QUBITS AND STU-BLACK HOLES WITH EIGHT MAGNETIC
AND EIGHT ELECTRIC CHARGES

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Abstract

In this work, we consider a STU black-hole model containing eight magnetic
and eight electric charges ((8+8)-signature). We show that such model admits
an extremal black holes solution in which (8+8)-signature can be associated
with the entropy via the Cayley’s hyperdeterminant.

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1.- Introduction

It is known that from string theory [1] one can obtain a extremal black-hole model that remains invariant under the group

$$SL(2, Z)_S \times SL(2, Z)_T \times SL(2, Z)_U, \tag{1}$$

associated with the dualities $S$, $T$ and $U$. It turns out that by using the Cayley hyperdeterminant mathematical concept a connection of the entropy associated with such model has been established [2]. In fact, it has been shown that when the signature of such extremal black-hole corresponds to four electric and four magnetic charges ($(4 + 4)$-signature), the entropy can be determined via the Cayley hyperdeterminant.

At this respect, it is convenient to recall that in string theory a duality transformation can be one of three types: $S$, $T$ and $U$ (see Ref. [2] and references therein). The $S$-duality, also called strong-weak duality, transforms a theory with coupling constant $g$ in a theory with coupling constant $1/g$. Meanwhile, $T$-duality establishes a symmetry between small and large radius $R$ corresponding to a compactification of the extra dimensions in Kaluza-Klein theory. Finally the $U$-duality combines the $S$-duality and the $T$-duality (see Ref. [1]).

In this work, we show that the entropy can also be connected with extremal black-hole corresponding to a $(8 + 8)$-signature. In this case, the relevant symmetry becomes

$$SL(2, Z)_{S1} \times SL(2, Z)_{T1} \times SL(2, Z)_{U1} \times SL(2, Z)_{S2} \times SL(2, Z)_{T2} \times SL(2, Z)_{U2}. \tag{2}$$

Our model may be of physical interesting because, using the Cayley hyperdeterminant, the procedure of $(4+4)$-signature can be extended in an analogue form to a solution of $(8 + 8)$-signature. This in turn demonstrate that the chosen signatures $(1 + 1)$, $(2 + 2)$, $(4 + 4)$ and $(8 + 8)$ in a $STU$ black-hole models looks very similar to the only possible dimensions; 1, 2, 4 and 8 of division algebras over the real numbers [3]-[4]. Moreover, we prove that just as $(4 + 4)$-signature is related to 2-qubits, the $(8 + 8)$-signature is connected to 3-qubits entanglement. At this respect, it has been mentioned in Refs. [5], [6] and [7] that for normalized qubits, the complex 1-qubit, 2-qubit and 3-qubit are deeply related to division algebras via the three Hopf maps.

Technically, this work is organized as follows. In section 2 we consider the $STU$-model associated with the $(8 + 8)$-signature ($(8 + 8)$-$STU$-model). In section 3, we determine the Bogomolny mass for this model. In section 4, we
find the entropy for extremal black-holes of \((8 + 8)\)-\(STU\)-model. In section 5, we study the quantum entanglement of 3-qubits and the corresponding black holes of the \((8 + 8)\)-\(STU\)-model. Finally, in section 6 we make some final remarks.

2.- The \((8 + 8)\)-\(STU\) model

In this section, with the idea of implementing the \((8 + 8)\)-signature, we shall briefly consider the \(STU\) model. For this purpose, let us introduce six complex scalar fields \(S(a)\), \(T(a)\) and \(U(a)\), namely two action/dilaton field \(S(1)\), \(S(2)\), two complex Kähler fields \(T(1)\), \(T(2)\) and two complex field structure \(U(1)\), \(U(2)\), defined by [2]:

\[
S(1) = S_1 + iS_2 = a + ie^{-\eta},
\]

\[
S(2) = S_3 + iS_4 = d + ie^{-\vartheta},
\]

\[
T(1) = T_1 + iT_2 = b + ie^{-\eta},
\]

\[
T(2) = T_3 + iT_4 = f + ie^{-\vartheta},
\]

\[
U(1) = U_1 + iU_2 = c + ie^{-\eta}
\]

and

\[
U(2) = U_3 + iU_4 = g + ie^{-\vartheta},
\]

respectively. Here, the quantities \(a, d, b, f, c\) and \(g\) are real scalar fields, while \(\eta\) and \(\vartheta\) are phase factors. This complex parameterization allows a natural transformation under the symmetries \(SL(2, Z)_1 \times SL(2, Z)_2\). In fact, the action of \(SL(2, Z)_S1 \times SL(2, Z)_S2\) is given by the modular group. This is the group of linear fractional transformations acting on the upper half of the complex plane \(\mathbb{C}\) having the form

\[
S \rightarrow \frac{aS + b}{cS + d} \times \frac{eS + f}{gS + h}.
\]

This is the known Möbius transformation. Here \(a, b, c, d, e, f, g\) and \(h\) are integers. It is evident that \(SL(2, Z)\) can be represented by \(2 \times 2\)-matrices. The transformation \(SL(2, Z)_S1 \times SL(2, Z)_S2\) can be represented by matrices \(4 \times 4\).
whose determinant leads to $ad - bc = eh - fg = 1$. One can have similar expressions for $SL(2, Z)_{T1} \times SL(2, Z)_{T2}$ and $SL(2, Z)_{U1} \times SL(2, Z)_{U2}$. Moreover, the Möbius transformation can be seen as the composition of a stereographic projection of the complex plane on a sphere, followed by a rotation or displacement of the area to a new location, and finally a stereographic projection from the sphere to the plane. Stereographic projection suggests that the complex plane can be thought of as a unit sphere in $\mathbb{R}^3$ without the north pole, which corresponds to a point in the infinity. This means that the points on the complex plane $\mathbb{C} \to \mathbb{C} \cup \{\infty\}$ form the extended complex plane. Moreover, the Möbius transformation can be understood as a sequence of simple transformations: a translation $f_1(S) = S + \frac{d}{c}$, followed by a reflection with respect to the real axis $f_2(S) = \frac{1}{S}$, after dilation $f_3(S) = \frac{bc-ad}{c^2}S$ and finally a rotation $f_4(S) = S + \frac{a}{c}$. Considering this sequence one can write the total action as

$$(f_4 \circ f_3 \circ f_2 \circ f_1)(S) = \frac{aS + b}{eS + d}. \tag{10}$$

The $STU$-model consider a matrix $M_S$, defined as

$$M_S = \frac{1}{S_2} \left( \begin{array}{cc} 1 & S_1 \\ S_1 & |S|^2 \end{array} \right). \tag{11}$$

For the group $SL(2, Z)_{S1} \times SL(2, Z)_{S2}$, this matrix can be extended in block form

$$M_S = \frac{1}{S_2} \left( \begin{array}{ccc} 1 & S_1 & 0 & 0 \\ S_1 & |S|^2 & 0 & 0 \\ 0 & 0 & 1 & S_2 \\ 0 & 0 & S_2 & |S|^2 \end{array} \right). \tag{12}$$

The action of $SL(2, Z)_{S1} \times SL(2, Z)_{S2}$ on the matrix is $M_S$ lead to $\omega_S^T M_S \omega_S$, where $\omega_S \in SL(2, Z)_{S1} \times SL(2, Z)_{S2}$ has the form

$$\omega_S = \left( \begin{array}{cccc} d & b & 0 & 0 \\ c & a & 0 & 0 \\ 0 & 0 & f & h \\ 0 & 0 & g & e \end{array} \right). \tag{13}$$

One can consider similar expressions for $M_T$ and $M_U$. The invariant tensor $\epsilon$ is also defined in the context of $SL(2, Z)_{1} \times SL(2, Z)_{2}$ through the matrix
\[ \epsilon_S = \epsilon_T = \epsilon_U = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \] (14)

Now, given the above definitions, one finds that it can be used in the bosonic action for the heterotic string STU-model which include the graviton \( g_{\mu\nu} \), dilaton \( \eta \), the two-form \( B_{\mu\nu} \), the four \( U(1) \) gauge fields \( A_S \) and the two complex scalar \( T \) and \( U \) [2]:

\[ I_{STU} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} e^{-\eta} [R_g + g^{\mu\nu} \partial_\mu \eta \partial_\nu \eta + \frac{1}{4} T_r((\partial M^b_T)^{-1}(\partial M^b_T)) + \frac{1}{4} T_r((\partial M^b_U)^{-1}(\partial M^b_U)) - \frac{1}{4} F_{S\mu\nu}(M_T^b M_U^b) F_{S}^{\mu\nu b}], \] (15)

where the superscript \( b \) runs from 1 to 8 and indicates the number of electric and magnetic charges. It turns out that this action contains different terms. First, the action for the gravitational field \( g_{\mu\nu} \) coupled with the dilatonic field \( \eta \) is given by

\[ I_g = \frac{1}{16\pi G} \int d^4x \sqrt{-g} e^{-\eta} [R_g + g^{\mu\nu} \partial_\mu \eta \partial_\nu \eta], \] (16)

Second, the action that describes the antisymmetric field \( H_{\mu\nu\rho}^b \), namely

\[ I_G = -\frac{1}{192\pi G} \int d^4x \sqrt{-g} e^{-\eta} g^{\mu\lambda} g^{\nu\tau} g^{\rho\sigma} H_{\mu\nu\rho}^b H_{\lambda\tau\sigma}^b. \] (17)

At this respect, one should recall that in string theory \( H_{\mu\nu\rho}^b \) allows a coupling with the 2-brane.

Also, for the \( TU \) scalar fields one has

\[ I_{TU} = \frac{1}{64\pi G} \int d^4x \sqrt{-g} e^{-\eta} [T_r((\partial M^b_T)^{-1}(\partial M^b_T)) + T_r((\partial M^b_U)^{-1}(\partial M^b_U))]. \] (18)

Finally, the action describing the electromagnetic field coupled with scalar field \( TU \) is given by

\[ I_E = -\frac{1}{64\pi G} \int d^4x \sqrt{-g} e^{-\eta} F_{S\mu\nu}^T (M_T M_U) F_S^{\mu\nu}. \] (19)
Moreover, the antisymmetric field $H_{\mu\nu\rho}$ is expressed in terms of complex scalar fields $T$ and $U$ in the form

$$H_{\mu\nu\rho} = 3(\partial_{[\mu}B_{\nu\rho]} - \frac{1}{2} A_{[\mu}^T F_{\nu\rho]}) .$$

(20)

It turns out that $H_{\mu\nu\rho}$ is invariant under the transformations $T$ and $U$.

It is not difficult to see that the action (15) is manifest invariant under the $T$ and $U$ dualities,

$$F_{S_{\mu\nu}}^a \to (\omega_T^{-1} \times \omega_U^{-1}) F_{S_{\mu\nu}}^b, \quad M_{T/\nu}^b \to \omega_T^{bT}/M_{T/\nu}^b \omega_U^{b} .$$

(21)

Note that the fields $\eta$, $g_{\mu\nu}$ and $B^b$ remain invariant under these transformations. The action (15) is also invariant under $S$-duality transformations. In fact, in this case the equations of motion and the Bianchi identities are interchanged.

For dualities $S$ and $T$ one considers the metric $g_{C\mu\nu} = e^{-\eta} g_{\mu\nu}$ and one finds that the key transformation looks like

$$\left( \begin{array}{c} F_{S_{\mu\nu}}^b \\ F_{S_{\mu\nu}}^{b^*} \end{array} \right) \to \omega S^{-1} \left( \begin{array}{c} F_{S_{\mu\nu}}^{bb} \\ F_{S_{\mu\nu}}^{b^{*}} \end{array} \right) ,$$

(22)

where $\tilde{F}_{S_{\mu\nu}}^b$ is given by

$$\tilde{F}_{S_{\mu\nu}}^b = -S_2 \left[ (M_T^{-1} \times M_U^{-1}) (\epsilon_T \times \epsilon_U) \right]^b_a * F_{S_{\mu\nu}}^a - S_1 F_{\mu\nu}^b .$$

(23)

In some cases the dilaton can be combined with axion to form a complex scalar field called axion/dilaton field, as in equation (3). In the expression $S = S_1 + i S_2 = a + ie^{-\eta}$ the axion $a$ is defined by

$$e^{\mu\nu\rho\sigma} \partial_\sigma a = \sqrt{-g} e^{-\eta} g^{\mu\sigma} g^{\nu\lambda} g^{\rho\tau} H_{a\lambda\tau} .$$

(24)

It turns out that, by permutations, instead of (15) one can also consider the action $I_{TUS}$ for type IIA theory, or the action $I_{UST}$ the type IIB theory. Both cases are obtained by cyclic permutation of fields $S$, $T$ and $U$.

Finally, one may consider a stock where the fields interact equally with the following general $STU$-prepotential [8]:

$$F = \frac{1}{2} d_{ABC} t^A t^B t^C = \frac{1}{2} t^1 \left[ (t^2)^2 - (t^3)^2 \right] = S \eta_{ij} t^i t^j ,$$

(25)

where the values of $d_{ABC}$ are $d_{ABC} = \{ d_{1ij} = \eta_{ij} \text{ or 0 otherwise} \}$, with $A$, $B$, $C = 1, 2, \ldots, n + 1$, and also $\eta_{ij} = diag(+, -, -, \ldots, -)$, indices $i, j$ running from 2 to $n + 1$. 

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For $t^0 = 1$, $t^1 = S^b$, $t^2 = \frac{1}{\sqrt{2}}(T^b + U^b)$ and $t^3 = \frac{1}{\sqrt{2}}(T^b - U^b)$ the expression (25) leads to

$$F = S^b T^b U^b.$$  \hspace{1cm} (26)

This corresponds to the so-called $STU$-prepotential.

3.- Bogomolny Mass

In the most general formulation the monopole solution in Yang-Mills theory can be defined as the generalization of the Dirac monopole in electrodynamics [9]. Roughly, the monopole mass emerges when one combines in the Lagrangian the Higgs and the Yang-Mills fields and breaks the original Higgs symmetry. The resultant mass is known as Bogomolny mass [10].

Here, we shall describe the Bogomolny mass in terms of the $STU$ model [2] extended to the $(8 + 8)$ symmetry. One associates the electric and magnetic charges $q^b_s$ and $p^b_s$ with the fields $F^b_{s0r}$ and $F^b_{s0r}$ through the expressions

$$F^b_{s0r} \sim \frac{q^b_s}{r^2},$$  \hspace{1cm} (27)

and

$$F^b_{s0r} \sim \frac{p^b_s}{r^2}.$$  \hspace{1cm} (28)

respectively. Here $F^a_{s0r}$ is the Hodge dual, defined by $F^a_{s0\mu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}_{s\nu}$. Moreover, the electric and magnetic charges can be defined in terms of vectors $\alpha^a_s$ and $\beta^a_s$ associated with fields $F^a_s$ and $\tilde{F}^a_s$ in the form

$$\left( \begin{array}{c} \alpha^b_s \\ \beta^b_s \end{array} \right) = \left( \begin{array}{cc} S_2^{(0)} M_T^{-1} \times M_U^{-1} & S_1^{(0)} \epsilon_T \times \epsilon_U \\ 0 & -\epsilon_T \times \epsilon_U \end{array} \right) \left( \begin{array}{c} Q^c_S \\ P^c_S \end{array} \right).$$  \hspace{1cm} (29)

The electric and magnetic charges acquire mass via its interaction with the Higgs field. As it is known, the Lagrangian for the Yang-Mills field has the property of being invariant under local gauge transformations. Moreover, since the Yang-Mills theory is a generalization of the electromagnetic Lagrangian, the associated gauge group is no longer $U(1)$ but a more general non-commutative group. For example, gluons of $QCD$ are described for Yang-Mills theory through non-commutative Lie group $SU(3)$ which corresponds to the color symmetry.

Explicitly, the Yang-Mills theory coupled to the Higgs field can be described by the Lagrangian density [10]-[11].
\[
L = -\frac{1}{4} \vec{G}^{ \mu \nu b} \cdot \vec{G}^{b}_{\mu \nu} + \frac{1}{2} D^{\mu b} \vec{\phi} \cdot D_{\mu b} \vec{\phi} - V(\vec{\phi}), \tag{30}
\]
where \(D^{\mu}\) is a covariant derivative and the field \(\vec{G}^{b}_{\mu \nu}\) is defined in terms of gauge potential \(\vec{W}\) in the form

\[
\vec{G}^{\mu \nu b} = \partial^{\mu} \vec{W}^{\nu b} - \partial^{\nu} \vec{W}^{\mu b} - e \vec{W}^{\mu b} \times \vec{W}^{\nu b}, \tag{31}
\]
with \(e\) denoting the analogue of the electrical charge. The potential for the Higgs field is given by \(V(\vec{\phi}) = \frac{1}{4} \lambda (\vec{\phi}^2 - a^2)^2\). Here, the Higgs field \(\vec{\phi}\) is a vector with components \(\vec{\phi} = (\phi_1, \phi_2, \phi_3)\) and is minimally coupled to the gauge field \(\vec{W}^{\mu a}\) through the covariant derivative \(D^{a}_{\mu} \vec{\phi} = \partial^{a}_{\mu} \vec{\phi} - e \vec{W}^{a}_{\mu} \times \vec{\phi}\). The Lagrangian (30) is invariant under the gauge transformations \(SO(3)\);

\[
\vec{\phi} \mapsto \vec{\phi}' = g(x) \vec{\phi} \tag{32}
\]
and

\[
\vec{W}^{a}_{\mu} \mapsto \vec{W}^{a}_{\mu}' = \vec{g}(x) \vec{W}^{-1}_{\mu} g^{-1}(x) + \frac{1}{e} \partial_{\mu} \vec{g}(x) \vec{g}^{-1}(x), \tag{33}
\]
where \(\vec{g}(x)\) is a orthogonal \(3 \times 3\)-matrix which generally depends on \(x\) and whose determinant is equal to unity.

The classical dynamics of the fields \(\vec{\phi}\) and \(\vec{W}^{\mu}\) is governed by the field equations \(D^{a}_{\nu} \vec{G}^{\mu \nu} = -e \vec{\phi} \times D^{a}_{\mu} \vec{\phi}\) and \(D^{a}_{\mu} D^{a}_{\mu} \vec{\phi} = -\lambda (\vec{\phi}^2 - a^2) \vec{\phi}\), and by the Bianchi identity \(D^{a}_{\mu} * \vec{G}^{\mu \nu} = 0\), where \(* \vec{G}^{\mu \nu} = \frac{1}{2} \epsilon^{\mu \nu \lambda \rho} \vec{G}_{\lambda \rho}\).

In contrast to the Dirac monopole, an important feature of the Bogomolny monopole type solution is that the mass is calculable. In the Higgs vacuum, the electromagnetic tensor \(F\) relates to \(G\) through \(\vec{G}^{\mu \nu} = \frac{\vec{F}^{\mu \nu b}}{a}\), while the solution of magnetic charge \(B_{n}^{b}\) is determined from the integral \(g = \int \vec{B}^{b} \cdot dS\) and the Bianchi identity \(D_{\mu} * \vec{G}^{\mu \nu} = 0\) [10]. Using electric and magnetic fields given as \(G_{a}^{0i} = -e_{a}^{ib}\) and \(G_{a}^{ijb} = -e_{ijk} B_{a}^{kib}\) (\(a\) run from 1 to 3), the dynamic field equations become \(D_{\mu} G_{m}^{\mu \nu} = -e e_{mbe} \phi_{b} (D^{a} \phi)^{c}\).

The energy density is given by

\[
H = \frac{1}{2} \vec{E}^{b} \cdot \vec{E}^{b} + \frac{1}{2} D_{0} \vec{\phi} \cdot D_{0} \vec{\phi} + \frac{1}{2} \vec{B}^{b} \cdot \vec{B}^{b} + \frac{1}{2} D_{i} \vec{\phi} \cdot D_{i} \vec{\phi} + V(\vec{\phi}), \tag{34}
\]
from which the mass can be calculated through the formula

\[
m = \int_{\mathbb{R}^3} \left\{ \frac{1}{2} \vec{E}^{b} \cdot \vec{E}^{b} + \frac{1}{2} D_{0} \vec{\phi} \cdot D_{0} \vec{\phi} + \frac{1}{2} \vec{B}^{b} \cdot \vec{B}^{b} + \frac{1}{2} D_{i} \vec{\phi} \cdot D_{i} \vec{\phi} + V(\vec{\phi}) \right\}. \tag{35}
\]
Since the minimum values of $V(\vec{\phi})$ and $D_0\vec{\phi}$ are zero, the above expression implies a mass limit:

$$m \geq \frac{1}{2} \int_{\mathbb{R}^3} \left[ \vec{E}_i^b \cdot \vec{E}_i^b + \vec{B}_i^b \cdot \vec{B}_i^b + D_i\vec{\phi} \cdot D_i\vec{\phi} \right]. \quad (36)$$

It turns out that such mass can be described differently. For this purpose, an angular parameter $\theta$ is introduced. Adding and subtracting $\sin \theta \vec{E}_i^b \cdot D_i\vec{\phi}$ and $\cos \theta \vec{B}_i^b \cdot D_i\vec{\phi}$ the integrand in (36) becomes (see Refs. [10]-[11] for details):

$$m \geq \frac{1}{2} \int_{\mathbb{R}^3} \left( \parallel \vec{E}_i^b - D_i\vec{\phi}\sin \theta \parallel^2 + \parallel \vec{B}_i^b - D_i\vec{\phi}\cos \theta \parallel^2 \right)$$

$$+ \sin \theta \int_{\mathbb{R}^3} D_i\vec{\phi} \cdot \vec{E}_i^b + \cos \theta \int_{\mathbb{R}^3} D_i\vec{\phi} \cdot \vec{B}_i^b$$

$$\geq \sin \theta \int_{\mathbb{R}^3} D_i\vec{\phi} \cdot \vec{E}_i^b + \cos \theta \int_{\mathbb{R}^3} D_i\vec{\phi} \cdot \vec{B}_i^b.$$ \quad (37)

Solutions of electric and magnetic charges are obtained from

$$\int_{\mathbb{R}^3} D_i\vec{\phi} \cdot \vec{E}_i^b = \int_{\mathbb{R}^3} \partial_i(\vec{\phi} \cdot \vec{E}_i^b) = \int_{\Sigma_\infty} \vec{\phi} \cdot \vec{E}_i^b dS_i = a \int_{\Sigma_\infty} \vec{E} \cdot d\vec{S} \equiv aq^b \quad (38)$$

and

$$\int_{\mathbb{R}^3} D_i\vec{\phi} \cdot \vec{B}_i^b = \int_{\mathbb{R}^3} \partial_i(\vec{\phi} \cdot \vec{B}_i^b) = \int_{\Sigma_\infty} \vec{\phi} \cdot \vec{B}_i^b dS_i = a \int_{\Sigma_\infty} \vec{B} \cdot d\vec{S} \equiv ap^b, \quad (39)$$

respectively.

Thanks to these two relationships, one can write the mass (37) for all angles $\theta$ as $m^b \geq a(p^b \cos \theta + q^b \sin \theta)$. One can give a more restraining condition when the right hand side of this equation is maximum. In this case one has $q^b \cos \theta = p^b \sin \theta$, or $\tan \theta = (\frac{q}{p})^b$. Taking the square root of the above expression for the mass, and inserting $q^b \cos \theta = p^b \sin \theta$, the limit for the Bogomolny mass leads to

$$m^b \geq a(p^{b2} + q^{b2})^{1/2}. \quad (40)$$

In the case of a Hoof’t-Polyakov monopole, this mass expression takes the form $m^b_p \geq a|p|$. Using the value of the magnetic charge $|p| = \frac{4\pi}{e}$, one finds that the monopole mass $m^b_p$ is related to the mass of the boson mass $m^b = ae^b\hbar$. Thus, one obtains the mass ratio [11]:

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\[ m_p^b \geq \frac{4 \pi \hbar}{q^b m_q^b} = \frac{1}{4 \alpha} m_q^b, \]  

(41)

where we considered that \( q^b = 2e^b \) and that the fine structure constant is given by \( \alpha = \frac{e^2}{4 \pi} \). (Here we used units such that \( c = \hbar = \epsilon_0 = 1 \).)

The mass formula associated with the scalar fields \( S^b T^b U^b \) which is invariant under the group \( SL(2, Z_1) \times SL(2, Z_2) \times SL(2, Z_3) \times SL(2, Z_4) \times SL(2, Z_5) \times SL(2, Z_6) \) and meets the transformation \( a^i_{jk} \rightarrow \omega^i_{Sl} \omega^j_{Tm} \omega^k_{Un} a^{lmn} \), has a similar form [2]:

\[ m_p^{b2} = \frac{1}{16 \alpha^2} a^T ((M_S^{-1b})(M_T^{-1b})(M_U^{-1b}) - (M_S^{-1b} \epsilon_T \epsilon_U - \epsilon_S M_T^{-1b} \epsilon_U - \epsilon_S \epsilon_T M_U^{-1b})) a. \]  

(42)

This mass expression can be considered as the Bogomolny \((8 + 8)\)-STU mass of an extended model [12].

4.- Extremal Black Hole Entropy

In this section, we shall see that the \((8 + 8)\)-STU-model contains a solution corresponding to the entropy of a extremal charged black hole. Moreover, we shall show that the entropy is given by a quarter of the area of the event horizon of such a black hole. However, the area computation requires evaluating the mass with no asymptotic values for charges in a moduli space. This is the space that represents the vacuum of the string theory, which is considered through the prepotential \( F = STU \).

It is worth mentioning that when the ten-dimensional heterotic string is compactified on a Calabi-Yau manifold one obtains an \( N = 1 \) supersymmetric effective low-energy action. Moreover, a feature of these models is that they contain massless scalar fields. These scalar fields are called moduli fields, because they are invariant under the modular group \( SL(2, Z)_1 \times SL(2, Z)_2 \).

The \( STU \) model is described by the prepotential [8]

\[ F(X) = \frac{d_{ijk} X^{ib} X^{jb} X^{kb}}{X^0}, \quad i, j, k = 1, 2, 3, \]  

(43)

where the terms \( X^i X^j X^k \) are holomorphic coordinates in a special Kähler’s manifold, with indices \( i, j \) and \( k \) labeling different multiplets [13]-[14]. Physically, these coordinates correspond to vector multiplets scalar components. Furthermore, the coefficients \( d_{ijk} = \int \omega_i \wedge \omega_j \wedge \omega_k \) corresponds to the num-
ber of Calabi-Yau intersections, where \( \varpi_i, \varpi_j \) and \( \varpi_k \) are local coordinates of the Calabi-Yau space (see Refs. [15]-[18]). The holomorphic section is determined by the one-form prepotential \( (X^\Lambda_b, F^b_\Lambda) \),

\[
F^b_\Lambda = \frac{\partial F^b}{\partial X^\Lambda_b}, \tag{44}
\]

where \( \Lambda = (0, 1, 2, 3) \).

The spatial coordinates \( z^b \) are determined by \( z^b = \frac{X^b}{X^0} \), which can take different values according to the \( S^b, T^b \) and \( U^b \) dualities. Thus, one may have \( z^1 = S^b = \frac{X^1b}{X^0}, z^2 = T^b = \frac{X^{2b}}{X^0} \) and \( z^3 = U^b = \frac{X^{3b}}{X^0} \). It is customary to set \( X^0 = 1 \). These coordinates define the potential in the Kähler form ([8] and [19]),

\[
K^b = -\ln(-i d_{ijk}(z^b - \bar{z}^b)^i(z^b - \bar{z}^b)^j(z^b - \bar{z}^b)^k). \tag{45}
\]

In terms of the \( z^b \)s, the quantity \( d_{ijk} \) allows for holomorphic sections. In fact, deriving \( F \) with respect to the terms \( \Lambda = (0, i = 1, 2, 3) \) according to equation (44) one gets

\[
X^\Lambda = \begin{pmatrix}
1 \\
1 \\
1 \\
\end{pmatrix}, \quad F^b_\Lambda = \begin{pmatrix}
-z^1z^2z^3 \\
z^2z^3 \\
z^1z^3 \\
z^1z^2 \\
\end{pmatrix}, \tag{46}
\]

Stabilization equations represent a mechanism by which the gauge fields acquires mass from the complex scalar field moduli space \( n_\nu \). This is the case for \( N = 2 \) supersymmetric black hole [20]. The relationship between charges and supersymmetric black holes near the horizon are established by [8]:

\[
\begin{pmatrix}
\rho^\Lambda_b \\
\eta^\Lambda_b \\
\end{pmatrix} = \Re \begin{pmatrix}
2i \tilde{Z}^b L^\Lambda_b \\
2i \tilde{Z}^b M^\Lambda_b \\
\end{pmatrix}. \tag{47}
\]

The value of \( \tilde{Z}^b \) is known as the central charge. Furthermore, the central charge can not be determined by considerations of symmetry [19]. The appearance of the central charge is related to a soft broken symmetry by introducing a macroscopic scale in the system [19]. Specifically, the central charge is defined as follows [21]-[22]:

\[
Z = -\frac{1}{2} \int_{S^2} (M^{\Lambda_b} F^{-\Lambda_b} - X^{\Lambda_b} G^b_\Lambda). \tag{48}
\]

Thus, from (47) one obtains
\[ p^{\Lambda b} = i(\bar{Z}^b X^{\Lambda b} - Z^b \bar{X}^{\Lambda b}) \] (49)

and

\[ q^b_\Lambda = i(\bar{Z}^b F^b_\Lambda - Z^b \bar{F}^{\Lambda b}). \] (50)

Considering \( \Pi = (L^\Lambda, M_\Lambda) = e^{k/2}(X^\Lambda, F_\Lambda) \) (see Ref. [23]), where \( L^\Lambda = e^{k/2}X^\Lambda \) and \( M_\Lambda = e^{k/2}F_\Lambda \) (see Ref. [24]), the central charge can be found by mean of \( Z = ie^{k/2}(X^\Lambda q_\Lambda - F_\Lambda p^\Lambda) = (L^\Lambda q_\Lambda - M_\Lambda p^\Lambda) \) [25]. One obtains

\[ p^{\Lambda b} = ie^{k/2}(\bar{Z}^b X^{\Lambda b} - Z^b \bar{X}^{\Lambda b}) \] (51)

and

\[ q^b_\Lambda = ie^{k/2}(\bar{Z}^b F^b_\Lambda - Z^b \bar{F}^{\Lambda b}). \] (52)

Eliminating the central charge \( \bar{Z} \) in the equations (51) and (52), lead to \( p^{\Lambda b} = ie^{k/2}(-Z^b \bar{X}^{\Lambda b}) \) and \( q^b_\Lambda = ie^{k/2}(-Z^b \bar{F}^{\Lambda b}) \), respectively. Thus, one may proceed by multiplying \( p^{\Lambda b} \) by \( F^b_\Lambda \) and \( q^b_\Lambda \) by \( X^{\Lambda b} \): the indices are renamed as \( p^{\Lambda b} F^b_\Sigma = ie^{k/2}(-Z^b X^{\Lambda b} F^b_\Sigma) \) and \( X^{\Lambda b} q^b_\Lambda = ie^{k/2}(-Z^b \bar{F}^{\Lambda b} X^{\Lambda b}) \). In this form one derives the expressions [8]

\[ X^{\Lambda b} q^b_\Lambda - p^{\Lambda b} F^b_\Sigma = ie^{k/2}Z^b(-\bar{F}^{\Lambda b} X^{\Lambda b} + \bar{X}^{\Lambda b} F^b_\Sigma) \] (53)

and

\[ X^{\Lambda b} q^b_\Lambda - p^{\Lambda b} F^b_\Sigma = ie^{k/2}Z^b(\bar{X}^{\Lambda b} F^b_\Sigma - \bar{F}^{\Lambda b} X^{\Lambda b}). \] (54)

This is a matrix system which can be used to solve the stabilized equations for the moduli charges. Moreover, one can solve for \( z^1 \) accordance with the corresponding charges. Since one has a three moduli symmetries solutions the results for \( z^2 \) and \( z^3 \) can be obtained in an analogous manner. In this case, the components \(((\Lambda, \Sigma) = (1, 0), (0, 1), (1, 1), (2, 3) \) and \((3, 2))\), are used respectively [2].

From the matrix system one can derive \( z^1 \). The key step is to use the two equations systems:

\[ q_0 + p^1 z^2 z^3 = ie^{k/2}Z(z^1 z^2 z^3 - \bar{z}^1 \bar{z}^2 \bar{z}^3), \] (55)

\[ q_1 - p^0 z^2 z^3 = ie^{k/2}Z(z^2 z^3 - \bar{z}^2 \bar{z}^3), \] (56)

\[ q_1 z^1 - p^1 z^2 z^3 = ie^{k/2}Z(z^1 z^2 z^3 - z^1 \bar{z}^2 \bar{z}^3), \] (57)
\[ q_3 z^2 - p^2 z^1 z^2 = i e^{k/2} Z (z^2 z^1 - z^2 z^1 ) \] (58)

and

\[ q_2 z^3 - p^3 z^1 z^3 = i e^{k/2} Z (z^3 z^1 z^1 - z^3 z^1 z^1 ) \] (59)

While for the second system of equations one has

\[ q_4 + p^4 z^6 z^7 = i e^{k/2} Z (z^5 z^6 z^7 - z^5 z^6 z^7 ) \] (60)

\[ q_5 - p^4 z^6 z^7 = i e^{k/2} Z (z^6 z^7 - z^6 z^7 ) \] (61)

\[ q_5 z^5 - p^5 z^6 z^7 = i e^{k/2} Z (z^5 z^6 z^7 - z^5 z^6 z^7 ) \] (62)

\[ q_7 z^6 - p^6 z^5 z^6 = i e^{k/2} Z (z^6 z^5 z^6 - z^6 z^5 z^6 ) \] (63)

and

\[ q_6 z^7 - p^7 z^5 z^7 = i e^{k/2} Z (z^7 z^5 z^7 - z^7 z^5 z^7 ) \] (64)

Dividing (55) and (56) and eliminating factors one obtains

\[ \bar{z}^1 = - \frac{q_0 + p^1 z^2 z^3}{q_1 - p^0 z^2 z^3} \] (65)

and dividing (60) and (61) one gets

\[ z^5 = - \frac{q_4 + p^5 z^6 z^7}{q_5 - p^4 z^6 z^7} \] (66)

While from (65) and (66) one gets value for \( z^2 z^3 \) and \( z^6 z^7 \) respectively

\[ z^2 z^3 = \frac{q_1 \bar{z}^1 + q_0}{p^0 \bar{z}^1 - p^1} \] (67)

and

\[ z^6 z^7 = \frac{q_5 \bar{z}^5 + q_4}{p^4 \bar{z}^5 - p^5} \] (68)

Furthermore, simplifying the equation (56) and setting \( \bar{z}^2 z^3 = \frac{1}{i e^{k/2} Z} (p^{0b} z^2 z^3 - q_0^b) + z^2 z^3 \) in (57) one finds that the expression \( i e^{k/2} Z \) acquire the form
\[ ie^{k/2}Z = \frac{p^0 z^1 - p^1}{z^1 - z^1}. \] (69)

Similarly using (62) and (63) one finds
\[ ie^{k/2}Z = \frac{p^4 z^5 - p^5}{z^5 - z^5}. \] (70)

Substituting (69) into (57) and (58) one obtains for \( \bar{z}^2 \) and \( \bar{z}^3 \) the formulae
\[ \bar{z}^2 = \frac{p^2 z^1 - q_3}{p^0 z^1 - p^1}, \] (71)
and
\[ \bar{z}^3 = \frac{p^3 z^1 - q_2}{p^0 z^1 - p^1}, \] (72)

respectively.

Similarly, substituting (70) into (62) and (63) one has for \( \bar{z}^6 \) and \( \bar{z}^7 \) the formulae
\[ \bar{z}^6 = \frac{p^6 z^5 - q_7}{p^4 z^5 - p^5}, \] (73)
and
\[ \bar{z}^7 = \frac{p^7 z^5 - q_6}{p^4 z^5 - p^5}, \] (74)

respectively.

Moreover, multiplying these last two expressions, and taking into account (67) and (69) as well as the equations (68) and (70) the quadratic equations are obtained
\[ (z^1)^2 + \frac{(p \cdot q)_1 - 2p^1 q_1}{(p^0 q_1 - p^2 p^3)}z^1 - \frac{(p^1 q_0 + q_3 q_2)}{(p^0 q_1 - p^2 p^3)} = 0 \] (75)

and
\[ (z^5)^2 + \frac{(p \cdot q)_2 - 2p^5 q_5}{(p^4 q_5 - p^6 p^7)}z^5 - \frac{(p^5 q_4 + q_7 q_6)}{(p^4 q_5 - p^6 p^7)} = 0, \] (76)

respectively. Here, one used the definitions
\[ (p \cdot q)_1 = (p^0 q_0) + (p^1 q_1) + (p^2 q_2) + (p^3 q_3) \] (77)
and

\[(p \cdot q)_2 = (p^4 q_4) + (p^5 q_5) + (p^6 q_6) + (p^7 q_7). \quad (78)\]

The equations (75) and (76) can be solved using the two solutions called roots. The two solutions for \(z^1\) and \(z^2\) moduli [8] are

\[z^1 = \frac{(p \cdot q)_1 - 2p^1 q_1 \mp i\sqrt{W}}{2(p^2 p^3 - p^0 q_1)} \quad (79)\]

and

\[z^5 = \frac{(p \cdot q)_2 - 2p^5 q_5 \mp i\sqrt{W}}{2(p^6 p^7 - p^4 q_5)}. \quad (80)\]

Here, for reasons of space has been simplified solution where \(W\) is [8]

\[W_1(p^\Lambda, q_\Lambda) = -(p \cdot q)_1^2 + 4((p^1 q_1)(p^2 q_2) + (p^1 q_1)(p^3 q_3) + (p^3 q_3)(p^2 q_2)) - 4p^0 q_1 q_2 q_3 + 4q_0 p^1 p^2 p^3 \quad (81)\]

and

\[W_2(p^\Lambda, q_\Lambda) = -(p \cdot q)_2^2 + 4((p^5 q_5)(p^6 q_6) + (p^5 q_5)(p^7 q_7) + (p^7 q_7)(p^6 q_6)) - 4p^4 q_5 q_6 q_7 + 4q_4 p^5 p^6 p^7. \quad (82)\]

The function \(W(p^\Lambda, q_\Lambda)\) is symmetric under the charges transformation \(p^1 \leftrightarrow p^2 \leftrightarrow p^3\) and \(p^5 \leftrightarrow p^6 \leftrightarrow p^7\) and as well as under \(q_1 \leftrightarrow q_2 \leftrightarrow q_3\) and \(q_5 \leftrightarrow q_6 \leftrightarrow q_7\). Finally the solution for the complex three modulis is

\[z^{i_1} = \frac{((p \cdot q)_1 - 2p^{i_1} q_{i_1}) \mp i\sqrt{W}}{2(3d_{ijk} p^j p^k - p^0 q_1)} \quad (83)\]

and

\[z^{i_2} = \frac{((p \cdot q)_2 - 2p^{i_2} q_{i_2}) \mp i\sqrt{W}}{2(3d_{ijk} p^j p^k - p^4 q_i)}. \quad (84)\]

Here, the index \(i\) in \(p^i q_i\) does not denote a sum. For consistency, the solution requires that \(W > 0\) and that the moduli charges be real.

Now a choice of signs in the imaginary part of moduli is done to preserve the symmetry [8]. Thus, Kähler potential given in equation (45), written as

\[e^{-K} = -id_{ijk}(z - \bar{z})^i(z - \bar{z})^j(z - \bar{z})^k,\]

can be calculated from equations (83) and (84). One gets the result
\[ e^{-K} = \pm \frac{W^{3/2}}{\varpi_1 \varpi_2 \varpi_3}. \quad (85) \]

And similarly one obtains

\[ e^{-K} = \pm \frac{W^{3/2}}{\varpi_5 \varpi_6 \varpi_7}, \quad (86) \]

with \( \Lambda = 0, 1, 2, 3 \). Here, one has

\[ \varpi_i = (3d_{ijk}p^jp^k - p^0q_i). \quad (1) \]

Thus, running the index \( i = 1, 2, 3 \), it is possible to obtain

\[ \varpi_1 = p^2p^3 - p^0q_1, \quad (88) \]

\[ \varpi_2 = p^1p^3 - p^0q_2 \quad (89) \]

and

\[ \varpi_3 = p^1p^2 - p^0q_3. \quad (90) \]

Multiplying by \( \varpi_1 \varpi_2 \varpi_3 = (p^2p^3 - p^0q_1)(p^1p^3 - p^0q_2)(p^1p^2 - p^0q_3) \), and using the equation for the expression \( W_1(p^\Lambda, q_\Lambda) \) given in (81), one sees that

\[ W_1(p^\Lambda, q_\Lambda) + (p \cdot q)^2 = 4((p^1q_1)(p^2q_2) + (p^1q_1)(p^3q_3) \]

\[ + (p^3q_3)(p^2q_2)) - 4p^0q_1q_2q_3 + 4q_0p^1p^2p^3. \quad (91) \]

In this form the substitution of the product \( \varpi_1 \varpi_2 \varpi_3 \) leads to

\[ \varpi_1 \varpi_2 \varpi_3 = \frac{1}{4}((p^0)^2W_1 + 4(p^1p^2p^3)^2 - 2p^0(p \cdot q)(p^1p^2p^3) + (p^0)^2(p \cdot q))^2), \quad (92) \]

and therefore one gets

\[ \varpi_1 \varpi_2 \varpi_3 = \frac{1}{4}((p^0)^2W_1 + [2p^1p^2p^3 - p^0(p \cdot q))^2]). \quad (93) \]

A similar relation results for \( \varpi_5 \varpi_6 \varpi_7 \):

\[ \varpi_5 \varpi_6 \varpi_7 = \frac{1}{4}((p^4)^2W_2 + [2p^5p^6p^7 - p^4(p \cdot q))^2]). \quad (94) \]

The solution for \( z^1 \) and \( z^5 \) moduli charges are
\[ z^1 = \frac{(p \cdot q)_1 - 2p^1q_1 - i\sqrt{W_1}}{2(3d_{ijk}p^ip^k - p^0q_i)} \] (95)

and

\[ z^2 = \frac{(p \cdot q)_2 - 2p^5q_5 - i\sqrt{W_2}}{2(3d_{ijk}p^ip^k - p^0q_i)} \] (96)

It is important to mention that one can have exponential positive value of \( e^{-K} \), namely

\[ e^{-K} = \frac{W_3^{3/2}}{\omega_1 \omega_2 \omega_3} > 0. \] (97)

Thus, now that we have obtained a value for \( e^{-K} \), will proceed with the calculation of the central charge \( \bar{Z} \). This shall allows to derive the black hole mass. Of course, for extremal black hole the mass is proportional to the area of the event horizon. Substituting therein (79) in (69), one obtains the value for \( Z \). Subsequently using the complex conjugate \( \bar{Z} \) one learns that [8],

\[ e^K \bar{Z} Z = \frac{(p^0)^2W_1 + [2p^1p^2p^3 - p^0(p \cdot q))^2]}{4W_1}, \] (98)

or

\[ Z \bar{Z} = e^{-K} \frac{(p^0)^2W_1 + [2p^1p^2p^3 - p^0(p \cdot q))^2]}{4W_1}. \] (99)

Following similar procedure one finds

\[ Z \bar{Z} = e^{-K} \frac{(p^4)^2W_2 + [2p^5p^6p^7 - p^4(p \cdot q))^2]}{4W_2}. \] (100)

Thus, one gets the relation

\[ Z \bar{Z} = M^2 = \frac{A}{4\pi} = (W(p^A, q_\Lambda))^{1/2}. \] (101)

Then, it has been fully described extremal black holes with moduli solutions in a theory of symmetrical STU [8]. This black hole corresponds to the extremal Reissner-Nordström metric, namely

\[ ds^2 = (1 + \frac{W(p^A, q_\Lambda)]^{1/4}}{r})^{-2} dt^2 - (1 + \frac{W(p^A, q_\Lambda)]^{1/4}}{r})^2 dr^2 + r^2 d\Omega^2. \] (102)
Considering the black hole area $A_{AN} = 4\pi M^2$ and the entropy $S_{BH} = \frac{k^2 A}{4\pi G}$ one discovers the expression
\[
S = \pi M^2 = \pi \sqrt{W(p^4, q_\Lambda)}.
\] (103)

Note that for consistency we must require $W > 0$.

Now, our goal is to describe (103) in terms of the so called rebits $b_{ijkl}$.

Using equation (22) and establishing the relation
\[
\begin{pmatrix}
  b_{0000} \\
  b_{0010} \\
  b_{0001} \\
  b_{0100} \\
  b_{1000} \\
  b_{0101} \\
  b_{0110} \\
  b_{1100} \\
  b_{0011} \\
  b_{0111} \\
  b_{1010} \\
  b_{1001} \\
  b_{1110} \\
  b_{1101} \\
  b_{1111}
\end{pmatrix}
= \begin{pmatrix}
  -p^0 \\
  -p^1 \\
  -p^2 \\
  -q_3 \\
  p^3 \\
  q_2 \\
  q_1 \\
  -q_0 \\
  -q_4 \\
  -p^4 \\
  -p^5 \\
  -p^6 \\
  -q_7 \\
  p^7 \\
  q_6 \\
  q_5 \\
  -q_4
\end{pmatrix},
\]
(104)

we shall attempt to write $W(p^4, q_\Lambda)$ in terms of a $2 \times 2 \times 2$-hyperdeterminant.
Note that (104) can also be written in terms of complex qubits structure
\[
\begin{pmatrix}
  a_{000} \\
  a_{001} \\
  a_{010} \\
  a_{011} \\
  a_{100} \\
  a_{101} \\
  a_{110} \\
  a_{111}
\end{pmatrix}
= \begin{pmatrix}
  -p^0 \\
  -p^1 \\
  -p^2 \\
  -q_3 \\
  p^3 \\
  q_2 \\
  q_1 \\
  -q_0
\end{pmatrix}
+ i \begin{pmatrix}
  -p^4 \\
  -p^5 \\
  -p^6 \\
  -q_7 \\
  p^7 \\
  q_6 \\
  q_5 \\
  -q_4
\end{pmatrix}.
\]
(105)

Considering the equations (81) and (82) one may obtain the correspondence
\[
W(p^4, q_\Lambda) = -\text{Det } b_4.
\] (106)
Hence, we have shown that the entropy for \((8 + 8)\) extremal black hole is given by the expression

\[
S = \pi \sqrt{-\text{Det } b_4}.
\]  

(107)

Of course, this expression is completely analogue to the case of \((4 + 4)\) extremal black hole in which one finds that the entropy \(S = \pi \sqrt{-\text{Det } b_3}\) is given in terms of the 3-rebits \(b_3 = b_{ijk}\).

5.- Quantum entanglement of 3-qubits and Black Holes

The Cayley hyperdeterminant is also an important mathematical notion in the quantum information theory [2]. In order to have a better understanding of this connection let us consider a 3-qubit system which is key concept in such a theory. This is a pure state \(|\Psi\rangle\) that can be written in the base \(|ijk\rangle\) as

\[
|\Psi\rangle = \sum_{ijk} a_{ijk}|ijk\rangle
\]  

(108)

or

\[
|\Psi\rangle = a_{000}|000\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{100}|100\rangle
\]

\[
+ a_{011}|011\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle.
\]  

(109)

In this context the quantities \(a_{ijk}\) are complex numbers. In this case, instead of integers symmetry \([SL(2, Z)^3]\) one must use the complex symmetry \([SL(2, C)^3]\). It turns out that the 3-way quantum entanglement of qubits \(A\), \(B\) and \(C\) state is given by the 3-entanglement [26]:

\[
\tau_{ABC} = 2|\epsilon^{i'j'k'}\epsilon^{nm'i'^n'm'}\epsilon^{m'i'j'} a_{ijk} a_{njk} a_{npk} a_{njp}| \]

(110)

or

\[
\tau_{ABC} = 4|\text{Det } a|.
\]  

(111)

where \(\text{Det } a\) denotes the Cayley hyperdeterminant applied to \(a_{ijk}\). Moreover, it is known that the Cayley hyperdeterminant gives a connection between string theory and quantum entanglement of information theory.

It is worth mentioning that some of the above notions have been used to explain the Bekenstein and Hawking (see Refs. [3], [27]-[33]) approach to black holes which establishes that at the quantum level a black holes should radiate energy. Thus, according to the no hair theorem, one should expect that the
Hawking radiation should be completely independent of the objects falling into the black hole. More precisely, if any quantum entanglement and part of the interlock system is thrown into the black hole one must determine that the other part is kept out. But all that falls into a black hole should reach the singularity in a finite time and then could completely disappear from the physical system (see Refs. [3], [27]-[33] for details).

6.-Concluding Remarks

In this work, we have shown that using the hyperdeterminant $\text{Det } b_4$ for a 4-qubit one can determine an entropy for a extremal black-hole with a $(8 + 8)$-signature associated to electric and magnetic charges. Since 8-dimensions is one of the allowed dimensions in both the Hopf maps and division algebras, one may expect that eventually our formalism may establish a connection with those remarkable mathematical structures.

Due to such a black-hole/qubit correspondence one is tempted to believe that qubit theory may be the key mathematical tool for quantum gravity. Of course this may lead eventually to a connection between qubit theory and string theory or M-theory. This idea is reinforced from the fact that our work suggests to consider quantum entanglement as an alternative for the study of micro black holes.

Finally, it is remarkable to see how different scenarios such as black holes thermodynamics, electromagnetism, quantum information, general relativity and string theory are closely related theoretically through the modular symmetry $\text{SL}(2, R)$. It turns out that, this symmetry is in agreement with the hyperdeterminant $\text{Det } a_4$ used in our formalism.

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