Quadrotor aircraft attitude control algorithm based on improved UKF

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Abstract. The quadrotor aircraft attitude estimation algorithm which is based on the adaptive attenuation memory unscented Kalman filter algorithm is proposed. The UKF algorithm uses the quaternion method to solve the attitude. The drift output of the gyroscope amplified as part of the state variable, and the attitude angle is used as the measurement variable to establish a filtering model, which effectively suppresses the attitude error caused by the gyro drift. The attenuation memory filter and adaptive algorithm are introduced based on the UKF algorithm to solve the problem of the accuracy degradation of the system model when it is abnormally interfered. The filter divergence is suppressed and the robustness and stability of the system are improved. The simulation results show that the algorithm can effectively improve the attitude estimation accuracy of the aircraft compared with the traditional UKF algorithm.

1. Introduction
In recent years, with the development of Micro-Electro-Mechanical System (MEMS) technology, aircraft technology has made great progress and has been widely used in civil and military fields. Attitude Control is an important part of aircraft systems, so it becomes critical to research the control of aircraft attitude. The attitude of the aircraft is got by an Inertial Measurement Unit (IMU) device, which mainly includes a three-axis gyroscope and a three-axis accelerometer. However, it increases the difficulty to control the accuracy of attitude due to system noise and drift errors of the IMU sensor, and these errors accumulate over time.

Attitude estimation of aircraft has always been a research hotspot as a typical nonlinear system. The literature [1-2] uses Kalman filter to combine the gyroscope and accelerometer data, and coments the drift error of the gyroscope output angular velocity. However, linear system and the Gaussian white noise are the requires of the system based on Kalman filtering, which is difficult for aircraft systems to meet. In order to solve the problem of nonlinear system, Bucy, Sunahara et al. proposed Extended Kalman Filter (EKF) for the further application of the Kalman filter theory to the nonlinear field. The basic idea of EKF is to linearize the nonlinear system and then perform Kalman filtering, so EKF is a suboptimal filter. In the literature[3–4], the EKF algorithm is used to achieve attitude fusion. However, since EKF discards higher-order terms above the second order in the linearization process, EKF is only suitable for the estimation of weak nonlinear systems, the stronger nonlinear the system is, the lager the resulting estimation error will be and may even cause filter divergence. In the literature[5], the EKF and complementary filtering algorithms are used for attitude calculation, which improves the filtering accuracy and improves the problem of filter divergence. However, the EKF algorithm needs to calculate the Jacobian matrix, which is tedious in calculation and inefficient in work. To solve these problems, S.J.Julier proposed an Unscented Kalman Filter method based on U
transform. The UKF designs a series of Sigma points on the distribution of original state. A set of function value points are obtained by propagation of nonlinear functions of these Sigma points. After transformation based on these point sets, the mean and covariance was calculated. The calculation results of UKF are more accurate than EKF, because the UKF is without linear processing and retain the high-order terms. In the literature [6-8], the sensor error variable is amplified to the system state variable, it overcomes the problems of EKF, rectifies the shortcoming of the sensor's own error, and improves the accuracy of data fusion by using the UKF algorithm to fuse the pose data. Although UKF improves the linear error of the system, UKF is sensitive to the initial value of the filter, and in practice, the noise is uncertain due to the error of the state model and the interference of the measurement information, which may lead to the reduction of the filtering accuracy or even filtering divergence. In order to solve this problem, Fagin and Sorenson proposed an attenuation memory filtering algorithm, which uses the attenuation factor to limit the memory length of the filter, so that the filtering process pays more attention to the use of information and reduces the dependence on pre-test information. The literature [9-10] proposed an adaptive attenuation memory filtering algorithm to solve the problem of filter divergence. However, these algorithms need to calculate numerous matrices, it affects the rapidity of the system.

Aiming at the current problems, this paper put forward an Adaptive Fading Memory Unscented Kalman Filter (AFM-UKF).

2. System model

2.1 Attitude description and equation of motion

The attitude of the aircraft is described by the pitch angle $\theta$, the roll angle $\gamma$, and the yaw angle $\phi$. The three-axis gyroscope, three-axis accelerometer, and three-axis magnetometer are used to collect the angular velocity, acceleration and magnetic field strength of the aircraft. The angular velocity $w_{ab} = [w_x, w_y, w_z]^T$ relative to the navigation coordinate system is obtained by the gyroscope. The equation of motion of the system can be obtained by the Euler angle method as shown in equation (1).

$$
\begin{bmatrix}
\dot{\theta} \\
\dot{\gamma} \\
\dot{\phi}
\end{bmatrix} = 
\begin{bmatrix}
0 & \sin \phi & \cos \phi \\
0 & \cos \phi / \cos \theta & -\sin \phi / \cos \theta \\
1 & -\tan \theta \cos \phi & \tan \theta \sin \phi
\end{bmatrix}
\begin{bmatrix}
w_x \\
w_y \\
w_z
\end{bmatrix}
$$

The Euler angle method cannot be in full attitude work since the singular points may come up at the pitch angle $\theta = \pm 90^\circ$ when integrates over equation (1) in Euler angle, it may cause values unstable. The quaternion method, by contrast, is widely used because of its fast and concise calculations, high precision, and full-feature operation. Specified $q$ as a normalized quaternion vector that:

$$
\tilde{q}(q_0, q_1, q_2, q_3) = q_0 + q_i \tilde{i} + q_j \tilde{j} + q_k \tilde{k}
$$

$$
|| \tilde{q} || = q_0^2 + q_i^2 + q_j^2 + q_k^2 = 1
$$

Therefore, the first-order differential equation can be obtained by using the quaternion method to re-describe the equation of motion shown in equation (1):

$$
\ddot{\tilde{q}} = \frac{1}{2} M(w) \dot{\tilde{q}}
$$

The direction cosine matrix as shown in equation (5) can be obtained through sloving the above equation yields a quaternion.
2.2 Sensor Mathematical Model
The gyroscope error is composed of random drift and noise. For better analyze of its characteristics, the mathematical model of the gyroscope is established as shown in equation (6).

\[ w_m = w_i + b_0 + b + w_a \]  

Among them, \( w_m \) is the angular velocity measurement, \( w_i \) is the true value of the angular velocity, \( b_0 \) is the constant value drift, \( b \) is the time-varying drift, \( w_a \) is the measurement noise, which can be regarded as white noise. Since the life time of the aircraft is generally around 0.5h, the model can be simplified to \( w_m = w_i + b_0 + w_a \).

The mathematical model of the accelerometer can also be established as shown in equation (7).

\[ a_m = a_b - g^b + e_a^a + v_a \]  

Among them, \( a_m \) is the accelerometer measurement value, \( a_b \) is the carrier acceleration, \( g^b \) is the projection of the gravity acceleration on the carrier, \( e_a^a \) is the accelerometer zero drift, \( v_a \) is white noise. Because the gravity field under the navigation system is: \( g^n = [0, 0, g]^T \), so \( g^b = C_n^b g^n \).

2.3 Equation of State and Measurement Equation
Assume that the state equation of the system model and the discrete form of the measurement equation are:

\[
\begin{align*}
X_{k+1} &= f(X_k) + W_k \\
Z_{k+1} &= h(X_k) + V_k
\end{align*}
\]  

Among them: \( X_{k+1} \) \( X_k \) are state variables at \( k+1 \) \( k \) respectively, \( Z_{k+1} \) is the measured value of the system at \( k+1 \) o'clock; \( f(*) \) is the nonlinear vector function of the state equation, and \( h(*) \) is the nonlinear vector function of the measurement equation; \( W_k \), \( V_k \) For uncorrelated zero-mean white noise, the variance matrix is \( Q_k \), \( R_k \) respectively.

The quaternion algorithm is used to realize the attitude calculation, and the quaternion is used as the system state variable. Because the gyroscope has drift error, in order to prevent the error accumulation, the state variable is amplified to \( X = [q_0, q_1, q_2, q_3, \Delta w_x, \Delta w_y, \Delta w_z]^T \), where \( \Delta w = [\Delta w_x, \Delta w_y, \Delta w_z]^T \) is the gyro drift. Combined with the attitude equation of the aircraft, the system state equation is:

\[ X_{k+1} = \Phi_d \cdot X_k + W_k \]  

Taking the output of the accelerometer and magnetometer as the measured value of the equation of the system, the measurement equation of the system is:

\[ Z_k = \begin{bmatrix}
    a_n \\
    m_n
\end{bmatrix} = \begin{bmatrix}
    C_n^b \\
    0
\end{bmatrix} \begin{bmatrix}
    a_n \\
    m_n
\end{bmatrix} = h(X_k) + V_k \]  

Among them, \( [a_n \ m_n] \) and \( [a_b \ m_b] \) are the values of acceleration and magnetic field strength measured under the navigation system and machine system respectively.
3.System model

3.1 UKF

UKF is a nonlinear Gaussian state estimator based on the minimum variance estimation criterion. It is based on the U transform and uses Kalman filter as the framework to approximate the posterior mean and the posterior covariance of the system state. A series of sample points (Sigma points) can be obtained with the UKF by deterministic sampling of the samples to approximate the probability density of the nonlinear system. Therefore, combined with the system model, the UKF algorithm can be summarized as:

(1) Initial state
\[ \hat{X}_0 = EX_0 \]  
\[ P_0 = E \left( (X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^T \right) \]

For \( k = 1, 2, 3, \ldots \), execution:

(2) Calculate Sigma point
\[ \chi_{k-1} = [\hat{X}_{k-1}, \hat{X}_{k-1} + \gamma(\sqrt{P_{k-1}}), \hat{X}_{k-1} - \gamma(\sqrt{P_{k-1}})] \]

It’s \( \gamma = \sqrt{n + \hat{\lambda}}, \ \hat{\lambda} = \alpha^2(n + \kappa) - n \).

(3) Time update
\[ \hat{X}_{k|k-1} = \frac{2}{2n + \hat{\lambda}} \sum_{i=0}^{2n} W_i^{(m)} Z_{k|k-1} \]

In \( W_i^{(m)} = \frac{1}{2(2n + \hat{\lambda})}, i = 1, 2, 3, \ldots, 2n; \ W_0^{(m)} = \frac{\hat{\lambda}}{2n + \hat{\lambda}} \).

(4) Measurement update
\[ \hat{X}_k = \hat{X}_{k|k-1} + K_k \left[ Z_k - \hat{X}_{k|k-1} \right] \]
\[ P_k = P_{k|k-1} - K_k P_{(zz)k|k-1} K_k^T \]

3.2 AFM-UKF

Although the attenuation memory filter can improve the filtering divergence caused by the initial value of the filtering to some extent, it is found in the simulation experiments that the selection of the attenuation factor has a great influence on the filtering effect, and the filtering precision of the attenuation memory will decreases as the attenuation factor increases. Moreover, the effect of estimation is not ideal when there is an unmeasurable disturbance in the system since the value of the attenuation factor is fixed throughout the estimation process. Therefore, an adaptive factor is introduced to form an adaptive attenuation memory UKF filter based on the attenuation memory filter. Adaptive filtering not only uses the measured value to correct the predicted value, also estimates and corrects the unknown or uncertain system model noise and statistical parameters of noise. Therefore, make the above UKF algorithm change, and (16) are changed to:

\[ P_{(zz)k|k-1}^* = \frac{1}{\alpha_k} \sum_{i=0}^{2n} W_i^{(c)} \left[ Z_{k|k-1} - \hat{Z}_{k|k-1} \right] \left[ Z_{k|k-1} - \hat{Z}_{k|k-1} \right]^T + R_k \]

\[ P_{(zz)k|k-1}^* = \frac{1}{\alpha_k} \sum_{i=0}^{2n} W_i^{(c)} \left[ X_{k|k-1} - \hat{X}_{k|k-1} \right] \left[ Z_{k|k-1} - \hat{Z}_{k|k-1} \right]^T \]

\[ P_k^* = \frac{1}{\alpha_k} P_k^{(c)} - K_k P_{(zz)k|k-1}^* K_k^T \]
The proper selection of the adaptive factor can not only balance the weight ratio of the state equation prediction and observation information, but also control the influence of the disturbance model of the state model on the filtering solution. The construction of $\alpha_k$ is:

$$
\alpha_k = \begin{cases} 
1 & \text{if } tr(V_k^{'T}V_k^{'T}) \leq tr(P_{zz_{kh-1}}) \\
\frac{tr(P_{zz_{kh-1}})}{tr(V_k^{'T}V_k^{'T})} & \text{if } tr(V_k^{'T}V_k^{'T}) > tr(P_{zz_{kh-1}})
\end{cases}
$$

(20)

$V_k^'$ is the prediction residual.

$$
V_k^' = Z_k - \sum_{i=0}^{2p} W_i^{(m)} Z_{k+i-1}
$$

(21)

It can be known from the above improvement of the UKF algorithm that when the initial value is unreasonable or the model has disturbance anomaly, $\alpha_k$ will be less than 1, that is, the contribution of the prediction information to the final filtering solution will be as small as possible; when the prediction information is obviously abnormal, $tr(V_k^{'T}V_k^{'T})$ will be very large. $\alpha_k$ will be close to 0, that is, the prediction information is completely discarded. Obviously, $\alpha_k$ can adaptively adjust the contribution of the $X_{k/k-1}$ pairs of filtering solutions based on the prediction residual $V_k^'$ using the measurement information $Z$.

4. Algorithm simulation experiment and result analysis

Simulation experiments were carried out by MATLAB in order to verify the effectiveness of the improved AFM-UKF algorithm. The initial conditions of the experiment are set as follows: the output sampling frequency of the gyroscope is 100Hz, the simulation time is 2500s, the pitch angle, roll angle and yaw angle are $[\theta, \phi, \psi] = [3^', 2', 3^']$, respectively, using the traditional UKF method and the AFM-UKF algorithm proposed in this paper and comparing them with the error angle of attitude.

![Figure 1. pitch angle error map](image)
Figure 2. roll angle error map

Figure 3 heading angle error map

Figure 1 shows the pitch angle estimation error, as shown in Figure 1, the accuracy of the two algorithms is equivalent when the error angle is a small angle. However, the AFM-UKF algorithm is more stable than the UKF algorithm with the accumulation of sampling time. Fig. 2 and Fig. 3 are the roll angle estimation error and the yaw angle estimation error respectively. It can be seen from Fig. 2 that the initial attitude angle error of the UKF algorithm is larger, and the improved algorithm of the filter divergence is more obvious. The filtering precision of the AFM-UKF algorithm in 3 is significantly higher and more stable.

The improved AFM-UKF algorithm also has better convergence speed than the UKF algorithm. It can be seen from Fig. 2 that the traditional UKF algorithm needs to achieve the AFM-UKF algorithm's roll angle and heading angle error under the premise of 100Hz sampling frequency. Left and right signal processing time. In Fig. 3, the heading angle error, the improved algorithm convergence speed and stability performance are obviously improved.

5. CONCLUSIONS
In view of the attitude calculation problem of the aircraft, the Euler angle method cannot be in full attitude work due to the existence of singular points, and the quaternion method is used instead of the Euler angle method to realize the attitude calculation; In order to improve the filtering precision caused by the gyroscope drift error and the filter divergence caused by the abnormal perturbation of the system model. The UKF algorithm is proposed to be the adaptive attenuation memory filtering algorithm. The simulation results show that under the given conditions, so the AFM-UKF algorithm improves the system filtering accuracy, suppresses the drift error caused by the gyroscope, improves the filter divergence caused by the model disturbance, and enhances the robustness of the system.
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References
[1] Shaohua Wang, Ying Yang. Quadrotor Aircraft Attitude Estimation and Control Based on Kalman Filter [C]// Proceedings of the 31st Chinese Control Conference. Hefei, China: IEEE, 2012: 5634-5639.
[2] Ye Weifeng, Feng Enxin. Two-wheel vehicle attitude stabilization method based on quaternion and Kalman filter[J]. Journal of Transduction Technology, 2012, 25(4): 524-528.
[3] Deng Hong, Guangbin Liu, Haoming Chen. Application of EKF for Missile Attitude Estimation based on "SINS/CNS" Integrated Guidance System [C] // Systems and Control in Aeronautics and Astronautics, 2010 3rd International Symposium on Harbin, China: 1101-1104.
[4] Jia Ruicai. Low-cost MEMS Attitude Estimation Algorithm Based on Quaternion EKF[J]. Journal of Transduction Technology, 2007, 27(1): 90-95.
[5] Guo Xiaohong, Yang Zhong, Chen Wei, et al. Application of EKF and complementary filter in flight attitude determination[J]. Sensors and Microsystems, 2011, 30(11): 149-152.
[6] Lin Zhao, Qi Nie, Qiafen Guo. Unscented Kalman Filtering for SINS Attitude Estimation [C]// IEEE International Conference on Control and Automation. Guangzhou, China: 2007:228-232.
[7] Yue Pan, Ping Song, Kejie Li. Attitude Estimation of Miniature Unmanned Helicopter Using Unscented Kalman Filter [C]// International Conference on Transportation, Mechanical and Electrical Engineering. Changchun, China: 2011:1548-1551.
[8] Chen Jizheng, Yuan Jianping, Fang Qun. Attitude Estimation Algorithm Based on Modified Rodrigues Parameters and UKF[J]. Journal of Astronautics, 2008, 29(5): 1622-1626.
[9] Xu Jingshuo, Qin Yongyuan, Peng Rong. Research on the selection method of adaptive Kalman filter fading factor[J]. System Engineering and Electronics, 2004, 26(11):1552-1554.
[10] Gao Qingwei, Zhao Guorong, Wu Fang, et al. Application of Attenuation Memory Adaptive Filtering in Transducer Alignment of Inertial Navigation System[J]. Systems Engineering and Electronics, 2010, 32(12): 2648-2651.