A first order dark $SU(2)_D$ phase transition with vector dark matter in the light of NANOGrav 12.5 yr data

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Abstract. We study a dark $SU(2)_D$ gauge extension of the standard model (SM) with the possibility of a strong first order phase transition (FOPT) taking place below the electroweak scale in the light of NANOGrav 12.5 yr data. As pointed out recently by the NANOGrav collaboration, gravitational waves (GW) from such a FOPT with appropriate strength and nucleation temperature can explain their 12.5 yr data. We impose a classical conformal invariance on the scalar potential of $SU(2)_D$ sector involving only a complex scalar doublet with negligible couplings with the SM Higgs. While a FOPT at sub-GeV temperatures can give rise to stochastic GW around nano-Hz frequencies being in agreement with NANOGrav findings, the $SU(2)_D$ vector bosons which acquire masses as a result of the FOPT in dark sector, can also serve as dark matter (DM) in the universe. The relic abundance of such vector DM can be generated in a non-thermal manner from the SM bath via scalar portal mixing. We also discuss future sensitivity of gravitational wave experiments to the model parameter space.

Keywords: cosmological phase transitions, gravitational waves / theory, dark matter theory

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1 Introduction

The NANOGrav collaboration has recently searched for a gravitational wave (GW) signal produced from a first order phase transition (FOPT) in 45 pulsars dataset spanning over 12.5 yr [1]. According to their analysis, the 12.5 yr data can be interpreted in terms of a FOPT occurring at a temperature below the electroweak (EW) scale. It should however be noted that similar effects at the NANOGrav experiment can also be caused by signals generated by supermassive black hole binary (SMBHB) mergers. In 2020, the NANOGrav collaboration also reported that they have found strong evidence of a stochastic process, modeled as a power-law spectrum having frequencies around the nano-Hz regime, with common amplitude and spectral slope across pulsars, in their 12.5 yr data [2]. Although the statistical significance for a stochastic GW background discovery claim was low, it still led to several interesting new physics explanations like cosmic string origins [3–5], FOPT origins [5–10], inflationary origin [11] etc. While more data are required to settle these issues, the pulsar timing arrays (PTAs) like NANOGrav sensitive to GW of extremely low frequencies do offer a complementary probe of GW background to future space-based interferometers like eLISA [12, 13]. Future data have the potential to probe many of the above-mentioned new physics explanations for such low frequency GW background. As pointed out recently, even different experiments like GAIA and its future upgrades have the potential to probe NANOGrav results [14]. Interestingly, the recent results from the PPTA collaboration, another PTA based GW experiment, have also indicated similar findings consistent with NANOGrav observations [15].

Motivated by the recent results from NANOGrav collaboration explained in terms of a FOPT characterized by the preferred ranges for strength ($\alpha_*$) as well as the phase transition temperature ($T_*$) as shown in [1], we study a simple model to achieve such a strong FOPT below electroweak scale. The idea of such strong FOPT at low scale has been explored within the context of Abelian gauge extended models with or without additional dark matter (DM) fields charged under the same gauge symmetry. More specifically, in our recent work [10], a complex scalar singlet charged under such Abelian gauge symmetry while simultaneously obeying classical conformal invariance led to a strong FOPT at bubble nucleation temperature much below electroweak scale. For earlier works on FOPT within such Abelian gauge extended scenarios, please refer to [16–22] and references therein. In this work, we consider the possibility of realizing such a low scale FOPT within a dark non-Abelian gauge model,
particularly focusing on a $SU(2)_D$ gauge extension. Several earlier works on such dark FOPT with non-Abelian gauge symmetries have already studied the consequences for GW at eLISA type experiments [23–25]. On the other hand, in this work we focus on the possibility of having a FOPT within such non-Abelian dark sectors at sub-EW scale such that the resulting stochastic GWs can have frequencies typically within the NANOGrav or other present as well as future PTA type experiments.

While such dark strong FOPT and resulting GWs have been investigated earlier as well, we study this possibility within a dark non-Abelian gauge sector for the first time after NANOGrav collaboration analysed their 12.5 year data in the context of GWs from the FOPT at a sub-EW scale [1]. Another motivation for such dark gauge symmetries like $U(1)_D$, as discussed in earlier work [10], is that it can also be motivated from DM point of view where a singlet field charged under this gauge symmetry plays the role of DM. On the other hand, a dark non-Abelian gauge symmetry like $SU(2)_D$, as we adopt in this work, naturally contains a DM candidate in the form of the massive vector boson.1 We consider such a minimal scenario of $SU(2)_D$ gauge symmetry with a complex scalar field responsible for symmetry breaking and constrain the parameter space from the requirement of providing one possible explanation of NANOGrav observations along with observed DM relic density from Planck [26]. We also show the future sensitivity of GW experiments [27] like SKA [28], IPTA [29] to the model parameter space.

2 The model

As demonstrated above, we study a $SU(2)_D$ extension of the standard model (SM). The newly introduced field in this model is a complex scalar doublet $\Phi$ required for spontaneous breaking of gauge symmetry. All the SM fields are neutral under this new gauge symmetry. The zero-temperature scalar potential at tree level is given by

$$V_{\text{tree}} = \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger \Phi) (H^\dagger H) + \lambda_3 (\Phi^\dagger \Phi)^2,$$

where $H$ is the SM Higgs doublet. As we impose classical conformal invariance, the scalar potential remains free from bare mass terms. The kinetic terms for the $SU(2)_D$ sector fields can be written as

$$L_{\text{kin}} \supset (D_\mu \Phi)^\dagger (D^\mu \Phi) - \frac{1}{4} (F_D)_{\mu\nu} (F_D)^{\mu\nu},$$

where $F_D$ is the field strength tensor for $SU(2)_D$ and $D_\mu \Phi = (\partial_\mu + ig_D \tau^i \cdot A_D^i / 2) \Phi$ is the covariant derivative. The dark scalar doublet $\Phi$ can be written in component form as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} G_2 + iG_3 \\ M + \phi + iG_1 \end{pmatrix}$$

The vacuum expectation value (VEV) of the dark scalar doublet, $\langle \Phi \rangle = M/\sqrt{2}$, acquired via the running of the quartic coupling $\lambda_3$ breaks the $SU(2)_D$ gauge symmetry leading to a massive gauge boson $M_{Z_D} = g_D M/2$. In order to realize the EW vacuum, the coupling $\lambda_2$ needs to be suppressed. Therefore, in our analysis we neglect the coupling $\lambda_2$. This assumption is for simplicity and also to make sure that the SM Higgs VEV does not affect

1Note that $U(1)_D$ vector boson can also be made a viable DM candidate by tuning the kinetic mixing $U(1)_D$ and $U(1)_Y$ of standard model, making it cosmologically long-lived.
the light singlet scalar mass. As we will see later, this assumption also helps in ensuring non-thermal production of light vector boson DM via scalar portal. Therefore, the suppressed coupling of SU(2) sector particles with the SM guarantees the latter’s contribution to be negligible as to suppress its role in the renormalisation group evolution (RGE) of the singlet scalar quartic coupling.

The total effective potential is schematically composed of the following three terms:

\[ V_{\text{tot}} = V_{\text{tree}} + V_{\text{CW}} + V_T, \]  

(2.4)

where \( V_{\text{tree}} \), \( V_{\text{CW}} \) and \( V_T \) denote the tree level scalar potential, the one-loop Coleman-Weinberg potential, and the thermal effective potential, respectively. In finite-temperature field theory, the effective potential, \( V_{\text{CW}} \) and \( V_T \), are calculated by using the standard background field method [30, 31]. While we assume Landau gauge in our analysis, issues related to gauge dependence in such conformal models can be found in [22]. Denoting the dark scalar doublet as above, the zero temperature effective potential is given as

\[ V_0 = V_{\text{tree}} + V_{\text{CW}}, \]  

(2.5)

where \( V_{\text{tree}} = \lambda_3 \phi^4 / 4 \) and \( V_{\text{CW}} \) is given by

\[ V_{\text{CW}} = \frac{1}{64\pi^2} \sum_i (2s_i + 1)m_i^4(\phi) \left[ \ln \left( \frac{m_i^2(\phi)}{\mu^2} \right) - C_i \right]. \]  

(2.6)

Here \( s_i \) is the spin with the index \( i \) runs over gauge boson and scalars. Their field dependent masses are

\[ m_D^2(\phi) = g_D^2 \phi^2 / 2, \quad m_\phi^2 = 3\lambda_3 \phi^2, \quad m_G^2 = \lambda_3 \phi^2. \]  

(2.7)

In the expression for \( V_{\text{CW}} \), the constant \( C_i = 5/2 \) for gauge bosons and \( C_i = 3/2 \) otherwise. The \( \overline{\text{MS}} \) renormalisation scale is denoted by \( \mu \).

The gauge coupling \( g_D(t) \) and quartic coupling \( \lambda_3(t) \) at renormalisation scale can be calculated by solving the corresponding RGE equations. In terms of \( \alpha_D = g_D^2 / 4\pi \) and \( \alpha_\lambda = \lambda_3 / 4\pi \), the RGEs are

\[ \frac{d\alpha_D(t)}{dt} = \frac{b}{2\pi} \alpha_D^2(t), \]  

(2.8)

\[ \frac{d\alpha_\lambda(t)}{dt} = \frac{1}{2\pi} \left( a_1 \alpha_\lambda^2(t) + 8\pi \alpha_\lambda(t) \gamma(t) + a_3 \alpha_D^2(t) \right), \]  

(2.9)

where \( t = \ln(\phi/\mu) \) and \( \gamma(t) = -a_2 \alpha_D(t)/(8\pi) \). For SU(2) gauge group, \( b = -43/3, \quad a_1 = 24, \quad a_2 = 9/2, \quad \text{and} \quad a_3 = 9/16 \). Taking the renormalisation scale \( \mu \) to be \( M \), the condition \( \frac{dV}{d\phi}|_{\phi=M} = 0 \) leads us to the relation,

\[ a_1 \alpha_\lambda^2(0) + a_3 \alpha_D^2(0) + 8\pi \alpha_\lambda(0) = 0, \]  

(2.10)

from which \( \alpha_\lambda(0) \) is determined in terms of \( \alpha_D(0) \). Using the analytic solutions of the above RGEs, the scalar potential can be written as [16]

\[ V_0(\phi, t) = \frac{\pi \alpha_\lambda(t)}{(1 - \frac{b}{2\pi} \alpha_D(0) t)^{a_2/b} \phi^4} \]  

(2.11)
\[
\alpha_D(t) = \frac{\alpha_D(0)}{1 - \frac{b}{2\pi\alpha_D(0)t}} 
\]

(2.12)

\[
\alpha_\lambda(t) = \frac{a_2 + b}{2a_1} \alpha_D(t) + \frac{A}{a_1} \alpha_D(t) \tan \left[ \frac{A}{b} \ln \left( \frac{\alpha_D(t)}{\pi} \right) + C \right] 
\]

(2.13)

and the coefficient \(C\) is determined by eq. (2.10).

The thermal effective potential \(V_T\) has two parts. Firstly, the usual thermal contributions to the effective potential are given by

\[
V_{\text{th}} = \sum_i \left( \frac{n_{B_i}}{2\pi^2 T^4} J_B \left[ \frac{m_{B_i}}{T} \right] \right), 
\]

(2.14)

where \(T\) is the temperature, and \(n_{B_i}\) denotes the degrees of freedom (dof) of the bosonic particles. In general, there exists a fermionic contribution too, but in our model only bosonic contributions exist. In the above expression, the function \(J_B\) is defined as follows:

\[
J_B(x) = \int_0^\infty dz z^2 \log \left[ 1 - e^{-\sqrt{z^2 + x^2}} \right]. 
\]

(2.15)

While calculating the thermal potential, we also include a contribution from the daisy diagrams, which constitute the second term in \(V_T\). Inclusion of such diagrams improves the perturbative expansion during the phase transition as discussed in earlier works [32–34]. There exist two prescriptions to find the daisy improved effective potential by inserting thermal masses into the zero-temperature field dependent masses. In one of these resummation prescriptions, known as the Parwani method [33], thermal corrected field dependent masses are used. In the other prescription, known as the Arnold-Espinosa method [34], the effect of the daisy diagram is included only for Matsubara zero-modes inside \(J_B\) function defined above. As we ignore the dark scalar doublet coupling to the SM Higgs, we calculate the field dependent and thermal masses as well as the daisy diagram contributions for \(SU(2)_D\) vector bosons only.

As there are two scales of evolution namely, the field \(\phi\) itself and temperature \(T\) of the universe, we consider the renormalisation scale parameter \(u\) instead of \(t\) as

\[
u \equiv \log(\Lambda/M) \quad \text{where} \quad \Lambda \equiv \max(\phi, T),
\]

(2.16)

where \(\Lambda\) represents the typical scale of the theory. Now, the one-loop level effective potential is given as:

\[
V_{\text{tot}}(\phi, T) = V_0(\phi, u) + V_T(\phi, T), 
\]

(2.17)

where

\[
V_T(\phi, T) = V_{\text{th}} + V_{\text{daisy}}(\phi, T), 
\]

(2.18)

\[
V_{\text{daisy}}(\phi, T) = -\sum_i \frac{g_i T}{12\pi} \left[ m_i^2(\phi, T) - m_i^2(\phi) \right], 
\]

wherein, \(V_{\text{th}}\) is the thermal correction and \(V_{\text{daisy}}\) is the daisy subtraction [32–34]. Denoting \(m_i^2(\phi, T) = m_i^2(\phi) + \Pi_i(T)\), the relevant thermal masses can be written as [35]

\[
\Pi_{Z_D} = \frac{5}{6} g_D^2 T^2, \quad \Pi_{\phi, G} = \left( \frac{\lambda_3}{2} + \frac{3}{16} g_D^2 \right) T^2.
\]

(2.19)

The parameter \(g_i = 1, 3, 3\) for \(\phi, Z_D, G\) respectively.
Figure 1. Shape of the potential at critical temperature (red curve) and nucleation temperature (blue curve) for chosen benchmark $\alpha = 0.45, T_* = 1.8 \text{ MeV}, g_D = 1.37, M_{ZD} = 8.23 \text{ MeV}$.

3 First order phase transition

As discussed before, we study the possibility of SU(2)$_D$ phase transition to be of first order. Such transitions proceed via tunnelling with the corresponding spherical symmetric field configurations known as bubbles being nucleated followed by expansion and coalescence. Recent reviews of cosmological phase transitions can be found in [36, 37]. The tunnelling rate per unit time per unit volume is given as [38]

$$\Gamma(T) = \mathcal{A}(T)e^{-S_3(T)/T},$$

(3.1)

where $\mathcal{A}(T) \sim T^4(S_3/(2\pi T))^{3/2}$ and $S_3(T)$ are determined by the dimensional analysis and by the classical configurations, called bounce, respectively. At finite temperature, the $O(3)$ symmetric bounce solution [39] corresponds to the solution of the differential equation given as follows,

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \frac{\partial V_{\text{tot}}}{\partial \phi}.$$  

(3.2)

This equation can be solved by using the boundary conditions given by

$$\phi(r \to \infty) = \phi_{\text{false}}, \quad \left. \frac{d\phi}{dr} \right|_{r=0} = 0,$$

(3.3)

where $\phi_{\text{false}}$ denotes the position of the false vacuum. Using $\phi$ governed by the above equation and boundary conditions, the bounce action can be written as

$$S_3 = \int_0^{\infty} dr 4\pi r^2 \left[ \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V_{\text{tot}}(\phi, T) \right].$$

(3.4)

The temperature corresponding to bubble nucleation is known as the nucleation temperature $T_*$ which is calculated by comparing the tunnelling rate to the Hubble expansion rate as

$$\Gamma(T_*) = H^4(T_*).$$

(3.5)
Figure 2. The key parameters identifying the SU(2)$_D$ phase transition in the $\alpha_*$ – $T_*$ plane where the SU(2)$_D$ gauge coupling $g_D$ is varied in a range corresponding to $\alpha_D \in 0.01 – 0.2$ while $M_{Z_D}$ is shown in colour bar. The solid and dashed contours correspond to 68% and 90% confidence levels respectively, according to the NANOGrav analysis [1] considering envelope approximation (top left panel), semi-analytic approximation (top right panel), and numerical results (bottom panel).

Considering a radiation dominated universe, the Hubble parameter is given by $H(T) \simeq 1.66\sqrt{g_\ast}T^2/M_{Pl}$ with $g_\ast$ being the dof of the radiation component. The critical temperature $T_c$ corresponds to the temperature where the two minima of the potential are degenerate. Now, if we have $\phi(T_c)/T_c > 1$ where $\phi(T_c)$ is the dark scalar VEV at the critical temperature $T = T_c$, the corresponding phase transition is conventionally called a strong first order phase transition (SFOPT). Sometimes, this criteria is also referred to as $\phi(T_\ast)/T_\ast > 1$, where $\phi(T_\ast)$ is the dark scalar VEV at the nucleation temperature, $T = T_\ast$. We consider $\phi(T_c)/T_c > 1$ in our work which usually guarantees the validity of the latter.

The free energy difference between the true and the false vacuum is given by

$$\Delta V_{tot} \equiv V_{tot}(\phi_{false}, T) - V_{tot}(\phi_{true}, T).$$  \hfill (3.6)

As a result of the bubble nucleation, the amount of vacuum energy released by the phase tran-
Figure 3. The SU(2)$_D$ parameter space in the plane of $g_D$ and $M_{Z_D}$ along with the strength of the FOPT $\phi_c/T_c \equiv \phi(T_c)/T_c$ represented in colour bar.

Figure 4. GW spectrum $\Omega h^2(f)$ in terms of frequency $f$ for a strong FOPT with benchmark parameters $\alpha_s = 0.36, T_* = 190 \text{ keV}, g_D = 1.37, M_{Z_D} = 817 \text{ keV}$. The red, orange, cyan and black curves correspond to the individual contribution from turbulence of the plasma, sound wave of the plasma, bubble collisions, and the total contribution respectively.

transition, in the units of radiation energy density of the universe, $\rho_{\text{rad}} = g_* \pi^2 T^4 / 30$, is given by

$$\alpha_s = \frac{\epsilon_*}{\rho_{\text{rad}}},$$

(3.7)

with

$$\epsilon_* = \left[ \Delta V_{\text{tot}} - \frac{T}{4} \frac{\partial \Delta V_{\text{tot}}}{\partial T} \right]_{T=T_*},$$

(3.8)

which is also related to the change in the trace of the energy-momentum tensor across the bubble wall [13, 40].

In figure 1, we show the shape of the potential in terms of $\phi/M$ at critical ($T_c$) and nucleation ($T_*$) temperatures. For illustrative purpose, we choose the relevant benchmark
values as $\alpha_*=0.45, T_* = 1.8\text{ MeV}, g_D = 1.37, \text{ and } M_{ZD} = 8.23\text{ MeV}$. The red coloured curve corresponds to the potential at $T = T_c$, while the one in blue colour corresponds to $T = T_*$. Clearly, $\phi = 0$ becomes a false vacuum below the critical temperature $T_c$. Also, from the shape of the potential at $T_c$, it can be clearly seen that there exists a potential barrier between the two vacua, an indication of a SFOPT. This eventually triggers bubble production and subsequent production of GW.

The SFOPT discussed above gets completed via the percolation of the growing bubbles. In order to determine the epoch of completion of the phase transition, one needs to estimate the percolation temperature $T_p$ at which significant volume of the universe is converted from the symmetric phase (false vacuum) to the broken phase (true vacuum). Adopting the prescription given in [41, 42], the percolation temperature $T_p$ is obtained from the probability of finding a point still in the false vacuum given by

$$P(T) = e^{-I(T)},$$

where

$$I(T) = \frac{4\pi}{3} \int_T^{T_c} dT' \Gamma(T') \left( \int_T^{T'} d\bar{T} \frac{d\bar{T}}{H(\bar{T})} \right)^3.$$  \tag{3.9}

The percolation temperature is then calculated by using $I(T_p) = 0.34$ [41] (implying that at least 34% of the comoving volume is occupied by the true vacuum). It is important to ensure that the physical volume of the false vacuum gets decreased around percolation for successful completion of the phase transition. This requirement gives rise to the following condition [41, 42]

$$\frac{1}{\mathcal{V}_{\text{false}}} \frac{d\mathcal{V}_{\text{false}}}{dx} = H(T) \left( 3 + T \frac{dI(T)}{dT} \right) < 0; \quad x \equiv \text{time} \tag{3.10}$$

ensuring which, at the percolation temperature $T_p$, can confirm the successful completion of the phase transition. For some chosen benchmark values, including the one shown in figure 1, we have confirmed the validity of the above condition at the percolation temperature $T_p$, as summarised in table 1.

4 Gravitational wave

A SFOPT can lead to the generation of stochastic GW background primarily due to three mechanisms namely, the bubble collisions [43–47], the sound wave of the plasma [48–51] and the turbulence of the plasma [52–57].

The amplitudes of such GW signal depend upon the amount of vacuum energy released by the phase transition in comparison to the radiation energy density of the universe, $\rho_{\text{rad}} = g_* \pi^2 T^4/30$, given by $\alpha_*$ defined in eq. (3.7). The amplitude of GW also depends upon the duration of the FOPT, denoted by the parameter $\beta$, defined as [12]

$$\frac{\beta}{H(T)} \simeq T \frac{d}{dT} \left( \frac{S_3}{T} \right). \tag{4.1}$$

Here, $\alpha_*$ and $\beta/H(T)$ are evaluated at the nucleation temperature $T = T_*$. While $S_3$ can be evaluated using eq. (3.4), the effective potential at sufficiently low temperatures i.e., $T \ll M,$
can be safely approximated as

\[ V_{\text{tot}} \simeq \frac{g_D^2(t')}{2} T^2 \phi^2 + \frac{\lambda_{\text{eff}}(t')}{4} \phi^4 , \]

with \( \lambda_{\text{eff}}(t') = \frac{4\pi\alpha(t')}{{\left(1 - \frac{b}{2\pi\alpha D(0)t'}\right)^{a_2/b}}^4} \),

\[ t' = \ln(T/M) . \]

With such approximation, the action can be written as [16, 38]

\[ S = \frac{S_3}{T} - 4 \ln(T/M) , \]

\[ \frac{S_3}{T} \simeq -9.45 \times \frac{g_D(t')}{\lambda_{\text{eff}}(t')} . \]

While estimating the GW amplitude we have used the expressions given by eq. (4.2) and eq. (4.3) to \( \alpha, \beta \) and the percolation temperature \( T_p \). We have cross-checked the validity of the approximated analytical expression for \( S_3/T \) mentioned above for the benchmark points mentioned in table 1 using numerical packages SimpleBounce [58] and BubbleProfiler [59] and found that the results from numerical analysis and those from the approximated expression differ by up to 10 %.

The NANOGrav collaboration, in their analysis [1], has estimated the required FOPT parameters using thin shell approximation for bubble walls (envelope approximation) [60], semi-analytic approximation [61] as well as full lattice results. Here, we present the predictions of our model (coloured points) against the backdrop of their estimates in figure 2. In the analysis, the gauge coupling \( g_D \) is varied in a range corresponding to \( 0.01 \lesssim \alpha_D \lesssim 0.2 \), while the gauge boson masses are shown in the colour bar. The solid (dashed) contour corresponds to the allowed region at 68(95)% confidence level obtained in [1] by using envelope approximation (top left panel), semi-analytic approximation (top right panel), and numerical results (bottom panel). Clearly, predictions based on light gauge boson with masses in
The numerical values of benchmark parameters used in the estimation of GW spectrum are given in Table 1.

| $\alpha_s$ | $(\beta/H_*)$ | $T_s$ | $v_w$ | $T_p$ | $\frac{1}{V_{\text{false}}}$ $\frac{\partial V_{\text{false}}}{dx}$ |
|------------|----------------|------|------|------|---------------------------------|
| 0.45       | 143            | 1.8 MeV | 0.887 | 8.23 MeV | $-1.3 \times 10^{-19}$ GeV |
| 0.36       | 151            | 190 keV | 0.872 | 817 keV | $-6.86 \times 10^{-18}$ GeV |

Table 1. Numerical values of benchmark parameters used in the estimation of GW spectrum.

In general, each contribution can be characterised by its own peak frequency and each GW spectrum can be parametrised, following [1], as

$$h^2 \Omega(f) = R \Delta(v_w) \left( \frac{\kappa \alpha_s}{1 + \alpha_s} \right)^p \left( \frac{H_*}{\beta} \right)^q S(f/f_0^2),$$

(4.5)

where the pre-factor $R \approx 7.69 \times 10^{-5} g_s^{-1/3}$ takes in account the red-shift of the GW energy density, $S(f/f_0^2)$ parametrises the shape of the spectrum and $\Delta(v_w)$ is the normalisation factor which depends on the bubble wall velocity $v_w$. The Hubble parameter at the nucleation temperature $T = T_*$ is denoted by $H_*$. Finally the peak frequency today, $f_0^2$, is related to the value of the peak frequency at the time of emission, $f_*$, as follows

$$f_0^2 \simeq 1.13 \times 10^{-10} \text{Hz} \left( \frac{f_*}{\text{Hz}} \right) \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{\text{MeV}} \right) \left( \frac{g_*}{10} \right)^{1/6},$$

(4.6)

where $g_*$ denotes the number of relativistic dof at the time of the FOPT. The values of the peak frequency at the time of emission, the normalisation factor, the spectral shape, and the exponents $p$ and $q$ are given in Table 1 of [1]. The efficiency factors namely, $\kappa_{\phi}$ is discussed in [16, 42] and $\kappa_{\text{sw}}$ is taken from [40, 63].

Using the above-mentioned prescription for estimating GW spectrum from a strong FOPT and by choosing a benchmark values of model as well as FOPT parameters shown in Table 1 consistent with NANOGrav data at 95% CL, we calculate the individual contributions to GW energy density spectrum $\Omega h^2(f)$ from bubble collisions, sound wave of the plasma, and turbulence of the plasma as well as the total contribution to $\Omega h^2(f)$. In figure 4, the red, orange, cyan and black curves correspond to the individual contribution from turbulence of the plasma, sound wave of the plasma, bubble collisions, and the total contribution to $\Omega h^2(f)$, respectively. Due to the small value of FOPT strength parameter $\alpha_s$, as anticipated from earlier studies [70, 71], the contribution from bubble collision remains suppressed compared to the other two contributions as can be seen in figure 4. In figure 5, we show the GW...
Figure 6. Left panel: comoving DM number density $Y$ as a function of temperature $T$ for DM mass $M_{Z_D} = 8.23 \text{ MeV}$ and $\lambda_2 = 1.90 \times 10^{-6}$. Right panel: the regions of $M_{Z_D} - \lambda_2$ parameter space giving rise to DM relic (shown in colour bar) for $g_D = 1.37$.

5 Dark matter

Origin of particle DM has been a longstanding puzzle. Ordinary or visible matter contributes only one-fifth ($\sim 20\%$) to the total matter content of the present universe. The rest of the matter remain in the form of a mysterious, non-luminous, non-baryonic component, often referred to as DM. While there have been astrophysical evidences for such non-baryonic matter for several decades [75–77], precision measurements of the cosmic microwave background (CMB) anisotropies at cosmology experiments like WMAP, Planck have confirmed its existence in a convincing way. The present abundance of DM is often quantified in terms of a dimensionless quantity as [26]:

$$\Omega_{DM} h^2 = 0.120 \pm 0.001 \quad (5.1)$$

at 68% confidence level (CL). Here $\Omega_{DM} = \rho_{DM}/\rho_{\text{critical}}$ is the density parameter of DM and $h = \text{Hubble Parameter}/(100 \text{ km s}^{-1}\text{Mpc}^{-1})$ is a dimensionless parameter of order unity. $\rho_{\text{critical}} = 3H^2/(8\pi G)$ is the critical density while $H$ is the Hubble parameter. Among different particle DM proposals in the literature, the weakly interacting massive particle (WIMP) paradigm is the most appealing one. In such a scenario, a stable or sufficiently long-lived DM particle having mass and interaction strength typically around the electroweak corner can get produced thermally from the SM bath in early universe, followed by freeze-out from the bath, leaving a relic similar to the observed DM abundance [78]. Apart from this remarkable coincidence referred to as the WIMP Miracle, such DM can be probed at
direct detection experiments by virtue of their sizeable interactions with SM particles like quarks [79]. However, due to absence of any such signals, alternatives to WIMP have also been discussed in recent times. One such appealing alternative is the non-thermal origin of DM, known as the feebly interacting (or freeze-in) massive particle (FIMP) DM [80, 81]. In such a scenario, DM has so feeble interactions with the SM particles that it never reaches thermal equilibrium, but gets produced non-thermally due to decay or scattering of SM bath particles.

As mentioned before, here we consider the SU(2)\text{D} vector bosons as DM candidates. This has been explored in several earlier works [24, 82–90]. While most of these works considered thermal vector boson DM, the non-thermal or FIMP possibility was discussed in [87]. The scenario in our present model is much more simpler as SM-DM interactions occur only via the Higgs portal. As we had assumed tiny Higgs portal coupling $\lambda_2$ between dark scalar $\Phi$ and SM Higgs doublet $H$ while discussing the FOPT details, it naturally provides the freeze-in portal. Additionally, constraints from CMB measurements disfavour light sub-GeV thermal DM production in the early universe through s-channel annihilations into SM fermions [26]. Therefore, we stick to the non-thermal DM scenario here.

In general, the Boltzmann equation for comoving DM density in FIMP scenarios can be written as

$$\frac{dY}{dx'} = \frac{\langle \sigma v \rangle s}{H x} (Y_{eq}^{SM})^2$$

with $x' = M_{DM}/T$ and $s = \frac{2\pi^2}{45} g_s T^3$ being the entropy density. In the above equation, we consider the freeze-in DM production from SM bath via scatterings of the type SM SM $\rightarrow$ DM DM with thermal averaged cross-section denoted by $\langle \sigma v \rangle$. Such a scattering is mediated by scalars via $H - \Phi$ mixing. In figure 6 (left panel), we show the evolution of comoving DM density $Y$ as a function of temperature for a benchmark choice of DM mass $M_{DM} = M_{Z_D} = 8.23$ MeV, $\lambda_2 = 1.99 \times 10^{-9}$, and $g_D = 1.37$. While dark gauge coupling is of order one, the smallness of $\lambda_2$ leads to a tiny scalar portal mixing required for realising FIMP scenario. It should be noted that the DM abundance rises sharply around a temperature close to its mass. This is because the scalar portal mixing arises dynamically only after the SU(2)\text{D} symmetry breaking at a scale $\sim M_{Z_D}/g_D$. On the right panel of figure 6, we show the parameter space in $\lambda_2 - M_{Z_D}$ plane with DM relic as shown in the colour bar. The parameter space consistent with correct DM relic abundance shows a linear relation between $\lambda_2$ and $M_{Z_D}$. This can be understood as follows. Since $g_D$ as well as $\Phi$ quartic coupling $\lambda_3$ are fixed, larger $M_{Z_D}$ implies larger VEV of $\Phi$ and hence larger scalar mass mediating SM-DM interactions. To compensate for this larger mediator mass, a larger mixing (and hence a larger $\lambda_2$) is required to generate the correct DM relic. The sharp discontinuity after DM mass crosses muon mass threshold arises as it corresponds to the FOPT occurring at temperatures above the muon mass threshold, allowing muon in the radiation bath to also contribute enhancing the contribution to DM freeze-in.

6 Conclusion

We have studied a minimal SU(2)\text{D} gauge extension of the SM with the possibility of a strong first order phase transition within the dark sector, specially at a low temperature below the EW scale such that resulting stochastic GWs can be observed at PTA based experiments like NANOGrav. Motivated by the recent results from the NANOGrav 12.5 yr data providing hints of such a cosmological phase transition at sub-EW scale, we constrain our model parameters from the requirement of fitting NANOGrav data. A SFOPT occurring
at sub-GeV scale can explain the NANOGrav data very well while also being sensitive to future experiments to be operating in this low frequency regime of GWs. The non-Abelian nature of our dark gauge symmetry also provides a natural vector boson DM candidate which is naturally stable due to the absence of any kinetic mixing with $U(1)_Y$ of the standard model at renormalisable level. Such light vector boson DM can be produced via freeze-in mechanism in the early universe. While such freeze-in mechanism is naturally realised due to tiny scalar portal couplings required to realise dark SFOPT without disturbing the EW vacuum and vice versa, it also helps in avoiding stringent CMB bounds on such light DM, if its production happens thermally. Depending upon the size of scalar portal couplings, we can have more non-trivial FOPT as well DM phenomenology which we leave for future studies.

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References

[1] NANOGrav collaboration, Searching for gravitational waves from cosmological phase transitions with the NANOGrav 12.5-year dataset, arXiv:2104.13930 [INSPIRE].

[2] NANOGrav collaboration, The NANOGrav 12.5 yr data set: search for an isotropic stochastic gravitational-wave background, Astrophys. J. Lett. 905 (2020) L34 [arXiv:2009.04496] [INSPIRE].

[3] S. Blasi, V. Brdar and K. Schmitz, Has NANOGrav found first evidence for cosmic strings?, Phys. Rev. Lett. 126 (2021) 041305 [arXiv:2009.06607] [INSPIRE].

[4] J. Ellis and M. Lewicki, Cosmic string interpretation of NANOGrav pulsar timing data, Phys. Rev. Lett. 126 (2021) 041304 [arXiv:2009.06555] [INSPIRE].

[5] L. Bian, R.-G. Cai, J. Liu, X.-Y. Yang and R. Zhou, Evidence for different gravitational-wave sources in the NANOGrav dataset, Phys. Rev. D 103 (2021) L081301 [arXiv:2009.13893] [INSPIRE].

[6] W. Ratzinger and P. Schwaller, Whispers from the dark side: confronting light new physics with NANOGrav data, SciPost Phys. 10 (2021) 047 [arXiv:2009.11875] [INSPIRE].

[7] A. Addazi, Y.-F. Cai, Q. Gan, A. Marciano and K. Zeng, NANOGrav results and dark first order phase transitions, Sci. China Phys. Mech. Astron. 64 (2021) 290411 [arXiv:2009.10327] [INSPIRE].

[8] Y. Nakai, M. Suzuki, F. Takahashi and M. Yamada, Gravitational waves and dark radiation from dark phase transition: connecting NANOGrav pulsar timing data and Hubble tension, Phys. Lett. B 816 (2021) 136238 [arXiv:2009.09754] [INSPIRE].

[9] R. Zhou, L. Bian and J. Shu, Probing new physics for $(g - 2)_\mu$ and gravitational waves, arXiv:2104.03519 [INSPIRE].

[10] D. Borah, A. Dasgupta and S.K. Kang, Gravitational waves from a dark $U(1)_D$ phase transition in light of NANOGrav 12.5 yr data, Phys. Rev. D 104 (2021) 063501 [arXiv:2105.01007] [INSPIRE].

[11] S. Vagnozzi, Implications of the NANOGrav results for inflation, Mon. Not. Roy. Astron. Soc. 502 (2021) L11 [arXiv:2009.13432] [INSPIRE].
[12] C. Caprini et al., Science with the space-based interferometer eLISA. Part II. Gravitational waves from cosmological phase transitions, JCAP 04 (2016) 001 [arXiv:1512.06239] [inSPIRE].

[13] C. Caprini et al., Detecting gravitational waves from cosmological phase transitions with LISA: an update, JCAP 03 (2020) 024 [arXiv:1910.13125] [inSPIRE].

[14] J. García-Bellido, H. Murayama and G. White, Exploring the early universe with Gaia and Thea, JCAP 12 (2021) 023 [arXiv:2104.04778] [inSPIRE].

[15] B. Goncharov et al., On the evidence for a common-spectrum process in the search for the nanohertz gravitational-wave background with the Parkes Pulsar Timing Array, arXiv:2107.12112 [inSPIRE].

[16] R. Jinno and M. Takimoto, Probing a classically conformal B-L model with gravitational waves, Phys. Rev. D 95 (2017) 015020 [arXiv:1604.05035] [inSPIRE].

[17] A. Mohamadnejad, Gravitational waves from scale-invariant vector dark matter model: probing below the neutrino-floor, Eur. Phys. J. C 80 (2020) 197 [arXiv:1907.08899] [inSPIRE].

[18] Y.G. Kim, K.Y. Lee and S.-H. Nam, Conformal invariance and singlet fermionic dark matter, Phys. Rev. D 100 (2019) 075038 [arXiv:1906.03390] [inSPIRE].

[19] T. Hasegawa, N. Okada and O. Seto, Gravitational waves from the minimal gauged $U(1)_{B-L}$ model, Phys. Rev. D 99 (2019) 095039 [arXiv:1904.03020] [inSPIRE].

[20] C. Marzo, L. Marzola and V. Vaskonen, Phase transition and vacuum stability in the classically conformal B − L model, Eur. Phys. J. C 79 (2019) 601 [arXiv:1811.11169] [inSPIRE].

[21] K. Hashino, M. Kakizaki, S. Kanemura, P. Ko and T. Matsui, Gravitational waves from first order electroweak phase transition in models with the $U(1)_X$ gauge symmetry, JHEP 06 (2018) 088 [arXiv:1802.02947] [inSPIRE].

[22] C.-W. Chiang and E. Senaha, On gauge dependence of gravitational waves from a first-order phase transition in classical scale-invariant $U(1)'$ models, Phys. Lett. B 774 (2017) 489 [arXiv:1707.06765] [inSPIRE].

[23] P. Schwaller, Gravitational waves from a dark phase transition, Phys. Rev. Lett. 115 (2015) 181101 [arXiv:1504.07263] [inSPIRE].

[24] I. Baldes and C. Garcia-Cely, Strong gravitational radiation from a simple dark matter model, JHEP 05 (2010) 190 [arXiv:1009.01198] [inSPIRE].

[25] T. Prokopec, J. Rezaeez and B. Świężewska, Gravitational waves from conformal symmetry breaking, JCAP 02 (2019) 009 [arXiv:1809.11129] [inSPIRE].

[26] PLANCK collaboration, Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6 [Erratum Ibid. 652 (2021) C4] [arXiv:1807.06209] [inSPIRE].

[27] K. Schmitz, New sensitivity curves for gravitational-wave signals from cosmological phase transitions, JHEP 01 (2021) 097 [arXiv:2002.04615] [inSPIRE].

[28] A. Weltman et al., Fundamental physics with the Square Kilometre Array, Publ. Astron. Soc. Austral. 37 (2020) e002 [arXiv:1810.02680] [inSPIRE].

[29] G. Hobbs et al., The international pulsar timing array project: using pulsars as a gravitational wave detector, Class. Quant. Grav. 27 (2010) 084013 [arXiv:0911.5206] [inSPIRE].

[30] L. Dolan and R. Jackiw, Symmetry behavior at finite temperature, Phys. Rev. D 9 (1974) 3320 [inSPIRE].

[31] M. Quirós, Finite temperature field theory and phase transitions, in ICTP summer school in high-energy physics and cosmology, (1999), pg. 187 [hep-ph/9901312] [inSPIRE].

[32] P. Fendley, The effective potential and the coupling constant at high temperature, Phys. Lett. B 196 (1987) 175 [inSPIRE].
[33] R.R. Parwani, *Resummation in a hot scalar field theory*, Phys. Rev. D **45** (1992) 4695 [Erratum ibid. **48** (1993) 5965] [hep-ph/9204216] [inSPIRE].

[34] P.B. Arnold and O. Espinosa, *The effective potential and first order phase transitions: beyond leading-order*, Phys. Rev. D **47** (1993) 3546 [Erratum ibid. **50** (1994) 6662] [hep-ph/9212235] [inSPIRE].

[35] J.M. Cline, M. Jarvinen and F. Sannino, *The electroweak phase transition in nearly conformal technicolor*, Phys. Rev. D **78** (2008) 075027 [arXiv:0808.1512] [inSPIRE].

[36] A. Mazumdar and G. White, *Review of cosmic phase transitions: their significance and experimental signatures*, Rept. Prog. Phys. **82** (2019) 076901 [arXiv:1811.01948] [inSPIRE].

[37] M.B. Hindmarsh, M. Lüben, J. Lumma and M. Pauly, *Phase transitions in the early universe*, SciPost Phys. Lect. Notes **24** (2021) 1 [arXiv:2008.09136] [inSPIRE].

[38] A.D. Linde, *Decay of the false vacuum at finite temperature*, Nucl. Phys. B **216** (1983) 421 [Erratum ibid. **223** (1983) 544] [inSPIRE].

[39] A.D. Linde, *Fate of the false vacuum at finite temperature: theory and applications*, Phys. Lett. B **100** (1981) 37 [inSPIRE].

[40] D. Borah, A. Dasgupta, K. Fujikura, S.K. Kang and D. Mahanta, *Observable gravitational waves in minimal scotogenic model*, JCAP **08** (2020) 046 [arXiv:2003.02276] [inSPIRE].

[41] J. Ellis, M. Lewicki and J.M. No, *On the maximal strength of a first-order electroweak phase transition and its gravitational wave signal*, JCAP **04** (2019) 003 [arXiv:1809.08242] [inSPIRE].

[42] J. Ellis, M. Lewicki and V. Vaskonen, *Updated predictions for gravitational waves produced in a strongly supercooled phase transition*, JCAP **11** (2020) 020 [arXiv:2007.15586] [inSPIRE].

[43] M.S. Turner and F. Wilczek, *Relic gravitational waves and extended inflation*, Phys. Rev. Lett. **65** (1990) 3080 [inSPIRE].

[44] A. Kosowsky, M.S. Turner and R. Watkins, *Gravitational radiation from colliding vacuum bubbles*, Phys. Rev. D **45** (1992) 4514 [inSPIRE].

[45] A. Kosowsky and M.S. Turner, *Gravitational waves from first order cosmological phase transitions*, Phys. Rev. Lett. **69** (1992) 2026 [inSPIRE].

[46] A. Kosowsky and M.S. Turner, *Gravitational radiation from colliding vacuum bubbles: envelope approximation to many bubble collisions*, Phys. Rev. D **47** (1993) 4372 [astro-ph/9211004] [inSPIRE].

[47] M.S. Turner, E.J. Weinberg and L.M. Widrow, *Bubble nucleation in first order inflation and other cosmological phase transitions*, Phys. Rev. D **46** (1992) 2384 [inSPIRE].

[48] M. Hindmarsh, S.J. Huber, K. Rummukainen and D.J. Weir, *Gravitational waves from the sound of a first order phase transition*, Phys. Rev. Lett. **112** (2014) 041301 [arXiv:1304.2433] [inSPIRE].

[49] J.T. Giblin and J.B. Mertens, *Gravitational radiation from first-order phase transitions in the presence of a fluid*, Phys. Rev. D **90** (2014) 023532 [arXiv:1405.4005] [inSPIRE].

[50] M. Hindmarsh, S.J. Huber, K. Rummukainen and D.J. Weir, *Numerical simulations of acoustically generated gravitational waves at a first order phase transition*, Phys. Rev. D **92** (2015) 123009 [arXiv:1504.03291] [inSPIRE].

[51] M. Hindmarsh, S.J. Huber, K. Rummukainen and D.J. Weir, *Shape of the acoustic gravitational wave power spectrum from a first order phase transition*, Phys. Rev. D **96** (2017) 103520 [Erratum ibid. **101** (2020) 089902] [arXiv:1704.05871] [inSPIRE].

[52] M. Kamionkowski, A. Kosowsky and M.S. Turner, *Gravitational radiation from first order phase transitions*, Phys. Rev. D **49** (1994) 2837 [astro-ph/9310044] [inSPIRE].
[53] A. Kosowsky, A. Mack and T. Kahniashvili, *Gravitational radiation from cosmological turbulence*, Phys. Rev. D **66** (2002) 024030 [arXiv:astro-ph/0111483] [SPIRE].

[54] C. Caprini and R. Durrer, *Gravitational waves from stochastic relativistic sources: primordial turbulence and magnetic fields*, Phys. Rev. D **74** (2006) 063521 [arXiv:astro-ph/0603476] [SPIRE].

[55] G. Gogoberidze, T. Kahniashvili and A. Kosowsky, *The spectrum of gravitational radiation from primordial turbulence*, Phys. Rev. D **76** (2007) 083002 [arXiv:0705.1733] [SPIRE].

[56] C. Caprini, R. Durrer and G. Servant, *The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition*, JCAP **12** (2009) 024 [arXiv:0909.0622] [SPIRE].

[57] P. Niksa, M. Schlederer and G. Sigl, *Gravitational waves produced by compressible MHD turbulence from cosmological phase transitions*, Class. Quant. Grav. **35** (2018) 144001 [arXiv:1803.02271] [SPIRE].

[58] R. Sato, *SimpleBounce: a simple package for the false vacuum decay*, Comput. Phys. Commun. **258** (2021) 107566 [arXiv:1908.10868] [SPIRE].

[59] P. Athron, C. Balázs, M. Bardsley, A. Fowlie, D. Harries and G. White, *BubbleProfiler: finding the field profile and action for cosmological phase transitions*, Comput. Phys. Commun. **244** (2019) 448 [arXiv:1901.03714] [SPIRE].

[60] R. Jinno and M. Takimoto, *Gravitational waves from bubble collisions: an analytic derivation*, Phys. Rev. D **95** (2017) 024009 [arXiv:1605.01403] [SPIRE].

[61] M. Lewicki and V. Vaskonen, *Gravitational waves from colliding vacuum bubbles in gauge theories*, Eur. Phys. J. C **81** (2021) 437 [arXiv:2012.07826] [SPIRE].

[62] P. Binetruy, A. Bohe, C. Caprini and J.-F. Dufaux, *Cosmological backgrounds of gravitational waves and eLISA/NGO: phase transitions, cosmic strings and other sources*, JCAP **06** (2012) 027 [arXiv:1201.0983] [SPIRE].

[63] J.R. Espinosa, T. Konstandin, J.M. No and G. Servant, *Energy budget of cosmological first-order phase transitions*, JCAP **06** (2010) 028 [arXiv:1004.4187] [SPIRE].

[64] P.J. Steinhardt, *Relativistic detonation waves and bubble growth in false vacuum decay*, Phys. Rev. D **25** (1982) 2074 [SPIRE].

[65] S.J. Huber and M. Sopena, *An efficient approach to electroweak bubble velocities*, arXiv:1302.1044 [SPIRE].

[66] L. Leitao and A. Megevand, *Hydrodynamics of phase transition fronts and the speed of sound in the plasma*, Nucl. Phys. B **891** (2015) 159 [arXiv:1410.3875] [SPIRE].

[67] G.C. Dorsch, S.J. Huber and T. Konstandin, *Bubble wall velocities in the Standard Model and beyond*, JCAP **12** (2018) 034 [arXiv:1809.04907] [SPIRE].

[68] J.M. Cline and K. Kainulainen, *Electroweak baryogenesis at high bubble wall velocities*, Phys. Rev. D **101** (2020) 063525 [arXiv:2001.00568] [SPIRE].

[69] A. Azatov and M. Vanvlasselaer, *Bubble wall velocity: heavy physics effects*, JCAP **01** (2021) 058 [arXiv:2010.02590] [SPIRE].

[70] D. Bödeker and G.D. Moore, *Electroweak bubble wall speed limit*, JCAP **05** (2017) 025 [arXiv:1703.08215] [SPIRE].

[71] J. Ellis, M. Lewicki, J.M. No and V. Vaskonen, *Gravitational wave energy budget in strongly supercooled phase transitions*, JCAP **06** (2019) 024 [arXiv:1903.09642] [SPIRE].

[72] M.A. McLaughlin, *The North American Nanohertz observatory for gravitational waves*, Class. Quant. Grav. **30** (2013) 224008 [arXiv:1310.0758] [SPIRE].
[73] R.N. Manchester et al., The Parkes Pulsar Timing Array project, *Publ. Astron. Soc. Austral.* **30** (2013) 17 [arXiv:1210.6130] [SPIRE].
[74] M. Krämer and D.J. Champion, The European Pulsar Timing Array and the Large European Array for Pulsars, *Class. Quant. Grav.* **30** (2013) 224009 [SPIRE].
[75] F. Zwicky, *Die Rotverschiebung von extragalaktischen Nebeln* (in German), *Helv. Phys. Acta* **6** (1933) 110 [Gen. Rel. Grav. **41** (2009) 207] [SPIRE].
[76] M. Krämer and D.J. Champion, The European Pulsar Timing Array and the Large European Array for Pulsars, *Class. Quant. Grav.* **30** (2013) 224009 [SPIRE].
[77] F. Zwicky, *Die Rotverschiebung von extragalaktischen Nebeln* (in German), *Helv. Phys. Acta* **6** (1933) 110 [Gen. Rel. Grav. **41** (2009) 207] [SPIRE].
[78] V.C. Rubin and W.K. Ford, Jr., Rotation of the Andromeda nebula from a spectroscopic survey of emission regions, *Astrophys. J.* **159** (1970) 379 [SPIRE].
[79] D. Clowe, M. Bradac, A.H. Gonzalez, M. Markevitch, S.W. Randall, C. Jones et al., A direct empirical proof of the existence of dark matter, *Astrophys. J. Lett.* **648** (2006) L109 [astro-ph/0608407] [SPIRE].
[80] E.W. Kolb and M.S. Turner, *The early universe*, *Front. Phys.* **69** (1990) 1 [SPIRE].
[81] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre et al., The waning of the WIMP? A review of models, searches, and constraints, *Eur. Phys. J. C* **78** (2018) 203 [arXiv:1703.07364] [SPIRE].
[82] L.J. Hall, K. Jedamzik, J. March-Russell and S.M. West, Freeze-in production of FIMP dark matter, *JHEP* **03** (2010) 080 [arXiv:0911.1120] [SPIRE].
[83] B. Barman, S. Bhattacharya, S.K. Patra and J. Chakrabortty, Non-Abelian vector boson dark matter, its unified route and signatures at the LHC, *JCAP* **12** (2017) 021 [arXiv:1704.04945] [SPIRE].
[84] B. Barman, S. Bhattacharya and M. Zakeri, Multipartite dark matter in SU(2)N extension of Standard Model and signatures at the LHC, *JCAP* **09** (2018) 023 [arXiv:1806.01129] [SPIRE].
[85] B. Barman, S. Bhattacharya and M. Zakeri, Non-Abelian vector boson as FIMP dark matter, *JCAP* **02** (2020) 029 [arXiv:1905.07236] [SPIRE].
[86] T. Abe, M. Fujiwara, J. Hisano and K. Matsushita, A model of electroweakly interacting non-Abelian vector dark matter, *JHEP* **07** (2020) 136 [arXiv:2004.00884] [SPIRE].
[87] T. Nomura, H. Okada and S. Yun, Vector dark matter from a gauged SU(2) symmetry, *JHEP* **06** (2021) 122 [arXiv:2012.11377] [SPIRE].
[88] T.A. Chowdhury and S. Saad, Non-Abelian vector dark matter and lepton g – 2, *JCAP* **10** (2021) 014 [arXiv:2107.11863] [SPIRE].