Weiqiang Yang,† Supplement Pan,‡ David F. Mota,† and Minghui Du§

†Department of Physics, Liaoning Normal University, Dalian, 116029, P. R. China
‡Department of Mathematics, Presidency University, 86/1 College Street, Kolkata 700073, India
†Institute of Theoretical Astrophysics, University of Oslo, 0315 Oslo, Norway
§Institute of Theoretical Physics, School of Physics, Dalian University of Technology, Dalian, 116024, P. R. China

We confront the dark energy anisotropic stress using the usual cosmological probes namely cosmic microwave background radiation, baryon acoustic oscillations, latest pantheon sample of supernovae type Ia and then make use of the simulated gravitational waves standard sirens (GWSS) data from Einstein Telescope with an aim to examine the constraining power of the simulated GWSS data over the standard cosmological probes. Our analyses show that GWSS can give better constraints on the model parameters compared to the usual cosmological probes, but the viscous sound speed appearing due to the dark energy anisotropic stress, is totally unconstrained even after the inclusion of GWSS.

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1. INTRODUCTION

The thrilling chapter of modern cosmology begun with the late-time accelerating phase of our universe [1]. This accelerated expansion is usually ascribed by the introduction of some exotic fluid with high negative pressure. This exotic fluid could be either some dark energy fluid (in the context of Einstein’s general theory of gravity) [2] or some geometrical dark energy (coming from the modified gravity theories) [3–5]. In this article we confine ourselves into the first approach, that means, dark energy fluid. In other words, our discussions will be restricted to the Einstein’s gravitational theory. Within the framework of general relativity, a cluster of dark energy models (see [2] for various dark energy models) have been introduced in the literature, however, most of them are basically the variations of either the cosmological constant or some scalar field theory [6]. These variations naturally include different kind of couplings between the matter components of the universe [7–9], couplings to the gravitational sector [10–11], or some non-canonical scalar field models, such as tachyons [12–13] or K-essence [14–15]. K-essence can be considered into the picture in order to have a wider universe evolution. The perturbations, respectively determine the background and perturbative evolution of the underlying cosmic fluid that is rotationally invariant, the anisotropic stress σ actually gives a quantification on how much the pressure of the cosmic fluid varies with the direction. In fact, the perturbation for anisotropic stress is very important for understand-

Moreover, aside from the equation of state w and the sound speed of perturbations $c_s^2$, one more important characteristic of a cosmic fluid is its anisotropic stress $\sigma$. Although for a class of cosmological models including minimally coupled scalar field and perfect fluids, the anisotropic stress vanishes, however, it is a generic property of realistic fluids with finite shear viscous coefficients [24–25]. Let us note that while w and $c_s^2$ respectively determine the background and perturbative evolutions of the underlying cosmic fluid that is rotationally invariant, the anisotropic stress $\sigma$ actually gives a quantification on how much the pressure of the cosmic fluid varies with the direction. In fact, the perturbation for anisotropic stress is very important for understand-
ing the evolution of inhomogeneities in the early universe [23, 26, 27]. Thus, undoubtedly it is a natural question to investigate whether the current observational data may indicate for a nonzero anisotropic stress perturbations in the dark energy dominated late-accelerating universe.

The effects of anisotropic stress, that may appear due to possible viscosity of dark energy, have not been paid much attention in the literature. The reason for neglecting the anisotropic stress is that, for the conventional dark energy fluids, such as the cosmological constant or canonical scalar field models, $\sigma = 0$. But, that should not be a logical case since there is no such fundamental theory available yet that could correctly describe the actual dynamics of dark energy; hence, the assumption of $\sigma = 0$ for any dark energy model, does not have any sense anymore. Therefore, from an unbiased scientific point of view, the presence of an anisotropic stress into the cosmic sector should be fairly considered and it is better to examine its non-null character, if any, with the help of recently available observational data. Some earlier analyses have shown that coupled scalar field models have a non-negligible anisotropic stress [24]. Additionally, dark energy vector field candidates (proposed in [28, 29]) also allow nonzero anisotropic stress. Therefore, it is fairly clear that the generalized dark energy models including the anisotropic stress should be investigated in detail.

In this article, our approach is more interesting. We want to probe the anisotropic stress using the gravitational waves data, detected recently [30, 31, 32, 33, 34]. In particular, we shall use the simulated gravitational waves standard sirens (GWSS) data to constrain the anisotropic stress with an aim to what future cosmological probe can tell us. The simulated GWSS data have already proved its super constraining power applied recently to various cosmological models, see for instance [35, 36, 37]. In fact, gravitational waves data are believed to offer more information about the nature of dark matter and dark energy. We refer to an incomplete list of recent works focusing on the effects of the GWSS data on various cosmological theories and related key parameters [38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75]. Thus, the consideration of GWSS has significant effects on the dynamics of our universe. In this article, we use the simulated GWSS data from Einstein Telescope [76] (see also a number of works focused on this specific telescope [77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92]), however, technically, one can equally consider other observatories like Laser Interferometer Space Antenna (LISA) [33], Deci-hertz Interferometer Gravitational wave Observatory (DECIGO) [34], TianQin [93]. In fact, it will be interesting to use different simulated GWSS data from all the above sources with an aim to a detailed investigations in this direction.

The work has been organized in the following manner. In section 2 we review the parameterization of a generalized cosmic fluid and introduce its connection with anisotropic stress. In section 3 we describe the observational data, namely both standard cosmological data and the simulated gravitational waves data. Then in section 4 we describe the extracted results obtained after using varieties of observational (real and forecasted) datasets. Finally, in section 5 we close the present work with the main findings and comment on some future work that should be performed along these lines.

2. PARAMETERIZING DARK ENERGY STRESS: FLRW BACKGROUND

The energy momentum tensor of a general cosmic fluid is defined as [26, 27]

$$T_{\mu\nu} = p u_{\mu} u_{\nu} + p h_{\mu\nu} + \Sigma_{\mu\nu},$$

(1)

where $p$, $\rho$ are respectively the pressure and energy density of the perfect fluid; $u_{\mu}$ is the four-velocity vector of this fluid, and $h_{\mu\nu}$, the projection tensor is defined as, $h_{\mu\nu} \equiv g_{\mu\nu} + u_{\mu}u_{\nu}$. The quantity $\Sigma_{\mu\nu}$ in (1) may include only spatial inhomogeneity which vanishes for a perfect fluid, that means, $\Sigma_{\mu\nu} \equiv 0$. Additionally, for a homogeneous and isotropic universe, $\Sigma_{\mu\nu}$ is also zero at the level of background; in fact, in such a case, it denotes the anisotropic perturbation at the first order. Thus, at the background level, since $\Sigma_{\mu\nu} = 0$, the evolution of the fluid (1) is determined by the continuity equation,

$$\dot{\rho} + 3H(p + \rho) = 0,$$

(2)

where $H$ is the Hubble rate of the Friedmann-Lemaître-Robertson-Walker (FLRW) universe. The condition for the adiabaticity of a fluid is $p = p(\rho)$, which tells us that the evolution of the sound speed is determined by the equation of state $w = p/\rho$, alone. However, being the most general, the sound speed is defined as the ratio of pressure to the density perturbations in the frame comoving with the dark energy fluid, defined as

$$c_{s,a}^2 \equiv \frac{\dot{\rho}}{\rho} = w - \frac{\ddot{w}}{3H(1+w)},$$

(3)

where an overhead dot represents the differentiation with respect to the conformal time $\tau$; $w$ being the equation of state defined as $w = p/\rho$; $H = \dot{a}/a$ is the conformal Hubble parameter. We note that the relation between the perturbations of $\delta p$ and $\delta \rho$ is, $\delta p = c_{s,a}^2 \delta \rho$. However, for an entropic fluid, the pressure $p$ may not only depend on the energy density $\rho$. In fact, there might have another degree of freedom in order to describe the microphysical properties of the general cosmic fluid and such microphysical property is usually encoded in the effective speed of sound $c_{s,\text{eff}}^2$ defined as

$$c_{s,\text{eff}}^2 \equiv \frac{\delta \rho}{\delta \rho} |_{\text{rf}},$$

(4)

in the rest-frame (‘rf’) of the underlying cosmic fluid. In absence of entropic perturbation, $c_{s,\text{eff}}^2 = c_{s,a}^2$. Therefore,
one may easily conclude that a perfect fluid is completely characterized by two quantities, one is its equation of state \( w \) and the other is its effective speed of sound \( c_{s, eff}^2 \).

However, aside from the previous quantities, namely, \( w \) and \( c_{s, eff}^2 \) associated with a cosmic fluid, one more important quantity is needed in order to understand the cosmic fluid at the level of background and perturbations. This quantity is the anisotropic stress \( \sigma \), and it should be considered into the cosmological framework even in an isotropic and homogeneous FLRW universe, where the anisotropic stress \( \sigma \) may be taken as the spatial perturbation. This anisotropic stress actually distinguishes between the Newtonian potential and curvature perturbation in the conformal Newtonian gauge.

So, now one can calculate the evolution equations at the level of perturbations for the above model considering any gauge. We choose the synchronous gauge in this article and using this gauge, the density perturbations and velocity perturbations can be written as

\[
\dot{\delta} = -(1 + w) \left( \dot{\theta} + \frac{\dot{h}}{2} \right) - 3 \mathcal{H} \left( \frac{\delta p}{\delta \rho} - \frac{8}{3} \right) \delta, \quad (5)
\]

\[
\dot{\theta} = -\mathcal{H} \left( 1 - 3 c_{s, eff}^2 \right) \theta + \frac{c_{s, eff}^2}{1 + w} k^2 \delta - k^2 \sigma, \quad (6)
\]

where the anisotropic stress \( \sigma \) is related to \( \Sigma_{\mu \nu} \) (see eqn. 1) via \( (p + \rho)\sigma = -(k_i k_j - \delta_{ij}/3)\Sigma^{ij} \). Now, using the effective speed of sound, one can recast the above equations as

\[
\dot{\delta} = -(1 + w) \left( \theta + \frac{\dot{h}}{2} \right) + \frac{\dot{\omega}}{1 + w} \delta - 3 \mathcal{H} \left( \frac{\delta p}{\delta \rho} - \frac{8}{3} \right) \left[ \delta + 3 \mathcal{H}(1 + w) \frac{\theta}{k^2} \right], \quad (7)
\]

\[
\dot{\theta} = -\mathcal{H} \left( 1 - 3 c_{s, eff}^2 \right) \theta + \frac{c_{s, eff}^2}{1 + w} k^2 \delta - k^2 \sigma, \quad (8)
\]

where following Hu \cite{Hu2001} we suppose that the anisotropic stress \( \sigma \) satisfies the evolution equation

\[
\dot{\sigma} + 3 \mathcal{H} \frac{c_{s, eff}^2}{w} \sigma = \frac{8}{3} \frac{c_{vis}^2}{1 + w} \left( \theta + \frac{\dot{h}}{2} + 3 \dot{\theta} \right), \quad (9)
\]

where \( c_{vis}^2 \) is the viscous speed of sound and it controls the relation between velocity/metric shear and the anisotropic stress. For a relativistic fluid, \( c_{vis}^2 = 1/3 \). For any dark energy fluid, \( c_{vis}^2 \) acts as a free model parameter to be determined by the observational data. One important remark is that, the value of \( c_{vis}^2/(1 + w) \) should be positive \cite{Pastor2002, Wang2001}. This forces us to consider different models, namely, (i) the dark energy equation of state is in the quintessence regime, that means, \(-1 < w < 0\) (Model I) and (ii) when the dark energy equation of state crosses the phantom divide line, that means, \( w < -1\) (Model II). Let us note that for all the analyses that will be described in section 4, we adopted the adiabatic initial conditions similar to the authors in Ref. \cite{Ma1995}.

3. STANDARD COSMOLOGICAL PROBES AND THE GWSS DATA

We now proceed to extract the cosmological constraints using a set of usual dark energy probes and the simulated gravitational waves standard sirens (GWSS). In this section we shall describe the standard cosmological data and then we will refer some works that describe the methodology to generate the mock GWSS.
The methodology to generate the GW data is described by the present authors in [35] (also see [36]). The present work deals with the same methodology as in [35, 36], thus, we avoid the repetition here and directly refer to [35, 36] for details. We would mark some important points here. Using the methodology here for this work we generate 1000 simulated GW data for our purpose. The generation of 1000 GW data is as follows. We first constrain the cosmological scenarios using the usual cosmological probes, such as CMB, BAO, Pantheon, CC. Then we use the best-fit values of all the free and derived parameters obtained from the standard cosmological probes and assuming the present anisotropic dark energy models as the fiducial models, and following exactly similar technique described in [35, 36] we generate the 1000 mock GW data. Next we add these 1000 mock GW data to these standard cosmological probes and constrain the underlying cosmological scenarios. For the entire statistical analysis we use the markov chain monte carlo package {	t cosmomc} [107, 108] equipped with a convergence diagnostic by Gelman-Rubin [109]. The {	t cosmomc} code also supports the Planck 2015 likelihood code [99]. Finally, in Table I we enlist the flat priors on the cosmological parameters used during the time of statistical analysis.

| Parameter | Prior (Model I) | Prior (Model II) |
|-----------|----------------|-----------------|
| $\Omega_b h^2$ | [0.005,0.1] | [0.005,0.1] |
| $\Omega_c h^2$ | [0.01,0.99] | [0.01,0.99] |
| $\tau$ | [0.01,0.8] | [0.01,0.8] |
| $n_s$ | [0.5,1.5] | [0.5,1.5] |
| $\log(10^{10} A_s)$ | [2.4,4] | [2.4,4] |
| $1000MC$ | [0.5,10] | [0.5,10] |
| $w$ | $[-1,0]$ | $[-3,-1]$ |
| $c^2_{vis}$ | [0,10] | $[-10,0]$ |

TABLE I: Flat priors set on various cosmological parameters for the statistical analysis.

4. RESULTS

In this section we describe the main results on the cosmological parameters for the two variants of the model, namely, Model I ($w > -1$) and Model II ($w < -1$) using various observational datasets. In particular, we focus on the effects of GW data on the cosmological parameters.

Before we present all the extracted cosmological constraints from the present cosmological scenarios, we wish to present the following. To generate GW catalogue, or mock GW data, it is essential to consider the fiducial model. Here, we consider Model I and Model II as the fiducial models. Now, assuming Model I as the fiducial model, we first constrain it using various observational data summarized in Table I. Then considering the best-fit values of all the free and derived parameters of this model (Model I), and following [35, 36], we generate the corresponding GW catalogue containing 1000 simulated GW events. In Fig. 1 we show $d_L(z)$ vs. $z$ (with error bars on $d_L(z)$) for simulated 1000 GW events. We use this catalogue as the forecasted dataset and include them with the standard cosmological datasets, namely, P15, BAO, Pantheon for the next step of the analysis. In a similar fashion we generate simulated 1000 GW events for the second model in this work and Fig. 4 shows the corresponding $d_L(z)$ vs. $z$ graphics. In what follows we describe the observational constraints on each model considering the usual cosmological probes and the inclusion of the simulated GWSS.

4.1. Model I: $c^2_{vis} > 0$, $w > -1$

In Table I we show the constraints on the model parameters for the usual cosmological probes and in Table III we display the constraints on the model parameters after the inclusion of the simulated GWs data with the usual cosmological probes. Thus, Table II and Table III summarize the main results on this model. In the following we become more explicit on the improvements of the constraints, if any, after the inclusion of GWs to the usual cosmological probes mentioned above.

Let us first focus on the constraints from P15 and P15+GW. From Table I we notice see that for P15 data alone the Hubble constant at present, i.e. $H_0$ takes lower value compared to $\Lambda$CDM based Planck but with high error bars compared to what we find in $\Lambda$CDM based Planck I. In particular, one finds that for P15 alone $H_0 = 64.11^{+3.22}_{-1.70}$ (68% CL). When the simulated GW data are added to P15, the Hubble constant rises up giving $H_0 = 66.94^{+0.39}_{-0.38}$ (68% CL, P15+GW). One can clearly see that the inclusion of GW to P15 significantly reduces the error bars on $H_0$. In fact, the error bars on $H_0$ are reduced at least by a factor of 5. This actually reflects the constraining power of GW. In a similar fashion when we look at the other derived parameters of this model, namely, $\Omega_{m0}, \sigma_8$, one can draw similar conclusion, that means the effects of GWs on the cosmological parameters is transparent. In fact, the free parameter, $w$, is also affected significantly after the addition of GW to P15. We see that the 68% upper bound on $w$ for P15 alone is $w < -0.854$, which is significantly changed to $w < -0.974$ after the addition of GW to P15. Now,
Table II: The table displays the constraints on various free and derived cosmological parameters at 68% and 95% CL for Model I using the usual cosmological probes, namely, P15, BAO and Pantheon. For the dark energy equation of state we present its upper limits at 68% and 95% CL.

| Parameters         | P15       | P15+BAO   | P15+Pantheon | P15+BAO+Pantheon |
|--------------------|-----------|-----------|--------------|------------------|
| \(\Omega_h^2\)     | 0.1190+0.0016 0.1188+0.0012+0.0017 | 0.1185+0.0014+0.0017 | 0.1184+0.0015+0.0017 | 0.1178+0.0016+0.0018 |
| \(\Omega_b^2\)     | 0.0229+0.0012+0.0025 | 0.0223+0.0014+0.0028 | 0.0223+0.0014+0.0025 | 0.0223+0.0012+0.0025 |
| \(\Delta T_0\)     | 1000/MC    | 1.0407+0.0030+0.0058 | 1.0409+0.0031+0.0059 | 1.0409+0.0030+0.0058 | 1.0495+0.0031+0.0056 |
| \(\alpha_{	ext{vis}}\) | unconstrained | unconstrained | unconstrained | unconstrained |
| \(\alpha_{	ext{vis}}\) | unconstrained | unconstrained | unconstrained | unconstrained |
| \(\sigma_8\)       | 0.791+0.022+0.026 | 0.820+0.016+0.027 | 0.823+0.014+0.028 | 0.824+0.014+0.028 |
| \(H_0\)            | 66.9+0.7+0.7 | 66.9+0.6+0.7 | 67.0+0.7+0.7 | 67.5+0.2+0.1 |

Table III: In this table we show the constraints on various free and derived cosmological parameters of Model I at 68% and 95% CL after the inclusion of GWSS data with the standard cosmological probes P15, BAO and Pantheon.

| Parameters         | P15+GW    | P15+BAO+GW | P15+Pantheon+GW | P15+BAO+Pantheon+GW |
|--------------------|-----------|------------|-----------------|---------------------|
| \(\Omega_h^2\)     | 0.1190+0.0016 0.1188+0.0012+0.0017 | 0.1185+0.0014+0.0017 | 0.1184+0.0015+0.0017 | 0.1178+0.0016+0.0018 |
| \(\Omega_b^2\)     | 0.0229+0.0012+0.0025 | 0.0223+0.0014+0.0028 | 0.0223+0.0014+0.0025 | 0.0223+0.0012+0.0025 |
| \(\Delta T_0\)     | 1000/MC    | 1.0407+0.0030+0.0058 | 1.0409+0.0031+0.0059 | 1.0409+0.0030+0.0058 | 1.0495+0.0031+0.0056 |
| \(\alpha_{	ext{vis}}\) | unconstrained | unconstrained | unconstrained | unconstrained |
| \(\alpha_{	ext{vis}}\) | unconstrained | unconstrained | unconstrained | unconstrained |
| \(\sigma_8\)       | 0.791+0.022+0.026 | 0.820+0.016+0.027 | 0.823+0.014+0.028 | 0.824+0.014+0.028 |
| \(H_0\)            | 66.9+0.7+0.7 | 66.9+0.6+0.7 | 67.0+0.7+0.7 | 67.5+0.2+0.1 |

Concerning the viscous speed of sound, \(c_{\text{vis}}^2\), we find that this parameter is neither constrained by P15 alone nor the addition of GW to P15 helps to constrain it. We refer to Fig. 2 showing the 1D marginalized posterior distributions for some key parameters of this scenario for P15 and P15+GW datasets. One may wonder that perhaps the increase of the prior on \(c_{\text{vis}}^2\) may help to constrain, however, this is not true in this case. We found that even if the prior varies in the interval [0, 100], this parameter remains unconstrained. We also show Fig. 3 displaying the dependence of \(c_{\text{vis}}^2\) with other parameters for P15 and P15+GW datasets.

We now discuss the next two datasets, namely P15+BAO and its companion P15+BAO+GW. For a quick view on the cosmological constraints we refer to the third columns of both Table II and Table III. Additionally, for graphical views, we refer to the upper panel of Fig. 4 which shows the 1D marginalized posterior distributions for some selected parameters. We find that the addition of GW to P15+BAO shifts the highest peak of \(H_0\) towards higher values and shifts \(\Omega_{m0}\) towards lower values. This is consistent since there is already a known correlation between \(H_0\) and \(\Omega_{m0}\). So, the addition of GW does not alter such correlation. A similar but very small shift of the \(\sigma_8\) parameter is also observed. Concerning the dark energy equation of state, \(w\), we have an interesting observation. From the 1D posterior distribution of \(w\) (upper panel of Fig. 1) we see that after the inclusion of GW to P15+BAO, we find the highest peak of \(w\) which was absent for the usual CMB+BAO analysis.

Finally, we notice that the parameter \(c_{\text{vis}}^2\) remains unconstrained for both the datasets, namely, P15+BAO and P15+BAO+GW. So, we see that the addition of GW to CMB+BAO does not alter the nature of this parameter.

We now consider the following two cases, namely CMB+Pantheon and CMB+Pantheon+GW. The results are summarized in the fourth columns of both Table II and Table III. And we refer to the middle panel of Fig. 4 for a quick look on the 1D posterior distributions of some important parameter before and after the inclusion of GW to the corresponding dataset (i.e., P15+Pantheon).

Our results are very clear and straightforward. In a similar fashion to the previous two analyses (i.e. P15+BAO and P15+BAO+GW), here too, we find that the addition of GW to P15+Pantheon shifts the highest peak of the Hubble constant towards higher values having an additional shift of \(\Omega_{m0}\) towards its lower values. However, the parameter space of both \(H_0\) and \(\Omega_{m0}\) is certainly improved due to GW. In addition we do not find any
FIG. 1: For the fiducial (Model I) model we first constrain the cosmological parameters using the datasets P15, P15+BAO, P15+Pantheon and P15+BAO+Pantheon and then we use the best-fit of the parameters for “each dataset” to generate the corresponding GW catalogue. Following this, in each panel we show $d_L(z) \text{vs} z$ catalogue with the corresponding error bars for 1000 simulated GW events. The upper left and upper right panels respectively present the catalogue $(z, d_L(z))$ with the corresponding error bars for 1000 simulated events derived using the P15 alone and P15+BAO dataset. The lower left and lower right panels respectively present the catalogue $(z, d_L(z))$ with the corresponding error bars for 1000 simulated events derived using the P15+Pantheon and P15+BAO+Pantheon datasets.

FIG. 2: 1-dimensional marginalized posterior distributions for some key parameters of Model I for the datasets P15 and P15+GW.

changes to the parameter space of $\sigma_8$, which is clear if one looks at the 1D posterior distribution of this parameter. Moreover, we have a different result when one looks at the 1D posterior of $w$ for both CMB+Pantheon and P15+Pantheon+GW. One could see that in contrary to the previous observation with P15+BAO+GW, the peak of $w$ disappears in this case. Finally, our conclusion regarding the viscous sound speed, $c^2_{\text{vis}}$, remains same, that means it is again unconstrained for both the datasets.

Lastly, we come to the last two datasets in this series, namely, P15+BAO+Pantheon and P15+BAO+Pantheon+GW. The last columns of both Table II and Table III summarize the constraints on the model parameters and the lower panel of Fig. 4 displays the 1D posterior distributions of some important parameters of this model. Looking at the lower panel of Fig. 4 specially for $H_0$ and $\Omega_{m0}$ we find their improvements after the inclusion of GW, however, we do not find
any shifts of the highest peaks of $H_0$ and $\Omega_{m0}$ in their posterior distributions in contrary to the earlier cases, such as P15+BAO+GW and P15+Pantheon+GW. Lastly, we again see that the parameter $c^2_{\text{vis}}$ is still unconstrained for both the datasets. So, GW data seem to be unable to constrain this particular parameter.

When we finished all the analyses of this paper, Planck released its final CMB data [100, 101]. We then wanted to check whether the new CMB data from Planck 2018 final release (P18 as referred in the text) could constrain the parameter, $c^2_{\text{vis}}$ of this scenario. We found that P18 data are also unable to constrain $c^2_{\text{vis}}$ which is also unconstrained by the earlier P15 data. To illustrate this nature, in Fig. 5 we have shown the 1-dimensional posterior distributions of some key parameters of this model showing the constraining power of P18 data alone and also the effects of GW on $c^2_{\text{vis}}$ after its inclusion to P18. From the present analyses, we have clearly visualized that the unconstrained nature of any parameter, here, $c^2_{\text{vis}}$ is not controlled by the external datasets, such as BAO, Pantheon, CC. We have found that if P15 data are unable to constrain a specific parameter, that parameter remains unconstrained by the external datasets. Thus, since P18 data remain unable to constrain $c^2_{\text{vis}}$, thus, other combinations, such as P18+any external dataset, such as BAO, Pantheon, CC, will also be unable to constrain $c^2_{\text{vis}}$. Hence, we do not consider these combinations for this work. The nature of $c^2_{\text{vis}}$ will actually be the same.

4.2. Model II: $c^2_{\text{vis}} < 0$, $w < -1$

In Table IV we show the constraints on the model parameters using the usual cosmological probes and in Table V we show the constraints on the model parameters after the inclusion of the simulated GWs data to the usual cosmological probes. Thus, Table IV and Table V summarize the main results on this model.

Now, following the similar pattern as with Model I we analyze this model as well. Thus, we first focus on the datasets namely P15 and P15+GW and discuss how the GWs data could improve the constraints on various free and derived parameters of this model. Looking at the constraint on $H_0$, one can quickly realize the effects of GWs onto it. From P15 alone, $H_0 = 86.69^{+12.21}_{-6.03}$ (68%
FIG. 4: 1-dimensional marginalized posterior distributions for some key parameters of Model I for the datasets P15+BAO, P15+BAO+GW (upper panel), P15+Pantheon, P15+Pantheon+GW (middle panel) and P15+BAO+Pantheon, P15+BAO+Pantheon+GW (lower panel).

FIG. 5: 1-dimensional marginalized posterior distributions for some key parameters of Model I for the datasets P18 and P18+GW.

| Parameters       | P15       | P15+BAO   | P15+Pantheon | P15+BAO+Pantheon |
|------------------|-----------|-----------|--------------|------------------|
| $\Omega_m h^2$   | $0.1191^{+0.0044+0.0027}_{-0.0041+0.0027}$ | $0.1191^{+0.0014+0.0022}_{-0.0011+0.0021}$ | $0.1195^{+0.0014+0.0026}_{-0.0013+0.0026}$ | $0.1189^{+0.0016+0.0022}_{-0.0011+0.0022}$ |
| $\Omega_b h^2$   | $0.02228^{+0.00016+0.00031}_{-0.00016+0.00030}$ | $0.02226^{+0.00014+0.00029}_{-0.00015+0.00028}$ | $0.02224^{+0.00014+0.00029}_{-0.00014+0.00029}$ | $0.02228^{+0.00014+0.00029}_{-0.00014+0.00027}$ |
| $100\theta_{MC}$ | $1.04080^{+0.00031+0.00062}_{-0.00031+0.00063}$ | $1.04080^{+0.00030+0.00062}_{-0.00031+0.00065}$ | $1.04073^{+0.00031+0.00064}_{-0.00032+0.00062}$ | $1.04083^{+0.00031+0.00062}_{-0.00031+0.00060}$ |
| $\tau$           | $0.076^{+0.018+0.034}_{-0.017+0.034}$ | $0.080^{+0.017+0.033}_{-0.017+0.032}$ | $0.078^{+0.017+0.033}_{-0.017+0.032}$ | $0.082^{+0.017+0.033}_{-0.017+0.032}$ |
| $\alpha$         | $0.9665^{+0.0046+0.0003}_{-0.0046-0.0009}$ | $0.9655^{+0.0040+0.00077}_{-0.0040-0.00079}$ | $0.9655^{+0.0044+0.00087}_{-0.0044-0.00086}$ | $0.9671^{+0.0042+0.00077}_{-0.0043-0.00078}$ |
| $\ln(10^{10}A_s)$| $3.085^{+0.034+0.066}_{-0.034-0.066}$ | $3.093^{+0.033+0.064}_{-0.033-0.062}$ | $3.091^{+0.033+0.064}_{-0.034-0.064}$ | $3.095^{+0.033+0.063}_{-0.032-0.065}$ |
| $w$              | $>-1.917>-1.973$ | $>-1.071>-1.133$ | $>-1.052>-1.088$ | $>-1.041>-1.078$ |
| $\epsilon_{vis}$ | unconstrained | unconstrained | unconstrained | unconstrained |
| $\Omega_m h^2$   | $0.195^{+0.020+0.088}_{-0.055-0.063}$ | $0.296^{+0.013+0.018}_{-0.008-0.020}$ | $0.303^{+0.009+0.018}_{-0.009-0.018}$ | $0.301^{+0.007+0.013}_{-0.007-0.014}$ |
| $\sigma_8$       | $0.983^{+0.094+0.116}_{-0.052-0.139}$ | $0.845^{+0.016+0.036}_{-0.019-0.034}$ | $0.841^{+0.015+0.030}_{-0.015-0.029}$ | $0.839^{+0.015+0.029}_{-0.015-0.029}$ |
| $H_0$            | $86.05^{+2.21+14.14}_{-6.5+16.53}$ | $69.25^{+1.80+2.33}_{-0.38-2.01}$ | $68.57^{+0.80+1.73}_{-0.97-1.76}$ | $68.64^{+0.76+1.32}_{-0.76-1.32}$ |

TABLE IV: The table presents the constraints on various free and derived cosmological parameters at 68% and 95% CL for Model II using the usual cosmological probes, namely, P15, BAO and Pantheon. For the dark energy equation of state we present its upper limits at 68% and 95% CL.
FIG. 6: For the fiducial (Model II) model we first constrain the cosmological parameters using the datasets P15, P15+BAO, P15+Pantheon and P15+BAO+Pantheon and then we use the best-fit of the parameters for “each dataset” to generate the corresponding GW catalogue. Following this, in each panel we show $d_L(z)$ vs $z$ catalogue with the corresponding error bars for 1000 simulated GW events. The upper left and upper right panels respectively present the catalogue ($z$, $d_L(z)$) with the corresponding error bars for 1000 simulated events derived using the P15 alone and P15+BAO dataset. The lower left and lower right panels respectively present the catalogue ($z$, $d_L(z)$) with the corresponding error bars for 1000 simulated events derived using the CMB+Pantheon and P15+BAO+Pantheon datasets.

FIG. 7: 1-dimensional marginalized posterior distributions for some parameters of Model II for the datasets P15 and P15+GW.

CL) while when GWs are added to P15, then $H_0$ is reduced both in its mean values as well as in its error bars, $H_0 = 69.63^{+0.48}_{-0.57}$ (68% CL, P15+GW). In fact, the error bars on $H_0$ are reduced by a factor of more than 10. Thus, a real effect on the $H_0$ parameter for the introduction of GWs data is clearly visible. Similar effects on other cosmological parameters are equally evident. As one can see that the equation-of-state for dark energy is significantly improved after the addition of GWs to P15. More explicitly, the upper limit (at 68% CL) of the dark energy equation of state changes from $w > -1.917$ (P15) to $w > -1.084$ (P15+GW). Thus, looking at the constraints on $w$, one can clearly conclude that the inclusion of GWs to P15 not only decreases the mean values of $w$ taken for P15 data alone, but also this reduces the error bars of $w$ that arise from P15 data only. Finally, we come to the most important part of this work, namely the behaviour of $c^2_{vis}$. We found that neither CMB nor P15+GW could constrain this parameter. Exactly similar conclusion has been found for Model I. So, irrespec-
TABLE V: In this table we show the constraints on various free and derived cosmological parameters of Model II at 68% and 95% CL after the inclusion of GWSS data with the standard cosmological probes P15, BAO and Pantheon.

For rest of the analyses with other cosmological datasets, we refer to Fig. 9 showing the 1D marginalized posterior distributions of some selected parameters. In particular, the upper panel of Fig. 9 compares the constraints on the model parameters for P15+BAO and its companion P15+BAO+GW. The lower panel of Fig. 9 similarly compares the constraints from P15+Pantheon and P15+Pantheon+GW and finally the last panel of Fig. 9 compares the constraints of some selective model parameters for P15+BAO+Pantheon and P15+BAO+Pantheon+GW. From Fig. 9 it is clear that
due to the inclusion of GW to the standard cosmological datasets, some of the model parameters are affected, for instance the effects on $H_0$ and $\Omega_{m0}$ are pretty clear while the effects on $w$ are not so pronounced much. However, the parameter on which we concentrate our focus in this work, namely, $c_{vis}^2$, is still unconstrained irrespective of either the usual cosmological probes or the inclusion of GW to them. So, effectively even if we include the GW data into the standard cosmological probes, this specific parameter remains unconstrained, that means GW fails to constrain it.

Finally, similar to Model I, here too, we consider the analyses with P18 and P18+GW. The results are shown in Fig. [10]. One can clearly see that P18 data are unable to constrain the parameter $c_{vis}^2$ which is unconstrained also by the P15 data.

5. CONCLUSION AND THE FINAL REMARKS

In this article we have considered a very general cosmological framework in which the dark energy component has an anisotropic stress. Our aim is to constrain the anisotropic stress of dark energy in terms of the viscous sound speed $c_{vis}^2$ using the usual cosmological probes, namely, P15, BAO, Pantheon and then measure the constraining power of GWs data. We have considered two different scenarios as follows: (i) Model I where $c_{vis}^2 > 0$, $w > -1$ and (ii) Model II in which $c_{vis}^2 < 0$, and $w < -1$.

We have performed several observational tests. Initially, we consider the analyses with P15, P15+BAO, P15+Pantheon and P15+BAO+Pantheon and afterwards we have measured the effects of the GWs data from Einstein Telescope. For Model I, the results of our analy-
GW data, namely, P18+GW could effectively constrain new CMB data, i.e., P18 and also its combination and applied to the models in order to see whether the considered the latest and final Planck data (P18) release.

Figure 3, we show the power of GW in constraining the cosmological parameters through the 1D marginalized posterior distributions of some important parameters.

We find that indeed simulated GW data are able to provide stringent constraints on the cosmological parameters, specifically, we find GWs data very powerful to constrain $H_0$, $\Omega_m$, and the dark energy equation of state, $w$. However, the parameter $c_{\text{vis}}^2$ remains degenerate with every parameter of the model and this does not alter even after the inclusion of the GWs data to the standard cosmological probes. This is a striking and surprising fact of this work where in one hand we see that GW data are extremely effective to reduce the parameter space, while on the other hand, this is not true when we consider the viscous sound speed. Thus, novel and complementary astrophysical probes need to be found to probe the value of $c_{\text{vis}}^2$.

Concerning the CMB data from Planck we have also considered the latest and final Planck data (P18) release and applied to the models in order to see whether the new CMB data, i.e., P18 and also its combination with GW data, namely, P18+GW could effectively constrain the parameter $c_{\text{vis}}^2$ which has been unconstrained for the previous datasets. We found that neither the final P18 data nor the combined data P18+GW are able to constrain this parameter (see Fig. 5 for Model I and Fig. 10 for Model II).

As a final comment we must mention that it is essential to confront other future experiments like EUCLID [110, 111], CMB Stage-4 [112], Dark Energy Spectroscopic Instrument (DESI) [113], Large Synoptic Survey Telescope (LSST) [114, 115, 116], Simons Observatory [117], in order to see whether the unconstrained parameter $c_{\text{vis}}^2$ for the present datasets, could be constrained well by them. Such an analysis is left for a future work.

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