Cartan’s Supersymmetry and the Decay of a $h^0$ with the mass $m_{h^0} \simeq 11\text{GeV}$ to $\Upsilon(1S)\gamma$ and to $\Upsilon(2S)\gamma$

Sadataka Furui

Graduate School of Teikyo University
2-17-12 Toyosatodai, Utsunomiya, 320-0003 Japan

E-mail: furui@umb.teikyo-u.ac.jp

Abstract: In the LHCb detector at CERN, decays of $\chi_b(3P)$ meson to $\Upsilon(1S)\gamma$ and $\Upsilon(2S)\gamma$ are reported at centre-of-mass energy of $\sqrt{s} = 7$ and 8 TeV. Following the success of the assignment of $\chi_b(1P) \rightarrow \Upsilon(1S)\gamma$ of the mass of $m(\chi_b(1P)) = 9.8923$ GeV and $\chi_b(2P) \rightarrow \Upsilon(1S)\gamma$ of the mass of $m(\chi_b(2P)) = 10.2547$ GeV, the new state $\chi_b(3P)$ of the mass of 10.5157 GeV was assigned, but its $J^P$ was not fixed.

We study the possibility that this boson is the light Higgs boson $h^0(0^+)$, and study its decay modes to a $b\bar{b}$ which reduces to an $\Upsilon(mS)$ ($m = 1$ or 2) and $\ell\ell$ which reduces to a $\gamma$, using the Cartan’s supersymmetry. The spin structure of $b\bar{b}$ and $\ell\ell$ and the interaction of $b\bar{b}$ and $\ell\ell$ can produce 3 energy states of the sum of $b\bar{b}$ and $\ell\ell$ energies.

Keywords: Supersymmetry Phenomenology, Hadronic Colliders
1 Introduction

In 2012, the ATLAS Collaboration presented a new \( \chi_b \) state in radiative transition to \( \Upsilon(1S) \) and \( \Upsilon(2S) \)[1, 2]. Since radiative transitions of \( \chi_b(1P) \) and \( \chi_b(2P) \) were observed near the energy of the new state, the state was assigned as \( \chi_b(3P) \), but its \( J^P \) could not be fixed by experimental analyses[3, 4].

We propose an assignment of the new state as the partner of the Higgs boson \( H^0 \) discovered by the ATLAS group[5], which has a lower mass and is called \( h^0[6] \). Higgs field \( H \) belongs to \( SU(2)_L \) doublet and can be expressed as

\[
H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix},
\]

The masses of the gauge bosons are

\[
\mathcal{L}_{MGB} = \frac{\nu^2}{4} g^2 (W_1^2 + W_2^2) + \frac{\nu^2}{4} (g W_3 - g' B)^2
\]

\[
= m_W^2 W^{+\mu} W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu,
\]

(1.1)

where minimum of the potential is

\[
\langle H_u \rangle_{\text{min}} = \begin{pmatrix} 0 \\ \nu_u \end{pmatrix}, \quad \langle H_d \rangle_{\text{min}} = \begin{pmatrix} \nu_d \\ 0 \end{pmatrix}
\]

and \( \nu^2 = \nu_u^2 + \nu_d^2 \).

The possible masses of the Higgs particles are

\[
m_{H^\pm}^2 = m_W^2 + m_A^2
\]

for charged massive states, and

\[
m_{H^0}^2 = \frac{m_{A^0}^2 + m_{A^0}^2}{2} + \frac{1}{2} \sqrt{(m_{A^0}^2 + m_{A^0}^2)^2 - 4 m_{A^0}^2 m_{A^0}^2 \cos^2 2\beta}
\]

\[
m_{H^0}^2 = \frac{m_{A^0}^2 + m_Z^2}{2} - \frac{1}{2} \sqrt{(m_{A^0}^2 + m_{A^0}^2)^2 - 4 m_{A^0}^2 m_{A^0}^2 \cos^2 2\beta},
\]

(1.2)
where $\tan \beta = \nu_u/\nu_d$, for neutral massive states.

When $\cos 2\beta = 0$, $m^2_{H^0} = 0$, $m^2_{H^0} = m^2_{\tilde{A}} + m^2_Z$, and $m_Z = 91.2 \text{GeV}$, $m_{H^0} = 125 \text{GeV}$[5] gives

$$m_{\tilde{A}} = 85.5 \text{GeV}.$$ 

and $m_W = 80.4 \text{ GeV}$ yields $m_{H^\pm} = 117 \text{ GeV}$.

There is a report of the search of $H^+ \gamma$ using the $t \rightarrow H^+b$ decay and $H^+ \rightarrow \tau \nu_\tau$, which yields $m_{H^+} = 120 \text{GeV}$, but in this analysis, the branching fraction $B(H^+ \rightarrow \tau \nu_\tau)$ could not be well determined, and it was assumed to be equal to 1. We expect that it is due to instability of the $H^+$ state. The requirement that $m^2_{H^\pm} = m^2_W + m^2_{\tilde{A}} = (120 \text{ GeV})^2$ gives $m_{\tilde{A}} = 78.0 \text{ GeV}$ and $m_{H^0}$ becomes $125 \text{ GeV}$, with

$$m_{\tilde{A}} = 78.0 \text{GeV} \quad \text{and} \quad \cos 2\beta = \pm 0.1878.$$ 

The fixed $\cos 2\beta$ gives the mass squared of $h^0$

$$m^2_{h^0} = \frac{m^2_{\tilde{A}} + m_Z^2}{2} - \frac{1}{2} \sqrt{(m^2_{\tilde{A}} + m_Z^2)^2 - 4m^2_{\tilde{A}}m_Z^2\cos^2 2\beta} = (11.2 \text{GeV})^2.$$ 

Near this energy region there are $\chi_{b0}(1P, J^{PC} = 0^{++}, 9.86 \text{ GeV})$, $\chi_{b1}(1P, J^{PC} = 1^{++}, 9.89 \text{ GeV})$ and $\chi_{b2}(2P, J^{PC} = 2^{++}, 10.23 \text{ GeV})$ which are expected to be made of $\bar{b}\bar{b}$ and a state which is called $\chi_b(3P, 10.53 \text{ GeV})$. The scalar boson $\chi_b(3P)$ decays radiatively to $\Upsilon(1S)$ and $\Upsilon(2S)$, and its $C = +$ but its $J^P$ is not well known[1, 2, 14].

The mass of $\chi_b(3P)$ is slightly below the $B\bar{B}$ threshold and there remains a possibility that the SUSY-breaking potential[6],

$$V_{SSB} = v(H^+_u H^0_u)\nu_2 \begin{pmatrix} H^0_d \\ H^+_d \end{pmatrix} + v^*(H^0_u H^+_d)(-i\tau_2) \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}$$

$$= v(H^+_u H^-_d - H^0_u H^0_d) + h.c. \quad (1.3)$$

where $v > 0$, makes a $H^\pm$ unstable, and a $h^0$ appears as a $\chi_b(3P)$.

2) $h^0(0^+) \rightarrow \Upsilon(mS)\ell\bar{\ell} \rightarrow \Upsilon(mS)\gamma$

In order to study the decay of $h^0(0^+)$ into $\Upsilon(mS)\gamma$, where $m = 1$ or 2, we adopt the model similar to that used in $H^0 \rightarrow \ell\bar{\ell}\ell\bar{\ell} \rightarrow 2\gamma$[7–12]. In the case of $H^0$ decay diagrams, the upper right circle represents the quark that propagates in the $H^0$, and the lower left circle represents the antiquark that propagates in the $H^0$, and a pair annihilation of the quark and the antiquark were considered.

In the present case we consider a $q\bar{q}$ and an $\ell\bar{\ell}$ are directly produced in $h^0(0^+)$ and the quark becomes a $b$ quark and a vector particle, which we denote $b + \gamma$ and the anti-quark becomes $\bar{b} + \gamma'$. The $b$ and the $\bar{b}$ becomes correlated to make $\Upsilon(1S, J^{PC} = 1^{--}, 9.46 \text{GeV})$, or $\Upsilon(2S, J^{PC} = 1^{--}, 10.02 \text{GeV})$. The $\gamma$ is absorbed by the $\ell$ which is produced in $h^0$ and the $\gamma'$ is absorbed by the $\ell'$, and in the final state there remains a $\bar{\ell}\ell'$ after absorption of $\gamma, \gamma'$ which pair annihilates to a $\gamma$. 

- 2 -
Typical diagrams in which helicity of the \( \bar{b} b \) are chosen along \( i \) or \( k \) are shown in Figures 1-12. There are other diagrams in which chosen helisities are along \( i \) and \( j \), or along \( j \) and \( k \).

In these diagrams, bosons that produce \( \psi C \psi \) s are denoted by \( h_0 \), and bosons that produce \( \phi C \phi \) s are denoted by \( h'_0 \). The quark \( \psi \) changes to \( C \phi \) and the antiquark \( \phi \) changes to \( C \psi \) by emitting a vector particle \( x0 \) or \( x2 \). We define, when the helicity of the quark and lepton are parallel, the exchanged vector particle is \( x0 \) and otherwise the exchanged vector particle is \( x2 \).

We first define the helicity of the produced quark on the upper circle \( \psi \) or \( C \psi \) in the case of the decay of \( h_0 \), and \( \phi \) or \( C \phi \) in the case of the decay of \( h'_0 \). Production of \( b \) or \( \bar{b} \) quark by emission of a vector particle \( x0 \) or \( x2 \) is assigned using the rule of \( \lambda \phi C X \psi \) of Cartan[8], which is defined as

\[
\mathcal{F} = \lambda \phi C X \psi = \lambda \phi \gamma_0 x^\mu \gamma_\mu \psi
\]

\[
= x^1 (\xi_{12} \xi_{14} - \xi_{31} \xi_{12} - \xi_{14} \xi_{12} + \xi_{1234} \xi_1) \\
+ x^2 (\xi_{23} \xi_{14} - \xi_{12} \xi_{24} - \xi_{24} \xi_{12} + \xi_{1234} \xi_2) \\
+ x^3 (\xi_{31} \xi_{23} - \xi_{23} \xi_{31} - \xi_{34} \xi_{12} + \xi_{1234} \xi_3) \\
+ x^4 (-\xi_{14} \xi_{23} - \xi_{24} \xi_{31} - \xi_{34} \xi_{12} + \xi_{1234} \xi_4) \\
+ x^4 (-\xi_{0} \xi_{234} + \xi_{234} \xi_4 - \xi_{243} + \xi_{342}) \\
+ x^4 (-\xi_{0} \xi_{314} + \xi_{314} \xi_4 - \xi_{341} + \xi_{341}) \\
+ x^4 (-\xi_{0} \xi_{124} + \xi_{124} \xi_4 - \xi_{142} + \xi_{241}) \\
+ x^4 (-\xi_{0} \xi_{123} - \xi_{232} \xi_4 - \xi_{312} - \xi_{123}). \tag{2.1}
\]

The quark on the lower part of the right circle is the antiquark of the quark that we considered in the decay of \( H \) to \( 2 \gamma \)[7].

When the \( b \) quark in the upper left corner of the Figure is defined as \( \psi \) or \( C \phi \), we choose the helicity of the antilepton as that of \( C \psi \) or \( \phi \) such that the physical pair creation from the vacuum becomes recovered when the antilepton is replaced by the corresponding antiquark. It corresponds to taking into account quark-antilepton or antiquark-lepton systems of \( ^3P_0 \) states. The helicity of the lepton on the lower part of the left circle becomes parallel as that of the corresponding diagram in the \( H^0 \) decay, and the helicity of the lepton on the upper part of the left circle is defined as the antilepton of the corresponding lower part of the left circle.

In addition to the diagrams in which the helicity of the \( b \) quark and that of the antilepton or the helicity of the \( \bar{b} \) and that of the lepton \( \ell \), or the helicity of \( b \) and that of the antilepton \( \bar{\ell} \) in the upper left corner of the diagrams are parallel, we consider diagrams in which the helicity of the antilepton is replaced by that of the lepton, and that of the lepton is replaced by that of the antilepton. By this method the helicity of the \( b \) quark and the lepton become parallel.

In Fig.1-3, the \( C \psi \) produced by \( h_0 \) changes to a \( \phi \) by emitting a vector particle \( x0 \) or \( x2 \), and the vector particle \( x0 \) produces a lepton \( C \phi \) and by interacting with the \( \phi \) produced
by $x0$ reduces to a $\gamma$. Helicity of the $b = C\phi$ and the antilepton $\phi$ are parallel, and helicity of $\bar{b} = \phi$ and the lepton $C\phi$ are parallel.

When a vector particle $x2$ is emitted, the helicity of $b = \phi$ becomes that of lepton $\phi$ and the helicity of $\bar{b}$ and that of lepton does not become parallel. When $x2$ is emitted, helicity of antilepton can be the parallel to that of $\bar{b}$ or that of the lepton.

As the third diagram, we consider the diagram in which the helicity of $\bar{b}$ quark and the lepton i.e. lines on the right lower corner are interchanged. By this method the helicity of the $b$ quark and $\bar{b}$ become parallel.

In Fig.7-9, the $\phi$ produced by $h_0'$ changes to a $\bar{b} = C\psi$ by emitting a vector particle $x0$ or $x2$ and the vector particles produce a lepton $\psi$. When an $x0$ is emitted, helicity of $\bar{b} = C\psi$ and that of the lepton $\psi$ are parallel, but when $x2$ is emitted, the helicity of the lepton $\psi$ can be parallel to that of antilepton $C\psi$.

### 3 Discussion and conclusion

We extended the analysis of $H^0 \to \ell\ell\ell \to 2\gamma$ based on Cartan’s supersymmetry[8, 13] to $h^0 \to q\bar{q}\ell\ell \to \Upsilon(b\bar{b})\gamma(\ell\ell)$. Different from the $H^0$ decay, $h^0$ produces a $q\bar{q}$ pair, and the $q$ changes to $b$ by emission of a vector particle $\gamma$, and $\bar{q}$ changes to $\bar{b}$ by emission of another vector particle which is denoted $\gamma'$. The $b$ and $\bar{b}$ makes a $\Upsilon(1S)$ or $\Upsilon(2S)$. The $\gamma$ interacts with the $\ell$ produced in the $h^0$ and becomes an $\ell$ and the $\gamma'$ interacts with the $\ell'$ produced in the $h^0$ and becomes a $\ell'$, and the $\ell$ and $\ell'$ pair annihilate and becomes a $\gamma$.

In our model, $b\bar{b}$ in two types of $\Upsilon$ have different helicity and $b\bar{b}$ in one type of $\Upsilon$ have parallel helicity. Since the initial state is assumed to be $0^+$, the helicity of $b\bar{b}$ in $\Upsilon$ and $\ell\ell'$ of $\gamma$ in the final states are parallel.

When the helicity of $b\bar{b}$ are not parallel, the product of coupling $bXq$ and $\bar{q}X\bar{b}$ becomes negative, but when they are parallel it becomes positive.

Coupling of the vector particle and leptons or antileptons is defined by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{(\ell) = e, \mu, \tau} \bar{\psi}^{(\ell)} ((i\gamma_{\mu} - ieA_{\mu}) - M^{(\ell)})\psi^{(\ell)} + h.c.$$  

where

$$\partial^{\mu} F_{\mu\nu}(x) = -\sum_{(\ell) = e, \mu, \tau} e\bar{\psi}^{(\ell)}(x)\gamma_{\nu}\psi^{(\ell)}(x) + h.c..$$

We took here $X = A$, and ignored mixing of the vector boson $Z$ and $\gamma$.

When the final state $\Upsilon(b\bar{b})$ have different helicities, the $\gamma$ produced by $\ell\ell'$ have different helicities, similarly. But there are two different states whether the exchanged vector particle is $x0$ and $b$ and $\ell$ have parallel helicities, or whether the exchanged vector particle is $x2$ and $b$ and $\ell$ have different helicities. We expect the $\chi(1P)$ and $\chi(2P)$ assigned in [2] correspond to the initial states which produces this $\Upsilon(b\bar{b})$.

When the final state $\Upsilon(b\bar{b})$ have parallel helicity, the $\gamma$ produced by $\ell\ell'$ have parallel helicity. We expect the $\chi(3P)$ corresponds to this initial state which produces this $\Upsilon(b\bar{b})$.

Cartan’s supersymmetry allows two types of leptons $\psi$ and $\phi$ and their charge conjugates, and the interaction with vector fields is fixed. We extended the model by introducing...
an effective interaction of antileptons and quarks or leptons and antiquarks with parallel helicities $x0$, and with different helicities $x2$, in the $3P_0$ states. In the quark pair creation in $pp$ annihilation into mesons, the $3P_0$ model was successful\cite{15}. By a detailed comparison of decay modes of $h^0$ and $H^0$, the structure of the Higgs field will become clarified.

Acknowledgement
The author thanks Dr. Fabian Cruz for sending the informaton of the ref.\cite{3}.

References

\cite{1} G. Aad et al., (ATLAS Collaboration), \textit{Observation of a New $\chi_b$ State in Radiative Transitions to $\Upsilon(1S)$ and $\Upsilon(2S)$ at ATLAS}, Phys. Rev. Lett.\textbf{108} (2012), 152001.

\cite{2} V.M. Abazov et al., (D0 Collaboration), \textit{Observation of a narrow mass state decaying into $\Upsilon(1S)\gamma$ in $pp$ collisions at $\sqrt{s}=1.96$ TeV}, Phys. Rev. D\textbf{86} (2012), 031103(R).

\cite{3} The CMS Collaboration, \textit{Search for a light Charged Higgs boson in Top quark decays in $pp$ collision at $\sqrt{s}=7$ TeV}, (2012) [hep-ex/1205.5736 v3]

\cite{4} The LHCb collaboration, \textit{Measurement of the $\chi_b(3P)$ mass and of the relative rate of $\chi_b1(1P)$ and $\chi_b2(1P)$ production}, doi:10.1007/JHEP 10(2014)088.

\cite{5} G. Aad et al., (ATLAS Collaboration), \textit{Combined search for the Standard Model Higgs boson in $pp$ collisions at $\sqrt{s}=7$ TeV with the ATLAS detector}, Phys. Rev. D\textbf{86} (2012), 032003.

\cite{6} P.Labelle, \textit{Supersymmetry Demystified}, McGraw Hill (2010).

\cite{7} S. Furui, \textit{Triality selection rules of Octonion and Quantum Mechanics}, [hep-ph/1409.3761

\cite{8} S. Furui S,\textit{Fermion Flavors in Quaternion Basis and Infrared QCD}, Few Body Syst. \textbf{52}, (2012) 171-187.

\cite{9} S. Furui, \textit{The Magnetic Mass of Transverse Gluon, the B-Meson Weak Decay Vertex and the Triality Symmetry of Octonion}, Few Body Syst. \textbf{53}, (2012) 343.

\cite{10} S. Furui, \textit{The flavor symmetry in the standard model and the triality symmetry}, Int. J. Mod. Phys. A\textbf{27} (2012) 1250158, [hep-ph/1203.5213].

\cite{11} S. Furui, \textit{Axial anomaly and triality symmetry of octonion}, Few Body Syst. DOI 10.1007/s0061-013-0719-9 (2013), [hep-ph/1301.2095].

\cite{12} S. Furui, \textit{Axial anomaly and triality symmetry of leptons and hadrons}, Few Body Syst. \textbf{55}, (2014) 1083, [hep-ph/1304.3776].

\cite{13} É. Cartan, \textit{The theory of Spinors}, Dover Pub.(1966).

\cite{14} K.A. Olive et al, (Particle Data Group) , \textit{Review of Particle Physics}, Chinese Physics \textbf{C38} (2014), 090001.

\cite{15} M.Maruyama, S.Furui, A.Faessler and R. VinhMau, \textit{p p Annihilation into Three Mesons in the $3P_0$ Model}, Nucl. Phys. A\textbf{473}(1987), 649.
Figure 1. The $h(0^+,11) \rightarrow \Upsilon(2S)\gamma(ik)$ with a vector particle $x0$ exchange.

Figure 2. The $h(0^+,11) \rightarrow \Upsilon(2S)\gamma(ik)$ with a vector particle $x2$ exchange.

Figure 3. The $h(0^+,11) \rightarrow \Upsilon(1S)\gamma(ikkk)$ with a vector particle $x2$ exchange.
Figure 4. The $h(0^+, jj) \rightarrow \Upsilon(2S)\gamma(ki)$ with a vector particle $x0$ exchange.

Figure 5. The $h(0^+, jj) \rightarrow \Upsilon(2S)\gamma(ik)$ with a vector particle $x2$ exchange.

Figure 6. The $h(0^+, jj) \rightarrow \Upsilon(1S)\gamma(ii)$ with a vector particle $x2$ exchange.
Figure 7. The $h'(0^+, 11) \rightarrow \Upsilon(2S)\gamma(ki)$ with a vector particle $x0$ exchange.

Figure 8. The $h'(0^+, 11) \rightarrow \Upsilon(2S)\gamma(ik)$ with a vector particle $x2$ exchange.

Figure 9. The $h'(0^+, 11) \rightarrow \Upsilon(1S)\gamma(ii)$ with a vector particle $x2$ exchange.
Figure 10. The $h'(0^+, jj) \to \Upsilon(2S)\gamma(ki)$ with a vector particle $x0$ exchange.

Figure 11. The $h'(0^+, jj) \to \Upsilon(2S)\gamma(ik)$ with a vector particle $x2$ exchange.

Figure 12. The $h'(0^+, jj) \to \Upsilon(1S)\gamma(ii \text{ or } kk)$ with a vector particle $x2$ exchange.