A non-parametric test of variability of Type Ia supernovae luminosity and CDDR

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Abstract. The first observational evidence for cosmic acceleration appeared from Type Ia supernovae (SNe Type Ia) Hubble diagram from two different groups. However, the empirical treatment of SNe Type Ia and their ability to show cosmic acceleration have been the subject of some debate in the literature. In this work we probe the assumption of redshift-independent absolute magnitude ($M_B$) of SNe along with its correlation with spatial curvature ($\Omega_{k0}$) and cosmic distance duality relation (CDDR) parameter ($\eta(z)$). This work is divided into two parts. Firstly, we check the validity of CDDR which relates the luminosity distance ($d_L$) and angular diameter distance ($d_A$) via redshift. We use the Pantheon SNe Ia dataset combined with the $H(z)$ measurements derived from the cosmic chronometers. Further, four different redshift-dependent parametrizations of the distance duality parameter ($\eta(z)$) are used. The CDDR is fairly consistent for almost every parametrization within a 2$\sigma$ confidence level in both flat and a non-flat universe. In the second part, we assume the validity of CDDR and emphasize on the variability of $M_B$ and its correlation with $\Omega_{k0}$. We choose four different redshift-dependent parametrizations of $M_B$. The results indicate no evolution of $M_B$ within 2$\sigma$ confidence level. For all parametrizations, the best fit value of $\Omega_{k0}$ indicates a flat universe at 2$\sigma$ confidence level. However a mild inclination towards a non flat universe is also observed. We have also examined the dependence of the results on the choice of different priors for $H_0$.

Keywords: supernova type Ia - standard candles, cosmological parameters from LSS, dark energy theory, cosmology of theories beyond the SM

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1 Introduction

The first observational evidence for the cosmic acceleration appeared from Type Ia supernovae (SNe Type Ia) observations performed by two different research groups [1, 2]. Over the decades, the number of SNe Type Ia catalogs have increased significantly. Even now, SNe Type Ia observations provide the most direct evidence for the current cosmic acceleration. In the context of Einstein’s General Theory of Relativity (GTR), SNe Type Ia observations support the existence of a mysterious form of energy called dark energy, that is either constant ($\Lambda$CDM) or slowly varying with time and space. (See reviews in refs. [3–5].)

Type Ia supernovae (SNe Type Ia) are extremely luminous explosions and these are observationally identified by the absence of hydrogen and presence of silicon (Si II) spectral lines in their spectra [6]. The use of SNe Type Ia as a reliable cosmological probe relies on two fundamental assumptions:

a) the first assumption is that the shape of the light curve of all type Ia supernovae (SNe Type Ia) is similar; hence these can be standardized [7, 8]. In practice, this task of standardizing light curve of SNe Type Ia is achieved by using several empirically derived light curve fitters like, for instance, MLCS/MLCS2k2 [9], SALT [10], SALT2 [10], SiFTO [11] etc. As stated, the generalized functional forms of the light curves obtained in these fitters are purely empirical and based on plausible physical explanations to explain the light curves of SNe Type Ia from the time of explosion to a few weeks after peak brightness [12].

b) The second basic assumption behind the SNe Type Ia analysis is that the intrinsic luminosity of a supernova is independent of the redshift and the host galaxy environment. In other words, it is theoretically assumed that two different SNe Type Ia in two different host galaxies have the same intrinsic luminosity, independent of masses and redshifts of the host galaxies. However, in recent years several dedicated studies have concluded that the light curve fitting analysis of SNe Type Ia depends on their
host galaxy masses [13]. Further, it is seen that SNe Type Ia events occurring in massive early-type, passive galaxies are brighter than those in late-type star forming galaxies [14–17].

The two main progenitor models of SNe Type Ia are the “single degenerate” and the “double degenerate” models. In the single degenerate model, a white dwarf accreting from a binary companion star is pushed over the Chandrasekhar mass limit, while in the double degenerate model of SNe Type Ia explosions, an orbiting pair of binary white dwarfs merge together and their mass eventually exceeds the Chandrasekhar mass limit. There is no established formalism to exactly differentiate between these two channels of SNe Type Ia explosion. Hence the observed supernova population catalog may have SNe Type Ia contributions coming from both these channels which will have a direct impact on the first and second assumptions used in SNe Type Ia analysis [18].

Further, the statistical treatment of SNe Type Ia, their dimming and their ability to prove the cosmic acceleration is still a topic of debate in the literature. This is because the process of obtaining the SNe Type Ia observations is not trivial but requires some priors for interpretations and corrections to convert an observed-frame magnitude to a rest-frame magnitude. Examples discussed in the literature are for instance: possible evolutionary effects in SNe Type Ia events [19, 20], local Hubble bubble [21, 22], modified gravity [23–25], unclustered sources of light attenuation [26–29] and the existence of Axion-Like-Particles (ALPs) arising in a wide range of well-motivated high-energy physics scenarios. All these could also lead to the dimming of SNe Type Ia brightness [30, 31] which would eventually affect the second assumption used in the SNe Type Ia analysis.

Given the above mentioned limitations, Tutusaus et al. (2019) [32] relaxed the standard assumption that SNe Type Ia intrinsic luminosity is independent of the redshift and examined its impact on the cosmic acceleration. The authors reconstructed the expansion rate of the Universe by fitting the SNe Type Ia observations with a cubic spline interpolation. They showed that a non-accelerated expansion rate of the Universe is able to fit all the main background cosmological probes. In addition, Tutusaus et al. (2017) [33] found that when SNe Type Ia intrinsic luminosity is not assumed to be redshift independent, a non-accelerated low-redshift power law model is able to fit the low-redshift background data as well as the $\Lambda$CDM model. It has also been found that a significant correlation exists between SNe Type Ia luminosity (after the standardization) and the stellar population age at a 99.5% confidence level [34] indicating that the light-curve fitters used by the SNe Type Ia community are not quite capable of correcting for the population age effect. More recently, Valentino et al. (2020) [35] have also raised the issue whether intrinsic SNe Type Ia luminosities might evolve with redshift. They analysed the impact of the latter on the inferred properties of the dark energy component responsible for cosmic acceleration. However, they find the evidence for cosmic acceleration to be robust to possible systematics. Along the same lines, Sapone et al. (2020) [36] also analyse the cosmological implications of an absolute luminosity of SNe Type Ia which could vary with respect to the redshift. Further, they study the impact of the latter on modified gravity models and non-homogeneous models.

These statements seem to offer enough plausible reasons to investigate the effects of variability of absolute luminosity of SNe Type Ia. However, these are not the sole reasons of concern about the variability of $M_B$. The cosmic acceleration rate and the cosmological parameters determined by the SNe Type Ia measurements are highly dependent on the possible dimming effect as well. Vavrycuk et al. (2019) [37] revived a debate about an origin of Type Ia supernova (SN Ia) dimming and showed that the standard $\Lambda$CDM model and
the opaque universe model (caused by light extinction by intergalactic dust) fit the SN Ia measurements at redshifts $z < 1.4$ fairly well. Then, there are still some possible loopholes in the current SNe Type Ia observations and alternative mechanisms contributing to the acceleration evidence or even mimicking the dark energy behaviour have been proposed. It is important to point out that a constant value of the absolute luminosity of SNe Type Ia at the peak of its light curve is not sensitive to Hubble constant ($H_0$). Nevertheless, a variable absolute magnitude will be sensitive to $H_0$.

These recent developments motivate us to carry out a model independent study of finding any correlation between variable luminosity of SNe Type Ia and observed SNe Type Ia dimming effect due to opacity of the environment. The most general methodology of testing the cosmic opacity of SNe Type Ia is based on the cosmic distance duality relation (CDDR) which connects the luminosity distance $d_L$ and angular diameter distance (ADD) $d_A$ at the same redshift and is defined as, $d_L(1 + z)^{-2}/d_A = \eta(z) = 1$. In order to look for the presence of some unknown physics phenomenon beyond the standard model or an inconsistency between cosmological data we check if $\eta(z) \neq 1$, that is CDDR is violated. This relation holds for general metric theories of gravity in any background, in which photons travel along null geodesics and the number of photons is conserved during cosmic evolution [38]. Briefly, the SNe Type Ia observations have been confronted with several other cosmological probes (strong gravitation lens systems, angular diameter distances, gas mass fractions, baryonic acoustic oscillations, cosmic microwave background, radio sources, gravitational waves, $H(z)$ measurements, gamma ray bursts etc.) in order to put limits on the redshift dependence of $\eta$, that is on $\eta(z)$ [39–65]. All these works conclude that CDDR is valid within a $2\sigma$ confidence level. However, it is worth stressing that current analysis could not distinguish which functional form of $\eta(z)$ best describes the data (see details in ref. [66]).

Another cosmological parameter which can be crucial to the variability of absolute luminosity of SNe Type Ia and its dimming effect is the cosmic curvature. Cosmic curvature ($\Omega_k$) is a fundamental geometric quantity of the Universe. It plays a crucial role in the evolution and dynamics of the universe. As, $\Omega_k$ is directly related to cosmological distances, a flat or non-flat space-time would obviously impact the path travelled by the photon and eventually the absolute magnitude of SNe Type Ia. Hence, in this paper, we also probe the variation of the absolute luminosity of SNe Type Ia and its correlation with the CDDR parameter ($\eta(z)$) and cosmic curvature ($\Omega_k$).

For clarity of exposition, this paper is divided into two parts. In the first part, we test the validity of CDDR. We propose a new cosmological non-parametric test for CDDR by using SNe Type Ia observations and $H(z)$ measurements from cosmic chronometers. The basic procedure is as follows: we obtain the angular diameter distances at SNe Type Ia redshifts by applying Gaussian Process integration method on cosmic chronometer $H(z)$ data. By using a deformed CDDR of the form $d_L = \eta(z)d_A(1 + z)^2$ and considering a flat universe, we impose limits on the SNe Type Ia absolute magnitude ($M_B$), and $\eta(z)$ functions, namely: $\eta(z) = \eta_0, \eta(z) = \eta_0 + \eta_1 z, \eta(z) = \eta_0 + \eta_1 z/(1 + z)$ and $\eta(z) = \eta_0 + \eta_1 \ln(1 + z)$.

After testing the validity of the CDDR parameter $\eta$ in the first part, in the second part, we assume the validity of CDDR and put limits on the possible evolution of SNe Type Ia absolute magnitude by assuming $M_B(z) = M_{B0}, M_B(z) = M_{B0} + M_{B1} z, M_B(z) = M_{B0} + M_{B1} z/(1 + z)$ and $M_B(z) = M_{B0} + M_{B1} \ln(1 + z)$ parametrizations in flat and non-flat cosmologies.

The structure of the paper is as follows: in section 2, we discuss the cosmological probes and the data sets used in the analysis along with their theoretical construction. In section 3,
we outline the methodology used. In section 4, the emphasis is on the results of both parts where in the first part we test the validity of CDDR and in the second part we test the variability of absolute luminosity of SNe Type Ia. Finally, in section 5, we discuss the final outcomes of both parts and also explore the impact of different $H_0$ priors on our analysis.

2 Cosmological probes and data sets

In this section we present the cosmological probes and data sets used in the analysis.

2.1 Type Ia supernovae measurement and Pantheon Sample

Type Ia supernovae are considered to be standard candles. Observations of these are at the core of establishing the validity of cosmic acceleration. The standard observable quantity in SNe Type Ia analysis is the distance modulus which is the difference between the apparent and absolute magnitude of the SNe Type Ia. The observational measurement of this quantity is given by the relation

$$
\mu_{SN} = m_{B}^{obs}(z) + \alpha \cdot X_1 - \beta \cdot C - M_B.
$$ (2.1)

where, $m_B$ is the rest frame B-band observed peak magnitude, $X_1$ and $C$ are time stretching of the light curve and SNe Type Ia color at maximum brightness respectively and $M_B$ is the absolute B-band magnitude. This relation indicates that the variability of the distance modulus is governed by two additional parameters $X_1$ and $C$. It must be noted here that we do have two nuisance parameters $\alpha$ and $\beta$ as well. In this paper we use the recent and largest database of SNe Type Ia known as the Pantheon data set. In this data set, these two nuisance parameters have been marginalized and eventually calibrated to be zero. This data set has 1048 SNe Type Ia measurements in the redshift range $0.01 \leq z \leq 2.26$ [13]. Hence, for the Pantheon data set, the observed distance modulus relaxes to the form $\mu_{SN} = m_{B}^{obs} - M_B$. Once we know the distance modulus, we can easily define the luminosity distance and uncertainty in observed luminosity distance as

$$
d_{L}^{th}(z; M_B) = 10^{(m_{B}^{th} - M_B - 25)/5}(\text{Mpc}),
$$ (2.2)

From eq. (2.2), it can be easily seen that once we estimate the value of $M_B$, we can find the luminosity distance at a given redshift. However, in this work, our aim is to study the variation of the absolute magnitude ($M_B$) of SNe Type Ia, so if we can get a model-independent estimate of the luminosity distance from other observational probes then we can constrain the variability of $M_B$.

We use the cosmic distance duality relation (CDDR) to reconstruct the luminosity distance theoretically which is given by

$$
d_{L}^{th}(z; \eta, \Omega_k) = \eta(z)d_A(z; \Omega_k)(1+z)^2.
$$ (2.3)

Using eq. (2.3) we can define, from eq. (2.2), $m_{B}^{th}$ as

$$
m_{B}^{th}(z; \eta, M_B, \Omega_k) = 5 \log \left(\eta(z)d_A(z; \Omega_k)(1+z)^2\right) + M_B + 25,
$$ (2.4)

Here, $\eta$ is the cosmic distance duality parameter which is a measure of the deviation from CDDR. CDDR holds for $\eta(z) = 1$. Here $d_{L}^{th}$ is the theoretical luminosity distance defined
in the terms of angular diameter distance $d_A$ and $\eta(z)$. The angular diameter distance is defined as

$$
\begin{aligned}
    d_A(z; H_0, \Omega_{k0}) &= \begin{cases}
        \frac{d_H}{(1+z)\sqrt{|\Omega_{k0}|}} \sinh \left[ \sqrt{|\Omega_{k0}|} \frac{d_C}{d_H} \right] & \text{for } \Omega_{k0} > 0, \\
        \frac{d_C}{(1+z)\sqrt{|\Omega_{k0}|}} & \text{for } \Omega_{k0} = 0, \\
        \frac{d_H}{(1+z)\sqrt{|\Omega_{k0}|}} \sin \left[ \sqrt{|\Omega_{k0}|} \frac{d_C}{d_H} \right] & \text{for } \Omega_{k0} < 0.
    \end{cases}
\end{aligned}
$$

Here $\Omega_{k0}$ is the cosmic curvature, where $\Omega_{k0}$ is greater, equal and less than zero for open, flat and closed universe respectively. $d_C$ is the comoving distance and $d_H = c/H_0$ is known as the Hubble distance where $c$ is the speed of light and $H_0$ is the Hubble constant.

### 2.2 Constraining angular diameter distance using $H(z)$ measurements

As is evident from eq. (2.5), if we can obtain a model-independent estimate of $d_C/d_H$ then we can easily estimate angular diameter distance $d_A(z; \Omega_{k0})$ as a function of $\Omega_{k0}$ and subsequently theoretical luminosity distance $d_L^{th}(z; \eta, \Omega_{k0})$ as a function of $\eta$ and $\Omega_{k0}$. In this section we will first discuss the data sets and the methodology to obtain angular diameter distances from them.

#### 2.2.1 Hubble data set

In cosmology, the Hubble parameter $H(z)$ is a crucial measured quantity which describes the dynamical properties of the universe such as the expansion rate and evolution history of the universe. It is also helpful to explore the nature of the dark energy. The most recent data compilation of Hubble parameter measurements [67] has 31 measurements of $H(z)$ which are obtained by using the differential ages of passively evolving galaxies. We now outline the steps needed to obtain the angular diameter distance that we need.

1. **Differential ages of passively evolving galaxies:** the Hubble parameter $H(z)$ can be expressed in the terms of the rate of change of cosmic time with the redshift, given by

$$
H(z) = -\frac{1}{(1+z)\Delta t} \frac{\Delta z}{\Delta t}.
$$

The change of cosmic time with redshift can be estimated from the ageing of the stellar population in the galaxies. However, one needs to be extremely careful in selecting the galaxies while calculating the $H(z)$ values using this method. In young evolving galaxies, the stars are being born continuously and the emission spectra will be dominated by the young stellar population. Hence to estimate accurately the differential ageing of the universe, passively evolving red galaxies are used as their light is mostly dominated by the old stellar population [68]. To find $H(z)$ at a given redshift, the ages of the early type passively evolving galaxies with similar metallicity and very small redshift interval is calculated. The redshift difference $\Delta z$ between two galaxies can be measured by using spectroscopic observations. For the estimation of $\Delta t$, [69] suggested the use of a direct spectroscopic observable (the 4000 Å break) which is known to be linearly related to the age of the stellar population of a galaxy at fixed metallicity. As the measure of $H(z)$ is estimated purely by using spectroscopic observations, it is independent of the cosmological model and has proved to be a strong probe to constrain cosmological models and assumptions. This method of calculating $H(z)$ is usually known as the “Cosmic Chronometers” and data points are generally referred to as CC $H(z)$. We have 31 data points estimated using this differential ages of passively evolving galaxies technique [69–74].
2.2.2 Comoving distance using Gaussian Process

In this analysis, we require a model-independent estimate of the angular diameter distance. For this, we first calculate the comoving distance $d_C$ using the $H(z)$ measurement of cosmic chronometers as

$$ \frac{d_C}{d_H} = \int_0^z \frac{dz'}{E(z')}.$$  \hspace{1cm} (2.7)

Here $E(z) = H(z)/H_0$. If we have the functional form of $E(z)$ then we can easily integrate eq. (2.7) by using any numerical integration method. However, in our case we have only $H(z)$ estimates at certain redshifts. In our analysis, to obtain continuous smooth values of $H(z)$ we use the Gaussian Process (GP), a well known hyper-parametric regression method [76]. GP has been widely used in the literature to reconstruct the shapes of physical functions and is very useful for such functional reconstructions due to its flexibility and simplicity. In this method, the complicated parametric relationship is replaced by parametrizing a probability model over the data. In mathematical terms, it is a distribution over functions, characterized by a mean function and covariance function, given by

$$ K(z, z') = \langle (H(z) - \mu(z)) (H(z') - \mu(z')) \rangle \hspace{1cm} (2.8)$$

where $\mu(z)$ is the prior mean. In order to avoid model dependence appearing through the choice of the prior mean function, we have chosen it to be zero. This is a common choice of mean function as one can always normalize the data so it has zero mean. We have also checked by taking different values of prior mean, $\mu(z)$ and observed that the result is independent of the choice of the prior mean function. This method, however, comes with a few inherent underlying assumptions- it is assumed that each observation is an outcome of an independent Gaussian distribution belonging to the same population and the outcomes of observations at any two redshifts are correlated with the strength of correlation depending on their nearness to each other. We reconstructed our data by using the square exponential kernel function, given by

$$ K(z, z') = \sigma_f^2 \exp \left( -\frac{(|z - z'|)^2}{2\ell^2} \right). \hspace{1cm} (2.9)$$

Here, $\sigma_f$ and $\ell$ are two hyperparameters which control the amplitude and length-scale of the prior covariance. The value of hyperparameters is calculated by maximizing the corresponding marginal log-likelihood probability function of the distribution. For maximization, we use flat priors for $\sigma_f$ and $\ell$ for all the choices of kernel function. In order to check the sensitivity of our analysis to the choice of the kernel function, we repeated our analysis with the Matérn-(3/2, 5/2, 7/2 & 9/2) kernel functions. Though the values of hyperparameters vary according to the choice of kernel function, the reconstructed curves do not show any significant deviation from the curve obtained from square exponential curve. Hence we choose to work with the square exponential kernel function only.

Once we obtain the reconstructed $H(z)$ at all possible redshifts in the range $0 < z < 2$ as shown in figure 1, we divide it by $H_0 = 67.66 \pm 0.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$ value to obtain $E(z)$. We use the Simpson 3/8 method for numerically integrating eq. (2.7) and obtain continuous values of $d_C/d_H$ at all redshifts in the range $0 < z < 2$ which is shown in figure 2. Further, we can use eq. (2.5) to obtain the angular diameter distance as a function of cosmic curvature $\Omega_k$. The uncertainty in $d_A$ i.e $\sigma_{d_A}$ is estimated by propagating the error obtained in $d_C/d_H$ using Gaussian Process.
Figure 1. In the left plot, 31 CC $H(z)$ vs. $z$ datapoints are shown. In the right plot, $H(z)$ is estimated at all intermediate redshifts in the range $0 < z < 2$ using a non-parametric smoothening technique, namely Gaussian Process. In the analysis, the Hubble constant value is taken to be $H_0 = 67.66 \pm 0.42$ km s$^{-1}$ Mpc$^{-1}$ estimated from CMB measurement [75]. The impact of other choices of $H_0$ values on analysis has been discussed in section 5, (C).

Figure 2. This plot represents the reconstructed values of $d_C/d_H$ in the redshift in the range $0 < z < 2$. The solid black line represents the best fit line while blue lines includes the 1$\sigma$ confidence region. The solid red line is the theoretical curve for the flat $\Lambda$CDM model with $\Omega_m = 0.3$. The value of $H_0$ used in reconstructing this plot is $H_0 = 67.66 \pm 0.42$ km s$^{-1}$ Mpc$^{-1}$.

3 Analysis

Generally, the Pantheon dataset comes in terms of SNe Type Ia apparent magnitudes and with a full covariance matrix, $C_{\text{sys}}$ correlating the apparent magnitudes at various redshifts. This covariance matrix is a non-diagonal matrix of systematic uncertainties that come from the bias corrections method [13, 77].

In this analysis, we have to fit simultaneously three parameters i.e. $M_B$, $\eta$ and $\Omega_k0$. These parameters are determined by maximizing the likelihood $\mathcal{L} \sim \exp \left( -\chi^2/2 \right)$, where chi-square ($\chi^2$) is a quantity summed over all the Pantheon SNe Type Ia Sample redshifts, and is defined as

$$
\chi_{\text{Pan}}^2 = \Delta m^T \cdot C^{-1} \cdot \Delta m
$$

1\text{http://github.com/dscolnic/Pantheon.}
where, \( C = D_{\text{stat}} + C_{\text{sys}} \). Here \( D_{\text{stat}} \) is the diagonal covariance matrix of the statistical uncertainties and \( \Delta m = m_B^{\text{obs}}(z_i) - m_B^{\text{th}}(z_i; \eta, M_B, \Omega_{k0}) \) which is given in eqs. (2.1), (2.4).

In this analysis, we have three unknown parameters, namely, the absolute magnitude of SNe Type Ia, \( M_B \), the cosmic distance duality parameter, \( \eta(z) \), and the cosmic curvature, \( \Omega_{k0} \). In order to investigate the variability of \( M_B \) we have to analyse its correlation with the remaining two parameters. In order to do so, we divide the work into two parts:

**Part I: test of CDDR.** In this part, we consider the distance duality relation parameter (\( \eta \)) to test CDDR and put constraints on \( \eta(z) \) simultaneously with the other two parameters i.e. \( M_B \) and \( \Omega_{k0} \). We take into account four parametrizations of \( \eta(z) \) which are as follows

- **P1:** \( \eta(z) = \eta_0 \).
- **P2:** \( \eta(z) = \eta_0 + \eta_1 z \).
- **P3:** \( \eta(z) = \eta_0 + \eta_1 z_1 + z \).
- **P4:** \( \eta(z) = \eta_0 + \eta_1 \ln(1 + z) \).

We included these to study the impact of different characterizations on the analysis. The motivation for this form of parametrizations comes from the commonly used parametrizations for the equation of state parameter, viz. the Chevallier-Polarski-Linder (CPL) Parametrization, Jassal-Bagla-Padmanabhan (JBP) Parametrization etc. [78–80].

In the first parametrization, we are choosing \( \eta \) to be redshift independent. In second parametrization, it is a simple Taylor series expansion around \( z = 0 \) but is not well behaved at higher \( z \) values. The third and fourth parametrizations are taken as these are well behaved even at high redshifts and are somewhat more slowly varying as compared to the second one.

Further in each parametrization of \( \eta(z) \), we discuss two cases. In the first case we choose a flat universe by considering \( \Omega_{k0} = 0 \) and then put constraints on \( M_B \) and \( \eta \). In the second case, we consider \( \Omega_{k0} \) as a free parameter corresponding to a non-flat universe.

**Part II: test of variability of SNe Type Ia absolute luminosity.** After testing the validity of CDDR in Part I, we solely focus on the variability of absolute magnitude of SNe Type Ia and use the following parametrizations of \( M_B \)

- **M1:** \( M_B(z) = M_{B0} \).
- **M2:** \( M_B(z) = M_{B0} + M_{B1} z \).
- **M3:** \( M_B(z) = M_{B0} + M_{B1} z_1 + z \).
- **M4:** \( M_B(z) = M_{B0} + M_{B1} \ln(1 + z) \).

In each parametrization of \( M_B(z) \), we discuss two cases. In the first case we choose a flat universe and then put constraints on \( M_B \) and in the second case, we consider a non-flat universe.

We use **emcee**, a Python based package, to perform the Markov Chain Monte Carlo (MCMC) analysis [81]. We find the best fit of all parameters and once the MCMC method is performed, the confidence level with their 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) uncertainties are computed with the Python package **corner** [82].

In this work, we assume a broad flat prior for all parameters (table 1). We use 100 walkers which take 10000 steps for exploring the parameter space with MCMC chains. The
| Parameter | Prior Range |
|-----------|-------------|
| $M_B$     | U$[-21, -17]$ |
| $\eta_0$ | U$[-3, 3]$ |
| $\eta_1$ | U$[-2, 2]$ |
| $\Omega_k_0$ | U$[-2.5, 2.5]$ |

Table 1. The prior range of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_k_0$.

| Parameter | Flat Universe | Non-Flat Universe |
|-----------|--------------|------------------|
| $M_B$     | $-19.300^{+0.812}_{-0.874}$ | $-19.653^{+0.587}_{-0.431}$ |
| $\eta_0$ | $0.960^{+0.300}_{-0.476}$ | $1.124^{+0.250}_{-0.265}$ |
| $\Omega_k_0$ | — | $0.076^{+0.110}_{-0.106}$ |

Table 2. The best fit values of $M_B$, $\eta_0$ and $\Omega_k_0$ with 1σ confidence level for P1 parametrization.

first 10% of the steps are discarded as burn-in period and the posterior distributions are analysed based on the remaining samples. To ensure that the chains are converging, an auto-correlation study is performed. For this we compute the integrated auto-correlation time $\tau_f$ using the `autocorr.integrated_time` function of the `emcee` package. For more details, please see ref. [81].

4 Results

In this paper our aim is to probe the variation of the absolute luminosity ($M_B$) of SNe Type Ia. This paper is divided into two parts. In the first part, we propose a new (cosmological) model-independent test for CDDR by using SNe Type Ia observations and $H(z)$ measurements from cosmic chronometers. In the second part we assume that CDDR is valid and check the dependency of absolute magnitude on redshift.

4.1 Test of CDDR

In this part, we consider the distance duality parameter ($\eta$) and take into account four parametrizations of $\eta(z)$. In each parametrization, we discuss two cases- flat and non-flat universe.

P1. $\eta(z) = \eta_0$. Taking $\eta(z)$ to be a constant, the best fit values of $M_B$, $\eta_0$ and $\Omega_k_0$ parameters are given in table 2.

The best fit value of $\eta(z)$ shown in table 2 for both flat as well as non-flat universe, indicates that the cosmic distance duality relation ($\eta(z) = 1$) holds at 1σ confidence level. In both flat and non-flat case, the value of $M_B$ remains same within 1σ confidence level which reflects that $M_B$ doesn’t have any strong dependence on the curvature parameter. The 1D and 2D posterior distributions of $M_B$, $\eta_0$ and $\Omega_k_0$ with 1σ, 2σ and 3σ confidence levels for flat and non-flat universe are shown in figure 3. The contour plots shown in figure 3 indicate
Table 3. The best fit values of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ with 1$\sigma$ confidence level for P2 parametrization.

| Parameter | Flat Universe | Non-Flat Universe |
|-----------|---------------|-------------------|
| $M_B$     | $-19.254^{+0.616}_{-0.778}$ | $-19.574^{+0.518}_{-0.495}$ |
| $\eta_0$  | $0.938^{+0.405}_{-0.232}$     | $1.097^{+0.279}_{-0.234}$   |
| $\eta_1$  | $0.002^{+0.010}_{-0.009}$     | $-0.098^{+0.047}_{-0.049}$  |
| $\Omega_{k0}$ | —               | $1.108^{+0.503}_{-0.531}$   |

a negative correlation between $M_B$ and $\eta_0$. Further, this parametrization supports a flat universe within 1$\sigma$ confidence level.

P2. $\eta(z) = \eta_0 + \eta_1 z$. In this parametrization, we consider $\eta(z)$ as a function of redshift. The best fit values of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ are given in table 3.

In case of a flat universe, the best fit value of $\eta_1$, suggests that the cosmic distance duality parameter holds within 1$\sigma$ confidence level. Similarly in the case of a non-flat universe, there is no violation of cosmic distance duality relation at 3$\sigma$ confidence level. The obtained value of $\Omega_{k0}$ can accommodate a flat universe at 3$\sigma$ confidence level. The best fit values of $M_B$ in both a flat universe and a non-flat universe remain the same within 1$\sigma$ confidence level which indicates that there is no strong impact of $\Omega_{k0}$ on $M_B$.

The 1D and 2D posterior distributions of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ in both flat and non-flat universes are shown in figure 4. The 2D posterior plots for two cases (flat and non-flat) show a correlation between $M_B$ and distance duality parameters.
Figure 4. The 1D and 2D posterior distributions of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ for P2 parametrization.

(a) Flat Universe.  
(b) Non-Flat Universe.

Table 4. The best fit values of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ with 1σ confidence level for P3 parametrization.

| Parameter | Flat Universe | Non-Flat Universe |
|-----------|---------------|-------------------|
| $M_B$     | $-19.34^{+0.489}_{-0.369}$ | $-19.368^{+0.432}_{-0.335}$ |
| $\eta_0$  | $0.976^{+0.179}_{-0.197}$   | $0.993^{+0.166}_{-0.178}$   |
| $\eta_1$  | $0.003^{+0.015}_{-0.015}$   | $-0.047^{+0.036}_{-0.031}$  |
| $\Omega_{k0}$ | —            | $0.312^{+0.194}_{-0.204}$   |

P3. $\eta(z) = \eta_0 + \eta_1 \frac{z}{1+z}$. In this parametrization, we choose $\eta$ as a function of redshift which converges at high redshift. The best fit values of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ are given in table 4.

In a flat universe, the CDDR holds within 1σ confidence level. Even, for the non-flat universe, results indicate that CDDR holds true at 2σ confidence level. The best fit value of $\Omega_{k0}$ prefer a flat universe at 2σ confidence level. The best fit values of $M_B$ in both a flat universe and a non-flat universe at 1σ confidence level indicates that there is no strong correlation between $\Omega_{k0}$ and $M_B$.

The 1D and 2D posterior distributions of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ for P3 parametrization of Part I are shown in figure 5. This figure shows strong correlation between $M_B$ and $\eta_0$, and $\Omega_{k0}$ and $\eta_1$.

P4. $\eta(z) = \eta_0 + \eta_1 \ln(1+z)$. In this parametrization, we choose $\eta$ as a function of redshift which varies with redshift logarithmic. The best fit value of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ are given in table 5.

In a flat universe, the best fit value of $\eta(z)$ supports the validity of CDDR at 1σ confidence level. Similarly, the results in a non-flat universe indicate that CDDR does hold
Table 5. The best fit values of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ with 1σ confidence level for P4 parametrization.

| Parameter  | Flat Universe | Non-Flat Universe |
|------------|---------------|-------------------|
| $M_B$      | $-19.209^{+0.469}_{-0.505}$ | $-20.000^{+0.582}_{-0.402}$ |
| $\eta_0$  | $0.916^{+0.241}_{-0.178}$    | $1.32^{+0.265}_{-0.306}$   |
| $\eta_1$  | $0.003^{+0.014}_{-0.012}$    | $-0.046^{+0.041}_{-0.032}$ |
| $\Omega_{k0}$ | $-$             | $0.333^{+0.215}_{-0.243}$ |

Figure 5. The 1D and 2D posterior distributions of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ for P3 parametrization.

true at 2σ confidence level. We find the best fit value of $\Omega_{k0}$ a flat universe at 2σ confidence level. The best fit values of $M_B$ in both a flat universe and a non-flat universe indicate that there is no strong correlation between $\Omega_{k0}$ and $M_B$ at 1σ confidence level.

The 1D and 2D posterior distributions of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ for P4 parametrization are shown in figure 6. This figure shows strong correlation between $M_B$ and $\eta_0$, and $\Omega_{k0}$ and $\eta_1$.

4.2 Test of variability of SNe Type Ia absolute luminosity

In the first part, we find that the cosmic distance duality relation is not violated at 2σ confidence level in four parametrizations of $\eta(z)$ (P1, P2, P3 and P4). Therefore, in this part we consider that the CDDR is valid and to check the dependency of absolute magnitude on redshift, we vary $M_B$ with redshift in four different ways in both a flat and a non-flat universe.

M1: $M_B(z) = M_{B0}$. In the first parametrization of $M_B$ we consider it as a constant parameter. The best fit values of $M_B$ and $\Omega_{k0}$ parameters are given in table 6.
Figure 6. The 1D and 2D posterior distributions of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ for P4 parametrization.

Table 6. The best fit values of $M_{B0}$ and $\Omega_{k0}$ with 1$\sigma$ confidence level for M1 parametrization.

| Parameter | Flat Universe | Non-Flat Universe |
|-----------|---------------|-------------------|
| $M_{B0}$  | $-19.390^{+0.015}_{-0.015}$ | $-19.393^{+0.015}_{-0.015}$ |
| $\Omega_{k0}$ | — | $0.075^{+0.104}_{-0.103}$ |

Table 7. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ with 1$\sigma$ confidence level for M2 parametrization.

| Parameter | Flat Universe | Non-Flat Universe |
|-----------|---------------|-------------------|
| $M_{B0}$  | $-19.391^{+0.016}_{-0.016}$ | $-19.376^{+0.018}_{-0.019}$ |
| $M_{B1}$  | $0.005^{+0.021}_{-0.021}$ | $-0.152^{+0.089}_{-0.091}$ |
| $\Omega_{k0}$ | — | $0.823^{+0.471}_{-0.450}$ |

From table 6, we find that the best fit value of $M_B$ is the same in both flat and non-flat universes within 1$\sigma$ confidence level. Further, the best fit value of $\Omega_{k0} = 0.075^{+0.104}_{-0.103}$ suggests a flat universe at 1$\sigma$ confidence level.

The 1D and 2D posterior distributions of $M_B$ and $\Omega_{k0}$ for M1 parametrization are shown in figure 7. This figure shows that there is no correlation between $M_B$ and $\Omega_{k0}$.

M2: $M_B(z) = M_{B0} + M_{B1}z$. In this parametrization, we consider $M_B$ as a function of redshift. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ are given in table 7.

In case of a flat universe, we don’t find any signal of the redshift evolution of absolute magnitude with 1$\sigma$ confidence level. Similarly, in a non-flat universe, there is no redshift
dependence of the absolute magnitude at 2\(\sigma\) confidence level. Further, the result supports a flat universe at 2\(\sigma\) confidence level.

Figure 8 illustrates the 1D and 2D posterior distributions of \(M_{B0}, M_{B1}\) and \(\Omega_{k0}\) for M2 parametrization. In this figure, the plot for a non-flat universe indicates a mild correlation between absolute magnitude and cosmic curvature parameter.
Parameter | Flat Universe | Non-Flat Universe
--- | --- | ---
$M_{B0}$ | $-19.390^{+0.017}_{-0.017}$ | $-19.380^{+0.018}_{-0.018}$
$M_{B1}$ | $0.001^{+0.038}_{-0.040}$ | $-0.111^{+0.082}_{-0.083}$
$\Omega_{k0}$ | — | $0.343^{+0.214}_{-0.225}$

Table 8. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ with $1\sigma$ confidence level for M3 parametrization.

Figure 9. The 1D and 2D posterior distributions of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ for M3 parametrization.

**M3:** $M_B(z) = M_{B0} + M_{B1} \frac{z}{1+z}$. In this parametrization, we choose $M_B$ as a function of redshift which converges at high redshift. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ are given in table 8.

In this parametrization, the best fit value of $M_{B1}$ in both a flat and a non-flat universe suggests that the absolute magnitude does not evolve with redshift at 2$\sigma$ confidence level. Furthermore, there is no indication of deviation from flat universe at 2$\sigma$ confidence level yet it mildly supports a non flat universe.

In both flat and non-flat universes, the 1D and 2D posterior distributions of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ are illustrated in figure 9. The plot for non-flat universe indicates a correlation between absolute magnitude and cosmic curvature parameter.

**M4:** $M_B(z) = M_{B0} + M_{B1} \ln(1 + z)$. In this last parametrization, we choose $M_B$ as a logarithmic function of redshift. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ are given in table 9.

In this parametrization, for both a flat and a non-flat universe, the absolute magnitude does not evolve with redshift at 2$\sigma$ confidence level. Furthermore, the best fit value of $\Omega_{k0}$ supports a flat universe at 2$\sigma$ confidence level.
| Parameter    | Flat Universe          | Non-Flat Universe        |
|--------------|------------------------|--------------------------|
| $M_{B0}$     | $-19.391^{+0.017}_{-0.016}$ | $-19.380^{+0.017}_{-0.018}$ |
| $M_{B1}$     | $0.005^{+0.030}_{-0.029}$  | $-0.110^{+0.079}_{-0.078}$ |
| $\Omega_{k0}$| —                      | $0.442^{+0.283}_{-0.287}$ |

Table 9. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ with 1σ confidence level for M4 parametrization.

Figure 10. The 1D and 2D posterior distributions of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ for M4 parametrization.

In both flat and non-flat universes, the 1D and 2D posterior distributions of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ are illustrated in figure 10. The plot for non-flat universe indicates a correlation between absolute magnitude and cosmic curvature parameter.

5 Discussion and conclusions

In this work, we test the validity of the cosmic distance duality relation which relates the luminosity distance to angular diameter distance via redshift. We use the recent database of 1048 Type Ia supernovae named Pantheon for the luminosity distance. For the angular diameter distance, we first reconstruct the Hubble parameter, $H(z)$ database of 31 datapoints from Cosmic Chronometers in a model-independent way using the Gaussian Process. Then by adopting the Planck prior on $H_0 = 67.66 \pm 0.42$ km s$^{-1}$ Mpc$^{-1}$, the angular diameter distance is obtained corresponding to the reconstructed $H(z)$. Further, in the luminosity distance we have a free parameter, i.e. absolute magnitude of Type Ia supernovae ($M_B$), and we put constraints on it simultaneously with other cosmological parameters i.e. $\Omega_{k0}$ and $\eta(z)$. Finally, we allow $M_B$ and $\eta(z)$ to evolve with redshift to check whether these are constant quantities or evolve with cosmic time.
We divide this work into two parts as follows.

(A) Part I: test of CDDR. In this part, we do not assume that cosmic distance duality is valid and to test this, we consider a free parameter i.e. $\eta(z)$. We fit this parameter simultaneously with $M_B$ and $\Omega_{k0}$. To check the dependency of the distance duality parameter on redshift, we assume four parametrizations of $\eta(z)$. A brief summary of the results is as follows:

- in the case of a flat universe, all parametrizations suggest that the cosmic distance duality relation holds at 1$\sigma$ confidence level.
- For the non-flat universe, P1, supports the $\eta(z) = 1$ at 1$\sigma$ confidence level, P3 & P4 parametrizations at 2$\sigma$ confidence level and, P2 parametrization at 3$\sigma$ confidence level. For this case, this variation in the confidence level for the validity of $\eta(z) = 1$ directly indicates the correlation between the $\eta(z)$ and $\Omega_{k0}$. This analysis also highlights that the results are sensitive to the choice of parametrization and hence, it justifies our use of multiple parametrizations.
- Consistently, for flat and non-flat cases, all the 2D contours between $\eta_0$ and $M_B$ in Part I of the analysis indicate a very strong negative correlation. This points to the need of considering the validity of cosmic distance duality relation in Part II to independently probe the variability of the absolute magnitude $M_B$.
- In non-flat universe case, the parametrization, P1, strongly suggests a flat universe within 1$\sigma$ confidence level while parametrization P2, P3 and P4 also supports the flat universe within 2$\sigma$ confidence level. Though the best fit value of $\Omega_{k0}$ mildly prefer a non-flat universe.

(B) Part II: test of variability of SNe Ia absolute luminosity. In the second part we assume that the cosmic distance duality relation is valid, i.e. $\eta=1$. We are thus left with two parameters that we have to fit simultaneously. These parameters are $M_B$ and $\Omega_{k0}$. To test the dependency of $M_B$ on redshift, we consider four parametrizations of $M_B$. All four parametrizations have two model parameters $M_{B0}$ and, $M_{B1}$. While analysing we choose $M_{B0}$ to be constrained in the uniform prior range $U[-21,-17]$ and, $M_{B1}$ in the uniform prior range $U[-2,+2]$. In each parametrization, we discuss two cases. In the first case we consider a flat universe i.e. $\Omega_{k0} = 0$ and in the second case we choose a non-flat universe.

Our main conclusions of this part are listed below:

- in the flat universe case, the parametrization $M1$, $M2$ and, $M4$, support no evolution of absolute magnitude with redshift with 1$\sigma$ confidence level. Even for the parametrization $M3$, our analysis does not show any signal of redshift evolution of absolute magnitude at 2$\sigma$ confidence level.
- Similarly, in the non-flat universe case, for all parametrizations, no redshift evolution has been found in our analysis in absolute luminosity $M_B$ at 2$\sigma$ confidence level. Hence from our analysis in Part I and Part II, we don’t find any indication towards a statistically significant variability of absolute magnitude ($M_B$) of type Ia supernova.
- For all parametrizations of the absolute magnitude $M_B$, the best fit value of $\Omega_{k0}$ suggests a flat universe at 2$\sigma$ confidence level. However, in the parametrizations $M2$, $M3$ and
M4, the best fit value of $\Omega_{k0}$ show mild preference for a non-flat universe. Further, from the 1D and 2D contours of all four parametrizations of $M_B(z)$ for non-flat case, we observed a negative correlation between the absolute magnitude and cosmic curvature which should be analysed further.

It is important to note that in Part I, we fit the distance duality parameter $\eta(z)$ along with $M_B$ and $\Omega_{k0}$. On the other hand, in Part II we assume the validity of the cosmic distance duality relation i.e; $\eta(z) = 1$ and are thus left with only $M_B$ & $\Omega_{k0}$ as free parameters. Thus, the number of free parameters to be probed decreases from four in Part I to three in Part II. In Part I, our analysis suggests a strong correlation between the number of free parameters to be probed decreases from four in Part I to three in Part II. In Part I, our analysis suggests a strong correlation between the $\eta(z)$ and $M_B$ values, which seems to have an impact on the 1$\sigma$ error bars of $M_B$. However, in Part II, the $\eta(z)$ parameter has been excluded, hence it results in tighter 1$\sigma$ constraints on $M_B$. This correlation between $\eta(z)$ and $M_B$ has also been highlighted using BAO and Cluster observations as well [83].

(C) Impact of different prior values of Hubble parameter ($H_0$). In this analysis, we consider the Planck prior and discuss our results in two parts. In the first part we test the CDDR and in the second part we test the evolution of $M_B$. However, it is important to check whether our results are sensitive to the chosen prior of $H_0$ or not. To check this sensitivity on the chosen prior for $H_0$, we make two more prior choices of $H_0$ while reconstructing the angular diameter distance. These two priors are

- No Prior on $H_0$. The reconstructed $H_0$ value using GP is $67.64 \pm 4.79 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- Planck Prior on $H_0 = 67.66 \pm 0.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- SH0ES Prior on $H_0 = 73.20 \pm 1.30 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

To check the impact of the prior in our analysis, we repeat the whole analysis for all parametrizations of $\eta(z)$ and $M_B$ with the three priors of $H_0$ i.e. No prior, Planck prior and SH0ES prior [84]. In table 10 and table 11, we show the results only for the third parametrization of $\eta(z)$ (i.e. $P3$) and $M_B(z)$ (i.e. $M3$) just to show their behaviour with different priors. The remaining parametrizations in all three priors of $H_0$ give the same conclusions as we find here in table 10 and table 11.

Through careful analysis, we find that while reconstructing the $H(z)$ data, the choice of the prior value of $H_0$ does not make a significant impact on correlations among the parameters. We also notice that the 1$\sigma$ error bars of the $M_B$ also get affected by the uncertainties of the $H_0$ prior chosen for the analysis. For example, the uncertainty in the value of $H_0$ from CMB is 0.42 while that from SH0ES is 1.3. We find that in the case of using the CMB prior on $H_0$, the error bars of $M_B$ are much smaller as compared to those obtained by using the SH0ES prior. It seems that the uncertainty in the prior propagates in $M_B$ during the fitting analysis and affects the error bars of $M_B$. Similar to cosmic probes like Cosmic Chronometers and Supernovae Type Ia, other independent probes like strong gravitational lensing (Time Delay angular distance measure) have also seen that the $M_B$ values and error bars are impacted by the associated uncertainties in the input priors, data and parameter values [85]. Given the above mentioned observations, we can conclude that along with the consideration of the $\eta(z)$ parameter, the uncertainties in the prior value of $H_0$ also impacts the 1$\sigma$ bounds on the $M_B$.

Finally, we observe that even if we adopt different priors for $H_0$, the absolute magnitude of SNe Type Ia does not show any redshift dependence in both flat and non-flat universes. This is consistent with the expected non-variability of $M_B$ up to 3$\sigma$ confidence level and shows no deviation from the observationally expected assumptions. Though our analysis
Table 10. The best fit values of $M_B$, $\eta_0$, $\eta_1$ and $\Omega_{k0}$ with 1σ confidence level for three $H_0$ priors for P3 parametrization of Part I.

| Parameter | No Prior | Planck Prior | SH0ES Prior |
|-----------|----------|--------------|-------------|
| $M_B$     | $-19.161^{+0.694}_{-0.590}$ | $-19.342^{+0.489}_{-0.369}$ | $-19.038^{+0.514}_{-0.454}$ |
| $\eta_0$  | $0.927^{+0.265}_{-0.254}$ | $0.976^{+0.179}_{-0.197}$ | $0.910^{+0.206}_{-0.193}$ |
| $\eta_1$  | $-0.016^{+0.008}_{-0.003}$ | $0.003^{+0.015}_{-0.015}$ | $-0.105^{+0.022}_{-0.031}$ |

Case 2: Non-Flat Universe

| Parameter | No Prior | Planck Prior | SH0ES Prior |
|-----------|----------|--------------|-------------|
| $M_B$     | $-19.493^{+0.433}_{-0.439}$ | $-19.368^{+0.332}_{-0.335}$ | $-19.133^{+0.563}_{-0.484}$ |
| $\eta_0$  | $1.085^{+0.256}_{-0.196}$ | $0.993^{+0.166}_{-0.178}$ | $0.948^{+0.226}_{-0.217}$ |
| $\eta_1$  | $0.077^{+0.024}_{-0.016}$ | $-0.047^{+0.036}_{-0.031}$ | $-0.089^{+0.017}_{-0.008}$ |
| $\Omega_{k0}$ | $0.132^{+0.152}_{-0.149}$ | $0.312^{+0.194}_{-0.204}$ | $-0.026^{+0.135}_{-0.135}$ |

Table 11. The best fit values of $M_{B0}$, $M_{B1}$ and $\Omega_{k0}$ with 1σ confidence level for three $H_0$ priors for M3 parametrization of Part II.

| Parameter | No Prior | Planck Prior | SH0ES Prior |
|-----------|----------|--------------|-------------|
| $M_{B0}$  | $-19.300^{+0.150}_{-0.155}$ | $-19.390^{+0.017}_{-0.017}$ | $-19.241^{+0.039}_{-0.039}$ |
| $M_{B1}$  | $-0.130^{+0.040}_{-0.040}$ | $0.001^{+0.038}_{-0.040}$ | $-0.268^{+0.040}_{-0.040}$ |

Case 2: Non-Flat Universe

| Parameter | No Prior | Planck Prior | SH0ES Prior |
|-----------|----------|--------------|-------------|
| $M_{B0}$  | $-19.286^{+0.147}_{-0.167}$ | $-19.380^{+0.018}_{-0.018}$ | $-19.229^{+0.039}_{-0.039}$ |
| $M_{B1}$  | $-0.238^{+0.079}_{-0.078}$ | $-0.111^{+0.082}_{-0.083}$ | $-0.407^{+0.079}_{-0.080}$ |
| $\Omega_{k0}$ | $0.342^{+0.213}_{-0.213}$ | $0.343^{+0.214}_{-0.225}$ | $0.382^{+0.192}_{-0.183}$ |

supports a flat universe but we observed the mild preference of best fit value of $\Omega_{k0}$ towards a non flat universe. It should be noted that there is a strong correlation between the absolute magnitude, the distance duality parameter. In Part II, we observed mild correlation between absolute magnitude and cosmic curvature as well. Any signal of deviation in one of these parameters will impact the rest of the parameters and the underlying assumptions. We expect that one can resolve these correlations among different parameters by performing a similar analysis with a larger data set of Supernovae and other observational probes. Upcoming surveys such as the Large Synoptic Survey, the Wide Field Infrared Survey and survey from Large Synoptic Survey Telescope (LSST) may assist us in detecting any deviations from the standard supernova type Ia light-curve modeling as well as deviation from the standard cosmological model, i.e. the $\Lambda$CDM model [86–89].
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