Maximal boost and energy of elementary particles as a manifestation of the limit of localizability of elementary quantum systems

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(Dated: April 10, 2015)

I discuss an upper bound on the boost and the energy of elementary particles. The limit is derived utilizing the core principle of relativistic quantum mechanics stating that there is a lower limit for localization of an elementary quantum system and the suggestion that when the localization scale reaches the Planck length, elementary particles are removed from observables. The limit for the boost and energy, $M_\text{Planck}/m$ and $M_\text{Planck}c^2 \approx 8.6 \cdot 10^{27}$ eV, is defined in terms of fundamental constants and the mass of elementary particle and does not involve any dynamic scale. These bounds imply that the cosmic ray flux of any flavor may stretch up to energies of order $10^{18}$ GeV and will cut off at this value.

Introduction

This note presents the scenario establishing the ultimate upper bound on the Lorentz boost and energy of elementary particles with a non-zero mass. The cosmic ray detectors measure higher and higher energies; recently the IceCube collaboration released data containing extra terrestrial neutrino events with $E_\nu \sim \text{PeV}$ ($10^{15}$ eV) [1]. The region of energies probed is steadily increasing: cosmic ray experiments as HiRes, Telescope Area, Auger report events with unprecedented energies of up to $10^{20}$ eV [2].

It is natural to ask whether there is no upper bound on Lorentz boosts/energies as it follows from the classical relativity. Accounting for quantum feature of elementary particles and gravity may result in saturating this unstoppable increase of boost and energy of elementary particles. Though no deviation from Lorentz invariance is observed, which implies that space-time symmetry is described by a non compact group and that boosts and energies can acquire arbitrarily large values, there have been numerous attempts to modify the untamed growth of the boost and energy, see e.g. [3], [4]. Present note addresses this question.

The suggested mechanism for the boost and energy cut-off is based on the fundamental concept of the limit of localizability of particle in relativistic quantum mechanics. This limit is combined with the conjecture that when the localization scale of the elementary particle reaches the Planck length, particles are removed from the observables and do not represent physical degrees of freedom any more. For brevity, let us call this assumption the quantum hoop conjecture, since it can be viewed as a quantum counterpart to the well-known hoop conjecture, which suggests that when the system is localized inside the volume of size of classical gravitational length, Schwarzschild radius, system undergoes gravitational collapse [5]. Combining the quantum hoop conjecture with the limit of localizability of the elementary particle leads to the upper bound on Lorentz boost and energy of elementary particles. Assuming that the mass of the elementary quantum system $m$ is less than Planck mass $M_P$, this presented model predicts value of maximal attainable boost $\Gamma_{\text{max}} = M_P/m$ and of maximal attainable energy $E_{\text{max}} = M_Pc^2$. When boost reaches $\Gamma_{\text{max}}$, the elementary particle is removed from the $S$-matrix observables, i.e. quanta of *in, out* fields disappear from a spectrum. Therefore, these limits can be considered as the ultimate bounds for the boost and energy of an elementary particle.

Maximal attainable boost and maximal attainable energy

To begin with, let us recall fundamental units. In this letter Planck mass $M_P$ is defined as the value of a mass parameter at which the Compton wave length of a system with the mass $m$, $\lambda_q(m) = h/mc$, is equal to the Schwarzschild radius, $\lambda_{gr}(m) = 2Gm/c^2$ (throughout a four dimensional space-time, no extra dimensions, is considered)

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where $h$ is the Planck’s constant, $G$ is the gravitational constant and $c$ is the speed of light. This results in

$$M_P = \sqrt{\frac{hc}{2G}} \approx 1.5 \cdot 10^{-8}\text{kg} \approx 8.6 \cdot 10^{27} \text{ eV}/c^2$$

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Planck length $\lambda_p$ is defined as a Compton wave length of system with the Planck mass, evidently coinciding with the Schwarzschild radius of a system with the Planck mass. From Eq. (1) it follows

$$\lambda_p \equiv \frac{\hbar}{M_p c} = \frac{2GM_p}{c^2} = \sqrt{\frac{2G\hbar}{c^4}} \approx 2.3 \cdot 10^{-35} \text{ m}$$

(3)

These values differ by a factor of $\sqrt{2}$ from the ones established on a purely dimensional basis. Note the relation for a mass-independent Planck length made up from the mass-dependent $\lambda_q(m)$ and $\lambda_{gr}(m)$:

$$\lambda_q(m) \lambda_{gr}(m) = \lambda_P^2$$

(4)

Let us consider a system at rest with mass $m$, localized inside volume with a linear size $L_0$. By localized, we understand that system is constrained in a finite volume; which means that probability of finding the system inside the ball of linear size $L_0$ is 1, and that the information between the system and the outside region may be exchanged in terms of field quanta. In other words, the observability is realized and described by exchanging quanta of fields between the localized system and the outside observer, separated from the system by distance much greater than $L_0$.

To find the maximal attainable boost, let us recall the hoop conjecture, which states that a black hole forms whenever the amount of energy $mc^2$ is compacted inside a region that in no direction extends outside a circle of circumference (roughly) equal to $2\pi \lambda_{gr}(m)$ [5]. In other words no signal from the ball radius $a$ can reach external observer when $a < \lambda_{gr}(m)$. Originally, hoop conjecture was put forward for astrophysical bodies, macroscopic objects which can be reasonably described by classical theory of gravity.

In this note we introduce and utilize a quantum counterpart to the hoop conjecture, stating that when elementary quantum system is localized inside a ball with a size less than Planck length $\lambda_p$, system is not observed as elementary particle, as the quantum of in or out fields. The idea that below the Planck length the notion of space-time and length ceases to exists is not new, see [6]-[9]. We just reformulate it in operational way in terms of elementary particles and will use the name quantum hoop conjecture for the phenomenon of removing elementary particles from observables when the localization scale is below $\lambda_P$.

In terms of quantum field theory, the quantum hoop conjecture is translated into the statement that when the localization scale reaches Planck length, $\langle \alpha | \sqrt{Z} \Psi_{in, out} | \beta \rangle$, the matrix element representing the one-particle solution, disappears from the solution of Heisenberg equations of motion [10], $\hat{\Psi} = \sqrt{Z} \hat{\Psi}_{in, out} + \hat{R}$, where the operator $\hat{\Psi}_{in, out}$ represents an incoming/outgoing free particle and $\hat{R}$ describes the rest of the solution, which in case of a classical source $j(x)$ is $\hat{R}(x) = \int d^4 y j(y) \Delta_{ret, adv} (x - y)$. The flux of the incoming/outgoing particles and consequently, the S-matrix elements are defined by $\hat{\Psi}_{in, out}$; if matrix element of $\hat{\Psi}_{in, out}$ vanishes, corresponding S-matrix element vanishes as well [10].

Maximal attainable boost for the elementary particle with mass $m$, $\Gamma_{max}(m)$, is derived from the requirement that the Lorentz-contracted localization scale is still larger than either Schwarzschild radius or Planck length and therefore, elementary particles are observable as a quanta of in, out fields.

From the classical special relativity it follows that when boosted with $\Gamma$, the localization scale is spatially contracted and becomes $L = L_0/\Gamma$. This relation holds in a relativistic quantum mechanics as well - position operator acquires overall factor $1/\Gamma$ in a reference frame boosted with $\Gamma$ [11]. We require that the Lorentz-contracted size of the system is still larger than the threshold value $L_{thr}$, given by either gravitational radius or Planck scale:

$$L = \frac{L_0}{\Gamma} \geq L_{thr} \equiv \text{max}(\lambda_p, \lambda_{gr}(m))$$

(5)

which results in

$$\Gamma \leq \Gamma_{max}(m) = \frac{L_0}{L_{thr}} = \frac{L_0}{\lambda_q(m)} \frac{\lambda_{q}(m)}{L_{thr}}$$

(6)

We utilize the well-known fact that in the framework of relativistic quantum mechanics, a particle at rest can not be localized with accuracy better than its Compton wave length $\lambda_q(m)$, i.e. $\min(L_0) = \lambda_q(m)$ [10, 12]. Then the inequality (6) turns into

$$\Gamma \leq \Gamma_{max}(m) = \frac{\lambda_q(m)}{L_{thr}}$$

(7)
In this letter we consider the case $m \leq M_P$; it seems natural that the mass of elementary particle is smaller than the Planck mass. In terms of spatial scales this constraint is realized as an inequality ordering scales as follows

$$\lambda_{gr}(m) \leq \lambda_P \leq \lambda_q(m)$$

(8)

and consequently $L_{thr} = \max(\lambda_P, \lambda_{gr}(m)) = \lambda_P$. Therefore, the maximal attainable boost for a particle with mass $m \leq M_P$ is

$$\Gamma_{max}(m) = \frac{\lambda_q(m)}{\lambda_P} = \frac{M_P}{m}$$

(9)

When $\Gamma$ exceeds $\Gamma_{max}(m)$, the one-particle matrix element vanishes, $(\alpha(\Gamma)|\Psi_{in, out}|\beta(\Gamma)) = 0$, where $|\alpha(\Gamma))$, $|\beta(\Gamma))$ are the states with the boost $\geq \Gamma_{max}(m)$. In other words, $S$-matrix element, corresponding to scattering with $\Gamma(m) > \Gamma_{max}(m)$, vanishes. One way to realize this disappearance of $in$, $out$ states from the spectrum could be by modifying canonical commutation relations for creation and annihilation operators: $[a_{in}(k), a_{in}^\dagger(p)] \sim \Theta(\Gamma_{max} - \Gamma) \delta(k - p)$, latter written in a preferred reference frame. As far as $\Gamma \leq \Gamma_{max}(m)$, a particle can be observed as a quantum of $in$, $out$ fields and the standard $S$-matrix can be defined. Equation (9) is the core observation of this letter.

This result is obtained by assuming that when the localization scale becomes less or equal than the Planck length $\lambda_P$, elementary particles are removed from observables, not represent the space of physical degrees of freedom any more. The hoop conjecture which may serve as a physical insight for the rearrangement of the space of physical degrees of freedom, appeals to the Schwarzschild radius $\lambda_{gr}$, as the minimal localization length. Connection between Planck length and gravity effects was established long ago, and it states that in presence of gravity it is impossible to measure the position of a particle with error less than $\lambda_P$ [4], [5], [6]. We have assumed that similar to hoop conjecture, suggesting that object collapses into black hole and disappears when localized in area with size less than classical gravitational radius, $2Gm/c^2$, elementary particle is removed from $S$-matrix observables when localized in area with size less than $\lambda_P$, latter determined by both $G$ and $\hbar$.

When the mass of elementary particle approaches $M_P$, as seen from Eq. (9), the Planck length and the Schwarzschild radius coincide

$$\lambda_P = \lambda_{gr}(M_P)$$

(10)

i.e. the quantum and the classical hoop conjectures merge into the same supposition.

As any other scenario predicting the maximum attainable boost and energy and thus postulating the existence of preferred reference frame, the present model also requires the existence of the reference frame to which the maximum boost is compared. We will choose for such a reference frame the cosmic rest frame, where cosmic microwave background (CMB) is at rest, and its temperature is homogeneous and is $2.73K$. Another good choice could be the rest frame of Local Group. The Earth rest reference frame moves relative to CMB with peculiar velocity $\sim 370 \text{ km/s} \approx 0.0012c$, as is follows from CMB dipole anisotropy measurements [13]. Because of the low value of $\Gamma_{Earth-CMB} \sim 1$, with a good approximation the Earth rest reference frame can be identified with the preferred reference frame, the one where as $\Gamma \rightarrow \Gamma_{max}$, no elementary particle can be observed.

According to Eq. (9), the value of maximal attainable boost changes from particle to particle, e.g. $\Gamma_{max}(\text{proton}) = M_P/m_{\text{proton}} \approx 9.2 \cdot 10^{14}$, $\Gamma_{max}(\text{electron}) \approx 1.7 \cdot 10^{22}$. Not so for the maximal attainable energy: $E_{max}$ is the same for all particles and is given by the Planck energy $E_P = M_pc^2$:

$$E_{max} = \Gamma_{max}(m)mc^2 = M_Pc^2 \approx 8.6 \cdot 10^{27} \text{ eV}$$

(11)

It is important to note that the above results are obtained and are valid for the systems with $m \leq M_P$ which we consider as elementary particles, i.e. quanta of $in$, $out$ fields.

For the macroscopic objects with $m \geq M_P$ the spatial scales are ordered as follows: $\lambda_{gr}(m) \geq \lambda_P \geq \lambda_q(m)$, therefore the reasoning based on a minimum localization scale and consequently Eqs. (4) - (7) are no longer valid. Thus, when $m \geq M_P$, premise leading to the upper limit on the boost and the energy is not justified, and the symmetry group of motion of classical objects (systems with $m \geq M_P$) is again the non-compact Poincare group of classical relativity.

The disappearance of $in$, $out$ terms from the solution of Heisenberg equations of motion, in other words, removing elementary particles from observables when $\Gamma > \Gamma_{max}$, seems to violate the $S$-matrix unitarity. Indeed, asymptotic completeness, according to which the $in$ and $out$ states span the same Hilbert space, which is also assumed to agree with the Hilbert space of interacting theory [10]

$$\mathcal{H}_{in} = \mathcal{H}_{out} = \mathcal{H}_{interacting}$$

(12)

is not satisfied. Condition of asymptotic completeness is not trivial already in the framework of standard quantum field theory: if particles can form bound states, the structure of space of states is modified, and the $S$-matrix unitarity
is restored only after bound states are accounted for in the relation for unitarity. Accounting for gravity brings in another reasoning for the violation of the S-matrix unitarity. It has previously been observed that in and out states, which are related by unitary transformation, cannot be be defined in the presence of an arbitrary metric. Applying the quantum hoop conjecture to elementary particles drives this observation to the extreme, stating that as soon as the localization region becomes smaller than $\lambda_p$, elementary particles are removed from the space of physical degrees of freedom. In this case, the unitarity condition has to be formulated not in terms of elementary particles, but in terms of new physical degrees of freedom of quantum gravity, task which is beyond the scope of this letter.

As for the signatures of a suggested scenario, the only clear one is the cut-off of beam of particles/flux of cosmic rays at limiting value $E_p \sim 10^{18}$ GeV, energy, which is not accessible by modern accelerators and cosmic rays observatories. Up to $E_p$, i.e. up to $\Gamma_{\nu}^{\text{max}}$ when the localization region is still larger than the Schwarzschild radius, physical degrees of freedom, observables, are elementary particles. When $E \gtrsim E_p$, particles are removed from observables, thus cosmic ray flux should vanish when $E \rightarrow 10^{18}$ GeV. As mentioned above, maximum attainable boost varies from particle to particle and is $M_p / m$, but maximum attainable energy for any type of elementary particle is the same $E_p$. These bounds are “kinematical” in a sense that no dynamic scale related with any particular interaction is involved in establishing the bound. Of course, some concrete conditions may alter the maximal observed energy. e.g. the well-known GZK limit on the energy of cosmic rays from distant sources, caused by the existence of the omnipresent target - cosmic microwave background. However the presented bounds are ultimate, derived from the quantum hoop conjecture and the basic principles of relativistic theory of quantum systems; in other words statement is that independently of dynamics the kinematic parameters describing elementary particles cannot exceed these bounds.

This is in contrast with the scenario for the boost and energy cut-off which was recently put forward. In it is suggested that the maximum attainable boost and maximum attainable energy for the neutrino are

$$\Gamma_{\nu}^{\text{max}} = \frac{M_p}{M_{\text{weak}}}, \quad E_{\nu}^{\text{max}} = \frac{M_p}{m_{\nu}} \frac{M_p}{M_{\text{weak}}}$$

(13)

where $M_p$ is a Planck scale and $M_{\text{weak}}$ is a scale of weak interactions ($\sim 100$ GeV). The main point of work is that the upper bound on energy is defined by weak scale; it follows from that neutrino spectrum cuts off at energies $\sim$ few PeV. The model in this letter predicts value $\Gamma_{\nu}^{\text{max}} = M_p / m_{\nu}$, i.e. much higher than one from Eq. (13). Regard neutrino energies, as an example, we quote the estimate from analysis of energetics of gamma-ray bursts - maximum neutrino energies may reach $10^{16} - 5\cdot10^{19}$ eV. This value does not contradict Eq. - $E_{\nu}^{\text{max}} = 8.6 \cdot 10^{27}$ eV, and exceeds the upper bound of a few PeV on neutrino energies suggested in .

Discussion

We have combined: the lower limit of localizability of an elementary quantum system, Lorentz contraction, and the quantum hoop conjecture to derive the upper bound on Lorentz boost and energy for massive particles. When the upper bound is reached, elementary particles disappear from a spectrum - they are not observable any more as quanta of $\text{in}$, $\text{out}$ fields. In derivation of this upper bound, we used the property of Lorentz contraction, i.e. validity of the theory of relativity for elementary particles up to $\Gamma_{\text{max}}$ is assumed. At the same time, the very idea of a maximal attainable boost and energy implies existence of a preferred reference frame to which these values are compared. Since the Planck length, and consequently the quantum hoop conjecture involves gravitational field, these two problems - violation of the S-matrix unitarity because of the disappearance of $\text{in}$, $\text{out}$ quanta and the existence of a preferred reference frame, must be resolved in the framework of a future theory of quantum gravity. The scenario is applicable only to elementary quantum systems with mass less than the Planck mass; for macroscopic objects classic relativity with the unbounded boost and energy is preserved. Though the limiting values of boost and energy are Lorentz invariant, assigning the physical meaning to $\Gamma_{\text{max}}$ and $E_{\text{max}}$ implies the existence of a preferred reference frame. This is only true for elementary quantum systems with $m < M_p$. For macroscopic systems with $m > M_p$ no bound can be obtained on $\Gamma$ and $E$; thus there is no need in a preferred reference frame.

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