Why the Water Bridge does not collapse

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In 2007 an interesting phenomenon was discovered: a thread of water, the so-called water bridge (WB), can hang between two glass beakers filled with deionized water if voltage is applied to them. We analyze the available explanations of the WB stability and propose a completely different one: the force that supports the WB is the surface tension of water and the role of electric field is not to allow the WB to reduce its surface energy by means of breaking into separate drops.

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After the WB (see Fig. 1) was rediscovered in 2007 [1] (it had been first time observed in 1893 [2]) it immediately captured attention and even entered some TV shows because the experiment is easy to reproduce and it can be treated as an evidence of some unique properties of water. What keeps WB stable against gravity? The first thing one can suppose is that the water in WB has properties similar to those of a polymer melt; i.e. in the electric field water molecules are arranged in quasi polymer chains that play the role of the WB load-carrying structure [3]. It has been also supposed that hydrogen bonds are the driving force of WB formation [4]. But in the computer simulation carried out in the work [4] the WB consisted of only $10^3$ molecules and it could be formed if the electric field was at least $\approx 10^3$ times stronger than the one necessary for formation of macroscopic WBs in real experiments [1, 5–9]. Some attempts have been made to reveal a specific structure of WB by means of neutron scattering and Raman scattering [5, 8], but no exhaustive explanation of the WB stability has been found on this way. An interesting feature of WB is the complicated spiral flow of water and formation of tiny bubbles inside it [6]. But it has not been proved yet that the dynamics of WB can be related to its stability. It has been even supposed that WB stability against gravity is a quantum effect [10].

However, the best explanation of a newly discovered phenomenon is the simplest one based on well known formulas. It has been stated already [7, 11] that not specific properties of water but just its high dielectric permittivity is likely to be the reason of the WB phenomenon. The convincing evidence of the statement is the "water bridge" (dielectric liquid bridge (DB)) formation of another low molecular polar dielectric liquid (DL): glycerine [9]. A good hint for the discovery with glycerine is the necessity to deionize water for forming WB. How can the high dielectric permittivity of a DL cause the DB stability? It is straightforward to assume that DB is kept stable against gravity by tension as a hanging flexible cable [11], the tension being somehow produced by electric field. Let us imagine a DL cylinder in a uniform electrostatic field (EF) $E$ parallel to its axis.

This is possible if the cylinder bases touch two infinite conducting planes to which voltage $\Delta \phi$ is applied (see Fig. 2). To simplify the explanation we have depicted in Fig. 2 gaps between the bases and the planes. The gaps are supposed to be infinitely thin, the pressure produced by the bases on the gaps is actually the pressure produced by the bases on the planes. The cylinder is the simplest model of DB. EF exerts pressure on a perpendicular to it DL interface [12, 13]. This pressure $\bar{P}$ on the cylinder bases is claimed in Ref. [7] to be the reason of the tension, which does not allow gravity to rupture DB. But this explanation of the stability is not correct, at least because EF parallel to a DL surface also produces a pressure $P$ on it [12, 13] (see Fig. 2) and only the effective DB tension $\tilde{\tau}$ [11] could be the reason of the stability. Assuming the pressure $P$ to be the tension holding DB, which is done in Ref. [9], is also a strange idea. More specifically, if $L$ and $A$ are the length and the
cross-section area of the cylinder, the work consumed for its small elongation is \( dW = \tilde{\tau} dA dL \). At the same time \( dW = -P dA dL - P dL dA \). In the first approximation the DL is incompressible, i.e. \( AdL + L dA = 0 \). Therefore, \( \tilde{\tau} = P - \tilde{P} \). A positive value of \( \tilde{\tau} \) could explain the DB stability. This explanation is proposed in Ref. [11] and seems at first to be the most consistent one. But let us calculate \( \tilde{\tau} \) carefully.

First, something strange is written in the Introduction of Ref. [11]. Namely, the normal component of the electrostatic displacement \( \mathbf{D} = \varepsilon \mathbf{E} \) (\( \varepsilon \) is the dielectric constant of the medium) is actually not discontinuous on a dielectric/dielectric interface. Besides, positive rather than negative charge is induced on the dielectric cylinders base of the "electric field arrow" (the right base of the cylinder in Fig. [2]). Moreover, it is essentially senseless to calculate the cylinder tension produced by the action of the external uniform electric field \( \mathbf{E} \) on this charge, because the field acts with a zero force on each dipole molecule of a dielectric; charges induced on a dielectric/dielectric interface are not free charges. \( P \) and \( \tilde{P} \) are obtained in Ref. [11] by using the expression for the Maxwell stress tensor (see Eqs. (15.9), (36.6), or (14.9) in Refs. [12, 13, 14] respectively) in the interior of the DL cylinder. But to obtain the electrostatic pressure on the interface of these DLs one must subtract the Maxwell stresses on both sides of the interface [13, 14]. As concerns DB, the Maxwell tension of vacuum is to be subtracted in the beginning from \( P \) and \( \tilde{P} \) and not in the end from \( \tilde{\tau} \) as has been done in Ref. [11]. Then the general expression for the pressures is the Eq. (36.11) in Ref. [13], and the pressure on air/DL interface is expressed by Eq. (15.11) in Ref. [12]:

\[
P = \frac{\varepsilon - 1}{8\pi} E^2 - \frac{\rho E^2}{8\pi} \left( \frac{\partial \varepsilon}{\partial \rho} \right)_T = \tilde{P} - \frac{\varepsilon - 1}{8\pi} E^2 \quad (1)
\]

where \( \rho \) and \( T \) are respectively the DL density and temperature, and the normal is directed outside DL.

Let us derive Eq. (1). It follows from the EF boundary conditions that 1) EF is same inside and outside the cylinder (see Fig. [2]) the surface densities of charges induced on the cylinder bases are \( \mp \sigma_{ind} \) while the densities on the corresponding adjacent areas of the conducting planes are \( \pm (\sigma_0 + \sigma_{ind}) \), and the densities on the corresponding rest parts of the planes are \( \pm \sigma_0 \). If the cylinder cross-section area is isothermally increased at constant voltage \( \Delta \varphi \) by \( dA \) while the length \( L \) is kept constant, the EF energy \( 1/(8\pi) \int \varepsilon E^2 d^2 \mathbf{r} \) is changed by \( dU^\perp \) and the voltage does the work \( dW^\perp \):

\[
dU^\perp = \frac{dW^\perp}{2} = \frac{\varepsilon - 1}{8\pi} E^2 \rho L dA - \frac{E^2}{8\pi} \rho \left( \frac{\partial \varepsilon}{\partial \rho} \right)_T L dA \quad (2)
\]

where it is taken into account that the cylinders dielectric permittivity and volume are changed due to the stretching, and the change of the permittivity and the cross-section area leads also to the change of the planes charges. It follows from the energy conservation law that \( dW^\perp = dU^\perp \), which gives the first part of Eq. (1). Let us now suppose that the planes are isothermally moved apart by \( dL \) at constant \( \Delta \varphi \), and the cylinders length \( L \) is increased respectively by \( dL \) while the cross-section area \( A \) is kept constant. For the part of the system outer to the cylinder the difference between the voltage work and the EF energy increase gives the mutual planes coulomb attraction force existing independently of the cylinder. In the cylinder part the EF energy is changed by \( dU^\parallel \) and the voltage does the work \( dW^\parallel \):

\[
dU^\parallel = \frac{dW^\parallel}{2} = -\frac{\varepsilon}{8\pi} E^2 \rho L dA - \frac{E^2}{8\pi} \rho \left( \frac{\partial \varepsilon}{\partial \rho} \right)_T L dA \quad (3)
\]

where it is taken into account that the cylinders dielectric permittivity and volume are changed, and also the EF is decreased by \( EdL/L \). In this case one should be careful to avoid a mistake: not only the pressure \( \tilde{P} \) does the work but also the coulomb attraction of the charges on the planes (see also the end of § 37 in Ref. [13]):

\[
dW^\parallel - dU^\parallel = \left( \tilde{P} - (\sigma_0 + \sigma_{ind}) \varepsilon E/2 \right) dL \quad (4)
\]

where \( \varepsilon E \) is the EF in the gaps between the cylinder bases and the planes. Eqs. (3) and (4) give the second part of Eq. (1).

So, DB pushes apart glass beakers with the pressure \( \tilde{\tau} = (\varepsilon - 1)^2 E^2/(8\pi) \) (see Eq. (1)) instead of pulling them towards each other as a suspension bridge pulls its pillars. Hence, the explanation proposed in Ref. [11] is not adequate. But the not existing tension of DB derived in Ref. [11] has an annoying feature: it is in some cases close in value to the one that really would hold DB. For this reason the theory in Ref. [11] even finds experimental "corroborations" [9]. We propose another experimental verification: if the theory is correct, a DB that is, say, two times longer (\( L \sim 3.5 - 4.5 \text{ cm} \)) is possible in a two times stronger EF. Seems to be not the case. What tension holds DB then? The EF \( \varepsilon E/2 \) in the gap, say, at the left plane (minus the plane EF) (see Fig. [2]) can be presented as the sum of the values \( E/2 \) and \( (\varepsilon - 1)/2 \). The \( E/2 \) is produced by the uniform charge density \( -\sigma_0 \) of the right plane, i.e. the attraction of the left plane by this EF is the force produced by the right plane. If \( L \ll \sqrt{A} \), EFs of the charges induced on the cylinder bases cancel out in the gap, and the field \( (\varepsilon - 1)/E/2 \) is produced only by the charge density \( -\sigma_{ind} \) on the right plane. In this case the DL "pancake" would really exert pressure on the planes. But in the case of a DB, \( L \gg \sqrt{A} \), the field of the \( -\sigma_{ind} \) circle on the right plane vanishes in the left gap, i.e. the EF \( (\varepsilon - 1)/E/2 \) is produced by the cylinder. The total coulomb interaction of the cylinder and the charge \( \sigma_0 \) on the left plane is equal to zero, because the uniform charge \( \sigma_0 \) produces a uniform EF and the total charge of the cylinder is zero. As to
the attraction between the $\sigma_{\text{ind}}$ circle of the left plane and the long cylinder, it is equal to the attraction $2\pi\sigma_{\text{ind}}^2 A$ between the circle and the opposite to it charge induced on the left cylinder base because the charge on the right base is far off. The Coulomb force subtracted in the right part of the Eq. (4) consists of the attraction $A\pi \varepsilon \mathbf{E}^2/(8\pi)$ between the $\sigma_0 + \sigma_{\text{ind}}$ circle on the left plane and the uniform charge $-\sigma_0$ on the right plane, of the attraction $A(\varepsilon - 1)\mathbf{E}^2/(8\pi)$ between the uniform charge $\sigma_0$ on the left plane and the $-\sigma_{\text{ind}}$ circle on the right plane, and of the attraction $2\pi\sigma_{\text{ind}}^2 A = A(\varepsilon - 1)\mathbf{E}^2/(8\pi)$ between the $\sigma_{\text{ind}}$ circle on the left plane and the cylinder. DB not only exerts pressure $-\tau_{\text{eff}}$ on a plane but also pulls it by EF. To which part of DB is the latter force applied? The force exerted by EF $\mathbf{E}$ on a small volume of dielectric is $(\mathbf{P} \nabla) \mathbf{E}$, where $\mathbf{P}$ is the dipole moment of the volume. The dipole moment density of the cylinder is uniform: $p = (\varepsilon - 1)\mathbf{E}/(4\pi)$. The EF $E_{\text{ind}}$ of the left plane $\sigma_{\text{ind}}$ circle is equal to $(\varepsilon - 1)\mathbf{E}/2$ on the left cylinder base, and it is equal to zero on the right base: all the EF lines go out through the cylinder lateral surface (see Fig. 2). Hence, the total force $\int (\mathbf{P} \nabla) \mathbf{E}_{\text{ind}} \, d^3r$ is equal to $2\pi\sigma_{\text{ind}}^2 A$ and it is applied to the left segment of the cylinder where the lines cross its lateral surface. The segment characteristic length is $\sqrt{A}$. The DB pressure $-\tau_{\text{eff}}$ on a plane and its attraction of it by EF cancel out.

The same is relevant to the interaction between two parts of DB. (The two parts of DB are to attract each other if the DB is in equilibrium as a suspension bridge.) A DL cylinder in a uniform EF $\mathbf{E}$ (see Fig. 3) is a stack of same and equally oriented one-dipolar-molecule-thick double electrostatic layers. Successive layers penetrate each other: the area of their overlapping is neutral, since the positive charge of one layer and the negative charge of the other are intermixed there. They are schematically distinguished in Fig. 3 by different rectangles (short and long) and by different colors of charges (white and black). The positive charge of the last layer at one base of the cylinder and the negative charge of the last layer at the other base are not neutralized. Surface densities of these charges are right equal to the charges induced on the cylinder bases: $\sigma_{\text{ind}}$ and $-\sigma_{\text{ind}}$. This means that each of the layers is a $\pm\sigma_{\text{ind}}$ double layer. The left and the right parts of a long cylinder, each consisting of an integer number of layers (see Fig. 3), interact as follows. (Dividing the cylinder by a plane into two not overlapping parts would have no sense because dipole molecules of one layer would be cut into pieces belonging to different parts). The last right layer (short and white) of the left part and the last left layer (long and black) of the right part overlap. The left cylindric part has charge density $\sigma_{\text{ind}}$ induced on its right base, while the right part has the charge density $-\sigma_{\text{ind}}$ induced on its left base. The EF produced in the right part by the left part is the EF of the charge $\sigma_{\text{ind}}$ of the left part right base because its left base is far off. Since the right part is long it is attracted by the Coulomb force $2\pi\sigma_{\text{ind}}^2 A$ to the left part like the whole cylinder is attracted to the left plane in Fig. 2. At the same time, the overlapping layers belonging to the two different parts repel each other with the same force: each of them consists of the $\sigma_{\text{ind}}$ and $-\sigma_{\text{ind}}$ charges, there are six different couples of these charges, in two of the couples charges repel each other with the force $2\pi\sigma_{\text{ind}}^2 A$, in another couple charges attract each other with the same force, in one more couple there is no parallel to the cylinder axis interaction between the charges because they overlap, the last two couples do not count as they are the two layers, i.e. rigidly bound parts of molecules. By the way, the same forces expulse from the cylinder its last layers at the bases. This is the origin of the difference between $P$ and $\bar{P}$.

So, the total tension of DB produced by EF is zero if DB is long enough, and we still have no explanation of the stability. But let us just estimate the tension of DB produced by surface tension (ST). Probably, it has not been done before because there was a strong belief in the electrostatic origin of DB tension. ST can hold DB if:

$$pgAL \approx 2l\gamma\Theta$$  \hspace{1cm} (5)$$

where $\gamma$ is the liquid ST, $\Theta$ is the small angle between an end of the DB and the horizontal (see Fig. 1), and $l$ is the DB cross-section perimeter. We have supposed that the WB cross-section is roughly an ellipse with the height about 1.5 times larger than the width, and analyzed using Eq. 5 the photos of WBs presented in Refs. [5–7, 9]. The obtained by us values of the WBs tensions caused by ST are 10-40% lower than the corresponding ones necessary for holding the WBs. The discrepancy may be caused by the low accuracy of our “experimental” investigation or of the approximation in Eq. 5 (one should take into account that the cross-section of WB is in fact not constant, especially at the ends). May be also, the reason is that the field between the beakers is in reality not uniform and therefore it slightly pulls WB up. Anyway, it is clear that ST is the main force holding DB. We have also analyzed the photos of glycerine DBs in the setup with the configuration producing a uniform EF [9]. If the cross-section of the glycerine DBs is a circle (the side views only are presented in the Ref. [9], according to the Eq. 5, the values of the tension are 10% lower and 40% higher than the ones necessary for holding the
DBs for respectively Figs. 7 left and middle in Ref. [9].

Why is a DB not possible then without electric field? Because ST plays actually an ambivalent role. On the one hand it does not allow gravity to tear DB. But on the other hand, as has been mentioned in Ref. [9], ST "wants" to break DB into separate round drops, because then the surface energy would decrease, i.e. DB is in a labile equilibrium without the outer longitudinal electric field. The latter provides the stable equilibrium: it does not allow the distortion of the DB shape to start, because the energy of electric field is the lowest if the shape is nonperturbed. The phenomenon has been extensively studied long ago [13,18]. In Ref. [10] energy change caused by small sinusoidal distortions of an infinite cylindrical jet of DL (an infinite DB in zero gravity, in other words) have been analyzed. It has been proved that the longitudinal EF $E_{cr}$ necessary for providing the stable equilibrium is $\sim \sqrt{\varepsilon}$ and it is the lower the larger is $A$ or $\varepsilon$. In Ref. [17] the equilibrium shape of a bridge of one DL surrounded by another DL of the same density has been studied. Existence of an equilibrium shape very close to the cylindrical one was used as the instability criterion, and same results have been obtained: $E_{cr}$ is proportional to the square root of the ST between the DLs and it is the lower the lower is $L/\sqrt{A}$ or the larger is the ratio of the DLs dielectric constants. Now we can explain why a long DB is hard to make: it must be thin to stay the gravity (see Eq. [3]) but a thinner DB needs a much stronger field to keep the shape. The model of Ref. [17] has been generalized in Ref. [18] for the case when the DB is vertical and there is a small difference in the two liquids densities. It has been shown that even the small axial gravity is an important factor destabilizing the equilibrium between the effects of the field and ST. This explains the lower stability of vertical WBs as compared to the horizontal ones [8].

Our speculations describe the basic role of ST and electric field in providing DB stability. They do not explain why the horizontal WB cross-section increases with the increase of the voltage between the beakers [8] and why the horizontal glycerine DB changes its shape [9] if the external uniform EF is altered. In the both cases the reason may be that EF, even a uniform one, affects the shape of DB if the shape is asymmetric (it is, actually), and if there are some free charges in the DL. In the first case nonuniformity of the EF between the beakers also may play a role.

Latterly, let us propose two small hints for experiment. 1) It has been reported that WB is possible in an oscillating electric field [8]. At the same time it is known that the water must be deionized, evidently because free charges relocate, thus screening the field. But if the field oscillates frequently enough the ions do not have time to relocate [17]. May be when using a high frequency oscillating voltage one does not need to deionize the water. It would be also possible then to measure the tension of DB and not the Coulomb attraction of electrodes. 2) It is interesting to make a DB of a liquid having dielectric permittivity higher than water has. May be one can obtain then a longer DB. Dielectric constant of N-Methylformamid (NMF) is around 200 [19,20]. The challenge is to make sure that NMF is really free of ion-producing contaminations: of water first of all. Otherwise the conductivity is too high [20].

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