Recent progress in exclusive charmless hadronic decays of the $B$ meson is discussed.

1. Introduction

Recently there has been a remarkable progress in the study of exclusive charmless $B$ decays, both experimentally and theoretically. On the experimental side, many new two-body decay modes were discovered by CLEO [1]:

$$B \rightarrow \eta' K^+, \eta' K_S^0, \pi^+ K_S^0, \pi^0 K^+, \omega K^+, \omega K_S^0, \omega \pi^+, \phi K^*.$$  

Moreover, CLEO has improved upper limits for many other channels. Therefore, it is a field whose time has finally arrived. On the theoretical aspect, there are two important issues to be addressed: (i) the renormalization scheme and scale dependence of hadronic matrix elements, and (ii) nonfactorizable effects in charmless $B$ decays. A fascinating progress in dealing with the above-mentioned theoretical issues has been made over the last few years. In this talk I’ll first discuss the theoretical progress and then proceed to elaborate the decay $B \rightarrow \eta' K$ which has received a lot of attention recently.

2. Renormalization scale and scheme dependence of hadronic matrix element

The relevant effective weak Hamiltonian for hadronic weak $B$ decay is of the form

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \lambda_u (c_1 O_1^u + c_2 O_2^u) + \lambda_c (c_1 O_1^c + c_2 O_2^c) - \lambda_t \sum_{i=3}^{10} c_i O_i \right],$$

where $\lambda_u = V_{ub} V_{uq}^*$. The Wilson coefficients $c_i(\mu)$ in Eq. (2) have been evaluated at the renormalization scale $\mu \sim m_b$ to the next-to-leading order. Beyond the leading logarithmic approximation, they depend on the choice of the renormalization scheme. The mesonic matrix elements are customarily evaluated under the factorization hypothesis. In the naive factorization approach, the relevant Wilson coefficient functions for color-allowed external $W$-emission (or so-called “class-I”) and color-suppressed (class-II) internal $W$-emission amplitudes are given by $a_1 = c_1 + c_2/N_c$, $a_2 = c_2 + c_3/N_c$, respectively, with $N_c$ the number of colors. Inspite of its tremendous simplicity, naive factorization encounters two major difficulties. First, it never works for the decay rate of class-II decay modes, though it usually operates for class-I transition. Second, the hadronic matrix element under factorization is renormalization scale $\mu$ independent as the vector or axial-vector current is partially conserved. Consequently, the amplitude $c_i(\mu) \langle O \rangle_{\text{fact}}$ is not truly physical as the scale dependence
of Wilson coefficients does not get compensation from the matrix elements. The first difficulty indicates that it is inevitable and mandatory to take into account nonfactorizable contributions, especially for class-II decays, to render the color suppression of internal $W$ emission ineffective. The second difficulty also should not occur since the matrix elements of four-quark operators ought to be evaluated in the same renormalization scheme as that for Wilson coefficients and renormalized at the same scale $\mu$.

To circumvent the aforementioned second problem, one should evaluate perturbative QCD and electroweak corrections to the hadronic weak matrix elements parametrized by the matrices $\hat{m}_s$ and $\hat{m}_c$, respectively, so that \[ c_i(\mu)\langle O_i(\mu) \rangle = c_i(\mu) \left[ I + \frac{\alpha_s(\mu)}{4\pi} \hat{m}_s(\mu) + \frac{\alpha}{4\pi} \hat{m}_c(\mu) \right] \langle O_j^{\text{tree}} \rangle \equiv c_i^{\text{eff}} \langle O_j^{\text{tree}} \rangle. \] (3)

Then factorization is applied to the matrix elements of tree operators; that is, before employing factorization to the four-quark operators, it is necessary to absorb all corrections to $O^{\text{tree}}$ into the effective coefficients $c_i^{\text{eff}}$. One-loop penguin corrections and vertex corrections to the operators $O_i$ have been calculated in [2, 3, 4]. One can explicitly check that the effective Wilson coefficients $c_i^{\text{eff}}$ are indeed renormalization scheme independent and approximately renormalization scale independent [5]. It should be stressed that the effective penguin coefficients $c_i^{\text{eff}}$ ($i = 3, \cdots, 10$) take into account the effect of penguin diagrams not only with top quark exchange characterized by the Wilson coefficients $c_i$ but also with internal $u$ and $c$ quarks induced by the tree operators $O_1^u$ and $O_1^c$, respectively.

3. Nonfactorizable effects in charmless hadronic $B$ decay

As stressed in the last section, it is mandatory to take into account the nonfactorizable effects, especially for class-II modes. For $B \to PP$ or $VP$ decays, nonfactorizable contributions can be lumped into the effective parameters $a_1$ and $a_2$ [6]:

\[ a_1^{\text{eff}} = c_1 + c_2 \left( \frac{1}{N_c} + \chi_1 \right), \quad a_2^{\text{eff}} = c_2 + c_1 \left( \frac{1}{N_c} + \chi_2 \right), \] (4)

where $\chi_i$ are nonfactorizable terms and receive main contributions from the color-octet current operators. Phenomenological analyses of two-body decay data of $D$ and $B$ mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters $a_{1,2}^{\text{eff}}$ do show some variation from channel to channel, especially for the weak decays of charmed mesons [3, 7]. An eminent feature emerged from the data analysis is that $a_2^{\text{eff}}$ is negative in charm decay, whereas it becomes positive in bottom decay [3, 8]:

\[ a_2^{\text{eff}}(D \to \bar{K}\pi) \sim -0.50, \quad a_2^{\text{eff}}(B \to D\pi) \sim 0.26, \] (5)

which in turn implies

\[ \chi_2(\mu \sim m_c; D \to \bar{K}\pi) \sim -0.36, \quad \chi_2(\mu \sim m_b; B \to D\pi) \sim 0.11. \] (6)

The observation $|\chi_2(B)| \ll |\chi_2(D)|$ is consistent with the intuitive picture that soft gluon effects become stronger when the final-state particles move slower, allowing more time for
significant final-state interactions after hadronization \[^3\]. Phenomenologically, it is often to
treat the number of colors \(N_c\) as a free parameter and fit it to the data. Theoretically, this
amounts to defining an effective number of colors by \(1/N^\text{eff}_c = (1/N_c) + \chi\). It is clear from
Eq. (6) that

\[
N^\text{eff}_c (D \to \bar{K} \pi) \gg 3, \quad N^\text{eff}_c (B \to D \pi) \sim 2. \tag{7}
\]

It is natural to ask that does the naive factorization approach also fail in charmless \(B\)
decays? If so, how large is the nonfactorizable effect? Since the energy release in charmless
two-body decays of the \(B\) meson is generally slightly larger than that in \(B \to D^{(*)} \pi\), it is
expected that \(N^\text{eff}_c\) for the \(B\) decay into two light mesons is close to \(N^\text{eff}_c (B \to D \pi) \approx 2\). It
is pointed out in \[^4\] that the parameters \(a_2, a_3\) and \(a_5\) are strongly dependent on \(N^\text{eff}_c\) and
the rates dominated by these coefficients can have large variation. For example, the decay
widths of \(B^- \to \omega K^{(*)-}\), \(B^0 \to \omega K^0\), \(\rho K^{(*)0}\), \(B_s \to \eta \omega\), \(\eta \phi, \omega \phi, \cdots\), etc. have strong \(N_c\)
dependence \[^4\]. We have shown recently in \[^1\] that the branching ratio of \(B^- \to \omega K^-\)
has its lowest value of order \(1 \times 10^{-6}\) near \(N^\text{eff}_c \sim 3 - 4\) and hence the naive factorization with
\(N^\text{eff}_c = 3\) is ruled out by experiment, \(\mathcal{B}(B^{\pm} \to \omega K^{\pm}) = (1.5^{+0.7}_{-0.6} \pm 0.3) \times 10^{-5}\) \[^1\].

However, it is not easy to discern between \(N^\text{eff}_c = \infty\) and \(N^\text{eff}_c = 2\) in \(B \to \omega K\) decays, and
it becomes important to have a more decisive test on \(N^\text{eff}_c\). For this purpose, we shall focus on
the decay modes dominated by the tree diagrams and sensitive to the interference between
external and internal \(W\)-emission amplitudes. The fact that \(N^\text{eff}_c = 2\) \((N^\text{eff}_c = \infty)\) implies
constructive (destructive) interference will enable us to differentiate between them. Good
examples are the class-III modes: \(B^{\pm} \to \pi^0 \pi^\pm, \eta \pi^\pm, \pi^0 \rho^\pm, \omega \pi^\pm, \cdots\). We found that \[^1\] the
averaged branching ratio of \(B^{\pm} \to \omega \pi^\pm\) has its lowest value of order \(2 \times 10^{-6}\) at \(N^\text{eff}_c = 3\) and
then increases with \(1/N^\text{eff}_c\). Since experimentally \[^1\] \(\mathcal{B}(B^{\pm} \to \omega \pi^\pm) = (1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}\),
it is evident that \(N^\text{eff}_c = \infty\) is disfavored by the data. Measurements of interference effects in
charged \(B\) decays \(B^- \to \pi^-(\rho^-) \pi^0(\rho^0)\) will help determine the parameter \(N^\text{eff}_c\). For example,
the ratio \(R_1 \equiv \mathcal{B}(B^- \to \pi^- \rho^0)/\mathcal{B}(B^- \to \pi^- \rho^0)\) is calculated to be 2.50 for \(N^\text{eff}_c = 2\) and 0.26
for \(N^\text{eff}_c = \infty\). Hence, a measurement of \(R_1\), which has the advantage of being independent
of the Wolfenstein parameters \(\rho\) and \(\eta\), will provide a decisive determination of the effective
number of colors \(N^\text{eff}_c\).

4. Exclusive charmless hadronic \(B\) decays to \(\eta'\) and \(\eta\)

The CLEO collaboration has recently reported the preliminary branching ratios for the
exclusive decay \(B \to \eta'K\) dominated by gluonic penguin daigrams \[^1\]:

\[
\mathcal{B}(B^{\pm} \to \eta' K^{\pm}) \equiv \frac{1}{2} \left[ \mathcal{B}(B^{+} \to \eta' K^{+}) + \mathcal{B}(B^{-} \to \eta' K^{-}) \right] = (7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5},
\]

\[
\mathcal{B}(B^{0} \to \eta' K^{0}) \equiv \frac{1}{2} \left[ \mathcal{B}(B^{0} \to \eta' K^{0}) + \mathcal{B}(B^{0} \to \eta' K^{0}) \right] = (5.3^{+2.8}_{-2.2} \pm 1.2) \times 10^{-5}. \tag{8}
\]

Early theoretical estimate of the \(B^{\pm} \to \eta' K^{\pm}\) branching ratio \[^2, 3, 3\] lies in the range of
\((1 - 2) \times 10^{-5}\). \[^1\] The CLEO result thus appears to be abnormally large. The question is

\[^1\] The prediction \(\mathcal{B}(B^{\pm} \to \eta' K^{\pm}) = 3.6 \times 10^{-5}\) given in \[^2\] is too large by about a factor of 2 because the
normalization constant (i.e. \(1/\sqrt{3}\)) of the \(\eta_0\) wave function was not taken into account in the form factor
\(F_0(B^{\eta_0})\). This negligence was also made in some recent papers on \(B \to \eta'K\).
then can the CLEO observation of $B \rightarrow \eta'K$ be accommodated in the standard model? Do the new data imply new physics? The theoretical interest and speculation in this subject has surged, as evidenced by the recent literature [4,14-20] that offer various interpretations on the unexpected large branching ratios.

In order to illustrate the problem clearly we choose the following parameters for calculation:

$$F_{0}^{BK}(0) = 0.34, \quad \sqrt{3F_{0}^{B_{0}}}(0) = 0.254, \quad f_{0} = f_{8} = f_{\pi}, \quad m_{s} = 150\,\text{MeV}, \quad \theta = -19.5^\circ, \quad (9)$$

and $N_{c}^{\text{eff}} = 2$, where $\theta$ is the $\eta-\eta'$ mixing angle. Using the renormalization scale and scheme independent effective Wilson coefficients $c_{i}^{\text{eff}}$ discussed in Sec. II, we find

$$B(B^{\pm} \rightarrow \eta'K^{\pm}) = \begin{cases} 1.4 \times 10^{-5}, & \text{for } \eta = 0.35, \quad \rho = 0.08, \\ 1.6 \times 10^{-5}, & \text{for } \eta = 0.34, \quad \rho = -0.12. \end{cases} \quad (10)$$

In the ensuing discussion, we will use (9) and (10) as the benchmarked values to be compared with. Since the choice of form factors and light quark masses is uncertain, one may argue that the CLEO data (8) can be fitted by choosing a small strange quark mass and/or large form factors $F_{0}^{BK}$ and $F_{0}^{B_{0}}$ [13]. For example, the above model estimate with $m_{s} = 55\,\text{MeV}$ or $F_{0}^{BK}(0) = 0.63$ will fit to the central value of $B(B^{\pm} \rightarrow \eta'K^{\pm})$. However it is dangerous to fit the parameters to a few particular decay modes. The point is that comparison between theory and experiment should be carried out using the same set of parameters for all decay channels. Indeed the measured branching ratio of $B \rightarrow \pi K$ puts a constraint on the strange quark mass and it indicates that $m_{s}$ cannot be too small. In the SU(3) limit we have the relation $F_{0}^{B_{\pi}\pm} = F_{0}^{BK}$. Most of the existing QCD-sum-rule and quark model calculations show that $F_{0}^{B_{\pi}\pm}(0) \lesssim 0.33$ (for a review, see [20]). We shall see below that a severe constraint on $F_{0}^{B_{\pi}\pm}(0)$ can be derived from the current limit on the decay $B^{+} \rightarrow \eta\pi^{+}$. Since SU(3) breaking is expected at most of 30% level, it is very unlikely that $F_{0}^{BK}(0)$ can deviate much from 0.33. Likewise, the nonet symmetry relation $\sqrt{3F_{0}^{B_{0}} = F_{0}^{B_{\pi}\pm}}$ implies that $\sqrt{3F_{0}^{B_{0}}}(0)$ cannot be too large than the model estimate, say 0.254 given in (9). In short, the parameters given in (9) cannot be modified dramatically without violating SU(3) symmetry relation and experimental observation of other decay modes.

Nevertheless, we can adjust the parameters in (9) slightly to improve the discrepancy between theory and experiment. The key point is that an accumulation of several small enhancement may eventually lead to a sizable enhancement. First, the current quark mass $m_{s} = 150\,\text{MeV}$ in (9) is defined at the renormalization scale $\mu = 1\,\text{GeV}$. For reason of consistency, one ought to apply the small running quark mass $m_{s}(m_{b}) \simeq 105\,\text{MeV}$ in calculation. Second, we use $f_{8}/f_{\pi} = 1.38 \pm 0.22$ and $f_{0}/f_{s} = 1.06 \pm 0.03$ to take into account SU(3) breaking effects in decay constants [22]. Third, for the form factor $F_{0}^{B_{0}}$ we follow [1] to use $\sqrt{3F_{0}^{B_{0}}}(0) = 0.33$, which is slightly larger than the value of 0.254 obtained in [21]. Fourth, previously we employed the mixing angle $\theta = -19.5^\circ$ so that the wave functions of the $\eta$ and $\eta'$ have the simple expressions: $\eta = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s})$ and $\eta' = \frac{1}{\sqrt{5}}(u\bar{u} + d\bar{d} + 2s\bar{s})$. Here we instead use the value $\theta = -22.0^\circ \pm 3.3^\circ$ extracted in [22]. Applying the new value for each of the aforementioned parameters individually, we find that the branching ratio of $B^{\pm} \rightarrow \eta'K^{\pm}$
is enhanced by 62%, 37%, 19%, and 5%, respectively. Hence, the dominant enhancement comes from the running strange quark mass and SU(3) breaking in decay constants. When all new parameters are employed, we obtain (see Table I)

\[
\mathcal{B}(B^\pm \rightarrow \eta' K^\pm) = \begin{cases} 
3.87 \times 10^{-5}, \\
4.04 \times 10^{-5}, 
\end{cases} \\
\mathcal{B}(B^0 \rightarrow \eta' K^0) = \begin{cases} 
3.70 \times 10^{-5}, \\
3.67 \times 10^{-5}, 
\end{cases}
\]  

(11)

for \( \eta = 0.35, \rho = 0.08 \) (upper entry) and \( \eta = 0.34, \rho = -0.12 \) (lower entry), respectively. Note that \( \mathcal{B}(B^\pm \rightarrow \eta' K^\pm) \) increases with \( 1/N_c^{\text{eff}} \) and it becomes as large as order of \( 6 \times 10^{-5} \) at \( N_c^{\text{eff}} = \infty \). However, as we have stressed before, it is very unlikely to have a large \( N_c^{\text{eff}} \) in the two-body charmless \( B \) decay. Therefore, we see that by adjusting the parameters in (9) within some reasonable range, the standard penguin contribution can account for the observed decay rate of \( B^0 \rightarrow \eta' K^0 \) but only marginally for \( B^\pm \rightarrow \eta' K^\pm \). Nevertheless, the current data allow for some new contributions (but not necessarily new physics) unique to the \( \eta' \). Of course, we have to await more new data to sort it out.

There are several mechanisms which are unique to the \( \eta' \) and may enhance the decay rate of \( B \rightarrow \eta' K \). (i) The \( b \rightarrow g^*g^* \) penguin followed by the transition \( g^* \rightarrow g\eta' \) via the QCD anomaly can in principle contribute to the exclusive decay \( B \rightarrow \eta' K \). This anomalous mechanism was originally advocated to explain the observed large inclusive \( B \rightarrow \eta' + X \) signal [23, 24]. However, since this mechanism involves a production of a gluon before hadronization, it will not play an essential role in low-multiplicity two-body exclusive decays unless the gluon is soft and absorbed in the wave function of the \( \eta' \). Another possibility is that the gluon produced from the penguin diagram and the gluon emitted from the light antiquark fuse into the \( \eta' \) [18, 19]. As the average momentum of the gluon emitted from the antiquark is in general less than 1 GeV, it is not clear if perturbative QCD is still applicable in this case. (ii) The process \( b \rightarrow s + g^*g^* \rightarrow s + \eta' \) involves two gluon production in the penguin-like diagram followed by the \( \eta' \)-gluon anomalous interaction. The decay \( b \rightarrow s + g^*g^* \) has been calculated in the literature [25]. It appears that the branching ratio arising from this mechanism is less than \( 1 \times 10^{-5} \) [26]. (iii) A new internal \( W \)-emission contribution comes from the Cabibbo-allowed process \( b \rightarrow \bar{c}c \gamma \) followed by a conversion of the \( \bar{c}c \) pair into the \( \eta' \) via two gluon exchanges. This new contribution is important since its mixing angle \( V_{cb}V_{cs}^{\ast} \) is as large as that of the penguin amplitude and yet its Wilson coefficient \( a_{2}^{\text{eff}} \) is larger than that of penguin operators. The decay constant \( f_{\eta'(cc)} \), defined by \( \langle 0|\bar{c}\gamma_\mu c\gamma_\nu|\eta'\rangle = if_{\eta'(cc)}q_{\mu}\), has been estimated to be \( |f_{\eta'(cc)}| = (50 - 180) \) MeV, based on the OPE, large-\( N_c \) approach and QCD low energy theorems [14]. It was claimed in [14, 27] that \( |f_{\eta'(cc)}| \sim 140 \) MeV is needed in order to exhaust the CLEO observation of \( B^\pm \rightarrow \eta' K^\pm \) and \( B \rightarrow \eta' + X \) by the mechanism \( b \rightarrow \bar{c}c + s \rightarrow \eta' + s \) via gluon exchanges. However, a large value of \( f_{\eta'(cc)} \) seems to be ruled out for three reasons. First, the decay constant \( f_{\eta'(uu)}^{(\text{fin})} \) is only of order 50 MeV. Second, suppose the pseudoscalar content of \( \bar{c}c \) is dominated by the \( \eta_c \). Then from the data of \( J/\psi \rightarrow \eta_c \gamma \) and \( J/\psi \rightarrow \eta' \gamma \), one can show that \( |f_{\eta'(cc)}^{(\text{fin})}| \geq 6 \) MeV, where the lower bound corresponds to the nonrelativistic quark estimate. (When the relativistic effect of the \( \eta' \) in \( J/\psi \rightarrow \eta' \gamma \) is taken into account, \( |f_{\eta'(cc)}^{(\text{fin})}| \) is larger than 6 MeV.) Even when contributions from e.g., \( \eta'_c, \eta''_c, \cdots \) to the \( \bar{c}c \) are included, it is argued in [28] that \( |f_{\eta'(cc)}^{(\text{fin})}| \leq 40 \) MeV. Third, based on the \( \eta' \) and \( \eta' \) transition form factor data, the range of allowed \( f_{\eta'(cc)}^{(\text{fin})} \) is estimated...
Table I. Averaged branching ratios for charmless $B$ decays to $\eta'$ and $\eta$, where “Tree” refers to branching ratios from tree diagrams only, “Tree+QCD” from tree and QCD penguin diagrams, and “Full” denotes full contributions from tree, QCD and electroweak (EW) penguin diagrams in conjunction with contributions from the process $c\bar{c} \rightarrow \eta_0$. Predictions are for $k^2 = m_{b^*}^2/2$, $N_c^{\text{eff}} = 2$, $f^{(cc)}_{\eta'} = -15$ MeV, $\eta = 0.35$, $\rho = 0.08$ (the first number in parentheses) and $\eta = 0.34$, $\rho = -0.12$ (the second number in parentheses).

| Decay               | Tree     | Tree+QCD | Tree+QCD+EW | Full      | Exp. [1]  |
|---------------------|----------|----------|-------------|-----------|-----------|
| $B^+ \rightarrow \eta' K^\pm$ | $1.41 \times 10^{-7}$ | $(4.00, 4.18) \times 10^{-9}$ | $(3.87, 4.04) \times 10^{-9}$ | $(5.48, 5.69) \times 10^{-9}$ | $(7.11^{+0.09}_{-0.10} \pm 0.9) \times 10^{-9}$ |
| $B^+ \rightarrow \eta K^\pm$  | $3.56 \times 10^{-7}$ | $(5.48, 3.37) \times 10^{-7}$ | $(3.40, 3.80) \times 10^{-7}$ | $(5.05, 8.68) \times 10^{-7}$ | $< 0.8 \times 10^{-5}$ |
| $B^+ \rightarrow \eta' K^{*\pm}$ | $2.11 \times 10^{-7}$ | $(2.69, 1.90) \times 10^{-6}$ | $(2.94, 2.11) \times 10^{-6}$ | $(5.90, 3.24) \times 10^{-7}$ | $< 29 \times 10^{-7}$ |
| $B^+ \rightarrow \eta K^{*\pm}$  | $5.25 \times 10^{-7}$ | $(0.92, 1.53) \times 10^{-6}$ | $(1.52, 2.42) \times 10^{-6}$ | $(2.49, 3.70) \times 10^{-6}$ | $< 24 \times 10^{-5}$ |
| $B^+ \rightarrow \eta' \pi^\pm$  | $1.94 \times 10^{-6}$ | $(1.13, 1.06) \times 10^{-5}$ | $(1.12, 1.05) \times 10^{-5}$ | $(1.29, 1.21) \times 10^{-5}$ | $< 4.5 \times 10^{-5}$ |
| $B^+ \rightarrow \eta \pi^\pm$   | $4.93 \times 10^{-6}$ | $(9.57, 6.02) \times 10^{-6}$ | $(9.82, 6.24) \times 10^{-6}$ | $(1.04, 0.67) \times 10^{-5}$ | $< 0.8 \times 10^{-5}$ |
| $B^+ \rightarrow \eta' \rho^\pm$  | $3.95 \times 10^{-6}$ | $(1.08, 1.84) \times 10^{-5}$ | $(1.08, 1.84) \times 10^{-5}$ | $(1.01, 1.71) \times 10^{-5}$ | $(1.19, 1.63) \times 10^{-5}$ |
| $B^+ \rightarrow \eta \rho^\pm$   | $7.92 \times 10^{-6}$ | $(1.21, 1.71) \times 10^{-5}$ | $(1.19, 1.66) \times 10^{-5}$ | $(1.19, 1.63) \times 10^{-5}$ | $(5.33^{+0.8}_{-0.7} \pm 1.2) \times 10^{-5}$ |

| Decay               | Tree     | Tree+QCD | Tree+QCD+EW | Full      | Exp. [1]  |
|---------------------|----------|----------|-------------|-----------|-----------|
| $B_d \rightarrow \eta' K^{0}$ | $5.09 \times 10^{-9}$ | $(3.82, 3.80) \times 10^{-5}$ | $(3.70, 3.67) \times 10^{-5}$ | $(5.22, 5.19) \times 10^{-5}$ | $< 0.8 \times 10^{-5}$ |
| $B_d \rightarrow \eta K^{0}$  | $1.93 \times 10^{-8}$ | $(1.23, 0.77) \times 10^{-7}$ | $(1.97, 3.11) \times 10^{-8}$ | $(2.49, 3.28) \times 10^{-7}$ | $< 4.2 \times 10^{-5}$ |
| $B_d \rightarrow \eta' K^{\ast 0}$ | $3.88 \times 10^{-9}$ | $(2.23, 2.08) \times 10^{-6}$ | $(2.47, 2.32) \times 10^{-6}$ | $(2.44, 2.96) \times 10^{-7}$ | $< 3.3 \times 10^{-5}$ |
| $B_d \rightarrow \eta K^{\ast 0}$  | $1.62 \times 10^{-8}$ | $(5.71, 6.64) \times 10^{-7}$ | $(1.23, 1.36) \times 10^{-6}$ | $(2.26, 2.44) \times 10^{-6}$ | $< 2.2 \times 10^{-5}$ |
| $B_d \rightarrow \eta' \pi^{0}$  | $3.31 \times 10^{-11}$ | $(4.83, 8.60) \times 10^{-6}$ | $(4.63, 6.52) \times 10^{-6}$ | $(5.32, 7.35) \times 10^{-6}$ | $(2.86, 3.83) \times 10^{-6}$ |
| $B_d \rightarrow \eta \pi^{0}$   | $6.70 \times 10^{-9}$ | $(2.71, 3.67) \times 10^{-6}$ | $(2.66, 3.60) \times 10^{-6}$ | $(2.86, 3.83) \times 10^{-6}$ | $(1.79, 2.93) \times 10^{-6}$ |
| $B_d \rightarrow \eta' \rho^{0}$  | $1.21 \times 10^{-7}$ | $(2.07, 3.33) \times 10^{-6}$ | $(2.01, 3.24) \times 10^{-6}$ | $(1.19, 1.89) \times 10^{-6}$ | $< 8.4 \times 10^{-5}$ |
| $B_d \rightarrow \eta \rho^{0}$   | $3.46 \times 10^{-7}$ | $(1.28, 2.20) \times 10^{-6}$ | $(1.16, 1.98) \times 10^{-6}$ | $(1.11, 1.89) \times 10^{-6}$ | $< 8.4 \times 10^{-5}$ |

To be $-65$ MeV $\leq f^{(cc)}_{\eta'} \leq 15$ MeV 29.

From Table I it is clear that for $f^{(cc)}_{\eta'} = -15$ MeV, which is consistent with above-mentioned constraints, the agreement between theory and experiment for $B \rightarrow \eta'K$ is substantially improved in the presence of large charm content in the $\eta'$. We conclude that no new physics is needed to account for the CLEO data of $B \rightarrow \eta'K$. We have also calculated the branching ratios of other exclusive charmless $B$ decays involving $\eta'$ and $\eta$ (see Table I), where use of $f^{(cc)}_{\eta'} = -\tan \theta f^{(cc)}_{\eta'}$ has been made. 3 Three comments are in order. (i) The effect of $c\bar{c}$ conversion into the $\eta'$ contributes destructively to $B \rightarrow \eta'K^*$. Consequently, the branching ratio of $B \rightarrow \eta'K^*$ is suppressed and $B(B \rightarrow \eta'K^*)/B(B \rightarrow \eta K^*) \sim \mathcal{O}(10^{-1})$. If $B \rightarrow \eta'K$ is assumed to be entirely accommodated by any of aforementioned new mechanisms, the decay rate of $B \rightarrow \eta'K^*$ will be predicted to be the same order of magnitude as $B \rightarrow \eta'K$. (ii) Contrary to $B \rightarrow \eta(\eta')K^*$ decays, we see from Table I that $B(B \rightarrow \eta'K)/B(B \rightarrow \eta K) \sim \mathcal{O}(10^2)$ due to the destructive interference in the penguin diagrams of $B \rightarrow \eta K$. (iii) The electroweak penguin effects are in general very small, but they become important for $B \rightarrow \eta K$ and

2In the two mixing angle parametrization scheme given in 28, the decay constant $f^{(cc)}_{\eta}$ is much smaller: $f^{(cc)}_{\eta} = -\tan \theta f^{(cc)}_{\eta'}$ with $\theta = -6^\circ \sim 9^\circ$.

3In 28 we have employed $f^{(cc)}_{\eta'} \sim -50$ MeV. In that case, $B \rightarrow \eta'K^*$ is dominated by the process $c\bar{c} \rightarrow \eta_0$ and its branching ratio is of order $10^{-5}$. 28
$B \rightarrow \eta K^*$ decays due to a large cancellation of QCD penguin contributions in these decay modes.

For $B \rightarrow \eta'(\eta)\pi(\rho)$ decays, the mechanism of $\bar{c}c \rightarrow \eta_b$ is much less dramatic since it does not gain mixing-angle enhancement as in the case of $B \rightarrow \eta'(\eta)K(K^*)$. Their branching ratios are sensitive to the light quark masses $m_u$, $m_d$ and form factors such as $F^{B\pi}_B$. The current experimental limit on the decay $B^\pm \rightarrow \eta\pi^\pm$ puts useful constraints on $m_q$ and $F^{B\pi}_B$. The predicted values presented in Table I are for $m_u(m_b) = 5$ MeV, $m_d(m_b) = 10$ MeV and $F^{B\pi}_B(0) = 0.30$. We find that even a slight increase of $F^{B\pi}_B(0)$ or decrease of $m_q$, say $m_u(m_b) \approx m_d(m_b)/2 \sim 3$ MeV, will make the decay rate of $B^\pm \rightarrow \eta\pi^\pm$ exceeding the present upper bound significantly. We also see that a negative $\rho$, which in turn implies a unitarity triangle $\gamma$ in the range $90^\circ < \gamma < 180^\circ$, is preferred [11]. By contrast, the present experimental value of the ratio $R^2 \equiv \Gamma(B^0 \rightarrow \pi^\pm K^\pm)/\Gamma(B^\pm \rightarrow \pi^\pm K^0)$ favors a positive $\rho$ [4]. Note that a positive $\rho$ is also preferred by the limit on the ratio $\Delta M_s/\Delta M_d$ [30]. Clearly more data of $B^\pm \rightarrow \eta\pi^\pm$ and $R^2$ are needed to pin down the sign of $\rho$.

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