CONSTRANTS ON DARK ENERGY FROM GALAXY CLUSTER GAS MASS FRACTION VERSUS REDSHIFT DATA

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ABSTRACT

We use the Allen et al. galaxy cluster gas mass fraction versus redshift data to constrain parameters of three different dark energy models: a cosmological constant dominated one (LCDM), the XCDM parameterization of dark energy, and a slowly rolling scalar field model with inverse power-law potential energy density. (Instead of using the Monte Carlo Markov chain method, when integrating over nuisance parameters we use an alternative method of introducing an auxiliary random variable.) The resulting constraints are consistent with, and typically more constraining than, those derived from other cosmological data. A time-independent cosmological constant is a good fit to the galaxy cluster data, but slowly evolving dark energy cannot yet be ruled out.

Subject headings: cosmological parameters — cosmology: observations — X-rays: galaxies: clusters

1. INTRODUCTION

Observations over the last decade have established that the cosmological expansion is accelerating. In the context of general relativity, this requires that the cosmological energy budget be dominated by dark energy (for reviews see, e.g., Copeland et al. 2006; Padmanabhan 2006; Ishak 2007; Uzan 2007; Linder 2007; Ratra & Vogeley 2008). This hypothetical construct—dark energy—is an enigma. It is not yet clearly established whether dark energy is Einstein’s cosmological constant $\Lambda$ (e.g., Peebles 1984) or whether it evolves slowly in time and varies weakly in space (e.g., Peebles & Ratra 1988; Ratra & Peebles 1988). While Type Ia supernova (SNIa) apparent magnitude measurements as a function of redshift indicate accelerated cosmological expansion (Riess et al. 1998; Perlmutter et al. 1999), SNIa data are as yet unable to unambiguously constrain dark energy (see, e.g., Alam et al. 2007; Nesseris & Perivolaropoulos 2007; Shafieloo 2007; Zhang et al. 2007; Wu et al. 2008; Ishida et al. 2008). Future SNIa data will improve the constraints (e.g., Podariu et al. 2001a) and could resolve some of the current differences in results from different SNIa data sets.

The results of the SNIa test are confirmed by a test based on cosmic microwave background (CMB) anisotropy data that must assume the cold dark matter (CDM) model for structure formation (see Peebles & Ratra 2003 and references therein for a discussion of apparent problems with the CDM model). CMB anisotropy data are consistent with the universe having negligible spatial curvature (see, e.g., Podariu et al. 2001b; Durrer et al. 2003; Mukherjee et al. 2003; Page et al. 2003; Spergel et al. 2007; Doran et al. 2007a), under the assumption that dark energy does not evolve in time (e.g., Wright 2006; Tegmark et al. 2006; Zhao et al. 2007; Ichikawa & Takahashi 2007; Wang & Mukherjee 2007). In combination with low measured nonrelativistic matter density (Chen & Ratra 2003b and references therein), negligible spatial hypersurface curvature indicates the presence of dark energy.

There are many different models of dark energy. In this Letter we consider three: standard LCDM, the XCDM parameterization, and a slowly rolling scalar field dominated one ($\phi$CDM). In the LCDM model the late-time universe is dominated by a cosmological constant $\Lambda$ with time-independent energy density $\rho_\Lambda$ (Peebles 1984). In $\phi$CDM the dark energy is a slowly rolling scalar field $\phi$; in the model we consider the scalar field potential energy density $V(\phi) \propto \phi^{-\omega}$, where $\omega$ is a nonnegative parameter (Peebles & Ratra 1988). We also consider the XCDM parameterization of the dark energy equation of state; here dark energy is modeled as a fluid with an equation of state that relates the fluid pressure $p = \omega \rho$ to its energy density $\rho$ where $\omega$ is a time-independent negative parameter. This approximation is inaccurate in the scalar field dominated epoch (Ratra 1991). In all three models, other contributors to the universe’s current energy budget include CDM and baryonic matter. In the $\phi$CDM and XCDM cases spatial hypersurfaces are taken to be flat, while spatial curvature is treated as a cosmological parameter in the LCDM model we consider.

It is important to confirm and strengthen the SNIa and CMB test results by using additional techniques. This will allow for consistency checks as well as possibly identifying systematic effects in a particular data set. Other promising current tests include the angular size of radio sources and quasars as a function of redshift (e.g., Chen & Ratra 2003a; Podariu et al. 2003; Daly & Djorgovski 2005; Daly et al. 2007), strong gravitational lensing (e.g., Chae et al. 2004; Alcaniz et al. 2005; Fedeli & Bartelmann 2007; Lee & Ng 2007; Oguri et al. 2008), measurements of the Hubble parameter as a function of redshift (e.g., Samushia & Ratra 2006; Sen & Scherrer 2008; Lazkoz & Majerotto 2007; Wei & Zhang 2007; Samushia et al. 2007), and large-scale structure baryon acoustic oscillation measurements (e.g., Doran et al. 2007b; Parkinson et al. 2007; Percival et al. 2007; Lima et al. 2007). For reviews of the observational situation, see Kurek & Szydlowsky (2008), Wang (2007b), and

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Lazkoz et al. (2007). Current data favor dark energy that does not evolve but do not yet strongly rule out evolving dark energy (see, e.g., Rapetti et al. 2005; Wilson et al. 2006; Davis et al. 2007).

In this Letter we use new galaxy cluster gas mass fraction versus redshift data (Allen et al. 2008, hereafter A08) to constrain parameters of the three dark energy models mentioned above. Earlier galaxy cluster gas mass fraction data (Allen et al. 2004) have been used to constrain these and other models (e.g., Chen & Ratra 2004; Rapetti et al. 2005; Alcaniz et al. 2005; Cannata & Kamenshchik 2006; Zhan 2006; Wei & Zhang 2007).

If rich galaxy clusters are large enough to have matter content that fairly samples that of the universe, then the baryonic-to-total mass ratio in clusters is equal to the cosmological baryonic mass density parameters. The baryonic mass in clusters is dominated by the X-ray-emitting gas that can be measured through X-ray observations. Combined with an estimate of the dark energy model, we can compare predicted values of the gas mass fraction with measurements for clusters at redshift $z$, by constructing a $\chi^2 = \sum_i \left( \frac{f_{\text{gas}}^i(z) - f_{\text{gas}}^i(z)}{\sigma^2} \right)$ function ($\sigma$ are the standard deviation measurement errors, and the summation is over the 42 A08 clusters) and so constrain parameters of dark energy models.

We construct a likelihood function $L = e^{-\chi^2/2}$, which depends on cosmological parameters like $\Omega_m$ and those describing the dark energy model, as well as on the nuisance parameters. We marginalize over the nuisance parameters by multiplying the likelihood by the probability distribution function for the nuisance parameters and then integrating (e.g., Ganga et al. 1997). The resulting probability distribution function depends on only two variables: $\Omega_m$ and a parameter $p$ describing the dark energy model. In the $\Lambda$CDM case $p$ is $\Omega_m$, in XCDM it is $\omega$, and in $\phi$CDM it is $\alpha$. Since we consider only spatially flat cosmologies for XCDM and $\phi$CDM models, two parameters $p$ and $\Omega_m$ completely describe the background evolution.

In addition to depending on cosmological parameters, in a given model the predicted gas mass fraction depends on a number of “nuisance” parameters that have to be marginalized over to derive the probability distribution function for the cosmological parameters of interest. We note that the likelihood function depends on certain functions of the “nuisance” parameters and so introduce auxiliary random variables to describe these functions. This technique helps us to significantly reduce the computational time.

In §2 we outline our computations. In §3 we present and discuss our results and conclude.

2. COMPUTATION

In our computations we follow A08. For a given cosmological model we compute predicted values of the gas mass fraction

$$f_{\text{gas}}(z, h, \Omega_m, p, \Omega_b, Q) = \frac{K A \gamma b_0 (1 + \alpha_e) \Omega_m}{1 + s_0 (1 + \alpha_e) \Omega_m} \left[ \frac{d_A(z)}{d_A(z)} \right]^{-1.5},$$

as a function of cluster redshift $z$, four cosmological parameters ($h, \Omega_m, \Omega_b$, and a parameter $p$, described below, that represents the dark energy model), and seven parameters ($s_0, b_0, \alpha_e, \alpha_b, K, \gamma, b_f$) represented by $Q$ and related to modeling the cluster gas mass fraction. Here $h$ is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $d_A(z)$ is the angular diameter distance computed in the reference spatially flat $\Lambda$CDM model (with $\Omega_m = 0.3, \Omega_b = 0.7, and h = 0.7$, where $\Omega_b$ is the cosmological constant density parameter), and $d_A(z)$ is the angular diameter distance computed in the cosmological model of interest and depends on the model and $h$. Since the angular diameter

$$d_A(z) = \frac{c}{H(z)} = \frac{1}{H_0} \frac{c}{(1+z)^2} \int_0^z \frac{dz'}{H(z')},$$

and so $f_{\text{gas}}$, depends on the assumed dark energy model, we can compare predicted values of the gas mass fraction with measurements for clusters at redshift $z$, by constructing a $\chi^2 = \sum_i \left( \frac{f_{\text{gas}}^i(z) - f_{\text{gas}}^i(z)}{\sigma^2} \right)$ function ($\sigma$ are the standard deviation measurement errors, and the summation is over the 42 A08 clusters) and so constrain parameters of dark energy models.

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Table 1: Prior Probability Distribution Functions for Nuisance Parameters

| Prior Parameter | Allowance          | Distribution |
|-----------------|--------------------|--------------|
| WMAP prior $\Omega_m h^2$ | $0.0223 \pm 0.0008$ | Gaussian     |
| WMAP prior $h$    | $0.73 \pm 0.03$    | Gaussian     |
| Alternative prior $\Omega_m h^2$ | $0.0205 \pm 0.0018$ | Gaussian     |
| Alternative prior $h$    | $0.68 \pm 0.04$    | Gaussian     |

Figures 1–3 show cluster gas mass fraction confidence level contours and best-fit values for the three dark energy models and the two sets of priors for $\Omega_m h^2$ and $h$ given in Table 1. Compared to the constraints derived from the earlier Allen et al. (2004) cluster gas mass fraction data (Chen & Ratra 2004), the difference between the contours corresponding to the two prior sets (for $\Omega_m h^2$ and $h$) is much reduced. The new constraints are almost as restrictive as the ones derived from SNIa data and more constraining than those derived from angular size versus redshift data or Hubble parameter versus redshift data.

Figure 1 shows constraints on $\Lambda$CDM. $\Omega_m$ is better constrained than $\Omega_\Lambda$, and the results are in good qualitative accord with previous analyses. The best-fit values are slightly away from a spatially flat model.

Figure 2 shows constraints on the XCDM parameterization. Again, the energy density of nonrelativistic matter is fairly well constrained while the equation-of-state parameter is less constrained. The best-fit values are again not exactly on the $\omega = -1$ line, which corresponds to the spatially flat $\Lambda$CDM case.

Figure 3 shows constraints on the $f$CDM model. $\Omega_m$ is better constrained than $\alpha$. For both sets of priors there is an upper limit on $\alpha$. The best-fit values are on the $\alpha = 0$ line, which
corresponds to the spatially flat $\Lambda$CDM case, but there is a large part of evolving dark energy parameter space that still is not ruled out.

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