Q-stars and charged q-stars

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Abstract

We present the formalism of q-stars with local or global $U(1)$ symmetry. The equations we formulate are solved numerically and provide the main features of the soliton star. We study its behavior when the symmetry is local in contrast to the global case. A general result is that the soliton remains stable and does not decay into free particles and the electrostatic repulsion preserves it from gravitational collapse. We also investigate the case of a q-star with non-minimal energy-momentum tensor and find that the soliton is stable even in some cases of collapse when the coupling to gravity is absent.

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1 Introduction

Boson stars were first discussed in the work of Kaup and Ruffini and Bonazzola [1, 2]. They investigated the case of a massive, free, complex scalar field. Complex scalar fields with a quartic self-interaction have been studied in [3, 4]. The case of charged boson stars was thoroughly investigated in [5]. The general result is that these boson stars are rather small, with radius of order of the inverse Compton wavelength. Gravity plays a stabilizing role.

Another sort of boson stars are the soliton stars. Their feature is that in the absence of gravity they remain stable, as simple solitonic solutions to highly non-linear Euler-Lagrange equations of scalar fields [6, 7]. These are very large solitons with typical radius $\sim M_P^2/\phi_0^3$ and mass $\sim M_P^4/\phi_0^3$, where $\phi_0$ is a typical value of the scalar field. These soliton stars resulted from non-topological solitons, earlier discussed in [8]. Another kind of non-topological solitons have been studied in the literature called q-balls [9]. They appear at the minimum of the $(U/|\phi|^2)^{1/2}$ quantity in Lagrangians with a global $U(1)$ symmetry. In non-topological solitons discussed in [6, 7], the fields rotate in their internal $U(1)$ symmetry space with an angular velocity $\omega$ which is very small compared to the mass of the free particles. So, the "kinetic" energy resulting from the temporal variation of the field can be neglected. This is not the case for q-balls or q-stars where this frequency is of the same order as the mass of the free particles, because it is related to the parameters of the potential (mass, trilinear parameters etc.) The surface of the non-topological soliton stars studied in [8, 9] contains an amount of energy which is of the same order as the energy stored in the interior. In contrast, the q-star surface contains a negligible amount of energy, because its thickness is of order of the inverse Compton wavelength. This condition gives a different relation for the total soliton mass and its radius, as we shall see. Q-stars with one and two bosons in the global case of the symmetry have been investigated in [10]. Also, q-stars including a boson and one or more fermions have been studied in [11, 12, 13, 14], mainly as a possible descriptive model for neutron stars.

We expect to observe q-balls in supersymmetric extensions of Standard Model [15, 16, 17], provided that the scalar potential decreases firstly slower than $|\phi|^2$ and then faster and there is a conserved quantum number concerning scalar supersymmetric partners of fermions. If this supersymmetric theory is an effective, low energy one, approximating a more general, probably superstring theory in more than four dimensions, we generally expect the scalar field to couple to gravity in a non-trivial way, usually with a $|\phi|^2 R$ term, [18, 19], where $R$ is the curvature scalar. Also, in a more realistic the-
oretical framework, scalar fields may not be governed by a global symmetry
but by a local one. So, it would be interesting to investigate the possibility
of having charged q-stars and to compare their properties when varying the
field strength or the frequency, which will be proved very important for the
formation of the field configuration.

2 The general framework

We consider a static, spherically symmetric metric:

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 d\alpha^2 - r^2 \sin^2 \alpha d\beta^2$$

where $\nu \equiv \nu(r)$ and the convention $g_{00} = e^\nu$ holds. The action in natural
units for a general scalar field with a local $U(1)$ symmetry coupled to gravity
is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + g^{\mu\nu}(D_\mu \varphi)^*(D_\nu \varphi) - U - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

where

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \quad D_\mu \varphi \equiv \partial_\mu \varphi + ieA_\mu \varphi.$$  

where $e$ is the charge (or the field strength). Einstein equations are

$$R_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu}R = 8\pi GT_{\mu\nu}$$

with the energy-momentum tensor:

$$T_{\mu\nu} = (D_\mu \varphi)^*(D_\nu \varphi) + (D_\mu \varphi)(D_\nu \varphi)^* - g_{\mu\nu}[g^{\alpha\beta}(D_\alpha \varphi)^*(D_\beta \varphi)]
+ g_{\mu\nu}U + \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} - g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}.$$  

The Euler-Lagrange equation for the matter field is

$$\left \{ \frac{1}{\sqrt{|g|}} D_\mu (\sqrt{|g|} g^{\mu\nu} D_\nu) + \frac{dU}{d|\varphi|^2} \right \} \varphi = 0.$$  

There is a Noether charge:

$$j^\mu = \sqrt{-g} g^{\mu\nu} \iota (\varphi^* D_\nu \varphi - \varphi D_\nu \varphi^*) = \sqrt{-g} g^{\mu\nu} \iota (\varphi^* \partial_\nu \varphi - \varphi \partial_\nu \varphi^* + 2eA_\nu |\varphi|^2).$$  

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This charge is conserved according to the equation:

\[ j^\mu_{,\mu} = 0. \]  

The total charge is defined:

\[ Q = \int d^3 x j^0. \]  

We want to study static solutions, which means that the metric and the energy-momentum tensor must be time-independent, though the matter field, \( \varphi \), may depend on time. A plausible ansatz is to choose

\[ \varphi = \sigma e^{i\omega t}. \]  

This is the ansatz used in order to take q-ball type solutions. Specifically, this type of non-topological solitons is stable when \( \sigma \) is a step-function and \( \omega = \omega_{\text{crit}} \equiv (U/\varphi^2)^{1/2}_{\text{min}}. \) We also choose \( A_\mu = (A_0, 0, 0, 0) \) in order not to have magnetic fields and in accordance with the static character of the configuration. As a simple self-consistency check, we can easily find that the energy-momentum tensor, and the metrics consequently, is time-independent. With these substitutions the Einstein equations are

\[ \lambda' = (1 - e^\lambda)/r + 8\pi Gr^2 e^\lambda \left[(\omega + eA_0)^2 e^{-\nu} \sigma^2 + U + \sigma^2 e^{-\lambda} + (1/2)A_0^2 e^{-\nu - \lambda}\right], \]  

\[ \nu' = (e^\lambda - 1)/r + 8\pi Gr^2 e^\lambda \left[(\omega + eA_0)^2 e^{-\nu} \sigma^2 - U + \sigma^2 e^{-\lambda} - (1/2)A_0^2 e^{-\nu - \lambda}\right]. \]

and the Euler-Lagrange equations for the scalar and the gauge field are

\[ \sigma'' + [2/r + (1/2)(\nu' - \lambda')]\sigma' + e^\lambda(\omega + eA_0)^2 e^{-\nu} \sigma - e^\lambda \frac{dU}{d\sigma} \sigma = 0, \]  

\[ A_0'' + [2/r - (1/2)(\nu' + \lambda')]A_0' - 2e\sigma^2 e^\lambda(\omega + eA_0). \]

We will now give the formulas for the energy and charge of the configuration. There are two alternative formulas for the charge:

\[ Q = 8\pi \int_0^\infty dr r^2 (\omega + eA_0) \sigma^2 e^{(\lambda - \nu)/2}, \]
Also, there are two alternative formulas for the total energy (the mass) of the soliton

\[ E = 4\pi \int_0^\infty dr r^2 [\omega + eA_0]^2 \sigma^2 e^{-\nu} + U + e^{-\lambda} \sigma^2 + (1/2)A_0^2 e^{-\nu-\lambda}]. \tag{16} \]

If we know the total charge of the soliton we can find the total mass from the asymptotic relation:

\[ e^\lambda = (1 - 2GE/r + GQ^2/4\pi r^2)^{-1}, \quad r \to \infty. \tag{17} \]

We will mainly use eq. 14, 16.

We have formulated the general framework concerning the case of local \( U(1) \) symmetry. In order to examine the global case we put \( e = 0 \) and \( A = 0 \) and the results of [10] can be obtained. It is also very convenient to define

\[ \theta \equiv \omega + eA_0. \tag{18} \]

In the case of the global symmetry \( \theta \to \omega \).

### 3 The q-star solution

For the sake of convenience we will use a simple potential of the form

\[ U = \sigma^2 \left( 1 - \sigma^2 + \frac{1}{3} \sigma^4 \right). \tag{19} \]

We will examine separately three different regions: The interior, the surface and the exterior of the soliton. At the interior the metric, gauge and matter fields vary very smoothly with respect to the radius, because, as we can see from Einstein equations, the metric derivatives are proportional to \( 8\pi G \) times \( \varphi^4 \) in rough estimate, which is a very small quantity. Let \( \sigma_0 \) be a value of the order of magnitude of the matter field values inside the soliton. We will make the rescallings:

\[ \tilde{r} = r\sigma_0, \quad \tilde{\sigma} = \sigma/\sigma_0, \quad \tilde{\theta} = \theta/\sigma_0, \quad \tilde{U} = U/\sigma_0^4. \tag{20} \]

We also define

\[ B \equiv e^{-\nu}, \quad A \equiv e^{-\lambda}. \tag{21} \]
The idea which the q-ball type soliton rely on, in contrast to other sorts of non-topological solitons, is the relation:

$$\omega \sim m \sim \varphi$$

The first part of the condition comes from the relation combining $$\omega_{\text{crit}}$$ to the potential parameters (i.e. $$\omega_{\text{crit}} = (U/|\varphi|^2)^{1/2}_{\text{min}}$$). The second is derived from solving the equation of motion of the matter field, so as to find its value. Remembering that the gravity becomes important when $$R \sim GM$$, where $$M$$ the soliton mass, we can easily find that the star radius is $$R \sim M_{\text{Pl}}/\sigma_0^2$$. We now define the quantity:

$$\epsilon = \sqrt{8\pi G \sigma_0} \sim m/M_{\text{Pl}}.$$ (22)

This means that $$\epsilon$$ is very small. It is easy to find that the radius of the soliton is the same order of magnitude as $$\epsilon^{-1}$$, or, to be more definite, if $$\tilde{r}_-$$ is the radius of the interior of the soliton, then for the interior holds

$$\tilde{r} = \tilde{r}_- x = \epsilon^{-1} \kappa x, \quad 0 \leq x \leq 1$$ (23)

where $$\kappa$$ is of order unity. We also define

$$\tilde{e}^2 = e^2 (\epsilon^{-1} \kappa)^2.$$ (24)

So, for the interior, dropping the tildes and the $$0(\epsilon)$$ terms the Einstein equations are

$$1 - A - x \frac{dA}{dx} = x^2 \kappa^2 \left[ \theta^2 \sigma^2 B + U + \frac{1}{2e^2} \left( \frac{d\theta}{dx} \right)^2 AB \right], \quad (25)$$

$$A - 1 - x \frac{dB}{B \ dx} = x^2 \kappa^2 \left[ \theta^2 \sigma^2 B - U - \frac{1}{2e^2} \left( \frac{d\theta}{dx} \right)^2 AB \right]. \quad (26)$$

which result from the $$G_0^0 = 8\pi GT_0^0$$ and $$G_1^1 = 8\pi GT_1^1$$ respectively, and the Euler-Lagrange equations:

$$\theta^2 B - \frac{dU}{d\sigma^2} = 0,$$ (27)

$$\frac{d^2 \theta}{dx^2} + \left[ \frac{2}{x} + \frac{1}{2} \left( \frac{1}{A \ dx} + \frac{1}{B \ dx} \right) \right] \frac{d\theta}{dx} - \frac{2e^2 \sigma^2 \theta}{A} = 0.$$ (28)
These equations are in a manageable form because the derivatives of the metrics with respect to $x$ are not too small as those with respect to $r$ and the soliton radius in units of $\sigma_0^{-1}$ is not "huge" any more. It is a matter of simple algebra to verify that eq. 27 has the correct limit when we study the global case (i.e.: $\theta \to \omega$) and the gravity is absent (so $B \to 1$).

The boundary conditions concerning the soliton interior are $A(0) = 1$, $A(1) = 1/B(1)$ and $\theta'(0) = 0$. The first condition results from the freedom to rescale the metrics, the second from the fact that outside the soliton (we will see that the surface region is extremely thin, so one can define as "matter region" the soliton interior) the solution to the Einstein equations should be Schwarzschild and the third condition reflects the absence of electric field at the center of the soliton.

Eq. 27 can provide an exact solution for the matter field, i.e.:

$$\sigma = \sqrt{1 + \theta \sqrt{B}}.$$  \hspace{0.5cm} (29)

Then:

$$U = \frac{1}{3}(1 + \theta^3 B^{3/2}).$$  \hspace{0.5cm} (30)

The surface is very thin in the case of q-stars and this is a crucial difference between them and other non-topological-soliton stars. The surface thickness is of order $\sigma^{-1}$. In this type of non-topological solitons $\sigma \sim \omega$, in contrast to other non-topological soliton. The surface energy is of order $\sigma^4 R^2 \sigma^{-1}$. We saw that $R \sim M_{Pl}/\sigma^2$. The energy stored in the soliton interior is $E_{int} \sim R^3 \sigma^4$. So the relation: $E_{sur} \sim \epsilon E_{int}$ hold. These results are numerically verified. So the surface energy is negligible, a result that not hold for the other kind of soliton stars. Within the surface region, the metric and the gauge field varies very slowly (their derivatives are of $0(\epsilon^{-1})$) and can be regarded as constant. The matter field varies rapidly (this is the meaning of a surface). Now we will find the "eigenvalue" equation for the gauge field, or the frequency in the global case. In eq. 12 $2/r + (1/2)(\nu' - \chi')$ are $0(\epsilon)$ terms and can be neglected. Eq. 12 can be straightforward integrated giving as result:

$$e^{-\chi} \left( \frac{d\sigma}{dr} \right)^2 + \theta^2 \sigma^2 e^{-\nu} - U = 0.$$  \hspace{0.5cm} (31)

In the interior the above equation, neglecting the $0(\epsilon)$ terms, will contain the second and third term. In order to confirm the continuity of the above
expression (and should be continuous as a first integral) we should set

\[(\theta^2 \sigma^2 e^{-\nu} - U)_{r_-} = 0.\]  

(32)

Switching off both gravity and the gauge field (i.e. making the symmetry global) the above expression arises from the Euler-Lagrange equation when the spatial derivative terms are neglected (i.e. the soliton is very large) and \(\omega = \omega_{\text{min}}\). Now \(\theta\) took the place of \(\omega\). Eq. 32 gives the eigenvalue condition for the frequency. Substituting equations 29, 30 into eq. 32 we find that for the surface

\[\theta^2_{\text{sur}} B_{\text{sur}} = 1/4.\]  

(33)

We can easily understand that different values of the potential parameters gives a different relation between \(\theta\) and the metrics. When both gravity and electric charge are absent the above relation takes the form \(\omega = 1/2\), which is really the value \((U/|\varphi|^2)^{1/2}\) for the potential of eq. 19. Eq. 33 gives a measure of the gravity strength when we know the frequency. When \(A(\text{sur}) < 1\), i.e. when we switch on gravity, the frequency of the large soliton (thin-wall approximation) is less than that corresponding to the absence of gravity. This fact summarizes the main effect of the gravity upon the soliton properties. The "potential" that enters the \(\omega_{\text{min}}\) relation now is not \(U\) but \(A(\text{sur}) U\) and is smaller than in the case that the gravity is absent.

In the exterior region the matter field \(\sigma\) is zero. The metric fields should obey the asymptotic relation

\[A(r) = 1/B(r), \quad r \to \infty\]

One can prove that the above condition holds for the entire exterior region, not only at infinity, using equations

\[1 - A - x \frac{dA}{dx} = x^2 \kappa^2 \left( \frac{d\theta}{dx} \right)^2 AB,\]

\[A - 1 - x \frac{dB}{B \frac{dx}{dx}} = -x^2 \kappa^2 \left( \frac{d\theta}{dx} \right)^2 AB, \quad x > 1\]

which concern the metrics outside the soliton, and the above boundary condition. This result is in agreement with the Schwarzschild type of the exterior solution. The boundary condition for the gauge field \(\theta\) results from the matching of the exterior end the interior solutions. The other condition for the gauge field is \(\theta \to \omega\) when \(r \to \infty\) which means that for large distances the "true" gauge field, \((\theta - \omega)/e\), tends to zero. Our results also hold in the global case of the \(U(1)\) symmetry studied in [10].
Figure 1: Soliton profiles for different values of the coupling constant, with \( A(\text{sur}) = 0.36 \) or, equivalently, \( \theta_{\text{sur}} = 0.3 \) for five different values of the field strength \( e \). We examine only the interior region because the surface is very thin.

4 Numerical results

The computation of energy (total mass) and particle number of the field configuration are based on eq. 14, 16. \( \kappa \) gives a measure of the soliton radius. The main features of the soliton are plotted as functions of the frequency, which is the most crucial parameter. Alternatively, we could use a parameter of the surface gravity (\( A \) or \( 1/B \)) in order to test the soliton behavior. But \( \theta_{\text{sur}} \), or \( \omega \), are not independent parameters, as we can see from equation 33. So, the value of the frequency gives a measure of the surface gravity strength as well.

The total particle number can be identified with the total charge if we take as unity the charge of each particle. Figure 1 shows the behavior of the soliton profile when decreasing slowly the field strength \( e \). We sketched only the interior region because the surface is very thin and the field varies rapidly. When the coupling constant increases, the soliton gets larger due to the electrostatic repulsion and, most interesting, the value at the origin is not the maximum one, due to the same effect.

Fig. 2 shows the behavior of the soliton charge as a function of the frequency. These results are intuitively expected. When \( \theta_{\text{sur}} \) or \( \omega \) decreases, the particle number increases because the gravity gets more important (eq. 33). The presence of the electric charge \( e \) makes the decrease of the particle number slower, due to the electrostatic repulsion between the different parts of the soliton.
Figure 2: The total charge (particle number) of the soliton as a function of $\theta_{sur}$, which equals to $\omega$ in the global case, for four different values of the field strength.

Figure 3: The mass of the field configuration as a function of $\theta_{sur}$ for four different values of the field strength.
Figure 4: The soliton radius as a function of $\theta_{\text{sur}}$ for four different values of the field strength.

The results of figure 3 are more interesting. In the case of global symmetry, the increase of frequency decreases the total soliton energy due to the decrease of the particle number. When the field strength $e$ is large, the decrease of the particle number is not enough to decrease the energy and the electrostatic energy should be taken into account. This energy becomes more important when gravity weakens, i.e. when frequency approaches the value that has when gravity is absent. For the potential of eq. 13 this value is $(U/|\varphi|^2)_{\text{min}}^{1/2} = 0.5$.

The soliton radius, figure 4, has a very interesting behavior. In general the radius gets larger with the coupling constant due to the electrostatic repulsion between the different parts of the soliton. The radius also increases when the frequency decreases, because the particle number gets larger. But in the global case and after a certain point the soliton radius decreases because the gravity becomes extremely strong. This is the boundary between stars and black holes, because when $\omega \to 0$, then $A(\text{sur}) \to 0$ (eq. 33 and the relation $A(\text{sur}) = 1/B(\text{sur})$ between the metrics) and the metric $B$ at the center of the soliton approaches infinity.

We will now study the soliton behavior when varying the coupling constant $e$. A general feature is the decrease of the soliton mass, radius and particle number when the coupling constant gets larger. We also observe the increase in the ratio soliton energy per particle number (which expresses the ratio soliton energy per free particles energy) which leads to an "electrostatic" instability of the soliton for very large values of the field strength $e$. The field
Figure 5: The value of the scalar field at center of the soliton as a function of $\theta_{sur}$.

Figure 6: The soliton charge as a function of the coupling constant for four different frequencies.
Figure 7: The total soliton mass as a function of the coupling constant $e$.

Figure 8: The soliton radius as a function of the coupling constant.
configurations described in the figures satisfy $E_{sol}/E_{free} < 1$, i.e. they are stable configurations. In order to simplify calculations we chose the mass of the free particles to be 1 (eq. 19), so the energy of the free particles with the same charge as the soliton equals the particle number. When the above ratio exceeds unity, the soliton decays into free particles. On the other hand the presence of non-zero electric charge prevents soliton from gravitational collapse. In the global case and when $\omega$ is very small, the metric $B$ at the center of the soliton tends to infinity, revealing a singularity at the center of the field configuration. Electrostatic repulsion removes this singularity and makes the soliton stable, even for lower frequencies.

The plot of charge as a function of the field strength, fig. 6, has a well-understood behavior. The increase in the electrostatic repulsion decreases the number of particles within the soliton. The plot of energy, fig. 7, is more interesting. When the frequency is small and, consequently the gravity is more important (eq. 33) the increase of the coupling constant leads to a decrease of the energy due to the decrease in the total charge. When gravity is weaker (for instance $\theta = 0.35$ or, equivalently $A(sur) = 0.49$), the increase of the coupling constant increases the electrostatic energy. The behavior of the soliton radius is interesting as well. Large coupling constant leads to bigger solitons when the strengthening of gravity has the opposite effect.

5 Q-stars with non-minimal energy-momentum tensor

We will now examine the case where the action has a new term concerning the coupling of the scalar field to gravity. Such terms can arise in effective Lagrangians when considering theories in more than four dimensions. We will use a simple, toy model for the action, namely:

$$S = \int d^4x \sqrt{-g} \left[ \left( \frac{1}{16\pi G} + \xi \varphi \varphi^* \right) R + g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi^* - U \right].$$

(34)

The energy-momentum tensor is

$$T_{\mu\nu} = \partial_\mu \varphi^* \partial_\nu \varphi + \partial_\nu \varphi^* \partial_\mu \varphi - g_{\mu\nu} (g^{\alpha\beta} \partial_\alpha \varphi^* \partial_\beta \varphi) + g_{\mu\nu} U + 2\xi \varphi^* \varphi (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) - 2 \xi g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \partial_\beta |\varphi|^2 + 2 \xi \partial_\mu \partial_\nu |\varphi|^2.$$ 

(35)

We have regarded only the case of global $U(1)$ symmetry for simplicity.
Using the ansatz of eq. 9 referring to the static configurations we take the Euler-Lagrange equation for the matter field:

\[ \sigma'' + \left[ \frac{2}{r} + \frac{1}{2}(\nu' - \lambda') \right] + e^{\lambda - \nu} \omega^2 \sigma + e^{\lambda} \xi R \sigma - e^{\lambda} \frac{dU}{d\sigma^2} \sigma = 0, \tag{36} \]

and the Einstein equations:

\[ (1 + 16\xi \pi G) \left( \frac{\lambda'}{r} + \frac{e^\lambda}{r^2} - \frac{1}{r^2} \right) = 8\pi G [e^{\lambda - \nu} \omega^2 \sigma^2 + U + \sigma'^2 (1 + 4\xi) - 2\xi \nu' \sigma' \sigma + 4e^\lambda \xi (U - \omega^2 \sigma^2 e^{-\nu} - \xi R \sigma^2)], \tag{37} \]

\[ (1 + 16\xi \pi G) \left( \frac{\nu'}{r} - \frac{e^\lambda}{r^2} + \frac{1}{r^2} \right) = 8\pi G [e^{\lambda - \nu} \omega^2 \sigma^2 - U + \sigma'^2 - 2\xi \sigma \sigma' (\nu' + 4/r)], \tag{38} \]

\[ [1 + 16\xi \pi G \sigma^2 (1 + 6\xi)] R = 8\pi G [(2 + 12\xi)(e^{-\lambda} \sigma'^2 - e^{-\nu} \omega^2 \sigma^2) + 4U (1 + 3\xi)], \tag{39} \]

where the last equation has been obtained by taking the trace of the Einstein equations and will be used for the calculation of the curvature scalar.

The total mass of the configuration is

\[ E = 4\pi \int_0^\infty dr r^2 \frac{1}{1 + 16\xi \pi G \sigma^2} [e^{-\nu} \omega^2 \sigma^2 (1 - 4\xi) + U (1 + 4\xi) + e^{-\lambda} \sigma'^2 (1 + 4\xi) - 2e^{-\lambda} \xi \nu' \sigma' \sigma - 4\xi^2 \sigma R]. \tag{40} \]

The total particle number is given by eq. 3, or, equivalently, 14. The potential is the same as in eq. 19. We also make the rescallings and the definitions 20 and 21. We also define

\[ \tilde{R} = R/\sigma_0^2. \tag{41} \]

We discriminate between three regions: The interior region, the very thin surface region and the exterior, where the metric is Schwarzschild. Following the steps of the previous case, we write the equations for the interior, dropping the tildes and the 0(\epsilon) quantities:

\[ 1 - A - x \frac{dA}{dx} = x^2 \kappa^2 \left[ \theta^2 \sigma^2 B (1 + 4\xi) + U (1 + 4\xi) \right], \tag{42} \]
Coupling to gravity factor

Central field value

Figure 9: The central field value as a function of the coupling $\xi$ of the scalar field to gravity for three different frequencies.

$$A - 1 - \frac{A dB}{B dx} = x^2 \kappa^2 [\theta^2 \sigma^2 B - U],$$  \hspace{1cm} (43)$$

resulting from the $G^0_0$ and $G^1_1$ Einstein tensors respectively. The rescaled curvature scalar, $\tilde{R}$ is of $0(\epsilon)$ order. So the total mass of the configuration is

$$E = 4\pi \int_0^\infty dr r^2 (\theta^2 \sigma^2 B (1 - 4\xi) + U (1 + 4\xi)).$$  \hspace{1cm} (44)$$

We also find that eq. 29 for the field value in the soliton interior remains valid.

We solve numerically eq. 42, 43 and we find the mass and particle number of the soliton using eq. 44 and 14 respectively. Some general features of the solutions for the $\xi \neq 0$ cases are: a)For small values of the parameter that describes the strength of the coupling of the scalar field to gravity the soliton becomes more ”stable” against gravitational collapse. We can find solutions for some values of the frequency that have $B(0) \to \infty$ when $\xi$ is zero. These solutions describe a real singularity at the soliton origin, which is removed when we ”switch on” the coupling to gravity. But this coupling, however strong, can not prevent soliton from gravitational collapse when $A(sur) \to 0$. b)Larger values of the coupling parameter correspond to solutions with lower energy and charge but larger radius, which means that the coupling of the matter field to gravity has an effect similar to the electrostatic repulsion. This effect has to do with the repulsive character of the $\xi \phi^2 R$ term.
Figure 10: The radius of the soliton as a function of the coupling $\xi$ for three different frequencies.

Figure 11: The soliton mass as a function of $\xi$. 
Figure 12: The soliton particle number as a function of $\xi$.

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