On the Group Structure of the Kalb-Ramond Gauge Symmetry.

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Abstract

The transformation properties of a Kalb-Ramond field are those of a gauge field; however, it is not clear which is the group structure these transformations are associated with. The purpose of this letter is to establish a basic framework in order to clarify the group structure underneath the 2-form gauge potential.

1 Introduction.

The so-called (Abelian) Kalb-Ramond field \( B_{\mu\nu} \), is a two-form field which appears in the low energy limit of String Theory and in several other frameworks in Particle Physics \([2]\); for instance, the most of the attempts to incorporate topological mass to the field theories in four dimensions take in account this object \([3]\). Its dynamics is governed by an action which remains invariant under transformations whose form are extremely similar to those of a one-form gauge field \([1]\). The Kalb-Ramond (KR) field transforms according with the following rule:

\[
B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu} \beta_{\nu]}.
\]

where \( \beta_\mu \) is a one form parameter. The question is, may this be systematically generated from a group element? In other words, how can we associate the parameter \( \beta_\mu \) to the manifold of some gauge group? The first observation one can do is that the set of parameters, namely \( \Gamma \equiv \{ \beta_\mu \} \), is not isomorphic to a group algebra in itself; however, we shall show here how it encodes this information. The main purpose of this letter is to shed some light to this question.

This work is organized as follows: in the Section 2, we deal with the conditions to parametrize a group with an one-form parameter. In Section 3; we considerably relax some apparently strong restrictions, extending our analysis to multi-tensorial parametrizations by considering a Clifford Algebra: similar conditions are obtained and the structure of Section 2 is recovered as a particular case. Concluding Remarks are collected in Section 4; and finally, in an Appendix we explain the technical details of our argument.

2 A One-Form Group Parameter.

Let us assume a four-dimensional Minkowski space-time \((M, \eta_{\mu\nu})\) and some Lie group denoted by \(G\), whose associated algebra is \( \mathfrak{g} \); \( \tau^a \) are the matrices representing the generators of the group with \( a = 1, \ldots, \text{dim}G \); \( \tau_{abc} \) are the structure constants. Let us take the gauge parameter to be an adjoint-one-form which can be expanded as below:

\[
\beta_\mu = \beta^a_\mu \tau^a.
\]
Consider also a vector space $\mathcal{S} = \{\psi_I\}$ being a representation for $G$, where $I, J$ denote the internal indices of this space. Which is its transformation law under a group element parametrized by the object $\beta_\mu^a$? Let us take an infinitesimal transformation $\epsilon$. We shall get

$$\psi'_I = g_{\beta} \psi = (I_{IJ} + i\epsilon^a_\mu \tau^a \rho^\mu_{IJ} + \delta^2(\epsilon))\psi_J$$

where $\{\rho^\mu\}_{\mu = 0}^3$ are four linearly independent matrices which must transform as space-time vectors under the Lorentz Group. For an arbitrary one form $\beta_\mu$, $\beta_\mu \rho^\mu$ must be a linear operator from $\mathcal{S}$ into itself. This remarkably implies that $\mathcal{S}$ is some spinor space.

With the first order expression (3), we may build up a group element corresponding to a non-infinitesimal parameter, $\beta$; by considering $\beta_\mu^a = N \epsilon^a_\mu$ for a large integer number $N$. Thus, we have

$$g_\beta = \exp(i\beta_\mu^a \tau^a \rho^\mu).$$

For the moment, we will not worry about the unitarity of these groups and their representations.

The key point in this construction arises from the analysis of the group property

$$g_{\beta_1} g_{\beta_2} = g_{\beta_3},$$

where $\beta_{1,2,3} \in \Gamma$.

The question here is to identify under which conditions the parameters $\beta_1$ and $\beta_2$ are such that $g_{\beta_3}$ is also a group element.

Thus, one must have that $[\beta_1 \rho^\mu; \beta_2 \rho^\nu]$ is some linear combination of the group generators $\tau^a$ (i.e., it remains in the algebra). This condition may be written explicitly as

$$[\beta_1, \beta_2] = \frac{1}{2} \beta^a_{1 \mu} \beta^b_{2 \nu} \{[\tau^a, \tau^b]\} \rho^\mu_{IJ} + \{[\tau^a, \tau^b]\} \rho^\nu_{IJ} = \beta^a_3 \tau^a,$$

where in principle, $\beta^a_3$ also has to be a one-form. Since we cannot extend the algebraic structure to be larger than that of the group $G$ and the space-time symmetries (Lorentz group); in general, the third term is not in the algebra, i.e., $[\rho^\mu, \rho^\nu]$ is not a linear combination of the matrices $\rho^\mu$, as well as the anti-commutators $\{\tau^a, \tau^b\}$ do not lie in $G$ in general.

The single non-trivial algebraic structure we can count on, compatible with the space-time symmetries, is the well-known Dirac’s matrices algebra $\{\gamma^\mu, \gamma^\nu\}$ $\{\gamma^\mu, \gamma^\nu\}$ $\{\gamma^\mu, \gamma^\nu\}$ $\{\gamma^\mu, \gamma^\nu\}$ $\{\gamma^\mu, \gamma^\nu\}$,$\{\gamma^\mu, \gamma^\nu\}$,$\{\gamma^\mu, \gamma^\nu\}$,$\{\gamma^\mu, \gamma^\nu\}$,$\{\gamma^\mu, \gamma^\nu\}$, $\{\gamma^\mu, \gamma^\nu\}$, so for concreteness, all we can do is to identify the $\rho^\mu$-matrices with the elements of this algebra and the fundamental spinorial representation, $\mathcal{S}$, with a Dirac spinor space. We shall carry out a more detailed discussion about these points in the next section.

Thus, it became clear that the group property (5) is not satisfied for arbitrary pairs of parameters $\beta_1, \beta_2$. Therefore, we must determine which are the sub-families (sections of $\Gamma$) of parameters, denoted by $P_G$, such that it satisfy two basic requirements:

(i) For all pair $\beta_1, \beta_2$ in the same $P_G$, eq. (5) is satisfied for some $b_3 \in P_G$.

(ii) Each $P_G$ is large enough to cover (parametrize) the group manifold.

Notice that due to relation (5), it is natural to introduce an additional scalar group parameter $\alpha$ associated with $I_{IJ}$; however, since the commutators $[\alpha, \beta] = \alpha^a \beta^b_{IJ} [\tau^a, \tau^b] \rho^\mu$ and $[\alpha, \alpha'] = \alpha^a \alpha'^b [\tau^a, \tau^b]$, remain in the algebra: no restrictions to this scalar parameters will arise. So, in order to simplify our analysis, we stand for (5), (6) and the following expressions involving the parameter $\beta = \beta_\mu^a \tau^a \rho^\mu_{IJ}$ up to the component associated to the identity.

For a better understanding of the conditions implied by the requirements (i) and (ii), let us first consider the case of a Abelian group: $G \sim U(1)$. Since no extra algebraic structure (of the type

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2Actually, $\psi_I$ is an $N$-component object, where $N$ is the the dimension of the group representation.

3Clearly, we must restrict our discussion to space-time point-preserving transformations.

4Where the matrices $\gamma^\mu$ are linear maps on the Dirac spinor space in itself.
\[ [\gamma^\mu, \gamma^\nu]_{IJ} = \epsilon^{\mu\nu} I_{IJ} + \epsilon^\mu_\rho \gamma^\rho_{IJ}, \text{ for certain tensors } \epsilon^{\mu\nu}, \epsilon^\mu_\rho \\Box \text{ can be required, the condition (6) may simply be expressed as,} \]

\[ \beta_1 \mu \beta_2 \nu [\gamma^\mu, \gamma^\nu]_{IJ} = 0. \]  

Then, we obtain a constraint on the parameters expressed by:

\[ \beta_1 |\mu \beta_2 \nu| = 0 \]  

This clearly implies that \( \beta_1 \mu \) and \( \beta_2 \nu \) are parallel.

Of course, this condition is also sufficient to satisfy (i), (ii): if \( \beta_1 \mu \propto \beta_2 \nu \), the algebra closes and the composition law (6) is satisfied. The other group properties are automatically satisfied too. Then, we get that, for each (fixed) direction in \( \Lambda^1 \) we choose, say \( v_\mu \), we have a group structure, and each straight line (a \( \text{dim}G \)-dimensional plane in \( \Gamma \)) passing through the origin is isomorphic to the group algebra \( \mathcal{G} \). Let us remark finally that the choice of \( v \) is completely arbitrary; so, this must not be interpreted as a sort of gauge choice nor a breaking of the Lorentz symmetry. For each direction \( v \), one have the full structure of the group algebra.

For a more general Lie group, \( G \), the second term of (6) must be vanish, therefore, the constraint on the group parameters reads:

\[ \beta_1^{(a} \beta_2^{b)} |\mu \beta_2 \nu| = 0. \]  

One can give a proof (see appendix) that there are two families of parameters which are solutions to these equations; the first one reads

\[ \beta_1^a \mu \propto \beta_2^a \mu. \]  

Nevertheless, this does not satisfy a basic condition of the group parametrization (requirement (ii)); this is a one-dimensional set of parameters, while \( \text{dim}G > 1 \). The second family of solutions is actually the appropriate one for us,

\[ \beta_1^a \mu = \beta_2^a v_\mu, \]  

where \( \beta^a \in \mathcal{G} \), while \( v_\mu \in \Lambda_1 \) has to be fixed for each group parametrization.

Recalling the presence of an arbitrary term proportional to \( I_{IJ} \), the general form of an element of the algebra is \( (\alpha^a I + \beta_1^a (= \beta_2^a v_\mu) \gamma^\mu) \) for a \( v \) fixed. This algebra is: \( \mathcal{G} = \mathcal{G} \oplus \mathcal{G} \), the direct sum of two copies of \( \mathcal{G} \) associated with the matrices \( I \) and \( \phi \) respectively.

So, our final result is that

\[ g(\alpha, \beta) = \exp i (\alpha^a + (\beta^a v_\mu) \gamma^\mu) \tau^a \]  

is a well-defined (generic) representative of a element of the Lie group \( \mathcal{G} \) which results from the exponentiation of \( \mathcal{G} \). In the Abelian case this simply reduces to \( \mathcal{G} \sim G(1) \circ G(\phi) \), the composition of two copies of \( G \) associated with \( I \) and \( \phi \) respectively.

For simplicity, it has been assumed up to now that only the five matrices \( \{I, \rho^\mu\} \), appearing in (6), can be used to close the algebra (eq 6); but, since only a structure compatible with the Lorentz symmetry may be considered, this constitutes a strong restriction to the parameters as it has been shown. However, one also count with the structure of the Clifford Algebra (CA) which closes by construction: so in principle, one may consider additional independent gauge parameters (with other tensorial ranks) apart from \( \beta_\mu \). We are going to see in the following section that, even if we start off with such a more general situation, one necessarily falls back to essentially the same results, and the framework described above is actually not too restrictive.

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5 In particular, \( I, \gamma^\mu, [\gamma^\mu, \gamma^\nu] \) are linearly independent.

6 Below, this shall be confirmed for general Lie groups.

7 Since this must not involve the generators, but only the parameters.
3 Multi-tensorial Lie parameters via Clifford’s Algebras and fixed directions.

Let us consider an ordered basis of the Clifford Algebra $C$:

$$\{\rho^A\} \equiv \{I,\gamma_5,\gamma^\mu,\gamma_5\gamma^\mu,\gamma^\mu,\gamma^\nu\}, \quad (14)$$

where $A$ is a multi-index running over this multi-tensorial basis. Thus we may represent infinitesimal group transformations as

$$\psi_I = g_\epsilon \psi = (I + e^a_\epsilon \tau^a_{IJ} + \sigma^2(\epsilon))\psi_J, \quad (15)$$

where $e^a_\epsilon = (\epsilon^a, \epsilon^a, c^a_\rho, \epsilon^a_\mu, \epsilon^a_\nu)$. The Clifford algebra satisfies a crucial property: there exist a set of tensors, $C^{AB}$, such that $(\rho^A)_1 K (\rho^B)_J = C^{AB} (\rho^C)_IJ$.

Notice once more that the component $\beta^a$, associated with the first element of the Clifford basis $(I_J)$ does need not some restriction, since $[I,\rho^B] \equiv 0$; thus, in order to simplify our analysis as before, we stand for all the expressions involving the multi-parameter $\beta = \beta^a \tau^a_{IJ}$ through this section, up to the component associated to the identity.

Let us first consider an Abelian (one-dimensional) group like $U(1)$: by construction, the commutator $[\beta_1;\beta_2]$ remains in $C$. So, in principle, one would have a $4 \times 4 (= 16)$-parametric representation of the group. However, an important point has to be taken in account: if no restrictions are imposed, these are not Abelian representations. In fact, $\exp \beta_1 \exp \beta_2 \neq \exp \beta_2 \exp \beta_1$, unless precisely $[\beta_1,\beta_2] = 0$, which leads to the extension of the conclusion of the previous section:

$$\beta_{(1,2)} = \beta_{(1,2)} V_A, \quad (16)$$

for a same (fixed) $V_A \in C$. This is a central result in this work: it states that, despite considering a full aggregate structure for the parameters, we may only consider families of parameters described by (16), such as it was understood in the first part of the article. In particular, if we wish to involve only a one-form parameter as in the initial formulation of the problem, one must to choose $V_A \equiv (0,0,v_\mu,0,0)$.

Recalling once more the presence of an arbitrary term proportional to $I_{IJ}$, the more general form of the multi-parameter involving a one-form is:

$$\beta = (\alpha I + \beta_\mu \gamma^\mu), \quad (17)$$

where the direction of $\beta_\mu$ is fixed, in coincidence with our previous result.

Notice that, if we consider an more general (non-Abelian) group $G$, the above conclusion cannot be stated in the same way because, by construction, the commutator

$$2[\beta_1,\beta_2] \equiv \beta^a_1 \beta^b_{J\mu} \left(\{\tau^a,\tau^b\}\{\rho^A,\rho^B\} + \{\tau^a,\tau^b\}[\rho^A,\rho^B]\right)_IJ \quad (18)$$

does not vanish. However, it may be extended also to the non-Abelian case by observing that $\{\tau^a,\tau^b\}$ is representation-dependent and in general, it is out of the algebra, thus the constraint must be imposed is similar to (16):

$$\beta^{(a}_{1[A} \beta^{b]}_{2B} = 0, \quad (19)$$

whose solutions are also similar (see Appendix):

$$\beta^a_A = \beta^a V_A, \quad (20)$$

where $\beta^a \in \mathcal{G}$, while $V_A \in C$ has to be fixed for each group parametrization. This is a $\dim G$-dimensional manifold of parameters such as it was understood in the first part of the article. Again, we may choose $V_A \equiv (0,0,v_\mu,0,0)$ and due to the presence of an arbitrary scalar gauge parameter, a generic group element writes as in (13).
3.1 Remarks and Outline.

The possible group structure associated with the KR symmetry was determined and well-defined together with its spinorial representations. We saw that, by considering the largest multi-tensorial parameter space (Clifford Algebra), the result is basically the same: the tensorial direction of the parameter is fixed, and tensorially separated and tensorially normally be built up. Then, the associated field strength is canonically defined and gauge theories may open up the possibility of defining a gauge massive model in four dimensions (even in the non-Abelian case), since almost every theory with topological mass involves a rank-two gauge field.

We conclude this letter by stressing that this construction is an initial step, the fundamental starting point in order to construct well-defined Abelian and non-Abelian gauge theories involving KR fields. This open up the possibility of defining a gauge massive model in four dimensions (even in the non-Abelian case), since almost every theory with topological mass involves a rank-two gauge field.

In a future work we shall construct the covariant derivative and extract from it, Kalb-Ramond two-form as a gauge field. Then, the associated field strength is canonically defined and gauge theories may normally be built up.

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4 Appendix: On the possible families of group parameters.

Here, we study the solutions to equation (19) which define a group parametrization. In the second section we found the eq. (19) whose form is identical and the solutions are of the same type, the single difference is that \( \Lambda_1 \) must be considered in behalf of the Clifford Algebra \( C \).

Let us denote by \( P_2 \) a family of parameters, \( \beta \), corresponding to a same group representation; as has been argued before, if two parameters are in \( P_2 \) then they are related by the equation (13). Here we shall show that only class of solutions to this equation actually constitutes a group parametrization.

Contracting this equation with generic objects \( \{e_{ij}\}_{i=1}^{dim(G)-2} \), a basis for it,

I- \( w^a, w^a \) are not proportional. Then, they may form, together with \((dim(G)) - 2\)-linearly independent elements of \( G \), say \( \{\epsilon_i\}_{i=1}^{dim(G)-2} \), a basis for it.

II- \( U_A, W_A \) are not proportional. Then, they may form, together with \((dim(C)) - 2(= 14)\)-linearly independent elements of \( C \), say \( \{C_{iA}\}_{i=1}^{14} \), a basis for it.

Clearly, we may write

\[
\beta_{2A}^a = b_2 b_1^{-1} \beta_{1A}^a + u^a U_A + w^a W_A, \tag{21}
\]

for certain \( u^a, w^a \in G \) and \( U_A, W_A \in C \). Here, we also have the complex numbers, \( b_{1,2} \equiv X_a Y^A \beta_{(1,2)A}^a \).

Next, we make two suppositions which shall define a particular class of solutions to (19):

I- \( w^a, w^a \) are not proportional. Then, they may form, together with \((dim(G)) - 2\)-linearly independent elements of \( G \), say \( \{\epsilon_i\}_{i=1}^{dim(G)-2} \), a basis for it.

II- \( U_A, W_A \) are not proportional. Then, they may form, together with \((dim(C)) - 2(= 14)\)-linearly independent elements of \( C \), say \( \{C_{iA}\}_{i=1}^{14} \), a basis for it.

Substituting this into the equation (19), and using the identity \( \beta_{1A}^a \beta_{1B}^b \equiv 0 \), we get the equation

\[
\Sigma_{i=1}^{14} [a_i^{(a}w^{b)}C_{i[A}U_{B]} + a_i^{(a}w^{b)}C_{i[A}W_{B]}] + [a_{W}^{(a}u^{b)} - a_{U}^{(a}u^{b)}]W_{[A}U_{B]} = 0, \tag{23}
\]
which, due to the linear independence implies $a_i^{(a)u^b} = 0, a_i^{(a)w^b} = 0$ and

$$
a_i^{(a)u^b} - a_i^{(a)w^b} = 0. \tag{24}
$$

Contracting the first two equations with an arbitrary element of $\mathcal{G}$, one obtains that $a_i^\alpha$ must be proportional to both $u^a, w^a$; since they are not parallel by hypothesis, we conclude $a_i^\alpha = 0$.

So, we shall finally solve only the equation $\beta_{1A}^\alpha$ Due to the assumption $I$, one may write

$$
a_i^{\alpha}\beta_{1A}^\alpha = \Sigma_{i=1}^{\text{dim} \mathcal{G}} \alpha_i^{(a)u^b} + a_i^{(a)u^a} + a_i^{(a)w^a} \tag{25}
$$

and

$$
a_i^{\alpha}\beta_{2A}^\alpha = \Sigma_{i=1}^{\text{dim} \mathcal{G}} \alpha_i^{(a)u^b} + a_i^{(a)u^a} + a_i^{(a)w^a} \tag{26}
$$

where $\alpha_i^{(a)u^b}, \alpha_i^{(a)u^a}, \alpha_i^{(a)w^a}, \alpha_i^{(a)w^a}$ are complex numbers. Putting these expressions back into eq. (24) we get

$$
\Sigma_{i=1}^{\text{dim} \mathcal{G}} \left[ \alpha_i^{(a)u^b} + \left[ \alpha_i^{(a)u^a} - \alpha_i^{(a)w^a} \right] u^a + \left[ a_i^{u^a} - a_i^{w^a} \right] u^a + a_i^{(a)w^a} w^a + a_i^{(a)w^a} u^a \right] = 0. \tag{27}
$$

Due to $I$, we conclude that $\alpha_i^{(a)u^b} = 0, \alpha_i^{(a)u^a} = 0, \alpha_i^{(a)w^a} = 0$ and $a_i^{(a)w^a} = 0$ and $a_i^{(a)w^a} = 0$. Then, our multi-parameter initial must have the form

$$
\beta_{1A}^\alpha = \beta_{1A}^a u^a \tag{28}
$$

which, by virtual of eq. (21), implies

$$
\beta_{2A}^\alpha \propto \beta_{1A}^a. \tag{29}
$$

However, as it has already been argued, this class of solutions do not constitute a group parametrization because it is a one-parametric family of parameters; therefore, it is not isomorphic to $\mathcal{G}$ (unless it is a one-dimensional group, which was excluded from this discussion).

So, we only can consider some of the hypotheses $I, II$ to be false; in both cases, eq. (21) may be rewritten as

$$
\beta_{2A}^\alpha = \beta_{1A}^a + \beta_{1A}^a u^a + v^a \,, \tag{30}
$$

for certain $v^a \in \mathcal{G}$ and $V_A \in \mathcal{C}$. Then, we may write

$$
\beta_{1A}^\alpha = \Sigma_{i=1}^{\text{dim}(C)(-1)(1=15)} \alpha_i^{a} C_i A + \beta_{1A}^a V_A \tag{31}
$$

where $\{ \{C_i A\}_{i=1}^{\text{dim}(C)(-1)(1=15)} ; V_A \}$ constitutes a basis for $\mathcal{C}$. Thus, using this in the equation (31), one gets

$$
\Sigma_{i=1}^{\text{dim}(C)(-1)(1=15)} \alpha_i^{(a)v^b} C_i (A V_B) = 0. \tag{32}
$$

Therefore,

$$
a_i^{(a)v^b} = 0 \tag{33}
$$

Contracting this equation with an arbitrary element of $\mathcal{G}$: $a_i^\alpha = a_i^{v^b}$; plugging this back into (33), we get $a_i^\alpha = 0$. Then, the complete solution results as below:

$$
\beta_{1A}^\alpha = \beta_{1A}^a V_A. \tag{34}
$$

From (15), $\beta_{2A}^\alpha = \beta_{2A}^a V_A$, which shows that all parameter in this family writes as $\beta_{1A}^\alpha = \beta_{2A}^a V_A$, where $V_A$ is fixed. This is a $\text{dim} \mathcal{G}$-parametric family of parameters, as expected.

This completes the argument.
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