COSMOLOGICAL-MODEL-INDEPENDENT TESTS FOR THE DISTANCE–DUALITY RELATION FROM GALAXY CLUSTERS AND TYPE Ia SUPERNOVA

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ABSTRACT

We perform a cosmological-model-independent test for the distance–duality (DD) relation $\eta(z) = D_L(z)/(1 + z) \eta(z)$, where $D_L$ and $D_A$ are the luminosity distance and angular diameter distance, respectively, with a combination of observational data for $D_L$ taken from the latest Union2 SNe Ia and that for $D_A$ provided by two galaxy cluster samples compiled by De Filippis et al. and Bonamente et al. Two parameterizations for $\eta(z)$, i.e., $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1 + z)$, are used. We find that the DD relation can be accommodated at 1σ confidence level (CL) for the De Filippis et al. sample and at 3σ CL for the Bonamente et al. sample. We also examine the DD relation by postulating two more general parameterizations: $\eta(z) = \eta_0 + \eta_1 z$ and $\eta(z) = \eta_0 + \eta_1 z/(1 + z)$, and find that the DD relation is compatible with the results from the De Filippis et al. and the Bonamente et al. samples at 1σ and 2σ CLs, respectively. Thus, we conclude that the DD relation is compatible with present observations.

Key words: cosmic background radiation – distance scale – galaxies: clusters: general – supernovae: general – X-rays: galaxies: clusters

Online-only material: color figures

1. INTRODUCTION

The distance–duality (DD) relation (Etherington 1933) between the luminosity distance $D_L$ and the angular diameter distance (ADD) $D_A$, i.e.,

$$\frac{D_L}{D_A}(1 + z)^{-2} = 1,$$

where $z$ is the redshift, plays an important role in modern observational cosmology (Schneider et al. 1999; Cunha et al. 2007; Mantz et al. 2010; Komatsu et al. 2011), and, actually, it has heretofore been applied to all analysis of the cosmological observations without any doubt. However, in reality, it is possible that one of the requirements in obtaining the DD relation may be violated. A violation of the DD relation may even be considered as a signal of the breakdown of the physics on which the DD relation is based (Csaki et al. 2002; Bassett & Kunz 2004a, 2004b).

Thus, it is desirable to perform a validity check on the DD relation by the astronomical observations. In this regard, Uzan et al. (2004) have tested it by using the observations from the Sunyaev–Zel’dovich effect (SZE) and X-ray surface brightness from galaxy clusters, and found that the DD relation is consistent with observations at 1σ confidence level (CL). With a different galaxy cluster sample provided by Bonamente et al. (2006), De Bernardis et al. (2006) also obtained a non-violation of the DD relation. In addition, by combining the Union Type Ia supernovae (SNe Ia; Kowalski et al. 2008) with the latest measurement of the Hubble expansion at redshifts between 0 and 2 (Stern et al. 2010), Avgoustidis et al. (2012) discussed this relation and determined that it is consistent with observations at 2σ CL. Recently, by assuming that the DD relation satisfies the following expression:

$$\frac{D_L}{D_A}(1 + z)^{-2} = \eta(z),$$

where $\eta(z)$ is parameterized as $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1 + z)$, Holanda et al. (2010a) discussed the validity of the DD relation with the ADD $D_A$ measurements from galaxy clusters provided by the De Filippis et al. (2005; elliptical $\beta$ model) sample and the Bonamente et al. (2006; spherical $\beta$ model) sample, and the luminosity distance $D_L$ given in the context of CDM. Here, the elliptical and spherical $\beta$ models are two different geometries used to describe the galaxy clusters. Their results showed that the elliptical model is more compatible with no violation of the DD relation. However, all the aforementioned analyses are model dependent since a cosmic concordance model (ΛCDM) is assumed in their discussions. It is worth noting that De Bernardis et al. (2006) have in fact tried to test the DD relation in a model-independent way. In their method, the ADD is given from galaxy clusters and the luminosity distance is from SNe Ia. To obtain the values of the ADD and the luminosity distances at the same redshift, De Bernardis et al. (2006) binned their data and found that the DD relation is not violated at 1σ CL. However, when they determine the ADD from observations, the relation $D_A^{\text{cluster}}(z) = D_A(z)$ is used, which holds under the condition with no violation of the DD relation.

Recently, a consistent cosmological-model-independent test for the reciprocity relation was proposed by Holanda et al. (2010b). The main idea is to test the DD relation directly with the observed luminosity and ADDs, provided by SNe Ia and galaxy cluster samples, respectively. They considered two specific different redshift-dependent parameterizations for $\eta(z)$: $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1 + z)$. The data sets used were given from the Constitution SNe Ia (Hicken et al. 2009) and two ADD samples (De Filippis et al. 2005; Bonamente et al. 2006) from galaxy clusters obtained through SZE and X-ray measurement with different geometry descriptions for the cluster; the elliptical $\beta$ model (De Filippis et al. 2005) and the spherical $\beta$ model (Bonamente et al. 2006). They found that the result from the elliptical model is consistent with the
DD relation at $2\sigma$ CL, while for the spherical model the relation is clearly incompatible with observations.

However, we find that six and twelve ADD data points should be removed, respectively, for the De Filippis et al. and Bonamente et al. samples, instead of only three ADD data points that were discarded for both samples in Holanda et al. (2010b). Therefore, in the present Letter, we first redo the same analysis as Holanda et al. (2010b) but with more data points removed to see how this would affect the result, and we obtain a more serious violation of the DD relation. We then consider the effect to see how this would affect the result, and we obtain a more improved DD relation at $2\sigma$ CL, while for the spherical model the relation is given by the galaxy clusters obtained by Bonamente et al. samples, instead of only three ADD data points.

For the luminosity distance $D_L$, which corresponds to $\eta = \frac{\Delta z}{z}$, at the same redshift. The DD relation is improved markedly for both samples of galaxy clusters.

2. DATA AND ANALYSIS METHOD

To obtain constraints on free parameters in the parameterizations of $\eta(z): \eta(z) = 1 + \eta_0 + \eta_1 z$ and $\eta(z) = 1 + \eta_0 + \eta_1 z/(1 + z)$, we first need to get $\eta_{obs}$, which can be determined by the observed luminosity distance $D_L$ and ADD $D_A$ at the same redshift. The observed ADD $D_A$ is given by the galaxy clusters obtained by combining the SZE+X-ray surface brightness measurements. It must be emphasized that, if a redshift-dependent expression for the DD relation is considered, the SZE+X-ray surface brightness observation technique gives $D_A^{\text{cluster}}(z) = D_A(z)\eta^2(z)$ (Sunyaev & Zel’dovich 1972; Cavaliere & Fusco-Fermiano 1978). So, we must replace the ADD $D_A(z)$ with $D_A^{\text{cluster}}(z)\eta^2$ when we try to test the reciprocity relation consistently with the SZE+X-ray observations from galaxy clusters. Thus, the observed $\eta_{obs}(z)$ is determined by the following expression:

$$\eta_{obs}(z) = \frac{D_A^{\text{cluster}}(z)(1 + z)^2}{D_L(z)}.$$ (3)

For $D_A^{\text{cluster}}$, we consider two samples. The first one, including a selection of 18 galaxy clusters in the redshift interval $0.14 < z < 0.8$ compiled by Reese et al. (2002) and a sample of 7 clusters compiled by Mason et al. (2001), was studied and corrected by De Filippis et al. (2005) with an isothermal elliptical $\beta$ model. The second is the Bonamente et al. (2006) sample. It consists of 38 ADD galaxy clusters analyzed by assuming the hydrostatic equilibrium model and spherical geometry for the cluster plasma and dark matter distributions (spherical $\beta$ model).

For the luminosity distance $D_L$, both the Constitution and Union2 SN Ia data sets are considered. For a given $D_A^{\text{cluster}}$ data point, theoretically, we should select an associated SN Ia data point at the same redshift to obtain an $\eta_{obs}$. However, in reality, it is almost impossible to have both $D_A^{\text{cluster}}$ and $D_L$ at

![Figure 1](image-url) Redshift subtraction for the cluster–SN Ia samples. The left panel shows the results from the De Filippis et al. (2005) sample, while the right panel from the Bonamente et al. (2006) sample. The blue points and red pentagrams represent the Constitution and Union2 data, respectively.

(A color version of this figure is available in the online journal.)
Figure 2. Left: likelihood distribution functions from the De Filippis et al. (2005) and re-selected Constitution SN Ia pairs for two parameterizations: \( \eta(z) = 1 + \eta_0 z \) and \( \eta(z) = 1 + \eta_0 z/(1+z) \). Right: likelihood distribution functions from the Bonamente et al. (2006) and Constitution SN Ia pairs for the same parameterizations. The top and bottom panels correspond to the cases without and with the errors of SNe Ia, respectively. (A color version of this figure is available in the online journal.)

Figure 3. Left: likelihood distribution functions from the De Filippis et al. (2005) and Union2 SN Ia pairs for two parameterizations: \( \eta(z) = 1 + \eta_0 z \) and \( \eta(z) = 1 + \eta_0 z/(1+z) \). Right: likelihood distribution functions from the Bonamente et al. (2006) and Union2 SN Ia pairs for the same parameterizations. The top and bottom panels correspond to the cases without and with the errors of SNe Ia, respectively. (A color version of this figure is available in the online journal.)
Elliptical $\beta$ model
Union2 without errors of SNe Ia

Spherical $\beta$ model
Union2 without errors of SNe Ia

Elliptical $\beta$ model
Union2 with errors of SNe Ia

Spherical $\beta$ model
Union2 with errors of SNe Ia

Figure 4. Left: likelihood distribution functions from the De Filippis et al. (2005) and Union2 SN Ia pairs for two more general parameterizations: $\eta(z) = \eta_0 + \eta_1 z$ and $\eta(z) = \eta_0 + \eta_1 z/(1+z)$. Right: likelihood distribution functions from the Bonamente et al. (2006) and Union2 SN Ia pairs for the same parameterizations. The top and bottom panels correspond to the cases without and with the errors of SNe Ia, respectively.

(A color version of this figure is available in the online journal.)

To get more reliable results, we use the latest Union2 SNe Ia (Amanullah et al. 2010) in our analysis. The main advantages of the Union2, as compared with the Constitution, are: (1) the selection criteria ($\Delta z = |z_{\text{Cluster}} - z_{\text{SNe Ia}}| < 0.005$) can be satisfied for all data points of both ADD samples except for the cluster CL J1226.9 + 3332 ($z = 0.890$) from the Bonamente et al. (2006) sample, which gives $\Delta z = 0.005$; and (2) points $z_{\text{SNe Ia}} - z_{\text{Cluster}}$ are more centered around the line $\Delta z = 0$, as shown in Figure 1, which plot the subtractions of redshifts between the clusters and the associated SNe Ia. Thus, with Union2, the accuracy of our test will be improved. In order to ensure the integrity of the ADD

exactly the same redshift. So, as in Holanda et al. (2010b), we use the criteria ($\Delta z = |z_{\text{Cluster}} - z_{\text{SNe Ia}}| < 0.005$) to select the SN Ia data. For Constitution SNe Ia, we find that there exist six data points (A2261, A2163, A520, A1689, A665, A2218) for the De Filippis et al. (2005) sample (elliptical $\beta$ model) and 12 data points (A68, A267, A586, A665, CL J1226.9+3332, A1689, A1914, A2111, A2163, A2218, A2259, A2261) for the Bonamente et al. (2006) sample (spherical $\beta$ model) that have to be discarded (listed in detail in Figure 1). This, however, differs from what is obtained by Holanda et al. (2010b), where only three data points are removed for both ADD samples.
samples, we keep the cluster CL J1226.9 + 3332 (Δz = 0.005) in our analysis. Using these observational data of galaxy clusters and SNe Ia and the selection criteria (Δz = |z_{Cluster} − z_{SNe Ia}| < 0.005), we can obtain η_{obs}. To estimate the model parameters of a given parameterized form, we use the minimum χ^2 estimator following standard route:

$$\chi^2(z; \mathbf{p}) = \sum_{z} \left[ \eta(z; \mathbf{p}) - \eta_{\text{obs}}(z) \right]^2 \sigma_{\text{obs}}^{-2}.$$

(4)

where σ_{\text{obs}} is the error of η_{obs} associated with the observational technique and \mathbf{p} represents the model parameter set. σ_{\text{obs}} comes from the statistical contributions and systematic uncertainties of the galaxy clusters which are combined in quadrature (D’Agostini 2004) and those of SNe Ia. In our analysis, we consider two cases, i.e., those with and without the errors of SNe Ia, respectively. When the errors of SNe Ia are not considered, we obtain that, for the De Filippis et al. (2005) sample (elliptical β model), the reciprocity relation is marginally consistent with observations at 3σ CL. While the results from the Bonamente et al. (2006) sample, where a spherical β model was assumed to describe the clusters, show a violation of the DD relation clearly at 3σ CL. Compared with the results by Holanda et al. (2010b), where they found that the result from the elliptical model is compatible with the DD relation at 2σ CL and that from the spherical model is incompatible with it, our results suggest a stronger violation. For the case with the errors of SNe Ia taken into account, we find that the results from both the elliptical and the spherical β models are compatible with the DD relation at 3σ CL, which means that the inclusion of the errors of SNe Ia improves the consistency between the DD relation and observations.

For the latest Union2 SN Ia data, from the results shown in Figure 3 and Table 1, we find that, for both the cases without and with the errors of SNe Ia, the DD relation can be accommodated at 1σ CL for the De Filippis et al. (2005) sample and at 3σ CL for the Bonamente et al. (2006) sample. The errors of SNe Ia do not affect the results significantly, although they tend to make the results be more compatible with the DD relation. Finally, we also examine the DD relation by assuming two more general expressions: \eta(z) = η_0 + η_1 z and \eta(z) = η_0 + η_1 z/(1 + z). The results are shown in Figure 4, which suggest that there is no violation of the DD relation for the elliptical geometry at 1σ CL and for the spherical β model at 2σ CL.

4. CONCLUSION

In this Letter, we test the DD relation by considering the ADDs given by two samples of galaxy clusters together with the luminosity distances provided by sub-samples of SNe Ia taken from the Constitution and the latest Union2 data sets. The Constitution sample has already been discussed by Holanda et al. (2010b), and they found that for both ADD samples three data points should be removed with the selection criteria (Δz = |z_{Cluster} − z_{SNe Ia}| < 0.005). However, we find that, with the same selection criteria, the number of data points that have to be removed are actually six and twelve, respectively, for the De Filippis et al. (2005) sample and the Bonamente et al. (2006) sample. A re-analysis with more data points discarded suggests a violation of the DD relation stronger than that given in Holanda et al. (2010b). In order to obtain a more reliable result, we investigate the DD relation by considering the latest Union2 SNe Ia. It is worth noting that with the Union2 SNe Ia all ADD data can be retained and the differences of the redshifts from ADD of the galaxy cluster and the associated luminosity distance from SNe Ia are much smaller. Thus, the accuracy of our test should be improved. Our results then show that the DD relation can be accommodated at 1σ CL for the elliptical β model (De Filippis et al. 2005) and at 3σ CL for the spherical β model (Bonamente et al. 2006). Finally, we examine the DD relation by postulating two more general parameterization forms: \eta(z) = η_0 + η_1 z and \eta(z) = η_0 + η_1 z/(1 + z), and we find that the consistencies between the observations and the DD relation are improved markedly for both samples of galaxy clusters. The DD relation is compatible with De Filippis et al. (2005) sample and Bonamente et al. (2006) sample at 1σ and 2σ CLs, respectively. Furthermore, with the inclusion of the errors of SNe Ia, the results become more consistent with the DD relation. Therefore, our results suggest that the DD relation is compatible with the observations. This differs from what is obtained by Holanda et al. (2010b), where the results from the Bonamente et al. (2006) sample give a clear violation of the DD relation.

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