On the Oscillatory Convective Instability in Nanofluid Layer with Temperature Dependent Viscosity

Joginder Singh Dhiman and Nivedita Sharma

Department of Mathematics, Himachal Pradesh University, Shimla 171005, India

Email: jsdhan66@gmail.com

Abstract. In the present paper, the problem of thermal convection in nanofluid layer heated from below with temperature dependent viscosity is studied mathematically for general boundary conditions, when the instability sets in as oscillatory convection. The expressions for Rayleigh numbers for exponential temperature dependent viscosity variation are derived using Galerkin method from the eigenvalue problem for each combination of rigid (the surfaces with non-slip condition) and dynamically free (the surfaces with stress free condition) boundaries. The effects of viscosity variation parameter, Lewis number, concentration Rayleigh number, modified diffusivity ratio and Prandtl number on the onset of oscillatory convection for each case of boundary conditions are investigated numerically. It is observed that the exponential variation of temperature dependent viscosity has stabilizing effect on the onset of oscillatory convection in nanofluid layer for each case of boundary conditions. However, the Lewis number and the concentration Rayleigh number have destabilizing effect on the onset of oscillatory convection.

1. Introduction

The convection in nanofluids has recently been an active field of research because of the greatly enhanced thermal properties of nanofluids. In view of this, the study of onset of thermal convection in nanofluid layer has been a subject matter of investigation for many authors in the recent years. The prominent among them are Chen et al [1], Tzou [2], [3], Nield and Kuznetsov [4] and Yadav et al. [5], who studied the problem of thermal instability of the nanofluid layer under varying assumptions of hydrodynamics. In most of the studies related to the investigation of thermal instability of ordinary fluid heated underside, much attention has been paid to the situations when the viscosity is constant. However, Rossby [6], Booker [7] and Booker and Stengel [8] have shown that for most of the classical fluids, the viscosity shows a rather pronounced variation with respect to temperature as viscosity is more sensitive to temperature than heat capacity and thermal conductivity. Therefore, it becomes important to consider the viscosity as temperature dependent in flow problems with heat transfer. Motivated by these considerations, Dhiman and Sharma [9] have recently studied the thermal convection of nanofluid layer heated underside with temperature dependent viscosity for each combination of rigid and dynamically free boundaries and established that principle of exchange of stabilities (PES) is valid for the problem when the modified diffusivity ratio \( N_d \leq \frac{1}{2} \). This clearly hints that when \( N_d > \frac{1}{2} \) (compliment of the condition for the existence of PES), the time-dependent oscillatory modes of instability can exist at the onset of thermal convection in nanofluids.

Motivated by the above discussion and the importance of the above discussed phenomenon in nanofluids, the aim of the present paper is to study the problem of thermal convection in a nanofluid...
layer heated underside for oscillatory instability and to investigate the effect of temperature dependent viscosity on the onset oscillatory convection for all cases of combinations of rigid and dynamically free boundaries. The present analysis thus extends the analysis of Dhiman and Sharma [9] for oscillatory convection in nanofluid layer heated underside with temperature dependent viscosity.

2. Physical configuration and eigenvalue problem

Consider an infinite horizontal layer of viscous, incompressible nanofluid of continuously varying density $\rho$ and viscosity $\mu$ statically confined between two horizontal boundaries $z^* = 0$ and $z^* = d$, maintained respectively at constant temperature and volumetric fraction of nanoparticles $T_0^*$ and $\phi_0^*$ at $z^* = 0$ and $T_u^*$ and $\phi_u^*$ at $z^* = d$. A constant temperature gradient ($T_0^* > T_u^*$) is maintained across the fluid layer parallel to the body force, which is gravity here. The viscosity of the fluid is taken to be an exponential function of temperature variation given as $\mu(T) = \mu_0 e^{-\gamma(T-T_0)}$, where $\gamma$ is the coefficient of viscosity variation.

Following the usual steps of linear stability theory (cf. Nield and Kuznetsov [4] and Chandrasekhar [10], Dhiman and Sharma [11] and [12]), the non-dimensional linearized perturbation equations governing the problem of thermal convection in nanofluids with variable viscosity as derived by Dhiman and Sharma [9] are given by:

\[
f(D^2 - a^2)^2w - \frac{n_f}{P_r}(D^2 - a^2)w + 2(Df)D(D^2 - a^2)w + D^2f(D^2 + a^2)w = Ra a^2 \theta - R_N a^2 \phi \tag{1}
\]

\[
(D^2 - a^2 - n)\theta + \left(\frac{2N_A N_B}{Le} - \frac{N_B}{Le}\right)D\theta + \frac{N_B}{Le}D\phi = -w \tag{2}
\]

\[
\left(\frac{1}{Le}(D^2 - a^2) - n\right)\phi + \frac{N_A}{Le}(D^2 - a^2)\theta = w \tag{3}
\]

together with either of the following cases of boundary conditions;

**Case 1: Both boundaries dynamically free**

\[
w = 0 = \theta = \phi = D^2w \quad \text{at } z = 0 \text{ and } z = 1
\]

**Case 2: Both boundaries rigid**

\[
w = 0 = \theta = \phi = Dw \quad \text{at } z = 0 \text{ and } z = 1
\]

**Case 3: Lower rigid and upper boundary free**

\[
w = 0 = \theta = \phi = Dw \quad \text{at } z = 0
\]

\[
w = 0 = \theta = \phi = D^2w \quad \text{at } z = 1
\]

In the above equations; $D \equiv \frac{d}{dz}$ is the differential operator; $a = \left(k_1^2 + k_2^2\right)^{\frac{1}{2}}$ is a dimensionless horizontal wave number; $n$ is a dimensionless complex growth rate (i.e. $n = n_r + i n_i$, where $n_r$ and $n_i$ are real); $w, \theta$ and $\phi$ are the respective perturbations in the initial vertical velocity, temperature and nanoparticle volume fraction; $f(z)$ is the non-dimensional temperature dependent viscosity variation function; $N_A$ is the modified diffusivity ratio; $N_B$ is the modified particle density increment; $Le$ is the Lewis number; $P_r$ is the Prandtl number; $Ra$ is the Thermal Rayleigh number and $R_N$ is the concentration Rayleigh number.

3. Mathematical analysis

Dhiman and Sharma [9] derived the above eigenvalue problem governing the thermal convection in a nanofluid represented by equations (1)-(3) together with boundary conditions (4)-(6) and studied the case of stationary instability. Here, in the present analysis, we have extended their analysis for the case of oscillatory convection and the expressions for Rayleigh numbers for each case of boundary conditions (4)-(6) are derived using the Galerkin method (cf. Finlayson [13]), by taking a single term in the expansions of $w, \theta$ and $\phi$. To proceed, we have considered suitably chosen trial functions; $w_1, \theta_1$ and $\phi_1$ which satisfy the respective boundary conditions given in (4)-(6), as

\[
w = Aw_1(z), \quad \theta = B\theta_1(z) \quad \text{and} \quad \phi = C\phi_1(z)
\]

(7)

Here, $A, B$ and $C$ are arbitrary constants.
Substituting these trial functions defined in (7) in equations (1)-(3) and multiplying the resulting equations (the *residuals*) respectively by \( w_1 \), \( \theta_1 \) and \( \phi_1 \) so that the expressions on the left-hand sides of these equations become orthogonal to the trial functions. Now, integrating these resulting equations over the vertical range of \( z \) and performing the integration by parts a suitable number of times using relevant boundary conditions (4)-(6), we obtain the following system of three linear homogeneous algebraic equations in the unknowns \( A \), \( B \) and \( C \):

\[
A \int_0^1 f \left(D^2w_1\right)^2 + 2a^2(Dw_1)^2 + a^4(w_1)^2 \, dz + Aa^2 \int_0^1 f(D^2f)(w_1)^2 \, dz + A \frac{n}{l_e} \int_0^1 [D(Dw_1)^2 + a^2(w_1)^2] \, dz - \nonumber \\
Ra a^2 B \int_0^1 \theta_1 w_1 \, dz + R_n a^2 C \int_0^1 \phi_1 w_1 \, dz = 0
\]  

\[
B \int_0^1 [(D\theta_1)^2 + (a^2+n)(\theta_1)^2] \, dz + B \left(\frac{(2N_A-1)N_B}{l_e}\right) \int_0^1 D\theta_1 \, dz + C N_B \int_0^1 \theta_1 \, dz - \int_0^1 \theta_1 w_1 \, dz = 0
\]  

\[
C \int_0^1 \left(\frac{1}{l_e} (D\phi_1)^2 + a^2(\phi_1)^2 + n(\phi_1)^2\right) \, dz - B \frac{n}{l_e} \int_0^1 [D(\phi_1)^2, \theta_1] - a^2 \phi_1, \theta_1\right] dz + A \int_0^1 \phi_1 w_1 \, dz = 0
\]  

For the existence of non-trivial solution of these equations, the vanishing of the determinant of coefficients of equations (8)-(10) yields an *eigenvalue equation* for the system in terms of Rayleigh number \( Ra \), for the case of oscillatory convection.

Thus, to obtain the values of the Rayleigh numbers \( Ra \) for each case of boundary conditions (4)-(6), and for the exponential non-dimensional viscosity variation, we have considered the following exponential viscosity variation law (cf. Dhiman and Kumar [14]):

\[
f = e^{\delta z}, \quad \text{where} \quad \delta = \text{the viscosity variation parameter.}
\]  

Now, proceeding in the analogous way as in the analyses of Dhiman and Sharma [9], [11] and [12], we shall derive the expressions for Rayleigh numbers for each of the cases of boundary conditions (4)-(6), separately.

**Case 1: Both boundaries dynamically free**

For the present case of boundary conditions, choosing the suitable polynomial trial functions satisfying the boundary conditions (4) given by:

\[
w = z^4 - 2z^3 + z, \quad \theta = z(z - 1) \quad \text{and} \quad \phi = z(z - 1)
\]  

Using viscosity variation law (11) and the trial functions (12) in equations (8)-(10), eliminating constants \( A \), \( B \) and \( C \) amongst the resulting equations and substituting \( n = i n_l \) for the case of oscillatory modes, we obtain the following dispersion relation:

\[
Ra \left( l_{11} + i n_l L e \right) + R_N \left[ l_{11} (N_A + L e) + i n_l L e \right] = \frac{1}{h_{11}} \left( M_{11} + i n_l \frac{\theta_{11}}{P_r} \right) \left[ l_{11} + i n_l L e \right] (l_{11} + i n_l)
\]  

Where,

\[
M_{11} = \frac{1}{\delta^9} \left\{ 144 \delta^4 \left( -12 - 6 \delta - \delta^2 + e^\delta (12 - 6 \delta + \delta^2) \right) + a^4 \left( -20160 - 1080 \delta - 1440 \delta^2 + 120 \delta^3 + 48 \delta^4 - 6 \delta^5 + e^\delta (20160 - 1008 \delta + 1440 \delta^2 + 120 \delta^3 - 48 \delta^4 + 6 \delta^5) \right) + 2 a^2 \delta^2 \left( -15840 - 7290 \delta - 1152 \delta^2 + 84 \delta^3 + 36 \delta^4 - 6 \delta^5 + e^\delta (15840 - 7290 \delta + 1152 \delta^2 + 84 \delta^3 - 36 \delta^4 + 6 \delta^5) \right) \right\}
\]  

and \( l_{11} = 10 + a^2; \quad \theta_{11} = \frac{306 + 31a^2}{630}; \quad h_{11} = \frac{8670 \ a^2}{176400} \)  

Now, equating the real and imaginary parts of both sides of equation (13), we obtain the following equations

\[
n_l^2 \left[ \frac{L e M_{11}}{l_{11}} + \frac{L e \theta_{11}}{P_r} + \frac{\theta_{11}}{P_r} \right] = M_{11} l_{11} - h_{11} \left[ Ra f + R_N (N_A + L e) \right]
\]  

and \( \frac{n_l^2 a^2 L e \theta_{11}}{P_r} = l_{11} M_{11} (1 + L e) + \frac{i n_l \theta_{11}}{P_r} - h_{11} \left[ L e(Ra f + R_N) \right] \)

If we take \( n_l = 0 \) (non-oscillatory case) in equation (16), we get an expression for Rayleigh number \( Ra ff = \frac{M_{11} l_{11}}{h_{11}} - R_N (N_A + L e) \)
which is the same expression for Rayleigh number for stationary case of instability as obtained by Dhiman and Sharma [9] for the case of both dynamically free boundaries for the problem under consideration. Expression (18) also corresponds to the boundary for non-oscillatory instability.

It is clear from the expression (14) that \( M_{11} \) is a positive constant for positive \( \delta \), whereas \( l_{11} \), \( g_{11} \) and \( h_{11} \) are clearly positive which can be verified from the expressions (15). Hence, equation (16) yields that for given \( n_{i} \) (real), \( R_{N} \) must be negative for large values of \( Le \).

Now, eliminating \( n_{i}^{2} \) from equations (16) and (17), we obtain the following equation:

\[
\left( \frac{M_{11}}{l_{11}g_{11}} + \frac{1}{Pr} \right) Ra_{f} + \left( \frac{M_{11}}{l_{11}g_{11}} + \frac{(1-N_{A})}{Le Pr} \right) R_{N} = \frac{M_{11}l_{11}}{h_{11}} \left[ \left( 1 + \frac{1}{Le} + \frac{1}{Pr} + \frac{l_{11}g_{11}}{LeM_{11}Pr} \right) \left( 1 + \frac{1}{Le Pr} + \frac{1}{Pr} + \frac{M_{11}}{l_{11}g_{11}} \right) - \frac{1}{Le Pr} \right]
\]

(19)

which yields the minimum (critical value) of Rayleigh number \( Ra \) for \( a^{2} = 2.22 \), when \( \delta \to 0 \), as

\[
Ra_{f}(c) = \left( \frac{664.525 P_{r}}{(0.991865 P_{r}+1)} \right) \left( 1 + \frac{1}{Le} + \frac{1.0082}{Le Pr} \right) \left( 0.991865 + \frac{1}{Le Pr} + \frac{1}{Pr} - \frac{1}{Le Pr} \right) - \frac{(0.991865 Le P_{r}+(1-N_{A}))}{Le(0.991865 P_{r}+1)} R_{N}
\]

(20)

This value corresponds to oscillatory instability boundary. Further, the value given in expression (20) for critical Rayleigh number when both boundaries are dynamically free is very close to the value obtained by Nield and Kuznetsov [4] for the same problem with constant viscosity.

**Case 2:** Both boundaries are rigid

For this case of boundary conditions, the suitable polynomial trial functions chosen are:

\[
w = z^{2} - 2z^{3} + z^{2}, \quad \theta = z(z - 1) \quad \text{and} \quad \phi = z(z - 1)
\]

(21)

Using viscosity variation law (11) and the trial functions (21) in equations (8)-(10), we obtain the following equation analogous to equation (13), with \( n = in_{i} \);

\[
Ra_{tr}(l_{22} + in_{i} Le) + R_{N}[l_{22}(N_{A} + Le) + in_{i} Le] = \frac{1}{h_{22}} \left( M_{22} + in_{i} \frac{g_{22}}{Pr} \right) (l_{22} + in_{i} Le) (l_{22} + in_{i})
\]

(22)

Where,

\[
M_{22} = \frac{1}{36} \left[ 6a^{4} \left( 1680 - 840 \delta - 180 \delta^{2} - 12 \delta^{3} - \delta^{4} + e^{\delta}(1680 + 840 \delta + 180 \delta^{2} + 12 \delta^{3} + \delta^{4}) + 2a^{2} \delta^{2} \left( 684 - 132 \delta - 96 \delta^{2} - 12 \delta^{3} - \delta^{4} + e^{\delta}(684 - 132 \delta - 96 \delta^{2} - 12 \delta^{3} + \delta^{4}) \right) \right] \]

(23)

\[
l_{22} = 10 + a^{2}; \quad g_{22} = \frac{1}{630} (12 + a^{2}); \quad h_{22} = \frac{3}{19600} a^{2}
\]

(24)

Now, proceeding analogously as in Case 1 above, we can obtain the expressions for real and imaginary parts of the equation (22) for Case 2 also. From these obtained expressions for real and imaginary parts, eliminating \( n_{i}^{2} \), we obtain the following dispersion relation analogous to expression (19);

\[
\left( \frac{M_{22}}{l_{22}g_{22}} + \frac{1}{Pr} \right) Ra_{tr} + \left( \frac{M_{22}}{l_{22}g_{22}} + \frac{(1-N_{A})}{Le Pr} \right) R_{N} = \frac{M_{22}l_{22}}{h_{22}} \left[ \left( 1 + \frac{1}{Le} + \frac{1}{Pr} + \frac{l_{22}g_{22}}{LeM_{22}Pr} \right) \left( 1 + \frac{1}{Le Pr} + \frac{1}{Pr} + \frac{M_{22}}{l_{22}g_{22}} \right) - \frac{1}{Le Pr} \right]
\]

(25)

**Case 3:** Lower rigid and upper free boundaries

For this case of boundary conditions, the suitable polynomial trial functions chosen are:

\[
w = 2z^{2} - 5z^{3} + 3z^{2}, \quad \theta = z(z - 1) \quad \text{and} \quad \phi = z(z - 1)
\]

(26)

Using relations (11) and (26) in equations (8)-(10), proceeding as in cases above, we have

\[
Ra_{tr}(l_{33} + in_{i} Le) + R_{N}[l_{33}(N_{A} + Le) + in_{i} Le] = \frac{1}{h_{33}} \left( M_{33} + in_{i} \frac{g_{33}}{Pr} \right) (l_{33} + in_{i} Le) (l_{33} + in_{i})
\]

(27)

Where,

\[
M_{33} = \frac{1}{s^{3}} \left[ 2 \left( -184 \delta^{4} \left( 384 + 240 \delta^{2} + 66 \delta^{2} + 10 \delta^{3} + \delta^{4} + 6e^{\delta}(64 - 24 \delta + 3 \delta^{2}) \right) + a^{2} \left( -36(2240 + 1400 \delta + 370 \delta^{2} + 50 \delta^{3} + 3 \delta^{4}) + e^{\delta}(80640 - 30240 \delta + 3240 \delta^{2} + 240 \delta^{2} - 72 \delta^{4} + 5 \delta^{4}) + 2a^{2} \delta^{2} \left( -18(3520 + 2200 \delta + 584 \delta^{2} + 80 \delta^{3} + 5 \delta^{4} + 6e^{\delta}(63360 - 23760 \delta + 259 \delta^{2} + 16 \delta^{3} - 54 \delta^{4} + 5 \delta^{4}) \right) \right) \right) - (N_{A} + Le) R_{N}
\]

(28)

\[
l_{33} = 10 + a^{2}; \quad g_{33} = \frac{1}{630} (216 + 19a^{2}); \quad h_{33} = \frac{5070}{17640} a^{2}
\]

(29)
Following the similar steps as in the earlier discussed two cases of boundary conditions, we obtain the following dispersion relation analogous to equation (19)
\[
\left( \frac{M_{33}}{\nu_{33}B_{33}} + \frac{1}{Pr} \right) Ra_f + \left( \frac{M_{33}}{\nu_{33}B_{33}} + \frac{(1-N_A)}{LePr} \right) Ra_f = M_{33} \left[ \frac{1}{Le} \frac{1}{Pr} + \frac{1}{LePr} \right] \left( 1 + \frac{1}{Le} \frac{1}{Pr} + \frac{1}{LePr} \right) \left( 1 + \frac{1}{Le} \frac{1}{Pr} + \frac{M_{33}}{\nu_{33}B_{33}} \right) - \frac{1}{LePr},
\]

(30)

4. Results and discussion

In the present analysis, the effect of temperature dependent viscosity on the onset oscillatory thermal convection in nanofluid layer heated from below has been studied for each combination of rigid and dynamically free boundary conditions. The expressions for the Rayleigh numbers for oscillatory convection are derived using Galerkin method and the values of the critical Rayleigh numbers for various values of critical wave numbers \( \alpha_c \), for certain fixed values of \( \delta \) (exponential variation of temperature dependent viscosity) and different values of other parameters; \( Le, R_N, N_A \) and \( Pr \) are calculated numerically and the obtained results are presented in Tables 1 and 2 for Case I of boundary conditions. The variation of Rayleigh number \( Ra_f \) with respect to \( \delta \) and \( Le \) for fixed value of \( N_A = 10, R_N = -2, Pr = 5 \) is presented in Figures 1(a)-(1(c) and the variation of \( Ra_f \) with respect to \( \delta \) and \( R_N \) for fixed value of \( N_A = 5, Le = 100, Pr = 5 \) is shown in Figures 1(d)-(1(f), for Case I of boundary conditions.

From the values of \( Ra_f \) presented in Tables 1 & 2 and from Figures 1(a)-(1(f), we note that the exponential variation of temperature dependent viscosity \( (\delta) \) has stabilizing effect on the onset of oscillatory convection in nanofluid layer heated from below. This is due to the fact that the increase in nanofluid viscosity increases the friction which results in the decrease of flow rate and consequently degrades the heat transfer performances. Also, from the values presented in Table 1 and Figures 1(a), 1(b) and 1(c) for Case I of the boundary conditions, we observed that for increasing values of Lewis number \( (Le) \), the values of \( Ra_f \) decreases, which implies that \( Le \) has destabilizing effect on the onset of oscillatory convection. Also, it can be observed from the values presented in Table 1 and Figures 1(d), 1(e), 1(f); that the values of critical Rayleigh numbers also decrease with the increasing value of the concentration Rayleigh number \( R_N \), hence has a destabilizing effect on the onset of oscillatory convection in nanofluids.

From the values presented in Table 2, we found that \( Ra_c \) show marginal variations with respect to increasing value of modified diffusivity ratio \( (N_A > \frac{1}{2}) \) and Prandtl number \( (Pr) \). This is due to the fact that the increase in \( N_A \), which defines the ratio of thermophoretic diffusion coefficient to Brownian diffusion coefficient, increase the cross-diffusion phenomena of thermophoresis and Brownian motion, while the increase in Prandtl number, which defines the ratio of kinematic viscosity to thermal diffusivity (heat transfer rate), thus increase the viscous force which causes the retardation in the velocity and hence the heat transfer rate and therefore have marginal effect on the onset of oscillatory convection. The analogous analysis for Case 2 and Case 3 of boundary conditions reveals the similar behaviour of Rayleigh numbers as observed in Case I boundary conditions and hence numerical values and graphical representation of the obtained results are both omitted here, for the sake of compactness.

### Table 1. Variation of \( Ra_f \) with respect to \( \delta \) and \( Le \) for different values of \( a_c \) and fixed value of \( N_A = 10, R_N = -2, Pr = 5 \) and for \( R_N \) with \( N_A = 5, Pr = 5, Le = 100, \) for Case I of boundary conditions

| \( \delta \) | \( Le \) | \( Ra_f^1 \) | \( Ra_f^2 \) | \( Ra_f^3 \) | \( Ra_f^4 \) |
|-------------|--------|-------------|-------------|-------------|-------------|
| 0.2         | 200    | 4.95501     | 742.382     | - 3         | 4.95503     | 747.483     |
| 0.5         | 200    | 4.93242     | 873.357     | -5          | 4.93244     | 879.232     |
| 0.9         | 200    | 4.87338     | 1103.32     | -10         | 4.87344     | 1110.37     |
| 0.2         | 500    | 4.95498     | 739.678     | -3          | 4.95499     | 753.365     |
| 0.5         | 500    | 4.93398     | 873.389     | -5          | 4.93424     | 885.257     |
| 0.9         | 500    | 4.87334     | 1099.64     | -10         | 4.87337     | 1116.57     |
| 0.2         | 800    | 4.95499     | 739.028     | -3          | 4.95497     | 757.559     |
| 0.5         | 800    | 4.93398     | 869.61      | -10         | 4.93237     | 889.561     |
| 0.9         | 800    | 4.87333     | 1098.72     | -10         | 4.87332     | 1121.01     |

### Table 2. Variation of \( Ra_f \) with respect to \( \delta \) and \( N_A \) for different values of \( a_c \) and fixed value of \( R_N = -2, Pr = 5, Le = 500 \) and for \( Pr \) with \( N_A = 5, R_N = -3, Le = 500 \) for Case I of boundary conditions

| \( \delta \) | \( N_A \) | \( a_c^1 \) | \( a_c^2 \) | \( a_c^3 \) | \( a_c^4 \) |
|-------------|--------|-------------|-------------|-------------|-------------|
| 0.2         | 10     | 4.95501     | 742.382     | - 3         | 4.95503     | 747.483     |
| 0.5         | 10     | 4.93242     | 873.357     | -5          | 4.93244     | 879.232     |
| 0.9         | 10     | 4.87338     | 1103.32     | -10         | 4.87344     | 1110.37     |
| 0.2         | 5      | 4.95498     | 739.678     | -3          | 4.95499     | 753.365     |
| 0.5         | 5      | 4.93398     | 873.389     | -5          | 4.93424     | 885.257     |
| 0.9         | 5      | 4.87334     | 1099.64     | -10         | 4.87337     | 1116.57     |
| 0.2         | 1      | 4.95499     | 739.028     | -3          | 4.95497     | 757.559     |
| 0.5         | 1      | 4.93398     | 869.61      | -10         | 4.93237     | 889.561     |
| 0.9         | 1      | 4.87333     | 1098.72     | -10         | 4.87332     | 1121.01     |

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Figures 1(a)-(c): Variation of $Ra_{ff}$ w. r. t. $a^2$ for $Le = 200, 500$ and $800$, respectively.

Figures 1(d)-(f): Variation of $Ra_{ff}$ w. r. t. $a^2$ for $R_N = -10, -5$ and $-3$, respectively.

5. References

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