Modeling and analysis of the friction in a non-linear sliding-mode triboelectric energy harvester

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Friction plays a pivotal role in the sliding-mode triboelectric energy harvester (TEH), which not only enables the charge transfer between two dielectrics, but also influences the energy harvesting performance by affecting the dynamic response of the TEH. How to evaluate the effects of the friction on TEHs is important for optimizing TEHs in engineering practices. In order to analyze the effects of the friction on the dynamic response and evaluate the energy harvesting performance of TEHs, the paper models the friction of a devised non-linear TEH based on the Coulomb friction model and the Macro-slip friction theory. The TEH equips a pair of magnets, rendering a switching between the bistability and the monostability by tuning the distance between two magnets. The dynamic model of the non-linear TEH is established by the extended Hamilton principle. The effects of friction in sliding-mode TEH are dissected in detail. The influences of parameters on both the mechanical and electrical responses are also systematically studied to explore an optimal energy harvesting performance in the low-frequency range. This work provides a guideline for designing and accurately analyzing a sliding-mode TEH.

Triboelectric energy harvester, Magnetic bistability, Macro-slip friction model, Coulomb friction model, Nonlinearity

1. Introduction

With the rapid growth of fifth-generation (5G) wireless communication globally [1,2], the application of tiny components and miniature sensors has become widely. It is becoming an urgent topic among researchers and engineers how to power the huge sensor network. Compared with traditional batteries, harvesting ambient energy (such as friction, mechanical vibration, water wave and human motion) has a limitless foreground of its huge reserves and pollution-free property. Owing to the dependence between the dynamical response and the energy harvesting performance, the essential issue becomes how to design a novel configuration to enhance the efficiency of converting ambient energy to electrical energy.

Ordinarily, the possible mechanisms for harvesting ambient energy include electromagnetic mechanism [3], piezoelectric mechanism [4,5], pyroelectric mechanism [6,7] and electrostatic mechanism [8,9]. The fundamental principle of the electromagnetic energy harvester can be boiled down to the law of electromagnetic induction. In terms of the piezoelectric energy harvester, the piezoelectric patches can harvest electric energy when they are extruded with mechanical strain [10]. It should be pointed out that the piezoelectric energy harvester possesses higher energy conversion efficiency at the high-frequency range. Unlike the electromagnetic and the piezoelectric energy harvester, the pyroelectric energy harvester achieves energy conversion by temperature fluctuation [11].

As an additional energy harvesting mechanism, electrostatic induction occurs in two triboelectric materials when they have opposite polarity [8]. In this state, positive and negative charges are on the two different materials. When the two triboelectric materials slide each other, charges transfer...
from one triboelectric material to another one, causing an electric current. In 2012, Fan et al. [12] proposed a triboelectric energy harvester (TEH) based on electrostatic induction. After that, a large number of scholars have been attracted by the TEHs due to their easy manufacture and widespread applications [13]. In order to obtain higher harvesting efficiency and energy output, the vertical contact separation mode TEH was presented [14]. Moreover, a variety of different modes TEHs were proposed by using the same theory of electrostatic induction, for instance, the in-plane sliding-mode [15] and the single-electrode mode [16]. According to this proposed triboelectric mechanism, a huge number of structures have been devised to broaden the frequency bandwidth [17-19] and improve the low-frequency vibration energy conversion efficiency [20,21]. However, it is found that the linear structures are still difficult to harvest low-frequency energy.

To address this issue, many researchers devoted themselves to designing non-linear structures that are conducive to harvesting energy from low-frequency vibration [22-24]. For instance, a tri-stable energy harvester was innovatively designed by Mei et al. [25], which can harvest energy from low-frequency rotational motions. A novel non-linear harvester was proposed by Cao et al. [26], which can effectively harvest low-frequency vibration energy. Additionally, a dual-function non-linear electrical energy harvester was devised by Lu et al. [27,28] to realize vibration isolation and low-frequency vibration energy harvesting simultaneously. Therefore, the non-linear energy harvesters have better performance of harvesting energy from low-frequency vibration than the linear ones [29-32].

Inspired by the successful usage of nonlinearity to enhance energy harvesting at low frequencies, nonlinearity was also introduced into the design of TEHs [33-36]. For example, Wang et al. [37] presented a TEH that could harvest ultra-low-frequency vibration energy by combining the Quasi-zero stiffness principle. Fu et al. [38] proposed a TEH using a three-degree-of-freedom vibro-impact oscillator. They dissected the relationship between the non-linear dynamic behavior and the electrical performance of the TEH. Based on the principle of multistability, Yang et al. [39] investigated a magnetic levitation-based bistable hybrid energy harvester for scavenging energy from low-frequency structural vibration. Wang et al. [40] presented a tri-hybrid energy harvester that can generate higher output power at lower frequencies by interwell oscillations. This work indicated that non-linear TEHs could improve the efficiency of converting energy from mechanical vibration to electrical energy by broadening the frequency bandwidth in the low-frequency range.

Generally, the energy harvesting performance of TEHs is closely related to structural vibration. In addition, friction plays a pivotal role in structural dynamics by affecting the dynamic response in TEHs. Originally, linear damping was used to describe the dissipation energy caused by friction, but the non-linear properties induced by friction are lost through this simplification. As a simple interface model, the Coulomb friction model is often used to describe the friction relationship between two contact surfaces, which assumes that the contact points do not move with respect to each other unless the friction force exceeds a critical limit [41]. Later, the Macro-slip friction model was presented based on the Coulomb friction model with considering contact stiffness [42]. Compared with the Coulomb friction model, the Macro-slip friction model is more accurate [43,44]. In order to simplify the analysis for TEHs, the friction between the two dielectrics is often simplified. For instance, the friction is simplified as linear viscous damping in Ref. [21]. Fortunately, Fu et al. [45] first investigated the friction in a sliding-mode TEH based on the Coulomb friction model, which pioneered the research about the effect of friction on the performance of TEHs. However, the systematic research about friction in TEHs is still sporadic, scarce and rudimentary, resulting in difficulty designing and evaluating a TEH more accurately.

This paper proposes a sliding-mode TEH consisting of a novel bi-stable bird-shaped cantilever. The Macro-slip friction model is employed to describe the slip-stick behavior, and the modeling of the TEH is established by using Hamilton principle. The structural dynamics are achieved to evaluate the energy harvesting performance at low-frequency by resolving this model numerically. The main contribution of this paper is attempting to reveal the fundamental principle of the slip-stick behavior of the friction between dielectric films in the sliding-mode TEHs, and thus find out how the friction affects the energy harvesting efficiency of the TEHs.

This paper is organized as follows. In Sect. 2, the TEH is designed and the working mechanism of the TEH is described. The modeling of the TEH system and the theory of Macro-slip friction model are also presented in this section. In Sect. 3, the dynamic behaviors of the TEH are investigated, and several factors impacting the dynamic behaviors, especially the friction model, are discussed. Based on the Macro-slip friction model, the electrical performances of the TEH are evaluated in Sect. 4. Finally, some conclusions are drawn in Sect. 5.

2. Theoretical modeling

2.1 Model and principle

The conceptual model of the TEH is shown in Fig. 1. The mechanical structure of the TEH includes two parts, namely a bird-shaped cantilever and a pair of magnets. The left-hand end of the cantilever is clamped and a cube magnet (Magnet 1) is adhered to its right free end. Another magnet (Magnet 2)
is fixed opposite to Magnet 1, to repel the Magnet 1 when the cantilever is in a horizontal position. The structure utilized to capture the vibration energy contains two dielectrics, two electrodes and two friction plates, as shown in Fig. 1b. One of the friction plates (Fiction plate 1) is fixed on a cantilever, forming a bird-shaped cantilever. Another friction plate (Fiction plate 2) is hinged at the base, and a normal force acts on this plate to enable friction between Dielectric 1 and Dielectric 2. The tangential contact stiffness $k_d$ is taken into account. For the Coulomb friction model, $k_d$ is infinite, while it is a constant for the Macro-slip model. When the cantilever is oscillated by the base excitation, reciprocating sliding appears between Friction plate 1 and Friction plate 2.

As shown in Fig. 1b, the dielectric and the electrode are attached to the friction plates. The operating principle of TEH [8] is given in Fig. 2. When the energy harvester is in the initial state, the contact area between Friction plate 1 and Friction plate 2 is depicted in Fig. 2a. In this state, Dielectric 1 and Dielectric 2 fully overlap each other. Owing to the different electron attracting capabilities charges transfer from Dielectric 2 to Dielectric 1, and thus Dielectric 1 gains electrons and Dielectric 2 loses electrons. As a consequence, the surfaces of Dielectric 1 and Dielectric 2 are negatively and positively charged, respectively.

When excitation is applied on the non-linear TEH, Friction plate 1, which is connected with the cantilever, repels Dielectric 2 and Electrode 2 far away from the initial position of the non-linear TEH. As illustrated in Fig. 2b, Dielectric 1 deviates from the initial position with an upward distance. An electric potential difference between Dielectric 1 and Dielectric 2, induced by the instability of the charge distribution, is generated and results in a flow of electrons from Electrode 1 to Electrode 2. When Dielectric 1 moves up to the maximum distance, it follows the oscillation of the cantilever and begins to slide down. Under this circumstance, the flow of electrons is from Electrode 2 to Electrode 1 until they are perfectly overlapped, as shown in Fig. 2c. After that, Dielectric 1 continues to slide down and reaches the maximum distance in the downward direction, as shown in Fig. 2d. After that, the Dielectric 2 will move upward and

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**Figure 1** Conceptual model of the TEH.

**Figure 2** The operating principle of TEH.
reaches the initial position.

2.2 Modeling of the mechanical system

The extended Hamilton principle is used to derive the equation of motion of the non-linear TEH under external excitations. The general variational equation of the TEH can be given as:

\[ \delta \int_{t_1}^{t_2} (T - U) dt + \int_{t_1}^{t_2} \delta W dt = 0, \]  \hspace{1cm} (1)

where \( \delta \) is the infinitesimal variation, \( T = T_b + T_{m1} \) where \( T_b \) and \( T_{m1} \) are the kinetic energy of cantilever and the kinetic energy of Magnet 1, respectively. \( U \) is the potential energy that can be written as \( U = U_b + U_m \) where \( U_b \) and \( U_m \) denote the elastic potential energy of cantilever and the potential energy in the magnetic field, respectively. \( W \) is the work done by external forces, including the friction force. \( t_1 \) is the start time and \( t_2 \) is the end time.

Since the mass of Friction plate 1 is negligible compared with the cantilever beam, the kinetic energy \( T_b \) can be given by

\[ T_b = \frac{1}{2} \int_0^{L_c} \rho_c b_c h_c \left( \frac{\partial w(x,t)}{\partial t} + \frac{da(t)}{dt} \right)^2 dx, \]  \hspace{1cm} (2)

where \( w(x,t) \) denotes the transverse displacement, \( L_c \) is the length of the cantilever beam, \( \rho_c \) is the density of the cantilever. \( b_c \) and \( h_c \) are the width and height of the cantilever beam’s cross-sectional area, respectively. The base displacement is defined as \( u(t) \).

The kinetic energy of Magnet 1 can be evaluated by

\[ T_{m1} = \frac{1}{2} \left[ m \left( \frac{\partial w(x,t)}{\partial t} + \frac{dw(t)}{dt} \right)^2 \right] + \frac{1}{2} \left[ I \frac{\partial^2 w(x,t)}{\partial x^2} \right]^2, \]  \hspace{1cm} (3)

where the mass of Magnet 1 is \( m \), and \( I \) is the rotational inertia of the mass of the tip magnet.

The elastic potential energy \( U_b \) is written as:

\[ U_b = \frac{1}{2} \int_0^{L_c} EI \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 dx, \]  \hspace{1cm} (4)

where \( EI \) is the bending stiffness of the cantilever beam.

In actual situation, the locations of the two magnets in the motion states are presented in Fig. 3 [46,47]. The tip magnet (Magnet 1) is subjected to the magnetic force of the fixed magnet (Magnet 2).

The elastic potential energy generated by the magnetic field can be expressed as:

\[ U_m = -B \cdot m, \]  \hspace{1cm} (5)

where \( m \) and \( B \) are the magnetic moment vector of the tip magnet and the magnetic field generated by the fixed magnet at the location of tip magnet, respectively. The magnetic field \( B \) can be evaluated by

\[ B = \frac{\mu_0}{4\pi} \nabla \frac{m_r \cdot r}{|r|}, \]  \hspace{1cm} (6)

where \( \mu_0 \) is the vacuum permeability, \( m \) denotes the magnetic moment vector of the fixed magnet, \( \nabla \) and \(|r|\) represent the vector gradient operator and 2-norm, respectively. \( r \) is the vector from the fixed magnet to that of the tip magnet and the corresponding expression is

\[ r = -(d + \delta_x) \hat{x} + w(L_c, t) \hat{y}, \]  \hspace{1cm} (7)

where \( d \) shows the distance between the center of the tip magnet in base status and the center of the fixed magnet, \( \hat{x} \) and \( \hat{y} \) represent the unit vectors along the \( x \)-axis and the \( y \)-axis, respectively. The longitudinal displacement of the tip magnet \( \delta_x \) can be expressed as

\[ \delta_x = \frac{1}{2} \int_0^{L_c} \left( \frac{\partial w(x,t)}{\partial x} \right)^2 dx. \]

In theory, gradient functions [48] are defined as follows:

\[ \nabla \frac{1}{r^n} = \begin{pmatrix} \frac{x}{r} \frac{1}{r^{n+1}} \\ \frac{y}{r} \frac{1}{r^{n+1}} \\ \frac{z}{r} \frac{1}{r^{n+1}} \end{pmatrix} = -\frac{n}{r^{n+1}} \mathbf{r}, \]  \hspace{1cm} (8)

\[ \nabla (\mathbf{v}_1 \cdot \mathbf{r}) = \begin{pmatrix} \frac{\partial}{\partial x} (\mathbf{v}_1 \cdot \mathbf{r}) \\ \frac{\partial}{\partial y} (\mathbf{v}_1 \cdot \mathbf{r}) \\ \frac{\partial}{\partial z} (\mathbf{v}_1 \cdot \mathbf{r}) \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} (x_1 x + y_1 y + z_1 z) \\ \frac{\partial}{\partial y} (x_1 x + y_1 y + z_1 z) \\ \frac{\partial}{\partial z} (x_1 x + y_1 y + z_1 z) \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \mathbf{v}_1, \]  \hspace{1cm} (9)

The magnetic field Eq. (6) can be derived by Eqs. (8) and (9), substituting Eq. (6) into Eq. (5) yield

\[ U_m = \frac{\mu_0}{4\pi} \int_{r_1}^{r_2} (\mathbf{m}_s \cdot \mathbf{r}) \left( \frac{m_s \cdot \mathbf{r}}{|r|^3} \right) dV, \]  \hspace{1cm} (10)

Figure 3  Geometries configuration of the two magnets.
where $r$ is the $L_2$-norm of the vectors $r$ and it can be written as
\[ r = \left( (d + \delta \dot{x})^2 + w^2(L_c, t) \right)^{1/2}. \]

In the magnetic field, $\mathbf{m}_i$ and $\mathbf{m}_f$ can be given by
\[ \mathbf{m}_i = M_f V_f (\cos \theta \vec{x} + \sin \theta \vec{y}), \]
\[ \mathbf{m}_f = -M_t V_t \vec{x}, \]
where $M_f$ and $M_t$ are the magnitude of the magnetization of the tip magnet and the fixed magnet, respectively. And $M_c$ can be described as $M_c = B_c / \mu_0$ by using the residual flux density $B_c$. $V_f$ and $V_t$ are the tip magnet’s volume and the fixed magnet’s volume, respectively. $\theta$ is the slope of the cantilever at the tip magnet.

Substituting Eqs. (7) and (11) into Eq. (10), one can get the expressions of the elastic potential energy as:
\[ U_m = \frac{\mu_0 M_c V_f M_t V_t}{4 \pi^2 r^3} \left\{ \cos \theta - \frac{3}{r^2} \left( (d + \delta_x)^2 \cos \theta + w(L_c, t)(d + \delta_x) \sin \theta \right) \right\}, \]
(12)
Assuming the cantilever beam is a uniform Euler Bernoulli beam, the transverse vibration displacement $w(x, t)$ of the model shape can be expressed as:
\[ w(x, t) = \sum_{i=1}^{n} \phi_i(x) q_i(t), \]
(13)
where $q_i(t)$ represents the temporal modal coordinate, $\phi_i(x)$ denotes the mode shape function. $n$ is the number of the mode shape.

The mode shape $\phi_i(x)$ can be given by
\[ \phi_i(x) = C_i \left( \cos \beta x - \cosh \beta x + \xi \left( \sin \beta x - \sinh \beta x \right) \right), \]
(14)
where $C_i$ is determined from the corresponding orthonormality conditions, $\beta$ and $\xi$ can be derived from the associated eigensystem. The boundary conditions of the cantilever beam can be given by
\[ w(x, t)|_{x=0} = 0, \quad \frac{\partial w(x, t)}{\partial x} \bigg|_{x=0} = 0, \]
\[ \frac{\partial^2 w(x, t)}{\partial x^2} \bigg|_{x=L_c} = 0, \quad \frac{\partial^3 w(x, t)}{\partial x^3} \bigg|_{x=L_c} = 0. \]
(15)

In this study, the natural frequency of the first mode of the beam without the tip mass is 131.7 Hz, which can be calculated by $\omega = \beta^2 \frac{E I}{\rho_c b_c h_c L_c^4}$. The excitation frequencies are considered from 5 to 25 Hz, which are much lower than the first natural frequency of the beam. Therefore, the effects of higher modes can be neglected, and it is reasonable to assume that the beam is deformed to the first mode shape in the subsequent theoretical analysis. The transverse vibration displacement of the first mode shape of the beam can be written as:
\[ w(x, t) = \phi_1(x) q_1(t). \]
(16)

Substituting Eq. (16) into Eqs. (2)-(5), kinetic energies of the cantilever beam and the tip magnet can be derived, potential energies of the cantilever beam and the magnetic field also can be obtained, and they are expressed as:
\[ T_b = \frac{1}{2} \int_0^{L_c} \rho_c b_c h_c \left( \frac{\partial \phi_1^2}{\partial t} + 2 \phi_1 \frac{\partial \phi_1}{\partial t} + u^2 \right) dx, \]
\[ T_{m1} = \frac{1}{2} m \left( \frac{\partial \phi_1^2}{\partial t} + 2 \phi_1 \frac{\partial \phi_1}{\partial t} + u^2 \right)^2 + \frac{1}{2} I_{01} \left( \frac{\partial \phi_1'(L_c)}{\partial t} \right)^2, \]
(17)
\[ U_b = \frac{1}{2} \int_0^{L_c} EI \left( \frac{\partial \phi_1^2}{\partial t} \right)^2 dx, \]
\[ U_m = \frac{\mu_0 M_c V_f M_t V_t}{4 \pi^2 r^3} \left\{ \cos \theta - \frac{3}{r^2} \left( (d + \delta_x)^2 \cos \theta + w(L_c, t)(d + \delta_x) \sin \theta \right) \right\}, \]
where
\[ \delta_x = \frac{1}{2} \int_0^{L_c} \left( \frac{\partial \phi_1^2}{\partial t} \right)^2 dx, \]
\[ \cos \theta = 1 - \frac{1}{2} \phi_1'^2(L_c)^2, \]
\[ \sin \theta = q \phi_1'(L_c). \]

The Lagrange function for the mechanical system can be written as:
\[ L = T_b + T_{m1} - U_b - U_m. \]
(19)

The dynamical equation of the mechanical system can be evaluated by
\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + F_{\phi}(s) = 0. \]
(20)

According to Eqs. (16)-(20), the dynamical equation can be derived as:
\[ \ddot{q} + 2 \zeta \omega q + \omega^2 q + \frac{F_m}{M} = \frac{a}{M} \omega^2 - \frac{F_{\phi}(s)}{M}, \]
(21)
where $\zeta$ is the structural damping ratio, $\omega$ is the undamped natural frequency of the cantilever beam without the tip magnet’s mass, $F_m$ is the magnetic force and it can be obtained by $\partial U_m / \partial q$, $u = \text{Asin} \omega t$ is the base excitation, $A$ is the acceleration amplitude, $\omega = 2 \pi f$ and $f$ is the frequency. $F_t$ is the friction force. The expression for $a$ and the first modal mass $M$ are given by
\[ a = \rho_c b_c h_c \int_0^{L_c} \phi_1(x) dx + m \phi_1(L_c), \]
\[ M = \rho_c b_c h_c \int_0^{L_c} \phi_1'(x)^2 dx + m \phi_1'(L_c) + I_{01} \phi_1'(L_c)^2. \]
(22)

Note that the electrostatic force between the two dielectric films can be expressed as $F_e = Q^2 / 2 \varepsilon_0 A$, where the variable $A$ denotes the contact area between the dielectric films. However, the electrostatic force is not considered in the dynamic modeling, since its low impact on the dynamic character-
istics of the TEH [45].

2.3 Marco-slip friction model

In this section, the mathematical relationship between the non-linear friction force and the relative displacement of the contact surface is described in detail based on the Macro-slip friction model, which is capable of describing the slip-stick behavior of the TEH [49-51]. To evaluate the accuracy of the Macro-slip friction model, the Coulomb friction model is also introduced to describe the friction and serve as a comparison with the Macro-slip friction model.

Assuming that the base excitation is harmonic, the relative displacement of the contact point on the friction plate can be given by

\[ w = X \cos(\omega t + \phi) = X \cos \vartheta, \]  

(23)

where \( X \) is the amplitude, \( \omega \) and \( \phi \) are the vibration angular frequency and initial phase, respectively.

As according to the concerned Marco-slip model, the non-linear friction force, \( F_{f_M} \), be expressed as:

\[
F_{f_M} = \begin{cases} 
 k_d (w - w_t), & \text{when } k_d |w - w_t| \leq \mu F_N, \\
\mu F_N \text{sgn}(w_t), & \text{when } k_d |w - w_t| \geq \mu F_N,
\end{cases}
\]

(24)

where \( w_t \) is the displacement of the friction plate, and \( k_d \) is the tangential contact stiffness of the friction plates. Figure 4 shows the relationship between the response and the friction force. The system is in stick state and slip state during the \( AB \) and \( BC \) sections, respectively. The velocity of the friction plate \( \dot{w}_t \) can be expressed as:

\[ \dot{w}_t = \begin{cases} 
 0, & \text{when } 0 \leq \vartheta \leq \vartheta^*, \\
\dot{w}, & \text{when } \vartheta^* \leq \vartheta \leq \pi,
\end{cases} \]

(25)

The displacement \( w_t \) can be obtained by integrating Eq. (24):

\[
w_t = \begin{cases} 
c_1, & \text{when } 0 \leq \vartheta \leq \vartheta^*, \\
\dot{w} + c_2, & \text{when } \vartheta^* \leq \vartheta \leq \pi.
\end{cases}
\]

(26)

In the stick state (as denoted by symbol \( AB \) in Fig. 4), the friction force is equal to zero when \( \vartheta \) is equal to \( \vartheta^* \). Thus, \( c_1 = X - \mu F_N/k_d \). During the slip state (as denoted by symbol \( BC \)), the difference between \( w \) and \( w_t \) is the constant \( c_2 \). When \( w_t \) is at point \( B \), one can find that \( c_2 \) is the horizontal distance between point \( B \) and the middle point of line \( AB \) in Fig. 4. Therefore, \( c_2 = \mu F_N/k_d \). From Eq. (24), the friction force can be given by

\[
F_{f_M} = k_d (w - w_t) = k_d (w - X + \mu F_N), \quad \text{when } 0 \leq \vartheta \leq \vartheta^*,
\]

(27)

\[
F_{f_M} = \mu F_N \text{sgn}(w_t) = -\mu F_N, \quad \text{when } \vartheta^* \leq \vartheta \leq \pi,
\]

(28)

when \( \vartheta = \vartheta^* \), one can yield

\[
\vartheta^* = \arccos \left[ 1 - \frac{2 \mu F_N}{k_d X} \right].
\]

(29)

Since the response is assumed to be a simple harmonic form, the response cycle can be divided into two symmetric half cycles, i.e., the half cycle \( ABC \) is equal to the other half cycle \( CDA \) in Fig. 4. Thus, one can find

\[
F_{f_M} = \mu F_N \text{sgn}(w_t) = -\mu F_N, \quad 0 \leq \vartheta \leq \pi.
\]

(30)

Therefore, the non-linear friction force corresponding to the sliding state between the contact surfaces of the friction plates can be expressed as:

\[
F_t = \begin{cases} 
 F - 2\mu F_N \left[ 1 - \left( 1 + \frac{X \cos \vartheta - X}{2X_{cr}} \right) \right], & 0 \leq \vartheta \leq \vartheta^*, \\
-\mu F_N, & \vartheta^* \leq \vartheta \leq \pi, \\
-F + 2\mu F_N \left[ 1 - \left( 1 - \frac{X \cos \vartheta + X}{2X_{cr}} \right) \right], & \pi \leq \vartheta \leq \pi + \vartheta^*, \\
\mu F_N, & \pi + \vartheta^* \leq \vartheta \leq 2\pi,
\end{cases}
\]

(31)

where \( F = \mu F_N \), \( \vartheta^* = \arccos (1 - 2X_{cr}/X) \) and \( X_{cr} \) can be given by \( \mu F_N \). When the system enters into a steady-state, periodically changed friction force on the contact surfaces is presented, and the friction force can be represented by the sum of the equivalent restoring and damping forces, as given by

\[
F_t = k_e w(s, t) + c_e \dot{w}(s, t),
\]

(32)

where \( k_e \) and \( c_e \) are the equivalent stiffness and equivalent damping coefficients, respectively, which can be given by

\[
\mu F_N = \arccos \left[ 1 - \frac{2 \mu F_N}{k_d X} \right].
\]
Substituting Eqs. (16), (32) and (33) into Eq. (21) yields the dynamical equation with the Macro-slip friction model:

\[
\ddot{\varphi} + 2\zeta \dot{\varphi} + \left(\frac{k_e}{M} \varphi - \frac{1}{2}\frac{k_d}{M} \sin 2\varphi\right) = \frac{F_0 - F_N}{M} \frac{\sin(\varphi + \phi)}{M},
\]

where \( k_e \) and \( k_d \) are the equivalent stiffness and damping coefficients, respectively.

The equivalent stiffness and damping coefficients are shown as Fig. 5. Because \( X/c \) is in direct proportion to the normal force, \( X/c \) rises with the decrease in the normal force. As the normal force decreases, the equivalent stiffness diminishes and the damping coefficients increases first and then decreases. When the normal force closes to infinity, \( k_e \) and \( c_e \) approach to \( k_d \) and zero, respectively.

Note that the Macro-slip model degrade to the Coulomb friction model when the tangential stiffness \( k_d \) is infinite. According to Coulomb’s law of friction, the kinetic friction force on the slider can be given as:

\[
F_{\text{f,C}} = \mu F_N \text{sgn}(\dot{\varphi}) = c_e \dot{\varphi},
\]

where \( \mu \) and \( F_N \) are the coefficient of friction and normal force on contact surfaces of friction plates, respectively. \( \text{sgn}(\cdot) \) is the signum function, \( c_e \) is the equivalent damping of Coulomb friction model.

In Eq. (35), the term \( \text{sgn}(\dot{\varphi}) \) is a periodic square wave function which can be expanded into a Fourier series. Including the fundamental harmonic term only, Eq. (35) can be rewritten as:

\[
F_{\text{f,C}} = \mu F_N \text{sgn}(\dot{\varphi}) \approx \frac{4\mu F_N}{\pi} \sin(\pi X/c) + \varphi.
\]

Substituting Eq. (36) into Eq. (35), one can get the equivalent damping of Coulomb friction model:

\[
c_e = \frac{4\mu F_N}{\pi \alpha X}.
\]

Table 1 The parameters employed for numerical simulations

| Parameter                                      | symbol | value  |
|-----------------------------------------------|--------|--------|
| Length of the cantilever beam                 | \( L_c \) | 200 mm |
| Width of the cross-sectional area of the beam | \( b_c \) | 10 mm  |
| Height of the cross-sectional area of the beam| \( h_c \) | 1 mm   |
| Material density of the beam                  | \( \rho \) | 7800 kg/m³ |
| Young’s modulus of cantilever                 | \( E \) | 210 Gpa |
| Poisson’s ratio                                | \( \nu \) | 0.3    |
| Damping ratio                                  | \( \zeta \) | 0.0038 |
| Mass of Magnet 1 or Magnet 2                  | \( m \) | 12.8 g |
| The vacuum permeability                       | \( \mu_0 \) | \( 4\pi \times 10^{-7} \) |
| Residual flux density of magnet               | \( B_r \) | 1.48   |
| Volume of Magnet 1                            | \( V_s \) | 8 mm × 8 mm × 8 mm |
| Volume of Magnet 2                            | \( V_f \) | 8 mm × 8 mm × 8 mm |
| Friction coefficient                          | \( \mu \) | 0.2    |

Figure 5 The variation of (a) equivalent stiffness, (b) equivalent damping coefficients.
3. Dynamic behaviors of the TEH

In this section, the equations of motion are solved by the Runge-Kutta method. The system parameters employed for numerical simulations are tabulated in Table 1.

3.1 Bistability of the TEH

In the TEH, only the restoring force of the cantilever beam and the magnetic force between the tip and fixed magnets are considered. The restoring force is merely affected by the deflection of the beam, and the magnetic force is influenced by the distance between the magnets, as illustrated in Fig. 6a. In effect, the strain energy of the cantilever beam and the magnetic potential energy depend on the restoring force and the magnetic force, respectively. Thus, the total force is composed of the restoring force and the magnetic force.

Figure 6c shows the relationship between the total potential energy of the TEH without friction and the tip displacement of the cantilever beam for different horizontal distances between the magnets (\(d = 0.012\) m, 0.016 m and 0.02 m). When the horizontal distance is small, the tip magnet and the fixed magnet are close, and the system exhibits significant bistability. The reason is that the potential energy barrier increases as the horizontal distance between two magnets decreases. In order to obtain a more intuitive variation of the total potential energy against the horizontal distance, a 3D surface plot is presented in Fig. 6d. Clearly, the bistability of the system gets weaker as the horizontal distance increases. If the magnets are far enough apart, the magnetic field has little effect on the tip magnet, and the system becomes monostable.

3.2 Effect of the friction

The TEH harvests energy primarily through electron transfer of triboelectrification, and friction is the main factor influencing triboelectrification. Meanwhile, friction causes energy dissipation by means of heat. Thus, friction is very complex in the non-linear TEH, and current researches on the TEH are sorely lack. In this section, Marco-slip friction model is applied to describe the slip-stick behavior of the system, which is also compared with the Coulomb friction model. In addition, the performance of the TEH is evaluated by investigating the effects of the system parameters on the dynamic characteristics.

3.2.1 Normal force

Based on the Coulomb model and Macro-slip model, the
effect of normal force on the vibration amplitude of the slider (Friction plate 1) in the TEH is reported in Fig. 7. Figure 7a, b show the vibration response of the Friction plate 1 changing against the excitation frequency for different normal force (such as I $F_N=0.1$ N, II $F_N=0.2$ N and III $F_N=0.3$ N) by using the Coulomb model and Macro-slip model, respectively.

As depicted in Fig. 7a-I and Fig. 7b-I, interwell oscillations coexists with intrawell oscillations when the normal force is equal to 0.1 N. The interwell oscillations occur in low-frequency region, because the friction force is small and the TEH is in slip state with large amplitudes. In addition, the motion pattern in Fig. 7a-I is the same as that presented in Fig. 7b-I. The reason is that the TEH crosses the critical condition easier in low-frequency region and the slip state is dominant in Macro-slip model case when the normal force is tiny (e.g., $F_N=0.1$ N). Thus, the Macro-slip model is equivalent to the Coulomb model as the normal force is small enough.

When the normal force increases from 0.1 to 0.2 N, however, the motion pattern of the TEH obtained by the Coulomb model is different from that acquired by the Macro-slip model. In terms of results calculated by the Coulomb model, as illustrated in Fig. 7a-II, the motion pattern of the TEH manifests in intrawell oscillations. That is, the TEH oscillates in one of the potential wells with small amplitude. The reason is that the friction force is so large that the restoring force cannot offset the friction force and the magnetic force simultaneously. However, as reported in Fig. 7b-II, the TEH oscillates between two potential wells in some special low-frequency ranges, resulting in interwell oscillations with large amplitude when the Macro-slip model is utilized to calculate the dynamic responses of the TEH. Actually, such a difference can be attributed to that the friction force calculated by the Macro-slip model is smaller than that by the Coulomb model under the same normal force. More interestingly, the TEH is in slip state in low-frequency range, but this phenomenon cannot be revealed by the Coulomb model.

When the normal force continues to increase to 0.3 N, the responses of the TEH obtained by the Coulomb model and the Macro-slip model are shown in Fig 7a-III and Fig 7b-III, respectively. From Fig 7a-III, it can be observed that the TEH cannot cross the potential energy barrier and undergoes intrawell oscillations, when the Coulomb model is utilized. However, the large-amplitude interwell oscillations appear in low-frequency range, when the Macro-slip model is employed, as shown in Fig. 7b-III. Moreover, it is different from Fig 7b-II that the band of interwell oscillations decreases, and the interwell oscillations disappears at high frequency. It is because that the sliders of the TEH are in the stick state in a broader frequency range as the normal force increases.

Generally, the large-amplitude interwell oscillations are expected for high-efficiency energy harvesting. Thus, the normal force corresponding to the occurrence of interwell oscillations is defined as critical normal force. For the Macro-slip friction model, the tangential contact stiffness has significant impact on the critical normal force, as shown in Fig. 8. Since the Coulomb friction model does not take into account the tangential contact stiffness, the critical normal force is invariable. Nevertheless, the Macro-slip friction model is different from Coulomb friction model. When the tangential contact stiffness is small, the Macro-slip friction

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**Figure 7** The effect of the normal force on the peak-peak value of dynamical response when I $F_N=0.1$ N, II $F_N=0.2$ N, III $F_N=0.3$ N, $A=0.3$ g, $d=0.016$ m, $a=0.2$, and $k_d=30$. 

---
model is approximately equal to Coulomb friction model. With the increases of the tangential contact stiffness, the critical normal force increases first and then decreases, as shown in Fig. 8. When tangential contact stiffness is sufficient large, the Macro-slip friction model is equivalent to Coulomb friction model again.

As a consequence, Macro-slip model is equivalent to Coulomb model when the normal force is sufficient large or quite small. Moreover, a broader frequency band for interwell oscillations in low-frequency range can be achieved by using small normal forces. Hence, a small normal force is conducive to harvesting energy in the TEH.

### 3.2.2 Friction coefficient

Figure 9 shows the vibration amplitudes (peak-peak value) of Friction plate 1 under frequency sweeps with different friction coefficients (I $\mu = 0.1$, II $\mu = 0.2$, III $\mu = 0.3$). The results showed in Fig. 9a, b are obtained based on the theory of Coulomb model and Macro-slip model, respectively.

As reported in Fig. 9a-I and Fig. 9b-I, the motion pattern of the TEH obtained by the Coulomb friction model is almost identical with that obtained by the Macro-slip friction model. When the friction coefficient is small, slip is easy to occur, according to Eq. (24). In such a case, the Macro-slip friction model is equivalent to the Coulomb friction model. Specifically, the TEH oscillates between two potential wells in the low-frequency range, and the large-amplitude responses appear. This is because the friction force between the films is sufficient small, and thus the tip magnet can easily cross the potential energy barrier. Generally, the large-amplitude responses are beneficial to converting the low-frequency vibration energy to electrical energy.

With the increase in the friction coefficient, as shown in Fig. 9a-II and Fig. 9b-II, the frequency band of interwell oscillations obtained by both the Coulomb friction model and the Macro-slip friction model get narrow. In addition, for the Coulomb friction model, the interwell oscillations in the low-frequency range disappear as the friction coefficient increases from 0.2 to 0.3, as delineated in Fig. 9a-III. However, for the Macro-slip friction model, the frequency band of interwell oscillations can still be observed, and the bandwidth gets narrow when the friction coefficient is increased to 0.3.

Consequently, the Coulomb friction model would lead to incorrect predictions, when the normal force and friction coefficient are large.
coefficients are in a certain range, owing to the neglect of tangential contact stiffness. The following dynamic analysis and electrical feature of the TEH are conducted by employing the Macro-slip model.

4. Parametric analysis of the TEH

To evaluate the vibration energy harvesting performance of the TEH and dissect the effect of the systemic parameters on the electric outputs, the electric modeling of the TEH is established firstly. As a parameter affecting the dynamic characteristics of the TEH, the distance between two magnets plays a pivotal role in the conversion from mechanical energy to an electrical one. Additionally, the nonlinearity and the amplitude of excitation are also two important factors that could influence the energy harvesting performance. Therefore, the dependency between parameters and the electric outputs is clarified in this section in detail.

4.1 Modeling of the electrical system

In the overlapped region of Dielectric 1 and Dielectric 2, the electric field $E_1$ and $E_2$ can be written as [9]:

$$
E_1 = \frac{\sigma x}{\varepsilon_0 \varepsilon_1 (l-x)},
$$

$$
E_2 = \frac{\sigma x}{\varepsilon_0 \varepsilon_2 (l-x)},
$$

where $\sigma$ is the charge density of both the Dielectric 1 and Dielectric 2; $l$ is the length of both Dielectric 1 and Dielectric 2; $x$ is the sliding distance of the Dielectric 1 relative to the Dielectric 2; $\varepsilon_0$ is the vacuum permittivity; $\varepsilon_1$ and $\varepsilon_2$ denote the relative permittivity of Dielectric 1 and Dielectric 2, respectively.

Based on the expressions of electric field presented in Eq. (38), the open-circuit voltage of the energy harvesting system can be given by

$$
V_{OC} = E_1 d_1 + E_2 d_2 = \frac{\sigma x}{\varepsilon_0 (l-x)} \left( \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right),
$$

where $d_1$ and $d_2$ stand for the thicknesses of Dielectric 1 and Dielectric 2, respectively.

The relationship between the voltage $V$ and the transferred charges $Q$ in the outer circuit of the TEH can be written as [8]:

$$
V = -\frac{1}{C_e} Q + V_{OC},
$$

where $C_e = \frac{\varepsilon_0 d (l-x)}{d_1 + d_2}$, and $b$ is the width of the dielectric membranes.

Substituting Eq. (39) into Eq. (40), the $V$-$Q$-$q$ relationship can be expressed as:

$$
V = -\frac{1}{b\varepsilon_0 (l-x)} \left( \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right) Q + \frac{\sigma x}{\varepsilon_0 (l-x)} \left( \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)
$$

(41)

According to the Ohm’s law $V = IR = R \frac{dQ}{dt}$, the electrical governing equation can be given by

$$
R \frac{dQ}{dt} = -\frac{Q}{b\varepsilon_0 (l-x)} \left( \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right) + \frac{\sigma x}{\varepsilon_0 (l-x)} \left( \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right).
$$

(42)

where $R$ is a load resistor.

The parameters utilized in electrical simulations are tabulated in Table 2.

4.2 Effect of the distance between magnets

As delineated in Sect. 3.1, the static characteristics of the non-linear TEH are related to the distance between two magnets. Therefore, the distance between the two magnets plays a pivotal role in the dynamic response of the non-linear TEH, and thus impacts the energy harvesting performance.

Figure 10a-c show the vibration responses of the Friction plate 1 change with the excitation frequency for different horizontal distance $d$ between the tip and fixed magnets by using the Macro-slip friction model. Clearly, when the distance is equal to 0.014 m, the TEH is a typical bistable configuration with a high energy barrier. The magnetic force and the friction force are greater than the restoring force of the cantilever beam, and the TEH cannot cross the potential energy barrier. Hence, the vibrations only exist in the potential well and the motion of the tip magnet is intrawell oscillation, as shown in Fig. 10a. In addition, as $d$ increases to 0.016 m, the magnetic force decreases. The TEH jumps out from the potential energy wells in the low-frequency range, and thus interwell oscillations coexist with intrawell oscillations in Fig. 10b. When the tip magnet is far apart from the fixed magnet, the structure of the TEH changes from bistability into monostability, due to the fact that the magnetic force is weak and the restoring force plays a dominant role, as shown in Fig. 10c.

The influences of the distance $d$ on the open-circuit voltage are exhibited in Fig. 10d. When the distance $d=0.014$ m, the open-circuit voltage is close to zero, since the TEH cannot

| Table 2 The parameters utilized in the electrical simulations |
|-------------------------------------------------------------|
| Parameter         | Value                           |
| Dielectric 1     | $\varepsilon_1 = 4, d_1 = 2.2 \times 10^{-4}$ m |
| Dielectric 2     | $\varepsilon_2 = 2, d_2 = 2.2 \times 10^{-4}$ m |
| Width of the dielectrics | $b = 0.01$ m                     |
| Length of the dielectrics | $l = 0.01$ m                     |
| Surface tribo-charge density | $\sigma = 6 \mu$C/m²             |
| Vacuum permittivity | $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m |
cross the high potential energy barrier, as shown in Fig. 10a. As the distance increases to 0.016 m, the large-amplitude interwell oscillations appear owing to the low potential energy barrier, as shown in Fig. 10b. Thus a high open-circuit voltage of the TEH can be realized in the low-frequency range. With the increase in the horizontal distance between the two magnets from 0.016 m to 0.018 m, the magnetic force decreases and the potential barrier of the system almost disappear. In this case, the TEH turns from bistability into monostability, and the open-circuit voltage with low amplitude shows in the whole frequency range.

Consequently, strong bistability can be fulfilled and a broad frequency band for interwell oscillations at low-frequency can be achieved by selecting proper distance \(d\) between the tip and fixed magnets, as shown in Fig. 10b. In such a case \((d = 0.016 \text{ mm})\), the TEH has the excellent ability of energy harvesting from interwell oscillations over a broader frequency band.

### 4.3 Effect of the base excitation

The level of base excitation has a significant contribution to the dynamic and electrical response of this proposed TEH. The influences of the excitation amplitude \(A\) on the dynamic behaviors and open-circuit voltage are exhibited in Fig. 11. Figure 11a-c report the vibration amplitudes of friction plate under frequency sweeps with the acceleration amplitude of the base excitation through using the Macro-slip friction model. As shown in Fig. 11a, the TEH cannot cross the potential energy barrier since the acceleration amplitude is tiny (i.e., \(A = 0.2 \text{ g}\)), thus, the vibrations only exist in a potential well. When the base excitations are increased to \(A = 0.3 \text{ g}\) and \(A = 0.4 \text{ g}\), the motion of the tip magnet can cross the potential energy barrier at some frequency ranges, and the responses are reported in Fig. 11b, c, respectively. It can be observed that interwell oscillations coexist with intrawell oscillations, and the frequency band of interwell oscillations increases with the increase of the excitation amplitude.

Figure 11d shows the relationship between the open-circuit voltage and the frequency for different acceleration amplitudes of the base excitation. Clearly, the open-circuit voltage of the TEH is low when the acceleration amplitude is weak, such as \(A = 0.2 \text{ g}\). In this case, the tip magnet cannot cross the potential energy barrier and the motion of the TEH is dominated by intrawell oscillations. However, the tip magnet can cross the potential energy barrier in the low-frequency range as the acceleration amplitude increases. Therefore, one can see from Fig. 11d that both the amplitude and frequency

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**Figure 10** Tip vibration responses of Friction plate 1 (peak-peak) under frequency sweeps with the horizontal distance \(d\) between the tip magnet and the fixed magnet: a \(d=0.014 \text{ m}\), b \(d=0.016 \text{ m}\), c \(d=0.018 \text{ m}\); \(A=0.3 \text{ g}\), \(F_N=0.15 \text{ N}\), \(k_d=30\). d The open-circuit voltage of TEH with the different horizontal distances.
bandwidth of open-circuit voltage increase with the increase of the acceleration amplitude in the low-frequency range. To sum up, high-level base excitation amplitude is beneficial to broaden the frequency band of interwell oscillations, and thus enhance the energy harvesting performance.

4.4 Influence of nonlinearity

The system dynamics can be sensitively influenced by the excitation frequency and the horizontal distance between the tip magnet and the fixed magnet. Thus it is imperative to study the bifurcation of the responses about the excitation frequency and the horizontal distance. Therefore, the influences of the nonlinearity on the dynamical response of the TEH are carried out in this section based on the Macro-slip model.

Figure 12a shows the bifurcation diagrams of the vibration amplitudes of the friction plate with respect to the excitation frequency when the acceleration excitation amplitude is \( A = 1 \) g and the horizontal distance is \( d = 0.016 \) m. As depicted in Fig. 12a, the motion is periodic when the excitation frequency is small. The responses of TEH turn into aperiodic/chaotic motion with the increase of the excitation frequency. Nonetheless, as the excitation frequency continues to increase, the system enters a periodic motion again. Figure 12b shows the bifurcation diagrams of the responses of the TEH for the horizontal distance between magnets when \( A = 1 \) g and \( f = 8 \) Hz. It can be observed that all the vibrations are periodic as the horizontal distance increases.

Figure 13 shows the phase diagram of the TEH at an acceleration excitation amplitude of \( A = 1 \) g and the horizontal distance \( d = 0.016 \) m between the tip and fixed magnets. As shown in Fig. 13a, the TEH cannot cross the potential barrier induced by the magnet pairs, resulting in periodic intrawell oscillations around one of the stable equilibria. With the increase of the excitation frequency, the TEH crosses the potential energy barrier, and the motion pattern becomes interwell periodic oscillations, as illustrated in Fig. 13b. Compared with the intrawell oscillation, the interwell one generates a larger response amplitude, which is beneficial for converting the mechanical energy from the ambient vibration. As the excitation frequency continues to increase, the TEH oscillates around one of the stable equilibrium again with small amplitude, as delineated in Fig. 13c. However, when the excitation frequency is at 16 Hz, the motion pattern of the system turns from periodic oscillations into aperiodic/chaotic interwell oscillations, as illustrated in Fig. 13d.

Figure 14 shows the time histories of the displacement

---

**Figure 11** Amplitudes of tip vibration (peak-peak) under frequency sweeps with the acceleration amplitude of the base excitation, \( a, A = 0.2 \) g, \( b, A = 0.3 \) g, \( c, A = 0.4 \) g; \( d = 0.016 \) m, \( F_d = 0.15 \) N, \( k_d = 30 \). **d** The open-circuit voltage of the TEH with different acceleration amplitudes.
response and the open-circuit voltage response of the TEH. As depicted in Fig. 14a, the displacement response of the TEH can be divided into four typical stages. When the excitation frequencies are 7 and 13 Hz, the displacement responses of TEH show up as negative and positive intrawell oscillations, owing to the potential energy barrier of the bistable system. In these two stages, tiny open-circuit voltage is generated, as shown in Fig. 14b. The TEH undergoes interwell periodic oscillations as the excitation frequency is 10 Hz, since the tip magnet can cross the potential energy barrier. In this stage, the open-circuit voltage of TEH becomes high. When the driving frequency is 16 Hz, the displacement response turns into aperiodic/chaotic interwell oscillations, and the open-circuit voltage response becomes chaotic corresponding to the complex dynamic behaviors. Compared with the intrawell oscillations, the interwell oscillations are more beneficial for energy harvesting.

In this energy harvesting system, the performance is strongly influenced by the horizontal distance between the tip magnet and the fixed magnet. Figure 15 depicts the phase diagram of the TEH for an excitation amplitude of $A = 1 \text{ g}$ and the excitation frequency of $f = 10 \text{ Hz}$. In all cases, the motion pattern of the TEH is periodic. When the horizontal distance is small, for example, $d = 0.014 \text{ m}$, the TEH undergoes intrawell oscillations, as shown in Fig. 14a. For such a motion pattern, the tip magnet cannot cross the potential energy barrier. In contrast, when the horizontal distance is large, the TEH undergoes interwell oscillations, as shown in Fig. 14b. For such a motion pattern, the tip magnet can cross the potential energy barrier.

**Figure 12** Bifurcation diagrams of the friction plate’s vibration amplitudes $w$ for a) the excitation frequency $f$ when the acceleration excitation amplitude $A = 1 \text{ g}$, and the horizontal distance $d = 0.016 \text{ m}$, b) the horizontal distance $d$ between magnets with the excitation amplitude $A = 1 \text{ g}$, and the excitation frequency $f = 8 \text{ Hz}$.

**Figure 13** The phase diagram of the TEH at an acceleration excitation amplitude of $A = 1 \text{ g}$, a horizontal distance $d = 0.016 \text{ m}$ and an excitation frequency of a) $f = 7 \text{ Hz}$, b) $f = 10 \text{ Hz}$, c) $f = 13 \text{ Hz}$, d) $f = 16 \text{ Hz}$.
barrier induced by the strong magnetic force. As the horizontal distance is increased to 0.016 m (Fig. 14b), the tip magnet crosses the potential barrier and periodically oscillates between the two equilibrium points. As the horizontal distance is further increased to 0.018 m and 0.02 m, the motion pattern turns to normal periodic oscillations with small-amplitude. This variation of the motion pattern can be attributed to the change of system state from strong bistability to weak bistability and then to monostability (as reported in Fig. 6). The tip magnet is hard to cross the potential barrier in the state of strong bistability, whereas it can easily cross the potential barrier in the state of weak bistability. Additionally, the potential barrier disappears as the state changes from bistability to monostability. Hence, the motion pattern of the tip magnet undergoes intrawell and interwell and then normal periodic oscillations as the horizontal distance is increased.

Figure 14. The time histories of a the displacement response, b the open-circuit voltage response for different excitation frequencies.

Figure 15. The phase diagram of the horizontal distance on the motion pattern when the excitation amplitude $A = 1$ g, and the excitation frequency of $f = 10$ Hz: a $d = 0.014$ m, b $d = 0.016$ m, c $d = 0.018$ m, d $d = 0.02$ m.

Figure 16 describes the time histories of the displacement response and the open-circuit voltage response with different horizontal distances. Clearly, when the horizontal distance is 0.014 m, the tip magnet cannot cross the potential barrier and the TEH is dominated by weak intrawell oscillations. Thus, the open-circuit voltage is almost zero. As the horizontal distance is increased to 0.016 m, the system enters into interwell oscillations from intrawell ones owing to the reduced potential barrier, and the open-circuit voltage response becomes much higher. As the horizontal distance is further increased, both the peak displacement and peak open-circuit voltage decrease, due to the disappearance of the potential barrier. Therefore, an optimal horizontal distance (0.016 m) is more helpful for energy harvesting.

Figure 17 shows the bifurcation diagrams of the vibration
amplitudes with respect to the normal force for different excitation frequencies when the acceleration excitation amplitude is $A = 1\, \text{g}$ and the horizontal distance is $d=0.016\, \text{m}$. As shown in Fig. 17a, the motion pattern of TEH is periodic in the whole range of the normal force. As the excitation frequency increases (such as $f = 14\, \text{Hz}$), the periodic motion (when the normal force is small) coexists with the aperiodic/chaotic oscillations (when the normal force is large), as presented in Fig. 17b. At excitation frequency of 17 Hz, the oscillations of the system are aperiodic/chaotic, as seen in Fig. 17c. In the end, the vibrations of the TEH become periodic as long as the excitation frequency is sufficiently large, as illustrated in Fig. 17d.

5. Conclusions

This paper proposed a non-linear sliding-mode TEH by equipping friction plate with a bird-shaped cantilever. Firstly, the conceptual design scheme of the non-linear TEH

![Figure 16](image1.png)

**Figure 16** The time histories when the excitation amplitude $A = 1\, \text{g}$, and the excitation frequency of $f = 10\, \text{Hz}$: a the displacement response, b the open-circuit voltage response.

![Figure 17](image2.png)

**Figure 17** Bifurcation diagrams of the vibration amplitudes $w$ for the normal force $F_N$ when the excitation frequency of a $f = 11\, \text{Hz}$, b $f = 14\, \text{Hz}$, c $f = 17\, \text{Hz}$, d $f = 20\, \text{Hz}$; $A = 1.0\, \text{g}$, $d = 0.016\, \text{m}$.
is elaborated, and the dynamic model is established based on the Macro-slip friction theory. Then, the effects of friction in sliding-mode TEH on dynamic behaviors and electrical performances are studied in detail. The effects of parameters on both the mechanical and electrical responses are also discussed. The main conclusions are given as follows.

(1) The Macro-slip friction model is equivalent to the Coulomb friction model when the normal force is sufficient small and sufficiently large. Nevertheless, the Coulomb friction model would be unable to predict the dynamic responses of the TEH when the normal force is in a certain range, owing to neglecting the tangential contact stiffness.

(2) With the increase of the normal force or the friction coefficient, the frequency band of the interwell oscillations decreases, leading to performance degradation of energy harvesting.

(3) When the magnets are close or far from each other, the system oscillates slightly in the potential energy wells. There exists a proper distance making the TEH oscillates between the potential energy wells to fulfill excellent performance of energy harvesting.

(4) When the base excitation is tiny, the TEH cannot cross the energy barrier and thus undergoes intrawell oscillations only. Interwell oscillations coexist with intrawell oscillations when the base excitation is sufficiently large. With the increase in the base excitation, the frequency band of interwell oscillations becomes broader.

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非线性滑动式摩擦发电机的摩擦建模与分析

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摘要 由于静电感应效应，滑动式摩擦发电机(TEH)在滑动摩擦过程中发生电荷转移，从而实现对机械能量的俘获。摩擦不仅直接影响滑动式TEH中介电质之间的电荷转移过程，还会改变TEH的动力学响应进而影响其能量收集性能。因此，研究摩擦对滑动式TEH的影响规律对评估其环境振动能俘获性能具有重要意义。本文设计了一种非线性滑动式TEH，并通过调整磁块之间的距离实现了双稳态和单稳态的自由切换，从而使其具有不同的动力学机制。基于库仑摩擦模型和宏观滑动摩擦模型，文章建立了滑动式TEH的理论摩擦模型，进一步，应用哈密顿原理建立了相应的动力学模型，分析了摩擦对滑动式TEH性能的影响规律。为进一步探究TEH在低频下的能量收集效率，文章还系统研究了关键参数对TEH动力学行为和电学输出响应的影响，研究结果为滑动式TEH的设计与分析提供了理论参考。