Ultrasonic elastic responses in a monopole lattice

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Abstract

The latest experimental advances have extended the scenario of coupling mechanical degrees of freedom in chiral magnets (MnSi/MnGe) to the topologically nontrivial skyrmion crystal and even monopole lattices. Equipped with a spin-wave theory highlighting the topological features, we devise an interacting model for acoustic phonons and magnons to explain the experimental findings in a monopole lattice with a topological phase transition, i.e. annihilation of monopole–antimonopole pairs. We reproduce the anisotropic magnetoelastic modulations of elastic moduli: drastic ultrasonic softening around the phase transition and a multi-peak-and-trench fine structure for sound waves parallel and orthogonal to the magnetic field, respectively. Comparison with experiments indicates that the magnetoelastic coupling induced by Dzyaloshinskii–Moriya interaction is comparable to that induced by the exchange interaction. Other possibilities such as elastic hardening are also predicted. The study implies that the monopole defects and their motion in MnGe play a crucial role.

1. Introduction

Topology is acquiring a more and more significant role in condensed matter physics. As well as an increasing focus on topological classification of quantum phases of matter, ideas concerning topology and magnetic stripes, vortices, domain walls, etc., have been under experimental and theoretical investigation for a long time \cite{1}. More recently, the realization of topologically nontrivial skyrmions or skyrmion crystal (SkX) in chiral magnets \cite{2–11} has revived the originally proposed hadron model \cite{12} and the scenario of interplay between the orbit and spin of electrons and ions, offering many of brand new phenomena \cite{13–22} together with the potential for application in magnetic storage \cite{23–29}. Symmetry breaking of spins in noncentrosymmetric chiral magnets, which show both Heisenberg exchange interaction \cite{27,29} and the Dzyaloshinskii–Moriya interaction (DMI) \cite{30–32} due to spin–orbit coupling, can bring about the skyrmion texture \cite{2,3}. A minimal Hamiltonian for isolated skyrmions or a skyrmion lattice in d spatial dimensions includes EXI, Bloch-type DMI and Zeeman energy \cite{33,34} (setting $\hbar = 1$ throughout this paper)

$$
\mathcal{H}_{\text{SkX}} = \int d^d x \left[ \frac{J}{a_0^2} (\nabla \vec{S})^2 + \frac{D}{a_0^d} S \cdot (\nabla \times \vec{S}) - \frac{1}{a_0^d} \mu_0 \vec{S} \cdot \vec{B} \right]
$$

(1)

in which $a_0$ is the microscopic lattice constant. In two-dimensional (2D) thin films, magnetic anisotropies usually play a key role in stabilizing the skyrmion phase. Unlike other modulated magnetic structures such as the conical and helical phases that can be realized therein, a skyrmion winds a sphere a certain number of times and is characterized by the topological skyrmion number \cite{35}. In such magnetic systems, the involvement of topology is often enhanced or even induced by a strong correlation effect. Indeed, based on an adiabatic approximation for the real-space Berry phase produced by the fixed-length spin texture of skyrmions, the constraint of strong coupling with itinerant electrons can be described by the emergent electromagnetic field (EEMF) \cite{15,36–39}.

Apart from the most common skyrmion lattice, a vertical magnetic field–induced triangular lattice of skyrmions observed in chiral magnets, the coalescence or bisection of columnar skyrmion tubes has been
observed experimentally in bulk Fe$_1$-$_x$Co$_x$Si [40]. Such points are in fact singularities or defects (the Bloch points [41, 42]) in the spin texture

$$\vec{n} = \vec{S} / |\vec{S}|$$

where the spin moment $\vec{S} = \vec{S}$. Specifically, they can be regarded as monopoles or antimonopoles [39, 40, 43] in terms of the EEMF and can create or annihilate skyrmions, which are tubes in three dimensions (3D). In a skyrmion lattice with such defects, the skyrmion tubes may not penetrate the sample and can end at monopole defects at depth. For simplicity, we henceforth may use the term 'monopole' to refer to both monopoles and antimonopoles. A two-skyrmion-merging model based on the nonlinear sigma model has been used to study the effect of monopole defects on transport [44]. Another solution for the end of such skyrmions has been investigated in the conical phase [45]. The dynamics and energetics of monopole defects have also been studied on the basis of the Landau–Lifshitz–Gilbert equation [46, 47].

Partly due to the intrinsic 3D nature of monopole defects, there is no experimental or theoretical evidence for the formation of a lattice of monopoles in thin films [2, 3, 48–50]. Nevertheless, it should be feasible to realize this in 3D bulk chiral magnets. First theoretically suggested as a superposition of multiple non–coplanar spin spirals [51], a 3D lattice of monopole defects was corroborated by subsequent thermodynamic calculation [52]. Experimental work, from an analysis of electric and thermal Hall effects and the Nernst effect in contrast to those of conventional MnSi-like defects, strongly inferred that 3D bulk MnGe under pressure possessed skyrmion tubes and a simultaneous lattice of the monopole–antimonopole defects [53–55]. Notably, with a lattice variant of the minimal Hamiltonian equation (1), Monte Carlo simulation confirmed the formation of a monopole–antimonopole lattice in 3D as long as the magnetic period is short [56], which is just the situation for MnGe observed in the experiments. This endorses our inclusion of EXI and DMI in section 2 with other possible magnetic interactions. Further, a recent study using Lorentz transmission electron microscopy clearly revealed the magnetic structure of MnGe in accordance with the three-orthogonal-spiral model anticipated [57].

It is necessary to note the distinction between two properties of the magnetic ordering background, i.e. the spin texture $\vec{n}(\vec{r})$ and the original spin moment field $\vec{S}(\vec{r})$, as we shall study the effect of fluctuations in these. $\vec{S}(\vec{r})$, which is used to construct multi-spiral spin density waves, is non-singular and topologically trivial because any configuration mapped onto a three-ball $B^3$ can be smoothly connected to $\vec{S}(\vec{r}) = \vec{0}$. However, this work aims to highlight the characteristic effect of topologically nontrivial defects and the orientation field $\vec{n}(\vec{r})$, which is inherently described by the second homotopy group of a two-sphere $S^2$ that cannot shrink to a point, and naturally appears to be physically more relevant. On the other hand, localized spin moments for MnGe in the strong correlation regime hardly vary in magnitude and the saturated magnetization is large and only vanishes at singular defects. Indeed, this choice of a unit-length field has already proved to be appropriate for describing the influence on conduction electrons in strongly correlated MnGe, where a 3D monopole lattice even exists in the absence of an external magnetic field [39, 43].

With the expectation of new effects due to monopole defects, we initiated our theoretical investigation to study the phenomena emerging from coupling between other components and the topologically nontrivial monopole lattice in the hosting material MnGe by calculating the influence of spin-wave excitations therein. Our first study focused on longitudinal electric transport in MnGe which is massively influenced by quantum fluctuations of the EEMF due to spin waves; we identified a topological phase transition arising from strong correlation [39]. One more intriguing possibility is coupling to phonons in the host solids. Indeed, unconventional ultrasonic responses of elastic stiffness have been observed recently. Longitudinal sound waves were propagated in the skyrmion phases in both MnSi [58, 59] and MnGe [43] with the sound propagating in a direction parallel ($k_z$-mode) or orthogonal ($k_y$-mode) to the applied external magnetic field along the $z$-axis, as shown in figure 1. The the SkX phase was recognized by a distinct elastic stiffness with anisotropy for the two modes of MnSi. In contrast, varying with the external magnetic field, MnGe showed not only a drastic softening of $k_z$-mode stiffness but also a multi-peak-and-trench fine structure of $k_y$-mode stiffness, which were much stronger than in the SkX phase of MnSi. The magnetic field range for the softening considerably overlaps with the aforementioned topological phase transition, namely the destruction of the monopole lattice. It is not surprising that fluctuations associated with such a qualitative structural transition produce a dramatic modification in mechanical properties. The EEMF not only helps describe itinerant electrons coupled with localized spins but also captures the essential feature of the nontrivial spin texture, hence we find it rather informative even for the magnetoelastic response, which will be discussed later. It is reasonable to suggest that the qualitatively different responses of MnGe compared with MnSi are due to the influence of the monopoles/antimonopoles on the magnon fluctuations therein.

We therefore reuse our spin-wave theory and propose in this paper an interaction theory for magnons and phonons induced by both EXI and DMI, which reproduces and explains the magnetoelastic features found in experiments. We refer the reader to other work [39, 43] for a detailed description of the monopole lattice consisting of three orthogonal spin spirals, the spin-wave theory of a skyrmion lattice or monopole lattice and...
the experimental confirmation. In section 2, we first devise the magnetoelastic interactions and derive the effective phonon theory by integrating out the magnon degrees of freedom. The influence of magnons on renormalized phonon excitations is discussed in section 3.1, with an emphasis on the renormalized phonon linear mode. In sections 3.2 and 3.3 we compare the experimental observations with our theoretical predictions and explain the physical origin of the magnetic field-dependent evolution of the hybridized excitation spectra. We emphasize the distinction between the monopole lattice in MnGe and the conventional SkX in MnSi in the rest of section 3 and conclude in section 4.

2. Theoretical models

2.1. Magnetoelastic coupling
As already mentioned, it is the unit-norm constrained spin texture \( \hat{n} = S/|S| \) that produces the EEMF and yields the nontrivial topology of the skyrmion and the monopole rather than the bare spin moment \( \hat{S} \) itself, which manifests strong electron correlation [39, 43]. If \( \hat{S} \) were used, the form factors to be calculated in the following would assume forms that are too simple to give rise to sufficiently complex magnetic field dependence and the correct theory to reproduce experiments. We thus make use of \( \hat{n} \) instead of \( \hat{S} \) to derive the appropriate magnetoelastic interactions from the Hamiltonian equation (1). This is the minimum model capable of producing a stable monopole lattice, as discussed previously. This study shows that EXI and DMI are adequate and essential to capture the correct physics.

When longitudinal sound waves are artificially generated in a material the lattice structure will be perturbed; this is described by a longitudinal phonon. However, the strengths of EXI and DMI depend on lattice bond lengths and hence the magnetoelastic coupling (figure 2). Such a perturbation to the magnetic interactions by mechanical degrees of freedom can be accounted for by expanding the strengths of EXI and DMI up to the linear order of phonon degrees of freedom

\[
J(\nabla \hat{n})^2 \rightarrow (J_0 + \alpha_{\text{EXI}}(\partial_j \mu_i)) \partial_j n_i \partial_i n_i \tag{2a}
\]

\[
D\hat{n} \cdot (\nabla \times \hat{n}) \rightarrow \epsilon \Phi(D_0 + \alpha_{\text{DMI}}(\partial_j \mu_i)) n_i \partial_j n_i \tag{2b}
\]

wherein \( \hat{n} \) is the lattice displacement from equilibrium in the continuum limit, \( \alpha_{\text{EXI/DMI}} \) is the coefficient of this expansion and summation over repeated indices is understood henceforth. We emphasize four features of this expansion:

(1) In conformity with translational symmetry, either \( J \) or \( D \) depends on \( \partial_j \mu_i \) rather than lattice displacement \( \hat{n} \) itself.
(2) Alongside, \( \partial_j \mu_i \) obviously concerns the longitudinal phonon.

Figure 1. Illustration of experimental settings for the \( k_{\parallel} \)-mode and \( k_{\perp} \)-mode. \( k \) and \( H \) stand for the sound wave and the external magnetic field, respectively.

Figure 2. Illustration of the mechanism of magnetoelastic coupling.
(3) All spatial derivatives match in direction, since any \( \partial_j \mathbf{n} \), reflecting magnetic interactions between two adjacent spins sitting at the endpoints of a lattice bond along the \( j \)-direction in a lattice model, should be affected by the longitudinal phonon propagating along the same direction through \( \partial_j u_i \).

(4) As stated in (3), these magnetoelastic couplings take anisotropy into account so as to reproduce the experimental observations.

The next step is to expand \( \mathbf{n} \) as \( n_i = n_i^{(0)} + \phi^\mu \partial_\mu n_i^{(0)} \) in equations \( (2a) \) and \( (2b) \) to incorporate the spin-wave degrees of freedom \( \phi^\mu = (\phi, \hat{b} \cdot \mathbf{m}) \), \( \mu = 1, \ldots, 6 \) where the superscript \( (0) \) means the ground state value or setting \( \phi^\mu = 0 \) after taking the derivative. For a spin spiral indexed by \( i \), this \( \phi^\mu \) is the phase of the constituent spin density wave, which also indicates the spatial shift of the skyrmion lattice or the position of the monopole defects. \( m_i \) is the magnetization along the propagation axis of spin spiral \( i \), and actually record the information about the complicated magnetic structure affecting the magnetoelastic interactions induced by EXI or DMI. After Fourier transformation to the momentum space, we take the spatial integration to be carried assuming that \( \partial^\mu \phi^\mu \) is uniform, resulting in zero for the total derivative. This uniform magnetization is the magnetization along the propagation axis of spin spiral \( i \), generating rotation around this axis. Note that we consider the situation where there are three orthogonal spin spirals and it is the deviations of those quantities away from their static mean field values that constitute the spin-wave fields \( \phi^\mu \) in the expansion [39].

Without loss of generality, we thus set the static value of any \( \phi^\mu \) to 0 and denote the fluctuation in \( m_i \) by \( \hat{b} m_i \).

Starting from the EXI and DMI induced magnetoelastic interaction parts in equations \( (2a) \) and \( (2b) \), up to terms bilinear in spin-wave and phonon fields, one arrives at

\[
\text{EXI part: } [\partial_\mu (\partial_\nu n_i^{(0)} \partial_j n_i^{(0)}) \phi^\mu + 2 \partial_\mu n_i^{(0)} \partial^j n_i^{(0)} \partial^j \phi^\mu ] \partial_j u_i \tag{3a}
\]

\[
\text{DMI part: } \varepsilon^{ijk} [\partial_\mu (n_i^{(0)} \partial_j n_k^{(0)}) \phi^\mu + n_i^{(0)} \partial_\nu n_k^{(0)} \partial^j \phi^\mu ] \partial_j u_i , \tag{3b}
\]

wherein we temporarily omit all the prefactors for simplicity. One crucial criterion is the translational invariance in the continuum model to be derived, i.e. for the spin-wave field \( \phi^\mu \); only its spatial derivative can enter simply because \( \phi^\mu \) is the (phase) displacement field of spin spiral \( i \). Therefore, when \( \mu = 1, 2, 3 \), i.e. for the \( \phi_1 \) field, \( \partial^\mu \phi^\mu \), applied upon the parenthesis in the first term in either equation \( (3a) \) or equation \( (3b) \) is equivalent to a spatial derivative. We note that the typical wave vector of the periodic function in the parenthesis is much larger than that of the slowly varying spin-wave field \( \phi_1 \). Since equations \( (3a) \) and \( (3b) \) will be part of a Lagrangian density, spatial integration can be carried assuming that \( \phi^\mu \) is uniform, resulting in zero for the total derivative. This procedure exactly eliminates the term linear in the \( \phi_1 \) field, otherwise translational invariance would be violated. However, this is unnecessary for \( \mu = 4, 5, 6 \), i.e. for the fluctuating field \( \hat{b} \cdot \mathbf{m} \). Therefore, in either equation \( (3a) \) or equation \( (3b) \), for \( \mu = 1, 2, 3 \), only the second term remains.

Thus for either EXI or DMI we attain two terms

\[
C^{i\mu} (\partial_j \phi^\mu \partial_j u_i) + D^{i\mu} (\phi^\mu, \partial_j u_i) \tag{4}
\]

in total, wherein \( \mu = 1, \ldots, 6 \) but \( D^{i\mu} \equiv 0 \) whenever \( \mu < 4 \). We call the functions preceding the bilinear fields form factors; these render the coupling between magnons and phonons nonuniform in space. They are defined as \( C^{i\mu} = 2 \partial_\mu n_i^{(0)} \partial_j n^{(0)} \), \( D^{i\mu} = \partial_\mu \partial_j \phi \), \( C^{j\mu} = 0 \), \( D^{j\mu} = 0 \), and actually record the information about the complicated magnetic structure affecting the magnetoelastic interactions induced by EXI or DMI. After Fourier transformation to the momentum space, we take the spatial average over a magnetic unit cell of the form factors, making the couplings dependent only on the variable uniform magnetization \( m_i \) along the \( z \)-axis. Therefore we have an exactly solvable theory without couplings between unequal momenta. This uniform magnetization \( m_i \) directly relates to the external magnetic field applied to the system along the \( z \)-axis. The magnetic field dependence of the ultrasonic responses is due to the fact that the magnetic texture of the monopole lattice varies with respect to \( m_i \) [39].

2.2. Effective theory of phonons

A standard free theory of the longitudinal phonon reads

\[
\mathcal{L}_{ph} = \frac{1}{2} [\kappa (\partial_i u)^2 + \kappa (\nabla \cdot u)^2] , \tag{5}
\]

where \( \kappa \) is an elastic constant and \( \kappa \) is the mass density and we work in imaginary time henceforth. For MnGe, the spin-wave \( \mathcal{L}_{SW} \) takes the form [39]

\[
\mathcal{L}_{SW} = \sum_{\alpha = x, y, z} [i e \alpha \beta A_{\alpha \beta} \partial_\alpha \phi_\beta - i e B_{\kappa} \phi_\kappa + \chi m_0 \phi_\alpha + \chi \hat{m}_0 \hat{u} + \rho (\nabla \phi_\alpha)^2] \tag{6}
\]

where \( A = -2 q \sum_{\kappa} \frac{1}{a^2} \kappa A_{\kappa} \frac{1}{a^2} = \frac{1}{a^2}, \chi = \frac{B_{\kappa}}{A_{\kappa}}, \rho = \frac{1}{a^2} \). Note that the \( m_\alpha \)-dependent \( B \) is the emergent magnetic field that characterizes the nontrivial topology of the skyrmions or monopoles in the system. We get the magnetoelastic interactions...
\[ \mathcal{L}_{\text{ME}} = \frac{\alpha_{\text{EXI}}}{\hbar c^2} \left( C_{\text{EXI}}^{ij} \partial_i \varphi_j \partial_j u + D_{\text{EXI}}^{ij} \varphi_i \partial_j u_j \right) + \frac{\alpha_{\text{DMI}}}{\hbar c^2} \left( C_{\text{DMI}}^{ij} \partial_i \varphi_j \partial_j u + D_{\text{DMI}}^{ij} \varphi_i \partial_j u_j \right) \] (7)

from section 2.1. Because the three parts, equations (5), (6) and (7), comprising the full theory are all bilinear, we can diagonalize the actions in the energy–momentum space. Below, \( k \) is shorthand for the argument \((k, z)\), where \( z \) is a generic complex frequency that can equal \( i\omega_n \) for instance. The longitudinal phonon action is transformed to

\[ S_{\text{ph}} = \int_0^\beta d\tau \int d^d \mathbf{r} \mathcal{L}_{\text{ph}} = \frac{1}{2} \sum_k u^T(k) M_{\text{ph}}(k) u(-k), \]

in which the \( 3 \times 3 \) matrix \((M_{\text{ph}})_{ij} = \kappa k_i k_j - c \delta_{ij} \) and \( u = (u_x, u_y, u_z)^T \). The spin-wave action is transformed to

\[ S_{\text{SW}} = \int_0^\beta d\tau \int d^d \mathbf{r} \mathcal{L}_{\text{SW}} = \frac{1}{2} \sum_k \varphi^T(k) M_{\text{SW}}(k) \varphi(-k), \]

in which the \( 6 \times 6 \) matrix \( M_{\text{SW}} \) can be solved and \( \varphi = (\varphi_0, \cdots, \varphi_5)^T \). The action of magnetoelastic interactions becomes

\[ S_{\text{ME}} = \int_0^\beta d\tau \int d^d \mathbf{r} \mathcal{L}_{\text{ME}} = \sum_k \varphi^T(k) \varphi(-k), \]

where the 6-vector \( F(k) = U(k) u(k) = (U_{\text{EXI}}(k) + U_{\text{DMI}}(k)) u(k) \) and \( 6 \times 3 \) matrices \((U_{\text{EXI}})^{ij} = -\frac{\alpha_{\text{EXI}}}{\hbar c^2} (k^i k^j) C_{\text{EXI}}^{ij} + \text{i} k D_{\text{EXI}}^{ij}, (U_{\text{DMI}})^{ij} = \frac{\alpha_{\text{DMI}}}{\hbar c^2} (k^i k^j) C_{\text{DMI}}^{ij} + \text{i} k D_{\text{DMI}}^{ij} \).

Now the full theory comprises three parts \( S = S_{\text{ph}} + S_{\text{SW}} + S_{\text{ME}} \), which are, respectively, the free phonon theory, the free magnon theory and their coupling due to the magnetoelastic interactions. Next, we can integrate out the harmonic magnon bath in the path-integral formalism, leading to the partition function expressed as

\[ Z[\varphi, u] = \int D\varphi D\mathbf{u} e^{-S} = Z[\varphi, u \equiv 0] \int D\mathbf{u} e^{-S_{\text{eff}}}. \]

Because of the bilinearity, after Gaussian functional integration we get the effective action for the phonons

\[ S_{\text{eff}} = \sum_k u^T(k) \left[ -\frac{1}{2} U^T(k) M_{\text{SW}}^{-1}(k) U(-k) + \frac{1}{2} M_{\text{ph}}(k) \right] u(-k). \] (8)

The expression in the bracket is nothing but \(-1\) multiplying the inverse of the \( 3 \times 3 \) Matsubara Green function matrix \( G_{\text{eff}}(k) \) of the effective phonon theory in energy–momentum space.

3. Methods and results

3.1. Renormalized phonon spectrum

Rather than use the barely accessible analytic dispersion relations, we resort to inspecting the spectral function \( A(\mathbf{k}, \omega) \) of this effective theory. In conformity with experimental investigations, we focus on the effects due to sound waves propagating along the x and z directions by looking at the diagonal \( A_{ij}(k, \omega) = -2\Im G_{ij}^R \) as a function of momentum \( k \) along the i direction and frequency \( \omega \), wherein the retarded Green’s function \( G_{ij}^R \) comes from the analytic continuation \( (G_{\text{eff}})_{ij}(k, \omega \rightarrow \omega + \text{i} \eta) \). First of all, we confirm the property of the bosonic spectral function [60] that it is always positive for \( \omega > 0 \) and negative for \( \omega < 0 \). Secondly, the renormalized phonon spectrum is reflected in an \( A_{ij}(k, \omega) \) plot by \( \delta \)-function-type ridge structures. Thirdly, setting a realistic but small enough \( \eta \), we can extract information about the phonon excitations from the \( A_{ij}(k, \omega) \) plot by identifying the ridge structures.

Equation (5) itself can only give banal longitudinal phonon excitations of linear dispersion \( \omega = \nu_z k \) with an acoustic velocity \( \nu_z = \sqrt{\kappa / \rho} \). On the other hand, magnon theory possesses an excitation spectrum composed of three distinct modes [39]. For MnGe there are two kinds of gapless excitation, one acoustic mode \( \omega_1 = 2 D_{\text{ac}} k \propto Dk \) and one quadratic mode \( \omega_2 = \frac{\omega_m}{\sqrt{\kappa}} k^2 \propto f^2 k^2 \) when \( k \) is small, and another excitation with an energy gap \( \Delta_{\text{mag}} = \frac{4 \kappa \nu_z}{B} \propto \frac{D}{f} \), as shown in figure 3. Because of the spin Berry phase of the topologically nontrivial spin textures of skyrmions/monopoles, it is possible that any two non-parallel spin spirals can mingle with each other. This gives rise to \( \phi \)-quadratic couplings like \( \phi \phi \) in equation (6), mixing up conjugate \( \phi \) fields and hence the quadratic mode [15, 39, 61]. An exception is that these three modes degenerate into the linear gapless mode \( \omega_1 \) when \( m_z = 0 \). These gapless excitations are the Nambu–Goldstone bosons due to broken translation symmetry. In the case of electric transport [39], the softest mode affects electron motion the most. However, all the gapless modes will play a crucial role in the low-energy physics describing the interplay with the linear acoustic phonons.
The salient point is that because of the magnetoelastic coupling, the three magnon modes will hybridize with the phonon mode, giving rise to rich possibilities for renormalized excitation spectra. One interesting example occurs when the phonon mode intersects with the magnon modes in the dispersion plot, resulting in mutual repulsion and reconstruction of the dispersion curves. Overall, despite possible reconstructions, one is in general still able to relate the new modes to their respective precedents before hybridization, which will henceforth be used as convenient tags for the new modes in effective phonon theory. Even without any intersection in the dispersion curves, the intensity of a new magnon mode (the sharpness of the ridge structure) in a spectral function plot directly reflects the degree of hybridization that influences the (approximately) linear renormalized phonon mode. We have the relation $V_{\text{renorm}}(m_z) = \sqrt{\kappa_{\text{renorm}}(m_z)} / c$ (seen from equation (5)) between the velocity of the renormalized phonon linear mode and the new stiffness $\kappa_{\text{renorm}}$. After hybridization this phonon linear mode is of major importance, because it is this stiffness $\kappa_{\text{renorm}}$ that is measured experimentally at different external magnetic fields as the ultrasonic response. Its velocity is the slope of the corresponding dispersion ridge extracted from scanning the spectral function, for example in the shaded region in figure 4.

Because of the experimental difficulty in determining an accurate value for $J$, $D$ and the magnetoelastic couplings $\alpha_{\text{EXI/DMI}}$, one has to search for the correct control parameters corresponding to the real materials. We consider two, $\alpha_{\text{REI}}$ and $R_{\text{velo}}$, for the effective theory. One is the ratio between strengths of different magnetoelastic couplings $\alpha_{\text{REI}}$ and $\alpha_{\text{EXI}}$, denoted as $\alpha_{\text{mag}}$. The other is the ratio of the velocity of the magnon linear mode, $v_{\text{mag}} = 2Da_0v_0$ to the original unperturbed acoustic velocity $v_0$, denoted as $R_{\text{velo}} = v_{\text{mag}} / v_0 = \frac{2Da_0}{\sqrt{\kappa / c}}$. We also give an estimate of the characteristic energy scales in the MnGe experiment. The sound wave propagating in MnGe...
crystal has a frequency
\[ f = 18 \text{ MHz} \]
and wavelength
\[ \lambda = 4.7 \times 10^3 \text{ m} \]. The original acoustic velocity is hence
\[ v_0 = \frac{\lambda}{f} = 4.7 \times 10^3 \text{ m s}^{-1} \]
and the phonon energy is
\[ \epsilon_0 = \hbar f_0 = 1.2 \times 10^{-26} \text{ J} \]. As for the material MnGe [54, 57], \( a_0 \) is about 4 Å and the melting temperature (~200 K) of the magnetic order can be used to estimate the strength of EXI \( J \). We set \( J = 10D \) for MnGe. Thus, it is straightforward to obtain \( v_{\text{mag}} = 12.0 \times 10^3 \text{ m s}^{-1} \) and
\[ \Delta_{\text{mag}} = 2.4 \times 10^{-23} \text{ J} \], using the typical value when \( m_z = 0.8 \). On the other hand, in our theoretical study we set \( \eta = 1 \times 10^{-6} \), \( a_{SKX} = 2\pi \), \( \bar{h} = \kappa = c = 1 \) and hence \( v_0 = 1 \). Consequently, \( J \) or \( D \) is fixed by \( R_{velo} \). In addition to the variable \( \alpha_{\text{EXI}} \), we set \( \alpha_{\text{DMI}} = 0.8 \) since too large a magnetoelastic coupling unrealistically alters the phonon spectrum while a tiny one renders the effect feeble and difficult to detect.

The first message from the above is that \( v_{\text{mag}} \) is of the same order as \( v_0 \). This implies that we had better tune \( R_{velo} \) not far from unity if we are to explain the experiment. The second message is that \( \epsilon_0 \ll \Delta_{\text{mag}} \), which means that the low-energy phonon excitations, and hence gapless magnons, around the long-wavelength limit play a major role. Because the \( U \) matrix in equation (8) contains two individual parts due to EXI and DMI, their combination becomes not simply a summation of the separate effects but one with inevitable interference between the two types of magnetoelastic interaction. Indeed, we have seen distinct \( V_{\text{renorm}}(m_z) \) profiles when changing \( \alpha_{\text{mag}} \) within a typical range \([-5, 5]\) in steps of 0.1. Note that a sign difference between two magnetoelastic interactions is possible as implied by the sign change in DMI [62–64].

### 3.2. Rich possibilities of magnetoelastic responses

The value of \( R_{velo} \) strongly affects the excitation spectrum of the effective phonon theory in a clearer manner than \( \alpha_{\text{out}} \). Imagine drawing the original unperturbed phonon dispersion line in figure 3, the slope of which might be smaller or larger than that of the blue magnon linear mode, providing a natural bipartite classification:

1. \( v_0 < v_{\text{mag}} \). The phonon mode lies below the blue magnon linear mode and intersects with the green magnon quadratic mode. On the one hand, if both the magnon quadratic mode itself and the magnetoelastic interactions have a strong enough intensity, depending on the details of the coupling, the intersection...
becomes an anticrossing of two reconstructed modes repelling each other in the effective phonon spectrum. On the other hand, near the structural phase transition where fluctuations become larger, the blue magnon mode strongly repels the phonon mode downward as a result of magnon–phonon interaction, serving as the major cause of the softening effect in figure 5 in agreement with the experimental data shown figure 6.

(2) $v_0 > v_{\text{mag}}$. The phonon mode lies above the blue magnon linear mode and intersects with the orange magnon gapped mode. Despite this, the high-energy scale of the gap renders itself irrelevant for the long-wavelength phonons. Similar to (1), near the phase transition the blue mode repels the phonon mode upward, which is an elastic hardening prediction from our theory. As seen in figure 7, the renormalized phonon velocity is in general larger than its original value of unity. It is expected to be realized by changing either $v_0$ or $v_{\text{mag}}$. Methods for controlling DMI in skyrmion lattices \cite{62, 65} or varying compositions \cite{64} can be candidates for realizing this. Last but not least, when $v_0 \approx v_{\text{mag}}$, the foregoing distinction becomes vague. As a minor reassurance we indeed saw evident, albeit complex, transitions from softening to hardening in $V_{\text{renorm}}(m_z)$ when $R_{\text{velo}}$ traverses the range $[0.7, 1.3]$. We now discuss in more detail the comparison between theory and experiment. Figure 5 shows the theoretical result that reproduces the experimentally observed signals in figure 6, including the drastic decrease in the velocity of the $k_{\parallel}$-mode and the multi-peak-and-trench fine structure for the $k_{\parallel}$-mode. In this model calculation we cannot produce a temperature dependence, simply because the spectral function in this case does not depend on temperature. However, as suggested by basically the same feature at different temperatures marked by the gray dots in figure 6, the current theory is able to illustrate the essential physics. The external magnetic field in figure 6 is in general not simply proportional to the parameter $m_z$ in figure 5, since $m_z$ should be regarded as an approximation in modeling the effect on the deformation of the magnetic structure due to the possibly complex magnetization process. Nevertheless, this comparison suffices to highlight the key magnetoelastic responses. The reliability of this result is supported by the fact that within the range $\alpha_{\text{EXI}} \in [0.8, 1.2]$, $R_{\text{velo}} \in [1.6, 4]$ of the control parameters, the basic characteristics hold. After all, we note that in figure 5 the most evident changes in the velocity (stiffness) always occur around the destruction of the monopole lattice, i.e. monopole–antimonopole pair annihilation at $m_z = \sqrt{2}$ \cite{39}, and this topological phase transition clearly manifests itself by the drastic softening in figure 5(b). As for the magnitude of the softening with respect to the original $v_0 = 1$ situation, our theory produces $\Delta V_{\text{renorm}} / V_{\text{renorm}} \approx 5$, which is a bit smaller than the experimental value between 6 and 10. Despite this discrepancy, we highlight the excellent consistency between the theoretical and experimental fine structures. Besides the clear match for the single-trench $k_{\parallel}$-mode case of figure 5(b), all three trenches and two peaks in figure 5(a) find their counterparts in the experiment. On the other hand, if we set $\alpha_{\text{EXI}}$ or $\alpha_{\text{DMI}}$ to zero, it becomes impossible to reproduce the experimental signals. Taken as a whole, these results strongly endore our theory and can be regarded as a new way to find some quantities that are temporarily outside the reach of experimental detection.
Another aspect worth discussing is why the experimental features are reproduced when $\alpha_{\text{mag}}$ is not far from unity although we have $J = 10D$. The resolution consists in the unique sensitivity of DMI to minute strains [62, 63, 65]. Some anticrossing points mixing up spin-up and spin-down bands in the band structure make a massive contribution to the spin–orbit interaction, and hence the DMI. The phonon induced strain modifies the band structure slightly; this, however, may considerably change the DMI because the position of the Fermi level with regard to the nearby anticrossing points can drastically change the contribution from these points. Experimentally, the relative modulation of DMI is typically ten times larger than that of EXI [65], which is consistent with our theoretical finding.

3.3. Magnetic field-dependent evolution of the hybridized excitation spectra

Obviously, noticing the range of $R_{\text{velo}}$, figure 5 belongs to the foregoing $v_{\text{velo}} < v_{\text{mag}}$ case. A careful inspection of the dispersion profiles, i.e. the spectral function plots in figure 8 (k$^\parallel$-mode) and figure 9 (k$^\perp$-mode), leads us to the following explanation. For the k$^\parallel$-mode, the dominant modes appear to be linear excitations of magnons (upper branch) and phonons (lower branch). With increasing $m_z$, the slope of the magnon mode remains almost the same while its intensity gradually increases ($m_z = 0.6–1.46$) to some maximum value around the phase transition and drops down afterwards ($m_z = 1.6–2.0$). The larger the intensity becomes, the stronger the repulsion exerted on the phonon mode underneath, explaining the drastic softening. The reason why this case lacks participation from the magnon quadratic mode lies in the magnetoelastic interaction in equation (4). The magnon quadratic mode originates from the $\phi$-quadratic term in our spin-wave theory in equation (6). Thus, if it were to largely affect the phonon linear mode in the effective phonon theory, the $\phi$-field must have adequate coupling with phonons. The relevant form factors here, $C_{\text{EXI/DMI}}^{ij}$ when $\mu = 1, 2, 3$, are diagonal, as seen from the calculation, making the k$^\parallel$-mode (k$^\perp$-mode) phonon primarily couple with $\phi_i$ ($\phi_j$). But only $\phi_i$ and $\phi_j$ in the $\phi$-quadratic term are important because $b_k$ are vanishing.

For the k$^\perp$-mode, the interplay takes place mainly between the original phonon linear mode and the magnon linear and quadratic modes, as seen from their high intensities in figure 9. We can first faintly recognize the three magnon modes (gapped, linear and quadratic, from left to right) together with the always dominant phonon linear mode when $m_z = 0.02$, which is in contrast to the $m_z = 0$ case with only one magnon linear dispersion is present. This is because of the degeneracy of magnon excitations when $m_z = 0$, mentioned in section 3.1. Note that the gapped magnon mode disappears in most plots since increasing $m_z$ enlarges the gap beyond the plot range. For $m_z = 0.02$ and $m_z = 0.1$, a typical (anti)crossing or reconstruction of the magnon and phonon linear...
modes occurs at some low-energy scale inside the plot range. The mutual repulsion between them makes the reconstructed phonon linear mode, especially the part right of the crossing point, move downward, explaining the first small trench in figure 5(a). Once the crossing point gets higher \( m_z = 0.3 \), the low-energy part of the phonon mode simply bounces back and gives rise to the first small peak near \( m_z = 0.25 \) in figure 5(a).

Thereafter, as \( m_z \) increases until the vicinity of 1.2, the magnon linear mode above the phonon linear mode becomes stronger and stronger while the magnon quadratic mode keeps moving down until \( m_z = 1.1 \), which constructively pushes the phonon mode downward, yielding the deepest trench. Then, abruptly, a transient reverse procedure is observed approximately from the \( m_z = 1.1 \) to \( m_z = 1.42 \) plots, hence the dramatic upsurge in figure 5(a). Next, the magnon quadratic mode recovers all the way back and crosses over the phonon mode at lower and lower energy scales while the magnon linear mode becomes stronger to \( m_z = 1.55 \), which is again a constructive effect of dragging the phonon mode downward. Finally, a fading reunion of the three magnon modes is observed from \( m_z = 1.7 \) to \( m_z = 2.2 \), which is natural for induced ferromagnetism with \( m_z \) too large.

In a nutshell, the experimentally observed magnetoelastic phenomena are the consequences of two aspects that vary with \( m_z \) or the external magnetic field. One is the magnon quadratic mode generated at nonzero \( m_z \) (together with the herein insignificant gapped mode), whose reciprocating shift in the spectral function plot in figure 9 is clearly controlled by the emergent magnetic field \( b_z (m_z) \), as explained in section 3.1 in terms of equation (6). The other is the family of form factors that directly affect the intensities of and hybridization between various modes. They show nonmonotonic behavior upon increasing \( m_z \) and vary strongly near the phase transition. Intricate integration of these two aspects leads to the rich experimental features. Detailed inspection of the underlying ground state spin configuration provides more insights [39]. Here we only recapitulate two key aspects. One is the nonmonotonic profile of \( b_z (m_z) \), whose maximum near \( m_z = 1.0 \) and fast dip around \( m_z = \sqrt{2} \) make the variation of the form factors more perceivable. Note that the realistic lattice cutoff introduced to the monopole defects in the calculation will also postpone the complete destruction of the

**Figure 9.** Logarithmic plots of spectral function \( A_{kx} (k_x, \omega) \) at various uniform magnetizations \( m_z \) for the \( k_x \)-mode sound wave.
monopole lattice to some value larger than the ideal value \( m_C = \sqrt{2} \). The other aspect is the nontrivial contribution from the (anti)monopoles and their characteristic collision-and-annihilation motion. In contrast to the SkX in MnSi discussed below, we have argued for a crucial role of topological defects, i.e. (anti)monopoles, in the magnetoelasticity, and especially the topological phase transition of the destruction of the monopole lattice. Here with regard to magnetoelastic phenomena, not only \( b_z \) but also the form factors reflect the rich facets of the spin textures. It is the coupling between phonons and the (anti)monopoles that causes all the complexities.

3.4. Relation to the triangular lattice of skyrmion tubes in MnSi

Now we briefly discuss the magnetoelastic couplings in the SkX of MnSi. Except for the modification for the spin-wave theory, we can apply a similar form of coupling to MnSi. Nevertheless, we observe vanishing couplings when the sound wave propagates along the cylindrically symmetric z-axis of skyrmion tubes in MnSi.

The form factors for the EXI induced coupling are simply zero due to translational symmetry along the \( z \) direction. Although this is not the case for the coupling induced by DMI, the spatial averages of corresponding form factors turn out to be zero in the end. As for the perpendicular propagating case, we observe nonvanishing magnetoelastic effects, albeit negligibly smaller than for MnGe. On the other hand, the experimental signals \( \left( \frac{\Delta \rho}{\rho} \approx 0.1\% \right) \) of the SkX in MnSi are somewhat smaller than those \( \left( \frac{\Delta \rho}{\rho} \approx 2\% \sim 10\% \right) \) detected in MnGe [43]. Therefore, our form of magnetoelastic coupling turns out to be the leading order effect in MnGe monopole lattices. Based on solid state mechanics and macroscopic thermodynamics, many more complex higher-order terms and fitting parameters are incorporated into the Ginzburg–Landau free energy calculation [66] for ultrasonic signals in MnSi [58, 59]. This comparison actually lends credence to the aforementioned essential role played by monopole defects that are absent in MnSi. In other words, the contribution from monopole defects to the magnetoelastic effects, if present at all, dominates and is captured by the formalism developed in this work.

4. Concluding remarks

The study of ultrasonic elastic responses is strongly motivated by experimental findings. We not only explain the observed softening effect but also predict new issues, e.g. hardening and other patterns of the dependence of stiffness on the magnetic field, which in turn provides a way to determine some experimentally inaccessible physical quantities. Based on a well-established spin-wave theory from a previous magnetoresistivity study, we are able to identify once again the nontrivial features of the monopole lattice, especially the topological phase transition signifying strong correlations. Thanks to the agreement with the experimental observations, this magnetoelasticity study, together with the magnetoelasticity one, has established the importance of the topological nature of the spin configuration in strongly correlated electronic systems and has emphasized the crucial role played by the monopole defects in chiral magnetic MnGe. In particular, these studies pave the way for even more intriguing scenarios of coupling topologically nontrivial objects with other systems. They show the rich physics therein and help us gain insights for further investigations into manipulation methods for skyrmionics applications.

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