Phonon-drag magnetothermopower in Rashba spin-split two-dimensional electron systems

Tutul Biswas and Tarun Kanti Ghosh

Department of Physics, Indian Institute of Technology-Kanpur, Kanpur-208 016, India
E-mail: tutulb@iitk.ac.in

Received 7 June 2013, in final form 2 August 2013
Published 18 September 2013
Online at stacks.iop.org/JPhysCM/25/415301

Abstract

We study the phonon-drag contribution to the thermoelectric power in a quasi-two-dimensional electron system confined in GaAs/AlGaAs heterostructure in the presence of both Rashba spin–orbit interaction and perpendicular magnetic field at very low temperature. It is observed that the peaks in the phonon-drag thermopower split into two when the Rashba spin–orbit coupling constant is strong. This splitting is a direct consequence of the Rashba spin–orbit interaction. We show the dependence of phonon-drag thermopower on both magnetic field and temperature numerically. A power-law dependence of phonon-drag magnetothermopower on the temperature in the Bloch–Gruneisen regime is found. We also extract the exponent of the temperature dependence of phonon-drag thermopower for different parameters like electron density, magnetic field, and the spin–orbit coupling constant.

1. Introduction

Low-temperature measurement of thermoelectric power provides an important tool for probing the electronic and transport properties of various low-dimensional systems. Extensive investigations on magnetothermopower measurement [1–9] in the two-dimensional electron system (2DES), formed at the interface of GaAs/AlGaAs heterostructure, were initiated in the mid-1980s. Electron diffusion and phonon-drag are the two additive contributions to the thermopower. The drift motion of electrons, due to external perturbations like temperature gradient or electric field, is entirely responsible for the diffusion thermopower. On the other hand, phonon-drag thermopower originates as an outcome of the interaction between electrons and phonons. In low-density semiconducting systems a tiny fraction of acoustic phonons with wavevector \( q \leq 2k_F \) (where \( k_F \) is the Fermi wavevector) interact with electrons below a certain characteristic temperature \( T_{BG} = 2\hbar v_s k_F/k_B \) (where \( v_s \) is the sound velocity) because of the phase space restriction. The temperature regime defined by \( T \leq T_{BG} \) is usually known as the Bloch–Gruneisen [10, 11] (BG) regime. In the BG regime the diffusion thermopower \( S_d \) varies linearly with temperature whereas the phonon-drag thermopower \( S_g \) shows a power-law dependence \( S_g \sim T^{\delta_e} \), where the effective exponent of the temperature dependence \( \delta_e \) varies for different systems as well as for different scattering mechanisms of electron–phonon interaction. Past experimental [12] and theoretical [13] works established that \( S_g(S_d) \) dominates over \( S_d(S_g) \) at temperature below(above) 1 K. In inversion asymmetric semiconducting heterostructures two different mechanisms are responsible for the electron–phonon interaction. They are known as deformation potential (DP) and piezoelectric (PE) scattering potential. Lattice deformation leads to the potential energy change of electrons to produce DP scattering potential. On the other hand, potential energy corresponding to the induced electric polarization due to crystal vibrations is known as PE scattering potential.

There are two equivalent methods available in the literature for the calculation of the phonon-drag contribution to the thermoelectric power. According to Herring [14], they are known as the ‘\( Q \)-approach’ and the ‘\( \Pi \)-approach’. The equivalency of the two approaches is confirmed by the Onsager symmetry and the fundamental relationships between these approaches have been established in the recent past [15]. In the ‘\( Q \)-approach’ a weak temperature
gradient $\nabla T$ is applied so that electrons and phonons move in directions opposite to each other. The flow of electrons causes the diffusion thermopower $S_d$. As a consequence of electron–phonon interaction a finite fraction of momentum is transferred from phonons to electrons which drags electrons in the opposite direction and the phonon-drag contribution to the thermoelectric power comes into the picture. Several authors [16–19] have calculated phonon-drag thermopower in various electronic systems in this approach by solving coupled Boltzmann equations for both electrons and phonons. On the other hand, in the ‘$\Pi$-approach’ a very weak electric field $E$ is applied to cause electrons drift. In this case, since no temperature gradient is applied (i.e. $\nabla T = 0$), phonons are in equilibrium. Interaction between electrons and phonons leads to a flow of momentum from electrons to phonons which produces a finite phonon heat current. In this way the phonon-drag contribution to the Peltier coefficient can be determined. Many authors [20, 8, 21–23] have calculated the phonon-drag thermopower of a 2DES in a perpendicular magnetic field using the $\Pi$-approach.

Recently, 2DES with spin–orbit interaction [24–26] (SOI) has become an emerging area of research due to its potential application for developing spintronic devices [27–29]. Two types of SOI, namely Rashba [30] and Dresselhaus [31], are present in low-dimensional semiconducting structures. Rashba spin–orbit interaction (RSOI) occurs due to the inversion asymmetry of hetero-interface. An external gate voltage can tune [32, 33] the strength of RSOI. Zero-field spin splitting [34–36] is an important consequence of RSOI. On the other hand, SOI of Dresselhaus type occurs in crystals which have bulk inversion asymmetry. The RSOI has many important consequences on various properties of 2DES. Electron–phonon interaction strength can be modified by RSOI [37, 38]. It causes an increase in polaron mass correction [39]. The temperature dependence of phonon-limited mobility [40] and resistivity [41] get modified by RSOI. The peak arising in longitudinal magnetoresistivity of a 2DES splits [42] into two due to RSOI. Very recently, a thermoelectric probe [43] has been used theoretically to calculate the strength of RSOI by analyzing the beating patterns obtained in the thermoelectric coefficients.

In this paper we calculate phonon-drag thermopower of a 2DES confined in a GaAs/AlGaAs heterostructure in the presence of both perpendicular magnetic field and RSOI at very low temperatures. An oscillatory behavior of phonon-drag thermopower with the applied magnetic field is found. It is found that at higher values of magnetic field strong Rashba coupling is able to split the peaks appearing in phonon-drag thermopower. This kind of splitting is considered to be as a direct effect of RSOI. The number of oscillations in the thermopower increases with electron density also. We study the behavior of phonon-drag thermopower with temperature for various values of magnetic field, electron density and Rashba spin–orbit coupling constant. At very low temperature (BG) regime a power-law dependence of phonon-drag thermopower is observed. The exponents of this temperature dependence have been evaluated numerically for different parameters. It is established that the RSOI causes a strong suppression in the effective exponent of the phonon-drag thermopower.

This paper is organized as follows. In section 2 we present detailed theoretical calculations. In section 3 numerical results and discussions are given. We summarize our work in section 4. Some calculations are shown in detail in the appendices A and B.

2. Theory

2.1. Basic information of the physical system

We consider a quasi-2DES confined at the interface of a GaAs/AlGaAs heterostructure. Electrons are restricted to move in the $x-y$ plane due to a confining potential of triangular type in the growth direction (say, the $z$-direction). We assume that only the lowest subband in the $z$-direction is occupied. The total wavefunction can be written as

$$\Psi(x, y, z) = \Psi(x, y)\xi_0(z),$$

where $\xi_0 = \sqrt{b^2/2e^2z^2}$ is the Fang–Howard [44] variational wavefunction in the $z$-direction. The variational parameter is given by $b = (4\pi m^*e^2/\varepsilon_0\hbar^2)^{1/3}(n_0 + 11n_0/32)^{1/3}$, where $m^*$, $\varepsilon_0$, $n_0$ and $n_1$ are effective mass of an electron in GaAs, dielectric constant of GaAs, permittivity of free space, depletion charge density and density of electron, respectively.

The single-particle Hamiltonian [42, 45] of a Rashba spin–orbit coupled 2DES in the presence of a perpendicular magnetic field along $z$-direction can be written as

$$H = \frac{P^2}{2m^*} + \frac{\alpha}{\hbar}(\sigma_x P_y - \sigma_y P_x) + \frac{1}{2}g^* \mu_B Bz,$$  \hspace{1cm} (1)

where $P = p + eA$ with $A$ as the vector potential, $\alpha$ is the Rashba spin–orbit coupling constant, $\sigma_i$ are the usual Pauli spin matrices, $g^*$ is the effective Lande $g$-factor and $\mu_B$ is Bohr magneton. To solve equation (1) we consider the Landau gauge $A = (0, Bx, 0)$.

The eigen spectrum is given by

$$E_n = n\hbar\omega_c + \lambda \sqrt{E_0^2 + 2n\hbar^2\alpha^2},$$ \hspace{1cm} (2)

where $\lambda = \pm, \omega_c = eB/m^*$ is the cyclotron frequency and $l_0 = \sqrt{\hbar/eB}$ is the magnetic length. For $n = 0$ there is only one level with energy $E_0 = (\hbar\omega_c - g^*\mu_B B)/2$.

The eigenfunctions corresponding to $E_n$ are respectively given by

$$\Psi_n^+(x, y) = \frac{\exp(ik_0x)}{\sqrt{2\pi A_n}} \begin{pmatrix} D_n\phi_{n-1}(x + x_0) \\ \phi_n(x + x_0) \end{pmatrix}_{l_0}$$  \hspace{1cm} (3)

and

$$\Psi_n^-(x, y) = \frac{\exp(ik_0x)}{\sqrt{2\pi A_n}} \begin{pmatrix} \phi_{n-1}(x + x_0) \\ -D_n\phi_n(x + x_0) \end{pmatrix}_{l_0}.  \hspace{1cm} (4)$$
Here $k_y$ is the $y$-component of electron wavevector $\mathbf{k}$, $x_0 = k_y l_0^2$ and $\phi_0(x + x_0)/l_0 = \sqrt{1/(2\pi n_0 \sqrt{\pi})} e^{-(x-x_0)^2}/(2\pi l_0^2)$. $H_n[x + x_0]/l_0$ is the harmonic oscillator wavefunction centered at $x = -x_0$. The coefficients $D_n$ and $A_n$ are given by $D_n = \sqrt{2n(\alpha/l_0)/(\epsilon_0 + \epsilon_{f0}^2 + 2\alpha^2/l_0^2)}$ and $A_n = 1 + D_n^2$. The eigenamplitude corresponding to $n = 0$ state is

$$\Psi_0^+(\mathbf{r}) = \frac{e^{\Phi_{\epsilon,\mathbf{r}}}}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \phi_0(x + x_0)/l_0 \end{pmatrix}.$$  \hspace{1cm} (5)

The density of states (DOS) is given by $D(\epsilon) = (1/(2\pi l_0^2)) \sum_{n,\lambda} \Gamma_L$, where $\Gamma_L$ is the Landau level weight. The electronic band structure of the DOS can be written as

$$D(\epsilon) = \frac{1}{2\pi^2 l_0^2} \sum_{n,\lambda} \Gamma_L \epsilon + \epsilon_{n,\lambda}^2 + \Gamma_L \epsilon.$$  \hspace{1cm} (6)

The chemical potential $\mu$ can be determined by the following condition:

$$n_e = \int d\epsilon D(\epsilon)f^{0}(\epsilon),$$  \hspace{1cm} (7)

where $f^{0}(\epsilon) = 1/(e^{\beta(\epsilon - \mu) + 1})$ with $\beta = 1/(k_B T)$ is the usual Fermi–Dirac distribution function.

### 2.2. Phonon-drag thermopower

To calculate phonon-drag thermopower we follow the ‘$\Pi$-approach’ as described in [23]. Since there is no temperature gradient ($i.e.$ $\nabla T = 0$) the transport equations are quite simple as $\mathbf{J} = \mathbf{S} \mathbf{E}$ and $\mathbf{U} = \mathbf{T} \mathbf{J}$, where $\mathbf{J}$, $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{T}$ are electron current density, phonon heat current density, conductivity tensor and Peltier coefficient tensor, respectively. According to Kelvin, the thermopower and the Peltier coefficient are related thermodynamically as $T \mathbf{S} = \mathbf{T}$.

The phonon heat current density is given by

$$\mathbf{U} = \frac{1}{L^2} \sum_{q,s} h\omega_{qs} v_{qs} N_{q,r}.$$  \hspace{1cm} (8)

where $L^2$ is the area of the sample, the index $s$ represents particular phonon mode, $v_{qs}$ is the phonon velocity, $\omega_{qs}$ is the phonon frequency, and finally the shift in phonon distribution is $N_{q,r} = N_q - N_q^r$.

The steady-state Boltzmann equation can be used to find $N_{q,r}$ as

$$\left(\frac{\partial N_{q,r}}{\partial t}\right)_{\text{coll}} + \left(\frac{\partial N_{q,r}}{\partial t}\right)_{\text{col}} = 0.$$  \hspace{1cm} (9)

Here, the first term represents the rate of change in phonon distribution due to electron–phonon interaction and the second term arises from various scattering processes such as phonon–phonon scattering, surface roughness scattering, etc. In the relaxation time approximation the second term of equation (9) can be written as

$$\left(\frac{\partial N_{q,r}}{\partial t}\right)_{\text{coll}} = -\frac{N_q - N_q^r}{\tau_p} = -\frac{N_{q,r}}{\tau_p}.$$  \hspace{1cm} (10)

where $\tau_p$ is the phonon relaxation time. Substitution of equation (10) into equation (9) yields

$$N_{q,r} = \tau_p \left(\frac{\partial N_{q,r}}{\partial t}\right)_{\text{ep}}.$$  \hspace{1cm} (11)

Now the rate of change in phonon distribution function due to electron–phonon interaction is given by

$$\left(\frac{\partial N_{q,r}}{\partial t}\right)_{\text{ep}} = \sum_{\nu,\nu'} \left[ P^{\text{em}}_{\nu\nu'} f^{\prime}_\nu(\epsilon^{\prime}_\nu) \left(1 - f_\nu(\epsilon_\nu)\right) - P^{\text{abs}} f_\nu(\epsilon_\nu) \left(1 - f^{\prime}_\nu(\epsilon^{\prime}_\nu)\right)\right],$$  \hspace{1cm} (12)

where $\nu \equiv (n, k_y, \lambda)$ represents the set of quantum numbers, $f^{\prime}_\nu(\epsilon^{\prime}_\nu)$ is the electron distribution functions, $P^{\text{em}}_{\nu\nu'}$ ($P^{\text{abs}}$) is the probability which is responsible for making transition of an electron from the initial state $\nu$ to the final state $\nu'$ with the absorption(emission) of a phonon.

According to Fermi’s golden rule we have

$$P^{\text{em}}_{\nu\nu'} = \frac{2\pi}{\hbar} |M_{\nu\nu'}(\mathbf{q})|^2 \left(N_{q,\nu}^0 + \frac{1}{2} + \frac{1}{2}\right) \delta \left(\epsilon_\nu - \epsilon^{\prime}_\nu + \hbar \omega_{q}\right).$$  \hspace{1cm} (13)

where $|M_{\nu\nu'}(\mathbf{q})|^2$ is square of the matrix element responsible for the electron–phonon interaction and $N_{q,\nu}^0 = 1/(e^{\beta(\omega_{q}\hbar) - 1})$ is the equilibrium Bose distribution function. Finally + and − signs in the parentheses represent emission and absorption, respectively. A detailed description of $|M_{\nu\nu'}(\mathbf{q})|^2$ is given in appendix A.

The applied electric field is vanishingly small so that one can linearize equation (11) about the equilibrium value. To do this we write the electron distribution function as $f^{\prime}_\nu(\epsilon^{\prime}_\nu) = f_{\nu,\nu'}(\epsilon^{\prime}_\nu) + f_{\nu,\nu'}(\epsilon^{\prime}_\nu)$. So equation (11) can be written as

$$N_{q,r} = \frac{\tau_p}{k_B T} \sum_{\nu,\nu'} \left(f_{\nu,\nu'}^{\prime} - f_{\nu,\nu'}\right) W_{\nu,\nu'},$$  \hspace{1cm} (14)

where $f_{\nu,\nu'} = \frac{\epsilon_{\nu,\nu'}}{\hbar \omega_{\nu,\nu'}}$ and $W_{\nu,\nu'} = f_\nu(1 - f^{\prime}_\nu(\epsilon^{\prime}_\nu))$. Since the applied electric field is very small, it can be treated as a perturbative term in the Hamiltonian. Using the first-order perturbation theory the energy eigenvalue of a state $\nu$ in the presence of $\mathbf{E}$ can be modified as $\epsilon_{\nu,\nu'}(\mathbf{E}) \simeq \epsilon_{\nu,\nu'}^{\prime} \mathbf{E}(x,\nu',\nu) + eE(x,\nu')$, where $\epsilon_{\nu,\nu'}$ is the unperturbed energy spectrum given by equation (2). The expectation values of $x$ are given by $\langle x \nu,\nu' \rangle = \langle \psi_{\nu}(\mathbf{r}) | x | \psi_{\nu'}(\mathbf{r}) \rangle = -i \hbar \epsilon_{\nu,\nu'}(\mathbf{E})$.

Therefore, equation (14) can be written as

$$N_{q,r} = \frac{\tau_p eE}{k_B T} \sum_{\nu,\nu'} \langle x \nu,\nu' \rangle - \langle x \nu \rangle W_{\nu,\nu'}.$$  \hspace{1cm} (15)
The components of the heat current density can be written as

\[
U_x = \frac{e \tau_p E}{k_B T L^2} \sum_{\nu,v',q,s} \hbar \omega_{q\nu} \langle \langle x \rangle \rangle_{\nu'} - \langle \langle x \rangle \rangle_{\nu} \rangle W_{\nu'\nu} q_x q_y
\]

(16)

and

\[
U_y = \frac{e \tau_p E}{k_B T L^2} \sum_{\nu,v',q,s} \hbar \omega_{q\nu} \langle \langle x \rangle \rangle_{\nu'} - \langle \langle x \rangle \rangle_{\nu} \rangle W_{\nu'\nu} q_y.
\]

(17)

Now the conservation of momentum in the y direction forces us to write \( \langle \langle x \rangle \rangle_{\nu'} - \langle \langle x \rangle \rangle_{\nu} \rangle = -q_y F_{\nu'\nu}^y \). We also have \( \hbar \nu_{q\nu} = v_s q_{\nu(y)}/q \). With these substitutions equations (16) and (17) become

\[
U_x = \frac{e \tau_p E}{k_B T L^2} \sum_{\nu,v',q,s} \hbar \omega_{q\nu} v_s W_{\nu'\nu} q_x q_y/q
\]

(18)

and

\[
U_y = -\frac{e \tau_p E}{k_B T L^2} \sum_{\nu,v',q,s} \hbar \omega_{q\nu} v_s W_{\nu'\nu} q_y^2/q
\]

(19)

Now \( \sum q \rightarrow (1/(2\pi)^3) \int dq_1 dq_2 dq_3 \alpha dq_z \), where \( q_1 = \sqrt{q_x^2 + q_y^2} \). If we evaluate all the summations involved in the above equations one can readily obtain \( U_z = 0 \) and this is quite obvious because the application of an electric field along the \( x \) direction causes a drift current in the \( y \) direction in the presence of a magnetic field in the \( z \) direction. For a particular phonon mode \( s \), \( U_z \) is given by (detailed calculations are given in appendix B)

\[
U_y = -\frac{e \lambda_s}{8\pi^2 k_B T} \sum_{\nu,v} \int dq_1 dq_2 dq_3 \hbar \omega_{q\nu} v_s \alpha (n^0 + 1) \times \left| C_{q\nu}^2 F_{\nu\nu}(q_1) I_{\nu,j}(q_2) \right| \left( \langle \epsilon_{\nu} - \epsilon_n^f + \hbar \omega_{q\nu} \rangle^2 + \Gamma_{\nu}^2 \right),
\]

(20)

where \( |C_{q\nu}|^2 \), \( F_{\nu\nu}(q_1) \) and \( I_{\nu,j}(q_2) \) are the square of the matrix element for various mechanisms of electron–phonon interaction, and the in-plane and out-of-plane form factors, respectively. The phonon mean free path is given by \( \lambda_{FP} = v_c \tau_p \) which is nearly 0.3 mm in the present case. In deriving equation (20) we have considered only the intra-Landau level (\( \nu = n' \)) and intra-branch (\( \lambda = \lambda' \)) scattering because at very low temperature inter-Landau level [8] and inter-branch contributions to the thermopower are negligible small.

Now the phonon-drag thermopower [21, 22] can be written as

\[
S_{xx} = S_{yy} = \frac{1}{T} \frac{U_y}{E} \rho_{xy},
\]

(21)

where the Hall resistivity in the presence of RSOI is given by [46] \( \rho_{xy} \approx (B/n_e)(1 + k_0^2 /k_F^2) \) with \( k_0 = m^* \alpha /h^2 \) and \( k_F = \sqrt{2\pi n_e} \).

3. Numerical results and discussions

In this section we present all the numerical results obtained by solving equation (21) numerically. For the numerical calculations we adopt the values of material parameters appropriate for GaAs as \( m^* = 0.06 m_0 \) where \( m_0 \) is the mass of a free electron, \( n_0 = 10^{15} \text{ m}^{-2}, \) \( \alpha_0 = 10^{-11} \text{ eV m}, \) \( n_d = 5 \times 10^{14} \text{ m}^{-2}, \) \( \kappa = 12.91, \) \( v_{sd} = 5.12 \times 10^3 \text{ ms}^{-1}, \) \( v_{sl} = 3.04 \times 10^3 \text{ ms}^{-1}, \) \( D = 12 \text{ eV}, \) \( h_{14} = 1.2 \times 10^9 \text{ V m}^{-1}, \) \( \rho_{\ell m} = 5.31 \times 10^7 \text{ kg m}^{-3}. \) Here, we have used a high value of \( \alpha \) in GaAs systems keeping in mind that \( \alpha \) can be enhanced by using the gate voltage as has been done for the InAs system [32]. The Landau level broadening parameter \( \Gamma_{\nu} \) depends on various parameters like magnetic field, temperature, etc. For simplicity, we have taken here a constant value of \( \Gamma_{\nu} = \Gamma_{L} = 1.2 \text{ meV}. \)

In figure 1 we present the dependence of the phonon-drag thermopower on the applied magnetic field at a fixed density \( n_e = 5n_0 \) and a fixed temperature \( T = 2 \text{ K}. \) The upper and lower panels are drawn for DP and PE scattering, respectively. Here, solid, dashed, dotted, dash-dotted and thick-solid lines represent \( \alpha = 0, \) \( \alpha_0, \) \( 2\alpha_0, \) \( 3\alpha_0 \) and \( 4\alpha_0, \) respectively. Contributions to \( S_{xx} \) from different branches of the Rashba spectrum for \( \alpha = 2\alpha_0 \) are shown in the inset.
Figure 2. Plots of the phonon-drag thermopower versus magnetic field due to DP scattering at a fixed $\alpha = \alpha_0$. Upper and lower panels are, respectively, for $T = 2$ and 6 K. Here, solid, dashed and dotted lines correspond to $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively.

In figure 2 we plot $S_{xx}$ due to DP scattering as a function of $B$ at a fixed $\alpha = \alpha_0$ for different densities: $n_e = 3n_0$, $5n_0$ and $7n_0$. Here, we consider two different temperatures $T = 2$ and 6 K. With increase of density, the number of oscillations increases. This is due to the fact that the oscillation frequency (in Tesla) of the density of states is directly proportional to the electron density. The magnitude of $S_{xx}$ is higher at higher value of temperature. In figure 3 we plot the same as in figure 2 for PE scattering. The rate of increment in $S_{xx}$ due to PE scattering with temperature is slower than that due to DP scattering.

The temperature dependence of phonon-drag thermpower for various densities is shown in figure 4. We have plotted $S_{xx}$ as a function of temperature $T$ for a fixed value of magnetic field $B = 1$ T and the Rashba spin–orbit coupling constant $\alpha = \alpha_0$. We consider both DP and PE scattering mechanisms of electron–phonon interaction separately. In the range of temperature up to $T \approx 3$ K the phonon-drag thermpower shows a power-law dependence $S_{xx} \sim T^\delta$. We extract the effective exponent of the temperature dependence from the log–log plot of $S_{xx}$ versus $T$ as shown in figure 4. We find that, due to DP scattering, $\delta_e = 3.15$, 3.65 and 3.45 for $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively. Due to PE scattering it is found that $\delta_e = 1.37$, 1.65 and 1.32 for $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively. The exponent $\delta_e$ depends on density. In figure 5 we describe the temperature dependence of $S_{xx}$ for different values of $\alpha$ at fixed magnetic field $B = 1$ T and fixed density $n_e = 5n_0$. We have found the exponent $\delta_e = 4.32$, 3.65 and 3.02 for $\alpha = 0$, $\alpha_0$ and $3\alpha_0$, respectively due to DP scattering. For PE scattering we find $\delta_e = 2.25$, 1.64 and 1.24 for $\alpha = 0$, $\alpha_0$ and $3\alpha_0$, respectively. Analyzing figure 5 we conclude that the presence of strong RSOI suppresses the exponents of the temperature dependence of $S_{xx}$ significantly. Similar results were found recently [47] in the temperature dependence of the phonon-drag thermpower of a Rashba spin–orbit coupled 2DES with zero magnetic field. In figure 6 we plot $S_{xx}$ as a function of $T$ for various values of $B$. In this case we fix $n_e = 5n_0$ and $\alpha = \alpha_0$. Exponents of the temperature dependence of $S_{xx}$ have been calculated in this case also. For DP scattering we find $\delta_e = 3.65$, 4.21 and 4.05 for $B = 1$, 5 and 8 T, respectively. We have $\delta = 1.64$, 1.94 and 1.84 for $B = 1$, 5 and 8 T, respectively, due to PE scattering.

In our previous study [47] with $B = 0$, $n_e = 5n_0$ and $\alpha = \alpha_0$ we found $\delta_e = 3.294$ for DP scattering and $\delta_e = 1.520$ for
Figure 5. Plots of the phonon-drag thermopower versus temperature due to both DP and PE scattering at a fixed density $n_e = 5n_0$ and $B = 1$ T.

Figure 6. Plots of the phonon-drag thermopower versus temperature due to both DP and PE scattering at a fixed $\alpha = \alpha_0$ and $n_e = 5n_0$.

PE scattering. So one can conclude that a finite amount of magnetic field is able to introduce a significant enhancement in the effective exponent of the temperature dependence of phonon-drag thermopower.

4. Summary

In summary, we have studied phonon-drag contribution to the thermoelectric power of a two-dimensional electron system confined in a GaAs/AlGaAs heterostructure in the presence of both Rashba spin–orbit interaction and perpendicular magnetic field. We have considered both deformation potential and piezoelectric scattering mechanisms of electron–phonon interaction. Interaction between electrons with two-dimensional wavevector and phonons of three-dimensional wavevector has been taken into consideration. Dependence of phonon-drag thermopower on an external magnetic field and temperature has been discussed thoroughly. At high values of magnetic field, splitting of peaks in phonon-drag thermopower is found at strong values of Rashba spin–orbit coupling constant. This splitting of peak is a direct effect of the Rashba spin–orbit interaction. We have also found a power-law dependence of phonon-drag thermopower on temperature. It is found that the exponent strongly depends on electron density, magnetic field and the spin–orbit coupling constant.

Appendix A

The Hamiltonian describing electron–phonon interaction can be written as

$$H_{ep} = \sum_{q,s} (C_{qs} e^{i\mathbf{q} \cdot \mathbf{r}} a_{qs} + C_{qs}^\dagger e^{-i\mathbf{q} \cdot \mathbf{r}} a_{qs}^\dagger).$$

where $a_{qs}$ ($a_{qs}^\dagger$) is the phonon annihilation (creation) operator and $C_{qs}$ is the matrix element responsible for electron–phonon interaction in a particular phonon mode ($\mathbf{q}, s$).

The square of the matrix element of $H_{ep}$ is given by

$$|M_{\nu,\nu'}(\mathbf{q})|^2 = |\langle \psi_{\nu'}(\mathbf{r})|H_{ep}|\psi_{\nu}(\mathbf{r})\rangle|^2 = |C_{qs}|^2 F_{m\nu'}^\parallel(\mathbf{q}) F_{\nu,q}(\mathbf{q}).$$

Here, the electron–phonon matrix elements ($C_{qs}$) are different for various scattering mechanisms. For DP and PE scattering the square of the matrix elements ($|C_{qs}|^2$) are respectively given by $|C_{qs}^{DP}|^2 = D^2 h_0 q / (2 \rho m \nu \nu)$ and $|C_{qs}^{PE}|^2 = (\hbar h_0)^2 h_{Al}(\mathbf{q}) \mathbf{q} / (4 \rho m \nu \nu)$, where $D$ is deformation potential constant, $h_{Al}$ is the relevant PE coupling tensor component, $\rho m$ is the mass density and $\nu \nu$ is the longitudinal(transverse) component of sound velocity. Finally the anisotropy factors in the longitudinal and transverse directions are given by $A_1 = 9q_y^2 q_z^2 / 2q^6$ and $A_1 = (8q_2^2 q_y^2 + q_0^6) / 4q^6$, respectively.

In equation (A.2), $F_{m\nu'}^\parallel(\mathbf{q}) = |\langle \psi_{\nu'}(\mathbf{r})| e^{i\mathbf{q} \cdot \mathbf{r}} |\psi_{\nu}(\mathbf{r})\rangle|^2$ is the in-plane form factor. For upper and lower branch we have, respectively,

$$F_{m\nu}^+(\mathbf{q}) = B_n^\nu(\xi) \left[ \sqrt{\frac{n}{n'}} \frac{D_{n'} D_{n} L_{n'-1}^{n'}(\xi) + L_{n}^{n'}(\xi)}{L_{n'}^{n'-1}(\xi) + L_{n}^{n'-1}(\xi)} \right]^2, \quad (A.3)$$

and

$$F_{m\nu}^-(\mathbf{q}) = B_{n'}^{\nu'}(\xi) \left[ \sqrt{\frac{n}{n'}} \frac{L_{n}^{n'-1}(\xi) + D_{n'} D_{n} L_{n'-1}^{n'}(\xi)}{L_{n'}^{n'-1}(\xi) + L_{n}^{n'-1}(\xi)} \right]^2, \quad (A.4)$$

where $B_n^\nu = (n'/n!)e^{-n'/4} \xi^{n'/2} \delta_{k_y,k_y+q_y}$, with $\xi = q_y^2 q_z^2 / 2$.

The out-of-plane form factor is given by $I_z(\mathbf{q}) = |\langle \psi_0(\mathbf{z}) | e^{i\mathbf{q} \cdot \mathbf{z}} |\psi_0(\mathbf{z})\rangle|^2 = b^2 / (q_z^2 + b^2)^3$.

Appendix B

The summations involved in equations (18) and (19) can be written as

$$\sum_{\nu,\nu',\mathbf{q},\mathbf{s}} \rightarrow \sum_{n,n',k_y,k_y',\lambda,\lambda',\mathbf{q},\mathbf{s}}.$$  

(B.1)

The summation over $k_y'$ can be easily done using the Kronecker delta symbol $\delta_{k_y,k_y+q_y}$ in $|M_{\nu,\nu'}|^2$ arising from the
momenntum conservation along the y direction. Again we have \( \sum_{\nu,v',\nu',q,s} \chi_{\nu'=\nu+q_s} = \frac{L^2}{2\pi^2k_B T} \). Then equation (B.1) is simplified to

\[
\sum_{\nu,v',\nu',q,s} \chi_{\nu'=\nu+q_s} = \frac{L^2}{2\pi^2k_B T} \sum_{n',\lambda',\lambda',q,s} \cdot \tag{B.2}
\]

Now one can convert the summation over \( q \) as \( \sum_q \rightarrow \frac{1}{(2\pi)^3} \int dq|q| \phi dq \), with \( q| = \sqrt{q_x^2 + q_y^2} \) and \( \phi = \cos^{-1}(q_y) \). Equation (19) can be written as

\[
U_y = -\frac{1}{8\pi^2k_B T} \sum_{n',\lambda',\lambda',q,s} v_y^2 \int dl'[q'] = f_0(\epsilon_{p}^{\nu}(q')|M_{\nu,v'}'(q)|^2 N_q^0 \times f_0(\epsilon_{p}^{\nu}(q) + h_{qy})| \delta(\epsilon_{p}^{\nu}(q) - \epsilon_{p}^{\nu}(q') + h_{qy})\right) \tag{B.3}
\]

Using \( \delta(\epsilon_{p}^{\nu}(q) - \epsilon_{p}^{\nu}(q) + h_{qy}) \cong \int f \delta(\epsilon - \epsilon_{p}^{\nu}(q) + h_{qy})f_0(\epsilon)|1 - f_0(\epsilon + h_{qy})| \). Equation (B.4) becomes

\[
U_y = -\frac{1}{8\pi^2k_B T} \sum_{n',\lambda',\lambda',q,s} v_y^2 \times \int dl'[q'] = f_0(\epsilon_{p}^{\nu}(q')|M_{\nu,v'}'(q)|^2 N_q^0 \times f_0(\epsilon_{p}^{\nu}(q) + h_{qy})\right) \tag{B.4}
\]

with \( G_{\nu,n}^{\nu'} = \int f \delta(\epsilon - \epsilon_{p}^{\nu}(q) + h_{qy})f_0(\epsilon)|1 - f_0(\epsilon + h_{qy})| \). Now in the presence of disorder, broadening of the Landau level occurs. Assuming Lorentzian broadening of width \( \Gamma_\nu \), the delta-function in the expression of \( G_{\nu,n}^{\nu'} \) can be written as \( \delta(\epsilon - \epsilon_{p}^{\nu}(q) = (1/\pi)\Gamma_\nu/((\epsilon - \epsilon_{p}^{\nu}(q)^2 + \Gamma_\nu^2) \) as described in equation (6) also. At very low temperature where phonon energy \( h_{qy} \) is comparable with the thermal energy \( k_B T \) and \( h_{qy} \ll k_B T \) we can make the following approximation:

\[
f_0(\epsilon)|1 - f_0(\epsilon + h_{qy})| \sim h_{qy}(N_q^0 + 1)\delta(\epsilon - \epsilon_{p}) \right) \]  

But in the BG regime the intra-level and intra-branch Landau level scatterings dominate over inter-level scattering. One can write \( n = n' \) and \( \lambda = \lambda' \). Now it is straightforward to arrive at equation (20) from equation (B.4) by taking all the approximations into consideration.

References

[1] Fletcher R, Maan J C and Weimann G 1985 Phys. Rev. B 32 8477
[2] Davidson J S, Dahlberg E D, Valois A J and Robinson G Y 1986 Phys. Rev. B 33 2941
[3] Obloh H, von Klitzing K, Ploog K and Weimann G 1986 Surf. Sci. 170 292
[4] Vuong T H H, Nicholas R J, Brumme M A, Portal J C, Alexandre F, Masson J M and Kerr T 1986 Solid State Commun. 57 381
[5] Vuong T H H, Nicholas R J, Brumme M A, Portal J C, Razeghi M, Alexandre F, Masson J M, Cheng K Y and Cho A Y 1986 Surf. Sci. 170 298
[6] Fletcher R, Maan J C, Ploog K and Weimann G 1986 Phys. Rev. B 33 7122
[7] Fletcher R, D’Iorio M, Sachrajda A S, Stoner R, Foxon C T and Harris J J 1988 Phys. Rev. B 37 3137
[8] Lyo S K 1985 Phys. Rev. B 30 6458
[9] Fletcher R, Coleridge P T and Feng Y 1995 Phys. Rev. B 52 2823
[10] Stormer H L, Pfeiffer L N, Baldwin K W and West K W 1990 Phys. Rev. B 41 1278
[11] Kawamura T and Das Sarma S 1992 Phys. Rev. B 45 3612
[12] Ruf C, Obloh H, Junge B, Gimelin E, Ploog K and Weimann G 1988 Phys. Rev. B 37 6377
[13] Sankeshwar N S, Kamatagi M D and Mulimani B G 2005 Phys. Status Solidi b 242 2892
[14] Herring C 1954 Phys. Rev. 96 1163
[15] Tsoucasidou M, Butcher P N and Tribleris G P 2001 Phys. Rev. B 64 165304
[16] Cantrell D G and Butler P N 1987 J. Phys. C: Solid State Phys. 20 1985
[17] Cantrell D G and Butler P N 1987 J. Phys. C: Solid State Phys. 20 1993
[18] Lyo S K 1988 Phys. Rev. B 38 6345
[19] Smith M J and Butcher P N 1989 J. Phys.: Condens. Matter 1 1261
[20] Jay-Gerin J P 1975 Phys. Rev. B 12 1418
[21] Kabukaddi S, Butcher P N and Mulimani B G 1989 Phys. Rev. B 40 1377
[22] Sankeshwar N S, Kabukaddi S S, Mulimani B G and Butcher P N 1990 J. Appl. Phys. 68 5919
[23] Fromhold T M, Butcher P N, Qin G, Mulimani B G, Oxley J P and Gallagher B L 1993 Phys. Rev. B 48 5326
[24] Fabian F, Matos-Abiague A, Ettler C, Stano P and Zatic I 2007 Acta Phys. Slovaca 57 565
[25] Winkler R 2003 Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Berlin: Springer)
[26] Bandypadhyay S and Cahay M 2008 Introduction to Spintronics (Boca Raton, FL: CRC Press)
[27] Wolf S A,Awschalom D D, Buhman R A, Daughton J M, von Molnar S, Roukes M L, Chichkovanova A Y and Treger D M 2001 Science 294 1488
[28] Awschalom D D and Flatté M E 2007 Nature Phys. 3 153
[29] Zatic I, Fabian J and Das Sarma S 2004 Rev. Mod. Phys. 76 323
[30] Rashba E 1960 Fiz. Tverd. Telu 8 1224
[31] Rashba E I 1960 Sov. Phys.—Solid State 2 1109 (Engl. transl.)
[32] Bychkov Y A and Rashba E I 1984 J. Phys. C: Solid State Phys. 17 6039
[33] Dresselhaus G 1955 Phys. Rev. 100 580
[34] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 Phys. Rev. Lett. 78 1335
[35] Matsuyama T, Kursten R, Meissner C and Merkt U 2000 Phys. Rev. B 61 15858
[36] Luo J, Munekata H, Fang F F and Stiles P J 1988 Phys. Rev. B 38 10142
[37] Das B, Datta S and Reifenberger R 1990 Phys. Rev. B 41 8278
[38] Hu C-M, Nitta J, Akazaki T, Takayanagi H, Osaka J, Pfeiffer P and Zawadzki W 1999 Phys. Rev. B 60 7736
[39] Cappelluti E, Grimalski C and Marsiglio F 2007 Phys. Rev. B 76 085334
[40] Cappelluti E, Grimalski C and Marsiglio F 2007 Phys. Rev. Lett. 98 167602
[41] Li Z, Ma Z, Wright A R and Zhang C 2007 Appl. Phys. Lett. 91 121120
[42] Chen L, Ma Z, Cao J C, Zhang Y and Zhang C 2007 Appl. Phys. Lett. 91 121121
[43] Biswas T and Ghosh T K 2013 J. Phys.: Condens. Matter 25 035300
[44] Wang X F and Vasilopoulos P 2003 Phys. Rev. B 67 085313
[45] Islam S K F and Ghosh T K 2012 J. Phys.: Condens. Matter 24 345301
[46] Fang F F and Howard W E 1966 Phys. Rev. Lett. 17 1109
[47] Langenbuech M, Suhrike M and Rossler U 2004 Phys. Rev. B 69 125303
[48] Islam S K F and Ghosh T K 2012 J. Phys.: Condens. Matter 24 185303
[49] Biswas T and Ghosh T K 2013 J. Phys.: Condens. Matter 25 265301