Dynamical Mass Generation in Graphene by Bicircular Laser Fields

M C Suster¹, J Derlikiewicz¹, K M Kotur¹, F Cajiao Vélez², K Krajewska² and J Z Kamiński²

¹ Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland
² Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland

E-mail: Felipe.Cajiao-Velez@fuw.edu.pl, Katarzyna.Krajewska@fuw.edu.pl, Jerzy.Kaminski@fuw.edu.pl

Abstract. Electron-hole creation in single-layer intrinsic graphene interacting with bicircular laser fields is investigated. Due to the linear dispersion relation near the degeneracy points, the electron and hole dynamics are treated according to the Dirac theory, i.e., it is assumed that the charge carriers behave as ultrarelativistic massless fermions. Of special interest is their dynamical mass generation induced by a THz electromagnetic radiation. The harmonic response from graphene and electron currents induced by the bicircular laser field are also analyzed.

1. Introduction

Graphene is widely considered as one of the most interesting and versatile materials in solid state physics. Since its discovery fifteen years ago [1], this two-dimensional crystal has already promised to revolutionize many fields in science and technology including nanoelectronics [2, 3, 4, 5], hydrogen storage [6, 7, 8, 9, 10], new generation of Li-ion batteries [11, 12, 13, 14], and bioengineering [15, 16, 17, 18], among others. The great interest in graphene is due, in part, to its planar configuration, high flexibility, large surface area, and biocompatibility [13, 15, 16]. Some of its unique properties stem from its electronic configuration and band structure.

Graphene is a zero-gap semiconductor (or zero-overlap semimetal) [19, 20, 21] with a hexagonal Brillouin zone (BZ) [19]. At the vertices of the hexagon in reciprocal space, known as Dirac (or degeneracy) points [22, 23], the energy difference between valence band (VB) and conduction band (CB) vanishes. In the vicinity of those points, electrons show a linear energy dispersion relation for low-energy excitations, \( E_p \approx v_F |p| \), where \( E_p \) is their kinetic energy and \( v_F \) is the Fermi velocity [19, 21, 22]. This is in contrast to standard 3D semiconductors, where the dispersion is parabolic [20]. Note that relativistic electrons exhibit the dispersion relation \( E_p = \sqrt{c^2 p^2 + m_e^2 c^4} \) and, for \( |p| \gg m_e c \), a linear relation \( E_p \approx c |p| \) is recovered. This suggested that charge carriers in graphene, near the Dirac points, behave as ultrarelativistic particles moving at the Fermi velocity [22]. Semenoff [23] demonstrated that in graphene electrons and holes behave as massless fermions and that their dynamics is governed by a Dirac-like equation in 2D+1 dimensions with a characteristic velocity \( v_F \approx c \) [20, 22, 23, 24]. However, if a bandgap \( \Delta \) is opened between CB and VB, charge carriers act as massive fermions with an effective mass \( m^* \) such that \( 2m^* v_F^2 = \Delta \) [25]. As it was also demonstrated in Ref. [23], electrons in heteroatomic...
hexagonal 2D structures, such as boron nitride, are characterized by an effective non-zero mass (see also Ref. [26]). Furthermore, Faisal [27] (see also Ref. [28]) has shown that it is possible to dynamically create a bandgap by exposing graphene to a linearly-polarized THz laser field. Thus, in such case, a nonzero mass ($m^* > 0$) is induced.

When graphene interacts with an oscillatory electric field, periodic electron transitions from the fully-occupied VB to the empty CB can take place. In particular, due to the small bandgap in the vicinity of the degeneracy points, graphene can be modelled as an effective two-level system for low-energy excitations. As charge carriers move in a 2D surface, the actual dispersion relation is conical in the neighborhood of the Dirac points, leading to a hyperbolic, momentum-dependent bandgap [29]. In this system, both electron interband and intraband transitions play important roles in the overall charge dynamics. Interference between electron wavefunctions partially transmitted (or reflected) through the bandgap lead to a phenomenon closely related to the Landau-Zener-Stückelberg (LZS) interferometry in a two-level system [25, 29]. (For a review about the LZS interferometry, we refer the reader to Ref. [30].) In Ref. [25] the time-dependent electron-hole creation (EHC), and the characteristic interference signatures, were studied from two different methods: the adiabatic-impulse model (electron dynamics is governed by repeated adiabatic and non-adiabatic transitions, see also [30]), and the Floquet analysis (for excitations created by a periodic and time-dependent electric field, see the reviews [31, 32]). In Ref. [33] Faisal calculated the transition probability rate due to the absorption of $N$ photons when the driving field is a linearly polarized sinusoidal wave. His analysis showed that, while in the perturbative regime the probability rates decrease rapidly with $N$, the non-perturbative regime is characterized by a plateau followed by a sharp cutoff.

Besides EHC, other nonlinear phenomena such as high-order harmonic generation (HHG) from graphene are of great interest. The first experimental evidence of HHG from single-layer intrinsic graphene (SLIG) was recently reported by Yoshikawa, Tamaya and Tanaka [34]. By directing an elliptically-polarized laser field towards a SLIG, the authors observed up to the 9th harmonic order in the optical response. From the theoretical perspective, Faisal analyzed the HHG from graphene by solving the corresponding Dirac equation under adiabatic conditions [35]. His study was carried out by considering two cases: i) low-energy excitations (called graphinos), where the electron momentum is close to the Dirac points; ii) general excitations, where regions of non-linear energy dispersions are also accounted for. As he has shown, the harmonic spectra in both cases obey different cutoff formulas and the low-energy excitation spectra show series of “revivals and plateaus” [35].

In this paper we will explore two main processes: dynamical electron-hole generation and HHG from SLIG. The driving field is a bicircular, THz laser field of infinite duration. Our treatment is based on the Floquet analysis of the Dirac-like equation governing the evolution of charge carriers. While at the beginning of Sec. 2 we describe the main properties of single-layer graphene, Sec. 2.1 contains a short theoretical description of our derivations. Moreover, in Sec. 2.2 we analyze the electron currents in graphene induced by the laser field. Such field is described in detail in Sec. 2.3. In Sec. 3 we present our numerical results. Conclusions are included in Sec. 4.

Throughout the paper, while we set $\hbar = 1$, we present the electron charge $e = -|e|$ explicitly. Furthermore, $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the Pauli matrices.

2. Theory

Graphene is a honeycomb-like structure with carbon atoms located at the vertices of each hexagon. Such a structure can be represented as a superposition of two equivalent triangular sub-lattices, say A and B (blue and red spheres in the left panel of Fig. 1, respectively) [19, 22, 23] displaced by a vector $d_0$ one from another. In this two-dimensional crystal, each lattice site,
here denoted as \( r_A \) (or \( r_B \)), is given by a (displaced) linear superposition of vectors \( a_1 \) and \( a_2 \),

\[
r_A = N_1 a_1 + N_2 a_2, \quad r_B = N'_1 a_1 + N'_2 a_2 + d_0, \tag{1}
\]

where \( N_1, N_2, N'_1 \) and \( N'_2 \) are integers. The two-dimensional vectors \( a_1 \) and \( a_2 \) are given by \( a_1 = \frac{2a_0}{3}(1, \frac{1}{\sqrt{3}}) \) and \( a_2 = \frac{2a_0}{3}(1, -\frac{1}{\sqrt{3}}) \), with \( a_0 \) being the bond length between carbon atoms (approximately equal to 1.42 Å) [19, 22, 23]. As it was initially demonstrated by Wallace [19], in the framework of the tight-binding model, and later discussed by Semenoff [23], the Brillouin zone of graphene in momentum space is also hexagonal and contains two non-equivalent Dirac points, denoted as \( K \) and \( K' \). The energy gap between VB and CB vanishes there and the energy dispersion is linear. For our further considerations, we make a transformation to the dimensionless coordinate system, defined as \( \kappa \equiv (\kappa_x, \kappa_y) = \frac{\sqrt{3}}{2}a_0(\sqrt{3}k_x, k_y) \). With this notation the \( K \)-points are located at \( \kappa_K = (0, 2\pi/3), (\pi, -\pi/3) \) and \( (-\pi, -\pi/3) \) (blue dots in the right panel of Fig. 1) [22]. On the other hand, the \( K' \) points are found at the remaining vertices of the hexagon, namely \( \kappa_{K'} = (0, -2\pi/3), (\pi, \pi/3) \) and \( (-\pi, \pi/3) \) (red dots in the same figure). The center of the BZ, known as the \( \Gamma \)-point, is located at the origin of coordinates, i.e., \( \kappa_\Gamma = (0, 0) \) (yellow dot).

The energy dispersion relation in graphene, according to the tight-binding model, is given by [19, 22]

\[
E_\pm(\kappa) = \pm \zeta \sqrt{1 + 4 \cos \kappa_y (\cos \kappa_x + \cos \kappa_y)}, \tag{2}
\]

where \( \zeta \) is the hopping parameter between nearest neighbours. Considering the energy dispersion at low excitations, i.e., when the momentum of electron is close to the Dirac points, we write

\[
\kappa_x = -\pi + \frac{3}{2}a_0 p_x, \quad \kappa_y = -\frac{\pi}{3} + \frac{\sqrt{3}}{2}a_0 p_y, \tag{3}
\]

with \( p_x, p_y \ll 1/a_0 \). Hence, after expanding the cosine functions in Eq. (2), we obtain the approximate dispersion relation

\[
E_{\pm}(p) \approx \pm \frac{3}{2}a_0\zeta \sqrt{p_x^2 + p_y^2} = \pm v_F|p|, \tag{4}
\]

where \( 3a_0\zeta/2 = v_F \approx c/300 \) is the Fermi velocity (see, e.g., [33]).

It was first shown by Semenoff in Ref. [23] that the equation governing the temporal evolution of charge carriers in graphene is similar to the Dirac equation for massless fermions (see also Ref. [20]). The two-component spinor describing the evolution of electrons and holes near to the degeneracy points \( K \) (or \( K' \)) is denoted as \( \psi(x, t) \) [or \( \psi'(x, t) \)], and satisfies the 2D+1 (two spatial dimensions plus one temporal) Dirac-type equation for massless particles [22, 25, 33, 36]

\[
i\partial_t \psi(x, t) = v_F \Sigma \cdot \hat{p} \psi(x, t), \tag{5}
\]

where \( x = (x_1, x_2) \) is a two-dimensional vector in position space and \( \Sigma = (\sigma_x, \sigma_y) \). A similar equation is found for the state \( \psi'(x, t) \), with \( \Sigma = -(\sigma_x, \sigma_y) \). Additionally, in Eq. (5), \( \hat{p} = -i\nabla \) is the momentum operator in two dimensions.

Let us assume now that graphene is irradiated by a laser field, defined by the vector potential \( A(x, t) \), which propagates perpendicularly to its surface. It is also assumed that such light field is not particularly strong and the dipole approximation holds, i.e., \( A(t) \equiv A(x, t) \). The influence of the electromagnetic radiation on the behavior of charge carriers can be taken into account in Eq. (5) by means of the Peierls substitution (minimal coupling) \( \hat{p} \rightarrow \hat{p} - eA(t) \) [23]. Hence, we arrive at the following expression [36],

\[
i\partial_t \psi(x, t) = v_F \Sigma \cdot (-i\nabla - eA(t)) \psi(x, t) \equiv \hat{H}(p, t)\psi(x, t). \tag{6}
\]
In the left panel we show the structure of graphene in configuration space. The hexagonal (honeycomb) structure can be obtained from the superposition of two triangular lattices A and B (blue and red spheres, respectively). While the sublattice A is defined by a linear combination of the vectors $a_1$ and $a_2$ [23], the lattice B is obtained by a translation by $d_0 = a_0(−1, 0)$ [see Eq. (1)], where $a_0$ is the C-C bond length. The vectors $d_i$, $i = 0, 1, 2$, connect an atom A with its three nearest neighbours. In the right panel we show the energy dispersion relation $E_\pm(\kappa)$ [Eq. (2)] in reciprocal space. At the center we see the hexagonal BZ for which, at its vertices, $E_+ = E_- = 0$ (Dirac points). Red and blue dots represent the $K$ and $K'$ degeneracy points, respectively. The $\Gamma$-point is marked as a yellow dot.

Here, $\hat{H}(p, t)$ is the total Hamiltonian of the system accounting for the interaction with the electromagnetic radiation. Such interaction has two main effects in graphene: i) excitation from valence to conduction bands leads to electron-hole generation; ii) positive and negative charges are accelerated by the electromagnetic forces. The resulting 2D charge and current densities ($\rho(x, t)$ and $j(x, t)$, respectively) are calculated in a standard way. Namely, we write

$$\rho(x, t) = e\psi^\dagger(x, t)\psi(x, t), \quad j(x, t) = ev_F\psi^\dagger(x, t)\Sigma\psi(x, t),$$

where $\psi^\dagger(x, t)$ is the Hermitian conjugate of the spinor $\psi(x, t)$.

### 2.1. Floquet analysis for charge carriers in graphene

In this paper we assume that the vector potential $A(t)$ is periodic in time with period $T$, namely $A(t + T) = A(t)$. Under such conditions, the Hamiltonian $\hat{H}(p, t)$ is also periodic as $\hat{H}(p, t + T) = \hat{H}(p, t)$; thus the Floquet analysis can be used for our treatment. We restrict our study to the carrier dynamics around the $K$-point. In this case, Eq. (6) can be rewritten as

$$i\partial_\phi\psi(x, \phi) = \frac{1}{p_0} \Sigma \cdot [−i\nabla − eA(\phi)]\psi(x, \phi),$$

where we have used the phase of the laser field $\phi = \omega t$ as the independent variable. Here, $\omega = 2\pi/T$ is the fundamental frequency of oscillations of the electromagnetic radiation and $p_0 = \omega/v_F$ is a characteristic unit of momentum. The function $\psi(x, \phi)$ is normalized over the quantization surface $S$, namely

$$\int_S d^2x \psi^\dagger(x, \phi)\psi(x, \phi) = 1.$$ 

In order to solve Eq. (8), we search for a solution of the type

$$\psi(x, \phi) = \frac{1}{\sqrt{S}} e^{ip \cdot x} \chi(p, \phi),$$
where $\chi(p, \phi)$ is a normalized unknown function, periodic in $\phi$. By inserting Eq. (10) into (8) we arrive at the following equation with periodic coefficients,

$$i\frac{d\chi(p, \phi)}{d\phi} = \frac{1}{p_0} \sum [p - eA(\phi)]\chi(p, \phi).$$

(11)

Thus, according to the Floquet theorem, we can write the two-component function $\chi(p, \phi)$ as

$$\chi(p, \phi) = e^{-i\varepsilon \phi} \chi_0(p, \phi).$$

(12)

$\varepsilon$ represents the dimensionless quasienergy, $\varepsilon = E/\omega$, and $\chi_0(p, \phi)$ is another periodic function of $\phi$ with period $2\pi$. From Eq. (12) we obtain the final homogeneous system

$$i\frac{d\chi_0(p, \phi)}{d\phi} + \left[\left(\varepsilon \hat{I} - \frac{1}{p_0} \sum [p - eA(\phi)]\right)\chi_0(p, \phi) = 0, \right.$$

(13)

which can be solved by standard numerical methods in order to obtain $\varepsilon$ and $\chi_0(p, \phi)$.

2.2. Electron currents and harmonic spectrum

In order to determine the harmonic spectrum we first need to calculate the electron currents generated in graphene due to the action of the electromagnetic radiation. In doing so, and taking into account Eqs. (7) and (10), we find out that the current density $j(p, \phi)$ is given by

$$j(p, \phi) = \frac{ev_F}{S} \chi^\dagger(p, \phi) \Sigma \chi(p, \phi).$$

(14)

In order to proceed, we introduce the Fourier decomposition of the current density,

$$j(p, \phi) = \sum_N j_N(p)e^{-iN\phi},$$

(15)

where $N$ is an integer number. The total current $J(p, \phi)$ is obtained by integrating $j(p, \phi)$ over the quantization surface, which leads to

$$J(p, \phi) = \int_S d^2 x j(p, \phi) = ev_F \chi^\dagger(p, \phi) \Sigma \chi(p, \phi) = \sum_N J_N(p)e^{-iN\phi}.$$  

(16)

Consider now the harmonic emission along a well-defined direction in space $n$. It can be shown, from elementary calculations, that the angle-resolved power spectrum for the $N$-th harmonic order as a function of the electron momentum, $W_N(p)$, is

$$W_N(p) = \frac{d^2W_N(p)}{d^2\Omega_n} = \frac{2(N\omega)^2\mu_0}{16\pi^2\varepsilon c}[J_N(p) \cdot J_N^*(p) - (n \cdot J_N(p))(n \cdot J_N^*(p))],$$

(17)

where $\Omega_n$ is the solid angle and $\mu_0$ is the magnetic permeability of free space. The factor of two accounts for the emission from electrons with two spin states near the Dirac point. It is now convenient to introduce the natural unit of power $W_0 = \omega^2/(2\pi)$, so the dimensionless spectrum reads

$$w_N(p) = \frac{W_N(p)}{W_0} = \alpha^2 \frac{N^2}{c^2} [J_N(p) \cdot J_N^*(p) - (n \cdot J_N(p))(n \cdot J_N^*(p))].$$

(18)

Here, $\alpha$ is the fine structure constant and $J_N = J_N/(ev_F)$. If we assume that the polarization of the resulting radiation is determined by two vectors $\varepsilon_1$ and $\varepsilon_2$ (which satisfy $\varepsilon_1 \times \varepsilon_2 = n$, $\varepsilon_i \cdot \varepsilon_j = \delta_{ij}$ and $\varepsilon_i \cdot n = 0$, for $i, j = 1, 2$), then for the polarization along the $j$ direction we have

$$w_{N,j}(p) = \alpha^2 \frac{N^2}{c^2} |J_{N,j}(p)|^2, \quad \text{with} \quad J_{N,j}(p) = J_N(p) \cdot \varepsilon_j^*.$$  

(19)

This equation allows us to determine the harmonic spectrum from graphene, once the currents $J_N(p)$ are obtained [see Eqs. (14) and (16)]. However, it becomes necessary to specify the form of the vector potential $A(\phi)$, such that $\chi_0(\phi)$ and $\varepsilon$ can be determined numerically from (13).
2.3. Laser field

In the following, we consider a bicircular laser field in the dipole approximation. It is assumed that such field was adiabatically turned on at \( t \to -\infty \) and it will be turned off at \( t \to \infty \). In particular, if the electromagnetic radiation has a frequency in the THz regime, we guarantee that the condition of ‘small excitations’ is satisfied, i.e., that the electron momentum is close to the Dirac point and the energy dispersion is linear. If \( \mathbf{e}_x \) and \( \mathbf{e}_y \) define the plane of polarization, the bicircular field is defined by the vector potential

\[
\mathbf{A}(\phi) = \mathbf{A}_1(\phi) + \mathbf{A}_2(\phi),
\]

(20)

with

\[
\mathbf{A}_1(\phi) = A_{10} \left[ \cos(N_1 \phi) \cos(\delta_1) \mathbf{e}_x + \sin(N_1 \phi) \sin(\delta_1) \mathbf{e}_y \right],
\]

(21)

\[
\mathbf{A}_2(\phi) = A_{20} \left[ \cos(N_2 \phi) \cos(\delta_2) \mathbf{e}_x + \sin(N_2 \phi) \sin(\delta_2) \mathbf{e}_y \right].
\]

(22)

Here, \( A_{10} \) and \( A_{20} \) are the amplitudes of each color, \( N_1 \) and \( N_2 \) are integers which define their relative frequencies, and \( \delta_1 \) and \( \delta_2 \) determine their polarization properties. In particular, for the bicircular radiation we use \( \delta_1 = \pi/4 \) and \( \delta_2 = \pm \pi/4 \) (positive for co-rotating and negative for counter-rotating components). For simplicity we also introduce here the dimensionless vector potential \( \mathbf{A}(\phi) \equiv -e\mathbf{A}(\phi)/p_0 \) and the corresponding dimensionless electric field \( \mathbf{E}(\phi)/E_G = -\partial_\phi \mathbf{A}(\phi) \), where \( E_G = -\omega p_0/e = \omega^2/(|e|v_F) \).

In our further calculations, we shall consider the bicircular laser field characterized by the integers \( N_1 = 1 \) and \( N_2 = 2 \). While the amplitude of the first component is \( A_{10} = 6p_0/|e| \), we set \( A_{20} = 3p_0/|e| \). The polarization is circular with counter-rotating components, meaning that \( \delta_1 = \pi/4 \) and \( \delta_2 = -\pi/4 \) [see Eqs. (20)-(22)]. The projection onto the \( xy \)-plane of the dimensionless vector potential \( \mathbf{A}(\phi) \) for such parameters is plotted in the left panel of Fig. 2. Furthermore, the corresponding dimensionless electric field \( \mathbf{E}(\phi)/E_G \) is shown in the right panel of the same figure. In obtaining those plots, the phase \( \phi \) changes from 0 to \( 2\pi \).
Figure 3. Dimensionless quasienergies $\varepsilon = E/\omega$, modulo 1, $(0 \leq \varepsilon < 1)$ obtained from the numerical solution of Eq. (13), as a function of $\mathbf{p}$. The momentum is calculated with respect (and in the vicinity) to the $K$ point, hence $p_x = p_y = 0$ defines the vertex of the unperturbed Dirac cone. While the left panel relates to the field-free case ($\mathbf{A}(\phi) = 0$), the right panel shows the quasienergies when graphene interacts with the bicircular laser field presented in Fig. 2.

3. Results

For our numerical illustrations, we have calculated the dimensionless quasienergies $\varepsilon = E/\omega$ as a function of the two dimensional momentum $\mathbf{p} = (p_x, p_y)$. This is done by solving numerically Eq. (13) for the laser field defined in Fig. 2. As it can be seen from the right panel of Fig. 3, the bicircular, counter-rotating laser field generates a gap opening at the position of the $K$ point in momentum space (i.e., at $\mathbf{p} = \mathbf{0}$). In the absence of the electromagnetic radiation (left panel of the same figure), valence and conduction bands meet at the degeneracy points. A similar effect was reported by Faisal in Ref. [27] while analyzing the electron dynamics in graphene irradiated by a linearly-polarized THz radiation. Moreover, in our calculations, 3 cone-type structures with non-vanishing gap are created for non-zero momenta. This means that charge carriers in the vicinity of those cones acquire dynamically a finite effective mass.

In Fig. 4 we present the squared harmonic density currents multiplied by $N^2$, i.e., we show $|N j_{N,x}|^2 + |N j_{N,y}|^2 = N^2(|\mathbf{j}_N \cdot \mathbf{e}_x|^2 + |\mathbf{j}_N \cdot \mathbf{e}_y|^2)$ as a function of the momentum $\mathbf{p}$ (logarithmic scale). Those quantities, calculated in the vicinity of the $K$ point, are shown for different harmonic orders $N$. The driving field is as described in Fig. 2, i.e., it is a counter-rotating bicircular laser field polarized in the $xy$-plane. In Fig. 4 we see that the induced currents present strong interference fringes. At low harmonic orders, such fringes are more intense in the vicinity of the Dirac point $(p_x = p_y = 0)$. However, from $N = 20$ up to $N = 45$ we notice that the dominant currents are further away from the $\mathbf{p} = \mathbf{0}$ region, i.e., electron transitions through the vertex of the Dirac cone contribute marginally to the harmonic emission. In contrast, for $N = 5$ to $N = 15$, small-momenta contributions are dominant.

The general shape of the harmonic currents can be understood intuitively by taking into account that the interaction with the laser field changes the electron momenta as $\mathbf{p} \rightarrow \mathbf{p} - e \mathbf{A}(\phi)$. Thus, it is expected that the currents flow along a trajectory (in momentum space) similar to the curve $-|e| \mathbf{A}(\phi)$ (analogously to the ionization spiral discussed in [37, 38, 39]). This is indeed the case, as for $N = 10$ up to $N = 45$ the most intense currents are found around a triangular star-like structure, which resembles the shape of $-\mathbf{A}(\phi)$, shown in the left panel of Fig. 2.

By looking at the first two rows of Fig. 4, we see that the currents are rather intense up to $N = 25$, before decreasing rapidly with the harmonic order. Beyond the 35-th harmonic an even
Figure 4. Harmonic current density squared times $N^2$ as a function of the two dimensional momentum near the Dirac $K$ point. Such currents are induced in graphene due to the interaction with the bicircular laser pulse described in Fig. 2. Each plot corresponds to a different harmonic order, as identified in the upper left corner of each panel.

Faster reduction is observed. In order to analyze this dependence, in Fig. 5 we plot the harmonic spectrum integrated over momentum. Namely, we show

$$W_N = \int d^2p W_N(p),$$

as a function of $N$. Here, $W_N(p)$ is the momentum-resolved harmonic spectrum [Eq. (17)]. In this figure we see that the maximum signal is reached at $N \approx 20$. Higher harmonic orders are emitted with decreasing power up to $N \approx 35$. There, a cutoff marks the beginning of a new region where the signal reduction is faster with increasing $N$.

4. Conclusions
By means of the Floquet analysis, we studied the interaction of graphene with bicircular laser fields of frequencies in the THz regime. As we have shown, the electromagnetic radiation creates a bandgap at the position of the Dirac points. This is equivalent to a dynamical mass generation for charge carriers, i.e., electrons and holes behave as massive fermions instead of massless particles.
Figure 5. Harmonic spectrum $W_N$ as a function of the harmonic order $N$ [Eq. (23)] from graphene irradiated by the THz laser pulse described in Fig. 2. While each point represents the power emitted by the $N$-th harmonic, the thin blue line describes the general trend. The data are scaled to their maximum value and presented in the logarithmic scale.

With respect to HHG we have analyzed the electric currents generated in graphene by the action of the laser field and the resulting harmonic emission. Such currents present intense interference effects while following trajectories close to $-|e|A(\phi)$ in momentum space. The resulting harmonic spectrum is characterized by a semi-plateau with non-constant envelope before a cutoff.

Acknowledgments

F C V, K K, and J Z K acknowledge the support by the National Science Centre (Poland) under Grant No. 2018/31/B/ST2/01251.

References

[1] Novoselov K S, Geim A K, Morozov S V, Jiang D, Zhang Y, Dubonos S V, Grigorieva I V and Firsov A A 2004 Science 306 666
[2] Berger C, Song Z, Li T, Li X, Ogbazghi Y O, Feng R, Dai Z, Marchenkov N, Conrad E H, First P N and de Heer W A 2004 J. Phys. Chem. B 108 19912
[3] Berger C, Song Z, Li X, Wu X, Brown N, Naud C, Mayou D, Li T, Hass J, Marchenkov A N, Conrad E H, First P N and de Heer W A 2006 Science 312 1191
[4] Xuan Y, Wu Y Q, Shen T, Qi M, Capano M A, Cooper J A and Ye P D 2008 Appl. Phys. Lett. 92 013101
[5] Westervelt R M 2008 Science 320 324
[6] Cabria I, López M J and Alonso J A 2005 J. Chem. Phys. 123 204721
[7] Ataca C, Aktürk E, Ciraci S and Ustunel H 2008 Appl. Phys. Lett. 93 043123
[8] Dimitrakakis G K, Tylanakis E and Froudakis G E 2008 Nano Lett. 8 3166
[9] Liu W, Zhao Y H, Nguyen J, Li Y, Jiang Q and Lavernia E J 2009 Carbon 47 3452
[10] Du A, Zhu Z and Smith C S 2010 J. Am. Chem. Soc. 132 2876
[11] Yoo E J, Kim J, Hosono E, Zhou H-S, Kudo T and Honma I 2008 Nano Lett. 8 2277
[12] Wang G, Shen X, Yao J and Park J 2009 Carbon 47 2049
[13] Wu Z-S, Ren W, Xu L, Li F and Cheng H-M 2011 ACS Nano 5 5463
[14] Li N, Chen Z, Ren W, Li F and Cheng H-M 2012 Proc. Natl. Acad. Sci. USA 109 17360
[15] Zhang H, Grüner G and Zhao Y 2013 J. Mater. Chem. B 1 2542
[16] Shin S R, Li Y-C, Jang H L, Khoshakhligh P, Akbari M, Nasajpour A, Zhang Y S, Tamayo A and Khademhosseini A 2016 Adv. Drug Del. Rev. 105 255
[17] Dasari Shareena T P, McShan D, Dasmahapatra A K, and Tchounwou P B 2018 Nano-Micro Lett. 10 53
[18] Hajian R et. al. 2019 Nat. Biomed. Eng. 3 427
[19] Wallace P R 1947 Phys. Rev. 71 622
[20] Katsnelson M I and Novoselov K S 2007 Solid State Commun. 143 3
[21] Novoselov K S 2009 ECS Trans. 19 3
[22] Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 Rev. Mod. Phys. 81 109
[23] Semenoff G W 1984 Phys. Rev. Lett. 53 2449
[24] Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Nature 438 197
[25] Fillion-Gourdeau F, Gagnon D, Lefebvre C and MacLean S 2016 Phys. Rev. B 94 125423
[26] Semenoff G W 2012 Phys. Scr. T146 014016
[27] Faisal F H M 2017 Mol. Phys. 115 1768
[28] Faisal F H M and Kamiński J Z 1997 Phys. Rev. A 56 748
[29] Higuchi T, Heide C, Ullmann K, Weber H B and Hommelhoff P 2017 Nature 550 224
[30] Shevchenko S N, Ashhab S and Nori F 2010 Phys. Rep. 492 1
[31] Potvliege R M and Shakeshaft R 1992 Atoms in Intense Laser Fields, ed Gavrila M (Boston: Academic)
[32] Chu S-I and Telenov D A 2004 Phys. Rep 390 1
[33] Faisal F H M 2013 Ann. Phys. (Berlin) 525 171
[34] Yoshikawa N, Tamaya T and Tanaka K 2017 Science 356 736
[35] Faisal F H M 2011 J. Phys. B: At. Mol. Opt. Phys. 44 111001
[36] Fillion-Gourdeau F and MacLean S 2015 Phys. Rev. B 92 035401
[37] Kamiński J Z, Cajiao Vélez F and Krajewska K 2017 Laser Phys. Lett. 14 075301
[38] Krajewska K, Cajiao Vélez F and Kamiński J Z 2017 Europhys. Lett. 119 13001
[39] Cajiao Vélez F, Krajewska K and Kamiński J Z 2018 Phys. Rev. A 97 043421