Mass Generation Mechanism in Supersymmetric Composite Model with Three Generations

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Abstract

We propose a supersymmetric composite model with three generations in which supersymmetry and electroweak symmetry are broken dynamically, and masses of quarks and leptons are generated without introducing any mass scales by hand. All the mass scales in the model are expected to be generated dynamically. The mechanism to have mass hierarchy is explicitly described, although the roughly estimated mass spectrum of quarks and leptons does not exactly coincide with the realistic one.

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I. INTRODUCTION

Recently, many supersymmetric composite models have been proposed towards a better understanding of the generation structure, the mass hierarchy of quarks and leptons, and the supersymmetry and electroweak symmetry breaking [1–8]. In spite of these many efforts, we still do not have satisfying understanding. In most of models, some mass scales must be introduced by hand to have higher dimensional (non-renormalizable) effective interactions in the superpotential for quark mass generation.

In this paper we propose a supersymmetric composite model with three generations of quarks and leptons, in which there is no mass scale introduced by hand. All the mass scales are expected to be generated dynamically. Although the resultant mass hierarchy of quarks and leptons does not exactly coincide with the realistic one, the mechanism for the generation of the mass hierarchy itself might be true.

We begin with a brief review of the compositeness structure of one generation which comes from the model proposed by Nelson and Strassler [1]. Consider the following particle content:

\[
\begin{array}{ccc}
SU(2)_H & SU(5) \\
\hline
P^a & 2 & 5 \\
N & 2 & 1 \\
\Phi_{1a} & 1 & 5^* \\
\Phi_{2a} & 1 & 5^*,
\end{array}
\]

where \(SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y\) contains the standard model gauge group and \(SU(2)_H\) is the additional hypercolor gauge interaction which becomes strong at some high scale \(\Lambda_H\) and confinement is expected. All fields are chiral superfields and \(a\) is the index of the \(SU(5)\) representation. The representation is vector-like, if we consider only one simple gauge group, and the anomaly cancellation is trivial. Although we use \(SU(5)\) representations to describe the quantum number of the standard model gauge group throughout this paper, we do not assume the grand unification. Using the technique developed by Seiberg et al. [9],
we can identify massless low energy effective fields as

\[ \Sigma^{ab} \sim [P^a P^b] \equiv \epsilon^{\alpha\beta} P^a_{\alpha} P^b_{\beta}, \quad \Phi^a \sim [P^a N], \]

(1)

where \( \Sigma^{ab} \) and \( \Phi^a \) are identified as the matter multiplet in \( \mathbf{10} \) representation and the Higgs multiplet in \( \mathbf{5} \) representation in the standard \( SU(5) \) grand unified theory, respectively. If we introduce the tree level superpotential

\[ W_{\text{tree}} = \bar{\Phi}^2 a \eta^a b [P^b N] \]

(2)

which is the general form in the given symmetry, the masses of Higgs particles are generated as

\[ W_{\text{tree}} \longrightarrow \Lambda_H \bar{\Phi}^2 a \eta^a b \Phi^b, \]

(3)

where \( \Lambda_H \) is the scale of the hypercolor dynamics, and the explicit \( SU(5) \) breaking effect is incorporated in the coupling matrix \( \eta^a_b \).

It is expected that the non-perturbative hypercolor dynamics generates the superpotential \[ \text{H} \]. If we consider the third generation, we can write it down as

\[ W_{\text{dyn}} = \frac{1}{2^3} \alpha \epsilon_{abcde} \Sigma^{ab} \Sigma^{cd} \Phi^e = \alpha (q_3 H \bar{t} + q_3 q_3 D + \bar{t} D \bar{\tau}) \]

(4)

where \( \alpha \) is the coupling constant expected to be of the order of unity, \( q_3, \bar{t} \) and \( \bar{\tau} \) are chiral superfields which are contained in the field \( \Sigma^{ab} \), and \( H \) and \( D \) are the chiral superfields which are contained in the field \( \Phi^a \). The first term of the above equation is the Yukawa coupling for the mass of the top quark. The exact magnitude of the coupling \( \alpha \) could be determined, if we have enough information of the Kähler potential for the effective fields. If we consider the explicit \( SU(5) \) breaking effect, the coupling constants for each three terms are not necessarily equal.

In the paper of Ref. \[ \text{H} \] the dynamical supersymmetry breaking, which is triggered by the strong \( SU(2)_S \) supercolor dynamics, was introduced in this simple model described above. It was shown that all gauginos (gluino, photino, wino, zino) and sfermions (squarks and
sleptons) in third generation obtain their masses, which are consistent with the experiment, and the electroweak symmetry is broken due to the strong Yukawa coupling of the top quark through the radiative breaking mechanism \[10\]. But the origin of the masses of the bottom quark and tau lepton was not specified.

In this paper we proceed further by including other two generations and specifying the mechanism of the mass generation for all quarks and leptons. In the next section we describe the particle contents and tree level superpotential, and explain how the strong gauge dynamics works. In section III the mechanism of the mass generation for quarks and leptons is described. The Yukawa couplings for the masses of up-type quarks are generated by virtue of the non-perturbative gauge dynamics as explained above, and their hierarchy comes from the mixing between composite Higgs particles. The Yukawa couplings for the masses of the bottom quark and tau lepton are generated through the exchange of a heavy particle which has strong relation with the dynamics of supersymmetry breaking. The mass of the heavy particle is expected to be generated dynamically. The masses of the strange and down quarks are generated through the kinetic mixing between the up-type quarks which is generated by the exchange of heavy particles whose masses are also expected to be generated dynamically. The flavor mixing in the up-type quark sector is strongly related with the diagonal masses in the down-type quark sector in this model. Unfortunately, the resultant masses of the strange and down quarks are too small to be realistic. The masses of muon and electron are also generated in the same way, but they are also too small to be realistic. In section IV we summarize the model, and describe some problems of this model.

II. THREE GENERATION MODEL

We introduce three generations as three copies of the same structure which is explained in the previous section. Therefore, there are three $SU(2)$ hypercolor gauge symmetry for each generation, namely, $SU(2)_1$, $SU(2)_2$ and $SU(2)_H$ for first, second and third generation, respectively. In addition to that, the supercolor gauge interaction, $SU(2)_S$, is introduced,
by which supersymmetry is dynamically broken through a mechanism which was proposed by Izawa et al. \[11\]. The scales of dynamics of each gauge interactions are assumed as

\[
\Lambda_{(1)} \gg \Lambda_{(2)} \gg \Lambda_S > \Lambda_H. \tag{5}
\]

We take \(\Lambda_S \simeq 10^{10} \text{ GeV}\) and \(\Lambda_H \simeq 10^9 \text{ GeV}\) following the analysis of Ref. \[4\].

The particle contents of the model is as follows.

| SU(2) | SU(2) | SU(2) | SU(2) | SU(5) | Z \(_2\) |
|-------|-------|-------|-------|-------|---------|
| \(P^{(1)}\) | 2 | 1 | 1 | 1 | 5 | + |
| \(N^{(1)}, N_{i}^{(1)}\) | 2 | 1 | 1 | 1 | 1 | − |
| \(P^{(2)}\) | 1 | 2 | 1 | 1 | 5 | + |
| \(N^{(2)}, N_{i}^{(2)}\) | 1 | 2 | 1 | 1 | 1 | − |
| \(P\) | 1 | 1 | 2 | 1 | 5 | + |
| \(N_{i}, N_{i}\) | 1 | 1 | 2 | 1 | 1 | − |
| \(\Phi_{A} / \Phi_{3+A}\) | 1 | 1 | 1 | 1 | 5* | − / + |
| \(Q\) | 1 | 1 | 2 | 2 | 1 | + |
| \(\tilde{Q}_{1} / \tilde{Q}_{2}\) | 1 | 1 | 1 | 2 | 1 | − / + |
| \(Z_{1} / Z_{2}\) | 1 | 1 | 2 | 1 | 1 | − / + |
| \(Z, X / Z', X^{(A)}\) | 1 | 1 | 1 | 1 | 1 | − / + |

Here, \(i = 1, 2\) and \(A = 1, 2, 3\). The discrete symmetry distinguishes six \(\mathbf{5}^*\) multiplets in \(SU(5)\) into three Higgs multiplets and three matter multiplets which include right-handed down-type quarks and left-handed lepton weak doublets. We introduce the following tree level superpotential which is consistent with the symmetry.

\[
W_{\text{tree}} = \left( [P^a N] \, [P^{(2)a} N^{(2)}] \, [P^{(1)a} N^{(1)}] \right) \eta \begin{pmatrix} \phi_{3a} \\ \phi_{2a} \\ \phi_{1a} \end{pmatrix} + [P^a N_i] \kappa^{(3)}_{iA} \phi_{Aa} + [P^{(2)a} N_{i}^{(2)}] \kappa^{(2)}_{iA} \phi_{Aa} + [P^{(1)a} N_{i}^{(1)}] \kappa^{(1)}_{iA} \phi_{Aa} + \lambda^{(3)}_{Z'} [N_1 N_2] + \lambda^{(2)}_{Z'} [N_{1'} N_{2'}^2] + \lambda^{(1)}_{Z'} [N_{1'}^2 N_{2'}^2] \]

5
where square brackets mean the contraction of the indexes of hypercolor \( SU(2) \) gauge groups (see Eq. (1)), and square brackets with subscript \( S \) mean the contraction of the indexes of the supercolor \( SU(2)_S \) gauge group. For simplicity, we do not include the explicit \( SU(5) \) breaking effect which can be easily incorporated whenever we want. Unfortunately, this superpotential is not the general form in the given symmetry. Although the unwanted interactions, like \( ZZZ \) and \( XXX \), are forbidden, several interactions, like \( Z'Z'Z' \) and \( Z[NZ_2] \), are dropped by hand. This may suggest the additional symmetry or the modification of the dynamics of the supersymmetry breaking. Since \( U(1)_R \) symmetry is explicitly broken by gauge anomaly, there is no R-axion problem. In the following, we regard all the coupling constants as the real ones, for simplicity.

Consider the confinement of the \( SU(2)_S \) gauge interaction and the supersymmetry breaking. Since there are four doublets of \( SU(2)_S \), the low energy effective fields are expected as

\[
\left( \begin{array}{c} V + V' \epsilon_{\alpha\beta} \\ V_{j\alpha} \\ -V_{i\beta} \\ (V - V')\epsilon_{ij} \end{array} \right) \sim \left( \begin{array}{cc} [Q_\alpha Q_\beta]_S & [Q_\alpha \tilde{Q}_j]_S \\ [\tilde{Q}_i Q_\beta]_S & [\tilde{Q}_i \tilde{Q}_j]_S \end{array} \right),
\]

with the constraint

\[
V^2 - V'^2 - [V_1 V_2] = \Lambda_S^4,
\]

where the effective fields have mass dimension two [9]. Under the condition that the Yukawa couplings \( \lambda, \lambda' \) and \( \lambda_i \) can be treated perturbatively, it is natural that the effective field \( V \sim \frac{1}{4} \{ \epsilon^{\alpha\beta}[Q_\alpha Q_\beta] + \epsilon^{ij}[\tilde{Q}_i \tilde{Q}_j] \} \) has vacuum expectation value due to the constraint. Namely,

\[
V = \pm \Lambda_S^2 \sqrt{1 + \frac{V'^2}{\Lambda_S^4} + \frac{[V_1 V_2]}{\Lambda_S^4 V'^2}} \rightarrow \Lambda_S^2 + \frac{1}{2} V'^2 + \frac{1}{2} [V_1 V_2].
\]
Here, we rescaled the fields $V'$ are $V_i$ by $\Lambda_S$ to have dimension one, and expanded by assuming that $V'/\Lambda_S$ and $V_i/\Lambda_S$ are small. (We take positive sign by using the anomalous $U(1)_R$ symmetry.) Then the superpotential of Eq.(6) effectively becomes

$$W_{eff} \simeq \left( [P^a N] [P^{(2)_a} N^{(2)}] [P^{(1)_a} N^{(1)}] \right) \eta \begin{pmatrix} \Phi_3a \\ \Phi_2a \\ \Phi_1a \end{pmatrix}$$

$$+ [P^a N] \kappa^{(3)}_{iA} \Phi_{Aa} + [P^{(2)_a} N^{(2)}_i] \kappa^{(2)}_{iA} \Phi_{Aa} + [P^{(1)_a} N^{(1)}_i] \kappa^{(1)}_{iA} \Phi_{Aa}$$

$$+ \lambda Z' \left[ N_1 N_2 \right] + \lambda Z' \left[ N_1^{(2)} N_2^{(2)} \right] + \lambda Z' \left[ N_1^{(1)} N_2^{(1)} \right]$$

$$+ \lambda X \left( [N_1 N_2] + \lambda X \left( [N_1^{(2)} N_2^{(2)}] + \lambda X \left( [N_1^{(1)} N_2^{(1)}] \right) \right. \right. \right)$$

$$+ \kappa_{1A} [P^a Z_1] \Phi_{Aa} + \kappa_{2A} [P^a Z_2] \Phi_{3+Aa}$$

$$+ \lambda Z \left[ Z_1 Z_2 \right] + \lambda X \left[ Z_1 Z_2 \right]$$

$$+ \lambda \left\{ \Lambda_S^2 + \frac{1}{2} V'^2 + \frac{1}{2} \left[ V_1 V_2 \right] - \Lambda_S V' \right\}$$

$$+ \lambda' \left\{ \Lambda_S^2 + \frac{1}{2} V'^2 + \frac{1}{2} \left[ V_1 V_2 \right] + \Lambda_S V' \right\}$$

$$+ \lambda i \Lambda_S [Z_i V_i]. \quad (10)$$

Here, we dare to leave the fields which couple with $SU(2)_1$ and $SU(2)_2$, although they are confined at the scale $\Lambda_S$. It can be shown that the $F$-component of $Z$ and $Z'$ have vacuum expectation values as

$$\langle F_Z \rangle = -\lambda \Lambda_S^2, \quad \langle F_{Z'} \rangle = -\lambda \Lambda_S^2,$$  \quad (11) 

and supersymmetry is spontaneously broken, where we set $\lambda' = \lambda$, for simplicity. We assume that vacuum expectation values of the scalar components of $Z_i$ and $V_i$, which parameterize the pseudo-flat direction, are fixed to zero by the effect of the Kähler potential. (Note that since both $Z_i$ and $V_i$ have the charge of $SU(2)_H$ gauge interaction, they should follow some non-trivial scalar potential which comes from the Kähler potential.) The vacuum expectation values of the scalar components of $Z$, $Z'$, $X$ and $X^{(A)}$ are also the parameters of the pseudo-flat direction. We simply expect that these fields have vacuum expectation values
of the order of $10^{14}$ GeV with $\lambda, \chi' \simeq 10^{-2}$ through the effect of the Kähler potential (see Refs. [1] and [12]). Once these assumptions are satisfied, the mechanism of the mediation of the supersymmetry breaking which is proposed by Ref. [4] works. All gauginos in the standard model have their masses which are compatible with experiments, and composite squarks and sleptons in the third generation have huge masses. The elementary squarks and sleptons in third generation and the squarks and sleptons in first and second generations have their masses through the radiative correction in the same way in the gauge-mediated supersymmetry breaking model [13].

If the scalar component of $Z'$ and/or $X^{(4)}$ have vacuum expectation values, fields $N_i, N_i^{(2)}$ and $N_i^{(1)}$ obtain their masses, and these fields can not be the component of massless composite fields at low energy. We expect that even if the masses of $N_i^{(1)}$ and $N_i^{(2)}$ are smaller than the scales $\Lambda_{(1)}$ and $\Lambda_{(2)}$, respectively, they decouple from the low energy physics. We integrate out these fields by using the conditions $\partial W_{\text{eff}}/\partial N_i = 0$ and the same conditions for $N_i^{(1)}$ and $N_i^{(2)}$. The same procedure can be applied for the field $Z_i$. The field $Z_i$ has mass, if $Z$ and/or $X$ have vacuum expectation values. The effective superpotential at the scale between $\Lambda_H$ and $\Lambda_S$ is obtained as

$$W_{\text{eff}} \simeq \left( \begin{array}{ccc} P^a N & \Lambda_{(2)} \Phi \Phi' & \Lambda_{(1)} \Phi \Phi^q \end{array} \right) \begin{pmatrix} \eta_{33} & 0 & 0 \\ \eta_{23} & \eta_{22} & 0 \\ \eta_{13} & \eta_{12} & \eta_{11} \end{pmatrix} \begin{pmatrix} \Phi_{3a} \\ \Phi_{2a} \\ \Phi_{1a} \end{pmatrix}$$

$$- \frac{\kappa_{1A}^{(1)} k_{2B}^{(1)}}{\lambda_{Z}^{(1)} \langle Z' \rangle + \lambda_{X}^{(1)} \langle X^{(1)} \rangle} \Phi_{1A} \Sigma_1 \Phi_{Bb} - \frac{\kappa_{1A}^{(2)} k_{2B}^{(2)}}{\lambda_{Z}^{(2)} \langle Z' \rangle + \lambda_{X}^{(2)} \langle X^{(2)} \rangle} \Phi_{1A} \Sigma_2 \Phi_{Bb}$$

$$- \frac{\kappa_{1A}^{(3)} k_{2B}^{(3)}}{\lambda_{Z}^{(3)} \langle Z' \rangle + \lambda_{X}^{(3)} \langle X^{(3)} \rangle} \Phi_{1A} [P^a P^b] \Phi_{Bb} - \frac{\kappa_{1A}^{(1)} k_{2B}^{(1)}}{\lambda_{Z} \langle Z \rangle + \lambda_{X} \langle X \rangle} \Phi_{1A} [P^a P^b] \Phi_{Bb}$$

$$+ \frac{\alpha^{(1)}}{2} \epsilon_{abcd} \Sigma_1 \Sigma_1 \Phi_1^{\epsilon} + \frac{\alpha^{(2)}}{2} \epsilon_{abcd} \Sigma_2 \Sigma_2 \Phi_2^{\epsilon},$$

where we neglect the supersymmetry breaking terms, for simplicity (see Ref. [4] for the supersymmetry breaking). Here, we introduced the low energy effective fields $\Phi_1 \sim [P^{(1)} N^{(1)}], \Sigma_1 \sim [P^{(1)} P^{(1)}], \Phi_2 \sim [P^{(2)} N^{(2)}]$ and $\Sigma_2 \sim [P^{(2)} P^{(2)}]$ which interact with each other through the dynamically-generated Yukawa interaction of the last line of the above superpotential.
The form of the coupling matrix $\eta$ in the above superpotential is the general one under the global $SU(3)$ rotation on $\Phi_{Aa}$ fields.

The confinement of $SU(2)_H$ occurs in succession, and the effective superpotential below the scale $\Lambda_H$ is obtained as follows.

$$W_{\text{eff}} \simeq \sum_{a=1,2,3} \Phi^a_{\bar{A}} M^D_{\bar{A}B} \bar{\Phi}_{Ba} + \sum_{a=4,5} \Phi^a_{\bar{A}} M^H_{\bar{A}B} \bar{\Phi}_{Ba}$$

$$- \frac{k^{(1)}_1 k^{(1)}_2 \Lambda_{(1)}}{\lambda^Z_{(1)} (Z') + \lambda^X_{(1)} (X')} \Phi_{Aa} \Sigma_{a1} \Phi_{Bb} - \frac{k^{(2)}_1 k^{(2)}_2 \Lambda_{(2)}}{\lambda^Z_{(2)} (Z') + \lambda^X_{(2)} (X')} \Phi_{Aa} \Sigma_{ab} \Phi_{Bb}$$

$$- \frac{k^{(3)}_1 k^{(3)}_2 \Lambda_{H}}{\lambda^Z_{(3)} (Z') + \lambda^X_{(3)} (X')} \Phi_{Aa} \Sigma_{a3} \Phi_{Bb} - \frac{k^{(3)}_1 k^{(2)}_2 \Lambda_{H}}{\lambda^Z_{(3)} (Z) + \lambda^X_{(3)} (X)} \Phi_{Aa} \Sigma_{ab} \Phi_{3+Bb}$$

$$+ \frac{\alpha^{(1)}}{2^3} \epsilon_{abcde} \Sigma_{a1} \Sigma_{cd} \phi^c_1 + \frac{\alpha^{(2)}}{2^3} \epsilon_{abcde} \Sigma_{ab} \Sigma_{cd} \phi^c_2 + \frac{\alpha^{(3)}}{2^3} \epsilon_{abcde} \Sigma_{ab} \Sigma_{3} \phi^c_3,$$

where we introduced the low energy effective fields $\Phi_3 \sim [PN]$ and $\Sigma_3 \sim [PP]$ which interact with each other through the dynamically-generated Yukawa interaction of the last term of the last line. The explicit $SU(5)$ breaking is considered in the first line of the above superpotential (the coupling matrix $\eta$ is decomposed to $\eta^D$ for $a = 1, 2, 3$ and $\eta^H$ for $a = 4, 5$), and the first and second terms in the first line are the mass terms for colored Higgs and Higgs fields, respectively. (Note that the indexes $\bar{A}$ and $\bar{B}$ run reverse way, namely, from 3 to 1.) The form of these mass matrixes are

$$M^D \simeq \begin{pmatrix} \eta^D_{33} \Lambda_{H} & 0 & 0 \\ \eta^D_{23} \Lambda_{(2)} & \eta^D_{22} \Lambda_{(2)} & 0 \\ \eta^D_{13} \Lambda_{(1)} & \eta^D_{12} \Lambda_{(1)} & \eta^D_{11} \Lambda_{(1)} \end{pmatrix}, \quad M^H \simeq \begin{pmatrix} \eta^H_{33} \Lambda_{H} & 0 & 0 \\ \eta^H_{23} \Lambda_{(2)} & \eta^H_{22} \Lambda_{(2)} & 0 \\ \eta^H_{13} \Lambda_{(1)} & \eta^H_{12} \Lambda_{(1)} & \eta^H_{11} \Lambda_{(1)} \end{pmatrix}.$$  

We assume that the matrix elements of $\eta^H$ are the same order of magnitude, except for 33-element. We also assume that the matrix elements of $\eta^D$ are the same order of magnitude, except for 33-element. The values of $\eta^H_{33} \Lambda_{H} \equiv \mu \simeq 100 \text{ GeV}$, namely $\eta^H_{33} \simeq 10^{-7}$, and $\eta^H_{33} \Lambda_{H} \equiv \mu_D \simeq 1000 \text{ GeV}$, namely $\eta^D_{33} \simeq 10^{-6}$ are necessary to have the electroweak symmetry breaking through the radiative breaking mechanism [3]. The eigenvalues of these mass matrixes are of the order of $\mu$, $\rho_{(2)} \sim \eta^H_{\bar{A}B} \Lambda_{(2)}$, $\rho_{(1)} \sim \eta^H_{\bar{A}B} \Lambda_{(1)}$ for $M^H$ and $\mu_D$, $\rho^D_{(2)} \sim \eta^D_{\bar{A}B} \Lambda_{(2)}$, $\rho^D_{(1)} \sim \eta^D_{\bar{A}B} \Lambda_{(1)}$ for $M^D (\bar{A}, \bar{B} \neq 3)$. The mass of the colored Higgs which couples with the
first generation particles through the dynamically-generated Yukawa interactions must be larger than $10^{17}$ GeV not to have rapid proton decay. Therefore, we assume $\rho_{(1)}^D \simeq 10^{17}$ GeV, namely $\eta_{AB}^D \simeq 1$ ($\tilde{A}$, $\tilde{B} \neq 3$) and $\Lambda_{(1)} \simeq 10^{17}$. This huge hierarchy in the coupling matrix $\eta$ is one of the problems of this model.

**III. MASS GENERATION MECHANISM**

First, we describe the generation of the mass hierarchy of up-type quarks. The interactions of the last line of Eq.(13) contain the Yukawa couplings for the masses of up, charm and top quarks, but there are three pairs of Higgs doublets in each generation. The Higgs fields in each generation mix with each other through the mass matrix $M^H$. The mass matrix is approximately diagonalized as $UM^H V^\dagger \simeq \text{diag}(\mu, \rho_{(2)}, \rho_{(1)})$, where the order of magnitude of the matrix elements of $U$ is

\[
U \sim \begin{pmatrix}
1 & \frac{\mu}{\rho_{(2)}} & \frac{\mu}{\rho_{(1)}} \\
\frac{\mu}{\rho_{(2)}} & 1 & \frac{\rho_{(2)}}{\rho_{(1)}} \\
\frac{\mu}{\rho_{(1)}} & \frac{\rho_{(2)}}{\rho_{(1)}} & 1
\end{pmatrix},
\]

(15)

and all the elements of the matrix $V$ are of the order of unity. According to Eq.(3), the hierarchy $\mu/\rho_{(2)}, \mu/\rho_{(1)}, \rho_{(2)}/\rho_{(1)} \ll 1$ is assumed. We can identify the lightest Higgs pair to the Higgs pair in the minimal supersymmetric standard model. The other Higgs pairs are heavy and decouple from the low energy physics. Therefore, Yukawa couplings for up-type quarks can be described as

\[
W_{Y}^{up} \simeq \frac{\alpha^{(1)}}{23} \frac{\mu}{\rho_{(1)}} \varepsilon_{abcde} \Sigma_1^{ab} \Sigma_1^{cd} \Phi^e + \frac{\alpha^{(2)}}{23} \frac{\mu}{\rho_{(2)}} \varepsilon_{abcde} \Sigma_2^{ab} \Sigma_2^{cd} \Phi^e + \frac{\alpha^{(3)}}{23} \varepsilon_{abcde} \Sigma_3^{ab} \Sigma_3^{cd} \Phi^e,
\]

(16)

where $\Phi$ denotes the lightest Higgs multiplet, and the index $e$ takes the values only 4 and 5. Then we have relations

\[
\frac{m_c}{m_t} \simeq \frac{\mu}{\rho_{(2)}}, \quad \frac{m_u}{m_t} \simeq \frac{\mu}{\rho_{(1)}}.
\]

(17)

Since we need to take $\mu \simeq 100$ GeV for the electroweak symmetry breaking, we have $\rho_{(2)} \simeq 10^4$ GeV and $\rho_{(1)} \simeq 10^7$ GeV, namely $\eta_{AB}^H \simeq 10^{-10}$ ($\tilde{A}$, $\tilde{B} \neq 3$) and $\Lambda_{(2)} \simeq 10^{14}$ GeV.
Next, we discuss the mass generation of down-type quarks. The interaction of the last term in the third line of Eq. (13) gives the following Yukawa couplings.

\[
\mathcal{L}_{\text{down}} = - \frac{\Lambda_H}{\lambda_Z \langle Z \rangle + \lambda_X \langle X \rangle} \kappa \left\{ \kappa_{21} \bar{H} b_R q_{3L} + \kappa_{22} \bar{H} s_R q_{3L} + \kappa_{23} \bar{H} d_R q_{3L} \right\} + \text{h.c.},
\]

where \( \bar{H} \) is the lightest \( SU(2)_L \)-doublet Higgs scalar field, \( b_R, s_R \) and \( d_R \) are right-handed quark fermion fields, and \( q_{3L} \) is the left-handed \( SU(2)_L \) doublet quark fields in third generation. We defined \( \kappa = \kappa_{13} V_{33} + \kappa_{12} V_{32} + \kappa_{11} V_{31} \), where \( V_{33}, V_{32} \) and \( V_{31} \) are matrix elements of the matrix \( V \). It is clear that the Yukawa coupling for the bottom quark mass is included in Eq. (18). Since it is natural to take \( \lambda_Z, \lambda_X \simeq \lambda \simeq 10^{-2} \) and \( \lambda_Z \langle Z \rangle + \lambda_X \langle X \rangle \simeq 10^{12} \), we have the Yukawa coupling for the bottom quark mass as

\[
g_b = \kappa \kappa_{21} 10^{-3}.
\]

It is expected that the Yukawa coupling for the top quark mass is of the order of unity and \( \tan \beta \simeq 4 \) (see Ref. [4]). Therefore, \( g_b = (m_t/m_t) \tan \beta \simeq 0.1 \) and \( \kappa \kappa_{21} \) must be of the order of \( 10^2 \). The mixing masses between \( b_L \) and \( s_R \) and between \( b_L \) and \( d_R \), which come from the second and third terms of Eq. (18), respectively, are expected to be the same order of the bottom quark mass. We can have the mass of the tau lepton which is the same order of the bottom quark mass.

The Yukawa couplings for the masses of the strange and down quarks are not included in the effective superpotential of Eq. (13). However, if we have mixing in the up-type quark sector, the masses of these quarks can be generated by the quantum correction through the diagram of Fig. 4. Although there is no mass mixing in the up-type quark sector, the kinetic mixing is generated through the diagram of Fig. 2.

Consider the generation of the strange quark mass. The kinetic mixing between top and charm quarks are estimated as follows.

\[
\mathcal{L}^{tc} = \varepsilon_{tc} \bar{f} i \gamma^\mu D_\mu c + \text{h.c.}
\]

\[
\varepsilon_{tc} \simeq \left( \frac{\kappa_{13}^{(3)} \kappa_{22}^{(3)} \Lambda_H^{(3)}}{\lambda_Z^{(3)} \langle Z' \rangle + \lambda_X^{(3)} \langle X^{(3)} \rangle} \right) \left( \frac{\kappa_{13}^{(2)} \kappa_{22}^{(2)} \Lambda_2^{(2)}}{\lambda_Z^{(2)} \langle Z' \rangle + \lambda_X^{(2)} \langle X^{(2)} \rangle} \right) \frac{1}{16\pi^2} \ln \left( \frac{\Lambda_H}{\rho^{(2)}} \right)^2,
\]
where we take $\Lambda_H$ as the physical ultraviolet energy-momentum cut off in Euclidean space.

If we naturally take the values of Yukawa couplings as $\lambda_{Z'}^{(2),(3)}, \lambda_X^{(2),(3)} \simeq \lambda \simeq 10^{-2}$, namely $\lambda_{Z'}^{(2),(3)} \langle Z' \rangle + \lambda_X^{(2),(3)} \langle X^{(2),(3)} \rangle \simeq 10^{12}$, and if we take $\kappa_{13}^{(2)} \kappa_{22}^{(3)} \kappa_{13}^{(3)} \kappa_{22}^{(3)} \simeq 10$, we have $\varepsilon_{tc} \simeq 0.1$.

We assume that the perturbative calculation is good for order estimation, even if the coupling constant of the vertexes in Fig.2 is very large.

The diagram of Fig.2 gives the strange quark mass as

$$m_s \simeq g_{s_{RtL}} g_{c_{RSL}} \varepsilon_{tc} \frac{\sin \beta \cos \beta}{16\pi^2} m_t \ln \left( \frac{\Lambda_H}{m_t} \right)^2,$$

where

$$g_{s_{RtL}} = \frac{\kappa \kappa_{22} \Lambda_H}{\lambda_Z \langle Z \rangle + \lambda_X \langle X \rangle}, \quad g_{c_{RSL}} \simeq \frac{\mu}{\rho(2)}$$

come from Eqs.(18) and (16), respectively. The main contribution in the diagram of Fig.2 is the would-be Nambu-Goldstone boson component of the Higgs fields in $R_\xi$-Landau gauge.

If we naturally take $g_{s_{RtL}} \simeq g_b \simeq 0.1$, then we have $m_s \simeq 0.1$ MeV. This value is too small for the strange quark mass which is usually considered as $100 < m_s < 300$ MeV. The reason of this small value is the smallness of $g_{c_{RSL}} \simeq \mu/\rho(2) \simeq 10^{-3}$ and loop suppression factors of the diagrams.

The mass of the down quark can also be generated in the same way. But the value is negligibly small, because the kinetic mixing coefficient $\varepsilon_{ut}$ have to be very small to have long life time of the proton ($\varepsilon_{ut} < 10^{-9}$).

The masses of muon and electron can be generated through the similar diagram of Fig.2 in which Higgs fields are replaced by colored Higgs fields. But the resultant masses are negligibly small, since the mixing angles between light colored Higgs and heavy colored Higgs are very small.

IV. CONCLUSION

In this paper we proposed a supersymmetric composite model in which the both supersymmetry and electroweak symmetry are dynamically broken, and the mass generation
mechanism for all quarks and leptons is explicitly described. The particle contents are very
simple. Only the particles which belong to the fundamental representation of each simple
unitary gauge group are considered. Since the representation is vector-like in each gauge
group, including the gauge group of the standard model, the anomaly cancelation is trivial.

The mass hierarchy in the up-type quark sector can be clearly understood as the result
of the mixing between the composite Higgs of each generations. The generation of the
mass hierarchy for down-type quarks and charged leptons is more complicated. The bottom
quark and tau lepton are special particles, since they can directly couple with the dynamics
of the supersymmetry breaking. Therefore, they have relatively large masses in comparison
with the masses of the corresponding particles in other generations. The generation of the
Yukawa couplings for the masses of down-type quarks and charged leptons in the other two
generations are forbidden at the tree level by the discrete $Z_2$ symmetry which distinguishes
the six multiplets in $5^*$ representation of $SU(5)$ into three Higgs multiplets and three matter
multiplets. (This discrete symmetry also forbids the Yukawa coupling at tree level which
contributes to the dimension-five operator \[14\] for proton decay.) But since the discrete
symmetry is spontaneously broken by the vacuum expectation value of $Z$ or $X$, their masses
can be generated through the quantum correction. Unfortunately, the resultant masses are
too small to be realistic, because of the many suppression factors and the constraint from
the long life time of proton. But it must be stressed that no mass scale is introduced by
hand.

There are many other open questions.

We put several assumptions on the dynamics. The most serious one is the unspecified
dynamics to have vacuum expectation values of the scalar components of the gauge singlet
fields $Z$, $X$, $Z'$ and $X^{(A)}$. Since they are parameters of the pseudo-flat direction, it could be
possible that they have vacuum expectation values through the quantum correction to the
Kähler potential \[12\]. It could also be true that there is more appropriate dynamics for
the supersymmetry breaking in which such vacuum expectation values are naturally generated.

Other problem is that we have to consider the hierarchical Yukawa coupling in the tree
level superpotential, especially in the coupling matrix $\eta$. It looks like very artificial and brings conceptual difficulty, because we are pursuing the origin of the mass hierarchy.

Although the model which is proposed in this paper is not the perfect one, we believe that it is a primitive one which is worth developing further.

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FIGURES

FIG. 1. The diagram for the strange quark mass. The factor $\varepsilon_{tc}$ denotes the coefficient of the kinetic mixing between top and charm quarks, and $B$ denotes the supersymmetry breaking mass for the lightest Higgs field. The diagram for the down quark mass is obtained by replacing the charm quark inside the loop with the up quark.

FIG. 2. The supergraph for the kinetic mixing between the up-type quarks.
Fig. 1
Fig. 2

\[ \Sigma_{1, 2, 3}^+ \quad \bar{\Phi}^+ \quad \bar{\Phi} \quad \Sigma_{1, 2, 3} \]