Testing the Accuracy of Data-driven MHD Simulations of Active Region Evolution

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Abstract

Models for the evolution of the solar coronal magnetic field are vital for understanding solar activity, yet the best measurements of the magnetic field lie at the photosphere, necessitating the development of coronal models which are “data-driven” at the photosphere. We present an investigation to determine the feasibility and accuracy of such methods. Our validation framework uses a simulation of active region (AR) formation, modeling the emergence of magnetic flux from the convection zone to the corona, as a ground-truth data set, to supply both the photospheric information and to perform the validation of the data-driven method. We focus our investigation on how the accuracy of the data-driven model depends on the temporal frequency of the driving data. The Helioseismic and Magnetic Imager on NASA’s Solar Dynamics Observatory produces full-disc vector magnetic field measurements at a 12-minute cadence. Using our framework we show that ARs that emerge over 25 hr can be modeled by the data-driving method with only ~1% error in the free magnetic energy, assuming the photospheric information is specified every 12 minutes. However, for rapidly evolving features, under-sampling of the dynamics at this cadence leads to a strobe effect, generating large electric currents and incorrect coronal morphology and energies. We derive a sampling condition for the driving cadence based on the evolution of these small-scale features, and show that higher-cadence driving can lead to acceptable errors. Future work will investigate the source of errors associated with deriving plasma variables from the photospheric magnetograms as well as other sources of errors, such as reduced resolution, instrument bias, and noise.

Key words: magnetohydrodynamics (MHD) – Sun: corona – Sun: magnetic fields – Sun: photosphere

1. Introduction and Main Result

In order to understand the complex interplay of magnetic, hydrodynamic, and thermal forces associated with the onset of various manifestations of solar activity, it is vital to know the magnetic field in the solar corona, where the storage and release of energy in non-potential (\(J = \nabla \times \mathbf{B}/\mu_0 \neq 0\)) magnetic fields is the cause of solar flares, coronal mass ejections (CMEs), and prominence eruptions. However, as yet no method exists to accurately achieve this goal. Fortunately, multiple missions such as the Solar Optical Telescope on Hinode (SOT, Tsuneta et al. 2008), the Helioseismic and Magnetic Imager on the Solar Dynamics Observatory (HMI/SDO, Scherrer et al. 2012), Big Bear Solar Observatory’s New Solar Telescope (Didkovsky et al. 2003), and the National Solar Observatory’s upcoming Daniel K. Inouye Solar Telescope (DKIST, Tritschler et al. 2015) are or will be able to observe the magnetic field at the photosphere. Currently, there are HMI observations of the Sun’s surface vector magnetic field at one optical depth every 12 minutes. These data can be used in a variety of ways to estimate the coronal magnetic field, using a number of assumptions.

One approach is to use measurements of the magnetic field at the photospheric surface, assuming that the constant optical depth surface can be transformed to a constant depth surface, and extrapolate the magnetic field into the domain above this surface, using either the potential, \(J = 0\), or more general force-free, \(J \times \mathbf{B} = 0\) (e.g., Wheatland et al. 2000; Wiegelmann 2004), approximations. These extrapolation methods can produce estimates of the three-dimensional (3D) magnetic field at discrete times during an active region’s (AR) evolution based on the magnetogram data, and are used to estimate useful indices such as the free magnetic energy (\(E_{\text{free}} \equiv E - E_{\text{pot}}\) where \(E\) is the total magnetic energy, and \(E_{\text{pot}}\) is the energy of the potential magnetic field). However, the approximations used in these extrapolations (namely \(\beta = 2\mu_0 P/B^2 = 0\), where \(P\) and \(B\) are the gas pressure and magnetic field strength, respectively) are not generally true for the lower solar atmosphere above where the photospheric data are observed. Furthermore, Peter et al. (2015) recently revisited the basic assumptions associated with the force-free field extrapolations, guided by analysis of the MHD equations, global energy considerations, and 3D MHD simulations. Their analysis suggests that the relative difference between the total energy in a volume of the atmosphere, and that obtained from a force-free extrapolation which assumes \(\beta = 0\), can be of the order of the plasma \(\beta\) averaged over that volume.

In recent years, the dynamic coronal magnetic field has been modeled by considering the field evolution as a succession of equilibria, with boundary conditions informed by the measurement of the photospheric magnetic field. These “data-driven” models have various assumptions. In the magneto-frictional approach (e.g., Cheung & DeRosa 2012; Weinzierl et al. 2016), the lower boundary of a zero-\(\beta\) (no plasma forces) coronal model is periodically updated with photospheric magnetic field observations, and the velocity everywhere is set proportional to the Lorentz force. Then an equilibrium is found before the next input of photospheric data. Relaxing the zero-\(\beta\) approximation, recent data-driven MHD approaches assume a low \(\beta\) atmosphere and update the lower boundary with the observed normal magnetic field and user-defined velocities. At \(t = 0\) a potential field extrapolation is used to fill the coronal volume, and after each update of the normal field and velocities on the boundary, the system is allowed to relax to a new equilibrium. The boundary velocities...
are derived from either (a) local correlation tracking methods (Bingert & Peter 2011; Bourdin et al. 2013; Galsgaard et al. 2015), (b) imposed density perturbations proportional to the horizontal magnetic field magnitude at the surface (Jiang et al. 2011, 2012), or (c) by the method of MHD characteristics (Wu et al. 2006).

While these data-driven models of the solar atmosphere represent an attractive approach to determining the magnetic field in the corona, there are many limitations that need to be evaluated: The lower boundary is assumed to be zero-\(\beta\) or low-\(\beta\) either by reducing the driving magnetic field strength, using coronal densities and temperatures, or both, and in general, only the normal component of the magnetic field vector is used. Furthermore, these methods use the photospheric magnetic field, with the other MHD quantities being inferred using various assumptions, to calculate a series of MHD equilibria, rather than to dynamically drive an MHD simulation at the photospheric boundary.

In this work it will be shown that this latter approach, that of driving a dynamic MHD model above the photosphere using observed MHD variables, is achievable with enough photospheric data. Furthermore, a framework is developed to test and validate such a method. There are multiple sources of error associated with the estimation of the coronal magnetic field using this data-driving approach. Two such sources are: (1) inferring the photospheric plasma velocity, density, and temperature when one only has measurements of the magnetic field at the photosphere, and (2) limited cadence of the photospheric data. This study focuses on the latter of these two issues here: suppose one has exact measurements of all of the necessary MHD variables to use as boundary conditions to drive a dynamic MHD model of the photosphere, chromosphere, and corona at a particular temporal frequency. How accurate can such a data-driven solution of the coronal magnetic field be, and how does the accuracy depend on the particular frequency of the driving data and on the temporal evolution of the AR’s surface magnetic field? These are important questions, as the answers can tell us whether the standard HMI time interval of 12 minutes is sufficient to accurately describe the coronal magnetic field above ARs using data-driven MHD modeling. It can also guide us as to which ARs are appropriate to study with such methods, and which evolve too quickly to be accurately modeled at such a driving interval.

Ideally, one would test the accuracy of such a data-driven simulation by comparing the estimated MHD solution with the actual temporal evolution of the MHD variables in the domain above the surface. Obviously, this latter ground-truth data set does not exist for solar observations. In this study, in lieu of this ground-truth, a dynamic MHD simulation of AR formation is run. This domain extends from below the surface into the corona, and models the emergence of magnetic flux from the upper layers of the convection zone through the stratified layers of the photosphere and chromosphere, and into the corona. Time-series of photospheric data are extracted from this “ground-truth” data set. These “pseudo-magnetograms” (which also include the velocity, density, and temperature) are used as a dynamic lower boundary condition for a new “data-driven” run that extends from the photosphere to the corona.

Using this framework, the accuracy of the data-driving method is calculated by comparing the ground-truth and data-driven runs using various error diagnostics, and inferences are made on the required temporal information required to obtain satisfactory results with real observational data.

Figure 1 shows the main results from our investigation, and plots the absolute value of the percentage error in the free magnetic energy, comparing the data-driven MHD solution to the ground-truth data set for a data-driving interval of 12 minutes and for varying emergence timescale (see Section 3.1 for the definition of this error measure). The range of emergence timescales is spanned by varying the flux, twist, and initial depth of the convection zone magnetic field in the ground-truth run. The figure highlights that, generally, the longer that an AR takes to emerge, the more accurate the data-driven MHD solution can be, when using a fixed driving interval of 12 minutes. For ARs that emerge over long timescales, the error in the free magnetic energy can be as low as 1%. The two red points are exceptions to this rule and, as will be shown, are results from ARs where small-scale features vary on timescales shorter than 12 minutes. The details of the procedures used to generate this figure are contained in the following sections, in which the methodology of using self-consistent dynamic MHD simulations of AR formation to test data-driven coronal models is prescribed, and results are presented on how the driving data interval affects the accuracy of the data-driven MHD solution.

2. Using MHD Simulations to Test the Accuracy of Data-driven Models of the Magnetic Field above Active Regions

2.1. Ground-truth MHD Simulations

Our ground-truth data set is a simulation of the dynamic emergence of a twisted flux tube from the convection zone into the corona. This type of simulation has been used previously to understand the emergence of magnetic flux through the convectively stable layers of the photosphere and above, the formation of bipolar ARs, the observed shearing and rotational motions seen in ARs, the build-up of coronal electrical current, and the creation of stable and unstable coronal flux ropes (see living review by Cheung & Isobe 2014). The equations solved and the details of the setup of the simulation can be found in Leake et al. (2014), and the details of the setup are summarized here. The simulations are performed using the Lagrangian Remap (LaRe) code (Arber et al. 2001), which evolves the
MHD variables on a staggered cartesian grid. The initial thermodynamic state is a hydrostatic atmosphere consisting of a model convection zone, photosphere/chromosphere, transition region, and corona, and this is the same in all the simulations in this study. A one-dimensional (1D) slice of this initial atmosphere can be seen in Figure 2. A buoyant twisted magnetic flux tube structure is inserted into the model convection zone, aligned along the \( \hat{y} \) direction:

\[
B = B_i(-q(z-z_t)e^{-(\alpha)^2}, e^{-(\alpha)^2}, qxe^{-(\alpha)^2})
\]  
(1)

where \( y \) is the axial direction of the tube, \( z \) is the vertical direction, \( B_i \) is the axial field strength at the center of the tube, \( a \) is the width of the Gaussian profile, \( q \) is the twist, and \( z_t \) is the tube depth. The domain is 87 Mm in the horizontal \((x)\) direction, and extends from \(-10 \) to \(50\) Mm in the vertical \((z)\) direction for simulations 1–4 and from \(-37 \) to \(50\) Mm for simulations 5–9, where \( z = 0 \) denotes the photosphere.

The majority of previous flux emergence simulations, including those in Leake et al. (2013, 2014), are relatively small, based on the axial flux in the tube and the size of the AR, with a typical flux \(<10^{20}\) Mx. Some larger-scale simulations have recently been performed, and suggest that the mechanism for emergence is similar to that in the smaller-scale scenarios, and that the emergence rate depends on the field strength, depth, width, and twist of the initial flux tube (Norton et al. 2016).

The simulations are run in 2.5D, which means that there are no variations in the axial \( (\hat{y}) \) direction but there is still a component of the vector fields in this direction. This allows us to run simulations for many realizations of the model parameter space and extensively investigate the accuracy of the “data-driven” models described below. Table 1 shows the initial model parameters used for nine ground-truth simulations, along with derived quantities in those simulations. Figure 2 shows the initial magnetic pressure as a function of height for a selection of those simulations.

The details of the general evolution of this initial condition can be seen in the top panels of Figure 3, and are discussed in Leake et al. (2010): as the tube rises it expands horizontally when it reaches the stable surface layers, and then emerges through these stable layers and into the corona by virtue of the magnetic buoyancy instability.

For each realization of the parameter space in Table 1, the emerging AR exhibits a different emergence rate. The emergence time, \( t_{\text{emerge}} \), is defined as the time elapsed between 10% and 90% of the maximal potential magnetic energy emerging above the surface. The potential energy above the surface is \( \int_{z>0} B_{\text{pot}}(x, z) dz \), where \( B_{\text{pot}} \) is the potential magnetic field with the same normal component on the boundary as the total magnetic field. Plots of the potential energy above the surface, normalized to its maximal value, are shown for Simulations 2, 5, and 9 in Figure 4. As can be seen in Table 1, our parameter study provides a range of emergence times from less than 1 to \(~25\) hr, and so allows us to make conclusions regarding ARs that form on a wide range of timescales.

2.2. Data-driven Simulations

The MHD simulations described in Section 2.1 model the evolution of the magnetic field as it buoyantly rises from the upper layers of the convection zone, through the stratified layers of the photosphere and chromosphere, and into the corona. Time-series of photospheric data are then extracted from this ground-truth data set. These “pseudo-magnetograms” (which also include the velocity, density, and temperature) are used as a dynamic lower boundary condition for a new “data-driven” run that extends from the photosphere to the corona. The ground-truth and data-driven runs are then compared above the surface to test the accuracy of the data-driving method, using certain error diagnostics. Provided that enough temporal and spatial resolution is applied, and provided that the lower boundary is driven consistently with the MHD equations, the magnetic field above the photosphere in the data-driven and ground-truth runs should be the same (within the typical accumulation of machine precision errors over space and time).

Figure 5 shows where the MHD variables are evolved on the staggered 2.5D LaRe grid. The cell center for the \( i \)th vertical cell is located at a height of \( z_i(1z) \) and the cell edge is located at \( z_i(1z) \). Density \((\rho)\), energy \((\epsilon)\), and the horizontal magnetic field components \((B_x, B_y)\) are defined at \( z_i(1z) \). The vertical magnetic field and the velocity vector \( V \) are defined at \( B_z(1z) \).

MHD only requires the specification of variables on the lower boundary of the data-driven runs at \( z_b(0) \). However, at present LaRe treats the boundary cells of the driven simulation with the same equations as the interior cells of the simulation. Therefore, because of the third-order gradients used in the remap step in LaRe, the integration method requires information from two surrounding cells in each direction to evolve the system. For example, the evolution of values at \( x_c(1) \) requires information from \( x_c(0) \) and \( x_c(1) \), which are formally exterior to the driven simulation. This approach, in a study focused on the effects of the temporal cadence of the driving data, allows for a proof of concept of the data-driving method and for the specification of errors associated with insufficient temporal information. This approach, however, may over-simplify the boundary information in certain conditions. Future work will investigate particular choices of boundary implementation, such as the method of characteristics (e.g., Wu et al. 2006), to determine which MHD variables are required at the boundary.

In the data-driven run, the boundary is supplied with all of the MHD variables (magnetic field, velocity, pressure, and temperature) at discrete intervals, and between those times interpolation is used to update the boundary conditions for each iteration of the numerical code. Figure 3 shows the result of this data-driving process at various times during the emergence of magnetic flux from beneath the surface, for Simulation 2. The
snapshots are taken at 1, 1.7, and 2.4 hr. The top panels show the ground-truth emergence run, and the bottom panels show the data-driven run, when the pseudo-magnetograms are applied to the data-driven boundary every 2.4 s, which is much smaller than the emergence timescale of 1.27 hr (Table 1) and is approximately equal to the initial CFL (Courant et al. 1928) limited timestep in the simulation. This timestep is defined as $\Delta t_{\text{CFL}} = \min(\Delta x/|V|, \Delta x/C_{\text{f}})$, where $C_{\text{f}} = \sqrt{C_s^2 + C_A^2}$, $\Delta x$ is the grid-cell resolution (170 km), and $C_s$ and $C_A$ are the local acoustic and Alfvén speed, respectively.

Figure 3 shows that the data-driving process is successful on a fundamental level: the magnetic flux appears at the photosphere in the data-driven run where no significant flux was initially present at all in the domain. Not only that, but the flux distribution it generates within the domain looks visually the same as what is generated in the ground-truth run, where it is self-consistently generated by flux emergence rather than by boundary driving.

In the next section, the simulations will be examined using quantitative error analysis, focusing on how the errors vary when one drives at slower cadences, which are more representative of cadences associated with standard photospheric observations.

### 3. Results

In this section results of quantitative error analysis are presented, first focusing in detail on one simulation (Simulation 2) that has an emergence timescale of 1.27 hr, and then expanding to the nine simulations listed in Table 1. For each simulation, the frequency at which the pseudo-magnetograms from the ground-truth run are used to drive data-driven runs is varied.

#### 3.1. Case-study of Simulation 2

For a better visual comparison of the 2.4 s data-driven and the ground-truth run for Simulation 2, Figure 6 shows the difference in the two magnetic field solutions at the same three moments in time (1, 1.7, and 2.4 hr). The difference is calculated at each pixel $(i)$ in the two-dimensional (2D)-driven domain as $\log_{10}|1 - b_i^2/B_i^2|$, where $B_i$ is the ground-truth magnetic field and $b_i$ is the data-driven magnetic field. The errors appear maximal at the edge of the emerging structure, with lower errors inside the emerging magnetic field.

To further compare the data sets of magnetic field between the ground-truth and data-driven runs, a set of error metrics based on some of the scalar functions used in Schrijver et al. (2006) is used:

$$1 - C_{\text{vec}} = 1 - \frac{\sum_i (b_i \cdot B_i)}{\sqrt{\sum_i b_i^2} \sqrt{\sum_j B_j^2}}$$

$$E_n = \sum_i |b_i - B_i|/\sum_i |B_i|$$

$$1 - \epsilon = 1 - \frac{\sum_i |b_i|^2}{\sum_i |B_i|^2}$$

$$1 - \phi_y = 1 - \frac{\sum_i b_y,i}{\sum_i B_y,i}$$

$$1 - \epsilon_{\text{free}} = 1 - \frac{\sum_i (|b_i|^2 - |b_{\text{pot}}|^2)}{\sum_i (|B_i|^2 - |B_{\text{pot}}|^2)},$$

where the subscript 'pot' represents the potential magnetic field. These functions represent, respectively, the vector correlation ($1 - C_{\text{vec}}$), the normalized vector error ($E_n$), the error in the total magnetic energy ($1 - \epsilon$), the errors in the out-of-plane ($y$) and horizontal ($x$) flux ($1 - \phi_y$, $1 - \phi_x$), and the error in the free magnetic energy ($1 - \epsilon_{\text{free}}$). The last metric is a new metric, equal to 1 minus the ratio of free magnetic energy in the data-driven run to that in the ground-truth run. Note that if $b_i = B_i$ everywhere, then all of these functions are identically zero.

To investigate the effect of the temporal frequency of the driving data on the accuracy of the data-driven solution, the frequency with which boundary data is supplied to the data-driven runs is varied, and linear interpolation is used at steps between data inputs. The data-driven and ground-truth runs are compared above the surface to test the accuracy of the data-driving. Primary driving timescales of $[0.04, 0.12, 0.4, 1.2, 4, 12]$ minutes are used, the largest being the standard timescale of the HMI magnetic field data. Two additional data-driven runs are performed. The first is driven every $\Delta t_{\text{CFL}}$, where $\Delta t_{\text{CFL}}$ is the CFL-limited timestep of the numerical scheme. Figure 7 shows $\Delta t_{\text{CFL}}$ as a function of time for Simulation 2. The timestep decreases significantly when magnetic field emerges into the low-density corona and the Alfvén speed increases. The second additional data-driven run is provided with boundary data at every substep within one CFL iteration of

### Table 1

| Sim. | $B$ Field $B_i$ (kG) | Tube Width $\delta$ (Mm) | Tube Twist $qa$ | Tube Depth $z_t$ (Mm) | Beta $\beta_{\text{hub}}$ | Emergence Time $t_{\text{emerge}}$ (hr) | Free Energy Error $1 - \epsilon_{\text{free}}$ (%) |
|------|-------------------|------------------|-------------|-----------------|-----------------|----------------------------|-----------------------|
| 1    | 3.90              | 0.85             | 1.00        | $-4.08$         | 51.06           | 0.83                      | $+$46.9               |
| 2    | 6.50              | 2.25             | 0.50        | $-5.44$         | 29.80           | 1.27                      | $+$19.7               |
| 3    | 5.00              | 2.25             | 0.50        | $-5.44$         | 51.06           | 1.91                      | $+$13.0               |
| 4    | 3.90              | 2.25             | 0.50        | $-5.44$         | 84.55           | 2.36                      | $-$6.27               |
| 5    | 13.0              | 4.25             | 0.50        | $-17.0$         | 144.0           | $-$4.98                   | $-$496.0              |
| 6    | 13.0              | 4.25             | 0.50        | $-20.4$         | 230.1           | 6.90                      | $+$1.50               |
| 7    | 13.0              | 4.25             | 0.40        | $-20.4$         | 230.5           | 8.05                      | $-$124.0              |
| 8    | 8.00              | 4.25             | 0.50        | $-20.4$         | 609.0           | 12.17                     | $+$1.30               |
| 9    | 8.00              | 4.25             | 0.50        | $-20.4$         | 609.0           | 24.92                     | $+$1.20               |

Note. The bold rows are particular simulations discussed later in this paper. The final column is results from analysis of the error in the associated data-driven run, discussed in Section 3.2. $\beta_{\text{hub}}$ is defined at the center of the tube.
the numerical code where an interior MHD variable is updated. This particular "sub-CFL" data-driven run should thus result in the same atmosphere above \( z = 0 \), with errors equal to spatially and temporally accumulated machine precision errors for the chosen error metric.

Figures 8–10 show the magnitude of the metrics of Equation (2) for Simulation 2, at 1, 1.7, and 2.4 hr into the emergence, as a function of the driving interval. The errors are shown on logarithmic (left axis, diamonds) and linear (right axis, squares) scales. The two additional data-driven runs, in which the boundary is driven with data on the sub-CFL and CFL-limited timestep, have driving timesteps that vary throughout the simulation but are plotted for illustrative purposes at times of .0003 and .0004 minutes, respectively.

Using the the linear scale in Figures 8–10, one can see that the errors are insignificant at driving intervals less than 1 minute, which is much less than the emergence timescale of 1.27 hr for Simulation 2 (Table 1). Above a driving interval of 1 minute, the errors increase, with, e.g., the error in the free energy (magenta line) reaching 30% at 1.7 hr into the emergence, when driven at a 12-minute cadence (Figure 9).

Using the logarithmic scale in Figures 8–10, and focusing on the metric \( 1 - C_{\text{vec}} \) at 2.4 hr (black lines of Figure 10), one can...
Figure 6. Spatial representation of the error in the data-driven run, given by $\log_{10}(1 - \beta^2/B^2)$, for Simulation 2 with a driving interval of 0.04 minutes, or 2.4 s.

Figure 7. CFL-limited timestep as a function of time for Simulation 2.

Figure 8. Magnitude of the error metrics for the data-driven runs, at 1 hr into the run, as a function of the driving interval, for the case of Simulation 2. Diamonds are on a log scale and the squares are on a linear scale. The data points at 0.003 and 0.004 minutes are for the sub-CFL and CFL timesteps, respectively.

Figure 9. Same as Figure 8 but for a time of 1.7 hr.

Figure 10. Same as Figure 8 but for a time of 2.4 hr.
see that with decreasing driving interval, the magnitude of the error plateaus at about $10^{-3}$ for Simulation 2, even at the CFL-limited timestep (represented by the data point at .0004 minutes). For the sub-CFL driving interval (represented by the data point at .0003 minutes) however, the error drops to $10^{-14}$. The other metrics show larger magnitudes at this sub-CFL interval, as their integrals are cumulative in nature. This analysis shows that the method of data-driving, when using all of the available boundary data as needed by the numerical algorithm, is able to reproduce the ground-truth solution to within errors associated with the accumulation of machine precision, and allows confident conclusions to be drawn at longer driving intervals.

The free magnetic energy is an important measure for space weather, as it gives an approximate measure for the amount of magnetic energy that can be released in a solar flare, CME, or filament eruption, and so warrants error analysis in these data-driven runs. However, the free magnetic energy is defined as the difference between the total magnetic energy and the energy in the potential magnetic field, where the potential field is the field having the same vertical field component at the surface as the reference field, but with no electric currents ($\mathbf{J} \sim \nabla \times \mathbf{B} = 0$). Before making conclusions regarding the accuracy of the data-driven runs based on the error in free magnetic energy, it must be established that these errors are not due to errors in the potential magnetic energy.

At specific times (e.g., every 12 minutes) when surface boundary data are supplied from the ground-truth run to the data-driven run, the potential field in the data-driven run should be identical to the potential field above the surface in the ground-truth run. In this case the error in the free magnetic energy will be equal to the error in the total magnetic energy multiplied by the ratio of total to free energy. This is shown below, denoting $E_{\text{dd}}$ for the data-driven solution and $E_{\text{gt}}$ for the ground-truth solution, and using the assumption that $E_{\text{pot}} = E_{\text{pot}}^{\text{dd}} = E_{\text{pot}}^{\text{gt}}$:

$$1 - \epsilon_{\text{free}} = 1 - \frac{E_{\text{free}}^{\text{dd}}}{E_{\text{free}}^{\text{gt}}} = \frac{E_{\text{free}}^{\text{gt}} - E_{\text{free}}^{\text{dd}}}{E_{\text{free}}^{\text{gt}}} = \frac{E_{\text{free}}^{\text{gt}} - E_{\text{pot}}^{\text{gt}} + E_{\text{pot}}^{\text{gt}}}{E_{\text{free}}^{\text{gt}}} = \frac{E_{\text{pot}}^{\text{gt}}}{E_{\text{free}}^{\text{gt}}} \left(1 - \frac{E_{\text{free}}^{\text{dd}}}{E_{\text{free}}^{\text{gt}}}\right) = \frac{E_{\text{gt}}^{\text{gt}}}{E_{\text{free}}^{\text{gt}}} (1 - \epsilon).$$

(3)

At times between the regular data-driving inputs, errors will grow in the potential energy, as the boundary data are not exactly equal to the ground-truth data at those times, but linearly interpolated. It is necessary to check that these errors in the vertical magnetic field do not result in significant errors in the potential field. Figure 11 shows the error in potential, total, and free energy in the data-driven run around the 3 hr mark for Simulation 2, when using driving data every 12 minutes. As predicted, the error in the potential energy is zero at data-input times, and maximal at points between these times. At these times, the absolute error in the potential energy is below 0.5%. Noting that the errors in the free energy are approximately 20%-30% during this period, it is clear that the errors in potential energy associated with the linear interpolation between driving time inputs do not significantly contribute to the errors in the free energy. Furthermore, the dashed line in Figure 11(b) is the total energy error multiplied by the ratio of total to free energy, as in Equation (3), and only deviates slightly from the free energy error at midpoints between data-driving inputs. This analysis shows that calculated errors in free magnetic energy can be reliably used to make conclusions about the accuracy of the data-driven runs in the following section.

### 3.2. Parameter Study

Figure 12(a) shows the emergence time for each of the nine ground-truth simulations as a specific function of the initial condition model parameters, $\sqrt{\beta_{\text{tube}}/qa}$. This function is chosen based on insight from previous simulations of flux emergence in the convection zone and atmosphere above: Murray et al. (2006) showed that as flux tubes rise in the convection zone, they fragment based on the amount of twist ($qa$ in these simulations) in the tube. The lower the twist, the more fragmented and distorted the tube becomes, and as a result, the less magnetic flux is able to build up at the top of the convection zone to drive the emergence. In addition, Longcope & Fisher (1996) and Emonet & Moreno-Insertis (1998) showed that the initial rise speed of the tube increases in proportion to $1/\sqrt{\beta_{\text{tube}}}$, and although there is no guarantee that the rise speed influences the emergence speed, this is a reasonable function to use to predict emergence rates. Relating these insights to the initial parameters of the ground-truth runs, and based on Figure 12(a), a monotonic dependence of the emergence time on $\sqrt{\beta_{\text{tube}}/qa}$ is suggested here, leaving a more complete parametric study of flux emergence to future publication.

Figure 12(b) shows the magnitude of the error in the free magnetic energy in the data-driven runs for each of the nine simulations from Table 1, as a function of the emergence time, when the data-driven run is supplied with MHD data every 12 minutes. The error calculation is performed toward the end of each emergence event, when the potential energy has reached its maximal value. From Figure 12(b), a general inverse relationship between the emergence timescale and the error in the free energy can be seen, with the slowest emerging Simulation (9) having a free energy error in the data-driven solution just above 1%. This result is encouraging for the validity of this method of data-driving to model the coronal
magnetic field above emerging ARs using HMI magnetograms, particularly as future data may be available at even smaller intervals (Sun et al. 2016).

However, from Figure 12(b), it is also clear that there are two exceptions in the parameter space, Simulations 5 and 7 (data points are colored red in both panels), which show very large error magnitudes (>100%). As seen in Table 1, the error in the free magnetic energy (1 – εfree) of the data-driven run for these two simulations is both large and negative, indicating that there is much more free magnetic energy in the data-driven run than in the ground-truth run. Note also from Table 1 that these two simulations only differ by relatively small values of the initial flux tube parameters and hence emergence time from other simulations that show small (<20%), positive errors in the free magnetic energy. To understand why and when such large errors might occur in data-driven models using observed magnetograms, it is instructive to examine these two simulations further.

Given that the free magnetic energy is related to the magnetic energy in the non-potential \( (J \neq 0) \) part of the magnetic field, it is worthwhile to analyze the amount of currents in the data-driven run compared to the ground-truth run. This is done for a simulation that shows small positive errors in free magnetic energy (Simulation 4), and one that has large negative errors (Simulation 5). The lower panels of Figures 13–14 show a measure of magnetic field variability \( dB \) (red triangles) for the ground-truth data for Simulations 4 and 5, the error in free energy for the data-driven run (solid black line), and the ratio of out-of-plane current between the data-driven and ground-truth run (solid red line). The ratio of out-of-plane current is given by \( \frac{\sum |I|_{\text{driven}}}{\sum |I|_{\text{ground}}} \), with the sum over the domain above \( z = 0 \). The variability measure \( dB \) is a measure of how much the ground-truth magnetic field changes between driving input times of \( dt = 12 \) minutes:

\[
dB(t) = \sum_{x_i} \left| \frac{B_z(x_i, t + dt) - B_z(x_i, t)}{B_z(x_i, t + dt) + B_z(x_i, t)} \right|
\]  

where \( x_i \) are all points on the model surface (\( z = 0 \)). The upper panels in Figures 13–14 show the potential energy in the ground-truth simulation to highlight where in time during the emergence the errors occur.

As can be seen in Figure 13, for Simulation 4 the variability measure \( dB \) remains less than unity, and the errors in free energy remain small (<20%). By contrast, in Simulation 5 (Figure 14) the variability measure has periods where it is larger than unity, as does the ratio of current, and the free energy error becomes increasingly negative during the emergence.
Figure 15. Panel (a): error in free energy for the data-driven run vs. the ratio of out-of-plane current (data-driven to ground-truth), for all the simulations in the parameter study. The red points are Simulations 5 and 7. Panel (b): the same but for in-plane current. Panel (c): the same but for a measure for the variability of the vertical magnetic field data on a 12-minute cadence, dB.

Figure 16. Panel (a): magnitude of current density (normalized by $B_0/\mu_0 L_0$ where $B_0 = 1300$ G and $L_0 = 170$ km) and contours of flux $A_y$ (white lines) for Simulation 4 at 108 minutes. Panel (b): the same but for Simulation 5 at 264 minutes.

Figure 15 shows plots of the free energy error against the ratio of out-of-plane current, the ratio of in-plane current $\sum |(J_x, 0, J_z)|_{\text{driven}}/\sum |(J_x, 0, J_z)|_{\text{ground}}$, and magnetic field variability $dB$ for all nine simulations. Those emergence events that exhibit large magnetic field variability (dB) over 12-minute intervals (5 and 7) have large currents generated in the data-driven runs and large, negative errors in the free magnetic energy. This is suggestive that the source of the large errors in the data-driven runs is the large variability of magnetic field and associated currents.

To investigate the origin of these large currents in Simulations 5 and 7, the nature of the emergence, in particular the morphology of the emerging magnetic field, is examined. Figure 16 shows the shape of the fieldlines in the 2D $(x, z)$ plane in Simulations 4 (small positive errors in free magnetic energy) and 5 (large, negative errors). The fieldlines are represented by contours of constant flux $A_y$. Also shown are the magnitude of the current density in normalized units. Simulation 4 (panel a) shows a simple bipolar emergence, with no undulations of the fieldlines crossing the surface.
Simulation 5 shows multiple undulations of fieldlines, and associated regions of larger current density.

To represent these two emergence events in time, the top two panels of Figure 17 show the vertical field along the surface, stacked in time, for the ground-truth runs of Simulations 4 and 5 at a cadence of 24 s. For the simple emergence, e.g., as that seen in Figure 16(a) for Simulation 4, one would expect two polarities slowly separating in time, as the magnetic flux associated with the original flux tube emerges through the surface. This is indeed seen for Simulation 4 in Figure 17(a).

However, for Simulation 5, shown in Figure 17(b), the signal of the vertical field is dominated by multiple pairs of polarities that appear on each side of the neutral line of the AR and move outwards. A similar morphology occurs, but is not shown here, for Simulation 7 (the second simulation with large negative errors in the free magnetic energy in the data-driven run). The emergence associated with Simulations 5 and 7 is more complicated than a simple bipole; the magnetic field is undular and fieldlines cross the surface multiple times, resulting in multipolar regions forming on small timescales.

The lower panels of Figure 17 show the same data as the top panels, but now sampled every 12 minutes, and interpolated to times (at a 24 s cadence) between those 12-minute inputs. This is the data that the data-driven run uses as a lower boundary condition. For Simulation 5 (panel d), with this interpolated data, the multipolar regions result in regions where the magnetic field changes sign significantly over a 12-minute period. This explains why large surface magnetic field variability is seen in Simulations 5 and 7 ($dB > 1$). In general, one would expect such variability and small-scale structure, such as that seen in Figure 17(b), in real ARs.

It is hypothesized here that this variability is the source of the spurious currents in the data-driven runs for Simulations 5 and 7, when the driving interval violates a sampling condition associated with the apparent motion of these small-scale bipoles along the photosphere. If $\tau$ is defined as the timestep at which the photospheric data is sampled, the apparent velocity of the small-scale bipoles is defined as $v_h$, and their spatial extent is characterized by $L$, then the condition for the sampling interval is $\tau < dt_{\text{sample}} \equiv L/v_h$. If the condition is violated, then between sampling times the bipoles move a distance $\tau v_h > L$, i.e., a distance larger than their spatial extent, and this under-sampling thereby effectively generates a strobe effect in the driving data, where the bipoles appear to jump across the photosphere rather than move continuously. Note that this condition is different from the CFL limitation of the numerical integration, $dt_{\text{CFL}} \equiv dx/\max(|V|, C_s, C_A)$.

For the ground-truth data set for Simulation 5 (seen at a cadence of 24 s in panel (b) of Figure 17), the spatial extent of the bipoles (half-width, half-max of one polarity of the bipole) is
approximately $L = 1$ Mm and the apparent horizontal velocity is $v_h = 20 \text{ km s}^{-1}$, which requires a sampling interval less than $dt_{\text{sample}} = L/v_h = 50$ s. The violation of this constraint, when the data-driven run uses photospheric data sampled every 12 minutes, is manifest in Figure 17(d), where one can see numerous small-scale features appearing and disappearing in the pseudo-magnetograms due to the strobe effect. To reconcile this time-dependent boundary condition, which has rapidly appearing and disappearing bipole, the data-driven run generates large electric currents which propagate into the domain. As was shown in Figure 14, this results in spurious electric currents in the data-driven model atmosphere, and a larger free magnetic energy, compared to the ground-truth run.

To highlight the importance of the driving timescale on the accuracy of the data-driven model for these multipolar emergence events, Figure 18 shows the error in the free magnetic energy in the data-driven runs for Simulations 5 and 7 as a function of the driving timescale. At the maximum driving timescale of 12 minutes, the errors are large and negative, as the large variability of the magnetic field in these multipolar emergence events cannot be captured on such a timescale (as shown above). However, as the driving timescale is reduced to about one minute, the errors in the data-driven run reduce to less than a few percent, as the driving timescale is now able to capture the magnetic field variability related to the undular, multipolar emergence. This is consistent with the estimate of the required driving interval of 50 s from above.

4. Discussion

This study aimed to assess the accuracy and feasibility of calculating the coronal magnetic field above ARs using dynamic data-driven MHD simulations with data provided at the photosphere. As there currently exist no observations that can provide us with a ground-truth data set for the magnetic field in the corona above the AR to test the data-driven MHD runs, we created multiple test cases using nine different dynamic MHD simulations of flux emergence from below the surface. Then we used the data at the surface from these ground-truth runs as synthetic magnetograms to drive data-driven runs of the atmosphere above the surface. To simplify our study, we assumed that all MHD variables are known at the surface, not just the magnetic field. This is not the case for current observations of the photosphere, where only vector magnetic field observations are reliably observed at high cadence over full AR scales. We then investigated the effect of the driving timescale on the errors produced in the coronal magnetic field solution for the data-driven runs. We investigated timescales across the range from the CFL-limited timescale ($dt_{\text{CFL}} \sim 0.004$ minute) all the way up to the 12-minute timescale typical of HMI observations. Furthermore, we conducted a parameter study by varying the model parameters in our ground-truth simulations to investigate the effect of the emergence timescale on the errors.

We found that, at a given emergence timescale, the error in the coronal magnetic field decreased with decreasing driving interval, as expected, and that, at the fundamental (sub-CFL) timescale of the numerical method, we were able to recover the ground-truth solution. Furthermore, we found that at the standard HMI interval of 12 minutes, the error in the coronal magnetic field solution decreased with emergence timescale, with the error in the free magnetic energy being as low as 1% for the AR that emerged over a day. This is encouraging for the future development of data-driven MHD models that use photospheric magnetic field measurements, and derived values for the other MHD variables, to model the dynamic evolution of the coronal magnetic field.

However, we also found exceptions to the above results, where large negative errors in the free magnetic energy were found in the data-driven solutions. Preliminary analysis indicates that this is due to the emergence and evolution of multiple sub-AR scale bipolar regions which evolve on small timescales, and are thus under-sampled by the 12-minute interval driving data. The data-driven runs of these undular, multipolar regions generated large electric currents which created spuriously large free energy in the atmosphere. We hypothesized that these currents were a result of the strobe effect generated by features moving at a rate faster than can be captured by the input cadence.

We defined a critical sampling condition $dt_{\text{sample}} = L/v_h$ for the data-driven runs in these complex emergence events, where $L$ and $v_h$ are the characteristic size and apparent velocity of the sub-AR scale bipolar regions. For the parameters we have considered, this sampling condition was approximately one minute. By reducing the driving timestep down to, and below, this critical value, we resolved the problem of spurious currents and free magnetic energy in the data-driven runs, and we recovered errors in the magnetic free energy (compared to the ground-truth) below a few percent. However, when using real photospheric data, an effort is clearly necessary to calculate this condition for the AR being studied. This study shows that extreme care must be taken when using solutions of magnetic field from data-driven MHD models of complex emerging ARs that contain rapidly evolving magnetic features. Fortunately, the potential availability of higher-cadence magnetic field measurements (90–135 s) from HMI-data (Sun et al. 2016) will allow for more analysis.

It is important to note that this investigation assumes the best-case scenario, a scenario that is required by MHD: that all of the MHD variables (including velocity, pressure and temperature), not just the magnetic field, are provided without error at the given cadence. This is currently beyond the state of the art in the observations, where, for example, the velocity must be inferred from the time-series of the photospheric magnetic field. The errors investigated in this study, which come from the lack of temporal information at the photosphere,
will always be present in data-driven models regardless of the methods used to mitigate the lack of information about the velocity and temperature. Future investigations will study the introduction of errors in data-driven coronal field models due to (1) this lack of information at the photosphere and methods used to mitigate it, (2) reduced spatial cadence of photospheric data, and (3) instrument noise and bias.

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