DYNAMICS AND NEUTRINO SIGNAL OF BLACK HOLE FORMATION IN NONROTATING FAILED SUPERNOVAE. I. EQUATION OF STATE DEPENDENCE

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ABSTRACT

We study black hole formation and the neutrino signal from the gravitational collapse of a nonrotating massive star of 40 $M_\odot$. Adopting two different sets of realistic equations of state (EOSs) for dense matter, we perform numerical simulations of general relativistic $\nu$-radiation hydrodynamics under spherical symmetry. We make comparisons of core bounce, shock propagation, evolution of nascent proto-neutron stars, and the resulting recollapse to a black hole to reveal the influence of EOSs. We also explore the influence of EOSs on neutrino emission during the evolution toward black hole formation. We find that the speed of contraction of the nascent proto-neutron star, whose mass increases quickly due to the intense accretion, is different depending on the EOS and that the resulting profiles of density and temperature differ significantly. The black hole formation occurs at 0.6–1.3 s after bounce, when the proto-neutron star exceeds its maximum mass, which is crucially determined by the EOS. We find that the average energies of neutrinos increase after bounce because of rapid temperature increase, but at different speeds depending on the EOS. The duration of neutrino emission up to black hole formation is found to be different according to different recollapse timing. These characteristics of neutrino signatures are distinguishable from those for ordinary proto-neutron stars in successful core-collapse supernovae. We discuss the idea that a future detection of neutrinos from a black hole–forming collapse will contribute to revealing the black hole formation and to constraining the EOS at high density and temperature.

Subject headings: black hole physics — equation of state — hydrodynamics — neutrinos — stars: neutron — supernovae: general

1. INTRODUCTION

Massive stars having more than $\sim$10 solar masses ($M_\odot$) end their lives when an iron core is formed after the stages of nuclear burning (Bethe 1990). For massive stars of $\sim$10–20 $M_\odot$, the spectacular events known as supernovae explosions occur due to the gravitational collapse of the iron core and the launch of a shock wave by the core bounce at high density. A successful explosion leaves a proto-neutron star, which emits a bunch of neutrinos during its formation and then cools down (Burrows 1988; Suzuki 1994). The duration ($\sim$20 s) of supernova neutrino emission is determined by the timescale of the diffusion of neutrinos in the dense matter inside the proto-neutron star. In the case of SN 1987A, the burst of neutrinos was detected by terrestrial detectors (Bionta et al. 1987; Hirata et al. 1987) and has proved the general scenario of supernova explosion and the formation of a dense compact object (Sato & Suzuki 1987).

Stars more massive than $\sim$20 $M_\odot$ may have different fates. They usually have larger iron cores and will be intrinsically too massive to have a stellar explosion. In this case, the outcome will be the formation of a black hole regardless of detailed scenarios. Although the number of such black hole–forming massive stars is uncertain, and depends on the mass range and the initial mass function, it definitely consists of a portion of the stellar mass distribution and could be a substantial fraction thereof. Therefore, it is important to clarify the general features of black hole–forming phenomena in the broader context of the gravitational collapse of massive stars. It is especially exciting to predict the neutrino signals from this type of event as a clear and unique identification of black hole formation. The calculated template of neutrino signals will be decisive in the identification of black hole formation in the Galaxy, if it occurs, by terrestrial neutrino research facilities.

Astronomically, the fate of massive stars beyond the mass limit ($\sim$20 $M_\odot$) for ordinary supernovae is currently attracting interest (Heger et al. 2003). Recent analyses of the observed light curves of supernovae exhibiting explosive nucleosynthesis suggest that there are novel categories of explosive phenomena where kinetic energies and ejected $^{56}$Ni masses are different from those of ordinary supernovae (Maeda & Nomoto 2003). In fact, faint supernovae and hypernovae might be the remnants of massive stars in the range of $\sim$20–60 $M_\odot$. They probably differ from each other in degree of rotation and fallback, but they are supposed to form a black hole regardless and are clearly distinguished from the ordinary neutron star–forming supernovae. Gamma-ray bursts are thought to be commonly associated with the hypernovae (collapsars; MacFadyen & Woosley 1999). The survey of their fate as a function of mass and metallicity is important in understanding the evolution of galaxies and the universe.

The scenarios for the formation of stellar-mass black holes have been argued in several different contexts, and the associated neutrino bursts have been studied in some cases. In the case of successful explosions from massive stars of $\sim$10–20 $M_\odot$, some of
the proto–neutron stars may evolve toward a black hole after their birth, even after the launch of a shock wave.

In most supernova explosions, the accretion of matter ceases within several 100 ms after bounce, and a proto–neutron star is born with a fixed total baryon mass. Since the maximum mass of proto–neutron stars, which are hot and lepton-rich, can be larger than the maximum mass of cold neutron stars, massive proto–neutron stars in a certain mass range can lead to black hole formation during cooling and deleptonization (Glendenning 1995). The scenario of metastable proto–neutron stars relies on the equation of state of dense matter having the appearance of exotica, such as hyperons, kaon condensates, and quarks, during the cooling through neutrino emission (Keil & Janka 1995; Pons et al. 1999, 2001a, 2001b). The emergence of neutrinos diffusing from proto–neutron stars has been calculated in a flux-limited diffusion approximation during quasi-hydrostatic evolution (Pons et al. 1999). In the case of metastable proto–neutron stars, termination of the neutrino burst is predicted to occur within 1–100 s, a timescale which largely depends on the internal composition. The disappearance occurs during an exponential decrease of neutrino luminosities, which is common in long-term proto–neutron star cooling. The delayed collapse of a proto–neutron star in this context has been studied in dynamical simulations with general relativistic treatment in order to study the final moment of black hole formation and the associated neutrino signals with leakage or diffusion scheme (Baumgarte et al. 1996a, 1996b).

The collapse to the black hole can be triggered if there is a significant fallback of material after a successful supernova explosion. The fallback in supernovae is used as a given ingredient in the studies of explosive nucleosynthesis, but has not been well established in the context of an explosion mechanism. The amount and timescale of fallback is uncertain, especially in the numerical simulations of supernova explosion, whose puzzle has not been solved yet. With aim of examining the supernova neutrinos from SN 1987A, the quasi-static evolution of proto–neutron stars, which leads in some cases to black hole formation, has been studied for the assumed accretion rate (Burrows 1988). Numerical simulations have been done by solving the quasi-static evolution with the flux-limited diffusion approximation; therefore, general relativistic instability was assumed to occur when the numerical code could not find a static solution. The characteristics of neutrino signals in the case of black hole formation were pointed out with the uncertainties in accretion rate and equation of state.

The current study focuses on black hole formation without supernova explosion from massive stars of $\gtrsim 20 M_\odot$, which is different from the delayed collapse of proto–neutron stars in successful explosions discussed above. Having large iron cores, the shock wave cannot propagate outward, and the standing accretion shock is settled above the proto–neutron star. The material of the outer layers falls down continuously from the beginning of gravitational collapse. The mass of the proto–neutron star increases accordingly toward the maximum mass and the dynamical collapse to the black hole takes place. Accordingly, there is no bright optical display associated with the collapse.

We aim to clarify the sequence of evolution, starting from the beginning of the gravitational collapse of a massive star of $40 M_\odot$ and ending with the final moment of black hole formation. In order to predict the time profile of the neutrino burst and the energy spectrum of neutrinos, as well as dynamical evolution, we perform elaborate simulations of $\nu$-radiation hydrodynamics in general relativity. We follow the evolution of a proto–neutron star with matter accretion for a long time ($\sim 1$ s), which is challenging numerically with the exact treatment of $\nu$-radiation hydrodynamics, to find when and how the black hole is formed.

The explosion energies and their dependence on progenitor models have been studied by two-dimensional simulations of hydrodynamics with a simplified neutrino treatment for 15, 25, and $40 M_\odot$ stars (Fryer 1999). Although the borderline for black hole formation without explosion has been estimated to be roughly $40 M_\odot$, the simplified treatment of neutrinos has allowed the author to provide only an approximate limit for the success of the explosion, and the evolution of the proto–neutron star toward a black hole has not been studied in his simulations. A simulation of general relativistic $\nu$-radiation hydrodynamics was first done by Liebendoerfer et al. (2004), and the numerical result for a $40 M_\odot$ star leading to black hole formation has been reported with the detailed description of a numerical code and test bed for core-collapse supernovae, but only for a single EOS.

Rapid formation of a black hole after the birth of a proto–neutron star can be crucially affected by the equation of state (EOS) of dense matter. The maximum mass of evolving proto–neutron stars is determined by the stiffness of the EOS and the degree of deleptonization. In addition, the initial profile of a proto–neutron star is determined right after the core bounce, which is also influenced by the EOS. Recently, different EOSs for supernova simulations have become available (Lattimer & Swesty 1991; Shen et al. 1998) and detailed comparisons of core collapse from a $15 M_\odot$ star, which is supposed to explode, forming a neutron star, have been made in $\nu$-radiation hydrodynamics (Sumiyoshi et al. 2005). It has become apparent that the profiles of a proto–neutron star long after bounce are significantly different depending on the EOS, and that the resulting spectra of neutrinos are distinct. Therefore, it is essential to examine the influence of the EOS on the evolution toward black hole formation and to predict the associated neutrino signals. We note again that the single EOS (Lattimer & Swesty 1991) has been adopted in the previous study by Liebendoerfer et al. (2004) and that the crucial influence of EOSs has not been explored yet.

In the current study, we make the first comparison of the dynamical formation of a black hole from a massive star with $\nu$-radiation hydrodynamics by adopting two sets of EOSs. We aim to clarify the difference of dynamical evolution due to the EOSs in order to probe the dense matter. In the black hole–forming collapse, the central core experiences higher densities and temperatures than in normal supernovae, as we will show. It will be especially interesting to reveal the differences in neutrino signals in order to extract the information regarding EOSs from terrestrial neutrino detections in the future. We will argue that the duration of the neutrino burst from the black hole–forming collapse can put a constraint on the stiffness of the EOS and the phase transition to exotic phases (Sumiyoshi et al. 2006).

We arrange this article as follows. We begin with descriptions of the numerical simulations in § 2, including the numerical code, the physical inputs, the initial setting, and the criterion for black hole formation. We report the numerical simulations in order in § 3, describing the evolution of core collapse, proto–neutron star evolution, and the second collapse to a black hole. The evolution of neutrino distributions and EOS dependences in neutrino signals are presented in §§ 3.5 and 3.6, respectively. Implications of current results for the EOS study are discussed in § 4. We end with a summary in § 5.

2. NUMERICAL SIMULATIONS

2.1. Numerical Code

We adopt the numerical code of general relativistic $\nu$-radiation hydrodynamics that solves the Boltzmann equation for neutrinos together with Lagrangian hydrodynamics under spherical
symmetry (Yamada 1997; Yamada et al. 1999; Sumiyoshi et al. 2005). The numerical code has been successfully applied to study the core collapse in the case of a 15 $M_\odot$ star as the progenitor (Sumiyoshi et al. 2005). It is a fully implicit code in order to follow the long-term evolution of supernova core for more than 1 s after bounce and has a rezoning feature to efficiently capture accretion of the outer layer onto the compact object. The same numerical treatment is applied to the case of a 40 $M_\odot$ star. We adopt 255 spatial zones for Lagrangian mass coordinates and 14 energy zones and six angle zones for the neutrino distribution function. The four species of neutrinos are treated separately as $\nu_e$, $\bar{\nu}_e$, $\nu_{\mu/\tau}$, and $\bar{\nu}_{\mu/\tau}$. Here $\nu_{\mu/\tau}$ denotes the neutrinos of $\mu$- and $\tau$-types and $\bar{\nu}_{\mu/\tau}$ denotes the antineutrinos of $\mu$- and $\tau$-types. More details about these methods can be found in Sumiyoshi et al. (2005).

We remark that the exact treatment of $\nu$-radiation hydrodynamics in general relativity is essential to describe the evolution including compact objects and to predict the neutrino emission in the dynamical situation. General relativity plays a dominant role in triggering the collapse of a proto–neutron star into a black hole. The hydrodynamical treatment is necessary to describe the accretion of material onto a shrinking central core and the neutrinos mainly emitted during this accretion phase being different from the quasi-static evolution of proto–neutron stars in the Henyey-type treatment.

The exact treatment of general relativistic $\nu$-radiation hydrodynamics is possible only under the spherical symmetry at the moment (see, however, Buras et al. [2003, 2006] and Livne et al. [2004] for efforts on multidimensional calculations). We note that the spherically symmetric models considered in this paper may correspond to a branch extending from the faint supernovae that are suggested to be the outcome of the collapse of slowly rotating massive stars (Nomoto 2005). On the other hand, multidimensional simulations of $\nu$-radiation hydrodynamics are required to reveal black hole formation in rapidly rotating massive stars and its possible consequence as hypernovae and/or gamma-ray bursts; these are, however, beyond the scope of the current study.

2.2.2. Input Physics

2.2.2.1. Equation of State

We use the two sets of supernova EOSs which are the standard in recent supernova simulations. The EOS by Lattimer & Swesty (LS-EOS; Lattimer & Swesty 1991), which has been a conventional choice, is based on the compressible liquid drop model for nuclei together with dripped nucleons. The density dependence of the EOS for infinite matter is assumed to take the form of Skyrme-type effective interactions, which are often used in nonrelativistic nuclear many-body calculations of nuclear structures. The parameters are determined by the bulk properties at saturation. The EOS by Shen et al. (SH-EOS; Shen et al. 1998), which is relatively new, is constructed by the relativistic mean field (RMF) theory along with a local density approximation. The RMF theory is based on the relativistic nuclear many-body frameworks (Brockmann & Machleidt 1990) and has been successfully applied to studies of nuclear structures and dense matter (Serot & Walecka 1986). It should be noted that nuclear interactions are constrained by the properties of unstable nuclei (Sugahara & Toki 1994), which recently became available in radioactive nuclear beam facilities around the world. For example, the neutron skin thickness of neutron-rich nuclei, which is sensitive to the nuclear symmetry energy, is well reproduced for isotopes of light nuclei (Suzuki et al. 1995; Sugahara et al. 1996).

The two sets of supernova EOSs are different from each other, reflecting the characteristics of nuclear theoretical frameworks and the inputs of nuclear data used to constrain the EOS. The relativistic frameworks tend to provide a stiff EOS as compared with the nonrelativistic frameworks. The density dependence of symmetry energy tends to be strong in the relativistic frameworks (see, e.g., Sumiyoshi et al. 1995b). The incompressibility and symmetry energy of the SH-EOS are 281 and 36.9 MeV, respectively. It should be noted that those values in the SH-EOS are not inputs to determine the interactions, but the outcome after fitting nuclear interactions to nuclear data, including neutron-rich nuclei. For the LS-EOS, we adopt the set of 180 MeV for the incompressibility among three choices. The EOS set of this value has been popularly used in most numerical simulations so far. The symmetry energy of the LS-EOS is 29.3 MeV. When those two sets of EOSs are adopted for cold neutron stars, the maximum masses are 1.8 and 2.2 $M_\odot$ for the LS-EOS and SH-EOS, respectively, due to the difference of stiffness. This difference is essential in discussing different maximum masses of proto–neutron stars in numerical simulations. Note that the proto–neutron stars contain more leptons and are hotter than ordinary neutron stars, and therefore the maximum mass of the former is not the same as for the latter.

As we will demonstrate in § 4, the density and temperature in proto–neutron stars collapsing to black holes become extremely high beyond the range of the EOS originally provided in the table. We have extended the EOS table by calculating the necessary quantities according to the original formulation of RMF for the SH-EOS (Sumiyoshi et al. 1995a). The corresponding EOS table for the LS-EOS is prepared by using the subroutine provided by Lattimer & Swesty for public use. The simple extrapolation of EOSs to these extremely high densities and temperatures, where the applicability of the nuclear many-body frameworks with only the nucleonic degree of freedom is doubtful, is apparently oversimplification. However, since this is the first comparison of different EOSs in rapid black hole formation from a massive star, we adopt the simplest extension of the currently available sets of EOSs. There may be the appearance of hyperons, possible condensations of mesons, and phase transitions to quarks and gluons at these high densities and temperatures. They are the targets of upcoming numerical simulations.

2.2.2.2. Weak Interaction Rates

We adopt the same set of weak interaction rates as in the previous study on a 15 $M_\odot$ star (Sumiyoshi et al. 2005) to facilitate comparisons and numerical checks. The basic set of reaction rates follows the standard formulation of Bruenn (1985). In addition, the plasmon process and the nucleon-nucleon bremsstrahlung are included (Sumiyoshi et al. 2005). The latter reaction has been shown to be an important process as a source of $\nu_{\mu/\tau}$ and $\bar{\nu}_{\mu/\tau}$ from the proto–neutron star (Burrows et al. 2000; Suzuki 1993). The recent developments of neutrino-matter interactions (Burrows et al. 2006) and electron-capture rates (Langanke & Martinez-Pinedo 2003) will be implemented in the future. One should be reminded that the details of neutrino signals from black hole formation may be modified by these updated neutrino reaction rates and further scrutiny. We will demonstrate, however, that the whole dynamics is controlled dominantly by the accretion, and the general feature of neutrino emission reflects largely the difference of EOSs rather than the minutes of neutrino rates.

2.3. Initial Model

We adopt the presupernova model of 40 $M_\odot$ by Woosley & Weaver (1995). This is the most massive model in the series of
the presupernova models and contains an iron core of 1.98 $M_\odot$. Their presupernova model of 15 $M_\odot$, which is the standard for recent supernova studies, contains an iron core of 1.32 $M_\odot$, by contrast. The large size of the iron core in the current model suggests that black hole formation without explosion will be its fate. We use the profile of the central part of the presupernova model up to 3.0 $M_\odot$ in baryon mass coordinates, large enough to describe the accretion of material for a long time.

2.4. Criterion for Black Hole Formation

The general relativistic formulation for the numerical code is based on the metric of Misner & Sharp (1964). In order to track down the formation of a black hole, we use the apparent horizon, following the numerical studies of black hole formations from supermassive stars by Nakazato et al. (2006). The apparent horizon exists when the relation

$$\frac{U}{c} + \Gamma \leq 0,$$

or equivalently,

$$r \leq r_g$$

is satisfied, where $U$ is the radial fluid velocity and $\Gamma$ is the general relativistic gamma factor (see Yamada 1997 for the definitions). The Schwarzschild radius is defined by

$$r_g = \frac{2GM_\odot(r)}{c^2},$$

where $M_\odot(r)$ denotes the gravitational mass inside the radius $r$. In the current study, we terminate the numerical simulations soon after the formation of the apparent horizon because of possible numerical errors and instability. In order to follow the evolution in a stable manner up to the final moment of the fade-out of neutrino emissions, we need to apply a singularity-avoiding scheme as in Baumgarte et al. (1995). We stress that the main aim of our study is to clarify the dynamics from the beginning of collapse up to the black hole formation through the accretion of outer envelopes as well as a major part of the neutrino signal during this period and, most importantly, their dependence on the EOS.

3. NUMERICAL RESULTS

We present the numerical results for the two models studied with Shen’s EOS and the Lattimer-Swesty EOS, which are hereafter denoted by SH and LS, respectively. We start with the outline of the whole evolutions of the two models ($\S$ 3.1) and discuss each stage from the core collapse, bounce ($\S$ 3.2), shock propagation, and proto–neutron star evolution ($\S$ 3.3) to the formation of a black hole ($\S$ 3.4). We present the distributions and emissions of neutrinos in $\S\S$ 3.5 and 3.6.

3.1. Outline of Evolution

Figures 1 and 2 show the radial trajectories of mass elements as a function of time after core bounce ($t_{pb}$) in the SH and LS models. The trajectories are plotted for each 0.02 $M_\odot$ in baryon mass coordinates. Thick lines denote the trajectories for 0.5, 1.0, 1.5, 2.0, and 2.5 $M_\odot$. After the gravitational collapse of the central iron core of the initial model, the core bounce occurs at central densities just above the nuclear matter density as in ordinary core-collapse supernova. The time at the core bounce from the start of the simulation is 0.357 and 0.333 s, respectively, in the SH and LS models.

The shock wave is launched by the core bounce and propagates beyond 100 km in both models. Because of the significant amount of matter accretion, the shock wave starts recession at $t_{pb} \sim 100$ ms and becomes the accretion shock above the central object just born at the center. The proto–neutron star is formed after the bounce and gradually contracts from the beginning. The whole proto–neutron star shrinks due to the increase of mass by the accretion. This tendency is striking in the LS model, which adopts a softer EOS, having large gradients of radial trajectories in the central part. Note that the rate of accretion is similar in the two models, as we can see from the trajectories in the outer core above 100 km.

The gravitational collapse of the proto–neutron star occurs when its mass exceeds the maximum mass for stable configurations of lepton-rich, hot neutron stars. The recollapse proceeds dynamically within a few milliseconds, and the central object becomes exceedingly compact, leading to the formation of a black hole. The dynamical collapse to the black hole occurs at $t_{pb} = 1.34$ and 0.56 s, respectively, in the SH and LS models.

3.2. Initial Collapse and Bounce

The gravitational collapse of the central iron core from the initial model occurs in a similar way to that in the ordinary core-collapse
supernovae (e.g., 15 \( M_\odot \) model). The electron captures on free protons and nuclei proceed to reduce the lepton fraction until neutrino trapping. The neutrinos (\( \nu_e \)) are trapped at high densities (\( \sim 10^{12} \text{ g cm}^{-3} \)) and contribute to build up the bounce core. During the collapse, the compositional difference between the two models can be seen as we have found in the case of a 15 \( M_\odot \) model. In fact, the SH model tends to reduce electron captures, having a smaller free proton fraction than the LS model.

We define the core bounce (\( t_{pb} = 0 \text{ s} \)) as the time when the central density reaches the maximum in this stage. The central density at bounce is \( 3.2 \times 10^{14} \text{ and } 4.1 \times 10^{14} \text{ g cm}^{-3} \) in the SH and LS models, respectively. After a slight expansion at bounce, the central densities gradually increase owing to the contraction of proto–neutron stars.

The profiles of lepton fractions at bounce are shown in Figure 3. The lepton and electron fractions at the center of SH are 0.35 and 0.30, respectively. The corresponding values in LS are smaller, but the difference between the two models is \( \sim 0.01 \). The modest difference leads to a small difference in the bounce cores. Figure 4 displays the velocity profiles at bounce in the two models. The size of the bounce core in the SH model is slightly larger than that in the LS model, but the difference is again small (\( \leq 0.02 M_\odot \)).

The radial position of the shock wave in SH is 11 km at formation, and that in LS is only \( \sim 2% \) smaller. Accordingly, the propagation of the shock wave in the early phase (up to \( t_{pb} \sim 100 \text{ ms} \)) is similar in the two models.

### 3.3. Rapid Formation of the Proto–Neutron Star

The shock wave propagates through the iron core, but it never overcomes the ram pressure of falling material. It stalls at \( \sim 150 \text{ km} \) and turns into a recession due to the substantial influence of accretion. In fact, material of about 1 \( M_\odot \) falls down within 100 ms after bounce in both models. The shock wave reaches 1.6 \( M_\odot \) at \( t_{pb} = 100 \text{ ms} \) and the central core below the shock wave already acquires a typical neutron star mass. Although the outer part of the central core is not yet hydrostatic, the proto–neutron star has already been formed in terms of mass. This is much faster than the case of ordinary core-collapse supernovae, in which it takes 300–400 ms to acquire enough mass (\( \sim 1.5 M_\odot \) in baryon mass) for proto–neutron stars.

We show in Figures 5 and 6 the profiles of density and temperature in the two models at \( t_{pb} = 0, 100, 300, \) and 500 ms. The

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**Fig. 3.**—Lepton, electron, and neutrino (\( \nu_e \)) fractions at bounce are shown as a function of baryon mass coordinate by solid, dashed, and dotted lines, respectively. The results for the SH and LS models are shown by thick and thin lines, respectively.

**Fig. 4.**—Velocity profiles at bounce are shown as a function of baryon mass coordinate. The results for the SH and LS models are shown by thick and thin lines, respectively.

**Fig. 5.**—Density profiles at \( t_{pb} = 0, 100, 300, \) and 500 ms are shown as a function of baryon mass coordinate. The results for the SH and LS models are shown by thick and thin lines, respectively.

**Fig. 6.**—Temperature profiles at \( t_{pb} = 0, 100, 300, \) and 500 ms are shown as a function of baryon mass coordinate. The results for the SH and LS models are shown by thick and thin lines, respectively.
rapid growth of proto-neutron star masses is evident with the mass over \(2 M_\odot\) at 500 ms after bounce. The difference of masses in the two models are within 0.03 \(M_\odot\), reflecting the initial difference in the size of the bounce cores and the similarity of the accretion rates in the two models. In both models, the density in whole proto-neutron stars increases within short times during the recollapse, while the baryon mass reaches \(2.10^{-5} M_\odot\) at \(t_{\text{pb}} = 0.56\) s and exceeds the maximum mass for the stable configurations, leading to the dynamical collapse to the black hole. In the SH model it increases further, up to \(2.66 M_\odot\) at \(t_{\text{bh}} = 1.34\) s. For comparison, we plot the corresponding baryon mass of the proto-neutron star taken from the numerical result of the gravitational collapse of a \(15 M_\odot\) star with the SH-EOS (Sumiyoshi et al. 2005) as an example of massive stars, which are supposed to lead to ordinary core-collapse supernovae.

The growth of the proto-neutron star is much faster in the case of the \(40 M_\odot\) star than in the case of the \(15 M_\odot\) star. This means that the accretion rate in the current models of \(40 M_\odot\) is considerably higher than that in the canonical model of \(15 M_\odot\). The accretion rate is \(\sim 1 M_\odot\) s\(^{-1}\) in the current models at \(t_{\text{pb}} = 0.4\) s while it is \(\sim 0.2 M_\odot\) s\(^{-1}\) for the \(15 M_\odot\) model. A large accretion rate makes the astrophysical events from the \(40 M_\odot\) stars unique, having a rapid contraction of proto-neutron stars and a quick formation of the black hole. This is clearly different from the delayed scenario of black hole formation after the cooling of proto-neutron stars from ordinary supernovae.

### 3.4. Recollapse and Formation of the Black Hole

In Figures 8, 9, and 10, we display the profiles of density, temperature, and velocity in the final stages leading up to black hole formation. Here \(t_{\text{bh}}\) denotes the time before the black hole formation. As sample snapshots, we plot the profiles at the beginning of recollapse (\(t_{\text{pb}} = -8.2\) and \(-1.0\) ms for SH and LS, respectively) with dashed lines and the profiles at the time \((t_{\text{pb}} = -0.14\) and \(-0.19\) ms for SH and LS, respectively) when the central density reaches \(\sim 2 \times 10^{15} \text{ g cm}^{-3}\) (near the maximum density in the original table of SH-EOS) with long-dashed lines. The timing of the snapshot of the recollapse is chosen to be when the velocity just behind the shock wave exceeds \(\sim 2 \times 10^8 \text{ cm s}^{-1}\) for practical reasons. The profiles at the formation of apparent horizon are shown by solid lines. For the SH model, we also plot the profile at \(t_{\text{pb}} = 1\) s with dot-dashed lines.

In both models, the density in whole proto-neutron stars increases within short times during the recollapse, while the baryon mass of proto-neutron stars does not change so much. The central density reaches \(\sim 10^{16} \text{ g cm}^{-3}\) at the end of computations. The temperature increases beyond 100 MeV during the recollapse and reaches \(\sim 200\) MeV at the peak position for the LS model and a lower value for the SH model. The recollapse proceeds dramatically, having a velocity increase from \(\sim 10^6 - 10^8 \text{ cm s}^{-1}\) at the beginning to \(\sim 10^9 \text{ cm s}^{-1}\) within 1–10 ms. The velocity eventually exceeds \(\sim 10^{10} \text{ cm s}^{-1}\) by the time of black hole formation.
Fig. 9.—Temperature profiles are shown as a function of baryon mass coordinate in the SH (left) and LS (right) models. The notation is the same as in Fig. 8.

Fig. 10.—Velocity profiles are shown as a function of baryon mass coordinate in the SH (left) and LS (right) models. The notation is the same as in Fig. 8.

Fig. 11.—General relativistic factors, $\Gamma$ and $e^\rho$, at the formation of apparent horizon are shown as a function of baryon mass coordinates by solid and dashed lines, respectively, in the SH (left) and LS (right) models.
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Fig. 12.—Number fractions of leptons, electrons, $\bar{\nu}_e$, $\bar{e}$, and $\nu_{\mu/\tau}$ are shown as a function of baryon mass coordinate by solid, dashed, dotted, dot-dashed, and double-dot–dashed lines, respectively, at $t_{pb} = 100$ ms (top), $t_{pb} = 500$ ms (middle), and $t_{bh} = 0$ (bottom) for the SH model.

The general relativistic factors, $\Gamma$ and $e^\beta$, at the time of formation of apparent horizon are shown in Figure 11. The general relativistic gamma factor, $\Gamma$, has a minimum around $1.5 M_\odot$. The large negative values for the velocity affect the location of apparent horizon, which is defined by the condition $U/c + \Gamma = 0$ (see the definition in § 2.4). The apparent horizon is formed at $\sim 1.2 M_\odot$ ($\sim 5$ km in radius) in the SH model and at $\sim 1.3 M_\odot$ ($\sim 5$ km in radius) in the LS model.

3.5. Neutrino Distributions

We show the distributions of leptons in the SH model at $t_{pb} = 100$ and 500 ms and $t_{bh} = 0$ (black hole formation) in Figure 12. The number fraction (number per baryon), $Y_i$, is defined by the ratio $n_i/n_B$, where $n_i$ and $n_B$ denote the number densities of particle $i$ and baryon, respectively. The (electron-type) lepton fraction, $Y_L$, which is defined by the sum of net electron fractions, $Y_e-e$, and net (electron-type) neutrino fractions, $Y_{\nu_e} - Y_{\bar{\nu}_e}$, is shown by a solid line. The (net) electron fraction, $Y_e$, and neutrino fractions for $\nu_e$, $\bar{\nu}_e$, and $\nu_{\mu/\tau}$ are shown by dashed, dotted, dot-dashed, and double-dot-dashed lines, respectively.

After the core bounce, the neutronization of matter proceeds after the passage of the shock wave and the electron fraction decreases drastically, like in the case of the proto–neutron star formation in ordinary supernovae. A trough of the electron fraction is formed between the outer edge of the inner core and the shock position by $t_{pb} = 100$ ms and persists until black hole formation. The profile of the lepton fraction is not changed much while the shape becomes wider according to the shock propagation. The fraction of $\nu_e$, which has been produced during the collapse and trapped inside the inner core, decreases during evolution. The $\bar{\nu}_e$ fraction increases initially in the outer core and prevails in the whole core at the end. Because of the appearance of $\bar{\nu}_e$, the lepton fraction is smaller than the electron fraction at $t_{bh} = 0$. The $\nu_e$ and $\bar{\nu}_e$ fractions are determined by the conditions for beta equilibrium including neutrinos, and their evolution is driven by the decrease of chemical potential, $\mu_e$, for electron-type neutrinos. Since the relation among the chemical potentials for electrons, protons, and neutrons holds as $\mu_e = \mu_p - \mu_n$ through the quasi-chemical equilibrium, the reduction of $\mu_e$ occurs by the compression of matter leading to the black hole formation. This is because the compression leads to the increase of $\mu_n$ and the decrease of $\mu_p$ by the increasing effect of symmetry energy at very high densities. The decrease of $\mu_e$ (actually to negative values) drives a shift from the dominance of $\nu_e$ to that of $\bar{\nu}_e$ under the beta equilibrium. The $\nu_{\mu/\tau}$ and $\bar{\nu}_{\mu/\tau}$ neutrinos are produced by pair processes, and therefore appear mainly in the region with nondegenerate electrons.

The diffusion of neutrinos trapped inside the proto–neutron star is actually slow and the emission of neutrinos is mainly driven by accretion. In Figure 13, we show the profiles of neutrino luminosities at $t_{pb} = 1$ s in the SH model as a function of radius, for example. The luminosities increase stepwise at $\sim 20$ km, which corresponds to the outer edge (surface) of the proto–neutron star.
This is because the cooling of material is maximal there, producing neutrinos by electron and positron captures on nucleons, and by pair (electron and positron) annihilation processes. The accreting matter falls down to the surface after being heated by shock waves and compression, and then cools down by emitting neutrinos while it settles down hydrostatically. In order to demonstrate the neutrino emitting region, we show in Figure 14 the positions of the neutrinosphere at \( t_{\text{ph}} = 1 \text{ s} \) for the SH model. We define the neutrinosphere as the location where the optical depth becomes 2/3 for a neutrino with a specific energy. The neutrinospheres are located at \( \sim 20 - 200 \text{ km} \), where the accreting matter cools down by emitting neutrinos. Some of the high-energy neutrinos are reabsorbed by falling material and are not free-streaming up to \( \sim 200 \text{ km} \). It should be noted that the neutrinosphere for \( \nu_{\mu/T} \) is located in the innermost part, while the ones for \( \nu_e \) and \( \bar{\nu}_e \) are in the outer part. This is because \( \nu_{\mu/T} \) interacts only through neutral currents, whereas \( \nu_e \) and \( \bar{\nu}_e \) interact through both neutral and charged currents. This difference leads to the hierarchy of the average energies for different neutrino species that we will see in neutrino signals, since the temperature becomes higher as one goes inward.

3.6. Neutrino Signals

In Figures 15 and 16, we show the average energies and luminosities of \( \nu_e \), \( \bar{\nu}_e \), and \( \nu_{\mu/T} \) as a function of time \( (t_{\text{ph}}) \) for the two models. These quantities are the ones measured at the outermost grid point \( (\sim 6000 \text{ km}) \). The average energy presented here is defined by the rms value, \( \langle E_{\nu}^2 \rangle^{1/2} \). The plots for \( \nu_{\mu/T} \) are not shown, since they show no significant difference from the ones for \( \nu_{e/\bar{e}} \). We will only discuss \( \nu_{e/\bar{e}} \) and will not mention \( \nu_{\mu/T} \) hereafter.

Around the core bounce \( (t_{\text{ph}} \approx \sim 0.1 - 0.1 \text{ s}) \), the time profiles of average energies and luminosities are similar to those for ordinary supernovae. There is a distinctive peak due to the neutronization burst in the luminosity of \( \nu_e \). The rise of the luminosities of \( \nu_e \) and \( \nu_{\mu/T} \) right after bounce occurs owing to the thermal production of neutrinos. The peak of average energies around the core bounce appears as a result of heating by the passage of the shock wave. The behaviors in two models, SH and LS, are similar to each other up to \( t_{\text{ph}} \sim 0.1 \text{ s} \).

After that, the average energies increase toward black hole formation, reflecting the temperature increase. This tendency is more evident in the LS model, since the contraction of the proto-neutron star is faster and the resulting temperature is higher. The average energy of \( \nu_{e/\bar{e}} \) increases most prominently among three species, having the neutrinosphere at the innermost. The average energies of \( \nu_e \) and \( \bar{\nu}_e \) are rather close to each other because their neutrinospheres are located at similar positions, which are determined by charged-current reactions on the mixture of neutrons and protons. It is remarkable to see that the average energies rise continuously up to the end and the increase from the value right after the bounce amounts to a factor of \( 2 - 3 \). This behavior of neutrino emission is different from the one usually seen in the numerical simulations of ordinary supernovae. In the latter, the proto-neutron star is more static and does not contract so much because the accretion ceases soon after the bounce. Therefore, the increase of average energies after the bounce is a clear signal that tells us about the evolution toward black hole formation.

The luminosities after the neutronization burst also increase toward black hole formation. This is in accord with the increase of accretion luminosity, especially for \( \nu_e \) and \( \bar{\nu}_e \). The accretion luminosity is proportional to \( \dot{M} \ell_{\text{Ross}} \), where \( \dot{M} \) and \( \ell_{\text{Ross}} \) denote the mass and radius, respectively, of the proto-neutron star discussed in § 3.3. While the accretion rate, \( \dot{M} \), stays roughly constant the radius decreases dramatically, like the shock positions in Figures 1 and 2 toward the end. The actual neutrino luminosities, which are a portion of the available energy obtained by the accretion, depend on the cooling processes. Cooling by \( \nu_e \) and \( \bar{\nu}_e \) emissions proceeds through the electron and positron captures on nucleons in hot accreting matter. Since the matter is nondegenerate and the electron fraction is \( \sim 0.5 \), containing both neutrons and protons equally, the \( \nu_e \) and \( \bar{\nu}_e \) luminosities are nearly equal. The \( \nu_{\mu/T} \)
Fig. 16.—Luminosities of $\nu_e$ (solid lines), $\bar{\nu}_e$ (dashed lines), and $\nu_{\mu/\tau}$ (dot-dashed lines) as a function of time ($t_{pb}$) in the SH (left) and LS (right) models.

Fig. 17.—Energy spectra shown as a function of neutrino energy for $\nu_e$, $\bar{\nu}_e$, and $\nu_{\mu/\tau}$ by solid, dashed, and dot-dashed lines, respectively, at $t_{pb} = 100$ ms, 500 ms, 1 s, and $t_{pb} = 0$ for the SH model.
luminosities are determined by the pair process (annihilation of electron-positron pairs), and therefore reflect the temperature change at the neutrinosphere for $C_{23}/C_{22}/C_{28}$.

It should be noted that the computations of the neutrino burst are terminated at the formation of apparent horizon in this study. After this moment, the neutrinosphere will be swallowed by the horizon in a fraction of a millisecond and the major neutrino emissions cease at this point. Since neutrinos emitted just at the neutrinosphere travel over $\sim 6000$ km to the observing point set at the outer boundary of the computation domain, the neutrino signals last for $\sim 20$ ms more, in reality, than the end point in the current figures. The last part of the neutrino signals will be affected more clearly by general relativity, e.g., as the energy redshift (Baumgarte et al. 1996a). Such detailed information of neutrino spectra is valuable for examining general relativistic effects and for constraining neutrino masses (Beacom et al. 2001). In order to predict the final moment of neutrino emission, however, one needs to implement a scheme to avoid both coordinate and real singularities (Baumgarte et al. 1995), which will be postponed to the future work. The scope of the current study is to reveal the general features of the evolution from the massive progenitor to the black hole. It is the difference in the duration of the proto–neutron star era and the associated neutrino bursts that we aim to clarify using the two realistic EOSs of dense matter.

In Figure 17, we show the time evolutions of energy spectra at the outermost grid point for the SH model. It is apparent that the spectra become harder during the evolution toward black hole formation as the average energies and the luminosities evolve (see also Figs. 15 and 16). At $t_{pb} = 100$ ms, the peak of the spectrum is located around $\sim 10$–20 MeV and has the same hierarchy as in the average energies. The peak is shifted to higher energies ($\sim 20$ MeV) and the spectral shapes become similar among three neutrino species at $t_{pb} = 500$ ms. The spectra for $\nu_e$ and $\bar{\nu}_e$ are similar to each other due to the comparable production mechanisms, as discussed above. The $\nu_{\mu/\tau}$ spectra become rapidly harder toward $t_{bh} = 0$, reflecting higher temperatures of the central object.

We compared the energy spectra for the SH and LS models and found that the difference is minor as long as we compare them at the same timing after the bounce. The spectra at $t_{pb} = 100$ ms for LS is quite similar to the ones shown in Figure 17a. We show the comparison at $t_{pb} = 500$ ms in Figure 18. The luminosities for the LS model are slightly higher than those for the SH model. The $\nu_{\mu/\tau}$ spectra for the LS model are harder than those for the SH model, while the $\nu_e$ and $\bar{\nu}_e$ spectra look similar for both models. These differences arise from the fact that the proto–neutron star in the LS model at $t_{pb} = 500$ ms is already close to the state of the maximum mass, whereas the SH model waits for further evolution.

4. IMPLICATIONS FOR EOS STUDIES

Here we describe the temperature and density regimes that stellar matter experiences during the evolution to black hole formation and discuss their implications for the further development of EOSs. In Figures 19 and 20, we plot the density and temperature of matter along the trajectory for a fixed baryon mass coordinate as a function of time after bounce for the two models. At the center (Fig. 19), the density increases quickly before the core bounce and gradually thereafter toward the recollapse. By the time of recollapse, the density reaches $10^{15}$ g cm$^{-3}$, which is 3 times the normal nuclear matter density. The temperature exceeds 10 MeV at bounce, and gradually increases beyond 30 MeV toward the recollapse. The final sharp increases we can see in the figures are due to the dynamical collapse to the black hole within a millisecond. We remark that the behavior before the sharp increase is
important to determining the trigger of the gravitational collapse at the maximum mass during the evolution of proto-neutron stars. In Figure 20, we show the case for 0.6 \( M_\odot \) in baryon mass coordinates, where the entropy peak is formed by the passage of a shock wave. In this case, the temperature increases quickly and becomes much higher than at the center. The temperature reaches 80 MeV for SH (90 MeV for LS) before the recollapse, and the density exceeds twice the normal nuclear matter density. These high density and temperature regimes are important to the study of the rapid collapse of proto-neutron stars.

As we described in § 2.2.1, we adopt the EOSs with only the nucleonic degree of freedom in the current study. For the high densities and temperatures we have seen above, more massive baryons including strangeness may appear and the quark degree of freedom becomes essential to describing the dense and hot matter. An emergence of new degrees of freedom will lead to the softening of the EOS, and therefore it makes the maximum mass of proto-neutron star smaller and the recollapse earlier. In this sense, the current result gives the maximum duration of the proto-neutron star era. Still, it is astonishing that we see a large difference even within the nucleonic EOSs. The current results are also the basis for exploring when the exotic phases of dense matter appear during the evolution. The determination of the end point of the neutrino burst will be important in putting a constraint on the phase transitions of dense matter. A shorter burst than the current prediction may suggest the existence of new phases on the density-temperature trajectories studied here. It is certainly important to constrain the nucleonic EOS further as a firm basis, as well as to perform numerical simulations with the extended EOS table with exotic phases. The systematic study on the hyperon and/or quark EOSs covering a wide range of environments and on the associated neutrino reactions is currently underway.

In the current study, we have used the incompressibility of 180 MeV among the three choices in the LS-EOS. If we use the LS-EOS with a higher incompressibility of 220 MeV, the duration of the neutrino signal will become longer than in the current case of 180 MeV and the difference between the LS-EOS and SH-EOS will be smaller than for the results we have shown. Since the characteristics of the EOS depends not only on the incompressibility but also on the symmetry energy, adiabatic index, and other factors at high densities, it would be interesting to extract the dependence on the incompressibility alone by numerical simulations using the LS-EOS with the higher incompressibility.

5. SUMMARY

We study the gravitational collapse of a nonrotating massive star of 40 \( M_\odot \) resulting in black hole formation. Adopting the two different sets of realistic equations of state (EOSs) of dense matter, we perform the numerical simulations of general relativistic \( \nu \)-radiation hydrodynamics. We reveal the influence of EOSs on the core bounce, the shock propagation, the formation of a proto-neutron star, and the ensuing recollapse to black hole, together with neutrino emissions during the evolution. Following the core bounce and shock stall, the accretion of material from outer layers causes the rapid increase of the mass of the nascent proto-neutron star. Accordingly, the proto-neutron star immediately starts shrinking within 100 ms after bounce. The speed of contraction depends predominantly on the adopted EOS, and the density and temperature of the proto-neutron star become very high as compared with those for ordinary proto-neutron stars in successful supernovae. We find that the duration of the proto-neutron star era is clearly different depending on the EOS, which determines the maximum mass of proto-neutron stars. Due to the gravitational instability beyond the maximum mass, the fat proto-neutron star collapses again at 0.56 and 1.34 s after the bounce for the models with the LS-EOS and the SH-EOS, respectively. The apparent horizon is formed within \( \sim 1 \) ms. The black hole is formed much more quickly after the initial collapse of a massive star than in the delayed scenarios. The resulting \( \nu \) emissions during the dynamical evolution are distinctive as compared with the neutrino signals not only from ordinary supernovae but also from the delayed collapse models. The ever-increasing average energies of neutrinos and the sudden termination of neutrino signals are the clear evidence of rapid black hole formation from massive stars considered here. Although the increasing feature of energy can be seen also in some models of successful explosions, it is more robust in this case due to the lasting intense accretion. In addition, the duration of \( \nu \) emissions can be used to constrain the stiffness of the EOS at high density and temperature. A softer EOS or an early occurrence of the phase transition to exotic phases may lead to a shorter neutrino burst. Further studies concerning the dependences on progenitor models and the EOS with hyperons or quarks are necessary to provide the templates for the neutrino signal of black hole formation and will be reported in separate papers. It would be interesting to study the black hole formation from rotating massive stars, since the rotation will modify the formation
time and the associated neutrino signal, which may contain the contributions from accretion disks. A terrestrial detection of neutrino bursts from this type of collapse of massive stars in the future will reveal the rapid black hole formation and will put a new constraint on the EOS at extreme conditions.

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