DERIVING SPIN OF THE BOSONIC STRING

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Abstract

Exploiting the strict analogy between the motion of strings and extended-like spinning particles, we propose an original kinematical formulation of the spin of bosonic strings and give, for the first time, an analytical derivation of an explicit expression of the string spin vector.

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1 Proper-time equation of motion for a bosonic string

By requiring the reparametrization and conformal invariances, the Polyakov Lagrangian describing the free motion of a bosonic string can be written in the conformal gauge (\(\dot{x} \equiv \partial x/\partial \tau, \ x' \equiv \partial x/\partial \sigma\))

\[
\mathcal{L} = \frac{1}{2} M \left( \dot{x}^2 - x'^2 \right),
\]

where the position \(x(\tau, \sigma)\) at the proper time \(\tau\) is parameterized by \(\sigma\). We have \(M \equiv LT\), where \(L \equiv 2\pi/\omega_0\) and \(T\) are the string “length” and the string tension, respectively*. The Euler-Lagrange equation is (\(\mu = 0, 1, 2, \ldots, N\), where \(N\) is the dimension of the embedding space)

\[
\ddot{x}_\mu = x''_\mu.
\]

Considering a closed string† we have to require the following boundary condition

\[
x^\mu(\tau, \sigma) = x^\mu(\tau, \sigma + L).
\]

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† In the literature it is often assumed \(\omega_0 = 2, M = \pi T, L = \pi\), with \(\omega_0, \tau\) and \(\sigma\) dimensionless. In this paper, having to perform a harmonic analysis in the \(\tau\)-sector, we prefer to assume time dimensions for \(\sigma\) and \(\tau\), with \(|\omega_0| = [t]^{-1}\).
† Equations and results quite analogous to the ones found in the present paper hold also for open strings.
The general solution of Eq. (2) satisfying constraint (3) is \( c = 1 \)

\[
x^\mu(\tau, \sigma) = x^\mu_0 + \frac{P^\mu}{M} \tau + 
\]

\[
+ i \sqrt{\frac{\hbar}{2M\omega_0}} \sum_{m=1}^\infty \frac{1}{m} \left[ \alpha^\mu_m e^{-im\omega_0(\tau-\sigma)} + \bar{\alpha}^\mu_m e^{-im\omega_0(\tau+\sigma)} + \alpha^\prime_m e^{im\omega_0(\tau-\sigma)} + \bar{\alpha}^\prime_m e^{im\omega_0(\tau+\sigma)} \right].
\]

(4)

Quantity \( p^\mu \) indicates as usual the constant mean momentum (whose spatial part \( p \), equal to \( M \) times the center-of-mass velocity, vanishes in the center-of-mass frame) which is different from the (non-constant) total canonical momentum \( P^\mu \) below defined. The dimensionless Fock operators \( \alpha^\mu_m, \bar{\alpha}^\mu_m \) obey the usual bosonic commutation rules. Let us define the operators \( a^\mu_m, \bar{a}^\mu_m \) according to

\[
a^\mu_m = \frac{1}{\sqrt{m}} \alpha^\mu_m \quad \bar{a}^\mu_m = \frac{1}{\sqrt{m}} \bar{\alpha}^\mu_m,
\]

(5)

and impose the usual (equal-\( \tau \)) canonical commutation relations

\[
[x^\mu(\tau, \sigma), x^\nu(\tau, \sigma)] = [v^\mu(\tau, \sigma), v^\nu(\tau, \sigma)] = 0,
\]

(6)

\[
[x^\mu(\tau, \sigma), P^\nu(\tau, \sigma)] = -i\hbar \delta(\sigma - \sigma') g^{\mu\nu},
\]

where \( P^\mu = \frac{\partial L}{\partial \dot{v}^\mu} = M v^\mu \) is the canonical momentum conjugate to \( x^\mu \). As a consequence we obtain for \( a^\mu, \bar{a}^\mu \) standard harmonic oscillator commutators

\[
[a^\mu_m, a^{\dagger \nu}_n] = [\bar{a}^\mu_m, \bar{a}^{\dagger \nu}_n] = -\delta_{m+n,0} g^{\mu\nu}.
\]

(7)

Differentiating Eq. (4) with respect to \( \tau \) we get the expression of the velocity

\[
v^\mu(\tau, \sigma) = \frac{p^\mu}{M} + \sqrt{\frac{\hbar\omega_0}{2M}} \sum_{m=1}^\infty \left[ \alpha^\mu_m e^{-im\omega_0(\tau-\sigma)} + \bar{\alpha}^\mu_m e^{-im\omega_0(\tau+\sigma)} - \alpha^\prime_m e^{im\omega_0(\tau-\sigma)} - \bar{\alpha}^\prime_m e^{im\omega_0(\tau+\sigma)} \right].
\]

(7)

With respect to the variable \( \tau \) the velocity is periodic with period \( T = \frac{2\pi}{\omega_0} \); then we have \( \forall \tau, \forall \sigma \)

\[
v_\mu(\tau + T, \sigma) - v_\mu(\tau - T, \sigma) = 0.
\]

(8)

Expanding in Taylor series (the notation \(^{(n)}\) indicates the \( n \)-th \( \tau \)-derivative)

\[
\sum_{n=0}^\infty \frac{v^{(n)}_\mu(\tau, \sigma)}{n!} T^n - \sum_{n=0}^\infty (-1)^n \frac{v^{(n)}_\mu(\tau, \sigma)}{n!} T^n = 0
\]

and then

\[
\sum_{n=0}^\infty \frac{v^{(2n+1)}_\mu(\tau, \sigma)}{(2n+1)!} T^{2n+1} = 0.
\]

(8)

By introducing, for convenience, coefficients \( k_n \) defined as

\[
k_n = M \left( \frac{(-1)^n}{(2n+1)!} \right) T^{2n} = M \left( \frac{(-1)^n}{(2n+1)!} \right) \left( \frac{2\pi}{\omega_0} \right)^{2n},
\]

(9)
we can re-write eq. (8) as follows
\[ \sum_{n=0}^{\infty} (-1)^n k_n v_n^{(2n+1)} = 0. \] (10)

Time-integrating both sides of the above equation we get
\[ \sum_{n=0}^{\infty} (-1)^n k_n v_n^{(2n)} = \text{constant}, \] (11)

which, being \( k_0 = M \) for (9), can be re-written as
\[ M v = \text{constant} + \sum_{n=1}^{\infty} (-1)^n k_n v_n^{(2n)}. \] (12)

By comparison between Eqs. (12) and (7) we infer that the constant is just the mean momentum \( p_\mu \). On the other hand inserting the expression of the velocity given by (7) in the above equation and exploiting definition (9), we get, after some algebra, just constant=\( p_\mu \): So that we finally get a “proper-time equation of motion” for a bosonic string in the form
\[ \sum_{n=0}^{\infty} (-1)^n k_n v_n^{(2n)} = p_\mu. \] (13)

For various applications in Section 3, let us introduce the dimensionless coefficients \( \bar{k}_n \) defined as follows
\[ \bar{k}_n \equiv k_n \frac{\omega_0^{2n}}{M}. \] (14)

Exploiting the explicit expression of \( \bar{k}_n \) we have for \( x \neq 0 \)
\[ \sum_{n=0}^{\infty} \bar{k}_n x^{2n} = \frac{1}{2\pi x} \sum_{n=0}^{\infty} \frac{(-1)^n (2\pi)^{2n+1}}{(2n+1)!} x^{2n+1} = \frac{\sin(2\pi x)}{2\pi x}. \]

Differentiating side by side the previous equation we get
\[ \sum_{n=0}^{\infty} n \bar{k}_n x^{2n} = \frac{\cos(2\pi x)}{2} - \frac{\sin(2\pi x)}{4\pi x}, \]

which for \( x = m \in \mathbb{N}^+ \) yields
\[ \sum_{n=0}^{\infty} n \bar{k}_n m^{2n} = \frac{1}{2}. \] (15)

We have also
\[ \sum_{n=0}^{\infty} \bar{k}_n m^{2n} = 0 \] (16)

holding for any positive integer \( m \): this relation can be obtained taking \( x \) equal to \( m \in \mathbb{N}^+ \) in
\[ \sum_{n=0}^{\infty} \bar{k}_n x^{2n} = \frac{\sin(2\pi x)}{2\pi x}. \]
2 Non-Newtonian mechanics: a short review

In recent papers [6] we proposed and developed a classical symplectic theory for extended-like‡ microsystems accounting for spin and Zitterbewegung. The classical motion of spinning particles was therein described without recourse to particular models or special formalisms, and without employing Grassmann variables, Clifford algebras, or classical spinors, but simply by generalizing the standard spinless theory. It was only assumed the invariance with respect to the Poincaré group; and only requiring the conservation of the linear and angular momenta we derived the Zitterbewegung and the other kinematical properties and motion constraints. Newtonian Mechanics is re-obtained as a particular case of that theory: namely for spinless systems with no Zitterbewegung.

We started with a Poincaré-invariant Lagrangian which generalizes the Newtonian Lagrangian \( \mathcal{L}^{(0)} = \frac{1}{2} m v^2 \) (as usual \( v^2 = v_\mu v^\mu \)) by means of proper-time derivatives of the velocity up to the \( N \)-th order (we take the scalar potential \( U = 0 \) considering only free particles)

\[
\mathcal{L}^{(N)} = \frac{1}{2} M v^2 + \frac{1}{2} k_1 \dot{v}^2 + \frac{1}{2} k_2 \ddot{v}^2 + \cdots - \equiv \sum_{n=0}^{N} \frac{1}{2} k_n^N v^{(n)2},
\]

(17)

where the \( k_n^N \) are scalar coefficients analogous to the \( k_n^N \) introduced in Section 1.

The coefficients \( k_i \)—which may be chosen equal to zero for \( i \) larger than a given integer—might be fixed by the self-interaction of the particle, and are often functions of mass and charge. Let us recall, for comparison, the infinite-terms equation of the self-radiating classical electron, Caldirola’s “chronon” theory of the electron[10], the so-called “dressed particles” described in [11]. In other theoretical frameworks, the coefficients \( k_i \) can be related to the underlying geometrical structure of spinning-particles or strings (or \( D \)-branes). Some authors [12, 13, 14] have proposed classical actions, describing “rigid particles” and “rigid strings”, in which appear additional terms dependent on the so-called “extrinsic curvature”, that is, on the 4-acceleration squared. Classical equations of the motion for a rigid particle or for a rigid \( n \)-dimensional worldsheet, either in flat or curved background spacetimes, have been derived from Lagrangians containing also terms dependent on higher derivatives of the 4-velocity (“torsion”-terms, etc.). In [15] the equations of motion are reformulated in terms of the principal worldline curvatures which turn out to be motion integrals, namely mass and spin. Another interesting result in [15] is the emergence of a particle maximal proper acceleration: which constitutes an example of a consistent relativistic dynamics obeying the principle of a superiorly limited value of the acceleration, recently advanced. In a sense most of the above-mentioned models—we refer to those approaches where the time parameter, i.e., the proper time, is just the time in the center-of-mass frame—are particular cases of the present theory: As a matter of fact we do not choose a particular (internal) geometry or kinematics, but include a priori time derivatives of any order, leaving the structure-coefficients \( k_i \) not fixed.

Coming back to (17), the Euler-Lagrange equation of motion

\[
\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{d\tau} \left( \frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{d^2}{d\tau^2} \left( \frac{\partial \mathcal{L}}{\partial \ddot{x}} \right) + \frac{d^3}{d\tau^3} \left( \frac{\partial \mathcal{L}}{\partial \dddot{x}} \right) - \cdots
\]

(18)

‡The term extended-like refers to spinning systems which, even if not “materially” extended as strings, nevertheless are something halfway between a point and a rotating rigid body (as, e.g., a top). In fact the center of mass and the center of charge are distinct points, velocity and momentum are not parallel, and we observe an internal microscopic motion besides the macroscopic external one (the so-called “Zitterbewegung”[7, 8, 9]).
gives a constant-coefficients $N$-th order differential equation, which appears as a generalization of Newton’s Law $F = Ma$, in which non-Newtonian Zitterbewegung terms appear (having assumed $U = 0$, here obviously $F = 0$):

$$0 = Ma^\mu + \sum_{n=1}^{N} (-1)^n k_n N a^{(2n)\mu}.$$  \hspace{1cm} (19)

The zero-th order canonical momentum

$$p^\mu_{[0]} = M v^\mu + \sum_{n=1}^{N} (-1)^n k_n N v^{(2n)\mu} = \sum_{n=0}^{N} (-1)^n k_n N v^{(2n)\mu}.$$  \hspace{1cm} (20)

This equation coincides with the proper time equation of motion for a bosonic string Eq. (13).

By satisfying the symmetry of the Lagrangian under 4-rotations, which implies the conservation of the total angular momentum, the classical spin can be unequivocally defined employing only classical kinematical quantities: Through the Nöther Theorem we derived the spin tensor and the spin vector, respectively

$$S_{\mu\nu} = \sum_{n=1}^{N} k_n N \sum_{l=0}^{n-1} (-1)^{n-l-1} \left( v^{(l)\mu} v^{(l)(2n-l-1)} - v^{(l)\nu} v^{(l)(2n-l-1)} \right);$$  \hspace{1cm} (21)

$$s = \sum_{n=1}^{N} k_n N \sum_{l=0}^{n-1} (-1)^{n-l-1} v^{(l)\times v^{(2n-l-1)}}.$$  \hspace{1cm} (22)

As an example let us take $N = 4$. We have

$$p = M v - k_1 a + k_2 a - k_3 a^{(V)} + k_4 a^{(VII)}.$$  \hspace{1cm} (23)

The spin is

$$s = k_1 (v \times a) + k_2 (a \times a - v \times a) + k_3 \left( a \times a - a \times a + v \times a^{(IV)} \right) +$$

$$+ k_4 \left( a \times a - a \times a^{(IV)} + a \times a^{(V)} - v \times a^{(VI)} \right);$$  \hspace{1cm} (24)

thus, after differentiating and simplifying,

$$\dot{s} = v \times \left( k_1 a - k_2 a + k_3 a^{(V)} - k_4 a^{(VII)} \right) = v \times (M v - p) = p \times v,$$

as expected [see Eq. (31) in the next section].

The Hamiltonian representation of the theory is performed by introducing, besides the (constant) zero-th order momentum $p^\mu_{[0]}$ given in (20), the other (non-constant) $l$-th order momenta $p^\mu_{[l]}$ canonically conjugate to $x_{[l]} = x^{(l)}$:

$$p^\mu_{[l]} = \sum_{n=l}^{N} (-1)^{n-l} \frac{d^n}{dx^{(n+1)}} \left( \frac{\partial L}{\partial x^{(n+1)}} \right) = \sum_{n=l}^{N} (-1)^{n-l} k_n N v^{(2n-l)}.$$  \hspace{1cm} (25)
[this definition contains also the $l = 0$ case, Eq. (20)]. Employing the high order momenta the spin vector (22) can be expressed also in a canonical way:

$$s = \sum_{l=1}^{N} x[l] \times p[l].$$

(26)

analogous to the orbital angular momentum $l = x \times p[0]$. The conserved scalar Hamiltonian, obtained imposing the $\tau$-reparametrization invariance of the Lagrangian, is

$$\mathcal{H} = \sum_{l=0}^{N} p[0][l] \dot{x}[l] - \mathcal{L} = \frac{1}{2} M v^2 + \sum_{n=1}^{N} k_n \left[ \frac{1}{2} v(n)^2 + \sum_{l=0}^{n-1} (-1)^n l v(l) \nu_\mu v(2n-l) \right].$$

(27)

It can be also shown that a couple of Hamilton equations

$$\dot{x}[l] = \frac{\partial \mathcal{H}}{\partial p[l] \mu} \quad \quad \quad \dot{p}[l] = -\frac{\partial \mathcal{H}}{\partial x[l] \mu}$$

(28)

holds for any couple of canonical variables $\left( x[l]; p[l] \right)$, and that the set of the Hamilton equations is globally equivalent to the Euler-Lagrange equation (19).

### 3 String motion constants

Let us turn back to bosonic strings. For a free spinning microsystem the angular momentum, defined as the sum of the orbital angular momentum $x^\mu p^\nu - x^\nu p^\mu$ and of the intrinsic angular momentum, conserves

$$\dot{J}^{\mu \nu} = \dot{L}^{\mu \nu} + \dot{S}^{\mu \nu} = 0.$$  

(29)

Since the momentum is constant, $\dot{p}^{\mu} = 0$, we get $\dot{L}^{\mu \nu} = v^{\mu} p^{\nu} - v^{\nu} p^{\mu}$: therefore we obtain

$$\dot{S}^{\mu \nu} = p^{\mu} v^{\nu} - p^{\nu} v^{\mu}.$$  

(30)

As a consequence, the equation of the motion for the spin 3-vector writes

$$\dot{s} = p \times v;$$  

(31)

and in the center-of-mass frame where $p = 0$ we have, as expected, a constant spin, $\dot{s} = 0$. Besides the trivial choice $s = -L = p \times x$, the only vector $s(\tau; \sigma)$ which can be builted up with the quantities at our disposal (that is $x$ and its derivatives) and satisfies (31) has the same formal expression (22) for the spin of an extended-like particle

$$s = \sum_{n=1}^{\infty} k_n \sum_{l=0}^{n-1} (-1)^{n-l-1} v(l) \times v(2n-l-1).$$  

(32)

This statement can be immediately realized taking into account that both the bosonic string and the extended-like particle obey the same proper time equation of motion, Eq. (13), provided that
coefficients $k_n$ be defined according to Eq. (9). Alternatively we can start time-differentiating side by side the previous equation

$$s = \sum_{n=1}^{\infty} k_n \sum_{l=0}^{n-1} (-1)^{n-l-1} \left[ v^{(l+1)} \times v^{(2n-l-1)} + v^{(l)} \times v^{(2n-l)} \right].$$

Putting $\hat{l} \equiv l + 1$ we have

$$s = \sum_{n=1}^{\infty} k_n \left[ \sum_{l=1}^{n-1} (-1)^n \hat{v}^{(\hat{l})} \times v^{(2n-\hat{l})} + \sum_{l=0}^{n-1} (-1)^{n-l-1} v^{(l)} \times v^{(2n-l)} \right] =$$

$$= \sum_{n=1}^{\infty} k_n \left[ \sum_{l=1}^{n-1} (-1)^n \hat{v}^{(\hat{l})} \times v^{(2n-\hat{l})} + \sum_{l=1}^{n-1} (-1)^{n-l-1} v^{(l)} \times v^{(2n-l)} +
+ v^{(n)} \times v^{(n)} + (-1)^{n-1} v \times v^{(2n)} \right].$$

In the above sum the first two terms mutually cancel, the third one is identically zero; the only contribute comes from the last term:

$$\dot{s} = -v \times \sum_{n=1}^{\infty} (-1)^n k_n v^{(2n)}.$$

Then, because of (13), we have

$$\dot{s} = p \times v.$$

By quite analogous arguments it can be proved that, not only for extended-like particles, but also for bosonic strings, the quantity appearing in Eq. (27)

$$E = \frac{1}{2} M v^2 + \sum_{n=1}^{\infty} k_n \left[ \frac{1}{2} v^{(n)}^2 + \sum_{l=0}^{n-1} (-1)^{n-l} v^{(l)} \mu \nu v^{(2n-l)} \right]$$

is the only 4-scalar integral of motion which can be builted through the quantities at our disposal ($p$, $x$ and its derivatives). Therefore, for any solution (7) it is

$$\dot{E} = 0.$$

For a bosonic string, because of Eqs. (9), (32), (33), we have the following expressions for the spin and $E$, respectively:

$$s = M \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left( \frac{2\pi}{\omega_0} \right)^{2n} \frac{1}{\omega_0} \sum_{l=0}^{n-1} (-1)^{l+1} v^{(l)} \times v^{(2n-l-1)}$$

$$E = \frac{1}{2} M v^2 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \left( \frac{2\pi}{\omega_0} \right)^{2n} \frac{1}{\omega_0} \left[ \frac{1}{2} v^{(n)}^2 + \sum_{l=0}^{n-1} (-1)^{n-l} v^{(l)} \mu \nu v^{(2n-l)} \right].$$
Let us choose the center-of-mass frame \((p = 0)\) as reference frame, and insert the solution (7) with \(p = 0\) in (32). After some algebra we are able to represent the \((\tau - \text{constant in the center-of-mass frame})\) spin vector in terms of operators (5)

\[
s(\sigma) = h \sum_{m=1}^{\infty} \left( \sum_{n=1}^{\infty} nk_n m^{2n} \right) i \left[ (a_m^\dagger \times a_m) + (\bar{a}_m^\dagger \times \bar{a}_m) + (a_m^\dagger \times \bar{a}_m) e^{-2im\omega_0\sigma} + (\bar{a}_m^\dagger \times a_m) e^{2im\omega_0\sigma} \right],
\]

which, accounting for (15), reduces to

\[
s(\sigma) = h \sum_{m=1}^{\infty} \frac{i}{2} \left[ (a_m^\dagger \times a_m) + (\bar{a}_m^\dagger \times \bar{a}_m) + (a_m^\dagger \times \bar{a}_m) e^{-2im\omega_0\sigma} + (\bar{a}_m^\dagger \times a_m) e^{2im\omega_0\sigma} \right].
\]

(37)

Thus the spin averaged over a closed string results to be

\[
\overline{s} = \frac{1}{L} \int_0^L s(\sigma) d\sigma = h \sum_{m=1}^{\infty} \frac{i}{2} \left[ (a_m^\dagger \times a_m) + (\bar{a}_m^\dagger \times \bar{a}_m) \right],
\]

(38)

which agrees with the usual expression (not explicitly derived but only postulated) in standard string theory.

Finally, inserting (7) in (33), after \(\sigma\)-integration we get

\[
\overline{E} = \frac{1}{L} \int_0^L E(\sigma) d\sigma = \frac{p^2}{2M} - \frac{h\omega_0}{2} \sum_{m=1}^{\infty} \frac{1}{m} \left[ (a_{m\mu}^\dagger a_m^\mu + a_{m\mu}^\dagger a_m^\mu) + (\bar{a}_{m\mu}^\dagger \bar{a}_m^\mu + \bar{a}_{m\mu}^\dagger \bar{a}_m^\mu) \right];
\]

or, exploiting (15) and (16),

\[
\overline{E} = \frac{p^2}{2M} - \frac{h\omega_0}{2} \sum_{m=1}^{\infty} \frac{1}{m} \left[ (a_{m\mu}^\dagger a_m^\mu + a_{m\mu}^\dagger a_m^\mu) + (\bar{a}_{m\mu}^\dagger \bar{a}_m^\mu + \bar{a}_{m\mu}^\dagger \bar{a}_m^\mu) \right].
\]

(39)

Taking into account that \(M = \frac{2\pi T}{\omega_0}\) we can put the above equation in the form

\[
\overline{E} = \frac{p^2}{2M} - \frac{hT}{2M} \sum_{m=1}^{\infty} \frac{1}{m} \left[ (a_{m\mu}^\dagger a_m^\mu + a_{m\mu}^\dagger a_m^\mu) + (\bar{a}_{m\mu}^\dagger \bar{a}_m^\mu + \bar{a}_{m\mu}^\dagger \bar{a}_m^\mu) \right],
\]

(40)

where we see that the quantum (due to the “internal” string motion) contribution to \(\overline{E}\) grows in modulus with the tension for mass unity \(T/M\).
The conserved quantity $E(\sigma)$ given by (36) can be put in comparison with the non-conserved Hamiltonian density found in the literature on bosonic string\cite{1,2,3,4,5}

$$\mathcal{H}(\tau; \sigma) = P_\mu \dot{x}^\mu - \mathcal{L} = \frac{1}{2} M \left( \dot{x}^2 + x'^2 \right),$$

which anyhow, taking the string-average, gives back just Eq. (40) (which is conserved):

$$\overline{\mathcal{H}} = \frac{1}{L} \int_0^L \mathcal{H} \, d\sigma = E.$$

4 Conclusions

In this paper we have analytically derived the spin vector of bosonic strings obtaining an explicit kinematical formulation of the intrinsic angular momentum through a sum over all string modes. We have exploited the common non-local character of strings and spinning extended-like particles. Actually one of the novelties of the present approach is describing a bosonic string through infinite time derivatives: the spin of the string arises as angular momentum of such a mechanical system in the center-of-mass frame.

The present derivation of the bosonic string spin through a Fourier expansion as in (38), or in a pure kinematical form as in (32), is, as far as we know, original and unpublished in the literature on strings.

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References

[1] D. Bailin and A. Love, *Supersymmetric Gauge Field Theory and String Theory* (Institute of Physics Publishing; Bristol and Philadelphia, 1994);

[2] P. West, *Introduction to Supersymmetry and Supergravity* (World Scientific Publishing; Singapore 1990)

[3] A. M. Polyakov, Phys.Lett.*103B*, 207 (1981)

[4] M. B. Green, J. H. Schwartz and E. Witten, *Superstring Theory* (Cambridge University Press; Cambridge 1987)

\footnote{Actually if we insert the general solution of the string motion equation given by Eq. (4) in the standard expression Eq. (36), we do not get a time-constant expression; as instead it is obtained if we insert the solution in Eq. (38).}
[5] J. Polchinski, *What is String Theory?* [arXiv:hep-th/9411028]

[6] G. Salesi, Int. J. Mod. Phys. A17, 347 (2002) [arXiv:quant-ph/0112052]; A20, 2027 (2005)

[7] P.A.M. Dirac, *The principles of Quantum Mechanics* (Claredon; Oxford, 1958), 4th edition, p.262; J. Maddox, Nature 325, 306 (1987)

[8] G. Salesi, Mod. Phys. Lett. A11, 1815 (1996); Int. J. Mod. Phys. A12, 5103 (1997); G. Salesi and E. Recami, Phys. Lett. A190, 137 (1994); A195, E389 (1994); Found. Phys. 28, 763 (1998); E. Recami and G. Salesi, Phys. Rev. A57, 98 (1998); Adv. Appl. Cliff. Alg. 6, 27 (1996); in *Gravity, Particles and Space-Time*, ed. by P. Pronin and G. Sardanashvily (World Scient.; Singapore, 1996), pp.345-368; M. Pavšić, E. Recami, W.A. Rodrigues, G.D. Maccarrone, F. Raciti and G. Salesi, Phys. Lett. B318, 481 (1993); W.A. Rodrigues, J. Vaz, E. Recami and G. Salesi, Phys. Lett. B318, 623 (1993); J. Vaz and W.A. Rodrigues, Phys. Lett. B319, 203 (1993)

[9] E. Schrödinger, Sitzunger. Preuss. Akad. Wiss. Phys.-Math. Kl. 24, 418 (1930); 25, 1 (1931)

[10] P. Caldirola, *Suppl. Nuovo Cim.* 3, 297 (1956); Nuovo Cimento 10, 1747 (1953); Lett. Nuovo Cim.16 (1976) 151; *Rivista Nuovo Cim.* 2 (1979), issue no.13, and refs. therein; P. Caldirola and E. Montaldi, Nuovo Cimento B53, 291 (1979); P. Caldirola, G. Casati and A. Prosperetti, Nuovo Cimento A43, 127 (1978); P. Caldirola, Nuovo Cimento A49, 497 (1979); A. Prosperetti, Nuovo Cimento B57, 253 (1980); L. Belloni, Lett. Nuovo Cim. 31, 131 (1981); V. Benza and P. Caldirola, Nuovo Cimento A62, 175 (1981)

[11] B.P. Kosyakov, Phys. Part. Nucl. 34, 808 (2003); B.P. Kosyakov, V.V. Nesterenko, Phys. Lett. B384, 70 (1996)

[12] M. Pavšić, Phys. Lett. B205, 231 (1988); B221, 264 (1989); Class. Quant. Grav. L7, 187 (1990)

[13] M.S. Plyushchay, *Comment on the relativistic particle with curvature and torsion of world trajectory*, [arXiv:hep-th/9810101] Phys. Lett. B262, 71 (1991); Mod. Phys. Lett. A3, 1299 (1988); A4, 837 (1989); A4, 2747 (1989); Int. J. Mod. Phys. A4, 3851 (1989); Phys. Lett. B243, 383 (1990); Phys. Lett. B236, 291 (1990); B235, 47 (1990); B253, 50 (1991)

[14] A.M. Polyakov, Nucl. Phys. B268, 406 (1986); Mod. Phys. Lett. A3, 325 (1988); Yu.A. Kuznetsov and A.M. Polyakov, Phys. Lett. B297, 49 (1992)

[15] V.V. Nesterenko, A. Feoli, G. Scarpetta, J. Math. Phys. 36, 5552 (1995); V.V. Nesterenko, Phys. Lett. B327, 50 (1994)