Radiation drag driven mass accretion in clumpy interstellar medium: implications for the supermassive black hole-to-bulge relation

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ABSTRACT

We quantitatively scrutinize the effects of the radiation drag arising from the radiation fields in a galactic bulge in order to examine the possibility that the radiation drag could be an effective mechanism to extract angular momentum in a spheroidal system like a bulge and allow plenty of gas to accrete onto the galactic center. For this purpose, we numerically solve the relativistic radiation hydrodynamical equation coupled with the accurate radiative transfer and quantitatively assess the radiation drag efficiency. As a result, we find that in an optically thick regime the radiation drag efficiency is sensitively dependent on the density distributions of interstellar medium (ISM). The efficiency drops according to $\tau_T^{-2}$ in an optically thick uniform ISM, where $\tau_T$ is the total optical depth of the dusty ISM, whereas the efficiency remains almost constant at a high level if the ISM is clumpy. Hence, if the bulge formation begins with a star formation event in a clumpy ISM, the radiation drag will effectively work to remove the angular momentum and the accreted gas may form a supermassive black hole. As a natural consequence, this mechanism reproduces a putative linear relation between the mass of a supermassive black hole and the mass of a galactic bulge, although further detailed modeling for stellar evolution is required for the more precise prediction.

Key words: galaxies: nuclei—galaxies: starburst—radiation drag

1 INTRODUCTION

Recently, Kormendy & Richstone (1995) have pioneeringly suggested that the mass of a supermassive black hole (BH) does correlate linearly with the mass of the hosting bulge. (It is noted that the term of a bulge is used to mean a whole galaxy for an elliptical galaxy in this paper as is often so.) Further high-quality observations of the galactic center using stellar dynamics, gasdynamics, and maser dynamics (Miyoshi et al. 1995; Magorrian et al. 1998; Richstone et al. 1998; Ho 1999; Wandel 1999; Kormendy & Ho 2000; Ferrarese et al. 2000; Gebhardt et al. 2000a; Sarzi et al. 2001; Merritt & Ferrarese 2001a) allow us to make a detailed demography of supermassive BHs. The recent findings are the following: (1) The BH mass exhibits a linear relation to the bulge mass for a wide range of BH mass with a median BH mass fraction of $f_{BH} \equiv M_{BH}/M_{bulge} = 0.001 - 0.006$ (Kormendy & Richstone 1995; Richstone et al. 1998; Magorrian et al. 1998; Gebhardt et al. 2000a; Ferrarese & Merritt 2000; Ferrarese & Ferrarese 2001a). (2) The BH mass correlates with the velocity dispersion of bulge stars with a power-law relation as $M_{BH} \propto \sigma^n$, $n = 3.75$ (Gebhardt et al. 2000a) or 4.72 (Ferrarese & Merritt 2000; Ferrarese & Ferrarese 2001a, 2001b). (3) $f_{BH}$ tends to grow with the age of youngest stars in a bulge until $10^9$ yr (Merrifield, Forbes & Terlevich 2000). (4) In disc galaxies, the mass ratio is significantly smaller than 0.001 if the disc stars are included (Salucci et al. 2000; Sarzi et al. 2001). (5) For quasars, the $f_{BH}$ is on a similar level to that for elliptical galaxies (Laor 1998; McLure & Dunlop 2001a; Wandel 2001). (6) The $f_{BH}$ in Seyfert 1 galaxies is under debate, which may be considerably smaller than 0.001 (Wandel 1999; Gebhardt et al. 2000a) or similar to that for ellipticals (McLure & Dunlop 2001a,2001b; Wandel 2001), while the BH mass-to-velocity dispersion relation in Seyfert 1 galaxies seems to hold good in a similar way to elliptical galaxies (Gebhardt et al. 2000b; Nelson 2000; Ferrarese et al. 2001). These BH-to-bulge correlations suggest that the formation of a supermassive BH is physically connected with the formation of a galactic bulge.

So far, very little is understood about the physical mechanism to produce such correlations, although some theoretical models has been proposed (Silk & Rees 1998; Ostriker 2000; Adams, Graff & Richstone 2001). Recently, as a possible mechanism to work in a spheroidal system, Umemura (2001) has considered the effects of radiation drag. The radiation drag is a relativistic effect, which may extract
the angular momentum effectively in a spheroidal system like a bulge, so that plenty of interstellar medium (ISM) could accrete on to the galactic center. Obviously, the radiation drag is inefficient in present-day elliptical galaxies or galactic bulges, since they possess little ISM. If the contents of a supermassive BH are initially in the form of ISM, however, the bulge must have been optically thick in the early stage:

\[
\tau \approx \chi \rho r_b = 1.0 \left( \frac{\chi}{100 \text{ cm}^2 \text{g}^{-1}} \right) \left( \frac{M_{\text{gas}}}{10^9 M_\odot} \right) \left( \frac{r_b}{3 \text{kpc}} \right)^{-2},
\]

where \(\chi\) is the mass extinction coefficient due to dust opacity, \(\rho\) is the density of the ISM, \(M_{\text{gas}}\) is the mass of the ISM, and \(r_b\) is the bulge radius. If a considerable amount of gas is expelled by a galactic wind at some stage, the optical depth should be still larger before the wind. If the radiation drag works efficiently in an optically thick medium, the rate of mass accretion induced by the radiation drag is maximal \(L_{\text{bal}}/c^2\) (Umemura, Fukue & Mineshige 1997, 1998; Fukue, Umemura & Mineshige 1997), where \(L_{\text{bal}}\) is the bolometric luminosity. Umemura (2001) has found that, if the maximal drag efficiency is achieved, the resultant BH-to-bulge mass ratio is basically determined by the energy conversion efficiency of the nuclear fusion from hydrogen to helium, i.e., 0.007. However, it is not very clear whether this mechanism really works efficiently in realistic situations.

In this paper, we investigate in detail the efficiency of the radiation drag in an optically thick ISM to test whether the radiation drag model is promising to account for the putative BH-to-bulge correlations. In particular, we concentrate our attention on the effects of the inhomogeneity in the ISM. The model for the chemical evolution of elliptical galaxies suggests that an elliptical galaxy is initiated by a starburst in its early stage of \(< 10^7\text{yr}\), and evolves passively after a galactic wind event at a few \(10^8\text{yr}\) (Arimoto & Yoshii, 1986, 1987; Kodama & Arimoto, 1997; Mori et al. 1997). Also, in nearby starburst galaxies that have been studied, the ISM is highly clumpy (Sanders et al. 1988; Gordon et al. 1997). Thus, if we consider the radiation drag in the early phase of bulge evolution, we should consider an inhomogeneous optically thick ISM. In this paper, we elucidate the effect of the inhomogeneity in the ISM on the radiation drag efficiency, we build up a simple model of the bulge system and accurately solve the radiation transfer in a clumpy ISM.

The paper is organized as follows. In Section 2, we construct the model of a galactic bulge. In Section 3, the basic equations for the ISM are provided. In Section 4, the angular momentum transfer efficiency is assessed in a uniform ISM. In Section 5, we investigate the angular momentum transfer efficiency by solving the radiation transfer in a clumpy ISM, and elucidate the relationship between the clumpiness of the ISM and the angular momentum transfer efficiency. In Section 6, we give implications for the correlation between the supermassive BH mass and the bulge mass. In addition, we discuss further effects that would give significant influence on the BH mass. Section 7 is devoted to our conclusions.

2 MODEL

We assume that a spherical galactic bulge consists of three components, that is, dark matter, stars, and a dusty ISM. The bulge radius \(r_b\) is set to be 1-10kpc by taking account of the observed sizes of elliptical galaxies or bulges in spiral galaxies. Dark matter is distributed uniformly inside the bulge. The mass of dark matter component, \(M_{\text{DM}}\), within the bulge size is equal to the stellar mass of the galactic bulge, \(M_{\text{bulge}}\). As for the stellar component, we assume star clusters with a specific stellar initial mass function (see below). The star clusters are distributed uniformly inside the galactic bulge. Furthermore, a star in this paper means a star cluster. Here, \(N_c(=100)\) stars are distributed randomly. For a dusty ISM, we consider the two cases: one is a uniformly distributed ISM, and the other is a clumpy ISM. In the case of clumpy ISM, \(N_c(=10^4)\) identical clouds are distributed randomly. The density \(\rho_{\text{gas}}\) in a cloud is assumed to be uniform. The size of a gas cloud, \(r_c\), is a parameter. Then, the optical depth of a gas cloud is \(\tau = \chi \rho_{\text{gas}} r_c\), where \(\chi\) is the mass extinction. We suppose that the stars and the ISM clouds corotate with the angular velocity corresponding to the angular momentum obtained by the tidal torque at the linear stage of density fluctuations. Quantitatively, the angular momentum is given by the spin parameter \(\lambda \approx (J/T_{E}|E|^{1/2})/(GM_B^{1/2}) = 0.05\), where \(J\), \(T_E\), and \(M_B\) are respectively the total angular momentum, energy, and mass (Barnes & Efstathiou 1987; Heavens & Peacock 1988). Here, the rigid rotation is assumed.

The mass range of galactic bulges is postulated to be \(10^{6-13}M_\odot\). (However, in the present analysis, it is not very important to specify \(M_{\text{bulge}}\), because the results are scaled with \(M_{\text{bulge}}\) as shown below.) The total mass of the ISM, \(M_{\text{gas}}\), and the mass of each gas cloud, \(m_c\), are parameters. If dust opacity as well as Thomson scattering is considered, the mass extinction is expressed by \(\chi = (n_e \sigma_T + n_d \sigma_d)/\rho_g (= \rho_k + \rho_d)\), where \(\sigma_T\) is the Thomson scattering cross section, \(n_e\) is the electron number density, \(\rho_g\) is the gas density, and \(n_d, \sigma_d,\) and \(\rho_d\) are respectively the number density, cross-section, and mass density of dust grains. If we take the dust-to-gas mass ratio \(f_{\text{dg}} \approx 10^{-2}\), then the opacity ratio is

\[
\frac{n_d \sigma_d}{n_e \sigma_T} = 1.9 \times 10^2 \left( \frac{a_d}{1 \mu\text{m}} \right)^{-1} \left( \frac{\rho_k}{\text{g cm}^{-3}} \right)^{-1} \left( \frac{f_{\text{dg}}}{10^{-2}} \right),
\]

where \(a_d\) is the grain radius and \(\rho_k\) is the density of solid material within the grain. Thus, we find \(n_d \sigma_d \gg n_e \sigma_T\) in the situations of interest. Hence, in this paper, we evaluate the mass extinction by \(\chi = n_e \sigma_T/\rho_g\).

Finally, as for the stellar evolution, we assume a Salpeter-type initial mass function (IMF) as \(d\phi = A(m_*/M_\odot)^{-1.35}\) for a mass range of \([m_i, m_u]\). In the present analysis, we consider an initial starburst and the subsequent passive evolution of stars. The upper mass limit is inferred to be around \(40M_\odot\) in starburst regions (Doyon, Puxley & Joseph 1992). As for the lower mass limit, some authors suggest that the IMF in starburst regions is deficient in low-mass stars, with a cut-off of about \(2-3M_\odot\) (Doane & Matthews 1993; Charlot et al. 1993; Hill et al. 1994). In this paper, we assume \(m_i = 2M_\odot\) and \(m_u = 40M_\odot\) in the early stage of bulge formation. However, it should be also kept in mind that the lower mass limit is under debate; the claimed lower cut-off could be due to the magnitude limit effect (Skelton et
al. 1999), and also recently sub-solar-mass stars are found in a starburst region in our galaxy, NGC 3603 (Brandl et al. 1999). In order to incorporate the stellar evolution, we adopt the mass-luminosity relation ($L/\mathcal{M}_\odot = (m_*/\mathcal{M}_\odot)^{3.77}$, and the mass-age relation $\tau_* = 1.1 \times 10^{10} (m_*/\mathcal{M}_\odot)^{-2.7}$ yr (Lang 1974), where $m_*$, $\tau_*$, and $\tau_*$ are respectively the stellar mass, luminosity, and age.

3 BASIC EQUATIONS

As a relativistic result of radiative absorption and subsequent re-emission, the radiation fields exert a drag force on moving material in resistance to its velocity. This radiation drag extracts angular momentum from the ISM, thereby allowing the gas to accrete on to the galactic center. The radiation drag is an effect of $O(c/v)$, but it could provide a key mechanism for angular momentum transfer in intense radiation fields. We put the origin at the center of the bulge, and adopt cylindrical coordinate $r$, $\phi$, and $z$, where $z$-axis is the rotation axis of stars and gas. The components of the specific radiation force that is exerted on moving fluid elements with velocity $v$ are given by

\begin{equation}
\dot{f}_r = \frac{\chi}{c} (F^r - v_r E - v_t P^\phi - v_\phi P^r - v_\phi P^z),
\end{equation}

\begin{equation}
\dot{f}_\phi = \frac{\chi}{c} (F^\phi - v_\phi E - v_\phi P^\phi - v_\phi P^r - v_\phi P^z),
\end{equation}

and

\begin{equation}
\dot{f}_z = \frac{\chi}{c} (F^z - v_z E - v_\phi P^z - v_\phi P^r - v_\phi P^\phi),
\end{equation}

(Mihalas & Mihalas 1984) in the $r$-, $\phi$-, and $z$- directions respectively. Here, $E$ is the radiation energy density, $F^\alpha$ is the radiation flux, and $P^{\alpha\beta}$ is the radiation stress tensor where the non-diagonal components are null owing to the present symmetry.

Using equations (1)–(3), we have the radiation hydrodynamical equations to $O(c/v)$ as

\begin{equation}
\frac{dv_r}{dt} = \frac{v_r^2}{r} - f_r^g + \frac{\chi}{c} [F^r - (E + P^\phi) v_\phi]
\end{equation}

\begin{equation}
\frac{1}{r} \frac{d(rv_\phi)}{dt} = \frac{\chi}{c} [F^\phi - (E + P^\phi) v_\phi]
\end{equation}

\begin{equation}
\frac{dv_z}{dt} = -f_z^g + \frac{\chi}{c} [F^z - (E + P^z) v_\phi]
\end{equation}

where $f^g_r$ and $f^g_z$ are respectively $r$- and $z$- component of the gravitational force.

The azimuthal equation of motion (5) is the equation of angular momentum transfer. This equation implies that the radiation flux force (the first term on the right-hand side) makes fluid elements tend to corotate with stars, whereas the radiation drag (the second term on the right-side) works to extract the angular momentum from gas. Therefore, the gain and loss of total angular momentum is determined by equation (5).

4 UNIFORM ISM

In this section, we consider the angular momentum transfer by radiation drag in a uniform ISM. First, we analytically calculate the radiation fields produced by spherically distributed stars in an optically thin regime, and assess the angular momentum loss rate $\dot{J}$ and the mass accretion rate $M$. Next, we extend the analysis to an optically thick regime. Then, the relationship between the optical depth of a dusty ISM and the angular momentum transfer efficiency is derived.

4.1 Optically thin regime

For uniform distributions of stars, the radiation fields inside the bulge are analytically integrated in an optically thin regime to be

\begin{equation}
cE = \frac{3L_{\text{bulge}}}{2\pi r_b^4} \left( 1 - \frac{\pi}{2} \frac{r}{r_b} \right),
\end{equation}

\begin{equation}
F^r = \frac{L_{\text{bulge}}}{4\pi r_b^4},
\end{equation}

\begin{equation}
cF^\phi = \frac{3L_{\text{bulge}}}{2\pi r_b^4} \omega r_b \left( \frac{225}{216} \right) \left( \frac{r}{r_b} \right)^2 + \frac{3L_{\text{bulge}}}{2\pi r_b^4} \omega r_b \left( \frac{1}{1 - \frac{r}{r_b}} \right),
\end{equation}

\begin{equation}
F^z = \frac{L_{\text{bulge}}}{4\pi r_b^4},
\end{equation}

\begin{equation}
cF^r = \frac{1}{3} \frac{L_{\text{bulge}}}{2\pi r_b^4} \left( 1 - \frac{3\pi}{16} \frac{r}{r_b} \right),
\end{equation}

\begin{equation}
cF^\phi = \frac{1}{3} \frac{L_{\text{bulge}}}{2\pi r_b^4} \left( 1 - \frac{9\pi}{32} \frac{r}{r_b} \right),
\end{equation}

\begin{equation}
cF^z = cF^\phi,
\end{equation}

where $L_{\text{bulge}}$ and $r_b$ are the luminosity and radius of the bulge and $\omega$ is the angular velocity of stars. These quantities are equivalent to those obtained by Fukue, Umemura & Mineshige (1997), except that the radiation flux $F^\phi$ in the azimuthal directions is generated by the rotation of the bulge. As seen in equation (5), the flux $F^r$ works to corotate the ISM with stars in contrast to the drag force. In Fig. 1, we compare both forces exerted on gas per unit mass, $f^{\text{drag}} = \chi (E + P^\phi) v_\phi/c$ and $f^\phi = cF^\phi/c$, where $v_\phi = r\omega$. It is found that $f^{\text{drag}}$ overwhelms $f^\phi$ everywhere. Thus, the optically thin ISM can always lose the angular momentum as a result of radiation drag. The angular momentum loss rate per unit volume per unit time $j$ is evaluated by

\begin{equation}
j = -\frac{L_{\text{bulge}}}{c^2} \frac{3\chi \omega}{2\pi} xf(x)
\end{equation}

at the point of $x = r/r_b$, where $f(x) = (\frac{225}{216} + \frac{45}{192})x^2 + \frac{1}{8}x$. Equation (14) is integrated over the volume of the bulge to give the total angular momentum loss rate $J$ as

\begin{equation}
j = -\frac{L_{\text{bulge}}}{c^2} \frac{3\chi \omega^2}{2\pi} r_b \rho_0,
\end{equation}

where $\rho_0$ is the density of the ISM, and $\beta = \int xf(x) dV$ with $dV$ being volume element in spherical coordinates. Noting that the initial angular momentum is

\begin{equation}
J_0 = \frac{2}{3} M_{\text{gas}} \omega r_b^3,
\end{equation}

the mass accretion rate is expressed as

\begin{equation}
\dot{M}_{\text{gas}} = -\frac{J}{J_0} = \eta \frac{L_{\text{bulge}}}{c^2} - \tau_r,
\end{equation}

Mass accretion in a clumpy ISM
where $\tau_T = \chi \rho_0 r_b = (3 \mathcal{M}_{\text{gas}})/\left(4 \pi r_b^2\right)$. The coefficient $\eta$ gives the radiation drag efficiency. By calculating the constant $\beta$ numerically, $\eta$ is found to be 0.32.

### 5.1 Extinction by a clumpy ISM

In the case of a clumpy ISM, $N_c (= 10^3)$ identical clouds with optical depth of $\tau = \chi \rho_{\text{gas}} r_c$ are distributed randomly. In an optically thin regime, the radiation fields produced by a star are

$$dE_0 = \frac{1}{c} \frac{\ell_\star}{4 \pi r^2}, \quad dF_0 = \frac{\ell_\star}{4 \pi r^2} r, \quad dF_0'' = dE_0, \quad dP_0^\phi = dP_0'^\phi \approx 0.$$  

As readily understood by these results, the angular momentum transfer efficiency by the radiation drag is maximum when the optical depth of the ISM is around unity.

### 4.2 Optically thick regime

In optically thick ISM, the regions where the optical depth $\tau_s$ from each star is less than unity are subject to the radiation drag (Tsuribe & Umemura 1997). In this situation, the flux for the region of $\tau_s \leq 1$ is given by

$$F^\phi = \int ER\omega dV,$$

and

$$F'^{\text{drag}} = \int \left(E + P^\phi\right) R_b \omega dV \approx \int ER_b \omega dV,$$

where $R$ are $R_b$ are respectively the distance from the rotational axis to a star and to one volume element, $dV = r^2 \sin \theta dr d\theta d\phi$ in spherical coordinates for the position of a star. $E = \ell_\star/4 \pi c r^2$, $P^\phi \approx 0$ with $\ell_\star$ being the luminosity of a star. Consequently, the local angular momentum transfer rate by equation (5) is

$$\dot{J} = -\frac{\ell_\star}{6} \chi \omega_r r_b^3 \rho_0.$$  

This is summed up over all stars to give

$$\dot{J} = -\frac{L_{\text{bulge}}}{6 c^2} \chi \omega_r r^3 \rho_0,$$

where $r_s$ is the size of the region of $\tau_s = 1$, and $L_{\text{bulge}} = N_c \ell_\star$. Then, the mass accretion rate is given as

$$\dot{M}_{\text{gas}} = -L_{\text{bulge}} \frac{\dot{J}}{J_0} = \frac{5}{12} \frac{L_{\text{bulge}}}{c^2} r^3 \tau_T^{-2}.$$  

So far, we have considered a rigidly rotating system. We can also analytically estimate the accretion rate in a system with a different rotation law as $\nu_\phi \sim n^\nu$ (e.g. $n = -0.5$ for the Keplerian rotation). Then, the mass accretion rate turns out to be

$$\dot{M}_{\text{gas}} = -M_{\text{gas}} \frac{\dot{J}}{J_0} = \frac{n^2 (n + 4)}{9 B} \left(\frac{\bar{R}_i}{r_b}\right)^{n-1} \frac{L_{\text{bulge}}}{c^2} \tau_T^{-2},$$

where $B = \sqrt{\Gamma(\frac{n+2}{2})/\Gamma(\frac{n+4}{2})}$, $\bar{R}_i$ is the mean distance from the rotational axis to each star, and $\Gamma(n)$ is the Gamma function. It is noted that the difference between (22) and (23) is merely a small factor for ordinary rotation laws. As a result, it is found that the mass accretion rate decreases according to $\tau_T^{-2}$ in an optically thick regime. The physical reason is that the larger is the optical depth of the bulge, the closer to zero is the difference between the velocity of stars and the gas velocity, so that the radiation drag efficiency falls.

Combined with the optically thin case, we have the mass accretion rate in the uniform case as

$$\dot{M}_{\text{gas}} \propto \begin{cases} L_{\text{bulge}} \tau_T^{-1} & (\tau_T < 1), \\ L_{\text{bulge}} \tau_T^{-2} & (\tau_T > 1). \end{cases}$$

As readily understood by these results, the angular momentum transfer efficiency by the radiation drag is maximum when the optical depth of the ISM is around unity.
Mass accretion in a clumpy ISM

Mass accretion in a clumpy ISM...
to consider further effects which have not been incorporated in this simple model. In the present analysis, we assumed an initial coeval event of star formation, and the subsequent passive evolution without further star formation episodes. The radiation drag efficiency is basically determined by the total number of photons which are emitted from sources and absorbed by clouds during the whole history of the bulge. The recycling of the ISM for star formation generates more photons and therefore could enhance the mass ratio roughly by a factor of 2 (Umemura 2001). Also, in realistic situations, the radiation drag is not likely to remove completely the angular momentum of stripped gas, and also stripped gas may be mixed with ISM having appreciable angular momentum. Although the detailed processes are not very clear at present, a little leftover angular momentum may lead to the formation of a viscous accretion disk around a BH, which could ignite QSO activity with nearly Eddington luminosity (Umemura 2001). If QSO activity is triggered, the radiation drag efficiency is considerably reduced. Recent observational results are plotted by symbols. The filled squares denote elliptical galaxies from Magorrian et al. (1998). The filled circles show elliptical galaxies from Ho (1999), Ferrarese & Merritt (2000), Kormendy & Ho (2000), and Sarzi et al. (2001). The filled circles denote elliptical galaxies from Merritt & Ferrarese (2001a). The small dots denote Seyfert galaxies from Ho (1999), Wandel (1999), and Gebhardt et al. (2000a), and the open triangles show QSOs from Lair (1998). The relation of Magorrian et al. (1998) is \( M_{\text{BH}} = 0.006 M_{\text{bulge}} \), which is shown by a dot-dashed line; the relation of Merritt & Ferrarese (2001a) is \( M_{\text{BH}} = 0.001 M_{\text{bulge}} \), which is shown by a dashed line; and the relation for Seyfert galaxies is \( M_{\text{BH}} = 2.5 \times 10^{-4} M_{\text{bulge}} \) (Sarzi et al. 2001), which is shown by a thin dashed line. The hatched area shows the prediction of this paper. The lower bound is a single-starburst model \( (M_{\text{BH}} = 4.1 \times 10^{-4} M_{\text{bulge}}) \). The upper bound is the model incorporating the effect of the recycling of star formation and AGN activity \( (M_{\text{BH}} = 1.4 \times 10^{-3} M_{\text{bulge}}) \).

Figure 3. The BH-to-bulge mass ratio \( (M_{\text{BH}}/M_{\text{bulge}}) \) against the total optical depth \( (\tau_T) \) of the bulge. The thick lines show the results for \( N_{\text{int}} \geq 1 \) and the thin lines are those for \( N_{\text{int}} < 1 \). Filled circles denote \( N_{\text{int}} = 20 \), filled squares \( N_{\text{int}} = 5 \), filled triangles \( N_{\text{int}} = 1 \), open circles \( N_{\text{int}} = 0.1 \), open squares \( N_{\text{int}} = 0.01 \), and open triangles \( N_{\text{int}} = 0.001 \). The dashed line is the analytic solution for a uniform ISM corresponding to \( N_{\text{int}} \rightarrow \infty \), where \( M_{\text{BH}}/M_{\text{bulge}} \propto \tau_T \) in an optically thin regime and \( M_{\text{BH}}/M_{\text{bulge}} \propto \tau_T^{-2} \) in an optically thick regime. The arrows show the points where the optical depth of a cloud (\( r \)) is unity. For \( N_{\text{int}} \approx 1 \), the radiation drag efficiency is maximal if \( \tau_T \geq 1 \). For \( N_{\text{int}} < 1 \), the radiation drag efficiency is saturated when \( \tau_T \geq 1 \).

Figure 4. The relation between the BH mass and the bulge mass. The vertical axis is the BH mass, and the horizontal axis is the bulge mass, in units of \( M_{\odot} \). Recent observational results are plotted by symbols. The filled squares denote elliptical galaxies from Magorrian et al. (1998). The filled circles show elliptical galaxies from Ho (1999), Ferrarese & Merritt (2000), Kormendy & Ho (2000), and Sarzi et al. (2001). The filled circles denote elliptical galaxies from Merritt & Ferrarese (2001a). The small dots denote Seyfert galaxies from Ho (1999), Wandel (1999), and Gebhardt et al. (2000a), and the open triangles show QSOs from Lair (1998). The relation of Magorrian et al. (1998) is \( M_{\text{BH}} = 0.006 M_{\text{bulge}} \), which is shown by a dot-dashed line; the relation of Merritt & Ferrarese (2001a) is \( M_{\text{BH}} = 0.001 M_{\text{bulge}} \), which is shown by a dashed line; and the relation for Seyfert galaxies is \( M_{\text{BH}} = 2.5 \times 10^{-4} M_{\text{bulge}} \) (Sarzi et al. 2001), which is shown by a thin dashed line. The hatched area shows the prediction of this paper. The lower bound is a single-starburst model \( (M_{\text{BH}} = 4.1 \times 10^{-4} M_{\text{bulge}}) \). The upper bound is the model incorporating the effect of the recycling of star formation and AGN activity \( (M_{\text{BH}} = 1.4 \times 10^{-3} M_{\text{bulge}}) \).

0.006 by Magorrian et al. (1998), while it is comparable to \( (M_{\text{BH}}/M_{\text{bulge}}) = 0.001 \) by Merritt & Ferrarese (2001a).

Another important effect is the geometrical dilution. In previous works, the radiation drag efficiency would be strongly subject to the effects of geometry (Umemura, Fukue, & Mineshige 1997, 1998; Ohnaga et al. 1999). A large fraction of emitted photons can escape from a disc-like system and thus the radiation drag efficiency is considerably reduced. Recent observation have revealed that the BH mass fraction is significantly smaller than 0.001 in disc galaxies (Salucci et al. 2000; Sarzi et al. 2001). Geometrical dilution may be a reason for the observational fact in disc galaxies, but the quantitative details are not clear before such an aspherical system is actually simulated.

7 CONCLUSIONS

By assuming a simple model of a bulge, we have investigated the mutual effect between the clumpiness of interstellar medium and the optical depth on the radiation drag efficiency for the angular momentum transfer. In a clumpy interstellar medium, we have accurately solved 3D radiation transfer to calculate the radiation drag force by the rotating bulge stars. We find that the radiation drag efficiency is sensitively dependent on the density distribution of the ISM in an optically thick regime. The efficiency drops by a
factor of $\tau_{\text{rad}}^{-2}$ in a uniform ISM, while the efficiency turns out to be almost constant at a high level in a clumpy ISM. Also, the radiation drag efficiency falls as the covering factor out to be almost constant at a high level in a clumpy ISM. The range of the predicted mass ratio is from the mass of a supermassive black hole and the mass of the galactic bulge. The present radiation hydrodynamical mechanism accounts for the linear relation between the mass ratio significantly: e.g. the realistic chemical evolution and geometrical dilution.

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