Investigations of granular specimen size effect in interface shear box test using a micro-polar continuum description

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Abstract
Parametric studies were carried out to investigate the effects of specimen size and interface friction angle on the mechanical behaviour of a cohesionless granular material in the interface shear box test under constant normal load and plane strain conditions. For a description of the pressure and density dependent properties of granular materials such as sand, a micro-polar hypoplastic continuum model is used. Numerical simulations were performed to obtain the evolution of the deformations and the shear resistance for three different specimen lengths and interface friction angles using the finite element method in the updated Lagrange frame. Due to the presence of lateral rigid boundaries of the shear box, the deformations and the stresses became significantly inhomogeneous within the granular specimen. Correspondingly, the mobilised interface shear resistance and the deformation field within the specimen were not uniform along the interface. In the case of higher interface friction angles, the evolution of the mechanical quantities along the interface can also be influenced by the specimen size. It is shown that the calculated average friction angle mobilised along a rough interface is influenced by the shear box length and it can be lower than the prescribed one. In addition, the influence of sidewall friction inside the shear box frame was explored in terms of the response of the specimen. The results of the micro-polar hypoplastic model were also compared with those obtained from the corresponding non-polar version of the model to clarify the role of polar effects within the granular body. The numerical results provide new insights into the microstructural effects of the granular material on the mobilisation of the interface friction angle and indicate the complexities and difficulties to precisely determine and interpret the interface friction angle, obtained from the classical interface shear box experiment.

KEYWORDS
granular materials, hypoplasticity, interface shear box test, micro-polar continuum, shear localization, sidewall friction, size effect
1 | INTRODUCTION

Shearing of granular materials such as sand, gravel or broken rock along a rough surface causes forced strain localization within the granular body close to the interface. Herein, the so-called interface friction angle is a key parameter not only for its role to indicate the maximum shear resistance along the interface, but it also affects the deformations in the neighbouring granular body. Corresponding examples of deformation mechanisms in geotechnical systems are for instance shear strain localization along retaining walls, in the vicinity of driven piles and buried pipes, and in reinforced soil structures. In the literature, the term “interface” often refers to a thin zone of soil grains where shear localization takes place. In the present paper, however, the term “interface” is defined for a hypothetical frictional surface of zero thickness between the soil body and the bounding structure. Thus, the interface friction angle reflects, in a phenomenological manner, the interaction between the granular material and the surface morphology of the neighbouring structure when shearing takes place. On the other hand, the term “shear band” is used for the zone of finite thickness where shear localization occurs within the granular material close to the surface of the bounding structure.

At the beginning of forced shearing, the evolution of the mechanical quantities can be very complex and strongly influenced by the test type. Uesugi and Kishida showed that from the onset of shearing up to the maximum shear resistance, the corresponding shear displacement is greater in the interface shear box test (ISBT) than in the simple shear test. Depending on the overall boundary conditions and the interaction between the granular material and the bounding surface, shear deformation can occur in various forms consisting of sliding along the interface, shearing within the granular body, or a combination of them both. While the soil moves like a rigid body along smooth bounding surfaces with negligible interface friction angle, the soil deformation can be relatively complex for higher interface friction angles. In this context, it is important to distinguish between the so-called interface friction angle, and the inter-granular friction angle of the material. The mobilisation of the latter strongly depends on the stress path and the morphology of granular material. Triaxial compression experiments under constant lateral stresses are standard tests to determine the evolution of the inter-granular friction angle. Alternatively, the simple shear apparatus or the direct shear box device can also be used to measure the inter-granular friction angle. Considering the limit shear resistance of the granular material, it is important to distinguish between the peak and the residual friction angles. The former is not a material constant while the later, also referred to as the critical friction angle, is a well-defined material parameter since it is independent on the pressure and density, provided that the influence of grain damage under large shearing can be neglected. The critical friction angle appears under large monotonic shearing and is defined for states, where the material can be deformed under constant volume and stress. On the other hand, the interface friction angle is related to the shear resistance of grains along the rough surface of the bounding structure, where the surface roughness, $R_{\text{max}}$, is defined as the vertical distance between the highest peak and the lowest valley along the surface profile over a length equal to mean grain diameter. The magnitude of the interface friction angle mainly depends on the interaction between $R_{\text{max}}$, the grain size distribution and the mechanical properties of granular material.

Uesugi and Kishida introduced the so-called “normalized roughness”, $R_n$, which is defined as the ratio of the surface roughness to the mean grain diameter, $d_{50}$, of granular material, that is, $R_n = R_{\text{max}}/d_{50}$. Below a certain critical value for $R_n$, a linear relation between the interface friction angle and the normalized roughness was obtained in sand-steel interface shear tests carried out by Uesugi and Kishida. Beyond this critical value, however, the interface friction angle reaches a plateau, which is related to the pressure and density dependent peak friction angle of the granular material. Direct sand-steel interface shear tests by Han et al. showed that for the same surface roughness and mean grain diameter, the interface friction angle is higher for uniform grain sizes than for graded sands. It can be noted in the context of the normalized roughness, that this quantity does not reflect the pattern of the surface profile and the orientation of shearing, so that a same $R_n$ can be pertinent for different interface friction angles.

In three-dimensional discrete element modelling (DEM) of interface shear tests with a high normalized interface roughness, interface friction angles larger than 40 degrees were obtained as well. For rough interfaces, the maximum mobilized interface friction angle is also higher for an initially dense material and a larger normal stress. The dilatancy in the localized zone is also higher for very rough interfaces and non-spherical particles.

Only limited shear deformations occur in a granular body at the onset of shearing for an interface friction angle lower than the inter-granular friction angle. For higher values of normalized roughness, however, small grains can be captured by the rough surface of the bounding structure so that the shear resistance is mainly dictated by the mechanical properties of the granular material. In this case, forced shear strain localization takes place within a zone of a specific thickness. In the localized zone, the displacement field is usually non-linear and influenced by the extent of the mobilised inter-granular friction angle and the structure of the rough surface. The latter governs the interface friction angle and the rotation resistance of grains along the interface. For very high interface friction angles, which indicate the presence of a very rough
bounding surface, shear strain localization accompanied with dilatancy and grain rotation develops within the granular material. The localized zone has usually a thickness of few grain diameters and acts as a transition zone between the granular body and the bounding structure. Under suppressed dilatancy and high pressures, grain abrasion and crushing also have an important influence on the evolution of the overall shear resistance. Thus, the shear resistance may also be influenced by strength, shape and size of the grains. Contrary discussions can be found in the literature for larger normalized roughness values whether the maximum magnitude of mobilised friction angle is determined by the peak value of inter-granular friction angle or by the critical friction angle of granular material. In addition to the quantity of the normalized surface roughness, the features of the topography of the rough surface can also have a notable influence on the maximum shear resistance. In this respect, experiments by Martinez and Martinez and Frost show that advanced shearing leads to a reduction of the interface friction angle after the maximum shear resistance is reached. Thus, the interface friction angle is not a material constant.

Various laboratory tests are proposed for experimental investigations into the evolution of the shear resistance along rough bounding structures, these include for example the interface shear box, ring shear and pull-out tests. The so-called ISBT is a standard laboratory test in experimental soil and rock mechanics and a widely used device to measure the interface friction angle. With standard ISBT apparatus, the evolution of shear stresses, volumetric strains and mobilised friction angles is determined in an average sense for the whole specimen while the local distribution of these state variables is difficult to quantify and usually remains unknown. In particular, the externally applied horizontal and vertical forces are measured, and the ratio of these quantities is assumed to be an indication of the shear resistance along the interface. The boundaries of the specimen frame for the interface shear box apparatus, however, do not usually reflect the actual field conditions under which the soil deforms.

Moreover, the surface roughness of the interior walls of the shear box frame also has an influence on the determination of the interface friction angle. As the sidewall friction restricts the free vertical movement of the specimen during pre-compression and subsequent shearing, the resulting normal stress acting on the interface can be either lower or higher than the measured vertical stress applied at the top surface of the specimen. This issue is one of the main sources of uncertainties in the determination of the interface friction angle in ISBTs (e.g., ). In particular, the calculated interface friction angle will be overestimated for a dilative behaviour and underestimated for a contractive behaviour of the soil specimen.

As deformations and stresses within the specimen are not homogeneously distributed, the ISBT is not an element test in the context of classical continuum mechanics. Consequently, doubt is cast upon the reliability of test results and the selection of appropriate stress parameters used in engineering design. Apart from the non-uniformity of stress and strain fields, other testing-related criticisms of ISBT are the size effect of specimen and the rotation of principal stresses when shear banding takes place. The problem of size effect is particularly important when the characteristics of reduced-scale laboratory tests are inevitably extrapolated to real-scale structures. The effect of specimen length and inhomogeneous distribution of the shear resistance along the interface in ISBTs can be overcome by using a ring shear device. The operation of ring shear apparatus is relatively time consuming and complex, however, and therefore it does not belong to the standard equipment of most soil mechanics laboratories. The ISBT is widely used today to estimate the interface friction angle due to its simplicity and lower testing cost comparing to the other sophisticated laboratory tests. For these reasons, more experimental and numerical studies have been conducted on the test in the past three decades.

The scale effect related to the specimen size in shear test devices has been studied since the 1930s. Parsons presented the variation of sand friction angle as a function of the shear box size. More insights into the general mechanisms of the ISBT can be obtained from particle image velocimetry and X-ray tomography and from numerical simulations with the discrete element method. An extensive review of the literature related to the box size as well as the scale effects on the shear band thickness was given by Cerato and Lutenegger.

The present paper re-examines the problem of ISBT scale effects using numerical simulations based on a micro-polar continuum description. In particular, when shear strain localization takes place, the additional kinematic variables referred to as micro-rotations and the additional static quantities referred to as couple stresses entered into the mathematical formulations of a micro-polar continuum allow a more refined modelling of properties related to the micro-structure of granular materials. Moreover, the boundary value problems in micro-polar continuum become mathematically well-posed at the onset of shear strain localization due to the presence of a characteristic length. The influences of the hypoplastic material parameters and micro-polar boundary conditions on the mechanical response of granular materials.
have been discussed for instance by Huang and Bauer,73 Tejchman,74 Tejchman and Bauer,75 Tejchman and Wu,76 Ebrahimian and Bauer.71

Plane shearing along a rough wall was numerically investigated with a micro-polar hypoplastic model by several authors.72,73,77 In these calculations, the assumption was made that no sliding of grains along the interface takes place, that is, relative displacements between the wall and the adjacent boundary grains were excluded so that the zone of shear localization was fully developed within the granular material. The results show that the rotation resistance of grains along the interface also has a strong influence on the location and thickness of the localized shear zone.71,73 In the paper by Ebrahimian and Bauer,71 the influence of the interface friction angle and the rotation resistance of particles was numerically investigated on the evolution of deformations within the granular body for the case of a theoretical infinite shear layer. Similar to the case in the ring shear device, where the state quantities are independent of the circumferential coordinates, the state quantities in the theoretical lateral infinite shear layer are also independent of the coordinate in the direction of shearing.73,78 In contrast to the infinite shear layer, the evolution of state variables in ISBT is affected by the specimen size and can vary along the interface. It is shown in the present paper that this effect is more pronounced in cases where the interface friction angle is greater than the inter-granular friction angle at the critical state.

The paper is organized as follows: In Section 2, the mechanical properties of the granular material described by the hypoplastic constitutive model and the boundary conditions for the numerical simulations of ISBT are outlined. The finite element results of ISBT simulations with ideally smooth sidewalls are discussed in Section 3.1 for different prescribed interface friction angles and specimen lengths. In order to keep the granulometric properties unchanged, it is assumed that grains are permanent which implies that a reduction of the mean grain diameter caused by grain abrasion and crushing is not taken into account. This assumption is relevant for stiff grains and lower pressure levels as considered in this study. As the friction of the inner sidewalls of the shear box frame cannot completely be avoided in real experiments, its influence on the volumetric behaviour and the interface response of the granular material is briefly addressed in Section 3.2. In addition, the results obtained from both the micro-polar and the non-polar hypoplastic material models are compared in Section 3.2 to specify the effects of micro-polar properties on the interface shear response of the granular materials. Finally, conclusions are given in Section 4.

Compression stress and shortening strain are taken as negative throughout the paper, as is usual in rational mechanics. For the representation of vector and tensor components, indices notation with respect to the rectangular Cartesian coordinate system, \( x_i \) \((i = 1, 2, 3)\), is used and summation over repeated indices is assumed. A superposed circle denotes an objective time derivation and a superposed dot denotes the material time derivation.

## 2 NUMERICAL MODELING OF ISBT

For numerical modelling, the rigid shear box and the coordinate system \((x_1, x_2)\) are assumed to be fixed in space while the bottom plate moves horizontally to the right as illustrated in Figure 1A. The granular specimen is considered under the conditions of plane strain and free vertical dilatancy. In order to avoid tilting of the rigid top plate caused by an inhomogeneous distribution of the reaction pressure of the specimen, the vertical translation of the top plate is constrained in the
numerical model. During the test, the vertical load, \( F \), applied on the top plate is kept constant, which leads to an average vertical pressure of \( p_0 = 100 \text{ kPa} \) at the top surface of the granular specimen. Thus, for different values of the length, \( l_0 \), and the constant value of the width, \( b_0 \), of the shear box frame, the vertical load applied on the top plate is \( F = p_0 \times b_0 \times l_0 \). The specimen is located within the fixed rigid shear box where different friction angles of the lateral walls are considered. In order to prevent the development of tensile stresses along the boundaries, the specimen is permitted to be detached from the shear box frame. This condition can occur for instance at the left boundary of the specimen when the bottom plate is moving to the right, that is, \( U_{IB} > 0 \).

To account for micro-structural properties such as grain rotations and couple stresses, a constitutive model based on a micro-polar continuum or the so-called Cosserat continuum is used. In this refined continuum description, rotational degrees of freedom are introduced in addition to translational degrees of freedom. Taking into account the mean grain diameter as the characteristic length, the micro-polar approach regularizes the ill-posedness of the governing mathematical equations when material softening and shear localization appear. Moreover, the numerical solutions obtained from finite element calculations do not suffer from pathological discretization sensitivity.\( ^{66,79–81} \) For the present simulation of ISBTs, plane strain condition is considered, in which the relevant kinematic quantities are the displacements, \( u_1 \) and \( u_2 \), and the micro-rotation, \( \omega_c ^{3} \), as illustrated in Figure 1B, and the static quantities are the normal stresses, \( \sigma_{ij} \), shear stresses, \( \sigma_{12} \), and the couple stresses, \( \mu_{31}, \mu_{32}, \) as shown in Figure 1C. The presence of couple stresses leads to non-symmetry of the stress tensor, which is distinctive in the zones of shear strain localization. When shearing takes place, the macro-rotation, \( \omega_s \), is different from the micro-rotation, \( \omega_c ^{3} \). In a micro-polar continuum, the components of the rate of deformation and the rate of curvature are defined as \( \dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij} + W_{ij} - W_{ij} ^{C} \) and \( \dot{\kappa}_{ij} = \dot{\sigma}_{ij} / \sigma_{kk} \), respectively. Herein, \( \dot{\varepsilon}_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2 \) and \( W_{ij} = -\varepsilon_{ijk} \dot{\sigma}_{k} \) are the symmetric and non-symmetric parts of the velocity gradient, \( \partial u_i / \partial x_j \), where \( \partial u_i / \partial x_j \) denotes the components of the velocity. \( W_{ij} ^{C} = -\varepsilon_{ijk} \omega_c ^{3} \) are the components of the micro-spin tensor and \( \varepsilon_{ijk} \) is the permutation symbol. In the present paper, the employed constitutive relations are of the rate type and based on the concept of hypoplasticity of Koliymbas type.\(^{82}\) The evolution of the stress components, \( \sigma_{ij} \), the couple stress components, \( \mu_{ij} \), and the void ratio, \( e \), are described by the following evolution equations proposed by Huang and Bauer:\(^{73}\)

\[
\sigma_{ij} = f_s \left[ \dot{\varepsilon}_{ij} ^{C} + (\sigma_{kl} \varepsilon_{kj} ^{C} + \mu_{kl} \delta_{kj}) \dot{\sigma}_{ij} + \int d \sigma \left( \dot{\varepsilon}_{ij} ^{C} + \dot{\sigma}_{ij} ^{C} \right) \sqrt{\dot{\varepsilon}_{pq} ^{C} \dot{\sigma}_{pq} ^{C} + \dot{k}_{pq} \dot{k}_{pq}} \right] \tag{1}
\]

\[
\mu_{ij} = f_s d_{50} \left[ \dot{\varepsilon}_{ij} ^{C} + (\sigma_{kl} \varepsilon_{kj} ^{C} + \mu_{kl} \delta_{kj}) \dot{\mu}_{ij} + 2 \int d \sigma \ a_c \left( \dot{\varepsilon}_{pq} ^{C} \dot{\sigma}_{pq} ^{C} + \dot{k}_{pq} \dot{k}_{pq} \right) \right] \tag{2}
\]

\[
e = (1 + e) \dot{\varepsilon}_{kk} ^{C} \tag{3}
\]

Herein, the objective measures, \( \sigma_{ij} \) and \( \mu_{ij} \), depend on the current void ratio, \( e \), the mean grain diameter, \( d_{50} \), the pressure and density dependent stiffness factor, \( f_s \), the relative density, \( f_d \), the rate of deformation, \( \dot{\varepsilon}_{ij} ^{C} \), the normalized components of the non-symmetric Cauchy stress tensor, \( \dot{\varepsilon}_{ij} = \sigma_{ij} / \sigma_{kk} \), its deviatoric part, \( \dot{\sigma}_{ij} ^{C} = \sigma_{ij} - \delta_{ij} / 3 \), the couple stress tensor, \( \dot{\mu}_{ij} = \mu_{ij} / (d_{50} \sigma_{kk}) \), and the rate of micro-curvature, \( \dot{\kappa}_{ij} = d_{50} \kappa_{ij} \). The mean grain diameter, \( d_{50} \), serves as the characteristic length and \( \delta_{ij} \) denotes the Kronecker delta. The evolution Equations (1, 2) are incrementally non-linear in terms of \( \varepsilon \) and \( \kappa \), which models inelastic behaviour. Factor \( a \) in Equations (1, 2) and factor \( a_c \) in Equation (2) are associated with the limit stress and the couple stress at the critical states, respectively. Factor \( a_c \) is related to the rotation resistance of particles and assumed to be constant. Factor \( d_{50} \) depends on the inter-granular friction angle, \( \varphi_e \), in the critical state. With respect to the symmetric part of the normalized stress deviator, that is, \( \dot{\sigma}_{pq} ^{ss} = (\dot{\sigma}_{pq} ^{s} + \dot{\sigma}_{qp} ^{s}) / 2 \), and the Lode-angle, \( \vartheta \), the adaptation of \( a \) to the stress limit condition by Matsuoka and Naikai can be represented as

\[
a = \frac{\sin \varphi_c}{3 - \sin \varphi_c} \left[ \sqrt{\frac{8/3 - 3 \dot{\sigma}_{pq} ^{ss} \dot{\sigma}_{pq} ^{ss} + \sqrt{3/2 (\dot{\sigma}_{pq} ^{ss} \dot{\sigma}_{pq} ^{ss}) ^{3/2} \cos (3\vartheta)}}{1 + \sqrt{3/2 (\dot{\sigma}_{pq} ^{ss} \dot{\sigma}_{pq} ^{ss}) ^{1/2} \cos (3\vartheta)}}} - \sqrt{\dot{\sigma}_{pq} ^{ss} \dot{\sigma}_{pq} ^{ss}} \right] \tag{4}
\]

where, \( \cos (3\vartheta) = -\sqrt{6} \dot{\sigma}_{kl} ^{ss} \dot{\sigma}_{lm} ^{ss} \dot{\sigma}_{mk} ^{ss} / (\dot{\sigma}_{pq} ^{ss} \dot{\sigma}_{pq} ^{ss}) ^{3/2} \).
Constitutive model characteristics: (A) critical stress state surface in the deviator plane in the principal stress space related to the symmetric part of the stress tensor; and (B) dependency of maximum void ratio, $e_i$, minimum void ratio, $e_d$, and critical void ratio, $e_c$, on normalized mean pressure, $\sigma_{kk}/h_s$.

### TABLE 1 Hypoplastic constitutive constants for medium quartz sand

| $\phi_c$ (°) | $h_s$ (MPa) | $n$ | $e_{i0}$ | $e_{d0}$ | $e_{c0}$ | $\alpha$ | $\beta$ | $d_{iso}$ (mm) | $a_c$ |
|-------------|-------------|-----|---------|---------|---------|---------|-------|--------------|------|
| 30          | 190         | 0.4 | 1.2     | 0.51    | 0.82    | 0.11    | 1.05  | 1.0          | 1.0  |

It is worth noting that factor $\hat{a}$ is active in the constitutive model for arbitrary states\(^84\) and only for critical stress states the value of $\hat{a}$ is related to the limit condition by Matsuoka and Nakai\(^83\) as sketched out in Figure 2A. In Equations (1, 2), factor $f_s$ is proportional to the solid hardness, $h_s$, and also depends on the current void ratio, $e$, the maximum void ratio, $e_i$, and the sum of normal stresses, $\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$, that is,

$$f_s = \frac{(e_i/e_{i0})h_s (1 + e_i)(-\sigma_{kk}/h_s)^{(1-n)}/ [n (\hat{\sigma}_{kl} \hat{\sigma}_{kl}) e_i]}{8 \sin^2 \phi_c (3 \sin \phi_c)^2} + 1 - 2\sqrt{2} \left( \frac{e_{i0} - e_{do}}{e_{co} - e_{do}} \right)^{\frac{\alpha}{\sin \phi_c (3 - \sin \phi_c)}}$$

In Equation (5), $\beta > 1$ and $\alpha < 0.5$ are constitutive constants. The dilatancy behaviour, the peak stress ratio and the strain-softening are dependent on the density factor, $f_d$, which represents a relation between the current void ratio, $e$, the critical void ratio, $e_c$, and the minimum void ratio, $e_d$, that is,

$$f_d = \left( (e - e_d) / (e_c - e_d) \right)^{\alpha}$$

According to the postulate proposed by Gudehus\(^85\), the maximum void ratio, $e_i$, the minimum void ratio, $e_d$, and the critical void ratio, $e_c$, decrease with an increase of the mean pressure, $\sigma_{kk}$, as illustrated in Figure 2B, that is,

$$e_i/e_{i0} = e_d/e_{d0} = e_c/e_{co} = \exp \left[ -(\sigma_{kk}/h_s)^{\phi_c} \right]$$

where, $e_{i0}$, $e_{d0}$, $e_{co}$ are the corresponding values for $\sigma_{kk} \approx 0$. With respect to the pressure dependent limit void ratios in Equation (7), the density factor, $f_d$, and the stiffness factor, $f_s$, depend on both the current void ratio and the mean pressure.

In order to specify the effect of micro-polar properties in ISBTs, particular numerical simulations are also carried out with the corresponding non-polar hypoplastic version. In a non-polar continuum, there are no polar effects, that is,\(^\phi\) $W_{ij} = W_{ij}$, so that $\xi_{ij}^c = \xi_{ij}$, and the couple stresses and their rates vanishes, that is, $\mu_{ij} = 0$ and $\kappa_{ij} = 0$. Then, the constitutive Equation (1) reduced to the one proposed by Bauer\(^84\) and Gudehus\(^85\):

$$\ddot{\sigma}_{ij} = f_s \left[ \hat{a}^2 \dot{\xi}_{ij} + (\hat{\sigma}_{kl} \hat{\xi}_{kl}) \ddot{\sigma}_{ij} + f_d \hat{a} \left( \ddot{\sigma}_{ij} + \hat{\sigma}_{ij}^* \right) \sqrt{\dot{\xi}_{pq} \xi_{pq}} \right]$$

The micro-polar hypoplastic constitutive equations, that is, Equations (1, 2), include ten constitutive constants, which can be calibrated based on the grain size distribution, isotropic and triaxial compression tests, the pressure dependent limit void ratios and the critical void ratio.\(^73\) The constitutive constants used in the present paper are relevant for a medium quartz sand and summarized in Table 1. For the numerical simulations, a four-node plane strain Cosserat element with
Computed average mobilised interface friction angle, $\varphi_{ave/mob} = \arctan(\sigma_{12}/\sigma_{22})$, against the normalized bottom displacement, $U_{IB}/h_0$, and for three different prescribed interface friction angles, that is, $\varphi_{int} = 17^\circ$, $28^\circ$, $40^\circ$ in ISBTs under constant average pressure of $\sigma_{22} = -100$ kPa, an initial void ratio of $e_0 = 0.6$ and a box length of $l_0 = 100$ mm

FIGURE 3 Computed average mobilised interface friction angle, $\varphi_{ave/mob} = \arctan(\sigma_{12}/\sigma_{22})$, against the normalized bottom displacement, $U_{IB}/h_0$, and for three different prescribed interface friction angles, that is, $\varphi_{int} = 17^\circ$, $28^\circ$, $40^\circ$ in ISBTs under constant average pressure of $\sigma_{22} = -100$ kPa, an initial void ratio of $e_0 = 0.6$ and a box length of $l_0 = 100$ mm

bilinear shape function is used to describe the displacements and the micro-rotation within the element. The Cosserat element and the constitutive model are implemented into the finite element program ABAQUS as outlined by Huang.

In order to investigate the influence of the size of the granular specimen on the evolution of the state quantities during interface shearing, three different lengths of the shear box are considered, that is, $l_0 = 50, 100$ and $200$ mm corresponding to small, medium and large boxes, respectively. In all simulations, an initial specimen height of $h_0 = 40$ mm is assumed. The granular specimen is discretized by plane strain Cosserat elements with an initial size of $1.25$ mm $\times$ $1.25$ mm. In order to obtain mesh independent results, the size of elements is chosen to be less than $5 \times d_{50}$ as proposed by Tejchman and Bauer. The micro-rotations of the element nodes along the top boundary are zero, while along the bottom interface, no restriction is considered for the micro-rotations.

The limit shear stress between the surface of the bottom plate and the specimen is prescribed by the classical friction law by Coulomb in the form of $\tau_{limit} = \mu_F \sigma_{22}$, where $\mu_F$ is the coefficient of interface friction and $\sigma_{22}$ denotes the local contact stress perpendicular to the interface. For the numerical simulations, the assumption is made that no relative displacement takes place between the granular body and the bottom plate as long as the mobilised interface friction angle, $\varphi_{mob} = \arctan(\sigma_{12}/\sigma_{22})$, is lower than the limit value, $\varphi_{int} = \arctan(\mu_F)$. When $\varphi_{mob} = \varphi_{int}$ meets, the method of contact surfaces and the finite sliding contact algorithm is employed. In order to consider large deformations, which may be important when shear localization takes place, an updated Lagrange formulation is employed. The influence of gravity of the granular specimen is not taken into account in the simulations. For the initial state, a homogeneous distribution of the initial void ratio of $e_0 = 0.6$ is assumed in all calculations.

## 3 RESULTS OF NUMERICAL SIMULATIONS

### 3.1 Influence of prescribed interface friction angle and shear box length

First, the results of numerical simulations of ISBTs are discussed for different interface friction angles, that is, $\varphi_{int} = 17^\circ$, $28^\circ$, $40^\circ$, prescribed on the surface of the bottom plate. The assumed interface friction angle of $\varphi_{int} = 40^\circ$ is higher than the critical friction angle of the granular material and should reflect a rough surface structure capable of mobilising additional passive shear resistances. For a shear box length of $l_0 = 100$ mm, the influence of the magnitude of the assumed interface friction angle on the evolution of the computed average value of the mobilised interface friction angle, $\varphi_{ave/mob}$, is shown in Figure 3. It is obvious that the amount of the prescribed bottom displacement, $U_{IB}$, to reach the maximum shear resistance, is larger for a higher interface friction angle.

For a prescribed interface friction angle lower than the critical friction angle of granular material, that is, $\varphi_{int} \leq \varphi_{critical} = 30^\circ$, the predicted maximum mobilised interface friction angle is equal to the prescribed one, that is,
Deformed granular specimen along with contour plot of the void ratio for $l_0 = 100$ mm and different applied bottom displacements: (A) $U_{IB} = 1.7$ mm, (B) $U_{IB} = 5$ mm and (C) $U_{IB} = 10$ mm

$\phi_{ave/mob} = \phi_{int}$, which is expected to be based on the experimental results reported by many researchers.\(^9,14,27,30,89,90\)

However, for a prescribed interface friction angle of $\phi_{int} = 40^\circ > \phi_{critical} = 30^\circ$, the predicted maximum average mobilised friction angle along the interface is $\phi_{ave/mob} = 38.7^\circ$ and thus, the value is lower than the prescribed one. This is because the mobilisation of shear stress, $\sigma_{12}$, and vertical stress, $\sigma_{22}$, is different along the interface. Moreover, the results of the simulations show that after the peak state, the average value of the mobilised interface friction angle first decreases down to $36.77^\circ$ and then it increases slightly up to $37.30^\circ$. This result is not obvious and it was the motivation for the authors to gain further insight into the complex interaction between the interface roughness, the mechanical properties of the granular material, and the evolution and the local distribution of the mobilised interface friction angle.

In order to demonstrate the interaction of pressure and density described by the hypoplastic constitutive model, the response of a single material element under plane shearing is represented in Appendix A for different vertical pressures and initial void ratios. In accordance with the concept of critical state soil mechanics, the hypoplastic model predicts a higher peak friction angle for a lower vertical pressure and a lower initial void ratio. After the peak state, strain softening can be detected and the mobilised friction angle tends toward the critical friction angle of the granular material. These properties indicate that the mobilisation of the inter-granular friction angle is strongly influenced by the evolution of the density and stress state, which can develop inhomogeneously during the ISBT as discussed in the following.

Figure 4 shows the evolution of void ratio for a specimen length of $l_0 = 100$ mm and at different bottom displacements, $U_{IB}$, of the ISBT. As a result of the applied vertical load, a compaction of the upper part of the specimen can be observed. The densification is more pronounced on the right hand side of the specimen. In Figure 4B, C, it is visible that densification grows from the right to the left with the advanced displacement of the bottom plate. As the bottom plate moves to the right, the resulting pressure acting on the shear box frame is greater at the right wall than at the left wall. Such a behaviour can also be observed in ISBT simulations using DEM, for example, Grabowski et al.\(^6\) In the present finite element simulations, a gap between the shear box frame and the granular specimen appears on the left side, which can be explained by the assumption made that gravity is neglected for the granular material. Close to the bottom, the void ratio is locally higher.
than the initial value which can be explained by dilatancy within the zone of shear localization. Under shearing, dilatancy is typical for cases in which the initial void ratio is lower than the critical one. The great differences of the pressures acting on the right and the left sides of the shear box frame causes a non-uniform distribution of stresses and deformations within the entire specimen. The numerical simulations show that the gradient of dilatancy of the material in the localized zone along the interface is not continuous, and for larger bottom displacements, the amount of dilatancy significantly increases. It is worth noting that higher void ratios are not necessarily an indicator of the current active zone of shear strain localization. Areas of higher void ratios reflect the history of shear strain localization as also discussed in more details for the case of an infinite shear layer by Huang and Bauer, and Ebrahimian and Bauer. Thus, the thickness of the shear band is not a material constant and strongly influenced by the interaction between the properties of the material model and the boundary conditions considered. In this context, the performance of interface elements with zero thickness used in finite element simulations, for example, Stutz et al., are restricted to cases when the evolution of the finite thickness of shear bands is not of interest.

Figure 5 shows the distribution of the void ratio, $e$, and the micro-polar rotation, $\omega^c_3$, across the vertical section in the middle of the specimen for different bottom displacements, $U_{IB}$. High gradients of the void ratio and the micro-polar rotation are indicators of the zones with pronounced shear localization, which can clearly be detected close to the interface. The thickness of the localized zone is six to eight times the mean grain diameter, which is much lower than the thickness of the shear bands developed within granular bodies. As shown in Figure 5A, the value of the void ratio within the localized zone increases with an increase of the bottom displacement. In contrary, outside the shear band, the value of the void ratio is about 0.59 and thus it is lower than its initial value of $e_0 = 0.60$ due to the densification at the beginning of shearing. For larger displacements of the bottom plate, that is, when the shear localization close to the interface becomes dominant, the void ratio and the micro-polar rotation outside the localized zone remain unchanged. These results are in agreement with experimental observations and numerical simulations.

In the following, the influence of the specimen size on the evolution of state variables during interface shearing is discussed for a prescribed interface friction angle of $40^\circ$. In particular, the distribution of void ratio, $e$, the local mobilised interface friction angle, $\varphi_{mob}$, the local relative displacement and the average interface friction angle, $\varphi_{ave/mob}$, for the lengths of the shear box frame of $l_0 = 50$, 100 and 200 mm are shown in Figures 6–12, respectively.

Figure 6 shows the deformed specimen with the contour plot of the void ratio after a bottom displacement of $U_{IB} = 10$ mm. Independent of the size of the specimen, the void ratio is higher close to the interface. The maximum value is little larger for a smaller shear box, that is, the development of shear localization and dilatancy depends on the specimen size. The shear zone thickness appears non-uniform along the interface. It is also confirmed by experiments that the propagation of the interface shear zone appears as a function of specimen length. 31,50
Figure 6 shows the evolution of the computed mobilised interface friction angle, $\varphi_{mob}$, and the relative displacement, $U_{1B} - u_1$, for different applied horizontal displacements of the bottom plate, $U_{1B}$. In order to compare the results of the different specimen sizes, the normalized location, $x_l/l_0$, of the particular specimens is used for the representation of the horizontal axis. Sliding along the interface is only possible for $\varphi_{mob} = \varphi_{int} = 40^\circ$. It is evident that the shear resistance is fully mobilised from the beginning of shearing at the interface on the right hand side of the specimen and it grows from the right to the left with an advanced displacement of the bottom plate. However, the mobilisation of the maximum shear resistance from the right to the left is not a monotonic process. For larger bottom displacements, for example, $U_{1B} = 10$ mm, the interface friction angle is also fully mobilised at the left hand side of the specimen while the behaviour in the middle part of the interface is determined by shear strain localization in the granular body adjacent to the interface. This explains the higher void ratio in the middle part as also shown in Figure 6.

A comparison between Figures 7A and 7C clearly indicates that the evolution of relative displacement along the interfaces is not the same for different specimen lengths. The process of mobilisation of the shear resistance in the middle part of the interface is relatively complex. At a specific shear box displacement, a reversed tendency can be observed and $\varphi_{mob}$ slightly decreases, which means that there are local changes of the interface behaviour from sliding to sticking. For instance, for the specimen with $l_0 = 50$ mm the part of the interface where the friction angle is fully mobilised decreases after a bottom displacement of $U_{1B} = 2$ mm and increases again after $U_{1B} = 5$ mm. The representations of the relative displacements show integral values, which do not allow an interpretation of whether the current state is in a sliding or a sticking mode. The behaviour is similar for the larger specimens but the particular course is size dependent. It can be concluded that during shearing a repeated change of the mode occurs within the certain parts of the interface.

The formation of inhomogeneous deformations is also related to an inhomogeneous evolution of the stresses. In contrast to the lateral infinite shear layer, the vertical and horizontal shear stresses are no longer independent of the coordinate along the direction of shearing as shown in Figures 8–11. Two different stress regions can be identified in the specimen, that is, a region with high vertical stresses at the right side where the specimen is laterally compressed to the right sidewall.
FIGURE 7  Distribution of mobilised interface friction angle, $\varphi_{mob} = \arctan(\sigma_{12}/\sigma_{22})$, and relative displacement, $U_{1B} - u_1$, against the normalized distance at the interface, $x_1/l_0$, for different applied bottom displacements, $U_{1B}$, and three different specimen lengths: (A) $l_0 = 50$ mm, (B) $l_0 = 100$ mm and (C) $l_0 = 200$ mm
of the shear box, and a region with low vertical stresses at the left sidewall where a gap between the specimen and the wall of the shear box can be formed. The comparison of Figure 8 and Figure 9 shows that the change of the vertical stress on the right hand side is more pronounced at an early stage of shearing, that is, for a bottom displacement of $U_{IB} = 0.05$ mm, than those for the larger shear displacement of $U_{IB} = 10$ mm. The contour plots of the horizontal shear stress clearly indicate that close to the interface, shear stresses decrease from the right to the left for a smaller bottom displacement, while a more complex distribution can be observed for larger displacements, which is also related to the distribution of the mobilised interface friction angle shown in Figure 7.

For a larger displacement, the distribution of the interface shear stresses also depends on the specimen length as clearly visible in Figure 11. Apparently, for the larger specimen length, the materials in the middle part of the box length experience less shear stress than those closer to the lateral boundaries in the small strain stage. In this context, it is worth noting that the amount of shear stress is not a measure of the maximum shear resistance as the mobilised interface friction also depends on the local normal stress.

Figure 12 shows the influence of the specimen size on the evolution of the average mobilised interface friction angle against the normalized bottom displacement. For the smaller specimen, the maximum average mobilised interface friction angle is higher and reached at a smaller bottom displacement, thus the incremental stiffness up to the peak state strongly depends on the shear box size. A similar trend was also observed by Wang and Jiang$^{32}$ and Cerato and Lutenegger.$^{55}$ After the peak state, an irregular course can be detected, which can be explained by local changes of the mobilised interface friction angle as discussed in Figure 7. It can be seen in Figure 12 that for a normalized displacement of $U_{IB} / h_0 = 0.25$, the average mobilised interface friction angle is lower for the larger specimen. It is evident that the calculated maximum average mobilised interface friction angle decreases with increasing box length as also mentioned by Jewell and Wroth$^{42}$ and Cerato and Lutenegger.$^{55}$ The effects of material softening become more pronounced for the larger length of the specimen, as it is also visible in Figure 12.

It is also worth noting that the experiments with very rough interfaces show that the mobilized interface friction angle decreases towards the critical friction angle of the granular material in larger shear displacements. In the present
numerical simulations, however, the reduction of the mobilized interface friction angle is small for a shear displacement up to 10 mm. This may indicate the fact that the influence of the surface structure of the interface cannot be well simulated with a prescribed constant interface friction angle.

### 3.2 Influence of sidewall friction and micro-polar properties

The numerical simulations carried out in the previous section are based on the assumption that the inner walls of the shear box frame are ideally smooth. As the sidewall friction inside the shear box frame cannot be completely avoided in real experiments, its influence on the response of the granular specimen is investigated in this section. When sidewall friction occurs, the true interface friction angle is difficult to determine in standard laboratory tests. This is justified by the fact that when calculating the interface friction angle, the applied vertical load, \( N_{\text{top}} \), measured at the top of the specimen is usually used instead of the resulting normal force, \( N_{\text{int}} \), acting on the interface. During pre-compression and subsequent shearing, sidewall friction mobilizes a reaction force, which is orientated against the vertical movement of the particles in contact with the lateral walls, so that the force \( N_{\text{int}} \) can be either smaller or larger than \( N_{\text{top}} \). Thus, the interface friction angle calculated with respect to \( N_{\text{top}} \), that is,

\[
\varphi_{\text{cal}} = \arctan \left( \frac{S}{N_{\text{top}}} \right)
\]

is different from the true interface friction angle calculated with respect to \( N_{\text{int}} \), that is,

\[
\varphi_{\text{int}} = \arctan \left( \frac{S}{N_{\text{int}}} \right)
\]

Herein, \( S \) denotes the horizontal shear force, which is also measured in experiments.
FIGURE 10  Deformed granular specimen along with contour plot of the horizontal shear stress, $\sigma_{12}$, at $U_{th} = 0.05$ mm for three different specimen lengths: (A) $l_0 = 50$ mm, (B) $l_0 = 100$ mm and (C) $l_0 = 200$ mm

TABLE 2  Influence of the sidewall friction on the response of the micro-polar hypoplastic granular specimen

| Sidewall roughness condition | Sidewall friction coefficient, $\mu_{SW}$ | $\varphi_{cal}^a$ ($^\circ$) | $\varphi_{int}^b$ ($^\circ$) | $u_{2 \text{ min}}$ (mm) | $u_{2 \text{ max}}$ (mm) |
|-----------------------------|-----------------------------------------|-----------------------------|-----------------------------|-------------------------|-------------------------|
| A: Smooth                   | 0.00                                    | 17.00                       | $-0.347$                    | $+0.280$                |
| B: Medium rough             | 0.47                                    | 17.00                       | $-0.164$                    | $+0.171$                |
| C: Medium rough             | 0.47                                    | 17.92                       | $-0.164$                    | $+0.171$                |
| D: Very rough               | 1.00                                    | 17.00                       | $-0.095$                    | $+0.149$                |
| E: Very rough               | 1.00                                    | 18.01                       | $-0.095$                    | $+0.149$                |

$^a$Average value of the maximum mobilised interface friction angle calculated based on Equation (9).
$^b$Average value of the maximum mobilised true interface friction angle calculated based on Equation (10).

In the following, the influence of sidewall friction on the determination of the interface friction angle is investigated for a shear box length of $l_0 = 100$ mm and for a medium rough interface with a prescribed friction angle of $\varphi_{int} = 17^\circ$. The numerical simulations are carried out under a plane strain condition, a constant average vertical pressure of $\sigma_{22} = -100$ kPa applied at the top surface, an initial void ratio of $e_0 = 0.6$ and a sidewall friction angle prescribed to the surface of the right and left sidewalls of the shear box frame. The evolution of the average mobilized interface friction angles obtained from the relations (9, 10) is compared in Figure 13A for the micro-polar hypoplastic material model (MPHM) and different prescribed sidewall friction angles. Although the amounts of the shear and the normal forces acting on the interface are influenced by the evolution of the sidewall friction, their ratios, however, are independent of the sidewall friction. Thus, the true quantity, $\varphi_{int}$, calculated based on relation (10) is the same as the prescribed interface friction angle. That is also clear for the computed quantities (A, B, D) for different sidewall friction angles, shown in Table 2. However, the friction angle, $\varphi_{cal}$, calculated based on relation (9), that is, (C, E), is higher for a higher sidewall friction angle. The vertical displacement of particles in contact with the lateral rough walls to upwards or downwards depends mainly on
FIGURE 11  Deformed granular specimen along with contour plot of the horizontal shear stress, $\sigma_{12}$, at $U_{1B} = 10$ mm for three different specimen lengths: (A) $l_0 = 50$ mm, (B) $l_0 = 100$ mm and (C) $l_0 = 200$ mm

FIGURE 12  Evolution of average mobilised interface friction angle, $\varphi_{ave/mob} = \arctan(\sigma_{12}/\sigma_{22})$, against the normalized shear box displacement, $U_{1B}/h_0$, for three different specimen lengths

the initial value of the state quantities and the loading history. In particular, the initial compaction caused by the applied vertical load leads to a downward movement of particles adjacent to the lateral walls. Thus, the resulting reaction force is directed upwards, so that $N_{int} < N_{top}$. During shear, the deformation of the specimen can be rather complex and the movement direction of the particles in contact with the sidewalls can change. Although, the vertical movement of the top plate in ISBTs is an indicator of the average volume change of the specimen, it does not allow a conclusion to the
Micro-polar hypoplastic simulation of the influence of the sidewall friction angle in the ISBT for a shear box length of $l_0=100$ mm, an average constant vertical pressure of $\sigma_{y_2}=-100$ kPa applied at the top surface of the specimen, an initial void ratio of $e_0=0.6$, a prescribed interface friction angle of $\phi_{\text{int}}=17^\circ$, and for different friction coefficients, $\mu_{sw}$, prescribed to the sidewalls: (A) evolution of the average mobilised interface friction angle, $\phi_{\text{ave/mob}}$, and (B) evolution of the vertical displacement, $u_2$, at the top surface of the specimen against the normalized shear box displacement, $U_{IB}/h_0$. 

Inhomogeneous volume change within the specimen, in particular in the zone close to the interface. The evolution of the vertical displacement of the top surface of the specimen is shown in Figure 13B. It can visibly be seen that the overall volume change of the specimen leads to an additional compaction at the beginning of shearing, while the movement of the top surfaces is directed upwards for a larger interface shearing. In the present investigation, the maximum heaving of the top surface is lower for a higher prescribed sidewall friction angle. A closer analysis of the volume change across the height of the specimen indicates that for an initially medium dense granular material, a larger shearing leads to a significant increase of the volume in the localized zone close to the interface while an additional compaction can be detected above the localized zone as shown in Figure 6. As a consequence of the inhomogeneous deformation field within the specimen, the movement of the particles in contact with the lateral walls can differ across the height of the specimen. For an assumed medium dense specimen and low vertical stress applied at the top surface, dilatancy and contractancy within the specimen are almost balanced so that the overall volume change is rather small. This is also the reason for the low value for the final heaving of the top surface. In the numerical simulations for plane strain condition, the contribution of the friction resistance acting on the sidewalls perpendicular to the plane strain direction is not taken into account and as a consequence the numerical results are of qualitative relevance only. Although the lateral boundary conditions of the shear box frame are such as in an oedometer device, it can be noted that the lateral stresses differ on both sides and do not allow an estimation of the lateral stresses using the coefficient of the earth pressure at rest.

The effect of employing the micro-polar material description can be investigated by comparing the results in Figure 13 and Table 2 obtained from the micro-polar hypoplastic material model (MPHM) with those in Figure 14 and Table 3 obtained from the non-polar hypoplastic material model (NPHM).

The residual interface friction angle computed from relation (10) is the same for the micro-polar material description, that is, (A, B, D), and the non-polar material description, that is, (F, G, I). The values computed from relation (9), that is, (C, E) and (H, J), are influenced by the sidewall friction and are a little higher for the micro-polar material description. In contrast to the NPHM description, the vertical movement of the top plate for the case of the shear box with rough sidewalls is significantly higher for the MPM. Although the basic material parameters and the values of the initial state variables are the same for both models, the computed average dilatancy evolved during interface shearing is larger for the MPM. This is due to a more pronounced dilatancy caused by a higher rotation resistance of particles in the localized zone close to the rough interface. Moreover, it can be noted that the thickness of the localized zone is controlled by the height of the finite elements in the NPHM description, that is, the shear band thickness is mesh dependent, while the thickness of the
FIGURE 14  Non-polar hypoplastic simulation of the influence of the sidewall friction angle in the ISBT for a shear box length of $l_0=100$ mm, an average constant vertical pressure of $\sigma_{22}=-100$ kPa applied at the top surface of the specimen, an initial void ratio of $e_0=0.6$, a prescribed interface friction angle of $\varphi_{\text{int}} = 17^\circ$, and for different friction coefficients, $\mu_{\text{sw}}$, prescribed to the sidewalls: (A) evolution of the average mobilised interface friction angle, $\varphi_{\text{ave/mob}}$, and (B) evolution of the vertical displacement, $u_2$, at the top surface of the specimen against the normalized shear box displacement, $U_{1b}/h_0$.

TABLE 3  Influence of the sidewall friction on the response of the non-polar hypoplastic granular specimen

| Sidewall roughness condition | Sidewall friction coefficient, $\mu_{\text{SW}}$ | $\varphi_{\text{cal}}^a$ ($^\circ$) | $\varphi_{\text{int}}^b$ ($^\circ$) | $u_2_{\text{min}}$ (mm) | $u_2_{\text{max}}$ (mm) |
|-----------------------------|-----------------------------------------------|-----------------------------------|---------------------------------|------------------------|------------------------|
| F: Smooth                   | 0.00                                          | 17.00                             | -0.321                          | +0.230                 |
| G: Medium rough             | 0.47                                          | 17.00                             | -0.155                          | +0.121                 |
| H: Medium rough             | 0.47                                          | 17.79                             | -0.155                          | +0.121                 |
| I: Very rough               | 1.00                                          | 17.00                             | -0.090                          | +0.103                 |
| J: Very rough               | 1.00                                          | 17.89                             | -0.090                          | +0.103                 |

$^a$Average value of the maximum mobilised interface friction angle calculated based on Equation (9).

$^b$Average value of the maximum mobilised true interface friction angle calculated based on Equation (10).

predicted localized zone obtained with the MPHM is independent of the size of the finite elements (e.g., Huang et al.,$^{95}$ Tejchman and Bauer,$^{75}$ Ebrahimian and Bauer$^{71}$).

4  | CONCLUSIONS

Parametric studies were conducted for ISBTs under constant vertical load and plane strain conditions to investigate the effects of specimen length and the interface friction angle on the evolution of the state quantities within the granular specimen. The paper provides new insights into both the effects of the micro-structural properties of the granular material on the mobilisation of the interface friction angle and the distribution of relative displacements along the interface. As the peak friction angle of dense granular materials is higher than the critical friction angle, the behaviour is investigated for both prescribed interface friction angles lower and higher than the inter-granular friction angle in the critical state. Numerical simulations are carried out with and without sidewall frictions assumed inside the shear box frame. A micro-polar hypoplastic constitutive model with pressure dependent limit void ratios is used to take into account micro-structure effects like grain rotations and couple stresses and to prevent mesh size dependency in the finite element simulations when shear strain localization takes place. In order to investigate the effect of the micro-polar description, the results obtained
from the micro-polar hypoplastic model (MPHM) are compared with those obtained from the corresponding non-polar hypoplastic model (NPHM).

According to the numerical results obtained from MPHM for three different specimen lengths, smooth sidewalls of the shear box frame, and different prescribed interface friction angles, the following conclusions can be drawn:

- In the ISBT, the deformation and stress of the granular body evolves inhomogeneously from the beginning of shearing. The mechanical behaviour strongly depends on the interface friction angle. In contrast to the theoretical infinite layer under shearing, the evolution of state variables is influenced by the specimen size. In particular, the size effect is more pronounced in cases where the interface friction angle is higher than the inter-granular friction angle in the critical state.
- Interface friction angles lower than the inter-granular friction angle, i.e. the so-called critical friction angle of the granular material, only lead to small shear deformations within the granular body close to the interface. Sliding against the interface already appears after a small shear displacement. The applied shear displacement to reach the maximum shear resistance is larger for a higher interface friction angle. When sliding occurs along the whole interface, the maximum shear resistance is directly related to the amount of the prescribed interface friction angle and normal stress. Accordingly, no size effect of the granular specimen on the maximum average mobilised interface friction angle can be detected, that is, the computed maximum mobilised interface friction angle is the same as the one prescribed.
- For a prescribed interface friction angle higher than the inter-granular friction angle, the maximum shear resistance is mobilised only within the part of the interface which is characterized by relative displacements between the bottom structure and the granular material. The location of zones of sliding and sticking changes in the course of shearing. In zones where no relative displacements take place, the mobilised friction angle is lower than the maximum one. This inhomogeneous distribution of the mobilisation of the maximum shear resistance along the interface leads to an average interface friction angle, which is lower than the prescribed one. The calculated maximum average mobilised interface friction angle is influenced by the length of the shear box and it is slightly higher for the smaller shear box. Although the maximum mobilised interface friction angle increases when the box length decreases, the shear displacement corresponding to the average peak interface friction angle decreases, that is, the incremental stiffness up to the peak shows a strong influence on the shear box size. It can be concluded that in cases when the calculated average mobilised interface friction angle is lower than the one prescribed, that is, the true one, difficulties and complexities arise in precisely determining and interpreting the interface friction angle obtained from the classical ISBT.

The main findings relevant to the investigations of the micro-polar effects and the influence of the sidewall friction inside the shear box frame on the interface shear behaviour of the granular specimen can be summarized as follows:

- Simulations of plane strain ISBTs for a prescribed interface friction angle, \( \varphi_{\text{int}} \), which is lower than the critical friction angle, \( \varphi_c \), show that the maximum mobilized friction angle at the interface is independent of the sidewall friction and is the same as that obtained from the simulations with the MPHM and the NPHM. However, rough sidewalls inside the shear box frame restrict the vertical movement of the specimen during pre-loading and shearing. For an initially dense specimen, shearing leads to dilatancy close to the interface and results in a heaving of the top surface which is lower for a higher sidewall friction. The computed maximum heaving is a little higher for the micro-polar material description than for the non-polar one. As a consequence of considering sidewall friction, the resulting normal force acting on the interface differs to the normal force applied at the top surface of the specimen. The amount of the resulting normal force and its orientation at the interface depends strongly on whether a contractive or dilative behaviour appears within the granular specimen close to the rough lateral walls of the shear box frame.
- In standard ISBTs, the mobilized friction angle, \( \varphi_{\text{cal}} = \arctan(S/N_{\text{top}}) \), is usually calculated from the measured horizontal shear force \( S \) and vertical load \( N_{\text{top}} \) applied at the top surface of the specimen. For rough sidewalls, the resulting normal force acting on the interface is different from the applied normal force on the top surface of the specimen, thus, the value of \( \varphi_{\text{cal}} \) is different to the true mobilized interface friction angle, \( \varphi_{\text{int}} \). In particular, the calculated friction angle, \( \varphi_{\text{cal}} \), depends on the dilatancy behaviour of the specimen and for an initially dense specimen, it is higher for a higher sidewall friction. The value of \( \varphi_{\text{cal}} \) may also be affected by other factors such as the initial density of the granular material, the applied normal load, grain crushing and the resulting sidewall friction force depending on the depth and height of the specimen. Such factors of influence, however, are out of the scope of the present study and will be investigated in a further paper.
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DATA AVAILABILITY STATEMENT
The data that supports the findings of this study are available from the corresponding author upon reasonable request.

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APPENDIX A: SIMULATION OF PLANE SHEARING OF A SINGLE MATERIAL ELEMENT

For the hypoplastic constitutive model and the given set of constitutive material parameters, the evolution of the mobilised inter-granular friction angle under plane shearing of a single material element is shown in Figure A1 for different vertical pressures and initial void ratios. It is worth noting that different definitions can be found for the mobilised friction angle of the granular material under plane shearing in the literature of soil mechanics. In particular, it can be distinguished between the definition based on the principal stresses, $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$, that is,

$$
\phi_{\text{mob}} = \arcsin \left( \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})}{(\sigma_{\text{max}} + \sigma_{\text{min}})} \right)
$$

as used in the representation of Figure A1A,B, and the definition with respect to fixed orientation of the stress components, $\sigma_{12}$ and $\sigma_{22}$, that is,

$$
\varphi_{\text{mob}} = \arctan \left( \frac{\sigma_{12}}{\sigma_{22}} \right)
$$

as used in the representation of Figure A1C,D.

FIGURE A1  Evolution of the mobilised friction angle for a single material element under shearing for different vertical pressures, $p_0 = 10, 100, 500$ kPa, and different initial void ratios, $e_0 = 0.55, 0.6, 0.85$: (A,B) mobilised friction angle obtained from $\phi_{\text{mob}} = \arctan \left( \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})}{(\sigma_{\text{max}} + \sigma_{\text{min}})} \right)$, and (C,D) mobilised friction angle obtained from $\varphi_{\text{mob}} = \arctan \left( \frac{\sigma_{12}}{\sigma_{22}} \right)$.
In the hypoplastic model employed, the predicted value of the peak friction angle is higher for an initially lower void ratio and a lower vertical pressure. No strain softening can be detected for an initial void ratio higher than the critical one. For large monotonic shearing, the mobilised friction angle tends toward the critical value independent of the initial void ratio and pressure level, which is in accordance with the concept of the critical state soil mechanics (e.g.,\textsuperscript{96}). For critical states, the hypoplastic model predicts a friction angle of $35^\circ$ and $30^\circ$ for the relations given in Equations (A1, A2), respectively. For the following relationship

$$\left( \frac{\tan \phi}{\tan \varphi} \right) = \left( \frac{\tan 35^\circ}{\tan 30^\circ} \right) \simeq 1.2$$ (A3)

the calculated value of 1.2 corresponds to the experimental results for sand reported by Stroud\textsuperscript{97} and also discussed in detail for the hypoplastic constitutive model by Bauer.\textsuperscript{98}