Research on the Contact State Method Considering the True Topography of the Part Surface

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Abstract. In aircraft manufacturing and other assembly scenarios that require high product assembly accuracy, the true topography of the mating surface of the part is one of the important factors that affect the aircraft’s aerodynamic shape, step difference, and other key characteristics. However, the existing Computer-Aided Tolerancing (CAT) system is difficult to express the true topography of the part surface and the mating simulation. To solve this problem, an analysis method that reflects the true contact state of the part is proposed—established a realistic assembly model containing the geometric information and the assembly force information of the part, designed the matching surface area segmentation and contact point search algorithm, and the contact state determination algorithm considering the assembly force. This paper takes the analysis of the contact state of step surface parts subjected to uniform assembly force as an example and compares the analysis results of commercial software packages and the difference surface method to verify the feasibility of this method.

Keywords. Contact state; skin model shape; realistic assembly model; TTPS.

1. Introduction

With the continuous development of intelligent manufacturing technology, physical models are increasingly using Computer-Aided Design (CAD) and other technologies to model in the digital space in the development of digital products. The purpose is to calculate the accuracy of the equipment without the need for a physical model by establishing a realistic assembly model of the product, including the True Topography of Part Surface (TTPS) of the part geometric deviation information and the assembly force information.

To analyze the contact state of assembly parts, a realistic assembly model considering TTPS must be established to overcome the shortcomings of most current studies that consider the geometric deviation of surface features as ignoring the form deviation and only perform translation and rotation processing on features.

The skin model is defined in reference [1] as the “model of the physical interface of the workpiece and its environment”, which is the projection of real physical features in the digital space. In spite of that the skin model is an infinite set [2], scholars have proposed the concept of Skin Model Shapes [3] that uses a finite description set to represent the infinite amount of information in the skin model. Obviously, the skin model shape can be measured and reconstructed, which enables it as an important method for reconstructing a realistic assembly model at present.
The application of skin model shape technology is shown in figure 1. The choice of route is depended on specific assembly scenario: Route A [4] is suitable for the stage that the actual assembly has not yet started, whereas the design of the assembly part has been completed, and the digital model of the part exists. The impact of the key characteristics of the part [5] on the key characteristics of the product will be analysed under the tolerance requirements; Route B [6] is suitable for the stage that the parts have been manufactured, whereas the physical model is reconstructed through measuring, and the assembly accuracy is verified.

**Figure 1.** Steps to create a skin model.

Before this concept was proposed and developed, tolerance representation models that were widely studied and applied mainly included: drift model [7], the degree of freedom model [8, 9], and the TTRS model [10]. These models cannot effectively consider the influence of shape deviation on TTPS. At present, commercial software packages such as 3DCS, VisVSA, etc., all adopt the variation model [11] yet do not take into account of the contact state under the influence of the assembly force.

In exploring the application of the skin model in coordination simulation, Andrea Corrado et al. [12] combined the skin model with finite element analysis and used the deviation of the finite element mesh nodes to construct the skin model. On this basis, the stress and deformation of the parts under the interference fit are analyzed, and the contact quality between the components is evaluated by the point projection method. The analysis process is mainly completed in the finite element software environment, which is time-consuming in the calculation. Benjamin Schleich et al. [13] did more exploration on this issue. They completed the complete process of CAT analysis based on the skin model, used the constraint registration method to complete the mating, and used the projection distance calculation in each simulation. The actual contact point of the mating feature. Jin Xin [14] used the NURBS-Brep data structure to build a skin model and used the difference surface method [15] to find the contact problem between two non-nominal surfaces. The contact problem between two equivalent planes was found using the convex hull of the surface shape error. Draw out the planes and contact points that represent the actual fit, and perform assembly error analysis based on this. Liu Ting [16] summarized the main methods of the skin model in coordination simulation-constrained registration and difference surface method and proposed a polyhedron-skin model to solve the error analysis of assemblies with arbitrary structures (serial, local parallel, and parallel, etc.).

This paper proposes a new matching simulation method, which is the method for analyzing the contact state of rigid parts based on the geometric deviation of parts under the influence of assembly force. It is divided into two parts, the former focuses on the algorithm design to determine the contact point between two surfaces, and the latter performs the contact state analysis considering the influence of assembly force.
2. Division of the Mating Surface Area and Search for Contact Points

2.1. Design of Mating Surface Area Division Algorithm

To better refer to the subject of related concepts, the following definitions are introduced:

[Definition 1] Characterizing surface S: Refers to the surface that contains TTPS information, that is, the reconstruction of the part surface using the skin model shape.

[Definition 2] Nominal surface NS: Refers to the key assembly feature surface on the nominal model of the part, that is, the target plane replaced by S.

[Definition 3] Surface datum normal n: Define the normal direction of NS replaced by S as the surface datum normal.

[Definition 4] Point height zij: The distance between any point on the S and NS along n is the point height, which can be expressed as: 

\[ z_{ij} = z_{ij}(x_i, y_j) - z_{nm}(x_i, y_j) \]

Then the point height field Z is:

\[
Z = \begin{bmatrix}
  z_{11} & z_{12} & \cdots & z_{1n} \\
  z_{21} & z_{22} & \cdots & z_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  z_{m1} & z_{m2} & \cdots & z_{mn}
\end{bmatrix}
\]

[Definition 5] Expansion domain and shrinkage domain: The curve that NS intersects with S during the entire movement along n composes the set \( L_A \), and all the closed intersection lines generated form the set \( L_C \). In set \( L_C \), the closed intersection line whose enclosing area gradually expands with the movement of NS is called the "expansion curve", and the maximum area enclosed by the expansion line during the movement of the nominal surface is the "expansion domain". Correspondingly, there are also "shrinkage curve" and "shrinkage domain". The expansion domain and shrinkage domain are collectively called "domain".

[Definition 6] Priority value PV: The priority value is defined for the sub-contact area where only two extreme points are mapped. Set \( \Delta h (\Delta h \geq 0) \) as the point height difference between the two extreme points, \( l \) is the projection distance of the two extreme points on the nominal surface, then the priority value is defined as \( \Delta h \times l \).

Using the method shown in figure 1 to reconstruct the model and determine the assembly status is cumbersome and inefficient. Therefore, using the related concepts in [Definition 4], the point height field Z is established from the surface containing the TTPS information, and the surface interpolation is used to reconstruct S. As shown in figure 2.

![Figure 2. Surface reconstruction using point height field.](image)

Common contact forms of rigid parts include plane-plane, stepped surface-stepped surface, cylindrical surface-cylindrical surface, and cone surface-cone surface. Using the method shown in figure 2, the above contact is transformed into the contact between the reconstructed representation surfaces generated by the point height field Z, that is, the plane-plane contact.

For the convenience of the following description, S also refers to the reconstruction of S. The following is a brief description of the above definition with S, as shown in figure 3.

For a specific surface, to determine the specific range of the expansion domains and shrinkage domains, S is cutting according to the point height values of all the surface extreme points, saddle points, and boundary extreme points to generate a set of intersection lines. The finite set of intersection
lines includes the birth, fusion, and rupture behaviors of all the intersections contained in the infinite set of the surface set $L_A$. The closed intersection with the enclosing area taken to a maximum defines all the expansion domains and shrinkage the domains. The following algorithm is shown in table 1.

![Image of intersection types](image)

**Figure 3.** A brief description of the above definition with $S$.

**Table 1.** Algorithm DOMAINDIVIDE.

| Algorithm DOMAINDIVIDE |
|------------------------|
| **Input:** $S_1$ and $S_2$ |
| **Output:** Representation surfaces $SD_1$ and $SD_2$ after domain division |
| 1. Calculate the position and height of all extreme points and saddle points on the surface |
| 2. Find the minimum and maximum height $h_{\text{min}}$ and $h_{\text{max}}$ of extreme points |
| 3. If $h_{\text{min}}$ < Surface boundary line height < $h_{\text{max}}$ then |
| Draw all contours of the surface at the height of the surface boundary line, and select the closed curve as the domain |
| 4. The saddle point height values are sorted, and the contours of the surface including the saddle point itself are drawn in turn |
| 5. If (The drawing result is two closed curves with one common points && the two curves have an inclusive relationship) then |
| Select the smaller closed curve contained therein as the domain |

Based on the concept of expansion domain and shrinkage domain, the contact of the two nominal surfaces of the representative surface in the surface datum normal direction is divided into the following four types: Type A: contact between shrinkage domain and expansion domain; Type B: contact between shrinkage domain and shrinkage domain; Type C: contact between expansion domain and expansion domain; Type D: contact between expansion domain and shrinkage domain. Figure 4 briefly illustrates these four contact types.

![Image of contact types](image)

**Figure 4.** Four contact types.

When two $S$ are in contact, the contact point is most likely to appear in the contact area A, followed by B and C. Therefore, the contact point search strategy can be defined as follows: firstly search for
contact points in contact areas A, secondly search for contact area B and C, and skip type contact area D. At the same time, the definition of ridge and groove is introduced. The expansion domains and the shrinkage domains are further divided, so that each sub-domain obtained by the final division contains only one extreme point.

After the division is completed, the original contact areas are divided into a large number of contact sub-areas. On this premise, it is necessary to continue to sort the contact sub-domains of the same type according to the priority value to obtain the search order of the contact points after the sub-areas are divided. Based on the algorithm DOMAINDIVIDE, a sub-domain division algorithm is shown in table 2.

Table 2. Algorithm DOMAINDIVIDE.

| Algorithm SUBDOMAINDIVIDE |
|----------------------------|
| **Input:** Representation surfaces SD$_1$ and SD$_2$ after domain division |
| **Output:** Contact sub-domains SR$_1$, SR$_2$...and search order after sub-domain division |
| 1. Overlap the surfaces SD$_1$ and SD$_2$ and divide the surfaces SD$_1$ and SD$_2$ with the boundary lines of the contraction domain and the expansion domain |
| 2. Identify and label each divided surface s$_1$, s$_2$... |
| If (divided surface s$_1$, s$_2$... ∈ expansion domains) then |
| label (s$_i$) = 1 |
| Else label (s$_i$) = 0 |
| 3. Perform logical AND operation on the labels of overlapping divided surfaces |
| If {label (s$_i$) && label (s$_j$)} == 1} then |
| This domain is a contact domain B or C |
| Else this domain is a contact domain A |
| 4. The groove line and the ridge line divide the contact area to form new contact sub-domains SR$_i$ |
| 5. Calculate PV of the extreme point in each contact sub-domain SR$_i$ |
| 6. Sort SR$_i$ by PV value and define SR search order |

2.2. Contact Point Search Algorithm Design

The domain division and search planning in the previous section has greatly reduced the search domains of contact points, only the divided contact sub-domains need to be searched for contact points in the search order, so contact point in the contact sub-domains search algorithm is proposed: using rectangular sampling method to discretize the contact sub-domains into points, the point pairs establish the global discrete height difference function between the two surfaces, and then retrieve the global minimum height difference function in the search order to determine the contact point position. The following algorithm is shown in table 3.

According to the search order, apply the algorithm POINTSEARCH to all the contact sub-domains A, B and C to search for all contact points, and take out the three with the smallest point height difference as the priority contact points. The contact status is mainly determined by these three priority contact points. Then all the remaining contact points are regarded as potential contact points, and their positions and height differences are recorded.

Take the contact of S as an example to illustrate the domains division rules. Figure 5 shows the contact domains A generated by the superposition of two S in shaded areas, in which the domain composed of the cyan closed curve is the shrinkage domain, and the red is the expansion domain. Figure 6 shows the search order after the divided contact sub-domains are sorted by PV.

3. Analysis of Contact State Considering the Influence of Assembly Force

If two surfaces want contact balance, at least three non-collinear contact points are required. Based on the search for contact points in the previous chapter, the external force on the contact surface should also be considered. Start the discussion from the classic two stress situations, namely the uniform force and concentrated force. It is assumed that friction is not considered when these two forces act.
Table 3. Algorithm POINTSEARCH.

**Algorithm** POINTSEARCH

**Input:** Contact sub-domains SR₁, SR₂… after sub-domain division

**Output:** Possible contact point locations in the contact domains (Xᵢ, Yᵢ)

1. Calculate the rectangular bounding box of SRᵢ
2. Scatter the square sampling points in the rectangular bounding box with the initial step length l, and store them in a matrix
3. Discard all sampling points beyond SRᵢ
4. Calculate the surface distance of all remaining sampling points
5. Randomly select a sampling point (xᵢ, yᵢ), compare it with the surface distances of adjacent sampling points in four directions, and find the minimum of the five distances
6. **If** [the sampling point corresponding to the minimum distance value is not (xᵢ, yᵢ)] **then**
   - Record it as (Xᵢ, Yᵢ) and go back to 5.
   **Else if** (Step length l is less than precision d) **then**
   - Output (Xᵢ, Yᵢ) and its surface distance
   **Else** l=l/2, Recalculate the sampling points with l in the four rectangles around (xᵢ, yᵢ) and return to 4.

The discussion in the previous chapter is that S receives a uniform force along n, and the three priority contact points calculated by the algorithm POINTSEARCH are the real contact points. Therefore, the follow-up will focus on the concentrated force. There is no doubt that the different positions of the force point on S will lead to different contact states. Figure 7 shows the contact point search results in the previous chapter. The solid point ABC is the priority contact point, and the hollow point DEF is the potential contact point.

The distribution of the concentrated force is divided into two types: inside and outside the triangle (priority triangle) ABC surrounded by the priority contact point. Different algorithms are used to analyze the contact state of the parts under the influence of the assembly force. It is not difficult to think that when the force F acts on the interior of the priority triangle ABC, the point ABC is the real contact point.

Considering the situation that F is not inside the priority triangle ABC, figure 8 shows the division of the outer area of the priority triangle ABC. The outer area is divided into six parts. According to the location of the area, it is divided into two types: Type I and Type II. Type I includes three areas Aᵦᵦ, Aᵦᵦ, and Aᵦᵦ, and type II includes three areas Aᵦᵦ, Aᵦᵦ, and Aᵦᵦ.

For the type I, take area ABC as an example, apply the algorithm TWOPOINTSSPIN which is shown in table 4 to further divide it into four areas D, Nᵦᵦ, Nᵦᵦ, and S, and analyze the contact state.
Table 4. Algorithm TWOPOINTSSPIN.

| Algorithm TWOPOINTSSPIN |
|-------------------------|
| 1. Use the following geometrical drawing method to divide the outer area of the priority triangle ABC into four areas D, N_B, N_C, and S. |
| Make the ray BF, delete the line segment BF, and keep the remaining part of the ray starting from F. |
| Make the ray CF, delete the line segment CF, and keep the remaining part of the ray starting from F. |
| Take the line segment BF as the diameter and the midpoint as the center to make a circle, and keep the arc connecting the point F and the line segment BC. |
| Take the line segment CF as the diameter and the midpoint as the center to make a circle, and keep the arc connecting the point F and the line segment BC. |
| 2. Rotate NS in the direction of the force F with the straight line BC as the axis, Search for the first potential contact point Pt in the part of the surface where the point F is located. |
| 3. If (Pt ∈ D) then |
| Report point Pt, forming a new triangle with the two initial points. |
| Else if (Pt ∈ N_B) then |
| Replace point B with Pt and go back to 1. |
| Else if (Pt ∈ N_C) then |
| Replace point C with Pt and go back to 1. |
| Else |
| Abandon the two initial points, use Pt as the new initial point, and apply the algorithm ONEPOINTSPIN. |

For the type II areas, take area AB as an example, apply the algorithm ONEPOINTSPIN which is shown in table 5 to analyze the contact state.

Through the algorithm TWOPOINTSSPIN and the algorithm ONEPOINTSPIN, the potential contact points near the force point are used to enclose the force point outside the priority triangle with a new triangle, and the new triangle vertex is the real contact point. That is, the contact of the part containing the TTPS information is transformed into the contact between the three non-collinear contact points on the surface of the part.

The pose relationship between the parts can be expressed by a homogeneous transformation matrix. In the matching process of parts A and B in figure 9, three non-collinear contact points are calculated to obtain the contact planes PL_A and PL_B. The contact between A and B is transformed into the PL overlap of A and B in different poses.
Table 5. Algorithm ONEPOINTSSPIN.

Algorithm ONEPOINTSSPIN

1. Take the initial point B and point F as a ray BF, and make a straight line perpendicular to the BF through the initial point B to divide the entire surface into two parts
2. Rotate NS in the direction of the force F with the straight line as the axis, Search for the first potential contact point Pt in the part of the surface where the point F is located
3. If (Pt ∈ D) then
   Report point Pt, forming a new triangle with the two initial points
   Else if (Point Pt is located on ray aF) then
   Replace point B with Pt and go back to 1.
   Else using Pt as the second initial point, apply the algorithm TWOPOINTSSPIN

Figure 9. Schematic representation of surface contact and nominal surface.

\[ P_{PL_A} = T_A \cdot R_A \cdot P_A \]  \hspace{1cm} (1)

In the formula, \( T_A = T_1 \cdot T_2 \cdot \ldots \cdot T_n \), \( R_A = R_1 \cdot R_2 \cdot \ldots \cdot R_n \), \( P_A \) is the nominal face pose corresponding to part A. \( P_{PL_A} \) is the PL_A pose of the contact plane.

Same as above, then

\[ P_{PL_B} = T_B \cdot R_B \cdot P_B \]  \hspace{1cm} (2)

The contact between parts A and B is transformed into the overlap of PL in different poses on A and B, so

\[ P_{PL_A} = P_{PL_B} \]  \hspace{1cm} (3)

Incorporating equations (1) and (2) into the above equation, we get

\[ P_A = R_A \cdot T_A \cdot T_B \cdot R_B \cdot P_B \]  \hspace{1cm} (4)

Therefore, the pose relationship between part A and part B can be obtained.

4. Case Studies and Discussions

The precise characterization of the TTPS model is the key issue of contact state analysis. A T-spline surface mathematical model is used to describe the TTPS. Taking the contact of two stepped surface parts as an example, consider the scenario where the geometric center of the outer surface of the part exerts a concentrated assembly force inward along \( n \). The geometric dimensions and tolerance requirements are shown in figure 10.

Generate point clouds with U (-0.2, 0.2) uniform distribution and N (0, 1/15) Gaussian distribution respectively for the two contact surfaces of the part. The 3×3 order T-spline surface generated by the interpolation lattice is used as the skin model shape of the TTPS. The Gaussian distribution is the reconstructed surface of part A, and the uniform distribution is the reconstructed surface of part B, as shown in figure 11.

Figure 12 shows the application of the algorithm in Chapter 3 to calculate the real contact points under one realistic assembly.
Figure 10. Requirements for geometric dimensions and geometric tolerances of two stepped surface parts.

Figure 11. The skin model shape of the TTPS after surface reconstruction.

The Monte-Carlo simulation method is used to randomly generate 1000 the skin model shapes of the TTPS according to the distributions, and the algorithms are used to calculate the contact state of each generated the skin model shape of the TTPS. The distance deviation with the geometric center of the TTPS as the measuring point is recorded. At the same time, the difference surface method and the commercial software package 3DCS were used to analyze and calculate the same model (figure 10). The results are shown in figure 13 and table 6.

Figure 12. Part B surface contact point.

Figure 13. Analysis result.
Table 6. Mean and variance of normal curve fitting.

|                      | Mean (mm) | Variance (mm²) |
|----------------------|-----------|----------------|
| Method of this article | 0.015     | 0.129          |
| The difference surface method | 0.018     | 0.124          |
| 3DCS                 | 0.000     | 0.110          |

It can be seen from the results that the results of the method in this article are similar to those of the difference surface method, but are quite different from the results of 3DCS analysis. This is because the method in this article is similar to the difference surface method in dealing with surface contact containing TTPS information. The method in this paper applies the related concepts of watershed to continuously reduce the contact areas, and then calculate the contact points. The difference surface method converts the contact between two surfaces into the contact between the nominal surface and the difference surface. The position of the contact points calculated by the Mento-Carlo simulation is different each time. 3DCS either uses discrete point cloud fitting least-squares surface for constraint fit or selects three points for deterministic mating.

5. Conclusion
Aiming at the difficulty of the existing CAT technology—that is, the inability to establish a realistic assembly model that expresses TTRS information and determines the surface contact state, this article proposes a new mating simulation method—a rigid part contact state analysis method that includes TTPS information. First, to simplify the search time of contact points, the matching surface area is divided and the search order is defined according to the priority value. Secondly, when the assembly force is considered as uniform force and concentrated force, analyze the contact state of parts. Finally, taking the assembly scene of step surface parts subjected to uniform force as an example, the analysis results of commercial software packages and the difference surface method are compared to verify the feasibility of the method in this paper.

However, the current work is only the first step in a complex assembly scenario. The feasibility of the method is verified by only the contact between two simple surfaces, without considering the interference between parts and the surface deformation under the action of the binding force. It will be gradually carried out in the follow-up works.

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