A little damping goes a long way: a simulation study of how damping influences task-level stability in running

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It is currently unclear if damping plays a functional role in legged locomotion, and simple models often do not include damping terms. We present a new model with a damping term that is isolated from other parameters: that is, the damping term can be adjusted without retuning other model parameters for nominal motion. We systematically compare how increased damping affects stability in the face of unexpected ground-height perturbations. Unlike most studies, we focus on task-level stability: instead of observing whether trajectories converge towards a nominal limit-cycle, we quantify the ability to avoid falls using a recently developed mathematical measure. This measure allows trajectories to be compared quantitatively instead of only being separated into a binary classification of ‘stable’ or ‘unstable’. Our simulation study shows that increased damping contributes significantly to task-level stability; however, this benefit quickly plateaus after only a small amount of damping. These results suggest that the low intrinsic damping values observed experimentally may have stability benefits and are not simply minimized for energetic reasons. All Python code and data needed to generate our results are available open source.

1. Introduction

Compliance is a defining characteristic of running motion [1], and models of running often include idealized springs. One of the most popular models is the spring-loaded inverted pendulum (SLIP) model [2]. Despite its simplicity, its parameters can be fit to closely emulate the ground reaction forces and centre-of-mass (CoM) kinematics of steady running in humans [3] and other animals [4]. Daley & Biewener [5] used the SLIP model to explain how well birds could run over an unexpected step-down perturbation. Guinea fowl were habituated to run over a level runway, after which a pothole camouflaged with tissue paper was introduced. Despite the unexpected perturbation, the birds did not stumble and fall. Because the duration of the step in the pothole was so short, the researchers concluded that the birds remained in open-loop control: they did not react or re-plan, but executed the step as if still running on level ground.

Even though damping effects are often observed in animal movement [6–8], they are only seldomly included in models. Birn-Jeffery \textit{et al.} [9] included damping in a modified SLIP model and found that this leads to more accurate predictions of...
ground reaction forces in running birds. Similar observations have also been made with other damped models [10].

Damping elements are sometimes added to SLIP-based models to study energy-injecting controllers [11–13]. These studies, however, focus on the stabilizing effects of the presented controllers and their potential applications in robotics. Shen & Seipel [14] studied the passive stability of a SLIP-like model with a viscous damper placed in parallel to the spring and driven by a constant hip-torque during stance to compensate for energy dissipation. They found that increasing damping and hip-torque tends to improve the stability of their model, though not at very small values of damping. This is likely owing to the unrealistic non-zero damping force at touch-down, which they addressed in a subsequent model [10]. Their study analyses the model’s open-loop stability, which may not capture important aspects of animal movement, as we discuss below.

Classical definitions of stability are based on the analysis of equilibrium points or limit-cycles and hinge on the notion of convergence. An equilibrium point (limit-cycle) is stable if nearby points (orbits) eventually converge towards it, and unstable if they diverge. Locomotion is often thought of as a limit-cycle, which allows a wealth of mathematical tools to be used for analysis [15–19]. However, Birn-Jeffery et al. [9] suggest that convergence may not be a task-level priority for running birds. Moore et al. [20] suggest that animals may even choose to prioritize unsteady motion to thwart be predators and also observe that bipedal Jerboas often switch between different gaits for the same speed. Humans are observed to show significant step-to-step variability even when moving at a fixed speed on a treadmill [21–23].

We define task-level stability as the ability to avoid falling. This definition of stability is less restrictive than definitions that require convergence and does not conflict with other task-level priorities. We show the influence that damping has on how robustly a model can maintain task-level stability. To quantify this robustness, we use a measure of viability, developed in Heim et al. [24] and presented below.

2. Materials and methods

(a) Measure of viability

Intuitively, the measure of viability quantifies how easy it is to avoid ever falling, which in our context corresponds to the body hitting the ground, i.e. falling.

A system is said to be in an unstable state if it is impossible to avoid falling within finite time, regardless of the control inputs chosen [25]. If the state is viable, on the other hand, there must exist at least one control input that takes the system to another viable state, and falling can be avoided forever by continuing to apply a viability-maintaining control input.

Because of this recursive property, viability can be evaluated on a single step, and heuristics such as steps-to-falling can be avoided [12,16]. Since viability only requires the ability to avoid falling, it does not impose strong requirements such as limit-cycle convergence [3,10,17,26].

The measure of viability is the n-dimensional volume of control inputs that keep the model viable. In our model, shown in figure 1, the leg angle-of-attack during flight is the only active control input; the measure is, therefore, the range of angles that keep the state viable. If the measure is zero, the state is not viable and the system will inevitably fall within a finite number of steps, no matter which angle is chosen. The greater the measure, the more robust the model is to imprecise control, regardless of whether this imprecision is owing to noisy motor-control, perturbations, or other causes [27–30]. Indeed, if the measure is too small, the model may not be able to robustly avoid falling, even if it is theoretically possible.

For a given system, we can pre-compute this measure for every state with an iterative algorithm. For nonlinear systems, the computational cost of this algorithm currently scales exponentially with the number of states and control inputs, and studies are therefore limited to low-dimensional models. The results presented here require roughly a full day of computation on a 24-core desktop. For mathematical and algorithmic details, see [24,31]. The Python code for computing this measure and all results in this paper is available in the electronic supplementary material [32] and online at github.com/sheim/vibly.

(b) Experiment

Inspired by the guinea fowl experiments of Daley and Biewener [5], we test for the ability to avoid falling after an unexpected ground-height perturbation. We start by finding a nominal limit-cycle to represent running over level ground. Then, we repeatedly simulate a single step from one flight apex to the next, keeping the damping coefficient and initial states fixed while sweeping through a range of ground-height perturbations. Since the ground-height perturbation is assumed to be unexpected, all control inputs are applied as if running on level ground. We then evaluate the pre-computed measure of viability at the apex state after the perturbed step and use this measure to compare different trajectories.

This battery of simulations is repeated for a range of damping values. Finally, we compare the viability measure after the perturbation, but with different amounts of damping.

(c) Model

We refer to our model, shown in figure 1, as the DASLIP model since it extends the SLIP model with a damper-actuator module.

As in the standard SLIP model, a point mass with coordinates (x, y) and mass m represents the body, and a massless leg with length ℓ represents the leg. During flight, the leg is held at a constant angle-of-attack α, and the motion of the joint mass is only affected by gravity. During stance, it is additionally affected by the leg force F_{leg}. The equations of motion of the CoM are given by:

\[ x = \frac{F_{leg}}{m} \sin \alpha \]

and

\[ y = \frac{F_{leg}}{m} \cos \alpha - mg. \]
In the SLIP model, the massless leg is composed of a spring with resting length \( \ell_0 \) and stiffness \( k \), such that the leg force is determined solely by the spring compression: 
\[
F_{\text{leg,SLIP}} = k(\ell - \ell_0).
\]
In our model, the system state is extended with the length \( \ell^A \) of the damper-actuator module. This module is placed in series with the spring, such that:
\[
F_{\text{leg}} = k(\ell - \ell^A - \ell_0).
\]
This module is composed of a time-dependent force source \( f(t) \) in parallel to a viscous damper with coefficient \( \beta \), such that the total force of the module, \( F_{\text{DA}} \), is given by:
\[
F_{\text{DA}} = f(t) - \beta \dot{\ell}^A.
\]
The dynamics of \( \ell^A \) are found by resolving the force balance between the module and the spring, \( F_{\text{DA}} = F_{\text{leg}} \):
\[
\dot{\ell}^A = \frac{f(t) - F_{\text{leg}}}{\beta}.
\]
We set the time-dependent force source \( f(t) \) such that the DASLIP model exactly follows the nominal limit-cycle of a SLIP model with the same initial states and parameters. This is achieved by matching \( f(t) \) to the force profile of the SLIP model. In the absence of perturbations, \( f(t) \) holds the module length \( \ell^A \) fixed, and the damper has no effect. When encountering a perturbation in ground-height \( h \), the spring force will no longer be matched by \( f(t) \), the module length \( \ell^A \) will change, and the damper will counteract this displacement.

The damping coefficient \( \beta \) must be greater than zero, but it can be set arbitrarily small. With infinitesimal damping, the total leg force will approach that of the open-loop actuator. By setting an infinitely high damping coefficient, the damper will resist all movement of the damper-actuator module, locking it in place. In this limit, the model behaves exactly like the SLIP model, also when perturbed.

While the SLIP model subsumes the entire leg behaviour into a massless spring, the DASLIP abstracts the muscles and tendons separately as the damper-actuator module and the spring, respectively. The damper-actuator module has two important properties of muscle: it develops little force unless activated and it has intrinsic damping [6]. We have modelled tendon as a massless spring, the DASLIP abstracts the muscles and tendons as a damper-actuator module in the leg, as an abstract representation of the forces produced by the tendons.

### Table 1. Parameters.

| name               | symbol | value  | normalized value |
|--------------------|--------|--------|------------------|
| mass               | \( m \) | 1.37 kg |                  |
| spring resting     | \( \ell_0 \) | 19.4 cm | 0.9 \( \ell_0 \) |
| length             |        |        |                  |
| spring stiffness   | \( k \) | 840.4 N m\(^{-1}\) | 13.6 mg/\( \ell_0 \) |
| landing stiffness  | \( \ell_0 \) | 3.42 cm | 0.1 \( \ell_0 \) |
| attack             | \( \alpha \) | 34.1° |                  |
| states             |        |        |                  |
| height at apex     | \( y_0 \) | 19.6 cm | 0.9 \( \ell_0 \) |
| velocity at apex   | \( x_0 \) | 2.7 m s\(^{-1}\) | 12.3 \( \ell_0 \) |
| damper-actuator    | \( \ell^A \) | 2.2 cm | 0.1 \( \ell_0 \) |
| module length      |        |        |                  |

3. Results

In figure 2, we visualize the simulated trajectories for two specific values of damping: in (i) \( \beta = 0.498k/\ell_0/g = 58.8 \) N s\(^{-1}\) and in (ii) \( \beta = 0.007k/\ell_0/g = 0.8 \) N s\(^{-1}\). In figure 2c, we visualize the viability measure for all simulations: each line corresponds to a battery of ground-height perturbations for a specific damping value \( \beta \).

At the right edge of figure 2c, the apparent cliff in viability measure is owing to the model stumbling on the raised ground. On the left side, the maximum step-down perturbation from which recovery is possible is smaller for lower damping values. More importantly, the overall viability measure is low, even for the nominal limit-cycle. As the damping value is increased from 0.007 to 0.498k/\( \ell_0/g \), the range of recoverable step-down perturbations roughly doubles from 1% \( \ell_0 = 2.0 \) cm to 21% \( \ell_0 = 4.5 \) cm. More importantly, the viability measure also increases substantially, for the nominal limit-cycle trajectory, from 9.4° to 19.5°.

Further increasing damping continues to increase the largest step-down perturbation. However, the increase in viability measure slows down rapidly. Indeed, when increasing the damping value up to 1.35 k/\( \ell_0/g = 159.7 \) N s\(^{-1}\), the maximum viability measure saturates at 18°, and the range of perturbations that reach this maximum, the ‘plateau’ in figure 2c, only increases marginally.

4. Discussion

The DASLIP model extends the classical SLIP model with a damper-actuator module in the leg, as an abstract representation of the forces produced by the tendons.
end of the step, we also visualize the viability-maintaining control inputs for the nominal limit-cycle, colourized according to the viability measure. In (a), the height for perturbations is also coloured starting from the point-mass position at touch-down until take-off. For clarity, unviable trajectories are not visualized. At the end of the step, we also visualize the viability-maintaining control inputs for the nominal limit-cycle, colourized according to the viability measure. In (c) we visualize the viability measure (vertical axis) at the apex reached after each ground-height perturbation (horizontal axis), where each line corresponds to a specific damping value $\beta$.

Figure 2. (a,b) The trajectories for two specific damping values, over a range of ground-height perturbations: (a) 0.498k $\sqrt{\ell_0/g}$ and (b) 0.007k $\sqrt{\ell_0/g}$. The limit-cycle trajectories over level ground are coloured in black. Trajectories for step-up and step-down perturbations are coloured blue and red, respectively. The ground-height for perturbations is also coloured starting from the point-mass position at touch-down until take-off. For clarity, unviable trajectories are not visualized. At the end of the step, we also visualize the viability-maintaining control inputs for the nominal limit-cycle, colourized according to the viability measure. In (c) we visualize the viability measure (vertical axis) at the apex reached after each ground-height perturbation (horizontal axis), where each line corresponds to a specific damping value $\beta$.

representation of muscle activation and damping. The damping coefficient allows us to smoothly blend between two different leg models: a feed-forward force source when damping tends to zero and an idealized spring when damping tends to infinity. For limit-cycle running on level ground, these two models (and all combinations) behave in the same way owing to the choice of the time-dependent force source $f(t)$.

Faced with unexpected ground-height perturbations, our model showed poor robustness when minimally damped. Increasing the damping coefficient increases both the viability measure and the range of perturbations that can be negotiated. However, the maximum viability measure quickly reaches a plateau, and further increasing damping only widens this plateau marginally. In other words, the benefit in robustness owing to increasing damping is limited.

Robustness is important because motor-control is inherently noisy and imprecise [27–30]. In studies of running guinea fowl and pheasants [35,36], the birds exhibited notable variability in their control of the landing angle-of-attack, with the standard deviation typically ranging between 3° and 8° depending on the presence and size of (visible) obstacles. In the context of our results, this would suggest that only the upper half of figure 2c represents perturbations that can be negotiated robustly: wherever the measure is small compared to the control input variability, recovery is possible, but not robust.

We believe a quantitative study to identify the minimum threshold for the viability measure (how ‘far down the mountain’ are animals willing to go?) would be of great interest, albeit challenging. In particular, it is not trivial to assess if motor-control variability is owing to a limit in capability, or simply because greater precision is not needed for a given task. Such a study would provide a further tool with which to compare task-level priorities [35] and, in particular, to study behaviour involving risk [37,38].

Damping is not free of cost, since it will also reduce muscle efficiency. In our model, the plateau in improved robustness can be reached with only a small cost in energy efficiency, which would likely be negligible for most animals. It would be interesting to study if this trade-off becomes relevant in specialized animals. In particular, if task-level stability is not a concern, but there is a strong evolutionary pressure to optimize efficiency, it may still be beneficial to further minimize damping. This may be the case in desert habitats, where the environment is relatively flat and does not require agility [39], and where falls do not determine predator–prey interactions [20,40].

These results may also be informative for robot design. There are few examples of legged robots that incorporate physical dampers in their design [41–43]. These efforts have, however, not focused on task-level stability, and the potential benefit of damping in this context remains to be explored.

Ethics. The locomotion data from running birds (Numida meleagris) underlying this study was previously published in [35], under the appropriate ethical conduct and kindly provided to us by one of the authors (Monica Daley). No additional experiments were conducted here, hence no ethical approvals or permits were required in this study. A project number is not applicable. [1]: doi:10.1371/journal.pone.0100399.

Data accessibility. All data and code for this submission are available online at https://github.com/sheim/vibly. A slimmed-down version of the repository (including only code and data relevant to the submission) has been uploaded to Dryad for permanence: https://doi.org/10.5061/dryad.44j0zpcbj.

Authors’ contributions. The study was conceived and designed by S.H. and M.M., with support from C.L.M. The DASLIP model was designed and implemented by M.M. The bird-data was processed and fit by M.M. and C.L.M. The experiment was implemented, carried out, and resulting data were processed by S.H. Analysis and interpretation of these results were done jointly by S.H., M.M., C.L.M. and A.B.-S. The manuscript was written by S.H. with support from all authors. All authors gave final approval for publication and agree to be held accountable for the work performed therein.

Competing interests. We declare we have no competing interests.

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