Numerical and experimental study on dynamics of the planar mechanical system considering two revolute clearance joints

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Abstract
Due to assembly, wear and manufacturing errors and clearance in the joints are inevitable. When the clearance is introduced into a mechanical system, the impact force in the clearance joint will cause undesirable vibration of the system. In this paper, the dynamic responses of the mechanical system with two revolute clearance joints are studied using computational and experimental methodology. The clearance joint is considered as force constraint. The normal contact force and tangential friction force between the journal and bearing in a clearance joint are modeled using a nonlinear contact force model considering energy loss and a modified Coulomb friction model considering a dynamic friction coefficient, respectively. A planar slider-crank mechanism with two revolute clearance joints is used to implement the study. The dynamic responses obtained from numerical simulation are compared with the experimental test. Numerical simulations and experimental tests for different clearance sizes and crank speeds are presented and discussed, respectively. The simulation results agree quite well with those of the experiment for different cases, which proves the accuracy and efficiency of the computational method for dynamics analysis of the mechanical system with two revolute clearance joints in this study. The investigation indicates that the clearances in revolute joints significantly affect the dynamic characteristics of mechanical systems, which must be considered in the precision analysis, design, and control of multibody systems, especially for high-speed machinery.

KEYWORDS
clearance joint, dynamic characteristics, experimental tests, mechanical system, modeling and simulation

1 | INTRODUCTION
In general, clearances in imperfect joints of mechanical systems are inevitable, which will cause contact and impact. Consequently, these contact forces will cause vibrations of interconnected bodies via the joints and affect the performances of multibody systems.1–5 The clearances eventually leads to deviations of real dynamic response of mechanical systems from expectations.6–10

The clearance effects on multibody systems have been investigated by many researchers. Flores11 presented a parametric...
investigation of clearance effects on dynamics of a planar linkage mechanism. The size of the clearance and the driving speed were analyzed and discussed. Erkaya and Uzmay proposed a study to reduce the deviations caused by a clearance joint in linkage mechanisms. Muvengei et al. studied the dynamics characteristics of a planar linkage mechanism with two revolute clearance joints. Bai and Sun analyzed the contact modes in multiclearance joints and three clearances were considered in a mechanical system. Wang et al. analyzed the dynamics of a rigid multibody system with clearances by using a nonpenetration approach of frictional contact. Salahshoor et al. studied the vibration of a planar flexible slider-crank mechanism with joint clearances considering joint stiffness. Pi and Zhang proposed a multiple patch-based model of revolute clearance joint to study dynamics characteristics of a planar slider-crank mechanism with multiple clearances. Zhan et al. investigated planar parallel manipulators with interval clearance variables. Guo et al. presented a dynamic model to study the position secondary motion of mechanical systems with clearances. Chen et al. analyzed the effect of multiple clearances and friction on the dynamics of mechanical systems. Additionally, many studies also focused on various kinds of mechanisms with clearance joints considering parameter uncertainty, joint wear, design optimization, lubrication in clearance joint, flexible body, and other topics. Also, recently, researchers are focusing on 3D revolute clearance joints. All the studies show that clearances in mechanism significantly affect the dynamic responses of mechanical systems. Therefore, investigation on joints with clearances is of great importance for the design and control of mechanical systems.

However, few work focused on the dynamics of mechanical systems with two clearances using the experimental method. Most of the previous literature studied the dynamics characteristics of various kinds of mechanisms with one clearance joint using experimental methods. The main goal of this paper is to present an experimental investigation of mechanism considering two revolute clearance joints and provide verification and validation of the dynamic modeling and analysis. A planar slider-crank mechanism with two revolute clearance joints is used for the numerical and experimental investigation. Dynamic simulation and experimental tests for different driving speeds as well as different clearance sizes are presented. The acceleration of the slider is measured by an accelerometer, and the measured signal is transmitted to the PC via an external data acquisition system. The experimental and simulation results are compared to validate the dynamics modeling. In the dynamics modeling, the clearance joint is considered as a force constraint. The normal and tangential forces between the journal and the bearing in a clearance joint are modeled using a nonlinear contact force model considering energy loss and a modified Coulomb friction model considering a dynamic friction coefficient, respectively. Then, the experimental investigation is presented. The dynamic responses obtained by the numerical simulation are compared with that of the experiment. The effects of two clearances on the dynamics characteristics of the mechanism are discussed.

This paper is organized as follows. Section 2 presents the models of a revolute joint with clearances and the contact force in the joint. Section 3 presents the experimental setup of a slider-crank mechanism with two revolute clearance joints. In Section 4, the experimental and simulation results of the clearance effects on the mechanism are discussed. Finally, Section 5 summarizes the paper.

2 | MODELING OF CLEARANCE IN REVOLUTE JOINT AND MULTIBODY DYNAMICS

2.1 | Definition of clearance joint

Usually, the radial clearance in a revolute joint is defined by the difference in radius of the bearing and journal elements, which is expressed by

\[ c = R_b - R_i, \tag{1} \]

where \( R_b \) and \( R_i \) are the radii of the bearing and journal.

The clearance joint limits the journal to move within the bearing, which provides kinematic restrictions but is not like the ideal revolute joint. Therefore, the dynamics of the clearance joint is governed by contact and impact forces between the journal and the bearing. Due to the contact and impact in the clearance joint, there are three motion modes of the journal inside the bearing for a revolute clearance joint, namely, the sustained contact mode, the free flight mode, and the impact mode.

2.2 | Contact force models of a clearance joint

The contact and impact are the typical phenomena for clearance joints. Figure 1 describes the contact between the bearing and the journal. The contact/impact forces in a real joint with clearance can be decomposed into a normal contact force and a friction force simultaneously, as shown in Figure 1A. The constraints of clearance joint are modeled as contact force constraints. The contact deformation caused by the contact and impact between the bearing and journal can be represented as:

\[ \delta = e - c, \tag{2} \]

where \( c \) is the clearance size and \( e \) the eccentricity of the journal center relative to the bearing center, as shown in Figure 1B.

To efficiently evaluate the contact forces, the contact force model of the clearance joint should be carefully established. Two methods are usually used: discrete analysis method and continuous contact analysis method. The continuous contact method assumes that the contact forces between the impact bodies are continuous and tallies with the real contact-impact behavior of objects. Hertz represented the contact force as a nonlinear function of
the penetration based on the pure elasticity theory and ignored the energy loss during the impact. Then, Hunt and Crossley34 proposed a nonlinear viscous-elastic model considering the energy transfer during the impact process. Further, to account for the energy dissipation in the contact process, Lankarani and Nikravesh38 proposed a continuous contact force model considering hysteretic damping factor, which separates the normal contact force into elastic and dissipative components. The Lankarani-Nikravesh contact force model is expressed as:

\[ F_n = K\delta^n + D\dot{\delta}, \tag{3} \]

where \( \delta \) is the deformation, \( \dot{\delta} \) is the relative deformation velocity, and \( K \) is the contact stiffness coefficient obtained from Equations (4) and (5):

\[ K = \frac{4}{3\pi(a_i + a_j)} \left[ \frac{R_i R_j}{R_i - R_j} \right]^{\frac{2}{3}}, \tag{4} \]

\[ a_i = 1 - \frac{v_i^2}{\pi E_i}, \]

\[ a_j = 1 - \frac{v_j^2}{\pi E_j}, \tag{5} \]

where \( v \) and \( E \) are the Poisson ratio and Young modulus, respectively. \( R_i \) and \( R_j \) are the radii of the joint elements.

Damping coefficient \( D \) in Equation (3) is expressed as:

\[ D = 3K(1 - c_e^2)\delta^0 \tag{6} \]

where \( c_e \) is the coefficient of restitution and \( \delta^0 \) is the initial relative velocity of the impact point.

Further, the contact force model of Equation (3) can be expressed as:

\[ F_n = K\delta^n + \frac{3(1 - c_e^2)\delta}{4\delta^0}. \tag{7} \]

The Lankarani and Nikravesh contact force model was widely applied to dynamics analysis of mechanical systems with revolute clearance joints and it shows good precision. Thus, in this paper, the Lankarani and Nikravesh contact force model is used in dynamics modeling and simulation.

The tangential contact of the clearance joint is represented using the tangential friction force model. The best-known friction model is the Coulomb friction model. However, there will be difficulties in the numerical calculation when relative tangential velocity is close to zero. Here, a modified Coulomb friction model with a dynamic friction coefficient is used for tangential contact of clearance joint, which can avoid numerical difficulties.2,40,41 The expression of the modified Coulomb friction model is:

\[ F_t = -\mu(v_t)\frac{v_t}{|v_t|}, \tag{8} \]

where the dynamic friction coefficient \( \mu(v_t) \) is a function of tangential velocity and can be expressed as:

\[ \mu(v_t) = \begin{cases} -\mu_d \text{sign}(v_t), & \text{for } |v_t| > v_b, \\
-\mu_d + (\mu_u - \mu_d)\left(\frac{|v_t| - v_b}{v_u - v_b}\right)^2, & \text{for } v_b \leq |v_t| \leq v_u, \\
\mu_u - 2\mu_d \frac{v_t + v_b}{2v_u}(3 - \frac{v_t + v_b}{v_u}), & \text{for } |v_t| < v_b.
\end{cases} \tag{9} \]

where \( v_t \) is relative sliding velocity, \( \mu_d \) and \( \mu_u \) are the dynamic and static friction coefficients, respectively. \( v_b \) and \( v_u \) are the given bounds for the tangential critical velocity.

### 2.3 Dynamics equations

Further, by using the Lagrange multiplier method, the dynamics equations of the mechanical system with clearances can be established as41,42:

\[ M\ddot{q} + C\dot{q} + Kq + \Phi_2^T \lambda = F + F_c \]

\[ \Phi(q, t) = 0. \tag{10} \]

where \( q \) is the generalized coordinate vector, \( M, C, \) and \( K \) are the generalized mass matrix, damping matrix, and stiffness matrix,
respectively, $\Phi_0$ and $\lambda$ are the Jacobian matrix of constraint equation and Lagrange multiplier column matrix, respectively. $F$ and $F_c$ are the generalized force and the contact force relative to $q$. It should be noted that the contact force $F_c$ contains both normal contact force $F_n$ and tangential friction force $F_t$. $F_c$ can be read as

$$F_c = \begin{cases} 0, & \text{if } \delta < 0, \\ F_n + F_t, & \text{if } \delta \geq 0. \end{cases} \quad (11)$$

3 | EXPERIMENTAL RESEARCH AND NUMERICAL MODEL

In this section, a flexible planar slider-crank mechanism with two revolute clearance joints, as shown in Figure 2, is set up for experimental investigation on the effects of clearances. The revolute joints B and C are imperfect joints with clearances, and the revolute joint A and translational joint D are ideal joints without clearance. The length and inertia properties of each body of the slider-crank mechanism are presented in Table 1.

To investigate the effects of the two clearances on the dynamic characteristics of the slider-crank mechanism, the general idea of the experiment is to obtain the acceleration responses of the slider using an accelerometer. And also, the clearance size and the driving speed can be adjusted to study different cases of the clearance effects. Therefore, the diameter of the bearing is fixed and the journal is changed with different diameters to obtain the different clearance sizes. The setup of this experiment, which is shown in Figure 3, includes a slider-crank mechanism, a motor speed control unit, and a system of data acquisition. The type of DC motor is JS-ZYT22 and the driving speed can be controlled by using a speed governor. The accelerometer is a three-axis piezoelectric acceleration sensor and the type is ZR412-36. The voltage sensitivity is 248.7 mV/ms in the $x$ direction.

4 | RESULTS AND DISCUSSION

Clearance size and driving speed are two important factors, which significantly influence the dynamics characteristics of the mechanical system. In this section, the effects of them on the slider are investigated respectively using numerical simulation and experiment. In the numerical simulation, the connecting rod is considered as a flexible body. The crank and slider are considered to be rigid. Therefore, in the experiment, the rigid crank is designed as a flywheel, as shown in Figure 3. The parameters used in the dynamic simulations are given in Table 2. The initial position of the crank and connecting rod is set horizontal. The flexible connecting rod is discretized using ANSYS and the first six-order frequencies are listed in Table 3. The corresponding vibration modes are shown in Figure 4.

4.1 | Clearance effects

To study the effects of clearance on the dynamic characteristics of the slider-crank mechanism, three cases with different clearance sizes
are presented and discussed. For each case, revolute joints B and C have the same clearance size. The clearance sizes are chosen to be 0.1, 0.2, and 0.5 mm, respectively, which is shown in Table 4. To obtain different clearance sizes in the experiment, the diameter of the bearing is fixed and, for each case, the journal is changed with a different diameter. Here, the crank is the driving body and rotates with a constant angular velocity 100 r/min. The experimental results and the simulation results for different cases are presented and compared in Figure 5. Please note that the results of the ideal joint without clearance are simulation results.

Figure 5 shows the acceleration of the slider for different cases obtained using experimental tests and numerical simulations. It should be noted that the simulation results for the ideal joint are presented and compared with that of the clearance joint to show the clearance effects. Both the experimental and simulation results show that the effects of clearances on the acceleration of the slider-crank mechanism are extremely obvious. The acceleration of the slider-crank mechanism with clearances is obviously oscillating and the amplitude increases compared to the slider-crank mechanism without clearance. The accelerations of the mechanism with clearances are oscillating with very high peaks, which indicates that the clearance will lead to the oscillation of the slider-crank mechanism and the motion stability of the mechanism is decreased. A larger size of clearance will lead to a significant oscillation and higher peaks of the mechanism, which deteriorates the dynamic performances of the mechanical system. Also, the numerical simulation results agree well with those of experimental tests with different clearance sizes, demonstrating that the computational method in this paper is reasonable and effective to model mechanical systems with revolute clearance joints.

The velocity and displacement of the slider for different cases are presented in Figures 6 and 7, respectively. The experimental results of velocity and displacement are obtained by integrating the measured acceleration over time once and twice, respectively. Figure 6 shows that clearance size has obvious effects on the velocity of the mechanism and a larger size of clearance leads to an obvious perturbation of the velocity. Figure 7 shows that the position of the slider is not significantly affected by the clearances. Therefore, compared with Figure 5, it can be found that clearances have much greater effects on the acceleration level of the mechanical system, thus leading to worse dynamic performances and undesirable

### Table 2 Parameters used in the dynamic simulation

| Parameter                  | Value  |
|----------------------------|--------|
| Elasticity modulus (GPa)   | 207    |
| Restitution coefficient     | 0.9    |
| Poisson’s ratio             | 0.29   |
| Dynamic friction coefficient| 0.1    |
| Static friction coefficient | 0.15   |
| Initial position           | 0      |
| Integration method         | Gear stiff |
| Integration time step (s)  | 0.0001 |
| Integration accuracy       | $1 \times 10^{-7}$ |

### Table 3 Frequencies of the flexible connecting rod

| Order | 1  | 2    | 3    | 4    | 5    | 6    |
|-------|----|------|------|------|------|------|
| Frequency (Hz) | 102.022 | 315.605 | 383.016 | 647.431 | 824.371 | 1086.400 |

**Figure 4** Vibration models of the flexible connecting rod: (A) the first vibration mode, (B) the second vibration mode, (C) the third vibration mode, (D) the fourth vibration mode, (E) the fifth vibration mode, and (F) the sixth vibration mode.
vibrations of mechanical systems. A larger size of clearances will decrease the dynamic performances of the mechanical system.

Further, the dimensionless root mean square (RMS) error is analyzed in this section, which can represent the effect of clearance on the movement stability of the mechanism and reflect the error of the response caused by joint clearance. The dimensionless RMS errors are defined as:

| Cases   | Clearance size (mm) |
|---------|---------------------|
| Case 1  | $c = 0.1$           |
| Case 2  | $c = 0.2$           |
| Case 3  | $c = 0.5$           |

**TABLE 4** Clearance sizes of revolute joints B and C in experiments

**FIGURE 5** Slider acceleration for different clearance sizes: (A), (C), (E) experimental results; (B), (D), (F) simulation results
\[ \text{DRMSP}(s) = \frac{\text{RMS}(s_c - s_i)}{\text{RMS}(s)} \times 100\%, \]  
\[ \text{DRMSP}(v) = \frac{\text{RMS}(v_c - v_i)}{\text{RMS}(v)} \times 100\%, \]  
\[ \text{DRMSP}(\alpha) = \frac{\text{RMS}(\alpha_c - \alpha_i)}{\text{RMS}(\alpha)} \times 100\%, \]

where \(s, v,\) and \(\alpha\) are position, velocity, and acceleration of the slider, respectively; the subscripts \(c\) and \(i\) represent the dynamic responses of the mechanism with clearance joints and ideal joints, respectively. \(\text{RMS}(s_c - s_i), \text{RMS}(v_c - v_i),\) and \(\text{RMS}(\alpha_c - \alpha_i)\) are the RMS errors of the position, velocity, and acceleration of the mechanical system with clearance joint. \(\text{RMS}(s), \text{RMS}(v),\) and \(\text{RMS}(\alpha)\) are the RMSs of the slider’s position, velocity, and acceleration of the mechanical system with the ideal joint, respectively.

The dimensionless RMS error indicators obtained by experimental tests and simulation results are presented in Tables 5 and 6.

### Table 5: Dimensionless RMS error indicator (experimental results)

| Clearance size (mm) | DRMSP\((s)\) | DRMSP\((v)\) | DRMSP\((\alpha)\) |
|---------------------|-------------|-------------|-----------------|
| \(c = 0.1\)         | 0.8233      | 1.8616      | 120.7596        |
| \(c = 0.2\)         | 1.3103      | 3.7328      | 158.4153        |
| \(c = 0.5\)         | 2.7026      | 5.6231      | 206.8026        |

### Table 6: Dimensionless RMS error indicator (simulation results)

| Clearance size (mm) | DRMSP\((s)\) | DRMSP\((v)\) | DRMSP\((\alpha)\) |
|---------------------|-------------|-------------|-----------------|
| \(c = 0.1\)         | 0.3609      | 1.4238      | 111.4949        |
| \(c = 0.2\)         | 0.6950      | 2.4383      | 144.7811        |
| \(c = 0.5\)         | 1.7567      | 4.3902      | 196.2675        |
FIGURE 8  Slider acceleration for different driving speeds: (A), (C), (E) (G) experimental results; (B), (D), (F), (H) simulation results
respectively. These values reflect how the movement of the mechanism is affected due to the joint with different clearances. Both the experimental and simulation results show that the dimensionless indicators increase as the clearance size increases. It demonstrates that clearances will lead to deviations of the motion response and undesirable vibrations of mechanical systems. A larger size of clearances will deteriorate the dynamic performances.

The difference of the response between the experiment and numerical simulation are due to the fact that the joint flexibility and the misalignment between journal and bearing were not considered in the numerical simulations, which, however, always exist in actual mechanical systems. Another reason is that the restitution coefficient and friction coefficient selected in the simulation are based on the published data, which will also lead to such a difference. In addition, the assembly and measurement errors in the experimental test can also lead to the difference.

Meanwhile, the numerical and experimental results of the clearance size effects obtained in this paper are compared to results from previous literature regarding the clearance effects using theoretical and experimental methods. The previous investigations also showed the same conclusions that clearance had significant effects on mechanical systems. A larger size of clearance will lead to higher vibration peaks and worse performance of the mechanisms. Therefore, the investigation results in this paper are also validated by the previous literature. The fact that clearances have important effects on the slider acceleration supports the idea that clearance effects must be considered in dynamics modeling and analysis of real mechanical systems.

4.2 Driving speed effects

In this section, the effects of the driving speeds on the dynamic characteristics of the slider-crank mechanism with two clearances are presented and discussed. Here, the initial angular velocities of the driving crank are set to 100, 150, 200, and 250 rpm, respectively. The initial clearance size for both joints B and C is 0.5 mm.

From Figure 8, it can be found that both the experimental and simulation results show that the effects of clearance on the accelerations of the slider-crank mechanism are slight when the driving velocity is low. However, as the driving velocity increases, the effects of clearance on the acceleration of the slider-crank mechanism become severe. Therefore, the higher driving speed will lead to a more significant oscillation and a higher amplitude of acceleration. Therefore, clearances have great effects on high-speed mechanisms. Also, the simulation results agree well with experimental tests. Therefore, the clearances in revolute joints must be considered in the precision analysis, design, and control of the mechanism systems, especially for the high-speed mechanisms.

The difference of the response between the experimental and numerical simulation is due to the assumptions that joints flexibility as well as the misalignment between journal and bearing elements were not considered in the numerical simulations, which, however, always exist in actual mechanical systems. Another important reason is the choice of the restitution and friction coefficients in the numerical simulations. In addition, the assembly and measurement errors in the experimental test can also lead to the difference between the simulation and experimental results.

Also, the numerical and experimental results regarding the driving speed effects are compared to previous literatures in which it is shown that higher driving speed caused higher peaks of impact force and vibration of the mechanical system with clearances. Moreover, the performance of the systems with clearances become much worse when the driving speed becomes higher. Therefore, the investigation results in this section are also validated by other published data.

5 CONCLUSIONS

This paper investigates the dynamic characteristics of a multibody system with two revolute clearance joints via computational and experimental methodology. A flexible slider-crank mechanism is used to implement the investigation. Dynamic tests for different crank speeds and different clearance sizes are performed, respectively. The simulation results agree well with the experimental results for different clearance sizes as well as different driving speeds.

(1) The computational method presented in this paper is reasonable, accurate, and efficient for modeling and simulation of mechanical systems with two revolute clearance joints.

(2) The investigation of this paper indicates that clearances have significant effects on the dynamic response of mechanical systems. The clearances lead to obvious oscillation of the dynamic responses and extremely high peaks compared with the mechanism without clearance.

(3) Clearance size and driving speed are two important factors that significantly influence the dynamic response of mechanical systems. Larger clearance size and higher driving speed will cause more severe oscillations of mechanisms.

Clearance deteriorates the motion stability and dynamic performances of mechanical systems. Therefore, the conclusions support that the clearances must be considered in the dynamics modeling, precision analysis, as well as design and control of the mechanical systems. The dynamics analysis of the mechanical system will be much closer to the reality by considering clearances. More experimental tests will be investigated in further research. Our future work will also focus on the nonlinear dynamic characteristics analysis of mechanisms with clearances.

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CONFLICT OF INTEREST
The authors declare that there are no conflict of interest.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES
1. Erkaya S. Effects of joint clearance on motion accuracy of robotic manipulators. Strojniški Vestnik – J Mech Eng. 2018;64(2):82-94.
2. Bai ZF, Zhao Y. Dynamic behaviour analysis of planar mechanical systems with clearance in revolute joints using a new hybrid contact force model. Int J Mech Sci. 2012;54:190-205.
3. Li Y, Wang Z, Wang C, Huang W. Effects of torque spring, CCL and latch mechanism on dynamic response of planar solar arrays with multiple clearance joints. Acta Astronaut. 2017;132:243-255.
4. Zhang X, Zhang R, Wang Q. Comparison and analysis of two Coulomb friction models on the dynamic behavior of slider-crank mechanism with a revolute clearance joint. Appl Math Mech Eng Ed. 2016;39(9):1239-1258.
5. Tian Q, Flores P, Lankarani HM. A comprehensive survey of the analytical, numerical and experimental methodologies for dynamics of multibody mechanical systems with clearance or imperfect joints. Mech Mach Theory. 2018;122:1-57.
6. Qian M, Qin Z, Yan S, Zhang L. A comprehensive method for the contact detection of a translational clearance joint and dynamic response after its application in a crank-slider mechanism. Mech Mach Theory. 2020;145:103717.
7. Guo J, Randall RB, Borghesani P, Smith WA, Haneef MD, Peng Z. A study on the effects of piston secondary motion in conjunction with clearance joints. Mech Mach Theory. 2020;149:103824.
8. Zhang X, Yan S, Wu J, Niu W. Dynamic response and sensitivity analysis for mechanical systems with clearance joints and parameter uncertainties using Chebyshev polynomials method. Mech Syst Signal Process. 2020;138:106596.
9. Wang G, Wang L. Dynamics investigation of spatial parallel mechanism considering rod flexibility and spherical joint clearance. Mech Mach Theory. 2019;137:83-107.
10. Li Y, Wang C, Huang W. Dynamics analysis of planar rigid-flexible coupling deployable solar array system with multiple revolute clearance joints. Mech Syst Signal Process. 2019;117:187-209.
11. Flores P. Parametric study on the dynamic response of planar multibody systems with multiple clearance joints. Nonlin Dyn. 2010;61:633-653.
12. Erkaya S, Uzmay I. Experimental investigation of joint clearance effects on the dynamics of a slider-crank mechanism. Multibody Syst Dyn. 2010;24:81-102.
13. Muvengei O, Kihiu J, Ikua B. Dynamic analysis of planar rigid-body mechanical systems with two-clearance revolute joints. Nonlin Dyn. 2013;73:259-273.
14. Bai ZF, Sun Y. A study on dynamics of planar multibody mechanical systems with multiple revolute clearance joints. Eur J Mech A/Solids. 2016;60:95-111.
15. Wang G, Qi Z, Wang J. A differential approach for modeling revolute clearance joints in planar rigid multibody systems. Multibody Syst Dyn. 2017;39:311-335.
16. Salahshoeh E, Ebrahimi S, Zhang Y. Frequency analysis of a typical planar flexible multibody system with joint clearances. Mech Mach Theory. 2018;126:429-456.
17. Pi T, Zhang Y. Simulation of planar mechanisms with revolute clearance joints using the multipatch based isogeometric analysis. Comput Methods Appl Mech Eng. 2019;343:453-489.
18. Zhan Z, Zhang X, Zhang H, Chen G. Unified motion reliability analysis and comparison study of planar parallel manipulators with interval joint clearance variables. Mech Mach Theory. 2019;138:58-75.
19. Chen X, Jiang S, Wang S, Deng Y. Dynamics analysis of planar multi- DOF mechanism with multiple revolute clearances and chaos identification of revolute clearance joints. Multibody Syst Dyn. 2019;47:317-345.
20. Xiang W, Yan S. Dynamic analysis of space robot manipulator considering clearance joint and parameter uncertainty: modeling, analysis and quantification. Acta Astronaut. 2020;169:158-169.
21. Zhao Q, Guo J, Hong J. Closed-form error space calculation for parallel/hybrid manipulators considering joint clearance, input uncertainty and manufacturing imperfection. Mech Mach Theory. 2019;142:103608.
22. Ordiz M, Cuadrado J, Cabello M, Retolaza I, Martinez F, Dopoco D. Prediction of fatigue life in multibody systems considering the increase of dynamic loads due to wear in clearances. Mech Mach Theory. 2021;160:104293.
23. Lai X, He H, Lai Q, et al. Computational prediction and experimental validation of revolute joint clearance wear in the low-velocity planar mechanism. Mech Sys Signal Process. 2017;85:963-976.
24. Erkaya S. Trajectory optimization of a walking mechanism having revolute joints with clearance using ANFIS approach. Nonlin Dyn. 2013;71:75-91.
25. Zhao B, Zhang ZN, Fang CC, Dai XD, Xie YB. Modeling and analysis of planar multibody system with mixed lubricated revolute joint. Tribol Int. 2016;98:229-241.
26. Daniel GB, Cavalca KL. Analysis of the dynamics of a slider-crank mechanism with hydrodynamic lubrication in the connecting rod slider joint clearance. Mech Mach Theory. 2011;46:1434-1452.
27. Bai ZF, Shi X, Wang PP. Effects of body flexibility on dynamics of mechanism with clearance joint. Lecture Notes Electr Eng. 2017;408:1239-1247.
28. Marques F, Isaac F, Dourado N, Flores P. An enhanced formulation to model spatial revolute joints with radial and axial clearances. Mech Mach Theory. 2017;116:123-144.
29. Yan S, Xiang W, Zhang L. A comprehensive model for 3D revolute joints with clearances in mechanical systems. Nonlin Dyn. 2015;80:309-328.
30. Isaac F, Marques F, Dourado N, Flores P. A finite element model of a 3D dry revolute joint incorporated in a multibody dynamic analysis. Multibody Syst Dyn. 2019;45:293-313.
31. Bai ZF, Jiang X, Li JY, Zhao JJ, Zhao Y. Dynamic analysis of mechanical system considering radial and axial clearances in 3D revolute clearance joints. J Vib Control. 2020.
32. Koshy CS, Flores P, Lankarani HM. Study of the effect of contact force model on the dynamic response of mechanical systems with dry clearance joints-computational and experimental approaches. Nonlin Dyn. 2013;73:325-338.
33. Khemili I, Romdhane L. Dynamic analysis of a flexible slider–crank mechanism with clearance. Eur J Mech A/Solids. 2008;27:882-898.
34. Hunt KH, Crossley FRE. Coefficient of restitution interpreted as damping in vibroimpact. J Appl Mech. 1975;42:440-445.
35. Bai ZF, Zhao Y. A Hybrid contact model of revolute joint with clearance for mechanical systems. Int J Non Linear Mech. 2013;48:15-36.
36. Skrinjar L, Slavič J, Boltíček M. A review of continuous contact-force models in multibody dynamics. Int J Mech Sci. 2018;171-187.
37. Flores P, Ambrosto J, Claro JCP, Lankarani HM. Influence of the contact-impact force model on the dynamic response of multibody systems. Proc Inst Mech Eng Part J Multibody Dyn. 2006;220:21-34.
38. Lankarani HM, Nikravesh PE. A contact force model with hysteresis damping for impact analysis of multibody systems. J Mech Design. 1990;112:369-376.
39. Corral E, Moreno RG, García MJG, Castejón C. Nonlinear phenomena of contact in multibody systems dynamics: a review. *Nonlin Dyn*. 2021;104:1269-1295.

40. Bai ZF, Zhao Y. Dynamics modeling and quantitative analysis of multibody systems including revolute clearance joint. *Precision Eng*. 2012;36:554-567.

41. Bai ZF, Zhao JJ, Chen J, Zhao Y. Design optimization of dual-axis driving mechanism for satellite antenna with two planar revolute clearance joints. *Acta Astronaut*. 2018;144:80-89.

42. Bai ZF, Liu YQ, Sun Y. Investigation on dynamic responses of dual-axis positioning mechanism for satellite antenna considering joint clearance. *J Mech Sci Technol*. 2015;29:453-460.

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