Sum Rules for Nucleon Spin Structure Functions

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Abstract

The uncertainties in the perturbative and higher twist corrections to the sum rules for $\Gamma_{p,n}$ are analyzed. The theoretical predictions for $\Gamma_{p,n}$ are compared with the whole set of experimental data and the restrictions on theoretical uncertainties were obtained. Suggestions for future experiments are given.
1 The sum rules for $\Gamma_{p,n}$. Theoretical Status.

I start with the consideration of the sum rules for the first moments of the spin structure functions

$$\Gamma_{p,n}(Q^2) = \int_0^1 dx g_{1p,n}(x, Q^2)$$

(1)

I will discuss the uncertainties in the theoretical predictions for $\Gamma_{p,n}$ and compare the theoretical expectations with experimental data. The aim of this consideration is to obtain from the experiment the restrictions on the uncertainties in the theoretical description of the problem.

Consider first the Bjorken sum rule [1]:

$$\Gamma_p(Q^2) - \Gamma_n(Q^2) = \frac{1}{6}g_A[1 - \frac{\alpha_s(Q^2)}{\pi}] - 3.6\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 \frac{b_{p-n}}{Q^2},$$

(2)

where $g_A$ is the axial $\beta$-decay coupling constant and the last term in (2) represents the twist-4 correction. The perturbation QCD corrections are known up to the third order $[2,3]$ (there is also an estimate of the fourth order term $[4]$). The coefficients in (2) correspond to the number of flavours $N_f = 3$.

In what follows I will so often transfer the data to the standard reference point $Q^2 = Q_0^2 = 10.5 GeV^2$ – the mean value of $Q^2$, at which EMC and SMC experiments were done.

Let us discuss the perturbative corrections in (2). Today there is a serious discrepancy in the values of $\alpha_s$ found from different experiments. The average value of $\alpha_s$, obtained at LEP is $[5]$

$$\alpha_s(m_Z^2) = 0.124 \pm 0.007$$

(3)

In two-loop approximation this value corresponds to the QCD parameter $\Lambda_3$ for three flavours

$$\Lambda_3 = 430 \pm 100 MeV$$

(4)

The data on the $\Upsilon \rightarrow$ hadrons decay give $[5]$ (the first $\alpha_s$ correction $[6]$ is accounted)

$$\alpha_s(m_b^2) = 0.178 \pm 0.010$$

(5)

from which it follows

$$\Lambda_3 = 170 \pm 30 MeV$$

(6)

The small error in (6) is caused by the fact that the partial width $\Gamma(\Upsilon \rightarrow 3g)$, from which (5) was determined, is proportional to $\alpha_s^3$ and the $\alpha_3$ correction to it is small. A strong contradiction of (4) and (6) is evident.

The overall fit $[7]$ of the data of deep inelastic lepton-nucleon scattering gives in the NLO approximation $\Lambda_3 = 250 MeV$ (the error is not given). New data in the domain $m_Z^2$ indicate lower $\alpha_s(m_Z^2)$ (SLD $[8]$ : 0.118 ± 0.013, 0.112 ± 0.004; OPAL $[9]$ : 0.113 ± 0.012), but the data of AMY $[10]$ on $e^+e^-$ annihilation at $\sqrt{s} = 57.3 GeV$ results in $\alpha_s(m_Z^2) = 0.120 \pm 0.005$. Finally, from $\tau$- decay it was obtained $[11]$:

$$\alpha_s(m_\tau^2) = 0.33 \pm 0.03$$

(7)

corresponding to

$$\Lambda_3 = 380 \pm 60 MeV$$
But the determination of $\alpha_s$ from $\tau$-decay can be criticized \cite{12} on the grounds that at such a low scale exponential terms in $q^2$ may persist in the domain of positive $q^2 = m^2_\tau$ besides the standard power-like terms in $q^2$ accounted in the calculation.

In such a confusing situation I will consider two options – of small and large perturbative corrections. In the first case I will take $\Lambda_3 = 200 MeV$. Then

$$\alpha_s (Q^2_0) = 0.180 \pm 0.010 \quad \Gamma_p (Q^2_0) - \Gamma_n (Q^2_0) = 0.194$$

(8)

In the second one $\Lambda_3 = 400 MeV$ and

$$\alpha_s (Q^2_0) = 0.242 \pm 0.025 \quad \Gamma_p (Q^2_0) - \Gamma_n (Q^2_0) = 0.187$$

(9)

The twist-4 contribution was disregarded in (8) and (9). There is also a discrepancy in its value. The value of $b_{p-n}$ was determined in the QCD sum rule approach by Balitsky, Braun and Koleshichenko (BBK) \cite{13}:

$$b_{p-n} = -0.015 GeV^2$$

(10)

On the other hand, the model \cite{14} based on connection \cite{15} of the Bjorken sum rule at large $Q^2$ with the Gerasimov, Drell, Hearn sum rule at $Q^2 = 0$ \cite{16} gives

$$b_{p-n} = -0.15 GeV^2$$

(11)

In what follows I will consider (10) and (11) as two options which correspond to small (S) and large (L) twist-4 corrections. I will discuss both approaches of determination of higher twist corrections in more details below. From my point of view, no one of these approaches is completely reliable and I will use them in comparison with experiments only as reference points.

I turn now to the sum rules for $\Gamma_p$ and $\Gamma_n$:

$$\Gamma_{p,n} (Q^2) = \frac{1}{12} \left[ \left( 1 - \frac{\alpha_s}{\pi} - 3.6\left( \frac{\alpha_s}{\pi} \right)^2 - 20\left( \frac{\alpha_s}{\pi} \right)^3 \right) (\pm g_A + \frac{1}{3} a_8) + \right.$$

$$\left. \frac{4}{3} \left[ 1 - \frac{\alpha_s}{3\pi} - 0.55\left( \frac{\alpha_s}{\pi} \right)^2 \right] \Sigma \right] - \frac{N_f}{18\pi} \alpha_s (Q^2) \Delta g (Q^2) + \frac{b_{p,n} Q^2}{Q^2}$$

(12)

According to the current algebra $g_A, a_8$ and $\Sigma$ are determined by the proton (or neutron) matrix elements of flavour octet and singlet axial currents

$$-2ms_\mu a_8 = <p, s | j_{5\mu}^{(8)} | p, s > \quad -2ms_\mu \Sigma = <p, s | j_{5\mu}^{(0)} | p, s >,$$  

(13)

$$-2ms_\mu g_A = <p, s | j_{5\mu}^{(3)} | p, s >$$

(13')

where

$$j_{5\mu}^{(8)} = \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2s \gamma_\mu \gamma_5 s, \quad j_{5\mu}^{(3)} = \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d - \bar{s} \gamma_\mu \gamma_5 s$$

In the parton model $g_A, a_8$ and $\Sigma$ are equal to

$$g_A = \Delta u - \Delta d \quad a_8 = \Delta u + \Delta d - 2\Delta S \quad \Sigma = \Delta u + \Delta d + \Delta s,$$

(14)

where

$$\Delta q = \int_0^1 [q_+ (x) - q_- (x)] \quad q = u, d, s$$

(15)
and $q_{\pm}$ are the quark distributions with the spin parallel (antiparallel) to the proton spin, which is supposed to be longitudinal (along the beam). $\Delta g$ in (12) has the similar meaning, but for gluons, as $\Delta q$ for quarks

$$\Delta g(Q^2) = \int_0^1 dx \left[ g_+(x, Q^2) - g_-(x, Q^2) \right]$$

(16)

Unlike $g_A, a_8$ and $\Sigma$ which in the approximation used above are $Q^2$ independent and have zero anomalous dimensions, $\Delta g$ anomalous dimension is equal to $-1$. This means that

$$\Delta g(Q^2)_{Q^2 \to \infty} \approx c \ln Q^2$$

(17)

The conservation of the projection of the angular momentum can be written as

$$\frac{1}{2} \Sigma + \Delta g(Q^2) + L_z(Q^2) = \frac{1}{2}$$

(18)

where $L_z$ has the meaning of the orbital momentum of quarks and gluons. As follows from (17),(18) at high $Q^2$ $L_z(Q^2)$ must compensate $\Delta g(Q^2)$, $L_z(Q^2) \approx -c \ln Q^2$. This means that the quark model, where all quarks are in $S$-state, failed with $Q^2$ increasing.

Now about the numerical values of the constants $g_A, a_8$ and $\Sigma$ entering eq.(12). $g_A$ is known with a very good accuracy, $g_A = 1.257 \pm 0.003$ [5].

Under assumption of SU(3) flavour symmetry in baryon decays $a_8$ is equal to

$$a_8 = 3F - D = 0.59 \pm 0.02$$

(19)

where $F$ and $D$ are axial coupling constants of baryon $\beta$-decays in SU(3) symmetry and the numerical value in the r.h.s. of (19) follows from the best fit to the data [17]. The combination $3F - D$ can be found from any pair of baryon $\beta$-decays. The comparison of the values of $3F - D$, obtained in this way shows, that the spread is rather narrow [18], $|\delta a_8| \leq 0.05$ and at least $a_8 > 0.50$. This may be an argument in the favour that SU(3) violation is not large here. (See, however, [19]). It should be mentioned that at fixed $\Gamma_{p,n}$ the uncertainty in $a_8$ only slightly influences the most interesting quantities $\Sigma$ and $\Delta s$. As follows from (12) and (14)

$$\delta \Sigma = -\delta (\Delta s) = \frac{1}{4} \delta a_8$$

(20)

$a_8$ was also determined by the QCD sum rule method [20]. In this approach no SU(3) flavour symmetry was assumed and the result

$$a_8 = 0.5 \pm 0.2$$

(21)

is in agreement with (19), although the error is large.

What can be said theoretically about $\Sigma$? In their famous paper [21] Ellis and Jaffe assumed that the strange sea in the nucleon is nonpolarized, $\Delta s = 0$. Then

$$\Sigma \approx a_8 \approx 0.60$$

(22)

This number is in a contradiction with the experimental data pointing to smaller values of $\Sigma$. On the other hand, Brodsky, Ellis and Karliner [22] had demonstrated that in the Skyrme model at large number of colours $N_c$, $\Sigma \sim 1/N_c$ and is small. From my point of view this argument is not
very convincing: the Skyrme model may be a good model for description of nucleon periphery, but not for the internal part of the nucleon determining the value of $\Sigma$ (see also [23]).

An attempt to calculate $\Sigma$ in the QCD sum rule approach in the way similar to the $a_8$ determination [20], failed. It was found [24], that the operator product expansion (OPE) breaks down in case of the polarization operator (and/or NNA vertex) for singlet axial current at the scale $\sim 1 GeV$. (The anomaly was properly accounted in the calculation). This is indicative of that the higher dimension operators (instantons?) are of importance in this problem. The physical consequence is that one may expect a violation of the Okubo-Zweig-Iizuka rule in the nonet of axial mesons. The similar trouble, perhaps, faces attempts to determine $\Sigma$ using the so called "$U(1)$ Goldberger-Treiman relation" (for a review see [25]). So, at this stage, the only way to find $\Sigma$ is from an experiment, exploiting eq.(12).

I dwell now on the determination of higher twist corrections. The twist-4 correction was calculated by BBK [13], using the QCD sum rule method for the vertex function in the external field [26,27]. The result for $b_{p-n}$ was given in (10). For $b_{p+n}$ it was obtained

$$b_{p+n} = -0.022 GeV^2$$  \hspace{1cm} (23)

This result, however, cannot be considered as reliable for the following reasons:

1. BBK use the same hypothesis as Ellis and Jaffe did, i.e. assume that s-quarks do not contribute to the spin structure functions and instead of singlet (in flavour) operator consider the octet one.

2. When determining the induced by external field vacuum condensates, which are very important in such calculations (see [26]), they saturate the corresponding sum rule by $\eta$-meson contribution, what is wrong. (Even the saturation by $\eta'$-meson would not be correct, since $\eta'$ is not a Goldstone).

3. One may expect that in the same way as in the calculation of $\Sigma$ by the QCD sum rule, in this problem the OPE series diverge at the scale $\sim 1 GeV$, where the BBK calculation proceeds.

Even in the case of the Bjorken sum rule, where the mentioned above problems are absent, the value $b_{p-n}$ (10) is questionable. $b_{p-n}$ was calculated by BBK using the OPE for the vertex function and accounting few terms in OPE with operator dimensions from 0 to 8. But the final result comes almost entirely from the last term of OPE – the operator of dimension 8. (There are other drawbacks in these calculations – see [28]). It is clear that such situation is unsatisfactory. Recently, Oganesian [29] had calculated the next term in OPE for $b_{p-n}$ determination – the dimension 10 term and found that by absolute value it is equal to (10), but has opposite sign, so the total result is zero. Of course, we cannot believe in this statement either: it means only that the results of the calculations are unstable and the value (10) characterizes the answer by the order of magnitude only.

The other way to determine higher twist corrections suggested in [14] is based on the connection of $\Gamma_{p,n}(Q^2)$ at high $Q^2$ with the Gerasimov-Drell-Hearn [16] (GDH) sum rule at $Q^2 = 0$. The idea is the following. For the real photon there is only the spin-dependent photon-nucleon scattering amplitude $S_1(\nu)$, for which we can write an unsubtracted dispersion relation

$$S_1(\nu) = 4 \int_0^\infty \nu' d\nu' \frac{G_1(\nu',0)}{\nu'^2 - \nu^2}$$ \hspace{1cm} (24)

Consider the limit $\nu \to 0$ in (24). According to the F.Low theorem the terms proportional to $\nu^0$ and $\nu^4$ in the expansion in powers of $\nu$ of the photon-nucleon scattering amplitude are expressed via static characteristics of nucleon.
The calculation gives
\[ S_1(\nu)_{\nu \to 0} = -\kappa^2, \]  
where \( \kappa \) is the nucleon anomalous magnetic moment: \( \kappa_p = 1.79, \kappa_n = -1.91. \)

From (24), (25) the GDH sum rule follows:

\[
\int_0^\infty \frac{d\nu}{\nu} G_1(\nu, 0) = -\frac{1}{4} \kappa^2
\]  

(26)

Till now only indirect check of (26) was performed, where in the l.h.s the parameters of resonances, obtained from the \( \pi N \) scattering phase analysis, where substituted. In this way with resonances up to 1.8 GeV it was obtained [30] (see also the second reference in [14] and [31])

| l.h.s of (26) | r.h.s. of (26) |
|-------------|----------------|
| proton      | -1.03          |
| neutron     | -0.83          |

The l.h.s. and the r.h.s. of (26) are not in a good agreement – a nonresonant contribution is needed. The direct check of the GDH sum rule would be very desirable!

An important remark: the forward spin dependent photon–nucleon scattering amplitude has no nucleon pole. This means that there is no nucleon contribution in the l.h.s. of GDH sum rule – all contributions come from excited states: the GDH sum rule is very nontrivial.

In order to connect the GDH sum rule with \( \Gamma_{p,n}(Q^2) \) consider the integrals [15]

\[ I_{p,n}(Q^2) = \int_{Q^2/2}^\infty \frac{d\nu}{\nu} G_{1,p,n}(\nu, Q^2) \]  

(28)

It is easy to see that at large \( Q^2 \)

\[ I_{p,n}(Q^2) = \frac{2m^2}{Q^2} \Gamma_{p,n}(Q^2) \]  

(29)

and at \( Q^2 = 0 \) (28) reduces to the GDH sum rule. For proton \( I_p \) is negative at \( Q^2 = 0 \) and positive at large \( Q^2 \), what indicates to large nonperturbative corrections. In [14] the VDM based interpolation model was suggested, describing \( I_{p,n}(Q^2) \) in the whole domain of \( Q^2 \). The model was improved in the second Refs. 14, where the contributions of baryonic resonances up to \( W = 1.8\text{GeV} \), taken from experiment, where accounted. The model has no free parameters, besides the vector meson mass, for which the value \( \mu_V^2 = 0.6\text{GeV}^2 \) was chosen. Using this model it is possible to calculate the higher twist contributions in (2),(12). The results are presented in Table 1, as the ratio of asymptotic \( \Gamma^{as} \) with power corrections excluded to the experimentally measurable \( \Gamma \) at given \( Q^2 \) [18].

**Table 1.**

Higher twist corrections in GDH sum rule + VDM inspired model.

| \( Q^2(\text{GeV}^2) \) | 2   | 3   | 5   | 10  |
|------------------------|-----|-----|-----|-----|
| \( \Gamma^{as}_p/\Gamma_p \) | 1.44 | 1.29 | 1.18 | 1.08 |
| \( \Gamma^{as}_n/\Gamma_n \) | 1.30 | 1.20 | 1.13 | 1.06 |
| \( \Gamma^{as}_{p-n}/\Gamma_{p-n} \) | 1.45 | 1.29 | 1.18 | 1.08 |
| \( \Gamma^{as}_{p+n}/\Gamma_{p+n} \) | 1.47 | 1.31 | 1.19 | 1.08 |
The power corrections, given in Table 1 are essentially larger (except for the case of neutron), than the values (10),(23) found in [13]. It must be mentioned that the accuracy of the model in the domain of intermediate \( Q^2 \), where it is exploited, is not completely certain.

2 Comparison with experiment.

When comparing the sum rules (2),(12) with experiment I consider two limiting variants of perturbative corrections: small with \( \Lambda_3 = 200\text{MeV} \) and large with \( \Lambda_3 = 400\text{MeV} \). \( \alpha_s \) is computed in 2-loop approximation, it is assumed that the number of flavours \( N_f = 3 \). For higher twist correction I also consider two limiting options: small (S), given by (10),(23) and large (L), determined by the data of Table 1. The contribution of gluons \( \Delta g(Q^2) \) in (12) will be found in the following way. Let us assume, that at \( 1 \text{GeV} \) the quark model is valid and \( L_z(1\text{GeV}^2) = 0 \) in eq.(18). Taking \( \Sigma = 0.3 \), what is a reasonable average of the data, we have from (18)

\[
\Delta g(1\text{GeV}^2) = 0.35 \tag{30}
\]

The \( Q^2 \) dependence of \( \Delta g \) can be found from the evolution equation \cite{32}

\[
\Delta g(Q^2) = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \left\{ 1 + \frac{2N_f}{b\pi} \left[ \alpha_s(Q^2) - \alpha_s(\mu^2) \right] \right\} \Delta g(\mu^2) + \\
+ \frac{4}{b} \left[ \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} - 1 \right] \Sigma(\mu^2), \tag{31}
\]

where \( b = 11 - (2/3)N_f = 9 \), \( \mu^2 = 1\text{GeV}^2 \) and \( \Sigma(1\text{GeV}^2) \approx 0.3 \). As the calculation shows, the change of scale at which the quark model is assumed to work (say \( 0.5\text{GeV}^2 \) instead of \( 1\text{GeV}^2 \)) or the of use slightly different \( \Sigma(\mu^2) \) in (31) only weakly influence the results for \( \Sigma \) and \( \Delta s \), obtained from experimental data.

I consider the following experimental data (Table 2).

| Experimental group | The target | \( \Gamma(p, n) \) | Mean \( Q^2(\text{GeV}^2) \) |
|--------------------|------------|------------------|-------------------|
| EMC \cite{33}      | \( p \)    | \( \Gamma_p = 0.126 \pm 0.010 \pm 0.015 \) | 10.7              |
| SMC \cite{34}      | \( p \)    | \( \Gamma_p = 0.136 \pm 0.011 \pm 0.011 \) | 10.5              |
| E143 \cite{35}     | \( p \)    | \( \Gamma_p = 0.127 \pm 0.004 \pm 0.010 \) | 3                 |
| E142 \cite{36}     | \( He^3 \) | \( \Gamma_n = -0.022 \pm 0.011 \) | 2                 |
| SMC \cite{37}      | \( d \)    | \( \Gamma_d = 0.034 \pm 0.009 \pm 0.06 \) | 10                |
| \( \Gamma_p + \Gamma_n = 0.073 \pm 0.022 \) | 10 |
| E143 \cite{38}     | \( d \)    | \( \Gamma_d = 0.042 \pm 0.003 \pm 0.004 \) | 3                 |
| \( \Gamma_p + \Gamma_n = 0.0908 \pm 0.006 \pm 0.008 \) | 3 |
In comparison with experiment the perturbative and higher twist corrections, as well as $\Delta g(Q^2)$ contributions are calculated for $Q^2$. Experimentally, $Q^2$ are different in different $x$-bins (higher $Q^2$ at larger $x$). This effect is not accounted in the calculation. The ratio of $\alpha_s^3$ term to $\alpha_s^2$ term in perturbative corrections is of order of 1 at $\Lambda_3 = 400\,\text{MeV}$ and $Q^2 \approx 2 - 3\,\text{GeV}^2$ (as well as $\alpha_s^4/\alpha_s^3$ estimate). For this reason we introduce in these cases an additional error equal to the $\alpha_s^3$. The values of $\Sigma$ and $\Delta s$ calculated from comparison of experimental data with eq.12 are shown in Table 3.(The errors are summed in quadrature).

| Experiment | $\Lambda_3$ MeV | High twist | $\Sigma$          | $\Delta s$          |
|------------|-----------------|------------|-------------------|---------------------|
| EMC p      | 200             | S          | 0.21 ± 0.17       | −0.13 ± 0.06        |
|            | 400             | S          | 0.29 ± 0.17       | −0.10 ± 0.06        |
|            | 200             | L          | 0.285 ± 0.17      | −0.10 ± 0.06        |
|            | 400             | L          | 0.37 ± 0.17       | −0.07 ± 0.06        |
| SMC p      | 200             | S          | 0.30 ± 0.14       | −0.10 ± 0.05        |
|            | 400             | S          | 0.39 ± 0.14       | −0.07 ± 0.05        |
|            | 200             | L          | 0.39 ± 0.14       | −0.07 ± 0.05        |
|            | 400             | L          | 0.47 ± 0.14       | −0.04 ± 0.05        |
| E143 p     | 200             | S          | 0.28 ± 0.10       | −0.10 ± 0.03        |
|            | 400             | S          | 0.42 ± 0.10       | −0.06 ± 0.03        |
|            | 200             | L          | 0.57 ± 0.10       | −0.006 ± 0.03       |
|            | 400             | L          | 0.71 ± 0.10       | 0.04 ± 0.03         |
| E142 n (3 He) | 200        | S          | 0.60 ± 0.12       | 0.003 ± 0.04        |
|            | 400             | S          | 0.57 ± 0.12       | −0.005 ± 0.04       |
|            | 200             | L          | 0.64 ± 0.12       | 0.016 ± 0.04        |
|            | 400             | L          | 0.61 ± 0.12       | 0.008 ± 0.04        |
| SMC d      | 200             | S          | 0.27 ± 0.10       | −0.11 ± 0.03        |
|            | 400             | S          | 0.33 ± 0.10       | −0.09 ± 0.03        |
|            | 200             | L          | 0.29 ± 0.10       | −0.10 ± 0.03        |
|            | 400             | L          | 0.34 ± 0.10       | −0.08 ± 0.03        |
| E143 d     | 200             | S          | 0.37 ± 0.06       | −0.07 ± 0.02        |
|            | 400             | S          | 0.44 ± 0.06       | −0.05 ± 0.02        |
|            | 200             | L          | 0.48 ± 0.06       | −0.04 ± 0.02        |
|            | 400             | L          | 0.54 ± 0.06       | −0.015 ± 0.02       |

Remark: the contribution to $\Sigma$ of the term proportional to $\Delta g$ is approximately equal to 0.06 in the case of $\Lambda_3 = 200\,\text{MeV}$ and 0.11 in the case of $\Lambda_3 = 400\,\text{MeV}$.

If we assume that all the analysed above experiments are correct in the limits of their quoted errors (or, may be, 1.5 st.deviations), then requiring for the results for $\Sigma$ and $\Delta s$ from various experiments to be consistent, we may reject some theoretical possibilities. A look at the Table 3 shows that the variant $\Lambda_3 = 400\,\text{MeV}, L$ (a contradiction of E143, p and SMC, d results for $\Sigma$) and, less certain, the variant $\Lambda_3 = 200\,\text{MeV}, S$ (a contradiction of E142, n and SMC, d) may be rejected.
Consider now the Bjorken sum rule. For comparison with theory I choose combinations of the SMC data - on proton and deuteron, the E143 data - on proton and deuteron and the E143 data on proton and the E142 on neutron ($^3$He). The results of the comparison of the experimental data with the theory are given in Table 4.

Table 4.

| Combination of experiments | $(\Gamma_p - \Gamma_n)_{exper.}$ | $\Lambda_3$(MeV) | High twist | $(\Gamma_p - \Gamma_n)_{th}$ |
|---------------------------|----------------------------------|------------------|------------|-----------------------------|
| SMC, p                    | $0.199 \pm 0.038$                | 200 S            | 0.193      |
| SMC, d                    |                                  | 400 S            | 0.186      |
| E143,d                    | $0.163 \pm 0.010 \pm 0.016$     | 200 S            | 0.182      |
| E143,p                    |                                  | 400 S            | 0.168      |
| E143,d rec.               | $0.147 \pm 0.015$                | 200 S            | 0.182      |
| E142 n($^3$He)            |                                  | 400 S            | 0.168      |

From Table 4 we see again some indications for rejection of variants $\Lambda_3 = 400 MeV$, L and, less certain, $\Lambda_3 = 200 MeV$, S.(In the first case there is a contradiction of the theory with the E143,p and d data, in the second - with the E143,p , E142, n data).

At existing experimental accuracy it is impossible to choose from the data the true values of $\Lambda_3$ and twist-4 correction. My personal preference is to the variant $\Lambda_3 = 200 MeV$ and to the value of twist-4 corrections 3 times smaller than given by the GDH sum rule + VDM inspired model and, correspondingly, $b_p = 0.04$ in (12), i.e., 2.2 times larger than the BBK result. The argument in the favour of such choice is that at larger $\Lambda_3$ there will arise many contradictions with the description of hadronic properties in the framework of the QCD sum rules. The recent preliminary SLAC data [39] on $g_1(x, Q^2)$ $Q^2$-dependence indicate that $\Gamma^{as}_p/\Gamma_p = 1 + c_p/Q^2$, $c_p = 0.25 \pm 0.15$ what is compatible with the estimate above. In this case all experimental data except for E142,n, are in a good agreement with one another and the values of $\Sigma$ and $\Delta s$ averaged over all experiments, except for E142,n are

$$\Sigma = 0.35 \pm 0.05 \quad \Delta s = -0.08 \pm 0.02$$  \hspace{1cm} (32)

(see also [42] where the values close to (32) were obtained).

The values of $\Sigma$ and $\Delta s$ obtained from the E142,n experiment at such a choice of $\Lambda_3$ and twist-4 corrections, are different:

$$\Sigma = 0.61 \pm 0.12 \quad \Delta s = 0.01 \pm 0.04$$  \hspace{1cm} (33)

It is impossible to compete (32),(33) by any choice of $\Lambda_3$ and higher twist correction. Perhaps, this difference is caused by inaccounted systematic errors in the E142 experiment.
A remarkable feature of the result (32) (as well as of the data in Table 3) is the large value of $|\Delta s|$ - the part of the proton spin projection carried by strange quarks. This value may be compared with the part of the proton momentum carried by strange quarks

$$V_2^s = \int dx \ x [s_+(x) + s_-(x)] = 0.026 \pm 0.006 \ [40], \ \ 0.040 \pm 0.005 \ [41] \ (34)$$

The much larger value of $|\Delta s|$ in comparison with $V_2^s$ contradicts the standard parametrization

$$s_+(x) + s_-(x) = A \ x^{-\alpha} (1 - x)^\beta$$

$$s_+(x) - s_-(x) = B \ x^{-\gamma} (1 - x)^\beta$$  \ (35)

and requirement of positiveness of $s_+$ and $s_-$, if $\alpha \approx 1$ (pomeron intercept) and $\gamma \leq 0$ ($a_1$ intercept). Large $|\Delta s|$ and small $V_2^s$ means that the transitions $\bar{s}s \rightarrow \bar{u}u + \bar{d}d$ are allowed in the case of the operator $j_{\mu\nu}$ and are suppressed in the case of the quark energy-momentum tensor operator $\Theta_{\mu\nu}$, corresponding to the matrix element $V_2$. Such situation can be due to nonperturbative effects and the instanton mechanism for its explanation was suggested $[43]$. Improvement of experimental accuracy is necessary in order to be sure that the inequality $|\Delta s| \gg V_2^s$ indeed takes place.

### 3 The calculations of the polarized structure functions by the QCD sum rule method

I mention here only some basing points of the calculations referring for details to $[44]$. The calculation was performed on the basis of OPE with the account of the unit operator and of the square of quark condensate $\sim \alpha_s < 0 \ | \bar{\psi} \psi | 0 >^2$. It was shown that for the bare quark loop the Bjorken and Burkhardt-Cottingham sum rules are fulfilled. It was also found that for the function $g_1(x)$ the results are reliable in a rather narrow domain of intermediate $x : 0.5 \leq x \leq 0.7$. In this domain the contribution of $u$-quarks $g_1^u$ is much larger than $d$-quarks, $g_1^u \gg g_1^d$. Therefore, $g_1 \approx (4/9) \ g_1^u$. $g_1^u$ was calculated and for the mean value of $g_1$ in this interval it was obtained (at $Q^2 \sim 5 - 10 GeV^2$):

$$g_1(0.5 < x < 0.7) = 0.05 \pm 50\% \ (36)$$

The large uncertainty in (36) results from the large contribution of nonleading term in OPE and from large background at the phenomenological side of the QCD sum rule. The E143 proton $[35]$ and deuteron $[38]$ data (the latter under assumption that $g_1^u$ is small in the interval $x$) give roughly the same values:

$$g_1 \ (0.5 \leq x \leq 0.7) = 0.08 \pm 0.02 \ (37)$$

This value is in a good agreement with the SMC result $[34]$

$$g_1 \ (0.4 \leq x \leq 0.7) = 0.08 \pm 0.02 \pm 0.01 \ (38)$$

and with our theoretical expectation (36).

For the case of the structure function $g_2$ only $g_2^u$ can be calculated at $0.5 < x < 0.8$, the calculation of $g_2^d$ fails because of large contribution of nonleading terms in OPE. If we assume that like in the case of $g_1$, $| g_2^d | \ll | g_2^u |$,

$$g_2(0.5 < x < 0.8) = -0.05 \pm 50\% \ (39)$$
The E143 data in this interval of $x$ are:

\[ g_2(0.5 < x < 0.8) = -0.037 \pm 0.020 \pm 0.003 \quad (40) \]

in a good agreement with (39).

4 What would be desirable to do experimentally in the near future?

1. To increase the accuracy by 2-3 times.
2. To study the $Q^2$-dependence (in separate bins in $x$).
3. To go to higher $Q^2$ (at HERA). The experiment at higher $Q^2$ will be informative if only its accuracy will not be worse than the existing ones.
4. To have better data in the domain of small $x$. (Probably, this also can be done only at HERA).
5. Measurements of two-jets events in polarized deep-inelastic scattering, which will give information about $\Delta g$.
6. To have more data on $s$-quark distribution in nonpolarized nucleon, particularly at $x < 0.1$.
7. To have better data for $g_2$.
8. To perform a direct check of the GDH sum rule.

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