Wave function perturbations propagation in multi-particle systems, Einstein-Podolsky-Rosen (EPR) paradox and entanglement

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Abstract

If a one-particle or multi-particle non-relativistic quantum system is initially in a stationary state, and its wave function field is locally perturbed, then perturbed and non-perturbed subregions appear in the region. According to Schrödinger equation, borders of the perturbed subregion propagate with infinite speed, and the perturbation instantaneously affects all infinite region. It means that Schrödinger equation predicts infinite speed of the wave function perturbations propagation. This feature of classical Schrödinger equation is traditionally interpreted as non-locality of quantum mechanics. In reality speed of propagation of the perturbed subregion borders is equal speed of light. On this basis we develop and analyse finite propagation speed concept for non-relativistic quantum systems. It results in local interpretation of EPR paradox and entanglement. Formulated theory agrees with classical experiments on electron matter waves diffraction.

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1 Introduction

If a one-particle or multi-particle non-relativistic quantum system is initially in a stationary state, and its wave function field is locally perturbed, then perturbed and non-perturbed subregions appear in the region. According to Schrödinger equation, borders of the perturbed subregion propagate with infinite speed, and the perturbation instantaneously affects all infinite region. It means that Schrödinger equation predicts infinite speed of the wave function perturbations propagation. This feature of classical Schrödinger equation is traditionally interpreted as non-locality of quantum mechanics. In reality speed of propagation of the perturbed subregion borders is equal speed of light. On this basis we develop and analyse finite propagation speed concept for non-relativistic
quantum systems. It results in local interpretation of EPR paradox and entanglement. Formulated theory agrees with classical experiments on electron matter waves diffraction.

In my previous work [1] this concept was presented for one-particle quantum systems. In the present work a general case of multi-particle quantum systems is considered.

2 Equations for multi-particle systems predicting finite speed of the wave function perturbations propagation

Classical time dependent Schrödinger equation for a multi-particle quantum system is [2]

\[
\frac{\text{i}}{\hbar} \frac{\partial \Psi}{\partial t} + \frac{\hbar}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla_i^2 \Psi - U \Psi = 0
\]  

(1)

where

\[
\nabla_i^2 \Psi = \frac{\partial^2 \Psi}{\partial x_i^2} + \frac{\partial^2 \Psi}{\partial y_i^2} + \frac{\partial^2 \Psi}{\partial z_i^2},
\]  

(2)

\(\hbar = h/2\pi, \ i = \sqrt{-1},\) and \(h, \ \Psi(A,t), \ t, \ U, \ n, \ m_i, \ x_i, \ y_i, \ z_i\) denote Planck’s constant, wave function, time, potential energy, total number of particles in the system, mass of the \(i\)-th particle and coordinates associated with the \(i\)-th particle, respectively. We define \(A = \{x_1, y_1, z_1, ..., x_i, y_i, z_i, ..., x_n, y_n, z_n\}\) as a set of \(3n\) independent coordinates of all particles in the system.

2.1 The wave function perturbations propagation in multi-particle systems

According to parabolic type equation (1), speed of the wave function perturbations propagation is infinite. To overcome this drawback, we will use method developed in our previous works [1], [3]-[6]. I suggest that initially the multi-particle system is in a stationary state. Then at a certain moment of time \(t = t_0\) wave function field is locally perturbed. As a result, perturbed and non-perturbed \(3n\) dimensional subregions appear, their boundary hypersurface \(S_P\) moves with speed of light \(v_P\) into the non-perturbed subregion, and the perturbed subregion gradually replaces the non-perturbed one.

As in papers [1], [3]-[6] we define perturbation traveltime \(t_P\) as a time moment when the perturbation reached a given point \(M\). Therefore the boundary hypersurface \(S_P\),
separating the perturbed from the non-perturbed subregion, is a surface of constant perturbation traveltime \( t_P = \text{const} \). For a multi-particle quantum system \( t_P, M, S_P \) are functions of \( A \). From definition of perturbation traveltime follows that \(|\nabla t_P| = \frac{1}{v_P} \) or

\[
\sum_{i=1}^{n} \left[ \left( \frac{\partial t_P}{\partial x_i} \right)^2 + \left( \frac{\partial t_P}{\partial y_i} \right)^2 + \left( \frac{\partial t_P}{\partial z_i} \right)^2 \right] = \frac{1}{v_P^2} \tag{3}
\]

The non-linear governing equation (3) with 3\( n \) independent variables is a generalisation of classical eikonal equation [7], [8]. Its primary wave solution \( t_P(A) \) defines perturbation traveltime field satisfying initial condition

\[
t_{0P}(A) = t_0 \tag{4}
\]

where \( t_{0P}(A) \) and \( t_0 \) denote initial values of traveltime and global time, respectively.

### 2.2 The modified Schrödinger equation for multi-particle systems

For any point \( M(A) \) of the region, can be introduced local time \( \vartheta(A,t) \) [1], [3]-[6] counted from a moment when the perturbation reached this point

\[
\vartheta(A,t) = t - t_P(A) \tag{5}
\]

Three characteristic cases are:

1. \( \vartheta = t - t_P(A) < 0 \), the perturbation has not reached the point \( M(A) \). Thus, the point belongs to the non-perturbed subregion.

2. \( \vartheta = t - t_P(A) = 0 \), the perturbation has reached the point \( M(A) \). Now the point \( M(A) \) is located on the border hypersurface \( S_P(A) \), separating the perturbed and the non-perturbed subregions.

3. \( \vartheta = t - t_P(A) > 0 \), the point \( M(A) \) is located inside the finite perturbed subregion with moving border \( S_P(A) \). All further analysis is related to this subregion where \( \vartheta > 0 \). Obviously, classical Schrödinger equation for multi-particle systems [1] does not describe such situation, and a new equation is needed.

According to the local time concept [1], [3]-[6], parabolic type partial differential equations predicting finite speed of the perturbations propagation and the respective equations with infinite speed of the perturbations propagation are identical, if local time \( \vartheta = t - t_P(A) \) is used as an independent variable instead of global time \( t \). When speed of the perturbations propagation is infinite \( v_P \to \infty \), local time in these equations is
identical to global time $\vartheta = t$ because $t_P = 0$. The same equations describe the finite $v_P$ case when $t_P > 0$, $0 < \vartheta < t$. All equations written in such universal form we call *modified equations*.

Application of local time concept leads to the following formulation of modified time dependent Schrödinger equation for multi-particle systems

$$i\hbar \frac{\partial \Psi(\vartheta, A)}{\partial \vartheta} + \frac{\hbar^2}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla_i^2 \Psi(\vartheta, A) - U \Psi = 0$$

(6)

When $n = 1$ equation (6) becomes identical to the modified one-particle Schrödinger equation derived in previous work [1].

### 3 Basic properties of the modified Schrödinger equation for multi-particle systems

Modified Schrödinger equation (6) is of the parabolic type as classical Schrödinger equation (1). In a classical case of infinite speed of the perturbations propagation $v_P \to \infty$ there is $t_P = 0$, $\vartheta = t$, and modified equation (6) becomes identical to classical equation (1).

It is easy to prove that modified Schrödinger equation (6) predicts finite speed of the perturbations propagation. Solutions of classical time dependent Schrödinger parabolic type equation (1) suggest that any local perturbation introduced at initial moment of time $t_0$, instantaneously affects all infinite space domain. The modified time dependent Schrödinger parabolic type equation (6) uses instead of global time $t$ local time $\vartheta$ as an independent variable. In this case, any local perturbation introduced at initial local time moment $\vartheta_0 = t_0 - t_{0P} = 0$, affects an arbitrary point $M(A)$ of the space domain at the same local time value $\vartheta_M = t_M - t_P(M) = 0$. Therefore the perturbation arrives to the point $M(A)$ at global time moment $t_M = t_P(M) > 0$, i.e. with global time delay. So the modified Schrödinger equation for multi-particle systems predicts finite speed of the perturbations propagation equal to speed of light, and the perturbations have a wave behaviour. These waves do not reflect and interfere, because only solutions of eikonal type equation (3) corresponding to primary perturbation waves propagating in the non-perturbed subregion, have physical meaning.

Let $\Psi(A, t)$ be a solution of the classical Schrödinger equation (1), satisfying certain boundary and initial conditions. For the same boundary and initial conditions, a solution of the modified Schrödinger equation (6) is the same function but with local time as an argument $\Psi(A, \vartheta)$, $\vartheta > 0$. For an arbitrary point $M(A)$, located inside the perturbed subregion, and corresponding perturbation traveltime value $t_P(M)$ difference of solutions
of classical and modified Schrödinger equations for multi-particle systems is

$$\Delta \Psi = \Psi(A, t) - \Psi(A, \vartheta = t - t_P) \approx \frac{\partial \Psi}{\partial \vartheta} t_P$$  \hspace{1cm} (7)$$

Absolute value of this difference is small for small values of perturbation traveltime $t_P$ and/or slow processes, when absolute value of $\frac{\partial \Psi}{\partial \vartheta}$ is small. In this case, solutions of classical time dependent Schrödinger equation for multi-particle systems are accurate enough.

It is interesting that for one-particle quantum systems studied in [1] and multi-particle systems the same formula determines difference of solutions of classical and modified Schrödinger equations.

For a case of constant energy of the quantum system $E = \text{const}$, potential energy $U$ not depending on time, and infinite speed of the perturbations propagation, wave function can be presented as $\Psi = \psi(A) \exp(-i \frac{E}{\hbar} t)$ [2], [9], [10]. From the local time concept follows that for finite speed of the perturbations propagation the right formula is $\Psi = \psi(A) \exp(-i \frac{E}{\hbar} \vartheta)$, and we obtain time independent version of modified Schrödinger equation (6) describing stationary state of multi-particle quantum system

$$\frac{\hbar^2}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla_i^2 \psi + (E - U) \psi = 0$$  \hspace{1cm} (8)$$

Obviously, time independent version of modified Schrödinger equation (8) for multi-particle systems is identical to classical time independent Schrödinger equation following from expression (1). Therefore all solutions of modified time independent Schrödinger equation are identical to solutions of classical equation. The only difference is: in the classical case multiplier $\exp(-i \frac{E}{\hbar} t)$ is a function of global time $t$, while in the case of modified Schrödinger equation multiplier $\exp(-i \frac{E}{\hbar} \vartheta)$ is a function of local time $\vartheta$.

4 Quantum interaction between the perturbed and non-perturbed subregions in multi-particle systems

Let $\Psi_0(\vartheta_0, A)$, $\Psi_p(\vartheta_P, A)$, $\vartheta_0 = t - t_{P0}$, $\vartheta_P = t - t_P$, $t_{P0}$, $t_P$ be wave function for the non-perturbed subregion, wave function for the perturbed subregion, local time for the non-perturbed subregion defined during its prehistory, local time for the perturbed subregion, traveltime for the non-perturbed subregion defined during its prehistory, and traveltime for the perturbed subregion, respectively.

The following two modified Schrödinger equations describe quantum states of the subregions

$$i\hbar \frac{\partial \Psi_0(\vartheta_0, A)}{\partial \vartheta_0} + \frac{\hbar^2}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla_i^2 \Psi_0(\vartheta_0, A) - U \Psi_0 = 0$$  \hspace{1cm} (9)$$

5
\[ i\hbar \frac{\partial \Psi_P(\vartheta_P, A)}{\partial \vartheta_P} + \frac{\hbar^2}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla^2_i \Psi_P(\vartheta_P, A) - U \Psi_P = 0 \] (10)

As the non-perturbed subregion is in stationary state and the perturbed subregion is in non-stationary state, we can present the wave functions \( \Psi_0 \) and \( \Psi_P \) in the following forms

\[ \Psi_0(\vartheta_0, A) = \psi_0(A) \exp \left( -i \frac{E_0}{\hbar} \vartheta_0 \right) \] (11)

\[ \Psi_P(\vartheta_P, A) = \psi_P(\vartheta_P, A) \exp \left( -i \frac{E_0}{\hbar} \vartheta_0 \right) \] (12)

where \( E_0 \) is total energy of the non-perturbed multi-particle system.

Now expression (11) is substituted in equation (9), and expression (12) in (10). As \( \frac{\partial \vartheta_0}{\partial \vartheta_P} = 1 \), we obtain

\[ \frac{\hbar^2}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla^2_i \psi_0 + (E_0 - U) \psi_0 = 0 \] (13)

\[ i\hbar \frac{\partial \psi_P}{\partial \vartheta_P} + \frac{\hbar^2}{2} \sum_{i=1}^{n} \frac{1}{m_i} \nabla^2_i \psi_P + (E_0 - U) \psi_P = 0 \] (14)

Equation (14) can be solved using initial condition \( \psi_P(0, A) = \psi_0(A) \) obtained from equation (13) and an additional function \( \psi_P(\vartheta_P, A_b) \) describing local perturbation of wave function at the point \( M(A_b) \). The solution of equation (14) \( \psi_P(\vartheta_P, A) \) and formula (12) give wave function of the perturbed subregion \( \Psi_P(\vartheta_P, A) \). Thus, becomes clear why the perturbed subregion gradually replaces non-perturbed subregion and not vice versa.

If the non-perturbed subregion is in non-stationary state, presentation (13) and (14) becomes impossible. It means that perturbed subregion does not appear, and in this case the perturbations propagation is non-observable.

In a particular classical case, quantum state of every particle is fully defined, i.e. wave function of every particle \( \Psi_i(t, x_i, y_i, z_i) \) is known. Therefore wave function of multi-particle system is a product of wave functions of all particles [2], [11]

\[ \Psi(t, A) = \prod_{i=1}^{n} \Psi_i(t, x_i, y_i, z_i) \] (15)

Expression (15) assumes infinite speed of the wave function perturbations propagation. For finite speed of the wave function perturbations propagation, according to the local
time concept the following formulae determine wave functions in the non-perturbed and perturbed subregions

\[ \Psi_0(\vartheta_0, A) = \prod_{i=1}^{n} \Psi_{0i}(\vartheta_0, x_i, y_i, z_i) \] (16)

\[ \Psi_P(\vartheta_P, A) = \prod_{i=1}^{n} \Psi_{Pi}(\vartheta_P, x_i, y_i, z_i) \] (17)

where \( \Psi_{0i}, \Psi_{Pi} \) is non-perturbed and perturbed wave function of \( i \)-th particle, respectively.

When the distance between particles is big enough and \( t_0 = 0 \), then for \( i \)-th particle approximate formula holds \( \vartheta_P = t - d_i/v_P \) (\( d_i \) denotes distance between the particle, where initial perturbation was introduced, and \( i \)-th particle). So the bigger local time delay \( \Delta \vartheta = \vartheta_P - t = -d_i/v_P \) will be observed for distant particles with bigger \( d_i \) as a result of finite speed of the wave function perturbations propagation.

5 Local interpretation of EPR paradox and entanglement

EPR paradox \[11], [12] reflects one of the most peculiar and principal features of classical non-relativistic quantum mechanics. The authors A. Einstein, B. Podolsky and N. Rosen considered two quantum systems I and II that initially interacted and then interaction stopped. Now certain quantity is measured in the system I. It appears that according to classical non-relativistic quantum mechanics, the measurement in the system I instantaneously affects state of the system II, isolated from the system I. If then another quantity is measured in system I, another state of the isolated system II instantaneously appears. Thus, classical quantum theory suggests paradoxical non-local superluminal action on distance. The authors wrote: ”We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave function is not complete” \[11\]. Today this conclusion is not supported by physical community \[13\], though the community does not reject the EPR paradox proof presented in \[11\].

Obviously, EPR paradox arises as a result of infinite speed of the wave function perturbations propagation suggested by classical non-relativistic quantum mechanics. The EPR paradox can be solved using developed concept of the wave function perturbations propagation with finite speed. After initial interaction, systems I and II create a two-particle system which is in stationary state described by equation (13). Any measurement in the system I is a local perturbation of the two-particle system wave function field. This perturbation propagates to system II with speed of light according to eikonal type equation (3), while perturbed field is described by equation (14). Thus, developed theory predicts, that as a result:
1. Quantum state of the system II will change when the wave function perturbation will arrive.

2. This change will occur with time delay.

3. Described interaction of the systems I and II has local character and is transferred with speed of light.

The same way of reasoning is applicable to any multi-particle system which is left in a stationary state after initial interaction of the particles. If then quantum state of one of the particles is perturbed, the perturbation wave according to equation (3) travels with speed of light to other particles affecting their state.

Developed theory does not use any hidden variables. Because of this it predicts that in experiments will be observed violation of the Bell’s inequalities. This feature was confirmed by numerous experiments.

6 Experimental evidence of finite speed of the wave function perturbations propagation

Classical experiments of C.J. Davisson [14] and G.P. Thomson [15]-[19] with their colleagues on electron matter waves diffraction were repeated many times and respective literature is enormous, however I could not find more detailed and reliable experimental data than published by these pioneers. This is the reason why I use their results for experimental check of my theoretical suggestions.

In our work [1] was shown that experimental data presented in [14] on electron matter wave diffraction by a monocrystal of nickel confirm our theoretical predictions relating to finite speed of the wave function perturbations propagation.

Now we will analyse experiments with electron matter waves diffraction by thin polycrystalline films [15]-[19]. The authors used Hull-Debye-Sherrer method initially developed for X-rays diffraction analyses of thin polycrystalline films. They sent an approximately homogeneous beam of accelerated electrons through a very thin polycrystalline film at normal incidence and studied a diffraction pattern registered by a photographic plate located on some distance from the film. The pattern is a family of concentric rings with diameters $D_\mu = a_\mu/k$, where $\mu = 1, 2, 3, ...$ is a number of the ring starting from the smallest $D_\mu$; $a_\mu$ denotes a coefficient, depending on material of the film, number $\mu$ of the diffraction ring and distance from the film to the photographic plate; while $k$ is wave number of electron matter wave. Therefore there is $a_\mu = D_\mu k$, and for the same polycrystalline film, the same distance from the film to photographic plate, and the same $\mu$, the product $D_\mu k$ does not depend on electric potential difference $U$ accelerating electrons and therefore affecting $k$. Authors of cited works [15]-[19] found experimentally that the product $D_\mu k$ does not vary significantly if $k$ is calculated using DeBroglie’s formula. From
this they concluded that classical quantum mechanics is correct, but we will show that our theory more accurately agrees with their experimental data.

In our previous work [1] was proved that for finite speed $v_P$ of the wave function perturbations propagation, real wave number $k_l$ for matter wave is different from classical value $k$ defined by DeBroglie’s formula

$$k_l = k + \xi \frac{\nu}{v_P}$$  \hspace{1cm} (18)

$$k = \frac{p}{h}$$  \hspace{1cm} (19)

$$\nu = \frac{E_k}{h}$$  \hspace{1cm} (20)

where $\nu$, $k$, $p$, $E_k$, $\xi = \pm 1$ denote frequency, classical DeBroglie’s wave number, the particle momentum, the particle kinetic energy, and a characteristic coefficient, respectively. In formula (18) there is $\xi = 1$ when the perturbation wave and the particle move in the same direction, and $\xi = -1$ corresponds to alternative case when they move in opposite directions.

Table 1: Main results of calculations using experimental data ($c$ is speed of light)

| Crystalline material, $\mu$ | Data source | $r_{\text{var}}$ | $v_P/c$ | $\var_{\mu_l} \cdot 10^{-9}$ | $\var_{\mu} \cdot 10^{-9}$ | $\xi$ |
|----------------------------|-------------|-----------------|---------|-----------------------------|-----------------------------|-------|
| Celluloid, $\mu = 1$       | [16]        | 0.58            | 2.16    | 1.43                        | 1.54                        | -1    |
| Gold, $\mu = 1$            | [16]        | 0.26            | 1.59    | 2.65                        | 3.02                        | -1    |
| Aluminium, $\mu = 1$       | [16]        | 0.21            | 1.39    | 2.66                        | 3.11                        | -1    |
| Platinum, $\mu = 1$        | [16]        | 0.43            | 1.07    | 2.72                        | 3.05                        | -1    |
| Silver, $\mu = 1$          | [19]        | 0.79            | 1.63    | 2.28                        | 2.63                        | -1    |
| Tin, $\mu = 1$             | [19]        | 0.46            | 1.07    | 2.69                        | 2.25                        | 1     |
| Copper, $\mu = 3$          | [19]        | 0.38            | 1.37    | 2.20                        | 2.60                        | -1    |
| Copper, $\mu = 4$          | [19]        | 0.45            | 2.97    | 3.24                        | 3.49                        | -1    |
| Copper, $\mu = 5$          | [19]        | 0.06            | 1.47    | 3.59                        | 4.18                        | -1    |
| Copper, $\mu = 6$          | [19]        | 0.06            | 1.05    | 3.95                        | 4.99                        | -1    |
| Copper, $\mu = 7$          | [19]        | 0.42            | 0.83    | 7.66                        | 5.99                        | 1     |

Expressions (18)-(20) were applied to experimental data presented in cited works [13]-[19]. For given material of the film, and for every value of $U$ and $\mu$ we calculated $E_k$, $p$, $k$, $a_\mu = D_\mu k$, $k_l$, $a_{\mu l} = D_\mu k_l$. Then for every set of $U$, corresponding to certain film material, the same distance of photographic plate from the film and $\mu = \text{const}$, we determined mean values $\overline{\mu}$, $\overline{\mu_l}$ and respective variances of experimental data $\text{var}_\mu$, $\text{var}_{\mu_l}$.
corresponding to infinite and finite speed of the perturbations propagation \( v_p \), respectively. As variance \( \text{var}_{\mu_\lambda} \) depends on \( v_p \), we take into account only such value of \( v_p \) that ensures minimal value of variance and therefore minimal ratio \( r_{\text{var}} = \frac{\text{var}_{\mu_\lambda}}{\text{var}_\mu} \). Main results of these calculations are presented in the Table 1. They do not include data from the source [15] because presented there \( U \) values are not accurate as the author claims in his next paper [16]. The table also does not present data for celluloid film from [18], aluminium film in short camera from [16] and lead film from [17], because all these data are not accurate enough. Experimental results for platinum film in short camera [16] include only two points, while for copper film [19] only three values of \( U \) for \( D_1 \) and \( D_2 \) are given, that is not enough for accurate conclusions, and these data are also not introduced in the table.

The table shows that in all analysed cases \( r_{\text{var}} < 1 \). It means that hypothesis of finite speed of the wave function perturbations propagation in non-relativistic quantum mechanics is confirmed experimentally. Mean value of dimensionless speed of the perturbations propagation calculated using data from the table and data from [1] is \( \frac{v_p}{c} = 1.4 \) with standard error of mean value equal 0.2. So 95% confidence limits for dimensionless speed of the perturbations propagation are \( 1.0 \leq \frac{v_p}{c} \leq 1.8 \). Therefore suggested in theory value \( \frac{v_p}{c} = 1 \) is within these 95% confidence limits. Thus, the cited experiments support our hypothesis that speed of the perturbations propagation is equal the speed of light though accuracy of the experiments was not high.

7 Conclusions

The following conclusions can be formulated:

1. The concept of finite speed of the wave function perturbations propagation in non-relativistic quantum mechanics is developed for multi-particle systems.

2. Introduced multidimensional eikonal type equation defining perturbation traveltime and modified multi-particle Schrödinger equation create theoretical basis for analyses of non-relativistic multi-particle quantum systems with finite speed of the wave function perturbations propagation.

3. The concept of finite speed of the wave function perturbations propagation in multi-component quantum systems leads to local interpretation of the Einstein-Podolsky-Rosen paradox and entanglement of quantum particles.

4. Classical experiments on electron matter waves diffraction by thin polycrystalline films and monocrystals support developed concept of the wave function perturbations propagation with speed of light.
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