A TREATISE ON FST LATTICE BASED MMI TRAINING

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ABSTRACT
Maximum mutual information (MMI) has become one of the two de facto methods for sequence-level training of speech recognition acoustic models. This paper aims to isolate, identify and bring forward the implicit modelling decisions induced by the design implementation of standard finite state transducer (FST) lattice based MMI training framework. The paper particularly investigates the necessity to maintain a pre-selected numerator alignment and raises the importance of determining FST denominator lattices on the fly. The efficacy of employing on the fly FST lattice determination is mathematically shown to guarantee discrimination at the hypothesis level and is empirically shown through training deep CNN models on a 18K hours Mandarin dataset and on a 2.8K hours English dataset. On assistant and dictation tasks, the approach achieves between 2.3-4.6% relative WER reduction (WERR) over the standard FST lattice based approach.

Index Terms— MMI training, lattices, FST

1. INTRODUCTION
Maximum mutual information (MMI) [1] is one of the two main approaches that has been experimentally shown to be the most effective in training state of the art acoustic models comprising of hidden Markov models (HMMs) [2] embedded with deep neural networks (DNNs) starting from a good initialisation [3, 4]. When context-dependent phone HMMs serve as the basic model to construct sentence level HMMs, it is computationally infeasible to train these hybrid models exactly with the MMI loss. In practice, models are trained within a computationally tractable lattice based framework [5, 6].

This paper brings forward the implicit modifications introduced to the MMI objective when implemented within the FST lattice based framework. In addition, this paper makes the following novel contributions:

1. This work presents a lower-bound proof that shows how dynamically choosing the Viterbi state alignment of each competing hypothesis in the denominator lattice leads to improvement in the original MMI objective. The efficacy of the proposed approach is shown through training deep CNN models on a 18K hours Mandarin dataset and on a 2.8K hours English dataset.

2. This work mathematically shows the effectiveness of using a fixed numerator alignment during MMI loss computation. The efficacy of using a fixed alignment against dynamically sampling the alignment for the MMI numerator loss computation is empirically verified on the 2.8K English dataset.

The rest of the paper is organised as follows: Sec. 2 reviews MMI training. Sec. 3 describes the FST lattice based MMI training framework. Sec. 4 presents the proposed approach with the proof given in the sec. 5. Sec. 6 presents the experimental setup followed by experimental results and conclusion.

2. MMI TRAINING
To facilitate understanding of the mathematical analysis undertaken in this work, let \((\mathbf{O}, \mathbf{W}^\text{ref})\) denote a sample where \(\mathbf{W}^\text{ref}\) is the reference word sequence associated with the observation feature sequence \(\mathbf{O}\). In the context of lattice based sequence training, a 3 state HMM topology [7, 8] equipped with emitting states serve as the underlying model for individual phones, the basic unit of the AM. An important advantage of using this modular form of HMM topology is that larger HMMs can be constructed by the concatenation or composition in the case of FSTs [9] of these basic models. This property allows sentence models to be constructed providing the flexibility to integrate LM scores as transition probabilities between states matching the start and end of words. In this work, the set of state alignments, indexed by \(i\), generated from the sentence HMM of the \(j\)th hypothesis \(\mathbf{W}^j\) w.r.t to \(\mathbf{O}\) of \(T\) frames will be denoted as \(\{y_{i,j}^T\}_i\). For a given \(\mathbf{O}\), the MMI loss is:

\[
\mathcal{L}_{\text{MMI}}(\theta) = \mathcal{G}(\theta) - \mathcal{F}(\theta) \quad \text{where} \quad (1)
\]

\[
\mathcal{G}(\theta) = \log \sum_j P_\theta(\mathbf{O}|\mathbf{W}^j)P(\mathbf{W}^j) = \log \sum_i \sum_j P_\theta(y_{i,j}^T, \mathbf{O}|\mathbf{W}^j)P(\mathbf{W}^j), \quad (2)
\]

\[
\mathcal{F}(\theta) = \log P_\theta(\mathbf{O}|\mathbf{W}^\text{ref})P(\mathbf{W}^\text{ref}) = \log \sum_i P_\theta(y_{i,j}^T, \mathbf{O}|\mathbf{W}^\text{ref})P(\mathbf{W}^\text{ref}). \quad (3)
\]
with \( P_0 \) being a function of model parameters \( \theta \). By minimising the difference between the two objective functions, optimising w.r.t the MMI loss not only maximises the probability of \( W_{\text{ref}} \), but also minimises the probability that of every competing hypothesis \( W^j \). This makes MMI a discriminatory loss.

## 3. FST LATTICE BASED MMI TRAINING

The computation of \( \mathcal{G}(\theta) \) involves summing over all possible sentence level state sequences which makes the computation expensive. To address the computational overhead, the standard FST lattice based approach employs a two step procedure: first a recognition pass on each training utterance is performed to collect only a subset of hypotheses that are most likely. The output of this process is then passed through a determinization algorithm [10] using a special semiring that only preserves the best state alignment for each word-sequence. The resultant alignments are stored in a special FST called a lattice. The standard approach is to generate these lattices once using a good initialised model, which will be referred to as the CE model going forward, and then proceed to use these lattices repeatedly at every iteration of discriminative sequence training. Although computing \( \mathcal{F}(\theta) \) is computationally less intractable, the standard recipe in FST lattice based MMI training involves sampling the Viterbi alignment once using the CE model and using it as the only target alignment [11]. For a given \( O \), these modifications lead to the following proxy function \( \hat{\mathcal{L}}_{\text{MMI}}(\theta) = \mathcal{G}(\theta) - \hat{\mathcal{F}}(\theta) \) being minimised where

\[
\hat{\mathcal{G}}(\theta) = \log \sum_j P_0 \left( y_{1:T}^{W_j^j}(\theta_{\text{CE}}), O|W^j \right) P(W^j), \quad (4)
\]

\[
\hat{\mathcal{F}}(\theta) = \log P_0 \left( y_{1:T}^{W_{\text{ref}}}(\theta_{\text{CE}}), O|W_{\text{ref}} \right). \quad (5)
\]

Here \( \theta_{\text{CE}} \) denotes the model parameters associated with CE model and \( y_{1:T}^{W_j^j}(\theta) = \arg \max_{y_{1:T}} P_0(y_{1:T}, O|W^j)P(W^j) \). FST lattice-based MMI training thus employs an alignment-level discriminative criterion where at each iteration of training, the model is updated to reduce the confusion between a CE model chosen numerator alignment and the CE model chosen Viterbi alignments from a set of competing hypothesis.

Using \( \hat{\mathcal{L}}_{\text{MMI}}(\theta) \) as a proxy to the true MMI loss has been shown to lead to consistent Word Error Rate reduction (WERR) from discriminative sequence training [11]. Although the procedure has been found to be experimentally effective, it is not clear why discriminating between a set of pre-selected alignments correlates with a hypothesis level discriminatory loss. Furthermore, it is biased to assume that a CE model chosen numerator alignment is the only feasible alignment for a given hypothesis. An utterance can have multiple correct alignments in its sentence level \( W_{\text{ref}} \) HMM.

The lower bound in eqn.(14) of the proof in Sec. 5 sheds some light in the effectiveness of such an approximation. The inequality shows how using a fixed subset of numerator alignments throughout training can still guarantee improvement w.r.t the MMI objective. This is verified in Sec. 7, where an empirical investigation on the necessity in using a fixed pre-selected numerator state alignment is conducted. The effect of dynamically sampling a numerator state alignment from a CE model generated numerator lattice using the current model parameter update is explored. In this work, two approaches to sampling have been considered. The first one employs the Viterbi algorithm [13] to select the best path at every iteration of training. The second approach employs ancestral sampling [14] to sample paths from the lattice. In this latter approach samples are drawn efficiently using the backward filtering forward sampling algorithm [15]. The algorithm performs local normalisation of the weight leaving each state by re-weighting the weight with the associated probability scores from the backward algorithm [2]. Samples from the re-weighted FST can then be drawn using simple ancestral sampling.

## 4. ON THE FLY LATTICE DETERMINIZATION

This work also proposes using the following candidate function as a proxy to \( \mathcal{G}(\theta) \):

\[
\bar{\mathcal{G}}(\theta) = \log \sum_j P_0 \left( y_{1:T}^{W_j^j}(\theta_{\text{CE}}), O|W^j \right) P(W^j). \quad (6)
\]

The loss is computed by using the latest model update to select the Viterbi path associated with each hypothesis captured by the initial recognition pass using the CE model. The motivation behind this approach is that at each iteration of training, the model parameters will be updated to reduce the confusion between a chosen numerator alignment and the alignment associated with each competing hypothesis that yields the greatest confusion. From an implementation point of view, this can be achieved by skipping the determinization process [10] during the initial lattice generation and performing it on the fly during training. Theorem 5 shows how such a modification in combination with \( \hat{\mathcal{F}}(\theta) \) guarantees discrimination over the subset of competing hypotheses captured in the lattice w.r.t the MAP decision rule under certain assumptions. The efficacy of this proposed modified objective is investigated in Section 7.

## 5. THEOREM

If the mapping \( f : \{ y_{1:T}^j \} \rightarrow \{ W^j \} \) is well defined then dynamically sampling the Viterbi paths of each competing hypothesis using the current model update in conjunction with using a fixed numerator alignment guarantees discrimination w.r.t maximum a posteriori (MAP) decision rule.
5.1. Discrete measure on the space of $\mathcal{Y}_{1:T}$

The mathematical proof presented relies on the concept of a measurable space and an associated measure [16]. Let $\mathcal{X}$ denote the set of all state alignments $\{y_{1:T}^i\}_i$, that are present in sentence level hybrid HMM-DNN models associated with a given $\mathcal{O}$ with $\mathcal{A}$ being the powerset of $\mathcal{X}$. Given that each state alignment corresponds to a hidden state sequence in some sentence level HMM, one can define as a positive real-valued function $m : (\mathcal{X}, \mathbb{R}^D) \rightarrow [0, \infty)$ as follows:

$$m(\mathcal{Y}_{1:T}, \theta) = \sum_j 1_{\mathbb{W}_j} (\mathcal{Y}_{1:T}) P_\theta(\mathcal{Y}_{1:T}, \mathcal{W}^j|\mathcal{O}),$$

(7)

where $\mathcal{Y}_{1:T} \in \{y_{1:T}^i\}_i$, $\theta \in \mathbb{R}^D$ denotes the $D$ dimensional parameter vector associated with the model and $1_{\mathbb{W}_j}$ is the indicator function which equates to 1 if $y_{1:T}^i$ belongs to the sentence level HMM of $\mathcal{W}^j$. To avoid notational clutter, the conditioning on $\mathcal{O}$ is not explicitly stated in the definition of $m(\mathcal{Y}_{1:T}, \theta)$.

For any vector $\theta$, it is easy to see that

$$\sum_i m(y_{1:T}^i, \theta) = 1.$$  

(8)

Hence, one can define a discrete probability measure $\mu_\theta : \mathcal{A} \rightarrow [0, \infty]$ can now be defined as follows:

$$\mu_\theta(\hat{A}) = \sum_i m(y_{1:T}^i, \theta) 1_{\hat{A}}(y_{1:T}^i) \forall \hat{A} \in \mathcal{A},$$

(9)

where $1_{\hat{A}}$ is the indicator function. It is easy to see that $\mu_\theta(\mathcal{X}) = 1$.

5.2. Proof

Let $\hat{A}$ denote the set of all state alignments $\{y_{1:T}^i, ref\}_i$, that belong to the sentence level HMM model of $\mathcal{W}^{ref}$, and let $\bigcup_j \hat{B}_j$ be a union of measurable sets with each $\hat{B}_j = \{y_{1:T}^i\}_i$, the set of state alignments belonging to the constructed sentence level HMM of $\mathcal{W}^j$. Thus, by construction

$$\hat{\mathcal{X}} = \left( \bigcup_j \hat{B}_j \right) \cup \hat{A},$$

and $\hat{A}$ being the power set of this set. Using $\mu_\theta$, the following probability measure $\hat{\mu}_\theta$ can now be defined on this space:

$$\hat{\mu}_\theta(C) = \frac{\mu_\theta(C)}{\sum_{j=1}^K \mu_\theta(\hat{B}_j)} \text{ for all } C \in \hat{A}. $$

(15)

Under such a construction,

$$\hat{\mu}_\theta(\hat{A}) = \frac{\mu_\theta(\hat{A})}{\sum_{j=1}^K \mu_\theta(\hat{B}_j)} = \frac{P_\theta(\mathcal{Y}_{1:T} = y_{1:T}^{k, ref}, \mathcal{O}|\mathcal{W}^{ref})P(\mathcal{W}^{ref})}{\sum_{j=1}^K P_\theta(\mathcal{Y}_{1:T} = y_{1:T}^j, \mathcal{O}|\mathcal{W}^j)P(\mathcal{W}^j)},$$

(17)

where $y_{1:T}^j = \arg\max_{\mathcal{Y}_{1:T}^j} P_\theta(\mathcal{Y}_{1:T}^j, \mathcal{O}|\mathcal{W}^j)P(\mathcal{W}^j)$. The negative log of $\hat{\mu}_\theta(\hat{A})$ corresponds to the proposed FST lattice-based MMI objective with the Viterbi path of each competing hypothesis chosen by the current model update. Since logarithmic function is monotonic, minimising $-\log(\hat{\mu}_\theta(\hat{A}))$ is equivalent to maximising $\hat{\mu}_\theta(\hat{A})$. For any parameter vector $\theta \in \mathbb{R}^D$, the associated measure $\mu_\theta$ will all satisfy the inequalities (13) and (14). Therefore maximising $\hat{\mu}_\theta(\hat{A})$ can be seen to maximise the probability of $\mathcal{W}^{ref}$ while simultaneously decreasing the probability of the competing hypothesis captured in the lattice as a consequence of lower bound (13). This ensures discrimination w.r.t MAP decision rule.
6. EXPERIMENTAL SETUP

The experiments relevant to this work were conducted on an internal anonymized Mandarin and British English dataset. For the Mandarin ASR experiment, 18K hours of training data was used from which roughly 10 hrs of training data was uniformly sampled to form the validation set. For the British English ASR experiment, systems were trained using a subset of 2.8K hours from the British English training data from which 15 hours of uniformly sampled data was used as the validation set. To estimate the generalisation performance of the candidate models, decoding of the resultant models was performed on language specific test sets using a pre-trained neural language model. In the Mandarin ASR task, 4 independent test sets: two comprising of assistant task data and two consisting of dictation data were used. While for the British English ASR task, an assistant task data set and dictation data test set were used respectively. Each of these sets composed roughly of 36 hrs of data.

The efficacy of the proposed modification to the FST lattice based MMI denominator loss computation and the investigation on the effect of dynamically sampling a numerator alignment is shown on training a 50-layer self-normalizing deep CNN (SNDCNN) model \[17\]. The input to the model was produced by splicing together 80 dimensional log-Mel filter bank (FBK) features using a context window of 41 frames. For all experiments, the input features were mean and variance normalised w.r.t the training data. For the Mandarin ASR experiment, the filter bank features were augmented with pitch information \[18\] and frame level spec-augmentation \[12\] was employed.

Prior to sequence training, the models were initialised with frame-level cross entropy training to serve as the CE models. These models were used to create the denominator lattices. Determinized lattices were used to get baseline WERs while to train models with the proposed modification, non-determinized lattices were generated. To investigate the effect of dynamically sampling a numerator alignment, numerator lattices were created under the same conditions used to generate the non-determinized denominator lattices. As there is an obvious mismatch between the various training criteria explored with the WER, over-fitting to the training criterion can occur \[19\]. To track how generalisation improvements w.r.t the training criterion correlates with WER reduction and to perform model selection, additional decoding was performed using a language specific separate held out development set after each epoch.

An epoch size of 500 hours was used in the British English ASR experiment. Models were trained in a decentralized distributed setting \[20\] using the ADAM optimiser \[21\] equipped with the new bob learning rate scheduler. For the much larger Mandarin ASR experiment, an epoch size of 200 hrs was used. To ensure fast and efficient training, the Block Model Update filtering (BMSGD) \[22\] algorithm was used to train the models in a centralized distributed setting.

7. SUMMARY OF RESULTS

7.1. Efficacy Of Using Fixed Numerator Alignment

The effect of dynamically sampling the numerator alignment during MMI training was evaluated on the British English ASR task. Table 1 summarizes the results where ‘baseline’ corresponds to the standard MMI recipe of using a fixed CE model chosen numerator alignment. The use of a fixed alignment can be seen to yield the greatest WERR from FST lattice based MMI training which corresponds to a relative improvement of 15% WERR on both the dictation and assistant task test sets. Over the two sampling based approaches, the fixed alignment approach achieves a relative WERR of 9.5-10% on the dictation task and 2.8% on the assistant task. Inequality \[14\] provides some insight into this behaviour: maximising the probability of fixed set of numerator alignments acts as a lower bound to the MMI objective’s numerator component.

| Numerator alignment selection | Assistant | Dictation |
|-------------------------------|-----------|-----------|
| Baseline                      | 6.16      | 6.57      |
| Ancestral sampling            | 6.34      | 7.33      |
| Viterbi                       | 6.33      | 7.26      |

Table 1. WERs on the British English test sets with different choices of numerator state alignment used during MMI training.

7.2. On The Fly Denominator Lattice determinization

The standard FST lattice based approach employs a two step procedure: first a recognition pass on each training utterance is performed using the CE model to collect only a subset of hypotheses that are most likely. As mentioned in Sec. 3 the output of this process is then passed through a determinization algorithm that only preserves the best state alignment for each word-sequence. In the proposed modification to MMI training, step 2 is deferred to the actual training stage. The current model update is used to select the best state alignment for each competing word-sequence during training. As a consequence, the lattices stored will maintain multiple alignments per hypothesis leading to increased lattice size. For the datasets used in this experiment, the storage overhead was found to increase by a factor of 5. The issue can be resolved by generating the lattices on the fly but this comes at the expense of increased training time. The proposed approach lies in between the lattice-based and on the fly lattice generation approach to MMI training. The non-necessity to do a recog-
nition pass makes it less computationally expensive than creating the lattices on the fly.

Table 2 compares the efficacy of the resultant Mandarin acoustic model trained with the proposed approach against an equivalent CE initialized model trained with the standard FST lattice based recipe on the 4 test sets. Sequence training using the standard approach can be seen to lead to a relative improvement of 12.7-13.8% on the assistant test sets and 12.4-13.6% WERR on the dictation test sets. Over the standard approach, the proposed approach achieves a further relative improvement of 2.3-3.1% on the assistant tasks and 3.4-4.6% WERR on the dictation test sets. The improvements seen accompanied by the mathematical proof in Sec 5 presents a strong argument in the efficacy of the proposed modification to the FST lattice based MMI loss computation.

Table 2. WERs with variants of FST lattice based MMI loss on the Mandarin test sets

| Approach                  | Assistant task 1 | Assistant task 2 | Dictation task 1 | Dictation task 2 |
|---------------------------|------------------|------------------|------------------|------------------|
| Baseline                  | 7.37             | 7.44             | 6.57             | 6.49             |
| Num path: Viterbi + on the fly determinization | 7.20             | 7.21             | 6.29             | 6.27             |

Table 3. WERs with variants of FST lattice based MMI loss on the British English dataset.

| Approach                  | Assistant | Dictation |
|---------------------------|-----------|-----------|
| Baseline                  | 6.16      | 6.57      |
| Num path: Viterbi + on the fly determinization | 6.18      | 6.90      |
| MMI + on the fly determinization | 6.02      | 6.53      |

8. CONCLUSION

This paper has highlighted the various implicit modifications introduced to the MMI objective when models are trained in an FST lattice based framework. To this end, one of the major goals of this work has been to bring forward these details to a larger audience in the ASR community and allow more people to work on the particular challenges of FST based MMI training. The paper in particular investigated the necessity to maintain a pre-selected numerator alignment and addressed the importance of on the fly FST denominator lattice determinization. The effectiveness of employing on the fly FST lattice determinization is mathematically supported to guarantee discrimination at the hypothesis level and is empirically shown on training a SNDCNN model on an 18K Mandarin dataset and 2.8K British English dataset respectively. On the Mandarin assistant and dictation test sets, the proposed approach achieved between 2.3-3.1% WERR on the assistant test sets and between 3.4-4.6% WERR on the dictation test sets over the standard FST lattice MMI training.

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