Disturbance-Observer Assisted Controller for Stand-Alone Four-Leg Voltage Source Inverter

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Abstract—In this paper, a newly disturbance-observer based control strategy for stand-alone four-leg VSI has been proposed and analyzed. The disturbance-observer operates together with a simple and effective resonant controller in order to regulate the inverter output voltage. In particular, the resonant controller regulates the output voltage fundamental harmonic, whereas the disturbance observer is designed for compensating output voltage harmonics when non-linear loads have to be fed. The new control strategy effectiveness and performance have been validated at first through a full MATLAB/Simulink model and then by experimental results on a dedicated 40 kVA three-phase 4-leg.

Keywords—resonant controllers, disturbance observer, three-phase four-leg inverter, non-linear loads, harmonic content.

I. INTRODUCTION

The harmonic content reduction of the current and voltage waveforms at the converters’ output is one of the fundamental objectives of research activities in power electronics. Many international standards, such as EN-50160 and EMC EN-61000, must be satisfied [1]. Consequently, the performances of voltage and current regulators play an important role in the modern applications of the power electronics. Regarding the four-leg Voltage Source Inverter (4L VSI), several control strategies have been investigated in literature [2]-[8].

As fully explained in [2], a possible implementation of the regulators is based on resonant controllers. Where, one of the resonant controllers is tuned at the fundamental frequency for the output voltage regulation, while the other several is required for compensating the harmonics induced by the non-linear load, such that the total harmonic distortion (THD) of the output voltage can be minimized. The advanced online tuning method is proposed in [2] for making the resonant controllers more adapted to the load variations.

Instead of using several resonant controllers, it is also possible to reduce all the periodic harmonics using a repetitive controller as in [3] and [4]. Where, the repetitive controller works only on the harmonics suppression to supplement the conventional feedback controller.

Taking the advantages of the both, the authors in [5] have combined a repetitive controller and a resonant controller, such that the reference voltage can be tracked with zero phase shift by the resonant controller, whereas the repetitive controller will fine regulate the output voltage till its THD is minimized.

Moreover, since the harmonics can be modelled as disturbances, disturbance-observer can also be used as a supplement to the feedback controller as in [6]. Interestingly, the authors in [7] have pointed out the similarity between the repetitive controller and the feed-forward disturbance-observer. On one hand, the new repetitive-controller-like disturbance-observer designed in [7] can work as a traditional repetitive controller for reducing periodic harmonics, but larger stability margin can be achieved without sacrificing its performance. On the other hand, this new disturbance-observer can be tuned as a conventional disturbance-observer. That is, following the separation principle, it can be tuned separately to the feedback controller.

Hence, this paper applies the new repetitive-controller-like Disturbance-Observer (DO) to stand-alone 4L VSI for the first time. The block scheme of the proposed new control strategy is shown in Fig. 1.

As it can be seen from Fig. 1, the regulator of the inverter control consists of a Resonant Controller (RC) plus the DO. Where, the RC is designed at the fundamental frequency for reference voltage tracking, while the DO should learn and cancel any periodic harmonics produced by the non-linearity of the inverter and the non-linear/unbalanced load.

Moreover, since the harmonics can be modelled as disturbances, disturbance-observer can also be used as a supplement to the feedback controller as in [6]. Interestingly, the authors in [7] have pointed out the similarity between the repetitive controller and the feed-forward disturbance-observer. On one hand, the new repetitive-controller-like disturbance-observer designed in [7] can work as a traditional repetitive controller for reducing periodic harmonics, but larger stability margin can be achieved without sacrificing its performance. On the other hand, this new disturbance-observer can be tuned as a conventional disturbance-observer. That is, following the separation principle, it can be tuned separately to the feedback controller.

Hence, this paper applies the new repetitive-controller-like Disturbance-Observer (DO) to stand-alone 4L VSI for the first time. The block scheme of the proposed new control strategy is shown in Fig. 1.

II. 4L VSI TOPOLOGY AND OUTPUT FILTER DESCRIPTION

In many microgrids applications, the trend is to eliminate the output transformer in order to improve the costs, size and weight. When the transformers are not used
in the output inverter and the currents flowing through the three-phase loads are not balanced, it is necessary to implement an alternative connection for the neutral terminal. The neutral terminal of the inverter can be connected in two different modes, called respectively three-phase four-wire inverter and three-phase four-leg inverter [8]-[9]. In the three-phase four-wire inverter the neutral point is connected to the split dc-bus capacitors [9], as shown in Fig. 2.

![Fig. 2. Three-phase four-wire inverter.](image)

In the three-phase four-leg inverter the middle-point of the added leg is directly connected to the neutral terminal. The three-phase 4L VSI topology is depicted in Fig. 3.

![Fig. 3. Three-phase four-leg inverter.](image)

On one hand the three-phase four-wire inverter is certainly the easiest solution, on the other hand it presents some problems. The first issue is the voltage unbalancing among the series connected DC-link capacitors. Furthermore, due to the deterioration of the harmonic content in the phase-to-neutral voltage, the sinusoidal PWM with third harmonic injection and vector machine techniques cannot be applied to the three-phase four-wire inverter. The three-phase four-leg inverter allows to overcome the mentioned problems; however, extra driving circuits are required due to the increased number of switches and power diodes. Considering the first-order approximation, the transfer function (TF) of the three-phase four-wire inverter can be written as in (1), where $V_{BUS}$ is the DC-bus voltage, $k$ is the factor depending on the modulation strategy and $f_{sw}$ is the switching frequency. Using the sinusoidal PWM with third harmonic injection, the $k$ factor is equal to 0.57.

$$G_{VL}(s) = k \frac{V_{BUS}}{s + \frac{2\pi f_{sw}}{s}}$$  \hspace{1cm} (1)

The prototype of the three-phase 4L VSI is depicted in Fig. 4. It is possible to recognize the DC-bus side, the driver circuit control board, the adapter board, the control board and the three-phase terminals $A, B, C$, as well as the neutral terminal $n$.

![Fig. 4. Three-phase 4L VSI prototype.](image)

Each leg is realized by the SEMIX module (SEMIX303GB12Vs) 300A-1200V rated. The rated power is 40kVA, line-to-line voltage $V_{L-L}=400V$, switching frequency $f_{sw}=12kHz$, DC-bus voltage $V_{BUS}=750V$ and efficiency at the rated power is about 97%. In order to remove the switching component from the output voltage and current waveforms, an appropriate output filter has been realized. The circuit diagram of the single-phase output filter is shown in Fig. 5.

![Fig. 5. Circuit diagram of the output power filter.](image)

| Table I. Output Power Filter Parameters |
|----------------------------------------|
| $L_f$ | 800 μH |
| $C_f$ | 5 μF  |
| $L_d$ | 810 μH |
| $C_d$ | 7.2 μF |
| $R_d$ | 20 Ω  |
| $L_t$ | 138.5 μH |
| $C_t$ | 1.2 μF |
| $R_t$ | 60 mΩ |

Performing some algebraic manipulations, the TF of the output filter can be derived as in (2).
\[ G_f(s) = \frac{1}{C_j \cdot L_f} \frac{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s^4 + b_5 s^3 + b_4 s^2 + b_3 s + b_1 s + b_0} \quad (2) \]

The TF coefficients of the output power filter are listed below.

\[ a_0 = \frac{1}{C_j \cdot L_j} \quad , \quad a_1 = \frac{1}{L_j \cdot L_j} \left( \frac{R_{d} \cdot R_{c}}{C_j} - \frac{R_{d}}{C_j} \right) \]

\[ a_2 = \frac{R_{d} \cdot R_{c}}{L_j \cdot L_j} + \frac{1}{C_j \cdot L_j} \quad , \quad a_3 = \frac{R_{d}}{L_j} + \frac{R_{c}}{L_j} \]

\[ b_0 = \frac{1}{C_j \cdot C_j \cdot L_j} \quad , \quad b_1 = \frac{1}{L_j \cdot L_j \cdot L_j} \left( \frac{R_{d}}{L_j} + \frac{R_{c}}{L_j} \right) \]

\[ b_2 = \frac{1}{C_j \cdot C_j \cdot L_j \cdot L_j} + \frac{1}{C_j \cdot L_j \cdot L_j} + \frac{1}{C_j \cdot C_j \cdot L_j} \quad + \quad \frac{1}{C_j \cdot L_j \cdot L_j} \]

\[ b_3 = \frac{R_{d}}{L_j} \quad , \quad b_4 = \frac{1}{C_j} \quad , \quad b_5 = \frac{1}{C_j} \quad , \quad b_6 = \frac{R_{d} \cdot R_{c}}{L_j} \quad , \quad b_7 = \frac{R_{d}}{L_j} + \frac{R_{c}}{L_j} \]

### III. PROPOSED CONTROL STRATEGY

The aim of the inverter control strategy is to obtain as much as possible sinusoidal waveform of the output voltage even in case of non-linear and unbalanced loads.

The simplified block diagram to be used for tuning the voltage control loop is shown in Fig. 6, where it is possible to identify the following transfer functions: \( G_{ds}(s) \) is the TF of the inverter (1), \( G_{ds}(s) \) is the TF of the output power filter (2), \( G_{ds}(s) \) is the TF of the second-order low-pass filter in the measurement chain, \( G_{ds}(s) \) and \( G_{ds}(s) \) are the TFs in the discrete domain of the resonant controller and of the disturbance-observer, respectively. The coefficient \( k_{gain} \) is the gain equal to \( 1/(k \cdot V_{Bus}) \), where \( k \) is the factor defined above, \( d_{mod} \) is the modulating signal which is then compared with the carrier to generate the gate signals. The TF of the second-order low-pass Butterworth filter \( G_{ds}(s) \) is defined in (3), where \( \omega_0 \) is filter cut-off frequency.

\[ G_p(s) = \frac{\omega_0^2}{s^2 + \sqrt{2 \omega_0 s + \omega_0^2}} \quad (3) \]

The RC is designed and tuned at 50 Hz in order to obtain an excellent output dynamic response and good stability margin of the system. The DO is used to compensate all the harmonics above the fundamental.

#### A. Resonant Controller

There are different analytical forms of the RC, i.e. ideal form, approximated form and real form with or without phase compensation [2]. In this case, the only task of the RC is to compensate the fundamental harmonic of the output voltage. For this reason, the real form of the RC without phase compensation is chosen (i.e. filter effects are negligible at the fundamental frequency) [11], [12]. The TF of the selected RC is given in (4), where \( k_{RC} \) is the gain, \( \omega_{r} \) is the width and \( \omega_0 \) is the resonance frequency.

\[ G_{RC}(s) = 2 k_{RC} \cdot \omega_0 \frac{s + \omega_0}{s^2 + 2 \omega_0 s + (\omega_0^2 + \omega_r^2)} \quad (4) \]

Taking into account that \( \omega_0 >> \omega_r \), the magnitude of the RC at the resonance frequency is given in (5), whereas the phase of the RC cannot be used as a design parameter, since it is locked at 0° at the resonance frequency.

\[ |G_{RC}(j \omega_r)| = \frac{2 k_{RC} \cdot \omega_0}{\sqrt{4 \omega_0^2 + \omega_r^2}} = \frac{2 k_{RC} \cdot \omega_0}{2 \omega_0} = k_{RC} \quad (5) \]

The discrete transfer function (DTF) of the RC \( G_{RC}(z) \) obtained by Tustin with pre-warping method [12] is given in (6), where \( K_T \) is pre-warping term defined as \( \omega_0 / tan(\omega_0 / 2 f_{sw}) \).

\[ G_{RC}(z) = k_{RC} \frac{z^2 + a_{RC1} z + a_{RC0}}{z^2 + b_{RC1} z + b_{RC0}} \quad (6) \]

\[ k_{RC} = \frac{2 k_{RC} \cdot \omega_r^2 + 2 k_{RC} \cdot K_T \cdot \omega_r}{K_T^2 + 2 K_T \cdot \omega_r + \omega_r^2 + \omega_T^2} \]

\[ a_{RC0} = \frac{2 k_{RC} \cdot \omega_r^2 - 2 k_{RC} \cdot K_T \cdot \omega_r}{2 k_{RC} \cdot \omega_r^2 + 2 k_{RC} \cdot K_T \cdot \omega_r} \]

\[ a_{RC1} = \frac{4 k_{RC} \cdot \omega_r^2}{2 k_{RC} \cdot \omega_r^2 + 2 k_{RC} \cdot K_T \cdot \omega_r} \]

\[ b_{RC0} = \frac{K_T^2 - 2 K_T \cdot \omega_r + \omega_r^2 + \omega_T^2}{K_T^2 + 2 K_T \cdot \omega_r + \omega_r^2 + \omega_T^2} \]

\[ b_{RC1} = \frac{2 \omega_r^2 - 2 K_T \cdot \omega_r + \omega_T^2}{2 K_T^2 + 2 K_T \cdot \omega_r + \omega_r^2 + \omega_T^2} \]

#### B. Disturbance-Observer

The DO proposed in [7] can be designed starting from the state-space model of the disturbance \( D(z) \). Assuming the disturbance \( D(z) \) as a periodic behavior, its dynamical model can be expressed as in (8), where \( X_d \) is the disturbance state vector, \( a_{ds} \) and \( c_{ds} \) are the matrix and the vector defined in (9). The disturbance vector \( X_d \) has \( N f_s / f_0 \) elements, where \( f_s \) is the sampling frequency and \( f_0 \) is the fundamental frequency. \( D(k) \) is the instantaneous disturbance at \( k \).
Overall, equations (8) and (9) denote a disturbance $D(k)$ that repeats itself every $N$ samples of measurement.

$$X_d(k+1) = a_{11}X_d(k)$$
$$D(k) = c_dX_d(k)$$

Thus, the observer for this disturbance during the $k$th sampling interval can be derived as in (10), where $U_{DO}(k)$ is the “equivalent measurement” of the disturbance, which equals the difference between the direct measured output voltage $k_{gain}V_{ph}$ at $t_k$ and the intended modulating voltage signal $d_{ph}$ required to be reached at $t_k$. $L$ is a vector of $N$ observer gains $L=[L_1, L_2, ..., L_N]$. $\dot{X}_d$ is the estimated disturbance state vector. $c_d\dot{X}_d(k)$ denotes the estimated disturbance at $t_k$. $A$ is the ratio between the “indirect measured” disturbance at $t_k$ and the estimated disturbance at the same time. $Q$ is the output matrix for the observer.

$$\dot{X}_d(k+1) = a_{11}\dot{X}_d(k) + L[U_{DO}(k) - A\dot{X}_d(k)]$$

$$Y_{DO}(k+1) = Q\dot{X}_d(k+1)$$

Based on equation(10), Fig. 7 shows the block diagram of the DO.

![Block diagram of the DO](image)

According to [7], thanks to the following three settings, the DO is able to attenuate periodic harmonics in the same way as a traditional repetitive controller.

1. The matrix $a_{11}$ is updated into $\bar{a}_{11}$ in order to include the forgetting factor $Q$ of the traditional repetitive controller.

$$\bar{a}_{11} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ Q & 0 & 0 & \cdots & 0 \end{bmatrix}$$

2. The observer gains $L$ is simplified by setting $L_1=L_2=...=L_{N-1}=0$. The only gain to tune is $L_{200}$.

3. The output matrix $Qf$ is functionally the stability filter that is commonly included in the repetitive controller for compensating any system delay seen by the repetitive controller. For the DO, $Qf$ is a $1\times N$ matrix with all zeros but the $i$th element equals one, where $i$ is the total number of delays between the output $Y_{DO}$ and the corresponding $U_{DO}$ becomes available in the feedback.

### IV. RESULTS

#### A. Simulation Results

The converter full digital-switching model has been realized in the Matlab/Simulink environment in order to validate the performances of the new control strategy. Preliminary results have been performed with reference to the system parameters listed in Table II.

| Table II. Inverter and Controller Parameters. |
|-----------------------------------------------|
| Parameter                                | Value |
| Rated Power [kVA]                        | 40    |
| Line-to-Line Voltage [V]                  | 400   |
| Output Fundamental Frequency $f_0$ [Hz]   | 50    |
| Switching Frequency $f_s$ [kHz]           | 12    |
| Dead-Time [$\mu$s]                        | 3.3   |
| DC-bus Voltage [V]                        | 750   |
| Controller gain $k_0$                     | 380   |
| Controller bandwidth $\omega_c$ [rad/s]   | 0.001 |
| Vector Length $N$                         | 240   |
| Forgetting Factor $Q$                     | 0.18  |
| DO gain $L_{200}$                         | 0.0025|
| DO coefficient $A$                       | 0.9   |

The output matrix $Qf$ of the observer is chosen according to the system delay and it is defined as $Qf=[0\ 0\ 0\ 0\ 0\ \cdots\ N]$. Fig. 8 and Fig. 9 show the phase-to-neutral output voltages $V_{An}, V_{Bn}, V_{Cn}$ and the related currents $I_{An}, I_{Bn}, I_{Cn}$ in case of resistive load when the DO is engaged or disabled (i.e. RC only). It is possible recognize in Fig. 8 that the output voltages and currents are distorted during the zero-crossing due to the dead-time, mainly 5th and 7th harmonics. The dead-time is fully compensated when the DO is enabled as depicted in Fig. 9.

![Phase-to-neutral Output Voltages](image)

![Currents](image)
Fig. 9. DO assisted RC under resistive load condition: a) Phase-to-neutral output voltages $V_a, V_b, V_c$; b) Currents $I_a, I_b, I_c$.

Considering the non-linear load as a three-phase diode bridge rectifier having the parameters shown in Fig. 10; Fig. 11, Fig. 12 and Fig. 13 show the phase-to-neutral output voltages $V_{An}, V_{Bn}, V_{Cn}$ and the currents $I_{An}, I_{Bn}, I_{Cn}$ when the DO is disabled and enabled, respectively. As it can be seen in Fig. 9 and Fig. 13, the combined RC-DO allows to compensate the harmonics of the output voltage.

![Fig. 10. Non-linear three-phase diode bridge rectifier load.](image)

Fig. 10. Non-linear three-phase diode bridge rectifier load.

![Fig. 11. RC without DO: Phase-to-neutral output voltages $V_a, V_b, V_c$.](image)

Fig. 11. RC without DO: Phase-to-neutral output voltages $V_a, V_b, V_c$.

![Fig. 12. RC without DO: Currents $I_a, I_b, I_c$.](image)

Fig. 12. RC without DO: Currents $I_a, I_b, I_c$.

![Fig. 13. DO assisted RC: a) Phase-to-neutral output voltages $V_a, V_b, V_c$; b) Current $I_a, I_b, I_c$.](image)

Fig. 13. DO assisted RC: a) Phase-to-neutral output voltages $V_a, V_b, V_c$; b) Current $I_a, I_b, I_c$.

### B. Experimental Results

The experimental tests have been performed in order to support the proposed control strategy. The inverter depicted in Fig. 4 is controlled by the National Instruments System-on-Module sbRIO-9651 with a dedicated board specifically designed for power electronics and drives applications, as described in [13]. The inverter control algorithm has been implemented in LabVIEW environment on the FPGA target. According to the operating condition listed in Table II, Fig. 14 shows the phase-to-neutral output voltages $V_{An}, V_{Bn}$ and the phase current $I_{An}$ under no load condition. Fig. 15 illustrates the phase-to-neutral output voltages $V_{An}, V_{Bn}$ and the phase current $I_{An}$, in the case that one of the three phases is loaded at $1\ kW$ and the other phases are at no-load condition. Finally, Fig. 16 shows the phase-to-neutral output voltages $V_{An}, V_{Bn}$ and the phase current $I_{An}$, considering the unbalanced load having the parameters depicted in Fig. 10.
In order to evaluate the computational effort of the control algorithm, the execution time of the control structure has been evaluated on the FPGA resulting about 10 μs.