Dictionary for the type II nongeometric flux compactifications

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Abstract

We study the $T$-dual completion of the four-dimensional $\mathcal{N} = 1$ type II effective potentials in the presence of (non-)geometric fluxes. First, we invoke a cohomology version of the $T$-dual transformations among the various moduli, axions and the fluxes appearing in the type IIA and type IIB effective supergravities. This leads to some useful observations about a significant mixing of the standard NS-NS fluxes with the (non-)geometric fluxes on the mirror side. Further, using our $T$-duality rules, we establish an explicit mapping among the $F$-terms, $D$-terms, tadpole conditions as well as the Bianchi identities of the two theories. Secondly, we propose what we call a set of “axionic flux polynomials”, which depend on all the axionic moduli and the fluxes. This subsequently helps in presenting the two scalar potentials in a concise and manifestly $T$-dual form, which can be directly utilized for various phenomenological purposes as we illustrate in a couple of examples.

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1 Introduction

The study of four-dimensional effective potentials arising from type II flux compactifications has been one of the most active research areas and it has received a tremendous amount of attention since more than a decade, especially in the context of moduli stabilization [1–7]. In this regard, non-geometric flux compactification has emerged as an interesting playground for model builders [8–22]. The existence of non-geometric fluxes is rooted through a successive application of $T$-duality on the three-form $H$-flux of the type II supergravities, where a chain with geometric and non-geometric fluxes appears in the following manner [23],

$$H_{ijk} \rightarrow \omega_{ij}^k \rightarrow Q_{t}^{jk} \rightarrow R^{ijk}.$$  

In addition, $S$-duality invariance of the type IIB superstring compactifications demands for including an additional flux, the so-called $P$-flux, which is $S$-dual to the non-geometric $Q$-flux [24–29]. Generically, such fluxes can appear as parameters in the four-dimensional effective theories, and subsequently can help in developing a suitable scalar potential for the various moduli and the axions. A consistent incorporation of various such fluxes makes the compactification background richer and more flexible for model building. In this regard, a continuous progress has been made towards the various phenomenological aspects such as moduli stabilization [9, 22, 30–33], constructing de-Sitter vacua [10, 11, 16, 17, 19, 34] and realisation of the minimal aspects of inflationary cosmology [18, 20, 35, 36]. Moreover, interesting connections among the toolkits of superstring
flux-compactifications, the gauged supergravities and the Double Field Theory (DFT) via non-geometric fluxes have given the platform for approaching phenomenology based goals from these three directions [8, 14, 15, 23, 30, 37–48].

In the conventional approach of studying four-dimensional type II effective theories in a non-geometric flux compactification framework, most of the studies have been centered around toroidal examples; or in particular with a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. A simple justification for the same lies in the relatively simpler structure to perform explicit computations, which have led toroidal setups to serve as promising toolkits in studying concrete examples. However, some interesting recent studies in [20, 32, 34, 34, 36, 49, 50] regarding the formal developments as well as the applications towards moduli stabilization, searching de-Sitter vacua as well as building inflationary models have boosted the interests in setups beyond toroidal examples, say e.g. in compactifications using Calabi Yaus (CY) threefolds. As the explicit form of the metric for a CY threefold is not known, while understanding the ten-dimensional origin of the 4D effective scalar potential, one should preferably represent the same in a framework where one could bypass the need of knowing the Calabi Yau metric. In this regard, the close connections between the symplectic geometry and effective potentials of type II supergravities [51–53] have been witnessed to be crucial. For example, in the context of type IIB orientifolds with the presence of standard NS-NS three-form flux ($H_3$) and RR three-form flux ($F_3$), the two scalar potentials, one arising from the $F$-term contributions while the other being derived from the dimensional reduction of the ten-dimensional kinetic pieces, could be matched via merely using the period matrices and without the need of knowing the CY metric [53, 54]. Similarly an extensive study of the effective actions in symplectic formulation have been performed for both the type IIA and the type IIB flux compactifications in the presence of standard fluxes using Calabi Yau threefolds and their orientifolds [53, 57].

In the context of non-geometric flux compactifications, there have been great amount of efforts for studying the 4D effective potentials derived from the Kähler- and super-potentials [9, 16–19, 58–61], while their ten-dimensional origin has been explored later on via Double Field Theory (DFT) [43, 62, 63] as well as in the supergravity theories [44, 46, 58, 54, 61, 64, 65]. In this regard, the symplectic approach of [53, 54] for the standard type IIB flux compactification with the $H_3/F_3$ fluxes, has been recently generalized by taking several iterative steps; first via including the non-geometric $Q$-flux in [67], and subsequently providing its $S$-dual completion via adding the non-geometric $P$-flux in [68]. In the meantime, a very robust analysis has been performed by considering the DFT reduction on the CY threefolds, and subsequently the generic $\mathcal{N} = 1$ type IIB results have been used to derive the $\mathcal{N} = 1$ effective potential with non-geometric fluxes [63]. An explicit connection between this DFT reduction formulation and the direct symplectic approaches of computing the scalar potential using the superpotential has been presented in [67] for the type IIB and in [68] for the type IIA non-geometric scenarios.

**Motivation and goals**

The crucial importance of the non-geometric flux compactification scenarios can be illustrated by the fact that generically speaking, one can stabilize all moduli by the tree level effects; for example this also includes the Kähler moduli in type IIB framework which, in conventional flux compactifications, are protected by the so-called “no-scale structure”. However, the complexity with introducing many flux parameters not only facilitates a possibility for the easier samplings to fit the values, but also backreacts on the overall strategy itself in a sense that it induces some inevitably hard challenges, which sometimes can make the situation even worse. For example the four-dimensional scalar potentials realised in the concrete models, say the ones obtained by using the type IIA/IIB setups with $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ toroidal orientifolds, are very often so huge that even it
gets hard to analytically solve the extremization conditions, and one has to look either for simplified ansatz by switching off certain flux components at a time, or else one has to opt for some highly involved numerical analysis \cite{15, 19, 24, 26}. In our opinion, this obstacle can be tackled if one could find some concise formulation of the scalar potential. Usually the convention is to start with the flux superpotential having several terms, and so it is natural to anticipate that the numerical computation will result in complicated scalar potential with no guaranteed hierarchy among the various terms, and so it would be hard to do anything analytically at that level. On these lines, we aim to provide a concise and concrete formulation of the scalar potentials of the two theories with a sense of distinctness among the axionic and saxionic sector, along with a manifestation of the T-duality between them. The details on the goals can be enumerated in the following points:

- The T-dual completions of type II effective theories by including the (non-)geometric fluxes have been studied in the toroidal context in \cite{10, 11, 20, 31, 71–73}, however a concrete connection between the (non-)geometric ingredients of the two theories is still missing in the beyond toroidal case. Although on these lines, a couple of interesting efforts have been initiated in \cite{57, 74}, however without having the full understanding of the T-duality at the level of NS-NS non-geometric flux components and the two scalar potentials, and we attempt to fill this gap.

- We present a cohomology version of the T-duality rules between the type IIA and type IIB theories, which subsequently enables us to read-off the T-dual ingredients from one theory to the other. This includes fluxes, moduli, axions, $F/D$-terms, tadpole cancellation conditions and the NS-NS Bianchi identities.

- For extending the understanding about the T-dual mapping from the level of flux superpotential and the $D$-terms to the level of total scalar potential, we invoke some interesting flux combinations with axions, which we call “axionic flux polynomials”, that are useful for writing down the full scalar potential in a few lines! Recalling the obstacle in moduli stabilisation and subsequent phenomenology with the toroidal model having around 2000 terms, it is remarkable that the generic scalar potential for the two theories could be so compactly formulated.

- With the above step, we present the generic formulation of the type IIA as well as type IIB scalar potential which can be explicitly written for a particular compactification by merely knowing (some of) the topological data (such as hodge numbers and intersection numbers) of the compactifying (CY) threefolds and their mirrors.

- We collect the T-duality rules for the fluxes, moduli, scalar potentials and the Bianchi identities in a concise dictionary in the form of 6 tables which present a one-to-one mapping among the various ingredients of the type IIA and type IIB theories.

The article is organised as follows: In section 2, we provide the basic ingredients for the non-geometric type II flux compactifications in some good detail. Section 3 is devoted to invoke the cohomology version of the T-duality rules and further for checking the consistency throughout the $F/D$-terms, tadpoles conditions and the Bianchi identities. In Section 4 we present axionic flux polynomials and a concise form of the scalar potentials for the two theories, which are manifestly T-dual to each other. Section 5 presents the illustration of the scalar potential formulation for two particular examples using toroidal orientifolds which subsequently also ensures the T-duality

\footnote{In this article we consider type II compactifications using non-rigid Calabi Yau threefolds. The study of scalar potentials arising in the rigid Calabi Yau compactifications will be presented elsewhere in \cite{75}.}
checks. While section 6 includes summary and the outlooks, we present a $T$-dual dictionary in the appendix A where we present 6 tables; namely table 7, table 8, table 9, table 10, table 11, table 12 which can be used for reading-off the relevant $T$-dual details of the two type II theories.

2 Non-geometric flux compactifications: preliminaries

In this section, we will review the relevant pieces of information regarding the type IIA-and type IIB-orientifold setups with the presence of (non-)geometric fluxes, in addition to the usual NS-NS and RR fluxes. In this regard, we will also revisit several standard things for setting up a consistent notation in order to fix any possible conflict in conventions, signs, or factors etc.

Considering the bosonic sector of the $\mathcal{N} = 1$ supergravity theory having one gravity multiplet, a set of complex scalars $\varphi^A$ and a set of vectors $A^\alpha$, the effective action can be given as

$$S^{(4)} = -\int_{M_4} \left( -\frac{1}{2} R + K_{AB} d\varphi^A \wedge *d\varphi^B + V * 1 \right) + \frac{1}{2} \left( \text{Re} f_{g\alpha\beta} F^\alpha \wedge *F^\beta + \frac{1}{2} \left( \text{Im} f_{g\alpha\beta} F^\alpha \wedge F^\beta \right) \right),$$

(2.1)

where $*$ is the four-dimensional Hodge star, and $F^\alpha = dA^\alpha$. There are three main ingredients, namely the Kähler potential ($K$), the superpotential ($W$) and the holomorphic gauge kinetic function ($f_g$) for determining the four-dimensional scalar potential ($V$) appearing in the above generic action. In fact, the total scalar potential can be simply expressed as a sum of $F$-term and $D$-term contributions as given below,

$$V = V_F + V_D,$$

(2.2)

where

$$V_F = e^K \left( K^{AB} D_A W D_B W - 3 |W|^2 \right), \quad V_D = \frac{1}{2} (\text{Re} f_{g\alpha\beta} D_\alpha D_\beta.$$

Note that the sum in the piece $V_F$ is over “all” the moduli, and the covariant derivative is defined through the relation $D_A W = d_A W + W \partial_A K$, and $D_\alpha$ is the $D$-term for the $U(1)$ gauge group corresponding to $A^\alpha$ as given below,

$$D_\alpha = (\partial_A K) \ (T_\alpha)^A_B \varphi^B + \zeta_\alpha,$$

(2.3)

where $T_\alpha$ is the generator of the gauge group and $\zeta_\alpha$ denotes the Fayet-Iliopoulos term. Now we come to the two specific $\mathcal{N} = 1$ supergravities, namely type IIA and type IIB including various fluxes.

2.1 Non-geometric Type IIA setup

We consider type IIA superstring theory compactified on an orientifold of a Calabi Yau threefold $X_3$. The orientifold is constructed via modding out the CY with a discrete symmetry $\mathcal{O}$ which includes the world-sheet parity $\Omega_p$ combined with the space-time fermion number in the left-moving sector $(-1)^{F_L}$. In addition $\mathcal{O}$ can act non-trivially on the Calabi-Yau manifold so that one has altogether,

$$\mathcal{O} = \Omega_p (-1)^{F_L} \sigma,$$

(2.4)

where $\sigma$ is an involutive symmetry (i.e. $\sigma^2 = 1$) of the internal CY and acts trivially on the four flat dimensions. The massless states in the four dimensional effective theory are in one-to-one
correspondence with various involutively even/odd harmonic forms, and hence they generate the equivariant cohomology groups $H^{p,q}_{\pm}(X_3)$. To begin with, we consider the following representations for the various involutively even and odd harmonic forms $[55]$, 

| Cohomology group | $H^{(1,1)}_+$ | $H^{(1,1)}_-$ | $H^{(2,2)}_+$ | $H^{(2,2)}_- | H^{(3)}_+ | H^{(3)}_-
| Basis | $\mu_\alpha$ | $\nu_\alpha$ | $\bar{\nu}^a$ | $\bar{\mu}^{\alpha}$ | $(\alpha^k_\lambda, \beta^\lambda)$ | $(\alpha_{\lambda}, \beta^k)$ |

Table 1: Representation of various forms and their bases

Here the dimensionality of bases $\mu_\alpha$ and $\bar{\mu}^{\alpha}$ are counted by the Hodge number $h^{(1,1)}_+(X_3)$ while those of the bases $\nu_\alpha$ and $\bar{\nu}^a$ are counted by $h^{(1,1)}_-(X_3)$. Moreover, the indices $k$ and $\lambda$ involved in the even/odd three-forms are such that summing over the same gives the total number of the real harmonic three-forms which is $2(h^{2,1}_+(X_3) + 1)$. The various field ingredients can be expanded in appropriate bases of the equivariant cohomologies. In order to preserve $N = 1$ supersymmetry, one needs the involution $\sigma$ to be anti-holomorphic, isometric and acting on the Kähler form $J$ as given below

$$\sigma^*(J) = -J,$$  \hfill (2.5)

which generically results in the presence of $O6$-planes. Given that the Kähler form $J$ and the NS-NS two-form potential $B_2$ are odd under the involution, the same can be expanded in the odd two-form basis $\nu_\alpha$ as,

$$J = t^a \nu_\alpha, \quad B_2 = -b^a \nu_\alpha,$$  \hfill (2.6)

where $t^a$ denotes the string-frame two-cycle volume while $b^a$ denotes axionic moduli. This leads to the following complexified Kähler class $J_c$ defining the chiral coordinates $T^a$ in the following manner,

$$J_c = B_2 + i J = -T^a \nu_\alpha, \quad \text{where} \quad T^a = b^a - i t^a.$$  \hfill (2.7)

Similarly, the nowhere vanishing holomorphic three-form $(\Omega_3)$ of the Calabi Yau can be expanded in the three-form basis using a prepotential $G^{(q)}_K$ of the quaternion sector in the $\mathcal{N} = 2$ theory in the following manner,

$$\Omega_3 = \mathcal{Z}^K \alpha_K - G^{(q)}_K \beta^K,$$  \hfill (2.8)

Now, the compatibility of the orientifold involution $\sigma$ with the Calabi Yau condition $(J \wedge J \wedge J) \propto (\Omega_3 \wedge \overline{\Omega}_3)$ demands the following condition,

$$\sigma^*(\Omega_3) = e^{2i\theta} \overline{\Omega}_3 \quad \implies \quad Im(e^{-i\theta} \mathcal{Z}^K) = 0, \quad Re(e^{-i\theta} G^{(q)}_K) = 0.$$  \hfill (2.9)

In addition, note that only one of these equations is relevant due to the scale invariance of $\Omega_3$ which is defined only up to a complex rescaling, and here we simply set $\theta$ in eqn. (2.9) to zero which leads to $\sigma^*(\Omega_3) = \overline{\Omega}_3$ and subsequently the following relations,

$$Im \mathcal{Z}^k = 0, \quad Re G^{(q)}_k = 0, \quad Re \mathcal{Z}^\lambda = 0, \quad Im G^{(q)}_\lambda = 0.$$  \hfill (2.10)
Kähler potential

The Kähler potential consists of two pieces and can be written as

\[ K_{\text{IIA}} \equiv K^{(k)} + K^{(q)}. \]  \hspace{1cm} (2.11)

Let us first consider the \( K^{(k)} \) part which encodes the information about the moduli space of the Kähler moduli, and can be computed from a prepotential of the following type \[ \text{[75, 76]}, \]

\[ G^{(k)} = -\frac{\kappa_{abc} T^a T^b T^c}{6 T^0} + \frac{1}{2} p_{ab} T^a T^b + p_a T^a T^0 - \frac{i}{2} p_0 (T^0)^2 + \ldots, \]  \hspace{1cm} (2.12)

where we have ignored the non-perturbative effects assuming the large volume limit. Here we have introduced \( T^0 = 1 \) as the parameter analogous to the complex structure homogeneous parameter on the mirror side. In addition, \( \kappa_{abc} \) denotes the classical triple intersection number determining the volume of the Calabi Yau threefold in terms of the two-cycle volume as \( V = \frac{1}{6} \kappa_{abc} t^a t^b t^c \), while the pieces with \( p_{ab}, p_a \) and \( p_0 \) correspond to the curvature corrections arising from different orders in the \( \alpha' \)-series. Although their origin from the 10D perspective is yet to be understood, the mirror symmetry arguments suggest that all the three quantities \( p_{ab}, p_a \) and \( p_0 \) are real numbers, and can be defined as \[ \text{[77, 78]}, \]

\[ p_{ab} = \frac{1}{2} \int_{\text{CY}} \hat{D}_a \wedge \hat{D}_b, \quad p_a = \frac{1}{24} \int_{\text{CY}} c_2(\text{CY}) \wedge \hat{D}_a, \quad p_0 = -\frac{\zeta(3) \chi(\text{CY})}{8 \pi^3}, \]  \hspace{1cm} (2.13)

where \( \hat{D}_a, c_2(\text{CY}) \) and \( \chi(\text{CY}) \) respectively denote the dual to the divisor class, the second Chern class and the Euler characteristic of the Calabi Yau threefold. Subsequently the Kähler potential is given as,

\[ K^{(k)} = -\ln \left( -i \left( \bar{T}^A G_A^{(k)} - T^A \bar{G}^{(k)}_A \right) \right) = -\ln (8V + 2p_0) \]

\[ = -\ln \left( \frac{i}{6} \kappa_{abc} (T^a - T^a^0) (T^b - T^b^0)(T^c - T^c^0) + 2p_0 \right). \]  \hspace{1cm} (2.14)

The second piece \( K^{(q)} \) encodes the information from the moduli space of the complex structure deformations, and for expressing it we start with defining a compensator field \( C \),

\[ C \equiv e^{-\varphi} e^{\frac{i}{2} K_{\text{IIA}}^{(cs)} - \frac{i}{2} K^{(k)}} = e^{-D_{4d}} e^{\frac{1}{2} K_{\text{IIA}}^{(cs)}}, \]  \hspace{1cm} (2.15)

where the ten-dimensional dilaton \( \varphi \) is related to the four-dimensional dilaton \( D_{4d} \) as,

\[ e^{D_{4d}} = \sqrt{8} e^{\varphi + \frac{i}{2} K_k} = \frac{e^{\varphi}}{\sqrt{V + P_0}}. \]  \hspace{1cm} (2.16)

With our normalizations, the piece \( K_{\text{IIA}}^{(cs)} \) can be determined from the prepotential \( G^{(q)} \) as,

\[ K_{\text{IIA}}^{(cs)} = -\ln \left( -\frac{i}{8} \int_{X_3} \Omega \wedge \overline{\Omega} \right) = -\ln \left[ \frac{1}{4} \left( \text{Re}(Z^k) \text{Im}(G^{(q)}_k) - \text{Im}(Z^k) \text{Re}(G^{(q)}_k) \right) \right]. \]  \hspace{1cm} (2.17)

Now using the compensator \( C \), we consider the following expansion of three-form,

\[ C\Omega = \text{Re}(C Z^k) \alpha_k + i \text{Im}(C Z^k) \alpha_k - i \text{Im}(C G^{(q)}_k) \beta_k - \text{Re}(C G^{(q)}_k) \beta_k^{\lambda}, \]  \hspace{1cm} (2.18)
where we have used the compensated orientifold constraints given in eqn. (2.10),

\[ Im(C\hat{Z}^k) = Re(C\hat{G}_{k}^{(q)}) = Re(CZ^\lambda) = Im(C\hat{G}_{\lambda}^{(q)}) = 0. \]  

Using the following expansion of the RR three-form which is even under the involution,

\[ C_3 = \xi^k \alpha_k - \xi_\lambda \beta^\lambda, \]  

we define a complexified three-form \( \Omega_c \) as,

\[ \Omega_c = C_3 + i Re(C\hat{\Omega}) \]

\[ = \left( \xi^k + i Re(C\hat{Z}^k) \right) \alpha_k - (\xi_\lambda + i Re(CG_\lambda)) \beta^\lambda \]

\[ \equiv N^k \alpha_k - U_\lambda \beta^\lambda. \]

Here the lowest components of the \( N = 1 \) chiral superfields \( N^k \) and \( U_\lambda \) are defined in the following manner,

\[ N^k \equiv \int_{X_3} \Omega_c \wedge \beta^k = \xi^k + i Re(C\hat{Z}^k), \]  

\[ U_\lambda \equiv \int_{X_3} \Omega_c \wedge \alpha_\lambda = \xi_\lambda + i Re(C\hat{G}_\lambda^{(q)}). \]

Now using these pieces of information, the second part of the Kähler potential, namely the \( K^{(q)} \) piece, can be written as,

\[ K^{(q)} \equiv -2 \ln \left[ \frac{1}{4} \int_{X_3} Re(C\hat{\Omega}) \wedge \ast Re(C\hat{\Omega}) \right] = 4 D_{4d}, \]  

where in the second step we have utilized the following identity,

\[ \int_{X_3} Re(C\hat{\Omega}) \wedge \ast Re(C\hat{\Omega}) = Re(C\hat{Z}^k) Im(C\hat{G}_{k}^{(q)}) - Im(CZ^\lambda) Re(CG_\lambda^{(q)}) = 4 e^{-2 D_{4d}}. \]

The above identity can be derived using the definitions of the four-dimensional dilaton \( D_{4d} \) through eqns. (2.16) and the \( K^{(cs)}_{IIA} \) given in eqn. (2.17). Moreover, the Kähler potential part \( K^{(q)} \) can be further rewritten in the following form having explicit dependence on a set of special coordinates defined as,

\[ Re(CZ^0) = y^0, \quad Re(CZ^k) = y^k, \quad Im(CZ^\lambda) = y^\lambda. \]

For knowing the explicit form of the prepotential \( G^{(q)} \) for the quaternion case we consider the following generic expression,

\[ G^{(q)}(Y) = \frac{k_{ABC} Y^A Y^B Y^C}{6 Y^0} + \frac{1}{2} \tilde{p}_{AB} Y^A Y^B + \tilde{p}_A Y^A Y^0 + i \frac{1}{2} \tilde{p}_0 (Y^0)^2, \]

which subsequently gives the following derivatives,

\[ \partial_{y^0} G^{(q)} = -\frac{k_{ABC} Y^A Y^B Y^C}{6 (Y^0)^2} + \tilde{p}_A Y^A + i \tilde{p}_0 Y^0, \]

\[ \partial_{y^A} G^{(q)} = \frac{1}{2} \frac{k_{ABC} Y^B Y^C}{Y^0} + \tilde{p}_{AB} Y^B + \tilde{p}_A Y^0. \]
Now, considering the identification of coordinates as $Y^0 = y^0$ and $Y^A = \{y^k, i_\lambda y^\lambda\}$, the pre-potential $G(q)$ takes the following form,

$$G(q) \left( y^0, y^k, i_\lambda y^\lambda \right) = -\frac{i}{6} y^0 k_{\lambda\rho\kappa} y^\lambda y^\rho z^\kappa + \frac{i}{2} y^0 \hat{k}_{\lambda\kappa\mu} y^\lambda y^\kappa y^\mu + i \tilde{p}_{k\lambda} y^k y^\lambda + i \tilde{p}_\lambda y^\lambda y^0 + \frac{i}{2} \tilde{p}_0 (y^0)^2, \quad (2.28)$$

and along with this we have the following expressions,

$$\text{Im}(C G_{0}^{(q)}) = \frac{1}{6} (y^0)^2 k_{\lambda\rho\kappa} y^\lambda y^\rho z^\kappa - \frac{1}{2} (y^0)^2 \hat{k}_{\lambda\kappa\mu} y^\lambda y^\kappa y^\mu + \tilde{p}_\lambda y^\lambda + \tilde{p}_0 y^0, \quad (2.29)$$

$$\text{Im}(C G_{k}^{(q)}) = \frac{1}{6} k_{\lambda\kappa\mu} y^\lambda y^\kappa y^\mu + \tilde{p}_{k\lambda} y^\lambda + \tilde{p}_k y^0,$n

$$\text{Re}(C G_{\lambda}^{(q)}) = -\frac{1}{2} k_{\lambda\rho\kappa} y^\rho y^\kappa + \frac{1}{2} k_{\lambda\kappa\mu} y^\lambda y^\kappa y^\mu + \tilde{p}_{k\lambda} y^k + \tilde{p}_\lambda y^0. \quad (2.30)$$

Further we define a new set of special non-homogeneous coordinates $z^0 = (y^0)^{-1}, z^k = y^k/y^0$ and $z^\lambda = y^\lambda/y^0$, and subsequently the pre-potential in eqn. (2.28) simplifies as,

$$G^{(q)}(z^0, z^k, z^\lambda) = (z^0)^{-2} g^{(q)}(z^k, z^\lambda), \quad (2.31)$$

where $g^{(q)}(z^k, z^\lambda)$ in special coordinates is given as,

$$g^{(q)}(z^k, z^\lambda) = -\frac{i}{6} k_{\lambda\rho\kappa} z^\lambda z^\rho z^\kappa + \frac{i}{2} \hat{k}_{\lambda\kappa\mu} z^\lambda z^k z^\mu + i \tilde{p}_{k\lambda} z^k z^\lambda + i \tilde{p}_\lambda z^\lambda + \frac{i}{2} \tilde{p}_0. \quad (2.32)$$

In addition, one has the following useful relations,

$$\text{Im}(C G_{0}^{(q)}) = (z^0)^{-1} \left( \frac{1}{6} k_{\lambda\rho\kappa} z^\lambda z^\rho z^\kappa - \frac{1}{2} \hat{k}_{\lambda\kappa\mu} z^\lambda z^k z^\mu + \tilde{p}_\lambda z^\lambda + \tilde{p}_0 \right),$$

$$\text{Im}(C G_{k}^{(q)}) = (z^0)^{-1} \left( \hat{k}_{\lambda\kappa\mu} z^\lambda z^\mu + \tilde{p}_{k\lambda} z^\lambda \right),$$

$$\text{Re}(C G_{\lambda}^{(q)}) = (z^0)^{-1} \left( -\frac{1}{2} k_{\lambda\rho\kappa} z^\rho z^\kappa + \frac{1}{2} \hat{k}_{\lambda\kappa\mu} z^\kappa z^\mu + \tilde{p}_{k\lambda} z^k + \tilde{p}_\lambda \right),$$

which give the following explicit forms for the chiral variables,

$$T^a = b^a - i t^a, \quad (2.33)$$

$$N^0 = \xi^0 + i (z^0)^{-1},$$

$$N^k = \xi^k + i (z^0)^{-1} z^k,$n

$$U_{\lambda} = \xi_{\lambda} - i (z^0)^{-1} \left( \frac{1}{2} k_{\lambda\rho\kappa} z^\rho z^\kappa - \frac{1}{2} \hat{k}_{\lambda\kappa\mu} z^\kappa z^\mu - \tilde{p}_{k\lambda} z^k - \tilde{p}_\lambda \right).$$

Moreover, we find that $K^{(q)}$ simplifies into the following form,

$$K^{(q)} \equiv 4 D_{ab} = -2 \ln \left[ \frac{1}{4} \left( \text{Re}(C Z^k) \text{Im}(C G_{k}^{(q)}) - \text{Im}(C Z^\lambda) \text{Re}(C G_{\lambda}^{(q)}) \right) \right]$$

$$= -4 (z^0)^{-1} - 2 \ln \left( \frac{1}{6} k_{\lambda\rho\kappa} z^\lambda z^\rho z^\kappa + \frac{\tilde{p}_0}{4} \right), \quad (2.34)$$


where the various moduli \( z^0, z^k, z^\lambda \) implicitly depend on the variables \( N^0, N^k \) and \( U_\lambda \). Subsequently the full Kähler potential can be collected as,

\[
K_{\text{IIA}} = -\ln \left( \frac{4}{3} \kappa_{abc} t^a t^b t^c + 2 p_0 \right) - 4 \ln(z^0)^{-1} - 2 \ln \left( \frac{1}{6} \kappa_{\lambda\rho\kappa} z^\lambda z^\rho z^\kappa + \frac{p_0}{4} \right),
\]

which can be thought of as a real function of the complexified moduli \( T^a, N^0, N^k \) and \( U_\lambda \). For the later purpose, we also define \( \mathcal{U} = \frac{1}{6} \kappa_{\lambda\rho\kappa} z^\lambda z^\rho z^\kappa \) for the complex structure side, an analogous quantity to the overall volume \( V \) of the CY threefold, and subsequently the Kähler potential can also be written as,

\[
K_{\text{IIA}} = -\ln (8 V + 2 p_0) - 4 \ln(z^0)^{-1} - 2 \ln \left( \mathcal{U} + \frac{p_0}{4} \right).
\]

Here we would like to convey to the readers that the forms and notations are being put in place keeping in mind the mirror symmetry arguments, to be illustrated/manifested after considering the type IIB side later on.

**Flux superpotential**

For getting the generalised version of GVW flux superpotential \( [79] \), we need to define the twisted differential operator given as \( [23] \),

\[
D = d - H \wedge . - w \ll . - Q \gg . - R \bullet.
\]

The action of operators \( \ll, \gg \) and \( \bullet \) on a \( p \)-form changes it into a \( (p + 1) \), \( (p - 1) \) and \( (p - 3) \)-form respectively, and the various flux actions can be given as \( [9] \),

\[
H \wedge \alpha_k = H_k \Phi_6 \quad \text{H \wedge \beta^\lambda = -H^\lambda \Phi_6 \quad H \wedge \alpha_\lambda = 0 = H \wedge \beta^k;}
\]

\[
w \ll \alpha_k = w_{ak} \tilde{\nu}^a \quad w \ll \beta^\lambda = -w^a \beta^\lambda \quad w \ll \alpha_\lambda = \tilde{\nu}_a \mu^a \quad w \ll \beta^k = -\tilde{\nu}_a \mu^a;
\]

\[
Q \gg \alpha_k = Q^a_k \nu_a \quad Q \gg \beta^\lambda = -Q^{\alpha_\lambda} \nu_a \quad Q \gg \alpha_\lambda = \tilde{Q}^{\alpha_\lambda} \mu_\alpha \quad Q \gg \beta^k = -\tilde{Q}^{\alpha_\lambda} \mu_\alpha;
\]

\[
R \bullet \alpha_k = R^\lambda_k 1 \quad R \bullet \beta^\lambda = -R^{\alpha_\lambda} 1 \quad R \bullet \alpha_\lambda = 0 = R \bullet \beta^k;
\]

\[
H \wedge 1 \equiv H \equiv -H^\lambda \alpha_\lambda - H^k \beta^k;
\]

\[
w \ll \nu_a = w^a \alpha_\lambda + w_{ak} \beta^k \quad w \ll \mu_\alpha = \tilde{\nu}_a \alpha_k + \tilde{\nu}_a \beta^\lambda;
\]

\[
Q \gg \tilde{\nu}^a = -Q^{\alpha_\lambda} \alpha_\lambda - Q^a_k \beta^k \quad Q \gg \tilde{\mu}^\lambda = -\tilde{Q}^{\lambda_\alpha} \alpha_k - \tilde{Q}^{\alpha_\lambda} \beta^\lambda;
\]

\[
R \bullet \Phi_6 = R^\lambda \alpha_\lambda + R_k^\beta \beta^k.
\]

Further, we take the following expansion for the multi-form RR fluxes \( F_{\text{RR}} \),

\[
F_{\text{RR}} = F_0 + F_2 + F_4 + F_6 = m^0 1 + m^a \nu_a + e_a \tilde{\nu}^a + e_0 \Phi_6.
\]

Now we consider the Kähler form expansion \( J_c = -T^a \nu_a \) to obtain the following multiform \( \Pi_{J_c} \) analogous to the period vectors on the mirror side,

\[
\Pi_{J_c} = \begin{pmatrix}
1 \\
(\frac{1}{2} \kappa_{abc} T^a T^b - p_{ab} T^b - p_a) \tilde{\nu}^c \\
- (\frac{1}{2} \kappa_{abc} T^a T^b T^c + p_a T^a + i p_0) \Phi_6
\end{pmatrix}.
\]
Note that usually in the absence of any $\alpha'$-corrections and the prepotential quantities such as $p_{ab}$, $p_a$, $p_0$, we usually denote $\Pi_{J_e}$ as,

$$\Pi_{J_e} = e^{J_e} = 1 + J_e + \frac{1}{2} J_c \wedge J_c + \frac{1}{3!} J_c \wedge J_c \wedge J_c$$

(2.41)

which gets modified after including the $\alpha'$-corrections. Now, the generalised flux superpotential having contributions from the NS-NS and RR fluxes can be given as [9, 30, 55, 57, 74],

$$W_{IIA} = W_{IIA}^R + W_{IIA}^{NS} := -\frac{1}{\sqrt{2}} \int_{X_3} \langle F_{RR} + D\Omega_c, \Pi_{J_e} \rangle,$$

(2.42)

where we have introduced a normalization factor of $\sqrt{2}$. Here the anti-symmetric multi-forms are defined through the following Mukai-pairings,

$$\langle \Gamma, \Delta \rangle_{\text{even}} = \Gamma_0 \wedge \Delta_6 - \Gamma_2 \wedge \Delta_4 + \Gamma_4 \wedge \Delta_2 - \Gamma_6 \wedge \Delta_0,$$

$$\langle \Gamma, \Delta \rangle_{\text{odd}} = -\Gamma_1 \wedge \Delta_5 + \Gamma_3 \wedge \Delta_3 - \Gamma_5 \wedge \Delta_1,$$

(2.43)

where $\Gamma$ and $\Delta$ denotes some even/odd multi-forms. Now utilizing the flux actions of various NS-NS and RR fluxes on various cohomology bases as given in eqns. (2.35) and (2.39), the superpotential takes the following form,

$$\sqrt{2} W_{IIA} = \left[ e_0 + T^a \overline{e}_a + \frac{1}{2} \kappa_{abc} T^a T^b T^c m^0 + \frac{1}{6} \kappa_{abc} T^a T^b T^c m^0 - i p_0 m^0 \right]$$

(2.44)

where we have introduced a shifted version of the flux parameters to absorb the effects from $p_{ab}, p_a$ in the following manner,

$$e_0 = e_0 - p_a m^a, \quad e_a = e_a - p_{ab} m^b + p_a m^0,$$

$$H_0 = H_0 - p_a Q^a_0, \quad w_a = w_a - p_{ab} Q^b_0 + p_a R_0,$$

$$H_k = H_k - p_a Q^a_k, \quad w_{ak} = w_{ak} - p_{ab} Q^b_k + p_a R_0,$$

$$H^\lambda = H^\lambda - p_a Q^a_\lambda, \quad w^\lambda_a = w^\lambda_a - p_{ab} Q^b_\lambda + p_a R_\lambda.$$

(2.45)

Thus we note that considering the $\alpha'$-corrected prepotential of the form (2.12) consistent with the mirror symmetry arguments generally results in some rational shifts via $(p_{ab}$ and $p_a$) for some of the conventional flux components. This has been earlier observed for the case of without having any non-geometric flux in [73]. Usually one doesn’t care about the quantities $p_{ab}$ and $p_a$ as it is only the $p_0$ which appears in the Kähler potential (and not $p_{ab}$ and $p_a$), however in that case, while doing phenomenology one should be careful with strictly considering the integral fluxes and using mirror symmetric arguments at the same time. In addition, let us also note that the analogous prepotential for the quaternionic sector given in eqn. (2.30) leads to a slight modification in the variable $U_\lambda$, and so does its mirror symmetric counterpart on the type IIB side as we will see later.

Utilising the generic form of the Kähler potential (2.31) and the superpotential (2.44), the $F$-term contribution to the four-dimensional scalar potential $V_{IIA}^F$ can be computed by using the eqn. (2.2) where the sum is to be taken over all the $T^a, N^0, N^k$ and $U_\lambda$ moduli.
Gauge kinetic couplings and the D-term effects

Let us quickly recollect the D-term contribution to the scalar potential by mostly following the ideas proposed in [60, 63, 65]. Keeping in mind that four-dimensional vectors can generically descend from the reduction on the three-form potential $C_3$ while the dual four-form gauge fields can arise from the reduction on five-form potential $C_5$, let us consider the following expansions of the $C_3$ and the $C_5$,

$$C_3 = \xi^k \alpha_k - \xi_\lambda \beta^\lambda + A^\alpha \mu_\alpha, \quad C_5 = A_\alpha \bar{\mu}^\alpha.$$  \hfill (2.46)

Now considering a pair $(\gamma^\alpha, \gamma_\alpha)$ to ensure the 4D gauge transformations of the quantities $(A^\alpha, A_\alpha)$, we have the following transformations,

$$A^\alpha \to A^\alpha + d\gamma^\alpha, \quad A_\alpha \to A_\alpha + d\gamma_\alpha.$$  \hfill (2.47)

Subsequently considering the twisted differential $D$ given in eqn. (2.76), we find the following transformation of the RR forms,

$$C_{RR} \equiv C_1 + C_3 + C_5 = \xi^k \alpha_k - \xi_\lambda \beta^\lambda + A^\alpha \mu_\alpha + A_\alpha \bar{\mu}^\alpha \to C_{RR} + D (\gamma^\alpha \mu_\alpha + \gamma_\alpha \bar{\mu}^\alpha).$$

where we have used the flux actions given in eqn. (2.38). Now the transformation given in eqn. (2.38) shows that the axions $\xi^k$ and $\xi_\lambda$ are not invariant under the gauge transformation, and this leads to the following shifts in the $N = 1$ coordinate $N^k$ and $U_\lambda$,

$$\delta N^k = -\gamma^\alpha \bar{w}_\alpha^k + \gamma_\alpha \hat{Q}^{\alpha k}, \quad \delta U_\lambda = \gamma^\alpha \bar{w}_\alpha^\lambda - \gamma_\alpha \hat{Q}^\alpha_\lambda.$$  \hfill (2.49)

In particular, this implies that if we define the following two type of fields,

$$\Xi^k = e^{iN^k}, \quad \Xi^\lambda = e^{iU_\lambda},$$  \hfill (2.50)

then these fields $\Xi^k$ and $\Xi^\lambda$ are electrically charged under the gauge group $U(1)_\alpha$ with charges $(-\bar{w}_\alpha^k)$ and $(\bar{w}_\alpha^\lambda)$ respectively while they are magnetically charged with charges $(\hat{Q}^{\alpha k})$ and $(-\hat{Q}^\alpha_\lambda)$ respectively. Now using the type IIA Kähler potential given in eqn. (2.33) and the variables in eqn. (2.33), we derive the following Kähler derivatives,

$$K^{N^0} = \frac{i}{2(z^0)^{-1}} \left( 1 - \frac{\hat{k}_{\lambda \kappa m} z^\lambda z^\kappa z^\mu}{2 (U + \frac{p_0}{4})} + \frac{3 p_0}{4 (U + \frac{p_0}{4})} \right),$$  \hfill (2.51)

$$K^{N^k} = \frac{i \hat{k}_{\lambda \kappa m} z^\lambda z^\kappa}{2 (z^0)^{-1} (U + \frac{p_0}{4})}, \quad K^{U_\lambda} = -\frac{i z^\lambda}{2 (z^0)^{-1} (U + \frac{p_0}{4})}.$$  \hfill (2.52)

Subsequently, one can compute the following two D-terms,

$$D_\alpha = -i \left[ (\partial_{N^k} K) \bar{w}_\alpha^k - (\partial_{U_\lambda} K) \bar{w}_\alpha^\lambda \right], \quad D^\alpha = i \left[ (\partial_{N^k} K) \hat{Q}^{\alpha k} - (\partial_{U_\lambda} K) \hat{Q}^\alpha_\lambda \right].$$  \hfill (2.53)

In addition, the gauge kinetic functions follow from the prepotential derivatives $G^{(k)}_{\alpha \beta}$ for the T-moduli written out by considering the even sector, which turns out to be given as,

$$(f_g^{ele})_{\alpha \beta} = -\frac{i}{2} (\hat{k}_{\alpha \beta} T^a - p_{\alpha \beta}),$$  \hfill (2.54)
where we also observe the presence of parameters $p_{\alpha\beta}$, which however will not appear in the “real” part and hence in the gauge kinetic couplings given as $Re(f^\text{ele}_g)_{\alpha\beta} = -\frac{1}{\hat{e}} \tilde{f}_{\alpha\beta}$. This leads to the following $D$-term contributions to the four-dimensional scalar potential,

$$V^D_{\text{IIA}} = \frac{1}{2} D\alpha \left[ Re(f^\text{ele}_g)_{\alpha\beta} \right]^{-1} D\beta + \frac{1}{2} D\alpha \left[ Re(f^\text{mag}_g)_{\alpha\beta} \right]^{-1} D\beta ,$$

(2.54)

where the explicit expressions of the $D$-terms given in eqn. (2.52) turn out to be given as,

$$D\alpha = \left( z_0^2 \right)^{-1} e^{K(\frac{q}{2})} \int X_3 \left[ \left[ \text{Im} \Omega^c \right], D\overline{F}_{\text{RR}} \right] ,$$

(2.55)

$$D\alpha = -\left( z_0^2 \right)^{-1} e^{K(\frac{q}{2})} \int X_3 \left[ \left[ \text{Im} N^k \right], \left( H^k m_0 - \omega_{ak} m^a + Q_{ak} e_a - R^k e_0 \right) \right] \right).$$

(2.56)

Tadpoles cancellation conditions and Bianchi identities

Generically, there are tadpole terms present due to the presence of $O6$-planes, and these can be canceled either by imposing a set of flux constraints or else by adding the counter terms which could arise from the presence of local sources such as (stacks of) $D6$-branes. These effects equivalently provide the following contributions in the effective potential to compensate the tadpole terms \[30\],

$$V^\text{tad}_{\text{IIA}} = \frac{1}{2} e^{K(q)} \int X_3 \left[ \left[ \text{Im} \Omega^c \right], D\overline{F}_{\text{RR}} \right] ,$$

(2.57)

where three-form $DF_{\text{RR}}$ can be expanded as \[69\],

$$DF_{\text{RR}} = -\left( H^k m_0 - \omega_{ak} m^a + Q_{ak} e_a - R^k e_0 \right) \alpha_k \right),$$

(2.58)

Subsequently, the eqn. (2.56) simplifies into the following form,

$$V^\text{tad}_{\text{IIA}} = \frac{1}{2} e^{K(q)} \left[ \left( \text{Im} N^k \right) \left( H^k m_0 - \omega_{ak} m^a + Q_{ak} e_a - R^k e_0 \right) \right] + \left( \text{Im} U^k \right) \left( H^k m_0 - \omega_{ak} m^a + Q_{ak} e_a - R^k e_0 \right) \right].$$

(2.59)

In the four-dimensional type IIA effective theory, the dynamics of various moduli is determined by the total scalar potential given as a sum of the $F$-term and the $D$-term contributions,

$$V^\text{tot}_{\text{IIA}} = V^F_{\text{IIA}} + V^D_{\text{IIA}} ,$$

(2.59)

where the various fluxes appearing in the scalar potential must be subjected to satisfying the full set of NS-NS Bianchi identities and RR tadpole cancellation conditions.
2.2 Non-geometric Type IIB setup

In this subsection, we present the relevant details about non-geometric type IIB orientifold setup. The allowed orientifold projections can be classified by their action $\mathcal{O}$ on the Kähler form $J$ and the holomorphic three-form $\Omega_3$ of the Calabi-Yau, which can be explicitly given as $[56]$:

$$\mathcal{O} = \begin{cases} \Omega_p \sigma & : \sigma^*(J) = J, \quad \sigma^*(\Omega_3) = \Omega_3, \\ (-)^{F_L} \Omega_p \sigma & : \sigma^*(J) = J, \quad \sigma^*(\Omega_3) = -\Omega_3. \end{cases} \quad (2.60)$$

Note that $\Omega_p$ is the world-sheet parity, $F_L$ is the left-moving space-time fermion number, and $\sigma$ is a holomorphic, isometric involution. The first choice leads to orientifold with $O5/O9$-planes whereas the second choice to $O3/O7$-planes.

As in the type IIA case, we denote the bases of even/odd two-forms as $(\mu_\alpha, \nu_\alpha)$ while four-forms as $(\tilde{\mu}_a, \tilde{\nu}_a)$ where $\alpha \in h^{1,1}(X)$, $a \in h_+^{2,1}(X)$ $[80–85]$. However for the type IIB setups, we denote the bases for the even/odd cohomologies $H_3^{\pm}(X)$ of three-forms as symplectic pairs $(a_K, b^I)$ and $(A_\lambda, B^\Delta)$ respectively, where we fix their normalization as,

$$\int_X a_K \wedge b^I = \delta^K_J, \quad \int_X A_\lambda \wedge B^\Delta = \delta^\lambda_\Delta. \quad (2.61)$$

Here, for the orientifold choice with $O3/O7$-planes, the indices are distributed in the even/odd sector as $K \in \{1, \ldots, h_+^{2,1}(X)\}$ and $\Lambda \in \{0, \ldots, h_+^{2,1}(X)\}$, while for $O5/O9$-planes, one has $K \in \{0, \ldots, h_-^{2,1}(X)\}$ and $\Lambda \in \{1, \ldots, h_-^{2,1}(X)\}$. In this article, our focus will be only in the orientifold involutions leading to the $O3/O7$-planes.

The various field ingredients can be expanded in appropriate bases of the equivariant cohomologies. For example, the Kähler form $J$, the two-forms $B_2$, $C_2$ and the RR four-form $C_4$ can be expanded as

$$J = l^a \mu_\alpha, \quad B_2 = - b^a \nu_\alpha, \quad C_2 = - c^a \nu_\alpha, \quad C_4 = c_a \tilde{\mu}_a + D_2^\lambda \wedge \mu_\alpha + V^K \wedge a_K - V_K \wedge b^K, \quad (2.62)$$

Note that $l^a$ is string-frame two-cycle volume moduli, while $b^a$, $c^a$ and $c_a$ are various axions. Further, $(V^K, V_K)$ forms a dual pair of space-time one-forms and $D_2^\lambda$ is a space-time two-form dual to the scalar field $c_a$. Also, since $\sigma^*$ reflects the holomorphic three-form $\Omega_3$, we have $h^{2,1}(X)$ number of complex structure moduli appearing as complex scalars.

Kähler potential

The generic form of the type IIB Kähler potential can be written as a sum of two pieces motivated from their underlying $\mathcal{N} = 2$ special Kähler and quaternionic structure, and the same is given as $[56]$,

$$K_{\text{IIB}} = K^{(c.s.)} + K^{(Q)}, \quad (2.63)$$

where the $K^{(c.s.)}$ piece depends mainly on the complex structure moduli, while the $K^{(Q)}$ part depends on the volume of the Calabi Yau threefold and the dilaton. For computing the $K^{(c.s.)}$ piece, we consider the involutively-odd holomorphic three-form $\Omega_3 \equiv X^{A} A_{\lambda} - F_{\lambda} B^{\Delta}$ which can be written out using a prepotential of the following form $[77, 80],$

$$F^{(c.s.)} = - \frac{l_{ijk} X^i X^j X^k}{6 A^0} + \frac{1}{2} \tilde{F}_{ij} X^i X^j + \tilde{F}_i X^i A^0 - \frac{i}{2} \tilde{F}_0 (A^0)^2 + i (A^0)^2 F_{\text{inst.}}(U^i), \quad (2.64)$$

$^3$For explicit construction of type-IIB toroidal/CY-orientifold setups with $h^{1,1}(X) \neq 0$, see $[80, 83]$. 

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where \( l_{ijk} \)'s are the classical triple intersection numbers on the mirror (Calabi Yau) threefold and we have defined the inhomogeneous coordinates \((U^i)\) as \( U^i = \frac{x^i}{\mathcal{X}} \) via further setting \( \mathcal{X}^0 = 1 \). Further, the quantities \( \tilde{p}_{ij}, \tilde{p}_i \) and \( \tilde{p}_0 \) are real numbers, and moreover \( \tilde{p}_0 \) is related to the perturbative \((\alpha')^3\)-corrections on the mirror type IIA side as we have argued before, and so is proportional to the Euler characteristic of the mirror Calabi Yau. In general, \( f(U^i) \) has an infinite series of non-perturbative contributions denoted as \( \mathcal{F}_{\text{inst.}}(U^i) \), however assuming the large complex structure limit we will ignore such corrections in the current work. The derivatives of the prepotential needed to explicitly determine the Kähler- and the super-potential terms are given as,

\[
\begin{align*}
\mathcal{F}^{(\text{c.s.})}_0 &= \frac{1}{6} l_{ijk} U^i U^j U^k + \tilde{p}_i U^i - i \tilde{p}_0, \\
\mathcal{F}^{(\text{c.s.})}_1 &= - \frac{1}{2} l_{ijk} U^j U^k + \tilde{p}_{ij} U^i U^j + \tilde{p}_i.
\end{align*}
\] (2.65)

Subsequently, the components of holomorphic three-form \( \Omega_3 \) can be explicitly rewritten as period vectors in terms of complex structure moduli \( U^i \) given as,

\[
\Pi_{\Omega_3} = \begin{pmatrix}
A_0 \\
U^a A_i \\
\left( \frac{1}{2} l_{ijk} U^j U^k - \tilde{p}_{ij} U^i U^j - \tilde{p}_i \right) B^i \\
\left( \frac{1}{2} l_{ijk} U^j U^k + \tilde{p}_{ij} U^i U^j - i \tilde{p}_0 \right) B^0
\end{pmatrix}.
\] (2.66)

Now the complex structure moduli dependent part of the Kähler potential can be simply given as,

\[
K^{(\text{c.s.})} = - \ln \left( -i \int_X \Pi_{\Omega_3} \wedge \overline{\Pi}_{\Omega_3} \right)
\] (2.67)

\[
= - \ln \left[ -i \left( \mathcal{X} A \mathcal{F}^{(\text{c.s.})}_A - \mathcal{X} A \mathcal{F}^{(\text{c.s.})}_{\overline{A}} \right) \right]
= - \ln \left( \frac{4}{3} l_{ijk} U^i U^j U^k + 2 \tilde{p}_0 \right)
= - \ln \left[ -i \frac{l_{ijk}}{6} (U^i - \overline{U}^i) (U^j - \overline{U}^j) (U^k - \overline{U}^k) + 2 \tilde{p}_0 \right],
\]

where we have used saxions/axions of the complex structure moduli via defining \( U^i \) as \( U^i = v^i - i u^i \). For the Kähler potential piece \( K^{(q)} \) which arises from the quaternion sector, we consider the Kähler form expansion \( \mathcal{F} = T^A \mu_A \), where \( \mu_A \) denotes the \((1,1)\)-form before orientifolding, and subsequently one can follow similar approach as was taken for the mirror type IIA case by considering the prepotential of the following form \[57\],

\[
\mathcal{F}^{(q)} = \ell_{ABC} T^A T^B T^C + \frac{1}{2} p_{AB} T^A T^B + p_A T^A T^0 + \frac{1}{2} i p_0 (T^0)^2,
\] (2.68)

where assuming the large volume limit we neglect the non-perturbative effects from the worldsheet instanton correction \[88\]. Now we define a multi-form \( \rho \) using the periods of the prepotential in the following manner,

\[
\rho = 1 + T^A \mu_A - \mathcal{F}^{(q)}_{\overline{A}} \tilde{\mu}^A + (2 \mathcal{F}^{(q)} - t^A \mathcal{F}^{(q)}_{\overline{A}}) \Phi_6
\] (2.69)

Now unlike the type IIA case, one can use a compensator field \( C = e^{-\phi} \) which does not depend on the volume, and further using the RR potential as \( C_{RR} = C_0 + C_2 + C_4 \) we consider a complex multi-form of even degree defined as \[57\],

\[
\Phi_{\text{even}}^{e.e.} = e^{B_2} \wedge C_{0}^{(0)} + i Re(C \rho)
\equiv S 1 - G^{\alpha} \nu_a + T_\alpha \tilde{\mu}^\alpha,
\] (2.70)
where the explicit forms for the above chiral coordinates in eqn. (2.40) are given as,

\[ S = C_0^{(0)} + i e^{-\phi} = c_0 + i s , \quad G^a = c^a + S b^a, \]  

\[ T_\alpha = c_\alpha + \ell_{aab} b^a b^b + \frac{1}{2} c_0 \ell_{aab} b^a b^b - i s \left[ \frac{1}{2} \ell_{\alpha\beta\gamma} t^\beta t^\gamma - \frac{1}{2} \ell_{aab} b^a b^b - p_{aa} b^a - p_a \right], \]

where we have rewritten the dilaton as \( e^{-\phi} = s \) and \( \{ \ell_{\alpha\beta\gamma}, \ell_{aab} \} \) represents the set of triple intersection numbers which survive under the orientifold action [67]. It is worth to note that there is a shift in the coordinates \( T_\alpha \) due to the presence of \( p_{aa} \) and \( p_a \) in the prepotential \( F(q) \), while the other variables remain the same. Now the Kähler potential can be computed in the following steps [87],

\[ K^{(Q)} = -2 \ln \left[ i \int_{CY} \langle \mathcal{C}_\rho, \mathcal{C}_\rho \rangle \right] \]

\[ = -2 \ln \left[ |\mathcal{C}|^2 \left( 2 \left( F(q) - \mathcal{F}(q) \right) - \left( F_a(q) + \mathcal{F}_a(q) \right) (T^a - T^a) \right) \right] \]

\[ = -4 \ln s - 2 \ln \left( V + \frac{p_0}{4} \right), \]  

where the overall internal volume of the CY threefold is written as \( V = \frac{1}{6} \ell_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma \) using the string-frame two-cycle volume moduli. Further, the string-frame \( V \) can be identified with the Einstein-frame volume \( V_E \) via \( V_E = s^{3/2} V \). Note that this \( \alpha' \)-correction in the Kähler potential has been used for naturally realising the LARGE volume scenarios [2]. To summarise, the full type IIB Kähler potential can be given by,

\[ K_{IIB} = - \ln \left( \frac{4}{3} l_{ijk} u^i u^j u^k + 2 \widetilde{p}_0 \right) - 4 \ln s - 2 \ln \left( \frac{1}{6} \ell_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma + \frac{p_0}{4} \right). \]  

Further, in order to compute the Kähler metric and its inverse for the scalar potential computations, one needs to rewrite the dilaton \( s \), the two-cycle volume moduli \( T_\alpha \) and the complex structure saxion moduli \( (u^i) \) in terms of the correct variables \( S, T_\alpha, G^a \) and \( U^i \) which in string-frame are defined as:

\[ U^i = v^i - i u^i, \]

\[ S = c_0 + i s, \]

\[ G^a = (c^a + c_0 b^a) + i s b^a, \]

\[ T_\alpha = \hat{c}_\alpha - i s \left[ \frac{1}{2} \ell_{\alpha\beta\gamma} t^\beta t^\gamma - \frac{1}{2} \ell_{aab} b^a b^b - p_{aa} b^a - p_a \right], \]

where \( \hat{c}_\alpha \) represents an axionic combination given as \( \hat{c}_\alpha = c_\alpha + \ell_{aab} b^a c^b + \frac{1}{2} c_0 \ell_{aab} b^a b^b \).

**Flux superpotential**

It is important to note that in a given setup, all flux-components will not be generically allowed under the full orientifold action \( \mathcal{O} = \Omega_p(-)^F L \sigma \). For example, only geometric flux \( \omega \) and non-geometric flux \( R \) remain invariant under \( \Omega_p(-)^F L \), while the standard fluxes \( (F, H) \) and non-geometric flux \( (Q) \) are anti-invariant [32, 60]. Therefore, under the full orientifold action, we can only have the following flux-components:

\[ F_3 \equiv (F_\Lambda, F^\Lambda), \quad H_3 \equiv (H_\Lambda, H^\Lambda), \quad \omega \equiv (\omega_\Lambda, \omega_\Lambda, \omega_K, \hat{\omega}_K), \quad Q \equiv (Q^{aK}, Q^{a\Lambda}, \hat{Q}^{a\Lambda}, \hat{Q}^{aK}), \quad R \equiv (R_K, R^K). \]  

(2.75)
In order to keep type IIB case distinct from the type IIA case, we define a new twisted differential $\mathcal{D}$ involving the actions from all the NS-NS (non-)geometric fluxes as \[ \mathcal{D} = d - H \wedge \cdot - \omega \triangleleft - Q \triangleright - R \bullet \cdot \] 

(2.76)

The action of operator $\triangleleft, \triangleright$ and $\bullet$ on a $p$-form changes it into a $(p+1)$, $(p-1)$ and $(p-3)$-form respectively, and we have the following flux actions \[ [60], \]

\[
 H \wedge A_\Lambda = - H_\Lambda \Phi_6, \quad H \wedge B^\Lambda = - H^\Lambda \Phi_6, \\
 H \wedge a_K = 0, \quad H \wedge b^K = 0, \quad H \wedge 1 = H = - H^\Lambda A_\Lambda + H_\Lambda B^\Lambda, \\
 \omega \triangleleft A_\Lambda = - \omega_\Lambda \phi^a, \quad \omega \triangleleft B^\Lambda = - \omega^a_\Lambda \phi^a, \quad \omega \triangleleft \nu_a = \omega^a_\Lambda A_\Lambda - \omega_\Lambda B^\Lambda, \\
 \omega \triangleleft a_K = - \omega^{a_K} \phi^a, \quad \omega \triangleleft b^K = - \omega^{b^K} \phi^a, \quad \omega \triangleleft \mu_a = \omega^{a_K} a_K - \omega_\Lambda b^K, \\
 Q \triangleright A_\Lambda = - Q^a_\Lambda \mu_\beta, \quad Q \triangleright B^\Lambda = - Q^a_\Lambda B^\Lambda, \quad Q \triangleright \nu_a = - Q^a_\Lambda A_\Lambda + Q^a_\Lambda B^\Lambda, \\
 Q \triangleright a_K = - Q^a_K \nu_b, \quad Q \triangleright b^K = - Q^a_K B^\Lambda, \quad Q \triangleright \tilde{\nu} = - Q^a_K a_K + Q^a_K b^K, \\
\]

(2.77)

\[
 R \bullet A_\Lambda = 0, \quad R \bullet B^\Lambda = 0, \quad R \bullet a_K = - R_K 1, \quad R \bullet b^K = - R^K 1, \\
 R \bullet \Phi_6 = R^K a_K - R_K b^K .
\]

Using the flux actions given in eqn. (2.77) for the NS-NS fluxes and the expansion of the RR-flux $F_3$ as $F_{RR} = - F^\Lambda A_\Lambda + F_\Lambda B^\Lambda$, one can present the following generic form for the flux superpotential \[ [24, 26, 30, 32], \]

\[
 W_{\text{IIB}} \equiv W^\text{IIB} + W^\text{NS} = - \frac{1}{\sqrt{2}} \int_{X_3} \left[ F_{RR} + \mathcal{D}_c \Phi^\text{even} \right] \wedge \Omega_3 \]

(2.78)

\[
 = \frac{1}{\sqrt{2}} \left( F_\Lambda - S H_\Lambda - G^a \omega_\Lambda - T_A Q^a_\Lambda \right) X^\Lambda \\
 - \frac{1}{\sqrt{2}} \left( F^\Lambda - S H^\Lambda - G^a \omega^\Lambda - T_A Q^a_\Lambda \right) F_\Lambda .
\]

Subsequently, using eqn. (2.65) leads to the following explicit form of the type IIB generalised flux superpotential,

\[
 \sqrt{2} W_{\text{IIB}} = \left[ F_0 + U^i \tilde{F}_i + \frac{1}{2} l_{ijk} U^i U^j U^k - \frac{1}{6} l_{ijk} U^i U^j U^k F^0 - i \tilde{p}_0 F^0 \right] \\
 - S \left[ \tilde{P}_0 + U^i \tilde{P}_i + \frac{1}{2} l_{ijk} U^i U^j H^k - \frac{1}{6} l_{ijk} U^i U^j U^k H^0 - i \tilde{p}_0 H^0 \right] \\
 - G^a \left[ \tilde{z}_{a0} + U^i \tilde{z}_{ai} + \frac{1}{2} l_{ijk} U^i U^j \omega_a^k - \frac{1}{6} l_{ijk} U^i U^j U^k \omega_a^0 - i \tilde{p}_0 \omega_a^0 \right] \\
 - T_A \left[ \tilde{Q}_0 + U^i \tilde{Q}_i + \frac{1}{2} l_{ijk} U^i U^j \hat{Q}^a_k - \frac{1}{6} l_{ijk} U^i U^j U^k \hat{Q}^a_0 - i \tilde{p}_0 \hat{Q}^a_0 \right],
\]

(2.79)
where because of the $\alpha'$-corrections on the mirror side, the complex structure sector is modified such that to induce rational shifts in the usual flux components given as,

$$
\begin{align*}
\bar{F}_0 &= F_0 - \tilde{p}_i F^i, \\
\bar{H}_0 &= H_0 - \tilde{p}_i H^i, \\
\bar{\omega}_{ai} = \omega_{ai} - \tilde{p}_i \omega^i, \\
\bar{Q}_0 &= \hat{Q}_0 - \tilde{p}_i \hat{Q}^{ni}, \\
\bar{Q}_i &= \hat{Q}_i - \tilde{p}_i \hat{Q}^{nj} - \tilde{p}_i \hat{Q}^{ni}.
\end{align*}
$$

\[\text{(2.80)}\]

**Gauge kinetic couplings and the D-term effects**

In the presence of a non-trivial sector of even (2,1)-cohomology, i.e. for $h^2_{\text{ax}}(X) \neq 0$, there are D-term contributions to the four-dimensional scalar potential. Following the strategy of \[60\], the same can be determined via considering the following gauge transformations of RR potentials $C_{RR} = C_0 + C_2 + C_4$,

$$
C_{RR} \rightarrow C_{RR} + D(\gamma K a_K - \gamma K b^K) \\

\begin{align*}
&\ni (C_0 + R_K \gamma K - R^K \gamma_K) - (c^a + Q^a_K \gamma K - Q^a_K \gamma_K) \nu_a \\
&+ (c^a + \hat{\omega}_a K \gamma K - \hat{\omega}_a K \gamma K) \tilde{\mu}_a,
\end{align*}
$$

\[\text{(2.81)}\]

which leads to the following flux-dependent shifts in the variables $S, G^a$ and $T_\alpha$ induced via the respective shifts in the $c_0$, $c^a$ and the $c_\alpha$ axionic components,

$$
\delta S = R_K \gamma K - R^K \gamma_K, \quad \delta G^a = Q^a_K \gamma K - Q^K a_K, \quad \delta T_\alpha = \hat{\omega}_a K \gamma K - \hat{\omega}_a K \gamma K.
$$

\[\text{(2.82)}\]

This leads to the following two D-terms being generated by the gauge transformations,

$$
\begin{align*}
D_K &= i \left[ R_K (\partial S K) + Q^a_K (\partial_a K) + \hat{\omega}_a K (\partial^K a) \right], \\
D^K &= -i \left[ R_K (\partial S K) + Q^K a_K (\partial_a K) + \hat{\omega}_a K (\partial^K a) \right].
\end{align*}
$$

\[\text{(2.83)}\]

Now using the Kähler potential in eqn. \[\text{2.73}\] and the variables given in eqn. \[\text{2.74}\], the Kähler derivatives can be given as,

$$
\begin{align*}
K_S &= \frac{i}{2 s} \left( 1 - \frac{\ell_{aabb} t^a b^b}{2 (V + \frac{p_0}{4})} + \frac{3 p_0}{4 (V + \frac{p_0}{4})} \right) = -K_S, \\
K_{G^a} &= \frac{i \ell_{aabb} t^a b^b}{2 s (V + \frac{p_0}{4})} = -K_{G^a}, \quad K_{T_\alpha} = -\frac{i t^{a}}{2 s (V + \frac{p_0}{4})} = -K_{T_\alpha},
\end{align*}
$$

\[\text{(2.84)}\]

which gives the following two explicit D-terms,

$$
\begin{align*}
D_K &= -\frac{s e^{K(Q)}}{2} \left[ R_K (V + p_0 - \frac{1}{2} \ell_{aabb} t^a b^b) + Q^a_K \ell_{aad} t^a b^d - t^a \hat{\omega}_a K \right], \\
D^K &= \frac{s e^{K(Q)}}{2} \left[ R_K (V + p_0 - \frac{1}{2} \ell_{aabb} t^a b^b) + Q^K a_K \ell_{aad} t^a b^d - t^K \hat{\omega}_a K \right].
\end{align*}
$$

\[\text{(2.85)}\]

Using these results in the $D$-term expression given in eqn. \[\text{2.83}\] leads to the following contributions in the four-dimensional scalar potential \[9\],

$$
V_{\text{D-term}} = \frac{1}{2} D J \left[ \text{Re}(f_{JK}) \right]^{-1} D_K + \frac{1}{2} D^J \left[ \text{Re}(f^{JK}) \right]^{-1} D^K.
$$

\[\text{(2.86)}\]
Here the gauge kinetic couplings for the electric and magnetic components can be computed from the orientifold even-sector of the holomorphic three-form. For that we consider the holomorphic three-form of the $\mathcal{N} = 2$ theory, and after the imposition of the orientifold involution it can be split in the even/odd sectors,

$$\Omega_3 = \Omega_3^{\text{odd}} + \Omega_3^{\text{even}} = \lambda^A A_A - F_\Lambda B^\Lambda + \lambda^K a_K - F_K b^K,$$

which leads to the following electric gauge kinetic coupling from the even-sector,

$$f_{JK} = -\frac{i}{2} F_{JK} |_{\lambda^K = 0}.$$

For the case of compactifications using rigid CYs and the cases of frozen complex structure moduli, the gauge coupling $f_{KJ}$ is just a constant, which otherwise can generically depend on the complex structure moduli $U^i$’s. Moreover, using mirror arguments and the prepotential, one can show that

$$f_{JK} = -\frac{i}{2} \left( \tilde{l}_{iJK} U^i - \tilde{p}_{JK} \right).$$

Here we recall that the index ‘$i$’ runs in odd (2, 1)-cohomology which counts the number of complex structure moduli $U^i$’s while the indices ‘$J$’ and ‘$K$’ run in the even (2, 1)-cohomology. Given that $\tilde{p}_{JK}$’s are real quantities, the same will not appear in the real gauge kinetic couplings, which is denoted as $\text{Re}(f_{JK}) = -\frac{1}{2} \tilde{l}_{iJK} U^i$.

Tadpoles cancellation conditions and Bianchi identities

Generically, there are tadpole terms present due to presence of $O3/O7$-planes, and these can be canceled either by imposing a set of flux constraints or else by adding the counter terms which could arise from the presence of local sources such as (stacks of) $D3/D7$-branes. These effects equivalently provide the following contributions in the effective potential,

$$V^\text{tad}_{\text{IIB}} = \frac{1}{2} \epsilon^{K(Q)} \int X_A \left( \langle \text{Im } \Phi^{\text{even}}_c \rangle , D_{RR} \right),$$

where the multi-form $D_{RR}$ can be expanded using the flux actions in the generalized twisted differential operator given as

$$D_{RR} = (F_\Lambda H^\Lambda - F^\Lambda H_\Lambda) \Phi_6 + (F_\Lambda \omega^\Lambda - F^\Lambda \omega_\Lambda) \tilde{p}^a + (F_\Lambda \hat{Q}^{\alpha\Lambda} - F^\Lambda \hat{Q}^\alpha) \mu_\alpha.$$

In addition, using the definition of $\Phi^{\text{even}}_c$ given in eqn. (2.70), the tadpole term given in the eqn. (2.90) simplifies into the following form,

$$V^\text{tad}_{\text{IIB}} = \frac{1}{2} \epsilon^{K(Q)} \left[ (F_\Lambda H^\Lambda - F^\Lambda H_\Lambda) [\text{Im } S] + (F_\Lambda \omega^\Lambda - F^\Lambda \omega_\Lambda) [\text{Im } G^a] + (F_\Lambda \hat{Q}^{\alpha\Lambda} - F^\Lambda \hat{Q}^\alpha) [\text{Im } T_\alpha] \right].$$

The moduli dynamics of the 4D effective theory is determined by the total scalar potential given as a sum of $F$- and $D$-term contributions,

$$V^\text{tot}_{\text{IIB}} = V^F_{\text{IIB}} + V^D_{\text{IIB}},$$

where the various fluxes appearing in the scalar potential must be subjected to satisfying the full set of NS-NS Bianchi identities and RR tadpole cancellation conditions.
3 Action of the T-duality transformations

In this section we invoke the $T$-duality rules in cohomology formulation by taking some iterative steps. We know that in the fluxless case, the mirror symmetry is present and hence type IIA and type IIB ingredients can be mapped to each other. After including the fluxes, this $T$-duality gets destroyed or restored if appropriate fluxes are included. So our plan to begin with, is to seek for the $T$-duality rules among the various moduli and axions in the fluxless case, and then to look at the superpotentials and $D$-terms to invoke the mapping between the various components of the type IIA and the type IIB fluxes.

Looking at the the two Kähler potentials given in eqn. (2.35) and eqn. (2.73) we observe that they are exchanged under a combined action of following set of transformations:

\[
(z^0)^{-1} \leftrightarrow s, \quad t^a \leftrightarrow u^i, \quad z^\lambda \leftrightarrow t^\alpha, \quad (3.1)
\]

In the above mapping, the quantities on the left side of the equivalence belong to type IIA while the respective ones on the right side belong to the type IIB theory. Moreover it is easy to observe that the complexified variables of type IIA given in eqn. (2.33) and those of type IIB in eqn. (2.74) are exchanged with the mapping details given in table 2.

| IIA  | $N^0$ | $N^k$ | $U_\lambda$ | $T^a$ | $\frac{1}{2} \beta$ | $z^k$ | $z^\lambda$ | $b^a$ | $t^\alpha$ | $\xi^0$ | $\xi^k$ | $\xi_\lambda$ |
|------|-------|-------|-------------|-------|-----------------|------|-------------|------|---------|--------|--------|-----------|
| IIB  | $S$   | $G^a$ | $T_\alpha$  | $U^i$ | $s$             | $b^a$| $t^\alpha$ | $v^i$| $u^i$   | $c_0$  | $c^a + \hat{\ell}_{a\lambda b} c^a b^b$ | + $\frac{1}{2} c_0 \hat{\ell}_{a\lambda b} c^a b^b$ |

Table 2: T-duality transformations for various type IIA and type IIB moduli

3.1 $F$-term contributions

Let us begin by summarising the various flux components which contribute to the effective four-dimensional potential via the $F$-term contributions. These are collected as:

**Type IIA** :

\[
\text{RR flux} \equiv (F_6 : e_0, F_4 : e_\alpha, F_2 : m^a, F_0 : m_0),
\]

\[
\text{NS flux} \equiv \left( H_0, H_k, H_\lambda, w_a, w_\alpha, w_\lambda, Q^a_0, Q^a_i, Q^{\alpha \lambda}, R_0, R_\alpha, R^\lambda \right),
\]

**Type IIB** :

\[
\text{RR flux} \equiv (F_3 : F_0, F_i, F^i, F^0),
\]

\[
\text{NS flux} \equiv \left( H_0, H_i, H^i, H^0, \omega_a, \omega_\alpha, \omega_\lambda, \omega^0_0, \hat{Q}^\alpha_0, \hat{Q}^\alpha_i, \hat{Q}^{\alpha \lambda}, \hat{Q}^{\alpha 0}, \hat{Q}^{\alpha i} \right).
\]

Now, it is interesting to observe that the explicit expressions of the type IIA and type IIB superpotentials as given in eqns. (2.34) and (2.79) respectively, are exchanged under a combined action of a set of $T$-duality transformations for the fluxes given in table 3 and table 4.
IIA | $H_0$ | $H_k$ | $H^\lambda$ | $w_{a0}$ | $w_{aK}$ | $Q^a_0$ | $Q^a_k$ | $R_0$ | $R_k$ | $R^\lambda$
---|---|---|---|---|---|---|---|---|---|---
IIB | $H_0$ | $\omega_{a0}$ | $\hat{Q}^a_0$ | $H_i$ | $\omega_{ai}$ | $\hat{Q}^a_i$ | $H^i$ | $\omega^i_0$ | $-H^0$ | $-\omega^0_a$ | $-\hat{Q}^a_0$

Table 3: T-duality transformations among the NS-NS fluxes appearing in the $F$-term effects

| IIA | $e_0$ | $e_a$ | $m^a$ | $m^0$
---|---|---|---|---
| *IIB* | $F_0$ | $F_i$ | $F^i$ | $-F^0$

Table 4: T-duality transformations among the RR-flux components

3.2 $D$-term contributions

In string-frame, the $D$-terms in both the (type IIA and type IIB) theories can be given as below,

**IIA**:

$$D_\alpha = (z^0)^{-1} e^{\frac{k_0}{2}} \left[ (U + \tilde{p}_0 - \frac{1}{2} \hat{k}_{\lambda k m} z^\lambda z^k z^m) \hat{w}^0_\alpha + \hat{k}_{\lambda k m} z^\lambda z^m \hat{w}^k_\alpha + z^\lambda \hat{w}^0_\alpha \right].$$  \hspace{1cm} (3.3)

$$D^\alpha = -(z^0)^{-1} e^{\frac{k_0}{2}} \left[ (U + \tilde{p}_0 - \frac{1}{2} \hat{k}_{\lambda k m} z^\lambda z^k z^m) \hat{Q}^0_\alpha + \hat{k}_{\lambda k m} z^\lambda z^m \hat{Q}^k_\alpha + z^\lambda \hat{Q}^0_\alpha \right].$$

**IIB**:

$$D_K = -s e^{\frac{k_0}{2}} \left[ R_K (V + p_0 - \frac{1}{2} \hat{\ell}_{aab} t^a b^a b^b) + Q^a_K \hat{\ell}_{aad} t^a b^c - t^a \hat{\omega}_a K \right],$$

$$D^K = s e^{\frac{k_0}{2}} \left[ R^K (V + p_0 - \frac{1}{2} \hat{\ell}_{aab} t^a b^a b^b) + Q^a_K \hat{\ell}_{aad} t^a b^c - t^a \hat{\omega}_a K \right].$$

Recalling that $\tilde{p}_0 \leftrightarrow p_0$ and $V \leftrightarrow U$ under the mirror symmetry, and subsequently after using the $T$-duality transformation listed for the moduli and the axions given in table 2, we find the $T$-duality transformation of $D$-term fluxes as presented in table 5.

| IIA | $\hat{Q}^a_\lambda$ | $\hat{w}_{a\lambda}$ | $\hat{Q}^a_K$ | $\hat{w}_{aK}$ | $\hat{Q}^a_0$ | $\hat{w}_{a0}$
---|---|---|---|---|---|---
| *IIB* | $\hat{w}_a K$ | $\hat{w}_a K$ | $-Q^a K$ | $-Q^a K$ | $-R^K$ | $-R_K$

Table 5: T-duality transformations among the NS-NS fluxes appearing in the $D$-term effects
3.3 Tadpole conditions

Now we compare the various tadpole terms generated in the type IIA and type IIB theories, which can be also compensated by appropriately adding the local effects from various $D_p$-brane and $O_p$-planes. In particular, for our current interest in this work, the tadpoles in the type IIA side can be compensated via the $D6/O6$ effects while the tadpoles in type IIB side can be compensated via $D3/O3$ and $D7/O7$ effects. These are given as,

\[
V_{\text{IIA}}^{\text{tad}} = \frac{1}{2} e^{K(q)} \left[ \left( \text{Im } N_k \right) \left( H_k m_0 - \omega_{ak} m^a + Q^a_k e_a - R_k e_0 \right) + \left( \text{Im } U_\lambda \right) \left( H^\lambda m_0 - \omega_\lambda m^a + Q^a_\lambda e_a - R^\lambda e_0 \right) \right],
\]

\[
V_{\text{IIB}}^{\text{tad}} = \frac{1}{2} e^{K(q)} \left[ \left( F_\Lambda H^\Lambda - F^\Lambda H_\Lambda \right) [Im S] + \left( F_\Lambda \omega_\alpha - F^\Lambda \omega_\alpha^\Lambda \right) Im(G^a) \right.

+ \left. \left( F^a_\Lambda \hat{Q}^\alpha - F^\Lambda \hat{Q}^{a\alpha} \right) Im(T_\alpha) \right].
\]

Now given that $K(q) \leftrightarrow K(q')$, $N^0 \leftrightarrow S$, $N^k \leftrightarrow G^a$, $U_\lambda \leftrightarrow T_\alpha$ under the explicit T-duality rules, it is simple to observe that the type IIA and type IIB tadpole terms are exchanged under the T-dual flux transformations given in tables 3 and 4.

3.4 Bianchi identities

As we have already established the exchange symmetry of the $F$-terms and the $D$-terms, now we check how our T-duality rules are applied to the flux constraints in the Bianchi identities of the two sides. This is necessary to prove the claim for the exchange symmetry between the actual effective potentials of the two type II theories, in the sense that if some pieces are killed by the Bianchi identities on one side then that should also be the case on the mirror dual side.

**Five classes of Bianchi identities for Type IIA**:  

Using the flux actions given in eqn. (2.35), the following five classes of the NS-NS Bianchi identities are obtained via demanding the nilpotency of the twisted differential operator $D$ as defined in eqn. (2.37) via imposing $D^2 = 0$ on the various harmonic forms,

(I) \hspace{1cm} H^\lambda \hat{w}_\alpha \hat{w}_\lambda = H_k \hat{w}_\alpha \hat{w}_k,  

(II) \hspace{1cm} H^\lambda \hat{Q}^\alpha = H_k \hat{Q}^\alpha k, \quad w_\alpha \hat{w}_\alpha = w_\alpha = w_\alpha \hat{w}_\alpha,  

(III) \hspace{1cm} \hat{Q}^\alpha \hat{w}_\alpha \hat{w}_\alpha = \hat{Q}^\alpha \hat{w}_\alpha \hat{w}_\alpha, \quad \hat{Q}^\alpha \hat{w}_\alpha \hat{w}_\alpha = \hat{Q}^\alpha \hat{w}_\alpha \hat{w}_\alpha,  

\hat{w}_\alpha \hat{w}_\alpha \hat{Q}^\alpha \hat{w}_\alpha \hat{Q}^\alpha \hat{w}_\alpha = \hat{Q}^\alpha \hat{w}_\alpha \hat{Q}^\alpha \hat{w}_\alpha, \quad \hat{w}_\alpha \hat{w}_\alpha \hat{Q}^\alpha \hat{w}_\alpha \hat{Q}^\alpha \hat{w}_\alpha = \hat{Q}^\alpha \hat{w}_\alpha \hat{Q}^\alpha \hat{w}_\alpha,  

H_{\left[ k R_{\bar{k}} \right]} + Q_{\left[ k w_{\bar{k}} \right]} = 0, \quad H^\lambda R_\rho + Q_{\left[ a \lambda w_\rho \right]} = 0,  

R^\lambda H_k - H^\lambda R_k + w_\alpha \Lambda Q^a_{\bar{\alpha}} - Q^\lambda \hat{w}_\alpha = 0,  

(IV) \hspace{1cm} R^\lambda \hat{w}_\alpha \hat{w}_\alpha \hat{w}_\alpha \hat{w}_\alpha = R_k \hat{w}_\alpha \hat{w}_\alpha \hat{w}_\alpha \hat{w}_\alpha,  

(V) \hspace{1cm} R^\lambda \hat{Q}^\alpha = R_k \hat{Q}^\alpha k.

These identities suggest that if one considers the anti-holomorphic involution such that the even $(1,1)$-cohomology is trivial, which is very often the case, then there will be no $D$-terms and the

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only Bianchi identities to worry about would be given as below,
\[ R^A H^A_k - H^A R^A_k + w_a^A Q^A_{k} - Q^a_{[k} w^A_{a]k} = 0, \]
\[ H^A [k R^A_{k'}] + Q^a_{[k} w^A_{a]k'} = 0, \quad H^A |R^A_{\rho|} + Q^a_{[\rho} w^A_{a]\rho} = 0. \]  

5. **Five classes of Bianchi identities for Type IIB:**

Similarly, using the flux actions given in eqn. (2.77), the following five classes of the NS-NS Bianchi identities are obtained via imposing \( D^2 = 0 \) on the various harmonic forms \[60\],

\[ (I). \quad H_A \omega^A_a = H^A \omega_{Aa}, \]
\[ (II). \quad H^A \dot{Q}^A_\alpha = H_A \dot{Q}^{A\alpha}, \quad \omega^A_\alpha \omega^A_{\beta \gamma} = \omega^A_\alpha \omega^A_{\beta \gamma}, \quad \omega^A_\beta \omega^A_{\alpha \gamma} = \omega^A_\beta \omega^A_{\alpha \gamma}, \]
\[ (III). \quad \omega^A_\alpha \dot{Q}^{A\alpha} = \omega^A_\alpha \dot{Q}^{A\alpha}, \quad Q^{A\alpha} \omega^A_{\alpha K} = Q^{A\alpha} \dot{Q}^{A\alpha K}, \]
\[ (IV). \quad R^A \dot{Q}^{A\alpha K} = R^A \dot{Q}^{A\alpha K} = 0, \]
\[ (V). \quad R^A K Q^{A\alpha} - R^A K Q^{A\alpha} = 0. \]

The above set of type IIB Bianchi identities suggests that if one chooses the holomorphic involution such that the even \((2,1)\)-cohomology is trivial, then only following Bianchi identities remain non-trivial,

\[ H_A \omega^A_\alpha = H^A \omega_{A\alpha}, \quad H^A \dot{Q}^{A\alpha} = H_A \dot{Q}^{A\alpha}, \quad \omega^A_\alpha \omega^A_{\beta \gamma} = \omega^A_\alpha \omega^A_{\beta \gamma}, \]
\[ \omega^A_\beta \omega^A_{\alpha \gamma} = \omega^A_\beta \omega^A_{\alpha \gamma}. \]  

In such a situation, there will be no \( D \)-term generated as all the fluxes with \( \{J, K\} \in h_+^{2,1} \) indices are projected out. Moreover, on top of this if the holomorphic involution is chosen to result in a trivial odd \((1,1)\)-cohomology, which corresponds to the situation of the absence of odd moduli \( G^a \) and is also very often studied case, then there are only two types of the Bianchi identities to worry about as given below,

\[ H^A \dot{Q}^{A\alpha} = H_A \dot{Q}^{A\alpha}, \quad \dot{Q}^{A\alpha} \dot{Q}^{A\beta} = \dot{Q}^{A\beta} \dot{Q}^{A\alpha}. \]

Using the \( T \)-duality transformations among the various NS-NS fluxes as listed in table \[\text{III}\] and table \[\text{IV}\] we find that indeed the 14 Bianchi identities on type IIA side are precisely mapped on to the 14 Bianchi identities on the type IIB side, and vice-versa. However, there is a quite significant mixing across the five classes of identities on the two sides. For example, the identity \( H^A \dot{Q}^{A\alpha} = H_A \dot{Q}^{A\alpha} \) corresponding to class (II) in the type IIB side, produces the identity \( (R^A H^A - H^A R^A + w_a^A Q^a_0 - Q^a_{[\alpha} w^A_{\alpha]} = 0 \) which corresponds to the class (III) on the type IIA side. To illustrate these features, we have presented a one-to-one correspondence among all the identities in table \[\text{II}\] of the appendix \[\text{A}\].

4. **Exchanging the scalar potentials under \( T \)-duality:**

In this section our first goal is to present a new set of axionic flux polynomials for both the type IIA and the type IIB theories which would include all the axionic fields appearing in those respective theories, and without having any saxions involved. This will be subsequently used to present the two scalar potentials completely in terms of these axionic flux polynomials and the moduli space metrics on the two theories.
4.1 Axionic flux polynomials

Type IIA

A careful look at the type IIA superpotential given in eqn. (2.44) and the D-terms given in eqn. (2.55), suggests to define some axionic flux combinations which we call as “axionic flux polynomials”, that can be useful for rewriting the generic complicated scalar potential with explicit dependence on the saxions/axions within a few lines. These axionic flux polynomials can be given by the following expressions,

\begin{align}
  f_0 &= G_0 - \xi^k H_k - \xi^\lambda H^\lambda, \\
  f_a &= G_a - \xi^k U_{ak} - \xi^\lambda U_{a^\lambda}, \\
  f^a &= G^a - \xi^k Q^a_k - \xi^\lambda Q^{a^\lambda}, \\
  f^0 &= G^0 - \xi^k R_k - \xi^\lambda R^\lambda, \\
  h_0 &= H_0 + H_k z^k + \frac{1}{2} \hat{h}_{\lambda mn} z^m z^n H^\lambda, \\
  h_a &= U_{a0} + U_{ak} z^k + \frac{1}{2} \hat{h}_{\lambda mn} z^m z^n U_{a^\lambda}, \\
  h^a &= Q^a_0 + Q^a_k z^k + \frac{1}{2} \hat{h}_{\lambda mn} z^m z^n Q^{a^\lambda}, \\
  h^0 &= R_{00} + R_k z^k + \frac{1}{2} \hat{h}_{\lambda mn} z^m z^n R^\lambda, \\
  h_{0k} &= H_k + \hat{h}_{\lambda kn} z^n H^\lambda, \\
  h_{ak} &= U_{ak} + \hat{h}_{\lambda kn} z^n U_{a^\lambda}, \\
  h^a_k &= Q^a_0 + \hat{h}_{\lambda kn} z^n Q^{a^\lambda}, \\
  h^0_k &= R_0 + R_k z^k + \frac{1}{2} \hat{h}_{\lambda kn} z^m z^n R^\lambda, \\
  h^\lambda_0 &= H^\lambda, \\
  h^\lambda_a &= U^\lambda_a, \\
  h^0 &= Q^{a^\lambda}, \\
  h^0_0 &= R^\lambda, \\
  \hat{h}_{\alpha^\lambda} &= \hat{h}_{\alpha^\lambda}, \\
  \hat{h}_a^\lambda &= \hat{h}_a^\lambda, \\
  \hat{h}_a^0 &= \hat{h}_a^0, \\
  \hat{h}_{\alpha^0} &= \hat{h}_{\alpha^0},
\end{align}

where the intermediate axionic flux polynomials appearing in the above eqn. (4.1) are given as,

\begin{align}
  G_0 &= \xi e_0 + b^a e_a + \frac{1}{2} \kappa_{abc} b^a b^b m^{c}, \\
  G_a &= \xi e_a + \kappa_{abc} b^b m^{c} + \frac{1}{2} \kappa_{abc} b^a b^b m_{0}, \\
  G^a &= m^a + m_{0} e^a, \\
  G^0 &= m_{0}, \\
  H_k &= \xi \hat{H}_k + \hat{\omega}_{ak} b^a + \frac{1}{2} \kappa_{abc} b^a b^b Q^c_{k} + \frac{1}{6} \kappa_{abc} b^a b^b b^c R_{k}, \\
  U_{ak} &= \xi \hat{U}_{ak} + \kappa_{abc} b^b Q^c_{k} + \frac{1}{2} \kappa_{abc} b^b b^c R_{k}, \\
  Q^a_k &= Q^a_0 + b^a R_k, \\
  R_k &= R_k.
\end{align}
\begin{align*}
\mathcal{H}^\lambda &= \Pi^\lambda + \pi_a^\lambda b^a + \frac{1}{2} \kappa_{abc} b^b b^c Q^{a\lambda} + \frac{1}{6} \kappa_{abc} b^a b^b R^\lambda, \\
\mathcal{U}_a^\lambda &= \pi_a^\lambda + \kappa_{abc} b^b Q^c^{a\lambda} + \frac{1}{2} \kappa_{abc} b^b b^c R^\lambda, \\
Q^{a\lambda} &= b^a R^\lambda, \\
R^\lambda &= R^\lambda,
\end{align*}

\begin{align*}
\hat{\mathcal{O}}_{a\lambda} &= \hat{w}_{a\lambda} + \hat{k}_{\lambda km} z^m \hat{w}_{a\lambda}^k - \frac{1}{2} \hat{k}_{\lambda km} z^k z^m \hat{w}_{a\lambda}^0, \\
\hat{\mathcal{O}}_{a}^k &= \hat{w}_{a}^k - z^k \hat{w}_{a}^0, \\
\hat{\mathcal{O}}_a^0 &= \hat{w}_a^0, \\
\hat{Q}^{a\lambda} &= \hat{Q}^a^{a\lambda} + \hat{k}_{\lambda km} z^m \hat{Q}^{a\lambda k} - \frac{1}{2} \hat{k}_{\lambda km} z^k z^m \hat{Q}^{a\lambda 0}, \\
\hat{Q}^{a}^k &= \hat{Q}^{a}^{a} k - z^k \hat{Q}^{a}^{a 0}, \\
\hat{Q}^0 &= \hat{Q}^0.
\end{align*}

Here we have utilized the shifted fluxes as defined in eqn. \eqref{2.45} due to the inclusion of \(\alpha\)-corrections in the Kähler moduli dependent prepotential. Some (partial) appearance of the type IIA axionic flux polynomials in eqn. \eqref{2.2} has been seen before in \cite{58, 63, 69}. In addition, the generalized RR flux polynomials defined as \(G^0, G_a, G_a^0\) have been used in \cite{75, 89–92} in the absence of (non-)geometric flux.

**Type IIB**

Similarly a careful look at the type IIB superpotential given in eqn. \eqref{2.79} and the \(D\)-terms given in eqn. \eqref{2.85}, suggests to define the following axionic flux polynomials, which would be directly in one-to-one correspondence with the \(T\)-dual fluxes on the type IIA side as we will see in a moment,

\begin{align*}
f_0 &= F_0 + v^i F_i + \frac{1}{2} l_{ijk} v^j v^k F^i - \frac{1}{6} l_{ijk} v^j v^k F^0, \\
f_i &= F_i + l_{ijk} v^j F^k - \frac{1}{2} l_{ijk} v^j v^k F^0, \\
f^i &= F^i - v^i F^0, \\
f^0 &= -F^0, \\
h_0 &= H_0 + v^i H_i + \frac{1}{2} l_{ijk} v^j v^k H^i - \frac{1}{6} l_{ijk} v^j v^k H^0, \\
h_i &= H_i + l_{ijk} v^j H^k - \frac{1}{2} l_{ijk} v^j v^k H^0, \\
h^i &= H^i - v^i H^0, \\
h^0 &= -H^0, \\
h_{a0} &= \hat{\mathcal{U}}_{a0} + v^i \hat{\mathcal{U}}_{ai} + \frac{1}{2} l_{ijk} v^j v^k \hat{\mathcal{U}}_{a}^i - \frac{1}{6} l_{ijk} v^j v^k \hat{\mathcal{U}}_{a}^0, \\
h_{ai} &= \hat{\mathcal{U}}_{ai} + l_{ijk} v^j \hat{\mathcal{U}}_{a}^k - \frac{1}{2} l_{ijk} v^j v^k \hat{\mathcal{U}}_{a}^0, \\
h_{a}^i &= \hat{\mathcal{U}}_{a}^i - v^i \hat{\mathcal{U}}_{a}^0, \\
h_{a}^0 &= -\hat{\mathcal{U}}_{a}^0,
\end{align*}
\[ h_{a0} = \hat{Q}^a_0 + v^i \hat{Q}^a_i + \frac{1}{2} l_{ijk} v^j v^k \hat{Q}^{ai} - \frac{1}{6} l_{ijk} v^j v^k \hat{Q}^{a0}, \]
\[ h_{ai} = \hat{Q}^a_i + l_{ijk} v^j \hat{Q}^{ak} - \frac{1}{2} l_{ijk} v^j v^k \hat{Q}^{a0}, \]
\[ h_{ai} = \hat{Q}^a_i - v^i \hat{Q}^{a0}, \]
\[ h_{a0} = -\hat{Q}^{a0}. \]

\[ \hat{h}_{aK} = \hat{\Omega}_{aK}, \]
\[ \hat{h}_{aK} = \hat{\Omega}_{aK}, \]
\[ \hat{h}_{K0} = -\hat{R}_K, \]
\[ \hat{h}_{K0} = -\hat{R}_K, \]

where the intermediate flux polynomials appearing in the above eqn. (4.3) are given as,

**F term fluxes**: (4.4)

\[ \mathcal{F}_\Lambda = \mathcal{F}_\Lambda - \hat{\omega}_{ab} c^a - \hat{Q}_\Lambda \left( c_a + \hat{\ell}_{ab} c^a b^b \right) - c_0 \mathcal{H}_\Lambda, \]
\[ \mathcal{F}_\Lambda = \mathcal{F}_\Lambda - \hat{\omega}_{ab} c^a - \hat{Q}_\Lambda \left( c_a + \hat{\ell}_{ab} c^a b^b \right) - c_0 \mathcal{H}_\Lambda, \]
\[ \mathcal{H}_\Lambda = \mathcal{H}_\Lambda + \hat{\omega}_{ab} b^a + \frac{1}{2} \hat{\ell}_{ab} b^a b^b \hat{Q}_\Lambda, \]
\[ \hat{\Omega}_{ab} = \hat{\omega}_{ab} + \hat{Q}_\Lambda \hat{\ell}_{ab} b^b, \]
\[ \hat{Q}_\Lambda = \hat{Q}_\Lambda, \]
\[ \mathcal{H}_\Lambda = \mathcal{H}_\Lambda + \hat{\omega}_{ab} b^a + \frac{1}{2} \hat{\ell}_{ab} b^a b^b \hat{Q}_\Lambda, \]
\[ \hat{\Omega}_{ab} = \hat{\omega}_{ab} + \hat{Q}_\Lambda \hat{\ell}_{ab} b^b, \]
\[ \hat{Q}_\Lambda = \hat{Q}_\Lambda, \]

**D term fluxes**: 

\[ \hat{\Omega}_{aK} = \hat{\omega}_{aK} - Q^a_K \hat{\ell}_{ab} b^b + \frac{1}{2} \hat{\ell}_{ab} b^a b^b R_K, \]
\[ Q^a_K = Q^a_K + R_K b^a, \]
\[ R_K = R_K, \]
\[ \hat{\Omega}_{aK} = \hat{\omega}_{aK} - Q^a_K \hat{\ell}_{ab} b^b + \frac{1}{2} \hat{\ell}_{ab} b^a b^b R_K, \]
\[ Q^a_K = Q^a_K + R_K b^a, \]
\[ R_K = R_K. \]

Note that we have utilized the shifted fluxes with bars at some places which are defined in eqn. (2.80). Recall that the axionic flux polynomials in eqn. (4.3) have been invoked as some peculiar flux combinations called as new generalized axionic flux polynomials by considering a deep investigation of the flux superpotential and the D-terms in the type IIB setting [64, 65]. Moreover, it is interesting to note that these flux polynomials are also useful in the sense that they collectively satisfy the generic Bianchi identities as presented in table 12.
It is worth to recall that all the axionic flux polynomials given in eqns. (4.1)-(4.2) for type IIA, and those given in eqns. (4.3)-(4.4) for type IIB case, involve fluxes and all the axions without having any dependence on the saxionic moduli. It is a tedious but straightforward computation to convince that under the $T$-duality transformations, the various axionic flux polynomials are exchanged as presented in Table 6.

| IIA | $f_0$ | $f_a$ | $f^a$ | $f^0$ | $h_0$ | $h_k$ | $h^k$ | $h^0$ | $h_{a0}$ | $h_{ak}$ | $h_{a^0}$ | $h_{a^k}$ | $h_{a^0}$ | $h_{a^k}$ | $h_{a^0}$ | $h_{a^k}$ | $h_{a^k}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|--------|---------|-----------|---------|---------|---------|---------|---------|---------|
| IIB | $f_0$ | $f_i$ | $f_i$ | $f^0$ | $h_0$ | $h_i$ | $h^i$ | $h^0$ | $h_{a0}$ | $h_{ai}$ | $h_{a^0}$ | $h_{a^i}$ | $h_{a^0}$ | $h_{a^i}$ | $h_{a^0}$ | $h_{a^i}$ | $h_{a^i}$ |

Table 6: Axionic flux polynomials under $T$-duality.

In order to prove that the axionic flux polynomials transform under $T$-duality as per the rules given in Table 6 one can use the following type IIB to type IIA transformations at the intermediate stage of computations,

\[
\begin{align*}
\mathbb{H}_0 & \rightarrow \mathbb{H}_0 + \mathbb{H}_k z^k + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \mathbb{H}^\lambda, \\
\mathbb{U}_{a0} & \rightarrow \mathbb{H}_k + \mathbb{H}^\lambda \hat{k}_{\lambda kn} z^n, \\
\hat{Q}^{a0} & \rightarrow \mathbb{H}^\lambda, \\
\mathbb{H}_i & \rightarrow \mathbb{W}_{a0} + \mathbb{W}_{ak} z^k + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \mathbb{W}_a^\lambda, \\
\mathbb{U}_{ai} & \rightarrow \mathbb{W}_{ak} + \mathbb{W}_a^\lambda \hat{k}_{\lambda kn} z^n, \\
\hat{Q}^{ai} & \rightarrow \mathbb{W}_a^\lambda, \\
\mathbb{H}^i & \rightarrow Q^a_0 + Q^a_k z^k + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n Q^\lambda, \\
\mathbb{U}_a^i & \rightarrow Q^a_k + Q^\lambda \hat{k}_{\lambda kn} z^n, \\
\hat{Q}^{ai} & \rightarrow Q^\lambda, \\
\mathbb{H}^0 & \rightarrow -R_0 - R_k z^k - \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n R^\lambda, \\
\mathbb{U}_a^0 & \rightarrow -R_k - R^\lambda \hat{k}_{\lambda kn} z^n, \\
\hat{Q}^{a0} & \rightarrow -R^\lambda, \\
F_0 & \rightarrow \mathbb{F}_0 - (\xi^k \mathbb{F}_k + \xi^\lambda \mathbb{F}^\lambda), \\
F_i & \rightarrow \mathbb{F}_a - (\xi^k \mathbb{F}_{ak} + \xi^\lambda \mathbb{F}_a^\lambda), \\
F^i & \rightarrow m^a - (\xi^k Q^a_k + \xi^\lambda Q^\lambda), \\
F^0 & \rightarrow -m^0 + (\xi^k R_k + \xi^\lambda R^\lambda), 
\end{align*}
\]
and the transformations for the D-term flux polynomials are given as:

\[
\hat{\Omega}_{aK} \rightarrow \hat{w}_{\alpha \lambda} + \hat{w}_{\alpha k} \hat{k}_{\lambda km} z^m - \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \hat{w}_{\alpha 0},
\]

\[
\hat{R}_K \rightarrow - \hat{w}_{\alpha 0}
\]

(4.6)

\[
\hat{\Omega}_a K \rightarrow \hat{Q}^a_{\lambda} + \hat{Q}^{nk} \hat{k}_{\lambda km} z^m - \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \hat{Q}^a_{0},
\]

\[
\hat{R}^K \rightarrow - \hat{Q}^a_{0}.
\]

Note that fluxes with bar on top are the shifted fluxes as defined in eqns. (2.40) and (2.80).

4.2 Scalar potentials

For the scalar potential computations we mainly need to focus on rewriting the \( F \)-term contributions arising from the type IIA and type IIB superpotentials as presented in eqn. (2.44) and eqn. (2.79) respectively. Also for our scalar potential computations we will ignore the effects of all the \( p_0 \)’s which depend on the Euler characteristics of the CY and its mirror, as it unnecessarily creates complexities in the various expressions in the respective scalar potentials, making it hard to enjoy the simple observations and its possibly easy utilities. However we will keep on considering the prepotential terms with coefficients \( p_{ab}, p_a, \tilde{p}_{ij} \) etc., which are linear and quadratic in the chiral variables (involving the saxions of the Kähler and the complex-structure moduli), and so may remain to be relevant in some regime of the moduli space even after imposing the large volume and large complex structure limit. In this limit, one has some estimates for the pieces with \( \chi(CY) \) given as,

\[
\mathcal{V} \gg \frac{p_0}{4} = - \frac{\zeta[3] \chi(CY)}{32 \pi^3} \propto 10^{-3} \chi(CY),
\]

\[
\mathcal{U} \gg \frac{\tilde{p}_0}{4} = - \frac{\zeta[3] \chi(C\bar{Y})}{32 \pi^3} \propto 10^{-3} \chi(C\bar{Y}).
\]

Therefore, for a trustworthy model building within a valid effective field theory description where one anyway demands \( \mathcal{V} \gg 1 \) and \( \mathcal{U} \gg 1 \), the above assumption we make is quite automatically justified, and it is very likely that the correction with \( p_0 \)’s will not be effective up to quite large value of the Euler characteristics of the CY and its mirror. Moreover \( p_0 \) appears at \((\alpha')^3\) order in type IIA, and we are keeping corrections till \((\alpha')^2\) through \( p_{ab} \) and \( p_a \), and therefore our assumption should be fairly justified. Given that all moduli should be present in the generic non-geometric scalar potential, and so it is natural to expect that all of them (at least the saxionic ones) would be dynamically fixed; in cases otherwise, the \((\alpha')^3\)-effects with \( \chi(CY) \) may get relevant at some sub-leading order.
Type IIB

With some tedious but conceptually straightforward computations using the axionic flux polynomials given in eqns. (4.3-4.4) and following the strategy of [64, 67, 68], the total scalar potential generated as a sum of the $F$-term and $D$-term contributions for the type IIB orientifold compactifications (in string frame) can be written as,

$$V_{\text{total}} = V_{\text{IB}} + V_{\text{IB}}^D = V_{\text{IB}}^{RR} + V_{\text{IB}}^{\text{NS}} + V_{\text{IB}}^{\text{loc}} + V_{\text{IB}}^{\text{D}},$$

where the four pieces are given as follows,

- **$V_{\text{IB}}^{\text{RR}}$**

$$V_{\text{IB}}^{\text{RR}} = \frac{\epsilon^{\alpha \beta}}{4 \sqrt{2} \mathcal{U}} \left[ f_0^2 + \mathcal{U} f^i G_{ij} f^j + \mathcal{U} f_i G^{ij} f_j + \mathcal{U}^2 (f^0)^2 \right],$$

- **$V_{\text{IB}}^{\text{NS}}$**

$$V_{\text{IB}}^{\text{NS}} = \frac{\epsilon^{\alpha \beta}}{4 \sqrt{2} \mathcal{U}} \left[ h_0^2 + \mathcal{U} h^i G_{ij} h^j + \mathcal{U} h_i G^{ij} h_j + \mathcal{U}^2 (h^0)^2 \right] + \mathcal{V} G^{\alpha \beta} \left( h_{a0} h_{b0} + \frac{l_i l_j}{4} h^i_a h^j_b + h_{ai} h_{bj} u^i u^j + \mathcal{U}^2 h^0_a h^0_b \right)$$

- **$V_{\text{IB}}^{\text{loc}}$**

$$V_{\text{IB}}^{\text{loc}} = \frac{\epsilon^{\alpha \beta} \ell_\alpha}{2 \sqrt{2}} \left[ (f^0 h_{a0} - f^i h^i_a - f_0 h^0_i) + (f^0 h_{a0} - f^i h^i_a - f_0 h^0_i) \frac{\ell_\alpha}{2} \right],$$

- **$V_{\text{IB}}^{\text{D}}$**

$$V_{\text{IB}}^{\text{D}} = \frac{\epsilon^{\alpha \beta}}{4 \sqrt{2} \mathcal{U}} \left( (\mathcal{V} \hat{h}^{J0} + t^a \hat{h}_{aJ}) G^{JK} (\mathcal{V} \hat{h}_K^0 + t^b \hat{h}_{bK}) \right)$$

where $\mathcal{U} = \frac{1}{6} \ell_{\alpha \beta \gamma} t^\alpha t^\beta t^\gamma$, $\mathcal{V} = \frac{1}{6} l_{ijk} u^i u^j u^k$ etc. as shorthand notations, we have the following form of the moduli space metrics,

$$G_{ij} = \frac{l_i l_j - 4 \mathcal{U} \ell_{ij}}{4 \mathcal{U}}, \quad G_{ij}^{\alpha} = \frac{2 u^i u^j - 4 \mathcal{U} \ell_{ij}}{4 \mathcal{U}} - i_{JK}, \quad G^{\alpha} = 2 t^\alpha t^\beta - 4 \mathcal{V} \ell^{\alpha \beta} - i_{ab}, \quad G_{ab} = - \ell_{ab}. \quad (4.10)$$
Type IIA

Although, it is equally tedious for type IIA case to compute the scalar potential from the flux superpotential, however one can show that using our axionic flux polynomials given in eqns. (1.1-1.2) and following the strategy of [69], the the total scalar potential for the type IIA orientifold compactifications (in string frame) can be written as,

\[ V_{\text{IIA}}^\text{tot} = V_{\text{IIA}}^R + V_{\text{IIA}}^D = V_{\text{IIA}}^R + V_{\text{IIA}}^\text{NS} + V_{\text{IIA}}^\text{loc} + V_{\text{IIA}}^\text{D} , \] (4.11)

where the four pieces can be explicitly given as follows,

\[ V_{\text{IIA}}^R = \frac{e^{4D_{4d}}}{4V} \left[ f_0^2 - \left( f^a \tilde{G}_{ab} f^b + \nabla f_a \tilde{g}_{ab} f_b + \gamma^2 (f^0)^2 \right) \right] , \] (4.12)

\[ V_{\text{IIA}}^\text{NS} = \frac{e^{2D_{4d}}}{4U V} \left[ h_{ab}^2 + \nabla h^a \tilde{g}_{ab} h^b + \nabla h_a \tilde{g}_{ab} h_b + \gamma^2 (h^0)^2 \right] + U \tilde{G}^{ij} \left( h_{i0} h_{j0} + \frac{\kappa_a \kappa_b}{4} h_i^a h_j^b + h_{ai} h_{bj} t^a t^b + \gamma^2 h_{ij}^0 h_{ij} \right) - \frac{\kappa_a}{2} h^a h^a_0 - \frac{\kappa_a}{2} h^a_0 h^a - \nabla (t^a h_{ai} h_{aj} - \nabla t^a h_{ai} h_{aj}) + U \tilde{G}^{ij} \left( h_{i0} h_{j0} + \frac{\kappa_a \kappa_b}{4} h_i^a h_j^b + t^a t^b h^a \lambda h^b \gamma + \gamma^2 h^\alpha h^\alpha \right) - \frac{\kappa_a}{2} h^a h^a_0 - \frac{\kappa_a}{2} h^a_0 h^a - \nabla t^a h^\alpha h^\alpha - t^a t^b h^\alpha h^\alpha_0 - \frac{\kappa_a \kappa_b}{4} h^a h^a_0 \right) - 2 \frac{\kappa_a}{2} \left( \nabla h^a \tilde{G}_{ab} h^b + \nabla h_a \tilde{G}_{ab} h_b + \nabla t^a h^\alpha h^\alpha + \gamma^2 h^\alpha h^\alpha \right) - \frac{\kappa_a}{2} h^a h^a_0 - \frac{\kappa_a}{2} h^a_0 h^a - \nabla (t^a h_{ai} h_{aj} - \nabla t^a h_{ai} h_{aj}) + U \tilde{G}^{ij} \left( h_{i0} h_{j0} + \frac{\kappa_a \kappa_b}{4} h_i^a h_j^b + t^a t^b h^a \lambda h^b \gamma + \gamma^2 h^\alpha h^\alpha \right) - \frac{\kappa_a}{2} h^a h^a_0 - \frac{\kappa_a}{2} h^a_0 h^a - \nabla t^a h^\alpha h^\alpha - t^a t^b h^\alpha h^\alpha_0 - \frac{\kappa_a \kappa_b}{4} h^a h^a_0 \right) , \]

\[ V_{\text{IIA}}^\text{D} = \frac{e^{3D_{4d}}}{2U} \left[ \left( f^a \hat{v}_a h^a - f^a h^a - f^0 h^0 \right) - \left( f^0 h^\alpha_0 - f^a h^\alpha_0 + f^a h^\alpha - f^0 h^\alpha \right) \right] \left( \frac{\kappa_a}{2} \right) , \]

where

\[ \tilde{G}_{ab} = \frac{\kappa_a \kappa_b}{4} - \frac{4}{V} f^a f^b , \quad \tilde{G}^{ab} = \frac{2 t^a t^b - 4 \nabla \kappa_{ab}}{4V} , \quad \tilde{G}^{a\beta} = - \kappa^{a\beta} , \quad \tilde{G}_{a\beta} = - \kappa_{a\beta} , \] (4.13)

Note that we have \( V = \frac{1}{2} \kappa_\alpha t^a t^b t^c , \ U = \frac{1}{4} \kappa_\lambda t^a t^b t^c \) for the type IIA case, and also here we have used \( e^{K_3} = e^{4D_{4d}} = \frac{1}{2} \kappa_\gamma \) from the eqn. (2.31) to restore the popular factor of \( e^{3D_{4d}} \) in the RR sector and \( e^{2D_{4d}} \) in the NS-NS sector and the D-term contributions, along with a factor of \( e^{3D_{4d}} \) in the local piece.
5 Applications

In this section we illustrate the utilities of our scalar potential formulation by considering two explicit toroidal examples. All we need to know is the orientifold even/odd hodge numbers and the some of the topological quantities such as non-vanishing triple intersection numbers etc., and the rest would subsequently follow from our formulation. Therefore it can be considered as a direct way of computing the scalar potential with explicit dependence on the saxionic and axionic moduli.

5.1 Type IIA on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$-orientifold

Considering the untwisted sector with the non-geometric type IIA setup having the standard involution (e.g. see [58, 69] for details), we can start extracting information from our formulation for this model just by starting with the following input,

$$h^{1,1}_1 = 3, \quad h^{1,1}_+ = 0, \quad h^{2,1} = 3.$$  \hspace{1cm} (5.1)

The hodge numbers show that there would be three $U_\lambda$ moduli and three $T_a$ moduli along with a single $N_0$-modulus. There are no $N_k$ moduli present as the even $(1,1)$-cohomology is trivial. Subsequently it turns out that all the fluxes with $k$ index are absent. There are four components for the $H_3$ flux (namely $H_0$ and $H^\lambda$) and the same for the non-geometric $R$-flux which are denoted as $R_0$ and $R^\lambda$ for $\lambda \in \{1, 2, 3\}$. In addition, there are 12 flux components for each of the geometric ($w$) flux and the non-geometric ($Q$) flux, denoted as $\{w_{a0}, w_{a\lambda}\}$ and $\{Q^{a\lambda}, Q^{a0}\}$ for $\alpha \in \{1, 2, 3\}$ and $\lambda \in \{1, 2, 3\}$. On the RR side, there are eight flux components in total, one from each of the $F_0$ and $F_6$ fluxes denoted as $m_0$ and $e_0$, while three from each of the $F_2$ and $F_4$ fluxes denoted as $m^a, e_a$ for $a \in \{1, 2, 3\}$. In addition, let us also note that there will be no $D$-terms generated in the scalar potential as the even $(1,1)$-cohomology is trivial which projects out all the relevant $D$-term fluxes. Having the above orientifold related ingredients in hand, one can directly read-off the scalar potential pieces from our generic formula in two steps,

- step1 - to work out all the axionic flux polynomials
- step2 - to work out the moduli space metric

step1:

The following eight types of the NS-NS axionic flux polynomials are trivial in this model,

$$h_k = 0, \quad h_{ak} = 0, \quad h_{a0}^k = 0, \quad h_k^0 = 0,$$

$$\hat{h}_{a0} = 0, \quad \hat{h}_{a0} = 0, \quad \hat{h}_{a0}^a = 0, \quad \hat{h}_{a0}^a = 0,$$  \hspace{1cm} (5.2)

where one can anticipate from the trivial cohomology indices that such fluxes would be absent. Further using eqn. (4.1), the eight classes of non-zero NS-NS axionic flux polynomials can be explicitly written out in terms of the 32 flux combinations, along with 8 flux polynomials coming from the RR sector given in the following manner,

$$f_0 = G_0 - \xi^0 H_0 - \xi_\lambda H^\lambda,$$
$$f_a = G_a - \xi^0 \tilde{Q}_{a0} - \xi_\lambda \tilde{Q}_{a0}^\lambda,$$
$$f_0^a = G^a - \xi^0 R_0 - \xi_\lambda R^\lambda,$$
$$h_0 = H_0, \quad h_a = \tilde{Q}_{a0}, \quad h^a = \tilde{Q}^{a0}, \quad h_0 = R_0,$$
$$h^\lambda_0 = H^\lambda, \quad h^\lambda_a = \tilde{Q}^{\lambda a}, \quad h^{a\lambda} = Q^{a\lambda}, \quad h^{\lambda0} = R^\lambda.$$  \hspace{1cm} (5.3)
where the axionic flux polynomials in the above eqn. are given as,

\[ \mathcal{H}_0 = H_0 + w_{10} b_1 + w_{20} b_2 + w_{30} b_3 + b_1 b_2 Q_3^0 + b_2 b_3 Q_1^0 + b_3 b_1 Q_2^0 + b_1 b_2 b_3 R_0, \]
\[ \mathcal{U}_{10} = w_{10} + b_1 \phi_3^0 + b_2 \phi_2^0 + b_3 \phi_1^0, \quad Q_1^0 = Q_1^0 + b_1 R_0, \]
\[ \mathcal{U}_{20} = w_{20} + b_1 Q_3^0 + b_2 Q_1^0 + b_3 Q_2^0 + b_1 b_2 R_0, \quad Q_2^0 = Q_2^0 + b_2 R_0, \]
\[ \mathcal{U}_{30} = w_{30} + b_1 Q_2^0 + b_2 Q_1^0 + b_1 b_2 R_0, \quad Q_3^0 = Q_3^0 + b_3 R_0, \quad \mathcal{R}_0 = R_0, \]

\[ \mathcal{H}^\lambda = H^\lambda + w_1^\lambda b_1 + w_2^\lambda b_2 + w_3^\lambda b_3 + b_1 b_2 Q_3^\lambda + b_2 b_3 Q_1^\lambda + b_3 b_1 Q_2^\lambda + b_1 b_2 b_3 R^\lambda, \]
\[ \mathcal{U}_1^\lambda = w_1^\lambda + b_1 Q_3^\lambda + b_2 Q_1^\lambda + b_3 Q_2^\lambda, \quad Q_1^\lambda = Q_1^\lambda + b_1 R^\lambda, \]
\[ \mathcal{U}_2^\lambda = w_2^\lambda + b_1 Q_3^\lambda + b_2 Q_1^\lambda + b_3 Q_2^\lambda, \quad Q_2^\lambda = Q_2^\lambda + b_2 R^\lambda, \]
\[ \mathcal{U}_3^\lambda = w_3^\lambda + b_1 Q_3^\lambda + b_2 Q_1^\lambda + b_3 Q_2^\lambda, \quad Q_3^\lambda = Q_3^\lambda + b_3 R^\lambda, \quad \mathcal{R}^\lambda = R^\lambda, \]

\[ G_0 = e_0 + b_1 e_1 + b_2 e_2 + b_3 e_3 + b_1 b_2 m_3 + b_2 b_3 m_1 + b_3 b_1 m_2 + b_1 b_2 b_3 m_0, \]
\[ G_1 = e_1 + b_2 m_3 + b_3 m_1 + b_1 b_2 m_0, \quad G_2 = m_1 + m_0 b_1, \]
\[ G_2 = e_2 + b_1 m_3 + b_3 m_1 + b_1 b_2 m_0, \quad G_3 = m_1 + m_0 b_1, \]
\[ G_3 = e_3 + b_1 m_3 + b_2 m_1 + b_1 b_2 m_0, \quad G_4 = m_3 + m_0 b_3, \quad G_5 = m_0. \]

In simplifying the axionic flux polynomials, we have used the fact that the only non-zero intersection number which survive in the Kähler moduli part of the prepotential is \( k_{123} = 1 \). The same thing happens on the complex structure moduli side also as we mention below,

\[ k_{123} = 1, \quad \hat{k}_{aa\beta} = 0, \quad k_{123} = 1, \quad \hat{k}_{\lambda mn} = 0. \]

**step2:**

In order for fully knowing the the scalar potential, now we only need to know the moduli spaces metrics to supplement with the axionic flux polynomials given as,

\[ \kappa_{ab} = \begin{pmatrix} 0 & t^3 & t^2 \\ t^3 & 0 & t^1 \\ t^2 & t^1 & 0 \end{pmatrix}, \quad -4 \mathcal{V} \kappa_{ab} = \begin{pmatrix} 2 (t^1)^2 & -2 t^1 t^2 & -2 t^1 t^3 \\ -2 t^1 t^2 & 2 (t^2)^2 & -2 t^2 t^3 \\ -2 t^1 t^3 & -2 t^2 t^3 & 2 (t^3)^2 \end{pmatrix}, \]
\[ \mathcal{V} \tilde{G}^{ab} = \begin{pmatrix} (t^1)^2 & 0 & 0 \\ 0 & (t^2)^2 & 0 \\ 0 & 0 & (t^3)^2 \end{pmatrix}, \quad \mathcal{U} \tilde{G}^{\lambda\rho} = \begin{pmatrix} (z^1)^2 & 0 & 0 \\ 0 & (z^2)^2 & 0 \\ 0 & 0 & (z^3)^2 \end{pmatrix}. \]

In addition, we also have the following useful shorthand quantities,

\[ \mathcal{V} = t^1 t^2 t^3, \quad \kappa_1 = 2 t^1 t^3, \quad \kappa_2 = 2 t^1 t^3, \quad \kappa_3 = 2 t^1 t^3, \]
\[ \mathcal{U} = z^1 z^2 z^3, \quad \kappa_1 = 2 z^1 z^3, \quad \kappa_2 = 2 z^2 z^3, \quad \kappa_3 = 2 z^1 z^2. \]

To verify our scalar potential formulation, first we compute it from the flux superpotential as given in eqn. which results in 2422 terms. Subsequently we show that our collection of pieces gives the same result after using the simplified axionic flux polynomials and the moduli space metrics as
On the RR side, there are eight flux components of the three-form \( H \). These scalar potential pieces are given as,

\[
V_{\text{IIB}}^{\text{RR}} = \frac{e^{4D_{\text{ld}}}}{4V} \left[ f_0^2 + \mathcal{V} f_a \tilde{g}_{ab} f_b + \mathcal{V} f_a \tilde{g}^{ab} f_b + \mathcal{V}^2 (f_0)^2 \right],
\]

\[
V_{\text{IIB}}^{\text{NS1}} = \frac{e^{4D_{\text{ld}}}}{4U V} \left[ h_0^2 + \mathcal{V} h^a \tilde{g}_{ab} h^b + \mathcal{V} h_a \tilde{g}^{ab} h_b + \mathcal{V}^2 (h_0)^2 \right]
\]

\[
V_{\text{IIB}}^{\text{NS2}} = \frac{e^{4D_{\text{ld}}}}{4U V} \left[ \mathcal{U} \tilde{G}_{\mathcal{L} \mathcal{P}} (h_0^4 + \frac{\kappa_a \kappa_b}{4} h_a^{\lambda} h_b^{\rho} + t_a t_b h_a^{\lambda} h_b^{\rho} + \mathcal{V}^2 h_{a0} h_{b0}) \right]
\]

\[
V_{\text{IIB}}^{\text{NS3}} = \frac{e^{4D_{\text{ld}}}}{4U V} \left[ -2 \times \frac{\kappa_a}{2} \left( \mathcal{V} h^a \tilde{g}_{ab} h^b + \mathcal{V} h_a \tilde{g}^{ab} h_b + \mathcal{V} t_a h_0^a h_0^b + \mathcal{V} t^a h_0^a h_0^b \right) \right]
\]

\[
V_{\text{IIB}}^{\text{loc}} = \frac{e^{3D_{\text{ld}}}}{2U} \left[ (f^0 h_0 - f^a h_a + f_a h^a - f_0 h_0) - (f^0 h_0 - f^a h_a + f_a h^a - f_0 h_0) \frac{\kappa_a}{2} \right]
\]

To appreciate the numerics, let us mention that the above pieces of the scalar potential matches with the following splitting of 2422 terms computed from the superpotential,

\[
\#(V_{\text{IIB}}^\text{RR}) = 1630, \quad \#(V_{\text{IIB}}^{\text{NS1}}) = 76, \quad \#(V_{\text{IIB}}^{\text{NS2}}) = 408, \quad \#(V_{\text{IIB}}^{\text{NS3}}) = 180, \quad \#(V_{\text{IIB}}^{\text{loc}}) = 128
\]

### 5.2 Type IIB on \( T^6 / (\mathbb{Z}_2 \times \mathbb{Z}_2) \)-orientifold

Considering the untwisted sector with the standard involution for the non-geometric type IIB setup (e.g. see \([31, 61, 67, 68]\) for details), we can start with the following input,

\[
h_{1,1}^1 = 3, \quad h_{1,1}^{-1} = 0, \quad h_{2,1}^1 = 0, \quad h_{2,1}^{-1} = 3
\]

The hodge numbers show that there would be three \( T_a \) moduli and three \( U^i \) moduli along with the universal axio-dilaton \( S \) in this setup. There are no odd-moduli \( G^a \) present in this setup as the odd \((1, 1)\)-cohomology is trivial. It turns out that the geometric flux \( \omega \) and the non-geometric \( R \) flux do not survive the orientifold projection in this setup, and the only allowed NS-NS fluxes are the three-form \( H_3 \) flux and the non-geometric \( Q \) flux. There are eight components for the \( H_3 \) flux while 24 components for the \( Q \) flux, denoted as \( H_\Lambda, H^\Lambda, Q_\Lambda, Q^\Lambda \) for \( \alpha \in \{1, 2, 3\} \) and \( \Lambda \in \{0, 1, 2, 3\} \). On the RR side, there are eight flux components of the three-form \( F_3 \) flux. In addition, there are no \( D \)-terms generated in the scalar potential as the even \((2, 1)\)-cohomology is trivial which projects out all the \( D \)-term fluxes. Now we repeat the two steps followed for the type IIA case before.

**step1:**

It turns out that the following eight NS-NS axionic flux polynomials are trivial in this model,

\[
h_a = 0, \quad h_{ai} = 0, \quad h_a^i = 0, \quad h_a^0 = 0, \quad \hat{h}_{ak} = 0, \quad \hat{h}_k = 0, \quad \hat{h}_K = 0, \quad \hat{h}^{K0} = 0
\]
where one can anticipate from the trivial cohomology indices that such fluxes would be absent. Further using eqn. \ref{14.13}, the eight classes of the non-zero NS-NS axionic flux polynomials can be explicitly written out in terms of the 32 flux combinations given as,

\[
\begin{align*}
 h_0 &= H_0 + v^1 H_1 + v^2 H_2 + v^3 H_3 + v^1 v^2 H^3 + v^2 v^3 H^1 + v^3 v^1 H^2 - v^1 v^2 v^3 H^0, \\
 h_1 &= H_1 + v^2 H^3 + v^3 H^2 - v^2 v^3 H^0, \\
 h_2 &= H_2 + v^1 H^3 + v^3 H^1 - v^1 v^3 H^0, \\
 h_3 &= H_3 + v^1 H^2 + v^2 H^1 - v^1 v^2 H^0, \\
 h^0 &= H_0 - H^0, \\
 h^i &= \hat{Q}^i_0 + v^1 \hat{Q}^i_1 + v^2 \hat{Q}^i_2 + v^3 \hat{Q}^i_3 + v^1 v^2 \hat{Q}^{i3} + v^2 v^3 \hat{Q}^{i1} + v^3 v^1 \hat{Q}^{i2} - v^1 v^2 v^3 \hat{Q}^{i0}, \\
 h^{i1} &= \hat{Q}^{i1} + v^2 \hat{Q}^{i3} + v^3 \hat{Q}^{i2} - v^2 v^3 \hat{Q}^{i0}, \\
 h^{i2} &= \hat{Q}^{i2} + v^1 \hat{Q}^{i3} + v^3 \hat{Q}^{i1} - v^1 v^3 \hat{Q}^{i0}, \\
 h^{i3} &= \hat{Q}^{i3} + v^2 \hat{Q}^{i1} + v^1 \hat{Q}^{i2} - v^1 v^2 \hat{Q}^{i0}, \\
 h^{i0} &= \hat{Q}^{i0} - v^3 \hat{Q}^{i0}, \\
 h^{i0} &= -\hat{Q}^{i0}.
\end{align*}
\]

In addition, there are eight axionic flux polynomials which also involve the RR axions \( c_0 \) and \( c_\alpha \) along with the complex structure axions \( v^1 \)'s, and the same can be given as,

\[
\begin{align*}
 f_0 &= F_0 + v^1 F_1 + v^2 F_2 + v^3 F_3 + v^1 v^2 F^3 + v^2 v^3 F^1 + v^3 v^1 F^2 - v^1 v^2 v^3 F^0, \\
 f_1 &= F_1 + v^2 F^3 + v^3 F^2 - v^2 v^3 F^0, \\
 f_2 &= F_2 + v^1 F^3 + v^3 F^1 - v^1 v^3 F^0, \\
 f_3 &= F_3 + v^1 F^2 + v^2 F^1 - v^1 v^2 F^0, \\
 f^0 &= F^0 - F^{i0} c_\alpha - c_0 H_0, \\
 f^i &= F^i - \hat{Q}^{i0} c_\alpha - c_0 H_i, \\
 f^0 &= F^0 - \hat{Q}^{i0} c_\alpha - c_0 H^0, \\
 f^i &= F^i - \hat{Q}^{i0} c_\alpha - c_0 H^i.
\end{align*}
\]

Here we have used the fact that the only non-zero intersection number are given as,

\[
l_{123} = 1, \quad \hat{l}_{iJK} = 0, \quad \ell_{123} = 1, \quad \hat{l}_{\alpha ab} = 0,
\]

which result in the following useful shorthand quantities,

\[
\begin{align*}
 V &= t^1 t^2 t^3, \\
 \ell_1 &= 2 t^2 t^3, \\
 \ell_2 &= 2 t^1 t^3, \\
 \ell_3 &= 2 t^1 t^2, \\
 U &= u^1 u^2 u^3, \\
 l_1 &= 2 u^2 u^3, \\
 l_2 &= 2 u^1 u^3, \\
 l_3 &= 2 u^1 u^2.
\end{align*}
\]

\[\text{step2:}\]

In order for fully knowing the the scalar potential, we now only need to know the moduli spaces metrics to supplement with the axionic flux polynomials which can be given as,

\[
\begin{align*}
 V G^{ij} &= \begin{pmatrix}
 (t^1)^2 & 0 & 0 \\
 0 & (t^2)^2 & 0 \\
 0 & 0 & (t^3)^2
 \end{pmatrix}, \\
 U G^{ij} &= \begin{pmatrix}
 (u^1)^2 & 0 & 0 \\
 0 & (u^2)^2 & 0 \\
 0 & 0 & (u^3)^2
 \end{pmatrix}.
\end{align*}
\]

To verify the scalar potential formulation, first we compute it using the flux superpotential as given in eqn. \ref{2.75}, which results in 2422 terms and subsequently we confirm that our following collection

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of pieces gives the same result after using the simplified axionic flux polynomials and the moduli space metrics,

\[ V_{\text{IIB}}^{\text{RR}} = \frac{e^{2\phi}}{4V^2} \left[ f_0^2 + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U}^2 (f^0)^2 \right], \quad (5.14) \]

\[ V_{\text{IIB}}^{\text{NS1}} = \frac{e^{2\phi}}{4V^2} \left[ h_0^2 + \mathcal{U} h^i \mathcal{G}_{ij} h^j + \mathcal{U} h_i \mathcal{G}^{ij} h_j + \mathcal{U}^2 (h^0)^2 \right] \]

\[ V_{\text{IIB}}^{\text{NS2}} = \frac{e^{2\phi}}{4V^2} \left[ V \mathcal{G}_{\alpha\beta} \left( h^{\alpha_0} h^\beta_0 + \frac{l_i l_j}{4} h^{\alpha_i} h^\beta_j + u^i u^j h^{\alpha_i} h^\beta_j + \mathcal{U}^2 h^{\alpha_0} h^\beta_0 \right. \right. \]
\[ \left. \left. - \frac{l_i}{2} h^{\alpha_0} h^\beta_i - \frac{l_i}{2} h^{\alpha_i} h^\beta_0 - 2 \mathcal{U} u^i h^{\alpha_0} h^\beta_i - \mathcal{U} u^i h^{\alpha_0} h^\beta_i \right) \right] \]

\[ \left. + \frac{\ell_0^2}{4} \left( \mathcal{U} h^{\alpha_i} \mathcal{G}_{ij} h^{\beta_j} + \mathcal{U} h^{\alpha_i} \mathcal{G}^{ij} h^{\beta_j} + \mathcal{U} u^i h^{\alpha_0} h^\beta_i + \mathcal{U} u^i h^{\alpha_0} h^\beta_i \right) \right] \]

\[ - u^i u^j h^{\alpha_i} h^\beta_i + \frac{l_i}{2} h^{\alpha_0} h^\beta_i + \frac{l_i}{2} h^{\alpha_i} h^\beta_0 - \frac{l_i}{4} \ell_0 \mathcal{U} h^{\alpha_i} h^\beta_j \right) \right], \]

\[ V_{\text{IIB}}^{\text{NS3}} = \frac{e^{2\phi}}{4V^2} \left[ - 2 \times \frac{\ell_0^2}{2} \left( \mathcal{U} h^i \mathcal{G}_{ij} h^{\alpha_j} + \mathcal{U} h_i \mathcal{G}^{ij} h^{\alpha_j} + \mathcal{U} u^i h^0 h^\alpha_i + \mathcal{U} u^i h_i h^\alpha_0 \right. \right. \]
\[ \left. \left. - u^i u^j h^{\alpha_i} h^\beta_i + \frac{l_i}{2} h^{\alpha_0} h^\beta_i + \frac{l_i}{2} h^{\alpha_i} h^\beta_0 - \frac{l_i}{4} \ell_0 \mathcal{U} h^{\alpha_i} h^\beta_j \right) \right] \],

\[ V_{\text{IIB}}^{\text{loc}} = \frac{e^{3\phi}}{2V^2} \left[ (f^0 h_0 - f^i h_i + f_i h^i - f_0 h^0) - (f^0 h^0 - f^i h_i + f_i h^0) \frac{\ell_0^2}{2} \right], \]

which matches with the following splitting of 2422 terms computed from the superpotential,

\[ \#(V_{\text{IIB}}^{\text{RR}}) = 1630, \quad \#(V_{\text{IIB}}^{\text{NS1}}) = 76, \quad \#(V_{\text{IIB}}^{\text{NS2}}) = 408, \quad \#(V_{\text{IIB}}^{\text{NS3}}) = 180, \quad \#(V_{\text{IIB}}^{\text{loc}}) = 128. \]

Thus we have explicitly verified our generic type IIA potential in eqn. (4.12) and type IIB potential in eqn. (4.13) for the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold setups, in which there are no $D$-terms present while the $F$-term contribution results in precisely the same number (2424) of terms in the scalar potential as it could be found by their respective flux superpotential computations! Needless to say that there is a perfect match for the two scalar potentials under our $T$-duality transformation for this canonical $T$-dual pair of models.

It is quite impressive to having written thousands of terms in just a few lines and keeping the information about the saxionic and axionic parts distinct! These generic toroidal type IIA and IIB setups have been found interesting in several numerical approaches [11, 17, 11, 17, 21], and our formulation certainly opens up the possibilities for making attempts towards non-supersymmetric moduli stabilising in an analytic approach.

6 Summary and conclusions

In this article, we have studied the $T$-dual completion of the four-dimensional type IIA and type IIB effective supergravity theories with the presence of (non-)geometric fluxes. In order to put things under a single consistent convention and notation with fixing signs, factors etc., first we have revisited the relevant ingredients for the type IIA and the type IIB setups in some good detail.

Considering an iterative approach, we have invoked the $T$-duality transformations among the various standard as well as (non-)geometric fluxes of the two theories. This connection has been
explicitly known for fluxes written in the non-cohomology formulation, mostly applicable to the toroidal examples \cite{10,11,30,71–73} but not in the cohomology formulation which could be directly promoted for the beyond toroidal cases such as with using CY compactifications. Given that in the absence of fluxes, mirror symmetry exchanges the two theories, first we considered the Kähler potential with explicit computations including $\alpha'$-corrections on the compactifying threefold and its mirror. This helps us in re-deriving the $T$-duality rules for the moduli, axions and hence for the chiral variables on the two sides \cite{53,57,74}. Subsequently, in the second step we investigate the fluxes in the superpotential where the moduli have explicit polynomial dependence through the chiral variables, and utilising the $T$-duality rules for the chiral variables fixed in the fluxless scenario, we derive the explicit transformations for the various fluxes on the two sides. This leads to some very interesting and non-trivial mixing among the (non-)geometric fluxes with the standard fluxes as we present in table 7. We repeat the same step for the $D$-term contributions to derive the $T$-dual connection among the relevant fluxes appearing in the scalar potentials through the $D$-term contributions. These are also presented in table 7.

As a genuine effective potential should be the one obtained after taking care of the tadpole conditions and the NS-NS Bianchi identities, which generically have the potential to nullify some terms in the respective scalar potentials and hence can influence the effectiveness of scalar potential pieces governing the moduli dynamics. Therefore in order to confirm the mapping one has to ensure that the $T$-duality rules invoked for the fluxes and moduli in the earlier steps are compatible with these constraints. We find that this is indeed the case, and on these lines we have confirmed a one-to-one mapping among all the Bianchi identities of the two theories. The explicit details have been presented in table 11 and table 12. It is worth to note that there is quite a non-trivial mixing among the flux identities, in the sense that, e.g. a “$HQ$-type” identity on the type IIB gets mapped on to a “$(HR + wQ)$-type” identity on the type IIA side. Nevertheless, the full set of constraints do have a perfect one-to-one correspondence under $T$-duality.

As the superpotentials can be directly useful only for the supersymmetric stabilization, we have extended our studies at the level of scalar potential to deepen our understanding of the $T$-dual picture in terms of explicit dependence on the saxions/axions, where it can be directly used for the non-supersymmetric moduli stabilization and other phenomenological purposes as well. In this regard, first we have invoked what we call “axionic flux polynomials” from the superpotentials and the $D$-terms of the two theories. These axionic flux polynomials include all the axions and the fluxes but do not include any saxions, which helps us rewriting the scalar potential in a concise form, and more importantly still keeping the saxionic/axionic dependence distinct and explicit. These relevant details are presented in table 8, table 9 and table 10. We have demonstrated how our scalar potential formulation can be used for reading-off the scalar potentials by applying the same for two explicit toroidal orientifolds.

The reason for reformulating the scalar potential is multi-fold. First, it is concise in the sense that the generic scalar potential could be written in a few lines making it possible to make attempts for model independent moduli stabilization. This step is quite non-trivial in itself as one can recall that toroidal $T^6/(Z_2 \times Z_2)$ orientifold gives more than 2000 terms arising from the flux superpotential in both the type IIA and type IIB 4D theories, and it is hard even to analytically solve the extremization conditions. Second, to make the exchange of the two potentials manifest under the $T$-duality transformations. Scalar potentials being the starting point or the building block for moduli stabilization, there can be several possible applications of our one-to-one proposed formulation. For example, this enables one to translate any useful findings in one setup into their $T$-dual picture. In this regard, one would note that there are several well-known de-Sitter No-Go theorems on the type IIA side, and subsequently there should be their $T$-dual counterparts on the
type IIB side, which have been of course not got into the due attention. We have made a detailed study along these lines in a companion work \cite{93} which illustrates the direct use of the concise pieces of information presented in this work.

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## A T-dual Dictionary for Type II non-geometric setups

| F-term fluxes | Type IIA with $D6/O6$ | Type IIB with $D3/O3$ and $D7/O7$ |
|---------------|-----------------------|-----------------------------------|
| $H_0$, $H_k$, $H^\lambda$, $w_{a0}$, $w_{ak}$, $w_a^\lambda$, $Q_a^0$, $Q^a_k$, $Q^a\lambda$, $R_0$, $R_k$, $R^\lambda$, $e_0$, $e_a$, $m^a$, $m_0$. | $H_0$, $\omega_{a0}$, $\hat{Q}^a_0$, $H_i$, $\omega_{ai}$, $\hat{Q}^a_i$, $H^i$, $\omega^a_i$, $\hat{Q}^{ai}$, $-H^0$, $-\omega^a_0$, $-\hat{Q}^{a0}$, $F_0$, $F_i$, $F^i$, $-F^0$. |

| D-term fluxes |                          |                                    |
|---------------|--------------------------|-----------------------------------|
| $\hat{w}_a^0$, $\hat{w}_a^k$, $\hat{w}_a\lambda$, $\hat{Q}^a_0$, $\hat{Q}^a_k$, $\hat{Q}^a\lambda$. | $-R_K$, $-Q^K_a$, $\hat{\omega}_a^K$, $-R^K$, $-Q^{aK}$, $\hat{\omega}^a^K$. |

| Complex Moduli |                        |                                    |
|---------------|------------------------|-----------------------------------|
| $N^0$, $N^k$, $U_\lambda$, $T^a$. | $S$, $G^a$, $T_\alpha$, $U^i$. | $T_\alpha = -\frac{i s}{2} (\ell_{\alpha\beta\gamma} t^\beta t^\gamma - \hat{\ell}_{aab} b^a b^b) + (c_\alpha + \ell_{aab} c^a b^b + \frac{1}{2} c_0 \hat{\ell}_{aab} b^a b^b)$. |
| $T^a = b^a - i t^a$, $N^0 = \xi^0 + i (z^0)^{-1}$, $N^k = \xi^k + i (z^0)^{-1} z^k$, $U_\lambda = -\frac{i}{2727} (k_{\lambda\rho\kappa} z^\rho z^\kappa - \hat{k}_{\lambda\kappa\mu} z^\lambda z^\mu) + \xi_\lambda$. | $S = c_0 + i s$, $G^a = (c^a + c_0 b^a) + i s b^a$, | |

| Axions | $z^k$, $b^a$, $\xi^0$, $\xi^k$, $\xi_\lambda$. | $b^a$, $v^i$, $c_0$, $c^a + c_0 b^a$, $c_\alpha + \ell_{aab} c^a b^b + \frac{1}{2} c_0 \hat{\ell}_{aab} b^a b^b$. |

| Saxions | $(z^0)^{-1}$, $z^\lambda$, $t^a$, $V$, $U$, $s \equiv e^{-\phi}$, $t^a$, $u^i$ $U$, $V$, | $s \equiv e^{-\phi}$, $t^a$, $u^i$ $U$, $V$, |

| Intersections | $k_{\lambda\mu}$, $\hat{k}_{\lambda\mu\nu}$, $\kappa_{abc}$, $\hat{\kappa}_{ab\beta}$, $\ell_{\alpha\beta\gamma}$, $\hat{\ell}_{aab}$, $l_{ijk}$, $\hat{l}_{iJK}$. | |

| **Table 7**: T-duality transformations among the various fluxes, moduli and the axions. | 38 |
| Function | Expression |
|----------|------------|
| \( f_0 \) | \( G_0 - \xi^k \mathcal{H}_k - \xi^\lambda \mathcal{H}^\lambda \) |
| \( f_a \) | \( G_a - \xi^k \mathcal{U}_{ak} - \xi^\lambda \mathcal{U}_a^\lambda \) |
| \( f^a \) | \( G^a - \xi^k \mathcal{Q}_k^a - \xi^\lambda \mathcal{Q}^a_\lambda \) |
| \( f^0 \) | \( G^0 - \xi^k \mathcal{R}_k - \xi^\lambda \mathcal{R}^\lambda \) |
| \( h_0 \) | \( \mathcal{H}_0 + \mathcal{H}_k z^k + \frac{1}{2} \mathcal{H}_{\lambda mn} z^m z^n \mathcal{H}^\lambda \) |
| \( h_a \) | \( \mathcal{U}_{a0} + \mathcal{U}_{ak} z^k + \frac{1}{2} \mathcal{U}_{\lambda mn} z^m z^n \mathcal{U}_a^\lambda \) |
| \( h^a \) | \( \mathcal{Q}_{a0}^a + \mathcal{Q}_{ak}^a z^k + \frac{1}{2} \mathcal{Q}_{\lambda mn}^a z^m z^n \mathcal{Q}^a_\lambda \) |
| \( h^0 \) | \( \mathcal{R}_0 + \mathcal{R}_k z^k + \frac{1}{2} \mathcal{R}_{\lambda mn} z^m z^n \mathcal{R}^\lambda \) |
| \( h_{k0} \) | \( \mathcal{H}_k + \mathcal{H}_{\lambda kn} z^n \mathcal{H}^\lambda \) |
| \( h_{ak} \) | \( \mathcal{U}_{ak} + \mathcal{U}_{\lambda kn} z^n \mathcal{U}_a^\lambda \) |
| \( h^{a\lambda} \) | \( \mathcal{Q}_{a k}^a + \mathcal{Q}_{\lambda kn}^a z^n \mathcal{Q}^a_\lambda \) |
| \( h^\lambda_0 \) | \( \mathcal{R}_k + \mathcal{H}_{\lambda kn} z^n \mathcal{R}^\lambda \) |

\[ F \text{-term fluxes} \]

\[ G_0 = \widetilde{e}_0 + b^a \widetilde{a}_a + \frac{1}{2} \kappa_{abc} b^a b^b m_c + \frac{1}{6} \kappa_{abc} b^a b^b b^c m_0, \]
\[ G_a = \widetilde{a}_a + \kappa_{abc} b^b m_c + \frac{1}{2} \kappa_{abc} b^b b^c m_0, \]
\[ G^a = m^a + m_0 b^a, \]
\[ G^0 = m_0, \]
\[ \mathcal{H}_k = \Pi_k + \Pi_{ak} b^a + \frac{1}{2} \kappa_{abc} b^b b^c \mathcal{Q}_{ak}^a + \frac{1}{6} \kappa_{abc} b^b b^c R_k, \]
\[ \mathcal{H}_k = \Pi_k + \Pi_{a k} b^a + \frac{1}{2} \kappa_{abc} b^b b^c \mathcal{Q}_k^a + \frac{1}{6} \kappa_{abc} b^b b^c R_k, \]
\[ \mathcal{U}_{ak} = \Pi_{ak} b^a + \kappa_{abc} b^b \mathcal{Q}_{ak}^a + \frac{1}{6} \kappa_{abc} b^b b^c R_k, \]
\[ \mathcal{U}_{a k} = \Pi_{a k} b^a + \kappa_{abc} b^b \mathcal{Q}_{ak}^a + \frac{1}{6} \kappa_{abc} b^b b^c R_k, \]
\[ \mathcal{Q}_{a k}^a = Q_{a k}^a + b^a R_k, \]
\[ Q_{a \lambda} = Q_{a \lambda} + b^a R_k, \]
\[ R_k = R_k, \]
\[ R^\lambda = R^\lambda. \]

\[ D \text{-term fluxes} \]

\[ \mathcal{h}_{a \lambda} \equiv \mathcal{h}_{a \lambda} = \mathcal{h}_{a \lambda} + \hat{k}_{\lambda km} z^m \mathcal{w}_{a k} - \frac{1}{2} \hat{k}_{\lambda km} z^k z^m \mathcal{w}_a^0, \]
\[ \mathcal{h}_a^k \equiv \mathcal{h}_a^k = \mathcal{h}_a^k - z^k \mathcal{w}_a^0, \]
\[ \mathcal{h}_a^0 = \mathcal{h}_a^0 = \mathcal{h}_a^0 = \mathcal{h}_a^0, \]
\[ \mathcal{h}_{a \lambda} \equiv \mathcal{h}_{a \lambda} = \mathcal{h}_{a \lambda} + \hat{k}_{\lambda km} z^m \mathcal{Q}_{a k}^a - \frac{1}{2} \hat{k}_{\lambda km} z^k z^m \mathcal{Q}_{a 0}^a, \]
\[ \mathcal{h}_{a k} \equiv \mathcal{h}_{a k} = \mathcal{h}_{a k} - z^k \mathcal{Q}_{a 0}^a, \]
\[ \mathcal{h}_{a 0} \equiv \mathcal{h}_{a 0} = \mathcal{h}_{a 0} = \mathcal{h}_{a 0}. \]

Table 8: Axionic flux polynomials for Type IIA side.
### Table 9: Type IIB axionic flux polynomials with their dual type IIA counterpart.

| Type IIB axionic flux polynomials | dual Type IIA flux polynomials |
|----------------------------------|--------------------------------|
| $f_0$                            | $f_0$                          |
| $f_i$                            | $f_i$                          |
| $f^i$                            | $f^i$                          |
| $f^0$                            | $f^0$                          |
| $h_0$                            | $h_0$                          |
| $h_i$                            | $h_i$                          |
| $h^i$                            | $h^i$                          |
| $h^0$                            | $h^0$                          |
| $h_{a0}$                         | $h_{a0}$                       |
| $h_{ai}$                         | $h_{ai}$                       |
| $h_{ai}^i$                       | $h_{ai}^i$                     |
| $h_{a0}^i$                       | $h_{a0}^i$                     |
| $h_a^0$                          | $h_a^0$                        |
| $h_i^0$                          | $h_i^0$                        |
| $h_i^i$                          | $h_i^i$                        |
| $h_i^0$                          | $h_i^0$                        |
| $h^0$                            | $h^0$                          |

**F-term fluxes**

$F^\Lambda = \overline{F}_\Lambda - \omega_a^e e^a - \overline{Q}_{e}^\Lambda (c_a + \hat{\ell}_{a}ab^b b^b) - c_0 \mathbb{H}_\Lambda$

$F^\Lambda = F^\Lambda - \omega_a^e e^a - \overline{Q}_{e}^\Lambda (c_a + \hat{\ell}_{a}ab^b b^b) - c_0 \mathbb{H}_\Lambda$

$\mathbb{H}_\Lambda = \overline{H}_\Lambda + \omega_a^e b^a + \frac{i}{2} \hat{\ell}_{a}ab^b b^b \overline{Q}_{e}^\Lambda$

$\mathbb{H}_\Lambda = H^\Lambda + \omega_a^e b^a + \frac{i}{2} \ell_{a}ab^b b^b Q_{e}^\Lambda$

$\overline{Q}_{a}^\Lambda = \overline{\omega}_{a}^e b^a + \overline{\ell}_{a}ab^b b^b \overline{Q}_{e}^\Lambda$

$Q_{a}^\Lambda = Q_{e}^\Lambda + Q_{e}^\Lambda \ell_{a}ab^b b^b$

$\overline{Q}_{a}^\Lambda = \overline{Q}_{a}^\Lambda$,  $\overline{Q}_{e}^\Lambda = \overline{Q}_{e}^\Lambda$

**D-term fluxes**

$\hat{h}_{aK} = \overline{\omega}_{aK} - \overline{Q}_{a}^e K \hat{\ell}_{a}ab^b b^b + \frac{i}{2} \ell_{a}ab^b b^b R_K$

$\hat{h}_{aK} = \overline{\omega}_{aK} - \overline{Q}_{e}^a K \hat{\ell}_{a}ab^b b^b + \frac{i}{2} \ell_{a}ab^b b^b R_K$

$\hat{h}_{aK} = \overline{Q}_{a}^e K = -Q_{e}^a K + R_K b^a$

$\hat{h}_{aK} = \overline{Q}_{e}^a K = -Q_{e}^a K + R_K b^a$

$\hat{h}_{K}^0 = \overline{R}_K = -R_K$,  $\hat{h}_{K}^0 = \overline{R}_K = -R_K$

$\hat{h}_{K}^0 = \overline{R}_K = -R_K$,  $\hat{h}_{K}^0 = \overline{R}_K = -R_K$
A one-to-one exchange of the scalar potentials under $T$-duality

\[ V_{\text{IIA}}^{\text{tot}} = \frac{e^{4D}}{4V} \left[ f_0^2 + \mathcal{V} f_a \tilde{G}_{ab} t_b + \mathcal{V} f_a \tilde{G}^{ab} f_b + \mathcal{V}^2 (f^0)^2 \right] + \frac{e^{3D}}{4V} \left[ h_0^2 + \mathcal{V} h^a \tilde{G}_{ab} h^b \right. \\
\left. + \mathcal{V} h_a \tilde{G}_{ab} h^b + \mathcal{V}^2 (h^0)^2 + \mathcal{U} \tilde{G}_{ij} \left( h_{00} h_{jj} + \frac{\kappa a \kappa b}{4} h_a h_j \right) + \mathcal{U} \tilde{G}_{\lambda \rho} \left( h_{00} h_0^\lambda + \frac{\kappa a \kappa b}{4} h^a h_0^b \right) \\
\left. + t^a \tilde{t}_a h_0^\lambda h_0^b + \mathcal{V}^2 h_0^\lambda h_0^b - \frac{\kappa a \kappa b}{2} h_0^\lambda h_0^a - \frac{\kappa a \kappa b}{2} h_0^\lambda h_0^b - \mathcal{V} t^a h_0^\lambda h_0^b - \mathcal{V} t^b h_0^\lambda h_0^b \right). \]

\[ + \frac{e^{3D}}{4V} \left[ (f_0^0 - f^0 h_a + f^0 h_0) - (f_0^0 h_0 - f^0 h_0^a + f^0 h_0^a - f^0 h_0^0) \right]. \]

\[ \tilde{G}_{ab} = \frac{\epsilon_a \epsilon_b - 4V \kappa_{ab}}{4V}, \quad \tilde{G}^{ab} = \frac{2 \epsilon_a \epsilon_b - 4V \kappa_{ab}}{4V}, \quad \tilde{G}_{ij} = \tilde{G}^{ij} = \tilde{G}_{\lambda \rho} = \tilde{G}^{\lambda \rho} = \frac{2 \epsilon_a \epsilon_b - 4V \kappa_{ab}}{4V}. \]

\[ Table 10: \] Scalar potentials for type IIA and IIB theories
A one-to-one exchange of the Bianchi identities under $T$-duality

| BIs   | Type IIB with $D3/O3$ and $D7/O7$ | Type IIA with $D6/O6$ |
|-------|----------------------------------|-----------------------|
| (1)   | $H_A \omega_a^\Lambda = H^A \omega^\Lambda_a$ | $H_{[0} R_{k]} + Q^a_{[0} w_{ak]} = 0$ |
| (2)   | $H^A \dot{Q}^A_\Lambda = H^A Q^{a\Lambda}$ | $R^A H_0 - H^A R_0 + w_a^\Lambda Q^{a_0} - Q^{a\Lambda} w_{a0} = 0$ |
| (3)   | $\omega_a^\Lambda \omega_{b\Lambda} = \omega_b^\Lambda \omega_{a\Lambda}$ | $H_{[k} R_{k']} + Q^a_{[k} w_{ak']} = 0$ |
| (4)   | $\hat{\omega}_a^K \hat{\omega}_b^K = \hat{\omega}_b^K \hat{\omega}_a^K$ | $\hat{w}_{a\Lambda} \hat{Q}^a_\rho = \hat{Q}^{a\Lambda} \hat{w}_{a\rho}$ |
| (5)   | $\omega_{a\Lambda} \hat{Q}^{a\Lambda} = \omega_a^\Lambda \hat{Q}^a_\Lambda$ | $R^A H_k - H^A R_k + w_a^\Lambda Q^{a_0} - Q^{a\Lambda} w_{a0} = 0$ |
| (6)   | $Q^aK \hat{\omega}_aK = Q^a K \hat{\omega}_a^K$ | $\hat{w}_{a\Lambda} \hat{Q}^{a_k} = \hat{Q}^{a\Lambda} \hat{w}_{ak}$ |
| (7)   | $H_0 R_k + \omega_{a0} Q^a K + \dot{Q}^{a_0} \hat{\omega}_aK = 0$ | $H^A \hat{w}_{a\Lambda} = H_k \hat{w}_{ak}$ |
|       | $H_1 R_k + \omega_{ai} Q^a K + \dot{Q}^{ai} \hat{\omega}_aK = 0$ | $w_a^\Lambda \hat{w}_{a\Lambda} = w_{ak} \hat{w}_{ak}$ |
| (8)   | $H^0 R_k + \omega_{a0} Q^a K + \dot{Q}^{a_0} \hat{\omega}_aK = 0$ | $R^A \hat{w}_{a\Lambda} = R_k \hat{w}_{ak}$ |
|       | $H^i R_k + \omega_{ai} Q^a K + \dot{Q}^{ai} \hat{\omega}_aK = 0$ | $Q^a_k \hat{w}_{ak} = Q^a \hat{w}_{a\Lambda}$ |
| (9)   | $H_0 R_k + \omega_{a0} Q^a K + \dot{Q}^{a_0} \hat{\omega}_aK = 0$ | $H^A \dot{Q}^{a\Lambda} = H_k \dot{Q}^{a_k}$ |
|       | $H_1 R_k + \omega_{ai} Q^a K + \dot{Q}^{ai} \hat{\omega}_aK = 0$ | $\dot{Q}^{a\Lambda} w_{a\Lambda} = w_{ak} \dot{Q}^{a_k}$ |
| (10)  | $H^0 R_k + \omega_{a0} Q^a K + \dot{Q}^{a_0} \hat{\omega}_aK = 0$ | $R^A \dot{Q}^{a\Lambda} = R_k \dot{Q}^{a_k}$ |
|       | $H^i R_k + \omega_{ai} Q^a K + \dot{Q}^{ai} \hat{\omega}_aK = 0$ | $Q^a \dot{Q}^{a\Lambda} = Q^a_k \dot{Q}^{a_k}$ |
| (11)  | $\dot{Q}^{a\Lambda} \dot{Q}^{b_\Lambda} = \dot{Q}^{a\Lambda} \dot{Q}^{b_\Lambda}$ | $H^{[A R]} + Q^a_{[A} w_{a\rho]} = 0$ |
| (12)  | $Q^a K \dot{Q}^{b K} = Q^b K \dot{Q}^{a K}$ | $\hat{w}_{ak} \dot{Q}^{a_{k'}} = \dot{Q}^{a_k} \hat{w}_{ak}$ |
| (13)  | $R^K \hat{\omega}_a K = R_K \hat{\omega}_a K$ | $\hat{w}_{a\Lambda} \dot{Q}^{a_0} = \dot{Q}^{a\Lambda} \hat{w}_{a0}$ |
| (14)  | $R_K Q^a K = R^K Q^a K$ | $\hat{w}_{a} \dot{Q}^{a_0} = \dot{Q}^{a_0} \hat{w}_{a}$ |

**Table 11**: One-to-one correspondence between the Bianchi identities under the T-dual flux transformations. Here we have considered $\Lambda = \{0, i\}$ on the type IIB side and $k = \{0, k\}$ on the type IIA side.
A one-to-one exchange of the Bianchi identities with flux polynomials having $b^a$ axions

| BIs | Type IIB with $D3/O3$ and $D7/O7$ | Type IIA with $D6/O6$ |
|-----|----------------------------------|-----------------------|
| (1) | $H_A Q^A_a = H_A Q^A_a$         | $H_{\{0 \mathcal{R}_k\}} + Q^a_{\{0 w_{ak}\}} = 0$ |
| (2) | $H^\Lambda Q^\Lambda_{a^r} = H^\Lambda Q^\Lambda_{a^r}$ | $R^\Lambda H_0 - H^\Lambda R_0 + \tilde{\omega}_a^\Lambda Q^a_0 - Q^a_{\Lambda a} \tilde{\omega}_a^0 = 0$ |
| (3) | $\tilde{\omega}_a^\Lambda \tilde{\omega}_a^\Lambda = \tilde{\omega}_b^\Lambda \tilde{\omega}_a^\Lambda$ | $H_{\{k \mathcal{R}_{ak}\}} + Q^a_{\{k \tilde{\omega}_{ak}\}} = 0$ |
| (4) | $\tilde{\omega}_{\alpha K} \tilde{\omega}_{\beta K} = \tilde{\omega}_{\beta K} \tilde{\omega}_{\alpha K}$ | $\tilde{\omega}_{\alpha a} \tilde{\omega}_{\rho} = \tilde{\omega}_{\alpha a} \tilde{\omega}_{\alpha \rho}$ |
| (5) | $\tilde{\omega}_{a a} \tilde{\omega}_{a a} = \tilde{\omega}_{a a} \tilde{\omega}_{a a}$ | $R^\Lambda H_k - H^\Lambda R_k + \tilde{\omega}_a^\Lambda Q^a_k - Q^a_{a a} \tilde{\omega}_{a k} = 0$ |
| (6) | $Q^a K \tilde{\omega}_{a K} = Q^a K \tilde{\omega}_{a K}$ | $\tilde{\omega}_{a a} \tilde{\omega}_{a a} = \tilde{\omega}_{a a} \tilde{\omega}_{a a}$ |
| (7) | $H^0 \mathcal{R}_K + \tilde{\omega}_{a 0} Q^a_0 + \tilde{\omega}_{a K} = 0$ | $H^a \tilde{\omega}_{a 0} = H^a \tilde{\omega}_{a 0}$ |
|     | $H^1 \mathcal{R}_K + \tilde{\omega}_{a K} + Q^a_{\alpha K} + \tilde{\omega}_{a 0} \tilde{\omega}_{a K} = 0$ | $Q^a_k \tilde{\omega}_{a k} = Q^a_{a a} \tilde{\omega}_{a a}$ |
| (8) | $H^0 \mathcal{R}_K + \tilde{\omega}_{a 0} Q^a_0 + \tilde{\omega}_{a K} + Q^a_{\alpha 0} \tilde{\omega}_{a K} = 0$ | $H^a \tilde{\omega}_{a 0} = H^a \tilde{\omega}_{a 0}$ |
|     | $H^1 \mathcal{R}_K + \tilde{\omega}_{a K} + Q^a_{\alpha K} + \tilde{\omega}_{a 0} \tilde{\omega}_{a K} = 0$ | $Q^a_k \tilde{\omega}_{a k} = Q^a_{a a} \tilde{\omega}_{a a}$ |
| (9) | $H^0 \mathcal{R}_K + \tilde{\omega}_{a 0} Q^a_0 + \tilde{\omega}_{a K} = 0$ | $H^a \tilde{\omega}_{a 0} = H^a \tilde{\omega}_{a 0}$ |
|     | $H^1 \mathcal{R}_K + \tilde{\omega}_{a K} + Q^a_{\alpha K} + \tilde{\omega}_{a 0} \tilde{\omega}_{a K} = 0$ | $Q^a_k \tilde{\omega}_{a k} = Q^a_{a a} \tilde{\omega}_{a a}$ |
| (10) | $H^0 \mathcal{R}_K + \tilde{\omega}_{a 0} Q^a_0 + \tilde{\omega}_{a K} = 0$ | $H^a \tilde{\omega}_{a 0} = H^a \tilde{\omega}_{a 0}$ |
|     | $H^1 \mathcal{R}_K + \tilde{\omega}_{a K} + Q^a_{\alpha K} + \tilde{\omega}_{a 0} \tilde{\omega}_{a K} = 0$ | $Q^a_k \tilde{\omega}_{a k} = Q^a_{a a} \tilde{\omega}_{a a}$ |
| (11) | $\tilde{\omega}_{a a} \tilde{\omega}_{a a} = \tilde{\omega}_{a a} \tilde{\omega}_{a a}$ | $H^a \mathcal{R}_k + Q^a_{\alpha 0} \tilde{\omega}_{a 0} = 0$ |
| (12) | $Q^a K \tilde{\omega}_{a K} = Q^a K \tilde{\omega}_{a K}$ | $\tilde{\omega}_{a a} \tilde{\omega}_{a a} = \tilde{\omega}_{a a} \tilde{\omega}_{a a}$ |
| (13) | $\mathcal{R}_K \tilde{\omega}_{a K} = \mathcal{R}_K \tilde{\omega}_{a K}$ | $\tilde{\omega}_{a a} \tilde{\omega}_{a a} = \tilde{\omega}_{a a} \tilde{\omega}_{a a}$ |
| (14) | $\mathcal{R}_K Q^a K = \mathcal{R}_K Q^a K$ | $\tilde{\omega}_{a a} \tilde{\omega}_{a a} = \tilde{\omega}_{a a} \tilde{\omega}_{a a}$ |

Table 12: One-to-one correspondence between the Bianchi identities with generalised flux polynomials having the NS-NS $b^a$ axions as presented in eqn. [13] for type IIB and in eqn. [12] for type IIA. Here we have considered $\Lambda = \{0, i\}$ on the type IIB side and $\hat{k} = \{0, k\}$ on the type IIA side.
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