On quantum effects in soft leptogenesis

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Abstract. It has been recently shown that quantum Boltzmann equations may be relevant for leptogenesis. Quantum effects, which lead to a time-dependent CP asymmetry, have been shown to be particularly important for resonant leptogenesis when the asymmetry is generated by the decay of two nearly degenerate states. In this work we investigate the impact of the use of quantum Boltzmann equations in the framework of ‘soft leptogenesis’ in which supersymmetry soft breaking terms give a small mass splitting between the CP-even and CP-odd right-handed sneutrino states of a single generation and provide the CP-violating phase to generate the lepton asymmetry.

Keywords: neutrino properties, baryon asymmetry, cosmology of theories beyond the SM

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1. Introduction

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1, 2]. In the standard type I seesaw framework [3], the singlet heavy neutrinos have lepton-number-violating Majorana masses and when decay out of equilibrium produce dynamically a lepton asymmetry which is partially converted into a baryon asymmetry due to fast sphaleron processes.

For a hierarchical spectrum of right-handed (RH) neutrinos, successful leptogenesis requires generically quite heavy singlet neutrino masses [4] of the order \( M > 2.4(0.4) \times 10^9 \) GeV for vanishing (thermal) initial neutrino densities [4, 5] (although flavour effects [6–9] and/or extended scenarios [11, 10] may affect this limit). Low-energy supersymmetry can be invoked to naturally stabilize the hierarchy between this new scale and the electroweak one. This, however, introduces a certain conflict between the gravitino bound on the reheat temperature and the thermal production of RH neutrinos [12]. A way out of this conflict is provided by resonant leptogenesis [13–15]. In this scenario RH neutrinos are nearly degenerate in mass which makes the self-energy contributions to the CP asymmetries resonantly enhanced and allows leptogenesis to be possible at much lower temperatures.

Once supersymmetry has been introduced, leptogenesis is induced also in singlet sneutrino decays. If supersymmetry is not broken, the order of magnitude of the asymmetry and the basic mechanism are the same as in the non-supersymmetric case. However, as shown in [16, 17], supersymmetry breaking terms can play an important role in the lepton asymmetry generated in sneutrino decays because they induce effects which are essentially different from the neutrino ones. In brief, soft supersymmetry breaking terms involving the singlet sneutrinos remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and provide new sources of lepton number and CP violation. As a consequence, the mixing between the sneutrino states generates a CP asymmetry in their decays. At zero temperature and at lowest order in the soft supersymmetry breaking couplings, the asymmetries generated in the sneutrino decays into fermions and scalars cancel out. However, thermal effects break this cancellation and once they are included the asymmetry can be sizable. In particular it is large for a relatively low RH neutrino mass scale, in the range \( 10^5–10^8 \) GeV, well below the reheat
temperature limits, which solves the cosmological gravitino problem. This scenario has been termed ‘soft leptogenesis’, since the soft terms and not flavour physics provide the necessary mass splitting and CP-violating phase.

In general, soft leptogenesis induced by CP violation in mixing as discussed above has the drawback that, in order to generate enough asymmetry, the lepton-violating soft bilinear coupling, responsible for the sneutrino mass splitting, has to be unconventionally small [16]–[18]. In this case, as for the case of resonant leptogenesis, the sneutrino self-energy contributions to the CP asymmetries are resonantly enhanced.

Until recently, the dynamics of thermal leptogenesis (both for the standard see-saw case, as well as for the soft leptogenesis scenario) has been studied using the approach of classical Boltzmann equations (BE). The possibility of using quantum Boltzmann equations (QBE) was first discussed in [22] and it has been recently derived in detail in [23]. In [23] QBE were obtained starting from the non-equilibrium quantum field theory based on the closed time-path formulation. They differ from the classical BE in that they contain integrals over the past times unlike in the classical kinetic theory in which the scattering terms do not include any integral over the past history of the system, which is equivalent to assuming that any collision in the plasma does not depend upon the previous ones. Quantitatively the most important consequence is that the CP asymmetry acquires an additional time-dependent piece, with its value at a given instant depending upon the previous history of the system. If the time variation of the CP asymmetry is shorter than the relaxation time of the particles’ abundances, the solutions to the quantum and the classical Boltzmann equations are expected to differ only by terms of the order of the ratio of the timescale of the CP asymmetry to the relaxation timescale of the distribution. This is typically the case in thermal leptogenesis with hierarchical RH neutrinos. However, as discussed in [24,25], in the resonant leptogenesis scenario, \((M_j - M_i)\) is of the order of the decay rate of the RH neutrinos. As a consequence the typical timescale to build up coherently the time-dependent CP asymmetry, which is of the order of \((M_j - M_i)^{-1}\), can be larger than the timescale for the change of the abundance of the RH neutrinos. This, as shown in [24,25], leads to quantitative differences between the classical and the quantum approach in the case of resonant leptogenesis and, in particular, in the weak washout regime they enhance the asymmetry produced.

Motivated by these results and the fact that in soft leptogenesis the CP asymmetry is produced resonantly, we perform a detailed study of the role of quantum effects in the soft leptogenesis scenario. Our results show that, because of the thermal nature of soft leptogenesis, the dependence of the quantum effects on the washout regime for soft leptogenesis is quantitatively different than in the see-saw resonant scenario. In particular, in the weak washout regime quantum effects do not enhance but suppress the produced baryon asymmetry. Quantum effects are most quantitatively important for extremely degenerate sneutrinos (that is, far away from the resonant condition), \(\Delta M \ll \Gamma_{\tilde{N}}\), and in the strong washout regime they can lead to an enhancement of the asymmetry produced, as well as a change of sign of the asymmetry produced. But altogether, for a given \(M\), the required values of the lepton-violating soft bilinear term \(B\) to achieve successful leptogenesis are not substantially modified.

3 Considering the possibility of CP violation also in decay and in the interference of mixing and decay of the sneutrinos [19], as well as extended scenarios [20,21], may alleviate the unconventionally small \(B\) problem.
2. Brief summary of soft leptogenesis: BE and CP asymmetry

The supersymmetric see-saw model could be described by the superpotential:

\[ W = \frac{1}{2} M_{ij} N_i N_j + Y_{ij} \epsilon_{\alpha \beta} N_i L_\alpha H^\beta, \]

where \( i, j = 1, 2, 3 \) are flavour indices, and \( N_i, L_i \) and \( H \) are the chiral superfields for the RH neutrinos, the left-handed (LH) lepton doublets and the Higgs doublets. The corresponding soft breaking terms involving the RH sneutrinos \( \tilde{N}_i \) are given by

\[ L_{\text{soft}} = -\tilde{m}_{ij}^2 \tilde{N}_i^* \tilde{N}_j - (A_{ij} Y_{ij} \epsilon_{\alpha \beta} \tilde{N}_i \ell \alpha h \beta + \frac{1}{2} B_{ij} M_{ij} \tilde{N}_i \tilde{N}_j + h.c.), \]

where \( \ell^T_i = (\tilde{\nu}_i, \tilde{\ell}^-_i) \) and \( h^T = (h^+, h^0) \) are the slepton and up-type Higgs doublets.

As a consequence of the soft breaking \( B \) terms, the sneutrino and antisneutrino states mix with mass eigenvectors:

\[ \tilde{N}_{+i} = \frac{1}{\sqrt{2}} (e^{i\Phi/2} \tilde{N}_i + e^{-i\Phi/2} \tilde{N}_i^*), \]

\[ \tilde{N}_{-i} = -\frac{i}{\sqrt{2}} (e^{i\Phi/2} \tilde{N}_i - e^{-i\Phi/2} \tilde{N}_i^*), \]

where \( \Phi = \text{arg}(BM) \) and with mass eigenvalues:

\[ M_{ii}^2 = M_{ii}^0 + \tilde{m}_{ii}^2 \pm |B_{ii} M_{ii}|. \]

In what follows, we will consider a single generation of \( N \) and \( \tilde{N} \) which we label as 1. We also assume a proportionality of soft trilinear terms and drop the flavour indices for the coefficients \( A \) and \( B \). As discussed in [16,17], in this case, after superfield rotations the Lagrangians (1) and (2) have a unique independent physical CP-violating phase, \( \phi = \text{arg}(AB^*) \), which we chose to assign to \( A \). Neglecting supersymmetry breaking effects in the right sneutrino masses and in the vertex, the total singlet sneutrino decay width is given by

\[ \Gamma_{\tilde{N}_+} = \Gamma_{\tilde{N}_-} = \Gamma_{\tilde{N}} = \frac{M (YY^\dagger)_{ii}}{4\pi} \equiv \frac{m_{\text{eff}} M^2}{4\pi v_u^2}, \]

where \( v_u \) is the vacuum expectation value of the up-type Higgs doublet, \( v_u = v \sin \beta \) (\( v = 174 \) GeV).

As discussed in [17], when \( \Gamma \gg \Delta M_{\pm} \equiv M_+ - M_- \), the two singlet sneutrino states are not well-separated particles. In this case, the result for the asymmetry depends on how the initial state is prepared. In what follows we will assume that the sneutrinos are in a thermal bath with a thermalization time \( \Gamma^{-1} \) shorter than the typical oscillation times, \( \Delta M_{\pm}^{-1} \). Therefore coherence is lost and it is appropriate to compute the CP asymmetry in terms of the mass eigenstates equation (3).

In this regime the relevant BE (following [27,28,18] in notation and detail) including the dominant \( \Delta L = 1 \) decays and inverse decays as well as the \( \Delta L = 1 \) scatterings with
top quark are
\[ sH \frac{dY_N}{dz} = - \left( \frac{Y_N}{Y_{eq}} - 1 \right) \left( \gamma_N + 4\gamma_t^{(0)} + 4\gamma_t^{(1)} + 4\gamma_t^{(2)} + 2\gamma_t^{(3)} + 4\gamma_t^{(4)} \right), \]
(6)
\[ sH \frac{dY_{\tilde{N}}}{dz} = - \left( \frac{Y_{\tilde{N},tot}}{Y_{eq}} - 2 \right) \left( \gamma_{\tilde{N}} + \gamma_{\tilde{N},tot}^{(3)} + 2\gamma_{22} + 2\gamma_{t}^{(5)} + 2\gamma_{t}^{(6)} + 2\gamma_{t}^{(7)} + \gamma_{t}^{(8)} + 2\gamma_{t}^{(9)} \right), \]
(7)
\[ sH \frac{dY_{L, tot}}{dz} = \left[ \epsilon(T) \left( \frac{Y_{\tilde{N},tot}}{Y_{eq}} - 2 \right) - \frac{Y_{L, tot}}{2Y_{eq}} \right] \gamma_{\tilde{N}} \]
\[ - Y_{L, tot} \left( \frac{1}{2} \gamma_{\tilde{N}} + \frac{Y_{\tilde{N},tot}}{Y_{eq}} \gamma_{\tilde{N}}^{(5)} + 2\gamma_{t}^{(6)} + 2\gamma_{t}^{(7)} + \frac{Y_{N}}{Y_{eq}} \gamma_{t}^{(3)} + 2\gamma_{t}^{(4)} \right) \]
\[ + \gamma_{\tilde{N}}^{(3)} + \frac{1}{2} \frac{Y_{\tilde{N},tot}}{Y_{eq}} \gamma_{\tilde{N}}^{(8)} + 2\gamma_{t}^{(9)} + 2 \frac{Y_{N}}{Y_{eq}} \gamma_{t}^{(0)} + 2\gamma_{t}^{(1)} + 2\gamma_{t}^{(2)} \]
\[ - \frac{Y_{L, tot}}{2Y_{eq}} \left( 2 + \frac{1}{2} \frac{Y_{\tilde{N},tot}}{Y_{eq}} \right) \gamma_{22}. \]
(8)

In the equations above, \( z = M/T \), \( Y_{\tilde{N},tot} \equiv Y_{\tilde{N},+} + Y_{\tilde{N},-} \) and \( Y_{L, tot} \equiv Y_{L_t} + Y_{L_s} \) with the fermionic and scalar lepton asymmetries defined as \( Y_{L_t} = (Y_{\tilde{t}} - Y_{t}) \) and \( Y_{L_s} = (Y_{\tilde{s}} - Y_{s}) \). The equilibrium abundances are given by \( Y_{eq} = 15/4\pi^2 g_*^s \) and \( Y_{N}^{eq}(T \gg M) = 90(3)/(4\pi^4 g_*^s) \), where \( g_*^s \) is the total number of entropic degrees of freedom, and \( g_*^s = 228.75 \) in the MSSM. In writing equations (6)–(8), fast equilibration between the lepton asymmetry in scalars and fermions due to supersymmetry conserving processes has been accounted for.

The different \( \gamma \)'s are the thermal widths for the following processes:
\[ \gamma_{\tilde{N}} = \gamma_{\tilde{N}}^t + \gamma_{\tilde{N}}^s = \gamma(\tilde{N}_\pm \leftrightarrow \tilde{h}\ell) + \gamma(\tilde{N}_\pm \leftrightarrow \tilde{h}\ell), \]
\[ \gamma_{\tilde{N}}^{(3)} = \gamma(\tilde{N}_\pm \leftrightarrow \tilde{\ell}\tilde{u}q), \]
\[ \gamma_{22} = \gamma(\tilde{N}_\pm \ell \leftrightarrow \tilde{u}q) = \gamma(\tilde{N}_\pm \tilde{q} \leftrightarrow \tilde{\ell}u) = \gamma(\tilde{N}_\pm \tilde{q} \leftrightarrow \tilde{\ell}q), \]
\[ \gamma_{t}^{(5)} = \gamma(\tilde{N}_\pm \ell \leftrightarrow \tilde{q}u) = \gamma(\tilde{N}_\pm \ell \leftrightarrow \tilde{q}u), \]
\[ \gamma_{t}^{(6)} = \gamma(\tilde{N}_\pm \tilde{u} \leftrightarrow \tilde{q}u) = \gamma(\tilde{N}_\pm \tilde{q} \leftrightarrow \tilde{u}u), \]
\[ \gamma_{t}^{(7)} = \gamma(\tilde{N}_\pm \tilde{u} \leftrightarrow \tilde{u}u) = \gamma(\tilde{N}_\pm u \leftrightarrow \tilde{u}q), \]
\[ \gamma_{t}^{(8)} = \gamma(\tilde{N}_\pm \tilde{\ell} \leftrightarrow \tilde{u}u), \]
\[ \gamma_{t}^{(9)} = \gamma(\tilde{N}_\pm \tilde{q} \leftrightarrow \tilde{\ell}q) = \gamma(\tilde{N}_\pm \tilde{u} \leftrightarrow \tilde{\ell}q), \]
\[ \gamma_{N} = \gamma(N \leftrightarrow \ell h) + \gamma(\tilde{N}_t \leftrightarrow \ell^* h), \]
\[ \gamma_{t}^{(0)} = \gamma(N \ell \leftrightarrow \tilde{q}u) = \gamma(N \ell \leftrightarrow \tilde{q}u), \]
\[ \gamma_{t}^{(1)} = \gamma(N \tilde{q} \leftrightarrow \tilde{\ell}u) = \gamma(N \tilde{q} \leftrightarrow \tilde{\ell}q), \]
\[ \gamma_{t}^{(2)} = \gamma(N \tilde{u} \leftrightarrow \tilde{\ell}q) = \gamma(N \tilde{q} \leftrightarrow \tilde{\ell}u), \]
\[ \gamma_t^{(3)} = \gamma(N\ell \leftrightarrow q\bar{u}), \]
\[ \gamma_t^{(4)} = \gamma(N \leftrightarrow \bar{\ell}q) = \gamma(N\bar{q} \leftrightarrow \bar{\ell}\bar{u}), \]

where in all cases a sum over the CP conjugate final states is implicit.

The final amount of \( B - L \) asymmetry generated by the decay of the singlet sneutrino states assuming no pre-existing asymmetry can be parameterized as

\[ Y_{B-L}(z \to \infty) = -Y_{L_{tot}}(z \to \infty) = -2\bar{\eta}\bar{\epsilon}Y_{\tilde{N}}^{eq}(T \gg M), \]

where \( \bar{\epsilon} \) is given in equation (14).\(^4\)

\( \eta \) is a dilution factor which takes into account the possible inefficiency in the production of the singlet sneutrinos, the erasure of the generated asymmetry by \( L \)-violating scattering processes and the temperature dependence of the CP asymmetry, and it is obtained by solving the array of BE above.

After conversion by the sphaleron transitions, the final baryon asymmetry is related to the \( B - L \) asymmetry by [26]

\[ Y_B = \frac{8}{23} Y_{B-L}(z \to \infty). \]

Without including quantum effects in the BE, the relevant CP asymmetry in equation (8) is

\[ \epsilon(T) \equiv \bar{\epsilon}(T) = \frac{\sum a_{k,k} \gamma(N_i \to a_k) - \gamma(N_i \to \bar{a}_k)}{\sum a_{u,k} \gamma(N_i \to a_k) + \gamma(N_i \to \bar{a}_k)}, \]

where \( a_k \equiv s_k f_k \) with \( s_k = \tilde{\ell}_k h \) and \( f_k = \ell_k \tilde{h} \) and we denote by \( \gamma \) the thermal averaged rates.

Neglecting supersymmetry breaking in vertices:

\[ \epsilon(T) = \bar{\epsilon} \frac{c_u(T) - c_{\ell}(T)}{c_u(T) + c_{\ell}(T)} \equiv \bar{\epsilon} \times \Delta_{BF}(T), \]

where

\[ \bar{\epsilon} = \frac{\text{Im} A}{M} \frac{4\Gamma B}{4B^2 + \Gamma^2} \]

is the zero-temperature CP asymmetry arising from the scalar (or minus the fermion) loop contribution to the \( \tilde{N} \) self-energy. The thermal factors are

\[ c_{\ell}(T) = (1 - x_\ell - x_{\tilde{\ell}}) \lambda(1, x_\ell, x_{\tilde{\ell}})[1 - f_{\ell}^{eq}][1 - f_{\tilde{\ell}}^{eq}], \]
\[ c_u(T) = \lambda(1, x_h, x_{\tilde{h}})[1 + f_{h}^{eq}][1 + f_{\tilde{h}}^{eq}], \]

where

\[ f_{h,\ell}^{eq} = \frac{1}{\exp[E_{h,\ell}/T] - 1}, \quad f_{h,\ell}^{eq} = \frac{1}{\exp[E_{h,\ell}/T] + 1} \]

\(^4\) The factor 2 in equation (10) arises from the fact that there are two right-handed sneutrino states while we have defined \( Y_{\tilde{N}}^{eq} \) for one degree of freedom. Defined this way, \( \eta \) has the standard normalization \( \eta \to 1 \) for perfect out-of-equilibrium decay.
are the Bose–Einstein and Fermi–Dirac equilibrium distributions, respectively, and

\[ E_{\ell h} = \frac{M}{2} (1 + x_{\ell h} - x_{h,\ell}), \quad E_{h,\ell} = \frac{M}{2} (1 + x_{h,\ell} - x_{\ell h}), \quad (18) \]

\[ \lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x}, \quad x_a \equiv \frac{m_a(T)^2}{M^2}. \quad (19) \]

We remind the reader that, as seen above, the relevant CP asymmetry in soft leptogenesis is \( T \)- (i.e. time-) dependent even in the classical regime. This is so because the CP asymmetry is generated by the supersymmetry breaking thermal effects which make the relevant decay CP asymmetries into scalars and fermions different. In the absence of these thermal corrections, no asymmetry is generated.

The inclusion of quantum effects (for technical details see [23]–[25]) introduces an additional time dependence in the CP asymmetry:

\[ \epsilon(T) = \epsilon \times \Delta_{BF}(T) \times QC(t), \quad (20) \]

where

\[ QC(t) = \left[ 2 \sin^2 \left( \frac{M_+ - M_-}{2} t \right) - \frac{\Gamma_N}{M_+ - M_-} \sin \left( (M_+ - M_-) t \right) \right]. \quad (21) \]

Now, we simply have to change the variable from time \( t \) of equation (21) to a more convenient variable \( z \), as we do when writing down the BE as in equations (6)–(8). As a reminder to the readers, we will write a few lines illustrating this change. For a universe undergoing adiabatic expansion, the entropy per comoving volume is constant, i.e. \( sR^3 = \text{constant} \). Since \( s \propto z^{-3} \), we have \( R \propto z \). Then, the Hubble constant is given by \( H \equiv R^{-1} dR/dt = z^{-1} dz/dt \). After integration, we get

\[ t = \frac{1}{H(M)} \frac{z^2 - z_0^2}{2}, \quad (22) \]

where \( z_0 \) is the temperature at \( t = 0 \) and \( H(M) \equiv H(z = 1) = 2/3\sqrt{g^* \pi^3 / 5 (M^2 / m_{\text{pl}})} \), with \( m_{\text{pl}} \) being the Planck mass. Substituting equation (22) into (21), we obtain

\[ QC(T) = \left[ 2 \sin^2 \left( \frac{1}{2} \frac{M_+ - M_-}{2H(M)} z^2 \right) - \frac{\Gamma_N}{M_+ - M_-} \sin \left( \frac{M_+ - M_-}{2H(M)} z^2 \right) \right] \\
= \left[ 2 \sin^2 \left( \frac{m_{\text{eff}}}{m_s} R \frac{z^2}{8} \right) - \frac{2}{R} \sin \left( \frac{m_{\text{eff}}}{m_s} R \frac{z^2}{4} \right) \right], \quad (23) \]

where we set \( z_0 = 0 \) (i.e. at very high initial temperature). In writing the second equality we have used that \( M_+ - M_- = B \) (see equation (4)) and we have defined the degeneracy parameter \( R \):

\[ R = \frac{2(M_+ - M_-)}{\Gamma_N} = \frac{2B}{\Gamma_N}, \quad (24) \]

and

\[ m_s = \frac{8 v_u^2}{3 m_{\text{pl}}} \sqrt{\frac{g^* \pi^5}{5}} \simeq 7.8 \times 10^{-4} \text{ eV}. \quad (25) \]
Thus the final CP asymmetry consists of three factors. The first one is $\bar{\epsilon}$ in equation (14):

$$\bar{\epsilon} = \frac{\text{Im} A}{M} \frac{2R}{R+1},$$

which is resonantly enhanced for $R = 1$. The second one is the thermal factor $\Delta_{BF}(T)$ which is only non-vanishing for $z \gtrsim 0.8$ [19, 33]. The third one is the quantum correction factor, $QC(T)$, which is composed of two oscillating functions.

Next we turn to quantify the impact of this last additional quantum time dependence of the CP asymmetry on the final lepton asymmetry.

Before doing so, let us notice that equation (8) corresponds to the one-flavour approximation. As discussed in [6, 7, 9, 11, 14, 25], [29]–[32] the one-flavour approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. This is not the case in soft leptogenesis since successful leptogenesis in this scenario requires a relatively low RH neutrino mass scale. Thus the characteristic $T$ is such that the rates of processes mediated by the $\tau$ and $\mu$ Yukawa couplings are not negligible, implying that the effects of lepton flavours have to be taken into account [18].

However, as shown in [23]–[25] quantum effects are flavour-independent as long as the damping rates of the leptons are taken to be flavour-independent. In this case the $QC(T)$ factor becomes the one given above (neglecting also the difference in the width between the two sneutrinos) which is the same for all flavours. Furthermore quantum flavour correlations can be safely neglected for soft leptogenesis because $M \lesssim 10^{-9}$ GeV and therefore there is no transition between three-to-two or two-to-one flavour regimes. So, following [18], it is straightforward to include flavour in the QBE given above in terms of flavour-dependent washout factors. For the sake of simplicity in the presentation we restrict here to the one-flavour approximation for the study of the relevance of the quantum effects.

3. Results

We show in figure 1 the evolution of the lepton asymmetry with and without the quantum correction factor in the CP asymmetry for several values of the washout factor $m_{\text{eff}}$ and for the resonant case $R = 1$ and the very degenerate case $R = 2 \times 10^{-4}$. The two upper panels correspond to strong and moderate washout regimes, while the lower two correspond to weak and very weak washout regimes. We consider two different initial conditions for the sneutrino abundance. In one case, one assumes that the $\tilde{N}$ population is created by their Yukawa interactions with the thermal plasma and set $Y_{\tilde{N}}(z \to 0) = 0$. The other case corresponds to an initial $\tilde{N}$ abundance equal to the thermal one, $Y_{\tilde{N}}(z \to 0) = Y_{\tilde{N}}^{\text{eq}}(z \to 0)$. The initial condition on the sneutrino abundances can lead to differences in the weak washout regime. In the strong washout regime the asymmetry generated in the $\tilde{N}$ production phase for $Y_{\tilde{N}}(z \to 0) = Y_{\tilde{N}}^{\text{eq}}(z \to 0)$ is efficiently washed out (contrary to what happens in the weak washout regime). Consequently, in the strong washout regime the generated asymmetry is independent of the initial conditions. This behaviour is explicitly displayed in the upper panel of figure 1. It can also be observed on the right-hand side of the upper panels as well as on the upper curves of the lower panels of figure 3, and on the right-hand side of figure 4.
Figure 1. Absolute value of the lepton asymmetry with the quantum time dependence of the CP asymmetry (solid) and without it (dashed) as a function of $z$ for different values of $m_{\text{eff}}$ as labelled in the figure. In each panel the two upper curves (black) correspond to the resonant case $R = 1$ while the lower two curves (red) correspond to the very degenerate case $R = 2 \times 10^{-4}$. The left (right) panels correspond to vanishing (thermal) initial $\tilde{N}$ abundance. The figure is shown for $M = 10^7$ GeV and $\tan \beta = 30$, though, as discussed in the text, the results as normalized in the figure are very weakly dependent on those two parameters.
First we notice that, as expected, for strong washout and large degeneracy parameter $R$ (see the upper curves in the upper panels), the quantum effects lead to the oscillation of the produced asymmetry till it finally averages out to the classical value.

The figure also illustrates that, for very small values of $R$ and in the strong washout regime, quantum effects enhance the final asymmetry. For small enough $R$ the arguments in the periodic functions in $QC(T)$ are very small for all relevant values of $z$ and $m_{\text{eff}}$. So the $\sin^2$ term in $QC(T)$ is negligible and expanding the sin term we get

$$QC(T) \simeq -\frac{m_{\text{eff}} z^2}{m_*^2}$$

which, in the strong washout regime, is always larger than 1.

Also we see that, independently of the initial conditions and of the value of the degeneracy parameter, $R$, the quantum effects always lead to a suppression of the final produced asymmetry in the weak washout regime. This is at difference with what happens in see-saw resonant leptogenesis in which quantum effects lead to an enhancement of the produced asymmetry in weak washout and $R \sim 1$ and for zero initial sneutrino abundances \[24\].

The origin of the difference is the additional time dependence of the asymmetry in soft leptogenesis due to $\Delta_{BF}$. In order to understand this, we must remember that in see-saw resonant leptogenesis, in the weak washout regime, the final lepton asymmetry results from a cancellation between the anti-asymmetry generated when RH neutrinos are initially produced and the lepton asymmetry produced when they finally decay. When the time-dependent quantum corrections are included, this near-cancellation does not hold or it occurs at earlier times. As a consequence the asymmetry grows larger once these corrections are included as discussed in \[24\].

But in soft leptogenesis, even in the classical regime the thermal factor $\Delta_{BF}$ already prevents the cancellation from occurring. Therefore the inclusion of the time-dependent quantum effects only amounts to an additional multiplicative factor which, in this regime, is smaller than one.

This behaviour is explicitly displayed in figure 2 where we compare the absolute value of the lepton asymmetry with the quantum time dependence of the CP asymmetry and without it in soft leptogenesis with what would be obtained if the thermal factor $\Delta_{BF}(z)$ was not included (so that the CP asymmetry takes a form similar to the one for resonant see-saw). As seen in figure 2, without the $\Delta_{BF}(z)$ the asymmetry starts being produced at lower $z$ and it changes sign in the classical regime. This change of sign is due to the cancellation between the anti-asymmetry generated when RH neutrinos are initially produced and the lepton asymmetry produced when they finally decay. Inclusion of the $QC(T)$ factor reduces the asymmetry at small $z$ and this makes the cancellation to occur at lower $z$ and consequently the final asymmetry is larger.

In the full calculation (left panel in figure 2) the asymmetry only starts being non-negligible for larger $z$, i.e. $z \gtrsim 0.8$, and it changes sign for $z \sim 1$, both features due to

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\[5\] We notice in passing that for standard see-saw resonant leptogenesis the weak washout regime is physically unreachable as long as flavour effects are not included. This is so because there is a lower bound on the washout parameter once the washout associated with the two quasi-degenerate heavy neutrinos contributes, which implies that $m_\nu > \sqrt{\Delta m^2_{\text{atm}}} \sim 8 \times 10^{-3}$ [34]. Such a bound does not apply to soft leptogenesis as long as, as assumed in this work, only the lightest sneutrino generation contributes.
the $\Delta_{BF}$ factor. Inclusion of the quantum correction, $QC(T)$, amounts for a suppression of the initial asymmetry by a factor given in equation (27). As a consequence the final asymmetry is suppressed (and it also has the opposite sign) after including the quantum corrections.

A more systematic dependence of the results with the washout and degeneracy parameters, $m_{\text{eff}}$ and $R$, is shown in figure 3 where we plot the efficiency factor $\eta$ as a function of $m_{\text{eff}}$ and $R$. We remind the reader that, within our approximations for the thermal widths, in the classical regime, $\eta$ is mostly a function of $m_{\text{eff}}$ exclusively\textsuperscript{6}. Inclusion of the quantum correction $QC(T)$ makes $\eta$ to depend both on $m_{\text{eff}}$ and $R$ but still remains basically independent of $M$.

From the figure we see that, for small enough values of the product of the washout parameter and the degeneracy parameter, the arguments of the periodic functions in $QC(T)$ are always small in the range of $z$ where the lepton asymmetry is generated. As explained above, in this regime the $\sin^2$ term in $QC(T)$ is negligible while the $\sin$ term is multiplied by an amplitude proportional to $1/R$. Therefore, the dependence on $R$ cancels in this limit and the resulting correction is given in equation (27). This explains the plateaux observed at low values of the degeneracy parameter $R$ in the lower panels of figure 3. Similar behaviour is found in [25] for the resonant leptogenesis scenario. Also, as seen in equation (27), the correction grows with $m_{\text{eff}}$, which leads to the considerable enhancement of the efficiency seen in the upper curves of the lower panel in figure 3. However, we must notice that this enhancement occurs in a regime where the CP asymmetry is very small due to the small value of $R$ since $\bar{\epsilon}$ is proportional to $R$.

\textsuperscript{6} There is a residual dependence on $M$ due to the running of the top Yukawa coupling as well as the thermal effects included in $\Delta_{BF}$ although it is very mild. For $\tan\beta \sim O(1)$ there is also an additional (very weak) dependence due to the associated change in the top Yukawa coupling.
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Figure 3. Efficiency factor as a function of $m_{\text{eff}}$ and $R$ for $M = 10^7$ GeV and $\tan \beta = 30$. The left (right) panels correspond to vanishing (thermal) initial $N$ abundance. In the upper panels the different curves correspond to $R = 1$ (black thick solid), $0.1$ (dashed), $10^{-2}$ (dotted), $10^{-3}$ (dashed-dotted) and $10^{-4}$ (thin solid). For comparison we also show the results without including the quantum effects (purple thick solid line). In the lower panels we plot the ratio of the efficiency factor with and without quantum corrections as a function of $R$. The different curves from top to bottom correspond to $m_{\text{eff}} = 10^{-1}$ eV (thick solid), $10^{-2}$ eV (dashed), $10^{-3}$ eV (dotted), $10^{-4}$ eV (dotted-dashed) and $10^{-5}$ eV (thin solid).

Finally, in figure 4 we compare the range of parameters $B$ and $m_{\text{eff}}$ for which enough asymmetry is generated, $Y_B \geq 8.54 \times 10^{-11}$, with and without inclusion of the quantum corrections. We show the ranges for several values of $M$ and for the characteristic value of $|\text{Im} A| = 1$ TeV. From the figure we see that, due to the suppression of the asymmetry for the weak washout regime discussed above, for a given value of $M$ the regions extend only up to larger values of $m_{\text{eff}}$ once the quantum corrections are included. Also, because of the enhancement in the very degenerate, strong washout regime, the regions tend to extend to lower values of $B$ and larger values of $m_{\text{eff}}$ for a given value of $M$. Furthermore,
Once quantum effects are included, $\eta$ can take both signs (depending on the value of $m_{\text{eff}}$), independently of the initial $\tilde{N}$ abundance. Thus it is possible to generate the right sign asymmetry with either sign of $\text{Im} A$ for both thermal and zero initial $\tilde{N}$ abundance. In contrast, without quantum corrections, for thermal initial conditions $\eta > 0$ and the right asymmetry can only be generated for $\text{Im} A > 0$.

4. Summary

In this paper we have performed a detailed study of the role of quantum effects in the soft leptogenesis scenario. We have studied the effects on the produced asymmetry as a function of the washout parameter $m_{\text{eff}}$ and the degeneracy parameter $R = 2\Delta M/\Gamma_{\tilde{N}}$. Our results show that, because of the thermal nature of soft supersymmetry, the characteristic
time for the building of the asymmetry is larger than in the see-saw resonant leptogenesis which leads to quantitative differences on the dependence of the effect on the washout regime between the two scenarios. In particular, in the weak washout regime, quantum effects do not enhance but suppress the produced lepton asymmetry in soft leptogenesis. Quantum effects are most quantitatively import for extremely degenerate sneutrinos $\Delta M \ll \Gamma_{\tilde{N}}$. In this case and in the strong washout regime quantum effects can enhance the absolute value of the produced asymmetry as well as induce a change of its sign. But, altogether, our results show that the required values of the Majorana mass $M$ and the lepton-violating soft bilinear coefficient $B$ to achieve successful leptogenesis are not substantially modified.

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