INTRODUCTION

The main purpose of aircraft engine compressors is to compress the air. This is due to the transfer of the kinetic energy from the rotating blades to the working medium. During the engine operation, air suction is generated in the course of the compression process [8]. As a result of this phenomenon, pickup and suction of a hard element into the engine may occur. Depending on the size, shape, and hardness, if the sucked element collides with the blade, the blade may be damaged (foreign object damage - FOD). The observed damage is often combined with plastic deformation and local change of the blade geometry.

The blades, themselves are critical elements with a complex state of loading. Mass forces combined with the pressure of the incoming working medium make the compressor blades vulnerable to destruction. The very geometry of the blade, i.e. its relatively small thickness concerning the other dimensions, means that the blades are characterized by low flexural rigidity. Low rigidity combined with harsh operating conditions and blade damage can contribute to rupture and breakage of the blade, which can result in engine damage or destruction.

The strength and durability of structural the aircraft and aircraft engines elements was the subject of many scientific papers [11, 15]. Unfortunately, most of the works focus either on the specific cases of damage and destruction [11] or refers to the geometric damage [15] (not including the stress associated with plasticizing the material).

The fatigue life of structural elements is influenced by many factors. Starting from the finite element mesh [14] in the case of the FEM analyses, the parameterization of the analysis and material data also have a significant influence. It is
particularly important to accurately and reliably determine the fatigue material data [12, 13] and also to consider cyclic hardening [9] (in the case of fatigue tests) and correction of mean stresses [3, 6]. The fatigue life results are also influenced by initial (residual) stresses and their concentration [5, 7] around the critical points of the structure. The impact of the material itself and its resistance to cracking [10] on the fatigue life should not be overlooked. Surface treatment also has an effect on durability [4, 5, 13], including shot peening and coating.

The main purpose of this work was to conduct numerical fatigue analysis of a first stage compressor blade from the PZL-10W turbine engine with the damage bearing signs of plastic deformation. This blade is made of EI-961 chromium steel. As part of the numerical tests, a simulation of collision with a hard object and exposure of the plasticized zone due to this impact was carried out. Then the obtained model was subjected to numerical harmonic analysis, thanks to which a load of a similar resonance nature (and the form of natural vibrations) was simulated. The analyzes was carried out for selected several values of vibration amplitudes. Stress distributions obtained in this way were used for numerical fatigue analysis.

The second purpose of the work was to compare the results of numerical fatigue analyses with the results of experimental studies [2]. In this work, a blade with the notch with a plastic nature (simulating FOD) were examined. Then, damaged blades were subjected to experimental fatigue tests under resonance conditions with different vibration amplitudes. Thanks to this approach, enables to assess the impact of vibration amplitude on the fatigue life of an aero-engine compressor blade and also to evaluate the influence of plastic deformation near the notch on fatigue life.

**NUMERICAL FATIGUE ANALYSIS**

The numerical fatigue analysis presented in the following publication requires simulation of a hard object impact (FOD) and harmonic analysis to determine the distribution of stress during vibration. Obtaining the residual stress distribution after the simulated impact allows determining the impact of the resulting plastic deformation on the fatigue life.

**Basic assumptions of the analysis**

The fatigue analysis itself was carried out for 9 different material fatigue models, three models of cyclic hardening, and three cases of mean stress correction (none, Morrow and Smith-Watson-Topper).

For e-N [12, 13] (low-cycle fatigue) analysis, 4 material data obtained analytically are needed. The analysis itself is based on the Manson-Coffin-Basquin equation (1). For the current analysis, 8 material fatigue models (Manson, 4-point Manson, Mitchell, Muralidharan-Manson, Baumel-Seeger, Ong, Roessle-Fatemi, and Median) were used and an additional - average value from all previous ones was created.

\[
\frac{\Delta \varepsilon}{2} = \frac{\varepsilon_e + \varepsilon_p}{2} = \frac{\sigma_f'(2N_f)^h + \varepsilon_f'(2N_f)^c}{E} \tag{1}
\]

where:

- $\Delta \varepsilon$ – total strain amplitude
- $\varepsilon_e$ – elastic strain
- $\varepsilon_p$ – plastic strain

![Fig. 1. Damaged first stage compressor blade of PZL-10W turbine engine (with a notch created under laboratory conditions)](image-url)
Because resonance analyzes are carried out based on a linear-elastic model, the Ramberg-Osgood hardening model (2) should be used, which works very well in the case of fatigue analyzes conducted for steel and metal elements [9].

\[
\Delta \varepsilon = \frac{\Delta \sigma}{E} + 2\left(\frac{\Delta \sigma}{2K'}\right)^{1/n'}
\]

(2)

where:
- \(\Delta \varepsilon\) – strain amplitude
- \(\Delta \sigma\) – the stress amplitude, MPa
- \(K'\) – cyclic strength coefficient, MPa
- \(n'\) – cyclic strain hardening exponent

Since the plastic deformation caused by impact with a hard object causes stresses on the surface of the notch bottom, the correction of mean stresses should be considered [3, 6], because of the cycle asymmetry factor (ratio of minimum and maximum stress in load cycle). The analysis used the modification of mean stresses based on the Morrow method (3) and the Smith-Watson-Topper method (4).

\[
\sigma_{alt} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f}}
\]

(3)

\[
\sigma_{alt} = \sqrt{\sigma_{max} \sigma_a}
\]

(4)

where:
- \(\sigma_{alt}\) – alternating stress, MPa
- \(\sigma_a\) – stress amplitude, MPa
- \(\sigma_m\) – mean stress, MPa
- \(\sigma_{max}\) – maximum stress, MPa

The tested blade is made of EI-961 alloy. As part of the static tensile test, Young’s modulus was determined and its value equaled 200 GPa, Poisson’s ratio was equal to \(v = 0.3\) and tensile strength UTS = 1200 MPa.

**Notch formation**

In order to determine the effect of plastic deformation on the distribution of principal stress in a notched blade made by impact, a two-stage numerical analysis was conducted. The first step involved denting a hard object (beater) into the leading edge of the compressor blade (the beater was modeled as a rigid body). The beater had a tip radius of 0.05 mm and an opening angle of 90°. The blade material was modeled as elastic-plastic with hardening. Frictional contact (\(\mu = 0.1\)) was defined between the beater and the blade. In the second step, the beater was withdrawn (Fig. 2a). After the beater retraction, it was possible to determine the values of pre-stressed (residual) (Fig. 3) and plastic deformations (Fig. 2) arising in the blade during the notch formation process.

The indentation resulted in a notch which was about \(b = 0.5\) mm deep (Fig. 2a) at a distance from the blade foot equal to \(h = 3\) mm. Plasticization of the material caused a local increase in the blade thickness by about 0.26 mm (Fig. 2b). The obtained plastic deformation is associated with the appearance of high-stress values in the material and on the surface of the notch.

Principal stresses after the notch formation process ranged from -316 to about 928 MPa. The lowest values were observed on the surface of the notch, at the concave surface of the blade (Fig. 3). In turn, the area of maximum stress...
values was located under the notch surface (Fig. 3a). The maximum stress on the notch surface was about 683 MPa (Fig. 3b).

The initial stresses determined in the paper and the change in the blade geometry in the notch zone have a significant impact on the fatigue life of the blade subjected to vibrations. The stress values in the model nodes obtained during calculations were saved in a text file and then imported in the next analysis (vibration simulation) as initial stresses. Similarly, the initial blade geometry was transferred for the vibration analysis. Owing to this approach, the subsequent analyses included both the geometry of the notched blade made by impact and the initial stress generated in the notch formation process.

**Results of numerical fatigue analysis**

This part of the work describes the results of stress distribution in a notched blade made by the impact of a hard object, exposed to resonance vibrations. The blade deflection that arose during the first form of resonance vibrations was determined by harmonic analysis in the ANSYS program [3].

As a result of the numerical analyses, stress values for the different amplitudes of vibrations (A) were determined. The results presented in Table 1 refer to the values of the highest stresses located near the notch bottom.

In the case of, the stress value increased from 415 MPa (at A = 1.8 mm) to 511 MPa (at A = 2.5 mm). The analyses show that the presence of compressive residual stresses caused by plasticizing the material caused a significant decrease in the main stresses observed on the surface of the blade subjected to resonance vibrations.

As in the case of the numerical analysis of e-N notched blades, as well as in the case of blades with a notch made by an impact with a hard object, a series of 27 simulations were carried out for various combinations of material fatigue models and hardening models. The same load history was used in the calculations (pendulum sinusoidal variables). In the presence of compressive stresses in the notch bottom, the mean value over the load cycle was negative.

The number of $N_{in}$ cycles for initiation was estimated for 4 different vibration amplitudes: $A = 1.8$ mm, $A = 2.1$ mm, $A = 2.3$ mm and $A = 2.5$ mm. All tested blades had the same initial stress condition resulting from the process of denting a hard object into the leading edge of the blade.

In the analysis of e-N notched blades, various models of mean stress correction $\sigma_m$ were used. The occurrence of initial stresses arising during the notch formation caused that the mean stress value $\sigma_m$ in the load cycle was different from zero. The calculations were carried out for one point where the maximum stress values were recorded. Due to the presence of compressive residual stresses in the notch zone, the modification of mean stresses was taken into account in the estimation of the number of $N_{in}$ cycles. By default, two different models of mean stress correction can be used in the e-N analysis carried out in ANSYS: Morrow and SWT (Smith-Watson-Topper).

In all analyses carried out, the highest $N_{in}$ value (regardless of the method of modifying mean stresses) was obtained for the Ong’s material model and Fatemi’s hardening model.

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Fig. 3. Maximum principal stress $\sigma_1$ distribution in the cross-section of the blade (a) and on the surface (b), MPa
The analysis took into account the limit (maximum) number of load cycles at 10° load cycles. For the test blade with an amplitude of \( A = 1.8 \) mm, the highest \( N_{\text{in}} \) value (at 10° cycles) was obtained for the following models (fatigue model/hardening model/correction model): Ong/Man/none, Ong/Man/Mor, Ong/Man/SWT, Ong/Fatem/none, Ong/Fatem/Mor, Ong/Fatem/SWT, Ong/Xian/none, Ong/Xian/Mor and Ong/Xian/SWT. On the basis of the above-mentioned analysis, it can be concluded that the highest blade durability was obtained in the case of the numerical analyses based on Ong’s model (using SWT stress correction).

The lowest durability (for \( A = 1.8 \) mm - when using mean stress correction) was obtained in the case of model sets: Man/Man/Mor, Man/Man/SWT, Man/Fatem/Mor, Man/Fatem/SWT, Man/Xian/Mor, and Man/Xian/SWT - the value obtained was 9.74 \( \times 10^6 \) cycles. The use of correction gave a nearly 50% higher result (in the case of Man4/Man/none, \( N_{\text{in}} = 5.96 \times 10^6 \) cycles was obtained).

For the blade analyzed at an amplitude of \( A = 2.3 \) mm, the maximum \( N_{\text{in}} \) value occurred for all types of correction of mean stresses, in the case of analyses based on the Ong’s material model (with Fatemi’s hardening).

The lowest values of \( N_{\text{in}} \) were obtained in the case of the analyses for the blade tested under amplitude conditions with \( A = 2.5 \) mm. This is the first case in which the maximum \( N_{\text{in}} \) blade value did not reach the limit life value for these analyses (1 \( \times 10^6 \) cycles) and amounted to 5.9 million cycles (Ong / Fatem, Ong / Man and Ong / Xian for Morrow and SWT stress correction).

To assess the impact of the selected test parameters on the value of the number of \( N_{\text{in}} \) cycles, some of the results are presented in a graphical form in Figure 4. This figure contains 9 graphs of the change in the number of \( N_{\text{in}} \) cycles as a function of vibration amplitude (\( A, \) mm). Each graph has a specific set of parameters: the type of mean stress correction and the type of result (maximum, average, minimum).

The graphs in the first row (Fig. 4) refer to the lack of correction of the mean stress. The second line presents the \( N_{\text{in}} \) results, including Morrow correction, while the third line presents the results obtained based on the Smith-Watson-Topper stress correction. In turn, the first column concerns only the maximum results (for various modifications of mean stresses), the second column presents the average values of \( N_{\text{in}} \) (from all 27 material variants), while the third column contains the minimum results for \( N_{\text{in}} \). Owing to such presentation of the obtained results, it is possible to qualitatively assess the impact of vibration amplitude and a given modification on fatigue life.

Observing the maximum \( N_{\text{in}} \) values (Fig. 4), it can be seen that up to an amplitude of \( A = 2.3 \) mm, the maximum possible value of the number of cycles for initiation was reached (10°). Further increasing the amplitude to 2.5 mm resulted in a decrease of \( N_{\text{in}} \) to about 5 \( \times 10^6 \) cycles. Initially, it can be stated that the modification of average stresses did not have a significant impact on these results (the waveforms presented in the first column are similar). A similar relationship can be observed when assessing the results of the minimum \( N_{\text{in}} \) values. In all compared cases of the vibration amplitude impact (third column of graphs in Figure 4), the observed changes are similar. In the beginning, in the range from 1.8 to 2.1 mm, a significant decrease in the number of \( N_{\text{in}} \) cycles is observed.

The effect of stress correction can be observed for the results related to the average score of \( N_{\text{in}} \). In the absence of this correction, an increase in amplitude above 2.1 mm caused a sharp decrease in the number of cycles to initiate, while in the case of using mean stress correction, this decrease occurred after exceeding the amplitude of 2.3 mm.

In the case of the analysis of the results based on the minimum values of \( N_{\text{in}} \), the use of mean stress correction resulted in an increase in blade life. Omitting of the stress correction resulted in obtaining \( N_{\text{in}} \) results at the level of 6 \( \times 10^6 \) (for vibration amplitude \( A = 1.8 \) mm). The introduction of SWT and Morrow mean stress correction increased durability of over 60%.

When analyzing the maximum \( N_{\text{in}} \) results, no effect of mean stress correction on the value of the number of cycles for crack initiation was observed (which is caused by taking into account the numerical limit).

| Amplitude A, mm | \( \sigma_{\text{in}}, \) MPa |
|-----------------|-----------------|
| 1.8             | 415.48          |
| 2.1             | 456.46          |
| 2.3             | 484.14          |
| 2.5             | 511.06          |
COMPARATIVE ANALYSIS

One of the main goals of the work was to compare the number of cycles for the initiation of fatigue crack \( (N_{in}) \) of the blade, obtained in experimental studies \([2]\) and numerical calculations. Based on the mentioned experimental results, for comparative purposes, a blade tested in the resonance range with an amplitude of \( A = 2.5 \) mm (blades tested in the range of amplitudes less than 2.5 mm did not crack under the given load conditions) was selected. The results discussion/comparison was summarized in Table 2.

In the cited work \([2]\), 8 blades were tested in conditions of resonance vibrations. As part of the experiment, it was determined that in the blade with a notch obtained by FOD (\( h = 3 \) mm, \( b = 0.5 \) mm), the crack was initiated only in the case of amplitude \( A = 2.5 \) mm. In the other tested blades (in smaller amplitudes) no cracks were observed despite 40 million load cycles. The very development of the crack in the mentioned cracked blades was too dynamic to control and monitor.

The results of the quoted experimental studies indicate that for the blade with notched by impact (\( h = 3 \) mm, \( b = 0.5 \) mm, \( A = 2.5 \) mm), the number of cycles for crack initiation was \( 5.94 \times 10^6 \). In the case of numerical calculations, 3 average \( N_{in} \) results were obtained (based on stress values). The number of load cycles for crack initiation was estimated (average value was \( 26.3 \times 10^6 \) (no stress correction) and \( 57.8 \times 10^6 \) (for mean stress correction according to the Morrow and SWT methods). Therefore, the

![Diagram](image-url)

**Fig. 4.** Results of the numerical fatigue analysis in order to different type of amplitude and mean stress correction
result was estimated by a numerical method (for the variant without stress correction) was about four times larger than the experimental result. In the case of mean stress correction, the numerical score $N_{in}$ was about 10 times greater than the experimental.

**CONCLUSIONS**

As a result of the work, the impact of the vibration amplitude on the fatigue life of the compressor blade was determined. The effect of the fatigue material model for the e-N analysis, cyclic hardening model, and correction of mean stress on the value of the number of cycles for fatigue crack initiation was also determined. The results of the cited experimental studies were adopted as a reference point. In the mentioned work [2], it was shown that a 0.2 mm long crack in a blade damaged by impact with a hard object (notch geometry similar in both numerical and experimental cases) was only initiated at a 2.5 mm amplitude (no crack was observed at lower amplitudes). Similar observations were obtained in most results from the numerical fatigue analysis. Also, it was found that the use of mean stress correction causes a nearly double increase in the number of cycles until the crack appears. It was also determined that the occurrence of the damage with plastic deformation (simulated FOD) causes the appearance of compressive stresses in the notch bottom, which causes an increase in the fatigue life of the element (comparison to results presented in previous work [1]).

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