A possible supersymmetric solution to the discrepancy between $B \to \phi K_S$ and $B \to \eta' K_S$ CP asymmetries

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We present a possible supersymmetric solution to the discrepancy between the observed mixing CP asymmetries in $B \to \phi K_S$ and $B \to \eta' K_S$. We show that due to the different parity in the final states of these processes, their supersymmetric contributions from the R-sector have an opposite sign, which naturally leads to $S_{\phi K_S} \neq S_{\eta' K_S}$. We also consider the proposed mechanisms to solve the puzzle of the observed large branching ratio of $B \to \eta' K$ and their impact on $S_{\eta' K_S}$.

Various measurements of CP violation in B factory experiments have been opening a new era for the phenomenology of supersymmetric (SUSY) models. While in the standard model (SM), all the CP violating phenomena have to be explained by a single phase in the Cabbibo-Kobayashi-Maskawa matrix, the SUSY models include additional new sources of CP violation. Since these new effects can manifest themselves in the CP asymmetries of various B-meson decays, the recent observed large discrepancy among the CP asymmetries of $B \to J/\psi K_S$, $B \to \phi K_S$ and $B \to \eta' K_S$ have raised high expectations for indirectly unveiling low energy SUSY [1].

The measurement of the angle of the unitarity triangle $\beta(\phi_1)$ by the so-called golden mode $B \to J/\psi K_S$ [2,3]:

$$S_{J/\psi K_S} = \sin 2\beta(\phi_1) = 0.734 \pm 0.054$$

is in a good agreement with the other measurements based on the standard model analysis. Flowingly, it has been shown that the effect from the SUSY particles in the box diagram which leads to the $B^0 - \bar{B}^0$ mixing is typically small [4]. On the contrary, in summer 2002, the B factory experiments reported a surprising result for the measurement of $\beta$ by using the $B \to \phi K_S$ process. Since in the SM, $B \to J/\psi K_S$ and $B \to \phi K_S$ have the same $B^0 - \bar{B}^0$ mixing part and do not have any additional CP violating phase in the decay process, the same value of $\sin 2\beta$ was expected to be extracted from them. Thus, the discovered large discrepancy [5,3]

$$S_{\phi K_S} = -0.39 \pm 0.41$$

has created quite a stir. Several efforts to explain this experimental data, in particular, by using SUSY models, have been made. In Ref. [6], it has been shown that this phenomena can be understood without contradicting the smallness of the SUSY effect on $B \to J/\psi K_S$ in the framework of the mass insertion approximation which allows us to perform a model independent analysis of the SUSY breakings [7]. In this approximation, SUSY contributions are proportional to the mass insertions $(\delta_i^{d^{(i)}})_{AB}$ where $i, j$ and $A, B$ are the generation and chirality indices, respectively. While the measurement of $B \to J/\psi K_S$ implies the smallness of $(\delta_{23}^{d^{(i)}})_{AB}$, $(A, B = L, R)$, the different generation mass insertion contributing to the $B \to \phi K_S$ process, $(\delta_{23}^{d^{(i)}})_{AB}$, can deviate $S_{\phi K_S}$ from $S_{J/\psi K_S}$. In this letter, we discuss another measurement of $\sin 2\beta$ [3,8]

$$S_{\eta' K_S} = 0.33 \pm 0.34$$

which has been thought to be problematic [1]. Since $B \to \eta' K_S$ gets contributions from $(\delta_{23}^{q})_{AB}$, $S_{\eta' K_S}$ and $S_{\phi K_S}$ were expected to display similar discrepancy from $S_{J/\psi K_S}$. We will first show that although the magnitude of the SUSY contributions to these processes are indeed similar, $B \to \eta' K_S$ has an opposite sign in the coefficient for the RL and RR mass insertions, which can naturally explain the experimental data. In fact, there is another open question on the $B \to \eta' K$ process, the observed unexpectedly large branching ratio [9]. We will further investigate the proposed new mechanisms to enhance the branching ratio of $B \to \eta' K$ and their impacts on $S_{\eta' K_S}$.

The Effective Hamiltonian for the $\Delta B = 1$ processes induced by gluino exchanges can be expressed as

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^{\ast} \sum_{i=3-6,g} \left[ C_i O_i + \tilde{C}_i \tilde{O}_i \right]$$

where the operators $\tilde{O}_i$ can be obtained from $O_i$ by exchanging $L \leftrightarrow R$. The Wilson coefficients $C_i$ and $\tilde{C}_i$ are proportional to $\delta_{LL,LR}$ and $\delta_{RR,RL}$, respectively. The definition of the operators and Wilson coefficients (and the effective Wilson coefficients below) can be found in [6]. Employing the naive factorisation approximation [10], where all the colour factor $N$ is assumed to be 3, the amplitude for the $B \to \phi K$ process can be expressed as

$$\mathcal{A}(\phi K) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^{\ast} \sum_{i=3}^{6} \left[ C_i^{\text{eff}} + \tilde{C}_i^{\text{eff}} \right] \langle \phi K_0 | O_i | B_0 \rangle$$

where the matrix element is given by

$$\langle \phi K_0 | O_i | B_0 \rangle = \left\{ \frac{4}{3} X, \frac{4}{3} X, \frac{1}{3} X \right\} (i = 3 - 6)$$

with $X = 2 F_i^{B\to K}(m_\phi^2) f_\phi m_\phi (p_K \cdot e_\phi)$. $F_i^{B\to K}(m_\phi^2) = 0.35$ GeV is the $B - \bar{K}$ transition form factor and $f_\phi = 0.233$ GeV is the decay constant of the $\phi$ meson. Since both $F_i^{B\to K}(m_\phi^2)$ and $f_\phi$ are insensitive to the chirality of the quarks, we used

$$\langle \phi K_0 | O_i | B_0 \rangle = \langle \phi K_0 | \tilde{O}_i | B_0 \rangle$$

(7)
to derive Eq. (5). On the other hand, the amplitude for $B \to \eta' K$ can be written by:

$$\mathcal{A}(\eta'K) = \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \left[ \sum_{i=1}^{2} C_i^\text{eff} \langle \eta' K^0 | O_i | B^0 \rangle \right]$$

$$- \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=3}^{6} C_i^\text{eff} - C_i \right] \langle \eta' K^0 | O_i | B^0 \rangle, \quad (8)$$

where we used

$$\langle \eta' K^0 | O_i | B^0 \rangle = -\langle \eta' K^0 | \bar{O}_i | B^0 \rangle \quad (9)$$

which is derived by the fact that the decay constant of $\eta'$ is sensitive to the chirality of the quarks. The matrix element is given by:

$$\langle \eta' K^0 | O_1 | B^0 \rangle = \frac{1}{3} X_2, \quad \langle \eta' K^0 | O_2 | B^0 \rangle = X_2, \quad (10)$$

$$\langle \eta' K^0 | O_3 | B^0 \rangle = \frac{1}{3} X_1 + 2 X_2 + \frac{4}{3} X_3, \quad \langle \eta' K^0 | O_4 | B^0 \rangle = X_1 + 2 \frac{2}{3} X_2 + \frac{4}{3} X_3, \quad (12)$$

$$\langle \eta' K^0 | O_5 | B^0 \rangle = R_1 \left( X_1 - 2 X_2 + (-1 + \frac{2}{3} R_2) X_3 \right), \quad (13)$$

$$\langle \eta' K^0 | O_6 | B^0 \rangle = R_1 X_1 - 2 \frac{3}{2} X_2 + (-1 + \frac{3}{2} R_2) X_3, \quad (14)$$

with

$$X_1 = (m_B^2 - m_{\eta'}^2) F_{1B \to \pi} (m_{K^0}^2) \frac{X_{\eta'}}{\sqrt{2}} f_K,$$

$$X_2 = - (m_B^2 - m_{\eta'}^2) F_{1B \to K} (m_{K^0}^2) \frac{X_{\eta'}}{\sqrt{2}} f_K,$$

$$X_3 = - (m_B^2 - m_{\eta'}^2) F_{1B \to K} (m_{K^0}^2) \sqrt{2 f_{K}^2 - f_{\eta'}^2},$$

$$R_1 = \frac{2 m_{K^0}^2}{(m_B - m_{\eta'})(m_{K^0} + m_{d})}, \quad R_2 = \frac{2 (m_{K^0}^2 - m_{d}^2)}{(m_B - m_{\eta'})(m_{K^0} + m_{s})},$$

where $f_{K} (q^2)$ is the $B \to \pi$ transition form factor and $f_{K_\pi} = 0.16 (0.13)$ GeV is the decay constant of $K(\pi)$ meson. $q$ is the momentum transfer of the $B \to s$ transition. $X_{\eta'} = 0.57$ and $Y_{\eta'} = 0.82$, which correspond to $\theta_p = -20^\circ$, represent the rate of the $u \bar{u} + d \bar{d}$ and $s \bar{s}$ component in the $\eta'$ [11,12]. We use the following quark masses, $(m_d, m_s, m_{\eta'}) = (0.0076, 0.122, 4.88)$ GeV. The tree contributions to $B \to \eta' K_S$ is found to be less than 1% and can be ignored.

Numerical results on the ratio between SM and SUSY amplitudes for $m_{3g} \simeq m_{\bar{q}} = 500$ GeV are obtained as [6]

$$\left( \frac{A_{\text{BUSY}}^{\phi_K}}{A_{\text{SM}}} \right)_{\eta'K_S} \simeq (0.23 + 0.04i)(\delta_{LL}^d + \delta_{RL}^d) \quad (15)$$

$$+ (95 + 14i)(\delta_{LL}^d + \delta_{RL}^d) \quad (16)$$

where $\delta_{LL}$ and $\delta_{RL}$ are CP violating and conserving phase differences between SM and SUSY, respectively. $\delta_L$ and $\delta_R$ include the contributions proportional to the mass insertions $(\delta_{LL,LR}^d)^{23}$ and $(\delta_{RL,LR}^d)^{23}$, respectively. Using these parameters, the mixing CP asymmetry is given as [6]

$$S_{\phi_{K_S}} = \sin \frac{2\beta + 2R_\phi \cos \delta_{12} \sin(\theta_\phi + 2\beta)}{1 + 2R_\phi \cos \delta_{12} \sin(\theta_\phi + 2\beta)} S_{\eta'K_S} = \sin \frac{2\beta + 2R_{\eta'} \cos \delta_{12} \sin(\theta_{\eta'} + 2\beta)}{1 + 2R_{\eta'} \cos \delta_{12} \sin(\theta_{\eta'} + 2\beta)},$$

where we use $\sin 2\beta = 0.73$ in our analysis. As can be seen from the above formulae, the strong phase enters only as $\cos \delta_{12}$ and the small strong phases found in Eqs. (15) and (16) lead to $\cos \delta_{12} = 0.99$. Thus, we use $\cos \delta_{12} = 1$ in the following.

Here let us recall our main conclusions on $S_{\phi_{K_S}}$ in Ref. [6]. $S_{\phi_{K_S}}$ as a function of $\theta_\phi$ behaves as a $\sin \theta_\phi$ curve taking the value $S_{\phi_{K_S}} = 0.73$ at the origin and bounded above by 1. A typical behaviour of $S_{\phi_{K_S}}$ with $R_\phi = 0.5$ and $\cos \delta_{12} = 1$ is shown as the solid line in Fig.1. In the following, we will use this result as a reference and fix $R_\phi = 0.5$ and also focus on the region $-3\pi/4 < \theta_\phi < 0$. $S_{\eta'K_S}$ becomes negative.

Now let us discuss the $B \to \eta' K_S$ process and see if we can explain out the puzzle of the observed mixing CP asymmetries: $S_{\phi_{K_S}} \leq 0$ while $S_{\eta'K_S} \lesssim S_{J/\psi K_S}$. First,
we shall show our result without including the contributions from these new mechanisms suggested to enhance the branching ratio of $B \to \eta'K$ in order to see explicitly the different behaviours of $S_{\phi KS}$ and $S'_{\eta'K}$ due to the minus sign in Eq. (18). Having some possible SUSY models in our mind, we perform a case-by-case study in the following.

**Case 1** $|\delta_R| \gg |\delta_L|$

Eqs. (17) and (18) lead to
\begin{align*}
R_{\phi}e^{i\delta_{\phi}} & = |\delta_R|e^{i\arg \delta_R}, \\
R_{\eta'}e^{i\delta_{\eta'}} & = |\delta_R|e^{i(\arg \delta_R + \pi)}.
\end{align*}

The CP asymmetry $S_{\eta'K}$, as a function of $\arg \delta_R (= \theta_\phi)$ is shown as a dashed line in Fig. 1. $|\delta_R|$ is fixed to have $R_{\phi} = |\delta_R| = 0.5$. As can be seen from this figure, $S_{\eta'K}$ is always larger than the experimental data in Eq. (3) where $S_{\phi KS}$ is within the experimental range. Note that the $|\delta_L|$ dominated models give a same curve as $S_{\phi KS}$.

**Case 2** $|\delta_L| = |\delta_R|$

In this case, Eqs. (17) and (18) are reduced to:
\begin{align*}
R_{\phi}e^{i\delta_{\phi}} & = 2|\delta_L|\cos \frac{\Delta \theta}{2}e^{i(\arg \delta_L + \arg \delta_R)/2} \\
R_{\eta'}e^{i\delta_{\eta'}} & = 2|\delta_L|\sin \frac{\Delta \theta}{2}e^{i(\arg \delta_L + \arg \delta_R + \pi)/2}
\end{align*}

where $\Delta \theta = \arg \delta_L - \arg \delta_R$. We depict $S_{\eta'K}$ as a function of $(\arg \delta_L + \arg \delta_R)/2(= \theta_{\eta'})$ for $\Delta \theta = \pi/10$ as the dotted line in Fig. 1. We fix $|\delta_L|$ so as to have $R_{\phi} = 0.5$. The $\pi/2$ shift appearing in Eq. (22) can be clearly seen in the plot. It is also remarkable that in this case, not only the phase shift between $\theta_\phi$ and $\theta_{\eta'}$ but also the amplitude difference which is given in terms of $\Delta \theta$ differentiate the behaviour of $S_{\phi KS}$ and $S_{\eta'K}$. In particular, for small $\Delta \theta$, no matter what the value of $|\delta_L|$ is, $S_{\eta'K}$ takes a value close to $\sin 2\beta$.

**Case 3** $\arg \delta_L = \arg \delta_R$

In this case, we have
\begin{align*}
R_{\phi}e^{i\delta_{\phi}} & = (|\delta_L| + |\delta_R|)e^{i\arg \delta_L} \\
R_{\eta'}e^{i\delta_{\eta'}} & = \Delta |\delta| e^{i\arg \delta_L}
\end{align*}

where $\Delta |\delta| = |\delta_L| - |\delta_R|$. We show our results for $S_{\eta'K}$ in terms of $\arg \delta_L (= \theta_{\phi})$ in Fig. 1 for $R_{\phi} = |\delta_L| + |\delta_R| = 0.5$ and $\Delta |\delta| = 0.2$ (dash-dotted line). We found that the experimental bound gives a constraint of $0 \lesssim \Delta |\delta| \lesssim 0.4$.

**Case 4** $\arg \delta_R = \arg \delta_L + \pi/2$

In this case, we have
\begin{align*}
R_{\phi}e^{i\delta_{\phi}} & = \sqrt{|\delta_L|^2 + |\delta_R|^2}e^{i(\arg \delta_L + \alpha)} \\
R_{\eta'}e^{i\delta_{\eta'}} & = \sqrt{|\delta_L|^2 + |\delta_R|^2}e^{i(\arg \delta_L - \alpha)}
\end{align*}

where $\tan \alpha = |\delta_R|/|\delta_L|$. In Fig. 1, we plot the result of $S_{\eta'K}$ as a function of $\arg \delta_R + \alpha(= \theta_{\phi})$ for $R_{\phi} = \sqrt{|\delta_L|^2 + |\delta_R|^2} = 0.5$ with $\alpha = 5\pi/4$ (dash-double-dotted line). With the phase shift of $2\alpha$, one can have both $S_{\phi KS}$ and $S_{\eta'K}$ within their experimental range.

We should comment that the above model independent analysis can be realised in well known SUSY models. For example, the SUSY models with Hermitian flavor structure which provide an interesting solution for the SUSY CP problem [16], have $\delta_{LR} = (\delta_{LL})^*$ with negligible ($\delta_{LL,RR}$), which is a realisation of Case 2 with $\Delta \theta = \pi$. Also in the SUSY seesaw models which are motivated by neutrino masses, $\delta_{RR}$ is much larger than $\delta_{LL}$ [17]. Therefore, Case 1 can accommodate these models.

As mentioned, another large discrepancy is observed in the branching ratio of the $B \to \eta'K$ process [9]:
\[ B_{\text{exp}}(B \to \eta'K) = (55^{+19}_{-16} \pm 8) \times 10^{-6} \]

which is 2 to 5 times larger than the standard model calculation [18]. Since such a large deviation is observed only in the $B \to \eta'K$ process, the new mechanisms based on the peculiarity of $\eta'$ meson, for instance intrinsic charm [19] or gluonium contents of $\eta'$ [20], have been investigated. We shall discuss in the following: the SUSY effects to the branching ratio and the impacts of those new mechanisms on the mixing CP asymmetry $S_{\eta'K}$.

Let us first discuss the SUSY contributions and also the uncertainties from various SM parameters. In general, the SUSY contributions can be written as
\[ B_{\text{Br}}(B \to \eta'K) = B_{\text{Br}}(B \to \eta'K) \times \left[ 1 + 2 \cos(\theta_{\eta'} - \delta_{12})R_{\eta'} + R_{\eta'}^2 \right] \]

Note that this equation can be applied to $B \to \phi K$ by replacing the indices. The input parameters which are used in our above analysis lead to $B_{\text{Br}}(B \to \eta'K) = 13 \times 10^{-6}$. In fact, this value is sensitive, especially to the $s$ quark mass and the value of $q^2$ in our calculation. For instance, $m_s = 0.08 \text{ GeV}$ and $q^2 = m_\pi^2/2$ give $B_{\text{Br}}(B \to \eta'K) = 36 \times 10^{-6}$. However, such a small value of $m_s$ enhances the branching ratio of some similar processes like $B \to \pi K$ [18] and also a larger $q^2$ is disfavoured by analysis of $S_{\phi KS}$ [6]. A maximum enhancement from SUSY contributions can be obtained by $\theta_{\eta'} = n\pi$, $n = 0, 1,$... and $\cos \delta_{12} = 1$, which lead to $B_{\text{Br}}(B \to \eta'K) = 2.25 \times B_{\text{Br}}(B \to \eta'K)$ for $R_{\eta'} \simeq 0.5$. Interestingly our solution to reproduce the experimental result of $S_{\phi KS}$ and $S_{\eta'K}$ requires a shift between $\theta_\phi$ and $\theta_{\eta'}$, which may suppress the leading SUSY contribution to the branching ratio for $B \to \phi K$. Thus, it is possible
to enhance $B \rightarrow \eta'K$ without changing the prediction for $B \rightarrow \phi K$ too much. On the other hand, the other similar processes such as $B \rightarrow \pi K$ require more attention. Apart from its tree contributions, $B \rightarrow \pi K$ obtains as large SUSY contributions as $B \rightarrow \eta'K$. Therefore, we must not ignore the limitation given by these similar processes, which would be revealed as soon as more precise experimental data from those processes will be available.

Now we turn to the new mechanisms proposed to enhance $Br(B \rightarrow \eta'K)$ and its impacts on $S_{\eta'K_S}$. We rewrite the amplitude in the following way,

$$A(\eta'K) = A_{\eta'K_S}^{SM} + A_{\eta'K_S}^{SUSY} + G^{SM} + G^{SUSY}$$

(29)

where $G^{SM}$ and $G^{SUSY}$ are the new mechanism contributions to SM and SUSY, respectively. Flowingly, the branching ratio including the contributions from both SUSY and new mechanisms is modified to

$$Br(B \rightarrow \eta'K) = Br_{SM}(B \rightarrow \eta'K)(1 + r)^2 \times [1 + 2 \cos(\delta \eta' - \delta_{12})R_{\eta'}^2 + R_{\eta'}^2].$$

(30)

where $r \equiv G^{SUSY}/G^{SM}$ and $R_{\eta'}e^{i\delta \eta'} = (A_{\eta'K_S}^{SUSY} + C_{\eta'K_S}^{SUSY})/(A_{\eta'K_S}^{SM} + C_{\eta'K_S}^{SM})$. Note that $Br_{SM}(B \rightarrow \eta'K)$ does not include the new mechanism contributions. Having the gluonium contributions in mind, we parametrise the SUSY contributions from new mechanism as:

$$G^{SUSY}/G^{SM} = a \left[|\delta_{LL}|_{23} + (\delta_{RR})_{23}\right] + b \left[|\delta_{LR}|_{23} + (\delta_{RL})_{23}\right].$$

where $(\delta_{LL})_{23}$ and $(\delta_{RR})_{23}$ have a same coefficient due to the penguin process and also a same sign since the amplitude is proportional to only the $B \rightarrow K$ transition form factor. Thus, Eq. (16) is modified to:

$$R_{\eta'}e^{i\delta \eta'} \simeq \frac{0.23 + a \, r}{1 + r} (\delta_{LL})_{23} + \frac{101 + b \, r}{1 + r} (\delta_{LR})_{23} - \frac{101 - b \, r}{1 + r} (\delta_{RL})_{23} - \frac{0.23 + a \, r}{1 + r} (\delta_{RR})_{23}$$

(31)

Although the quantitative estimation of $r$ is difficult at the moment, the parameters $a$ and $b$ could be computed for a given mechanism. For the intrinsic charm contribution, we have $a = b = 0$ since it come from a tree diagram. For the spectator gluonium contribution ($G^{SUSY}/G^{SM} \simeq \sqrt{2} / (V_{tb} V_{ts}^{*} V_{cb} V_{cs}^{*}) C_{g}/C_{s}$), we obtain $a = -1.2$ and $b = -585$ at the LL order. The spectator gluonium process means that the weak $b \rightarrow sg$ transition (chromo-magnetic operator $O_2$) accompanied by one gluon emission from spectator is followed by two gluon fusion into gluonium in $\eta'$ [21,22]. Using these values in Eq. (31), we find that as $r$ increases $|\delta_{LL}|_{\eta'}$ is reduced and $|\delta_{RR}|_{\eta'}$ is enlarged. In fact, this does not disturb our previous explanation for the discrepancy between $S_{\eta'K_S}$ and $S_{\eta'K_S}$ especially because the sign in front of $|\delta_{RR}|_{\phi}$ and $|\delta_{RL}|_{\eta'}$ remain different, which was a crucial point. In Fig. 2, we show the result for the branching ratio versus $S_{\eta'K_S}$ for Case 1–4 including the spectator gluonium contribution. We fix $R_{\phi} = 0.5$ and $\theta_{\phi} = -5\pi/8$ in order to have $S_{\phi K_S} \simeq -0.2$. As can be seen from this figure, we can have both of the CP asymmetry of $B \rightarrow \eta'K$ and its branching ratio within the experimental limits in a significant range of SUSY parameters space.

To conclude, we have considered possible supersymmetric contributions to the CP asymmetry $S_{\eta'K_S}$ and $S_{\eta'K_S}$. We showed that the discrepancy between their measurements can be naturally resolved by considering the different parity sensitivity of these processes to the SUSY contributions from R-sector that lead to $R_{\eta'} < R_{\phi}$ and/or a phase shift between $\theta_{\eta'}$ and $\theta_{\phi}$. We also studied the observed large branching ratio of $B \rightarrow \eta'K$. We have considered the new mechanisms proposed to enhance $Br(B \rightarrow \eta'K)$ and their impacts on $S_{\eta'K_S}$ correlation. We have shown that a simultaneous solution for discrepancy between the CP asymmetries of $B \rightarrow \phi K_S$ and $B \rightarrow \eta'K_S$ and the puzzle of the large branching ratio is possible for some SUSY models. More prices experimental data would allow us to draw more definite conclusions and shed light on compatible SUSY models with this solution.

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which would provide a significant contribution in the SUSY analysis, we need QCDF results at one order higher in $\alpha_s$. In order to avoid a misleading result, we did not include any annihilation contributions in our QCDF estimate.

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