Angular momentum and clustering properties of early dark matter halos

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ABSTRACT

In this paper we study the angular momentum properties of simulated dark matter halos at high redshift that likely host the first stars in the Universe. Calculating the spin distributions of these $10^6 - 10^7 M_{\odot}$ halos in redshift slices from $z = 15 - 6$, we find that they are well fit by a log-normal distribution as is found for lower redshift and more massive halos in earlier work. We find that both the mean value of the spin and dispersion are largely unchanged with redshift for all halos. Our key result is that subsamples of low and high spin $10^6 M_{\odot}$ and $10^7 M_{\odot}$ halos show difference in clustering strength. In both mass bins, higher spin halos are more strongly clustered in concordance with a tidal torquing picture for the growth of angular momentum in dark matter halos in the CDM paradigm.

Key words: cosmology: dark matter – cosmology: early Universe – galaxies: high-redshift – galaxies: formation

1 INTRODUCTION

In the standard $\Lambda$CDM paradigm of structure formation, dark matter over-densities accrete mass, forming deep potential wells into which baryons condense and cool to eventually form stars. The details of the cooling and star formation process are quite complex, and the physics of the first episode of star formation and that of subsequent generations is not completely understood. There have been detailed numerical studies of the formation of the first stars which suggest that the masses of these stars were skewed to the massive end (Abel, Bryan, & Norman 2002; Bromm, Coppi, & Larson 2002; Omukai & Palla 2003; O'Shea & Norman 2007). Numerical simulations are challenging owing to the large dynamical range needed as well as the complexities of hydrodynamical modeling required to capture the essential physics. Current models of galaxy formation in the early Universe are able to make some simplifying assumptions to make the simulations of subsequent star formation easier (see, e.g., Grief et al. 2003; Johnson, Grief, & Bromm 2008; Ricotti, Gnedin, & Shull 2002; Wise & Abel 2007; Wise, Turk, & Abel 2008). However, much work still needs to be done to fully uncover the details of galaxy formation, including a better understanding of the properties of the dark matter halos and how the dark matter properties affect baryonic structure formation.

In numerical simulations, three basic properties of dark matter are followed: mass, position, and velocity. From these three properties, much has been learned about the buildup and assembly of early dark matter halos, including halo mass functions, galaxy clustering, halo shapes, and halo spins (Reed et al. 2007; Jang-Condell & Hernquist 2001; Reed et al. 2003, 2008; Maccio, Dutton, & van den Bosch 2008). Refinement in our understanding of dark matter halos is needed to gain insight into how the baryonic component of galaxies couples with the dark matter (e.g., angular momentum transfer, stellar and AGN feedback). A key missing piece is the role of angular momentum in the formation and modulation of structure formation. This paper focuses on the angular momentum and clustering properties of dark matter halos in the early ($z = 15 - 6$) Universe, roughly 2 Gyr after the Big Bang in the concordance model. The goal of this work is to understand more about the early, low mass dark matter halos that likely host the earliest stars and galaxies in a statistically large sample.

2 ORIGIN OF ANGULAR MOMENTUM IN DARK MATTER HALOS

In the standard $\Lambda$CDM paradigm, angular momentum is acquired by dark matter halos due to tidal torquing by their neighbors (Hodil 1949). During the linear phases of protogalactic evolution the angular momentum grows to first order in proportion to time $t$. Peebles (1969) found from the study of collapsing spherical regions that the growth in the angular momentum $\mathbf{J}$ occurs in the second order only as
In such regions growth occurs purely as a result of convective effects on the baryonic surface. This was calculated using linear theory to find the growth rate of the angular momentum within a co-moving spherical region of an expanding Friedmann Universe. Doroshkevich (1970) showed that while the angular momentum of a proto-galaxy did grow $\propto t$ for a flat Universe, the Peebles result was a consequence of the imposed spherical symmetry and was verified numerically by White (1984). We briefly outline the process of angular momentum acquisition by a dark matter halo in what follows.

Consider the growth of fluctuations in an expanding Friedmann Universe filled with pressurized matter with density, $\rho(\vec{r}, t)$, and let

$$\delta(\vec{r}, t) = \frac{\rho(\vec{r}, t)}{\rho_0(\vec{r}, t)} - 1; \quad \delta(\vec{x}, t) = b(t) \delta_0(\vec{x}). \quad (1)$$

The local overdensity can be written as a separable function of time and a comoving coordinate $\vec{x}$ related to $\vec{r}$ by $\vec{r} = a(t)\vec{x}$, where $a(t)$ is the cosmological expansion factor. The trajectory of a dust particle in linear theory is given by:

$$\vec{r}(\vec{q}, t) = a(t) \vec{x}(\vec{q}, t) = a(t) [\vec{q} - b(t) \nabla \phi(\vec{q})] \quad (2)$$

where $\vec{q}$ is the Lagrangian coordinate defined as the $\vec{x}$ position of the particle as $t \to 0$ and $\vec{q}$ the peculiar gravitational potential. If $\langle \delta^2 \rangle \ll 1$, a condition that is satisfied over the period of time preceding the collapse only when the initial density field has a coherence length of the proto-galactic scale. In general, this will cease to describe the detailed evolution of the matter distribution long before galaxies form. The spin angular momentum of the material that makes up a proto-galaxy can be written as,

$$\vec{J}(t) = \rho_0 a^5 \int_{V_L} (\vec{x} - \vec{x}) \times T \, d^3 q \quad (3)$$

Since the leading term in $\vec{x}$ is first order and is parallel to $\vec{x}$, the 2nd order term gives the non-zero contribution to the above integral. Writing it explicitly as an integral over the surface (using Gauss's theorem), we have:

$$\vec{J}(t) = -a^6 b \epsilon_{ijk} T_{jl} I_{lk} \quad (4)$$

where $T$ is the tidal tensor and describes the local deformation at $\vec{q}$ and $I$ is the inertia tensor of the matter in $V_L$. This tensor product is used to calculate the tidal torque on an extended body in the tidal field. Angular momentum is acquired as the first order tidal field couples to the zeroth order quadrupole moment of the irregular boundary of the proto-galaxy. The principal axes in general do not coincide as $T$ defines the shape and disposition of the neighboring perturbations and $I$ depends only on the shape of the proto-galaxy. This outlines in principle the origin and offers insight into the calculation of angular momentum of simulated dark matter halos.

3 METHOD

We ran a series of N-body simulations to follow the growth of dark matter halos from $z \approx 100$ down to $z = 6$. In order to study halos that likely host Pop III stars, we chose the particle mass such that a $10^6 M_\odot$ dark matter halo would have $\approx 100$ particles. For $512^3$ particles, this requirement sets the box size at 2.46 Mpc/h and the particle mass at $M_{DM} = 7.3 \times 10^4 M_\odot$. The initial conditions were generated using a parallelized version of Grafic (Prunet et al. 2003), which calculates the Gaussian random field for the dark matter particles. We used Gadget-2 (Springel 2003) to follow dark matter particles down to a redshift of $z = 6$, with output snapshots at $z = 15, 12, 11, 10, 9, 8, 7, 6$. We used the WMAP3 (Spergel et al. 2007) cosmological parameters for the runs presented here, $[\Omega_M, \Omega_L, \Omega_b, h, n, \sigma_8] = [0.238, 0.762, 0.0416, 0.732, 0.958, 0.761]$. For each snapshot, we used a friends-of-friends code to find individual halos, using a linking length of 0.2 times the initial particle spacing. Once the halos were identified, each particle in the halo was tested to see if it was actually bound to the halo. We used SKID (Stadel 2001) to do the unbinding. SKID finds the potential and kinetic energies for all particles in a given halo and removes the most unbound particle from the halo. Successive iterations are performed until all particles are either bound or there are no more particles in the halo. Without unbinding, there was excess power in both tails of the spin distribution.

Once unbound particles were removed, for each halo that had more than 50 dark matter particles, we calculated the dimensionless spin parameter,

$$\lambda = \frac{J/E}{GM^3/2},$$

where $J$ is the total angular momentum, $E$ the total energy (kinetic plus potential), and $M$ the total mass of the halo. For the most massive halos, with more than $10^4$ particles, we used the hierarchical force calculator Treecoq v1.4 (Barnes & Hut 1986) to get the potential energy, inducing an error of 0.2% in the potential energy as compared to direct summation.

4 RESULTS

Figure 1 shows the spin distribution at four redshifts $(z = 6, 10, 12, 15)$. As has been found for more massive halos at low redshift (Bett et al. 2007; Ballin & Steinmetz 2005, Cole & Lacey 1996, Steinmetz & Bartelmann 1996, Warren et al. 1992, e.g.), the distribution is log-normal, with a slight excess at low spin values. At $z = 10$, Jang-Condell & Hernquist (2001) also found a log-normal distribution, with $\langle \lambda \rangle = 0.033$ and $\sigma = 0.52$, while we find $\langle \lambda \rangle = 0.04$ and $\sigma = 0.55$ at the same redshift. These differences may be due to the increased resolution, volume, and number of particles in our study. In addition, Jang-Condell & Hernquist (2001) used a different halo finding algorithm (SKID’s density maxima finder), and found a Press-Schechter mass function. However, our mass function matches the Reed et al. (2007) modification to the Sheth-Tormen mass function. Finally, the differences between their results and our work may be due to different cosmological parameters chosen, particularly $\sigma_8$. In future work, we plan to study the effect of mass resolution on the derived angular momentum properties as well as the effect of changing $\sigma_8$.
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Figure 1. Spin parameter distribution for all halos at four redshifts. The solid curve is a log-normal fit to the data. The right two panels show the change in mean $\lambda$ and the standard deviation for the log-normal fit for all redshift outputs used in this study.

found only slight variation in the mean spin, with $\lambda = 0.0378$ at $z = 15$ increasing to $\lambda = 0.0426$ at $z = 6$. The small increase in spin over time is most likely due to stronger tidal torques from more massive halos that collapse at later times. The standard deviation decreases slightly from $\sigma = 0.573$ at $z = 15$ to $\sigma = 0.553$ at $z = 6$. We show in Figure 2 the spin distribution for halos with mass within $\pm 0.2$ dex of $10^7 M_\odot$ and $10^6 M_\odot$ at the same redshifts as Figure 1. For the $10^7 M_\odot$ halos, we find that $\langle \lambda \rangle$ does not monotonically increase with redshift as it did for the whole sample, and $\langle \lambda \rangle$ is systematically larger than the sample mean at any given redshift. There is also a sharp increase in $\langle \lambda \rangle$ at $z = 15$, possibly because these halos were still in the process of forming and may have significant infalling mass clumps. However, there are only 55 halos in this mass bin at $z = 15$, making it difficult to reliably fit the sample with a log-normal curve. The $10^6 M_\odot$ halos show spin distributions more similar to the entire sample, with a slowly increasing mean spin from $z = 15$ to $z = 6$.

5 CORRELATION FUNCTION OF HIGH-REDSHIFT DARK MATTER HALOS

We also studied the clustering properties of dark matter halos selected by their angular momentum. It is well known that more massive halos are strongly clustered (e.g., Mo & White 2002; Sheth & Tormen 1999; Lemson & Kauffmann 1999). Lemson & Kauffmann (1999) found no correlation between $\lambda$ and environment for halos with $M_{tot} > 10^{12} M_\odot$ at $z = 0$. (see also Cervantes-Sodi et al 2008; Maccio et al 2007; Berta et al 2008; Bett et al 2007). This diagnostic is potentially important as a halo of mass $M$ with larger angular momentum may take longer to collapse and therefore longer to form stars than a lower spin halo of the same mass. Thus, low spin halos could preferentially host star formation earlier. This in turn may allow feedback processes such as ionizing photons, stellar winds, and supernovae to become important earlier than in their high spin counterparts. These are important consequences for galaxy formation, and the detailed role of the spin of the halo on the collapse of baryons will be explored in a subsequent paper.

We calculate the two-point correlation function, $\xi(r)$, using the standard definition,

$$ \xi(r) = \frac{DD(r)RR(r)}{RD(r)^2} - 1, $$

where $DD(r)$ is the number of halo-halo pairs within a separation of $r - dr/2$ and $r + dr/2$, $RR(r)$ the same for a random distribution, and $RD(r)$ is the cross-correlation between the data set and the random distribution. The quantity $\xi(r)$ represents the excess probability of finding a halo at distance $r$ compared to a random distribution.

Figure 3 shows the correlation function, $\xi(r)$, for halos grouped by spin parameter for a given mass. The top panel is for halos with mass $10^6 M_\odot$, and the bottom panel for $10^7 M_\odot$ halos. We separated the halos out by mass to delineate the effect of spin from that of mass. We first computed the overall correlation function for all halos in the given mass bin at $z = 15$. We then used this as the reference distribution and compared it to the spin selected halos.
range, and then used two sub-samples based on their spin parameter. The cuts in spin parameter were chosen so that one third of the halos were in the high and low spin bins respectively. The error bars show Poisson errors in $DD(r)$, 

$$\sigma_\xi = \frac{RR(r)}{RD(r)^2} \sigma_{DD(r)}.$$ 

When counting halo pairs for the spin cut samples, we employed two methods. First, we counted only pairs where both halos were in the given bin, and these results are shown in Figure 3. We also counted the pairs where only one halo was in the high or low spin bin. This method of counting produced similar results as the first method.

We find that there is a distinction in the correlation function when the data are separated by the spin parameter. High spin halos are more strongly clustered than low spin halos, similar to the result found by Bett et al. (2007) in the Millennium Simulation. We also find that the difference in clustering is preserved over a range of redshifts. The excess clustering is due to the fact that in a denser environment, halos feel stronger tidal torques, and thus have larger angular momentum and spin parameters. The correlation function can be fit to a power-law, $\xi(r) = (r/R)^\gamma$, where $R$ is the correlation length. We find that the high spin halos have a correlation length on average 25% larger than the low spin halos over the various mass and redshift slices used here. The slope, $\gamma$, did not show any correlation with spin, mass, or redshift.

6 DISCUSSION

In this paper, we have examined the angular momentum distribution of simulated high redshift dark matter halos and their correlation functions. While the angular momentum distributions for this population are well fit by log-normal distributions as is the case for lower redshift and more massive halos, we find that the clustering properties of the high redshift population depend on the value of spin. For a given mass bin selecting by spin, we find that the correlation function is higher for high spin halos. This is an important trend and appears to be robust. Although in this study we have restricted ourselves to the analysis of dark matter only simulations, the different correlation lengths of high and low spin halos are likely to have an important impact in feedback from these early galaxies. Pop III stars appear to have a large impact on their environment due to radiative (Johnson, Grief, & Bromm 2007; Whalen et al. 2008) and supernova (Grief et al. 2007; Whalen et al. 2008) feedback.

We note here that in a recent paper O'Shea & Norman (2007) tracked the spin parameters of halos that formed the first stars in 12 cosmological random realizations and did not find a correlation between halo spin and collapse time. The resolution and analysis pursued by them differ from ours as they zoom in and track (by explicit selection) the properties of the dark matter halo that hosts the first star. Our analysis is distinct by construction as we are measuring the spin parameters of an ensemble of dark matter halos and arguing
Figure 3. Correlation function for $10^6 \, M_\odot$ (top panel) and $10^7 \, M_\odot$ (bottom panel) halos as a function of comoving separation, $r$. Each plot shows the correlation function for all halos in the mass bin (solid), as well as for a cut based on the halo spin parameter. The cuts were chosen so that one third of the halos would fall in the high (dotted) and low (dashed) bins.
that the higher spin halos might in general commence the
collapse and formation of its first star later. As shown in
Figure 2, the distribution of halo spin for the high mass bin
at \( z = 15 \) is shifted to higher spins when compared to the
general population. Thus, the larger halos that would host
Pop III stars will have larger spins, but the largest spin that
O’Shea & Norman (2007) followed was for \( \lambda \approx 0.1 \), which
is at the half-max of the log-normal fit. One halo with this
spin may not be representative of all high spin, massive ha-
los, which may show a delayed baryonic collapse. While this
is beyond the scope of this paper, we intend to pursue the
consequences of our results for baryonic collapse.

Our results are in agreement with what Bett et al. (2007)
found for the correlation of dark matter halos at
\( z = 0 \) in the Millenium Run. Berta et al. (2008) found trends
in the SDSS spectroscopic sample of local galaxies relating
the estimated dark matter spin parameter with the galaxy’s
baryonic properties, such as stellar mass and recent star for-
mation histories. Thus, angular momentum may be a key to
determining the epoch of formation, size, and structure of the
stellar and gaseous disks in galaxies. In the early Uni-
veme however, it remains unclear how much effect angular
momentum will have on the first generation of galaxy for-
mation, although our results are suggestive.

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