Research on the Co-occurrence Characteristics of the Floods Between Lower Reaches of Jinsha River and the Chuanjiang River

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Abstract. In order to provide a theoretical basis for real-time flood control, reduce the flood disaster in the lower reaches of Jinsha River and the upstream Yangtze River (the section in Sichuan Province, also known as the Chuanjiang River), as well as alleviate flood control pressure of the downstream Three Gorges Project flood encounter characteristics between the lower reaches of Jinsha River and the Chuanjiang River were discussed in detail based on Copula Function Method. AIC criterion and OLS criterion were applied in the paper to evaluate the fitting effect of four Archimedean Copula function models, and Frank Copula function with the best effect was selected for the flood encounter analysis subsequently. The result shows that: the proportion of the maximum 3d, 7d, 15d, 30d flood volume of the lower reaches of Jinsha River in the composition of the Chuanjiang River increases in sequence. Frank Copula function has very small errors in the flood encounter problems of the lower reaches of Jinsha River and Chuanjiang River, with higher credibility, its calculation results are consistent with the flood encounter laws of the two rivers. The frequency of occurrence of the largest flood peak in the two rivers in different months is significantly different. The co-occurrence probability of floods with peak flow greater than 50-year flood for the two rivers is 0.15%. Therefore, by taking full advantages of the flood co-occurrence characteristics, it’s extremely practicable to reduce not only the flood control pressure of Chuanjiang River but also the downstream Yangtze with the cascade reservoir operation in the lower reaches of Jinsha River.

1. Introduction

With global climate change, extreme weather disasters such as floods, high temperatures and droughts occur frequently [1,2]. The fifth assessment report released by the Intergovernmental Panel on Climate Change (IPCC) in 2013 pointed out that the flood disaster has an increasing frequency around the world [3], which has caused a serious impact on the sustainable development of social economy and people’s life safety [4]. Flood is one of the most common disasters. According to statistics, nearly more than 100 large and medium-sized cities, 3.3×10⁷ km² of cultivated land are threatened by floods [5].

In recent decades, China has invested a large amount of money in the construction of water conservancy projects and has become the country with the largest number of reservoirs in the world. By
by flood disasters, but also promotes the sustainable development of China's social economy. With the completion of the Wudongde hydropower Station in the lower reaches of Jinsha River, four world-class cascade hydropower stations will benefit flood prevention during flood season and rational allocation of water resources in the Yangtze River basin.

The study of the flood characteristics and Co-occurrence problems in the lower reaches of Jinsha River and Chuanjiang River provides a scientific basis for the safe operation and optimal management of cascade reservoirs in the lower reaches of Jinsha River, and is of great significance for ensuring the safety of flood control objects and rational allocation of flood resources in the lower reaches.

2. Overview of the study area
The main research area of this article is from the lower reaches of Jinsha River (Panzhihua Station) to Chuanjiang River (Cuntan Station). The watershed area of the lower reaches of the Jinsha River is 214,000 km², and the average ratio of the river reaches 0.93‰. The Chuanjiang River starts at the junction of the Jinsha River and the Min River in Yibin City, Chuanjiang Province, and reaches Nanjin Pass in Yichang City, Hubei Province, with a total length of 1030km. Figure 1 is a map of the lower reaches of the Jinsha River and the Chuanjiang River Basin.

3. Flood characteristics and regional composition
3.1. Analysis of flood characteristics
The frequency statistics of the largest flood peaks in the two rivers in each month are shown in Figure 2. It can be seen from Figure 2 that the frequency of the largest flood peaks in the two rivers Basin in different months is significantly different. The maximum frequency of the peak floods in the lower reaches of the Jinsha River in August is 42%, and the highest frequency in the Chuanjiang River in July is 47.6 %.

The characteristics of the floods in the two rivers are shown in Table 1. It can be seen from Table 1 that the flood peak height of the Chuanjiang River is large, and the average maximum flood peak is 7 times that of the lower reaches of the Jinsha River, and the maximum 3, 7, 15, 30d floods are 6.49, 5.93, 5.62, and 5.62 times, respectively. The flood duration is short, and the average flood history is 0.6 times that of the lower reaches of the Jinsha River. The average flood rate of the Chuanjiang River floods is larger, and the average rate of rise and fall is 10.02 and 10.30 times that of the lower reaches of the Jinsha River respectively, and the flood process line is steeper.
Table 1. Features of sub-flood of Jinsha River and Chuanjiang River

| Station       | Panzhihua | Cuntan | Cuntan/Panzhihua |
|---------------|-----------|--------|------------------|
| Average maximum flood peak (m³) | 6800      | 47578  | 7.00             |
| Average maximum 3d flood volume (10⁸ m³) | 16.66     | 108.11 | 6.49             |
| Average maximum 7d flood volume (10⁸ m³) | 36.44     | 216.02 | 5.93             |
| Average maximum 15d flood volume (10⁸ m³) | 71.21     | 399.99 | 5.62             |
| Average maximum 30d flood volume (10⁸ m³) | 126.70    | 711.64 | 5.62             |
| Duration of flood (d) | 30.00     | 18.00  | 0.60             |
| Average flood rise rate (m³ s⁻¹ h⁻¹) | 2.138     | 218.73 | 10.02            |
| Average flood fall rate (m³ s⁻¹ h⁻¹) | 15.35     | 158.09 | 10.30            |

3.2 Regional composition of the flood in Chuanjiang River

It can be seen from Figure 3 that the lower reaches of the Jinsha River account for 15.18%, 16.46%, 17.25%, and 17.4% of the composition of the Chuanjiang River's largest floods of 3, 7, 15, and 30 days, respectively.

![Figure 3. Regional composition of maximum 3d, 7d, 15d, 30d flood volume](image)

Combined with the research on the characteristics and regional composition of the floods in the two rivers, considering that the floods in the two rivers are mainly floods, the impact of the floods in the lower reaches of the Jinsha River on the Chuanjiang River increases as the time period increases, and the average flood duration of the Chuanjiang River is The characteristics of 18d and so on, the maximum 15d flood volume is selected for the flood analysis of the downstream of Jinsha River and Chuanjiang River.

4. Copula function of floods in lower reaches of Jinsha River and Chuanjiang River

The Copula function can be divided into three types: Archimedes, ellipses, and quadratic. At present, Archimedes is the most widely used in the field of hydrology, and its structural form is [7]:

\[
C(\mu, v) = \varphi^\alpha(-1) [\varphi(\mu) + \varphi(v)], \quad 0 \leq \mu, v \leq 1
\]

where \(\mu\) and \(v\) are the edge distribution functions; \(\varphi\) is the generator.

According to Sklar quantification and Copula function related theory, Copula function \(C(\mu, v)\)
makes the joint probability distribution function of \(X_1\) and \(X_2\):

\[
F(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2) = \int_{-\infty}^{x_2} \int_{-\infty}^{x_1} f(\mu, \nu) d\mu d\nu = C(\mu, \nu)
\]

(2)

Where, \(F_1(x_1)\) and \(F_2(x_2)\) are edge distribution functions \(\mu\) and \(\nu\) respectively.

### 4.1. Marginal distribution

In this paper, the Pearson type III distribution, which is the most widely used in hydrological analysis \[10\], is used, and its probability density expression is formula (3):

\[
f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} \times (x - a_0)^{(\alpha-1)} \times \exp \left[ -\beta \times (x - a_0) \right]
\]

(3)

Where, \(\Gamma(\alpha)\) is the gamma function of \(\alpha\), \(a_0\), \(\alpha\), \(\beta\) are the position, shape, and scale parameters.

The statistical results of the P-III distribution function parameters and design values of the maximum 15-day flood volume of the two rivers floods are shown in Table 2.

| Station     | Average value (10^8 m³) | Cv  | Cs/Cv | Design flood volume (10^8 m³) | Time        |
|-------------|-------------------------|-----|-------|-------------------------------|-------------|
|             |                         |     |       |                               |             |
| Panzhihua   | 70.86                   | 0.32| 4     | 143.39 131.14 114.44 101.23  | 1965~2018   |
| Cuntan      | 399.98                  | 0.23| 4     | 673.25 630.61 571.3 523.19    | 1965~2018   |

### 4.2. Archimedean Copula function and parameter estimation

At present, the most commonly used Archimedean Copula in the field of hydrology includes Clayton, Frank, Gumbel-Hougaard (GH) and Ali-Mikhail-Haq (AMH), etc. \[8\], and their structures are shown in Table 3.

| Copula function | Function expression | The functional relationship of \(\theta\) and \(\tau\) |
|-----------------|---------------------|-----------------------------------------------|
| Clayton         | \(C(\mu, \nu) = (\mu^\theta + \nu^\theta - 1)^{-\frac{1}{\theta}}\); \(\theta \in (0, \infty)\) | \(\theta = \frac{1}{1-\tau}\) |
| Frank           | \(C(\mu, \nu) = -\frac{1}{\theta} \ln \left[ 1 + \frac{\exp(-\theta \times \mu) - 1}{\exp(-\theta) - 1} \times \exp(-\theta \times \nu) - 1) \right] ; \theta \in \mathbb{R}\) | \(\tau = 1 + \frac{4}{\theta} \times \int_{0}^{\theta} \frac{t}{\exp(t) - 1} dt - 1\) |
| GH              | \(C(\mu, \nu) = \exp\left[ -\left( -\ln \mu \right)^\theta + (-\ln \nu)^\theta \right] \); \(\theta \in [1, \infty)\) | \(\theta = \frac{1}{1-\tau}\) |
| AMH             | \(C(\mu, \nu) = \frac{\mu \times \nu}{1 - \theta(1-\mu)(1-\nu)}\); \(\theta \in [-1, 1]\) | \(\tau = 1 - \frac{2}{3} \times \frac{2}{\theta} \times \left( \frac{1}{\theta} - \frac{1}{2} \right)^2 \times \ln(1-\theta)\) |

where: \(\mu\) and \(\nu\) are marginal distribution functions; \(\theta\) is undetermined parameter; \(\tau\) is the correlation coefficient. The calculation results of Kendall rank correlation coefficient \(\tau\) and parameter \(\theta\) are shown in Table 4.

| \(\tau\) | The estimated value of \(\theta\) |
|----------|---------------------------------|
| GH       | 1.77                            |
| Clayton  | 0.77                            |
| AMH      | —                               |
| Frank    | 4.66                            |

### 4.3. Inspection and evaluation of goodness of fit

The Clayton, Frank, and GH Copula functions are tested, and the theoretical and empirical joint probability values are linearly fitted to obtain the correlation between the theoretical and empirical joint probability values, the slope and intercept of the fitted straight line. Two hydrological variables can use the Gringorten experience frequency to calculate the joint cumulative experience frequency, the formula is shown in (4):
\[ F(x_1, x_2) = P(X_1 \leq x_{i1}, X_2 \leq x_{i2}) = \sum_{m=1}^{i} \sum_{n=1}^{i} \frac{N_{mn} - 0.44}{N + 0.12} \] (4)

(a) GH Copula function (b) Clayton Copula function (c) Frank Copula function

Figure 4. Theoretical value and empirical value of three types Copula functions

It can be seen Figures 4 that the theoretical and empirical joint probability points of the Clayton, Frank and GH Copula functions fit the data well, but the Frank Copula function has the best theoretical value and empirical value.

In order to select the Copula function with the best fitting effect in the two rivers flood, the above three Copula functions were evaluated using the AIC criterion and the OLS criterion [9]. The values of AIC and OLS calculated are shown in Table 5.

|       | GH            | Clayton       | Frank         |
|-------|---------------|---------------|---------------|
| AIC   | -433.73       | -343.89       | -458.32       |
| OLS   | 0.01256       | 0.03084       | 0.00982       |

It can be seen from Table 5 that Frank Copula function should be selected to analyze the flood encounter between the two rivers.

The joint probability distribution function of flood in the two rivers is shown in formula (5):

\[ C(\mu, \nu) = \frac{-1}{4.66} \ln \left[ 1 + \frac{\exp(-4.66 \times \mu) \times \exp(-4.66 \times \nu) - 1}{\exp(-4.66) - 1} \right] \] (5)

5. Analysis of flood encounters in the lower reaches of Jinsha River and Chuanjiang River

Under natural conditions, the two rivers have encountered diverse flood situations. Therefore, it is necessary to analyze the probability of occurrence of various flood combinations under different conditions. In this paper, two models of co-occurrence and conditions are selected to analyze and calculate the probability of encountering floods in the two rivers.

The co-occurrence probability of the two rivers is the probability of simultaneous occurrence of floods, the formula is shown in (6):

\[ F_1(x_1, x_2) = P(X_1 > x_1, X_2 > x_2) = 1 - \mu - \nu + C(\mu, \nu) \] (6)

Under the condition of X1>x1 (that is, flooding occurs in the lower reaches of Jinsha River), the probability of flooding in the Chuanjiang River is calculated by formula (7) [10]:

\[ F_2(X_2 > x_2 | X_1 > x_1) = \frac{P(X_2 > x_2, X_1 > x_1)}{P(X_1 > x_1)} = \frac{1 - \mu + u + C(\mu, \nu)}{1 - \mu} \] (7)

Figure 5 and Table 6 shows the probability of flood co-occurrence the two rivers.
Figure 5. Association, co-occurrence and conditional probability distribution of the flood encounter of lower reaches of Jinsha River and Chuanjiang River

Table 6. Encounter probability of flood in lower reaches of Jinsha River and Chuanjiang River

| Lower reaches of Jinsha River | Chuanjiang River | Probability (%) |
|------------------------------|-----------------|----------------|
| maximum 15d flood volume (10^8 m³) | maximum 15d flood volume (10^8 m³) | Association probability | co-occurrence probability | conditional probability |
| Recurrence period (a) | Recurrence period (a) | | | |
| 143.39 | 100 | 630.61 | 50 | 97.82 | 0.06 | 5.00 |
| 143.39 | 100 | 571.3 | 20 | 94.01 | 0.15 | 13.33 |
| 143.39 | 100 | 523.19 | 10 | 88.24 | 0.38 | 34.30 |
| 131.14 | 50 | 630.61 | 50 | 95.92 | 0.15 | 4.78 |
| 131.14 | 50 | 571.3 | 20 | 92.41 | 0.40 | 12.81 |
| 131.14 | 50 | 523.19 | 10 | 87.01 | 1.04 | 33.26 |
| 114.44 | 20 | 630.61 | 50 | 94.01 | 0.23 | 4.58 |
| 114.44 | 20 | 571.3 | 20 | 90.78 | 0.63 | 12.31 |
| 114.44 | 20 | 523.19 | 10 | 85.74 | 1.65 | 32.25 |
| 101.23 | 10 | 630.61 | 50 | 90.17 | 0.38 | 4.21 |
| 101.23 | 10 | 571.3 | 20 | 87.44 | 1.04 | 11.38 |
| 101.23 | 10 | 523.19 | 10 | 83.07 | 2.76 | 30.30 |

It can be seen from Figure 5 and Table 6, the probability of occurrence of flooding of the two rivers is low, and the probability of simultaneous flooding for more than once-in-50-years of the two rivers is 0.15%, and the probability of simultaneous flooding for more than once-in-20-years is 0.63%, and the probability of simultaneous flooding for more than once-in-10-years is 2.76%. When the flood in the lower reaches of the Jinsha River occurs in 50 years, the probability of occurrence in the Chuanjiang River in 10 and 50 years is 0.333 and 0.048, respectively.

6. Conclusion

(1) The frequency of occurrence of the largest flood peaks in the two rivers in different months is significantly different. Among them, the largest flood peaks in the lower reaches of Jinsha River usually occur in August and the largest flood peaks in Chuanjiang River usually occur in July. Compared with floods in the lower reaches of Jinsha River, floods in Chuanjiang River are characterized by large peak and volume, short flood duration, rapid flood rising and falling rates.

(2) The composition of the maximum 3d, 7d, 15d, 30d flood volume of Chuanjiang River accounts for 15.18%, 16.46%, 17.25% and 17.4%. With the increase of time period, the contribution rate of the lower reaches of Jinsha River to the composition of Chuanjiang River flood area increases.

(3) The Clayton Copula, Frank Copula and GH Copula functions have good effects on the study of the lower Jinsha River and the Chuanjiang River flood. The Frank Copula function has the smallest AIC
and OLS values of -458.32 and 0.00982 respectively, with the best fitting effect.

(4) The probability of simultaneous flooding of the two rivers is low, and the probability of simultaneous flooding for more than once-in-50-years of the two rivers is 0.15%, and the probability of simultaneous flooding for more than once-in-10-years is 2.76%. This provides more favorable conditions for reducing the flood control pressure of the Chuanjiang River by the cascade reservoir operation in the lower reaches of Jinsha River.

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