Intermediate Scale Supersymmetric Inflation, Matter and Dark Energy

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Abstract

We consider supersymmetric inflation models in which inflation occurs at an intermediate scale and which provide a solution to the $\mu$ problem and the strong CP problem. Such models are particularly attractive since inflation, baryogenesis and the relic abundance of cold dark matter are all related by a set of parameters which also affect particle physics collider phenomena, neutrino masses and the strong CP problem. For such models the natural situation is a universe containing matter composed of baryons, massive neutrinos, lightest superpartner cold dark matter, and axions. The present day relic abundances of these different forms of matter are (in principle) calculable from the supersymmetric inflation model together with a measurement of the CMB temperature and the Hubble constant. From these relic abundances one can deduce the amount of the present day dark energy density.

March 25, 2022
1 Introduction

Recent data on the cosmic microwave background (CMB) radiation [1] provides strong support for inflation, by measuring the first, second and third peaks of the angular power spectrum. Inflation therefore seems to be increasingly well established. Low energy supersymmetry (SUSY) is perhaps less well established (though there is considerable indirect evidence for it [2]), but is certainly desirable from many points of view, and has the advantage when combined with inflation of helping to ensure that the inflaton potential is sufficiently and naturally flat [3]. The CMB data also supports a $\Lambda CDM$ universe in which the energy density is dominated by dark energy (DE) (corresponding possibly to a cosmological constant $\Lambda$), and cold dark matter (CDM) [4].

In this paper we consider supersymmetric inflation models in which inflation occurs at an intermediate scale [5, 6, 7, 8], and which provide an intermediate scale solution to the $\mu$ problem and a solution to the strong CP problem via the Peccei-Quinn (PQ) mechanism [9]. When right-handed neutrinos are included, such models may give baryogenesis via leptogenesis. They will have calculable CDM and DE relic densities. And they may have supergravity (SUGRA) mediated SUSY breaking with no moduli or gravitino problems. A main point of the paper is to show that in such models in which all these features are simultaneously present there will be fewer parameters than in models in which these problems are separately addressed. We will discuss an explicit example of such a model in order to demonstrate the connections between the physics of each of the separate sectors, and the resulting enhanced predictivity leading for example to connections between collider physics experiments and cosmological observations.

Within such a framework one may expect on general grounds that our Universe contains sizeable relic abundances of baryons (from leptogenesis), axions (a) (from the solution to the strong CP problem), as well as weakly interacting massive particles (WIMPS). In R-parity conserving SUSY the WIMP is identified as the lightest supersymmetric particle (LSP), and this is often assumed to be the lightest neutralino of the minimal supersymmetric standard model (MSSM). We shall argue from this perspective that the LSP could equally well be a lighter stable singlet (singlino) [10] identified with an axino [11] or inflatino [12]. Neutrino masses provide smaller amounts of hot dark matter. Within this approach all these forms of matter in the universe will have calculable relic abundances, given a measurement of the CMB temperature and the Hubble constant, which are related to the parameters of the underlying supersymmetric theory which may be determined from particle physics experiments. This amounts to a generalisation of the observation made a few years ago that the relic abundance of neutralinos is related to the parameters of the super-
symmetric theory. Further, a given parameter typically is relevant to more than one relic abundance, so the total number of parameters is fewer than when the various forms are considered separately. From the calculated present day relic abundances of matter one can deduce the amount of the present day DE density, even without specifying the physics of DE.

2 The Omega Problem in Supersymmetric Inflation

In general the ratio of the total density of the universe $\rho_{\text{tot}}$ to critical density $\rho_{\text{crit}}$ is given by $\Omega_{\text{tot}}$ where

$$\Omega_{\text{tot}} = \Omega_\gamma + \Omega_{\text{matter}} + \Omega_{\text{DE}}$$

and $\Omega_\gamma$, $\Omega_{\text{matter}}$, $\Omega_{\text{DE}}$ are the ratios of radiation density $\rho_\gamma$, matter density $\rho_{\text{matter}}$ and DE density $\rho_{\text{DE}}$ to critical density $\rho_{\text{crit}}$, and the radiation density is unimportant $\rho_\gamma \ll \rho_{\text{matter}}$. Note that the critical density is a function of time and in the present epoch $\rho_{\text{crit}} = 3M_P^2H_0^2 = (3h^{1/2} \times 10^{-3} \text{ eV})^4 \sim (M_W^2/M_P)^4$ where $M_W$ is the weak scale, $M_P$ is the Planck scale, and $H_0 = 100h\text{km.s}^{-1}\text{Mpc}^{-1}$ is the present day Hubble constant, with $h = 0.7 \pm 10\%$ \[13\]. From observation \[1\] $\Omega_{\text{DE}} \sim 2/3$ while $\Omega_{\text{matter}} \sim 1/3$ and $\Omega_{\text{tot}}$ is very close to unity. Inflation predicts $\Omega_{\text{tot}} = 1$, for all times after inflation. The matter contributions consist of (at least)

$$\Omega_{\text{matter}} = \Omega_b + \Omega\nu + \Omega_{\text{LSP}} + \Omega_a$$

The most recent data \[1\] is consistent with nucleosynthesis estimates of $\Omega_b \sim 0.04$, where the baryons (b) in the universe are mainly to be found in dark objects. The determined value of $\Omega_{\text{CDM}} \sim 0.3$ contains unknown relative contributions from $\Omega_{\text{LSP}}$ and $\Omega_a$. Super-Kamiokande sets a lower limit on neutrino masses $\sum_i m_{\nu_i} \geq 0.05 \text{ eV}$ which corresponds to $\Omega_{\nu} \geq 0.003$. In hierarchical neutrino mass models the lower bound is saturated and interestingly the neutrino density is then comparable to the visible baryon density $\Omega_{\text{stars}} \sim 0.005$.

We now discuss how to calculate the relic densities from our present rather primitive state of knowledge about the comprehensive theory needed to really do that. \[1\] In the following it is important to keep in mind that our present inability to calculate the ratios of different forms of matter should be distinguished from the ability to calculate them in principle. Later we discuss a simple model which illustrates these ideas.

\[1\]The calculation will involve non-perturbative cosmological effects during the reheating process, such as preheating \[14, 15, 16, 17\].
Of the three likely dominant densities $\Omega_b$, $\Omega_{LSP}$ and $\Omega_a$, the latter arises from non-relativistic axions being produced at the QCD scale by the usual misalignment mechanism, and is the most difficult of the three to estimate since it depends on a randomly selected angle. Therefore we shall focus mainly on the question of how to calculate $\Omega_b$ and $\Omega_{LSP}$.

The present day value of $\Omega_b$ corresponds to the ratio of baryon number density to entropy density of the universe $Y_b = n_b/s \approx 0.7 \times 10^{-10}$, assuming $n_\bar{b} = 0$. For the calculation of $Y_b$, we shall restrict ourselves here to the so-called leptogenesis mechanism. The basic idea of leptogenesis [18, 19] is that right-handed neutrinos (or possibly sneutrinos) are copiously produced in the early universe, then decay to produce lepton number (and hence $B - L$) asymmetry. The lepton number asymmetry is subsequently converted into baryon number asymmetry by sphaleron interactions. Concerning the three Sakharov conditions: the CP-violation originates from complex Yukawa coupling constants [3]; lepton number violation originates from the Majorana mass of the right-handed neutrinos, and baryon number violation from sphalerons. Concerning the out-of-equilibrium condition, in the conventional approach to leptogenesis it is assumed that the right-handed neutrinos are produced by their couplings to other particles in the thermal bath, but that these couplings are sufficiently weak that the decays occur out-of-equilibrium, leading to a narrow range of couplings [20, 21]. From the perspective of inflation the conventional leptogenesis picture will change if the reheat temperature is below the mass of the lightest right-handed neutrino. In this case right-handed neutrinos may be produced during reheating via (direct or indirect) couplings to the inflaton field, and may then be produced with masses greatly exceeding the reheat temperature, providing only that they are lighter than the inflaton field. In this case the out-of-equilibrium condition is automatically satisfied during reheating. This second mechanism is preferred from the point of view of the gravitino constraint, since in this case if the reheat temperature is below the limit $T_R < 10^9$ GeV then thermally produced gravitinos are not a problem.

In the conventional case the LSP is regarded as the lightest neutralino $\tilde{\chi}_1$ (a linear combination of neutral bino $\tilde{B}$, wino $\tilde{W}_3$, and higgsinos $\tilde{H}_1, \tilde{H}_2$) and it is produced in thermal equilibrium at temperatures above its mass $\sim M_W$. According to the following very crude argument when their annihilation rate $\Gamma \sim \sigma n_f \sim n_f/M_W^2$ becomes smaller than the expansion rate of the universe $H \sim T_f^2/M_P$ (at a temperature $T_f \sim M_W$), they freeze out of the thermal bath, and the present day matter density is then $\rho_\chi \sim M_W n_f(T/M_W)^3 \sim M_W^2 T^3/M_P$ including the dilution

\footnote{Any relativistic axions produced during (p)reheating will be red-shifted away.}

\footnote{Complex SUSY mass parameters are not relevant for leptogenesis if its scale is above that of SUSY breaking.}

\footnote{If the gravitino is the LSP then this limit on the reheat temperature may be relaxed [22].}
factor \( (R_f/R)^3 \sim (T/M_W)^3 \). The current temperature is obtained by roughly equating \( \rho_{\chi} \sim \rho_{\gamma} \approx T^4 \) which gives \( T \sim M_W^2/M_P \). Inserting this temperature we find \( \rho_{\chi} \sim (M_W^2/M_P)^4 \sim \rho_{\text{crit}} \). Many more careful analyses have been performed in order to obtain precise estimates for \( \rho_{\chi} \) by considering the detailed annihilation channels within different regions of SUSY parameter space. What concerns us more here is how including the effects of inflation will change this simple framework. As in the discussion of leptogenesis (above) from the perspective of inflation and baryogenesis, and in general a broader picture, the conventional picture will change if the reheat temperature is below the mass of the lightest neutralino, and they are produced by non-thermal processes during the reheating process after inflation. Another way in which the physics might differ is if the LSP is not the lightest neutralino, but instead some lighter singlino associated with an axino or inflatino. For example, if the lightest neutralino freezes out then decays into an axino (\( \tilde{a} \)) then the present day axino density is suppressed by the ratio of axino mass to lightest neutralino mass \( \rho_{\tilde{a}} = (m_{\tilde{a}}/m_{\chi})\rho_{\chi} \) [11]. In order to learn the actual relic density of LSPs (or axions or any candidate) it is necessary to actually calculate it; detecting LSPs or axions is possible even if the relic density is well below \( \Omega_{\text{matter}} \) of order 1/3 [23].

3 An Example

3.1 Why Intermediate Scale Inflation?

We now wish to consider a specific example of a model which addresses the particle physics issues mentioned earlier, in order to illustrate many of the general features that we have discussed above. The brand of inflation most closely related to particle physics seems to be hybrid inflation which may occur at a scale well below the Planck scale, and hence be in the realm of particle physics [24]. The next question is what is the relevant scale at which hybrid inflation takes place? One obvious possibility is to associate the scale of inflation with some grand unified theory (GUT) symmetry breaking scale, as originally conceived by Guth [25]. However it is somewhat ironic that hybrid inflation at the GUT scale faces the magnetic monopole problem, which was precisely one of the original motivations for considering inflation in the first place! Although in certain cases this problem may be resolved, there are typically further symmetry breaking scales below the GUT scale at which discrete symmetries are broken, leading to problems with cosmological domain walls.

As an example of the difficulties faced by GUT scale inflation models, consider the breaking of the Pati-Salam symmetry group \( SU(4) \times SU(2) \times SU(2) \) down to the standard model gauge group. The minimal symmetry breaking potential in this
model [26] contains a singlet which could be a candidate for the inflaton of hybrid inflation. However it was immediately realised that such a scenario would face a magnetic monopole problem since the gauge group is unbroken during inflation, and only broken by the choice of vacuum after inflation [26]. A possible solution to this problem is to consider the effect of higher dimension operators in the superpotential which for a range of parameters have the effect of breaking the gauge symmetry during inflation [27]. Such a scheme can also address the \( \mu \) problem and the strong CP problem, at the expense of introducing additional singlets which develop vacuum expectation values (VEVs) at the PQ symmetry breaking scale [9]. However in this scheme it turns out that the vacuum in which PQ symmetry is unbroken is preferred below the reheat temperature of the model so that PQ symmetry is not broken after inflation [27]. Moreover PQ symmetry breaking is associated with the breaking of discrete symmetries in the model, so that it would lead to the domain wall problem in any case [27]. The only solution to these problems seems to be to assume that PQ symmetry is also broken during inflation, but since the inflaton has zero PQ charge, and since the inflation scale of order \( 10^{14} \text{GeV} \) [27] and hence much larger than the PQ symmetry breaking scale, this assumption seems rather questionable.

Intermediate scale hybrid inflation immediately solves both the magnetic monopole problem and the domain wall problem. The idea is simply that there is a period of hybrid inflation occurring below the GUT scale at the PQ symmetry breaking scale itself, in which the inflaton carries PQ charge and so the choice of domain is fixed during inflation. The universe therefore inflates inside a particular domain, and the magnetic monopole relics produced by the GUT scale symmetry breaking are inflated away. This provides a powerful motivation for intermediate scale inflation, which is the subject of this paper. We now turn to an explicit example of an intermediate scale inflation model.

3.2 Intermediate Scale Supersymmetric Inflation Model

The model we consider [6] is a variant of the NMSSM. This model has a SUGRA foundation [8], and leptogenesis and reheating has been studied [7], and preheating [17, 28] has been demonstrated not to lead to over-production of either axions or gravitinos. The model provides a solution to the strong CP problem and the \( \mu \) problem, with inflation directly solving the monopole and domain wall problems at the inflation scale. It is therefore a well motivated, successful model that has been well studied and does not appear to suffer from any embarrassing problems, and is therefore a suitable laboratory for our discussion here. This variant of the NMSSM has the following superpotential terms involving the standard Higgs doublets \( H_u, H_d \)
and two gauge singlet fields \( \phi \) (inflaton) and \( N \),

\[
W = \lambda NH_u H_d - k\phi N^2
\]  

(3)

where \( \lambda, k \) are dimensionless coupling constants. Notice that the standard NMSSM is recovered if we replace the inflaton \( \phi \) by \( N \). However this leads to the familiar domain wall problems arising from the discrete \( Z_3 \) symmetry. In this new variant, the \( Z_3 \) becomes a global Peccei-Quinn \( U(1)_{PQ} \) symmetry that is commonly invoked to solve the strong CP problem \[9\]. This symmetry is broken in the true vacuum by intermediate scale \( \phi \) and \( N \) VEVs, where the axion is the pseudo-Goldstone boson from the spontaneous symmetry breaking and constrains the size of the VEVs. With such large VEVs this model should be regarded as giving an intermediate scale solution to the \( \mu \) problem, and as such will have the collider signatures discussed in \[10\].

We can make the \( \phi \)-field real by a choice of the (approximately) massless axion field. We will now regard \( \phi \) and \( N \) to be the real components of the complex singlets in what follows. When we include soft SUSY breaking mass terms, trilinear terms \( A_k k \phi N^2 + h.c. \) (for real \( A_k \)) and neglect the \( \lambda NH_u H_d \) superpotential term, we have the following potential:

\[
V = V_0 + \frac{1}{2}m^2(\phi)N^2 + \frac{1}{2}m^2_{\phi}\phi^2
\]  

(4)

where \( m^2(\phi) = m_{\text{N}}^2 + 4k^2\phi^2 - 2kA_k\phi \). We can identify the various elements of the potential: \( V_0 \) arises from some other sector of the theory, SUGRA for example, and dominates the potential \[3\]; the soft SUSY breaking parameters \( A_k \) and \( m_N \) are generated through some gravity-mediated mechanism with a generic value of \( O(\text{TeV}) \).

The basic idea of hybrid inflation is very simple \[3\]. For the field dependent mass of the \( N \) scalar positive, \( m^2(\phi) > 0 \), then \( N = 0 \) since its potential has positive curvature. With \( N = 0 \) the potential becomes very simple indeed,

\[
V = V_0 + \frac{1}{2}m^2_{\phi}\phi^2
\]  

(5)

We shall assume that \( m_{\phi} \) comes from no-scale SUGRA, and vanishes at the Planck scale \[3\], so that it is generated through radiative corrections such that \( m_{\phi}^2 \sim -k^2A_k^2 \). Since \( m_{\phi}^2 \) is negative, during inflation \( \phi \) is slowly rolling away from the origin, and therefore we have inverted hybrid inflation. When \( \phi \) exceeds a critical value

\[
\phi_c = (A_k/4k)(1 - \sqrt{1 - 4m_{\text{N}}^2/A_k^2})
\]  

(6)

the sign of the field dependent mass of \( N \) will become negative, \( m^2(\phi) < 0 \), and the \( N \) field will no longer be pinned to zero, but will roll out to the global minimum of
the potential,

$$\langle \phi \rangle = A_k/4k, \quad \langle N \rangle = (A_k/2\sqrt{2}k)\sqrt{1 - 4m_N^2/A_k^2}. \quad (7)$$

Together with our assumptions about the SUSY parameters, this implies

$$A_k \sim k \phi_c \sim k \langle N \rangle \sim k \langle \phi \rangle \sim \lambda \langle N \rangle \equiv \mu \sim 10^3 \text{GeV} \quad (8)$$

During inflation the inflaton field $\phi$ must satisfy the slow roll conditions $\epsilon, \eta \ll 1$.

The COBE normalisation $\delta_H = 1.95 \times 10^{-5}$ requires the value of the inflaton mass from radiative corrections $m_\phi^2 \sim -k^2 A_k^2 \sim -(100eV)^2$ and from Eq.8 this implies that $\lambda, k \sim 10^{-10}$ and $\langle N \rangle \sim \langle \phi \rangle \sim 10^{13} \text{GeV}$. We address the smallness of $\lambda, k$ in the next section. The spectral index $n$ which relates to the power spectrum $P_k \propto k^{n-1}$, is given by $|n - 1| = 2\eta - 6\epsilon \sim 2\eta \sim 10^{-12}$ which provides a basic prediction of the model. The present value of the spectral index in the range $0.80 < n < 1.05$ at 68% C.L. slightly disfavours the accurate prediction that $n = 1.00$ but only at the $1\sigma$ level, and we may have to wait for the results from the Planck satellite which will measure it to an accuracy of $\Delta n = \pm 0.01$ to definitively test this prediction [29].

It is non-trivial that a set of parameters exists that is consistent with axion and SUSY physics and allows the correct COBE perturbations to be achieved by radiative corrections. Without SUSY one would be free to add soft scalar masses at will, but with SUSY one must rely on the theory which either generates soft masses of order a TeV, or sets them equal to zero as in no-scale SUGRA, in which case the radiative corrections, which are under control in the case of SUSY predict the relevant value of the soft parameters, without any further adjustable parameters. Thus SUSY is playing a crucial role in the model which is why we refer to it as a Supersymmetric Inflation Model.

### 3.3 Role of Non-renormalisable Operators

The couplings $\lambda, k$ should be thought of as effective couplings arising from non-renormalizable operators [3, 6] so they are actually couplings of order unity times small factors arising from ratios of VEVs to the Planck mass to some power, as can occur in models, and are not unnaturally small. In the original model it was suggested that the superpotential in Eq.3 arose as an effective theory from a non-renormalisable

\[^5\]Recall that $\epsilon \equiv \frac{1}{2}M_P^2(V''/V)^2$ with $|\eta| \equiv |\tilde{M}_P^2V''/V|$, where $V'(V'')$ are the first (second) derivatives of the potential and $\tilde{M}_P^2 = M_P^2/8\pi$ is the reduced Planck mass. The COBE normalisation is given by $\delta_H^2 = (1/150\pi^2)V_0/(M_P^2\epsilon)$. 

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superpotential which at leading order is given by

\[ W_{NR} = \lambda' N H_u H_d \frac{MM}{M_P^2} - k' \phi N^2 \frac{M^2}{M_P^2} + c \frac{(MM)^3}{M_P^8} + d \frac{(NM)^5(MM)^2}{M_P^{11}} + \cdots \]  

(9)

where two extra singlets \( M \) and \( \overline{M} \) have been introduced which develop VEVs by a radiative mechanism \( < M > = < \overline{M} > \sim 10^{14} \text{GeV} \), as a result of which we recover the original superpotential in Eq.3 and we reinterpret our couplings \( \lambda, k \) as effective couplings given by

\[ \lambda \equiv \lambda' \frac{< M >}{M_P^2} \sim \lambda' 10^{-10}, \quad k \equiv k' \frac{< \overline{M} >^2}{M_P^2} \sim k' 10^{-10}, \]  

(10)

Thus the underlying coupling constants \( \lambda', k' \) are of order unity, although for the most part we find it convenient to discuss the model in terms of effective couplings \( \lambda, k \). The underlying theory respects a \( Z_3 \times Z_5 \) symmetry, and the \( U(1)_{PQ} \) symmetry arises as an approximate effective symmetry, leading to an explicit contribution to the axion mass from the term proportional to \( d \) which tilts the axion potential slightly, and perturbs the \( \theta \) angle by an amount \( \Delta \theta \approx 10^{-11} \), thereby preserving the PQ solution to the strong CP problem, but providing a prediction for the next generation of electric dipole moment (EDM) experiments.

### 3.4 The Cosmological Constant Problem

Notice that the SUGRA-derived potential contribution \( V_0 \) exactly cancels with the other terms (by tuning) to provide agreement with the observed small cosmological constant. Thus we assume that at the global minimum \( V(\langle \phi \rangle, \langle N \rangle) = 0 \) which implies that \( V_0 = k^2 \langle N \rangle^4 \). The height of the potential during inflation is therefore \( V_0^{1/4} = k^{1/2} \langle N \rangle \sim 10^8 \text{GeV} \). Since the approach has a consistent way to set the large cosmological constant to zero, the absence of a real solution to this problem may not be an obstacle to the implications of the approach.

### 3.5 Parameter Counting and Singlino Mixing

A relevant parameter count at this stage reveals two superpotential effective parameters (\( \lambda \) and \( k \)), the two soft SUSY breaking parameters (\( A_k \) and \( m_N \)), plus the constant energy density \( V_0 \). From these five parameters we have inflated the universe with the correct COBE perturbations, provided a \( \mu \) term of the correct order of magnitude and solved the strong CP problem. They also govern the phenomenology of
the singlet Higgs and Higgsino components of $\phi$ and $N$ which may weakly mix with the MSSM superfields $H_u, H_d$. For example the Higgsino mixing matrix in the basis $\tilde{H}_d, \tilde{H}_u, \tilde{N}, \tilde{\phi}$ is

$$
\begin{pmatrix}
0 & -\lambda < N > & -\lambda < H_u > & 0 \\
-\lambda < N > & 0 & -\lambda < H_d > & 0 \\
-\lambda < H_u > & -\lambda < H_d > & 2k < \phi > & 2k < N > \\
0 & 0 & 2k < N > & 0
\end{pmatrix}, \quad (11)
$$

The LSP will be the lightest eigenvalue of the full “ino” matrix, extended in the usual way to include gaugino-higgsino mixing. Clearly if $k < \lambda/2$ then a singlino will be the LSP. In our case the singlino may be regarded as a linear combination of axino and inflatino. The coupling of the superfield $S$ containing the singlino $\tilde{S}$ is given from the usual result $[10]$ $W = \mu (1 + \epsilon S/v) H_u H_d$ with $\mu = \lambda < N >$. Here $\epsilon \sim v/f_a$ where the $f_a$ is the axion decay constant and $v$ is an electroweak VEV. Thus we have $f_a \sim < N >$ so $\epsilon \sim \lambda$, and hence

$$
W \sim \lambda < N > (1 + \lambda S/v) H_u H_d. \quad (12)
$$

As usual in models based on an intermediate scale solution to the $\mu$ problem $[3, 8, 10]$ the coupling of the singlino to the neutralinos means that $\tilde{S}$ nearly decouples. However the conservation of R-parity means that eventually the lightest neutralino produced in colliders must decay into the singlino, and all the collider signatures discussed in $[10]$ may apply. In the case that the lightest neutralino leaves the detector before it decays into the singlino, there will be no unconventional collider signature. In this case the knowledge concerning a lighter singlino will come from cosmology since the LSP relic density gets diluted by the ratio of the singlino to lightest neutralino masses, and direct dark matter searches will not see anything since the singlino LSP will not scatter off nuclei.

One of the main things we want to emphasize is the connection between the calculation of relic densities and the other physics, via their common parameters. This is summarised in Table 1 for the particular model we have been discussing. The same parameters that control the ino mass matrix will also be involved in reheating of the universe after inflation, and giving the relic densities of LSP and in leptogenesis as we discuss in the next section. Different models may have different mechanisms to solve some of the problems, different reheating and preheating, and so on, but will still lead to a version of Table 1.
### Table 1: This table illustrates the fact that a particular parameter of the model (columns) simultaneously controls several different aspects of particle physics and cosmology (rows) which are thereby related. $L_{\text{soft}}$ contains $A_k, m_N^2$ and the other soft parameters, $L_{\text{Yuk}}$ contains the Yukawa coupling constants controlling all fermion masses and mixing angles.

| Parameter                              | $V_0$ | $k$ | $\lambda$ | $L_{\text{soft}}$ | $L_{\text{Yuk}}$ |
|----------------------------------------|-------|-----|-----------|-------------------|------------------|
| Inflation and COBE                      | ✓     | ✓   | -         | ✓                 | -                |
| MSSM $\mu$ parameter                   | -     | -   | ✓         | ✓                 | -                |
| Fermion Masses, Mixings                | -     | -   | -         | ✓                 | ✓                |
| SUSY collider physics                   | -     | ✓   | ✓         | ✓                 | ✓                |
| Strong CP, axion abundance ($\Omega_a$) | -     | ✓   | ✓         | ✓                 | ✓                |
| Leptogenesis ($\Omega_b$)              | -     | ✓   | ✓         | ✓                 | ✓                |
| LSP abundance ($\Omega_{\text{LSP}}$)  | -     | ✓   | ✓         | ✓                 | -                |

#### 3.6 Preheating/Reheating

It is usually assumed that inflation ends with the singlets $\phi, N$ oscillating about their global minimum. Although the final reheating temperature is estimated to be of order 1 GeV [8], during the reheating process the effective temperature of the universe, as determined by the radiation density, can better be viewed as rapidly rising to $V_0^{1/4} = k^{1/2}\langle N \rangle \sim 10^8 \text{GeV}$ then slowly falling to the final reheat temperature [7] during the reheating process. This reheating gives entropy to the Universe. Non-perturbative effects can produce particles with masses up to the potential height, i.e. $m \leq V_0^{1/4} \sim 10^8 \text{GeV}$ (preheating). The preheating and reheating process in this model is quite complicated, but the essential physics is as follows. To begin with the potentially problematic axions and gravitinos are not over produced during preheating [17, 28]. Higgs scalars are copiously produced through preheating via the couplings $\lambda$ and $k$ to the oscillating inflaton fields. Although the neutralinos are produced in Higgs decays via preheating, once the Higgses decay they go into thermal equilibrium, and subsequently freeze out while the universe is radiation dominated, similar to the usual hot big bang scenario. However, for a range of parameters the singlino is lighter than the lightest neutralino, and in this case after freeze-out the lightest neutralino decays into the singlino thereby reducing the LSP relic density by the ratio of their masses.

Recently it has been realised that reheating in all hybrid models, including the SUSY motivated ones of interest to us here, goes through very effective tachyonic preheating [16]. As a result the stage of the background oscillations of the scalars around the minimum of the potential will never be reached. This picture is dramatically different from the early papers on the reheating in hybrid models [14, 15].
and will probably affect the results in \cite{17}. On the other hand the new picture of preheating implies that exciting one field (for example $\phi$ or $N$) is sufficient to rapidly drag all other light fields with which it interacts into a similarly excited state. This strengthens our claim that the Higgs doublets which interact with $N$ will be efficiently preheated.

Once the Yukawa couplings are included in the superpotential, right-handed sneutrinos are also expected to be produced during the initial period of preheating via their couplings to the Higgs doublets, and decay out-of-equilibrium into leptons and Higgses giving rise to leptogenesis. We have already remarked that, unlike the usual hot big bang scenario, the out-of-equilibrium condition is automatically satisfied during reheating, and furthermore the production mechanism of right-handed neutrinos is totally different. In the standard scenario the baryon asymmetry is given by $Y_b \sim d\epsilon_1/g^*$ where $\epsilon_1$ is the lepton number asymmetry produced in the decay of the lightest right-handed neutrino of mass $M_1$, $g^*$ counts the effective number of degrees of freedom (for the SM $g^* = 106.75$ while for the Supersymmetric SM $g^* = 228.75$) and $d$ is the dilution factor which takes into account suppressions from either the couplings being too small to thermally produce right-handed neutrinos, or too large to satisfy the out-of-equilibrium condition. Typically $d \ll 1$ except for a narrow range of couplings \cite{20}. In the inflationary picture of reheating outlined above, the baryon asymmetry is given by $Y_b \sim \gamma \epsilon_1 (cV_0)^{1/4}/M_1$ where $c$ is the fraction of the total vacuum energy converted into right-handed neutrinos due to preheating, and $\gamma$ accounts for dilution due to entropy production during reheating. Since $\epsilon_1 \sim 10^{-6}(M_1/10^{10}\text{GeV})$ \cite{21} we find that $Y_b \sim 10^{-8}\gamma(c)^{1/4}$, apparently independently of $M_1$ (although $c$ will depend on $M_1$, for instance $c = 0$ for $M_1 > 10^8$ GeV.)

Solving the Boltzmann equations for a particular choice of parameters in this model the densities of the neutralinos, radiation, relativistic axions and baryons were calculated at reheating time, defined as the time at which the oscillating singlet energy density rapidly decayed to zero \cite{7}. This time represents the start of the hot big bang. The important point to emphasise is that at this time $t_{RH}$, for a given model the Boltzmann equations enable us to calculate the energy density of the different types of matter $\rho_{\text{matter}}(t_{RH})$, as well as the radiation energy density $\rho_r(t_{RH})$. In the present model the details of this calculation are discussed in \cite{7}, and a similar calculation may be performed for any other model.
How To Calculate the Size of the Dark Energy Density in Supersymmetric Inflation

We now turn to the question of dark energy, which is not addressed by our supersymmetric particle physics based model of inflation. The origin of DE might be a traditional cosmological constant with equation of state \( w = -1 \) or some time-varying smooth energy (quintessence) with \(-1 < w < -0.6\) where the upper bound is from current observations, and may be in conflict with some quintessence models \([30]\). Quintessence models assume a zero cosmological constant and add some arbitrary field to account for DE. From the point of view of our inflation model, the simplest possibility is to assume that at the global minimum (after inflation) the height of the potential is not zero but about \(10^{-3}\)eV. This possibility, which just corresponds to a standard cosmological constant with \( w = -1 \), can be arranged (though not explained) by tuning \( V_0 \) in Eq.4; \( V_0 \) has to be tuned in any case to give a zero cosmological constant, so this possibility requires no additional tuning to that already required in the model. In the no-scale SUGRA model \([8]\) \( V_0 \) arises from the moduli fields in the string theory, and is determined by the non-perturbative physics of moduli stabilisation which is not yet understood from a fundamental point of view but may nevertheless be parametrised. Of course such a procedure raises the cosmic coincidence question: why should we have \( \rho_{\text{DE}} \sim \rho_{\text{matter}} \) at the present epoch? Until the cosmological constant problem is resolved, there is no way that this question can be answered. We reject recent claims to the contrary which are based on setting the cosmological constant to zero by hand to start with, since there is always the danger in this approach that one has thrown away the baby along with the bathwater. In the absence of anything better, some authors have turned to anthropic arguments, but many anthropoids reject this approach also as long as alternatives may be possible.

Is there anything that we can say about DE at the current time from the perspective of our supersymmetric particle physics based model of inflation? Perhaps surprisingly the answer is positive: we shall show that we can deduce the present day value of dark energy density from the model, together with the measured CMB temperature and the Hubble constant, even if the model does not yet specify the physics of dark energy!

A key point of our approach is that a supersymmetric particle physics based model of inflation enables us to calculate (in principle at least) the (energy or number) densities of all forms of radiation and matter (but excluding dark energy) at some early

\[ w = \frac{p}{\rho} = \frac{(KE - PE)}{(KE + PE)} \]

\( w < -1 \) would require negative \( KE \) which corresponds to the scalar being a ghost field, and a loss of unitarity \([1]\).
time $t_{RH}$ after inflation and reheating has taken place, corresponding to the start of the standard hot big bang. For simplicity we consider only one type of matter energy density $\rho_{\text{matter}}(t_{RH})$ (which may readily be obtained from the calculated number density) and radiation energy density $\rho_{\gamma}(t_{RH})$. The argument may be straightforwardly generalised to the case of several components of radiation and matter. Now, using the equations of the standard hot big bang, we wish to obtain their values at the present time $t_0$, $\rho_{\gamma}(t_0)$ and $\rho_{\text{matter}}(t_0)$. Without further information this is impossible since we need something to tell us when the present time $t_0$ is, and moreover the model does not specify either $\rho_{DE}(t_{RH})$, or its equation of state, both of which will influence the evolution of the universe. Therefore let us input into our analysis the present day observed CMB temperature $T_0 = 2.725$K, which corresponds to a photon density $\rho_{\gamma}(t_0) = (2.115 \times 10^{-4}\text{eV})^4$, a photon number density $n_\gamma = 410\text{cm}^{-3}$, and, assuming three families of light neutrinos, an entropy density $s = 7.04n_\gamma$. Then, ignoring additional sources of entropy between $t_{RH}$ and $t_0$ (such as electron-positron annihilation), since we know the equations of state for photons and matter, $\rho_{\gamma} \sim R^{-4}$ and $\rho_{\text{matter}} \sim R^{-3}$, where $R$ is the scale factor of the universe, using the initial values of $\rho_{\gamma}(t_{RH})$ and $\rho_{\text{matter}}(t_{RH})$ predicted by the model and the present value of $\rho_{\gamma}(t_0)$ from observation, we find 

$$\rho_{\text{matter}}(t_0) \approx \rho_{\text{matter}}(t_{RH})[\rho_{\gamma}(t_0)/\rho_{\gamma}(t_{RH})]^{3/4}. $$

We emphasise that this determination of $\rho_{\text{matter}}(t_0)$ is independent of the unknown dark energy. Allowing for entropy production, which will increase the photon energy density relative to the matter energy density, it is usually convenient to consider the ratio $n_{\text{matter}}/s$ which is equal to the number of particles of each species per comoving volume. From the model we can calculate $n_{\text{matter}}/s$ at $t_{RH}$, and by particle number conservation the value of this ratio at the present time $t_0$ is unchanged. Using the present value of $s$ (above) we therefore immediately find $n_{\text{matter}}(t_0)$ and from the mass of the particle type we readily find $\rho_{\text{matter}}(t_0)$, again independently of the dark energy.

Once we have obtained $\rho_{\text{matter}}(t_0)$, from a combination of our model calculation and the observed CMB temperature, as outlined above, we now use the observed Hubble constant $H_0$, or equivalently the present day critical density $\rho_{\text{crit}}$, to convert $\rho_{\text{matter}}(t_0)$ into the various $\Omega_{\text{matter}} = \rho_{\text{matter}}(t_0)/\rho_{\text{crit}}$. Once $\Omega_{\text{matter}}$ is predicted within some supersymmetric particle physics based model of inflation, supplemented by measurements of the CMB temperature and the Hubble constant, then it is clear that $\Omega_{DE}$ is also predicted to be

$$\Omega_{DE} = 1 - \Omega_{\text{matter}} \quad \text{(13)}$$

Thus a model of inflation that is capable of predicting $\Omega_{\text{matter}}$ using measurements of the CMB temperature and the Hubble constant, is also capable of predicting $\Omega_{DE}$ from Eq.13. This sum rule was written down in ref.32, including a curvature term and it was discussed there how to determine each of the components $\Omega_{DE}$ and $\Omega_{\text{matter}}$. 

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from observation. What we are saying here is quite different from the empirical approach to determining the components of this equation discussed in ref. [32], and should not be confused with it. To begin with we are assuming inflation, so that the curvature contribution is zero. Secondly we are only taking two inputs from observation, namely the CMB temperature and the value of the Hubble constant. Given these inputs we have shown how an inflation model allows us to then calculate $\Omega_{\text{matter}}$, and hence deduce $\Omega_{DE}$ from Eq. [13].

At first sight our result appears surprising: how can we have deduced $\Omega_{DE}$ from a model in which the dark energy is unspecified? In order to answer this it is useful to compare two slightly different models of inflation, one in which the dark energy density is zero and one in which it is non-zero, but which otherwise predict identical values of $\rho_\gamma(t_{RH})$ and $\rho_{\text{matter}}(t_{RH})$ (in our example this just corresponds to tuning $V_0$ slightly differently in the two cases leaving all the other parameters unchanged.) In both cases this will result in the same values of $\rho_{\text{matter}}(t_0)$, once $\rho_\gamma(t_0)$ is inputted. The only difference is that the Friedmann equations, with zero curvature term due to the inflation assumption, will determine two different values of the Hubble constant, corresponding to two different values of critical density. The first is given by $\rho_{\text{crit}1} = \rho_{\text{matter}}(t_0)$, and the second involving the sum of two contributions $\rho_{\text{crit}2} = \rho_{\text{matter}}(t_0) + \rho_{DE}(t_0)$. Since we input the Hubble constant from observation, we know the true value of $\rho_{\text{crit}}$ in our universe, and so we can discriminate between the two cases from a measurement of the Hubble constant. More generally, it is clear that, once the correct supersymmetric particle inflation model is known, and the present day value of $\rho_{\text{matter}}(t_0)$ is calculated from it (using the CMB temperature as input), that the present day value of $\rho_{DE}(t_0)$ may be deduced from the Hubble constant $H_0$ which is telling us information about dark energy by telling us the critical density. From this example it is clear that our argument gives us no new insight into the cosmic coincidence question, since a universe without dark energy would simply correspond to having a different Hubble constant.

Once the importance of the Hubble constant $H_0$ in our argument is realised, the next question is whether our argument contains any content at all? The answer must be yes, since the conclusion relies on non-trivial information coming from the model, namely the initial condition for $\rho_{\text{matter}}(t_{RH})$ and $\rho_\gamma(t_{RH})$ without which it would be impossible to find $\rho_{\text{matter}}(t_0)$ from the CMB temperature alone, and without $\rho_{\text{matter}}(t_0)$ it would be impossible to deduce $\rho_{DE}(t_0)$ from the Hubble constant $H_0$. A related question, is whether our argument is actually useful in practice, given that at the present time we do not know the correct model, and even if the model were known and all the parameters of that model were accurately specified, we still would need to know the physics of preheating and reheating very well. Also when the argument is generalised to all the forms of matter and radiation we would need to have a
good understanding of the physics of baryogenesis and a way of calculating the axion misalignment angle, and so on in order to be able to specify the present day relic densities of all the component forms of matter. One could criticise the argument on the grounds that the accuracy of the deduced density of dark energy is therefore limited by the accuracy with which the matter density can be calculated. While this is true, it would seem remarkable to us to suppose otherwise: while it would be nice to be able to calculate the DE density much more accurately than the matter density, this possibility hardly seems very likely. What our argument gives is a way of calculating \( \Omega_{DE} \) to the same precision as \( \Omega_{\text{matter}} \), and this we believe is the best that one can hope for.

Whether the DE is due to scalar fields, or an incomplete vanishing of a cosmological constant (corresponding perhaps to the universe ending up temporarily in a vacuum state slightly above a global minimum at zero), perhaps some of the parameters that determine it will be related to parameters that also determine the forms of matter. In the present paper we do not specify any particular model for DE and so we must therefore rely on observation to determine the equation of state for the DE. If observation eventually tells us that \( w = -1 \) and that the DE is equivalent to a cosmological constant, then it will be a tremendous challenge to theorists to explain this (see for example the approach of Bastero-Gil, Mersini, and Kanti [33]). Explaining a vanishing cosmological constant is already proving a very difficult question for string theorists, and explaining a very small one does not apparently make this any easier. In this case it is possible that the DE question will be around for a long time. In the meantime progress may be forthcoming on determining the supersymmetric particle inflation model and in determining its parameters and in being able to use those parameters to calculate the relic matter densities with increasing accuracy. In such a scenario, the one consolation may be that our argument enables one to then calculate the size of the DE density (i.e. the cosmological constant) even in the absence of any theory of it.

5 Summary and Conclusion

We have considered the class of supersymmetric inflation models in which inflation occurs at the intermediate PQ symmetry breaking scale. Such models are better motivated than GUT scale inflation models which face the problems with magnetic monopoles and domain walls. In intermediate scale supersymmetric inflation the same theory which is responsible for inflation is also responsible for the solution to the \( \mu \) problem, and the strong CP problem. As a consequence one would generally expect CDM to contain an axion component in addition to an LSP component. The
LSP itself need not be the lightest neutralino, but may be a singlino associated with the singlet fields which control hybrid inflation and resolve the $\mu$ problem. The present day relic densities of the CDM components comprising LSPs and axions may be calculated in a given model, using the observed CMB temperature and Hubble constant, although the axion density will be subject to the usual uncertainties of the unknown misalignment angle.

Once right-handed neutrinos are added, as the recent confirmations of neutrino masses suggests that they should be, then the possibility of baryogenesis via leptogenesis seems well motivated, and then the baryon density may be calculated in a given model. The various relic densities are in principle calculable in a given model, and are related to each other and to other phenomena since the number of parameters involved is generally smaller than would be the case without a theory. The same parameters control cosmology on the one hand and collider physics on the other hand. For example common soft SUSY breaking parameters are involved in both inflation and collider physics. In order to illustrate all these ideas we have described an explicit intermediate scale supersymmetric inflation model which already exists and is quite well studied in the literature [3-8], and many of the general ideas above are made very explicit in the model, for example the role of the underlying parameters in determining different phenomena is demonstrated for this model in Table 1.

Over the next few years there will be considerable progress in cosmology from the Map and Planck explorer satellites, and in SUSY from the Tevatron and LHC. We believe the time is ripe for a new closer synthesis of SUSY and inflation, and that the most promising scenario will involve these theories meeting at the intermediate scale. We have shown that in this case one may hope to relate different phenomena in cosmology and in particle physics in a much closer and more predictive way than ever before. Finally we have made the original observation that, given the value of the CMB temperature and Hubble constant from observation, an intermediate scale supersymmetric inflation model allows the present day matter relic density to be calculated, and hence the present day DE relic density to be determined from Eq.13 even in the absence of any theory of DE.

Acknowledgements

GK appreciates helpful conversations with S. Carroll, D. Chung, I. Maor, M. Turner and L.-T. Wang, and SK is similarly grateful to J. Cline, A. Kusenko, A. Liddle and D. Rayner, and appreciates the support and hospitality of the Michigan Center for Theoretical Physics and PPARC for a Senior Fellowship.
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