Electroweak structure of light nuclei within chiral effective field theory

Laura E. Marcucci

University of Pisa
INFN-Pisa

EFB22, September 11, 2013

Thanks to my collaborators:
A. Kievsky, M. Viviani, S. Rosati [Univ. of Pisa & INFN-Pisa]
L. Girlanda [Univ. of Salento]
R. Schiavilla, M. Piarulli [ODU & Jefferson Lab., USA]
S. Pastore [ANL, USA]
Outline

- Introduction: the $\chi$EFT framework for the EW transition operators
- The electromagnetic sector:
  - electromagnetic structure of $A = 2, 3$ nuclei
  - electromagnetic moments and transitions in $A \leq 9$
- The weak sector:
  - muon capture on $A = 2, 3$ nuclei
  - the $pp$ reaction ($p + p \rightarrow d + e^+ + \nu_e$)
- Outlook
Introduction

Until \( \simeq 15 \) years ago: POTENTIAL MODEL APPROACH (PMA)

- Accurate \textbf{phenomenological} potentials: \( V_{NN} + V_{NNN} \) (see AV18+UIX)
- \textbf{Realistic} electroweak currents: Meson-Exchange Currents (MEC) + \( \Delta \)

\( \Rightarrow \) very successful \textbf{BUT} no simple connection to QCD

Chiral Effective Field Theory (\( \chi \)EFT): very short summary

- Nuclear physics \( \simeq \) QCD at low-energy
  \( \rightarrow \) nucleons \( (N) \), pions \( (\pi) \), EW fields \( (A_\mu) \)

- Chiral Lagrangian \( \mathcal{L}(N; \pi; A_\mu) = \sum_\nu \mathcal{L}_\nu \); \( \mathcal{L}_\nu \propto \mathcal{O}(Q/\Lambda_{QCD})^\nu \)
  \( \rightarrow \) regularization with cutoff function \( - \Lambda \simeq 500 - 600 \text{ MeV} + \text{LECs} \)

Disadvantage: limited to processes with \( Q \simeq 1 - 2 m_\pi \)

\textbf{Advantages:}
- nuclear force "hierarchy" \( \rightarrow \) accurate \( V_{NN} + V_{NNN} \)
- consistent framework for interactions + currents
Nuclear EW currents in $\chi$EFT

EW operators: $\rho^\gamma, j^\gamma; \rho^{V/A}, j^{V/A}$

$\text{CVC } \Rightarrow \rho^V/j^V \rightarrow \rho^\gamma/j^\gamma$

History

- Park et al. in heavy-baryon $\chi$PT (HB$\chi$PT) $\rightarrow$ since $\simeq$ 1995
- Pastore et al. in time-ordered perturbation theory (TOPT) $\rightarrow$ since 2009
- Kölling et al. with the unitary transform method $\rightarrow$ in parallel since 2009

To be remarked:

- Park et al. currents ready BEFORE the $\chi$EFT potentials
- $\chi$EFT currents + phenomenological potentials = “hybrid” $\chi$EFT approach
Power counting for $j^\gamma$

$\mathcal{O}(Q^{-2})$  

$O(Q^{-1})$  

$\mathcal{O}(Q^0)$  

$\mathcal{O}(Q^1)$

\[ j^{(-2)} \propto [e_N(1)(p'_1 + p_1) + i\mu_N(1)\sigma \times q] \times \delta(p'_2 - p_2) + 1 \leftrightarrow 2 \]

"standard" one – pion – exchange

= relativistic corrections

Note: vanishing contribution from diagrams like
Similar results between Pastore et al. and Kölling et al.

Differences with Park et al.

for the box-diagrams

for the terms

Park et al. → $J(ij) \propto q \times [g_{4S}(\sigma_i + \sigma_j) + g_{4V}(\tau_i \times \tau_j)^z (\sigma_i \times \sigma_j)]$

Pastore et al. →

$J_{\text{min}}(ij)$ [LECs from $NN$ scatt. data]

$J_{\text{nm}}(ij) \propto q \times [d_1^S \sigma_i + d_1^V (\tau_i^z - \tau_j^z)\sigma_i]$

$J_{\text{OPE}}(ij) \propto \frac{\sigma_j \cdot k_j}{(m_\pi^2 + k_j^2)} q \times [(d_2^S \tau_i \cdot \tau_j + d_2^V \tau_j^z)k_j$

$+ d_3^V (\tau_i \times \tau_j)^z \sigma_i \times k_j]$
- \( d_1^S \) & \( d_2^S \) → \( \mu_d \) & \( \mu_S(A = 3) \)

- \( d_3^V = \frac{d_2^V}{4} \) (\( \Delta \)-resonance saturation picture) → 

- \( d_1^V \) and \( d_2^V \):

  SET I → \( \sigma_{np} \) & \( \mu_V(A = 3) \)

|                  | \( d_1^V \) | \( d_2^V \) |
|------------------|------------|------------|
| N3LO+N2LO-500    | 10.36      | 17.42      |
| N3LO+N2LO-600    | 41.84      | 33.14      |
| AV18+UIX-500     | 45.10      | 35.57      |
| AV18+UIX-600     | 257.50     | 75.00      |

⇒ \( d_2^V \) fixed by \( \Delta \)-resonance saturation

SET II → \( \sigma_{np} \) ⇒ prediction for \( \mu_V(A = 3) \)

SET III → \( \mu_V(A = 3) \) ⇒ prediction for \( \sigma_{np} \)
| \( \Lambda \) [MeV] | \( \mathcal{O}(Q^{-2}) \) | \( \mathcal{O}(Q^{-1}) \) | \( \mathcal{O}(Q^0) \) | \( \mathcal{O}(Q^1) \)-TPE | \( \mathcal{O}(Q^1) \)-min | \( \mathcal{O}(Q^1) \)-nm | \( \mathcal{O}(Q^1) \)-OPE | \( \mathcal{O}(Q^1) \)-OPE |
|---|---|---|---|---|---|---|---|---|
| | \( \sigma_{np} \) [mb] | | | | | | | |
| | 500 | 600 | 500 | 600 | 500 | 600 | 500 | 600 |
| \( \mathcal{O}(Q^{-2}) \) | 305.8 | 304.6 | -2.193 | -2.182 | | | | |
| \( \mathcal{O}(Q^{-1}) \) | 320.6 | 318.9 | -2.408 | -2.392 | | | | |
| \( \mathcal{O}(Q^0) \) | 319.2 | 317.6 | -2.384 | -2.370 | | | | |
| \( \mathcal{O}(Q^1) \)-TPE | 321.3 | 320.5 | -2.403 | -2.432 | | | | |
| \( \mathcal{O}(Q^1) \)-min | 321.3 | 320.5 | -2.413 | -2.415 | | | | |
| \( \mathcal{O}(Q^1) \)-nm \( d_1^V \) – SET I | 315.2 | 305.7 | -2.297 | -2.142 | | | | |
| \( \mathcal{O}(Q^1) \)-OPE \( d_2^V \) – SET I | \textbf{332.6} | \textbf{332.6} | \textbf{-2.553} | \textbf{-2.553} | | | | |
| \( \mathcal{O}(Q^1) \)-nm \( d_1^V \) – SET II | 329.1 | 328.5 | -2.562 | -2.561 | | | | |
| \( \mathcal{O}(Q^1) \)-OPE \( d_2^V \) – SET II | \textbf{332.6} | \textbf{332.6} | \textbf{-2.612} | \textbf{-2.622} | | | | |
| \( \mathcal{O}(Q^1) \)-nm \( d_1^V \) – SET III | 326.0 | 324.7 | -2.502 | -2.491 | | | | |
| \( \mathcal{O}(Q^1) \)-OPE \( d_2^V \) – SET III | 329.4 | 328.8 | \textbf{-2.553} | \textbf{-2.553} | | | | |
| Exp. | 332.6±0.7 | | | | | | | -2.553 |
### Static properties for $A = 2, 3$ nuclei

|                  | Theory          | Exp.                     |
|------------------|-----------------|--------------------------|
| $r_c(d)$ [fm]    | $1.972 \pm 0.004$ | $1.9733 \pm 0.0044$      |
| $Q(d)$ [fm$^2$]  | $0.2836 \pm 0.0016$ | $0.2859 \pm 0.0003$      |
| $Q(d)$ [fm$^2$] (PMA-AV18) | $0.275$ |                       |
| $r_c(^3\text{He})$ [fm] | $1.962 \pm 0.004$ | $1.959 \pm 0.030$       |
| $r_c(^3\text{H})$ [fm] | $1.756 \pm 0.006$ | $1.755 \pm 0.086$       |
| $r_m(^3\text{He})$ [fm] | $1.905 \pm 0.022$ | $1.965 \pm 0.153$       |
| $r_m(^3\text{H})$ [fm] | $1.791 \pm 0.018$ | $1.840 \pm 0.181$       |

Piarulli et al., PRC 87, 014006 (2013)
A = 2 results:
current operator

Piarulli et al., PRC 87, 014006 (2013)
\( A = 3 \) results:
charge operator

Piarulli et al., PRC 87, 014006 (2013)
$A = 3$ results: current operator

Piarulli et al., PRC 87, 014006 (2013)
Magnetic moments for $A = 6 - 9$ nuclei

hybrid $\chi$EFT = \textit{AV18/UIX GFMC w.f.} + Pastore et al. $\chi$EFT currents

|          | PMA$^{[1]}$ | $\chi$EFT$^{[2]}$ | Exp.  |
|----------|-------------|-------------------|-------|
| $\mu_S(A = 7)$ | 0.83        | 0.91              | 0.929 |
| $\mu_V(A = 7)$ | -4.57       | -4.66             | -4.654|
| $\mu_S(A = 8)$ | 1.18        | 1.30              | 1.345 |
| $\mu_V(A = 8)$ | -0.18       | -0.19             | -0.309|
| $\mu_S(A = 9; 3/2^-)$ | 0.89        | 1.01              | 1.023 |
| $\mu_V(A = 9; 3/2^-)$ | -1.41       | -1.57             | -1.609|
| $\mu_S(A = 9; 3/2^+)$ | 0.78        | 0.88              |       |
| $\mu_V(A = 9; 3/2^+)$ | 4.17        | 4.35              |       |

[1] Marcucci et al., PRC 78, 065501 (2008)
[2] Pastore et al., PRC 87, 035503 (2013)
EM transitions widths in $A = 6 - 9$ nuclei

- $^9$Be($^{5/2}^- \rightarrow ^3/2^-$) $B(E2)$
- $^9$Be($^{5/2}^- \rightarrow ^3/2^-$) $B(M1)$
- $^8$B($^3^+ \rightarrow 2^+$) $B(M1)$
- $^8$B($^1^+ \rightarrow 2^+$) $B(M1)$
- $^8$Li($^3^+ \rightarrow 2^+$) $B(M1)$
- $^8$Li($^1^+ \rightarrow 2^+$) $B(M1)$
- $^7$Be($^{1/2}^- \rightarrow ^3/2^-$) $B(M1)$
- $^7$Li($^{1/2}^- \rightarrow ^3/2^-$) $B(E2)$
- $^7$Li($^{1/2}^- \rightarrow ^3/2^-$) $B(M1)$
- $^6$Li($0^+ \rightarrow 1^+$) $B(M1)$

EXPT

GFMC(IA)

GFMC(TOT)

Ratio to experiment

Pastore et al., PRC 87, 035503 (2013)
Power counting for $j^A$

Note:

- $O(Q^1)$: loop and two-pion-exchange contributions (not yet calculated)

- Park et al. only available model at $O(Q^0)$ → one LEC - $d_R$

$$d_R = \frac{M_N}{\Lambda_{\chi EFT}} c_D + \frac{1}{3} M_N (c_3 + 2 c_4) + \frac{1}{6}$$

Gårdestig and Phillips, PRL 96, 232301 (2006)
Gazit et al., PRL 103, 102502 (2009)

- fit $c_D$ and $c_E$ (in TNI at N2LO) to $B(A = 3)$ and $GT_{Exp}$

Laura E. Marcucci (Univ. of Pisa & INFN) EW structure of light nuclei within $\chi$EFT EFB22, September 11, 2013
\( \Rightarrow \{c_D; c_E\}_{\text{MAX}} \) and \( \{c_D; c_E\}_{\text{MIN}} \)

| Model       | \( \Lambda \) [MeV] | \( c_D \)  | \( c_E \)  | \( B(^{4}\text{He}) \) [MeV] | \( ^2a_{nd} \) [fm] |
|-------------|---------------------|------------|------------|----------------------------|-----------------|
| N3LO/N2LO*  | 500                 | 1.0        | -0.029     | 28.36                      | 0.675           |
| N3LO/N2LO   | 500                 | -0.12      | -0.196     | 28.49                      | 0.666           |
| N3LO/N2LO   | 600                 | -0.26      | -0.846     | 28.64                      | 0.696           |
| Exp.        |                     |            |            | 28.30                      | 0.645(10)       |

Marcucci et al., PRL 108, 052502 (2012); Viviani et al., arXiv:1307.5167, submitted to PRL
Elastic $p-d$ scattering $E_{lab} = 3$ MeV

- NN (AV18 + $\chi$EFT)
- NN+NNN − $\chi$EFT
- AV18+IL7
Elastic $p-^3\text{He}$ scattering $E_p = 5.54$ MeV
Results: muon capture on $A = 2, 3$ nuclei

- $\mu^- + d \rightarrow n + n + \nu_\mu \rightarrow$ capture rate in the doublet iperfine state $\Gamma^D$
- $\mu^- + ^3\text{He} \rightarrow ^3\text{H} + \nu_\mu \rightarrow$ total capture rate $\Gamma_0$

|                         | $\Gamma^D(1S_0)$ | $\Gamma^D$ | $\Gamma_0$ |
|-------------------------|-------------------|-------------|------------|
| IA $- \Lambda = 500$ MeV | 238.8             | 381.7       | 1362       |
| IA $- \Lambda = 600$ MeV | 238.7             | 380.8       | 1360       |
| FULL $- \Lambda = 500$ MeV | 254.4(9)          | 399.2(9)    | 1488(9)    |
| FULL $- \Lambda = 600$ MeV | 255(1)            | 399(1)      | 1499(9)    |

$\Gamma^D = 399(3)$ s$^{-1}$ & $\Gamma_0 = 1494(21)$ s$^{-1}$

vs. $\Gamma^D(\text{exp}) \cdots$ & $\Gamma_0(\text{exp}) = 1496(4)$ s$^{-1}$

Marcucci et al., PRL 108, 052502 (2012)
Marcucci et al. (2011) [PMA+χEFT*]

Wang et al. (1965)

Bertin et al. (1973)

Bardin et al. (1986)

Cargnelli et al. (1986)

Marcucci et al. (2012) [χEFT]
The proton-proton weak capture: where do we stand

\[ S(E) = S(0) + S'(0) E + \frac{1}{2} S''(0) E^2 + \cdots \]

- Gamow peak: \( E \approx 6 \text{ keV} \) in the Sun, \( E \approx 15 \text{ keV} \) in larger stars
- Latest review: SFII: E.G. Adelberger et al., RMP 83, 195 (2011)

\[ S(0) = 4.01(1 \pm 0.009) \times 10^{-23} \text{ MeV fm}^2 \]
(PMA\[^1\], \( \chi \text{EFT*}[2] \) and \( \chi \text{EFT}[3] \) calculations)

\[ S'(0) = S(0) (11.2 \pm 0.1) \text{ MeV}^{-1} \]
(only a PMA calculation)

No realistic calculation of \( S''(0) \)

[1] Schiavilla et al., PRC 58, 1263 (1998)
[2] Park et al., PRC 67, 055206 (2003)
[3] Chen et al., PRC 67, 025801 (2003)
Very recently . . .

\( S(E) \) in \( \chi \text{EFT} \) and PMA

- Energy range 2 keV – 100 keV
- PMA [AV18] or \( \chi \text{EFT} \) [N3LO] + FULL EM interaction
- \( pp \ L \leq 1 \) partial waves: \( ^1S_0 \) + all \( P \)-waves

\[
S(0) - ^1S_0 \quad \text{(in } 10^{-23} \text{ MeV fm}^2) \]

|                  | \( V_{\text{nucl}} + V_{\text{Coul}} \) | \( V_{\text{nucl}} + V_{\text{EM}} \) |
|------------------|-----------------------------------------|-------------------------------------|
| PMA-IA           | 3.99                                    | 3.96                                |
| PMA-FULL         | 4.03                                    | 4.00                                |
| \( \chi \text{EFT}(500) \)-IA | 3.96                                    | 3.94                                |
| \( \chi \text{EFT}(500) \)-FULL | 4.03                                    | 4.01                                |
| \( \chi \text{EFT}(600) \)-IA | 3.94                                    | 3.93                                |
| \( \chi \text{EFT}(600) \)-FULL | 4.01                                    | 4.01                                |

- agreement with \( S^{\text{SFII}}(0) = 4.01(1 \pm 0.009) \)
- \( V_{\text{EM}} - V_{\text{Coul}} \rightarrow \leq 1 \% \) effect
- agreement PMA-\( \chi \text{EFT} \)
- very small cutoff dependence (\( \leq 1 \% \))

Marcucci et al., PRL 110, 192503 (2013)
Cumulative contributions to $S(0)$

|                  | $^1S_0$  | $\cdots + ^3P_0$ | $\cdots + ^3P_1$ | $\cdots + ^3P_2$ |
|------------------|---------|------------------|------------------|------------------|
| PMA              | 4.000(3)| 4.003(3)         | 4.015(3)         | 4.033(3)         |
| $\chi$EFT(500)  | 4.008(5)| 4.011(5)         | 4.020(5)         | 4.030(5)         |
| $\chi$EFT(600)  | 4.007(5)| 4.010(5)         | 4.019(5)         | 4.029(5)         |

- $P$-waves contribution to $S(0) \approx 1\%$
- theoretical uncertainty very small

$$S(0) = 4.03(1 \pm 0.006) \times 10^{-23} \text{ MeV fm}^2$$

vs.

$$S(0)^{\text{SFII}} = 4.01(1 \pm 0.009) \times 10^{-23} \text{ MeV fm}^2$$
Energy dependence of $S(E)$
Polynomial fit of \( S(E) \)

\[
\text{Fit 1: } S(E) = S(0) + S'(0) E + \frac{1}{2} S''(0) E^2
\]

\[
\text{Fit 2: } S(E) = S(0) + S'(0) E + \frac{1}{2} S''(0) E^2 + \frac{1}{6} S'''(0) E^3
\]

| \( S'/S(0) \) [MeV\(^{-1}\)] | \( S''/S(0) \) [MeV\(^{-2}\)] | \( S'''/S(0) \) [MeV\(^{-3}\)] | \( \chi^2 \) |
|-------------------------------|-------------------------------|-------------------------------|------------|
| \( S + P - \) Fit 1          | 12.59(1)                      | 199.3(1)                      | 8.8 \times 10^{-4} |
| \( S + P - \) Fit 2          | 11.94(1)                      | 248.8(2)                      | 1.9 \times 10^{-4} |
| \( ^1S_0 - \) Fit 1          | 12.23(1)                      | 178.4(3)                      | 1.2 \times 10^{-3} |
| \( ^1S_0 - \) Fit 2          | 11.42(1)                      | 239.6(5)                      | 1.9 \times 10^{-4} |
| \( ^1S_0 - \chi\text{EFT}^{[1]} \) | 11.3(1)                       | 170(2)                        | 3.4 \times 10^{-1} |

\[
S'(0)/S(0)^{\text{SFII}} = (11.2 \pm 0.1) \text{ MeV}^{-1}
\]

\[^{[1]}\] Chen et al., PLB 720, 385 (2013)
Now that EW processes can be studied in $\chi$EFT ⋯

- Study other EM processes of interest in $\chi$EFT (as $p + d \rightarrow ^3\text{He} + \gamma$)
- Develop weak current operators in $\chi$EFT-TOPT
- Repeat $pp$ reaction and muon captures studies
- Study other weak processes of interest in $\chi$EFT (as $p + ^3\text{He} \rightarrow ^4\text{He} + e^- + \nu_e$)
- ⋯