\textbf{$\rho$-Decay Widths of Excited Heavy Mesons From Light-Cone QCD Sum Rules in the Leading Order of HQET}

Shi-Lin Zhu and Yuan-Ben Dai

\textit{Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing 100080, China}

Abstract

The couplings and decay widths of the processes $(1^+, 2^+) \rightarrow (0^-, 1^-) + \rho$ are studied with the light-cone QCD sum rules in the leading order of heavy quark effective theory. These processes are employed to estimate the two pion transition widths of the $(1^+, 2^+)$ doublet through the low mass tail of the $\rho$ resonance. The ambiguity due to the presence of two distinct $1^+$ states is solved. Our calculation shows that the two pion decay widths of the $(1^+, 2^+)$ doublets are much smaller than the single pion decay widths. However, the $B_1, B_2^*$ mesons should also have significant two pion decay widths around 1.5MeV.

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1 Introduction

The heavy quark effective theory (HQET) \cite{1} provides a systematic expansion of the heavy hadron spectra and transition amplitude in terms of $1/m_Q$, where $m_Q$ is the heavy quark mass. The spectrum of the ground state heavy meson has been studied with the QCD sum rules \cite{2} in HQET in \cite{3}. In \cite{4} the mass of the lowest excited heavy meson doublets $(1^+, 2^+)$ and $(0^+, 1^+)$ were studied with QCD sum rules in the heavy quark effective theory (HQET) up to the order of $\mathcal{O}(1/m_Q)$.

The QCD sum rules was used to analyse the exclusive radiative $B$-decays with the help of the light-cone vector meson wave function in \cite{5}. With the same formalism the off-shell $g_{B^*B\rho}$ and $g_{D^*D\rho}$ couplings were calculated in \cite{6}. In this work we employ the light-cone QCD sum rules (LCQSR) in HQET to calculate the decay widths of the processes $(1^+, 2^+) \rightarrow (0^-, 1^-) + \rho$ to the leading order of $1/m_Q$. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be treated quite nicely.

One difficult problem encountered in studying the decay widths of excited heavy mesons is due to the degeneracy of the spectra. There are a pair of states for any spin-parity $j^P$ with close values in their masses but quite different in magnitudes of their decay widths, except for the ground states. Only in the $m_Q \rightarrow \infty$ limit, is there a conserved quantum number...
$j_\ell$, the angular momentum of the light component, which can be used to differentiate the two states. Therefore, HQET has important and unique advantage for this purpose. These are the motivation for our approach of using LCQSR in HQET.

The proper interpolating current $J_{\alpha_1^{\ldots} \alpha_j}^{\gamma_1^{\ldots} \gamma_j}$ for the states with the quantum number $j, P, j_\ell$ in HQET was given in [3]. They were proved to satisfy the following conditions

$$
\langle 0|J_{j_\ell,P,j_\ell}^{\alpha_1^{\ldots} \alpha_j}(0)|j_\ell', P', j_\ell' \rangle = \delta_{j,j'}\delta_{P,P'}\delta_{j_\ell,j_\ell'}\eta_{\alpha_1^{\ldots} \alpha_j},
$$

$$
i \langle 0|T\left(J_{j_\ell,P,j_\ell}^{\alpha_1^{\ldots} \alpha_j}(x)J_{j_\ell',P',j_\ell'}^{\gamma_1^{\ldots} \gamma_j}(0)\right)|0\rangle = \delta_{j,j'}\delta_{P,P'}\delta_{j_\ell,j_\ell'}(-1)^j S_{\eta_{\alpha_1^{\ldots} \gamma_j}} \times \int dt \delta(x - vt) \Pi_{P,j_\ell}(x)
$$

in the $m_Q \to \infty$ limit, where $\eta_{\alpha_1^{\ldots} \alpha_j}$ is the polarization tensor for the spin $j$ state, $v$ is the velocity of the heavy quark, $g_{\mu}^{\alpha \beta}$ is the transverse metric tensor, $S$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1^{\ldots} \alpha_j)$ and $(\beta_1^{\ldots} \beta_j)$, $f_{P,j_\ell}$ and $\Pi_{P,j_\ell}$ are a constant and a function of $x$ respectively which depend only on $P$ and $j_\ell$. Because of equations (1) and (2), the sum rule in HQET for decay widths derived from a correlator containing such currents receive no contribution from the unwanted states with the same spin-parity as the states under consideration in the $m_Q \to \infty$.

## 2 Sum rules for decay amplitudes

In the present work we shall confine ourselves to the leading order of $1/m_Q$ expansion. Denote the doublet $(1^+, 2^+)$ with $j_\ell = 3/2$ by $(B_1, B_2^*)$. From covariance and conservation of the angular momentum of the light component in the $m_Q \to \infty$ limit, there are three independent decay amplitudes for the decays of $B_1, B_2^*$ to the rho meson and the ground states $B, B^*$ in the doublet $(0^-, 1^-)$ with $j_\ell = 1/2$. They are characterized by $(j_h, l) = (2, 2), (1, 2), (1, 0)$, where $l$ and $j_h$ are the orbital and total angular momentum of the rho meson respectively [4]. The S-wave amplitude is dominant.

The amplitudes read as follows:

$$M(B_1 \to B^* \rho) = I \eta_\beta^* \left\{ i \epsilon_{\mu \nu \alpha \beta} v_\sigma (\eta^\alpha \epsilon_\beta + \eta_\beta \epsilon^\alpha) (g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho} - \frac{1}{3} g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho}) (B_1, B^*)
+ i \epsilon_{\mu \nu \alpha \beta} v_\sigma (\eta^\alpha \epsilon_\beta - \eta_\beta \epsilon^\alpha) (g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho} - \frac{1}{3} g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho}) (B_1, B^*)
+ i \epsilon_{\mu \nu \alpha \beta} \eta_\alpha \epsilon_\beta v_\sigma g_{\mu \nu} (B_1, B^*) \right\},
$$

$$M(B_1 \to B \rho) = I \eta_\beta^* \left\{ (g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho} - \frac{1}{3} g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho}) (B_1, B) + g_{\mu}^{\alpha \beta} g_{\nu} (B_1, B) \right\},
$$

$$M(B_2^* \to B^* \rho) = I \eta_\alpha^* \epsilon_{\mu \nu \alpha \beta} (g_{\mu}^{\alpha \rho} g_{\nu}^{\beta \rho} - \epsilon_\beta^* g_{\mu \nu} g_{\alpha \beta}) (B_2^*, B^*)
+ i \epsilon_{\mu \nu \alpha \beta} \eta_\alpha \epsilon_\beta g_{\mu \nu} (B_2^*, B^*) \right\},
$$

$$M(B_2^* \to B \rho) = I \eta_\alpha^* \epsilon_{\mu \nu \alpha} \epsilon_{\mu \nu}^* (B_2^*, B),
$$

where $\eta_\mu \nu$, $\eta_\mu$, $\epsilon_\mu$ and $\epsilon_{\mu}^{(\lambda)}$ are polarization tensors for the $2^+, 1^+, 1^-$ heavy mesons and the rho meson respectively. $q_{\mu} = q_\mu - v \cdot q v_\mu$. $I = 1, \frac{1}{\sqrt{2}}$ for the charged and neutral.
rho meson respectively. The amplitude \( g_{(2,2)} \), \( g_{(1,2)} \) and \( g_{(1,0)} \) in (3)-(5) corresponds to the \((j_n, l) = (2,2), (1,2), \) and \((1,0) \) decay amplitude respectively.

Due to the heavy quark symmetry the D-wave and S-wave amplitudes in these four processes are related. From vector current conservation we have another constraint. For example, vector current conservation requires that the tensor structure be \( (q_t^a q_t^b - q_t^2 g_t^{\alpha \beta}) \) for the process \( B_1 \to B^* \rho \). From this condition we have \( g_{(1,0)}(B_1, B) = -\frac{2}{3} \frac{g_t^2}{g_{(2,2)}}(B_1, B) \). In HQET there exist only two independent coupling constants \( g_d = g_{(1,2)}(B^*_2, B) \) and \( g_s = g_{(1,0)}(B^*_2, B^*) \), corresponding to the D-wave and S-wave decays respectively.

In order to derive the sum rules for the coupling constants \( g_d \) and \( g_s \) we consider the correlators

\[
J^{(B^*_2 B)}(\omega, \omega') = \frac{g_d}{\mp \omega_3}(\omega, \omega') G^{B^*_2 B}(\omega, \omega') + \frac{g_s}{\mp \omega_3}(\omega, \omega') G^{B^*_2 B}(\omega, \omega'),
\]

(7)

\[
J^{(B^*_2 B)}(\omega, \omega') = \int d^4 x \ e^{-ik \cdot x} \langle \rho(q)|T\{\gamma_\mu \gamma_5 \rho(q)|\{\gamma_\nu \gamma_5 \rho(q)\}\} \rangle \Omega(q, q') \rangle = I \ e^{i \omega' (\omega - \omega') - \frac{1}{3} \omega_3 (\omega - \omega')} \left( J^{(B^*_2 B)}(\omega, \omega') + \frac{g_d}{\mp \omega_3}(\omega, \omega') \right),
\]

(8)

\[
J^{(B^*_2 B)}(\omega, \omega') = \int d^4 x \ e^{-ik \cdot x} \langle \rho(q)|T\{\gamma_\mu \gamma_5 \rho(q)|\{\gamma_\nu \gamma_5 \rho(q)\}\} \rangle \Omega(q, q') \rangle = I \ e^{i \omega' (\omega - \omega') - \frac{1}{3} \omega_3 (\omega - \omega')} \left( J^{(B^*_2 B)}(\omega, \omega') + \frac{g_d}{\mp \omega_3}(\omega, \omega') \right),
\]

(9)

where \( k' = k - q, \omega = 2v \cdot k, \omega' = 2v \cdot k', q^2 = q'^2 - (q \cdot v)^2 \) and \( q^2 = m_r^2 \).

The interpolation currents are given in [4] as

\[
J^{(B^*_2 B)}(\omega, \omega') = \int d^4 x \ e^{-ik \cdot x} \langle \rho(q)|T\{\gamma_\mu \gamma_5 \rho(q)|\{\gamma_\nu \gamma_5 \rho(q)\}\} \rangle \Omega(q, q') \rangle = I \ e^{i \omega' (\omega - \omega') - \frac{1}{3} \omega_3 (\omega - \omega')} \left( J^{(B^*_2 B)}(\omega, \omega') + \frac{g_d}{\mp \omega_3}(\omega, \omega') \right),
\]

(10)

where \( h_v \) is the heavy quark field in HQET and \( \gamma_{\mu \nu} = \gamma_\mu - \nu_\mu \hat{v} \).

Let us first consider the function \( G_d(\omega, \omega') \equiv G^{B^*_2 B}(\omega, \omega') \) in (10). As a function of two variables \( \omega \) and \( \omega' \), it has the following pole terms from double dispersion relation

\[
G_d(\omega, \omega') = \frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} g_d}{(2 \Lambda_{-\frac{1}{2}} - \omega')(2 \Lambda_{+\frac{1}{2}} - \omega') + \frac{c'}{2 \Lambda_{+\frac{1}{2}} - \omega'} + \frac{c}{2 \Lambda_{-\frac{1}{2}} - \omega'}},
\]

(14)

where \( f_{P, j\xi} \) are constants defined in [4], \( \Lambda_{P, j\xi} = m_{P, j\xi} - m_Q \).
For deriving QCD sum rules we calculate the correlator \([10]\) by the operator expansion on the light-cone in HQET to the leading order of \(1/m_Q\). The expression with the tensor structure reads:

\[
- \int_0^\infty dt \int dx e^{-ikx} \delta(-x-vt) \text{Tr}\{(i\gamma_5)\frac{1+\hat{\nu}}{2}(-i\gamma_5)(D_t^\alpha - \frac{1}{3}\gamma_t^\alpha \hat{D}_t)\langle \rho(q)|u(x)d(0)|0\rangle\},
\]

(15)

The rho wave function is defined as the matrix elements of nonlocal operators between the vacuum and rho meson state. Up to twist four the Dirac components of the rho meson structure reads:

\[
\text{P}\text{u}
\]

where \(N\) and \(g\) contribute two and three contributions. The twist-two contributions to the wave functions the transversely and longitudinally polarised rho meson respectively. Both wave functions the leading twist distributions in the fraction of total momentum carried by the quark in the transversely and longitudinally polarised rho meson respectively. Both wave functions \(g^\rho_a(u,\mu)\) and \(g^\nu(u,\mu)\) correspond to transverse spin distribution. They contain both twist-two and -three contributions. The twist-two contributions to the wave functions \(g^\rho_a(u,\mu)\) and \(g^\nu(u,\mu)\) can be expressed in terms of the leading twist longitudinal wave function \(\varphi^L_\rho(u,\mu)\):

\[
g^\nu_*\text{twist-2}(u) = \frac{1}{2}\{\int_0^u dv \frac{\varphi^L_\rho(u)}{1-v} + \int_u^1 dv \frac{\varphi^L_\rho(u)}{v}\},
\]

(19)

\[
\frac{d}{du} g^\nu_{\text{twist-2}}(u) = 2\{-\int_0^u dv \frac{\varphi^L_\rho(u)}{1-v} + \int_u^1 dv \frac{\varphi^L_\rho(u)}{v}\}.
\]

(20)

All the four wave functions \(f = \varphi^T, \varphi^L, g^\nu, g^a\) are normalised such that \(\int_0^1 f(u)du = 1\). We shall keep only the leading twist contributions, i.e., we use the asymptotic forms for the following wave functions:

\[
\varphi^L(u,\mu) = 6u(1-u),
\]

(21)

\[
g^\nu(u,\mu) = \frac{3}{4}(1+\xi^2),
\]

(22)

\[
g^a(u,\mu) = 6u(1-u),
\]

(23)

with \(\xi = 2u - 1\). We use the wave function in \([10]\) for \(\varphi^T(u)\) with the mock hadron mass replaced by the average total quark energy \([11]\):

\[
\varphi^T(u) = Nexp\left(-\frac{m^2}{8\beta^2u(1-u)}\right)[\mu^2 + \mu\tilde{\mu} + \tilde{\mu}^2u(1-u)],
\]

(24)

where \(N = 0.338, \mu = \frac{m}{\beta}, \tilde{\mu} = \frac{\bar{m}}{\beta}\) with \(m = 330\text{MeV}, \bar{m} = 1130\text{MeV},\) and \(\beta = 320\text{MeV}\). The wave function \([24]\) satisfies the constraint from the QCD sum rule analysis of the
moments of the rho meson wave function [9, 5, 12]. The rho meson wave function is close to the asymptotic form, which was pointed out in [9, 10, 5]. The rho meson coupling constants were determined from QCD sum rule analysis: $f_\rho^2 = f_\rho = 200$ MeV [9].

Expressing (13) with the rho light-cone wave functions, we arrive at:

$$G_d(\omega, \omega') = -\frac{1}{4} \int_0^\infty dt \int_0^1 du e^{(1-u)\frac{2}{\Lambda}} e^{iu\frac{\Lambda}{4}} u \left\{ f_\rho^T \varphi_\rho^T(u) - \frac{it}{4} f_\rho m_\rho g_\rho^a(u) \right\}. \quad (25)$$

Similarly for the other D-wave functions in (14)-(16), we have:

$$G_{B_{1/2}^1 B_{1/2}^1}(\omega, \omega') = \frac{\sqrt{6}}{6} G_d(\omega, \omega'), \quad (26)$$

$$G_{B_{1/2}^3 B_{1/2}^3}(\omega, \omega') = \frac{\sqrt{6}}{6} G_d(\omega, \omega'), \quad (27)$$

$$G_{B_{1/2}^1 B_{1/2}^3}(\omega, \omega') = \frac{\sqrt{6}}{6} G_d(\omega, \omega'), \quad (28)$$

$$G_{B_{1/2}^3 B_{1/2}^1}(\omega, \omega') = G_d(\omega, \omega'). \quad (29)$$

For the S-wave functions, we have:

$$G_{B_{1/2}^1 B_{1/2}^1}(\omega, \omega') = \frac{\sqrt{6}}{6} G_s(\omega, \omega'), \quad (30)$$

$$G_{B_{1/2}^3 B_{1/2}^3}(\omega, \omega') = \frac{\sqrt{6}}{6} G_s(\omega, \omega'), \quad (31)$$

where we define $G_s(\omega, \omega') \equiv G_{B_{1/2}^1 B_{1/2}^1}(\omega, \omega')$.

$$G_s(\omega, \omega') = \frac{g_\rho^2}{3} G_d(\omega, \omega') = \frac{1}{3} [m_\rho^2 - \frac{(\omega - \omega')^2}{4}] G_d(\omega, \omega'). \quad (32)$$

From the above relations we know there are two independent coupling constants only. This is in agreement with the discussions above.

In order to extract $g_d$ we first make Wick rotation, then make double Borel transformation to eliminate the single-pole terms in (14). Subtracting the continuum contribution which is modeled by the dispersion integral in region $\omega, \omega' \geq \omega_c$, we arrive at:

$$g_d f_{-\frac{1}{2}, \frac{1}{2}} f_{+\frac{1}{2}, \frac{1}{2}} = \frac{1}{2} e^{-\frac{\Lambda}{4} + \frac{A}{4}} u_0 \left\{ f_\rho^T \varphi_\rho^T(u_0) T(1 - e^{-\frac{\Lambda}{4}}) - \frac{1}{2} f_\rho m_\rho g_\rho^a(u_0) \right\}, \quad (33)$$

where $u_0 = \frac{T_1}{T_1 + T_2}$, $T = \frac{T_1 T_2}{T_1 + T_2}$, $T_1, T_2$ are the Borel parameters corresponding to the variables $\omega, \omega'$ respectively. In obtaining (33) we have used the Borel transformation formula: $B_\omega^T e^{\alpha \omega} = \delta(\alpha - \frac{1}{T})$.

For the coupling constant $g_s$, we have:

$$g_s f_{-\frac{1}{2}, \frac{1}{2}} f_{+\frac{1}{2}, \frac{1}{2}} = \frac{1}{6} e^{-\frac{\Lambda}{4} + \frac{A}{4}} u_0 \left\{ f_\rho^T \varphi_\rho^T(u_0) T(1 - e^{-\frac{\Lambda}{4}}) - \frac{1}{2} f_\rho m_\rho g_\rho^a(u_0) \right\}$$

$$- \frac{1}{24} e^{-\frac{\Lambda}{4} + \frac{A}{4}} \frac{d^2}{du^2} \left( f_\rho^T u \varphi_\rho^T(u) T^3 f_2(\frac{\Lambda}{4}) - \frac{1}{2} f_\rho m_\rho u g_\rho^a(u) T^2 f_1(\frac{\Lambda}{4}) \right) |_{u=u_0}, \quad (34)$$

where $f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}$ is the factor used to subtract the continuum. The derivative in (34) arises from the factor $(q \cdot v)^2$ in (32). We have used integration by parts to absorb the factor $(q \cdot v)^2$. In this way we arrive at the simple form after double Borel transformation.
3 Determination of the parameters

In order to obtain the coupling constants from (33) and (34) we need to use the mass parameters $\bar{\Lambda}$'s and the coupling constants $f$'s of the corresponding interpolating currents as input. $\bar{\Lambda}_{-1/2}$ and $f_{-1/2}$ can be obtained from the results in [13] as $\bar{\Lambda}_{-1/2} = 0.5 \text{ GeV}$ and $f_{-1/2} \approx 0.25 \text{ GeV}^{3/2}$ at the leading order of $\alpha_s$. Notice that the coupling constant $f_{-1/2}$ defined in the present work is a factor $1/\sqrt{2}$ smaller than that defined in [13]. $\bar{\Lambda}_{+3/2}$ is given in [4]. $f_{+3/2}$ can be determined from the formulas (34) of reference [4] derived from sum rules for two point correlators. The results are

$$\bar{\Lambda}_{+3/2} = 0.82 \text{ GeV} \quad f_{+3/2} = 0.19 \pm 0.03 \text{ GeV}^{5/2},$$

(35)

We choose to work at the symmetric point $T_1 = T_2 = 2T$, i.e., $u_0 = \frac{1}{2}$ as traditionally done in literature [7]. The mass difference between $(1^+,2^+)$ and $(0^-,1^-)$ doublets is about 0.5 GeV in the leading order of HQET, which is much smaller than the large value of $T_1, T_2$ used below, $T_1, T_2 \sim 3.6 \text{ GeV}$. So such a choice is reasonable. The values of the various functions appearing in (33), at $u_0 = \frac{1}{2}$, are: $\varphi^T_\rho(u_0) = 1.552, g^a_\rho(u_0) = 1.5, \varphi''_\rho(u_0) = -11.55$ and $g''_\rho(u_0) = -12$.

4 Numerical results and discussion

We now turn to the numerical evaluation of the sum rules for the coupling constants. The upper limit of $T$ is constrained by the requirement that the continuum contribution is less than 40%. This corresponds to $T < 2.5 \text{ GeV}$. The lower limit of $T$ is at the point of $T$ where stability develops. This leads to $T > 1.0 \text{ GeV}$ for the sum rules (33).

Stability develops for the sum rule (33)-(34) in the region $1.0 \text{ GeV} < T < 2.5 \text{ GeV}$. With the values of pion wave functions at $u_0 = \frac{1}{2}$ we obtain the sum rules for $g_d f_{-1/2} f_{+3/2}$ and $g_s f_{-1/2} f_{+3/2}$ as functions of $T$ and $\omega_c$. The results are plotted as curves in Fig. 1 and 2 with $\omega_c = 3.2, 3.0, 2.8 \text{ GeV}$.

Numerically we have:

$$g_d f_{-1/2} f_{+3/2} = (0.17 \pm 0.02) \text{ GeV}^2,$$

(36)

$$g_s f_{-1/2} f_{+3/2} = (0.095 \pm 0.02) \text{ GeV}^4,$$

(37)

where the errors refers to the variations with $T$ and $\omega_c$ in this region. The central value corresponds to $T = 1.8 \text{ GeV}$ and $\omega_c = 3.0 \text{ GeV}$.

With the values of $f$'s in (35), we get:

$$g_d = (3.8 \pm 0.4 \pm 0.8) \text{ GeV}^{-2},$$

(38)

$$g_s = 2.1 \pm 0.4 \pm 0.4,$$

(39)

where the second error takes into account the uncertainty in $f$'s. The inherent uncertainties due to the method of QCD sum rules and the choice of the rho meson wave functions are not included here.
The decay width formulas in the leading order of HQET are

\[ \Gamma(B_1 \to B \rho) = \frac{1}{144 \pi} g_s^2 |q|^5 + \frac{1}{8 \pi} g_s^2 |q| , \]
\[ \Gamma(B_1 \to B^* \rho) = \frac{1}{16 \pi} \left( \frac{14}{9} + \frac{1}{27} \frac{|q|^2}{m_\rho^2} \right) g_s^2 |q|^5 + \frac{2}{9} \frac{|q|^5}{m_\rho^2} g_s g_s + \left( 1 + \frac{1}{3} \frac{|q|^2}{m_\rho^2} \right) g_s^2 |q| , \]
\[ \Gamma(B_2^* \to B \rho) = \frac{3}{80 \pi} g_s^2 |q|^5 , \]
\[ \Gamma(B_2^* \to B^* \rho) = \frac{1}{16 \pi} \left( \frac{4}{3} + \frac{1}{9} \frac{|q|^2}{m_\rho^2} \right) g_s^2 |q|^5 + \frac{2}{3} \frac{|q|^5}{m_\rho^2} g_s g_s + \left( 3 + \frac{|q|^2}{m_\rho^2} \right) g_s^2 |q| , \]

where \( |q| = \sqrt{(m_1^2 - (m_2 + s)^2)(m_1^2 - (m_2 - s)^2)/2m_1} \), \( m_1, m_2 \) is the parent and decay heavy meson mass, \( s \) is the \( \rho \) meson mass and \( m_\rho = 770 \text{MeV} \) is the \( \rho \) meson central mass.

In (40) the sum over charged and neutral rho meson final states has been included. In order to estimate the two-pion transition widths of the \((1^+, 2^+)\) doublet, we assume these transitions are dominated by the low-mass tail of the \( \rho \) resonance. Note \( 2m_\pi \leq s \leq m_1 - m_2 \).

We apply the leading order formulas obtained above to the excited states of charmed mesons assuming the HQET holds well for the charm system. The value \( |\vec{q}| \) for these processes in (40) are calculated from the experimental mass values of the relevant particles [14]. We have to smear the expression (40) with a Breit-Wigner form to take into account of the 150-MeV width of the \( \rho \) meson, \( f(s^2) = \frac{1}{\pi} \frac{m_\rho^2}{(s^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} \). Our final results are:

\[ \Gamma(D_1 \to D \rho) = 2.32 \text{MeV} , \]
\[ \Gamma(D_1 \to D^* \rho) = 0.08 \text{MeV} , \]
\[ \Gamma(D_2^* \to D \rho) = 0.05 \text{MeV} , \]
\[ \Gamma(D_2^* \to D^* \rho) = 0.51 \text{MeV} \]

(41)

The masses of \( B_1, B_2^* \) are not known experimentally. If we use \( m_{B_1} = 5.755 \text{GeV} \) and \( m_{B_2^*} = 5.767 \text{GeV} \) from the quark model [8], the decay widths are:

\[ \Gamma(B_1 \to B \rho) = 1.11 \text{MeV} , \]
\[ \Gamma(B_1 \to B^* \rho) = 0.34 \text{MeV} , \]
\[ \Gamma(B_2^* \to B \rho) = 0.01 \text{MeV} , \]
\[ \Gamma(B_2^* \to B^* \rho) = 1.16 \text{MeV} \]

(42)

In reference [8], the above decay widths were estimated based on the potential model and the assumption that the strange quark satisfies the heavy quark symmetry:

\[ \Gamma(D_1 \to D \rho) = 6 \text{MeV} , \]
\[ \Gamma(D_1 \to D^* \rho) =< 1 \text{MeV} , \]
\[ \Gamma(D_2^* \to D \rho) =< 1 \text{MeV} , \]
\[ \Gamma(D_2^* \to D^* \rho) = 3 \text{MeV} \]

(43)

\[ \Gamma(B_1 \to B \rho) = 3 \text{MeV} , \]

(44)
\[ \Gamma(B_1 \to B^\ast \rho) = 1 \text{MeV}, \]
\[ \Gamma(B_2^* \to B\rho) = <1 \text{MeV}, \]
\[ \Gamma(B_2^* \to B^\ast \rho) = 3 \text{MeV} \] (44)

In this work we have calculated the two-pion transitions of the \((1^+, 2^+)\) doublets through the low-mass tail of the rho resonance with the method of the light cone QCD sum rules in the leading order of \(1/m_Q\). The derived two-pion decay widths are smaller than the estimated values in [8]. Although we may conclude that the total width of \(D_2^\ast\) and \(D_1\) is dominated by one pion decay, the two-pion decay channels are not negligible. Experimental data is not yet available for \(B_1\) and \(B_2^\ast\) mesons. From the above discussions we know that they should also have significant two-pion decay widths.

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Figure Captions

Fig. 1. The sum rule for $f_{-\frac{1}{2}} f_{+\frac{3}{2}} g_d$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

Fig. 2. The sum rule for $f_{-\frac{1}{2}} f_{+\frac{1}{2}} g_s$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.
