Nonlinear analysis of the efficiency and saturation length of free electron laser with realistic helical wiggler and ion-channel guiding

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Abstract
A nonlinear study of the efficiency, power, and the saturation length of a free-electron laser with realistic helical wiggler and ion-channel guiding using numerical simulation is presented. A set of coupled nonlinear differential equations is derived using the Lorentz force equation for the ensemble of electrons and Maxwell’s equations for radiation waves. To calculate the efficiency, power, and the saturation length of the amplifier, these equations are solved simultaneously using a code developed in this study. By choosing different values of the plasma frequency of ion-channel guiding, the dominant mode for the group I and group II orbits is found. The effect of ion-channel guiding on efficiency and saturation length was studied, and the results show that ion-channel guiding increases efficiency and reduces saturation length. Moreover, the efficiency increases, and the saturation length decrease when the annular electron beam is substituted for a solid electron beam. Finally, the results show that the efficiency of a realistic wiggler free-electron laser is higher than that of an idealized one, while the saturation length is lower.

Keywords Free electron laser · Nonlinear · Efficiency · Power · Saturation

1 Introduction
A free-electron laser (FEL) can be defined as a high-power, continuously tunable source of coherent radiation over a broad spectrum of wavelengths (Takayama and Hiramatsu 1988; Naveen et al. 2022). Ion-channel guiding can be considered as a positive plasma channel that protects the beam’s electrons against the self-repulsive forces. Ion-channel guiding in an FEL is cheaper than axial magnetic guiding and permits the use of higher beam currents than the vacuum limit (Williams 2002; Seo et al. 1989). Also, the transverse beam breakup instability is effectively suppressed by ion-channel guiding (Jha and Kumar 1998). Under Budker conditions, the ion-channel guide is applicable (Budker 1956). (i.e., $n_b / \gamma^2 \ll n_i \ll n_b$)
Ion-channel guiding in an FEL was first proposed by Ramian (1992). Theoretical investigations of electron trajectories, self-fields, and gains in an FEL with ion-channel guiding have been conducted (Jha and Kumar 1996, 2003, 2004, 2006; Mahdizadeh 2018; Esmaeilzadeh et al. 2002). The nonlinear analysis of an FEL with an idealized helical wiggler and axial guide magnetic field was studied by Freund (1983). Also, nonlinear analyses of FELs with realistic helical wiggler and axial guide magnetic fields have been carried out in order to find the optimized efficiency via analysis of efficiency enhancement and multimode analysis (Ganguly and Freund 1985; Freund and Ganguly 1986a, 1986b, 1987; Freund 1988). Nonlinear analyses of an FEL with planar wiggler (Freund et al. 1985; Wang et al. 2006), coaxial hybrid wiggler (Freund et al. 1994), and quadrupole wiggler (Scharer et al. 1988; Chang et al. 1988) have been performed. Recently, the nonlinear analysis of an FEL with idealized helical wiggler and ion-channel guiding, as well as an FEL with idealized planar wiggler and ion-channel guiding, has been performed (Raghavi et al. 2008; Rouhani and Maraghechi 2010). In these recent works, which were performed in the presence of ion-channel guiding, the Budker condition has not been considered.

The purpose of the present article is to perform the nonlinear study of the efficiency, power, and saturation length of an FEL with realistic (three-dimensional) helical wiggler and ion-channel guiding using a simulation code that was developed in this work. An analysis is performed within a Compton (high-gain) scattering regime so that the space-charge fields can be ignored.

A set of 3-D coupled nonlinear differential equations is derived using Maxwell’s equations and the Lorentz force equations. This set of equations, which describes the self-consistent evaluation of an ensemble of electrons and electromagnetic fields, is solved numerically by a developed simulation code. An electron beam is injected adiabatically in the interaction region. An adiabatic injection allows the wiggler field amplitude to grow slowly from zero to a constant level after \( N_w \) wiggler periods.

This paper is organized as follows. In section II, a set of nonlinear differential equations, including electron orbit equations for the ensemble of electrons and Maxwell’s equation for radiation fields, are derived. In section III, the results of numerical simulations for efficiency and power are presented. Finally, a summary and discussion are given in section IV.

## 2 General equations

A realizable (three-dimensional) helical wiggler magnetic field, which is produced by the bifilar current windings, can be described as in Eq. (1) (Diament 1981; Freund and Ganguly 1985; Fajans et al. 1985).

\[
B_w = 2B_w(z)\left\{I_1(\lambda)\hat{e}_r \cos(\chi) - I_1(\lambda)\left[\lambda^{-1}\hat{e}_\theta \sin(\chi) - \hat{e}_z \sin(\chi)\right]\right\}
\]

(1)

In cylindrical coordinates, where \( B_n \) is the amplitude of the wiggler magnetic field, \( k_w \equiv 2\pi/\lambda_w \). \( \lambda_w \) is the wiggler period, \( \chi \equiv \theta - k_w z \), \( I_1 \) and \( I_1' \) are the modified Bessel functions of the first kind (of order 1) and its derivative respectively.

The electrostatic field generated by the ion-channel guiding with positive charge +e and density number \( n_i \) is given in Eq. (2).

\[
\vec{E}_i = 2\pi e n_i (\hat{e}_x x + \hat{e}_y y)
\]

(2)
In order to inject the electron beam adiabatically to the FEL interaction region, the wiggler amplitude \( B_w(z) \) can be written as in Eq. (3):

\[
B_w(z) = \begin{cases} 
  B_w \sin^2 \left( \frac{k_w z}{4N_w} \right) & 0 \leq z \leq N_w \lambda_w \\
  B_w & z > N_w \lambda_w
\end{cases}
\]  

(3)

where \( N_w \) represents the number of wiggler periods.

The fluctuating radiation electromagnetic field is described using the magnetic vector potential \( \delta A \). Since the analysis is performed in the Compton regime, the space-charge effects can be neglected, while the vector potential of the radiation field, \( \delta A \), can be expanded in terms of the orthogonal eigenfunctions of the cylindrical vacuum waveguide. Thus, the vector potential of the radiation field for transverse electric (TE) modes in the Coulomb gauge can be written as in Eq. (4):

\[
\delta \bar{A}(x, t) = \sum_{l=0}^{\infty} \sum_{n=1}^{\infty} \delta A(z) \left[ \frac{l}{k_{ln} r} J_l(\kappa_{ln} r) \hat{e}_r \sin \alpha_l + J'_l(\kappa_{ln} r) \hat{e}_\theta \cos \alpha_l \right]
\]  

(4)

\[
\alpha_l \equiv \int_0^z \kappa(z') + \beta \cos \omega t
\]  

(5)

\( \omega \) is the frequency and \( \kappa(z) \) is the wave number of the radiation field, \( \kappa_{ln} \), \( \kappa_{ln}' \) is zero of \( J'_l(\kappa_{ln}) = 0 \), \( R_g \) is the waveguide radius, and \( J_l \) and \( J'_l \) are regular Bessel functions of the first kind and its derivative.

In order to obtain the nonlinear differential equations of amplitude and the wavenumber of the radiation field, the magnetic vector potential in Eq. (4) is substituted for the Maxwell equations. The method used by Ganguly and Freund is utilized (Ganguly and Freund 1985). As a result, the equation is as presented in Eqs. (6–9):

\[
\frac{d^2}{dz^2} a_{ln} + \left[ \overline{\omega}^2 - \kappa^2 - \kappa_{ln}^2 \right] a_{ln} = \overline{\omega}^2 \beta_{\kappa_{ln}^2} \frac{\beta_{\kappa_{ln}^2}}{(\kappa_{ln}^2 - \beta_{\kappa_{ln}^2}) J_l^2(\kappa_{ln})} \left( \frac{\beta_1 s^{(+)}}{\beta_1} + \frac{\beta_2 R^{(+)}}{\beta_2} \right),
\]

(6)

\[
2\kappa^{-1/2} \frac{d}{dz} \left( \kappa^{-1/2} a_{ln} \right) = \overline{\omega}^2 \beta_{\kappa_{ln}^2} \frac{\beta_{\kappa_{ln}^2}}{(\kappa_{ln}^2 - \beta_{\kappa_{ln}^2}) J_l^2(\kappa_{ln})} \left( \frac{\beta_1 R^{(-)} - \beta_2 S^{(-)}}{\beta_1} \right),
\]

(7)

\[
a_{ln} = \frac{e \delta A_{ln}}{mc^2}, \quad \beta_{\kappa_{ln}^2} \equiv \frac{v_{\kappa_{ln}^2}}{c}, \quad \overline{\omega}^2 = \frac{4\pi e^2 n_b}{k_w mc^2}, \quad \overline{\omega} = \frac{\omega}{ck_w}, \quad \overline{\kappa} = \frac{\kappa}{k_w}, \quad \overline{z} = zk_w,
\]

(8)

\[
S^{(\pm)} \equiv J_{l-1}(\kappa_{ln} r) \sin^{(\pm)}(\xi^{(\pm)}), \quad R^{(\pm)} \equiv J_{l+1}(\kappa_{ln} r) \cos^{(\pm)}(\xi^{(\pm)}),
\]

(9)

\( \beta_1, \beta_2 \) are the normalized transverse components of the electron velocity relative to the wiggler frame, which is defined by basis vectors in Eq. (10).
\begin{align}
\dot{\varepsilon}_1 &= \dot{\varepsilon}_x \cos (k_w z) + \dot{\varepsilon}_y \sin (k_w z) \\
\dot{\varepsilon}_2 &= -\dot{\varepsilon}_x \sin (k_w z) + \dot{\varepsilon}_y \cos (k_w z) \\
\dot{\varepsilon}_3 &= \dot{\varepsilon}_z, \\
\end{align}

In addition, the particle phase relative to the ponderomotive wave is shown in Eqs. (11–12).

\[ \xi^{(\pm)} \equiv \psi_t + (l \pm 1) \chi, \]

\[ \psi_t \equiv \psi_0 + \int_0^z \left( \kappa + k_w - \frac{\omega}{v_z} \right) dz \]

To study the interaction between each mode of the radiation field and the electron beam, the electron orbit equation is required. Thus, the Lorentz force equation is derived for each electron [in the form of Eqs. (13–15)] to specify these equations in the presence of an ion channel, wiggler field, and radiation field:

\begin{align}
\frac{d}{dz} (u_1) &= -\frac{1}{\beta_z} \left\{ \frac{\beta_z}{a_{\ln} [\bar{S}_I^-]} - 2 \bar{K}_{\ln} \beta_2 I_1 (l_{\ln} r) \cos \alpha_l + \beta_z \left( \bar{K}_{\ln} R_l^{(+)} + \bar{K}_I S_I^{(+)} \right) \right\}, \\
\frac{d}{dz} (u_2) &= -\frac{1}{\beta_z} \left\{ \frac{\beta_z}{a_{\ln} [\bar{S}_I^-]} - 2 \bar{K}_{\ln} \beta_2 I_1 (l_{\ln} r) \cos \alpha_l + \beta_z \left( \bar{K}_{\ln} R_l^{(+)} + \bar{K}_I S_I^{(+)} \right) \right\}, \\
\frac{d}{dz} (u_3) &= \frac{1}{\beta_z} \left\{ a_{\ln} [I_0 (l) + I_2 (l) \cos (2 \chi)] - a_{\ln} I_2 (l) \frac{\sin (2 \chi)}{\omega} \right\} - \frac{1}{2} \frac{\beta_z}{a_{\ln} [\bar{K}_{\ln} R_l^{(+)} + \bar{K}_I S_I^{(+)}]},
\end{align}

where \( \bar{u} \equiv \gamma \beta \bar{K}_{\ln} \equiv \frac{\bar{u}_{\ln}}{k_w}, \ a_{\ln} \equiv \frac{\alpha}{\partial_{\ln}}, \bar{\Gamma} \equiv \frac{\Gamma}{k_w}. \) Here \( \bar{u} \) is the momentum-like parameter relative to the frame rotating with the wiggler field.

\[ \Gamma_{\ln} = \frac{d}{dz} (\ln a_{\ln}). \]

In Eq. (16) \( I_{\ln} \) represents the growth rate and \( l, n \) display characteristics of the wave mode. The differential equations governing the transverse electron motion in Cartesian coordinates are in Eqs. (17–19).

\begin{align}
\frac{d}{dz} \bar{x} &= \frac{1}{\beta_z} \left( \beta_1 \cos (\bar{z}) - \beta_2 \sin (\bar{z}) \right), \\
\frac{d}{dz} \bar{y} &= \frac{1}{\beta_z} \left( \beta_1 \sin (\bar{z}) + \beta_2 \cos (\bar{z}) \right)
\end{align}
Equations (6), (7), and (16) for electromagnetic fields and Eqs. (13)–(15) and (17)–(19) for an ensemble of electrons define a set of three-dimensional nonlinear, self-consistent differential equations that qualify the interaction. Maxwell’s equations (5) can be simplified as three first-order differential equations for \( k, a, \ln \Gamma \).

Therefore, \( 6N_T + 3 \) differential equations should be solved via numerical simulation where \( N_T \) is the number of electrons that contributed to the analysis. The power is calculated by integrating the axial component of the Poynting vector into the cross-section of the guide is as Eq. (20) (Ganguly and Ahn 1990; Fares and Mahmoud 2020; Pourali et al. 2018; Song et al. 2020):

\[
P_{\text{out}} = \frac{1}{2} \text{Re} \int (E \times H^*) \cdot \hat{e}_z \, dx \, dy
\]

Also, the efficiency is given by \( \eta = \frac{P_{\text{out}} - P_{\text{in}}}{V_0 I_0} \) where \( V_0, I_0 \) and \( P_{\text{in}} \) are the electron beam voltage, current and input signal power, respectively.

3 Numerical results and discussion

The FEL amplifier’s characterization (efficiency, power, and saturation length) is investigated by simultaneously solving the nonlinear differential equations derived in Sec. II, subject to initial conditions. As a result, a simulation code was developed in this study to solve these equations. The Adams–Moulton predictor/corrector and Runge–Kutta techniques are used in this code. In addition, in order to calculate the averages of the radiation field equations [Eqs. (6)–(7)], the electron beam is divided into identical beamlets using a quasi-static assumption (Li et al. 2010; Sprangle et al. 1980): (i.e. \( \vec{v}_{\text{i}}(z;x_0,y_0,t_0) + 2\pi N/\omega) = \vec{v}_{\text{i}}(z;x_0,y_0,t_0) \) for integer \( N \)). Thus, the study is performed only for one beamlet containing 1000 electrons, and the averages are treated using the Nth-order Gaussian quadrature technique (Ganguly and Freund 1985). Input power, growth rate, and wavenumber are initially set to 10 watts, zero, and \( k_0 \), respectively, where \( k_0 \) is the wavenumber under vacuum conditions: (i.e. \( k_{\text{in}}(z = 0) = \left( \omega^2/c^2 - \kappa_{\text{in}}^2 \right)^{1/2} \)). The electron beam at \( z = 0 \) is assumed to be monoenergetic and continuous, without radiation and with a uniform cross-section. The initial electron distribution is determined using the Gaussian algorithm within the range of \( -\pi \leq \psi_0 \leq \pi, 0 \leq \theta_0 \leq 2\pi, 0 \leq t_0 \leq R_b \).

The wiggler field amplitude and wavelength are assumed to be \( B_w = 1.5 \, \text{KG} \) and \( \lambda_w = 1.2 \, \text{cm} \), respectively. The wiggler amplitude is adiabatically increased from zero to a constant value over 10 wiggler periods [C, see Eq. (30)]. Also, the electron beam with 256 keV of energy, a current of 4000 A, and a radius of 0.14 cm is assumed to propagate through a waveguide with a radius equal to 0.37 cm. The density of the ion channel and electron beam are chosen to satisfy the Budker condition.

In the absence of a radiation field, theoretical studies of electron trajectories in the combined realizable helical wiggler and ion-channel guiding have been carried out by Esmaeilzadeh et al. (2005). Meanwhile, in this article, steady-state orbits, which contain helical trajectories of electrons about the axis of symmetry, have been characterized by Eq. (21):
\[ \vec{v} = v_1 \hat{e}_1 + v_2 \hat{e}_2 + v_3 \hat{e}_3 \] (21)

where \( v_1, v_2, v_3, \lambda \) and \( \chi \) are constants and have been obtained from Eqs. (22–26).

\[ v_1 = v_w = \frac{2\Omega_w v_1^2 I_1(\lambda_0)}{\omega_i^2 \pm 2\Omega_w v_1 I_1(\lambda_0) - v_1^2}, \] (22)

\[ v_2 = 0, \] (23)

\[ v_3 = v_1 = \text{const}, \] (24)

\[ \lambda = \lambda_0 = \pm \frac{v_w}{v_1}, \] (25)

\[ \chi = \pm \frac{\pi}{2}. \] (26)

When \( \omega_i^2 \pm 2\Omega_w v_1 I_1(\lambda_0) - v_1^2 = 0 \) in Eq. (22), a resonance enhancement in transverse electron velocity occurs, and the electron orbits are divided into two groups for different plasma frequencies of ion-channel guiding. Group I and Group II are defined by \( \omega_i \leq v_1 - 2\Omega_w v_1 I_1(\lambda_0) \) and \( \omega_i > v_1 - 2\Omega_w v_1 I_1(\lambda_0) \) respectively. Substituting Eqs. (22), (23), and (24) into \( v_1^2 + v_2^2 + v_3^2 = (1 - \gamma^{-2}) c^2 \) gives an expression for axial velocity, where \( \gamma \) is the relativistic factor.

In Fig. 1, the normalized axial velocity \( \frac{v_1}{c} \) is represented as a function of ion-channel frequency. As shown in the figure, there are two types of orbits. Group I orbits are found for low ion channel densities, and group II orbits occur when the ion channel density is high. The dashed line represents unstable trajectories.

Numerical studies have shown that the efficiency and output power of TM modes are minimal—hence, only the TE modes are considered here. In Fig. 2, the efficiency is plotted as a function of the frequency of injected radiation fields for group I orbits (\( \omega_i = 0.4 \)). Also, TE_{11}, TE_{21}, and TE_{31} find the maximum efficiency and optimum frequency at which the maximum efficiency for different modes. The maximum efficiencies for TE_{11}, TE_{21}, and TE_{31} modes are 38.06%, 26%, and 14.13%, respectively. The efficiencies of other TE modes are so small that they are ignored. TE_{11} is the dominant mode for the group I orbits and has the highest efficiency.

Also, Fig. 3 presents the efficiency as a function of the frequency of injected waves to the system for group II orbits (\( \omega_i = 0.6 \)) and considering different modes (TE_{11}, TE_{21}, and TE_{31}). As shown in the figure, the maximum efficiencies for TE_{11}, TE_{21}, and TE_{31} modes are 43.7%, 26%, and 14.13%, respectively. Again, other TE modes are ignored because of their low efficiency. TE_{11} is also the dominant mode for the group II orbits. Also, the TE_{21} mode has a higher efficiency than the TE_{31} mode, both for group I and group II orbits.

Figure 4 shows the maximum efficiency, saturation length, and optimum frequency as a function of the ion channel frequency \( \omega_i \), thus illustrating the effect of ion-channel guiding on these parameters. In this figure, ion channel frequency varied, and the results for group I and II orbits are presented. In the absence of ion-channel guiding, the maximum efficiency and saturation length are 22.6% and 95, respectively, as shown in Fig. 4a, b.
On the other hand, in the presence of an ion channel (e.g., when $\omega_i = 0.1$), these quantities are 27.7% and 91, respectively. Thus, it is concluded that ion-channel guiding increases efficiency and shortens saturation length. This is because the ion-channel guiding weakens the self-field effects and reduces the energy loss within a system.

Moreover, as shown in Fig. 4a, b, enhancing the ion channel frequency increases the maximum efficiency and decreases the saturation length for group I orbits. However, Fig. 4a, b also show that increasing the ion channel frequency causes a decrease in maximum efficiency and an increase in saturation length for group II orbits. This is because increasing the ion channel frequency for group I orbits and decreasing the ion channel frequency for group II orbits brings the system closer to the resonant region, where $\omega_i^2 \pm 2\Omega_{n, v} I_1(\lambda_0) - v_{||}^2 = 0$, and resonance enhancement occurs in the transverse electron velocity.

Figure 4a also depicts that the maximum efficiency for group II orbits near the resonance region is higher than for group I orbits. Also, group II orbits have a shorter saturation length.

The optimum frequency as a function of ion channel frequency is plotted in Fig. 4c. As shown in the figure, increasing the ion channel frequency increases the optimum frequency for group I orbits, whereas decreasing the ion channel frequency increases the optimum frequency for group II orbits.

The output power for group I orbits ($\omega_i = 0, .2, .4$) and group II orbits ($\omega_i = .6, .8, 1$) as a function of the normalized axial position are shown in Fig. 5. As seen in the figure, the ion channel increases the output power and decreases the saturation length. Also, the output power is increased when the ion channel frequency approaches the resonance region.

So far, it has been assumed that the electron beam is solid. In this section, the effect of the annular electron beam is investigated. Figure 6 plots the efficiency versus normalized axial position for an annular electron beam considering group I and II orbits. The efficiency of a solid beam is also shown for comparison. It is observed that the saturation length of the annular electron beam is shorter and that its efficiency is higher than that of the solid beam. The former result is because of the ratio of the self-magnetic and self-electric field for annular beam and solid beam given by Eq. (27):

$$\frac{B_s^{\text{solid}}}{B_s^{\text{annular}}} = \frac{E_s^{\text{solid}}}{E_s^{\text{annular}}} = \frac{r^2}{r^2 - r^2_{\text{min}}} > 1 \tag{27}$$

It is evident from Eqs. (27) that the self-magnetic and self-electric fields of the annular beam are weaker than those of the solid beam. Therefore, less energy is lost from a system with an annular beam. It follows that an annular beam is more efficient than a solid electron beam and that the saturation length of the annular beam is shorter.

If the radial displacement of electrons from the wiggler’s center is smaller than in the wiggler period (i.e., $r \ll \lambda_w$), the wiggler magnetic field is reduced to zero.

$$B_w = B_w (\hat{e}_x \cos (k_w z) + \hat{e}_y \sin (k_w z)) \tag{28}$$

This indicates the presence of an idealized helical wiggler magnetic field. The efficiency of an FEL with an idealized helical wiggler and ion-channel guiding, both for group I and II orbits, is depicted in Fig. 7. The efficiency of the realistic wiggler is also shown in this figure for comparison. As the figure shows, the maximum efficiency of an FEL with a
realistic wiggler is higher than that with an idealized wiggler; its saturation length is also shorter.

4 Summary and conclusion

In this paper, nonlinear theory and numerical simulation are used to study a free electron laser with a realistic (three-dimensional) helical wiggler and ion-channel guiding. The system of coupled nonlinear differential equations explains the self-consistent evolution of the trajectories of an ensemble of electrons and radiation fields. Space-charge fields have been neglected, and the analysis is valid for the high gain Compton regime.

In this study, the TM modes were ignored because of their small gain and efficiency. An analysis was performed only for TE modes. The results revealed that TE_{11} is the dominant mode both for group I and II orbits. The effect of ion-channel guiding on maximum efficiency and saturation length was also studied. The results demonstrate that ion-channel guiding increases maximum efficiency while decreasing saturation length. Also, near the resonance region for group I and II orbits, ion-channel guiding has a stronger effect than in other regions.

The annular electron beam was more efficient than a solid beam and had a shorter saturation length. A comparison between the realistic helical wiggler and an idealized wiggler showed that the efficiency of the realistic wiggler is higher than that of the idealized one and that its saturation length is shorter.

Fig. 1 The normalized axial velocity as a function of the ion channel frequency. The dash lines indicate unstable orbits
Fig. 2  Variation of efficiency as a function of normalized axial distance $k_w z$ for group I orbits and different TE mods

Fig. 3  Variation of efficiency as a function of normalized axial distance $k_w z$ for group II orbits and different TE mods
Fig. 4  

(a) The maximum efficiency as a function of ion channel frequency $\omega_i$ for group I and II orbits, 

(b) the saturation length as a function of ion channel frequency $\omega_i$ for group I and II orbits, 

(c) the optimum frequency at which the maximum efficiency occurs as a function of ion channel frequency $\omega_i$ for group I and II orbits.

\[ B_i = 1.5\text{KG} \]
\[ \lambda_w = 1.2 \text{ cm} \]
\[ I_b = 4\text{KA} \]
\[ V_b = 256 \text{ Kev} \]
\[ R_b = 14 \text{ cm} \]
\[ R_g = 37 \text{ cm} \]
Fig. 5 The power as a function of axial position: (a) for group I orbit, (b) for group II orbit
Fig. 6 The efficiency as a function of axial position $k_wz$ for solid electron beam and annular electron beam:

(a) for group I orbit, (b) for group II orbit
Fig. 7 The efficiency as a function of axial position $k_w z$ for an idealized helical wiggler (dotted line). The efficiency of the realistic one is also shown for compression (continuous line): (a) for group I orbit, (b) for group II orbit.

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