Research of Multi-objective Optimization with Time Restriction

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Abstract. Most of the operating strategies for modern companies are depending on scientific management methods. The research of 0-1 program can provide valuable and efficient solutions for companies. The purpose of this paper is analyzing the properties between teams and projects like quality, remuneration, cost, and revenue. Confirming the relationship between those properties by linear regression and Back Propagation Neural Network. Then, calculate the best Assignment scheme with a time restriction to provide a reference for companies.

Statement of Problem

As the development of management theories and the progress of social technology, efficient management method become an important factor for the company. Improving the whole production and assigning teams with an efficient model to have to promote efficiency for companies.

In recent years, great achievements of multi-objective assignment have been made in China. 0-1 program model can increase resource utilization. There are some related researches like literature [1] used ant colony optimization, provide an optimized model by data of time, cost and quality. [2] Use Genetic Algorithm to offer options for different cooperative partners by two properties: task fitness and task coordination rate. [3] Realized task allocations by minimized Euclidean distance of ideal points. [4] Analyzed Hungarian method, then solved the problem of staff assignment. Most of those mentioned literature refers scientific Multi-objective assignment method, build mathematic model then provide solutions. When considering degrees of importance for different variables, they use AHP or Entropy method. However, for practical problems, the degrees of importance are not constant but changing by the value of those variables. According to the above analysis, this paper survey the relationship between variables and the objective function, then calculate the solution.

Model Creation

Parameter Introduction

I means the set of tasks, i=1,2,…m;
J means the set of teams, j=1,2,…n;
t_{ij} means the period if assign team j to perform task i;
w_{ij} means the remuneration for team j to perform task I;
q_{ij} means the quality of task i if assigning team j to perform task i;
c_{ij} means the cost of team j to perform task i;
r_{ij} means the revenue if assign team j to perform task I;
v means the time restriction;

x_{ij} is 0-1 decision variable, if assign team j to perform task i, x_{ij}=1. If not, x_{ij}=0.

Designing Model

Maxf = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij} x_{ij} \tag{1}

s.t.
\[
\text{Max}\{t_{ij}x_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\} \leq v \tag{2}
\]
\[
\sum_{j=1}^{n} x_{ij} \leq 1, i = 1,2, \ldots m \tag{3}
\]
\[
\sum_{i=1}^{m} x_{ij} \leq 1, j = 1,2, \ldots n \tag{4}
\]
\[x_{ij}=0 \quad \text{or} \quad 1; \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n \tag{5}\]

Equation (1) is the objective function, and it shows maximize the total revenue. Equation (2) to (5) is the restriction. (2) Means all the task should be finished no longer than \(v\) days. Equation (3) means each task is performed by one team. Equation (4) means each team can only perform one task. Equation (5) means the range of decision variable.

**Solution of the Model**

**Testing Hypothesis of Linear Relationship**

Using ‘a’ to show the quantity of the company’s historical data of projects. Each data have ‘h’ number of variable \(Z_h(h=1,2,3\ldots b, \text{b is 3 in this paper})\) and \(R_g(g=1,2,3\ldots a)\) means the value of the project. \(z_{gh}\) means the value of variable \(Z_h\) in project ‘g’. Using linear regression for variables and the objective function of those historical data. If the condition of fitting looks well, it can be assumed that the relationship between objective function and variables is linear. If not, the relationship is nonlinear.

**Analyzing Nonlinear Variable**

For the condition that the relationship between objective function and variables is nonlinear, using Matlab to calculate. Choosing some of \(R_g\) and \(z_{gj}\) as a test set, then, create a BP Neural Network called ‘net’ for the other \(R_g\) and \(z_{gj}\). Putting the \(z_{gj}\) of the test set into ‘net’ and compare those results with \(R_g\) of the test set. If the standard deviation is low, it can be assumed that the ‘net’ fit the nonlinear relationship.

**Forecasting Revenue**

When confirming the relationship between \(R_g\) and \(z_{gj}\), put the information of each task(\(w_{ij}, q_{ij}, c_{ij}\)) into the linear regression or BP Neural Network. Then getting the array \(W, Q,\) and \(C\) which means the contribution of quality, remuneration, and cost for revenue. At last, calculating and forecasting the revenue as array \(R\). The formula is:

\[
R = W + Q + C \tag{6}
\]

**Assigning with Time Restriction**

When getting the array \(R\), maximize total revenue with time restriction array \(T\). Common method are Hungarian methods and ant colony optimization. This paper decides to use Excel Solver to get the result directly. Finally, the result is the best assignment decision.

**Example**

**Background Introduction**

The company has 7 teams. Managers need to arrange them into 7 tasks. The time, remuneration, and cost for each team to perform each task are shown by matrixes. Time matrix and quality matrix are shown in table 1. The remuneration matrix and cost matrix are shown as table 2 and table 3. The time restriction is 25 days. There are 100 history data about similar tasks. Forecasting the revenue and making a decision to maximize the total revenue. For two conditions of the relationships between variables and objective function, there are two kinds of solutions. The calculation procedure of the two methods given above is briefly described below.
Table 1. Table of Time and Score of quality

| Time (day) | Score of quality |
|------------|------------------|
| $j_1$      | 3.1 3.7 2.6 3.5 2.2 2.7 |
| $j_2$      | 4.3 2.1 3.2 2.8 2.3 |
| $j_3$      | 2.5 3.6 3.8 3.3 2.3 3.5 |
| $j_4$      | 2.1 3.7 2.4 2.3 3.5 2.7 |
| $j_5$      | 3.7 3.3 2.1 3.2 2.7 3.3 3.1 |
| $j_6$      | 3.9 3.9 3.6 3.1 2.7 2.2 2.2 |
| $j_7$      | 2.8 3.9 3.1 2.5 3.8 2.1 2 |

Table 2. Remuneration for each team

| Remuneration | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ | $l_7$ |
|--------------|-------|-------|-------|-------|-------|-------|-------|
| $j_1$        | 56309 | 63244 | 64605 | 53440 | 58455 | 68696 | 53709 |
| $j_2$        | 63936 | 66736 | 52606 | 61633 | 63695 | 60811 | 51885 |
| $j_3$        | 60963 | 59820 | 63323 | 67880 | 52307 | 51969 | 66965 |
| $j_4$        | 63756 | 66542 | 63463 | 62176 | 51158 | 65443 | 55921 |
| $j_5$        | 60445 | 64169 | 51125 | 57464 | 62164 | 58590 | 62344 |
| $j_6$        | 55432 | 66648 | 68773 | 53741 | 51258 | 56762 | 61258 |
| $j_7$        | 51083 | 52192 | 61149 | 67318 | 54846 | 61881 | 57244 |

Table 3. Cost table

| Cost     | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ | $l_7$ |
|----------|-------|-------|-------|-------|-------|-------|-------|
| $j_1$    | 120974| 129460| 136529| 117231| 136380| 138313| 117464|
| $j_2$    | 113825| 111733| 117542| 117030| 112386| 123675| 125482|
| $j_3$    | 132347| 120720| 129731| 109015| 111542| 124359| 128730|
| $j_4$    | 124707| 118791| 120183| 101403| 132465| 130992| 101801|
| $j_5$    | 108665| 137683| 136362| 130878| 120607| 136059| 119964|
| $j_6$    | 111943| 110253| 122162| 112899| 122709| 113120| 132554|
| $j_7$    | 119943| 116180| 121719| 135406| 106042| 136371| 101075|

**Condition 1: Linear Relationship**

Using linear regression for 100 history data. When the fitness is well, record the coefficient of quality, remuneration and cost are 1952.447, 2.929 and 1.022. Multiply those three coefficients with matrixes of quality, remuneration, and cost. Then add them together we can get the table of forecasting revenue, shown as table 4.

Combine table 4 with time matrix, using Excel Solver to get the best decision in 25 days. Shown in table 5:

**Condition 2: Nonlinear Relationship**

Using linear regression for 100 history data. When the fitness is not good, it can be assumed that the relationship between variables and objective function is nonlinear. Thus, using Matlab to calculate. Choosing 10 data randomly as a test set. Creating and training a BP Neural Network for the other 90 data. Called the BP Neural Network as ‘net’. Put test variables into ‘net’, compare the results with test objective function, the result is shown in Figure 1. The Standard Deviation is low. So, ‘net’ can describe the nonlinear relationship.
Put all the data of quality matrix, remuneration matrix, and cost matrix into ‘net’. The result is the revenue of different tasks and teams. Shown in table 6.

Combine table 7 and time matrix, using Excel Solver to get the best decision. Shown in table 7.

Table 4. Revenue of linear relationship

|   | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ |
|---|-------|-------|-------|-------|-------|-------|-------|
| $j_1$ | 464285 | 508890 | 465432 | 452056 | 462886 | 520239 | 441367 |
| $j_2$ | 485175 | 485197 | 447979 | 469991 | 443950 | 482184 | 430552 |
| $j_3$ | 460253 | 478214 | 468397 | 501574 | 450733 | 439413 | 473336 |
| $j_4$ | 468435 | 478359 | 500050 | 463420 | 466799 | 489562 | 453316 |
| $j_5$ | 438437 | 475096 | 480447 | 491457 | 496679 | 478573 | 471167 |
| $j_6$ | 420957 | 452372 | 474672 | 430938 | 427834 | 432291 | 467186 |
| $j_7$ | 420590 | 429755 | 461650 | 522994 | 419357 | 467054 | 450591 |

Table 5. Decision of linear relationship

|   | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ |
|---|-------|-------|-------|-------|-------|-------|-------|
| $j_1$ | 0   | 0   | 0     | 0   | 0   | 1     | 0     |
| $j_2$ | 1   | 0   | 0     | 0   | 0   | 0     | 0     |
| $j_3$ | 0   | 1   | 0     | 0   | 0   | 0     | 0     |
| $j_4$ | 0   | 0   | 1     | 0   | 0   | 0     | 0     |
| $j_5$ | 0   | 0   | 0     | 1   | 0   | 0     | 0     |
| $j_6$ | 0   | 0   | 0     | 0   | 0   | 0     | 1     |
| $j_7$ | 0   | 0   | 0     | 1   | 0   | 0     | 0     |

Table 6. Revenue of Nonlinear relationship

|   | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ |
|---|-------|-------|-------|-------|-------|-------|-------|
| $j_1$ | 1223630 | 1495703 | 1135623 | 1169739 | 1135829 | 1510218 | 1083707 |
| $j_2$ | 1372876 | 1367111 | 1143526 | 1249361 | 1076456 | 1319468 | 986643 |
| $j_3$ | 1121306 | 1309618 | 1178185 | 1512478 | 1200589 | 1317377 | 1241901 |
| $j_4$ | 1195406 | 1281264 | 1468183 | 1275559 | 1237141 | 1317377 | 1241901 |
| $j_5$ | 1062620 | 1195414 | 1334212 | 1389466 | 1447496 | 1250521 | 1240776 |
| $j_6$ | 1007423 | 1135268 | 1236665 | 1027160 | 982067 | 1017957 | 1166964 |
| $j_7$ | 944396 | 1015294 | 1169178 | 1555249 | 968708 | 1151529 | 1211487 |

Table 7. Decision of nonlinear relationship

|   | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ |
|---|-------|-------|-------|-------|-------|-------|-------|
| $j_1$ | 0   | 0   | 0     | 0     | 0     | 0     | 1     |
| $j_2$ | 0   | 0   | 0     | 1     | 0     | 0     | 0     |
| $j_3$ | 0   | 0   | 0     | 0     | 0     | 0     | 0     |
| $j_4$ | 0   | 0   | 0     | 0     | 0     | 0     | 0     |
| $j_5$ | 0   | 0   | 0     | 0     | 0     | 0     | 0     |
| $j_6$ | 0   | 0   | 0     | 0     | 0     | 0     | 0     |
| $j_7$ | 0   | 0   | 0     | 0     | 0     | 0     | 0     |

Conclusion

The assignment problem is an important part of Multi-objective optimization. Companies should maximize their revenue as this method. This paper uses the Multi-objective assignment model with a time restriction, refer to linear regression and BP Neural Network, and avoid the traditional thinking like constant weight and normalization. Clearer and more practical objective function make this paper more valuable. This paper provides a model and method for the company to arrange teams and tasks.
Reference

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