Nonlinear Mechanisms for Cyclotron-Resonance Accelerations by an Alfvén-Wave Pulse

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Abstract. Particle accelerations by an Alfvén-wave “pulse” are investigated analytically and numerically. It is found that as a function of pulse amplitude, there may exist three types of distinct phases in cyclotron-resonance interactions. First, when the amplitude is small the acceleration is linear with a single resonance peak. As the pulse amplitude increases, a weakly nonlinear regime characterized by multiple resonances emerges. Further above this regime, the interaction transforms into a strongly nonlinear one, which is characterized by unique velocity-dependent regions, i.e., multiple resonances at relatively small velocities and reflections at larger velocities.

1. Introduction
Particle accelerations by an electromagnetic wave/pulse have been investigated extensively for various objectives [1]. Furthermore, systems far from equilibrium are known to produce localized structures such as solitons, shock waves and wave packets, in which particles are accelerated and/or heated, and thus some entropy is produced locally [2]. Such localized dissipative structures lead the system to equilibrium. Here, accelerations of beam-like particles initially traveling along an external magnetic field due to an Alfvén wave-pulse, propagating also along the field are studied. The pulse has a Gaussian envelope with the left circularly polarized electric field (and its induced magnetic field) directed perpendicular to the z-axis along which it propagates at group velocity \(v_g\):

\[
E_0 \exp\left[-\frac{(z-v_g t)/L}{2}\right] \left\{ \hat{x} \cos(k_0 z - \omega_0 t + \theta) - \hat{y} \sin(k_0 z - \omega_0 t + \theta) \right\}; \quad (1)
\]

here, \(E_0\), \(z\), \(t\), \(L\), \(\hat{x}\), \(\hat{y}\) are the amplitude, the longitudinal position, the time, and a measure of the pulse-length, unit vectors in the \(x\) and \(y\) directions, respectively, and \(\theta\) is the phase constant; furthermore, \(k_0\) (\(\omega_0\)) is the wave number (angular frequency) of the carrier wave.

2. Results
2.1 Linear cyclotron-resonance due to a dispersive pulse
If the pulse amplitude is efficiently small, the interaction is categorized as linear-transit time cyclotron-resonance acceleration, and it was previously found by the authors that initially beam-like particles with only a longitudinal velocity \(v_0\) experience after interaction with an em pulse well-defined perpendicular velocity shifts [3]. To check the validity of the linear theory some numerical
solutions to the ion equation of motion [3] are presented and analyzed below, increasing $E_0$. To obtain numerical solutions the relativistic equation of motion is solved with the use of the 4th order Runge-Kutta scheme.

Plotted in Figure 1(a) are perpendicular velocity shifts of initially beam protons propagating at velocity $v_0$ along the external magnetic field after interacting with an Alfvén wave-pulse with $E_0= eE_0/(mc\omega_0) = 0.0001$, $v_p = v \approx 0.01c$ and $L_n= L/(c/\omega_0) = 0.2$ and 0.5, computed as a function of $v_0$. Since the magnetic field strength will be held constant at $\Omega_p = 10 \omega_0$, where $\Omega_p$ is the proton cyclotron frequency, the protons with $v_0 \approx (\omega_0 - \Omega_p)/k = -0.09c$ are cyclotron-resonant. Overall agreement between the theory(circles) and numerical results(solid curves) is excellent. Dominant features of linear cyclotron-resonance acceleration by an Alfvén-wave pulse are, as demonstrated in Figure 1(a), that the resultant velocity shifts form a single peak at the cyclotron resonance velocity $v_0 \approx -0.09$, and the peak velocity is greater than the quiver velocity $cE_n$ by a factor of 100-200. In addition, as the shifts in longitudinal velocity in the linear regime are negligible and the perpendicular velocity shifts are dominant, the entire process is similar to transit-time accelerations, in which particles penetrate an entire electromagnetic pulse [3]. As the pulse length is reduced further to $L_n = 0.1$, approaching somewhat to the electromagnetic solitons [4] in shape the resonance velocity range is made quite wide; it shows the fact eloquently that such short pulses are highly dissipative. Meanwhile, as $L_n$ is increased, the analytic values gradually deviate from numerical counterparts. Specifically, peaks shift to smaller velocities due to nonlinear effects as follows: though perpendicular velocities are increased due to the interaction with the pulse, parallel velocities are increased because the protons move approximately along the weakly relativistic Hamiltonian surface [3] due to the interaction; the Hamiltonian surface approximately forms a semi-circle with radius $V=|v_0 - v_p|$ with its center located at $v_z = v_p$. Hence, though the initial parallel velocities $v_0$’s of the most resonant protons are slightly slower than the resonance velocity $v \sim 0.09c$, during the interaction their time-averaged parallel velocities $\bar{v}_z$ approximately equal the resonance velocity, $i.e., v_0 \sim \bar{v}_z$. Such effects are negligible for pulses of small amplitudes, and resultant velocity shifts are practically perpendicular ones. When, however, such discrepancies between the theory and numerical results become no longer negligible another interaction regime of “weak nonlinearity” emerges.

![Figure 1](image1.png) **Figure 1.** Initial velocity dependence of cyclotron resonance due to linear Alfvén pulses

![Figure 2](image2.png) **Figure 2.** Perpendicular velocity shifts of protons after interacting with pulses with various amplitudes.
2.2 Weakly nonlinear cyclotron acceleration and multiple resonances

As the pulse amplitude is further increased to $E_n=0.001$ and above with a fixed length $L_n=0.5$, however, the number of resonance peaks keeps increasing as if some harmonics are generated. Figure 2 depicts such a transition of linear cyclotron resonance to weakly nonlinear one with multiple peaks and an expanded resonance range as a result of interactions with pulses of various amplitudes $E_n=0.0001$ (solid curve), $E_n=0.001$ (dotted curve), $0.01$ (broken curve), respectively. The multiple resonances are caused by the finite pulse length as well as proton trapping. As soon as protons enter a pulse they are trapped by the pulse field, moving back and forth in the velocity space along the Hamiltonian surface. Consequently, their perpendicular velocities are modulated in an oscillatory manner with longitudinal velocities nearly fixed. The multiple peaks in Figure 2 are formed if the protons exit the pulse attaining maximum perpendicular velocity shifts, and the valleys between the peaks are formed if the protons return to the original velocity at the time of departure from the pulse. The peak velocity shifts attained in this case is one order of magnitude greater than the quiver velocity. This acceleration is still the transit-time-type acceleration.

As mentioned earlier, because of the cyclotron resonance and trapping, some protons exit the pulse when accelerated, while others do when not. The trapping period is given by [3]

$$\omega_0 T_n = 2\pi v_p / \sqrt{c E_n \Delta v_\perp}. \quad (2)$$

If this becomes shorter than the transit time $=3.3L/|v_y - v_0|$, the trapping becomes important, and the resonance peaks emerge. Hence, the finite extent of the pulse combined with trapping is causing the multiple peaks. Then, the same structure should emerge for a pulse even with a smaller amplitude as the pulse is made longer. This is indeed so was numerically confirmed (not shown). Multiple resonances, which are a sign of the weakly nonlinear regime, are also observed for whistler wave-pulses that are dispersive unlike Alfvén waves.

2.3 Strongly nonlinear acceleration

As the pulse amplitude is further increased so that the wave magnetic field dominates the external one, another type of interactions emerges. Figure 3(a) depicts perpendicular velocity shifts of protons driven by a large-amplitude pulse with $E_n=0.09$ with $L_n=0.5$. In this case, the pulse magnetic field dominates the external magnetic field. Now as the relative effect of the external field is reduced, the attained velocity shifts are nearly comparable to the quiver velocity $cE_n$. Protons with a wide range of velocities may be accelerated. A dominant feature in Figure 3(a) is the broad plateau located at between $-0.08c$ and $-0.2c$ caused by reflections driven by particle trapping. The interactions between 0 and $-0.1c$ are transit-time type super-multiple resonances with tens of peaks as magnified in Figure 3(b).

Figure 3. Strongly nonlinear interactions are presented in (a), and part of them is magnified in (b).
3. Conclusions
First, a linear perturbation theory is successfully applied to particles interacting with a small-amplitude Alfvén-wave pulse. Then, weakly nonlinear interactions of particles with Alfvén pulses yield quantized multiple resonances when the pulse amplitude is intermediate, and reflections when it is stronger. The overall results of this research reveal that the wave-particle interactions may show dramatically different features as the waves evolve from linear waves to nonlinear pulses during the development of turbulence. In general, as the wave amplitude increases, the resonance velocity region is expanded accordingly. The reflections driven by phase trapping, which is equivalent to Fermi acceleration [5], is a sign of strongly nonlinear large-amplitude pulses. It is not an accident that one of the most energetic wave activities in the universe, i.e., shock waves efficiently reflects particles.

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