Adiabatic exponent in isentropic convective zone: a heavy elements abundance and seismic inversion

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Abstract. Profile of adiabatic exponent along isentropic curve is considered in context of determination of the CO and Ne contents in the solar convective zone. Tiny variations of the adiabatic exponent are caused by ionizations of elements. Isolated ionization stage leads to a narrow gap of the adiabatic exponent. Position of the gap depends on the ionization potentials and partition function of ions, which are essential parts of an equation of state. With the extended version of SAHA-S EOS we are able to study traces of elements in the adiabatic exponent profile. We give preliminary estimations on the abundance according to the inverted profile of the adiabatic exponent. Results do not reveal an indication of low-Z abundances in the adiabatic part of the convection zone.

1. Introduction

We focus on tiny variations of the adiabatic exponent $\Gamma_1$ profile. In the lower part of the solar convection zone a function $\Gamma_1 (\lg T)$ is following to adiabatic stratification. The function $\Gamma_1 (\lg T)$ appears is determined by the EOS and depends on a model only via a value of entropy of the adiabatic curve.

Adiabatic exponent is determined by several factors. First, due to almost full ionization of hydrogen and helium, $\Gamma_1$ is close to 5/3 as in the case of monatomic perfect gas. Second, $\Gamma_1$ slightly increases over 5/3 due to the Coulomb screening, which causes some sort of “Coulomb elasticity” ([1]). Third, value of $\Gamma_1$ decreases if a degree of ionization and specific capacity are increased. Hydrogen and helium may not be fully ionized, but reach limits of adiabatic ionization, and they do not contribute to lowering of $\Gamma_1$. Heavy elements gradually ionized with depth mainly contribute to $\Gamma_1$ lowering. Profiles of $\Gamma_1$ depression may be considered as an adiabatic spectrum of the element. A sum of the elements contributions gives final $\Gamma_1$ profile, what may serve for estimations of the heavy elements content. We present $\Gamma_1$ profile variations due to several factors. One of them is a value of the specific entropy which specifies an adiabatic curve. The helium content and specific contribution to $\Gamma_1$ from the heavy elements ionization, included to SAHA-S EOS, i.e. C, N, O, Ne, Si, Fe, are considered. In the conclusion we roughly estimate the heavy element content from the high-precision helioseismic inversion of $\Gamma_1$.

The results are obtained with the newly developed SAHA-S EOS ([2],[3]). The SAHA-S EOS characterized by the extended set of included elements and some details in physics (see [4]). Together with basic thermodynamic varietes, the set of the ionic distributions and some other values are available. At the moment an updated version SAHA-S2 EOS is beeing developed.
Figure 1. The value of $\Gamma_1$ along three adiabatic curves with the specific entropy $s/R_g = 25, 27, 29$ (dotted, solid, dashed curves accordingly). Thin curve shows the $\Gamma_1$ in the model $S$ calculated with OPAL96 EOS.

Figure 2. $\Gamma_1$ for different mixtures. Solid curve is for pure hydrogen. Dashed – for hydrogen-helium mixture with $X = 0.7$. Dot-dashed and dotted are for SAHA-S2 and SAHA-S ($X = 0.7$ and $Z = 0.02$). Thin dashed is for OPAL96 EOS.

2. Effect of the specific entropy
Actual adiabatic curve of the solar convection zone is unknown a priory. Generally, the specific entropy of the convection zone (together with the envelope helium abundance) is defined in calibration of the model to the solar radius and luminosity. The model calibration provides the convective entropy difference between solar models with similar opacities is rather small. Alternatively, the convective entropy may be found in a course of the helium ionization zone calibration (see e.g. [5]), which is essentially independent of the model calibration. Accordance of these calibrations is a necessarily condition for the adequate model of the Sun. Fig. 1 shows $\Gamma_1$ along three adiabatic curves, which are calculated for the selected values of the specific entropy - $s/R_g = 25, 27, 29$. The curves and $\Gamma_1$ have been calculated with SAHA-S EOS for the mixture with $X = 0.7$ and $Z = 0.02$. Even if the effect of entropy is rather large, the plotted variations of entropy are much larger then the model or helioseismic calibration uncertainties. The thin curve corresponds to $\Gamma_1$ in the model $S$ [6]. The adiabatic exponent $\Gamma_1$ rises with lowering of the entropy (i.e. increase of density at the same temperature) because of higher Coulomb effect. At the same time, the ionization depression of $\Gamma_1$ (and the radiative lowering) becomes smaller with smaller entropy, so a maximum of $\Gamma_1$ may exceed of 5/3.

3. Chemical composition effect
Fig. 2 shows effects of main components of solar mixture on general $\Gamma_1$ profile, calculated for the model set of points. The maximum of the adiabatic exponent $\Gamma_1$ is most remarkable on pure hydrogen and determined by the nonideal Coulomb ”excess” of $\Gamma_1$. The depression due to helium ionization reduces the maximum noticeably and shifts it downward. The $Z$-ionization depression of the maximum is comparable with the effect of helium. It is interesting that the $Z$-depression increases with the helium content due to smaller total particle concentration. So exact value of $\Gamma_1$ depends on accurate helium content in the convection zone. Fig. 2 shows effect of EOS, calculated on the same set of points. The difference with a ”classic” version of OPAL96 EOS [7] is rather remarkable. Comparison with OPAL2001 and OPAL2005 is considered in [4]. But reason of general excess in $\Gamma_1$ of OPAL EOS over SAHA-S EOS is still unclear. $\Gamma_1$ plotted from SAHA-S2 EOS, which differs from a predecessor with newly added heavy elements ($S$ and $Mg$) and revised system of exited states of ions.
4. Heavy elements contributions to the $\Gamma_1$ depression

![Figure 3.](image1.png) ![Figure 4.](image2.png)

**Figure 3.** The distribution of the oxygen ions in solar models. The relative parts of ions marked with the degree of ionization are plotted together with the mean charge of ions.

**Figure 4.** The depression of the adiabatic exponent due to ionization of oxygen (solid curve, left axis) and the mean charge of the oxygen ions (dashed curve, right axis).

Last considered effect is the ionization depression of adiabatic exponent induced by the heavy elements. The depression of adiabatic exponent in a region of ionization arises due to an increase of the specific capacity $c_V$. Heavy elements are considered as small impurities on the background of hydrogen and helium. The technique of calculations of $\Gamma_1$-contribution in this approximation is described in [8]. We present distributions of the elements ions along model points for two examples, oxygen and neon. On Fig. 3 and Fig. 5 the relative parts of ions $n_j^Z/N^Z$ in $j$-stages of ionization are plotted for oxygen and neon. The mean charge of ion divided by atomic number $\langle Z \rangle/\langle Z_{el} \rangle$ is plotted as the monotonically increasing solid curve. With enlarge of atomic number $Z_{el}$, the regions of ionized ions are overlapped and an "jumping" over some ionization stage becomes possible. It can be clearly seen on example of neon and iron (see figures for the iron in [4]). The effect appears when the maximum relative part of ion is smaller then neighbors and causes a stepper rise of the mean charge. Fig. 4 and 6 give contributions to $\Gamma_1$ depressions due to ionization of oxygen and neon. Also, the mean charge $\langle Z_j \rangle$ of ions are plotted as dashed curves versus the right axis. The main purpose is searching a relationship between a course of ionization and "gaps" $\Gamma_1$ depression profiles from individual element. The $C$, $N$ and $O$ are very similar and do not show ionization "jumping" in the solar convection zone. But "gaps" of $\Gamma_1$ do exist. They are connected with ionization of the last two electrons from K-shell. The potentials of K-ionization are large and these ionizations are well separated from others. Both two K-ionizations are overlapped to one gap within the convective zone. Moreover, all three main contributions from CNO are overlapped and appear as common depression near the bottom of convection zone. Interestingly, that the neon $\Gamma_1$-depression also demonstrate a narrow gap, although the neon does not reach K-shell ionizations inside the convection zone. The neon gap is due to ionization "jumping" and localizes at $\lg T \approx 5.6$, where coinciding with the local maximum of the $\Gamma_1$. So increase of the neon content affects the peak of $\Gamma_1$ and may lead to "camel humps" on $\Gamma_1$ profile. We can conclude, that the elements specifically contribute to $\Gamma_1$, what allow analyse a content of a element or a group of elements.
5. Conclusion
Detailed analysis of ionization of heavy elements reveals specific variations $\Gamma_1$ from the individual elements. The gaps of lowering have specific position at temperature and are highly narrow in the case of the neon and iron. We explain them as result of ionization "jumping". After comparison of the result of helioseismic inversion of $\Gamma_1$ (see [9], and figure in [4]) and SAHA-S profiles for different abundances, we preliminary find that the neon abundance may be somewhat less than assumed in [10], but the content of CNO is hardly less than supposed in [10]. That is low-Z content [11] is not revealed in $\Gamma_1$ profile. These conclusions are preliminary and based on tiny features of $\Gamma_1$ profile. Also, it is based on the high-accuracy inversion results, which is extremely difficult problem.

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References
[1] Baturin V A, Däppen W, Gough D O and Vorontsov S V 2000 Mon. Not. R. Astron. Soc. 316 71-83
[2] Gryaznov V K, Ayukov S V, Baturin V A, Isosilevskiy I L, Starostin A N and Fortov V E 2004 Equation-of-state and phase-transition issues in models of ordinary astrophysical matter (Leiden) ed V Celebonovic, W Däppen and D Gough (Melville, New York: AIP conference proceedings 731) 147
[3] Gryaznov V K, Ayukov S V, Baturin V A, Isosilevskiy I L, Starostin A N and Fortov V E 2006 J. Phys. A, Math. General 39 4459
[4] Baturin V A 2010 it Astrophys. Space Sci. 328 147-151
[5] Baturin V A 2006 Beyond the spherical Sun (Sheffield) ed K Fletcher and M J Thompson (Noordwijk: ESA Publications Division SP-624)
[6] Christensen-Dalsgaard J et al. 1996 Science 272 1286
[7] Rogers F, Swenson F and Iglesias C 1996 Astrophys. J 456 902 (OPAL96)
[8] Baturin V A 2007 Unsolved problems in stellar physics (Cambridge) ed R J Stancliffe, J Dewi, G Houdek, R G Martin and C A Tout (Melville, New York: AIP conference proceedings 948) 213
[9] Vorontsov S V 2004 Equation-of-state and phase-transition issues in models of ordinary astrophysical matter (Leiden) ed V Celebonovic, W Däppen D Gough (Melville, New York: AIP conference proceedings 751) 47
[10] Grevesse N and Noels A 1993 Origin and Evolution of the Elements ed N Prantzos, E. Vangioni-Flam and M Casse (Cambridge: Cambridge Univ. Press) pp. 14-25 (GN93)
[11] Asplund M, Grevesse N, Sauval A J and Scott P 2009 Annu. Rev. Astron. Astrophys. 47(1), 481 (AGSS09)