Prognostics for an actuator with the combination of support vector regression and particle filter

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Abstract. The accurate prognostics for actuator malfunctions is a challenging task. Developing reliable prognostic methods is vital for providing reasonable preventive maintenance schedules and preventing unexpected failures. Particle filter has been proved to be a traditional approach to deal with actuator prognostic problems. However, the measurement function in the particle filter algorithm cannot be obtained in the prediction process, this paper presents a hybrid framework combining support vector regression (SVR) and particle filter (PF). The SVR output prediction results are employed as the “measurements” for the subsequent PF algorithm. To accomplish the accurate prognostics for actuator fault of civil aircraft, an improved PF based on Kendall correlation coefficient is put forward to solve the problem of particles’ degeneracy. The experimental results are presented, demonstrating that the SVR-PF hybrid approach has satisfactory performance with better prognostics accuracy and higher fault resolution than traditional approaches.

1 Introduction

Developing a satisfactory fault prognostics technique for the aircraft automatic flight control system (AFCS) is of great importance for the civil aviation safety. Generally, the actuator is strongly coupled, severely nonlinear and electro-hydraulic, which greatly increases the difficulty of prognostics. In particular, with the development of AFCS, the focus of the contradiction is gradually shifting to the actuators. How to ensure the smooth running of the actuator is of great interest in both the aerospace industry and the theoretical research.

The essential fault prognostics approaches can be categorized into model-based methods and data-driven methods. A typical model-based fault prognostics approach generally starts with the internal working mechanism of the object system, and predicts the trend of development by using an analytical model that can reflect the physical laws of the system. However, in most practical applications, the establishment of models from the internal working mechanism of the target system or the component requires a lot of expert knowledge, especially for complex electronic systems (actuator, autopilot, etc.), building

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effective physical models are usually not possible. Data-driven fault prognostics technology does not require the prior knowledge of the target system (analytical model or expert experience), while directly putting the test or state monitoring data as an object to estimate the future evolution of the target system’s trend, thus the drawback of model-based fault prognostics techniques can be avoided. Data-driven methods use several instruments, most of which are originated from artificial intelligence or statistical fields, and consist of neural networks (NN), support vector machine (SVM) and particle filter (PF) etc. SVR is a generalization of SVM in the case of regression; see [1]. SVR can get smaller errors based on limited training samples, and also ensure that small errors are maintained on independent test sets. A direct remaining useful life estimation method based on SVR is proposed in [2]. Recently, nonlinear filtering has become an increasingly active subject. The PF algorithm has been proven to be an effective approach for solving nonlinear and non-Gaussian problems [3]. PF is capable of achieving state estimation that can reflect more detailed and sufficient information of the target component when compared to the SVR’s pure output prediction (Typically, SVR can only embody the fitting of the input and output relations, and can’t learn the state of the system.). Therefore, the prognostics method based on PF has higher resolution than the output prediction based on SVR, i.e., the prognostics results may contain more abundant information. Unfortunately, the measurement function in the PF algorithm might be inaccessible in the long-term prediction process. It doesn’t have the ability to predict the future state accurately in the absence of the latest measurement value. Moreover, a common phenomenon of PF is the sample degeneracy. Resampling is the main technique for solving this problem, which will always cause another noticeable problem: the loss of particle diversity.

The main contributions of this paper lie in two aspects: 1) From the perspective of data-driven approach, a novel framework combining SVR and PF is proposed to accurately estimate the state variables inside the actuator that are not visible (can’t be directly measured), while is capable of achieving fault prognostics based on state variables with higher resolution; 2) An improved method for PF is developed to alleviate the problem of particle degeneracy to achieve better prediction accuracy.

The structure of this paper is organized as follows. In Section 2, the basic principle of PF is introduced. The proposed SVR-PF method is also described in detail in Section 2, where we present an improved PF algorithm. Section 3 illustrates the experiment and results analysis. Conclusions and future works are summarized in Section 4.

2 Prognostic method

2.1 Classical particle filter

Consider a discrete-time nonlinear state-space model with additive noise, given by:

\[ x_k = f(x_{k-1}, v_{k-1}) \]

\[ z_k = h(x_k, w_k) \]

where \( x_k \in \mathbb{R}^{n_x} \) and \( z_k \in \mathbb{R}^{n_z} \) are the state vector and the measurement vector, respectively. \( f(\cdot) \) and \( h(\cdot) \) are nonlinear functions that describe the system and measurement models, respectively. \( v_{k-1} \) is the process noise and \( w_k \) is the measurement noise.

To approximate the true posterior probability density function (PDF) \( p(x_k | z_{1:k}) \), new samples and their corresponding weights are generated from the Sequential Importance
Sampling (SIS) approach. Firstly, drawing samples \( x_k^i \) from the importance density
\( q \left( x_k \mid x_{k-1}, z_{1:k} \right) \) for every \( i = 1, \ldots, N \). Secondly, using the latest measurement \( z_k \), the weights \( \omega_k^i \) are updated by the Bayes rule
\[ \omega_k^i = \omega_{k-1}^i \cdot p \left( z_k \mid x_k^i \right) p \left( x_k^i \mid x_{k-1}^i \right) \left[ q \left( x_k \mid x_{k-1}, z_{1:k} \right) \right]^{-1} \]
(3)
to approximate the target distribution and normalized
\[ \omega_k^i = \omega_k^i \cdot \left( \sum_{j=1}^{N} \omega_k^j \right)^{-1} \]
(4)

The posterior PDF is conventionally represented by a set of weighted samples, denoted by:
\[ p \left( x_k \mid z_{1:k} \right) = \sum_{i=1}^{N} \omega_k^i \delta \left( x_k - x_k^i \right) \]
(5)
where \( \delta \) denotes the Dirac delta function. When the importance density is selected as \( p \left( x_k \mid x_{k-1} \right) \), the formula (3) changes to the form as follows:
\[ \omega_k^i = \omega_{k-1}^i \cdot g_k \left( z_k \mid x_k^i \right) \]
(6)

In addition, the PF monitors the degeneracy phenomenon by using effective sample \( N_{eff} \). Resampling step is triggered if \( N_{eff} \leq N_{thres} \), with \( N_{thres} \) a pre-defined threshold.

### 2.2 Improved particle filter based on Kendall correlation coefficient

Clearly, the standard PF algorithm avoids degeneracy phenomenon by eliminating small weight particles and retaining large weight particles, but this can also cause loss of particle diversity. Choosing the appropriate importance density is a way to solve the problem of samples’ degeneracy. The selection of the importance density function has a very obvious influence on the efficiency of the algorithm and the rate of weight degradation. When the likelihood is too narrow or it is located in the tails of the prior distribution, most particles which extracted from the prior distribution have small weights, and the estimation accuracy can be significantly decreased. Due to the neglect of the latest measurement information, the importance density only contains the model information, failed to overcome the problem of sample weight degeneracy. To conquer the mentioned drawback, a novel technique of sampling from prior importance density is proposed. Then particles are pushed in to the regions of highly likelihood by using the measurement information. We use the Kendall correlation coefficient (KCC) incorporated into the standard particle filter as a criterion to determine whether the particle is similar to the real state and then decide that the particle should be ignored or should be considered in the next step of the process.

#### 2.2.1 Kendall correlation coefficient

The Kendall correlation coefficient (KCC) is a non-parametric statistic used to measure the relevance of two random variables (real-valued vectors), and it is one of the most widely used methods for similarity measurement. More detailed explanations about this topic can be found in [4]. The Kendall correlation coefficient \( \tau \) can be calculated as follows:
\[ \tau = 2 \left( N_c - N_d \right) \cdot N_b \left( N_b - 1 \right)^{-1} \]
(7)
where $-1 \leq \tau \leq 1$, $N_c$ represents the logarithm of elements that satisfy consistency and $N_d$ represents the logarithm of elements that don’t satisfy consistency. The coefficient $\tau$ ranges from $-1$ to $1$ and it is invariant to linear transformations of either variables.

### 2.2.2 KCC-based particle filter

Suppose that a particle’s measurement path is close to the measurement path of the true state, then its path is more likely to be close to the path of the true state. The degree of similarity between the paths of the two measurements can be computed by the coefficient $\tau$. When the degree of similarity is high, it indicates that the particle is closer to the real state. On the contrary, the similarity between the particle and the real state of the system is low. The particles are potentially pushed into the significant areas of the state space depend upon the KCC correction term.

Let $X^* = \{x^*_j(i)\}_{j=k-L+1}^{k}$ represent all the $N$ particles from time $k - L + 1$ to time $k$, where $1 < L \leq k$ is a constant that is pre-determined, and these particles are sampled from the importance density. $Z^* = \{z^*_j(i)\}_{j=k-L+1}^{k}$ represent the corresponding measurement paths of the $N$ particles, where $z^*_j(i)$ can be computed by using the measurement model (2), denoted by $z^*_j(i) = h(x^*_j(i), 0)$.

Let $Z_k = \{z_j\}_{j=k-L+1}^{k}$ represent the measurement path of the system state that is actually measured from time $k - L + 1$ to $k$, and the latest observations are provided by the SVR output prediction. In this case, an exponential function that measures the degree of similarity between the two measured values can be defined as:

$$\beta_k = J\left(\frac{z^*_j(i)}{z_j}\right) = e^{\alpha\tau}$$  \hspace{1cm} (8)

where $\beta_k > 0$ represents the degree of similarity between the measured value’s path of a particle and the measured value path of the true system state from time $k - L + 1$ to time $k$. $\tau$ can be computed by the formula (7). Here, $\alpha > 0$ is a scaling factor, which needs to be set before the algorithm runs. The value $\alpha$ can be adjusted according to the magnitude of the measurement noise to change the relative size of the value $\tau$ corresponding to the different value $\beta_k$, so as to improve the estimation accuracy. For instance, when the measurement noise is small, the value $\alpha$ can be appropriately increased. When the measurement noise is large, the value $\alpha$ can be appropriately reduced. It is noted that we select the particles that is the most similar to the real state as the final particles $x^*_k$ to perform state estimation. Unlike standard particle filter, we pick out the particles that are closest to the true states in the prediction step.

### 2.3 The hybrid framework of SVR-PF

Our aim is to combine the improved SVR and the improved PF algorithm in depth to achieve actuator state prediction. On the one hand, owing to the monitoring data generated from the actuator is available, while the actuator is operating, the proposed SVR-PF method could be used to achieve the prognostics for the actuator. On the other hand, due to the real
measurement value might be inaccessible, the output prediction value could be used as the approximated value of the real value of the actuator. Therefore, the output prediction from the improved SVR time series model could be used as the state measurement equation to improve the precision of prognosis. Meanwhile, the improved PF algorithm could make further fine estimate of the invisible future state of the system, and obtain the uncertainty representation. Firstly, the improved SVR model is established to achieve output prediction of the actuator data, its output is named “SVRpredict(k).” This prediction result is used as the measurement value for the state updating equation of the improved PF algorithm. The final state prediction result is output by the improved PF algorithm, denoted as “PFpredict(k).” Furthermore, the uncertainty representation can be obtained by the improved PF algorithm. If the “PFpredict(k)” reaches the fault threshold xthr from the component maintenance manual (CMM) of the actuator, the fault warning is performed. If the fault threshold is not reached, the model is updated and the forward prediction is continued.

This hybrid framework is used to emphasize the fact that the SVR algorithm and the PF algorithm are complementary. The output of the SVR algorithm is the input of the PF algorithm, which provides the “data excitation” for the subsequent state estimation of the PF algorithm, while the PF algorithm is a further enrichment and refinement of the SVR algorithm, i.e., on the basis of obtained the prediction values of the actuator output, the prediction values of the state variables which are not visible in the actuator are further obtained.

3 Experiment and results analysis

The actuator is a mechanical part that connects the actuator body with the corresponding rudder surface. When the internal oil leakage of the actuator occurs, due to the insufficient of oil supply pressure, the actuator will not carry out normal piston movement. Thus the position of the actuator cylinder is selected as the variable to predict. To fully validate the performance of the designed prognostics approach, we choose an actuator cylinder movement model as the research object, and its parameters are analyzed as the following expression:

\[
\begin{align*}
    x_k &= x_{k-1} + \Delta T \phi(x_{k-1}) + \nu_{k-1} \\
    z_k &= x_k + w_k
\end{align*}
\]

where \( x_k \) denotes the actual actuator cylinder position and

\[
\phi(x_{k-1}) = K_{ci} K(u_{k-1} - x_{k-1}) \left\{ \frac{\Delta P - \nu F_{aero} S^{-1}}{\Delta P_{ref} + K_a [K_{ci} K(u_{k-1} - x_{k-1})]^2 S^{-1}} \right\}^{1/2}
\]

The process noise \( \nu_{k-1} \sim N(0,0.2) \) and measurement noise \( w_k \sim N(0,0.4) \) are assumed to be Gaussian white noise.

The actuator is a fast identification system. According to the actual data sampling rate of the airborne equipment, the airborne data acquisition module (SDCU) extracts a running data at intervals of 250 milliseconds. In addition, in order to improve the computational efficiency and satisfy the requirements of practical applications, we properly control the size of the data set. Therefore, the sampling time interval is set to 250 milliseconds. In other words, four data are collected within one second. The flight control input \( u_k \) is set to a sine function, and its form is \( u_k = A \sin(2 \pi \xi k) \). The experimental time is set to 20 seconds and
the time interval $\Delta T$ between two consecutive measurement values is 250 milliseconds. The oscillation frequency $\omega$ is 1.2 Hz. The rest of the selected initial parameters in the experiments are listed in Table 1. We use the root mean square error (RMSE) and mean absolute percentage error (MAPE) to evaluate the performance of the proposed methods.

**Table 1.** List of parameter values.

| Term | Physical significance | Model value |
|------|-----------------------|-------------|
| $K_{ci}$ | A simple or double slope gain | 2.326 |
| $K$ | Servo control gain | 1.085 |
| $\Delta P$ | Hydraulic pressure delivered to actuator | 3045.79 psi |
| $F_{aero}$ | Aerodynamic force applying on the control surface | $\approx 0$ |
| $K_a$ | Actuator damping coefficient with maximum rod speed | 0.8 |
| $\Delta P_{ref}$ | Constant differential pressure | 2581.67 psi |
| $S$ | Actuator piston surface area | 4.38 sq.in |
| $A$ | Amplitude of the actuator rod oscillation | 16.68 in |
| $\alpha$ | Tuning parameter | 1.645 |

$a$ It is worth noting that psi is an acronym for pounds per square inch.

$b$ $F_{aero}$ is set to zero, which means that the aerodynamic forces exerted on the control surface is negligible.

The performance of the proposed SVR-PF approach will be evaluated, and also compared with the standard PF, the radial basis function particle filter (RBF-PF) [5] and the particle filtering technique combined with a kernel smoothing method (PF-KS) [6]. In the improved PF algorithm, due to the particles with the most similarity of the true state are chosen as the final particles according to the KCC, and the distribution of these particles are adjusted by the improved SVR. Both the particle degeneracy problem and the computational complexity reduction problem have been solved. Therefore, the proposed SVR-PF method can improve the efficiency of calculation and reduce the burden of calculation. We compare the SVR-PF approach with standard PF, RBF-PF, and PF-KS. The sample number is set to be 200 particles. As shown in Figure.1, the introduced SVR-PF derives a satisfactory prediction result. Table 2 gives a comparison of the prediction performance of standard PF, RBF-PF, PF-KS, and SVR-PF in terms of RMSE and MAPE. It can be found from Figure.1 that the tracking performance of the proposed SVR-PF approach is better than other approaches. Therefore, the proposed SVR-PF can be considered as an excellent tool to be applied to the state prediction of the actuator.

**Table 2.** Comparison for the four methods.

| Algorithm   | Time steps | RMSE | MAPE |
|-------------|------------|------|------|
| PF (200)    | 120s       | 5.2713 | 12.83% |
| RBF-PF (200) | 120s     | 4.6942 | 8.71% |
| PF-KS (200)  | 120s       | 2.5211 | 6.04% |
| SVR-PF (200) | 120s      | 1.1875 | 1.32% |
the time interval $T$ between two consecutive measurement values is 250 milliseconds. The oscillation frequency $\omega$ is 1.2Hz. The rest of the selected initial parameters in the experiments are listed in Table 1. We use the root mean square error (RMSE) and mean absolute percentage error (MAPE) to evaluate the performance of the proposed methods.

### Table 1.

| Term | Physical significance | Model value |
|------|-----------------------|-------------|
| $K_{ci}$ | A simple or double slope gain | 2.326 |
| $K_s$ | Servo control gain | 1.085 |
| $p_{act}$ | Hydraulic pressure delivered to actuator | 3045.79 psi |
| $F_{act}$ | Aerodynamic force applying on the controlsurface | $\approx 0$ |
| $K_a$ | Actuator damping coefficient with maximum rods speed | 0.8 |
| $p_{ref}$ | Constant differential pressure | 2581.67 psi |
| $S$ | Actuator piston surface area | 4.38 sq.in |
| $A$ | Amplitude of the actuator rod oscillation | 16.68 in |
| $T$ | Tuning parameter | 1.645 |

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![Fig. 1. Comparison of state tracking capabilities.](image)

### 4 Conclusion

In this paper, starting from a data-driven perspective, we explored a set of new deep hybrid fault prediction method for the actuator. Satisfactory experimental results have been achieved by using the real actuator’s data, and the SVR-PF demonstrated improved prediction capability over the standard PF. The main contributions of this paper are summarized as: 1) A novel state prediction approach of actuator is proposed, which provides a good foundation for multi-step forward prediction; 2) The improved PF algorithm based on Kendall correlation coefficient effectively alleviates the degeneracy problem. Future work is devoted to reducing the computational complexity of the proposed SVR-PF algorithm and further improving the prediction accuracy of the SVR algorithm.

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