Critical Casimir force in the presence of random local adsorption preference

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Abstract – We study the critical Casimir force for a film geometry in the Ising universality class. We employ a homogeneous adsorption preference on one of the confining surfaces, while the opposing surface displays quenched random disorder, leading to a random local adsorption preference. Disorder is characterized by a parameter $p$ which measures, on average, the portion of the surface which prefers one component, so that $p = 0, 1$ correspond to homogeneous adsorption preference. By means of Monte Carlo simulations of an improved Hamiltonian and finite-size scaling analysis, we determine the critical Casimir force and the associated universal scaling function. We show that by tuning the disorder parameter $p$ the system exhibits a crossover between an attractive and a repulsive force. At $p = 0.5$ disorder allows to effectively realize Dirichlet boundary conditions, which are generically not accessible in classical fluids. Our results are relevant for the experimental realizations of the critical Casimir force in binary liquid mixtures.

Introduction. – When a fluid close to a critical point is confined between two surfaces, an effective force between them arise [1]. The resulting fluctuation-induced force is called critical Casimir force and has attracted numerous experimental and theoretical investigations, see Refs. [2,3] for recent reviews and also the updated list of references in Ref. [4]. The critical Casimir force is characterized by a universal scaling function, which depends on the universality class (UC) of the bulk phase transition, as well as on the shape of the confinement and on the boundary conditions (b.c.) therein, through the so-called surface universality classes [5,6]. Critical Casimir forces have been first indirectly measured by studying the thickness of wetting layers of $^4$He [7], and of classical [8] and quantum [9] binary mixtures, close to the critical point. More recently, a direct measure of the critical Casimir force has been obtained by monitoring individual colloidal particles immersed into a critical binary mixture and exposed to a substrate [10–12]. The critical Casimir force has also been studied through its influence on colloidal aggregation [13–15]. Binary liquid mixtures at the critical concentration undergo a continuous demixing phase transition in the Ising UC, where the order parameter is given by the deviation of the concentration of one of the two species with respect to the critical concentration. For a classical binary liquid mixture the surfaces involved generically prefer one component of the mixture, giving rise to an increase of the absolute value of the order parameter close to the surfaces. The force is found to be attractive (resp. repulsive) when the two surfaces prefer the same (resp. opposite) component of the mixture [10,11,17,18]. Experimental realizations of the critical Casimir force for colloidal particles immersed into binary liquid mixtures have proven to be very flexible in creating different b.c. for the surfaces involved. Beside surfaces with a homogeneous adsorption preference [10–12], the critical Casimir force has been investigated in the presence of a chemically structured substrate [12], leading to a laterally varying adsorption preference, as well as in the presence of a substrate with a gradient in the adsorption preference [13].

The influence of defects and quenched disorder on surface critical phenomena has attracted much interest, see Ref. [19] for a recent review. In this context, an important issue is whether disorder is a relevant or irrelevant perturbation. This problem has been extensively investigated in Ref. [20], where Harris-type criteria have been formulated. In agreement with early results on random-field surface disorder [21], uncorrelated random surface field with null expectation value and uncorrelated random surface couplings are found to be irrelevant at the ordinary tran-
For the latter type of disorder the irrelevance on the surface magnetization critical exponent can be rigorously established, provided that the surface bonds do not exceed the threshold of the special transition \[22\]. In this letter we study the critical Casimir force for a system in the Ising UC and in the film geometry, such that one confining surface displays a homogeneous adsorption preference while the opposite surface exhibits quenched disorder, leading to a random local adsorption preference. To this end, we combine numerical integration with Monte Carlo (MC) simulations of an improved spin model on a three-dimensional lattice. We introduce a parameter \(\alpha\) which controls the fraction of the disordered surface that one confining surface displays a homogeneous adsorption preference, while for \(\alpha = 0.5\) on average there is no preferential adsorption for one of the components. This setup is equivalent to the presence of an infinitely strong random field on the disordered surface, such that for \(\alpha = 0\) and \(\alpha = 1\) we recover a homogeneous adsorption preference, while for \(\alpha = 0.5\) on average there is no preferential adsorption for one of the components. This setup is equivalent to the presence of an infinitely strong random field on the disordered surface, such that for \(\alpha = 0.5\) the expectation value of the surface field vanishes. In agreement with the aforementioned Harris-type criterion, we find that in this case the surface effectively realizes a Dirichlet b.c., which corresponds to the ordinary UC. For \(\alpha \neq 0.5\), we observe a crossover to the limiting cases of homogeneous adsorption preference \(\alpha = 0\) and \(\alpha = 1\).

**Model and Method.** – We study an improved lattice model, whose critical behavior belongs to the Ising UC. As in recent studies \[1, 22–26\], we consider the Blume-Capel model \[27, 28\], which is defined on a simple cubic lattice, where the spin variables \(S_i\) on each site can take values \(+1, 0, -1\). The reduced Hamiltonian \(H\) is

\[
H = -\beta \sum_{\langle ij \rangle} S_i S_j + D \sum_i S_i^2, \quad S_i = -1, 0, 1, \quad (1)
\]

so that the Gibbs weight is \(\exp(-H)\). As done in previous investigations of this model \[1, 23, 26, 29\], in the following we shall keep \(D\) constant, treating it as a part of the integration measure over the spin configurations, while we vary the parameter \(\beta\) which controls the distance to the critical point. In the limit \(D \rightarrow -\infty\), the model reduces to the usual Ising model. Starting from \(D \rightarrow -\infty\), the phase diagram of the model displays a line of second-order phase transitions in the Ising UC, which ends at a tricritical point \(D_{\text{tri}}\). For \(D > D_{\text{tri}}\) the transition is of first order. In three dimensions, \(D_{\text{tri}}\) has been recently determined as \(D_{\text{tri}} = 2.0313(4)\) \[30\]. At \(D = 0.656(20)\) \[24\] the model is improved, i.e., the leading scaling correction \(\propto L^{-\omega}\), with \(\omega = 0.832(6)\) \[29\], vanishes. In the simulations presented here we fixed \(D = 0.655\), which is the value used also in recent numerical investigations of the critical Casimir force \[4, 23, 26\]. For such a value of \(D\), the model is critical for \(\beta = \beta_c = 0.387721735(25)\) \[29\].

We consider a three-dimensional film geometry, with lateral extension \(L\) and thickness \(L\), with \(L \gg L\). We impose periodic b.c. on the two lateral directions, and fixed b.c. on the two confining surfaces. On the upper surface we fix the spins to \(S_i = 1\), mimicking a homogeneous adsorption preference. The spins on the lower surface \(S_i\) are fixed to \(\pm 1\) according to the probability distribution

\[
P(S_i) = p\delta (S_i - 1) + (1 - p)\delta (S_i + 1). \quad (2)
\]

This choice of b.c. corresponds to a random local adsorption preference where, locally, the substrate prefers one component with probability \(p\). We note that for \(p = 0\) (resp. \(p = 1\), the b.c. reduce to that of a homogeneous adsorption preference, with opposite (resp. identical) adsorption preference for the two confining surfaces; these b.c. are usually denoted with \((+,-)\) and \((+,+)\). In the presence of quenched random disorder, one distinguishes between two averages, the thermal average \((\ldots)\), i.e., the average over the Gibbs measure at a given disorder configuration, and the average \([\ldots]\) over the disorder realizations.

The reduced free-energy density \(F(\beta, L, L, p)\), i.e., the free energy per volume \(V\) and in units of \(k_B T\) is given by

\[
F(\beta, L, L, p) = -\frac{1}{V} \left[ \ln \left( \frac{Z(\beta, L, L; \{S_i\})}{Z(\beta = 0, L, L; \{S_i\})} \right) \right], \quad (3)
\]

where \(V = L^2, Z(\beta, L, L; \{S_i\})\) is the partition function at a given disorder realization \(\{S_i\}\)

\[
Z(\beta, L, L; \{S_i\}) = \sum_{\{S_i\}} e^{-H}, \quad (4)
\]

and the denominator in eq. (3) ensures the normalization of \(F(\beta, L, L, p)\) such that \(F(\beta = 0, L, L, p) = 0\). In line with the prescription of quenched random disorder, in eq. (3) the average over the disorder distribution \([\ldots]\) is done after taking the logarithm of the partition function. The reduced bulk free-energy density \(F_{\text{bulk}}(\beta)\) is obtained by taking the thermodynamic limit of \(F(\beta, L, L, p)\):

\[
F_{\text{bulk}}(\beta) = \lim_{L, L \to \infty} F(\beta, L, L, p). \quad (5)
\]

Since \(F_{\text{bulk}}(\beta)\) is independent of the b.c., it does not depend on \(p\) either. The reduced excess free-energy \(F_{\text{ex}}(\beta, L, L, p)\) is defined as the remainder of \(F(\beta, L, L, p)\) after having subtracted \(F_{\text{bulk}}(\beta)\).

\[
F_{\text{ex}}(\beta, L, L, p) = F(\beta, L, L, p) - F_{\text{bulk}}(\beta). \quad (6)
\]

The critical Casimir force \(F_C\) per area \(L^2\) and in units of \(k_B T\) is defined as

\[
F_C(\beta, L, L, p) = \frac{\partial (L F_{\text{ex}})}{\partial L} \bigg|_{\beta, L, p} \bigg|_{\beta, L, p} = \frac{\partial (L F)}{\partial L} \bigg|_{\beta, L, p} + F_{\text{bulk}}(\beta). \quad (7)
\]

According to Renormalization-Group (RG) theory \[31\], the leading scaling behavior of \(F_C\) can be expressed as

\[
F_C(\beta, L, L, p) = \frac{1}{L^3} \theta (\tau, p), \quad (8)
\]

\[
\tau \equiv t (L/\xi_a^\beta), \quad t \equiv (\beta_c/\beta - 1),
\]
where $\theta(\tau, p)$ is a universal scaling function and $\xi_0^+ \approx 0.1415(4)$ is the non-universal amplitude of the correlation length $\xi$ in the high-temperature phase, which fixes the normalization of $\xi_0^-$. From Ref. [23] we infer $\xi_0^+ = 0.4145(4)$.

The determination of the critical Casimir force proceeds in two steps. We first replace the partial derivative on the r.h.s. of eq. (11) with a finite difference, computing the free energy difference $\Delta F(\beta, L, L||, p)$ per area $L||$, between a film of thickness $L$ and a film of thickness $L - 1$

$$\Delta F(\beta, L, L||, p) \equiv L F(\beta, L, L||, p) - (L - 1) F(\beta, L - 1, L||, p). \quad (9)$$

By using the definition of the critical Casimir force given in eq. (1), one finds [4]

$$F_C \left( \beta, L - \frac{1}{2}, L||, p \right) = -\Delta F(\beta, L, L||, p) + F_{\text{bulk}}(\beta), \quad (10)$$

where the choice of computing $F_C$ at the intermediate thickness $L - 1/2$ ensures that no additional corrections $\propto L^{-1}$ are generated in the Finite-Size Scaling (FSS) limit [4]. Two methods for computing $\Delta F(\beta, L, L||, p)$ have gained popularity in recent numerical investigations. In the coupling parameter approach introduced in Ref. [17], one defines a crossover Hamiltonian $H_\lambda$, which depends on a parameter $\lambda \in [0, 1]$ and is a convex combination of the Hamiltonians for films with thicknesses $L$ and $L - 1$. Then $\Delta F(\beta, L, L||, p)$ is obtained by a numerical integration over $\lambda$ of the thermal average of a suitable observable $O_\lambda$, determined by standard MC simulations of the Gibbs ensemble described by $H_\lambda$. An alternative approach, introduced in Ref. [26], consists in evaluating $F(\beta, L, L||, p)$ through a numerical integration over $\beta$, where the integral $\int d\beta F/\partial \beta$ can be determined by standard MC simulations, and subsequently calculating the free energy difference in eq. (11). Finally, the determination of the critical Casimir force requires the subtraction of the bulk free energy density $F_{\text{bulk}}(\beta)$. In Ref. [26] we have determined $F_{\text{bulk}}(\beta)$ for the present model, using periodic b.c. and achieving a precision of $10^{-7}$.

Eq. (8) describes only the leading scaling behavior of $F_C$. In order to extract the universal scaling function $\theta(\tau, p)$, it is important to take into account corrections to scaling. In a finite size, and for the lattice model considered here one expects that the leading scaling correction is due to the presence of non fully periodic b.c. and can be absorbed by the substitution $L \rightarrow L + c$, where $c$ is a non-universal, temperature–independent length $\xi_0^-$. Recently, this property has been checked numerically in Refs. [3][23][25]. In the present case, upon employing the substitution $L \rightarrow L + c$ in eq. (8) and using it in eq. (10), we obtain the following FSS Ansatz for $\Delta F(\beta, L, L||, p)$:

$$\Delta F(\beta, L, L||, p) \approx F_{\text{bulk}}(\beta) - \frac{1}{(L - 1/2 + c)^{1/\nu}} L \chi(\ell L - 1/2 + c), \quad (11)$$

A detailed discussion on the type of scaling corrections and possible modifications to eq. (11) can be found in Ref. [26].

Results. – We have first determined the Casimir force at criticality. To this end, we have used the coupling parameter approach introduced in Ref. [17] and also used in Refs. [3][23][26][33]; see Ref. [4] for a detailed discussion on the implementation of the algorithm. In a series of MC simulations we have computed $\Delta F(\beta = \beta_c = 0.387721375, L, L||, p)$ for $L = 8, 12, 16, 24, 32$, and $48$, and for three different aspect ratios $\rho = L/L|| = 1/8, 1/12, \text{and } 1/16$. We have sampled the distribution of the randomly frozen spins given in eq. (2) to the statistical error bars are essentially determined by the number of disorder samples (see, e.g., App. B of Ref. [23]). We have checked that within the statistical precision our results are independent of the aspect ratio $\rho$, thus we consider them as reliably describing the limit $\rho \rightarrow 0$. By setting $\beta = \beta_c$, in eq. (11) we obtain the expected leading FSS behavior of $\Delta F(\beta = \beta_c, L, L||, p)$:

$$\Delta F(\beta_c, L, L||, p) \approx F_{\text{bulk}}(\beta_c) - \frac{\Theta(p)}{(L - 1/2 + c)^{1/\nu}}, \quad (12)$$

where $\Theta(p) \equiv \theta(0, p)$ is the Casimir amplitude at criticality. In Table I we report the results of the fits of $\Delta F(\beta_c, L, L||, p)$ to eq. (12) leaving $F_{\text{bulk}}(\beta_c), \Theta(p)$, and $c$ as free parameters, as a function of the smallest thickness $L_{\text{min}}$ used in the fits. We observe that $\Theta(p)$ changes sign with $p$: the force is attractive for $p > 0.5$, and repulsive for $p < 0.5$. Furthermore the fit results for $p < 0.5$ (resp. $p > 0.5$) appear to approach a common value $\Theta \approx 5 - 5.5$ (resp. $\Theta \approx -0.8$). Inspection of the fit results for $p = 0.5$ reveals a good $\chi^2/\text{DOF}$ ($\text{DOF}$ denotes the degrees of freedom); however, the fitted values of $\Theta(p = 0.5)$ display a small dependence on $L_{\text{min}}$ which is larger than the statistical error bars. In order to assess the size of possible subleading or competing scaling corrections, we have fitted our MC data for $\Delta F(\beta_c, L, L||, p)$ to the alternative Ansatz $F_{\text{bulk}}(\beta_c) - \Theta(p)(1 - C(L - 1/2)^{-\omega}) / ((L - 1/2)^{3})$, leaving $F_{\text{bulk}}(\beta_c), \Theta(p), C, \text{and } \omega$ as free parameters. For this fit we obtain $\Theta(p = 0.5) = 0.52(2)$ and $\omega = 1.1(1)$, which is consistent with eq. (12), where the leading scaling corrections are $\propto L^{-1}$. Nevertheless, we cannot exclude the presence of additional irrelevant operators with an RG-dimension close to $-1$. By judging conservatively the fit results reported in Table I we can infer $\Theta(p = 0.5) = 0.51(2)$, which agrees with the results for $L_{\text{min}} = 12, 16,
including a variation of one error bar, and it also agrees with the fit result for \(L_{\text{min}} = 8\) and the one obtained with a free correction-to-scaling exponent \(\omega\). As a further check of the reliability of the value of \(\Theta(p = 0.5) = 0.51(2)\), we observe that the fitted values of \(F_{\text{bulk}}(\beta_c)\) fully agree with the result reported in Ref. \[23\]. \(F_{\text{bulk}}(\beta_c) = -0.0757368(4)\) and with the values obtained in Ref. \[26\] for the various b.c. considered therein. Our result for \(p = 0.5\) can be compared with the critical Casimir amplitude for a system in the film geometry and in the Ising UC for the so-called \((+,o)\) b.c., where one of the confining surfaces has uniformly fixed spins, and the opposite surface has open b.c., thus realizing Dirichlet b.c. which corresponds to the ordinary surface UC. Recent numerical studies have reported \(\Theta_{(+,o)} = 0.497(3)\) \[24\] and \(\Theta_{(+,o)} = 0.492(5)\) \[26\]. These values are in full agreement with our result for \(p = 0.5\) and thus suggest an effective “averaging” mechanism: the resulting critical Casimir force is equivalent to the force for a system where the disordered surface is substituted by a surface where the spin variables are fixed to their expectation value \(\langle s_i \rangle = 2p - 1 = 0\), thus realizing open b.c. Dirichlet b.c. are generically not accessible in fluids; such b.c. can also be obtained with a chemically striped surface, in the limit of narrow stripes \[26\]. The disorder setup for \(p = 0.5\) is equivalent to a random surface field of vanishing expectation value and in the limit of infinite amplitude. According to a Harris-type criterion for surface disorder \[20\], random surface field with null expectation value is an irrelevant perturbation at the ordinary transition. Thus, our MC results confirm the validity of this Harris-type criterion in the limit of infinite amplitude of the random field. The fit results for \(p = 0.2, 0.3\) are less satisfactory. The fitted values of \(\Theta(p)\) exhibit a systematic drift as \(L_{\text{min}}\) is increased and a good \(\chi^2/\text{DOF}\) is reached for \(L_{\text{min}} = 16\) only. It is therefore not possible to independently assess the reliability of the fit results. Nevertheless, at least for \(p = 0.2\), \(L_{\text{min}} = 16\) the fitted value of \(F_{\text{bulk}}(\beta_c)\) agrees with the previous determination \(F_{\text{bulk}}(\beta_c) = -0.0757368(4)\) \[23\] and the resulting value of \(\Theta(p = 0.2) = 0.578(14)\) is in marginal agreement with a previous determination for \((+,−)\) b.c. \(\Theta_{(+−)} = \Theta(p = 0) = 5.613(20)\) \[23\], suggesting that the critical behavior is controlled by the \(p = 0\) fixed point. Also the fit results for \(p = 0.3\) are compatible with a slow approach to the \((+,−)\) fixed point. The fit results for \(p = 0.7, 0.8\) are more stable. Aside from the fits for \(p = 0.7\) and \(L_{\text{min}} = 8\), we observe a good \(\chi^2/\text{DOF}\), the fitted values are stable upon increasing \(L_{\text{min}}\). \(F_{\text{bulk}}(\beta_c)\) agrees with the available determination \(F_{\text{bulk}}(\beta_c) = -0.0757368(4)\) \[23\]. By judging conservatively the fit results, we infer \(\Theta(p = 0.7) = -0.82(4), \Theta(p = 0.8) = -0.81(3)\). These values agree with the critical Casimir amplitude for \((+,+)\) b.c. \(\Theta_{(++)} = \Theta(p = 1) = -0.820(15)\) \[23\], suggesting that for \(p > 0.5\) the critical behavior is controlled by the \(p = 1\) fixed point.

The discussion concerning the various fixed points reported above is further confirmed by the computation of the critical Casimir scaling functions \(\theta(\tau, p)\). To this end, we have used the integration scheme introduced in Ref. \[35\], with the optimization discussed in Ref. \[26\]. In a series of MC simulations we have determined \(\Delta F(\beta, L, L_{\beta}, p)\) for \(L = 8, 12, 16\), aspect ratios \(\rho = 1/8, 1/12, 1/16\), and for a range of temperatures around the critical point. We have averaged over \(N_s = 100 - 1000\) disorder samples. We have checked that our results are a reliable extrapolation of the \(\rho \rightarrow 0\) limit and we have taken the average over the three aspect ratios. The universal scaling function \(\theta(\tau, p)\) has been obtained by inverting eq. \[14\], using the recent accurate determination of \(F_{\text{bulk}}(\beta)\) of Ref. \[26\]. The value of the non-universal length \(c\) has been extracted from the fit results reported in Table I. In Fig. I we show the resulting scaling func-

### Table 1: Fits to eq. (12). \(L_{\text{min}}\) is the minimum lattice thickness taken into account. \(\text{DOF}\) denotes the degrees of freedom.

| \(L_{\text{min}}\) | \(F_{\text{bulk}}(\beta_c)\) | \(\Theta\) | \(c\) | \(\chi^2/\text{DOF}\) |
|----------------|-----------------|-----|-----|------------------|
| 8              | -0.07573884(9)  | 5.342(4) | 0.845(3) | 789.8/15        |
| 12             | -0.0757374(1)   | 5.519(8) | 1.018(7) | 32.0/12         |
| 16             | -0.0757370(1)   | 5.578(14) | 1.09(2)  | 5.1/9           |
| 8              | -0.0757432(1)   | 4.523(5) | 1.390(4) | 4508.9/15       |
| 12             | -0.0757394(1)   | 5.02(1)  | 2.00(1)  | 228.0/19        |
| 16             | -0.0757382(2)   | 5.25(2)  | 2.33(3)  | 9.8/9           |
| 8              | -0.07573673(8)  | 0.531(3) | -0.89(2) | 13.4/15         |
| 12             | -0.0757368(1)   | 0.525(7) | -0.94(5) | 11.1/12         |
| 16             | -0.0757369(1)   | 0.505(12)| -1.2(1)  | 4.7/9           |
| 8              | -0.0757379(1)   | -0.98(1) | 4.20(5)  | 111.8/15        |
| 12             | -0.0757371(2)   | -0.84(2) | 3.2(1)   | 13.1/12         |
| 16             | -0.0757370(2)   | -0.81(3) | 2.9(3)   | 10.2/9          |
| 8              | -0.0757368(1)   | -0.832(7)| 1.21(3)  | 12.9/15         |
| 12             | -0.0757368(1)   | -0.83(1) | 1.20(7)  | 12.6/12         |
| 16             | -0.0757367(2)   | -0.81(2) | 1.0(2)   | 9.5/9           |
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...tion $\theta(\tau, p)$, for $p = 0.2$ and $p = 0.3$. We also compare our results with the scaling function $\theta_{(+,-)}(\tau) = \theta(\tau, p = 0)$, as computed in Ref. [23]. Indeed, we observe that the MC results for $p = 0.2$ and the largest film thickness $L = 16$ agrees with the $\theta_{(+,-)}(\tau)$ curve for $\tau \gtrsim -5$, while a systematic deviation is observed for lower values of $\tau$. Nevertheless, a comparison of the curves for $L = 12$ and $L = 16$ suggests that our results approach the $\theta_{(+,-)}(\tau)$ curve for increasing values of $L$. As in the case of the critical Casimir amplitude, the comparison between $\theta_{(+,-)}(\tau)$ and $\theta(\tau, p = 0.3)$ is less satisfactory: for $L = 16$, $\theta(\tau, p = 0.3)$ agrees with $\theta_{(+,-)}(\tau)$ for $\tau \gtrsim 5$, but exhibits a systematic deviation for $\tau \lesssim 5$. In Fig. 2, we show our results for the scaling function $\theta(\tau, p = 0.5)$. We also compare our determinations with those obtained for $(+, o)$ b.c. in Refs. [24,26,53].

Our results for $L \geq 12$ agree very well with the curves for $\theta_{(+, o)}(\tau)$ as determined in Refs. [24,26]. The small, but significant, deviation from the curve of Ref. [53] may be due to residual scaling corrections [24] in the data of Ref. [53]. In Fig. 3 we show the resulting scaling function $\theta(\tau, p)$, for $p = 0.7$, 0.8 and $\theta_{(+, +)}(\tau) = \theta(\tau, p = 1)$, as determined in Ref. [23]. Our results for $p = 0.8$, $L = 16$ agrees well with $\theta_{(+, +)}(\tau)$, showing only a small deviation for $\tau \gtrsim 10$. The data for $p = 0.7$ exhibit a larger deviation from the $\theta_{(+, +)}(\tau)$ curve. Nevertheless, the data for $L = 16$ agrees with the scaling function $\theta_{(+, +)}(\tau)$ for $\tau \lesssim 4$, and a comparison of the curves for $L = 12$ and $L = 16$ suggests an approach to the $\theta_{(+, +)}(\tau)$ curve upon increasing $L$. Interestingly, we observe that the MC curve for $L = 8$ becomes positive for $\tau \lesssim -3$; the same feature is observed also for the $L = 12$ curve and $\tau \lesssim -6$, although with a smaller amplitude.

The observed $L$-dependence of the resulting $\theta(\tau, p)$ scaling functions for $p \neq 0.5$ implies that the Ansatz of eq. [11] does not completely capture the FSS behavior of the model. Our results clearly show that the critical behavior for $p \neq 0.5$ differs from the critical behavior at $p = 0.5$, thus $p - 0.5$ is a relevant perturbation to the $p = 0.5$ fixed point. For $p \to 0.5$ and neglecting for simplicity scaling corrections, the critical Casimir force is expected to exhibit a crossover behavior:

$$F_C(\beta, L, L_{\|}, p) = \frac{1}{L^5} \theta_{cr}(\tau, (p - 0.5) L^\eta), \quad p \to 0.5,$$

where $y > 0$ is the RG dimension of the relevant perturbation. If indeed the critical behavior for $p > 0.5$ is controlled by the $p = 1$ fixed point, then for $0.5 < p < 1$ and a finite lattice size $L$, the system is expected to exhibit a crossover behavior from the $p = 0.5$ fixed point to the $p = 1$ fixed point which is more significant for values of $p$ closer to $p = 0.5$, and for smaller lattice sizes. An analog crossover behavior is expected for $0 < p < 0.5$ and a finite lattice size $L$ if the critical behavior for $p < 0.5$ is controlled by the $p = 0$ fixed point. This is consistent with the observation that our results for $p = 0.3$ (resp. for $p = 0.7$) show a larger deviation from the $\theta_{(+, +)}(\tau)$ (resp. $\theta_{(+, +)}(\tau)$) curve with respect to the corresponding results for $p = 0.2$ (resp. $p = 0.8$), see Figs. [14]. Also the change of sign observed for $p = 0.7$ and $L = 8, 12$ is consistent with a crossover from the $p = 0.5$ repulsive fixed point to the $p = 1$ attractive fixed point. The observed crossover behavior is analogous to the crossover effect induced on the critical Casimir force by the presence of finite surface fields [24,53,56]. In view of this analogy, one may expect to identify the RG dimension $y$ in eq. [19] with the RG dimension of the surface field at the ordinary transition $y_{h_1} = 0.7249(6)$ [24].

**Summary.** We have numerically investigated the critical Casimir force in a film geometry, where one surface exhibits a homogeneous adsorption preference, and the opposing surface displays a random local adsorption preference, characterized by a parameter $p$ which measures, on average, the portion of the surface which prefers one component. When $p = 0.5$, on average there is no preferential adsorption for one component and the surface
effectively realizes Dirichlet b.c., which generically do not hold for fluids. The resulting critical Casimir force belongs to the (+, o) UC. Our results suggest that when \( p > 0.5 \) (resp. \( p < 0.5 \)) the critical Casimir force belongs to the (+, +) (resp. (+, −)) UC, albeit with large crossover effects. To further strengthen this picture, larger lattice sizes would be required. The present setup can be experimentally realized by monitoring the thickness of a wetting layer of a binary liquid mixture close to its critical point on a disordered substrate. Another possibility is provided by considering a spherical colloidal particle in front of such a disordered substrate, provided that the radius of the particle is much larger than its distance to the wall.

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