Universal role of migration in the evolution of cooperation

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We study the role of unbiased migration in cooperation in the framework of spatial evolutionary game on a variety of spatial structures, involving regular lattice, continuous plane and complex networks. A striking finding is that migration plays a universal role in cooperation, regardless of the spatial structures. For high degree of migration, cooperators cannot survive due to the failure of forming cooperator clusters to resist attacks of defectors. While for low degree of migration, cooperation is considerably enhanced compared to static spatial game, which is due to the strengthening of the boundary of cooperator clusters by the occasionally accumulation of cooperators along the boundary. The cooperator cluster thus becomes more robust than that in static game and defectors nearby the boundary can be assimilated by cooperators, so the cooperator cluster expands, which facilitates cooperation. The general role of migration will be substantiated by sufficient simulations associated with heuristic explanations.

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Cooperation is fundamental to biological and social systems. Understanding factors that facilitate and hamper cooperation is a significant issue. In the framework of evolutionary games, a number of mechanisms in favor of cooperation have been found, such as costly punishment [1, 2], reputation [3, 4] and social diversity [5–7]. Quite recently, the role of migration in cooperative behavior has drawn growing interests [8–13], because of the fact that migration is a common feature in nature and society. For example, millions of animals migrate in the savannas of Africa every year, and thousands of people travel among different countries every day. In this regard, Vainstein et al. [10] considered a scenario that individuals can move to neighboring sites on a two-dimensional lattice randomly with some probability. In particular, it is found that such movement can maintain and even enhance cooperation compared to the absence of migration. More recently, Meloni et al. studied evolutionary games composed of mobile players on a continuous plane [13]. Their results showed that cooperation can survive provided that both the temptation to defect and the velocity at which individuals move are not too high. Beyond random migration, Helbing and Yu proposed a success-driven migration strategy which is spurred by the pursuit of profit as a nature of individuals [11]. Specifically, individuals tend to move to neighboring site with the highest estimated payoffs. Interestingly, such migration results in the outbreak of cooperation in a noisy environment.

Although it has been demonstrated that migration can promote cooperation in evolutionary games on some regular spatial structures [10–12], the role of migration on other kinds of structures, for instance, complex networks, is unknown yet. A natural concern is then whether migration plays some general role in cooperation, regardless of underlying structures, or there exists dependence of the role of migration on structures? To address this issue, in this paper we incorporate random migration in evolutionary games on a variety of spatial structures, involving continuous space, regular structure, and typical complex topologies. Strikingly, we find that the role of random migration in promoting cooperation is universal, regardless of different structures. This is somewhat counterintuitive in the sense that mobility of individuals may weaken the stability of cooperation clusters which are key for the survival of cooperators. However, we will substantiate the positive effect of random migration on cooperation by intensive simulations and provide convinced explanations for the underlying mechanisms.

To be concrete, we use the Prisoner’s Dilemma [14] to carry out our researches. In principle, the Prisoner’s Dilemma is a game played by two players, each of whom chooses one of two strategies, cooperation or defection. They both receive payoff $R$ upon mutual cooperation and $P$ upon mutual defection. If one defects while the other cooperates, cooperator receives $S$ while defector gets $T$. The ranking of the four payoff values is: $T > R > P > S$. Thus in a single round of the Prisoner’s Dilemma it is best to defect regardless of the opponent’s decision. The Prisoner’s Dilemma has attracted much attention in theoretical and experimental studies of cooperative behavior. Following common practice [15], we set $T = b (1 < b < 2)$, $R = 1$, and $P = S = 0$, where $b$ represents the temptation to defect.

To explore the role of migration, we resort to the spatial game in which individuals are placed on some spatial structures. Since the combination of spatial structures into evolutionary games by Nowak and May [15],
there has been much interest in revealing the influence of population structures on cooperation, ranging from regular lattices to complex networks [16–30]. In the spatial games, interactions among individuals are restricted within immediate neighbors and usually neighbors of an arbitrary individual keep fixed. While in the presence of migration, neighboring individuals can be changed by encountering different partners as time goes on. In the seminal works of Vainstein et al. [10] and Meloni et al. [13], random migration has been considered in spatial games on a two-dimensional lattice and on a continuous plane, respectively. Inspired by these original researches, we extend migration to regular structure to complex networks and uncover the general role of migration in promoting and hampering cooperation.

Let us first consider individuals moving on a continuous square plane (SP) with periodic boundary conditions. Initially, individuals are randomly located on a square plane and cooperators and defectors with equal percentage are randomly distributed on the plane. At each time step, each individual plays the game with individuals falling in a circle of radius \( q \) that centered at his/her current position. Individuals synchronously update their strategies according to a best-takes-over reproduction, that is, each individual compares his/her payoff with his/her neighbors and update his/her strategy by following the one (including himself/herself) with the greatest payoff. We have examined that the qualitative results shown below are robust, regardless of detailed updating rules, such as the finite population analogue of the replicator dynamics [14] and Fermi update rule [31, 32]. After the strategy updating process, individuals move to new locations with random directions of motion in migration speed \( v \). The absolute value of \( v \) defines the distance an individual can move in a typical time step. Simulation results for the fraction of cooperators for different migration speeds are shown in Fig. (a). We can see that compared to the static case \((v=0)\), cooperation is enhanced in a wide range of temptation to defect when individuals move slowly \((v=0.04)\). On the other hand, fast moving \((v=1)\) leads to complete extinction of cooperators, analogous to the situation arising in the well-mixed population.

Next, we study individuals migrating on various network models, including square lattices (SL), random graphs (RG) [33], small-world networks (SW) [34] and scale-free networks (SF) [35]. Initially, each node of the network is occupied by an individual and individuals with two strategies (cooperators or defectors) are randomly distributed. At each time step, each individual plays the game with individuals sitting on the same node and neighboring nodes. Individuals synchronously update their strategies according to the best-takes-over reproduction and then each individual jumps to a randomly chosen neighboring node with probability \( p \) (a node can be occupied by more than one individual). Results are shown in Figs. (b)-(e). As compared to the static case \((p=0)\), low migration probabilities (e.g. \( p = 0.001 \)) promote cooperation in a wide range of the temptation to defect \( b \) (except for large \( b \) on square lattices and ran-
the migration speed \( v \)

**FIG. 2:** (Color online) Fraction of cooperators as a function of \( L \) \( N \) individuals migrate on the square plane (SP) of linear size \( L \). We have also investigated the dependence of fraction of cooperators on the migration speed \( v \) or the migration probability \( p \). (a) Individuals migrate on the square plane (SP) of linear size \( L = 20 \). The temptation to defect \( b = 1.35 \) and the interaction radius \( q = 1 \). (b) Individuals migrate on various networks. The temptation to defect \( b = 1.5 \). Average connectivity \( z = 4 \) for SL and SF, \( z = 6 \) for RG and SW. The population size is 1024. Each data point depicted corresponds to an average over 1,000 simulations; that is, 100 runs for 10 different realizations of the same class of graph.

dom graphs, where the fraction of cooperators is lower than that of \( p = 0 \), similar to the results on the continuous plane. While for high migration probability \( p = 1 \), defectors dominate the whole population.

From Fig. 1 we can find that by the comparison with spatial game in the absence of migration, low migration speeds/probabilities can considerably promote cooperation whereas high migration speed/probability facilitates defection, which are qualitatively regardless of underlying structures. We have also investigated the dependence of fraction of cooperators on the migration speed \( v \) and probability \( p \) with fixing the value of temptation to defect \( b \). As exhibited in Fig. 2 as \( v \) and \( p \) increase, the fraction of cooperation monotonously decreases. It has been known that in spatial games, cooperators can survive by forming clusters \([31, 32, 37]\), in which the benefits of mutual cooperation can outweigh losses against defectors, thus enable cooperation to be maintained. Combining Figs. 1 and 2 we can find that the effect of migration on cooperation is twofold. For high migration speed/probability, cooperation is inhibited since cooperator clusters can be hardly formed induced by the frequent change of neighbors. Without the protection of cluster structures, cooperator can hardly survive. For the low degree of migration, it is not easy to figure out the influence of migration to cooperation. A heuristic explanation is that after the construction of cooperator clusters, small perturbation along the boundary by migration can trigger the expansion of cooperator clusters and enhance the fraction of cooperation.

To intuitively understand the effect of perturbation around the cooperator cluster on cooperation, we construct a crossed cooperator cluster (including 5 cooperators) surrounded by defectors on a square lattice (see Fig. 3). The temptation to defect \( b = 1.4 \). If individuals are immobile, the crossed cooperator cluster is stable and keeps unchanged. In the presence of migration, situations arising at the cluster boundary can be classified into four types: (1) a cooperator at the boundary enters the defector cluster; (2) a defector at the boundary intrudes into the cooperator cluster; (3) a defector moves away from boundary within its defector cluster and (4) a cooperator moves away from boundary within its cooperator cluster. In case (1), the irruptive cooperator transfers to a defector; In case (2), the irruptive defector changes to a cooperator; In case (3), nothing happens. Cases (1) to (3) do not drastically affect fraction of cooperator in the system (not shown here). However, in case (4), the territory of the cooperator cluster expands to other regions of the square lattice and the number of cooperators increases from 5 to 12, as shown in Fig. 3b-3e. It is thus the rising of case (4) that promotes the prevalence of cooperation in the population. In general, this scenario is representative of the strengthening of the cooperator cluster boundary by multi-cooperators at the same node (the density of cooperators is augmented along the boundary). A direct result is that the payoffs of cooperator along the boundary are increased and defectors nearby the boundary will be assimilated. As a result, cooperator clusters expand and cooperation is enhanced.

To visually observe how low degree of migration affects the evolution of cooperator clusters and defector clusters, we initially set some cooperators in the middle region of a square plane, while defectors are located on other regions. From Figs. 4a-c, one can find that, for low migration speed, the cooperator cluster gradually expands as time step \( t \) increases and cooperators dominate the whole population in the end. For the static case in which individuals do not move, the cooperator cluster keeps almost unchanged [see Fig. 4d].

In summary, we have studied the role of random migration in cooperation in the framework of spatial prisoner’s dilemma game \([39]\) on a variety of spatial structures. Our findings are that although high degree of migration by disabling the formation of cooperator clusters results in the extinction of cooperation, low degree of migration
can considerably enhance cooperation by increasing the cooperator density along the boundary of the cooperator cluster. Due to the accumulation of cooperators along the boundary, the benefits of mutual cooperation outweigh losses against defectors nearby the boundary, this thus not only enables cooperation within the cluster to be maintained, but also induces the expansion of the cooperator cluster, in contrast to the static spatial game. The strengthening at the boundary of cooperator clusters induced by the small degree of migration plays the key role in the enhancement of cooperation, regardless of the underlying structure on which the evolutionary game takes place. Our work may inspire further effort in exploring the effect of migration behavior on not only game-based cooperation but also other dynamical processes, such as epidemic spreading and information routing in ad-hoc networks.

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[38] Note that for low migration probability $p$, the system has sufficient time to evolve to another stable state after migration.

[39] We have found that, the role of migration is the same for the snowdrift game.