Recent lattice QCD results on charmonium properties are reviewed. I comment on molecules and hybrid states as well as on future studies of states near strong decay thresholds.

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1. New States

Recently, for the first time in over twenty years, several narrow hadronic resonances have been discovered. Examples are an $\Upsilon D$-wave, the $B_c$, the $\eta_c'$, the $h_c$ and also new charmed baryons. Some findings were particularly embarrassing for theorists, such as the $D_{s0}^{*}(2317)$, the $D_{s1}^{*}(2460)$ and the $X(3872)$, whose masses and properties were not anticipated. Of course, in retrospect everyone can explain anything. The power of postdiction is symptomatic for our lack of understanding of strong interaction dynamics. This is further illustrated by the now fading pentaquark-hype. What are the relevant degrees of freedom for the classification of hadronic excitations? Are angular and radial excitations of quark model states sufficient to obtain a qualitative picture or do we have to include hybrid- and molecules (i.e. meson-meson and/or tetraquark states) as well? (How) can we make quantitative predictions of masses and strong decay rates?

Charmonia offer an exciting arena for improving theoretical tools and experimental techniques. In some aspects charm quarks can be classed as “heavy”. Nonetheless, with relative valence quark velocities $v > 1/2$ and binding energies $\Lambda \gtrsim m_c/3$, it is clear that corrections to an effective heavy quark field theory description can be large: ultimately there is no short-cut to the inclusion of non-perturbative low energy QCD effects. What we learn from charmonia will to a large extent also apply to light hadron spectroscopy. However, (potential) non-relativistic QCD ((p)NRQCD) expectations can still serve as a starting point. Also potential models with phenomenologically fitted parameters appear to describe many features reasonably well.
and it needs to be clarified to what extent these are connected to QCD as the fundamental theory.

Due to the spatial compactness of charmonium wavefunctions we encounter the luxury of finding many narrow resonances below strong decay thresholds. Also above such thresholds the situation is still much cleaner than it is in the light quark sector. While this applies even more so to bottomonia, in the Υ case, production in $e^+e^-$ machines is limited to vector states and their decay products. Not even the ηb meson has been detected thus far! Furthermore, the $B\overline{B}$ threshold region is elusive. Some information could be gained by scans above Υ(4S) and runs at Υ(10860)|Υ(11020) in a future super-$B$ factory. Nonetheless, the prospects in Υ spectroscopy are quite bleak due to the small hadronic production cross sections and the unavailability of a top-meson that could produce Υ states in its decay. In the charm-sector however many new states have been discovered recently, some in $p\overline{p}$ collisions at FNAL, others in $\psi$ production and decays at CLEO and many by the $B$ factories, mostly in $B$ decays. A summary of these recent findings is presented in Fig. 1. 10 resonances have been discovered 1974 – 1977, none in 1978 – 2001 and eight (!) since 2002. For recent reviews of the properties of these new states, see e.g. Ref. 1
2. Spectroscopy: $n_f = 0$ and $n_f > 0$

Most hadronic resonances undergo strong decays. This situation is hard to study on the lattice and requires repeating the simulation on different (and large) physical volumes. Fortunately, most charmonia are very narrow. The states above 3900 MeV however have widths ranging from $O(10)$ MeV to $O(100)$ MeV. A model with strong decays switched off can serve as a starting point for investigations of the more involved realistic situation. This can be achieved by switching sea quark effects off completely. Studies in this so-called quenched approximation have been performed by a variety of collaborations, with an anisotropy parameter $\xi = a_s/a_t > 1$ and keeping spatial and temporal lattice resolution the same, $a_s = a_t$.

For charmonia, $\langle r^{-1} \rangle \gg M$, where the mass $M$ is calculated in units of $a_t^{-1}$ and the wavefunction extent $r$ in units of $a_s$. Therefore, introducing an anisotropy appears to allow for a smaller number of lattice points and hence a reduction in the computational effort. However, quantum corrections considerably reduce the naive gain factor. Also there is some overhead from the nonperturbative matching of fermionic and gluonic anisotropies, which is particularly delicate when sea quarks are included. Nevertheless, a pilot study with $n_f = 2$ and $\xi \approx 6$ already exists.

We summarize the quenched situation in Fig. 2. The Columbia, CP-PACS, and CP-PACS results are extrapolated to the continuum limit. Similar results for a smaller number
of states have been obtained by the Guangzhou group\textsuperscript{5,6} and QCD-TARO\textsuperscript{11}. It is reassuring that the latter study on isotropic lattices confirms the anisotropic results. The scale $r_0^{-1} \approx 394$ MeV is set from potential models. A systematic scale error of about 10\% should be assumed, for stable states that do not mix! One often thinks of the charm quark as heavy in the sense that its mass is bigger than typical QCD binding energies. However, the spectrum of states entirely made out of glue covers a similar energy range. To illustrate this we have superimposed these glueball\textsuperscript{4} onto the Figure.

Note that for the lattice results displayed, effects of diagrams with disconnected quark lines have been neglected. One might expect OZI violating contributions from these, for instance for states that lie close to glueballs with the same quantum numbers or close to decay thresholds. Also (tiny) effects from the axial anomaly or from $\eta_c - \eta'$ mixing cannot be excluded. The impact of such diagrams on the vector and pseudoscalar channels has been studied by McNeile and Michael\textsuperscript{12} and by CP-PACS\textsuperscript{13} and found to be insignificant within errors of about 20 MeV.

The Figure also contains the lightest two spin-exotic $c\bar{c}$-gluon hybrids.\textsuperscript{2} Again, the results agree with other studies.\textsuperscript{5,6} It is also possible to employ hybrid sources with non-exotic quantum numbers. States prepared in this way are found to decay rapidly in Euclidean time into the respective non-hybrid ground states which is hardly surprising, given the fact that one expects an energy gap of about 1.4 GeV when solving the Schrödinger equation with ground state and hybrid lattice potentials as input. However, the extent of spectroscopic hybrid components in charmonium wavefunctions deserves further study.\textsuperscript{16} Similarly, it is difficult to disentangle $S$ and $D$ waves in lattice simulations of the $1^{--}$ sector.\textsuperscript{2}

When sea quarks are switched on, would-be glueballs, standard charmonia and $D\bar{D}$ molecules will mix (as will exotic hybrid charmonia and molecules). This has not yet been studied on the lattice. In addition, one would expect the fine structure to be particularly sensitive to the sea quark content. For instance, the $S$ wave singlet-triplet splitting in NRQCD reads, $\Delta M = c_F/(6m_c^2)\langle \psi|V_4|\psi \rangle + \cdots$ where $c_FV_4 = (32\pi/3)\alpha_s\delta^3(r)$ to leading order in perturbative QCD. Due to the short-distance nature of this term one would expect significant finite $a$ effects. Moreover, with sea quarks included, the running of $\alpha_s(q)$ will slow down such that at high momenta/short distances $\alpha_s$ will maintain a relatively larger value. In Ref.\textsuperscript{17} this effect is estimated to amount to 30–40\%.

Indeed, setting the scale from $r_0$, a splitting of only 77(2)(6) MeV is found\textsuperscript{11}. When adjusting the lattice spacing to the $1\overline{P} - 1\overline{S}$ splitting instead, the central value increases to $\Delta M \approx 89$ MeV, still short of the experimental $\Delta M \approx 117$ MeV. This result is consistent with Refs.\textsuperscript{2,3} while in small volume studies at one lattice spacing\textsuperscript{14} and at a too small charm quark mass\textsuperscript{4} higher results have been obtained. The FNAL and MILC\textsuperscript{18} collaborations indeed find the $J/\psi - \eta_c$ splitting to increase to $\Delta M = 94(1), 101(2)$ and 107(3) MeV at lattice spacings $a^{-1} \approx 1.1, 1.6$ and 2.3 GeV, respectively, in studies with $n_f = 2 + 1$ sea quark flavours. Note however
that there are theoretical doubts about the validity of MILC's fermion approach. Also, disconnected quark line diagrams have not yet been included. While their impact is likely to be small effects of up to 20 MeV cannot be excluded.\textsuperscript{12}

3. Radiative Transitions

In order to match experimental resonances to theoretical predictions ultimately one has to go beyond the mass spectrum. For instance one would expect hybrids and molecules to couple differently to decay and production channels than states with quark model dominance. Gaining information on strong decay rates from lattice simulations, without additional assumptions, is complicated. However, calculating electromagnetic (EM) decay and transition rates is in principle straightforward. For narrow states such predictions can directly be confronted with experiment. For broad states above threshold, where data on EM transitions is lacking, a knowledge of the wavefunction will reveal qualitative information on possible production mechanisms.

QCD-TARO\textsuperscript{11} calculated electric quarkonium wavefunctions. In the non-relativistic approximation, the EM decay constant is to leading order proportional to the wavefunction at the origin. Dudek and Edwards directly calculated two-photon decay rates\textsuperscript{8} as well as the decay constants\textsuperscript{7} $f_{J/\psi}, f_{\psi'}, f_{\eta_c}$ and $f_{\eta_c'}$. These studies have been performed at one lattice spacing on small volumes in the quenched approximation and with a charm quark mass about 5\% smaller than the physical one. In particular the calculations of this pilot study of EM transition rates are extremely promising. For instance the M1 transition rate...
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\[ \Gamma(J/\psi \rightarrow \eta_c \gamma) \propto q^3/(m_{J/\psi} + m_{\eta_c})^2 \alpha_s |\hat{V}(0)|^2 \]

contains the term \( \hat{V}(Q^2) \) at \( Q = 0 \). This decomposes into a kinematical factor, multiplied by the three point function, \( \langle \eta_c(p')|j^\mu(0)|J/\psi(p) \rangle \), that can be calculated on the lattice. In Fig. 3 we display the lattice prediction \( \hat{V} \) of \( \hat{V} \), obtained for different momentum transfers \( Q^2 = (p' - p)^2 \), together with the \( Q = 0 \) value derived from the experimentally measured transition rate. This expectation was phenomenologically corrected for the mis-adjusted lattice charm quark mass (lat. mass) to facilitate the comparison: the lattice \( Q^2 \)-extrapolation appears very reasonable.

The transition \( \chi_{c1} \rightarrow J/\psi \gamma \) was studied too as were \( \chi_{c0} \rightarrow J/\psi \gamma \) and \( \Gamma(h_c \rightarrow \eta_c \gamma) \approx 600 \text{ keV} \). The latter rate is a prediction since CLEO only detected the cascade \( \psi' \rightarrow \pi_0 h_c, h_c \rightarrow \eta_c \gamma \) where both individual rates are unknown.

4. Mixing and Strong Decays

It is evident that states close to decay thresholds like the \( X(3872) \) or the \( Y(3940) \) (see Fig. 1) will couple to molecular \( D\bar{D} \) or \( D_s\bar{D}_s \) components, respectively. At present there exist no meaningful direct lattice studies of such \( c\bar{c} \leftrightarrow c\bar{q}q\bar{q} \) mixing phenomena. Ideally one would create and destroy states using an extended basis of operators, including molecules and hybrids, to study their mixing.

In the absence of such an investigation, one might benefit from the non-relativistic approximation. At least qualitatively charmonia can be described by potential models. The \( Q\bar{Q} \) sector can be extended to include \( D\bar{D} \) potentials. \( Q \) denotes a static (heavy) quark and \( D = Q\bar{q} \). This situation has recently been studied on the lattice, albeit only at one lattice spacing \( a \approx 0.083 \text{ fm} \) and with \( n_f = 2 \) mass-degenerate sea quark flavours, slightly lighter than the strange quark. We
denote the physical eigenstates by $|n\rangle$. These can be decomposed,

$$
|1\rangle = \cos \theta |\bar{Q}Q\rangle + \sin \theta |\bar{D}D\rangle + \cdots ,
$$

$$
|2\rangle = -\sin \theta |\bar{Q}Q\rangle + \cos \theta |\bar{D}D\rangle + \cdots .
$$

The resulting energy levels and mixing angle $\theta$ are displayed in Fig. 4 as a function of the $\bar{Q}Q$-separation $r$.

At $r > r_c \approx 1.25$ fm the $\bar{Q}Q$ string breaks and the ground state is dominated by the $\bar{D}D$ configuration. Interestingly, also at $r < r_c$ there is a significant four-quark admixture in the ground state. This 4% probability is consistent with the large four-quark components found in phenomenological models of charmonia and bottomonia. In Ref. 22 the action and energy density distributions around the static sources have been investigated: string breaking appears to be an instantaneous process with no spatial localization of the light $q\bar{q}$ pair that is created. This suggests an $2 \times 2$ Hamiltonian with a transition term $(E_2 - E_1) \sin \theta \cos \theta$ that only depends on the distance $r$.

One can include $\bar{D}D$ effects as quantum mechanical perturbations, starting from an unperturbed $\bar{Q}Q$ wave function $\psi_0$. By summing over all possible intermediate states $i$ where $\psi_i$ can be in both sectors, $\bar{D}D$ and $\bar{Q}Q$, one ends up with complex Hamiltonian for each state. The solution of the corresponding Schrödinger problem yields both: the respective energy levels and decay widths.

A study of the sea quark mass dependence of the static energy levels is as yet lacking. The bands in the left plot of Fig. 4 represent an $n_f = 2 + 1$ speculation. In this case string breaking will occur at a somewhat smaller distance, $r_c = 1.13(10)(10)$ fm, the gap $E_2(r_c) - E_1(r_c)$ will be larger and moreover there will be a second string breaking threshold $\bar{Q}Q \leftrightarrow D_s\bar{D}_s$. It would be interesting to see, once the sea quark mass dependence has been established, if properties of $X(3872)$ and $Y(3940)$ can be reproduced or if the present overpopulation of the $1^{--}$ sector can be explained by coupled channel models, based on such lattice potentials.

5. Summary

We have arrived at a good understanding of the spectrum of states in the mass region, 2.5 – 5 GeV, in the quenched approximation to QCD, including glueballs and $c\bar{c}$-glue hybrid states, besides standard charmonia. However, information on four-quark states is still lacking and there is some uncertainty regarding the size of contributions due to diagrams with disconnected quark lines. Electromagnetic decay and transition rates have been calculated successfully in a pilot study.

Studies of $n_f = 0$ QCD set the stage for more realistic $n_f > 0$ simulations. Promising $n_f \geq 2 + 1$ results have been obtained, indicating a convergence of low lying states towards experiment, once sea quarks are included. Over the next few years such simulations will be extended to include more realistic sea quarks and a broader basis of operators to create states, including four-quark and hybrid operators. In the meantime one has to resort to coupled channel models to understand
threshold effects. These can be refined by using input from lattice simulations. Ultimately, one would wish to arrive at QCD predictions of strong decay rates. This is very hard to achieve by a direct lattice computation but again, the predictive power of models can benefit from dedicated lattice tests and comparisons. Last but not least I hope that these exciting times will continue with further experimental discoveries.

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