Charge and Spin Currents of the 1D Hubbard Model at Finite Energy

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Abstract

The transport of charge and spin at finite energies is studied for the Hubbard chain in a magnetic field by means of the pseudoparticle perturbation theory. In the general case, this involves the solution of an infinite set of Bethe-ansatz equations with a flux. Our results refer to all densities and magnetizations. We express the charge and the spin-diffusion currents in terms of elementary currents associated with the charge and spin carriers. We show that these are the $\alpha, \gamma$ pseudoparticles (with $\alpha = c, s$ and $\gamma = 0, 1, 2, 3, ...$) and we find their couplings to charge and spin. We also study the ratios of the pseudoparticle charge and spin transport masses over the corresponding static mass. These ratios provide valuable information on the effects of electronic correlations in the transport properties of the quantum system. We show that the transport of charge and spin in the Hubbard chain can, alternatively, be described by means of pseudoparticle kinetic equations. This follows from the occurrence of only forward-scattering pseudoparticle interactions at all energies.

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I. INTRODUCTION

The transport properties of strongly correlated electron models for low-dimensional conductors has been a subject of experimental and theoretical interest for over twenty years. Low-dimensional conductors show large deviations in their transport properties from the usual single-particle description. This suggests that electronic correlations might play an important role in these systems [1], even if they are small [2]. Solvable one-dimensional many-electron models such as the Hubbard chain are often used as an approximation for the study of the properties of quasi-one-dimensional conductors [1,2]. Although the Hubbard chain has been diagonalized long ago [3,4], the involved form of the Bethe-ansatz (BA) wave function has prevented the calculation of dynamic response functions, these including the charge-charge and spin-spin response functions and their associate conductivity spectra.

Information on low-energy expressions for correlation functions can be obtained by combining BA with conformal-field theory [5]. On the other hand, several approaches using perturbation theory [6], bosonization [7,8], the pseudoparticle formalism [9], scaling methods [10], and spin-wave theory [11] have been used to investigate the low-energy transport properties of the model away from half filling and at the metal – insulator transition [3]. Unfortunately, only limited information on the transport properties at finite energies has been obtained by numerical methods [12,14].

Recently, a pseudoparticle description of all the BA Hamiltonian eigenstates [15] has allowed the evaluation of analytical expressions for correlation functions at finite energy [16]. From these results one can obtain expressions for the absorption band edges of the frequency-dependent electronic conductivity, $\sigma(\omega)$ [17]. The pseudoparticle theory of Ref. [15] introduces new branches of pseudoparticles and generalizes for all energy scales previous low-energy studies [18]. The new pseudoparticle branches are associated with heavy pseudoparticles. These are the quantum objects needed for the description of Hamiltonian eigenstates showing an energy gap relatively to the ground state. (This justifies why they are called heavy.) As in the case of the low-energy properties of the Hubbard model [18],
it is possible to write the Hamiltonian in terms of a set of antimutting pseudoparticle operators. Importantly, all the model eigenstates can be generated from the $SO(4)$ ground state \[19\].

In this paper we use the above generalized pseudoparticle theory to study the charge and spin currents of the Hubbard chain at finite energy. For that we solve the BA equations with a twist angle for all densities and magnetizations. Moreover, we express the charge and the spin-diffusion currents in terms of the elementary currents of the charge and spin carriers. It is shown that the latter carriers are the $\alpha, \gamma$ pseudoparticles of the pseudoparticle-perturbation theory (PPT) \[15\]. We evaluate their couplings to charge and spin and define the charge and spin pseudoparticle transport masses. The ratios of these masses over the corresponding static mass provide important information on the role of electronic correlations in the transport of charge and spin in the 1D quantum liquid. Furthermore, we find that the transport of charge and spin can be described by means of pseudoparticle kinetic equations. Our results are a generalization to finite energies of the low-energy results on transport of charge and spin presented in Ref. \[9\]. This is possible by means of the generalized pseudoparticle representation introduced in Ref. \[15\] which is a generalization to finite-energy scales of the usual low-energy operator representation \[18\] in terms of pseudoparticles \[20–24\].

The paper is organized as follows. In Sec.\[II\] we present the PPT description of the Hilbert space associated with the zero-temperature transport of charge and spin. In Sec.\[III\] we introduce the general BA equations with a spin dependent twist angle, $\phi_\sigma$, valid for all energy scales, and solve these equations by use of the PPT. The pseudoparticle transport and static masses and the transport of charge and spin are studied and discussed in Sec.\[IV\]. In Sec.\[V\] we show that the transport of charge and spin can be described by means of kinetic equations. Finally, the concluding remarks are presented in Sec.\[VI\].
II. PSEUDOPARTICLE PERTURBATION THEORY

Let us consider the Hubbard-chain Hamiltonian with $N$ electrons in a magnetic field $H$ and with chemical potential $\mu$

\[
\hat{H} = -t \sum_{j,\sigma} [c_{j\sigma}^\dagger c_{j+1\sigma} + h.c.] + U \sum_j [\hat{n}_{j,\uparrow} - 1/2][\hat{n}_{j,\downarrow} - 1/2] - \mu(N_a - \sum_{j,\sigma} \hat{n}_{j,\sigma}) - \mu_0 H \sum_{j,\sigma} \sigma \hat{n}_{j,\sigma},
\]

where $c_{j\sigma}^\dagger (c_{j\sigma})$ creates (annihilates) one electron with spin $\sigma$ (here and when used as operator index, $\sigma = \uparrow, \downarrow$, and $\sigma = \pm 1$ otherwise), $\hat{n}_{j,\sigma} = c_{j\sigma}^\dagger c_{j\sigma}$ is the number operator at site $j$, $N_a$ is the number of sites of the chain (since we are using periodic boundary conditions it is rather a ring), and $c_{N_a+1\sigma} = c_{1\sigma}$. In general, we use units such that $\hbar = 1$, the lattice spacing $a = 1$, and the electron charge $-e = 1$. The form of the interaction term accounts for the particle–hole symmetry of the model at half filling [3,25]. For simplicity, we consider electronic and magnetization densities in the domains $0 \leq n \leq 1$ and $0 \leq m \leq n$, respectively, where $n = N/N_a = [N\uparrow + N\downarrow]/N_a$, $m = [N\uparrow - N\downarrow]/N_a$, and $N_\sigma$ is the number of $\sigma$ electrons.

Although the pseudoparticle description refers to all Hamiltonian eigenstates [15], in this paper we restrict our study to the Hilbert subspace involved in the zero-temperature charge and spin frequency-dependent conductivity [17]. This is spanned by all the Hamiltonian eigenstates contained in the states $\hat{j}^\zeta |GS\rangle$, where $|GS\rangle$ denotes the ground state and $\hat{j}^\zeta$ are the charge ($\zeta = \rho$) and spin ($\zeta = \sigma_z$) current operators (given by Eqs. (27) and (28) below, respectively). Since these current operators commute with the six generators of the $\eta$-spin and spin algebras [26], our Hilbert subspace is in the present parameter space spanned only by the lowest-weight states (LWS’s) of these algebras [19]. This refers to the Hilbert subspace directly described by the BA solution [15]. [Therefore, following the studies and notations of Ref. [15], we can use the $\alpha, \gamma = 0$ pseudoparticles instead of the $\alpha, \beta$ pseudoholes (with $\beta = \pm 1$) required for the description of the non-LWS’s outside the BA solution.]

The PPT introduced in detail in Ref. [13] involves infinite branches of pseudoparticles labelled by the quantum numbers $\alpha$ and $\gamma$. Here $\alpha = c, s$ and $\gamma = 0, 1, 2, 3, \ldots \infty$. Fortu-
nately, for most Hamiltonian eigenstates of physical interest only a small number out of the infinite available $\alpha, \gamma$ bands have pseudoparticle occupancy. The possible pseudomomentum occupancies correspond to the different LWS’s of the model. The pseudomomentum values $q$ are such that $q^{(-)}_{\alpha,\gamma} \leq q \leq q^{(+)}_{\alpha,\gamma}$, where the expressions of the pseudo-Brillouin-zone limits, $q^{(\pm)}_{\alpha,\gamma}$, are given in Ref. [15]. They are of the form $q^{(\pm)}_{\alpha,\gamma} = \pm q_{\alpha,\gamma} + O(1/N_a)$, where for the case of a ground state $q_{\alpha,\gamma}$ is given by

\begin{align*}
q_{c,0} &= \pi; & q_{c,\gamma} &= \pi - 2k_F \quad \gamma > 0, \\
q_{s,0} &= k_{F\uparrow}; & q_{s,\gamma} &= k_{F\uparrow} - k_{F\downarrow} \quad \gamma > 0.
\end{align*}

(2)

The branches $c,0$ and $s,0$ have been called in previous low-energy studies $c$ and $s$, respectively [9,18]. They were shown to describe the low-energy excitations of the Hubbard chain and to determine the low-energy behavior of its charge and spin transport properties [9,18]. (In the limit of low energy the heavy-pseudoparticle branches are empty.) On the other hand, description of the LWS’s of the model that have a finite-energy gap, $\omega_0$, relatively to the ground state involves the heavy pseudoparticle branches $c, \gamma > 0$ and $s, \gamma > 0$ [15].

A very useful concept in this theory is that of generalized ground state (GGS). In Ref. [15] it was defined as the Hamiltonian eigenstate(s) of lowest energy in the Hilbert subspace associated with a given sub-canonical ensemble. The concept of sub-canonical ensemble follows from the conservation laws of the $\alpha, \gamma$ pseudoparticle numbers, $N_{\alpha,\gamma}$. Each Hamiltonian eigenstate has constant values for these numbers and a sub-canonical ensemble refers to a given choice of constant $N_{\alpha,\gamma}$ numbers.

On the other hand, in Ref. [16] the concept of GGS was extended to (i) filled $\alpha, \gamma$ pseudoparticle seas with compact occupations around $q \approx 0$, i.e. for $q^{(-)}_{F_{\alpha,\gamma},+1} \leq q \leq q^{(+)}_{F_{\alpha,\gamma},+1}$, where the pseudo-Fermi points are given by $q^{(\pm)}_{F_{\alpha,\gamma},+1} = \pm \frac{\pi N_{\alpha,\gamma}}{N_a} + O(1/N_a)$, and (ii) filled $\alpha, \gamma$ pseudoparticle seas with compact occupations for $q^{(-)}_{\alpha,\gamma} \leq q \leq q^{(-)}_{F_{\alpha,\gamma},-1}$ and for $q^{(+)}_{F_{\alpha,\gamma},-1} \leq q \leq q^{(-)}_{\alpha,\gamma}$, where the pseudo-Fermi points are given by $q^{(\pm)}_{F_{\alpha,\gamma},-1} = \pm [q_{\alpha,\gamma} - \frac{\pi N_{\alpha,\gamma}}{N_a}] + O(1/N_a)$. From the studies of Refs. [16,17], it will be shown elsewhere that the creation of one $\alpha, \gamma$ pseudoparticle from the ground state involves, to leading order, a number $2\gamma$ of electrons.
Since the currents are two-electron operators, it follows that the creation of single \(\alpha,1\) pseudoparticles from the ground state are the most important contributions to the transport of charge and spin at finite energies. On the other hand, since the states \(\hat{\mathcal{C}}|\text{GS}\rangle\) which define our Hilbert space have zero momentum (relatively to the ground state) and the creation from the ground state of single \(\alpha,1\) pseudoparticles of type (ii) is a finite-momentum excitation, for simplicity in this paper we restrict our study to GGS’s of type (i). Therefore, in order to simplify our notation we denote the pseudo-Fermi points \(q_{F\alpha,\gamma}^{(\pm)}\) simply by \(q_{F\alpha,\gamma}\). These are given by \(q_{F\alpha,\gamma}^{(\pm)} = \pm q_{F\alpha,\gamma} + O(1/N_a)\) where the pseudo-Fermi momentum (3) appears in several expressions below. Note, however, that the generalization of our results to GGS’s of type (ii) is straightforward. We emphasize that Ref. [15] definition of GGS refers to the above choice (i) for the \(c,0\) and \(s,\gamma\) branches and to the choice (ii) for the \(c,\gamma\) branch with \(\gamma > 0\). Therefore, in the case of the \(c,\gamma > 0\) pseudoparticles our GGS choice differs from the choice of that reference.

The ground state is a special case of a GGS where there is no \(\alpha,\gamma > 0\) heavy-pseudoparticle occupancy [15] and the pseudo-Fermi points (3) are of the form

\[
q_{Fc,0} = 2k_F; \quad q_{Fs,0} = k_{F\downarrow}; \quad q_{F\alpha,\gamma} = 0 \quad \gamma > 0.
\]

The PPT consists in expanding the Hamiltonian (1) in the density of excited pseudoparticles relatively to the initial ground state. This allows us to write Hamiltonian (1) in normal order relatively to that ground state as [15]

\[
\hat{H} := \hat{H}_0 + \hat{H}_{\text{Landau}},
\]

where up to second order in the density of excited pseudoparticles, \(\hat{H}_{\text{Landau}}\) is of the form

\[
\hat{H}_{\text{Landau}} = \hat{H}^{(1)} + \hat{H}^{(2)},
\]

with
\[ \hat{H}^{(1)} = \sum_{q,\alpha,\gamma} \epsilon_{\alpha,\gamma}(q) : \hat{N}_{\alpha,\gamma}(q) : , \]  

\[ \epsilon_{\alpha,0}(q) = \epsilon_{\alpha,0}^{(0)}(q) - \epsilon_{\alpha,0}^{(0)}(q_{F,\alpha,0}) , \quad \epsilon_{\alpha,\gamma}(q) = \epsilon_{\alpha,\gamma}^{(0)}(q) - \epsilon_{\alpha,\gamma}^{(0)}(0) , \]  

and \( \hat{H}^{(2)} \) is given by

\[ \hat{H}^{(2)} = \frac{1}{N_0} \sum_{q,\alpha,\gamma} \sum_{q'} \frac{1}{2} f_{\alpha,\gamma,\alpha',\gamma'}(q, q') : \hat{N}_{\alpha,\gamma}(q) :: \hat{N}_{\alpha',\gamma'}(q') : . \]

Here \( \hat{N}_{\alpha,\gamma}(q) = b_{q,\alpha,\gamma}^\dagger b_{q,\alpha,\gamma} \) is the \( \alpha, \gamma \)-pseudoparticle operator number at pseudomomentum \( q \) and the operators \( b_{q,\alpha,\gamma}^\dagger \) and \( b_{q,\alpha,\gamma} \) obey the usual anticommuting algebra \[15\]. The Hamiltonian eigenstates described by the BA are also eigenstates of \( \hat{N}_{\alpha,\gamma}(q) \) with eigenvalue \( N_{\alpha,\gamma}(q) \) and eigenstates of \( : \hat{N}_{\alpha,\gamma}(q) : \) with eigenvalue \( \delta N_{\alpha,\gamma}(q) \equiv N_{\alpha,\gamma}(q) - N_{\alpha,\gamma}^0(q) \), where \( N_{\alpha,\gamma}^0(q) \) is the ground-state pseudomomentum distribution \[13\]. These distributions characterize the occupancy configurations of the pseudomenta in the \( \alpha, \gamma \)-pseudoparticle bands.

The physical meaning of the Hamiltonian terms \( \hat{H}_0 \) and \( \hat{H}_{\text{Landau}} \) is explained in Ref. \[15\]. These Hamiltonian terms are such that \([ \hat{H}_0, \hat{H}_{\text{Landau}} ] = 0 \) and \( \hat{H}_0 \) has eigenvalue \( \omega_0 \) given by \[15,16\]

\[ \omega_0 = 2\mu \sum_{\gamma > 0} \gamma N_{c,\gamma} + 2\mu_0 H \sum_{\gamma > 0} (1 + \gamma) N_{s,\gamma} + \sum_{\alpha, \gamma > 0} \epsilon_{\alpha,\gamma}^{(0)}(0) N_{\alpha,\gamma} , \]

where \( N_{\alpha,\gamma} \) are the numbers of \( \alpha, \gamma \) heavy pseudoparticles created in the transition from the ground state to the GGS.

The set of energies \( \omega_0 \), Eq. \( (10) \), play a central role in the theory. This is because for a given initial ground state the PPT is associated with a final Hilbert subspace characterized by a set of finite \( N_{\alpha,\gamma} \) numbers. The states which span such subspace differ from the initial ground state by a small density of pseudoparticles and have small positive \( (\omega - \omega_0) \) energy.

The main point of the PPT is that for low-excitation energy, \( (\omega - \omega_0) \), only the first two Hamiltonian terms, Eq. \( (7) \), are relevant \[15,16\]. Therefore, the truncated Hamiltonian \( (5) - (6) \) describes the physics for energies just above the set of finite-energy values \( \omega_0 \) of the form \( (10) \). [This includes \( \omega_0 = 0 \).]
Since the conservation of the electron numbers imposes the following sum rules on the numbers $N_{\alpha,\gamma}$ \[N_{\downarrow} = \sum_{\gamma > 0} \gamma N_{c,\gamma} + \sum_{\gamma} (1 + \gamma) N_{s,\gamma}, \quad (11)\]
and
\[N = N_{c,0} + 2 \sum_{\gamma > 0} \gamma N_{c,\gamma}, \quad (12)\]
the creation of heavy pseudoparticles from the ground state at constant electron numbers requires the annihilation of $\alpha,0$ pseudoparticles. It follows from Eqs. (11) and (12) that the changes $\Delta N_{\alpha,0}$ associated with a corresponding ground-state – GGS transition read
\[\Delta N_{s,0} = -\sum_{\gamma > 0} \gamma N_{c,\gamma} - \sum_{\gamma > 0} (1 + \gamma) N_{s,\gamma}, \quad (13)\]
and
\[\Delta N_{c,0} = -2 \sum_{\gamma > 0} \gamma N_{c,\gamma}. \quad (14)\]
For instance, the creation of one $c,\gamma$ heavy pseudoparticle from the ground state requires the annihilation of a number $2\gamma$ of $c,0$ pseudoparticles and of a number $\gamma$ of $s,0$ pseudoparticles, whereas the creation of one $s,\gamma$ pseudoparticle involves the annihilation of a number $1 + \gamma$ of $s,0$ pseudoparticles and conserves the number of $c,0$ pseudoparticles.

Although, following Eq. (10), $\omega_0$ can be large, we emphasize that the PPT is always a low $(\omega - \omega_0)$ energy theory. This is because within the PPT the densities of removed $\alpha,0$ pseudoparticles, $-\Delta N_{\alpha,0}/N_a$, of added $\alpha,\gamma > 0$ heavy pseudoparticles, $\Delta N_{\alpha,\gamma}/N_a = N_{\alpha,\gamma}/N_a$, and of their pseudoparticle – pseudohole processes are always kept small. Moreover, for each set of finite $N_{\alpha,\gamma > 0}$ numbers there is one PPT and one value of energy (10).

The energy bands $\epsilon_{\alpha,\gamma}^0(q)$ and the $f$-functions $f_{\alpha,\gamma;\alpha',\gamma'}(q,q')$ are given, respectively, by
\[\epsilon_{c,0}^0(q) = -\frac{U}{2} - 2t \cos K^{(0)}(q) + 2t \int_{-Q}^{Q} dk \tilde{\Phi}_{c,0;0,c,0} \left(k, K^{(0)}(q)\right) \sin k, \quad (15)\]
\[ \epsilon_{c,\gamma}^0(q) = -\gamma U + 4t \Re \sqrt{1 - u^2[R_{c,\gamma}^0(q) - i\gamma]}^2 + 2t \int_{-Q}^Q dk \Phi_{c,0,c,\gamma}(k, R_{c,\gamma}^0(q)) \sin k, \] (16)

\[ \epsilon_{s,\gamma}^0(q) = 2t \int_{-Q}^Q dk \Phi_{c,0,s,\gamma}(k, R_{s,\gamma}^0(q)) \sin k, \] (17)

and

\[ \frac{1}{2\pi} f_{\alpha,\gamma;\alpha',\gamma'}(q, q') = v_{\alpha,\gamma}(q) \Phi_{\alpha,\gamma;\alpha',\gamma'}(q, q') + v_{\alpha',\gamma'}(q') \Phi_{\alpha',\gamma';\alpha,\gamma}(q', q) + \sum_j \sum_{\alpha''} \sum_{\gamma''} \theta(N_{\alpha'';\gamma''}) v_{\alpha'';\gamma''} \Phi_{\alpha'',\gamma'';\alpha,\gamma}(jq_{\alpha'';\gamma''}, q) \Phi_{\alpha'',\gamma'';\alpha',\gamma'}(jq_{\alpha'';\gamma''}, q'). \] (18)

Here \( \theta(x) = 1 \) for \( x > 0 \) and \( \theta(x) = 0 \) otherwise is the usual theta function and \( v_{\alpha,\gamma} = v_{\alpha,\gamma}(q_{F\alpha,\gamma}) \) is the velocity at the pseudo-Fermi point. The phase-shift functions \( \Phi_{\alpha,\gamma;\alpha',\gamma'} \) and the phase shifts \( \Phi_{\alpha,\gamma;\alpha',\gamma'} \) are defined in Ref. [15]. \( \Phi_{\alpha,\gamma;\alpha',\gamma'}(q, q') \) measures the shift in the phase of the \( \alpha', \gamma' \) pseudoparticle of pseudomomentum \( q' \) due to a zero-momentum forward-scattering collision with the \( \alpha, \gamma \) pseudoparticle of pseudomomentum \( q \). It is useful to introduce the function \( W^0(q) \) such that \( W = K, R_{c,\gamma}, R_{s,\gamma} \). (Here, \( \gamma = 1, 2, \ldots, \infty \) for \( R_{c,\gamma} \) and \( \gamma = 0, 1, 2, \ldots, \infty \) for \( R_{s,\gamma} \).) It represents any of the three ground-state rapidity functions \( K^0(q), R_{c,\gamma}^0(q), \) and \( R_{s,\gamma}^0(q) \), whereas the functional \( W(q) \) represents any of the three general functional rapidity functions \( K(q), R_{c,\gamma}(q), \) and \( R_{s,\gamma}(q) \) [15]. These functionals are obtained from Eqs. (A7), (A8), and (A9) with \( \phi = 0 \). (In that Appendix we solve the BA equations with twist angles.) The ground-state functions \( W^0(q) \) are obtained by taking the particular choice \( N_{\alpha,\gamma}(q) = N_{0,\gamma}(q) \) in the latter equations. It is useful to introduce the pseudo-Fermi rapidity parameters [15]

\[ Q = K^0(q_{F\alpha,0}); \quad r_{c,0} = \frac{\sin Q}{u}; \quad r_{\alpha,\gamma} = R_{\alpha,\gamma}^0(q_{F\alpha,\gamma}), \] (19)

where \( Q \) appears in the integrals of Eqs. (13) - (17) and in the last expression \( r_{\alpha,\gamma} \) refers to all \( \alpha, \gamma \) branches except \( c, 0 \).

**III. CHARGE AND SPIN CURRENTS: SOLUTION OF THE BA EQUATIONS**

Within linear response theory the charge and spin currents of the 1D Hubbard model can be computed by performing a spin-dependent Peierls-phase substitution in the hopping
integral of Hamiltonian (1), \( t \rightarrow te^{i\phi_\sigma/N_a} \) [6,27].

It has been possible to solve the Hamiltonian (1) with the additional hopping phase \( e^{\pm i\phi_\sigma/N_a} \) by means of the coordinate BA both with twisted and toroidal boundary conditions, both approaches giving essentially the same results [27,28]. One obtains the energy spectrum of the model parameterized by a set a numbers \( \{k_j, \Lambda_\delta\} \) which are solution of the BA interaction equations given by

\[
e^{ik_jN_a} = e^{i\phi_\uparrow} \prod_{\delta=1}^{N_4} \frac{\sin(k_j) - \Lambda_\delta + iU/4}{\sin(k_j) - \Lambda_\delta - iU/4}, \quad (j = 1, \ldots, N), \tag{20}
\]

and

\[
\prod_{j=1}^{N} \frac{\sin(k_j) - \Lambda_\delta + iU/4}{\sin(k_j) - \Lambda_\delta - iU/4} = e^{i(\phi_\uparrow - \phi_\downarrow)} \prod_{\beta=1, \beta \neq \delta}^{N_4} \frac{\Lambda_\beta - \Lambda_\delta + iU/2}{\Lambda_\beta - \Lambda_\delta - iU/2}, \quad (\delta = 1, \ldots, N_4). \tag{21}
\]

However, previous studies of the \( \phi_\sigma \neq 0 \) problem [21,28] have only considered the real BA rapidities of Eqs. (20) and (21) which refer to low energy. Here we follow the same steps as Takahashi [4] for the \( \phi_\sigma = 0 \) interaction Eqs. (20) and (21) and consider both real and complex rapidities. We then arrive to the following \( \phi_\sigma \neq 0 \) equations which refer to the real part of these rapidities

\[
k_jN_a = 2\pi I^c_j + \phi_\uparrow - \sum_{\gamma}^{N_\gamma} \sum_{j'=1}^{N_\gamma} 2 \tan^{-1} \left( \frac{\sin(k_j)/u - R_{c,\gamma,j'}}{\gamma + 1} \right)
- \sum_{\gamma > 0}^{N_\gamma} \sum_{j'=1}^{N_\gamma} 2 \tan^{-1} \left( \frac{\sin(k_j)/u - R_{c,\gamma,j'}}{\gamma} \right), \tag{22}
\]

\[
2N_aRe \sin^{-1}([R_{c,\gamma,j} + i\gamma]u) = 2\pi I^{c,\gamma}_j + \gamma(\phi_\uparrow + \phi_\downarrow) - \sum_{j'=1}^{N_\gamma} 2 \tan^{-1} \left( \frac{\sin(k_j')/u - R_{c,\gamma,j'}}{\gamma} \right)
+ \sum_{\gamma' > 0}^{N_\gamma} \sum_{j'=1}^{N_\gamma} \Theta_{\gamma,\gamma'}(R_{c,\gamma,j} - R_{c,\gamma',j'}), \tag{23}
\]

and

\[
\sum_{j'=1}^{N_\gamma} 2 \tan^{-1} \left( \frac{R_{s,\gamma,j} - \sin(k_j')/u}{1 + \gamma} \right) + (1 + \gamma)(\phi_\uparrow - \phi_\downarrow)
= 2\pi I^{s,\gamma}_j + \sum_{\gamma' = 1}^{N_\gamma} \sum_{j'=1}^{N_\gamma} \Theta_{\gamma+1,\gamma'+1}(R_{s,\gamma,j} - R_{s,\gamma',j'}). \tag{24}
\]
The functions $\Theta_{\gamma',\gamma}(x)$ [and $\Theta_{\gamma+1,\gamma'+1}(x)$] of Eqs. (22), (23), and (24) are defined in Ref. [15]. The following definitions for the real part of the rapidities, $\Lambda^{\alpha+1}_n/u = R_{s,\gamma,j}$ (with $n + 1 = \gamma$ and $\alpha = j$), $\Lambda^{\alpha}/u = R_{c,\gamma,j}$ (with $n = \gamma$ and $\alpha = j$), and $\gamma = 1, 2, \ldots, \infty$ for the $N_{c,\gamma}$ sums and $\gamma = 0, 1, 2, \ldots, \infty$ for the $N_{s,\gamma}$ sums, allows us to recover Takahashi’s formulae for $\phi = 0$ [4]. Here and often below we use the notation $c \equiv c, 0$, which allows the $c, \gamma$ sums to run over $1, 2, 3, \ldots, \infty$. Whether we are using this notation or the previous one will be obvious from the context.

The important numbers $I^c_j$, $I^{c,\gamma}_j$, and $I^{s,\gamma}_j$ which appear in going from Eqs. (20) and (21) to Eqs. (22), (23), and (24) are the quantum numbers which describe the Hamiltonian eigenstates. Depending on the parity of the numbers $[\sum_{\gamma=0} N_{s,\gamma} + \sum_{\gamma=1} N_{c,\gamma}]$, $[N_a - N + N_{c,\gamma}]$, and $[N - N_{s,\gamma}]$, respectively, they are consecutive integers or half-odd integers [15]. All the LWS’s of the model are described by the different occupancies of these quantum numbers.

For example, the ground state is described by a compact symmetric occupancy around the origin of the numbers $I^c_j$ and $I^{s,0}_j$, and by zero occupancy for the numbers $I^{c,\gamma}_j$ and $I^{s,\gamma>0}_j$ [15]. It is convenient to describe the eigenstates of the model in terms of pseudomomentum $\{q^{\alpha,\gamma}_j = 2\pi I^{\alpha,\gamma}_j/N_a\}$ distributions, where $I^{c,0}_j \equiv I^c_j$.

The energy and momentum eigenvalues are given by [15]

$$E = -2t \sum_{j=1}^{N_a} \cos(k_j) + \sum_{\gamma>0} \sum_{j=1}^{N_{c,\gamma}} 4t Re[1 - u^2 (1 - R^{c,\gamma,j})^2] + N_a(U/4 - \mu) + N(\mu - U/2) - \mu_0 H(N^\uparrow - N^\downarrow),$$

and

$$P = 2\pi \frac{N_c}{N_a} \left[ \sum_{j=1}^{N_c} \sum_{\gamma} I^{c,\gamma}_j + \sum_{j=1}^{N_{s,\gamma}} \sum_{\gamma>0} I^{s,\gamma}_j - \sum_{j=1}^{N_{c,\gamma}} \sum_{\gamma>0} I^{c,\gamma}_j \right] + \pi \sum_{\gamma>0} N_{c,\gamma},$$

respectively. We emphasize that the rapidity dependence on $\phi_\sigma$ is defined by Eqs. (22)-(24) and determines the energy-functional (23) dependence on $\phi_\sigma$. The corresponding $\phi_\sigma = 0$ expressions recover the rapidity and energy expressions of Ref [15].

In the limit of a large system ($N_a \to \infty$, $N/N_a$ fixed) we can develop a generalization of the low-energy pseudoparticle-Landau-liquid description of the Hubbard model [20–22].
and of its operational representation \[18\]. This generalization refers to energies just above the set of energies $\omega_0$, Eq. (10), where the Hamiltonian (3) describes the quantum-problem physics. Note that the choice $\omega_0 = 0$, which refers to $N_{\alpha,\gamma} = 0$ for $\gamma > 0$, recovers the usual low-energy theory of Refs. \[18,20–24\]. On the other hand, when $\omega_0 > 0$, in addition to finite occupancy of the usual $c, 0 \equiv c$ and $s, 0 \equiv s$ pseudoparticle bands, there is finite occupancy for some of the branches of the heavy $c, \gamma$ and $s, \gamma$ pseudoparticles \[13\].

In the above thermodynamic limit the rapidity real parts $k_j = k_j(q_j)$, $R_{s,\gamma,j} = R_{s,\gamma,j}(q_j)$, and $R_{c,\gamma,j} = R_{c,\gamma,j}(q_j)$ proliferate on the real axis. As in Refs. \[20–22\], equations (22), (23), and (24) can be rewritten as integral equations with an explicit dependence on the pseudomomentum distribution functions $N_{\alpha,\gamma}(q)$. These are Eqs. (A7), (A8), and (A9) of Appendix A which refer to the case $\phi_\sigma \neq 0$. In that Appendix we derive ground-state normal-ordered expressions for the rapidities and charge and spin currents.

The combination of Eqs. (25), (A7), (A8), and (A9) allows the evaluation of several interesting transport quantities. This includes the charge and spin currents and the charge and spin pseudoparticle transport masses. The charge and spin current operators $\hat{j}^\zeta$ (with $\zeta = \rho$ for charge, and $\zeta = \sigma_z$ for spin) are for the 1D Hubbard model given by \[4\]

\[
\hat{j}^\rho = -eit \sum_{\sigma}^{N_a} \sum_{j=1}^{N_a} (c_{j+1,\sigma}^\dagger c_{j,\sigma} - c_{j,\sigma}^\dagger c_{j+1,\sigma}),
\]

and

\[
\hat{j}^{\sigma_z} = -(1/2)it \sum_{\sigma}^{N_a} \sum_{j=1}^{N_a} \sigma (c_{j,\sigma}^\dagger c_{j+1,\sigma} - c_{j+1,\sigma}^\dagger c_{j,\sigma}).
\]

Importantly, the discrete nature of the model implies that the commutators of the Hamiltonian (1) and of the current operators $\hat{j}^\zeta$, Eqs. (27) and (28), are non-zero. It follows that the BA wave function does not diagonalizes simultaneously the Hamiltonian (1) and the current operators (27) and (28). Since the BA solution alone only provides the diagonal part in the Hamiltonian-eigenstate basis of the physical operators \[13\], we can only evaluate expressions for the diagonal part of the currents which provide the mean values of the charge and spin currents. These refer to all LWS's and are important quantities for they allow us
to compute the transport masses of the charge and spin carriers of the system. In addition, our formalism defines the charge and spin carriers. These are found to be the $c$ (i.e., $c, 0$) and $c, \gamma$ pseudoparticles for charge, and the $c$ and $s, \gamma$ pseudoparticles for spin. This follows from Eqs. (A7)-(A9) and also from the Boltzmann transport analysis of Sec. V.

We emphasize that combining the generalized pseudoparticle representation [13] with a low-energy ($\omega - \omega_0$) conformal-field theory [16], leads to finite-energy current - current correlation function expressions which are determined by the non-diagonal terms (in the Hamiltonian-eigenstate basis) of the current operators. This is a generalization of the low-energy correlation-function studies of Refs. [5,23]. However, these expressions cannot be derived within the BA solution alone. Therefore, these studies go beyond the scope of the present paper and here we consider the diagonal part of the charge and spin current operators only.

The mean value of the current operator $\hat{j}^\zeta$ in a given LWS, $|m\rangle$, is given by

$$\langle m | \hat{j}^\zeta | m \rangle = -\frac{d(E_m/N_a)}{d(\phi/N_a)} \bigg|_{\phi=0},$$

where $E_m$ is the energy of the Hamiltonian eigenstate $|m\rangle$ and [27]

$$\phi = \phi_\uparrow = \phi_\downarrow, \quad \zeta = \rho,$n
$$\phi = \phi_\uparrow = -\phi_\downarrow, \quad \zeta = \sigma_z.$$ (30)

In our basis the LWS’s are simply obtained by considering all the possible occupation distributions of the pseudomomenta $q_j = 2\pi I_{j,\alpha,\gamma}/N_a$. Therefore, it is convenient to describe the matrix elements $\langle m | \hat{j}^\zeta | m \rangle$ in terms of the pseudomomentum occupation distributions $N_{\alpha,\gamma}(q)$. This leads to a functional form for the current mean values. The computation of $\langle m | \hat{j}^\zeta | m \rangle$ involves the expansion of Eqs. (25) and (A7)-(A9) up to first order in the flux $\phi$. Writing Eq. (25) in the limit of $N_a \rightarrow \infty$, expanding it up to first order in the flux $\phi$, and using Eq. (29) we obtain

$$\langle m | \hat{j}^\zeta | m \rangle = -2t \frac{1}{2\pi} \int_{-q_c}^{q_c} dq N_c(q) K^\phi(q) \sin(K(q))$$
where the important functions $W^\phi(q)$ (with $W = K, R_{a,\gamma}$, and $R_{c,\gamma}$) are the derivatives of the rapidity functions defined by Eqs. (A7) - (A9) in order to the flux $\phi$ at $\phi = 0$. They obey a set of integral equations obtained from differentiation of Eqs. (A7) - (A9).

It is convenient to write $\langle m|\hat{j}^\zeta|m\rangle$ in normal order relatively to the ground state. To achieve this goal we expand all the rapidities $W(q)$ and the functions $W^\phi(q)$ as

\begin{align}
W(q) &= W^0(q) + W^1(q) + \ldots, \\
W^\phi(q) &= W^{0,\phi}(q) + W^{1,\phi}(q) + \ldots,
\end{align}

respectively. In these equations the functions $W^0(q)$ and $W^{0,\phi}(q)$ are both referred to the ground state, and the functions $W^1(q)$ and $W^{1,\phi}(q)$ are first-order functionals of the deviations $\delta N_{\alpha,\gamma}(q)$. In Appendix [A] we show that the above expansions lead to a ground-state normal-ordered representation. To first order in the deviations the normal-ordered expression for the matrix element (31) simply reads

\begin{equation}
\langle m|\hat{j}^\zeta|m\rangle = \sum_\alpha \sum_\gamma \int_{-q_{\alpha,\gamma}}^{q_{\alpha,\gamma}} dq \delta N_{\alpha,\gamma}(q) j^\zeta_{\alpha,\gamma}(q),
\end{equation}

where the elementary-current spectrum $j^\zeta_{\alpha,\gamma}(q)$ is given by

\begin{equation}
j^\zeta_{\alpha,\gamma}(q) = \sum_{\alpha'} \sum_{\gamma'} \theta(N_{\alpha',\gamma'}) C^\zeta_{\alpha',\gamma'} [v_{\alpha,\gamma}(q) \delta_{\alpha,\alpha'} \delta_{\gamma,\gamma'} + F^1_{\alpha,\gamma;\alpha',\gamma'}(q)].
\end{equation}

Here

\begin{equation}
F^1_{\alpha,\gamma;\alpha',\gamma'}(q) = \frac{1}{2\pi} \sum_{j=\pm 1} j f_{\alpha,\gamma;\alpha',\gamma'}(q,jq_{\alpha',\gamma'}),
\end{equation}

and $C^\zeta_{\alpha,\gamma}$ are the coupling constants of the pseudoparticles to charge and spin given by

\begin{equation}
C^\zeta_{\alpha,\gamma} = \delta_{\alpha,c} \delta_{\gamma,0} + K^\zeta_{\alpha,\gamma},
\end{equation}

where

\begin{equation}
K^\rho_{\alpha,\gamma} = \delta_{\alpha,c} 2\gamma; \quad K^\sigma_{\alpha,\gamma} = -\delta_{\alpha,s} 2(1 + \gamma).
\end{equation}
As in a Fermi liquid \[31,32\], the expressions of the elementary currents (35) involve the velocities \(v_{\alpha,\gamma}(q)\) and the interactions [or \(f\)-functions] \(f_{\alpha,\gamma;\alpha',\gamma'}(q,q')\). However, the pseudoparticle coupling constants to charge and spin, Eqs. (37) and (38), are very different from the corresponding couplings of the Fermi-liquid quasiparticles. We emphasize that at low energy Eq. (34) recovers the expression already obtained in Ref. [9] which only contains the \(c \equiv c,0\) and \(s \equiv s,0\) elementary currents. The coupling constants (37)-(38) play an important role in the description of charge and spin transport and are a generalization for \(\gamma > 0\) of the couplings introduced in Ref. [9]. They define the \(\alpha, \gamma\) pseudoparticles as charge and spin carriers. We emphasize that when \(C_{\alpha,\gamma}^\zeta = 0\) the corresponding \(\alpha, \gamma\) pseudoparticles do not couple to \(\zeta\) (i.e. charge or spin). Therefore, for \(\gamma > 0\) the \(c, \gamma\) and \(s, \gamma\) pseudoparticles do not couple to spin and charge, respectively. This is related to the charge and spin separation of one-dimensional quantum liquids which in the case of the present model was studied in Refs. [18,24]. Importantly, when \(C_{\alpha,\gamma}^\zeta = 0\) the \(\alpha, \gamma\) pseudoparticle – pseudohole processes do not contribute to the \(\zeta\) correlation functions. It will be shown elsewhere that this provides a powerful selection rule which implies that some of the terms obtained from the small \((\omega - \omega_0)\) conformal-field theory [16] for the correlation functions vanish.

In contrast to the general current expression (31), expression (34) is only valid for Hamiltonian eigenstates which differ from the ground-state pseudoparticle occupancy by a small density of pseudoparticles. This is because in expression (34) we are only considering the first-order deviation term.

The velocity term of current-spectrum expression (35) is what we would expect for a non-interacting gas of pseudoparticles and the second term takes account for the dragging effect on a single pseudoparticle due to its interactions with the other pseudoparticles. (This is similar to the Fermi-liquid quasiparticle elementary currents [31,32].) We remind that Eq. (35) is valid for finite energies \(\omega\) just above the energy \(\omega_0\) corresponding to the suitable set of finite \(N_{\alpha,\gamma}\) numbers. These numbers characterize the state \(|m\rangle\). Therefore, the sum over \(\gamma\) is in Eq. (35) restricted to the \(\alpha, \gamma\) bands that have non-zero occupancy of pseudoparticles, as is imposed by the presence of the step-function. Within the PPT, the deviation second-order
pseudoparticle energy expansion corresponds to the deviation first-order current expansion (34) which refers to small positive values \((\omega - \omega_0)\) of the excitation energy. In contrast to Fermi liquid theory, our PPT is valid for finite energies [just above the energy values \(\omega_0\), Eq. (10)] because (i) there is only forward scattering among the pseudoparticles at all energy scales and (ii) at small \((\omega - \omega_0)\) energy values only two-pseudoparticle forward-scattering interactions are relevant. (In a Fermi liquid this is only true for \(\omega_0 = 0\) and \(\omega \rightarrow 0\) \([31,32]\).)

We emphasize that the current expression (31) includes all orders of scattering and, therefore, applies to all energies without restrictions. Elsewhere it will be shown to be useful in the study of charge and spin transport at finite temperatures.

IV. PSEUDOPARTICLE TRANSPORT AND STATIC MASSES

In reference [9] the charge and spin transport masses of the \(c, 0\) and \(s, 0\) pseudoparticles were defined and were shown to play an important role in the transport of charge and spin. For instance, they were shown to fully determine the charge and spin stiffnesses \([9,10,13,27,29,30]\). Here we generalize the mass definitions of Ref. [9] to \(\gamma > 0\) and define the charge and spin transport masses, \(m_{\alpha,\gamma}^\zeta\), as

\[
m_{\alpha,\gamma}^\zeta = \frac{q_{F,\alpha,\gamma}}{C_{\alpha,\gamma} j_{\alpha,\gamma}^\zeta},
\]

where \(j_{\alpha,\gamma}^\zeta = j_{\alpha,\gamma}^\zeta(q_{F,\alpha,\gamma})\). They contain important physical information. As in a Fermi liquid \([31,32]\), the ratio

\[
r_{\alpha,\gamma}^\zeta = m_{\alpha,\gamma}^\zeta / m_{\alpha,\gamma}^*,
\]

of the transport mass over the static mass provides a measure of the correlations importance in transport. Similarly to the \(\gamma = 0\) case \([9]\), the latter is in general defined as

\[
m_{\alpha,\gamma}^* = \frac{q_{F,\alpha,\gamma}}{v_{\alpha,\gamma}}.
\]

In Appendix B we define the mass \(m_{\alpha,\gamma}^\zeta\) in terms of suitable functions and find some limiting expressions.
It can be shown from the transport- and static-mass expression that the ratio $m^{ζ}_{α,γ}/m^*_{α,γ}$ involves the Landau parameters

$$F^i_{α,γ;α',γ'} \equiv F^i_{α,γ;α',γ'}(q_{FA,γ}); \quad i = 0, 1,$$

(42)

with $F^i_{α,γ;α',γ'}(q)$ given by Eq. (39). These parameters can be written as follows

$$F^i_{α,γ;α',γ'} = \delta_{α,α'}\delta_{γ,γ'}v_{α,γ} + \sum_{α''}θ(N_{α'',γ''})v_{α'',γ''}[ξ^i_{α'',γ'';α,γ}ξ^i_{α'';α',γ'}],$$

(43)

where the quantities $ξ^i_{α,γ;α',γ'}$ are given by

$$ξ^i_{α,γ;α',γ'} = \delta_{α,α'}δ_{γ,γ'} + \sum_{l=±1} l^iΦ_{α,γ;α',γ'}(q_{FA,γ}, lq_{FA',γ')},$$

(44)

We find for the ratios $m^{ζ}_{α,γ}/m^*_{α,γ}$ the following expressions

$$\frac{m^{ζ}_{α,0}}{m^*_{α,0}} = \frac{v_{α,0}}{C^ζ_{α,γ}(∑_{α',α''}C^ζ_{α',0}v_{α'',0}ξ^1_{α'',0;α,0}ξ^1_{α,0})}, \quad γ = 0,$$

(45)

and

$$\frac{m^{ζ}_{α,γ}}{m^*_{α,γ}} = \frac{1}{C^ζ_{α,γ}(C^ζ_{α,γ} + ∑_{α'}C^ζ_{α',0}ξ^1_{α';0,0})}, \quad γ > 0.$$

(46)

In the Table analytical limiting values for the mass ratios of form (40) are listed. Obviously, since for $γ > 0$ the $c, γ$ and $s, γ$ pseudoparticles do not couple to spin and charge, respectively, the ratios $m^{σ,z}_{c,γ}/m^*_{c,γ}$ and $m^{ρ}_{s,γ}/m^*_{s,γ}$ are infinite.

As was referred previously, it can be shown from the results of Refs. [15–17] that the creation of one $α, γ$ pseudoparticle from the ground state is a finite-energy excitation which, to leading order, involves a number $2γ$ of electrons. Therefore, and since the current operators are of two-electron character and couple to charge and spin according to the values of the constants (37) and (38), at finite energies the $c, 1$ and $s, 1$ heavy pseudoparticles play the major role in charge and spin transport, respectively. On the other hand, the $α, γ > 1$ heavy pseudoparticles contribute very little to charge and spin transport. It follows that in the present section we restrict our study to the ratios (40) involving $γ = 1$ heavy pseudoparticles. We consider the ratios $m^{ρ}_{c,1}/m^*_{c,1}$ and $m^{ρ}_{s,1}/m^*_{s,1}$. (Note that $m^{σ,z}_{c,1}/m^*_{c,1} = m^{ρ}_{s,1}/m^*_{s,1} = ∞$.)
We also consider the case of the \( c, 0 \) charge-mass ratio which is closely related to the charge stiffness studied in detail in Ref. [9]. In Figs. 1-12 these ratios are plotted as functions of the onsite repulsion \( U \) in units of \( t \), electronic density \( n \), and magnetic field \( h = H/H_c \). Note that the ratios of the figures are smaller than one. Moreover, the \( \alpha, 1 \) mass ratios never achieve the value 1 whereas the \( \alpha, 0 \) mass ratios tend to one in some limits because of the generalized adiabatic principle of Ref. [9].

Combined analysis of Figs. 1 and 2 reveals that the charge-mass ratio for the \( c, 1 \) pseudoparticle is, for large \( U \), fairly independent both of the band filling \( n \) and magnetic field \( h \). It is a decreasing function of \( U \) and as a function of the density, \( n \), goes through a maximum for a density which is a decreasing function of \( U \). Moreover, figures 3 and 4 show that this ratio is a decreasing function of the magnetic field.

In contrast, Figs. 5 - 8 reveal that the charge-mass ratio for the \( c, 0 \) pseudoparticle is an increasing function of \( U \) and of \( h \) and as a function of the density, \( n \), goes through a minimum for a density which is a decreasing function of \( U \). Note that from Fig. 5 the evolution of the \( c, 0 \) pseudoparticles to free spinless fermions as \( U \) increases is clear. This is signaled by the ratio going to one as \( U \to \infty \). This behavior follows from the generalized adiabatic principle of Ref. [9] and agrees with the well known decoupling of the BA wave function in free spinless fermions (in the low-energy sector [10]) and localized antiferromagnetic spins [41]. Figures 7 and 8 also reveal that in the fully-polarized ferromagnetic limit, \( h \to 1 \), the ratio goes to one. This mass-ratio behavior also follows from the generalized adiabatic principle [9] and confirms that in that limit the onsite Coulomb interactions play no role in charge transport (they are frozen by the Pauli principle).

Note that in the large-\( U \) Figs. 2 - (c) and 3 - (c) the ratios \( m^e_{c,1}/m^e_{c,1} \) and \( m^e_{c,0}/m^e_{c,0} \), respectively, are almost symmetric around the density \( n = 0.5 \). This implies that for large \( U \) the charge transport properties show similarities in the cases of vanishing densities and of densities closed to one.

Figure 9 shows that the spin-mass ratio of the \( s, 1 \) pseudoparticles is a decreasing function of \( U \) but that it depends little on \( U \) for \( U > 6 \). For large \( U \) this ratio almost does not depend
on the density $n$, as revealed by Fig. 10 - (c). Figures 11 show that, in general, it is an increasing function of $n$ but that for $h \to 1$ it has a maximum for an intermediate density. In figures 11 and 12 this spin-mass ratio is plotted as a function of $h$. It is a decreasing function of $h$.

The transport masses are very sensitive to the effects of electronic correlations, as for instance to the metal-insulator transition which occurs at zero temperature when $n \to 1$ \[3\]. As a direct result of this transition, $m^{\rho}_{c,0} \to \infty$ as $n \to 1$, as was shown and discussed in Ref. \[3\]. Moreover, the zero-temperature charge and spin stiffnesses, $D^\zeta$, \[10,13,27,29,30\], defined as

$$D^\zeta = \frac{1}{2} \frac{d^2(E_0/N_a)}{d(\phi/N_a)^2} \bigg|_{\phi=0}, \tag{47}$$

where $\phi$ is defined for charge $\zeta = \rho$ and spin $\zeta = \sigma_z$ in Eq. \[3\], are such that $2\pi D^\rho = \sum_\alpha q_{F\alpha,0}/m^\zeta_{\alpha,0}$ \[3\]. For charge, $m^\rho_{s,0} = \infty$, and the latter expression reads $2\pi D^\rho = q_{Fc,0}/m^\rho_{c,0}$ and is such that $D^\rho \to 0$ as $n \to 1$, satisfying Kohn criterion \[33\]. These quantities can be computed within the pseudoparticle formalism by direct evaluation of Eq. (47). They can also be obtained by combining a pseudoparticle Boltzmann transport description with linear response theory, as in Ref. \[3\]. In order to confirm the validity and correctness of our formalism, in Appendix C we have recovered the charge and spin stiffness expressions (135) - (137) of Ref. \[3\] by direct use of Eq. (47).

Equations (37) and (38) show that the Hubbard-chain charge carriers are the $c, \gamma$ pseudoparticles. In contrast to the zero-temperature limit where the $c, 0$ pseudoparticles fully determine the charge stiffness, we expect that the $c, \gamma$ heavy pseudoparticles play an important role in the charge-transport properties at finite temperatures \[34,35\]. Moreover, elsewhere it will be shown that the limiting behavior of the $s, \gamma$ and $c, \gamma$ heavy-pseudoparticle bands as $H \to 0$ and $n \to 1$, respectively, will have effects on the charge- and spin-transport properties at finite temperatures. In order to obtain some information on that behavior, it is useful to consider limiting values for the quantities whose general expressions we have introduced in previous sections.
In Appendix D we present simpler equations to define the pseudoparticle bands and phase shifts in the limit of zero magnetic field. These results show that for \( H \to 0 \) and \( \gamma > 0 \) the bands \( \epsilon_{s,\gamma}^0(q) \) collapse to a point. This is because both the bandwidth [see Eq. (D1)] and the momentum pseudo-Brillouin zone width [see Eq. (2)] go to zero as \( H \to 0 \). This behavior is also present in the Heisenberg chain and, therefore, in that model the triplet and singlet excitations are degenerated at zero magnetic field \[39\]. This also holds true for the Hubbard chain at \( H = 0 \) and in the limit \( U \gg t \), where the BA wave function factorizes in a spinless-fermion Slater determinant and in the BA wave function for the 1D anti-ferromagnetic Heisenberg chain. On the other hand, in the limit \( n \to 1 \) the bands \( \epsilon_{c,\gamma}^0(q) \) (for \( \gamma > 0 \)) collapse to a point also because both the bandwidth and the momentum pseudo-Brillouin zone width [see Eq. (2)] go to zero in that limit.

V. KINETIC EQUATIONS FOR THE PSEUDOPARTICLES

In the previous sections the quantum-liquid physics for energies just above the \( \omega_0 \) values, Eq. (10), was described in terms of homogeneous pseudoparticle distributions. The pseudoparticles experience only zero-momentum forward-scattering interactions at all energy scales. This property is absent in Fermi-liquid theory where it holds true only at low excitation energy when the quasiparticles are well defined quantum objects \[31,32,37\]. This unconventional character of integrable models \[38\] allows us to extend the use of the kinetic equations to energy scales just above the \( \omega_0 \) energy values, Eq. (10), and not only to low energies \[3\]. The results presented in this section are a generalization of the kinetic-equation low-energy study presented in Ref. \[3\].

In the final Hilbert subspace of energy \( \omega \) relative to the initial ground state the Hubbard model can be mapped onto a continuum field theory of small energy \((\omega - \omega_0)\) \[16\]. The time coordinate \( t \) of such theory is the Fourier transform of the small energy \((\omega - \omega_0)\) which corresponds to a finite energy \( \omega \) in the original Hubbard model. The validity of this approach is confirmed by the fact that it fully reproduces the rigorous results of Section \[11\].
Let us consider excitations described by space and time dependent pseudoparticle distribution functions, \(N_{\alpha,\gamma}(q, x, t)\), given by
\[
N_{\alpha,\gamma}(q, x, t) = N^0_{\alpha,\gamma}(q) + \delta N_{\alpha,\gamma}(q, x, t),
\]
where \(N^0_{\alpha,\gamma}(q)\) is the ground-state distribution. It follows from the PPT introduced in Ref. \[15\] and discussed in the previous sections that the single-pseudoparticle local energy is given, to first order in the deviations \(\delta N_{\alpha,\gamma}(q, x, t)\), by
\[
\tilde{\varepsilon}_{\alpha,\gamma}(q, x, t) = \varepsilon_{\alpha,\gamma}(q) + \frac{1}{2\pi} \sum_{\alpha',\gamma'} \int_{q_{\alpha',\gamma'}}^{q_{\alpha',\gamma'}} dq' \delta N_{\alpha',\gamma'}(q', x, t) f_{\alpha,\gamma;\alpha',\gamma'}(q, q').
\]
(49)

Let \(\mathcal{A}^\zeta\) represent the total charge, \(\zeta = \rho\), or spin, \(\zeta = \sigma_z\). It follows from the relations (11) and (12) involving the pseudoparticle and electron numbers that \(\mathcal{A}^\zeta\) depends linearly on the pseudoparticle deviation numbers. Thus, in the case of inhomogeneous excitations described by Eq. (48) the corresponding expectation value at point \(x\) and time \(t\), \(\langle \mathcal{A}^\zeta(x, t) \rangle\), can be written as
\[
\langle \mathcal{A}^\zeta(x, t) \rangle = \langle \mathcal{A}^\zeta \rangle_0 + \frac{N_\alpha}{2\pi} \sum_{\alpha',\gamma'} \int_{q_{\alpha',\gamma'}}^{q_{\alpha',\gamma'}} dq' \delta N_{\alpha',\gamma'}(q', x, t) C^\zeta_{\alpha',\gamma'} \times a^\zeta,
\]
where \(a^\rho = -e\) and \(a^{\sigma_z} = 1/2\).

In this “semi-classical” approach the response to a scalar field, \(V^\zeta(x, t)\), is proportional to the conserved quantity \(\mathcal{A}^\zeta\). As for low energy \[3\], in the presence of the inhomogeneous potential the force \(\mathcal{F}^\zeta(x, t)_{\alpha,\gamma}\) that acts upon the \(\alpha, \gamma\) pseudoparticle is given by \(\mathcal{F}^\zeta_{\alpha,\gamma}(x, t) = -[\partial V^\zeta(x, t)/\partial x] C^\zeta_{\alpha,\gamma} \times a^\zeta\). It follows that the deviations \(\delta N_{\alpha,\gamma}(q, x, t)\) are determined by the solution of a system of kinetic equations (one equation for each occupied \(\alpha, \gamma\) branch) which reads
\[
0 = \frac{\partial N_{\alpha,\gamma}(q, x, t)}{\partial t} + \frac{\partial N_{\alpha,\gamma}(q, x, t)}{\partial x} \frac{\partial \varepsilon_{\alpha,\gamma}(q, x, t)}{\partial q} - \frac{\partial N_{\alpha,\gamma}(q, x, t)}{\partial q} \frac{\partial \varepsilon_{\alpha,\gamma}(q, x, t)}{\partial x} - \frac{\partial N_{\alpha,\gamma}(q, x, t)}{\partial q} \frac{\partial V^\zeta(x, t)}{\partial x} C^\zeta_{\alpha,\gamma} \times a^\zeta.
\]
(51)

Introducing Eq. (48) in Eq. (51), expanding to first order in the deviations \(\delta N_{\alpha,\gamma}(q, x, t)\), and using Eq. (49) we obtain the following set of linearized kinetic equations.
\[
0 = \frac{\partial \delta N_{\alpha,\gamma}(q, x, t)}{\partial t} + v_{\alpha,\gamma}(q) \frac{\partial \delta N_{\alpha,\gamma}(q, x, t)}{\partial x} \\
- \frac{\partial \delta N_{\alpha,\gamma}(q, x, t)}{\partial q} \left\{ \frac{\partial V^\zeta(x, t)}{\partial x} C^\zeta_{\alpha,\gamma} \times a^\zeta + \sum_{\alpha',\gamma'} \frac{1}{2\pi} \int_{q_{\alpha',\gamma'}}^{q_{\alpha',\gamma'}} dq' \frac{\partial \delta N_{\alpha',\gamma'}(q', x, t)}{\partial x} f_{\alpha,\gamma;\alpha',\gamma'}(q, q') \right\}.
\]  
\tag{52}

The conservation law for \( \langle \mathcal{A}^\zeta(x, t) \rangle \) leads in one dimension to
\[
\frac{\partial \langle \mathcal{A}^\zeta(x, t) \rangle}{\partial t} + \langle \mathcal{J}^\zeta(x, t) \rangle \frac{\partial x}{\partial x} = 0,
\tag{53}
\]
where \( \langle \mathcal{A}^\zeta(x, t) \rangle \) is given by Eq. (50) and \( \langle \mathcal{J}^\zeta(x, t) \rangle \) is the associate current. Multiplying Eq. (52) by \( C^\zeta_{\alpha,\gamma} \times a^\zeta \), summing over \( \alpha \) and \( \gamma \), and integrating over \( q \) we find for \( V^\zeta(x, t) = 0 \) and by comparing the result with Eq. (53) that the current spectrum \( j_{\alpha,\gamma}^\zeta(q) \) is given by \( a^\zeta \) times expression (35). (This expression has been derived from the solution of the BA equations with \( a^\zeta = 1 \).)

This agreement confirms the validity of the above low-\((\omega - \omega_0)\) continuum-field theory. The unusual spectral properties associated with the zero-momentum forward-scattering character of the pseudoparticle interactions follow from the integrability of the Hubbard chain \[15,38\].

VI. CONCLUDING REMARKS

In this paper we have generalized the finite-energy PPT \[15\] to the case of the Hubbard chain with a spin dependent Peierls substitution. This has allowed the evaluation of the charge and spin currents in terms of the elementary currents of the charge and spin carriers. We have shown that at all energy scales these carriers are the \( \alpha, \gamma \) pseudoparticles of the PPT. We have evaluated their couplings to charge and spin and introduced the associate charge and spin transport masses. Our results are also a generalization for finite energies of the low-energy studies of Ref. \[9\], our charge and spin current expressions recovering the expressions already obtained in that reference in the limit of low energy.
The obtained heavy-pseudoparticle transport masses are important quantities. They are believed to control the charge and spin stiffnesses at finite temperatures. Moreover, the pseudoparticle couplings to charge and spin obtained in the present paper provide important selection rules concerning the ground-state transitions which contribute to the finite-energy charge – charge and spin – spin correlation functions. Expressions for these functions can be derived by combining the BA solution with a low-energy \((\omega - \omega_0)\) generalized conformal-field theory [16]. The above selection rules will be shown elsewhere to provide important information on the finite-energy correlation functions which cannot be extracted from conformal-field theory alone.

Finally, the \(\phi_\sigma\)-dependent charge and spin current expressions of general form (31) will be used elsewhere to find out whether the half-filling Hubbard model is or is not an insulator at all temperatures [34,35].

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**APPENDIX A: NORMAL-ORDERED SOLUTION OF THE BA EQUATIONS WITH THE FLUX \(\phi\)**

In this appendix we derive the normal-ordered BA equations required for the evaluation of Eqs. (34) and (35). Writing \(W^{\phi}(q)\) from Eq. (33) as

\[
W^{\phi}(q) = \frac{dW(q)}{dq}L^{\phi}(q),
\]

where \(L\) equals \(L_{c,0}\), \(L_{c,\gamma}\), or \(L_{s,\gamma}\) when \(W\) equals \(K\), \(R_{s,\gamma}\), or \(R_{c,\gamma}\), respectively, we find that \(W^{1,\phi}(q)\) obeys the following equality.
\[
W^{1,\phi}(q) = \frac{dW^0}{dq}L^{1,\phi}(q) + \frac{dW^1}{dq}L^{0,\phi}(q). \tag{A2}
\]

Introducing the above equation in Eq. (31) and writing the distributions functions \(N_{\alpha,\gamma}(q)\) as \(N^0_{\alpha,\gamma}(q) + \delta N_{\alpha,\gamma}(q)\), we can expand \(J \equiv \langle m|\hat{J}\zeta|m\rangle\) in terms of the pseudomomentum deviations as

\[
J = J^0 + J^1 + J^2 + \cdots, \tag{A3}
\]

where the first term, \(J^1\), of the current normal-ordered expansion (A3) can after some algebra be written as

\[
J^1 = J^1_0 - 2t \sum_{j=\pm 1} \frac{L^{0,\phi}(jq_{\text{F}c})L^{1,\phi}_c(jq_{\text{F}c})}{2\pi \rho_{c,0}(Q)} \sin(Q) +
\]

\[
+ \sum_{\gamma > 0} \theta(N_{c,\gamma}) Re 4t \sum_{j=\pm 1} \frac{u^2[jr_{c,\gamma} - i\gamma]}{1 - u^2[jr_{c,\gamma} - i\gamma]^2} \frac{L^{0,\phi}(jq_{\text{F}c,\gamma})L^{1,\phi}_c(jq_{\text{F}c,\gamma})}{2\pi \rho_{c,\gamma}(r_{c,\gamma})} \]

\[
- 2t \int_{-q_{\text{F}c}}^{q_{\text{F}c}} dq \frac{dK^{(0)}(q)}{dq} \sin(K^{(0)}(q))L^{1,\phi}_c(0), \tag{A4}
\]

where the functions \(2\pi \rho_{c,0}(k)\) and \(2\pi \rho_{\alpha,\gamma}(r)\) were defined in Ref. [15] and

\[
J^1_0 = -2t \int_{-q_c}^{q_c} dq \delta N_c(q) \frac{dK^{(0)}(q)}{dq} \sin(K^{(0)}(q))L^{0,\phi}_c(q)
\]

\[
+ \sum_{\gamma > 0} \theta(N_{c,\gamma}) Re 4t \int_{-q_{c,\gamma}}^{q_{c,\gamma}} dq \delta N_{c,\gamma}(q) \frac{u^2[R^{(0)}_{c,\gamma}(q) - i\gamma]}{\sqrt{1 - u^2[R^{(0)}_{c,\gamma}(q) - i\gamma]^2}} \frac{dR^{(0)}_{c,\gamma}(q)}{dq} L^{0,\phi}_{c,\gamma}(q). \tag{A5}
\]

The function \(L^{1,\phi}(q)\) is defined as

\[
L^{1,\phi}(q) = L^{1,\phi}(q) - W^1(q) \frac{L^{0,\phi}(q)}{dq}. \tag{A6}
\]

In order to obtain the integral equations for \(L^{0,\phi}(q)\) and \(L^{1,\phi}(q)\) (with \(\mathcal{L} = \mathcal{L}_{c,0}, \mathcal{L}_{c,\gamma}, \mathcal{L}_{s,\gamma}\)), we start from the continuum limit of Eqs. (22), (23), and (24) which reads

\[
K(q) = q + \phi_{\uparrow}/N_a - \sum_{\gamma'} \frac{1}{2\pi} \int_{-q_{s,\gamma'}}^{q_{s,\gamma'}} dq' N_{s,\gamma'}(q') 2\tan^{-1}\left(\frac{(\sin(K(q))/u - R_{s,\gamma'}(q'))}{(\gamma' + 1)} \right)
\]

\[
- \sum_{\gamma' > 0} \frac{1}{2\pi} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' N_{c,\gamma'}(q') 2\tan^{-1}\left(\frac{(\sin(K(q))/u - R_{c,\gamma'}(q'))}{\gamma'} \right), \tag{A7}
\]

24
\[ 2R \text{e} \sin^{-1}[(R_{c,\gamma}(q) - i\gamma)u] = q + \gamma(\phi_{\uparrow} + \phi_{\downarrow})/N_a - \]
\[ -\frac{1}{2\pi} \int_{-q_c}^{q_c} dq' N_{c}(q') \tan^{-1}\left(\frac{\sin(K(q'))/u - R_{c,\gamma}(q')}{\gamma}\right) \]
\[ + \sum_{\gamma'>0} \frac{1}{2\pi} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' N_{c,\gamma'}(q') \Theta_{\gamma,\gamma'}(R_{c,\gamma}(q) - R_{c,\gamma'}(q')) , \tag{A8} \]

and

\[ q = (\gamma + 1)(\phi_{\uparrow} - \phi_{\downarrow})/N_a + \frac{1}{2\pi} \int_{-q_c}^{q_c} dq' N_{c}(q') \tan^{-1}\left(\frac{R_{s,\gamma}(q) - \sin(K(q'))/u}{(\gamma + 1)}\right) \]
\[ - \sum_{\gamma'} \frac{1}{2\pi} \int_{-q_{s,\gamma'}}^{q_{s,\gamma'}} dq' N_{s,\gamma'}(q') \Theta_{\gamma+1,\gamma'+1}(R_{s,\gamma}(q) - R_{s,\gamma'}(q')) . \tag{A9} \]

It is convenient to write the function \( \Theta_{\gamma,\gamma'}^{[1]}(x) \), defined by Eq. (B7) of Ref. [1], as follows
\[ \Theta_{\gamma,\gamma'}^{[1]}(x) = \sum_l \frac{2b_l^{\gamma,\gamma'}}{1 + [x/l]^2} . \tag{A10} \]

We emphasize that comparison term by term of expression (B7) of Ref. [1] with expression (A10) fully defines the coefficients \( b_l^{\gamma,\gamma'} \) and the corresponding set of integer numbers \( l \).

Following equation (30), we have \( \phi_{\uparrow} = \phi_{\downarrow} \) for a charge-probe current and \( \phi_{\uparrow} = -\phi_{\downarrow} \) for a spin probe. With the above equations written in terms of \( \phi_{\uparrow} \) and \( \phi_{\downarrow} \), Eq. (A4) provides both the charge and spin currents. In what follows, we introduce in the functions \( L^\phi(q) \) the index \( \zeta = \rho, \sigma_z \) to label the equations for either the charge or the spin current, respectively. We start by expanding Eqs. (A7), (A8), and (A9) up to first order in \( \phi \). This procedure reveals that the functions \( L^\phi(\phi)(q) \) obey the following integral equations
\[ L^\phi_{c,0}(q) = C_{c,0}^{\phi} + \sum_{\gamma'} \frac{1}{(\gamma' + 1)\pi} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' \frac{N_{s,\gamma'}(q')}{1 + \left[\sin(K(q'))/u - R_{s,\gamma'}(q')\right]^2} \frac{dR_{s,\gamma'}(q')}{dq'} L_{c,\gamma'}^{\phi,\zeta}(q') \]
\[ + \sum_{\gamma'>0} \frac{1}{\pi\gamma'} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' \frac{N_{c,\gamma'}(q')}{1 + \left[\sin(K(q'))/u - R_{c,\gamma'}(q')\right]^2} \frac{dR_{c,\gamma'}(q')}{dq'} L_{c,\gamma'}^{\phi,\zeta}(q') , \tag{A11} \]
\[ L^\phi_{c,\gamma}(q) = C_{c,\gamma}^{\phi} + \frac{1}{\pi u\gamma} \int_{-q_c}^{q_c} dq' \frac{N_{c}(q')}{1 + \left[\sin(K(q'))/u - R_{c,\gamma}(q')\right]^2} \frac{dK(q')}{dq'} \cos(K(q')) L_{c,\gamma}^{\phi,\zeta}(q') \]
\[ + \sum_{\gamma'>0} \sum_l \frac{1}{\pi l} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' \frac{N_{c,\gamma'}(q')b_l^{\gamma,\gamma'}}{1 + \left[R_{c,\gamma}(q) - R_{c,\gamma'}(q')\right]^2} \frac{dR_{c,\gamma'}(q')}{dq'} L_{c,\gamma'}^{\phi,\zeta}(q') , \tag{A12} \]

and
\[ L^\delta_{s,\gamma}(q) = C^\xi_{s,\gamma} + \]
\[ + \frac{1}{u(\gamma + 1)\pi} \int_{-q_c}^{q_c} dq' \frac{N_c(q')}{1 + \sin(K(q'))/u - R_{s,\gamma}(q')} \sum_{\gamma'} \frac{dK(q')}{dq'} \cos(K(q')) L^{\delta,\xi}_{c,0}(q') \]
\[ - \sum_{\gamma'} \frac{1}{\pi l} \int_{-q_{s,\gamma'}}^{q_{s,\gamma'}} dq' \frac{N_{s,\gamma'}(q')L^{\gamma+1,\gamma'+1}_{s,\gamma'}(q')}{1 + [R_{s,\gamma}(q') - R_{s,\gamma'}(q')]^2} \sum_{\gamma'} \frac{dR_{s,\gamma'}(q')}{dq'} L^{\delta,\xi}_{s,\gamma'}(q'), \]  
(A13)

where the coupling constants \( C^\xi_{s,\gamma} \) are defined by Eqs. (37) and (38). We again write the distributions functions \( N_{a,\gamma}(q) \) of Eqs. (A11), (A12), and (A13) as \( N_{a,\gamma}(q) + \delta N_{a,\gamma}(q) \). This allows us to obtain integral equations for \( L^0,\phi,\xi(q) \) and \( L^1,\phi(q) \) (we remark that the functions \( L^1,\phi(q) \) are the same both for \( \zeta = \rho, \sigma_z \)). It is then straightforward to find the integral equations obeyed by \( L^0,\phi,\xi(q) \) and show that \( L^0,\phi,\xi(q) \) can be simply expressed in terms of linear combinations of phase shifts. The final result is

\[ L^0,\phi,\xi_{a,\gamma}(q) = C^\xi_{a,\gamma} + \sum_{\alpha',\gamma', j=\pm 1} j \theta(N_{a',\gamma'}) C^\xi_{a',\gamma',\gamma'} F_{a,\gamma;a',\gamma'}(q, j q_{a',\gamma'}). \]  
(A14)

The integral equations obeyed by \( L^1,\phi(q) \) are related to the integral equations obeyed by \( \tilde{L}^1,\phi(r) \), where \( r \) equals \( \sin(K^{(0)}(q))/u, R_v^{(0)}(q) \), and \( R_{s,\gamma}^{(0)}(q) \) for \( L = L_{c,0}, L_{c,\gamma}, \) and \( L_{s,\gamma} \), respectively. The functions \( \tilde{L}^1,\phi(r) \) obey the following integral equations

\[ \tilde{L}^1,\phi_{c,0}(r) = \tilde{L}^1,\phi_{c,0}(r) + \frac{1}{\pi} \int_{-r_c}^{r_c} dr' \frac{\tilde{L}^1,\phi_{c,0}(r')}{1 + (r - r')^2}, \]  
(A15)

\[ \tilde{L}^1,\phi_{c,\gamma}(r) = \tilde{L}^1,\phi_{c,\gamma}(r) - \frac{1}{\pi \gamma u} \int_{-r_c}^{r_c} dr' \frac{\tilde{L}^1,\phi_{c,0}(r')}{1 + (r - r')^2}, \]  
(A16)

and

\[ \tilde{L}^1,\phi_{s,\gamma}(r) = \tilde{L}^1,\phi_{s,\gamma}(r) - \frac{1}{\pi (\gamma + 1) u} \int_{-r_c}^{r_c} dr' \frac{\tilde{L}^1,\phi_{c,0}(r')}{1 + (r - r')^2}, \]
\[ - \sum_{\gamma'} \frac{1}{\pi l} \int_{-r_{s,0}}^{r_{s,0}} dr' \frac{L^{\gamma+1,\gamma'+1}_{s,\gamma'}(r')}{1 + [r - r_{s,0}]^2} \tilde{L}^1,\phi_{s,\gamma'}(r'), \]  
(A17)

where the free terms \( \tilde{L}^1,\phi_{c,0}(r), \tilde{L}^1,\phi_{c,\gamma}(r), \) and \( \tilde{L}^1,\phi_{s,\gamma}(r) \) are, respectively, given by

\[ \tilde{L}^1,\phi_{c,0}(r) = \sum_{\gamma'} \frac{1}{\pi \gamma} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' \delta N_{c,\gamma'}(q') \frac{R^{(0)}_{c,\gamma'}(q')}{1 + [r - R^{(0)}_{c,\gamma'}(q')]^2} \frac{dR_{c,\gamma'}^{(0)}(q')}{dq'}. \]
+ \sum_{\gamma'} \frac{1}{\pi (\gamma' + 1)} \int_{-q_{s,\gamma'}}^{q_{s,\gamma'}} dq' \delta N_{s,\gamma'}(q') \frac{L^0_{s,\gamma'}(q')}{1 + \left[ \frac{r - R^0_{s,\gamma'}(q')}{\gamma' + 1} \right]^2} dR^0_{s,\gamma'}(q')

+ \sum_{\gamma'} \theta(N_{c,\gamma'}) \frac{1}{\gamma' \pi} \sum_{j = \pm 1} j L^1_{c,\gamma'}(jq_{FC,\gamma'}) \frac{L^0_{c,\gamma'}(jq_{FC,\gamma'})}{1 + \left[ \frac{r - j q_{c,\gamma'}}{\gamma'} \right]^2}

+ \sum_{\gamma'} \theta(N_{s,\gamma'}) \frac{1}{(\gamma' + 1) \pi} \sum_{j = \pm 1} j L^1_{s,\gamma'}(jq_{FS,\gamma'}) \frac{L^0_{s,\gamma'}(jq_{FS,\gamma'})}{1 + \left[ \frac{r - j q_{s,\gamma'}}{\gamma'} \right]^2}, \quad (A18)

\tilde{\mathcal{L}}^{1,\phi,0}(r) = - \frac{1}{\pi u \gamma} \int_{q_e}^{q_c} dq' \delta N_c(q') \frac{L^0_{c,\phi}(q')}{1 + \left[ \frac{\sin(K^{(0)}(q'))/u - r}{\gamma} \right]^2} \cos(K^{(0)}(q')) \frac{dK^{(0)}(q')}{dq'}

- \sum_{\gamma'} \sum_{l} \frac{1}{\pi l} \int_{-q_{c,\gamma'}}^{q_{c,\gamma'}} dq' \delta N_{c,\gamma'}(q') - b_l^{\gamma,\gamma'} L^0_{c,\gamma'}(q') \frac{dR^0_{c,\gamma'}(q')}{dq'}

- \frac{1}{\gamma' \pi} \sum_{j = \pm 1} j L^1_{c,0}(jq_{FC}) \frac{L^0_{c,0}(jq_{FC})}{1 + \left[ \frac{r - j q_{c,0}}{\gamma} \right]^2} \cos(Q)

- \sum_{\gamma'} \theta(N_{c,\gamma'}) \sum_{l} \frac{1}{\pi l} \sum_{j = \pm 1} j b_l^{\gamma,\gamma'} L^1_{c,\gamma'}(jq_{FC,\gamma'}) \frac{L^0_{c,\gamma'}(jq_{FS,\gamma'})}{(1 + \left[ \frac{r - j q_{c,\gamma'}}{\gamma} \right]^2)}, \quad (A19)

and

\tilde{\mathcal{L}}^{1,\phi,0}(r) = - \frac{1}{\pi u (\gamma + 1)} \int_{q_e}^{q_c} dq' \delta N_c(q') \frac{L^0_{c,\phi}(q')}{1 + \left[ \frac{\sin(K^{(0)}(q'))/u - r}{\gamma + 1} \right]^2} \cos(K^{(0)}(q')) \frac{dK^{(0)}(q')}{dq'}

- \sum_{\gamma'} \sum_{l} \frac{1}{\pi l} \int_{-q_{s,\gamma'}}^{q_{s,\gamma'}} dq' \delta N_{s,\gamma'}(q') b_l^{\gamma+1,\gamma' + 1} L^0_{s,\gamma'}(q') \frac{dR^0_{s,\gamma'}(q')}{dq'}

+ \frac{1}{(\gamma + 1) \pi} \sum_{j = \pm 1} j L^1_{c,0}(jq_{FC}) \frac{L^0_{c,0}(jq_{FC})}{1 + \left[ \frac{r - j q_{c,0}}{\gamma + 1} \right]^2} \cos(Q)

- \sum_{\gamma'} \theta(N_{s,\gamma'}) \sum_{l} \frac{1}{\pi l} \sum_{j = \pm 1} j b_l^{\gamma+1,\gamma' + 1} L^1_{s,\gamma'}(jq_{FS,\gamma'}) \frac{L^0_{s,\gamma'}(jq_{FS,\gamma'})}{(1 + \left[ \frac{r - j q_{s,\gamma'}}{\gamma + 1} \right]^2)}, \quad (A20)

Introducing the functions \( \mathcal{L}^{1,\phi}(q) \) obtained from Eqs. (A13), (A16), and (A17), in Eq. (A4) and keeping terms only up to second order in the density of heavy pseudoparticles, we obtain Eq. (B1) with \( j^\xi_{\alpha,\gamma}(q) \) given by

\[ j^\xi_{\alpha,\gamma}(q) = v_{\alpha,\gamma}(q) L^0_{\alpha,\gamma}(q) + \sum_{\alpha',\gamma'} \sum_{j = \pm 1} j \theta(N_{\alpha',\gamma'}) v_{\alpha',\gamma'} L^0_{\alpha',\gamma'}(jq_{FS,\gamma'}) \Phi_{\alpha',\gamma';\alpha,\gamma}(jq_{FS,\gamma'}, q). \quad (A21) \]

Inserting Eq. (A14) in Eq. (A21) we obtain Eq. (B3).
APPENDIX B: STATIC MASSES FOR THE HEAVY PSEUDOPARTICLES

The static mass $m_{\alpha,\gamma}^*$ is defined in Ref. [15] as

$$ \frac{1}{m_{\alpha,\gamma}^*} = \frac{2t}{(2\pi\rho_{\alpha,\gamma}(r))^2} \left. \frac{d\eta_{\alpha,\gamma}(r)}{dr} \right|_{r=r_0} - \frac{2t\eta_{\alpha,\gamma}(r)(2\pi d\rho_{\alpha,\gamma}(r)/dr)}{(2\pi\rho_{\alpha,\gamma}(r))^3} \bigg|_{r=r_0}, $$

where the functions $2t\eta_{\alpha,\gamma}(r)$ and $2\pi\rho_{\alpha,\gamma}(r)$ are defined in Ref. [15] and $r_0$ is $W_{\alpha,\gamma}(qF_{\alpha,\gamma})$ which represents $Q$, $r_{c,\gamma}$, and $r_{s,\gamma}$.

After some straightforward algebra, the general expressions (B1) lead to the following simple expressions for $1/m_{\alpha,\gamma}^*$

$$ \frac{1}{m_{c,\gamma}^*} = -\frac{4tu^2/(1 + u^2\gamma^2)^{3/2} + \Lambda_{\gamma}^\eta}{(2u/\sqrt{1 + u^2\gamma^2} - \Lambda_{c,\gamma}^\rho)^2}, \quad \gamma > 0, $$

and

$$ \frac{1}{m_{s,\gamma}^*} = \frac{\Lambda_{c,\gamma}^\rho - \Lambda_{s,\gamma} - \Lambda_{s,\gamma+2}}{(\Lambda_{c,\gamma}^\rho + 1 - \Lambda_{s,\gamma} - \Lambda_{s,\gamma+2})^2}, \quad \gamma > 0. $$

In Eqs. (B2) and (B3) the functions $\Lambda_{\alpha,x}^\eta$ and $\Lambda_{\alpha,x}^\rho$ read

$$ \Lambda_{\alpha,x}^\eta = 2 \int_{-qF_{\alpha,0}}^{qF_{\alpha,0}} dq \frac{v_{\alpha,0}(q)R_{\alpha,0}^{(0)}(q)}{\pi x^3 \left[ 1 + (R_{\alpha,0}^{(0)}(q)/x)^2 \right]^2}, $$

and

$$ \Lambda_{\alpha,x}^\rho = \int_{-qF_{\alpha,0}}^{qF_{\alpha,0}} \frac{dq}{\pi x} \frac{1}{1 + \left[ R_{\alpha,0}^{(0)}(q)/x \right]^2}, $$

with

$$ R_{c,0}^{(0)}(q) = \frac{\sin(K^{(0)}(q))/u}{u}. $$

In the limit of fully polarized ferromagnetism, these expressions lead to the following closed-form expressions for the static masses

$$ \frac{1}{m_{c,\gamma}^*} = \frac{t\pi}{8[\eta_{1,\gamma}]^2} \left( -\frac{\pi + 2[\eta_{1,\gamma}]}{\sqrt{1 + [u\gamma]^2}} - \frac{u\gamma \sin(2n\pi)}{[u\gamma]^2 + \sin^2(n\pi)} \right), $$

$$ \frac{1}{m_{s,\gamma}^*} = \frac{t\pi}{[\eta_{2,\gamma+1}]} \left( \frac{1}{\sqrt{1 + [u(\gamma + 1)]^2}} - \frac{u(\gamma + 1) \sin(2n\pi)}{2[\eta_{2,\gamma+1}][u(\gamma + 1)]^2 + \sin^2(n\pi)} \right), $$

28
where

$$\eta_{1,x} = \tan^{-1}\left[ \cot(n\pi) \frac{ xu}{ \sqrt{1 + u^2x^2}} \right], \quad (B9)$$

and $\eta_{2,x} = \pi/2 - \eta_{1,x}$. We remark that the static masses of the $c, \gamma$ pseudoparticles are, in general, negative. The static masses of the $\alpha, 0$ pseudoparticles have been studied in Ref. [22].

**APPENDIX C: CHARGE AND SPIN STIFFNESSES AT ZERO TEMPERATURE**

In this Appendix we show that the direct use of Eq. (A7) leads to the stiffness expressions (135) - (137) of Ref. [9].

The calculation of the charge and spin stiffnesses (A7) requires the expansion of Eq. (25) and of Eqs. (A7), (A8), and (A9) up to second order in $\phi$. As in the case of the charge and spin current, both the charge and spin stiffnesses can be computed from Eq. (A7), and we obtain one or the other depending on the coupling constants we choose in Eqs. (A7), (A8), and (A9). Expanding the ground-state energy up to second order in $\phi$, we obtain

$$\left. \frac{d^2(E_0/N_a)}{d(\phi/N_a)^2} \right|_{\phi=0} = \frac{1}{2\pi} \int_{q_{FC}}^{q_{FC}} dq \left[ 2tK^{0,0}(q) \sin(K^{(0)}(q)) + 2t[K^{0,0}(q)]^2 \cos(K^{(0)}(q)) \right], \quad (C1)$$

where the function $K^{0,0}(q)$ is the second derivative of the rapidity function defined by Eq. (A7) in order to $\phi/N_a$ at $\phi = 0$. The functions $K^{0,0}(q)$ and $K^{0,0,0}(q)$ can be written as

$$K^{0,0}(q) = \frac{dK^{(0)}(q)}{dq} L^{0,0}(q), \quad (C2)$$

and

$$K^{0,0,0}(q) = \frac{d}{dq} \left( \frac{dK^{(0)}(q)}{dq} [L^{0,0}(q)]^2 \right) + 2 \frac{dK^{(0)}(q)}{dq} L^{0,0,0} + \frac{1}{2\pi} \sum_{j=\pm 1} 2t \sin(Q)[L^{0,0}(jq_{FC})]^2, \quad (C3)$$

respectively. The use of Eqs. (C2) and (C3) in Eq. (C1) leads then to

$$\left. \frac{d^2(E_0/N_a)}{d(\phi/N_a)^2} \right|_{\phi=0} = \frac{1}{2\pi} \int_{q_{FC}}^{q_{FC}} dq 2t \sin(K^{(0)}(q)) \frac{dK^{(0)}(q)}{dq} L^{0,0,0}(q) + \frac{1}{2\pi} \sum_{j=\pm 1} 2t \sin(Q)[L^{0,0}(jq_{FC})]^2, \quad (C4)$$
where the function $L_{c,0}^{0,\phi}(jq_{Fc})$ is defined in Appendix A. The function $L_{c,0,s}^{0,\phi}(q)$ obeys the following integral equation

$$
L_{c,0,s}^{0,\phi}(q) = \frac{1}{2\pi} \sum_{j=\pm 1} \frac{j[L_{c,0}^{0,\phi}(jq_{Fs,0})]^2}{2\pi \rho_{s,0}(r_{s,0})(1 + [\sin(K^{(0)}(q))/u - j r_{s,0}]^2)}
+ \frac{1}{\pi} \int_{-q_{Fs,0}}^{q_{Fs,0}} dq' \frac{dR_{s,0}^{(0)}(q')}{dq'} \frac{L_{s,0,s}^{0,\phi}(q')}{1 + [\sin(K^{(0)}(q))/u - R_{s,0}^{(0)}(q')]^2},
$$

which was obtained by performing the type of expansions developed in Appendix A. Moreover, $L_{s,0,s}^{0,\phi}(q)$ is given by

$$
L_{s,0,s}^{0,\phi}(q) = \frac{1}{2u\pi} \sum_{j=\pm 1} \frac{j[\cos(Q)L_{c,0}^{0,\phi}(jq_{Fc})]^2}{2\pi \rho_{c,0}(Q)(1 + [R_{s,0}^{(0)}(q) - j r_{c,0}]^2)}
- \frac{1}{4\pi} \sum_{j=\pm 1} \frac{j[L_{s,0}^{0,\phi}(jq_{Fs,0})]^2}{2\pi \rho_{s,0}(r_{s,0})(1 + [(R_{s,0}^{(0)}(q) - j r_{s,0})/2]^2)}
- \frac{1}{2\pi} \int_{-q_{Fs,0}}^{q_{Fs,0}} dq' \frac{dR_{s,0}^{(0)}(q')}{dq'} \frac{L_{s,0,s}^{0,\phi}(q')}{1 + [(R_{s,0}^{(0)}(q) - R_{s,0}^{(0)}(q'))/2]^2}
+ \frac{1}{\pi u} \int_{-q_{Fc}}^{q_{Fc}} dq' \frac{dK^{(0)}(q')}{dq'} \frac{\cos(K^{(0)}(q'))L_{s,0,s}^{0,\phi}(q')}{1 + (\sin(K^{(0)}(q'))/u - R_{s,0}^{(0)}(q'))^2}.
$$

Introducing Eqs. (C5) and (C6) in Eq. (C4) we obtain, after some algebra, the following expression for the (charge and spin) stiffness $D^\zeta$

$$
4\pi D^\zeta = \sum_{j=\pm 1} \nu_{c,0}[L_{c,0}^{0,\phi,\zeta}(jq_{Fc,0})]^2 + \sum_{j=\pm 1} \nu_{s,0}[L_{s,0}^{0,\phi,\zeta}(jq_{Fs,0})]^2,
$$

where the functions $L_{c,0,s}^{0,\phi,\zeta}(jq_{Fo,0})$ are defined by Eq. (A14). After some simple algebra, expression (C7) can be shown to be the same as expressions (135) - (137) of Ref. [3].

**APPENDIX D: THE ZERO MAGNETIC-FIELD CASE**

For the case of zero magnetic field it is possible to cast the equations for the energy bands, phase shifts, and rapidities in a simpler form. After some algebra, the $\epsilon_{s,\gamma}(q)$ band (with $\gamma = 0, 1, 2, \ldots, \infty$) and the $\epsilon_{c,\gamma}(q)$ band with $(\gamma = 1, 2, \ldots, \infty)$ can at zero magnetic field be rewritten as

$$
\epsilon_{s,\gamma}^{0}(q) = -\delta_{\gamma,0} \left[ 2t \int_{0}^{\infty} d\omega \frac{\cos(\omega R_{s,0}^{(0)}(q))}{\omega \cosh(\omega)} T_{1}(\omega) \right],
$$

$$(D1)$$
and
\[ e_{c,\gamma}^0(q) = \text{Re} \ 4t \sqrt{1 - u^2 (R_{c,\gamma}^0(q) + i\gamma)^2} - 4t \int_0^\infty d\omega \frac{e^{-\gamma\omega}}{\omega} \cos(\omega R_{c,\gamma}^0(q)) \Upsilon_1(\omega), \tag{D2} \]

where \( \Upsilon_1(\omega) \) obeys the integral equation
\[ \Upsilon_1(\omega) = \Upsilon_1^0(\omega) + \int_{-\infty}^\infty d\omega' \Upsilon_1(\omega') \Gamma(\omega', \omega). \tag{D3} \]

Here the free term and the kernel read
\[ \Upsilon_1^0(\omega) = \frac{1}{2\pi} \int_Q^{-Q} dk \sin(k) \sin(\omega \sin(k)/u), \tag{D4} \]

and
\[ \Gamma(\omega', \omega) = \frac{\sin((\omega - \omega')r_{c,0})}{\pi(\omega - \omega')(1 + e^{2|\omega'|})}, \tag{D5} \]

respectively. The kernel \( \Upsilon_1^0(\omega) \) was already obtained in Ref. \[12\] (see Eq. (A5) of that reference). Equation \( \Upsilon_1(\omega) \), together with the fact that in the limit of zero magnetic field the width of the \( s, \gamma > 0 \) momentum pseudo-Brillouin zone vanishes [see Eq. (2)], shows that the \( s, \gamma \) bands collapse for \( \gamma > 0 \) and all values of \( U \) and \( n \) to the point zero.

In this limit it is also possible to cast the integral equations for the phase shifts, whose expressions are given in Ref. \[13\], in the following alternative form
\[ \tilde{\Phi}_{c,0,c,0}(r, r') = -B(r - r') + \int_{-r_{c,0}}^{r_{c,0}} dr'' \tilde{\Phi}_{c,0,c,0}(r'', r') A(r - r''), \tag{D6} \]
\[ \tilde{\Phi}_{c,0,c,\gamma}(r, r') = -\frac{1}{\pi} \tan^{-1}\left(\frac{r - r'}{\gamma}\right) + \int_{-r_{c,0}}^{r_{c,0}} dr'' \tilde{\Phi}_{c,0,c,\gamma}(r'', r') A(r - r''), \tag{D7} \]

\[ \tilde{\Phi}_{c,0,s,\gamma}(r, r') = -\delta_{0,\gamma} \frac{1}{2\pi} \tan^{-1}[\sinh(\pi/2(r - r'))] + \int_{-r_{c,0}}^{r_{c,0}} \frac{dr''}{\gamma' + 1} \tilde{\Phi}_{c,0,s,\gamma}(r'', r') A(r - r''), \tag{D8} \]

\[ \tilde{\Phi}_{c,\gamma;c,0}(r, r') = \frac{1}{\pi} \tan^{-1}\left(\frac{r - r'}{\gamma}\right) - \int_{-r_{c,0}}^{r_{c,0}} \frac{dr''}{\gamma' + 1} \frac{\tilde{\Phi}_{c,0,c,0}(r'', r')}{\gamma'}\left(\frac{r - r''}{\gamma'} + \frac{r - r''}{\gamma'}\right), \tag{D9} \]
\[ \Phi_{c,\gamma}(r, r') = \frac{1}{2\pi} \Theta_{\gamma \gamma}(r - r') - \int_{-r,0}^{r,0} \frac{dr''}{\pi r' \gamma} \Phi_{c,0;\gamma}(r'', r') , \]  
(D10)

\[ \Phi_{c,\gamma}(r, r') = - \int_{-r,0}^{r,0} \frac{dr''}{\pi r' \gamma} \Phi_{c,0;\gamma}(r'', r') , \]  
(D11)

\[ \Phi_{s,\gamma;0}(r, r') = 0 , \]  
(D12)

\[ \Phi_{s,\gamma;0}(r, r') = \Phi_{c,0;\gamma}(r'', r') \]  
(D13)

\[ \Phi_{s,\gamma;0}(r, r') = \frac{1}{4} \int_{-r,0}^{r,0} \frac{dr''}{\pi \gamma + 1 \cosh(\pi/2(r'' - r))} , \]  
(D14)

and

\[ \Phi_{s,\gamma;0}(r, r') = \frac{1}{2\pi} \Theta_{\gamma+1,\gamma'}(r - r') - \frac{1}{4} \int_{-r,0}^{r,0} \frac{dr''}{\pi \gamma + 1 \cosh(\pi/2(r'' - r))} , \]  
(D15)

The functions \( A(r) \) and \( B(r) \) are defined as

\[ A(r) = \frac{1}{\pi} \int_{0}^{\infty} d\omega \frac{\cos(r\omega)}{1 + e^{2|\omega|}} , \]  
(D16)

and

\[ B(r) = \frac{1}{\pi} \int_{0}^{\infty} d\omega \frac{\sin(r\omega)}{\omega(1 + e^{2|\omega|})} , \]  
(D17)

respectively.

At \( n = 1 \) we have that \( \Upsilon_1(x) = \Upsilon_1^0(x) = J_1(x/u) \), where \( J_1(x/u) \) is the Bessel function of order one, and the bands \([D1]\) and \([D2]\) are obtained in closed form.
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FIGURES

FIG. 1. The ratio $m_{c,1}^p / m_{c,1}^*$ as function of $U$ at electronic density $n = 0.7$ and for values of the magnetic field $h = H / H_c = 0.1$ (full line), $h = 0.5$ (dashed line) and $h = 0.9$ (dashed-dotted line). For other electronic densities, the plots follow the same trends as for $n = 0.7$.

FIG. 2. The ratio $m_{c,1}^p / m_{c,1}^*$ as function of the electronic density $n$ and for values of the magnetic field $h = 0.1$, $h = 0.3$, $h = 0.5$, $h = 0.7$, and $h = 0.9$. The onsite Coulomb interaction is (a) $U = 1$, (b) $U = 5$, and (c) $U = 20$.

FIG. 3. The ratio $m_{c,1}^p / m_{c,1}^*$ as function of the the magnetic field $h$ and for values of the electronic density $n = 0.1$, $n = 0.3$, $n = 0.5$, $n = 0.7$, and $n = 0.9$. The onsite Coulomb interaction is (a) $U = 1$, (b) $U = 5$, and (c) $U = 20$.

FIG. 4. The ratio $m_{c,1}^p / m_{c,1}^*$ as function of the the magnetic field $h$ and for values of the onsite Coulomb interaction $U = 1$, $U = 2$, $U = 3$, $U = 5$, $U = 10$, and $U = 20$. The electronic density is (a) $n = 0.5$ and (b) $n = 0.9$.

FIG. 5. The ratio $m_{c,0}^p / m_{c,0}^*$ as function of $U$, for electronic density $n = 0.7$, and for values of the magnetic field $h = 0.1$, $h = 0.3$, $h = 0.5$, $h = 0.7$, and $h = 0.9$. For other electronic densities, the plots follow the same trends as for $n = 0.7$.

FIG. 6. The ratio $m_{c,0}^p / m_{c,0}^*$ as function of the electronic density $n$ and for values of the magnetic field $h = 0.1$, $h = 0.3$, $h = 0.5$, $h = 0.7$, and $h = 0.9$. The onsite Coulomb interaction is (a) $U = 1$, (b) $U = 5$, and (c) $U = 20$.

FIG. 7. The ratio $m_{c,0}^p / m_{c,0}^*$ as function of the magnetic field $h$ and for values of the electronic density $n = 0.1$, $n = 0.3$, $n = 0.5$, $n = 0.7$, and $n = 0.9$. The onsite Coulomb interaction is (a) $U = 1$, (b) $U = 5$, and (c) $U = 20$. 

37
FIG. 8. The ratio $m_{e,0}/m_{c,0}^*$ as function of the the magnetic field $h$ and for values of the onsite Coulomb interaction $U = 1$, $U = 2$, $U = 3$, $U = 5$, $U = 10$, and $U = 20$. The electronic density is (a) $n = 0.5$ and (b) $U = 0.9$.

FIG. 9. The ratio $m_{s,1}/m_{s,1}^*$ as function of $U$, for electronic density $n = 0.7$, and for values of the magnetic field $h = 0.1$, $h = 0.3$, $h = 0.5$, $h = 0.7$, and $h = 0.9$. For other electronic densities, the plots follow the same trends as for $n = 0.7$.

FIG. 10. The ratio $m_{s,1}^*/m_{s,1}^*$ as function of the electronic density $n$ and for values of the magnetic field $h = 0.1$, $h = 0.3$, $h = 0.5$, $h = 0.7$, and $h = 0.9$. The onsite Coulomb interaction is (a) $U = 1$, (b) $U = 5$, and (c) $U = 20$.

FIG. 11. The ratio $m_{s,1}/m_{s,1}^*$ as function of the magnetic field $h$ and for values of the electronic density $n = 0.1$, $n = 0.3$, $n = 0.5$, $n = 0.7$, and $n = 0.9$. The onsite Coulomb interaction is (a) $U = 1$, (b) $U = 5$, and (c) $U = 20$.

FIG. 12. The ratio $m_{s,1}/m_{s,1}^*$ as function of the the magnetic field $h$ and for values of the onsite Coulomb interaction $U = 1$, $U = 2$, $U = 3$, $U = 5$, $U = 10$, and $U = 20$. The electronic density is (a) $n = 0.3$, (b) $n = 0.5$, and (c) $U = 0.9$. 

38
Mass ratios in several limits of physical interest. The function \( \eta_\gamma \) is defined as \( \eta_\gamma = 2/(\pi)\tan^{-1}[(\sin(n\pi))/((u\gamma + 1))] \). The equations for the static masses \( m^*_{c,\gamma} \) are given in Appendix B. In the case \( H \to 0 \), simple expressions for the parameters \( \xi_{0,\gamma}^{1,0} \), Eq. (44), can be obtained from the results of Appendix D. The ratios \( m^\rho_{c,\gamma}/m^*_{c,\gamma} \) and \( m^\rho_{s,\gamma}/m^*_{s,\gamma} \) are infinite. The dependence on \( U \) and \( n \) of the parameter \( \xi_0 \) has been studied in Refs. [5,36].

| \( H \to H_c \) | \( H \to 0 \) | \( n \to 1 \) |
|-----------------|-----------------|-----------------|
| \( m^\rho_{c,\gamma}/m^*_{c,\gamma} \) | \( \frac{1}{2\gamma(2\gamma-\eta_\gamma-1)} \) | \( \frac{1}{2\gamma(2\gamma+\xi_{0,\gamma,c,0})} \) | \( \frac{1}{4\gamma^2} \) |
| \( m^\rho_{c,0}/m^*_{c,0} \) | \( \frac{\eta_\gamma}{2\sin(\pi n\gamma)} \) | \( \frac{1}{(\xi_0)^2} \) | \( \infty \) |
| \( m^\rho_{s,\gamma}/m^*_{s,\gamma} \) | \( \frac{1}{2(\gamma+1)(2\gamma+2-\eta_\gamma)} \) | \( \frac{1}{2(\gamma+1)(2\gamma+2+2\xi_{0,\gamma,s,0})} \) | \( \frac{1}{2(\gamma+1)(2\gamma+2+2\xi_{0,\gamma,s,0} - \xi_{0,\gamma,c,0})} \) |
$n = 0.7$

$\rho_{c,1}^\rho$ vs $U$

Legend:
- Solid line: $h = 0.1$
- Dotted line: 0.5
- Dashed line: 0.9
The diagram illustrates the relationship between $h$ and $r_{c,1}^\rho$ for different values of $U$ and $n$. The curves are labeled with values of $U = 1$ and $n = 0.1$. The x-axis represents $h$, ranging from 0.0 to 1.0, and the y-axis represents $r_{c,1}^\rho$, ranging from 0.0 to 1.0.
