Deformations of a piezo-actuated bearing ring in journal bearings

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An elastically deformable bearing ring is investigated. The piezo-actuated shaping of the elastic bearing ring as well as the journal motion are coupled and determine the height of the lubrication gap which takes a crucial part in the theory of hydrodynamic lubrication. The influence of the active-dynamic shaping of the bearing ring on the rotordynamic behaviour has been investigated in [1], especially for a two-lobe journal bearing. The present investigations are aiming at the active-static shaping of a bearing ring, i.e. a finite number of piezo actuators is used to cause deformations of the bearing ring. Moreover, the realisation of different shapes by the introduced piezo actuators is examined, whereas the desired shapes are not necessarily limited to two-lobe geometries.

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1 Introduction

Nowadays, fluid film bearings are used frequently in high-speed turbomachinery. However, such bearings are subject to various difficulties, e.g. self-excited vibrations and instabilities. To ensure the safe operation of the rotordynamic system, the influence of passively/actively varying bearing geometries has been investigated [1]. One way to change the shape of a bearing ring is the use of piezoelectric actuators. These piezoelectric elements can also be used as sensors in order to measure displacements at distinct points. With that measured displacements it is possible to estimate the shape of the bearing ring. Hence, with piezoelectric sensors and actuators smart, controllable bearings can be designed for the condition monitoring and control of rotordynamic systems.

2 Model and analytical approach

In the following a model of the bearing ring deformed by piezo actuators will be introduced. A scheme of such a bearing is depicted in figure 1 for instance with three evenly distributed actuators. The realisation of a desired shape requires the solution of an inverse problem in order to determine the voltages $U_i$ to be applied.

The bearing ring is modelled as a weakly curved Euler-Bernoulli beam. The radial displacement $w(\varphi)$ and tangential displacement $u(\varphi)$ are assumed to be small. Due to the inextensibility assumption the radial displacement $w(\varphi)$ is described by an ordinary differential equation, whereas the tangential displacement $u(\varphi)$ is coupled by a kinematic constraint equation, namely

$$\frac{EI}{r^2} \left( \frac{d^4 w}{d\varphi^2} + \frac{w}{r^2} \right) = -M(\varphi) \quad \text{and} \quad \frac{du}{d\varphi} + w = 0. \quad (1)$$

The piezoelectric actuators acting at distinct angles $\varphi_i$ are taken into account by radial and tangential forces

$$F_{i,r} = k_3 w(\varphi_i) + k_2 U_i \quad \text{and} \quad F_{i,t} = k_3 u(\varphi_i)$$

for $i = 1, \ldots, n$ where the finite number of actuators is given by $n$.

The ring equations (1) were integrated piecewise in the regions $0 \leq \varphi < \varphi_1$, $\varphi_1 \leq \varphi < \varphi_{i+1}$, $\ldots$, $\varphi_n \leq \varphi < \varphi_{n+1} = 2\pi$. Both the integration constants $C_1, C_2, C_3, C_{n+1,1}, C_{n+1,2}, C_{n+1,3}$ as well as the unknown stress resultants $N_0$, $Q_0$, $M_0$...
can be determined by the following boundary conditions
\begin{align*}
\varphi = \varphi_i : & \quad w_i(\varphi_i) = w_{i+1}(\varphi_i), \\
\varphi = 2\pi : & \quad w_n(2\pi) = w_n(0),
\end{align*}

and the static equilibrium equations
\begin{equation}
\sum_{i=1}^n F_i \cos\varphi_i - F_i \sin\varphi_i = 0, \quad \sum_{i=1}^n F_i \sin\varphi_i + F_i \cos\varphi_i = 0, \quad \sum_{i=1}^n F_i = 0.
\end{equation}

As a result the radial displacement \( w \) is given as a piecewise function with respect to the coordinate \( \varphi \) and appears linearly dependent on the applied voltages \( U_i \). Hence, it is possible to formulate a linear least-squares problem in order to minimize the total sum of residuals between the analytic radial displacement \( w \) and desired radial displacement \( \bar{w} \) at distinct angles \( \varphi_j \)
\begin{equation}
j = 1, \ldots, m : \quad r_j = w(\varphi_j, U_1, U_2, \ldots, U_n) - \bar{w}(\varphi_j) \iff r = AU - \bar{w}
\end{equation}
where \( r, \bar{w} \in \mathbb{R}^m, U \in \mathbb{R}^n \) and \( A \in \mathbb{R}^{m \times n} \). This inverse problem has a unique solution given by \( U = (A^TA)^{-1}A^T\bar{w} \).

### 3 Results

Results are shown for a bearing ring with 8 attached piezo actuators. The desired radial displacement is given by
\begin{equation}
\bar{w}(\varphi) = \frac{w_0}{3} \sum_{k=1}^3 \sin(\lambda_k(\varphi - \varphi_0))
\end{equation}
with \( w_0 = 20 \mu m, \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3 \) and \( \varphi_0 \in [0, 2\pi] \). Analytic displacements and desired displacements in fig. 2 are in very good agreement for all values of \( \varphi_0 \), i.e. for any tilted position of the bearing ring. The maximum error between analytic and desired displacements is about 1%. The voltages that have to be applied are plotted in fig. 3 with respect to the parameter \( \varphi_0 \).

![Fig. 2: displacements of the bearing ring for \( \varphi_0 = 0 \)](image)

![Fig. 3: actuator voltages](image)

The other parameters used in the calculations are given by \( n = 8, m = 360, r = 0.05 \text{ m}, h = 0.0025 \text{ m}, d = 0.01 \text{ m}, E = 205 \text{ GPa}, I = \frac{h^3r}{12}, n_{disc} = 25, b = 0.001 \text{ m}, d_{33} = 400 \cdot 10^{-12} \text{ C} \cdot \text{N}^{-1}, S_{33}^e = 27.7 \cdot 10^{-12} \text{ m}^2 \text{N}^{-1}, k_1 = -\frac{\pi^2}{n_{disc}S_{33}^e}, k_2 = \frac{n_{disc}^{1/3}}{6S_{33}^e}, k_3 = 10^{-9}k_1 \).

### 4 Conclusions

In order to achieve a desired shape specific voltages \( U_i \) must be applied. These voltages can uniquely be determined by a linear least-squares problem. Changing the number of piezo actuators reveals that the shape ability is limited by a cutoff frequency \( \lambda_c < \frac{\pi}{2} \) so that not all shapes can be realized in any tilted position. By increasing the number of actuators the cutoff frequency can be increased and the errors between the analytic and desired displacements can be decreased. Determining the right number of actuators also depends on deformations that are caused by the pressure and shear forces present in fluid film bearings. This aspect will be considered in future investigations.

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References

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