Mass and Spin Measurement with $M_{T2}$ and MAOS Momentum

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Outline

1 Motivation: New Physics Events with Missing Transverse Momentum

2 $M_{T2}$-Kink Method for Mass Measurement

3 $M_{T2}$-Assisted On-Shell (MAOS) Momentum and Its Application to Spin Measurement

4 Conclusion
Motivations for new physics at the TeV scale:

- Hierarchy Problem

\[
\delta m_H^2 \sim \frac{g^2}{8\pi^2} \Lambda_{SM}^2 \sim M_Z^2 \quad \implies \quad \Lambda_{SM} \sim 1 \text{ TeV}
\]

- Dark Matter

Thermal WIMP with

\[
\Omega_{DM} h^2 \sim \frac{0.1}{g^4} \left( \frac{m_{DM}}{1 \text{ TeV}} \right)^2 \sim 0.1
\]

\[\implies m_{DM} \sim 1 \text{ TeV} \]

Many new physics models solving the hierarchy problem while providing a DM candidate involve a $Z_2$-parity symmetry under which the new particles are $Z_2$-odd, while the SM particles are $Z_2$-even:

**SUSY with $R$-parity, Little Higgs with $T$-parity, UED with $KK$-parity, ...**

* At colliders, new particles are produced always in pairs.
* Lightest new particle is stable, so a good candidate for WIMP DM.
LHC Signal

Pair-produced new particles \((Y + \bar{Y})\) decaying into visible SM particles \((V)\) plus invisible WIMPs \((\chi)\):

\[
pp \rightarrow Y + \bar{Y} \rightarrow \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)
\]

\((\text{multi-jets} + \text{leptons} + p_{T})\)

\(U \equiv \text{Upstream momenta carried by the visible SM particles not from the decay of } Y + \bar{Y} \quad (\bar{Y} \text{ is not necessarily the antiparticle of } Y.)\)
Mass measurement of these new particles is quite challenging:
  * Initial parton momenta in the beam-direction are unknown.
  * Each event involves two missing WIMPs.

**Kinematic methods of mass measurement:**
  i) Endpoint Method
  ii) Mass Relation Method
  iii) $M_{T2}$-Kink Method

Spin measurement appears to be even more difficult:
  * It often requires a more refined event reconstruction and/or polarized mother particle state.

**MAOS momentum** provides a controllable approximation for the unmeasurable WIMP momentum, which can be useful for spin measurement in some case.
Mass Measurement Methods

Endpoint Method  Hinchliffe et. al.; Allanach et. al.; Gjelsten et. al.; ...

Endpoint value of the invariant mass distribution of visible (SM) decay products depend on the new particle masses.

* 3-step squark cascade decay when $m_{\tilde{q}} > m_{\chi_2} > m_{\tilde{\ell}} > m_{\chi_1}$

\[
\begin{align*}
    m_{\ell\ell}^{\text{max}} &= m_{\chi_2} \sqrt{(1 - m_{\ell}^2/m_{\chi_2}^2)(1 - m_{\chi_1}^2/m_{\tilde{\ell}}^2)} \\
    m_{q\ell\ell}^{\text{max}} &= m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2/m_{\tilde{q}}^2)(1 - m_{\chi_1}^2/m_{\chi_2}^2)} \\
    m_{q\ell}^{\text{max (high)}} &= m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2/m_{\tilde{q}}^2)(1 - m_{\chi_1}^2/m_{\tilde{\ell}}^2)} \\
    m_{q\ell}^{\text{max (low)}} &= m_{\tilde{q}} \sqrt{(1 - m_{\chi_2}^2/m_{\tilde{q}}^2)(1 - m_{\tilde{\ell}}^2/m_{\chi_2}^2)}
\end{align*}
\]
Input masses: \((m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (540, 177, 143, 96)\) GeV

Fitted masses: \((543 \pm 13, 180 \pm 9, 146 \pm 11, 98 \pm 9)\) \((\int \mathcal{L} = 100 \text{ fb}^{-1})\)
Reconstruct the missing momentum with on-shell constraints.

* A pair of symmetric 3-step cascade decays of squark pair

- 16 unknowns: $k^\mu, l^\mu, k'^\mu, l'^\mu$
- 12 mass-shell constraints:
  \begin{align*}
  k^2 &= l^2 = k'^2 = l'^2, \\
  (k + p_3)^2 &= (l + q_3)^2 = (k' + p'_3)^2 = (l' + q'_3)^2, \\
  (k + p_2 + p_3)^2 &= (l + q_2 + q_3)^2 = (k' + p'_2 + p'_3)^2 = (l' + q'_2 + q'_3)^2, \\
  (k + p_1 + p_2 + p_3)^2 &= (l + q_1 + q_2 + q_3)^2 = (k' + p'_1 + p'_2 + p'_3)^2 = (l' + q'_1 + q'_2 + q'_3)^2,
  \end{align*}

- 4 $p_T$-constraints: $k_T + l_T = p_T$, \quad $k'_T + l'_T = p'_T$
* 8 (complex) solutions for each event-pair, some of which are real.
* Many wrong solutions from wrong combinatorics.

For given set of event-pairs, number of real solutions shows a peak at the correct masses: Cheng et. al.

![Graph showing mass versus solutions per GeV]

Input masses: \((m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (568, 180, 143, 97) \text{ GeV}\)
Fitted masses: \((562 \pm 4, 179 \pm 3, 139 \pm 3, 94 \pm 3) \quad (\int L = 300 \text{ fb}^{-1})\)
Remarks

- Mass relation method and endpoint method require a long cascade decay, at least 3-steps, to determine the involved new particle masses.

- However, there are many cases (including a large fraction of popular scenarios) that such a long cascade decay is not available:
  
  A simple example: mSUGRA with $m_0^2 > 0.6 M_{1/2}^2 \Rightarrow m_{\tilde{\ell}} > m_{\chi_2}$

- SUSY with $m_{\text{sfermion}} \gg m_{\text{gaugino}}$:
  (Focus point scenario, Loop-split SUSY, Some string moduli-mediation, ...)

  * Mass relation method simply can not be applied.
  * Endpoint method determines only the gaugino mass differences.
  * $M_{T2}$-kink method can determine the full gaugino mass spectrum.
**MT2-Kink Method** Cho, Choi, Kim, Park; Barr, Gripaios, Lester

$M_{T2}$ is a generalization of the transverse mass to an event with two missing particles.

**Transverse mass of $Y \rightarrow V(p) + \chi(k)$:**

\[
M_T^2 = m_V^2 + m_\chi^2 + 2 \sqrt{m_V^2 + |p_T|^2} \sqrt{m_\chi^2 + |k_T|^2} - 2p_T \cdot k_T
\]

One can use an arbitrary trial WIMP mass $m_\chi$ to define $M_T$. (True WIMP mass $= m_\chi^{\text{true}}$).

* For each event, $M_T$ is an increasing function of $m_\chi$.
* For all events, $M_T(m_\chi = m_\chi^{\text{true}}) \leq m_Y^{\text{true}}$ in the zero width limit.
$M_{T2}$ Lester and Summers

\[ M_{T2}(\text{event}; m_\chi) = \min_{k_T + l_T = p_T} \left[ \max \left( M_T(p_T, m_{V_1}, k_T, m_\chi), M_T(q_T, m_{V_2}, l_T, m_\chi) \right) \right] \]

* For each event, $M_{T2}$ is an increasing function of $m_\chi$.
* For all events, $M_{T2}(m_\chi = m_\chi^{\text{true}}) \leq m_Y^{\text{true}}$ in the zero width limit.
**$M_{T2}$-Kink**

If the event set has an enough variety,

\[
M_{T2}^{\text{max}}(m_\chi) = \max_{\text{all events}} \left[ M_{T2}(\text{event}; m_\chi) \right]
\]

has a kink-structure at $m_\chi = m_\chi^{\text{true}}$ with $M_{T2}^{\text{max}}(m_\chi = m_\chi^{\text{true}}) = m_Y^{\text{true}}$.
What kind of variety?

- The visible decay products of $Y \rightarrow V + \chi$ have a wide range of invariant mass, which would be the case when $V$ is a multi-particle state. Cho, Choi, Kim, Park

- There are events with large upstream transverse momentum, which would be the case when $Y$ is produced with a large ISR or produced through the decay of heavier particle. Gripaios; Barr, Gripaios, Lester

For cascade decays, $M_{T2}$-kink method can be applied to generic sub-event:
Gluino $M_{T2}$-Kink in heavy sfermion scenario

$M_{T2}$ of hard 4-jets (no $b$, no $\ell$) which are mostly generated by the gluino-pair 3-body decay: $\tilde{g}\tilde{g} \rightarrow q\bar{q}\chi q\bar{q}\chi$, where $m_{\tilde{g}} \lesssim 1$ TeV and $m_{\tilde{q}} \sim$ few TeV.

Input masses: $(m_{\tilde{g}}, m_{\chi_1}) = (780$ GeV, $98$ GeV) (Wino-like $\chi_1$)

Fitted masses: $(776 \pm$ few, $97 \pm$ few) ) ($\int \mathcal{L} = 300$ fb$^{-1}$)
\textbf{$M_{T2}$-kink for mixed or inclusive event set:}

In some cases, it is difficult to select an exclusive event set:

multi-jet events (no $b$, no $\ell$) generated by

\[ \tilde{g}\tilde{g} \rightarrow q\bar{q}\chi q\bar{q}\chi, \quad \tilde{q}\tilde{g} \rightarrow qq\bar{q}\chi q\bar{q}\chi, \quad \tilde{q}\tilde{q}^* \rightarrow qq\bar{q}\chi q\bar{q}\chi, \]

where $m_{\tilde{q}_{L,R}} = 820, 800 \text{ GeV}$, $m_{\tilde{g}} = 740 \text{ GeV}$, $m_{\chi_1} = 120 \text{ GeV}$.

$M_{T2}^\text{max}$ of the hardest 4-jets (hemi-sphere method for combinatorics) still shows a kink at $m_{\chi} = m_{\chi}^\text{true}$ with $m_{\tilde{g}} < M_{T2}^\text{max}(m_{\chi}^\text{true}) < m_{\tilde{q}}$: 

\begin{tabular}{l|l}
$\chi^2$ / ndf & 11.4 / 10 \\
p0 & 906.8 ± 12.7 \\
p1 & 123.8 ± 2.006 \\
p2 & 0.8703 ± 0.0129 \\
\end{tabular}
$M_{T2}$-Assisted-On-Shell (MAOS) Momentum

MAOS momentum is a collider variable which approximates systematically the missing WIMP momenta in generic new physics events producing a pair of WIMPs.
Construction of the MAOS WIMP momenta $\tilde{p}^\mu$ and $\tilde{l}^\mu$

i) Choose appropriate trial WIMP and mother particle masses: $m_\chi$, $m_Y$.

ii) Determine the transverse MAOS momenta with $M_{T2}$:

$M_{T2}$ selects unique $\tilde{k}_T$ and $\tilde{l}_T$ under the constraint $\tilde{k}_T + \tilde{l}_T = \tilde{p}_T$.

Barr, Lester, Stephens

iii) Determine the longitudinal components with the on-shell constraints:

$$\tilde{k}^2 = \tilde{l}^2 = m_\chi^2, \quad (\tilde{k} + p)^2 = (\tilde{l} + q)^2 = m_Y^2$$
\[ \Rightarrow \quad \tilde{k}_z^\pm = \frac{1}{p^2 + |\mathbf{p}_T|^2} \left( A p_z \pm p_0 \sqrt{A^2 - (p^2 + |\mathbf{p}_T|^2)(m_\chi^2 + |\tilde{k}_T|^2)} \right) \]

\[ \tilde{l}_z^\pm = \frac{1}{q^2 + |\mathbf{q}_T|^2} \left( B q_z \pm q_0 \sqrt{B^2 - (q^2 + |\mathbf{q}_T|^2)(m_\chi^2 + |\tilde{l}_T|^2)} \right) \]

\[ A = \frac{(m_Y^2 - m_\chi^2 - p^2)/2 + \mathbf{p}_T \cdot \tilde{k}_T}{2} \]

\[ B = \frac{(m_Y^2 - m_\chi^2 - q^2)/2 + \mathbf{q}_T \cdot \tilde{l}_T}{2} \]
Some features of MAOS momentum:

- For each event, MAOS momenta are real iff \( m_Y \geq M_{T2}(\text{event}; m_\chi) \)

\[ \implies \text{MAOS momenta are real for all events if} \]

\[ m_Y \geq M_{T2}^{\text{max}}(m_\chi) \equiv \max_{\{\text{events}\}} \left[ M_{T2}(\text{event}; m_\chi) \right] \left( m_Y^{\text{true}} = M_{T2}^{\text{max}}(m_Y^{\text{true}}) \right) \]

* If \( m_\chi^{\text{true}} \) and \( m_Y^{\text{true}} \) are known, use \( m_\chi = m_\chi^{\text{true}} \) and \( m_Y = m_Y^{\text{true}} \).

* Unless, one can use \( m_\chi = 0 \) and \( m_Y = M_{T2}^{\text{max}}(0) \).

- \( k_\text{maos}^\mu = k_\text{true}^\mu \) for the \( M_{T2} \) endpoint events constructed with \( m_\chi = m_\chi^{\text{true}} \) and \( m_Y = m_Y^{\text{true}} \).

\[ \implies \text{One can systematically reduce} \left( \frac{\Delta k}{k} \right)_{m_\chi^{\text{true}}, m_Y^{\text{true}}} - \left( \frac{\Delta k}{k} \right)_{m_Y = M_{T2}^{\text{max}}(0)} = \mathcal{O} \left( \frac{m_\chi^{\text{true}2}}{m_Y^{\text{true}2}} \right) \]

- Precise knowledge of \( m_\chi^{\text{true}} \) and \( m_Y^{\text{true}} \) is not essential:
\[
\frac{\Delta k_T}{k_T} = \frac{\tilde{k}_T - k_{true}^{\text{true}}}{k_{true}^{\text{true}}} \quad \text{distribution for } \tilde{q}\tilde{q}^* \rightarrow q\chi\bar{q}\chi:
\]

\[
\tilde{k}_T = \frac{1}{2} p_T \quad (\tilde{k}_T + \tilde{l}_T = p_T)
\]

\[
\tilde{k}_T = k_T^{\text{maos}} \quad \text{for full events}
\]

\[
\tilde{k}_T = k_T^{\text{maos}} \quad \text{for the top 10 \% of near endpoint events}
\]
Example 1: Gluino/KK-gluon 3-body decay for SPS2 point and its UED equivalent:

\[ s = (p_q + p_{\bar{q}})^2, \quad t_{\text{true}} = (p_{q,\bar{q}} + k_{\text{true}})^2, \quad t_{\text{maos}} = (p_{q,\bar{q}} + k_{\text{maos}}^\pm)^2 \]

Without \( k_{\text{maos}}^\mu \), one may consider the s-distribution to distinguish gluino from KK-gluon: Csaki, Heinonen, Perelstein
With $k_\mu^{\text{maos}}$, one can use the $s-t_{\text{maos}}$ distribution:

$$\frac{d\Gamma}{d\sigma d\sigma_{\text{true}}} \quad \quad \quad \frac{d\Gamma}{d\sigma d\sigma_{\text{maos}}}$$

- **Gluino 3-body decay**
- **KK-gluon 3-body decay**
Employ appropriate event cut with hemi-sphere method to deal with combinatoric errors, and include detector smearing effects.

For universal gaugino mass scenario at $M_{\text{GUT}} (\ni \text{SPS2})$, which gives $m_{\tilde{g}} / m_\chi \simeq 6$, the s-distribution ($d\Gamma / ds$) can not distinguish SUSY from UED even with $\mathcal{L} = 300 \text{ fb}^{-1}$.

On the other hand, \[ \frac{d\Gamma}{dsdt_{\text{maos}}} \] can clearly discriminate SUSY from UED.
Example 2: Drell-Yan pair production of **slepton** or **KK-lepton** for SUSY SPS1a point and its UED equivalent:

\[
\frac{d\Gamma}{d \cos \theta_Y} \quad \text{and} \quad \frac{d\Gamma}{d \cos \theta_\ell} \quad \text{of} \quad q\bar{q} \rightarrow Z^0/\gamma \rightarrow Y\bar{Y} \rightarrow \ell\chi\bar{\ell}\chi
\]

\[Y = \text{slepton or KK-lepton}, \quad \chi = \text{LSP or KK-photon},\]

\[\cos \theta_Y = \hat{p}_Y \cdot \hat{p}_{\text{beam}} \quad \text{in the CM frame of} \quad Y\bar{Y},\]

\[\cos \theta_\ell = \hat{p}_\ell \cdot \hat{p}_{\text{beam}} \quad \text{in the CR(rapidity) frame of} \quad \ell\bar{\ell}\]
Without MAOS, one may look at the lepton angle \( \cos \theta_\ell \) distribution to distinguish the slepton pair production from the KK-lepton pair production: Barr

With MAOS momentum, the mother particle production angle \( \cos \theta_Y \) can be reconstructed: Cho, Choi, Kim, Park

\[
Y(p \pm k_{\text{maos}}) \bar{Y}(q \pm l_{\text{maos}}) \rightarrow \ell(p) \chi(k_{\text{maos}}) \bar{\ell}(q) \chi(l_{\text{maos}})
\]

\[
\frac{d\Gamma}{d \cos \theta_Y^{\text{maos}}} \equiv \sum_{\alpha=\pm} \sum_{\beta=\pm} \frac{d\Gamma}{d \cos \theta_{\alpha\beta}}
\]

\[
( \cos \theta_{\pm\pm} = \hat{p}_Y \cdot \hat{p}_{\text{beam}} \text{ for } k_{\text{maos}} \text{ and } l_{\text{maos}} )
\]
\[ \frac{d\Gamma}{d \cos \theta_\ell} \quad \text{vs} \quad \frac{d\Gamma}{d \cos \theta_Y^{\text{maos}}} \]

with appropriate event cut (\( \exists \) the \( M_{T2} \)-cut selecting the top 30 \%) while including the detector smearing effect for SUSY SPS1a and its UED equivalent: (Knowledge of the mass is not essential.)
Summary

- $M_{T2}$-kink method might be able to determine new particle masses (with a good accuracy) even when a long cascade decay is not available.

$M_{T2}$-kink for appropriate (inclusive) multi-jet events with missing $E_T$ could be a good starting point to determine the missing WIMP mass.

- MAOS momentum provides a controllable approximation for the unmeasurable WIMP momentum, which can be quite useful for spin measurement in some case.

- MAOS momentum can be useful also for some SM process with two missing neutrinos (works in progress):

  * $\bar{t}t \rightarrow bW^+\bar{b}W^- \rightarrow b\ell^+\nu\bar{b}\ell\bar{\nu}$
  * $h \rightarrow W^+W^- \rightarrow \ell^+\nu\ell\bar{\nu}$