Using Linear Equations Approach to Formulate Balanced and Least-Cost Ration for Dairy Cattle

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Abstract

The main objective of this study was to find an effective way to use linear equations for the purpose of creating a balanced ration at the lowest possible cost for dairy cattle by using linear equations and solving these equations through the Solver tool provided by MS-Excel. Samples of barley, corn, wheat bran, soybean meal and wheat straw were collected from the local markets and the necessary chemical analyzes were performed for them. After that the mathematical formulas of the linear equations were developed according to the specified constraints for crude protein ratio, value of metabolizable energy and the percentage of each calcium and phosphorus, which meets the needs of a medium-production dairy cow (15 kg) and weighting (650 kg), then the data was entered into Microsoft Excel and the equations were solved by Solver tool. The results showed a superior ability of linear equations to solve the problems consisting of several variables where the feed was formed by mixing barley, Corn, wheat bran, soybean meal, wheat hay, calcium phosphate and salt in proportions (5, 17.91, 50, 10.76, 13.66, 1.64 and 1) respectively, The cost of the feed mixture was (268.6 $/ton), which is the lowest possible cost for a ration that meets the required needs, linear programming will provide the animal breeders efficiency with the highest production by reducing the costs and balancing of the ration through the steps described in the search.

Keywords: Animal Nutrition, Dairy Cattle, Linear Equations, Linear programming, Ration Cost, Solver.

1. Introduction

The cost of feeding cows is the main factor that determines the value of profits or losses during the production process in dairy cattle, as it constitutes about 60 - 80% of the total cost of milk production [1]. This means that forming of ration at the lowest possible cost will have the largest impact on increasing profits compared to the rest of the costs and it will encourage more investments in dairy cattle as well., but reducing the cost of the diet should be determined on the condition that the nutritional requirements of the cow are met. [2] and [3], stated that milk production in dairy cows needs a delicate balance between protein and energy in the ration, in addition to the milk production it has a big effect on subfertility [4,5]. In addition to the importance of providing protein and energy in correct ratios in the diets of dairy cattle, calcium and phosphorus come second in importance [6], as the low level of calcium in the blood causes a disease known as milk fever, especially in the early stage of lactation after birth. This disease is of critical importance for dairy cattle welfare and economy[7-8], [9], indicated that the energy and protein balance have a wide impact on milk production and fertility, as well as that the proportion of calcium and phosphorous in the blood serum is highly sensitive and responds to a different ratio in the rations and the amount of produced milk. So it is clear that creating a diet with several variables at the lowest possible cost requires mathematical equations of a certain type to solve them. in this case it will be the linear equations which is a mathematical procedure that can be used for obtaining a value-weighting solution to a several of simultaneous equations [10]. In linear programming the equations can be solved using the simplex method, GRG nonlinear method and Evolutionary method according to the types of the equations, simplex is a method invented by George Bernard Dantzig in 1947 [11] and it is considered the appropriate method to solve the equations in this research. These equations could be solved with solver tool from MS-Excel which is demonstrated on [12] which is a computerized linear programming.

2. Material and Method

2.1 Chemical analysis

Samples of barley, Maize, wheat bran, soybean meal and wheat straw were collected from local markets of Erbil, Sulaymaniyah and Halabja, and mixed to obtain a representative sample(note: most of the feedstuffs are imported). Crude Protein (CP) ratio was analyzed according to [13], while the value of metabolite energy (ME) and the ratio of both of calcium (Ca) and phosphorous (P) were according to [14]. The chemical analyses of feed stuffs are shown in Table (1).
Table 1. The chemical analyses and prices of feed stuffs and cow requirements.

| Feed stuff | ME | CP | Ca | P | Pr | X |
|------------|----|----|----|---|----|---|
| F1         | ME1| CP1| Ca1| P1| Pr1| X1|
| F2         | ME2| CP2| Ca2| P2| Pr2| X2|
| F3         | ME3| CP3| Ca3| P3| Pr3| X3|
| F4         | ME4| CP4| Ca4| P4| Pr4| X4|
| F5         | ME5| CP5| Ca5| P5| Pr5| X5|
| F6         | ME6| CP6| Ca6| P6| Pr6| X6|
| F7         | ME7| CP7| Ca7| P7| Pr7| X7|
| UL         | UL1| UL2| LL3| LL4| Minimum | X total = 100 |

2.2 The prices of feed stuffs

Prices represent the average of the prices of feed stuffs in a number of different places in the Kurdistan Region – Iraq (Erbil, Sulaymaniyah and Halabja), the prices of feed stuffs are shown in table (1).

2.3 Nutritional requirements

The requirements estimated on the basis of the cow’s weight 650 kg and 15 kg/day milk production which represents the average production of local cattle. The metabolizable energy is 11 MJ/kg of ration and the crude protein level is 16% as shown in Table (1).

1. According to [14].
2. According to actual chemical analyses.
3. Actual prices $ in the local markets of Kurdistan Region-Iraq (Erbil, Sulaymaniyah and Halabja)
4. These are the requirements of a cow that weights 650 kg and produces 15 kg of milk/day according to [14].

2.4 Linear modelling

The following table (2) shows the abbreviations that will be used to express the feed stuffs, their chemical analysis, prices and the limits of their use.

Table 2. The abbreviations that represent chemical analysis, feed prices, and the upper and lower limits of cow’s requirements.

| Feed stuff      | ME (MJ/kg) | CP1 % | Ca1% | P1% | price$/kg feed stuff |
|-----------------|------------|-------|------|-----|----------------------|
| Barly           | 12.8       | 11.2  | 0.05 | 0.4 | 0.35                 |
| Maize           | 14.2       | 9.4   | 0.03 | 0.27| 0.35                 |
| wheat bran      | 10.1       | 17    | 0.16 | 0.136| 0.25                |
| SBM             | 13.3       | 44.5  | 0.35 | 0.68| 0.45                 |
| wheat straw     | 6.1        | 3.4   | 0.45 | 0.07| 0.1                  |
| Calcium phosphate| 0         | 0     | 22   | 18  | 0.02                 |
| Salt            | 0          | 0     | 0    | 0   | 0.1                  |
| Cow requirements/ kg | 11  | 16    | ≥ 0.55 | ≥ 0.4 |                     |

F1 = barly, F2 = maize, F3 = wheat bran, F4 = SBM, F5 = wheat straw, F6 = Calcium phosphate, F7 = salt, ME = Metabolizable energy, CP = Crude protein Ca = Calcium P = Phosphorus, Pr = Price, x = Quantity of feed stuff, UL= Upper limit, LL= Lower limit.
2.4.1 The objective

The objective is set to get the lowest ration price

\[ Pr1 \times X1 + Pr2 \times X2 + Pr3 \times X3 + Pr4 \times X4 + Pr5 \times X5 + Pr6 \times X6 + Pr7 \times X7 = \text{minimum} \]

2.4.2 The constraints

The 1st constraint: The aim of this constraint is to set the maximum acceptable level of metabolizable energy in the ration.

\[ ME1 \times X1 + ME2 \times X2 + ME3 \times X3 + ME4 \times X4 + ME5 \times X5 + ME6 \times X6 + ME7 \times X7 \leq UL1 \]

The 2nd constraint: The aim of this constraint is to set the minimum acceptable level of metabolizable energy in the ration.

\[ ME1 \times X1 + ME2 \times X2 + ME3 \times X3 + ME4 \times X4 + ME5 \times X5 + ME6 \times X6 + ME7 \times X7 \geq LL1 \]

The 3rd constraint: The aim of this constraint is to set the maximum acceptable level of protein in the ration.

\[ CP1 \times X1 + CP2 \times X2 + CP3 \times X3 + CP4 \times X4 + CP5 \times X5 + CP6 \times X6 + CP7 \times X7 \leq UL2 \]

The 4th constraint: The aim of this constraint is to set the minimum acceptable level of protein in the ration.

\[ CP1 \times X1 + CP2 \times X2 + CP3 \times X3 + CP4 \times X4 + CP5 \times X5 + CP6 \times X6 + CP7 \times X7 \geq LL2 \]

The 5th constraint: The aim of this constraint is to set the minimum acceptable level of calcium in the ration.

\[ Ca1 \times X1 + Ca2 \times X2 + Ca3 \times X3 + Ca4 \times X4 + Ca5 \times X5 + Ca6 \times X6 + Ca7 \times X7 \geq LL3 \]

The 6th constraint: The aim of this constraint is to set the minimum acceptable level of phosphorus in the ration.

\[ P1 \times X1 + P2 \times X2 + P3 \times X3 + P4 \times X4 + P5 \times X5 + P6 \times X6 + P7 \times X7 \geq LL4 \]

The 7th constraint: The aim of this constraint is for the summation of the quantities to equal one hundred.

\[ X1 + X2 + X3 + X4 + X5 + X6 + X7 = 100 \]

The 8th constraint: The aim of this constraint is that the quantity of any feed item not to be more than 50%.

\[ X1, X2, X3, X4, X5 \leq 50 \]

The 9th constraint: The aim of this constraint is that the quantity of any feed item not to be less than 5%.

\[ X1, X2, X3, X4, X5 > 5 \]

The 10th constraint: The aim of this constraint is that the quantity of calcium phosphate not to be more than 3%.

\[ X6 \leq 3 \]

2.5 Setting up the data and the equations

The following is the steps of setting up the data and equations in MS-Excel tables, Figure (1) illustrates how the table looks like:

Feed stuffs listed from A2:A8
Metabolizable energy listed from B2:B8
Crude protein ratio listed from C2:C8
Calcium ratio listed from D2:D8
Phosphorus ratio listed from E2:E8
Price/Kg of feed stuffs listed from F2:F8
Quantities of feed stuffs in ration listed from G2:G8
Upper limits of (ME, CP, Ca, P) listed in (B9, C9, D9, E9) respectively
Lower limits of (ME, CP, Ca, P) listed in (B10, C10, D10, E10) respectively
Cost/Quantities:

Barley (H2) = F2*G2
Maize (H3) = F3*G3
Wheat bran (H4) = F4*G4
SBM (H5) = F5*G5
Wheat straw (H6) = F6*G6
Calcium Phosphate (H7) = F7*G7  
Salt (H8) = F8*G8  

Total quantities (G11) = SUM(G2:G8)  
Total cost/100 Kg (H12) = SUM(H2:H8)  

Ration information after formulation:

- Metabolizable energy (B13) = \( \frac{B2*G2+B3*G3+B4*G4+B5*G5+B6*G6+B7*G7+B8*G8}{100} \)  
- Crude protein ratio (B14) = \( \frac{C2*G2+C3*G3+C4*G4+C5*G5+C6*G6+C7*G7+C8*G8}{100} \)  
- Calcium ratio (B15) = \( \frac{D2*G2+D3*G3+D4*G4+D5*G5+D6*G6+D7*G7+D8*G8}{100} \)  
- Phosphorus ratio (B16) = \( \frac{E2*G2+E3*G3+E4*G4+E5*G5+E6*G6+E7*G7+E8*G8}{100} \)

**Figure 1.** Illustrates setting up the data and the equations cells in MS-Excel.

### 2.6 Working on Solver tool

At first the user must make sure that the Solver tool is available under Data menu bar, otherwise, you need to go through file menu > Options > Add-Ins > Go > select Solver Add-Ins and press OK. Open Solver tool from Data menu bar and set (H12) cell as an objective and press on (Min) option in order to formulate least cost ration, then select the variable cells (G2:G8) as shown in the Figure (2).

**Figure 2.** Illustrates setting up the data in Solver tool.

At last add the constraints through pressing on add button, each time the add button is pressed a new dialog box will appear and it will be possible to add one constraint only as shown in the Figure (3).
After setting up all of objectives, variable cells, and constraints as shown in the Figure (2), We must select simplex LP from solving method as a linear programming method for the equations, the equations will be solved through pressing solve button and the final results will be shown as in Figure (4) which include the quantities of each feed stuff, cost/quantities, total quantities, total cost/100kg, the value of each of CP%, ME, Ca% and P%.

![Figure 3. Illustrates setting up the constraints.](image)

![Figure 4. Illustrates the final results.](image)

2.7 comparison with alternative approach

In the second part, in order to compare the results obtained from the computerized linear equations with what the human brain can accomplish, a form was given to two lecturers (experts) of the Animal Production Department / College of Agricultural Engineering Sciences / University of Duhok who hold a doctorate in the animal nutrition field. The given form to the experts includes the same chemical analysis and prices of feed stuffs, as well as the same eleven constraints used in preparing the ration by using computerized linear equations and they were asked to prepare the ration according to these constraints.

3. Results and Discussion

3.1 Computerized linear programming

The results in the Figure (4) show that to formulate a balanced ration with least cost using the given feed stuffs with the given chemical analyses and prices, the ratios of Barley, Mize, Wheat bran, Soybean meal, Wheat straw, Calcium phosphate and Salt should be (5, 17.91, 50, 10.76, 13.66, 1.64, 1) respectively. The cost in this formulation will be 268.6 $/Ton which is the lowest possible price. Feed stuff ratios were between (5-50%) because they were subject to the eighth and ninth constraints. Salt ratio was exactly 1% because it was subject to the eleventh constraint while calcium phosphate ratio (1.64%) was sufficient and within the range of constraint. The total amounts of feed stuffs were (100 kg) exactly which was subject to the seventh constraint. Here we note the value of the metabolized energy (subject to the first and second constraints) and crude protein ratio (subject to the third and fourth constraints) were exactly in the lower limits of the constraints range, this is due to the prices of feed stuffs, as the feed stuffs that contain high protein ratios and energy are expensive. Calcium and phosphorus ratio were more than (0.55 and 0.35) respectively because they were subject to the fifth and sixth constraint respectively. These results show a superior ability of linear programming to solve problems consisting of several variables and it can be widely used in animal nutrition field.
3.2 Comparison of the results

The two experts used the trial-and-error method using the Excel program to facilitate the calculation process. Table (3) shows that each of the first and second experts met all eleven constraints required of them. Through the value of energy and the ratios of protein, calcium and phosphorous we notice that the diet, which calculated by the first expert, contained more energy and protein than the rest of the rations, while it hardly achieved the required calcium level. This is what was confirmed by the expert through feedback. As for the second expert, the use of the Excel program gave him the opportunity to make many trials and error in a short time, therefore, he was closer to the ideal state according to the results shown and as confirmed by his feedback. But despite all of the above and when looking at the most important point and aim of the research, which is forming a balanced ration with least possible cost, we see that there is a clear advantage of computerized linear programming over what the human brain can do especially if we take into consideration that we are talking about specialists who hold a doctorate in the animal nutrition field. These results show a superior ability of linear equations to solve problems consisting of several variables and it can be widely used in animal nutrition field.

| Feed stuff         | Computerized linear equations | First expert | Second expert |
|--------------------|--------------------------------|--------------|---------------|
| Barly              | 5                              | 30           | 10            |
| Maize              | 17.91                          | 13           | 20            |
| wheat bran         | 50                             | 35           | 31            |
| SBM                | 10.79                          | 14           | 16            |
| wheat straw        | 13.66                          | 5            | 20            |
| Calcium phosphate  | 1.64                           | 2            | 2             |
| Salt               | 1                              | 1            | 1             |
| Total              | 100                            | 100          | 100           |
| Price $/Ton        | 268.6                          | 307.4        | 275.9         |

### Conclusion

The use of linear programming in the simplex method has a great possibility of creating balanced ration at the lowest possible cost, and animal breeders can use the Solver tool with the steps we have explained, which will provide him efficiency with the highest production by reducing the costs and balancing of the ration.

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### References

[1] Webster, A.J.F. (1993). Understanding the Dairy Cow. 2nd Edition. Blackwell Scientific Publication.
[2] Church, D.C. (1993). The Ruminant animal digestive physiology and nutrition. Waveland press, Inc
[3] National Research Council. Nutrient Requirements of Dairy Cattle. 5th revised edition, national academy of science. Washington DC, 1975.
[4] Drackley J., & Cardoso F. (2014). Prepartum and postpartum nutritional management to optimize fertility in high-yielding dairy cows in confined TMR systems. Animal, 8, 5–14.
[5] Walsh, S., Williams E., & Evans A. (2011). A review of the Causes of oor pfertility in high milk producing dairy cows. Animal Reproduction Science, 123, 127–138.
[6] Thilsing-Hansen, T., R.J. Jørgensen & S. Østergaard (2002). Milk Fever Control Principles: A Review Acta Veterinaria Scandinavica. 2002, 43, 1-19.
[7] Curtis CR., Erb HN., Sniffen CJ., Smith RD., Powers PA.,Smith MC., White ME., Hillman RB., & Pearson EJ (1983). Association of Parturient hypocalcaemia with eight periparturient disorders in Holstein cows. Journal of the American Veterinary Medical Association, 183, 559-561.

[8] Curtis CR., Erb HN., Sniffen CJ. & Smith RD (1984). Epidemiology of parturient paresis: Predisposing factors with emphasis on dry cow feeding and management. Journal of Dairy Science, 67, 817-825.

[9] Humer, E., Leonhard G., & Qendrim Z. (2018). Effects of Meeting the Requirements in Energy and Protein, and of Systemic Inflammation on the Interval from Parturition to Conception in Dairy Cows, Czech Journal of Animal Science, 63, (6). 201-211.

[10] Danzing, G. B. & Thapa, M. N. (2003). Liner programming: Theory and extensions. Springer-Verlag, New York

[11] Thie, P. R. & G. E. Keough. (2008). An introduction to linear programming and game theory. Third Edition, A john wiley & sons, inc., publication.

[12] www.solver.com

[13] A.O.A.C. (2002). Official Methods of Analysis. 17th Ed. Association of Official Analytical Chemists. Washington, DC.

[14] MacDonald,P, Greenhalph J.F.D., Edwards R.A., Morgan C.A. Sinclair L.A., & Wilkinson R.G,( 2011) Animal nutrition. Pearson.