Application of Pollution Index Method Based on Dynamic Combination Weight to Water Quality Evaluation

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Abstract. In view of the shortage of the current single-factor assessment and the common pollution index method, a new water quality comprehensive pollution index method was proposed to achieve the water quality evaluation. The combination of AHP and entropy method is used, and the dynamic adjustment of S type function is used to enlarge the influence of standard exceeded pollutants. It solved the problem that the average pollution index method is too loose and the single factor evaluation method is too strict. The evaluation results were compared with single-factor assessment, average pollution index method and Nemerow index method. We effectively proved the rationality of the evaluation method.

1. Introduction
Water is a resource that people rely on for survival. Today, water resources are very urgent with the absence of water resources and serious water pollution. The quantitative evaluation of water quality is the prerequisite for the protection of water resources, and it is the basis for the management of water source. The common water quality evaluation method included single-factor assessment, pollution index method, comprehensive evaluation method and so forth. Water quality evaluation method Chinese government used in water quality monitoring report is the single-factor assessment, using the measured data and standards for comparison and selecting the worst water quality category to get the evaluation results. However, the evaluation result is quite conservative and exaggerated the effects of the most serious pollution index and do not consider the influence of other index. Pollution index method is a method to compare the measured value of the water quality detection index with the standard value, and finally get the water pollution index. At present, the commonly used pollution index method is the Nemerow index method. Nemerow index method used the maximum and average value of the index to examine the pollution degree of the water quality. In addition to considering the worst index, it also takes into account the effect of other indicators. This method is simple and easy to describe, but it overemphasizes the contribution of the worst indicators to water pollution, and makes the evaluation result highly correlated with the worst indicators, that is, the water pollution index is largely determined by the worst indicators. The comprehensive evaluation method is a method of evaluating water quality by using the measured value of the index and evaluating the water body according to the score.

According to the national water quality monitoring weekly (the 5th issue of 2018) published by the Ministry of environmental protection of People's Republic of China, we selected 143 samples of water
quality monitoring data to analyze. We selected 4 indexes of each sample, being the pH value, dissolved oxygen, ammonia concentration and permanganate index, getting the pollution index of each index, and determining their weights. There are three main methods determining the weight, the first is the subjective weighting method, such as the analytic hierarchy process (AHP), least-square method, Delphi method and so forth. The subjective weighting method can determine the weight according to the experience and knowledge, and will not contradict the actual situation, but the result of the evaluation contains more subjective components. The second is objective weighting method, such as principal component analysis (PCA), entropy method, multi-object planning method, etc. Objective weighting method is to determine weight by mathematical analysis, and the weight is more objective and has theoretical support. The third is combination weighting method, which combines the two methods above to determine the weight. In this paper, we used the third method to combine the subjective weight obtained by AHP method and the objective weight obtained by entropy weight method, and get the combination weight.

Most of the current evaluation methods suppose that the impact of pollutants on the environment is linear with the concentration of pollutants, but the fact is not the case. In addition, for different water bodies, it is obviously unreasonable to choose the same weight for the pollution index of the corresponding index, that is, we need to reflect the main pollution impact. At the same time, the gap between the indexes of the index is different, such as the little influence of the index from class I to class III. However, the change of water body from class III to class IV is very important, and it is a qualitative change (from no exceeding the standard to exceeding the standard and from being drinkable to not being drinkable). Based on the above considerations, we introduced a "S" type function to dynamically adjust the weight, and finally get the comprehensive pollution index of water. The results are compared with the results obtained from other water quality evaluation methods.

2. Pollution index of each index
The pollution index of each index is attained as follows.

If the value of the index is smaller, the better, as permanganate index and ammonia concentration, then

\[ P_i = c_i / x_i \]

If the value of the index is bigger, the better, as dissolved oxygen, then

\[ P_i = x_i / c_i \]

If the value of the index is closer to a certain value, the better as pH value, then

\[ P_i = \begin{cases} c_i / x_i, & pH \leq 7 \\ x_i / c_i, & pH > 7 \end{cases} \]

where

pollution index is \( P_i \), limit of water quality protection target is \( c_i \) (Here we use the boundary value of class III and IV to be get the threshold value), and the measured value of the index is \( x_i \).

3. Dynamic combination weight of each pollution index

3.1. Subjective weights obtained through AHP
In 143 samples, monitoring data from the permanganate index, dissolved oxygen, ammonia concentration, respectively 5, 9, 16 samples did not meet the water quality standard (class I-III). According to the relevant literature, the relationship between the importance of each index is obtained as \( pH < \text{dissolved oxygen} < \text{permanganate index} < \text{ammonia concentration} \). The difference between the importance of each index is not very obvious, so we get judgment matrix as
The eigenvector corresponding to the maximum eigenvalue is

\[ w_j = (0.095, 0.160, 0.278, 0.467) \]

The maximum eigenvalue is \( \lambda = 4.031 \). Consistency index is \( CR = 0.011 < 0.1 \), so the consistency check is passed.

The weight vector of the pollution index obtained by AHP is

\[ w_j = (0.095, 0.160, 0.278, 0.467) \]

### 3.2. Subjective weights obtained through entropy method

Entropy method is a method of determining weight using the amount of effective information contained in the data. The weight of the pollution index is determined by the difference of the data of pollution index.

First of all, the pollution index is standa

\[ y_j = \frac{p_y - \min(p_i)}{\max(p_i) - \min(p_i)} \]

The information entropy of each index is

\[ E_i = -\ln(1/n) \sum_z z_i \ln(z_i) \]

where

\[ z_i = y_i / \sum_i y_i \]

The weight of each index is

\[ w_{oi} = \frac{1 - E_i}{4 - \sum_i E_i} (i = 1, 2, \ldots, 4) \]

| Pollution index | pH   | DO   | COD(Mn) | NH3-N |
|-----------------|------|------|---------|-------|
| information entropy | 0.970 | 0.991 | 0.966  | 0.823  |
| entropy weight       | 0.121 | 0.036 | 0.136  | 0.707  |

The weight vector of the pollution index obtained by entropy method is

\[ w_o = (0.120, 0.041, 0.135, 0.703) \]

### 3.3. Combination weight

Comparing the weight vectors respectively obtained by the AHP method and entropy method, we can see that the result of the weight obtained by two methods are that ammonia concentration is greater than the permanganate index, and permanganate index is larger than the other two. However, the two methods have the opposite results in the sequencing of pH and dissolved oxygen. Entropy method is based on the amount of information contained in the data to get the weight of the index. However, when the index value fluctuates sharply (suddenly becomes bigger or smaller) or changes slightly, the result of entropy method will be distorted, resulting in a large difference between weights. According to the actual situation, it is not reasonable that the weight of pH is far greater than the weight of dissolved oxygen. AHP can get the result consistent with the fact in order to determine the degree of importance among the indexes. But it has greater subjectivity in determining the relative size of the weight. In order to get more reliable results, we adopt the subjective and objective combination weighting method, combining
entropy method with AHP. The commonly used combination weighting method has the multiplication set method and the addition set method, and the multiplication set method is used here as

\[ w_{ci} = \frac{w_{si} + w_{oi}}{2} \]

So the combination weight is

\[ w_c = (0.107, 0.101, 0.207, 0.585) \]

3.4. Dynamic adjustment of weight and comprehensive pollution index

The relationship between the index concentration of sample and the water quality is nonlinear, and the pollution index of the corresponding index of different samples has the same weight, which is obviously not reasonable. What’s more, the gap between the indexes of the index is different, such as the little influence of the index from class I to class III. However the change of water body from class III to class IV is very conspicuous, and it is a qualitative change (from no exceeding the standard to exceeding the standard and from being drinkable to not being drinkable). The dynamic adjustment of weight needs to consider the above factors, so the S type function is used to dynamically adjust the weight. According to the GB3838-2002《Surface Water Environment Quality Standard》, There is no limit on the pH value of all classes of water. The pH values in the 143 samples all meet the requirements. So we make a dynamic adjustment to the pollution index except pH value. The expression of the S type function is

\[ f(x) = \frac{2b}{1 + e^{k(b-x)}} + c \]

where the central point of this function is \((a, b+c)\), the maximum rate of change is \(k\) which is at the center point, the left and right positions of the function center point is determined by \(a\), the amplitude of the change of the function is determined by \(b\), the lower limit of the function is determined by \(c\).

For convenience, the definition of \(x\) is a category boundary. For a certain pollution index, if the index is in the boundary of class III and IV, then \(x=3\). When the pollution index is not in the boundary, we can use linear interpolation to obtain \(x\).

![Fig1. The function image of S type function](image)

After many experiments, we found that it can achieve better results when \(a=3.08\), \(b=0.1\), \(c=1\) and \(k=5\).

\[ f(x) = \frac{0.2}{1 + e^{20(3.08-x)}} + 1 \]

The dynamic combination weight of each index is

\[ w_{di} = \begin{cases} w_{ci}, & i = 1 \\ f(x)w_{ci}, & i = 2, 3, 4 \end{cases} \]

The comprehensive pollution index of the water quality of each sample is

\[ P_j = \sum_{i=1}^{n} w_{di}P_{ij} \]
4. Analysis of results
We use single-factor assessment, average pollution index method, Nemero index method, combination weight pollution index method and dynamic combination weight pollution index method to evaluate the water quality of 143 samples respectively. The calculation method of single-factor assessment, average pollution index method, Nemero index method is described in detail in the relevant literature, so I won't go into much detail here. The results of water quality classification obtained by the various evaluation methods are shown in the following table.

From the table above, we can see that the evaluation results of various methods may vary obviously, but the high quality water quality ratio (the first 3 categories of the total percentage) were similar, indicating that these methods are reasonable and practical to some extent. On the whole, the results of the single factor evaluation method are more rigorous and conservative, so the impact of other indexes cannot be reflected for only considering the influence of worst indexes. While average pollution index method is difficult to reflect the influence of overweight indicators, and a standard index may be diluted when combining with other better index, making the distortion of the evaluation results and the conclusion not rigorous. When the few index selected, Nemrow index method may enlarge the limit of larger pollution index, making evaluation too loose and lead to evaluation results anamorphic, if the value of pollution index varies a lot. If there is a large difference between different index values of the same sample of water, the evaluation of this sample may be distorted. So do the average pollution index method. Combination weighting method due to its reasonable choice of the weights of evaluation result is superior to the average pollution index method and Nemero index method. In general, the result of combination weight method is close to that of single factor evaluation. But the evaluation results of water of class III are very different. In the following, there are some reasons for this phenomenon.

Comparing evaluation results of single-factor assessment and combination weight method, we find that the results of single-factor assessment at 29 samples are class III, while the results of combination weight method are class II. The combination weight method and the dynamic combination weight method are the same for the evaluation results of 29 samples. There are 21 samples of class III evaluated by the single factor evaluation method as class III for permanganate index does not meet the requirements of class II (2-4). Among them, 9 samples of the permanganate index only slightly exceed class II requirement, from 4.1 to 4.3.

| Location of the sample                  | pH     | DO   | COD(Mn) | NH3-N | single-factor assessment | combination weight method |
|----------------------------------------|--------|------|---------|-------|--------------------------|--------------------------|
| Xuzhou red ring                        | 7.85   | 15.2 | 4.2     | 0.15  | III                      | II                       |
| Wusu Town, Fuyuan                     | 6.51   | 9.92 | 4.3     | 0.2   | III                      | II                       |
| Yanbian Nanping                        | 7.51   | 7.59 | 4.2     | 0.22  | III                      | II                       |
| Songhua River village, Changchun       | 7.13   | 11.6 | 4.2     | 0.3   | III                      | II                       |
| West Mountain in Suzhou                | 7.71   | 16.1 | 4.1     | 0.33  | III                      | II                       |
| Jilin River Estuary                    | 8.5    | 8.64 | 4.1     | 0.31  | III                      | II                       |
| Linyi Heavy Mill Bridge                | 8.28   | 13.7 | 4.1     | 0.38  | III                      | II                       |
| Zhoukou Shenqiu sluice                 | 8.09   | 13.21| 4.1     | 0.48  | III                      | II                       |
| Jilin Dunhua meadow                    | 7.5    | 9.25 | 4.1     | 0.53  | III                      | II                       |
The 9 samples come from 4 different rivers in the Liaohe river basin (the Jilin River and the Songhua River village are both in Songhua River) and 4 different rivers in the Huaiehe river basin. The dissolved oxygen at all these 9 points meets class I requirements. The permanganate index at 8 of these points meets class II requirements. Therefore, in the Liaohe River Basin and Huaiehe river basin water quality assessment, the handle of permanganate index determine the boundary between class II and class III of water bodies. Single-factor assessment is too strict and unreasonable. By comparison, the results obtained by the combination weight method are more reasonable.

The results of the dynamic combination weight method and the combination weight method are basically the same. The results of the 6 samples are different as the following table.

Table 4 The comparison of combination weight method and dynamic combination weight method

| Location of the sample | pH   | DO   | COD(Mg/L) | NH3-N | combination weight method | dynamic combination weight method | single-factor assessment |
|------------------------|------|------|-----------|-------|---------------------------|-----------------------------------|-------------------------|
| South of Dianchi, Kunming | 8.92 | 8.53 | 6.6       | 0.35  | II                        | III                               | IV                      |
| Little Wang Qiao, Huaibe | 8.33 | 9.77 | 7.5       | 0.97  | III                       | IV                                | IV                      |
| Huaibin hydrology station, Xinyang | 7.59 | 11.7 | 4.1       | 1.09  | III                       | IV                                | IV                      |
| Weinan Tongguan suspension bridge | 7.54 | 9    | 3.3       | 1.81  | IV                        | V                                 | V                       |
| Haidong people and Bridge | 8.4  | 9.52 | 1.9       | 1.91  | IV                        | V                                 | V                       |
| Jixi Bridge             | 7.21 | 6.15 | 6.3       | 2.74  | V                         | VI                                | VI                      |

Except for the South of Dianchi, Kunming, the results of the other 5 samples dynamic combination weight method are higher than the combination weight method, which is consistent with the single factor evaluation method. Due to the enlargement of the S type function to the standard exceeded pollutants, the evaluation results can reflect the pollution of water quality more, and good results have been obtained. The permanganate index of South of Dianchi, Kunming is type IV while the remaining index is quite good. Therefore, the evaluation result of combination index method is class II and do not conform to the fact. The dynamic combination weight method can enlarge the evaluation index, make the evaluation result close to the real situation, but there are still some shortcomings. We can consider to use the multi-level adjustment of the weight to optimize it.

5. Conclusions
According to the evaluation results, the reliability of single-factor assessment, combination weight method and dynamic combination weight method is better than the average pollution index method and the Nemerow index method. The average pollution index is very difficult to make a reliable assessment. For the less indexes we choose in this text, the difference in the limit value of each pollution index is too large, so the evaluation result of the internal Nemerow index method is seriously distorted. The sensitivity of single-factor assessment to standard exceeded pollutants leads to too strict evaluation result, which makes the evaluation of some rivers in Liaohe river basin and Huaiehe river basin unreasonable. Dynamic combination weighting method due to magnify the impact of standard exceeded pollutants, making the evaluation results more reasonable compared with the combination weighting method. But on some samples, the evaluation is not very reasonable, and it can adjust of the weight compound adjustment not only in III class near IV class boundaries and, in many limits for multistage adjustment, which will get a more reliable evaluation result.

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