RESEARCH PAPER

Generation and application of sub-kilohertz oscillatory flows in microchannels

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Abstract
We present an accessible and versatile experimental technique that generates sub-kilohertz sinusoidal oscillatory flows within microchannels. This method involves the direct interfacing of microfluidic tubing with a loudspeaker diaphragm, which allows independent control of oscillation frequency and amplitude. Oscillatory flows were generated with frequencies ranging from 10 to 1000 Hz and amplitudes ranging from 10 to 600 μm. Fourier spectral analysis of particle trajectories, obtained by particle tracking velocimetry, was used to evaluate the oscillatory displacement in microchannels and shown to accurately represent simple harmonic motion specified by the loudspeaker. Oscillatory flow profiles in microchannels of square and rectangular cross-sections were characterized as a function of oscillation frequency, or Womersley number, and compared to theoretical benchmarks, such as Stokes flow and Stokes’ second problem. To highlight the versatility and effectiveness of the experimental method, prototypical applications were demonstrated utilizing pulsatile flow in microfluidic devices, such as inertial focusing and enhanced mixing at low Reynolds numbers.

1 Introduction
The addition of oscillatory flow to steady unidirectional flow in microfluidic devices have been shown to be useful in a range of applications such as mixing at low Reynolds numbers (Phelan et al. 2008; Ahmed et al. 2009; Frommelt et al. 2008), particle sorting and focusing (Thameem et al. 2016; Schmid et al. 2014; Marmottant and Hilgenfeldt 2003; Mutlu et al. 2018), enhancement of heat transfer (Qu et al. 2017), flow control (Leslie et al. 2009; Phillips et al. 2016), microrheology (Vishwanathan and Juarez 2019a, b), and chemical extraction (Lestari et al. 2016; Xie et al. 2015). Nevertheless, the widespread use and study of the oscillatory flow component in microchannels remains uncommon due to challenges of implementation.

At low frequencies (0.1 ≤ f ≤ 10 Hz), oscillatory flows are usually achieved with a programmed syringe pump, electromechanical relay valves (Abolhasani and Jensen 2016) or a pneumatic pressure controller (Zhou and Schroeder 2016). The fidelity of the desired waveform is limited by inertia of the oscillatory driver. For low frequencies, the response time of syringe pumps and actuators in electromechanical valves and pneumatic pressure controllers is on the order of Ω(10 ms), therefore preventing the realization of sinusoidal oscillations at higher frequencies. Some purely fluidic oscillators using a constant pressure head have also been developed in this range of frequencies (Dang and Kim 2017).

At high frequencies (10³ ≤ f ≤ 10⁶ Hz), piezoelectric transducers are used since they typically possess resonant frequencies in this range (Rallabandi et al. 2017; Phillips et al. 2016; Xie et al. 2015; Lieu et al. 2012; Morris and Forster 2003). The utility of piezoelectric transducers in 10–1000 Hz range, however, are limited by the small amplitudes generated. The amplitudes may be partially increased through the use of designed features, such as membrane cavities in the channel on to which the piezoelectric elements need to be bonded to be used properly (Vázquez-Vergara et al. 2017). More recent designs of microfluidic oscillators primarily aim to achieve oscillatory flows free of external actuators with a focus on miniaturization and integration with other lab-on-chip modules. This is typically done by exploiting non-linear fluid-elastic interactions with a membrane or diaphragm unit as a steady flow is driven through it. Therefore, a time dependent response is obtained even with a steady input at low Reynolds numbers (Xia et al. 2012; Leslie
et al. 2009; Kim et al. 2013; Mosadegh et al. 2010). Other possibilities that have been explored are the use of non-Newtonian fluids for switching (Groisman et al. 2003), generation of oil droplets as an oscillatory source (Basilio et al. 2019), and the Coanda effect (Yang et al. 2007). Although these micro-oscillators are highly miniaturized, modular and in some cases, capable of producing frequencies in the audible range (Xia et al. 2012), they mostly require the fabrication of complex MEMS devices. Therefore, potentially discouraging their use in research attempting to explore the utility of oscillatory flows. Further, for microfluidic oscillators that function based on fluid-elastic interactions, the amplitudes and frequencies are coupled; hence cannot be independently controlled.

Here, we describe the operation and performance of a simple plug and play apparatus capable of producing harmonic oscillatory flow in microchannels. This setup allows the user to independently control the oscillation frequency in the range of 10–1000 Hz and amplitude in the range of 10–600 μm. The aim of this method is simplicity and accessibility; hence allowing researchers to implement oscillatory flow at the microscale in a convenient and cost-effective manner without the need for prior design constraints and sophisticated microfabrication.

2 Experimental setup

The apparatus is set up as displayed in the schematic, shown in Fig. 1a. A PDMS microchannel is bonded to a glass slide and observed through an inverted microscope. A loudspeaker (DROK TDA7297B, 15 W, 90 dB) is mounted next to the microscope stage. The oscillation frequency and amplitude (volume) of the loudspeaker diaphragm are controlled by a computer via an auxiliary cable. One end of microfluidic polyethylene tubing (PE60 Intramedic 427416, 0.76 mm ID × 1.22 mm OD) is directly attached to the diaphragm of loudspeaker while the other end is inserted into the microchannel inlet. The tubing is maintained taut and its boundary conditions correspond to a fixed end at the microchannel inlet and a forced oscillatory displacement at the diaphragm. The microchannel and the tubing are filled with liquid during operation. The oscillatory displacement of the diaphragm is transduced into elastic deformations of the microfluidic tubing at the fixed end of the device outlet. The stress induced by tubing deformation generates a time-varying pressure at the inlet of the channel, resulting in oscillatory displacement of the liquid at the same frequency (f) as the loudspeaker diaphragm. The streamwise displacement of a tracer particle in the channel is described by $x = s \cos(\omega t + \phi)$ and illustrated in Fig. 1(b). Here, s is the oscillation amplitude, $\omega$ is the angular frequency, and $\phi$ is the initial phase.

![Fig. 1](image-url)  
**Fig. 1** a Schematic of the experimental setup. The loudspeaker diaphragm is directly interfaced with microfluidic tubing (maintained taut and filled with liquid) to generate sinusoidal liquid oscillations in a PDMS microchannel. b Schematic of tracer particle (and liquid) displacement in a microchannel described by the oscillation amplitude and angular frequency. c Experimental measurements of the streamwise displacement of 0.93 μm diameter tracer particles suspended in water over a number of oscillation cycles obtained with micro-particle tracking velocimetry.
To characterize the oscillatory flow in microchannels, tracer particles at the midplane of the channel were observed using brightfield microscopy with 10× and 20× objective lenses with a depth of field of 8.5 μm and 5.5 μm, respectively. To ensure that the tracers accurately represented the flow, polystyrene tracer particles with a mean diameter of 0.93 μm and density of 1.08 g/cm³ were suspended in deionized (DI) water, unless otherwise mentioned. The response time associated with these tracer particles (τ = ρd³/18μ ≈ 50 × 10⁻⁹ s) is much smaller than the oscillatory timescales considered in this study (τ ≪ 1/f). This ensures that the measured particle velocity faithfully reflects the fluid velocity, except within 2 μm of the channel walls where weak lateral migration is expected (Fischer et al. 2005). The particle positions were recorded using a high-speed scientific CMOS camera with frame rates exactly twenty times larger than the driving oscillation frequency (20f), therefore, acquisition rates ranged from 500 to 16000 fps for the data shown here. The displacement and velocity fields are then obtained from 2D particle tracking velocimetry algorithms.

The microchannel geometries used were the following: (1) a square channel of width and height of 110 μm and length of 5 cm, (2) a rectangular channel with a width of 5 mm, height of 200 μm, and length of 2 cm, and (3) a cross-slot channel with square cross section with a width and height of 110 μm.

The dimensionless groups considered here are the Womersley number, the Reynolds number, and the non-dimensional oscillation amplitude. The Womersley number, defined as α = W√ρoμ/μ, is the ratio of the transient inertial force to the viscous force. Equivalently, it represents the ratio of length scales between the characteristic channel width to the Stokes boundary layer thickness (Landau and Lifshits 1959), and of time scales between the viscous time scale of the liquid to the oscillation period. Here, W is the channel width, ρ and μ are the liquid density and dynamic viscosity, respectively. The Reynolds number, defined as Re = psωW/μ, is the ratio of inertial to viscous forces of the fluid within the microchannel (Thomas et al. 2011). Here, psω is taken to be the characteristic liquid velocity. The non-dimensional oscillation amplitude may be expressed in terms of the Womersley and Reynolds number as ε = s/W = Re/α². For the operational conditions considered in this study, a microchannel with characteristic width of 200 μm, and DI water as the working liquid, the values of α, Re, and ε range from 1.5 < α < 15, 0.4 < Re < 80, and 0.1 < ε < 5, respectively.

3 Results

3.1 Oscillatory displacement in microchannels

Examples of streamwise displacement of individually tracked particles from their mean position over a number of cycles during a 20 ms interval are shown in Fig. 1c. The ratio of sampling frequency (camera framerate) to liquid oscillation frequency is kept constant at 20. That is, for oscillations at 200, 400, and 16000 Hz, a framerate of 4000, 8000 and 16000 Hz is used, respectively. The corresponding amplitudes are 100, 27 and 14 μm.

The independent operational range between amplitude and frequency is shown in Fig. 2a. For a given frequency, the displacement of a tracer particle is influenced by the volume setting of the loudspeaker. Three volume settings are considered here as example cases: low (30%), intermediate (60%), and high (90%). The percentages correspond to the maximum speaker volume as determined by the computer. At 100 Hz, the amplitude ranges from 50 μm at the low setting to 800 μm at the high volume setting. The amplitude swept by a tracer particle over a single oscillation period, for a given volume setting, shows a non-monotonic variation with frequency. Owing to the performance characteristics of the speaker, the maximum oscillation amplitude occurs at 200 Hz, which corresponds to the resonant frequency (∼ 230 Hz) of the loudspeaker.

![Fig. 2 a The amplitude of oscillatory displacement in microchannels for a range of frequencies and three amplitudes, or speaker volume settings, of low (30%, blue), intermediate (60%, magenta) and high (90%, red). b Fourier spectrum analysis of tracer particle displacement in the streamwise direction at three different frequencies (50, 200, and 800 Hz) and amplitudes (low, intermediate, and high) (colour figure online)](image)
diaphragm. The horizontal black line indicates the maximum particle oscillation amplitude of 800 μm that can be measured due to the field of view limited by the camera when using a 10x microscope objective lens.

A Fourier spectrum analysis of multiple particle trajectories at different oscillation amplitudes and frequencies is shown in Fig. 2b. The spectra have been obtained for 50 oscillation cycles for frequencies of 50, 200 and 800 Hz and at volume settings of low, intermediate, and high. For all volume settings, the principal peaks in the spectral intensity are equally narrow and correspond to the input driving frequency of the loudspeaker. For high volume settings, contributions due to higher harmonics are of visible strength.

A quantitative measure of harmonic distortion present in the signal as compared to the fundamental driving frequency is obtained by calculating the total harmonic distortion (THD). The THD is defined as 

\[ \text{THD} = \sqrt{\sum_{i=2}^{N} V_i^2 / V_1}, \]

where \( V_i \) is the power of the spectral intensity at the \( i \)th harmonic (Shmilovitz 2005). A low THD value is associated with a more accurate representation of the original driving signal. For low volume settings, the THD at 50, 200, and 800 Hz are 1.4%, 3.7%, and 3.2%, respectively. For intermediate volume settings, the THD at 50, 200, and 800 Hz are 1.5%, 4.3%, and 5.5%, respectively. For high-volume settings, the THD at 50, 200, and 800 Hz are 1.5%, 7.6%, and 9.2%, respectively. Based on this analysis, operation at high frequencies (> 500 Hz) should be limited to low or intermediate speaker volumes in order to maintain harmonic oscillatory displacement. At low frequencies, the speaker volume should be tuned such that the maximum amplitude is chosen to avoid damage to the microchannel or unfastening of the outlet tube from the speaker cone. Therefore, sinusoidal oscillations with amplitudes ranging from 10 < s < 200 μm can be reliably achieved throughout the entire range of frequency.

The maximum pressure inside the square channel may be estimated from the modified Poiseuille formula:

\[ \Delta P = \frac{64 \mu L_s f}{D_h^2}, \]  

where \( D_h \) and \( L \) are the hydraulic diameter and length of the channel, respectively. For the square channel \( (D_h = 110 \mu m \) and \( L = 5 \text{ cm} \)) filled with DI water (\( \mu = 1.002 \text{ mPa s} \)) and settings for maximum oscillatory displacement \( (f = 200 \text{ Hz}, s = 600 \mu m) \), the pressure inside the channel is calculated to be approximately equal to 31 kPa. At these maximum pressure values, we do not expect any significant deformation of our microfluidic elements. This includes the polystyrene tracer particles which have a bulk modulus of 3 GPa and exhibit deformations of 0.01% at most, and hence can be considered rigid spheres throughout. Therefore, accounting for the variables in Eq. (1), the user can ensure that minimally safe operating pressures are maintained throughout the microchannel.

### 3.2 Oscillatory flow in microchannels

The small length scales of \( O(100 \mu m) \) associated with microchannels imply that most microscale flows are laminar and governed by the Stokes equation. An important feature of microscale oscillatory flows in 10–1000 Hz range is that transient effects associated with the unsteady Stokes equation become significant. An example of departure from Stokes flow is illustrated by the comparison of the steady Stokes flow velocity profile (O’Brien 1975) at the midplane (black solid curve) against those obtained experimentally for oscillatory flow at different frequencies (symbols), shown in Fig. 3a. To obtain the amplitude of velocity in the square channel midplane, 50–200 particles (0.93 μm diameter) are tracked for one hundred oscillation cycles and their respective velocities are computed. The amplitude of each velocity series \( (U_{\text{max}}(y)) \) is obtained and superposed in the streamwise direction. The resulting spread of speeds is filtered for outliers and averaged. The associated statistical error bars are smaller than the data markers shown.

The velocity profile for the 100 Hz and 400 Hz cases are similar to the Stokes laminar flow profile, shown in Fig. 3a. At 800 Hz; however, there is considerable deviation from the steady velocity profile. This is due to increasing the Womersley number \( (\alpha) \) with increasing frequency. For the cases of 100, 400, and 800 Hz the \( \alpha \) values are 2.75, 5.51, and 7.78, respectively. For these \( \alpha \) values, the analytical series solution for the amplitude of the midplane oscillatory velocity profile \( (U_{\text{max}}(y)) \) was evaluated correct to one hundred terms (O’Brien 1975). The series solutions are found to be in good agreement with experimental data, even at 800 Hz, shown by the continuous lines in Fig. 3a.

In contrast, deviations from the unsteady Stokes equation are demonstrated by comparing the amplitude of measured oscillatory flow speed (symbols) in a rectangular channel \( (W \gg H) \) with those obtained theoretically from the solution to oscillatory flow over an infinite flat plate, or Stokes’ second problem (Landau and Lifshits 1959; Wang 1989). The maximum temporal flow velocity as a function of distance from a flat plate is given by the expression:

\[ U_{\text{max}}(y) / \sqrt{x_0} = \sqrt{2e^{-y/2\delta}} \sqrt{\cosh(y/\delta) - \cos(y/\delta)}, \]  

where \( \delta \) is the Stokes boundary layer length and equal to \( \sqrt{\mu / \rho \omega} \).

At 800 Hz, normalized experimental data (symbols) is in good agreement with Eq. (2), shown in Fig. 3b. At 50 Hz, and to a lesser extent at 200 Hz, deviations from the theory occur where the velocity amplitudes are larger than those at the far field and is detailed in the inset of Fig. 3b. This is due...
to the relatively short channel height (200 μm) and the effect of boundary layers at the top and bottom walls of the channel affecting flow at the midplane. The corresponding values of the Womersley number (\(\alpha\)) for the cases of 50, 200, and 800 Hz are 3.57, 7.14, and 14.3, respectively. Therefore, at the midplane, effects of channel side-walls may be neglected for distances larger than \(\frac{4}{\alpha}\) into the channel. Further, three dimensional flow effects can be ignored for \(\alpha \geq 7.5\) when the shorter dimension is used.

3.3 Applications to microfluidics

Purely oscillatory flow is seldom used in applications; however, it can be a powerful supplement to steady flows. The more general class of flows involving a steady flow superposed with an oscillatory flow is referred to as pulsatile flow (Riley 2001). This category is useful in mixing (Ahmed et al. 2009), hydrodynamic manipulation of particles and cells (Lutz et al. 2006; Lee et al. 2019; Thameem et al. 2016), and more recently, in micro rheology (Vishwanathan and Juarez 2019a, b). Here, we demonstrate two specific applications of the oscillatory driver from each category, namely, inertial focusing from the former and enhanced mixing from the latter, using simple prototypical microfluidic configurations and a superposed constant steady flow.

3.3.1 Inertial focusing

Inertial focusing in microchannels is a passive technique where suspended particles undergoing unidirectional flow migrate across streamlines to equilibrium positions because of particle inertia (Di Carlo et al. 2007; Di Carlo 2009; Martel and Toner 2014; Stoecklein and Di Carlo 2019). In straight channels, of rectangular or circular cross section, the competing forces that lead to particle migration are the wall interaction force, which directs the particle away from the channel wall, and the shear gradient force, which directs the particle toward the channel wall. The summation of these forces is termed the inertial lift and the equilibrium position of the particle is determined once the opposing forces are balanced, leading to particles being focused at the channel outlet. Factors influencing the inertial lift force are the channel geometry, flow rate, and particle size. Since it is a high-throughput method for non-contact manipulation at the microscale, inertial focusing has been utilized in numerous applications ranging from flow cytometry (Hur et al. 2010; Bhagat et al. 2010; Oakey et al. 2010), size sorting (Kuntaegowdanahalli et al. 2009; Wu et al. 2012; Nivedita and Papautsky 2013), mixing (Amini et al. 2012), and filtration (Seo et al. 2007). An important parameter when designing channels for applications is the length required to reach the equilibrium focus position, estimated (Di Carlo 2009) to be equal to

\[
L_f = \frac{\pi \mu D_k^3}{\rho U_{max} \alpha^2 C_f},
\]

where \(U_{max}\) is the maximum flow velocity and \(C_f\) is the lift coefficient, which typically varies in the range of 0.02–0.05. From

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**Fig. 3** a Oscillatory velocity flow profile in a square channel (symbols) showing deviation from Stokes flow (black solid line) with increasing frequency. The solid lines are theoretical curves obtained for unsteady flow profiles (O’Brien 1975). b Oscillatory velocity profile with position (symbols) near a solid channel wall in a semi-infinite rectangular channel \((W \gg H)\) showing agreement with the theoretical solution (black solid curve) to Stokes’ second problem (Landau and Lifshits 1959) with increasing frequency. (Inset) Close-up of the region from \(1.5 \leq y/\delta \leq 4.5\) to demonstrate agreement with Eq. (2) at different frequencies.
this relation, it is apparent that sufficiently high velocities and large particles are required to minimize the focusing length. Recent work, however, demonstrated the use of oscillatory flows for inertial focusing, where oscillatory flow at relatively low frequencies ($< 20$ Hz) results in a channel of practically infinite length (Mutlu et al. 2018). Thus, it becomes possible to focus particles with far smaller particle Reynolds numbers corresponding to $Re_p = (a/W)^2Re < 0.01$.

Inertial focusing of a $10$ μm polystyrene particle in an oscillatory flow of $400$ Hz with an amplitude of $22$ μm is demonstrated in the square channel, for which, $Re_p = 0.050$ and $a/W = 0.091$. Micrographs of a single polystyrene particle at regular time intervals as it migrates to the equilibrium position are shown in Fig. 4a. Using stroboscopic imaging, the lateral migration toward the center of the channel is observed. Note that the apparent blurring of the particle for $t > 0$ is due to the streaking artifact caused by large instantaneous velocities relative to the camera exposure time. The corresponding vertical position of the particle centroid, as determined by particle tracking, is shown as a function of the approximate path length traversed by the particle in Fig. 4b. The marked points (symbols) correspond to the instances shown in the micrographs. The path length is estimated as $4sft$, where $4$ s is the distance covered by the particle at the center of the channel in a single oscillation cycle and $t$ is the time elapsed after the start of oscillations. The focusing behavior of another particle at $25$ Hz and an amplitude of $36$ μm is also shown. The corresponding path length for focusing (or focus length) is found to be about $0.15$ m, comparable to the $400$ Hz case and in good agreement with previous results for similar conditions ($Re_p = 0.050$ and $a/W = 0.125$) (Mutlu et al. 2018). The time required for focusing, however, is about $3$ s for the $400$ Hz case compared to $35$ s for the $25$ Hz case. Therefore, access to a wider range of oscillation amplitudes and frequencies could vastly impact fundamental studies and applications to the area of inertial focusing and particle manipulation at the microscale.

The advantages of oscillatory flow for inertial focusing include decreased channel lengths, lower pressure drops, and lower shear rates. Because there is no net displacement, the particle remains in the field of view as it migrates to the equilibrium position. In contrast with unidirectional flow, the approximate channel length required for the particle to reach its equilibrium position as determined by Eq. (3), where $U_m$ is given by the characteristic fluid velocity $s w_a$, is $L_f \approx 1.37$ m, which is impractical. The lower pressure drop allows for the convenient fabrication and use of PDMS microchannels. The combination of lower pressure drop and lower shear rates is of particular interest in biomedical application where cells are susceptible to damage induced by fluid stresses.

### 3.3.2 Microscale mixing

Low Reynolds number ($Re \leq 1$) flow in microchannels present a significant challenge to applications involving mixing (Ottino and Wiggins 2004). For flows with $Re \gg 1$, mixing is rapid and efficient because of chaotic advection. At low $Re$, chaotic advection is negligible and the dominant mechanism of mixing is diffusion. The effectiveness of a micromixer is quantified by the length required to achieve mixing ($L_m$). For purely diffusive mixing, this length is estimated by

$$L_m \geq D \cdot Pe .$$

Above, $Pe$ is the Peclet number and defined as the ratio of $U_a W/D$, where $U_a$ is the average flow velocity and $D$ is the diffusion coefficient. For $Re \leq 1$ and diffusion coefficients of $D \approx 100 \mu m^2 s^{-1}$, the Peclet number would be $Pe \geq 10^4$, resulting in a channel length for sufficient mixing to be $L_m \geq 1$ m, which is undesirable.
To overcome this challenge, a variety of microscale mixers have been developed to enhance mixing at low Re, and are categorized as either passive or active mixers (Hessel et al. 2005; Nguyen and Wu 2005; Lee et al. 2011, 2016; Ward and Fan 2015; Cai et al. 2017). Passive mixers make use of the channel geometry, usually incorporating repeating complex or 3D channel features to enhance mixing of two streams flowing together at a constant rate. On the other hand, active mixers rely on externally applied forces and are further categorized based on the nature of the external actuation (Ober et al. 2015). One such category of mixers are acoustic micromixers (Liu et al. 2002; Ahmed et al. 2009; Bachman et al. 2019) that rely on external acoustic actuation to generate steady rectified flows that mix different liquids.

Here, mixing of two aqueous glycerol solutions (30% w/w, $\mu = 2$ mPa s), one with colored dye and one without, is demonstrated in the cross-slot channel using pulsatile flows of varying frequency. As seen from Fig. 5a, the dyed solution enters the cross-slot with a flow rate of 0.3 $\mu$L/min (red arrow) while the undyed solution enters the cross-slot from either side with identical flow rates of 0.15 $\mu$L/min (green arrows). This results in a total flow rate of 0.6 $\mu$L/min at the outlet.

When no oscillatory flow is imposed, minimal diffusive mixing at the interface is observed within the field of view. When an oscillatory flow of 400 Hz is imposed on otherwise identical conditions, steady vortices are generated near the corners of the cross-slot, which facilitate mass transfer across the interface through advection. The result is a well mixed downstream profile as seen in Fig. 5b. In order to obtain repeatable results, an exposure time of 20 ms was used so that variations over a single oscillation period are averaged. The vortex structures responsible for the mixing are visualized using tracer particles and stroboscopic imaging in Fig. 5c.

The mixing performance is quantified by first obtaining the image gray-scale intensity profile across the channel at a distance $2.5W$ downstream from the center of the cross slot, indicated by the yellow dashed lines in Fig. 5a and b. The standard deviation of the mixture fraction profile, derived from the intensity values, is used as the mixing index and is defined as $\text{MI} = \sqrt{\Sigma (I_i - I_m)^2/N}$, where, $I_i$ is the pixel intensity value and $I_m$ is the pixel intensity of a completely streaming vortices that cause mixing at the center region of the cross-slot device.

![Fig. 5](image-url)

**Fig. 5** a Mixing of co-flowing streams of 30% (w/w) aqueous glycerol solutions without oscillations applied. b Mixing of co-flowing streams of 30% (w/w) aqueous glycerol solutions with oscillations applied at a frequency of 400 Hz. c Stroboscopic visualization of mixing at the center region of the cross-slot device. d Mixing index as a function of frequency for constant amplitude and constant volume settings measured at a distance 2.5W downstream from the cross-slot region.
mixed solution, and $N$ is the number of sampling points (Liu et al. 2000). The values of the index range from MI = 0.5 for completely unmixed to MI = 0 for completely mixed solutions. A value of MI ≤ 0.1 indicates sufficient mixing.

The variation of the mixing index as a function of frequency for both constant oscillation amplitude and constant volume setting are shown in Fig. 5d. For constant amplitude settings, the mixing index decreases monotonically with increasing frequency, implying sufficient mixing for $f \geq 400$ Hz. The improved mixing with increasing frequency is due to the increase in magnitude of the steady rectified flow velocities, which scale as $\vec{v}_0/W$. For a constant volume (intermediate) setting, however, the mixing index is non-monotonic, with sufficient mixing occurring in the range of 100–200 Hz. Based on the amplitude characteristics, shown in Fig. 2a, for constant volume settings, the largest amplitudes occur near the resonance frequency of the loudspeaker diaphragm.

For the specific case presented here, the length demonstrated to achieve good mixing with high-frequency pulsatile flows ($L_m \leq 250 \mu$m) is less than the length required for diffusive mixing by two orders of magnitude according to Eq. (4), which is calculated to be $L_m = 2.5$ cm. Although the nature of forces involved are purely hydrodynamic, oscillatory flows with independently controllable amplitude and frequency allow for the decoupling of flow rate from the rate of mixing which is not possible for passive micromixers. Additionally, for the range of frequencies and amplitudes achieved here, strong rectified flows are produced near solid boundaries in the microchannel. This is in contrast with the boundaries of bubbles used with ultrasonic frequencies elsewhere (Liu et al. 2002; Ahmed et al. 2009) which are unstable at long operation times. Lastly, the implementation of this method in combination with any other passive or active technique can further enhance mixing at the microscale by increasing the number of passes without affecting flow rate.

4 Discussion

We have discussed an accessible, effective, and versatile plug and play technique to generate oscillatory flows over a range of amplitudes and frequencies in microchannels. By directly interfacing microfluidic tubing with a loudspeaker diaphragm, sub-kilohertz oscillatory flows in the range of 10–1000 Hz with amplitudes in the range of 10–600 μm are produced. The corresponding wavelengths lie between 1 and 100 m and are far larger than the dimensions of a typical microchannel. Thus, nearly unattenuated flows of a uniform phase can be achieved throughout the device. This is in contrast with flows in the frequency range of $10^3$–$10^7$ Hz, where attenuation is significant and effects are usually local to the transducer. The resulting velocity profiles can also be tuned from Stokes-like flow ($\alpha \leq 4$) at low frequencies to plug-like unsteady Stokes flow ($\alpha \geq 7.5$) at high frequencies allowing for manipulation of the flow profile for a given micro-scale geometry. In addition to coherent oscillatory flows, strong and well defined rectified flows near curved boundaries and interfaces are also made possible in this frequency and amplitude range.

The guiding principles for applications are two-fold. First, oscillatory flows permit an increase of the net distance travelled by the fluid without an accompanied increase in flow rate, shear rate, pressure drop, or particle displacement seen in long channel steady flows and high throughput applications. More generally, such conditions are particularly useful when the desired sample or analyte response to the flow environment is directionally invariant. In theory, the resulting effect can be increased indefinitely by simply increasing the frequency of oscillation. Although in practice, attenuation and secondary flows which grow stronger with increasing frequency prevent this from being realized. Owing to the large wavelengths in this range of flows, attenuation is limited while secondary flows usually become significant only for frequencies larger than 100 Hz implying that an optimum frequency is likely encountered in the range of frequencies realized here. The resulting lower steady pressure drop and steady shear rates are particularly useful for cells and other suspended biological matter that are sensitive to or potentially damaged by prolonged exposure to excessive shear rates or pressures. The absence of particle net displacement is useful in decreasing the footprint of microfluidic devices and in situations where continuous observation is needed to track the evolution in processes such as cell growth or chemical synthesis in dynamic environments.

Second, the steady rectified flow speeds are of considerable magnitude despite the maximum frequency considered being much smaller than the typical resonant frequencies of piezoelectric transducers (1–100 kHz). This is because of the large amplitudes accessed by the loudspeaker diaphragm and the dependency of flow speed on amplitude, which scales as $\vec{v}_0/W$. Further, the Stokes boundary layer thickness is accurately controlled and varies in size from $10 \leq \delta \leq 100 \mu$m for the highest and lowest frequencies accessed here, respectively. Therefore, the flow pathlines are less sensitive to manufacturing defects and feature surface quality, making this approach more amenable to precision applications such as sorting, trapping and manipulation of particles and cells, viscometry, and other controlled mass transfer applications.

References

Abolhasani M, Jensen KF (2016) Oscillatory multiphase flow strategy for chemistry and biology. Lab Chip 16:2775
Thameem R, Rallabandi B, Hilgenfeldt S (2016) Particle migration and sorting in microbubble streaming flows. Biomicrofluidics 10:014124

Thomas C, Bassom AP, Blennerhassett PJ, Davies C (2011) The linear stability of oscillatory Poiseuille flow in channels and pipes. Proc R Soc A Math Phys Eng Sci 467:2643

Vázquez-Vergara P, Rojas AMT, Guevara-Pantoja PE, Poiré EC, Caballero-Robledo GA (2017) Microfluidic flow spectrometer. J Micromech Microeng 27:077001

Vishwanathan G, Juarez G (2019a) Steady streaming viscometry of Newtonian liquids. Phys Fluids 31:041701

Vishwanathan G, Juarez G (2019b) Steady streaming flows in viscoelastic liquids. J Non Newton Fluid Mech 271:104143

Wang CY (1989) Exact solutions of the unsteady Navier-Stokes equations. Appl Mech Rev 42:269

Ward K, Fan ZH (2015) Mixing in microfluidic devices and enhancement methods. J Micromech Microeng 25:094001

Wu L, Guan G, Hou HW, Bhagat AAS, Han J (2012) Separation of leukocytes from blood using spiral channel with trapezoid cross-section. Anal Chem 84:9324

Xia HM, Wang ZP, Fan W, Wijaya A, Wang W, Wang ZF (2012) Converting steady laminar flow to oscillatory flow through a hydroelasticity approach at microscales. Lab Chip 12:60

Xie Y, Chindam C, Nama N, Yang S, Lu M, Zhao Y, Mai JD, Costanzo F, Huang TJ (2015) Exploring bubble oscillation and mass transfer enhancement in acoustic-assisted liquid-liquid extraction with a microfluidic device. Sci Rep 5:12572

Yang J, Chen C, Hu I, Lyu P (2007) Design of a self-flapping microfluidic oscillator and diagnosis with fluorescence methods. J Micromech Microeng Syst 16:826

Zhou Y, Schroeder CM (2016) Single polymer dynamics under large amplitude oscillatory extension. Phys Rev Fluids 1:053301

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