RISK ANALYSIS OF CHINA STOCK MARKET DURING ECONOMIC DOWNTURNS–BASED ON GARCH-VAR AND WAVELET TRANSFORMATION APPROACHES

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ABSTRACT

Accurately measuring the risk of stock market is important for improving the risk management level of stock market and reducing losses for the investors, especially during economic downturns. In this paper, to study the stock market risk in economic downturns, we first adopted wavelet transformation analysis to detect and define those phases. This paper selected the Shanghai Composite Index and eleven representative sector indexes from January 1, 2008 to January 31, 2020, established GARCH models based on student-t and GED distributions, and found the volatility aggregation and the leverage effect of the stock market. This paper measured the risk of stock market by the parameter method, using conditional volatility estimated by the GARCH models, and carried out back-testing analysis and the Kupiec failure rate test on measurement accuracy of VaR. To study the stock market risk during economic downturns, this paper calculated and compared the risks of different sectors during the corresponding phases, and the results showed that the risks in real estate, industrial, telecom service sectors are much higher during financial downturns. Finally, this paper summarized the conclusion and put forward suggestions for investors from the perspective of risk management.

Contribution/Originality: This study is one of few to have investigated the stock market risks based on the division of economic period. By combining GARCH and VAR models, we studied the commonalities of China stock market risks during economic downturns, which were detected by Wavelet transformation analysis.

1. INTRODUCTION

Since the market economy system reform in 1990s, China’s economy transformed from an old-fashioned and hermetic economy to a modernized global participator. Due to the deep development of economy’s globalization and intergradation, the financial system of China including the stock market has also been adversely affected by the risks all over the world (Qureshi, Kutan, Khan, & Qureshi, 2019; Zhao, Chen, & Zhang, 2019). Thus, how to prevent and control the financial risks have become critical concerns of the government (Bieszk-Stolorz & Markowicz, 2017; Huang, 2019) and analysis of the fluctuations and risks of the stock market has become one of the most important concerns of researchers (Nordin, 2020).

Extant research on the analysis of the financial market risks is diverse. Some research tries to use the sensitive analysis to estimate the risks (Banda, 2019; Inanoglu & Jacobs, 2009) while some researchers use the VaR (Value at Risk), which is one of the most popular methods. With the development of models that estimate volatility, more and
more research combines the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model and VaR together to study the fluctuations of the financial market and to find the law of capital market risk (Ranković, Drenovak, Urošević, & Jelic, 2016; Ren, Shan, & Liang, 2015). Researchers such as, Andersen, Bollerslev, and Christoffersen (2017) systematically studied the risk of the financial market by using the GARCH-VaR model. Chen and Lian (2008) and Xintong (2015) applied the GARCH-VaR model to calculate the risks of investment funds area. Chen, Zhang, and Deng (2018) adopted the GARCH-VaR model to evaluate the risk of China bond market.

Another branch in the area of analyzing financial market risk mainly focuses on the financial market risk during or after a crisis (Huang, Su, & Tsui, 2015; Sun & Sun, 2018). Zhao et al. (2019) used the stock transaction data to explore the systemic risk of China’s stock market during crashes in 2008 and 2015. They compared the market risks in 2008 crash and 2015 crash, and found that the volatility of systemic risk rose in 2015 compared to the one in 2008. Wu and Bai (2018) studied the risks of investment and stock market after the 2007 financial crisis, and the empirical results show that CSI 300 index appeared significant volatility before and during the financial crisis. In addition, in order to study the risk during different periods, there are some extended works that examine how to identify or detect “bull” and “bear” phases in financial market (Frøystad & Johansen, 2017; Gonzalez, Hoang, Powell, & Jing, 2006). Hanna (2018) proposed a top-down approach to identify bull and bear market stage. Kole and Van Dijk (2017) compared and summarised the methods used to identify or forecast the different states of the financial market.

By reviewing the existing literature, we found that most of studies concern the risk of financial market in a general condition or during/after a certain crisis or event. There are very few studies that concern the financial market risk based on the division of economic period, especially the commonalities during economic downturns. During an economic downturn, the volatility of stock market is much higher than the normal phase, and may cause huge losses to investors, which is closely related to the vital interests and the life of ordinary investors. If we could find some commonalities in stock market risk during economic downturns, then we can provide guidance on investment strategies, thus reducing their losses. All in all, the analysis of the fluctuations and risks of the financial market during economy downturns is really needed and has an important research significance.

Therefore, in this paper, we focused on the volatility and market risks of the stock market during economic downturns, and compare the market risks of different sectors, hoping to provide advice to investors. We first calculated the volatilities of Shanghai Composite Index and different sectors’ indices through the GARCH models. Based on the calculated volatilities, we compared the cross-sector market risks by using the VaRs. We focused on the performances of different sectors during economic downturns, which are detected by the wavelet transformation analysis.

The remainder of this paper is arranged as follows. In section 2, the models and methods used in this article as well as the selected stock market data are introduced. In section 3, we used wavelet transformation analysis to detect the peaks and troughs of Chinese stock market, and to define the economic downturns. The data processing is also introduced in this section. In section 4, we conduct the empirical study and present the results and findings, which is followed by a section that briefly concludes the paper by summarizing the findings.

2. MODELS AND DATA

This section briefly introduces the models and methods used in this paper.

2.1. Model Specification

2.1.1. The GARCH Model

The ARCH model (the Autoregressive Conditional Heteroskedasticity model), which is developed by Engle (1995); Engle, Hendry, and Trumble (1985) is widely used in modeling time-varying volatility of financial data series, for it offers a more real-world context than other models when trying to predict the prices and returns of
financial instruments. In 1986, Engle and Bollerslev (1986) proposed a new model named GARCH model to make the model more universal. The GARCH model assumes the variance of the error term follows an autoregressive moving average process, thus can be a better fit for modeling time series data when the data exhibits heteroskedasticity and volatility clustering.

Equation 1 and Equation 2 are the basic form of the GARCH model:

$y_t = x_t \beta + \varepsilon_t \hspace{1cm} (1)$

$\sigma_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \hspace{1cm} (2)$

Where the $\alpha \varepsilon_{t-1}^2$ is the ARCH term and $\beta \sigma_{t-1}^2$ is the GARCH term. They represent the autoregressive and the move average components of GARCH respectively (Bollerslev, 1986). To ensure the stability of the GARCH model, the basic requirements for the establishment of the model are: $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$. If $\alpha + \beta \rightarrow 1$ it means the volatility exists for a long time and has the characteristics of high durability.

Note that, since the GARCH model is built based on the ARCH model, it was necessary to examine whether the time series had the ARCH effect before using it. The detailed process will be described in section 3. In addition, the distribution of residuals in GARCH model mainly includes normal distribution, student-t distribution and generalized error distribution (GED).

2.1.2. VaR—Value at Risk

VaR is a measure that describe the risk of investments (Love & Zichino, 2006; Luintel & Khan, 1999). It emerges as a suitable and remarkable tool to quantify risk and becomes substantially popular and prevalent due to its simplicity. It is widely used to assess the risk exposure of investments and is a measure for market risks especially after 1990s. The definition of VaR (Barone-Adesi, Finta, Legnazzi, & Sala, 2019; Dees, Mauro, Pesaran, & Smith, 2007) is described as the maximum expected loss of an investment at a prespecified confidence level and holding time period. The historical simulation, variance–covariance and Monte Carlo simulation methods are three universal ways to calculate a VaR. Similar to other research (Alexander, Lazar, & Stanescu, 2013; Ruqi, 2019) we used Equation 3 to assess the VaR in the following section.

$$VaR_t = -(\mu_t + \Phi^{-1}(a) \sqrt{\nu_t}) \hspace{1cm} (3)$$

Where $\mu_t$ represents the conditional mean series, $\Phi^{-1}(a)$ represents the critical value corresponding to the confidence level of 1-$\alpha$, and $\nu_t$ represents the conditional variance of the yield series.

In short, yield series often has obvious peak and thick tail characteristics, and another significant feature of yield series is the phenomenon of clustering of volatility. Therefore, finding a suitable method that could explain the volatility phenomenon is important, and the GARCH model was a suitable approach and could satisfy this requirement. Thus, in this paper, we used the GARCH model to find the volatility of the time series and then proceeded to use the VaR to analyze the risk of the financial market.
2.1.3. Wavelet Transformation—Detect Economic and Financial Downturns

With the development of the signal processing technology, scholars tried to use the wavelet transformation analysis to capture the changes of signal. Similarly, wavelet transformation (Nobre & Neves, 2019; Struzik, 2001; Wee, Grayden, Zhu, Petkovic-Duran, & Smith, 2008) also has been used to analyze and predict time series in both time and frequency domain. The global economy is a complex system and changeable, which leads to the fluctuating nature of the transaction data in the financial market. Therefore, how to capture the fluctuations and detect the changes of the financial market was also a main concern of this paper.

According to Percival and Walden (2000) a wavelet is a function \( \psi(t) \) in \( L^2(R) \) such \( \psi(t) \) satisfies Equation 4. It can express the oscillatory nature of the generating function and its frequency domain characteristics:

\[
\int_{-\infty}^{\infty} \psi(t) \, dt = 0
\]  
(4)

A CWT (Continuous Wavelet Transform) on a function \( f(t) \) at point \((t_0, s)\) is described as Equation 5:

\[
W[f, \psi](t_0, s) = \langle f, \psi_{t_0,s} \rangle = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \psi\left(\frac{t-t_0}{s}\right) dt
\]  
(5)

Where \( \psi_{t_0,s}(t) \) is \( \psi(t) \) translated by \( t_0 \) and scaled by \( s \), and we can use Equation 6 to calculate.

\[
\psi_{t_0,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-t_0}{s}\right)
\]  
(6)

The resulting CWT coefficients in Equation 5 contain patterns of peaks and troughs which can be used to detect the position and strength of peaks in \( f(t) \) of similar size to \( \psi_{t_0,s}(t) \). While, we could set different \( s \) in \( \psi_{t_0,s}(t) \) to obtain different-width wavelets, thus all peaks in \( f(t) \) can be detected, no matter what their width is.

Compared with time series analysis in pure time domain, wavelet transformation analysis can provide new perspective and feature robustness for prediction. Unlike the traditional method -- Fourier decomposition technique -- wavelet transformation analysis also has the advantages of locality without global data, and disorderly setting of specific data structures (Mensi, 2019). Therefore, in this paper, we chose the wavelet transformation for analysis and prediction based on the non-stationary and nonlinear characteristics of stock market data.

2.2. Data

This paper focused on analyzing the volatility and the risk of China’s stock market during the economic downturns (“Bear Market”) from 2008 to 2020. To more accurately reflect the volatility of China’s stock market, the Shanghai Composite Index and eleven representative sector indices that best reflect China’s stock market were selected as samples, and the time of the data ranges from January 1st, 2008 to January 30th, 2020. We collected the data from Wind database and the eleven sector indexes were daily consumption, energy, financial, healthcare, industrial, information technology, material, optional consumption, public utility, real estate and telecom service.
sectors. In this paper, we used Python, MATLAB and EViews 10.0 to process the data. Figure 1 shows examples of the daily series of Shanghai composite index and some selected sector indices.

![Figure 1. The daily series of Shanghai Composite Index and some sector indexes from 2008 to 2020.](image)

3. ECONOMIC DOWNTURN DEFINITION AND DATA PREPROCESS

3.1. Economic Downturn Definition Based on Wavelet Transformation

In this paper, we used wavelet transformation analysis to identify the economic and financial downturns. To find the downturn episodes (Bear market) of China stocks market and reflect the whole economy's development and change process, we chose the Shanghai Composite Index as the representative of the Chinese stock market.

Then, we used wavelet transformation analysis method to detect peaks and troughs of the time series and the detected results are showed in Figure 2 (the peaks and troughs are marked in red). To define the economy downturns from 2008 to 2020, we checked the global and domestic economic environment during the selected period and also considered the definitions of “bull” and “bear” phases in Hanna (2018)’s work. He explains that the “bull” and “bear” phases alternate, and exhibit a significant rise (fall) in prices between start and end. Besides, he emphasizes that the prices of each phase should be bounded by the values achieved at the phase end points. We detected and defined four economic downturn periods. The first period was the global financial crisis in 2008 (from 14 Jan. 2008 to 04 Nov. 2008); the second phase was from 08 Nov. 2010 to 27 Jun. 2013, and during this period investors suffered a lot, due to the US Treasury crisis in 2011 and the scarcity of money in 2013; the third stage corresponded to the Chinese stock market crash in 2015 (from 12 Jun. 2015 to 28 Jan 2016); and the last stage was the stock fluctuation during the 2018-2019 (from 24 Jan. 2018 to 03 Jan. 2018). Table 1 shows the specific division of different periods.

3.2. Descriptive Statistical Analysis and Stationarity Test

From Figure 2 and Figure 3a, we can see that the sector index series was not stable. Therefore, before analyzing we had to perform a stationary treatment on the data samples first. Generally, the log-return is often used for the analysis of time series due to its stationary feature. It is defined as show in Equation 7.

$$r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) = \ln(p_t) - \ln(p_{t-1})$$ (7)
Where $p_t$ and $p_{t-1}$ represent the prices of time $t$ and $t-1$ respectively.

Figure 2. The results of the peak-find in the wavelet transformation.

Table 1. Details of the economic downturn division.

| Down cycle of the stock market | Duration/month |
|--------------------------------|----------------|
| 2008.01.14—2008.11.04          | 11             |
| 2010.11.08—2013.06.27          | 19             |
| 2015.06.12—2016.01.28          | 7              |
| 2018.01.24—2019.01.03          | 12             |

Similarly, in this paper, we also adopted the log-return to process the time series. Figure 3b is an example of the processed results. By observing the yield fluctuations of the Shanghai Composite Index and the selected eleven sectors indices, we found that although they all followed a similar trend, the volatilities were indeed different.

Table 2 summarizes the descriptive statistical analysis results of daily return series of the Shanghai Stock Composite Index and the selected eleven sectors indices, and it can be seen that the kurtosis statistics of the twelve time series were significantly greater than 5 during the observation period and the skewness of the sequences were all negative, and with a left-skew distribution. The minimum value of the J-B test statistic of the twelve series was 1138.261, indicating that the J-B test statistic of the yield time series were all significantly larger than the critical value, which means they all do not obey the normal distribution, and have an obvious spike and thick tail feature.

The ADF test results showed that the t-statistic of each series was less than the critical value at the three confidence levels, indicating that the processed yield time series were stable at a 99% significance level. The ARCH effect test results showed that there was an obvious heteroscedasticity in the selected indices when lag order equals 1, and there was an ARCH effect, which met the basic conditions for establishing the GARCH models. In addition, sometimes, researchers also combined the autocorrelation graph and partial autocorrelation graph to judge the stability of a time series. From Figure 3c and 3d, we could see that both the autocorrelation and the partial autocorrelation coefficient were always equal to one when lag order equaled zero, and rapidly decreased from one to zero, and then fluctuated slightly along the zero axis with the increase of lag values, which basically met the requirements of stability.
4. EMPIRICAL RESULTS

In this paper, we selected the GARCH model to analyze the fluctuations of the Shanghai Composite Index and the eleven sectors indices. To save space, this article does not list the estimation results of the mean equations, but only gives the final modeling results of the GARCH models. Based on the student-t and GED distribution hypothesis, we respectively calculated the VaR of each yield series, and carried out back-testing analysis and the Kupiec failure rate test on measurement accuracy of VaR. Based on the definition of economic downturns, we calculated and compared the VaRs of different sectors in corresponding economic downturn phases.

4.1. The GARCH Model

From the analysis results of descriptive statistics, we saw that the yield series of Shanghai Composite Index and the selected sectors have obvious peak and thick tail features, and the normal distribution could not accurately describe their characteristics. Therefore, in this paper, we only considered building the GARCH models under the student-t and GED distributions.

From the estimation results in Table 3, we saw that under both student-t and GED distributions, most parameters in variance equations of the Shanghai Composite Index and the selected sector indices passed the significance test, and met $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta <1$. It meant that the GARCH models established in this paper were stable. To ensure the validity of the GARCH model, this paper also conducted LM tests on the residual items of each model. The test results showed that the twelve sequences all eliminated the heteroscedasticity, and the results are shown in Table 4.
Table 2. Descriptive statistics and analysis of test results.

| Index          | Mean     | Standard deviation | Skewness  | Kurtosis | J-B statistics | ARCH-LM | ADF inspection |
|----------------|----------|--------------------|-----------|----------|----------------|---------|----------------|
| Shanghai composite | -0.000219 | 0.016010            | -0.585703 | 7.954864 | 3216.595       | 117.3422***  | -53.4839***    |
| Daily consumption | 0.000305  | 0.018187            | -0.555789 | 6.823576 | 1967.383       | 229.6385***  | -59.78272***   |
| Energy           | -0.000421 | 0.019165            | -0.433599 | 7.384518 | 2487.015       | 154.1880***  | -53.4839***    |
| Financial        | 0.000263  | 0.018511            | -0.303130 | 7.518423 | 2578.911       | 105.5378***  | -53.98868***   |
| Healthcare       | 0.000370  | 0.019008            | -0.556752 | 6.876825 | 2018.794       | 322.2067***  | -39.32134***   |
| Industrial       | -0.000841 | 0.019783            | -0.739689 | 7.511728 | 2979.366       | 276.9307***  | -50.34560***   |
| Information technology | 0.000293 | 0.022794            | -0.616162 | 5.766720 | 1138.261       | 279.2433***  | -49.91658***   |
| Material         | -0.00158  | 0.020678            | -0.726542 | 6.695710 | 1956.760       | 276.7685***  | -49.91481***   |
| Optional consumption | -0.0000638 | 0.019546            | -0.762057 | 7.267829 | 2548.335       | 276.5523***  | -50.49290***   |
| Public utility   | -0.000110 | 0.017623            | -0.838108 | 9.426440 | 5437.171       | 491.9187***  | -51.29733***   |
| Real estate      | -0.000394 | 0.021921            | -0.563722 | 6.092994 | 1343.783       | 106.1976***  | -51.83188***   |
| Telecom service  | -0.000202 | 0.022803            | -0.277930 | 6.607006 | 1652.724       | 202.8931***  | -53.32408***   |

Note: *** indicates that it is significant at a 1% confidence level, the value in parentheses indicates the probability p corresponding to the statistic, and the ARCH effect test is a 1-order lag test.
| Index                  | Distribution | GARCH (1,1)-t distribution | GARCH (1,1)-GED distribution |
|-----------------------|--------------|----------------------------|------------------------------|
|                       |              | \(\omega\)                 | \(\alpha\)                  | \(\beta\)                   | \(\omega\)     | \(\alpha\) | \(\beta\)       |
| Shanghai composite    |              | 0.045549*** (0.0000)        | 0.955451*** (0.0000)         |                              | 0.0000000833*** (0.0000) | 0.058395*** (0.0000) | 0.940313*** (0.0000) |
| Daily consumption     |              | 0.00000326*** (0.0005)     | 0.081701*** (0.0000)        | 0.910857*** (0.0000)        | 0.000000298*** (0.0004) | 0.077226*** (0.0000) | 0.915029*** (0.0000) |
| Energy                |              | 0.00000172*** (0.0032)     | 0.049461*** (0.0000)        | 0.947551*** (0.0000)        | 0.000000292*** (0.0023) | 0.052624*** (0.0000) | 0.941462*** (0.0000) |
| Financial             |              | 0.00000112** (0.0153)      | 0.050348*** (0.0000)        | 0.945423*** (0.0000)        | 0.000000126*** (0.0000) | 0.055231*** (0.0000) | 0.941976*** (0.0000) |
| Healthcare            |              | 0.00000121** (0.0180)      | 0.068526*** (0.0000)        | 0.911709*** (0.0000)        | 0.00000153*** (0.0000) | 0.063177*** (0.0000) | 0.934124*** (0.0000) |
| Industrial            |              | 0.00000229*** (0.0098)     | 0.064821*** (0.0000)        | 0.931646*** (0.0000)        | 0.00000273*** (0.0000) | 0.062623*** (0.0000) | 0.930525*** (0.0000) |
| Information technology|              | 0.00000326*** (0.0067)     | 0.067144*** (0.0000)        | 0.920917*** (0.0000)        | 0.00000393*** (0.0000) | 0.061854*** (0.0000) | 0.931329*** (0.0000) |
| Material              |              | 0.00000298*** (0.0024)     | 0.065163*** (0.0000)        | 0.928560*** (0.0000)        | 0.00000341*** (0.0000) | 0.063355*** (0.0000) | 0.928131*** (0.0000) |
| Optional consumption  |              | 0.000000152*** (0.0109)    | 0.065970*** (0.0000)        | 0.935888*** (0.0000)        | 0.00000177*** (0.0004) | 0.063130*** (0.0000) | 0.933657*** (0.0000) |
| Public utility        |              | 0.00000111*** (0.0027)     | 0.076139*** (0.0000)        | 0.925477*** (0.0000)        | 0.00000136*** (0.0000) | 0.073821*** (0.0000) | 0.922769*** (0.0000) |
| Real estate           |              | 0.00000213*** (0.0132)     | 0.064407*** (0.0000)        | 0.934795*** (0.0000)        | 0.00000241*** (0.0000) | 0.062595*** (0.0000) | 0.933787*** (0.0000) |
| Telecom service       |              | 0.00000374*** (0.0000)     | 0.057465*** (0.0000)        | 0.942535*** (0.0000)        | 0.00000374*** (0.0000) | 0.069110*** (0.0000) | 0.925507*** (0.0000) |
From Table 3, the sum of the ARCH and GARCH coefficients of the sequences selected were close to 1, indicating whether part of the Shanghai Composite Index or the other sectors indices, they were deeply influenced by the external information. Note that, because the telecom service sector and the Shanghai Composite indices under student-t distribution could not pass the GARCH model, we used IGARCH to improve the models. In Table 3, these two items were calculated by IGARCH models.

The degrees of freedom of the GARCH-t models were between 4.2 and 7.3, and the degrees of freedom of the GARCH-GED models were all less than 2, which proved once again that the data selected in this paper had a thick-tailed feature. Among them, the ARCH coefficient of public utility, healthcare and daily consumption were relatively large, indicating that those sectors respond more quickly to new information. The GARCH coefficients of the real estate, financial and energy sectors were larger than other sectors, meaning that old information has a greater impact on those sectors.

| Index                  | Distribution | GARCH (1,1)-t distribution | P-Value | GARCH (1,1)-GED distribution |
|------------------------|--------------|---------------------------|---------|-------------------------------|
| GARCH (1,1)-t distribution LM statistics | 6.314602 | 0.2120 | 4.464196 | 0.1347 |
| GARCH (1,1)-GED distribution LM statistics | 0.054795 | 0.8149 | 0.172679 | 0.6778 |
| GARCH (1,1)-t distribution LM statistics | 3.274487 | 0.1705 | 2.365511 | 0.1241 |
| GARCH (1,1)-GED distribution LM statistics | 2.087484 | 0.1486 | 1.612979 | 0.2042 |
| GARCH (1,1)-t distribution LM statistics | 0.277395 | 0.5985 | 0.628070 | 0.4281 |
| GARCH (1,1)-GED distribution LM statistics | 0.705650 | 0.4010 | 0.665502 | 0.4147 |
| GARCH (1,1)-t distribution LM statistics | 0.011309 | 0.9153 | 0.095982 | 0.7567 |
| GARCH (1,1)-GED distribution LM statistics | 0.977017 | 0.3230 | 0.886811 | 0.3464 |
| GARCH (1,1)-t distribution LM statistics | 2.523380 | 0.1123 | 2.724656 | 0.1989 |
| GARCH (1,1)-GED distribution LM statistics | 1.664793 | 0.1971 | 1.582667 | 0.2085 |
| GARCH (1,1)-t distribution LM statistics | 0.034298 | 0.8531 | 0.049512 | 0.8239 |
| GARCH (1,1)-GED distribution LM statistics | 6.314604 | 0.1120 | 4.464196 | 0.2347 |

4.2. Calculation of VaR

This paper used the GARCH-t and the GARCH-GED models to generate conditional volatility and used Matlab’s inverse cumulative distribution function to obtain the quantiles of the student-t and the GED distributions respectively. Figure 4 shows an example of the volatility.

Equations 8 and Equation 9 show the probability density of the t-distribution and GED-distribution respectively:

\[ f(x, r) = \frac{\Gamma((r+1)/2)}{\sqrt{\pi r} \Gamma(r/2)} \left[ 1 + \left( x^2 / r \right)^{(r-1)/2} \right] \]
Finally, this paper measured the VaR of each sector index by Equation 3, using conditional volatility estimated by the GARCH models. The results are shown in Table 5.

### Table 5. VaR calculation results.

| Index           | 95% confidence level | 99% confidence level |
|-----------------|----------------------|----------------------|
|                 | GARCH (1,1)-t model | GARCH (1,1)-GED model | GARCH (1,1)-t model | GARCH (1,1)-GED model |
| Shanghai composite | 0.03086              | 0.02406              | 0.05264              | 0.03896              |
| Daily consumption  | 0.03200              | 0.02777              | 0.05062              | 0.04260              |
| Energy           | 0.03649              | 0.02882              | 0.06171              | 0.04612              |
| Financial        | 0.03671              | 0.02775              | 0.08369              | 0.04525              |
| Healthcare       | 0.03353              | 0.02894              | 0.05306              | 0.044556             |
| Industrial       | 0.03616              | 0.02958              | 0.05794              | 0.04689              |
| Information Technology | 0.04029          | 0.03514              | 0.06339              | 0.05375              |
| Material         | 0.03747              | 0.03129              | 0.06093              | 0.04890              |
| Optional consumption  | 0.03552             | 0.02944              | 0.05807              | 0.04615              |
| Public utility    | 0.03048              | 0.02530              | 0.04983              | 0.03991              |
| Real estate       | 0.04109              | 0.03364              | 0.06773              | 0.05286              |
| Telecom service   | 0.04543              | 0.03523              | 0.07728              | 0.05645              |

From Table 5, it can be seen that under both 95% and 99% confidence levels, VaRs calculated by the GARCH-t models were always larger than the calculation results of the GARCH-GED models. By comparing the VaRs of different sectors, we found that VaRs of the telecom service, real estate, financial and information technology were relatively larger than the VaRs of others. Although the nature of the stock market is volatile, it indeed can reflect the true situation of the economy. During those years, the telecom service, real estate, financial and information technology sectors had much higher risks in the investment market.

### 4.3. Far Test Based on Failure Rate

To verify the effectiveness of the models built in the paper, it was necessary to check the coverage of the VaR on the actual loss. Therefore, in this paper, we carried out a back-testing analysis and used Kupiec's failure frequency test (Halilbegovic & Vehabovic, 2016; Wang, 2019) to examine the effectiveness of the models. Here, we used the rate of return in time t ($r_t$) to calculate the overflow of days N. Equation 10 shows the detailed conditions of the back-testing analysis.

$$N = \sum_{t=1}^{T} N_{\bar{r}} \begin{cases} 1, & \text{if } VaR \leq \tau \bar{r} \\ 0, & \text{if } VaR > \tau \bar{r} \end{cases}$$

And then, we used the following Equation 11 to calculate the statistic LR of Kupiec test:

$$LR = 2\ln((1 - p^*)^T) - 2\ln((1 - p)^T)$$

Where $T$ represents the number of samples, $N$ is the number of failure days, and $p = 1 - \alpha p^* = N/T \alpha$ is the confidence level.
This paper considered the 95% confidence level and compared the actual failure rate \( p \) with the expected failure rate \( p^* \). If \( p > p^* \), it means that the model underestimated the risk of China’s securities market; otherwise, it means that the model overestimated the risk. The results are shown in Table 6.

### Table 6. The results of failure frequency test.

| Index                     | Distribution | t distribution | GED distribution |
|---------------------------|--------------|----------------|------------------|
|                           | Failure rate | LR statistics  | Failure rate     | LR statistics  |
| Shanghai Composite        | 0.01443      | 108.97060      | 0.04196          | 4.27788        |
| Daily consumption         | 0.02632      | 41.41714       | 0.04363          | 2.64679        |
| Energy                    | 0.01874      | 80.00190       | 0.04215          | 4.08177        |
| Financial                 | 0.02282      | 57.55570       | 0.04833          | 0.17501        |
| Healthcare                | 0.02249      | 59.19543       | 0.04230          | 3.91876        |
| Industrial                | 0.01141      | 134.05402      | 0.02652          | 41.41715       |
| Information Technology    | 0.01913      | 77.35190       | 0.03491          | 15.89248       |
| Material                  | 0.01477      | 106.43569      | 0.03155          | 24.42072       |
| Optional consumption      | 0.01309      | 119.59356      | 0.03189          | 23.47257       |
| Public utility            | 0.01342      | 116.86299      | 0.03155          | 24.42071       |
| Real estate               | 0.02283      | 57.55570       | 0.04431          | 2.10780        |
| Telecom Service           | 0.0214       | 64.30          | 0.0469           | 0.5771626      |

From the estimation results in Table 6, we can see that the actual failure rates of both Shanghai Composite Index and selected sectors under t-distribution were significantly lower than the expected failure rate \( p^*=5\% \), which indicated that the VaR estimated by GARCH (1,1)-t model did not work that well. Compared with the student-t distribution, the actual failure rate under the GED distribution was much closer to the expected rate \( p^* \). The smaller the value of the statistic LR was, the higher the prediction accuracy of the model was. The LR statistic of each section under GED distribution was less than the one under t-distribution. It means that the GARCH model could describe the situation of China’s stock market more accurately by using GED distribution, and the calculation and prediction accuracy were relatively high.

#### 4.4. The Risk of Different Sectors during the Economic Downturn

In last subsection, we summarize the risks in different sectors during the whole period. Here, we mainly focus on the commonalities during different economic downturns.

Table 7 and Table 8 summarise the VaRs of different sectors under student-t and GED distributions at 95% confidence level. From the estimated values, we found that due to the different background of economic downturns, different sectors vary in performance in different phases.

During the 2008 financial crisis, real estate, financial, information technology, industrial and telecom service sectors showed higher risk than the other sectors. During the second period (which contains the US Treasury crisis in 2011 and the scarcity of money in 2013), real estate, information technology, telecom service, healthcare and industrial sectors were relatively risky. During the stock crash in 2005, we should have paid much more attention to the information technology, industrial, telecom service sectors. At the last stage, real estate, telecom service, information technology sectors showed more risks during the economic fluctuations in 2018.

But all in all, we found that in the Chinese stock market, when the economic downturn comes, the real estate, industrial, telecom service sectors in stock market were big risks and the investors should avoid holding such shares.
Table-7. Comparison of VaR in different sectors during economic downturn periods under GED distribution.

| Index                      | 2008.01.14--2008.11.04 | 2010.11.08--2013.06.27 | 2015.06.12--2016.01.28 | 2018.01.24--2019.01.03 |
|----------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| Shanghai Composite         | 0.04622                 | 0.01976                 | 0.04467                 | 0.02094                 |
| Daily consumption          | 0.04725                 | 0.02318                 | 0.05327                 | 0.02748                 |
| Energy                     | 0.04745                 | 0.02133                 | 0.05379                 | 0.02535                 |
| Financial                  | 0.05437                 | 0.02142                 | 0.04565                 | 0.02395                 |
| Healthcare                 | 0.04965                 | 0.02526                 | 0.06065                 | 0.02843                 |
| Industrial                 | 0.05092                 | 0.02463                 | 0.06368                 | 0.02392                 |
| Information Technology     | 0.05113                 | 0.02898                 | 0.07194                 | 0.03313                 |
| Material                   | 0.05203                 | 0.02738                 | 0.06282                 | 0.02606                 |
| Optional consumption       | 0.05141                 | 0.02437                 | 0.06229                 | 0.02406                 |
| Public utility             | 0.04940                 | 0.02094                 | 0.06131                 | 0.01802                 |
| Real estate                | 0.06069                 | 0.02924                 | 0.05955                 | 0.02871                 |
| Telecom Service            | 0.06178                 | 0.02677                 | 0.06294                 | 0.03351                 |

Table-8. Comparison of VaR in different sectors during economic downturn periods under t-distribution.

| Index                      | 2008.01.14--2008.11.04 | 2010.11.08--2013.06.27 | 2015.06.12--2016.01.28 | 2018.01.24--2019.01.03 |
|----------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| Shanghai Composite         | 0.05911                 | 0.02530                 | 0.05754                 | 0.02673                 |
| Daily Consumption          | 0.05445                 | 0.02674                 | 0.06127                 | 0.03168                 |
| Energy                     | 0.05987                 | 0.02692                 | 0.06845                 | 0.03203                 |
| Financial                  | 0.07194                 | 0.02824                 | 0.06065                 | 0.03160                 |
| Healthcare                 | 0.05806                 | 0.02921                 | 0.07126                 | 0.03309                 |
| Industrial                 | 0.06207                 | 0.02999                 | 0.07870                 | 0.02897                 |
| Information Technology     | 0.05927                 | 0.03303                 | 0.08357                 | 0.03798                 |
| Material                   | 0.06276                 | 0.03274                 | 0.07590                 | 0.03111                 |
| Optional Consumption       | 0.06243                 | 0.02934                 | 0.07570                 | 0.02895                 |
| Public Utility             | 0.05999                 | 0.02516                 | 0.07477                 | 0.02156                 |
| Real Estate                | 0.07446                 | 0.03567                 | 0.07311                 | 0.03501                 |
| Telecom Service            | 0.08130                 | 0.03389                 | 0.08299                 | 0.04296                 |

5. CONCLUSION

In this paper, the risk of the stock market from 2008 to 2020 was analyzed. We not only focused on the risk of stock market during the normal periods, but also emphasized the stock market risks during economic downturns, which were detected by wavelet transformation analysis. Through data analysis results, we have a more clear and rational understanding of the risks in China’s financial market. First, from the fitting results of the GARCH (1,1) models, we saw that the sum of the ARCH term coefficient and the GARCH term coefficient was close to 1 under the assumption of the student-t and GED distributions, which indicated that external information will have a lasting impact on China's stock market. While verifying that the GARCH (1, 1) model could effectively fit the stock yield series, we also have a clearer understanding of the long-term of China economic development. Second, from the results of the GARCH (1,1) model, we found that public utility, healthcare and daily consumption sectors in the stock market respond more quickly to new information; while the real estate, financial and energy sectors were influenced much more by old information. Third, from the overall results, telecom service, real estate, financial and information technology sectors in the stock market have relatively higher risks, and the VaR values calculated under the t-distribution were all greater than those calculated under the GED distribution.

Fourth, Kupiec test results showed that the actual failure rate under t-distribution was much lower than the expected failure rate, indicating that the GARCH-t model overestimated the risk in the stock market. In comparison, the VaRs calculated under the assumption of the GED distribution could more realistically describe the risk situation of China’s stock market. By detecting economic downturns in the study period, we found that although the risks of different sectors vary over time, real estate, industrial, telecom service sectors in stock market could be big risks during all economic downturns, and investors should avoid holding such shares during that period.
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