A Comparison of Tomographic SAR Reconstruction Methods Using Spaceborne Data

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Abstract — The main objective of the paper is to give a review and comparison of the reconstruction techniques in SAR tomography. The algorithms are compared with a real dataset obtained from TerraSAR-X. The paper explains the building of Tomographic data for high resolution (HS) Spotlight mode. Further, a comparative analysis of MUSIC and Compressive sensing (CS) based Slimmer will be presented.

Keywords — Synthetic aperture radar (SAR), SAR Tomography, TerraSAR-X, high resolution spotlight, reconstruction algorithms, compressive sensing.

I. INTRODUCTION

Synthetic Aperture Radar has established itself as an advanced method in the field of remote sensing. It allows Earth’s surface to be mapped from a moving platform such as an airplane or satellite. The imaging geometry of SAR is hereby different to the optical geometry, as the imaging plane is rather spanned by azimuth and range instead of azimuth and elevation. Hence a single SAR image projects a three dimensional scene onto a two dimensional plane. To retrieve the real three dimensional (3-D) localization and motion of scattering objects, advanced interferometric methods, like persistent scatterer interferometry (PSI) or SAR tomography, are required, which exploit stacks of complex-valued SAR images with diversity in space and time [1]. The recent advances of the space technology involving constellations of satellites, and the planning for future missions involving formation of satellites, have the ability to acquire simultaneous (multi-baseline) SAR images repeatedly over the time. This, in turn, has pushed the research and development of new techniques capable of processing, jointly and coherently, the stacks of SAR images [2]. SAR tomography (TomoSAR) is a technique that allows resolving scatterer densities in the third native radar co-ordinate “elevation (s)” by exploiting its ability to acquire multipass acquisitions at different orbital positions to form a virtual array [3]. TomoSAR is a major contributor to monitor and explore solutions for urban area developments as they play an important role in tackling the global climate change. Over the last two decades SAR Tomography and differential SAR Tomography has been extensively researched [4], [5], [6], [7], [8], [9], [10].

Many imaging techniques have been invented which can provide high resolution 3-D mapping and reconstruction of an urban area. However, some of the requirements for urban monitoring are maintaining the meter azimuth-range resolution, improving the elevation resolution, achieving high 3-D localization accuracy even in the presence of unmodeled non-Gaussian noise and retrieving nonlinear motion [1].

Taking the requirements into consideration, this paper explores the different methods involved in the reconstruction of Tomographic SAR spaceborne data.

II. TOMOSAR IMAGING MODEL

The TomoSAR data stacks are formed by multipass acquisitions of an area of interest at different times and across different orbital positions.

The Fig.1 depicts the geometry of TomoSAR for different positions \( P_n \forall n \in [1, N] \) with \( L_x \) as the cross-range aperture and \( \vartheta \) being the look angle. Height profile of the scatterers fall in the same range-azimuth resolution cell for a single path. The range resolution \( \rho_r \) depends on the system bandwidth \( B \) and the azimuth resolution \( \rho_z \) depends on the length of the synthetic aperture \( L_x \).
where, $\gamma(s)$ is the reflected signal along elevation $s$, $r_n(s)$ is the distance of the scatterer from the $n^{th}$ antenna at elevation $s$. The distance at elevation point $O$, $r_n(0)$, is subtracted from data at each antenna, $r_n(s) - r_n(0)$. Therefore, the data processed via deramping is given by the focused complex value,

$$g_n = h_n \exp \left( -j \frac{4\pi}{\lambda} r_n(s) \right) = \int_{\Delta s} \gamma(s) \exp(-j 2\pi \xi_n s) ds$$

where, $\xi_n$ is the spatial frequency in elevation given by,

$$\xi_n = - \frac{2b_m}{\lambda r}$$

(3)

The imaging model can be formulated as with the noise parameter, $\epsilon$ is given as,

$$g = R\gamma + \epsilon$$

where, $g = [g_1, g_2, \ldots, g_n]^T$, $R$ is a $N \times L$ matrix given as $R_{nl} = \exp(-j2\pi \xi_n s_l)$, $\gamma$ is a discrete reflectivity vector for $L$ elements, $s_l \forall l \in [1, L]$ is the discrete elevation positions. $s$ profiles for every $x \rightarrow r$ pixel can be obtained by SAR tomographic processing of the data stacks. Reconstruction of these $s$ profiles is the spectrum estimation problem and valid scatterers can be obtained from the focused reflectivity function of the model.

### III. OVERVIEW OF EXISTING ALGORITHMS

Based on the standard linear equations for tomographic SAR system, as given in Eq.(4), the reconstruction of the $s$ profile can be given as the inversion of the system. The inversion must be carefully implemented because: (i) the Fourier samples are irregularly spaced at $\xi_n$, (ii) their number $N$ may be small, (iii) the SNR may be low for the majority of the pixels, (iv) the data may contain non-Gaussian phase noise due to uncompensated atmospheric delay and unmodeled motion and (v) the orbit tube of modern SAR satellites is tight leading to a low elevation resolution [11].

There are many reconstruction techniques and the most relevant ones are beamforming, Singular Value Decomposition (SVD), Capon, MUSIC, Nonlinear Least Squares and SL1MMER. The paper explores MUSIC and SL1MMER technique.

#### A. Multiple Signal Classification (MUSIC)

A prominent model based spectral estimation method is the Multiple Signal Classification (MUSIC). The covariance matrix $C_{gg}$ is separated into eigenvalues and eigenvectors. The $K$ number of scatterers can be estimated from the eigenvalues or can be given as a priori. $N \geq K$ eigenvectors determine the noise subspace, $\hat{G}$. Hence, the spectrum at each elevation position, $s_l$ is given as

$$|\hat{\gamma}|^2_{MUSIC} = \frac{1}{r_l^H \hat{G} \hat{G}^H r_l}$$

(5)

MUSIC offers better resolution and sidelobe suppression in comparison to beamforming and Capon [12]. It should also be noted that the performance of MUSIC is good for uncorrelated signals but the estimation precision deteriorates when the signals are highly correlated [13].

#### B. SL1MMER

A CS-based TomoSAR algorithm known as Scale-down by L1 norm Minimization, Model selection and Estimation Reconstruction (SL1MMER), is a spectral estimation algorithm and was first demonstrated by [12]. The algorithm explores sparsity in the elevation direction. The algorithm can be divided into 3 main parts: 1) a dimensionality scale down by $L_1$ norm minimization, 2) model selection and 3) linear parameter estimation. Assuming that the elevation profile is sparse, there can be infinite number of solutions for such a system and the sparest and most probable outcome is given by the $L_0$ norm of the reflectivity vector. Since the problem is NP-hard, the $L_0$ norm is replaced by the $L_1$ norm. Since the raw data measurement is noisy, the $L_1$ norm is minimized with a residual data term.

$$\hat{\gamma} = \arg \min_{\gamma} \| g - R\gamma \|_2^2 + \lambda_\gamma \| \gamma \|_1$$

(6)

The sparsest estimate of reflectivity can be given by the Eq.(6) given that the matrix $R$ satisfies the two main properties of compressive sensing: Restricted Isometry Property (RIP) and incoherence [14]. Nevertheless, these properties are not completely satisfied in TomoSAR for several reasons such the matrix $R$ is not optimum and is predetermined by the measurement system and oversampling of the elevation points can make $R$ overcomplete. However, the step of dimensionality scale down by $L_1$ norm minimization can shrink $R$ and give the sparsest estimate of $\gamma$ which may contain outliers.

The second step of SL1MMER is the model order selection which removes the estimate of $\gamma$ of insignificant, spurious scatterers and gives an estimate of the number of scatterers, $\hat{K}$ in the elevation profile. Assuming $\theta(K)$ as the unknown phases, amplitudes and elevation of each of the scatterers, the best model fit can be given by the likelihood as: $p(g|\theta(K), K)$. The likelihood criteria is then given by:

$$\hat{K} = \arg \min_K \left\{ -2 \ln p(g|\theta(K), K) + 2C(K) \right\}$$

(7)

where, $C(K)$ is the complexity penalty and is proportional to the number of scatterers, $K$, i.e., $C(K) \propto ||\gamma||_0$. Eq.(7) is a $L_2 - L_0$ norm minimization problem with a few known $K$ candidate elevation positions.

The final step in the algorithm is the is the linear parameter estimation where $R(\hat{s})$ is the $N \times \hat{K}$ mapping matrix with $R_{nl,k} = \exp(-j2\pi \xi_n s_k), s_k$ is the estimated elevation value of $k^{th}$ scatterer. The final estimate of reflectivity is obtained by solving the following linear system of equation:

$$g = R(\hat{s})\gamma(\hat{s}) + v'$$

where $v'$ involves the model error and measurement noise. Hence, the estimate of complex valued reflectivity is given by the following least squares solution

$$\hat{\gamma}(\hat{s}) = (R(\hat{s})^H R(\hat{s}))^{-1} R(\hat{s})^H g$$

(9)
The final step is performed as the $L_2 - L_1$ norm minimization tends to underestimate the value of $\gamma$. Since amplitude fidelity is not the main purpose of TomoSAR makes the SL1MMER an unbiased CS algorithm [1][12].

IV. FIRST RESULTS USING SPACEBORNE DATA

The main goal of our project is to compare different methods for reconstructing tomographic SAR data. For verification purposes, we intend to check the robustness of the methods using the spaceborne dataset.

A. Geometry and data analysis

The scene of investigations is the city center of Cologne including different types of buildings and infrastructure. The data obtained from TerraSAR-X, covered the area in a high number of repetitions within a period from 08/05/2012 to 12/10/2014 in High Resolution Spotlight (HS) mode using an operating bandwidth of 300 MHz at a center frequency of 9.65 GHz and is of single VV polarization.

Fig. 2 shows the interferometric phase of the city center using a baseline of 373 m resulting in height ambiguity of approx. 18 m per fringe. The range direction is from top to bottom.

A data set of 40 processed SAR images is available for the study. In general the satellite orbit is kept in a tube of 250 m (standard deviation) in relation to a reference orbit [15], a different across-track angle is available for each acquisition, which enables SAR interferometry and SAR tomography. Fig. 3 gives the used perpendicular and temporal baselines for the stack. The perpendicular baseline varies irregularly from $-200$ to $467$ m which leads according to

$$\rho_n = \frac{\lambda}{2d_n}$$

(10) to an expected elevation resolution [12] of 15.175 m for our scene chosen.

Fig. 2. Interferogram subset of the TerraSAR-X high resolution spotlight data, 22/07/2013 - 11/11/2012 (Cologne city center, image ©DLR 2014).

B. Data pre-processing stages

The main tasks involved in SAR tomography are the pre-processing stages which involve coregistration, interferogram formation and the generation of Persistent Scatterer Interferometry (PSI) (see Fig. 4). These stages are carried out using the European Space Agency’s (ESA) SNAP 8.0.0.

Image registration is one of the preliminary steps before performing InSAR processing. The spotlight coregistration ia performed by firstly creating a stack of the data, i.e., collocating two spatially overlapping data. The main components in coregistration are cross-correlation and warping. An optimal master image is chosen with the condition that the dispersion of the perpendicular baseline is minimum which in turn maximizes the coherence of the interferometric stack. This aids in improved visual interpretation of the interferogram.

The interferometric processing for the case of spotlight mode requires attention because of the increased spatial resolution in the azimuth direction caused due to backward sweeping of the azimuth beam during acquisition [15]. This results in a doppler drift, hence the azimuth bandpass filtering and the resampling of the slave image must be taken into consideration during interferometry.

The interferogram is generated with the consideration of the elevation model to obtain the persistant scatterers. The scatterers are obtained via the StaMPS tool [16]. This data is exported to form the imaging model and apply MUSIC and Sl1mmer for data reconstruction.

V. CONCLUSION

The results produced in this paper are partial. The evaluation of PSI for the TomoSAR dataset and the comparison of the mentioned algorithms will be updated soon. The
Fig. 4. Block diagram for TomoSAR data reconstruction.

comparison includes the computation time, robustness and verification of the height results with respect to the ground truth of dataset.

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