Bayesian Analysis of Record Statistics Based on Generalized Inverted Exponential Model

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Abstract— In some situations, only observations that are more extreme than the current extreme value are recorded. This kind of data is called record values which have many applications in a lot of fields. In this paper, the Bayesian estimators using squared error and LINEX loss functions for the generalized inverted exponential distribution parameters are considered depending on upper record values and upper record ranked set sampling. The Bayes estimates and credible intervals are derived by considering the independent gamma priors for the parameters. The Markov Chain Monte Carlo (MCMC) method is developed due to the lack of explicit forms for the Bayes estimates. A Simulation study is implemented to compute and compare the performance of estimators in both sampling schemes with respect to relative absolute biases, estimated risks and the width of credible intervals.

Keywords— upper record ranked set sample; Bayesian estimator; squared error (SE) loss function; linear exponential (LINEX) loss function; Markov Chain Monte Carlo.

I. INTRODUCTION

Record data are very important in many situations when the observations are difficult to obtain or are destroyed in experimental tests. Record data arise in a wide variety of practical situations including industrial stress testing, meteorology, sports, hydrology and economics. Record values can be viewed as order statistics from a sample that is determined by the values and the order of occurrence of the observations. A record value of some phenomenon is the largest (smallest) observation anyone has ever made. The theoretical contributions and inference for record values have been studied extensively in the literature. The reader may refer to [1], [2], [3] and [4].

According to [3], record values can be classified into lower record values (LRV) and upper record values (URV).

Let \( X_j, j \geq 1 \) be a sequence of independent and identically distributed (iid) random variables, an observation \( X_j \) is called URV (LRV) if its value exceeds (lower than) all of the previous observations, i.e., \( X_j > X_i (X_j < X_i) \) for every \( i < j \).

Reference [5] presented a new sampling scheme, called record ranked set sampling (RRSS), for generating record data. The new scheme helps the scientists in situations where the only observations that are going to be used are the last record data as in athletic, weather and Olympic data.

The upper record ranked set sampling (URRSS) can be described as follows: Suppose that there exist \( n \) independent sequential sequences of continuous random variables, the \( i^{th} \) sequence sampling is ceased when the \( i^{th} \) record value is observed. The only observations that are used for analysis are the last record value in each sequence. The last record value of the \( i^{th} \) sequence in this plane is denoted by \( U_{i,.} \), then the available observations are \( U = (U_{1,1}, U_{2,2}, \ldots, U_{n,n})' \), i.e.

1. \( U_{(\bar{1})} \) \( \rightarrow \) \( U_{1,1} = U_{(\bar{1})} \)
2. \( U_{(2)} \) \( \rightarrow \) \( U_{1,2} = U_{(2)} \)
   \( \vdots \) \( \vdots \)
   \( n: \) \( U_{(n)} \) \( \rightarrow \) \( U_{n,n} = U_{(n)} \),

where, \( U_{(i)} \) is the \( i^{th} \) record in the \( j^{th} \) sequence. It is recognized that \( U_{i,.} \)'s are independent random variables but not ordered.

Bayesian estimation based on record values has been considered by several researchers. Among them, [6]-[19]. Reference [20] discussed the Bayesian estimation problem...
for the shape parameter of the weighted exponential distribution based on URRSS.

The applicability of the one parameter exponential distribution is the simplest and the most widely discussed distribution for lifetime data but it is restricted to a constant hazard rate. Most generalizations of the exponential distributions possess the constant, non-increasing, non-decreasing and bathtub hazard rates. But in many practical situations, the data shows the inverted bathtub hazard rate (initially increase and then decrease, i.e., unimodal). For such data types, another extension of the exponential distribution known as the one parameter inverted exponential distribution is provided, which have inverted bathtub hazard rate [21]. Reference [22] introduced the two-parameter generalized inverted exponential distribution (GIED) by adding a shape parameter to the inverted exponential distribution.

The probability density function (pdf) of the GIED with the shape parameter \( \alpha \) and the scale parameter \( \lambda \) takes the following form

\[
f(x; \alpha, \lambda) = \frac{\alpha \lambda}{x^\alpha} (1 - e^{-\frac{x}{\lambda}})^{\alpha - 1} e^{-\frac{x}{\lambda}}
\]

\[ : x > 0, \alpha, \lambda > 0. \tag{1}\]
The cumulative distribution function (cdf) is as follows

\[
F(x; \alpha, \lambda) = 1 - (1 - e^{-\frac{x}{\lambda}})^\alpha.
\]

Recently, there has been a growing interest in the study of inference problems associated with record values and record ranked set samples via the Bayesian approach. The Bayesian estimation for the GIED based on URRSS hasn't been studied in the literature yet. The goal of this paper is to study the applicability of the one parameter exponential distribution based on URRSS. These Bayes estimates and credible intervals width of the posterior 95% credible intervals for the unknown parameters of GIED on the basis of URV and URRSS. These Bayes estimates and credible intervals width are obtained using independent gamma priors under symmetric (squared error (SE)) and asymmetric (linear exponential (LINEX)) loss functions through MCMC method. The procedures are illustrated through analysing a simulated data. The rest of the paper is organized as follows.

Section II gives the Bayes estimates based on URV, the Bayes estimates under URRSS MCMC approach and a simulation. Discussion and results of the simulation study appear in Section (III). Concluding remarks appear in Section (IV).

II. MATERIAL AND METHOD

A. Bayesian Estimators Based on URV

In this section, Bayesian estimators of the unknown parameters of the GIED under the assumption of independent gamma priors on both the shape and scale parameters are considered. Based on URV, the Bayes estimators cannot be obtained in explicit forms. Hence the MCMC technique is carried out to generate samples from the posterior distributions and consequently computing the Bayes estimators and construct the corresponding credible intervals. Here, two types of loss functions are considered for Bayesian computation; symmetric one (SE) and asymmetric one (LINEX).

Let \( \xi = (r_1, \ldots, r_m) \) be a set of URV from GIED(\( \alpha, \lambda \)), the likelihood function according to [3], is given by

\[
L = f(r) \prod_{i=1}^{m} \frac{f(r_i)}{1 - F(r_i)} \quad 0 < r_1 < \ldots < r_m < \infty, \tag{3}\]

where, \( f(., \theta) \) and, \( F(., \theta) \) are respectively the pdf and the cdf of GIED(\( \alpha, \lambda \)). The likelihood function of the observed URV is obtained, as follows

\[
L_i = (1-e^{-\frac{r}{\lambda}}) \prod_{i=1}^{m} \frac{\alpha \lambda}{r_i^\alpha} e^{-\frac{r}{\lambda}} (1 - e^{-\frac{r}{\lambda}})^{-1} \tag{4}\]

Further, assuming that the prior of parameters \( \alpha \) and \( \lambda \) has a gamma distribution with parameters \((a_1, b_1)\) and \((a_2, b_2)\) respectively. Hence, assuming independence of parameters, the joint prior distribution of parameters, denoted by \( \pi(\alpha, \lambda) \), is as follows

\[
\pi(\alpha, \lambda) = \frac{1}{\Gamma(a_1)\Gamma(a_2)} \alpha^{a_1-1} \lambda^{a_2-1} e^{-\alpha a_1 b_1} \tag{5}\]

The expression for the joint posterior can be written as

\[
\pi(\alpha, \lambda | \xi) \propto \alpha^{m a_1+1} \lambda^{m a_2+1} e^{-\lambda b_2 a_2} (1 - e^{-\frac{r}{\lambda}})^\alpha \prod_{i=1}^{m} r_i^{-a_2} e^{\frac{r_i}{\lambda}} (1 - e^{-\frac{r_i}{\lambda}})^{-1} \tag{6}\]

Hence, the marginal posterior distributions of \( \alpha \) and \( \lambda \), based on URV, take the following forms

\[
\pi_\alpha(\alpha | \xi) = C \alpha^{m a_1+1} e^{-\lambda \alpha} \int_0^{\infty} \lambda^{m a_2+1} e^{-\lambda b_2 a_2} (1 - e^{-\frac{r}{\lambda}})^\alpha \prod_{i=1}^{m} r_i^{-a_2} e^{\frac{r_i}{\lambda}} (1 - e^{-\frac{r_i}{\lambda}})^{-1} d\lambda, \tag{7}\]

\[
\pi_\lambda(\lambda | \xi) = C \lambda^{m a_2+1} e^{-\lambda b_2 a_2} \int_0^{\infty} \alpha^{m a_1+1} e^{-\alpha a_1 b_1} (1 - e^{-\frac{r}{\lambda}})^\alpha \prod_{i=1}^{m} r_i^{-a_2} e^{\frac{r_i}{\lambda}} (1 - e^{-\frac{r_i}{\lambda}})^{-1} d\alpha, \tag{8}\]

where,

\[
C^{-1} = \int_0^{\infty} \alpha^{m a_1+1} e^{-\alpha a_1 b_1} (1 - e^{-\frac{r}{\lambda}})^\alpha \prod_{i=1}^{m} r_i^{-a_2} e^{\frac{r_i}{\lambda}} (1 - e^{-\frac{r_i}{\lambda}})^{-1} d\alpha d\lambda . \tag{9}\]

Therefore, based on URV the Bayes estimates of the unknown parameters under SE loss function, denoted by \( \hat{\alpha}_{SE} \) and \( \hat{\lambda}_{SE} \), can be obtained as posterior mean as follows

\[
\hat{\alpha}_{SE} = E(\alpha | \xi) = \int_0^{\infty} \alpha \pi_\alpha(\alpha | \xi) d\alpha, \tag{10}\]

and

\[
\hat{\lambda}_{SE} = E(\lambda | \xi) = \int_0^{\infty} \lambda \pi_\lambda(\lambda | \xi) d\lambda. \tag{11}\]
Additionally, the Bayesian estimators of \( \alpha \) and \( \lambda \) under LINEX loss function, denoted by \( \hat{\alpha}_{\text{LINEX}} \) and \( \hat{\lambda}_{\text{LINEX}} \), are given by

\[
\hat{\alpha}_{\text{LINEX}} = \frac{-1}{v} \log E[e^{-\alpha}]
= \frac{-1}{v} \log \left\{ \int_0^\infty e^{-\alpha} F_i(x|\alpha) \, d\alpha \right\},
\]
and
\[
\hat{\lambda}_{\text{LINEX}} = \frac{-1}{v} \log E[e^{-\lambda}]
= \frac{-1}{v} \log \left\{ \int_0^\infty e^{-\lambda} F_i(x|\lambda) \, d\lambda \right\}.
\]

Generally, as observed, the analytical solution of integrations given by (7) and (8) is very difficult to obtain due to the complicated mathematical form. Therefore, the MCMC technique is employed to approximate these integrations. Therefore Metropolis-Hastings (M-H) method will be implemented which is a powerful MCMC technique to compute the Bayes estimates and credible intervals width.

### B. Bayesian Estimators Based on URRSS

This section discusses the Bayes estimates of the unknown shape and scale parameters of the GIED under the assumption of independent gamma priors defined in (5) based on URRSS using SE and LINEX loss functions. Let \( r_i = (r_{i1}, \ldots, r_{im}) \) be a set of observed URRSS, then the joint density function denoted by \( L_2 \), according to [5], is given by

\[
L_2 = \prod_{i=1}^m \frac{(-log F(r_i; \theta))^{i-1}}{(i-1)!} f(r_i; \theta) ; \theta \in \Theta,
\]
where, \( F(\cdot; \theta) = 1 - F(\cdot; \theta) \). \( \theta \) is real valued parameter, \( \Theta \) is the parameter space. Inserting the pdf (1) and cdf (2) in (9), then the likelihood function is obtained as follows

\[
L_2 = \prod_{i=1}^m \frac{(-log F(r_i; \theta))^{i-1}}{(i-1)!} \frac{1}{r_i} e^{-\frac{r_i}{\alpha}} (-log(1-e^{-\frac{r_i}{\alpha}}))^{i-1} (1-e^{-\frac{r_i}{\alpha}})^{\alpha-1}.
\]

Under the assumption that \( \alpha \) and \( \lambda \) are independent, the expression for the joint posterior using gamma priors (5) can be written in the following form

\[
\pi_i^2(\alpha, \lambda) = \pi_1^2(\alpha) \lambda^{m_{\alpha+1}} e^{-h_{\alpha-b_i\lambda}} \prod_{i=1}^m \frac{e^{-\frac{r_i}{\alpha}}}{(i-1)! r_i} (-log(1-e^{-\frac{r_i}{\alpha}}))^{i-1} (1-e^{-\frac{r_i}{\alpha}})^{\alpha-1}. \]

Hence, the marginal posterior distributions of \( \alpha \) and \( \lambda \), based on URRSS, can be expressed as follows

\[
\pi_i(\alpha | r_i) = K \alpha^{\frac{\sum_{j=1}^{m_{\alpha+1}}}{m_{\alpha+1}}} e^{-h_{\alpha-b_i\lambda}} \prod_{i=1}^m \frac{1}{(i-1)! r_i} e^{-\frac{r_i}{\alpha}} (-log(1-e^{-\frac{r_i}{\alpha}}))^{i-1} (1-e^{-\frac{r_i}{\alpha}})^{\alpha-1} \, d\lambda,
\]
and

\[
\pi_i(\lambda | r_i) = K \lambda^{m_{\alpha+1}} e^{-\lambda^{b_i - \frac{r_i}{\alpha}}} \prod_{i=1}^m \frac{1}{(i-1)! r_i} e^{-\frac{r_i}{\alpha}} (-log(1-e^{-\frac{r_i}{\alpha}}))^{i-1} (1-e^{-\frac{r_i}{\alpha}})^{\alpha-1} \, d\alpha,
\]

where,

\[
K^{-1} = \prod_{i=1}^m \frac{1}{(i-1)! r_i} e^{-\frac{r_i}{\alpha}} (-log(1-e^{-\frac{r_i}{\alpha}}))^{i-1} (1-e^{-\frac{r_i}{\alpha}})^{\alpha-1} \, d\lambda \, d\lambda.
\]

Therefore, based on URRSS, the Bayes estimates of the unknown parameters under SE loss function, denoted by \( \hat{\alpha}_{\text{SE}} \) and \( \hat{\lambda}_{\text{SE}} \), can be obtained as the posterior mean as follows

\[
\hat{\alpha}_{\text{SE}} = \frac{1}{v} \int_0^\infty \pi_i(\alpha | r_i) \, d\alpha,
\]
and

\[
\hat{\lambda}_{\text{SE}} = \frac{1}{v} \int_0^\infty \pi_i(\lambda | r_i) \, d\lambda.
\]

Similarly, the Bayes estimators of \( \alpha \) and \( \lambda \) under LINEX loss function, denoted by \( \hat{\alpha}_{\text{LINEX}} \) and \( \hat{\lambda}_{\text{LINEX}} \), are given by

\[
\hat{\alpha}_{\text{LINEX}} = \frac{-1}{v} \int_0^\infty e^{-\alpha} \pi_i(\alpha | r_i) \, d\alpha,
\]
and

\[
\hat{\lambda}_{\text{LINEX}} = \frac{-1}{v} \int_0^\infty e^{-\lambda} \pi_i(\lambda | r_i) \, d\lambda.
\]

where, \( v \) is a real number.

Again, the integrals (10) and (11) cannot be reduced to a closed form due to its difficult mathematical form. So, M-H algorithm is used to compute the Bayes estimator under the SE and LINEX loss functions.

### C. Simulation Study

In this section, a numerical study is performed in order to examine and compare the behaviour of the Bayes estimators for the two parameters of the GIED \((\alpha, \lambda)\) based on URV and URRSS. The Bayes estimates are obtained using gamma priors under SE and LINEX loss functions. The major difficulty in the implementation of the Bayesian procedure is that of obtaining the posterior distribution. MCMC is simply an iterative procedure drawing values from the posterior distributions of the parameter in the concerned model. The M-H algorithm is one of the most famous subclasses of
MCMC method in Bayesian literature to simulate the
deviates from the posterior density and produce the good
approximate results. Here, M-H algorithm will be used via R
program.

To compare the estimators, MCMC simulations are
performed for different sample sizes under SE and LINEX
loss functions. For each simulation, the number of records
are selected as \( n = 3,4,5,6,7,8 \) and the parameter values
are selected as \((\alpha, \lambda) = (2,2),(0.5,0.5),(0.5,1.5),(1.5,0.5),
(1,2) \) and \((2,1)\). The hyper-parameters for gamma priors are
selected as \( a_1 = 4 \) and \( b_1 = 1 \). Also, we take
\( v = -2,2 \). All the results are based on the number of
replications \( NR = 5000 \). Evaluating the performance of
the estimates is considered through some measurements of
accuracy, so it is convenient to use the relative absolute
biases (Rabs) and estimated risks (ERs) of the Bayes
estimates which are computed as follows:

\[
\text{Rabs(estimator)} = \frac{\text{estimator} - \text{true value}}{\text{true value}}
\]

\[
\text{ERs(estimator)} = \frac{\sum_{i=1}^{NR} (\text{average} - \text{population parameter})^2}{NR}
\]

Here is how M–H algorithm works; let \( g(.) \) be the
density of the subject distribution. The M–H algorithm
proceeds as follows

Initialize a starting value \( x_0 \) and the number of samples \( N \)
for \( i = 2 \) to \( N \)
set \( x = x_{i-1} \)
generate \( u \) from \( U(0,1) \)
generate \( y \) from \( g(.) \)
if \( u \leq \frac{\pi(g(y))g(x)}{\pi(g(y))g(x)} \) then
set \( x = y \)
else
set \( x_i = x \)
end if
end for

III. RESULTS AND DISCUSSION
The simulation results are summarized in Tables (I-VI)
and represented through Fig. (1-6). From these tables and
figures, the following observations can be made

![Fig. 1 ERs of \( \alpha \) under SE and LINEX loss functions based on URV and URRSS at \((\alpha, \lambda) = (2,2)\)

Fig. (1) shows that the ERs of \( \hat{\alpha}_{SE} \) are less than the ERs
of the corresponding \( \hat{\alpha}_{LINEX} \) for all the number of records
except at \( n = 8 \). Also, the ERs of \( \tilde{\alpha}_{LINEX} \) when \( v = 2 \) are
less than the ERs of the corresponding; \( \tilde{\alpha}_{LINEX} \) for
\( n = 3,5,6 \) and when \( v = -2 \) the ERs of \( \tilde{\alpha}_{LINEX} \) are less than
the ERs of the corresponding \( \tilde{\alpha}_{LINEX} \) for all the number of records except at \( n = 8 \).

![Fig. 2 ERs of \( \lambda \) under SE and LINEX loss functions based on URV and URRSS at \((\alpha, \lambda) = (2,2)\)

Fig. (2), shows that the ERs of \( \hat{\lambda}_{SE} \) are less than the ERs
of the corresponding \( \hat{\lambda}_{LINEX} \) for all the number of records
except at \( n = 4,6 \). Also, the ERs of \( \tilde{\lambda}_{LINEX} \) when \( v = 2 \) are
less than the ERs of the corresponding \( \tilde{\lambda}_{LINEX} \) for \( n = 4,5,8 \),
but when \( v = -2 \), ERs of \( \tilde{\lambda}_{LINEX} \) are less than the ERs of the
corresponding \( \tilde{\lambda}_{LINEX} \) for all the number of records except at \( n = 4,5 \).
| Number of records/parameters | 3          | 4          | 5          | 6          | 7          | 8          |
|-----------------------------|------------|------------|------------|------------|------------|------------|
|                             | α          | λ          | α          | λ          | α          | λ          |
| Estimates                   | 1.9983     | 1.9998     | 1.9987     | 2.0001     | 2.0007     | 1.9993     |
| Rab                         | 0.0008     | 0.0001     | 0.0006     | 0.0008     | 0.0001     | 0.0000     |
| ER                          | 1.09E-03   | 2.57E-04   | 7.06E-04   | 8.55E-04   | 2.82E-04   | 2.68E-04   |
| Width                       | 0.0039     | 0.0016     | 0.0025     | 0.0022     | 0.0018     | 0.0020     |
| Estimates                   | 1.9998     | 2.0003     | 2.0000     | 2.0009     | 2.0018     | 1.9984     |
| Rab                         | 0.0006     | 0.0002     | 0.0000     | 0.0005     | 0.0009     | 0.0008     |
| ER                          | 2.65E-10   | 1.85E-11   | 1.10E-13   | 1.68E-10   | 6.59E-10   | 4.85E-10   |
| Width                       | 0.0018     | 0.0015     | 0.0014     | 0.0021     | 0.0028     | 0.0024     |
| Estimates                   | 1.9999     | 2.0001     | 2.0000     | 1.9992     | 1.9961     | 1.9990     |
| Rab                         | 0.0001     | 0.0000     | 0.0000     | 0.0004     | 0.0004     | 0.0005     |
| ER                          | 3.00E-12   | 1.52E-12   | 7.50E-14   | 1.42E-10   | 1.60E-10   | 1.95E-10   |
| Width                       | 0.0012     | 0.0010     | 0.0013     | 0.0017     | 0.0013     | 0.0018     |
| Estimates                   | 1.9990     | 1.9990     | 2.0006     | 2.0002     | 2.0002     | 1.9994     |
| Rab                         | 0.0005     | 0.0005     | 0.0003     | 0.0011     | 0.0001     | 0.0003     |
| ER                          | 2.20E-10   | 1.87E-10   | 6.32E-11   | 5.69E-12   | 5.31E-12   | 7.90E-11   |
| Width                       | 0.0019     | 0.0013     | 0.0018     | 0.0020     | 0.0012     | 0.0012     |
| Estimates                   | 2.0000     | 2.0005     | 2.0007     | 2.0005     | 2.0000     | 2.0020     |
| Rab                         | 0.0000     | 0.0002     | 0.0004     | 0.0002     | 0.0001     | 0.0003     |
| ER                          | 7.98E-14   | 4.19E-11   | 1.07E-10   | 4.22E-11   | 4.94E-12   | 5.76E-11   |
| Width                       | 0.0021     | 0.0010     | 0.0013     | 0.0009     | 0.0012     | 0.0016     |
| Estimates                   | 2.0007     | 2.0007     | 2.0022     | 1.9996     | 2.0021     | 1.9996     |
| Rab                         | 0.0003     | 0.0004     | 0.0011     | 0.0002     | 0.0010     | 0.0002     |
| ER                          | 9.54E-11   | 1.08E-10   | 5.33E-12   | 3.43E-11   | 8.80E-10   | 3.96E-11   |
| Width                       | 0.0012     | 0.0015     | 0.0015     | 0.0016     | 0.0039     | 0.0015     |

**TABLE I**

Bayes Estimates, Rab and ER of \(\alpha\) and \(\lambda\) based on URV and URRSS for \((\alpha, \lambda) = (2, 2)\).
| Number of records/parameters | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------------|---|---|---|---|---|---|
| \( \alpha \) | 0.4997 | 0.5003 | 0.5006 | 0.4991 | 0.4996 | 0.5005 |
| \( \lambda \) | 0.5007 | 0.5009 | 0.5000 | 0.5007 | 0.4985 | 0.5002 |
| \( \alpha \) | 0.0006 | 0.0015 | 0.0012 | 0.0018 | 0.0008 | 0.0010 |
| \( \lambda \) | 0.0115 | 0.0118 | 0.0101 | 0.0114 | 0.0030 | 0.0014 |
| \( \alpha \) | 2.07E-11 | 1.10E-10 | 1.58E-11 | 1.57E-10 | 9.99E-11 | 4.59E-11 |
| \( \lambda \) | 1.10E-10 | 1.56E-10 | 2.57E-13 | 9.99E-11 | 4.56E-10 | 8.09E-12 |
| \( \alpha \) | 0.0009 | 0.0017 | 0.0021 | 0.0022 | 0.0100 | 0.0025 |
| \( \lambda \) | 0.0019 | 0.0011 | 0.0015 | 0.0013 | 0.0033 | 0.0019 |
| \( \alpha \) | URV | SE | LINEX \( v=(2) \) | LINEX \( v=(-2) \) | URRSS | LINEX \( v=(2) \) | LINEX \( v=(-2) \) |
| \( \alpha \) | 0.5009 | 0.4988 | 0.5002 | 0.5008 | 0.5009 | 0.5015 | 0.5008 |
| \( \lambda \) | 0.5000 | 0.4988 | 0.5003 | 0.5012 | 0.5008 | 0.4999 | 0.5008 |
| \( \alpha \) | 0.0018 | 0.0023 | 0.0004 | 0.0016 | 0.0005 | 0.0012 | 0.0015 |
| \( \lambda \) | 0.0015 | 0.0002 | 0.0005 | 0.0008 | 0.0003 | 0.0000 | 0.0010 |
| \( \alpha \) | 1.55E-10 | 2.72E-10 | 1.15E-10 | 1.10E-10 | 9.49E-13 | 1.28E-10 | 1.86E-11 |
| \( \lambda \) | 2.05E-12 | 1.15E-10 | 3.09E-10 | 1.33E-12 | 2.35E-10 | 5.10E-10 | 6.43E-11 |
| \( \alpha \) | 0.0015 | 0.0018 | 0.0026 | 0.0018 | 0.0009 | 0.0013 | 0.0015 |
| \( \lambda \) | 0.0019 | 0.0025 | 0.0017 | 0.0016 | 0.0009 | 0.0013 | 0.0015 |
| \( \alpha \) | LINEX \( v=(-2) \) | SE | LINEX \( v=(2) \) | LINEX \( v=(-2) \) | URRSS | LINEX \( v=(2) \) | LINEX \( v=(-2) \) |
| \( \alpha \) | 0.4987 | 0.4991 | 0.5015 | 0.4999 | 0.5008 | 0.5005 | 0.5008 |
| \( \lambda \) | 0.4989 | 0.5000 | 0.5002 | 0.5008 | 0.5008 | 0.5005 | 0.5008 |
| \( \alpha \) | 0.0026 | 0.0018 | 0.0010 | 0.0001 | 0.0015 | 0.0001 | 0.0015 |
| \( \lambda \) | 0.0021 | 0.0001 | 0.0016 | 0.0026 | 0.0002 | 0.0003 | 0.0010 |
| \( \alpha \) | 3.43E-10 | 1.55E-10 | 2.24E-10 | 1.10E-10 | 1.12E-10 | 2.35E-10 | 2.81E-13 |
| \( \lambda \) | 3.21E-13 | 3.21E-13 | 3.21E-13 | 3.21E-13 | 3.21E-13 | 3.21E-13 | 3.21E-13 |
| \( \alpha \) | 0.0022 | 0.0012 | 0.0023 | 0.0015 | 0.0027 | 0.0011 | 0.0013 |
| \( \lambda \) | 0.0023 | 0.0010 | 0.0023 | 0.0015 | 0.0018 | 0.0011 | 0.0013 |
| \( \alpha \) | LINEX \( v=(2) \) | SE | LINEX \( v=(-2) \) | LINEX \( v=(2) \) | URRSS | LINEX \( v=(2) \) | LINEX \( v=(-2) \) |
| \( \alpha \) | 0.5015 | 0.4999 | 0.5005 | 0.4999 | 0.5005 | 0.5015 | 0.5008 |
| \( \lambda \) | 0.4999 | 0.5002 | 0.4985 | 0.4999 | 0.4995 | 0.5008 | 0.5005 |
| \( \alpha \) | 0.0030 | 0.0002 | 0.0012 | 0.0004 | 0.0029 | 0.0001 | 0.0015 |
| \( \lambda \) | 0.0010 | 0.0004 | 0.0005 | 0.0009 | 0.0007 | 0.0021 | 0.0011 |
| \( \alpha \) | 4.60E-10 | 2.69E-12 | 5.49E-11 | 1.00E-11 | 4.34E-10 | 2.20E-11 | 1.95E-10 |
| \( \lambda \) | 2.69E-12 | 5.49E-11 | 4.48E-11 | 4.48E-11 | 4.48E-11 | 4.48E-11 | 5.56E-11 |
| \( \alpha \) | 0.0019 | 0.0013 | 0.0018 | 0.0017 | 0.0035 | 0.0021 | 0.0026 |
| \( \lambda \) | 0.0013 | 0.0012 | 0.0012 | 0.0016 | 0.0016 | 0.0014 | 0.0014 |
### TABLE III
BAYES ESTIMATES, Rab and ER of $\alpha$ and $\lambda$ BASED on URV and URRSS for $(\alpha, \lambda) = (0.5, 1.5)$.

| Number of records/parameters | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------------------|---|---|---|---|---|---|
| SE Estimates                 | 0.4974 | 1.5006 | 0.5002 | 1.5006 | 0.4996 | 1.4990 | 0.4987 | 1.5000 | 0.5006 | 1.5006 | 0.5001 | 1.5014 |
| Rab                         | 0.0051 | 0.0004 | 0.0003 | 0.0004 | 0.0008 | 0.0007 | 0.0026 | 0.0000 | 0.0012 | 0.0004 | 0.0002 | 0.0009 |
| ER                           | 1.32E-09 | 6.64E-11 | 4.89E-12 | 6.39E-11 | 2.87E-11 | 2.12E-10 | 3.42E-10 | 1.59E-17 | 7.43E-11 | 8.07E-11 | 2.50E-12 | 3.85E-10 |
| Width                        | 0.0037 | 0.0009 | 0.0014 | 0.0018 | 0.0010 | 0.0013 | 0.0021 | 0.0022 | 0.0018 | 0.0015 | 0.0014 | 0.0021 |
| LINEX $v=(2)$ Estimates      | 0.4989 | 1.5001 | 0.4988 | 1.4990 | 0.5001 | 1.4999 | 0.4993 | 1.4994 | 0.5009 | 1.4998 | 0.5005 | 1.5015 |
| Rab                         | 0.0021 | 0.0001 | 0.0024 | 0.0006 | 0.0002 | 0.0001 | 0.0015 | 0.0004 | 0.0018 | 0.0001 | 0.0011 | 0.0010 |
| ER                           | 2.23E-10 | 2.26E-12 | 2.76E-10 | 1.88E-10 | 1.45E-12 | 2.19E-12 | 1.11E-10 | 8.35E-11 | 1.68E-10 | 8.47E-12 | 5.77E-11 | 4.66E-10 |
| Width                        | 0.0021 | 0.0015 | 0.0017 | 0.0010 | 0.0015 | 0.0018 | 0.0017 | 0.0014 | 0.0027 | 0.0022 | 0.0014 | 0.0027 |
| LINEX $v=(-2)$ Estimates     | 0.4992 | 1.4990 | 0.4994 | 1.4996 | 0.4992 | 1.5006 | 0.4985 | 1.5004 | 0.4984 | 1.4981 | 0.5001 | 1.5001 |
| Rab                         | 0.0015 | 0.0007 | 0.0012 | 0.0003 | 0.0017 | 0.0004 | 0.0029 | 0.0003 | 0.0031 | 0.0013 | 0.0001 | 0.0001 |
| ER                           | 1.19E-10 | 1.98E-10 | 6.76E-11 | 3.35E-11 | 1.42E-10 | 6.20E-11 | 4.34E-10 | 3.54E-11 | 4.92E-10 | 7.31E-10 | 8.82E-13 | 2.70E-12 |
| Width                        | 0.0011 | 0.0027 | 0.0012 | 0.0015 | 0.0016 | 0.0019 | 0.0023 | 0.0022 | 0.0038 | 0.0034 | 0.0015 | 0.0023 |
| URV Estimates                | 0.4989 | 1.5007 | 0.4990 | 1.4988 | 0.4999 | 1.4990 | 0.4993 | 1.5006 | 0.4979 | 1.5000 | 0.4999 | 1.4985 |
| Rab                         | 0.0022 | 0.0005 | 0.0020 | 0.0008 | 0.0001 | 0.0007 | 0.0014 | 0.0004 | 0.0041 | 0.0000 | 0.0002 | 0.0010 |
| ER                           | 2.46E-10 | 9.53E-11 | 1.99E-10 | 2.97E-10 | 1.06E-12 | 2.03E-10 | 9.30E-11 | 6.61E-11 | 8.43E-10 | 1.04E-13 | 2.51E-12 | 4.25E-10 |
| Width                        | 0.0024 | 0.0019 | 0.0012 | 0.0023 | 0.0017 | 0.0015 | 0.0020 | 0.0020 | 0.0031 | 0.0022 | 0.0035 | 0.0022 |
| LINEX $v=(2)$ Estimates      | 0.5005 | 1.5002 | 0.4994 | 1.5010 | 0.4995 | 1.5014 | 0.5002 | 1.4994 | 0.5008 | 1.4999 | 0.4992 | 1.4989 |
| Rab                         | 0.0010 | 0.0002 | 0.0012 | 0.0007 | 0.0009 | 0.0010 | 0.0003 | 0.0004 | 0.0015 | 0.0000 | 0.0016 | 0.0008 |
| ER                           | 5.01E-11 | 1.10E-11 | 6.90E-11 | 1.99E-10 | 4.16E-11 | 4.11E-10 | 4.74E-12 | 6.84E-11 | 1.14E-10 | 1.10E-12 | 1.35E-10 | 2.57E-10 |
| Width                        | 0.0013 | 0.0011 | 0.0021 | 0.0015 | 0.0012 | 0.0017 | 0.0026 | 0.0021 | 0.0014 | 0.0011 | 0.0017 | 0.0021 |
| LINEX $v=(-2)$ Estimates     | 0.5014 | 1.5005 | 0.5004 | 1.4998 | 0.5001 | 1.5002 | 0.4996 | 1.5009 | 0.5008 | 1.5006 | 0.4997 | 1.4992 |
| Rab                         | 0.0028 | 0.0003 | 0.0008 | 0.0001 | 0.0002 | 0.0001 | 0.0008 | 0.0006 | 0.0017 | 0.0004 | 0.0006 | 0.0006 |
| ER                           | 4.06E-10 | 4.89E-11 | 3.59E-11 | 5.33E-12 | 2.51E-12 | 7.06E-12 | 3.35E-11 | 1.49E-10 | 1.43E-10 | 7.51E-11 | 1.96E-11 | 1.38E-10 |
| Width                        | 0.0022 | 0.0021 | 0.0014 | 0.0020 | 0.0013 | 0.0019 | 0.0023 | 0.0024 | 0.0017 | 0.0017 | 0.0023 | 0.0020 |
| Number of records/parameters | 3                | 4                | 5                | 6                | 7                | 8                |
|-----------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                             | $\alpha$         | $\lambda$        | $\alpha$         | $\lambda$        | $\alpha$         | $\lambda$        |
| URV                         | Estimates        | SE               | Estimates        | SE               | Estimates        | SE               |
|                             | 1.0006           | 2.0008           | 1.0006           | 1.9999           | 1.0002           | 2.0020           |
|                             | Rab              | 0.0006           | 0.0004           | 0.0006           | 0.0006           | 0.0000           |
|                             | ER               | 6.27E-11 - 1.41E-10 | 6.74E-11 - 9.29E-13 | 7.23E-12 - 7.83E-10 | 2.37E-11 - 2.17E-10 | 4.07E-10 - 1.52E-10 |
|                             | Width            | 0.0014           | 0.0014           | 0.0018           | 0.0016           | 0.0012           |
|                             | URV              | 1.0006           | 1.9984           | 1.0005           | 2.0004           | 1.0013           |
|                             | Estimates        | 1.0006           | 1.9988           | 1.0002           | 2.0011           | 0.9985           |
|                             | Linex v=(2)      | Rab              | 0.0007           | 0.0006           | 0.0008           | 0.0002           |
|                             | Estimates        | 9.38E-11 - 2.92E-10 | 1.05E-11 - 2.41E-10 | 2.45E-10 - 4.57E-11 | 4.73E-10 - 1.39E-12 | 3.93E-11 - 1.19E-12 |
|                             | Linex v=(-2)     | Rab              | 0.0007           | 0.0006           | 0.0002           | 0.0005           |
|                             | Estimates        | 6.01E-11 - 3.46E-11 | 1.19E-10 - 2.23E-11 | 3.70E-12 - 5.38E-10 | 2.51E-11 - 1.35E-11 | 1.15E-11 - 7.43E-13 |
|                             | Linex v=(2)      | Rab              | 0.0005           | 0.0002           | 0.0008           | 0.0008           |
|                             | Estimates        | 4.18E-13 - 4.58E-10 | 2.01E-10 - 1.23E-10 | 3.86E-12 - 1.24E-10 | 1.92E-10 - 9.47E-11 | 1.57E-11 - 9.78E-11 |
|                             | Linex v=(-2)     | Rab              | 0.0000           | 0.0000           | 0.0004           | 0.0004           |
|                             | Estimates        | 3.94E-11 - 7.27E-11 | 1.82E-10 - 4.13E-12 | 2.64E-11 - 2.24E-12 | 7.93E-10 - 8.32E-11 | 2.10E-10 - 1.34E-10 |
|                             | Linex v=(-2)     | Rab              | 0.0004           | 0.0003           | 0.0001           | 0.0001           |

**TABLE IV**

BAYES ESTIMATES, Rab and ER of $\alpha$ and $\lambda$ BASED on URV and URRSS for $\alpha, \lambda = (1, 2)$. 
| Number of records/parameters | 3 | 4 | 5 | 6 | 7 | 8 |
|----------------------------|---|---|---|---|---|---|
| **α**<br>Estimates        | 1.4996 | 1.5003 | 1.4998 | 1.4988 | 1.4995 | 1.5012 |
| Rab                       | 0.0003 | 0.0002 | 0.0001 | 0.0008 | 0.0003 | 0.0008 |
| ER                        | 3.36E-11 | 1.40E-11 | 5.27E-12 | 2.79E-10 | 4.43E-11 | 2.98E-10 |
| Width                     | 0.0015 | 0.0015 | 0.0014 | 0.0029 | 0.0014 | 0.0021 |
| **λ**<br>Estimates        | 0.4993 | 0.5007 | 0.5009 | 0.5003 | 0.5004 | 0.4996 |
| Rab                       | 0.0014 | 0.0013 | 0.0017 | 0.0005 | 0.0007 | 0.0008 |
| ER                        | 1.03E-10 | 8.64E-11 | 1.53E-10 | 1.40E-11 | 2.72E-11 | 3.34E-11 |
| Width                     | 0.0017 | 0.0017 | 0.0015 | 0.0008 | 0.0021 | 0.0015 |
| **URV**<br>SE<br>v=(2)    | Estimates | 1.4990 | 1.5011 | 1.4999 | 1.4981 | 1.5004 | 1.5001 |
| Rab                       | 0.0006 | 0.0002 | 0.0001 | 0.0013 | 0.0003 | 0.0000 |
| ER                        | 1.82E-10 | 2.22E-10 | 4.45E-12 | 7.45E-10 | 3.68E-11 | 5.87E-13 |
| Width                     | 0.0019 | 0.0020 | 0.0030 | 0.0015 | 0.0012 | 0.0009 |
| **ER**<br>Estimates       | 0.5001 | 0.4999 | 0.5013 | 0.5010 | 0.5010 | 0.5006 |
| Rab                       | 0.0002 | 0.0003 | 0.0027 | 0.0011 | 0.0020 | 0.0004 |
| ER                        | 1.73E-12 | 6.80E-13 | 1.39E-11 | 2.18E-10 | 4.68E-10 | 6.54E-11 |
| Width                     | 0.0020 | 0.0025 | 0.0025 | 0.0015 | 0.0026 | 0.0011 |
| **URRSS**<br>SE<br>v=(2)  | Estimates | 1.5001 | 1.5006 | 1.4989 | 1.4996 | 1.5004 | 1.4988 |
| Rab                       | 0.0001 | 0.0004 | 0.0007 | 0.0003 | 0.0005 | 0.0008 |
| ER                        | 1.72E-12 | 6.48E-11 | 2.41E-10 | 3.03E-11 | 5.9E-12 | 3.01E-10 |
| Width                     | 0.0011 | 0.0001 | 0.0018 | 0.0021 | 0.0026 | 0.0022 |
| **ER**<br>Estimates       | 0.4999 | 0.5000 | 0.5001 | 0.4994 | 0.4998 | 0.5014 |
| Rab                       | 0.0017 | 0.0001 | 0.0002 | 0.0012 | 0.0004 | 0.0008 |
| ER                        | 7.77E-13 | 1.50E-13 | 1.66E-12 | 7.13E-11 | 9.5E-12 | 3.01E-10 |
| Width                     | 0.0011 | 0.0001 | 0.0018 | 0.0021 | 0.0026 | 0.0019 |
| **URRSS**<br>SE<br>v=(-2) | Estimates | 1.4985 | 1.5007 | 1.4992 | 1.5003 | 1.5004 | 1.4992 |
| Rab                       | 0.0010 | 0.0005 | 0.0005 | 0.0002 | 0.0005 | 0.0005 |
| ER                        | 4.31E-10 | 1.08E-10 | 1.31E-10 | 8.34E-11 | 1.96E-12 | 2.73E-10 |
| Width                     | 0.0022 | 0.0020 | 0.0029 | 0.0013 | 0.0014 | 0.0015 |
| **ER**<br>Estimates       | 0.5009 | 0.4992 | 0.4993 | 0.4994 | 0.5001 | 0.4988 |
| Rab                       | 0.0017 | 0.0016 | 0.0013 | 0.0013 | 0.0002 | 0.0005 |
| ER                        | 1.50E-10 | 8.90E-11 | 8.34E-11 | 1.96E-12 | 2.73E-10 | 2.73E-11 |
| Width                     | 0.0020 | 0.0029 | 0.0023 | 0.0013 | 0.0014 | 0.0015 |
| **URRSS**<br>SE<br>v=(-2) | Estimates | 1.5003 | 1.5002 | 1.5002 | 1.5003 | 1.5001 | 1.4981 |
| Rab                       | 0.0004 | 0.0031 | 0.0001 | 0.0008 | 0.0005 | 0.0012 |
| ER                        | 2.15E-11 | 9.32E-12 | 5.54E-10 | 2.94E-11 | 1.17E-10 | 7.00E-10 |
| Width                     | 0.0008 | 0.0025 | 0.0029 | 0.0010 | 0.0019 | 0.0027 |
| Number of records/parameters | \( 3 \) | \( 4 \) | \( 5 \) | \( 6 \) | \( 7 \) | \( 8 \) |
|-----------------------------|---------|---------|---------|---------|---------|---------|
| \( \alpha \) Estimates SE | 1.0006  | 2.0008  | 1.0006  | 1.9999  | 1.0002  | 2.0020  |
| \( \lambda \) Estimates SE | 0.0006  | 0.0004  | 0.0006  | 0.0000  | 0.0002  | 0.0101  |
| \( \text{Rab} \) ER Width | 3.18E-10 | 9.90E-11 | 3.19E-12 | 6.61E-11 | 3.07E-11 | 4.74E-12 |
| \( \text{URV} \) Linex \( v=2 \) | 1.0006  | 1.9984  | 1.0005  | 2.0004  | 1.0009  | 1.9998  |
| \( \text{ER} \) Linex \( v=-2 \) | 2.65E-11 | 5.90E-10 | 8.24E-11 | 9.90E-13 | 8.54E-12 | 1.16E-10 |
| \( \text{URRSS} \) Linex \( v=2 \) | 1.0005  | 2.0004  | 1.0008  | 2.0003  | 1.0001  | 1.9984  |
| \( \text{ER} \) Linex \( v=-2 \) | 5.35E-10 | 1.46E-11 | 1.33E-12 | 2.53E-10 | 2.77E-10 | 5.24E-10 |
| \( \text{URRSS} \) Linex \( v=2 \) | 1.0000  | 1.9985  | 1.0010  | 2.0008  | 0.9999  | 2.0008  |
| \( \text{ER} \) Linex \( v=-2 \) | 7.36E-11 | 2.86E-11 | 1.05E-10 | 5.31E-12 | 2.41E-11 | 4.21E-13 |
| \( \text{URRSS} \) Linex \( v=2 \) | 0.9996  | 1.9994  | 1.0010  | 1.9999  | 1.0004  | 2.0001  |
| \( \text{ER} \) Linex \( v=-2 \) | 4.74E-10 | 4.83E-11 | 2.55E-11 | 5.51E-11 | 6.88E-12 | 1.17E-10 |
| \( \text{URRSS} \) Linex \( v=2 \) | 0.0004  | 0.0003  | 0.0010  | 0.0001  | 0.004  | 0.0001  |
| \( \text{ER} \) Linex \( v=-2 \) | 4.74E-10 | 4.83E-11 | 2.55E-11 | 5.51E-11 | 6.88E-12 | 1.17E-10 |
| \( \text{URRSS} \) Linex \( v=2 \) | 0.0012  | 0.0026  | 0.0023  | 0.0019  | 0.0018  | 0.0009  |
| \( \text{ER} \) Linex \( v=-2 \) | 4.74E-10 | 4.83E-11 | 2.55E-11 | 5.51E-11 | 6.88E-12 | 1.17E-10 |
From Table I one can observe that the width of the Bayes credible intervals for \( \hat{\alpha}_{SE} \) is shorter than the corresponding \( \hat{\alpha}_{LINEX} \) for all the number of records except at \( n = 8 \). Also the width for \( \hat{\alpha}_{LINEX} \) when \( v = 2 \) is shorter than that of the corresponding \( \hat{\alpha}_{LINEX} \) for \( n = 4,5,6 \) but when \( v = -2 \), the width of the Bayes credible intervals for \( \hat{\alpha}_{LINEX} \) is shorter than that of the corresponding \( \hat{\alpha}_{LINEX} \) in case of \( n = 4,5,6 \).

From Fig. (3) one can observe that the ERs of \( \hat{\alpha}_{SE} \) are less than the ERs of corresponding \( \hat{\alpha}_{LINEX} \) when \( \nu = 6,7,8 \), also ERs of \( \hat{\alpha}_{LINEX} \) when \( \nu = 2 \) are less than the ERs of corresponding \( \hat{\alpha}_{LINEX} \) for all the number of records except at \( n = 4,7 \), but when \( \nu = -2 \), ERs of \( \hat{\alpha}_{LINEX} \) is shorter than that of ERs of corresponding \( \hat{\alpha}_{LINEX} \) for all number of records except at \( n = 7 \).

From Table II one can observe that, the width of the Bayes credible intervals for \( \hat{\lambda}_{SE} \) is shorter than that of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 6,7 \). While the width of credible intervals for \( \hat{\lambda}_{SE} \) is shorter than that of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 5,8 \). Also the width of the Bayes credible intervals for \( \hat{\lambda}_{LINEX} \) when \( \nu = 2 \) is shorter than that corresponding of \( \hat{\lambda}_{LINEX} \).

For \( n \leq 5 \) the Rabs of \( \hat{\alpha}_{SE} \) are less than that of the corresponding \( \hat{\alpha}_{LINEX} \), but the Rabs of \( \hat{\lambda}_{SE} \) is less than that of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 4 \). However the Rabs for \( \hat{\lambda}_{LINEX} \) when \( \nu = -2 \) are less than that of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 4 \) according to Table II.

Table III shows that the ERs of \( \hat{\alpha}_{SE} \) are less than the ERs of the corresponding \( \hat{\lambda}_{SE} \) for all number of records except at \( n = 5,7 \), the ERs of \( \hat{\lambda}_{LINEX} \) when \( \nu = -2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 6,8 \). The ERs of \( \hat{\lambda}_{LINEX} \) when \( \nu = 2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 5,8 \) and the ERs of \( \hat{\lambda}_{LINEX} \) when \( \nu = -2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 3,8 \).

As shown in Table III, the width of the Bayes credible intervals for \( \hat{\lambda}_{SE} \) is shorter than that of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 6 \). The width of the credible intervals for \( \hat{\lambda}_{LINEX} \) when \( \nu = 2 \) is shorter than that width of the credible intervals for the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 4,6 \). But when \( \nu = -2 \) the width of the credible intervals for \( \hat{\lambda}_{LINEX} \) is shorter than that of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 5,7 \).

Table IV illustrates that the ERs of \( \hat{\lambda}_{SE} \) are less than the ERs of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 4 \). The ERs of \( \hat{\lambda}_{SE} \) are less than the ERs of the corresponding \( \hat{\lambda}_{SE} \) for all number of records except at \( n = 4,6 \). While, the ERs of \( \hat{\lambda}_{LINEX} \) when \( \nu = -2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{LINEX} \) for all number of records except at \( n = 6,8 \) and the ERs of \( \hat{\lambda}_{LINEX} \) when \( \nu = 2 \)
are less than the ERs of the corresponding \( \hat{\alpha}_{\text{LINEX}} \) for all number of records except at \( n = 4,7 \).

The width of the Bayes credible intervals for \( \hat{\lambda}_{\text{SE}} \) is shorter than that of the corresponding \( \hat{\lambda}_{\text{SE}} \) for all number of records except at \( n = 4 \). While, the width of the credible intervals for \( \hat{\lambda}_{\text{LINEX}} \) when \( v = 2 \) is shorter than that of the corresponding \( \hat{\lambda}_{\text{LINEX}} \) for all number of records except at \( n = 4,8 \) according to Table IV.

Table V shows that the ERs of \( \hat{\alpha}_{\text{SE}} \) are less than the ERs of the corresponding \( \hat{\alpha}_{\text{SE}} \) for all number of records except at \( n = 3,6 \). While, the ERs of \( \hat{\lambda}_{\text{SE}} \) are less than the ERs of the corresponding \( \hat{\lambda}_{\text{SE}} \) for all number of records except at \( n = 6,8 \). The ERs of \( \hat{\lambda}_{\text{LINEX}} \) when \( v = 2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{\text{LINEX}} \) for all number of records except at \( n = 6,7 \). While, when \( v = -2 \) the ERs of \( \hat{\lambda}_{\text{LINEX}} \) are less than the ERs of the corresponding \( \hat{\lambda}_{\text{LINEX}} \) for all number of records except at \( n = 6,7 \).

![Fig. 5 ERs of α under SE and LINEX loss functions based on URV and URRSS at (α,λ)=(2,1)](image)

From Fig. (5), the ERs of \( \hat{\alpha}_{\text{SE}} \) are less than the ERs of the corresponding \( \hat{\alpha}_{\text{SE}} \) for all the number of records except for \( n = 4,6 \). Also, when \( v = 2 \) the ERs of \( \hat{\alpha}_{\text{LINEX}} \) are less than of the corresponding \( \hat{\alpha}_{\text{LINEX}} \) for all number of records except at \( n = 7 \). But when \( v = -2 \), the ERs of the \( \hat{\alpha}_{\text{LINEX}} \) are less than the ERs of the corresponding \( \hat{\alpha}_{\text{LINEX}} \) for all the number of records except at \( n = 3,6 \).

![Fig. 6 ERs of λ under SE and LINEX loss functions based on URV and URRSS at (α,λ)=(2,1)](image)

Fig. (6) shows that the ERs of \( \hat{\lambda}_{\text{SE}} \) are less than of the corresponding \( \hat{\lambda}_{\text{SE}} \) for all number of records except at \( n = 3,8 \). Also, the ERs of \( \hat{\lambda}_{\text{LINEX}} \) when \( v = 2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{\text{LINEX}} \) for \( n = 3,5,6 \). The ERs of \( \hat{\lambda}_{\text{LINEX}} \) when \( v = -2 \) are less than the ERs of the corresponding \( \hat{\lambda}_{\text{LINEX}} \) for \( n = 5,7,8 \).

Table VI shows that, the width of the Bayes credible intervals for \( \hat{\alpha}_{\text{SE}} \) is shorter than that corresponding \( \hat{\alpha}_{\text{SE}} \) for all number of records except at \( n = 7,8 \). Also, The width of the Bayes credible intervals for \( \hat{\alpha}_{\text{LINEX}} \) when \( v = 2 \) is shorter than that corresponding \( \hat{\alpha}_{\text{LINEX}} \) for all number of records except at \( n = 3,8 \).

**IV. CONCLUSION**

In this paper, we presented how to develop Bayes estimates in the context of upper record values and upper record ranked set sampling from generalized inverted exponential distribution under symmetric and asymmetric loss functions.

Based on the URV and URRSS, it is observed that the Bayes estimators cannot be obtained in explicit forms. Therefore, the MCMC technique has been used to generate posterior samples.

We observe from the numerical study that the relative absolute biases, estimated risks and widths of confidence intervals are very small based on the two sampling schemes for both SE and LINEX loss functions.

Generally, the Bayes estimates under LINEX loss function when \( v = -2 \) perform better than the Bayes estimates under LINEX loss function when \( v = 2 \) in case of URV in approximately most of situations. While the Bayes estimates under LINEX loss function when \( v = 2 \) perform better than estimates under LINEX loss function when \( v = -2 \) in case of URRSS in approximately most of situations.
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