When Wireless Hierarchical Federated Learning Meets Physical Layer Security: A Finite Blocklength Approach

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Abstract—In this paper, the wireless hierarchical federated learning (HFL) is revisited by considering physical layer security (PLS). First, we establish a framework for this new problem. Then, we propose a practical finite blocklength (FBL) coding scheme for the wireless HFL in the presence of PLS, which is self-secure when the coding blocklength is larger than a certain threshold. Finally, the study of this paper is further explained via numerical examples and simulation results.

Index Terms—Finite blocklength coding, physical layer security, privacy-utility trade-off, wireless federated learning

I. INTRODUCTION

The wireless federated learning (FL), which uses mobile devices to produce and store data that is for the training of machine learning (ML) models [1], has been extensively studied in the literature. Currently, the wireless FL mainly focuses on improving efficiency [2]-[3], enhancing privacy and security [4]-[6], power control of wireless devices [7]-[8] and beamforming design [9]. Recently, with the use of edge computing, the client-edge-cloud hierarchical federated learning (HFL) systems receive much attention. Compared with FL with single server, the use of edge servers in HFL can reduce the computing load [10]-[11], the communication cost between users and cloud server [12]-[15], the running time of FL [12], and it enhances privacy and security [13]-[14] in FL.

Due to the broadcast nature of wireless communications, the wireless FL is susceptible to eavesdropping. Different from the privacy requirement of the FL that the information leakage between users and servers does not need to be arbitrarily small due to the accuracy of data analysis, the information leakage to the eavesdropper should vanish, which is also known as the physical layer security (PLS) requirement. When wireless FL meets the PLS, the fundamental limit in the utility-privacy-secrecy trade-off was characterized in [16]. However, note that in [16], random binning coding scheme [17]-[18] is applied, which indicates that to achieve PLS requirement, the coding blocklength should tend to infinity, and this is non-constructive for practical systems.

In this paper, we study the wireless HFL in the presence of eavesdropping, see Figure 1. In Figure 1, users, edge servers and the cloud server cooperate with each other to jointly train a learning model, and in the meanwhile, the malicious cloud server may infer the presence of an individual data sample from a learnt model by various attacks. Differential privacy (DP) [19] has been proved to be an effective way to protect the individual data against such attacks, and hence before aggregation of all users’ gradients to the edge servers, the Gaussian noise which is used as local differential privacy (LDP) mechanism [20] is added to the gradient of each user. Moreover, each edge server communicates with the cloud server via a duplex fading channel, and due to the broadcast nature of wireless communication, this channel is eavesdropped by an external eavesdropper. The main object of Figure 1 is to minimize the information leakage to the malicious cloud server subject to a certain mount of utility of the polluted data gradients (added by the Gaussian noises), and protect the transmitted data in wireless channels from eavesdropping, i.e., the information leakage to the eavesdroppers should be arbitrarily small.

A straightforward way to achieve the above goal is that the edge servers securely encode the polluted data gradients as codewords and transmit them into wireless duplex channels, the cloud server can be viewed as a legitimate receiver and hence can successfully decode the polluted data gradients, while the eavesdropper obtains arbitrarily small amount of information about the polluted data gradients. By this way, the PLS and the privacy of the data can simultaneously be guaranteed since the actual data gradients are protected by the LDP mechanism. However, note that the currently used PLS techniques (e.g.) mainly follow from the classical random binning coding scheme in [17]-[18], which is non-constructive for practical wireless systems. Then it is natural to ask: can we design a constructive coding scheme for the edge server, which not only consists of codewords with finite coding blocklength, but also can achieve PLS (arbitrarily small amount of leaked information to the eavesdropper)?

In this paper, first, we propose a practical finite blocklength (FBL) coding scheme for wireless HFL in the presence of eavesdropper, and then show that the proposed scheme achieves PLS under a given secrecy level when the coding blocklength
satisfies a certain threshold. Next, we derive an achievable rate according to the proposed scheme. Finally, we show the relationship between utility, privacy, secrecy and other parameters via numerical examples.

II. NOTATIONS, PRELIMINARY, MODEL FORMULATION AND THE MAIN RESULTS

A. Notations

Bold letters represent a vector. The superscript $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and conjugate transpose, respectively. Unif$[a, b]$ denotes a uniformly distributed in $[a, b]$, $\mathcal{C}N(0, \sigma^2)$ denotes a circularly symmetric complex Gaussian (CSCG) distribution with mean 0 and covariance $\sigma^2$, $\mathcal{N}(0, \sigma^2)$ represents a Gaussian distribution with mean 0 and covariance $\sigma^2$, and $\sim$ represents "distributed as". $\mathbb{C}$ and $\mathbb{R}$ denote the complex and real number. $E(\cdot)$ and $\text{Var}(\cdot)$ represent statistical expectation and variance for random variables, respectively. $||\mathbf{X}||$ represents the $l_2$-norm of the vector $\mathbf{X}$. $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ denote the real and imaginary parts of a complex element, respectively. $T^n_q$ represents the set of $\varepsilon$-typical $n$-sequences. The Gaussian Q-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{u^2}{2})du$, and its inverse function is $Q^{-1}(\cdot)$. The log function takes base 2.

B. Preliminary: learning protocol

In Figure 1, there are a cloud server, $L$ edge servers indexed by $l$, and $K_{\text{total}}$ users indexed by $k$ and $t$. $C_l^{(k)} \subseteq \mathbb{R}^m$ represents the disjoint user set and $|C_l^{(k)}|$ is the number of users in edge domain $l$. $S_k^{(\ell)}$ represents the distributed datasets and $S_k = \bigcup_{\ell=1}^L S_k^{(\ell)}$ is the cardinality of $S_k^{(\ell)}$, where $S_k^{(\ell)} = \{ (\mathbf{u}_{k,j}, v_{k,j}) \}_{j=1}^{S_k^{(\ell)}}$, $\mathbf{u}_{k,j} \in \mathbb{R}^m$ is the $j$-th vector of covariates with $q$ features and $v_{k,j} \in \mathbb{R}$ is the corresponding associated label at user $k$. Denote the aggregated dataset in edge $\ell$ domain by $S^{(\ell)}$, and each edge server aggregates gradients from its users. The global loss function $F(\mathbf{m})$ is given by

$$F(\mathbf{m}) = \frac{1}{S_{\text{total}}} \sum_{k=1}^{K_{\text{total}}} S_k^{(\ell)} F_k(\mathbf{m}),$$  \hspace{1cm} (2.1)$$

where $\mathbf{m} \in \mathbb{R}^m$ is the model vector, and $S_{\text{total}} = \sum_k S_k^{(\ell)}$. $F_k(\cdot)$ is the local loss function for user $k$, where

$$F_k(\mathbf{m}) = \frac{1}{S_k^{(\ell)}} \sum_{(\mathbf{u}_{k,j}, v_{k,j}) \in S_k^{(\ell)}} f(\mathbf{m}; \mathbf{u}_{k,j}, v_{k,j}) + \lambda R(\mathbf{m}),$$  \hspace{1cm} (2.2)$$

and $f(\mathbf{m}; \mathbf{u}_{k,j}, v_{k,j})$ is the sample-wise loss function. $R(\mathbf{m})$ is a strongly convex regularization function and $\lambda \geq 0$. The model training by minimizing the global loss function as

$$\mathbf{m}^* = \arg\min_{\mathbf{m}} F(\mathbf{m}).$$  \hspace{1cm} (2.3)$$

To minimize $F(\mathbf{m})$, we use a distributed gradient descent iterative algorithm. Specifically, in the $t$-th ($t \in \{1, 2, ..., T\}$) communication round, the cloud server broadcasts the current global model vector $\mathbf{m}_t$ to all users by edge servers and all users know the perfect $\mathbf{m}_t$. Each user $k$ computes its own local gradient $\nabla F_k(\mathbf{m}_t) = \frac{1}{S_k^{(\ell)}} \sum_{(\mathbf{u}_{k,j}, v_{k,j}) \in S_k^{(\ell)}} \nabla f(\mathbf{m}_t; \mathbf{u}_{k,j}, v_{k,j}) + \lambda \nabla R(\mathbf{m}_t)$ by using $S_k^{(\ell)}$ and the current model $\mathbf{m}_t$, and the users send the corrupted local gradients (added by Gaussian noises for LDP) to the edge servers. After receiving users’ information about the local gradients, the edge server $\ell$ computes its estimation $\widehat{\nabla F}(\mathbf{m}_t)$ about the partial gradient

$$\nabla F^{(\ell)}(\mathbf{m}_t) = \frac{1}{S^{(\ell)}} \sum_{k \in C^{(\ell)}} \nabla f(\mathbf{m}_t; \mathbf{u}_{k,j}, v_{k,j}),$$  \hspace{1cm} (2.4)$$

where $S^{(\ell)} = |S^{(\ell)}|$ is the total number of $S^{(\ell)}$, after receiving all the estimations about partial gradients from edge servers, the cloud server computes its estimation $\widehat{\nabla F}(\mathbf{m}_t)$ about the global gradient

$$\nabla F(\mathbf{m}_t) = \frac{1}{S_{\text{total}}} \sum_{t=1}^T S^{(\ell)} \nabla F^{(\ell)}(\mathbf{m}_t),$$  \hspace{1cm} (2.5)$$

and the global model $\mathbf{m}_{t+1}$ updated by the cloud server is given by

$$\mathbf{m}_{t+1} = \mathbf{m}_t - \mu \nabla \widehat{F}(\mathbf{m}_t).$$  \hspace{1cm} (2.6)$$

where $\mu$ is the learning rate. The process in (2.4) and (2.6) are iterated till convergence. For convenience, in the $t$-th ($t \in \{1, 2, ..., T\}$) communication round, we denote $\mathbf{W}_k = S_k^{(\ell)} \nabla F_k(\mathbf{m}_t)$.

C. Model formulation

In this paper, we assume that each edge server communicates with the cloud server without interference from other edge servers. Hence we only focus on the communication between one of the edge servers and the cloud server. An information-theoretic approach of Figure 1 is illustrated in Figure 2. For simplification, we make the following assumptions:

- Similar to [6], we assume that the channels are flat-fading, i.e., the channel coefficients are time invariant during the transmission.
- We assume that the cloud server and the edge server have perfect channel state information (CSI) of the feed-forward channel and feedback channel, and this assumption is in line with that in [6, 8]-[9].
- From similar arguments in [21], we assume that the edge server is an active user but it is un-trusted by the cloud server, which indicates that the perfect CSI of eavesdropper's
channel is known by the eavesdropper and the edge server. Moreover, we assume that the eavesdropper also knows the perfect CSI of the edge server–cloud server’s channels, and this assumption is similar to that in [22], [25].

**Definition 1** (Privacy by mutual information [26]): If the mutual information between \( W^v \) and \( W'^q \) satisfies \( \frac{1}{q} I(W^v; W'^q) \leq \epsilon \), we say the LDP mechanism satisfies \( \epsilon \)-mutual-information privacy for some \( \epsilon > 0 \).

**Definition 2** (Utility by rate distortion [27]): The utility of \( W'^q \) is characterized by the quadratic distortion between \( W^v \) and \( W'^q \), i.e., 

\[
d(W^v, W'^q) = \| W^v - W'^q \|^2. \]

If \( \frac{1}{q} E(d(W^v, W'^q)) \leq D \), we say the utility of \( W'^q \) is up to \( D \).

**Channels**: At time instant \( i \in \{1, 2, ..., N\} \), channel inputs and outputs are given by 

\[
Y_i = h_1X_i + \eta_{i,1}, \tilde{Y}_i = h_2X_i + \eta_{i,2}, Z_i = h_kX_i + \eta_{i,k}, \quad (2.9)
\]

where \( X_i \) is the feedforward channel input subject to an average power constraint \( \frac{1}{N} \sum_{i=1}^{N} E[X_i] \leq P \), \( \tilde{X}_i \) is the feedback channel input subject to an average power constraint \( \frac{1}{N} \sum_{i=1}^{N} E[\tilde{X}_i] \leq P \), \( h_1, h_2, h_k \in \mathbb{C} \) are the CSI of the cloud server’s channel, feedback channel and the wiretap channel, respectively, here note that \( |h_1|, |h_2| \) and \( |h_k| \) represent the modulus of \( h_1, h_2, \) and \( h_k \). \( Y_i \) and \( \tilde{Y}_i \) are the channel output of the cloud server and the edge server, \( Z_i \) is the channel output of the eavesdropper, \( \eta_{i,1}, \eta_{i,2} \) and \( \eta_{i,k} \) are i.i.d. CSCG noises across the time index \( i \), \( \eta_{i,1} \sim \mathcal{CN}(0, \sigma_1^2) \), \( \eta_{i,2} \sim \mathcal{CN}(0, \sigma_2^2) \) and \( \eta_{i,k} \sim \mathcal{CN}(0, \sigma_k^2) \). The signal-to-noise ratios of the feedforward and feedback channels are denoted by \( \text{SNR} = \frac{P}{\sigma_1^2} \) and \( \text{SNR} = \frac{P}{\sigma_2^2} \).

**Source encoder**: Once the source encoder receives the input \( W'^q \), \( W'^q \) is quantified to \( [W'^q] \). The quantization interval is small enough, which indicates that \([W'^q]\) approaches \( W'^q \) [27], Chapter 3.4.1, pp. 50-51]. Similar to [17], there exists a source encoder mapping \( \phi : [W'^q] \rightarrow \{1, 2, ..., 2^{hq(W'^q(i))}\} \) (\( i \in \{1, 2, ..., q\} \)), it compresses \( W'^q \) into an index \( W''^q \) which is uniformly distributed in \( \mathcal{W}''^q = \{1, 2, ..., 2^{hq(W'^q(i))}\} \).

**Channel encoder**: The input \( W''^q \) of the channel encoder is uniformly drawn in the set \( \mathcal{W}''^q \), and it is encoded as a codeword of length \( N \). At time \( i \in \{1, 2, ..., N\} \), the transmitted codeword \( X_i \) is a stochastic function of \( W''^q \), \( h_1, h_2 \) and \( Y_{i-1}^- = (Y_1, ..., Y_{i-1}) \), i.e., \( X_i = f_j(W''^q, h_1, h_2, Y_{i-1}^-). \)

**Channel decoder**: After the cloud server receives \( Y^N \), the channel decoder’s estimation \( \hat{w}'' = \phi(Y^N) \), where \( \phi \) is the channel decoder’s decoding function. The average decoding error probability \( P_e^d \) of message \( w'' \) sent is given by

\[
P_e^d = \frac{1}{|W''^q|} \sum_{w'' \in W''^q} Pr\{\phi(Y^N) \neq w'' | w'' \text{ sent}\}. \quad (2.10)
\]

**Source decoder**: For the source decoder, after receiving \( W''^q \), a source decoding mapping maps \( W''^q \) to \( W'^q \). The average error probability \( P_e^s \) for the source code is \( P_e^s = Pr\{[W'^q] \notin \mathcal{T}_q^d\} \). Based on the property of typical sequences [27], Chapter 2.4, pp. 25-26], the average error probability \( P_e^s \) for the source code tends to zero if \( q \rightarrow \infty \).

Here note that the overall decoding error probability is given by

\[
P_e = \frac{1}{q} \sum_{i=1}^{q} Pr\{W'^q(i) \neq W''^q(i)\} \leq P_e^e + P_e^d. \quad (2.11)
\]
\[ \Delta = \frac{h(W^n)}{(W^n)} \left( Z^n, h_1, h_2, h_3 \right), \quad (2.12) \]

where \( 0 \leq \Delta \leq 1 \).

The rate \( R \) is said to be \((N, \epsilon_1, D, \epsilon)\)-achievable with \( \delta \)-secrecy, if for a given blocklength \( N \), decoding error probability \( \epsilon_1, \frac{1}{2}I(W^n; W^n) \leq \epsilon \) and \( \frac{1}{2}E(d(W^n; W^n)) \leq D \), there exists an \((N, W^n, \epsilon_1, D, \epsilon, \delta)\) code such that

\[ \frac{h(W^n)}{N} = R, \quad P_x \leq \epsilon_1, \quad \Delta \geq \delta. \quad (2.13) \]

The secrecy capacity \( C(N, \epsilon_1, D, \epsilon) \) is composed of all achievable \((N, \epsilon_1, D, \epsilon)\) rates defined above. Here note that \( \delta \in [0, 1] \), and \( \delta = 1 \) corresponds to perfect secrecy.

\[ \text{III. A FBL APPROACH ACHIEVING } R(N, \epsilon_1, D, \epsilon) \text{ IN } \text{THEOREM[1]} \]

\[ \text{A. Channel splitting} \]

At time \( i \in \{1, 2, \ldots, N\} \), since the elements in \( \{2.9\} \) are complex numbers, \( \{2.9\} \) can be re-written as

\[ Y_{R,i} + jY_{I,i} = (h_{R,i} + jh_{I,i})(X_{R,i} + jX_{I,i}) + \eta_{R,i} + j\eta_{I,i}, \]

\[ \bar{Y}_{R,i} + j\bar{Y}_{I,i} = (h_{R,i} + jh_{I,i})(\bar{X}_{R,i} + j\bar{X}_{I,i}) + \eta_{R,i} + j\eta_{I,i}, \quad (3.1) \]

where \( j = \sqrt{-1} \), \( Y_{R,i} = \text{Re}(Y_{i}), \quad Y_{I,i} = \text{Im}(Y_{i}), \quad h_{R,i} = \text{Re}(h_{i}), \quad h_{I,i} = \text{Im}(h_{i}), \quad X_{R,i} = \text{Re}(X_{i}), \quad X_{I,i} = \text{Im}(X_{i}), \quad \eta_{R,i} = \text{Re}(\eta_{i}), \quad \eta_{I,i} = \text{Im}(\eta_{i}), \quad \bar{Y}_{R,i} = \text{Re}(\bar{Y}_{i}), \quad \bar{Y}_{I,i} = \text{Im}(\bar{Y}_{i}), \quad \bar{X}_{R,i} = \text{Re}(\bar{X}_{i}), \quad \bar{X}_{I,i} = \text{Im}(\bar{X}_{i}), \quad \eta_{R,i} = \text{Re}(\eta_{i}), \quad \eta_{I,i} = \text{Im}(\eta_{i}). \)

Here note that \( X_{R,i}, X_{I,i}, \bar{X}_{R,i}, \bar{X}_{I,i} \) and \( \eta_{R,i}, \eta_{I,i}, \bar{Y}_{R,i}, \bar{Y}_{I,i} \) are the respective subject to the power constraints \( E(X_{R,i}^2) = P_R \), \( E(X_{I,i}^2) = P_I \), \( E(\bar{X}_{R,i}^2) = \bar{P}_R \) and \( E(\bar{X}_{I,i}^2) = \bar{P}_I \), due to \( \eta_{R,i} \sim CN(0, \sigma_{\eta}^2) \) and \( \eta_{I,i} \sim CN(0, \sigma_{\eta}^2) \), \( \eta_{R,i}, \eta_{I,i} \sim N(0, \frac{1}{2} \sigma_{\eta}^2) \). In addition, \( \eta_{R,i}, \eta_{I,i}, \bar{Y}_{R,i}, \bar{Y}_{I,i} \) are independent of each other.

Moreover, \( \{3.1\} \) can be further re-written as

\[ Y_{R,i} = h_{R,i} X_{R,i} - h_{I,i} X_{I,i} + \eta_{R,i}, \quad Y_{I,i} = h_{R,i} X_{I,i} + h_{I,i} X_{R,i} + \eta_{I,i}, \quad \bar{Y}_{R,i} = h_{R,i} \bar{X}_{R,i} - h_{I,i} \bar{X}_{I,i} + \eta_{R,i}, \quad \bar{Y}_{I,i} = h_{R,i} \bar{X}_{I,i} + h_{I,i} \bar{X}_{R,i} + \eta_{I,i}, \quad (3.2) \]

From \( \{3.2\} \), we have

\[ X_{R,i} = Y'_{R,i} - \eta'_{R,i}, \quad X_{I,i} = Y'_{I,i} - \eta'_{I,i}, \quad \bar{X}_{R,i} = \bar{Y}'_{R,i} - \eta'_{R,i}, \quad \bar{X}_{I,i} = \bar{Y}'_{I,i} - \eta'_{I,i}, \quad (3.3) \]

where

\[ Y'_{R,i} = h_{R,i} Y_{R,i} + h_{I,i} Y_{I,i}, \quad Y'_{I,i} = \frac{h_{R,i}^2}{|h_{I,i}|^2} Y_{R,i} - h_{I,i} Y_{R,i}, \quad \bar{Y}'_{R,i} = h_{R,i} \bar{Y}_{R,i} + h_{I,i} \bar{Y}_{I,i}, \quad \bar{Y}'_{I,i} = \frac{h_{R,i}^2}{|h_{I,i}|^2} \bar{Y}_{R,i} - h_{I,i} \bar{Y}_{R,i}, \quad \eta'_{R,i} = \frac{h_{R,i} \eta_{R,i} + h_{I,i} \eta_{I,i}}{|h_{I,i}|^2}, \quad \eta'_{I,i} = \frac{h_{R,i} \eta_{I,i} + h_{I,i} \eta_{R,i}}{|h_{I,i}|^2}, \quad \eta'_{R,i} = \frac{h_{R,i} \eta_{R,i} + h_{I,i} \eta_{I,i}}{|h_{I,i}|^2}, \quad \eta'_{I,i} = \frac{h_{R,i} \eta_{I,i} + h_{I,i} \eta_{R,i}}{|h_{I,i}|^2}. \quad (3.4) \]

Hence, \( \{3.3\} \) is equivalent to \( \{3.1\} \), which indicates that the feedforward and feedback channels are divided into the two sub-channels. In addition, from \( \{3.2\} \), we conclude that \( \text{Var}(\eta_{R,i}) = \text{Var}(\eta_{I,i}) = \frac{\sigma_{\eta}^2}{|h_{I,i}|^2} \) and \( \text{Var}(\eta_{R,i}) = \text{Var}(\eta_{I,i}) = \frac{\sigma_{\eta}^2}{|h_{I,i}|^2} \), which show that the distributions of the noises in the two sub-channels of the feedforward/feedback channel are the same. Hence, the maximum sum rates of messages in the two sub-channels of feedforward channel and the two sub-channels of feedback channel are achieved, if

\[ P_R = P_I = \frac{1}{2} P, \quad \bar{P}_R = \bar{P}_I = \frac{1}{2} \bar{P}. \quad (3.5) \]

From \( \{3.5\} \), we conclude that \( E[X_i X_i^H] = E[(X_{R,i})^2] + E[(X_{I,i})^2] = P \) and \( E[X_i X_i^H] = E[(\bar{X}_{R,i})^2] + E[(\bar{X}_{I,i})^2] = \bar{P} \), which indicates that the power constraints of \( X_i \) and \( \bar{X}_i \) are satisfied.
B. Coding scheme

First, the message $W''$ is divided into two independent parts $(W''_1, W''_2)$, where $W''_r (r = 1, 2)$ takes values in $W''_r = \{1, 2, ..., 2^{NR_r(N, \epsilon_1, D, \epsilon)}\}$. Divide the interval $[-\sqrt{3}, \sqrt{3}]$ into $2^{NR_r(N, \epsilon_1, D, \epsilon)}$ equally spaced sub-intervals, and the center of each sub-interval is mapped to a message value in $W''_r$. Let $\theta_r$ be the center of the sub-interval with respect to (w.r.t) the message $W''_r$ (the variance of $\theta_r$ approximately equals 1, i.e., $E(\theta^2_r) = 1$).

For simplification, we only describe the coding scheme of message $W''_1$, and the coding scheme of message $W''_2$ is similar to the coding scheme of message $W''_1$.

Initialization: At time instant 1, the channel encoder maps the messages $W''_1$ to $\theta_1$, and sends

$$X_{R,1} = \sqrt{P_R} \theta_1,$$

(3.6)

at the end of time 1, the channel decoder of cloud server receives $\hat{Y}_{R,1}$, and it knows the perfect CSI $h_1$. Using (3.2)-(3.4), the decoder obtains $\hat{Y}'_{R,1}$ and computes the first estimation $\hat{\theta}_{1,1}$ of $\theta_1$ by

$$\hat{\theta}_{1,1} = \frac{\hat{Y}'_{R,1}}{\sqrt{P_R}} = \theta_1 + \frac{\eta_{R,1,1}}{\sqrt{P_R}} = \theta_1 + \epsilon_{R,1},$$

(3.7)

where $\epsilon_{R,1} = \hat{\theta}_{1,1} - \theta_1 = \frac{\eta_{R,1,1}}{\sqrt{P_R}}$ is the decoding error of decoder at time instant 1. Define $\alpha_{R,1} = Var(\epsilon_{R,1}) = \frac{\sigma_{\epsilon_{R,1}}^2}{2P_R}$.

Iteration: From the second time instant, we first introduce the dither signal sequences $V_N = (V_{R,1}, ..., V_{R,N})$ and $V_I = (V_{I,1}, ..., V_{I,N})$, which follows from the extended SK-type feedback scheme in (28). We assume that the two i.i.d generated sequences $V_N$ and $V_I$ are perfectly known by all parties, where $V_{R,i} \sim \text{Unif}[-\frac{d_R}{2}, \frac{d_R}{2}]$, $V_{I,i} \sim \text{Unif}[-\frac{d_I}{2}, \frac{d_I}{2}]$, $d_R = \sqrt{12P_R}$ and $d_I = \sqrt{12P_I}$. The dither signals ensure that the encoded codeword of cloud server satisfies the power constraint.

At time instant $i (2 \leq i \leq N)$, the channel decoder of cloud server computes and sends

$$\tilde{X}_{R,i-1} = M_{d_R}[\gamma_{R,i-1} \hat{\theta}_{i-1} + V_{R,i-1}],$$

(3.8)

where $\gamma_{R,i-1}$ is the modulation coefficient of cloud server, $M_{d_R}$ is the modulo-$d_R$ function and it is defined in (28). From property 1 of proposition 1 in (28), we have $E(\tilde{X}^2_{R,i-1}) = P_R$. The channel encoder receives $Y_{i-1}$ and knows the perfect CSI $h_2$. According to (3.2)-(3.4), the channel encoder obtains $\tilde{Y}_{R,i-1}$ and computes the noisy version of decoding error $\tilde{\epsilon}_{R,i-1} = \hat{\theta}_{i-1} - \theta_1$ by

$$\tilde{\epsilon}_{R,i-1} = \frac{1}{\gamma_{R,i-1}} M_{d_R}[\tilde{Y}_{R,i-1} - \gamma_{R,i-1} \theta_1 - V_{R,i-1}].$$

(3.9)

(3.9)

$$\tilde{\epsilon}_{R,i-1} = \frac{1}{\gamma_{R,i-1}} M_{d_R}[\gamma_{R,i-1} \tilde{\epsilon}_{R,i-1} + \eta_{R,2,i-1}],$$

(3.9)

where $g$ is due to property ii of proposition 1 in (28). The modulo-aliasing errors do not occur in channel encoder, if $\gamma_{R,i-1}$ is chosen such that $\gamma_{R,i-1} \theta_1 + \eta_{R,2,i-1} \in [-\frac{d_R}{2}, \frac{d_R}{2})$. Hence, the channel encoder obtains $\tilde{\epsilon}_{R,i-1} = \epsilon_{R,i-1} + \eta_{R,2,i-1}$.

Then, the channel encoder sends

$$X_{R,i} = \lambda_{R,i-1} \gamma_{R,i-1} \tilde{\epsilon}_{R,i-1},$$

(3.10)

where $\lambda_{R,i-1}$ is chosen to satisfy the input power constraint $P_R = \lambda_{R,i}$. Analogously, the channel decoder receives $Y_i$ and computes $Y_{R,i}$. Then, the channel decoder updates $\theta_{1,i}$ by computing

$$\hat{\theta}_{1,i} = \hat{\theta}_{1,i-1} - \tilde{\epsilon}_{R,i-1} = \hat{\theta}_{1,i-1} - \beta_{R,i} Y_{R,i},$$

(3.11)

where $\beta_{R,i} = \frac{E(Y_{R,i} \epsilon_{R,i-1})}{E(Y_{R,i})^2} = \frac{E((Y_{R,i} + \eta_{R,i,1}) \epsilon_{R,i-1})}{E(Y_{R,i} + \eta_{R,i,1})^2}$.

(3.12)

is the Minimum Mean Square Error (MMSE) estimation coefficient, which ensures that $\epsilon_{R,i-1}$ is correctly estimated from $Y_{R,i}$. Define $\epsilon_{R,i} = \tilde{\theta}_{1,i} - \hat{\theta}_{1,i}$, (3.11) yield that

$$\epsilon_{R,i} = \epsilon_{R,i-1} - \beta_{R,i} Y_{R,i},$$

(3.13)

and define $\alpha_{R,i} = Var(\epsilon_{R,i})$.

Decoding: At time instant $N$, the channel decoder receives $Y_N$, computes $Y_{R,N}$ and obtains the final estimation $\hat{\theta}_{1,N} = \theta_1 + \epsilon_{R,N}$. Then the channel decoder declares the center of sub-interval which $\hat{\theta}_{1,N}$ belongs to as the final estimation of the $\theta_1$. The channel decoder successfully decodes the message $W''$ if $\hat{\theta}_{1,N}$ is in the sub-interval of $\theta_1$, i.e., $\epsilon_{R,N} \in [-\frac{d_R}{2N_{r_1}(N, \epsilon_1, D, \epsilon)}, \frac{d_R}{2N_{r_1}(N, \epsilon_1, D, \epsilon)}]$.

The following algorithm further explains the encoding-decoding scheme described above.

Algorithm 1 Encoding-decoding procedure

**Input:** $W''_1, \epsilon_1, N$  
**Output:** $X_{R,N}$

**Initialization:** 
Map $W''_1 \rightarrow \theta_1$, 
Encode $X_{R,1} = \sqrt{P_R} \theta_1$. 
Compute $\theta_{1,1}$ from (3.7).

**Iteration:**

1: for $2 \leq i \leq N$ do
2: Compute $\lambda_{R,i-1}$, $\gamma_{R,i-1}$, $\beta_{R,i}$ from (5.4)-(5.9).
3: Encode $\tilde{X}_{R,i-1} = M_{d_R}[\gamma_{R,i-1} \hat{\theta}_{i-1} + V_{R,i-1}].$
4: Compute $\tilde{\epsilon}_{R,i-1}$ from (3.9).
5: Encode $X_{R,i} = \lambda_{R,i-1} \gamma_{R,i-1} \tilde{\epsilon}_{R,i-1}$.
6: Compute $\hat{\theta}_{1,i}$ from (3.11).

end for

IV. NUMERICAL AND SIMULATION RESULTS

A. Numerical results

The following Figure 2 plots the trade-off between privacy $\epsilon$ and utility $D$ for $S^T = 10^3$, $K = 10$ and $\sigma^2_w$ taking several values. In Figure 1, for a fixed error probability $\epsilon_1$, the secrecy
rate $R(N, \epsilon_1, \delta, D, \epsilon)$ asymptotically equals $R^*(N, \epsilon_1, D, \epsilon)$ as $N$ is increasing, and there still exists a gap between the Shannon limit and $R^*(N, \epsilon_1, D, \epsilon)$. Figure 5 shows the relationship between error probability of cloud server, secrecy level and self-secure blocklength threshold. From Figure 5 we conclude that when the error probability is decreasing and the secrecy level is increasing, the self-secure blocklength threshold is increasing. Moreover, when $\epsilon_1 = 10^{-7}$ and $\delta = 0.99$, the self-secure blocklength threshold is 17, which is extremely short compared with existing LDPC/Polar codes.

![Figure 5: The relationship between error probability $\epsilon_1$ of cloud server, secrecy level $\delta$ and self-secure blocklength threshold for $K = 10$, $\sigma^2_w = 10^{-4}$, $\sigma^2 = 1$, $S^\ell = 10^4$, $q = 200$, $D = 10$, $\epsilon = 1$, $P = 10$, $P_i = 10^3$, $\sigma^2_i = \sigma^2_2 = \sigma^2_3 = 1$, $h_1 = \frac{1}{2} + j\frac{1}{2}$, $h_2 = \frac{5}{6} + j\frac{5}{6}$ and $h_e = \frac{1}{8} + j\frac{1}{8}$](image)

Figure 5: The relationship between error probability $\epsilon_1$ of cloud server, secrecy level $\delta$ and self-secure blocklength threshold for $K = 10$, $\sigma^2_w = 10^{-4}$, $\sigma^2 = 1$, $S^\ell = 10^4$, $q = 200$, $D = 10$, $\epsilon = 1$, $P = 10$, $P_i = 10^3$, $\sigma^2_i = \sigma^2_2 = \sigma^2_3 = 1$, $h_1 = \frac{1}{2} + j\frac{1}{2}$, $h_2 = \frac{5}{6} + j\frac{5}{6}$ and $h_e = \frac{1}{8} + j\frac{1}{8}$.

In Figure 6 it has been shown that for a fixed secrecy level, when the feedback power is increasing, the self-secure blocklength threshold is decreasing. Figure 7 plots the relationship between secrecy level $\delta$, utility $D$ and privacy $\epsilon$. From Figure 7 we conclude that the secrecy level $\delta$ is increasing while the utility $D$ is increasing and the privacy $\epsilon$ is decreasing.

![Figure 6: The relationship between secrecy level $\delta$, self-secure blocklength threshold and SNR for $K = 10$, $\sigma^2_w = 10^{-4}$, $\sigma^2 = 1$, $S^\ell = 10^4$, $q = 200$, $D = 10$, $\epsilon = 1$, $P = 10$, $P_i = 10^3$, $\sigma^2_i = \sigma^2_2 = \sigma^2_3 = 1$, $h_1 = \frac{1}{2} + j\frac{1}{2}$, $h_2 = \frac{5}{6} + j\frac{5}{6}$ and $h_e = \frac{1}{8} + j\frac{1}{8}$](image)

Figure 6: The relationship between secrecy level $\delta$, self-secure blocklength threshold and SNR for $K = 10$, $\sigma^2_w = 10^{-4}$, $\sigma^2 = 1$, $S^\ell = 10^4$, $q = 200$, $D = 10$, $\epsilon = 1$, $P = 10$, $P_i = 10^3$, $\sigma^2_i = \sigma^2_2 = \sigma^2_3 = 1$, $h_1 = \frac{1}{2} + j\frac{1}{2}$, $h_2 = \frac{5}{6} + j\frac{5}{6}$ and $h_e = \frac{1}{8} + j\frac{1}{8}$.

In Figure 4 it was shown that for a fixed secrecy level, when the feedback power is increasing, the self-secure blocklength threshold is decreasing. Figure 7 plots the relationship between secrecy level $\delta$, utility $D$ and privacy $\epsilon$. From Figure 7 we conclude that the secrecy level $\delta$ is increasing while the utility $D$ is increasing and the privacy $\epsilon$ is decreasing.

**B. Simulation results**

The following Figure 8 shows the performance of our proposed FBL scheme in the wireless HFL. Specifically, let the number of users be 10 ($K = 10$), $\sigma^2_w = 10^{-4}$, $\sigma^2 = 10^{-2}$, $\sigma^2_1 = \sigma^2_2 = \sigma^2_3 = 1$, $S^\ell = 10^3$, $q = 10$, $D = 1$ and $\epsilon = 1$, the signal-to-noise ratios of the cloud server’s channel be 10dB, the signal-to-noise ratios of the feedback channel taking some values, and the length of transmitted information bits is determined by $q_h(W^\prime(i)) (i \in \{1, 2, ..., q\})$. Moreover, let the channel gains $h_1 = \frac{1}{2} + j\frac{1}{2}$, $h_2 = \frac{5}{6} + j\frac{5}{6}$ and $h_e = \frac{1}{8} + j\frac{1}{8}$.

From Figure 8 we see that the error probability of the cloud server decreases with the increasing of $N$, and the error probability of the cloud server is $P_e = 10^{-5}$ at blocklength $N = 11$ and $SNR = 20$dB. Besides, Figure 8 shows the error probability of the eavesdropper equals 0.5, which indicates no information about the corrupted gradients is leaked to the eavesdropper, and the PLS is guaranteed by using our proposed coded transmission.

![Figure 8: The performance of the proposed FBL scheme](image)

Figure 8: The performance of our proposed FBL scheme in the wireless HFL.
which can be re-written as

$$\sigma^2 \geq \frac{S^2 \sigma_w^2}{K(2^m - 1)}.$$  \hfill (5.2)

Combining (5.2) and (5.3), (2.21) in Theorem 1 is proved.

B. Performance analysis

In our proposed scheme, we define the parameters $\lambda_{R,i}, \lambda_{I,i}, \beta_{R,i}, \beta_{I,i}, \gamma_{R,i}$ and $\gamma_{I,i}$ are the parameters for transmitting message $W_{q_i}^A$ as follows, and the transmission performance of our scheme is determined by these parameters.

$$\lambda_{R,i} = \lambda_{I,i} = \sqrt{L \cdot \frac{P}{P}}, \quad (5.4)$$

$$\gamma_{R,i} = \frac{1}{\alpha_{R,i}} \left( \frac{\tilde{P}}{2L} - \frac{\sigma_2^2}{2|h_2|^2} \right), \quad (5.5)$$

$$\gamma_{I,i} = \frac{1}{\alpha_{I,i}} \left( \frac{\tilde{P}}{2L} - \frac{\sigma_2^2}{2|h_2|^2} \right), \quad (5.6)$$

$$\beta_{R,i} = \sqrt{2\lambda_{R,i-1}} \frac{\sqrt{\text{SNR}(1 - L \cdot \text{SNR}^{-1})|h_2|^2}}{\sqrt{\text{SNR} + |h_1|^{-2}}}, \quad (5.7)$$

$$\beta_{I,i} = \sqrt{2\lambda_{I,i-1}} \frac{\sqrt{\text{SNR}(1 - L \cdot \text{SNR}^{-1})|h_2|^2}}{\sqrt{\text{SNR} + |h_1|^{-2}}}, \quad (5.8)$$

where

$$\alpha_{R,i} = \alpha_{I,i} = |h_1|^{-2} \text{SNR}^{-1} \left( 1 + \frac{\text{SNR}|h_1|^2}{\Psi_1 \Psi_2} \right)^{1-i}, \quad (5.9)$$

and $\Psi_1, \Psi_2$ and $L$ are defined in (2.18)-(2.20). Combining the above definitions of parameters, and through the error probability analysis, the achievable rate of this FBL scheme is given by

$$R(N, \epsilon, D, \epsilon) = \frac{1}{N} \log \left( \frac{3|\text{SNR}|h_1|^2}{|Q^{-1}(\frac{1}{\epsilon})|} \left( 1 + \frac{\text{SNR}|h_1|^2}{\Psi_1 \Psi_2} \right)^{N-1} \right). \quad (5.10)$$

The parameters derivation and error probability analysis of the FBL scheme are similar to those in [28], which are shown in Appendix A.

C. Equivocation analysis

The general term for the transmitted codewords is shown in Lemma 1 which will be applied in the equivocation analysis.

**Lemma 1.** From above coding procedure, the transmitted codewords can be expressed as

$$X_i = f_i(h_1, h_2, \eta_{i,1}, ..., \eta_{i,-1}, \eta_{2,1}, ..., \eta_{2,i-1}). \quad (5.11)$$

**Proof:** See Appendix B.

Then, we will perform a security analysis of our FBL scheme within the FBL regime. Since $V^N_R$ and $V^N_I$ are known by all
Δ = H(W^n|Z^N, h_1, h_2, h_e, V_R^N, V_I^N) / h(W^n)

(1) H(W''|Z^N, h_1, h_2, h_e, V_R^N, V_I^N)

= H(W''_1, W''_2|Z^N, h_1, h_2, h_e, V_R^N, V_I^N)

(2) H(θ_1, θ_2|Z^N, h_1, h_2, h_e, V_R^N, V_I^N)

= 1 / H(θ_1, θ_2)

≥ 1 / H(θ_1, θ_2)

= H(θ_1, θ_2) + h(h(e)) + h(η_1, e) - h(h_e(√PRθ_1 + j√PT θ_2) + η_1, e) + 1 / H(θ_1, θ_2)

≥ 1 / H(θ_1, θ_2)

= H(θ_1, θ_2) + h(h(e)) + h(η_1, e) - h(h_e(√PRθ_1 + j√PT θ_2) + η_1, e) + 1 / H(θ_1, θ_2)

is guaranteed when blocklength N satisfies

NR(N, e_1, D, ε) ≥ 1 / (1 - δ) log \left(1 + |h_e|^2P / σ_e^2\right). (5.15)

Finally, combining the 5.2, 5.3, 5.10 and 5.15, we complete the proof of Theorem 1.

VI. CONCLUSION AND FUTURE WORK

This paper establishes a framework for the wireless HFL in the presence of PLS, and proposes a practical FBL coding scheme, which is self-secure when the coding blocklength is larger than a certain threshold. Possible future work includes:

- Investigating the impact of multi-antennas on the wireless HFL in the presence of PLS.
- Studying the impact of interference among edge servers on the wireless HFL in the presence of PLS.

APPENDIX A

PARAMETERS DERIVATION AND ERROR PROBABILITY ANALYSIS

We bound the decoding error probability \( P_e \) of W'' as

\[ P_e \leq P_e^{d1} + P_e^{d2}, \]

where \( P_e^{d1} \) and \( P_e^{d2} \) are the decoding error probabilities of \( W''_1 \) and \( W''_2 \), respectively. We first analyze the error event of message \( W''_1 \) transmitted as the real part of the complex channel codeword. There are two kinds of decoding errors for \( W''_1 \), consisting of:

1. At time instant \( i + 1 (1 \leq i \leq N - 1) \), a demodulation error occurs in the channel encoder is defined as

\[ E_{R,i} = \left\{ ε_R,i \in R : \phi \notin \frac{d_R - \sqrt{3}}{2}, \frac{d_R + \sqrt{3}}{2} \right\}. \] (A1)

2. At time instant \( N \), a decoding error occurs in the channel decoder is defined as

\[ E_{R,N} = \{ ε_R,N \notin \frac{\sqrt{3}}{2N R_i(N,e_1,D,ε)}, \sqrt{3} \}. \] (A2)

Thus error probability is bounded by

\[ P_e^{d1} \leq \sum_{i=1}^{N} Pr \left( \bigcup_{i=1}^{N} E_{R,i} \right) = Pr \left( \bigcup_{i=1}^{N} E_{R,i} \right) + Pr \left( \bigcap_{i=1}^{N} E_{R,i} \cap E_{R,N} \right) \]

\[ = \sum_{i=1}^{N-1} Pr \left( \bigcap_{i=1}^{N-1} E_{R,i} \cap E_{R,N} \right) + Pr \left( \bigcap_{i=1}^{N-1} E_{R,i} \cap E_{R,N} \right) \]

\[ = \sum_{i=1}^{N-1} Pr \left( \tilde{E}_{R,i} \right) + Pr \left( \tilde{E}_{R,N} \right), \] (A3)

where \( E_{R,i}^c \) denotes the complement of the set \( E_{R,i} \), \( \tilde{E}_{R,i} = \bigcap_{j=1}^{i-1} E_{R,j} \cap E_{R,N} \), \( Pr \left( \tilde{E}_{R,i} \right) \) is the error probability that a demodulation error occurs at time instant \( i + 1 \) and no error occurring in all previous times. \( Pr \left( \tilde{E}_{R,N} \right) \) is the error probability of the final decoding and no demodulation error occurring in all times.

Since \( P_e \leq ε_1 \), we choose \( P_e^{d1} \leq ε_2 \) and let \( Pr \left( \tilde{E}_{R,N} \right) = \sum_{i=1}^{N-1} Pr \left( \tilde{E}_{R,i} \right) = \frac{ε_1}{4} \). (A4)

for simplification, define

\[ Pr \left( \tilde{E}_{R,i} \right) = ... = Pr \left( \tilde{E}_{R,N-1} \right) = p_m, \] (A5)
substituting (A5) into (A4), we have
\[ p_m = \frac{\epsilon_1}{4(N-1)}. \] (A6)

For event $\tilde{E}_{R,i}$, since no demodulation error occurs before time instant $i + 1$, according to (3.13) and the fact that $\eta_{R,2,i}$ is Gaussian distributed, we can conclude that $\gamma_{R,i} \varepsilon_{R,i} + \eta_{R,2,i} \sim \mathcal{N}(0, \frac{\sigma^2_{R,i}}{2|h_2|^2})$. From (A1) and $d_R = \sqrt{12P_R}$, we have
\[ p_m = 2Q\left(\frac{3P_R}{E(\gamma_{R,i} \varepsilon_{R,i} + \eta_{R,2,i})^2}\right), \] (A7)
to simplify the calculation, let
\[ L = \frac{1}{3} \left[Q^{-1}(\frac{P_m}{2})^2\right] = \frac{1}{3} \left[Q^{-1}(\frac{\epsilon_1}{8(N-1)})\right]^2, \] (A8)
substituting (A7) into (A8), we have
\[ E(\gamma_{R,i} \varepsilon_{R,i} + \eta_{R,2,i})^2 = \frac{\sigma^2_{R,i}}{2|h_2|^2} = \frac{\tilde{P}}{2L}. \] (A9)
From (A9), we can conclude that
\[ \gamma_{R,i} = \left[\frac{1}{\alpha_{R,i}} \left(\frac{\tilde{P}}{2L} - \frac{\sigma^2_{R,i}}{2|h_2|^2}\right)\right], \] (A10)
and due to the $X_{R,i+1}$ subject to the power constraint $P_R$, we have
\[ E[(X_{R,i+1})^2] = \lambda^2_{R,i}, E(\gamma_{R,i} \varepsilon_{R,i} + \eta_{R,2,i})^2 = \frac{P}{2}. \] (A11)
According to (A9) and (A11), we conclude that
\[ \lambda_{R,i} = \sqrt{L \cdot \frac{P}{\tilde{P}}}. \] (A12)

From (3.12), (A10), (A12) and $Y_{R,i+1} = X_{R,i+1} + \eta_{R,1,i+1} = \lambda_{R,i}(\gamma_{R,i} \varepsilon_{R,i} + \eta_{R,2,i}) + \eta_{R,1,i+1}$, (A13)
we have
\[ \beta_{R,i+1} = \frac{E(Y_{R,i+1})^2}{E(Y_{R,i+1})^2} = \lambda_{R,i} \gamma_{R,i} \alpha_{R,i} + \frac{\sigma^2_{R,i}}{2|h_2|^2} = \frac{\beta_{R,i}}{2L}, \] (A14)
According to (3.13), (A10), (A12), (A13) and (A14), we have
\[ \alpha_{R,i+1} = E(\varepsilon_{R,i+1})^2 = E(\varepsilon_{R,i})^2 - \frac{E(Y_{R,i+1} \varepsilon_{R,i})}{E(Y_{R,i+1})^2} = \alpha_{R,i} \left(1 + \frac{SNR|h_1|^2}{\frac{1 + L \cdot SNR \cdot |\varepsilon_{R,i}|^2}{1 + L \cdot SNR \cdot |h_1|^2}}\right)^{-1} = |h_1|^2 SNR^{-1} \left(1 + \frac{SNR|h_1|^2}{\Psi_{R,i}^2}\right)^{-i}, \] (A15)
From (A2), we have
\[ Pr(\tilde{E}_{R,N}) = Pr(\varepsilon_{R,N} \notin \left[-\sqrt{\frac{\sqrt{2}}{2N R_R(N,k_1,D,s)}, \sqrt{\frac{\sqrt{2}}{2N R_R(N,k_1,D,s)}}}\right], \] (A16)
According to (A4) and (A16), (A3) can be re-written as
\[ P_{e,1} = \frac{e_1}{4} + 2Q\left(\frac{\sqrt{3}}{2N R_R(N,k_1,D,s)} \cdot \frac{1}{\sqrt{\alpha_{R,N}}}. \right) \] (A17)
From (A17), $P_{e,1} \leq \frac{e_1}{2}$ is guaranteed by
\[ Q\left(\frac{\sqrt{3}}{2N R_R(N,k_1,D,s)} \cdot \frac{1}{\sqrt{\alpha_{R,N}}}. \right) = \frac{e_1}{8}. \] (A18)
Substituting (A15) into (A18), we have
\[ R_1(N, \epsilon_1, D, \epsilon) = \frac{1}{2N} \log \left(\frac{3SNR|h_1|^2}{Q^{-1}(\frac{\sqrt{2}}{2})^2} \left(1 + \frac{SNR|h_1|^2}{\Psi_1^2}\right)^{-1}\right). \] (A19)
Analogously, we have
\[ R_2(N, \epsilon_1, D, \epsilon) = R_1(N, \epsilon_1, D, \epsilon). \] (A20)
According to (A19) and (A20), we have
\[ R(N, \epsilon_1, D, \epsilon) = R_1(N, \epsilon_1, D, \epsilon) = \frac{1}{2N} \log \left(\frac{3SNR|h_1|^2}{Q^{-1}(\frac{\sqrt{2}}{2})^2} \left(1 + \frac{SNR|h_1|^2}{\Psi_1^2}\right)^{-1}\right). \] (A21)
From the above analysis, according to (2.11), for a sufficiently large $N (q = \frac{SNR(K,F,D)}{h(W,q)})$ and $i \in \{1, 2, ..., q\}$, we have
\[ P_e = \sum_{i=1}^{q} Pr(\tilde{W}^n(i) \neq W^q(i)) \leq \epsilon_1, \] (A22)
and the parameters derivation and error probability analysis are completed.

**APPENDIX B**

**PROOF OF LEMMA II**

From Subsection II-D we conclude that
\[ X_{R,i} = \sqrt{P_R} \theta_i, \] (B1)
we have
\[ X_{R,i} = \lambda_{R,i} \gamma_{R,i} \varepsilon_{R,i} + \eta_{R,2,i} + \eta_{R,1,i+1}, \] (B2)
According to (B1) and the fact that $\lambda_{R,i-1}, \lambda_{R,i+1}, \beta_{R,i}, \gamma_{R,i-1}$ and $\gamma_{R,i+1}$ (2 \leq i \leq N) are independent of the transmitted messages (see (5.4)-(5.8), we conclude that for 2 \leq i \leq N, $X_i = X_{R,i} + \eta X_{R,i}$ is a function of ($h_1, h_2, \eta_{R,1,i}, \eta_{R,1,i+1}, \eta_{R,2,i}, \eta_{R,2,i+1}$), (B3)
From (3.4), it is not difficult to see that $X_i$ is a function of ($h_1, h_2, \eta_{R,1,i}, \eta_{R,1,i+1}, \eta_{R,2,i}, \eta_{R,2,i+1}$), for 2 \leq i \leq N. For convenience, we define
\[ X_i = f_i(h_1, h_2, \eta_{R,1,i}, \eta_{R,1,i+1}, \eta_{R,2,i}, \eta_{R,2,i+1}). \] (B2)
which completes the proof of Lemma II.
REFERENCES

[1] G. Zhu, D. Liu, Y. Du, C. You, J. Zhang, and K. Huang, “Toward an Intelligent Edge: Wireless Communication Meets Machine Learning,” IEEE Communications Magazine, vol. 58, no. 1, pp. 19-25, 2020.

[2] H. H. Yang, A. Arafa, T. Q. S. Quek and H. Vincent Poor, “Age-Based Scheduling Policy for Federated Learning in Mobile Edge Networks,” ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 8743-8747, 2020.

[3] S. Yang and Y. Liu, “Training Efficiency of Federated Learning: A Wireless Communication Perspective,” 2020 International Conference on Wireless Communications and Signal Processing (WCSP), pp. 922-926, 2020.

[4] A. Elgabli, J. Park, C. B. Issaid and M. Bennis, “Harnessing Wireless Channels for Scalable and Privacy-Preserving Federated Learning,” IEEE Transactions on Communications, vol. 69, no. 8, pp. 5194-5208, 2021.

[5] R. F. Schaefer, A. Khisti and H. V. Poor, “Secure Broadcasting Using Wiretap Channel With Secure Rate-Limited Feedback,” IEEE Transactions on Information Theory, vol. 55, no. 12, pp. 5353-5361, 2009.

[6] W. Wang, L. Ying and J. Zhang, “When Wireless Federated Learning Meets Physical Layer Security: The Fundamental Limits,” IEEE INFOCOM 2020 - IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS), pp. 653-658, 2020.

[7] E. Ardestanizadeh, M. Frateschetti, T. Javidi and Y. Kim, “Wiretap Channel With Secure Rate-Limited Feedback,” IEEE Transactions on Information Theory, vol. 55, no. 12, pp. 5353-5361, 2009.

[8] A. Ben-Yishai and O. Shayevitz, “Interactive Schemes for the AWGN Tap Channel,” IEEE Transactions on Information Theory, vol. 63, no. 4, pp. 2409-2427, 2017.

[9] A. Wainakh, A. S. Guinea, T. Grube and M. M. Oberhuber, “Enhancing Privacy via Hierarchical Federated Learning,” 2021 IEEE Global Communications Conference (GLOBECOM), Madrid, Spain, pp. 1-7, 2021.

[10] M. S. H. Abad, E. Ozfatura, D. GUndUz and O. Ercetin, “Hierarchical Federated Learning ACROSS Heterogeneous Cellular Networks,” ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Barcelona, Spain, pp. 8866-8870, 2020.

[11] R. Ahlswede and N. Cai, “Transmission, identification and common randomness capacities for wire-tap channels with secure feedback from the decoder,” book chapter in General Theory of Information Transfer and Combinatorics, LNCS 4123, pp. 258-275, Berlin: Springer-Verlag, 2006.

[12] A. Mukherjee, S. A. A. Fakoorian, J. Huang and A. L. Swindlehurst, “Principles of Physical Layer Security in Multiuser Wireless Networks: A Survey,” IEEE Communications Surveys & Tutorials, vol. 16, no. 3, pp. 1550-1573, Third Quarter 2014.