Consistent Decoupling of Heavy Scalars and Moduli in $\mathcal{N} = 1$ Supergravity

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We consider the conditions for integrating out heavy chiral fields and moduli in $\mathcal{N} = 1$ supergravity, subject to two explicit requirements. First, the expectation values of the heavy fields should be unaffected by low energy phenomena. Second, the low energy effective action should be described by $\mathcal{N} = 1$ supergravity. This leads to a working definition of decoupling in $\mathcal{N} = 1$ supergravity that is different from the usual condition of gravitational strength couplings between sectors, and that is the relevant one for inflation with moduli stabilization, where some light fields (the inflaton) can have long excursions in field space. It is also important for finding de Sitter vacua in flux compactifications such as LARGE volume and KKLT scenarios, since failure of the decoupling condition invalidates the implicit assumption that the stabilization and uplifting potentials have a low energy supergravity description.

We derive a sufficient condition for supersymmetric decoupling, namely, that the Kähler invariant function $G = K + \log |W|^2$ is of the form $G = L(\text{light}, H(\text{heavy}))$ with $H$ and $L$ arbitrary functions, which includes the particular case $G = L(\text{light}) + H(\text{heavy})$. The consistency condition does not hold in general for the ansatz $K = K(\text{light}) + K(\text{heavy})$, $W = W(\text{light}) + W(\text{heavy})$ and we discuss under what circumstances it does hold.

The viability of theories based on extra dimensions, in particular string theory, relies on being able to stabilize and integrate out the fields (moduli) that describe the shapes and sizes of those extra dimensions, for which so far there is no observational evidence. In flux compactifications some moduli are stabilized at a high energy scale and decouple from the low energy theory. From that moment on we never see them in the effective low energy description.

Unlike in global supersymmetry, complete decoupling is of course impossible in supergravity—even in principle—because gravity couples to all fields; so at low energies one is usually satisfied with gravitational strength couplings between the heavy, stabilized, fields and the low energy fields. However such interaction terms are of order $O(G_{\text{Newton}} E^2) = O(E^2/M_p^2)$, where $E$ is the energy scale and $M_p \approx 2.4 \times 10^{18}\text{GeV}$ the reduced Planck mass. Even if they are strongly suppressed at low energy and in particle accelerators, these couplings become sizeable at the energy scales relevant to the early Universe, and one must look for a more robust definition of decoupling that can be extrapolated over a wide range of energy scales.

The purpose of this note is to explain some of the difficulties encountered in supergravity models of inflation with moduli stabilization. The problem essentially disappears for consistently decoupled moduli (see \cite{2,3}).

**NOTATION AND CONVENTIONS**

We will use units in which $M_p = 1$. We start by recalling that the $\mathcal{N} = 1$ supergravity action involving scalars and gauge fields (chiral and gauge superfields)

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} R + T - V + \mathcal{L}_{\text{gauge}} \right) \quad (1)$$

is entirely described by three functions of the scalars: the Kähler potential $K(z, \bar{z})$, the holomorphic superpotential $W(z)$ and the gauge kinetic functions $f_{ab}(z)$. The action and the supersymmetry transformations are invariant under Kähler transformations,

$$K \rightarrow K + h(z) + \bar{h}(\bar{z}) \quad W \rightarrow W e^{-h(z)} \quad (2)$$

with $h(z)$ an arbitrary holomorphic function. Actually, if $W \neq 0$, they only depend on the Kähler invariant function $G = K + \log |W|^2$ and the gauge kinetic functions.
We are interested in the case in which the resulting effective theory is also described by $N = 1$ supergravity. In this case, there should be an effective $K$ and $W$ (or $G$) depending only on the light fields, from which to compute the low energy action $S$ and supersymmetry transformations

$$G[L, \mathcal{T}] = \hat{G}[L, \mathcal{T}, H_0, \mathcal{P}_0]$$

$$\delta_e L = \hat{\delta}_e L|_{H_0} = f[L, G(L, \mathcal{T})]$$

$$\hat{\delta}_e H|_{H_0} = 0.$$  

Notice that the F-terms (eq. 6m) of the heavy fields must vanish because the supersymmetry transformations read,

$$\hat{\delta}_e H \sim \chi \epsilon, \quad \hat{\delta}_e \chi \sim \phi H \epsilon - \frac{1}{2} F \epsilon$$

and if the F-terms are non-zero a supersymmetry transformation will generate light fermions that are not in the low energy effective action. Thus, the heavy fields cannot contribute to supersymmetry breaking, leading to

$$\partial_H \hat{G}|_{H_0} = 0 \quad \text{or} \quad \hat{D}_H \hat{W}|_{H_0} = 0,$$  

(see also [4]) where $\hat{D}_H \hat{W} = \partial_H \hat{W} + (\partial \hat{K}) \hat{W}$ is the Kähler covariant derivative that transforms as $\hat{D}_H \hat{W} \rightarrow e^{-h(z)} \hat{D}_H \hat{W}$ under Kähler transformations. Note that $\hat{D}_H \hat{W} = 0$ is the condition used in flux compactifications [1] and by extension in KKLT [2] and LARGE volume scenarios [3], where the complex structure moduli are stabilized at a supersymmetric point before uplifting.

The Kähler metric should be block diagonal in the light and heavy fields when evaluated at $H_0$, otherwise propagators will mix these two sets of fields. Additionally, the truncation $H = H_0$ must of course be a consistent truncation. This means that the equations of motion of the light fields derived from the effective theory are the same as the equations of motion obtained from the full theory. To zeroth order in the fluctuations of the heavy fields:

$$\frac{\delta \hat{S}}{\delta L}|_{H_0} = \frac{\delta \hat{S}|_{H_0}}{\delta L} = \frac{\delta S}{\delta L}. $$

ensuring that the fluctuations of $H$ are not sourced by the light fields. In particular, the heavy fields should be singlets under the surviving gauge group at low energies (otherwise they remain coupled to the light fields by the gauge interaction). In what follows we will consider $f_{ab}$ independent of the heavy fields. In that case they do not contribute to the D-terms, which will only involve light fields.
ANALYSIS OF THE CONSISTENCY CONDITIONS

The heavy fields thus need to be stabilized at an expectation value \( H_0 \), where \( H_0 \) is the solution to eq. \( \ref{eq:const} \)

\[
\left. \partial_H \hat{W}(H, L) + \partial_H \hat{K}(H, \overline{H}, L, \overline{L}) \hat{W}(H, L) \right|_{H_0} = 0. \tag{15}
\]

which implies \( \partial_H \hat{W} \big|_{H_0} = 0 \). The LHS is some function of both the heavy and the light fields, let us call it \( \Phi(H, \overline{H}, L, \overline{L}) \). In general, the condition \( \Phi = 0 \) (together with its complex conjugate \( \overline{\Phi} = 0 \)) relate the heavy and light fields. If we can solve for \( H \) we obtain an expression of \( H_0 \) as a function of the light fields,

\[
H = H_0(L, \overline{L}), \tag{16}
\]

which can be substituted back into \( \hat{K}, \hat{W} \) to give an effective action for the light fields

\[
S(L, \overline{L}) = \hat{S}(H_0(L, \overline{L}), \overline{H_0}(L, \overline{L}), L, \overline{L}). \tag{17}
\]

An immediate concern with the consistency of this procedure, pointed out in \[4\], is that in general this leads to a non-holomorphic expression for the would-be effective superpotential \( W = \hat{W}(H_0(L, \overline{L}), L) \). However, this problem is easily avoided: it does not arise if \( \hat{W} \) is independent of \( H \). The case \( \hat{W} = 0 \) is obvious, so consider \( \hat{W} \neq 0 \). It is always possible to perform a Kähler transformation that makes \( \hat{W} \) constant

\[
\hat{W} \to 1 \tag{18a}
\]

\[
\hat{K} \to \hat{K} + \log \hat{W} + \log \hat{W} = \hat{G}. \tag{18b}
\]

In this so called Kähler gauge, eq. \( \ref{eq:const} \) reads

\[
\partial_H \hat{G}(H, \overline{H}, L, \overline{L}) = 0, \tag{19}
\]

from which we can extract \( H = H_0(L, \overline{L}) \) and make the previous substitution directly into the Kähler invariant function without any inconsistency (see also \[10\]):

\[
G = \hat{G}(H_0(L, \overline{L}), \overline{H_0}(L, \overline{L}), L, \overline{L}). \tag{20}
\]

In fact, the issue is not whether \( H_0(L, \overline{L}) \) is holomorphic but rather whether it is a (non-trivial) function at all. The assumption that the heavy fields are stabilized at \( H = H_0 \) is simply the condition that \( H_0(L, \overline{L}) \) is constant. Any other dependence on the light moduli would translate into a constraint on the light fields which would have to be accounted for explicitly in the low energy action \[3\]. This is what we have to avoid.

To summarize: the (rather obvious) mathematical condition for the heavy fields to be integrated out consistently with an expectation value \( H_0 \) and to decouple from the low energy fields is that the system of equations

\[
\partial_H \hat{G} \equiv \Phi(H, \overline{H}, L, \overline{L}) = 0, \tag{21}
\]

which is the same as \( \ref{eq:const} \) defined in the Kähler gauge (eq. \( \ref{eq:const} \)), admits the constant solution

\[
H = H_0(L, \overline{L}) = \text{const} \quad \overline{H} = \overline{H_0}(L, \overline{L}) = \text{const}. \tag{22}
\]

In spite of being obvious, this condition is not empty. For instance, we will see below that it fails generically for standard couplings of the form \( K = K_1 + K_2 \) and \( W = W_1 + W_2 \). But let us first consider two specific situations in which the decoupling condition does hold.

1. The consistency condition is trivially satisfied if the function \( \Phi(H, \overline{H}, L, \overline{L}) \) has no explicit dependence on the light fields. In this case integrating eq. \( \ref{eq:const} \) recovers the condition found in \[6\]

\[
\partial_H \hat{G} = \Phi(H, \overline{H}) \rightarrow \hat{G} = \hat{G}_1(H, \overline{H}) + \hat{G}_2(L, \overline{L}) \tag{23}
\]

and it is obvious that the Kähler metric is block diagonal in this case. This ansatz has a long history \[11\] and allows a detailed stability analysis of the heavy fields \[3\ \[12\], in particular in the context of F-term uplifting of flux compactifications.

2. On the other hand, this requirement is too restrictive. It is sufficient if the function \( \Phi(H, \overline{H}, L, \overline{L}) \) factorizes:

\[
\Phi(H, \overline{H}, L, \overline{L}) = \Phi_1(H, \overline{H}) \Phi_2(L, \overline{L}) = 0 \tag{24}
\]

in which case we just solve \( \Phi_2 = \overline{\Phi}_2 = 0 \) to get constant \( H_0, \overline{H}_0 \). We cannot give the general form of \( G \) for which this factorization occurs, but it will certainly hold if \( G \) has the following functional form:

\[
\hat{G} = f(L, \overline{L}, g(H, \overline{H})) \tag{25}
\]

since in that case eq. \( \ref{eq:const} \) is replaced by

\[
\partial_H g(H, \overline{H}) = 0. \tag{26}
\]

The first situation, eq. \( \ref{eq:const} \), is a special case of eq. \( \ref{eq:const} \), with \( \Phi_1 \) constant. In both cases, the same condition that makes \( \hat{G}(H)|_{H_0} = 0 \) also implies that the Kähler metric and the Hessian of \( V \) are block diagonal for any \( \Phi_1 \). Indeed, from equation \( \ref{eq:const} \) we find that

\[
\hat{G}_{LH}|_{H_0} = \partial_L \partial_g f(L, \overline{L}, g(H, \overline{H})) \partial_H g(H, \overline{H})|_{H_0} = 0 \tag{27}
\]

and further all mixed derivatives with only one derivative with respect to the heavy field vanish. As \( V_{LH} \) always contains terms \( \propto \hat{G} \) or \( \propto (\partial_L)^n \hat{G} \), which vanish at \( H_0 \), the Hessian of \( V \) is block diagonal. \footnote{Note that it is always possible to diagonalize the Kähler metric or the Hessian of \( V \) at one point, but it is not necessarily the case that both diagonalizations are compatible, as we have here.}
CONSISTENT DECOUPLING VERSUS STANDARD GRAVITATIONAL COUPLINGS

Finally, we stress that the condition derived here has no direct relation to the condition usually associated with gravitational strength coupling. In fact, the ansatz

\[
\hat{K} = K_1(H, \overline{H}) + K_2(L, \overline{L}) \quad (28a)
\]

\[
\hat{W} = W_1(H) + W_2(L) \quad (28b)
\]
does not satisfy the decoupling condition in general. Suppose eq. (13) admits a constant solution \( H = H_0 \). Then

\[
0 = \partial_H W_1|_{H_0} + \partial_H K_1|_{H_0} [W_1(H_0) + W_2(L)] , \quad (29)
\]

which only holds if

\[
\partial_H K_1|_{H_0} = 0 \Rightarrow \partial_H W_1|_{H_0} = 0
\]

\[
\partial_H K_1|_{H_0} \neq 0 \Rightarrow W_2(L) = - \frac{\partial_H W_1|_{H_0}}{\partial_H K_1|_{H_0}} - W_1(H_0) = \text{const.} \quad (30)
\]

Another way to see this: since \( \hat{D}_H \hat{W} = 0 \) does not factorize, the (Kähler-gauge covariant) requirement that it is independent of the light fields is (see also [13])

\[
\hat{D}_L(\hat{D}_H \hat{W}) = 0 . \quad (31)
\]

Inserting the ansatz (eq. 28) then gives

\[
\partial_H K_1|_{H_0} \partial_L W_2 = 0 . \quad (32)
\]

Unless \( K_1(H, \overline{H}) \) has no linear terms or \( W_2(L) = \text{constant} \), the condition will not be met. However, if \( W_2(L) = \text{constant} \) (e.g. no scale models [1, 14]) then equation (28) holds and \( \hat{W} \) is trivially a product. On the other hand, we can always expand \( K_1(H, \overline{H}) \) around \( H_0 \) and remove the linear terms by a Kähler transformation (eq. 2), but this spoils the separability of the superpotential (eq. 28).

In other words, if two sets of fields have separable Kähler functions \( K = K_1(\text{heavy}) + K_2(\text{light}) \), the addition of their superpotentials does not respect the decoupling condition except in special cases (and, incidentally, neither does it guarantee gravitational strength couplings if \( K_1(\text{heavy}) = O(M_p^2) \), as is usual for moduli).

DISCUSSION

In this Letter we have studied how to integrate out heavy scalars and moduli and their superpartners in \( \mathcal{N} = 1 \) supergravity, subject to two explicit requirements. First, the expectation values of the heavy fields should be unaffected by low energy phenomena, in particular supersymmetry breaking. Second, the low energy effective action should be described by \( \mathcal{N} = 1 \) supergravity. This is what we call consistent decoupling.

If the heavy fields are stabilized at a critical point of the potential, integration of the whole superfield requires that the F-terms should be zero [6]. The criterion for consistent decoupling is that the expectation value of the heavy scalars \( H \) should not depend on the light fields \( L \) [5]. Our main result is a class of Kähler invariant functions that satisfy the condition, given in eq (25):

\[
\tilde{G} = f(L, \overline{L}, g(H, \overline{H})) .
\]

This functional form guarantees that the Kähler metric and Hessian of \( V \) are simultaneously block diagonal in the heavy and light fields. It also allows the embedding of BPS solutions of the low energy effective theory into the full theory without destroying their BPS character (if the F-terms of the heavy fields are zero and in the absence of constant Fayet-Iliopoulos terms, the supersymmetric transformation of the gravitino depends only on the light fields). We would expect the BPS character to survive quantum corrections - now in the full theory - So at least in this special case it would seem possible to “screen” the heavy, decoupled fields from the effects of (partial) supersymmetry breaking in the low energy sector.

We only have experimental access to \( G \), the effective low energy theory, and there is a large class of supergravity models (read a landscape of compactifications), characterized by \( G \), in which the low energy theory could be embedded. Here, \( \tilde{G} \) includes all stringy, perturbative and non-perturbative effects. The decoupling condition restricts the allowed functional form of \( \tilde{G} \) and therefore the class of models that are consistent with the assumption of decoupling that is implicit in our use of \( G \). From the point of view of model building, it provides a simple test that has not been considered before. There are string compactifications which approximately satisfy the decoupling condition in the form [23], such as some LARGE volume scenarios (LVS) [2, 15, 16, 17].

To see this, note first of all that the tree level or GKP limit [1] of \( \tilde{G} \) satisfies eq. (23) with the complex structure moduli and the dilaton \( S \) playing the role of the heavy fields. Assume the usual form for the leading non-perturbative and \( s' \) corrections, \( \hat{W} = W_{\text{GKP}}(H) + W_{\text{np}}(L) \). However, \( G^{\text{KPS}} \sim 2(S + \overline{S})^{3/2} / \text{vol} \). Ignoring for a moment the dilaton dependence of \( \delta \hat{K} \), we find for the complex structure moduli

\[
\partial_H \hat{G} = \partial_H K_{\text{heavy}}(H) + \frac{\partial_H W_{\text{GKP}}(H)}{W_{\text{GKP}}(H)} \left[ 1 + \delta(L, H) \right]^{-1} ,
\]

where \( \delta = W_{\text{np}}(L)/W_{\text{GKP}}(H) \). Including dilaton effects adds a correction \( \delta \sim (S + \overline{S})^{3/2} / \text{vol} \) (whichever is larger). The condition of consistent decoupling is violated by the \( L \)-dependence of \( \delta \). It is negligible, \( \delta \sim O(10^{-10}) \), for an LVS vacuum with parameters \( A \sim 1, W_{\text{GKP}}(H_0) \sim 10, \).
vol \sim 10^{10}, A e^{-a_4 \tau_4} \sim 1/vol \text{ (see } \text{[9]).} \\text{2 In the mirror mediation scenarios } \delta \text{ is even smaller. By constrast, } \\
\delta \sim O(1) \text{ in a KKLT vacuum with parameters } A \sim O(1), \\
W_{\text{GKP}}(H_0) \sim O(10^{-4}), aL \sim O(10) \text{ (see } \text{[2]).} \\
\text{Finally, we emphasize that the condition (eq. } 25 \text{) is} \\
\text{not easily expressed in terms of } K \text{ and } W, \text{ in particular it} \\
\text{has nothing to do with gravitational strength couplings.}

\text{When } K = K_1(\text{heavy}) + K_2(\text{light}), \text{ the addition of superpotentials does not lead to consistent decoupling in} \\
\text{general (whereas the product always does). The problem} \\
\text{considered here illustrates once again the dangers of ex-} \\
\text{trapolating our low energy, weak gravity intuition, based} \\
\text{on } K \text{ and } W, \text{ to the very high energy regimes encoun-} \\
\text{tered in the early Universe. Inflation model building is} \\
\text{hard enough as it is without these unnecessary compli-} \\
\text{cations.}

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\text{2 [Refs. } 18, 19 \text{] suggest that string loop corrections to } K \text{ scale} \\
\text{as } (\text{vol})^{-2/3} \text{ and would lead to } \delta < 10^{-6}. \text{ We thank M. Cicoli} \\
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\text{[1] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev.} \\
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