Stabilizing the Axion by Discrete Gauge Symmetries

K. S. Babu, Ilia Gogoladze and Kai Wang

Department of Physics, Oklahoma State University
Stillwater, OK 74078, USA

Abstract

The axion solution to the strong CP problem makes use of a global Peccei–Quinn $U(1)$ symmetry which is susceptible to violations from quantum gravitational effects. We show how discrete gauge symmetries can protect the axion from such violations. PQ symmetry emerges as an approximate global symmetry from discrete gauge symmetries. Simple models based on $Z_N$ symmetries with $N = 11, 12$, etc are presented realizing the DFSZ axion and the KSVZ axion. The discrete gauge anomalies are cancelled by a discrete version of the Green–Schwarz mechanism. In the supersymmetric extension our models provide a natural link between the SUSY breaking scale, the axion scale, and the SUSY–preserving $\mu$ term.

1E-mail address: babu@okstate.edu
2On a leave of absence from: Andronikashvili Institute of Physics, GAS, 380077 Tbilisi, Georgia. E-mail address: ilia@hep.phy.okstate.edu
3E-mail address: wk@okstate.edu
1 Introduction

One of the challenges facing the Standard Model is an understanding of the strong CP problem [1]. The Lagrangian of quantum chromo-dynamics (QCD) contains a term

\[ L_{QCD} \supset \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \]

where \( G_{\mu\nu} \) is the gluon field strength tensor, \( g_s \) the QCD gauge coupling, and \( \theta \) a dimensionless parameter. This term violates both P and CP invariance. In the full QCD Lagrangian involving massive quarks, \( \theta \) is not a physical observable, but \( \bar{\theta} \) defined as

\[ \bar{\theta} = \theta + \arg(\det M_U \det M_D), \]

is. Here \( M_U \) and \( M_D \) are the up–quark and the down–quark mass matrices. The observed CP violation in weak interactions indicates that the complex phase in the Cabibbo–Kobayashi–Maskawa matrix is of order unity. Without additional symmetries one would expect that the quark mass matrices, and hence their determinants, also contain large complex phases. Therefore one would naturally expect from Eq. (2) that \( \bar{\theta} \) is order one. However, this is in gross violation of the experimental limits arising from the neutron electric dipole moment (EDM). Eq. (2) leads to a neutron EDM of order \( d_n \sim 5 \times 10^{-16} \bar{\theta} \) e cm. Current experimental limit on \( d_n \) is \( d_n < 10^{-25} \) e cm [2], which puts a constraint \( \bar{\theta} < 10^{-10} \). Why \( \bar{\theta} \) is so small, when its natural values if of order one, is the strong CP problem.

An elegant and popular solution to the strong CP problem is provided by the Peccei–Quinn mechanisms [3]. It is based on on a global axial symmetry, \( U(1)_{PQ} \), which is spontaneously broken. In this mechanism \( \bar{\theta} \) is promoted to a dynamical field, \( \bar{\theta} = a/f_a \), with an effective potential for this field induced by non–perturbative QCD effects. Here \( a \) stands for the pseudo–Goldstone mode of the spontaneously broken \( U(1) \) symmetry, the axion [4], and \( f_a \) the axion decay constant (equal to the VEV of the scalar that breaks the PQ symmetry). Minimizing the axion potential would fix the vacuum expectation value (VEV) of \( a \), or equivalently \( \bar{\theta} \) to zero, thus providing a natural solution to the strong CP problem.

The axion decay constant \( f_a \) is model–dependent. In the original Weinberg–Wilczek axion [4], the PQ symmetry is broken spontaneously at the electroweak scale. This model contains two Higgs doublets \( \Phi_1 \) and \( \Phi_2 \) with VEVs \( v_1 \) and \( v_2 \). Without the QCD instanton effects, spontaneous breaking of the global \( U(1) \) symmetry would imply that the axion is massless. Including non–perturbative QCD effects which break the global \( U(1) \) explicitly the axion would acquire a mass given by [4]

\[ m_a \sim \frac{f_\pi m_\pi}{v} \simeq 100 \text{ keV} \]

where \( m_\pi \) and \( f_\pi \) are the pion mass and decay constant, and \( v = \sqrt{v_1^2 + v_2^2} \sim 246 \) GeV. The axion decay constant is \( f_a \sim v \) in this model. Laboratory data, especially from the non–observation of the rare decay \( K \to \pi + a \), excludes this simple weak scale realization of the axion [1, 2].

The laboratory bound on the axion coupling can be evaded by introducing a singlet scalar \( S \) into the theory carrying a non–zero PQ charge. When this scalar acquires a VEV, the
global $U(1)$ symmetry will break spontaneously. The axion decay constant in this case is $f_a \sim \langle S \rangle$, which can be much larger than the weak scale. With $f_a \gg v$, the couplings of the axion with the Standard Model fields will be highly suppressed, by a factor $v/f_a$, and the laboratory limits are evaded. This class of models are the “invisible axion models”. If the spectrum of the theory contains the scalars $S$ and two doublets $\Phi_1$ and $\Phi_2$, we have Dine–Fischler–Srednicki–Zhitnitskii (DFSZ) axion [5]. If the scalar spectrum contains a single Higgs doublet and a singlet $S$ but the fermionic spectrum is enlarged to contain a pair of quark–antiquark fields, we have the Kim–Shifman–Vainshtein–Zakharov (KSVZ) axion [6]. In the DFSZ axion model the SM fermions carry PQ charges and the axion potential is induced by the QCD chiral anomaly associated with the SM fermions. In the KSVZ axion model only the exotic vector–like quarks carry PQ charges and their QCD anomaly induces the axion potential.

The PQ symmetry breaking scale $f_a$ is constrained by a combination of laboratory, astrophysical, and cosmological limits to be in the range $10^{10}$ GeV $\leq f_a \leq 10^{12}$ GeV. If $f_a$ is much above the weak scale, axions produced in the interior of stars can escape freely, draining the star of its energy too rapidly. The limit $f_a \leq 10^{10}$ GeV arises from the stability of stars and supernovae [1]. If $f_a$ exceeds about $10^{12}$ GeV, the energy density in the coherent oscillations of the axions will over-close the universe, contradicting cosmological limits. The mass of the axion in these invisible models is extremely small, $m_a \sim f_\pi m_\pi/f_A \sim 10^{-4}$ eV. Axion with a mass in this range can constitute a good cold dark matter candidate [1].

Quantum gravitational effects can potentially violate the global PQ symmetry which is needed for the axion solution. These effects, associated with black holes, worm holes, etc are believed to violate all global symmetries, while they respect all gauge symmetries [7]. When a gauge symmetry is spontaneously broken, often a discrete remnant survives [8]. Such discrete gauge symmetries are left intact by quantum gravity. Any such quantum gravity violations of the PQ symmetry should be extremely small for axion to be a viable solution to the strong CP problem. For example, in the invisible axion models, the gauge symmetries would allow for a term in the scalar potential of the form

$$V \supset \frac{S^n}{M_{Pl}^{n-1}},$$

where $S$ is the singlet field carrying PQ charge, $M_{Pl}$ is the Planck scale, and $n$ is a positive integer. This term would induce a non–zero $\bar{\theta}$ given by

$$\bar{\theta} \simeq \frac{f_a^n}{M_{Pl}^{n-4}\Lambda_{QCD}^4}$$

where $\Lambda_{QCD} \simeq 0.2$ GeV is the QCD scale. Since the neutron EDM sets the limit $\bar{\theta} \leq 10^{-10}$, using $f_a \sim 10^{11}$ GeV one finds that $n \geq 10$ is necessary in Eq. (4) [9]. This is indeed a severe constraint on axion models.

A possible way to avoid the problems associated with quantum gravity is to identify Peccei–Quinn symmetry as an approximate global symmetry associated with a gauge symmetry. Attempts have been made in the past with some success to achieve this by extending the low energy (below the Planck scale) particle content of the invisible axion models [9].
In this paper we show how it is possible to use discrete gauge symmetries to stabilize the axion solution without enlarging the low energy particle content. This is made possible by a discrete version of the Green–Schwarz anomaly cancellation mechanism [10]. Superstring theory when compactified to four dimensions generically contains an anomalous $U(1)$ symmetry with its anomalies cancelled by a shift in the pseudoscalar partner of the dilaton field (the string axion field). This anomalous $U(1)$ is broken near the string scale when some scalar fields carrying $U(1)_A$ charges are shifted to cancel the Fayet–Iliopoulos term associated with the $U(1)_A$ (so that supersymmetry is left unbroken near the string scale) [11]. Often a discrete $Z_N$ subgroup of the $U(1)_A$ symmetry is left unbroken to low energies. This $Z_N$ symmetry, being of a gauge origin, is protected against quantum gravitational violations.

From an effective theory point of view the conditions for Green–Schwarz anomaly cancellations are

$$\frac{A_i}{k_i} = \frac{A_{\text{gravity}}}{12} = \delta_{\text{GS}},$$

where $A_i$ are the anomaly coefficients associated with $G_i^2 \times U(1)_A$ with $G_i$ being one of the factors of the surviving gauge symmetry. $\delta_{\text{GS}}$ is a constant that is not specified by the low energy theory alone. $k_i$ are the levels of the Kac–Moody algebra. For non–Abelian groups $k_i$ are (positive) integers, although for $U(1)$ factors $k_i$ need not be integers. All other anomaly coefficients such as $G_i G_j G_k$ and $[U(1)_A]^2 \times G_i$ should vanish. In the case of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_A$ being the surviving symmetry, the GS anomaly cancellation conditions are

$$\frac{A_3}{k_3} = \frac{A_2}{k_2} = \delta_{\text{GS}},$$

where $A_3$ and $A_2$ stand for the $[SU(3)_C]^2 \times U(1)_A$ and $[SU(2)_L]^2 \times U(1)_A$ anomaly coefficients. There is an analogous expression involving the hypercharge $Y$, however since $Y$ is not quantized, the associated level $k_1$ is not an integer in general and this condition is not very useful from an effective low energy theory point of view [12]. The $[U(1)_A]^3$ anomaly can be cancelled by the GS mechanism, but this condition also has an arbitrariness from the normalization of $U(1)_A$. The anomaly in $[U(1)_A]^2 \times U(1)_Y$ also does not give any useful low energy constraint.

When the $U(1)_A$ symmetry breaks down to a $Z_N$ discrete symmetry, the effective low energy theory would satisfy the following discrete Green–Schwarz anomaly cancellation condition [12, 13, 14, 15]:

$$\frac{A_3 + mN/2}{k_3} = \frac{A_2 + m'N/2}{k_2},$$

where $m$ and $m'$ are integers. Heavy particles which are chiral under $U(1)_A$ can acquire masses of order the $U(1)_A$ breaking scale through $Z_N$–invariant Yukawa couplings, the contributions proportional to the integers $m$ and $m'$ account for these heavy fermions.

In what follows, we shall impose the conditions of Eq. (8), which should be sufficient for the surviving $Z_N$ symmetry to be protected from quantum gravitational effects.
2 Stabilizing the DFSZ Axion

The non–supersymmetric DFSZ axion model [5] introduces two Higgs doublets $H_u$ and $H_d$ and a Standard Model singlet scalar $S$. The Lagrangian of the model relevant for the discussion of axion physics is

$$L = Q u^c H_u + Q d^c H_d + L e^c H_d + L \nu^c H_u + \nu^c \nu^c - \lambda (H_u H_d S^2 + h.c).$$

Here we have used a standard notation that easily generalizes to our supersymmetric extension as well. $Q, L$ are the left–handed quark and lepton doublets, $u^c, d^c, e^c, \nu^c$ are the quark and lepton singlets. The seesaw mechanism [16] for small neutrino masses has been incorporated in Eq. (9).

Eq. (9) has three $U(1)$ symmetries, as can be inferred by solving the six conditions imposed by Eq. (9) on nine parameters. These three $U(1)$ symmetries can be identified as the SM hypercharge $U(1)_Y$, baryon number $U(1)_B$ and a Peccei–Quinn symmetry $U(1)_{PQ}$. If we denote the charges of $(Q, u^c, d^c)$ as $(q, u, d)$, the symmetries can be realized as $B = q - u - d$, $PQ = -d$, $Y/2 = q/6 - 2u/3 + d/3$. The $U(1)$ charges of the various particles under these symmetries are listed in Table 1.

|       | $Q$ | $u^c$ | $d^c$ | $L$ | $e^c$ | $H_d$ | $H_u$ | $S$ |
|-------|-----|-------|-------|-----|-------|-------|-------|-----|
| $Y/2$ | 1/6 | -2/3  | 1/3   | -1/2| 1     | 0     | -1/2  | 1/2 |
| $B$   | 1   | -1    | -1    | 0   | 0     | 0     | 0     | 0   |
| $PQ$  | 0   | 0     | -1    | 0   | -1    | 0     | 1     | 0   | -1/2 |

Table 1: $Y/2$, $B$ and $PQ$ symmetries corresponding to hypercharge, baryon number and PQ charge respectively. The charges are assumed to be generation independent.

After $H_d, H_u$ and $S$ fields develop VEVs, the global PQ symmetry is broken and the light spectrum contains a Goldstone boson, the axion. Non–perturbative QCD effects induce an axion mass [1] given by

$$m_a^{DSFZ} \simeq 0.6 \times 10^{-4} \text{ eV } \frac{10^{11}}{f_a} \text{ GeV},$$

where $f_a \sim \langle S \rangle$ is the axion decay constant.

We now apply the Green–Schwarz mechanism for discrete anomaly cancellation to stabilize the axion from quantum gravity corrections. Even though the model under discussion is non-supersymmetric, the GS mechanism for anomaly cancellation should still be available, since SUSY breaking in superstring theory need not occur at the weak scale in principle. Since baryon number has no QCD anomaly, any of its subgroup will be insufficient to solve the strong CP problem via the PQ mechanism. On the other hand, the PQ symmetry does have a QCD anomaly, although with the charges listed in Table 1 it has no $SU(2)_L$ anomaly. Since hypercharge $Y$ is anomaly free, we attempt to identify the anomalous $U(1)_A$ symmetry as a linear combination of $PQ$ and $B$:

$$U(1)_A = PQ + \gamma B.$$
According to the Eq. (11) and the charge assignment presented in Table 1, we have for the anomaly coefficients for the $U(1)_A$,

$$
A_3 \equiv [SU(3)]^2 \times U(1)_A = -\frac{3}{2} \\
A_2 \equiv [SU(2)]^2 \times U(1)_A = \frac{9}{2} \gamma .
$$

(12)

If we identify $\gamma = -k_2/(3k_3)$, the anomalies in $U(1)_A$ will be cancelled by GS mechanism. Thus we have

$$
U(1)_A = PQ - \frac{1}{3} \frac{k_2}{k_3} B.
$$

(13)

The simplest possibility is $k_2 = k_3 = 1$, corresponding to the levels of Kac–Moody algebra being one. Normalizing the charge of the singlet field $S$ to be an integer, Eq. (13) can be rewritten as

$$
U(1)_A = 6(PQ) - 2(B).
$$

(14)

The corresponding charge assignment is given in Table 2. As discussed earlier, since hypercharge $Y$ is anomaly free, one can add a constant multiple of $Y/2$ to the $U(1)_A$ charges, and still realize GS anomaly cancellation mechanism. The charges listed in Table 2 assumes the combination $-\frac{5}{3}(6PQ - 2B + 4Y)$. As can be seen from Table 2, this choice of charges is compatible with $SU(5)$ grand unification.

Suppose that the $U(1)_A$ symmetry is broken near the string scale by the VEV of a scalar field which has a $U(1)_A$ charge of $N$ in a normalization where all $U(1)_A$ charges have been made integers. A $Z_N$ symmetry will then be left unbroken to low scales. Two examples of such $Z_N$ symmetries are displayed in Table 2 for $N = 11, 12$. Invariance under these $Z_N$ symmetries will not be spoiled by quantum gravity, it is this property that we use to stabilize the axion.

Potentially dangerous terms that violate the $U(1)_{PQ}$ symmetry of Eq. (9) are $S^n/M_{Pl}^{n-3}$, $H_u H_d S^m/M_{Pl}^{m-2}$ etc, for positive integers $n, m$. For the induced $\theta$ to be less than $10^{-10}$, the integers $n, m$ must obey $n \geq 10, m \geq 5$. The choice of $N = 11, 12$ satisfy these constraints. Note that a $Z_{10}$ discrete symmetry would have allowed a term $S^2$, which would be inconsistent with the limit on $\theta$. $Z_N$ symmetries with $N$ larger than 12 can also provide consistent solutions. Since by construction, the $U(1)_A$ symmetry in Table 1 is anomaly-free by GS mechanism, any of its $Z_N$ subgroup is also anomaly-free by the discrete GS mechanism, as can be checked directly. In the $Z_{11}$ model, for example, we have $A_3 = A_2 = 4$. Consistent with the $Z_{11}$ invariance, terms that violate the $U(1)_{PQ}$ symmetry and give rise to an axion mass are $S^{11}/M_{Pl}^{11}$, $H_u H_d S^{*0}/M_{Pl}^{10}$ etc, all of which are quite harmless. We conclude that the DFSZ axion can be stabilized against potentially dangerous non–renormalizable terms arising from quantum gravitational effects in a simple way.

### 3 Stabilizing the KSVZ Axion

The scalar sector of the non–supersymmetric KSVZ axion model [6] contains the SM doublet and a singlet field $S$. All the SM fermions are assumed to have zero PQ charge under the
global $U(1)_{PQ}$ symmetry. The Yukawa sector involving the SM fermions is thus unchanged. An exotic quark–antiquark pair $\Psi + \bar{\Psi}$ is introduced, which transform vectorially under the SM (so the magnitude of its mass term can be much larger than the electroweak scale), but has chiral transformations under $U(1)_{PQ}$. The QCD anomaly needed for the axion potential arises from these exotic quarks. The Lagrangian involving the singlet field and these vector quarks is given by

$$\Delta L = S\Psi \bar{\Psi} + h.c. \quad (15)$$

When $S$ field develops a VEV, the PQ symmetry is spontaneously broken leading to the axion in the light spectrum.

The global PQ $U(1)$ symmetry is susceptible to unknown quantum gravity corrections. We shall attempt to stabilize the KSVZ axion by making use of discrete gauge symmetries with anomaly cancellation by the Green–Schwarz mechanism. The most dangerous non–renormalizable term in the scalar potential that can destabilize the axion is $S^n/M_{Pl}^{n-4}$, as in the case of the DFSZ axion. We seek a discrete gauge symmetry that would forbid such terms.

In order for the Green–Schwarz mechanism for anomaly cancellation to be viable, the anomaly coefficients $A_2$ and $A_3$ corresponding to the $[SU(2)_L]^2 \times U(1)_A$ and $[SU(3)_C]^2 \times U(1)_A$ should equal each other at the $U(1)$ level. This would imply that the $\Psi + \bar{\Psi}$ fields can not all be singlets of $SU(2)_L$. The simplest example we have found is the addition of a $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$ to the SM spectrum. Such a modification is clearly compatible with grand unification. The $\mathbf{5}$ contains a $(\mathbf{3}, \mathbf{1})$ and a $(\mathbf{1}, \mathbf{2})$ under $SU(3)_C \times SU(2)_L$. We allow the following Yukawa coupling involving these fields:

$$\mathcal{L} \supset \lambda \mathbf{5}\mathbf{5}S + h.c. \quad (16)$$

If we denote the PQ charges of $\mathbf{5}$ and $\bar{\mathbf{5}}$ as $\phi$ and $\bar{\phi}$, invariance of Eq. (16) under a surviving discrete $Z_N$ symmetry would imply

$$\phi + \bar{\phi} + s = pN \quad (17)$$

where $p$ is an integer. In this simple model, all the SM particles are assumed to be trivial under the PQ symmetry. The discrete anomaly coefficients are then $A_3 = A_2 = \frac{3}{2}(\phi + \bar{\phi}) = \frac{3}{2}(pN - s)$. Since $A_2 = A_3$, the gauge anomalies are cancelled by the GS mechanism. As

|      | $Q$ | $u^c$ | $d^c$ | $L$  | $e^c$ | $\nu^c$ | $H_u$ | $H_d$ | $S$ |
|------|-----|-------|-------|------|-------|---------|-------|-------|-----|
| $U(1)_A$ | 2   | 2     | 4     | 4    | 2     | 0       | $-4$  | $-6$  | 5   |
| $Z_{11}$ | 2   | 2     | 4     | 4    | 2     | 0       | 7     | 5     | 5   |
| $Z_{12}$ | 2   | 2     | 4     | 4    | 2     | 0       | 8     | 6     | 5   |

Table 2: The anomalous $U(1)$ charge assignment for the DFSZ axion model. Also shown are the charges under two discrete subgroups $Z_{11}$ and $Z_{12}$ which can stabilize the axion.
long as $N \geq 10$, all dangerous couplings that would destabilize the axion through non-renormalizable terms will be sufficiently small. We see that the KSVZ axion can be made consistent in a simple way.

We have also examined the possibility of stabilizing the axion by introducing only a single pair of fermions under the SM gauge group, rather than under the grand unified group. Let us consider a class of models with a pair of fermions transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_A$ as

$$\Psi(\mathbf{3}, n, y, \psi) + \bar{\Psi}(\bar{\mathbf{3}}, \bar{n}, -y, \bar{\psi}),$$  

along with a scalar field $S(1, 1, 0, s)$. The Lagrangian of this model contains a term $\Psi \bar{\Psi} S$ as in Eq. (15) and its invariance under an unbroken $Z_N$ symmetry imposes the constraint

$$\psi + \bar{\psi} + s = pN$$

where $p$ and $N$ are integers. Since the SM particles all have zero anomalous $U(1)$ charge, the anomaly coefficients arise solely from the $(\Psi + \bar{\Psi})$ fields. They are

$$A_3 = \frac{1}{2} (n \psi + n \bar{\psi}) = \frac{n}{2} (pN - s)$$

$$A_2 = \frac{(n-1)n(n+1)}{12} (3\psi + 3\bar{\psi}) = \frac{(n-1)n(n+1)}{4} (pN - s).$$

The Green-Schwarz discrete anomaly cancellation condition, Eq. (8), implies

$$s = pN + \frac{2(-m + bm')}{n(b(n^2 - 1) - 2)}$$

where $b \equiv k_3/k_2$.

By choosing specific values of the Kac–Moody levels, one can solve for $s$, the singlet charge. For instance, in the simple case when $k_3 = k_2 \Leftrightarrow b = 1$,

$$s = \frac{2(m' - m)}{n^3 - 3n} N.$$  

We have normalized all $U(1)_A$ charges to be integers, including $s$, so the unbroken $Z_N$ symmetry will be transparent.

When $n = 2$, $\Psi$ and $\bar{\Psi}$ are $SU(2)$ doublets. One can calculate the charge of $S$ from Eq. (22) and determine the allowed discrete symmetries. For $b = 1$, the solution is $s = 0 \mod N$. This solution would imply that $S^n$ terms in the potential are allowed for any $n$, in conflict with the axion solution. A similar conclusion can be arrived at for $b = 1/2$. For other values of $b$, the $Z_N$ symmetry typically turns out to be too small to solve the strong CP problem. For example, if $b = (2, 3, 1/3, 3/2)$, the allowed discrete symmetries are $(Z_4, Z_7, Z_3, Z_5)$. A special case occurs when $b = 2/3$, in which case $s$ is undetermined, since $A_3/k_3 = A_2/k_2$. If one chooses $s \geq 10$, the KSVZ axion can be stabilized in this case.

If the quarks $\Psi$ and $\bar{\Psi}$ are triplets of $SU(2)_L$, stability of the DSVZ axion solution can be guaranteed in a simple way. For $b \equiv k_3/k_2 = (1, 2, 3, 1/2, 1/3, 2/3, 3/2)$, which are the allowed possibilities if we confine to Kac–Moody levels less than 3, we have from Eq. (21), the unbroken discrete symmetries to be $(Z_9, Z_{21}, Z_{33}, Z_6, Z_3, Z_{15}, Z_{30})$ respectively. For all $Z_N$ with $N \geq 10$, the axion solution will be stable against quantum gravitational corrections.
4 SUSY extensions of invisible axions and a natural link between $M_{\text{SUSY}}, f_a$ and the $\mu$ term

The models of the previous sections can be easily generalized to incorporate supersymmetry. The DFSZ axion is a natural extension of SUSY, since supersymmetry requires the existence of two Higgs doublets. For the axion to be weakly coupled (or invisible), we also need a pair of singlet scalars $S$ and $\tilde{S}$. As we shall see, there is a natural link between the SUSY breaking scale $M_{\text{SUSY}}$, the PQ symmetry breaking scale $f_a$, and the supersymmetric $\mu$–term.

4.1 SUSY DSFZ axion

The superpotential of the model is taken to be

$$W = Qu^c H_u + Qd^c H_d + Le^c H_d + L\nu^c H_u + M_R \nu^c \nu^c + \lambda_1 \frac{H_u H_d S^2}{M_{\text{Pl}}} + \lambda_2 \frac{(S \tilde{S})^2}{M_{\text{Pl}}}.$$  \hspace{1cm} (23)

As discussed earlier, the VEVs of the singlet fields $S$ and $\tilde{S}$ set $f_a \sim 10^{11} \text{ GeV}$. Consequently the $\lambda_1$ term in Eq. (23) will induce a $\mu$–term for the MSSM Higgsinos given by \[18\]

$$\mu = \lambda_1 \frac{\langle S \rangle^2}{M_{\text{Pl}}} \approx 10^2 \text{ GeV}. \hspace{1cm} (24)$$

Furthermore, we have the following relation for the SUSY breaking mass terms in terms of the axion decay constant:

$$M_{\text{SUSY}} \simeq \frac{f_a^2}{M_{\text{Pl}}}. \hspace{1cm} (25)$$

Thus, three a priori unrelated quantities get linked in these models.

To see the link between $M_{\text{SUSY}}$ and $f_a$, we need to minimize the scalar potential\[4] which contains both the soft SUSY breaking terms and the F-terms:

$$V = (\lambda_2 C \frac{(S \tilde{S})^2}{M_{\text{Pl}}} + h.c) + m_s^2 |S|^2 + m_{\tilde{S}}^2 |\tilde{S}|^2 + 4\lambda_2 \frac{|S \tilde{S}|^2}{M_{\text{Pl}}} (|S|^2 + |\tilde{S}|^2). \hspace{1cm} (26)$$

Here $C$ is the soft SUSY breaking term corresponding to the non–renormsizable coupling in the superpotential of Eq. (23).\[5] Let us assume for simplicity that $m_S = m_{\tilde{S}}$. Relaxing this assumption does not have any major effects.

By minimizing the potential of Eq. (26) \[17\] we obtain

$$x^2 = \frac{C \pm \sqrt{C^2 - 12m_S^2}}{12\lambda_2} M_{\text{Pl}} \hspace{1cm} (27)$$

\[4\] As far as the VEVs of the $S$ and $\tilde{S}$ are concerned, we can ignore the $H_u H_d S^2/M_{\text{Pl}}$ term, which can affect the VEVs by only $10^{-14} \text{ GeV}$.

\[5\] Notice that $C$ can be either positive or negative. We shall follow a notation where $C$ is positive. By setting $S = |S|e^{i\alpha}$ and $\tilde{S} = |\tilde{S}|e^{i\beta}$, $C$ would appear in the potential as $2\lambda_2 C \frac{|S \tilde{S}|^2}{M_{\text{Pl}}} \cos 2(\alpha + \beta)$. Minimizing this potential we obtain $\cos 2(\alpha + \beta) = -1$. 

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where \( x^2 = \langle |S| \rangle^2 = \langle |\tilde{S}| \rangle^2 \). This is the desired relation between \( f_a \) and \( M_{\text{SUSY}} \) since Eq. (27) implies

\[
f_a \sim \langle |S| \rangle \sim \sqrt{M_{\text{Pl}}M_{\text{SUSY}}} \sim 10^{11} \text{ GeV.} \tag{28}
\]

The scalar spectrum of this model contains two Higgs bosons from \( S \) and \( \tilde{S} \), besides those in the MSSM, with masses given by

\[
m_{S_1}^2 = -\frac{2\lambda_2 x^2 C M_{\text{Pl}} + 24\lambda_2^2 x^4}{M_{\text{Pl}}^2}, \quad m_{S_2}^2 = \frac{6x^2\lambda_2 x^2 C M_{\text{Pl}} + 40\lambda_2^2 x^4}{M_{\text{Pl}}^2} \tag{29}
\]

where \( x \) is given in Eq. (27). Note that both scalars have masses of order \( M_{\text{SUSY}} \). Their mixings with the doublet Higgs is very small, being suppressed by a factor \( v/x \sim 10^{-8} \).

Among the pseudoscalars in \( S, \tilde{S} \), one combination remains massless and is identified as the axion. (QCD anomaly induces a tiny mass for the axion.) The orthogonal combination has a mass of order \( M_{\text{SUSY}} \), given by

\[
m_{S_3}^2 = \frac{4\lambda_2 C x^2}{M_{\text{Pl}}} \tag{30}
\]

The fermionic partner of the axion, the axino \( (\tilde{a}) \), receives a mass after SUSY breaking from two sources [19]. The superpotential terms induce an axino mass of order \( v_u v_d/M_{\text{Pl}} \sim 10^{-5} \) eV. A second source of its mass is through a Lagrangian term

\[
\mathcal{L} \supset \int d^4 \theta (S^1 S)^2/M_{\text{Pl}}^2 \rightarrow \tilde{a} \tilde{F}_S^c \langle S^* \rangle /M_{\text{Pl}}^2 \sim m_{\text{SUSY}}^2/M_{\text{Pl}} \sim 10^{-3} \text{ eV.} \tag{31}
\]

The fermionic component of the heavy pseudoscalar \( S_3 \) obtains a mass from the superpotential, of order \( M_{\text{SUSY}} \):

\[
\int d^2 \theta (S\tilde{S})^2/M_{\text{Pl}} \rightarrow S_F S_F \langle \tilde{S} \tilde{S} \rangle /M_{\text{Pl}} \sim M_{\text{SUSY}} S_F S_F. \tag{32}
\]

The axino is then the lightest SUSY particle in this model. The lightest neutralino (the LSP in conventional SUSY models) will decay eventually into an axino. To estimate this decay lifetime, we first note that the doublet Higgsino–axino mixing angle \( \theta \) is of order \( M_{\text{SUSY}}/M_{\text{Pl}} \)^{1/2} \sim 10^{-8}. Suppose that the lightest neutralino is mostly a Bino and that its mass is larger than the lightest Higgs boson mass. The decay \( \tilde{B} \rightarrow \tilde{a} + h^0 \) can then proceed with a rate estimated to be \( \Gamma \sim (3\alpha_1/20)(\theta/\tan \beta)^2 M_{\tilde{B}} \). For reasonable values of the parameters we see that the lifetime is of order \( 10^{-7} \) seconds, so the decay will likely occur outside the detector. For some other values of the model parameters, the decay length could be of order cm, which should be measurable experimentally.

Turning now to the stability of the SUSY DFSZ axion, as in the non–SUSY case we make use of a discrete \( Z_N \) symmetry. We start with the same \( U(1)_A \) charge assignment as in Table 2. One difference in the SUSY extension\(^6\) is that there is an extra \( S \) singlet field.

\(^6\) The Lagrangian actually has four \( U(1) \) symmetries, hypercharge, Baryon number, Peccei-Quinn and \( R \) symmetries. However, the \( U(1)_R \) is broken by the gaugino mass term to a discrete subgroup. Conventionally, the gauginos are assigned \( R \) charge of 1, after the \( F \)-component of a spurion field \( Z \) acquires a VEV, to generate gaugino masses, the \( R \) symmetry is reduced to a \( Z_2 \). This \( Z_2 \) will be related to a subgroup of \( B-L \) symmetry [15].
The charge assignment under a $Z_N$ subgroup is displayed in Table 3 for $N = 22$. The $Z_{11}$ symmetry of Table 2 has been embedded into a bigger group $Z_{22}$ in order to prevent a bare mass term $S\bar{S}$ in the superpotential, while allowing a term $(S\bar{S})^2$. This enables us to relate the $\mu$ term, $M_{SUSY}$ and $f_a$. This $Z_{22}$ symmetry forbids to high orders all dangerous terms of the type
\[
\frac{S^m\bar{S}^{n-m}}{M_{Pl}^{n-3}}. \tag{33}
\]
that can potentially destabilize the axion.

A second point that distinguishes the SUSY DFSZ axion model from non-SUSY case is that the anomaly coefficients now receive contributions from the Higgsino and the gaugino. (The absence of $S\bar{S}$ term in $W$, with $(S\bar{S})^2$ allowed implies that the Grassman variable $\theta$ must transform non-trivially under the $Z_N$ symmetry.) Note that the anomalies of $Z_{22}$ are cancelled by Green–Schwarz mechanism.

In Table 3 we have also displayed a $Z_2$ symmetry that turns out to be an unbroken subgroup of $B - L$, which is clearly anomaly free. This $Z_2$ symmetry serves as $R$–parity needed for the stability of the proton in MSSM [15].

|     | $Q$ | $u^c$ | $d^c$ | $L$ | $e^c$ | $\nu^c$ | $H_u$ | $H_d$ | $S$ | $\tilde{S}$ | $\alpha$ | $(A_2, A_3)$ |
|-----|-----|-------|-------|-----|-------|---------|-------|-------|----|----------|----------|-------------|
| $Z_{22}$ | 3  | 19    | 1     | 11  | 15    | 11       | 22    | 18    | 13 | 20       | 11       | (19, 8)     |
| $Z_2$   | 1  | 1     | 1     | 1   | 1     | 1        | 2     | 2     | 2  | 2        | 2        | (0, 0)      |

Table 3: The charges of particle corresponding SUSY DSFZ model with a $Z_{22}$ discrete gauge symmetry.

4.2 SUSY KSVZ axion model and its stability

In the SUSY version of the KSVZ axion model we assume the superpotential to be

\[
W = Qu^cH_u + Qd^cH_d + Le^cH_d + L\nu^cH_u + M_R\nu^c\nu^c + \Psi\bar{\Psi}S + \frac{(S\bar{S})^2}{M_{Pl}}. \tag{34}
\]

Here $\Psi + \bar{\Psi}$ are the new vectorlike fields with QCD anomaly. We use the same mechanism as in the DFSZ model to relate $f_a$ with $M_{SUSY}$ and $\mu$ terms. It requires that the mass term $S\bar{S}$ be absent in $W$, but the coupling $(S\bar{S})^2$ be present.

The simplest possibility is to assume that the fields $\Psi + \bar{\Psi}$ belong to $5 + \bar{5}$ of $SU(5)$. Gauge coupling unification that seems to occur within MSSM can be maintained with the introduction of complete multiplets of $SU(5)$. The charge assignment under a $Z_{12}$ discrete symmetry are shown in Table 4. As before, the anomalies of $Z_{12}$ vanish via the discrete GS mechanism. The lowest order term that breaks the PQ symmetry is $S^{12}$, which is harmless for the axion solution.

In Table 5 we have displayed a variant KSVZ axion model where the Green–Schwarz anomaly cancellation occurs at a higher Kac–Moody level with $k_3 = 3$, and $k_2 = 2$. We have
extended the solution obtained for non–SUSY case to the SUSY case here. There is a single \( \Psi + \bar{\Psi} \) field, belonging to a \((3, 3)\) representation. Exact \( R \)-parity is also realized as a \( Z_2 \) symmetry, as shown in Table 5.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
Q & u^c & d^c & L & e^c & \nu^c & H_u & H_d & \Psi + \bar{\Psi} & S & \tilde{S} & \alpha & (A_2, A_3) \\
\hline
Z_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 11 & 1 & 5 & 6 & & (5, 11) \\
Z_2 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & & (0, 0) \\
\hline
\end{array}
\]

Table 4: SUSY KSVZ axion model realized from \( Z_{12} \times Z_2 \) symmetry.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
Q & u^c & d^c & L & e^c & \nu^c & H_u & H_d & \Psi + \bar{\Psi} & S & \tilde{S} & \alpha & (A_2, A_3) \\
\hline
Z_{30} & 0 & 0 & 0 & 0 & 0 & 0 & 29 & 1 & 14 & 15 & & (24, 57/2) \\
Z_2 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & & (0, 0) \\
\hline
\end{array}
\]

Table 5: SUSY KSVZ axion realized at higher Kac–Moody level from \( Z_{30} \times Z_2 \) symmetry.

In conclusion, we have shown how the axion solution of the strong CP problem can be stabilized against potential quantum gravitational corrections via discrete gauge symmetries. Green–Schwarz anomaly cancellation mechanism plays a crucial role in our models. Both the DFSZ axion model and the KSVZ axion model are realized in rather simple ways without enlarging the low energy particle content. In the supersymmetric extension of these models, we have found an interesting link between the SUSY breaking scale, the axion decay constant, and the SUSY–preserving \( \mu \)–term.

5 Acknowledgement

We thank Ts. Enkhbat and J. Lykken for useful discussion. This work is supported in part by DOE Grant # DE-FG03-98ER-41076, a grant from the Research Corporation and by DOE Grant # DE-FG02-01ER-45684.

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