Optimization and selection of Galileo triple-frequency carrier linear combination

Jun Wang1, Xurong Dong1, Wei Fu2, Di Yan3 and Zengkai Shi1

Abstract
The triple-frequency linear combination with a low noise, a long wavelength, and a weak ionosphere is beneficial to effectively eliminate or weaken the common errors, advance the reliability of cycle slip detection and repair, and speed up the convergence time of fixed ambiguity. By establishing the Galileo triple-frequency carrier linear combination model, three types of linear combinations are derived: Geometry-free (GF) combinations, minimum noise (MN) combinations, and ionosphere-free (IF) combinations. The geometric relationships of these linear combinations are displayed in the form of image. The results indicate that the angle formed by the IF combinations and the MN combinations is between 75.02° and 86.01°, which also illustrates that it is more difficult to meet the carrier phase combinations with a low noise and a weak ionosphere. Moreover, to guarantee the integer cycle characteristics of ambiguity, the combination coefficient must be an integer. Galileo triple-frequency linear combination is solved utilizing the extremum method. To sum up, the sum of the coefficients of the extra wide lane (EWL) combinations and wide lane (WL) combinations is zero, and the sum of the coefficients of the narrow lane (NL) combinations is one. \((0, 1, -1)\) is the optimal triple-frequency linear combination in Galileo. Three independent linear combinations are selected separately from the EWL, WL, and NL to jointly solve the integer ambiguity. Further, it creates a prerequisite for high-precision and real-time kinematic positioning.

Keywords
Galileo, extra wide lane, extremum method, ionospheric delay amplification factor, triple-frequency, linear combination, integer ambiguity resolution

Date received: 17 September 2020; accepted: 24 November 2020

Introduction
With the stable operation of Galileo and BDS systems and the promotion of the modernization and transformation of GPS and GLONASS systems, global satellite navigation system (GNSS) has developed multi-system and multi-frequency integrated positioning and navigation. The development of GNSS technology also provides a broad space for the prosperity of smart city, traffic control, CORS, and other applications.1,2 GNSS broadcasts more than three frequency carrier signals at present. On July 25, 2018, Europe successfully launched four Galileo satellites with full operational capability. So far the number of Galileo space constellation satellites has reached 26, the 26 satellites expand the global coverage of Galileo and achieve full operational capacity.3 The multi-frequency carrier phase of the Galileo system can form triple-frequency linear combination with smaller ionospheric delay error, lower observation noise error and longer wavelength through certain linear combination.4 Using these excellent performance linear combination, the different linear combination can be selected according to the different application scenarios, which can effectively eliminate or weaken many kinds of error, and speed up the convergence time of fixed ambiguity resolution.5,6

Considering the advantages of linear combination observations and the continuous development of multi-frequency technology, Researchers have performed many studies on triple-frequency carrier linear combination theory. Han and Chris7 proposed to use the new civil frequency signal L5 of GPS to constitute the triple-frequency linear combination. Some useful linear

1School of Space Information, Space Engineering University, Beijing, China
2Unit 32039, Beijing, China
3Beijing Institute of Remote Sensing Information, Beijing, China

Corresponding author:
Jun Wang, School of Space Information, Space Engineering University, Bayi Road 1, Huairou District, Beijing 101416, China.
Email: 42199718@qq.com

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
combinations are given. Feng⁸ presented that three optimal carrier linear combinations of GNSS achieved a more reliable triple-frequency ambiguity resolution strategy. Cocard et al.⁹ proposed to use GPS new carrier signal L5 for three-frequency carrier linear combination, and analyzed the characteristics of the triple-frequency linear combination. Urquhart¹⁰ presented to obtain the optimal three-frequency linear combination, the derivation of the characteristics for the linear combinations is performed including the second and third order ionospheric delay amplification factors. Zhang and He⁵ and Richert and El-Shemy¹¹ studied many kinds of triple-frequency carrier linear combinations. The authors explored different types of linear combinations and verified the geometric relationship between them. Wu¹² focused on error characteristics, and listed a series of the typical combinations. All the above studies established a theoretical basis to calculate ambiguity of multi-frequency applications. Liu et al.¹³ applied multi-frequency combination observation with excellent GPS performance to solve cycle slip. Li et al.¹⁴,¹⁵ studied GF and IF triple-frequency combination observations and solved the problem of the triple-frequency AR of the medium and long baseline scenarios. Yu et al.¹⁶ resolve the triple-frequency combination coefficient based on the least square method. Li et al.¹⁷,¹⁸ proposed to use the function extremum method to solve the GNSS triple-frequency optimal linear combination. Huang et al.¹⁹ investigated the self-adaptive clustering algorithm, and experiment verified that the algorithm offers a lower workload and great efficiency and can still find the best combination. Wang and Liu²⁰ and Duong et al.²¹ produced a comprehensive study of the inter-frequency combination of four Galileo carrier phase observations, in which several typical linear combinations were calculated. Wei et al.²²,²³ Zhang et al.²³ and Fan et al.²⁴ utilized the function extremum method to develop integral coefficients of triple-frequency linear combinations for BDS-2, GPS, and BDS-3 and verified the optimal triple-frequency carrier linear combinations of BDS-3 for four civil frequencies. He et al.²⁵ Meng and Li.²⁶ and Fanjun et al.²⁷ adopted the fuzzy C-means clustering algorithm to optimize data for BDS triple-frequency carrier combination observations. Then, through the matrix transformation method and measured data, the integer ambiguity of the optimized combination was resolved.

Different linear combination coefficients correspond to different combination observation wavelength, ionospheric delay effect and noise characteristics. The extremum method studies systematically the four triple-frequency carrier combination in Galileo (E1, E6, E5a, and E5b) through mathematical derivations. This method can obtain the Galileo linear combinations of optimal integral coefficients, which can guarantee the integer characteristics of ambiguity and improve the efficiency of fixed ambiguity resolution.²⁸

Galileo triple-frequency linear combination model

The original carrier phase observations can form new combination observations via mathematical operations.²⁹ Some linear combination observations can have the characteristics of a long wavelength, low noise, weak ionospheric delay, which are convenient simplifying data processing model and fixing the integer ambiguity.

Since the Galileo satellite navigation system began its initial operation service on December 15, 2016, as of August 10, 2020, 4 in-orbit validation satellites and 22 full operational capability satellites have been deployed in the Galileo constellation. Among them, PRN E20 and E22 cannot be used, E14 and E18 are under testing, and the other satellites are in normal operation. All Galileo satellites in normal operation broadcast signals at four civil frequencies: E1, E6, E5a, and E5b. Therefore, four groups of triple-frequency carrier linear combination can be acquired. In December 2016, the European Commission released the European GNSS (Galileo) Open Service Signal-In-Space Interface Control Document (OS SIS ICD) Issue 1.3.³⁰ The satellite navigation signals of Galileo are displayed in Figure 1.

Taking the triple-frequency carrier signals as a group, four groups of triple-frequency carrier signals can be obtained. To unify the number of the following formula, the triple-frequency linear combination is arranged in the frequency values from largest to smallest, satisfying $f_1 > f_2 > f_3$ and $f_1 - f_2 > f_2 - f_3$. Four civil frequencies signals of Galileo comprise four groups of triple-frequency carrier combinations. Each group of carrier frequency is expressed by an integer multiple of the reference frequency $f_0 = \gcd(f_1, f_2, f_3) = 10.23\text{MHz}$, where $\gcd()$ is the

![Figure 1. Satellite navigation signal from Galileo.](image-url)

Figure 1. Satellite navigation signal from Galileo.
greatest common divisor operator, reference wavelength \( \lambda_0 = c/f_0 = 29.31 \text{m} \). Three frequency values \((f_1, f_2, \text{and } f_3)\) of Galileo are integral multiple of the reference frequency \( f_0 \), with \( f_1 = n_1 f_0 \), \( f_2 = n_2 f_0 \) and \( f_3 = n_3 f_0 \). \((n_1, n_2, n_3)\) is positive integers. Here, \( \text{gcd}(n_1, n_2, n_3, n_1 n_2) \) obtains different values according to different linear combinations. Table 1 lists the corresponding parameters for the Galileo triple-frequency linear combination. If there is no special statement, combination 1 (E1, E6, E5b) is usually taken as the research object.

When the pseudo-range neglects the ionospheric high order term and multipath effect, the pseudo-range \( \rho \) contains non-dispersive errors (tropospheric delay error, receiver clock error). The general expression of the single frequency carrier measurement in cycles is as follows:

\[
\varphi_i = \frac{\rho}{\lambda_i} = \left(\frac{f_i}{f_0}\right)^2 \frac{f_i}{\lambda_i} + N_i + \varepsilon_i \tag{1}
\]

Where \( \lambda_i \) is the combination wavelength in frequency \( f_i \); \( \lambda_i \) is the wavelength in frequency \( f_i \); \( q_i = (f_i/f_0)^2 = (\lambda_i/\lambda_1)^2 \) is the ionospheric effect coefficient; \( N_i \) is the integer ambiguity in frequency \( f_i \); and \( \varepsilon_i \) is the combination observation noise in frequency \( f_i \). After the linear combinations, each component can be obtained by the following formulas:

\[
\varphi_c = \sum_{i=1}^{3} a_i \varphi_i \tag{2}
\]

\[
N_c = \sum_{i=1}^{3} a_i N_i \tag{3}
\]

\[
f_c = \sum_{i=1}^{3} a_i f_i = \left(\sum_{i=1}^{3} a_i n_i\right) f_0 = k f_0 \tag{4}
\]

\[
\lambda_c = \frac{c}{f_c} = \lambda_0 \left(\sum_{i=1}^{3} a_i n_i\right)^{-1} = \lambda_0 / k \tag{5}
\]

\[
q_c = a_1 + a_2 \frac{f_1}{f_2} + a_3 \frac{f_1}{f_3} \tag{6}
\]

\[
\hat{q}_c = \frac{\lambda_c}{\lambda_1} \left( a_1 + a_2 \frac{f_1}{f_2} + a_3 \frac{f_1}{f_3} \right) \tag{7}
\]

\[
n_{\text{noise}} = \sqrt{\sum_{i=1}^{n} a_i^2} \tag{8}
\]

Table 1. The corresponding parameters of the Galileo triple-frequency signal.

| Triple-frequency combination number | Signal \((f_1, f_2, f_3)\) | Reference frequency \(f_0\) (MHz) | Frequency coefficient \((n_1, n_2, n_3)\) | Reference wavelength \(\lambda_0\) (m) | Greatest common divisor operator \(\text{gcd}(n_1 n_2, n_1 n_3, n_1 n_2)\) |
|-----------------------------------|-----------------|-----------------|-----------------|-----------------|-------------------------------|
| 1                                 | E1, E6, E5b     | 10.23           | (154, 125, 118) | 29.31           | 2                             |
| 2                                 | E1, E6, E5a     | 10.23           | (154, 125, 115) | 29.31           | 2                             |
| 3                                 | E1, E5b, E5a    | 10.23           | (154, 118, 115) | 29.31           | 2                             |
| 4                                 | E6, E5b, E5a    | 10.23           | (125, 118, 115) | 29.31           | 5                             |

The relationship between the wavelength \( \lambda_c \) and the lane number \( k \) of the Galileo triple-frequency linear combination is shown in Figure 2. In Figure 2, the maximum combination wavelength of Galileo is 29.31 m, and the asterisk in the figure indicates the corresponding combination wavelength when the lane number takes different integers. The combination wavelength \( \lambda_c \)

\[
n_{\text{noise}} = \lambda_c \sqrt{\sum_{i=1}^{n} a_i^2} \tag{9}
\]

Where \( \varphi_c \) is the combination carrier observation in cycles; \( a_1, a_2 \) and \( a_3 \) are the combination coefficients; \( N_c \) is the integer ambiguity; \( f_c \) is the combination frequency corresponding to the combination observation \( \varphi_c \); \( k \) is the lane number; \( q_c \) is the ionospheric delay amplification factor in cycles; \( \hat{q}_c \) is the ionospheric amplification factor in meters; \( n_{\text{noise}} \) is the noise amplification factor in cycles; and \( \hat{n}_{\text{noise}} \) is the noise amplification factor in meters.

**Characteristic analysis of triple-frequency linear combinations**

The purpose of multi-frequency linear combination is to acquire the excellent performance linear combination with a long combination wavelength, a weak ionospheric effect, and a low observation noise. However, there are a fewer linear combinations that can satisfy the above the three conditions. To obtain the optimal triple-frequency linear combinations of Galileo, the geometric relations of the four types of linear combinations are studied.

**Long wavelength combinations**

Taking combination 1 as an example, the coefficients of the carrier signal E1, E6, and E5b are \( n_1 = 154, n_2 = 125, \) and \( n_3 = 118 \).

\[
\lambda_c = \lambda_0 / (a_1 n_1 + a_2 n_2 + a_3 n_3) = \lambda_0 / k \tag{10}
\]

The lane number is a special linear combination that characterizes the wavelength.\(^9\) Formula (10) shows that the wavelength \( \lambda_c \) is inversely proportional to the lane number \( k \). \( k = 1 \) indicates that the maximum wavelength \( \lambda_c = \lambda_0 = 29.31 \text{m} \) for the integral coefficients linear combination.

The relationship between the wavelength \( \lambda_c \) and the lane number \( k \) of the Galileo triple-frequency linear combination is shown in Figure 2. In Figure 2, the maximum combination wavelength of Galileo is 29.31 m, and the asterisk in the figure indicates the corresponding combination wavelength when the lane number takes different integers. The combination wavelength \( \lambda_c \)
and the lane number \( k \) have an inverse proportional constraint relationship, and the smaller the lane number, the larger the wavelength. The wide lane combinations signify a smaller lane number, while the wide lane combinations indicate combinations with longer wavelength.

### Geometry-free (GF) combinations

Non-dispersive errors (tropospheric delay error, receiver clock error) is unrelated to the frequency. However, the non-dispersive error of the triple-frequency carrier combination in cycles is

\[
T = l \frac{c}{k}.
\]

This formula illustrates that the longer the combination wavelength, the smaller the tropospheric effect. The geometry-free combination is obtained by Formula (5):

\[
T_c = T = l \frac{c}{k} = l \frac{c}{l_0} = T_c\text{gcd}(n_2n_3, n_1n_3, n_1n_2) = 0
\]

When \( T_c = 0 \), the data can be substituted into the following formula:

\[
k = 154a_1 + 125a_2 + 118a_3 = 0
\]

The GF combinations are a three-dimensional space plane determined by satisfying Formula (12). The plane is determined by the combination coefficient \((a_1, a_2, a_3)\). The closer the combination coefficient is to the plane formed by the GF linear combinations, the smaller the tropospheric delay error.

By comparing Formulas (10) and (11), it can be concluded that the combinations of GF and long wavelength can establish a close relationship through the lane number \( k \). This GF combination is positively proportional to the lane number, while the wavelength combination is inversely proportional to the lane number. There is a positive correlation between the GF combinations and the long wavelength combinations.

### Ionosphere-free (IF) combinations

In high precision positioning, the ionospheric effect is the main factor for the fixed ambiguity. To eliminate or mitigate the ionospheric delay error, it is important to find a group of linear combination observations that can eliminate the influence of the ionosphere. The triple-frequency linear combination can accelerate the fixed efficiency of ambiguity resolution. Below, we discuss the Galileo triple-frequency IF combinations. When the ionospheric amplification factor \( q_c \) in cycles is zero:

\[
q_c = a_1 + a_2f_1/f_2 + a_3f_1/f_3 = (a_1n_2n_3 + a_2n_1n_3 + a_3n_1n_2)/(n_2n_3)
\]

\[
Q = q_c\text{gcd}(n_2n_3, n_1n_3, n_1n_2) = 0
\]

Where \( Q \) is the ionospheric layer number; \( \text{gcd}(n_2n_3, n_1n_3, n_1n_2) \) is the greatest common divisor operator; and \( Q = \text{gcd}(n_2n_3, n_1n_3, n_1n_2) = a_1n_2n_3 + a_2n_1n_3 + a_3n_1n_2 \). The combination satisfying Formula (13) is a set of three-dimensional space planes that is determined by the combination coefficients \((a_1, a_2, a_3)\). These planes are parallel to each other. Next, we insert the corresponding values of \( f_1, f_2 \) and \( f_3 \) into Formula (13):

\[
q_c = a_1 + 1.232a_2 + 1.305a_3 = 0
\]

The IF combination for Galileo system combination 1 is a spatial plane determined by the combination coefficients \((a_1, a_2, a_3)\). If the combination coefficients cannot meet the requirements for being in the plane, the carrier linear combination closes to the plane as far as possible.
In this way, the influence of the ionospheric delay error can be effectively reduced.

**Minimum noise (MN) combinations**

The noise of the original carrier phase observation for Galileo is generally \( \varepsilon_0 = 0.01 \text{cycle} \) of its wavelength. The noise of the three frequencies in cycles is \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_0 = 0.01 \text{cycle} \). The combination observation noise in meters can be attained from Formulas (8) and (9):

\[
e_c = n_{\text{noise}} \varepsilon_0 = \sqrt{a_1^2 + a_2^2 + a_3^2 \varepsilon_0} \tag{15}
\]

\[
\hat{e}_c = \hat{n}_{\text{noise}} \varepsilon_0 = \lambda_c/\lambda_1 \sqrt{a_1^2 + a_2^2 + a_3^2 \varepsilon_0} \tag{16}
\]

As shown in Formula (16), when the wavelength \( \lambda_c \) of the combination observation is larger than the carrier wavelength \( \lambda_1 \), there are some linear combination observations that the noise of the combination in meters is reduced. The expression of the minimum observation noise in meters can be acquired by substituting \( \lambda_c \) into Formula (16):

\[
\hat{n}_{\text{noise}} = \frac{\sqrt{a_1^2 + a_2^2 + a_3^2}}{a_1n_1 + a_2n_2 + a_3n_3} n_1 \rightarrow \min \tag{17}
\]

The combination satisfying Formula (17) is a straight line, and the expression satisfying that line is

\[
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\end{bmatrix} = t
\begin{bmatrix}
154 \\
125 \\
118 \\
\end{bmatrix} \tag{18}
\]

where \( t \) is any real number and Formula (18) is a three-dimensional line determined by the minimum noise.

**Geometric relationships of the three types of linear combinations**

To comprehensively understand the characteristics of the real coefficients linear combinations for Galileo, we next proceed a detailed analysis of the long wavelength combinations, IF combinations, GF combinations, and MN combinations and then discuss the geometric relationship between these linear combinations.

1. The angle formed by IF combinations and GF combinations

\[
\varphi = \cos^{-1} \left( \frac{q_c \cdot T_c}{||q_c|| ||T_c||} \right) = 13.01^\circ \tag{19}
\]

2. The angle formed by GF combinations and MN combinations

\[
\gamma = \cos^{-1} \left( \frac{\hat{n}_{\text{noise}} \cdot T_c}{||\hat{n}_{\text{noise}}|| ||T_c||} \right) = 90^\circ \tag{20}
\]

Table 2. The angle between the three types of combinations.

| Combination number | IF and TF \( \varphi \) | TF and MN \( \gamma \) | IF and MN \( \theta \) |
|-------------------|-----------------|--------------------|------------------|
| 1                 | 13.01°          | 90°                | 76.99°           |
| 2                 | 13.95°          | 90°                | 76.05°           |
| 3                 | 14.98°          | 90°                | 75.02°           |
| 4                 | 3.99°           | 90°                | 86.01°           |

Table 2 shows that the angle between the plane composed of the IF combination and the GF combination is small, with a value between 3.99° and 14.98°. According to the principle of correlation, it is easy to simultaneously obtain a combination of the ionospheric delay and non-dispersive error with low influence.

To illustrate the geometric relationship of two types linear combinations more intuitively, Figure 3 displays the distribution of the two planes in a spatial rectangular coordinate system. The angle between the two planes is 13.01°. The angle between the two planes is small, which indicates that there is a strong correlation between them, and the changing trend of the combinations is basically the same. Because there is a positive correlation between the GF combinations and long wavelength combinations, it is easy to acquire the combinations of weak ionospheric delay and long wavelength at the same time.

Figure 4 indicates the spatial distribution of the IF combinations and the MN combinations of Galileo system combination 1 (E1, E6, E5b); the angle between the two is 76.99°. Because this angle is very large, it is difficult to meet the linear combination of the weak ionospheric effect and low noise at the same time. The ideal combination can only be achieved near the intersection of the IF plane and the MN line. For the other three
combinations of Galileo, Table 2 shows that the angle formed by IF combinations and MN combinations lies between 75.02° and 86.01°, which is similar to that of combination 1.

Formulas (12) and (18) calculate that the angle formed by the GF combinations and the MN combinations is 90°, which is orthogonal. This conclusion is also verified in Table 2, which indicates that there is no geometric correlation between them. Similarly, the long wavelength combinations accompany the greater observation noise.

Among the four groups of Galileo triple-frequency linear combinations, the angle formed by IF combinations and the MN combinations of combination 4 (E6, E5b, E5a) is 86.01°, which is almost vertical. It is more difficult to satisfy the linear combination with a weak ionospheric effect and low noise. Therefore, the combination 4 is not an ideal Galileo triple-frequency linear combination.

Figures 3 and 4 show results of the analysis of the relationship between the three types of linear combinations from the perspective of spatial geometry. Since there is a positive correlation between the long wavelength combinations and the GF combinations, the characteristics of the four types of carrier phase combinations are clearly constructed based on the spatial geometric distribution. These characteristics can help us choose different carrier phase linear combinations according to different baseline lengths, application purposes, and application occasions.

Optimization and selection triple-frequency linear combinations

To understand the general characteristics of the Galileo triple-frequency combination, the paper analyzed four types of real coefficients linear combination in the previous section, and obtained the geometric relations of the long wavelength combinations, IF combination, GF combination, and MN combination. It is conducive to optimize the selection of triple-frequency linear combination. To guarantee the integer cycle characteristics of ambiguity resolution \( N_c = a_1N_1 + a_2N_2 + a_3N_3 \), the combination coefficient \( (a_1, a_2, a_3) \) must be an integer. The paper discuss the optimization and selection of the Galileo triple-frequency integral coefficients linear combination.

Selecting the linear combination using the extremum method

Galileo triple-frequency carrier phase observations can form infinite linear combinations according to different conditions \( (a_1, a_2, a_3) \). However, only some linear combinations have practical significance in application, it can speed up the convergence time of fixed ambiguity. These linear combinations should meet the following four conditions:

1. The combination coefficients \( (a_1, a_2, a_3) \) must be an integer to maintain the integer characteristic of ambiguity \( n_c \) and thus determine the integer ambiguity correctly;
2. The combination observation \( \varphi_c \) must have a longer wavelength \( \lambda_c \)—that is, the lane number \( k \) must be smaller;
3. The combination observation \( \varphi_c \) must have a small ionospheric delay error \( I_c \)—that is, the ionospheric amplification factor \( q_c \) or the ionospheric layer number \( Q \) must be small;
4. The combination observation \( \varphi_c \) must have smaller observation noise \( e_c \)—that is, the noise amplification factor \( n_{noise} \) must be smaller.

Contradiction and unity go hand in hand, and this applies to linear combinations. In a perfect world, we would look for combinations that have all of the desirable characteristics—that is, a combination that reduces wide lane, receiver noise, and ionospheric refraction. This is represented by the green area in Figure 5, which illustrates an ideal linear combination.
The above four conditions make a qualitative analysis of the selection of meaningful carrier phase linear combination observations, and then make a quantitative analysis of all possible linear combinations of the integer coefficients. As shown in Formula (22), the above four conditions are stated in the form of a function expression.

\[
\begin{align*}
S_1 &= 3.7 \times 10^{-3} k + 5.9383 \times 10^{-4} Q \\
S_2 &= 3.7 \times 10^{-3} k + 1.5030 \times 10^{-4} Q \\
S_3 &= 3.7 \times 10^{-3} k + 6.3398 \times 10^{-4} Q \\
S_4 &= 4.2 \times 10^{-3} k + 1.7698 \times 10^{-4} Q
\end{align*}
\]

Then, take the derivative of Formula (25):

\[
\hat{S} = -z_1^T b = -z_1^T z_1 b - z_1^T z_2 Q
\]

The nearest integer operator \(\text{round}(S)\) means that the noise amplification factor \(n_{\text{noise}}\) is minimized under a specific \(k\) and \(Q\). Next, the four groups of the Galileo carrier frequency values are inserted into Formula (26) to obtain:

\[
\begin{align*}
\hat{S}_1 &= 0.4379 q_{c1}, \hat{S}_2 = 0.4321 q_{c2} \\
\hat{S}_3 &= 0.4302 q_{c3}, \hat{S}_4 = 0.4803 q_{c4}
\end{align*}
\]

Since the coefficient of the lane number \(k\) in Formula (27) is small,

\[
\begin{align*}
\hat{S}_1 &= 2.28 S_1, \hat{S}_2 = 2.31 S_2, \\
\hat{S}_3 &= 2.32 S_3, \hat{S}_4 = 2.08 S_4
\end{align*}
\]

If a weak ionospheric combination is acquired, the ionospheric amplification factor is \(|q_c| < 1\), the combination wavelength is \(\lambda_c > \lambda_1\), and the sum of the combination coefficients is \(S_c = a_1 + a_2 + a_3 = \text{round}(S) = 0\). According to Formula (29), the ionospheric amplification factor \(q_c\) increases with an increase in \(S_c\). Combination 1, combination 2 and combination 3 are about 2.3 times the value of \(S_c\), and combination 4 is about 2.0 times. Therefore, to obtain excellent performance of linear combinations, the sum of the combination coefficients should be smaller.

To better present the linear relationship between \(q_c\) and \(k\), Formula (26) is transformed into the following form:

\[
q_c = \frac{Q \cdot \gcd(n_{2n3})}{n_{2n3}} = -\left(\frac{\gcd}{n_{2n3}}\right)(z_1^T z_1 k + z_1^T z_2 \text{round}(S))
\]

The above formula represents the relationship between \(k\) and \(q_c\), when \(s\) is taken as a specific integer \(\text{round}(S)\). According to Formula (30), the minimum noise line corresponding to a specific value \(S_c\) can be drawn in the plane determined by \(k - q_c\). In the four groups of Galileo linear combinations, when \(S_c\) is equal to \(-1, 0, 1\), Figure 6(a)–(d) display the minimum noise line distribution in the plane determined by \(k - q_c\).

Among the four groups of Galileo linear combinations, when the wavelength and the minimum noise take a fixed value, the analysis in Figure 6 indicates that the ionospheric delay amplification factor in cycles increases with an increase of the \(S_c\). Meanwhile, when the ionospheric delay amplification factor and minimum noise take a fixed value, the wavelength decreases with an increase of the \(S_c\).

The optimal triple-frequency linear combination satisfying the requirements of a low noise, a long
wavelength, and a weak ionosphere is located in the wide lane area marked in Figure 6(a), and the sum of the combination coefficients $S_c = a_1 + a_2 + a_3$ from the WL and the EWL is zero. For the narrow lane combination, the sum of the combination coefficients $S_c$ is one.

When $S_c = a_1 + a_2 + a_3 = 0$, there are only two independent linear combination observations, and any other combination observations can be expressed linearly by these two independent combination observations. In Figure 6(a), when $q_c = 0$, the lane number $k$ is greater than 250, which signifies that the ionospheric effect of the combination observations is small, and the wavelength is short. Therefore, when $s = 1$, the optimal triple-frequency linear combination cannot be satisfied.

**Optimal linear combination of the Galileo system**

The errors of the short baseline have a strong correlation. Therefore, the noise effect is paid more attention for a short baseline. The correlation of ionospheric delay is weak at the medium and long baseline. Thus, we should pay more attention to combination observations with weak ionospheric delay. For different purposes, the selection of Galileo multi-frequency linear combination has different constraints.

The principle of linear combination needs to meet the four basic conditions of Formula (22). The four groups of Galileo linear combinations correspond to the EWL, WL, and NL, respectively according to the wavelength. The EWL combinations should satisfy $1 \leq k \leq 10$ and $S_c = 0$, the WL combinations should satisfy $11 \leq k \leq 40$ and $S_c = 0$, and the NL combinations should satisfy $k \geq 40$ and $S_c = 1$. For the conditions of a weaker ionospheric delay and lower observation noise, the carrier phase combination should satisfy $|q_c| \leq 1$ and $0 \leq n_{\text{noise}} \leq 8$. Because there are many NL combinations that meet the above conditions, stricter constraints are imposed on the NL combinations conditions, which are only listed the $|q_c| \leq 0.1$ or $0 \leq n_{\text{noise}} \leq 3$ combinations.
The wavelength, ionospheric delay, and observation noise have different effects at different baseline lengths. It is more sensitive to the ionospheric delay in the medium and long baseline, and it is more sensitive to the wavelength and observation noise in the short baseline. Therefore, it is necessary to select three independent optimal linear combination for triple-frequency ambiguity resolution according to different application scenarios.

Table 3 lists the Galileo triple-frequency carrier signals (E1, E6, E5b) that meet the constraints of the EWL, WL, and NL. The linear combinations with better performance are displayed in bold in Table 3. For the EWL and WL combinations with the sum of combination coefficients zero, it stand for the EWL and WL combinations with longer wavelength. Meanwhile, the $q_c$ and $n_{noise}$ are small too, which represent the optimal triple-frequency linear combination for Galileo combination 1. However, when $S_c = 0$, there are only two independent optimal linear combinations. Other EWL and WL combinations can be expressed linearly by the two independent combinations. The third independent linear combination can obtain from the narrow lane combination when $S_c = 1$.

By analyzing and comparing the above statistics results, we can see the following:

1. First combination: No matter what a short baseline or a long baseline scenario, the triple-frequency linear combination $(0, 1, -1)$ can be selected. Because the longer wavelength is 4.186m, the weaker ionospheric delay amplification factor is $-0.073$, and the lower noise amplification factor is 1.414. The combination $(0, 1, -1)$ is the optimal EWL combination. Moreover, Table 3 shows that the EWL combination $(1, -5, 4)$ and $(1, -4, 3)$ is also an excellent phase combination.

2. Second combination: In the WL combination, the first four combinations have multiple relationships with the EWL combination $(0, 1, -1)$. They have a linear correlation and cannot be used as an independent second linear combination to solve integer ambiguity. In the long baseline scenario, a WL combination $(1, -3, 2)$ with a smaller ionospheric delay amplification factor ($q_c = -0.086$) can be chosen, and in the short baseline scenario, a WL combination $(1, -1, 0)$ with the observation noise ($n_{noise} = 1.414$) can be chosen.

3. Third combination: The third independent combination should use a NL combination with the sum of the combination coefficients $S_c = 1$, and in the long baseline scenario, the narrow lane combination $(5, -3, -1)$ with an ionospheric delay amplification factor approaching zero ($q_c = -0.001$) should be selected. The ionospheric delay error of the meter level has less than 0.1 cycles on its ambiguity resolution, and its impact on the positioning solution is less than 0.1m. Moreover, its wavelength is greater than $10cm$, so it is suitable for geometric mode ambiguity resolution and precise positioning in the long distance application or the ionospheric active region. The noise amplification factor should be as small as possible in the short baseline scenario. Since the original observations $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ have the minimum noise amplification factor ($n_{noise} = 1$) compared to the other NL combinations, these original observation values can be used as the third independent combination to solve the integer ambiguity.

According to the above analysis and the different application scenarios, the four groups of optimal triple-frequency linear combination in the Galileo system is listed in Table 4.

### Table 3. Optimal linear combination for Galileo system combination 1 (E1, E6, E5b).

| $k$ | $a_1$ | $a_2$ | $a_3$ | $S_c$ | $\lambda_c$ | $q_c$ | $n_{noise}$ |
|-----|-------|-------|-------|-------|-------------|-------|-------------|
| Extra wide lane combination | | | | | | | |
| 6   | -1    | 6     | -5    | 0     | 4.884       | -0.133 | 7.874       |
| 7   | 0     | 1     | -1    | 0     | 4.186       | -0.073 | 1.414       |
| 1   | 1     | -5    | 4     | 0     | 29.305      | 0.060  | 6.481       |
| 8   | 1     | -4    | 3     | 0     | 3.663       | -0.013 | 5.099       |
| Wide lane combination | | | | | | | |
| 14  | 1     | 0     | -2    | 0     | 2.093       | -0.146 | 2.828       |
| 21  | 0     | 3     | -3    | 0     | 1.395       | -0.219 | 4.243       |
| 28  | 0     | 4     | -4    | 0     | 1.047       | -0.292 | 5.657       |
| 35  | 0     | 5     | -5    | 0     | 0.837       | -0.365 | 7.071       |
| 15  | 1     | -3    | 2     | 0     | 1.954       | -0.086 | 3.742       |
| 22  | 1     | -2    | 1     | 0     | 1.332       | -0.159 | 2.449       |
| 29  | 1     | -1    | 0     | 0     | 1.011       | -0.232 | 1.414       |
| 36  | 0     | 0     | -1    | 0     | 0.214       | -0.537 | 2.141       |
| 30  | 2     | -6    | 4     | 0     | 0.777       | -0.172 | 7.483       |
| 37  | 2     | -5    | 3     | 0     | 0.792       | -0.245 | 6.164       |

| Narrow lane combination | | | | | | | |
| 154 | 1     | 0     | 0     | 1     | 0.190       | 1.000  | 1.000       |

Note: The optimal linear combination is shown in bold in the table.
As can be seen from Table 4, the combination coefficient \((0, 1, -1)\) is the common EWL combination in Galileo system. This combination has the minimum noise amplification factor \((n_{\text{noise}} = 1.414)\), a longer wavelength, and a weaker ionospheric delay, which is the optimal triple-frequency carrier linear combination in Galileo system. Meanwhile, the combination \((0, 1, -1)\) is also suitable for the long baseline and short baseline application scenarios. In the WL combination, the independent linear combination should be selected according to different applications. In the process of data statistics, it is found that some wide lane combinations have multiple relationships with the EWL combinations, such as \((0, -1, 1), (0, 2, -2), \) and \((0, 3, -3)\), which should be excluded when selecting the WL combinations. The third independent combination should be selected from the NL combinations. In the short baseline applications, the original carrier phase combination \((0, 0, 1)\) should be selected because the minimum noise is beneficial for ambiguity resolution. The combination with the smallest ionospheric delay amplification factor should be selected in the long baseline.

To more intuitively display the characteristics of the optimal triple-frequency linear combination in Galileo system, Table 5 listed all EWL combinations without

| Combination number | Combination coefficient | Type | Baseline scenario | \(k\) | \(\lambda_c/m\) | \(q_c\) | \(n_{\text{noise}}\) |
|---------------------|-------------------------|------|------------------|------|----------------|-------|-------------------|
| Combination 1 (E1, E6, E5b) | \((0, 1, -1)\) | EWL | Both | 7 | 4.186 | -0.073 | 1.414 |
| | \((1, -1, 0)\) | WL | Short | 29 | 1.011 | -0.232 | 1.414 |
| | \((1, -3, 2)\) | WL | Long | 15 | 1.954 | -0.086 | 3.742 |
| | \((0, 0, 1)\) | NL | Short | 118 | 0.248 | 1.000 | 1.000 |
| | \((5, -3, -1)\) | NL | Long | 277 | 0.016 | -0.001 | 5.916 |
| Combination 2 (E1, E6, E5a) | \((0, 1, -1)\) | EWL | Both | 10 | 2.931 | -0.107 | 1.414 |
| | \((1, -1, 0)\) | WL | Short | 29 | 1.011 | -0.232 | 1.414 |
| | \((1, -2, 1)\) | WL | Long | 19 | 1.542 | -0.125 | 2.449 |
| | \((0, 0, 1)\) | NL | Short | 115 | 0.255 | 1.000 | 1.000 |
| | \((3, -5, 1)\) | NL | Long | 262 | 0.112 | 0.000 | 6.557 |
| Combination 3 (E1, E5b, E5a) | \((0, 1, -1)\) | EWL | Both | 3 | 9.768 | -0.034 | 1.414 |
| | \((1, -1, 0)\) | WL | Short | 36 | 0.814 | -0.305 | 1.414 |
| | \((1, -6, 5)\) | WL | Long | 21 | 1.395 | -0.135 | 7.874 |
| | \((0, 0, 1)\) | NL | Short | 115 | 0.255 | 1.000 | 1.000 |
| | \((4, -1, -2)\) | NL | Long | 268 | 0.109 | 0.017 | 4.583 |
| Combination 4 (E6, E5b, E5a) | \((0, 1, -1)\) | EWL | Both | 3 | 9.768 | -0.028 | 1.414 |
| | \((1, 1, -2)\) | WL | Short | 13 | 2.254 | -0.115 | 2.449 |
| | \((2, -3, 1)\) | WL | Long | 13 | 2.254 | -0.115 | 2.449 |
| | \((0, 0, 1)\) | NL | Short | 115 | 0.255 | 1.000 | 1.000 |
| | \((8, 1, -8)\) | NL | Long | 198 | 0.148 | 0.364 | 11.36 |

1 The baseline scenario includes long baseline (Long), short baseline (Short), and both baseline (Both).

| Combination number | Combination coefficient | Combination type | \(k\) | \(\lambda_c/m\) | \(q_c\) | \(n_{\text{noise}}\) |
|---------------------|-------------------------|------------------|------|----------------|-------|-------------------|
| Combination 1 (E1, E6, E5b) | \((-1, 6, -5)\) | EWL | 6 | 4.884 | -0.133 | 7.874 |
| | \((0, 1, -1)\) | EWL | 7 | 4.186 | -0.073 | 1.414 |
| | \((1, -1, 0)\) | EWL | 1 | 29.305 | 0.060 | 6.481 |
| | \((1, -5, 4)\) | EWL | 8 | 3.663 | -0.013 | 5.099 |
| Combination 2 (E1, E6, E5a) | \((-1, 4, -3)\) | EWL | 1 | 29.305 | -0.089 | 5.099 |
| | \((0, 1, -1)\) | EWL | 10 | 2.931 | -0.107 | 1.414 |
| | \((-1, 3, 2)\) | EWL | 9 | 3.256 | -0.018 | 3.742 |
| Combination 3 (E1, E5b, E5a) | \((-1, 4, -3)\) | EWL | 3 | 9.768 | -0.034 | 1.414 |
| | \((-1, 5, -4)\) | EWL | 5 | 5.861 | -0.051 | 6.481 |
| | \((-1, 6, -5)\) | EWL | 8 | 3.663 | -0.079 | 7.874 |
| | \((0, 1, -1)\) | EWL | 3 | 9.768 | -0.028 | 1.414 |
| | \((1, -3, 2)\) | EWL | 1 | 29.305 | -0.004 | 3.742 |
| | \((1, -2, 1)\) | EWL | 4 | 7.326 | -0.032 | 2.449 |
| | \((1, -1, 0)\) | EWL | 7 | 4.186 | -0.059 | 1.414 |
| | \((1, 0, -1)\) | EWL | 10 | 2.931 | -0.087 | 1.414 |
| | \((2, -5, 3)\) | EWL | 5 | 5.861 | -0.036 | 6.164 |
multiple relationships. All EWL combinations in Galileo system are drawn on the plane constructed by \( n_{\text{noise}} - q_c \). These EWL combinations are the optimal triple-frequency carrier linear combinations. All the EWL combinations in Galileo system are represented in Table 5.

In the \( n_{\text{noise}} - q_c \) determined plane, all EWL combinations are shown in Figure 7. Each circle represents a linear combination, and the color of the circle represents the combination number of Galileo. Here, red signifies combination 1, magenta signifies combination 2, blue signifies combination 3, and green signifies combination 4. The radius of the circle is determined by the combination wavelength \( \lambda_c \). The combination near the origin of coordinates is an ideal combination with long wavelength, weak ionosphere and low noise, and the combination characteristics are optimal.

By analyzing the distribution of each combination circle in Figure 7, the conclusion is as follows:

- The combination \((0, 1, -1)\) of Galileo system is the closest to the origin of coordinates. The combination \((0, 1, -1)\) is the optimal triple-frequency linear combination of Galileo system.
- Generally speaking, the larger the combination coefficients \((a_1, a_2, a_3)\) is, the worse the characteristics of the combination is. The large combination coefficients can amplify the observation noise obviously.
- The linear combination increases the noise amplification factor \( n_{\text{noise}} \) in cycles. The minimum noise amplification factor after combination is 1.414.
- The ionospheric amplification factor of all the EWL combinations is within the range \([-0.15, 0.10]\), which can help solve the ambiguity in the long baseline application and the ionospheric active region. Furthermore, the ionospheric amplification factor \( q_c \) of the combination \((1, -3, 2)\) is \(-0.004\), which is conducive to the constitute of an ionosphere-free combination.

In order to further verify the superiority of the linear combination observations selected in this paper, we have provided an example in the updated manuscript. The example analyzes the performance of the EWL of Galileo combination 2 \((E1, E6, E5a)\). This paper uses the Second-order Time-difference Phase Ionospheric Residual (STPIR) to calculate the integer ambiguity. By comparing the values of the integer ambiguity rounding error in the long baseline and short baseline scenarios, the advantages of the above EWL are verified.

In the short baseline scenario, the static data of IGS reference station WHU2 (Wuhan) and JFNG (Jiufeng) on June 15, 2020 are used. The sampling interval is 1 s. The observation time is about 4 h, and the baseline length is about 13.1 km. In the long baseline scenario, the static data of IGS reference station WHU2 and HKWS (Hongkong) on July 18, 2020 are used. The sampling interval is also 1 s. The observation time is about 3 h, and the baseline length is about 902.9 km. The observation data for ambiguity resolution are all from PRN08 and PRN15 of Galileo. Figures 8 and 9 are the EWL ambiguity rounding errors of Galileo combination 2 over time under the long and short baselines.

It can be seen from Figures 8 and 9, the linear combination of EWL \((0, 1, -1)\) and \((1, -3, 2)\) have an ambiguity rounding error of \(\pm 0.05\) cycles under most epochs in the short baseline scenario. The integer ambiguity of combination 2 \((E1, E6, E5a)\) is easy to be determined. In the long baseline scenario, the ambiguity rounding error changes greatly, which is mainly caused by the increase of baseline length and ionospheric delay. The ambiguity rounding error of \((0, 1, -1)\) and \((1, -3, 2)\) is within \(\pm 0.4\) cycles, so the integer ambiguity of linear combination can be estimated by rounding. According to the statistical data in Table 6,
the parameters of combination (0, 1, −1) are better than combination (1, −3, 2) in both long baseline and short baseline scenarios, which also confirm the conclusion: combination (0, 1, −1) is the optimal triple-frequency linear combination.

**Conclusion**

This paper uses the four civil frequencies of the Galileo system as the object of study. Firstly, the model of the triple-frequency carrier linear combination of Galileo is analyzed. The combination model can effectively eliminate or weaken the common observation error, and theoretically derive long wavelength combinations, IF combinations, GF combinations, and MN combinations. According to the different spatial distribution of them, the relationships between them are analyzed from a geometric perspective. Secondly, the optimal triple-frequency linear combination of integer coefficients was solved based on the extremum method, and the relationships between the lane number $k$ and the ionospheric delay amplification factor $q_\text{S}_k$ are established, when the noise amplification factor acquires a minimum value. Thirdly, to obtain three independent optimal triple-frequency linear combinations of Galileo, their corresponding optimal combinations are acquired from the EWL, WL, and NL combinations according to different constraints. Finally, the distribution of all extra wide lane combinations in Galileo system is presented in the form of circles to more intuitively display the optimal Galileo triple-frequency linear combination. Through the study of the above contents, the summary is as follows:

- The angle formed by IF combinations and GF combinations is relatively small, which indicates that there is a strong correlation between them, and the changing trend of the combinations is basically the same. Meanwhile the GF combinations have a positive correlation with the long wavelength combinations. Thus, there are many combinations that can satisfy both long wavelength and weak ionospheric delay combinations.
- The angle formed by the MN combinations and the other two combinations is relatively large, even the angle formed by GF combinations and MN combinations is 90°. Therefore, it is very difficult to simultaneously satisfy the optimal triple-frequency linear combination with a low noise, a long wavelength, and a weak ionosphere.
- When the wavelength and the minimum noise take a fixed value, the ionospheric delay amplification factor in cycles increases with an increase of the sum of the combination coefficients. The ionospheric delay amplification factor is more than twice the value of $S_\text{M}$.
- The sum of the EWL and WL combination coefficients is zero. The sum of the NL combination coefficients is one. There are only two independent linear combinations from the EWL and WL. The third independent combination needs to be selected from the NL combinations.
- All linear combinations can amplify the observation noise in cycles; the minimum noise amplification factor after a linear combination is 1.414.
- The EWL combination (0, 1, −1) is the optimal triple-frequency carrier phase linear combination in Galileo system.

**Acknowledgments**

This research was performed with technology and equipment support from the Satellite Navigation Laboratory. We wish to express our sincere gratitude to Dr Dong and MS Fu for their valuable help.

**Author contributions**

Conceptualization, X.D., W.F., and J.W.; methodology, X.D. and D.Y.; software, J.W. and W.F.; validation, J.W. and

---

**Table 6.** Data statistics of ambiguity rounding error in the EWL combination 2.

| Scenario     | EWL combination | Minimum (cycles) | Maximum (cycles) | Median (cycles) | Standard deviation (STD) |
|--------------|-----------------|------------------|------------------|-----------------|--------------------------|
| Short baseline | (0, 1, −1)      | −0.0673          | 0.0354           | −0.0007         | 0.0076                   |
|              | (1, −3, 2)      | −0.0828          | 0.2835           | −0.0002         | 0.0125                   |
| Long baseline | (0, 1, −1)      | −0.3634          | 0.2430           | −0.0313         | 0.0977                   |
|              | (1, −3, 2)      | −0.3734          | 0.2530           | −0.0335         | 0.0992                   |

**Figure 9.** Ambiguity rounding error varies with time under long baseline.
Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD
Jun Wang https://orcid.org/0000-0002-4598-2875

References
1. Liu F, Andrienko G, Andrienko N, et al. Citywide traffic analysis based on the combination of visual and analytic approaches. J Geovis Spatial Anal 2020; 4(2): 1–17.
2. Botsyo S, Bortei BB and Ayer J. CORS usage for GPS survey in the greater accra region: advantages, limitation, and suggested remedies. J Geovis Spatial Anal 2020; 4(2): 1–12.
3. Jurisica L, Ducheň F, Kaštan D, et al. High precision gns guidance for field mobile robots. Int J Adv Robot Syst 2012; 9(5): 169.
4. Qin H, Liu P, Qu J, et al. Optimal carrier-phase integer combinations for modernized triple-frequency BDS in precise point positioning. IEEE Access 2019; 7: 177449–177459.
5. Zhang XH and He XY. BDS triple-frequency carrier-phase linear combination models and their characteristics. Sci China Earth Sci 2015; 58(6): 896–905.
6. Ferreira A, Matias B, Almeida J, et al. Real-time GNSS precise positioning: RTKLIB for ROS. Int J Adv Robot Syst 2020; 17(6): 1–8.
7. Han SW and Chris R. The impact of two additional civilian GPS frequencies on ambiguity resolution strategies. In: 55th national meeting US institute of navigation, navigation technology for the 21st century, Cambridge, MA, 1999, pp.28–30.
8. Feng Y. GNSS three carrier ambiguity resolution using ionosphere-reduced virtual signals. J Geod 2008; 82(12): 847–862.
9. Cocard M, Bourgon S, Kamali O, et al. A systematic investigation of optimal carrier-phase combinations for modernized triple-frequency GPS. J Geod 2008; 82(9): 555–564.
10. Urquhart L. An analysis of multi-frequency carrier phase linear combinations for GNSS. Techn. Report 263, 2009.
11. Richert T and El-Shemy N. Optimal linear combinations of triple frequency carrier phase data from future global navigation satellite systems. GPS Sol 2007; 11(1): 11–19.
12. Wu Y. The theory and application on multi-frequency data processing of GNSS 2. Wuhan: Wuhan University, 2005.
13. Liu X, Wu Y, Huang X, et al. Application of GPS multi-frequency carrier phase combinations for preprocessing of original carrier phase observations. Bull Survey Mapp 2007; (2): 14–17.
14. Li B, Shen Y and Zhou Z. A new method for medium and long range three frequency GNSS rapid ambiguity resolution. Acta Geodaetica Cartogr Sin 2009; 38(4): 296–301.
15. Li J, Yang Y, He H, et al. Benefits of BDS-3 B1C/B1I/ B2a triple-frequency signals on precise positioning and ambiguity resolution. GPS Sol 2020; 24(4): 1–10.
16. Yu X, Zhang X, Liu J, et al. Performance assessment of long-baseline integer ambiguity resolution with different observation models. In: Proceedings of the 24th international technical meeting of the satellite division of the institute of navigation (ION GNSS 2011), Portland, OR, 2001, pp.688–698.
17. Li J, Yang Y, He H, et al. Optimal carrier phase combinations for triple-frequency GNSS derived from an analytical method. Acta Geodaetica Cartogr Sin 2012; 41(06): 797–803.
18. Li J, Yang Y, He H, et al. An analytical study on the carrier-phase linear combinations for triple-frequency GNSS. J Geod 2017; 91(2): 151–166.
19. Huang L, Song L and Liu X. Optimization and selection of GPS triple-carries phase combination observations based self-adaptive clustering algorithm. J Geod Geodyn 2011; 4: 99–102.
20. Wang Z and Liu J. Model of inter-frequency combinations of Galileo GNSS. Geomat Inform Sci Wuhan Univ 2003; 28(6): 723–727.
21. Duong V, Harima K, Choy S, et al. An optimal linear combination model to accelerate PPP convergence using multi-frequency multi-GNSS measurements. GPS Sol 2019; 23(2): 49.
22. Wei F, Dong X, Wang M, et al. Research on the model of BDS-3 triple-frequency carrier-phase combination observations. GNSS World China 2018; 43(05): 1–8.
23. Zhang C, Guo C and Zhang D. Data fusion based on adaptive interacting multiple model for GPS/INS integrated navigation system. Appl Sci 2018; 8(9): 1682.
24. Fan X, Tian R, Dong X, et al. Cycle slip detection and repair for BeiDou-3 triple-frequency signals. Int J Adv Robot Syst 2020; 17(3): 1–14.
25. He W, Tao T and Wang Z. Selection on improved fuzzy C-means algorithm of BDS triple-frequency combination observations. Chin Space Sci Technol 2014; 34(4): 24.
26. Meng F and Li S. BDS triple-frequency carrier phase combination smoothing pseudo-range algorithm. MS&E 2020; 780(3): 032044.
27. Fanjun M, Shujun LI, Zongpeng P, et al. Optimization and selection of BDS triple-frequency carrier phase combination observations based on fuzzy C-means algorithm. J Geod Geodyn 2019; 39(3): 246–251.
28. Wang J, Huang G, Zhang Q, et al. GPS/BDS-2/galileo precise point positioning and ambiguity resolution based on the uncombined model. Remote Sens 2020; 12(11): 1853.
29. Hu Y, Bian S, Cao K, et al. GNSS spoofing detection based on new signal quality assessment model. GPS Sol 2018; 22(1): 1–13.
30. European GNSS Service Centre. European GNSS (Galileo) Open Service Signal-In-Space Interface Control Document Issue 1.3, https://www.gsc-europa.eu/sites/default/files/sites/all/files/Galileo-OS-SIS-ICD.pdf (2016, accessed on 30 July 2020).
31. Zhang X and Cai W. BDS four frequency carrier phase combination models and their characteristics. Surv Rev 2020; 52(371): 97–106.