MANIFESTLY INVARIANT ACTIONS
FOR HARMONIC SELF-DUAL GAUGE THEORY

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Abstract

We discuss alternative descriptions of four-dimensional self-dual Yang-Mills fields in harmonic space with additional commuting spinor coordinates. In particular, the linear analyticity equation and nonlinear covariant harmonic-field equations are studied. A covariant harmonic field can be treated as an infinite set of ordinary four-dimensional fields with higher spins. We analyze different constructions of invariant harmonic-field actions corresponding to the self-dual harmonic equations.

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1 Introduction

It is well known that the self-dual Yang-Mills (SDYM) equations in four-dimensional Euclidean space $E_4$ have equivalent sigma-model-type representations [1, 2]. In contrast with the original self-duality equation, these representations are not manifestly invariant under $SO(4) \simeq SU(2)_L \times SU(2)_R$ rotations. These remarkable 4D field models have been intensively studied [3, 4, 5, 6], and their applications as effective field models for the open $N=2$ string have also been discussed [7, 8, 9]. Different sigma-model representations of the SDYM equations are equivalent classically; however, the consistency and compatibility of these models in the quantum region are not evident.

The problem of constructing an $SO(4)$-invariant action for self-dual gauge fields was also investigated. The simplest invariant action for the SDYM equations contains propagating Lagrange multiplier [10]. An improved invariant action based on an infinite set of non-propagating auxiliary fields has been considered in ref. [11].

The geometric description of the self-dual solutions is connected with twistor methods [12, 13]. We shall use the harmonic approach to the SDYM equations [14, 15, 16], which is a covariant version of the twistor method. The (anti)self-dual solutions are described in harmonic-twistor space using additional harmonic coordinates $u^\pm_\alpha$ which are $SU(2)_L$ spinors. The space of self-dual fields is parametrized by the set of harmonic connections $V^{++}(x, u)$ satisfying a linear analyticity equation. Fields in harmonic space are equivalent to infinite sets of ordinary 4D fields (harmonic components) of higher spins (helicities). The analyticity condition on $V^{++}$ translates to an infinite set of linear equations for the covariant component fields. Note that analogous infinite multiplets of fields appear in the bosonic variant of superspace proposed in ref. [17] for describing the effective field theory of the $N=2$ string.

After briefly reviewing non-covariant formulations of self-dual Yang-Mills theory in sect.2, we shall consider in sect.3 harmonic representations of the SDYM equations. These generalize the non-covariant self-dual equations [1, 2]. We then discuss real forms of the harmonic fields in different representations. In sect.4, the dual harmonic connection $V^{--}(x, u)$ is used to propose a harmonic generalization of the Leznov equation. Also, a harmonic analog of the Yang equation can be written in terms of a harmonic field $L(x, u)$ which contains odd spins only. On-shell all these equations again produce the SDYM equations for the Yang-Mills field, which can depend on the harmonic coordinates. However, one can entertain the speculation that the higher-spin harmonic components propagate off-shell and generate new effects in the quantum theory.

An off-shell description of self-dual harmonic fields has been proposed in ref. [18] but was shown to lead to a perturbatively trivial $S$-matrix [19]. In sect.5, we discuss alternative constructions of invariant harmonic actions using different harmonic-field variables. One can consider a modification of the simplest action for the analyticity equation by adding polynomial terms which produce on-shell relations between the Lagrange multiplier and the prepotential. It is possible that such a modification allows for nontrivial quantum corrections, although the precise consequences for quantization are unclear.
Second-order self-dual harmonic equations can be treated as nonlinear generalizations of the ordinary Laplace equation and can be quantized with the help of standard methods. In sect. 3, we consider the quadratic part of an invariant action for the harmonized Leznov equation, since we do not know the exact form of the corresponding interactions. The most simple action is constructed for the harmonic analog of the Yang equation.

Useful relations for $SU(2)_L/U(1)$ harmonics are collected in an appendix.

Possible applications and quantum calculations with harmonic self-dual actions are not considered in this paper. However, we hope that the harmonic approach will be useful for revealing the role of self-dual fields in quantum field theory.

2 Non-covariant formulations of self-dual theories

The four-dimensional equations for self-dual Yang-Mills (SDYM) fields are consistent in 4D Euclidean space $E_4$ as well as in 4D Kleinian space $K_4$ (of signature (2,2)). The spinor covering groups for these spaces are $SU(2)_L \times SU(2)_R$ and $SL(2, R)_L \times SL(2, R)_R$, respectively. Since we shall consider only the formal problem of constructing actions for SDYM theory, we may restrict our analysis to the Euclidean case. Let $\alpha, \beta \ldots$ and $\dot{\alpha}, \dot{\beta} \ldots$ be 2-spinor indices of the $SU(2)_L$ and $SU(2)_R$ groups, respectively, and denote the Euclidean Weyl matrices by

$$(\sigma^m)_{\alpha \dot{\beta}} \quad \text{and} \quad (\sigma^m)^{\dot{\beta} \alpha} = \varepsilon^{\alpha \rho} \varepsilon^{\dot{\beta} \dot{\gamma}} (\sigma^m)_{\rho \dot{\gamma}} . \quad (2.1)$$

Bispinor representations for the 4D coordinates and partial derivatives are

$$x^{\alpha \dot{\beta}} = \frac{1}{\sqrt{2}} (\sigma_m)^{\dot{\beta} \alpha} x^m \quad \text{and} \quad \partial_{\alpha \dot{\beta}} = \frac{1}{\sqrt{2}} (\sigma^m)_{\alpha \dot{\beta}} \partial_m . \quad (2.2)$$

The (anti)self-duality equation for the non-Abelian gauge field $A_{\alpha \dot{\beta}} = (\sigma^m)_{\alpha \dot{\beta}} A_m$ has the following form:

$$[\nabla_{\alpha \dot{\beta}}, \nabla_{\gamma \dot{\rho}}] = \varepsilon_{\alpha \gamma} F_{\dot{\beta} \dot{\rho}} \quad , \quad (2.3)$$

where $\nabla_{\alpha \dot{\beta}} = \partial_{\alpha \dot{\beta}} + A_{\alpha \dot{\beta}}$ is the covariant derivative. It is useful to rewrite these equations in an $SU(2)_L$ non-covariant form,

$$[\nabla_{1 \dot{\alpha}}, \nabla_{1 \dot{\beta}}] = 0 \quad , \quad (2.4)$$

$$[\nabla_{2 \dot{\alpha}}, \nabla_{2 \dot{\beta}}] = 0 \quad , \quad (2.5)$$

$$\varepsilon^{\dot{\alpha} \dot{\beta}} [\nabla_{1 \dot{\alpha}}, \nabla_{2 \dot{\beta}}] = 0 \quad . \quad (2.6)$$

The Yang ansatz $[1]$ solves the first two equations of this system via

$$A_{1 \dot{\beta}} = h^{-1} \partial_{1 \dot{\beta}} h \quad \text{and} \quad A_{2 \dot{\beta}} = \bar{h} \partial_{2 \dot{\beta}} \bar{h}^{-1} \quad , \quad (2.7)$$

where $h$ and $\bar{h}$ are independent $SL(N, C)$ matrices. The reality condition on the gauge potential reads

$$(A_{1 \dot{\beta}})^\dagger = -A_{2 \dot{\beta}} \quad \text{in } E_4 \quad \text{or} \quad (A_{\alpha \dot{\beta}})^\dagger = -A_{\alpha \dot{\beta}} \quad \text{in } K_4 \quad . \quad (2.8)$$

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Consequently, one must choose $\bar{h} = h^\dagger$ in the $E_4$ case but take independent unitary matrices $h$ and $\bar{h}$ in $K_4$. The remaining equation (2.6) within this ansatz,

$$\partial_1^\alpha \left[(h\bar{h})\partial_{2\bar{a}}(h\bar{h})^{-1}\right] = 0 \quad ,$$

(2.9)

resembles the equation for the 2D principal chiral model. In terms of the matrix variable $l = \ln(h\bar{h})$, this equation can be obtained from the action [3, 4]

$$S[l] = \int d^4x \int_0^1 ds \, \text{Tr} l \partial_1^\alpha \left[e^{sl} \partial_{2\bar{a}}e^{-sl}\right] = \int d^4x \, \text{Tr} \left(l \partial_1^\alpha \partial_{2\bar{a}} l - \frac{1}{3} l \left[ \partial_1^\alpha l , \partial_{2\bar{a}} l \right] + O(l^4)\right) \quad ,$$

(2.10)

where $s$ is some auxiliary parameter. Note that this nonlinear action for SDYM theory is not invariant with respect to $SU(2)_L$.

The alternative Leznov ansatz [2] for the SDYM equations reads

$$A_{1\dot{a}} = 0 \quad \text{and} \quad A_{2\bar{a}} = \partial_{1\dot{a}} A \quad ,$$

(2.11)

where $A$ is some Lie-algebra-valued function. This ansatz trivially solves eq.(2.6), but now eq.(2.5) becomes dynamical,

$$\partial_1^\alpha \left(\partial_{2\bar{a}} A + \frac{1}{2} [A , \partial_{1\dot{a}} A] \right) = 0 \quad .$$

(2.12)

Clearly, it is equivalent to the following first-order equation [2]

$$\partial_{2\bar{a}} A + \frac{1}{2} [A , \partial_{1\dot{a}} A ] = \partial_{1\dot{a}} \Omega \quad ,$$

(2.13)

where $\Omega$ is some arbitrary function.

It is interesting that Yang’s equation (2.9) is non-polynomial in $l$ while Leznov’s equation (2.12) is quadratic in $A$. The simplest corresponding action is cubic,

$$S[A] = \int d^4x \, \text{Tr} \left( A\partial_1^\alpha \partial_{2\bar{a}} A + \frac{1}{6} A \left[ \partial_1^\alpha A , \partial_{1\dot{a}} A \right] \right) = 0 \quad .$$

(2.14)

Alternatively, the first-order equation (2.13) can be obtained from

$$S(P , A , \Omega) = \int d^4x \, \text{Tr} P^{\dot{a}} \left\{ \partial_{2\bar{a}} A + \frac{1}{2} [A , \partial_{1\dot{a}} A ] - \partial_{1\dot{a}} \Omega \right\} + S'(A) \quad ,$$

(2.15)

where $P^{\dot{a}}$ is a dynamical Lagrange multiplier. The additional term $S'(A)$ only affects the equation for $P^{\dot{a}}$ and does not modify eq.(2.13).

The two given representations of the on-shell self-dual gauge field are related by

$$e^l \partial_{2\bar{a}} e^{-l} = \partial_{1\dot{a}} A \quad .$$

(2.16)

This relation will not be valid off-shell, and so the quantum relation between different representations requires further analysis.
3 Twistor-harmonic solutions of SDYM equation

The connection of the SDYM equation with the inverse-scattering method has been considered in ref. [12]. The corresponding equation of the auxiliary linear problem has the following form:

\[(\nabla_{1\dot{\alpha}} + \lambda \nabla_{2\dot{\alpha}}) g(\lambda, x) = 0\]

where \(\lambda\) is the complex spectral parameter, and \(g(\lambda, x)\) is the auxiliary matrix function, usually chosen as a meromorphic function of \(\lambda\) with the specific rule of conjugation

\[g^\dagger(-1/\bar{\lambda}) = g^{-1}(\lambda)\]

In the twistor interpretation of the SDYM equation [13] the projective line \(CP(1)\) is described by complex coordinates \(\eta^\alpha\). The anti-self-dual solutions can be written with the help of a flat connection in this special direction,

\[\eta^\alpha \nabla_{\alpha\dot{\beta}} = g^{-1}(\eta, x)\eta^\alpha \partial_{\alpha\dot{\beta}} g(\eta, x)\]

The harmonic description of the SDYM equation [16] can be viewed as a covariant version of the twistor approach, using the isomorphism between \(CP(1)\) and the coset \(SU(2)_L/U(1)\). The harmonics \(u_{\alpha\dot{\alpha}}^\pm\) can be introduced as additional coordinates of the harmonic-twistor space. They transform covariantly with respect to \(SU(2)_L\) and an extra \(U(1)\) [14, 15]. In the special complex parametrization of harmonics [1,13] the \(SU(2)_L\) transformations correspond to rational transformations of the complex plane. It should be stressed that the practitioners of the harmonic approach treat harmonics as global coordinates on the sphere \(SU(2)_L/U(1)\) and employ special representations only when comparing with other methods. All covariant functions of harmonics possess regular series expansions on the sphere, and conservation of the extra \(U(1)\) charge is a necessary condition in this approach.

Using the \(SU(2)_L\)-invariant harmonic projections of the covariant derivatives,

\[(\nabla_c)^+_\beta = u^{+\alpha} \nabla_{\alpha\dot{\beta}} = \partial^+_\beta + C^+_\beta\]
\[(\nabla_c)^-_\beta = u^{-\alpha} \nabla_{\alpha\dot{\beta}} = \partial^-_\beta + C^-_\beta\]

one can obtain the harmonic decomposition of the SDYM equation [23],

\[F^{++}_{\dot{\alpha}\dot{\beta}} \equiv [(\nabla_c)^+_\dot{\alpha}, (\nabla_c)^-_\dot{\beta}] = 0\]
\[F^{--}_{\dot{\alpha}\dot{\beta}} \equiv [(\nabla_c)^-_\dot{\alpha}, (\nabla_c)^-_\dot{\beta}] = 0\]
\[F^{+-}_{\dot{\alpha}\dot{\beta}} \equiv [(\nabla_c)^+_\dot{\alpha}, (\nabla_c)_\dot{\beta}] = 0\]

In addition to these equations of dimension \(d=-2\), we may use the following \(d=-1\) equations of the harmonic approach:

\[F^{++}_{\dot{\alpha}\dot{\beta}} \equiv [(\nabla_c)^{++}, (\nabla_c)^{+\dot{\alpha}}] = 0\]
\[F^{--}_{\dot{\alpha}\dot{\beta}} \equiv [(\nabla_c)^{-\dot{\alpha}}, (\nabla_c)^{-\dot{\beta}}] = 0\]
\[F^{+-}_{\dot{\alpha}\dot{\beta}} \equiv [(\nabla_c)^{+\dot{\alpha}}, (\nabla_c)^{-\dot{\beta}}] = 0\]

where covariant harmonic derivatives \(\nabla^{\pm\pm}\) are introduced. Until now, we worked in the so-called central basis (CB) with \(u\)-independent gauge group and flat harmonic derivatives.
\((\nabla_c)^{\pm\pm} = \partial^{\pm\pm}\). The properties of harmonics and partial harmonic derivatives \([14, 15]\) are considered in the appendix.

One can define two types of conjugations for the harmonic matrix functions, namely \(C(u) \rightarrow \bar{C}(u)\) and \(C(u) \rightarrow \tilde{C}(u)\) according to (7.3) and (7.4). In the case of a unitary gauge group, the harmonic projections \(C^\pm_\beta\) of the anti-Hermitean CB connections \(A^\pm_\alpha\beta\) satisfy the following reality conditions with respect to these conjugations,

\[
\begin{align*}
\bar{C}^\pm_\beta &= \pm C^\beta_\pm \quad , & \bar{C}^\pm_\beta &= -C^\beta_\pm .
\end{align*}
\] (3.10)

The general harmonic solution of equation (3.5) has the following form:

\[
C^+_\beta = g^{-1}\partial^+_\beta g ,
\] (3.11)

where the bridge matrix \(g(x, u)\) satisfies the analyticity condition

\[
[\partial^+_\beta, V^{++}] = 0 \quad \text{with} \quad V^{++} = -(\partial^{++} g)g^{-1} .
\] (3.12)

The bridge \(g\) provides a transformation between the geometric quantities of the CB and a new analytic basis (AB), defined by the relations

\[
\begin{align*}
\nabla &= g(\nabla_c)g^{-1} , & \mathcal{F} &= gF_c g^{-1} , \\
\nabla^+_\beta &= \partial^+_\beta , & \nabla^{++} &= \partial^{++} + V^{++} , \\
\nabla^-_\beta &= \partial^-_\beta + A^-_\beta , & A^-_\beta &= -\partial^-_\beta V^{--} , \\
\nabla^{--} &= D^{--} + V^{--} , & \partial^{++}V^{--} - \partial^{--}V^{++} + [V^{++}, V^{--}] &= 0 .
\end{align*}
\] (3.13-3.16)

The gauge group in the AB is analytic,

\[
\delta V^{\pm\pm} = D^{--}\omega + [V^{\pm\pm}, \omega] , & \partial^+_\alpha\omega = 0 ,
\] (3.17)

while the bridge matrix enjoys a mixed transformation law

\[
\delta g = \omega g - g\tau ,
\] (3.18)

where \(\tau\) is the CB gauge parameter.

The standard harmonic SDYM solution \([16]\) uses the analytic connection \(V^{++}\) as initial data and reduces the full problem to solving the linear harmonic equation

\[
(\partial^{++} + V^{++}) g = 0 .
\] (3.19)

In the representation \((7.19)\) one has \(\partial^{++} g \sim g/\partial\bar{\lambda}\), so this equation generalizes the meromorphicity condition on the auxiliary matrix function in the standard twistor approach. (See, however, ref. \([20]\) for an analogous equation in the twistor formalism.) The harmonic equation possesses a simple one-instanton solution \([16]\) regular in harmonics. Note that the reality condition becomes simply

\[
\bar{g}(u, x) = g^{-1}(u, x) , & \bar{V}^{++} = -V^{++} .
\] (3.20)
The other conjugation is not used in this analytic representation of the self-dual solutions.

It is useful to introduce analytic coordinates and derivatives

\[ x^{\dot{\alpha} \pm} = u^{\pm \alpha} x^{\alpha \dot{\beta}} , \quad \partial_{\dot{\beta}}^{\pm} = u^{\pm \alpha} \partial_{\alpha \dot{\beta}} , \quad \partial_{\dot{\beta}}^{\pm} x^{\alpha \dot{\beta}} = \pm \delta_{\dot{\beta}}^{\alpha} . \]  

(3.21)

Analytic functions do not depend on the \( x^{\alpha -} \) coordinates, like \( V^{++} = V^{++}(x^+, u^+) \). Note that in analytic coordinates we should use the covariant harmonic derivatives

\[ D^{\pm \pm} = \partial^{\pm \pm} \pm x^{\alpha \dot{\beta}} \partial^{\pm \alpha} \partial_{\dot{\beta}} , \quad [D^{++}, \partial_{\dot{\beta}}^{-}] = \partial_{\dot{\beta}}^{\alpha} , \quad [D^{--}, \partial_{\dot{\beta}}^{+}] = \partial_{\dot{\beta}}^{-} . \]

(3.22)

The anti-self-dual gauge field in the central basis and corresponding field strength can be written in terms of the harmonic quantities as

\[ A_{\alpha \dot{\beta}}(V^{++}) = u^{\alpha} C_{\dot{\beta}}^{+} - u^{\dot{\beta}} C_{\alpha}^{-} = g^{-1}(\partial_{\alpha \dot{\beta}} - u^{\dot{\beta}} A_{\alpha}^{-}) g , \]

(3.23)

\[ F_{\dot{\alpha} \dot{\beta}}(V^{++}) = g^{-1}\partial_{\dot{\alpha}}^{+} \partial_{\dot{\beta}}^{\alpha} V^{-} g . \]

(3.24)

This wide class of self-dual fields does not depend manifestly on harmonics,

\[ \partial^{++} A_{\alpha \dot{\beta}}(V^{++}) = 0 \quad , \quad \partial^{++} F_{\dot{\alpha} \dot{\beta}}(V^{++}) = 0 \]  

(3.25)

if eqs. (3.12)-(3.19) are satisfied. In the following section we shall discuss SDYM configurations \( A_{\dot{\alpha} \dot{\beta}} \) which depend parametrically on the harmonics and cannot be expressed via the analytic connection.

The definition of the analytic basis is formally independent of the analyticity condition (3.12), so we may consider the nonlinear zero-curvature equation

\[ [\nabla_{\dot{\beta}}^{-}, \nabla_{\dot{\beta}}^{-}] \equiv F_{\dot{\alpha} \dot{\beta}}^{-} - (V^{--}) = \partial_{\dot{\beta}}^{-} V^{--} + D^{--} \partial_{\dot{\beta}}^{+} V^{--} + [V^{--}, \partial_{\dot{\alpha}}^{+} \partial_{\dot{\beta}}^{-} V^{--}] = 0 \]  

(3.26)

as an alternative analyticity condition in the AB. Equation (3.26) is equivalent to the linear equation (3.12), and its perturbative solution can be written in terms of the analytic prepotential \( V^{++} \) as

\[ V^{--}(x,u) = \sum_{n=1}^{\infty} (-1)^{n} \int du_{1} \ldots du_{n} \frac{V^{++}(x,u_{1}) \ldots V^{++}(x,u_{n})}{(u_{1}^{*} u_{1}^{+}) (u_{2}^{*} u_{2}^{+}) \ldots (u_{n}^{*} u_{n}^{+})} \]  

(3.27)

where \((u_{1}^{*} u_{2}^{+})^{-1}\) is the harmonic distribution \([5]\). Conversely, one may treat \( V^{--} \) as a basic harmonic field and obtain the connection \( V^{++} \) as the solution of (3.16). The bridge matrix of this formulation satisfies the equation

\[ (\partial^{--} + V^{--}) g = 0 \]  

(3.28)

### 4 Generalized harmonic self-dual equations

The partial derivative of the alternative analyticity equation (3.26) produces a harmonic generalization of the Leznov equation (2.12),

\[ [\partial^{\dot{\alpha} \dot{\beta}}, F_{\dot{\gamma} \dot{\delta} \dot{\alpha} \dot{\beta}}^{--}(V^{--})] = 2 \partial^{\dot{\alpha} \dot{\beta}} \partial_{\dot{\gamma} \dot{\delta}}^{+} V^{--} + [\partial^{\dot{\alpha} \dot{\beta} \dot{\delta}}^{+} V^{--}, \partial_{\dot{\gamma} \dot{\delta}}^{\alpha} \partial_{\dot{\beta}}^{-} V^{--}] = 0 . \]  

(4.1)
Note that this second-order equation is invariant under the additional transformation 
\[ \delta V^- = \omega^- \] with an analytic parameter \( \omega^- \). In distinction with (3.26), this equation does not contain harmonic derivatives, and so the dependence of \( V^{++} \) on harmonics is merely parametric. In the non-covariant approach, one can treat the Leznov field \( A_{(2.12)} \) as a special configuration of the covariant harmonic field \( V^- \).

Note that the second-order equation (4.1) is equivalent to the first-order equation
\[
2 \partial^- \alpha V^- + [V^-, \partial^- \alpha V^-] = \partial^- \Omega^--- ,
\]
where \( \Omega^---(x, u) \) is an arbitrary function. Thus, the second-order harmonic equation (4.1) is in some sense more general than the first-order equations (3.12) or (3.26). The general solutions \( V^- \) of (4.1) correspond to non-analytic connections \( V^{++} \). Using the bridge matrix \( g (3.28) \) for these solutions one can obtain the CB-connection \( A_{\alpha\dot{\beta}}(x, u) (3.23) \) which manifestly depends on harmonics in this case. Nevertheless, the study of this representation can be useful for the construction of self-dual gauge fields.

It should be underlined that the solutions of the 4D Laplace equation, arising as the linear approximation of the self-dual equations (2.9) and (2.12), have the general harmonic-twistor representation
\[
f = \int du \, F(x^{\dot{\alpha}+}, u_{\alpha}^{\pm}) , \quad \Box f = 0 ,
\]
where \( F \) is an analytic harmonic function. This representation connects solutions of the non-covariant self-dual equations and their harmonic generalizations with the space of analytic functions. Each solution of the abelian equation \( \partial^{\dot{\alpha}+} A_{\alpha}^+ = 0 \) can be written in the different equivalent forms
\[
A_{\alpha}^+ = \partial_{\alpha}^+ v_0(x, u) = \partial_{\alpha}^+ a_{0}^{++}(x, u) ,
\]
with \( a_{0}^{++}(x, u) \) solving the 4D-Laplace equation. The proof follows from eq.(4.3) or can be obtained by the expansion in \( x^{\dot{\alpha}-} \). Let us consider the harmonic generalization of the Yang representation in the central basis
\[
C_{\dot{\beta}}^+ = g^{-1} \partial_{\dot{\beta}}^+ g , \quad C_{\dot{\beta}}^- = (-\partial_{\dot{\beta}}^- \bar{g}) \bar{g}^{-1} ,
\]
where the matrix \( \bar{g} \) is defined via transposition and ordinary conjugation (7.5). The simultaneous use of two different conjugations is an interesting peculiarity of this harmonic representation. Using the transformation \( A_{\alpha}^+ = g(\partial_{\alpha}^+ + C_{\alpha}^+) g^{-1} \) to the analytic basis, one can obtain the following representation:
\[
A_{\dot{\beta}}^+ = 0 , \quad A_{\dot{\beta}}^- = -(\partial_{\dot{\beta}}^- \bar{g}) (\bar{g} \bar{g})^{-1} .
\]
Now introduce the new matrix variable \( L(x, u) \)
\[
g \bar{g} = e^L , \quad L = \dot{v} + \bar{v} + \frac{1}{2} [\dot{v}, \bar{v}] + \ldots ,
\]
where \( v = \ln g \). This matrix satisfies specific reality properties with respect to the two conjugations,

\[
\tilde{L}(x, u) = -L(x, u) \quad \text{and} \quad \bar{L}(x, u) = L(x, u) \, .
\]  

(4.8)

Note that these conditions are compatible for the following harmonic decomposition

\[
L(x, u) = \sum_{k=0}^{\infty} (u^{+(2k+1)} u^{-/(2k+1)})_{\alpha_1 \cdots \alpha_{4k+2}} L_{\alpha_1 \cdots \alpha_{4k+2}}(x) \, .
\]  

(4.9)

It is not difficult to show that these \( U(1) \)-neutral harmonics with odd spins \( 2k + 1 \) are imaginary or real with respect to the different conjugations, and so all fields \( L_{\alpha_1 \cdots \alpha_{4k+2}}(x) \) are anti-Hermitean. This representation trivially solves eqs. (3.5) and (3.6) and produces the harmonic generalization of the Yang equation (2.9):

\[
\partial^+ \left[ e^L(\partial^- e^{-L}) \right] = 0 \, .
\]  

(4.10)

Finally, let us relate the harmonic SDYM configurations in different representations. Equation (4.7) connects the field \( L \) with the solutions of the harmonic bridge equation (3.28) corresponding to the self-dual field \( V^{--} \). The inverse construction of the fields \( g \) and \( V^{--} \) in terms of \( L \) is not so straightforward. One considers the equation

\[
e^L[\partial^- e^{-L}] = -\partial^+ V^{--} \, ,
\]  

(4.11)

which is the non-abelian analog of the relation (4.4). The explicit relation between \( V^{--} \) and \( L \) has the following nonlocal form:

\[
V^{--}(L) = \frac{1}{\Box} \partial^+ \left[ e^L(\partial^- e^{-L}) \right] + \Lambda^{--} \, ,
\]  

(4.12)

where \( \Box = \partial^+ \partial^- \) and \( \Lambda^{--} \) is an analytic function. This expression can be used in the harmonic equation (3.28) for the construction of the bridge \( g(L) \) corresponding to the solution \( L \) of eq.(4.10).

Thus, the harmonic space may be employed to formulate various covariant equations which are equivalent to the self-duality equation in ordinary space. These formulations allow us to describe more precisely the structure of the moduli space for the SDYM equation.

5 Invariant actions for the analyticity equations

In the non-covariant formalisms of SDYM theory one may analyze the actions for the fields \( l \) (2.10) or \( A \) (2.14) off-shell and construct the corresponding functional integral. These field theories are used for the analysis of \( N=2 \) string amplitudes and for calculations of the Yang-Mills \( S \)-matrix in the self-dual sector.

Attempts to construct an invariant action for the self-duality equation lead to the problem of doubling of states or even to the appearance of an infinite number of auxiliary
fields \[11\]. An infinite number of physical states also arise in the effective field theory for the \(N=2\) string \[17\].

In the harmonic approach the analytic prepotential \(V^{++}\) or its alternative cousins \(V^{--}\) and \(L\) are equivalent to the on-shell self-dual fields in Euclidean space. We may try to treat the covariant harmonic equations as basic field equations in the extended harmonic space \(E_4 \times S_2\) and use unconstrained harmonic fields to define the theory off-shell. In contrast with the Kaluza-Klein approach, the harmonic-field theory identifies the symmetry group of \(S^2\) with the \(SU(2)_L\) subgroup of \(SO(4)\). It is evident that this covariant harmonic theory contains an infinite number of ordinary 4\(D\) fields by construction, but its connection with the non-covariant theories containing a finite number of fields is not clear. Here we shall analyze invariant actions for harmonic equations, hoping to find a consistent description of the nontrivial interactions of higher-spin fields.

In order to preserve manifest covariance off-shell we treat one of the dimensionless harmonic fields as the basic field variable in the action. The purely harmonic equations (3.16) and (3.19) with \(d=0\) are independent of the analyticity conditions, and so we can use off-shell the relations

\[
V^{\pm \pm} = -(\partial^{\pm \pm} e^v) e^{-v}.
\]

For a unitary gauge group one should also keep off-shell the conjugation rules

\[
\bar{V}^{\pm \pm} = -V^{\pm \pm}, \quad \bar{v} = -v.
\]

We covariantly decompose the off-shell fields into an infinite set of fields with arbitrary spins \(k\),

\[
V^{\pm \pm}(x, u) = \sum_{k=1}^{\infty} \left( u^{+ (k+1)} u^{- (k-1)} \right)_{\alpha_1 \cdots \alpha_{2k}} V^{\alpha_1 \cdots \alpha_{2k}}_k(x),
\]

\[
v(x, u) = v_0(x) + \sum_{k=1}^{\infty} \left( u^{+k} u^{-k} \right)_{\alpha_1 \cdots \alpha_{2k}} v^{\alpha_1 \cdots \alpha_{2k}}_k(x).
\]

Note that these harmonic component fields radically differ from the infinite set of bispinor auxiliary fields \(G_\alpha^\beta\) of ref.\[11\], which are covariant with respect to the ordinary gauge group and have \(d=-2\). On the other hand, the invariant self-dual action of ref.\[10\] contains only one additional propagating field \(G_0^\alpha^\beta\),

\[
\int d^4 x \ Tr G_0^\alpha^\beta \left( \partial^\gamma A_{\beta \gamma} + [A_{\alpha \gamma}^\gamma, A_{\beta \gamma}^\gamma] \right).
\]

A general formulation of the harmonic gauge theory in the AB requires a choice of basic parametrization of the connection \(A_\alpha^\gamma\) in terms of the independent harmonic variables \(V^{++}, \ V^{--},\) or \(v\). The analytic on-shell gauge group of the harmonic self-dual equations is unessential for the off-shell harmonic fields, and we do not know the appropriate generalization of the gauge group in the harmonic space. However, we shall present several possible actions for the harmonic equations considered in sect.\[3\]. The off-shell geometry in the analytic basis automatically guarantees the relation \(F^{+++}_{\alpha \beta} = 0\), but we might want to
include additional off-shell constraints. The formal use of the relations (3.23) determines the gauge field, which depends on the harmonics off-shell.

We should like to discuss shortly the problem of truncating the infinite set of harmonic component fields satisfying the various types of self-duality equations. The linear analyticity equation \( \partial^2_{\pm} V^{++} = 0 \) is compatible with an arbitrary truncation of the harmonic expansion of \( V^{++} \). Nevertheless, it is difficult to use truncated harmonic fields in the action. The nonlinear equations (4.1) and (4.10) are not consistent with simple truncations of the harmonic expansion for \( V^{--} \) and \( L \).

The bilinear action with the Lagrange multiplier \( P^{\dot{\alpha}---}(x, u) \)

\[
S_P = \int dx \, du \, \text{Tr} \, P^{\dot{\alpha}---} \partial^{++}_{\alpha} V^{++} \quad (5.6)
\]
is trivial if one treats \( V^{++}(x, u) \) as the basic off-shell field. The authors of ref.[18] have instead used the field \( v(x, u) \) (see (3.11)) as the independent off-shell harmonic field in this action. However, their approach produces a perturbatively trivial quantum theory [19].

Let us then deform the bilinear action (5.6) by terms depending on \( V^{++} \), defining a new class of harmonic-field models. For example, one can choose such terms by analogy with the harmonic action of \( N=2 \) Yang-Mills theory[21]

\[
S(V^{++}) = c \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \int dx \, du_1 \ldots du_n \, \frac{V^{++}(x, u_1) \ldots V^{++}(x, u_n)}{(u_1^+ u_1^-) \ldots (u_n^+ u_n^-)} \quad (5.7)
\]
where \( c \) is a constant of \( d=-4 \) and \( (u_1^+ u_2^-)^{-1} \) is the harmonic distribution [13]. The action \( S_P + S(V^{++}) \) produces the equations

\[
\partial^{++}_{\dot{\alpha}} V^{++} = 0 \quad , \quad \partial^{++}_{\dot{\alpha}} P^{\dot{\alpha}---} = e V^{--}(V^{++}) \quad , \quad (5.8)
\]
where the perturbative solution \( V^{--}(V^{++}) \) of the harmonic equation (3.10) is to be used. The second equation describes some restriction on the analytic fields, namely

\[
\int (d^2 x)^-- V^{--}(V^{++}) = 0 \quad . \quad (5.9)
\]

Since the importance of analytical group is not evident for the off-shell theory, we may consider an arbitrary quadratic term in the full harmonic space with coordinates \( z = (x, u) \):

\[
S_2(V^{++}) = \frac{1}{2} \int dz_1 dz_2 \, K^{-----}(z_1, z_2) \, \text{Tr} \, V^{++}(z_1) V^{++}(z_2) \quad , \quad (5.10)
\]
where \( K^{-----}(z_1, z_2) \) is some local or nonlocal kernel (e.g. \( (u_1^+ u_2^-)^{-2} \delta(x_1 - x_2) \) for eq.(5.7)). Higher terms contain polynomials in \( V^{++} \). The auxiliary field possesses an additional gauge symmetry, \( \delta P^{\dot{\alpha}---} = \partial^{++}_{\dot{\alpha}} a^{---}(x, u) \). In the quantum version of this model, one can thus introduce the gauge

\[
\partial^{++}_{\dot{\alpha}} P^{\dot{\alpha}---} = 0 \quad . \quad (5.11)
\]

Propagators of the harmonic fields in this gauge are

\[
< P^{\dot{\alpha}---}(z_1) | V^{++}(z_1) > = -\frac{1}{\Box} \partial^{\dot{\alpha}} \delta(x_1 - x_2) \delta^{--++}(u_1, u_2) \quad , \quad (5.12)
\]
\[
< P^{\dot{\alpha}---}(z_1) | P^{\dot{\alpha}---}(z_2) > = -\frac{1}{\Box^2} \partial^{\dot{\alpha}} \partial^{\dot{\alpha}} \delta^{(2)} K^{-----}(z_1, z_2) \quad . \quad (5.13)
\]
An action with Lagrange multiplier can be written also for the nonlinear analyticity equation (3.26),

$$ S(V^{--}, P^{+++}) = \int dx \, du \, \text{Tr} \, P^{+++} F^{--}_{-\alpha}(V^{--}) \ . $$

(5.14)

Of course, this action can be deformed as well by terms depending on $V^{--}$.

Finally, we discuss shortly a completely different off-shell description of self-dual harmonic fields, using functional integrals over the analytic fields $V^{++}(\zeta)$ where $\zeta = (x_{\alpha}^+, u_{\beta}^\pm)$. Candidate actions for this approach can be written directly in the analytic space. The quadratic term, for instance, have the following form:

$$ \int d\zeta^+ d\zeta^+ A^{--} A^{--} (\zeta_1, \zeta_2) V^{++}(\zeta_1) V^{++}(\zeta_2) \ , $$

(5.15)

where the analytic kernel relates to the kernel in the general harmonic space (5.10) via

$$ A^{--} (x_{1}^+, u_{1}^\pm, x_{2}^+, u_{2}^\pm) = \int (d^2 x_1)^--(d^2 x_2)^-- K^{--} (x_{1}^+, u_{1}^\pm, x_{2}^+, u_{2}^\pm) \ . $$

(5.16)

Let us mention that an arbitrary analytic prepotential $V^{++}$ describes solutions of the SDYM equations with unbounded value of the classical action $\int d^4 x \, \text{Tr} \left( \frac{1}{2} F^{--}_{\alpha\beta} (V^{--})^2 \right)$. Hence, the relation of the functional integrals over analytic fields $V^{++}(\zeta)$ with the instanton ideology is unclear.

6 Actions for generalized self-dual equations

Now we shall consider invariant actions for the generalized self-dual harmonic equations of sect. 4. An action for the first-order equation (4.2) can be written by analogy with the non-invariant action (2.15),

$$ S(P^{+++}, V^{--}, \Omega^{---}) = \int dx \, du \, \text{Tr} \, P^{+++} \{ 2 \partial^{-\alpha} V^{--} - [V^{--}, \partial^{+\alpha} V^{--}] - \partial^{+\alpha} \Omega^{---} \} + S'(V^{--}) $$

(6.1)

where the last term may contain, for instance, polynomial functions of $D^{++} V^{--}$. This action exhibits no invariance with non-analytic parameters and generates simple Feynman rules.

It is easy to show that the second-order equation (4.1) can indeed be derived from a harmonic action with non-covariant measure:

$$ \int dx \, du \, f^{+++}(u) \, \text{Tr} \left( V^{--} \partial^{+\alpha} \partial^{-\alpha} V^{--} + \frac{1}{3} V^{--} [\partial^{+\alpha} V^{--}, \partial^{+\alpha} V^{--}] \right) $$

(6.2)

where $f^{+++}(u)$ denotes an arbitrary harmonic function, which breaks the manifest $SU(2)_L$ invariance. This action generates a nontrivial quantum perturbation theory completely local in harmonic variables. For instance, the generating functional of the free Green’s functions has the following form:

$$ W(J) = \ln Z(J) = \frac{1}{2} \int dx \, du \, f^{+++}(u) \, \text{Tr} \left[ J^{--}(x, u) \frac{1}{2} J^{--}(x, u) \right] $$

(6.3)
where $J^{- -}(x,u)$ is the classical source.

The construction of a bilinear action for the harmonic equation $\Box V^{- -} = 0$ is very simple:

$$S_2(V^{- -}) = \frac{1}{f^2} \int \! d^4x \, du \, \text{Tr} \, (D^{++})^2 V^{- -} \partial^+ \partial^{-} V^{- -} \ ,$$

(6.4)

where $f$ is a coupling constant of $d=1$. Unfortunately, the cubic term local in harmonics should contain an infinite number of terms, carrying arbitrary powers of the harmonic derivatives $D^{++}$ and $D^{- -}$. We are only in the position to construct these terms iteratively. The starting trilinear term in this infinite harmonic construction is

$$\frac{1}{f^2} \int \! d^4x \, du \, \text{Tr} \, (D^{++})^2 V^{- -} \left[ \partial^+ V^{- -}, \partial^+ V^{- -} \right] \ .$$

(6.5)

The variation of this term produces not only the unique quadratic term in eq.(4.1), but also some additional terms which are undesirable and should be cancelled by the infinite sum of variations of higher terms. Thus, we do not know the invariant action for eq.(4.1) in closed form. Of course, one may use the term (6.5) for describing some self-interaction of $V^{- -}$ which is not directly connected with the self-duality equation.

Off-shell harmonic fields in our treatment contain an infinite series of higher-spin fields. In this approach, the harmonic equation (4.1) yields an infinite set of equations for the component fields $V^{\alpha_1 \cdots \alpha_{2k}}(x)$ (5.3),

$$\partial^+ F^{- - \alpha_1 \alpha_2 \cdots \alpha_{2k}} = \sum_{k=1}^{\infty} \left( u^{+(k-1)} u^{-(k+1)} \right)_{\alpha_1 \cdots \alpha_{2k}} E^{\alpha_1 \cdots \alpha_{2k}} = 0 \ .$$

(6.6)

Each equation contains the linear Laplace term and the infinite number of bilinear terms describing the interaction of higher spin fields. Using the harmonic identities (7.10-7.14) one can calculate, in principle, any term of these equations. Consider, for example, the lowest terms of the equation with $k = 1$,

$$F^{\alpha_1 \alpha_2} = \Box V^{\alpha_1 \alpha_2} + \frac{2}{9} [\partial^\rho V^{\beta \alpha_1}_{\rho \alpha_2}, \partial^\beta V^{\beta \alpha_2}_{\rho \alpha_1}] +$$

$$+ \frac{1}{6} [\partial^{(\alpha_1 \alpha_2} V^{\beta \alpha_3)}_{1,\beta \alpha_3}, \partial^\beta V^{(\alpha_1 \alpha_2 \beta \alpha_3)}_{1,\beta \alpha_3}] - \frac{1}{10} [\partial^{(\alpha_1 \alpha_2} V^{\beta_1 \beta_2 \beta_3)}_{1,\beta_1 \beta_2 \beta_3}, \partial^{\alpha_3} V^{(\alpha_1 \alpha_2 \beta_1 \beta_2 \beta_3)}_{1,\beta_1 \beta_2 \beta_3}] + O(V_2) \ ,$$

(6.7)

where all bilinear terms with $k \geq 2$ are omitted. It is clear that an invariant component action with an infinite number of bilinear terms exists for the linearized infinite system of equations. The analysis of the trilinear terms can be performed, in principle, step by step for interactions of the fields $V_k$.

We shall now demonstrate that the harmonic field $L$ (4.3) is in fact more convenient for the construction of an invariant action. The action (local in $x$ and $u$) for the harmonic non-polynomial equation (4.10) can be written by analogy with eq.(2.10):

$$S(L) = \frac{1}{f^2} \int \! d^4x \, du \, \int_0^1 ds \, \text{Tr} \, L \, \partial^+ e^{s L} \partial^- e^{-s L} =$$

$$\int \! d^4x \, du \, \text{Tr} \left( L \, \partial^+ \partial^- L - \frac{1}{3} L \left[ \partial^+ L , \partial^- L \right] + O(L^4) \right) \ .$$

(6.8)
This action contains formally non-renormalizable derivative terms and may serve as effective action for the infinite set of fields $L^{\alpha_1 \cdots \alpha_{4k+2}}(x)$ with odd spins $2k+1$. The action is nondegenerate and does not require gauge fixing for quantization.

Remarkably, the non-covariant field theory for the analogous action (2.10) is one-loop finite \cite{5, 6}. It is conceivable that a covariant sigma model for the harmonic field $L(x, u)$ has such interesting quantum properties, as well.

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### 7 Appendix: Relations for $SU(2)/U(1)$ harmonics

The harmonics $u^\pm_\alpha$ ($\alpha = 1, 2$) can be interpreted as projective coordinates of the sphere $SU(2)/U(1)$ \cite{14, 15}. The basic relations for the harmonics and the harmonic derivatives are

\begin{align}
  u^+_\alpha u^-_\beta - u^-_\beta u^+_\alpha &= \varepsilon_{\alpha\beta} , \\
  [\partial^+, \partial^-] &= \partial^0 , \\
  [\partial^0, \partial^\pm] &= \pm 2\partial^\pm , \\
  \partial^+ u^+_\alpha &= 0 , \\
  \partial^- u^-_\alpha &= 0 .
\end{align}

These relations can be understood as $SU(2)$ covariant conjugations on the sphere $SU(2)/U(1)$. The alternative harmonic conjugation conserves $U(1)$ charges.

\begin{align}
  \bar{u}^\pm_\alpha = \pm u^\mp_\alpha , \\
  \bar{\partial}^\pm_\alpha = \mp \partial^\pm_\alpha .
\end{align}

For the case of $SL(2, R)_L \times SU(2, R)_R$ all coordinates and harmonics are real.
We shall use the notation \((u^p u^q)_{\alpha_1 \cdots \alpha_{(p+q)}}\) for the irreducible symmetric combinations of harmonics with \(p\) positive and \(q\) negative charges, for example,

\[
(u^+ u^-)_{\alpha_1 \alpha_2} = \frac{1}{2}(u^+_{\alpha_1} u^-_{\alpha_2} + u^+_{\alpha_2} u^-_{\alpha_1}) \quad , \tag{7.8}
\]

\[
(u^+ u^2)_{\alpha_1 \alpha_2 \alpha_3} = \frac{1}{3}(u^+_{\alpha_1} u^-_{\alpha_2} u^-_{\alpha_3} + u^+_{\alpha_2} u^-_{\alpha_3} u^-_{\alpha_1} + u^+_{\alpha_3} u^-_{\alpha_1} u^-_{\alpha_2}) \quad . \tag{7.9}
\]

Multiplication rules for irreducible harmonics can be obtained with the help of basic relations (7.1):

\[
u^+(u^-)_{\alpha_1 \alpha_2} = (u^+ u^-)_{\rho \alpha_1 \alpha_2} + \frac{2}{3} \varepsilon_{\rho \alpha_1} u^-_{\alpha_2} \quad , \tag{7.10}
\]

\[
u^- (u^+ u^-)_{\alpha_1 \alpha_2 \alpha_3} = (u^+ u^-)_{\rho \alpha_1 \alpha_2 \alpha_3} - \frac{1}{2} \varepsilon_{\rho \alpha_1} (u^-)_{\alpha_2 \alpha_3} \quad , \tag{7.11}
\]

\[
u^+(u^+ u^-)_{\alpha_1 \alpha_1 \alpha_2 \alpha_3} = (u^+ u^-)_{\rho \alpha_1 \alpha_2 \alpha_3} - \frac{3}{5} \varepsilon_{\rho \alpha_1} (u^+ u^-)_{\alpha_2 \alpha_3} \quad , \tag{7.12}
\]

\[
u^+(u^+ u^-)_{\alpha_1 \alpha_1 \alpha_2 \alpha_3 \alpha_4} = (u^+ u^-)_{\rho \alpha_1 \alpha_2 \alpha_3 \alpha_4} + \frac{3}{5} \varepsilon_{\rho \alpha_1} (u^+ u^-)_{\alpha_2 \alpha_3 \alpha_4} \quad , \tag{7.13}
\]

\[
u^+(u^+ u^-)_{\alpha_1 \alpha_2} (u^+ u^-)_{\beta_1 \beta_2 \beta_3} = (u^+ u^-)_{\alpha_1 \beta_1 \beta_2 \beta_3} - \frac{1}{6} \varepsilon_{\alpha_1 \beta_1 \beta_2 \beta_3} (u^+ u^-)_{\alpha_1 \beta_1 \beta_2 \beta_3} \quad . \tag{7.14}
\]

where parentheses denote complete symmetrization of indices. Note that in the last formula we use separate symmetrization of \(\alpha_1\) and \(\beta_1\) indices.

The harmonic distributions \([15]\) satisfy the following relations,

\[
\partial_1^{++} \frac{1}{(u_1^+ u_2^+)^n} = \frac{1}{(n-1)} (\partial_1^{-})^{n-1} \delta^{(n,-n)} (u_1, u_2) \quad , \tag{7.15}
\]

\[
\partial_1^{-} \frac{1}{(u_1^- u_2^-)^n} = \frac{1}{(n-1)} (\partial_1^{++})^{n-1} \delta^{(-n,n)} (u_1, u_2) \quad , \tag{7.16}
\]

where indices 1 and 2 parametrize the independent sets of harmonics. One can prove directly a useful integral relation for the harmonic distributions,

\[
\int du_2 \frac{1}{(u_1^- u_2^-)^2} \frac{1}{(u_2^+ u_3^+)^2} = \delta^{(2,-2)} (u_1, u_3) \quad . \tag{7.17}
\]

Finally, there exists a useful representation of harmonics in terms of a real variable \(\varphi\) and a complex spectral variable \(\lambda\),

\[
\begin{pmatrix}
  u_1^- & u_1^+ \\
  u_2^- & u_2^+
\end{pmatrix} = \frac{1}{\eta(\lambda)} \begin{pmatrix} 1 & -\lambda \\ \frac{1}{\bar{\lambda}} & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \quad . \tag{7.18}
\]

where \(\eta(\lambda) = \sqrt{1 + \lambda \bar{\lambda}}\). The harmonic derivatives then take the following form:

\[
\partial^{++} = e^{2i\varphi} [\eta^2(\lambda) \partial_\lambda - (i/2) \lambda \partial_\varphi] \quad , \tag{7.19}
\]

\[
\partial^{--} = - e^{-2i\varphi} [\eta^2(\lambda) \partial_\lambda + (i/2) \bar{\lambda} \partial_\varphi] \quad , \tag{7.20}
\]

\[
\partial^0 = - i \partial_\varphi \quad . \tag{7.21}
\]
References

[1] C.N. Yang, Phys. Rev. 38 (1977) 1377.

[2] A.N. Leznov, Th. Math. Phys. 73 (1988) 1233;
    A.N. Leznov and M.A. Mukhtarov, J. Math. Phys. 28 (1987) 2574;
    A.N. Leznov and M.V. Saveliev, Acta Applic. Math. 16 (1989) 1.

[3] S. Donaldson, Proc. Lond. Math. Soc. 50 (1985) 1.

[4] V.P. Nair and J. Schiff, Phys. Lett. B246 (1990) 423; Nucl. Phys. B371 (1992) 329.

[5] A. Losev, G. Moore, N. Nekrasov and S. Shatashvili,
    Nucl. Phys. Proc. Suppl. 46 (1996) 130.

[6] S.V. Ketov, Phys. Lett. B383 (1996) 390.

[7] H. Ooguri and C. Vafa, Nucl. Phys. B361 (1991) 469.

[8] N. Marcus, Nucl. Phys. 387 (1992) 263.

[9] O. Lechtenfeld and W. Siegel, Phys. Lett. B405 (1997) 49.

[10] G. Chalmers and W. Siegel, Phys. Rev. D54 (1996) 3515.

[11] N. Berkovits and C. Hull, Manifestly covariant actions for D=4 self-dual Yang-Mills
    and D=10 super-Yang-Mills, [hep-th/9712007].

[12] A.A. Belavin and V.E. Zakharov, Phys. Lett. B73 (1977) 53.

[13] R.S. Ward, Phys. Lett. A61 (1977) 81.

[14] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev,
    Class. Quant. Grav. 1 (1984) 469.

[15] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev,
    Class. Quant. Grav. 2 (1985) 601.

[16] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Ann. Phys. 185 (1988) 1;
    Conformal invariance in harmonic superspace in: Quantum field theory and quantum
    statistics, ed. I. Batalin et al., v.2, 233-248, Adam Hilger, Bristol 1987.

[17] Ch. Devchand and O. Lechtenfeld, Nucl. Phys. B516 (1998) 255.

[18] S. Kalitzin and E. Sokatchev, Phys. Lett. B257 (1991) 151.

[19] N. Marcus, Y. Oz and S. Yankielowicz, Nucl. Phys. B379 (1992) 121.

[20] S. Chakravarty and E.T. Newman, J. Math. Phys. 28 (1986) 334.

[21] B.M. Zupnik, Phys. Lett. B183 (1987) 175; ibid. B209 (1988) 513;
    Sov. J. Nucl. Phys. 48 (1988) 744.