The performance analysis of sonar target tracking based on pressure hydrophone arrays

Zhe Chen, Qijun Liu and Zhizhong Li
Underwater Acoustics Center, Navy Submarine Academy, Qingdao, 266100, China

Abstract. For the linear array sonar that consists of pressure hydrophones, it is difficult to solve the problem of port/starboard ambiguity. To estimate the target’s real azimuth accurately, the conventional beam forming methods of different arrays which include linear array, arc array, cross array and Y-shaped array were analysed. Based on the port/starboard discrimination ability and beam width, the sonar target tracking performance of different arrays was compared. It is shown that all arrays except the linear array could discriminate the real target, and the arc array’s effect is the best.

1 Introduction
The traditional sonar system is usually a linear array which consists of pressure hydrophones, and it could estimate the target bearing using array signal processing methods[1-2]. However, the biggest problem of linear array is that it is unable to discriminate the target’s port/starboard. It will seriously affect the sonar to track the real target, especially when the number of targets is large.

There are two ways to solve this problem, one is to improve the structure of pressure hydrophone array, such as hydrophone triplets[3-4] and twin-line array[5-6], they could discriminate the targets’ bearing under certain limiting conditions. The performance of hydrophone triplets is related to the correlation intensity of the space and line-spectrum[3-4], while the twin-line array has the blind area[6].

The second way is to use the vector hydrophone array. Compared with pressure hydrophone array, vector array could suppress the noise better and discriminate the target port/starboard. Recently, many researchers have studied on this topic, and the key points are different beam forming algorithms, such as the MVDR algorithm based on quaternion model[7], the adaptive beam forming based on vector hydrophone linear array[8], and the improved bartlett beam forming based on the acoustic vector sensor array[9]. Although the vector array has more advantages, its high cost make it uneconomical to be used widely in practice.

Furthermore, some researchers have used different arrays including cross array[10] and Y-shaped array[11] to estimate the target bearing, but they only considered the single target, and they didn’t compare which array is better to track targets.

This paper introduces the principles of four arrays, including linear array, arc array, cross array and Y-shaped array, and compares their performance to track multi-target based on the port/starboard discrimination ability and beam width.

2 Manuscript preparation

2.1 Linear array
Suppose that \( N \) is the number of elements in linear array, \( d \) is the length between two elements, \( \theta \) is the angle between signal incident direction and linear array horizontal direction, its range is \( \theta \in [0, 360) \). The model is shown in Fig. 1.

![Figure 1: Linear array model](image)

The delay between signal source and referenced element could be ignored, and point \( S_i \) is taken as the time reference, so that the delay between \( S_i \) and \( S_j \) is

\[
\tau_k = (k - 1) \frac{d}{c} \cos \theta, \quad k = 1, \ldots, N
\]  

(1)

If the signal model is \( s(t) \), the received signal of \( S_i \) is

\[
s_k(t) = s(t - \tau_k) + n_k(t)
\]  

(2)

where \( n_k(t) \) is the noise received by \( S_k \).

As pressure hydrophone has no directivity, if the source is from symmetrical bearing, each \( s_k(t) \) is invariable. Therefore, linear array can not discriminate the real target.

### 2.2 Arc array

Suppose that \( r \) is the radius of arc array, \( N \) is the number of elements, the length between two elements is \( d \), \( \alpha \) is the central angle between two elements, and \( \alpha = \frac{2\pi}{N} - 1 \), \( \theta \) is the angle between signal incident direction and arc array horizontal direction, its range is \( \theta \in [0, 360) \). The model is shown in Fig. 2.

![Figure 2: Arc array model](image)

Take point \( O \) as time reference, the delay of received signal between \( S_i \) and \( O \) is

\[
\tau_k = \frac{r \cos[(\theta - (k - 1)\alpha)]}{c}, \quad k = 1, \ldots, N
\]  

(3)

If the signal model is \( s(t) \), the received signal of \( S_i \) is

\[
s_k(t) = s(t - \tau_k) + n_k(t)
\]  

(4)

### 2.3 Cross array

Cross array is composed of two perpendicular linear arrays. Suppose that \( N \) is the number of elements in cross array, \( d \) is the length between two elements, \( \theta \) is the angle between signal incident direction
and linear array horizontal direction, its range is $\theta \in [0,360)$ . The cross array is divided into four parts, and each part is the uniform linear array with $N/4$ elements. The model is shown in Fig. 3.

![Cross array model](image)

Figure 3: Cross array model

Take point $S_i$ as the time reference, the delay of received signal between $S_k$ and $S_i$ is

$$\tau_k = \begin{cases} 
(k - 1) \frac{d}{c} \cos \theta, & k = 1, \ldots, \frac{N}{4} \\
(k - \frac{N}{4}) \frac{d}{c} \sin \theta, & k = \frac{N}{4} + 1, \ldots, \frac{N}{2} \\
(k - \frac{N}{2}) \frac{d}{c} \cos \theta, & k = \frac{N}{2} + 1, \ldots, \frac{3N}{4} \\
(k - \frac{3N}{4}) \frac{d}{c} \sin \theta, & k = \frac{3N}{4} + 1, \ldots, N 
\end{cases} \quad (5)$$

If the signal model is $s(t)$, the received signal of $S_k$ is

$$s_k(t) = \begin{cases} 
s(t + \tau_k) + n_k(t), & k = 1, \ldots, \frac{N}{2} \\
s(t - \tau_k) + n_k(t), & k = \frac{N}{2} + 1, \ldots, N 
\end{cases} \quad (6)$$

2.4 Y-shaped array

Y-shaped array is composed of three linear arrays which are arrayed uniformly with the angle of 120 degree. Suppose that $N$ is the number of elements in Y-shaped array, $d$ is the length between two elements, $\theta$ is the angle between signal incident direction and linear array horizontal direction, its range is $\theta \in [0,360)$ .

The Y-shaped array is divided into three parts, and each part is the uniform linear array with $N/3$ elements. The model is shown in Fig. 4.

![Y-shaped array model](image)

Figure 4: Y-shaped array model

Take point $S_i$ as the time reference, the delay of received signal between $S_k$ and $S_i$ is
\[
\tau_k = \begin{cases} 
(k - 1) \frac{d}{c} \sin(\theta + \frac{\pi}{3}), & k = 1, \ldots, \frac{N}{3} \\
(k - \frac{N}{3}) \frac{d}{c} \sin(\frac{\pi}{3} - \theta), & k = \frac{N}{3} + 1, \ldots, \frac{2N}{3} \\
(k - \frac{2N}{3}) \frac{d}{c} \sin(\theta), & k = \frac{2N}{3} + 1, \ldots, N 
\end{cases} \quad (7)
\]

If the signal model is \( s(t) \), the received signal of \( S_k \) is

\[
s_k(t) = \begin{cases} 
s(t + \tau_k) + n_k(t), & k = 1, \ldots, \frac{N}{3} \\
s(t - \tau_k) + n_k(t), & k = \frac{N}{3} + 1, \ldots, N 
\end{cases} \quad (8)
\]

According to the models of cross array and Y-shaped array, it could be seen that both of them are composed of linear array with different arrangement, so they also have the feature of linear array in a certain extent.

The output signals of different arrays can be calculated by

\[
s(t) = \sum_{k=1}^{N} s_k(t) \quad (9)
\]

3 Algorithm simulation and performance comparison

In order to contrast the performance of four arrays, suppose the number of elements is 32, and the length between two elements is 2.5 meters. If the sound source sends a broadband signal, each element will receive the signal and gaussian noise at the same time, and both of them are independent. The conventional beam forming patterns of different arrays are shown in Fig. 5.

(a) The target bearing is 60 degree
As shown in Fig.1(a), when the target bearing is 60 degree, linear array can’t discriminate the real target, while arc array can do it well. Cross array and Y-shaped array can discriminate the real target according to the amplitude, but they will also bring false targets in other directions. As shown in Fig.1(b), when the target bearing is 170 degree, linear array can’t discriminate the real target. Moreover, due to the bad azimuth resolution of parallel direction, the linear array has blind area where targets can’t be detected. By contrast, the other arrays’ performance is better.

It can be seen that for the single target, only linear array could not discriminate real target bearing, but the cross array and Y-shaped array will bring false targets. At this point, the arc array’s performance is the best.

In order to compare the beam width and spatial gain of different arrays, the conventional beam outputs are shown in Fig. 6.
The array gains of linear array and cross array are relatively high, and the arc array’s is the lowest, but it has no false targets. The calculation results of half power beam width are shown in Table 1.

| Bearing | Linear array | Arc array | Cross array | Y-shaped array |
|---------|--------------|-----------|-------------|----------------|
| 60°     | 5.8037       | 7.2753    | 13.353      | 10.7013        |
| 170°    | 40.006       | 13.959    | 14.975      | 11.1125        |

Table 1. The half power beam width of targets

The result shows that when the target bearing is 60 degree, the beam width of linear array is the smallest, so its azimuth resolution is much better than other arrays. But when the target bearing is 170 degree, due to the mix of real and false targets, the beam width of linear array is the biggest. Comparing with the other arrays’ bearing resolution, Y-shaped array is the best, and cross array is the worst.

In order to research the multi-target tracking performance of different arrays, suppose that there are five targets whose azimuths are 30, 120, 190, 259 and 269 degree. The beam patterns are shown in Fig. 7.

As shown in Fig. 7, linear array is still unable to discriminate the real targets, while arc array’s effect is the best. Cross array and Y-shaped array bring more false targets, and they can’t discriminate the targets whose azimuth are 268 and 278 degree. The beam outputs of different arrays are shown in Fig. 8.
It can be seen that the spatial gain of linear array is the highest, while the arc array’s is the worst. However, arc array is the only one that bring no false targets.

The calculation results of half power beam width are shown in Table 2.

| Bearing | Linear array | Arc array | Cross array | Y-shaped array |
|---------|--------------|-----------|-------------|---------------|
| 30º     | 9.2996       | 11.2527   | 13.7036     | 11.8817       |
| 120º    | 5.4981       | 7.4978    | 13.5459     | 12.8185       |
| 190     | 39.8273      | 14.7738   | 15.7223     | 11.2187       |
| 259     | 5.0298       | 10.5818   | 25.8243     | 19.3355       |
| 269     | 4.53         | 7.2054    | 25.8243     | 19.3355       |

Table 2. The half power beam width of targets

The result shows that when the target is not on the parallel direction, the beam width of linear array is the smallest. But when targets are near the parallel direction of linear array, they will enter the blind area because of the big beam width and the mix of real and false targets. For the other arrays’ bearing resolution, the arc array is the best.

In order to compare the tracking performance of different arrays in a long time, suppose the time length is 1800 seconds, and the target initial azimuths are 10, 71, 98, 174, 189, 235, 241, 317 degree.

The bearing-time plot of four arrays are shown in Fig. 9.

![Bearing-time records of multi-target](image)

Figure 9. Bearing-time records of multi-target

It can be seen that all but the arc array have the false targets. Linear array’s beam width is the smallest on the vertical direction, but it has blind area and can’t discriminate the real target. For cross
array and Y-shaped array, the real targets can be discriminated by the amplitude, but they will also bring false targets. Therefore, the arc array’s performance is the best.

4 Conclusion
This paper compares the multi-target tracking performance of four types of pressure hydrophone arrays based on the beam width and port/starboard discrimination ability. According to the simulation results, the conclusions could be summarized as follows:

1. Linear array has the highest spatial gain and the best bearing resolution on the vertical direction, but it has blind area, and it can’t discriminate the real target.

2. Cross array and Y-shaped array could solve the problem of port/starboard ambiguity, but they will bring false targets, which will affect sonar to track the real target when the number of targets is large.

3. Arc array could discriminate the real target, and moreover, it has no blind area or false target, so its effect is the best.

References
[1] Chen Guiying, Liu Huimin, Xing Liping, et al. “Scalar and vector hydrophone array’s directional performance comparison research”, Instrument Technique & Sensor, 4, pp. 91-97, (2014)
[2] Chen Hongyu, Wu Weiwei, Wang Mingyu. “Research of sound field localization based on near-field focusing beam forming”, Computer Simulation, 31, pp. 30-34, (2014)
[3] Wang Pingbo, Zhang Mingmin, Shen Degang. “Simulation research on the port/starboard discrimination performance of hydrophone triplets towed line sonar by the optimal array processing algorithm in the case of broad band”, Signal Processing, 22, pp. 554-558, (2006)
[4] Zhu Daizhu, Du Xuanmin. “The port/starboard discrimination based on line-spectrum character in towed line array with hydrophone triplets”, Acta Acustica, 34, pp. 296-302, (2009)
[5] Li Qihu, Chen Xinhua, Li Min. “Effect of shape distortion in solving left-right ambiguity of towed twin-line array”, UDT Eur. 2006, (amburg, Germany, 2006)
[6] Li Qihu. “Preliminary analysis of left-right ambiguity resolution performance for twin-line array”, Acta Acustica, 31, pp. 3-6, (2006)
[7] He Guangjin, Cheng Jinfang, Liu Yi, et al. “DOA estimation algorithm and its improvement in vector hydrophone array”, Computer Simulation, 29, pp. 162-165, (2012)
[8] Li Min, yang Xiuting, Li Qihu. “The performance of port/starboard beam-forming using vector hydrophone linear arrays”, Technical Acoustics, 26, pp. 1135-1139, (2007)
[9] Yao Zhixiang, Yu Guishui, He Fuqiang. “Improved bartlett beam forming for vector sensor array”, Acta Armamentarii, 32, pp. 176-180, (2011)
[10] Wang Peng, Zhang Guojun, Xiong Jijun. “Performance simulation and analysis of DOA estimation based on non-uniform vector array”, Computer Simulation, 28, pp. 388-392, (2011)
[11] Wang Wei, Cao Xiangyu, Wang Rui, et al. “A Y-like adaptive antenna array and its performance simulation”, Journal of Microwaves, 24, pp. 36-40, (2008)