Towards Scalable Security in Interference Channels
With Arbitrary Number of Users

Parisa Babaheidarian
Qualcomm Technologies
pbabahei@qti.qualcomm.com

Somayeh Salimi
Cygate AB
somayeh.salimi@cygate.se

Panos Papadimitratos
KTH Royal Institute of Technology
papadim@kth.se

Abstract—In this paper, we present an achievable security scheme for an interference channel with arbitrary number of users. In this model, each receiver should be able to decode its intended message while it cannot decode any meaningful information regarding messages intended for other receivers. Our scheme achieves individual secure rates which scale linearly with \( \log(\text{SNR}) \) and achieves sum secure rates which is within constant gap of sum secure capacity. To design the encoders at the transmitters side, we combine nested lattice coding, random i.i.d. codes, and cooperative jamming techniques. Asymmetric compute-and-forward framework is used to perform the decoding operation at the receivers. The novelty of our scheme is that it is the first asymptotically optimal achievable scheme for this security scenario which scales to arbitrary number of users and works for any finite-valued SNR. Also, our scheme achieves the upper bound sum secure degrees of freedom of 1 without using external helpers.

I. INTRODUCTION

Wireless communication channels are susceptible to leakage and interception by illegitimate users. Oftentimes, cryptographic algorithms such as the public key systems (PKI) are used to provide confidentiality. Many of such techniques rely on trapdoor functions whose security are threatened by advances in quantum computers and artificial intelligence. On the other hand, information theoretic tools such as i.i.d. random codes in [1], [2], promise unconditional security. These techniques have been vastly studied in different communication models including interference channels [3]. In the last decade, studies showed that despite promising performance of random codes in achieving reliable transmission, these codes perform poorly in achieving high secure rates in high SNR regime. In [4], [5] it was shown that the i.i.d. Gaussian random codes achieve zero sum secure degrees of freedom as SNR approaches infinity. To combat this limitation, structured codes have been incorporated in several security scenarios in which they outperformed Gaussian random codes [4], [6], [7]. In [8], Babaheidarian et al., presented an achievable scheme using structured lattice codes which was shown to provide weak secrecy, defined in [9], in a two-user interference channel with weak or moderately weak interference power levels. The advantage of their scheme compared to prior research based on real alignment [4] is that the scheme in [8] maintains security at any finite SNR value and the secure rates linearly scale with \( \log(\text{SNR}) \). Furthermore, they showed their scheme is asymptotically optimal. However, the scheme in [8] assumed only a two-user scenario and the direct generalization to arbitrary number of users is not straightforward.

In this work, we present a new achievable secure scheme for an interference channel with arbitrary number of users specifically \( (K > 2) \) users in which interference level is within weak or moderately weak regimes. Inspired by [8], [10], [11], our scheme utilizes the compute-and-forward decoding framework to handle finite SNR regimes as opposed to real-alignment schemes in [6], [7] which applied a maximum likelihood decoder. Our scheme takes advantage of a two-layer codebook structure in which the inner layer uses a set of nested lattice codebooks and the outer layer uses i.i.d. repeated codes. The novelty of our scheme is that the proposed scheme scales to any number of users \( (K > 2) \) and works at any finite SNR value. Also, we show that our scheme achieves optimal sum secure degrees of freedom of 1 asymptotically. Thus, our achievable sum secure rate is within constant gap from sum secure capacity in finite SNR regime. It is worth to mention that unlike prior schemes in [4] and [12], in our scheme, transmitters collectively ensure confidentiality of their messages at every unintended receiver without using any external helper.

The rest of the paper is organized as follows: Section II defines the problem statement and our assumptions, Section III introduces our achievability results, Section IV provides proof of achievability and finally conclusion remarks are presented in Section V.

II. PROBLEM STATEMENT

In this paper, we focus on the problem of simultaneous transmission of secure messages to their intended receivers in a \( K \)-user interference channel where \( K \) is an arbitrary even number and \( K > 2 \). For the case with odd number of users, one dummy user is added. At receiver \( i \) \( (1 \leq i \leq K) \), the channel output is denoted as \( y_i \) and at transmitter \( j \) the input to the channel is denoted as \( x_j \). The channel gain between transmitter \( j \) and receiver \( i \) is denoted as \( h_{ji} \), and lastly, the noise at receiver \( i \) is modeled as an i.i.d. random Gaussian vector with zero mean and identity covariance matrix and it is denoted as \( z_i \). The relation between input and output of the channel is defined as

\[
y_i = h_{ii}x_i + \sum_{j \neq i} h_{ji}x_j + z_i \quad \forall i \in \{1, \ldots, K\}
\]  

Our assumption is that the channel gains are real valued and
known by the transmitters. Fig. 1 illustrates the communication model. We assume that the transmitted and received codewords, i.e., $X_j$ and $y_i$, are of length $N$, for all $i, j \in \{1, \ldots, K\}$. Transmitter $j$ has an independent confidential message for receiver $j$ which is denoted as $W_j$ and is uniformly distributed over the set $\{1, 2, \ldots, 2^{N_1(W_j)}\}$. Transmitter $j$ encodes its message to codeword $X_j$ through a stochastic encoder $E_j$ subject to a power constraint $\|X_j\|^2 \leq NP$, where $P$ is a positive number. Also, receiver $i$ is equipped with decoder $D_i$ which maps codeword $y_i$ to an estimate of its message: $\hat{W}_i = D_i(y_i)$.

**Definition 1 (Achievable secure rates):** For the $K$-user $(K > 2)$ Gaussian interference channel with independent confidential messages, a non-negative secure-rate tuple $(R_1, R_2, \ldots, R_K)$ is achievable with weak secrecy, if for any $\epsilon > 0$ and sufficiently large $N$, there exist encoders $\{E_j\}_{j=1}^K$ and decoders $\{D_i\}_{i=1}^K$ such that $\forall i, j \{1, 2, \ldots, K\}$:

\[
\text{Prob}(D_i(y_i) \neq W_i) < \epsilon, \quad (2)
\]

\[
R_j \leq \frac{1}{N} H(W_j | y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_K) + \epsilon \quad (3)
\]

**III. MAIN RESULTS**

In this section, we present the achievable secure rates the defined interference channel model.

We define a few notations to present the secure rates in closed form. Assume $R_{\text{comb}}^\ell$ is an achievable rate at which confidential message $W_\ell$ can be reliably decoded at Receiver $\ell$ without any security constraint. Also, assume $P_{\ell,m} \geq 0$ is the power allocated by Transmitter $\ell$ to encode the $m$-th component of confidential message $W_\ell$ where total number of components is set to a positive integer $M$. Assume $m^*$ is the index of the component with the densest lattice codebook. Additionally, the power allocated by Transmitter $\ell$ to encode the $m$-th component of the jamming codeword is denoted by $P_{\ell,m}^j$. Also, assume $S \subset \{1, 2, \ldots, M\}$ where $\frac{|S|}{M} \to 1$ for large enough $M$. Then, we have

**Theorem 1 (Achievable secure rates):** A non-negative rate tuple $(R_1, R_2, \ldots, R_K)$ which satisfies the following inequalities is achievable with weak secrecy for the defined interference channel model:

\[
R_\ell < R_{\text{comb}}^\ell - \max_{i \neq \ell} \left( \log \left( \sum_{m \in S} h_{\ell,i}^2 P_{\ell,m} + h_{\ell-1,i}^2 P_{\ell-1,m} \right) \right) \quad (4)
\]

Note that the supremum of all such rates over power allocations $P_{\ell,m}$ and $P_{\ell,m}^j$, for all $(\ell, m)$, are also achievable as long as the power allocations satisfy the power constraints in Section [IV] are satisfied.

The codebook structure and the encoding-decoding algorithms are described in Section [IV] Our achievable scheme utilizes nested lattice codebooks and random i.i.d. repetitions to generate two-layered lattice codewords. Transmitters apply beamforming operation on message codewords as well as jamming codewords to ensure the security of the confidential messages at any unintended receiver. Note that despite the cooperative jamming scheme, no online communication among the transmitters is required. Proof of reliable decoding and analysis of weak secrecy are presented in the following section.

**Corollary 1:** The optimal sum secure degrees of freedom (s.s.d.f.) of 1 for an interference channel with arbitrary $K > 2$ users is achievable following our scheme for weak and moderately weak interference power is 1, i.e.,

\[
s.s.d.f = \frac{\sum_{i=1}^K R_i}{2 \log(1 + P)} \leq 1 \quad (5)
\]

Proof of Corollary 1 is presented in Subsection [IV-E]

**IV. ACHIEVABILITY SCHEME**

We prove the achievability result presented in Theorem 1 by describing the codebook construction followed by the encoding and decoding operations as well as analysis of secrecy.

**A. Codebook construction**

Our codebook and encoding process is based on the idea of passive cooperation among the transmitters. The passive cooperation happens when each transmitter is a sender of its own message but also acts as a helper to protect the confidentiality of another user’s message at the illegitimate receivers. For instance, in the $K$-user setting, transmitter 1 helps protecting Transmitter $K$’s message at all the receivers except receiver $K$, and transmitter 2 does the same job for user $K - 1$ and so forth. The reason we call this cooperation passive is that it does not require users to exchange messages among each other so long as they know the channel state and the index of the user they need to help which can be agreed on in the initial acquisition and prior to online secure transmission. Broadly speaking, Transmitter $i$ protect its confidential message with the help of Transmitter $j = K - i + 1$ which generates a random jamming codeword which is beam-formed to align

\[
\begin{align*}
W_1 & \rightarrow E_1 X_1 \rightarrow D_1 \rightarrow \hat{W}_1 | W_j \{j=2\} \\
W_i & \rightarrow E_i X_i \rightarrow D_i \rightarrow \hat{W}_i | W_j \{j \neq i\} \\
W_K & \rightarrow E_K X_K \rightarrow D_K \rightarrow \hat{W}_K | W_j \{j=K-1\}
\end{align*}
\]

Fig. 1: The $K$-user Gaussian interference channel model with confidential messages.
To construct the corresponding outer codebook, Transmitter generates \( \bar{P}(m,i) \) inner codeword probability distribution.\footnote{Note that the lattice set associated with the jamming codwords are denser than the message codewords in all dimensions.} Similarly, the outer codebook generated to encode the\( m \)-th component of the jamming signal at Transmitter \( i \) is denoted as \( C_{Jm,i} \) and its outer codeword element is denoted as \( \bar{u}_{m,i} \). The constructed outer codebooks are partitioned to emulate the wiretap code [2]. To do this, Transmitter \( i \) randomly partitions codebook \( C_{m,i} \) into \( 2^{NR_{m,i}} \) bins of equal sizes. Each bin \((m,i)\) is given an index \( w_{m,i} \) where \( w_{m,i} \in \{1,\ldots,2^{NR_{m,i}}\} \). These indices are essentially independent submessages of the confidential message \( W_i \). The transmitter chooses the non-negative rates \( R_{m,i} \) such that \( R_i = \sum_{m=1}^{M} R_{m,i} \), where the secure rate \( R_i \) is set to

\[
R_i \triangleq R_{comb} - \max_{\vec{\ell}_i} \left( \log \sum_{m=1}^{M} \frac{h_i^2 P_{1,m} + h_{K-i+1,m}^2 P_{K-i+1,m}}{h_{K-i+1,m}^2 \ell P_{K-i+1,m}} \right)
\]

where for all \( m, m' \in M \), and \( i \in \{1,2,\ldots,K\} \), quantities \( P_{m,i} \) and \( P_{j,m} \) are positive values that represent power allocation among confidential message and jamming signal components, respectively. These quantities are formally defined in Subsection \( \text{IV-B} \).

Additionally, for each component of the confidential message \( i \) and the jamming signal, transmitter \( i \) generates a random dither codeword \( \bar{d}_{m,i} \) and \( \bar{d}_{Jm,i} \), respectively. Dithers are drawn uniformly random from the corresponding Voronoi regions, \( \mathcal{V}_{m,i} \) and \( \mathcal{V}_{Jm,i} \). Dithers are public information and after selection are provided to all parties. In the following we establish the details of the codebook construction at Transmitter 1 which protects Transmitter \( K \)'s confidential message at receivers \( 1,2,\ldots,K-1 \). Codebook construction is performed at the other transmitters in a similar fashion.

### Encoding

Transmitter \( i \) splits the confidential message \( w_i \in \{1,\ldots,2^{NR_i}\} \) into \( M_i \triangleq T^{2K-2} \) independent sub-messages, where \( T \) is a large number. The \( m \)-th sub-message is denoted as \( w_{m,i} \in \{1,\ldots,2^{NR_{m,i}}\} \). To encode this sub-message, Transmitter \( i \) randomly picks an outer codeword \( \bar{t}_{m,i} \) from the corresponding outer codebook \( C_{m,i} \). Next, the selected codeword is mixed with a random dither \( \bar{d}_{m,i} \) according to the following equation

\[
\bar{x}_{m,i} \triangleq \left[ \bar{t}_{m,i} + \bar{d}_{m,i} \right] \mod \Lambda_i^m
\]

The modular operation in dithering step is done blockwise over each \( n \)-length block, separately. Similarly, the jamming codeword \( \bar{x}_{Jm,i} \) is defined. Note that the lattice set associated with the jamming codewords are denser than the message codewords. In the next step, beam-forming operation is performed over each sub-message codeword. Note that each component is sent over a different beam-forming dimension where the number of all the dimensions is \( M \). The idea is to align the jamming signal and the confidential message codeword across many such dimensions at unintended receivers. The precoder
applied to codeword $x_{m,i}$ is denoted as $f(m, i, K-1+1, H)$, where $H = (h_1, h_2, \ldots, h_K)$ is the matrix of channel gains from all transmitters to all receivers. The precoder $f$ is a mapping that takes sub-message indices $(m, i)$ and channel gains $H$ as inputs and outputs a scalar value. This mapping ensures that the resulting codewords are rationally independent for all channel gains expect for a small Lebesgue measure. Transmitter $i$ applies the individual precoders over each component codeword and transmits the superposition codeword $x_i$ over the channel, where

$$x_i \triangleq \sum_{m=1}^{M} \hat{x}_{m,i} f(m, i, K-1+1, H) \tag{9}$$

Similarly, a precoder is applied to the jamming codeword to protect the confidential message of user $K-1 + 1$, i.e.,

$$x_i^J \triangleq \sum_{m=1}^{M} \hat{x}_{m,i}^J g(m, i, K-1+1, H) \tag{10}$$

The transmitted codeword is denoted as $X_i \triangleq x_i + x_i^J$ and it satisfies the power constraint. Let us define $P_{m,i} \triangleq \sigma^2_{m,i} f(m, i, K-1+1, H)^2$ and $P_i = \sum_{m=1}^{M} P_{m,i}$. Similarly, for the jamming codeword, define $P_{m,i}^J \triangleq \sigma^2_{m,i} f(m, i, K-1+1, H)^2$ and $P_i^J = \sum_{m=1}^{M} P_{m,i}^J$. Transmitter $i$ allocates power between jamming power and message power such that $P_i + P_i^J \leq P$. Furthermore, the coarse lattice sets associated with every jamming codeword is scaled such that for all $i \in \{1, \ldots, K\}$, we have

$$h_{K-1+1,i}^2 P_{K-1+1}^J \leq 1 \tag{11}$$

Note that the above condition is essential to achieve sum secure degrees of freedom of 1 and without this condition, the achievable sum secure degrees of freedom would reach $\frac{K}{2}$. The precoder mapping $f(m, i, K-1+1, H)$ is a product of powers of channel gains between both Transmitter $i$ and Transmitter $K-1 + 1$ and the receivers, i.e.,

$$f(m, i, K-1+1, H) = (h_{1,i}^2 h_{2,i}^2 \ldots h_{i-1,i}^2 h_{i,i+1}^2 \ldots h_{K-1+1,i}^2)\times (h_{K-1+1,i} h_{K-1+1,i+1} h_{K-1+1,i+2} \ldots h_{K,i-1,i} h_{K-1,i+1,i} h_{K-1,i+2,i} \ldots h_{K,i,K}) \tag{12}$$

and

$$g(m, i, K-1+1, H) = (h_{1,i}^2 h_{2,i}^2 \ldots h_{i-1,i}^2 h_{i,i+1}^2 \ldots h_{K-1+1,i}^2)\times (h_{K-1+1,i} h_{K-1+1,i+1} h_{K-1+1,i+2} \ldots h_{K,i-1,i} h_{K-1,i+1,i} h_{K-1,i+2,i} \ldots h_{K,i,K}) \tag{13}$$

The powers $(r_1, r_2, \ldots, r_{2K-2})$ are computed using a one-to-one mapping $\phi(m)$ that takes the $m$-th beam-forming dimension to the $2K-2$-length tuple power where each power is one of the possible $T$ dimensions. In other words, we have:

$$\phi(m) : \{1, \ldots, M\} \rightarrow \{1, \ldots, T\} \times \{1, \ldots, T\} \times \ldots \{1, \ldots, T\} \tag{14}$$

and for every $m \in \{1, \ldots, M\}$ there exists a non-negative $2K-2$ length tuple such that

$$(r_1, r_2, \ldots, r_{2K-2}) = \phi(m) \; r_j \in \{1, 2, \ldots, T\} \tag{15}$$

### C. Decoding

Decoding at each receiver follows asymmetric compute-and-forward technique used in [12]. In the following, we describe the decoding process at Receiver $i$. Other receivers act in a similar manner.

Receiver $i$ observes the scaled lattice codeword associated with its own message plus a set of unintended codewords aligned with jamming codewords plus effective noise as

$$y_i = h_i x_i + \sum_{\ell=1}^{K} (h_i x_{\ell} + h_{K-\ell+1,i} x_{K-\ell+1}) \tag{16}$$

Due to asymptotic alignment [13] along many beam-forming dimensions, the collections of the confidential and the jamming codewords participating in the second term in (16) are mutually aligned. Also, note that due to the constraint in (11), the power of the third term falls below noise power (assuming all noise powers are normalized). Also, this term includes only a jamming signal which is of no use to Receiver $i$. Note that the condition in (11) is aligned with weakly and moderately weak interference definition in [8]. Therefore, Receiver $i$ treats the third term as an additional noise term and the normalized effective noise term $\tilde{z}_i$ is defined as

$$\tilde{z}_i \triangleq \frac{1}{\sqrt{1 + h_{K-1+1,i}^2 P_{K-1+1}^J}} (h_{K-1+1,i} x_{K-1+1} + z_i) \tag{17}$$

As a result, Receiver $i$ effectively observes a $K$-user Multiple Access Channel (MAC) at its end, i.e.,

$$\tilde{y}_i = \frac{h_i}{\sqrt{1 + h_{K-1+1,i}^2 P_{K-1+1}^J}} x_i + \frac{1}{\sqrt{1 + h_{K-1+1,i}^2 P_{K-1+1}^J}} \sum_{\ell=1}^{K} (h_i x_{\ell} + h_{K-\ell+1,i} x_{K-\ell+1}) + \tilde{z}_i \tag{18}$$

The effective MAC channel gain vector at Receiver $i$ is denoted as $h_{eff,i}$ and it is defined as

$$h_{eff,i} \triangleq \left( \frac{h_i}{\sqrt{1 + h_{K-1+1,i}^2 P_{K-1+1}^J}}, \frac{1}{\sqrt{1 + h_{K-1+1,i}^2 P_{K-1+1}^J}}, \ldots \right)$$

The ratio between the power of each effective codeword in the effective MAC equation (18) and the power constraint $P$ is defined as power scaling vector $b_{eff,i}$ and

$$b_{eff,i} \triangleq \left( \frac{P_i}{P}, \frac{P_i^J}{P}, \ldots, \frac{P_i^J}{P} \right)$$

Now, Receiver $i$ applies the compute-and-forward technique used for a MAC channel in [10]. Receiver $i$ finds the nearly optimal set of linear independent integer-valued coefficient vectors which maximize the achievable MAC sum-rate for that Receiver. The receiver constructs $K$-linearly independent equations using these integer-valued coefficient vectors and decode each equation successively. The first equation is to
aim to decode the effective lattice codeword with the highest data rate, i.e., the lattice codeword that belongs to the densest fine lattice set. Upon decoding the codeword, it is canceled out from the second equation and so forth. Therefore, the least achievable equation rate is associated with the $K$-th combination equation.

Let us denote the optimal set of integer-valued coefficient vectors that construct the $K$ combination equations with $a_1, a_2, \ldots, a_K$. Also, let us denote the rates at which the combination equations are decoded at Receiver $i$ as $R_{\text{comb},i}^{(1)} < R_{\text{comb},i}^{(2)} \leq \cdots \leq R_{\text{comb},i}^{(K)}$. The set of coefficient vectors is computed such that the rate at which combination equations are decoded is non-increasing, i.e., $R_{\text{comb},i}^{(1)} \geq R_{\text{comb},i}^{(2)} \geq \cdots \geq R_{\text{comb},i}^{(K)}$. Basically, the effective codeword with the highest achievable rate is decoded first and canceled out and then the second highest rate effective codeword is decoded and so forth. Let us denote the first combination equation as $v_1 \triangleq a_1(\ell) x_{\text{eff},1}$. For instance, the effective codewords at Receiver $i$ are defined as $x_{\text{eff},1} \triangleq x_i$, $x_{\text{eff},2} \triangleq h_i x_i + h_K x_K^I$ and so forth so it would match the corresponding gain order in the effective MAC observed by Receiver $i$. Decoding equation $v_1$ is performed using the compute-and-forward technique [10, 15] by scaling the noisy observation and canceling out the public dithers as

$$\beta_1 \tilde{y}_i - \sum_{\ell=1}^{K} a_1(\ell) d_{\text{eff},\ell} = v_1 + z_{\text{eff},1}$$

where

$$z_{\text{eff},1} \triangleq \sum_{\ell=1}^{K} (\beta_1 h_{\text{eff},1}(\ell) - a_1(\ell)) x_{\text{eff},1} + \beta_1 \tilde{z}_i$$

Let us denote the second moment of the effective noise term associated with combination equation $v_1$ in (19) as $\sigma_{\text{eff},1}^2$. Following Theorem 2 in [15] equation $v_1$ can be decoded at an achievable rate $R_{\text{comb},i} = \frac{1}{2} \log(\frac{P_i}{\sigma_{\text{eff},1}^2})$, where $j$ is the index of the effective lattice codeword with densest lattice sets among participating codewords in combination equation $v_1$. Similarly, combination equations $v_2, \ldots, v_K$ are constructed and decoded. Assume that the mapping between effective codeword indices and the order at which they get decoded at Receiver $i$ is determined by a one-to-one permutation function $\pi(\cdot) : \{1, 2, \ldots, K\} \rightarrow \{1, 2, \ldots, K\}$. Therefore, the achievable combination rates are derived as

$$R_{\text{comb},i}^{(\ell)} = \frac{1}{2} \log \left( \frac{P_{\text{eff},i,\ell}}{\sigma_{\text{eff},i,\ell}^2} \right)$$

We already established that the combination equations are constructed such that the codeword with the densest lattice set gets decoded first therefore $\sigma_{\text{eff},1}^2 \leq \sigma_{\text{eff},2}^2 \leq \cdots \leq \sigma_{\text{eff},K}^2$. As a result, the lowest achievable rate to decode effective codeword $x_{\text{eff},1}$ at Receiver $i$, i.e., $x_i$ is

$$R_{\text{comb}}^{(\ell)} = \frac{1}{2} \log \left( \frac{P_{\text{eff},i,\ell}}{\sigma_{\text{eff},i,\ell}^2} \right)$$

Similarly, the achievable combination rates are determined at the other receivers. Since $R_i \leq R_{\text{comb}}^{(\ell)}$ for $i \in \{1, \ldots, K\}$, the achievability proof is completed. In the following subsection, we show that $R_i$ is achievable with weak secrecy.

D. Analysis of Security

So far we have shown that the lower bound stated in Theorem 1 are achievable in terms of reliably getting decoded at intended receivers. In this part, we proceed with showing the presented rates provide weak secrecy for every confidential message at all unintended receivers. To do this, it suffice to show the following for an arbitrary Receiver $i \in \{1, \ldots, K\}$:

$$\frac{1}{N} I(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i) \leq \epsilon$$ (23)

in which $\epsilon > 0$ approaches to zero as $N = nB$ tends to infinity. Note that $\frac{1}{nB} I(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i) \leq \sum_{i=1}^{K} R_i - \frac{1}{nB} H(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i)$. Since conditioning does not increase entropy, we have

$$\frac{1}{nB} H(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i) \leq \sum_{i=1}^{K} R_i - \frac{1}{nB} H(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i)$$ (24)

Next step is to obtain a lower bound on the second term in (24). Let us define notation $[\hat{t}]_{i=1}^{K}$ which represents the set of outer lattice codewords of all transmitters expect Transmitter $i$. We have

$$\frac{1}{nB} H(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i, \hat{t})$$

$$= \frac{1}{nB} H(W_i, \ldots, W_i, W_i, \ldots, W_i; y_i, [\hat{t}]_{i=1}^{K})$$

$$- \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, y_i, W_i, \ldots, W_i)$$

$$\geq \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, y_i, \hat{t})$$

$$\geq \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, y_i, \hat{t}, [\hat{d}]_{i=1}^{K}, x_{K-i+1}^I, z_i) - 2\epsilon_i$$

$$\geq \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, y_i, \hat{t}, [\hat{d}]_{i=1}^{K}, x_{K-i+1}^I, z_i) - 2\epsilon_i$$

$$= \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, \sum_{\ell=1}^{K} (h_{i,\ell} x_i + h_{K-i+1} x_{K-i+1}^I, \hat{t}), [\hat{d}]_{i=1}^{K}, x_{K-i+1}^I, z_i) - 2\epsilon_i$$

$$= \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, \sum_{\ell=1}^{K} (h_{i,\ell} x_i + h_{K-i+1} x_{K-i+1}^I, \hat{t}), [\hat{d}]_{i=1}^{K}, x_{K-i+1}^I, z_i) - 2\epsilon_i$$

$$= \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, \sum_{\ell=1}^{K} (h_{i,\ell} x_i + h_{K-i+1} x_{K-i+1}^I, \hat{t}), [\hat{d}]_{i=1}^{K}, x_{K-i+1}^I, z_i) - 2\epsilon_i$$

$$= \frac{1}{nB} H([\hat{t}]_{i=1}^{K}, \sum_{\ell=1}^{K} (h_{i,\ell} x_i + h_{K-i+1} x_{K-i+1}^I, \hat{t}), [\hat{d}]_{i=1}^{K}, x_{K-i+1}^I, z_i) - 2\epsilon_i$$
mod $\Lambda_{m^*,\ell}^J$, $\ell \sum_{\ell=1}^{K} Q_{\Lambda_{m^*,\ell}^J} \left( \sum_{m=1}^{M} h_{\ell\ell,m} P_{\ell,m} + h_{K-\ell+1,m} P_{K-\ell+1,m} \right) - 2\epsilon_i$

(c) $\frac{1}{nB} H \left( \{ k \} \sum_{\ell=1}^{K} \left[ \sum_{m \in S}^M \left( h_{\ell\ell,m} + h_{K-\ell+1,m} P_{K-\ell+1,m} \right) \right] \right) - 2\epsilon_i - \delta$

\begin{align*}
\sum_{\ell=1}^{K} \log \left( \sum_{m \in S}^M \frac{h_{\ell\ell,m}^2 P_{\ell,m} + h_{K-\ell+1,m}^2 P_{K-\ell+1,m}}{h_{K-\ell+1,m}^2 P_{K-\ell+1,m^*}} \right) - 2\epsilon_i - \delta
\end{align*}

\hspace{1cm} (25)

In the above inequalities the followings hold: inequality (a) is
due to i.i.d. random repetitions of the outer codewords and is
deduced by using Packing Lemma as shown in [14]. Inequality
(b) holds since conditioning does not increase entropy. Also,
we defined the set of all public dither codewords from all
users with notation $\{ d \}_{\ell=1}^{K}$. Inequality (c) is concluded from
the principal that joint entropy is not bigger than the sum
of individual entropies. Equality (d) comes from definition
of codewords described in Subsection [V-A] Equality (e)
holds after subtracting off the dithers from outer codewords.
Inequality (f) comes applying Crypto Lemma [16] to the
first term. Note that the jamming codewords are constructed from
denser lattice sets and have uniform distribution over their
codebooks, therefore the aligned codeword belongs to the
same codebook and it is also uniformly distributed hence
it is independent of the confidential message codewords.
Inequality (g) is concluded from Lemma 1 in [17] and power
allocation among message and jamming codewords described
in Subsection [V-B] Lastly, equality $h$ comes from the rate of
constructed outer lattice codewords explained in Subsection
[V-A]

E. Proof of Corollary 1

We present the proof in two steps. Step 1 is to prove that the
first term in expression (3) achieves $\frac{1}{K}$ degrees of freedom for
each user. Step 2 is to show that the second term in expression
(3) is constant with respect to power constraint $P$ as power $P$
approaches infinity.

Step 1: Following Corollary 5 in [10], all $K$ optimal combination
rates in (3) offer $\frac{1}{K}$ degrees of freedom for almost every
real-valued channel gain vector.

Step 2: This step is to show that for each user the second
term in expression (3) is constant with respect to power $P$ as
$P$ tends to large values, i.e., the following holds:

\begin{align*}
\lim_{P \to \infty} \left[ \log \left( \sum_{m \in S}^M \frac{h_{\ell\ell,m}^2 P_{\ell,m} + h_{K-\ell+1,m}^2 P_{K-\ell+1,m}}{h_{K-\ell+1,m}^2 P_{K-\ell+1,m^*}} \right) \right] \times \frac{1}{\log(1 + P)} \to 0
\end{align*}

Note that partial powers $P_{\ell,m}$ and $P_{K-\ell+1,m}$ are allocated power
to encode the $m$-th component of the confidential message $\ell$
and the jamming signal at Transmitter $\ell$, respectively. These
powers are tuned to satisfy the power constraint $P$. Therefore, at each Transmitter $\ell$ and for all $m \in \{1,2,\ldots,M\}$ we have $P_{\ell,m} = \alpha_{\ell,m}P$ and $P_{\ell,m}^{\prime} = \beta_{\ell,m}P$ for some positive factors $0 < \alpha_{\ell,m} < 1$ and $0 < \beta_{\ell,m} < 1$. Therefore, we have

\[
\begin{align*}
\frac{h_{\ell}^{2} P_{\ell,m} + h_{K-\ell+1,i}^{2} P_{\ell,m}^{\prime}}{h_{K-\ell+1,i}^{2} k^{\prime}} &= \frac{P \left(h_{\ell}^{2} \alpha_{\ell,m} + h_{K-\ell+1,i}^{2} \beta_{\ell,m}ight)}{P \left(h_{K-\ell+1,i}^{2} eta_{K-\ell+1,m}ight)} \\
&= \frac{h_{\ell}^{2} \alpha_{\ell,m} + h_{K-\ell+1,i}^{2} \beta_{K-\ell+1,m}}{h_{K-\ell+1,i}^{2} \beta_{K-\ell+1,m}}
\end{align*}
\]

(27)

where all the terms in expression (27) are considered constant with respect to power $P$ and hence, the following is deduced

\[
\lim_{P \to \infty} \left[ \log \left( \frac{h_{\ell}^{2} \alpha_{\ell,m} + h_{K-\ell+1,i}^{2} \beta_{K-\ell+1,m}}{h_{K-\ell+1,i}^{2} \beta_{K-\ell+1,m}} \right) \times \frac{1}{\log(1 + P)} \right] \to 0
\]

(28)

Also, note that in [4] it was shown that sum degrees of freedom of 1 is the upper bound for an arbitrary $K$-user interference channel with confidential messages. Therefore, our achievable sum secure degrees of freedom is optimal. This concludes the proof of Corollary 1.

V. CONCLUSION

We provided a new achievable security scheme to transmit confidential messages over an asymmetric interference channel with arbitrary number of users ($K > 2$) so long as interference is within weak and moderately weak interference regimes. Our achievable scheme utilizes the nested lattice codebooks, i.i.d. repetitive codes, cooperative jamming, superposition coding, and the compute-and-forward decoding strategy.

We showed that following our scheme, users achieve secure rates which scale linearly with log(SNR) and a sum secure rate that is within constant gap of sum capacity. Our cooperative scheme shared among transmitters achieves the sum secure degrees of freedom of 1 without any online communication among transmitters or using external helpers.

REFERENCES

[1] I. Csiszár and J. Korner, “Broadcast channels with confidential messages,” IEEE transactions on information theory, vol. 24, no. 3, pp. 339–348, 1978.
[2] A. D. Wyner, “The wire-tap channel,” Bell system technical journal, vol. 54, no. 8, pp. 1355–1387, 1975.
[3] Y. Liang, H. V. Poor, S. Shamai, et al., “Information theoretic security,” Foundations and Trends in Communications and Information Theory, vol. 5, no. 4–5, pp. 355–580, 2009.
[4] J. Xie and S. Ulukus, “Secure degrees of freedom of one-hop wireless networks,” IEEE Transactions on Information Theory, vol. 60, no. 6, pp. 3359–3378, 2014.
[5] J. Xie and S. Ulukus, “Secure degrees of freedom of $K$-user gaussian interference channels: A unified view,” IEEE Transactions on Information Theory, vol. 61, no. 5, pp. 2647–2661, 2015.
[6] X. He and A. Yener, “Secure degrees of freedom for gaussian channels with interference: Structured codes outperform gaussian signaling,” in GLOBECOM 2009-2009 IEEE Global Telecommunications Conference, pp. 1–6, IEEE, 2009.
[7] G. Bagherikaram, A. S. Motahari, and A. K. Khandani, “On the secure degrees-of-freedom of the multiple-access-channel,” arXiv preprint arXiv:1003.0729, 2010.
[8] P. Babaheidarian, S. Salimi, and P. Papadimitratos, “Security in the gaussian interference channel: Weak and moderately weak interference regimes,” in 2016 IEEE International Symposium on Information Theory (ISIT), pp. 2434–2438, IEEE, 2016.
[9] U. Maurer and S. Wolf, “Information-theoretic key agreement: From weak to strong secrecy for free,” in International Conference on the Theory and Applications of Cryptographic Techniques, pp. 351–368, Springer, 2000.
[10] O. Ordentlich, U. Erez, and B. Nazer, “The approximate sum capacity of the symmetric gaussian $k$-user interference channel,” IEEE Transactions on Information Theory, vol. 60, no. 6, pp. 3450–3482, 2014.
[11] P. Babaheidarian, S. Salimi, and P. Papadimitratos, “Finite-snr regime analysis of the gaussian wiretap multiple-access channel,” in 2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton), pp. 307–314, IEEE, 2015.
[12] P. Babaheidarian, S. Salimi, and P. Papadimitratos, “Preserving confidentiality in the gaussian broadcast channel using compute-and-forward,” in 2017 51st Annual Conference on Information Sciences and Systems (CISS), pp. 1–6, IEEE, 2017.
[13] A. S. Motahari, S. Oveis-Gharan, M.-A. Maddah-Ali, and A. K. Khandani, “Real interference alignment: Exploiting the potential of single antenna systems,” IEEE Transactions on Information Theory, vol. 60, no. 8, pp. 4799–4810, 2014.
[14] A. El Gamal and Y.-H. Kim, Network information theory. Cambridge university press, 2011.
[15] B. Nazer and M. Gastpar, “Compute-and-forward: Harnessing interference through structured codes,” IEEE Transactions on Information Theory, vol. 57, no. 10, pp. 6463–6486, 2011.
[16] G. D. Forney Jr, “On the role of mmse estimation in approaching the information-theoretic limits of linear gaussian channels: Shannon meets wiener,” arXiv preprint cs/0409053, 2004.
[17] P. Babaheidarian and S. Salimi, “Compute-and-forward can buy secrecy cheap,” in 2015 IEEE International Symposium on Information Theory (ISIT), pp. 2475–2479, IEEE, 2015.