Genesis of Dark Energy: Dark Energy as Consequence of Release and Two-stage Tracking of Cosmological Nuclear Energy

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Abstract

Recent observations on Type-Ia supernovae and low density (Ω\textsubscript{m} = 0.3) measurement of matter including dark matter suggest that the present-day universe consists mainly of repulsive-gravity type ‘exotic matter’ with negative-pressure often said ‘dark energy’ (Ω\textsubscript{e} = 0.7). But the nature of dark energy is mysterious and its puzzling questions, such as why, how, where and when about the dark energy, are intriguing. In the present paper the authors attempt to answer these questions while making an effort to reveal the genesis of dark energy and suggest that ‘the cosmological nuclear binding energy liberated during primordial nucleo-synthesis remains trapped for a long time and then is released free which manifests itself as dark energy in the universe’. It is also explained why for dark energy the parameter \( w = -\frac{2}{3} \). Noting that \( w = 1 \) for stiff matter and \( w = \frac{1}{3} \) for radiation; \( w = -\frac{2}{3} \) is for dark energy because “−1” is due to ‘deficiency of stiff-nuclear-matter’ and that this binding energy is ultimately released as ‘radiation’ contributing “+\( \frac{1}{3} \)”, making \( w = -1 + \frac{1}{5} = -\frac{2}{3} \). When dark energy is released free at \( Z = 80 \), \( w = -\frac{2}{3} \). But as on present day at \( Z = 0 \) when radiation strength has diminished to \( \delta \to 0 \), \( w = -1 + \delta \frac{1}{3} = -1 \). This, thus almost solves the dark-energy mystery of negative pressure and repulsive-gravity. The proposed theory makes several estimates/predictions which agree reasonably well with the astrophysical constraints and observations. Though there are many candidate-theories, the proposed model of this paper presents an entirely new approach (cosmological nuclear energy) as a possible candidate for dark energy.

Key words : Cosmology, Dark energy, Nucleo-synthesis.

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1 Introduction

Recently observed astronomical phenomena have revolutionized the understanding of cosmology. Consequences of combined analysis of Ia Supernovae (SNe Ia) observations \[1, 2\], galaxy cluster measurements \[3\] and cosmic microwave background (CMB) data \[4\] have shown that dark energy causing cosmic acceleration dominates the present universe. This acceleration is observed at a very small red-shift showing that it is a recent phenomenon in the late universe. Observations of 16 type Ia Supernovae made by Hubble Space Telescope (HST) \[5\] modified earlier astronomical results and provided conclusive evidence for deceleration prior to cosmic acceleration caused by dark energy in the recent past. The acceleration is realized with negative pressure and positive energy density that violates strong energy condition (SEC) \[6\]. This violation of SEC gives reverse gravitational effect. Due to this effect, universe gets a jerk and transition from deceleration to acceleration takes place. Phantom energy has appeared as a potential dark energy candidate in this arena, which violates both weak and strong energy conditions \[7, 8\]. These data suggest 73% content of the universe in the form of dark energy (DE), 23% in the form of non-baryonic dark matter (DM) and the rest 4% in the form of baryonic matter as well as radiation.

It is an irony of nature and is a puzzling phenomenon that most abundant form of matter-energy in the universe is most mysterious. The simplest dark energy candidate is the cosmological constant \(\Lambda\), but it needs to be extremely fine-tuned to satisfy the current value of the dark energy. Alternatively, to explain decay of the density, many dynamic models have been suggested, where \(\Lambda\) varies slowly with cosmic time (t) \[9, 10\]. In addition to models with dynamic \(\Lambda(t)\), many hydrodynamic models with or without dissipative pressure have been proposed in which barotropic fluid is the source of dark energy \[10\]. Chaplygin gas as well as generalized Chaplygin gas have also been considered as possible dark energy sources due to negative pressure \[8, 11, 12, 13\]. Other than these approaches, some authors have considered modified gravitational action by adding a function \(f(R)\) (R being the Ricci scalar curvature) to Einstein-Hilbert lagrangian, where \(f(R)\) provides a gravitational alternative for dark energy causing late-time acceleration of the universe \[14, 15, 16, 17\]. A review on modified gravity as an alternative to dark energy is available in \[18\]. A more comprehensive review is provided in \[19\]. All these models are phenomenological in the sense that an idea of dark energy is introduced a priori either in term of gravitational field or non-gravitational field. In spite of these attempts, still cosmic acceleration is a challenge.

Dark energy and the accelerated expansion of the universe have been the direct prediction of the distant supernovae Ia observations which are also supported, indirectly, by the observations of the CMB anistropies, gravitational lensing and the studies of galaxy clusters. It is generally believed that the distant SNe Ia observations predict an accelerating expansion of the universe powered by some hypothetical source with negative pressure generally termed
as ‘dark energy’. Supernovae at relatively high red shift are found fainter than that predicted for an earlier-thought slowing-expansion and indicate that expansion of universe is actually speeding up [1, 2, 20, 21]. Recent studies [21, 22] establish firmly that universe is now undergoing an acceleration; with repulsive gravity of some strange energy-form i.e., dark-energy at work. Dark energy, a ‘mysterious substance’ pressure of which is ‘negative’ and accounts for nearly 70% of total matter-energy budget of the universe, but has no clear explanation.

To understand the origin of dark energy and its nature is one of the greatest problems of the 21st century. In the present paper, a modest approach has been made to explain the genesis of dark energy, suggesting that dark energy is a result of released-free cosmological nuclear-energy in the universe; the origin of dark energy lies within the nucleus.

The outline of the paper is as follows. In section 2, cosmo-dynamics of FRW model is reviewed for completeness and further use in the paper. Section 3 presents a fresh approach towards understanding the genesis of dark energy. Wherein we have suggested that the dark energy is the released binding energy during the primordial nucleo-synthesis. This dark energy decays in two stages: in constraint way as stage-1 and liberated way in stage-2. In section 4, values of $\Omega_x$ at different values of Z have been estimated and are found to be well in accordance with the astrophysical constraints. In addition, theoretical estimates of deceleration parameter, with conventional and modified method, have been made in section 5. After making critical discussions in section 6 about the genesis of dark energy, it is noticed that the present model is in good agreement with the astrophysical constraints. Finally, conclusions are summarized in section 7.

2 Cosmo-Dynamics

The standard Friedman-Robertson-Walker (FRW) cosmological model prescribes a homogeneous and an isotropic distribution for its matter in the description of the present state of the universe. The FRW equations and the analysis for cosmo-dynamics are briefly summarized as follows for clarity and completeness sake and for its subsequent uses:

$$\left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho,$$

$$\frac{2\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = -8\pi G p,$$

where scale factor or universe-size at a particular time is $a(t)$ or simply as $a$. The dot denotes differentiation with respect to cosmic time $t$. Here, $G$ is gravitational constant, $p$ is pressure and $\rho$ is the total density i.e., $\rho = \rho_r + \rho_m + \rho_x$, where subscripts $r$ is for radiation, $m$ is for matter (including dark matter), and $x$ is for dark energy. The curvature factor is $k$ and depending on $k$ being 1, 0, $-1$.
the universe is closed, flat or open.

Eqs. (1) and (2) lead to
\[ \frac{2\ddot{a}}{a} = -\frac{8\pi G}{3}(\rho + 3p), \]
(3)

Differentiating Eq. (1) and using (3) in it, we get
\[ \dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}. \]
(4)

Eq. (1) can be written as
\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \]
(5)

where \( H \), being Hubble constant, is defined by
\[ H = \frac{\dot{a}}{a}. \]
(6)

Critical density \( \rho_c \) is defined as that for which the universe is flat (\( k = 0 \)), thus Eq. (5) leads to
\[ \rho_c = \frac{3H^2}{8\pi G}. \]
(7)

The deceleration parameter \( q \) is defined as
\[ q = -\frac{a}{a^2} = -\frac{\ddot{a}}{aH^2}. \]
(8)

Using the equation of state parameter \( w \) given by
\[ p = w\rho, \quad \text{where} \quad -1 \leq w \leq 1, \]
(9)
in (4), we obtain
\[ \frac{\dot{\rho}}{\rho} = -3(1 + w)\frac{\dot{a}}{a}, \]
(10)

which leads to
\[ \rho \propto \frac{1}{a^{3(1+w)}}. \]
(11)

Eqs. (1) and (11), for flat universe, lead to
\[ a \propto t^{\frac{2}{3(1+w)}}. \]
(12)

Usually, a parameter called cosmological red-shift (\( Z \)) is often referred to as a measure of the era (time \( t \)) of the universe. For example; \( Z_0 \) corresponds to the present time \( t_0 \) with present universe size (scale) \( a_0 \), and \( Z \) corresponds to some time \( t \) in the past with a smaller universe size \( a \). In fact the cosmological red-shift for spectral-line (wavelength \( \lambda \)) is defined as
\[ Z = \frac{\Delta \lambda}{\lambda} = \frac{(a_0-a)}{a} = \frac{a_0}{a} - 1. \]
Thus, $Z$ also gives a measure of universe size at a particular time, higher $Z$ means smaller universe size in the past and is expressed as

$$\frac{a_0}{a} = 1 + Z.$$  

(13)

**Particular Cases:**

(i) For $w = +1$, i.e., for stiff matter

$$\rho \propto \frac{1}{a^6} \quad \text{and} \quad a \propto t^{\frac{1}{3}}.$$  

(14)

(ii) For $w = +\frac{1}{3}$, i.e., for radiation dominated universe

$$\rho \propto \frac{1}{a^4} \quad \text{and} \quad a \propto t^{\frac{1}{2}}.$$  

(15)

(iii) For $w = 0$, i.e., for matter dominated (dust) universe

$$\rho \propto \frac{1}{a^3} \quad \text{and} \quad a \propto t^{\frac{1}{3}}.$$  

(16)

(iv) For $w = -\frac{2}{3}$, i.e., for dark energy (quintessence)

$$\rho \propto \frac{1}{a} \quad \text{and} \quad a \propto t^2.$$  

(17)

(v) For $w = -1$, i.e., for vacuum energy.

$$\rho = \text{constant} \quad \text{(like cosmological constant)}.$$  

(18)

These variations ‘$\rho$ versus $a$’ and ‘$a$ versus $t$’ of the particular cases are easily obtained from Eqs. (11) and (12) and are also shown in Figures 1 and 2.

### 3 Nuclear Genesis of Dark Energy

#### 3.1 Negative Pressure Due to Release of Cosmological Nuclear Energy

It is suggested that dark energy is a consequence of nuclear energy. The fusion nuclear binding energy begins to be liberated (but trapped) during primordial nucleo-synthesis; and much later ($Z = 80$) after decoupling, the nuclear binding energy is released free and appears effectively as dark energy. As shown (Figure 3) in subsequent section of the paper; this dark energy is produced earlier but released free later, of course quantity-wise ($\Omega_x = 0.0003$) insignificant; remains low during galaxy formation (say, at $Z = 3$, $\Omega_x = 0.1$), becomes 50% at transition ($Z = 0.5$, $\Omega_x = 0.5$) and emerges as dominant at present time ($Z = 0$, $\Omega_x = 0.7$), satisfying all the astrophysical constants [23].
Figure 1: Density $\rho$ decay with space size $a$
Figure 2: Space size $a$ expansion with time $t$

- $a \sim t^2$ (dark energy $w = -2/3$)
- $a \sim t^2$ (matter/dust $w = 0$)
- $a \sim t^{3/2}$ (radiation $w = 1/3$)
- $a \sim t^{3/3}$ (stiff matter $w = 1$)
The equation of the state parameter \( w \) for dark energy (quintessence) is taken as \( w = -\frac{2}{3} \). The reason for \( w = -\frac{2}{3} = -1 + \frac{1}{3} \) is explained as follows. Noting that \( w = +1 \) for stiff matter and \( w = \frac{1}{3} \) for radiation; ‘\(-1\)’ is due to ‘deficiency of stiff (nuclear) matter’ which is manifested as mass-defect (binding-energy) and ‘\( \frac{1}{3} \)’ is because this released binding-energy though initially-trapped but is finally released as ‘radiation’. Thus, considering that the binding-energy due to mass-defect (deficiency) in stiff-nucleus finally comes out as radiation; the equation of state parameter \( w = -1 + \frac{1}{3} = -\frac{2}{3} \). When dark energy is released free at \( Z = 80, w = -\frac{2}{3} \). But as on present day at \( Z = 0 \) when radiation strength has diminished to \( \delta \to 0, w = -1 + \delta \frac{1}{3} = -1 \), that is why dark energy appears to be like the vacuum energy due to the cosmological constant \( \Lambda \) [9, 10]. The puzzle of negative pressure (or repulsive gravity) is, thus, almost solved; the negative pressure (or dark energy) is produced due to released cosmological nuclear energy.

### 3.2 Variation of Matter-Density and Dark-Energy-Density Much After Decoupling Era Between \( Z = 80 \) to \( Z = 0 \)

Stiff-matter density decays very fast and vanishes very soon. Radiation density also decays fast and vanishes soon much after decoupling era. Although, as explained later in subsection 3.3 and in Figure 3, the dark energy is born much earlier in the past (nucleo-synthesis era) at about \( Z = 10^{10} \) but remains trapped dormant and decays in constrained way along with the matter and is released free only after \( Z = 80 \) to be effective and dominant. For flat universe \((\rho = \rho_c)\), the expressions for the density ratios can be written as follows.

\[
\rho = \rho_c = \rho_r + \rho_m + \rho_x
\]

and

\[
\Omega_c = \frac{\rho}{\rho_c}, \quad \Omega_r = \frac{\rho_r}{\rho_c}, \quad \Omega_m = \frac{\rho_m}{\rho_c}, \quad \Omega_x = \frac{\rho_x}{\rho_c}.
\]

Eqs. (19) and (20) lead to

\[
\Omega_c = 1 = \Omega_m + \Omega_r + \Omega_x.
\]

It is noticed that between \( Z = 80 \) to \( Z = 0 \), taking the present values of densities \( \rho_{m0} \approx 10^{-28} \text{gram/cm}^3, \rho_{x0} \approx 2 \times 10^{-28} \text{gram/cm}^3, \rho_{r0} \approx 10^{-31} \text{gram/cm}^3 \) or \( \Omega_{m0} \approx 0.3 \) and \( \Omega_{x0} \approx 0.7 \) and negligibly small \( \Omega_{r0} \approx 0.0003 \), the variations of radiation-density, matter-density and dark-energy-density can be obtained, ratio-wise, from Eqs. (19) – (17) as follows.

\[
\rho_r = \frac{\rho_{r0}a_0^4}{a^4} = 0.0003\rho_c(1 + Z)^4,
\]

\[
\rho_m = \frac{\rho_{m0}a_0^3}{a^3} = 0.3\rho_c(1 + Z)^3,
\]
Figure 3: Genesis of Dark Energy $\rho_x$ and its two stage (Dominant and Free) variations/tracking along with variations of $\rho_r$ and $\rho_m$.
\[
\rho_x = \frac{\rho_x(0)a_0}{a} = 0.7\rho_c(1 + Z),
\]

where the subscript 0 indicates the value at present time \((Z = 0)\).

### 3.3 Biography of Dark Energy and Its Two-Stage Tracking

Biography of mysterious dark-energy is even more mysterious. Nevertheless, a brief biographical description of dark energy is presented here for interesting clarity.

Dark energy in fact is the released nuclear binding energy of cosmos. Dark energy is born during primordial nucleo-synthesis epoch \((Z \approx 10^{10})\) in the early universe out of ‘mother’ matter present there, while the nucleo-synthesis (of say, Helium) liberates the binding energy as the ‘child’ the child is trapped in or around mother’s lap and remains almost dormant for a long period and moves parallel to mother’s foot-steps. Approximate weight of this dormant dark-energy (child) is roughly estimated as 1% of binding energy, of 25% of primordial helium-synthesis, from 13% \((0.13 \approx \frac{0.24}{0.3})\) of baryonic-matter \((\Omega_b = 0.04)\) out of total mass of (mother) matter \((\Omega_m = 0.3)\); this comes out to be \(\frac{\rho_m}{\rho_{m,0}} = 0.000325\) and since this era is radiation dominated era \(\rho_r\) being extremely high, gives \(\Omega_r\) very close to zero. This ‘child’ i.e., dark-energy (nuclear-binding-energy liberated) though has come out of mother’s womb (nucleus) yet remains trapped/dormant for quite a long time at least up to decoupling era \(Z = 1000\) and even beyond. When positive radiation \((\rho_r)\) pressure is much more dominant than the negative pressure of dark energy radiation \((\rho_x)\), the free-release of this dark energy is thus prohibited till then. While the universe is expanding, the dark energy (the child) decays parallel to matter (mother) foot-step at the same rate \((\rho \propto \frac{1}{a^3})\) from \(Z = 10^{10}\) to \(Z = 1000\) and even beyond \(Z < 100\) up to \(Z = 80\) (Figure 3). At \(Z = 80\), though dark energy \(\Omega_x\) still being less than \(\Omega_r\) (which opposes liberation of the trapped dark energy) but \(\Omega_x\) is sufficient enough to fight with \(\Omega_r\), the dark energy is ultimately released. Thus at \(Z = 80\), the dark energy (child) is released free from mother’s constrained-protection. Then onwards \((Z < 80)\) the matter density continues to decay at the same rate \((\rho \propto \frac{1}{a^3})\), but the dark energy (with \(w = -\frac{2}{3} = -1 + \frac{1}{3}\)) at release time, as explained earlier in subsection 3.1) density decays at much slower rate \((\frac{1}{a})\) then onwards. Between \(Z = 80\) to \(Z = 0\), the dark energy density curve \((\rho_x \propto \frac{1}{a})\) crosses radiation density curve \((\rho_r \propto \frac{1}{a})\) at \(Z = 12\) and crosses matter density curve \((\rho_m \propto \frac{1}{a^3})\) at \(Z = 0.5\). Briefly as summary (Figure 3): ‘Dark energy is born as a result of liberated binding energy during primordial nucleo-synthesis at \(Z \approx 10^{10}\) as a very small fraction \((\frac{\rho_m}{\rho_m,0} \approx 0.0003, \Omega_x \approx 0)\) of total matter-mass, initially (stage-1) it remains dormant and decays fast \((\frac{1}{a})\) along with matter till \(Z = 80\) \((\frac{\rho_m}{\rho_m,0} \approx 0.0003, \Omega_x \approx 0.0003)\) and finally it is released free (stage-2) and decays slowly \((\frac{1}{a})\) between \(Z = 80\) to \(Z = 0\) and becomes more effective and even dominant today \((\frac{\rho_m}{\rho_m,0} \approx 2, \Omega_x \approx 0.7)\).’ In stage-1: \(\Omega_x\) is almost equal to
zero; whereas in stage-2: $\Omega_x$ increases as $Z$ decreases; $\Omega_x = 0.0003$ at $Z = 80$; $\Omega_x = 0.01$ at $Z = 12$; $\Omega_x = 0.1$ at $Z = 3$; $\Omega_x = 0.5$ at $Z = 0.5$ and $\Omega_x = 0.7$ at $Z = 0$. The 'mantra' of dark energy hidden in nucleus comes out free and speaks aloud as $-1 + \frac{1}{3} = -\frac{2}{3}$, and orients itself towards acceleration of the universe. Presently (as also mentioned in section 3.1), due to the fast decay of radiation strength, the current value of $w$ could be close to 'minus one'.

4 Estimation of $Z$ and $\Omega_x$

(i) For present time; $Z = Z_0 = 0$, $\Omega_x = 0.7$, $\Omega_m = 0.3$ (Present-day known values).

(ii) For matter/dark-energy transition ($\rho_m = \rho_x$); $Z$ can be estimated from Eqs. (23) and (24) as $Z_T = 0.5$.

(iii) For galaxy formation (say at $Z = Z_g = 3$); $\Omega_x$ can be estimated from Eqs. (19) – (24) as $\Omega_{xg} = 0.1$.

(iv) For radiation/dark-energy transition ($\rho_r = \rho_x$); $Z$ can be estimated from Eqs. (22) and (24) as $Z = Z_t = 12$.

(v) At $Z = Z_f = 80$ when dark-energy is released 'free', $\Omega_x$ can be estimated (in stage-2) from Eqs. (19) – (24) as $\Omega_{xf} = 0.0001$. The time $Z = 80$ is the joining point for stage-1 and stage-2 (Figure 3). In stage-1, initial $\frac{\rho_x}{\rho_m} \approx 0.0003$ (see Section 3.3) remains constant as both $\rho_x$ and $\rho_m$ decay in same way ($\frac{1}{1+z}$), and for stage-2 $\frac{\rho_x}{\rho_m} \approx \frac{0.7/0.3}{(1+Z)^2}$ (see Eqs. (23) and (24)); equating these $0.0003 = \frac{0.7/0.3}{(1+Z)^2}$, gives $Z \approx 80$ as the meeting point for the two stages. At the meeting point ($Z = 80$); negative pressure of dark energy is much less than positive pressure of matter and radiation energy, but appears to be strong enough so as to enable the trapped dark energy to tunnel-through to come-out as released-free.

(vi) At decoupling epoch ($\rho_m = \rho_r$) $Z = Z_{dec} = 1000$, $\Omega_x$ will be nearly half of the initial value of $\frac{\rho_x}{\rho_m}$ in stage-1 (Figure 3), i. e., $\Omega_x = 0.00015$.

(vii) At about $Z = Z_n \approx 10^{10}$ during nucleo-synthesis era, birth of dark energy takes place. The fusion nuclear binding energy (= 1%) begins to be liberated during primordial Helium synthesis (25%) from 13% ($0.13 = 0.04$) of baryonic-matter ($\Omega_{b0} = 0.04$) out of total matter-mass ($\Omega_{m0} = 0.3$). Considering the liberated nuclear-binding-energy as dark-energy, the dark-energy-density is roughly estimated (1% of 25% of 13%) at 0.000325 of the total matter-density. However, since radiation density at that high value of $Z$ (at nucleo-synthesis time $Z = Z_n \approx 10^{10}$) would be much higher than that of matter, the estimated value for $\Omega_{xn}$ would be close-to-zero.
The liberated nuclear-energy as dark-energy, however, is trapped (as mentioned in subsection 3.3 and in Figure 3) in two stages. In trapped stage-1 (between $Z = 10^{10}$ to $Z = 80$) the dark-energy is trapped and dormant and varies ($\frac{1}{a^3}$) in a constrained way along with its parent matter; whereas in free stage-2 (between $Z = 80$ to $Z = 0$) the dark-energy is released free and becomes effective/dominant and varies ($\frac{1}{a}$) at slower rate during universe expansion. At $Z = 10^{10}$, $\Omega_x \approx 0$; at $Z = 80$, $\Omega_x = 0.0003$; at $Z = 12$, $\Omega_x = 0.01$; at $Z = 3$, $\Omega_x = 0.1$; at $Z = 0.5$, $\Omega_x = 0.5$ and at $Z = 0$, $\Omega_x = 0.7$. It may be noted that these estimates are quite reasonable and satisfy necessary astrophysical constraints [23]; as shown in Figure 4, the density ratio $\Omega_x$ is initially too low and remains low till galaxy formation.

Figure 4: Variation of density ratio for dark energy ($\Omega_x$) versus Red shift ($Z$) showing that $\Omega_x$ is too low making galaxy formation feasible.
5 Estimating Deceleration-Parameter $q$

The space dimension for universe with (i) matter, (ii) dark-energy and (iii) both these combined are written as follows:

(i) For matter ($w = 0$) universe

$$a_m = a = K_m t^2,$$

where $$K_m = \frac{a_0}{t_0^2}.$$

(ii) For dark-energy ($w = -\frac{2}{3}$) universe

$$a_x = a = K_x t^2,$$

where $$K_x = \frac{a_0}{t_0^2}.$$

(iii) For the universe combined with matter ($\Omega_m$) plus dark-energy ($\Omega_x$)

$$a = \Omega_m a_m + \Omega_x a_x = \Omega_m K_m t^2 + \Omega_x K_x t^2.$$  

The deceleration-parameter ($q = -\frac{\ddot{a}}{\dot{a}}$) can be estimated at present time $t_0$ ($\Omega_m = 0.3$, $\Omega_x = 0.7$) using Eqs. (25) - (27) and is found as $q_0 = -0.52$ (as also shown in subsection 5.2) which indicates that the expansion of the universe is accelerating. The current known value [21] is, however, $q_0 = -0.67$.

In fact, there could be following two approaches to find the deceleration-parameter which are described as follows:

5.1 Conventional Method

The ‘deceleration parameter’ for ‘only matter’ universe can easily be evaluated as $q_m = \frac{1}{2}$ and that for ‘only dark-energy’ universe as $q_x = \frac{1}{2} + \frac{3}{2} w$. So a simple conventional method usually followed assumes that the actual ‘deceleration parameter $q$ for the matter plus dark-energy universe’ is ‘weighted sum of $q_m$ and $q_x$’. Thus,

$$q_0 = \frac{1}{2} \Omega_m 0 + \left( \frac{1}{2} + \frac{3}{2} w \right) \Omega_x 0,$$

which gives for $w = -\frac{2}{3}$, $\Omega_m 0 = 0.3$, $\Omega_x 0 = 0.7$; $q = -0.20$ a value much different from actual known [21] value of $q_0 = -0.67$ necessitating modifications in evaluation as follows.

5.2 Modified Method

The authors suggest a modified method to evaluate the deceleration-parameter. This modified method assumes that actual ‘space scale factor (a) for matter plus dark-energy universe’ is ‘weighted sum of $a_m$ and $a_x$’. Thus, the equation for total actual space-size (a) can be written (similar to Eq. (27)) as

$$a = \Omega_m a_m + \Omega_x a_x = \Omega_m K_m t^2 + \Omega_x K_x t^2 \left( 1 + \frac{3}{2} w \right).$$
The deceleration parameter \( q = -\frac{\ddot{a}a^2}{\dot{a}^2} \) is evaluated, using Eqs. (25) and (12) in Eq. (29) at present time \( t_0 \) as

\[
q_0 = -\frac{\frac{3}{2} \Omega_{m0} - \frac{2(1+3w)}{3(1+w)} \Omega_{x0}}{\frac{3}{2} \Omega_{m0} + \frac{2}{3(1+w)} \Omega_{x0}}.
\]

(30)

which gives for \( w = -\frac{2}{3}, \Omega_{m0} = 0.3, \Omega_{x0} = 0.7; q_0 = -0.52 \) (as found before using Eq. (27)) which is reasonably near to the known observed value of \( q_0 = -0.67 \).

Modified equation (30) seems to be more correct than the conventional method’s expression given by Eq. (28) since the expansion (accelerating) of the universe due to dark-energy \( (a \sim t^2) \) is much faster than the expansion (decelerating) of universe due to matter \( (a \sim t^\frac{2}{3}) \). This fact is taken into account in the modified-method (in Eqs. (27), (29) and (30); the second term is more dominant). So, the modified method predicts more nearer to value of \( q_0 = -0.52 \), instead of \(-0.20 \) from conventional method, to the known observed value of \(-0.67 \).

6 Discussions

The exact time of birth \( Z = 10^{10} \) or \( 10^9 \) cannot be specified very precisely. It is suggested by the authors that nuclear binding energy manifests itself as dark energy but the beginning of nuclear energy and its completion is not exactly defined/known. There is two-stage tracking (decay) of dark energy (Figure 3). Initially in stage-1 the decay is constrained and fast during which the dark energy remains almost trapped/dormant for a long time. Finally in stage-2 the dark energy is released free at \( Z = 80 \) and decays slow and thus becomes more effective and dominant in due course. The free released nuclear binding energy plays its role as dark energy (with \( w = -\frac{2}{3} \)) creating negative-pressure or repulsive-gravity. Due to the fast decay of the radiation strength, the current value of \( w \), however, could be \( w = -1 \); indicating that as if vacuum energy is dark energy. Dark energy begins with close-to-zero value of \( \Omega_x \), remains low till galaxy formation \( (Z = 3, \Omega_x = 0.1) \), becomes 50\% at transition \( (\Omega_x = 0.5, Z = 0.5) \) and thereafter leads to accelerated-universe becomes dominant at present time \( (Z = 0, \Omega_x = 0.7) \). All these stand in accordance with the astrophysical constraints. The dark-energy variations, in stage-2 \( (\rho_x \propto \frac{1}{a^2}) \) is governed by final/present condition (at \( Z = 0 \)) and in stage-1 \( (\rho_x \propto \frac{1}{a^3}) \) is governed by initial conditions (at \( Z = 10^{10} \)). The meeting point for the two stages (dotted and firm curves in Figure 3) is found at \( Z = 80 \). There could be some error in it \( (Z = 80) \) due to approximations. Though the meeting point is a point of interest in view of the fact that at this time the dark-energy is finally released-free, however, any future change in its estimate (other than \( Z = 80 \)) will hardly have any effect on other values (estimated in section 4) or on the theme and
philosophy of the model which will remain unchanged.

For estimating the deceleration parameter, the authors suggest a more realistic modified method (Eq. \ref{eq:30}) which gives an estimate of \( q_0 = -0.52 \) against the observed known value of \( q_0 = -0.67 \). The agreement is not bad. The discrepancy in the value, however, may be due to following reason: the dark energy as a result of liberation of nuclear binding energy not only takes place during primordial nucleo-synthesis \((Z = 10^{10} \text{ or } 10^{9})\) era but also due to fusion reaction taking place within stars after or around galaxy formation \((Z = 4 \text{ to } 0)\). It may be noted that only ‘primordial nucleo-synthesis’, which indeed is more significant, has been taken into consideration; but the ‘star nucleo-synthesis’ will also have some effect on acceleration of universe i.e., on the deceleration parameter in the right direction. Also, this means that nucleo-synthesis is more or less a continuous process.

Though there are many suspects (candidates) such as \cite{24, 25} cosmological constant, vacuum energy, scalar field, brane world etc. as reported in the vast literature for the dark energy, the proposed model in this paper at least presents a new candidate (cosmological nuclear-energy) as a possible suspect (candidate) for the dark energy.

7 Conclusions

The authors propose a novel model for genesis of dark energy, indicating its origin in nucleus and suggesting that it is the released-free nuclear binding energy of cosmos which manifests itself as dark energy causing negative pressure and repulsive gravity in the universe. It explains why for dark energy \( w = -\frac{2}{3} \) at release time and \( w = -1 \) at present time. Also, the model describes the biography of the dark energy and tells that \( \Omega_x \) begins with a close-to zero value, remains low as necessary till galaxy formation say at \( Z = 3, \Omega_x = 0.1 \); transition at \( Z = 0.5, \Omega_x = 0.5 \) and becomes dominant at present time \( Z = 0, \Omega_x = 0.7 \) satisfying very well all the astrophysical constraints. The authors also propose a modified method to estimate deceleration parameter and find that with the proposed model \( q_0 = -0.52 \) in reasonable agreement with known observed value. It can be metaphorically said that ‘dark energy is a fossil of nuclear reaction that had taken place in the early universe and that the fossil has recently been noticed when the matter-cover over it has diminished in due course’. The secret of acceleration of big-universe is hidden in the small-nucleus.

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