Electroweak vacuum stability and inflation via non-minimal derivative couplings to gravity

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based on work with Cristiano Germani, Phys.Rev. D93 (2016) no.4, 045005 [arXiv:1508.04777]

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For this talk I assume that the diphoton excess recently reported by the ATLAS and CMS collaboration is just a statistical fluctuation.
Aim of the talk

1. Show that the (probably metastable) SM vacuum can be stabilized if all the SM fields are coupled to gravity through non-minimal kinetic terms.

2. Show that the Higgs boson can act as inflaton in this framework. Still, compatibility with CMB data requires the SM parameters (especially $M_t^{\text{pole}}$) to lie approximately in the same region allowing for vacuum stability.
Observation of the Higgs boson at the LHC Run I

Most recent ATLAS+CMS combination \( \delta M_H / M_H \sim 2 \times 10^{-3} \) [PRL 2015]

\[
M_H = 125.09 \pm 0.21 \text{(stat.)} \pm 0.11 \text{(syst.)} \text{ GeV}
\]
SM symmetry-breaking sector

Higgs potential

\[ V(\Phi) \sim \Lambda^4 - \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 + Y_{ij} \bar{\psi}_L^i \psi^j \Phi + \frac{g_{ij}}{\Lambda} \psi^i_L \psi^{jT}_L \Phi \Phi^T \]

- Cosmological constant problem (worst fine tuning problem ever!)
- Quadratic sensitivity to heavy dof’s when matching onto UV theory (do heavy dof’s exist?)
- Vacuum instability at large field values if \( \lambda < 0 \iff M_H < M_H^{\text{stability}} \)
- Loss of perturbativity if \( \lambda > 4\pi \iff M_H > M_H^{\text{triviality}} \)
- SM flavor problem + \( M_\nu \):
  - large unexplained hierarchy \( M_t/M_e \sim 3 \times 10^5 \)
  - \( U(3)^5_F \mathrel{\xrightarrow{Y_{ij}}} U(1)_B \otimes U(1)_L^{(3)} \)
If your mexican hat turns out to be a dog bowl you have a problem...
SM vacuum stability and RGE evolution

For $\phi$ bigger than the Higgs mass term, the effective $V$ (choosing $\mu \sim \phi$)

- can be approximated by $V_{\text{eff}} \simeq \lambda(\phi)\phi^4$

This means ignoring the non-logarithmic loop contributions. Still, it shows the instability for $\phi \sim 10^{10-11}$ GeV $\ll M_{\text{Pl}}$

- better: one can always cast it as $V_{\text{eff}} = \lambda_{\text{eff}}(\phi)\phi^4$

NNLO with prev. world average $m_t$

NNLO with $m_t : \lambda(M_{\text{Pl}}) = \beta_\lambda(M_{\text{Pl}})$
SM vacuum stability and RGE evolution

- $\lambda(M_{Pl}) \leq 0$ crucially depends on $M_t$: no stability for central value. What about error bands?
- $\lambda$ runs fast and then slows down, never too negative
- Around $M_{Pl}$ both $\lambda$ and $\beta_\lambda$ are $\sim 0$. Any meaning? But no RGE fixed point

NNLO with prev. world average $m_t$

NNLO with $m_t : \lambda(M_{Pl}) = \beta_\lambda(M_{Pl})$
SM vacuum stability at NNLO

Higgs mass $M_h$ in GeV

Pole top mass $M_t$ in GeV

Instability

Stability

Meta-stability

$10^7$

$10^{12}$

$10^{10}$

$M_t$, $M_h$

1, 2, 3 $\sigma$

Degrassi, Elias-Mirò, Espinosa, Giudice, Isidori, Strumia, DV 12

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 13

S. Di Vita (DESY) Non-minimal derivative couplings to gravity
Standard Model: quite a few shortcomings

- Describes 4.9% of universe
- Not enough $\mathcal{CP}$ for baryogenesis
- Neutrino oscillations $\Rightarrow \Delta m_\nu$
- Gravity?
- Inflation?

[Planck 15]

[Non-minimal derivative couplings to gravity]
The Higgs boson as inflaton? (1)

Minimal coupling to gravity

\[ S_{\text{min}} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - g^{\mu\nu} (\nabla_\mu H^\dagger)(\nabla_\nu H) - \frac{\lambda}{4} (H^\dagger H)^2 \right] \equiv V(H^\dagger H) \]

- \( R \) Ricci scalar, \( M_p \simeq 2.435 \text{ GeV} \) reduced Planck mass
- Unitary gauge \( \rightarrow \) Higgs radial mode \( \phi \)
- Slow roll, potential parameters \( \epsilon_V \equiv \frac{V' M_p^2}{2 V^2} \) and \( \eta_V \equiv \frac{V'' M_p^2}{V} \)
- Power spectrum of primordial perturbations \( P \simeq \frac{H^2}{8\pi^2 \epsilon_V M_p^2} \)
- Spectral index \( n_s = 1 - 6\epsilon_V + 2\eta_V \)
- \( P \sim 2 \times 10^{-9} \) and \( n_s \sim 0.968 \) (Planck)
Planck + BICEP2 + KeckArray 2015

[The Keck Array and BICEP2 Collaborations, 15]

Non-minimal derivative couplings to gravity
The Higgs boson as inflaton? (2) “New Higgs Inflation”

Non-minimal derivative coupling to gravity [Germani, Kehagias 10]

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \left( g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) \left( \mathcal{D}_\mu \mathcal{H}^\dagger \right) \left( \mathcal{D}_\nu \mathcal{H} \right) - V\left( \mathcal{H}^\dagger \mathcal{H} \right) \right] \]

- \( G^{\mu\nu} \) Einstein’s tensor, \( M^2 > 0 \) unknown mass scale
- **Unique** non-minimal kinetic term that does not propagate any other dof’s than Einstein-Hilbert + scalar sector

1. For \( M \ll H \), the kinetic term generates **enhanced friction** in EoM for \( \phi \) making it slow-roll even if \( V \) is not flat / convex

   [Germani, Kehagias 10; Germani, Watanabe 11; + Wintergerst 14]

2. At large \( \phi \), if also the other SM fields are non-minimally coupled, quantum effects leading to vacuum destabilization are suppressed can be seen as non-trivial normalization of the fields \[ \text{[Germani, DV 15]} \]
Non-canonical normalization

Canonical momentum on a spacelike hypersurface. $n_\mu$ is an arbitrary timelike unit 4-vector with zero vorticity

$$\pi_\mathcal{H} \equiv \frac{\delta \mathcal{L}}{\delta \dot{\mathcal{H}}} = -2\sqrt{-g} \left( g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) n_\mu \partial_\nu \mathcal{H}$$

Standard equal time commutation rules:

$$\left[ \mathcal{H}(x)^\dagger, \dot{\mathcal{H}}(y) \right] = \frac{1}{2\sqrt{-g} N} i\hbar \delta^{(3)}(x - y)$$

where, in some coordinates adapted to $n_\mu$,

$$N \equiv - \left( g^{tt} - \frac{G^{tt}}{M^2} \right)$$

Two strategies:

- work with the non-canonical fields [De Simone, Hertzberg, Wilczek 09]
- work with canonical fields [Bezrukov, Shaposhnikov 08]
Disentangling the graviton fluctuations

In terms of the original metric $\bar{g}_{\mu \nu}$

$$S = \int d^4x \sqrt{-\bar{g}} \left[ \frac{M_p^2}{2} \bar{R} - \left( \bar{g}^{\mu \nu} - \frac{\bar{G}^{\mu \nu}}{M^2} \right) \left( D_\mu \bar{H}^\dagger \right) \left( D_\nu \bar{H} \right) - \bar{V}(\bar{H}^\dagger \bar{H}) \right]$$

- disformal transformation $g_{\alpha \beta} = \bar{g}_{\alpha \beta} - \frac{D_\alpha \bar{H}^\dagger D_\beta \bar{H}}{M^2 M_p^2}$
- expand and neglect higher-$D$ terms we want to study vacuum properties and inflation
- term with $\bar{G}^{\mu \nu}$ is absorbed by the variation of $\sqrt{-\bar{g}} R$
- a new term $\propto \bar{V}$ is generated by the variation of $\sqrt{-\bar{g}}$

In terms of the disformal metric $g^{\mu \nu}$

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \left( 1 + \frac{\bar{V}(\bar{H}^\dagger \bar{H})}{M^2 M_p^2} \right) g^{\mu \nu} \left( D_\mu \bar{H}^\dagger \right) \left( D_\nu \bar{H} \right) - \bar{V}(\bar{H}^\dagger \bar{H}) \right]$$
Disentangling the graviton fluctuations

In terms of the original metric $\bar{g}_{\mu\nu}$

$$S = \int d^4x \sqrt{-\bar{g}} \left[ \frac{M_p^2}{2} \bar{R} - \left( \bar{g}_{\mu\nu} - \bar{G}_{\mu\nu} \right) \left( \mathcal{D}_\mu \mathcal{H}^\dagger \right) \left( \mathcal{D}_\nu \mathcal{H} \right) - V(\mathcal{H}^\dagger \mathcal{H}) \right]$$

- disformal transformation $g_{\alpha\beta} = \bar{g}_{\alpha\beta} - \frac{\mathcal{D}_\alpha \mathcal{H}^\dagger \mathcal{D}_\beta \mathcal{H}}{M^2 M_p^2}$
- expand and neglect higher-$\mathcal{D}$ terms we want to study vacuum properties and inflation
- term with $\bar{G}_{\mu\nu}$ is absorbed by the variation of $\sqrt{-g} R$
- a new term $\propto V$ is generated by the variation of $\sqrt{-g}$

In the zero-momentum limit (neglecting all the other SM fields), using the classical Einstein equations

$$\bar{G}_{\mu\nu} = \frac{T_{\mu\nu}}{M_p^2} \xrightarrow{p \to 0} - \frac{V}{M^2 p^2} g_{\mu\nu} \Rightarrow N \xrightarrow{p \to 0} 1 + \frac{V}{\Lambda_t^4} , \quad \Lambda_t \equiv \sqrt{M M_p} \lambda^{-1/4}$$
Tree-level unitarity

\[ \frac{\bar{G}^{\mu\nu}}{M^2} (D_\mu \mathcal{H}^\dagger)(D_\nu \mathcal{H}) \leftrightarrow \frac{V(\mathcal{H}^\dagger \mathcal{H})}{M^2 M_p^2} g^{\mu\nu} (D_\mu \mathcal{H}^\dagger)(D_\nu \mathcal{H}) \]

- Expanding \( \bar{G}^{\mu\nu} \) around the Minkowski background,
  \( \bar{g}^{\mu\nu}(x) \rightarrow \eta^{\mu\nu} + M_p^{-1} h^{\mu\nu}(x) \), \( \bar{G}^{\mu\nu} \sim \partial^2 h^{\mu\nu}/M_p \), so that one obtains just derivative interactions, suppressed by \( \Lambda_M^3 \equiv M^2 M_p \)

- This is **wrong in a large scalar background**: the large \( V \) sources Einstein’s equations, generating a non-trivial gravitational background \( \Rightarrow \) mixing with graviton, background-dependent cutoff, \( \Lambda_M \rightarrow \sim M_p \)

- Go to canonical field \( \chi \) and compute \( \Lambda_n \sim (d^n U/d\chi^n)^{-1/(n-4)} \)

Recall Higgs inflation:
- background-dependent effective \( M_p \) in Jordan frame
- background dependent cutoff in Einstein frame
Coupling also fermions and gauge bosons (1)

Fermions

\[
- \frac{\mathcal{L}_{\text{kin}}^{\psi}}{\sqrt{g}} = \left( \bar{g}^{\mu \nu} - \frac{\bar{G}^{\mu \nu}}{M^2} \right) \bar{\psi} \gamma_\mu D_\nu \psi
\]

- Unique non minimal kin. term that does not introduce extra dof’s
- If \( \psi \)'s have SUSY partners, they must have the same coupling

Gauge bosons

\[
- \frac{\mathcal{L}_{\text{kin}}^{A}}{\sqrt{g}} = \frac{1}{4} \left( \bar{g}^{\alpha \nu} \bar{g}^{\beta \nu} + \frac{\mathcal{H}^\dagger \mathcal{H}}{M^2 M_p^2} \frac{** \bar{R}^{\mu \nu \alpha \beta}}{M^2} \right) \text{Tr} \ F_{\alpha \beta} F_{\mu \nu}
\]

- Unique non minimal kin. term that does not introduce extra dof’s
- Prevents trans-Planckian gauge boson masses during inflation
Coupling also fermions and gauge bosons (2)

As before, in the zero-momentum limit,

\[
\left( g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) \bar{\psi} \gamma_\mu D_\nu \psi \to \left( 1 + \frac{V}{\Lambda^4_t} \right) \bar{\psi} \gamma_\mu D_\nu \psi
\]

\[
\left( g^{\alpha\mu} g^{\beta\nu} + \frac{\mathcal{H}^\dagger \mathcal{H} ** R^{\mu\nu\alpha\beta}}{\Lambda^2_t} \right) \text{Tr} F_{\alpha\beta} F_{\mu\nu} \to \left( 1 + \frac{\mathcal{H}^\dagger \mathcal{H} V}{\Lambda^2_t \Lambda^4_t} \right) \text{Tr} F^2
\]

**Small field ⇒ SM**

\[
\phi \ll \Lambda_t \Rightarrow \begin{cases} \mathcal{N}, \mathcal{N}_\psi & \approx 1 \\ \mathcal{N}_A & \approx 1 \end{cases}
\]

**Large field**

\[
\phi \gg \Lambda_t \Rightarrow \begin{cases} \mathcal{N}, \mathcal{N}_\psi & \approx \frac{V}{\Lambda^4_t} \\ \mathcal{N}_A & \approx \frac{\mathcal{H}^\dagger \mathcal{H} V}{\Lambda^2_t \Lambda^4_t} \end{cases}
\]
Large Higgs-background limit: Higgs

- Convenient to switch to the chiral formalism

\[
(D_\mu \mathcal{H}^\dagger)(D_\nu \mathcal{H}) \rightarrow \frac{1}{4} \text{Tr} \left( D_\mu \mathcal{H}^\dagger \right) \left( D_\nu \mathcal{H} \right), \quad \mathcal{H} = h \times \exp \left( i \pi^a T^a \right) \equiv U
\]

where the \( \pi^a \) are the (non-canonical) Goldstone bosons.

- The canonically normalized Higgs boson \( \chi \) is

\[
\chi = \int \sqrt{N} dh \simeq \frac{\sqrt{\lambda}}{6} \frac{h^3}{\Lambda_t^2}
\]

- The potential for the canonical field

\[
U(\chi) \equiv V(h(\chi)) = \lambda \frac{h(\chi)^4}{4} \simeq (m^2 \chi)^{4/3}
\]

where \( m = (9/2)^{1/4} \Lambda_t \lambda^{1/8} = (9/2)^{1/4} \Lambda \lambda^{3/8} \).
Large Higgs-background limit: all together

The canonically normalized fields are

\[ \chi \simeq \frac{\sqrt{\lambda}}{6} \frac{h^3}{\Lambda_t^2} , \quad \pi^a_{can} \simeq \frac{\sqrt{\lambda}}{2} \frac{h^3}{\Lambda_t^2} \pi^a , \quad \psi_{can} \simeq \frac{\sqrt{\lambda} h^2}{2 \Lambda_t^2} \psi , \quad A_{can} \simeq \frac{\sqrt{\lambda} h^3}{2 \Lambda_t^3} A \]

In the chiral representation, for large \( h \) and small momenta,

\[ L_{\text{chiral}} \simeq -\frac{1}{2} (\partial \chi)^2 - \frac{1}{g^2} H_1 - \frac{1}{g't^2} H_2 - L_{W/Z} + L_Y - U(\chi) \]

\[ H_1 = \frac{1}{2} \text{Tr} W_{\mu \nu}^2 , \quad H_2 = \frac{1}{4} B_{\mu \nu}^2 , \quad L_{W/Z} = \frac{\Lambda_t^2}{4} \text{Tr} V_{\mu}^2 , \quad L_Y = -\bar{\psi}^L R \psi^L R , \]

\[ V_{\mu} = i W_{\mu} - i U B_{\mu}^Y U^\dagger , \quad W_{\mu} = 2 W^a_{\mu} \tau^a , \quad W_{\mu \nu} = 2 \partial_{[\mu} W_{\nu]} + i [W_{\mu} , W_{\nu}] , \quad B_{\mu}^Y = B_{\mu} T^3 , \quad B_{\mu \nu} = 2 \partial_{[\mu} B_{\nu]} . \]

First approximation: quantum corrections negligible

- Scale of gauge boson masses set not by \( \bar{\chi} \) but by \( \Lambda_t \simeq 0 \)
- Yukawa interactions suppressed by large \( \bar{\chi} \)
- Field-dependent Higgs mass \( d^2 U / d\chi^2 \simeq (m^4 / \chi)^{2/3} \simeq 0 \)
Matching and vacuum stabilization

- non-canonical kinetic term, $\mathcal{N} \sim 1 + \frac{h^4}{4\Lambda^4}$, $\Lambda = \Lambda_t \lambda^{-1/4}$
- assume $\Lambda_M \gtrsim \mathcal{O}(1 \text{ TeV})$ $\Rightarrow \Lambda \gtrsim 10^7 \text{ GeV}$
- for small $h$ the theory is $\sim \text{SM}$, the state-of-the-art NNLO analyses apply [Degrassi et al 12; Buttazzo et al 13; Bednyakov et al 15]
- for large $h$ the theory is well approximated by $U_{\text{tree}}(\chi)$
- $\mathcal{N}$ grows as $h^4$, the transition will be fast (and $\lambda$ runs slowly)

assume sharp transition at $h_* = \sqrt{2}\Lambda$ (i.e. $\chi_* \approx 0.47\Lambda$) $\Rightarrow m = (9/2)^{1/4} \Lambda \lambda_*^{3/8}$

EW vacuum stabilized if transition happens before $\lambda$ turns negative, $h_0 \approx 6 \times 10^9 \text{ GeV}$ (for central values of $m_h, m_t, \alpha_s$): one can always choose $M$ such that $h_* \ll h_0$ can be accommodated
The Higgs as the inflaton (1)

- $\mathcal{P} \simeq \frac{H^2}{8\pi^2\epsilon M_p^2} \simeq 2 \times 10^{-9} \quad n_s = 1 - 5\epsilon \quad r = 16\epsilon$

- high friction (HF) regime (in which $V(h(\chi)) \gg \Lambda_t^4$)
  \begin{align*}
  \epsilon &= \frac{8 M^2 M_p^2}{3 H^2 h_I^2}, \\
  H^2 &= \frac{V}{3M_p^2}, \\
  N &= \frac{1}{3} \left( \frac{1}{\epsilon} - 1 \right)
  \end{align*}

- $n_s = \begin{cases} 
  0.966 & \text{if } N = 50 \\
  0.972 & \text{if } N = 60
  \end{cases}$, \quad $r = \begin{cases} 
  0.106 & \text{if } N = 50 \\
  0.088 & \text{if } N = 60
  \end{cases}$

BICEP $r < 0.07$ at 2-sigma level $\leftrightarrow N \simeq 75$ (still being within 2-sigma level from the central value of $n_s$ from Planck)

- cosmological parameters indep of $\lambda_*$, but $M$ and $\chi_I$ in general will depend. However, in the HF regime, $m$ and $\chi_I$ are completely fixed by the CMB

  \[ m \simeq \frac{5.38 \times 10^{15}}{(1 + N)^{5/8}} \text{GeV} \quad \chi_I \simeq 3.96 \times 10^{18} \text{GeV} \sqrt{N + 1} \gg \chi_* \]

chaotic inflation scenario
The Higgs as the inflaton (2)

$(m_h, m_t)$ plane  
$(\alpha_s, m_t)$ plane

- purple = successful inflation
- green = SM vacuum stability

CMB ⇒ $m \simeq 4.55 \times 10^{14}$ GeV
- bounds ∼ overlap
Conclusions

- A SM-like Higgs with $M_H \sim 125\text{ GeV}$ does not allow us to infer, in a model independent way, the scale of NP.
- The SM vacuum is probably metastable, but the tunneling is slow enough that the vacuum has a lifetime longer than the age of the universe. We are left with a bunch of issues to solve, anyways.
- If the SM fields have non-minimal kinetic couplings to gravity, one can
  1. choose the new parameter so that the SM vacuum is stabilized even for the central values of $m_h, m_t, \alpha_s$
  2. get successful inflation via the Higgs, due to the gravitationally enhanced friction mechanism, for the SM parameters leading to a stable vacuum
Thanks for your attention!
The $m_t$ issue

- position in the SM phase diag. $\leftrightarrow m_t$
- top mass used is the Tevatron+LHC average $m_t^{MC} = 173.34 \pm 0.76 \text{ GeV}$
- $m_t^{MC}$ extracted with template methods (Pythia mass) from decay products. Event modeling is delicate!
- if one extracts $y_t(\mu)$ from $m_t^{pole}$:
  $\mathcal{O}(\Lambda_{QCD})$ uncert. + is $m_t^{pole} = m_t^{MC}$?
- stay on the safe side: use $\overline{m}_t(m_t) = 162.3 \pm 2.3 \text{ GeV}$ from $t\bar{t}$ inclusive $\sigma$. But can't say much on the SM vacuum until ILC . . .
- exploit high precision in $m_t^{MC}$ determination with new methods
- e.g. $m_t^{MC} \Rightarrow m_t^{pole} = 173.39^{+1.12}_{-0.98} \text{ GeV}$ [Moch 14]