Introducing Deeper Nulls and Reduction of Side-Lobe Level in Linear and Non-Uniform Planar Antenna Arrays Using Gravitational Search Algorithm

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Abstract—Array antennas synthesis is one of the most important problems in the optimization of antenna and electromagnetics. In this paper, a recently developed metaheuristic algorithm, known as the Gravitational Search Algorithm (GSA), is employed for the pattern synthesis of linear and non-uniform planar antenna arrays with desired pattern nulls in the interfering directions and minimum side-lobe level (SLL) by position-only optimization. Like other nature-inspired algorithms, GSA is also a population-based method and uses a population of solutions to proceed to a global solution. The results of GSA are validated by comparing them with the results obtained using particle swarm optimization (PSO) and some other algorithms reported in literature for linear and planar array. The side-lobe level and null depth obtained from gravitational search algorithm for planar array are improved up to $-30\text{ dB}$ and $-200\text{ dB}$, respectively. The results reveal the superior performance of GSA to the other techniques for the design of linear and planar antenna arrays.

1. INTRODUCTION

Antenna arrays [1], which contain major developments in directional characteristic rather than a single element, are widely used in personal, commercial and military applications such as radio, television, mobile, radar, and sonar. Hence, designing array antennas with a desired radiation pattern has been extensively studied over the last few decades [2–21]. In these problems, we need to control parameters which depend on the structure of an antenna, and they are required in order to achieve the desired radiation characteristics. Among these parameters, we can refer to the number of elements [2–5], geometry of arrays [6–8], determination of the excitation of elements [9–12] and distance between the elements [13].

In literature, the synthesis of antenna array can be divided into two categories: traditional analytical/deterministic synthesis method and stochastic/evolutionary method. Some of the most important traditional analytical/deterministic methods are used for array antennas, such as Woodward-Lawsen method, Chebyshev method, Schelkunoff method and Levenberg-Marquardt algorithm [14], steepest decent [15], gradient-minimax [16], and intersection approach [17, 18], each of which is used to fulfill various purposes in designing array antennas patterns. Many drawbacks are associated with using deterministic methods such as shortcomings in designing array antennas with complex geometrical structures, consuming more computationally time as the number of the elements in the array increases, and some of these methods are from testing and trying type, having a large gap with the most efficient conditions. In contrast, evolutionary algorithms due to their versatility in an extensive range of problems without major changes in their algorithms have been applied in research work regarding electromagnetics. 

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and array antennas [19–26]. They are capable of radiating pattern synthesis due to many restrictions which are difficult to treat by analytical/deterministic technique.

Genetic algorithm (GA) is one of the first algorithms used in array synthesis and electromagnetics. In [9], the synthesis of a linear array antenna for the purpose of side-lobe level (SLL) reduction with the fixation of half power beam width is presented. GA is used to determine a set of parameters of antenna elements which provide the required radiation pattern. Particle swarm optimization is another algorithm used in this field [20, 25, 27]. Pattern synthesis of linear arrays using Accelerated PSO (APSO) is presented to minimize the SLL with a constrain on beam width for isotropic linear antenna arrays in [7]. Besides these techniques, there are other evolutionary algorithms used in this manner [28–30]. In [6], the design of circular and linear antenna arrays using cuckoo search optimization algorithm is regarded. This algorithm is used to determine the position of the elements in order to obtain a desired radiation pattern. The firefly algorithm is used for the thinning of multiple concentric circular ring arrays in [28]. In [29], Wind Driven Optimization (WDO) is applied to three electromagnetics optimization problems, including the synthesis of a linear antenna array, a double sided magnetic conducting surface, and an E-shaped microstrip patch antenna.

With the development of industry and technology, the pollution of electromagnetics environment is increased and leads to the study of array pattern nulling techniques [31–34]. On the other hand, we need an antenna with desired nulls directions to reduce unintentional interferences. Reducing the effects of intentional interference or jamming is another benefit of null. Also the broad nulls are needed when the direction of arrival of the unwanted interference may vary slightly with time or may not be known precisely. Here, several methods for synthesis of array antenna patterns with prescribed nulls are reviewed. [11] demonstrates the pattern synthesis of planar array with prescribed pattern nulls and side-lobe reduction by optimizing the amplitude-only and position-only of elements using invasive weed optimization (IWO). In [21], pattern synthesis of linear arrays using PSO algorithm is presented to minimize the SLL and imposing null in specific angles by optimizing the position of the elements. In [30], the application of Composite Differential Evolution algorithm, called CoDE, for designing a linear antenna array, having suppressed side lobe and efficient null control in a certain direction is shown.

In the current paper, two types of array antenna are chosen to receive the desired radiation pattern. The first type is a synthesis of the radiation pattern of a symmetric linear array, and the second one is a synthesis of a planar array antenna. The goal is to suppress SLL and steer the single, multiple and broad nulls in desired directions by using Gravitational Search Algorithm (GSA) with position only optimization. To reduce the mutual coupling between elements in a planar array, we define the distance between every two elements as greater than or equal to 0.2λ.

The rest of the paper is arranged as follows. In the second section, GSA is briefly explained, and in the third one, several array antenna syntheses are conducted, including linear array antenna. In Section 4, some simulation results of the planar array antenna are assessed, and finally in Section 5, the summary of the paper is given.

2. GRAVITATIONAL SEARCH ALGORITHM

The Standard GSA is developed by Rashedi et al. in 2009 and uses the Newtonian laws of gravity and motion to find optimum solutions [35]. In the proposed method, they considered an isolated universe in which only gravity force played the role. Each particle in such an isolated universe attracts other particles by a gravitational force as shown in Equation (1).

$$F = \frac{G (M_1M_2)}{R^2}$$

Based on Equation (1), the magnitude of gravitational force (F) between two agents is directly proportional to the product of their masses (M1 and M2), and inversely proportional to their distance squared (r^2). G represents the gravitational constant which changes during the course of time.

According to Newton’s second law, when a force is applied to an object its acceleration, a, depends only on the force and its mass:

$$a = \frac{F}{M}$$

Based on Equations (1) and (2), there is an attracting gravity force among all particles of the universe. Among the particles, the effects of bigger and closer ones are higher than the others.

Before explaining the process of the GSA, we need to be familiar with some key expressions used in this algorithm to avoid misunderstanding. Table 1 shows the key terms in the GSA.

**Table 1.** Terminology used in GSA.

| Term                  | Description                                                                 |
|-----------------------|-----------------------------------------------------------------------------|
| Agent                 | Each individual in an isolated universe containing a value of each optimization variables |
| Fitness               | A value representing the goodness of the solution for each agent            |
| Mass                  | One agent after evaluating its fitness                                      |
| Gravitational force   | Each mass in such isolated universe attracts other mass by a gravitational force |
| Isolated universe     | The space of entire agents in which only gravity force plays a role         |
| Population size       | The number of masses in the universe                                        |

Regarding these expressions, the process of the GSA optimization is explained in the following steps.

1. Initializing parameters.
2. Agents initializing: after defining the required parameters, the position of each agent is determined randomly.
3. Evaluate the fitness of each agent: the fitness function is defined to represent the goodness of the solution.
4. Find the best and worst fitness: For a minimization problem we have:
   \[ best(t) = \min_{j \in [1,\ldots,N]} fit_j(t) \]
   \[ worst(t) = \max_{j \in [1,\ldots,N]} fit_j(t) \]
   where \( fit_i(t) \) represents the fitness value of agent \( i \) at the time of \( t \).
5. Mass calculation: Particles masses are a map from their fitness function so that better solutions have heavier masses. The masses are updated by the following equation:
   \[ m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \]
   where \( worst(t) \) and \( best(t) \) are calculated in the previous step.
6. Gravitational constant: In physics, the gravitational constant is not actually constant, instead it depends on the age of universe and is decreased by elapsing time:
   \[ G(t) = G(t_0) \times \left( \frac{t_0}{t} \right)^\beta, \quad \beta < 1 \]
   where \( G(t) \) is the value of gravitational constant at the time of \( t \), \( G(t_0) \) the value of the gravitational constant at the first cosmic quantum-interval of the time of \( t_0 \), and \( \beta \) a constant less than one.
   In GSA algorithm, the gravitational constant has an exponential behavior and can be stated as follows:
   \[ G(t) = G_0e^{-\alpha \frac{t}{T}} \]
   where \( G_0 = 100, \alpha = 20 \), \( t \) is the number of iterations and \( T \) the total number of iterations.

(i) Calculation of the total force in different directions: At a specific time \( t \), we define the force acting on mass \( i \) from mass \( j \) as follows:
\[
F^d_{ij} = G(t) \times \frac{M_i \times M_j}{R_{ij}(t)} + \epsilon \left( x^d_j(t) - x^d_i(t) \right)
\]  
(8)

where \(F^d_{ij}\) is the force acting on mass \(i\) from mass \(j\) in dimension \(d\); \(M_i\) and \(M_j\) are the masses of particles \(i\) and \(j\) respectively; \(\epsilon\) is a small constant which is used in order to avoid Equation (8) becoming ambiguous when two masses are in the same position; \(R_{ij}(t)\) is the Euclidean distance between two particles which can be calculated as follows:

\[
R_{ij}(t) = |X_i(t) - X_j(t)|_2
\]  
(9)

To give a stochastic characteristic to the GSA algorithm, the total force which acts on agent \(i\) in a dimension \(d\) can be a random weighted sum of \(d\)th components of the forces exerted from other agents:

\[
F_i^d(t) = \sum_{j=1, j \neq i}^{N} rand_j F_{ij}^d(t)
\]  
(10)

where \(F_i^d(t)\) is the total applied force on agent \(i\) in dimension \(d\), \(rand_j\) a random number generator with uniform distribution in the interval \([0, 1]\), and \(N\) the total number of agents acting on agent \(i\).

8. Calculation of acceleration and update agents’ velocity and position: The acceleration of agent \(i\) at the time of \(t\), and in the direction of \(d\) can be stated as follows:

\[
a^d_i(t) = \frac{F_i^d(t)}{M_i}
\]  
(11)

By calculating the acceleration in different directions, the particles velocity and position can be updated as follows:

\[
v^d_i(t+1) = v^d_i(t) + rand \times a^d_i(t)
\]  
(12)

\[
x^d_i(t+1) = x^d_i(t) + v^d_i(t+1)
\]  
(13)

where \(rand\) is a random number generator with uniform distribution in the interval \([0, 1]\). This random number plays an important role in increasing the exploration power of the search algorithm.

9. The termination of the algorithm: The process is repeated at step 3 till either the maximum number of iterations is reached or the fitness criterion is obtained.

Figure 1 pictorially shows a flowchart which can be used for GSA.
3. LINEAR ANTENNA ARRAY SYNTHESIS

Linear array is an appropriate way to achieve desired radiation characteristics. This problem has been extensively studied in the past decades by using various optimizers [36–38]. In this section, GSA is proposed to be used to synthesize two different types of linear antenna arrays, targeting side-lobe suppression and null controlling. Then, an appropriate comparison between the GSA and other common optimizers is provided for each example. The programming has been written in MATLAB language using MATLAB 8.3.0 (R 2014a) version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

Figure 2 shows the geometry of a linear antenna array in which the elements are placed symmetrically along the $x$ axis. The array factor for an even number of isotropic elements is defined as follows:

$$AF(\varphi) = 2 \sum_{n=1}^{N} a_n \cos[kd_n \cos(\varphi) + \varphi_n]$$ (14)

where $k$ is the wave number, and $a_n$, $\varphi_n$ and $d_n$ are the excitation amplitude and phase and the position of the $n$th element of array, respectively. Based on the type of the problem, $a_n$ or $\varphi_n$ or $d_n$ (or some of them) can be defined as the optimization variables.

Figure 2. Geometry of the $2N$-element symmetric linear array placed along the $x$-axis.

By assuming a uniform excitation of amplitude and phase, the array factor is simplified as follows:

$$AF(\varphi) = 2 \sum_{n=1}^{N} \cos[kd_n \cos(\varphi)]$$ (15)

All the optimizations are carried out only using the elements positions as optimizing variables, keeping a uniform excitation with $a_n = 1$ and $\varphi_n = 0$.

3.1. Side-Lobe Suppression

The first example is the synthesis of a 10-element array antenna with the reduction of SLL in regions $[0^\circ \ 82^\circ]$ and $[98^\circ \ 180^\circ]$, but the emphasis is on the first inner side lobe. In this example, no angle is prescribed for null. A similar problem in [21] and [29] is considered which applied PSO and WDO algorithms respectively. The objective function can be defined as follows:

$$fitness_1 = \sum_{i=1}^{M} \frac{1}{\Delta \varphi_i} \int_{\varphi_{l_i}}^{\varphi_{u_i}} |AF(\varphi)|^2 d\varphi$$ (16)

where $[\varphi_{l_i}, \varphi_{u_i}]$s are the regions of SLL suppression, $\Delta \varphi = \varphi_{u_i} - \varphi_{l_i}$, and $M$ is the number of regions.

GSA algorithm uses 40 population size in each search and a maximum of 500 iterations which approximately take 17 sec. In order to show a good comparison, the obtained radiation pattern is compared with PSO and some other evolutionary algorithms as well as conventional array which has a uniform spacing of $\frac{\lambda}{2}$ between neighboring elements, whose results are shown in Figure 3. The element
Figure 3. Normalized array factor of the 10-element linear array obtained using GSA algorithm compared with PSO [21], Comprehensive Learning Particle Swarm Optimization (CLPSO) [42], Ant Colony Optimization (ACO) [37], Spider Monkey Optimization (SMO) [39], Self-adaptive Hybrid Differential Evolution algorithm (SHDE) [38] and conventional array. The region of suppressed SLL is $[98^\circ \ 180^\circ]$.

Table 2. Element position of the 10-element linear array antenna obtained using seven different methods. The numbers are normalized with respect to $\lambda/2$.

| Method     | Position 0 | Position 1 | Position 2 | Position 3 | Position 4 |
|------------|------------|------------|------------|------------|------------|
| Conv. array| $\pm 0.50$ | $\pm 1.50$ | $\pm 2.5$  | $\pm 3.5$  | $\pm 4.5$  |
| PSO        | $\pm 0.503$| $\pm 1.11$ | $\pm 2.13$ | $\pm 3.00$ | $\pm 4.22$ |
| CLPSO      | $\pm 0.443$| $\pm 1.422$| $\pm 2.416$| $\pm 3.670$| $\pm 5.117$|
| ACO        | $\pm 0.50$ | $\pm 1.10$ | $\pm 2.10$ | $\pm 3.10$ | $\pm 4.30$ |
| SMO        | $\pm 0.472$| $\pm 1.056$| $\pm 2.014$| $\pm 2.942$| $\pm 4.252$|
| SHDE       | $\pm 0.430$| $\pm 1.1998$| $\pm 2.122$| $\pm 3.174$| $\pm 4.50$ |
| GSA        | $\pm 0.2684$| $\pm 1.3310$| $\pm 2.0852$| $\pm 3.4942$| $\pm 4.9964$|

Table 3. Comparative results for 10-element linear array between GSA and other optimization algorithm.

| Algorithm | Conventional array | PSO [21] | CLPSO [42] | ACO [37] | SMO [38] | SHDE [38] | GSA |
|-----------|---------------------|----------|------------|----------|----------|-----------|-----|
| SLL (dB)  | $-12.97$            | $-17.4$  | $-19.11$   | $-18.57$ | $-20.49$ | $-19.71$  | $-26.33$ |

positions found by GSA and PSO as well as other algorithms are shown in Table 2 and normalized with respect to $\frac{\lambda}{2}$ where $\lambda$ is the free-space wavelength.

For a better comparison, the obtained SLL from GSA and some other evolutionary algorithms are shown in Table 3. As can be seen from Table 3, the GSA provides further reduction in SLL.

3.2. Null Control

The second example is related to the synthesis of 32-element array antennas with the purpose of SLL suppression and desired null at $99^\circ$. In this example, the cost function is:

$$fitness = M_1 \times \sum_{i=1}^{M} \frac{1}{\Delta \theta_i} \int_{\theta_{li}}^{\theta_{ui}} |AF(\varphi)|^2 d\theta + M_2 \times \sum_{i=1}^{N} |AF(\varphi_k)|^2$$  (17)
Figure 4. Normalized array factor of the 32-element linear array obtained using GSA and Conventional array with prescribed null at 99°.

Table 4. Element position of the 32-element linear array antenna obtained using two different methods. The numbers are normalized with respect to $\lambda/2$.

|          | Conv. array | GSA           |
|----------|-------------|---------------|
| ±0.50    | ±1.50       | ±0.2374 ±1.4578 ±1.8582 ±3.1758 ±3.3854 ±4.4970 ±4.9432 ±6.2236 |
| ±1.50    | ±2.50       | ±1.8582 ±3.1758 ±3.3854 ±4.4970 ±4.9432 ±6.2236 |
| ±2.50    | ±3.50       | ±3.1758 ±3.3854 ±4.4970 ±4.9432 ±6.2236 |
| ±3.50    | ±4.50       | ±3.3854 ±4.4970 ±4.9432 ±6.2236 |
| ±4.50    | ±5.50       | ±4.4970 ±4.9432 ±6.2236 |
| ±5.50    | ±6.50       | ±4.9432 ±6.2236 |
| ±6.50    | ±7.50       | ±6.2236 |

Table 5. Comparative results for 32-element linear array between GSA and other optimization algorithm.

| Algorithm | PSO [21] | CLPSO [42] | CPSO [25] | ILPSO [41] | FA $^1$ [6] | BBO $^2$ [6] | CS $^3$ [40] | DE $^4$ [6] | COA $^5$ [6] | GSA |
|-----------|----------|------------|-----------|-------------|-------------|--------------|--------------|-------------|-------------|-----|
| SLL (dB)  | −18.80   | −22.73     | −23.17    | −23.75      | −22.64      | −16.93       | −22.8        | −22.81      | −23.81      | −20.16 |
| Null at 99° (dB) | −62.12 | −60        | −63.16    | −73         | −60.59      | −61.73       | −62.63       | −60.03      | −70.85      | −84.64 |

$^1$ Firefly Algorithm (FA)
$^2$ Biogeography Based Optimization (BBO)
$^3$ Cuckoo Search algorithm (CS)
$^4$ Differential Evolutionary algorithm (DE)
$^5$ Cuckoo Optimization Algorithm (COA)

where $\phi_k$ is the direction of the $k$th null, $N$ the number of nulls, and $M_1$ and $M_2$ are penalty coefficients set to $10^5$ and $10^3$, respectively. As clear in Equation (17), the first term is related to SLL reduction, and the second one is used for null controlling.

Because of the symmetry, only the positions of 16 elements are used in the optimization. 2000 iterations are considered with 40 primary populations for GSA algorithm with runtime of 103.83 [sec]. The position of this algorithm and the best radiation pattern after 10 separate implementations are shown in Table 4 and Figure 4, respectively. The null depth obtained by GSA is $-84.64$ dB which is lower than the one obtained by other evolutionary algorithms mentioned in Table 5.

The last problem in this section is related to the optimization of the 28-element array antenna with the purpose of imposing three nulls at the angles of 120°, 122.5° and 125°, and the reduction of
138 Hesari and Ebrahimzadeh

Figure 5. Normalized array factor of the 28-element linear array obtained using GSA and conventional array with three desired nulls at 120°, 122.5° and 125° angles.

Table 6. Element position of the 28-element linear array antenna obtained using two different methods. The numbers are normalized with respect to λ/2.

| Conv. array | ±0.50 | ±1.50 | ±2.50 | ±3.50 | ±4.50 | ±5.50 | ±6.50 |
|-------------|-------|-------|-------|-------|-------|-------|-------|
| array       | ±7.50 | ±8.50 | ±9.50 | ±10.50| ±11.50| ±12.50| ±13.50|
| GSA         | ±0.3396 | ±0.9688 | ±1.5404 | ±2.3078 | ±2.8394 | ±3.6142 | ±4.4684 |
|             | ±5.5530 | ±6.4996 | ±7.4446 | ±8.7498 | ±9.9222 | ±11.0100 | ±12.3466 |

Table 7. Comparative results for 28-element linear array between GSA and other optimization algorithm.

| Algorithm | PSO | CLPSO | FA | BBO | CS | DE | COA | GSA |
|-----------|-----|-------|----|-----|----|----|-----|-----|
| SLL       | −13.22 | −21.60 | −18.16 | −17.13 | −15.30 | −18.05 | −21.86 | −26.37 |
| Null at 120° | −52.73 | −60.45 | −88.58 | −63.66 | −62.27 | −60.14 | −60.08 | −73.21 |
| Null at 122.5° | −51.65 | −60.00 | −68.82 | −61.68 | −61.09 | −60.00 | −60.05 | −77.30 |
| Null at 125° | −61.46 | −60.61 | −61.01 | −61.36 | −63.98 | −60.08 | −60.10 | −72.09 |

the SLL. The same synthesis problem was addressed in [21] using PSO, in [42] using CLPSO, in [40] using CS and in [6] using COA. Because of the symmetry, only the positions of 14 elements are used in optimization process. 2000 iterations with 40 population size are used in this problem. It takes 103.43 sec to finish.

Objective function in this example is like the previous one. Figure 5 illustrates the desired radiation pattern by utilizing GSA optimization technique. The corresponding array geometry is given in Table 6. Table 7 shows the performance values of different algorithms for the 28-element antenna. As seen from the results in Table 7, the SLL obtained by GSA is −26.37 dB, and the null depth in 122.5° and 125° are respectively −77.30 dB and −72.09 dB which are the least among the listed results of other algorithms.

4. PLANAR ANTENNA ARRAY SYNTHESIS

In this section, both the GSA and PSO are applied to the problem of synthesizing the far-field radiation patterns of planar array antennas. A planar array can provide deeper null with smaller number of elements and steered electronically in both azimuth and elevation. Figure 6 shows a planar antenna array (PAA) that consists of N elements. Assuming that the elements are located over the x-y plane,
the far-field pattern can be calculated using the following expression:

$$AF(\theta, \phi) = \sum_{i=1}^{N} a_i e^{j\beta_i} e^{j2\pi dx_i \sin \theta \cos \phi} e^{j2\pi dy_i \sin \theta \sin \phi}$$  (18)

where \(i\) is the number of elements, and \(a_i\) and \(\beta_i\) are the amplitude and phase of excitation of the \(i\)th element. \(dx_i\) and \(dy_i\) are the locations of elements in \(x\) and \(y\) directions, respectively. \(\theta\) is the elevation angle with respect to the \(z\)-axis and \(\phi\) the azimuth angle with respect to the \(x\)-axis.

**Figure 6.** Geometry of \(N\) elements planar array.

There are three parameters controlling the \(AF\): the amplitude, phase and position of the elements. In this paper, we achieve the desired radiation pattern by optimizing the position of the elements. The distance between every two elements is constrained to greater than 0.2\(\lambda\), so that the minimum distance is never less than 0.2\(\lambda\), in order to reduce the mutual coupling between elements.

To demonstrate the possibilities of the GSA to steer single, multiple and broad nulls with the imposed directions, four examples of a planar array have been implemented. In all these four examples, a 10-isotropic element with the aperture of 1.5\(\lambda\) \times 1.5\(\lambda\) and a 50 population size as well as maximum of 2000 iterations are attempted.

The objective function, “Cost function” (CF) to be minimized with algorithms for introducing deeper nulls and the relative SLL reduction are given in the following equation:

$$f = \left[ \sum_{k=1}^{K} w_k |AF_k| - ND_k |^2 + w_d |d - d_{\text{min}}|^2 + w_s |SLL - SLL_{\text{max}} |^2 \right]^{1/2}$$  (19)

where \(K\) is the number of desired nulls, \(AF_k\) the value of the array factor for the \(k\)th direction to be suppressed and \(ND_k\) the desired null depth for the \(k\)th null. The weight \(w_k(k = 1, \ldots, K)\) is set to zero (i.e., \(w_k = 0\)) if the condition \(|AF_k| \leq ND_k\) is satisfied. \(d\) and \(d_{\text{min}}\) are the minimum distance and the desired minimum distance between the array elements, respectively. If the condition \(d \geq d_{\text{min}}\) is satisfied, then the weight \(w_d\) is set to zero. The third term in the CF is added to reduce the side lobe up to a desired level. \(SLL\) and \(SLL_{\text{max}}\) are side-lobe level and desired maximum side-lobe level for the overall pattern of the antenna array, respectively. The weight \(w_s\) is set to zero if the obtained \(SLL\) is smaller than \(SLL_{\text{max}}\).

**Table 8.** Particle swarm optimization parameters.

| Name                  | Cognitive rate \((c_1)\) | Social rate \((c_2)\) | Inertial weight \((w)\) | Maximum velocity \((V_{\text{max}})\) |
|-----------------------|--------------------------|-----------------------|-------------------------|-------------------------------------|
| value                 | 2.00                     | 2.00                  | 0.9–0.4                 | 4                                   |
In the first example, the planar array pattern with one symmetric null imposed at 40° is considered. A uniform amplitude excitation \( a_i = 1 \) with no phase differences \( \beta_i = 0 \) is assumed in the array factor for a comparison between GSA and one of the famous meta-heuristic techniques, namely PSO. We test various sets of parameters and also the parameters as other works in the literature and finally compare the results with each other. The achieved results show that choosing parameters the same as other works in the literature is the best choice. Table 8 shows the primary parameters of PSO algorithm.

**Figure 7.** The produced radiation pattern with one symmetric null imposed at 40°.

**Figure 8.** Normalized positions of the elements in wavelengths synthesized for one null pattern using GSA & PSO.

**Figure 9.** The produced radiation pattern with three nulls imposed at 40°, 60° and 80°.
used in this test as it was used in the research work done previously [21, 31].

The corresponding pattern is shown in Figure 7. The maximum null depth obtained by using the GSA is $-201$ dB, which is better than the $-81.86$ dB by the PSO. The positions of the array elements (normalized to lambda) are shown in Figure 8 for imposing only one symmetric null.

In the next example, we set three separate symmetric nulls at $40^\circ$, $60^\circ$ and $80^\circ$. Figure 9 obviously shows that the placement is successful with nulls depth at least $191.7$ dB down the main beam and the SLL value of $-34.72$ dB. The element positions found by GSA and PSO are recorded in Figure 10 and normalized with respect to wavelength.

In the third example, four desired symmetric nulls at $20^\circ$, $40^\circ$, $60^\circ$ and $80^\circ$ are assumed. The average CF convergence rate over 20 trials is given in Figure 11. Figure 12 is concerned with the placing of four separate symmetric nulls at $20^\circ$, $40^\circ$, $60^\circ$ and $80^\circ$. As can be seen in Table 9, GSA has a better function in comparison with PSO algorithm in order to achieve deeper nulls.

![Figure 10](image10.png)  
**Figure 10.** Normalized positions of the elements in wavelengths synthesized for three nulls pattern using GSA & PSO.

![Figure 11](image11.png)  
**Figure 11.** The produced radiation pattern with four symmetric nulls imposed at $20^\circ$, $40^\circ$, $60^\circ$ and $80^\circ$.

| Algorithm | PSO (dB) | GSA (dB) |
|-----------|---------|----------|
| Null at 20$^\circ$ | $-61.3$ | $-190.2$ |
| Null at 40$^\circ$ | $-34.65$ | $-186.6$ |
| Null at 60$^\circ$ | $-62.23$ | $-188.2$ |
| Null at 80$^\circ$ | $-200$ | $-199.1$ |

*Table 9. **Comparitive results for planar array with four nulls between GSA and PSO.***
Figure 12. Normalized positions of the elements in wavelengths synthesized for four nulls pattern using GSA & PSO.

Figure 13. The produced radiation pattern with broad null at 41° to 47°.

Figure 14. Normalized positions of the elements in wavelengths synthesized for broad null pattern using GSA and PSO.

In the last example, the pattern with a broad null sector centered 45° with Δθ = 6° is considered. The resulting pattern is shown in Figure 13. The desired broad null is achieved by GSA with a null depth of −201.5 dB at 41° and 47°, and the wide null is below −60 dB, but with PSO algorithm, the null depth is −68 dB at 41° and 45°, and also the wide null is below −60 dB. As seen in Figure 13, the SLL attained with GSA is better than PSO. The geometry of the array for GSA and PSO is presented in Figure 14.
5. CONCLUSION

The present paper demonstrates the use of GSA optimization in the pattern synthesis of antenna arrays. GSA is a new optimization technique which is inspired by the law of gravity and mass interactions. The GSA method effectively evaluates the design of two types of antenna arrays to generate a radiation pattern with desired characteristics. In the first type, synthesis of linear antenna arrays with deeper nulls and SLL reduction by adjusting the element positions is considered. The numerical results show that the GSA method produces minimum SLL and deeper nulls compared with other algorithms such as PSO, CLPSO, ILPSO, BBO, CS, DE and COA. The GSA method is also employed to create an efficient position, assisting the planar array elements in order to achieve the above-mentioned purposes. The results obtained by this algorithm illustrate that maximum SLL is lower than the PSO. The SLL and null depth obtained from GSA for planar array have been improved up to $-30\,\text{dB}$ and $-200\,\text{dB}$, respectively. The results reveal the superior performance of GSA to the other techniques reported in literature. To reduce the coupling effects between elements in the planar array, we define the desired minimum distance between array elements. In this work, isotropic elements are implemented to synthesize the linear and planar arrays, but it is not solely limited to this case. It can easily be implemented to array antennas with non-isotropic elements and different geometrics for the design of various array patterns.

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