COLLIDER JETS IN PERTURBATION THEORY†

Stephen D. Ellis‡

Theoretical Physics Division, CERN
CH - 1211 Geneva 23
Switzerland

Abstract

Recent progress in the perturbative analysis of hadronic jets, especially in the context of $pp$ and $p\bar{p}$ colliders, is discussed. The characteristic feature of this work is the emergence of a level of precision in the study of the strong interactions far beyond that previously possible. Inclusive cross sections for high energy jets at the Tevatron are now perturbatively calculable with a reliability on the order of 10%. At present this theoretical precision is comparable to (if not somewhat better than) the quoted experimental errors. Progress has also been made towards understanding both the internal structure of jets and the influence of the details of the jet-defining algorithm.

†) Talk presented at the XXVIIIth Rencontres de Moriond, QCD and High Energy Hadronic Interactions, March 1993.

‡) Permanent address: Department of Physics FM-15, University of Washington, Seattle, WA, USA.
1 Goals

Why study jets? The basic goal is to be able to perform “precision” studies of the strong interactions, a prospect unthinkable just a few years ago. In this context the term “precision” implies a theoretical uncertainty no larger than 10%. Intuitively the study of jets is a natural arena for precision since the jets can be thought of as the “footprints” of the underlying partons. It is at the parton level that we have the best control of the strong interactions through perturbative QCD. This perturbative approach applies not only to hadron-hadron collisions, as discussed here, but also to $ep$ collisions and $e^{+}e^{-}$ collisions.

With such a precise tool one can attack issues such as looking for deviations from the Standard Model at very short distances due, for example, to compositeness. One can study jet production to learn more about the structure of the hadrons, especially the gluon distribution function. Finally one can study the jets in detail in order to possibly differentiate quark-initiated and gluon-initiated jets so as to control triggers and backgrounds in the search for $p\bar{p} \rightarrow H + X \rightarrow W^{+}W^{-}(ZZ) + X \rightarrow 4$ jets + $X'$ at the SSC and LHC.

2 What is a Jet?

Clearly the first question that we must answer is how to define (and find) jets. The qualitative goal is clear. The relatively isolated sprays of energetic hadrons observed in the final states of high energy collisions are naively (and correctly) associated with the production of isolated, energetic partons via the scattering of small numbers of partons. These large angle scatterings involve only short distance interactions where the strong interactions are relatively weak and perturbative techniques are appropriate. Further, since we will deal with inclusive definitions of features of the final state, one anticipates that nonperturbative corrections to the perturbative results are small. However, although we would like to associate a unique subset of the final hadrons with the jet from a single scattered parton, we know that such a mapping cannot, in principle, be precise. The partons (quarks and gluons) carry color charge and are (treated as) essentially massless in the theoretical calculation. On the other hand a jet of hadrons has no color charge and often large invariant mass. Jets must arise from the coherent, collaborative activities of several ($\geq 2$) partons. Thus jets are necessarily somewhat ambiguous objects and we wish to treat them in such a way that these unavoidable ambiguities do not play an important role.

We need a jet definition or algorithm that, while it is a priori arbitrary, must still satisfy certain well defined constraints. It must be reliable and easy to use for both theorists and experimentalists. For the former this means “infrared finite” order by order in perturbation theory, while the latter demand an algorithm that is straightforward, efficient and well suited to the experimental situation. In the $p\bar{p}$ case this means jets defined in terms of the natural variables of longitudinal phase space, the pseudo-rapidity $\eta = \ln(\cot(\theta/2))$, the azimuthal angle $\phi$, and the “transverse component” of the energy $E_T = E \sin \theta$. We choose to use the cone algorithm outlined in the so-called “Snowmass Accord.” To a good approximation this algorithm is employed by the CDF Collaboration, although we will see below that there are still some issues to be resolved in practice.

---

1 This can be (half-seriously) contrasted with studies performed with Monte Carlo simulations, where we note that one of the definitions of the noun “simulation” is “the assumption of a false appearance.”
The jet from the cone algorithm is typically defined in terms of the particles \( n \) whose momenta \( p_n \) lie within a cone centered on the jet axis \((\eta_J, \phi_J)\) in pseudo-rapidity \( \eta \) and azimuthal angle \( \phi \), \( \sqrt{(\eta_n - \eta_J)^2 + (\phi_n - \phi_J)^2} < R \). The jet angles \((\eta_J, \phi_J)\) are the averages of the particles’ angles,

\[
\eta_J = \sum_{n \in \text{cone}} p_{T,n} \eta_n / E_{T,J}, \quad \phi_J = \sum_{n \in \text{cone}} p_{T,n} \phi_n / E_{T,J}; \quad E_{T,J} = \sum_{n \in \text{cone}} p_{T,n}.
\]

This process of finding the center and then recalculating the cone is iterated until the cone center matches the jet center \((\eta_J, \phi_J)\).

This definition implies that, for a given hadronic final state, one identifies all jets that satisfy the algorithm (typically with jet \( E_{T,J} \) above some lower threshold \( E_{T,\text{min}} \)). This process can and does lead to situations where individual hadrons are members of more than one jet, i.e. the cones are found to overlap. Hence the cone algorithm must be augmented to handle this situation and we will return to this point below. It will not have a large numerical impact on the jet cross section but will affect the observed internal structure of jets.

### 3 Uncertainties

Recall that the general theoretical structure of the perturbative jet cross section calculation for \( A \+ B \rightarrow \text{jet} + X \) has the schematic form

\[
\frac{d\sigma}{dE_T d\eta}(\eta; E_T, s; \Lambda_{QCD}; \mu; R) \sim \int \int dx_a dx_b d(k_i) G_{i/A}(x_a; \Lambda_{QCD}; \mu) G_{b/B}(x_b; \Lambda_{QCD}; \mu) \times \\
\frac{d\hat{\sigma}}{dk_i}(a + b \rightarrow i; x_a, x_b, s, k_i; \Lambda_{QCD}; \mu) \times \ S_{\text{jet}}(k_i; E_T, \eta; R).
\]

The various components include the parton distribution functions, \( G_{i/I}(x_i; \Lambda_{QCD}; \mu) \), the parton-parton scattering cross section, \( d\hat{\sigma}/dk_i \), and the jet algorithm or “projection” function \( S_{\text{jet}} \) that identifies the jet in the final state partons. Note the explicit dependence on the \textit{a priori} arbitrary theoretical parameter \( \mu \), the factorization/renormalization scale, which is an unavoidable feature of finite order perturbation theory. The inclusive jet cross section evaluated\(^3\) at order \( \alpha_s^3 \) is compared with CDF data\(^1\) in Fig. 1a. Note the outstanding agreement between theory and data over nearly ten orders of magnitude. These results are for \( \sqrt{s} = 1800 \text{ GeV} \) and are averaged over the rapidity range \( 0.1 \leq |\eta| \leq 0.7 \) with a cone of size \( R = 0.7 \). The theoretical parameter \( \mu \) is set to \( E_T/2 \) as discussed below and the HMRS(B) parton distribution functions\(^2\) are employed. This comparison is pursued in more detail in Fig. 1b, where the difference between the data and the theory, scaled by the theory, is exhibited (the reference theoretical result is the dotted line). Unlike Fig. 1a where the full experimental systematic errors are indicated, in Fig. 1b the error bars include only the \( E_T \) dependent systematic uncertainties while the \( E_T \) independent uncertainty is indicated by the dashed lines (\( \sim \pm 20\% \)). The curves correspond to \( \mu = E_T/4 \) (solid) and \( \mu = E_T \) (dashed). (The dot-dashed curve will be discussed below.) It is, in fact, difficult to distinguish the various theoretical curves in the figure and this feature is precisely the point.

From the consideration of Fig. 1b we learn that the absolute agreement between data and theory, at least for \( E_T > 50 \text{ GeV} \), satisfies our 10\% goal (actually better agreement than required by the stated overall normalization uncertainty in the data of order 20\%).
We also see that the theoretical result is reassuringly independent of the arbitrary scale $\mu$, varying by only 10% in the “physically relevant” range $E_T > \mu > E_T/4$. Note that both of these limit values yield cross sections below that for $E_T/2$, which is a local extremum. If we accept this stability in $\mu$ as a reliable measure of the desired 10% theoretical uncertainty, we learn the important fact that the reliability of the order $\alpha_s^3$ degrades dramatically for $E_T$ below 50 GeV as indicated by the divergence of the solid and dashed curves in this regime. It is troubling to note that this behavior essentially scales in $x_T = 2E_T/\sqrt{s}$. Thus this order in perturbation theory is not to be considered reliable (at the 10% level) to describe jet physics at the SSC or LHC at $E_T$ values much below 1000 GeV!

The parton distribution functions, $G_{i/I}(x_i; \Lambda_{QCD}; \mu)$, especially the gluon component, are not precisely known and until recently were a “major” source of uncertainty at the level of 20%. However, the more recent fits[13, 14] exhibit jet cross sections with differences at the 10% level (and good agreement with the HMRS(B) distributions used here).

Since Eq. (2) represents a purely perturbative result, it contains no explicit nonperturbative effects, either from fragmentation smearing or from the underlying event. The former effect, which is intended to characterize how the partons interact coherently to form the final color singlet hadrons, is thought to involve some amount of momentum transfer transverse to the original parton direction. Thus, while the process must conserve overall $E$ and $p_T$, the $E_T$ of the final hadrons can be somewhat redirected from that of the partons. If the characteristic momentum transfer is of order 0.5 GeV, this effect should result in a smearing of angles of the magnitude $\Delta \Theta \leq 0.5 \text{ GeV}/\langle E_{\text{Hadron}} \rangle$. This will be unimportant for energetic hadrons ($E_{\text{Hadron}} > 10$ GeV) and reasonably sized jets ($0.4 < R < 1.0$). The second and, perhaps, more important effect not included in the calculations is the underlying event. This term is intended to describe the soft interactions of the remaining partons in the initial hadrons that, while not participating in the short-distance, large $p_T$ interaction described by Eq. (2), still can contribute final hadrons and thus $E_T$ to the jet of interest. To the extent that such soft interactions generate a fairly uniform distribution of particles in the variables $\eta$ and $\phi$, as in usual minimum bias events, the contribution to the jet is essentially a geometric effect. This is one of the advantages of such a cone definition for the jet. With an observed $E_T$ density, in $\eta, \phi$ units, of order 1 GeV (see below) in minimum bias events, jet sizes characterized by $\pi R^2 \approx \pi (0.7)^2 \approx 1.5$ and a logarithmic derivative for the differential cross section with respect to $E_T$ of order 6 (i.e. $d\sigma/dE_T \propto E_T^{-6}$ in the range of interest) we have $\Delta \sigma/\sigma \leq 9$ GeV/$E_T \leq 10\%$ for $E_T > 100$ GeV. This is essentially the same range of reliability as defined by the perturbative effects discussed earlier. We will return to the issue of the underlying event later.

Finally there is expected to be some uncertainty involving the specific jet algorithm, $S_{\text{jet}}$, itself. While the “Snowmass Accord” was intended to constitute an agreement by both theorists and experimentalists to use the identical jet algorithm, in the event, “real-life physics” is somewhat more complicated. In particular the “Snowmass Accord” does not treat the issue of how overlapping jets identified by the algorithm are “merged” and we will discuss this point below. However, it is important to recognize that the effect of this issue on the inclusive jet cross section is numerically small, within our working limit of 10% (see the dot-dashed curve in Fig. 1b, which is explained below).
4 Jet Shape Dependence and the Jet $E_T$ Profile

Coupled to the increased numerical reliability of the order $\alpha_s^3$ cross section, is the attractive feature that, like the data, the theoretical result depends on the specific details of the jet algorithm. In the present context this implies a dependence on the jet “size” parameter $R$. While such dependence appears only at “lowest order”, in some sense, in the $\alpha_s^3$ calculation, it is of interest to compare the dependence with that observed in the data.[11]

In a careful study of this issue in Ref. [2] two features stand out. First the region of stability in $\mu$, as defined above, is also correlated with the $R$ dependence. At both large $R$ ($R > 1$) and small $R$ ($R < 0.4$), the theoretical cross section exhibits a monotonic dependence on $\mu$, reminiscent of the Born cross section. This feature is indicated by the solid and dashed curves in Fig. 2a showing the inclusive jet cross section versus $R$ with various $\mu$ values (solid = $E_T/2$, long dashed = $E_T/4$, short dashed = $E_T$, the dot-dashed curve will be explained below). Note that the three curves intersect in the region of $R \sim 0.7$ but that the cross section increases monotonically with $\mu$ at small $R$ and decreases at large $R$. Thus, at least at this order of perturbation theory, the theory itself gives a hint as to the optimum value of $R$ at which to compare theory and data, $R \sim 0.7$. Luckily this is just where CDF has been working!

The second point contained in Ref. [2] involves the actual dependence on $R$ of the cross section. As indicated in Fig. 2a the reference value $\mu = E_T/2$ yields a cross section that does not vary as rapidly with $R$ as the data of CDF. In some crude sense the data are suggesting the need for “fatter” jets. Changing to a smaller $\mu$ value (here $\mu = E_T/4$) leads to a larger $\alpha_s$ value, more radiated gluons, “fatter” jets and more rapid $R$ variation.

The $E_T$ distribution within the cone of the jet can be analysed more directly by studying the fractional $E_T$ profile, $F(r, R, E_T)$. Given a sample of jets of transverse energy $E_T$ defined with a cone radius $R$, $F(r, R, E_T)$ is the average fraction of the jets' transverse energy that lies inside an inner cone of radius $r < R$ (concentric with the jet-defining cone). Thus the quantity $1 - F(r, R, E_T)$ describes the fraction of $E_T$ that lies in the annulus between $r$ and $R$. It is this quantity that is most easily calculated in perturbation theory as it avoids the collinear singularities at $r = 0$. The results for $F$ with the three $\mu$ values are plotted in Fig. 2b versus the inner radius $r$ with $R = 1.0$ for $E_T = 100$ GeV and compared to CDF data.[15] (The dot-dashed curve will be explained below.) As with the $R$ dependence discussed above, $F$ is being calculated to lowest nontrivial order and thus exhibits monotonic $\mu$ dependence. While there is crude agreement between theory and experiment, the theory curves are systematically below the data. This situation suggests that the theoretical jets have too large a fraction of their $E_T$ near the edge of the jet ($r \simeq R$).

We have seen that the $R$ dependence of the cross section suggests the importance of higher-order contributions to increase the level of associated radiation, at least near the center of the cone. At the same time our considerations of $F$ suggest that the data favor a reduction of the $E_T$ fraction near the edge of the cone. Although these conclusions seem initially to be contradictory, the likely consistent explanation is based on a detailed but important physical point concerning how the jets are defined. The issue, as mentioned earlier, is that of merging, i.e. how close in angle should two partons be in order to be associated as a single jet. In a real experiment such a situation is presumably realized as two sprays of hadrons, each with a finite angular size that arises from both fragmentation effects and real experimental angular resolution effects. If the angular separation is large enough, there is a valley in the $E_T$ distribution between the two sprays and experimental jet-finding algorithms will tend to recognize this situation as two
distinct jets. Recall that we expect for jets of $E_T > 100$ GeV that the angular extent of fragmentation effects will be small compared to the defined jet cone sizes. However, the theoretical jet algorithm defined in strict adherence to the “Snowmass Accord” will merge two partons into a single jet whenever it is mathematically possible. Thus the limiting configuration with two equal transverse energy partons 2$R$ apart will be counted as a single jet with its cone centered in the “valley” between the two partons. A “real experimental” treatment of this configuration is unlikely to identify it as a single jet. To simulate the experimental algorithm in a simple way we add an extra constraint in our theoretical jet algorithm. When two partons, $a$ and $b$, are separated by more than $R_{sep}$, $R_{ab} = [(\eta_a - \eta_b)^2 + (\phi_a - \phi_b)^2]^{1/2} \geq R_{sep}(\leq 2R)$, we no longer merge them into a single jet. As an example, the results of calculating both the $R$ dependence and the $E_T$ fraction $F$ with $R_{sep} = 1.3R$ and $\mu = E_T/4$ are illustrated by the dot-dashed curves in Figs. 2a and b. Clearly the extra constraint of $R_{sep}$ has ensured that there is approximately the observed fraction of $E_T$ near the edge of the cone while the reduced $\mu$ value has increased the amount of associated radiation near the center of the cone and produced a larger variation with $R$. The specific choice $R_{sep} = 1.3R$ is also in good agreement with the detailed CDF study[10] of this issue. It is important to note that, while these limiting configurations of the partons make important contributions to $F$ for $r \sim R$, they constitute only a small contribution to the jet cross section itself. Hence the cross section is relatively insensitive to the parameter $R_{sep}$, decreasing by $\leq 10\%$ as $R_{sep}$ is reduced from $2R$ to 1.3$R$ with fixed $\mu$ for $E_T = 100$ GeV. This point is illustrated in Fig. 1b by the dot-dashed curve that indicates the small change in the cross section from the reference result.

5 Scaling in Jet Cross Sections

One of the most interesting new measurements from the CDF Collaboration[16] involves the comparison of jet cross sections at two different center-of-mass energies, $\sqrt{s} = 1800$ and 546 GeV. By comparing the two cross sections, multiplied by $E_T^3$ to obtain dimensionless quantities, at fixed $x_T$ values one has a check on the scaling violation as predicted by QCD. The resulting ratio is illustrated in Fig. 3. The data are clearly inconsistent not only with pure scaling (a ratio of unity) but also with the cross section calculations we have discussed up to now. It is important to note that the data and theory at the two different energies agree within the full systematic errors (as we saw in detail above for the 1800 GeV case) but in the ratio much of the systematic uncertainty cancels and the deviation between theory and data appears to be of order 2$\sigma$ at low $x_T$. Note that the region of comparison, $0.1 < x_T < 0.3$, is where the theory was argued above to be reliable to 10%. Also note that, while the individual cross sections are sensitive to the specific choice of parton distribution functions, the ratio is remarkably insensitive to that choice. This point is illustrated by the dashed curve in Fig. 3 corresponding to the quite different parton distributions constructed by Berger and Meng[17]. While there is evidently not a large deviation between data and theory, I suspect that at least one contributing issue has physics interest and I will discuss it briefly here.

The important point is how the data are corrected for the contribution from the underlying event. As suggested earlier it is presumably a good approximation to treat the underlying event as an essentially uniform (in $\eta$ and $\phi$) distribution of $E_T$ with only minimal correlation with the hard scattering process (for earlier discussion of this issue, both experimental and theoretical, see Refs. [18] and [19]). This “splash-in” effect will
contribute to the jet $E_T$ simply on the basis of geometry. The issue here is then to evaluate the level of this background contribution. In the correction of the CDF data\cite{16} this underlying level is determined at each $\sqrt{s}$ by measuring the $E_T$ density in jet events but at $90^\circ$ in $\phi$ to the observed jet direction, the “pedestal” of the jet. This yields an $E_T$ density of about 1.6 GeV/$R^2$ (1.0 GeV/$R^2$) at $\sqrt{s} = 1800$ GeV (546 GeV). This value is to be contrasted with the corresponding $E_T$ density observed in minimum bias events and found to be approximately 0.7 GeV/$R^2$ (0.5 GeV/$R^2$). Although there is some selection bias in the jet trigger process towards underlying events with larger than average local $E_T$ densities, the suggestion here is that what is seen at $90^\circ$ to the jet can be thought of as a fairly standard minimum bias event plus the contribution of bremsstrahlung explicitly associated with the hard scattering, i.e. associated “splash-out”. This latter contribution is meant to be accounted for in the theoretical calculation (the $E_T$ density observed outside of the jet cone in the theoretical calculation is about half of the pedestal height observed experimentally in jet events). Thus to compare theory and data in first approximation, the experimental jet $E_T$ should be corrected for a contribution from an underlying event just equal to a minimum bias event. This conclusion implies that the current data have been over-corrected. A “plausible” scenario is that the jet $E_T$ in the 1800 GeV data have been over-corrected by 1.25 GeV ($\sim(1.6 \text{ GeV} - 0.7 \text{ GeV}) \times \pi R^2$) while for those at 546 GeV the over-correction is 1.0 GeV ($\sim(1.0 \text{ GeV} - 0.5 \text{ GeV}) \times \pi R^2$ plus 0.25 GeV to account for non-perturbative splash-out effects being more important at the lower $E_T$ values). Instead of removing this correction from the data it is easier (for me) to add it to the theoretical calculation. The result is the dot-dashed curve in Fig. 3 where we see that the disagreement is now at about the $1\sigma$ level. While this improvement is perhaps not overwhelming, it is relevant and I believe that the physics issues involved are now more correctly treated, i.e. the underlying event contribution is in reality more like a minimum bias event than the pedestal observed in jet events. (Note that this difference between these two scenarios for the corrections is meant to be spanned by the CDF error bars\cite{16} and this feature presumably explains why switching the central value from one to the other improves the agreement by approximately $1\sigma$.)

6 New Jet Algorithms

With no extra space left (either in the talk or this contribution), I will just note that there has been recent work\cite{20,21} on the question of replacing the cone algorithm with $e^+e^-$ successive combination style algorithms for the study of jets in hadron collisions. While there are positive indications of qualitative improvement (e.g. the merging issue per se is removed), it is not yet clear whether there is a quantitative improvement.

7 Summary

Let us briefly summarize what (I hope) we have learned.

- The theoretical calculations of one (and two) jet cross sections at order $\alpha_s^3$ in perturbation theory are reliable at essentially the 10% level and now allow very precise comparisons with data. These comparisons enhance our confidence in perturbative QCD in the large $E_T$ regime. Unfortunately the calculations suggest much larger higher order contributions for $x_T \leq 0.05$ corresponding to $E_T$ as large as 1 TeV at the SSC (0.4 TeV at the LHC)!
• At this order in perturbation theory the results are most reliable for cone sizes around $R \sim 0.7$. For cones sizes much smaller or much larger than 0.7 higher orders must play an important role.

• The analyses of the $R$ dependence of the cross section and the $E_T$ profile $F$ yield an even more detailed understanding of the structure of jets and suggest that the “merging” issue must be taken into account. These studies may also yield an avenue for attacking the problem of differentiating quark-initiated jets from gluon-initiated jets.

• Further work is urgently required on the issues of scaling violations and the role of the underlying event; of order $\alpha_s^4$ contributions for the study of $E_T \sim M_W$ scale jets at the SSC/LHC; of $e^+ e^-$ jet studies with cone algorithms to check the role of the underlying event; of studies of $e^+ e^-$ style successive combination jet algorithms at hadron colliders.
Acknowledgements

It is a pleasure to acknowledge the essential contributions of my two collaborators, Z. Kunszt and D.E. Soper, to much of the work discussed here. A special thanks also goes to the CDF QCD group, especially J. Huth, N. Wainer and S. Behrends, for repeatedly and patiently explaining their measurements to me. This work was supported in part by the U.S. Department of Energy under grant DE-FG06-91ER-40614 and by the CERN TH Division, who are warmly thanked for their hospitality.

References

[1] The American Heritage Dictionary, Copyright 1986,1987, Houghton Mifflin Company, CD version.

[2] S.D. Ellis, Z. Kunszt and D.E. Soper, Phys. Rev. Lett. 69, 3615 (1992).

[3] S.D. Ellis, Z. Kunszt and D.E. Soper, Phys. Rev. Lett. 69, 1496 (1992); Phys. Rev. Lett. 64, 2121 (1990); Phys. Rev. D40, 2188 (1989); Phys. Rev. Lett. 62, 726 (1989).

[4] See also F. Aversa, M. Greco, P. Chiappetta and J. Ph. Guillet, Phys. Rev. Lett. 65, 401(1990); Z. Phys. C46, 253(1990); Nucl. Phys. B327, 105(1989); Phys. Lett. 211B, 465(1988) and 210B, 225(1988).

[5] See also, W.T. Giele, E.W.N. Glover and D.A. Kosower, preprint CERN-TH-6750/92.

[6] See, for example, the contributions to these Proceedings from E. Bareiro, K. Küster and H. Melanson.

[7] See, for example, the contributions to these Proceedings from S. Catani and P. Burrows.

[8] See, for example, the contribution to these Proceedings from N. Watson and also J. Pumplin, Phys. Rev. D44, 2025 (1991); “Variables for Distinguishing Between Quark Jets and Gluon Jets”, Proceedings of Research Directions For The Decade, Snowmass 1990, Snowmass, July 1990, ed. E.L. Berger (World Scientific, Singapore, 1992), p. 174.

[9] J. Huth, N. Wainer, K. Meier, N. Hadley, F. Aversa, M. Greco, P. Chiappetta, J.Ph. Guillet, Z. Kunszt, S.D. Ellis and D. Soper, “Toward a Standardization of Jet Definition”, Proceedings of Research Directions For The Decade, Snowmass 1990, Snowmass, July 1990, ed. E.L. Berger (World Scientific, Singapore, 1992), p. 134.

[10] CDF Collaboration, F. Abe et al., Phys. Rev. D45, 713 (1992).

[11] See the contribution to these Proceedings from A.M. Zanetti and also F. Abe et al., Phys. Rev. Lett. 68, 1104 (1992).

[12] P.N. Harriman et al., Phys. Rev. D42, 798 (1990).

[13] A.D. Martin, W.J. Stirling and R.G. Roberts, preprint DTP-92-80; Phys. Rev. D47, 867 (1993).

[14] J. Botts et al., preprint FERMILAB-PUB-92-371.
[15] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 70, 713 (1993).
[16] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 70, 1376 (1993).
[17] E.L. Berger and R. Meng, preprint CERN-TH 6785/93.
[18] UA1 Collaboration, C. Albajar et al., Nucl. Phys. B309, 405 (1988).
[19] G. Marchesini and B.R. Webber, Phys. Rev. D38, 3419 (1988).
[20] See the contribution from M.H. Seymour to these Proceedings and also S. Catani, Yu.L. Dokshitzer, M.H. Seymour and B.R. Webber, preprint CERN-TH.6775/93.
[21] S.D. Ellis and D.E. Soper, CERN preprint CERN-TH.6860/93.

Figure captions

Figure 1: a) Inclusive jet cross section versus $E_T$ at $\sqrt{s} = 1800$ GeV with $R = 0.7$ comparing data[11] with the order $\alpha_s^3$ result for HMRS(B) parton distributions with $\mu = E_T/2$; b) Scaled difference from theory result in (a) comparing data and theory with $\mu = E_T/2$ (dots), $\mu = E_T/4$ (solid), $\mu = E_T$ (dashes), and $\mu = E_T/4, R_{sep} = 1.3R$ (dot-dashed, as explained in the text).

Figure 2: a) Inclusive jet cross section data[11] versus the cone size $R$ at $E_T = 100$ GeV and $\sqrt{s} = 1800$ GeV compared with the standard order $\alpha_s^3$ result for HMRS(B) parton distributions with $\mu = E_T/2$ (solid), $\mu = E_T/4$ (long dashed), $\mu = E_T$ (short dashed) and also with $\mu = E_T/4, R_{sep} = 1.3R$ (dot-dashed, as explained in the text); b) $E_T$ fraction $F(r, R, E_T)$ versus the inner radius $r$ for $R = 1.0$, $E_T = 100$ GeV and $\sqrt{s} = 1800$ GeV with the four curves defined as in (a).

Figure 3: Ratio of the scaled cross sections at $\sqrt{s} = 546$ and 1800 GeV versus $x_T$ showing the CDF data[16] (with the overall systematic uncertainty suggested by the dotted box) and the theoretical results for $\mu = E_T/4$ and $R_{sep} = 1.3R$ with HMRS(B) parton distributions (solid curve), Berger-Meng(A) parton distributions[17] (dashed curve) and the situation correcting for the underlying event contribution as discussed in the text (dot-dashed curve).
Figure 1
Figure 2

(a) 

\[ \langle \frac{d\sigma}{dE_T d\eta} \rangle \text{ (nb/GeV)} \]

(b) 

\[ F(r, R, E_T) \]
