Non linear conductivity and collective charge excitations in the lowest Landau level

Assa Auerbach\textsuperscript{1} and Daniel P. Arovas\textsuperscript{2}

\textsuperscript{1}Physics Department, Technion, 32000 Haifa, Israel

\textsuperscript{2}Department of Physics, University of California at San Diego, La Jolla, California 92093, USA

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For weakly disordered fractional quantum Hall phases, the non linear photoconductivity is related to the charge susceptibility of the clean system by a Floquet boost. Thus, it may be possible to probe collective charge modes at finite wavevectors by electrical transport. Incompressible phases, irradiated at slightly above the magneto-roton gap, are predicted to exhibit negative photoconductivity and zero resistance states with spontaneous internal electric fields. Non linear conductivity can probe composite fermions’ charge excitations in compressible filling factors.

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Fractional quantum Hall (FQH) phases exhibit exotic ground states and many-body collective modes. For example, the Magneto-Roton (MR) mode in Laughlin states was predicted by Girvin, MacDonald and Platzman (GMP\textsuperscript{[1]}) to have a minimal gap $\Delta_0$ at wavevector $k_0$. (See Fig[1]). This mode was seen by Raman scattering,\textsuperscript{[2]} resistivity activation gap with phonon pulses\textsuperscript{[3]} and microwave absorption with surface acoustic wave.\textsuperscript{[4]}

Some of these probes are based on the powerful relation between conductivity $\sigma_{xx}(\omega)$ and charge excitations in weakly disordered quantum Hall gases.\textsuperscript{[5,6]}

$$\sigma_{xx}(\Omega) = \frac{n e^2}{\hbar B^2} A \sum_k \Gamma_k^2 \kappa_y^2 \frac{\chi_0''(k,\Omega)}{\Omega} + O(V^4). \quad (1)$$

$V(r) = A^{-1/2} \sum_k V_k e^{i k \cdot r}$ is the random disorder potential taken from a Gaussian ensemble with fluctuation spectrum $\langle V_k^2 V_{k'}^2 \rangle = \Gamma_k^2 \delta_{kk'}$, $\chi_0''(k,\Omega)$ is the charge susceptibility of the “clean” ($V = 0$), but fully interacting, system, at wavevector $k$ and frequency $\Omega$. $B$, $c$, $n$, and $A$ are the perpendicular magnetic field ($B = Bz$), speed of light, electron density, and system area, respectively.

In this paper we extend Eq. (1) to the non linear current versus field response. We propose new and independent experiments to probe the collective charge excitations, which would complement information given by the linear response probes. Our derivation incorporates strong electric fields into the current reponse function. To go beyond linear Kubo formula, we use a Floquet boost transformation.\textsuperscript{[3]}

We propose to measure the DC photoconductivity $\sigma_{xx}^{\text{photo}}(\Omega)$ in the presence of an AC radiation field at frequency $\Omega$. Within GMP theory, at frequency $\Omega > \Delta_0$, $\sigma_{xx}^{\text{photo}}$ should become singularly large and negative. As a consequence, the system will be unstable toward a “zero resistance state” (ZRS)\textsuperscript{[8]} where internal electric field domains are spontaneously created.\textsuperscript{[9,10]} ZRS phases, and spontaneous internal fields, were previously observed at weak magnetic fields (high Landau levels)\textsuperscript{[11–13]}. The photoconductivity has been calculated by various methods.\textsuperscript{[12,14–17]} At high Landau levels, the dark conductivity is metallic, and negative photocurrent can be induced by strong microwave power, which is positively detuned from the cyclotron frequency $\omega_c$. In contrast, we show that the ZRS in the lowest Landau level (LLL) arises from different phenomena. It involves intra-Landau Level excitations of MR mode which is expected at $\Omega \ll \omega_c$. In addition, negative photoconductivity appears already at leading order in radiation power, since the dark conductivity is zero.

For the compressible phases, such as around $\nu = \frac{1}{2}$, the dark non linear current $j_x^{\text{dc}}[E_x]$ is expressed in terms of $\chi''_0(k,\omega)$. Thus, predictions of composite fermions theory for $\chi''_0(k,\omega)$ could be tested by transport measurements. We
conclude by discussing experimentally relevant frequency and field scales, and regime of validity of the weak disorder expansion.

**Quantum Hall Hamiltonian**: Two dimensional electrons in a uniform magnetic field, and a time dependent vector potential \( \mathbf{a}(t) \), are described by the Hamiltonian \( H = H_0 + V \), where

\[
H_0[a] = \frac{1}{2m} \sum_{i=1}^{N} \left( \pi_i - \frac{q}{c} \mathbf{a}(t) \right)^2 + \sum_{i<j} \Phi(\mathbf{r}_i - \mathbf{r}_j) \tag{2}
\]

and \( V = A^{-1/2} \sum_k V_k \rho_k \). \( N \) is the number of particles of charge \( q \), and \( \Phi \) are the two-body interactions. The density operator is \( \rho_k = \sum_i e^{-i \mathbf{k} \cdot \mathbf{r}_i} \), and the cyclotron momentum for particle \( i \) is \( \pi_i = \mathbf{p}_i - \frac{qB}{2m} \hat{z} \times \mathbf{r}_i \), obeying \( [\pi_i, \pi_j] = i \sin(q) \hbar^2 \ell^2 e^{i \alpha} \delta_{ij} \), where \( \ell = (hc/|qB|)^{1/2} \) is the Landau length. For electrons, \( q = -e \). The current density at time \( t \) is

\[
j(t) = \frac{q}{m} \sum_{i=1}^{N} \left( U^\dagger(t) \pi_i U(t) \right) - \frac{n q^2}{mc} \mathbf{a}(t), \tag{3}
\]

where \( n = N/A \), and \( \langle \cdots \rangle \) denotes thermal and disorder averaging at \( t = 0 \). \( U(t) \) is the time evolution operator.

The calculation of the nonlinear photocurrent generalizes the single electron approach of Auerbach and Pai, while using the underlying Galilean symmetry of \( H_0 \). We proceed in two steps.

(i) **Floquet Boost**: We decompose \( U(t) \) according to

\[
U(t) = W(t) \mathcal{T} \exp \left\{ -\frac{i}{\hbar} \int_0^t dt' \tilde{H}(t') \right\}, \tag{4}
\]

where \( W(t) = \prod_i e^{-i g(t) \cdot \pi_i/\hbar} \) is the unitary Floquet boost. To cancel \( \mathbf{a}(t) \) from \( H_0[a] \), \( g(t) \) must satisfy

\[
g + \omega_c \hat{z} \times g(t) = -\frac{\omega_c}{B} \mathbf{a}(t), \tag{5}
\]

where \( \omega_c = \frac{eB}{mc} \) and \( g(0) = 0 \). \( g(t) \) can be readily solved for any \( \mathbf{a}(t) \).

\( \tilde{H} = W^\dagger HW - i \hbar W^\dagger \mathcal{W} \) is the Hamiltonian in the boosted frame, given by \( \tilde{H} = \tilde{H}_0 + \tilde{V} + f(t) \), with

\[
\tilde{H}_0 = \sum_i \frac{\pi_i^2}{2m} + \sum_{i<j} \Phi(\mathbf{r}_i - \mathbf{r}_j),
\]

\[
\tilde{V} = A^{-1/2} \sum_k V_k \rho_k e^{i \mathbf{k} \cdot \mathbf{g}(t)},
\]

and \( f(t) \) is an irrelevant c-number.

(ii) **Expansion in the disorder potential**: The evolution operator is \( U(t) = U_0 \mathcal{T} \exp \left[-\frac{i}{\hbar} \int_0^t dt' V(t') \right] \), where \( U_0 = W(t) e^{-i \tilde{H}_0 t/\hbar} \) is the clean evolution operator, and \( V(t) = A^{-1/2} \sum_k V_k \rho_k(t) e^{i \mathbf{k} \cdot \mathbf{g}(t)} \), where \( \rho_k(t) \equiv e^{i \tilde{H}_0 t/\hbar} \rho_k e^{-i \tilde{H}_0 t/\hbar} \).

The evolution of the global Landau operator \( \Pi^I = \sum_i (\pi_i^x - i \pi_i^y) \) for the disorder-free Hamiltonian is given by Kohn’s theorem:

\[
U_0^I(t) \Pi^I U_0^I(t) = \exp(i \omega_c t) \Pi^I - \frac{i N \hbar^2}{2 \ell^2} g^I(t), \tag{7}
\]

where \( g(t) = g_x(t) + ig_y(t) \). For the clean system, by setting \( U \to U_0 \) in [3], it is easy to see from [7] that for a DC field \( E \), (i) the longitudinal current vanishes at all times, and (ii) the Hall current is the Galilean result \( j = (nec/B) \hat{z} \times E \).

The longitudinal current [3] is expanded in powers of the disorder \( (V)^n \). The leading order is for \( n = 2 \), whose Fourier transform is given by

\[
j_x(\omega) = \frac{q n}{\hbar A} \sum_k \frac{k_y^2}{\omega + \omega_c} \langle \mathbf{R}_k \rangle \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} R_{-k}(\omega - \omega') \chi''_{\omega}(\mathbf{k}, \omega') R_k(\omega') . \tag{8}
\]

The electric field factors are given by

\[
R_k[\mathbf{E}(t), \omega] = \int_{-\infty}^{\infty} dt' e^{i \mathbf{k} \cdot \mathbf{E}(t')} e^{i \omega t} . \tag{9}
\]

\( \chi''_{\omega}(\mathbf{k}, \omega') \) is the dynamical charge susceptibility of the clean system,

\[
\chi''_{\omega}(\mathbf{k}, \omega) = \frac{1}{N Z_0} \text{Re} \int_0^\infty dt e^{i \omega t} \text{Tr} \left\{ e^{-\frac{\tilde{H}_0}{\hbar}} \left[ \rho_{-k}(t), \rho_k(0) \right] \right\} . \tag{10}
\]

Eq. [8] applies (to second order in \( V \)) for an arbitrary time dependent electric field.

**DC Photocurrent**. We consider an electric field with two components: a DC field \( E_x \hat{x} \), and a circularly polarized AC field \( \mathcal{E} \) of frequency \( \Omega \), such that \( \mathbf{a}(t) = -c \mathcal{E} \left( \sin(\Omega t) \hat{x} - \cos(\Omega t) \hat{y} \right) - c E_x t \hat{x} \). Solving Eq. [5] yields

\[
g(t) = \frac{c E_x}{\hbar B \omega_c} \left( \hat{x} - \omega_c t \hat{y} \right) \tag{11}
\]

\[
- \frac{c \mathcal{E}}{B (\omega_c - \Omega)} \left( \cos(\Omega t) \hat{x} + \sin(\Omega t) \hat{y} \right) .
\]

and

\[
R_k(\omega) = \sum_{l=-\infty}^{\infty} e^{i \phi_k J_l(k\lambda)} \delta(\omega + l\Omega - \frac{c k_y}{B} E_x) . \tag{12}
\]
where $\Delta(k) = \Delta_0 (1 + a_2 (\delta k \ell)^2 + a_3 (\delta k \ell)^3 + \ldots)$.

Using Eq. (14), the photocurrent (at radiation powers $\lambda \varepsilon / \ell \ll 1$) is

$$j^\text{dc}(\Omega, E_x) \simeq \frac{n c |\lambda \varepsilon|^2}{\hbar B} \int_0^\infty dk e^{-\frac{1}{2} (k \ell)^2} k^4 \Gamma(k) \int_0^{2\pi} d\theta \sin \theta \times \delta \left( \Omega - \Delta(k) + \frac{E_x}{B} c k \sin \theta \right).$$

The *detuning frequency* is defined as $\delta \Omega = \Omega - \Delta_0$. The DC electric field defines the electric frequency scale

$$f_x = c k_0 E_x / B.$$

In the regime $f_x, |\delta \Omega| \ll \Delta_0$, the DC photoconductivity (PC) is

$$\sigma_{xx}^\text{photo} \equiv \frac{dj^\text{dc}(\Omega)}{dE_x} \bigg|_{E_x=0} \approx \left| \lambda \varepsilon \right|^2 \left( \frac{n c^2}{\hbar B^2} \right) \int_0^{\infty} dk \Gamma^2(k) s(k) \frac{d}{d\Omega} \delta (\Omega - \Delta(k))$$

$$\approx -\left| \lambda \varepsilon \right|^2 \left( \frac{n c^2 \Gamma^2 \delta_k^2 \delta_s(k_0)}{4\hbar B^2 \sqrt{\alpha_2 \Delta_0}} \right) |\delta \Omega|^{-3/2} \Theta(\delta \Omega),$$

The dependence of $\sigma_{xx}^\text{photo}$ on radiation frequency is depicted in Fig. 2.

At finite driving field $E_x > 0$, the photocurrent is

$$j^\text{dc} = j_0 \left( \frac{\delta \Omega}{f_x} \right) \text{sgn}(f_x),$$

where $\delta k = k - k_0$, and $a_2 > 0$. In the regime $f_x, |\delta \Omega| \ll \Delta_0$, the photocurrent is unstable in the shaded region toward a ZRS state with spontaneous electric field $E_x^\text{sr}$ given by Eq. (22).

FIG. 2: Photoconductivity in an incompressible Laughlin phase, given by Eq. (19). $\Delta_0$ is the MR gap depicted in Fig. 1. $\Omega$ is the radiation frequency. At $\Omega > \Delta_0$, large negative conductivity is predicted.

FIG. 3: Nonlinear photocurrent for an incompressible Laughlin phase, given by Eq. (19). $f_x(E_x)$ is defined in Eq. (17). For positive detuning $\delta \Omega > 0$, (blue curve), the negative photocurrent is unstable in the shaded region toward a ZRS state with spontaneous electric field $E_x^\text{sr}$ given by Eq. (22).
where the current scale is

\[ j_0 \equiv \frac{2|\lambda_x|^2 n \ell k_x^3 s(k_y) F_{k_y}^2}{\sqrt{\alpha_x^2 \Delta_x^2}} |\delta \Omega|^{-\frac{1}{2}} \]  

(20)

and \( h(u) \) is the universal function,

\[ h(u) = |u|^{1/2} \int_{(u-1)_+}^{u+1} \frac{ds}{\sqrt{s}} \frac{s - u}{\sqrt{1 - (u-s)^2}} \]  

(21)

where \((u-1)_+ = \max(0, u-1)\). Photocurrent versus electric field at fixed detuning frequency is depicted in Fig. 3 for both positive and negative detuning frequencies.

**ZRS spontaneous field.** Negative uniform conductivity signals a thermodynamic instability toward formation of a ZRS state, with spontaneous internal electric fields. For a homogeneous quantum Hall phase these fields are fixed by the minima of the Lyapunov functional condition\(^ {11} \) \( \sigma \propto E^* = 0 \). Near the detuning threshold, we see in Fig. 3 that the current vanishes \( j_x^* = 1.6 \delta \Omega \), which by Eq. (17) yields the magnitude of the spontaneous electric fields as,

\[ E^*[V/cm] = 1.6 \times \delta \Omega \frac{B}{c k_0} \Theta(\delta \Omega) \]  

(22)

Notice that the spontaneous field is an independent measure of the MR wavevector \( k_0 \). For inhomogeneous systems with spatially varying Hall conductivity, there is no Lyapunov functional. As a result, spontaneously generated electric fields can fluctuate in time, which may be the source of the experimentally observed telegraph noise\(^ {12} \).

**Compressible FQH phases.** At \( \nu \approx \frac{1}{2} \), the electronic states have been described by composite Fermions (CF)\(^ {20,22} \), which see an effective weak magnetic field. Their effective “Fermi energy scale” \( \epsilon_F^* \) is determined by intra-LLL coulomb interactions. Read\(^ {21} \) computed the long wavelength dynamical charge susceptibility to go as

\[ \chi_{CF}(k, \omega) \sim e^{-k^2 \epsilon_F^*/2} \frac{h^2 \omega}{\epsilon_F^*} \frac{\epsilon_F^*}{(k \ell)^3} \]  

(23)

Near \( \nu = \frac{1}{2} \), Park\(^ {23} \) has proposed to search for ZRS by photoconductivity above the CF effective Landau level spacing. Since the dark system is metallic, negative conductivity could be achieved at high radiation power if CF Landau levels are sharp modes at finite wavevectors.

Since the excitations are gapless, we can use the nonlinear current as a probe to the low energy excitations. In the “dark” DC case, i.e. \( E(t) = E_x \hat{x} \), Eq. (5) yields \( g^{dc}(t) = (c E_x / B \omega_c) (\hat{x} - \omega_c t \hat{y}) \). Thus,

\[ R^{dc}_{k}(\omega) = 2\pi \exp \left( \frac{ik_x E_x}{B \omega_c} \right) * \left( \omega - \frac{c k_y}{B} E_x \right) \]  

(24)

and Eqs. (8) and (24) yield the dark nonlinear longitudinal current,

\[ j_x^{dc} = \frac{nc}{h B A} \sum_k i k_x^2 k_y \chi^\prime\prime_{k}(k, \frac{c k_y}{B} E_x) \]  

(25)

The low field limit yields the linear conductivity

\[ \sigma_{xx}(0) = \frac{d}{dE_x} j_x^{dc} = \frac{nc^2}{h B^2 A} \sum_k i k_x^2 k_y \frac{d}{d\omega} \chi^\prime\prime_{k}(k, 0) \]  

(26)

which coincides with the DC limit of Eq. (1).

The characteristic wave vectors which dominate \( k_x \) in the LLL are of the order of \( \ell^{-1} \). The dark DC conductivity at the compressible filling fraction \( \nu \) is of the scale

\[ \sigma_{xx}(0) \sim \frac{\nu q^2}{\hbar} \frac{\Gamma^2}{(\epsilon_F^*)^2} \left( 1 + O(\Gamma/\epsilon_F^*) \right) \]  

(27)

Away from \( \nu = \frac{1}{2} \), the composite fermion theory predicts resonances in \( \chi_{k}^{\prime\prime}(\omega) \) corresponding to the spectrum of “CF Landau levels”. Such resonances should appear as oscillations in the current by Eq. (25).

**Experiments.** Photoconductivity measurements in FQH samples were carried out by some time ago\(^ {20,22} \). Oscillatory magnetoresistance was reported without comparison to theoretical calculations. The published data did not show indications of ZRS effects in the Laughlin phases. However, the frequency scale may have been below the magnetoroton threshold: For filling fraction \( \nu=1/3 \) at carrier density \( n=7.6 \times 10^{10} \) cm\(^{-2} \), the magnetic field is \( B=9.5 \) T, and \( h \omega_c \sim 100 \) K. The resistivity activation gap\(^ {26} \), which is expected to be similar to \( \Delta_0 \), was about 210 GHz, which was higher than the microwave frequencies used in these experiments. Recent advances in terahertz spectroscopy may open the door to photoconductivity measurements in the FQH regime. It would be instructive to compare e.g. the magnetoroton gap by photoconductivity to the activation gap of resistivity.

For the compressible phases, we propose to measure Eq. (25). The relation between the charge excitation frequency \( \tilde{\omega}_{\text{charge}} \) and the corresponding electric field \( \tilde{E}_x \) is

\[ \tilde{E}_x = B \tilde{\omega}_{\text{charge}} / c \]

\[ = 0.26 \tilde{\omega}_{\text{charge}} \text{[GHz]} \left( B \text{[T]} \right)^{\frac{3}{2}} \text{[V/cm]}, \]  

(28)

where we have used \( \ell = 26 \text{nm} \sqrt{B / |T|} \).

**Validity of weak disorder expansion.** Eqs. (8), (13) and (25) are the second order expansion of the current in the disorder potential \( V \). Note that these expressions have finite LLL limits for \( \omega_c \to \infty \), keeping \( \nu < \infty \). The weak disorder expansion of the longitudinal current is valid under the following conditions:

1. \( \sigma_{xx} = 0 \) in the clean limit. Note that this condition fails for an ordinary clean metal at zero field.
2. Landau level broadening. In the absence of interactions the Landau level are infinitely degenerate. The degeneracy is lifted by Coulomb interactions which introduce the intra-Landau level charge excitation scales $\omega = \Delta_0, \epsilon_p^*/\hbar$ we have seen above. These energy scales control the higher order corrections by powers of $\Gamma/\hbar \omega \ll 1$.

3. Density matrix is at equilibrium. It is expected that strong time dependent electric fields modify the density matrix at long times, and produce strong effects on the DC photocurrent\cite{22}. Our analysis above did not take non equilibrium effects on the density matrix into account, which is justified in cases of rapid thermalization. In other words, we assume short non radiative relaxation time by phonons $\tau_{\text{rel}} \ll \tau_{\text{tr}}$, where the transport time $\tau_{\text{tr}} = \sigma_{xx} m/\epsilon^2 n$ is long in the weak disorder limit.

4. By applying the derivation of Eq. (25) to the transverse current $j_y(E_x)$, the Hall conductivity $\sigma_{xy}^{\text{dc}} = n q e/B$ gains no corrections at any finite order in $V$. Thus, plateaux of $\sigma_{xy} = \nu e^2/\hbar$ in the incompressible phases are necessarily non perturbative effects in $V$ such as nucleation of localized quasiparticles or motion of domain edges in the long range potential landscape.

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