Free expansion of a quasi-2D Bose-Einstein condensate with quantized vortices

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We observe concentric density ripples in mechanically perturbed and freely expanding quasi-two-dimensional Bose-Einstein condensates, and show that the ripples develop from quantized vortices in the condensates. Free expansion of a condensate containing a vortex is numerically simulated under the assumption of no atom-atom interactions, and the experimental data are found to be in good quantitative agreement with the numerical results for a singly charged vortex, in particular, including the defocus effect in imaging due to the gravitational freefall of the condensate. In the defocused image, the vortex core becomes magnified and appears to be filled after a certain expansion time. We show that the vortex charge number can be determined from the onset time of the core-filling in the defocused image.

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Quantized vortices are topological defects in a superfluid, possessing quantized circulation of the superfluid around them. They play an important role in the transport phenomena in the superfluid such as the critical superflow and quantum turbulence [1]. Furthermore, in two dimensions they are responsible for the superfluid phase transition, where pairing of vortices with opposite circulation occurs below a critical temperature [2]. Studying quantized vortices defines one of the active frontiers in quantum gas experiments [3]. Since quantized vortices have been observed first in rotating Bose-Einstein condensates (BECs) [4,5], many aspects of their dynamic behavior have been investigated, including line defect deformation [6], instability of multiply charged vortices [7,8], and vortex dipole dynamics [9,10]. Recently, thermally activated vortices have been observed in two-dimensional (2D) Bose gases [11,12].

At the core of a quantized vortex, superfluid density is depleted because of the diverging centrifugal potential. This distinctive feature is typically used for detecting the quantized vortices. However, in a trapped BEC, the spatial extent of the density-depleted vortex core, which is determined by the healing length of the condensate, is generally smaller than the wavelength of imaging light, and so its in situ optical detection is experimentally challenging. A conventional method to circumvent this difficulty is releasing the trap to let the condensate freely expand, where the vortex core also expands. Free expansion of a BEC containing a vortex was theoretically studied, showing that the vortex core may expand faster than the condensate during the initial stage of the expansion [13], and further faster in the case of an oblate BEC [14].

In this paper, we study expansion dynamics of a quasi-2D, i.e., highly oblate BEC containing quantized vortices. In our recent experiment [15], we observed that ring-shaped density ripples appear in a mechanically perturbed and freely expanding quasi-2D condensate. Based on the observation of their apparent ring symmetry and long lifetime, we interpreted them as vortex excitations induced by the mechanical perturbation. Because the condensate expands fast along the tight confinement direction when the trap is released, the atom density and consequently the atom-atom interaction effects rapidly reduce in the expanding condensate. For this sudden quenching of the interactions, the density-depleted vortex core, which is initially small in the trapped condensate, would evolve into the concentric ripples. Here, we confirm this interpretation by numerically simulating free expansion of a non-interacting BEC with a singly charged vortex, and find that the ripple profiles observed in the experiment are in good quantitative agreement with the numerical results, in particular, when we include the defocus effect in imaging caused by the free fall of the condensate under gravity.

The experimental procedure was described in our previous publication [14]. Bose-Einstein condensates of $^{23}$Na atoms in the $|F = 1, m_F = -1\rangle$ state are generated in an optically plugged magnetic quadrupole trap [16], and transferred into a single optical dipole trap whose trapping frequencies are $(\omega_x, \omega_y, \omega_z) = 2\pi \times (3.0, 3.9, 370)$ Hz, where the $z$ direction is along the gravity. For a typi-

![FIG. 1: Images of quasi-2D Bose-Einstein condensates after (a) $t_e = 12$ ms and (b) $18$ ms of time-of-flight. The condensates are mechanically perturbed and prepared in nonequilibrium states (see text for detail), and ring-shaped ripples are observed in the freely expanding condensates.](image-url)
FIG. 2: (Color online) (a) Schematic diagram of the imaging system. (b)-(d) Images of ring-shaped ripples for various expansion times $t_e$. Each image is obtained by averaging several experimental data. (e)-(g) Numerically calculated density distributions of a freely expanding, non-interacting condensate containing a singly-charged vortex and (h) their radial profiles. The dotted line is the initial density profile with a healing length of $\xi = 1\,\mu m$. (i)-(k) Defocused images are numerically constructed from the density distributions in (e)-(g), respectively, for the defocus distance $d = \frac{1}{2}gt_e^2$ due to the free fall of the sample under gravity, illustrated in (a). (l)-(n) The radial profiles from the experimental data [(b)-(d), blue square] and the defocused images [(i)-(k), red line]. For comparison, the experimental data are scaled to have the same total atom number as in the corresponding numerical results. (o) Radius $r_1$ ($r_2$) of the first bright (dark) ring versus expansion time. The dashed line is the radius $r_p$ in (h).

cal atom number of $1.0 \times 10^6$, the chemical potential is $\mu \approx \hbar \times 260$ Hz in the Thomas-Fermi (TF) approximation, which is less than the confining energy $\hbar\omega_z$, and the healing length is $\xi = \hbar/\sqrt{2m\mu} \approx 1\,\mu m$, where $m$ is the atomic mass. At low temperatures, our sample constitutes a weakly interacting quasi-2D BEC.

In order to induce vortex excitations, we intentionally displace the optical trap from the center of the plugged magnetic trap by about $40\,\mu m$ in the horizontal direction, and carry out a rapid transfer of the condensate to the optical trap. The optical trap and the magnetic potential are simultaneously ramped up and down, respectively, for $500\,ms$, where the optical plug beam is kept on to let the condensate experience an obstacle as it flows into the optical trap. Finally, the plug beam is linearly turned off for $400\,ms$. Free expansion is initiated by suddenly turning off the optical trap, and the column density distribution $n(x, y)$ is measured by taking an absorption image after an expansion time $t_e$.

The perturbed condensate initially shows very complicated and random density modulations after expansion, indicating strong phase fluctuations induced in the transfer process. The relaxation dynamics of the non-equilibrium state has been investigated by measuring the power spectrum of the density modulations [15]. Ring-shaped ripples become discernible after long relaxation times over $15\,s$ as the background modulations diminish (Fig. 1). At this moment, the temperature of the thermal component is about $50\,nK$, and we note that a condensate prepared in thermal equilibrium at this temperature shows no such ripples.

Figure 2(b)-(d) display the images of the ring-shaped ripples for various expansion times, which are obtained by averaging those of the ripples located in the center region of the condensate. The ripples have a concentric multiple-ring structure and propagate outward. One remarkable observation is that the center region of the ripples becomes filled up after a certain expansion time. This is quite unexpected in vortex expansion dynamics because of the phase singularity at the vortex core.

To understand the observed evolution of the concentric ripples, we first carry out numerical simulation of free expansion of a BEC containing a quantized vortex at its center. Here, we assume that the atom-atom interactions...
immediately vanish at the start of the expansion. In this non-interacting case, the axial motion in the $z$ direction is separable from the radial motion in the $x$–$y$ plane, so we consider a cylindrically symmetric situation. The initial condensate wave function is set as

$$\psi_0(r, \phi) = \frac{r \ e^{i \phi}}{\sqrt{2 \pi c^2 + r^2}} \times \sqrt{n_0 \max[1 - \frac{r^2}{r_{TF}^2}, 0]},$$

(1)

where the first part describes the vortex core structure with phase winding number $l$ [17] and the second part corresponds to the TF density profile in a harmonic potential with central density $n_0$ and radius $r_{TF}$. Because of the cylindrical symmetry, the non-interacting evolution of the condensate wave function can be expressed as

$$\psi(r, \phi, t_e) = e^{i \phi} \sum_{n=1}^{\infty} c_{nl} J_l(\beta_{nl} r) \exp\left(-i \frac{\hbar^2}{2m} \frac{\beta_{nl}^2 r^2}{R^2} t_e\right),$$

(2)

where $J_l$ is the Bessel function of the first kind of order $l$, and $\beta_{nl}$ is the $n$th zero of $J_l$. The coefficients $c_{nl}$ are determined by

$$c_{nl} = \frac{2e^{i \phi}}{\pi^2 (n+1)(\beta_{nl} R)} \int_0^R \psi_0(r) J_l(\beta_{nl} r) r \ dr.$$  

In our calculation, we set $r_{TF} = 150 \ \mu m$ and $R = 3 r_{TF}$. Fig. 2(c)-(g) show the numerical results including the defocus effect for the density distribution $n(r, t_e) = |\psi(r, t_e)|^2$ with $\xi = 1 \ \mu m$ and $l = 1$. A multiple-ring structure develops and radially expands as observed in the experiment. However, the growth rate of the first ring is much slower than that in the experiment [Fig. 2(o)], and furthermore, the core-filling behavior is absent.

This qualitative discrepancy can be attributed to the defocus effect in our imaging. We use a 4$f$ imaging system set along the $z$ direction [Fig. 2(a)]. Since the condensate falls due to the gravity during free expansion, the relative distance of the condensate to the imaging plane increases as $d = \frac{1}{2} g t_e^2$. The depth of field is about $80 \ \mu m$ in the imaging system, so the defocus effect would not be negligible for $t_e > 4 \ \text{ms}$. To check this effect, we numerically construct the defocused image from the calculated density distribution $n(r, t_e)$. The probe beam after the sample is set to have an intensity distribution $I(r) = I_0 \exp[-\sigma n(r)]$ with the central optical density $\sigma n_0 = 1.1$, and a uniform phase, ignoring the sample thickness which is $\sim \frac{\hbar}{2 \max(1 + r^2)} = 60 \ \mu m$ for $t_e < 23 \ \text{ms}$. From the Fresnel diffraction theory, the beam intensity distribution $I_d(r)$ after propagating by distance $d$ is given as

$$I_d(r) = \frac{4 \pi^2}{\lambda^2 d^2} \int_0^\infty \sqrt{I(r)} J_0(2 \pi r_1 r / \lambda d) \exp(-\pi r_1^2 / \lambda d) r_1 dr_1^2,$$

(3)

where $\lambda = 589 \ \text{nm}$ is the wavelength of the probe beam. Finally, to take into account the finite imaging resolution, we convolute $I_d(r)$ with a Gaussian function with $1/e^2$ width of $5 \ \mu m$.

The numerical results including the defocus effect reveal the core-filling behavior consistent with the experimental observation [Fig. 2(i)-(k)]. Remarkably, the radial profiles in the calculated defocused images show quite good quantitative agreement with the experiment data over the whole range of the expansion time $5 \ \text{ms} < t_e < 25 \ \text{ms}$ [Fig. 2(i)-(n)]. We have experimentally confirmed this defocus effect by taking in-focus images with adjusting the axial position of the CCD camera. This demonstrates that the concentric ripples observed in the freely expanding quasi-2D BEC develop from singly charged vortices, and in particular, indicates that atom-atom interaction effects are negligible in the free expansion.

Next, we numerically investigate the expansion dynamics of a quasi-2D condensate containing multiply charged vortices with $l > 1$. A concentric, multiple-ring structure develops as in the $l = 1$ case, and the expansion rate increases with higher $l$. We characterize the time evolution of the concentric ripples with the radius $r_\text{p}$ of their first density-peak ring [Fig. 3(g)], and find that the ring radii are nicely fit to $r_\text{p} = (3.9, 4.7, 5.4) \times \sqrt{T_e} \ \mu m/\text{ms}^{1/2}$ for $l = 1, 2, 3$, respectively. The time dependence might be understood from the continuity equation for 2D propagation, keeping $r_\text{p} d \text{r}_\text{p} / d\text{t}$ constant. Remarkably, the expansion rate shows weak dependence on the initial vortex core
size, i.e. $\xi$. This seems to reflect the dependence of the kinetic energy of the vortex state on the parameters, $E_0 \propto \hbar^2 \ln(\pi a/\xi)$ and suggests that the vortex charge number $l$ can be unambiguously determined from the radius $r_p$.

One interesting method for determining the ripple radius is using the core-filling behavior in the defocused image as observed in our experiment. It is well known in the diffraction theory that when a light propagates through an aperture of size $a$, its diffraction pattern after the aperture is determined by the so-called Fresnel number, $\pi a^2/\lambda d$, where $d$ is the propagation distance from the aperture. Since the shape of the concentric ripples is almost preserved during expansion, we may expect that the core-filling behavior would occur when the normalized propagation distance $D = \lambda d/\pi r_p^2 > D_c$, which we numerically find the case. For the defocus distance $d = \frac{1}{2} g t_e^2$, the core region starts to be filled at $t_e \approx 6.8, 9, 11$ ms for $l = 1, 2, 3$, respectively, giving the critical value $D_c \approx 0.36$. Thus, the vortex charge number can be determined from the onset time of the core-filling in the defocused image.

In our previous work [15], we have measured the power spectrum of the density fluctuations in a freely expanding quasi-2D Bose gas and demonstrated the thermal nature of the phase fluctuations in a 2D Bose gas. The peak positions in the measured spectra, however, showed different scaling behavior from the theoretical prediction that the $n$th peak position $q_n$ ($n = 1, 2, 3, \ldots$) satisfies the relation $\hbar q_n^2 t_e/2\pi m = n - 1/2$ [20]. Recently, numerical simulations were performed, taking into account the experimental conditions such as the finite size of the sample and the atomic interactions [21], but the discrepancy is still unresolved. The defocus effect might explain the observed scaling behavior of the spectral peak positions [19]. For an object of spatial size $\Delta r \sim \pi/q_n$, the normalized defocus distance due to the gravitational freefall is $D = \lambda g t_e^2/\pi \Delta r^2 = \frac{\lambda g m}{\pi \hbar^2} (n - \frac{1}{2}) t_e$, which is already comparable to $3D_c$ for $n = 1$ and $t_e = 10$ ms. The defocus effect on the power spectrum is currently under our experimental investigation.

In conclusion, we have demonstrated that the concentric ripples observed in a freely expanding quasi-2D BEC is a manifestation of quantized vortices present in the condensate. The quantitative agreement between the experiment data and the numerical results reveals that the free expansion of the quasi-2D condensate is well described as non-interacting expansion. Finally, we have suggested that the core-filling behavior in the defocused image can be used for determining the vortex charge number.

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