Sampled-Data Adaptive ESO Based Composite Control for Nano-Positioning Systems

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ABSTRACT This work studies a novel sampled-data adaptive extended state observer (ESO) based composite control approach for the digital nano-positioning systems with sampling restrictions. In the proposed control architecture, a sampled-data ESO with an output predictor is designed to estimate states and disturbances within a continuous period of time, where the observer gains are adaptively updated. To deal with the inter-sample disturbances, a multirate disturbance compensator is then worked out by upsampling, with which a discrete composite feedback controller with linear and nonlinear parts is employed aiming at improving the transient performance of positioning. Moreover, the estimation convergence and the stability are analyzed by means of Lyapunov methods. The proposed control method is then applied to the simulations and experiments in a piezoelectric nano-positioning system so as to validate its effectiveness.

INDEX TERMS Nano-positioning, disturbance rejection, extended state observer, sampled-data.

I. INTRODUCTION

Nano accuracy positioning technology successfully emerges from fundamental research to advanced engineering areas, such as semiconductor equipment, biology engineering and optical devices [1]–[3]. In particular, piezoelectric actuated nano-positioning systems are explored in these applications thanks to the features of high resolution and high bandwidth, which, consequently, has raised increasing concerns over mechanism design, modeling analysis and control of such systems [4]–[8].

It has been noted that external disturbances and internal uncertainties have major adverse impacts on the positioning performance of such piezoelectric actuated motion systems. To deal with these uncertainties and disturbances, various robust control methods, e.g. \( H_\infty \) controllers, sliding mode controllers and disturbance observer based controllers [9]–[12] have been worked out and applied to nano-positioning systems. However, the control performance of these robust control methods depends largely on the accurate model information. Recently, extended state observer (ESO) based control, which requires less model information [13], [14], has gradually become a new tendency in high precision control design [15], [16]. Nevertheless, the estimation and anti-disturbance capabilities of the ESOs with fixed observer gains are degraded by rate/amplitude dependent uncertainties, such as hysteresis and creep [17]. Despite the fact that adaptive ESO based controllers are primarily investigated to improve the disturbance rejection performance so as to better handle different working conditions [17]–[21], inter-sample ripples (disturbances beyond Nyquist frequency of output) induced by low sampling rate still significantly deteriorate the control performance in digital implementations because of the sampling hardware restrictions [16]. On the other hand, for most ESO based controllers, the linear control design in the feedback loop also limits the transient positioning performance [20]. Thus, the ESO based control method design with better robustness and transient performance is still a challenging issue for digital nano-positioning systems.

To deal with these issues, we propose a sampled-data adaptive ESO based composite control method aimed at the digital nano-positioning systems. In this control architecture, a sampled-data ESO with an inter-sample output predictor is designed to reconstruct the system states and existing disturbances within a continuous period of sampling time, where
the observer gains are adjusted adaptively for the purpose of improving the estimation performance. Moreover, a multirate disturbance compensator with fast sampling is designed to compensate the inter-sample ripples. Inspired by [22], [23], a discrete composite controller is designed in the feedback loop so as to improve the transient positioning performance and stabilize the system. Furthermore, the estimation convergence and the stability are analyzed by means of Lyapunov methods. Then we comprehensively applied the proposed control method to a piezoelectric actuated nano-positioning system in order to validate its effectiveness based on simulations and real-time experiments.

The remainder of the paper is organized as follows. Firstly, the problem formulation is presented in Section II. Secondly, the proposed sampled-data adaptive ESO based composite control design is discussed in Section III, which is followed by simulations and experimental validation in Section IV. Finally, a conclusion is reached in Section V.

II. PROBLEM FORMULATION

In this work, we study the sampled-data nano-motion system subject to external disturbances and internal uncertainties depicted in Fig.1, where the positioning performance suffers from the inter-sample ripples caused by limited hardware measurement sampling rate. To address this problem, a sampled-data adaptive ESO based control diagram is proposed in Fig.2, where adaptive gain design is introduced for performance scheduling and a multirate disturbance compensator is employed to deal with inter-sample disturbances. In addition, linear and nonlinear composite digital controller will be applied to improve transient positioning performance.

As to the positioning system in Fig.1, the single-axis dynamical model of the motion stage with ZOH (zero-order-hold) sampling method can exactly be formulated as the following general form

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Ef, \\
y(t) &= Cx(k), \quad t \in [kT_y, (k+1)T_y)
\end{align*}$$

where $u(t) \in \mathbb{R}$ refers to the control input, $x(t) \in \mathbb{R}^n$ denotes the system state, and the system output $y(t) \in \mathbb{R}$ is measured at sampling instants, $kT_y$ for $k = 0, 1, 2, \cdots$ refers to the set of measurement sampling instants and $T_y$ represents the sampling period of measurement such that $x[k] = x(kT_y)$.

The matrices in (1) are

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n \times n}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b \\ 1 \end{bmatrix}_{n \times 1}, \quad E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{n \times 1}.$$ 

where $b$ denotes the equivalent input gain.
In addition, \( f \) is total disturbance including internal uncertainties and external disturbances, and the derivative of \( f \) to time is defined as \( h = \dot{f} \). The objective of this work is to design a novel sampled-data adaptive ESO based multirate composite feedback control method for system (1).

III. SAMPLED-DATA ADAPTIVE ESO BASED CONTROL
A. INTER-SAMPLE OUTPUT PREDICTOR BASED ADAPTIVE ESO

The adaptive ESO based multi-rate control diagram for the nano-positioning systems, as shown in Fig.2, will be presented in detail in this part. First, \( f \) is defined as an extended state such that \( \bar{x} = [\bar{x}^T, \bar{f}^T]^T \). Thus, the extended system from system (1) can be obtained as:

\[
\begin{align*}
\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{E}h, \\
y(t) &= \bar{C}\bar{x}[k], \quad t \in [kT_y, (k + 1)T_y)
\end{align*}
\]  

where the matrices are

\[
\bar{A} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C^T & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} 0 & E \end{bmatrix}.
\]

With generality, we propose the following assumption for \( f \) and \( h \) in systems (1) and (2):

Assumption 1: We assume that the total disturbance \( f \) and its derivative to time \( h \) are bounded such that

\[
||f|| \leq f_{\text{max}}, \quad ||h|| \leq h_{\text{max}}.
\]

(3)

where \( f_{\text{max}} \) and \( h_{\text{max}} \) are positive constants.

Before presenting the ESO, a lemma is first given:

Lemma 2 [24], [25]: For matrix \( \bar{A} \) in (2), we define \( \bar{A}_e = \bar{A} - LC \), where \( L = [l_1, \ldots, l_{n+1}]^T \) and \( l_i, i = 1, \ldots, n + 1 \) are constants, such that

\[
\bar{A}_e = \begin{bmatrix}
-l_1 & 1 & 0 & \cdots & 0 & 0 \\
-l_2 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
-l_n & 0 & \cdots & 0 & 1 \\
-l_{n+1} & 0 & \cdots & 0 & 0 & 0
\end{bmatrix}.
\]

There exists a \( \alpha \in \mathbb{R}^+ \) and a symmetric positive definite matrix \( P \) satisfying

\[
P\bar{A}_e + \bar{A}_e^TP \leq -I, \quad PD + DP \geq \alpha I.
\]

(4)

(5)

where

\[
D = \begin{bmatrix}
n + 1 & 0 & \cdots & 0 & 0 \\
0 & n & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 2 & 0 \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix}.
\]

Under Assumption 1, the sampled-data adaptive ESO are proposed as:

\[
\begin{align*}
\begin{cases}
\dot{\hat{x}}_1 &= \hat{x}_2 + l_1\phi^{-1}(w(t) - \hat{x}_1(t)), \\
\dot{\hat{x}}_2 &= \cdots \\
\dot{\hat{x}}_n &= \hat{x}_{n+1} + bu + l_n\phi^{-n}(w(t) - \hat{x}_1(t)), \\
\dot{\hat{x}}_{n+1} &= l_{n+1}\phi^{-(n+1)}(w(t) - \hat{x}_1(t)), \quad t \in [kT_y, (k + 1)T_y),
\end{cases}
\end{align*}
\]

(6)

where \( \hat{x}_i \) denote the estimates of \( x_i \) for \( i = 1, \ldots, n + 1 \) and \( l_i, i = 1, \ldots, n + 1 \) refer to observer gains determined by Lemma 2. Note that \( w(t) \) used in ESO (6) is the inter-sample prediction of system output and generated by

\[
\begin{align*}
\dot{w}(t) &= \bar{C}\hat{x}(t), \quad t \in [kT_y, (k + 1)T_y), \\
w[k] &= y[k] = \bar{C}\bar{x}[k].
\end{align*}
\]

(7)

In addition, \( \phi(t) \) is a dynamic gain with \( \phi(0) \geq 1 \) which can be updated by

\[
\dot{\phi}(t) = \frac{1}{\alpha \phi(t)} \max\{4\lambda_{\text{max}}(P) + 1 - \frac{\phi(t)}{2}, 0\},
\]

(8)

where \( \lambda_{\text{max}}(P) \) is the maximum eigenvalue of matrix \( P \). From (8), we have the following relations for the dynamic gain:

\[
\dot{\phi}(t) \geq 0,
\]

(9)

To study the convergence of ESO (6), we set the estimation error as \( \hat{e}_i(t) = x_i(t) - \hat{x}_i(t) \) and the prediction error as \( e_w(t) = w(t) - y(t) \), such that the estimation error system can be obtained from (2) and (6) as:

\[
\begin{align*}
\begin{cases}
\dot{\hat{e}}_1 &= \hat{e}_2 - l_1\phi^{-1}(e_w(t) + \hat{e}_1(t)), \\
\dot{\hat{e}}_2 &= \cdots \\
\dot{\hat{e}}_n &= \hat{e}_{n+1} - l_n\phi^{-n}(e_w(t) + \hat{e}_1(t)), \\
\dot{\hat{e}}_{n+1} &= -l_{n+1}\phi^{-(n+1)}(e_w(t) + \hat{e}_1(t)) + h(t),
\end{cases} \quad t \in [kT_y, (k + 1)T_y),
\end{align*}
\]

(10)

Letting

\[
\begin{align*}
\hat{e}_i(t) &= \frac{\hat{e}_i(t)}{\phi^{n+1-i}}, \quad i = 1, \ldots, n + 1, \\
e_w(t) &= \frac{e_w(t)}{\phi^{n+1}}.
\end{align*}
\]

(11)

(12)

Then we get

\[
\dot{\hat{e}}(t) = \frac{1}{\phi(t)}\hat{e}(t) - D \frac{\dot{\phi}(t)}{\phi(t)}\hat{e}(t) + \Upsilon + \frac{1}{\phi(t)}Le_w(t),
\]

(13)

where

\[
\hat{e}(t) = [\hat{e}_1(t), \ldots, \hat{e}_{n+1}(t)], \quad \Upsilon = [0, \ldots, h(t)/\phi(t)].
\]

And we can have

\[
\dot{e}_w(t) = \frac{1}{\phi(t)}e_w(t) - (n + 1)\frac{1}{\phi(t)}e_w(t).
\]

(14)
The convergence of estimation error system (13) can be demonstrated as:

**Theorem 3:** Consider sampled-data adaptive ESO (6) with inter-sample output predictor (7). If Assumption 1 holds and the sampling time is less than the maximal allowable sampling period \( T_{\text{max}} \), we can select appropriate matrix \( P \), constants \( \sigma_i \) with \( i = 1, \cdots, n + 1 \), \( \alpha \) and the dynamic gain \( \phi(t) \) satisfying Lemma 2 and (9), such that the estimation error will converge to a small bounded region.

**The Proof of Theorem 4:** We select a Lyapunov candidate as \( V_1(t) = \varepsilon^T(t)L\varepsilon(t) + \theta(t)E^2(t) \), where \( \theta \) is a positive constant. In addition, the positive and bounded function \( \varphi(t) \) satisfies:

\[
\begin{align*}
\varphi[k] &= \mu, \\
\dot{\varphi}(t) &= 0, \quad t \in [kT_y, (k + 1)T_y), \\
\varphi[k + 1] &= \mu^{-1},
\end{align*}
\]

where \( \mu > 1 \).

Using (13) and (14), the following differential equality can be obtained for \( t \in [kT_y, (k + 1)T_y) \) as:

\[
\begin{align*}
\dot{V}_1 &= \left( \frac{1}{\phi(t)} \tilde{A}\varepsilon(t) - D \frac{\dot{\phi}(t)}{\phi(t)} \varepsilon(t) + \frac{1}{\phi(t)} L\varepsilon(t) \right)^T \varepsilon(t) \\
&\quad + \varepsilon^T(t) \left( \frac{1}{\phi(t)} \tilde{A}\varepsilon(t) - D \frac{\dot{\phi}(t)}{\phi(t)} \varepsilon(t) + \frac{1}{\phi(t)} L\varepsilon(t) \right) + \theta(t)E^2(t),
\end{align*}
\]

We set

\[
\begin{align*}
U_{11} &= \left( \frac{1}{\phi(t)} \tilde{A}\varepsilon(t) - D \frac{\dot{\phi}(t)}{\phi(t)} \varepsilon(t) + \frac{1}{\phi(t)} L\varepsilon(t) + \Gamma \right)^T \varepsilon(t) \\
&\quad + \varepsilon^T(t) \left( \frac{1}{\phi(t)} \tilde{A}\varepsilon(t) - D \frac{\dot{\phi}(t)}{\phi(t)} \varepsilon(t) + \frac{1}{\phi(t)} L\varepsilon(t) \right) + \theta(t)E^2(t),
\end{align*}
\]

\[
U_{12} = \theta(t)E^2(t) + 2\theta(t)E(t) - 2\theta(t)E(t)L^2(t).
\]

Using Young’s inequality, we can get

\[
\begin{align*}
U_{11} &\leq \left( \frac{1}{\phi(t)} - \frac{\phi(t)}{\phi(t)} + \frac{4\lambda_{\text{max}}(P)}{\phi^2(t)} \right)|\varepsilon(t)||\varepsilon(t)|^2 \\
&\quad + 2\lambda_{\text{max}}(P)|h^2(t)| + 2\lambda_{\text{max}}(P)|L||e_w(t)||^2, \\
U_{12} &\leq \left( \theta(t)E(t) + \frac{\phi(t)}{\phi(t)} \right)|\varepsilon(t)||\varepsilon(t)|^2,
\end{align*}
\]

Combining (19) and (20), we have

\[
\begin{align*}
\dot{V}_1 &\leq \left( \frac{1}{\phi(t)} - \frac{\phi(t)}{\phi(t)} + \frac{4\lambda_{\text{max}}(P)}{\phi^2(t)} + 1 \right)|\varepsilon(t)||\varepsilon(t)|^2 + (\theta(t)E(t) + \frac{\phi(t)}{\phi(t)})|\varepsilon(t)||\varepsilon(t)|^2 \\
&\quad + \theta(t)E(t) + (\theta(t)E(t) + \frac{\phi(t)}{\phi(t)})|\varepsilon(t)||\varepsilon(t)|^2 + 2\lambda_{\text{max}}(P)|L||e_w(t)||^2, \quad (21)
\end{align*}
\]

By selecting \( \hat{\phi}(t) = - \left( \theta(t)E^2(t) + \frac{2\lambda_{\text{max}}(P)}{\phi(t)} \right), \) we get

\[
\begin{align*}
\dot{V}_1 &\leq \left( -\frac{1}{\phi(t)} - \frac{\phi(t)}{\phi(t)} + \frac{4\lambda_{\text{max}}(P)}{\phi^2(t)} \right)|\varepsilon(t)||\varepsilon(t)|^2 \\
&\quad - 2\theta(t)E(t) + \phi(t)|\varepsilon(t)||\varepsilon(t)|^2 + 2\lambda_{\text{max}}(P)|h^2(t)| \\
&\leq \left( -\frac{1}{\phi(t)} - \frac{\phi(t)}{\phi(t)} + \frac{4\lambda_{\text{max}}(P)}{\phi^2(t)} \right)|\varepsilon(t)||\varepsilon(t)|^2 \\
&\quad + 2\lambda_{\text{max}}(P)|h^2(t)|. \quad (22)
\end{align*}
\]

It can be seen from equations (8) and (9) that the dynamic gain \( \phi(t) \) satisfies

\[
\frac{\phi(t)}{2} \leq 4\lambda_{\text{max}}(P) + 1. \quad (23)
\]

When \( \phi(t) < 2(4\lambda_{\text{max}}(P) + 1) \), we select the adaptive update as \( \phi(t) = \frac{1}{\phi(t)} \left( 4\lambda_{\text{max}}(P) + 1 - \frac{\phi(t)}{\phi(t)} \right) \) from (8) and get

\[
\dot{V}_1 \leq \left( -\frac{1}{\phi(t)} - \frac{4\lambda_{\text{max}}(P) + 1}{\phi^2(t)} \right)|\varepsilon(t)||\varepsilon(t)|^2 + 2\lambda_{\text{max}}(P)|h^2(t)|. \quad (24)
\]

When \( \phi(t) = 2(4\lambda_{\text{max}}(P) + 1) \), we have \( \phi(t) = 0 \) and

\[
\dot{V}_1 \leq \left( -\frac{1}{\phi(t)} - \frac{4\lambda_{\text{max}}(P) + 1}{\phi^2(t)} \right)|\varepsilon(t)||\varepsilon(t)|^2 + 2\lambda_{\text{max}}(P)|h^2(t)|. \quad (25)
\]

If the error \( \varepsilon(t) \) satisfies

\[
||\varepsilon(t)|| \leq \frac{2\lambda_{\text{max}}(P)|h^2(t)|}{\frac{\phi(t)}{2} - 4\lambda_{\text{max}}(P) + 1},
\]

then the estimation error \( \varepsilon(t) \) will convergent to a bounded region according to the boundary theorem and satisfies

\[
||\varepsilon(t)|| \leq \frac{2\lambda_{\text{max}}(P)|h^2(t)|}{\frac{\phi(t)}{2} - 4\lambda_{\text{max}}(P) + 1}. \quad (26)
\]

**Remark 5:** From the proof above, designing the adaptive law (8) can sufficiently guarantee the convergence of the estimation dynamics (13). After system (13) reaches steady state, the following relationships hold:

\[
\phi(t) = 8\lambda_{\text{max}}(P) + 2, \quad \dot{\phi}(t) = 0. \quad (27)
\]

Considering Assumption 1 and (26)-(27), the estimation error is bounded by

\[
||\varepsilon(t)|| \leq h_{\text{max}} \sqrt{4\lambda_{\text{max}}^2(P) + \lambda_{\text{max}}(P)}, \quad (28)
\]

and

\[
||\hat{\varepsilon}(t)|| \leq ||\phi(t)|^2 - 1(t)||\varepsilon(t)|| \leq 2^{n+1} \sqrt{\lambda_{\text{max}}(P)\Pi^{n-i-3/2}}, \quad (29)
\]

where \( \Pi = 4\lambda_{\text{max}}(P) + 1. \)
B. COMPOSITE MULTIRATE FEEDBACK CONTROL DESIGN

Based on the estimation of a continuous-discrete adaptive ESO, a digital composite controller with multirate disturbance compensator is proposed as follows:

\[ u[k, m] = u_d[k, m] + u_c[k] = u_d[k, m] + u_l[k] + u_N[k], \]

\[ k = 0, 1, 2, \cdots, \quad m = 0, 1, \cdots, N - 1, \quad (30) \]

where \( u[k, m] \) denotes the \( m \)-th control input change at \( t = (k + m)/N T_y \) between two consecutive sampling instants \( k T_y \) and \( (k + 1) T_y \).

In the present work, the double-rate condition, namely \( N = 2 \), for disturbance commentator will be used to simplify the design such that system (1) shows the following lifted single-rate discrete form with the frame sampling period as \( T_f = T_y \):

\[
\begin{aligned}
&\begin{cases}
x[k + 1] = A_d x[k] + B_d u_c[k] + \left[ \begin{array}{c} \tilde{B}_{11} \\ \tilde{B}_{12} \end{array} \right] \begin{bmatrix} u_d[k, 0] \\ u_d[k, 1] \end{bmatrix} \\
y(t) = C_d x[k], \quad t \in [kT_f, (k + 1)T_f)
\end{cases}
\end{aligned}
\]  

(31)

where

\[ A_d = e^{AT_f}, \quad B_d = \int_{0}^{T_f} \exp(\alpha t) B \, d\tau, \]

\[ \tilde{B}_{11} = \int_{0}^{T_f/2} \exp(\alpha t) B_d \, d\tau, \quad \tilde{B}_{12} = \int_{T_f/2}^{T_f} \exp(\alpha t) B_d \, d\tau, \]

\[ \tilde{E}_{11} = \int_{T_f/2}^{T_f} \exp(\alpha t) E_d \, d\tau, \quad \tilde{E}_{12} = \int_{0}^{T_f} \exp(\alpha t) E_d \, d\tau. \]

This digital controller can be divided into three parts:

(I) the linear term \( u_l[k] \) is designed as:

\[ u_l[k] = F \hat{x}[k] + Gr, \]

(32)

where \( r \) is the reference, and control gain \( F \) is selected to make \( A_d + B_d F \) have all its eigenvalues in the open unit disc. In addition, gain \( G \) is chosen as:

\[ G = [C_d(I - A_d - B_d F)^{-1} B_d]^{-1}. \]

(33)

For controller (32), the following lemma can guarantee its stability.

**Lemma 6:** For a given positive definite matrix \( Q \in \mathbb{R}^{n \times n} \), we can choose a positive definite symmetric matrix \( P_2 \) satisfying:

\[ P_2 = (A_d + B_d F)^T P_2 (A_d + B_d F) + Q. \]

(34)

Let \( \epsilon = (I - A_d - B_d F)^{-1} B_d G \) and \( \hat{x}[k] = x[k] - x_e \). Then we can get the exact form of controller (32) as:

\[ u_l[k] = F \hat{x}[k] - \epsilon[k] + I + (I - A_d - B_d F)^{-1} B_d G \epsilon \]

\[ = F (\hat{x}[k] - x_e) + H \epsilon, \]

(35)

where \( \epsilon = [\epsilon_1, \cdots, \epsilon_n] \).

(II) Nonlinear control term \( u_N[k] \) is designed as:

\[ u_N[k] = \rho(r, y)B_d^T P_2 (A_d + B_d F) \hat{x}[k] - \epsilon[k] \]

\[ = \rho(r, y)B_d^T P_2 (A_d + B_d F) (\hat{x}[k] - x_e), \]

(36)

where the nonpositive scalar function \( \rho(r, y) \), locally Lipschitz in \( y \), is selected to tune the damping ratio.

(III) Multirate disturbance compensator \( u_d[k, m] \) has the following form:

\[ u[k, m] = -\frac{1}{b} \hat{x}_{n+1}[k, m]. \]

(37)

Combining (35), (36) and (37), the exact form of controller (30) is obtained:

\[ u[k, m] = u_c[k] + u_d[k, m] \]

\[ = H \epsilon + F \hat{x}[k] - x_e + \rho(r, y)B_d^T P_2 (A_d + B_d F) (\hat{x}[k] - x_e) \]

\[ - \frac{1}{b} \hat{x}_{n+1}[k, m]. \]

(38)

Then, the following main results is presented to demonstrate the stability of system (31).

**Theorem 7:** Consider the system (31) with the multirate composite feedback controller (38). Then for any function \( \rho(r, y) \) satisfying \( |\rho(r, y)| \leq \rho^* = 2(B_d^T P_2 B_d)^{-1} \), if Assumption 1 holds and the sampling time is less than the maximal allowable sampling period \( T_{\text{max}} \), the closed loop system will be stable, making \( y[k] \) asymptotically approach to the step reference \( r \) and the tracking error will be bounded.

**The Proof of Theorem 8:** Combining (31) and (38), the error system can be obtained as:

\[ \tilde{x}[k + 1] = (A_d + B_d F) \tilde{x}[k] + B_d u_N[k] - B_d F \epsilon[k] + \tilde{E}_{11} \hat{e}_{n+1}[k, 0] + \tilde{E}_{12} \hat{e}_{n+1}[k, 1]. \]

(39)

Consider the following Lyapunov candidate:

\[ V_2[k] = \tilde{x}^T[k] P_2 \tilde{x}[k] + \tilde{\epsilon}^T[k] P_1 \tilde{\epsilon}[k]. \]

(40)

Thus, the increment of the Lyapunov function (40) along the trajectories of system (39) is get as:

\[ \nabla V_2 = V_2[k + 1] - V_2[k] \]

\[ = \tilde{x}^T[k+1] P_2 \tilde{x}[k + 1] - \tilde{x}^T[k] P_2 \tilde{x}[k] + V_1[k + 1] - V_1[k] \]

\[ = [(A_d + B_d F) \tilde{x}[k] + B_d u_N[k] - B_d F \epsilon[k] + \tilde{E}_{11} \hat{e}_{n+1}[k, 0] + \tilde{E}_{12} \hat{e}_{n+1}[k, 1] + \tilde{E}_{11} \hat{e}_{n+1}[k, 0]] - \tilde{x}^T[k] P_2 \tilde{x}[k] \]

\[ = \tilde{x}^T[k] (A_d + B_d F)^T P_2 (A_d + B_d F) \tilde{x}[k] - \tilde{x}^T[k] P_2 \tilde{x}[k] + 2 \tilde{x}^T[k] (A_d + B_d F)^T P_2 \tilde{\epsilon}[k] 
\]

\[ - 2 \tilde{x}^T[k] (A_d + B_d F)^T P_2 \tilde{\epsilon}[k] + 2 \tilde{x}^T[k] (A_d + B_d F)^T P_2 \tilde{\epsilon}[k] + 2 \tilde{x}^T[k] (A_d + B_d F)^T P_2 \tilde{\epsilon}[k], \]

\[ = \tilde{x}^T[k] (A_d + B_d F)^T P_2 (A_d + B_d F) \tilde{x}[k] - \tilde{x}^T[k] P_2 \tilde{x}[k] + 2 \tilde{x}^T[k] (A_d + B_d F)^T P_2 \tilde{\epsilon}[k] + 2 \tilde{x}^T[k] (A_d + B_d F)^T P_2 \tilde{\epsilon}[k], \]

(41)
\begin{align*}
\times P_2\hat{e}_{n+1}[k, 1] + u_N^T[k]B_d^T P_2B_d u_N[k] \\
- 2u_N^T B_d^T P_2B_d F\hat{e}[k] \\
+ 2u_N^T B_d^T P_2\hat{e}_{n+1}[k, 0] + 2u_N^T B_d^T P_2\hat{e}_{n+1}[k, 1] \\
+ \hat{e}[k]F^T B_d^T P_2B_d F\hat{e}[k] \\
- 2\hat{e}[k]F^T B_d^T P_2\hat{e}_{n+1}[k, 0] \\
- 2\hat{e}[k]F^T B_d^T P_2\hat{e}_{n+1}[k, 1] \\
+ \hat{e}[k+1][k, 0]E_{n+1}^T P_2\hat{e}_{n+1}[k, 0] \\
+ 2\hat{e}[k+1][k, 0]E_{n+1}^T P_2\hat{e}_{n+1}[k, 1] \\
+ \hat{e}[k+1][k, 1]E_{n+1}^T P_2\hat{e}_{n+1}[k, 1].
\end{align*}

Consider the solution $P_2 = (A_d + B_d F)^T P_2 A_d + B_d F) + Q$ in Lemma 6, the exact form of $u_N$ in (36) and the condition for non positive $\rho$ in Theorem 7. Using Young's inequality, we can get

$$
\nabla V_2 = -\hat{X}^T[k]Q\hat{X}[k] + \rho\hat{X}^T[k](A_d + B_d F)^T P_2B_d(2 + \rho B_d^T P B_d) \\
- B_d^T P_2(A_d + B_d F)^T k\hat{X}[k] - \hat{X}^T[k](A_d + B_d F)^T P_2B_d \\
+ [F + \rho B_d^T P B_d B_d^T P_2(A_d + B_d F)]k\hat{e}[k] \\
+ 2\hat{X}^T[k](A_d + B_d F)^T P_2(1 + \rho B_d B_d^T P_2)E_{n+1}\hat{e}_{n+1}[k, 0] \\
+ 2\hat{X}^T[k](A_d + B_d F)^T P_2(1 + \rho B_d B_d^T P_2)E_{n+1}\hat{e}_{n+1}[k, 1] \\
+ \hat{e}[k]k\rho(A_d + B_d F)^T P_2B_d B_d^T P_2(A_d + B_d F) \\
+ F^T B_d^T P_2 + 2\hat{e}^T[k+1][k, 0]B_d F\hat{e}[k] \\
+ \hat{e}^T[k+1][k, 0]E_{n+1}^T P_2\hat{e}_{n+1}[k, 0]E_{n+1}^T \\
+ \hat{e}^T[k+1][k, 1]E_{n+1}^T P_2\hat{e}_{n+1}[k, 1] \\
+ \hat{e}^T[k+1][k, 1]E_{n+1}^T P_2\hat{e}_{n+1}[k, 1] \\
- 2\hat{e}^T[k][k\rho P_2B_d B_d^T + F^T B_d^T P_2]E_{n+1}\hat{e}_{n+1}[k, 0] \\
- 2\hat{e}^T[k][k\rho P_2B_d B_d^T + F^T B_d^T P_2]E_{n+1}\hat{e}_{n+1}[k, 1] \\
+ \Delta V_1 \\
\leq -3\hat{X}^T[k]Q\hat{X}[k] + (-\sigma_1 + \sigma_2 + \sigma_3)||\hat{X}[k]||^2 + \sigma_4 h_{max}^2.
$$

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. EXPERIMENTAL SET-UP AND CONTROLLER DESIGN

In this part, the proposed sampled-data adaptive ESO based multirate composite controller is implemented in the X-Y nano positioning system depicted in Fig.3. More specifically, the positioner in Fig.3 is a parallel X-Y nano motion stage based on a flexure mechanism, with a stroke of 100 $\mu m$ on each axis. The motion of each axis is driven by a piezoelectric actuator, with the actuator being driven by a voltage amplifier with high bandwidth in each direction. In addition, a linear encoder with resolution of 1.2 $nm$ (from MicroE Systems) is used in each direction to measure real time displacement signals for feedback control, along with the application of a Matlab/RTW rapid prototyping system with the National Instruments PCI-6259 I/O hardware to facilitate the implementation of high-speed real-time experiments.

For the parallel X-Y nano-positioner, the dynamics on $X$ and $Y$ axes can be constructed as equation (43), by referring to [10]

$$
\begin{align*}
m_X\ddot{y}_X(t) + c_X\dot{y}_X(t) + k_Xy_X(t) + \Phi_{XY} = b_XV_X + d_X, \\
m_Y\ddot{y}_Y(t) + c_Y\dot{y}_Y(t) + k_Yy_Y(t) + \Phi_{YX} = b_YV_Y + d_Y, \\
\end{align*}
$$

where $y_j$ and $V_j$ denote displacement and input voltage on $j$-axis, respectively, $m_j$ represents the moving mass, $c_j$ and $k_j$ refer to the damping ratio and stiffness on $j$-axis, respectively, and $b_j$ denotes the control gain, where $j = X, Y$. In addition, $d_j$ denotes disturbance including hysteresis and creep non-linearities, uncertainties and external disturbances on $j$-axis., with cross-coupling effects being denoted by $\Phi_{XY}$ and $\Phi_{YX}$, respectively.

It is noted that motions on $X$ and $Y$ axes share similar dynamics from equation (43), helping explain that only the controller design for $X$-axis will be investigated as follows. By defining system state $x = [y_X, \dot{y}_X]^T \in \mathbb{R}^2$, control input

\[-\sigma_1 + \sigma_2 + \sigma_3 < 0.\]
TABLE 1. Parameters of the nano-positioning system.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \(m_X\)   | 1.20  | \(m_Y\)  | 1.20  |
| \(c_X\)   | 224.13| \(c_Y\)  | 302.42|
| \(k_X\)   | \(6.61 \times 10^6\) | \(k_Y\)  | \(9.13 \times 10^6\) |
| \(b_X\)   | 53.96 | \(b_Y\)  | 44.8  |

\(u = V_X \in \mathbb{R}\) and system output \(y = y_X \in \mathbb{R}\), the state space model of the dynamics of \(X\)-axis can be obtained from (43) as:

\[
\begin{align*}
\dot{x}_1 &= A_x x + B_x u_x + E_x f_x, \\
y &= C_x x,
\end{align*}
\]

where total disturbance \(f_x\) is

\[
f_x = -\frac{c_x}{m_x} \dot{y}_x(t) - \frac{k_x}{m_x} y_x(t) - \Phi_{XY} + H_x[V_x + dX] - bV_x,
\]

and matrices \(A, B, C\) and \(E\) are:

\[
A_x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_x = \begin{bmatrix} 0 \\ b \end{bmatrix}, \quad C_x = \begin{bmatrix} 1^T \\ 0 \end{bmatrix}, \quad E_x = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]

where \(b\) is the equivalent control input gain and determined as \(b = 44.97\).

Based on the sampled-data adaptive ESO (6) and controller (38), we can design a multirate composite controller for system (44) as:

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + l_1 \phi^{-1}(w(t) - \dot{x}_1(t)), \\
\dot{x}_2 &= \dot{x}_3 + bu + l_2 \phi^{-2}(w(t) - \dot{x}_1(t)), \\
\dot{x}_3 &= l_3 \phi^{-3}(w(t) - \dot{x}_1(t)), \quad t \in [kT_s, (k + 1)T_s], \\
u[k, m] &= F(\hat{x}[k] - x_r) + Hr - \frac{1}{2} x_{k+1}[k, m] + \rho(r, y)B_d^T P_2(A_d + B_d F)(\hat{x}[k] - x_r).
\end{align*}
\]

where control parameters are selected as:

\[
T_s = 2T_u = 1 \times 10^{-4}s, \quad l_1 = 30, \quad l_2 = 550, \quad l_3 = 82000, \quad \alpha = 2.5, \quad \lambda_{\text{max}}(P) = 5, \quad F = [-502.4, -20.3], \quad P_2 = diag(0.01, 10^{-4}), \quad \rho(r, y) = -2.374(e^{-11-y/r} - 0.214).
\]

B. SIMULATION RESULTS

Simulations are first conducted using Matlab/Simulink Software with the proposed method. In the simulation, the system parameters are listed in TABLE 1. For the purpose of performance comparison, discrete ADRC proposed in [15] is also applied, where the control parameters are selected as \(\omega_0 = 200\pi \text{ rad/s}\) and \(\omega_0 = 600\pi \text{ rad/s}\).

The proposed control method, as shown in Fig.4, shows a good tracking and transient performance, which achieves about 49.2% improvement on settling time over the discrete ADRC (refer to Fig.4 (d) for more details). As demonstrated in Fig.4 (b)-(c), the proposed method has shorter convergence time of estimation than the discrete ADRC, thus illustrating a better estimation performance of the proposed method.

Then reference dependent disturbances are added to testify the robustness of the control methods. In such simulations, the amplitudes of reference signals are selected as 1, 3 and 5 \(\mu m\), and disturbances are selected as \(d = 2 \times 10^6 \text{Amp}(r)\). Larger deviation and oscillations, as depicted in Fig.5, are observed with the discrete ADRC when disturbances are added at \(t = 1.5s\). By contrast, the proposed method needs shorter recovery time, with smaller deviation in each simulation.

FIGURE 4. Simulation results of step response. (a) tracking a step signal at 1 \(\mu m\) amplitude, (b) estimation error \(x_1 - \hat{x}_1\), (c) estimation error \(x_2 - \hat{x}_2\), (d) tracking a step-wise signal.

FIGURE 5. Step response comparison with amplitude dependent disturbances. (a) reference \(r = 1 \mu m\), (b) Reference \(r = 3 \mu m\), (c) reference \(r = 5 \mu m\), (d) performance comparison of disturbance rejection.
In order to test the robustness to inter-sample disturbances (disturbances beyond Nyquist frequency), 7000Hz periodical disturbances are then added at \( t = 1.5s \). The proposed method in Fig.6 shows a better disturbance rejection performance in both time and frequency domain analysis while oscillations are increased with the discrete ADRC.

**C. EXPERIMENTAL RESULTS**

Then the proposed method will be completely applied to the nano-positioning experimental system, as depicted in Fig.3.

We first conduct single-axis positioning experiments. Fig.7 shows that the system output is gradually and rapidly tracking the reference position with faster transient performance, which improves settling time over the discrete ADRC by around 56.8%. The detailed performance comparisons are referred to Fig.8. Steady-state performance comparison display that, the root mean square (RMS) values of position error signal (PES) are improved by at least 17.4% by using the proposed method. It is noted that the improvement is more obvious with the increase in step amplitude.

7000 Hz periodical disturbances, to evaluate the robustness to high frequency disturbances, are added to the experiments to track step and stair signals. Increasing oscillations, as shown in Fig.9 and Fig.10, are observed when disturbances are added to the positioning system with the discrete ADRC, while the proposed method shows better disturbance rejection capability, where RMS values of tracking error improve by around 71.0% and 58.7% (see TABLE2 for more details), respectively.
Moreover, we conduct dual-axes experiments to track a desired pattern. As demonstrated in Fig. 11, the proposed method shows rapid positioning performance on both X and Y axes, making the motion stage track the desired trajectory well. By contrast, the system output tracks the reference signal slowly with the discrete ADRC on each axis. Then, 7000 Hz and 6000 Hz are added to the positioning system in the time intervals $t = 5 - 6s$ and $t = 9 - 11s$, respectively. It can be observed from Fig. 12 that larger deviations and oscillations appear with the discrete ADRC, while the proposed method shows a better positioning and disturbance rejection performance. TABLE 2 illustrates that the proposed method can help improve by around 67.3%, thus demonstrating its better anti-disturbance capability and positioning performance.

V. CONCLUSION
A multi-rate composite control approach based on sampled-data adaptive ESO was proposed for digital nano-positioning systems in the present work. In such control architecture, a novel sampled-data adaptive ESO based on output predictor was designed to estimate system states and total disturbances within a continuous period of sampling time, where the observer gains were adaptively updated. Then, a multi-rate disturbance compensator was designed with a fast sampling so as to deal with inter-sample disturbances, along with the proposal of a discrete composite controller with linear and nonlinear parts to control the sampled-data system and improve the transient positioning performance. The estimation convergence and system stability are analyzed by means of Lyapunov methods. Finally, simulation and real-time experimental results further verify the effectiveness of the proposed control method on a digital piezoelectric actuated nano-positioning system, thus demonstrating its good positioning performance.

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C. Tian, P. Yan: Sampled-Data Adaptive ESO Based Composite Control for Nano-Positioning Systems

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