Analysis of geometric and weibull queuing model (case study: customer service and electronic ID card recording counters at Dispendukcapil of Semarang City)

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Abstract. Semarang is the Capital City of Central Java which has a large population. For 2019, the population has increased from the previous year so that it will cause a change in population identity data. Dispendukcapil of Semarang City is one of the executors of public administration services in Central Java whose task is to carry out affairs in the field of population and civil registration. However, the queue has become one of the problems that occur there because of the arrival of many customers at the same time and is random. So one way to reduce queuing is to apply queuing theory to the system. The queue theory was developed to determine the queuing model so that it can be used to calculate system performance. In measuring the performance of the system determined using Graphical User Interface (GUI) R. The queue model that has been obtained at Dispendukcapil counters in Semarang is customer service counter (Geometric/Weibull)/1):(GD//∞/∞) and electronic ID Card recording counter (Geometric/Weibull)/1):(GD//∞/∞).

1. Introduction
The increase of population at Semarang City in 2019 will cause changes in population identity data. According to data obtained from the Semarang City Administration for 2019, the population reached 1,674,358 million people, an increase from the previous year, which was 1,668,578 [3]. Population events carried out by the community begin when they are born, during their life until the end of their lives, including, among others, events of birth, death, marriage, divorce, and other important events. All population events experienced by the community need to be registered and reported because every incident that occurs will cause changes in identity data so that it has an impact on the accuracy of population data.

Dispendukcapil of Semarang City is one of the implementers of public administration services in Central Java which has the task of carrying out government affairs in the field of population and civil registration. Changes in community population data will cause more resident to come to the Dispendukcapil Office along with the increase in population with the aim of resolving all interests related to population problems.

The large number of visitors who come to the Dispendukcapil Office every day causes high queues because the number of service facilities and time provided to serve visitors is limited so that Dispendukcapil officers cannot serve visitors who come optimally. One way that can be done to reduce the occurrence of queues is to apply queuing theory to the system. Therefore, this queuing system should be designed more effectively and efficiently by using queuing theory in order to improve customer
2. Literature review

2.1. Profile Dispendukcapil of Semarang City
Dispendukcapil of Semarang City is one of the Regional Government agencies in charge of serving the community in population affairs and civil registration. In serving the community, the Semarang City Dispendukcapil has several types of counters. The Dispendukcapil is led by a Head of Service in exercising the authority of the Semarang City Government [2].

2.2. Notation Kendall
A queuing model in the arrival and service process can be written in standard notation [5]:

\[(a / b / c): (d / e / f)\]

with:
- \(a\): arrival distribution
- \(b\): service time distribution
- \(c\): server
- \(d\): service discipline
- \(e\): the maximum number of customers allowed in the system
- \(f\): the source of the call

2.3. Steady state
Steady state conditions are met if the average number of customers who come does not exceed the average number of customers who have been served, in other words \(\lambda < \mu c\) or \(\rho < 1\). For example, \(\lambda\) is the average number of customers arriving to the service place per certain time unit, \(\mu\) is the average customer that has been served per unit of time, and \(c\) is the number of service facilities (servers), then \(\rho\) or the utility factor can be defined as follows: \(\rho = \frac{\lambda}{\mu c}\) [7].

2.4. Poisson process and the exponential distribution
An arrival counting process \(\{N(t), t \geq 0\}\), where \(N(t)\) denotes the total number of arrivals up to time \(t\), with \(N(0) = 0\), and which satisfies the following three assumptions [4]:

i. The probability that an arrival occurs between \(t\) dan \(t + \Delta t\) is equal \(\lambda \Delta t + o(\Delta t)\). We write this as \(P_{t}\{N(\Delta t) = 1\} = \lambda \Delta t + o(\Delta t)\), where \(\lambda\) is a constant independent of \(N(t)\), \(\Delta t\) is an incremental element, and \(o(\Delta t)\) denotes a quantity that becomes negligible when compared to \(\Delta t\) as \(\Delta t \to 0\), that is:

\[\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0\]

ii. \(P\{more\ than\ one\ arrival\ between\ t\ and\ t + \Delta t\} = \lambda \Delta t + o(\Delta t)\)

iii. The numbers of arrivals in nonoverlapping intervals are statistically independent.

2.5. Kolmogorov Smirnov test
The most commonly used method of alignment is the Kolmogorov-Smirnov test (Daniel, 1989). The Kolmogorov-Smirnov test procedure is as follows [1]:

1) Determine the Hypothesis:
   \(H_0 = \) The sample distribution follows a defined distribution
   \(H_1 = \) The sample distribution doesn’t follow a defined distribution

2) Determine the Significance Level: \(\alpha\)

3) Test Statistics:
\[ D = \max_{1 \leq i \leq r} \{ \max \{ |S(x_i) - F_0(x_i)|, |S(x_{i-1}) - F_0(x_i)| \} \} \]

With:
- \( S(x_i) \): the cumulative distribution of the sample from the population
- \( F_0(x_i) \): the cumulative distribution of the theoretical data from the hypothesized distribution

4) Test Criteria:
- \( H_0 \) is rejected if \( D > D^*(\alpha) \) or if the p-value < \( \alpha \). The value of \( D^*(\alpha) \) is the critical value obtained from the Kolmogorov-Smirnov table.

2.6. Queue model \((G/G/c) : (GD/\infty/\infty)\)

By assuming \( r = \lambda / \mu \) and \( \rho = \lambda / c \mu \), obtained the probability for 0 customers can be written [4]:

\[ P_0 = \left( \frac{1}{n!} + \frac{r^c}{c! (1 - \rho)} \right)^{n-1} \]

While the probability for \( n \) customers can be written:

\[ P_n = \begin{cases} \left( \frac{r^n}{n!} \right) P_0, & \text{for } n < c \\ \left( \frac{r^n}{c! n! - c} \right) P_0, & \text{for } n \geq c \end{cases} \]

So that the formula of system performance measures in \((M / M / c) : (GD / \infty / \infty)\) is obtained as follows:

1. The estimated number of customers in the queue
   \[ L_q = \left( \frac{r^c \rho}{c! (1 - \rho)^2} \right) P_0 \frac{\mu^2 v(t) + v(t') \lambda^2}{2} \]
   with: \( v(t) = \left( \frac{1}{\mu^2} \right)^2 \) and \( v(t') = \left( \frac{1}{\lambda^2} \right)^2 \)

2. The estimated number of customers in the system
   \[ L_s = L_q + r \]

3. The estimated total time in the queue
   \[ W_q = \frac{L_q}{\lambda} \]

4. The estimated total time in the system
   \[ W_s = \frac{L_s}{\lambda} = W_q + \frac{1}{\mu} \]

2.7. Geometric distribution

If repeated independent trials can result in a success with probability \( p \) and a failure with probability \( q = 1 - p \), then the probability distribution of the random variable \( X \), the number of the trial on which the first success occurs is [9]

\[ f(x; p) = pq^{x-1}, \ x = 1,2,3, ... \]

The mean and variance of a random variable following the geometric distribution are
\[ \mu = \frac{1}{p} \text{ and } \sigma^2 = \frac{1-p}{p^2} \]

2.8. Weibull distribution (3 parameters)
A random variable \( X \) has a three-parameter Weibull distribution with parameters \( a, b \) and \( c \) if its density function is given by [6]:

\[ f_X(x|a,b,c) = \frac{c}{b} \left( \frac{x-a}{b} \right)^{c-1} \exp \left\{ -\left( \frac{x-a}{b} \right)^c \right\}, \quad x \geq a \]

While the cumulative distribution function is

\[ F(x|a,b,c) = 1 - \exp \left\{ -\left( \frac{x-a}{b} \right)^c \right\} \]

With \( a \) is a location parameter, \( b \) is a scale parameter, and \( c \) is a shape parameter.

2.9. Graphical user interface (GUI) R
R studio is a software that can be obtained free of charge and is open source. One of the programs in R that can create a web-based user interface menu is R-shiny. R-shiny is a toolkit from the R program that can be used to create interactive web pages so that R capabilities which are basically CLI (Command Line Interface) can be accessed through the web menu using a GUI (Graphical User Interface). R-shiny has components that are divided into two groups, namely [8]:

a. User Interface
   The benefits of the user interface are as a control panel, inputting input values and presenting output related to the results of the analysis

b. Server. This section is the brain of the program whose task is to simulate, analyze data and send the results to the output section.

3. Research methods
The steps taken to implement the research are as follows:
1. Determine the place of research and methods to be used.
2. Research directly at the Dispendukcapil Office by observing and recording directly to obtain data on the number of arrivals and service time data in the time unit set by the researcher.
3. Check steady state conditions \( \rho = \frac{\lambda}{c \mu} < 1 \) are the average number of customers who come does not exceed the average number of customers served. If the data does not meet steady state conditions, it will be followed up by increasing the number of services or speeding up service time according to existing conditions.
4. Fit the distribution for data on the number of arrivals and time of service using the Kolmogorov Smirnov test.
5. Determine the queue model according to the Kendall notation format \( (a/b/c):(d/e/f) \).
6. Determine the performance measure of the queuing system that occurs at each Dispendukcapil counter, including the estimated number of customers in the system \( (L_s) \), the estimated number of customers in the queue \( (L_q) \), the estimated total time in the system \( (W_s) \), and time waiting for the estimated queue \( (W_q) \).
7. Analysis based on the results of the calculation of system performance measures.

4. Results and discussion

4.1. Steady state
Steady state conditions are met if the value of the level of use ($\rho$) < 1 means that the average rate of arrival of counter visitors is smaller than the average rate of service. Following are the results of the steady state size.

Table 1. Steady state conditions

| Counter                     | $c$  | $\lambda$ | $\mu$ | $\rho = \frac{\lambda}{c:\mu}$ |
|-----------------------------|------|-----------|-------|-------------------------------|
| Customer Service            | 1    | 7.68      | 17.48 | 0.439                         |
| ID Card Recording           | 1    | 8.16      | 19.39 | 0.421                         |

Based on the calculation results from the table 1, the value of $\rho$ for both counters are less than 1, so it can be concluded that it has met the steady state conditions. It means that the average number of customers who come does not exceed the average number served, so the queuing system for customer service and ID card recording counter have been stable. Therefore, customers who come are still able to be served as a whole.

4.2. Distribution fit test

Based on the output of R Studio, it is obtained:

1. Customer service
   Number of Arrivals

   Figure 1. Kolmogorov Smirnov test of number of arrivals

   Service Time

   Figure 2. Kolmogorov Smirnov test of service time

The table above shows the results of distribution fit test for the variable number of customers using the Poisson test, while the service time variable uses the exponential test.
2. ID card recording
   Number of Arrivals

   ![Identification of Queuing Models and System Performance](image)

   **Figure 3.** Kolmogorov Smirnov test number of arrivals

   Service Time

   ![Identification of Queuing Models and System Performance](image)

   **Figure 4.** Kolmogorov Smirnov of service time

   **Table 2.** Kolmogorov Smirnov test

   | Counter             | Variable       | P-Value | D       | Decision         |
   |---------------------|----------------|---------|---------|------------------|
   | Customer Service    | Number of Arrivals | 0.006145 | 0.2796  | Ho was rejected  |
   |                     | Service Time    | 1.1x10^-47 | 0.17159 | Ho was rejected  |
   | ID Card Recording   | Number of Arrivals | 0.00489  | 0.28509 | Ho was rejected  |
   |                     | Service Time    | 2.6x10^-46 | 0.14973 | Ho was rejected  |

   Based on the results of the steady-state analysis and distribution fit test of the number of arrivals and service times of customers to the counters at Dispendukcapil. It is known that the $p$-value < $\alpha = 5\%$, so the Ho was rejected, it means that the two counters have General distribution. For customer service counters and electronic ID card recording, the queuing model obtained is (G / G / 1) : (GD / $\infty$ / $\infty$). Then a further test was carried out to determine the specific distribution at each counter using EasyFit software.

   **Table 3.** Distribution fit test with EasyFit software

   | Counter             | Variable       | P-Value | D       | Distribution | Decision         |
   |---------------------|----------------|---------|---------|--------------|------------------|
   | Customer Service    | Number of Arrivals | 0.10115  | 0.19607 | Geometric    | Ho was accepted  |
   |                     | Service Time    | 0.07996  | 0.07468 | Weibull      | Ho was accepted  |
   | ID Card Recording   | Number of Arrivals | 0.21738  | 0.16863 | Geometric    | Ho was accepted  |
   |                     | Service Time    | 0.47907  | 0.04785 | Weibull      | Ho was accepted  |
Based on the results of the distribution fit test in the table 3, that the \( p\)-value > \( \alpha = 5\% \), so the Ho was accepted. And the final model for queuing at the customer service and ID Card Recording counter at Dispendukcapil are \((\text{GEOM/WEIB}/1):(\text{GD/}\infty/\infty)\). The model shows that the number of customers is geometric distributed, for service time distributed in Weibull with the number of service facilities is 1, for general discipline is the first in first out, with the maximum number of customers and unlimited call sources.

4.3. System performance measures

Based on the output of R Studio, system performance were obtained for each counter in the Dispendukcapil of Semarang City.

1. Customer service

![Image](Figure 5. System performance of customer service counter)

![Image](Figure 6. System performance of ID card recording counter)

**Table 4. System performance of Dispendukcapil counter**

| Counter            | Lq     | Ls     | Wq     | Ws     | Po     |
|--------------------|--------|--------|--------|--------|--------|
| Customer service   | 0.003482 | 0.44267 | 0.000454 | 0.057672 | 0.560811 |
| ID card recording  | 0.002703 | 0.42363 | 0.000331191 | 0.051902 | 0.579073 |

Based on the calculation of system performance from the table 4, it is known that the system performance for customer service counters and electronic ID card recording has a small waiting time in the system and queues, so that in the system, there is no pile of customer queues and the two counters have a good performance in serving customers, so there is no need to add service facilities.
5. Conclusions
After the results of the above analysis, the following conclusions can be drawn that the queue system for customer service and ID Card Recording counters at Dispendukcapil of Semarang is stable because it has a utility value of less than 1. The final queue model that fits at the customer service and ID Card Recording Counters at Dispendukcapil is (Geometric/Weibull/1):(GD/∞/∞). Based on the system performance of Dispendukcapill is running well, seen from the waiting line time in the queue and the low system so that services at that counters do not need to add more serves.

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