Bounds on $M_W$, $M_t$, $\sin^2 \theta_{\text{eff}}^{\text{lept}}$

A. Ferroglia$^a$\textsuperscript{1}, G. Ossola$^b$\textsuperscript{2}, and A. Sirlin$^b$\textsuperscript{3}

$^a$Fakultät für Physik, Universität Freiburg, D-79104 Freiburg, Germany
and
Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

$^b$Department of Physics, New York University, 4 Washington Place, New York, NY 10003, USA

Abstract

Assuming that the Standard Model is correct and taking into account the lower bound on $M_H$ from direct searches, we discuss bounds on $M_W$, $M_{\text{top}}$, and $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ at various confidence levels. This permits to identify theoretically favored ranges for these important parameters in the Standard Model framework, regardless of other observables. As an illustration of possible future developments, a hypothetical benchmark scenario, involving shifts $\lesssim 1\sigma$ in the experimental central values, is discussed.

\textsuperscript{1}e-mail: andrea.ferroglia@physik.uni-freiburg.de
\textsuperscript{2}e-mail: giovanni.ossola@physics.nyu.edu
\textsuperscript{3}e-mail: alberto.sirlin@nyu.edu
1 Introduction

The general consensus at present is that the Standard Model (SM) gives a very good description of a multitude of phenomena from atomic energies up to the electroweak scale. On the other hand, a fundamental pillar of the theory, the Higgs boson, has not been found so far and some experimental observables put sharp constraints on its mass. This is particularly true of the $M_W$ measurement. For instance, it was pointed out in Ref. [1] that the 2002 average value $(M_W)^{\text{exp}} = 80.451 \pm 0.033$ GeV, in conjunction with $(M_t)^{\text{exp}} = 174.3 \pm 5.1$ GeV, $\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036$, $\alpha_s(M_Z) = 0.118 \pm 0.002$, led to the prediction $M_H = 23^{+49}_{-23}$ GeV, and the 95% C.L. upper bound $M_H^{95} = 122$ GeV. The first value is embarrassingly low relative to the 95% C.L. lower bound $(M_H)_{\text{L.B.}} = 114.4$ GeV from direct searches [2], while the second one is only slightly larger. Since then the situation has changed significantly: repeating this analysis with the new experimental value $(M_W)^{\text{exp}} = 80.426 \pm 0.034$ GeV [2], we find, on the basis of the simple formulae of Ref. [1], the predictions $M_H = 45^{+69}_{-36}$ GeV, $M_H^{95} = 184$ GeV, which are much less restrictive. The predictions are further relaxed if one uses as inputs both $(M_W)^{\text{exp}}$ and the current average value $(s_{eff}^2)^{\text{exp}} = 0.23150 \pm 0.00016$ [2], where $s_{eff}^2$ is an abbreviation for $\sin^2 \theta_{e eff}$. This analysis leads to

$$M_H = 112^{+69}_{-45} \text{ GeV}; \quad M_H^{95} = 243 \text{ GeV},$$ (1)

which are not far from the values currently derived from the global fit: $M_H = 96^{+69}_{-38}$ GeV, $M_H^{95} = 219$ GeV [2].

There are three factors that single out the $M_W$ determination as particularly significant: i) as illustrated in the above remarks, it places sharp restrictions on $M_H$; ii) The LEP2 and collider measurements of $M_W$ are in excellent agreement with $\chi^2$/Dof = 0.3/1; iii) the relevant electroweak correction $\Delta r$ [3] has been fully evaluated at the two-loop level [4], an important theoretical achievement.

The aim of this paper is to derive bounds on $M_W$, $M_{\text{top}}$, and $\sin^2 \theta_{e eff}^{\text{lept}}$ in the SM framework by comparing the experimental measurements of these three basic parameters at various confidence levels with the theoretical functions $M_W = M_W(M_H, M_t)$ and $s_{eff}^2 = s_{eff}^2(M_H, M_t)$, for fixed values of $M_H$. The lower bound $(M_H)_{\text{L.B.}}$ restricts the available parameter space and this leads to bounds on $M_W$, $M_{\text{top}}$, and $\sin^2 \theta_{e eff}^{\text{lept}}$ that are significantly sharper.

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than those derived from the experimental measurements. Of course, this approach assumes the validity of the SM and makes use of the \((M_H)_{L.B.}\). Thus, the derived bounds may be regarded as theoretically favored domains for these parameters in the SM framework, regardless of other observables. As such, they may suggest plausible ranges of variability in future, more precise experimental determinations.

In Section 2 we examine the bounds derived from the theoretical functions \(M_W = M_W(M_H, M_t)\) using the simple formulae from Ref. [1], as well as the new theoretical expressions presented in Ref. [5]. In Section 3 we extend the analysis to the functions \(s_{eff}^2 = s_{eff}^2(M_H, M_t)\) using the results of Ref. [1]. In Section 4 we present the conclusions and, as an illustration of possible future developments, we discuss a hypothetical benchmark scenario involving shifts of \(\lesssim 1\sigma\) in the experimental central values. Appendix A discusses the effect on the analysis of the bounds due to the estimated errors in the \(\Delta\alpha_h(5)\) determinations and Appendix B extends the analysis of Section 4 to the case of a “theory driven” calculation of \(\Delta\alpha_h(5)\).

2 \(M_W\) and \(M_t\)

In this Section we compare the experimental values of \(M_W\) and \(M_t\) with the theoretical SM curves \(M_W = M_W(M_H, M_t)\) for fixed values of \(M_H\), taking into account the lower bound \((M_H)_{L.B.}\) on \(M_H\) from the direct searches. To simplify the analysis we take the restriction \(M_H \geq 114.4\) GeV to be a sharp cutoff rather than a 95% C.L. bound. The theoretical curves depend also on \(\Delta\alpha_h(5)\), the contribution from the first five quark flavors to the running of \(\alpha\) at the \(M_Z\) scale. We use as inputs \(\Delta\alpha_h(5) = 0.02761 \pm 0.00036\) [6] and the “theory driven” calculation \(\Delta\alpha_h(5) = 0.02747 \pm 0.00012\) [7]. We also employ as inputs \(\alpha, G_F, M_Z,\) and \(\alpha_s(M_Z) = 0.118 \pm 0.002\) [12].

Fig. 1 shows the theoretical SM curves \(M_W(M_H, M_t)\) for \(M_H = 114.4\) (dashed line), 139, 180, 224 GeV, \(\Delta\alpha_h(5) = 0.02761\), and \(\alpha_s(M_Z) = 0.118\), evaluated with the simple formulae of Ref. [1], in the effective scheme of renormalization [18], as well as the 68%, 80%, 90%, 95% C.L. contours derived from the current experimental values \((M_W)_{exp} = 80.426 \pm 0.034\) GeV, \((M_t)_{exp} = 174.3 \pm 5.1\) GeV. An interesting feature is that the theoretical curves are nearly linear over the range of \(M_t\) values considered. At a given C.L. the allowed region lies within the corresponding ellipse and below the \(M_H = 114.4\) GeV SM theoretical curve (dashed line), which we call
the boundary curve (B.C.). As shown in Fig. 1, the B.C. barely misses intersecting the 68% C.L. ellipse, so that strictly speaking this region is not allowed when the \((M_H)_{L.B.}\) restriction is imposed. It turns out that, to a good approximation, the maximum and minimum \(M_W\) and \(M_t\) values in a given allowed region are determined by the intersections of the B.C. with the associated ellipse. This interesting feature can be understood by a glance at Fig. 1. The allowed \(M_W\) and \(M_t\) ranges determined by such intersections are shown in Table 1 for the 80%, 90%, 95% C.L. domains. As \(M_H\) increases beyond 114.4 GeV, the allowed ranges decrease in size. At a given C.L. domain, the maximum allowed \(M_H\) corresponds to the theoretical curve \(M_W(M_H, M_t)\) that just touches the associated ellipse. From Fig. 1 we can see that these values are \(M_H \approx 139\) GeV, \(180\) GeV, \(224\) GeV corresponding to the 80%, 90%, 95% C.L. domains.

In the above analysis \(\Delta \alpha_h^{(5)}\) has been kept fixed at the central value \(\Delta \alpha_h^{(5)} = 0.02761\). If it is allowed to vary according to \(\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036\), the analysis is somewhat more involved (see Appendix A). However, the conclusion is that the \(M_W, M_t\) ranges reported in Table 1 are at most affected by minor shifts.

Table 2 presents the \(M_W, M_t\) ranges evaluated with \(\Delta \alpha_h^{(5)} = 0.02747\). In this case we see that there is a very narrow window of compatibility with the 68% C.L. domain. Otherwise, the \(M_W, M_t\) ranges are very similar to those in Table 1. It is interesting to note that compatibility with the SM improves as \(\Delta \alpha_h^{(5)}\) decreases.

Tables 3 and 4 repeat the analysis of Tables 1 and 2 on the basis of the SM theoretical formulae presented in Ref. [5], which are based on a complete two-loop calculation of \(\Delta r\) [4] in the on-shell scheme of renormalization [3,9]. These Tables employ \(\Delta \alpha_h^{(5)} = 0.02761\) and \(\Delta \alpha_h^{(5)} = 0.02747\), respectively. Again the 68% C.L. domain is not compatible with the SM curves subject to the \((M_H)_{L.B.}\) restriction. The allowed \(M_W, M_t\) ranges in the 80%, 90%, 95% C.L. domains are similar but somewhat more restrictive than in Tables 1 and 2. In particular, although the minimum \(M_W\) values are nearly the same, the maximum \(M_W\) values are from 10 to 7 MeV smaller than in Tables 1 and 2.

In Table 5 we present the mid-points of the \(M_W\) and \(M_t\) ranges in Table 2 with variations that cover the full intervals. At a given C.L., these are compared with the domains in \(M_W\) and \(M_t\) derived from \((M_W)_{exp} = 80.426 \pm 0.034\) GeV and \((M_t)_{exp} = 174.3 \pm 5.1\) GeV. As expected, the SM
allowed $M_W$, $M_t$ ranges are significantly restricted. To very good approximation, the mid-points $(80.402, 177.7)$ GeV are independent of the C.L. and are shifted from the experimental central values $(M_W)_{\text{exp}}$ and $(M_t)_{\text{exp}}$ by $\Delta M_W = -0.71\sigma_{M_W}$ and $\Delta M_t = +0.67\sigma_{M_t}$.

In the case of Tables 1, 3, and 4, to very good approximation the mid-points are also independent of the C.L. and are given by $(80.401, 177.9)$ GeV, $(80.397, 178.3)$ GeV, and $(80.398, 178.1)$ GeV, respectively. The largest shifts occur for Table 3, where the $M_W$ mid-point is $0.85\sigma_{M_W}$ below $(M_W)_{\text{exp}}$ and the $M_t$ mid-point is $0.78\sigma_{M_t}$ above $(M_t)_{\text{exp}}$.

The theoretical formulae employed in this analysis are of course subject to errors associated with the truncation of the perturbative series and the QCD corrections. A relevant question is what is the effect of these errors in the determination of the allowed $M_W$, $M_t$ ranges. We give two specific examples concerning the 90% C.L. domains in Tables 1 and 3. Using the estimated theoretical errors discussed in Ref. [1], we find that the 90% C.L. ranges in Table 1 become $(60 \pm 5)$ MeV in $M_W$ and $(9.5^{+0.8}_{-0.9})$ GeV in $M_t$. Using the estimated theoretical errors of Ref. [5], we find that the corresponding 90% C.L. intervals in Table 3 become $(53^{+3}_{-4})$ MeV in $M_W$ and $(8.8^{+0.6}_{-0.8})$ GeV in $M_t$. Thus, the effect of the errors in the theoretical formulae is to change the size of the allowed intervals by less then 10%. We also note that these modifications are significantly smaller than the experimental errors of $M_W$ and $M_t$ expected at TeV-LHC (cf. Section 4).

3\hspace{2cm} \text{sin}^2 \theta_{\text{eff}}^{\text{lept}} \text{ and } M_t

On the experimental side, we consider two possibilities: the current world average $(\text{sin}^2 \theta_{\text{eff}}^{\text{lept}})_{\text{exp}} = 0.23150 \pm 0.00016$ [2] and the average derived from the leptonic observables $(\text{sin}^2 \theta_{\text{eff}}^{\text{lept}})(t) = 0.23113 \pm 0.00021$ [2]. The difference between these values reflects the well-known dichotomy between the leptonic and hadronic determinations, which differ by $\approx 3\sigma$. On the theoretical side, the relevant electroweak correction is $\Delta r_{\text{eff}}$ [8,10,11]. Unlike $\Delta r$, it has not been fully evaluated at the two-loop level. For this reason, we simply employ the formulae of Ref. [1] in the effective scheme of renormalization. They contain two-loop electroweak effects enhanced by powers $(M_t^2/M_W^2)^n$ ($n = 1, 2$), as well as QCD corrections. For $M_t$, $\Delta \alpha_h^{(5)}$, and $\alpha_s(M_Z)$ we employ the same inputs as in Section 2.

Fig. [2] shows the 68%, 80%, 90%, and 95% C.L. domains derived from
the world average \((\sin^2 \theta_{\text{eff}})_{\text{exp}} = 0.23150 \pm 0.00016\) and \((M_t)_{\text{exp}} = 174.3 \pm 5.1\) GeV, as well as the SM theoretical curves \(s^2_{\text{eff}}(M_H, M_t)\) for \(M_H = 114.4\) (dashed line), 193, 218, 253, 289 GeV, evaluated with \(\Delta \alpha_h^{(5)} = 0.02761\) and \(\alpha_s(M_Z) = 0.118\). At a given C.L. the allowed region lies within the corresponding ellipse and above the B.C.. Since in this case the center of the ellipses lies in the allowed regions, the situation is very different from that in Fig. 1. In fact, the reduction in parameter space is much less radical than in the \((M_W, M_t)\) analysis. In particular, as shown in Fig. 2, the maximum \(s^2_{\text{eff}}\) and \(M_t\) values are not affected by the \((M_H)_{\text{L.B.}}\) restriction and the minimum values are increased by relatively small amounts.

In the case of the leptonic average \((\sin^2 \theta_{\text{eff}})_{\text{lept}} = 0.23113 \pm 0.00021\), the situation is depicted in Fig. 3. The allowed regions lie again within the C.L. ellipses and above the B.C. (dashed line). The 68% C.L. domain is clearly forbidden.

In the \((\sin^2 \theta_{\text{eff}})_{\text{lept}}, M_t\) analysis, the effect of varying \(\Delta \alpha_h^{(5)}\) according to \(\Delta \alpha_h^{(5)} = (\Delta \alpha_h^{(5)})^c \pm \sigma_{\Delta \alpha}\) is more pronounced than in the \(M_W, M_t\) case and, accordingly, we have derived the allowed intervals using the \(\chi^2\)-analysis discussed in Appendix A. They are shown in Tables 6 and 7 for \(\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036\) and \(\Delta \alpha_h^{(5)} = 0.02747 \pm 0.00012\), respectively. To good approximation, the mid-points are again independent of the C.L. and are given by \((0.23129; 177.3\) GeV) in Table 6 and by \((0.23129; 177.4\) GeV) in Table 7. For a latter application in Section 4, we list also the mid-points of the \((\sin^2 \theta_{\text{eff}})_{\text{lept}}, M_t\) ranges defined by the intersections of the \(\Delta \alpha_h^{(5)} = 0.02761\) and \(\Delta \alpha_h^{(5)} = 0.02747\) B.C. with the C.L. ellipses. They are \((0.23133; 178.1\) GeV) and \((0.23130; 177.5\) GeV), respectively. In the \(\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036\) case, they differ by relatively small amounts from the mid-points in Table 6 mainly because the latter take into account the effect of the \(\Delta \alpha_h^{(5)}\) variation discussed before. In the \(\Delta \alpha_h^{(5)} = 0.02747 \pm 0.00012\) case, because of the smallness of the error, the effect is less significant and they almost coincide with the mid-points in Table 7. The shifts of the B.C. mid-points \((0.23133; 178.1\) GeV) and \((0.23130; 177.5\) GeV) from the experimental central values amount to \(+0.95 \sigma_{s_{\text{eff}}}, +0.75 \sigma_{M_t}\) and \(+0.81 \sigma_{s_{\text{eff}}}, +0.63 \sigma_{M_t}\), respectively.

In analogy with the discussion at the end of Section 2, we now study the effect of the theoretical errors on the determination of the \((\sin^2 \theta_{\text{eff}})_{\text{lept}}, M_t\) ranges. We consider, as an illustration, the 90% C.L. intervals in Table 7.
Taking into account the theoretical errors discussed in Ref. [1], we find that these ranges become $0.00033 \pm 0.00005$ in $(s_{\text{eff}})^2_t$ and $(10.5^{+1.0}_{-1.9})$ Gev in $M_t$. Thus, we see that the effect of the theoretical errors is more pronounced than in the $M_W$, $M_t$ case, changing the size of the allowed domains by less than 15% in $(s_{\text{eff}})^2_t$ and 18% in $M_t$.

4 Discussion

As is well known, global analyses of the electroweak observables in the SM framework have frequently led to the derivation of preferred domains in the $M_W$, $M_t$ plane at various C.L. [2,12]. Although such studies are very valuable on general grounds, the focus of the present paper is quite different. Specifically, in Section 2 we have addressed the following question: given the present experimental values of $M_W$ and $M_t$, and irrespective of other observables, what are the allowed ranges for these important parameters in the SM framework when the $(M_H)_{L.B.}$ is taken into account? In Section 3 the same question is addressed in the case of $s_{\text{eff}}$ and $M_t$. This approach conforms with the idea that, aside from the global fits, it is also important to compare the theory separately with the precise observables on which $M_H$ depends most sensitively [13]. In fact, it is in principle possible that striking discrepancies of crucial observables and important information may be blurred in the global analysis. An obvious observation is that, if the SM is correct, the central values of $M_W$, $M_t$, and $(s_{\text{eff}})^2_t$ must approach the allowed regions as the errors decrease, irrespective of what happens to other observables. We also note that in cases such as $(M_W, M_t)$ and $((s_{\text{eff}})^2_t, M_t)$, in which the central values lie well outside the allowed regions (cf. Fig.1 and Fig.3), the bounds derived in this approach are significantly more restrictive than those obtained in the indirect global analysis (cf. Fig.16.2 of Ref. [2]). Finally, it is worth pointing out that analyses of the kind carried out in this paper are particularly simple. For instance, the ellipses in Figs.1 and 3 are obtained from the experimental values by purely statistical means and the theory essentially enters in the derivation of the theoretical curves.

In order to implement this approach, in Section 2 we have compared the experimental values for $M_W$ and $M_t$ at various C.L. with the SM theoretical curves $M_W(M_H, M_t)$ for fixed $M_H$, imposing the restriction $M_H \geq 114.4$ GeV. We have employed both the theoretical formulae of Ref. [1] and Ref. [2] and considered two values of $\Delta \alpha_h^{(5)}$. As expected from the discussion in
the Introduction, the $M_W$ and $M_t$ ranges are significantly reduced in the SM theoretical framework when the bound $M_H \geq 114.4$ GeV is taken into account. Compatibility with the experimental 68% C.L. region only occurs in one of the alternatives we have considered and is at best marginal. In the experimental 80% C.L. domain, the current allowed $M_W$ and $M_t$ ranges vary from $(M_W = 80.402 \pm 0.020; M_t = 177.7 \pm 3.1)$ GeV to $(M_W = 80.397 \pm 0.012; M_t = 178.3 \pm 2.0)$ GeV, depending on the value of $\Delta \alpha_h^{(5)}$ and whether one employs the theoretical expressions of Ref. [1] or Ref. [5].

In order to belong to the allowed region, it is understood that pairs of $M_W$ and $M_t$ values from these intervals should be chosen so that they lie within the 80% C.L. domain. At the 90% and 95% C.L., the allowed $M_W$ and $M_t$ intervals are of course wider and can be read from Tables 1-4; however, to a very good approximation, in each Table the mid-points are independent of the C.L..

The allowed $M_W$ and $M_t$ domains derived in this manner may be regarded as theoretically favored in the SM framework when the $(M_H)_{L.B.}$ is taken into account, irrespective of other observables. Qualitatively, they indicate that compatibility with the theory would improve if $(M_W)_{exp}$ would decrease and $(M_t)_{exp}$ would increase. Assuming the validity of the SM, the central values of $M_W$ and $M_t$ must approach the allowed regions as the errors decrease. Of course, the precise end-point of this trajectory is not known, nor is it very clear what is the optimal C.L. to select the allowed region. On the other hand, the mid-points of the allowed regions provide natural representative examples. The fact that to very good approximation they are independent of the C.L. used in selecting the allowed regions (provided the C.L. are sufficiently large that there are allowed regions), makes them particularly attractive benchmarks. Therefore, we will consider, as an illustration of plausible future developments, a hypothetical, but representative scenario in which the experimental central points move to the mid-points of the current allowed intervals.

This would require a shift of $-0.71$ to $-0.85\sigma_{M_W}$ in $(M_W)_{exp}$ and of $+0.67$ to $0.78\sigma_{M_t}$ in $(M_t)_{exp}$. It is interesting to note that a change in $(M_W)_{exp}$ of the same direction and magnitude occurred in the recent past: namely, the shift of $(M_W)_{exp} = 80.451$ GeV to $(M_W)_{exp} = 80.426$ GeV represented a $-0.76\sigma_{M_W}$ effect. Also, it is worthwhile to observe that the most precise $M_W$ measurement, the LEP2 determination $(M_W)_{LEP2} = 80.412 \pm 0.042$ GeV [2] has a central value that is significantly closer than $(M_W)_{exp}$ to the mid-
points mentioned above. Finally, there is a very recent preliminary value $M_t = 180.1 \pm 5.4$ GeV from the D0 collaboration that suggests that $(M_t)_{\text{exp}}$ may significantly increase in the near future [14].

Assuming that the Higgs boson remains undiscovered, a natural question is: what would be the $M_H$ prediction in this hypothetical scenario? We use $\sigma_{M_W} = 15$ MeV and $\sigma_{M_t} = 2$ GeV, which are projected for Tev-LHC [15]. $\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036$, and $\alpha_s (M_Z) = 0.118 \pm 0.002$. When the formulae of Ref. [11] are employed, the mid-points are (80.401, 177.9) GeV and we obtain the prediction

$$M_H = 114^{+46}_{-35} \text{ GeV}; \quad M_H^{95} = 195 \text{ GeV}. \quad (2)$$

Instead, using Ref. [5], the mid-points are (80.397, 178.3) GeV, and this leads to

$$M_H = 114^{+47}_{-37} \text{ GeV}; \quad M_H^{95} = 198 \text{ GeV}. \quad (3)$$

Eqs. (2,3) have been obtained without taking into account the $(M_H)_{L,B}$. Including its effect we find that $M_H^{95}$ is shifted to 214 GeV in Eq. (2) and 218 GeV in Eq. (3).

In Section 3, we compared the experimental value for $s_{eff}^2$ and $M_t$ at various C.L. with the SM theoretical curves $s_{eff}^2 = s_{eff}^2 (M_H, M_t)$ for fixed $M_H$, taking into account the $(M_H)_{L,B}$ restriction. Here we considered two alternatives: the world average value for $s_{eff}^2$ and the average $(s_{eff}^2)_{(l)}$ derived from the leptonic observables. In the first case, there is very good compatibility with $(M_H)_{L,B}$ and, in fact, the allowed $s_{eff}^2, M_t$ intervals are only reduced by relatively small amounts. In the second case, the 68% C.L. is forbidden by the $(M_H)_{L,B}$ and the allowed $(s_{eff}^2)_{(l)}$ and $M_t$ intervals are significantly reduced. As in the case of the $M_W, M_t$ analysis, we may consider a hypothetical scenario in which the experimental $(s_{eff}^2)_{(l)}$, $M_t$ central values move in the future to representative points of the allowed region. In the $(s_{eff}^2)_{(l)}, M_t$ case, it is convenient to use as benchmarks the mid-points of the ranges defined by the intersection of the B.C. with the C.L. ellipses (cf. Section 3). In order to illustrate how this shift in the central values would affect the $M_H$ prediction, we assume again $\sigma_{M_t} = 2$ GeV, an error $\sigma_{s_{eff}^2} = 0.00001$ for $(s_{eff}^2)_{(l)}$, as projected for $s_{eff}^2$ in the GigaZ application of the NLC, and employ $\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036$, for which the mid-points discussed in Section 3 are (0.23133; 178.1 GeV). These inputs lead to

$$M_H = 115^{+37}_{-28} \text{ GeV}; \quad M_H^{95} = 180 \text{ GeV}. \quad (4)$$
Including the effect of \((M_H)_{L.B.}, M_H^{95}\) in Eq. (1) is shifted to 196 GeV.

The central values in Eqs. (2,3,4) reflect the interesting feature that the mid-points are close to the \(M_H = 114.4\) GeV boundary curves.

The fact that the benchmark scenario we have considered involves a decrease in \((M_W)_{c^{\exp}}\) and an increase in \((M_t)_{c^{\exp}}\) and \((s_{eff}^2)_{(l)}\) can be readily understood qualitatively by observing the relative positions of the C.L. domains and the allowed theoretical curves in Figs. 1 and 3.

If instead we assume that the current central values for \(M_W, M_t,\) and \((s_{eff}^2)_{(l)}\) remain unaltered while the errors decrease to \(\sigma_{M_W} = 15\) MeV, \(\sigma_{M_t} = 2\) GeV, and \(\sigma_{s_{eff}^2} = 0.00001\), the estimates of Eqs. (2,3,4) are replaced by

\[
\begin{align*}
M_H &= 45^{+25}_{-18} \text{ GeV; } M_H^{95} = 90 \text{ GeV,} \\
M_H &= 36^{+23}_{-17} \text{ GeV; } M_H^{95} = 79 \text{ GeV,} \\
M_H &= 59^{+21}_{-16} \text{ GeV; } M_H^{95} = 96 \text{ GeV,}
\end{align*}
\]

respectively. Clearly, Eqs. (5-7) would indicate a sharp disagreement with the \((M_H)_{L.B.}\)!

An alternative possibility that would circumvent the incompatibility of Eqs. (5-7) would be an increase of \((M_t)_{c^{\exp}}\) with \((M_W)_{c^{\exp}}\) and \([(s_{eff}^2)_{(l)}]_{c^{\exp}}\) kept fixed. As pointed out in Ref. [14], such a shift would improve in general the compatibility with the SM. This can be readily understood from Figs. 1-3 since the C.L. ellipses would move towards the allowed region.

If \((M_t)_{c^{\exp}}\) increases by the current error 5.1 GeV, as discussed in Ref. [14], and we again employ \(\sigma_{M_W} = 15\) MeV, \(\sigma_{M_t} = 2\) GeV, we would obtain from the \(M_W\) input

\[
\begin{align*}
M_H &= 87^{+38}_{-29} \text{ GeV; } M_H^{95} = 155 \text{ GeV,} \\
M_H &= 74^{+36}_{-27} \text{ GeV; } M_H^{95} = 138 \text{ GeV,}
\end{align*}
\]

using Ref. [1], and

\[
\begin{align*}
M_H &= 83^{+29}_{-22} \text{ GeV; } M_H^{95} = 134 \text{ GeV.}
\end{align*}
\]
Unlike Eq. (5,6,7), Eq. (8,9,10) are marginally compatible with the \((M_H)_{L.B.}\). On the other hand, Eqs. (8,9,10), based on the \((M_t)_{\text{exp}}^C = 179.4\) GeV assumption, are significantly more restrictive than Eqs. (2,3,4) corresponding to the benchmark scenario discussed in this paper.

A qualitative difference between the two scenarios is that in Eqs. (2,3,4) the central experimental points reach the allowed region, while this does not happen in Eqs. (8,9,10). In fact, in order to reach the allowed region by varying \((M_t)_{\text{exp}}^C\) alone, one would need a shift of +7.5 GeV or 1.5 times the current error if one employs the \(M_W\) input to predict \(M_H\), and of 9.8 GeV or 1.9 times the current error if one uses \((s_{\text{eff}}^2)^C(l)\).

Throughout this Section we have employed \(\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036\). The corresponding \(M_H\) estimates using the “theory driven” calculation \(\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012\) are presented in Appendix B.

In summary, if the SM is correct, the experimental central values should approach the allowed region as the errors shrink, regardless of other observables, and on that basis the analysis of this paper suggests, as illustrated in Figs. 1 and 3, the possibility that \((M_W)_{\text{exp}}^C\) will decrease, while \((M_t)_{\text{exp}}^C\) and \((s_{\text{eff}}^2)^C(l)\) will increase. In the hypothetical benchmark illustration that we have described, all these changes are < 1\(\sigma\) in magnitude, so they are certainly not extreme. The fact that shifts of this magnitude have recently occurred gives some plausibility to this scenario. Thus, as the accuracy increases, it will be very interesting to see whether the central values approach the allowed regions preferred by the SM, remain where they are, or move in the opposite direction. In the last two cases a sharp disagreement with the SM would emerge, thus providing strong evidence for new physics!

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**Appendix A**

In the discussions of Section 2 and 3 we have kept \(\Delta\alpha_h^{(5)}\) fixed at their central values. This has the advantage that the analysis is particularly simple and
can be readily illustrated in terms of C.L. ellipses and the SM theoretical curves, as shown in the figures. If $\Delta \alpha^{(5)}$ is allowed to vary according to $\Delta \alpha^{(5)}_h = (\Delta \alpha^{(5)}_h)^c \pm \sigma_{\Delta \alpha}$, the simplest procedure in the $M_W, M_t$ case (Section 2) is to consider the $\chi^2$ function:

$$
\chi^2 = \frac{(M_W(M_H, M_t, \Delta \alpha^{(5)}_h) - M_W^c)^2}{\sigma_{M_W}^2} + \frac{(M_t - M_t^c)^2}{\sigma_{M_t}^2} + \frac{(\Delta \alpha^{(5)}_h - (\Delta \alpha^{(5)}_h)^c)^2}{\sigma_{\Delta \alpha}^2},
$$

(A1)

where $M_W^c$, $M_t^c$, and $(\Delta \alpha^{(5)}_h)^c$ are the central values of $(M_W)_{exp}$, $(M_t)_{exp}$ and $\Delta \alpha^{(5)}_h$, respectively, and $M_W(M_H, M_t, \Delta \alpha^{(5)}_h)$ is the SM theoretical curve that now depends on $M_W$, $M_t$ and $\Delta \alpha^{(5)}_h$. For fixed $\chi^2$ and $M_H$, Eq. (A1) defines an implicit function $f(M_t, \Delta \alpha^{(5)}_h) = \chi^2$ relating $M_t$ and $\Delta \alpha^{(5)}_h$. Varying $\Delta \alpha^{(5)}_h$ over an appropriate finite interval, one finds numerically the range spanned by $M_t$. Using $M_W(M_H, M_t, \Delta \alpha^{(5)}_h)$ one then obtains the domain of variability of $M_W$. The resulting $M_W, M_t$ ranges agree, up to at most minor changes, with those presented in the Tables in Section 2.

In the $s^2_{eff}, M_t$ case, the effect of the $\Delta \alpha^{(5)}_h = (\Delta \alpha^{(5)}_h)^c \pm \sigma_{\Delta \alpha}$ variation is more pronounced and, accordingly, we have derived the allowed intervals in the Tables of Section 3 from a $\chi^2$-analysis analogous to that explained above.

**Appendix B**

In this Appendix we present the $M_H$ estimates discussed in Section 4 when one employs the “theory driven” calculation $\Delta \alpha^{(5)}_h = 0.02747 \pm 0.00012$ [7], instead of $\Delta \alpha^{(5)}_h = 0.02761 \pm 0.00036$ [6].

We find that Eqs. (2,3,4), corresponding to our benchmark scenario with future projected errors, are replaced by

$$
M_H = 115^{+43}_{-34} \text{ GeV}; \quad M_H^{95} = 191 \text{ GeV}, \quad (B1)
$$

$$
M_H = 114^{+45}_{-35} \text{ GeV}; \quad M_H^{95} = 194 \text{ GeV}, \quad (B2)
$$

$$
M_H = 115^{+19}_{-17} \text{ GeV}; \quad M_H^{95} = 148 \text{ GeV}, \quad (B3)
$$

respectively.
Eqs. (5,6,7), corresponding to the current central values with future projected errors, are replaced by

\[ M_H = 48^{+25}_{-18} \text{ GeV}; \quad M_H^{95} = 92 \text{ GeV}, \] (B4)

\[ M_H = 38^{+23}_{-17} \text{ GeV}; \quad M_H^{95} = 80 \text{ GeV}, \] (B5)

\[ M_H = 65^{+12}_{-10} \text{ GeV}; \quad M_H^{95} = 86 \text{ GeV}. \] (B6)

Finally, instead of Eqs. (8,9,10), corresponding to the \((M_t)^c_{exp} = (174.3 + 5.1) \text{ GeV}\) assumption, with current \((M_W)^c_{exp}\) and \([s_{eff}^2(l)]^c_{exp}\) values, and future projected errors, we have

\[ M_H = 91^{+38}_{-28} \text{ GeV}; \quad M_H^{95} = 157 \text{ GeV}, \] (B7)

\[ M_H = 78^{+35}_{-27} \text{ GeV}; \quad M_H^{95} = 141 \text{ GeV}, \] (B8)

\[ M_H = 92^{+16}_{-14} \text{ GeV}; \quad M_H^{95} = 120 \text{ GeV}. \] (B9)

We see that the most significant change is between Eq. (1) and Eq. (B3) derived from the \((s_{eff}^2(l))\) input in the benchmark scenario discussed in this paper.

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Figure 1: 68%, 80%, 90%, 95% C.L. domains derived from $(M_W)_{exp} = 80.426 \pm 0.034 \text{GeV}$ and $(M_t)_{exp} = 174.3 \pm 5.1 \text{GeV}$, together with the SM theoretical curves $M_W(M_H, M_t)$ for $M_H = 114.4$ (dashed line), 139, 180, 224 GeV from Ref. [1] with $\Delta \alpha_h^{(5)} = 0.02761$. At a given C.L. the allowed region lies within the corresponding ellipse and below the dashed boundary curve B.C.
Figure 2: 68%, 80%, 90%, 95% C.L. domains derived from \( \sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23150 \pm 0.00016 \) and \( M_t = 174.3 \pm 5.1 \) GeV, together with the SM theoretical curves \( s_{\text{eff}}^2(M_H, M_t) \) for \( M_H = 114.4 \) (dashed line), 193, 218, 253, 289 GeV from Ref. \[\text{[1]}\] and \( \Delta \alpha_{h}^{(5)} = 0.02761 \). The allowed regions lie above the dashed B.C..
Figure 3: 68%, 80%, 90%, 95% C.L. domains derived from \((\sin^2 \theta_{\text{eff}}^{\text{lept}})(t) = 0.23113 \pm 0.00021\) and \(M_t = 174.3 \pm 5.1\) GeV, together with the SM theoretical curves \(s_{\text{eff}}^2(M_H, M_t)\) for \(M_H = 114.4\) (dashed line), 119, 143, 169 GeV from Ref. [1] and \(\Delta \alpha_{\text{h}}^{(5)} = 0.02761\). The allowed regions lie above the dashed B.C..
Table 1: Comparison of the experimental values \((M_W)_{\text{exp}} = 80.426 \pm 0.034 \text{ GeV}\) and \((M_t)_{\text{exp}} = 174.3 \pm 5.1 \text{ GeV}\), at various C.L., with SM theoretical expressions based on Ref. \[1\] and \(\Delta \alpha_h^{(5)} = 0.02761\). The Table shows the ranges for \(M_W\) and \(M_t\) that, according to the SM, are compatible with the restriction \(M_H \geq 114.4 \text{ GeV}\). In order to belong to the allowed regions, pairs of \(M_W\) and \(M_t\) values from these intervals should be chosen so that they lie within the corresponding C.L. domains. Within each C.L. domain, the \(M_W, M_t\) ranges decrease as \(M_H\) increases from 114.4 GeV.

| EFF scheme  | range \(M_W\) [GeV] | range \(M_t\) [GeV] |
|-------------|---------------------|--------------------|
| \(\Delta \alpha_h^{(5)} = 0.02761\) | 80% C.L. 80.383 – 80.419 | 175.0 – 180.7 |
|             | 90% C.L. 80.371 – 80.431 | 173.1 – 182.6 |
|             | 95% C.L. 80.362 – 80.441 | 171.6 – 184.1 |

Table 2: Same as in Table 1 for \(\Delta \alpha_h^{(5)} = 0.02747\).

| EFF scheme  | range \(M_W\) [GeV] | range \(M_t\) [GeV] |
|-------------|---------------------|--------------------|
| \(\Delta \alpha_h^{(5)} = 0.02747\) | 68% C.L. 80.396 – 80.408 | 176.7 – 178.7 |
|             | 80% C.L. 80.383 – 80.422 | 174.6 – 180.8 |
|             | 90% C.L. 80.371 – 80.434 | 172.7 – 182.6 |
|             | 95% C.L. 80.362 – 80.443 | 171.3 – 184.0 |
|                  | range $M_W$ [GeV] | range $M_t$ [GeV] |
|------------------|------------------|------------------|
| **Awramik et al. [5]** \[
\Delta \alpha_h^{(5)} = 0.02761
\] |                  |                  |
| 80% C.L.         | 80.385 – 80.409  | 176.3 – 180.3    |
| 90% C.L.         | 80.370 – 80.423  | 173.9 – 182.7    |
| 95% C.L.         | 80.361 – 80.433  | 172.3 – 184.2    |

Table 3: Same as in Table 1 with SM theoretical expressions from Ref. [5].

|                  | range $M_W$ [GeV] | range $M_t$ [GeV] |
|------------------|------------------|------------------|
| **Awramik et al. [5]** \[
\Delta \alpha_h^{(5)} = 0.02747
\] |                  |                  |
| 80% C.L.         | 80.384 – 80.413  | 175.7 – 180.5    |
| 90% C.L.         | 80.371 – 80.426  | 173.6 – 182.7    |
| 95% C.L.         | 80.361 – 80.435  | 172.0 – 184.2    |

Table 4: Same as in Table 3 for $\Delta \alpha_h^{(5)} = 0.02747$.  

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Table 5: Allowed $M_W$, $M_t$ ranges from Table 21 expressed as mid-points and variations covering the corresponding intervals. They are compared with the ranges extracted from the experimental values.

| C.L.    | $(M_W)_{exp}$ [GeV] | Allowed $M_W$ [GeV] | Allowed $M_t$ [GeV] | $(M_t)_{exp}$ [GeV] |
|---------|---------------------|---------------------|---------------------|---------------------|
| 68% C.L.| 80.426 ± 0.034       | 80.402 ± 0.006      | 177.7 ± 1.0         | 174.3 ± 5.1         |
| 80% C.L.| 80.426 ± 0.044       | 80.402 ± 0.020      | 177.7 ± 3.1         | 174.3 ± 6.5         |
| 90% C.L.| 80.426 ± 0.056       | 80.402 ± 0.032      | 177.7 ± 5.0         | 174.3 ± 8.4         |
| 95% C.L.| 80.426 ± 0.067       | 80.402 ± 0.041      | 177.7 ± 6.4         | 174.3 ± 10.0        |
\[ \Delta \alpha^{(5)}_h = 0.02761 \]

| EFF scheme | range \( \left( \sin^2 \theta_{\text{eff}}^{\text{lept}} \right)_l \) | range \( M_t \) [GeV] |
|------------|---------------------------------|------------------|
| 80% C.L.   | 0.23119 – 0.23139                | 174.7 – 179.9    |
| 90% C.L.   | 0.23111 – 0.23147                | 172.1 – 182.6    |
| 95% C.L.   | 0.23105 – 0.23153                | 170.4 – 184.3    |

Table 6: Comparison of the experimental values \( (s_{\text{eff}}^2)_l = 0.23113 \pm 0.00021 \) and \( M_t = 174.3 \pm 5.1 \) GeV at various C.L. with SM theoretical expressions from Ref. [1] and \( \Delta \alpha^{(5)}_h = 0.02761 \pm 0.00036 \). The Table shows the ranges for \( s_{\text{eff}}^2 \) and \( M_t \) that, according to the SM, are compatible with the restriction \( M_H \geq 114.4 \) GeV. In order to belong to the allowed regions, pairs of \( s_{\text{eff}}^2 \) and \( M_t \) values from these intervals should be chosen so that they lie within the corresponding C.L. domains.

\[ \Delta \alpha^{(5)}_h = 0.02747 \]

| EFF scheme | range \( \left( \sin^2 \theta_{\text{eff}}^{\text{lept}} \right)_l \) | range \( M_t \) [GeV] |
|------------|---------------------------------|------------------|
| 80% C.L.   | 0.23119 – 0.23140                | 174.2 – 180.6    |
| 90% C.L.   | 0.23113 – 0.23146                | 172.2 – 182.7    |
| 95% C.L.   | 0.23107 – 0.23151                | 170.6 – 184.2    |

Table 7: Same as in Table 6 for \( \Delta \alpha^{(5)}_h = 0.02747 \pm 0.00012 \).