Determination of $J/\psi$ Leptonic Branching Fraction via $\psi(2S) \to \pi^+ \pi^- J/\psi$

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1 Institute of High Energy Physics, Beijing 100039, People’s Republic of China
2 California Institute of Technology, Pasadena, California 91125
3 Colorado State University, Fort Collins, Colorado 80523
4 Hangzhou University, Hangzhou 310028, People’s Republic of China
5 Shandong University, Jinan 250100, People’s Republic of China
6 Shanghai Jiaotong University, Shanghai 200030, People’s Republic of China
7 Stanford Linear Accelerator Center, Stanford, California 94309
8 University of Hawaii, Honolulu, Hawaii 96822
9 University of California at Irvine, Irvine, California 92717
10 University of Science and Technology of China, Hefei 230026, People’s Republic of China
11 University of Texas at Dallas, Richardson, Texas 75083-0688
Abstract

A comparison of the rates for $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow l^+l^-$ and $J/\psi \rightarrow$ anything is used to determine the $J/\psi$ leptonic branching fractions. The results are $B(J/\psi \rightarrow e^+e^-) = (5.90 \pm 0.05 \pm 0.10)\%$ and $B(J/\psi \rightarrow \mu^+\mu^-) = (5.84 \pm 0.06 \pm 0.10)\%$, where the first error is statistical and the second is systematic. Assuming lepton universality, the leptonic branching fraction of the $J/\psi$ is $B(J/\psi \rightarrow \ell^+\ell^-) = (5.87 \pm 0.04 \pm 0.09)\%$ per species. This result is used to estimate the QCD scale factor $\Lambda_{\overline{MS}}^{(4)}$ and the strong coupling constant $\alpha_s$.

I. INTRODUCTION

The branching fractions for the leptonic decays $J/\psi \rightarrow e^+e^- (B_e)$ and $\mu^+\mu^- (B_\mu)$ are basic parameters of the $J/\psi$ resonance. They can be used to determine the strong coupling constant $\alpha_s$ or, equivalently, the fundamental scale parameter of QCD, $\Lambda_{\overline{MS}}$. The ratio $B_e/B_\mu$ provides a test of lepton universality. In addition, these branching fractions are used to determine the total number of $J/\psi$ events in a wide variety of measurements that take advantage of the clean experimental $J/\psi \rightarrow \ell^+\ell^- (\ell = e$ or $\mu)$ signature.

The first reported measurement of the leptonic $J/\psi$ branching fractions has a precision of about 15% and is based on an energy scan across the resonance performed by the Mark-I group [2]. A subsequent measurement by the Mark-III group [3] is based on a comparison of the rates for $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow \ell^+\ell^-$ and $J/\psi \rightarrow$ anything and is, thus, independent of the luminosity determination. The Mark-III measurement has a precision of 4% and is about one (Mark I) standard deviation below the Mark-I result. More recently, BES performed an energy scan measurement and obtained results in good agreement with Mark-III, but with larger errors [4]. In this paper, we report the results of a measurement of the leptonic branching fractions using a sample of $\psi(2S)$ decays measured in the BES detector at the BEPC storage ring. We apply a technique similar to that used by the Mark-III group to a larger data sample.

II. THE BES DETECTOR

The Beijing Electron Spectrometer, BES, is a conventional cylindrical magnetic detector that is coaxial with the BEPC colliding $e^+e^-$ beams. It is described in detail in Ref. [5]. A four-layer central drift chamber (CDC) surrounding the beampipe provides trigger information. Outside the CDC, the forty-layer main drift chamber (MDC) provides tracking and energy-loss ($dE/dx$) information on charged tracks over 85% of the total solid angle. The momentum resolution is $\sigma_p/p = 0.017\sqrt{1+p^2}$ (p in GeV/c), and the $dE/dx$ resolution for hadron tracks for this data sample is $\sim 9\%$. An array of 48 scintillation counters surrounding the MDC provides measurements of the time-of-flight (TOF) of charged tracks with a resolution of $\sim 450$ ps for hadrons. Outside the TOF system, a 12 radiation length lead-gas
barrel shower counter (BSC), operating in self-quenching streamer mode, measures the energies of electrons and photons over 80% of the total solid angle. The energy resolution is \( \sigma_E/E = 0.22/\sqrt{E} \) (E in GeV). Surrounding the BSC is a solenoidal magnet that provides a 0.4 Tesla magnetic field in the central tracking region of the detector. Three double layers of proportional chambers instrument the magnet flux return (MUID) and are used to identify muons of momentum greater than 0.5 GeV/c. Endcap time-of-flight and shower counters extend coverage to the forward and backward regions.

III. TECHNIQUE

Our measurement is based on a data sample corresponding to an integrated luminosity of about 6.1 pb\(^{-1}\) accumulated at the \( \psi(2S) \) resonance. The \( \psi(2S) \) is a copious source of \( J/\psi \) decays: the branching fraction \( \psi(2S) \to \pi^+\pi^- J/\psi = 0.324 \pm 0.026 \) \cite{1}, is the largest single \( \psi(2S) \) decay channel. We determine the \( J/\psi \) leptonic branching fraction from a comparison of the exclusive and inclusive processes:

\[
\psi(2S) \to \pi^+\pi^- J/\psi \to \text{l}^+\text{l}^- \quad (I) \\
\text{and} \quad \psi(2S) \to \text{anything} \quad (II)
\]

The \( J/\psi \) leptonic branching fraction is determined from the relation

\[
B(J/\psi \to l^+l^-) = \frac{N_{\ell}^{\text{obs}}}{N_{J/\psi}^{\text{obs}}} \frac{\varepsilon_{\ell}}{\varepsilon_{J/\psi}},
\]

where \( N_{\ell}^{\text{obs}} \) and \( N_{J/\psi}^{\text{obs}} \) are observed numbers of events for processes I and II, and \( \varepsilon_{\ell} \) and \( \varepsilon_{J/\psi} \) are the respective acceptances.

IV. EVENT SELECTION

For both processes I and II, we use only runs of good quality. We require at least one pair of oppositely charged candidate pion tracks that each satisfy the following criteria:

1. \( P_\pi < 0.5 \text{ GeV}/c \), where \( P_\pi \) is the pion momentum.
2. \( P_{\pi xy} > 0.1 \text{ GeV}/c \), where \( P_{\pi xy} \) is the momentum of the pion transverse to the beam direction. This removes tracks that circle in the Main Drift Chamber.
3. \( |\cos \theta_\pi| < 0.75 \). Here \( \theta_\pi \) is the polar angle of the \( \pi \) in the laboratory system.
4. \( \cos \theta_{\pi\pi} < 0.9 \). \( \theta_{\pi\pi} \) is the laboratory angle between the \( \pi^+ \) and \( \pi^- \). This cut is used to eliminate contamination from misidentified \( e^+e^- \) pairs from \( \gamma \) conversions, as shown in Fig. \( \text{[4]} \).

The invariant mass recoiling against the candidate \( \pi^+\pi^- \) pair, \( m_{\pi^+\pi^-}^{\text{recoil}} = [(m_{\psi(2S)} - E_{\pi^+} - E_{\pi^-})^2 - (P_{\pi^+} + P_{\pi^-})^2]^{1/2} \), is required to be in the range \( 3.0 \leq m_{\pi^+\pi^-}^{\text{recoil}} \leq 3.2 \text{ GeV}/c^2 \).
FIG. 1. The distribution of the cosine of the angle between the $\pi^+$ and $\pi^-$, $\cos \theta_{\pi\pi}$, in $\psi' \to \pi^+\pi^- J/\psi$, $J/\psi \to e^+e^-$ events (dots with error bars) compared with the distribution of Monte Carlo data (histogram). The data has background near $\cos \theta_{\pi\pi} = 1$.

A. $J/\psi \to \ell^+\ell^-$

For leptonic decay candidate events (process-1), the number of charged tracks is required to be at least four with a 4-track combination of net charge zero [7]. Lepton pair candidates must satisfy the following selection criteria:

1. $P_\ell > 0.5$ GeV/c. Here $P_\ell$ is the three-momenta of the candidate lepton track.

2. $P_{\ell^+} > 1.3$ GeV/c or $P_{\ell^-} > 1.3$ GeV/c or $(P_{\ell^+} + P_{\ell^-}) > 2.4$ GeV/c. This cut selects events consistent with $J/\psi$ decay, while rejecting background.

3. $|\cos \theta_\ell| < 0.75$, $|\cos \theta_\mu| < 0.60$. Here $\theta_\ell$ and $\theta_\mu$ are the polar angles of the electron and muon, respectively. This cut ensures that electrons are contained in the BSC and muons in the MUID system.

4. $\cos \theta_{\ell^+\ell^-}^{cm} < -0.975$, where $\theta_{\ell^+\ell^-}^{cm}$ is the angle between the two leptons in the rest frame of the $J/\psi$.

5. For $e^+e^-$ candidate pairs: $SCE_+ > 0.6$ GeV/c, where $SCE$ is the energy deposited in the BSC, or, if one of the tracks goes through a BSC rib or has $P_\ell < 0.8$ GeV/c, the $dE/dx$ information of both tracks in the MDC must be consistent with that expected for electrons. The rib region of the BSC is not used because the Monte Carlo does not model the energy deposition well in this region.

6. For $\mu^+\mu^-$ pair candidates at least one track must have $N^{\text{hit}} > 1$, where $N^{\text{hit}}$ is the number of MUID layers with matched hits and ranges from 0 to 3. If only one track is identified in this fashion, then the invariant mass of the $\mu\mu$ pair must also be within 250 MeV/c$^2$ of the $J/\psi$ mass.

Fig. 2a shows the $m_{\pi^+\pi^-}^{\text{ recoil}}$ distribution for the $\psi' \to \pi^+\pi^- J/\psi$, $J/\psi \to l^+l^-$ events.
FIG. 2.  a) Number of events versus $m_{\pi^+\pi^-}^{\text{recoil}}$, the mass recoiling against the two $\pi$'s, for $\psi(2S) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow l^+l^-$ events. The histogram is data, and the smooth curve is a BWG (“signal”) plus a third order polynomial (“background”).  b) Number of events versus $m_{\pi^+\pi^-}^{\text{recoil}}$ for inclusive events. The histogram is data, and the smooth curve is a BWG (signal) with parameters determined from a) plus a fourth order polynomial (background).

B. Fitting

The number of process-II events ($J/\psi \rightarrow$ anything) is determined from a fit to the $\pi^+\pi^-$ recoil mass spectrum, using a $J/\psi$ line shape that is determined from the recoil mass spectrum for the leptonic decays (Fig. 2a). This is fit with a Breit-Wigner function folded with a Gaussian (BWG) and a third order background polynomial [8]. The parameters thus obtained for the BWG fit are then used together with a fourth order polynomial background to fit the $\pi^+\pi^-$ recoil mass for the inclusive $J/\psi$ decays, as shown in Fig. 2b. The resulting number of inclusive decays is $N_{J}^{\text{obs}} = 530423 \pm 1270$, where the error is the statistical uncertainty combined with the uncertainty in the fitting procedure. The same BWG parameters and a third order background are used to obtain the number of $e^+e^-$ and $\mu^+\mu^-$ events separately and yield a total of $18118 \pm 150 J/\psi \rightarrow e^+e^-$ events and $14611 \pm 134 J/\psi \rightarrow \mu^+\mu^-$ events. The results are summarized in Table I.

| $N_{\text{obs}}$ | $J/\psi \rightarrow$ anything | $J/\psi \rightarrow e^+e^-$ | $J/\psi \rightarrow \mu^+\mu^-$ |
|------------------|-------------------------------|-----------------------------|-------------------------------|
| $\epsilon_{\text{MC}}$ | 530423 $\pm$ 1270 | 18118 $\pm$ 150 | 14611 $\pm$ 134 |
| $N_{\text{obs}}/\epsilon_{\text{MC}}$ | 44.67% | 25.85% | 21.07% |

TABLE I. Summary of Results.
V. ACCEPTANCE

The acceptances are obtained from Monte Carlo simulations. According to ref. [4], the orbital angular momenta between the \( \pi^+\pi^- \) system and the \( J/\psi \), as well as that between the \( \pi^+ \) and \( \pi^- \) is zero, and the \( \pi^+\pi^- \) mass, \( m_X \), distribution is

\[
\frac{d\sigma}{dm_X} \propto (\text{Phase Space}) \times (M_X^2 - 4m_{\pi}^2)^2
\]

\[
\propto (M_X^2 - 4m_{\pi}^2)^2 (M_{J/\psi}^2 - 4m_{J/\psi}^2)^{1/2} \times
\]

\[
[(M_{\psi(2S)}^2 - m_{J/\psi}^2 - m_X^2)^2 - 4m_{J/\psi}^2m_X^2]^{1/2}.
\]

We generate process-I as sequential two-body decays \( \psi(2S) \rightarrow X + J/\psi \), \( X \rightarrow \pi^+\pi^- \), and \( J/\psi \rightarrow l^+l^- \). Isotropic angular distributions are used for the \( J/\psi \) in the laboratory and for the charged pions in the \( X \) rest frame [10]. Leptons are generated with a \( 1 + \cos^2 \theta \) angular distribution in the \( J/\psi \) rest frame and with order \( \alpha^3 \) final state radiative corrections [11]. The \( \pi^+ \) and \( \pi^- \) decay in the detector according to the PDG [6] life time and branching ratios. Initial state radiation is not included in the \( \psi(2S) \) generation for data taken at the \( \psi(2S) \) peak energy. See Fig. 3 for a comparison of the \( m_X \) distribution for data and Monte Carlo generated data.

![FIG. 3. Number of events versus \( m_X \). The points with error bars are data, and the histogram is Monte Carlo data.](image)

Monte Carlo samples of about eight times the number of events produced in our data samples via process-I are generated. After application of the same selection criteria and fitting procedure as used for the data, we get \( \varepsilon_e = 25.85\% \) and \( \varepsilon_\mu = 21.07\% \).

For process-II, the \( \pi\pi \) acceptance, \( \varepsilon_{J/\psi} \), depends on the charged particle multiplicity produced in the \( J/\psi \) decay. The acceptances obtained from generating different multiplicity \( J/\psi \) decay events are listed in Table II. Also shown is the multiplicity distribution obtained from our data [12]. Using this distribution and the acceptances, we obtain \( \varepsilon_{J/\psi} = (44.67 \pm 0.20)\% \), where the error comes from varying the multiplicity values over the range of values.
reported by other experiments [13], as well as the Monte Carlo statistical uncertainties. The acceptance $\epsilon_{J/\psi}$ is not sensitive to the number of $\pi^0$'s or $K^0$'s accompanying the charged $\pi$'s in the $J/\psi$ decay channel or to replacing the charged $\pi$'s with charged kaons.

TABLE II. Acceptance (ππ) for different charged multiplicities.

| Charged Multiplicity | MC decay channel | Percent of $J/\psi$ | Acceptance (%) |
|----------------------|------------------|----------------------|----------------|
| 0                    | all neutrals     | 0.5                  | 48.34          |
| 2                    | $\pi^+\pi^-\pi^0$ | 35.6                | 46.47          |
| 4                    | $2(\pi^+\pi^-)$ | 40.9                | 44.26          |
| 6                    | $3(\pi^+\pi^-)$ | 17.7                | 43.03          |
| 8                    | $4(\pi^+\pi^-)$ | 5.3                 | 41.32          |

VI. ERROR ESTIMATION

The statistical branching fraction errors, propagated from the statistical errors on $N_{l}^{obs}$ and $N_{\mu}^{obs}$, are $\sigma_{B_e} = 0.0005$ and $\sigma_{B_{\mu}} = 0.0006$.

The effect of changing cuts has been studied. The results are shown in Table III. Other systematic contributions come from the acceptance uncertainties, particle identification (PID) uncertainties, and the fitting method. For $ee$ events, electron particle identification backgrounds and efficiencies can be measured by comparing the BSC results and the $dE/dx$ information from the MDC for events outside the BSC ribs. The backgrounds found and the difference in efficiency between that determined from the data and that predicted by the Monte Carlo are small and are used in the estimate of the electron PID systematic error. The PID systematic error for the $\mu\mu$ events is obtained by determining the background allowed by the $\mu\mu$ invariant mass cut for events where only one $\mu$ track is identified by the MUID system and the estimated efficiency of this cut. The systematic error associated with the fitting procedure is determined by using an alternative fitting method. The total systematic error is taken as the sum in quadrature of all the individual systematic errors. The relative systematic errors on both the $J/\psi \rightarrow e^+e^-$ and $\mu^+\mu^-$ branching fractions are 1.7%.

Contamination from hadronic events has been checked by using a sample of kinematically selected $\psi(2S) \rightarrow \pi^+\pi^-J/\psi$, $J/\psi \rightarrow \rho\pi$ events. The number of these events satisfying the cuts used in this analysis was negligible. In addition, a simulation of the potential background process $\psi(2S) \rightarrow \eta J/\psi$ with $\eta$ going to $\pi^+\pi^-\pi^0$ or $\pi^+\pi^-\gamma$ indicates a negligible contribution.
TABLE III. Branching Ratio Systematic Errors (%) 

| Variable | Variation | $B(\text{ee})$ | $B(\text{\(\mu\mu\)})$ |
|----------|-----------|----------------|-----------------|
| $|\cos\theta_{\pi}|$ cut | $0.75 \rightarrow 0.70$ | 0.21 | 0.74 |
| $\cos\theta_{\pi\pi}$ cut | $0.9 \rightarrow 0.85$ | 0.07 | 0.07 |
| $P_{\pi xy}$ cut | turn off | 0.46 | 0.56 |
| $|\cos\theta_{\mu}|$ cut | $0.6 \rightarrow 0.65$ | 0.05 | 0.23 |
| $|\cos\theta_{\ell}|$ cut | $0.75 \rightarrow 0.7$ | 0.19 | 0.10 |
| $\cos\theta_{\ell\ell}$ cuts | turn off | 0.46 | 0.28 |
| SCE cut | $0.6 \rightarrow 0.7 \text{ GeV}$ | 0.18 |
| $P_+^+$ or $P_-^-$ cut | $>1.3 \rightarrow 1.4 \text{ GeV/c}$ | 0.30 | 0.01 |
| $P_+^+$ and $P_-^-$ cut | $>0.5 \rightarrow 0.8 \text{ GeV/c}$ | 0.89 | 0.09 |
| Use only best tracks | | 0.14 | 0.43 |
| Fitting Method | | 0.85 | 1.21 |
| Efficiency Uncertainty | | 0.41 | 0.45 |
| PID Uncertainties | | 0.84 | 0.17 |
| Combined | | 1.69 | 1.71 |

VII. RESULTS AND DISCUSSION

The numbers of events obtained are summarized in Table I [14], and the final results for the branching fractions are:

$$B(J/\psi \rightarrow e^+e^-) = (5.90 \pm 0.05 \pm 0.10)\%$$

and

$$B(J/\psi \rightarrow \mu^+\mu^-) = (5.84 \pm 0.06 \pm 0.10)\%.$$  

The close equality of $B_\mu$ and $B_e$ is a verification of $e$-$\mu$ universality:

$$\frac{B(J/\psi \rightarrow e^+e^-)}{B(J/\psi \rightarrow \mu^+\mu^-)} = 1.011 \pm 0.013 \pm 0.016.$$  

Assuming $B_\mu = B_e$ [14], we find a combined leptonic branching fraction of

$$B(J/\psi \rightarrow \ell^+\ell^-) = (5.87 \pm 0.04 \pm 0.09)\%$$

Our results are compared with previous experiments in Table IV. They are consistent with and improve on the precision of the MARK-III measurement [3]. They are also consistent with BES results determined from $e^+e^-$ cross section measurements in the vicinity of $J/\psi$ resonance [4]. If we combine the values of $B(ll)$, we obtain a new world average

$$B(ll) = (5.894 \pm 0.086)\%,$$

which has an error about half that in the 1996 PDG [3].
The experimental ratio of the quarkonium annihilation rates

\[
R_2 = \frac{\Gamma(\text{quarkonium} \to ggg)}{\Gamma(\text{quarkonium} \to \mu^+\mu^-)}
\]

allows the determination of the strong coupling constant \(\alpha_s\) or, equivalently, the QCD scale parameter \(\Lambda_{MS}^{(n_f)}\). Following the notation of ref. [10], one obtains

\[
(1 + R_1)R_2 = \left[1 - 2B(J/\psi \to l^+l^-) - B(J/\psi \to \gamma^* \to q\bar{q}) - B(J/\psi \to \gamma\eta_c)\right]/B(J/\psi \to l^+l^-)
\]

where,

\[
R_1 = \frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16\alpha}{5\alpha_s}(1 - 3.0\frac{\alpha_s}{\pi}),
\]

\[
R_2 = \frac{\Gamma(J/\psi \to ggg)}{\Gamma(J/\psi \to l^+l^-)} = \frac{5(\pi^2 - 9)\alpha_s^3}{18\pi\alpha^2}(1 + 1.59\frac{\alpha_s}{\pi})\gamma_{\psi},
\]

and

\[
\gamma_{\psi} = \frac{\Gamma(c\bar{c} \to ggg)}{\Gamma_0(c\bar{c} \to ggg)} = 0.31 \pm 0.03.
\]

The factor \(\gamma_{\psi}\) is the reduction factor of the three gluon decay caused by the finite size effect in the matrix element. For \(R_1\), we use the theoretically calculated rather than the measured value because of the large error associated with the latter [14].

Combining the above \(J/\psi\) decay parameters and the values for \(B(J/\psi \to \gamma^* \to q\bar{q})\) and \(B(J/\psi \to \gamma\eta_c)\) listed in PDG [3], we obtain

\[
\alpha_s(m_c = 1.5 GeV/c^2) = 0.28 \pm 0.01 \quad \text{and} \quad \Lambda_{MS}^{(4)} = (209 \pm 21) \text{MeV},
\]

where only the experimental errors are included. Our results are in good agreement with the PDG [3] values.

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‡ Present address: Beijing University, Beijing, China.
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[7] Approximately 4% of the events have more than four prongs. These are usually due to very low energy \( \pi \) track being reconstructed into two tracks. The Monte Carlo does not model this effect well, so it is necessary to include these events, even though the \( m^{\text{recoil}}_{\pi^+\pi^-} \) distribution is not as clean for these events. We rely on our fitting procedure to extract the signal.
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[12] For process-II, \( n^{\text{obs}}_i \), the total number of events observed with charged multiplicity \( i \) is related to the \( N^{\text{prod}}_k \), the number generated with charged multiplicity \( k \), by the relation

\[
n^{\text{obs}}_i = \sum_k \varepsilon_{ki} N^{\text{prod}}_k
\]

with \( k = 0, 2, 4, \ldots \), \( i = 0, 1, 2, 3, \ldots \),

where the matrix element \( \varepsilon_{ki} \) is the probability of an observed \( i \)-prong event coming from a \( k \)-type event. Summing over \( i \) and using

\[
\frac{n^{\text{obs}}}{N^{\text{prod}}} = \varepsilon_{J/\psi},
\]

we have

\[
\varepsilon_{J/\psi} = \sum_k \varepsilon_k \omega_k
\]

where \( \varepsilon_k \) is the acceptance for and \( \omega_k \) the fraction of multiplicity-\( k \) events. By solving the overdetermined equations above, we estimated \( J/\psi \) charged-track-multiplicity distribution shown in Table [11].
Using variations like those in Table III, we can determine the systematic errors associated with $N_{J/\psi}$. We obtain $N_{J/\psi} = (1.188 \pm 0.003 \pm 0.016) \times 10^6$. Using the branching ratio of $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ given by [6], the produced number of $\psi(2S)$ events corresponding to the data used in this analysis is estimated to be $(3.67 \pm 0.01 \pm 0.30) \times 10^6$. Other analyses may use, in addition, runs of fair quality. Using good and fair quality runs, yields $(1.227 \pm 0.003 \pm 0.017) \times 10^6 J/\psi$ events and $(3.79 \pm 0.01 \pm 0.31) \times 10^6 \psi(2S)$ events.

The difference in the branching ratios due to the difference in phase space for the two modes is small compared to the experimental errors.

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[14] Using variations like those in Table III, we can determine the systematic errors associated with $N_{J/\psi}$. We obtain $N_{J/\psi} = (1.188 \pm 0.003 \pm 0.016) \times 10^6$. Using the branching ratio of $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ given by [6], the produced number of $\psi(2S)$ events corresponding to the data used in this analysis is estimated to be $(3.67 \pm 0.01 \pm 0.30) \times 10^6$. Other analyses may use, in addition, runs of fair quality. Using good and fair quality runs, yields $(1.227 \pm 0.003 \pm 0.017) \times 10^6 J/\psi$ events and $(3.79 \pm 0.01 \pm 0.31) \times 10^6 \psi(2S)$ events.
[15] The difference in the branching ratios due to the difference in phase space for the two modes is small compared to the experimental errors.
[16] H.C. Chiang et al., Phys.Lett. B324 (1994) 482
[17] D.L. Scharre et al., Phys.Rev. D23 (1981) 43