High-\(T_c\) superconductors have Fermi energies \(E_F\) much smaller than conventional metals comparable to phonon frequencies. In such a situation nonadiabatic effects are important. A generalization of Eliashberg theory in the nonadiabatic regime has previously been shown to reproduce some anomalous features of the high-\(T_c\) superconductors as for instance the enhancement of \(T_c\) or the isotopic effects on \(T_c\) and \(m^*\). In this contribution we address the issue of the symmetry of the gap in the context of nonadiabatic superconductivity. We show that vertex corrections have a momentum structure which favours \(d\)-wave superconductivity when forward scattering is predominant. An additional increase of \(T_c\) is also found.

1. Introduction

High temperature superconductors (HTSC) show many “anomalous” features which are not reported in conventional materials. One of these peculiarities is the strong dependence of the order parameter on the momentum \(k: \Delta = \Delta(k)\). Josephson tunneling experiments and photoemission spectroscopy show that the superconducting gap of many cuprates has a dominant \(d\)-wave symmetry: \(\Delta(k) \approx \Delta[\cos(k_x) - \cos(k_y)]\).

The origin of the \(d\)-wave symmetry in HTSC is still controversial. On one hand, the observed \(d\)-wave symmetry is regarded as an evidence against any purely electron-phonon pairing interaction so that the mechanism responsible for super-
conductivity should be sought among pairing mediators of electronic origin (like antiferromagnetic fluctuations) with eventually a minor electron-phonon component. On the other hand, several theoretical studies have shown that the el-ph interaction could give arise, under some quite general circumstances, a d-wave symmetry of the condensate. This could happen when for example charge carriers experience an on-site repulsive interaction together with a phonon induced attraction for large inter-electrons distances. The on-site repulsion inhibits the isotropic s-wave superconducting response leading the system to prefer order parameters of higher angular momenta.

Another important feature of the HTSC compounds is the low density of the charge carriers. As a consequence, the Fermi energy $E_F$ in these materials is extremely small and can be comparable with the phononic frequency scale $\omega_{ph}$. In such a situation one of the assumptions of the conventional theory of superconductivity, Migdal’s theorem, does not hold and a generalization of the standard equation in nonadiabatic regime ($\omega_{ph} \approx E_F$) is necessary.

The small density of charge carriers moreover implies a reduced electronic screening of eventual charge fluctuations like as the ones induced by el-ph interaction. This leads to a momentum modulation of the el-ph interaction: the long range contribution (small momenta) is characterized by the unscreened el-ph interaction and is therefore attractive while the local scattering at short range (large momenta) is mainly dominated by the stong electronic correlation and is repulsive. This structure of the interaction favours a d-wave symmetry of the order parameter.

The experimental evidence of a d-wave component of the superconducting order parameter and the breakdown of Migdal’s theorem are thus two unavoidable aspects to be taken into account.

2. The Model

In this section we introduce a simple model interaction suitable for our investigation beyond Migdal’s limit and capable of providing for s- or d-wave symmetries of the order parameter. To define the model interaction $V_{\text{pair}}(k - k')$ we have made use of a number of informations gathered from previous studies. First, in order to obtain order parameters with higher angular momenta than s-wave, it is sufficient to consider a pair interaction made of a repulsive part at short distances and an attractive one at higher distances. In momentum space, this interaction corresponds to an attractive coupling for small $q$ and a repulsive one for large $q$, where $q = k - k'$ is the momentum transfer (Fig. 1):

$$V_{\text{pair}}(q, \omega_m) = \left[ -|g(q)|^2 \theta(q_c - |q|) + U \theta(|q| - q_c) \right] D(\omega_m),$$

where $D$ represents the retarded interaction.

Let us now try to interpret this strong momentum modulation in terms of e-ph and electron-electron interactions. In strongly correlated systems, as HTSC materials, the el-ph interaction acquires an important momentum dependence in such a way that for large values of the momentum transfer $q$ the el-ph interaction...
is suppressed, whereas for small values of $q$ it is enhanced. A physical picture to justify this momentum modulation is the following. In many-electron systems a single charge carrier is surrounded by its own correlation hole of size $\xi$ which can be much larger than the lattice parameter $a$ in the strongly correlated regime. This means that any electron interacts only with lattice distortions with wavelength larger than $\xi$, leading to an effective upper cut-off $q_c \simeq \xi^{-1}$ in the momenta space. Thus we have a non zero electron-phonon interaction when $|q| < q_c$. The cut-off momentum $q_c$ can also be regarded as a measure of the correlation in the system: $aq_c \ll 1$ in strongly correlated systems while $aq_c \simeq 1$ in the case of free electrons. From the above consideration, the attractive part at small $q$ of our model pairing interaction (the first term of Eq. (1)) finds a natural interpretation in $el-ph$ coupling modified by the strong electron correlations.

Having defined the nature of the attractive part of the total pairing interaction, we offer now a possible interpretation for the remaining repulsive part acting at large $q$ (second term in Eq. (1)). This repulsion is given by the residual $e-e$ interaction and its momentum dependence can be obtained, in analogy with the renormalization of the $e-ph$ interaction, by using the above picture of correlation holes. In this picture, the residual $e-e$ interaction represents the impossibility to overlap two correlation holes.

3. d-Wave Nonadiabatic Superconductivity

The breakdown of Migdal’s theorem in high-$T_c$ superconductors inevitably calls for a generalization beyond the Migdal-Eliashberg (ME) scheme to include the no longer negligible vertex corrections. A possible way to accomplish this goal is to rely on a perturbative scheme by truncating the infinite set of vertex corrections at a given order. In previous works, we have proposed a perturbative scheme in which the role of small parameter is played roughly by $\lambda \omega_{ph}/E_F$ ($\lambda$ is the $el-ph$ coupling) leading to a generalized ME theory which includes the first nonadiabatic
A diagramatic representation of the effective interaction (i.e. with the inclusion of vertex corrections) in the Cooper’s channel is shown in Fig. 2, where we only include the terms that give a finite contribution for \( T = T_c \).

A systematic study of the several gap symmetries involves the projection of the order parameter on the different harmonic functions:

\[
\Delta(\phi) = \sum_{l = -\infty}^{+\infty} \Delta^{(l)} Y_l(\phi),
\]

where \( \phi \) is the polar angle on the Fermi’s surface and \( Y_l(\phi) = e^{il\phi}/\sqrt{2\pi} \) are eigenfunctions of the operator \( L = -id/d\phi \). In this paper we focus only on the interplay between \( s \)- and \( d \)-wave symmetries.

A purely \( s \)-wave contribution is recovered when interaction is totally attractive, namely for \( q_c \leq 2k_F \). In the limit \( q_c \to 0 \) the \( s \)-wave component is suppressed and only the \( d \) symmetry survives. From qualitative point of view we expect a crossover from \( s \)-wave superconductivity to \( d \)-wave superconductivity as function of the parameter \( q_c \) and therefore as function of the degree of electronic correlation of the system.

The generalized ME equations including the nonadiabatic corrections are numerically solved to calculate the critical temperature \( T_c \) as function of the dimensionless parameter \( Q_c = q_c/2k_F \). In Fig. 3 we show the dependence of \( T_c \) on \( Q_c \) in the weak repulsion case \( U/g^2 = 0.1 \) for an Einstein phonon spectrum with frequency \( \omega_0 = 0.2E_F \). The inclusion of the nonadiabatic contributions produces a marked enhancement of the critical temperature for small values of \( Q_c \), where vertex corrections yield an increase of the effective electron-phonon interaction.
Figure 3: Behaviour of the critical temperature $T_c$ as function of $Q_c$ in the $s$- and $d$-wave symmetry channels. The case shown refers to the parameters $\lambda = 1$ and $\mu^* = 0.1$. In the inset it is shown the case without nonadiabatic corrections.

4. Conclusions

We have studied a model interaction in which the el-ph interaction is dominant at small values of $q$ and the residual repulsion of electronic origin is instead important at larger momentum transfers, we have shown that also when the solution has $d$-wave symmetry, the inclusion of nonadiabatic corrections enhances $T_c$ compared to the case without corrections. This result is a consequence of the weak dependence at small $q$ on the particular symmetry averages, $s$- or $d$-wave, of the vertex and cross corrections.

It is finally interesting to compare the present results with the phenomenology of the oxide SC, which show $d$-wave, and the fullerides, which instead show $s$-wave. In our perspective there are important differences between the two materials. The main one is that the oxides have their largest values of $T_c$ when the Fermi surface is strongly influenced by Van Hove singularities. Then correlation effects can be estimated to be larger in the oxides and, finally, fullerides seem to have rotational disorder which would favour $s$-wave. Therefore, in principle, it could happen that in the oxides, going into the overdoped phase might lead to a crossover from $d$-wave to $s$-wave depending on the parameters. Also in this perspective one can understand the different behavior of the Nd electron doped compounds which seem to show $s$-wave. We conjecture that this is due to the possibly large distance of the Fermi surface with the respect to the Van Hove singularity.

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