Modeling of heat transfer in an element with anisotropic porosity

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Abstract. The paper presents the results of modeling the heat transfer process in a channel filled with a porous medium. Moreover, the porosity changes according to the specified law along one of the coordinates. It is shown that an element with anisotropic porosity has better heat transfer characteristics in comparison with an element with a homogeneous porous structure.

1. Introduction
The use of porous structures makes it possible to intensify the heat exchange process in the channels of power plants for various purposes, for example, in turbine blades, in devices for cooling systems for radio electronics, satellite communication systems and many other technical devices that require compactness and high thermal efficiency [1, 2]. A significant contribution to the heat transfer process is made by the heat transfer surface area, which is increased in comparison with the empty channel. Unfortunately, this leads to an increase in the hydraulic resistance in the channel, and, accordingly, in the pressure drop, which does not affect the energy consumption in the best way during the transportation of the coolant. This aspect is one of the most important in deciding on the use of porous elements for thermal stabilization of heat-stressed elements.

It is possible to reduce the hydraulic resistance in a porous medium in several ways: the use of interchannel transpiration of the coolant, the use of a porous medium with a local or uniform change in the permeability tensor [3, 4]. In this work, the anisotropy of a porous medium is considered in the form of a smoothly varying porosity. Moreover, its value at the base of the channel exposed to thermal action is large compared to the area in the central part of the channel. This solution will allow to increase the speed of the coolant near the heated surface, therefore, to intensify the heat transfer process. But, on the one hand, an increase in the height of the channel region with higher porosity leads to a decrease in the heat exchange surface area, and on the other hand, to a decrease in the pressure drop. In this regard, we will consider several functions responsible for the law of porosity variation near the heat-stressed boundary.

2. Statement of the problem
Consider a channel 40×5 mm in size, filled with a porous medium, through which a coolant flows (figure 1). The flow regime is laminar. The lower part of the channel is exposed to heat flow. The upper part of the channel is thermally insulated. The internal space of the channel is a structure, the
porosity of which changes along the ordinate axis according to the specified piecewise continuous function with smoothing using a continuous second derivative [5]. Let us consider 2 variants of such a function, determined by the systems of equations (1), (2). Function graphs are shown in figure 2.

\[
e(y)|_{1}^{2} = 0.227y + 1.027, \quad \varepsilon(y)|_{1}^{2} = 0.3; \quad (1)
\]

\[
e(y)|_{1}^{6} = -0.5y + 1.3, \quad \varepsilon(y)|_{1}^{6} = 0.3. \quad (2)
\]

![Function graphs](figure2.png)

**Figure 2.** Piecewise continuous functions of porosity change.

For a mathematical description of the hydraulic subproblem, we use a system consisting of the Brinkman equation and continuity

\[
\nabla \cdot \left[ -pI + K \right] - \left( \mu \kappa^{-1} + \beta \varepsilon \rho \|u\| + Q_m \varepsilon^{-2} \right) u + F = 0, \quad (3)
\]

\[
\rho \nabla \cdot u = Q_m, \quad (4)
\]

where \( p \) – pressure, Pa; \( \mu \) – dynamic viscosity coefficient, kg/(m·s); \( \kappa \) – porous medium permeability; \( \beta \) – inertia coefficient, 1/m; \( \varepsilon \) – porosity; \( \rho \) – density; \( u \) – velocity vector, m/s; \( Q_m \) – mass source; \( F \) – additional force vector.

Permeability was determined according to the Kozeny-Karman ratio by the formula

\[
\kappa = \frac{1}{180} \varepsilon^3 d_p^2 (1 - \varepsilon)^{-2}. \quad (5)
\]

Here \( d_p \) – diameter of spherical particles that make up a porous structure, m.

The thermal subproblem was solved, like the hydraulic one, in a stationary two-dimensional formulation. For the solution, the temperature difference between the porous matrix and the coolant was taken into account, i.e. a two-temperature heat transfer model was used. In addition, the problem

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**Figure 1.** Formulation of the problem.
took into account the non-isothermality of the coolant flow, i.e., its density was a function of pressure and temperature. Below are the equations used in general terms for:

- heat transfer in a porous matrix

\[ d_z \rho c_p u \cdot \nabla T + \nabla \cdot \mathbf{q} = d_z Q + q_0 + d_z Q_{red}; \]

\[ \mathbf{q} = -d_z k \nabla T; \tag{6} \]

- heat transfer agent

\[ d_z \rho c_p u \cdot \nabla T_2 + \nabla \cdot \mathbf{q} = d_z Q + q_0 + d_z Q_{ad}; \]

\[ \mathbf{q} = -d_z k \nabla T_2; \tag{7} \]

- accounting for the two-temperature model

\[ Q_s = \frac{q_{sf}}{\theta_p} (T_f - T_s); \]

\[ Q_f = \frac{q_{sf}}{1 - \theta_p} (T_s - T_f); \]

\[ q_{sf} = a_{sf} h_{sf}. \tag{8} \]

To determine the thermal characteristics, the value of the element depth is taken as mm. The material of the porous matrix is aluminum. The heat carrier is water. Boundary conditions at the channel entrance: temperature 20 °C, speed - was in the range from 0.03 to 0.06 m / s. The excess pressure at the outlet was taken to be zero. To determine the regularity of heat transfer between the porous matrix and the coolant, the following expressions were used to determine the specific surface area

\[ a_{sf} = \frac{6 (1 - \varepsilon)}{d_p}; \tag{9} \]

and the interstitial heat transfer coefficient between the porous matrix and the heat carrier

\[ q_{sf} = \frac{\lambda_l}{d_p} \left[ 2 + 1.1 Pr^{1/3} \left( \frac{v_0 d_p}{\mu \rho} \right)^{0.6} \right], \tag{10} \]

\[ Pr = \frac{\mu c_p \ell}{\lambda_l}. \tag{11} \]

The components for equations (8) - (11) were determined as follows. So, the values of porosity, thermal conductivity, dynamic viscosity were calculated as average values over the entire area of the element. Pressure - as an average value at the boundary of the coolant inlet.

3. Solution

3.1. Modeling the operating modes of the element

The solution of equations (3) - (8) and the obtaining of thermohydraulic characteristics for an element with anisotropic porosity was carried out using one of the engineering analysis packages based on a numerical method for solving differential equations. At the beginning of the calculation, for each of the porosity options, the operating modes were determined in which the Reynolds number did not exceed 2300, and the coolant temperature was 100 °C. This was controlled in order to consider the solution of the problem within the framework of formulating a laminar flow regime and preventing a
phase transition inside the porous element, as well as in order to adequately compare the proposed options for anisotropic porosity.

The hydraulic performance was estimated based on the relationship between the drag coefficient and the Reynolds number. For this, in each of the 5 modes, both of these parameters are determined by the following formulae

\[ \xi = \frac{2d\Delta p}{l\rho'_lv'_{\infty}}, \]  
\[ \text{Re} = \frac{vd\rho'}{\mu_e}. \]  

Here \( d \) – hydraulic channel diameter; \( \Delta p \) – pressure drop; \( l \) – porous channel length.

The results of the study are presented as a relation \( \xi(\text{Re}) \) in figure 3. Relationships \( \varepsilon = f_1(y) \) and \( \varepsilon = f_2(y) \) correspond to the laws of porosity change (1) and (2), respectively.

\[ \text{Figure 3.} \text{ Relationship between the drag coefficient and the Reynolds number for different variants of the porous element.} \]

Porosity values \( \varepsilon = 0.41 \) and \( \varepsilon = 0.35 \) determined as averages over the computational domain with a change in porosity according to (1) and (2), respectively. A \( \varepsilon = 0.41 \) and \( \varepsilon = 0.35 \) taken as the extreme values of the range of porosity variation when determining relations (1) and (2).

To analyze the thermal characteristics of an anisotropic element for each operating mode, the Nusselt number was determined by the formula

\[ \text{Nu} = \frac{\alpha_m d}{\lambda_y}. \]  

Here \( \alpha_m = \frac{q_s}{t_w - t_{\text{mf}}} \). \( t_w \) – average temperature of the channel base, °C; \( t_{\text{mf}} \) – average temperature of the coolant in the channel, °C.
The relationship between the Nusselt number and the Reynolds number for two variants of anisotropic porosity and several values of uniform porosity is shown in figure 4.

![Graph of Nu(Re) for different types of porous element.](image)

**Figure 4.** Relationship graph of $\text{Nu}(\text{Re})$ for different types of porous element.

3.2. **Findings**

Analysis of the relationships shown in figures 3 and 4 shows that the drag coefficient for the first variant of anisotropy is lower than for the second. While the intensity of heat transfer in the channel with the first option of anisotropy is lower compared to option 2. Comparing the difference between the drag coefficient and the heat transfer rate for one of the flow regimes corresponding, for example, to the value $\text{Re} = 800$, you can see that $\xi$ differs by 2.5 times. In this case, the difference in the intensity of heat transfer is 1.2. The use of isotropic porosity in the channel leads to an increase in $\xi$ for $\varepsilon = 0.35$ and $\varepsilon = 0.3$ compared to anisotropic. When $\varepsilon = 0.41$ value $\xi$ comparable to the porosity anisotropy corresponding to law (2). The channel has the lowest resistance coefficient with $\varepsilon = 0.8$, but such a configuration is inferior in heat exchange to a channel in which the porosity corresponds to relationships (1) or (2). Heat transfer for a channel with any variant of isotropic porosity is less intense than for a channel with anisotropic porosity.

4. **Conclusion**

The results of the study of two models of anisotropic porosity for a flat channel show that with a sharper shift of the porosity gradient to the heat-stressed boundary, the heat transfer intensity increases, and the ratio of the values of the resistance coefficients in this case exceeds the ratio of the heat transfer intensity. In this case, for small values of the Reynolds number ($\text{Re} < 500$), the difference in the intensity of heat transfer is smaller in comparison with the regimes corresponding to $\text{Re} > 500$. The use of porosity anisotropy leads to a more significant intensification of heat transfer into the channel as compared to an isotropic porous medium. Thus, the use of anisotropic porous media makes it possible to increase the efficiency of heat exchange elements.
References

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