Dynamical control of matter-wave tunneling in periodic potentials

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We report on measurements of dynamical suppression of inter-well tunneling of a Bose-Einstein condensate (BEC) in a strongly driven optical lattice. The strong driving is a sinusoidal shaking of the lattice corresponding to a time-varying linear potential, and the tunneling is measured by letting the BEC freely expand in the lattice. The measured tunneling rate is reduced and, for certain values of the shaking parameter, completely suppressed. Our results are in excellent agreement with theoretical predictions. Furthermore, we have verified that in general the strong shaking does not destroy the phase coherence of the BEC, opening up the possibility of realizing quantum phase transitions by using the shaking strength as the control parameter.

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Quantum tunneling of particles between potential wells connected by a barrier is a fundamental physical effect. While typically quantum systems decay faster when they are perturbed, if the wells are periodically shaken back and forth (or a time-varying potential is applied in a different way), the tunneling rate can actually be reduced and, for certain shaking strengths, even completely suppressed [1,2].

Modifications of the dynamics of quantum systems by applying periodic potentials have been investigated in a number of contexts including the renormalization of Landé g-factors in atoms [3], the micromotion of a single trapped ion [4] and the motion of electrons in semiconductor superlattices [5]. In particular, theoretical studies of double-well systems and of periodic potentials have led to the closely related concepts of coherent destruction of tunneling and dynamical localization [1,6]. In the latter, tunneling between the sites of a periodic array is inhibited by applying a periodically varying potential, e.g. by shaking the array back and forth (see Fig. 1), and as a consequence the tunneling parameter \( J \) representing the gain in kinetic energy in a tunneling event is replaced by \(|J_{\text{eff}}| < |J|\). In a number of experiments signatures of this tunneling suppression have been observed [6,7,8], and recently dynamical localization and coherent suppression of tunneling have been demonstrated using light propagating in coupled waveguide arrays [9,10]. Also, the predictions of the Bose-Hubbard model in a moving frame were recently tested [11]. So far, however, an exact experimental realization of the intrinsically nonlinear Bose-Hubbard model driven by a time-periodic potential has not been reported.

In this Letter, we report on the observation of the dynamical tunneling suppression predicted in refs. [2,12] using Bose-Einstein condensates (BECs) in strongly driven periodic optical potentials [13]. In contrast to other systems, the characteristics of such optical lattices - potential depth, lattice spacing, driving strength and frequency - can be freely chosen and allow us to control the tunneling over a wide range of parameters. In this way we were able to experimentally confirm theoretical predictions with great accuracy. Also, our system allows us to observe the effects of the shaking both by monitor-

![Figure 1: Suppression of tunneling by strong driving.](image)

The dynamics of a Bose-Einstein condensate in a periodic potential is governed by the tunneling matrix element \( J \) and the on-site interaction energy \( U \). If the potential is strongly shaken, tunneling between the wells is dynamically suppressed, leading to a renormalized tunneling matrix element \( J_{\text{eff}} \) below but leaving the interaction energy \( U \) unaffected.

Furthermore, BECs have an intrinsic nonlinear on-site interaction energy (represented by \( U \) in Fig. 1), the interplay of which with the tunneling parameter \( J \) has been shown to lead to the Mott-insulator quantum phase transition for a critical value of the ratio \( U/J \). It has been theoretically predicted that for a BEC in a shaken optical lattice, this ratio can be replaced by \( U/J_{\text{eff}} \) and hence that it should be possible to drive the system across the quantum phase transition by varying the shaking parameter. In this work, we demonstrate the feasibility of the key ingredients of this scheme. In particular, we show that when tunneling in the shaken lattice is
completely suppressed, the phase coherence of the BEC is lost in agreement with the physical picture of a sudden ‘switch-off’ of the inter-well coupling and a subsequent independent evolution of the local phases due to collisions between the atoms [16, 17].

Our system consisting of a Bose-Einstein condensate inside a (sinusoidally) shaken one-dimensional optical lattice is approximately described by the Hamiltonian

$$\hat{H}_0 = -J \sum_{\langle i,j \rangle} (\hat{c}_i \hat{c}_j^\dagger + \hat{c}_j \hat{c}_i^\dagger) + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) + K \cos(\omega t) \sum_j \hat{n}_j,$$

(1)

where $\hat{c}_i^{\dagger}$ and $\hat{c}_i$ are the boson creation and annihilation operators on site $i$, $\hat{n}_i = \hat{c}_i^\dagger \hat{c}_i$ are the number operators, and $K$ and $\omega$ are the strength and angular frequency of the shaking, respectively. The first two terms in the Hamiltonian describe the Bose-Hubbard model [14] with the tunneling matrix element $J$ and the on-site interaction term $U$. The shaking of the lattice is expected to renormalize the tunneling matrix element $J$, leading to an effective tunneling parameter [2]

$$J_{\text{eff}} = J \mathcal{J}_0(K_0),$$

(2)

where $\mathcal{J}_0$ is the zeroth-order ordinary Bessel function and we have introduced the dimensionless parameter $K_0 = K/\hbar \omega$.

In our experiment we created BECs of about $5 \times 10^4$ 87-rubidium atoms using a hybrid approach in which evaporative cooling was initially applied in a magnetic time-orbiting potential (TOP) trap and subsequently in a crossed dipole trap. The dipole trap was realized using two intersecting gaussian laser beams at 1030 nm wavelength and a power of around 1 W per beam focused to waists of 50 μm. After obtaining pure condensates of around $5 \times 10^4$ atoms the powers of the trap beams were adjusted in order to obtain elongated condensates with the desired trap frequencies ($\approx 20$ Hz in the longitudinal direction and 80 Hz radially). Along the axis of one of the dipole trap beams a one-dimensional optical lattice potential was then added by ramping up the power of the lattice beams in 50 ms (the ramping time being chosen such as to avoid excitations of the BEC). The optical lattices used in our experiments were created using two counter-propagating gaussian laser beams ($\lambda = 852$ nm) with 120 μm waist and a resulting optical lattice spacing $d_L = \lambda/2 = 0.426 \mu m$. The depth $V_0$ of the resulting periodic potential is measured in units of $E_{\text{rec}} = \hbar^2 \pi^2/(2md_L^2)$, where $m$ is the mass of the Rb atoms. By introducing a frequency difference $\Delta \nu$ between the two lattice beams (using acousto-optic modulators which also control the power of the beams), the optical lattice could be moved at a velocity $v = d_L \Delta \nu$ or accelerated with an acceleration $a = d_L \Delta \nu / \omega$. In order to periodically shake the lattice, $\Delta \nu$ was sinusoidally modulated with angular frequency $\omega$, leading to a time-varying velocity $v(t) = d_L \Delta \nu_{\text{max}} \sin(\omega t)$ and hence to a time-varying force

$$F(t) = m \omega d_L \Delta \nu_{\text{max}} \cos(\omega t) = F_{\text{max}} \cos(\omega t).$$

(3)

The peak shaking force $F_{\text{max}}$ is related to the shaking strength $K$ appearing in Eq. (1) by

$$K = F_{\text{max}} d_L,$$

(4)

and hence the dimensionless shaking parameter

$$K_0 = \frac{\Delta \nu_{\text{max}}}{\hbar \omega} = \frac{\pi^2 \Delta \nu_{\text{max}}}{2 \omega_{\text{rec}}}.$$

(5)

The spatial shaking amplitude $\Delta x_{\text{max}}$ can then be written as

$$\Delta x_{\text{max}} = \frac{2 \omega_{\text{rec}}}{\pi^2} K_0 d_L,$$

(6)

so for a typical shaking frequency $\omega/2\pi = 3$ kHz we have $\Delta x_{\text{max}} \approx 0.5 d_L$ at $K_0 = 2.4$.

After loading the BECs into the optical lattice, the frequency modulation of one of the lattice beams creating the shaking was switched on either suddenly or using a linear ramp with a timescale of a few milliseconds. Finally, in order to measure the effective tunneling rate $|J_{\text{eff}}|$ between the lattice wells (where the modulus indicates that we are not sensitive to the sign of $J$), in contrast to the time-of-flight experiments described below), we then switched off the dipole trap beam that confined the BEC along the direction of the optical lattice, leaving only the radially confining beam switched on (the trap frequency of that beam along the lattice direction was on the order of a few Hz and hence negligible on the timescales of our expansion experiments, which were typically less than 200 ms). The BEC was now free to expand along the lattice direction through inter-well tunneling and its in-situ width was measured using a resonant flash, the shadow cast by which was imaged onto a CCD chip. The observed density distribution was then fitted with one or two gaussians.

In a preliminary experiment without shaking ($K_0 = 0$), we verified that for our expansion times the growth in the condensate width $\sigma_x$ along the lattice direction was to a good approximation linear and that the dependence of $d \sigma_x/dt$ on the lattice depth (up to $V_0/E_{\text{rec}} = 9$) followed the expression for $J(V_0/E_{\text{rec}})$ in the lowest energy band [18]

$$J \left( \frac{V_0}{E_{\text{rec}}} \right) = \frac{4E_{\text{rec}}}{\sqrt{16 \pi}} \left( \frac{V_0}{E_{\text{rec}}} \right)^{3/4} e^{-2 \sqrt{V_0/E_{\text{rec}}}}.$$

(7)

This enabled us to confirm that $d \sigma_x/dt$ measured at a fixed time was directly related to $J$ and, in a shaken lattice, to $|J_{\text{eff}}(K_0)|$. The results of our measurements of $|J_{\text{eff}}(K_0)/J|$ for various lattice depths $V_0$ and driving frequencies $\omega$ are summarized in Fig. 2. We found a universal behaviour of $|J_{\text{eff}}/J|$ that is in very good agreement with the Bessel-function re-scaling of Eq. (4). We were
able to measure $|J_{\text{eff}}/J|$ for $K_0$ up to 12, albeit agreement with theory beyond $K_0 \approx 6$ was less good, with the experimental values lying consistently below the theoretical curve. For the zeroes of the $J_0$ Bessel function at $K_0 \approx 2.4$ and 5.4, complete suppression of tunneling was observed (within our experimental resolution, we could measure a suppression by at least a factor of 25).

We also checked the behaviour of $|J_{\text{eff}}/J|$ as a function of $\omega$ for a fixed value of $K_0 = 2$ (see insert in Fig. 2) and found that over a wide range of frequencies between $\hbar \omega/J \approx 0.3$ and $\hbar \omega/J \approx 30$ the tunneling suppression due to the shaking of the lattice works, although for $\hbar \omega/J \lesssim 1$ we found that $|J_{\text{eff}}(K_0)/J|$ as a function of $K_0$ deviated from the Bessel function near the zero points, where the suppression was less efficient than expected. In the limit of large shaking frequencies ($\omega/2\pi \gtrsim 3\text{kHz}$, to be compared with the typical mean separation of $\approx 15\text{kHz}$ between the two lowest two energy bands at $V_0/E_{\text{rec}} = 9$), we observed excitations of the condensate to the first excited band of the lattice. In our in-situ expansion measurements, these band excitations were visible in the condensate profile as a broad gaussian pedestal below the near-gaussian profile of the ground-state condensate atoms. From the widths of those pedestals we inferred that $|J_{\text{eff}}/J|$ of the atoms in the excited band also followed the Bessel-function rescaling of Eq. (2), and that the ratios of the tunneling rates in the two bands agreed with theoretical models.

![FIG. 2: Dynamical suppression of tunneling in an optical lattice. Shown here are the values of $|J_{\text{eff}}/J|$ as calculated from the expansion velocities as a function of the shaking parameter $K_0$. The lattice depths and shaking frequencies were: $V_0/E_{\text{rec}} = 6$, $\omega/2\pi = 1\text{kHz}$ (squares), $V_0/E_{\text{rec}} = 6$, $\omega/2\pi = 0.5\text{kHz}$ (circles), and $V_0/E_{\text{rec}} = 4$, $\omega/2\pi = 1\text{kHz}$ (triangles). The dashed line is the theoretical prediction. Insert: Dependence of the tunneling suppression $|J_{\text{eff}}/J|$ on the shaking frequency $\omega$ for $K_0 = 2.0$ and $V_0/E_{\text{rec}} = 9$ corresponding to $J/h = 90\text{Hz}$.](image)

![FIG. 3: Phase coherence in a shaken lattice. (a) Dephasing time $\tau_{\text{deph}}$ (decay time of the visibility) of the condensate as a function of $K_0$ for a lattice with $V_0/E_{\text{rec}} = 9$ and $\omega/2\pi = 3\text{kHz}$. The vertical dashed line marks the position of $K_0 = 2.4$ dividing the regions with $J_{\text{eff}} > 0$ (left) and $J_{\text{eff}} < 0$ (right). In both regions, a typical (vertically integrated) interference pattern of a time-of-flight experiment without final acceleration to the zone edge is shown (on the $x$-axis, the spatial position has been converted into the corresponding momentum in units of the recoil momentum $p_{\text{rec}} = h/d_L$). Insert: Rephasing time after dephasing at $K_0 = 2.4$ and subsequent reduction of $K_0$. (b) Dephasing time as a function of the normalized driving frequency $\hbar \omega/J$ for $K_0 = 2.2$.](image)

We now turn to the phase coherence of the BEC in the shaken lattice. In order to quantify the degree of phase coherence, after shaking the condensate in the lattice for a fixed time between 1 and $\approx 200\text{ms}$ we accelerated the lattice for $\approx 1\text{ms}$ so that at the end of the acceleration the BEC was in a staggered state at the edge of the Brillouin zone. After switching off the dipole trap and
lattice beams and letting the BEC fall under gravity for 20 ms, this resulted in an interference pattern featuring two peaks of roughly equal height. In Fig. 3 (a), we found an interference pattern (see Fig. 3 (a)) that was shifted by half a Brillouin zone, in agreement with theoretical predictions. We then measured the visibility $V = (h_{\text{max}} - h_{\text{min}})/(h_{\text{max}} + h_{\text{min}})$ of the interference pattern as a function of the time the condensate spent inside the shaken lattice, where $h_{\text{max}}$ is the mean value of the condensate density at the position of the two interference peaks and $h_{\text{min}}$ is the condensate density in a region of width equal to about 1/4 of the peak separation centered about the halfway point between the two peaks. For a perfectly phase-coherent condensate $V \approx 1$, whereas for a strongly dephased condensate $V \approx 0$. For $K_0 \lesssim 2.2$, the BEC phase coherence was maintained for several tens of milliseconds, demonstrating that the tunneling could be suppressed by a factor of up to 10 over hundreds of shaking cycles without significantly disturbing the BEC. This result is expressed more quantitatively in Fig. 3 (a). Here, the condensate was held in the lattice $(V_0/E_{\text{rec}} = 9)$, and the shaking was switched on suddenly at $t = 0$ (we found no significantly different behaviour when $K_0$ was linearly ramped in a few milliseconds). Thereafter, the visibility was measured as a function of time and the decay time constant $\tau_{\text{deph}}$ of the resulting near-exponential function was extracted. Apart from a slow overall decrease in the dephasing time for increasing $K_0$, a sharp dip around $K_0 = 2.4$ is visible. In this region, $J$ is suppressed by a factor of more than 20 and hence the effective tunneling rate $|J_{\text{eff}}/h| \lesssim 10 \text{ Hz}$, which for our experimental parameters is comparable to the on-site interaction energy $U$ expressed in frequency units (we checked that the widths of the on-site wavefunctions and hence $U$ were independent of $K_0$ by analyzing the side-peaks in the interference pattern). This means that neighbouring lattice sites are effectively decoupled and the local phases evolve independently due to interatomic collisions, leading to a dephasing of the array $13, 17$. By increasing the dipole trap frequency (and hence $U$), we verified that the timescale for this dephasing decreases as expected. We also studied a re-phasing of the BEC when, after an initial dephasing at $K_0 = 2.4$, the value of the shaking parameter was reduced below 2.4. The time constant $\tau_{\text{reph}}$ of the subsequent rephasing of the condensate (mediated by inter-well tunneling and on-site collisions) increased with decreasing $J_{\text{eff}}$ (see the insert of Fig. 3 (a), where we compare $\tau_{\text{reph}}$ as a function of $K_0$ with the inverse of the generalized Josephson frequency $\omega_{\text{Josephson}} \propto J_{\text{eff}}^{-1/2}$ predicted by the two-well model $17, 20$).

Finally, we investigated the dependence of the dephasing time on the shaking frequency $\omega$ (see Fig. 3 (b)). Interestingly, while the tunneling suppression as observed in-situ works even for $h\omega/J \approx 1$, in order to maintain the phase coherence of the condensate, much larger shaking frequencies are needed. Indeed, for our system there exists an optimum shaking frequency of $h\omega/J \approx 30$.

In summary, we have measured the dynamical suppression of tunneling of a BEC in strongly shaken optical lattices and found excellent agreement with theoretical predictions. Our results show that the tunneling suppression occurs in a phase-coherent way and can, therefore, be used as a tool to control the tunneling matrix element while leaving the on-site interaction energy unchanged (in contrast to the usual technique of increasing the lattice depth, which changes both) and without disturbing the condensate. This might ultimately lead to the possibility of controlling quantum phase transitions by strong driving of the lattice. In this context, it will be important to investigate the question of adiabaticity when dynamically changing the shaking parameter. Furthermore, our system also opens up other avenues of research such as the realization of exact dynamical localization using discontinuous shaking waveforms $3, 21$ or tunneling suppression in superlattices $22$.

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