Fission Barriers of Neutron-rich and Superheavy Nuclei calculated with the ETFSI Method.*

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Abstract

Using the ETFSI (extended Thomas-Fermi plus Strutinsky integral) method, we have calculated the fission barriers of nearly 2000 exotic nuclei, including all the neutron-rich nuclei up to $A = 318$ that are expected to be relevant to the r-process, and all superheavy nuclei in the vicinity of $N = 184$, with $Z \leq 120$. Our calculations were performed with the Skyrme force SkSC4, which was determined in the ETFSI-1 mass fit. For proton-deficient nuclei in the region of $N = 184$ we find the barriers to be much higher than previously believed, which suggests that the r-process path might continue to mass numbers well beyond 300. For the superheavy nuclei we typically find barrier heights of 6–7 MeV.

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1 Introduction

The r-process of stellar nucleosynthesis depends crucially on the masses and fission barriers (among other quantities) of nuclei that are so neutron-rich that there is no hope of being able to measure them in the laboratory (see Refs. [1, 2] for reviews discussing the nuclear data required for an understanding of the r-process). It is thus of the greatest importance to be able to make reliable extrapolations of these quantities away from the known region, relatively close to the stability line, out towards the neutron-drip line. Until recently the masses and barriers used in all studies of the r-process were calculated on the basis of one form or another of the liquid-drop(let) model (LDM). However, in an attempt to put the extrapolations on as rigorous a footing as possible we have developed a mass formula that is based entirely on microscopic forces, the ETFSI-1 mass formula [3, 4, 5, 6, 7]. Calculations of the r-process using the ETFSI-1 masses have already been performed [8, 9], but they are incomplete in that fission had to be neglected, barriers not yet having been calculated in the ETFSI model. Of course, recourse could have been made to the extensive barrier calculations [10, 11] based on the LDM that had been used in earlier r-process studies, but it would have been inconsistent to use one model for the masses (required for the neutron-separation energies $S_n$ and the beta-decay energies $Q_\beta$), and another for the barriers.

Here we remedy this deficiency by presenting the results of ETFSI-method calculations of the fission barriers of all of the nearly 2000 nuclei lying in the region of the $(N,Z)$ plane shown in Fig. 1. This region covers the range $84 \leq Z \leq 120$, and extends from moderately neutron-deficient to extremely neutron-rich nuclei, including thereby not only a large fraction of the nuclei whose barriers are known experimentally, but also all nuclei with $A \leq 318$ whose barriers can reasonably be expected to be relevant to the r-process. Towards the upper limit of our range of $Z$ values the r-process path lies at much higher values of $A$ (if it has not already been terminated by fission), and we restrict ourselves to nuclei lying close to the stability line in the long-expected “island” of stability that is now becoming experimentally accessible [12, 13, 14].

The ETFSI method is a high-speed approximation to the Skyrme-Hartree-Fock (SHF) method, with pairing correlations generated by a $\delta$-function force, treated in the usual BCS approach (with blocking). There are two parts to the total energy calculated by the ETFSI method, the first consisting of a purely semi-classical approximation to the SHF method, the
full fourth-order extended Thomas-Fermi (ETF) method, while the second part, which is based on what we call the Strutinsky-integral (SI) form of the Strutinsky theorem, constitutes an attempt to improve this approximation perturbatively, and in particular to restore the shell corrections that are missing from the ETF part. The way in which we extended the ETFSI method, developed originally for the calculation of ground-state energies, to the large-scale calculation of fission barriers is described in Ref. [15], which should be consulted for all details of our calculational methods. However, it is important to recall here that our fission paths are optimized with respect to the elongation parameter $c$, the necking parameter $h$, and the asymmetry parameter $\tilde{\alpha}$ (see Ref. [15], and in particular its App. A, for the definition of these quantities, and a description of the nuclear shapes allowed by our parametrization in the case of fission; note that as in that reference we are assuming axial symmetry).

In Ref. [15] we calculated all the barriers that have been measured in nuclei with $Z \geq 81$. The comparison of our results with the available data showed that if the primary, i.e., highest, barrier is lower than 10 MeV the error never exceeds 1.4 MeV, with either sign being possible, while for primary barrier heights lying between 10 and 15 MeV we probably overestimate the height by about 1.5 MeV; barrier heights in excess of 15 MeV are overestimated by 2 MeV or more. Since the primary barrier is the only one that is really relevant to the r-process we see that our method is sufficiently accurate for the calculation of barriers that are low enough to be of interest to the r-process (see Section 3 below). The calculations here are performed in exactly the same way, and in particular we use the same force parameters, set SkSC4, which, it should be noted, were determined entirely by the mass fit [6], and have in no way been modified for the barrier calculations.

## 2 Results

We summarize the results of our calculations in Fig. 1, and in Table 1, where for each nucleus we present the primary barrier. In order to keep its size within reasonable bounds, Table 1 does not show the three deformation parameters of the barriers, $c$, $h$, and $\tilde{\alpha}$. However, these parameters can be found on our web site (http://www-astro.ulb.ac.be/), which gives also the value of the secondary (i.e., second-highest) barrier, when present, as well as its deformation parameters. As mentioned in Ref. [15], it is possible to
Figure 1: Schematic representation in the $(N,Z)$ plane of the primary fission barriers displayed in Table 1. The energy bins are described in the figure. Thin solid lines represent r-process paths for constant values (in MeV) of $S_a$ (defined in text). The neutron-drip line is shown by the thick solid line.
distinguish without ambiguity between “inner” and “outer” barriers from their value of \(c\), each of the two highest barriers belonging to a different category. A typical situation is illustrated in Fig. 2 for the uranium isotope chain. For smaller values of \(Z\) the separation between primary and secondary barriers, expressed in terms of \(c\), tends to increase, and the external barrier becomes the primary one for all isotopes, as it is already in Fig. 2 for large neutron excesses (this can be explained by the fact that nuclei with smaller fissility need higher deformation to fission). For larger \(Z\) this separation generally decreases and the inner barrier is always the highest, while the outer one progressively disappears, so that for \(Z > 100\) only one barrier (the inner one) is left. We label the primary barriers displayed in Table [1] with the superscript \(i\) or \(o\) to show whether they belong to the inner or to the outer category, respectively.

The general trends of the results of Table [1] are displayed in Fig. 1 as well as by the curves labelled “ETFSI-1” in Figs. 3–5, where we show three isotope chains, \(Z = 84, 92,\) and \(100\), respectively (note that in Fig. 3 the ETFSI-1 barriers for \(A = 207–212\), not included in Table [1], are taken from Ref. [15]).

The curve labelled “ETF” denotes the results we find for pure ETF calculations of the barriers without either shell or pairing corrections. In these latter calculations we search anew the positions of the ground states and saddle points, as defined by their three deformation parameters, and find different deformations for these points than obtained in the ETFSI calculation. We emphasize that if shell and pairing corrections had simply been subtracted from the ETFSI saddle-point and ground-state energies, the ETF curve would not be as smooth as it is in Figs. 3–5.

Also seen on these graphs are the results of the LDM calculations of Howard and Möller (labelled “HM”) [10], and the results of Myers and Swiatecki (labelled “MS”) [16], based on zeroth-order Thomas-Fermi calculations [17]. Both these latter sets of results include shell corrections calculated, in one way or another, by the Strutinsky method. Actually, Ref. [16] gives just a smooth formula representing the main trends of Thomas-Fermi barrier calculations, which we reproduce here as the curve “MS.0”. We have constructed the curve “MS” ourselves, following the prescription of Ref. [16], i.e., we have added the shell corrections quoted in that paper (see their Ref. [4]) to the ground-state energy only, assuming there are no shell corrections at the saddle points (this is their “topographic” theorem).
Figure 2: (a) Inner (solid curve) and outer (dot-dashed curve) barriers for the $Z = 92$ isotopic chain. The (single) ETF barrier is also shown (dashed curve). (b) The corresponding elongation parameter $c$. 
Table 1: Heights (in MeV) of primary barriers (\(^i\) denotes inner, \(^o\) outer – see text)

| Z | N | B | Z | N | B | Z | N | B | Z | N | B | Z | N | B | Z | N | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 84 | 130 | 22.5° | 84 | 160 | 21.5° | 85 | 136 | 15.4° | 85 | 166 | 21.2° | 86 | 143 | 13.4° | 86 | 173 | 24.6° |
| 131 | 21.6° | 161 | 21.3° | 137 | 15.1° | 167 | 21.6° | 144 | 13.8° | 174 | 24.5° |
| 132 | 20.9° | 162 | 21.5° | 138 | 14.9° | 168 | 22.5° | 145 | 14.2° | 175 | 25.3° |
| 133 | 20.0° | 163 | 21.5° | 139 | 14.8° | 169 | 22.9° | 146 | 14.7° | 176 | 25.8° |
| 134 | 19.4° | 164 | 22.1° | 140 | 14.7° | 170 | 24.4° | 147 | 15.2° | 177 | 24.8° |
| 135 | 18.5° | 165 | 22.3° | 141 | 14.8° | 171 | 25.2° | 148 | 16.3° | 178 | 26.7° |
| 136 | 18.3° | 166 | 23.5° | 142 | 15.2° | 172 | 26.4° | 149 | 15.1° | 179 | 27.2° |
| 137 | 17.2° | 167 | 23.8° | 143 | 15.6° | 173 | 26.2° | 150 | 16.2° | 180 | 28.5° |
| 138 | 17.6° | 168 | 24.8° | 144 | 16.3° | 174 | 27.0° | 151 | 15.7° | 181 | 29.7° |
| 139 | 16.6° | 169 | 25.4° | 145 | 16.0° | 175 | 27.4° | 152 | 16.3° | 182 | 31.5° |
| 140 | 16.8° | 170 | 26.9° | 146 | 16.7° | 176 | 28.1° | 153 | 16.4° | 183 | 32.9° |
| 141 | 17.4° | 171 | 27.2° | 147 | 16.5° | 177 | 28.2° | 154 | 16.6° | 184 | 33.5° |
| 142 | 17.8° | 172 | 28.9° | 148 | 16.7° | 178 | 29.1° | 155 | 17.3° | 133 | 12.4° |
| 143 | 17.9° | 173 | 29.5° | 149 | 17.3° | 179 | 29.8° | 156 | 17.1° | 134 | 12.3° |
| 144 | 18.7° | 174 | 29.3° | 150 | 17.2° | 180 | 31.2° | 157 | 16.8° | 135 | 12.0° |
| 145 | 18.9° | 175 | 30.0° | 151 | 17.5° | 181 | 32.3° | 158 | 16.9° | 136 | 11.8° |
| 146 | 19.2° | 176 | 30.1° | 152 | 17.8° | 182 | 34.0° | 159 | 16.9° | 137 | 12.0° |
| 147 | 18.6° | 177 | 30.5° | 153 | 18.4° | 183 | 35.6° | 160 | 17.1° | 138 | 10.4° |
| 148 | 18.9° | 178 | 31.8° | 154 | 18.6° | 184 | 36.3° | 161 | 17.1° | 139 | 10.4° |
| 149 | 18.8° | 179 | 32.2° | 155 | 19.0° | 86 | 132 | 14.8° | 162 | 17.5° | 140 | 10.4° |
| 150 | 19.9° | 180 | 34.0° | 156 | 19.4° | 133 | 13.9° | 163 | 17.9° | 141 | 10.4° |
| 151 | 19.3° | 181 | 35.5° | 157 | 19.1° | 134 | 13.4° | 164 | 17.7° | 142 | 10.4° |
| 152 | 20.7° | 182 | 37.3° | 158 | 19.1° | 135 | 12.8° | 165 | 18.1° | 143 | 10.8° |
| 153 | 20.9° | 183 | 38.7° | 159 | 18.9° | 136 | 12.7° | 166 | 18.7° | 144 | 11.1° |
| 154 | 21.2° | 184 | 39.0° | 160 | 19.1° | 137 | 12.6° | 167 | 19.5° | 145 | 11.5° |
| 155 | 22.0° | 85 | 131 | 18.4° | 161 | 19.2° | 138 | 12.2° | 168 | 20.0° | 146 | 12.0° |
| 156 | 22.0° | 132 | 17.7° | 162 | 19.6° | 139 | 12.2° | 169 | 20.4° | 147 | 12.4° |
| 157 | 22.0° | 133 | 16.9° | 163 | 19.5° | 140 | 12.1° | 170 | 21.5° | 148 | 13.0° |
| 158 | 21.6° | 134 | 16.3° | 164 | 19.7° | 141 | 12.2° | 171 | 23.4° | 149 | 13.5° |
| 159 | 21.4° | 135 | 15.5° | 165 | 20.2° | 142 | 12.8° | 172 | 23.8° | 150 | 14.5° |
| Z  | N   | B   | Z  | N   | B   | Z  | N   | B   | Z  | N   | B   | Z  | N   | B   |
|----|-----|-----|----|-----|-----|----|-----|-----|----|-----|-----|----|-----|-----|
| 87 | 151 | 14.5°| 87 | 181 | 26.9°| 88 | 160 | 12.8°| 89 | 140 | 8.0°| 89 | 170 | 14.0°|
| 152 | 14.0°| 182 | 28.6°| 161 | 12.2°| 141 | 8.1°| 171 | 14.5°| 152 | 9.2°|
| 153 | 13.9°| 183 | 30.1°| 162 | 14.1°| 142 | 8.3°| 172 | 15.4°| 153 | 9.0°|
| 154 | 14.1°| 184 | 30.9°| 163 | 13.6°| 143 | 8.6°| 173 | 16.4°| 154 | 8.9°|
| 155 | 14.4°| 88  | 134 | 9.3°| 164 | 14.4°| 144 | 8.5°| 174 | 17.3°| 155 | 8.9°|
| 156 | 14.5°| 135 | 9.3°| 165 | 14.4°| 145 | 8.8°| 175 | 18.0°| 156 | 8.3°|
| 157 | 14.4°| 136 | 9.1°| 166 | 14.1°| 146 | 8.7°| 176 | 19.4°| 157 | 8.3°|
| 158 | 14.6°| 137 | 9.0°| 167 | 15.1°| 147 | 9.2°| 177 | 19.0°| 158 | 8.8°|
| 159 | 14.4°| 138 | 8.8°| 168 | 15.9°| 148 | 9.3°| 178 | 18.7°| 159 | 8.5°|
| 160 | 14.7°| 139 | 8.9°| 169 | 15.9°| 149 | 9.7°| 179 | 18.4°| 160 | 9.7°|
| 161 | 15.5°| 140 | 9.0°| 170 | 16.1°| 150 | 9.7°| 180 | 20.1°| 161 | 8.2°|
| 162 | 15.1°| 141 | 9.4°| 171 | 16.6°| 151 | 9.7°| 181 | 21.5°| 162 | 8.3°|
| 163 | 16.0°| 142 | 9.2°| 172 | 19.3°| 152 | 9.9°| 182 | 22.6°| 163 | 10.1°|
| 164 | 16.4°| 143 | 9.5°| 173 | 20.3°| 153 | 10.8°| 183 | 24.6°| 164 | 10.3°|
| 165 | 15.8°| 144 | 9.5°| 174 | 20.2°| 154 | 9.9°| 184 | 25.9°| 165 | 8.7°|
| 166 | 17.2°| 145 | 9.4°| 175 | 20.9°| 155 | 10.8°| 90  | 136 | 8.2°| 166 | 10.5°|
| 167 | 17.5°| 146 | 9.7°| 176 | 21.1°| 156 | 9.8°| 137 | 6.4°| 167 | 9.4°|
| 168 | 18.3°| 147 | 9.9°| 177 | 22.1°| 157 | 11.0°| 138 | 6.4°| 168 | 10.0°|
| 169 | 18.5°| 148 | 10.0°| 178 | 21.2°| 158 | 9.7°| 139 | 6.9°| 169 | 11.7°|
| 170 | 18.9°| 149 | 10.9°| 179 | 21.2°| 159 | 9.6°| 140 | 6.8°| 170 | 11.5°|
| 171 | 19.3°| 150 | 11.2°| 180 | 23.7°| 160 | 9.5°| 141 | 7.2°| 171 | 12.1°|
| 172 | 21.6°| 151 | 11.4°| 181 | 24.4°| 161 | 9.9°| 142 | 7.1°| 172 | 13.7°|
| 173 | 21.8°| 152 | 12.7°| 182 | 26.0°| 162 | 12.0°| 143 | 6.8°| 173 | 14.0°|
| 174 | 23.0°| 153 | 12.5°| 183 | 27.4°| 163 | 10.0°| 144 | 7.5°| 174 | 15.0°|
| 175 | 22.9°| 154 | 12.5°| 184 | 28.5°| 164 | 10.5°| 145 | 7.6°| 175 | 15.3°|
| 176 | 23.4°| 155 | 12.1°| 89  | 135 | 8.2°| 165 | 10.7°| 146 | 7.9°| 176 | 16.1°|
| 177 | 22.2°| 156 | 12.3°| 136 | 8.0°| 166 | 11.3°| 147 | 8.2°| 177 | 16.8°|
| 178 | 23.6°| 157 | 12.4°| 137 | 7.9°| 167 | 11.7°| 148 | 8.0°| 178 | 16.1°|
| 179 | 23.5°| 158 | 12.4°| 138 | 7.7°| 168 | 12.4°| 149 | 8.6°| 179 | 18.1°|
| 180 | 26.2°| 159 | 12.6°| 139 | 8.1°| 169 | 13.0°| 150 | 9.6°| 180 | 17.4°|
| $Z$ | $N$ | $B$ | $Z$ | $N$ | $B$ | $Z$ | $N$ | $B$ | $Z$ | $N$ | $B$ | $Z$ | $N$ | $B$ |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 90  | 181 | 18.3 | 91  | 163 | 7.4  | 92  | 144 | 5.2  | 92  | 174 | 9.9  | 93  | 154 | 5.8i |
| 182 | 20.5 | 164 | 6.3  | 145 | 5.7i | 175 | 10.6 | 155 | 5.4i | 185 | 14.8 |
| 183 | 22.0 | 165 | 8.5  | 146 | 5.7i | 176 | 10.5 | 156 | 5.2i | 186 | 14.0 |
| 184 | 23.5 | 166 | 8.2  | 147 | 6.1i | 177 | 11.2 | 157 | 5.3i | 187 | 12.7 |
| 91  | 137 | 4.6  | 167 | 8.4  | 148 | 6.3  | 178 | 11.8 | 158 | 4.8i | 188 | 12.8 |
| 138 | 5.3  | 168 | 8.8  | 149 | 6.3i | 179 | 12.5 | 159 | 5.2i | 189 | 10.9 |
| 139 | 6.0  | 169 | 9.2  | 150 | 6.0  | 180 | 13.0 | 160 | 4.5i | 190 | 11.8 |
| 140 | 5.8  | 170 | 7.9  | 151 | 6.4  | 181 | 13.8 | 161 | 4.8i | 191 | 9.8  |
| 141 | 5.9  | 171 | 8.8  | 152 | 5.9i | 182 | 14.9 | 162 | 4.3i | 192 | 8.9  |
| 142 | 6.0  | 172 | 10.3 | 153 | 5.9i | 183 | 15.8 | 163 | 4.4i | 193 | 8.9i |
| 143 | 5.4i | 173 | 11.0 | 154 | 5.6i | 184 | 17.7 | 164 | 4.4i | 194 | 9.1  |
| 144 | 5.7i | 174 | 12.1 | 155 | 5.8i | 185 | 17.7 | 165 | 4.4i | 195 | 9.4  |
| 145 | 5.7  | 175 | 12.3 | 156 | 5.4  | 186 | 16.6 | 166 | 4.6i | 196 | 9.4  |
| 146 | 6.7  | 176 | 13.0 | 157 | 5.6  | 187 | 16.2 | 167 | 4.6i | 197 | 9.9  |
| 147 | 5.8i | 177 | 14.0 | 158 | 5.4i | 188 | 15.2 | 168 | 3.8i | 198 | 11.0 |
| 148 | 7.9  | 178 | 14.3 | 159 | 5.7i | 139 | 4.4i | 169 | 3.9i | 94  | 140 | 4.2i |
| 149 | 6.1i | 179 | 15.5 | 160 | 5.4i | 140 | 4.3i | 170 | 3.9i | 141 | 4.7i |
| 150 | 7.9  | 180 | 14.8 | 161 | 5.5  | 141 | 5.0i | 171 | 4.2i | 142 | 4.8i |
| 151 | 7.7  | 181 | 15.5 | 162 | 6.2i | 142 | 4.9i | 172 | 4.2i | 143 | 5.5i |
| 152 | 7.6  | 182 | 16.9 | 163 | 5.9i | 143 | 5.7i | 173 | 6.0i | 144 | 5.4i |
| 153 | 7.5  | 183 | 19.1 | 164 | 6.1i | 144 | 5.4i | 174 | 7.2i | 145 | 5.8i |
| 154 | 7.2  | 184 | 20.5 | 165 | 6.8i | 145 | 6.0i | 175 | 7.7i | 146 | 5.8i |
| 155 | 7.0  | 185 | 19.8 | 166 | 6.3i | 146 | 5.9i | 176 | 7.9i | 147 | 6.4i |
| 156 | 6.9  | 186 | 18.7 | 167 | 6.3i | 147 | 6.3i | 177 | 8.9i | 148 | 6.2i |
| 157 | 6.4i | 92  | 138 | 3.9i | 168 | 6.7i | 148 | 6.1i | 178 | 9.0i | 149 | 6.7i |
| 158 | 6.5  | 139 | 4.3i | 169 | 7.2i | 149 | 6.4i | 179 | 10.3i | 150 | 6.4i |
| 159 | 6.5  | 140 | 4.2i | 170 | 5.3i | 150 | 6.1i | 180 | 10.9i | 151 | 6.7i |
| 160 | 6.2i | 141 | 4.7i | 171 | 6.0i | 151 | 6.5i | 181 | 12.0i | 152 | 6.2i |
| 161 | 7.5i | 142 | 4.8i | 172 | 7.6i | 152 | 6.1i | 182 | 12.4i | 153 | 6.3i |
| 162 | 7.5i | 143 | 5.4i | 173 | 9.5i | 153 | 6.2i | 183 | 12.7i | 154 | 5.9i |
| Z | N | B | Z | N | B | Z | N | B | Z | N | B |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 94 | 155 | 6.2i | 94 | 185 | 11.1° | 95 | 149 | 6.9i | 95 | 179 | 7.2i |
| 186 | 5.5i | 186 | 9.6° | 150 | 6.6i | 180 | 7.4i | 143 | 5.5i | 173 | 4.7i |
| 187 | 5.6i | 187 | 9.4° | 151 | 6.9i | 181 | 8.4i | 144 | 5.5i | 174 | 4.0i |
| 188 | 5.2i | 188 | 8.9° | 152 | 6.5i | 182 | 8.5i | 145 | 6.2i | 175 | 5.7i |
| 189 | 5.5i | 189 | 8.7° | 153 | 6.5i | 183 | 9.5i | 146 | 6.1i | 176 | 5.5i |
| 190 | 5.0i | 190 | 8.0° | 154 | 6.2i | 184 | 9.4° | 147 | 6.6i | 177 | 6.1i |
| 191 | 4.8i | 191 | 8.5i | 155 | 6.1i | 185 | 9.0° | 148 | 6.4i | 178 | 5.8i |
| 192 | 4.5i | 192 | 7.7i | 156 | 5.8i | 186 | 7.8i | 149 | 6.7i | 179 | 6.6i |
| 193 | 4.6i | 193 | 7.7i | 157 | 5.8i | 187 | 7.9i | 150 | 6.5i | 180 | 6.6i |
| 194 | 4.2i | 194 | 7.1i | 158 | 5.4i | 188 | 7.0i | 151 | 6.7i | 181 | 7.6i |
| 195 | 4.3i | 195 | 8.0i | 159 | 5.3i | 189 | 7.6i | 152 | 6.5i | 182 | 7.8i |
| 196 | 3.8i | 196 | 8.0i | 160 | 4.7i | 190 | 6.9i | 153 | 6.7i | 183 | 8.6i |
| 197 | 3.6i | 197 | 8.6i | 161 | 4.8i | 191 | 7.3i | 154 | 6.1i | 184 | 8.3i |
| 198 | 3.8i | 198 | 9.1i | 162 | 4.6i | 192 | 6.4i | 155 | 6.4i | 185 | 8.3i |
| 199 | 4.2i | 199 | 9.3i | 163 | 4.5i | 193 | 6.5i | 156 | 5.9i | 186 | 6.6i |
| 200 | 3.8i | 200 | 9.8i | 164 | 4.1i | 194 | 5.8i | 157 | 5.5i | 187 | 7.4i |
| 201 | 3.7i | 201 | 12.1° | 165 | 4.3i | 195 | 6.7i | 158 | 5.0i | 188 | 6.4i |
| 202 | 4.1i | 202 | 10.5° | 166 | 3.9i | 196 | 6.9i | 159 | 5.2i | 189 | 6.7i |
| 203 | 6.0i | 203 | 10.9° | 167 | 4.0i | 197 | 7.5i | 160 | 4.7i | 190 | 6.0i |
| 204 | 5.9i | 204 | 11.1° | 168 | 3.6i | 198 | 7.6i | 161 | 4.9i | 191 | 6.4i |
| 205 | 6.8° | 205 | 11.6° | 169 | 3.8i | 199 | 7.9i | 162 | 4.5i | 192 | 5.6i |
| 206 | 6.9° | 206 | 11.5° | 170 | 3.5i | 200 | 8.3i | 163 | 4.5i | 193 | 5.6i |
| 207 | 7.9° | 95 | 141 | 4.9i | 171 | 3.9i | 201 | 9.1i | 164 | 4.2i | 194 | 4.9i |
| 172 | 5.1i | 172 | 3.4i | 202 | 8.9i | 165 | 4.2i | 195 | 5.3i | 179 | 4.7i |
| 180 | 5.7i | 180 | 5.7i | 174 | 5.3i | 204 | 9.5i | 167 | 3.6i | 197 | 6.3i |
| 181 | 6.3i | 181 | 6.1i | 175 | 6.1i | 205 | 9.9i | 168 | 3.2i | 198 | 6.7i |
| 182 | 6.3i | 182 | 6.0i | 176 | 6.0i | 206 | 9.9i | 169 | 3.6i | 199 | 7.3i |
| 183 | 6.8i | 183 | 6.7i | 177 | 6.7i | 207 | 10.2° | 170 | 3.1i | 200 | 7.4i |
| 184 | 6.5i | 184 | 6.3i | 178 | 6.3i | 208 | 10.1° | 171 | 3.3i | 201 | 8.0i |
| Z | N | B | Z | N | B | Z | N | B | Z | N | B |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 96 | 202 | 8.0i | 97 | 164 | 4.2i | 97 | 194 | 3.8i | 98 | 156 | 5.7i |
| 204 | 8.3i | 166 | 3.8i | 195 | 4.8i | 157 | 5.8i | 187 | 5.0i | 149 | 7.3i |
| 205 | 8.5i | 167 | 3.8i | 197 | 5.1i | 159 | 5.2i | 189 | 4.4i | 151 | 7.6i |
| 206 | 8.4i | 168 | 3.3i | 198 | 6.0i | 160 | 4.8i | 190 | 3.6i | 152 | 7.3i |
| 207 | 9.0i | 169 | 3.2i | 199 | 7.8o | 161 | 4.9i | 191 | 3.9i | 153 | 6.9i |
| 208 | 8.5i | 170 | 3.0i | 200 | 6.0o | 162 | 4.5i | 192 | 2.9i | 154 | 6.5i |
| 209 | 8.9i | 171 | 2.9i | 201 | 7.9o | 163 | 4.3i | 193 | 3.0i | 155 | 6.5i |
| 210 | 8.6i | 172 | 2.8i | 202 | 7.5o | 164 | 4.0i | 194 | 2.0i | 156 | 6.0i |
| 97 | 143 | 6.0i | 173 | 3.2i | 203 | 7.9o | 165 | 4.1i | 195 | 3.3i | 157 | 6.1i |
| 144 | 5.9i | 174 | 3.1i | 204 | 7.7o | 166 | 3.6i | 196 | 3.8i | 158 | 5.6i |
| 145 | 6.3i | 175 | 5.0i | 205 | 7.8o | 167 | 3.6i | 197 | 4.3i | 159 | 5.4i |
| 146 | 6.4i | 176 | 4.6i | 206 | 8.0o | 168 | 3.1i | 198 | 5.0i | 160 | 5.0i |
| 147 | 6.8i | 177 | 5.4i | 207 | 8.4o | 169 | 2.9i | 199 | 5.5i | 161 | 4.8i |
| 148 | 6.7i | 178 | 5.3i | 208 | 8.0o | 170 | 2.5i | 200 | 5.7i | 162 | 4.6i |
| 149 | 7.2i | 179 | 5.6i | 209 | 8.1o | 171 | 2.7i | 201 | 6.1i | 163 | 4.5i |
| 150 | 6.9i | 180 | 5.6i | 210 | 8.1o | 172 | 2.4i | 202 | 6.2i | 164 | 4.1i |
| 151 | 7.1i | 181 | 6.6i | 211 | 8.4o | 173 | 2.7i | 203 | 6.8i | 165 | 4.0i |
| 152 | 6.9i | 182 | 6.6i | 98 | 144 | 5.5i | 174 | 2.8i | 204 | 6.6i | 166 | 3.6i |
| 153 | 7.1i | 183 | 7.3i | 145 | 6.0i | 175 | 4.2i | 205 | 7.2o | 167 | 3.6i |
| 154 | 6.6i | 184 | 7.5i | 146 | 6.1i | 176 | 4.2i | 206 | 7.3o | 168 | 3.0i |
| 155 | 6.4i | 185 | 7.1i | 147 | 6.7i | 177 | 5.0i | 207 | 7.4o | 169 | 2.9i |
| 156 | 5.9i | 186 | 5.8i | 148 | 6.5i | 178 | 5.0i | 208 | 6.8o | 170 | 2.3i |
| 157 | 5.8i | 187 | 5.3i | 149 | 6.9i | 179 | 5.3i | 209 | 7.4o | 171 | 2.3i |
| 158 | 5.2i | 188 | 4.5i | 150 | 6.7i | 180 | 5.3i | 210 | 7.0o | 172 | 2.0i |
| 159 | 5.3i | 189 | 4.7i | 151 | 6.9i | 181 | 6.1i | 211 | 7.3o | 173 | 2.4i |
| 160 | 4.9i | 190 | 3.8i | 152 | 6.7i | 182 | 6.1i | 212 | 7.1o | 174 | 2.1i |
| 161 | 5.0i | 191 | 5.5i | 153 | 7.0i | 183 | 7.0i | 99 | 145 | 6.6i | 175 | 3.6i |
| 162 | 4.5i | 192 | 4.4i | 154 | 6.2i | 184 | 6.8i | 146 | 6.5i | 176 | 3.9i |
| 163 | 4.5i | 193 | 4.7i | 155 | 6.3i | 185 | 6.5i | 147 | 7.1i | 177 | 4.4i |
| Z | 178 | 3.6 | 99 | 208 | 6.3 | 100 | 170 | 2.2 | 100 | 200 | 4.0 | 101 | 162 | 4.5 | 101 | 192 | 1.6 |
|---|-----|-----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 179 | 4.4 | 209 | 6.6 | 171 | 2.2 | 201 | 4.7 | 163 | 4.3 | 193 | 1.2 |
| 180 | 4.4 | 210 | 6.2 | 172 | 1.8 | 202 | 4.7 | 164 | 4.0 | 194 | 1.7 |
| 181 | 5.1 | 211 | 6.4 | 173 | 2.3 | 203 | 5.1 | 165 | 3.6 | 195 | 1.9 |
| 182 | 5.0 | 212 | 6.5 | 174 | 1.8 | 204 | 5.3 | 166 | 3.2 | 196 | 2.8 |
| 183 | 6.2 | 213 | 6.8 | 175 | 3.1 | 205 | 5.4 | 167 | 3.1 | 197 | 3.1 |
| 184 | 5.9 | 100 | 146 | 5.9 | 176 | 2.6 | 206 | 5.9 | 168 | 2.6 | 198 | 3.6 |
| 185 | 5.3 | 147 | 6.5 | 177 | 4.0 | 207 | 5.8 | 169 | 2.2 | 199 | 4.2 |
| 186 | 4.8 | 148 | 6.3 | 178 | 3.3 | 208 | 6.0 | 170 | 1.9 | 200 | 4.3 |
| 187 | 4.7 | 149 | 6.8 | 179 | 4.2 | 209 | 6.4 | 171 | 1.7 | 201 | 4.7 |
| 188 | 3.7 | 150 | 6.7 | 180 | 4.3 | 210 | 7.3 | 172 | 1.4 | 202 | 5.0 |
| 189 | 4.0 | 151 | 6.8 | 181 | 5.2 | 211 | 6.3 | 173 | 1.8 | 203 | 5.3 |
| 190 | 3.2 | 152 | 6.3 | 182 | 5.1 | 212 | 6.4 | 174 | 1.3 | 204 | 5.7 |
| 191 | 3.6 | 153 | 6.4 | 183 | 6.3 | 213 | 8.3 | 175 | 2.4 | 205 | 5.6 |
| 192 | 2.6 | 154 | 5.8 | 184 | 6.0 | 214 | 8.4 | 176 | 1.9 | 206 | 6.4 |
| 193 | 2.4 | 155 | 6.0 | 185 | 5.5 | 101 | 147 | 6.4 | 177 | 3.2 | 207 | 6.1 |
| 194 | 2.0 | 156 | 5.5 | 186 | 4.3 | 148 | 6.5 | 178 | 3.0 | 208 | 7.0 |
| 195 | 2.1 | 157 | 5.6 | 187 | 4.0 | 149 | 6.6 | 179 | 3.9 | 209 | 7.5 |
| 196 | 2.5 | 158 | 5.1 | 188 | 3.4 | 150 | 6.4 | 180 | 3.4 | 210 | 7.3 |
| 197 | 2.8 | 159 | 5.2 | 189 | 3.5 | 151 | 6.8 | 181 | 4.7 | 211 | 6.6 |
| 198 | 3.6 | 160 | 4.8 | 190 | 2.7 | 152 | 6.4 | 182 | 4.5 | 212 | 6.5 |
| 199 | 4.5 | 161 | 4.7 | 191 | 2.9 | 153 | 6.5 | 183 | 5.7 | 213 | 8.5 |
| 200 | 4.6 | 162 | 4.4 | 192 | 2.1 | 154 | 6.0 | 184 | 5.3 | 214 | 7.8 |
| 201 | 5.1 | 163 | 4.1 | 193 | 1.9 | 155 | 6.2 | 185 | 5.3 | 215 | 8.2 |
| 202 | 5.1 | 164 | 3.8 | 194 | 1.6 | 156 | 5.7 | 186 | 4.2 | 102 | 148 | 5.8 |
| 203 | 5.4 | 165 | 3.8 | 195 | 1.7 | 157 | 6.1 | 187 | 3.4 | 149 | 6.0 |
| 204 | 5.4 | 166 | 3.3 | 196 | 2.1 | 158 | 5.3 | 188 | 2.7 | 150 | 5.8 |
| 205 | 5.7 | 167 | 3.2 | 197 | 3.1 | 159 | 5.3 | 189 | 3.0 | 151 | 6.2 |
| 206 | 5.8 | 168 | 2.7 | 198 | 3.4 | 160 | 5.0 | 190 | 2.2 | 152 | 5.7 |
| 207 | 6.3 | 169 | 2.6 | 199 | 4.1 | 161 | 4.8 | 191 | 2.4 | 153 | 5.8 |
| Z  | N  | B  | Z  | N  | B  | Z  | N  | B  | Z  | N  | B  | Z  | N  | B  |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 102 | 154 | 5.6i | 102 | 184 | 5.6i | 102 | 214 | 7.6i | 103 | 176 | 1.2i | 103 | 206 | 6.1i |
| 155 | 5.8i | 185 | 5.5i | 215 | 8.0i | 177 | 2.4i | 207 | 6.6i | 171 | 1.5i |
| 156 | 5.3i | 186 | 4.1i | 216 | 7.7i | 178 | 2.5i | 208 | 6.7i | 172 | .8i |
| 157 | 5.5i | 187 | 4.2i | 103 | 149 | 6.0i | 179 | 3.7i | 209 | 7.0i | 173 | 1.3i |
| 158 | 5.0i | 188 | 2.4i | 150 | 5.9i | 180 | 3.3i | 210 | 7.1i | 174 | 1.0i |
| 159 | 5.1i | 189 | 2.5i | 151 | 6.3i | 181 | 4.2i | 211 | 7.6i | 175 | 1.5i |
| 160 | 4.8i | 190 | 1.7i | 152 | 5.9i | 182 | 4.1i | 212 | 7.5i | 176 | 1.6i |
| 161 | 4.6i | 191 | 2.2i | 153 | 6.1i | 183 | 5.3i | 213 | 7.9i | 177 | 2.7i |
| 162 | 4.3i | 192 | 1.2i | 154 | 5.8i | 184 | 4.9i | 214 | 7.7i | 178 | 2.7i |
| 163 | 4.0i | 193 | .9i  | 155 | 6.1i | 185 | 4.8i | 215 | 8.1i | 179 | 3.9i |
| 164 | 3.7i | 194 | 1.1i | 156 | 5.6i | 186 | 4.2i | 104 | 150 | 5.3i | 180 | 3.7i |
| 165 | 3.6i | 195 | 2.2i | 157 | 5.7i | 187 | 4.0i | 151 | 5.6i | 181 | 5.0i |
| 166 | 3.0i | 196 | 2.6i | 158 | 5.2i | 188 | 1.7i | 152 | 5.3i | 182 | 4.7i |
| 167 | 2.5i | 197 | 2.8i | 159 | 5.4i | 189 | 2.1i | 153 | 5.6i | 183 | 5.8i |
| 168 | 2.5i | 198 | 3.3i | 160 | 4.9i | 190 | 1.2i | 154 | 5.2i | 184 | 5.6i |
| 169 | 2.3i | 199 | 3.8i | 161 | 4.9i | 191 | 1.3i | 155 | 5.3i | 185 | 4.8i |
| 170 | 1.7i | 200 | 3.9i | 162 | 4.4i | 192 | .3i  | 156 | 5.0i | 186 | 4.0i |
| 171 | 1.7i | 201 | 4.4i | 163 | 4.0i | 193 | .3i  | 157 | 5.0i | 187 | 3.9i |
| 172 | 1.3i | 202 | 4.6i | 164 | 3.7i | 194 | 1.9i | 158 | 4.7i | 188 | 2.9i |
| 173 | 1.7i | 203 | 5.0i | 165 | 3.7i | 195 | 2.2i | 159 | 4.6i | 189 | 2.9i |
| 174 | 1.3i | 204 | 5.4i | 166 | 3.1i | 196 | 2.6i | 160 | 4.4i | 190 | 1.9i |
| 175 | 1.7i | 205 | 5.7i | 167 | 2.9i | 197 | 2.9i | 161 | 4.4i | 191 | 1.9i |
| 176 | 1.7i | 206 | 6.0i | 168 | 2.4i | 198 | 3.3i | 162 | 4.0i | 192 | .4i |
| 177 | 3.0i | 207 | 5.7i | 169 | 2.2i | 199 | 3.8i | 163 | 3.6i | 193 | 1.0i |
| 178 | 2.8i | 208 | 6.5i | 170 | 1.6i | 200 | 4.0i | 164 | 3.3i | 194 | 1.5i |
| 179 | 4.6i | 209 | 7.0i | 171 | 1.4i | 201 | 4.4i | 165 | 3.0i | 195 | 1.9i |
| 180 | 3.8i | 210 | 7.1i | 172 | 1.1i | 202 | 4.7i | 166 | 2.5i | 196 | 2.1i |
| 181 | 4.8i | 211 | 6.2i | 173 | 1.2i | 203 | 4.9i | 167 | 2.6i | 197 | 2.3i |
| 182 | 4.9i | 212 | 7.3i | 174 | .9i  | 204 | 5.3i | 168 | 1.9i | 198 | 2.9i |
| 183 | 6.0i | 213 | 7.8i | 175 | 1.3i | 205 | 5.8i | 169 | 2.0i | 199 | 3.3i |

13
|   | Z N B |   | Z N B |   | Z N B |   | Z N B |   | Z N B |   | Z N B |   |
|---|-------|---|-------|---|-------|---|-------|---|-------|---|-------|---|
|104| 200 3.4|105| 166 2.8|105| 196 1.9|106| 164 3.3|106| 194 1.0|107| 164 3.7|
|201| 4.1i |167| 2.8i |197| 2.1i |165| 3.0i |195| 1.3i |165| 3.7i |
|202| 4.3i |168| 1.8i |198| 2.7i |166| 2.3i |196| 1.5i |166| 2.4i |
|203| 4.5i |169| 2.0i |199| 2.9i |167| 2.5i |197| 2.1i |167| 2.7i |
|204| 5.0i |170| 1.4i |200| 3.3i |168| 1.4i |198| 2.3i |168| 1.9i |
|205| 5.4i |171| 1.2i |201| 3.9i |169| 1.7i |199| 2.6i |169| 2.1i |
|206| 5.6i |172| 1.9i |202| 4.0i |170| 1.0i |200| 2.9i |170| 1.3i |
|207| 6.1i |173| 1.9i |203| 4.4i |171| 1.2i |201| 3.4i |171| 1.4i |
|208| 6.2i |174| 1.5i |204| 5.1i |172| .7i |202| 3.7i |172| 1.1i |
|209| 6.6i |175| 1.9i |205| 5.6i |173| 1.4i |203| 4.2i |173| 1.5i |
|210| 6.6i |176| 1.5i |206| 5.6i |174| 1.2i |204| 4.6i |174| 1.4i |
|211| 7.0i |177| 2.0i |207| 6.0i |175| 1.8i |205| 4.8i |175| 1.8i |
|212| 7.0i |178| 2.1i |208| 6.0i |176| 1.6i |206| 5.0i |176| 1.6i |
|213| 7.5i |179| 3.0i |209| 6.5i |177| 2.7i |207| 5.5i |177| 2.3i |
|214| 7.3i |180| 3.3i |210| 6.5i |178| 2.5i |208| 5.5i |178| 2.2i |
|105| 151 5.7i|181| 4.5i |211| 6.9i |179| 3.5i |209| 6.0i |179| 3.0i |
|152| 5.5i |182| 4.2i |212| 6.9i |180| 3.7i |210| 6.0i |180| 3.1i |
|153| 5.6i |183| 5.3i |213| 7.2i |181| 5.1i |211| 6.3i |181| 4.3i |
|154| 5.2i |184| 4.6i |106| 152 4.7i|182| 4.8i |212| 6.2i |182| 4.0i |
|155| 5.3i |185| 4.7i |153| 4.9i |183| 5.7i |107| 153 4.8i|183| 5.2i |
|156| 5.1i |186| 3.7i |154| 4.6i |184| 5.1i |154| 4.6i |184| 4.6i |
|157| 5.2i |187| 3.8i |155| 4.7i |185| 4.8i |155| 4.8i |185| 4.6i |
|158| 4.9i |188| 2.7i |156| 4.3i |186| 3.8i |156| 4.5i |186| 3.5i |
|159| 4.9i |189| 2.9i |157| 4.4i |187| 4.2i |157| 4.5i |187| 4.0i |
|160| 4.6i |190| 1.8i |158| 4.2i |188| 3.4i |158| 4.4i |188| 2.7i |
|161| 4.5i |191| 1.7i |159| 4.3i |189| 2.9i |159| 4.6i |189| 2.8i |
|162| 4.2i |192| .7i |160| 3.9i |190| 2.0i |160| 4.5i |190| 2.0i |
|163| 4.1i |193| 1.0i |161| 3.8i |191| 1.9i |161| 4.3i |191| 1.6i |
|164| 3.7i |194| 1.5i |162| 3.7i |192| .8i |162| 3.9i |192| .7i |
|165| 3.1i |195| 1.8i |163| 3.8i |193| .5i |163| 4.0i |193| .2i |
| Z  | N   | B  | Z  | N   | B  | Z  | N   | B  | Z  | N   | B  | Z  | N   | B  |
|----|-----|----|----|-----|----|----|-----|----|----|-----|----|----|-----|----|
| 107| 194 | .9i| 108| 166 | 2.5i| 108| 196 | 1.2i| 109| 170 | 1.6i| 109| 200 | 2.0i| 110| 176 | 3.1i|
| 195| 1.3i| 167 | 2.1i| 197 | 1.2i| 171 | 1.7i| 201 | 2.5i| 177 | 4.2i| 196| 1.6i| 168 | 1.9i| 198 | 1.4i| 202 | 2.2i|
| 197| 1.8i| 169 | 1.6i| 199 | 1.9i| 173 | 1.8i| 203 | 3.0i| 179 | 5.1i| 198| 2.2i| 170 | 1.3i| 200 | 2.3i| 174 | 1.6i|
| 199| 2.6i| 171 | 1.4i| 201 | 2.7i| 175 | 2.1i| 205 | 3.6i| 181 | 6.0i| 200| 2.8i| 172 | 1.2i| 202 | 2.9i| 176 | 1.7i|
| 201| 3.3i| 173 | 1.6i| 203 | 3.4i| 177 | 2.7i| 207 | 4.0i| 183 | 6.6i| 202| 3.5i| 174 | 1.3i| 204 | 3.5i| 178 | 2.6i|
| 203| 4.1i| 175 | 2.1i| 205 | 3.9i| 179 | 4.4i| 209 | 4.3i| 185 | 5.9i| 204| 4.3i| 176 | 1.9i| 206 | 4.1i| 180 | 4.2i|
| 207| 5.3i| 179 | 4.0i| 209 | 4.8i| 183 | 5.8i| 159 | 2.3i| 189 | 3.2i| 208| 5.3i| 180 | 3.8i| 210 | 5.0i| 184 | 5.5i|
| 209| 5.6i| 181 | 5.0i| 109 | 155 | 3.1i| 185 | 5.3i| 161 | 2.2i| 191 | 2.1i| 209| 5.8i| 182 | 4.9i| 156 | 2.4i|
| 210| 6.2i| 183 | 6.2i| 157 | 3.4i| 187 | 4.4i| 163 | 2.8i| 193 | .8i| 211| 6.2i| 183 | 6.2i| 157 | 3.4i|
| 108| 154 | 3.6i| 184 | 5.6i| 158 | 2.8i| 188 | 3.3i| 164 | 1.8i| 194 | .2i| 155| 3.9i| 185 | 5.3i| 159 | 3.1i|
| 156| 3.7i| 186 | 4.2i| 160 | 3.2i| 190 | 2.2i| 166 | 2.1i| 195 | .1i| 157| 3.9i| 187 | 4.4i| 161 | 3.4i| 191 | 2.0i|
| 158| 3.5i| 188 | 3.1i| 162 | 3.3i| 192 | 1.2i| 168 | 1.7i| 198 | .9i| 159| 3.5i| 189 | 3.2i| 163 | 3.0i| 193 | .5i|
| 160| 3.4i| 190 | 2.2i| 164 | 2.7i| 194 | 1.6i| 170 | 1.6i| 200 | 1.4i| 161| 3.5i| 191 | 1.9i| 165 | 3.1i| 195 | .5i|
| 162| 3.4i| 192 | .9i| 166 | 2.6i| 196 | .7i| 172 | 1.5i| 202 | 1.8i| 163| 3.6i| 193 | .3i| 167 | 2.3i| 197 | .8i|
| 164| 2.6i| 194 | .1i| 168 | 2.1i| 198 | 1.4i| 174 | 1.7i| 204 | 2.5i| 165| 2.9i| 195 | .9i| 169 | 1.8i| 199 | 1.8i|

15
| Z  | N  | B  | Z  | N  | B  | Z  | N  | B  | Z  | N  | B  |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 110| 206| 3.0| 111| 184| 6.4| 112| 164| 1.6| 112| 194| .7 |
| 207| 3.3| 185| 5.7| 165| 2.0| 195| -.5| 177| 6.3| 161| 2.1|
| 208| 3.5| 186| 5.0| 166| 1.9| 196| -.2| 178| 6.3| 162| 2.2|
| 111| 157|.9 | 187| 4.4| 167| 1.8| 197| -.3| 179| 7.2| 163| 2.0|
| 158| 1.2| 188| 3.8| 168| 1.6| 198| -.1| 180| 7.3| 164| 1.8|
| 159| 1.7| 189| 3.4| 169| 1.9| 199| .1 | 181| 7.4| 165| 2.3|
| 160| 2.2| 190| 2.6| 170| 1.8| 200| .4 | 182| 7.1| 166| 1.9|
| 161| 1.5| 191| 2.2| 171| 2.2| 201| .8 | 183| 7.4| 167| 2.2|
| 162| 1.7| 192| 1.5| 172| 2.2| 202| .9 | 184| 6.7| 168| 2.2|
| 163| 2.3| 193|.9 | 173| 2.7| 203| 1.4| 185| 6.3| 169| 2.4|
| 164| 2.0| 194|.4 | 174| 3.6| 204| 1.5| 186| 5.9| 170| 2.4|
| 165| 2.4| 195|.0 | 175| 4.3| 205| 2.0| 187| 5.5| 171| 2.7|
| 166| 1.6| 196| -.6| 176| 4.8| 206| 1.9| 188| 4.7| 172| 4.1|
| 167| 2.1| 197|.4 | 177| 5.6| 159| 1.6| 189| 4.2| 173| 4.8|
| 168| 2.0| 198|.4 | 178| 5.8| 160| 1.9| 190| 3.4| 174| 6.1|
| 169| 1.9| 199|.7 | 179| 6.5| 161| 1.9| 191| 2.7| 175| 6.7|
| 170| 1.7| 200| 1.0| 180| 6.4| 162| 2.1| 192| 2.1| 176| 6.6|
| 171| 2.1| 201| 1.3| 181| 7.1| 163| 2.6| 193| 1.6| 177| 7.3|
| 172| 1.8| 202| 1.3| 182| 6.9| 164| 1.8| 194| .9 | 178| 7.2|
| 173| 2.4| 203| 2.0| 183| 6.9| 165| 2.5| 195| .6 | 179| 7.6|
| 174| 2.4| 204| 2.0| 184| 6.5| 166| 2.4| 196| .4 | 180| 7.5|
| 175| 4.1| 205| 2.5| 185| 6.3| 167| 2.3| 197| .1 | 181| 8.2|
| 176| 3.8| 206| 2.5| 186| 5.7| 168| 2.2| 198| .1 | 182| 7.8|
| 177| 4.9| 207| 2.9| 187| 5.2| 169| 2.4| 199| -.8| 183| 8.1|
| 178| 4.8| 112| 158|.8 | 188| 4.5| 170| 2.5| 200| -.6| 184| 7.5|
| 179| 5.6| 159| 1.1| 189| 3.8| 171| 3.2| 201| .5 | 185| 7.2|
| 180| 5.5| 160| 1.3| 190| 3.1| 172| 2.8| 202| -1.3| 186| 6.5|
| 181| 6.5| 161| 1.6| 191| 2.5| 173| 3.2| 203| 1.1| 187| 6.1|
| 182| 6.2| 162| 2.0| 192| 1.9| 174| 4.4| 204| 1.0| 188| 5.3|
| 183| 6.9| 163| 2.3| 193| 1.2| 175| 5.0| 205| 1.4| 189| 4.4|
|    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 114| 190| 3.9i| 115| 175| 7.7i| 116| 174| 6.9i| 117| 175| 6.6i| 118| 178| 7.0i|
| 191| 3.5i| 176| 7.8i| 175| 7.5i| 176| 6.8i| 179| 7.6i| 184| 7.4i|
| 192| 3.0i| 177| 7.3i| 176| 6.5i| 177| 7.4i| 180| 7.4i| 185| 7.3i|
| 193| 2.5i| 178| 7.4i| 177| 7.0i| 178| 7.3i| 181| 7.9i| 186| 6.6i|
| 194| 1.8i| 179| 8.1i| 178| 7.2i| 179| 8.0i| 182| 7.7i| 187| 6.0i|
| 195| 1.1i| 180| 7.9i| 179| 7.7i| 180| 7.8i| 183| 7.7i| 120| 174| 5.7i|
| 196| .8i| 181| 8.5i| 180| 7.6i| 181| 8.4i| 184| 7.4i| 175| 6.1i|
| 197| .4i| 182| 8.4i| 181| 8.2i| 182| 8.1i| 185| 7.1i| 176| 6.2i|
| 198| .4i| 183| 8.7i| 182| 7.8i| 183| 8.5i| 186| 6.5i| 177| 6.7i|
| 199| .2i| 184| 8.2i| 183| 8.3i| 184| 7.9i| 187| 5.9i| 178| 6.6i|
| 200| .1i| 185| 7.7i| 184| 7.7i| 185| 7.9i| 188| 5.6i| 179| 7.1i|
| 201| .1i| 186| 7.3i| 185| 7.4i| 186| 7.1i| 119| 173| 7.1i| 180| 6.8i|
| 202| .1i| 187| 7.0i| 186| 6.9i| 187| 6.5i| 174| 7.2i| 181| 7.2i|
| 203| .2i| 188| 6.0i| 187| 6.1i| 188| 6.3i| 175| 6.8i| 182| 7.2i|
| 204| .0i| 189| 5.7i| 188| 5.8i| 189| 5.7i| 176| 6.7i| 183| 7.2i|
| 115| 169| 2.6i| 190| 4.9i| 189| 5.1i| 118| 172| 6.0i| 177| 7.2i|
| 170| 2.6i| 191| 4.4i| 190| 4.5i| 173| 7.1i| 178| 7.0i| 185| 6.2i|
| 171| 3.6i| 116| 170| 3.9i| 117| 171| 5.4i| 174| 6.8i| 179| 7.5i|
| 172| 6.0i| 171| 4.9i| 172| 6.6i| 175| 7.4i| 180| 7.4i| 186| 5.8i|
| 173| 6.9i| 172| 6.0i| 173| 6.5i| 176| 6.6i| 181| 8.0i|
| 174| 7.2i| 173| 6.8i| 174| 6.3i| 177| 7.1i| 182| 7.7i|

|    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 |
3 Discussion

3.1 Comparison with experiment

The solid circles in Figs. 3–5 denote the few experimental barrier heights that have been measured for these isotope chains [18]. In general, all three of the calculations “ETFSI-1”, “HM”, and “MS” are in good agreement with the data, although in the case of \( Z = 84 \) the “ETFSI-1” barriers are between 2 and 4 MeV too high. This discrepancy reflects the general tendency noted above and in Ref. [15]: we overestimate all barriers higher than 15 MeV by 2 MeV or more. Possible reasons for this are discussed in Ref. [15]; another point not mentioned there is that barriers will be much more sensitive than masses to the droplet-model curvature coefficient \( a_{cv} \) corresponding to the Skyrme force. Thus it is conceivable that despite the good mass fit our \( a_{cv} \) is sufficiently overestimated to lead to serious errors at extreme deformations. (The Thomas-Fermi calculations of Ref. [17] encountered the same problem, but resolved it by adjusting the so-called “congruence energy”; we stress that we do not make use of this feature in our calculations, and indeed if we had done so then the good agreement with experiment that we find for the lower barriers would have been destroyed.) In any case, these large errors are of no practical consequence for the r-process, since nuclei with such high barriers will be effectively stable against fission in this context. On the other hand, for all barriers lower than 15 MeV the ETFSI-1 results never disagree with experiment by more than 1.7 MeV, and usually by much less: for \( Z \geq 86 \) the rms error is 717 KeV, while for \( Z \geq 88 \) it is as small as 698 KeV (this includes a few measured nuclei that are given in Table 2 of Ref. [15] but not here).

3.2 Highly neutron-rich nuclei

While all three methods agree reasonably well over the narrow experimentally known region, they give widely different extrapolations to the highly neutron-rich region. In particular, the ETFSI-1 calculations predict exceptionally high barriers in the region of \( N = 184 \) for the lower values of \( Z \), although as \( Z \) increases, i.e., as the stability line is approached, these barriers become lower, and the various sets of calculations tend to converge. Actually, a similar trend in the vicinity of \( N = 184 \) can be observed in the case of the MS calculations, although on a much lesser scale, while the HM calculations
Figure 3: Primary fission barriers for the $Z = 84$ isotopic chain, calculated with the various models considered in text. Neutron closed shells are indicated by vertical dashed lines, experimental values by solid circles.
predict no enhancement of barrier heights at all close to $N = 184$. Because of our tendency to overestimate barriers that are high anyway, the trend in the ETFSI-1 results shown in Figs. 3–5 may be somewhat exaggerated. However, we can estimate these errors by referring to Table I of Ref. [13], and we find that our calculated barrier for nucleus $Z = 84, N = 184$ is probably no more than 12 MeV too high, while that of $Z = 92, N = 184$ is only 2 MeV too high at the most. Thus the tendency we have reported for ETFSI-1 is at least qualitatively real, and it is certainly stronger than in the MS case.

The rapid fall-off beyond $N = 184$ for all three isotope chains is indicative of a shell effect. In fact, the behaviour of the ETFSI-1 barriers in the vicinity of $N = 184$ must be at least partially related to the fact that the force SkSC4 leads to a strong magic gap at $N = 184$ for proton-deficient nuclei, the gap becoming much smaller as the stability line is approached with increasing $Z$. This point is illustrated in Fig. 6, where we show the variation of the gap $\Delta = S_{2n}(Z, N = 186) - S_{2n}(Z, N = 184)$ with $Z$. (It must be recalled that we have calculated pairing in the BCS approximation, and that with the more realistic Bogolyubov treatment much weaker neutron shell gaps will be found in general for large neutron excesses [20]. However, unpublished results of Dobaczewski, discussed by Pearson et al. [21], show that this “Bogolyubov quenching” of shell gaps is much less pronounced for the $N = 184$ magic number, and we therefore neglect it here.) We also show in Fig. 6 the variation of this same gap for the FRDM (“finite-range droplet model”) mass formula [22]. Since the shell corrections of the MS calculations consist entirely of those of the FRDM (applied exclusively to the ground state) we can understand why the MS barriers peak much less strongly near $N = 184$.

Sensitivity of barrier heights to symmetry coefficient

However, the very high barriers found for ETFSI-1 near $N = 184$ cannot be the result entirely of shell effects, since the pure ETF calculations displayed in Figs. 3–5 show that the macroscopic part plays a role also, there being a steady increase of barrier height with $N$, for constant $Z$. Referring to Table 2, one sees in fact that the barrier heights of these highly neutron-rich nuclei are strongly anti-correlated with the symmetry coefficient $J$ of the model in question, or equivalently, strongly correlated with the surface-symmetry coefficient $a_{ss}$, defined by

$$a_{ss} = \frac{(2L/K_v) a_{sf} - 9J^2/4Q}{},$$

where $L$ is the density-symmetry coefficient, $K_v$ the incompressibility, $a_{sf}$ the surface coefficient, and $Q$ the surface-symmetry stiffness coefficient [19].
Figure 4: Same as Fig. 3 for $Z = 92$. Also shown on this figure are some results for forces SkSC10 and SkSC15.
Figure 5: Same as Fig. 3 for $Z = 100$
anti-correlation between \( J \) and \( a_{ss} \) holds for all entries in Table 2, and is, in fact, a quite general property of all models that have been fitted to nuclear masses, whether they are of the droplet type or are based on microscopic forces \[23, 24\].

This result can be easily understood in terms of the following gross approximation to the droplet model, which holds best close to the stability line,

\[
e = a_v + a_{sf}A^{-1/3} + (J + a_{ss}A^{-1/3})I^2 + a_{coul}Z^2A^{-4/3} + \ldots ,
\]

where \( e \) is the energy per nucleon and \( I = (N - Z)/A \). We see now that for ground-state masses of nuclei relatively close to the stability line an increase in \( J \) can be roughly compensated by a decrease in \( a_{ss} \) over a large range of values of \( A \). Now the term in \( a_{ss}A^{-1/3} \) is really a surface term, so we can write the fissility parameter as

\[
x = \frac{a_{coul}Z^2}{2a_{sf}(I)A} ,
\]

where

\[
a_{sf}(I) = a_{sf} + a_{ss}I^2 .
\]

Thus, while a decrease in \( J \) will be compensated by an increase in \( a_{ss} \) as far as ground-state masses are concerned (at least for nuclei relatively close to the stability line), the result will be an increased barrier height for nuclei of large neutron excess \( I \).

This interpretation of our very high barriers as being at least in part related to macroscopic symmetry properties is strengthened by recalculating the barrier of U\(_{184}\) with force SkSC10, a force that has been fitted in exactly the same way as force SkSC4, except that it is constrained to \( J = 32 \) MeV \[25\], rather than 27 MeV: it will be seen from Fig. 4 that this barrier is lowered by more than 6 MeV. At the same time, for a nucleus much closer to the stability line, U\(_{146}\), the barrier is lowered by only 1 MeV on replacing force SkSC4 by SkSC10, as we would expect for a symmetry-related effect, and we see that the agreement with the experimental barriers that we found in Ref. \[15\] is more or less \( J \)-independent. However, our new barrier for U\(_{184}\) is still considerably higher than that given by the MS calculation, even though the latter has almost the same value for \( J \) (32.65 MeV). This suggests that shell effects are still playing a major role in our high barriers, an indication that
Figure 6: The $N = 184$ pairing gap as a function of $Z$ for SkSC4, SkSC10 and FRDM.
is confirmed in Fig. 6, where it will be seen that the shell effects for our two Skyrme forces are more or less the same.

Nevertheless, it is clear that there is a strong macroscopic symmetry effect, at least within the ETFSI framework, and it is obviously essential that we tie down much better the value of $J$ (and thus of $a_{ss}$). Now the rms error of the mass fit for force SkSC10 is much worse than for the ETFSI-1 force, SkSC4 ($\sigma = 0.893$ MeV rather than 0.736 MeV), and we now find that the highest value of $J$ for which acceptable mass fits can be found in the ETFSI framework is 28 MeV. On the other hand, this is the lowest value for which the known stability of neutron matter can be assured, so we now regard 28 MeV as our best value for $J$ [26]. Recent HF calculations [27] reach an identical conclusion, and it now seems that 28 MeV is a quite robust value within the very general framework of Skyrme forces. This conclusion stands in contradiction with the value of $J = 32.73$ MeV given by the FRDM [22], the best droplet-model fit to masses; the almost identical value adopted in the zeroth-order Thomas-Fermi calculations of MS is presumably dictated by the requirements of self-consistency, the shell corrections of these calculations coming from the FRDM. Ref. [26] suggests one way in which the FRDM could lead to a spuriously large value of $J$, but in any case we have no option in an ETFSI-model calculation of barriers but to choose lower values of $J$ if we wish to simultaneously fit masses.

Table 2: Volume- and surface-symmetry coefficients of the forces and models used in this paper (see text)

|       | SkSC4 | MS  | HM  | SkSC10 | SkSC15 |
|-------|-------|-----|-----|--------|--------|
| $J$ (MeV) | 27.0  | 32.65 | 36.5 | 32.0   | 28.0   |
| $L$ (MeV) | -9.29 | 49.9 | 100 | 55.82  | 6.73   |
| $K_v$ (MeV) | 234.7 | 234 | 240 | 235.8  | 234.9  |
| $a_{sf}$ (MeV) | 17.7  | 18.63 | 20.76 | 18.11  | 17.78  |
| $Q$ (MeV) | 75*   | 35.4 | 17.0 | 34     | 56     |
| $a_{ss}$ (MeV) | -23.3 | -59.9 | -159 | -59.2  | -30.5  |

Ref. [37] [17] [38] [37] [37]

*This newly calculated value replaces the one given in Ref. [3].

The question now arises as to how much our barriers would be lowered if we changed $J$ from the value of 27 MeV corresponding to the force SkSC4
with which they were calculated (and which leads to a unphysical collapse of neutron matter) to our newly preferred value of 28 MeV. We have accordingly constructed a new force, SkSC15, that has been fitted to the same mass data as SkSC4, but under the constraint of $J = 28$ MeV, rather than 27 MeV. Repeating the calculation of $U_{184}$, we see from Fig. 4 that the barrier is lowered by 3.2 MeV, which still leaves it much higher than the MS and HM values. Moreover, this is an extreme case, and closer to the stability line the effect of changing $J$ by 1 MeV will be negligible.

The r-process

The fact that the barriers we find for very neutron-rich nuclei are much higher than hitherto believed has significant implications for the r-process, some of which have been presented in Ref. [28]. Here we confine ourselves to a few general remarks.

For fission to occur on a timescale that is short compared to the beta-decay lifetimes of the nuclei found on the r-process path, and the r-process path thereby terminated, the nucleus in question will have to be excited close to or over the top of the barrier. Such excitation of a nucleus during the r-process can occur as the result of either neutron capture (neutron-induced fission) or beta-decay (beta-delayed fission), so for rapid fission to occur the $S_a$ or the $Q_\beta$ (of the parent nucleus), respectively, must be close to or higher than the height of the primary barrier. Now Fig. 1 displays r-process paths corresponding to different values of $S_a$, defined as half the two-neutron separation energy $S_{2n}$, as well as the neutron-drip line (all calculated with the ETFSI-1 mass formula). With typical values of $S_a$ lying between 1 and 2 MeV, we see from this figure that neutron-induced fission will certainly not occur in the r-process below $A = 318$, and probably not below much higher values. On the other hand, referring to the ETFSI-1 mass tables [7] for the relevant $Q_\beta$'s, we find that beta-delayed fission will certainly be possible for some r-process nuclei with $A < 300$, but the extent to which this occurs depends on the beta-decay strength function of the precursor being sufficiently concentrated towards the upper end of the spectrum. But some fraction of r-process nuclei will always escape beta-delayed fission, and with neutron-induced fission no longer being operative we now have to entertain the possibility of the r-process path extending to values of $A$ considerably in excess of 300. This conclusion is not vitiated by a possible overestimation of our barrier heights, the limits of which are discussed in the previous subsection.

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3.3 Superheavy nuclei

We have already pointed out how our barriers in the vicinity of $N = 184$, while very high on the r-process path, become lower as $Z$ increases, but even as the stability line is approached they become higher again, confirming all previous expectations of an “island” of stability in this region. (Actually, isofar as one speaks of the “valley of stability”, it would be more appropriate to speak of a “basin of stability” than of an “island”.) Thus for nuclides in the range $112 \leq Z \leq 120$ and $177 \leq N \leq 186$ our calculated barriers are always at least 5 MeV, and occasionally nearly 9 MeV, high, assuring thereby a large measure of stability, at least with respect to fission. As for the recently discovered superheavies, $^{289}114$ \cite{14} and $^{293}118$ \cite{13}, our calculated barrier heights are 6.7 and 7.4 MeV, respectively. The barriers of these nuclei have not yet been measured, but the very fact that they are stable enough to have been observed at all constitutes a qualitative verification of the high barriers that our calculations predict.

As for the extent to which we agree with other microscopic calculations in this region we note that in a SHF calculation on $^{288}112$ Ćwiok et al. find a barrier height of about 6.5 MeV \cite{29}, as compared with our value of 4.8 MeV. This difference is not very large, but their barrier is a double one, with both peaks having about the same height, whereas we have a single, inner, barrier. All in all, it looks very much as though this nucleus is more deformable in our calculation than in that of Ref. \cite{29}, despite the fact that the force they use, SLy7 \cite{30}, has a lower surface coefficient $a_{sf}$ than our own SkSC4 (17.0 MeV rather than 17.7 MeV). The reason could be that the force SLy7 \cite{30} has a low effective mass $M^*$, 0.69$M$, which will lead to too low a density of single-particle states, and thus an unrealistically high resistance to deformation. (The fact that Ref. \cite{29} imposes reflection symmetry while we do not cannot be a factor, since in our calculations this nucleus turns out to be reflection-symmetric anyway.)

The only other microscopic calculations in this region of which we are aware are those of Berger et al. \cite{31}, who use the HF-Bogolyubov (HFB) method with the finite-range Gogny force. They find a barrier height of 10 MeV for $^{294}112$, of 11 MeV for $^{298}114$, and of 7 MeV for $^{306}118$; these results are to be compared with our own values of 6.9, 7.5, and 5.6 MeV, respectively. The consistently higher barriers obtained by Ref. \cite{31} could be accounted for in the same way that we have suggested for the case of Ref. \cite{29}: too low an effective mass for the Gogny force. However, an additional
contribution could now come from the surface coefficient, since this is higher for the Gogny force (20.1 MeV \cite{32}) than for SkSC4. (The fact that Ref. \cite{31} imposes reflection symmetry while we do not can be a factor only in the case of $^{306}\text{118}$, since we find the other two nuclei to be reflection-symmetric anyway.)

As for the MS procedure of Ref. \cite{16}, it cannot be applied to nuclei with $Z > 112$, since this corresponds to a limiting fissility beyond which no prescription is given in that paper for calculating the macroscopic barriers. Of course, the original TF method \cite{17} on which Ref. \cite{16} was based is just as applicable in the superheavy region as elsewhere, but no such calculations seem to have been published.

4 \hspace{1em} Conclusions

We have extended the ETFSI-1 mass formula, based on the Skyrme force SkSC4, to the calculation of the fission barriers of all nuclei that can be expected to play a role in the r-process of nucleosynthesis; we recall that the force SkSC4 gives an excellent fit to the mass data, and to the known primary-barrier heights that are lower than 15 MeV. The results that we present here are radically different from the only other such calculations that have been made \cite{10, 16}, in that we obtain much higher barriers for proton-deficient nuclei in the region of $N = 184$, a consequence of which is that the r-process path might continue to mass numbers considerably in excess of 300 before being brought to a halt by neutron-induced fission. In view of the importance of this result we summarize here the reasons why we believe that our calculations are essentially correct. Our high barriers on the r-path are related both to the shell effects associated with our Skyrme force, and to our much lower value of the symmetry coefficient $J$ (27 MeV as opposed to 36.5 MeV for HM \cite{11} and 32.65 MeV for MS \cite{16}); we now examine each of these points in turn.

Our shell effects appear to be fairly robust within the ETFSI framework, being difficult to change significantly while maintaining the fit to the mass data. At the same time, we recall that in extrapolating far from the known region of the nuclear chart out to the highly neutron-rich region that is relevant to the r-process, the isospin dependence of the ETFSI spin-orbit field conforms well to the predictions of relativistic mean-field theory \cite{33, 34}, adding thereby to our confidence in the reliability of the extrapolation.
As for the value of the symmetry coefficient $J$, we have found, after the completion of these calculations, that 28 MeV would have been a better value than 27 MeV, essentially because it allows us to avoid an unphysical collapse of neutron matter, while maintaining a high-quality mass fit. A complete new mass fit, ETFSI-2, is currently being undertaken with the constraint $J = 28$ MeV, and in principle when this is completed we should repeat the calculations of this paper with the new force determined by this fit. However, we have shown here that the maximum effect of the change of force will be to lower the barriers of the most neutron-rich nuclei by about 3 MeV, and that for nuclei closer to the stability line the effect will be correspondingly smaller. In any case, one can be reasonably sure that the ETFSI-2 fission barriers would lie considerably closer to the ETFSI-1 barriers presented here than to the barriers of either MS [16] or HM [10].

We have also extended our barrier calculations of nuclei with $N$ in the vicinity of 184 down to the stability line, exploring thereby the much studied “island” of stability. Here our barriers are in reasonable agreement with other microscopic calculations that have been made in this region: this constitutes an additional check on the overall validity of our calculations, and there is certainly no indication of a tendency for our calculational procedure to overestimate barrier heights.

Nevertheless, we have not yet discussed triaxiality, which could be expected to lower fission barriers to some extent. Several studies of this question have been made, the most extensive probably being that of Ref. [35] (see that paper for other references), in which the barriers of 15 heavy and superheavy nuclei, some close to the stability line, others highly neutron-rich, were investigated within the framework of the ETFSI method and found, when triaxiality was taken into account, to be lowered by 0.6 MeV on average, with a maximum shift of 1.3 MeV. It was concluded that this effect is probably negligible, given the overall discrepancy between calculation and experiment (see Section 1). Nevertheless, two counter-examples in both of which triaxiality is claimed to lower the barriers by the enormous amount of 4 MeV are to be noted: $^{310}$Zr [23] and $^{258}$Fm [36]. The latter case, which was regrettably overlooked in Ref. [35], is particularly disturbing, since it occurs in a region of the nuclear chart where the barriers of neighbouring nuclei are much less sensitive to triaxiality. The former of these two nuclei is too heavy for us to check with our codes, but we have repeated the latter case using the ETFSI method, and find that triaxiality lowers the barrier by 1.6 MeV. This is a significantly smaller effect than the 4.1 MeV claimed in Ref. [36].
but it is still the largest triaxiality shift that we have found, and really too large to be neglected. One is thus forced to the conclusion that while in most cases triaxiality can reasonably be neglected, as far as barriers are concerned, there will be isolated cases where this is not possible. The situation is most unsatisfactory, since such cases can only be identified by first performing the full triaxial calculation, and doing this for the nearly 2000 nuclei considered here would be prohibitively time-consuming. It seems that the best that one can do at the present time is to proceed as we have done here, neglecting triaxiality, and bear in mind that a few of our barriers, including some low enough to be relevant to the r-process, will be considerably overestimated because of this approximation. However, the barriers of our proton-deficient nuclei in the vicinity of $N = 184$ will remain much higher than previously believed.

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