LIMITED DEFORMABILITY DESIGN OF HIGH-STRENGTH CONCRETE BEAMS IN LOW TO MODERATE SEISMICITY REGIONS

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Abstract. In the design of reinforced concrete (RC) beams located in low-moderate seismicity regions, adequate flexural deformability apart from flexural strength to cater for the imposed seismic demand should be designed. As per the existing RC design codes, this is achieved by restricting the maximum neutral axis depth or tension steel ratio, or limiting the minimum confining steel. However, these deemed-to-satisfy rules were derived many years ago based on normal-strength concrete and steel, which would impair the deformability when applied directly to RC beams made of high-strength materials. To resolve the problem, a new design method based on a prescribed deformability is advocated. In this study, the authors proposed that instead of complying with the deemed-to-satisfy rules, a consistent deformability derived based on the design requirements of Eurocode 2 should be provided to all RC beams located in low-moderate seismicity regions. Using the theoretical formulas developed previously by the authors, two different sets of design values expressed in terms of maximum tension steel ratio and neutral axis to beam effective depths for different concrete and steel yield strengths are evaluated. Finally, simplified guidelines for designing RC beams satisfying the proposed deformability requirement are developed for practical design application.

Keywords: beams, confinement, curvature, deformability, design formulas, high-strength concrete, high-strength steel, low-moderate seismicity, reinforced concrete, rotation capacity.

1. Introduction

In the traditional design of reinforced concrete (RC) beams not located in seismic regions, more attention has been put on the design of sufficient flexural strength than flexural deformability. The provision of flexural deformability only relies on some empirical deemed-to-satisfy rules that control the maximum tension steel area or neutral axis depth. This is understandable because the deformability demand in non-seismic region is not very large and that provided by the existing deemed-to-satisfy rules should be sufficient for RC beams made of normal-strength concrete (NSC) and normal-strength steel (NSS) (Park and Ruitong 1988; Kwan et al. 2006). However, for RC beams located in regions of low to moderate seismicity, the design for sufficient deformability to cater for the imposed seismic demand (Kuang and Atanda 2005; Djebbar and Chikh 2007; Seifi et al. 2008; Tsang et al. 2009) is as crucial as the design of sufficient flexural strength. Furthermore, the deformability provided to these beams should be larger than that provided to those in non-seismic regions. Therefore, the existing empirical deemed-to-satisfy rules should not be applied for designing RC beams located in low to moderate seismicity regions.

Apart from the above, the existing deemed-to-satisfy rules are not able to provide a consistent deformability to RC beams made of high-strength concrete (HSC) and/or high-strength steel (HSS). Except in Eurocode 2 (ECS 2004) that a set of more stringent requirements are specified for HSC beams, these empirical rules are not dependent on concrete and steel yield strength. However, as reported in a series of theoretical studies conducted on flexural deformability carried out previously by the authors (Ho et al. 2010a,b; Zhou et al. 2010), it is evident that at a given tension steel ratio or neutral axis depth, the deformability of RC beams varies significantly with the concrete and steel yield strength. Therefore, it is apparent that the deformability provided to RC beams made of HSC and/or HSS as per the deemed-to-satisfy rules would be smaller than that provided to RC beams made of NSC and NSS. More critically, the deformability would decrease to an unacceptably low level if these rules are adopted for HSC beams. Considering nowadays that the adoption of HSC and HSS, which reduces the amount of construction materials under the same design load and hence lower the embodied energy and carbon level in the structures (Bilodeau and Malhotra 2000; Kosior-Kazberuk and Lelusz 2007; Scrivener and Kirkpatrick 2008; Xu et al. 2008), are getting more popular in tall buildings construction, the existing empirical rules for deformability design of RC beams in low to moderate seismicity regions should be revised to incorporate the adoption of HSC and/or HSS.

From performance-based design point of view, adequate flexural deformability design would prevent the beams from immediate collapse under earthquake attack (Vaidogas 2005; Zareian et al. 2010). During an
earthquake attack, RC beams with sufficient deformability could resist the required seismic deflection without suffering severe inelastic damage and collapse (Wu et al. 2004). The enormous energy imposed by earthquake can subsequently be dissipated by redistributing moment to other parts of the beams through formation of plastic hinges (Bae and Bayrak 2008). To achieve this purpose, the reinforcement within the critical regions (Pam and Ho 2009) should be designed carefully such that a certain level of deformability would be provided and plastic hinges can be formed successfully (Ho and Pam 2003; Havaei and Keramati 2011; Yan and Au 2010; Ho 2011). The deformability could also be increased by providing sufficient confining pressure to the concrete core within critical region in the following ways by: (1) confining the concrete member using circular or rectangular hollow steel tube (Ellobody and Young 2006; Kuranovas and Kvedaras 2007; Salma and Marcikaitis 2007; Szmigiera 2007; Soundararajan and Shanmugasundaram 2008; Kuranovas et al. 2009); (2) using external steel plate (Su et al. 2009); (3) wrapping the concrete member with fibre reinforced polymer (Kamiński and Trapko 2006; Vallivonis and Skuturna 2007; Benzaid et al. 2008; Lam and Teng 2009; Wu and Wei 2010). These methods are commonly adopted in the design of low to medium rise buildings. For very tall building structures, the huge amount of energy induced by earthquake can in addition be dissipated by installing dampers (Matsagar and Jangid 2005; Lewandowski and Grzymislawski 2009; Chen and Han 2010) and adopting base isolation (Takewaki and Fujita 2009).

To evaluate the deformability of RC beams, the authors have carried out a series of theoretical studies to investigate the critical factors affecting their deformability (Ho et al. 2010a; Zhou et al. 2010). In these studies, it was proposed to use the “normalised rotation capacity” – defined as the product of ultimate beam curvature and effective depth — to evaluate the deformability of RC beams. Based on the results, it was found that the deformability of RC beams increases as the degree of reinforcement decreases or confining pressure increases. The use of HSC would decrease the deformability at a constant degree of reinforcement, but increase the deformability at a constant tension steel ratio. On the other hand, the use of HSS would decrease the deformability at a constant tension steel ratio, but it increases the deformability at a constant degree of reinforcement. Furthermore, the addition of confining steel would always increase the deformability of RC beams. Based on the results obtained, a formula for direct evaluation of deformability of RC beams based on the above parameters was developed. In a separate study the authors have also investigated the effects of these parameters on the limits of both flexural strength and deformability that can be achieved by a given beam section. From the results, it was found that for a given concrete strength and confining pressure, there is a maximum and minimum limits of flexural strength and deformability that can be achieved simultaneously. Moreover, for a given pair of concrete strength and deformability, there is a maximum allowable limit of the degree of reinforcement or tension steel ratio, beyond which the deformability can never be achieved apart from increasing the confining pressure or beam size.

In this paper, the deformability required for designing RC beams located in low to moderate seismicity regions will be derived based on the design requirements of Eurocode 2 (ECS 2004). The flexural design of RC beams possessing this deformability is named by the authors as the “Limited Deformability Design”. As per Eurocode 2, the derived deformability will have sufficient rotation capacity at ultimate limit state for the formation of plastic hinge and hence allow moment redistribution to occur. Based on this, the maximum degree of reinforcement and tension steel ratio for designing RC beams with the prescribed deformability would be derived for different concrete and steel yield strength (Zhou et al. 2010). Lastly, for practical design application, a set of simplified design guidelines that depend on concrete and steel yield strength are developed for limited deformability design.

2. Nonlinear moment-curvature analysis

The deformability of RC beams is studied using the method of nonlinear moment-curvature analysis developed previously by the authors Pam et al. (2001) and Ho et al. (2003). The stress-strain curves of concrete as per Attard and Setunge (1996) were adopted while that of steel reinforcement follows the model given by Eurocode 2 (ECS 2004) but with the stress-path dependence incorporated to take account of the unloading properties. The unloading path is having the same initial elastic modulus until it reaches zero steel stress. The stress-strain curves of concrete and steel are shown in Fig. 1.

There were five assumptions made in the analysis: (1) plane sections before bending remain plane after bending; (2) the tensile strength of the concrete may be neglected; (3) there is no relative slip between concrete and steel reinforcement; (4) the concrete core is confined while the concrete cover is unconfined; (5) the confining pressure provided to the concrete core by confinement is assumed to be constant throughout the concrete compression zone. Assumptions (1) to (4) are commonly accepted and have been adopted by various researchers (Park et al. 2007; Au et al. 2009; Bai and Au 2009; Lam et al. 2009; Kwak and Kim 2010). Assumption (5) is not exact but however a fairly reasonable assumption in the sense that: (i) at small concrete strains, the variation of confining pressure would not have significant effect on the confined concrete stress (Attard and Setunge 1996); (ii) when the extreme fibre of confined concrete reaches about 0.003–0.004 before concrete cover spalls off entirely, there will be some variations of confining pressure within the concrete compression zone due to strain gradient. However, as this happens within a narrow range of concrete strain, the differences in the confined concrete compressive force and moment capacity of column are not significant; (iii) after the concrete cover had spalled off completely at large concrete strain, the Poisson’s ratio of concrete increases abruptly that causes the confining steel to yield. The confining pressure becomes a constant equal to $0.5 k_p f_{c}^\prime$, where $k_p$ is the confinement effectiveness factor.
Fig. 1. Stress-strain curves of concrete and steel reinforcement

a) Stress-strain curves of concrete

b) Stress-strain curve of steel with stress-path dependence considered

Fig. 2. Beam sections analysed

(Mander et al. 1988), $\rho$, and $f_y$ are respectively the volumetric ratio and yield strength of confining steel. In the analysis, the moment-curvature curve of the beam section is analysed by applying prescribed curvatures incrementally starting from zero. At a prescribed curvature, the stresses developed in the concrete and the steel are determined from their stress-strain curves. Then, the neutral axis depth and resisting moment are evaluated from equilibrium conditions. The above procedure is repeated until the resisting moment has increased to the peak and then decreased to below 80% of the peak moment. Fig. 2 describes a typical beam sections adopted in the nonlinear moment-curvature analysis.

3. Parametric study for deformability

3.1. Definition of deformability

In this study, the flexural deformability of beam sections are expressed in terms of normalised rotation capacity $\theta_{pl}$ defined as follows (Ho et al. 2010a):

$$\theta_{pl} = \frac{\phi_u d}{f_y}$$

where $\phi_u$ is the ultimate curvature, $d$ is the effective depth. The ultimate curvature is taken as the curvature when the resisting moment has dropped to 0.8 $M_p$ after reaching $M_p$, where $M_p$ is the peak moment. The value of $\phi_u$ represents the rotation capacity of beam with plastic hinge length $\ell_p$ equal to its effective depth. For concrete beams subjected to pure flexure, the plastic hinge length remains relatively constant at about $0.4d$ (Mendis 2001) to $0.5d$ (Standards New Zealand 2006). Therefore, it is fairly reasonable to use the proposed normalised rotation capacity to compare the deformability of different RC beams.

A comprehensive parametric study on the effects of various factors on the normalised rotation capacity has been conducted previously (Ho et al. 2010a). The studied factors are: (1) degree of reinforcement – which measures the degree of beam section being under- or over-reinforced (Eq. 4); (2) concrete strength; (3) steel area ratios – which is defined as the tension or compression steel area divided by the effective area of beam section (i.e. breadth $\times$ effective depth); (4) steel yield strength; and (5) confining pressure. The beam sections analysed was shown in Fig. 2. The concrete strength $f_c$ was varied from 40 to 100 MPa, the confining pressure $f_r$ evaluated using the method recommended by Mander et al. (1988) was varied from 0 to 4 MPa, the tension steel ratio $\rho$ was varied from 0.4 to 2 times the balanced steel ratio, the compression steel ratio $\rho_c$ was varied from 0 to 2%, and the tension $f_{yt}$ and compression $f_{yc}$ steel yield strength were varied from 400 to 800 MPa.

3.2. Definition of balanced steel ratio

The balanced steel ratio of a beam section provides the area of tension steel which causes the steel with maximum tensile stress to yield during failure. It is defined as $\rho_{bo} = A_{sb}/bd$, where $A_{sb}$ is the balanced steel area. For beam section containing tension steel area less than the balanced steel area, the steel will yield during failure and the section is under-reinforced. Otherwise, the steel will not yield during failure and the section in over-reinforced. For beam sections containing compression steel ratio $\rho_c$, the balanced steel ratio $\rho_b$ is given by:

$$\rho_b = \rho_{bo} + \left(\frac{f_{yc}}{f_{yt}}\right)\rho_c$$

The values of $\rho_{bo}$ for various concrete strengths and confining pressure are listed in Table 1 for different yield strength of tension steel (Ho et al. 2003).
Table 1. Balanced steel ratios $\rho_{bo}$ for different tension steel yield strength

| $f_{co}$ (MPa) | $f'_t = 0$ MPa | $f'_t = 1$ MPa | $f'_t = 2$ MPa | $f'_t = 3$ MPa | $f'_t = 4$ MPa |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 40             | 4.74           | 5.98           | 6.90           | 7.73           | 8.56           |
| 50             | 5.63           | 6.91           | 7.86           | 8.78           | 9.60           |
| 60             | 6.46           | 7.79           | 8.77           | 9.70           | 10.59          |
| 70             | 7.29           | 8.62           | 9.61           | 10.54          | 11.50          |
| 80             | 8.06           | 9.38           | 10.37          | 11.35          | 12.29          |
| 90             | 8.77           | 10.11          | 11.13          | 12.11          | 13.03          |
| 100            | 9.42           | 10.80          | 11.82          | 12.78          | 13.76          |

Balanced steel ratios $\rho_{bo}$ (% for different compression steel yield strength $f_{yc}$)

| $f_{co}$ (MPa) | $f'_c = 0$ MPa | $f'_c = 1$ MPa | $f'_c = 2$ MPa | $f'_c = 3$ MPa | $f'_c = 4$ MPa |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 40             | 2.74           | 3.60           | 4.23           | 4.83           | 5.37           |
| 50             | 3.23           | 4.12           | 4.78           | 5.40           | 6.00           |
| 60             | 3.69           | 4.61           | 5.29           | 5.93           | 6.55           |
| 70             | 4.13           | 5.06           | 5.76           | 6.41           | 7.04           |
| 80             | 4.56           | 5.50           | 6.19           | 6.85           | 7.49           |
| 90             | 4.94           | 5.90           | 6.59           | 7.28           | 7.91           |
| 100            | 5.29           | 6.27           | 6.97           | 7.67           | 8.29           |

Balanced steel ratios $\rho_{bo}$ (% for different tension steel yield strength $f_{yt}$)

| $f_{co}$ (MPa) | $f'_t = 0$ MPa | $f'_t = 1$ MPa | $f'_t = 2$ MPa | $f'_t = 3$ MPa | $f'_t = 4$ MPa |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 40             | 1.82           | 2.48           | 2.96           | 3.42           | 3.84           |
| 50             | 2.13           | 2.82           | 3.33           | 3.80           | 4.25           |
| 60             | 2.43           | 3.14           | 3.66           | 4.14           | 4.61           |
| 70             | 2.70           | 3.43           | 3.96           | 4.45           | 4.93           |
| 80             | 2.97           | 3.69           | 4.22           | 4.75           | 5.21           |
| 90             | 3.22           | 3.95           | 4.50           | 5.00           | 5.49           |
| 100            | 3.44           | 4.19           | 4.74           | 5.22           | 5.74           |

The following empirical equation was also derived for easy practical design application:

$$\rho_{bo} = 0.005 \left( f_{co} \right)^{0.58} \left( 1 + 1.2 f'_t \right)^{0.3} \left( f_{yt} / 460 \right)^{-1.35} .$$  \hspace{1cm} (3)

All strengths are in MPa, $400 \text{ MPa} \leq f_{yt} \leq 800 \text{ MPa}$ and $0 \leq f'_t \leq 4 \text{ MPa}$.

3.3. Effects of degree of reinforcement, concrete and steel yield strength

The degree of reinforcement $\lambda$, which accounts for the degree of section being under- or over-reinforced, is expressed in Eq. (4):

$$\lambda = \frac{f_{yt} \rho_t - f_{yc} \rho_c}{f_{yt} \rho_{bo}} .$$  \hspace{1cm} (4)

By definition, the beam section is classified as under-reinforced, balanced and over-reinforced sections when $\lambda$ is less than, equal to and larger than 1.0 respectively. To investigate the effects of $\lambda$ on the deformability of RC beams, the normalised rotation capacity $\theta_{pl}$ is plotted against $\lambda$ and $\rho_t$ in Figs 3a and 4b respectively for different tension steel yield strength $f_{yt}$. On the other hand, Figs 3b and 4a plot the $\theta_{pl}$ against $\lambda$ and $\rho_t$ for different compression steel yield strength $f_{yc}$. Generally, it is observed from Fig. 4 that at a constant $\lambda$, the deformability decreases as $\lambda$ increases until reaching $\lambda = 1.0$. After that, the deformability remains relatively constant. On the other hand, it can be seen from Fig. 3a that at a constant $\lambda$, the deformability decreases as the concrete strength increases. However, if HSC is used at the same tension steel ratio $\rho_t$, it is evident from Fig. 3b that the deformability increases as concrete strength increases albeit that HSC is less deformable per se. This is because the balanced steel ratio increases as concrete strength increases, and hence for a given $\rho_t$, $\lambda$ decreases and the deformability increases.

To investigate the effects of steel yield strengths on the deformability of RC beams, $\theta_{pl}$ is plotted against $\lambda$ and $\rho_t$ in Figs 4a and 4b respectively for different tension steel yield strength $f_{yt}$. On the other hand, Figs 5a and 5b plot the $\theta_{pl}$ against $\lambda$ and $\rho_t$ for different compression steel yield strength $f_{yc}$. Generally, it is observed from Fig. 3a that at a constant $\lambda$, the deformability increases as the tension steel yield strength increases, notwithstanding that HSS is less deformable per se. However, it decreases as the tension steel yield strength increases at a given $\rho_t$. From Figs 5a and 5b, it is seen that the deformability increases only very slightly as the compression steel yield strength increases at constant $\lambda$. Nonetheless, the deformability increases significantly as the compression steel yield strength increases at constant $\rho_t$.
3.4. Effects of confining pressure

To study the effect of the confining pressure $f_c$, $\theta_{pl}$ is plotted against confining pressures $f_c$ for different concrete strength $f_{con}$, degree of reinforcement $\lambda$ and tension steel ratios $\rho_t$ in Fig. 6. It is evident from Fig. 6a that at a given $\lambda$, $\theta_{pl}$ increases as the $f_c$ increases for all concrete strength. It is also seen from Fig. 6b that at a fixed $f_{con}$, $\theta_{pl}$ increases as $f_t$ increases for all $\lambda$. In Fig. 6c, it is seen that at a fixed $f_{con}$, $\theta_{pl}$ increases as the confining pressure $f_c$ increases for all $\rho_t$.

3.5. Effects of neutral axis depth

Alternatively, the degree of beam section being under- or over-reinforced may be expressed in terms of $x_u/x_{ub}$, where $x_u$ and $x_{ub}$ are the neutral axis depths of beam section and the balanced section, respectively. As per the existing RC design codes (Ministry of Construction 2001; ECS 2004; Standards New Zealand 2006; ACI Committee 2008), the value of $x_u$ is measured at the ultimate limit state at maximum moment. To be consistent, the values of $x_u$ and $x_{ub}$ neutral axis depths presented in this study are all taken at the maximum moment point. To study the effect of $x_u/x_{ub}$ and the neutral axis depth itself (expressed in dimensionless form of $x_u/d$) on the deformability of beams, $\theta_{pl}$ is plotted against $x_u/x_{ub}$ and $x_u/d$ in Figs 7 and 8 for different concrete strength and tension steel yield strength respectively. It can be observed from Fig. 7 that
the deformability decreases as $x_u/x_{ub}$ or $x_u/d$ increases until $x_u/x_{ub}$ is equal to 1.0, after which the deformability remains relatively constant. At a given ratio of $x_u/x_{ub}$ or $x_u/d$, the deformability decreases as concrete strength increases. Therefore, adopting HSC would decrease the deformability of concrete beam at a specified ratio of $x_u/x_{ub}$ or $x_u/d$. It is now evident that the empirical deemed-to-satisfy rules stipulated in the existing RC design codes, which restrict the maximum neutral axis depth for all concrete strength, would provide a smaller deformability to beam when HSC is adopted. For the effects of steel yield strength, it is apparent from Fig. 8a that at a given $x_u/x_{ub}$, the deformability increases as the tension steel yield strength increases. Nevertheless, at a given $x_u/d$, it can be seen from Fig. 8b that the deformability is insensitive to the tension steel yield strength.
4. Limited deformability design of concrete beams

4.1. Required deformability for RC beams in low-moderate seismicity regions

In the existing RC design codes, the design of deformability for concrete beams subjected to seismic risk is usually governed by some sets of empirical deemed-to-satisfy rules that limit the maximum neutral axis depth or tension steel ratio. This ensures that a large tensile strain would be developed in the tension steel at the ultimate limit state such that adequate rotation capacity is provided to the RC beams for the formation of plastic hinge and moment redistribution to occur. The respective rules of some existing RC design codes are extracted and highlighted as follows:

1. American Code ACI 318 (ACI Committee 2008): In addition to Clause 10.3.5 that limits the maximum tension steel strain to not less than 0.004, confinement of minimum diameter of 10 mm and maximum spacing of not larger than 0.25$d$, $8d_o$, $24d$, or 300 mm (whichever is the smallest), where $d_o$ and $d$ are the diameter of longitudinal and confining steel respectively, should be provided within the critical region as per Clause 7.1.0.5.1.

2. Chinese Code GB50011 (Ministry of Construction 2001): Clause 6.3.3 of the code requires the neutral axis depth to be not larger than 0.35 $d$. Confinement of minimum diameter of $8$ mm and maximum spacing not larger than 0.25 $d$, $8d_o$, or $100$ mm (whichever is the smallest) should be provided within the critical region as per Clause 6.3.3.3.

3. European Code EC2 (ECS 2004): Clause 5.6.2.2 of the code limits the neutral axis depth to not more than 0.25 $d$ when $f'_t \leq 50$ MPa or 0.15 $d$ when $f'_t > 50$ MPa, in which $f'_t$ is the concrete cylinder strength.

4. New Zealand Code NZS3101 (Standards New Zealand 2006): In addition to Clause 9.3.8.1 that restricts the neutral axis depth to not more than 0.75 of that in the balanced section, Clause 9.4.3.3 further restricts the maximum tension steel ratio should not be larger than $(f'_t+10)/6f'_t \leq 2.5\%$ for reinforcement design within critical region. Compression steel ratio of not less than 0.38 times the tension steel ratio should also be provided as per Clause 9.4.3.4.

From the above, the deformability expressed in terms of normalised rotation capacity $\theta_{pl}$ provided by various existing design codes may be evaluated by their respective values of $\theta_{pl}$ at different concrete strength and steel yield strength. To reflect the ranges of concrete and steel that are commonly adopted in practical construction, the deformability at $f_{co} = 30$, 50 and 100 MPa and $f_{st} = 400$ and 800 MPa are calculated using nonlinear moment-curvature analysis and summarised in Table 2. Alternatively, the deformability could be calculated using the following formulas previously developed by the authors (Zhou et al. 2010):

$$\theta_{pl} = 0.03m(f_{co})^{-0.3}(\lambda)^{1.0n}$$

$$m = \frac{1+10(f_{co})^{-1.1}(f_{st}P_c)^{0.3}}{f_{st}P_r^{1.5}P_c^{0.6}}$$

$$n = 1 + 3 f_{co}^{-0.2}(f_r/f_{co})^0$$

The validity of Eq. (5) has been verified by comparing with the measured deformability of beams tested by other researchers. The comparison is shown in Tables 3 and 4 for NSC and HSC beams respectively.

4.2. Derivation of limited deformability

For RC beams located in low to moderate seismicity regions, the beam should be designed to have adequate rotation capacity to enable the formation of plastic hinges, and hence allow moment redistribution to occur. According to Eurocode 2 (ECS 2004), no direct check of rotation capacity is needed and the required deformability is deemed to satisfy if the ratio of $x_d/d \leq 0.25$ for $f'_t \leq 50$ MPa and $x_d/d \leq 0.15$ if $f'_t > 50$ MPa. As seen from Table 2, the normalised rotation capacity calculated as per the deemed-to-satisfy requirements of Eurocode 2 is about 0.03 rad in all circumstances. The authors have also carried out an independent check of this value against the recommended maximum allowable tension steel ratio stipulated in Eurocode 8 Part 1 (2004). Using Clauses 5.2.3.4.3(3) and 5.4.3.1.2(4), as well as Table 5 of EC8, it can be calculated that for $f_{co} = 500$ MPa, the maximum allowable tension steel ratios are 1.15%, 1.73% and 2.3% for beam design as per medium ductility class. These steel ratios can be converted to the normalised rotation capacity using Eq. (5.4), and the respective values are...
\( \theta_{pl,\text{lim}} = 0.033 \text{ rad}, 0.028 \text{ rad} \) and 0.023 rad. The average value of these normalised rotation capacities is 0.028 rad, which is very close to the previous obtained value of \( \theta_{pl,\text{lim}} = 0.03 \text{ rad} \). Therefore, the authors suggest in this study to adopt this normalised rotation capacity \( \theta_{pl,\text{lim}} = 0.03 \text{ rad} \) as the benchmark for providing limited deformability to RC beams in low to moderate seismicity regions.

### Table 2. Deformability provided in design codes for RC beams subjected to seismic risk

| Design codes     | Normalised rotation capacity \( \theta_{pl} \) (rad) |
|------------------|-----------------------------------------------------|
|                  | \( f_{co} = 30 \text{ MPa} \) | \( f_{co} = 50 \text{ MPa} \) | \( f_{co} = 100 \text{ MPa} \) |
|                  | \( f_{yt} = 400 \text{ MPa} \) | \( f_{yt} = 800 \text{ MPa} \) | \( f_{yt} = 400 \text{ MPa} \) | \( f_{yt} = 800 \text{ MPa} \) |
| American Code ACI318 | 0.0157 | 0.0183 | 0.0195 | 0.0147 | 0.0253 | 0.0118 |
| Chinese Code GB50011 | 0.0410 | 0.0286 | 0.0363* | 0.0271 | 0.0433* | 0.0184* |
| Eurocode EC2 | 0.0304 | 0.0304 | 0.0218 | 0.0218 | 0.0270 | 0.0270 |
| New Zealand Code NZS3101 | 0.0442 | 0.0286 | 0.0363* | 0.0209 | 0.0427* | 0.0153* |
| Average | 0.0328 | 0.0296 | 0.0285 | 0.0211 | 0.0346 | 0.0181 |

*Deformability is calculated for beam section containing the largest allowable tension steel ratio of 2.5% instead of complying with the empirical deemed-to-satisfy rules.

### Table 3. Comparison with experimental results on rotation capacities of NSC beams

| Code | \( f_c' \) (MPa) | \( f_c \) (MPa) | \( f_t' \) (MPa) | \( \rho_t \) (%) | \( \rho_c \) (%) | \( \theta_{pl} \) by Eq. (5) (rad) [1] | \( \theta_{pl} \) by others (rad) [2] | \( \theta_{pl} \) by EC2 (rad) [3] | [1] [2] [3] |
|------|-----------------|----------------|----------------|----------------|--------------|---------------------------------|---------------------------------|---------------------------------|------|
| Navy et al. (1968) | | | | | | | | | |
| P9G1 | 33.6 | 0.00 | 328 | 1.73 | 0.71 | 0.0870 | 0.0650 | 0.0330 | 1.34 | 0.51 |
| P11G3 | 35.1 | 0.50 | 328 | 1.73 | 0.71 | 0.1536 | 0.1110 | 0.0320 | 1.38 | 0.29 |
| P3G4 | 37.5 | 1.30 | 452 | 1.73 | 0.71 | 0.1232 | 0.1340 | 0.0260 | 0.92 | 0.19 |
| P4G5 | 39.1 | 1.30 | 452 | 1.73 | 0.71 | 0.1217 | 0.1360 | 0.0265 | 0.89 | 0.19 |
| Pece and Fabbocino (1999) | | | | | | | | | |
| A | 41.3 | 0.98 | 471 | 2.60 | 0.05 | 0.0255 | 0.0220 | 0.0100 | 1.16 | 0.45 |
| B | 41.3 | 0.94 | 454 | 1.10 | 0.05 | 0.0736 | 0.1220 | 0.0265 | 0.60 | 0.22 |
| Debernardi and Taliano (2002) | | | | | | | | | |
| T1A1 | 27.7 | 0.46 | 587 | 0.67 | 0.30 | 0.1433 | 0.1035 | 0.0310 | 1.38 | 0.30 |
| T3A1 | 27.7 | 0.46 | 587 | 2.00 | 0.59 | 0.0270 | 0.0290 | 0.0080 | 0.93 | 0.28 |
| T5A1 | 27.7 | 0.35 | 587 | 0.63 | 0.22 | 0.0978 | 0.1130 | 0.0300 | 0.87 | 0.27 |
| T6A1 | 27.7 | 0.35 | 587 | 1.28 | 0.22 | 0.0311 | 0.0245 | 0.0160 | 1.27 | 0.65 |
| Haskett et al. (2009) | | | | | | | | | |
| A1 | 38.2 | 0.67 | 315 | 1.47 | 0.0 | 0.0313 | 0.0360 | 0.0269 | 0.87 | 0.75 |
| A2 | 42.3 | 0.32 | 318 | 1.47 | 0.0 | 0.0226 | 0.0205 | 0.0280 | 1.10 | 1.37 |
| A3 | 41.0 | 0.31 | 336 | 1.47 | 0.0 | 0.0209 | 0.0168 | 0.0270 | 1.24 | 1.61 |
| A4 | 42.9 | 1.29 | 315 | 2.95 | 0.0 | 0.0222 | 0.0305 | 0.0172 | 0.73 | 0.56 |
| A5 | 39.6 | 0.59 | 314 | 2.95 | 0.0 | 0.0136 | 0.0207 | 0.0154 | 0.66 | 0.74 |
| A6 | 41.1 | 0.31 | 328 | 2.95 | 0.0 | 0.0103 | 0.0118 | 0.0153 | 0.87 | 1.30 |
| B1 | 43.0 | 0.65 | 329 | 1.47 | 0.0 | 0.0293 | 0.0277 | 0.0278 | 1.06 | 1.00 |
| B2 | 41.8 | 0.31 | 322 | 1.47 | 0.0 | 0.0222 | 0.0152 | 0.0277 | 1.46 | 1.82 |
| B3 | 42.9 | 1.29 | 321 | 2.95 | 0.0 | 0.0217 | 0.0218 | 0.0168 | 1.00 | 0.77 |
| B4 | 42.9 | 0.64 | 323 | 2.95 | 0.0 | 0.0138 | 0.0120 | 0.0166 | 1.15 | 1.38 |
| C2 | 26.0 | 0.39 | 329 | 1.47 | 0.0 | 0.0219 | 0.0258 | 0.0203 | 0.85 | 0.79 |
| C3 | 25.6 | 0.32 | 330 | 1.47 | 0.0 | 0.0201 | 0.0187 | 0.0200 | 1.07 | 1.07 |
| C4 | 25.9 | 1.23 | 325 | 2.95 | 0.0 | 0.0205 | 0.0297 | 0.0080 | 0.69 | 0.27 |
| C5 | 23.4 | 0.64 | 328 | 2.95 | 0.0 | 0.0126 | 0.0130 | 0.0080 | 0.97 | 0.62 |
| C6 | 27.4 | 0.34 | 319 | 2.95 | 0.0 | 0.0102 | 0.0125 | 0.0080 | 0.82 | 0.64 |
| Average | | | | | | 1.01 | 0.72 | | |
| Standard deviation | | | | | | 0.24 | 0.47 | | |
Table 4. Comparison with experimental results on rotation capacities of HSC beams

| Code    | \( f'_c \) (MPa) | \( f'_r \) (MPa) | \( f_{ri} \) (MPa) | \( \rho_t \) (%) | \( \rho_c \) (%) | \( \theta_{pl,\text{by others}} \) by Eq. (5) (rad) | \( \theta_{pl,\text{by EC2 (rad)}} \) | \( [1] \) | \( [2] \) | \( [3] \) |
|---------|----------------|----------------|----------------|--------------|--------------|---------------------------------|---------------------------------|------|------|------|
| Pecce and Fabbicino (1999) | | | | | | | | | | |
| AH      | 93.8           | 0.98           | 471            | 2.60         | 0.05         | 0.0271                          | 0.0220                          | 0.0170 | 1.23 | 0.77 |
| CH      | 95.4           | 1.11           | 534            | 2.20         | 0.04         | 0.0300                          | 0.0380                          | 0.0170 | 0.79 | 0.45 |
| Ko et al. (2001) | | | | | | | | | | |
| 6-65-1  | 66.6           | 2.26           | 415            | 3.59         | 0.79         | 0.0547                          | 0.0472                          | 0.0150 | 1.16 | 0.32 |
| 6-75-1  | 66.6           | 2.33           | 427            | 4.27         | 0.77         | 0.0399                          | 0.0412                          | 0.0100 | 0.97 | 0.24 |
| 8-50-1  | 82.1           | 2.42           | 443            | 3.35         | 0.80         | 0.0580                          | 0.0482                          | 0.0160 | 1.20 | 0.33 |
| 8-65-1  | 82.1           | 2.33           | 427            | 4.27         | 0.77         | 0.0398                          | 0.0450                          | 0.0100 | 0.88 | 0.22 |
| 8-75-1  | 82.1           | 2.15           | 394            | 4.97         | 0.79         | 0.0338                          | 0.0484                          | 0.0080 | 0.70 | 0.17 |
| 7-62\text{\%}-1 | 70.8           | 1.91           | 408            | 3.16         | 0.00         | 0.0403                          | 0.0530                          | 0.0135 | 0.76 | 0.25 |
| 7-62\text{\%}-1 | 70.8           | 1.91           | 408            | 3.16         | 0.00         | 0.0587                          | 0.0510                          | 0.0160 | 1.15 | 0.31 |
| Lopes and Bernardo (2003) | | | | | | | | | | |
| A(64.9-2.04) | 64.9           | 0.59           | 555            | 2.04         | 0.20         | 0.0248                          | 0.0200                          | 0.0210 | 1.24 | 1.05 |
| A(63.2-2.86) | 63.2           | 0.62           | 575            | 2.86         | 0.20         | 0.0161                          | 0.0180                          | 0.0110 | 0.89 | 0.61 |
| A(65.1-2.86) | 65.1           | 0.62           | 575            | 2.86         | 0.20         | 0.0161                          | 0.0150                          | 0.0110 | 1.07 | 0.73 |
| B(82.9-2.11) | 82.9           | 0.59           | 555            | 2.11         | 0.20         | 0.0243                          | 0.0210                          | 0.0180 | 1.16 | 0.86 |
| B(83.9-2.16) | 83.9           | 0.59           | 555            | 2.16         | 0.20         | 0.0237                          | 0.0200                          | 0.0180 | 1.19 | 0.90 |
| B(83.6-2.69) | 83.6           | 0.62           | 575            | 2.69         | 0.20         | 0.0178                          | 0.0210                          | 0.0150 | 0.85 | 0.71 |
| B(83.4-2.70) | 83.4           | 0.62           | 575            | 2.70         | 0.20         | 0.0177                          | 0.0200                          | 0.0150 | 0.89 | 0.75 |
| Average | 1.01           | 0.54           | 1.01           | 0.54         | 0.18         | 0.28                           |

5. Methods of providing limited deformability

5.1. By controlling the maximum degree of reinforcement

Based on this specified value of normalised rotation capacity \( \theta_{pl,\text{lim}} = 0.03 \) rad, it can be seen from Eq. (5a) that a corresponding maximum allowable value of \( \lambda \), denoted by \( \lambda_{\text{max}} \), exists for each chosen \( f_{co} \) and \( f_{ri} \). The expression for \( \lambda_{\text{max}} \) is shown in Eq. (6a):

\[
\lambda_{\text{max}} = \frac{0.03 m(f_{co})^{-0.3} \left(\frac{f_{yt}}{460}\right)^{0.3}}{1 + 110(f_{co})^{-1} \left(\frac{f_{yt} \rho_c}{f_{yt} \rho_t}\right) \left(\frac{\rho_{pl,\text{lim}}}{\theta_{pl,\text{lim}}}\right)^{1.0} \pi} \tag{6a}
\]

the values of \( \lambda_{\text{max}} \) with respect to the specified deformability \( \theta_{pl,\text{lim}} = 0.03 \) rad are evaluated rigorously by nonlinear moment-curvature analysis and are summarised in Tables 5 to 7 for different combination of concrete and steel yield strengths of \( f_{co} = 30 – 100 \) MPa and \( f_{ri} = 400, 600 \) and \( 800 \) MPa. Alternatively, the value of \( \lambda_{\text{max}} \) for unconfined singly RC beams can be calculated by substituting \( \theta_{pl,\text{lim}} = 0.03 \) rad into Eq. (6a):

\[
\lambda_{\text{max}} = \left(\frac{f_{co}}{460}\right)^{-0.3} \left(\frac{f_{yt}}{460}\right)^{0.3} \tag{6b}
\]

It can be seen from the above tables that the value of \( \lambda_{\text{max}} \) decreases significantly as the concrete strength increases from 30 to 100 MPa for a given steel yield strength. On the contrary, the value of \( \lambda_{\text{max}} \) increases slightly as the steel yield strength increases from 400 to 800 MPa for a given concrete strength. It is apparent that a lower value of \( \lambda_{\text{max}} \) should be set for designing RC beams when HSC is adopted in order to achieve the provision of limited deformability of \( \theta_{pl,\text{lim}} = 0.03 \) rad. And a slightly higher value of \( \lambda_{\text{max}} \) should be set for the design of RC beams when HSS is adopted. However, since the effects of concrete strength is not the same as that of steel yield strength, the value of \( \lambda_{\text{max}} \) may or may not decrease when both HSC and HSS are adopted.

The corresponding maximum allowable tension steel ratio \( \rho_c,\text{max} \) for singly-reinforced concrete beam section (i.e. \( \rho_c = 0 \)) having different concrete and steel yield strength can be determined by multiplying the value of \( \lambda_{\text{max}} \) with the respective balanced steel ratio \( \rho_t \). The evaluated values of \( \rho_c,\text{max} \) have been listed in Tables 5 to 7. It can be observed from these tables that although the value of \( \lambda_{\text{max}} \) decreases substantially as the concrete strength increases, the value of \( \rho_c,\text{max} \) increases as the concrete strength increases because the value of \( \rho_c,\text{max} \) increases considerably with the concrete strength. Therefore, the use of HSC has the major advantage of increasing the maximum design flexural strength and providing limited deformability. On the other hand, it is seen that the value of \( \rho_c,\text{max} \) decreases significantly as the steel yield strength increases because the balanced steel ratio \( \rho_c,\text{max} \) decreases as the steel yield strength increases. Nevertheless, since higher strength steel is adopted, the provision of a lower tension steel ratio may or may not lead to a reduction in the maximum design limit of flexural strength.

In order to investigate numerically the flexural strength that can be achieved by the beam sections designed for the proposed deformability, the maximum moment capacity expressed in terms of \( M_p/(bd^2) \) for singly RC beam section (\( \rho_c = 0 \)) having different concrete and
steel yield strength are calculated. The results are tabulated in Tables 8 to 10. It can be observed from these tables that the maximum flexural strength achieved by the beam section designed with \( \theta_{\text{pl,lim}} = 0.03 \) rad increases significantly as the concrete strength increases. However, the flexural strength remains relatively constant as tension steel yield strength increases. The advantages of using higher strength materials are now obvious. The use of HSC with/without HSS in RC beams would allow a higher flexural strength to be achieved at limited deformability level, despite that HSC and HSS are less deformable per se. And the use of HSS solely in RC beams would not increase the flexural strength at the limited deformability level, but nevertheless reduce significantly the amount of tension steel and hence avoid steel congestion in the critical regions and beam-column joints.

### Table 5. Values of \( \lambda_{\text{max}} \) and \( \rho_{t,\text{max}} \) for \( \theta_{\text{pl,lim}} = 0.03 \) rad when \( f_{\text{yd}} = 400 \) MPa

| \( f_{\text{yc}} (\text{MPa}) \) | \( \rho_{t,\text{c}} (\%) \) | \( \lambda_{\text{max}} \) | \( \rho_{t,\text{max}} (\%) \) |
|---|---|---|---|
| 30 | 3.815 | 0.366 | 1.396 |
| 40 | 4.735 | 0.320 | 1.513 |
| 50 | 5.625 | 0.289 | 1.627 |
| 60 | 6.455 | 0.269 | 1.734 |
| 70 | 7.285 | 0.252 | 1.838 |
| 80 | 8.055 | 0.237 | 1.907 |
| 90 | 8.765 | 0.227 | 1.994 |
| 100 | 9.415 | 0.221 | 2.083 |

### Table 6. Values of \( \lambda_{\text{max}} \) and \( \rho_{t,\text{max}} \) for \( \theta_{\text{pl,lim}} = 0.03 \) rad when \( f_{\text{yd}} = 600 \) MPa

| \( f_{\text{yc}} (\text{MPa}) \) | \( \rho_{t,\text{c}} (\%) \) | \( \lambda_{\text{max}} \) | \( \rho_{t,\text{max}} (\%) \) |
|---|---|---|---|
| 30 | 2.225 | 0.418 | 0.929 |
| 40 | 2.735 | 0.369 | 1.008 |
| 50 | 3.225 | 0.336 | 1.083 |
| 60 | 3.685 | 0.314 | 1.156 |
| 70 | 4.125 | 0.297 | 1.225 |
| 80 | 4.555 | 0.279 | 1.270 |
| 90 | 4.935 | 0.269 | 1.328 |
| 100 | 5.285 | 0.262 | 1.387 |

### Table 7. Values of \( \lambda_{\text{max}} \) and \( \rho_{t,\text{max}} \) for \( \theta_{\text{pl,lim}} = 0.03 \) rad when \( f_{\text{yd}} = 800 \) MPa

| \( f_{\text{yc}} (\text{MPa}) \) | \( \rho_{t,\text{c}} (\%) \) | \( \lambda_{\text{max}} \) | \( \rho_{t,\text{max}} (\%) \) |
|---|---|---|---|
| 30 | 1.495 | 0.466 | 0.697 |
| 40 | 1.815 | 0.417 | 0.757 |
| 50 | 2.125 | 0.383 | 0.813 |
| 60 | 2.425 | 0.358 | 0.869 |
| 70 | 2.695 | 0.341 | 0.919 |
| 80 | 2.965 | 0.321 | 0.953 |
| 90 | 3.215 | 0.309 | 0.995 |
| 100 | 3.435 | 0.302 | 1.036 |

(Note \( f_{\text{yd}} = 0 \) MPa and \( \rho_{t,\text{c}} = 0\% \) in Tables 5 to 7)

5.2. By controlling the maximum neutral axis depth

As an alternative method, the provision of limited deformability to RC beams can be achieved by restricting the maximum neutral axis depth at ultimate state. This method of limiting the neutral axis depth for the provision of flexural deformability has been adopted by some of the existing RC design codes. The maximum limits of neutral axis depth are normally expressed in a dimensionless form in the ratio of \( x_{d}/d \), e.g. EC 2 and GB50011, where \( x_{d} \) and \( d \) are the neutral axis and effective beam depths; or \( x_{d}/x_{ub} \), e.g. New Zealand Code, where \( x_{ub} \) is the neutral axis depth of balanced section at ultimate state. However, since not all of these limits are dependent on the concrete and steel yield strength, the deformability provided to HSC beam will be lower than that of beam section made of NSC and/or NSS.

To ensure that a consistent level of deformability is provided to RC beams with different concrete and steel yield strength, the maximum limits of neutral axis depth should depend on the strengths of concrete and steel reinforcement. The maximum limit of neutral axis depth is expressed in this study by the ratio \( x_{d}/d \), which is more commonly adopted by the existing RC design codes. For designing beam section having the proposed limited deformability of \( \theta_{\text{pl,lim}} = 0.03 \) rad, the respective maximum limits of \( x_{d}/d \) for different concrete and steel yield strength are derived using moment-curvature analysis, which are summarised in Table 11. It is noted that the maximum value of \( x_{d}/d \) decreases significantly as the concrete strength increases from 30 to 100 MPa. Thus, a lower maximum limit should be set to the value of \( x_{d}/d \) when HSC is used. On the other hand, that the maximum value of \( x_{d}/d \) is constant when steel yield strength is varied.

5.3. Improving flexural strength and deformability by adding compression steel

From Tables 8 to 10, it is easily observed that at a prescribed concrete strength, there is a maximum design limit of flexural strength that can be achieved in association with the proposed limited deformability \( \theta_{\text{pl,lim}} = 0.03 \) rad. To increase the flexural strength of the beam section at the same concrete strength, the beam dimensions should be enlarged appropriately. However, in real construction practice especially in the design of building structures, it is often not practicable to increase the beam size because of maximising the usable floor space to the users. Under such a circumstance, some compression steel could be added to the beam to increase the maximum design limit of flexural strength while maintaining the provision of limited deformability. The enhanced maximum allowable tension steel ratios and design limit of flexural strength are calculated and presented in Tables 8 to 10 for \( \rho_{t,\text{c}} = 1\% \) and \( 2\% \). It is evident from the table that adding compression steel can increase both the maximum allowable limits of tension steel ratio and flexural strength to achieve limited deformability. It is also observed that the rate of increase in both maximum limits decreases as concrete strength increases.
### Table 8. Values of $\rho_{t,\text{max}}$ and $M_p/(bd^2)$ for $\theta_{pl,\text{lim}} = 0.03$ rad when $f_{yt} = f_{yc} = 400$ MPa (at different compression steel ratios $\rho_c$)

| $f_{co}$ (MPa) | $\rho_c = 0\%$ | $\rho_c = 1.0\%$ | $\rho_c = 2.0\%$ | $\rho_c = 0\%$ | $\rho_c = 1.0\%$ | $\rho_c = 2.0\%$ |
|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 30           | 1.396          | 2.601          | 3.785          | 5.030          | 9.301          | 13.501         |
| 40           | 1.513          | 2.702          | 3.869          | 5.557          | 9.783          | 13.969         |
| 50           | 1.627          | 2.798          | 3.970          | 6.043          | 10.214         | 14.417         |
| 60           | 1.734          | 2.903          | 4.075          | 6.494          | 10.665         | 14.860         |
| 70           | 1.838          | 2.976          | 4.148          | 6.918          | 10.996         | 15.196         |
| 80           | 1.907          | 3.066          | 4.198          | 7.217          | 11.380         | 15.442         |
| 90           | 1.994          | 3.160          | 4.269          | 7.573          | 11.769         | 15.757         |
| 100          | 2.083          | 3.193          | 4.333          | 7.932          | 11.940         | 16.039         |

### Table 9. Values of $\rho_{t,\text{max}}$ and $M_p/(bd^2)$ for $\theta_{pl,\text{lim}} = 0.03$ rad when $f_{yt} = f_{yc} = 600$ MPa (at different compression steel ratios $\rho_c$)

| $f_{co}$ (MPa) | Maximum value of $\rho_{t}$ (%) | $M_p/(bd^2)$ (MPa) |
|--------------|-------------------------------|-------------------|
| $\rho_c = 0\%$ | $\rho_c = 1.0\%$ | $\rho_c = 2.0\%$ | $\rho_c = 0\%$ | $\rho_c = 1.0\%$ | $\rho_c = 2.0\%$ |
| 30           | 0.929                        | 1.733             | 2.527             | 5.022          | 9.294          | 13.517         |
| 40           | 1.008                        | 1.799             | 2.582             | 5.552          | 9.772          | 13.982         |
| 50           | 1.083                        | 1.868             | 2.646             | 6.037          | 10.227         | 14.416         |
| 60           | 1.156                        | 1.933             | 2.711             | 6.494          | 10.652         | 14.833         |
| 70           | 1.225                        | 1.983             | 2.767             | 6.917          | 10.991         | 15.203         |
| 80           | 1.270                        | 2.042             | 2.838             | 7.211          | 11.364         | 15.442         |
| 90           | 1.328                        | 2.107             | 2.888             | 7.568          | 11.769         | 15.716         |
| 100          | 1.387                        | 2.140             | 2.888             | 7.923          | 12.000         | 16.036         |

### Table 10. Values of $\rho_{t,\text{max}}$ and $M_p/(bd^2)$ for $\theta_{pl,\text{lim}} = 0.03$ rad when $f_{yt} = f_{yc} = 800$ MPa (at different compression steel ratios $\rho_c$)

| $f_{co}$ (MPa) | Maximum value of $\rho_{t}$ (%) | $M_p/(bd^2)$ (MPa) |
|--------------|-------------------------------|-------------------|
| $\rho_c = 0\%$ | $\rho_c = 1.0\%$ | $\rho_c = 2.0\%$ | $\rho_c = 0\%$ | $\rho_c = 1.0\%$ | $\rho_c = 2.0\%$ |
| 30           | 0.697                        | 1.300             | 1.894             | 5.020          | 9.293          | 13.512         |
| 40           | 0.757                        | 1.349             | 1.936             | 5.556          | 9.772          | 13.979         |
| 50           | 0.813                        | 1.403             | 1.985             | 6.042          | 10.240         | 14.417         |
| 60           | 0.869                        | 1.448             | 2.033             | 6.500          | 10.644         | 14.834         |
| 70           | 0.919                        | 1.489             | 2.075             | 6.918          | 11.002         | 15.203         |
| 80           | 0.953                        | 1.531             | 2.117             | 7.212          | 11.284         | 15.444         |
| 90           | 0.995                        | 1.578             | 2.128             | 7.557          | 11.759         | 15.712         |
| 100          | 1.036                        | 1.605             | 2.165             | 7.894          | 12.000         | 16.030         |

(Note $f_{yt} = f_{yt} = f_{yc}$ in Table 11)

### Table 11. Maximum value of $x_u/d$ for $\theta_{pl,\text{lim}} = 0.03$ rad

| $f_{co}$ (MPa) | Maximum value of $x_u/d$ |
|--------------|--------------------------|
| $f_{yt} = 400$ MPa | $f_{yt} = 600$ MPa | $f_{yt} = 800$ MPa |
| 30           | 0.251                     | 0.251                     | 0.251                     |
| 40           | 0.214                     | 0.214                     | 0.214                     |
| 50           | 0.190                     | 0.190                     | 0.190                     |
| 60           | 0.172                     | 0.172                     | 0.172                     |
| 70           | 0.159                     | 0.159                     | 0.159                     |
| 80           | 0.151                     | 0.151                     | 0.151                     |
| 90           | 0.142                     | 0.142                     | 0.143                     |
| 100          | 0.136                     | 0.136                     | 0.136                     |

5.4. Improving flexural strength and deformability by adding confinement

As an alternative method of extending the limits of maximum allowable tension steel ratios and flexural strength for limited deformability design of RC beams, additional confinement could be added. The enhanced limits are calculated and presented in Tables 12 to 14 for $f_r = 1, 2$ and 3 MPa at different steel yield strengths. Similar to Tables 5 to 7, it is seen that adding confinement can increase both the maximum allowable limits of tension steel ratio and flexural strength to achieved limited deformability. It is also noted that the rate of increase in both maximum limits decreases as concrete strength increases. At a fixed concrete strength and steel yield strength, it is found that the increase in the maximum limit of design flexural strength decreases as the confinement increases. Therefore, the effectiveness of adding confinement decreases as confining pressure increases. It should also note that for beam section with $f_{co} = 30$ MPa and $f_{yt} = 600$ or 800 MPa, the maximum allowable tension steel ratio is larger than the respective balanced steel ratio. In these cases, the maximum tension steel ratio equal to the balanced steel ratio is proposed instead of the actual tension steel ratio in order to avoid the design of over-reinforced beam.
6. Simplified design guidelines

The maximum limits of the degree of reinforcement and neutral axis to effective depths ratio have been derived and presented in Tables 5 to 14. For incorporation into RC design codes, more simplified guidelines are preferred to the above tables. Referring to the maximum allowable values of the degree of reinforcement $\lambda_{\text{max}}$ summarised in Tables 5 to 7, it can be observed that the variation of $\lambda_{\text{max}}$ with steel yield strength $f_y$ can be represented fairly accurately by the following equation:

$$\left( \frac{\lambda_{\text{max}}}{\lambda_{\text{max}}^*} \right) = \left( \frac{f_y}{f_y^*} \right)^{0.35},$$

(7)

where $(\lambda_{\text{max}}^*)$ is the maximum allowable value of degree of reinforcement at limited deformability level with respect to tension steel yield strength $(f_y^*)$. Therefore, it is recommended herein only to propose the design values of $\lambda_{\text{max}}$ for $f_y^* = 400$ MPa, and use Eq. (7) to evaluate the values of $\lambda_{\text{max}}$ for other steel yield strength. The guidelines for $f_y^* = 400$ MPa are shown below:

In case of $f_y^*$ and $f_y^*$ = 400 MPa: $\lambda_{\text{max}}$ should not exceed $0.35$ when $f_y^* \leq 30$ MPa, should not exceed $0.25$ when $30$ MPa $\leq f_y^* < 60$ MPa, and should not exceed $0.2$ when $60$ MPa $\leq f_y^* < 100$ MPa.

### Table 12. Values of $\rho_{\text{max}}$ and $M_p/(bd^2)$ for $\theta_{\text{pl,lim}} = 0.03$ rad when $f_y^* = f_y^* = 400$ MPa (at different confining pressure $f_c$)

| $f_c$ (MPa) | $f_y^*$ (MPa) | $\rho_{\text{max}}$ (%) | $M_p/(bd^2)$ (MPa) |
|------------|---------------|-------------------------|---------------------|
| $f_y^*$ = 1.0 | $f_y^*$ = 2.0 | $f_y^*$ = 3.0 | $f_y^*$ = 1.0 | $f_y^*$ = 2.0 | $f_y^*$ = 3.0 |
| 30 | 2.843 | 4.233 | 6.017 | 9.176 | 12.325 | 14.925 |
| 40 | 3.049 | 4.379 | 5.930 | 10.237 | 13.620 | 16.905 |
| 50 | 3.205 | 4.486 | 5.908 | 11.048 | 14.554 | 17.863 |
| 60 | 3.332 | 4.568 | 5.889 | 11.702 | 15.271 | 18.656 |
| 70 | 3.440 | 4.637 | 5.893 | 12.251 | 15.844 | 19.263 |
| 80 | 3.520 | 4.702 | 5.896 | 12.682 | 16.335 | 19.732 |
| 90 | 3.588 | 4.754 | 5.907 | 13.042 | 16.730 | 20.133 |
| 100 | 3.620 | 4.772 | 5.925 | 13.265 | 16.990 | 20.493 |

### Table 13. Values of $\rho_{\text{max}}$ and $M_p/(bd^2)$ for $\theta_{\text{pl,lim}} = 0.03$ rad when $f_y^* = f_y^* = 600$ MPa (at different confining pressure $f_c$)

| $f_c$ (MPa) | $f_y^*$ (MPa) | Maximum value of $\rho_{\beta}$ (%) | $M_p/(bd^2)$ (MPa) |
|------------|---------------|-----------------------------------|---------------------|
| $f_y^*$ = 1.0 | $f_y^*$ = 2.0 | $f_y^*$ = 3.0 | $f_y^*$ = 1.0 | $f_y^*$ = 2.0 | $f_y^*$ = 3.0 |
| 30 | 1.896 | 2.899 | 4.145* | 9.178 | 12.336 | 14.774 |
| 40 | 2.033 | 2.961 | 4.110 | 10.238 | 13.656 | 16.692 |
| 50 | 2.137 | 2.994 | 4.089 | 11.048 | 14.558 | 17.954 |
| 60 | 2.222 | 3.045 | 4.042 | 11.705 | 15.271 | 18.782 |
| 70 | 2.293 | 3.091 | 3.934 | 12.252 | 15.845 | 19.270 |
| 80 | 2.347 | 3.135 | 3.931 | 12.684 | 16.335 | 19.733 |
| 90 | 2.392 | 3.170 | 3.938 | 13.042 | 16.731 | 20.133 |
| 100 | 2.417 | 3.181 | 3.950 | 13.290 | 16.990 | 20.493 |

Note: The mark * indicates that evaluated value is larger than the respective balanced steel ratio and consequently the latter is used.

### Table 14. Values of $\rho_{\text{max}}$ and $M_p/(bd^2)$ for $\theta_{\text{pl,lim}} = 0.03$ rad when $f_y^* = f_y^* = 800$ MPa (at different confining pressure $f_c$) (Confining pressure $f_c$ is expressed in MPa in Tables 12 to 14)

| $f_c$ (MPa) | $f_y^*$ (MPa) | Maximum value of $\rho_{\beta}$ (%) | $M_p/(bd^2)$ (MPa) |
|------------|---------------|-----------------------------------|---------------------|
| $f_y^*$ = 1.0 | $f_y^*$ = 2.0 | $f_y^*$ = 3.0 | $f_y^*$ = 1.0 | $f_y^*$ = 2.0 | $f_y^*$ = 3.0 |
| 30 | 1.432 | 2.293 | 2.965* | 9.184 | 12.306 | 14.792 |
| 40 | 1.525 | 2.295 | 3.399 | 10.235 | 13.717 | 16.530 |
| 50 | 1.602 | 2.311 | 3.190 | 11.047 | 14.656 | 17.996 |
| 60 | 1.666 | 2.297 | 3.156 | 11.702 | 15.294 | 18.908 |
| 70 | 1.720 | 2.320 | 3.128 | 12.251 | 15.852 | 19.527 |
| 80 | 1.760 | 2.351 | 3.023 | 12.682 | 16.335 | 19.863 |
| 90 | 1.794 | 2.377 | 2.953 | 13.042 | 16.730 | 20.131 |
| 100 | 1.816 | 2.386 | 2.962 | 13.309 | 16.990 | 20.487 |

Note: The mark * indicates that evaluated value is larger than the respective balanced steel ratio and consequently the latter is used.
Furthermore, by substituting Eqs. (3) and (4) into Eq. (7), it can be seen from Eq. (8) that the maximum allowable tension steel ratio for singly-reinforced beam (i.e., $\rho_c = 0$) is inversely proportional to the yield strength of tension steel at a given concrete strength for providing limited deformability:

$$\frac{\rho_{t,\text{max}}}{\rho_{t,\text{max}}^2} = \frac{f_{yt}}{f_{yt}^2},$$

(8)

where $\rho_{t,\text{max}}$ is the maximum allowable value of tension steel ratio at limited deformability level with respect to its yield strength ($f_{yt}$). For beam sections with or without compression steel, Eq. (9) is derived that correlates the maximum allowable tension steel ratio to concrete and steel yield strength:

$$\rho_{t,\text{max}} = 4(f_{co} + 100 + 100\rho_c)/f_{yt},$$

(9)

where $\rho_{t,\text{max}}$ and $\rho_c$ are in %, $f_{co}$ and $f_{yt}$ are in MPa.

As seen in Table 11, the effects of steel yield strength on the maximum allowable neutral axis to effective depth ratio are considered insignificant. Hence, it is proposed to ignore the effect of steel yield strength in the simplified design guidelines. Accordingly, the following guidelines are developed:

In the case of $400 \leq f_{co} = f_{yt} \leq 800$ MPa, $x_u/d$ should not exceed 0.25 when $f_{co} \leq 30$ MPa, should not exceed 0.17 when $30$ MPa $\leq f_{co} < 60$ MPa, and should not exceed 0.13 when $60$ MPa $\leq f_{co} < 100$ MPa.

7. Conclusions

The flexural deformability of RC beams in terms of normalised rotation capacity was studied by nonlinear moment-curvature analysis. From the study, it was found that the variation of deformability with degree of reinforcement $\lambda$, confining pressure $f_c$, and the neutral axis depth at maximum moment (expressed in dimensionless ratio of $x_u/x_d$ or $x_u/d$) are not unique and dependent on the concrete and steel yield strength. Because of such dependence, the current empirical deemed-to-satisfy rules stipulated in most of the RC design codes, which are concrete and/or steel yield strength dependent, are not able to provide a consistent level of deformability to RC beams. Most importantly, the deformability provided by the existing empirical rules to HSC beams with HSS is much lower than that provided to NSC beams containing NSS (~55%).

In order to provide a consistent level of deformability to RC beams to cater for the seismic demand in low to moderate seismicity regions, it is proposed to set a consistent level of deformability in the design of RC beams. The design of RC beams possessing this required level of deformability $\rho_{t,\text{lim}}$ is named the limited deformability design. This proposed deformability is set at the normalised rotation capacity provided to NSC beams in accordance with the plastic design method stipulated in Eurocode 2. To achieve the provision of limited deformability to RC beams, the maximum degree of reinforcement $\lambda_{\text{max}}$ or tension steel ratio $\rho_{t,\text{max}}$ or neutral axis to effective depth ratio $x_u/d$ should be limited. In this study, these maximum allowable values were derived for different combination of concrete strength (30–100 MPa) and steel yield strength (400–800 MPa). From the results, it is evident that maximum allowable values of $\lambda_{\text{max}}$, $\rho_{t,\text{max}}$, and $x_u/d$ decrease significantly as the concrete strength increases. Moreover, it was also found that there exists a maximum flexural strength of a singly-reinforced beam section for the provision of limited deformability at a given concrete strength.

To improve the maximum flexural strength limit of RC beams designed for limited deformability, compression steel and/or confinement can be added without the need of enlarging beam size. The maximum tension steel ratio and the design limit of flexural strength for different compression steel ratio from 0 to 2% and confining pressure from 0 to 3 MPa have also been derived in this study. From the results, it can be concluded that the use of HSC with or without HSS would improve the maximum design limit of flexural strength of RC beams at limited deformability. And the use of HSS solely would not increase the flexural strength of RC beam section at limited deformability level, but nevertheless reduce significantly the amount of tension steel and hence avoid steel congestion. Lastly, simplified guidelines for incorporation into RC design codes that limit the value of $\lambda_{\text{max}}$, $\rho_{t,\text{max}}$ and $x_u/d$ to ensure provision of the proposed limited deformability have been developed.

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