General description of New Physics in $t$, $b$ and boson
Interactions and its Unitarity Constraints

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Abstract

We list all possible $\text{dim} = 6$ CP conserving and $SU(3) \times SU(2) \times U(1)$ gauge invariant
interactions, which could be generated in case no new particles would be reachable in the
future Colliders, and the only observable New Physics would be in the form of new inter-
actions affecting the scalar sector and the quarks of the third family. These interactions
are described by operators involving the standard model scalar field, the quarks of the
third family and the gauge bosons. Subsequently, we identify those operators which do
not contribute to LEP1 (and lower energy) observables at tree level and are not purely
gluonic. Since present measurements do not strongly constrain the couplings of these
operators, we derive here the unitarity bounds on them. Finally, in order to get a feeling
on the possible physical meaning of the appearance of any of these operators, we identify
the operators generated in a class of renormalizable dynamical models which at the TeV
scale, are fully described by the $SU(3) \times SU(2) \times U(1)$ gauge group.

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1 Introduction

Up to now the Standard Model (SM) has passed all tests and stands in an amazing agreement with the experimental data [1, 2]. Minor discrepancies which are occasionally announced tend to disappear as the statistics is increasing [2]. Nevertheless it is widely believed that there is some New Physics (NP) to be learned which is beyond SM and which will help clarifying the mysterious mechanism of the spontaneous gauge symmetry breaking. In other words, it is quite commonly expected that the NP which may be discovered some day, will be related to the way the Higgs particle(s) is generated and interacts [3, 5].

It may turn out that no scalar particles really exist and that the Higgs induced New Physics takes the form of a new strong interaction among the longitudinal $W$ and $Z$ bosons [4]. This possibility is widely studied these days, but it will not concern us here [6, 7]. Instead, the present work is within the alternative option that one or more Higgs particles exist having masses of the order of the electroweak scale $v = (\sqrt{2}G_{\mu})^{-1/2} = 0.246 \text{ TeV}$. One example of such a philosophy is of course SUSY, where the desire to invent a mechanism which ”naturally” accommodates $m_H \sim v$, leads to the intriguing consequence that every known particle should have a supersymmetric partner, some of which must be reachable in the future and may be even the present Colliders.

We should remark though, that even if NP remains perturbative, it may turn out that the underlying cause of spontaneous breaking is much more contrived than the present realizations of SUSY suggest. One viable possibility for the future could be that no new particles will be reachable at the future Colliders, apart of course from the theoretically well known standard model Higgs particle. If this turns out to be the case, then NP could only appear in the form of slight modifications of the SM interactions among the known particles and of course the SM Higgs [3].

How we could parameterize these NP interactions in a rather general way, provided that no particles beyond those already present in SM, will be reachable in the future Colliders? To achieve such a description, we subscribe to the idea that NP stems from the scalar sector and observe that the Higgs particle in SM couples appreciably only to the singlet $t_R$ and the doublet $(t_L, b_L)$ fields, and of course to the appropriate electroweak gauge bosons. Motivated by this, we assume that the NP hidden in the scalar sector is somehow able to discriminate among the families (e.g. through some kind of a horizontal symmetry), and that it predominantly couples to the quarks of the third family. Assuming in addition that NP is CP invariant and that its scale is large, we conclude that a reasonably general description of NP is in terms of a linear combination of all possible $dim = 6 \times SU(3) \times SU(2) \times U(1)$ gauge invariant and CP symmetric operators involving the scalar doublet of SM, together with the quarks of the third family and of course the gauge bosons [4]. Thus, the first aim of the present work is to establish the complete list of all possible such operators. Conservatively, we have included in this list also the operators containing the singlet $b_R$ field, in spite of the fact that in SM it does not couple appreciably.
bly to the Higgs. We should keep in mind though, that the $b_R$ involving operators might be less likely to appear.

The complete list of all $\text{dim} = 6$ $SU(3) \times SU(2) \times U(1)$ gauge invariant operators involving all families of fermions has been known for a long time [8]. In establishing this list, the equations of motion have traditionally been used in order to eliminate depended operators. Within our philosophy though, in which the scalar doublet and the quarks of the third family have a very special role in generating NP, only those equations of motion which do not mix-in light quarks or leptons are allowed to be used. Thus, in the approximation that we neglect all fermion masses except the top, only the equations of motion for the $t$, $b$ quarks and the scalar fields are to be used. Under such assumptions, the purely bosonic CP conserving operators have to a large extent already been classified in [9]; while two classes of CP conserving operators involving the quarks of the third family and the gluon have been given in [10].

Here we complete this list. Thus, we first add to the list of the purely bosonic operators one more operator involving Higgs-gluon interactions and two purely gluonic ones. Secondly, for the operators involving quarks of the third family, we complete the list of [10], by adding to it also the operators involving the covariant derivative of the gauge field strengths. Of course, after having identified the NP operators, the gauge boson equations of motion may always be used when doing calculations involving operators containing the covariant derivative of the gauge field strengths. Because of this, the NP generated by the properties of the third family, actually also induces NP couplings for the light fermions as well, [11, 12].

So in the first part of the present work we give the complete list of all possible such NP operators. It involves 14 purely bosonic operators and 34 operators containing quarks of the third family. We then proceed to identify those that give no tree level contributions to the present LEP1 and low energy observables. These are the operators for which the present constraints are most mild, and which have therefore the best chance to describe any possible kind of NP that may exist. They are 9 bosonic and 25 quark operators. Two of the bosonic operators involve only gluon fields, and their study and compatibility with experiment necessitates a somewhat detail QCD treatment. Therefore we postpone their consideration for the future, and in the present paper concentrate on the 7 purely bosonic and the 25 quark operators.

The next step consists in studying the couplings associated to each NP operator at an energy scale smaller than the NP scale $\Lambda_{NP}$; ($s \lesssim \Lambda_{NP}^2$). Remembering that the $\text{dim} = 6$ nature of the operator requires a dimensionful coupling constant (i.e. involvement of a scale ), we remark that there are two possible ways to introduce this effective Lagrangian:

(a) either from some theoretical prejudice one knows the NP scale $\Lambda_{NP}$ and then one writes the interaction

$$\mathcal{L}_{\text{eff}} = \frac{f_i}{\Lambda_{NP}^2} \mathcal{O}_i,$$  \hspace{1cm} (1)

(b) or, if this is not the case, one chooses an arbitrary mass scale $M$, (like e.g. $M_W$ or
and writes

$$\mathcal{L}_{\text{eff}} = \frac{f_i}{M^2} \mathcal{O}_i.$$  \hfill (2)

If we follow the way "(a)" , then an ambiguity often appears in the definition of $\Lambda_{NP}$ and thus in the normalization of $f_i$. So it is difficult to accurately define the strength of the interaction. If the underlying renormalizable dynamical theory which induces the above effective NP Lagrangian were known, then the "matching conditions" could be used to determine $f_i(\Lambda_{NP}^2)$ \cite{14}, and the renormalization group equations would subsequently determine $f_i(s)$ at any lower scale $s < \Lambda_{NP}^2$. On the other hand, if the underlying renormalizable dynamical theory is not known, then we have no theoretical means to determine $f_i(\Lambda_{NP}^2)$.

Nevertheless, as proposed in Ref. \cite{14}, it is possible to determine for any given value of $f_i$ in (1) or (2), the energy scale at which the interactions described by the corresponding operator $\mathcal{O}_i$, becomes "strong". The fact that these interactions have to become strong at some energy, is an inevitable consequence of the $\mathcal{O}_i$ dimensionality being larger than four. Thus, for operators with $\text{dim} = 6$ the tree level two-body amplitudes generally grow like $s$, inevitably approaching the unitarity limit at some energy scale. One can then define the NP scale $\Lambda_{NP}$ as the energy at which this happens. This is a natural definition, as at this energy one precisely expects that the residual "effective" description ceases to be valid. In other words, for $s \sim \Lambda_{NP}^2$ or $s < \Lambda_{NP}^2$, new particles, resonances, or substructures,..., typical of NP should appear. These are the phenomena that should restore unitarity by changing the form of the interaction Lagrangian $\mathcal{L}_{\text{eff}}$.

To achieve this procedure, we use the way "(b)" mentioned above, and compute the unitarity constraint for each interaction $\frac{f_i}{M^2} \mathcal{O}_i$, using two-body amplitudes. One then gets relations of the type

$$\frac{f_i}{M^2} \frac{s}{C_i(s)} = 1 , \hfill (3)$$

where $C_i(s)$ is a well-defined (generally) energy dependent coefficient. Defining then $\Lambda_{NP}$ as

$$\Lambda_{NP} \equiv \left[ C_i(\Lambda_{NP}^2) \cdot \frac{M^2}{f_i} \right]^{1/2},$$  \hfill (4)

we rewrite the unitarity constraint in (3), as

$$f_i = C_i(\Lambda_{NP}^2) \cdot \frac{M^2}{\Lambda_{NP}^2}. \hfill (5)$$

Note that there is no ambiguity in the definition of $\Lambda_{NP}$. In other words, for any given $f_i$ and $\mathcal{O}_i$, $\Lambda_{NP}$ is exactly determined, at least at the level of using tree level amplitudes.

For the six purely bosonic operators appearing in \cite{14}, the unitarity relations have been established in \cite{14,15,16}. Here we present the results for the new bosonic operator $\mathcal{O}_{GG}$ inducing anomalous Higgs-gluon interactions, and the set of the quark operators selected as explained above.
The contents of the paper is the following. In Sect.2 the complete list of the gauge invariant operators is established. The unitarity constraints are established in Sect.3. In Sect.4 the implications of various renormalizable dynamical models on the appearance and strength of the various operators is presented, while the final discussion is given in Sect.5.

2 The list of $dim = 6$ gauge-invariant operators

The complete list of the CP conserving purely bosonic $dim = 6$ and $SU(3)_c \times SU(2) \times U(1)$ gauge invariant operators is given by

$$\mathcal{O}_{DW} = 2 (D_\mu \tilde{W}^{\mu \rho}) (D^\nu \tilde{W}_{\nu \rho}) ,$$

$$\mathcal{O}_{DG} = 2 (D_\mu \tilde{G}^{\mu \rho}) (D^\nu \tilde{G}_{\nu \rho}) ,$$

$$\mathcal{O}_G = \frac{1}{3!} f_{ijk} G^{i \mu \nu} G^j_{\rho \lambda} G^{k \lambda \mu} ,$$

$$\mathcal{O}_{DB} = 2 (D_\mu B^{\mu \rho}) (D^\nu B_{\nu \rho}) ,$$

$$\mathcal{O}_{BW} = \frac{1}{2} \Phi^\dagger B_{\mu \nu} \cdot \tilde{W}^{\mu \nu} \Phi ,$$

$$\mathcal{O}_{\Phi_1} = (D_\mu \Phi^\dagger \Phi) (D^\mu \Phi) ,$$

$$\mathcal{O}_{\Phi_2} = 4 D_\mu (\Phi^\dagger \Phi) D^\mu (\Phi^\dagger \Phi) ,$$

$$\mathcal{O}_{\Phi_3} = 8 (\Phi^\dagger \Phi)^3 ,$$

$$\mathcal{O}_W = \frac{1}{3!} \left( \tilde{W}^\mu_\nu \times \tilde{W}^\nu_\lambda \right) \cdot \tilde{W}^{\mu \lambda}_\nu ,$$

$$\mathcal{O}_{WW} = i (D_\mu \Phi)^\dagger \cdot \tilde{W}^{\mu \nu} (D_\nu \Phi) ,$$

$$\mathcal{O}_{BB} = i (D_\mu \Phi)^\dagger B^{\mu \nu} (D_\nu \Phi) ,$$

$$\mathcal{O}_{GG} = (\Phi^\dagger \Phi) \tilde{G}^{\mu \nu} \cdot \tilde{G}_{\mu \nu} .$$

Except of $\mathcal{O}_{GG}$, $\mathcal{O}_{DG}$ and $\mathcal{O}_G$, these operators were first enumerated in [9], following the general classifications in [8]. In connection with this we also note that in [9], instead of $\mathcal{O}_{DW}$ and $\mathcal{O}_{DB}$ the operators

$$\mathcal{O}_{DW} = (D_\mu \tilde{W}_{\nu \rho}) (D^\mu \tilde{W}^{\nu \rho}) , \quad \mathcal{O}_{DB} = (D_\mu B_{\nu \rho}) (D^\mu B^{\nu \rho})$$

are used, which satisfy

$$\mathcal{O}_{DW} = \overline{\mathcal{O}}_{DW} + 12 g \mathcal{O}_W , \quad \mathcal{O}_{DB} = \overline{\mathcal{O}}_{DB} .$$
In the preceding formulae the usual definitions

\[ \Phi = \left( \phi^+ + \frac{1}{\sqrt{2}} (v + H + i\phi^0) \right), \]

\[ D_\mu = (\partial_\mu + ig Y B_\mu + i \frac{g}{2} \tau^\mu \cdot \vec{W}_\mu + i \frac{g_s}{2} \lambda^\mu \cdot \vec{G}_\mu), \]

are used where \( v \approx 246 \text{ GeV} \), \( Y \) is the hypercharge of the field on which the covariant derivative acts, and \( \vec{\tau} \) and \( \vec{\lambda} \) are the isospin and colour matrices applicable whenever \( D_\mu \) acts on iso-doublet fields and quarks respectively.

We next turn to the operators containing also quarks of the third family which first appeared in \([10, 8]\). As in \([10]\) we put in Class 1 those operators which involve at least one \( t_R \) field, but do not contain the covariant derivative of the gauge boson field strength. Correspondingly in Class 2 we put the operators which contain neither \( t_R \) nor any covariant derivative of the gauge boson field strengths. In both cases, the operators in each class are further divided into two groups containing four-quark and two-quark fields respectively. Finally in Class 3 (which was not included in \([10]\)) we put the operators involving covariant derivatives of gauge boson field strengths. It may be useful to remark that all quark fields in (24-57) below, should be considered as weak eigen-fields for the third family. Therefore, the left-handed of them should eventually be mixed by the usual CKM matrix, in order to give the mass eigen-state fields. We thus have:

### Class 1.

**A1) Four-quark operators**

\[ O_{qt} = (\bar{q}_L t_R)(\bar{t}_R q_L), \]

\[ O_{q8}^{(8)} = (\bar{q}_L \lambda t_R)(\bar{t}_R \lambda q_L), \]

\[ O_{tt} = \frac{1}{2} (\bar{t}_R \gamma_\mu t_R)(\bar{t}_R \gamma^\mu t_R), \]

\[ O_{tb} = (\bar{t}_R \gamma_\mu t_R)(\bar{b}_R \gamma^\mu b_R), \]

\[ O_{tb}^{(8)} = (\bar{t}_R \gamma_\mu \lambda t_R)(\bar{b}_R \gamma^\mu \lambda b_R), \]

\[ O_{qq} = (\bar{t}_R t_L)(\bar{b}_R b_L) + (\bar{t}_R t_L)(\bar{b}_L b_R) - (\bar{b}_R b_L)(\bar{t}_L t_R) - (\bar{b}_L b_R)(\bar{t}_L t_R), \]

\[ O_{q8}^{(8)} = (\bar{t}_R \lambda t_L)(\bar{b}_R \lambda b_L) + (\bar{t}_R \lambda t_R)(\bar{b}_L \lambda b_R) - (\bar{b}_R \lambda b_L)(\bar{t}_L \lambda t_R) - (\bar{b}_L \lambda t_R)(\bar{t}_L \lambda b_R). \]

**B1) Two-quark operators.**

\[ O_{t1} = (\Phi^\dagger \Phi)(\bar{q}_L t_R \Phi + \bar{t}_R \Phi^\dagger q_L), \]

\[ O_{t2} = i \left[ \Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi \right] (\bar{t}_R \gamma^\mu t_R). \]
B2) Two-quark operators.

\[ O_{t3} = i (\Phi^\dagger D_\mu \Phi)(i R \gamma^\mu b_R) - i (D_\mu \Phi^\dagger \Phi)(\bar{b}_R \gamma^\mu t_R) \quad , \]
\[ O_{Dt} = (\bar{q}_L D_\mu t_R)D^\mu \Phi + D^\mu \Phi^\dagger (D_\mu t_R \bar{q}_L) \quad , \]
\[ O_{bW\Phi} = (\bar{q}_L \sigma^{\mu\nu} \gamma^\tau t_R)\Phi \cdot \bar{W}_{\mu\nu} + \Phi^\dagger (i R \sigma^{\mu\nu} \gamma^\tau q_L) \cdot \bar{W}_{\mu\nu} \quad , \]
\[ O_{tB\Phi} = (\bar{q}_L \sigma^{\mu\nu} t_R)\Phi B_{\mu\nu} + \Phi^\dagger (i R \sigma^{\mu\nu} q_L)B_{\mu\nu} \quad , \]
\[ O_{tG\Phi} = [ (\bar{q}_L \sigma^{\mu\nu} \chi a t_R)\Phi + \Phi^\dagger (i R \sigma^{\mu\nu} \chi a q_L) ] G_{\mu\nu}^a \quad . \]

Class 2.
A2) Four quark operators

\[ O_{qq}^{(1,1)} = \frac{1}{2} (\bar{q}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu q_L) \quad , \]
\[ O_{qq}^{(1,3)} = \frac{1}{2} (\bar{q}_L \gamma_\mu \gamma^\tau q_L) \cdot (\bar{q}_L \gamma^\mu \gamma^\tau q_L) \quad , \]
\[ O_{bb} = \frac{1}{2} (\bar{b}_R \gamma_\mu b_R)(\bar{b}_R \gamma^\mu b_R) \quad , \]
\[ O_{qb} = (\bar{q}_L b_R)(\bar{b}_R q_L) \quad , \]
\[ O_{qb}^{(8)} = (\bar{q}_L \chi b_R) \cdot (\bar{b}_R \chi q_L) \quad . \]

B2) Two-quark operators.

\[ O_{\Phi q}^{(1)} = i (\Phi^\dagger D_\mu \Phi)(\bar{q}_L \gamma^\mu q_L) - i (D_\mu \Phi^\dagger \Phi)(\bar{q}_L \gamma^\mu q_L) \quad , \]
\[ O_{\Phi q}^{(3)} = i \left[ (\Phi^\dagger \gamma^\tau D_\mu \Phi) - (D_\mu \Phi^\dagger \gamma^\tau \Phi) \right] \cdot (\bar{q}_L \gamma^\mu \gamma^\tau q_L) \quad , \]
\[ O_{\Phi b} = i \left[ (\Phi^\dagger D_\mu \Phi) - (D_\mu \Phi^\dagger \Phi) \right] (\bar{b}_R \gamma^\mu b_R) \quad , \]
\[ O_{Db} = (\bar{q}_L D_\mu b_R)D^\mu \Phi + D^\mu \Phi^\dagger (D_\mu b_R q_L) \quad , \]
\[ O_{bW\Phi} = (\bar{q}_L \sigma^{\mu\nu} \gamma^\tau b_R)\Phi \cdot \bar{W}_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} \gamma^\tau q_L) \cdot \bar{W}_{\mu\nu} \quad , \]
\[ O_{bB\Phi} = (\bar{q}_L \sigma^{\mu\nu} b_R)\Phi B_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} q_L)B_{\mu\nu} \quad , \]
\[ O_{bG\Phi} = (\bar{q}_L \sigma^{\mu\nu} \chi a b_R)\Phi G_{\mu\nu}^a + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} \chi a q_L)G_{\mu\nu}^a \quad , \]
\[ O_{b1} = (\Phi^\dagger \Phi)(\bar{q}_L b_R \Phi + b_R \Phi^\dagger q_L) \quad . \]

Class 3.

\[ O_{qB} = \bar{q}_L \gamma^\mu q_L (\partial^\nu B_{\mu\nu}) \quad , \]
\[ O_{qW} = \frac{1}{2} (\bar{q}_L \gamma_\mu \overrightarrow{\lambda} q_L) \cdot (D_\nu \overrightarrow{W}^{\mu\nu}) , \]

\[ O_{bB} = \bar{b}_R \gamma_\mu b_R (\partial^\nu B_{\mu\nu}) , \]

\[ O_{tB} = \bar{t}_R \gamma_\mu t_R (\partial^\nu B_{\mu\nu}) , \]

\[ O_{tG} = \frac{1}{2} (\bar{t}_R \gamma_\mu \overrightarrow{\lambda} t_R) \cdot (D_\nu \overrightarrow{G}^{\mu\nu}) , \]

\[ O_{bG} = \frac{1}{2} (\bar{b}_R \gamma_\mu \overrightarrow{\lambda} b_R) \cdot (D_\nu \overrightarrow{G}^{\mu\nu}) , \]

\[ O_{qG} = \frac{1}{2} (\bar{q}_L \gamma_\mu \overrightarrow{\lambda} q_L) \cdot (D_\nu \overrightarrow{G}^{\mu\nu}) . \]

where \( \lambda^a \) are the eight usual colour matrices.

Concerning the above list, a few remarks must be made. As mentioned already, we have used the equations of motion for the quark and scalar fields, but not for the gauge bosons, since the latter mix in light fermions. This attitude leads to including in the list also the operators \( O_{qB}, O_{qW}, O_{bB}, O_{tB}, O_{tG}, O_{bG} \) and \( O_{qG} \), collected in Class 3 and defined through (51-57). Eqs. (51-57) constitute one possible definition for these operators though. Another possibility is to substitute for them in (51-57) the gauge boson equations of motion

\[ D_\mu \overrightarrow{G}^{\mu\nu} = g_s J_3^{\nu} \]

\[ D_\mu \overrightarrow{W}^{\mu\nu} = g J_2^{\nu} - i \frac{g_2}{2} [D_\nu \Phi^\dagger \overrightarrow{\Phi} - \Phi^\dagger \overrightarrow{\Phi} D_\nu \Phi] \]

\[ \partial_\mu B^{\mu\nu} = g t J_1^{\nu} - i \frac{g}{2} [D_\nu \Phi^\dagger \overrightarrow{\Phi} - \Phi^\dagger \overrightarrow{\Phi} D_\nu \Phi] \]

where \( J_3^{\nu}, J_2^{\nu}, J_1^{\nu} \) are the \( SU(3), SU(2) \) and hypercharge fermionic currents respectively. Such a substitution provides another possible definition for these operators which is in fact more convenient for higher order calculations [13]. At tree level and to linear order in the NP couplings both definitions give identical results in Feynman diagram calculations. Differences start appearing at higher order, for which the definition using directly (51-57) implies a different (and more involved) structure of counter-terms\(^2\) [13]. We come back to this below.

Compared to [10] we should also mentioned that we have dropped the operators

\[ O_{qq}^{(8,1)} = \frac{1}{2} (\bar{q}_L \gamma_\mu \overrightarrow{\lambda} q_L) \cdot (\bar{q}_L \gamma_\mu \overrightarrow{\lambda} q_L) \]

\[ O_{qq}^{(8,3)} = \frac{1}{2} (\bar{q}_L \gamma_\mu \lambda^a \tau^j q_L) (\bar{q}_L \gamma_\mu \lambda^a \tau^j q_L) \]

since they are related to \( O_{qq}^{(1,1)}, O_{qq}^{(1,3)} \) in [38, 39] through the Fierz identities

\[ O_{qq}^{(8,1)} = O_{qq}^{(1,3)} + \frac{1}{3} O_{qq}^{(1,1)} \]

\(^2\)These remarks are of course also valid for \( O_{DW}, O_{DB} \) and \( O_{DG} \) given in (66) respectively.
Finally we have also added for completeness the 2nd Class operator \( \mathcal{O}_{b1} \) (analogous to \( \mathcal{O}_{t1} \)) which is \( \Phi^\dagger \Phi \) times the standard model Yukawa mass term for the \( b \)-quark. We should keep in mind though, that the \( b_R \) involving NP operators are on a somewhat weaker basis, since SM suggests that \( b_R \) couples very weakly to the Higgs field, which is assumed to be the source of NP.

In the framework explained so far, NP is described in terms of 14 purely bosonic CP conserving \( \text{dim} = 6 \) operators, and 34 operators involving quarks of the third family. To proceed further we reduce the number of operators to be studied by excluding those contributing at tree level to LEP1 and lower energy observables. Thus, we remark that \( \mathcal{O}_{DW} \) contributes at tree level to \( \epsilon_{1,2,3} \), \( \mathcal{O}_{DB} \) to \( \epsilon_1 \), \( \mathcal{O}_{BW} \) to \( \epsilon_3 \) and \( \mathcal{O}_{\Phi1} \) to \( \epsilon_1 \) \[17, 9, 18\]. Moreover, the 2nd and 3rd Class operators \( \mathcal{O}_{qq}^{(1)}, \mathcal{O}_{qq}^{(3)}, \mathcal{O}_{\Phi q}, \mathcal{O}_{Db}, \mathcal{O}_{bW\Phi}, \mathcal{O}_{bB\Phi}, \mathcal{O}_{qB}, \mathcal{O}_{qW} \) and \( \mathcal{O}_{bB} \) give tree level contributions to \( Z\bar{b}b \) and they are thus also very strongly constrained \[10\]; so far as we consider the action of one operator at a time. In addition, we also exclude from any further consideration the operator \( \mathcal{O}_{\Phi3} \) \[13\], since it gives no contribution to LEP1 physics and its experimental study in the future Colliders looks almost impossible. Therefore the operators which are not already very strongly constrained by existing measurements are the 9 bosonic ones in \{4, 5, 13, 14 -19\} and the 25 quark operators in \{24-42, 49, 50, 54-57\}. In the following we study the unitarity constraints on the couplings of all these operators except for the purely gluonic ones \( \mathcal{O}_{DG} \) and \( \mathcal{O}_{G} \), which are left for a future work as this deserves a special treatment of QCD effects (see the final discussion).

### 3 Unitarity constraints

Following the procedure ”(b)” explained in the Introduction, we define the New Physics couplings through the effective lagrangian

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_t + \mathcal{L}_{\text{bos}} \quad ,
\]

where the contribution from the 25 \((i = 1...25)\) ”quark” operators is written as

\[
\mathcal{L}_t = \sum_i \frac{f_i}{m_t^2} \mathcal{O}_i \quad .
\]

As in \[2\], the \( m_t \) in the denominators in \( \text{(66)} \) are simply normalization factors. Correspondingly, the contribution from the 7 purely bosonic operators is written as

\[
\mathcal{L}_{\text{bos}} = \lambda_W \frac{g}{M_W^2} \mathcal{O}_W + f_W \frac{g}{2M_W^2} \mathcal{O}_{W\Phi} + f_B \frac{g'}{2M_W^2} \mathcal{O}_{B\Phi} + \\
\frac{d}{v^2} \mathcal{O}_{WW} + \frac{d_B}{v^2} \mathcal{O}_{BB} + \frac{f_{\Phi2}}{v^2} \mathcal{O}_{\Phi2} + \frac{d_G}{v^2} \mathcal{O}_{GG} \quad .
\]

The operators \( \mathcal{O}_{WW}, \mathcal{O}_{BB} \) are analogous to the \( \mathcal{O}_{UW} \) and \( \mathcal{O}_{UB} \) introduced in \[3, 14, 15, 16\], but the definition of their couplings is exactly the same.
The unitarity constraints for the first 6 purely bosonic operators has been done in \[14, 15, 16\]. The results are

\[
|\lambda_W| \approx 19 \frac{M_W^2}{\Lambda_{NP}^2}, \quad |f_B| \approx 98 \frac{M_W^2}{\Lambda_{NP}^2}, \quad |f_W| \approx 31 \frac{M_W^2}{\Lambda_{NP}^2}, \quad (68)
\]

\[
d \approx \frac{104.5 \left(\frac{M_W}{\Lambda_{NP}}\right)^2}{1 + 6.5 \left(\frac{M_W}{\Lambda_{NP}}\right)} \quad \text{for } d > 0,
\]

\[
d \approx -\frac{104.5 \left(\frac{M_W}{\Lambda_{NP}}\right)^2}{1 - 4 \left(\frac{M_W}{\Lambda_{NP}}\right)} \quad \text{for } d < 0, \quad (69)
\]

\[
d_B \approx \frac{195.8 \left(\frac{M_W}{\Lambda_{NP}}\right)^2}{1 + 200 \left(\frac{M_W}{\Lambda_{NP}}\right)} \quad \text{for } d_B > 0,
\]

\[
d_B \approx -\frac{195.8 \left(\frac{M_W}{\Lambda_{NP}}\right)^2}{1 + 50 \left(\frac{M_W}{\Lambda_{NP}}\right)} \quad \text{for } d_B < 0, \quad (70)
\]

while for $O_{\Phi_2}$ we refer to \[16\].

For the bosonic operator $O_{GG}$, (which induces a custodial $SU(2)_c$ invariant Higgs-gluon coupling), the strongest unitarity constraint arises from the colour singlet $J = 0$ channels $|gg++>$, $|gg-->$ and $|HH>$. Diagonalizing the transition matrix we get for it

\[
|d_G| \approx \frac{4\pi}{\sqrt{1 + \frac{60\pi^2}{\Lambda_{NP}^2}}} \left(\frac{v}{\Lambda_{NP}}\right)^2 \approx \frac{119}{\sqrt{1 + \frac{1782M_W^2}{\Lambda_{NP}^2}}} \left(\frac{M_W}{\Lambda_{NP}}\right)^2. \quad (71)
\]

We next turn to the selected 25 operators, involving quarks of the third family. As said above, 14 of them belong to the first class, 7 to the second class and 4 to the third one. We start from the 14 1st Class operators. We first give the unitarity relations and subsequently we discuss them. These are

\[
|f_{qt}| \approx \frac{16\pi}{3} \left(\frac{m_t^2}{\Lambda_{NP}^2}\right), \quad (72)
\]

\[
|f_{qt}^{(8)}| \approx \frac{9\pi}{\sqrt{2}} \left(\frac{m_t^2}{\Lambda_{NP}^2}\right), \quad (73)
\]

\[
|f_{tt}| \approx 6\pi \left(\frac{m_t^2}{\Lambda_{NP}^2}\right), \quad (74)
\]

\[
|f_{tb}| \approx 8\pi \left(\frac{m_t^2}{\Lambda_{NP}^2}\right), \quad (75)
\]

\[
|f_{tb}^{(8)}| \approx \frac{9\pi}{2} \left(\frac{m_t^2}{\Lambda_{NP}^2}\right), \quad (76)
\]
\[ |f_{qq}| \approx \frac{32\pi}{7} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (77) \]
\[ |f^{(8)}_{qq}| \approx 6\pi \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (78) \]
\[ |f_{t1}| \approx \frac{16\pi}{3\sqrt{2}} \left( \frac{m_t^2}{v\Lambda_{NP}} \right), \quad (79) \]
\[ |f_{t2}| \approx 8\pi\sqrt{3} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (80) \]
\[ |f_{t3}| \approx 8\pi\sqrt{6} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (81) \]
\[ |f_{Dt}| \approx 10.4 \left( \frac{m_t^2}{\Lambda_{NP}^2} \right) \text{ for } f_{Dt} > 0 \]
\[ |f_{Dt}| \approx -6.4 \left( \frac{m_t^2}{\Lambda_{NP}^2} \right) \text{ for } f_{Dt} < 0, \quad (82) \]
\[ |f_{tW}\Phi| \approx \frac{61.6}{\sqrt{1 + 645 \frac{m_t^2}{\Lambda_{NP}^2}}} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (83) \]
\[ |f_{tB}\Phi| \approx \frac{61.6}{\sqrt{1 + 645 \frac{m_t^2}{\Lambda_{NP}^2}}} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (84) \]
\[ |f_{tG}\Phi| \approx \frac{m_t^2\sqrt{\pi}}{v\Lambda_{NP}\sqrt{1 + \frac{2}{3}\alpha_s}} \quad \text{for } \Lambda_{NP} \lesssim 10 TeV \]
\[ |f_{tG}\Phi| \approx \frac{75(m_t/\Lambda_{NP})^2}{\sqrt{1 + 591 (m_t/\Lambda_{NP})^2}} \quad \text{for } \Lambda_{NP} \gtrsim 10 TeV. \quad (85) \]

These results arise as follows:
- \(\mathcal{O}_{qq}\): The dominant unitarity constraint arises from the \(J = 0\) transition amplitude affecting the colour singlet channel \(|t\bar{t}++\rangle\) or \(|t\bar{b}++\rangle\).
- \(\mathcal{O}^{(8)}_{qq}\): The dominant constraint comes from the \(J = 1\) transition matrix affecting the colour singlet channels \(|t\bar{t}+-\rangle, |t\bar{t}--\rangle, |b\bar{b}--\rangle\).
- \(\mathcal{O}_{tt}\): From the \(J = 1\) transition amplitude affecting the colour singlet \(|t\bar{t}+\rangle\).
- \(\mathcal{O}_{tb}\): From the \(J = 1\) transition matrix affecting the colour singlet \(|t\bar{t}+\rangle, |b\bar{t}+-\rangle\) channels.
- \(\mathcal{O}^{(8)}_{tb}\): From the \(J = 1\) transition amplitude affecting the colour singlet \(|t\bar{b}+-\rangle\) channel.
- \(\mathcal{O}_{qq}\): From the \(J = 0\) transition matrix affecting any of the colour singlet sets of channels \((|t\bar{t}++\rangle, |b\bar{b}--\rangle), (|t\bar{t}--\rangle, |b\bar{b}++\rangle)\) or \((|b\bar{t}++\rangle, |b\bar{t}--\rangle)\).
- \(\mathcal{O}^{(8)}_{qq}\): From the \(J = 0\) transition matrix affecting any of the colour singlet sets of
channels (|t\bar{t}++>, |b\bar{b}-->) or (|b\bar{t}++>, |b\bar{t}-->).

\(O_{t1}\): This operator is of the form \(\Phi^{\dagger}\Phi\) times the standard Yukawa top mass term. Although \(O_{t1}\) is formally a \(dim = 6\) operator, it actually behaves as a lower-dimension one when restricting to two-particle channels. When this is nevertheless done, the dominant constraint comes from the \(J = 0\) transition matrix affecting the colour singlet channels \(|t\bar{t}++>, |t\bar{t}-->, |HH>, |W^+W^- LL>, |ZZ LL>\). As seen from (79), this leads to a unitarity constraint in which the anomalous coupling does not behave like \(\sim 1/\Lambda_{NP}^2\) for large \(\Lambda_{NP}\) [19]. A constraint of the form \(\sim 1/\Lambda_{NP}^2\) could only be obtained by considering transitions affecting channels containing more particles. The unitarity properties of \(O_{t1}\) are not further investigated here, since this operator gives no contribution either to LEP1 observables [10], or to observables in \(e^-e^+ \rightarrow t\bar{t}\) and \(t \rightarrow bW\) [24].

\(O_{t2}\): The dominant constraint arises from the \(J = 1\) transition matrix affecting the colour singlet channels \(|t\bar{t}-->, |ZH L>, |W^+W^- LL>\).

\(O_{t3}\): From the \(J = 1\) transition matrix affecting the colour singlet channels \(|b\bar{t}++, |W^-H L>, |W^-Z LL>\). The transition matrix elements may be obtained from the corresponding ones for \(O_{t2}\) by dividing them by \(-\sqrt{2}\).

\(O_{Dt}\): From the \(J = 0\) transition amplitude affecting the colour singlet channel \(|t\bar{t}++>, |t\bar{t}-->\).

\(O_{tW\Phi}, O_{tB\Phi}\): From the \(J = 1\) transition matrices affecting the colour singlet channels \(|t\bar{t}++>, |t\bar{t}-->\).

\(O_{tG\Phi}\): For \(\Lambda_{NP} \lesssim 10TeV\), the dominant unitarity constraint arises from the \(J = 0\) transition matrix affecting the colour singlet channels \(|t\bar{t}++>, |t\bar{t}-->, |gg++>, |gg-->\). For \(\Lambda_{NP} \gtrsim 10TeV\), the \(J = 1\) transition matrix affecting the colour-octet channels \(|t\bar{t}++>, |t\bar{t}-->, |gH++>, |gH-->\), dominates.

At this point, we have finished with the first Class operators and we turn to the 2nd Class ones given in (38-42, 49, 50) which imply

\[
|f_{q\bar{q}}^{(1,1)}| \approx \frac{24\pi}{7} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right),
\]

\[
|f_{q\bar{q}}^{(1,3)}| \approx \frac{24\pi}{5} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right),
\]

\[
|f_{bb}| \approx 6\pi \left( \frac{m_t^2}{\Lambda_{NP}^2} \right),
\]

\[
|f_{q\bar{b}}| \approx \frac{16\pi}{3} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right),
\]

\[
|f_{q\phi}^{(8)}| \approx \frac{9\pi}{\sqrt{2}} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right),
\]

\[
|f_{bG\Phi}| \approx \frac{m_t^2\sqrt{\pi}}{v\Lambda_{NP}\sqrt{1 + \frac{2}{3}\alpha_s}} \text{ for } \Lambda_{NP} \lesssim 10TeV
\]
\[ |f_{BG}\Phi| \simeq \frac{75(m_t/\Lambda_{NP})^2}{\sqrt{1+591 (m_t/\Lambda_{NP})^2}} \quad \text{for} \quad \Lambda_{NP} \gtrsim 10 \text{TeV}, \quad (91) \]
\[ |f_{b1}| \simeq \frac{16\pi}{3\sqrt{2}} \left( \frac{m_t^2}{v\Lambda_{NP}} \right). \quad (92) \]

These results arise as follows:

\( \mathcal{O}_{qq}^{(1,1)} \): From the \( J = 1 \) transition matrix affecting the colour singlet channels \(|t\bar{t}--\rangle, |b\bar{b}--\rangle\).

\( \mathcal{O}_{qq}^{(1,3)} \): From the \( J = 1 \) transition matrix affecting the colour singlet channels \(|t\bar{t}--\rangle, |b\bar{b}--\rangle\), or the channel \(|t\bar{b}+--\rangle\).

\( \mathcal{O}_{bb} \): From the \( J = 1 \) transition amplitude affecting the colour singlet \(|b\bar{b}++\rangle \) or \(|b\bar{t}++\rangle \); (similar to \( \mathcal{O}_{tt} \)).

\( \mathcal{O}_{qb} \): From the \( J = 0 \) transition amplitude affecting the colour singlet channel \(|b\bar{t}++\rangle \) or \(|b\bar{t}++\rangle \); (similar to \( \mathcal{O}_{qt} \)).

\( \mathcal{O}_{bG} \Phi \): It is similar to \( \mathcal{O}_{tG} \Phi \). For \( \Lambda_{NP} \lesssim 10 \text{TeV} \), the dominant constraint comes from the \( J = 0 \) transition matrix affecting the colour singlet channels \(|b\bar{b}++\rangle, |b\bar{b}--\rangle, |gH++\rangle \) and \(|gH--\rangle\). For \( \Lambda_{NP} \gtrsim 10 \text{TeV} \), the \( J = 1 \) transition matrix affecting the colour-octet channels \(|b\bar{b}++\rangle, |b\bar{b}--\rangle, |gH+\rangle \) and \(|gH-\rangle\), dominates.

\( \mathcal{O}_{b1} \): This operator is of the form \( \Phi^\dagger \Phi \) times the standard Yukawa \( b \)-quark mass term. \( \mathcal{O}_{b1} \) is similar to \( \mathcal{O}_{t1} \) and the same remarks apply. The dominant constraint, generated from two-particle channels, comes from the colour singlet states \(|b\bar{b}++\rangle, |b\bar{b}--\rangle, |HH+\rangle, |WW--LL_+\rangle, |ZZ LL_\rangle\).

The last set of unitarity constraints concerns the four 3rd Class operators \( \mathcal{O}_{tB}, \mathcal{O}_{tG}, \mathcal{O}_{bG}, \mathcal{O}_{G} \) in (54-57). As already stated, we could define these operators either directly by eq (54-57), or alternatively by substituting in them the covariant derivative of the gauge boson field strengths appearing in (58-60). Since these two definitions are only identical to linear order in the anomalous couplings and in general induce a different structure of counter terms at loop calculations, they can in principle also imply different unitarity relations. As we will see below, this actually happens only for the \( \mathcal{O}_{tB} \) case though. To explain these, we start from the second definition (utilizing the gauge boson equations of motion), which leads to the unitarity constraints

\[ |f_{tB}| \simeq 25 \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (93) \]
\[ |f_{tG}| \simeq 6\pi \left( \frac{m_t^2}{g_s \Lambda_{NP}^2} \right), \quad (94) \]
\[ |f_{bG}| \simeq 6\pi \left( \frac{m_t^2}{g_s \Lambda_{NP}^2} \right), \quad (95) \]

13
Concerning (93-96), the following comments are in order:

\( \mathcal{O}_{tB} \): When the \( B_\mu \) equation of motion (see (60)) is substituted in (54), then the dominant constraint arises from the \( J = 1 \) transition matrix affecting the colour singlet channels \( |ff+−>, |ff−+>, \) where \( f \) is any fermion (quark or lepton), and \( |ZH_L> \).

All transition matrix elements obtained this way, depend linearly on \( f_tB \) and lead to the constrain given in (93). If instead (60) is not used, then there is an additional \( f_{tB}^2 \) contribution to the amplitude \( <tt−|\mathcal{T}^{J=1}tt+−>, \) which changes the result of (93) as

\[
|f_{tB}| \simeq \frac{6\pi}{g_s} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right) .
\] (96)

At this point we would not like to enter arguments on whether we should use (60) or not. Instead, we simply take the weaker constraint (93) as an indication on what could be the largest allowed value of \( f_{tB} \) for any given \( \Lambda_{NP} \).

\( \mathcal{O}_{bG} \): The dominant unitarity constraint arises from the \( J = 0 \) transition amplitude affecting the colour singlet channel \( |tt+−> \).

\( \mathcal{O}_{qG} \): Similar to \( \mathcal{O}_{bG} \). The dominant unitarity constraint arises from the \( J = 0 \) transition amplitude affecting the colour singlet channel \( |bb+−> \).

The results on the last three operators \( \mathcal{O}_{tG}, \mathcal{O}_{bG}, \mathcal{O}_{qG} \) do not depend on whether we define them through (54-57) directly, or after having substituted in them the gauge boson equations of motion appearing in (58-60).

4 Dynamical Scenarios

The aim of the present section is to investigate which of the above operators will be generated in a wide class of renormalizable dynamical models. Considerations of this type have already been presented in [20, 15] for dynamical models inducing only purely bosonic operators and in [21] where fermionic operators are also generated, but no special role is assigned to the quarks of the third family.

Here we concentrate on the NP operators observable through new anomalous couplings at the 0.5 – 1 TeV scale. We assume that any possible new gauge bosons that might exist have much heavier masses. Therefore the relevant gauge group in the 0.5 – 1 TeV region is simply \( SU(3) \times SU(2) \times U(1) \). Based on this, we consider the most general renormalizable dynamical models containing SM and involving in addition any number of scalars or fermions which are singlets or doublets under weak isospin, and singlets or triplets under...
colour. We always assume the new particles to get their masses independently of the spontaneous breaking of the electroweak gauge symmetry, which then leads to the natural expectation that these masses should be sufficiently heavier than $v \simeq 246 \text{ GeV}$. For the new bosons this is always possible. In order to be possible for the new fermions also we assume them to be (in general) left-right symmetric, which also guarantees that no anomalies are introduced. Finally we note that the new scalars and fermions couple to the gauge bosons according to the gauge principle, while their couplings among themselves and the Higgs is determined by new, often unknown, Yukawa-type couplings.

When integrating out the aforementioned heavy fields, we find that

- the purely bosonic operators $O_{W\Phi}, O_{B\Phi}$
- the four-quark operator $O_{q^4}^{(8)}$, and
- the two-quark operators $O_{Dt}, O_{Db}, O_{bW\Phi}, O_{bB\Phi}$ and $O_{bG\Phi}$

are never generated in any such model up to the 1-loop order.

For the rest of the operators listed above, we find that they are generated in at least some of the models. In many cases though, the generation or not of certain operators depends directly on the quantum numbers of the heavy particles we have integrated out. Thus, $O_W$ is generated only when one of these heavy particles carries isospin, while the appearance of $O_G$ is possible only when it carries colour. Corresponding remarks apply also for the generation of $O_{WW}$ and $O_{GG}$ respectively, with the additional caveat that the responsible heavy particle which has been integrated out must be scalar.

Concerning the couplings of the generated operators, we find that those involving only gauge bosons like $e.g.$ $O_W$ or $O_G$, have strengths proportional to $g^n/\Lambda_{NP}^2$, with $g^n$ being some positive power of the related gauge coupling and $\Lambda_{NP}$ describing the order of magnitude of the mass of the new heavy states. On the contrary, the operators involving also the Higgs and/or the quark fields have their couplings determined by the new Yukawa couplings and $1/\Lambda_{NP}^2$ of course. Usually there is no principle to constrain the magnitude of these couplings, which can therefore be large. It appears therefore that the Higgs or quark involving operators, often have a better chance to be generated by NP at an observable level, than the purely gauge boson ones.

As a concrete and rather special example of these models we consider below the case where the only relevant heavy degrees of freedom which need to be considered at the $0.5 - 1 \text{ TeV}$ region is a heavy isosinglet colour-triplet boson $\Psi$ with hypercharge $-\frac{2}{3}$, and a Majorana fermion $F$ with vanishing $SU(3) \times SU(2) \times U(1)$ quantum numbers. The most general CP conserving and renormalizable lagrangian describing this model is obtained by adding to the SM lagrangian the interaction

$$L_i = \frac{1}{2} (v D^\mu F - M_F F F) + D_\mu \Psi^\dagger D^\mu \Psi - M_\Psi^2 \Psi^\dagger \Psi + 2 g_\Phi (\Psi^\dagger \Psi)(\Phi^\dagger \Phi) + f (\bar{T}_R \Psi^\dagger F + \text{h.c.}) ,$$

where $g_\Phi$ and $f$ are unknown real Yukawa couplings determined by NP. The masses of $\Psi$ and $F$ are generated before the electroweak breaking and they can be naturally assumed
to be large. Below we assume for simplicity

$$ M_\Psi \sim M_f \sim \Lambda_{NP} \ . $$

(99)

Such a model could be generated e.g. in a supersymmetric theory, where only a right stop and a singlet $B$-gaugino, identified with $\Psi$ and $F$ respectively, are sufficiently light to be relevant. Thus, in such a SUSY model we assume that all other new (s)particles, including all Higgses except the lightest one usually called $h$, are (say) above 3 $\text{TeV}$ and that they are are ignored. We also remark that in such a context $g_\Psi$ still remains a free Yukawa parameter, while the other Yukawa coupling $f$, is actually fixed by SUSY to

$$ f = - \frac{2\sqrt{2}}{3} g' \ . $$

(100)

After integrating out the new heavy particles, we find that at a scale just below $\Lambda_{NP}$, NP is described by the effective lagrangian

$$ L_{NP} = \frac{1}{(4\pi \Lambda_{NP})^2} \left\{ - \frac{g_s^3}{60} \mathcal{O}_G - \frac{g f^2}{90} \mathcal{O}_{DB} - \frac{g^2}{240} \mathcal{O}_{DG} 
- g_\Psi \frac{2g'^2}{9} \mathcal{O}_{BB} - g_\Psi \frac{g_s^2}{12} \mathcal{O}_{GG} + \frac{g^3}{2} \mathcal{O}_{\Psi 3} + \frac{g^2}{4} \mathcal{O}_{\Psi 2} + \frac{f^4}{12} \mathcal{O}_{tt} 
+ f^2 \frac{g m_t}{36 \sqrt{2} v} \mathcal{O}_{tB} + f^2 \frac{g_s m_t}{48 \sqrt{2} v} \mathcal{O}_{tG} + f^2 \frac{m_t}{6 \sqrt{2} v} \left[ 4 g_\Psi + \left( \frac{m_t}{v} \right)^2 \right] \mathcal{O}_{t1} 
+ f^2 \frac{m_t^2}{12 v^2} \mathcal{O}_{tq}^{(1)} - f^2 \frac{g f}{36} \mathcal{O}_{tB} - f^2 \frac{g_s m_t}{24} \mathcal{O}_{tG} \right\} . $$

(101)

We note that only 14 operators, out of the total number of 48, are generated in this particular model. We also note that only the Higgs or quark involving operators have their strengths affected by the Yukawa couplings $g_\Psi$ and $f$.

## 5 Final discussion

In this paper we have first established the full list of CP conserving $\text{dim} = 6$ and $SU(3)_c \times SU(2) \times U(1)$ gauge invariant operators, which should describe any kind of a high scale New Physics induced by the Higgs and the particles most strongly coupled to it. Taking into account the groupings implied in the Standard Model, we identified these later particles as being the gauge bosons and the quarks of the third family. This list of NP operators consists of 14 purely bosonic ones involving photon, $Z$, $W$, Higgs and gluon fields, as well as 34 operators containing in addition quarks of the third family.

Subsequently, we have restricted the above list by excluding the ”non-blind” operators, which are defined as the operators contributing at tree level to observables measurable at LEP1/SLC and low energy experiments. Such ”non-blind” operators are excluded
because present experiments imply values for their couplings that would be too weak to produce measurable effects at future colliders. On the other hand, operators contributing to $Z$ peak processes at 1-loop order, are retained, because the resulting constraints are so mild that they leave room for various effects observable at higher energies. The study of the purely gluonic operators $\mathcal{O}_{DG}$ and $\mathcal{O}_G$ is postponed for the future, since it needs somewhat detail QCD considerations. The corresponding constraints should come from the effect of these operators on multijet production and also on the running of $\alpha_s(q^2)$.

For each of the retained operators, we have computed the two-body scattering amplitudes and established the unitarity constraints. This allows us to associate without any ambiguity a NP scale to the coupling constant of each of these operators. This is the scale where the strongest tree level amplitude approaches the unitarity bound. Close to this scale, we expect various manifestations of NP to appear, like e.g. creation of new particles, resonances,....

This result is useful in various respects. If the NP scale is not known a priori, one can derive upper bounds for it, using the observability limits for the NP couplings obtained in present experiments, or expected at future colliders. For example, using the observability limits established in [23], one obtains that the study of anomalous 3-gauge boson couplings in $e^+e^- \rightarrow W^+W^-$ allows to reach NP scales up to about $1.5 \text{ TeV}$ at LEP2 and $10 \text{ TeV}$ at an $e^-e^+$ Linear Collider (LC) at 0.5 TeV. With $e^+e^- \rightarrow HZ, H\gamma$, if the value of $m_H$ allows the Higgs to be produced, one reaches in the study of anomalous Higgs-gauge boson couplings, scales of about $7 \text{ TeV}$ at LEP2 and $20 \text{ TeV}$ at LC [10]. With $\gamma\gamma$ processes, realizable through the laser backscattering method at LC, one should have independent informations on these various couplings and reach NP scales up to $20 \text{ TeV}$ in $\gamma\gamma \rightarrow VV$ and $65 \text{ TeV}$ in $\gamma\gamma \rightarrow H$ [4]. Finally anomalous top quark couplings contributing at tree level in $e^+e^- \rightarrow tt$ should be visible for NP scales less than about $35 \text{ TeV}$ at LC [24].

This procedure allows to make useful comparisons between different types of operators and between different sectors in the processes accessible to experiment. It is free from ambiguities in the normalizations of the coupling constants associated to different types of fields (scalars, spinors, vectors,...). We can more safely discuss the hierarchy that appears in the various sectors, either from theoretical reasons, or from purely experimental considerations.

Conversely, if one knows the NP scale from some theoretical prejudice, one can use the unitarity constraints to predict the values of the NP coupling associated to a given operator contributing in the NP effective Lagrangian. For example, specific New Physics schemes can fix the NP scale in some dynamical way, either from the mass of the new heavy particles, or from the strength of the underlying interaction.

Finally we have also investigated which of the above operators would be generated by NP in the $0.5 - 1 \text{ TeV}$ region, under the assumption that in this energy range the relevant group is just $SU(3) \times SU(2) \times U(1)$, and that the relevant nearby heavy particles whose integration out creates the effective NP operators, are just scalars and fermions. For their quantum numbers we have considered any combination of singlets and doublets under isospin, and singlets and triplets under QCD colour. This way we have identified 8 operators which are never generated. Whenever possible, we have also mentioned the
conditions determining the appearance or not of some of the operators that can in principle be generated. A specific model was presented in which NP is described by only 14 of the above operators.

We conclude that experimental limits on NP effects from present and future data when expressed in terms of NP scales and compared with theoretical landscapes such as those suggested by these dynamical models should be instructive when looking for hints about the origin and the basic structure of NP.

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