Three-dimensional transient simulation of water filling and draining of gravity flow based on Lattice Boltzmann Method

Yanhui Li*, Jin Jiang, Rui Ying, Yucheng Wang, and Wensheng Zhao

Key Lab of Hydraulic Machinery Transients, Key Lab of Hydraulic Machinery Transients, MOE, Wuhan University, Wuhan, China.

*Corresponding author e-mail: lee_yanhui@whu.edu.cn

Abstract. The traditional one-dimensional method of characteristics is available to analyze the transient flow of system level in full pipe flow. As for water filling of empty pipeline and water draining of water-filled pipeline, it is difficult to analyze with this method as the boundary conditions are changing along with the location and time. Meshless Lattice Boltzmann Method was used in this investigation to simulate these two circumstances in the gravity flow water-delivery system respectively. We visually obtained, by this method, the changing process of liquid level, total time of the filling and draining process, flow state around the butterfly valves in the pipe inlet and outlet respectively. Meanwhile, time history data of water flow rate and submersed fraction on the cross section behind the valves were extracted as well. CFD method used in analysis of three-dimensional pipeline flow in system level is an innovation of this research that it not only makes the result more accurate, but also details and visualizes the local transient flow state in pipeline system of gravity flow.

1. Introduction

Pump flow and gravity flow are two kinds of de-livery modes commonly in water-delivery pressurized pipeline system. Water-delivery system of pump flow mainly consists of pump stations, pipelines and other hydraulic structures. And in that system, water will be delivered far away with pumps as energy source. Whereas the gravity flow pipeline system delivers water completely depending on the overall drop and its own gravity without pressurized pump. Delivering water with mode of gravity flow is low-cost, economical, and ideal as it maximizes the use of potential energy of overall drop. Ensuring the safe operation of gravity flow pipeline is an engineering challenge including flow rate controlling, water-hammer protection and filling and draining arrangements [1]. Gravity flow is usually simulated with method of characteristics [2]. However, the radial flow field will change obviously with relatively large pipe diameter. And by method of characteristics, the simulation results are not reliable when the pipe are simplified to one-dimensional line. Moreover, the one-dimensional simulation is not available to analyze the transient flow field in the specific parts of pipeline, such as elbows, valves and pumps.

Lattice Boltzmann Method is an efficient algorithm for single-phase and multiphase fluid flow simulation. With simple form, it is suitable for calculation of locality and especially flexible to deal with complicated boundary conditions and multiphase interfaces [3]. As well, this algorithm is such perfectly parallel that it is available to run on computing cluster of large scale. In recent years, the Lattice Boltzmann Method has been widely used in investigation of fluid flow and heat transfer. For instance,
it is used in the in-depth study of basic phenomenon of fluid mechanics, such as turbulence [4] and multiphase flow [5]. In addition, by this method, transient conduction and radiation heat transfer problems can be solved [6] as well as bubble growth and detachment investigation [7]. Besides, Lattice Boltzmann Method is also applicable to analyze the mechanism of microscale flow [8].

In this research, simulations of water filling of empty pipeline and water draining of water-filled pipeline in gravity flow pipe system were carried out with meshless Lattice Boltzmann Method. It is challenging and innovative to simulate the transient flow in such a system level.

2. Mathematical models and simulation settings

2.1. Mathematical model of gravity flow

Figure 1 shows the common gravity flow system in which water flows from cross section V-1 to V-2. Pressure and temperature are represented by $P_0$ and $T_0$ in V-1, $P_1$ and $T_1$ in pipe inlet, $P_2$ and $T_2$ in V-2 (horizontal plane of pipe outlet) respectively. The basic level is set on the ground and heights are $H$, $H_1$ and $H_2$ respectively. Driven by overall drop and its own gravity, the water flows from V-1 to V-2 while valves are opened. To simplify the analysis, some assumptions are made for the system that there are not flash evaporation and other complex phenomena during the gravity flow meanwhile the pipe diameter and fluid density are constant. Also, it does not involve heat transfer, i.e. $T_0$, $T_1$ and $T_2$ are equivalent.

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![Figure 1. Common gravity flow system.](image)

During the gravity flow, the mechanical energy of system is conserved. That can be expressed as:

$$\left(\frac{u_1^2}{2g} + H_1 + \frac{P_1}{\rho g}\right)m_1 - \left(\frac{u_2^2}{2g} + H_2 + \frac{P_2}{\rho g}\right)m_2 = E$$

(1)

Where $m_1$ and $m_2$ are mass flow rates in inlet and outlet respectively; $u_1$ and $u_2$ the fluid velocities; and $E$ the energy loss of system.

As for full pipe flow, the mass flow rates along the pipeline are equivalent as well as the fluid velocities, while the pipe diameter and fluid density are supposed to be constant.

$$m_1 = m_2 = m$$

(2)

$$u_1 = u_2 = u$$

(3)
And the energy loss of system involves the head losses of inlet, pipe and outlet. It is expressed as:

\[ E = m\left(\frac{u^2}{2g}\right)(k_1 + \sum \lambda_i \frac{L_i}{D} + \sum \zeta_j + k_2) \]  

(4)

Where \( k_1 \) and \( k_2 \) are resistance coefficients in inlet and outlet respectively; \( L_i \) the equivalent length of each pipe; \( \lambda_i \) the linear resistance coefficient; and \( \zeta_j \) the coefficient of local resistance.

The total driving force of gravity flow in the form of head is:

\[ X = (H_1 - H_2) + (P_1 - P_2)/\rho g \]  

(5)

2.2. Mathematical model of Lattice Boltzmann Method

There are three levels of fluid flow description in physics: macroscopic, microscopic and mesoscopic [9]. Based on the kinetic theory of fluid, the Lattice Boltzmann Method describes the basic characteristics of micro movement and establishes the simplified kinetics lattice model accompanied with completely discrete time and space. Lots of mesoscopic fluid particles are dispersed among lattices and migrate along the lattice lines according to certain rule and collide with each other in lattice points. Their complicated motion obeys the law of dynamics and statistical law. Therefore the macroscopic motion variables of fluid are obtained by statistically averaging the large quantities of particles in flow field.

Compared with the traditional CFD algorithm, the Lattice Boltzmann Method is characterized by meshless solution. It just needs the definition of lattice size. Furthermore, advantages of this method includes: (a) It can perform simulation from aspect of microscopic particles that it is easy to deal with the complex inter-actions between different components and fluid inter-faces without the empirical and semi-empirical formulas. (b) Compared with the nonlinear convective term of N-S equations, the spatial convection process of Lattice Boltzmann Method is linear such that the evolution process is clearer and easier to solve. (c) The pressure is expressed as the equation of state and not iteratively solved with velocity so that this algorithm is relatively simple.

The Lattice Boltzmann equation with single relaxa tion time operator and volume force term is expressed as follows:

\[ f_i(r + e_i dr, t + dt) - f_i(r, t) = -\frac{1}{\tau}[f_i(r, t) - f_i^{eq}(r, t)] + dtF_i \]  

(6)

The equation above is decomposed into two types of evolution: collision step and migration step. Collision step:

\[ f_i(r + e_i dr, t + dt) = f_i^+(r, t) \]  

(7)

Migration step:

\[ f_i^+(r, t) = f_i(r, t) - \frac{1}{\tau}[f_i(r, t) - f_i^{eq}(r, t)] + dtF_i \]  

(8)

Where \( f_i(r, t) \) is the density distribution function of particles along the direction of velocity \( e_i \) at time \( t \) and on location \( r \). And \( dt \) is the time step, \( \tau \) the dimensionless relaxation time, \( f_i^{eq} \) the local equilibrium distribution function, and \( F_i \) the volume force term.

The local equilibrium distribution function is described as follow:

\[ f_i^{eq} = w_i \rho(1 + \frac{3e_i \mu}{C^2} + \frac{9(e_i \mu)^2}{C^2} - \frac{3\mu^2}{2C^2}) \]  

(9)

As an efficient mesoscopic numerical algorithm in CFD, the Lattice Boltzmann Method is available to simulate the three-dimensional gravity flow with higher level of accuracy.
2.3. Geometric model and simulation settings
Gravity flow system analyzed in this investigation consists of upper water tank, pipeline (500 m long) and lower water tank. Overall drop between these two tanks are 300 m and the diameter of the pipeline is constant as 1.5 m. In addition, two butterfly valves are respectively set in the inlet and outlet of pipeline. In simulation of water filling of empty pipeline, the initial height of liquid level in upper tank was set as 15 m, as shown in Fig. 2(a) where the red part represents the water phase and blue part the air phase. In this circumstance, the butterfly valve in the outlet kept open all the time and the one in the inlet opened linearly in 5 s from closure. Meanwhile, in simulation of water draining, the pipeline was full of water while the two tanks were empty at initial moment as shown in Fig. 2(b). Similarly, in this draining circumstance, the butterfly valve in the inlet kept continually open and the one in the outlet also opened linearly in 5 s. Monitoring cross sections were set 3 m downstream away from both valves to extract the changing fluid parameters.

The total simulation times of both circumstances were set as 60 s. Particle-based multiphase model was utilized along with the surface tension set as 0.072 N/m. Besides, the inlet and outlet boundary conditions were set as atmospheric pressure on the top surfaces of upper and lower tank respectively.

3. Results and discussion
3.1. Transient water filling process
The changing process of liquid level in water filling of empty pipeline is indicated in Fig. 3. As the water is filling the pipe, the residual air exhausts downstream to the lower tank and atmosphere outside. Meanwhile the replenishing air automatically flows into the upper tank where liquid level declines gradually.

Flow state changing around the butterfly valve in the pipe inlet in water filling process is shown in Fig. 4. As the valve opens linearly in 5 s, water fills in the entrance pipe gradually and keeps accelerating. The maximum flow velocity is approximately 9.5 m/s when the valve fully opens. Meanwhile flow of local high velocity mainly concentrates on the elbow, butterfly valve and pipe inlet.

Time history data of water flow rate and submerged fraction on the monitoring cross section downstream away from the upper butterfly valve are indicated in Fig. 5 and Fig. 6 respectively. As for the flow rate, it increases rapidly with large slope between 4 s and 5 s when the valve is almost fully open. This is determined by the characteristics of the butterfly valve whose flow area does not increase linearly though it opens linearly. Afterwards, the flow rate almost increase linearly before 31 s. At this stage, water in pipe, driven by its own gravity and the overall drop, accelerates its flow velocity and the liquid level declines quickly in the upper tank. As shown in the curve of flow rate, total time of the filling process is approximately 31 s after which the water flow changes from pure pipe flow to orifice flow in the outlet. Therefore the flow rate decreases and fluctuates. Finally, it maintains stable a level subsequently as the orifice flow becomes steady. Corresponding to the stable level of the flow rate, liquid level in the upper tank also declines steadily.
Figure 3. The changing liquid level in water filling process: (a) t=5s; (b) t=15s; (c) t=25s; (d) t=35s.

Figure 4. Velocity distribution around upstream valve in filling: (a) t=2s; (b) t=3s; (c) t=4s; (d) t=5s.

Submersed fraction on monitoring cross section, as depicted in Fig. 6, indicates the volume fractions of water and air phases respectively changing on that location. 0 value means it is full of air on this cross section and 100 % full of water otherwise. The increasing stage of the submersed fraction is similar to that of the flow rate approximately between 4 s and 5 s.
3.2. Transient water filling process

The changing process of liquid level in water draining of water-filled pipeline is shown in Fig. 7. As the water is draining from the pipe, the replenishing air automatically flows into the upper tank and pipe while the liquid level declines gradually. And the residual air in lower tank exhausts to atmosphere outside.

Flow state changing around the butterfly valve in the pipe outlet in water draining process is shown in Fig. 8. As the valve opens linearly in 5 s, water accelerates its flow velocity while draining from the pipe and discharging into the lower tank. The maximum flow velocity is approximately 28 m/s concentrating on the flow area of butterfly valve. Obviously, in the opening process of valve, magnitude of flow velocity in water draining is larger integrally than that in water filling. It is caused by the initial head pressure in water draining (300 m head) which is larger considerably than that in water filling (15 m head).

![Figure 5](image1.png)  
![Figure 6](image2.png)  

**Figure 5.** Water flow rate in filling process.  
**Figure 6.** Submersed fraction in filling process.

![Figure 7](image3.png)  

**Figure 7.** The changing liquid level in water draining process: (a) t=6s; (b) t=12s; (c) t=18s; (d) t=24s.
Figure 8. Velocity distribution around downstream valve in draining: (a) t=2s; (b) t=3s; (c) t=4s; (d) t=5s.

Time history data of water flow rate and submersed fraction on the monitoring cross section downstream away from the lower valve are shown in Fig. 9 and Fig. 10, respectively. In particular, the flow rate increases and decreases with parabolic trend up to 66.4 m³/s between 2 s and 28 s. At this stage, it also fluctuates slightly based on the main trend because of the unsteady orifice flow. And the flow rate maintains subsequently stable a level of near-zero valve as the residual water in horizontal outlet pipe flows slowly.

In Fig. 10, the submersed fraction increases rapidly from 0 to 100 % between 1.7 s and 1.9 s, which also demonstrates the larger magnitude of flow velocity in water draining process. And total time of the draining process is approximately 43 s at which moment the submersed fraction decreases rapidly from 100 % to low value. Finally, the submersed fraction fluctuates constantly at low value because of the slowly flowing of residual water in horizontal outlet pipe.

Figure 9. Water flow rate in draining process. Figure 10. Submersed fraction in draining process.

4. Conclusion
Pipeline system of gravity flow was simulated by CFD algorithm of Lattice Boltzmann Method in this investigation. Two circumstances, water filling of empty pipeline and water draining of water-filled pipeline, were analyzed respectively. Detailed results include the changing process of liquid level, flow field of velocity, total time of the filling and draining process and time history data of water flow rate and submersed fraction as well. It is an innovative investigation to analyze pipeline system with three-dimensional CFD method other than traditional one-dimensional method of characteristics. And simulation results with high accuracy are available by this Lattice Boltzmann Method.
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