Abstract. Modern CDCL SAT solvers learn clauses rapidly, and an important heuristic is the clause deletion scheme. Most current solvers have two (or more) stores of clauses. One has “valuable” clauses which are never deleted. Most learned clauses are added to the other, with an aggressive deletion strategy to restrict its size. Recent solvers in the MapleSAT family, have comparatively complex deletion scheme, and perform well. Many solvers store only binary clauses permanently, but MapleL-CMDistChronoBT stores clauses with small LBD permanently. We report an experimental study of the permanent clause store in MapleL-CMDistChronoBT. We observe that this store can get quite large, but several methods for limiting its size reduced performance. We also show that alternate size and LBD based criteria improve performance, while still having large permanent stores. In particular, saving clauses up to size 8, and adding small numbers of high-centrality clauses, both improved performance, with the best improvement using both methods.

Keywords: Learned Clause Database Management · Permanent Clauses.

1 Introduction

High-performance CDCL SAT solvers often learn millions of clauses during a run. Retaining all of these clauses permanently seems impractical, so most must be deleted. The scheme for deleting learned clauses is an important CDCL solver heuristic, usually based on measures of “clause quality” such as size or literal block distance (LBD) to estimate future utility. Most solvers store some “high quality” clauses permanently and review the others periodically to delete some of lower quality. We will call the set of clauses that are never deleted PERM, and the set which will be reviewed for deletion TEMP. (Some PERM clauses are deleted because they are satisfied by a learned unit clause, but these could not be used again anyway.) PERM and TEMP are often, but not always, stored in distinct data structures.

We have performed many experiments on decision and deletion heuristics in solvers of the MapleSAT family, which have performed very well in recent SAT solver competitions. This experience convinced us that the clause deletion scheme is important to their performance, but the complexity of the scheme makes it hard to understand why. Recent MapleSAT-based solvers
have three distinct stores of clauses, use at least two dynamic clause quality measures (LBD and activity), and a number of heuristic rules to move clauses between the stores. In contrast, it is possible to build quite good solvers with much simpler schemes [13]. Many solvers place only binary clauses in PERM and use a simple measure to delete from TEMP. For example, Glucose and Cadical use LBD with ties broken by size [3,5]. (Cadical also retains some TEMP clauses based on two bits of recent use information.)

Here we report an empirical study of PERM in MapleLCMDistChronoBT. We show that:

- Usually PERM is of moderate size, but sometimes it grows very large;
- At least some ways of restricting PERM size for formulas where it gets large did not help.
- Alternate LBD and size based criteria for PERM can improve performance (with similar-sized PERM). In particular (perhaps surprisingly) sending all clauses of size up to 8 to PERM was very effective.
- Adding a few very high-centrality (HC) clauses to PERM improved performance on formulas for which centrality computation is fast. Our results indicate that very high-centrality clauses are valuable even if they are long.
- The best improvement in our experiments comes from a combination of size \( \leq 8 \) and adding HC clauses to PERM. This version solved 197 instances, 13 more than the 184 that MapleLCMDistChronoBT solved.
- High-centrality clauses are used more often in conflict analysis.
- There are small clauses that are easy to derive, and which help when added to TEMP, but hurt when added to PERM.

The base solver in all reported experiments is MapleLCMDistChronoBT, the first-place solver in the 2018 SAT solver competition [17,10]. We denote simply “maple” in keys of some figures, to keep names short. The MapleLCMDistChronoBT deletion scheme was originally adopted from COMiniSatPS [14,19]. Our data are for 400 formulas from the Main Track of the 2020 competition with a 5000 second timeout [4]. Computations were performed on the Cedar compute cluster [7] operated by Compute Canada [8]. The cluster consists of 32-core, 64 GB nodes with Intel “Broadwell” CPUs running at 2.1Ghz. We use clause quality measures Size, LBD and Centrality. Size is the number of literals in the clause. LBD [3] is the number of decision levels of literals in the clause at the time it is computed (at which time all literals must be assigned). Clause Centrality [12] is the average betweenness centrality of its variables in the primal graph of the formula [11].

2 Size and Value of PERM in MapleLCMDistChronoBT

We begin by considering the make-up and value of PERM in MapleLCMDistChronoBT. The clause database is partitioned into three sets called Core, Tier2 and Local, stored in distinct databases. Core stores the PERM clauses, and Tier2 and Local comprise TEMP. A new learned clause is sent to Core if LBD \( \leq 3 \), to
Local if $\text{LBD} > 6$, and to Tier2 otherwise. If after the first 100,000 conflicts $|\text{Core}| < 100$, the core threshold is changed from $\leq 3$ to $\leq 5$. The LBD of a clause is re-computed each time it is used in conflict analysis. When the LBD of a clause is reduced, it is moved from Local to Tier2 or Core, or from Tier2 to Core, in accordance with the relevant thresholds. Every 10,000 conflicts, any clause in Tier2 that has not been used for 30,000 conflicts is moved to Local. Deletion is carried out only on clauses in Local, using a “Delete Half” scheme based on a VSIDS-like clause activity measure [18,19].

MapleLCMDistChronoBT solved 184 instances from the benchmark set with the 5000 second time-out. Figure 1 is a histogram showing the number of clauses in PERM at the end of the run on these formulas. For about half of the formulas the final size of PERM is 20,000 or less, which is moderate compared to the average TEMP size of about 30,000. However for nearly a quarter of the instances (22%) the final PERM size is more than 150,000.

We report two experiments to evaluate the usefulness of PERM in MapleLCMDistChronoBT. The first compares the default solver with two modified versions, one with PERM empty, and one with only binary clauses sent to PERM. Figure 2 shows that both modifications reduce performance substantially.
ventional wisdom suggests 150,000 is very large for a learned clause set, and we hypothesized it might be better to limit its size. In the second experiment, we use various schemes to restrict the size of PERM, with the goal of keeping it less than 100,000. We applied these schemes to the formulas for which PERM grew to more than 150,000 clauses. As Figure 3 shows, even for these formulas most of the methods were damaging to performance, and even the best do not seem beneficial. The schemes are as follows:

- Maple-PERM-LBD2: Require $\text{LBD} \leq 2$.
- Maple-PERMset-max100K: If size of PERM reaches 100,000, send no more clauses to PERM and send all new clauses to TEMP.
- Maple-PERMset-DelHalf-Act-max100K: If size of PERM reaches 100,000, invoke a “delete-half” deletion scheme on PERM, based on clause activity.
- Maple-PERMset-DelHalf-LBD-Save-X-max100K: If size of PERM reaches 100,000, invoke a “delete-half” deletion scheme on PERM, based on LBD (with ties broken by clause age), but never deleting clauses with property $X$, for $X$ in \{size $\leq 2$, size $\leq 3$, LBD $\leq 2$\}.

3 Varying Size and LBD Criteria for PERM

The MapleLCMDistChronoBT criterion for putting in PERM is LBD $\leq 3$. (Sometimes it is changed to LBD $\leq 5$ during the run but for few formulas.) We report an experiment in which we vary the criteria over two ranges: Size $\leq k$, for $k \in \{2, \ldots, 15\}$ and LBD $\leq k$, for $k \in \{2, \ldots, 8\}$. Figure 4 shows the effect on the fraction of learned clauses sent to PERM, and on the final size of PERM. Figure 5 shows the effect of these variations on Par-2 performance scores \[1\]. The best Par-2 score is with Size $\leq 8$. As Figure 4 shows, the number of clauses saved to PERM with Size $\leq 8$ falls between that for LBD $\leq 3$ and LBD $\leq 4$, and is only slightly larger than for the default version. Figure 6 compares the performance of the Size $\leq 8$ version with the default LBD $\leq 3$ in a cactus plot.

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**Fig. 3**: Effect of limiting size of PERM on “LC” formulas.
Fig. 4: Fraction of learned clauses sent to PERM (upper), and final size of PERM (lower) with varied PERM criteria.

Fig. 5: Effect of PERM criterion on Par-2 Scores

Fig. 6: Performance of solvers with small clauses in Core
4 Adding High-Centrality Clauses to PERM

Clause betweenness centrality as defined in [12] has shown to be a useful clause quality measure. Here we add a limited number of high-centrality (HC) clauses to PERM in MapleLCMDistChronoBT. We computed variable centralities using the Brandes algorithm [6] in the solver. The centrality computation sometimes takes too long, and we limited it to 150 seconds, obtaining centralities for 168 of the 400 instances. For the other formulas we did not use centrality. We normalize the centrality values by \( \frac{1}{(n-1)(n-2)} \) where \( n \) is the number of variables, so they fall in \([0,1]\). The HC clauses can be large and result in computation and memory overhead so care is needed when adding them to PERM. We aimed to include at least the 0.02\% of learned clauses with highest centrality. We set an initial centrality threshold of \( CT \geq 0.008 \) which was chosen empirically. Every 100,000 conflicts, if the number HC clauses in PERM is less than 0.02\% of all learned clauses, \( CT \) is reduced by 0.001, but it is never reduced below 0.001. We report three versions:

- Maple-PERM-HC-max10K: Add at most the first 10K HC clauses to PERM.
- Maple-PERM-HC-max25K: Add at most the first 25K HC clauses to PERM.
- Maple-PERM-HC-Size15-max10K: Add HC clauses to PERM only if they have size \( \leq 15 \), adding at most the first 10K.

Figure 7 compares the performance of these versions, including centrality computations, against the default. The two version with no size limit on the HC clauses performed noticeably better. The version with the size limit performed only slightly better than the default. This indicates long HC clauses are valuable. The average number of additional HC PERM clauses (that would not have been placed in PERM because of LBD) was 8,200 in the version with the limit of 10K, 16,000 with the limit of 25K, and 7050 with the limit of 10K and size \( \leq 15 \).

Often the combination of two heuristics that are beneficial does not improve over the best of the two. However, in the case of our criteria for PERM, the following combination did improve overall performance: For each instance, if did not get centralities within the 150 seconds time limit, we used PERM criterion of Size\( \leq 8 \); otherwise we used LBD\( \leq 3 \) and added HC clauses. Figure 8 compares this version with the original. It solves 13 more instances.

![Figure 7: Performance of with high-centrality clauses in PERM](image-url)
An Experimental Study of Permanently Stored Learned Clauses

Long clauses are generally less valuable than short clauses. They eliminate fewer truth assignments, and are used less frequently in CDCL solvers. However, our data show that the value of HC clauses added to PERM comes from the longer ones. Thus, we would like to understand the usage of HC clauses better.

We define the Clause Usage to be the number of times a clause is used during conflict analysis. We ran MapleLCMDistChronoBT with clause deletion turned off, and stopped the execution after 500,000 conflicts (so 500,000 clause have been learned). Then looked at clause usage as a function of centrality. Figure 9 is a histogram of the usage rates for all learned clauses obtained from 10 formulas randomly chosen from our benchmark. The figure shows clearly that HC clauses were used more than others, suggesting they may be more useful in generating new conflicts.

5 Small good clauses not to add to PERM

In Section 3, we showed that performance improved when we saved all learned clauses of size up to 8 to PERM. Here, we show that there are small clauses we can derive simply which help when added to TEMP but not when added to PERM. Standard conflict analysis schemes derive one clause, called the 1-UIP (for First Unique Implication Point) clause, at each conflict. Various other
schemes have been tried, but most reports confirm that the 1-UIP scheme is best [21][9][20]. An example of adding more clauses is in [9], but these clauses require significant additional reasoning.

Here, we introduce a simple scheme to learn additional small clauses which are very cheap to obtain but still improve performance. Regarding the focus of this paper, the interesting observation is that they have length less than 8, but adding them to PERM reduces performance while adding them to TEMP improves performance. (In contrast, adding all small 1-UIP clauses to PERM improves performance, as we showed above.)

Assume a conflict at level $x$, meaning after assigning $x$ literals $l_1, l_2, ... , l_x$ to true, a conflict is reached. After conflict analysis the solver backjumps to a level $b$ and learns a 1UIP clause $C = \{m_1, m_2, ..., m_{i-1}, m_i\}$. Only one literal $m_i$ from $C$ belongs to level $x$, and $b < x$, so after the first $b$ decisions, if we had $C$ in the clause database unit propagation we would prevent this conflict by assigning $m_i$ true. Therefore, we can also learn clause $C_2 = \{\neg l_1, \neg l_2, ..., \neg l_b, m_i\}$. If $b < 6$, this clause has size $\leq 6$, so we have a new small clause with little work. Here the last two literals of $C_2$, $\neg l_b$ and $m_i$, are glued together so $C_2$ has LBD $|b|$ and size $|b| + 1$. We added clauses of this kind and added them either to PERM or TEMP. Figure 10 compares performance of these version with MapleLCMDistChronoBT. In the formulas of this experiment, an average of 8500 additional clauses were generated using this scheme most of which had LBD 4 or 5. This can be a factor in making them less interesting for PERM.

![Figure 10: Performance of solvers with new learned clauses](image)

6 Discussion

We have performed a number of experiments on criteria for learned clauses in MapleLCMDistChronoBT to be saved permanently (the PERM set). Our experiments confirm that even a large PERM set helps performance, using several criteria related to size and LBD, which are widely used as clause quality measures. We also showed that adding a small number of very high centrality clauses, when centralities could be computed, improved performance. A solver version in which the PERM criteria was either size $\leq 8$ or LBD $\leq 3$ plus HC clauses, depending on availability of centrality information, solved 13 more formulas than the original MapleLCMDistChronoBT on a benchmark of 400 competition instances.
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