Possibilities of application of the swirling flows in cooling systems of laser mirrors

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Abstract. The paper presents analytical investigations into advanced cooling systems of the laser mirrors with heat exchange intensification by methods of ordered vortex impact on a coolant flow structure. Advantages and effectiveness of the proposed cooling systems have been estimated to reduction displacement of an optical mirror surface due to a flexure.

1. Introduction
Cooled laser mirrors consist of a thin substrate, a cooling system and a thick base. The task of the laser mirror is to reflect as much as possible the incident radiation and minimize the distortions caused by the heating of the mirror. Thermal expansion and bending movement of the optical surface depend on the temperature level and its distribution over the thickness of the mirror. The task of the cooling system is to reduce the overtemperature level and to protect the mirror base from the penetration of heat into it. The cooling system is characterized by two integral parameters: the reduced heat-transfer factor αpr taken to a heat exchange surface of a substrate and "thermal insulation coefficient" of base Kti – the relation of excess temperatures of heat exchange surfaces of a base and a substrate. For improvement of mirror operability it is necessary to maximize αpr and to minimize Kti, but these requirements are contradictory. The value of αpr is influenced by both the surface heat-transfer coefficient and the degree of finning of the surface of the cooling system. For increase in αpr it is necessary to intensify the heat transfer so that its growth would exceed hydroresistance growth. One of the effective intensification methods is impact on structure of the heat carrier flow by well-ordered vortex formation.

In [1] it is shown that for channel cooling systems of laser mirrors (at Biot's numbers of Bi=α0d⌘/λ<1, where d⌘ - the hydraulic diameter of the channel) the value of the reduced heat transfer coefficient αpr and the temperature distribution over the mirror thickness are practically independent of the relations coefficients of heat transfer on finned α0 and unfinned α surfaces of the cooling system (within 0.7<α/α0<1.5), but is determined only by the average coefficient of heat transfer. In [2] the results of an experimental study of the hydraulic resistance and heat transfer are presented in the trial application of some methods of vortex intensification of heat transfer in cooling systems of laser mirrors.

In this work perspective cooling systems with heat transfer intensification were analyzed, and the advantages and efficiency of the proposed cooling systems are estimated. Estimated calculations were performed using programs written in mathematical editor Mathcad.
2. Results of estimated calculations of the proposed cooling systems

The cooling systems taken for the calculated analysis are schematically given in figure 1. In comparison with the models of cooling systems [2] tested earlier it is offered:

a) make smaller the structure of the cooling system following the path of reducing the hydraulic diameter and the increase of the heat exchange surface (compactness of the system);

b) to use the processes of vortex formation intensifying heat transfer in the energetically favorable region of Reynolds numbers (as a rule, this is the transient flow and the initial development of the turbulent flow);

c) to take for the comparative variant of the porosity-optimized cooling system, which ensures maximum $\alpha_p$ and "to subject" its intensification of heat transfer.

For the calculations of the hydraulic resistance and heat transfer both literature data and our results [2] were taken.

![Figure 1. Schemes of the studied cooling systems: a) the slit channel with spiral tapes; b) the crossing (coplanar) channels [3] - $2\beta$ - angle of the relative crossing of channels; $h_h=h_k$ - height of the rib, channel; $a=\delta_k$ - width of the channel; $\delta_r$ - thickness of the rib; $w$ – heat carrier speed; $q$ – specific heat flux; c) a spring insert and a spiral tape in a square channel; d) corrugation with holes.](image)

2.1. Slit channels with spiral tapes

In the case of technological impossibility of obtaining narrow high channels with thin fins, it is proposed to increase the surface heat transfer of the finned surface by installation to the rectangular canal of several spiral tapes on height of the channel. Calculations were made for two cases of slotted channels made in copper ($\lambda=385$ W·m$^{-1}$·K$^{-1}$):

1) in the channel with a width and height of 1.5×3.0 mm$^2$ and a rib thickness of 0.5 mm are inserted at each other two spiral tapes from a copper strip 0.25 mm thick with two different steps - 12.5 and 6.25 mm, which are placed along the entire length of the channel;

2) in the channel 1.5×4.5 mm$^2$ with rib thickness of 0.5 mm are inserted at each other two spiral tapes identical stated above.

For the two cases considered, the porosity of the initial cooling system was optimized in the Reynolds number range $Re=(2\cdot10^3)$ and was $\varepsilon=0.75$. At the same time, the coefficient of heat transfer intensification ($K_{in}=\alpha_{pr}/\alpha_0$) was: 1) for the first case $K_{in}=4.0$ ($Re=2\cdot10^3$) and $K_{in}=2.7$ ($Re=1\cdot10^4$); 2) for the second case $K_{in}=4.8$ ($Re=2\cdot10^3$) and $K_{in}=2.9$ ($Re=1\cdot10^4$). The insert of tapes leads to double decrease of $d_\varepsilon$, but at the same time the tape surface practically does not develop the heat exchange area of fins.

For calculated estimates of hydraulic resistance and heat transfer in the channel with tapes are used:

a) our results [2] obtained on a square channel of 2.65×2.65 mm$^2$ with an inserted spiral tape; b) literary data for round channels [4, 5].

For the first case and a step of the tape $t=12.5$ mm increase in relation to the smooth channel is received: 1) hydraulic resistance from 2 to 1.6 times ($Re=2\cdot10^3$); 2) $\alpha_{pr}$ from 1.4 to 1.1 times; 3)
\( \alpha_0 \) from 1.7 to 1.36 times; 4) the energy efficiency calculated from the ratio of the increase in the relative reduced heat transfer to the increase in the relative hydraulic resistance was less than 1 (0.44-0.71); 5) reduction in the thermal insulation coefficient base 1.09-1.16 times. The coefficient of thermal insulation in the channel without tapes varied from 0.68 \((Re=2 \times 10^3)\) to 0.36 \((Re=1 \times 10^4)\).

For the first case and a step of the tape \( t=6.25 \) mm increase in relation to the smooth channel is received: 1) hydraulic resistance from 3.2 to 2.3 times \((Re=(2-10) \times 10^3)\); 2) \( \alpha_{pr} \) from 1.6 to 1.2 times; 3) \( \alpha_0 \) from 2.3 to 1.7 times; 4) the energy efficiency was less than 1 \((\sim 0.5)\); 5) reduction in the thermal insulation coefficient base 1.26-1.44 times.

For the second case and a step of the tape \( t=12.5 \) mm increase in relation to the smooth channel is received: 1) hydraulic resistance from 2.9 to 1.55 times \((Re=(2-10) \times 10^3)\); 2) \( \alpha_{pr} \) from 1.36 to 1.14 times; 3) \( \alpha_0 \) from 1.9 to 1.53 times; 4) the energy efficiency was less than 1 \((0.47-0.73)\); 5) reduction in the thermal insulation coefficient base 1.27-1.52 times. The coefficient of thermal insulation in the channel without tapes varied from 0.53 \((Re=2 \times 10^3)\) to 0.213 \((Re=1 \times 10^3)\).

For the second case and a step of the tape \( t=6.25 \) mm increase in relation to the smooth channel is received: 1) hydraulic resistance from 3.2 to 2.3 times \((Re=(2-10) \times 10^3)\); 2) \( \alpha_{pr} \) from 1.54 to 1.24 times; 3) \( \alpha_0 \) from 2.6 to 1.9 times; 4) the energy efficiency was less than 1 \((0.48-0.54)\); 5) reduction in the thermal insulation coefficient base 1.62-2.13 times.

The calculated increase in the surface heat transfer coefficient was 1.5-2.6 times. In this case, the maximum relative increase of \( \alpha_{pr} \) with decreasing Reynolds number decreased from 1.7 to 1.14.

2.2. Cooling system with coplanar (crossing) channels

An attempt was made to construct an optimal cooling system with a perpendicular arrangement of channels (coplanar) square channels, a kind of a two-storeyed cooling system. As a basis channels of square section \( 1.5 \times 1.5 \) mm\(^2\) were assumed made in a copper blank with 2.1 mm pitch (thickness of a rib \( \delta_s=0.6 \) mm, porosity \( \varepsilon=0.75 \), \( d_g=1.5 \) mm). As one of the limiting cases of crossing of channels (intersection angle \( 2\beta=0^\circ \)) the system with the doubled height of channels (the size of channel \( 1.5 \times 3.0 \) mm\(^2\), \( d_g=2.0 \) mm) is considered. Channels in this case are "stacked" on each other, a continuous rib. The case of mutually perpendicular crossed channels \((2\beta=90^\circ)\) was also calculated.

In the calculations of a single square channel and a "double" channel, formulas were used for the hydraulic resistance and heat transfer in smooth channels for laminar and turbulent flow of water at room temperature in them:

- for a laminar flow regime \((Re<2300)\)

\[
\xi = \frac{A}{Re}, \quad (A=64 \text{ for } h_b/\delta_b=1; \ A=62 \text{ for } h_b/\delta_b=2), \quad \text{Nu}_{f} = 1.4 \text{Pr}^{0.33} \left( \frac{Re}{L} \right). \tag{1}
\]

- for turbulent flow regime \((Re>2300)\)

\[
\xi = 0.316/Re^{0.25}, \quad \text{Nu}_{f} = 0.021Re^{0.8} \text{Pr}^{0.43}. \tag{2}
\]

For crossed channels, generalizations of other researchers using generalized geometric characteristics for hydraulic tracts have been used \([3, 6, 7]\). In \([3]\) to hydraulic resistance and heat transfer in the turbulent flow regime proposed the following formulas:

\[
\xi = \left( \frac{2512}{Re^{1.32}} \right)^{+} + 0.137, \quad \text{Nu} = 0.16 Re^{0.68} \text{Pr}^{0.4}. \tag{3}
\]

In \([6,7]\) as the characteristic dimension at generalization of results the equivalent diameter of \( d_e=V_h/F_b \) is accepted (where \( V_h \) – the volume occupied by the heat carrier in a path, \( F_b \) – the heat exchange surface of the hydraulic path). In relation to our configuration formulas will take a form:
\[ V_b = \delta_k \left( \frac{\delta_k - \delta_h}{\delta_k + \delta_h} \right) \left[ 2h_k \left( 1 - \frac{\delta_h}{\delta_k + \delta_h} \right) \right], F_b = 2\delta_k \left( \frac{\delta_k - \delta_h}{\delta_k + \delta_h} \right) \left[ 1 + \frac{2h_k}{(\delta_k + \delta_h)^2} + \frac{2h_i\delta_k}{(\delta_k + \delta_h)^2} \right] \]  

and the equivalent diameter is equal \( d_b = 1.44 \text{ mm} \).

For hydraulic resistance and heat transfer in the turbulent flow regime in the [6, 7] was proposed the following formulas:

- for the coefficient of hydraulic resistance:
  \[ \xi = B/Re^n, \]  
  where \( n = H/(6S) + 1.8\beta^2 - 2\beta + 0.55, \quad B = 3.65H/S + 150\beta^2 - 120\beta + 21.15, \quad H = h_{k1} + h_{k2} = 2h_k, \quad S = \delta_i + \delta_h \) for \( 2\beta = 90^\circ \) (\( \beta = 0.5 \)).

- for surface heat transfer:
  \[ \text{Nu}_e = \left( 0.24 - 0.185 \frac{H}{S} \right) \text{Re}^{0.16} \text{Pr}^{0.62}. \]  

and in our case \( \text{Nu}_e = 0.07 \text{Re}^{0.765} \).

Also for calculation of the surface heat transfer the formula for heat exchange of a turbulent flow in short channels [8] was used. As applied to our conditions it was transformed to the form:

\[ \text{Nu}_{k\delta} = 0.22 \cdot 1.57 \left( \frac{\delta_i}{d_\delta} \right)^{-0.12} \text{Re}^{0.8} \text{Pr}^{0.43}. \]

Earlier we in [2] received the following formula for reduced and surface heat transfer of coplanar channels with sizes \( \delta_i \times h_k = 1.5 \times 1.0 \text{ mm}^2 \) which are crossed at an angle \( 2\beta = 120^\circ \), porosity of a cooling system \( \varepsilon = 0.5 \):

\[ \alpha_{\text{pre}} = 587.8 \text{Re}^{0.594}, \quad \alpha_c = 63 \text{Re}^{0.7}. \]  

Our data on surface heat transfer were closer to the data obtained in [3, 8].

The results of the calculations on the resistivity showed that the relative excess of the resistance in the developing turbulent flow regime \( \text{Re} = (2.3-10) \cdot 10^3 \) is \( \sim 5 \) times for [3], 13 times for [6], and is practically independent of the Reynolds number.

The data on surface heat transfer calculations showed that the relative excess of heat transfer in the regime of developing turbulent flow \( \text{Re} = (2.3-10) \cdot 10^3 \) at application the formulas of different authors is 1.5 to 3 times and decreases with increasing \( \text{Re} \).

In the future, in order to find the best energy efficiency of the system (within a certain range of Reynolds numbers) different crossing angles, and dimensions of the channels, the porosity and the material can be chosen for calculations. To calculate the thermal and hydraulic characteristics of the systems can be recommended the calculated dependences given in [3, 8].

### 2.3. Spring insert and spiral tape in a square channel

In this subsection on the basis of ours [2] and literary data [5] calculations and comparisons of the springs and spiral tapes installed in the square channel are carried out. In calculations, the diameter of the spring and its step, as well as the step of the tape and the dimensions of the channel, were varied.

At the beginning, to compare the spiral and spring swirls, a square-section channel was taken with a spring inserted into it [2], for which there were data on the hydro resistance and heat transfer. A spring with an external diameter \( D_n = 3.5 \text{ mm} \) was wound from a nichrome wire \( [\lambda = 13.4 \text{ W m}^{-1} \text{K}^{-1}] \) with a diameter \( d_{sp} = 0.5 \text{ mm} \) with step \( h = 1.0-1.1 \text{ mm} \). It is inserted into the cooling system model with square shaped channels \( \delta_i = h_i = 3.5 \text{ mm} \) chopped cutter with step \( t = \delta_i + \delta = 4.4 \text{ mm} \) in the copper blank, and acted as an intensifying element. In the calculations it was assumed that a spiral tape was inserted...
into such a square channel with the maximum possible degree of its twist (step \(H=6.25 \text{ mm}\)). This case corresponded to one of the experimental cases considered in [2], with a square channel size of 2.65 mm.

Experimental and calculated data showed that the hydraulic resistance of the channel with a spring is 16-26 times greater than the channel resistance without a spring (in the interesting range of Reynolds numbers \(\text{Re}=(1-10)\times10^3\), \(\alpha_{pr}\) is 2-3 times more, and the energy efficiency is at the level of \(\eta=0.14-0.1\). The hydraulic resistance of the channel with the tape at the step \(H=12.5 \text{ mm}\) is 2-1.5 times greater than the resistance of the channel without the tape (in the range of numbers \(\text{Re}=(2.3-10)\times10^3\), \(\alpha_{pr}\) is 1.1-0.82 times more, and the energy efficiency is at the level \(\eta=0.56-0.53\). The hydraulic resistance of the channel with the tape at the step \(H=6.25 \text{ mm}\) is 3.1-2.3 times greater than the resistance of the channel without the tape (in the range of numbers \(\text{Re}=(2.3-10)\times10^3\), \(\alpha_{pr}\) is 1.4-1.0 times more, and the energy efficiency is at the level \(\eta=0.45-0.43\).

A case of a smaller channel with insertion of a tape and a spring was also considered. In an optimized square channel of \(2\times2 \text{ mm}^2\) (\(\delta_s=0.5 \text{ mm}, \epsilon=0.8, d_s=2 \text{ mm}\), a spring "was inserted" from the wire \(d_{ps}=0.2 \text{ mm}\) and a step \(t=1.5 \text{ mm}\). The increase in the surface heat transfer coefficient \(K_{ps}\) was estimated from the formula [5] proposed for the turbulent flow regime in round tubes:

\[
K_{pr} = 1.85 + 2.5 \left( \frac{2d_{ps} / d_s}{H/H_s} \right) \left( \frac{0.85 + 2.5 \left( 2d_{ps} / d_s \right)}{2.8 + 12.6 \left( 2d_{ps} / d_s \right)} \right) t / d_s. \tag{9}
\]

For our case \(K_{pr}=2.16\) takes place. At the same time the increase in the reduced heat transfer in comparison with the channel without insert at 3.4-3.1 times was observed.

For a case when the spiral tape 0.3 mm thick "was inserted" into the same channel diameter of \(D_s=\delta_s=2 \text{ mm}\), and step of \(H=4.7 \text{ mm}\) (at this \(d_s=1.13 \text{ mm}\), and the relative step of the tape \(2\pi H/D_s\) corresponds to one of experiments in [2]). For estimates of hydraulic resistance and the surface heat transfer the formulas [5] proposed for turbulent flow regime in circular pipes were used. Calculations at change of Reynolds numbers in the range of \(\text{Re}=(2.3-10)\times10^3\) established excess of heat transfer:

- channel with a spring above a smooth channel - 2.16 times (see \(K_{pr}=2.16\));
- in the channel with the tape in 6-5.1 times with respect to the smooth channel.

2.4. A cooling system from corrugations with the small holes applied on them

This system "was formed" hypothetically of a corrugated insert on which the small holes of necessary optimum depth and density located in chess packing are previously applied. The corrugation from the high-temperature material had the optimum step in terms of the porosity of the system, providing a maximum of \(\alpha_{pr}\) in the region of interesting Reynolds numbers. In calculations are used our experimental [2] and literary data.

To carry out the calculations a corrugation was taken for which there were experimental data [9]: the height of the corrugation \(h_k=1.73 \text{ mm}\), the average width of the channel \(\delta_k=1.73 \text{ mm}\), the thickness of the corrugation \(\delta_s=0.1 \text{ mm}\), the porosity of the cooling system \(\epsilon=0.89\).

This corrugation was "coated" with holes of diameter \(D=1 \text{ mm}\) and depth \(h=0.1 \text{ mm}\) (the characteristic ratios \(H/D=1.73\), \(H=\delta_k, h/D=0.2\), and \(f=0.7\) is the relative area of a covering of a corrugation the small holes). The calculation was taken our data slit channel obtained in [2]. For the heat exchanged surface coated with the holes \((f=0.7)\) was established an increase in heat transfer (it was normalized to the heat transfer of an identical reference specimen without holes) to 1.5 times (with \(\text{Re} \approx 10^3\)). This value was increased surface heat transfer for the hypothetical calculated cooling system. At the same time, because of the small contribution of the finning in \(\alpha_{ps}\) (which decreases rapidly with increasing Reynolds number), it has increased approximately by the same amount. The coefficient of thermal insulation is reduced by 2-7 times (with \(\text{Re} \text{ changing from 2300 to } 10^3\)). This system in the Reynolds number region \(4\times10^3 \leq \text{Re} \leq 14\times10^3\) is energetically favorable, since in it the heat
transfer exceeded the hydraulic resistance (by 1.4 times (at Re=1.4x10^3)) with a drop to 1 (at Re=14x10^3)).

**Conclusion**
The calculations and analytical comparison demonstrate the usefulness of some methods of heat transfer enhancement in the proposed designs of cooling systems.

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