Pathways to Naturally Small Dirac Neutrino Masses

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Abstract

If neutrinos are truly Dirac fermions, the smallness of their masses may still be natural if certain symmetries exist beyond those of the standard model of quarks and leptons. We perform a systematic study of how this may occur at tree level and in one loop. We also propose a scotogenic version of the left-right gauge model with naturally small Dirac neutrino masses in one loop.
**Introduction:**

If neutrinos are Majorana fermions, then it has been known [1] since 1979 that they are described by a unique dimension-five operator beyond the standard model of quarks and leptons, i.e.

\[
L_5 = -\frac{f_{ij}}{2\Lambda}(\nu_i\phi^0_l - l_i\phi^+)(\nu_j\phi^0_l - l_j\phi^+) + H.c. \tag{1}
\]

Neutrino masses are then proportional to \(v^2/\Lambda\), where \(v = \langle \phi^0 \rangle\) is the vacuum expectation value of the Higgs doublet \((\phi^+, \phi^0)\). This formula is necessarily seesaw because \(\Lambda\) has already been assumed to be much greater than \(v\) in the first place. It has also been known [2] since 1998 that there are three specific tree-level realizations (denoted as Types I,II,III) and three generic one-particle-irreducible one-loop realizations.

If neutrinos are Dirac fermions, then the term \(m_D\bar{\nu}_L\nu_R\bar{\phi}^0\) is desired but \((m_N/2)\nu_R\nu_R\) must be forbidden. This requires the existence of a symmetry, usually taken to be global \(U(1)_L\) lepton number. This may be the result of a spontaneously broken \(U(1)_{B-L}\) gauge symmetry where the scalar which breaks the symmetry carries three [3] and not two units of \(B - L\) charge. On the other hand, global \(U(1)_L\) is not the only possibility. The notion of lepton number itself may in fact be discrete. It cannot of course be \(Z_2\), then \(m_N\) would be allowed and neutrino masses are Majorana. However, it may be \(Z_3\) [4, 5] or \(Z_4\) [6, 7, 8], but then new particles must appear to legitimize this discrete lepton symmetry. Since there are three neutrinos, a flavor symmetry may also be used to forbid the \(\nu_R\nu_R\) terms [9].

To obtain a naturally small \(m_D\), there must be another symmetry which forbids the dimension-four \(\bar{\nu}_L\nu_R\bar{\phi}^0\) term, but this symmetry must also be softly or spontaneously broken, so that an effective \(m_D\) appears, at tree level or in one loop, suppressed by large masses. The symmetry used to achieve this is model-dependent. Nevertheless generic conclusions may be obtained regarding the nature of the necessary particles involved, as shown below.
*Four specific tree-level realizations*:

Assume a symmetry $S$ under which $\nu_L$ and $\phi^0$ do not transform, but $\nu_R$ does. There are then four and only four ways to connect them at tree level through the soft breaking of this symmetry.

- Insert a Dirac fermion singlet $N$ which does not transform under $S$, then break $S$ softly by the dimension-three $\bar{\nu}_R N_L$ term.

\[
\begin{align*}
\phi^0 \\
\downarrow \\
\nu_L \times N_R \times N_L \times \nu_R 
\end{align*}
\]

Figure 1: Dirac neutrino mass with a Dirac singlet fermion insertion.

- Insert a Dirac fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ which does not transform under $S$, then break $S$ and $SU(2)_L \times U(1)$ together spontaneously to obtain the dimension-three $\bar{\nu}_R \Sigma^0_L$ term.

\[
\begin{align*}
\phi^0 \\
\downarrow \\
\nu_L \times \Sigma^0_R \times \Sigma^0_L \times \nu_R 
\end{align*}
\]

Figure 2: Dirac neutrino mass with a Dirac triplet fermion insertion.

- Insert a Dirac fermion doublet $(E^0, E^-)$ which transforms as $\nu_R$ under $S$, then break $S$ softly by the dimension-three $(\bar{E}^0 \nu_L + E^+ e^-)$ term.
• Insert a scalar doublet \((\eta^+, \eta^0)\) which transforms as \(\nu_R\) under \(S\), then break \(S\) softly by the dimension-two \((\eta^- \phi^+ + \bar{\eta}^0 \phi^0)\) term.

In Figs. 1 to 3, the mechanism which makes \(m_D\) small is the Dirac seesaw \([10]\). The 2\(\times\)2 mass matrix linking \((\bar{\nu}_L, \bar{\psi}_L)\) to \((\nu_R, \psi_R)\), where \(\psi = N, \Sigma^0, E^0\), is of the form

\[
M_{\nu\psi} = \begin{pmatrix} 0 & m_1 \\ m_2 & M_{\psi} \end{pmatrix}.
\]

Since \(M_{\psi}\) is an invariant mass, it may be assumed to be large, whereas \(m_{1,2}\) come from either electroweak symmetry breaking or \(S\) breaking and may be assumed small in comparison. Hence \(m_D \simeq m_1 m_2 / M_{\psi}\) is naturally small as desired.

In Fig. 4, the mechanism is also seesaw but in the scalar sector, as first pointed out in Ref. \([11]\). Using the small soft \(S\) breaking term \(\bar{\eta}^0 \phi^0\) together with a large mass for \(\eta\), a small vacuum expectation value \(\langle \eta^0 \rangle\) is induced to obtain \(m_D\) \([12]\). This may also be accomplished by extending the gauge symmetry \([13, 14]\).
Two generic one-loop realizations:

Suppose the new particles considered previously for connecting $\nu_L$ with $\nu_R$ at tree level are not available, then a Dirac neutrino mass may still occur in one loop. Assuming that this loop consists of a fermion line and a scalar line, then the external Higgs boson must couple to either the scalar line or the fermion line, yielding two generic diagrams.

- Consider the one-loop connection shown below. Since $\nu_R$ transforms under $S$ and $\nu_L$ and $\phi^0$ do not, a Dirac neutrino mass is only generated if $S$ is broken softly by either the dimension-three $\bar{\psi}_L\psi_R$ term or the dimension-three $\bar{\eta}\chi\phi^0$ term. There are an infinite number of solutions for the new fermion $\psi$ and the new scalars $\eta$ and $\chi$. Under the electroweak $SU(2)_L \times U(1)_Y$, the three simplest solutions are listed in Table 1. Note that solutions also exist with $\psi, \chi, \eta$ all carrying color. Let $S$ be $Z_2$ as an example, then the assignments of $\eta, \psi_R, \psi_L$, and $\chi$ under $S$ are given in Table 2. The

![Diagram](image)

**Figure 5:** Dirac neutrino mass in one loop with trilinear scalar coupling.

| solution | $\psi$   | $\eta$       | $\chi$       |
|----------|----------|---------------|---------------|
| A        | (1,0)    | (2, $-1/2$)  | (1,0)         |
| B        | (2, $1/2$) | (1,0)        | (2, $1/2$)    |
| C        | (2, $-1/2$) | (1, $-1$)    | (2, $-1/2$)  |

Table 1: $SU(2)_L \times U(1)_Y$ assignments of $\psi$, $\eta$, and $\chi$.  

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Table 2: $S = Z_2$ assignments of $\eta$, $\psi_R$, $\psi_L$, and $\chi$.

| solution | $\eta$ | $\psi_R$ | $\psi_L$ | $\chi$ | $\bar{\eta}\chi\phi^0$ |
|----------|--------|----------|----------|--------|-----------------|
| A1       | 0      | -        | -        | +      | +               |
| A2       | -      | -        | +        | -      | -               |
| B1       | +      | +        | +        | -      | -               |
| B2       | -      | -        | -        | -      | +               |
| C1       | +      | +        | +        | -      | -               |
| C2       | -      | -        | +        | -      | +               |

solutions A1 and B1 must be discarded, because $\chi$ and $\eta$ are neutral scalar singlets which are even under $Z_2$ respectively. As such, they will acquire vacuum expectation values from interactions with $\Phi$. From the trilinear coupling $\bar{\eta}\chi\phi^0$, this in turn would induce a vacuum expectation value for $\eta$ and $\chi$ in A1 and B1 respectively. Hence the loop of Fig. 5 would collapse to a tree as shown in Figs. 1 and 3. The solutions A2(B2) should also be discarded because $\psi_{R,L}$ transform exactly as $\nu_{R,L}$, thus collapsing to Fig. 4. However, these solutions could be reinstated with the scotogenic mechanism to be discussed later.

- Consider now the other possible connection. The only soft term here is the quadratic

$$\bar{\eta}\chi$$

term which must be odd under $S = Z_2$. If $\eta$ and $\chi$ are neutral, then again they

\[ \bar{\eta}\chi \]

Figure 6: Dirac neutrino mass in one loop with quadratic scalar mixing.
must have vacuum expectation values, thus collapsing the loop of Fig. 6 to a tree. Hence \( \eta \) and \( \chi \) must be charged or colored, and if \( \chi \sim \pm \) under \( S \), then \( \psi_{L,R}, \eta \sim \mp \). An example of such a model is Ref. [15]. It has also been implemented in left-right gauge models many years ago [16, 17].

In the above, there must be of course also a symmetry which maintains lepton number. This symmetry may propagate along the fermion line in the loop, which is the conventional choice, but it may also propagate along the scalar line in the loop. If the latter, then lepton number may serve as the stabilizing symmetry of dark matter [18]. The reason is very simple. For the lightest scalar, say \( \eta \), having lepton number which is conserved, it can only decay into a lepton plus a fermion which has no lepton number, say \( \psi \), and vice versa. Hence the lightest \( \psi \) or the lightest \( \eta \) is dark matter. This means that the loop diagrams of Figs. 5 and 6 could be naturally scotogenic, from the Greek ‘scotos’ meaning darkness. This mechanism was invented 10 years ago [19]. The unconventional assignment of lepton number to scalars and fermions also reinstates the solutions A,B considered earlier, because now the particles in the loop have odd dark parity, as discussed in Ref. [20, 21]. This application of the one-loop diagram for Dirac neutrino mass using scalars carrying lepton number is actually well-known in supersymmetry, where the exchange of sleptons and neutralinos contributes to charged-lepton masses. Here we show that the generic idea is also applicable without supersymmetry.

**Scotogenic Dirac neutrino mass in left-right model**:

The absence of a tree-level Dirac neutrino mass may be due to the underlying gauge symmetry and the scalar particle content. Consider the following left-right gauge mode based on \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \) together with a discrete \( Z_2 \) symmetry. It extends the standard model (SM) to include heavy charged quarks and leptons which are odd under \( Z_2 \), but no scalar bidoublet [22] as shown in Table 3. Its fermion content is identical to a
The breaking of $SU(2)_{L,R}$ is accomplished by the Higgs doublets $\Phi_{L,R}$. The SM quarks and charged leptons obtain masses from $\Phi_L$. The heavy quarks and charged leptons obtain masses from $\Phi_R$. They are separated by the $Z_2$ symmetry and do not mix. Both $\nu_L$ and $\nu_R$ are massless and separated by $Z_2$. To link them with a Dirac mass, this $Z_2$ has to be broken.
This is implemented as shown in Table 4 using the unbroken symmetry $Z_2^D$ for dark matter, under which $N, \eta_{L,R}, \chi_{L,R}$ are odd and all others are even. The symmetry $Z_2$ is assumed to be respected by all dimension-three terms as well, so there is no $\bar{Q}_L q_R$ or $\bar{E}_L e_R$ term. Hence $V_{CKM}$ remains unitary as in the SM. It is broken only by the unique dimension-two term $\chi_L \chi_R$. The resulting scotogenic diagram for Dirac neutrino mass is shown in Fig. 7. The connection between the heavy fermions of the $SU(2)_R$ sector and the SM fermions is

\[
\begin{align*}
&\phi^0_L \quad \chi_L \quad \chi_R \quad \phi^0_R \\
&\eta^0_L \quad \eta^0_R \\
&\nu_L \quad N_R \quad N_L \quad \nu_R
\end{align*}
\]

Figure 7: Scotogenic Dirac neutrino mass in left-right symmetry.

$\chi_0$ with the allowed dimension-four Yukawa couplings $\chi_0 \bar{Q}_L q_R$ and $\chi_0 \bar{E}_L e_R$. Now $\chi_0$ mixes only radiatively with the SM Higgs boson, a phenomenon discovered only recently [24], and decays to SM particles but its lifetime may be long. At the Large Hadron Collider, the heavy $SU(2)_R$ quarks are easily produced if kinematically allowed. The lightest will decay to a SM quark and $\chi_0$ which may escape the detector as missing energy. This has the same signature as dark matter. The true dark matter is of course the lightest neutral fermion or boson with odd $Z_2^D$.

Concluding remarks:
The notion that neutrino masses are Dirac is still viable in the absence of incontrovertible experimental proof of the existence of neutrinoless double beta decay. The theoretical challenge is to understand why. In this paper we study systematically how the smallness of Dirac neutrino masses may be achieved at tree level (four specific cases) and in one loop (two generic cases). We also propose a scotogenic left-right gauge model.
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