Weak Measurement and (Bohmian) Conditional Wave Functions

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It was recently pointed out (and demonstrated experimentally) by Lundeen et al. that the wave function of a particle (more precisely, the wave function possessed by each member of an ensemble of identically-prepared particles) can be “directly measured” using weak measurement. Here it is shown that if this same technique is applied to one particle from a (perhaps entangled) multi-particle system, the result is precisely the so-called “conditional wave function” of Bohmian mechanics. Thus, a plausibly operationalist method for defining the wave function of a quantum mechanical sub-system corresponds to the natural definition of a sub-system wave function which Bohmian mechanics (uniquely) makes possible. An experimental arrangement which could demonstrate the non-local dependence of the sub-system wave function on distant interventions is suggested and discussed.

I. INTRODUCTION

The notion of “weak measurement”, first introduced in Ref.[1], has recently become an important tool for exploring foundational questions in quantum mechanics. For example, the recent theorem of Pusey, Barrett, and Rudolph [2] – according to which the quantum state or wave function must be understood as having an ontological (as opposed to merely epistemic) character – is nicely supported and supplemented by the rather different recent work of Lundeen et al. showing that the quantum wave function can be “directly measured”. [3] (This is “direct” in contrast to the indirect or reconstructive approaches involved in quantum state tomography – but see also Ref.[4].) At the risk of treading onto dangerously thin philosophical ice, one is tempted to think that, if the wave function can be determined experimentally in this way, it is surely, in some sense, physically real. [5]

The experimental determination goes as follows. In a weak measurement, one lets a system in state $|\psi\rangle$ couple weakly to a pointer variable which, if the coupling were stronger, would unambiguously register the value associated with observable $A$. With the weak coupling, however, the pointer’s registration remains quite ambiguous; but this can be made up for by repeating the process many times (on identically-prepared systems) and averaging. After the system couples weakly to the pointer, one may also make a (normal, strong) measurement of some other observable $B$ and post-select on the outcome. In the weak-coupling limit, the expectation value of the pointer variable (when the final measurement has outcome $b$) is the real part of the (here, complex) “weak value”

$$\langle A \rangle_W = \frac{\langle b | \hat{A} | \psi \rangle}{\langle b | \psi \rangle}. \quad (1)$$

In the scheme introduced by Lundeen et al., one lets $\hat{A} = \hat{p}_x = |x\rangle \langle x|$ and $\hat{B} = \hat{p}_x$. We then have that

$$\langle \pi_x \rangle_W = \frac{\langle p_x | x \rangle \langle x | \psi \rangle}{\langle p_x | \psi \rangle} = e^{-ip_x x/\hbar} \frac{\psi(x)}{\psi(p_x)}. \quad (2)$$

For the particular case $p_x = 0$ we thus have that the weak value is proportional to the particle’s wave function:

$$\langle \pi_x \rangle_{W=0} \sim \psi(x). \quad (3)$$

Lundeen et al. used this technique to directly measure the transverse wave function of an ensemble of identically-prepared photon(s). [6]

Another important and relevant example of the use of weak measurements involves Bohmian mechanics. Wiseman pointed out that a certain naively plausible operational approach to experimentally determining the trajectory of a quantum particle – namely, defining the velocity of a particle at a certain position in terms of the difference between the weak value of its position at time $t$ and the strong value at $t + dt$ – yields precisely the Bohmian expression for the particle’s velocity. [7] Steinberg et al. implemented this scheme to reconstruct the average trajectories for photons in the 2-slit experiment. [8] The beautiful experimentally-reconstructed trajectories are indeed congruent with the iconic images of 2-slit Bohmian trajectories. [9]

The goal of the present work is to address the following seemingly natural question: what happens if the Lundeen et al. technique, for “directly measuring” the wave function of a particle, is applied to a particle which does not, according to ordinary quantum theory, have a wave function of its own, because it is entangled with some other particle(s)? The answer turns out to be that the “directly measured” one-particle wave function corresponds exactly to the so-called “conditional wave function” of Bohmian mechanics. [9] Since this is undoubtedly an unfamiliar concept to most physicists, we briefly review it here before proceeding with the analysis at hand.
II. BOHMIAN CONDITIONAL WAVE FUNCTIONS

Consider for simplicity a system of two particles (coordinates \(x\) and \(y\)) each moving in one spatial dimension. According to ordinary quantum mechanics (OQM) the wave function \(\Psi(x, y, t)\), obeying an appropriate two-particle Schrödinger equation, provides a complete description of the state of the system. According to Bohmian mechanics (BM), however, the description provided by the wave function alone is decidedly incomplete; a complete description requires specifying in addition the actual particle positions \(X(t)\) and \(Y(t)\). For BM the wave function \(\Psi(x, y, t)\) obeys the usual Schrödinger equation, while the particle positions evolve according to

\[
\frac{dX(t)}{dt} = \frac{\hbar}{2m_1}\left[\frac{\partial}{\partial x}\Psi^*\frac{\partial}{\partial y}\Psi - \frac{\partial}{\partial y}\Psi^*\frac{\partial}{\partial x}\Psi\right]_{x=X(t), y=Y(t)}
\]

and similarly for \(Y(t)\). It is a joint property of the time-evolution laws for the wave and particles that, if the particle positions \(X\) and \(Y\) are random and \(|\psi|^2\)-distributed at some initial time (this is the so-called quantum equilibrium hypothesis, QEH), they will remain \(|\psi|^2\) distributed for all times. This so-called “equivariance” property is crucial for understanding how BM reproduces the statistical predictions of OQM. \[9\]

The Bohmian “conditional wave function” (CWF) – for, say, the first particle – is simply the (“universal”) wave function \(\Psi(x, y, t)\) evaluated at \(y = Y(t)\):

\[
\chi_1(x, t) = \Psi(x, y, t)|_{y=Y(t)}.
\]

This is the obvious and natural way to construct a “single particle wave function” given the resources that BM provides. (OQM, with fewer resources at hand, provides no such natural – or even an unnatural – construction.) Note that the evolution law for the position \(X(t)\) of particle 1, Equation \[10\], can be re-written in terms of particle 1’s CWF as follows:

\[
\frac{dX(t)}{dt} = \frac{\hbar}{2m_1}\frac{\partial}{\partial x}\chi_1 - \frac{\partial}{\partial x}\chi_1 \frac{\partial}{\partial x}\chi_1\bigg|_{x=X(t)}.
\]

It is thus appropriate to think of \(\chi_1(x, t)\) as the guiding-or pilot-wave that directly influences the motion of particle 1.

It is important to appreciate that \(\chi_1(x, t)\) depends on time in two different ways – through the \(t\)-dependence of \(\Psi\) and also through the \(t\)-dependence of \(Y\). Thus, in general, \(\chi_1(x, t)\) does not obey a simple one-particle Schrödinger equation, but obeys instead a more complicated pseudo-Schrödinger equation. \[9\] \[10\] In particular, it is easy to see that, under the appropriate measurement-like circumstances, \(\chi_1(x, t)\) will collapse. Suppose for example that particle 1 has initial wave function

\[
\psi(x) = \sum_n c_n \psi_n(x)
\]

where the \(\psi_n(x)\) are eigenstates of some observable \(\hat{A}\) with eigenvalues \(a_n\). And suppose that particle 2 is the pointer on an \(\hat{A}\)-measuring device, initially in the state

\[
\phi_0(y) \sim e^{-y^2/2w^2}.
\]

Now suppose the particles experience a (for simplicity, impulsive) interaction

\[
\hat{H}_{int} = -\lambda \delta(t) \hat{A} \hat{p}_y.
\]

The usual unitary Schrödinger evolution of the initial wave function \(\Psi(x, y, 0^-) = \psi(x) \phi_0(y)\) then takes it into

\[
\Psi(x, y, 0^-) = \sum_n c_n \psi_n(x) \phi_0(y - \lambda a_n).
\]

That is, the two-particle wave function after the interaction can be understood as an entangled superposition of

\[\text{FIG. 1: Illustration of the collapse of the Bohmian CWF during an energy measurement (}\hat{A} = \hat{H}\text{) on a particle in a box. The initial two-particle wave function }\Psi(x, y, 0^-) = \psi(x)\phi_0(y)\text{ has support in the blueish region of the configuration space. Since this initial state factorizes, the CWF at }t = 0^-\text{ is (up to a multiplicative constant) just }\chi_1(x) = \sum_n c_n \psi_n(x)\text{, indicated with the bolded lower curve. The Schrödinger evolution from }0^-\text{ to }0^+\text{ produces a two-particle wave function with localized islands of support in the configuration space, indicated by the yellowish regions. Each of the yellowish blobs is a Gaussian in }y\text{ (centered at one of the possible post-interaction pointer positions }\lambda a_n\text{) multiplied by one of the energy eigenfunctions (here }\psi_n(x) \sim \sin(n\pi x/L)\text{). And so if (for example, as shown) the actual configuration point }\{X(0^-), Y(0^-)\}\text{ ends up in the support of the yellowish blob at }y = \lambda a_2\text{ – something which will occur with probability }|c_2|^2\text{ with random initial configuration }\{X(0^-), Y(0^-)\}\text{ in accord with the QEH – then the post-interaction CWF of particle }1\text{ will be }\chi_1(x, 0^+) \sim \psi_2(x)\text{ (as shown).}\]

\[\]
terms, each of which has particle 1 in an eigenstate of $\hat{A}$ and particle 2 in a new position that registers the corresponding value $a_n$. (Note that we assume here that $\lambda$ is sufficiently large that the separation between adjacent values of $\lambda_n$ is large compared to the width $w$ of the pointer packet. This is thus a "strong" measurement.) From the point of view of OQM, Equation (10) exhibits the standard problem of Schrödinger’s cat: instead of resolving the superposition of distinct $a$-values, the measuring device itself gets infected with the superposition. In OQM (where there is nothing but the wave function at hand) one thus needs to introduce additional dynamical ("collapse") postulates to account for the observed (apparently-nonsuperposed) behavior of real laboratory equipment.

In BM, however, there is no such problem. The outcome of the measurement is not to be found in the wave function, but instead in the actual position $Y(0^+)$ of the pointer after the interaction. It is easy to see that (with appropriate random initial conditions) this will, with probability $|c_n|^2$, lie near the value $\lambda_n$ which indicates that the result of the measurement was $a_n$. Furthermore, it is easy to see that if $Y(0^+)$ is near the value $\lambda_n$, then the CWF of particle 1 will be (up to a multiplicative constant) the appropriate eigenfunction:

$$\chi_1(x, 0^+) \sim \psi_n(x). \quad (11)$$

That is, the CWF of particle 1 collapses (from a superposition of several $\psi_n$s to the particular $\psi_n$ which corresponds to the actually-realized outcome of the measurement) as a result of the interaction with the measuring device, even though the dynamics for the “universal” wave function $\Psi$ is completely unitary. See Figure 1 and its caption for an illustration.

We have here explained the idea of (and one important and perhaps surprising property of the dynamical evolution of) Bohmian CWFs as if the particle of interest were interacting with the particle or particles constituting a measuring device. That is more important for our purposes here is the fact that the Bohmian CWFs (for a single particle) is perfectly well-defined at all times for any particle that is part of a larger (multi-particle) quantum system. Indeed, BM only really provides a solution of the measurement problem because it treats “measurements” as just ordinary physical interactions, obeying the same universal dynamical laws as all interactions. It should thus be clear that, according to BM, collapses (like the one we just described happening as a result of an interaction with a measuring device) will actually be happening all the time, as particles interact with each other. It is the goal of the following analysis to show how this feature of the Bohmian theory can be experimentally manifested using weak measurement.

### III. DIRECT MEASUREMENT OF SINGLE PARTICLE WAVE FUNCTIONS

Let us then turn to the main result of the present paper. Suppose we carry out the Lundeen-type “direct measurement of the wave function” procedure on one particle of a two-particle system. As a reality check, suppose to begin with that the two-particle system has a factorizable quantum state

$$|\Psi \rangle = |\psi \rangle |\phi \rangle \quad (12)$$

where the first and second factors on the right refer to particles 1 and 2 respectively. The Lundeen-type procedure involves post-selecting on the final momentum $p_x$ of the particle whose wave function we are trying to measure (here, particle 1). Let us also post-select on the final position $Y$ of particle 2. \[11\] It is then straightforward to calculate that

$$\langle \hat{\pi}_x \rangle_W = \frac{\langle Y|\phi \rangle \langle p_x|x|\psi \rangle}{\langle Y|\phi \rangle \langle p_x|\psi \rangle} = \frac{e^{-ip_x x/\hbar} \psi(x)}{\psi(p_x)} \quad (13)$$

which is, as expected, identical to Equation (2).

If, however, particle 1 is in a general, entangled state with particle 2, as in

$$|\Psi \rangle = \int d\chi \, e^{ipy} \chi(x, y') |y' \rangle \quad (14)$$

then the operational determination of particle 1’s wave function (post-selected on the final strongly-measured position $Y$ of particle 2) yields

$$\langle \hat{\pi}_x \rangle_W = \frac{\langle p_x|x, Y|\Psi \rangle}{\langle p_x, Y|\Psi \rangle} = \frac{e^{-ip_x x/\hbar} \Psi(x, Y)}{\int d\chi \Psi(x', Y) e^{-ip_x x'/\hbar}} \quad (15)$$

Restricting, as before, our attention to the cases in which the final measured momentum $p_x$ is zero, we have that

$$\langle \hat{\pi}_x \rangle_W |_{p_x=0, y=0} \sim \Psi(x, Y) = \chi_1(x) \quad (16)$$

where the right hand side is precisely the Bohmian CWF for particle 1. Note that we have tacitly relied on the fact that, for Bohmian mechanics, position is a non-contextual (“hidden”) variable. Thus the final position measurement on particle 2 simply reveals, for Bohmian mechanics, the actual pre-existing location $Y$ of that particle. In short, the two $Y$s in the analysis – the one representing the outcome of the final position measurement of particle 2, and the one, used in the definition of the Bohmian CWF, representing the actual position of particle 2 – are, for Bohmian mechanics, the same.

### IV. ELABORATION AND ILLUSTRATION

In the last section we basically ignored the issue of the exact timing of the various measurements on the two-particle system. Let us then slow down and think some
enters the device is entangled with a second photon:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\psi_1\rangle |\phi_1\rangle + |\psi_2\rangle |\phi_2\rangle).$$  \hspace{1cm} (17)

It is important (for the proper functioning of the wave function measurement) that $|\psi_1\rangle$ and $|\psi_2\rangle$ have the same (say, linear) polarization. But let them have distinct (transverse) spatial profiles – for example, and most simply, suppose that $|\psi_1\rangle$ has transverse spatial support just above the z-axis (i.e., for $x > 0$) while $|\psi_2\rangle$ has transverse spatial support just below the z-axis ($x < 0$). See Figure 2(a).

As to photon 2, suppose that (at least initially) $|\phi_1\rangle$ and $|\phi_2\rangle$ have identical transverse profiles and completely overlap spatially, but are distinct in some way (for example, they could be orthogonally polarized, or could have different energies) that allows the two parts of the beam to be separated by some type of beam splitter (BS). See Figure 2(b).

Let us now consider several different spatial locations for, and time-orderings involving, the final measurement of the position $Y$ of photon 2.

To begin with, let us first imagine that the measurement/post-selection on photon 2’s transverse coordinate $y$ occurs (in, say, the lab frame) before the weak measurement on photon 1 and at a plane like A in Figure 2 (i.e., before photon 2 has passed any beam splitter). According to OQM, this measurement of $Y$ will collapse the 2-particle wave function and leave photon 1 with a definite (non-entangled) wave function of its own. Because $|\phi_1\rangle$ and $|\phi_2\rangle$ overlap at A, however, the position measurement gives no information about $|\phi_1\rangle$ vs. $|\phi_2\rangle$ and so leaves photon 1 in the state $\frac{1}{\sqrt{2}} (|\psi_1\rangle + |\psi_2\rangle)$.

On the other hand, if the measurement/post-selection on photon 2 is performed at plane B (after photon 2 has passed the beam splitter, but still before the measurement protocol has been carried out on photon 1) then according to OQM the wave function of photon 1 will collapse to either $|\psi_1\rangle$ (if $Y$ is found in the support of $|\phi_1\rangle$) or $|\psi_2\rangle$ (if $Y$ is found in the support of $|\phi_2\rangle$).

It is thus perhaps not terribly surprising, from the point of view of OQM, that these same results should be obtained if (say) the optical distance from the particle source to the beam splitter (BS) in Figure 2(b) is increased – so that measurement/post-selection at A or B occurs simultaneously with the weak measurement on photon 1. To see this, we need simply calculate that, e.g., for measurement/post-selection at A

$$\langle Y | \psi_1(x) \rangle = \psi_1(x) \langle Y | \psi_1 \rangle + \psi_2(x) \langle Y | \psi_2 \rangle \sim \frac{1}{\sqrt{2}} \langle \psi_1(x) \rangle + \langle \psi_2(x) \rangle$$  \hspace{1cm} (18)

where we have used the fact that, for detection at the A plane, $\langle Y | \psi_1 \rangle = \langle Y | \psi_2 \rangle$.

On the other hand, for detection at the B plane, one or the other of $\langle Y | \psi_1 \rangle$ and $\langle Y | \psi_2 \rangle$ will be zero. We will
FIG. 3: In the \((x,y)\) configuration space, the entangled two-photon wave function initially has support in the blueish region and the CWF for photon 1 \(-\chi_1(x,t_1)\) – looks something like the two-hump curve shown. The passage of photon 2 through the beam-splitter (BS) separates the two-photon wave function into the two yellowish islands. The actual configuration point (shown here as a black dot) ends up (depending on its random initial position for each photon pair) in one of the two yellowish islands, and the CWF at \(t_2\) will thus have collapsed to either \(\langle x|\psi_1\rangle\) or \(\langle x|\psi_2\rangle\). (The latter case is shown here.)

thus find that

\[
\langle \pi_x \rangle^{P_{x=0, y=Y}}_W \sim \psi_m(x)
\]

(19)

for \(Y \in \text{supp}(|\psi_m\rangle)\). As shown in the previous section, in both of these cases the operationally-determined one-particle wave function of photon 1 exactly matches the natural one-particle wave function – that is, the Bohmian CWF – that would be assigned to it by Bohmian mechanics at the time in question. We have tried to suggest here that, for the two cases considered, these results are not terribly surprising from the point of view of OQM.

The most interesting case, then, involves measurement/post-selection of photon 2’s position at plane C (such that this measurement takes place well after the weak measurement on photon 1 has already gone fully to completion). It is trivial to see that the results of the weak measurement will again be given by Equation (19) – that is, dramatically different (“collapsed”) wave functions for photon 1 will be found depending on whether particle 2 is (later!) found in the support of \(|\phi_1\rangle\) or \(|\phi_2\rangle\). From the point of view of OQM, however, it is rather difficult to understand why, prior to any actual measurement that would trigger a collapse, one should find collapsed one-particle wave functions. On the other hand, this is perfectly natural from a Bohmian point of view: as sketched in Figure 3 the conditional wave function (CWF) for particle 1 collapses as soon as the packets separate in the two-dimensional configuration space. This happens when the components \(|\phi_1\rangle\) and \(|\phi_2\rangle\) are split at the BS; no actual position measurement is required. (Note that the claim here is not that OQM makes the wrong predictions. Undoubtedly it makes the right predictions – indeed we have used nothing but orthodox quantum ideas to calculate what should be observed in the experiment. The point is rather only that, from the OQM perspective, it is at best obscure what one ought to expect the operationalist determination of photon 1’s “one-particle wave function” to yield at times when photon 1 remains entangled with photon 2. Whereas, from the Bohmian point of view, the results are the obvious, natural thing.)

The proposed experiment, then, should involve a fixed detection plane, like plane C in the Figure, at a greater optical distance from the two-particle source than that of the measuring apparatus for particle 1. The removable beam splitter BS should, on the other hand, ideally be slightly closer to the two-particle source than the particle 1 apparatus. With the BS in place, the arrangement would be like that shown in the Figure and the “direct measurement” on photon 1 would yield (after appropriate post-selection on \(Y\)) collapsed photon 1 wave functions (even though the actual position measurement on photon 2 would occur well after photon 1 was already measured). On the other hand, with the BS removed, the situation would be equivalent to detecting photon 2 at plane A (except that this too would only occur later, after the measurements on photon 1 had occurred) and the measurement of photon 1 would reveal the uncollapsed wave function. One could thus observe the collapse of the Bohmian CWF of photon 1 as a direct result of the insertion (perhaps at space-like separation) of the BS into the path of photon 2.

This proposed setup should be realizable in practice along the following lines. Type II spontaneous parametric down-conversion yields a pair of photons in an entangled polarization state \(\langle(H)|H⟩+⟨V)|V⟩\rangle/\sqrt{2}\) with the individual photons being coupled into single-mode optical fibres. The first photon should then be split by a PBS, with, say, the \(|H⟩\) component being shunted into the +\(z\) direction with \(x > 0\) and the \(|V⟩\) component passed through a \(\lambda/2\) plate (rotating its polarization to \(|H⟩\) before being shunted into the +\(z\) direction with \(x < 0\). This prepares photon 1 as suggested in Figure 2a and the subsequent measurements may then be carried out exactly as in Ref. 8. The second photon can be directly shunted into the –\(z\) direction so that the two orthogonal polarization components have identical and perfectly overlapping transverse spatial profiles, as in Figure 2b. The overall (transverse spatial and polarization) two-photon state after such preparation can thus be written

\[
|\Psi⟩ = \frac{1}{\sqrt{2}} \left( |+H⟩|0H⟩ + |−H⟩|0V⟩ \right)
\]

(20)

where + indicates that the transverse state has support for \(x > 0\), – indicates that the support lies in \(x < 0\), and 0 indicates that the support is centered at \(x = 0\). (It is also of course understood that photon 1 is moving in the +\(z\) direction and photon 2 in the –\(z\) direction.) The
perfect correspondence between Equations (20) and (17) should be clear. Note that for this type of implementation, the (generic) “BS” in Figure 2(b) can be a standard polarizing beam splitter (PBS).

V. DISCUSSION

If the procedure of Ref. [3] for making a “direct measurement of the quantum wavefunction” is applied to one particle from an entangled pair (and regarded as a plausible operationalist definition of the “single particle wave function” for such a particle) the result is precisely the “conditional wave function” (CWF) of Bohmian mechanics – that is, the natural theoretical concept of a “single particle wave function” that Bohmian mechanics (uniquely) makes possible. This is particularly interesting in cases like that elaborated in the previous section, in which the weak measurement (that reveals the wave function of particle 1) is carried out prior to the strong position measurement on particle 2 on which post-selection (of particle 1) is carried out prior to the strong position measurement on particle 2 on which post-selection will be based. As explained, one can thus observe how the CWF of one particle is dramatically affected by interventions – e.g., the insertion of the BS in Figure 2 at the location of a distant (but entangled) particle. The type of experiment suggested here would thus in principle allow the collapse of the CWF (due to distant interventions) to be empirically observed (but with, of course, the usual caveats associated with weak measurement).

Looked at this way, the proposal here is very much in the spirit of – and indeed this work was directly inspired by – the recent proposal of Braverman and Simon “to observe the nonlocality of Bohmian trajectories with entangled photons.” [12] From the perspective suggested earlier, in which each particle’s CWF is regarded as the object which (via Equation (6), for the non-relativistic case) directly guides or pilots the Bohmian particle trajectory, the present work can be seen as digging yet one level deeper: instead of merely observing how the particle trajectories change as a result of some distant interventions, one may also observe how the “field” responsible for those changes itself changes. A successful experimental demonstration of this effect would thus in some sense reveal the non-local character of Bohmian mechanics in an unprecedentedly fundamental way.

Finally, at the risk of diminishing the reader’s interest in the proposal, it is worth explicitly pre-empting a possible confusion that might naturally be produced by the way certain things – both here and in the conceptually similar Braverman/Simon proposal – are put. It would be easy to get the impression, for example, that by inserting (or refraining from inserting) the BS in the path of photon 2, one can instantaneously affect the observable CWF of the distant particle. That is, it would be easy to get the impression that one could (in principle, if impractically) send a superluminal signal by running many copies of the experiment in parallel. But this, of course, should be impossible (whether one believes in BM or OQM or any other such empirically-equivalent theory).

There are two points to be understood here, one rather obvious and one more subtle. The obvious point is that Alice (on the right) must learn the outcome of Bob’s position measurement (on the left) before she can know how to properly bin her data. And this information will have to be sent to her through a “classical” (i.e., here, sub-luminal) communication channel. So it is already clear that no actual superluminal signalling will be possible.

The more subtle, and more interesting, point is that the statistical relationship between Alice’s and Bob’s measurement results is actually independent of the exact temporal sequence of the measurements. This is certainly not surprising from the point of view of relativity, given that Alice’s and Bob’s measurements may well occur at spacelike separation. But it is somewhat surprising from the point of view of Bohmian mechanics, which involves a hidden (but dynamically relevant) privileged reference frame.

Essentially for reasons of drama, we have described the setup above so that the temporal sequence is as follows: first Bob (or his assistant) decides whether to insert or not insert the BS into the path of particle 2; then Alice’s measurement protocol on particle 1 occurs; and then finally Bob measures the final transverse position Y of particle 2. From the point of view of Bohmian mechanics, then, we may say the following. If this is the true temporal sequence – in the dynamically privileged reference frame posited by the theory – then things develop causally in the way we have suggested: the insertion of the BS (if and only if it is inserted) causes the two-particle wave function to divide as shown in Figure 3 and thus causes the CWF for photon 1 to collapse; the (here, subsequent) measurement protocol by Alice then simply reveals the true CWF of photon 1 at the time of that measurement; the final measurement/post-selection by Bob then plays the (dynamically) purely passive role of revealing, to Bob, what was already physically definite, in order that the already-acquired data can be properly binned.

But since the privileged frame is, for Bohmian mechanics, hidden, it is entirely possible that the “true” temporal sequence (i.e., the temporal sequence in the privileged frame) is instead as follows: Alice’s measurement protocol on photon 1 occurs first; then comes Bob’s (assistant’s) decision to insert the BS or not, followed by measurement of the transverse position Y. If this is the “true” temporal sequence, the statistics will be unchanged, but the causal story will be somewhat different. To wit: instead of the passage (or not) of photon 2 through the BS affecting the CWF of photon 1, now it will be the measurement protocol on photon 1 which (at least sometimes) affects the CWF of photon 2 and thus influences where, for a given Y, it will go, should it encounter the BS. Concretely, there will exist possible initial conditions for the 2-particle system which have the following property: had the measurement protocol
on particle 1 not been carried out, particle 2 would definitely have gone “up” at the BS (and would hence have been found in the support of |φ1⟩), but given that the measurement protocol on particle 1 was carried out, particle 2 instead went “down” at the BS (and was hence found in the support of |φ2⟩).

With this second possible “true” temporal sequence, it is no longer really the case that the weak measurement protocol on particle 1 is simply revealing the structure of particle 1’s CWF at the time of the measurement. Instead, the measurement on particle 1 may actively affect the state (in particular, the CWF) of particle 2, making the subsequent post-selection on particle 2’s position rather less benign, less passive. And this makes clear in principle why, despite the presence in the theory of a dynamically privileged reference frame in which instantaneous action-at-a-distance occurs, one is not only prevented from sending signals faster than light, but also prevented from putting any experimental limits on the speed of the laboratory with respect to the presumed underlying privileged reference frame. The statistical patterns in the data will—presumably—remain the same as the “true” temporal sequence is varied between the two possibilities discussed here, even as the Bohmian causal story changes rather dramatically.

It does, however, remain absolutely valid to say that the experimental setup suggested here would allow for a direct empirical observation of the non-local dependence of a single particle’s (Bohmian) CWF on distant interventions. (It’s just that—compared to the way we initially explained things—it is rather ambiguous whether one is observing the effect, on particle 1’s CWF, of inserting or not inserting the BS in front of particle 2... or instead observing the effect, on particle 2’s CWF, of carrying out the Lundeen-style “direct measurement” of particle 1’s “one-particle wave function”.) It thus remains appropriate to conclude that a realization of the proposed experiment would be quite interesting and would in particular help bring the Bohmian CWF into the light of experimental reality.

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[13] One can most easily understand how this comes about by considering the conceptually-simplifying case in which the pointer variable for the weak measurement is a third particle, with say an initially Gaussian wave function, whose position (Z) is ultimately measured. Then the relevant configuration space (cf. Figure 5) is three-dimensional. In particular, one can understand the weak coupling of photon 1 with the pointer as ever-so-slightly “raising” (up out of the page, i.e., in the +z direction) a certain narrow sliver, at a value of x corresponding to the location x′ of the weak probe. (“Ever-so-slightly” here means, of course, by an amount that is small compared to the width of the z-packet.) Now if the actual position X of particle 1 happens to have been sufficiently near x′, and if the actual pointer position Z happens to have been “extremal” (meaning, right at one edge of the z-packet), then the CWF for the original 2-particle system, χ(x, y) = Ψ(x, y, Z), will collapse as a result of the interaction with the pointer. And in particular, the CWF of particle 2 will collapse to either ϕ1(y) or ϕ2(y) (depending on whether x′ > 0 or x′ < 0). Particle 2 will then be guaranteed to go a certain way (regardless of its transverse position Y) should it encounter the BS, whereas, had its CWF not collapsed, which way it would have gone would have depended on Y. Thus the weak measurement protocol on particle 1 has the potential to non-locally influence the state of particle 2.