Emerging ordinary superhumps as the standard candle for WZ Sge stars

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Abstract

In Kato (2015, arXiv:1507.07659), I suggested that the magnitude when ordinary superhumps appear can be a standard candle for WZ Sge stars. Using Gaia EDR3 parallaxes, I studied 53 WZ Sge stars to examine this suggestion. The analysis indicated that the absolute magnitudes when ordinary superhumps appear are strongly dependent on orbital inclinations, which is consistent with what is expected for the projected area for optical thick accretion disks. I showed that there is a linear relation between these absolute magnitudes and logarithmic amplitudes of early superhumps, which are also dependent on the inclinations. I confirmed that the magnitude when ordinary superhumps appear can be used as the standard candle for WZ Sge stars particularly when the amplitude of early superhumps is observationally known. I showed that the resultant median absolute magnitude when ordinary superhumps appear was $+5.4$ (for an average inclination of 1 radian). A few objects with multiple rebrightenings, which are good candidates for period bouncers, showed slightly fainter absolute magnitudes, although the majority of the same class of objects follows the relation. Using the relation, I derived an empirical relation between the inclination and the amplitude of early superhumps. I applied the relation to MASTER OT J030227.28+191754.5, which was initially considered as a possible optical counterpart of the high-energy neutrino event IceCube-211125A, and concluded that this WZ Sge star was one of the faintest before the outburst and one of the brightest around the optical peak in terms of absolute magnitudes.

1 Introduction

WZ Sge stars are a subclass of dwarf novae and they usually show rare (typically once in a decade) and large-amplitude (typically 8 mag) superoutbursts [for general information of cataclysmic variables and dwarf novae, see e.g. Warner (1995)]. Although WZ Sge stars were originally defined as dwarf novae showing rare, large-amplitude superoutbursts and almost lacking normal outbursts (see e.g. Bailey 1979; Downes and Margon 1981; Downes 1990), this definition remained somewhat ambiguous. Following the dramatic superoutburst of WZ Sge in 2001 (Patterson et al. 2002; Ishioka et al. 2002; Baba et al. 2002), our understanding of WZ Sge stars has been refreshed. It was known that periodic variations having the orbital period were observed in WZ Sge stars during the early phase of their superoutbursts. Patterson et al. (1981) considered that they are orbital humps and that they reflect a greatly enhanced mass-transfer from the secondary. At that time, the only example was the 1978–1979 superoutburst of WZ Sge (Patterson et al. 1981). The number of objects showing this feature gradually increased: HV Vir in 1992 (Kato et al. 2001) [although Barwig et al. (1992); Mendelson et al. (1992); Leibowitz et al. (1994) reported the same feature, they considered the variations to be usual superhumps] and AL Com in 1995 (Kato et al. 1996; Patterson et al. 1996; Howell et al. 1996; Nogami et al. 1997). Following the outburst of AL Com, the presence of double-wave modulations during the early stage of the superoutburst was established. These variations are currently referred to as early superhumps. Osaki and Meyer (2002) identified early superhumps as the manifestation of the 2:1 resonance, in addition to the 3:1 resonance which causes superhumps and superoutbursts in SU UMa stars (Whitehurst 1988; Osaki 1989; Hirose and Osaki 1990; Lubow 1991) and in WZ Sge stars later during superoutbursts. WZ Sge stars are currently defined as dwarf novae in which the 2:1 resonance plays a role during their superoutbursts (Kato 2015).

In Kato (2015), I suggested in its subsection 7.10 that the brightness of WZ Sge stars when ordinary superhumps appear could be used as the standard candle since the disk is expected to have a size close to the radius of the 3:1 resonance. This could be equally applied to SU UMa stars. There are, however, unavoidable large uncertainties arising from inclinations. Paczynski and Schwarzenberg-Czerny (1980) derived a formula for an optically thick disk (as in dwarf-nova outbursts) considering the projected area and limb darkening of the disk

$$
\Delta M_v(i) = -2.5 \log_{10} \left( 1 + \frac{3}{2} \cos i \cos \alpha \right),
$$

(1)

where $\Delta M_v$ and $i$ are the correction to produce the visual absolute magnitude and the inclination, respectively. These corrections can become large (Warner 1987; Patterson 2011). This relation is shown in figure 1. Pole-on systems ($i=0$) are observed 1.0 mag brighter than the average and systems with $i=80^\circ$ are nearly 2 mag fainter.
Figure 1: Dependence of the apparent brightness of an optically thick disk on the orbital inclination using equation (1). Fainter magnitudes are displayed lower.

The use of the accretion disk as the standard candle is somewhat limited since $i$ is relatively difficult to measure other than in eclipsing systems.

In WZ Sge stars, however, the amplitude of early superhumps is considered to depend on the inclination since early superhumps arise from a geometric effect (to the observer) of vertical structures of the disk (Osaki and Meyer 2002). Uemura et al. (2012) succeeded in reproducing the profile of early superhumps by considering self-eclipse of a vertically extended disk. Kato (2015) used the code by Uemura et al. (2012) and reasonably reproduced the distribution of the observed amplitudes of early superhumps except objects with very large amplitudes (in its subsection 5.5). The amplitudes of early superhumps could thus be used instead of $i$.

At the time of Kato (2015), WZ Sge stars with known parallaxes were only three (WZ Sge, GW Lib and V455 And). Now that Gaia parallaxes are available (GaiaEDR3: Gaia Collaboration et al. 2021), I calibrated this “standard candle” and studied the dependence on the amplitude of early superhumps.

2 The Data

2.1 Data source

The data were mostly taken from table 5 in Kato (2015). The quiescent magnitudes were replaced by average $G$ magnitudes in Gaia Collaboration et al. (2021). The amplitudes of early superhumps were taken from table 2 in Kato (2015). The full amplitudes of early superhumps ($A_{ESH}$) correspond to $A_2$ (mean amplitude) in table 2 in Kato (2015). Some objects in table 5 in Kato (2015) had more recent superoutbursts. If the values were improved, I supplied the data from these superoutbursts. As stated in Kato (2015), the accuracy of the magnitudes at which ordinary superhumps appear is $\sim0.1$ mag. In some cases, zero-point calibrations of unfiltered CCD magnitudes caused more uncertainties (particularly in the past). In such cases, I used All-Sky Automated Survey (ASAS-3: Pojmański 2002) $V$ data and All-Sky Automated Survey for Supernovae (ASAS-SN) Sky Patrol $V$ data (Shappee et al. 2014; Kochanek et al. 2017) to obtain well-calibrated magnitudes. Observations from the AAVSO International Database\(^1\) were sometimes used.

Additional objects were taken from Kato et al. (2017, 2020). Some objects (mainly since 2017) reported to VSNET (Kato et al. 2004) were also included to increase the sample. The detailed source of the data of each object is given later in this section. The objects having $1\sigma$ errors in Gaia parallaxes typically less than 20% were

\(^1\)\url{http://www.aavso.org/data-download}.\)
included in the table. When drawing figures and making statistical analysis, I limited objects with parallax errors less than 10%, corresponding to errors of 0.2 mag. Dwarf novae whose parallaxes are measured to this accuracy are nearby objects and I ignored interstellar extinction. The lack of reddening was confirmed by the blue colors \((BP - RP)\) in Gaia magnitudes. In examining the light curves, I also used Public Data Releases of the Zwicky Transient Facility (Masci et al. 2019) observations\(^2\).

2.2 Notes on individual objects

Simply referring to “magnitude”, here I mean the magnitude when ordinary superhumps appeared.

**AL Com**: updated using the 2019 April data (Tampo et al. 2021).

**EG Cnc**: updated using the 2018 October data (Kimura et al. 2021).

**HV Vir**: the same magnitude was obtained using the 2016 March data (Imada et al. 2018, and also VSNET data).

**RZ Leo**: updated using the 2022 January data (Outburst detection by Tadashi Kojima. H. Maehara, vsnet-alert 26522\(^3\); T. Kato, vsnet-alert 26529\(^4\) and Y. Tampo, vsnet-alert 26532\(^5\)). The main observers were Kyoto U. team, Hiroshi Itoh, Seiichiro Kiyota, Osaka Kyokou U. team, Shawn Dvorak, Tamás Tordai, Stephen M. Brincat, Vihorlat Observatory team and Filipp Romanov.

**UZ Boo**: magnitude updated using snapshot \(V\) measurements.

**ASAS J102522−1542.4**: value updated using ASAS-3 \(V\) magnitudes. The amplitude of early superhumps was taken from Kato et al. (2009a).

**EZ Lyn**: magnitude updated based on \(V\) data used in Kato et al. (2009a).

**V455 And**: magnitude updated based on \(V\) data used in Kato et al. (2009a).

**OT J111217.4−353829**: the epoch of the appearance of ordinary superhumps was somewhat uncertain. The magnitude was from AAVSO \(V\) observations.

**V1838 Aql**: The amplitude of early superhumps was taken from Kato et al. (2014). See also Echevarría et al. (2019).

**ASASSN-14cv**: early superhumps were reanalyzed in this work. The magnitude was confirmed during the 2020 July–August superoutburst.

**V529 Dra**: magnitude updated using snapshot \(V\) data. First case of double superoutburst (Kato et al. 2013a) and is one of the best candidates for period bouncers.

**GS Cet**: data from Kato et al. (2017). The magnitude was from ASAS-SN \(V\) data.

**ASASSN-16eg**: data from Wakamatsu et al. (2017). WZ Sge star with an unusually long orbital period.

**ASASSN-16js**: data from Kato et al. (2017). The magnitude was from ASAS-SN \(V\) data.

**ASASSN-17el**: data from Kato et al. (2020). The amplitude of early superhump was corrected in this work. The magnitude was from ASAS-SN \(V\) data.

**PNV J20205397+2508145**: data from Kato et al. (2020).

**HO Cet**: data from Kato et al. (2009a). The magnitude was from ASAS-SN \(V\) data.

**V627 Peg**: data from Kato et al. (2010).

**PNV J17144255−2943481**: data from C. Nakata et al. in preparation; reanalyzed in this paper. This object showed five post-superoutburst rebrightenings (Kato 2015).

**TCP J18154219+3515598**: superoutburst in 2017 June\(^6\) (H. Maehara, vsnet-alert 21098\(^7\); K. Isogai, vsnet-alert 21105\(^8\); spectrum by P. Berardi\(^9\) and T. Kato, vsnet-alert 21109\(^10\)).

\(^{11}\)The ZTF data can be obtained from IRSA <https://irsa.ipac.caltech.edu/Missions/ztf.html> using the interface <https://irsa.ipac.caltech.edu/docs/program_interface/ztf_api.html> or using a wrapper of the above IRSA API <https://github.com/MickaelRigault/ztfquery>.

\(^{12}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/26522>.

\(^{13}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/26529>.

\(^{14}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/26532>.

\(^{15}\)<http://www.cbat.eps.harvard.edu/unconf/followups/J18154219+3515598.html>.

\(^{16}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/21098>.

\(^{17}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/21100>.

\(^{18}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/21105>.

\(^{19}\)<http://quasar.teoth.it/html/spectra/tcp18154219+3515598_PB.png>.

\(^{20}\)<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/21109>.
This object showed 10 post-superoutburst rebrightenings (June–September), nine of which were announced on VSNET: R. J. Modic, vsnet-alert 21175; R. J. Modic, vsnet-alert 21224; E. de Miguel, vsnet-alert 21244; H. Machara, vsnet-alert 21268; T. Kato, vsnet-alert 21278; K. Isogai, vsnet-alert 21302; H. Machara, vsnet-alert 21322; R. J. Modic, vsnet-alert 21334 and H. Machara, vsnet-alert 21347. See also Zubareva et al. (2018). The amplitude of early superhumps is based on re-analysis of the data reported to VSNET. The main observers were Geoff Stone, Tamás Tordai, Enrique de Miguel, Tonny Vamnumster, Lisnyky Observatory team, Kyoto U. team, Vihorlat Observatory team, Stephen M. Brincat, Hiroshi Itoh, Rudolf Novák, Terskol Observatory team, Seiichiro Kiyota, Kiyoshi Kasai, Roger D. Pickard, Natalia Katysheva, Ian Miller, Alexandra M. Zubareva, Javier Ruíz, William Stein, Sergey Yu. Shugarov, Lewis M. Cook, TSHAO Observatory team and Crimean Astrophysical Observatory team.

ASASSN-17pm: also known as PNV J05580574−001115521. Superoutburst in 2017 November–December (T. Vamnumster, vsnet-alert 21624; T. Kato, vsnet-alert 21627 and T. Kato, vsnet-alert 21646). The amplitude of early superhump was re-analyzed in this work. There was a short rebrightening, a shallow dip and a long rebrightening. The main observers were Hiroshi Itoh, Crimean Astrophysical Observatory team, Franz-Josef Hambsch, Seiichiro Kiyota, Tonny Vamnumster, Kyoto U. team, Tamás Tordai, Ian Miller and Domenico Licchelli.

ASASSN-18do: the data were not very good around the appearance of ordinary superhumps. Superoutburst in 2018 February–March (T. Vamnumster, vsnet-alert 21906; T. Kato, vsnet-alert 21921). This object is eclipsing (T. Kato, vsnet-alert 21933). The main observers were Tonny Vamnumster, Hiroshi Itoh, Jochen Pietz, Tamás Tordai, Crimean Astrophysical Observatory team, Sergey Yu. Shugarov and Kyoto U. team.

ASASSN-18wa: superoutburst in 2018 September–October (T. Vamnumster, vsnet-alert 22560; Y. Wakanatsu, vsnet-alert 22565; Y. Wakanatsu, vsnet-alert 22591). There was a short gap in the observations after the superoutburst and it was unclear whether there was a post-superoutburst rebrightening. The ZTF data showed a smooth fading tail characteristic to a WZ Sge star. The main observers were Tamás Tordai, Osaka Kyoiku U. team, Tonny Vamnumster, Hiroshi Itoh and Geoff Stone.

ASASSN-19ag: superoutburst in 2019 January (T. Vamnumster, vsnet-alert 22927). Although brightening and growth of ordinary superhumps were recorded, the profile before the growth of ordinary superhumps was not similar to that of early superhumps (T. Kato, vsnet-alert 22937). I therefore did not give the amplitude of early superhumps. The object showed at least one post-superoutburst rebrightening in the ZTF data.

TCP J06373299−0935420 = ASASSN-19de: superoutburst in 2019 February–March (spectrum by R. Leadbeater, vsnet-alert 23014 and T. Kato, vsnet-alert 23036). There was one post-superoutburst rebrightening (T. Kato, vsnet-alert 23083). The main observers were Osaka Kyoiku U. team, Hiroshi Itoh, Stephen M. Brincat, Berto Monard, Franz-Josef Hambsch, Seiichiro Kiyota, Arto Oksanen, Masanori Mizutani, Tonny Vamnumster and Yasui Sano. According to vsnet-alert postings, Gianluca Masi also reported observations.
but I could not receive the data and are not included in this analysis.

**TCP J05390410+4748030 = ASASSN-19hh**: superoutburst in 2019 March–April\(^{37}\) (T. Vanmunster, vsnet-alert 23068\(^{38}\); T. Kato, vsnet-alert 23072\(^{39}\); T. Kato, vsnet-alert 23120\(^{40}\) and T. Kato, vsnet-alert 23124\(^{41}\)). The main observers were Tonny Vanmunster, Seiichiro Kiyota, Hiroshi Itoh, Crimean Astrophysical Observatory team, Vihorlat Observatory team, Tamás Tordai, Stephen M. Brincat, Ian Miller and Lewis M. Cook.

**ASASSN-19hl**: superoutburst in 2019 March–April (D. Denisenko, vsnet-alert 23090\(^{42}\); T. Kato, vsnet-alert 23103\(^{43}\) and T. Kato, vsnet-alert 23144\(^{44}\)). The main observers were Berto Monard, Franz-Josef Hambsch and Hiroshi Itoh.

**V3101 Cyg**: data from Tampo et al. (2020). The amplitude of early superhumps was re-analyzed in this work.

**EQ Lyn**: data from Tampo et al. (2021).

**GY Cet**: superoutburst in 2020 July–August (P. Schmeer, vsnet-alert 24446\(^{45}\); T. Kato, vsnet-alert 24486\(^{46}\) and T. Kato, vsnet-alert 24498\(^{47}\)). No post-superoutburst rebrightening was present. The main observers were Franz-Josef Hambsch and Berto Monard.

**VX For**: superoutburst in 2021 January–February (Y. Maeda, vsnet-alert 25264\(^{48}\); P. Schmeer, vsnet-alert 25265\(^{49}\) and Y. Tampo, vsnet-alert 25288\(^{50}\)). There were five post-superoutburst rebrightenings (P. Schmeer, vsnet-alert 25355\(^{51}\); P. Schmeer, vsnet-alert 25386\(^{52}\); R. Stubbings, vsnet-outburst 26651\(^{53}\); P. Schmeer, vsnet-alert 25414\(^{54}\) and ASAS-SN \(g = 14.3\) on 2021 February 28). This object also showed five post-superoutburst rebrightenings in 2009 (Kato et al. 2010) and has been suggested as a period bouncer (Kato 2022). The main observers were Hiroshi Itoh, Berto Monard, Peter Nelson and Franz-Josef Hambsch.

**ASASSN-21et = TCP J06154200−2756220**: superoutburst in 2021 April–May\(^{55}\) (P. Schmeer, vsnet-alert 25637\(^{56}\), T. Kato, vsnet-alert 25658\(^{57}\) and T. Kato, vsnet-alert 25780\(^{58}\)). There was no post-superoutburst rebrightening in the ASAS-SN data. The main observers were Berto Monard and Franz-Josef Hambsch.

I only deal with statistics in this paper and the full analysis and figures for these unpublished objects are planned to appear in separate paper(s).

### 3 Result and Discussion

#### 3.1 Absolute magnitudes when ordinary superhumps appear

The absolute magnitudes when ordinary superhumps appear [hereafter \(M_V(\text{SH})\)] are listed in table 1. Note that this table includes objects with relative large errors in parallax. For example, the bright \(M_V(\text{SH})\) in HO Cet is likely a result of the uncertainty in the parallax. In this table, I gave 1\(\sigma\) errors estimated from the errors in parallax. They are the main cause of the overall errors of \(M_V(\text{SH})\).

\(^{37}\)\text{http://www.cbat.eps.harvard.edu/unconf/followups/J05390410+4748030.html}.
\(^{38}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23068}.
\(^{39}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23072}.
\(^{40}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23120}.
\(^{41}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23124}.
\(^{42}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23090}.
\(^{43}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23103}.
\(^{44}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/23144}.
\(^{45}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/24446}.
\(^{46}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/24486}.
\(^{47}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/24498}.
\(^{48}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25264}.
\(^{49}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25265}.
\(^{50}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25288}.
\(^{51}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25358}.
\(^{52}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25386}.
\(^{53}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-outburst/26651}.
\(^{54}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25414}.
\(^{55}\)\text{http://www.cbat.eps.harvard.edu/unconf/followups/J06154200−2756220.html}.
\(^{56}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25637}.
\(^{57}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25658}.
\(^{58}\)\text{http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-alert/25780}.
Table 1: Brightness when superhumps appear

| Object       | Year | P$^*$ | Mag1$^†$ | Mag2$‡$ | A$_{ESH}$ | $\omega^3$ | $\omega_{error}$ | $M_V$(SH)$§$ |
|--------------|------|-------|----------|---------|-----------|------------|----------------|--------------|
| WZ Sge       | 2011 | 0.05669 | 15.2 | 9.9 | 0.14 | 22.104 | 0.030 | 6.6(0) |
| AL Com       | 2019 | 0.05667 | 19.7 | 13.7 | 0.04 | 1.903 | 0.565 | 5.1(8) |
| EG Cnc       | 2018 | 0.05997 | 18.8 | 12.7 | 0.018 | 5.361 | 0.199 | 6.3(1) |
| HV Vir       | 1992 | 0.05707 | 19.0 | 13.3 | 0.044 | 3.152 | 0.261 | 5.8(2) |
| RZ Leo       | 2022 | 0.07603 | 18.2 | 12.8 | 0.00 | 3.564 | 0.149 | 5.9(1) |
| QZ Lib       | 2004 | 0.06460s | 18.8 | 12.2 | – | 5.023 | 0.254 | 5.7(1) |
| UZ Boo       | 2013 | 0.0620s | 19.9 | 12.8 | – | 3.086 | 0.511 | 5.3(2) |
| ASAS J102522−1542.4 | 2006 | 0.06136 | 19.3 | 12.5 | – | 5.023 | 0.254 | 5.7(1) |
| EZ Lyn       | 2010 | 0.05901 | 17.8 | 12.7 | 0.067 | 7.004 | 0.110 | 6.9(0) |
| GW Lib       | 2007 | 0.05332 | 16.5 | 10.2 | 0.00 | 8.846 | 0.061 | 4.9(0) |
| V455 And     | 2010 | 0.05706 | 17.9 | 11.7 | 0.011 | 4.940 | 0.142 | 5.2(1) |
| OT J111217.4−353829 | 2012 | 0.05973 | 20.0 | 15.2 | 0.050 | 1.754 | 0.493 | 6.4(7) |
| V1838 Aql    | 2013 | 0.05706 | 17.9 | 11.7 | 0.011 | 4.940 | 0.142 | 5.2(1) |
| ASASSN-14cl  | 2014 | 0.05838 | 18.2 | 11.9 | 0.018 | 3.825 | 0.188 | 4.8(1) |
| ASASSN-14cv  | 2014 | 0.05992 | 19.1 | 12.9 | 0.03 | 3.407 | 0.171 | 5.1(6) |
| PNV J03052314−0225455 | 2014 | 0.05456 | 19.2 | 13.4 | 0.035 | 2.103 | 0.299 | 5.0(3) |
| PNV J03093063+2638031 | 2014 | 0.05615 | 18.7 | 12.2 | 0.018 | 4.132 | 0.188 | 5.3(1) |
| ASASSN-14jv  | 2014 | 0.05442 | 18.8 | 12.5 | 0.017 | 3.333 | 0.147 | 5.1(1) |
| ASASSN-15bp  | 2014 | 0.05563 | 19.8 | 13.7 | 0.014 | 1.792 | 0.325 | 5.0(4) |
| V1251 Cyg    | 2008 | 0.07433 | 20.2 | 13.4 | 0.018 | 1.965 | 0.517 | 4.9(7) |
| BC UMa       | 2003 | 0.06261 | 18.3 | 12.6 | 0.04 | 3.392 | 0.126 | 5.3(1) |
| V529 Dra     | 2011 | 0.07168 | 20.4 | 13.7 | 0.005 | 1.661 | 0.488 | 4.8(8) |
| CRTS J122221.6−311524 | 2013 | 0.07649 | 18.9 | 12.3 | – | 4.136 | 0.222 | 5.4(1) |
| PNV J06000985+1426152 | 2014 | 0.06331s | 20.0 | 12.9 | – | 1.833 | 0.706 | 4.2(11) |
| GS Cet       | 2016 | 0.05957 | 20.7 | 14.2 | 0.065 | 3.846 | 1.388 | 7.1(10) |
| ASASSN-16eg  | 2016 | 0.07548 | 19.4 | 13.4 | 0.05 | 2.260 | 0.204 | 5.2(2) |
| ASASSN-16js  | 2016 | 0.06034 | 20.5 | 15.3 | 0.18 | 2.475 | 0.350 | 7.3(3) |
| ASASSN-17cl  | 2017 | 0.05434 | 18.7 | 12.6 | 0.025 | 2.889 | 0.120 | 4.9(1) |
| PNV J20205397+2508145 | 2017 | 0.05651 | 20.1 | 14.2 | 0.06 | 1.561 | 0.526 | 5.2(9) |
| HO Cet       | 2006 | 0.05490 | 19.3 | 13.5 | 0.035 | 1.134 | 0.392 | 3.8(9) |
| V627 Peg     | 2010 | 0.05452 | 15.6 | 10.3 | 0.045 | 10.079 | 0.037 | 5.3(0) |
| PNV J17144255−2943481 | 2014 | 0.05956 | 17.2 | 12.3 | 0.05 | 5.665 | 0.098 | 6.1(0) |
| OV Boo       | 2017 | 0.04626 | 18.2 | 13.5 | 0.27 | 4.715 | 0.091 | 6.9(0) |
| TCP J18154219+3515598 | 2017 | 0.06168s | 19.1 | 12.4 | 0.015 | 4.600 | 0.171 | 5.7(1) |
| ASASSN-17pm  | 2017 | 0.05973 | 17.8 | 14.2 | 0.14 | 3.363 | 0.120 | 6.8(1) |
| ASASSN-18do  | 2018 | 0.05498 | 20.5 | 15.9 | 0.6 | 1.593 | 0.671 | 6.9(12) |
| ASASSN-18wa  | 2018 | 0.05441 | 20.2 | 14.4 | 0.02 | 2.068 | 0.949 | 6.0(13) |
| ASASSN-19ag  | 2019 | 0.06616 | 20.0 | 13.0 | – | 2.512 | 0.635 | 5.0(6) |
| TCP J06373299−0935420 | 2019 | 0.06642s | 19.2 | 12.1 | 0.01 | 3.432 | 0.275 | 4.8(2) |

$^*$Orbital or superhump (with a suffix 's') period (d).
†Quiescent brightness.
‡Brightness when ordinary superhumps appear.
§Gaia EDR3 parallax (mas).
∥Absolute magnitude when ordinary superhumps appear. The 1σ error estimated from $\omega_{error}$ is given.
Figure 2: Distribution of absolute magnitudes when ordinary superhumps appear \([M_V(\text{SH})]\). Objects with parallax errors less than 10% were selected.

Table 1: Brightness when superhumps appear (continued).

| Object               | Year | \(P^\ast\) | \(\text{Mag}^1\) | \(\text{Mag}^2\) | \(A_{\text{ESH}}\) | \(\varpi^3\) | \(\varpi_{\text{error}}\) | \(M_V(\text{SH})^\parallel\) |
|---------------------|------|-------------|-------------------|-------------------|--------------------|----------------|-----------------|---------------------|
| TCP J05390410+4748030 | 2019 | 0.05534     | 19.6              | 13.7              | 0.10               | 2.957          | 0.318           | 6.1(2)              |
| ASASSN-19hl          | 2019 | 0.05415     | 19.5              | 12.9              | 0.03               | 2.889          | 0.274           | 5.2(2)              |
| V3101 Cyg            | 2019 | 0.05352     | 17.7              | 10.9              | 0.045              | 9.226          | 0.072           | 5.7(0)              |
| EQ Lyn               | 2019 | 0.0528      | 18.9              | 12.0              | –                  | 3.223          | 0.222           | 4.5(2)              |
| GY Cet               | 2020 | 0.05663     | 18.5              | 11.9              | 0.015              | 3.712          | 0.181           | 4.7(1)              |
| VX For               | 2021 | 0.06133s    | 20.4              | 13.2              | –                  | 2.060          | 0.574           | 4.8(7)              |
| ASASSN-21et          | 2021 | 0.05820     | 20.2              | 14.5              | 0.12               | 2.816          | 0.508           | 6.7(4)              |

\(^\ast\)Orbital or superhump (with a suffix 's') period (d).
\(^\dagger\)Quiescent brightness.
\(^\ddagger\)Brightness when ordinary superhumps appear.
\(^\S\)Gaia EDR3 parallax (mas).
\(^\parallel\)Absolute magnitude when ordinary superhumps appear. The 1\(\sigma\) error estimated from \(\varpi_{\text{error}}\) is given.

The distribution of absolute magnitudes when ordinary superhumps appear is shown in figure 2. The mean value is \(\langle M_V(\text{SH})\rangle=+5.57\) and the standard deviation is 0.67 mag. This standard deviation is too large to be directly used as the standard candle. Furthermore, the mean value is not adequate since \(M_V(\text{SH})\) does not follow a Gaussian distribution as explained later. The dispersion in \(M_V(\text{SH})\) is largely due to the inclination effect as seen in equation (1).

3.2 Correlation with the amplitude of early superhumps

Considering that \(A_{\text{ESH}}\) is expected to be a function of \(i\), the relation between \(A_{\text{ESH}}\) and \(M_V(\text{SH})\) is shown in figure 3. Systems with large \(A_{\text{ESH}}\) have fainter \(M_V(\text{SH})\) as expected. The relation becomes linear using log \(A_{\text{ESH}}\) (figure 4). The solid line in the figure corresponds to

\[M_V(\text{SH}) = 7.5(3) + 1.28(20) \log_{10} A_{\text{ESH}}.\] (2)
Figure 3: Dependence of absolute magnitudes when ordinary superhumps appear \( M_M^{(SH)} \) on amplitudes of early superhumps. Objects with parallax errors less than 10\% were selected. The error bars reflect the errors from parallax measurements.

In drawing the figure and making a regression, \( M_M^{(SH)}=0.01 \) was given for objects with \( M_M^{(SH)} < 0.01 \) (The value of 0.01 mag is a realistic upper limit of the detection of early superhumps; use the same value in applying the formula to WZ Sge stars without detectable early superhumps). The standard deviation from this relation is 0.42 mag.

### 3.3 Special objects and the updated relation

The two most deviating objects were EG Cnc (1.1 mag fainter than this relation) and EZ Lyn (0.9 mag fainter than this relation). One of the possible reason for EG Cnc is that the observations of EG Cnc did not start early enough (Kimura et al. 2021) and \( A_{ESH} \) might have been underestimated. EZ Lyn is an eclipsing system and is suggested to be a period bouncer (Zharikov et al. 2008; Pavlenko et al. 2007; Kato et al. 2009b; Kato and Osaki 2013; Amantayeva et al. 2021).

Although TCP J18154219+3515598 with 10 rebrightenings is also 0.5 mag fainter than this relation, ASASSN-14cv with 5–8 rebrightenings [8 in 2014 (Sklyanov et al. 2016; Kato 2015); 5 in 2020, VSNET data] does not show a strong deviation. QZ Lib with four rebrightenings (Kato et al. 2009a; Pala et al. 2018) and PNV J17144255–2943481 with five rebrightenings are also on this relation. The very unusual object V3101 Cyg, which showed multiple rebrightenings and superoutbursts following the initial superoutburst (Tampo et al. 2020; Hameury and Lasota 2021), is also on the relation. The most extreme period bouncer in this sample CTRS J122221.6−311524 (Kato et al. 2013b; Neustroev et al. 2017, 2018) is not in figure 4 since \( A_{ESH} \) is unknown, but has an ordinary \( M_M^{(SH)}=5.4 \).

There is a possible reason of the deviation for (some) period bouncers if the deviation is real. They are likely to have small mass ratios (Kato 2022, and references therein) and the tidal torque on the disk is expected to be weak. This may cause a weaker effect of the 2:1 resonance and \( A_{ESH} \) may be smaller than in other WZ Sge stars.
Figure 4: Dependence of absolute magnitudes when ordinary superhumps appear $[M_V(SH)]$ on amplitudes of early superhumps in logarithmic scale. Objects with parallax errors less than 10% were selected. Several unusual objects are plotted with different colors. Red marks represent objects with multiple post-superoutburst rebrightenings. (See text for the details). The solid and dashed lines represent equations (2) and (3), respectively. The error bars reflect the errors from parallax measurements.
Since a number of WZ Sge stars with multiple rebrightenings are on the relation, this explanation does not seem to apply to all period bouncers. This possibility needs to be studied further both from the observational and theoretical sides.

The only population II object below the period minimum OV Boo (Littlefair et al. 2007; Patterson et al. 2008; Uthas et al. 2011; Patterson et al. 2017; Ohnishi et al. 2019) is on the relation. This object is located near the period minimum of population II cataclysmic variables and may not be as unusual as more evolved period bouncers. ASASSN-16eg is a WZ Sge star with a very long orbital period (Wakamatsu et al. 2017). The deviation to the brighter side may reflect the large disk size, but the error in the parallax is still large to draw a definite conclusion.

Disregarding the objects deviating by more than 0.6 mag from equation (2), I obtained

$$M_V(SH) = 7.5(2) + 1.31(15) \log_{10} A_{ESH}.$$  

The standard deviation from this regression is 0.31 mag. This equation can currently be regarded as the best relation for WZ Sge stars with potential exceptions of evolved period bouncers and systems with very long orbital periods.

3.4 Standard candle for the average inclination

The median value of $M_V(SH)$ for all the objects is +5.38, which can be considered as a value for the average inclination ($\approx$1 radian = 57°). This value can be used as the standard candle if $A_{ESH}$ is unknown, but with a 1σ error of 0.67 mag. Although, errors may be even larger in high-inclination systems, $i$ can probably be estimated using eclipses in such systems and there would be no need for the use of the median $M_V(SH)$.

3.5 Estimation of the inclination from the amplitude of early superhumps

By equating the equation (1) and the equation (3) and using the value of the standard candle of $M_V(SH)=+5.38$ for $i=1$ radian, one can obtain an experimental relation between the amplitude of early superhumps and the inclination (figure 5). Note that this figure is based on an assumption that the relation between $\log A_{ESH}$ and $M_V(SH)$ is linear. This figure suggests that $A_{ESH}$ is 0.02 mag for the average inclination ($\approx$1 radian). It is likely that early superhumps are not detectable for $i < 40°$. There is only one object (OV Boo) with $i > 80°$ in our sample. It is possible that the amplitude of early superhumps is saturated for very large $i$ and the linear relation would break. The relation in figure 5 would be helpful for estimating $i$ for objects with moderate $A_{ESH}$.

4 Application to MASTER OT J030227.28+191754.5

MASTER OT J030227.28+191754.5 = PNV J03022732+1917552 was reported as a possible counterpart (Zhirkov et al. 2021) of IceCube-211125A high-energy neutrino event (IceCube Collaboration 2021). This object had a large outburst amplitude of 10 mag (Zhirkov et al. 2021). It was independently discovered by Y. Nakamura. Due to the large outburst amplitude and the possible association with the neutrino event, it was suspected to be a nova. Early spectroscopy indeed showed narrow emission lines and it was suggested to be either a narrow-lined He/N nova or a dwarf nova with an exceptionally large amplitude (Taguchi et al. 2021). The initial spectrum did not resemble that of a dwarf nova and the object was initially favored to be a nova (Paliya 2021). Follow-up observations did not detect very-high-energy gamma-ray flux (Quinn et al. 2021; Ayala 2021a,b). The optical spectrum on the second night clarified that the object is a very high-amplitude dwarf nova (Isogai et al. 2021). CCD images taken 8.5 hr before the neutrino event indicated that the object was already 1 mag above the preoutburst level (Sarneczky et al. 2021). Although this observation suggested the association with the neutrino event less likely, the magnitude might have been too faint to be considered as the ignition of the outburst. Early superhumps with an amplitude of 0.03 mag were detected (T. Kato, vsnet-alert 26477). The object eventually started to show ordinary superhumps on 2021 December 30, 30–32 d after the outburst detection (T. Kato, vsnet-alert 26501). This delay of the appearance of ordinary superhumps was the longest ever observed (Kato 2015).
Due to the unusual nature and the extremely slow development of ordinary superhumps in this object, I applied the present method to estimate the distance and luminosity. Ordinary superhumps appeared at a magnitude of 14.9 (VSNET data; Y. Tampo et al. in preparation). Using the equation (3), $M_V$ (SH) is expected to be $+5.6$ and the distance modulus is estimated to be 9.3, which corresponds to $\sim 720$ pc. The SDSS magnitude $g=22.0$ (Abazajian et al. 2009) corresponds to the quiescent absolute magnitude of $+12.7$, which is one of the faintest recorded in WZ Sge stars (Tampo et al. 2020). The peak $V$ magnitude 11.8 corresponds to $M_V=+2.6$, which is also among the brightest (Tampo et al. 2020).

To examine the possibility if any nuclear reaction was involved in the outburst of this object, I made a comparison with the faintest measurement of an outbursting nova. The recurrent nova T Pyx was recorded on the rise at a magnitude of 13.0 during the 2011 eruption by M. Linnolt (Waagan et al. 2011; Schaefer et al. 2013) [the initial spectrum was taken several hours later by Arai et al. (2015), confirming the nova eruption]. This corresponds to $M_V=+0.7$ and the maximum $M_V$ of MASTER OT J030227.28+191754.5 is still 6 times below this value. The faint brightness of T Pyx during the rise was probably due to the large bolometric correction and the actual upper limit of the luminosity of the white dwarf due to nuclear reaction should be much lower than this estimate (compared to T Pyx) based on the $V$-band data.

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List of objects in this paper

V455 And, V1838 Aql, UZ Boo, OV Boo, GS Cet, HU Cet, HO Cet, EG Cnc, AL Com, V1251 Cyg, V3101 Cyg, V529 Dra, VX For, RZ Leo, GW Lib, QZ Lib, EQ Lyn, EZ Lyn, V624 Peg, V627 Peg, T Pyx, BW Scl, WZ Sge, SU UMa, BC UMa, V355 UMa, HV Vir, ASAS J102522−1542.4, ASASSN-14cl, ASASSN-14cv, ASASSN-14jv, ASASSN-15bp, ASASSN-16eg, ASASSN-16js, ASASSN-17el, ASASSN-17pm, ASASSN-18do, ASASSN-18wa, ASASSN-19ag, ASASSN-19hl, ASASSN-21et, CTRS J104411.4+211307, CTRS J122221.6−311524, IceCube-211125A, MASTER OT J030227.28+191754.5, MASTER OT J211258.65+242145.4, OT J012059.6+325545, OT J111217.4−353829, PNV J030930.63+2638031, PNV J060009.85+1426152, PNV J17144255−2943481, PNV J20205397+2508145, PNV J23052314−0225455, SDSS J161027.61+090738.4, TCP J05390410+4748030, TCP J06373299−0935420, TCP J18154219+3515598

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I provide two forms of the references section (for ADS and as published) so that the references can be easily incorporated into ADS.

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