Double electromagnetically induced transparency and narrowing of probe absorption in a ring cavity with nanomechanical mirrors

Sumei Huang

Department of Physics, University of California, Merced, California 95343, USA
Department of Electrical and Computer Engineering, National University of Singapore,
4 Engineering Drive 3, 117583 Singapore
E-mail: elehuasu@nus.edu.sg

Received 25 November 2013, revised 20 January 2014
Accepted for publication 23 January 2014
Published 20 February 2014

Abstract
We study the effect of a strong coupling field on the absorptive property of a ring cavity with two mirrors oscillating at slightly different frequencies to a weak probe field. We observe double electromagnetically induced transparency windows separated by an absorption peak at line centre in the output probe field under the action of a strong coupling field. We find that increasing driving power can broaden the two transparency windows, which results in narrowing of the central absorption peak. At high driving power, the linewidth of the sharp central absorption peak is approximately equal to the mechanical linewidth. We show the normal mode splitting in both the output probe field and the generated Stokes field. We also find that the suppression of the four-wave mixing process can be achieved on resonance.

Keywords: optomechanical systems, electromagnetically induced transparency, narrowing of probe absorption, normal mode splitting

(Some figures may appear in colour only in the online journal)

1. Introduction

It is well-known that a \( \Lambda \)-type three-level atomic medium can become transparent to a weak probe field by applying a strong coupling field, which is the result of the destructive interference between two different excitation pathways to the upper level. This is the phenomenon of electromagnetically induced transparency (EIT) [1–4]. The EIT has been shown to be important for various applications such as slow light [5], light storage [6], and so on. Besides, the studies of EIT have been extended to multi-level atomic systems. The double EIT windows separated by a narrow absorption peak in the probe absorption spectrum have been observed in the four-level atomic systems [7–11]. Recently, the EIT effect has been reported in the macroscopic optomechanical systems. The analogy of EIT in optomechanical systems has been shown theoretically [12], and observed in a number of experiments in optical cavities [13–15] and microwave cavities [16, 17]. Moreover EIT in optomechanical systems in the nonlinear regime was analysed [18–20]. Additionally, the electromagnetically induced absorption, the opposite effect to EIT, was discussed in a two-cavity optomechanical system [21]. In addition, it has been proven that a strong dispersive coupling between the optical mode and the mechanical mode when the effective optomechanical coupling rate exceeds the optical and mechanical decay rates leads to normal mode splitting [22–24]. The effective optomechanical coupling rate can be enhanced by increasing the power of the driving laser.

In this paper, we investigate the nonlinear response of a ring cavity with two moving mirrors having two close frequencies to a weak probe field in the absence and the presence of a strong coupling field. We find that there are two transparency windows and an absorption peak in the transmitted probe field in the presence of the
A strong coupling field at frequency \( \omega_0 \) and a weak probe field at frequency \( \omega_p \) are injected into the ring cavity via the fixed mirror and interact with two movable mirrors whose resonance frequencies \( (\omega_1, \omega_2) \) are a little different.

The pump-induced broadening of the two EIT dips leads to narrowing the central absorption peak. And the narrow absorption peak associated with double EIT windows may have potential application in high-resolution laser spectroscopy [7-9]. We also observe the normal mode splitting in the output probe field at high driving power. In addition, we show that the normal mode splitting occurs in the Stokes field generated by means of the four-wave mixing process. And the four-wave mixing process is completely suppressed on resonance. However, if two movable mirrors in a ring cavity are oscillating at identical frequencies, the EIT-like dip can be observed in the output probe field, and the four-wave mixing process is not suppressed on resonance.

The paper is organized as follows. In section 2, we introduce the system, give the time evolutions of the expectation values of the system operators, and solve them. In section 3, the expressions for the components of the output field at the probe frequency and the Stokes frequency are given. In section 4, we present the numerical results for the output probe field without or with the coupling field, and compare it with that from a ring cavity with two movable mirrors having equal frequencies.

In section 5, we show the numerical results for the output Stokes field from a ring cavity with two moving mirrors having different or equal frequencies, and compare them. Finally in section 6, we conclude the paper.

2. Model

We consider a ring cavity with round-trip length \( L \) formed by three mirrors, as shown in figure 1 [25]. One of them is not movable and partially transmitting, while the other two are allowed to vibrate and assumed to have 100% reflectivity. The cavity field at the resonance frequency \( \omega_0 \) is driven by a strong coupling field with amplitude \( \epsilon \) at frequency \( \omega_c \). Meanwhile a weak probe field with amplitude \( \epsilon_p \) at frequency \( \omega_p \) is sent into the cavity. The coupling field and the probe field are treated classically here. The mechanical motions of both movable mirrors are coupled to the cavity field through the radiation pressure force exerted by the photons in the cavity. The movable mirrors’ dynamics can be approximated as those of a single harmonic oscillator, with resonance frequency \( \omega_j \), effective mass \( m_j \), damping rate \( \gamma_j \) (\( j = 1, 2 \)).

We assume that the system is in the adiabatic limit where the mechanical frequencies \( \omega_j \) (\( j = 1, 2 \)) are much smaller than the cavity free spectral range \( c/L \) (\( c \) is the speed of light in vacuum). The adiabatic limit \( \omega_j \ll c/L \) (\( j = 1, 2 \)) implies that the mechanical frequencies \( \omega_j \) (\( j = 1, 2 \)) are very small compared to the cavity resonance frequency \( \omega_0 \), so the moving mirrors are moving so slow that the retardation effect, the Casimir effect, and the Doppler effect become completely negligible [26–28]. Hence the radiation pressure force does not depend on the velocity of the movable mirrors. Assuming that \( k \) is the wave vector of the cavity field with \( k = \omega_0/c \), the radiation pressure force exerted by the light field on the movable mirror can be calculated from the momentum exchange between the cavity field and the movable mirror, which is \( F = \frac{2\omega_0^2\epsilon}{c} c \cos(\theta/2) \), where \( \theta \) is the angle between the incident light and the reflected light at the surfaces of the movable mirrors, \( c_\epsilon \) is the photon number operator of the cavity field, \( c_\epsilon^\dagger \) and \( c_\epsilon \) are the creation and annihilation operators of the cavity field, obey the standard commutation relation \[ [c_\epsilon, c_\epsilon^\dagger] = 1 \]. Note that the radiation pressure forces acting on the movable mirrors vary linearly with the instantaneous photon number in the cavity. Under the action of the radiation pressure force, the movable mirrors make small oscillations. Then the small motion of the mirrors changes the length of the optical cavity, and alters the intensity of the cavity field, which in turn modifies the radiation pressure force acting on the mirrors. Therefore the interaction between the cavity field and the mechanical motion in the ring cavity is nonlinear.

In a frame rotating at the driving frequency \( \omega_c \), the Hamiltonian of the whole system takes the form

\[
H = \hbar (\omega_0 - \omega_c) c_\epsilon^\dagger c_\epsilon + \frac{\hbar \omega_0}{2} (Q_1^2 + P_1^2) + \frac{\hbar \omega_2}{2} (Q_2^2 + P_2^2) \\
+ i\hbar (g_1 Q_1 - g_2 Q_2) c_\epsilon^\dagger c_\epsilon \cos \theta + i\hbar \kappa (c_\epsilon^\dagger - c_\epsilon) \\
+ i\hbar \epsilon_p (c_\epsilon^\dagger e^{-i\delta} - e^{i\epsilon_p} c_\epsilon e^{i\delta}).
\]

(1)

Here the first three terms are the free energies of the cavity field and two mechanical oscillators, respectively. \( (Q_j, P_j) \) denote the dimensionless position and momentum quadratures of the two mirrors, \( Q_j = \sqrt{\frac{m_j \hbar}{\kappa}} q_j \), \( P_j = \frac{1}{\sqrt{m_j \hbar}} p_j \) (\( j = 1, 2 \)), and \( [Q_j, P_k] = i\delta_{jk} \). The fourth term describes the nonlinear optomechanical interactions between the cavity field and the two movable mirrors, \( g_j = \frac{\hbar}{L} \sqrt{\frac{\kappa}{m_j \hbar}} \) (\( j = 1, 2 \)) is the optomechanical coupling strength. The last two terms give the interactions of the cavity field with the coupling field and the probe field, respectively. \( \epsilon \) is related to the power \( \gamma_0 \) of the coupling field by \( \epsilon = \sqrt{2k \hbar \gamma_0} \), where \( k \) is the decay rate of the cavity due to the transmission losses through the fixed mirror. \( \epsilon_p \) is related to the power \( \gamma_p \) of the probe field by \( |\epsilon_p| = \sqrt{\frac{2k \hbar \gamma_p}{\omega_0}} \).

Starting from the Heisenberg equations of motion, taking into account the dissipations of the cavity field and the
mechanical oscillators, and neglecting quantum noise and thermal noise, we obtain the time evolutions of the expectation values of the system operators
\[
\langle \hat{Q}_1 \rangle = \omega_1 \langle P_1 \rangle, \\
\langle \hat{P}_1 \rangle = -\omega_1 \langle Q_1 \rangle - g_1 (c')^\dagger (c) \cos \frac{\theta}{2} - \gamma_1 \langle P_1 \rangle, \\
\langle \hat{Q}_2 \rangle = \omega_2 \langle P_2 \rangle, \\
\langle \hat{P}_2 \rangle = -\omega_2 \langle Q_2 \rangle + g_2 (c')^\dagger (c) \cos \frac{\theta}{2} - \gamma_2 \langle P_2 \rangle, \\
\langle \epsilon \rangle = -i \left[ \omega_0 - \omega_k + (g_1 \langle Q_1 \rangle - g_2 \langle Q_2 \rangle) \cos \frac{\theta}{2} \right] (c) + \epsilon_p \epsilon^{i \omega t} - \kappa \langle \epsilon \rangle.
\]
(2)
Here we are interested in the strong driving regime so that \( \epsilon \gg 1 \), thus the interaction between the cavity field and each movable mirror is bilinear [12], so we have used the mean field assumption \( \langle c' c \rangle \approx \langle c'^2 \rangle \approx \langle c \rangle \) and \( \langle Q_j c \rangle \approx \langle Q_j \rangle \langle c \rangle \) for \( j = 1, 2 \) in equation (2). In the weak driving regime, especially in the single-photon strong coupling regime [29, 30], the nonlinear interaction between the cavity field and each mechanical oscillator affects the response of the system to the probe field [18-20]. Since the probe field is much weaker than the coupling field \( \langle |\epsilon| \rangle \ll \epsilon \), the steady-state solution to equation (2) can be approximated to the first order in the probe field \( \epsilon_p \). In the long time limit, the solution to equation (2) can be written as
\[
\langle s \rangle = s_0 + s_1 \epsilon_p \epsilon^{i \omega t} + s_2 \epsilon_p^2 \epsilon^{2 i \omega t},
\]
(3)
where \( s = Q_1, P_1, Q_2, P_2, \) or \( c \). The solution contains three components, which in the original frame oscillate at \( \omega_k, \omega_p, 2\omega_k - \omega_p \), respectively. Substituting equation (3) into equation (2), evauating coefficients of \( \epsilon^0 \) and \( \epsilon^{\pm i \omega t} \), we find the following analytical expressions
\[
Q_{10} = \frac{G_1}{\omega_1} c_0, \quad P_{10} = 0, \\
Q_{20} = \frac{G_2}{\omega_2} c_0, \quad P_{20} = 0, \\
c_0 = \frac{\epsilon}{\kappa + i \Delta'},
\]
(4)
\[
c_+ = \frac{1}{d(\delta)} \left\{ [\kappa - i (\Delta' + \delta)] (\omega_k^2 - \delta^2 + i \gamma_1 \delta) \times (\omega_k^2 - \delta^2 + i \gamma_2 \delta) + i [G_1^2 \omega_1 (\omega_k^2 - \delta^2 - i \gamma_1 \delta) \\
+ G_2^2 \omega_2 (\omega_k^2 - \delta^2 - i \gamma_1 \delta)] \right\},
\]
\[
c_- = \frac{i \epsilon_0^2}{|c_0|^2 d(\delta)} \left\{ [G_1^2 \omega_1 (\omega_k^2 - \delta^2 + i \gamma_1 \delta) \\
+ G_2^2 \omega_2 (\omega_k^2 - \delta^2 + i \gamma_1 \delta)] \right\},
\]
(5)
where \( G_1 = g_1 |c_0| \cos \frac{\theta}{2} \) and \( G_2 = g_2 |c_0| \cos \frac{\theta}{2} \) are the effective optomechanical coupling rates, \( \Delta' = \omega_0 - \omega_k + (g_1 Q_{10} - g_2 Q_{20}) \cos \frac{\theta}{2} = \Delta + (g_1 Q_{10} - g_2 Q_{20}) \cos \frac{\theta}{2} \) is the effective detuning of the coupling field from the cavity resonance frequency, including the frequency shift induced by the radiation pressure, note that \( \Delta' \) depends on itself through \( c_0 \). And
\[
d(\delta) = [\kappa + i (\Delta' - \delta)] [\kappa - i (\Delta' + \delta)] (\omega_k^2 - \delta^2 + i \gamma_1 \delta) \times (\omega_k^2 - \delta^2 + i \gamma_2 \delta) - 2 \Delta [G_1^2 \omega_1 (\omega_k^2 - \delta^2 - i \gamma_1 \delta) \\
+ G_2^2 \omega_2 (\omega_k^2 - \delta^2 - i \gamma_1 \delta)].
\]
3. The output field
According to the input-output relation [31], the expectation value \( \langle \epsilon_{\text{out}} \rangle \) of the output field satisfies
\[
\langle \epsilon_{\text{out}} \rangle + \frac{\epsilon_c}{\sqrt{2 \kappa}} + \frac{\epsilon_p}{\sqrt{2 \kappa}} \epsilon^{-i \omega t} = \sqrt{2 \kappa} \langle \epsilon \rangle,
\]
(6)
which can be rewritten as
\[
\sqrt{2 \kappa} \langle \epsilon_{\text{out}} \rangle + \epsilon_c + \epsilon_p \epsilon^{-i \omega t} = 2 \kappa \langle \epsilon \rangle.
\]
(7)
Then we define \( \sqrt{2 \kappa} \langle \epsilon_{\text{out}} \rangle + \epsilon_c + \epsilon_p \epsilon^{-i \omega t} \) as a new variable \( \epsilon_{\text{out}}(t) \), we have \( \epsilon_{\text{out}}(t) = 2 \kappa \langle \epsilon \rangle \). In analogy with equation (3), we expand the output field to the first order in the probe field \( \epsilon_p \),
\[
\epsilon_{\text{out}}(t) = \epsilon_{\text{out}0} + \epsilon_{\text{out}+} \epsilon_p e^{-i \omega t} + \epsilon_{\text{out}-} \epsilon_p e^{i \omega t},
\]
(8)
where \( \epsilon_{\text{out}0}, \epsilon_{\text{out}+}, \) and \( \epsilon_{\text{out}-} \) are the components of the output field oscillating at frequencies \( \omega_c, \omega_p, 2\omega_c - \omega_p \). Here \( \epsilon_{\text{out}-} \) is called a Stokes field, and it is generated via the nonlinear four-wave mixing process, in which two photons at frequency \( \omega_c \) interact with a single photon at frequency \( \omega_p \) to create a new photon at frequency \( 2\omega_c - \omega_p \). Thus we find that the components of the output field at the probe frequency and the Stokes frequency are
\[
\epsilon_{\text{out}+} = 2 \kappa \epsilon_c, \\
\epsilon_{\text{out}-} = 2 \kappa \epsilon_c,
\]
(9)
respectively. In the absence of the coupling field \( \langle \rho = 0 \rangle \), the components of the output field at the probe frequency and the Stokes frequency are given by
\[
\epsilon_{\text{out}+} = \frac{2 \kappa}{\kappa + i (\Delta' - \delta)}, \\
\epsilon_{\text{out}-} = 0.
\]
(10)
These are not unexpected results. Let us write the real part of \( \epsilon_{\text{out}+} \) as \( \nu_p \), which exhibits the absorption characteristic of the output field at the probe frequency. It can be measured by the homodyne technique [31].

4. Numerical results of the output probe field
In this section, we numerically evaluate how the coupling field modifies the absorption of the ring cavity to the probe field.

The values of the parameters chosen are similar to those in [13]: the wavelengths of the coupling field \( \lambda = 2 \pi c / \omega_k = 775 \text{ nm} \), the coupling constants \( g_1 = 2 \pi \times 12 \text{ GHz} \) \( \times \sqrt{\hbar / (m_1 \omega_k)} \), \( g_2 = 2 \pi \times 12 \text{ GHz} \) \( \times \sqrt{\hbar / (m_2 \omega_2)} \), the masses of the movable mirrors \( m_1 = m_2 = 20 \text{ ng} \), the frequencies of the movable mirrors \( \omega_1 = \omega_{m1} + 0.1 \omega_{m1}, \omega_2 = \omega_{m2} - 0.1 \omega_{m2} \), where \( \omega_{m1} = 2 \pi \times 51.8 \text{ MHz} \), the cavity decay rate \( \kappa = 2 \pi \times 15 \text{ MHz} \), \( \kappa / \omega_{m1} \approx 0.289 \approx 1 \) (the system is placed in the resolved sideband regime), the mechanical damping rates \( \gamma_1 = \gamma_2 = \gamma = 2 \pi \times 4.1 \text{ kHz} \), the mechanical quality factors \( Q_1 = \omega_1 / \gamma_1 \approx 13897, Q_2 = \omega_2 / \gamma_2 \approx 11370, \) the angle \( \theta = \pi / 3 \). And the coupling field is tuned close to the red sideband of the cavity resonance \( \Delta' = \omega_{m2} \). The parameters chosen ensure the system is operating in the stable regime.

We note that the structure of the quadrature of the output probe field \( \nu_p \) is determined by \( d(\delta) \). The roots \( \delta \) of \( d(\delta) \) are complex values. The real parts \( \text{Re}(\delta) \) of the roots determine the
positions of the normal modes of the optomechanical system; the imaginary parts $\text{Im}(\delta)$ of the roots describe their widths. Figure 2 shows the real parts of the roots of $d(\delta)$ in the domain $\text{Re}(\delta) > 0$ versus the coupling beam power. It is seen that the real parts of the roots of $d(\delta)$ are $\text{Re}(\delta) = 0.9\omega_m, \omega_m, 1.1\omega_m$ at low driving power. At high driving power, one of the real parts is still $\omega_m$ (dot–dashed curve), not changing with increasing the power of the coupling field, the difference between the other two (dotted curve and dashed curve) increases with increasing the power of the coupling field, which implies the normal mode splitting [22–24] in the output probe field. Figure 3 shows the imaginary parts of the roots of $d(\delta)$ versus the coupling beam power. We see that the imaginary parts of the roots of $d(\delta)$ have three different values at low driving power. At high driving power, the imaginary parts of the roots of $d(\delta)$ have three values, two of them are identical, the other one is small. Moreover, in figures 2 and 3, the dot–dashed curves represent the real part and the imaginary part of one of the roots of $d(\delta)$, respectively, similarly for dotted curves and dashed curves. Hence, at high driving power, the central peak in the quadrature $\nu_p$ is narrow, two side peaks in the quadrature $\nu_p$ have the same broad linewidths.

In figure 4, we plot the quadrature of the output probe field $\nu_p$ as a function of the normalized probe detuning $\delta/\omega_m$ for three different powers of the coupling field. In the absence of the coupling field, $\nu_p$ (solid curve) has a standard Lorentzian absorption peak with a full width at half maximum (FWHM) of $2\kappa$ at the line centre $\delta/\omega_m = 1$. However, in the presence of the coupling field with power $2\text{ mW}$, it is seen that the dot–dashed curve exhibits two symmetric narrow EIT dips centred at $\delta/\omega_m = 0.9, 1.1$ and a broad absorption peak centred at $\delta/\omega_m = 1$. The FWHM of the two EIT dips are about $|\gamma + \frac{G_1}{4}|$ and $|\gamma + \frac{G_1}{4} + \frac{G_2}{4}|$, respectively. The FWHM of the central absorption peak is about $|\omega_1 - \omega_2| - |\frac{G_1}{4} + \frac{G_2}{4}|$. The two transparency dips display that the input probe field could be simultaneously transparent at two symmetric frequencies.

From the level diagram of the system in figure 5, it is seen that the EIT dip at $\delta = \omega_j$ is the result of the destructive interference between the anti-Stokes field at frequency $\omega_1 + \omega_j$ generated by the interaction of the coupling field with the movable mirror with frequency $\omega_j$ and the input probe field at frequency $\omega_p$. The absorption peak at the line centre implies that the incident probe field is almost fully absorbed by the optomechanical system. Moreover, increasing the power $\varphi$ of the coupling field, the two EIT dips become broader, which results in narrowing of the central absorption peak. Further, from the dotted curve in figure 4, one can see that increasing the power of the coupling field to $15\text{ mW}$ results in a larger splitting between the left and right side peaks with the same linewidths, it also results in a narrow central peak. Our calculations show that in the strong
coupling limit $2(G_1^2 + G_2^2) \gg (\kappa - \delta)^2$, the three dressed modes of the system corresponding to the three absorption peaks are 
\[ \delta \approx \omega_m - i\gamma/2, \text{ and } \delta \approx \omega_m \pm \frac{1}{2} \sqrt{2(G_1^2 + G_2^2)} - \frac{i}{2} (\kappa - \delta). \]

Note that the FWHM for the narrow central peak at $\delta = \omega_m$ is about $\gamma$. In addition, the FWHM for the right and left peaks at $\delta \approx \omega_m \pm \frac{1}{2} \sqrt{2(G_1^2 + G_2^2)}$ are the same $(\kappa - \delta)$, the splitting between the side peaks is about $\sqrt{2(G_1^2 + G_2^2)}$, which is proportional to the power of the coupling field. These results are consistent with those in figures 2 and 3.

For comparison, we consider the same previous system, but the two mechanical oscillators have the same frequencies $\omega_1 = \omega_2 = \omega_m$, thus their effective optomechanical coupling rates are equal, we assume $G_1 = G_2 = G$. We plot the quadrature of the output probe field $\nu_p$ as a function of the normalized probe detuning $\delta/\omega_m$ for several values of the driving power when the frequencies of the two movable mirrors are not equal ($\omega_1 \neq \omega_2$), as shown in figure 6. When the coupling field is not present, the quadrature $\nu_p$ (solid curve) has a Lorentzian absorption lineshape. However, the presence of the control field with power 2 mW leads to a narrow EIT-like dip at the line centre (dotted–dashed curve). Thus the probe field can almost completely propagate through the ring cavity on resonance with almost no absorption. The FWHM of the EIT-like dip is about $\gamma + 2G$. The EIT-like dip is attributed to the destructive interference between the input weak probe field and the scattering quantum fields at the probe frequency $\omega_p$ generated by the interactions of the coupling field with two mirrors having identical frequencies. Moreover, in the strong coupling limit $2G \gg \kappa$, the normal mode splitting exhibits in the quadrature $\nu_p$ (dotted curve), the two peaks have the same FWHM of $\kappa + \delta$, their positions are $\delta \approx \omega_m \pm G$, the separation between them is about $2G$.

In order to understand the narrow central peak in figure 4 and the EIT-like dip in figure 6, let us introduce the relative coordinates $(Q_a, P_a)$ and centre of mass coordinates $(Q_s, P_s)$ of the two movable mirrors as
\[ Q_a = \frac{g_1Q_1 - g_2Q_2}{\sqrt{g_1^2 + g_2^2}}, \quad P_a = \frac{g_1P_1 - g_2P_2}{\sqrt{g_1^2 + g_2^2}}. \]

With these new coordinates we can write the Hamiltonian equation (1) as
\[ H = \hbar(\omega_1 - \omega_c)c^\dagger c + \hbar \frac{\kappa}{2} (Q_1^2 + P_1^2) + \hbar \frac{\kappa}{2} (Q_2^2 + P_2^2) + \hbar \frac{\hbar}{4} (\frac{g_1^2 + g_2^2}{g_1}) \omega_1 \frac{\omega_2}{g_2} (Q_1^2 + P_1^2) + \hbar \frac{\hbar}{2} \sqrt{g_1^2 + g_2^2} Q_a e^{i\epsilon} \cos \theta + \hbar \frac{\hbar}{2} \sqrt{g_1^2 + g_2^2} P_a \epsilon e^{i\epsilon}, \]
\[ + \hbar \frac{\hbar}{2} \sqrt{g_1^2 + g_2^2} \epsilon e^{i\epsilon} - \epsilon^\dagger e^{i\epsilon}, \]
\[ Q_s = \frac{g_1Q_1 + g_2Q_2}{\sqrt{g_1^2 + g_2^2}}, \quad P_s = \frac{g_1P_1 + g_2P_2}{\sqrt{g_1^2 + g_2^2}}. \]

5. Numerical results of the output Stokes field

In this section, we numerically examine the effect of the coupling field on the output Stokes field generated via the four-wave mixing process.

Figure 7 shows the intensity of the Stokes field $|\nu_{out}|^2$ as a function of the normalized probe detuning $\delta/\omega_m$ for several values of the driving power when the frequencies of the two mirrors are not equal ($\omega_1 = 1.1\omega_m$ and $\omega_2 = 0.9\omega_m$).
It is seen that $|\varepsilon_{\text{out}}|^2 = 0$ (solid curve) in the absence of the coupling field. However, in the presence of the coupling field with power $\varphi = 2 \, \text{mW}$, the dot–dashed curve shows two peaks at $\delta/\omega_m = 0.9, 1.1$. The peak at $\delta = \omega_f$ represents the intensity of the Stokes field at frequency $\omega_m - \omega_f = (\omega_m - \omega_p - \omega_c)$ produced by the coupling field interacting with the movable mirror oscillating at frequency $\omega_f$. From figure 7, one can see that the peak separation increases with the driving power. Moreover, the maximum value of $|\varepsilon_{\text{out}}|^2$ is bigger for larger driving power, the maximum value of $|\varepsilon_{\text{out}}|^2$ is about 0.19 when $\varphi = 15 \, \text{mW}$. Note that the intensity of the Stokes field goes to zero when the detuning $\delta/\omega_m = 1$. Hence, the four-wave mixing effect is completely suppressed on resonance so that there is only the probe field stored inside the optomechanical system.

For comparison, we also consider the case of a ring cavity in which the frequencies of the two mirrors are equal ($\omega_1 = \omega_2 = \omega_m$), the intensity $|\varepsilon_{\text{out}}|^2$ of the output Stokes field versus the normalized probe detuning $\delta/\omega_m$ for several values of the driving power is shown in figure 8. The significant difference between figures 8 and 7 is that the intensity of the Stokes field in figure 8 is non-zero at $\delta/\omega_m = 1$ in the presence of the coupling field. Thus the four-wave mixing process is not suppressed at $\delta/\omega_m = 1$, which arises from the constructive interference between the Stokes fields produced by the coupling field interacting with the two movable mirrors oscillating at the same frequencies $\omega_1 = \omega_2 = \omega_m$. So there is no apparent normal mode splitting in this case.

6. Conclusions

To summarize, we have demonstrated double EIT windows separated by an absorption peak in an optomechanical system formed by a ring cavity with two movable mirrors whose frequencies are close to each other. We have shown that increasing the pump power gives rise to broadening two EIT dips and narrowing the central absorption peak. Further, the narrow central absorption peak has approximately the mechanical linewidth at high driving power. Moreover, the normal mode splitting appears in the output probe field and the output Stokes field. However, when two movable mirrors in a ring cavity have identical frequencies, the system behaves like a standard optomechanical setup of a Fabry–Perot cavity with one movable end mirror. The results of this paper are applicable to other optomechanical systems containing two nearly degenerate mechanical modes, such as two mechanical modes in a membrane [32], two membranes [33], a Bose–Einstein condensate and an optomechanical mirror [34], and so on.

Acknowledgments

The author thanks Professor G S Agarwal for helpful discussions. The author also thanks Professor Lin Tian and Professor Mankei Tsang for support. This work is supported by the Singapore National Research Foundation under NRF Grant No. NRF-NRFF2011-07.

References

[1] Harris S E 1997 Phys. Today 50 36
[2] Harris S E, Field J E and Imamoglu A 1990 Phys. Rev. Lett. 64 1107
[3] Boller K J, Imamoglu A and Harris S E 1991 Phys. Rev. Lett. 66 2593
[4] Fleischhauer M, Imamoglu A and Marangos J P 2005 Rev. Mod. Phys. 77 633
[5] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 Nature 397 594
[6] Liu C, Dutton Z, Behroozi C H and Hau L V 2001 Nature 409 490
[7] Lukin M D, Yelin S F, Fleischhauer M and Scully M O 1999 Phys. Rev. A 60 3225
[8] Gavra N, Rosenbluh M, Zigdon T, Wilson-Gordon A D and Friedmann H 2007 Opt. Commun. 280 374
[9] Goren C, Wilson-Gordon A D, Rosenbluh M and Friedmann H 2004 Phys. Rev. A 69 063802
[10] Ye C Y, Zibrov A S, Rostovtsev Y V and Scully M O 2002 Phys. Rev. A 65 043805
[11] Paspalakis E and Knight P L 2002 J. Mod. Opt. 49 87
[12] Agarwal G S and Huang S 2010 Phys. Rev. A 81 041803
[13] Weis S, Rivière R, Deleglise S, Gavartin E, Arcizet O, Schliesser A and Kippenberg T J 2010 Science 330 1520
[14] Lin Q, Rosenberg J, Lang D, Camacho R, Eichenfield M, Vahala K J and Painter O 2010 Nature Photon. 4 236
[15] Safavi-Naeini A H, Alegre T P M, Chan J, Eichenfield M, Winger M, Lin Q, Hill J T, Chang D and Painter O 2011 Nature 472 69
[16] Teufel J D, Li D, Allman M S, Cicak K, Sirois A J, Whittaker J D and Simmonds R W 2011 Nature 471 204
[17] Massel F, Cho S U, Pirkalakainen J M, Hakonen P J, Heikkinen T T and Sillanpää M A 2012 Nature Commun. 3 987
[18] Lemond M-A, Didier N and Clerk A A 2013 Phys. Rev. Lett. 111 053602
[19] Bokje K, Nunnkenkamp A, Teufel J D and Girvin S M 2013 Phys. Rev. Lett. 111 053603
[20] Kromwald A and Marquardt F 2013 Phys. Rev. Lett. 111 133601
[21] Qu K and Agarwal G S 2013 Phys. Rev. A 87 031802
[22] Marquardt F, Chen J P, Clerk A A and Girvin S M 2007 Phys. Rev. Lett. 99 093902
[23] Dobrindt J M, Wilson-Rae I and Kippenberg T J 2008 Phys. Rev. Lett. 101 263602
[24] Grölscher S, Hammerer K, Vanner M and Aspelmeyer M 2009 Nature 460 724
[25] Huang S and Agarwal G S 2009 New J. Phys. 11 103044
[26] Giovannetti V and Vitali D 2001 Phys. Rev. A 63 023812
[27] Mancini S and Tombesi P 1994 Phys. Rev. A 49 4055
[28] Mancini S, Vitali D and Tombesi P 1998 Phys. Rev. Lett. 80 688
[29] Rabl P 2011 Phys. Rev. Lett. 107 063601
[30] Nunnenkamp A, Børkje K and Girvin S M 2011 Phys. Rev. Lett. 107 063602
[31] Walls D F and Milburn G J 1994 Quantum Optics (Berlin: Springer)
[32] Shkarin A B, Flowers-Jacobs N E, Hoch S W, Deutsch C, Reichel J and Harris J G E 2014 Phys. Rev. Lett. 112 013602
[33] Hartmann M J and Plenio M B 2008 Phys. Rev. Lett. 101 200503
[34] Singh S, Jing H, Wright E M and Meystre P 2012 Phys. Rev. A 86 021801