Density saturation and the decrease of the normalised width of the multiplicity distribution in high energy \( pp \) collisions

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Abstract

It is experimentally observed that the width of the KNO multiplicity distribution—or the negative binomial parameter, \( 1/k \)—for \( pp \) collisions, in the energy region \( 10 \lesssim \sqrt{s} \lesssim 1800 \) GeV, is an increasing function of the energy. We argue that in models with parton or string saturation such trend will necessary change: at some energy the distribution will start to become narrower. In the framework of percolating strings, we have estimated the change to occur at an energy of the order of 5–10 TeV.

1 The problem

Experimentally, in \( pp \) and \( p\bar{p} \) collisions, not only the average particle multiplicity is increasing with energy but, as well, the normalised width of the multiplicity distribution is an increasing function of energy [1]. In other words, both \( \langle n \rangle \) and \( D^2/\langle n \rangle^2 \equiv (\langle n^2 \rangle - \langle n \rangle^2)/\langle n \rangle^2 \) increase with energy.

The increase of \( \langle n \rangle \) is expected: more particles are produced when more energy is available. The growth of \( D^2/\langle n \rangle^2 \) is, however, not so obvious. The general argument seems to be that the increase of energy implies phase-space opening. There is more room for partons, strings, clusters or clans, more room for multiple collisions and different impact parameters contributions, more room for harder collisions and additional QCD branching [2]. In all these approaches,
if there are no limitations, fluctuations increasingly dominate over average multiplicity: \( D^2 / \langle n \rangle^2 \) increases and, asymptotically, it may reach a constant value (KNO scaling [3]). This kind of behaviour, as pointed out in [4], has been observed long ago in cosmic ray physics [5], with the negative binomial parameter \( k \) stabilising at a value of the order of 2.5–3, for \( \sqrt{s} \geq 1 \) TeV.

However, if saturation phenomena occur [6,7], i.e., too many objects occupy a finite region of interaction, then fluctuations are severely limited and \( D^2 / \langle n \rangle^2 \) will start to decrease. On general grounds, this is not unreasonable. As the number of intermediate objects (or final particles) is bounded by \( \sqrt{s} \) (conservation of energy), while the region of interaction is constrained by the stronger Froissart bound, \( \ln^2 \sqrt{s} \), there exists a real possibility for saturation. It is also well known that, at fixed energy, when triggering on rare events [8] or selecting central impact parameter, \( b = 0 \), collisions [9], the KNO distribution, due to the higher density, is narrower.

### 2 Theoretical framework

We shall work in the framework of the dual string model [10]. Saturation, in this case, results from clustering and percolation of strings. In a collision, \( N_s \) strings are produced and they may form clusters with different number of strings (or different sizes). These clusters act as random, particle emitting, sources. Following [11], we write for the multiplicity distribution, \( P(n) \),

\[
P(n) = \int W(X)P(n; X\bar{n}_1) dX, \tag{1}
\]

where \( W \) is a normalised weight function, \( X \) is a clustering variable, \( P(n; X\bar{n}_1) \) a convolution function for a cluster of strings, \( \bar{n}_1 \) being the single string average multiplicity. We assume for \( P(n; X\bar{n}_1) \) a Poisson distribution, this being consistent with the single string distribution of low energy \( e^+e^- \) annihilations. Note that, as emphasised in [11], an equation similar to (1), with the same function \( W(X) \), can be written for the inclusive \( p_T \) distribution, \( f(p_T) \).

From (1) we obtain

\[
\langle n \rangle = \langle X \rangle \bar{n}_1 \quad \tag{2}
\]

and

\[
\frac{1}{k} \equiv \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2} - \frac{1}{\langle n \rangle} = \frac{\langle X^2 \rangle - \langle X \rangle^2}{\langle X \rangle^2}. \quad \tag{3}
\]

Defining now \( N_c \) as the number of clusters, \( N \) the number of strings in a cluster and \( \bar{N}_s \) the average number of strings, which must satisfy the sum rule
\langle N_c \rangle \langle N \rangle = \bar{N}_s, \text{ we shall write}

X \equiv \langle N_c \rangle N, \tag{4}

such that, finally, we obtain

\langle n \rangle = \langle N_c \rangle \langle N \rangle \bar{n}_1 = \bar{N}_s \bar{n}_1, \tag{5}

and

\frac{1}{k} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2}. \tag{6}

The relevant parameter in string clustering (and percolation) is the transverse density \( \eta \),

\eta = \left( \frac{r_0}{R} \right)^2 \bar{N}_s, \tag{7}

where \( r_0 \) is the transverse radius of the string (we shall use \( r_0 = 0.2 \text{ fm} \)) and \( R \) is the effective radius of the interaction region, with \( R \approx 1 \text{ fm in pp collisions} \).

It is the parameter \( \eta \) that controls clustering: when \( N \) strings cluster, the effective colour charge is not proportional to \( N \) but, due to the vectorial nature of colour summation [12], is proportional to \( F(\eta)N \), where [13]

\[ F(\eta) = \sqrt{\frac{1-e^{-\eta}}{\eta}}, \tag{8} \]

such that, instead of (5), one should write

\[ \langle n \rangle = F(\eta)\bar{N}_s \bar{n}_1. \tag{9} \]

Equation (6) for \( 1/k \) remains, of course, unchanged. Note that \( F(\eta) \) is a decreasing function of \( \eta \).

3 The toy model

In order to show, clearly, the problem of the competition between the opening of new possibilities and limitations due to saturation, let us introduce a very simple, and known to everybody, toy model. Imagine that one has \( N_s \) identical coins to distribute over \( M \) identical boxes. The reader may imagine the distribution of \( N_s \) partons or strings over the interaction region of size \( M \), or \( R^2 \) (in units of \( r_0^2 \)). This is a purely combinatorial problem, without percolation.

What is the distribution of coins in the boxes? In Ref. [14] it was shown that
in the thermodynamical limit only the transverse density
\[ \eta \equiv \frac{N_s}{M}, \tag{10} \]
is relevant, and that the first moments of the distribution \( P(N) \) of the number of strings per cluster (the probability of having a \( N \)-cluster)
\[ \langle N \rangle = \frac{\eta}{1 - \exp(-\eta)}, \tag{11} \]
and
\[ K = \frac{\langle N \rangle^2}{\langle N^2 \rangle - \langle N \rangle^2} = \frac{\eta}{1 - (1 + \eta) \exp(-\eta)}. \tag{12} \]
If we use this toy model as input for \( W(X) \) in (1), and take, as mentioned before, the Poisson distribution for \( P(n; \tilde{n}_1) \), then, see (6), one obtains
\[ K = k. \tag{13} \]

In Figure 1 we present plots of (11) and (12). The average multiplicity is, naturally, a growing function of the density. The parameter \( k \) goes to infinity at small \( \eta \) (one just has 1-clusters), decreases and reaches a minimum, \( k \approx 3.3 \), at \( \eta \approx 1.8 \), then moves to infinity again as \( \eta \to \infty \) (the distribution is dominated by \( N_s/M \)-clusters). We have checked by Monte Carlo simulation that the distribution \( P(N) \) of the model is similar to a gamma distribution.
4 The physics

In order to make contact with real physics, we need to estimate the dependence of the parameter $k$, experimentally measured in $pp$ collisions as a function of the energy, on the density parameter $\eta$. In other words, we have to find the energy dependence of $\eta$. As $\eta$, see (7), depends on $N_s$ and $R^2$, we need to find the energy dependence of $N_s$ and $R^2$.

For the $pp$ average charged particle multiplicity in the dual string model, we shall write [15]:

$$\langle n(\sqrt{s}) \rangle = F(\eta)\bar{n}_1 \left[ 2 + (N_s(\sqrt{s}) - 2)\alpha \right],$$  \hspace{1cm} (14)

where $\bar{n}_1$ is the single valence string average multiplicity (assumed constant), $\alpha$ the sea string multiplicity reduction factor ($\alpha < 1$). Notice that in (14) one always has 2 valence strings and $N_s - 2$ sea strings. At low energy, $\sqrt{s} \approx 10$ GeV, experimentally $\langle n \rangle \approx 4$, and theoretically $N_s \approx 2$ (the two valence strings) and of course $F(\eta) \approx 1$ and $\bar{n}_1 \approx 2$. Notice that (14) is a natural generalisation of (9) when there are two kinds of strings.

As the slope parameter $B(\sqrt{s})$ is a direct measure of the square of the interaction radius $R^2(\sqrt{s})$, we write, from (7),

$$\frac{N_s(\sqrt{s})}{\eta} = \left( \frac{R(\sqrt{s})}{r_0} \right)^2 = cB(\sqrt{s}),$$ \hspace{1cm} (15)

with $c \approx 1.6$ GeV$^2$fm$^2$, such that at $\sqrt{s} \approx 10$ GeV, $R \approx 1$ fm and $B \approx 10$ GeV$^{-2}$.

By using equations (14) and (15), and experimental information on $\langle n(\sqrt{s}) \rangle$ [16] and on $B(\sqrt{s})$ [17], we can in principle construct $N_s(\sqrt{s})$ and $\eta(\sqrt{s})$: from $\eta(\sqrt{s})$ we obtain $k(\sqrt{s})$, Eq. (12).

In Fig. 2a we show our fit to $B(\sqrt{s})$ and in Fig. 2b our fit to $\langle n(\sqrt{s}) \rangle$. These parameterisations are then used in Eq. (15) and Eq. (14).

Intuitively, one expects the number of strings, $N_s$, to increase with $\sqrt{s}$. The results from our fits are shown in Fig. 3: $N_s$ increases with energy and, within a factor 2, is not substantially different from Monte Carlo direct calculations [23]. Naively, as $\eta \sim N_s$, one also expects $\eta$ to grow with energy. However, as the interaction area also grows with energy, the situation is not so clear. From (14) and (15) it is easily seen that the $\eta$ dependence on $\sqrt{s}$, $\eta(\sqrt{s})$, is determined by the ratio $\langle n(\sqrt{s}) \rangle / B(\sqrt{s})$. In Figure 3b we present $\eta(\sqrt{s})$; in the figure is shown as well the percolation threshold: $\eta_c \approx 1.15$. 
Finally, in Fig. 3c, we show the dependence of $k$ on energy, see (12) and Fig. 3b, in comparison with experimental points. If the “coins in boxes” model reflects the physical situation, we expect $k$ to reach a minimum at $\sqrt{s} \approx 6$ TeV, in the LHC energy region. The behaviour we predict is qualitatively different from what is expected on the basis of extrapolations of other phenomenological models: for instance, consider the Wróblewski regularity [19]: $D = a\langle n \rangle + b$; this leads to an asymptotic ($\langle n \rangle \to \infty$) KNO scaling, thus to a levelling off of $k$ (see dashed line in Fig. 3c.). Other proposals (e.g. $k^{-1} = a + b \ln \langle n \rangle$ in [20], and the Pythia Monte Carlo [22], respectively dotted line and cross in Fig. 3c) give a monotonically decreasing $k$; for a discussion of the situation in which $k$ continues to decrease at high energy and reaches values smaller than 1, see [21].

5 Conclusions

We have argued that the energy behaviour of the parameter $k$ (or $\langle N \rangle^2/D_N^2$) of the multiplicity distribution in $pp$ collisions is sensitive to the energy dependence of the parton/string transverse density $\eta$. If impact parameter overcrowding never occurs ($\eta$ is always small) $k$ will decrease with energy, reaching, eventually, some asymptotic constant value. If overcrowding takes place ($\eta$ increases rapidly with energy) then $k$ necessarily will change its behaviour and start increasing with energy. The turning point may occur at an energy of the
Fig. 3. (a) $N_s$ vs c.m. energy; (b) $\eta$ vs c.m. energy; (c) $k$ vs c.m. energy; experimental points from ISR and UA5 (filled points, [16]) and E735 (open squares, [18]). The values of the parameters used [see (14) and (15)] are: $\alpha = 0.45$, $\bar{n}_1 = 2.9$ and $c = 1.9$ GeV$^2$fm$^2$. The dashed line is a fit using Wróblewski regularity, $D = 0.59\langle n \rangle - 1.4$; the dotted line is a fit using $k^{-1} = 0.154\ln\langle n \rangle - 0.25$ (both fits have been carried out considering only data with $\sqrt{s} \geq 30$ GeV.) The cross is the prediction by the Pythia Monte Carlo with the most recent parameters optimisation [22].

It was shown in [25,11] that the parameter $k$ of the multiplicity distribution is the parameter $k$ of the $p_T$-distribution: $(1 + bp_T^2)^{-k}$. This parameter $k$ is also a decreasing function of energy in the $\sqrt{s} \leq 1.8$ TeV region. In Figure 4 we have compared experimental data on $p_T$ distributions to the mentioned parameterisation, with $k$ given by the multiplicity distribution values at the order of 5–10 TeV.
Fig. 4. $p_T$ distributions (not normalised) at several c.m. energies, (from [16,24]) fitted with the power-law $A(1 + bp_T^2)^{-k}$, using, for each energy, the respective value of $k$ obtained from the multiplicity distribution (from Fig. 3), $A$ and $b$ being free parameters.

same energy. The agreement is reasonable, and improving with energy.
6 Acknowledgements

R. Ugoccioni would like to thank for discussions A. Prokudin on elastic collisions and M. Monteno on Monte Carlo tuning. E.G. Ferreiro and C. Pajares thank the financial support of the CICYT of Spain through the contract FPA2002-01161 and of Xunta de Galicia through the contract PGIDIT03PXIC20612PN

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