**Fig. 10.** A-phase current and DC voltage at steady state.

### CONCLUSIONS AND FUTURE WORK

A Modulated Optimal Model Predictive Control for the control of an active rectifier that is supplied by a permanent magnet synchronous generator has been presented. The proposed strategy ensures both the fast dynamics of the Finite-Set MPC and good phase currents quality at steady state which is typical of the control typologies characterized by the presence of a modulator. It has been individuated a parameter $T_t$ which allows to modify the transient behavior of the control action and the phase current quality at steady state. Future experimental campaigns should point to evaluate the most suitable value for this parameter on the basis of the system characteristics and of the working conditions.

### REFERENCES

[1] M. J. Melfi, S. Evon, and R. McElveen, “Induction versus permanent magnet motors: For power density and energy savings in industrial applications,” *IEEE Ind. Appl. Mag.*, vol. 15, no. 6, pp. 28–35, 2009.

[2] A. Cavagnino, Z. Li, A. Tenconi, and S. Vaschetto, “Integrated generator for more electric engine: Design and testing of a scaled-size prototype,” *IEEE Trans. Ind. Appl.*, vol. 49, no. 5, pp. 2034–2043, 2013.

[3] M. Cheng and Y. Zhu, “The state of the art of wind energy conversion systems and technologies: A review,” *Energy Convers. Manage.*, vol. 88, pp. 332–347, Dec. 2014.

[4] Q. Wang and S. Niu, “Design, Modeling, and Control of a Novel Hybrid-Excited Flux-Bidirectional-Modulated Generator-Based Wind Power Generation System,” *IEEE Trans. Power Electron.*, vol. 33, no. 4, pp. 3086–3096, Apr. 2018.

[5] M. Abdelrahem, C. M. Hackl, and R. Kennel, “Finite Position Set-Phase Locked Loop for Sensorless Control of Direct-Driven Permanent-Magnet Synchronous Generators,” *IEEE Trans. Power Electron.*, vol. 33, no. 4, pp. 3097–3105, Apr. 2018.

[6] T. R. S. de Freitas, P. J. M. Menegaz, and D. S. L. Simonetti, “Rectifier topologies for permanent magnet synchronous generator on wind energy conversion systems: A review,” *Renew. Sustain. Energy Rev.*, vol. 54, pp. 1334–1344, Feb. 2016.

[7] Z. Zhang, X. Feng, H. Fang, and R. Kennel, “Ripple-reduced model predictive direct power control for active front-end power converters with extended switching vectors and time-optimised control,” *IET Power Electron.*, vol. 9, no. 9, pp. 1914–1923, Jul. 2016.

[8] Z. Song, Y. Tian, W. Chen, Z. Zou, and Z. Chen, “Predictive Duty Cycle Control of Three-Phase Active-Front-End Rectifiers,” *IEEE Trans. Power Electron.*, vol. 31, no. 1, pp. 698–710, Jan. 2016.

[9] M. Gendrin, J.-Y. Gauthier, and X. Lin-Shi, “A Predictive Hybrid Pulse-Width-Modulation Technique for Active-Front-End Rectifiers,” *IEEE Trans. Power Electron.*, vol. 32, no. 7, pp. 5487–5496, Jul. 2017.

[10] Y. Zhang, Y. Peng, and C. Qu, “Model Predictive Control and Direct Power Control for PWM Rectifiers With Active Power Ripple Minimization,” *IEEE Trans. Ind. Appl.*, vol. 52, no. 6, pp. 4909–4918, Nov. 2016.

[11] E. Fuentes, C. A. Silva, and R. M. Kennel, “MPC Implementation of a Quasi-Time-Optimal Speed Control for a PMSM Drive, With Inner Modulated-FS-MPC Torque Control,” *IEEE Trans. Ind. Electron.*, vol. 63, no. 6, pp. 3897–3905, Jun. 2016.

[12] L. Tarisciotti et al., “Model Predictive control for Shunt Active Filters with Fixed Switching Frequency,” *IEEE Trans. Ind. Appl.*, vol. 53, no. 1, pp. 297–304, 2016.