Does the weak trace show the past of a quantum particle?

Jonte R. Hance,1 John Rarity,1 and James Ladyman2

1Quantum Engineering Technology Laboratories, Department of Electrical and Electronic Engineering, University of Bristol, Woodland Road, Bristol, BS8 1US, UK
2Department of Philosophy, University of Bristol, Colham House, Bristol, BS6 6JL, UK

We investigate the weak trace approach to determining the path of a quantum particle. Specifically, obtaining the weak value of the position operator necessarily perturbs the system away from the situation it is claimed to describe (e.g. unbalancing a balanced interferometer and so allowing light to leak through), showing weak coupling and no coupling are not equivalent. Despite appeals to the ubiquity of weak coupling between particle and environment, these weak values only give the first-order term in this coupling, suggesting the approach neglects relevant information. Furthermore, there is no reason to associate non-zero weak values of the spatial projection operator with the classical idea of ‘particle presence’, especially when the situation does not have features associated with the classical idea of a particle being present (e.g., travelling on a continuous path, or still being present on coarse-grainings of its location).

I. INTRODUCTION

According to the weak trace approach to particle presence, particles leave traces along the paths they traverse, due to interaction with the environment. The approach claims such a trace is left even when this interaction with the environment is vanishing. A non-zero weak value for the spatial projection operator for a given particle along a given path supposedly indicates the particle left a trace along the path, in this vanishing limit of interaction (a weak trace). Therefore, such a non-zero weak value is claimed to show that particle travelled along that path. Like all weak values, these spatial weak values are obtained via weak measurement and pre- and post-selection.

The weak trace approach further claims evaluating this trace, by identifying these non-zero weak values, lets us see where a quantum particle has been, even when this would normally be impossible due to the pre- and post-selected states not being eigenstates of these spatial projection operators [3, 4]. Therefore, he suggests the weak trace approach gives us more information about the particle than von Neumann measurement of this spatial projection operator would allow. This weak trace approach even allows discontinuous particle trajectories, hinting at a mechanism by which seemingly disconnected events can affect one another (such as in counterfactual communication [5]).

The weak trace approach has been controversial [6–15]. In this paper, we analyse how one obtains the weak value of an operator, then give a case where this weak value of the spatial projection operator is unexpectedly non-zero (and so, by the weak trace approach, a particle has left an unexpected trace in its environment). We then point out three major problems with the weak trace approach.

First, we show that the weak trace approach to the path of a quantum particle is not consistent. We do this by showing that even weak measurements disturb a system, so any approach relying on such a perturbation to determine the location of a quantum particle will only describe this disturbed system, not a hypothetical undisturbed state. We highlight this using the case of a balanced interferometer tuned to have destructive interference (i.e. no light exiting at its dark port) where such a perturbation changes the nature of the system. The unperturbed state is vacuum, while the perturbed state has light present. While the measurement effect can be made arbitrarily small, this is not the same as removing it entirely. Further, we show Peleg and Vaidman’s attempt to respond to this line of argument [16], by saying there are no completely unperturbed systems as weak interactions are ubiquitous in nature, fails to vindicate the use of these non-zero weak values as a criterion for particle presence.

Secondly, we show that, even assuming no disturbance, there is no reason to associate only the element of the trace given by the non-zero weak value of the spatial projection operator (but not elements of the trace given by higher-order powers of the coupling constant) with the classical idea of ‘particle presence’. Indeed, in some situations, just taking the element of the trace given by the weak value (the element proportional to the first-order term in the expansion of the coupling) gives features inconsistent with classical ideas associated with a particle being present (e.g., giving discontinuous particle trajectories, or particles appearing absent when we coarse-grain their location).

Finally, we show the weak trace approach to the path of a particle is not a useful tool - it does not imply any new physics beyond standard quantum theory [17], to explain the causes of counterintuitive quantum effects (or even the paradoxes the approach itself creates). It simply assumes the particle was present wherever these traces exist. Hence it adds a claim of particle presence but contributes nothing testable to our ontology.
II. THE WEAK TRACE APPROACH

We first go over the derivation of a weak value of an operator, and how it is argued this relates to the trace left by a particle on its environment.

Aharonov et al defined the weak value $O_w$ of an operator $\hat{O}$, where

$$O_w = \langle \hat{O} \rangle_w = \frac{\langle \psi_f | \hat{O} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} \tag{1}$$

As this weak value increases as $\langle \psi_f | \psi_i \rangle$ goes to zero, weak value protocols are used in metrology to amplify signals from delicate results so they can be observed experimentally. This is at the expense of postselection reducing success probability. Weak values however also lead to a range of paradoxes.

To derive Eq. (1) we first couple our initial system $|\psi_i\rangle$ to our initial pointer state $|\phi\rangle$, by weakly measuring them with the probe Hamiltonian $\hat{H} = \lambda \hat{O} \otimes \hat{P}_d/T$ for small coupling constant $\lambda$ and state-probe interaction time $T$. This produces the state

$$|\psi_w\rangle = e^{-\frac{i}{\hbar} \hat{O} \otimes \hat{P}_d} |\psi_i\rangle \otimes |\phi\rangle = e^{-\frac{i}{\hbar} \hat{O} \otimes \hat{P}_d} |\psi_i\rangle \otimes |\phi\rangle \tag{2}$$

where $\hat{P}_d$ is the momentum of that pointer. We then strongly measure this weak-measured state $|\psi_w\rangle$ with the operator

$$\hat{F}_1 = |\psi_f\rangle \langle \psi_f | \otimes \hat{I}_d \tag{3}$$

Assuming $\hat{P}_d$ has Gaussian distribution around 0 with low variance (so $X_d$, the position of the pointer, has Gaussian distribution with high variance), we can say

$$e^{-\frac{i}{\hbar} \hat{O} \otimes \hat{P}_d} = \sum_{k=0}^{\infty} \left( -\frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d \right)^k / k! \tag{4}$$

$$= 1 - \frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d + O(\lambda^2) \approx 1 - \frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d$$

so this strong measurement gives the result

$$|\psi_f\rangle \langle \psi_f | e^{-\frac{i}{\hbar} \hat{O} \otimes \hat{P}_d} |\psi_i\rangle \otimes |\phi\rangle \approx |\psi_f\rangle \langle \psi_f | (1 - \frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d) |\psi_i\rangle \otimes |\phi\rangle \tag{5}$$

$$= |\psi_f\rangle \otimes \langle \psi_f | \langle \psi_i \rangle (1 - \frac{i\lambda}{\hbar} O_w \hat{P}_d) |\phi\rangle$$

$$\approx |\psi_f\rangle \otimes \langle \psi_f | \psi_i \rangle e^{-\frac{i}{\hbar} O_w \hat{P}_d} |\phi\rangle$$

This means the position of our initial pointer state $|\phi\rangle$, acting as our “readout needle”, shifts, with measurement of the pointer position giving a read-out value $(\langle x \rangle - a)$. For an initial Gaussian pointer state, this shift causes the average position to move from $\langle x \rangle$ to $(\langle x \rangle - \pi)$ due to the application of the effective operator $1 - i\lambda O_w \hat{P}_d/\hbar$. $a$ is distributed over a wide range of values for many repeats, but the distribution of “a”s will be a Gaussian centred on (the real part of) $O_w$. Peculiarly, this weak value $O_w$ can be very far from any of the eigenvalues of $\hat{O}$, or even imaginary. This is odd, given this weak value appears in exactly the place in the equation that an eigenvalue of $\hat{O}$ would for a Von Neumann measurement (where the variance of $X_d$ on the measured state is vanishingly small). This led to the belief that the weak value $O_w$ represented some fundamental value of the operator $\hat{O}$ between measurements.

Consider a case where the weak value of the spatial projection operator along a certain path was non-zero for a given $|\psi_i\rangle$ and $|\psi_f\rangle$. Vaidman argued we should interpret this as meaning a particle was on that path and left a weak trace along it, on two ground [1]. Firstly, if it were a strong measurement, the spatial projection operator having non-zero eigenvalues would determine if a quantum particle was present. Secondly, if a particle was present in a region, he argues, it should have non-zero interaction with the local environment, due to decoherence, and so leave a “trace” along its local path. While this interaction would necessarily be weak, as we can see by the environmental coupling not collapsing position superpositions in the way a projective measurement would, supporters of the weak trace approach claim it would still be in principle detectable (i.e. only the first order in the expansion of the weak coupling constant), and so would be equivalent to the weak value of the spatial projection operator along that path (which is also only to first order in $\lambda$).

This interpretation is what we refer to as “the weak trace approach”—that is, the weak trace approach interprets the weak value of the spatial projection operator for a particle along a given path as being equivalent to the trace left by that particle interacting with its environment as it travelled along that path, as the particle’s interaction with the environment becomes vanishingly small. By the weak trace approach, a non-zero weak value of the spatial projection operator for a particle on a given path indicates the presence of that particle on that path. Therefore, in this paper, we use the term “weak trace” as Vaidman does—to signify the spatial projection operator for a given path having a non-zero weak value. We however distinguish this from the broader operational idea of a particle leaving a “trace” where it has travelled and interacted with its environment.

Due to their interrelatedness, the presence claims of this weak trace approach can be presented easily in a graphical format through the Two-State Vector Formalism (TSVF). Aharonov et al developed the TSVF, which considers both the backwards and forwards-evolving quantum states, rather than just the forward as in standard quantum mechanics. (A similar intuition led Watanabe to develop the Double Inferential-state Vector Formalism.) For some operator $\hat{O}$, the forwards travelling initial state $|\psi_i\rangle$, and the backwards travelling final state $|\psi_f\rangle$, the TSVF gives out a conditional prob-
ability amplitude $\langle \psi_f | \hat{O} | \psi_i \rangle$, where $\langle \psi_f | \psi_i \rangle$ is referred to as the Two-State Vector. This conditional probability amplitude is of the same form as the numerator of the weak value, and so the TSVF is useful as a tool to graphically plot where the weak value of the spatial projection operator is nonzero. This involves plotting the forward-evolving state (possible paths the particle could have travelled via from its original position) and the backwards-evolving state (possible locations the particle could have come from to reach its final position). The weak trace approach asserts a quantum particle is present wherever these states visibly overlap (as shown in Fig. 1).

The nested-interferometer model, which we show in Fig. 1 was pointed out as a case where the Two-State Vector Formalism contradicted “common-sense” continuous-path approaches [1]. The inner interferometer is balanced such that a photon entering from arm $D$ exits to detector $D3$. Consequently, the outer interferometer is unbalanced, so there is an equal probability of a photon introduced from the source ending in $D1$ or $D2$. When the photon ends at $D2$, common sense would tell you it must have travelled via path $A$. This is as, had it travelled via $D$ into the inner interferometer, it could not have exited onto path $E$, or reached $D2$. Supporters of the weak trace approach however claim, while the photon never travelled paths $D$ or $E$, it travelled along paths $B$ and $C$ as well as along path $A$.

III. ISSUES WITH THE WEAKNESS ASSUMPTION

The process of weak measurement and strong postselection given above leads to a small shift in the position $X_d$ of our pointer system $|\phi(x)\rangle$, with an average value

$$\bar{\sigma} = \lambda \text{Re}(O_w) \ll \Delta X_d$$

Performing the measurement on a pre-and-postselected ensemble of $N$ particles will allow the measurement of the shift to precision $\Delta X_d / \sqrt{N}$. Therefore, supporters of the weak trace approach claim, as long as $N > (\Delta X_d)^2 \mathcal{O}(\lambda^{-2})$, the presence of the particle will be revealed. This is still however a coupling— so long as $\lambda \neq 0$, the system still involves measurement. This makes sense as, per Busch’s Theorem, we cannot gain information about the state of the system without disturbing the system [24].

This is an issue with the “weakness assumption” in weak measurement - weak interaction is not equivalent to non-interaction. In an un-postselected scenario, the relationship between the amplitude and the intensity of light (and so energy, and so the probability of particle detection at a location) is polynomial - therefore, while reduction of amplitude in coupling/transmission may polynomially reduce the intensity (and so chance of a quantum particle coupling/being transmitted), this is not the same as making the intensity zero. This means, so long as we are taking a non-zero coupling/transmission amplitude, as all weak value experiments do, we cannot claim we are looking at the case when any interaction/measurement is actually zero - just how it is when this interaction is infinitesimally small. Treating infinitesimal interaction as if it is no interaction presupposes that this limit is not singular - which, given the difference in observable effects between small coupling and no coupling, is unlikely (see [27] for an example of such a difference).

Popular interpretations of weak values claim that they exist in the absence of measurement and perturbation [17] [28] [29]. However, it has been shown this is clearly not the case [27] [30] [37]. To defend against this point Peleg and Vaidman say, “in a hypothetical world with vanishing interaction of the photon with the environment, Vaidman’s definition is not applicable, but in the real world there is always some non-vanishing local interaction. Unquestionably, there is an unavoidable interaction of the photon with the mirrors and beam splitters of the interferometer” [16].

This statement contradict the approach taken practically in obtaining the weak trace. If the purpose of the weak trace approach is to identify the non-vanishing interaction a quantum particle has with its environment, to trace its path, then a definition of this trace which neglects some of these non-vanishing local interactions is only telling us part of the story. Proponents of the weak trace only consider the first-order trace, as per the approximation given in Eq. 4 due to the supposedly vanishing nature of the terms of second-order or higher in $\lambda$. Despite this, Peleg and Vaidman are specifically saying in [16] that these environmental terms provide non-vanishing local interaction. A typical photon interaction with mirrors for instance transfers of order $10^{-55}$
of its energy per reflection (for a 1500nm-wavelength photon and a 1 gram mirror), a suitably weak interaction that most of us safely ignore. Hence even the higher order terms are many orders of magnitude larger than the strongest perturbation from the environment in optics experiments and ignoring higher-order terms while invoking the weak trace naturally left by the quantum object on its surroundings, is misleading.

It could be argued that this is more an objection to the experimental methods used to detect the weak trace than the application of the concept itself - however, it is from consideration of these experimental methods that supporters of the weak trace approach claim we should neglect the higher-order terms (due to their comparative undetectability) - so therefore, if we are instead looking more theoretically at the non-vanishing local interactions, we should associate presence with all terms, rather than just the first order terms in $\lambda$.

IV. THE WEAK TRACE APPROACH AND INTUITIVE IDEAS OF PRESENCE

Next, we show that the weak trace approach does not reveal the path of a quantum particle.

First, consider the classical idea of a particle being present at a certain place at a certain time; this requires:

i). It is only at that location at that time - it cannot be at both that location and another location simultaneously;

ii). It has travelled to that location via a continuous path, and will travel from that location via a continuous path;

iii). It can interact with other objects/fields local to that location; and,

iv). If a particle’s location at a time is within some region, then the particle is also located in that region in that time (e.g. if a particle’s location at a time is on path $B$, then its location is also on the union of paths $B$ and $C$ at that time).

Note that to require that there is always some location at every time where these criteria are satisfied, is equivalent to advocating a hidden variable approach to quantum mechanics, where the hidden variable is the particle’s location. This is because prima facie these criteria are not satisfied in physics. Condition (i) is debatable at best given quantum particles can be in superpositions of position, in the same way that their other properties can be superposed. However, despite quantum tunnelling and other phenomena, condition (ii) still applies for quantum particles in superposition of position evolving according to the Schrödinger equation, since the evolution of the superposition is simply the superposition of the classical evolutions of the classical components of the superposition, each of which has a continuous trajectory. (iii) and (iv) are compatible with quantum mechanics.

A possible condition for a particle being present at a location in a quantum context is that it has a non-zero weak value for the spatial projection operator at that location. Whenever a particle satisfies condition (ii), it also satisfies this condition, as its forwards and backwards-travelling states always overlap. This can only be a necessary condition for particle presence, rather than a sufficient condition, as we have shown above cases where:

1). A particle has a non-zero weak value of the spatial projection operator at a location, but has no continuous path to/from this location;

2). A particle has a non-zero weak value of the spatial projection operator at a location, but we cannot say for certain that the particle would interact with other objects/fields in that location, as any such effect would cause a breakdown in interference and so change the system; and,

3). A particle having a non-zero weak value of the spatial projection operator at a location, but a weak value of zero for the spatial projection operator at a coarse-graining of that location (e.g. having a non-zero weak value of the spatial projection operator on path $B$, and a non-zero weak value of the spatial projection operator on path $C$, but a weak value of zero for the spatial projection operator for the space composed of paths $B$ and $C$).

Point (2) could be challenged by Peleg and Vaidman’s claim that quantum objects leave a ubiquitous weak trace wherever they travel, which would have an effect on the environment there. Dziewior et al. [38] however show there are locations where “no effect on local external systems can be observed, even when the forward-evolving wavefunction did not vanish”, and further claim that certain key cases where an effect on local systems (“coupling to the external degrees of freedom”) can be thought of “as being due to misalignment of the interferometer.” This reinforces the idea that there is a difference between a non-zero weak trace and presence in the setup as claimed, rather than a misaligned version (with a different actual state to that claimed through the “weakness’ assumption).

Using the two-state vector formalism and weak value tools, we can quantitatively analyse the nested interferometer set-up to show Point (3). We first define the forwards-travelling initial vector and backwards-travelling final vector by the paths via which they evolve:

$$\langle \psi_f \rangle = \frac{\sqrt{2} |A\rangle + |B\rangle + |C\rangle}{2}$$

$$\langle \psi_i \rangle = \frac{\sqrt{2} |A\rangle + |B\rangle - |C\rangle}{2}$$

Using these, and defining the spatial projection operator for arm $A$ as $P_A = |A\rangle\langle A|$ (similarly for $B$ and $C$),
we get the weak values Vaidman does in \([1]\):

\[
\langle \hat{P}_A \rangle_w = \frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} = 1
\]

\[
\langle \hat{P}_B \rangle_w = \frac{\langle \psi_f | \hat{B} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} = \frac{1}{2}
\]

\[
\langle \hat{P}_C \rangle_w = \frac{\langle \psi_f | \hat{C} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} = -\frac{1}{2}
\]  \hspace{1cm} (8)

If we want to see however if the particle was present in the inner interferometer as a whole (as in either arm B or C), we define \( \langle \hat{P}_{BC} \rangle_w = |B\rangle\langle B| + |C\rangle\langle C| \) as we are allowed to do, given projectors in standard quantum mechanics are linear. We find

\[
\langle \hat{P}_{BC} \rangle_w = \frac{\langle \psi_f | (|B\rangle\langle B| + |C\rangle\langle C|) | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} = \frac{1}{2} - \frac{1}{2} = 0
\]  \hspace{1cm} (9)

If we assumed a non-zero weak trace implies particle presence, this would mean the photon was never in the inner interferometer (made up of arms B or C) overall. This seems incoherent, given, taken separately, it claims the particle was in arm B, and was in arm C. This illustrates the importance of the sign of the weak values, which the weak trace approach, and the graphical form of the TSVF (as in Fig.1), neglect.

For weak value experiments aiming to distinguish which-path information, we can link this to the Visibility-Distinguishability Inequality \([10, 11]\).

\[ D^2 + V^2 \leq 1 \]  \hspace{1cm} (10)

where \( D \) is the distinguishability of which path the light travelled, and \( V \) is the fringe-visibility at the output of the interferometer. Therefore, doing anything which would increase the distinguishability between the two paths (e.g. placing different tags on B and C) will affect the interference pattern at the BCE beamsplitter. Given perfect interference is required to ensure all light that enters the inner interferometer from D exits into D3, anything causing distinguishability between paths B and C will allow light to leak through onto E, and cause a trace to show in D2. Therefore, it will never be showing what would happen in an unperturbed system.

Given Eq. \(10\) \( \langle \hat{P}_{BC} \rangle_w \) is a far better measure of whether light reaching D2 was ever in the undisturbed inner interferometer. This is as measuring \( \langle \hat{P}_{BC} \rangle_w \) does not cause distinguishability between paths B and C in the inner interferometer, and so does not affect the interference pattern required to output all inner interferometer light into D3. \( \langle \hat{P}_{BC} \rangle_w \) being 0 provides support for a ‘common-sense’ path (i.e. light only travelling via A to reach D2) in an unperturbed system.

From these three points, we see that the “nonzero weak value for the spatial projection operator” condition can be satisfied while other necessary conditions for a particle being present at a location are not satisfied. Therefore, it is not a sufficient condition for particle presence.

FIG. 2. Two-State Vector Formalism analysis of Salih et al’s polarisation-based single-outer-cycle protocol for counterfac-
tual communication. Bob communicates with Alice by turning off/on his switchable mirrors to determine whether the photon goes to D1 or D0 respectively. We specify the polarisation, given it determines direction of travel through the polarising beamsplitters (PBSs). The forwards- and backwards-travelling states do not overlap anywhere on the inner interferometer chain when there is a detection at D0, meaning, by the weak trace approach, the particle detected at D0 was never at Bob. This means Bob’s ability to communicate with Alice is not explained by the weak trace approach any more than it is by standard quantum mechanics.

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\( ^1 \) Vaidman explicitly says that weak values obey the sum rule, and so allows us to say \( \langle \hat{P}_{BC} \rangle_w \) must equal \( \langle \hat{P}_B \rangle_w + \langle \hat{P}_C \rangle_w \) \([17, 39]\).

\( ^2 \) Aharonov et al consider a similar scenario in their three-box experiment, and also discuss this idea of negative weak value in the equivalent of arm C cancelling the positive weak value in the equivalent of arm B \([52]\).
V. USEFULNESS OF THE WEAK TRACE APPROACH

Supporters of the weak trace approach argue that it provides more (interpretational) information about the underlying state of the system than standard quantum mechanics. They claim that an issue with Wheeler’s “common-sense” approach to particle trajectories [42], was that it was entirely operational: not telling us anything about the underlying mechanisms at work, just the final result [1]. Similarly, Vaidman claims “the von Neumann description of the particle alone is not sufficient to explain the weak trace” [1] - implying the weak trace provides something more than the standard von Neumann approach does. We however turn this criticism back on these supporters - the weak trace approach does not tell us anything about the underlying system either, beyond standard quantum mechanics.

A key motivation for the weak trace approach was to explain how the results of interference are affected by changes to disconnected regions. It was intended to show that phenomena such as Wheeler’s Delayed Choice, or Salih et al’s Counterfactual Communication protocol [5, 43] and related effects [44-48] aren’t as “spooky” as they appear [49-52]. Salih et al however have shown that in their communication protocol, there is no weak trace by Vaidman’s criterion at Bob when Alice receives the quantum particle (see Fig. 2) [53]. Aharonov and Vaidman have also given an altered protocol for counterfactual communication where there is no weak trace at Bob [54] [55]. These both show that the weak trace does not explain this phenomenon.

While suggesting a time-symmetry to quantum processes through the TSVF, the weak trace approach does not imply any new physics beyond standard quantum theory [17], to explain the causes of counterintuitive quantum effects when there is a weak trace. The weak trace approach simply assumes the particle was present wherever the weak value of the spatial projection operator is nonzero. Hence it adds a claim of particle presence but contributes nothing testable to our ontology. This label confuses matters by oversimplifying a complex concept: what it means for a specific particle to be sequentially “present” at a two specific places in quantum field theory. It also causes a number of paradoxes by itself, such as the “jumping” photon in the nested interferometer set-up above.

VI. CONCLUSION

We have shown that not only does the weak trace approach to particle presence give incoherent results (claiming that a particle can be in B, or in C, but not in B or C), but that it also rests on the faulty assumption that weak coupling is equivalent to no coupling. Regardless of disturbance, a non-zero weak value of the spatial projection operator is insufficient evidence of particle presence, especially when other criteria for particle presence are not met.

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In this Appendix, we describe issues with Danan et al’s experiment, which claims to provide evidence for the weak trace approach to the past of a quantum particle. Danan et al had mirrors MA, MB, MC, MD and ME on Fig. 1 each oscillate at a different frequency, and sent weak coherent pulses through the arrangement, to probe where, supposedly, a photon had been. This involved recording the position over time of the beam centroid detected at D2, taking the Fourier transform of this, and cross-referencing each frequency peak with its respective mirror. Zhou et al repeated the experiment and obtained the same results. Both Danan et al and Zhou et al showed only peaks corresponding to the frequencies of MA, MB and MC, which they claim shows photons never travel by paths D or E, and proves Vaidman’s hypothesis. It is debatable however if this is a weak value experiment, or if it shows the validity of the weak trace approach.

The nested-interferometer experiment in Fig. 4 requires perfect interference at the beamsplitters splitting and recombining paths B and C, to ensure no photons travel along paths D or E. Experimentally, it is vital to preserve the interference pattern (i.e., to ensure paths B and C negatively interfere at the ports leading to path D and E) to ensure photons do not travel along path D and E in the postselected case. We can see this by considering the protocol by standard quantum mechanics (not including postselection) where half of the photons will travel via path D and enter the inner interferometer. In this situation, if B and C don’t interfere destructively for port E on the BCE beamsplitter, photons will leak out via path E, and enter detectors D1 and D2. Obviously photons will then have travelled via paths D and E to get to D2. Danan and Vaidman both claim we know no photons have travelled via paths D and E when we observe no trace from MD or ME on the output at D2, and the intensity detected at D3 remains constant.

This ignores however the difference in scale between the effects of oscillations on the combined beam travelling through/from the inner interferometer (that caused by MD or ME), and the interference-altering oscillations when the beam is split (that caused by MB and MC). By definition, the raw oscillations have to be small, in order for the perturbation to count as a weak measurement, but small compared to the beam diameter at D2 (O(10^{-3}m)) is very different to small compared to the wavelength of the photon used (O(10^{-6}m)).
detection). Therefore, the trace from the path mirrors $MD$ and $ME$ is not of the scale of the oscillations from $MB$ and $MC$ - but this does not mean photons didn’t travel along paths $D$ or $E$.

To quantify this, Danan et al say the oscillating mirrors each caused an angular change of 300nrad. They do not give the dimensions of the inner interferometer, but assuming it is of $O(10^{-2}m)$, the change in position on the recombining beamsplitter in the inner interferometer is roughly $10^{-8}m$. Given the beam had wavelength 785nm, this is a comparatively large change in interference, allowing 1/40 of the light going into the inner interferometer via $D$ to exit via $E$. One would expect this to show up on detector $D3$ - Danan et al however do not show us the detected spectrum from $D3$, which leads us to question their results.

Further, by showing that classical optics also predicts to first order oscillations from $MA$, $MB$ and $MC$, but not from $MD$ or $ME$, if light went via $D$ and $E$, Danan et al make our point for us - their experiment gives us no reason to believe light can discontinuously “hop” into the inner interferometer without travelling through arms $D$ and $E$. Saldanha, and Potöchek and Ferenczi, both give the same analysis of Danan et al’s results being obtainable using classical optics, with light slipping through when the oscillations at $MB$ and $MC$ unbalance the interferometer [57–59].

This analysis is reinforced by that of Salih [32, 60], Więśniak [31], and Svensson [30, 33, 34, 36], who all independently show that, if $MB$ and $MC$ oscillate at the same rate, their respective traces on $D2$ disappear, as the interferometer is kept balanced. Similarly, Alonso and Jordan show that adding a Dove prism to the set-up along path $C$, while still balancing interferometer $CD$, allows the oscillations at $MD$ to be seen. This yet again proves the visibility of $MB$ and $MC$, but not $MD$ and $ME$, to be the result of differences in scale of oscillation effect, rather than light never passing along $D$ or $E$ [61].

Supporters of the weak trace approach attempt to say that this invalidates the experiment, by conceding that the Danan et al experiment isn’t a direct measurement of the trace left by the photon [62]. This however makes our point for us: the experiment isn’t a real measurement of weak value, and so it contributes little to confirming or denying the weak trace at a given point.