On the Power of False Negative Awareness in Indicator-based Caching Systems

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Abstract—Distributed caching systems such as content distribution networks often advertise their content via lightweight approximate indicators (e.g., Bloom filters) to efficiently inform clients where each datum is likely cached. While false-positive indications are necessary and well understood, most existing works assume no false-negative indications. Our work illustrates practical scenarios where false-negatives are unavoidable and ignoring them has a significant impact on system performance. Specifically, we focus on false-negatives induced by indicator staleness, which arises whenever the system advertises the indicator only periodical, rather than immediately reporting every change in the cache. Such scenarios naturally occur, e.g., in bandwidth-constraint environments or when latency impedes each client’s ability to obtain an updated indicator.

Our work introduces novel false-negative aware access policies that continuously estimate the false-negative ratio and sometimes access caches despite negative indications. We present optimal policies for homogeneous settings and provide approximation guarantees for our algorithms in heterogeneous environments. We further perform an extensive simulation study with multiple real system traces. We show that our false-negative aware algorithms incur a significantly lower access cost than existing approaches or match the cost of these approaches while requiring an order of magnitude fewer resources (e.g., caching capacity or bandwidth).

I. INTRODUCTION

Caches are extensively used in networking environments such as Content Delivery Networks [11–13], Named Data Networks [4], 5G networks [5], and Information Centric Networks [6]. In such networks, accessing caches often incurs some overhead in terms of latency, bandwidth, or energy [3, 7]. On the other hand, fetching a datum without caches usually incurs a larger miss penalty, e.g., for retrieving the requested item from a remote server [2].

In large distributed systems, caches often further optimize performance by advertising their content [11–13, 14, 15]. Such advertisements allow clients to minimize costs by selecting which cache to access for a requested datum. Ideally, the advertisement policy would always accurately reflect the up to date content at every cache in the network. However, such a solution requires a prohibitive amount of memory, computation, and bandwidth resources. Hence, systems often compromise some accuracy for efficiency by advertising periodical approximate indicators. Indicators are data structures that trade accuracy for space efficiency. Common embodiment of such indicators are Bloom filters [7, 9–11], and fingerprint hash tables [12].

Such approximations commonly introduce the risk of false-positive errors, i.e., the indicator sometimes wrongly indicates that a datum is stored in the cache. In such a case, accessing the cache results in an unnecessary cache access, which translates to an excessive cost. Consequently, the problem of advertising space-efficient indicators while keeping a low false-positive rate has attracted a bulk of research effort [7, 9–11]. Other works addressed the cache selection problem, namely, selecting which cache to access when there exist one (or more) several positive indications, where some of them may actually be false-positives [7, 14].

Most previous works [3, 7, 14] assume that there are no false-negative indications. Indeed, there exist indicators that theoretically guarantee a false-negative ratio of zero (e.g., simple Bloom filter [2]), or a negligible false-negative ratio (e.g., a Counting Bloom Filter (CBF) [10]). However, to manifest this guarantee in practical distributed environment, every cache should advertise its indicator to all the clients in the network upon every change in the cached content, usually resulting in prohibitive bandwidth consumption. For instance, a leading CDN provider reports using Bloom filters of about 70MB in size in every cache [2]. Insisting on sending an update upon every change in the cached content in such a system may result in having the advertised indicators consume more bandwidth than the cached content itself. Henceforth, caches commonly advertise their content only periodically.

When using periodical updates, the advertised content gradually becomes stale. Namely, it takes time for the indicator available at the clients to reflect changes in the cached content. Unfortunately, such staleness may lead to a significant increase in false-negative indications. To illustrate the problem, consider a cache that advertises a fresh indicator, and later admits a new item x. When the client tests for x, the indicator is likely to wrongly indicate that x is not in the cache (as it wasn’t in the cache when the advertisement was sent), resulting in a false-negative error. Such scenarios are quite common in highly dynamic networks, such as 5G-networks [5].

To explore the significance of false-negatives caused due to staleness, consider Fig. 1. The figure presents the false-negative ratio indications as a function of the time between subsequent advertisements, referred to as the update interval. We measure the update interval by the number of cache changes (insertions of new items). The indicator used is an optimally configured simple Bloom filter [9], where the figure
shows distinct indicators with varying number of Bits Per cached Element (bpe); a higher bpe implies a larger indicator, that is guaranteed to provide a lower false-positive ratio [13]. Notice that the X-axis and the Y-axis are in logarithmic scale. Fig. 1 shows that the false-negative ratio dramatically increases for all indicator sizes. Furthermore, this phenomenon is manifested for various types of workloads, where Fig. 1 shows this for two specific traces, Wiki and Gradle (which represent significantly distinct workloads, as described in Sec. V-A). For instance, it is not uncommon to have a false-negative ratio as high as 10% when the update interval is above 1K. Most interestingly, using a larger indicator, which guarantees a lower inherent false-positive ratio [13], results in a higher false-negative ratio. We discuss and explain this phenomenon with more detail in Sec. V-C.

Designing an access strategy that considers both false-positives, and false-negatives, is a challenging task. In particular, it is unclear whether, or when, it may be beneficial to access a cache despite a negative indication. However, to the best of our knowledge, despite its importance, this problem has never been studied.

A. Our Contribution

We consider the problem of accessing a multi-cache system while using indicators that exhibit both false-positive and false-negative indications. We challenge the common practice which assumes that it is always better not to access caches with negative indications. Specifically, we develop a framework that supports false-negative awareness, and design policies that actively access caches with negative indications, aiming at minimizing the overall access cost.

We first present an algorithm for fully-homogeneous environments, and show that it is optimal in terms of the overall access cost. These results appear in Sec. III. In Sec. IV we develop a strategy for heterogeneous environments where both the cache and the client estimate some of the underlying distributions (which may depend inter alia on the system configuration and the workload being served). Our suggested false-negative aware (FNA) strategy makes deliberate accesses also to caches with negative indications. Furthermore, we show that any approximation guarantee provided by a false-negative oblivious (FNO) access strategy (in our model), can be used for our false-negative aware strategy. In particular, we show how to employ known FNO strategies as subroutines, which induce their performance guarantees on our proposed FNA solution.

Finally, in Sec. V we present the results of our in-depth simulation study, where we evaluate the performance of our proposed solution in varying system configurations. We show that our FNA strategy implies a significant reduction in access costs in many real-life scenarios, compared to state-of-the-art FNO approaches. Furthermore, our results show that our FNA strategy with minimal resources obtains comparable results to those obtained by FNO strategies that use considerably more resources. For instance, our results indicate that in order to match the performance of our FNA strategy, an FNO approach might require as much as an order of magnitude more resources (e.g., in terms of system caching capacity or the bandwidth required for indicator advertisement).

B. Related Work

As described in the previous section, indicators are commonly used to periodically advertise the content of caches in an efficient manner. Since indicators are of bounded size, they usually fail to provide a precise representation of the cache content and exhibit false-positive indications [9], [13]. The pioneering work of [7] shows that due to these false-positives, sometimes naively relying on an indicator for accessing even a single cache may do worse than not using an indicator at all. Subsequent work [14] tackles a distributed scenario where multiple caches advertise indicators, and develops access strategies that take into account both the access cost, and the false-positive rate in each cache, to minimize the overall expected cost. However, [7], [14] disregard false-negative indications.

The work of [15] studies the problem of false-negatives in practical deployment of Counting Bloom Filters [10]. Other techniques to reduce the false-negative ratio in numerous variants of Bloom filters are surveyed in [11]. These works address false-negatives that stem from architectural design, i.e., from concrete data structures used to implement indicators. Consequently, these works focus on developing enhanced data structures that reduce such false-negatives. In contrast, we focus on false-negatives caused by staleness, i.e., false negatives that follow from the operational usage of the system. Such false-negatives may occur in any indicator, even if its design is false-negative-free, such as a simple Bloom filter [9], [13]. In this sense our approach is orthogonal to previous work targeting the reduction of false-negatives [11], [15], and these approaches may be seamlessly combined with our proposed solutions.

Since constantly advertising a fresh indicator might be prohibitively costly, in practice caches commonly advertise fresh indicators only periodically [11], [16–18], where one usually refers to the period between the advertisements of fresh indicators as the update interval. Several works addressed the interplay between the update interval and performance by.
II. SYSTEM MODEL AND PRELIMINARIES

This section formally defines our system model and notations, which are summarized in Table I. We consider a set $N$ of $n = |N|$ caches, containing possibly overlapping sets of items. We associate each time $t$ with a unique item request $x_t$ issued at time $t$, and we refer to the entire sequence of requests as $\sigma$. Let $S_{j,t}$ denote the set of items stored in cache $j$ at time $t$, prior to handling request $x_t$. For every request $x_t$ issued at time $t$, drawn from some distribution, we let $h_{j,t}$ denote the probability that $x_t \in S_{j,t}$. This probability depends both on the distribution of the requests, as well as on the cache policy. The average $h_{j,t}$ over the entire sequence is commonly referred to as the hit ratio, i.e., the fraction of requests in $\sigma$ that were available in cache $j$ upon being issued. Similarly to previous works, we assume that the past hit ratio is a reasonable estimate of $h_{j,t}$ [24], [25]. We refer to this estimation as the probability that the next accessed item $x_t$ is available in $S_{j,t}$.

Each cache $j$ maintains an indicator $I_{j,t}$, which approximates the set of items in cache $j$ at time $t$; given an item $x$, $I_{j,t}(x) = 1$ is referred to as a positive indication while $I_{j,t}(x) = 0$ is considered a negative indication.

The false positive ratio of $I_{j,t}$ is defined by $FP_{j,t} = \Pr(I_{j,t}(x) = 1 | x \notin S_{j,t})$. It captures the probability that given a request $x$ issued at time $t$, the indicator would mistakenly indicate that it is in $S_{j,t}$.

Similarly, the false negative ratio of $I_{j,t}$ is defined by $FN_{j,t} = \Pr(I_{j,t}(x) = 0 | x \in S_{j,t})$. It captures the probability that given a request for $x$, issued at time $t$, the indicator would mistakenly indicate that it is not in $S_{j,t}$.

For every cache $j$, and every time $t$, we denote by $\pi_{j,t} = \Pr(x \notin S_{j,t} | I_{j,t}(x) = 1)$ the positive exclusion probability, that is, the probability that a requested item $x$ is not in the cache, despite a positive indication. Similarly, we let $\nu_{j,t} = \Pr(x \notin S_{j,t} | I_{j,t}(x) = 0)$ denote the negative exclusion probability, that is, the probability that a requested item $x$ is not in the cache, given a negative indication. We denote by $q_{j,t}$ the probability of a positive indication for an item requested from cache $j$, and refer to $q_{j,t}$ as the positive indication ratio. When clear from the context, we abuse notation and omit the time $t$.

Since a positive indication occurs when either $x \in S_j$ and no false-negative occurs; or $x \notin S_j$, and a false-positive occurs,
we have
\[ q_j = \Pr(I_j(x) = 1) = h_j \cdot (1 - FN_j) + (1 - h_j) \cdot FP_j. \] (1)
Using Bayes’ theorem it follows that
\[ \pi_j = \Pr(x \notin S_j | I_j(x) = 1) = FP_j \cdot (1 - h_j) / q_j \] (2)
\[ \nu_j = \Pr(x \notin S_j | I_j(x) = 0) = (1 - FP_j) \cdot (1 - h_j) / (1 - q_j), \] (3)
for \( q_j \) as defined in Eq. (1).

We say that a system is sufficiently-accurate if for every indicator of cache \( j \), \( FP_j + FN_j < 1/2 \). We note that in most real-life scenarios, both the false-positive ratio \( FP_j \), and the false-negative ratio \( FN_j \), are well below 0.5, and therefore such systems are sufficiently-accurate.

The following simple condition characterizes sufficiently-accurate systems (proof omitted due to space constraints).

**Proposition 1.** A system is sufficiently-accurate if and only if for every \( j \) it holds that \( \nu_j > \pi_j \).

For any queried item \( x \), let \( N_x \) denote the set of caches with positive indications, i.e., \( N_x = \{ j | I_j(x) = 1 \} \). We let \( n_x \) denote the number of caches with positive indications, i.e., \( n_x = |N_x| \).

A request for datum \( x \) triggers a data access which consists of (i) querying for \( x \) in all the \( n \) indicators, (ii) selecting a subset \( D \subseteq N \) of caches, and (iii) accessing all the \( |D| \) selected caches in parallel. Accessing each cache incurs some predefined access cost, \( c_j \). For ease of presentation, we assume without loss of generality that \( \min_j c_j = 1 \). The overall access costs of accessing a set \( D \) of caches is \( c_D = \sum_{j \in D} c_j \).

A multi-cache data access is considered a hit if the item \( x \) is found in at least one of the accessed caches, and a miss otherwise. A miss incurs a miss penalty \( M \), for some \( M \geq 1 \).

In our model, we do not assume any specific sharing policy among the caches. Yet, in the analysis of our system (Sections III-V) we assume that the exclusion probabilities are mutually independent. Under this assumption, our analysis provides a baseline for understanding the performance of such systems. However, in the evaluation of our algorithms, we also consider environments where the exclusion probabilities need not be mutually independent (Section IV).

For a subset of caches \( D \), we define its (expected) miss cost for a query \( x \) by \( M \cdot \prod_{j \in D} \pi_j \cdot \prod_{j \in D} \nu_j \).

The (expected) service cost of a query is the sum of the access cost and the miss cost, namely,
\[ \phi_x(D) = \sum_{j \in D} c_j + M \prod_{j \in D} \pi_j \cdot \prod_{j \in D} \nu_j. \] (4)

The Cache Selection (CS) problem is to find a subset of caches \( D \subseteq N \) that minimizes the expected cost \( \phi_x(D) \).

In what follows we refer to an access to a cache with a positive indication as a positive access, and refer to an access to a cache with a negative indication as a negative access. In particular, we consider two types of approaches to solving the CS problem: (i) false-negative oblivious (FNO) schemes, which only perform positive accesses, and (ii) false-negative aware (FNA) schemes, which may also perform negative accesses. While the former may be viewed as the traditional manner in which access strategies are designed, the latter can be viewed as a more speculative approach, which sometimes accesses a cache even with no positive indication, risking an increased access cost.

## III. The Fully Homogeneous Case

In this section, we focus on a simplified fully-homogeneous case. In such settings, the access cost of all caches is the same, and is normalized to one (\( c = 1 \)). The per-cache hit ratio, false-positive ratio, and false-negative ratio, are identical for all caches. I.e., for each cache \( j \), \( h_j = h \), \( FP_j = FP \) and \( FN_j = FN \) for some constants \( h, FP, FN \in [0, 1] \).

We explore the challenges and potential benefits arising from developing a false-negative aware cache selection strategy. We first describe the aspects specific to such homogeneous settings, and then describe and analyze our false-negative aware Homogeneous Cache Selection policy, HoCSFNA. Our analysis shows that HoCSFNA minimizes the service cost in the fully-homogeneous case. Later on, we use HoCSFNA to derive insights as to when it is beneficial to access a cache despite a negative indication.

### A. Preliminaries

In the fully-homogeneous settings, the task of selecting a subset of the caches \( D \subseteq N \) that minimizes the service cost is reduced to selecting two integers: \( 0 \leq r_1 \leq n_x \), the number of caches with positive indication to access; and \( 0 \leq r_0 \leq n - n_x \), the number of caches with negative indication to access. The objective function \( \phi \) (Eq. (4)) is reduced to
\[ \hat{\phi}(r_0, r_1) = r_0 + r_1 + M \cdot \nu^{r_0} \cdot \pi^{r_1}. \] (5)

We let \( r_0^* \) and \( r_1^* \) denote the values of \( r_0 \) and \( r_1 \) that minimize the service cost, namely
\[ \hat{\phi}(r_0^*, r_1^*) = \min_{0 \leq r_0 \leq n_x, 0 \leq r_1 \leq n - n_x} \hat{\phi}(r_0, r_1). \] (6)

In our analysis of \( \hat{\phi} \), we use the extension of \( \hat{\phi} \) over the reals, which we hereafter denote by \( \hat{\phi} \). Observe that for any fixed constants \( a, b \), the functions \( \hat{\phi}(a, r_1) \), and \( \hat{\phi}(r_0, b) \) are strictly convex. The following proposition will be used throughout our analysis in this section (proof omitted due to space constraints).

**Proposition 2.** Let \( \tilde{f} : \mathbb{R} \to \mathbb{R} \) be a strictly convex function, and let \( f \) be its restriction over the integers. If \( f \) obtains its minimum value at some integer \( \tilde{a} \geq 1 \), then \( f(1) < f(0) \).

We recall that the function \( \hat{\phi}(r_0, r_1^*) \) is strictly convex. By applying Proposition 2 we therefore obtain the following corollary.

**Corollary 3.** If \( r_0^* \geq 1 \), then \( \hat{\phi}(1, r_1^*) < \hat{\phi}(0, r_1^*) \).
Algorithm 1 HoCSFNA

1: \( r_0^* = 0; r_1^* = \arg \min_{r_1 \in [0,n_x]} [r_1 + M \cdot \pi r_1] \)
2: if \( M \cdot \pi r_1^* > 1 \) then
3: \( r_0^* = \arg \min_{r_0 \in [0,n-n_x]} [r_0 + M \cdot \pi r_1^* \nu r_0] \)
4: return \( r_0^*, r_1^* \)

B. An Optimal Strategy

Our algorithm for the fully-homogeneous settings, HoCSFNA, is formally defined in Algorithm 1. The algorithm first calculates the number of caches with positive indication to access, \( r_1^* \), assuming no cache with negative indication is accessed (line 1). Next, if the expected miss cost is still higher than accessing an additional cache (the condition in line 2), the algorithm also considers caches with negative indications (line 4). The following theorem shows that HoCSFNA is optimal in the fully-homogeneous case.

Theorem 4. If a fully-homogeneous system is sufficiently-accurate, then HoCSFNA minimizes the service cost.

Proof. By Proposition 1 we have \( \nu > \pi \). In addition, by the definition of \( \phi \) (Eq. 5), the objective function \( \hat{\phi}(r_0, r_1) \) is symmetric in \( r_0, r_1 \). It follows that assigning \( r_0 > 0 \) may reduce cost only if \( r_1 \) is maximized, namely, if \( r_1 = n_x \). Hence, line 1 Indeed calculates the optimal value of \( r_1^* \).

If \( M \cdot \pi r_1^* > 1 \), then the algorithm sets \( r_0^* \) to a value which minimizes the total cost (line 3). Else (namely, if \( M \cdot \pi r_1^* \leq 1 \)), accessing cache(s) with negative indications can only increase the total cost, as it increases the aggregate access cost by at least 1, which is at least as high as the potential marginal decrease in the miss cost, which is at most \( M \cdot \pi r_1 \). Hence, when the if-condition (line 2) is not met, it is best to keep the default value \( r_0^* = 0 \).

HoCSFNA implies that sometimes it is better to access a cache in spite of a negative indication. The following proposition characterizes such cases.

Proposition 5. Assume \( \nu, \pi \in (0,1) \). (i) If \( n_x = 0 \), then accessing at least one cache with negative indication strictly reduces the service cost iff \( \nu < 1 - \frac{1}{M} \). (ii) If \( n_x > 0 \), then accessing at least one cache with negative indication strictly reduces the service cost iff \( \nu < 1 - \frac{1}{M \pi n_x} \) and \( M \cdot \pi n_x - 1 < (1 - \pi) > 1 \).

Proof. We first provide some intuition as to the validity of the claim. Intuitively, \( \nu \) is a proxy to the true negative rate. Hence, the conditions in cases (i) and (ii) reflect the fact that if the expected miss cost after accessing all the caches with a positive indication is high, and the true-negative rate is low, then it might be beneficial to access a cache with a negative indication. We now turn to prove the claim.

(i) Assume \( n_x = 0 \), implying that \( r_1^* = 0 \). By the definition of \( \hat{\phi} \), the inequality \( \nu < 1 - \frac{1}{M} \) holds iff \( \hat{\phi}(1,0) = 1 + M \cdot \nu < M = \hat{\phi}(0,0) \). Hence, it suffices to prove that accessing at least one cache with negative indication strictly reduces the service cost iff \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \).

For one direction, if \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \), then indeed accessing at least one cache with negative indication strictly reduces the service cost. For the other direction, if accessing at least one cache with negative indication strictly reduces the service cost, we have \( r_0^* > 0 \). Hence, by Corollary 3 \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \), thus completing the proof of this case.

(ii) Assume \( n_x > 0 \). By the definition of \( \hat{\phi} \) the condition \( M \cdot \pi n_x - 1 > 1 \) holds true iff \( n_x > 1 + M \pi n_x - 1 \), which is equivalent to \( \phi(0,n_x) = \hat{\phi}(0,0) = 1 \). Similarly, the condition \( \nu < 1 - \frac{1}{M \pi n_x} \) holds true iff \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \). We therefore have to prove that accessing a cache with a negative indication strictly reduces the service cost iff \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \).

For the first direction, assume that accessing a cache with negative indication strictly reduces the service cost, namely, \( r_0^* > 0 \). By the proof of Theorem 1, having \( r_0^* > 0 \) implies that \( r_1^* = n_x \). Hence, by Corollary 3 \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \). Furthermore, HoCSFNA calculates \( r_1^* \) while using \( r_0 = 0 \) (line 1). We can therefore conclude that \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \), thus completing the proof for this direction.

For the other direction, assume that \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \). The function \( \hat{\phi}(0,1) \) is convex in \( r_1 \), and therefore does not have a local maximum within the domain \( r_1 \in [0,n_x] \). As \( \hat{\phi}(0, n_x) < \hat{\phi}(0,0) \), it follows that \( \hat{\phi}(0, r_1) \) is monotonically decreasing in the range \( r_1 \in [0,n_x] \). Hence, \( \hat{\phi}(0, r_1) \) is minimized when \( r_1 = n_x \). As HoCSFNA calculates \( r_1^* \) by assigning \( r_0 = 0 \) (line 3), we know that \( r_1^* = n_x \). By assigning \( r_1^* = n_x \) in our assumption \( \hat{\phi}(1,0) < \hat{\phi}(0,0) \), we obtain \( \hat{\phi}(1,r_1^*) < \hat{\phi}(0,r_1^*) \). Hence, accessing at least one cache with a negative indication strictly reduces the service cost.

HoCSFNA also implies that sometimes it is better to access no cache, even if there exist positive indications. The following proposition characterizes such cases.

Proposition 6. If there exist positive indications but \((1-h)FP < h(1-FN)(M-1)\), then the best policy is to access no cache.

Proof. We first provide some intuition as to the validity of our claim. Observe that in a system where \((1-h)FP\) is large, a positive indication is very likely to be false-positive. Hence, it may be beneficial not to access a cache despite a positive indication. This is true especially if the miss penalty is small, as reflected by the right-hand-side of the condition.

We now turn to prove the claim. To prove the claim, it suffices to show that if there exist positive indications, and the best policy is to access at least one cache, then \((1-h)FP < h(1-FN)(M-1)\).

Assume that the best policy is to access at least one cache. Namely, either \( r_0^* > 0 \), or \( r_1^* > 0 \). By the proof of Theorem 4 assigning \( r_0 > 0 \) may reduce the cost only if \( r_1^* = n_x \). As it is given that there exist positive indications (namely, \( n_x > 0 \), it follows that \( r_0^* > 0 \) only if \( r_1^* > 0 \). Hence, the best policy is to access at least one cache only if \( r_1^* > 0 \).

Algorithm HoCSFNA calculates \( r_1^* \) by setting \( r_0 = 0 \) (line 1). As HoCSFNA minimizes the service cost (Theorem 4), we have that \( \hat{\phi}(0,r_1^*) < \hat{\phi}(0,0) \). As the function
\(\hat{\phi}(0, r_1)\) is strictly convex, by Proposition 2 we know that 
\(\hat{\phi}(0, 1) < \hat{\phi}(0, 0)\). By the definition of \(\hat{\phi}\) (Eq. 5), we have 
\(1 + M \cdot \pi < M\). Using the expression for \(\pi\) from Eq. (2) (recall that in the homogeneous case, we omit the subscript \(j\)), we obtain 
\(FP(1-h) < \frac{M-1}{M}.\) Assigning the value of \(q\) from Eq. (1), we have 
\(M \cdot FP(1-h) < (M-1) [h(1-FN) + (1-h) FP]\). By algebraic simplification this implies that 
\(FP(1-h) < h(1-FN)(M-1)\), thus completing the proof.

IV. The Heterogeneous Case

This section focuses on the heterogeneous case, where both cache access costs and exclusion probabilities can be arbitrary. The main challenge in such settings is to evaluate the exclusion probabilities \(\pi_j\) and \(\nu_j\). In order to evaluate these terms, we collect recent statistics of the various parameters governing system behavior.

In what follows, we first provide an overview of the relevant aspects of Bloom filters, which we use as indicators (Sec. IV.A). Subsequently, we detail how we estimate and use the statistics we collect, both by the cache (Sec. IV.B), and by the client (Sec. IV.C), to evaluate the exclusion probabilities, \(\pi_j\), and \(\nu_j\). Later on, we show how approximation algorithms devised for false-negative oblivious settings (e.g., in [13]) can be used as a building block in algorithms for solving the CS problem.

A. Bloom Filters and False Negatives

For completeness, we provide a brief overview of simple Bloom filters [9], and their structural properties which are relevant for our proposed solutions. A Bloom filter is a randomized data structure that approximately represents a set of items. Bloom filters are composed of a bit array of size \(|I|\), as well as \(k\) independent hash functions. When adding an item to the filter, each of the \(k\) hash functions is applied to the item, and the corresponding bit in the array is set. When testing for an item’s existence, we apply the \(k\) hash functions and test the corresponding bits. If all bits are set, the Bloom filter replies with a positive indication. Otherwise, the indication is negative.

In a fresh Bloom filter, which is updated upon every insertion of an item to the set, positive indications may be false due to hash collisions, but negative indications are guaranteed to be correct. However, in a stale Bloom filter, negative indications may also be erroneous. Such false-negatives occur, e.g., when indicators are advertised to the users only periodically. In such a scenario, when a new item is admitted, but the updated indicator is not yet advertised, the stale indicator available to the user fails to represent this change.

To allow a meaningful analysis of the trade-off between accuracy and memory footprint, it is useful to express the size of Bloom filters using the notion of Bits Per Element (bpe). Intuitively, optimally configured Bloom filters of the same bpe have the same false-positive accuracy, regardless of the size of the set being approximated by the Bloom filter. More formally, given the value of bpe, one can calculate the optimal number of hash functions \(k\), that minimizes the false-positive ratio (see [13]).

| updated BF | \(B_0(t)\) | \(B_1(t)\) |
|-----------|----------|---------|
| 0...0     | 0...0    | 1...1   |
| 1...1     | 0...0    | 1...1   |

\[\Delta_0(t) = B_0(t) - \Delta_1(t)\]

\[\Delta_1(t) = B_1(t) - \Delta_1(t)\]

| stale BF | \(\Delta_0(t)\) | \(\Delta_1(t)\) |
|---------|----------------|---------|

Fig. 2. An example of an updated Bloom filter and a stale Bloom filter at time \(t\).

We consider the usage of Bloom filters as indicators of cache content. In particular, we let \(C_j\) denote the size of cache \(j\), i.e., the maximum number of elements that can be stored in cache \(j\), where we assume that all elements have the same size.

The actual size of the bloom filter indicator \(I_j\) associated with cache \(j\) is therefore \(|I_j| = bpe \cdot C_j\). Each cache manages its own Bloom filter, and occasionally advertises the indicator to the client. At any time \(t\), we consider the updated Bloom filter maintained by the cache, and let \(B_1(t)\) and \(B_0(t)\) denote the number of bits set (i.e., with value 1) and reset (i.e., with value 0), at time \(t\), respectively, in the Bloom filter approximating the content of the cache. A client uses a replica of the cache’s Bloom filter, which represents a snapshot of the Bloom filter that the cache advertised at some time \(t' \leq t\). We refer to this replica as the stale Bloom filter. Let \(\Delta_1(t)\) denote the number of bits that are set in the updated Bloom filter but are reset in the stale Bloom filter. Similarly, we let \(\Delta_0(t)\) denote the number of bits that are reset in the updated Bloom filter, but are set in the stale Bloom filter. Fig. 2 illustrates this situation. For clarity of presentation, and WLOG, we group together all the bits contributing to \(\Delta_0(t)\), and also group together the bits accounted for by \(\Delta_1(t)\).

The false-negative ratio: Consider a query for an item \(x\) that is stored in the cache at time \(t\). Recall that an updated Bloom filter never exhibits false negatives. Hence we know that all the \(k\) hashes of \(x\) are mapped to the bits that are set in the updated Bloom filter. The query for \(x\) in a stale indicator is a true positive iff all the hashes are mapped to the set of \(B_1(t) - \Delta_1(t)\) bits that are also in the set of the stale indicator; by the fact that the \(k\) hash functions are independent, and uniformly distributed over their range, this happens with probability \(\left(\frac{B_1(t) - \Delta_1(t)}{B_1(t)}\right)^k\). Otherwise, the query for \(x\) is a false-negative. It follows that the false-negative ratio of the cache at time \(t\) can be estimated by

\[FN_t = 1 - \left(\frac{B_1(t) - \Delta_1(t)}{B_1(t)}\right)^k. \tag{7}\]

\[\text{The false-positive ratio:}\] Consider a query for an item \(y\) that is not stored in the cache at time \(t\). For uniformly distributed and independent hash functions, the hashes of \(y\) are mapped to arbitrary locations in the Bloom filter. The stale Bloom filter exhibits a false positive iff all the \(k\) hashes of \(y\) map to bits that are set in the stale Bloom filter. Hence, the probability of a false positive in the stale indicator at time \(t\) can be estimated by:

\[FP_t = \left(\frac{B_1(t) - \Delta_1(t) + \Delta_0(t)}{|I_j|}\right)^k. \tag{8}\]

We note that Eqs. (7) and (8) are only estimations, while the exact miss probabilities strongly depend upon the workload, and the cache policy. For instance, consider the case where immediately after cache \(j\) sends an update, it caches an item \(x\).
Then, any subsequent request for \( x \), until the cache advertises the next update, are false-negatives. However, until the cache advertises the next updated indicator, \( x \) may be accessed many times, or not accessed at all, according to the concrete workload.

The analysis above assumes that the indicator is a simple Bloom filter \([9]\). However, one may apply a similar technique of estimating the false-positive ratio, and the false-negative ratio, also to other indicators (e.g., TinyTable \([12]\)).

Estimating the false-negative ratio and the false-positive ratio can be done periodically to reduce the computational overhead of comparing the stale and updated bloom filters. In what follows, we show how these estimations can be harnessed for solving the CS problem in heterogeneous and dynamic settings.

### B. Cache-side Algorithm

The cache maintains both the stale Bloom filter (i.e., the most recently advertised Bloom filter, which is also available at the client) and the updated Bloom filter. Along a sequence of requests \( \sigma \), each cache \( j \) will estimate the false-negative ratio, and the false-positive ratio, according to Eqs. (7) and (8), by comparing the stale and updated Bloom filters. These estimates will be sent (periodically) to the client.

### C. Client-side Algorithm

The client has two main tasks when solving the CS problem: (i) estimating the exclusion probabilities \( \pi_j \) and \( \nu_j \) for every cache \( j \), and (ii) deciding which of the caches should be accessed for every request. The algorithm executed by the client, which we dub \( \text{CS}_{FNA} \), is formally defined in Algorithm 2. We now turn to describe and discuss the two main tasks performed by \( \text{CS}_{FNA} \).

1) **Exclusion probabilities estimation**: For estimating the exclusion probabilities of some cache \( j \), the client makes use of Eqs. (1)–(3). First, recall that the client periodically receives estimations of \( \text{FN}_j \), \( \text{FP}_j \) from the cache. Next, for evaluating \( q_j \), the client periodically estimates the probability \( \Pr(I_j(x) = 1) \) **empirically**, using a weighted exponential moving average. Formally, consider a sequence of requests \( \sigma \), and consider epochs of \( T \) requests. Let \( a_j(s,t) \) denote the number of positive indications of indicator \( I_j \) for requests \( s + 1, \ldots, t \) made by the client. For any \( t \leq T \) we let the estimated positive indication ratio after handling request \( t \) be \( q_{j,t} = \frac{a_j(0,t)}{t} \). For every \( i = 1, 2, \ldots \) and every \( iT < t < (i + 1)T \), we let \( q_{j,t} \) be the most recent estimate over epochs of \( T \) requests, i.e., \( q_{j,t} = q_{j,(t/T) \cdot T} \), and the estimate is updated at \( t = (i + 1)T \) such that

\[
q_{j,(i+1)T} = \delta \cdot \frac{a_j(iT, (i+1)T)}{T} + (1 - \delta) \cdot q_{j,iT},
\]

where \( \delta \in (0,1) \) is some constant governing the dynamics of the estimate change. We note that only the client can perform such an estimation since it requires knowing all the requests in \( \sigma \), and not only requests for which the cache has been accessed.

Finally, given the current values for \( \text{FP}_j, \text{FN}_j \), and \( q_j \), for every item being requested in the sequence \( \sigma \), the client estimates the hit ratio \( h_j \) (line 6), and the exclusion probabilities \( \pi_j \) (line 8) and \( \nu_j \) (line 10) using Eqs. (1), (2), and (3), respectively.

2) **Choosing the caches to access**: For any set of caches \( D \), the client’s estimations of the exclusion probabilities essentially determine the expected miss cost. By setting \( \rho_j = \pi_j \) if \( I_j(x) = 1 \), and \( \rho_j = \nu_j \) if \( I_j(x) = 0 \), the expected miss cost can be expressed by \( M \cdot \prod_{j \in D} \rho_j \), and the objective function defined in Eq. (4) translates to finding the set of caches \( D \) minimizing

\[
\phi_x(D) = \sum_{j \in D} e_j + M \cdot \prod_{j \in D} \rho_j.
\]
where \( D \) (i.e., \( N \)) we abuse notation, and refer to \( \phi \) variety of scenarios, using traces of real-life workloads. We are inputs. The theorem follows. is also the optimal solution to the CS problem with \( \alpha \) an \( \tilde{\nu} \) set of all caches, \( N \).

We define algorithm Alg\(^*\) such that Alg\(^*\) returns the output of Alg for the inputs of \( \tilde{\nu}^* \) (for the positive exclusion probabilities), \( \tilde{\nu}^\ast \) for the negative exclusion probabilities), and the set of all caches with a positive indication according to \( \tilde{I}^\ast \) (i.e., \( N \)). We now show that the solution returned by Alg\(^*\) is an \( \alpha \)-approximate solution for the CS problem with negative exclusion probabilities \( \nu_j \).

By the assumption on Alg, its output \( D \) satisfies

\[
\phi_{x,\tilde{\nu}^*,\tilde{\pi}^*,\tilde{I}^*}(D) \leq \alpha \cdot \phi_{x,\tilde{\nu}^*,\tilde{\pi}^*,\tilde{I}^*}(D^\ast)
\]

where \( D^\ast \) is the optimal solution to the CS problem with \( \tilde{\nu}^*, \tilde{\nu}^\ast, \) and the set of all caches generated by \( \tilde{I}^\ast \) as inputs. Since by the definition of \( \tilde{\nu}^*, \tilde{\nu}^\ast, \) and \( \tilde{I}^\ast \) it follows that

\[
\phi_{x,\tilde{\nu}^*,\tilde{\pi}^*,\tilde{I}^*}(\tilde{D}) = \phi_{x,\tilde{\nu}^*,\tilde{\pi}^*,\tilde{I}^*}(\tilde{D})
\]

for every set of caches \( \tilde{D} \), we are guaranteed to have that \( D^\ast \) is also the optimal solution to the CS problem with \( \tilde{\pi}^*, \tilde{\nu}^\ast, \) and \( \tilde{I}^\ast \) as inputs. The theorem follows.

The proof of Theorem 7 implies the following corollary.

**Corollary 8.** If the estimations of \( \pi_j \) and \( \nu_j \) produced by CS\(_{FNA}\) are precise, and the algorithm Alg used by CS\(_{FNA}\) is an \( \alpha \)-approximation algorithm for the restricted CS problem, then CS\(_{FNA}\) produces an \( \alpha \)-approximate solution to the CS problem.

Combining Corollary 8 with the results of [14], we obtain a myriad of tradeoffs and possible approximation guarantees for CS\(_{FNA}\). In particular, in Sec. V we consider the performance of one specific realization of CS\(_{FNA}\), which uses algorithm DS\(_{PGM}\) for the restricted CS problem presented in [14].

V. SIMULATION STUDY

In this section, we evaluate the performance and trade-offs of our proposed false-negative aware algorithm, CS\(_{FNA}\), in a variety of scenarios, using traces of real-life workloads. We begin by describing our evaluation settings and parameters.

A. Simulation Settings

**Traces:** We use the following real workloads, which are commonly used when evaluating caching systems: (i) Wiki: Read requests to Wikipedia pages [27]. (ii) Gradle: Gradle is a build tool that reduces the compilation time of large projects by caching build parts, and they compile from scratch upon a cache miss. This trace [28] was provided by the Gradle project. (iii) Scarab: A trace from Scarab Research, a personalized recommendation system for e-commerce sites [28]. (iv) F2: Traces taken from a financial transaction processing system [29]. For each of the traces above, we use the first 1M requests in the trace.

**Caches:** We consider a system-wide request distribution where a missed item is placed in a single cache that is chosen by the controller. Such an approach is common in large distributed systems, in an attempt to obtain good load balancing, and maximize the amount of content being cached [30].

Each cache applies the Least Recently Used (LRU) policy, which is arguably the most common policy used in practice. LRU maintains items ordered by their last access time and evicts the least recently used item when admitting an item to a full cache.

**Indicators:** Each cache \( j \) of size \( C_j \) periodically advertises an indicator \( I_j \) of size \( bpe \cdot C_j \). For computing the indicator, cache \( j \) maintains a Counting Bloom Filter (CBF) [10] with three-bit counters, where the number of counters is \( bpe \cdot C_j \). The CBF uses the optimal number of hash functions so as to minimize the false positive probability. The CBF is maintained for bookkeeping where we add an item to the CBF upon item insertion to the cache, and remove an item from the CBF upon item eviction. The cache uses the CBF for constructing the advertised indicator by compressing the CBF to a simple (1 bit) indicator where a bit is set if the respective counter in the CBF is strictly positive.

**Algorithms compared:** Recall that CS\(_{FNA}\) makes use of an algorithm for solving the CS problem for the case where indicators exhibit no false-negatives. In our evaluation, we make use of the DS\(_{PGM}\) algorithm from [14]. This strategy was shown to produce a \((\log M)\)-approximation for the CS problem with no false-negatives. By Theorem 7 this guarantee also applies to the general CS problem. Furthermore, this algorithm exhibits close-to-optimal results in practice, when tested on real-world workloads [14].

We consider two benchmarks for evaluating the performance of CS\(_{FNA}\): (i) applying the vanilla DS\(_{PGM}\) algorithm, which only considers accessing caches with a positive indication (albeit stale), using only the estimates of \( \bar{\pi}_j \) for every cache \( j \), and using \( \nu_j = 1 \) for all \( j \), and (ii) the ideal strategy that uses perfect information (PI), i.e., a strategy that always has access to the precise cache content, which accesses the cheapest cache containing an item if such a cache exists, and doesn’t access any cache otherwise. We refer to the former algorithm which is false-negative oblivious as the CS\(_{FNO}\) algorithm, and to the latter ideal strategy as the PI strategy.

Throughout our evaluation both CS\(_{FNA}\) and CS\(_{FNO}\) evaluate \( q_j \) with a time horizon of \( T = 100 \) requests and using \( \delta = 0.25 \) for the weighting of the moving average. Furthermore, each cache \( j \) re-estimates the false-positive ratio \( FP_j \) and the false-negative ratio \( FN_j \) once every 50 insertions to the cache.

**Evaluation metric:** We consider the mean service cost per request for each algorithm over the entire input. In order to obtain a qualitative metric for comparing the performance of the various algorithms and scenarios, we consider the normalized cost of each algorithm, where we normalize the
mean service cost by the mean service cost of the ideal PI strategy. We recall that the PI strategy is infeasible in real-life, as it requires the client to have an accurate representation of the cache content at any time. However, it is instructive to use it as the baseline comparison, and its performance can be viewed as a lower bound on the cost of any policy for solving the CS problem.

**Baseline scenario:** Unless stated otherwise, our evaluation considers three 10K elements caches whose access costs are 1, 2, and 3, and a miss penalty of 100 (chosen to be 50 times the average cache access cost). Each cache advertises an updated indicator once the number of insertions performed since the previous update reaches 10% of its capacity. For our baseline scenario, this translate to an indicator advertisement once every 1K insertions. The advertised indicator of each cache $j$ uses $b_{pe} = 14$, which implies an indicator size of $14 \cdot C_j$, where the number of hash functions is optimized to minimize the false-positive ratio. In particular, in our baseline scenario, this translates to a designed false-positive ratio of 0.1% [13]. In each of our evaluations, we explore the impact of varying one of the systems parameters, where the remaining parameters are set according to our baseline scenario.

Our Python code used for performing the evaluation is available in [31].

### B. Impact of Miss Penalty and Workload Diversity

We first compare the performance of $\text{CS}_{FNO}$ and $\text{CS}_{FNA}$ when varying the miss penalty values $M$ in the range $\{50, 100, 500\}$. The results in Figure 3 show that while the performance of the false-negative oblivious policy $\text{CS}_{FNO}$ degrades as the miss penalty increases, the performance of our proposed false-negative aware algorithm $\text{CS}_{FNA}$ improves significantly. Furthermore, the performance of $\text{CS}_{FNA}$ tends to the optimal performance as the miss penalty increases. This behavior follows from the fact that a higher miss penalty accentuates the impact of false-negative events. In particular, ignoring negative indications (as is done by $\text{CS}_{FNO}$) might reduce the access cost but is severely penalized by an increased expected miss cost in cases where the miss penalty is large.

Fig. 3 also demonstrates significant differences across distinct workloads. $\text{CS}_{FNO}$ worst performance is exhibited for the Gradle trace, whereas its best performance is obtained for the Wiki trace. To understand this phenomena, we observe that Gradle exhibits a high recency-bias, where items are requested shortly after their first appearance. As false-negatives occur since the indicator takes time to reflect the insertion of new items, $\text{CS}_{FNO}$, which never accesses caches with a negative indication, fails to take advantage of this recency-bias. In contrast, the Wiki trace is more frequency-biased, which implies that popular items do not rapidly change over time and that the impact of false-negatives is less pronounced.

As the Wiki trace and the Gradle trace exhibit the most extreme scenarios in terms of the impact of false-negatives, due to space constraints, in subsequent sections, we focus our attention on these traces alone.

### C. Impact of Advertisement Policy and Indicator Parameters

1) **Update interval:** We now turn to study the effect of staleness on the performance of our algorithm. To this end, we let the update interval, which is the number of insertions between indicator advertisement, vary between 16 and 8K (8192), and consider the normalized cost of both $\text{CS}_{FNA}$ and $\text{CS}_{FNO}$. These results are presented in Figure 4, where we consider the performance for the Gradle and Wiki workloads.

Our results show that both algorithms’ performance degrades as the update interval increases. When updates are relatively frequent (i.e., up to 128), the performance of $\text{CS}_{FNA}$ and $\text{CS}_{FNO}$ is similar. However, a significant gap emerges between the performance of both algorithms for larger update intervals. In particular, the performance of $\text{CS}_{FNO}$, which ignores negative indications, quickly degrades, whereas $\text{CS}_{FNA}$ shows a considerably milder degradation. This phenomenon is directly related to the fact that when the update interval is large, the false-negative ratio increases significantly (as demonstrated in Fig. 1). Under such regimes, $\text{CS}_{FNO}$ fails to access a cache even when the item is available at the cache, whereas $\text{CS}_{FNA}$ relies on its false-negative awareness to make accesses even in cases of negative indications, taking into account the false-negative ratio estimation provided by the caches. Our results imply that $\text{CS}_{FNA}$ match the performance of $\text{CS}_{FNO}$ while using a significantly lower bandwidth overhead for cache advertisements. For instance, for the Wiki workload $\text{CS}_{FNA}$ obtains the same service cost as $\text{CS}_{FNO}$ while using x16 less bandwidth for indicator advertisements. To observe this notice...
that CS\textsubscript{FNA}'s cost using an update interval of 8K is on par with the service cost of CS\textsubscript{FNO} with an update interval of 512.

2) \textit{Indicator size:} Fig. 5 illustrates our results for varying the size of the indicator being used and advertised by the cache. We vary the number of indicator bits per cached element (bpe) and study the impact of the indicator's size on the service cost. In our evaluation, we compare the performance of CS\textsubscript{FNO} and CS\textsubscript{FNA} with update intervals of 256 and 1024.

Not surprisingly, in most cases, the performance improves when increasing the indicator size, which can be attributed to the fact that larger indicators exhibit fewer false-positive errors. Interestingly, and somewhat counter-intuitively, there exist cases where the performance of CS\textsubscript{FNO} does not improve when increasing the indicator size, and in some cases performance actually \textit{degrades}. To understand this anomaly, it is instructive to understand the impact of false-positive indications, false-negative indications, and the interplay between such errors. First, note that the false-positive rate is often inversely proportional to the false-negative rate. I.e., a constant decrease in the false-positive ratio is usually associated with an increase in the false-negative ratio. An extreme such case occurs, for example, when all indications are negative, thus exhibiting a false-positive ratio of 0 and a large false-negative ratio. Next, note that a false-positive event typically translates to an unnecessary cache access, resulting in a relatively small penalty (e.g., an access cost of 1, 2, or 3 in our evaluation). However, a false-negative event typically translates to a “non-compulsory” miss, translating to a high miss penalty (e.g., 100, in our evaluation). It follows that even a mild decrease in the false-positive ratio may easily result in a non-negligible increase in the false-negative ratio that may nullify the contribution of a lower false-positive ratio. Such effects are especially significant when the miss penalty is high. High miss penalties are common in systems as cache misses often imply accessing slower memories whose access time may be orders of magnitude higher than that of the cache [24]. [32]. Still, our proposed false-negative aware algorithm CS\textsubscript{FNA} handles such scenarios seamlessly, extracting the benefits of the reduced false-positive ratio without any adverse impact on performance.

D. Impact of Caching Capacity

In this section, we study the effect of the caching architecture on the performance of our algorithm. In particular, we study the effect of having a larger, or more diverse, caching capacity on system performance. To this end, we use a longer trace of Wiki to ensure that sufficiently many elements are indeed accessed, and caches are not underutilized. Specifically, we make use of a Wiki trace containing 4.3M requests and 394K unique elements. One should note that the cost of the ideal PI strategy usually drops when increasing the caching capacity as it has perfect information, and the requested items are more likely to be stored in at least one of the caches. Therefore, in order to provide better insight as to the performance of the system, throughout this section we consider the actual mean cost per request (and not the normalized service cost, as done in previous sections).

1) \textit{Scaling the cache size:} We study the impact of the cache size on the performance of CS\textsubscript{FNO} and CS\textsubscript{FNA} with update interval of 256 and 1024. The results in Fig. 6 show that, as could be expected, for every given setting, scaling-up the caches' capacities decreases the service cost due to the improved hit ratio. Our results show that when updates are relatively frequent (e.g., the case where the update interval is 256), the performance of both CS\textsubscript{FNO} is comparable to that of CS\textsubscript{FNA} and they both exhibit a performance close to that of the ideal PI strategy. However, once the updates are less frequent (e.g., the case where the update interval is 1024), CS\textsubscript{FNO} exhibits a significant degradation in performance. CS\textsubscript{FNA}, on the other hand, is far less affected by the increase in the update interval, and is still quite comparable to PI. In general, CS\textsubscript{FNA} shows up to 25\% reduction in cost compared to CS\textsubscript{FNO}. The differences
between CS\textsubscript{FNO} and CS\textsubscript{FNA} become more accentuated when one considers the cache size required to maintain a certain level of cost; CS\textsubscript{FNA} performs better in a system where caches are of size 4K, than CS\textsubscript{FNO} performs with caches of size 32K.

2) Scaling the number of caches: To study the effect of varying the number of caches in the system, we focus our attention on homogeneous cache access costs to obtain results comparable with the results presented in previous sections. In particular, we assume all caches have an access cost of 2, ensuring that the average access cost is the same as in other scenarios examined in our evaluation. Fig. 7 shows the results for update intervals of 256 and 1024. The results show that having more caches is not necessarily beneficial to either CS\textsubscript{FNA} or CS\textsubscript{FNO}. Intuitively, for CS\textsubscript{FNA}, having more caches with negative indications makes it harder to identify which of them are actually false-negative. Similarly, for both CS\textsubscript{FNA} and CS\textsubscript{FNO}, having more caches with false-positive indications makes it harder to reliably select a cache without a false-positive indication. However, CS\textsubscript{FNA} consistently outperforms CS\textsubscript{FNO}, and significantly more so as the update interval increases.

VI. CONCLUSIONS

This work studies the cache selection problem while using approximate indicators exhibiting both false-positive and false-negative errors. The client in such a system selects a subset of the caches to minimize the expected service cost. While there is extensive work in this field, all previous access strategies do not access caches with negative indications. While reasonable at first glance, our work shows that such an omission severely hinders the system performance. We argue that caches that periodically advertise their content indicators inherently introduce false-negative indications, and the rate of such indications is non-negligible. In particular, we show that it is sometimes advisable to access caches with a negative indication, as it may reduce the overall system cost.

We devise false-negative-aware access strategies in two main scenarios: (i) fully-homogeneous settings, where we show a policy that attains the optimal (minimal) access cost, and (ii) general heterogeneous environments, where we present a strategy for which we can bound its approximation guarantee compared to the optimal solution. We complete our study through an extensive evaluation based on real system traces. Our results show that our proposed methods perform significantly better than the state-of-the-art in diverse settings. Furthermore, our false-negative aware solutions can match the cost of competitive false-negative oblivious approaches while requiring an order of magnitude fewer resources (e.g., caching capacity or bandwidth required for indicators advertisement).

Our results demonstrate the potential benefits of embracing false-negative awareness into the algorithmic design space. We expect our work to further induce both analytic and experimental research on the role of false negatives in large distributed systems.

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