Analysis of Slotted ALOHA with an Age Threshold

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Abstract

We present a comprehensive steady-state analysis of threshold-ALOHA, a distributed age-aware modification of slotted ALOHA proposed recently in [1]. In threshold-ALOHA each terminal suspends its transmissions until the age of its status flow exceeds a certain threshold \( \Gamma \), and once age exceeds \( \Gamma \), it attempts transmission with some constant probability \( \tau \), as in standard slotted ALOHA. We analyze the time-average expected Age of Information (AoI) attained by this policy, and explore its scaling with network size, \( n \). We derive the probability distribution of the number of active users at steady state, and show that as network size increases the policy converges to one that runs slotted ALOHA with fewer sources: on average about one fifth of the users is active at any time. We obtain an expression for steady-state expected AoI in the network and use this to optimize the parameters \( \Gamma \) and \( \tau \), resolving the conjectures in [1] by confirming that the optimal age threshold and transmission probability are \( 2.2n \) and \( 4.69/n \), respectively. We find that the optimal AoI scales with the network size as \( 1.4169n \), which is almost half the minimum AoI achievable using slotted ALOHA, while the loss from the maximum achievable throughput of \( e^{-1} \) remains below 1%.

Index Terms

Slotted ALOHA, Age of Information, AoI, threshold policy, random access, age-based thinning, stabilized ALOHA

I. INTRODUCTION

Age of Information (AoI) emerged almost a decade ago [2, 3] as a useful new metric facilitating the characterization and control of information freshness in status-update based networked
systems, including most Internet-of-Things (IoT) and Machine-type Communications (MTC) scenarios. Many classical networking formulations have since been revisited from the perspective of AoI analysis and optimization. Among these revised formulations, the addressing of random access with an AoI objective is relatively new [4], particularly motivated by applications such as industrial automation, networked control systems, environmental monitoring, health and activity sensing, where multiple sensor nodes send updates of sensed data a common access point on a shared channel.

A series of recent works [1, 4, 5, 6, 7] studied basic abstractions that capture the essence of information aging in this random access environment: (1) time is slotted and nodes are synchronized to the slot timing, (2) concurrent transmissions result in packet loss, (3) nodes make distributed transmission decisions, (4) the longer it takes a node to successfully transmit a packet, the more its corresponding data flow ages. Accordingly, in order to keep the time-average age in the network under control, the distributed decision mechanism needs to strike a balance between each node attempting transmission sufficiently often, and more than one transmission attempts at a time being unlikely. This problem is related to the classical problem of distributed stabilization of slotted ALOHA (see, e.g., [8]), revisited here through the lens of AoI, which is a fundamentally different performance objective. Throughput optimality and age optimality in channel access scheduling often do not coincide [9]- a throughput optimal mechanism can be arbitrarily poor in terms of average AoI, however, age optimality requires high throughput, and is often attained at an operating point that is nearly throughput-optimal, an example of which we will demonstrate in this paper in the context of random access.

There have been a number of prior studies that study AoI optimization in scheduled access [6, 10, 11, 12]. These, however are require centralized decisions which may be restrictive for most IoT scenarios. Alternatively, MaxWeight type strategies where the transmission probability of users are controlled by their ages have been explored in [5, 9, 13] and CSMA-type policies were proposed and studied in [14, 15]. The work in [5] also considers a random access channel where packets arrive according to exogenous arrival processes. Among several policies studied in [5], the SAT policy is closest to the threshold-ALOHA policy studied in this paper, as it employs a fixed age threshold, resulting in an age-thinning that reduces the number of active users, combined with an adaptive transmission probability equal to $1/\hat{n}$ (where $\hat{n}$ is the estimate of the number of active users) akin to the mechanism in Rivest’s stabilized slotted ALOHA [16].
Interestingly, the optimal AoI under the SAT policy scales as $\frac{\pi}{2} n$, similarly to threshold-ALOHA.

Stationary and distributed policies where new packets are generated “at will” (where nodes generate a new sample when they decide to transmit) were considered in [4,17,18]. [4] analyzes slotted ALOHA and Round-robin policies for the random access channel. The ratio between the minimum achievable AoI under these two policies is determined as $2e$. In [18], an AoI expression is derived for slotted ALOHA allowing up to a certain number of retransmissions. We remark that age is not an inherent parameter influencing transmission decisions in any of the policies studied in the aforementioned papers. In an effort to construct a random access policy that directly bases its decisions on age, [1] suggested a modified slotted ALOHA policy: sources with an age above a certain threshold attempt transmission with a fixed probability. The problem was shown to be equivalent to the optimization of a finite state Markov Chain and a closed-form formula for AoI was obtained. The optimal age threshold and AoI were conjectured to be around $2.2n$ and $1.4n$, respectively, where $n$ is the number of users, as a result of simulations. An independent analysis of the same algorithm was performed in [7], in which the network is analyzed over the states of a single source. A closed-form solution is derived along with sub-optimal findings, which were justified by hardware experiments in [19]. The results of [7,19], however, are limited by a cap on transmission probabilities of the sources that hinder the potential of the policy to reach optimal AoI.

In this paper, we build on the model in [1] and provide a detailed steady-state analysis of the threshold-based slotted ALOHA policy introduced therein (called the Lazy Policy, in [1]). This policy, which we will refer to as threshold-ALOHA in the rest of this paper, and precisely describe in Section II, differs from ordinary slotted ALOHA only in that users back-off for a deterministic amount of time (an age threshold) after a successful transmission. A partial analysis of this algorithm was also performed in [7] albeit under limited cases as we will elaborate in Section IV of this paper.

Our main contributions are the following:

- We find the explicit steady state solution of the DTMC model established in [1] and derive the distribution of number of active users, for any network size (Lemma 1).
- We show, in the limit of an infinitely large network, (Corollary 1) that the number of active sources is independent of the state of a particular source and use it to establish the limiting probability, $q_0$, of a successful transmission at steady state.
• We analyze the behavior of the policy in a large network and show that the policy converges to a slotted ALOHA policy with fewer users (Theorem 2 and 3), as the number of users grows. This limiting behavior is similar to Rivest’s stabilized slotted ALOHA, or the age-thinning policy introduced in [5], albeit with much lower computational complexity.

• We derive an expression relating the time average AoI to the network size \( n \), the transmission threshold \( \Gamma \), and the transmission probability \( \tau \), and show that the optimal achievable time average AoI with this policy scales with \( n \) as \( 1.4169n \) (Theorem 4), which is close to half the minimum value \( en \) achievable by ordinary slotted ALOHA [4]. Moreover, at this AoI-optimized operating point, the loss in throughout is below 1% with respect to the maximum achievable throughput.

The rest of the paper is organized as follows: Section II presents the system model. Section III-A contains the derivation of the steady state distribution of DTMC defined in [1]. Section III-B analyzes the system in the large network limit and formulates the distribution of number of active users, conditioned on the age of a single source. Sections III-C and III-D characterize the two possible steady-state behaviors of the policy. Section III-E derives the AoI expression and presents optimal results. Section IV gives simulation results and compares the performance of threshold-ALOHA with other related policies. We conclude in section V by summarizing our contributions and discussing future directions.

II. System Model

We consider a wireless network containing \( n \) sources (alternatively, users) and a common access point (AP). The sources wish to send occasional status updates to their (possibly remote) destinations reached through the AP. Nodes are synchronized with a common time reference (obtained through a control channel), and there is a slotted time-frame structure. We adopt the “generate-at-will” model [20] such that each source that decides to transmit generates a fresh sample just before transmission. We disallow collision resolution, such that if two or more users attempt transmission in the same slot, all transmitted packets are lost. There are no re-transmissions. When a failed source attempts transmission again, it will generate a new packet. If there is no collision, the transmission of the packet is successfully completed within a single time slot.
For simplicity, we will have each source generate a single data flow. The Age of information (AoI) of user \(i \in \{1, \ldots, n\}\) (equivalently, that of flow \(i\)) at time slot \(t\), \(A_i[t]\), is defined as the number of time slots that have elapsed since the freshest packet of this flow thus far received by the AP was generated. Due to the generate-at-will model we imposed, \(A_i[t]\) is equal to the number of slots since the most recent successful transmission of source \(i\), plus one. In the case of a successful transmission, the sender receives a 1-bit acknowledgement (possibly piggybacked on a back-channel packet.), and resets the age of its flow to 1. Accordingly, the age process \(\{A_i[t], t = 1, 2, \ldots\}\) evolves as:

\[
A_i[t] = \begin{cases} 
1, & \text{source } i \text{ transmits successfully at time slot } t - 1 \\
A_i[t - 1] + 1, & \text{otherwise}
\end{cases}
\]

The long term average AoI of source \(i\) is defined as:

\[
\Delta_i = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} A_i[t]
\]

on each sample path where the limit exists. In the next section, we define the threshold-ALOHA policy.

### III. Problem Definition and Analysis

In slotted ALOHA, users initiate transmission attempts with a fixed probability \(\tau\) in each time slot. When buffering and re-transmissions are allowed, this algorithm is unstable. Stabilization can be achieved through modification of the probability \(\tau\) according to the state of the network, which is often inferred through feedback about successful transmission. In the same vein, feedback about successful transmissions can be used by each source to determine its instantaneous age. In [1], a simple modification of slotted ALOHA was proposed, which we shall refer to as threshold-ALOHA in the rest of this paper. (This algorithm was called Lazy Policy in [1], we modify the name here to one that may be more descriptive of the nature of the policy.)

Threshold-ALOHA is a simple age-aware extension of slotted ALOHA: sources will wait until their age reaches a certain threshold \(\Gamma\), before they turn on their slotted ALOHA mechanism, and only then start to attempt transmission with a fixed probability \(\tau\) at each time slot. Hence, sources, who have successfully sent an update not more than \(\Gamma - 1\) time slots ago, stay idle and allow others with larger ages contend for the channel. It was numerically observed, without proof, in [1] that this policy is an improvement over slotted ALOHA in the sense that it achieves around half the long term average age achieved by regular slotted ALOHA, without significantly compromising network throughput. Furthermore, it was hypothesized that the optimal threshold
scales with the network size as $\Gamma = 2.2n$. These will be confirmed to be essentially correct, as part of the results of our precise analysis of the various convergence modes of this policy.

From the above description of threshold-ALOHA, it is clear that the decision of each source at timeslot $t$ is determined by its age at the beginning of this timeslot: if the age is below threshold, the node will stay idle, and if not, it will transmit with probability $\tau$. In [1] it was established that the age vector of the sources can be used to denote the state of the network, and for any value of $n$, this state evolves as a Markov Chain (MC):

$$A[t] \triangleq \langle A_1[t] \ A_2[t] \ldots \ A_n[t] \rangle$$

(3)

It was also shown in [1] that for the purpose of age analysis, it suffices to consider a truncated version of this MC, which constitutes a Finite State Markov Chain (FSMC), with a unique steady-state distribution. The truncated model is based on the observation that once the age of a source exceeds $\Gamma$, it becomes an active source, and its behavior remains same regardless of how much further its age increases. In most of the remainder of our analysis, unless stated otherwise, the ages of active sources will be truncated at $\Gamma$. Due to the ergodicity of the FSMC, and due to the symmetry between the users, the time average AoI (2) of each user can be found by computing the expectation over the steady-state distribution of the age, which is equal for all $i$:

$$\Delta_i = \lim_{t \to \infty} \mathbb{E}[A_i[t]]$$

(4)

In the rest, we explore this steady-state distribution and exploit its asymptotic characteristics.

A. Steady State Solution

As in [1], we define the truncated state vector:

$$A^\Gamma[t] \triangleq \langle A_1^\Gamma[t] \ A_2^\Gamma[t] \ldots \ A_n^\Gamma[t] \rangle$$

(5)

where $A_i^\Gamma[t] \in \{1,2,\ldots,\Gamma\}$ is the AoI of source $i$ at time $t \in \mathbb{Z}^+$ truncated at $\Gamma$ and evolves as:

$$A_i^\Gamma[t] = \begin{cases} 1, & \text{source } i \text{ updates at time } t-1, \\ \min\{A_i^\Gamma[t] + 1, \ \Gamma\}, & \text{otherwise}. \end{cases}$$

(6)

The resulting state space is $S = \{1,2,\ldots,\Gamma\}^n$. As shown in [1], $\{A^\Gamma[t], t \geq 1\}$ is a finite state Markov Chain (MC) with a unique steady state distribution. We first describe the recurrent class of this MC.

**Proposition 1.** If a state $\langle s_1 \ s_2 \ldots \ s_n \rangle$ in the truncated MC $\{A^\Gamma[t], t \geq 1\}$ is recurrent, then for distinct indices $i$ and $j$, $s_i = s_j$ if and only if $s_i = s_j = \Gamma$. 
Proof. Suppose at time $t > 1$, there exist two entries of the state vector that are equal to 1, i.e. there is a pair of sources $(i,j)$ such that $s_i = s_j = 1$. This would imply two simultaneous successful transmissions at $t-1$. However, this is impossible due to the assumption that colliding packets are lost. We extend this argument to cases where $s_i = s_j = s < \Gamma$ and $t > s$. The existence of such an $(i,j)$ pair implies two simultaneous transmissions at $t-s$. As this is impossible, such $(i,j)$ pairs cannot exist. Finally, if the system started in a state where there are two (or more) users that have the same age, $a < \Gamma$, at $t = 1$, these ages will grow to $\Gamma$ in $\Gamma - a$ time slots after which they will be decoupled, because only one can get reset to 1 at a time. Therefore, if the initial state of the MC is one that contains non-distinct below-threshold values, the chain will leave this state in at most $\Gamma$ time slots, and it will never return. This implies that such states are transient.

According to Prop. 1, states where distinct users have equal below-threshold age are transient. So, without loss of generality, the steady-state analysis that follows will be limited to the remaining states, where $s_i = s_j$ if and only if $s_i = s_j = \Gamma$. It will later be proved that all the remaining states are recurrent, moreover, as there is a unique steady state (from (11)) those states are all in the same recurrent class in the truncated MC. So in the rest, we refer to the remaining states as recurrent states.

We define the type of a recurrent state in the following way:

$T\langle s_1 \ s_2 \ldots \ s_n \rangle = (M, \{u_1, u_2,\ldots, u_{n-M}\})$, \hspace{1cm} (7)

where $M$ is the number of entries equal to $\Gamma$ (i.e., the number of active sources), and the set $\{u_1, u_2,\ldots, u_{n-M}\}$ is the set of entries smaller than $\Gamma$ (i.e., the set of ages below the threshold).

**Proposition 2.** States of the same type have equal steady state probabilities.

**Proof.** Follows from the symmetry between users.

In the next lemma, we further show that, for a given $M$, the set $\{u_1, u_2,\ldots, u_{n-M}\}$ has no effect on the steady state probability. That is, the steady state probability of a state is determined by $M$, the number of active sources. This facilitates the derivation of the distribution of the number of active sources.

**Lemma 1.** The truncated MC $\{A^{\Gamma}[t], t \geq 1\}$ has the following properties:
i Given a state vector \( \langle s_1, s_2, \ldots, s_n \rangle \), its steady state probability depends only on the number of entries that are equal to \( \Gamma \).

ii Let \( P_m \) be the total steady state probability of states having \( m \) active users. Then

\[
\frac{P_m}{P_{m-1}} = \frac{(1 - (m - 1)\tau(1 - \tau)^{m-2})(n - m + 1)}{m\tau(1 - \tau)^{m-1}(\Gamma - 1 - n + m)}
\]

iii The steady state probability of having no active sources is

\[
P_0 = \frac{1}{1 + \sum_{m=1}^{n} \prod_{i=1}^{m} \frac{1}{i\tau(1 - \tau)^{i-1}(\Gamma - 1 - n+i)}}
\]

**Proof.** First, suppose that the given state vector has no entry equal to 1. Let the type of this state vector be \( \mathcal{T}_1 \triangleq (M, \{u_1, u_2, \ldots, u_{n-M}\}) \), where \( M \in \{0, 1, \ldots, n\} \) is the number of entries equal to \( \Gamma \) and \( u_i > 1, i = 1, 2, \ldots, u - M \). As there is no source whose age is 1 at the current time, \( t \), there has been no successful transmission in the previous time slot, \( t - 1 \). Hence, the number of active users at \( t - 1 \) cannot have been \( M + 1 \) or larger. So the state at \( t - 1 \) must be one of the following types:

- \( \mathcal{T}_2 \triangleq (M, \{u_1 - 1, u_2 - 1, \ldots, u_{n-M} - 1\}) \)
- \( \mathcal{T}_3 \triangleq (M - 1, \{\Gamma - 1, u_1 - 1, u_2 - 1, \ldots, u_{n-M} - 1\}) \)

If, on the other hand, there was a successful transmission whilst in types \( \mathcal{T}_2 \) and \( \mathcal{T}_3 \), the resulting state would have been of type \( \mathcal{T}_0 \triangleq (M - 1, \{u_1, u_2, \ldots, u_{n-M}, 1\}) \).

Alternatively, if the given state vector has an entry that is equal to 1 at current time, \( t \), it indicates a successful transmission at \( t - 1 \). In this case, the given state vector is of type \( \mathcal{T}_0 \) and the state at \( t - 1 \) must be of types \( \mathcal{T}_2 \) or \( \mathcal{T}_3 \), as defined above.

Let \( \mathcal{C}_t \) be the set of states that are of type \( \mathcal{T}_0 \) or type \( \mathcal{T}_1 \). Let \( \mathcal{C}_{t-1} \) be the set of states that are of type \( \mathcal{T}_2 \) or type \( \mathcal{T}_3 \). If the system is in a state that is in \( \mathcal{C}_t \) at time \( t \), then its state at time \( (t - 1) \) must be in \( \mathcal{C}_{t-1} \). This follows from the fact that there can be at most 1 transmission at each time slot and due to Prop. \( \square \) all source states except \( \Gamma \) are unique. Similarly, if the system is in a state that is in \( \mathcal{C}_{t-1} \) at time \( (t - 1) \), then its state at time \( t \) must be in \( \mathcal{C}_t \).

Any given state of type \( \mathcal{T}_2 \) evolves into a state of type \( \mathcal{T}_0 \) with probability \( M\tau(1 - \tau)^{M-1} \) and into a state of type \( \mathcal{T}_1 \) with probability \( 1 - M\tau(1 - \tau)^{M-1} \). A state of type \( \mathcal{T}_3 \) evolves into a state of type \( \mathcal{T}_0 \) with probability \( (M - 1)\tau(1 - \tau)^{M-2} \) and into a state of type \( \mathcal{T}_1 \) with probability \( 1 - (M - 1)\tau(1 - \tau)^{M-2} \). Let \( \pi_{\mathcal{T}_j} \) be the steady state probability of a single state of type \( \mathcal{T}_j \). By the arguments above, the steady-state probabilities are related to each other by the following
\[
\pi_{T_1} = \pi_{T_3} (1 - M \tau (1 - \tau)^{M-1}) + \pi_{T_3} M (1 - (M - 1) \tau (1 - \tau)^{M-2}) \tag{8}
\]

\[
\pi_{T_0} = \pi_{T_2} \tau (1 - \tau)^{M-1} + \pi_{T_3} (M - 1) \tau (1 - \tau)^{M-2} \tag{9}
\]

As \(A^{\Gamma}\) has a unique steady state, a solution set satisfying the above steady state equations shall yield the steady state probabilities. As (8) and (9) stand for all the incoming and outgoing transition probabilities of all recurrent states, this set of equations fully describes the steady state probabilities. Part (i) of our claim can be tested by assigning \(\pi_m\) as the steady state probabilities of system states that have \(m\) sources at state \(\Gamma\). Noting that \(\pi_{T_1} = \pi_{T_2} = \pi_{M}\) and \(\pi_{T_0} = \pi_{T_3} = \pi_{M-1}\), with appropriate substitutions (8) becomes:

\[
\pi_M = \pi_M (1 - M \tau (1 - \tau)^{M-1}) + \pi_{M-1} M (1 - (M - 1) \tau (1 - \tau)^{M-2}), \tag{10}
\]

and (9) becomes:

\[
\pi_{M-1} = \pi_M \tau (1 - \tau)^{M-1} + \pi_{M-1} (M - 1) \tau (1 - \tau)^{M-2}. \tag{11}
\]

Both of these equations are reduced to the same equation below that holds for all \(m\):

\[
\frac{\pi_m}{\pi_{m-1}} = \frac{1 - (m - 1) \tau (1 - \tau)^{m-2}}{\tau (1 - \tau)^{m-1}}. \tag{12}
\]

Therefore, part (i) holds and this can be used to calculate the steady state probability of having \(m\) active users. The total number of states corresponding to \(\pi_m\) are the number of recurrent system states with \(m\) sources at truncated age \(\Gamma\):

\[
N_m = \binom{n}{m} \frac{(\Gamma - 1)!}{(\Gamma - n - 1 + m)!}. \tag{13}
\]

Recall that \(P_m\) was defined as the total probability of all states with \(m\) active sources. By Lemma [1](i), each of these states are equiprobable with steady state probability \(\pi_m\). Hence,

\[
P_m = N_m \pi_m \tag{14}
\]

\[
\frac{P_m}{P_{m-1}} = \frac{(1 - (m - 1) \tau (1 - \tau)^{m-2})(n - m + 1)}{\tau (1 - \tau)^{m-1} m (\Gamma - 1 - n + m)} \tag{15}
\]

\[
\sum_{m=0}^{N} P_m = 1 \tag{16}
\]

From (15) and (16),

\[
P_{0}(1 + \sum_{m=1}^{n} \prod_{i=1}^{m} \frac{(1 - (i - 1) \tau (1 - \tau)^{i-2})(n - i + 1)}{\tau (1 - \tau)^{i-1} i (\Gamma - 1 - n + i)}) = 1 \tag{17}
\]

\[
P_m = P_{0} \prod_{i=1}^{m} \frac{(1 - (i - 1) \tau (1 - \tau)^{i-2})(n - i + 1)}{\tau (1 - \tau)^{i-1} i (\Gamma - 1 - n + i)} \tag{18}
\]

provides the steady state solution.
B. Pivoted MC

In this part, we make our analysis over a single source, which we refer to as the pivot source. Any source in the network can be selected as pivot. After selecting a source a pivot, we modify the truncated MC of previous subsection, \( \{A^\Gamma[t], t \geq 1\} \), to create pivoted MC \( \{P^\Gamma[t], t \geq 1\} \), where the states of all the sources except the pivot are truncated at \( \Gamma \).

We extend our definitions and arguments from the proof of Lemma 1 to \( P^\Gamma \), in particular extend the definition of types of states. The type of a state in \( P^\Gamma \) is defined as:

\[
T^{P}(S^{P}) \triangleq (s, M, \{u_1, u_2, \ldots, u_{n-M-1}\})
\]  

(19)

where \( s \in \mathbb{Z}^+ \) is the state of the pivot source, \( M \) is the number of entries equal to \( \Gamma \) (i.e., the number of active sources not including the pivot), and the set \( \{u_1, u_2, \ldots, u_{n-M-1}\} \) is the set of entries smaller than \( \Gamma \) (i.e., the set of ages below the threshold, not including \( s \)). With a slight abuse of notation, we will refer to such a state as type \( M \)-state where it is clear from the context.

**Proposition 3.** (i) \( P^\Gamma \) has a unique steady state distribution.

(ii) Steady state probability of a type-\( m \) state in \( P^\Gamma \) is equal to \( \pi_m \), obeying (12), if \( s \in \{1, 2, \ldots, \Gamma - 1\} \).

**Proof.** States in \( P^\Gamma \) where \( s = 1, 2, \ldots, \Gamma - 1 \) have one-to-one correspondence with the related states in the truncated MC \( A^\Gamma \). The system visiting these corresponding states in \( P^\Gamma \) and \( A^\Gamma \) constitutes the same event hence these have identical steady state probabilities and identical transition probabilities, by construction. Therefore, they follow (12).

Next, we shall establish the existence of a steady state probability for the states in \( P^\Gamma \) for which \( s \geq \Gamma \). For a given \( s \), we augment \( A^\Gamma \) to form the augmented truncated MC \( \{A^{s,\Gamma}[t], t \geq 1\} \) where the pivot is truncated at \( s + 1 \) and all other sources are truncated at \( \Gamma \). Truncation of the pivot source is illustrated in Fig. [1] Let us the call the state where the state of the pivot source is \( s + 1 \) and state of all other sources is \( \Gamma \) the unlucky state. The unlucky state can be reached by all the states in the MC, including the unlucky state itself, if there are no successful transmissions in the network for \( s \) consecutive time slots, which can happen with non-zero probability. This means that there is a single recurrent class in this MC and a unique steady state distribution. Finally, there is a one-to-one correspondence between the states of \( A^{s,\Gamma} \) and \( P^\Gamma \) for which the state of the pivot source is \( s \). Existence of steady state probabilities for the states in \( A^{s,\Gamma} \) entails the existence of steady state probabilities for the states in \( P^\Gamma \). \( \square \)
Definition 1. Let \( S^p \) be a state in \( P^\Gamma \) of type \( T^p(S^p) = (s, m, \{u_1, u_2, \ldots, u_{n-m-1}\}) \), where the \( \{u_i\} \) are ordered from largest to smallest. \( Q(S^p) \), preceding type of \( S^p \), is defined as follows:

\[
Q(S^p) = \begin{cases} 
T^p(S^p), & \text{if } s = 1 \\
(s - 1, m, \{\Gamma - 1, u_1 - 1, u_2 - 1, \ldots, u_{n-m-2} - 1\}), & \text{if } s \neq 1, u_{n-m-1} = 1 \\
(s - 1, m, \{u_1 - 1, u_2 - 1, \ldots, u_{n-m-1} - 1\}), & \text{if } s \neq 1, u_{n-m-1} \neq 1 
\end{cases}
\]

(20)

The reasoning behind \( Q(S^p) \) is that if current state is \( S^p \) and number of active sources did not change in the previous time slot (excluding pivot source), then the type of previous state must be \( Q(S^p) \). This does not hold for case \( s = 1 \), but we are not interested in such a characterization for this case; nevertheless, we choose \( Q(S^p) \) to be the type \( S^p \) itself, so that we do not have to exclude this special case in what follows. Finally, we denote the steady state probability of \( S^p \) as \( \pi(S^p) \) or \( \pi(s, m, \{u_1, u_2, \ldots, u_{n-m-1}\}) \).

Lemma 2. Let \( S^p_1 \) and \( S^p_2 \) be two arbitrary states in \( P^\Gamma \) where the state of the pivot source is equal for both states. Let the types of \( S^p_1 \) and \( S^p_2 \) be:

\[
T^p(S^p_1) = (s, m_1, \{u_1, u_2, \ldots, u_{n-m_1-1}\}) \\
T^p(S^p_2) = (s, m_2, \{v_1, v_2, \ldots, v_{n-m_2-1}\})
\]

i) Let \( Q^p_1 \) be any state satisfying \( T^p(Q^p_1) = Q(S^p_1) \). Then,

\[
\lim_{n \to \infty} \frac{\pi(S^p_1)}{\pi(Q^p_1)} = 1
\]

(21)

ii) If \( m_1 = m_2 \), then

\[
\lim_{n \to \infty} \frac{\pi(S^p_1)}{\pi(S^p_2)} = 1
\]

(22)

iii) If \( m_1 = m_2 + 1 \), then

\[
\lim_{n \to \infty} \frac{\pi(S^p_1)}{n \pi(S^p_2)} = \frac{e^{k\alpha}}{\alpha} - k
\]

(23)

where \( \lim_{n \to \infty} \frac{m_1}{n} = k \) and \( \lim_{n \to \infty} \tau n = \alpha \). (\( k, \alpha \in \mathbb{R^+} \))
Theorem 1. For some $r, \alpha \in \mathbb{R}^+$, such that $\lim_{n \to \infty} \frac{\Gamma}{n} = r$ and $\lim_{n \to \infty} \tau n = \alpha$, define $f : (0, 1) \to \mathbb{R}$:

$$ f(x) = \ln(e^{x\alpha}) + \ln(r - 1) - \ln(r + r - 1) - 1 \tag{24} $$

Then, for all $m$ such that $\lim_{n \to \infty} \frac{m}{n} = k \in (0, 1)$ and $s \in \mathbb{Z}^+$

$$ \lim_{n \to \infty} \ln \frac{P_m^{(s)}}{P_{m-1}^{(s)}} = f(k) \tag{25} $$

where $P_m^{(s)}$ is the steady state probability of having $m$ active sources (excluding the pivot source), given that state of the pivot source is $s$.

Proof. The term $P_m^{(s)}$ is the total steady state probability of states in which there are $m$ active users and the state of the pivot source is $s$. The number of such recurrent states is:

$$ N_m = \binom{n - 1}{m} \frac{(\Gamma - 1)!}{(\Gamma - n + m + 1)!} \tag{26} $$

Meanwhile, the number of recurrent states containing $m - 1$ active users is:

$$ N_{m-1} = \binom{n - 1}{m - 1} \frac{(\Gamma - 1)!}{(\Gamma - n + m - 1)!} \tag{27} $$

Let $B_m = \{S_1^{(m)}, S_2^{(m)}, \ldots, S_{N_m}^{(m)}\}$ be the set of all recurrent type-$m$ states where the state of the pivot source is $s$. Similarly, we define the set $B_{m-1} = \{S_1^{(m-1)}, S_2^{(m-1)}, \ldots, S_{N_{m-1}}^{(m-1)}\}$ as the set of all recurrent type-$(m - 1)$ states where the state of the pivot source is $s$. Then,

$$ \lim_{n \to \infty} \frac{P_m^{(s)}}{P_{m-1}^{(s)}} = \lim_{n \to \infty} \frac{n \sum_{i=1}^{N_m} \pi(S_i^{(m)})}{n \sum_{j=1}^{N_{m-1}} \pi(S_j^{(m-1)})} \tag{28} $$

where (a) is obtained by dividing both sides of the fraction by the steady state probability of any element of $B_{m-1}$, which was arbitrarily chosen as the first element, and (b) follows from
Lemma 2 (ii) and (iii). Hence,
\[
\lim_{n \to \infty} \ln \frac{P_m^{(s)}}{P_{m-1}^{(s)}} = \ln \left( \frac{e^{k\alpha}}{k\alpha} - 1 \right) + \ln \left( \frac{r}{r + k - 1} - 1 \right) = f(k) \tag{29}
\]

The above argument shows that as \( n \to \infty \) the relation \( P_m^{(s)}/P_{m-1}^{(s)} \) determines the PMF of \( m \) regardless of the state \( s \) of the pivot source. Consequently, the number of active sources (excluding the pivot), \( m \), is independent of the state of the pivot source. We record this in the following corollary:

**Corollary 1.** In the case of a large network (\( n \to \infty \)),

(i) The number of active sources, \( m \), (excluding the pivot) is independent of the state \( s \) of the pivot source.

(ii) As long as \( s \geq \Gamma \), the probability of a successful transmission being made by the pivot source is \( \tau(1 - \tau)^m \) which has no dependence on \( s \).

(iii) The probability of the pivot state of \( s \geq \Gamma \) being reset to 1 is \( q_s = \lim_{l \to \infty} \sum_{m=0}^{l} P_m^{(s)} \tau(1 - \tau)^m \).

**Proof.** Parts (i) and (ii) follow from the proof of Lemma 1. Every time the state of the pivot source reaches a particular value \( s \), it observes an identical distribution in terms of number of active users. Therefore, the transition probabilities from \( s = i \) to \( s = i + 1 \) for \( i < \Gamma \) and, and the transition probability from \( s \geq \Gamma \) to 1 depends only on the number of active users, hence, the evolution of the state of the pivot can be represented by the state diagram in Fig. 2.

![Fig. 2: State diagram of the pivot source](image)

The transition probabilities \( q_s \) marked on Fig. 2 refer to the probability of a successful transmission made by the pivot source. In the rest, we will consider the asymptotic case as the network size \( n \) grows. We will show that in the limit as \( n \to \infty \), \( q_s \) is equal to some \( q_o \) for all values of \( s \) as long as the pivot source is active.

**C. Large network asymptotics**

In this part, we investigate the PMF of \( m \), number of active sources in the network. Function \( f \) of Lemma 1 gives valuable insight on the distribution of \( m \) and we will derive some properties
of $f$ with the eventual goal of proving that the ratio of active users, $k$, converges to the root of $f$ in probability, presented in Theorem 2.

To facilitate the asymptotic analysis in the network size $n$, we replace the main parameters of the model, $\tau$ and $\Gamma$, with the following that control the scaling of these parameters with $n$. As the number of active sources, $m$, take values between 0 and $n$, the fraction of active sources, $k$, will vary between 0 and 1.

$$\alpha = n\tau, \quad r = \Gamma/n, \quad k = m/n$$

(30)

**Proposition 4.** Roots of $f$ for which $f$ is decreasing correspond one-to-one to the local maxima of $P_m$, with a scale of $n$.

In this context, $\alpha$ and $r$ are fixed system parameters while $k$, the fraction of active users,
is a variable indicating the instantaneous system load. As the change in \( P_m \) is determined by \( f(k) \), the roots of \( f(k) \) provide the local extrema of \( P_m \). Local maxima of \( P_m \) are the points where both \( \ln P_m/P_{m-1} \) and \( \ln P_m/P_{m+1} \) are positive, corresponding to roots of \( f(k) \) for which \( f \) is decreasing. The following proposition restricts the number of roots \( f(k) \), and therefore the number of local maxima \( P_m \) can have.

**Proposition 5.** The number of distinct roots of \( f \) is at least 1 and at most 3.

**Proof.** See Appendix B.

Since \( f(k) \) has at most three roots, there can be at most 2 roots of \( f \) where \( f \) is decreasing and consequently at most two local maxima. Cases of one local maximum and two local maxima are analyzed separately, however they lead to a similar discussion. Theorem 2 is given for the case where \( f(k) \) has only one root and a single local maximum. The case with 2 local maxima is discussed in section III-D.

**Theorem 2.** Let \( k_0 \) be the only root of \( f(k) \) and \( m \) be the number of active sources. For the sequence \( \epsilon_n = cn^{-1/3} \) where \( c \in \mathbb{R}^+ \),

\[
\Pr(\left| \frac{m}{n} - k_0 \right| < \epsilon_n) \to 1
\]  

(31)

**Proof.** See Appendix C.

This theorem establishes that the fraction of active users converges in probability to \( k_0 \) as the network size grows. Loosely speaking, threshold-ALOHA gradually converts the system to one with \( nk_0 \) users with a slotted ALOHA analysis. At steady state, approximately \( nk_0 \) sources will be making transmission attempts while remaining \( n - nk_0 \) sources with small age will be idle. For this reason, it resembles a stabilized ALOHA algorithm. For large \( N \), throughput of the channel remains close to \( e^{-1} \) while average age can be dramatically improved through optimal parameters, as will be shown in the section III-E.

**D. Double Peak Case**

In this section, we extend the single peak analysis of the previous section to the case with 2 peaks. Theorem 3 gives the same result as in Theorem 2 although it imposes an additional integral constraint to be applicable.

So far, it has been argued that roots of \( f(k) \) where \( f \) is decreasing correspond to the peaks in the probability distribution of the number of active sources. If there are two such roots, then
there will be two possible values of $m$ where the number of active sources are concentrated around. Accordingly, we define the following state sets:

$$S_0 \triangleq \left\{ S \mid T(S) = (m, \{u_1, u_2, \ldots, u_{n-m}\}) \text{ where } \frac{m}{n} \leq \frac{k_0 + k_1}{2} \right\}$$  \hspace{1cm} (32)

$$S_1 \triangleq \left\{ S \mid T(S) = (m, \{u_1, u_2, \ldots, u_{n-m}\}) \text{ where } \frac{k_0 + k_1}{2} < \frac{m}{n} < \frac{k_1 + k_2}{2} \right\}$$  \hspace{1cm} (33)

$$S_2 \triangleq \left\{ S \mid T(S) = (m, \{u_1, u_2, \ldots, u_{n-m}\}) \text{ where } \frac{k_1 + k_2}{2} \leq \frac{m}{n} \right\}$$  \hspace{1cm} (34)

$S_0$ corresponds to the states where number of active users are around the smaller root and $S_2$ corresponds to the states where number of active users are around the larger root. States in between are grouped as $S_1$ and thresholds are set at the mid-points between consecutive roots.

![Fig. 5: State sets](image)

In the proof of Theorem 3 it is shown that, if the integral is negative, probability of $S_1$ and $S_2$ state sets diminishes as $n$ goes to infinity. By showing that $S_0$ happens with probability 1, basic principles used for the single peak case can be used again to derive similar results.

**Theorem 3.** Let $f(k)$ have three distinct roots and $k_0, k_1, k_2$ be the roots in increasing order and $m$ be the number of active sources.

1) If

$$\int_{k_0}^{k_2} f(k) dk < 0$$  \hspace{1cm} (35)

then for the sequence $\epsilon_n = cn^{-1/3}$ where $c \in \mathbb{R}^+$,

$$\Pr\left(\frac{m}{n} - k_0 < \epsilon_n\right) \rightarrow 1$$  \hspace{1cm} (36)
ii) If
\[ \int_{k_0}^{k_2} f(k) dk > 0 \] (37)

then for the sequence \( \epsilon_n = cn^{-1/3} \) where \( c \in \mathbb{R}^+ \),
\[ \Pr\left( \left| \frac{m}{n} - k_2 \right| < \epsilon_n \right) \to 1 \] (38)

**Proof.** See Appendix D.

The ratio of active users converges to either \( k_0 \) or \( k_2 \), depending on the sign of the integral above. If the integral result is positive, this ratio will converge to the larger root, however, this is not desired since larger root is equivalent to more active users at the same time. In order to fully benefit from the age threshold, parameters should be chosen such that \( k \) converges to \( k_0 \).

Even though Theorem [3] yields a similar result as in Theorem [2], double peak cases may not be as practical as single peak cases in networks with fewer users. For \( n \) values that are not large enough, steady state probabilities of \( S_1 \) and \( S_2 \) may not be small enough to yield useful results. As \( k \) values for state sets \( S_1 \) and \( S_2 \) are larger than that for \( S_0 \), these states have more active users and this may lead to the congestion of the channel with collisions by having too many users trying to transmit at the same time. This negates the benefit of threshold-ALOHA and should be avoided. Single peak cases do not have \( S_1 \) and \( S_2 \) sets and system converges more quickly to \( k_0 \).

In networks with a large number of users, initial conditions must be selected properly to achieve good results. Selecting all users active initially leads to the aforementioned congestion scenarios, slowing down the convergence in Theorem [3]. As \( n \) increases, the transition probabilities between state sets decrease exponentially. If the initial state of the system is in \( S_2 \), it may be nearly impossible for the network to reach a state in \( S_0 \) in a reasonable time period. Initial state of users can be randomized to prevent initial congestion.

Despite all these drawbacks, the double peak cases produce asymptotically optimal values and are preferable in systems with large number of users.
E. Steady state average AoI in the large network limit

**Theorem 4.** Optimal parameters for threshold-ALOHA in an infinitely large network satisfy the following:

\[
\lim_{n \to \infty} \frac{\Gamma^*}{n} = 2.21 \tag{39}
\]

\[
\lim_{n \to \infty} n \tau^* = 4.69 \tag{40}
\]

Moreover, the optimal expected AoI at steady state scales as:

\[
\lim_{n \to \infty} \frac{\Delta^*}{n} = 1.4169 \tag{41}
\]

**Proof.** As can be recalled from the ending of section III-B, \( q_0 \) was defined as successful transmission probability of an active source and it has been argued that \( q_0 \) is independent of the age of the active source. Alternatively, \( q_0 \) can be expressed as:

\[
q_0 = \mathbb{E}[\tau(1 - \tau)^{M-1}] \tag{42}
\]

where the expectation is over the distribution of \( M \), the number of active sources at steady state, which was characterized earlier. We firstly prove that

\[
\lim_{n \to \infty} n q_0 = \alpha e^{-k_0 \alpha} \tag{43}
\]

Let \( \gamma_n \) be defined as:

\[
\gamma_n \triangleq \Pr(m_0 - cn^{2/3} < M < m_0 + cn^{2/3}) \tag{44}
\]

where \( m_0 = k_0 n \). From Theorem 2 and 3, \( \gamma_n \to 1 \) as \( n \to \infty \). When \( M \) is within the bounds given in (44), the successful transmission probability is also bounded from both sides. This is used to obtain the following bound:

\[
\gamma_n [\tau(1 - \tau)^{m_0}(1 - \tau)^{cn^{2/3}}] < q_0 < \gamma_n [\tau(1 - \tau)^{m_0}(1 - \tau)^{cn^{2/3}}] + (1 - \gamma_n) \tag{45}
\]

As \( n \) goes to infinity, both upper and lower bounds converge to \( \tau(1 - \tau)^{m_0} \). Finally,

\[
\lim_{n \to \infty} n q_0 = \lim_{n \to \infty} n \tau(1 - \tau)^{m_0} = \alpha e^{-k_0 \alpha} \tag{46}
\]

Value of \( q_0 \) can be used to compute steady state probabilities of a single source using the model in Fig. 2. In this model, states are not truncated and age is equivalent to state. Steady state probability of state \( j \) is:

\[
\pi_j = \frac{(1 - q_0)^{\max\{j - \Gamma, 0\}}}{\Gamma - 1 + 1/q_0}, \quad j = 1, 2, \ldots \tag{47}
\]
Steady state probabilities are used to derive the following expected time-average AoI expression:

\[ \Delta = \frac{\Gamma(\Gamma - 1)}{2(\Gamma - 1 + 1/q_0)} + 1/q_0 \quad (48) \]

Limiting behavior of average AoI is found as:

\[ \lim_{n \to \infty} \frac{\Delta}{n} = \frac{r^2}{2(r + e^{k_0\alpha}/\alpha)} + e^{k_0\alpha}/\alpha \quad (49) \]

(49) can alternatively be expressed in terms of \( r \) and \( k_0 \):

\[ \lim_{n \to \infty} \frac{\Delta}{n} = \frac{k_0^2 + 1}{2(1 - k_0)} \quad (50) \]

Average AoI can be optimized by searching values of \( r \) and \( \alpha \) that minimizes (49).

Optimal parameters and steady-state characteristics (expected fraction of active users, expected avg. AoI and throughput) of threshold-ALOHA derived from (49) are summarized in Table I and contrasted with those of regular slotted ALOHA as a reference. Note that as threshold-ALOHA has two possible operating regimes, results for these, namely the single peak case and double peak case are separately provided. Note that slotted ALOHA is a special case of threshold-ALOHA where the age threshold is \( \Gamma = 1 \) and all users are active regardless of their ages, and thus \( r = 1/n \) goes to 0, from (30).

|                  | \( r^* \) | \( \alpha^* \) | \( k_0^* \) | \( G \)    | \( \Delta^*/n \) | Throughput |
|------------------|-----------|----------------|-------------|---------|-----------------|------------|
| Threshold-ALOHA  (single peak) | 2.17      | 4.43           | 0.2052      | 0.9090  | 1.4226          | 0.3658     |
| Threshold-ALOHA  (double peak)  | 2.21      | 4.69           | 0.1915      | 0.8981  | **1.4169**      | 0.3644     |
| Slotted ALOHA    | 0         | 1              | 1           | 1       | \( e \approx 2.7182 \) | \( e^{-1} \approx 0.3678 \) |

**TABLE I**: A comparison of optimized parameters of ordinary slotted ALOHA and threshold-ALOHA, and the resulting AoI and throughput values. \( r^* \): age-threshold/\( n \); \( \alpha^* \): transmission probability \( \times n \); \( k_0^* \): expected fraction of active users; \( G \): expected number of transmission attempts per slot; \( \Delta^* \): avg. AoI

In Table I, \( G \) refers to the expected number of transmission attempts in a single slot. Under threshold-ALOHA, \( G \) is equal to the the product of \( \tau \), probability of a transmission attempt, and \( nk_0 \), number of active users. As a result, \( G = k_0\alpha \) holds. Value of \( G \) can be used to compare the throughput of basic slotted ALOHA and threshold-ALOHA. \( Ge^{-G} \) is the probability of a successful transmission under both of these policies, since

\[ \lim_{n \to \infty} nk_0q_0 = k_0\alpha e^{-k_0\alpha} = Ge^{-G} \quad (51) \]

Hence, the probability of a successful transmission is upper bounded by \( e^{-1} \), with equality if
$G = 1$. Under an AoI-optimized selection of $\Gamma$ and $\tau$ for threshold-ALOHA, $G$ is equal to 0.8981, for which the throughput is 0.3644. Note that the throughput drop from the upperbound is below 1 percent, in return for reduction in AoI to almost one half of that achievable with slotted ALOHA.

The AoI in slotted ALOHA under optimal parameters is [4]:

$$\Delta = \frac{1}{2} + \frac{1}{\tau(1-\tau)^{n-1}}$$

(52)

The expression in (52) can be minimized by setting $\tau = 1/n$. Hence, optimal AoI under slotted ALOHA has the following limit [18]:

$$\lim_{n \to \infty} \frac{\Delta_{SA}^n}{n} = \lim_{n \to \infty} \frac{1}{2n} + \frac{1}{(1 - \frac{1}{n})^{n-1}} = e$$

(53)

Finally, we observe a similarity between threshold-ALOHA and Rivest’s stabilized slotted ALOHA [16, Sec. 4.2.3]. Rivest’s algorithm uses collision feedback to estimate the number of active sources, $\hat{m}(t)$, in each time slot and uses this estimate to optimize the probability of transmission, $\tau(t)$, such that $\hat{m}\tau = 1$. Rivest’s algorithm has also been exploited in [5] to achieve age-based thinning. Even though threshold-ALOHA does not track the number of active users, we have showed that the number of active users converges in probability to some $m_0 = nk_0$ (from (31)), and that under optimized parameter settings, $m_0\tau$ is close to 1, similarly to what Rivest’s stabilized ALOHA tries to achieve.

IV. Numerical Results and Discussion

In this section, we present numerical plots and simulation results to illustrate our theoretical findings and to perform comparisons with related policies. In Fig. 6 optimal AoI results can be observed under threshold-ALOHA, slotted ALOHA and stationary age-based thinning (SAT) policy presented in [5]. Simulations of SAT and threshold-ALOHA were performed under different $n$ values ranging from 50 to 1000 and run for $10^7$ time slots. Initial states of the users were randomized so that a bias from the initial congestion of having too many active users could be prevented and the decentralized structure of the algorithm could be preserved. Note that avg. AoI of threshold-ALOHA rises with slope 1.4169 with network size which is almost the same as SAT and roughly half the slope of slotted ALOHA.

We showed above that threshold-ALOHA keeps the number of active users at any time at steady state at about one-fifth of all users (see Table I), with optimal parameter settings. This enables the users to utilize the channel more efficiently, approaching throughput of $e^{-1}$ packets
Fig. 6: Optimal time average $AoI$ vs $n$, number of sources, under Slotted ALOHA (computed from (52)), threshold-ALOHA (simulated) and SAT Policy[5] (simulated).

Fig. 7: Plot of $Ge^{-G}$ per slot. Fig. 7 plots $Ge^{-G}$, where $G = 1$ has been marked as the throughput optimal operating point of ordinary slotted ALOHA and $G = 0.89$ has been marked for threshold-ALOHA. The corresponding throughput values are $e^{-1}$ and $0.3658$, respectively, which differ by less than 1%. Hence, threshold-ALOHA nearly halves avg. AoI while maintaining a near-optimal throughput.

SAT policy[5] has been established on a random access channel model and proposes a fixed age threshold before users can become active, like age-threshold slotted ALOHA. Probability of the active users making a transmission attempt is updated at each time slot by the collision feedback that is available to all users. This, however, is in contrast to the age-threshold slotted ALOHA where each active user makes a transmission attempt with a fixed probability, despite both policies employing a fixed age threshold. In age-threshold slotted ALOHA, feedback information is used only as a means of updating the user ages after successful transmissions. Hence, SAT policy needs more costly feedback and utilizes a varying transmission probability to increase performance, but it is similar to age-threshold slotted ALOHA in all other aspects. Hence, SAT policy makes a good benchmark to evaluate the performance of age-threshold slotted ALOHA. In [5], it has been shown that optimal AoI of SAT policy converges to $\frac{e}{2}n$, whereas age-threshold slotted ALOHA is able to achieve AoI of $1.4169n$. age-threshold slotted ALOHA performs slightly worse, by 4%, than SAT policy. It is worth remarking that the up to 4% loss in the simple threshold-AoI policy is offset by the computational ease of this algorithm, as the SAT policy...
relies on a computationally complex operation to be done at each time-slot, whose cost in terms of power and time may not warrant the 4% gain in AoI, especially in massive networks.

A recent analysis of threshold-ALOHA was presented in [7]. This analysis was based on two major simplifying approximations: (1) there is a constant successful transmission probability at all times, and (2) the states of the sources are independent of each other. However, Lemma 1 characterizes $P_m / P_{m-1}$, the ratio between steady state probabilities of there being $m$ and $m - 1$ active sources, and shows that users being active are not independent of each other and in fact the probability of a user being active is heavily influenced by the number of active users in the network. Nevertheless, the initial intuition of [7] that successful transmission probability of a user is independent of its state has been confirmed in section III-B. The analysis in [7] was limited to $\tau < 2/n$, which can be far from the optimal choice of $\tau$ as we showed above, which is consistent with the simulation results presented in [7] that indicate an AoI (2$n$, or 1.6$n$ in two different simulation plots), which are above the optimal value of 1.41$n$. Our analysis, which included the entire possible range of $\tau$ has shown the optimal $\tau$ to be $4.69/n$.

V. CONCLUSION AND FUTURE DIRECTIONS

We have presented a comprehensive steady-state analysis of threshold-ALOHA, which is an age-aware modification of slotted ALOHA proposed in [1]. In threshold-ALOHA each terminal suspends its transmissions until its age exceeds a certain threshold, and once age exceeds the threshold, it attempts transmission with constant probability $\tau$, just as in standard slotted ALOHA. We have analyzed time-average expected age attained, and explored its scaling with network size. We adopted the generate-at-will model where each time a user attempts transmission, it generates a fresh packet, accordingly every time a successful transmission occurs, the age of the corresponding flow is reset to 1. We have firstly derived the steady state solutions of DTMC that was formed in [1] and subsequently found the distribution of number of active users. We have shown that the policy converges to running slotted ALOHA with fewer sources: on average about one fifth of the users is active at any time. We then formulated an expression for avg. AoI and derived optimal parameters of the policy. This resolved the conjectures in [1] by confirming that the optimal age threshold and transmission probability are $2.2n$ and $4.69/n$, respectively. We have found optimal avg. AoI to be $1.4169n$, which is half of what is achievable using slotted ALOHA while the loss from the maximum achievable throughput of $e^{-1}$ is below 1%.
The novel methodology developed in this paper can be extended to analyze the performance of threshold ALOHA under conditions such as exogeneous arrival processes, lossy channels (nonzero probability of decoding error), or the availability of advanced physical layer techniques including contention resolution [21] where the channel encoder/decoder facilitates the mutual decoding of a certain number of colliding packets.

APPENDIX A

PROOF OF LEMMA

We firstly prove that properties of Lemma hold for \( s = 1, 2, \ldots, \Gamma - 1 \). Property (i) and (ii) follows from Prop. 3 (i), \( \pi(S^p_1) = \pi_{m_1} \) and \( \pi(S^p_2) = \pi_{m_2} \). Property (iii) follows from the same property, albeit not directly:

\[
\lim_{n \to \infty} \frac{\pi(S^p_1)}{n \pi(S^p_2)} = \lim_{n \to \infty} \frac{\pi_{m_1}}{n \pi_{m_1-1}} = \frac{1 - (m_1 - 1)\tau(1 - \tau)^{m_1-2}}{n\tau(1 - \tau)^{m_1-1}} = e^{\alpha - k}
\]

where (a) follows from (12).

Next, we calculate the steady state probabilities of the states in \( P^\Gamma \) where \( s = \Gamma \). We firstly show that \( \pi(S^p_1) = \pi_{m_1} \). Assuming that the current state is \( S^p_1 \), if \( 1 \not\in \{ u_1, u_2, \ldots, u_{n-m_1-1} \} \), then previous state must be of one of the following types:

- \( (\Gamma - 1, m_1, \{ u_1 - 1, u_2 - 1, \ldots, u_{n-m_1-1} - 1 \} ) \)
- \( (\Gamma - 1, m_1 - 1, \{ \Gamma - 1, u_1 - 1, u_2 - 1, \ldots, u_{n-m_1-1} - 1 \} ) \)

Steady state probability expression for states of these types are given in Prop. 3 (ii). Steady state probabilities for states of the first type and second type are \( \pi_{m_1} \) and \( \pi_{m_1-1} \), respectively. Steady state probability of \( S^p_1 \) can be derived using the steady state probabilities of preceding states along with their transition probabilities:

\[
\pi(S^p_1) = \pi_{m_1}(1 - m_1\tau(1 - \tau)^{m_1-1}) + \pi_{m_1-1}m_1(1 - (m_1 - 1)\tau(1 - \tau)^{m_1-2}) = \pi_{m_1}
\]

Resulting \( \pi_{m_1} \) is obtained through the ratio given in (12). Now, we calculate the steady state probability for the case \( 1 \in \{ u_1, u_2, \ldots, u_{n-m_1-1} \} \), following similar steps. W.l.o.g., assume that \( u_{n-m_1-1} = 1 \). Then previous state must be one of the following types:

- \( (\Gamma - 1, m_1 + 1, \{ u_1 - 1, u_2 - 1, \ldots, u_{n-m_1-2} - 1 \} ) \)
- \( (\Gamma - 1, m_1, \{ \Gamma - 1, u_1 - 1, u_2 - 1, \ldots, u_{n-m_1-2} - 1 \} ) \)
Steady state probabilities for states of the first type and second type are \(\pi_{m+1}\) and \(\pi_m\), respectively. Steady state probability of \(S_1^P\) is derived as:

\[
\pi(S_1^P) = \pi_{m+1} \tau (1 - \tau)^{m_1} + \pi_m (m_1) \tau (1 - \tau)^{m_1-1} = \pi_m
\] (56)

Due to symmetry, \(\pi(S_2^P) = \pi_m\). Property (i) and (ii) follows from Prop. 3(i) and Property (iii) follows from (54).

Finally, we prove that properties of the Lemma hold for \(\forall s \geq \Gamma\) by induction. Initial case \(s = \Gamma\) has been covered above. We assume \(s > \Gamma\) and that above properties hold for all states of \(P^\Gamma\) in which age of the pivot source is smaller than \(s\). Then we prove property (i) in two separate cases:

**Case 1.** If \(1 \not\in \{u_1, u_2, \ldots, u_{n-m-1}\}\)

In order to make the equations easier to read, we shorten steady state probability expressions in the following way:

\[
\pi_m^{(s)} = \pi(s, m, \{u_1, u_2, \ldots, u_{n-m-1}\}) = \pi(S_1^P)
\] (57)

\[
\pi_m^{(s-1)} = \pi(s - 1, m, \{u_1 - 1, u_2 - 1, \ldots, u_{n-m-1} - 1\}) = \pi(Q_1^P)
\] (58)

\[
\pi_m^{(s-1)} = \pi(s - 1, m - 1, \{\Gamma - 1, u_1 - 1, u_2 - 1, \ldots, u_{n-m-1} - 1\})
\] (59)

Steady state probabilities of the states that can precede a state of type \((s, m, \{u_1, u_2, \ldots, u_{n-m-1}\})\) are \(\pi_m^{(s-1)}\) or \(\pi_m^{(s-1)}\). Value of \(\pi_m^{(s)}\) is calculated as:

\[
\pi_m^{(s)} = \pi_m^{(s-1)} (1 - (m + 1) \tau (1 - \tau)^{m}) + \pi_m^{(s-1)} (m + 1)(1 - m \tau (1 - \tau)^{m-1})
\] (60)

Then,

\[
\lim_{n \to \infty} \frac{\pi_m^{(s)}}{\pi_m^{(s-1)}} = \lim_{n \to \infty} \frac{\pi_m^{(s-1)} (1 - (m + 1) \tau (1 - \tau)^{m}) + \pi_m^{(s-1)} (m + 1)(1 - m \tau (1 - \tau)^{m-1})}{\pi_m^{(s-1)}}
\]

\[
= \lim_{n \to \infty} 1 - (m + 1) \tau (1 - \tau)^{m} + \frac{\pi_m^{(s-1)} (m + 1)(1 - m \tau (1 - \tau)^{m-1})}{\pi_m^{(s-1)}}
\]

\[
= \lim_{n \to \infty} 1 - \frac{m + 1}{n} (n \tau)(1 - \tau)^{m} + \frac{\pi_m^{(s-1)} m + 1}{\pi_m^{(s-1)}} \frac{1}{n} (1 - \frac{m}{n} (n \tau)(1 - \tau)^{m-1})
\]

\[
\overset{(a)}{=} \lim_{n \to \infty} 1 - k \alpha e^{-k \alpha} + \frac{1}{e^{k \alpha} - \frac{1}{k}} k (1 - k \alpha e^{-k \alpha}) = 1
\]

where (a) follows from property (iii).

**Case 2.** If \(1 \in \{u_1, u_2, \ldots, u_{n-m-1}\}\), then w.l.o.g. \(u_{n-m-1} = 1\)

In order to make the equations easier to read, we shorten steady state probability expressions in the following way:

\[
\pi_m^{(s)} = \pi(s, m, \{u_1, u_2, \ldots, u_{n-2}, 1\}) = \pi(S_1^P)
\] (62)
\[ \pi^{(s-1)}_m = \pi(s-1, m, \{\tau - 1, u_1 - 1, u_2 - 1, \ldots, u_{m-2} - 1\}) = \pi(Q^p_1) \] (63)

\[ \pi^{(s-1)}_{m+1} = \pi(s-1, m + 1, \{u_1 - 1, u_2 - 1, \ldots, u_{m-2} - 1\}) \] (64)

Steady state probabilities of the states that can precede a state of type \( (s, m, \{u_1, u_2, \ldots, u_{m-2}\} \) are \( \pi^{(s-1)}_m \) or \( \pi^{(s-1)}_{m+1} \). Value of \( \pi^{(s)}_m \) is calculated as:

\[ \pi^{(s)}_m = \pi^{(s-1)}_{m+1}(1 - \tau)^m + \pi^{(s-1)}_m m \tau (1 - \tau)^{m-1} \] (65)

Then,

\[
\lim_{n \to \infty} \frac{\pi^{(s)}_m}{\pi^{(s-1)}_m} = \lim_{n \to \infty} \frac{\pi^{(s-1)}_{m+1}(1 - \tau)^m + \pi^{(s-1)}_m m \tau (1 - \tau)^{m-1}}{\pi^{(s-1)}_m}
\]

\[
= \lim_{n \to \infty} \frac{\pi^{(s-1)}_{m+1}}{\pi^{(s-1)}_m} (1 - \tau)^m + m \tau (1 - \tau)^{m-1}
\]

\[
= \lim_{n \to \infty} \frac{\pi^{(s-1)}_{m+1}}{n \pi^{(s-1)}_m} (n \tau)(1 - \tau)^m + \frac{m}{n} (n \tau)(1 - \tau)^{m-1}
\]

\[
= \lim_{n \to \infty} \left( \frac{e^{k\alpha}}{\alpha} - k \right) \alpha e^{-k\alpha} + k \alpha e^{-k\alpha} = 1
\]

Thus, the proof of property (i) is completed. Next, for the case \( m_1 = m_2 \),

\[
\lim_{n \to \infty} \frac{\pi(S^p_1)}{\pi(S^p_2)} = \lim_{n \to \infty} \frac{\pi(S^p_1) \pi(Q^p_2) \pi(Q^p_2)}{\pi(Q^p_1) \pi(S^p_2) \pi(Q^p_2)}
\]

\[
= \lim_{n \to \infty} \frac{\pi(Q^p_1)}{\pi(Q^p_2)} \ (a) = 1
\]

where (a) follows from property (i) and (b) follows from property (ii) since state of the pivot source for states \( Q^p_1 \) and \( Q^p_2 \) is \( s-1 \) and number of active sources is \( m_1 \) and \( m_2 \) respectively.

Similarly, for the case \( m_1 = m_2 + 1 \),

\[
\lim_{n \to \infty} \frac{\pi(S^p_1)}{\pi(n S^p_2)} = \lim_{n \to \infty} \frac{\pi(S^p_1) \pi(Q^p_2) \pi(Q^p_2)}{\pi(Q^p_1) \pi(S^p_2) \pi(Q^p_2)}
\]

\[
= \lim_{n \to \infty} \frac{\pi(Q^p_1)}{\pi(Q^p_2)} \ (a) = \frac{e^{k\alpha}}{\alpha} - k
\]

where (a) follows from property (i), (b) follows from property (iii) since state of the pivot source for states \( Q^p_1 \) and \( Q^p_2 \) is \( s-1 \) and number of active sources is \( m_1 \) and \( m_2 \) respectively.

**APPENDIX B**

**PROOF OF PROPOSITION 5**

To prove that \( f(k) \) has at least 1 root, it is sufficient to observe that \( f(0^+) = +\infty \) and \( f(1^-) = -\infty \). Since \( f(k) \) is continuous in \((0,1)\) domain, \( f(k) \) has at least one root.
To prove that \( f(k) \) has at most 3 roots, we formulate \( r \) in terms of \( \alpha \) and \( k \) when \( f(k) = 0 \).

\[
f(k) = \ln\left(\frac{e^{k\alpha}}{k\alpha} - 1\right) + \ln\left(\frac{r}{k + r - 1}\right) - 1 = 0
\]

(69)

\[
r = \frac{e^{k\alpha}(1-k)}{k\alpha}
\]

(70)

\[
\frac{dr}{dk} = \frac{e^{k\alpha}}{k^2\alpha}(-\alpha k^2 + \alpha k - 1)
\]

(71)

Since \( \frac{dr}{dk} \) has at most two roots, there can be at most 3 different values of \( k \) that satisfy (70). These are the only possible roots of \( f(k) \). Hence, \( f(k) \) has at most 3 roots.

**APPENDIX C**

**PROOF OF THEOREM 2**

We shall prove the following Lemma, from which Theorem 2 will follow as a special case for \((a, b) = (0, 1)\).

**Lemma 3.** For \((a, b) \subseteq (0, 1)\), let \( k_0 \) be the only root of \( f(k) \) in the interval \((a, b)\) and \( f'(k_0) < 0 \), \( \lim_{k \to a} f(k) \neq 0 \), \( \lim_{k \to b} f(k) \neq 0 \). Then for the sequence \( \epsilon_n = cn^{-1/3} \) where \( c \in \mathbb{R}^+ \),

i)

\[
\Pr \left( \left| \frac{m}{n} - k_0 \right| \geq \epsilon_n, \frac{m}{n} \in (a, b) \right) \to 0
\]

(72)

ii)

\[
\Pr \left( \left| \frac{m}{n} - k_0 \right| < \epsilon_n \mid \frac{m}{n} \in (a, b) \right) \to 1
\]

(73)

**Proof.** Firstly, we make the observation that if \( f(k) \) satisfies above conditions, then we can find a positive \( \epsilon \) small enough such that for \( \forall k \in (k_0 + \epsilon, b) \), \( f(k) < f(k_0 + \epsilon) \) holds.

From this, for \( b > k = \frac{m}{n} > k_0 + \epsilon \)

\[
\ln\left(\frac{P_m}{P_{m-1}}\right) = f(k) < f(k_0 + \epsilon)
\]

(74)

\[
P_m < P_{m-1} \exp(f(k_0 + \epsilon))
\]

(75)

\[
P_m < P_{m-1} \exp(f(k_0 + \epsilon))^t
\]

(76)

\[
\sum_{i=n(k_0+\epsilon)}^{nb} P_i < \sum_{i=n(k_0+\epsilon)}^{nb} P_{n(k_0+\epsilon)} \exp(f(k_0 + \epsilon))^{i-n(k_0+\epsilon)} < \frac{P_{n(k_0+\epsilon)}}{1 - \exp(f(k_0 + \epsilon))}
\]

(77)

\[
\Pr\left(\frac{m}{n} - k_0 \geq \epsilon, \frac{m}{n} \in (a, b)\right) < \frac{P_{n(k_0+\epsilon)}}{1 - \exp(f(k_0 + \epsilon))}
\]

(78)

Similar approach can be used to derive

\[
\Pr\left(\frac{m}{n} - k_0 \leq -\epsilon, \frac{m}{n} \in (a, b)\right) < \frac{P_{n(k_0-\epsilon)}}{1 - \exp(f(k_0 - \epsilon))}
\]

(79)
From the Riemann sum over $f(k)$, $(m_0 \triangleq nk_0)$

$$\ln P_{n(k_0+\varepsilon)} = \sum_{i=m_0+1}^{n(k_0+\varepsilon)} \ln P_i - \ln P_{i-1} = \sum_{i=m_0+1}^{n} f(i/n) \leq n \int_{k_0}^{k_0+\varepsilon} f(k) \, dk$$

(80)

As a result, the following bound is derived:

$$\Pr\left(\frac{m}{n} - k_0 \geq \varepsilon, \frac{m}{n} \in (a,b)\right) \leq \frac{P_{m_0} \exp(n \int_{k_0}^{k_0+\varepsilon} f(k) \, dk)}{1 - \exp(f(k_0 + \varepsilon))}$$

(81)

The above analysis can be repeated for the negative part to obtain the following bound:

$$\Pr\left(\frac{m}{n} - k_0 \leq -\varepsilon, \frac{m}{n} \in (a,b)\right) < \frac{P_{m_0} \exp(n \int_{k_0-\varepsilon}^{k_0} f(k) \, dk)}{1 - \exp(f(k_0 - \varepsilon))}$$

(82)

Next, Taylor series expansion is used to linearize $f(k_0 + \varepsilon)$.

$$f(k_0 + \varepsilon) = f(k_0) + f'(k_0)\varepsilon + o(\varepsilon)$$

(83)

For small $\varepsilon$, $f(k_0 + \varepsilon) \approx f'(k_0)\varepsilon$. The bound from (81) becomes,

$$\Pr\left(\frac{m}{n} - k_0 \geq \varepsilon, \frac{m}{n} \in (a,b)\right) < \frac{P_{m_0} \exp(f'(k_0)n\varepsilon^2/2)}{1 - \exp(f'(k_0)\varepsilon)}$$

(84)

We want to choose an $\varepsilon_n$ sequence such that both the sequence and the above bound converges to 0. $\varepsilon_n = cn^{-1/3}$ satisfies this condition since,

$$\lim_{n \to \infty} \frac{\exp(f'(k_0)n\varepsilon^2/2)}{1 - \exp(f'(k_0)\varepsilon)} = \lim_{n \to \infty} \frac{\exp(c^2f'(k_0)n^{1/3}/2)}{1 - \exp(cf'(k_0)n^{-1/3})} = 0$$

(85)

Similar arguments can be used for the negative side and sum of (82) and (84) gives the following.

$$\Pr\left(\left|\frac{m}{n} - k_0\right| \geq \varepsilon_n, \frac{m}{n} \in (a,b)\right) \to 0$$

(86)

Then, since $\Pr\left(\frac{m}{n} \in (a,b)\right) \geq P_{m_0}$,

$$\Pr\left(\left|\frac{m}{n} - k_0\right| \geq \varepsilon_n \mid k \in (a,b)\right) \to 0$$

(87)

Finally, the equation in second property is obtained:

$$\Pr\left(\left|\frac{m}{n} - k_0\right| < \varepsilon_n \mid k \in (a,b)\right) \to 1$$

(88)

\[\square\]

**Appendix D**

**Proof of Theorem 3**

We only give the proof for the first part of the theorem. Second part follows similarly, by switching $S_0$ and $k_0$ with $S_2$ and $k_2$. Under the conditions given in part (i), we will first prove that

$$\Pr(S_0) \to 1, \Pr(S_1) \to 0, \Pr(S_2) \to 0$$

(89)
To show that $\Pr(S_2) \to 0$, we use Lemma 3. Lemma 3 can be used for $S_0$ and $S_2$ regions since $k_0$ and $k_2$ satisfy the conditions of the Lemma over regions $(0, \frac{k_0+k_1}{2})$ and $(\frac{k_1+k_2}{2}, 1)$ respectively. Using property (i) of Lemma 3,

$$\Pr\left(\frac{m}{n} - k_2 \geq \epsilon_n, S_2\right) \leq P_{m_2} o(1)$$  \hfill (90)

Since $P_{m_2}$ is the local maxima, we can use it as an upper bound over all $P_m$ values in the region between $k_2 - \epsilon_n$ and $k_2 + \epsilon_n$, which will also be inside $S_2$.

$$\Pr\left(\frac{m}{n} - k_2 < \epsilon_n, S_2\right) \leq P_{m_2} 2n\epsilon_n = P_{m_2} 2cn^{2/3}$$  \hfill (91)

$$\Pr(S_2) \leq P_{m_2} (2cn^{2/3} + o(1))$$  \hfill (92)

Now we define $k_3$ such that $\int_{k_3}^{k_2} f(k')dk' = 0$ and $k_3 \in (k_0, k_2)$ holds. Such $k_3$ exists since $\int_{k_0}^{k_2} f(k')dk' < 0$ and $f(k)$ is continuous. Then,

$$\ln \left( \frac{P_{m_3}}{P_{m_2}} \right) \to n \int_{k_3}^{k_2} f(k')dk' = 0$$  \hfill (93)

$P_{m_3}$ can be used as a lower bound in interval between $k_0$ and $k_3$, similar to how $P_{m_2}$ was used as an upper bound. Furthermore, $f(k_3)$ must be negative and thus $k_3 \in (k_0, k_1)$. Hence, $k_3$ does not lie in the region $S_2$ and regions $(k_0, k_3)$ and $S_2$ are disjoint:

$$1 - \Pr(S_2) \geq \Pr \left( \frac{m}{n} \in (k_0, k_3) \right) \geq P_{m_3} n(k_3 - k_0)$$  \hfill (94)

Ratio of (92) and (94) results in the following:

$$\frac{\Pr(S_2)}{1 - \Pr(S_2)} \leq \frac{P_{m_2}}{P_{m_3}} \left( \frac{c}{k_3 - k_0} n^{-1/3} + o(1/n) \right)$$  \hfill (95)

Upper bound of (95) goes to 0, so $\Pr(S_2)/(1 - \Pr(S_2))$ goes to 0 as well. As a result, $\Pr(S_2) \to 0$.

Next, we derive $\Pr(S_1)$. Region $S_1$ corresponds to the local minima or the valley of the PMF over the number of active sources. The point with maximum probability (in PMF) in $S_1$ will be one of the endpoints. We use this probability as an upper bound over $S_1$.

$$\Pr(S_1) < n \left( \frac{k_2 - k_0}{2} \right) \max \{P_{n \frac{k_0+k_k}{2}}, P_{n \frac{k_1+k_2}{2}}\}$$  \hfill (96)

$$\ln \left( \frac{P_{n \frac{k_0+k_1}{2}}}{P_{nk_0}} \right) \to n \int_{k_0}^{k_0+k_1} f(k')dk'$$  \hfill (97)

$$\ln \left( \frac{P_{n \frac{k_1+k_2}{2}}}{P_{nk_2}} \right) \to -n \int_{k_1+k_2}^{k_2} f(k')dk'$$  \hfill (98)

Since $\int_{k_0}^{k_0+k_1} f(k')dk' < 0$ and $\int_{k_1+k_2}^{k_2} f(k')dk' > 0$, both $P_{n \frac{k_0+k_1}{2}}$ and $P_{n \frac{k_1+k_2}{2}}$ decay exponentially as $n$ grows, hence $\Pr(S_1) \to 0$. Since $\Pr(S_0) + \Pr(S_1) + \Pr(S_2) = 1$, we finally obtain
\( \Pr(S_0) \to 1 \). Following bound originates from the conditional probability:

\[
\Pr\left( \left\lvert \frac{m}{n} - k_0 \right\rvert < \epsilon_n \right) \geq \Pr\left( \left\lvert \frac{m}{n} - k_0 \right\rvert < \epsilon_n \mid S_0 \right) \Pr(S_0) \tag{99}
\]

From property (ii) of Lemma 3

\[
\Pr\left( \left\lvert \frac{m}{n} - k_0 \right\rvert < \epsilon_n \mid S_0 \right) \to 1 \tag{100}
\]

Finally, \( \Pr(S_0) \to 1 \) is used along with (99) and (100), to conclude the proof:

\[
\Pr\left( \left\lvert \frac{m}{n} - k_0 \right\rvert < \epsilon_n \right) \to 1 \tag{101}
\]

ACKNOWLEDGMENT

This work was supported in by TUBITAK grants 117E215 and 119C028, and by Huawei. We thank Mutlu Ahmetoglu for his assistance with simulations.

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