Numerical simulation of an axisymmetric discharge in a supersonic air co-flow

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Abstract. The integral and local characteristics of an electric discharge initiated between two coaxial relatively thin conical electrodes are considered. The electrodes are located in an incoming uniform stationary supersonic flow of cold air (M=2, T_g = 167 K, p = 22 kPa) The axis of symmetry of the electrode system is oriented along the flow velocity vector. Numerical simulation of a two-dimensional unsteady flow of thermochemically nonequilibrium air in a self-consistent electric field is performed using the MHD configuration of the PlasmAero package [1] for a nonequilibrium quasi-neutral partially ionized plasma. The evolution of the discharge with a stepwise change in the discharge current from 10 mA to 10 a with the duration of each stage until the practical stabilization of the discharge is achieved is presented. In this case, the discharge passes through stages from a classical glow discharge of moderate pressure to an almost arc discharge, in which, however, the role of field non-thermal ionization is still great, especially near current-carrying spots on the electrodes. The important role of chemical disequilibrium is also noted.

1. Introduction
Studies of electric discharge characteristics in high-speed flow conditions have been widely developed since the early 60s, largely due to the urgent need to search new effective technologies in the field of aerospace and energy conversion [2-4]. In the last couple of decades, one of the most dynamically developing industries has developed a specific direction of research on plasma actuators - means and methods for using electric discharges (sometimes in the presence of an additional magnetic field) by creating thermal and dynamic effects on the gas-dynamic and physical-chemical characteristics of the flow. Various aspects of this problem have been studied in numerous papers over the past 10-20 years: discharge dynamics due to high-speed flow of the working medium [5], the effect of discharge on the shock-wave structure [6, 7], boundary layer control [8], momentum and heat release effects [9], spectral characteristics of mixtures of various compositions [10]. This study was initiated by research on ignition control and combustion maintenance in supersonic flows of fuel-air mixtures [11-14] and focuses, in particular, on determining detailed information about physicochemical and thermochemical processes inside and near the current channel in high-speed flows of the working fluid. The specific task of this work is to analyze in details the channel structure and the quasi-stationary behavior of its integral characteristics with a sufficiently "slow" change in load parameters within the limits of practical interest of experimental studies. Due to the usual size restrictions (tens of mm for laboratory conditions), the quasi-stationary nature of the process is provided at times of the ten microseconds...
order. The considerable complexity of the research object makes us look for acceptable simplifications in the problem statement, for example, by eliminating the usual configuration difficulties for experiments with a complex spatial shape of the electrodes, the inevitable influence of the chamber walls, and so on. At the preliminary stages a significant influence of the electrode geometry was established. At this (initial) stage, the results of the integral and local (profile) distributions of the current-carrying discharge channel, the evolution of the current-voltage characteristic with a step change in the integral discharge current, as well as preliminary estimates of the effects of chemical disequilibrium will be presented.

2. Statement of the problem

Following [12], we consider a homogeneous stationary air flow with parameters: Mach number = 2, static pressure \( P = 22 \text{ kPa} \), static temperature \( 157 \text{ K} \). The interelectrode gap is the area between the vertices of thin (\( r_{\text{max}} = 200 \) microns) conical electrodes oriented strictly along the flow and separated from each other by a distance of 10 mm.

It is assumed that the discharge power system has the necessary power to provide the required discharge current at any given time. A mixture of \( 0.78 \text{ N}_2 + 0.22 \text{ O}_2 \) and its chemical transformation during heating in the electric discharge zone is considered as the working medium.

The process is described in a cylindrical coordinate system. The mathematical formulation is determined by the following system of equations

2.1. Basic equations

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{U}) &= 0 \\
\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla (\rho \mathbf{U} \cdot \mathbf{U}) + \nabla \tau &= -\frac{\partial P}{\partial r} + \mathbf{j} \times \mathbf{B} \\
\frac{\partial \rho e^0}{\partial t} + \nabla (\rho (e^0 + P) \mathbf{U}) + \nabla \mathbf{q} + \nabla (U \tau) &= \mathbf{j} E \\
\frac{\partial \rho_i}{\partial t} + \nabla (\rho_i \cdot \mathbf{U}) + \nabla \Gamma_i &= \omega_i \\
\nabla \mathbf{j} &= 0
\end{align*}
\]

2.2. Fluxes

\[
\begin{align*}
\Gamma_i &= -\rho D_i \frac{\partial Y_i}{\partial r} \\
\mathbf{q} - \lambda \frac{\partial T}{\partial r} + \sum_{i \neq e} h_i \Gamma_i \\
\tau_{ij} &= \frac{2}{3} \eta \delta_{ij} \nabla \mathbf{U} - \eta \left( \frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} \right) \\
\mathbf{j} + \frac{\beta}{B} (\mathbf{j} \times \mathbf{B}) &= \sigma (E + (\mathbf{U} \times \mathbf{B})) \\
E &= -\frac{\partial \phi}{\partial r}
\end{align*}
\]

2.3. Equations of States

\[
e^0 = e + \frac{u^2}{2}; \quad P = \sum \rho_i R_i T; \quad e = \sum_{i=1}^{N_1} \epsilon_i e_i^0 + \sum_{i=1}^{N_v} v_i e_i + \sum_{i=1}^{N_1} \epsilon_i^1 + \sum_{i=1}^{N_2} \epsilon_i^2 + e_e \\
\rho = \sum \rho_i; \quad c_i = \frac{\epsilon_i}{\omega_i}
\]

2.4. Chemical kinetics
\[ \omega_i = W_i \sum_{r=1}^{N_r} \left[ \nu_{i,r}^n - \nu_{i,r}^f \right] \cdot \left( \lambda_{fr} \prod_{l=1}^{N} \left[ \nu_{i,l}^v - \nu_{i,l}^n \right] \right) \]

\[ k_{fr} = a \cdot T^b \cdot \exp(-E_a/T) \]

\[ k_{fr} = a \cdot T^b \cdot \exp(-E_a/T_e) \]

2.5. Electrophysics

\[ \sigma = e n_e \mu_e; \quad \mu_e = e \frac{\tau_e}{m_e} = \frac{\tau_e}{B}; \quad \tau_e = v_e^{-1}; \quad v_e = \sum v_{ej}; \quad v_{ej} = n_j u_e Q_{je} \]

This paper uses a widely recognized 11-component model of air in physical gas dynamics – (N2, O2, NO, N, O, N2+, O2+, NO+, N+, O+, e), tested in a wide temperature range of 0 – 20000 K [15-18]. The scheme of thermochemistry is borrowed from the work of K. Park [16], and the set of plasma chemical reactions is formed by N. A. Popov [19].

The used kinetic scheme is shown in Table 1.

| Table 1 Thermo-Chemistry and Plasma-Chemistry Kinetics |
|------------------------------------------------------|
| 1) O2+O2=O+O2+O2+O2  | 17) N+=Np+e+e  | 33) N+NOp=Np NO  |
| O+O+O+O2+O  | N+Np+e+e  | Np+NO=N+NO+Op  |
| 2) O2+N2+O+O2+N2  | 18) N+O+Np+O  | 34) O+O2p+Op NO  |
| O+NO+O2+O2+N2  | Np+O+O+N  | Op+NO=O+Op NO  |
| 3) O2+O+O+O+O  | 19) N+Np+e+N  | 35) O2+NOp=O2p NO |
| O+N+O2+O2+N  | Np+e+Np+N  | O2p+NO=O2+Op NO  |
| 4) O2+N+N+O2  | 20) O+O2p+Op  | 36) N2+O2p=Np+O2  |
| O+O+O+O2+N  | Op+e+O+O2  | N2p+O2=N2+O2p  |
| 5) O2+N+O+O2+N  | 21) O+O2p+Op  | 37) N2p+O2=Np+O2  |
| O+O+O+N+O  | Op+e+O+O  | Np+O2=Np+O2p  |
| 6) N2+N2+O+O2+N2  | 22) N+O2p+Op  | 38) N2p+O2=Np+O2  |
| N+O+O2+O2+N2  | Op+e+O+O  | O2p+O2=N2p+O2 |
| 7) N2+O2+N+N2+N2  | 23) O2p+e+O2p  | 39) N2p+O2p=Np+O2  |
| N+O2+N+N2+N2  | Op+e+O2p  | O2+Op=O2+Op NO  |
| 8) N2+N+N2+N  | 24) NO+O2p+e+e  | 40) N+Np+e+NO  |
| N+N+O2+N  | NOp+e+e+NOe  | Np+O+O2  |
| 9) N2+N+N+N+N  | 25) N+O=O+O+N  | 41) N2p+O=Np+O2  |
| N+N+N+N+N  | Op+e+O+O  | N2p+N+2Np  |
| 10) N2+N+N+N+N  | 26) NO+O2p+e+e  | 42) NO+O2p+e NO  |
| N+N+N+N+N  | O2p+e+O2p  | N2p+N+2Np NO  |
| 11) NO+O2+O+O2+N  | 27) O+O2p+e+e  | 43) O+O2p+e+e  |
| O+O+O2+N  | O2p+e+O2p  | O2p+N+O+O  |
| 12) NO+N+N+O+N2  | 28) N+Np+2p+e+e  | 44) N+Np+2p+e+e  |
| N+O+N+N+O2  | N2p+e+e+N  | N2p+N+2Np  |
| 13) NO+O+O+O+N2  | 29) N+O+O2p+e+e  | 45) NO+O2p+e+e N2  |
| N+O+O+N  | O2p+e+O  | O2p+N+2Np NO  |
| 14) NO+O+N+N+N  | 30) N+O2p+e+e  | 46) N2p+O+Np+O  |
| N+O+N+N+N  | O2p+e+O2p  | Np+O2+N+O  |
| 15) NO+O+N+N+N  | 31) N+O2p+e+e  | 47) NO+O2p+e+e  |
| N+N+N+N+N  | O2p+N+2Np NO  | O2p+N+2Np NO  |
| 16) N2p+O2p+e+e  | 32) N2p+O2p+e+e  | 48) N2p+O2p+e+e  |
| N2p+e+e+N2  | N2p+e+e+N2  | N2p+e+e+N2  |

3. Boundary and initial conditions

At the left boundary, conditions are set for a homogeneous supersonic flow of a mixture of nitrogen and oxygen with a Mach 2 number, a static temperature of 167 KB, and a static pressure of 22 kPa.

On the lower boundary between the tips of the electrodes, the conditions of axial symmetry are assumed, on conical surfaces, the usual conditions on solid walls in viscous heat-conducting gas flow.

\[ \sigma = e n_e \mu_e; \quad \mu_e = e \frac{\tau_e}{m_e} = \frac{\tau_e}{B}; \quad \tau_e = v_e^{-1}; \quad v_e = \sum v_{ej}; \quad v_{ej} = n_j u_e Q_{je} \]
The equipotential current-collecting sections (~1mm and ~ 10mm at the output) are shifted about 100 mcm deep from the tips to the bottoms of the cones electrodes to prevent "non-physical" extreme values of electrical and, as a result, gas-dynamic values. Soft free outflow conditions are used on the upper and right borders.

To regularize the initial calculation process, the initial background ionization is set in the form of a small molar fraction of electrons and a positive ion $\zeta_{{NO}} = \zeta_{{e}} = 10^{-18}$. In addition, in the near-axial zone of the interelectrode gap with a radius of 40 microns, the initial concentrations of electrons and ions are increased by another ten orders of magnitude to the level of $10^{-8}$, which is still significantly lower than the concentration of electrons and ions in the entire range of discharge current considered below. For at least a qualitative account of viscous effects and heat transfer at the walls, the additional wall portion of the length of about 10 mm upstream the electrode region is introduced, at the lower end of which the conditions of solid insulating wall is specified. In the active electric discharge zone of the region an almost square grid with a step of ~ 10 mcm is used, and outside this zone a logarithmic grid is used to adapt to long-range boundary conditions.

4. Simulation result

**Figure 1. Voltage-Current Curve**

Figure 1 shows the current-voltage characteristic for a stepwise variation of the discharge current from 10 mA to 10 A. As can be seen from the figure, it takes several microseconds to stabilize the integral characteristics of the discharge. As a rule, the transition to the next value of the discharge current is carried out at a constant rate in the interval of 1 microsecond. At this interval, the discharge voltage behaves as if linearly – with a growth of current the voltage is also growing. After the current is stabilized, the discharge channel resistance decreases as a result of a rapid increase in the degree of ionization. Then, over a time interval of about 5 microseconds, the discharge is stabilized at a new voltage level that is lower than the previous one which corresponds to a lower current level. Thus, the falling characteristic of the channel discharge in the gas is built up.
The following figures show the longitudinal axial and radial distributions of the main gas-dynamic and physical-chemical parameters of the discharge at a current of 10 mA. The parameters correspond to the conditions of moderate pressure glow discharges, and "field" ionization plays a decisive role in maintaining the required level of ionization.

**Figure 4.** Distributions of velocity, pressure, temperature, potential, and current density in the meridional section of the discharge. A current of 10 mA. Load Time 15 µs

**Figure 5.** Distributions of velocity, pressure, temperature, potential, and current density in the meridional section of the discharge current of 3.33 A. Load Time 60 mcs
The temperature rise is due to the "excess" heat of Joule dissipation. As the discharge current increases, the temperature in the near-axial zone of the discharge quickly reaches a level at which the contribution of thermal ionization also seems to be significant. A qualitative illustration of this phenomena is similar data for the case with a discharge current of 3.33 A. The nature of the change in the transverse distributions in the middle cross-section with a stepwise change in the discharge current (current-voltage characteristic in Figure 1) can be clearly seen in the following Figure 8 and Figure 9. As the current grows, the zone of high (sufficient for the passage of a given current) electron concentration expands with a relatively uniform growth of the maximum values (see Figure 8). Gas temperature profiles behave very regularly (Figure 9, solid curves). An interesting feature is found in the evolution of electron temperature profiles - the behavior of the electron temperature maximum is non-monotonic and passes its maximum somewhere in the vicinity of 0.3 A currents, which apparently may indicate an intensification of the transition of the discharge mode from glow to arc.

5. Discussion

The results obtained already at the first stage allow us to clarify some generally accepted ideas about channel discharges in the air at a pressure close to atmospheric, and currents in the range from ten milliamperes to ten amperes. As can be seen from the above results, in this range of currents, the discharge under consideration is in the transition from the glow to the arc interval [20,21]. The...
numerical model used in this analysis does not contain special processing of near-electrode regions, which usually plays an important role in the classification and interpretation of both experimental and theoretical parameters of this type of discharge. This simplification is related to the main goal of our work – to obtain reliable data on the discharge parameters in its main regular part outside the near-electrode regions. Using the geometry of electrodes with "sharpeners" such as conical tips allows us to reach the lower limit of the current density in the framework of the diffusion-drift approximation of a quasi-neutral partially ionized plasma.

The second important aspect is the use of a reduced but fully functional model of chemical kinetics for a model 11-component medium based on \((0.22O_2 + 0.78N_2)\) using the simplest field ionization model [19]. Apparently, the approach used provides a reliable description of the discharge parameters beyond small near-electrode regions with localization of current binding to the electrode.

In conditions of supersonic longitudinal flow around the discharge channel, the most important mechanism of thermal stabilization is convective-conductive heat exchange, which provides heat removal from the high-temperature zone. The energy exchange process is particularly intense near the surface of the electrodes in areas with extreme current density. The use of an isothermal boundary condition on the electrode leads to the appearance of extreme densities of the heat flow into the wall, which, in principle, requires a special analysis of the possibility of heat removal entering the body of the electrode in the framework of solving the problem of conjugate heat exchange. This problem has not been investigated in this paper. On the flow side, limited heat transfer to the wall leads to a sharp increase in the gas temperature and, as a result, to an increase in pressure and a decrease in the gas density. As a result, a high-speed superheated gas jet is formed near this zone of extreme energy flows, which balances the heat exchange in this zone. On a smaller scale, this process of forming high-speed and, most importantly, subsonic jets occur in the vicinity of the axis of symmetry and downstream. The deformation of the velocity field is clearly visible in the two-dimensional distributions given above. It should be added that from the point of view of gas dynamics, the fact of a sharp decrease in the \(y\)-pressure in the high-temperature discharge regions and the possible occurrence of additional problems with the adaptation of these subsonic jets to the cold supersonic external flow outside the electrode system is important. In addition, the question arises about the degree of disequilibrium of the chemical composition under conditions of intensive transfer of components by an uneven velocity field, which is especially important in the vicinity of strong spatial inhomogeneities (for example, in high-temperature anode and cathode spots). Note also that the longitudinal velocity in the near-axial high-temperature region of the discharge almost everywhere, with the exception of only small near-electrode sections, exceeds the rate of ion drift.

The evolution of the radial profile of the electron temperature in the average cross-section, shown in Figure 8, is a definite sign of the transition of the discharge type from glow at low currents to almost arc at the maximum values up to 10 A considered here. It can be seen that at low currents of less than one ampere, the electronic temperature on the discharge axis increases with the load current, and at large it begins to fall (the falling volt-ampere characteristic of the discharge [20]. It should be noted that the electron temperature does not directly participate in this discharge model, but is only an information characteristic that is restored from approximation expressions as a function of the reduced field [19]. It can be shown that in the high-temperature core of the channel, with the exception of again small near-electrode regions, the profile field normalized to the maximum value also coincides with the normalized gas temperature profile, and the same is true with the corresponding profile fields, taking into account the normalized electron temperature. The specified profiles of temperature and electric field follows from the fact that the constancy of the electric field in the cross section of the discharge channel and is almost constant in the cross-section static pressure, so that the total density of particles is inversely proportional to the temperature and, accordingly, a given field is proportional to temperature. The similarity of the profile of the electron temperature value used here may mean that a certain effective total cross-section of total electron energy losses in collisions is also constant in the cross-section.
Concluding remarks
A number of numerical simulation of the evolution of an axisymmetric electric discharge in the supersonic air flow of a satellite with a stepwise change in the load current from 10 mA to 10 a is performed under conditions close to complex experimental and computational-theoretical studies [11-14].

It is noted that the discharge characteristics in the satellite stream strongly depend on the configuration of the electrodes. When developed separation zones occur behind the upper electrode, the discharge is localized in the separation zone and contributes to its significant expansion up to the formation of a closed cavity.

When using thin conical electrodes, the effect of the discharge on the flow is characterized, on the one hand, by a significant increase in the flow rate, but, on the other hand, by a sharp decrease in the Mach number to low subsonic values. This, in turn, leads to a significant reduction in the braking pressure and problems with setting boundary conditions at the downstream boundary.

In the considered range of load currents, the discharge is located on the falling VAC, the movement of which is accompanied by a gradual transition from the glow discharge mode to the arc one.

The role of field ionization and the degree of non-equilibrium of the discharge decreases as the load increases from the dominant one at currents of several milliamperes to a significant one at the maximum, considered here 10 A.

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