Virialization-induced curvature versus dark energy

Jan J. Ostrowski\textsuperscript{1,2}, Boudewijn F. Roukema\textsuperscript{1,2}, Thomas Buchert\textsuperscript{2}

\textsuperscript{1}Toruń Centre for Astronomy, Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University, ul. Gagarina 11, 87-100 Toruń, Poland

\textsuperscript{2}Université de Lyon, Observatoire de Lyon, Centre de Recherche Astrophysique de Lyon (CRAL), CNRS UMR 5574: Université Lyon 1 and École Normale Supérieure de Lyon, 9 avenue Charles André, F–69230 Saint-Genis-Laval, France

The concordance model is successful in explaining numerous observable phenomena at the price of introducing an exotic source of unknown origin: dark energy. Dark energy dominance occurs at recent epochs, when we expect most cosmological structures to have already formed, and thus, when the error induced by forcing the homogeneous Friedmann–Lemaître–Robertson–Walker (FLRW) metric onto the data is expected to be the most significant. We propose a way to quantify the impact of deviations from homogeneity on the evolution of cosmological parameters. Using a multi-scale partitioning approach and the virialization fraction estimated from numerical simulations in an Einstein–de Sitter model, we obtain an observationally realistic distance modulus over redshifts $0 < z < 3$ by a relativistic correction of the FLRW metric.

1 Introduction

The weakness of the anisotropy observed in the cosmic microwave background (CMB), i.e. the weakness in deviations from angular homogeneity, is widely accepted as one of the strongest justifications for using the Friedmann–Lemaître–Robertson–Walker (FLRW) metric to describe the expanding Universe. However, from first principles, the non-commutativity of averaging and time differentiation is key to the unresolved issue of the impact of inhomogeneities on average properties of our Universe, which in reality is strongly inhomogeneous at late epochs. Explaining observations within the standard $\Lambda$CDM cosmology requires a new type of energy that violates the weak energy condition. What is normally seen as a “very successful theory” is in fact putting us in the very uncomfortable position of dealing with a new type of energy of unknown origin. There is no direct evidence of the existence of what has recently become the dominating component of the cosmic triangle—“crisis” seems to be a more adequate word than “success” for the situation in today’s standard cosmology. Not being subject to any physical constraints, the cosmological constant comes out as a fitting parameter rather than an actual feature of the real Universe. We propose a simpler model (Roukema, Ostrowski & Buchert 2013\textsuperscript{8}) that matches the observations to a satisfactory initial level of accuracy without the need for modifying the well-tested theory of general relativity.

\textsuperscript{a}Long-term visit to CRAL

\textsuperscript{b}Short-term visit to CRAL
2 Theoretical framework

2.1 Scalar averaging

In this section we will briefly present the scalar averaging procedure (see e.g., Buchert 2000) treating the scalar quantities derived from the Einstein equations for a dust matter model in a 3 + 1 comoving-synchronous slicing of space-time.

We take the spatial average of scalar fields $\Psi$, where $\langle \Psi(t, X^k) \rangle_D := \frac{1}{V_D} \int_D d\mu_g \Psi(t, X^k)$, with the Riemannian volume element of the spatial metric $g_{ij}$, $d\mu_g = J d^3X$ in local coordinates $X^i$, $J := \sqrt{\text{det}(g_{ij})}$, and note the non-commutativity relation, $\partial_t \langle \Psi(t, X^k) \rangle_D - \langle \partial_t \Psi(t, X^k) \rangle_D = \langle \theta \Psi \rangle_D - \langle \theta \rangle_D \langle \Psi \rangle_D$, where $\theta$ denotes the trace of the expansion tensor. The volume scale factor $a_D$, defined via the domain’s volume $V_D(t) = |D|$, and the initial volume $V_{D_i} = V_D(t_i) = |D_i|$, $a_D(t) := \left( \frac{V_D(t)}{V_{D_i}} \right)^{1/3}$, obeys the well-known equations:

$$3 \frac{\dot{a}_D}{a_D} + 4\pi G \frac{M_{D}}{V_D a_D^2} - \Lambda = Q_D ;$$

$$3 \left( \frac{\dot{a}_D}{a_D} \right)^2 + 3k_{D_i} a_D^{-2} - 8\pi G \frac{M_{D_i}}{V_{D_i} a_D^2} + \frac{\langle W \rangle_D}{2} - \Lambda = -Q_D^2 ,$$

where the total rest mass $M_{D_i}$ and the backreaction variables $\langle W \rangle_D$ and the $Q_D$ are domain-dependent and, except for the mass, are time dependent. The backreaction source terms are defined by:

$$Q_D := \frac{2}{3} \left( \langle \theta \rangle_D - \langle \theta \rangle_D^2 \right)_D - 2 \langle \sigma^2 \rangle_D ; \quad W_D = \langle R \rangle_D - \frac{6k_{D_i}}{a_D^2} ,$$

with $\langle R \rangle_D$ the averaged spatial 3-Ricci scalar, and $\sigma^2 := 1/2\sigma_{ij}\sigma^{ij}$ the squared rate of shear. The backreaction variables satisfy the integrability condition:

$$\partial_t Q_D + 6H_D Q_D + \partial_t \langle W \rangle_D + 2H_D \langle W \rangle_D = 0 ; \quad H_D := \frac{\dot{a}_D}{a_D} .$$

2.2 Multi-scale partitioning

We model the large-scale structure of the Universe by introducing a volume-partitioning into two main components, defined by dividing a spatial slice into disjoint unions of two complementary subregions: voids and overdense regions. In order to quantify their behavior, we introduce three characteristic scales, $L_\varepsilon$ for voids, $L_M$ for a typical galaxy cluster and $L_D$ for the largest scale, at which we assume statistical homogeneity on a given spatial hypersurface. Defining $\lambda_M$, the fraction of volume occupied by virialized matter as a proportion of the total volume, and using the partitioning rule, $\langle f \rangle_D = (1 - \lambda_M) \langle f \rangle_\varepsilon + \lambda_M \langle f \rangle_M$, we obtain relations between averaged quantities on different domains (Buchert and Carfora 2008):

$$\langle \theta \rangle_D = \lambda_M \langle \theta \rangle_M + (1 - \lambda_M) \langle \theta \rangle_\varepsilon ,$$

$$\langle R \rangle_D = \lambda_M \langle R \rangle_M + (1 - \lambda_M) \langle R \rangle_\varepsilon ,$$

$$H_D = \lambda_M H_M + (1 - \lambda_M) H_\varepsilon ,$$

$$Q_D = \lambda_M Q_M + (1 - \lambda_M) Q_\varepsilon + 6\lambda_M (1 - \lambda_M) (H_M - H_\varepsilon)^2 .$$

3 Virialization Approximation

The virialization approximation (VA) is a hybrid model that uses observational, numerical and phenomenological inputs to evaluate the underlying analytical model. This approach is far from perfect and needs to be treated as a step towards a more refined model. In this section we briefly list the assumptions used in the VA.
3.1 Observational inputs

Observational inputs (Table 1) are taken from publicly available catalogues and sky surveys, where \( h := \frac{H_0}{100 \text{ km/s/Mpc}} \) is the dimensionless zero-redshift FLRW Hubble parameter.

| parameter                                      | value                  |
|------------------------------------------------|------------------------|
| observational inputs: low-redshift limit of \( H_{\text{eff}} \) | 74.0 ± 1.6 km/s/Mpc    |
| comoving void radius                          | 25 ± 2h\(^{-1}\) Mpc  |
| infall vel. around rich cluster               | 1200 ± 30 km/s         |
| inferred value: zero-redshift value of \( H_{\text{pec}} \) | 36 ± 3 km/s           |

See Roukema, Ostrowski & Buchert (2013) for details including references.

3.2 Analytical assumptions

We employ scalar averaging together with multi-partitioning, supplemented by some reasonable simplifications. We use the (mass-based) virialization fraction \( f_{\text{vir}} \) estimated from Einstein–de Sitter (EdS) \( N \)-body simulations and the density contrast of non-linear collapse \( \delta_{\text{vir}} \approx 200 \) from the scalar virial theorem to determine the \( \lambda_M \) parameter. Adopting the stable clustering hypothesis (Peebles 1980\(^9\)), i.e. \( H_M \approx 0 \), considering the kinematical backreaction term \( Q_D \) as subdominant, and defining \( \lambda_M := \frac{f_{\text{vir}}}{\delta_{\text{vir}}} \), results in a closed set of balance equations:

\[
\begin{align*}
\Omega_D^R &= \lambda_M \Omega_M^M + (1 - \lambda_M) \Omega_R^E - \frac{\lambda_M}{1 - \lambda_M}, \\
\Omega_m^D &= \lambda_M \Omega_m^M + (1 - \lambda_M) \Omega_m^E, \\
\Omega_m^M + \Omega_R^M &= 0, \quad \Omega_m^D + \Omega_R^D = 1.
\end{align*}
\]

4 Results

The main result obtained with the VA is the effective metric [(2.37), Roukema, Ostrowski & Buchert 2013]. Here we present only one possible use of the effective metric, namely the redshift–distance-modulus relation—a standard tool in observational cosmology, shown in Fig. 1.
5 Summary and prospects

It is widely believed that the effect of inhomogeneities on large-scale dynamics is negligible and cannot explain away dark energy. Moreover, Newtonian perturbations added on top of uniformly expanding space are commonly assumed to describe the structure formation process accurately. It has even been suggested that under certain conditions on a class of solutions of the Einstein equations, the contributions to the stress-energy tensor would be traceless (as tensors) and obey the weak energy condition, thus being unable to mimic dark energy with its unusual equation of state (Wald & Green 2011). However, exact, inhomogeneous, dark-energy–free cosmological solutions of the Einstein equations have long been shown to match the observed redshift–luminosity-distance relation (the Stefani model, Dabrowski & Hendry 1998; the Lemaître–Tolman–Bondi model, e.g., Célérié, Bolejko, & Krasiński 2010).

We present a somewhat different, though still general-relativistic and dark-energy–free, approach focusing on scalar quantities derived from a generic metric and trying to determine their evolution (cf. Wiltshire et al. 2012). The VA model is designed to be a rough working model based on replacing missing puzzles from a still developing analytical approach with a combination of observable parameters and N-body simulations estimates. Since observationally realistic values result from this framework, there is a strong motivation towards replacing the unphysical dark energy with realistic GR-based properties. Detailed calculations involving relativistic Lagrangian perturbation theory provide yet another argument supporting this case (Buchert & Ostermann 2012; Buchert, Nayet & Wiegand 2013). Applying the relativistic Zel’dovich approximation to the VA will allow us to strengthen this model and provide an important voice in the ongoing debate on the relevance of inhomogeneities in the Universe.

Acknowledgments

Part of this work consists of research conducted in the scope of the HECOLS International Associated Laboratory. Some of JJO’s contributions to this work were supported by the Polish Ministry of Science and Higher Education under “Mobilność Plus II edycja”. A part of this project has made use of Program Obliczeń Wielkich Wyzwań nauki i techniki (POWIEW) computational resources (grant 87) at the Poznań Supercomputing and Networking Center (PCSS). This work was conducted within the “Lyon Institute of Origins” Grant No. ANR-10-LABX-66.

References

1. T. Buchert, On Average Properties of Inhomogeneous Fluids in General Relativity I: Dust Cosmologies, Gen. Rel. Grav. 32, 105, (Jan., 2000) [gr-qc/9906015].
2. T. Buchert and M. Carfora, On the curvature of the present-day Universe, Class. Quant. Grav. 25, 195001 (October, 2008) [arXiv:0803.1401].
3. T. Buchert and M. Ostermann, Phys. Rev. D 86, 023520, (July, 2012) [arXiv:1203.6263].
4. T. Buchert, C. Nayet, and A. Wiegand, Phys. Rev. D 87, 123503, (June, 2013) [arXiv:1303.6193].
5. M. Célérié, K. Bolejko, and A. Krasiński, A&A 518, A21 (July, 2010) [arXiv:0906.0905].
6. M. P. Dabrowski and M. A. Hendry, ApJ 498, 67 (May, 1998) [astro-ph/9704123].
7. S. R. Green, R. M. Wald, Phys. Rev. D, 83, 084020 (2011) [arXiv:1011.4920]
8. B. F. Roukema, J. J. Ostrowski, and T. Buchert, JCAP, 10, 043, (2013) [arXiv:1303.4444].
9. P. J. E. Peebles, Large-Scale Structure of the Universe, Princeton University Press, Princeton, 1980.
10. D. L. Wiltshire, P. R. Smale, T. Mattsson, and R. Watkins, ArXiv e-prints (Jan., 2012) [arXiv:1201.5371].