Behaviour of Steel Arch Stabilized by a Textile Membrane

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**Abstract.** Behaviour of the slender steel arch supporting textile membranes in a membrane structure with respect to in-plane and out-of-plane stability is investigated in the paper. In the last decades the textile membranes have been widely used to cover both common and exclusive structures due to progress in new membrane materials with eminent properties. Nevertheless, complex analysis of such membranes in interaction with steel structure (carbon/stainless steel perimeter or supporting elements) is rather demanding, even with specialized software. Laboratory model of a large membrane structure simulating a shelter roof of a concert stage was tested and the resulting stress/deflection values are presented. The model of a reasonable size was provided with prestressed membrane of PVC coated polyester fabric Ferrari® Précontraint 702S and tested under various loadings. The supporting steel structure consisted of two steel arch tubes from S355 grade steel and perimeter prestressed cables. The stability behaviour of the inner tube was the primary interest of the investigation. The SOFiSTiK software was used to analyse the structural behaviour in 3D. Numerical non-linear analysis of deflections and internal forces of the structure under symmetrical and asymmetrical loadings covers various membrane prestressing and specific boundary conditions. The numerical results are validated using test results. Finally, the preliminary recommendations for appropriate numerical modelling and stability design of the supporting structure are presented.

1. Introduction

Tensioned Fabric Structures (TSF) are becoming more and more popular not only because of their visual attractiveness but also because of their ability to be a fully load-bearing component in a complex structure. Various membrane types are available for both single layer double curved shapes (hypars, cones, barrels) and pneumatic constructions (inflatable cushions, air supported pneumatic structures). The PVC coated polyester fabric (e.g. Précontraint FERRARI®) seems to be a rational choice for common single layer structures with lifetime up to 20 years, acceptable costs and with good joining possibilities (welding, sewing). When longer lifetime is needed, a glass fabric coated by PTFE (Teflon), silicon rubber or titanium dioxide could be used but at higher expenses. Rather expensive but...
excellent material is expanded PTFE coated 2 sides by fluoropolymer film (TENARA®), joint by welding. The membranes may also be thermally insulated using Nanogel Aerogel™ and 2 sides coated with PTFE (in result translucent, with total thickness 9 mm, CABOT Cor.). Plastic foils of low thickness 50÷500 μm are used mainly as inflatable cushions, nowadays predominantly from ETFE (TEXLON®) or THV materials.

Whereas the geometrically and materially non-linear analysis with imperfections (GMNIA) is necessary for proper designing and material properties of the used membranes are essential inputs into the software, various models of material behavior have been established. The most sophisticated model with 15 input parameters obtained from laboratory tests was proposed by Kato et al. [1]. This model demonstrates excellent agreement with the test data but because of its complexity and time requirements is rather unsuitable for practical utilization. A non-linear material model with 5 parameters based on experimental results and depending on load ratios in warp and fill direction was proposed by Galliot and Luchsinger [2], [3]. Model parameters are: warp and fill Young’s modules for load ratio 1:1, the change in warp and fill Young’s modules and the Poisson’s ration. Recently a simplified stress-strain model for coated plain-weave fabrics was developed by Pargana and Leitao [4]. The model consists of three nonlinear elements to model the yarns and an isotropic plate to model the coating.

Outstanding and unique structures using either common or sophisticated textile fabrics were designed in last few decades which indicate new ways and directions of novel structure design. Allianz Arena in Munich (design by Herzog & de Meuron, 2005), SkySong ASU in Scotsdale (FTL Design Engineering Studio, 2009) or Union Station in Denver (Skidmore, Owings & Merrill LLP (SOM), 2014) are just some examples of excellent projects realized in the last decade.

For these structures it is necessary to use supporting components that are as slender as possible to follow the concept of the delicate, light and attractive structure with the membrane surface and to avoid any visual intervention of supporting steelworks into the membrane area. While membrane surface is exclusively tensioned, supporting construction is most often exposed to compression or bending. This type of loading, in combination with slender elements, results in stability problems and design must be done with respect to these effects. Different situations occur, when membrane surface is joined with the supporting steel structure continually. In this case the membrane represents a spring support for the supporting structure, the critical length of individual elements is changing, and both parts of the entire system cannot be investigated separately but as one complex structure using proper software package, which allows integrated modeling and computing (e.g. EASY [5], FORTEN [6], SOFiSTiK™, Rhino Membrane [8], NDN [9], etc.). Detailing of membrane structures and erection methods are well described by Seidel [10] and the basic design is discussed in the European Design Guide [11].

In this paper a significant stabilizing effect of membranes with respect to supporting steelwork is presented based on the data obtained from testing of a structure representing a reduced concert stage structure with two supporting arches. Influence of various membrane prestressing on the structure stabilization is analyzed and results are validated with tests.

2. Description of the model under experimental investigation

2.1. Geometry and material properties of the used components
Model of a reasonable size representing a roofing of a concert stage was designed, built and tested in the lab of the Faculty of Civil Engineering of CTU in Prague. The final shape of the tested
configuration resulted from several analytical models examined using Formfinder software [12], which provides user friendly design interface and various possibility of shape designing. Floor projection of the model is $L \times B = 4500 \times 2250$ [mm], with height $H = 1200$ [mm] (figure 1). From 8 configurations of loading (5 on arch alone, i.e. without membrane and 3 on the complex structure with membrane) two were used as representative. Both symmetrical and asymmetrical loading were applied on the stand-alone arch and on the arch connected with the membrane.

Inner arch, which was of primary interest within the investigation, was incurved from a hot formed CHS profile $\varnothing 26.9 \times 3.2$ [mm] made of steel with yield strength (based on coupon tests) $f_y = 475$ MPa, ultimate strength $f_u = 595$ MPa and elongation 27.1%. Young's modulus of elasticity was considered in accord with Eurocode 3 as $E = 210$ GPa. Outer arch, incurved from hot formed CHS profile of $\varnothing 88.9 \times 3.2$ [mm] cross section, proved to have yield strength $f_y = 460$ MPa and ultimate strength $f_u = 574$ MPa. Young's modulus was also considered as $E = 210$ GPa.

The membrane employed was Précontraint FERRARI® 702S with opaque surface (weight 830 g/m²). Basically the material is polyester scrim coated on both sides with liquid PVC and PVDF topcoat, weldable for joining. The fabric, developed and patented by the French manufacturing group Serge Ferrari, is unique due to pre-stressing of the base fabric before and during the coating operations. The operation ensures in both warp and fill directions similar elongation behavior and minimum creep. The thick coating also supplies the membrane with longevity and extraordinary resistance against soiling. Concerning material characteristics, the biaxial test performed by Lab BLUM Stuttgart [13] is available. Both warp and fill braking strength was considered as $s_{ult} \approx 56$ kN/m, while working stress $s_{max} = s_{ult}/5 \approx 11.2$ kN/m to exclude tearing. The test prestressing was designed as approx. $s_p \approx 0.4$ kN/m. However, the material due to its structure is non-homogeneous, orthotropic (warp and fill directions) and non-linear. Based on the material model investigated by Galliot and Luchsinger [2] [3] dealing with five parameters, i.e. warp and fill Young's modules for 1:1 load ratio, the change in warp and fill Young's moduli and the Poisson’s ratio, the mechanical properties for FERRARI® 702 used in the FE analysis of the test were established as 635.3 kN/m, 661.9 kN/m, 295.0 kN/m, 168.5 kN/m and 0.196, respectively. The formulas for orthotropic linear elastic behavior are given in [2] and appropriate shear modulus 14.3 MPa in [3].

The membrane was joined to the outer arch using riveted aluminum keder profile while to the inner tube via alternating pockets. Common 7x7 wire peripheral rope from CarlStahl Company with diameter of 6 mm in curved cuff fastened the membrane to the corner plates and anchors.

Figure 1. Geometry (left) and view of the loaded model at the laboratory (right).
2.2. Loading schemes and test results

The investigated arch was fitted with transducers (electrical potentiometers) in vertical (V), transverse (H) and longitudinal (L) directions to measure deflections. In supports and middle of the arch 12 strain gauges were placed always in four mutually perpendicular positions (figure 2).

![Figure 2. Position of transducers (left), strain gauges (right) and loading (P).](image)

The loading schemes were performed with respect to the inner investigated arch in two phases. First, the arch standing alone was tested, loaded with calibrated pouches with steel pellets. Second, the membrane was assembled and prestressed by tightening of the perimeter ropes. The aim was to achieve a uniform prestress. Control measurement with strain gauges placed on the surface of the membrane was carried out to check a prestress uniformity. However, due to non-homogeneous structure of the membrane (perpendicular net of yarns and additional coating layer), the measurement was informative and real value of prestress could not be established. After membrane was assembled, the complete membrane structure was loaded in the same way as the arch standing alone. Sum of the total loading in individual loading schemes is shown in diagrams (figure 3) for both symmetrical and asymmetrical loading. Loadings were terminated when maximal permissible deflection of transducers or yield strength of steel was achieved.

![Figure 3. Maximal symmetrical (left) and asymmetrical (right) loading imposed on the complete membrane structure and the stand alone arch (without membrane).](image)

The process of sagging of the inner arch under increasing loading in vertical direction is shown in figure 4. Unloading path is not included and in general it was fully elastic. Arch standing alone collapsed with out-of-plane buckling mode under total loading approaching $F_0 = 5.5 \text{ kN}$ (transverse...
deflection at the top of the arch was $\eta_0 = 79.7$ mm and vertical one $\delta_0 = 35.1$ mm) while the complex structure with the membrane resisted till the total load of $F_M = 8.3$ kN, with approaching the arch yield stresses. However, even at this level of loading the transverse deflection oscillated around the initial unloaded form.

The influence of the textile membrane on the slender steel tube behavior is even more significant under asymmetrical loading. Vertical deflection at load level $F_0 = 2.37$ kN of the arch alone at point P6 was $\delta_0 = 41.3$ mm, while the deflection at the same loading $F_M = 2.37$ kN of the arch with membrane $\delta_M = 18.5$ mm. At maximal loading of the structure with membrane $F_{M,max} = 3.47$ kN the vertical deflection was $\delta_{M,max} = 30.4$ mm and horizontal one $\eta_{M,max} = 1.7$ mm.

3. Numerical analysis

Software package SOFiSTiK was chosen for a detailed numerical analysis of both the arch standing alone and complete structure with the membrane. SOFiSTiK enables complete setting of all material parameters of the used membranes including their pretension in standard units [kN/m']. For correct analysis, it is necessary to set proper material characteristics of all used materials, especially for orthotropic material of the used membrane Précontraint FERRARI® 702S.

SOFiSTiK is compatible with another software package AutoCAD, where the exact geometry with initial deformations was determined. Initial deformations were measured in the laboratory on the model just before first loading and were found to be around 5 mm in horizontal direction at the top of the arch and around +/-10 mm in vertical direction at the quarters of the arch. Initial shape of the membrane surface was generated automatically by the AutoCAD as a surface with minimal area. This approximation is sufficiently accurate for further computing in SOFiSTiK, to which the model from AutoCAD was exported. For each level of membrane prestress, exact value of cable pretension was calculated based on formula (1) [14].

$$S_S = R \cdot S_M$$

Where, $S_S$ is a tension force in a rope,
$R$ is a diameter of the rope,
$S_M$ is prestress in membrane perpendicular to the rope [kN/m'].

Figure 4. Comparisons of vertical (left) and horizontal (right) deflections under symmetrical loading of the inner arch standing alone and with the membrane.
Formfinding and all other calculations were done using geometrically non-linear analysis with Newton-Raphson method [15] and also using the theory of large deformation (in Germany called third order theory) which is necessary for the calculation of the membrane structure.

3.1. Comparison of numerical analysis and test results

Primary goal of the laboratory testing was to validate the numerical analysis using SOFiSTiK software. Subsequent work would head towards further modeling of different types of structures and a parametrical study. Comparison of the test results and the analysis of arch alone and the complex structure using geometrically nonlinear analysis with imperfections (GNIA) are shown in figure 5 and figure 6, respectively. The measured initial deflections were introduced in the analysis.

Figure 5. Symmetrical loading of the isolated arch, i.e. arch alone (left). Comparison of experimental horizontal deflections and numerical GNIA (right).

Excellent agreement between numerical analysis and test data is obtained for the arch standing alone. Agreement is presented by the diagram with symmetrical loading showing a transverse deflection uY out of the arch plane. Till sixth loading step, values from numerical analysis are lower than those from test. After sixth loading step, the material hardening of the real structure is noticeable and gives rather higher value. Under total load of 4.85 kN, the transverse test deflection of the middle of the arch was 33.4 mm and 42.2 mm at numerical analysis. Vertical deformation at the same point and at the same load level was 17.3 mm during experiment, respectively 13.1 mm in numerical analysis.

Table 1. Comparison of experimental vertical deflections and numerical ones with various pretension.

| experiment | Pret. 0.2 kN/m | Pret. 0.4 kN/m | Pret. 0.6 kN/m | Pret. 0.8 kN/m | Pret. 1.0 kN/m |
|------------|---------------|---------------|---------------|---------------|---------------|
| Total load | Defl. [mm]    | Defl. [mm]    | Defl. [mm]    | Defl. [mm]    | Defl. [mm]    |
| [kN]       | [mm]          | [mm]          | [mm]          | [mm]          | [mm]          |
| 0.00       | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          |
| 2.08       | 3.69          | 3.43          | 3.22          | 3.10          | 3.02          |
| 4.17       | 8.33          | 8.14          | 7.60          | 7.27          | 7.05          |
| 6.25       | 13.86         | 13.50         | 12.50         | 11.90         | 11.50         |
| 8.33       | 20.77         | 20.10         | 18.70         | 17.60         | 17.00         |
Several levels of membrane pretension were investigated using software SOFiSTiK at this step as shown in the diagram (figure 6, right). It is obvious that level of pretension is a crucial parameter for the arch stability and resistance capacity. Higher pretension in membrane logically means higher stability and smaller deflection of the investigated arch. It should be noted that the real pretension during the testing was due to alternating “pockets” connection of the membrane and arch rather non-uniform and slacked. This is reflected in numerical analysis with pretension close to zero (see the value of 0.2 kN/m in table 1.) which is near but still lower than the experimental one. From measured values of stresses in the membrane during test a real value of membrane pretension was assessed roughly as 0.4 kN/m. Nevertheless, taking all mentioned notes into account, the received results from the numerical analysis are sufficient for justification of the described SOFiSTiK model to be used in planned wider parametric studies and investigations.

![Figure 6](image-url)

**Figure 6.** Symmetrical loading of complex structure, i.e. with membrane (left). Comparison of experimental vertical deflections and numerical ones with various pretension (right).

### 4. Conclusion

The paper deals with the detailed description of an experiment concerning an assembly of a textile membrane with supporting steelwork. Relevant values of deflection, stresses and pretension of the membrane and behavior of the supporting inner arch are presented and compared. The results confirmed the enormous influence of the textile membrane on stability of the slender arch which can be considered in practical design and should be involved in the complex analysis. Separate modeling of the membranes and supported steelwork is not recommended and in large structures leads to incorrect design and finally can lead to the collapse of the whole structure.

Based on the obtained data, software SOFiSTiK is a credible software package for further investigations and parametrical studies of crucial parameters of membrane structures.

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