Neutrino Oscillations from String Theory

A. Halprin\textsuperscript{(a)} and C. N. Leung\textsuperscript{(a),(b)}

\textsuperscript{(a)} Department of Physics and Astronomy, University of Delaware
Newark, DE 19716

\textsuperscript{(b)} Institute of Physics, Academia Sinica, Taipei, Taiwan

Abstract

We derive the character of neutrino oscillations that results from a model of equivalence principle violation suggested recently by Damour and Polyakov as a plausible consequence of string theory. In this model neutrino oscillations will take place through interaction with a long range scalar field of gravitational origin even if the neutrinos are degenerate in mass. The energy dependence of the oscillation length is identical to that in the conventional mass mixing mechanism. This possibility further highlights the independence of and need for more exacting direct neutrino mass measurements together with a next generation of neutrinoless double beta decay experiments.
Damour and Polyakov [1] have argued that string theory may very well lead to a violation of the equivalence principle through interactions of the string dilaton field which may be massless. They have shown that the resultant effective theory of gravity is a variant of Brans-Dicke [2] scalar-tensor type and leads to the following two-particle static gravitational potential energy,

\[ V(r) = -G_N m_A m_B (1 + \alpha_A \alpha_B) / r, \quad (1) \]

where \( G_N \) is Newton’s gravitational constant. For vanishing \( \alpha_j \) the interaction energy is the usual universal spin-2 exchange contribution, while the \( \alpha \) dependent piece arises from spin-0 exchange. The remarkable features of the Damour-Polyakov (DP) scenario are that the spin-0 field remains massless and that the \( \alpha_j \) are species dependent. It is the species dependence that violates the equivalence principle.

An interaction lagrangian density that gives rise to this spin-0 exchange contribution to the static gravitational energy is, of course, given by

\[ \mathcal{L} = m_j \alpha_j \bar{\psi}_j \psi_j \phi, \quad (2) \]

where \( \psi_j \) is a matter field of type \( j \). The dilaton field, \( \phi \), is coupled to the trace of the energy-momentum tensor through

\[ g_{\mu\nu} \partial^\mu \partial^\nu \phi = -4\pi G_N \alpha_l g_{\mu\nu} T^\mu_\nu_l, \quad (3) \]

where \( T^\mu_\nu_l \) stands for the energy-momentum tensor for \( l \) type matter. This interaction embraces the DP scenario up to higher derivatives of the gravitational fields and is sufficient for our purposes.

If gravity is treated classically (i.e., as a static background) and linearized (i.e., treated as a weak perturbation), the evolution of a fermion in an external gravitational field will be governed by a Dirac equation with an effective mass \( m^* \) given by

\[ m^* = m - m \alpha \phi_c. \quad (4) \]
The classical value of the dilaton field, $\phi_c$, is characterized by the $\alpha$ value of the bulk matter producing it and, for a static matter distribution, is proportional to the Newtonian potential, $\Phi_N$, viz.

$$\phi_c = \alpha \Phi_N.$$  \hfill (5)

There is also a modification to the metric in the Dirac equation due to the spin-2 gravitational field. But unlike recent alternative considerations [3–12], the tensorial gravity is universal in the DP approach. It will therefore play no role in the neutrino mixing phenomenology of interest here and so we dispense with it.

The interaction above can easily be applied to the case of two neutrino mixing (e.g., $\nu_e$ and $\nu_\mu$) by replacing $m$ and $m_\alpha$ by $2 \times 2$ matrices, $M$ and $M_\alpha$, respectively. Let us call the eigenstates of $M$ the mass eigenstates and those of $M_\alpha$ the gravitational eigenstates.

To illustrate the possible outcome of the DP scenario in neutrino physics, we consider here the special case in which the mass and gravitational eigenstates are identical and shall be referred to as the $m^*$-eigenstates. (The more general case in which the mass eigenstates are distinct from the gravitational eigenstates is similar to the situation discussed in Section II.C of Ref. [3] and will be dealt with elsewhere.) Neutrino flavor oscillations will therefore take place if the $m^*$-eigenstates differ from the neutrino flavor eigenstates and if the $m^*$-eigenvalues are not degenerate. In this case the evolution equations governing the oscillation phenomenon of relativistic neutrinos are given in the flavor basis by

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^*}{4E} \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix},$$  \hfill (6)

where $\theta$ is a mixing angle and

$$\Delta m^* \equiv m_2^2(1 - \alpha_2 \phi_c)^2 - m_1^2(1 - \alpha_1 \phi_c)^2 \
\simeq \Delta m^2 - 2\phi_c (m_2^2\alpha_2 - m_1^2\alpha_1).$$  \hfill (7)

Here $\Delta m^2 \equiv m_2^2 - m_1^2$ denotes the difference in neutrino vacuum masses (squared) and only terms up to first order in $\phi_c$ are kept in the above approximation for $\Delta m^*$. If the vacuum
mass squared difference dominates $\Delta m^2$, a violation of the equivalence principle (VEP) will not be observed in neutrino oscillations. On the other hand, even if the neutrinos are completely degenerate but not massless, the VEP term will still produce oscillations, and in that case

$$\Delta m^2 \simeq -2m^2\alpha_{\text{ext}}\Phi_N\Delta\alpha, \quad (8)$$

where $m$ is the degenerate neutrino mass and $\Delta\alpha \equiv \alpha_2 - \alpha_1$ is the difference between the $\alpha$ values of the two neutrino species.

While the physics of the usual spin-2 gravitational field is not dependent upon the absolute value of $\Phi_N$, we see that the same is not true for the scalar contribution. Anywhere in our solar system, the dominant contribution to the local gravitational potential appears to come from the great attractor which is about $3 \times 10^{-5}$ [13,14]. For earthbound experiments, we can regard $\Phi_N$ as essentially constant. In this case, the survival probability for an electron neutrino that has travelled a distance $L$ is given by

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta) \sin^2\left(\frac{L\Delta m^2}{4E}\right). \quad (9)$$

This is analogous to the situation when flavor oscillations of neutrinos are caused by their vacuum mass differences. Consequently, $\Delta m^2$ is subject to the same constraints derived for $\Delta m^2$ in the mass mixing mechanism. For instance, according to the analyses in Ref. [13], the solar neutrino data constrain $\Delta m^2$ to be in the range

$$4 \times 10^{-6}\text{eV}^2 < |\Delta m^2| < 10^{-4}\text{eV}^2 \quad (10)$$

if the MSW transitions [16,17] are assumed; and in the range

$$5 \times 10^{-11}\text{eV}^2 < |\Delta m^2| < 10^{-10}\text{eV}^2 \quad (11)$$

if one assumes vacuum transitions.

To see if the dilaton-induced VEP can have anything to do with the solar neutrino deficit, we use the Newtonian potential due to the great attractor and the limit on $\alpha_{\text{ext}}$ coming from
solar-system gravitational experiments, \(\alpha_{\text{ext}}^2 < 10^{-3}\). Since electron neutrinos are necessarily involved here, we use the 10 eV limit on its mass as the degenerate value. These numbers illustrate that the VEP mechanism considered here is unlikely to contribute to the solar neutrino deficit if the flavor transitions are through the MSW effect. On the other hand, if the transitions occur in vacuo, current solar neutrino data probe the string theory violation of the equivalence principle at the level of

\[
2 \times 10^{-7} < |\Delta \alpha| < 5 \times 10^{-5}.
\]

Although not as good as the most restrictive limit for ordinary matter (which may have little to do with neutrinos), this is better than the limits on neutrinos obtained from SN1987A. The above limit should not be compared with those obtained in Refs. [8] and [9], since the equivalence principle violation considered there arises from the tensorial gravitational couplings whereas the source of VEP here resides in the couplings to the string dilaton.

Turning to other neutrino processes, we note that we have considered the DP scenario only for the case in which the neutrino vacuum mass terms are of the Dirac type. It follows that the dilatonic contribution to the effective mass is also of the Dirac type, so that it will not generate neutrinoless double beta decays. In the more general case in which the vacuum mass terms are of the Majorana plus Dirac type, neutrinoless double beta decays as well as neutrino-antineutrino oscillations become possible. We plan to study the richer phenomenology for this case in a future communication. Finally, it should be noted that beta-decay spectrum end-point measurements will see the mass \(m^*\), which in this scenario could be of order eV rather than the scale of \(\Delta m^*\).

We therefore conclude there is the distinct possibility that the solar neutrino deficit may be telling us about a nonuniversal scalar gravitational interaction rather than the existence of a neutrino mass difference. This oscillation mechanism is phenomenologically distinguished from the conventional mass mixing mechanism by providing a rationale for the possibility that effective neutrino mass differences pertinent to solar neutrinos are small while true
neutrino masses are orders of magnitude larger - with degeneracy protected by a family symmetry. This adds yet another contender to the list of alternatives [28,29] to a neutrino mass difference and emphasizes further the independence of and need for more exacting direct measurements of neutrino masses as well as the effective mass arising in neutrinoless double beta decays. Our study here shows that these neutrino experiments are important not only for studying the physical properties of neutrinos but also as a means to test the low energy phenomenology of string theory.

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