Experimental demonstration of a non-destructive controlled-NOT quantum gate for two independent photon-qubits

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(Dated: April 1, 2022)

Universal logic gates for two quantum bits (qubits) form an essential ingredient of quantum information processing. However, the photons, one of the best candidates for qubits, suffer from the lack of strong nonlinear coupling required for quantum logic operations. Here we show how this drawback can be overcome by reporting a proof-of-principle experimental demonstration of a non-destructive controlled-NOT (CNOT) gate for two independent photons using only linear optical elements in conjunction with single-photon sources and conditional dynamics. Moreover, we have exploited the CNOT gate to discriminate all the four Bell-states in a teleportation experiment.

PACS numbers: 03.67.Lx, 42.50.Dv

The controlled-NOT (CNOT) or similar logic operations between two individual quantum bits (qubits) are essential for various quantum information protocols such as quantum communication [1, 2, 3] and quantum computation [4]. In recent years, certain quantum logic gates have been experimentally demonstrated, for example, in ion-traps [5, 6] and high-finesse microwave cavities [7]. These achievements open many possibilities for future quantum information processing (QIP) with single atoms. Another promising system for QIP is to use single photons. This is due to the photonic robustness against decoherence and the availability of single-qubit operation. However, it has been very difficult to achieve the necessary logic operations for two individual photon-qubits since the physical interaction between photons is much too small.

Surprisingly, Knill, Laflamme and Milburn (KLM) has shown that nondeterministic quantum logic operations can be performed using linear optical elements, additional photons (ancilla) and postselection based on the output of single-photon detectors [8]. The original proposal by KLM, though elegant, is not economical in its use of optical components. Various schemes have been proposed to reduce the complexity of the KLM scheme while improve its theoretical efficiency [8, 10, 11]. Remarkably, a recent scheme proposed by Nielsen [12] suggests that without using the elaborate teleportation and Z-measurement error correction in the KLM scheme, any non-trivial linear optical gate that succeeds with finite probability is sufficient to obtain efficient quantum computation. Hence, this scheme significantly simplifies the experimental implementation of linear optical quantum computation (LOQC).

A crucial requirement in the schemes of LOQC is the so-called classical feedforwardability, that is, it must be in principle possible to detect when the gate has succeeded by performing some appropriate measurement on ancilla photons [8, 12]. This information can then be feedforwarded for conditional future operations on the photonic qubits to achieve efficient LOQC.

Recently destructive CNOT operations have been realized using linear optical elements [13, 14, 15, 16]. However, as they necessarily destroy the output state such logic operations are not classically feed-forwardable and have little practical significance. Fortunately, it has been suggested [16] that a destructive CNOT gate together with the quantum parity check can be combined with...
a pair of entangled photons to implement a nondestructive (conventional) CNOT gate that satisfies the feed-forwardability criterion.

In this paper, we present for the first time a proof-of-principle demonstration of a non-destructive CNOT gate for two independent photons, realizing the proposal of Pittman, Jacobs, and Franson [10]. The quality of such a CNOT gate is further demonstrated by discriminating all the four Bell-states in an experiment on quantum teleportation.

In our experiment, we consider qubits implemented as the polarization states of photons. We define the horizontal polarization state \( |H \rangle \) as logic 1, and the vertical one \( |V \rangle \) as logic 0. As shown in Fig. 1a, one can achieve the desired nondestructive CNOT gate for photons 2 and 5 by performing a quantum parity check on photons 2 and 3 and a destructive CNOT operation on photons 4 and 5, where photons 3 and 4 are in the state \( |\Psi^-\rangle_{34} \), which is one of the four Bell states

\[
|\Phi^\pm\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|H\rangle_j \pm |V\rangle_i|V\rangle_j), \\
|\Psi^\pm\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|V\rangle_j \pm |V\rangle_i|H\rangle_j).
\]

Here \( i \) and \( j \) index the spatial mode of the photons. Then, according to ref. [10] the nondestructive CNOT gate for photons 2 and 5 can be accomplished conditioned on detecting a \( |\rangle \) photon in mode 3' and a \( |H\rangle \) photon in mode 4'. The logic table of the CNOT operation is given by

\[
|V\rangle|V\rangle \rightarrow |V\rangle|V\rangle, |V\rangle|H\rangle \rightarrow |H\rangle|H\rangle, |H\rangle|V\rangle \rightarrow |H\rangle|H\rangle \text{ and } |H\rangle|H\rangle \rightarrow |H\rangle|V\rangle.
\]

One immediate application of the proposed CNOT gate is that it can be used to generate entanglement between the control qubit and target qubit [18]. For example, by setting the control bit to be in the state \( |\rangle \) and the target qubit in the state \( |H\rangle \), one can utilize the non-destructive CNOT gate to prepare photons 2 and 5 in the entangled state \( |\Psi^-\rangle_{25} \).

Another important application is that the non-destructive CNOT gate can be used to simultaneously identify all the four Bell states [18] in a quantum teleportation protocol [2]. For example, suppose photon 5, which Alice wants to teleport to Bob, is in an unknown polarization state \( |\Phi\rangle_5 = \alpha |H\rangle_5 + \beta |V\rangle_5 \), and the pair of the photons 1 and 2 shared by Alice and Bob is in the entangled state \( |\Psi^-\rangle_{12} \) (Fig. 1b). It is necessary to discriminate the four Bell-states of photons 2 and 5, \( |\Psi^\pm\rangle_{25} \) and \( |\Phi^\pm\rangle_{25} \) in order to realize the complete quantum teleportation [2].

Under the CNOT operation the four Bell-states of photons 2 and 5 will evolve into one of the four orthogonal separable states

\[
|\Psi^\pm\rangle_{25} \rightarrow |\pm\rangle_{25} |H\rangle_5', \\
|\Phi^\pm\rangle_{25} \rightarrow |\pm\rangle_{25} |V\rangle_5'.
\]

Therefore, the crucial Bell-state measurement can be accomplished probabilistically by applying a nondestructive CNOT operation and performing a subsequent polarization analysis on photons 2 and 5. Depending on Alice’s measurement results, Bob can then perform a unitary transformation, independent of \( |\Phi\rangle_5 \), on photon 1 to convert its state into the initial state of photon 5.

A schematic of the experimental setup used to demonstrate both the CNOT gate and the identification of the four Bell states required for quantum teleportation is shown in Fig. 2. To realize the CNOT gate, it is necessary to overlap photons 2 and 3 at the PBS1 and photons 4 and 5 at the PBS2. In the experiment the PBS2, i.e.
the desired 45-degree oriented polarizing beamsplitter, is accomplished by inserting one half-wave plate (HWP) in each of the two inputs and two outputs of an ordinary polarizing beamsplitter. Note that, all the four HWP were oriented at 22.5° with respect to the horizontal direction, which corresponds to a 45° polarization rotation. The good temporal overlap was achieved by adjusting the two delay mirrors, Delay 1 and Delay 2. Experimentally, we first adjust the position of Delay 1 such that photons 2 and 3 arrive at the PBS1 simultaneously, and then adjust the position of Delay 2 to achieve the temporal overlap of photons 4 and 5 at the PBS2. Furthermore, the use of narrow-band interference filters (F) with $\Delta \lambda_{FWHM} = 3$ nm for all five photons makes the photons at the same PBS indistinguishable [21]. The temporal overlap between photons 2 and 3 was verified by observing a four-particle interference visibility of 0.82 among photons 1, 2, 3 and 4 by removing the PBS2; and the temporal overlap of photons 4 and 5 was verified by observing a three-particle interference visibility of 0.68 among photons 3, 4 and 5 by removing the PBS1 [22, 23].

To experimentally demonstrate that the CNOT gate has been successfully implemented, we first prepare the input control and target qubits in the following specific states $|H\rangle_2 |H\rangle_5$, $|H\rangle_2 |V\rangle_5$, $|V\rangle_2 |H\rangle_5$ and $|V\rangle_2 |V\rangle_5$. If the CNOT gate works properly, then conditioned on detecting a $|\rangle$ polarized photon in mode $3'$ and a $|H\rangle$ polarized photon in mode $4'$ the two qubits in modes $2'$ and $5'$ would be, respectively, in the states $|H\rangle_2 |V\rangle_5'$, $|H\rangle_2 |V\rangle_5'$ and $|V\rangle_2 |V\rangle_5$. After the non-destructive CNOT operation the output components corresponding to the above specific input states, which were measured in the $H/V$ basis, are shown in Fig. 3a, respectively. The experimental fidelity of achieving the CNOT logic table is estimated to be $0.78 \pm 0.05$.

Second, to show the CNOT gate also works for an arbitrary superposition of the control qubit, we now prepare the control qubit in the state $\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ and the target qubit in the state $|H\rangle$ to entangle these two independent photons. Then, after the CNOT operation the two output qubits would be in the state $\frac{1}{\sqrt{2}}(|H\rangle |V\rangle - |V\rangle |H\rangle)$, i.e. the Bell state $|\Psi^-\rangle$. To verify the expected Bell state has implemented successfully, we first measured the four possible polarization combinations of the control and target qubits in the $H/V$ basis. The signal-to-noise ratio between the desired ($|H\rangle |V\rangle$ and $|V\rangle |H\rangle$) and unwanted ($|H\rangle |H\rangle$ and $|V\rangle |V\rangle$) were measured to be $4.2 : 1$. This confirms that the $|H\rangle |V\rangle$ and $|V\rangle |H\rangle$ terms are the dominant components. Furthermore, to prove the two terms are indeed in a coherent superposition, we also perform a conditional coincidence measurement as a function of the orientation of polarizer P2 as polarizer P5 was fixed at $+45^0$. As shown in Fig. 3b, the experimental results of the polarization correlation exhibit an interference fringe with a visibility of $0.58 \pm 0.09$, which is in consistent with the prediction of the interference fringe for the Bell state $|\Psi^-\rangle$.

The CNOT gate can be used not only to entangle two independent photons, it can also be used to disentangle two entangled photons. To demonstrate the latter, let us now exploit the CNOT gate to simultaneously discriminate all the four Bell-states in an quantum teleportation experiment. As described in equation (2), conditioned on detecting a $|\rangle$ photon in mode $3'$ and a $|H\rangle$ photon in mode $4'$ the required joint Bell-state measurement can be achieved by performing a polarization measurement both on photon $2'$ in the $+/-$ basis and on photon $5'$ in the $H/V$ basis. For example, registering a $|+/\rangle_2' |H\rangle_5'$ coincidence implies a projection onto the Bell-state $|\Psi^+\rangle_{25}$. In this way, we can identify all the four Bell-states. According to teleportation protocol [2] it is obvious that,
throughout the whole experiment the fidelities were observed to be $F \approx 0.79 \pm 0.05$, which clearly surpasses the classical limit of $2/3$. Note that the fidelities were obtained without performing any background subtraction.

Finally, we emphasize that in the above teleportation experiment we only verified the corresponding relation between the initial state of photon 5 and the final state of photon 1 after the Bell-state analysis is complete, but did not perform a conditional operation on the photon 1 to convert its final state into the original state of photon 5. We plan to address this challenging task in a forthcoming experiment.

In summary, we have for the first time experimentally demonstrated a probabilistic nondestructive CNOT gate for two independent photons using only linear optics. Furthermore, we demonstrated that such a device can be used not only to entangle two independent photons, but also to discriminate all the four Bell-states for quantum teleportation. We believe that the methods developed for this experiment would have various novel applications in quantum information processing with linear optics.

This work was supported by the NSF of China, the CAS and the National Fundamental Research Program (under Grant No. 2001CB309303), and the Alexander von Humboldt Foundation.

FIG. 4: The experimental results for quantum teleportation with complete Bell-state analysis. The data clearly confirm the expected phase shift, hence demonstrating that the four Bell-states have been identified successfully in the teleportation experiment.

if the photons 2 and 5 are measured to be in the state $\ket{\Psi^-}_{25}$, then photon 1 will be projected into the state $\alpha \ket{H} + \beta \ket{V}$; if the photons 2 and 5 are measured to be in the state $\ket{\Phi^+}_{25}$, then photon 1 will be left in the state $\alpha \ket{H} - \beta \ket{V}$. In these two cases, the two corresponding states of photon 1 would, in general, have a relative phase shift of $\pi$. Similarly, for the projections onto the state $\ket{\Phi^+}_{25}$ or $\ket{\Phi^-}_{25}$, photon 1 will be correspondingly left in the state $\alpha \ket{V} - \beta \ket{H}$ or $\alpha \ket{V} + \beta \ket{H}$. Again, the two states of photon 1 have a relative phase shift of $\pi$.

To experimentally verify the above analysis, we decided to teleport the left-hand circular polarization state $\frac{1}{\sqrt{2}}(\ket{H} - i \ket{V})$ from photon 5 to photon 1. The output circular polarization states of photon 1 are analyzed by inserting a QWP and a polarizer in front of the detector D1. As shown in Fig. 4a and b, the five-fold coincidences are recorded as polarizer 1 was rotated. The experimental results are consistent with the prediction of the phase shift, hence confirming that the four Bell-states have been successfully discriminated. Following the same definition of ref. [24], our experimental average fidelity of teleportation was estimated to be $F = 0.79 \pm 0.05$, which clearly surpasses the classical limit of $2/3$. Note that throughout the whole experiment the fidelities were obtained without performing any background subtraction.

Finally, we emphasize that in the above teleportation experiment we only verified the corresponding relation between the initial state of photon 5 and the final state of photon 1 after the Bell-state analysis is complete, but did not perform a conditional operation on the photon 1 to convert its final state into the original state of photon 5. We plan to address this challenging task in a forthcoming experiment.

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