Research on Buckling State of Prestressed Fiber-Strengthened Steel Pipes

Ruheng Wang¹ and Kunchang Lan²
Southwest University of Science and Technology, Mianyang 621010, China
E-mail: 475650168@qq.com

Abstract. The main restorative methods of damaged oil and gas pipelines include welding reinforcement, fixture reinforcement and fiber material reinforcement. Owing to the severe corrosion problems of pipes in practical use, the research on renovation and consolidation techniques of damaged pipes gains extensive attention by experts and scholars both at home and abroad. The analysis of mechanical behaviors of reinforced pressure pipelines and further studies focusing on “the critical buckling” and intensity of pressure pipeline failure are conducted in this paper, providing theoretical basis to restressed fiber-strengthened steel pipes. Deformation coordination equations and buckling control equations of steel pipes under the effect of prestress is deduced by using Rayleigh Ritz method, which is an approximation method based on potential energy stationary value theory and minimum potential energy principle. According to the deformation of prestressed steel pipes, the deflection differential equation of prestressed steel pipes is established, and the critical value of buckling under prestress is obtained.

1. Introduction
In the application of prestressed reinforcement repair technology, excessive prestressing force may result in buckling failure of steel pipes when the force reaches to the limitation of external pressure of buckling. Partial buckling pressure of oil and gas pipelines under external pressure is defined as instability pressure. Whereas the way to determine the instability of oil and gas pipelines is a critical point. Thus, theoretical and experimental analysis were conducted by scholars to form the evaluation standard of the pipeline stability, and several criterions were obtained as fellows: Initial buckling criterion, fast growth strain criterion, maximum moment strain criterion and cross section deformation criterion.

(1) Initial buckling criterion
An external pressure test, in which full size specimens were adapted to measure ultimate strain along the circumferential direction of the outer side of the pipeline, was carried out by Bouwkamp [1]. However, the way to determine the initial buckling was not explicitly put forward in this study. Instead, the initial buckling was described through experimental phenomenon.

(2) Fast growth strain criterion
Based on the research of Bouwkamp, a simulation analysis towards pipes under external pressure was conducted by Corona [2]. A fast growth strain criterion, which is regarded as a way to assess whether pipelines are buckling or not, is put forward. Lara regards that: The rapid growth of pipeline strain in the radial direction is detected and pipes have not been into failure, yet pipelines have entered into the yield stage.

(3) Maximum moment strain criterion
Buckling can be observed when pipes bear bending moment. The deformation caused by buckling develops rapidly when the external bending moment reaches the maximum value. According to the
research of Gresnigt [3], the maximum bending moment was considered as a key factor to judge whether pipelines enter into buckling stage or not. The method is suitable for the circumstance that pipes under bending moment or concentrated loads. Dama [4] conducted a test on pipelines which were made of aluminum material, the results shows that: the strain of pipes under buckling is more suitable for the control of deformation.

(4) Cross section deformation criterion

Elliptical phenomenon can be observed when pipeline bears inhomogeneous external pressure. Generally, due to the weight of soil, construction technology and inhomogeneous deformation caused by other effects of the external loads, large deformation may result in buckling.

In the researches of pipeline buckling, Fairnaim [5] conducted an experiment towards the buckling of thin-wall steel tube under even external pressure, the experiment showed that diameter to thickness ratio plays a main role in “the critical buckling”; The Small deformation theory of pipeline was adapted in the research of Mahan [6], cross-section ring of pipes were intercepted to analyze the stability problems of pipes under external pressure, and a buckling formula of the pipeline was obtained. The principle of conservation of energy and the mechanics of plates and shells were adapted by Li Zhaochao [7] to analyze “the critical buckling” formula of pipes under external pressure. Jiang Kebin [8] et al. conducted a research on prestress-strengthened steel structure, a calculation formula of prestressing steel structure is put forward, and the simulation method is verified. The results show that the application of prestressed technology in the reinforcement of steel structure is valid. Yang Yongxin [9] et al. adapted the theory of mechanics of elasticity to study the circumferential stress distribution of the pipe under prestressing. Li Xiaqin [10] et al. adapted the theory of mechanics of elasticity and three-dimensional elastic anisotropy to analyze pipes under external pressure, a stress calculation formula of pipeline is obtained. Jia Bin [11] et al. studied the bonding principle between the fiber material and pipes, a pressure formula of contact surface was obtained.

2. Premise and Hypothesis

(1) In shell theory, the Gauss curvature of the neutral plane of shell and plate is zero.

(2) The displacement of axial and circumferential directions in the neutral plane is minute. Thus, the influence of displacement of axial and circumferential directions towards curvature and torsion can be ignored.

(3) The influence of shear stress in the neutral plane can be ignored, and the thickness of the pipeline is equal thickness.

3. Geometric Deformation Equation

If the large deflection is caused by external force, every point in the neutral layer will displace. The displacement components are $u$, $v$, $\omega$. Corresponding strain are $\varepsilon_x$, $\varepsilon_y$, $\gamma_{yx}$, and torsion ratio are $\chi_x$, $\chi_y$, $\chi_{xy}$. The variable of curvature is caused by deflection $\omega$.

$$
\chi_y = -\frac{\partial^2 \omega}{\partial x^2}, \chi_x = -\frac{\partial^2 \omega}{\partial y^2}, \chi_{xy} = -\frac{\partial^2 \omega}{\partial x \partial y}
$$

The neutral plane strain is related to $u$, $v$, $\omega$, and the neutral plane strain can be divided into two parts. The first part is caused by displacement $u$, $v$, and the second part is caused by the deflection in the radial direction, and the in-plane deformation is shown in figures 1 and figures 2.

$$
\varepsilon_{x1} = \frac{\partial u}{\partial x}, \varepsilon_{y1} = \frac{\partial v}{\partial y}, \gamma_{xy1} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
$$
According to neutral plane strain equation and the Saint Venant theorem, deformation coordination equation is obtained.

\[
\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} - \frac{\partial^2\gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2\omega}{\partial x \partial y}\right)^2 - \frac{\partial^2\omega}{\partial x^2} \frac{\partial^2\omega}{\partial y^2}
\]

(3)

4. Physical Equation
According to Hooke's law and the physical equation of elastic shell:
\[ N_x = \frac{E h}{1 - \nu^2} (\varepsilon_x + \nu \varepsilon_y) \quad M_x = -D \left( \frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2} \right) \quad \varepsilon_x = \frac{1}{E h} (N_x - \nu N_y) \]

\[ N_y = \frac{E h}{1 - \nu^2} (\varepsilon_y + \nu \varepsilon_x) \quad M_y = -D \left( \frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2} \right) \quad \varepsilon_y = \frac{1}{E h} (N_y - \nu N_x) \]

\[ N_{xy} = \frac{E h}{2(1 + \nu)} \gamma_{xy} \quad M_{xy} = -D (1 - \nu) \frac{\partial^2 \omega}{\partial x \partial y} \quad \gamma_{xy} = \gamma_{yx} = \frac{2(1 + \nu)}{E h} (N_y - \nu N_x) \]

where \( h \) is shell thickness, and \( D \) is the rigidity of the pipe section.

5. Equilibrium Equation
Since the shell structure belongs to the thin shell, the shell thickness is much smaller than diameter, the film force can’t be ignored. The film force is partly caused by surface load, the other part is relative to the radial displacement \( \omega \) in radial direction. Therefore, when buckling of pipeline under external loads is under research, the theory of large deflection is adapted. And pipeline structure belongs to shell structure, thus pipes can be considered as elastic cylindrical shell. The shell thickness is \( h \) and the radius is \( R \). The corresponding coordinate establishment and the internal force element diagram of the cylindrical shell are described in Figure 3 and figure 4.

**Figure 3.** The reference coordinates

**Figure 4.** The internal forces in cylindrical shell
The differential equation with large deflection of the upper shell is obtained:

\[
\frac{D}{h} \Delta^2 \Delta^2 \omega = L(\omega, \Phi) + \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} + \frac{q}{h}
\]

(5)

\[
\frac{1}{E} \Delta^2 \Delta^2 \Phi = \left(\frac{\partial^2 \omega}{\partial x \partial y}\right)^2 - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial \omega}{\partial y}^2 + \frac{1}{R} \frac{\partial \omega}{\partial x^2}
\]

(6)

The buckling model of the shell is branch point instability model. Tsien Hsueshen and Carmen [12] put forward the nonlinear theory of large deflection for the imperfect structure, pointing out that there is a far lower process of buckling than the limit load of linear buckling process in the implementation process of linear buckling of the structure, the postbuckling equilibrium is very close to the experimental phenomenon observed, and it can explain why test value is 1/5 to 1/2 of the value of the linear theory. Structure appear buckling when buckling state of shell jump from pre-buckling to postbuckling, the critical load obtained from linear buckling theory is called upper critical load, which is denoted as \( P_{cl} \), the critical load obtained from nonlinear buckling jump theory is called lower critical load, which is denoted as \( P_{cr} \), they are shown in figure 5.

![Figure 5. The buckling critical load](image)

### 6. Buckling State of Shell

#### 6.1. Pre-Buckling State of Shell Instability

Shell does not have any moment before buckling, according to the static criterion, when the shell is close to buckling state, the stress and displacement are \( \sigma_{xII}, \sigma_{yII}, \tau_{xyII}; u_{II}, v_{II}, \omega_{II} \). Therefore, the equilibrium state is changed after disturbance:

\[
\sigma_x = \sigma_{x,I} + \sigma_{x,II}, \sigma_y = \sigma_{y,I} + \sigma_{y,II}, \tau_{xy} = \tau_{xy,I} + \tau_{xy,II}
\]

(7)

the stress function increment of the additional state II is \( \phi(x, y) \),and

\[
\sigma_{x,II} = \frac{\partial^2 \phi_{II}}{\partial y^2}, \sigma_{y,II} = \frac{\partial^2 \phi_{II}}{\partial x^2}, \tau_{xy,II} = \frac{\partial^2 \phi_{II}}{\partial x \partial y}
\]

(8)

buckling equations of cylindrical shells is obtained:

\[
\frac{D}{h} \nabla^4 \omega + \frac{E}{R^4} \frac{\partial^4 \omega}{\partial x^4} + p \nabla \left( \frac{\partial^2 \omega}{\partial x^2} \right) + p \nabla \left( \frac{\partial \omega}{\partial x} \right) = 0
\]

(9)
shell under the external load and boundary condition is simply supported, differential equation of
deflection which Simple supported boundary conditions of shells can be satisfied is obtained.

\[ \omega = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{R} \]  \hspace{1cm} (10)

where \( m \) is the displacement wave number in the axial direction, \( n \) is the displacement wave number in
the circumferential direction, \( L \) is the calculation distance of axial length in pipeline, \( R \) is the
the calculation radius of the pipeline.

Input the differential equation of deflection into the buckling equation of the pipe under external
pressure.

\[ \frac{D}{h} \left( \frac{m^2 \pi^2}{L^2} + \frac{n^2 \pi^2}{R^2} \right) + \frac{E}{R^2} \cdot \frac{m^2 \pi^2}{L^2} - \frac{qR}{h} \left( \frac{m^2 \pi^2}{L^2} + \frac{n^2 \pi^2}{R^2} \right)^2 \cdot \frac{n^2}{R^2} = 0 \]  \hspace{1cm} (11)

\[ q = \bar{q} \cdot \frac{R^2}{h^2} = \frac{1}{12(1-\nu^2)} (1+\theta^2)^4 \eta + \frac{\theta^4}{(1+\theta^2)^4 \eta} \]  \hspace{1cm} (12)

where \( h \) is the thickness of pipeline and \( \theta = \frac{mnR}{nL}, \eta = \frac{n^2 h}{R} \).

6.2. Post buckling State of Shell Instability

The nonlinear large deflection buckling theory is shown in Figure 6. Under the external pressure, “the
critical buckling” of perfect structure is obviously higher than that of imperfect structure, and the
nonlinear theory of large deflection can explain the instability phenomenon in shell with original
defect. The critical load of a cylindrical shell with initial imperfections is much lower than the critical
load obtained by linear theory in the case of instability. The nonlinear large deflection theory can be
used to describe the deformation of plates and shells under external loads, but it is difficult to solve the
exact solutions. Ritz method is a kind of approximate solution, postbuckling can be analysed base on
Ritz method, the key to solve the buckling of pipeline is assume a suitable deflection function
according to deflection when pipeline is under external pressure, the appropriate deflection function
can simplify, Simplification can provide convenience to calculation, and the results obtained is
more precise.

\[ \text{Figure 6. The instability diagram of the perfect structure and imperfect structure} \]

Nonlinear large deflection buckling theory is a theory considering the geometrical nonlinearity of
plate and shell structures and under external loads. Through the test of pipe under external pressure,
the buckling deflection function of pipe contains the sine wave and the deep diamond wave, the
former wave express linear term and the later wave express nonlinear term.

It can be assumed that the buckling deflection function is equation (13).
\[ \omega = f_0 + f_1 \sin \frac{m\pi x}{L} \sin \frac{ny}{R} + f_2 \sin^2 \frac{m\pi x}{L} \sin^2 \frac{ny}{R} \]  

(13)

where the first item represents the uniform transverse waveform, the second term represents the linear transverse waveform, and the third term represents the transverse waveform of the diamond.

The differential equations and strain coordination equations for large deflection bending of shells is obtained.

\[ \frac{D}{h} \Delta^2 \Delta \omega = L(\omega, \Phi) + \frac{1}{R} \alpha \beta \frac{\partial^2 \Phi}{\partial x \partial y} + \nu \]  

(14)

\[ \frac{1}{E} \Delta^2 \Delta \Phi = \left( \frac{\partial^2 \omega}{\partial x \partial y} \right)^2 - \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \frac{1}{R} \frac{\partial \omega}{\partial x} \frac{\partial \omega}{\partial y} \]  

(15)

input the buckling deflection equation into the deformation coordination equation.

\[ \frac{1}{E} \nabla^2 \nabla \Phi = \frac{\alpha^2 \beta^2}{2} \left( f_1^2 + f_2^2 \right) \left( \cos 2\alpha x + \cos 2\beta y \right) + \alpha^2 \beta^2 \left( \frac{1}{R} - \beta^2 f_2 \right) \left( \cos 2\alpha x + \cos 2\beta y \right) \]  

\[ - \frac{\alpha^2 \beta^2}{2} f_2^2 \left( \cos 4\alpha x + \cos 4\beta y \right) + \frac{\alpha^2 \beta^2}{2} f_2^2 \left( \cos 4\alpha x \cos 2\beta y + \cos 2\alpha x \cos 4\beta y \right) \]  

\[ + \alpha^2 f_1 \left( \frac{1}{R} - \beta^2 f_2 \right) \sin \alpha x \sin \beta y - \frac{\alpha^2}{R} f_2 \cos 2\alpha x \]  

\[ + \frac{3\alpha^2 \beta^2}{2} f_1 f_2 \left( \sin 3\alpha x \sin \beta y + \sin \alpha x \sin \beta y \right) \]  

(16)

where \( \alpha = \frac{m\pi}{L} ; \beta = \frac{n}{R} \). The homogeneous equation of the linear equation is related to the external load. The specific solution is same shape as left end of equation, and only the coefficients of the equation are equal. Thus the stress function is solved as equation (17).

\[ \Phi = \gamma_1 \cos \alpha x + \gamma_2 \cos \beta y + \gamma_3 \sin \alpha x \sin \beta y + \gamma_4 \alpha x \sin \beta y - \frac{N x^2}{E 2} \]  

(17)

where \( g \) is a function related to \( \alpha, \beta \). \( P \) is stress in the axial direction and \( N \) is stress in radial direction of shell structure, axial compressive stress can be obtained by the load, the stress of the radial direction depends on radial deformation, condition for closure of shell can deduce the periodicity of circumferential displacement, so, circumferential displacement won’t get any increment while increace of \( y \) coordinate value. So the expression of the circular displacement is equation (18).

\[ \int_0^{2\pi R} \frac{\partial \nu}{\partial y} dy = 0 \]  

(18)

according to the strain equation and physical equation.

\[ \frac{\partial \nu}{\partial y} = \frac{1}{E} \left( \frac{\partial^2 \Phi}{\partial x^2} - \nu \frac{\partial^2 \Phi}{\partial y^2} \right) - \frac{1}{2} \left( \frac{\partial \omega}{\partial y} \right)^2 + \frac{\omega}{R} \]  

(19)

\[ \frac{N}{E} = -\frac{1}{8} f_1^2 \beta^2 + \frac{1}{2R} f_2 + \frac{1}{R} f_0 \]  

(20)

by Gauss Codazzi formula[13] and deformation coordination equation, formula of bending strain energy \( U_b \) and film strain energy \( U_m \) is obtained.
\[ U_b = \frac{1}{2} \int \frac{d}{d\lambda} \left( \frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \eta^2} \right)^2 - 2(1 - \mu) \left( \frac{\partial \Phi}{\partial \xi} \frac{\partial \Phi}{\partial \eta} - \frac{\partial \omega}{\partial \xi \eta} \right)^2 \right) d\xi d\eta \]  
\[ U_m = \int \frac{h}{2E} \left\{ (\nabla^2 \Phi)^2 - 2(1 + \nu) \left[ \frac{\partial^2 \Phi}{\partial \xi^2} \frac{\partial \Phi}{\partial \eta} - \frac{\partial^2 \Phi}{\partial \xi \eta} \right] - \frac{\partial \Phi}{\partial \xi} \right\} d\xi d\eta \]

input the buckling deflection function and the stress function into the bending strain energy and the film strain energy.

\[ U_b = \frac{D}{2 \pi RL} \frac{1}{2} \beta \left( 1 + \theta \right) \sum f_i^2 + 4 \beta \theta f_i \]

\[ U_m = \frac{E}{2} \left( \frac{\beta^4}{64} (1 + \theta^4) f_1^4 + \frac{1}{2} \beta \left[ \frac{\theta^4}{(1 + \theta^2)^2} + \frac{\theta^4}{(1 + 9 \theta)^2} \right] f_2^2 \right) - \frac{\beta^2}{8R} (1 + \theta^2) f_1^2 - \frac{1}{4R} f_2^2 \]

\[ + 2 \frac{1}{R} \left( f_0 + f_2 \right) - \frac{1}{2R} \beta^2 f_1^2 \left( f_0 + f_2 \right) + 4 \beta \xi f_2 \]

the work done by an external force depends on the displacement or strain in the axial direction of both ends of the cylindrical shell. The work done by the external force can be obtained from the strain equation and the physical equation.

\[ W = - \frac{ph}{E} \int \int \frac{1}{2} \left( \frac{\partial^2 \Phi}{\partial \xi^2} - 2 \frac{\partial \omega}{\partial \xi} \right)^2 \right) d\xi d\eta = 2q\pi RL \left( f_0 + \frac{f_2^2}{2} \right) \]

because the total potential energy \( \Pi \) is the difference work between done by the film strain energy and the bending strain energy and done by the external force, the expression of the total potential energy is expressed as equation (26).

\[ \Pi = U_b + U_m - W \]  

system potential energy functional variation.

\[ \frac{\partial \Pi}{\partial f_0} \delta f_0 + \frac{\partial \Pi}{\partial f_1} \delta f_1 + \frac{\partial \Pi}{\partial f_2} \delta f_2 \]

if \( \frac{\partial \Pi}{\partial f_0} = 0 \), the potential energy of the system is minimum.

\[ \frac{\partial \Pi}{\partial f_0} = 0, \quad \frac{\partial \Pi}{\partial f_1}, \quad \frac{\partial \Pi}{\partial f_2} \]

\[ - \frac{1}{8} \theta^2 f_2^2 + \frac{1}{2R} f_2 + \frac{1}{R} f_0 - \frac{qR}{Eh} = 0 \]

\( N = \frac{qR}{h} \) can be obtained from equations (28) and (29).

Dimensionless expression of total potential energy is obtained.

\[ \Pi = \frac{1}{2} C_1 \zeta^4 + C_2 \zeta^2 \xi^2 - C_3 \zeta^2 \xi^2 - C_4 \xi^2 + C_5 \xi^2 + C_6 \xi^2 \]

where \( H \) is geometric scale parameter of pipe, \( H = \frac{L^2}{h}, \bar{q} = \frac{qR^2}{Eh^2} \)
\[ C_1 = \frac{\pi^4 (1 + \theta^4)}{64 \theta^4 H^2}, \quad C_2 = \frac{\pi^2}{4H^2} \left[ \frac{1}{(1 + \theta^2)^2} + \frac{1}{(1 + 9 \theta^2)^2} \right], \quad C_3 = \frac{\pi^2}{16 \theta^2 H} \left[ 1 + \frac{8 \theta^4}{(1 + \theta^2)^2} \right] \]
\[ C_4 = \frac{\theta^4}{4 (1 + \theta^2)^2} + \frac{\pi^2 (1 + \theta^2)}{48 \theta^3 H}, \quad C_5 = \frac{1}{8H} + \frac{\pi^4}{6(1 - \nu^2)H^2}, \quad C_6 = \frac{\pi^2}{4 \theta^2 H} \]

(31)

partial derivative of \( f_1 \) and \( f_2 \) for the total potential energy \( \Pi \).

\[ C_6 q = C_1 \xi^2 + C_2 \xi^2 - C_3 \xi + C_4 \]

(32)

\[ C_2 \xi^2 \xi - \frac{1}{2} C_3 \xi^2 + C_5 \xi = 0 \]

(33)

critical buckling expression of pipeline under external pressure is obtained.

\[ \ddot{q} = \frac{2C_1 C_5 \xi}{C_6 (C_3 \xi - \xi \xi)} + \frac{C_5^2 - C_5}{C_6} + C_4 \]

(34)

For a known geometric scale parameter of pipe, set different wavelength ratios \( \theta \), curve of can be obtained, make the envelopes line of these curves, the lowest point of the envelope line is the critical value of the pipeline to bear the external pressure.

7. Test

Utilizing anchorage to produce prestressing force in fiber, the measuring points of strain gauge and the test layout of the anchor are shown as figure 7.

Figure 7. The layout of strain gauge
8. Conclusion

Figure 8. The torque-strain curve
Figure 9. The torque strain curve of fiber ring direction and axial
According to figure 8, two Conclusions can be draw.
1. The strain law of the steel tube is the same with fiber. In the circumferential direction, the strain value is the largest near the anchor, and in the axial direction, the strain value is the largest at the center of the load symmetry.

2. The strain of fiber and steel pipe increase along with the increase of torque, torque and strain is nearly linear. The strain of fiber is significantly greater than that of steel. That's because there is slip between fiber and steel tube, so the test results shows that the strain of fiber is obviously higher than steel tube.

According to figure 9 and figure 10, two Conclusions can be draw.
3. Under the uniform external pressure, the fiber strain reaches the maximum at the symmetry of the load center.

4. The strain of steel tube and fiber increase along with the increase of torque, and there are prestress losses in the circumferential direction. Strain value decreases along with the increase of distance away from the anchor, which is exerted torque. Under prestressing, the strain of steel and fiber reach to maximum value at symmetrical center of load, and strain value decrease along with the increase of distance away from the symmetrical center of load.

9. References
[1] Bouwkamp J G. Large diameter pipe under combined loading [J]. Journal of the Trasportation Engineering Division.1973, 99(3):521-536.
[2] Corona E, kyriakides S. On the collapse of inelastic tubes under combined bending and pressure [J]. International Journal of Solides and structures.2006, 24(5):505-535.
[3] Gresnigt VF. Collapse of VOE manufactured steel pipes. International offshore andploar Engineering conference [J]. ISOPE, seatle, USA, 2000:170-181.
[4] Dama E. Failure of Locally Buckled Pipelines [J]. Journal of Pressure Vessel Technology. 2007, 129(2):272-279.
[5] W. Fairbrain, on the resistaince of tubes to collapse, philophical transactions of the royal society of London, 2012, 148:389-413.
[6] F. A. Mahan, D. H. Mahan. Hydraulic motors. Tr. from the French Cours de mecanique appliquée par M. Bresse,[M]. Scholarly Publishing Office, University of Michigan Library, 1866.
[7] Li Zhaochao. Study on buckling stability of underground pipeline [D] Zhejiang University, 2012.
[8] Jiang Kebin. Analysis of improving bearing capacity of steel structure by prestressed carbon fiber technology [J]. Journal of PLA University of Science and Technology, 2003, 4 (4): 58-60.
[9] Yang Yongxin, Peng Fuming, Yue Qing Rui. Stress analysis of composite Reinforcement of fiber reinforced metal pipe [J]. Port Engineering Technology, 2005 (4): 17-19.
[10] Li Xiaoqin. Stress analysis of filament wound composite pipes [J]. China Chemical Preparation, 2008.
[11] Jia bin. Analysis of influence factors on reinforcing effect of steel pipe strengthened with fiber cloth [J]. Industrial Construction, 2015,45 (3): 174-179.
[12] Karmen, Th. Von, and Tsien, H.S., The bucking of thin cylindrical shells under axial compression, J. Of Aero. Scie., 8, 303 (1941).
[13] He Fubao, Shen Yapeng. Theory of plates and shells [M]. Xi'an Jiaotong University, 1993.