VELOCITY PEAKS AND CAUSTIC RINGS

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The late infall of cold dark matter onto an isolated galaxy produces flows with definite local velocity vectors throughout the galactic halo. It also produces caustic rings, which are places in the halo where the dark matter density is very large. The self-similar model of halo formation predicts that the caustic ring radii follow the approximate law \( a_n \approx 1/n \). I interpret bumps in the rotation curves of NGC 3198 and of our own galaxy as due to caustic rings of dark matter. In this model of our halo the annual modulation effect in direct searches for WIMPs has the opposite sign from that predicted by the isothermal sphere model.

1 Late infall

There are compelling reasons to believe that the dominant component of the dark matter of the Universe is non-baryonic collisionless particles, such as axions, weakly interacting massive particles (WIMPs) and massive neutrinos. The word “collisionless” signifies that the particles are so weakly interacting that they have moved purely under the influence of gravity since their decoupling at a very early time (of order \( 10^{-4} \) sec for axions, of order 1 sec for neutrinos and WIMPs). In the limit where the primordial velocity dispersion of the particles is neglected, they all lie on the same 3-dimensional (D) ‘sheet’ in 6-D phase-space. Their phase-space evolution must obey Liouville’s theorem. This implies that the sheet cannot tear and hence that it satisfies certain topological constraints.

Because their phase-space sheet cannot tear, collisionless dark matter (CDM) particles must be present everywhere in space, including specifically intergalactic space. The space density may be reduced by stretching of the sheet, but it cannot vanish. Moreover, the average space density is recovered as soon as the average is taken over distances larger than the distance CDM may have moved locally away from perfect Hubble flow. In a region that is sparsely populated with galaxies, this distance is much smaller than the distance between galaxies. The implication is that isolated galaxies are surrounded by unseen CDM and hence, because of gravity, CDM keeps falling continuously onto such galaxies from all directions. If the galaxy joins other galaxies to form a cluster,
infall onto the galaxy gets shut off because of lack of material, but infall onto the cluster continues assuming that the cluster is itself isolated. In an open universe \( (\Omega < 1) \), the infall process eventually turns off because the universe becomes very dilute. However, even if our universe is open, we are far from having reached the turn-off time.

2 Velocity peaks

Consider the infall of CDM onto an isolated galaxy. Let us at first neglect the velocity dispersion of the infalling particles. In practice it is sufficient that their velocity dispersion is much smaller than the rotation velocity of the galaxy. Consider the time evolution of all CDM particles that are about to fall onto the galaxy for the first time in their history at time \( t \). For the sake of definiteness, we may consider all particles which have zero radial velocity \( (\dot{r} = 0) \) for the first time then. Such particles are said to be at their ‘first turnaround’; they were receding from the galaxy as part of the general Hubble flow before \( t \) and will be falling onto the galaxy just after \( t \). They form a closed surface, enclosing the galaxy, called the turnaround 'sphere' at time \( t \). The present turnaround sphere of the Milky Way galaxy has a radius of order 2 Mpc. The turnaround sphere at time \( t \) falls through the central parts of the galaxy at a time of order \( 2t \). Particles falling through the galactic disk (assuming the galaxy is a spiral) get scattered through an angle \( \Delta \theta \sim 10^{-3} \) by the gravitational fields of various inhomogeneities such as molecular clouds, globular clusters, and stars. However, most particles carry too much angular momentum to reach the luminous parts of the galaxy and are scattered much less. Because the scattering is small, the particles on the turnaround sphere at time \( t \), after falling through the galaxy, form a new sphere which reaches its maximum radius \( R' \) at some time \( t' \). The radius \( R' \) at the second turnaround is smaller than the radius \( R \) at the first turnaround because the galaxy has grown by infall in the meantime. The sphere continues oscillating in this way although it gets progressively fuzzier because of scattering off inhomogeneities in the galaxy.

Fig. 1 describes the fall of one turnaround sphere through the galaxy as a succession of time frames. No particular symmetry is assumed. It is assumed, for the sake of definiteness, that the particles on the turnaround sphere carry net angular momentum about the vertical axis. The particles which are near the top in frame a) carry little angular momentum. They fall through the center of the galaxy and end up near the bottom of the turnaround sphere in frame f). Similarly the particles which are near the bottom in frame a) end up near the top in frame f). The particles which are near the equator carry
Figure 1: Infall of a turnaround sphere. The sphere has net angular momentum about the vertical axis. It crosses itself between frames b) and c). After frame e) the sphere has completed the process of turning itself inside out. The cusps in frames d) and e) are at the intersection of a ring caustic with the plane of the figure.
the most angular momentum. They form a ring whose size decreases to some minimum value and then increases again. In the process of falling through the galaxy the turnaround sphere turns itself inside out. Note also that the sphere crosses at least twice each point which is inside the sphere both at the initial time of frame a) and at the final time of frame f).

Fig. 1 shows the motion of just one sphere in a continuous flow of such spheres. Moreover, there are many flows in and out of the galaxy going on at the same time. To each flow in and out of the galaxy is associated a pair of peaks in the velocity distribution of CDM particles at every physical point in the galactic halo. One peak is due to particles falling onto the galaxy for the first time, one peak is due to particles falling out of the galaxy for the first time, one peak is due to particles falling onto the galaxy for the second time, and so on. In particular this is true on Earth. Igor Tkachev, Yun Wang and I obtained estimates of the velocity magnitudes and the local density fractions associated with these peaks using the self-similar infall model of galactic halo formation. We generalized the existing version of the model to take account of the angular momentum of the dark matter particles. We find that the first twenty peaks contribute each between one to four percent of the local density. This implies a large non-thermal component to the galactic CDM distribution.

3 Caustic rings

There is a caustic ring associated with each flow in and out of the galaxy. A caustic is a place in physical space where the density is large because the phase-space sheet ‘folds back’ there. At the caustic, the space density diverges in the limit of zero velocity dispersion. In Fig. 1, the caustic ring intersects the plane of the figure at the location of the cusps in frames d) and e). One can prove mathematically that the density diverges at these points in the limit of zero velocity dispersion of the infalling particles. In reality the infalling particles have some velocity dispersion. However only when this velocity dispersion is as large as 30 km/s do the caustic rings in our galaxy get washed out. It is generally believed that the velocity dispersion of infalling CDM particles is at most 10 km/s.

For an arbitrary angular momentum distribution on the turnaround sphere, the ring is a closed loop of arbitrary shape. However, if the angular momentum distribution is dominated by a smooth component that carries net angular momentum, the ring resembles a circle. If there is no angular momentum at all, the ring reduces to a point at the galactic center.

The caustic ring is located in physical space near where the particles with the largest amount of angular momentum are at their distance of closest ap-
proach to the galactic center. In the particular case where the turnaround sphere is initially rotating with a definite angular velocity as if it were a rigid body, the density distribution near the caustic is:

$$d(a; \rho, z) \simeq \frac{dM}{d\Omega dt} \frac{2}{v} \frac{1}{\sqrt{(r^2 - a^2)^2 + 4a^2z^2}}$$

where $(\rho, z, \theta)$ are cylindrical coordinates, $a$ is the caustic ring radius assumed to be much smaller than $R$, $r = \sqrt{\rho^2 + z^2}$, $dM/d\Omega dt$ is the rate at which mass falls in per unit time and unit solid angle, and $v$ is the velocity magnitude of the particles at the caustic. Near the caustic, the density diverges as the inverse distance to the ring:

$$d(a; \rho, z) \simeq \frac{dM}{d\Omega dt} \frac{1}{v a \sigma}$$

with $\sigma = \sqrt{(\rho - a)^2 + z^2}$.

The self-similar infall model mentioned earlier predicts the values of successive radii to be:

$$\{a_n : n = 1, 2, 3...\} \simeq (39, 19.5, 13, 10, 8...)kpc \left(\frac{j_{max}}{0.25}\right) \left(\frac{0.7}{h}\right) \left(\frac{v_{rot}}{220\text{km/s}}\right)$$

for $\epsilon = 0.3$. $\epsilon$ is a parameter which is predicted by theories of large scale structure formation to be in the range 0.2 to 0.35, $j_{max}$ is the maximum value of the angular momentum of the particles on the turnaround sphere in the dimensionless units defined in Ref.[2], $h$ is the Hubble rate in units of 100 km/sec.Mpc and $v_{rot}$ is the rotation velocity of the galaxy. For $\epsilon = 0.2$, $a_1 \simeq 36kpc \left(\frac{j_{max}}{0.25}\right) \left(\frac{0.7}{h}\right) \left(\frac{v_{rot}}{220\text{km/s}}\right)$, but the ratios $a_n/a_1$ are almost the same as in the $\epsilon = 0.3$ case. Thus in the range of interest $0.2 \leq \epsilon \leq 0.35$, the ratios of ring radii are nearly $\epsilon$ independent and follow the approximate law: $a_n \sim 1/n$. The self-similar model also predicts for each ring the prefactor $dM/d\Omega dt \frac{2}{v}$ that appears in Eq.(1). For example, for $\epsilon = 0.2$, $\left\{\frac{dM}{d\Omega dt} \frac{2}{v} : n = 1, 2...\right\} = (26, 11, 7, 5, 4, ...) 10^{-2} \frac{v_{rot}^2}{4\pi G}$.

The amount of angular momentum is related to the effective core radius of the galactic halo. For galaxies like our own, the average of the dimensionless angular momentum distribution $j \simeq 0.2$. For a given $j$ distribution, $j$ and $j_{max}$ are related by some numerical factor. If the turnaround sphere is initially rigidly rotating, $j_{max} = \frac{4}{\pi} j$.

If the caustic rings lie close to the galactic plane, they may manifest themselves as bumps in the rotation curve. Galactic rotation curves often do have bumps. Of special interest here are those which occur at radii larger than the
disc radius because they cannot readily be attributed to inhomogeneities in the luminous matter distribution. Consider the rotation curve of NGC 3198, one of the best measured and often cited as providing compelling evidence for the existence of dark halos. It appears to have bumps near 28, 13.5 and 9 kpc, assuming $h = 0.75$. Although the statistical significance of these bumps is not great, let’s assume for the moment that they are real effects. Note then that their existence is inconsistent with the assumption that the dark halo is a perfect isothermal sphere. On the other hand, the radii at which they occur are in close agreement with the ratios predicted by the self-similar model assuming that the bumps are caused by the gravitational fields of the first three caustic rings of NGC 3198. Since $v_{\text{rot}} = 150$ km/sec, we find that $j_{\text{max}} = 0.28$ in this case if $\epsilon = 0.3$. The uncertainty in $h$ drops out. A fit of the infall model to our own galaxy produced $j \simeq 0.2$ for $\epsilon = 0.2$ to 0.3. If the turnaround sphere is initially rigidly rotating, one has $j_{\text{max}} = \frac{4}{3} j$. Thus the values of $j$ for our own halo and that of NGC 3198 are found to be similar.

Let’s discuss the implications of the self-similar model for our own halo. As an example, we use the model parameters $\epsilon = 0.28$, $j_{\text{max}} = 0.25$ and $h = 0.7$. Table 1 shows the caustic ring radii $a_n$, the local velocities $v_n$, and the local densities $d_\pm^n$, at the position of the Sun, associated with the first 20 pairs of flows. $\hat{z}$ is the direction perpendicular to the galactic plane, $\hat{r}$ is the radial direction in the galactic plane and $\hat{\phi}$ is the direction of galactic rotation. The velocities are given in the rest frame of the Galaxy. $d_\pm^n$ is the density of each of the two $n$th flows. We at 8.5 kpc are between the 4th and 5th ring. The local densities of the 4th and 5th flows are large because of our proximity to the corresponding rings.

There is evidence for the 6th through 13th caustic rings in that there are sudden rises in the inner rotation curve of our galaxy at radii very near those listed in the Table. Similarly there is some evidence for the 2d and 3d caustic rings in the averaged outer rotation curve. This will be discussed in detail in a future paper.

This study was motivated by the axion and WIMP dark matter searches. These experiments may some day measure the energy spectrum of CDM particles on Earth. I would like to stress here the relevance of the model for the annual modulation of the signal in direct searches for WIMP dark matter. The $n = 2, 3, ..., 8$ pairs of flows have velocities exceeding the 220 km/s velocity of the Sun in the direction $\hat{\phi}$ of the Sun’s motion. A simple calculation shows that they dominate the annual modulation because of their large contributions to the local halo density. Thus the annual modulation of the WIMP signal has the opposite sign in this model from that predicted by the isothermal sphere model.
Table 1: Caustic ring radii, local velocities and local densities of the first 20 pairs of flows in the self-similar infall model with $\epsilon = 0.28, j_{max} = 0.25$ and $h = 0.7$.

| n  | $a_n$ (kpc) | $v_n$ (km/s) | $v_{n\theta}$ (km/s) | $v_{nz}$ (km/s) | $v_{nr}$ (km/s) | $n_0^n$ (10^{-26} gr/cm^3) |
|----|-------------|--------------|----------------------|----------------|----------------|--------------------------|
| 1  | 38.9        | 620          | ±605                 | 0              | 0              | 4.4                      |
| 2  | 19.6        | 565          | 255 ±505             | 0              | 0              | 1.0                      |
| 3  | 13.3        | 530          | 350 ±390             | 0              | 0              | 2.0                      |
| 4  | 9.7         | 500          | 440 ±240             | 0              | 0              | 6.3                      |
| 5  | 7.8         | 480          | 440 ±190             | 0              | 0              | 9.2                      |
| 6  | 6.5         | 460          | 355 0                | ±295           | 2.9            |
| 7  | 5.6         | 445          | 290 ±330             | 0              | 1.9            |
| 8  | 4.9         | 430          | 250 ±350             | 0              | 1.4            |
| 9  | 4.4         | 415          | 215 ±355             | 0              | 1.1            |
| 10 | 4.0         | 400          | 190 ±355             | 0              | 1.0            |
| 11 | 3.6         | 390          | 170 ±355             | 0              | 0.9            |
| 12 | 3.3         | 380          | 150 ±350             | 0              | 0.8            |
| 13 | 3.1         | 370          | 135 ±345             | 0              | 0.7            |
| 14 | 2.9         | 360          | 120 ±340             | 0              | 0.6            |
| 15 | 2.7         | 350          | 110 ±330             | 0              | 0.6            |
| 16 | 2.5         | 340          | 100 ±325             | 0              | 0.55           |
| 17 | 2.4         | 330          | 90 ±320              | 0              | 0.50           |
| 18 | 2.2         | 320          | 85 ±310              | 0              | 0.50           |
| 19 | 2.1         | 315          | 80 ±305              | 0              | 0.45           |
| 20 | 2.0         | 310          | 75 ±300              | 0              | 0.45           |

The model is also relevant to calculations of the sky-map of gamma radiation from WIMP annihilation in the galactic halo. The annihilation rate is proportional to the square of the density. If the diffuse gamma ray component seen by EGRET is attributed to annihilations in the halo away from the disk, there ought to be hot spots in the disk at the location of the rings.

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