Finite Temperature Phase Diagram of QCD with improved Wilson Fermions *

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We present results of an ongoing study of two flavour QCD with Wilson fermions at finite temperature. We have used tree level Symanzik improvement in both the gauge and fermion part of the action. The phase diagram was previously determined on an $8^3 \times 4$ lattice and the existence of Aoki's phase demonstrated. On our current lattice of $12^2 \times 24 \times 4$ we have extended the set of observables and studied chiral and thermal properties at light quark masses.

1. Introduction

The study of the finite temperature phase diagram of 2 flavour QCD with Wilson quarks has revealed a rather intricate picture. As was pointed out already some time ago [1], the pion can become massless despite the fact that the Wilson term breaks chiral symmetry, because of the existence of a second order phase transition into a parity and flavour symmetry violation phase. At finite temperature this phase is expected to pinch into a cusp that meets with the $m = 0$ line coming in from weak coupling [2]. The thermal line of the deconfinement phase transition crosses the $m_q = 0$ line and in [3] it was argued that it should bounce back towards weaker coupling due to a symmetry under the change of sign of the mass term in the continuum theory. The relation of this crossing point to the tip of the cusp of the Aoki phase is somewhat subtle. First of all the thermal line should not run into the Aoki phase, since this would mean, that one has a massless pion in the deconfined phase. The thermal line can now either touch the cusp of the Aoki phase or not. In the latter case one would have a transition line (probably first order) from a confined phase with $m_q > 0$ to a confined phase with $m_q < 0$, without the pion actually becoming massless [4].

2. Results

Utilizing a tree level $O(a^2)$ improved action for the gauge fields and the tree level clover action for the fermion fields, we have, in a previous study on an $8^3 \times 4$ lattice, mapped out the phase diagram using the pion norm and the Polyakov loop as observables. In the present study we have increased the lattice size to $12^2 \times 24$ to check finite size effects and to measure the pion screening mass and quark mass accurately. From these we can also determine the properly subtracted order parameter of chiral symmetry breaking for Wilson fermions.

2.1. Quark mass

The current quark mass is defined via the axial Ward identity [5]:

$$2m_q \equiv \frac{\nabla_\mu \langle 0 | A^\mu | \pi \rangle}{\langle 0 | P | \pi \rangle}$$

(1)

Our results are shown in figure 1. For $\beta = 2.8$ we find some curvature for the quark mass as a function of $1/\kappa$, though a linear fit produces a reasonable $\chi^2$. For $\beta = 3.1$ we have also explored a region of hopping parameters where the quark mass becomes negative. There we find a rather peculiar behaviour. The quark mass decreases as one lowers $\kappa$ towards $\kappa_c$ (see section [3]), which is in contrast to simulations with unimproved Wilson fermions that have shown

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the same qualitative behaviour for positive and negative quark masses \[^2\].

2.2. Pion screening mass

We have also accurately measured the pion screening mass. Our results are depicted in figure 2. For \(\beta = 2.8\) the decrease of the pion mass is consistent with a linear behaviour \(m_\pi^2 \propto 1/\kappa - 1/\kappa_c\) down to small values of \(m_\pi^2\). This also applies for \(\beta = 3.1\) for \(\kappa\) values that correspond to positive quark masses. For negative quark masses the behaviour is quite different and inconsistent with a linear behaviour for the points measured. We will argue below that for \(\beta = 3.1\) the pion mass does not go to zero as the quark mass becomes zero, because one crosses the finite temperature transition line before the quark mass becomes zero.

2.3. Locating the critical line

To locate the critical hopping parameter \(\kappa_c\) we have performed the following fits to our data. We have fitted \(m_\pi^2\) linearly and \(m_q\) quadratically as a function of \(1/\kappa\). Since the pion norm near \(\kappa_c\) is expected to behave like \(\Pi \approx 1/m_\pi^2\), we have also extracted a value for \(\kappa_c\) from this observable. Since the data show quite some nonlinear behaviour as a function of \(1/\kappa\), we have used a quadratic fit ansatz. As the results from the different fits turned out to be consistent, we took confidence in using our data for the pion norm from our simulation at \(\beta = 3.0\) on the smaller lattice to get an estimate of \(\kappa_c\) for \(\beta = 3.0\). Our results are summarized in the following table:

| \(\kappa_c\) | \(\beta = 2.8\) | \(\beta = 3.0\) | \(\beta = 3.1\) |
|-------------|----------------|----------------|----------------|
| \(m_\pi\)   | 0.1860(2)     | 0.1783(2)     |                |
| \(m_q\)     | 0.1853(3)     | 0.1770(3)     |                |
| \(\Pi\)     | 0.1859(3)     | 0.1823(10)    | 0.1800(5)      |

2.4. Chiral order parameter

Because of the explicit breaking of chiral symmetry by the Wilson action, one has to define a properly subtracted order parameter to obtain the correct continuum limit. Using axial Ward identities the order parameter is defined as follows\[^5\]:

\[
\langle \bar{\psi}\psi \rangle_{sub} = 2m_q \cdot Z \cdot \sum_x \langle \pi(x)\pi(0) \rangle
\]

Here \(Z\) is a renormalization factor for which we take its tree level value \(Z = (2\kappa)^2\). The sum over the pion correlation function is just the pion norm. For \(\beta = 2.8\) the data extrapolate to a finite intercept at \(m_q = 0\). For \(\beta = 3.1\) the data show more curvature and we expect \(\langle \bar{\psi}\psi \rangle_{sub}\) to go to zero, though a definitive statement has to await further data at smaller quark masses.

2.5. Polyakov loop

Figure 4 shows our results for the Polyakov loop as a function of \(\kappa\) including data from both
lattices. The vertical lines indicate the position of the extrapolated $\kappa_c$, which decreases as $\beta$ increases. For $\beta = 2.8$ all our data for the pion mass lie in the confined region. We therefore conclude that the pion mass vanishes as the quark mass goes to zero, i.e. for $\beta = 2.8$ we hit the Aoki phase as we increase $\kappa$. For $\beta = 3.0$ and 3.1 this is no longer so clear. The Polyakov loop already shows a high temperature behaviour where the extrapolated pion mass would be small. For $\beta = 3.0$ the situation is less prominent and one could still argue that the pion becomes massless, but since the Polyakov loop is in the high temperature phase immediately after one crosses $\kappa_c$, the finite temperature line and the line of vanishing quark mass come very close together. For $\beta = 3.1$ it becomes evident, that one crosses the finite temperature transition line before the line of vanishing quark mass. The pion will therefore not become massless as the quark mass vanishes. Because the Polyakov loop continues to rise past the $m_q = 0$ line, one can exclude that the finite temperature line bounces back towards weaker coupling immediately.

3. Summary and Conclusions

From the vanishing of the pion mass at zero quark mass we conclude that at $\beta = 2.8$ there exists a parity and flavour symmetry breaking phase. At $\beta = 3.0$ the finite temperature transition line and the line of vanishing quark mass come at least very close and for $\beta = 3.1$ the Aoki phase ceases to exist and one crosses the finite temperature transition line before the line of vanishing quark mass and therefore the pion does not become massless. To make these claims even stronger it would be desirable to lower the quark mass further for $\beta = 3.1$ and to see the pion mass rising again. Once the phase diagram is known one can start to investigate the thermodynamic behaviour at small pion masses.

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