Discovery Cycle as Strategy of Science Teaching

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Abstract. The discovery cycle is a strategy for reaching the scientific knowledge trough of discovery. We are applying this strategy with physics students of Mechanics Laboratory of the first year of university. The cycle is focused on research the dynamical behaviors of mechanical systems modeled with Newton’s Laws. It consists of seven stages: exploration, experimentation, empirical modeling, theoretical modeling, numerical modeling, model results versus experiment and prediction. Students carry out this strategy in a specific context of a mechanics system. In this paper, we analyze this strategy with a group of 24 students in the study of the motion of an object, sliding on a tilted plane, and present the results we got in the application of this strategy.

1. Introduction
The experimental mechanics is taught in the Science Faculty of the National Autonomous University of Mexico in the second semester of the first year of the physics career. Our course of Laboratory of Mechanics is a calculus-based physics course, with two three-hour sessions each week. Each session has a teacher, a teaching assistant, and up to 24 students. The Laboratory of Mechanics course is independent of the theoretical course of mechanics that are taught in the same semester. In the Laboratory of Mechanics course, we teach the experimental point of view of the Newtonian mechanics.

Because this course is the first physics laboratory that students take in the physics career, we need to introduce our students to the technics and methods of experimental science and to achieve deep learning in mechanics concepts and their experimental applications, we developed the Discovery Cycle, which has shown to be an effective learning strategy.

This strategy uses two methods of reasoning as are the inductive and deductive approaches divided into four main phases: exploration, experimentation, modeling, and prediction and the objective of the Discovery Cycle is to create an environment of learning in the laboratory for the discovery of Newton’s dynamics by students.

Many phenomena of movement modeled by the second law of Newton are new for our students and the methods to study these phenomena are also unknown to them. Thus, the Discovery Cycle is a scientific strategy to guide students to know the processes for discovering the dynamical behaviors of mechanics’ systems.

Thus, this teaching strategy develops scientific skills in students to discover the behaviors of mechanical systems that are modeled by Newton’s Second Law.

2. Formation of concepts
2.1 Evolution of the learning cycles
In 1962, Robert Karplus[1] et al developed a teaching theory based on the invention and discovery of concepts, considering the initial introduction of new concepts as invention and its subsequence verification or extension of these concepts as a discovery. In 1967, Karplus included the new phase of exploration to the previous phases of invention and discovery, and the term of learning cycle appeared
in 1970 in the teacher guide of the units of the program SCIS (Science Curriculum Improvement Study). The term of learning cycle with the phases of exploration, invention, and discovery was used until 1975 and in 1977, but Karplus changed the phases of the learning cycle to exploration, the introduction of concepts and application of concepts to improve its conception by teachers.

Lawson[2] identifies a general pattern between processes of concept formation and conceptual change. These processes are not different, on the contrary, they are two ends of the same continuum. The pattern begins with a question which has been raised due to some experience, follows with the formulation of an alternative hypothesis discernible from features of the problem situations (induction) or analogical reasoning (abduction) from the own memory or other sources as books. The alternative hypothesis must be tested experimentally to deduce logical consequences (predictions) that must be compared with the real results. If the comparison between predicted results and the real results is shown essentially the same, the truth of the hypothesis is inferred, but if this comparison is not valid, the hypothesis is weakened, another hypothesis must be generated and be tested until getting a reasonable agreement. This type of process is now known as inquiry.

In 1992 White and Gunstone proposed POE (Predict, Observe, Explain) as another learning cycle which nowadays is used many times in a lecture which includes a demonstrative experiment[3,4].

The cycle used in this work, Discovery Cycle, differs from the previous ones because it starts with exploration and ends with a prediction and in the central part it includes the experimentation and modeling carried out by the students. This process includes cognitive strategies like combinations of alternatives hypothesis (combinatorial reasoning), control of variables (experimenting with the variations of one independent variable), comparing ratios that confirm or disconfirm events (correlational reasoning), data analysis, graphical interpretation and comprehension of the relation between variables (graphical reasoning[5,6]).

2.2 Dynamical systems of mechanics
A conceptual frame to learn in Physics is the mechanical system[7]. The importance of the dynamics of a system in mechanics is the understanding of the movement of objects that are driven by forces modeled by Newton’s laws. The dynamical behaviors of movements of these systems are well explained by the solution of the equations of motion deduced from Newton’s second law.

Multiple concepts are involved in each problem of the movements of a mechanical system and students must assimilate all these concepts to have the best understanding of the dynamical behavior of the system. The understanding of all these concepts is a complex task for students. Thus, the teaching of them needs a scientific strategy that guides the learning of students for a successful understanding.

Certainly, when students are confronted with an unknown mechanical system, they are put in a new concept environment where they must discover these concepts and their connections between them to get the best comprehension of this system. Students are not alone for reaching this purpose, they are supported by their teachers in this double process of teaching and learning.

2.3 Critical Thinking
An efficient teaching strategy must develop a process for improving the critical thinking or to promote the “good critical thinking” of science’s students. For critical thinking, Burmester[8] identifies eight types of skills of thinking: skills to recognize problems, skills to delimit a problem, skills to recognize and accumulate facts related to the solution of a problem, skills to recognize a hypothesis, skills to plan experiments to test hypothesis, skills to carry out experiments, skills to handle certain basic skills necessary to the interpretation of data and skills to apply generalizations to new situations.

Ennis[9] considers the critical thinking as the correct assessment of statements; Bailin[10] considers that critical thinking is frequently conceptualized in terms of processes or skills and the idea of critical thinking is constituted by processes interpreted in one of two ways: either mental processes or as a series of procedural moves.

The American Association of Physics Teacher[11] considers that Physics is a way of approaching problem-solving and requires direct observation and physical experiment and to be successful in this purpose they recommends that students synthesize and use a broad spectrum of knowledge and skills,
as mathematical, computational, experiment and practical skills for the developing of particular habits of mind, similar as "thinking like a physicist", necessary to construct knowledge of the physical universe.

3. Discovery Cycle
The formal formation of concepts needs the conceptual change and we need to provide a proper ambient where students can generate their own ideas for confronting them with the accepted scientific thinking. As Lawson comments of Piaget’s theory, this process takes the mind of students to a state from equilibrium to a state of disequilibrium and back to a new equilibrium. The new conceptual equilibrium needs: (a) confront students with data inconsistent with their prior ways of thinking, (b) an alternative conception/hypothesis and c) enough time, motivation and the developing of thinking skills of students for comparing the alternative hypothesis with their predicted consequences with the evidence.

A proper strategy must shape to link all the processes for moving the mind of students of previous ideas to the scientific thinking and the new challenge is to create a new learning cycle for being successful in the teaching of Newtonian mechanics for undergraduate students.

For this goal, we have tested a learning cycle for many years for students to discover Newtonian mechanics’ dynamics. We consider this strategy of teaching as a Discovery Cycle. So, the Discovery Cycle is a teaching strategy with the purpose to develop high thinking skills in the undergraduate students for the study of Newtonian mechanics’ dynamics. Why discovery?

We can consider that Newtonian mechanics’ dynamics is a collection of concepts with very strong interrelations among them and it corresponds to the physics teacher to facilitate students the assimilation of these concepts and their relationships. In the sense of Karplus, students must discover these concepts and their relationships in a learning environment that stimulate the expression of their own ideas of students to be confronted with Newtonian concepts.

But a discovery process must be a process that needs a learning strategy to carry it out. For us, this strategy is a learning cycle that we have called the Discovery Cycle for the learning of Newtonian dynamics. Discovery Cycle is a learning cycle with four main phases: Exploration, Experimentation, Modeling (empirical modeling, theoretical modeling, numerical modeling, model results versus experiment), and Prediction, we can see a scheme of this cycle in Figure 1. Before starting the Discovery Cycle, a question of the research is posed by the teacher to students. This question is related to the dynamical behavior of the phenomenon selected and the students must discover this behavior.

3.1. Exploration phase. The Discovery cycle begins with the exploration phase, where students, working in teams of three or four of them, enter in contact with a variety of objects and equipment of laboratory, but not instruments of measurement to become familiar with the phenomenon by experimenting intuitively with these objects. In this phase, the students must observe and describe the movements produced carefully and the most accuracy. Then, questions about the observed movements must make by students and they must try to answer tentatively these questions given explanations that can be taken as a hypothesis. Students select one of their best hypotheses and they must test the selected hypothesis experimentally.

3.2. Experimental phase. The Discovery Cycle continues with the experimental phase. Student design an experiment to test the selected hypothesis by using the same experimental equipment and measuring instruments, where they must select proper experimental parameters. The teacher takes care that hypotheses of the different teams of the group will be well posed and students proceed to perform the experiment by using experimental techniques of data capture seen previously.
Figure 1. Discovery Cycle of four phases for the learning of Newtonian dynamics: exploration, experimentation, modeling (empirical modeling, theoretical modeling, numerical modeling, model results versus experiment), and prediction.

The technique uses to collect experimental data is video techniques, because this technique of data capture is more intuitive for concrete and abstract students. The video technique is well appreciated by students because the use of these techniques is new and interesting for them. The software recommended to analyze videos of movements in a computer is Tracker that is a free software part of the Open Source Physics (OSP) Collection hosted by the comPADRE Digital Library\cite{12} and a spreadsheet.

By making a plot of data, and fitting an empirical equation to the curve in the graph, students are testing the validity of the hypothesis. If the hypothesis is true, the students confirm their tentative explanation that was done before, but if the hypothesis is not according with the experimental evidence, the students must accept the hypothesis is false, even if students enter in conflict with this result.

3.3. Modeling phase. The modeling phase is divided into four parts: empirical modeling, theoretical modeling, numerical modeling, and the confrontation between the model and the experiment.

The empirical modeling results of the analysis of experimental data and graphs. Theoretical modeling consists of the introduction of theoretical concepts by the teacher. In this part, students and teacher make the analysis of forces on the object with free object diagram to deduce the equation of motion of the object from the application of Newton’s second law.

In numerical modeling, students with the support of the teacher solve the equation of motion of the object with numerical methods. The solution of the equation of motion of an object is an important objective of the Discovery Cycle because students discover the complete dynamical behavior of the movement in many ways of relationships between the dynamical variables of the movement.

The numerical method is programmed in a spreadsheet. In the spreadsheet, the algebraic equations of this numerical method (Table 5) are the same for all the motions studied and this is a simplification of the mathematical problem arises from using different analytical solutions.

One of the most important results of the numerical solutions is the dependence of the position of the object with the time that students compare with the experimental data to validate the theoretical and numerical model if the match between them is accurate.
3.4. Prediction phase. Students can predict the complete dynamical behavior of the motion with the numerical solution of the equation of motion, so they have many ways to understand well this dynamic. By changing the parameters of the mechanical system, students predicted new features of this system and tested experimentally the predictions to confirm its validity.

4. Dynamics of a sliding object on an inclined plane with friction

As an example of applications of the Discover Cycle, we applied it to the study of sliding motion of an object on an inclined plane with friction on two dimensions\(^{[13]}\) that is a phenomenon of motion unknown for students generally. The teacher proposes the students to study the two-dimensional sliding of an object on an inclined plane and he makes the research question: which is the trajectory of the object if it is thrown with an initial velocity \(\vec{v}_0\)?

4.1. Exploring the trajectory of the object. Students begin the exploration phase of the Discovery Cycle exploring the movement of the object on a frictional inclined plane when the initial velocity of the object is zero and the object goes down the plane. Students realize that the motion occurs only when the inclination angle of the plane is big enough to have a kinetic coefficient of friction. In this point, the students know about the static and kinetic coefficient friction and they realize the importance of the experiment because it is the way to measure the kinetic coefficient of friction.

This phase continues with the exploration of motion with a different velocity of launch on the inclined plane by observing different trajectories of the parabolic trajectory, but they confuse these trajectories with a parabolic trajectory because it is more familiar for them.

Questions about their observations are: Is the acceleration constant? Is friction force independent of the angle of the launch of the object? Does friction force disturb the movement of the object? Is the trajectory of the object a parabola? If the trajectory of the object is not a parabola, what curve is it?

The common answers of this questions are relating with a parabolic trajectory with constant acceleration because the students are strongly influenced by the parabolic path of a projectile launch in the vacuum described in the textbooks and it is difficult for them to change these ideas. So, the hypothesis more shared between the students is that the trajectory of the object is a parabola, but some other students don’t suppose it, although they cannot identify the trajectory they observed.

4.2. Experimenting with different trajectories. The students build an experimental setup with an inclined square table of 70 x 70cm, supported by rods to give a precise inclination, for example of 45°. Then, they launch a small object manually with the fingers of the hand, given him a sudden hit in any direction with an angle of launch regarding the horizontal line from 0° to 90°. The students get the trajectory of the movement with Tracker (Figure 2) and copy and paste the data to a spreadsheet. They graph the trajectory again in the spreadsheet and try to fit a parabola to the curve, but they realize that it is not possible, because they find incompatible differences between the experimental points with the fitted parabola. This creates doubts in students.

![Figure 2](image.png)

*Figure 2.* The trajectory of an object moving on an inclined plane with friction. The red points are the marks frame by frame of a video of a movement displayed at 300 fps.
4.3. Theoretical modeling of the dynamics of the object. Students with the support of the teacher make a free object diagram to find the forces applied on the object (Figure 3).

![Free object diagram](image)

**Figure 3.** Free object diagram. Two forces are exerted on the object parallel to the plane, the component of the weight parallel to the plane and the friction force opposite to the velocity of the object.

Then, they can write the equation of motion, as:

\[ \vec{F} = m \frac{d\vec{v}}{dt} = -mg \sin \alpha - \mu mg \cos \alpha \frac{\vec{v}}{\left| \vec{v} \right|} \]  

(1)

with \( \vec{v} = \frac{d\vec{r}}{dt} \), \( m \) the mass of the object, \( g \) the acceleration of gravity, \( \alpha \) the inclination angle of the plane, \( \mu \) is the kinetic coefficient of friction and \( \frac{\vec{v}}{\left| \vec{v} \right|} \) a unitary vector in direction of the velocity.

Force in equation (1) is a vector, with the next two components:

\[ F_x = m \frac{dv_x}{dt} = -\mu mg \cos \alpha \frac{v_x}{\sqrt{v_x^2 + v_y^2}} \]  

(2)

\[ F_y = m \frac{dv_y}{dt} = -mg \sin \alpha - \mu mg \cos \alpha \frac{v_y}{\sqrt{v_x^2 + v_y^2}} \]  

(3)

with \( v_x = \frac{dx}{dt} \) and \( v_y = \frac{dy}{dt} \).

In equations (2) and (3), the components \( f_x \) and \( f_y \) of the friction are not constant because depend of the quantities \( \cos \beta = \frac{v_x}{\sqrt{v_x^2 + v_y^2}} \) and \( \sin \beta = \frac{v_y}{\sqrt{v_x^2 + v_y^2}} \) respectively, where \( \beta \) is the angle of the velocity respect the horizontal. As \( \beta \) change with the time, \( f_x \) and \( f_y \) change with the time too.

4.4. Numerical solution of the equation of motion. The analytical solutions of equations (2) and (3) are not simple\(^{14, 15} \) and they are out of the scope of the students of the first year. A good option is to solve this equation with numerical methods.

| Table 1. Parameters of the system |
|----------------------------------|
| Parameter       | value     | \( \delta v \) |
| \( m \) (kg)     | 0.0103    | 0.0001          |
| \( g \) (m/s\(^2\)) | 9.78      | 0.05            |
| \( \mu \)        | 0.31      | 0.01            |
| \( \alpha \) (grades) | 46        | 1               |
| \( \alpha \) (rad)   | 0.436     | 0.02            |
| \( \Delta t \) (s)  | 0.000075  | 0.000005        |
Before starting the numerical solution, it is necessary to know the parameters of the system and the initial conditions of the movement. These values are given in Table 1 and Table 2.

Table 2. Initial conditions

| Initial condition | value | \( \delta v \) |
|-------------------|-------|---------------|
| \( t_0(s) \)      | 0     |               |
| \( x_0(m) \)      | -0.270| 0.005         |
| \( y_0(m) \)      | -0.114| 0.005         |
| \( v_0(m/s) \)    | 2.8   | 0.3           |
| \( \beta_0(grades)\) | 60.1  | 5             |
| \( \beta_0(rad) \) | 1.10  | 0.09          |
| \( v_{xo}(m/s) \)  | 1.27  | 0.05          |
| \( v_{yo}(m/s) \)  | 2.50  | 0.05          |

The numerical solution is derived from the numerical method of Euler for the differential equation (2), and (3) and consisted of six pairs of coupled algebraic equations that are solved iteratively with the method given in Table 3. The iterative process needs the parameters of the system in table 1, the initial conditions of table 2, and the specific form of the force or its components in equations (1), (2) or (3).

Table 3. Numerical solution of the equation of motion

| #   | \( x \) - component | \( y \) - component |
|-----|-----------------------|---------------------|
| 0   | \( t = 0 \)          |                     |
| 1   | \( F_{x,i} = -\mu mg \cos \alpha \frac{v_{x,i}^2 + v_{y,i}^2}{\sqrt{v_{x,i}^2 + v_{y,i}^2}} \) |
| 2   | \( v_{x,i+1} = v_{x,i} + \frac{F_{x,i}}{m} \Delta t \) |
| 3   | \( v_{x(2i+1)/2} = \frac{v_{x,i} + v_{x,i+1}}{2} \) |
| 4   | \( x_{i+1} = x_i + \frac{v_{x(2i+1)/2}}{2} \Delta t \) |
| 5   | \( F_{x(2i+1)/2} = -\mu g \cos \alpha \frac{v_{x(2i+1)/2}^2 + v_{y(2i+1)/2}^2}{\sqrt{v_{x(2i+1)/2}^2 + v_{y(2i+1)/2}^2}} \) |
| 6   | \( v_{x,i+1} = v_{x,i} + \frac{F_{x(2i+1)/2}}{m} \Delta t \) |
| 7   | \( t = t + 1 \) |

4.5. Comparison of the experiment with the numerical solution

This numerical solution is programmed in a spreadsheet and the numerical solutions for \( x \) as a function of \( t \), \( y \) as a function of \( t \) and \( y \) as a function of \( x \) are plotted with the experimental data in the corresponding plots as is shown in Graph 1 and Graph 2.

Graph 1. Comparison between (a) the experimental data \( x \) as a function of \( t \) and (b) the experimental data \( y \) as a function of \( t \) with the results of the numerical solutions (continuous lines).
Graph 2. Comparison between the experimental data $y$ as a function of $x$ with the results of the numerical solution (continuous line).

4.6. Predictions
The total time of the experiment was at least 1 s, but with the numerical model, it is possible to extrapolate the results of the numerical solutions to very large time, for example, 10 s. The purpose is to explore limit situations of the movement. In this case, the graph $a_x$ vs $a_y$ disclose a bound behavior of the movement for a large time as it is shown in graph 3.

Graph 3. The graph $a_x$ vs $a_y$ shows a bound behavior of the movement.

In graph 3, the intersection of the curve with $y$-axe is a limit behavior. The horizontal acceleration $a_x$ goes to zero and the acceleration $a_y$ parallel to the inclined plane reaches the constant value of 4.91 $m/s^2$.

5. Assessment
The phenomenon presented here and analyzed by students was a complex problem in two dimensions. However, students carried out the pretty well the experiment, the analysis of videos with Tracker, the building of graphs and their interpretation, including the graphical comparison between experimental data with numerical results. In general, the empirical model was well understood and this understanding opened the minds of students to have a better comprehension of the phenomenon theory and for formulating the equation of motion (Newton's differential equation) that was solved numerically efficiently. The numerical modeling was not easy for students, because this phenomenon was the first in two dimensions analyzed by them, for which they had applied the numerical method twice to solve two coupled differential equation (previous applications were only in one dimention), but in general, all the students programmed the double solution in Excel and get the kinematical graphs shown in Graphs 1 to 3 successfully.

The assessment of the Discovery Cycle in the Dynamics of a sliding object on an inclined plane with friction is made from the evaluation of the reports done by students. The report is divided into twelve sections: objective, exploration, hypothesis, experimentation, experimental data, graphs, graphs' interpretation, numerical solution, results, analysis, prediction, and conclusions.
Graph 4. Histogram of items evaluated from written reports of 24 students.

The histogram in Graph 4 is the result of this evaluation for 24 written reports. The twelve items are grouped in the different phases of the Discovery Cycle.

In this complex phenomenon, the goals of the study are discussed in class with the students, but as their writing skills are also in full developing, 30% of the redaction of the reports are not so clear. The performance in exploration was high but hypotheses given by students were a focus on a parabolic trajectory, where effects of the friction were difficult to see for them. The experimental setup was difficult to build for some students because they needed to reach high accuracy for a good experiment, but when they applied a good variables control they solved the practical problems they found in the experimental setup. The data capture was easier to do for students, due to the technology used (a fast camera of 300 fps and an automatic trace of data points with Tracker), but graphs and their interpretations were not so easy for them, however when they applied techniques of graph analysis as interpolation and extrapolation they improved the interpretation of graphs.

Teacher discusses with all the students the theoretical modeling and the general numerical solutions of equation (2) and (3), but the application of the theory to solve these equations numerically was no easy for students when they tried to solve the equations with the numerical method by itself, but when they realized that applications of the numerical method in the analysis of previous phenomena were analogous with the present analysis, students quite improved its application.

The interpretation of graph solutions had some difficulties at the beginning because students explored a range not large enough in Excel spreadsheet and they needed to add not only some dozen of programmed lines like they did but hundreds or maybe thousands of lines for extrapolations of the experimental results.

The spreadsheet turned for students in a dynamical spreadsheet where they could simulate in addition to modeling situations that they did not see in the experiment to predict end behaviors of the phenomenon. The Excel spreadsheets contain valuable analysis made by students, but they were not part of the assessment of graph 4, because these spreadsheets were a lot to be evaluated and are always changing, but in class the teacher and the assistant are always reviewing this spreadsheet with the students until they have the best spreadsheet that they can program and they can report the results of this spreadsheet the most efficiently possible.

With the analysis of the spreadsheet results, the students concluded that the hypothesis of parabolic trajectory with general consensus at the beginning of the investigation was not true, because the movement of the body on an inclined plane with friction had an X-component that is slowing down until it reaches to zero velocity as it is extrapolated from Graph 1.a in the spreadsheet and a Y-component with constant acceleration. Thus, students successfully concluded that the terminal movement of the body was only a vertical accelerated movement without horizontal movement, with a non-parabolic trajectory with a terminal rectilinear accelerated movement of the body. So students discovered the real behavior of the movement that they were confronted with their initial false hypothesis, and that they explained later as the impossibility of model at the beginning of the investigation the effect of the friction in a movement in two dimensions.
6. Conclusions
Although the knowledge of many mechanical systems has already been discovered and one can find these discoveries in textbooks or other papers, it is very important that students follow a scientific strategy to do their own discoveries.

The Discovery Cycle is a strategy to guide the students in their investigations on the dynamics of mechanical systems and we have applied it with students for the study of the movement of an object on an inclined plane with friction because they are not familiar with this motion. For this, all the new knowledge learned (including new concepts) will be new discoveries for students.

The general performance of students in the application of the Discovery Cycle, as shown in graph 4, was of 72% with a standard deviation of 15%, but only in the strict assessment of reports. According to this result, the success of this strategy was a medium result because the emphasis of the assessment was made in the last four sections that include results, analysis, prediction, and conclusions, but these sections are the most difficult for students.

Students developed many thinking skills in the application of this Discovery Cycle as those described by Burmester, but for brevity of this paper we will describe them in a future work, however if we add the discussions with students in the analysis of their Excel spreadsheet during the classes, we can conclude that the Discovery cycle is a strategy that helps the students to think like a physicist.

7. References.
[1] Fuller R G ed. 2002 A Love of Discovery: Science Education. The Second Career of Robert Karplus (NewYork: Kluwer/Plenum).
[2] Lawson A E, Abraham M R, and Renner J W 1989 A theory of instruction: Using the learning cycle to teach science concepts and thinking skills. NARST Monograph, 1 (Cincinnati: The National Association for Research Science Teaching).
[3] White R and Gunstone R 1992 Probing Understanding. Falmer Press (London).
[4] Sokoloff D R, and Thornton R K 1997 Using interactive lecture demonstrations to create an active learning environment. The Physics Teacher 35, 340.
[5] Klein P, Müller A and Kuhm, J. 2017 Assessment of representational competence in kinematics. Physical Review Physics Education Research 13, 010132.
[6] Anderson M, Meyer B, and Oliver P eds. 2002 Diagrammatic representation and reasoning (London: Springer-Verlag).
[7] Josephs H and Huston R L 2002 Dynamics of Mechanical System (Boca Raton: CRC Press).
[8] Burmester M A (1952) Behavior involved in the critical aspects of scientific thinking. Science Education 36 259.
[9] Ennis R H 1964 A Definition of Critical Thinking. The Reading Teacher, 17, 599.
[10] Bailin S 2002 Critical Thinking and Science Education. Science & Education 11 361.
[11] Kozminski J, Beverly N, Deardorff D, Dietz R, Eblen-Zayas M, Hobbs R, Lewandowski H, Lindas S, Reagan A, Tagg R, Williams J, and Zwickl B 2014 “AAPT Recommendations for the Undergraduate Physics Laboratory Curriculum” American Association of Physics Teachers.
[12] Tracker (2018). Video Analysis and Modeling Tool. Retrieved from https://physlets.org/tracker/
[13] Aghamohammad C and Aghamohammad A 2011. Slipping and rolling on an inclined plane. Eur. J. Phys., 32, 1049.
[14] Shunyakov V M and Lavrik V 2010. Analytical solutions of curvilinear motion on an inclined plane. Am. J. Phys., 78, 1406.
[15] Wang, X 2014. The trajectory of a projectile on a frictional inclined plane. Am. J. Phys., 82, 764.