Single photon wave-front characterization by optical mixing

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Abstract

Optical mixing experiments show the ability of amplifying a weak optical signal by superposing it with a stronger one. This principle has been demonstrated also for weak signals at the quantum level, down to a single photon. In the present communication it is suggested that the sensitivity of optical mixing between a strong macroscopic source and a single photon can be further enhanced as to allow the sensing the wavefront of the photon’s mode simultaneously at two or more locations. Key conditions for that detection is reducing the active size of the detectors below the typical size of the transverse modes, and performing an optical intensity correlation measurement of the Hanbury Brown and Twiss type. Due to the inherent amplification effect of the mixing process, a macroscopic signal is extracted, out of which the photon wave-front characterization at more than one location is achievable with good fidelity even for a single photon emission event. A basic scheme is proposed for the demonstration of the effect, which is analyzed based on a simple quantum model. The validity of the model is confirmed by comparison with previous theoretical and experimental reports involving single photon sources.

Single photons are nowadays a preferred building block for an increasing number of quantum-information related technologies, and as a consequence, their generation, propagation and detection are being vigorously pursued [1]. The question of location of a photon in more than one place simultaneously has driven plenty of research wherever the wave-particle nature of photons is manifested. In most reported situations, if a truly single photon source was implemented, the final experimental detection involved also a
single photon detection event, and in order to characterize spatially the photon’s field
distribution at an extended area, the statistical monitoring of multiple events was required
[2-4]. An exception to this rule seems to be the proposition by Zagoskin et Al. [5], who
suggested the entangling interaction of an incoming photon with a quantum metamaterial
sensor array. In the present communication an alternative method for single photon
transverse wave-front profiling is suggested based on a Hanbury Brown and Twiss
(HBT)-type interferometer in which a single photon field is mixed with a strong
macroscopic optical field. It is theoretically demonstrated that in such an arrangement, a
macroscopic correlation signal can be generated in which the wavefront is manifested at
two or more different points. The correlation signal depends on the local amplitude value
of the field at the detectors’ position, implying that an entire transverse mode can be
quantitatively delineated in a single photon event.

The basic proposed scheme is depicted in Fig.1. This arrangement shares common
features with many optical mixing setups reported, and it is immediately associated with
a photonic correlation setup involving a single or multiple photon sources [6]. A
prominent previously reported arrangement, closest to the one presented here was
demonstrated by Rarity et Al. [7], in which photons from different sources were
combined to show a non-classical anti-correlation Ou-Hong-Mandel (HOM) dip. The fact
that quantum interference effects are present for photon from independent sources was
predicted back in 1983 by Mandel [8], and further-on demonstrated in many following
reports [6,9]. Choosing [7] as a reference setup, in which a single-photon field was mixed
with an highly attenuated Coherent State (CS) ($|\alpha|^2 < 5$), the setup proposed here differs
from it in several aspects: First the attenuated CS port is replaced by a general optical state eventually highly populated, that will be called as customary Local Oscillator (LO). The second modification is that the transverse spatial mode excited by the single photon is converted from a basic Gaussian into a higher one (e.g. TEM$_{1,0}$) this is proposed in order to emphasize the local character of the detection, since as stated, the main goal here is to actually show that a single correlation measurement involves local information on the photon’s spatial mode. The third difference is that in the scheme proposed here the detectors’ area is small as compared to the mode cross-section of the beams involved, i.e. the detectors are set to be intentionally and inherently inefficient. The fact that the addition of a single photon to a highly populated CS can drastically modify its properties has been reported many years ago [10] and is also exploited here.
Figure 1: The proposed basic scheme. Two mutually coherent optical sources with different modal profiles are mixed by a beam-splitter. One source emits a single photon and the second is a higher intensity Local Oscillator (LO) optical field. At the two output ports, detectors are placed exposing small apertures situated at arbitrary transverse locations \((x_1,y_1)\) and \((x_2,y_2)\). The detectors are connected to an intensity correlator.

In the following, the main detectability effect is theoretically demonstrated by a simple quantum model showing explicitly that the intensity correlation signal carries information on the local electric field of the two interacting modes, meaning that if the reference mode is known, the photon field at the apertures can be determined. The result is then generalized, by replacing one detector by an array, and performing the correlation measurements simultaneously rendering eventually the entire transverse profile of the photon’s mode. Finally the formalism is adapted to calculate the correlation signal for the case the detectors are wide but the optical beams are axially misaligned. These outcomes are connected with previous results in the literature.
For the description of the scheme a standard quantum approach is adopted [4, 11, 12]:

Starting by defining field operators for the positive frequency part of the two electric fields:

\[
\hat{E}_{ph}^+(\vec{r}, t) = A_{ph}(\vec{r}) \cdot \hat{a}_{ph} \quad \hat{E}_{LO}^+(\vec{r}, t) = A_{LO}(\vec{r}) \cdot \hat{a}_{LO}
\]  

(1)

Where \(\hat{a}_{ph}\) and \(\hat{a}_{LO}\) are the usual annihilation operators for the photon and local oscillator fields and \(A_{ph}(\vec{r})\), \(A_{LO}(\vec{r})\) are vector functions carrying the spatial modal and polarization information of the respective modes, namely:

\[
A_{ph}(\vec{r}) = \varepsilon_{ph} U_{ph}(x, y) e^{-ik_{ph}z}, \quad A_{LO}(\vec{r}) = \varepsilon_{LO} U_{LO}(x, y) e^{-ik_{LO}z}
\]  

(2)

The modal functions \(U_{ph,LO}(x, y)\) express the transversal coordinates' dependences which are assumed in general explicitly different for the photon and LO fields. \(\varepsilon_{ph,LO}\) are the basic electric field units associated with one photon. A monochromatric propagating mode was assumed for both waves and their time dependence is implicit in the annihilation operators \(\hat{a}_{ph}, \hat{a}_{LO}\) (Heisenberg Picture). We further assume the two polarizations to be identical and that the fields also share the same angular frequency \(\omega\). The two source fields are combined in the beam splitter:

\[
\begin{bmatrix}
\hat{E}_1^+ \\
\hat{E}_2^+
\end{bmatrix} = 
\begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{E}_{LO} \\
\hat{E}_{ph}
\end{bmatrix}
\]  

(3)

Here the BS is represented as a general S-matrix. The fourth-order field correlation rate is next calculated for partial detectors of area \(dS\) located in the detectors’ planes at positions \((x_1, y_1)\) and \((x_2, y_2)\) respectively:

\[
\eta^{(2)} = \eta^2 \cdot (dS)^2 \left\langle \hat{E}_1^+(r_1) \hat{E}_2^+(r_2) \hat{E}_1^+(r_2) \hat{E}_2^+(r_1) \right\rangle 
\]  

(4)
Where $\eta$ is a common efficiency factor, defined for the case the detecting elements are fully open and uniformly illuminated, and $dS$ is the active area of each detector. This last expression represents the photocurrent cross-correlation for square-law photodetectors and in a photon-counting scenario it is proportional to the photon coincidence count rate. Considering the base function for the expectation-value calculation $|\psi_{LO}\rangle|\phi_{ph}\rangle$ and replacing (1) and (2) into (4), $w^{(2)}$ is straightforwardly evaluated furnishing:

$$w^{(2)} = \eta^2 \cdot (dS)^2 \{ s_{11} s_{22} A_{LO}(\vec{r}_1) A_{LO}(\vec{r}_2) \hat{\langle} n_{LO} (n_{LO} - 1) \hat{\rangle} + s_{11} s_{22} A_{LO}(\vec{r}_1) A_{ph}(\vec{r}_2) + s_{12} s_{21} A_{LO}(\vec{r}_1) A_{ph}(\vec{r}_1) \hat{\langle} n_{LO} \hat{\rangle} \}$$

(5)

For ease of examination, the simplest case of a fully symmetric BS will be replaced namely: $s_{11} = s_{22} = -1/\sqrt{2}$, $s_{12} = s_{21} = i/\sqrt{2}$ rendering:

$$w^{(2)} = \frac{\eta^2}{2} \cdot (dS)^2 \{ A_{LO}(\vec{r}_1) A_{LO}(\vec{r}_2) \hat{\langle} n_{LO} (n_{LO} - 1) \hat{\rangle} + A_{LO}(\vec{r}_1) A_{ph}(\vec{r}_2) - A_{LO}(\vec{r}_2) A_{ph}(\vec{r}_1) \hat{\langle} n_{LO} \hat{\rangle} \}, \quad (6)$$

where $\langle n_{LO} \rangle = \langle \psi_{LO} | \hat{a}_{LO}^\dagger \hat{a}_{LO} | \psi_{LO} \rangle$. Eq (6) is the main outcome of this report and in the following will be carefully examined. It can be immediately stated that the second term in the brackets depends on the local values of the modal functions $A_{ph}(\vec{r}_1)$ and $A_{ph}(\vec{r}_2)$ in which the coordinates $\vec{r}_1$ and $\vec{r}_2$ correspond to locations of detectors, arbitrarily separated, each at different arms away from the BS and at different distances from the respective optical axes. The term is furthermore of macroscopic nature, as being amplified by $\langle n_{LO} \rangle$. The derivation of Eq. (6) contains no approximations beyond the basic assumptions of the model and is therefore expected to be valid for any arbitrary LO
In the following some special cases are inspected critically: If the LO input state contains no photons (vacuum), only the single photon is incident on the BS, and no correlation signal is predicted by Eq. (6) as expected [2,4]. The case where the LO state contains also exactly one photon ($n_{LO} = 1$), deserves more attention: According to the HOM effect [11] no correlation signal is expected for single photons incident at different ports even if the photons originated at different sources [9]. This apparently contradicts the outcome of Eq. (6), since the anti-correlation expression contained in the second term will in general not cancel. It will cancel however if the detectors are wide enough to encompass the entire area of the incoming beams at both detectors. Then, Eq. (6) needs to be integrated over the variables $(\vec{r}_1, \vec{r}_2)$, and for perfect alignment of the modes involved, the integrals will cancel if the modes are identical. This is actually the situation encountered in many experiments, since in general experimentalists seek to maximize the efficiency of the photo-detection. Strikingly, the incomplete cancellation of the HOM dip, when encountered, was attributed to non-perfect overlap between the modes [11-13]. In ref [13], the misalignment effect for the $n_{LO} = 1$ case, was explicitly calculated and associated with measurements. Non perfect overlap in time domain was analyzed in [14] rendering similar expressions. This issue is further discussed at the end of this article.

Returning now to analyze the more general situation depicted in Fig. (1) with partial apertures, the first term within the brackets of Eq. (6) corresponds to the case where only the LO field is present. In the limit $\langle n_{LO} \rangle \gg 1$ one gets a classical geometrical beam-splitting ratio for the LO depending of the product of the field power flux at positions $\vec{r}_i$. 
and \( \tilde{r}_2 \). One should notice that at that limit the first term overvalues the second one as being of the order of \( \langle n_{LO}^2 \rangle \). The point of distinguishing between the two terms is a key issue and it is addressed at the end of the article.

For further analysis of the effect of observing a single photon’s wavefront in multiple points, we isolate the second (heterodyne) term of eq. (6) and annotate it in a slightly different form:

\[
W_{\text{het}}^{(2)}(x_1, y_1; x_2, y_2) = \frac{\eta^2 (dS)^2}{2} |U_{\text{LO}}(x_1, y_1)U_{\text{ph}}(x_2, y_2) - U_{\text{LO}}(x_2, y_2)U_{\text{ph}}(x_1, y_1)|^2 \langle n_{LO} \rangle \tag{7}
\]

We note here first that this term has no explicit time dependence as expected after assuming \( \omega_{\text{ph}} = \omega_{\text{LO}} \). Furthermore, we approximated \( k_{\text{ph}} = k_{\text{LO}} \), cancelling also the \( z \) dependence for travelling wave modes of the same frequency. In practical terms, this means spatial longitudinal beat length [11] is neglected as being long with respect to lengths involved in an experimental situation. The strongest dependence remains on the transversal positions of both detectors \((x_1, y_1)\) and \((x_2, y_2)\). This correlation term will cancel if \( x_1 = x_2, y_1 = y_2 \) meaning the two points are equivalently located in their respective planes with respect to the modes’ profiles (see Fig. 2). It will also cancel if the positions are symmetrically located with respect to their respective optical axes and the modes have the same symmetry, and it would find its maximal expression if modes \( U_{\text{ph}} \) and \( U_{\text{LO}} \) have opposite symmetries. The main point of emphasis is that both partial detectors simultaneously contribute to the mixed signal expressed in eq. (7). If we leave one aperture fixed before detector \( D_1 \) at \((x_1, y_1)\) and scan the second point \((x_2, y_2)\) a graph can
be generated as illustrated in Fig.2 from which the profile of the photon’s mode can be uniquely inferred.

Moreover one can envisage a situation as depicted in Fig. 3, in which instead of the scan, an array of partial detectors $D_m$ is distributed in front of the optical beam at the branch location of detector D2, at positions $\{x_m, y_m\}$ and their output is fed individually to the correlator from which multiple correlation functions $w^{(2,m)}_{H\ell} = w^{(2)}_{H\ell}(x_0, y_0; x_m, y_m)$ can be simultaneously determined. Since all these signals are scaled-up into the macroscopic regime by the factor $\langle n_{LO} \rangle$, one can conclude that the entire mode profile $U_{ph}(x, y)$ of the single photon can be measurably retrieved by means one photon emission event.
FIG. 3. Proposed scheme for the simultaneous profiling of the entire photon mode in a single measurement event. The partial detector at one arm of the intensity interferometer is replaced by a detector array. Each detector in the array is independently fed into the correlator.

Regarding the time dependence or spectral content of the electromagnetic fields involved, a monochromatic (infinite) running wave was assumed for simplicity for both waves. The main results however predicted here are expected to be valid for photons emitted in finite-time packets as long as they remain mutually coherent during the detection process.
The same considerations would apply for sources with slightly different frequencies $\omega_{ph} \neq \omega_{LO}$, or finite bandwidth [14], as long as time coherence is preserved along the relevant measurement time. A main point regarding the macroscopic observability of the predicted effects is the resolution of the second term in Eq. (6) from the dominating first one in the limit $\langle n_{LO} \rangle >> 1$. A customary solution would be the synchronization of the detection event with a twin photon simultaneously emitted by the single-photon source.

As an additional implementation of the formalism developed, the correlation signal for widely opened detectors, as encountered in most experimental situations is calculated by integrating Eq. 6 is over the transverse coordinates. This will allow comparison with established reports on few-photon correlation measurements [7,12,11]. Moreover, the integration allows the introduction of mutual misalignment [13] between the modal profiles of the photon and local oscillator. In Fig. 4 the effect of misalignment is simulated: the integrated correlation signal is calculated for a shifting profile of the photon’s mode while the LO mode is kept fixed at a centered position of the detector window. The width of the detector is $2d$ and the mode center displacement relative to the detector’s center is designated by $x_{d(ph)}$. In Fig 4(a), the HOM configuration is reproduced, namely a single photon state is simulated also for LO port. A zero-dip is attained for the ideal case and the graph resembles reported cases [8,12,21], there however the variable was the delay time between the photon pair and not the mutual displacement between the modes. The effect non-perfect overlap (misalignment), has been reported to be responsible for the non-complete cancelation of the correlation at the center of the dip [11, 12, 13]. In Fig. 4(a) also the case of the photon's mode being
orthogonal to the one of the LO is shown as a separated trace. This situation has not been considered previously in the literature. When the two orthogonal modes are centered and aligned \((x_{o\phi h})=0\), the HOM dip is fully filled and the correlation assumes the uncorrelated value. At slightly misaligned positions however, \((x_{o\phi h})\neq 0\), quantum interference effects are still apparent as adjoin dips even for orthogonal modes. The case of a LO field being in a coherent state represented by \(\alpha_{lo}\), is seen in Fig. 4(b). The photon expectation values in Eq. (7) can be then straightforwardly calculated to be

\[
\langle n_{lo} \rangle = |\alpha_{lo}^2| \quad \text{and} \quad \langle n_{lo}(n_{lo}-1) \rangle = |\alpha_{lo}^4|.
\]

Furthermore the interference visibility between a coherent state and a single photon was calculated for \(0 < |\alpha_{lo}^2| < 5 \) was calculated and Fig. 2 of ref. [7] confirmed (see Appendix). Worth remarking is that although the visibility is reduced as a function of the LO strength, the absolute depth of the HOM dip increases.
FIG. 4. Effect of misalignment in fully open detectors: the integrated correlation signal is calculated for a shifting profile of the photon (PH) mode, while the LO mode is kept fixed at a centered position of its detector window. The full (red) lines correspond to interfering TEM$_{00}$ modes for both beams, and the dotted (blue) line corresponds to a TEM$_{10}$ mode for the PH mode. In (a), a single photon is placed also in the LO port, emulating the HOM effect. In (b), the single photon is mixed with a coherent state of intensity $|\alpha|^2 = 4$, showing the amplification effect. The vertical lines denote the detectors’ aperture. In regions I the PH mode is outside the detector. In II, both modes are inside the detector not overlapping, and in III, partial and full overlap effects are evident.

Resuming, the simultaneous partial measurement of a single photon event at different spatial locations was theoretically demonstrated. Key enabling features for its observation are exposing the fields involved to small area detectors, mixing and correlating of the photon filed with a strong LO field. An extension of the scheme to a detector array would allow the entire modal profiling of a single-photon in a single event. The formalism also was used to calculate the correlation signal for widely opened detectors with misaligned modal shapes. Quantum intensity interference effects for single-photon orthogonal modes were predicted here too. Misalignment effects reported in the literature, which were commonly regarded as undesirable, are actual evidence of the spatially distributed correlation contribution of single photons.

Appendix: Interference visibility calculations for mixing between a single photon and a coherent state.

The fringe intensity visibility has been defined in both the time and space dimensions as the fractional reduction in correlation value at full overlap compared to its value at a non-overlapping situation [7,9]. Here the calculation for the case of mixing between a single photon and a coherent state is supplied, mainly with the purpose of validating the model
applied in the article and associate the output of partial-aperture detectors with reported calculations for partial time overlap. Visibility for partial time overlap, as a function of the intensity was shown in Fig. 2 of ref. [7] and was calculated there by photon statistics considerations. Here it is simulated by the integration of Eq. (7) and depicted in Fig. A1 below, this time for lateral mode varying overlap. Worth remarking is that although the visibility is reduced as a function of the LO strength, the absolute depth of the HOM dip increases, as also shown in Fig. 1.
Fig. A1. Visibility (V) and depth (D) of the HOM dip for the mixing of a single photon with a coherent state as a function of the CS intensity parameter. The HOM depth was arbitrarily set to 1 for $|\alpha| = 4.5$.

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