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Dissipative Dynamics of a Fermionic Superfluid with Two-Body Losses

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We study the dissipative dynamics of a fermionic superfluid in presence of two-body losses. We use a variational approach for the Lindblad dynamics and obtain dynamical equations for Anderson's pseudo-spins where dissipation enters as a complex pairing interaction as well as effective, density-dependent, single particle losses which break the conservation of the pseudo-spin norm. We show that this latter has key consequences on the dynamical behavior of the system. In the case of a sudden switching of the two-body losses we show that the superfluid order parameter decays much faster than then particle density at short times and eventually slows-down, setting into a power-law decay at longer time scales driven by the depletion of the system. We then consider a quench of the pairing interaction, leading to coherent oscillations in the unitary case, followed by the switching of the dissipation. We show that losses wash away the dynamical BCS synchronization by introducing not only damping but also a renormalization of the frequency of coherent oscillations, which depends strongly from the rate of two-body losses.

\textbf{Introduction} - The nonequilibrium dynamics of superfluids and superconductors has attracted fresh interest in recent years. In condensed matter physics there has been substantial progress in controlling quantum materials with ultrafast pump-probe techniques and Floquet engineering \cite{1, 2}. The reports of light-induced superconductivity in variety of materials \cite{3, 4, 5, 6} represented among the most striking demonstration of this effort, that spurred large theoretical interests \cite{7, 8, 9, 10, 11}. Similarly, the experimental developments in non-linear optical spectroscopy have renewed the interest on dynamical signature of collective modes in the superconducting phase \cite{12, 13, 14}. In atomic physics, on the other hand, earlier investigations motivated by the realization of fermionic superfluids \cite{15, 16, 17} have focused on the dynamics after sudden changes of the pairing interaction and revealed characteristic dynamical transitions \cite{18, 19, 20, 21, 22, 23, 24, 25}. More recently the spectroscopy of driven superfluids has been performed \cite{26}. In most cases, theoretical investigations of these phenomena have focused on the dynamics of closed isolated systems. Dissipation is however not only unavoidable in realistic experimental contexts, such as in the solid-state, but can sometime be controlled with high-degree of flexibility, as in certain ultracold atoms experiments, and used as a tool to control the dynamical long-time behavior of the system.

Dissipative quantum many-body systems represent a fresh platform where novel dynamical phenomena and phase transition can appear as result of the competition between unitary evolution and dissipative couplings \cite{24, 25, 26}. For bosonic or fermionic particles these can model both single particle processes such as pump and losses as well as correlated effects, such as heating due to stimulated emission \cite{27, 28, 29}, spontaneous emission \cite{30} or two-particle losses \cite{31–35}. These types of dissipative inelastic scattering processes naturally arise for example in experiments with ultracold fermions made of Alkali-Earth atoms \cite{36–38}. Their role for the dynamics has recently attracted large interest in the context of Dicke states \cite{34, 39, 40} and Quantum Zeno Effect (QZE) \cite{41} where the effective dissipation decreases as the loss rate is increased \cite{42, 43}. The interplay of Zeno physics with other many-body phenomena has recently attracted large interest \cite{44–51}.

In this Letter we study the dissipative dynamics of a fermionic superfluid, modelled as an attractive Hubbard model \cite{52} in presence of weak local two-body losses. Recent works in this context have focused on simplified descriptions of dissipation in terms of a non-Hermitian Bardeen-Cooper-Schrieffer (BCS) problem \cite{53} or an effective unitary dynamics with complex pairing potential \cite{54}. Here we use a variational approach for Lindblad dynamics to show that a complete dissipative BCS theory also includes an effective, density-dependent, single particle loss term, which corresponds to decoupling the two-body losses in the particle-particle and particle-hole channels. We show that this term completely controls the long-time dynamics of the system and leads to a breakdown of the conservation of Anderson’s pseudo-spins norm, which is a signature of the dissipative nature of the problem. We first consider the dynamics after a sudden switching of the dissipation, where we show that the order-parameter displays a crossover from a short-time exponential decay to a long-time power law decay controlled by the depletion of the system. Then we study the interplay between quench of pairing and dissipation, revealing that dissipation washes away the dynamical synchronization transition \cite{19}. Surprisingly we show that dissipation not only damps out the synchronization dynamics, but also dramatically changes the coherent oscillations of the order parameter which become faster for increasing dissipation rate. Our results can
be experimentally tested in experiments with ultracold fermionic superfluids [55, 56], where two-body losses can be introduced through photoassociation [33, 35].

Model - We consider a system of spinful fermions hopping on a lattice, in presence of a local pairing interaction as described by the attractive Hubbard model whose Hamiltonian reads

\[ H = \sum_{\langle i,j \rangle \sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - |U| \sum_i n_i^\uparrow n_i^\downarrow \]  

(1)

where \(-|U|\) is the attraction and the \(t_{ij}\) the nearest neighbor hopping. The hopping gives rise to a single particle band of width \(W\), and characterized by a semi-circular density of states. This model has been studied in thermal equilibrium [57] in the context of the BCS to BEC superfluidity crossover [58–61], while its dynamics has received attention recently [62–68] and revealed a variety of dynamical phase transitions. Here we focus on an open quantum system setting in which the evolution of the system density matrix \(\rho\) is described by a Lindblad master equation [69], \((\hbar = 1)\),

\[ \partial_t \rho = -i[H, \rho] + \sum_i \left( L_i \rho L_i^\dagger - \frac{1}{2} \left( L_i^\dagger L_i - I \right) \rho \right) \]  

(2)

with local, on-site, jump operators describing Markovian dissipation. We note that the Lindblad dynamics can be seen also to arise from a continuous monitoring of the system and subsequent average over the quantum trajectories [70]. Here we consider dissipative processes in which pairs of fermions on the same site and with opposite spins escape from the system to the environment, leading to a jump operator of the form \(L_i = c_{i\uparrow} c_{i\downarrow}\). The resulting dissipative dynamics does not conserve the total number of particles. In absence of any driving term to counterbalance the loss of particles into the environment the system evolves at long times towards the zero density limit. We note that two-body losses conserve instead the total spin which would prevent from reaching complete depletion [40], unless the system is initially prepared in a total singlet state as it is our case here. While the stationary state properties of the model are therefore trivial its depletion dynamics can still reveal intriguing features and give rise to different dynamical regimes, as we are going to discuss.

Method - To study the dynamics of the system we use a time-dependent variational approach. While for unitary system the Dirac’s variational principle is a standard and much used result, both for gaussian and for correlated wave functions, its generalisation to the open system case pose some challenges. Recent work [71] has proposed a variational principle for the stationary state which is however not of direct use here, where the long-time limit is the vacuum. To focus on dynamics we proceed along a different line, directly inspired by work on unitary quantum dynamics. We note that stating that a density matrix \(\rho\) evolves according to Eq. 2

is equivalent to say that the functional \(S[\rho_0, \rho_{aux}] = \int \text{Tr} [\rho_{aux}(i\partial_t \rho_0 - L[\rho_0])]\) is stationary with respect to any given density matrix \(\rho_{aux}\). Using this condition on a Gaussian density matrix \(\rho_0\) for which Wick’s theorem applies, including normal and anomalous contractions, allow us to obtain the following variational dynamics [72].

\[ \partial_t \rho_0 = -i \left( \tilde{H}_{BCS}, \rho_0 \right) + \Gamma n \sum_\sigma L_\sigma^{\text{1p-loss}}[\rho_0] \]  

(3)

Namely the density matrix evolution has a unitary part that comes from the usual BCS mean-field Hamiltonian plus a complex pairing field \(U + i \Gamma\)

\[ \tilde{H}_{BCS} = H_{0,BCS} + i \Gamma \Delta \sigma_{i\downarrow} c_{i\downarrow} - i \Gamma \Delta^* \sigma_{i\dagger} c_{i\dagger}^\dagger \]  

(4)

with a self-consistent pairing field

\[ \Delta(t) = \sum_k \text{Tr} \left( \rho_0 c_{k\dagger} c_{-k\dagger} \right) \]  

and an effective single-particle loss dissipator that will play a key role in the following. Its form reads

\[ L_\sigma^{\text{1p-loss}}[\rho_0] = \sum_i \left( c_{i\sigma} \rho_0 c_{i\sigma}^\dagger - \frac{1}{2} \left\{ c_{i\sigma}^\dagger c_{i\sigma}, \rho_0 \right\} \right) \]  

(5)

with a strength \(n\Gamma\), with \(n\) the time-dependent particle density. The variational dynamics associated to the above effective Lindbladian reads

\[ \sigma_k^{\text{x}} = -2 \varepsilon_k \sigma_k^{\text{y}} + 2 \text{Im}(\Phi) \sigma_k^{\text{z}} - \Gamma n \sigma_k^{\text{z}} \]  

(6)

\[ \sigma_k^{\text{y}} = 2 \varepsilon_k \sigma_k^{\text{z}} - 2 \text{Re}(\Phi) \sigma_k^{\text{x}} - \Gamma n \sigma_k^{\text{x}} \]  

(7)

\[ \sigma_k^{\text{z}} = 2 \text{Re}(\Phi) \sigma_k^{\text{y}} - 2 \text{Im}(\Phi) \sigma_k^{\text{x}} - \Gamma n (\sigma_k^{\text{z}} + 1) \]  

(8)

where we have introduced the Anderson’s pseudo-spin \(\sigma_k = \text{Tr} (\rho_0 \Psi_k^{\dagger} \sigma^\alpha \Psi_{-k})\) with \(\sigma^\alpha = x, y, z\) given by the Pauli matrices, where \(\varepsilon_k\) is the bare energy dispersion of the lattice, and \(\Phi(t) = (U + i \Gamma) \Delta(t)\) is the self-consistent pairing field. This dynamics describe the competition between precession of Anderson’s pseudo-spin around an effective magnetic field, as in the unitary case, and losses-induced decoherence towards the steady state \(\sigma_k^{\text{x}} = \sigma_k^{\text{y}} = 0\) and \(\sigma_k^{\text{z}} = -1\), corresponding to vanishing order parameter and density. We note that the length of the pseudo spin \(S = \sum_{\alpha k} (\sigma_k^{\alpha})^2\) is not conserved due to the presence of the single particle loss term proportional to the density. Furthermore the purity of the variational state \(P = \text{Tr} (\rho_0^2)\) is also not conserved, as expected for a dissipative Lindblad dynamics. The dynamical equations above differ therefore from those that can be obtained by Hubbard-Stratonovich decoupling [54], which essentially take the form of a unitary dynamics with a complex pairing term \(U + i \Gamma\). This difference arises due to the presence of the effective single particle loss term in Eq. (5), that couples the Keldysh contours. We will discuss below the consequences of this term for the physics of the problem.
We note that instead the equations above coincide with those that can be obtained through a direct mean-field decoupling of Hamiltonian and dissipator, including both contributions coming from particle-particle and particle-hole channels.

Results - Dissipation quench We begin our discussion from the dynamics after a sudden switching of the two-body losses $\Gamma$, starting from the ground-state of the attractive Hubbard model with $|U|/W = 1.0$. 

In Fig. 1 we plot the time evolution of the order parameter $\Delta(t)$ and particle density $n(t)$ for different values of the dissipation measured with respect to the interaction $\Gamma/|U|$. We see in the right panels that the density remains constant at short times while above a time scales which depends weakly on the loss rate it displays a power-law decay towards zero, corresponding to the vacuum state, with an exponent $\sim t^{-1}$ which is independent of $\Gamma$.

On the other hand the dynamics of the superfluid order parameter $\Delta(t)$ is richer and shows a crossover from an exponential decay at short times followed by a slower power law decay on longer time scales. We argue that this crossover is a key dynamical signature of a dissipative superfluid with two-body losses. The power law regime is controlled in fact by the slow depletions of the system, which drags the order parameter down towards zero. One can verify that by imposing an additional external single particle pump to keep the density constant in time changes the long-time behavior of the order parameter which only features an exponential decay (dot-dashed lines). The crossover between the two dynamical regimes for $\Delta(t)$, exponential at short times and power-law on longer times, occur on a crossover time $\tau$ which decreases with the two-body losses (see inset in Fig. 1). We understand this behavior by considering the fact that stronger dissipation gives rise to a faster depletion of particles, and, in turn, to a faster crossover to the regime in which the order parameter decay gets slowed down by the density decay.

The fact that order parameter and density display different dynamical regimes arise naturally from the variational dynamics of Anderson’s pseudospins and in particular from the lack of conservation of the norm of the pseudospin, due to the decoupling in both the particle-particle and particle-hole channels included in our treatment. To see this more clearly we can write down the dynamics of the density $n = \sum_k \sigma^z_k + 1$ and obtain using Eq. (6-8)

$$\frac{dn}{dt} = -2\Gamma|\Delta|^2 - \Gamma n^2.$$  

(9)

This result highlights how the decay of density and order-parameter amplitude in a dissipative superfluid are intertwined. The first term in Eq. (9) describes the depletion due to losses of Cooper pairs $[54]$, while the second one accounts for the contribution of non-condensed pairs, which is always present even when the system is in the normal phase and that becomes dominant at long-times. In fact this second term gives rise to the power-law decay of the density, as one can readily understand from disregarding the order parameter, which gives $n \sim -n^2$, implying $n \sim t^{-1}$. We remark that the power law decay of the density is a hallmark of the many-body nature of the dissipation term $[42, 46]$. For comparison, we show in the dashed lines of Fig. 1 the behavior obtained with only single particle losses for which the dynamics of the density displays an exponential decay. Similarly, in presence of single particle losses the depletion dynamics of $\Delta(t)$ is again exponential (dashed lines in Fig. 1).

Results - Double quench We now consider the dynamics after a double quench, where first at some negative time the pairing interaction is suddenly changed $U_i \rightarrow U_f$ and then the two-body losses are suddenly switched on at time $t = 0$. This dynamical protocol allows to discuss the effect of correlated dissipation on the dynamical synchronization transition $[18–21]$ that is known to occur in the isolated case.

In Fig. 2 we plot the dynamics of the order param-
depends strongly on $\Gamma$. For $\Gamma/|U_f| \gg 1$ shown in the inset of Fig. 3 we plot frequency $\omega/|U_f|$ and plot it in Fig. 3 as a function of $\Gamma/|U_f|$ parameter, obtained by Fourier transforming the real-time signal, and leads to a non-conservation of the norm of Anderson's pseudo spin norm that to the many-body nature of the dissipative process. In fact we have obtained qualitatively similar results in presence of only single particle losses (not shown).

Conclusions - In this work we have studied the dissipative dynamics of a fermionic superfluid with two-body losses. We have used a time-dependent variational method for open quantum systems, from which the resulting dynamics takes the form a BCS problem with complex pairing interactions and effective single particle losses that were disregarded in previous works \[53, 54\]. We show that the latter play a key role for the dynamics and leads to a non-conservation of the norm of Anderson’s pseudospin. We consider first the case of a sudden switch-on of the dissipation, where we show the order parameter decays exponentially at short times and with a characteristic power-law due to depletion at long times. By considering a double quench of pairing and dissipation, we show that the losses wash away the dynamics of the order parameter and leads to a non-conservation of the norm of Anderson’s pseudospin. We consider first the case of a sudden switch-on of the dissipation, where we show the order parameter decays exponentially at short times and with a characteristic power-law due to depletion at long times. By considering a double quench of pairing and dissipation, we show that the losses wash away the dynamics of the order parameter, with a period essentially set by the ratio between initial and final gap \[19\]. We see that the switching of the dissipation at $t = 0$ drastically changes the time evolution, inducing not only a damping of coherent oscillations but also a substantial renormalization of their frequency, which increases with $\Gamma$. Remarkably we note from the upper panel of Fig. 2 that even a tiny dissipation, corresponding to $\Gamma/|U_f| = 10^{-7}$, has a sizable effect on the oscillation frequency. To highlight this point we extract the dominant frequency $\omega_*$ of the coherent oscillations of the order parameter, obtained by Fourier transforming the real-time signal, and plot it in Fig. 3 as a function of $\Gamma/|U_f|$. We see that $\omega_*$ depends strongly on the losses, with a non-analytic behavior at small $\Gamma$ compatible with a power-law $\omega_* \sim (\Gamma/|U_f|)^\alpha$, where $\omega_0^\|\|$ is the oscillation frequency in the unitary case and where the exponent $\alpha$ depends weakly on the final value of the interaction. As the dissipation is increased, even though remaining a small fraction of the interaction $|U_f|$, we see that the frequency $\omega_*$ tends to saturate to a value of order $|U_f|$. This is clearly shown in the inset of Fig. 3 we plot $\omega_*$ as a function of the final interaction $|U_f|$ and show that the frequency increases with $|U_f|$ almost linearly with a prefactor that depends strongly on $\Gamma$. For $\Gamma/|U_f| \gtrsim 10^{-2}$ the coherent oscillation become overdamped, and the frequency $\omega_*$ becomes ill defined. The fact that the dissipation changes so dramatically the frequency of oscillations of the order parameter is the second important result of this work. We note that this effect seems to be due more to the lack of conservation of Anderson’s pseudo spin norm that to the many-body nature of the dissipative process.

Figure 2. Dynamics after a sudden quench of the interaction $|U_i|/W = 0.125 \rightarrow |U_f|/W = 1.0$. For $-200 < t < 0$ the dynamics is unitary. For positive times we switch on a finite dissipation, $\Gamma/|U_f| = 10^{-7}, 10^{-4}$, and 0.005, from top to bottom. In all the panels, the light grey lines represent the corresponding unitary dynamics.
BEC type of superfluidity and large dissipation corresponding to the Zeno regime. For this goal it will be crucial to extend the time-dependent variational principle to correlated wave-functions such as the Gutzwiller one.

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Supplemental Material to ‘Dissipative Dynamics of a Fermionic Superfluid with Two-Body Losses’

In this Supplemental Material, we provide details on the derivation of the BCS dissipative dynamics by using a time dependent variational principle for the density matrix.

The time evolution of a density matrix according to a given Liouvillian reads

\[ i\dot{\rho} = L[\rho], \]

Eq. S1 can be cast in term of a variational principle, by introducing an auxiliary density matrix \( \rho_{\text{aux}} \), and requiring stationarity of the functional

\[ \mathcal{S}[\rho, \rho_{\text{aux}}] = \int dt \text{Tr} \left[ \rho_{\text{aux}} (i\dot{\rho} - L[\rho]) \right] \frac{\delta \mathcal{S}[\rho, \rho_{\text{aux}}]}{\delta \rho_{\text{aux}}} = 0. \]

We now consider a density matrix of the BCS type, \( \rho = \rho_0 \), and compute the functional S2 for generic auxiliary density matrix \( \rho_{\text{aux}} \). This is straightforwardly computed by using Wick theorem. In particular, for any operator \( O \), a trace of the type \( \text{Tr} (\rho_0 \rho_{\text{aux}} O) \) can be expressed as

\[ \text{Tr} (\rho_0 \rho_{\text{aux}} O) = \sum_{\text{contractions}} \text{Tr} \left( \rho_0 \left( \rho_{\text{aux}} O \right) \right), \]

where the symbol \( \left( \rho_{\text{aux}} O \right) \) means the contractions using single fermionic lines of the operator \( \rho_{\text{aux}} O \). By further singling out the contractions only involving terms terms in the operator \( O \), it is possible to reconstruct the expectation value S3 in terms of the contractions of the operator \( O \) times the expectation value of the non-contracted part of \( \rho_{\text{aux}} O \),

\[ \text{Tr} (\rho_0 \rho_{\text{aux}} O) = \sum_{\text{contractions}} \text{Tr} \left( \rho_0 \left( \rho_{\text{aux}} O \right) \right) \text{Tr} \left( \rho_0 \rho_{\text{aux}} \delta O \right), \]

where \( \delta O \) indicates the part of \( O \) not included in the contraction \( \text{Tr} \left( \rho_0 \rho_{\text{aux}} O \right) \). By applying S4 to the dissipator in the main text, and contracting in both the normal and anomalous channels, we get

\[ L[\rho_0, \rho_{\text{aux}}] = \text{Tr} \left( \rho_{\text{aux}} [H_{\text{BCS}}, \rho_0] \right) + \left( i\Gamma \Delta \text{Tr} \left( \rho_{\text{aux}} [c_i^\dagger c_i, \rho_0] \right) \right) - i\Gamma \Delta^* \text{Tr} \left( \rho_{\text{aux}} [c_i^\dagger c_i, \rho_0] \right) \]

\[ - i\frac{\Gamma}{2} \sum_{\sigma} \text{Tr} \left( \rho_{\text{aux}} \xi^{\text{loss}}_{\sigma} [\rho_0] \right), \]

where \( H_{\text{BCS}} \) is the unitary BCS Hamiltonian with pairing \(-|U|\), i.e.

\[ H_{\text{BCS}} = \sum_{<ij>} t_{ij} c_i^\dagger c_j - |U| \left( \Delta c_i^\dagger c_i + \Delta^* c_i^\dagger c_i \right), \]

and \( \Delta = \text{Tr} \left( \rho_0 c_i^\dagger c_i \right) \). Plugging S5 into the variational principle S2 we get the variational dynamics for \( \rho_0 \) reported in Eq. 3 of the main text.