Proposal for a Simple Model of Dynamical SUSY Breaking

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We discuss supersymmetric $SU(2)$ gauge theory with a single matter field in the $I = 3/2$ representation. This theory has a moduli space of exactly degenerate vacua. Classically it is the complex plane with an orbifold singularity at the origin. There seem to be two possible candidates for the quantum theory at the origin. In both the global chiral symmetry is unbroken. The first is interacting quarks and gluons at a non-trivial infrared fixed point – a non-Abelian Coulomb phase. The second, which we consider more likely, is a confining phase where the singularity is simply smoothed out. If this second, more likely, possibility is realized, supersymmetry will dynamically break when a tree level superpotential is added. This would be the simplest known gauge theory which dynamically breaks supersymmetry.
Only supersymmetric gauge theories with chiral matter content can dynamically break supersymmetry with a stable vacuum. The reason for this is that the matter fields in a non-chiral gauge theory can be given mass terms and decoupled, leaving as the low energy limit a supersymmetric pure gauge theory which is known to have non-zero Witten index \( \text{Tr} (-1)^F \neq 0 \) and, hence, unbroken supersymmetry [1]. As long as the theory has a stable vacuum, \( \text{Tr} (-1)^F \) is independent of the magnitude of the added mass terms and so the theory with non-chiral matter does not break supersymmetry [1]. Chiral models with dynamical supersymmetry breaking were given in [2]. Here we will discuss a simpler model, based on an \( SU(2) \) gauge theory with chiral matter content, which may well break supersymmetry upon adding a tree level superpotential.

Consider supersymmetric \( SU(2) \) gauge theory with a single matter field \( Q \) in the \( I = 3/2 \) representation. The quadratic index \( \mu \) of the \( I = 3/2 \) representation of \( SU(2) \) is \( \mu = 10 \) (for general \( I \) it is \( \mu = 2(2I + 1)(I(I + 1)/3) \)). Since this index is even, the theory is well-defined; it does not suffer from the global anomaly of [3]. In addition, this theory is asymptotically free because the matter content satisfies the relevant inequality: \( \mu < 12 \). Therefore, this is a sensible theory containing a single pseudo-real field. By Bose statistics in the superfield \( Q \), there is no non-zero gauge singlet which is quadratic in \( Q \) – it is impossible to give the pseudo-real field \( Q \) a mass. In other words, this theory is chiral.

The basic gauge singlet superfield is \( u = Q^4 \), with a totally symmetric contraction of the gauge indices. At the classical level, this theory has a moduli space of degenerate vacua labeled by the expectation value of \( u \). For \( u \neq 0 \) the \( SU(2) \) gauge group is completely broken by the Higgs mechanism. The classical Kahler potential for \( u \) is \( K_{cl} = QQ^\dagger \sim (uu^\dagger)^{1/4} \), which is singular at \( u = 0 \). This is a \( Z_4 \) orbifold singularity. It reflects the fact that the gauge bosons become massless at \( u = 0 \) and hence must be included in the low energy effective theory.

To analyze the theory at the quantum level, note that there is an anomaly free \( U(1)_R \) symmetry under which the scalar component of \( Q \) has charge \( q = 3/5 \). This symmetry, along with holomorphy, restricts the form of an effective superpotential to be \( W_{eff} = au^{5/6}\Lambda^{-1/3} \), where \( \Lambda \) is the dynamically generated scale of the theory and \( a \) is some constant. However, this superpotential does not have sensible behavior in the weak coupling \( u \gg \Lambda \) limit and, hence, \( a = 0 \). Therefore, the quantum theory also has a moduli space of degenerate vacua.

The most interesting question about the quantum moduli space is the nature of the classical singularity at \( u = 0 \) in the quantum theory. Similar spaces in other theories have
been analyzed and several possibilities found. First, a classical singularity can be smoothed out in the quantum theory \cite{4,5}. Alternatively, the singularities in the quantum theory are associated with new massless fields which are collective excitations of the elementary fields \cite{4,5}. Finally, the quantum theory at the singularity can be similar to the classical theory with massless interacting quarks and gluons \cite{4} – a non-Abelian Coulomb phase. (Of course, other as yet unknown possibilities might also exist.)

In the example at hand there seem to be two plausible alternatives for the quantum behavior at \( u = 0 \). First, there could be a non-Abelian Coulomb phase. The beta function is

\[
\beta(\alpha) = -\frac{\alpha^2}{2\pi} \left( 1 - \frac{71}{4\pi} \alpha + \mathcal{O}(\alpha^2) \right) \tag{1}
\]

where \( \alpha = g^2/4\pi \). Ignoring the higher order corrections, there is a non-trivial infrared stable fixed point at \( \alpha^* = 4\pi/71 \). The appropriate effective coupling might well be larger because of the large matter Casimir. This coupling may be small enough for perturbation theory to be reliable and the nontrivial fixed point and its non-Abelian Coulomb behavior to be physical. We have no compelling argument that this is indeed the case.

There is second more interesting dynamical possibility for the behavior at \( u = 0 \) which we find more likely. The topology of the space of vacua could remain the entire \( u \) plane and the classical singularity in the Kahler potential (metric) at the origin could be smoothed out. The massless spectrum on the whole moduli space would consist just of the \( u \) quanta. This possibility passes the following highly non-trivial consistency test. At the point \( u = 0 \) the global \( U(1)_R \) symmetry is unbroken. The massless spectrum – the \( u \) quanta – should saturate the 't Hooft anomaly conditions \cite{6}. The fermions in the microscopic theory are the 3 gluinos, each with \( R \) charge 1, and the 4 quark components of \( Q \), each with \( R \) charge \(-2/5\). In the conjectured macroscopic theory there is a single fermion, the fermionic component of \( u \), which has \( R \) charge \( 7/5 \). There are two equations to check:

\[
\begin{align*}
\text{Tr } R &= 3 + 4(-2/5) = 7/5 \\
\text{Tr } R^3 &= 3 + 4(-2/5)^3 = (7/5)^3,
\end{align*}
\tag{2}
\]

which are indeed satisfied. This seems too miraculous to be a coincidence. Therefore in the remainder of this paper we will assume that the theory behaves in this manner and proceed to explore the consequences.
By the R symmetry the Kahler potential is of the form $K = |\Lambda|^2 k(uu^\dagger/|\Lambda|^8)$ for some function $k$. The smooth behavior near the origin and the semiclassical behavior at infinity constrain $K$ to satisfy

$$K = |\Lambda|^2 k(uu^\dagger/|\Lambda|^8) \sim \begin{cases} uu^\dagger|\Lambda|^{-6} & \text{for } uu^\dagger \ll \Lambda^8 \\ (uu^\dagger)^{1/4} & \text{for } uu^\dagger \gg \Lambda^8. \end{cases}$$

Since the low energy superpotential vanishes and the Kahler potential is independent of the phase of $u$, the low energy theory has, in addition to the $U(1)_R$ symmetry, an accidental global $U(1)$ symmetry under which the chiral superfield $u$ has, say, charge 1. The corresponding symmetry in the microscopic theory is anomalous. In the macroscopic theory it is violated by higher dimensional terms which are not determined by the Kahler potential and the superpotential.

The fact that the massless spectrum at $u = 0$ consists only of $u$ means that there is confinement there even though the chiral $U(1)_R$ symmetry is unbroken. Confinement without chiral symmetry breaking is in contrast to the prediction of the most attractive channel (MAC) arguments. These arguments would have suggested that a bilinear condensate is formed, breaking the gauge symmetry $SU(2) \rightarrow U(1)$ with chiral symmetry breaking at a scale much larger that $\Lambda$. However, we see no sign of a massless photon associated with an unbroken $U(1)$. Other counterexamples to the MAC intuition have also been observed in other supersymmetric gauge theories [4].

We now consider perturbing the theory by a tree level superpotential

$$W_{\text{tree}} = \lambda u.$$  

This term is non-renormalizable; perhaps it can be realized as the low energy effective superpotential obtained from a larger renormalizable gauge theory at a scale $m \sim \lambda^{-1}$, or as a low energy limit of string theory with $\lambda \sim 1/M_p$. In any event, our effective theory (as well as the microscopic gauge theory) is then only sensible for $|u| < |\lambda|^{-4}$. For $|u| > |\lambda|^{-4}$ we would have to include the fields in a more fundamental renormalizable theory and the physics of this region would contain non-universal dependence on the choice of the higher energy theory. We can take $\lambda \ll \Lambda^{-1}$ so as to have room to take $u$ outside the region of strong coupling gauge dynamics, $|u| \gg |\Lambda|^4$, without running into the scale of the non-renormalizable term.

Using symmetries and holomorphy as in [4], the exact superpotential of the low energy theory is constrained to be

$$W_{\text{exact}} = \lambda uf(t = \lambda^6 \Lambda^2 u).$$

3
Perturbative behavior in $\lambda$ and the requirement of no cuts in $u$ leads to $f = \sum_{n=0}^{\infty} a_n t^n$; the $n$-th term has the quantum numbers of a $2n$ instanton contribution. In our allowed range of $u$, $|t| \ll 1$ and so we have $W \approx \lambda u$. Therefore, the scalar potential in our region is

$$V_{\text{eff}} = (K_{uu^t})^{-1} |W_u|^2 = (K_{uu^t})^{-1} |\lambda|^2.$$  \hfill (6)

Using the asymptotic behavior (3), it is clear that supersymmetry is broken with vacuum energy

$$E \sim |\Lambda|^6 |\lambda|^2.$$  \hfill (7)

Note that the superpotential $W = \lambda u$ preserves an $R$ symmetry which is a combination of the anomaly free $U(1)_R$ symmetry and the accidental $U(1)$ symmetry. Therefore, the supersymmetry breaking found here is in accord with the analysis of [8]. If the minimum energy vacuum is at $u = 0$, this $R$ symmetry is unbroken. Otherwise, there is a massless Goldstone boson. If we include the higher terms in (5), the theory will have a supersymmetric minimum, but outside the domain $|u| < |\lambda|^{-4}$ where this description is valid. Again, the existence of such a minimum is in accord with [8] because with the higher terms in (5) included there will no longer be an $R$ symmetry.

To summarize, classically the superpotential $\lambda u$ does not lead to spontaneous supersymmetry breaking because of the singularity of the Kahler potential at $u = 0$. Quantum mechanically the singularity is smoothed out and supersymmetry is dynamically broken. We would like to conclude by stressing that this mechanism for supersymmetry breaking is more general than the specific model studied here. It can no doubt exist in other models and perhaps even in Nature.

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