Stark broadening of hydrogen/deuterium spectral lines by a relativistic electron beam: analytical results and possible applications to magnetic fusion edge plasmas

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Abstract
We present a theory of the Stark broadening of hydrogen/deuterium spectral lines by a relativistic electron beam (REB). The theory is developed analytically by using an advanced formalism. We discuss the possible application of these analytical results to magnetic fusion edge plasmas, taking into account also the major outcome of the interaction of a REB with plasmas: the development of strong Langmuir waves.

1. Introduction
The interaction of a relativistic electron beam (REB) with plasmas has both the fundamental importance for understanding physics of plasmas and practical applications. The latter include (but not limited to) plasma heating, inertial fusion, generation of high-intensity coherent microwave radiation, acceleration of charged particles in plasmas—see, e.g., papers [1–3] and references therein.

The latest (though negative) application relates to magnetic fusion and deals with runaway electrons. In some discharges in tokamaks, the plasma current decays and is partly replaced by runaway electrons that reach relativistic energies: this poses danger to the mission of the next generation tokamak ITER—see, e.g., papers [4–6] and references therein. At various discharges at different tokamaks, such as, e.g., those presented in papers [7–9], the energy of the runaway electrons was measured in the range \(\sim(0.2–10)\) MeV and the ratio of their density to the density of the bulk plasma electrons was measured in the range \(\sim(10^{-1}–10^{-4})\).

Therefore developing diagnostics of a REB and its interaction with plasmas should be important. In the particular case of tokamaks, the development of a REB should be timely detected to allow the mitigation of the problem.

Diagnostics based on the analysis of spectral line shapes have known advantages over others. They are not intrusive and allow measuring plasma parameters and parameters of various fields in plasmas without perturbing the parameters to be measured—see, e.g., books [10–15].

In the current paper we present a theory of the Stark broadening of hydrogen/deuterium spectral lines by a REB. The theory is developed analytically by using an advanced formalism. We discuss the possible application of these analytical results to magnetic fusion edge plasmas, taking into account also the major outcome of the interaction of a REB with plasmas: the development of strong Langmuir waves\(^1\).

2. Analytical results and applications to magnetic fusion
The presence of a REB introduces anisotropy in the process of the Stark broadening of spectral lines in plasmas. A different kind of anisotropic Stark broadening was first considered by Seidel in 1979 [17] for the following

\(^1\) We note that Rosato et al. [16] attempted studying the Stark broadening of hydrogen line by a REB in magnetic fusion edge plasmas. However, they used the quasistatic approximation, which is totally inappropriate for the broadening by fast electrons of a REB (it is inappropriate even for the broadening by thermal electrons in such plasmas).
situation. If hydrogen atoms radiate from a plasma consisting mostly of much heavier ions, then in the reference frame moving with the velocity \( v \) of the radiating hydrogen atom, the latter ‘perceives’ a beam of the much heavier ions moving with the velocity \( v \). Seidel \([17]\) treated this situation by applying the so-called standard (or conventional) theory of the impact broadening of hydrogen lines, also known as Griem’s theory \([18]\). Therefore, while Seidel \([17]\) should be given credit for pioneering the anisotropic Stark broadening, his specific calculations had a weakness that plagues the standard theory: the inherent divergence at small impact parameters causing the need for a cutoff defined only by an order of magnitude.

Later in paper \([19]\) the authors considered the same situation as Seidel \([17]\), but applied a more advanced theory of the Stark broadening called the generalized theory developed in paper \([20]\) and presented also in book \([12]\). (It should be emphasized that in paper \([19]\) it was the application of the ‘core’ generalized theory from paper \([20]\) without the additional effects that were introduced later and were the subject of discussions in the literature.) The authors of paper \([19]\) took into the exact account (in all the orders of the Dyson expansion) the projection of the dynamic, heavy-ion-produced electric field onto the velocity of the radiator exactly. As a result, there was no divergence at small impact parameters and thus no need for the imprecise cutoff.

In the present paper we use the formalism from paper \([19]\) to treat the Stark broadening of hydrogen/deuterium spectral lines by a REB in plasmas. There are two major distinctions from paper \([19]\): (1) the broadening is by a beam of electrons rather than ions; (2) the electrons are relativistic.

Following paper \([19]\) we choose the \( z \)-axis in the direction of the REB and represent the Hamiltonian \( H(t) \) perturbed by the field \( E(t) \) of the REB in the form:

\[
H(t) = H_0(t) + V(t), \quad H_0(t) \equiv H_0 - dz E_z(t), \quad V(t) \equiv -dE_z - d_y E_y.
\]

(1)

The partial time-dependent Hamiltonian \( H_0(t) \) is diagonalized here in the parabolic quantization and is allowed for exactly. The residual interaction \( V(t) \) is taken into account via the Dyson perturbation expansion.

The starting expression for the lineshape \( I(\omega, v) \) depends on the velocity \( v \) of the REB:

\[
I(\omega, v) = -\frac{1}{\pi} \text{Re} \sum_{\sigma} \sum_{\alpha'\beta'} \langle \beta | d_\sigma | \alpha \rangle \langle \alpha' | d_\sigma | \beta' \rangle \langle \langle \alpha | \beta | G^{-1} | \alpha' \beta' \rangle \rangle.
\]

(2)

Here \( \alpha, \alpha' \) and \( \beta, \beta' \) label the Stark sublevels of the upper \( (a) \) and lower \( (b) \) states involved in the radiative transition; \( d_\sigma \) are components of the dipole moment operator; the spectral operator \( G \) is

\[
G = i\Delta \omega + \Phi_{ab}(v),
\]

(3)

where the impact operator \( \Phi_{ab}(v) \) is

\[
\Phi_{ab} = N_b \int_0^{\infty} 2\pi \rho \, d\rho \{ S_\alpha S_{\beta} - 1 \}. \]

(4)

Here \( N_b \) is the electron density of the REB.

The operator \( \Phi_{ab}(v) \) is subdivided into adiabatic \( \Phi_{ad,ab}(v) \) and nonadiabatic \( \Phi_{na,ab}(v) \) contributions

\[
\Phi_{ab}(v) = \Phi_{ad,ab}(v) + \Phi_{na,ab}(v),
\]

(5)

where \( \Phi_{ad,ab}(v) \) contains only the following combination of the diagonal matrix elements of the dipole moment operator: \( e^2(z_{\alpha\alpha'} - z_{\beta\beta'})^2 \). An important feature of the impact Stark broadening by a beam of ions or electrons is that the adiabatic part \( \Phi_{ad,ab}(v) \) vanishes—in distinction to the impact Stark broadening by randomly moving thermal ions or electrons \([19]\).

The scattering matrix \( S \) entering equation \((4)\) is represented in the form:

\[
S = \exp \left( i/\hbar \int_{-\infty}^{\infty} dt_1 E_z(t) \right) \hat{T} \exp \left( -i/\hbar \int_{-\infty}^{\infty} dt Q^* (d_z E_z + d_y E_y) Q \right).
\]

(6)

For Lyman lines the scattering matrix \( S_\alpha = 1 \), what simplifies calculations. Then in the second order of the modified Dyson expansion \((6)\), the matrix elements of the nonadiabatic part of the operator \( \Phi_{ab}(v) \) are:

\[
\Phi_{na} = -4\pi N_e \frac{e^2}{\hbar v} \sum_{\nu', \nu} \sum_{\alpha', \alpha} d_{\alpha \nu'}^* d_{\alpha' \nu} \int_{0}^{\infty} C_{\nu}(Z) \frac{dZ}{Z}.
\]

(7)

Here

\[
Z = 2m_e v \rho / (3n \hbar),
\]

(8)

where \( n \) is the principal quantum number of the upper level and \( \rho \) is the impact parameter. So, physically the quantity \( Z \) is the scaled, dimensionless impact parameter and the integration over \( Z \) in equation \((7)\) corresponds to the integration over impact parameters.
If the electron beam would be a non-relativistic, so that the electric field produced by the beam electron at the location of the radiating atom would be

\[ E(t) = \frac{1}{r^3(t)} \]

where \( r(t) \) is the radius vector from the beam electron to the radiating atom, then the broadening functions \( C_+ \) and \( C_- \) entering equation (7) for nondiagonal and for diagonal matrix elements, respectively, would be the following double integrals:

\[
C_{\pm}(Z) = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx_1 dx_2}{g(x_1)g(x_2)} \exp \left[ \frac{i}{Z} \left( \frac{1}{g(x_1)} \pm \frac{1}{g(x_2)} \right) \right],
\]

\[ g(x) = \sqrt{1 + x^2}. \]  

However, for the REB, equation (9) has to be modified to (see, e.g., equation (38.8) from book [21]):

\[
E(t) = \frac{1}{r^3(t)} \gamma^2 (\cos^2 \theta + \sin^2 \theta / \gamma^2)^{3/2} \]

Here

\[ \gamma = 1 / (1 - v^2 / c^2)^{1/2} \]

is the relativistic factor and \( \theta(t) \) is the angle between the beam velocity \( v \) and vector \( r(t) \), so that

\[ \cos^2 \theta = v^2 t^2 / (\rho^2 + v^2 t^2), \quad \sin^2 \theta = \rho^2 / (\rho^2 + v^2 t^2), \]

the instant \( t = 0 \) corresponding to the closest approach of the beam electron to the radiating atom.

The relativistic counterparts \( C_{\pm} \) of the broadening functions \( C_{\pm} \) become as follows:

\[
C_{\pm}(Z) = \frac{1}{2\gamma^4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx_1 dx_2}{g_s(x_1)g_s(x_2)} \exp \left[ \frac{i}{Z} \left( \frac{1}{g_s(x_1)} \pm \frac{1}{g_s(x_2)} \right) \right],
\]

\[ g_s(x) = \sqrt{1 / \gamma^2 + x^2}. \]  

For the real parts \( A_{\pm} = \Re C_{\pm} \), the double integral in equation (14) can be calculated analytically. It yields:

\[
A_{\pm} = (\pi / 2)^2 [H_{-1}(1/s) + J_1(1/s)], \quad A_{\pm} = (\pi / 2)^2 [H_{-1}(1/s) - J_1(1/s)], \quad s = Z / \gamma,
\]

where \( H_{-1}(1/s) \) and \( J_1(1/s) \) are Struve and Bessel functions, respectively. Below we omit the suffix 'r' for brevity.

The width of spectral line components is controlled by the subsequent integral over the scaled impact parameter \( Z \):

\[ a_{\pm} = \int_0^{\infty} A_{\pm}(Z) dZ / Z = \int_0^{\gamma Z} \frac{A_{\pm}(s) ds}{s}, \quad s = Z / \gamma. \]

Figure 1 shows the plot of the integrand \( A_{\pm}(s) / s \) versus \( s \). It is seen that the corresponding integral \( a_{\pm} \) does not diverge at small impact parameters.

Figure 2 presents the plot of the integrand \( A_{\pm}(s) / s \) versus \( s \) and figure 3 shows a magnified part of this plot at small impact parameters. It is seen that the corresponding integral \( a_{\pm} \) also does not diverge at small impact parameters.

Thus, the integrals over the scale impact parameter \( Z \) in equation (16) converge at small impact parameters —in distinction to what would have resulted from the standard theory. At large \( Z \) the integral diverge (just as what would have resulted from the standard theory), which is physically because of the long-range nature of the Coulomb interaction between the charged particles. However, due to the Debye screening in plasmas, there is a natural upper cutoff \( Z_{\text{max}} \):
Here

$$\rho_D = [T_e/(4\pi e^2 N_e)]^{1/2}$$

(18)

is the Debye radius; $T_e$ and $N_e$ are the temperature and the density of bulk electrons, respectively.

The integration in equation (16) can be performed analytically because the integrals in equation (16) have the following antiderivatives

$$j_{\pm}(s) = \int A_{\pm}(s) \, ds = (\pi^2/8)(2/\pi) \text{MeijerG}[[\{0\}, \{1\}], \{(0,0), \{-1/2, 1/2\}], 1/4s^2]
+ H_2^2/(1/s) + H_2^2/(1/s) \pm [1 - \text{i} F_2(1/2; 1, 2; -1/s^2)],$$

(19)

where MeijerG[...] and $F_2(\ldots)$ are the MeijerG function and the generalized hypergeometric function, respectively. Thus, we obtain analytical results for the width functions:

$$a_{\pm} = j_{\pm}(Z_{\max}/\gamma) - j_{\pm}(0).$$

(20)

Below, as an example, we calculate explicitly the shape $I(\Delta \omega, \gamma)$ of the spectral line Ly-alpha broadened by a REB, where $\Delta \omega$ is the detuning from the unperturbed frequency of the spectral line. Similarly to paper [19], after inverting of the spectral operator, we obtain:

$$I(\Delta \omega, \gamma) = \frac{1}{3\pi} \left( \frac{\Gamma_{\pi}}{\Delta \omega^2 + \Gamma_{\pi}^2} + \frac{2\Gamma_{\sigma}}{\Delta \omega^2 + \Gamma_{\sigma}^2} \right),$$

(21)

where $\Gamma_{\pi}$ and $\Gamma_{\sigma}$ are the half-widths at half-maximum of the $\pi$- and $\sigma$-components of the Ly-alpha line, respectively. They are expressed as follows:

$$\Gamma_{\sigma} = \left[\eta_0/(1 - 1/\gamma^2)^{1/2}\right]j_{\pm}(Z_{\max}/\gamma) - j_{\pm}(0),$$

(22)

$$\Gamma_{\pi} = \left[\eta_0/(1 - 1/\gamma^2)^{1/2}\right]\int_0^\infty [A_{\pi}(s) - A_{\pi}(s)] \, ds/s,$$

(23)

where

$$\eta_0 = 4\pi \hbar^2 N_e/(m_e^2 c) = 5.618 \times 10^{-10} N_e (\text{cm}^{-3}) s^{-1}. $$

(24)
It is worth noting that in equation (23), the upper limit of the integration is infinity. This is because for the \( \pi \)-component of the Ly-alpha line the width in equation (23) is proportional to the difference of diagonal and nondiagonal matrix elements of the broadening operator, so that the corresponding integral converges not only at small, but also at large impact parameters, yielding the following relatively simple expression for the width:

\[
\Gamma_x = \pi^2 \eta_0 / [4(1 - 1/\gamma^2)^{1/2}].
\]  

(25)

Figure 4 shows the plot of the scaled width of the \( \sigma \)-component \( \Gamma_x / \eta_0 \) (upper curve) and of the scaled width of the \( \pi \)-component \( \Gamma_x / \eta_0 \) (lower curve) of the Ly-alpha line broadened by a REB versus the relativistic factor \( \gamma \) at \( N_e = 10^{15} \text{ cm}^{-3} \) and \( T_e = 2 \text{ eV} \).

Figure 5 presents the ratio \( \Gamma_x / \Gamma_\sigma \) of the widths of the \( \sigma \)- and \( \pi \)-components of the Ly-alpha line versus the relativistic factor \( \gamma \) at \( N_e = 10^{15} \text{ cm}^{-3} \) and \( T_e = 2 \text{ eV} \). It is seen that as \( \gamma \) increases from unity, this ratio increases, then reaches the maximum, and then decreases. The maximum ratio \( \Gamma_x / \Gamma_\sigma = 5.39 \) corresponds to \( \gamma = 2^{1/2} \).

Separate measurements of the widths of the \( \sigma \)- and \( \pi \)-components (and thus of the ratio \( \Gamma_x / \Gamma_\sigma \)) can be performed for the observation perpendicular to the REB velocity by placing a polarizer into the optical system: when the axis of the polarizer would be perpendicular or parallel to the REB velocity, then one would be able to measure \( \Gamma_\sigma \) or \( \Gamma_\pi \), respectively. By monitoring the dynamics of the ratio \( \Gamma_x / \Gamma_\sigma \), it would be possible, at least in principle, to detect the development of a REB in tokamaks and to engage the mitigation of the problem.

Figure 6 shows the theoretical profiles of the entire Ly-alpha line, corresponding to the observation perpendicular to the REB velocity without the polarizer, at \( N_e = 10^{15} \text{ cm}^{-3} \) and \( T_e = 2 \text{ eV} \). The profiles were calculated using equations (21)–(24) and presented versus the scaled detuning \( \Delta \omega / \Gamma_\sigma \) denoted as \( d \). Due to the scaled detuning, the profiles are ‘universal’ in the sense that they are independent of the beam electron density. The solid curve corresponds to \( \gamma = 2^{1/2} \), while the dashed curve—to \( \gamma = 1 \). In the case of \( \gamma = 2^{1/2} \), the profile is by 12% narrower than for the case of \( \gamma = 1 \). Detecting the development of a REB via such relatively small decrease of the width seems to be less advantageous compared to the polarization analysis of the width discussed above, where the widths ratio \( \Gamma_x / \Gamma_\sigma \) could increase by an order of magnitude as a REB develops in the plasma.
The above theoretical results represented the Stark broadening of hydrogen/deuterium spectral lines only by a REB without allowing for other factors affecting the line shapes. This was done for presenting the effect of a REB on the lineshape in the ‘purest’ form. Below we remove this restriction.

The major outcome of the interaction of a REB with plasmas is the development of strong Langmuir waves—see, e.g., book [22]. The maximum amplitude $E_0$ of the Langmuir wave electric field is [22]:

$$E_0 = \left[8\pi m_e c^2 \gamma^2 N_b^{4/3} / N_e^{1/3} \right] / 2, \quad \gamma N_b^{1/3} / N_e^{1/3} \ll 1.$$  \hfill (26)

For the case of $N_e = 10^{15}$ cm$^{-3}$, $N_b = 6 \times 10^9$ cm$^{-3}$, and $\gamma = 2^{1/2}$, corresponding to an early stage of the development of a REB in tokamaks, equation (26) yields $E_0 = 20$ kV cm$^{-1}$.

The primary manifestation of Langmuir waves in the profiles of hydrogen/deuterium or hydrogenlike spectral lines is the appearance of some local structures (called L-dips) at certain locations of the spectral line profile. This phenomenon arises when radiating atoms/ions are subjected simultaneously to a quasistatic field $F$ and to a quasimonochromatic electric field $E(t)$ at the characteristic frequency $\omega$, where $E < F$. In the heart of this phenomenon is the dynamic resonance between the Stark splitting of hydrogenic spectral lines and the frequency $\omega$ or its harmonics. There is a rich physics behind the L-dip phenomenon: even when the applied electric field is monochromatic, there occurs a nonlinear dynamic resonance of multifrequency nature involving all harmonics of the applied field—as it was explained in detail in paper [23]. Further details on the theory of the L-dips can be found in book [15].

As for the experimental studies of the L-dips, book [15] and later reviews [24–26] summarize all such studies with applications to plasma diagnostics. The practical significance of studies of the L-dips is threefold. First, they provide the most accurate passive spectroscopic method for measuring the electron density $N_e$ in plasmas, e.g., more accurate than the measurement from the line broadening. This passive spectroscopic method for measuring $N_e$ does not differ in its high accuracy from the active spectroscopic method—more complicated experimentally—using the Thompson scattering [27]. Second, they provide the only one non-perturbative method for measuring the amplitude of Langmuir waves in plasmas [15]. Third, in laser-produced plasmas they facilitate revealing physics behind the laser-plasma interaction [28–30].

According to the theory [15], L-dips originate from a dynamic resonance between the Stark splitting

$$\omega_{\text{stark}}(F) = 3n\hbar F / (2Z e)$$  \hfill (27)

of hydrogenic energy levels, caused by a quasistatic field $F$ in a plasma, and the frequency $\omega_{\text{L}}$ of the Langmuir wave, which practically coincides with the plasma electron frequency $\omega_{pe} = (4\pi e^2 N_e / m_e)^{1/2}$:

$$\omega_{\text{stark}}(F) = k \omega_{pe}(N_e), \quad s = 1, 2, \ldots$$  \hfill (28)

Here $Z$ is the nuclear charge of the radiating hydrogenic atom/ion (radiator), $k$ is the number of quanta (Langmuir plasmons) involved in the resonance.

The resonance condition (28) translates into specific locations of L-dips in spectral line profiles, which depend on $N_e$ since $\omega_{pe}$ depends on $N_e$. In particular, for relatively low density plasmas (like in magnetic fusion machines) or in the situation, where the quasistatic field $F$ is dominated by the low-frequency electrostatic

2 So far we used, as an example the Ly-alpha line just to get the message across (since we obtained relatively simple analytical expressions for the shape of this line). We note that at $N_e \sim 10^{15}$ cm$^{-3}$, the Stark width of the Lyman-alpha line calculated by equations (22)–(25) would be by about two and a half orders of magnitude below the natural width. However, the dynamical Stark width scales $\sim n^4$, while the natural width scales $\sim 1/n^3$ ($n$ being the principal quantum number). Therefore, for the lines originating from the level of $n = 4$ (such as Ly-gamma, Balmer-beta, Paschen-alpha) and higher levels, the corresponding dynamical Stark width would exceed the natural width.
turbulence (e.g., the ion acoustic turbulence), for the Ly-lines, the distance of an L-dip from the unperturbed wavelength $\lambda_0$ can be expressed as

$$\Delta \lambda_{\text{dip}}(q, N_e) = [\lambda_0^2/(2\pi\varepsilon)] q\omega_d(N_e). \quad (29)$$

Here $\lambda_0$ is the unperturbed wavelength of the spectral line and $q = n_1 - n_2$ is the electric quantum number expressed via the parabolic quantum numbers $n_1$ and $n_2 : q = 0, \pm 1, \pm 2, \ldots, \pm (n - 1)$. The electric quantum number labels Stark components of Ly-lines. Equation (29) shows that for a given electron density $N_e$, the locations of L-dips are controlled by the product $qk$. It should be emphasized that the abbreviation 'L-dip' refers to a local structure consisting of the central minimum and (generally) two adjacent bumps surrounding the central minimum—the latter is called 'dip' for brevity. Equation (29) specifies the locations of the central minima (dips) of these structures: it is from the locations of the central minima that the electron density can be determined experimentally. The dip-bump separation is controlled by the Langmuir field amplitude $E_0$ and thus allows the experimental determination of $E_0 [15]$. For finishing this brief excerpt from the L-dip theory necessary for understanding the next paragraphs, it should be also noted that when a bump-dip-bump structure is superimposed with the inclined part of the spectral line profile, this might lead to the appearance of a secondary minimum of no physical significance. Also, when the L-dip is too close to the unperturbed wavelength, its bump nearest to the unperturbed wavelength might have zero or little visibility. These subtleties were observed numerous times [15, 24–26] and will also be relevant below.

So, we will use the Ly-delta line of deuterium as an illustrative example of possible diagnostics of the early stage of the development of a REB in tokamaks. The Ly-delta line has four Stark components in each wing, corresponding to $q = \pm 1, \pm 2, \pm 3, \pm 4$. Therefore, according to equation (29), the L-dip in the profile of the component of $q = 1$ due to the four-quantum resonance $(k = 4)$ coincides by its location with the L-dip in the profile of the component of $q = 2$ due to the two-quantum resonance $(k = 2)$ and with the L-dip in the profile of the component of $q = 4$ due to the one-quantum resonance $(k = 1)$. The superposition of three different L-dips at the same location results in the L-super-dip of the significantly enhanced visibility.

Also, according to equation (29), the L-dip in the profile of the component of $q = 1$ due to the two-quantum resonance $(k = 2)$ coincides by its location with the L-dip in the profile of the component of $q = 2$ due to the one-quantum resonance $(k = 1)$. The superposition of two different L-dips at the same location results also enhances the visibility of the resulting structure.

For diagnostic purposes it is important to choose the spectral line where superpositions of several L-dips at the same location in the profile are expected. This is because due to competing broadening mechanisms (such as, e.g., the dynamical broadening by electrons and some ions, as well as the Doppler broadening), a single L-dip could be washed out, but a superposition of two or especially three L-dips at the same location could 'survive' the competition.

Figure 7 presents the theoretical profile of the Ly-delta line of deuterium, calculated with the allowance for all broadening mechanisms and for the effect of strong Langmuir waves (in distinction to the Ly-alpha profile in figure 6 that illustrated the pure effect of the broadening by the REB only), at the following parameters: $N_e = 10^{13} \text{ cm}^{-3}, N_r = 6 \times 10^7 \text{ cm}^{-3}, \gamma = 2^{1/2}$ (corresponding to the beam kinetic energy of 210 keV), and $T_e = 2 \text{ eV}$. The solid curve corresponds to the presence of the strong Langmuir waves of $E_0 = 20 \text{ kV cm}^{-1}$.
caused by a REB (according to equation (26)), while the dashed curve corresponds to the absence of the REB. The detuning Δλ (denoted ‘\( \lambda \)’ in figure 7) is in Angstrom.

The theoretical profile shown by the solid curve exhibits two L-dip structures at both the red and blue parts of the profile. The central minimum of the L-super-dips of \( qk = \pm 4 \) is at \( \Delta \lambda = \pm 0.338 \text{ A} \). This L-super-dip structure is very pronounced: the central minimum is relatively deep and both of the adjacent bumps are clearly visible. (Being superimposed with the inclined part of the profile, it creates also secondary minima of no physical significance at \( \Delta \lambda = \pm 0.275 \text{ A} \).

The L-dip structure of \( qk = \pm 2 \), whose central minimum is at \( \Delta \lambda = \pm 0.169 \text{ A} \), is also visible. However, it is less pronounced (compared with the L-super-dip of \( qk = \pm 4 \)) and its bump closest to the unperturbed wavelength has practically zero visibility. This is due to the fact that because of the proximity of this L-dip to the unperturbed wavelength, the ion dynamical broadening is more significant than for the L-super dip at \( \Delta \lambda = \pm 0.338 \text{ A} \).

In this example, the ratio of the energy density of the Langmuir waves to the thermal energy density of the plasma (the ratio called sometimes the ‘degree of the turbulence’) is \( E_0^2 / (8\pi N_e T_e) \sim 0.06 \). Since \( E_0^2 / (8\pi N_e T_e) \approx m_e / M \sim 0.0003 \) (where \( M \) is the mass of deuterium atoms), these Langmuir waves qualify as the strong turbulence.

Thus, the monitoring the shape of deuterium spectral lines (such as, e.g., Ly-\( \delta \)-le, or Balmer-\( \beta \)-le, or Paschen-\( \beta \)-le, or Paschen-\( \delta \)-le) and the observation of the formation of the L-dips in the experimental profile would constitute the detection of the early stage of the development of a REB in tokamaks. The detection of the early stage of the development of a REB would allow mitigating the problem in a timely manner.

3. Conclusions

We developed an advanced analytical theory of the Stark broadening of hydrogen/deuterium spectral lines by a REB. We showed that the final stage of the development of the REB (where the beam electron density \( N_B \) could become just of one or two orders of magnitude below the electron density \( N_e \) of bulk electrons), would be manifested—and thus could be detected, at least in principle—by a decrease of the width of hydrogen/deuterium spectral lines. We demonstrated that especially sensitive to the final stage of the development of the REB would be the ratio of widths of \( \sigma \)- and \( \pi \)-components, which could be determined by the polarization analysis.

We also showed that the early stage of the development of the REB could be detected by observing the formation of the L-dips in spectral line profiles. The observation of the L-dips, which manifest the development of strong Langmuir waves caused by the REB, could be an important tool for the early detection and the mitigation of the problem of REB in tokamaks.

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