Radial pulsation as a function of hydrogen abundance

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ABSTRACT
Using linear non-adabatic pulsation analysis, we explore the radial-mode (p-mode) stability of stars across a wide range of mass (0.2 ≤ M ≤ 50M☉), composition (0 ≤ X ≤ 0.7, Z = 0.001,0.02), effective temperature (3000 ≤ Teff ≤ 40000K), and luminosity (0.01 ≤ L/M ≤ 100,000 solar units). We identify the instability boundaries associated with low- to high-order radial oscillations (0 ≤ n ≤ 16). The instability boundaries are a strong function of both composition and radial order (n). With decreasing hydrogen abundance we find that i) the classical blue edge of the Cepheid instability strip shifts to higher effective temperature and luminosity, and ii) high-order modes are more easily excited and small islands of high radial-order instability develop, some of which correspond with real stars. Driving in all cases is by the classical κ-mechanism and/or strange modes. We identify regions of parameter space where new classes of pulsating variable may, in future, be discovered. The majority of these are associated with reduced hydrogen abundance in the envelope; one has not been identified previously.

Key words: stars: oscillations, stars: interiors, stars: chemically peculiar, stars: variables: general

1 INTRODUCTION
Since the discovery of periodic light variations in the luminous giant δ Cephei, the study of stellar pulsations has transformed our understanding of how stars work, as well as establishing a distance scale whereby the cosmos can be measured. The fact that the light variations in δ Cep represented a major discovery testifies to the fact that not all stars are variable. However, as telescopes and detectors have become more sensitive, pulsations have been identified in diverse groups of stars of all masses and across the Hertzsprung-Russell diagram. Such discoveries continue to the present day, with pulsations in low-mass white dwarfs and pre-white dwarfs being the latest additions to the pulsating star zoo (Fig. 1) (Maxted et al. 2013; Hermes et al. 2013).

Jeffery & Saio (2013) demonstrated that pulsation instability in the low-mass pre-white dwarf J0247-25B would arise in a high-order overtone if the envelope was depleted in hydrogen. The principal reason for this, demonstrated previously by Saio & Jeffery (1988) and Jeffery & Saio (2007), is that hydrogen acts as a poison, suppressing the positive opacity gradient around an opacity peak which would otherwise drive pulsations if located at an appropriate depth beneath the stellar surface.1

The question therefore arose whether it would be possible to predict the properties of other hitherto undiscovered pulsating variables, especially those in which hydrogen has been depleted as a consequence of prior evolution. Consequently, we have carried out a parametric survey in order to identify locations where new classes of variable star await discovery.

2 RADIAL PULSATION MODELS
The investigation commenced by computing a grid of 258,000 models of stellar envelopes covering a range of chemical mixtures and for masses on the range 0.2 ≤ M/M⊙ ≤ 50, effective temperatures log Teff/K = 3.50(0.02)4.60, and luminosity-to-mass ratios log(L/L⊙)/(M/M⊙) = −2.0(0.2)5.0. The linear nonadiabatic analysis of stability

1 The condition for driving an oscillation by the κ-mechanism in such a region is generally understood to be that the spatial derivative d(κT + κρ/(Γ3 − 1))/dr > 0, where κT and κρ are the temperature and density derivatives of the opacity κ, and Γ3 is the usual adiabatic exponent (Unno et al. 1989, p. 243).

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against pulsation was carried out following methods described by Saio et al. (1983) and Jeffery & Saio (2006a,b).

The OPAL95 (Iglesias & Rogers 1996) opacities were adopted, except at low temperatures, where Alexander & Ferguson (1994) opacities were used. As a test of sensitivity to opacity, additional calculations were made with OP opacities (Badnell et al. 2005). Convection is treated assuming a standard mixing-length theory with the ratio mixing-length to pressure scale height \(l/H_p = 1.5\). Any convection/pulsation interaction is neglected by setting the divergence of the convective flux perturbation to zero. Therefore results for \(T_{\text{eff}} < 4000\) K should be treated with caution.

The outer boundary for the envelope model is set at the Rosseland mean optical depth \(\tau = 10^{-3}\). The integration is carried out with pressure \(\log_{10} P\) as the independent variable, with initial stepsize \(\delta \log_{10} P = 0.02\), which is adjusted to maintain increments in radius \(\delta r/r < 0.01\), density \(\delta \rho/\rho < 0.1\) and electron pressure \(\delta P_e/P_e < 0.08\) at each step. The integration is halted at a fractional mass \(m = M_e/10\) or fractional radius \(r = R_e/100\), whichever occurs first.

For each model envelope, the first 17 eigenfrequencies of pulsation were located and stored, including the real and imaginary components \(\omega_r\) and \(\omega_i\), the period \(\Pi\) and the number of nodes \(n\). Approximate spectral types are described by small numbers (\(M_\odot\)). The zero-age main sequence and horizontal branch, the Cepheid instability strip, \(\delta\) Scuti stars and \(\gamma\)-dwarf-like variables are highlighted on the contour plots. Details may be verified from figures in Appendix A.

For high \(M_\odot\) stars, the second overtone pulsations become significant, but are only slowly dependent on the mass of the star over a range from 0.2 to 50 \(M_\odot\) (cf. Fig. 2: top row). Nevertheless there are significant features to note; details may be verified from figures in Appendix A.

3 RADIAL-MODE INSTABILITY

The use of \(L/M\) to parameterize the model grid exploits the fact that the radial pulsation properties of stellar envelopes are only slowly dependent on the mass of the star over a range from 0.2 to 50 \(M_\odot\) (cf. Fig. 2: top row). Nevertheless there are significant features to note; details may be verified from figures in Appendix A.

At \(X = 0.70, Z = 0.02\) (Figs. A.1 and A.9):

i) The most familiar feature is the classical Cepheid instability strip running diagonally toward high \(L/M\) and low \(T_{\text{eff}}\) at \(\log L/M > 0\). The principal driving is provided by the second ionization of singly-ionized helium (He\(^+\)).

ii) This is supplemented by an adjacent and parallel strip at lower \(T_{\text{eff}}\) in which higher-order modes are excited (cf. Fig. 3: top row). This may be related to the solar-like oscillations detected by Corot and Kepler in many less luminous red giants (Miglio et al. 2009; Bedding et al. 2010; Huber et al. 2010) and marked as \(\xi\) Hya variables in Fig. 1. Although the latter are generally thought to be excited stochastically by turbulence, the order of the \(p\)-modes detected in red giants decreases as the luminosity increases, which is consistent with our prediction from the kappa-mechanism. Xiong & Dong (2007) report from a non-adiabatic analysis including coupling between convection and pulsations that these modes are excited. We hence find that evidence exists for a contribution of the kappa-mechanism from ionization of H and He\(^0\) to the driving of small-amplitude oscillations in red giants.

iii) The low \(T_{\text{eff}}\) strip extends to very low \(L/M\) values and is responsible in low-mass main-sequence stars for \(\delta\) Scuti variables. The principal driving comes from H and He\(^0\) ionisation, although the high overtone pulsations in hotter \(\delta\) Sct stars are probably associated with He\(^+\) ionization (i.e. the classical instability strip), which extends to the main sequence with \(\log g < 5\), \(\log L/M > 0\).

iv) At high \(L/M\), models show instability in one or...
Figure 2. Unstable pulsation modes in stars with homogeneous envelopes for selected compositions and masses, as labelled. Full grids are shown in Appendix A. The number of unstable radial modes is represented by grey scale contours, with the lightest shade marking the instability boundary (one unstable mode), and the darkest shade representing ten or more unstable modes. Broken (maroon online) diagonal lines represent contours of constant surface gravity at log g = 8, 7, 6, . . . , 1. Pale green areas denote regions where envelope models encountered convergence difficulties. Red symbols with error bars shown on selected panels represent the observed positions of pulsating low-mass hydrogen-deficient stars, including extreme helium stars and R Coronae Borealis variables (Jeffery 2008b), shown on panels with $X \leq 0.1$ and $M = 1.0 \, M_\odot$, and $\alpha$ Cyg variables (Crowther et al. 2006; Searle et al. 2008; Firnstein & Przybilla 2012), shown on panels with $X = 0.7$, $Z = 0.02$ and $M \geq 10 \, M_\odot$. Figs. 2 and 3 are best viewed online and expanded; one grid is enlarged in Fig. 4.

more modes; these are the so-called strange modes (Gautschy & Glatzel 1990; Saio et al. 1998). The general confusion in the mode boundary diagram (Fig. A.9) is accounted for by a breakdown in the strict 1-1 correspondence between mode eigenfrequency and node number for strange modes$^2$ $\alpha$ Cygni variables can be explained by strange

$^2$ Strange modes in high $L/M$ stars are essentially as defined by Gautschy & Glatzel (1990); Saio et al. (1998), and first identified by Wood (1976). Other modes of somewhat different character have also been described as strange, e.g. by Buchler et al. (1997);
modes, if considerable mass has been lost (Saio et al. 2013).
v) At log \( L/M > 3 \) and log \( T_{\text{eff}} \approx 4.4 \) there is a weak instability finger caused by iron-group opacities at temperatures \( T \approx 2 \times 10^5 \) K. This can be identified with \( \beta \) Cepheid variables on the upper main sequence. The shape of this finger is sensitive to mass, and also to the iron and nickel abundances (Jeffery & Saio 2007).

At \( X = 0.30, Z = 0.02 \) (Figs. A.2 and A.10), a modest reduction in hydrogen abundance has the effect of increasing the driving effect of helium in the classical instability strip and iron in the ‘Z-bump’ instability finger. The chief consequences are:
i) As the mean molecular weight in the envelope increases, the \( \text{He}^+ \) instability strip shifts to higher \( T_{\text{eff}} \). With the excitation of higher-order radial modes, the strip also

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**Figure 3.** As Fig. 2, but showing the boundaries for individual radial modes as coloured contours, with the darkest red representing the boundary of the fundamental \((n = 0)\) mode, with increasing higher orders \((n = 1-10)\) represented progressively by colours of increasing frequency (orange, yellow, green, blue...). Solid blue lines represent contours of equal fundamental radial-mode period in seconds spaced at decadal intervals.

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Buchler & Kolláth (2001); Smolec (2016). The current analysis makes no distinction between strange and normal modes; it only identifies which modes are stable or unstable.
extends to lower $L/M$.

ii) For $\log L/M < -1$ the high-order strip narrows significantly.

iii) With increasing contrast between iron-group and hydrogen opacity, the ‘Z-bump’ finger becomes significantly stronger.

iv) At high mass, ‘Z-bump’ excited fundamental modes extend to $\log T_{\text{eff}} \approx 4.0$, forming a ‘spur’ to the ‘Z-bump’ finger. Close inspection of these modes show an unrealistically large amplitude propagating deep into the interior (to fractional mass $m/M < 0.001$); we do not consider these to be real.

At $X = 0.10, Z = 0.02$ (Figs. A.3 and A.11), the consequences of reducing the hydrogen abundance seen at $X = 0.3$ continue. In addition:

i) The width of the high-order He$^+$ instability strip at low $L/M$ increases. As mass increases, the region becomes
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Iglesias & Rogers 1996

Kilkenny et al. 2014;

A.12

A.13

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been unstable, as previously shown by

evidence that radial modes in some DB white dwarfs may

been identified with pulsations observed in J0247-25B

are sensitive to details of the precise chemical mixture and

diminishing in extent with increasing mass. This island has

been no successful detection of p-mode pulsations in any

white dwarfs (Silvotti et al. 2011; Kilkenney et al. 2014).

At X = 0.70, Z = 0.001 (Figs. A.5 and A.13), the principal

consequence of reducing the metallicity is that the Z-bump

finger disappears. In addition, the classical instability strip,

and the high-overtone strip to the red are both narrowed.

At X = 0.002, Z = 0.001 (Figs. A.8 and A.16), the high-

overtone instability island identified at X = 0.002, Z = 0.02

persists but, as there were substantial difficulties computing

envelope models with M < 2 M⊙ at this composition for

other ranges of L/M and Teff, these models are not shown.

At X = 0.70, Z = 0.02 and log L/M ≤ 0 (Fig. A.1), unstable

radial modes initially form a single narrow strip, extending

the classical Cepheid instability strip to very low L/M. With

X = 0.30 (Fig. A.2), a second strip develops substantially to

the blue of the first. Again, this is the low L/M extension of

the He+ -driven strip already seen. We note that the excited

modes are dominated by high-order modes (Fig. A.10).

At X = 0.10 both strips broaden to form a single region with

a complicated mode structure (Figs. A.3,A.11). Finally, at

X = 0.002, the redward strip stabilizes, to leave only a

broad blue instability strip (Fig. A.12). Similar behaviour

is replicated at Z = 0.001. Predictions of radial instability

in both DA and DB white dwarfs are well established

(Saio et al. 1983; Kawaler 1993). However there has so far

been no successful detection of p-mode pulsations in any

white dwarfs (Silvotti et al. 2011; Kilkenney et al. 2014).

4 CONCLUSION

We have made an extensive survey of the stability against

radial pulsations for the envelopes of stars having masses in

the range 0.2 − 50 M⊙ and hydrogen abundances (by

mass fraction) from X = 0.70 to 0.002, considering both

metal-rich (Z = 0.02) and metal-poor (Z = 0.001) mix-

tures. The grid of models ranges in effective temperature

from Teff = 3000 − 40 000 K, and in luminosity-to-mass ratio

from L/M = 0.01 − 100,000 (in solar units), covering most

of parameter space occupied by stars, excepting only the

hottest and coolest supergiants, the hottest subdwarfs, the

most massive white dwarfs and the coolest supergiants and

dwarfs. By considering overtones up to n = 16, we identify

nearly all stars likely to be unstable to p-mode oscillations

driven by the κ−mechanism.

We demonstrate that the Hertzsprung-Russell diagram

for pulsation instability expressed as a plot of L/M versus

Teff is only slowly variant with mass M, but is much more

sensitive to composition, especially the hydrogen abundance

since the latter normally acts to damp pulsations.

Within a single computational framework, we recover

all hitherto known regions of radial and non-radial p-mode

instability due to the κ−mechanism and/or strange modes.

The detailed boundaries are likely to vary for specific cases

Figure 6. Work integrals W as a function of temperature for the

9th overtone in the neighbourhood of the instability island

at X = 0.002, Z = 0.02, with M = 0.5 M⊙, log L/M = 2.7, and

log Teff as shown. Positive values of W at the surface indicate

instability. At log Teff ≤ 3.9, driving is due to He triple ionization –

essentially the classical Cepheid mechanism. For log Teff = 3.95, 4.0 and 4.2, the mode is stable. For log Teff = 4.1, the mode is

unstable due to driving by both He triple and He + ionization.

ii) For log L/M < −1, there are virtually no unstable modes

in the high-order strip.

iii) The Z-bump finger extends to log L/M > 2.0; the

high-mass spur persists, but see above.

At X = 0.002, Z = 0.02 (Figs. A.4 and A.12), the conse-

quences of reducing the hydrogen abundance seen at

X = 0.1 continue. In addition:

i) At M < 5 M⊙, an additional narrow strip of high-

overtone instability is seen immediately to the blue of the

He + instability strip at log L/M ≈ 1. This strip has

been identified with pulsations observed in J0247-25B

(Jeffery & Saio 2013).

ii) The Z-bump finger is very strong and broad, but only

low-order (n < 4) modes are excited; the high mass n = 0

Z-bump spur persists.

iii) The width of the high-order He + instability strip at

low L/M remains large. At low mass, the models give

evidence that radial modes in some DB white dwarfs may

be unstable, as previously shown by

Kawaler (1993). iv) At M < 2 M⊙, an island of high-overtone instability is

seen around log Teff ≈ 4.1 and log L/M ≈ 2 − 3 (Fig. 4),

diminishing in extent with increasing mass. This island has

appeared in previous calculations (Jeffery & Saio 2013),

but only here identified as a persistent feature over a range

of masses. Driving is due to a combination of He triple and

He + ionization (Fig. 6). Since pulsation instability boundaries

are sensitive to details of the precise chemical mixture and

to the overall opacity calculation, a few of these models

were recalculated using OP opacities (Badnell et al. 2005)

instead of OPAL95 opacities (Iglesias & Rogers 1996). Mi-

nor differences may be seen (Fig. 5), but the new instability

island persists and the differences are small compared with

the large variations in X which are of primary interest. We

suggest that stars having these characteristics, should they

actually occur in nature, would be good candidates to show

high-order radial (or non-radial) p-mode pulsations.

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owing to other properties such as internal composition gradients and long-term evolution effects.

We identify one new region of pulsation instability for low-mass hydrogen-deficient stars with $\log L/M \approx 2 - 3$ and $\log T_{\text{eff}} \approx 4.0 - 4.2$; $\kappa$-mechanism driving is by He$^0$ and He$^+$ ionization. No stars are currently known to exhibit these pulsations.

In addition, we conclude that solar-like oscillations in red giants may be at least partially driven by the $\kappa$–mechanism, supporting Xiong & Deng (2007). To investigate this and other questions, a more detailed study of the relative roles of convection and the $\kappa$-mechanism in exciting pulsations across the H-R diagram will follow.

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APPENDIX A: RADIAL PULSATION MODEL GRIDS

Figures A.1 to A.16 contain the complete grids showing the numbers of unstable radial modes and the instability boundaries for each unstable mode.
Figure A.1. Unstable pulsation modes in stars with homogeneous envelopes with hydrogen content $X = 0.70$, $Z = 0.02$, and OPAL opacities, and mass $0.20 < M/M_\odot < 50$, as labelled. The number of unstable radial modes is represented by grey scale contours, with the lightest shade marking the instability boundary (one unstable mode), and the darkest shade representing ten or more unstable modes. Broken (maroon online) diagonal lines represent contours of constant surface gravity at $\log g = 8.7, 7, \ldots, 1$. Models with $X = 0.7$ and $\log T_{\text{eff}} < 3.6$ were excluded. Red symbols with error bars shown on selected high-mass panels represent the observed parameters of pulsating $\alpha$ Cygni variables (Crowther et al. 2006; Searle et al. 2008; Firnstein & Przybilla 2012).
Figure A.2. As Fig. A.1 with $X = 0.30, Z = 0.02$. 
Figure A.3. As Fig. A.1 with $X = 0.10, Z = 0.02$. Red symbols with error bars shown on selected low-mass panels represent the observed parameters of pulsating low-mass hydrogen-deficient stars, including extreme helium stars and R Coronae Borealis variables (Jeffery 2008b).
Figure A.4. As Fig. A.3 with $X = 0.002, Z = 0.02$. 
Figure A.5. As Fig. A.1 with $X = 0.70$, $Z = 0.001$. 
Figure A.6. As Fig. A.1 with $X = 0.30$, $Z = 0.001$. 
Figure A.7. As Fig. A.1 with $X = 0.10, Z = 0.001$. 
Figure A.8. As Fig. A.1 with $X = 0.002$, $Z = 0.001$, for models with $M \geq 4 M_\odot$. Lower-mass models encountered numerical problems at high $L/M$ ratios.
Figure A.9. Unstable pulsation mode boundaries in stars with homogeneous envelopes with hydrogen content $X = 0.70$, $Z = 0.02$, and OPAL opacities, and mass $0.20 < M/M_\odot < 50$, as labelled. The boundaries of unstable radial modes are represented by coloured contours, with the darkest red representing the boundary of the fundamental ($n = 0$) mode, with increasing higher orders represented progressively by colours of increasing frequency (orange, yellow, green, blue ...). Models with $X = 0.7$ and $\log T_{\text{eff}} < 3.6$ were excluded. Solid blue lines represent contours of equal fundamental radial-mode period in seconds spaced at decadal intervals.
Figure A.10. As Fig. A.9 with $X = 0.30, Z = 0.02$. 

Pulsation instability
Figure A.11. As Fig. A.9 with $X = 0.10$, $Z = 0.02$. 
Figure A.12. As Fig. A.9 with $X = 0.002, Z = 0.02$. 

Pulsation instability
Figure A.13. As Fig. A.9 with $X = 0.70$, $Z = 0.001$. 

Figure A.14. As Fig. A.9 with $X = 0.30, Z = 0.001$. 
Figure A.15. As Fig. A.9 with $X = 0.10, Z = 0.001$. 
Figure A.16. As Fig. A.9 with $X = 0.002$, $Z = 0.001$, for models with $M \geq 4 M_\odot$. Lower-mass models encountered numerical problems at high $L/M$ ratios.