Mathematical simulation of bearing ring grinding process

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Abstract. The paper suggests the method of forming a solid finite element model of the bearing ring. Implementation of the model allowed one to evaluate the influence of the inner cylindrical surface grinding scheme on the ring shape error.

1. Introduction

Rolling bearings are the main types of support for revolving and rolling parts of machines and mechanisms of various purposes. Machine improvements such as an increase of service life, speeds and productivity, reduction of sizes and weight and increase of precision requires further improvement of rolling bearings construction and improvement of technological processes of their manufacture.

Performance of mechanical devices is defined by the quality of the elements of which they are assembled. The most critical component of the quality is the manufacture precision.

Study of the strained condition of the outer bearing ring was conducted in order to evaluate the influence of the inner cylindrical surface grinding scheme on the ring shape error.

2. Materials and methods

Three grinding schemes were examined: internal chuck grinding (Figure 1, a), centerless grinding by means of cross-feed roller (Figure.1, b), centerless grinding on fixed support (Figure.1, c).

Figure 1. The grinding schemes: a – internal chuck grinding; b – centerless grinding by means of cross-feed roller; c – centerless grinding on fixed support.
The influence of the inner cylindrical surface grinding scheme on the ring shape error was studied by the numerical method of finite elements (FEM) and can be found in various papers, [1, 2, 3, 4]. The main quality of FEM is the possibility to account for real surface shapes under any loading conditions, freedom to choose position for nodal points, uniformity of design diagrams, which significantly extends the scope of solvable tasks and reduces time spent on calculation [5].

For the study the authors chose a bearing ring with the nominal outer diameter of the ring equal to $D = 210$ mm, inner diameter $d = 177$ mm, width – $152$ mm. Ring material is steel 100Cr6 according to Euronorm (52100 -for US), $\sigma_s = 740$ MPa , $\sigma_f = 420$ MPa , HB=185. The ring material was considered as isotropic Hookean substance. Temperature loads were not considered in calculations.

Bearing ring surface was evenly divided into variety of three-dimensional octagonal elements constituting a frame. Frame curves on the interval of change of borders of one frame cell $(i, i+1)$ were described by parametric lines $Y = Y(z)$ when $x = const$ , $Y = Y(x)$ when $z = const$ in the $XYZ$ coordinates system. Polynomials of the third order were used for defining frame curves as described in [6, 7]:

$$Y = a_0 + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3$$
$$Y = a_0 + a_1 \cdot z + a_2 \cdot z^2 + a_3 \cdot z^3$$

Factors are: $a_{jx} = \frac{\Delta a_{jx}}{\Delta x}$, $a_{jz} = \frac{\Delta a_{jz}}{\Delta z}$, $j=0,1,2,3$

$$\Delta x = \begin{bmatrix} x_{i-1}^2 & x_{i-1}^3 \\ x_i^2 & x_i^3 \\ x_{i+1}^2 & x_{i+1}^3 \\ x_{i+2}^2 & x_{i+2}^3 \end{bmatrix}, \Delta z = \begin{bmatrix} z_{i-1}^2 & z_{i-1}^3 & z_{i-1}^4 \\ z_i^2 & z_i^3 & z_i^4 \\ z_{i+1}^2 & z_{i+1}^3 & z_{i+1}^4 \\ z_{i+2}^2 & z_{i+2}^3 & z_{i+2}^4 \end{bmatrix}$$ (1)

$i = 1,...,n-2$, where $n$ is a number of frame line points of division.

Determiners $\Delta a_{jx}$, $\Delta a_{jz}$ are defined by substitutions of corresponding values of $y_i$ into (1), for example:

$$\Delta a_{jx} = \begin{bmatrix} y_{i-1} & x_{i-1}^2 & x_{i-1}^3 \\ y_i & x_i^2 & x_i^3 \\ y_{i+1} & x_{i+1}^2 & x_{i+1}^3 \\ y_{i+2} & x_{i+2}^2 & x_{i+2}^3 \end{bmatrix}, \Delta a_{jz} = \begin{bmatrix} y_{i-1} & z_{i-1}^2 & z_{i-1}^3 & z_{i-1}^4 \\ y_i & z_i^2 & z_i^3 & z_i^4 \\ y_{i+1} & z_{i+1}^2 & z_{i+1}^3 & z_{i+1}^4 \\ y_{i+2} & z_{i+2}^2 & z_{i+2}^3 & z_{i+2}^4 \end{bmatrix}$$ (2)

Because of calculations, a coordinate system of 720 equally-spaced nodal elements was obtained. Groups of 8 nodal elements were united into octagonal finite elements. To increase calculation precision, each of finite elements was consequently split into 5 tetrahedral elements with rectilinear sides like Figure 2.

Maximum error in receiving the required surface corresponds to nodal elements $(x_i, z_{i+1})$ and $(x_{i+1}, z_i)$. The deviation value in the specified nodal elements is defined in the following way:

$$\Delta y_{i,i+1} = (a_{i1} + 2a_{i2} \cdot z + 3a_{i3} \cdot z^2) \cdot \Delta z - (y_{i,i+1} - y_{i,i})$$
$$\Delta y_{i+1,i} = (a_{i1} + 2a_{i2} \cdot x + 3a_{i3} \cdot x^2) \cdot \Delta x - (y_{i+1,i} - y_{i,i})$$ (3)
Figure 2. The division of the finite element into 5 tetrahedral elements with rectilinear sides.

Figure 3. The finite element model of the ring.

The finite element model of the ring shown in Figure 3 is formed as a result of gluing a variety of tetrahedral and octagonal elements.

Displacement of nodal elements was calculated according to the matrix equation of the following type according to [5]:

\[ \{U\} = [K]^{-1} \{F\}, \]  

(4)

where \( \{U\} \) is a column vector of ring nodal elements displacement; \( \{F\} \) is a column vector of external force in the nods; \([K]^{-1}\) - ring pliability matrix.

A technological sequence of a grinding process consists of the following stages: installing, locating, fastening and external disturbing effect. As a result, ring elements contact elements of the grinding technological system in some areas. The assumption was made for the calculation that technological system surface elements were given without considering shape deviation, position and relief condition. The system of external force action (ef) affects nodal (i) elements of the bearing ring and defines stages when ring strained state appears.

\[ \{F_{ef}\} = \{P_i\}, \{P_{i+1}\}, \ldots, \{P_{i+m}\}, \{Q_i\}, \{F_i\}, \{M_i\} \]  

(5)

where \( F_i \) is a sum of external forces, \( M_i \) is a sum of moments, \( P_i \) is mass, \( Q_i \) is the retaining pressure, affecting nodal element of the ring; \( m \) is the total number of nodal elements of the ring.

The stiffness matrix for a tetrahedral finite ring element represents an integral described in [3, 8]:

\[ [K] = \int_B [B]^T [D][B]dV \]  

(6)

where \( V \) is a volume of tetrahedron; \( [B] \) is the matrix of tetrahedron gradients; \( [D] \) is the matrix of tetrahedron elastic constants:

\[
[B] = \frac{1}{6V} \begin{bmatrix}
    b_i & 0 & 0 & b_j & 0 & 0 & b_k & 0 & 0 & b_l & 0 & 0 \\
    0 & c_i & 0 & 0 & c_j & 0 & 0 & c_k & 0 & 0 & c_l & 0 \\
    0 & 0 & d_i & 0 & 0 & d_j & 0 & 0 & d_k & 0 & 0 & d_l \\
    c_i & b_i & 0 & c_j & b_j & 0 & c_k & b_k & 0 & c_l & b_l & 0 \\
    d_i & 0 & b_i & d_j & 0 & b_j & d_k & 0 & b_k & d_l & 0 & b_l \\
    0 & d_i & c_i & 0 & d_j & c_j & 0 & d_k & c_k & 0 & d_l & c_l
\end{bmatrix}
\]
where \( v_1 = \frac{\mu}{1-\mu} \), \( v_2 = \frac{1-2\mu}{2(1-\mu)} \), \( E \) and \( \mu \) are modulus of elasticity and tetrahedron material Poisson's ratio correspondingly. Physical-mechanical properties of finite elements were considered constant within the limits of each finite element.

To evaluate adequacy of the finite element model of the ring, a series of numerical experiments was conducted. Workpiece-retaining pressure \( Q \) and cutter force \( F \) were the variable factors.

3. Results and Discussion

Numerical values of extent of the error of the shape of the inner ring surface were obtained as a result of calculations. Maximum extent of the error was obtained during chuck grinding of the internal ring surface. Minimum extent of the error corresponds to the scheme of grinding on fixed support. Calculation results are shown in Figure 4.

4. Conclusion

Results of numerical experiments correlate with experimental study of the centerless grinding error according to [9, 10]; therefore the developed finite element model can be used for prediction of the processing error depending on structural, construction and operational parameters of the technological system of grinding on the design phase of grinding technological operation for the inner cylindrical surface of the bearing ring.

Figure 4. Calculation results: a – inner cylindrical surface shape error depending on the retaining pressure; b – inner cylindrical surface shape error depending on the cutter force.

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