Post-injection aseismic slip plays an important role in a broad range of human-made and natural systems, from the exploitation of geo-resources to the understanding of earthquakes. Recent studies have shed light on how aseismic slip propagates in response to continuous fluid injections. Yet much less is known about the response of faults after the injection of fluids has stopped. In this work, we investigate via a hydro-mechanical model the propagation and ultimate arrest of aseismic slip during the so-called post-injection stage. We show that after shut-in, fault slip propagates in pulse-like mode. The conditions that control the propagation as a pulse and notably when and where the ruptures arrest are fully established. In particular, critically stressed faults can host rupture pulses that propagate for several orders of magnitude the injection duration and reach up to nearly double the size of the ruptures at the moment of shut-in. We consequently argue that the persistent stressing of increasingly larger rock volumes caused by post-injection aseismic slip is a plausible mechanism for the triggering of post-injection seismicity—a critical issue in the geo-energy industry. Our physical model shows quantitative agreement with field observations of documented cases of post-injection induced seismicity.

1. Introduction

It has been long recognized that subsurface fluid injections can induce fast fault slip [1] that radiates detectable elastodynamic waves, and also slow, sometimes called aseismic slip [2], which is generally more difficult to detect by classical monitoring networks. Evidence
for injection-induced aseismic slip dates back to the 1960s, when a slow surface fault rupture was causally linked to fluid injection operations of an oil field in LA, USA [2]. Since then, an increasing number of studies have inferred the occurrence of aseismic slip episodes as a result of anthropogenic subsurface fluid injections [3–6], with recent in situ experiments of fluid injection into natural fault systems demonstrating by direct measurements of fault deformation that slow slip was systematically the dominant style of fault motion [7,8]. Injection-induced aseismic slip is thought to play an important role in seismicity induced by industrial fluid injections, a phenomenon that has become critical in ensuring the sustainable development of unconventional hydrocarbon reservoirs [9] and deep geothermal resources [10,11]. It is understood that fluid-driven slow ruptures transmit stresses quasi-statically to unstable fault patches and trigger instabilities at distances that can be far from the region affected by the pressurization of pore-fluid [12,13]. Moreover, a similar mechanism might be also operating behind some natural episodes of seismicity such as seismic swarms and aftershock sequences. In fact, both phenomena are commonly interpreted to be driven by either the diffusion of pore pressure [14,15] or the propagation of aseismic slip [16,17], with recent studies suggesting that both mechanisms might be indeed coupled and responsible for the observed spatio-temporal patterns of seismicity [18–20]. Likewise, tectonic tremors and low-frequency earthquakes are often considered to be driven by slow slip events in subduction zones [21,22], at depths where systematic evidence of over-pressurized fluids is found [22,23]. Metamorphic dehydration reactions [24] and fault-valving behaviour [25] are the two common candidates to explain not only this inferred over-pressurization, but also the very nature of pore pressure and aseismic slip transients in these zones [26].

The seemingly relevant role of fluid-driven aseismic slip in the previous phenomena has motivated the development of physical models that, in recent years, have contributed to a better understanding of the mechanics of this hydro-mechanical problem. Some recent advances include a better notion of how the initial state of stress, the fluid injection parameters, the fault hydraulic properties, and the possible rate-strengthening dependence of rock friction, may affect the dynamics of fluid-driven aseismic slip transients in two- [27–30] and three-dimensional media [31]. The three-dimensional case is the relevant one for field applications and has been shown to be not only quantitatively but also qualitatively very different from two-dimensional configurations [31]. One common aspect of all prior studies is that they focus on fluid sources that are continuous (uninterrupted) in time. Yet much less is known about aseismic ruptures after the injection of fluids has stopped. In particular, the conditions that control the further propagation and ultimate arrest of fluid-driven aseismic slip are fairly unknown, despite a recent investigation for a two-dimensional plane-strain configuration [32]. This is of broad interest since, ultimately, any kind of fluid source and accompanying rupture will have to stop. Understanding the mechanics of post-injection aseismic slip in a realistic three-dimensional scenario is thus of major importance. It corresponds to the first goal of this study.

Our second goal is to understand how and to what extent post-injection aseismic slip can be considered as a possible mechanism for the triggering of seismicity after the end of subsurface fluid injections. It is well known since the Denver earthquakes in the 1960s [1] that upon shutting off the wells, seismicity might continue to occur. Multiple observations suggest that, indeed, it is not rare that the largest events of injection-induced seismic sequences happen during the post-injection stage. Examples of those cases are the 2006 $M_L > 3$ Basel earthquakes in Switzerland, and the 2017 $M_w$ 5.5 Pohang earthquake in South Korea, both causally linked to hydraulic stimulation operations of deep geothermal reservoirs [10,11]. Because the shut-in of the wells does not guarantee the cessation of seismic events, current efforts to manage the seismic risk in the geo-energy industry such as the so-called traffic light systems [33] might be subjected to important limitations in their effectiveness. Understanding the physical mechanisms underpinning post-injection seismicity is thus of paramount importance for the successful development of geo-energy projects. We therefore aim in this study at understanding the combined effect of pore pressure diffusion and solid stress changes due to aseismic slip in the triggering of post-injection seismicity.
2. Governing equations and scaling analysis

(a) Governing equations

We consider a planar fault in an infinite, linearly elastic, isotropic, impermeable and three-dimensional solid, as depicted in figure 1. The initial stress tensor is assumed to be uniform and is characterized by a shear stress \( \tau_0 \) resolved on the fault plane that acts along the x-direction and a total normal stress \( \sigma_0 \) (acting along the z-direction). We assume a uniform initial pore pressure field of magnitude \( p_0 \) that is perturbed by the sudden injection of fluids into a poroelastic fault zone of width \( w \). We model such fluid injection via a line source that is located along the z-axis and crosses the entire fault width. The fault zone permeability \( k \) and storage coefficient \( S \) are assumed to be uniform and constant. As a result, fluid flow is axisymmetric with regard to the z-axis and occurs only within the fault zone. We further assume that the shear modulus \( \mu \) and Poisson’s ratio \( \nu \) of the poroelastic fault zone and impermeable elastic host rock are the same. Under such conditions, the displacement field induced by the fluid injection is irrotational and the pore pressure diffusion equation of poroelasticity reduces to its uncoupled version \[34\], the same. Under such conditions, the displacement field induced by the fluid injection is irrotational and the pore pressure diffusion equation of poroelasticity reduces to its uncoupled version \[34\], the same.

By neglecting any poroelastic coupling within the fault zone upon activation of slip, the quasi-static elastic equilibrium that relates fault slip to the shear stress components along the fault can be written as the following boundary integral equations,

\[
\tau_i(x, y, t) = \tau_i^0 + \int_{\Gamma} K_{ij}(x - \xi, y - \zeta; \mu, \nu) \delta_i(\xi, \zeta, t) \, d\xi \, d\zeta, \quad \text{with} \quad i, j = x, y, \tag{2.1}
\]

where \( \tau_i^0 = (\tau_0, 0)^T \) is initial shear traction vector, \( K_{ij} \) is the hypersingular (of order \( 1/r^3 \)) elastostatic traction kernel (see [35] for example), and \( \delta_i = u_i^+ - u_i^- \) is the fault slip vector, with \( u_i^\pm \) the displacement vector at the upper (+) and lower (−) faces of the slip surface \( \Gamma \).

The fault interface is assumed to obey a Mohr–Coulomb shear failure criterion with a constant friction coefficient \( f \) and no cohesion expressed as the following local inequality:

\[
||\tau_i(x, y, t)|| \leq f(\sigma_i^0 - \Delta p(r, t)), \tag{2.2}
\]

where \( \sigma_i^0 = \sigma_0 - p_0 \) is the initial effective normal stress, and \( \Delta p(r, t) = p(r, t) - p_0 \) is the axisymmetric pore pressure perturbation, with \( r \) the radial coordinate.

Owing to the unidirectionality of the initial shear traction vector and the axisymmetry property of the equivalent shear load in terms of magnitude, both fault slip and maximum shear stress are characterized by an approximately uniform direction along the x-axis (see the electronic supplementary material). The latter implies that for all practical purposes in this paper, the differences between \( ||\delta_i|| \) and \( ||\delta_x|| \), and \( ||\tau_i|| \) and \( ||\tau_x|| \), are negligible (an approximation that was implicit in [31]). We therefore refer interchangeably hereafter to fault slip as \( \delta \equiv \delta_x \) and fault shear stress as \( \tau \equiv \tau_x \). Moreover, for the particular case in which \( \nu = 0 \), \( K_{xy} = K_{yx} = 0 \) (see, for instance, eqn 7.4 in [35]) and thus the direction of slip (or maximum shear) occurs exactly along the x direction.

The injection of fluid starts at \( t = 0 \) and consists of two subsequent stages: a continuous injection stage characterized by a constant rate of injection \( Q \), and a post-injection stage in which the injection rate drops instantaneously to zero at \( t = ts \) (figure 1c). Hereafter, we refer to \( ts \) as the shut-in time. The solution of the linear diffusion equation before shut-in (when \( 0 < t \leq ts \)) is known as \( \Delta p(r, t) = \Delta p_s E_1(r^2/4at) \) (section 10.4, eqn 5, [36]), where

\[
\Delta p_s = \frac{\Delta p_c}{4\pi} \quad \text{and} \quad \Delta p_c = \frac{Q\eta}{kw}. \tag{2.3}
\]

In the previous equations, \( \Delta p_s \) is the intensity of the injection with units of pressure, \( \Delta p_c \) is a characteristic pressure that is of the order of magnitude of the overpressure at the fluid source and \( E_1(\cdot) \) is the exponential integral function. The solution after shut-in (when \( t > ts \)) is obtained
simply by superposition as

$$\Delta p(r, t) = \Delta p_c \left\{ E_1 \left( \frac{r^2}{4\alpha t} \right) - E_1 \left( \frac{r^2}{4\alpha (t - t_s)} \right) \right\}.$$  \hspace{1cm} (2.4)

Note that the infinitesimal size of the fluid source in our model provides a proper finite volume of injected fluid $V = Q t_s$, but infinite pressure at the injection point. In this regard, our model must be understood as the late-time asymptotic solution of a more general model where the fluid source possesses a finite characteristic size, say $\ell_s$. Our results will be thus applicable when $t_s$ is much larger than the characteristic diffusion time $\ell_s^2/\alpha$ over the finite source lengthscale. This is indeed the case of most borehole fluid injections, where the borehole radius is $\ell_s \sim 10$ cm. By assuming values of hydraulic diffusivity in the range $10^{-4}$ to $1$ m$^2$ s$^{-1}$, the characteristic time $\ell_s^2/\alpha$ takes values between 100 down to 0.01 s that are much smaller than typical fluid injection duration in geo-energy applications. Moreover, the asymptotic behaviour of the exponential integral function for small values of its argument, $E_1(x) \approx -\gamma - \ln(x)$, where $\gamma$ is the Euler–Mascheroni’s constant, allows us to identify $\Delta p_c$ as the pressure scale that is of the order of magnitude of the overpressure at $r = \ell_s$ when $t \gg \ell_s^2/\alpha$, as previously stated.

Equations (2.1), (2.2) and (2.4) constitute a complete system of equations to solve for the spatio-temporal evolution of fault slip $\delta(x, y, t)$ and, particularly, the moving boundary representing the slipping region $S(t)$. The slipping patch $S(t)$ may be defined mathematically as the region in which the equality of equation (2.2) holds: $S(t) = \{(x, y) \in \Gamma : |\tau(x, y, t) = f(\sigma_0 - \Delta p(r, t))\}$. The initial conditions are naturally taken as $\delta(x, y, 0) = 0$ and $\dot{\delta}(x, y, 0) = 0$ (fault initially at rest and fully locked). The numerical methods used to solve the problem are described in the electronic supplementary material.

(b) Scaling analysis and limiting regimes

The shut-in of the injection provides the natural characteristic time scale of the problem, the shut-in time $t_s$. This latter introduces, via the solution of the pore-pressure diffusion equation (2.4), a characteristic diffusion lengthscale $\sqrt{4\alpha t_s}$. Alternatively, one may choose (as suggested in [31])
to scale the spatial coordinates by a characteristic rupture lengthscale at the moment of shut-in, say \( R_s^* \). We normalize the pore pressure perturbation \( \Delta p(r, t) \), equation (2.4), by the intensity of the injection \( \Delta p_s \). Introducing the foregoing characteristic scales in the Mohr–Coulomb shear failure criterion, equation (2.2), gives the normalized shear stress, \( \frac{(\tau - f\sigma_0')f\Delta p_s}{\Delta p_s} \), which in turn is introduced in the balance of momentum, equation (2.1), allowing us to close the scaling of the problem by normalizing the fault slip by \( (f\Delta p_s/\mu)\sqrt{4\alpha t_s} \), or alternatively by \( f\Delta p_s/\mu R_s^* \) if the characteristic rupture lengthscale is chosen for the spatial scale.

It can be shown by using the previous scales (see the electronic supplementary material) that the model depends in addition to dimensionless space and time on one single dimensionless number

\[
T = \frac{f\sigma_0' - \tau_0}{f\Delta p_s},
\]

and Poisson’s ratio \( \nu \).

The so-called stress-injection parameter \( T \) is identical to the one presented recently by Sáez et al. [31] for the problem of continuous injection, and similar to the one introduced first by Bhattacharya & Viesca [13] in their two-dimensional plane-strain model. \( T \) is defined as the ratio between the amount of stress necessary to activate fault slip \( f\sigma_0' - \tau_0 \), and \( f\Delta p_s \), which quantifies the effect of the fluid injection on the reduction of fault strength near the injection point. The stress-injection parameter can vary theoretically between 0 and \( +\infty \) [31]. However, for practical purposes, we note here that \( T \) is upper bounded. Since \( \Delta p_c = 4\pi f\Delta p_s \) is of the order of magnitude of the overpressure at the fluid source, the minimum amount of overpressure that one needs to guarantee to activate fault slip under such a line-source approximation is \( f\Delta p_c \sim f\sigma_0' - \tau_0 \). Substituting the previous relation into (2.5) leads to an order-of-magnitude upper bound \( T \lesssim 10 \) (the factor \( 4\pi \) indeed). Note that the introduction of \( \Delta p_c \) and the resulting upper bound for \( T \) in this work put now the results of the continuous-injection problem [31] in a more practical perspective. The limiting values of \( T \) are associated with two end-member scenarios that were first introduced by Garagash & Germanovich [37]. When \( T \) is close to zero, \( \tau_0 \rightarrow f\sigma_0' \) and thus the fault is critically stressed (about to fail before the injection starts). On the other hand, when \( T \sim 10 \), the fault is ‘marginally pressurized’ as the injection has provided just the minimum amount of overpressure to activate fault slip, \( f\Delta p_c \sim f\sigma_0' - \tau_0 \). We will make extensive use of the terms critically stressed/fault/region and marginally pressurized fault/region to refer to these limiting cases throughout this article.

As shown recently by Sáez et al. [31], an important aspect of the foregoing two limiting regimes is that they follow different scales during the stage of continuous injection, which is particularly valid in the post-injection problem right at the moment of shut-in \( t_s \). Indeed, the proper slip scale \( \delta_s \) in the critically stressed limit comes from choosing \( \sqrt{4\alpha t_s} \) as the characteristic lengthscale,

\[
\delta \sim \delta_s = \frac{f\Delta p_s}{\mu} \sqrt{4\alpha t_s}.
\]

The proper scale for the slip rate follows from differentiation of the slip scale with respect to time,

\[
v \sim v_s = \frac{f\Delta p_s}{\mu} \sqrt{\frac{\alpha}{t_s}}.
\]

The slip and slip rate scales of the marginally pressurized limit can be obtained likewise by choosing the characteristic rupture scale \( R_s^* \) as the spatial lengthscale of the problem, i.e. \( \delta \sim (f\Delta p_s/\mu)R_s^* \) and \( v \sim (f\Delta p_s/\mu)R_s^* \sqrt{t_s} \). Nonetheless, the latter scales will be seen later to be somewhat irrelevant in the post-injection stage, because marginally pressurized faults will host ruptures that after shut-in arrest almost immediately.

Finally, the second dimensionless parameter of the model, Poisson’s ratio \( \nu \), is expected to have only an effect on the rupture shape. As shown by Sáez et al. [31] for the continuous-injection stage, Poisson’s ratio modifies the aspect ratio of the resulting rupture fronts, which become more elongated for increasing values of \( \nu \). The position of the elongated rupture fronts is nevertheless determined primarily by the position of circular ruptures, that correspond to a limiting case in which Poisson’s ratio of the solid \( \nu \) is zero. The case of circular ruptures is thus particularly
insightful and notably simpler, since in that limit, we can take advantage of the axisymmetry property of the mechanical problem [13,31].

3. Pore-pressure diffusion: the pore-pressure back front

The pore-pressure perturbation $\Delta p(r,t)$ is the only external action driving the rupture after shut-in. It is thus essential to examine in detail its spatio-temporal evolution. Various normalized spatial distributions of pore pressure are plotted in figure 2a at different dimensionless times. We first observe that pore pressure decreases quickly after the stop of injection near the fluid source in the sense $r \ll \sqrt{4\alpha t}$, asymptotically as $\Delta p(r,t)/\Delta p_s \approx \ln(t/(t - 1))$ (red dashed curve), with $\iota = t/t_s$ the dimensionless time. Moreover, at large times ($\iota \gg 1$), it decreases simply as $\Delta p(r,t)/\Delta p_s \approx 1/\iota$ (blue dashed curve) for points that are close to the injection now in the sense $r \ll \sqrt{4\alpha t}$. The latter indicates that the region in which the previous asymptotic behaviour is approximately satisfied grows diffusively ($\propto \sqrt{\alpha t}$), and has the shape of a plateau in figure 2a.

Moreover, figure 2a displays that after shut-in the pore pressure keeps increasing away from the injection line. To better understand such a process, it may be convenient to look at the temporal evolution of pore pressure at fixed normalized positions over the fault plane. Figure 2b displays such a plot. Here, we observe that pore pressure remains increasing for some time after injection stops ($\iota > 1$) and eventually reaches a maximum (red circles). After this instant of maximum pressure, the pore pressure decreases following ultimately the same asymptotes of figure 2a, meaning that the aforementioned diffusely expanding plateau has reached the corresponding fixed radial positions in figure 2b.

The relation between a given position over the fault plane and the time at which the maximum pressure is reached, defines a 'back front' of pore pressure, $P(t) = \{r(t) \in \Gamma : \partial \Delta p(r,t)/\partial t = 0\}$. Owing to the axisymmetry property of pore pressure diffusion in our model, $P(t)$ is circular and is thus fully defined by its radius $P(t)$. Differentiating equation (2.4) with respect to time and solving for $\partial \Delta p/\partial t = 0$ leads to the following expression for the normalized radius of the pore-pressure back front,

$$P(t) = \sqrt{4\alpha t_s} \approx \sqrt{t},$$

which further reduces to $P(t)/\sqrt{4\alpha t_s} \approx 1$, or simply $P(t) \approx \sqrt{4\alpha t}$, at large times. Note that the large-time asymptote of $P(t)$ is the same expression than that of the radius of the pore-pressure perturbation front $L(t) = \sqrt{4\alpha t}$ for the continuous-injection stage, albeit each expression describes different processes.

On the other hand, the maximum pore pressure increase undergone historically at a given position $r$, say $\Delta p_m(r)$, is obtained by simply evaluating the pore pressure perturbation $\Delta p(r,t)$ at the time $t_m$ in which the maximum occurs, i.e. $\Delta p(r,t_m(r))$. $t_m$ comes from solving for time in (3.1) with $P(t) = r$. $\Delta p_m(r)$ is displayed in figure 2a (grey dashed curve) representing the pore pressure back front $P(t)$ in this plot, which corresponds to the envelope of all curves (for all times) associated with instantaneous spatial distributions of pore pressure. Note that $\Delta p_m(r)$ cannot be obtained in closed form since equation (3.1) is not invertible for time, nevertheless, in the large-time limit, $t_m(r)/t_s \approx \bar{r}^2$, such that $\Delta p_m(r)/\Delta p_s \approx E_1(1) - E_1(1/(\bar{r}^2 - 1))$. The previous asymptotic approximation is also plotted in figure 2a (green dashed curve). Similarly, an expression for $\Delta p_m$ as a function of time $t$ may be derived, which is indeed obtainable in closed form as $\Delta p_m(t) = \Delta p(P(t),t)$ (grey and green dashed curves in figure 2b).

4. Pulse-like circular ruptures

Circular ruptures occur in the limit of a Poisson’s ratio $\nu = 0$. As discussed in previous sections, a Poisson’s ratio different than zero is expected to affect mainly the shape of the resulting ruptures, and less significantly other relevant quantities of the fault response [31]. Circular ruptures are thus a particularly insightful case, in which the axisymmetry property of the mechanical problem
[13,31] simplifies the analysis of results. The effect of Poisson’s ratio on the rupture shape will be quantified in §5.

(a) Recall on the self-similar solution before shut-in

The solution for the continuous-injection stage ($t < t_s$) of the same problem was presented recently by Sáez et al. [31]. We briefly summarize some of their results as they provide the starting point for the understanding of the post-injection phase. Sáez et al. showed that fault slip induced by injection at constant volume rate (from an infinitesimal source) is self-similar in a diffusive manner. The rupture radius $R(t)$ evolves simply as $R(t) = \lambda L(t)$, where $L(t) = \sqrt{4\alpha t}$ is the nominal position of the pore-pressure perturbation front, and $\lambda$ is the so-called amplification factor, for which an analytical solution as function of the stress-injection parameter $T$ exists (eqn (21) in [31]). The asymptotic behaviour of $\lambda$ for the limiting values of $T$ is particularly insightful. For critically stressed faults ($T \ll 1$), the amplification factor turns out to be large ($\lambda \gg 1$) and thus the rupture front outpaces largely the fluid pressure front ($R(t) \gg L(t)$), with $\lambda$ obeying the simple asymptotic relation $\lambda \approx 1/\sqrt{2\pi T}$. On the other hand, for marginally pressurized faults ($T \sim 10$), the amplification factor is small ($\lambda \ll 1$), such that the rupture front lags significantly the fluid pressure front ($R(t) \ll L(t)$), with $\lambda \approx (1/2) \exp[(2 - \gamma - T)/2]$, where $\gamma = 0.577216 \ldots$ is the Euler–Mascheroni’s constant. These simple closed-form expressions provide important insights into the response of faults during continuous fluid injections. They allow us to estimate the slip front position in a compact and convenient way that also determines whether aseismic-slip stress transfer ($\lambda > 1$) or pore pressure increase ($\lambda < 1$) is the dominant mechanism in the possible triggering of seismicity associated with coupled fluid flow and fault slip processes.

(b) Transition from crack-like to pulse-like rupture: the locking front

In §3, we showed that the shut-in of the injection is characterized by the emergence of the so-called pore-pressure back front, which corresponds to the instantaneous position at which the pressure has reached its maximum historically. With reference to figure 2c, we consequently see that pore pressure is instantaneously increasing at every point on the fault plane located beyond...
Given by equation (2.2) establishes that, within the slipping patch, the pulses decrease monotonically with time. On the other hand, the Mohr–Coulomb inequality and rupture fronts are highlighted. Note that both the magnitude (peak slip rate) and width of the amount of slip accumulated through the passage of the slip pulse (shown in figure 3) can be seen clearly in figure 3. The aseismic pulses are prominent a feature of post-injection aseismic slip. Similarly to the rupture front $R(t)$, the locking front $B(t)$ must be circular when $v = 0$, and is thus fully defined by its radius $B(t)$. Figure 3 shows the transition from crack-like to pulse-like propagation as seen in the normalized slip $δ$, slip rate $v$, and shear stress $τ$ and shear strength $τ_s$, spatial distributions, at times right before shut-in and at some time after shut-in. The aseismic pulses can be seen clearly in figure 3b for the slip rate distribution, where the positions of the locking and rupture fronts are highlighted. Note that both the magnitude (peak slip rate) and width of the pulses decrease monotonically with time. On the other hand, the Mohr–Coulomb inequality given by equation (2.2) establishes that, within the slipping patch ($B(t) < r < R(t)$), the shear stress must be equal to the fault shear strength, whereas in the re-locked region ($r < B(t)$), the shear stress must be lower than the fault shear strength. Both cases can be clearly seen in figure $3c$, where the positions of the rupture and locking fronts are also highlighted. Note that the normalized shear strength in equation (3c) is equal to $$(τ_s - f_0 \sigma_{33}' - f \Delta p, r) = -\Delta p/\Delta p_s,$$ which is minus the normalized pore pressure perturbation. In addition, we observe that at some distance away from the locking front and towards the injection point, the shear stress after shut-in is slightly greater than the shear stress right before shut-in. This amplification of fault shear stress after shut-in is due to the amount of slip accumulated through the passage of the slip pulse (shown in figure 3a) and the non-local redistribution of stresses associated with it via the quasi-static non-local integral operator of equation (2.1). We further elaborate on the amplification of shear stress behind the locking front when analysing the characteristics of slip rate and shear stress rate along the fault in the electronic supplementary material.

Figure 3. Normalized spatial profiles of (a) slip $δ$, (b) slip rate $v$ and (c) shear stress $τ$ and shear strength $τ_s$, for a mildly critically stressed fault with $T = 0.1$, at two dimensionless times, one right before shut-in ($t_1 = 1$) and the other one at some moment after shut-in ($t_2 = 3$). The positions of the locking $B(t)$ and rupture $R(t)$ fronts are highlighted throughout (a–c). In (b), grey curves correspond to aseismic pulses at times $\bar{t} = 1.54$ and 6.25.
Note that the rupture front of radius \( r \) and locking front of radius \( B(t) \) are located behind the rupture front in order to guarantee the sustained propagation of the rupture: \( P(t) < R(t) \). Similarly, the propagation of the locking front is driven by the existence of a region within the pulse where the pore pressure decreases (and thus the shear strength increases to effectively re-lock the fault), such that \( P(t) > B(t) \). Hence, the following inequalities must be satisfied during pulse-like propagation in the post-injection stage, \( B(t) < P(t) < R(t) \), which are verified in all our numerical solutions, such as the one displayed in figure 4a.

On the other hand, since Coulomb’s shear stress on the fault plane is always bounded, the shear stress developed at the front of the rupture pulse must be bounded as well, to effectively allow both quantities to be equal. In other words, the slipping patch must propagate with no stress singularity at the ‘fracture’ front of such a cohesive-like crack [38]. Hence, both stress intensity factors (SIF) in mode II and mode III must be equal to zero at \( r = R(t) \). This fracture-mechanics-like propagation condition was derived in appendix A of Sáez et al. [31] for any axisymmetric circular shear rupture, and is still valid for pulse-like (annular) ruptures as long as the ‘fracture’ is understood not as the current slipping patch, but rather as the region that has ever experienced any amount of slip since the start of the injection (i.e. for all \( r < R(t) \)). The condition reads as [31]

\[
\int_0^{R(t)} \frac{\Delta \tau(r,t)}{\sqrt{R(t)^2 - r^2}} r \, dr = 0 \iff \int_0^{B(t)} \frac{\tau_0 - \tau^+(r,t)}{\sqrt{B(t)^2 - r^2}} r \, dr + \int_0^{R(t)} \frac{\tau_0 - f(\sigma'_0 - \Delta p(r,t))}{\sqrt{R(t)^2 - r^2}} r \, dr = 0. \tag{4.1}
\]

Note that \( \Delta \tau(r,t) = \tau_0 - \tau(r,t) \), with \( \tau_0 \) the initial shear stress and \( \tau(r,t) \) the instantaneous shear stress distribution. In the right-hand side of equation (4.1), we have split the integral into two parts recognizing that within the slipping patch, the shear stress must be equal to the fault shear strength: \( \tau(r,t) = f(\sigma'_0 - \Delta p(r,t)) \), whereas in the re-locked region, \( \tau^+(r,t) \) corresponds to the instantaneous shear stress distribution which depends on the current slip distribution through the quasi-static non-local integral operator of equation (2.1).

The second propagation condition for the post-injection aseismic pulses comes from the analysis of Garagash [39] for seismic and aseismic pulses driven by thermal pressurization. As

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![Figure 4](image-url)
we discuss in detail in the electronic supplementary material, our aseismic pulses are akin to an annular ‘crack’ of inner radius \( B(t) \) and outer radius \( R(t) \), as long as the dislocation density is replaced by the slip rate gradient \( \partial v/\partial r \), and the change of shear stress by the shear stress rate \( \partial \tau/\partial t \). The near-tip asymptotic behaviour of classical cracks [40] is thus valid but in terms of \( \partial \tau/\partial t \) and \( v \). Having this result in mind, we can now invoke Garagash’s healing condition [39], which states that the shear stress rate \( \partial \tau/\partial t \) must be non-singular at the locking front to effectively allow the fault to re-lock. This is because within the slipping patch and, particularly, at the locking front, the shear stress rate must equal the shear strength rate (the plastic consistency condition, see for example, appendix A in [31]), in our case, \( \partial \tau/\partial t = -\tau \partial \Delta p/\partial t \). Since \( \partial \Delta p/\partial t \) is bounded all over the fault plane, it follows that \( \partial \tau/\partial t \) cannot be singular at the locking front.

Garagash obtained an integral equation similar to (4.1) based on the non-singularity of shear stress rate at the locking front (eqn (19) in [39]). Such an integral equation is valid under the assumptions of his study in two-dimensional elasticity for a solitary steady pulse travelling in anti-plane (III) or in-plane (II) mode of sliding. Indeed, if our pulse that is non-steady would be solved in two-dimensional elasticity, a similar expression can be derived for the two symmetric pulses (travelling in opposite directions) that would emerge in that case, from known SIF formulae [41]. To the best of our knowledge, the SIF formulae for an annular crack under axisymmetric loading are unknown (numerical procedures have been proposed to calculate them in mode I [42]). Nevertheless, one can still formalize this second propagation condition based on the near-tip asymptotic behaviour of classical cracks as [38,40]

\[
K_i = 0, \quad \text{with} \quad \frac{\partial \tau(r,t)}{\partial t} \simeq \frac{K_i}{\sqrt{2\pi(B(t) - r)}} \quad \text{when} \quad r \rightarrow B(t)^{-}. \tag{4.2}
\]

\( K_i \) is the so-called ‘stress-rate intensity factor’ [39] with units of (pressure/time) × length\(^{1/2}\). It quantifies the intensity of the leading-order (singular) term of \( \partial \tau/\partial t \) near the locking front. Note that \( K_i \) may be also written in terms of the mode II, \( K_{2,i} \), and mode III, \( K_{3,i} \), stress-rate intensity factors, as \( K^2_i = K^2_{2,i} + K^2_{3,i} \). Indeed, since the two components of the shear stress rate on the fault plane must be non-singular along the locking front, equation (4.2) can be stated in the stronger form, \( K_{2,i} = K_{3,i} = 0 \). Note also that the stress-rate intensity factor is not the same as the SIF rate, at least along the locking front. For the latter, there is no SIF whatsoever, since there is no fracture tip and thus no potential singular term for \( r \) when \( r \rightarrow B(t)^{-} \). Hence, although it could have been tempting to write the healing condition (4.2) as \( dG/dt = 0 \), where \( G \) is the energy release rate, such a statement is not true.

Equations (4.1) and (4.2) are, therefore, the necessary conditions for the propagation of post-injection aseismic slip as an annular pulse. Together, these two equations describe the motion of the rupture front \( R(t) \) and locking front \( B(t) \) that are at quasi-static equilibrium with a given \( \Delta \tau(r,t) \). Yet as already mentioned, we cannot solve these two equations without solving the entire moving boundary value problem. However, they provide additional insights into the mechanics of the problem and, as shown in the electronic supplementary material, define important characteristics of the slip rate and shear stress rate distributions along the fault plane.

We now seek for the conditions that characterize the final arrest of the rupture pulses. Figure 4a shows that as expected at the moment of arrest, the locking front \( B(t) \) catches up the rupture front \( R(t) \). Since in that instant the rupture pulse effectively vanishes, equation (4.2) has no relevance. However, the ‘fracture’ (slipped surface) is still present and, to be at mechanical equilibrium, equation (4.1) must be satisfied. Since the second term of the right-hand side of (4.1) approaches zero at the time of arrest, equation (4.1) takes now the simpler form

\[
\int_{0}^{R_a} \frac{\tau_0 - \tau^*(r,t_a)}{\sqrt{R_a^2 - r^2}} r \, dr = 0, \tag{4.3}
\]

where \( t_a \) is the time of arrest and \( R_a = R(t_a) \) is the corresponding rupture radius at the arrest time (the maximum run-out distance). In the previous equation, \( R_a \) may be equivalently replaced by \( B_a = B(t_a) \). Furthermore, since the rupture after shut-in is driven uniquely by the further increase
of pore pressure that occurs in the region $r > P(t)$, the rupture arrest will be also characterized by the moment at which the pore-pressure back front catches up the rupture front (figure 4a). Therefore, the following conditions must be also met exactly at the time of arrest,

$$R(t_a) = B(t_a) = P(t_a), \quad (4.4)$$

and more importantly, by combining the previous equation with equation (3.1), we obtain an analytic relation between the radius $R_a$ and time $t_a$ of arrest,

$$R_a = \sqrt{4 \pi \alpha t_a \left( \frac{t_a}{t_s} - 1 \right) \ln \left( \frac{t_a}{t_a - t_s} \right)^{1/2}}, \quad (4.5)$$

which in the large-time limit ($t_a \gg t_s$) becomes simply $R_a \approx \sqrt{4 \pi \alpha t_a}$. Equations (4.3) and (4.5) provide a complete system of equations to solve for the time at arrest $t_a$ and the maximum run-out distance $R_a$. However, the shear stress profile left by the arrested rupture $\tau^*(r, t_a)$ in equation (4.3) is unknown analytically. Hence, we must determine at least one of these two quantities numerically.

(d) Arrest time and maximum run-out distance

We now determine numerically the normalized arrest time $t_a/t_s$ and normalized maximum run-out distance $R_a/R_s$ by spanning the relevant parameter space with 40 values of $T$ ranging from $10^{-3}$ to 10, equally spaced in logarithmic scale. The results are presented in figure 5. Marginally pressurized faults ($T \sim 10$) produce slip pulses that arrest almost immediately after the stop of the injection and practically do not grow further than the size of the ruptures at the moment of shut-in ($R_s$), whereas critically stressed faults ($T \ll 1$) host events that can last up to $\approx 10^3$ times the injection time $t_s$ and grow approximately up to 1.8 times the rupture radius at the shut-in time (for the smallest value of $T$ considered). Critically stressed faults are, therefore, the asymptotic regime of more practical interest. We thus focus on determining its behaviour precisely.

For the normalized arrest time, figure 5a shows a clear power law between $t_a/t_s$ and $T$. Using a linear least square regression, we obtain

$$\frac{t_a}{t_s} \approx aT^{-b}, \quad (4.6)$$
with $a = 0.946876$ and $b = 1.084361$. In solving the least-squares problem, we have considered data points satisfying $T \lesssim 0.01$ ($\ll 1$) and obtained an $R^2$ equal to 0.999993.

We can now use the analytic relation between $R_a$ and $t_a$, equation (4.5), to construct a critically stressed asymptotic approximation for the normalized maximum run-out distance using the power-law relation (4.6). In particular, at large arrest times ($t_a \gg t_s$), (4.5) is approximately equal to $R_a/R_s \approx (1/\lambda)\sqrt{t_a/t_s}$. Substituting $\lambda \approx 1/\sqrt{2T}$ (see §4a) and (4.6) into the previous expression, leads to the sought critically stressed approximation,

$$
\frac{R_a}{R_s} \approx \left( \frac{2a}{Tb^{-1}} \right)^{1/2} \quad \text{or} \quad R_a \approx \left( \frac{4a\nu t_s}{Tb} \right)^{1/2}.
$$

In addition, we can substitute equation (4.6) directly into (4.5) to provide an improved asymptotic approximation for the critically stressed regime that is approximately valid over a broader range of values of $T$ (green curve in figure 5b). Note that in figure 5b, we have also plotted the values of $R_a$ not from the numerical solution, but rather by evaluating the analytic relation (4.5) (grey crosses) given the numerical solution of $t_a$. The latter is just to illustrate the exactness of the relation between the radius of arrest and the time of arrest (4.5).

5. Pulse-like non-circular ruptures

In this section, we examine the effect of Poisson’s ratio different than zero on the propagation and ultimate arrest of the post-injection aseismic pulses. For this purpose, we consider a fixed value of $\nu = 1/4$ (Poisson’s solid).

(a) Recall on the self-similar solution before shut-in

Sáez et al. [31] also solved the continuous-injection problem for the case of $\nu \neq 0$. They showed that the rupture front is well-approximated by an elliptical shape that becomes more elongated for increasing values of $\nu$ and decreasing values of $T$: the more critically stressed the fault is, the more elongated the rupture becomes. The aspect ratio of the quasi-elliptical rupture fronts is upper bounded by $1/(1 - \nu)$ in the critically stressed limit ($T \ll 1$), and lower bounded by $(3 - \nu)/(3 - 2\nu)$ in the marginally pressurized limit ($T \sim 10$). Furthermore, the rupture area $A(t)$ evolves simply as $A(t) = 4\pi a(t^2)$ (linear with time), and interestingly it does not depend on Poisson’s ratio $\nu$ (see fig. 6 in [31]). The previous numerical observations led to closed-form approximate expressions for the entire evolution of the quasi-elliptical rupture fronts in the terms of the semi-major $a(t)$ and semi-minor $b(t)$ axes of an ellipse, as $a(t) = R(t)/\sqrt{1 - \nu}$ and $b(t) = R(t)/\sqrt{3 - \nu}$ in the critically stressed regime, and $a(t) = R(t)/\sqrt{3 - \nu}/\sqrt{3 - 2\nu}$ and $b(t) = R(t)/\sqrt{3 - 2\nu}/\sqrt{3 - \nu}$ in the marginally pressurized regime, with $R(t)$ the rupture radius of a circular rupture for the same value of $T$, which is known analytically (see §4a).

(b) Effect of $\nu$ on the propagation of the aseismic pulses

We summarize the main characteristics of the propagation of pulse-like non-circular ruptures in figure 6. This figure is composed by four snapshots of the normalized slip rate distribution for a mildly critically stressed fault with $T = 0.1$. Each snapshot represents a remarkably different propagation phase. The first one, figure 6a, corresponds to a moment right before stopping the injection, specifically at $\tilde{t} = 0.99$, where $\tilde{t} = t/t_s$. At this time, the rupture is still propagating in crack-like mode. For this value of $T$, the quasi-elliptical rupture front outpaces the pore-pressure perturbation front. Note that the spatial coordinates in figure 6 are normalized by the rupture lengthscale $R_s = \lambda\sqrt{4a\nu t_s}$ (radius of a circular rupture with the same value of $T$, see §4a). The second snapshot, figure 6b, displays the rupture after shut-in at approximately 4.75 times the injection duration. In this figure, we can already observe the locking front and thus the propagation of the rupture as an annular pulse. Interestingly, although the rupture front is clearly
Figure 6. Snapshots of normalized slip rate distribution for a non-circular rupture with $T = 0.1$ and $\nu = 0.25$. The spatial coordinates $x$ and $y$ are scaled by the rupture radius at the shut-in time $R_s$ for the same $T$ but $\nu = 0$, whereas dimensionless time $\bar{t} = t/t_s$ as usual. (a) Crack-like propagation right before shut-in ($\bar{t} = 0.99$). (b) Propagation as an annular pulse after shut-in ($\bar{t} = 4.75$). (c) Moment in which the locking and rupture fronts are about to coalesce ($\bar{t} \approx 9.44$). (d) Propagation after coalescence of the fronts as two ‘moon-shaped’ pulses travelling in opposite directions ($\bar{t} = 11.75$). The maximum run-out distance of the rupture $\ell_c$ that will be eventually reached at some later time is depicted. The positions at which the locking, rupture and pore-pressure back fronts intersect each other are indicated with red circles.

elongated along the mode II direction of sliding (same as it is before shut-in), the locking front seems to be slightly elongated along the mode III direction instead.

Figure 6c and 6d show the most remarkable effects of considering a Poisson’s ratio different than zero. Figure 6c corresponds to the instant at which the locking front is about to coalesce with the rupture front along the mode III direction of sliding. Note that at this time of coalescence $t_c$, approximately equal to 9.44 times the injection duration in this case, the pore-pressure back front is also intersecting both the rupture and locking fronts. Let us denote the maximum run-out distance of the rupture $\ell_c$ that will be eventually reached at some later time is depicted. The positions at which the locking, rupture and pore-pressure back fronts intersect each other are indicated with red circles.

The previous observation, which was somewhat expected from the analysis of circular ruptures (see equation (4.4)), means that there is a unique relation between $\ell_c$ and $t_c$, in the form $\ell_c = P(t_c)$, where $P(t)$ is the instantaneous radius of the pore-pressure back front known analytically from equation (3.1).
Moreover, after the coalescence of the fronts, figure 6d displays that the original annular pulse is split into two symmetric ‘moon-shaped’ pulses that travel in opposite directions. The moving positions at which the locking and rupture fronts are intersecting each other are shown with red circles in figure 6d. These positions are again such that the pore-pressure back front is located exactly at the same place. The condition (4.4) for circular ruptures at the time of arrest is thus still valid for non-circular ruptures but now at any time \( t_c \leq t \leq t_a \). Finally, the instantaneous rupture area of each moon-shaped pulse decreases continuously with time and will eventually collapse into a point located along the \( x \)-axis at some distance \( \ell_a \) as depicted in figure 6d. \( \ell_a \) corresponds to the maximum run-out distance of the entire rupture and, similarly to the case of circular ruptures, equation (4.5) will now take the form \( \ell_a = P(t_a) \), where \( t_a \) is the time of arrest.

(c) Effect of \( \nu \) on the arrest time and maximum run-out distance

We calculate the time of arrest \( t_a \) and maximum run-out distance of the rupture \( \ell_a \), as well as the time of coalescence \( t_c \) and maximum run-out distance in the mode III direction \( \ell_c \), for 10 values of the stress-injection parameter \( T \) ranging from \( 10^{-3} \) to \( \approx 0.59 \). This range allows us to span the most relevant part of the parameter space associated with critically stressed faults. Figure 7a summarizes the results for the normalized arrest time \( t_a/t_s \) and normalized time of coalescence \( t_c/t_s \), including the results for the arrest time of circular ruptures \( t_a^{\nu=0} \). We observe that the coalescence of the fronts occurs at earlier times than the arrest of circular ruptures. Moreover, Poisson’s ratio \( \nu \neq 0 \) has the clear effect of delaying the arrest of the aseismic pulses with regard to the circular case. To better quantify this delayed arrest, we plot the same results in the inset but normalizing the arrest and coalescence times by the arrest time of circular ruptures. It can be seen that the more critically stressed the fault is, the longer it takes for non-circular ruptures to arrest compared with circular ones. Furthermore, the ratio \( t_a/t_a^{\nu=0} \) seems to approach an asymptotic value of \( \approx 1.21 \) in the critically stressed limit (when \( T \to 0 \)).

Figure 7b shows the results for the normalized maximum run-out distance of the rupture \( \ell_a/R_a \) and normalized maximum run-out distance in the mode III direction \( \ell_c/R_s \). In accordance with the results for \( t_c \) and \( t_a \), the distance \( \ell_c \) is lower than the arrest radius of circular ruptures \( R_a \) for the same value of \( T \), whereas the maximum run-out distance of non-circular ruptures \( \ell_a \) is greater than the maximum run-out distance of its circular counterpart \( R_a \). In addition, the relations \( \ell_c = P(t_c) \) and \( \ell_a = P(t_a) \) discussed in the previous section are shown to be in good agreement (grey crosses in figure 7b) with the numerical results up to numerical discretization errors. Moreover,
the inset shows the run-out distances $\ell_a$ and $\ell_c$ but normalized by the arrested radius of circular ruptures $R_a$. The maximum run-out distance $\ell_a$ seems to approach an asymptotic value in this case of $\approx 1.1R_a$ in the limit of critically stressed faults.

6. Discussion

(a) Critically stressed regime versus marginally pressurized regime

Figures 5 and 7 summarize one of the most important results of this article. Marginally pressurized faults ($T \sim 10$) produce aseismic pulses that arrest almost immediately after shut-in, whereas critically stressed faults ($T \ll 1$) are predicted to host ruptures that propagate for several orders of magnitude the injection duration and grow up to approximately twice the size of the ruptures at the moment of shut-in (for the smallest value of $T$ considered). Critically stressed faults are therefore the relevant propagation regime that is able to sustain seismicity after shut-in. Under this regime, the long-lived aseismic pulses provide a longer exposure of the surrounding rock mass to continuous stressing due to aseismic slip and, at the same time, their longer run-out distances perturb a larger volume of rock mass up to $\approx 2^3 = 8$ times larger than the volume affected at the moment of shut-in, therefore increasing the likelihood of triggering earthquakes during the post-injection stage compared with the marginally pressurized case.

To illustrate under what conditions these two regimes can occur, let us consider some characteristic values of a fault undergoing fluid injection. Consider that water is injected into a fault zone at a constant volume rate $Q \sim 30 \text{ [l s}^{-1}]$, which is typical of hydro-shearing treatments of deep geothermal reservoirs at around 3–4 km depth [11]. Assume a fluid dynamic viscosity $\eta \sim 10^{-3}$ [Pa s]. The fault hydraulic transmissivity can vary over several orders of magnitude. Consider, for instance, a plausible value of $kw \sim 10^{-12}$ m$^3$ [43]. This yields a characteristic pressure at the fluid source of $\Delta p_c \approx 30$ [MPa] and an intensity of the injection $\Delta p_s \approx 2.4$ [MPa] (equation (2.3)). Let us consider for the initial effective normal stress a characteristic value of $\sigma'_0 \sim 60$ [MPa], that is somewhat consistent with a depth of 3–4 km under gravitational loading and hydrostatic conditions. Assuming a friction coefficient $f = 0.6$, the initial fault shear strength is $f\sigma'_0 = 36$ [MPa], while the intensity of the strength reduction due to fluid injection is $f\Delta p_s \approx 1.4$ [MPa]. The amount of initial shear stress $\tau_0$ acting on the fault will finally determine the regime of the fault response in this example. Consider first a fault that is close to failure, say $\tau_0 = 35.9$ [MPa] (0.1 [MPa] to failure). This leads to a small value of the stress-injection parameter $T \approx 0.07$ (equation (2.5)), and therefore to a fault responding in the so-called critically stressed regime ($T \ll 1$). Before shut-in, the fluid-induced aseismic slip front in this example is predicted to outpace the pore pressure perturbation front by a constant factor $\lambda \approx 1/\sqrt{2T} \approx 2.7$ (§4a), for the case of circular ruptures. Consider that the injection was conducted during a period of $t_s = 1$ day. Then, after shut-in, the post-injection aseismic pulse is expected to propagate for $t_a - t_s \approx 16$ days (equation (4.6)) and reach a maximum run-out distance of $\approx 1.54$ times the run-out distance at the moment of shut-in (equation (4.7)). To close this dimensional example, let us assume that the fault has a hydraulic diffusivity of, say, $\alpha = 0.01$ m$^2$ s$^{-1}$. The radius of the rupture at the moment of shut-in is $R_c = \lambda\sqrt{4\alpha t_s} \approx 159$ m (see again §4a), and the maximum run-out distance of post-injection aseismic slip will be $R_a \approx 1.54 \times 159$ m $\approx 245$ m. If the hydraulic diffusivity were 10 times lower, $R_c$ and $R_a$ would decrease by approximately 30%.

Consider the same example of the previous paragraph, but now with a fault that is further away from failure. Assume, for instance, a lower value of the initial shear stress $\tau_0 = 25$ [MPa] (11 [MPa] to failure). The stress-injection parameter for this case becomes $T \approx 7.9$, well within the marginally pressurized regime ($T \sim 10$). Prior to shut-in, the slip front is predicted to lag the pore pressure perturbation front by a factor $\lambda \approx (1/2) \exp[(2 - \gamma - T)/2] \approx 0.02$ (§4a). Moreover, upon the stop of the injection, the rupture is expected to arrest almost immediately (figure 5), such that $t_a \approx t_s \approx 1$ [day] and $R_a \approx R_s = \lambda\sqrt{4\alpha t_s} \approx 1.2$ m. In the two previous examples, the end-member regimes were achieved by changing only the initial shear stress acting on the fault or, equivalently, the distance to failure. This highlights the importance of the pre-injection stress state
in determining the response of the fault. However, there is another relevant quantity that may equally change the fault response regime, namely, the intensity of the injection $\Delta p_s$. So far, we have assumed hydraulic parameters resulting in $\Delta p_s \approx 2.4$ [MPa] (and $\Delta p_c = 30$ [MPa]). Imagine now that the fault is still relatively close to failure as in the first example ($t_0 = 35.9$ [MPa]) but a much smaller amount of fluid is being injected, say $Q = 1$ [l s$^{-1}$]. This is unlikely the case of a hydraulic stimulation but could instead occur in the case of a natural source of fluids occurring at the same depth. The intensity of the injection (equation (2.3)) is now $\Delta p_s \approx 0.08$ [MPa] (with $\Delta p_c = 1$ [MPa]), and the corresponding stress-injection parameter $T \approx 2.1$. This will lead equivalently to a rupture that lags the fluid pressure front during continuous injection ($\lambda \approx 0.37$) and that arrests almost immediately after shut-in. The latter, despite the fact that the fault was ‘just’ 0.1 [MPa] away from failure under ambient conditions. On the other hand, a more intense injection than the one with $Q = 30$ [l s$^{-1}$] will act in the direction of moving the fault response towards the critically stressed regime (decreasing values of $T$). Those injections might however sometimes open the fault (if $\Delta p_c \gtrsim \sigma_0^c$), a mechanism that is not accounted for in this work.

(b) Conceptual model of post-injection seismicity

In the next sections, we use our model to investigate to what extent post-injection aseismic slip can be considered as a mechanism for the delayed triggering of seismicity. However, before going into the details of the discussion, it seems convenient to establish first a few general concepts. First, we note that any model that aims at explaining observations of post-injection seismicity must produce spatio-temporal changes of pore pressure or solid stresses after shut-in. Our model produces both of them. In fact, the pore pressure changes alone are essentially equal to the ones already considered by Parotidis et al. [44] to explain the so-called back front of seismicity that is sometimes observed after the termination of fluid injections. Note that Parotidis et al.’s model assumes the increase of pore pressure as the only triggering mechanism of seismicity. We, instead, elaborate on a mechanism based on the combined effect of transient changes in solid stresses due to aseismic slip and pore pressure after shut-in. For the sake of simplicity, we carry out all discussions in terms of circular ruptures only. Rupture non-circularity does not modify the order of magnitude of the results.

Let us define now conceptually what seismicity means in the context of our model. We consider that a single fault plane undergoes some transient fluid injection that may be approximated as a pulse of injection rate (figure 1c). It is assumed that as a result of the injection, the fault slides predominantly aseismically. Seismic events are thought to be triggered on unstable patches of the same fault plane (due to, for instance, heterogeneities in rock frictional properties) or other pre-existing discontinuities in the surroundings of the slowly propagating rupture. The former and latter events are commonly denominated as on-fault and off-fault seismicity, respectively, a terminology that we use later on. Note that an important underlying assumption of this conceptual model is that the unstable patches of the aseismic fault do not represent a sufficiently large area to change its predominantly stable mode of sliding to a large dynamic rupture.

We define a Mohr–Coulomb failure function in the form $F(x; t; \mathbf{n}) = |\tau(x; t; \mathbf{n}) - f(\sigma(x; t; \mathbf{n}) - p(x; t))|$, where $\tau$ and $\sigma$ are the shear and total normal stresses acting on a certain unstable patch with unit normal vector $\mathbf{n}$ at a given position $x$ and time $t$, $p$ is the pore-fluid pressure and $f$ is a constant (static) friction coefficient. The failure function is such that $F \leq 0$ always. The inequality $F < 0$ holds when no frictional failure occurs, whereas the equality $F = 0$ is valid whenever frictional sliding is activated. The initial stress state is assumed such that $F(x, t < 0; \mathbf{n}) < 0$ at all pre-existing discontinuities before injection starts. Once injection begins, the failure function may approach zero by changes in the solid stresses $\tau$ and $\sigma$ and the pore pressure $p$. Upon failure of a certain patch, slip may evolve seismically only if the friction coefficient is allowed to weaken. The time-dependent nucleation of instabilities and, moreover, the explicit modelling of seismicity, are out of the scope of our study. Instead, we consider a conceptual model in which any positive change of the failure function at a certain position and time may lead to frictional sliding and, subsequently, to the possibility of triggering an instability.
We introduce two types of changes of the failure function that will prove to be useful for our analysis. The first one, a static change $\Delta F$ between two times, and the second one, an instantaneous change $\dot{F}$. Their mathematical expressions are

\[
\Delta F(x; n) = \Delta \tau(x; n) - f(\Delta \sigma(x; n) - \Delta p(x)), \tag{6.1}
\]

\[
\dot{F}(x; t; n) = \dot{\tau}(x; t; n)\text{sgn}(\tau(x; t; n)) - f(\dot{\sigma}(x; t; n) - \dot{p}(x; t)). \tag{6.2}
\]

$\Delta F$ is widely known in seismology as the Coulomb stress change [45], a terminology we adopt hereafter. Note that in equation (6.1), we assume that the change of shear stress $\Delta \tau$ is positive if it occurs in the same direction as the shear stress at the selected initial time. On the other hand, in equation (6.2), $\text{sgn}(\cdot)$ is the sign function and the dot over the scalar fields represents a partial derivative in time. $\dot{F}$ is commonly denominated as the Coulomb stressing rate, a quantity that is sometimes correlated to seismicity rates [46]. Finally, we highlight that positive contributions to failure $\Delta F > 0$ ($\dot{F} > 0$) are given by both positive changes of pore pressure $\Delta p > 0$ ($\dot{p} > 0$) and negative changes of total normal stress $\Delta \sigma < 0$ ($\dot{\sigma} < 0$). The effect of the shear stress is however less straightforward and knowledge about the direction of $\tau$ and thus about the absolute state of stress is required to evaluate its relative contribution to $\Delta F (\dot{F})$.

(c) On-fault seismicity

(i) Theoretical considerations

On fault, both aseismic-slip stress transfer and pore pressure changes are active during the post-injection stage. We use the Coulomb stressing rate $\dot{F}$ as an indicator of the regions where seismicity is expected. Let us first note that due to the planarity of the fault, the total normal stress rate $\dot{\sigma}$ is zero. Hence, the only component of stress rate that is active along the fault plane is the shear one $\dot{\tau}$. Moreover, since the absolute state of stress of the fault is known, we can assume by convention that $\tau > 0$ and consequently that positive rates of shear stress are the ones that contribute to frictional failure. Along the fault plane, equation (6.2) thus further simplifies to

\[
\dot{F}(x; t) = \dot{\tau}(x; t) + f\dot{p}(x; t). \tag{6.3}
\]

We first analyse the stress-transfer effect, $\dot{\tau}$ in equation (6.3). With reference to the characteristics of slip rate and shear stress rate that we describe in detail in the electronic supplementary material, it can be readily shown that the shear stress rate acting on possible unstable patches is positive everywhere along the fault plane. For the region outside of the pulse, this can be derived after differentiating equation (2.1) with respect to time and noting that seismicity should be triggered predominantly in the proximity of both the locking and rupture fronts, as a consequence of the amplification of shear stress rate concentrated near the tips of the aseismic pulses (see the electronic supplementary material). Regarding the inside of the pulse, the value of $\dot{\tau}$ in our model, which is simply equal to the fault shear strength rate, is somewhat irrelevant. The actual stress-transfer effect comes rather from considering that unstable patches may be effectively locked and surrounded by aseismic slip. The latter would increasingly load the locked patches in shear until eventually a dynamic event could be triggered.

With regard to the pore-pressure effect, $f\dot{p}$ in equation (6.3), its contribution to seismicity can be understood readily from the mere definition of the pore-pressure back front. At distances $r > P(t)$, where $P(t)$ is the radius of the front, the pore pressure rate is positive. Therefore, the region ahead of the pore-pressure back front is where fluid-induced instabilities are expected, a result that was already introduced by Parotidis et al. [44]. Conversely, at distances $r < P(t)$, the opposite holds $\dot{p} < 0$, and thus the triggering of instabilities is here inhibited.

We are now ready to superimpose the effects of $\dot{\tau}$ and $f\dot{p}$ to notably determine the sign of $\dot{F}$. This is schematized in figure 8a, where four different regions in which the results of the superposition operate differently, are delineated by the three relevant fronts of the problem, namely, the rupture front $R(t)$, the pore-pressure back front $P(t)$ and the locking front $B(t)$. In region $R1$ for distances ahead of the pulse $r > R(t)$, the Coulomb stressing rate $\dot{F}$ is always positive.
In this region, seismicity is expected to be triggered by the contribution of both a positive shear stress rate $\dot{\tau} > 0$ and a positive pore pressure rate $\dot{\Delta p} > 0$. Let us further analyse the magnitude and distribution of $\dot{F}$ here. This is shown in figure 8b, where the spatial profile of normalized Coulomb stressing rate $\dot{F}$ is plotted at various dimensionless times for an exemplifying case with $T = 0.1$. Dimensionless times in the legend box are in the format $\bar{t} = (t - t_s)/(t_\alpha - t_s)$. (c) Application of our model to the 1993 hydraulic stimulation of the GPK1 well at the Soultz-sous-Forêts geothermal site in France [47]. The migration of seismicity from the injection point is well represented by our model based on realistic field parameters as described in the main text. *The injection point has been taken at 2925 m depth as in [44].
whereas at intermediate times and notably at large times, the pore pressure rate becomes the most dominant quantity ($\Delta p \gg \dot{\tau}$) from some distance ahead of the rupture front that gets increasingly closer to the slip front with time. However, at the rupture front itself and very close to it, the shear stress rate will always dominate the magnitude of $\dot{\tilde{F}}$ due to the square-root singularity of $\dot{\tau}$ discussed in the electronic supplementary material.

Let us now consider the regions R2 and R3 that compose the slipping patch. Here, seismic events are promoted by the increase of shear stress $\dot{\tau} > 0$ acting on locked unstable patches that are loaded by surrounding aseismic slip, a mechanism that is denoted by $\dot{\delta} > 0$ in figure 8a. Moreover, in region R2 ahead of the pore pressure back front $r > P(t)$, the pore pressure rate is also positive, meaning that the Coulomb stressing rate $\dot{F}$ is strictly positive too. The triggering of instabilities in region R2 is thus expected. Conversely, in the region R3 behind the pore pressure back front $r < P(t)$, the pore pressure rate is negative. As such, its effect counterbalances the positive contribution due to aseismic slip ($\dot{\delta} > 0$). Whether $\dot{F}$ is positive or negative in this region is not possible to know from our calculations, because we do not explicitly model the effect of any potential locked patch on the shear loading. Nevertheless, by assuming that the shear loading $\dot{\tau}$ is some unknown but positive quantity, the solitary effect of the pore pressure changes provides a lower bound for $\dot{F}$ within the slipping patch. Even more, the pore-pressure effect $f \Delta p$ is a lower bound of $\dot{F}$ over the entire fault plane since as already discussed, the stress-transfer effect $\dot{\tau}$ is positive everywhere. This can be clearly observed in figure 8b when looking at the regions behind (R4) and ahead (R1) of the rupture pulse.

Finally, in the region R4 behind the slip pulse $r < B(t)$, seismicity is promoted by the amplification of the shear stress rate near the locking front (see the electronic supplementary material), albeit such amplification is going to be neutralized to some extent by the negative pore pressure rate operating in this region. In fact, figure 8b shows that the negative pore pressure rate appears to completely neutralize the stress rate amplification behind the locking front for the particular case shown in this figure. Furthermore, a general proof of this numerical observation can be developed as follows. First, we recall that $\dot{F} = 0$ within the slipping patch and, particularly, when $r \to B(t)^+$. Note that we have just referred to $\dot{F}$ with the same notation as the Coulomb stressing rate acting on unstable locked patches (equation (6.3)). However, we are rather referring by $\dot{F}$ in this particular case to the rate of the failure function on the portion of the slipping patch that is aseismically sliding. Since $\dot{\tau}$ and $\Delta p$ are both continuous at the locking front (the former due to the healing condition discussed in §4c), the previous limit is also valid when approaching $B(t)$ from the outside of the pulse: $\dot{F} = 0$ when $r \to B(t)^-$. Because both $\dot{\tau}$ and $\Delta p$ decrease monotonically with increasing distances measured from the locking front and towards the injection point $r = 0$, we conclude that $\dot{F} < 0$ for all points located behind the locking front, $r < B(t)$. We highlight that this statement is valid at any time after shut-in and for any value of $T$. More importantly, it allows us to establish unequivocally that seismicity is not expected to occur behind the locking front, despite the positive increase of shear stress operating behind the rupture pulse.

The final region along the slip plane where seismicity is theoretically expected after shut-in ($\dot{F} > 0$) is highlighted in figure 8a by the white points that represent the location and time of possible seismic events. Note that in this figure, we intentionally draw increasingly less events as time goes by. This is due to the Coulomb stressing rate decreasing by many orders of magnitude at intermediate and large times compared with early times (figure 8b) and, as such, the seismicity rate is expected to decrease in a similar manner. Moreover, the lower limit of the seismically active region resembles the back front of seismicity proposed by Parotidis et al. [44]. Indeed, Parotidis et al.’s back front is equal to the pore-pressure back front $P(t)$ of this work (eqn 5 in [44], equation (3.1) here). Whether some instabilities can be triggered or not in region R3 seems quite irrelevant from an observational point of view, since such small spatial differences between the locking front and the pore pressure back front are very likely indistinguishable in seismic catalogues and, in many cases, possibly even smaller than their location errors. Therefore, the back front of on-fault seismicity of post-injection aseismic slip is for practical purposes equal to the one of Parotidis et al. [44].
Finally, unlike the back front of seismicity that corresponds to a sharp front ($\Delta \dot{p} = 0$), the upper limit of the seismically active region or seismicity front is not sharply defined. This is because the Coulomb stressing rate $\dot{F}$ vanishes theoretically only at infinity. The definition of a seismicity front thus requires some degree of arbitrariness likely associated with a chosen threshold to trigger instabilities. In our case, to be somehow consistent with the definition of the back front in terms of rates, one possible choice is to define the front of seismicity as a small percentage of the Coulomb stressing rate undergone in the proximity of the rupture front. Since the scale $\dot{F}_s = f \Delta p_s / t_s$ represents such a quantity at least at early times, we calculate and display curves associated with 1% and 10% of $\dot{F}_s$ in figure 8a. We recognize that other choices are possible, notably in terms of Coulomb stress rather than in terms of rates.

(ii) The 1993 hydraulic stimulation at the Soultz geothermal site, France

With the previous theoretical considerations in mind, we aim now at testing our model against field observations of post-injection seismicity. We choose the well-documented case of the 1993 hydraulic stimulation at the Soultz geothermal site in France, where direct evidence of significant aseismic slip induced by the fluid injection exists [48]. Our focus is the first injection test of the GPK1 well, where 25 000 m$^3$ of water were injected into granite over a period of about 15 days. The injection was conducted with a step incremental flow rate that reached a maximum of 36 [l s$^{-1}$] through a 550 m open-hole section located at depths between 2850 m and 3400 m [48]. Figure 8c shows the spatio-temporal evolution of seismicity of the more than 10 000 events recorded during the injection test, before and after shut-in [47].

Previous studies have suggested that this injection test stimulated a network of fractures [49], whereas our model considers the stimulation of only one single planar fault in three dimensions. Nevertheless, recent simulations of injection-induced aseismic slip in a two-dimensional discrete fracture network (DFN) have shown that the same patterns predicted by a single fracture in two dimensions emerge collectively for the DFN [50]. This is notably the case of a set of fractures operating under critically stressed conditions [50]. Since the fractures intersecting the open-hole section of the borehole in this test are known to be critically stressed [51], we make the seemingly reasonable assumption that the response of the fracture network can be approximated by an equivalent single planar fault in three dimensions.

To compute the evolution of the relevant fronts associated with the signature of seismicity predicted by our model, we must estimate two quantities independently: the fault hydraulic diffusivity $\alpha$ and the stress-injection parameter $T$. Thanks to the estimates of aseismic slip based on borehole-wall deformation by Cornet et al. [48], we may estimate the fault hydraulic diffusivity based on the formula of accrued slip at the injection point in the critically stressed regime, $\delta(t = 0, t = t_s) \approx 3.5(f \Delta p_s / \mu) \sqrt{4 \alpha t_s}$ (equation (2.6) here, with the pre-factor in equation 28 of [31]), where $t_s = 370$ h is the shut-in time. Considering a shear modulus $\mu = 20$ [GPa] [48] and a constant friction coefficient $f = 0.6$, we just need to estimate $\Delta p_s$ (equation (2.3)). For the latter, we approximate the injection history as a pulse of injection characterized by the same duration of 370 h and a constant injection rate that equals the actual volume of injected fluid, which yields $Q \approx 19$ [l s$^{-1}$]. The hydraulic transmissivity $kw$ has been estimated by others at the highest pressures of the test in approximately $1.7 \times 10^{-12}$ [m$^3$] [43]. Assuming a water dynamic viscosity of $\eta = 8.9 \times 10^{-4}$ [Pa s], we can finally calculate the pressure intensity $\Delta p_s \approx 0.8$ [MPa] and the characteristic overpressure at the fluid source $\Delta p_c \approx 10$ [MPa]. The latter is indeed very close to the actual downhole overpressure measured at 2850 m depth, equal to 9.1 [MPa] [48]. Finally, based on the estimates by Cornet et al. [48] who reported fracture slip along the open-hole section up to 4.7 cm, we estimate a fault hydraulic diffusivity of $\alpha \approx 0.06$ m$^2$ s$^{-1}$. This value of $\alpha$ is of the same order of magnitude as the one estimated through a different method by others [44].

With this estimate of diffusivity, we compute the evolution of the pore pressure back front after shut-in (equation (3.1)), and the evolution of the overpressure front $L(t) = \sqrt{4 \alpha t}$ before shut-in, both displayed in figure 8c. The next step is to compute the evolution of the rupture and locking fronts, for which we must estimate the stress-injection parameter $T$ (equation (2.5)). From...
the previous analysis of determining $\alpha$, we already know that $f \Delta p_s \approx 0.48$ [MPa], so that we just need to estimate the initial distance to failure, $f_{\alpha} - \tau_0$. This is perhaps the most difficult quantity to estimate in our model. Although in this case we do know that the fault is critically stressed [51], the value of $T$ in the critically stressed regime is very sensitive to changes of the order of tenths or hundredths of one megapascal, which is quite challenging to constrain confidently. It seems then more reasonable to assume a priori that the amplification of shear stress near the slip front contributes to some significant extent to drive the seismicity front before shut-in—since the fault is critically stressed and the overpressure front slightly lags the seismicity front—and consequently determine the amplification factor $\lambda$ that explains well the evolution of seismicity. We find $\lambda = 1.45$ to explain well the data before shut-in (see figure 8c). In the previous estimate, we considered the fact that some seismicity is expected to be triggered ahead of the slip front and thus the rupture front must lag the seismicity front to some extent. Finally, this value of $\lambda$ yields a value of $T \approx 0.279$ (§4a), which in turn gives a distance to failure of approximately 0.1 [MPa]. The latter is within the range of initial distances to failure of fractures intersecting the open-hole section as estimated by others [51].

The corresponding rupture and locking fronts resulting from $T \approx 0.279$ are shown in figure 8c. We observe that our model seems capable of explaining relatively well the migration of seismicity during this episode of combined fluid flow and aseismic slip, both before and after shut-in. Our hydro-mechanical model enriches previous analyses of the migration of seismicity [44] by considering the previously overlooked effect of stress transfer due to aseismic slip, which was previously suggested to be important in this field case [52] but never tested likely due to the lack of physical models. Notably, the theoretical signature of post-injection seismicity (figure 8a) seems to be present in the seismicity cloud. Moreover, the arrest time of the aseismic pulse for this value of $T$ is predicted as $t_a \approx 3.78 \times 3 = 58$ days (equation (4.6)), which is $\approx 43$ days after shut-in. Seismicity was however recorded only within the first 5 days upon the stop of the injection, which is $t_{\text{stop}} \approx 0.12$ of normalized time as defined in figure 8a. This seems again consistent with our theoretical reasoning that most of the seismicity should be expected at early times after shut-in, when the Coulomb stressing rate is the highest in the post-injection problem (figure 8b).

(d) Off-fault seismicity

Seismicity can be triggered not only on unstable patches laying along the aseismically sliding fault, but also on unstable patches present in other pre-existing discontinuities nearby the propagating slow rupture. Because fluid flow is assumed to occur only within the permeable fault zone that hosts the reactivated slip plane, off-fault seismicity can be triggered uniquely by the mechanism of stress transfer due to aseismic slip in our model. Equations (6.1) and (6.2) thus take the following forms that exclude the null pore pressure changes, $\Delta F(x; n) = \Delta \tau(x; n) - f \Delta \sigma(x; n)$ and $\tilde{F}(x, t; n) = \tilde{\tau}(x, t; n) \text{sgn}(\tau(x, t; n)) - f \tilde{\sigma}(x, t; n)$, respectively. Evaluating the previous expressions requires knowledge of the normal vectors $n$ of pre-existing discontinuities and the absolute state of stress, both of which are obviously site-specific and always partially uncertain. Given the site-specific nature of the required information, we address the challenge of exploring the off-fault triggering mechanism by analysing a specific field example. Moreover, in the electronic supplementary material, we discuss some further implications of our model such as the Coulomb stressing rate in the vicinity of the reactivated fault and the off-fault counterpart of the seismicity signature shown in figure 8a.

(i) The 2013 hydraulic stimulation at the Rittershoffen geothermal site, France

We focus on the 2013 hydraulic stimulation of the GRT-1 well in the crystalline basement of the Rittershoffen geothermal field in France, at a depth of about 2 km [53,54]. This stimulation lasted for about 1 day and was accompanied by swarm-like seismicity that illuminated a single planar structure that was reactivated in shear due to the fluid injection [53]. Our model of a single planar fault in three dimensions seems therefore a good approximation for this field case. Upon shut-in,
seismicity stopped immediately. However, after 4 days in which no seismic activity was detected, a short-lived second swarm began and developed along a nearby sub-parallel structure likely part of an en-echelon fault system [53]. Several hypotheses were discussed by Lengliné et al. [53] to explain what possibly led the second fault to failure as well as the 4-days delay of this second swarm. The favoured hypothesis for the failure of the second fault was in fact the stress transfer due to injection-induced aseismic slip associated with the first fault. However, the authors discarded post-injection aseismic slip as a mechanism for explaining the delayed triggering of the second swarm. They argued that slip on the first fault would be difficult to promote after shut-in due to the fault pressure dropping quickly to zero, thus moving the fault interface away from failure [53]. In light of our results, this statement is valid only locally at the injection point, where the fault interface re-locks immediately upon the stop of the injection. However, as we have seen, pore pressure keeps increasing away from the injector after shut-in, which drives the propagation of a pulse-like frictional rupture that further accumulates slip on the fault. Hence, injection-induced aseismic slip can potentially explain as a unique mechanism both the failure of the second fault and the delayed triggering of the swarm of this particular case for which the reactivated fault is indeed thought to be critically stressed [53].

Again, as in the 1993 Soultz case, we want to estimate the fault hydraulic diffusivity $\alpha$ and the stress-injection parameter $T$. In the absence of reliable estimates of both aseismic slip and hydraulic diffusivity, we assume for the purpose of this illustration a characteristic value of $\alpha = 0.05 \text{ m}^2 \text{s}^{-1}$ for the reactivated fault. This is of the order of magnitude of diffusivities estimated from other fluid injections in nearby reservoirs [44], as well as in this same geothermal field via pressure transient analysis [54]. Yet the latter must be considered with care since the estimates are based on production tests that are strongly influenced by near-well processes [54]. To estimate the stress-injection parameter $T$, we follow a similar approach to the one used in the previous section for the 1993 Soultz case. We assume the aseismic slip front to play an important role in explaining the migration of seismicity, which consists in this case of more than 1300 events [53]. Considering a rupture radius at the moment of shut-in $R_s = 250 \text{ m}$ (see fig. 9 in [53]), we obtain from $R_s = \lambda \sqrt{4 \alpha t_s}$ ($\S$A4) with $t_s = 19 \text{ h}$ in this case, an amplification factor $\lambda \approx 2.14$, which yields a stress-injection parameter $T \approx 0.117$.

This value of $T$ gives an arrest time of aseismic slip $t_a \approx 9.70 \times t_s \approx 184 \text{ h}$ (equation (4.6)), which is equivalent to approximately 7 days after the stop of the injection. The latter is indeed longer than the 4-days delay of the second swarm. Yet the relatively short time between the estimated arrest time and the occurrence of the second swarm suggests that most of the post-injection stress transfer from the first to the second fault must have occurred shortly after stopping the injection. To better quantify this, we calculate the Coulomb stress change associated with the reactivation of the main fault in our model. The Coulomb stress calculations are based on the following combination of parameters: $Q = 38 \text{ [l s}^{-1}], \eta = 8.9 \times 10^{-4} \text{ [Pa s}], kw = 10^{-11} \text{ [m}^3], \mu = 20 \text{ [GPa]}, f = 0.6, \sigma_0' = 10 \text{ [MPa]}$ and $\tau_0 = 5.981 \text{ [MPa]}$. The constant injection rate $Q$ is obtained by equating the injected volume of fluid of the actual injection history considering the same injection duration. The fault hydraulic transmissivity $kw$ is calculated such that the characteristic overpressure at the fluid source $\Delta p_f$ (equation (2.3)) approximately matches the maximum downhole overpressure measured at a depth of 1920 m during the injection, approximately equal to 3 [MPa] [54]. Finally, the initial stress state ($\sigma_0'$ and $\tau_0$) is chosen to be consistent with the value of $T$, the injection intensity $\Delta p_s$, and the partially constrained stress state in the zone [53].

Following Lengliné et al. [53], we further consider that the fluid injection induced pure left-lateral aseismic slip on a strike-slip fault oriented N25$^\circ$ E and dipping 70$^\circ$ W. As the second swarm occurred over a fault with variable strike, we focus for the purpose of our illustrative example on the northern branch of the second fault only. We thus consider a receiver fault oriented N55$^\circ$ E with the same dip angle as the main fault [53] (figure 9a). We calculate the slip distribution over the reactivated fault at three times: right at the moment of shut-in, 1 day after shut-in, and 4 days after shut-in, as displayed in figure 9b. In this figure, we can observe that 1 day after stopping the injection most of the post-injection aseismic slip accumulated at the time the second swarm began, has already taken place. The Coulomb stress change associated with the foregoing
Figure 9. Application example of our model to the 2013 hydraulic stimulation of the GRT-1 well at the Rittershoffen geothermal site in France. (a) Coulomb stress change $\Delta F$ due to injection-induced aseismic slip on the reactivated fault for discontinuities oriented as the receiver fault. The plane $x'-y'$ is perpendicular to the reactivated fault and crosses at its origin the injection point at a depth of about 2370 m. Yellow and red circles correspond to the injection point and the nearest point of the receiver fault to the injection, respectively. (b) Slip distributions over the reactivated fault at different times after shut-in. (c) Coulomb stress change along the receiver fault after shut-in. The distance along the receiver fault is measured from the red circle indicated in (a) and in the direction of the black arrow.

slip distributions are shown in the top and bottom panels of figure 9a, at the shut-in time and 4 days after shut-in (when the second swarm started), respectively. Our Coulomb stress analysis indicates that the location of the second swarm is in a region of positive Coulomb stress change, which is consistent with the calculations in [53]. Note that our distribution of Coulomb stress change is different to the one in [53] since the slip distribution of our model peaked around the injection point and is not uniform.

Finally, figure 9c displays the spatio-temporal evolution of the Coulomb stress change along the receiver fault at the same three times as before, with the origin at the red circle indicated in figure 9a. Note that the magnitude of the Coulomb stress change is about 2 kPa. This value is quite low and suggests that if aseismic-slip stress transfer is responsible for the delayed triggering of the second swarm, such a fault must be very critically stressed under the assumptions of our example. Given that pore pressure diffusion can sometimes weaken unstable areas even off fault, we can postulate a hydraulic connection between the two faults, possibly created by a wing crack in a step-like en-echelon fault system, and estimate the resulting overpressure 4 days after shut-in. By assuming $\alpha = 0.05 \text{ m}^2 \text{s}^{-1}$ still as a representative value of diffusivity, the overpressure at the nearest point (red circle in figure 9a) of the receiver fault with regard to the injection point (yellow circle in figure 9) is approximately 6 kPa (equation (2.4)). The latter is of the same order of magnitude as the Coulomb stress change due to aseismic slip, suggesting that both mechanisms may be equally important in our example. Note that 4 days after the stop of the injection, the increment of Coulomb stress with regard to the moment of shut-in, is just about 0.5 kPa (figure 9c). Hence, most of the post-injection increment has already happened within the first day. Although our analysis has oversimplified many aspects of this field case, our physical model seems to have the potential to reproduce field observations of post-injection off-fault seismicity, as illustrated by this application example. However, an in-depth analysis would be needed to fully clarify the role of post-injection aseismic slip in this particular field case.
(e) Possible evidence for post-injection aseismic slip and other relevant field cases

In addition to the foregoing field cases, there are apparently a few other cases of geo-energy projects where observations of seismicity after shut-in suggest the occurrence of post-injection aseismic slip. One of these cases is the 2016 long-lived seismic swarm that persisted for more than 10 months after completion of hydraulic fracturing operations of a hydrocarbon reservoir in western Canada [55]. In this case study, the authors attributed indeed the delayed swarm activity to post-injection aseismic slip. Unlike the off-fault setting of the Rittershoffen case in France, the configuration here is a clear case of on-fault seismicity. Specifically, aseismic slip is thought to be induced by hydraulic fractures that intersect (and pressurize) frictionally stable segments of a nearby fault at the reservoir level (shales), which in turn transmits solid stresses to distal segments of the same fault but in carbonate units that are thought to slide unstably [12]. One key aspect of the proposed conceptual model for post-injection seismicity in this case is the assumption of heterogeneities in fault permeability to explain a remarkable characteristic of the swarm, namely, a nearly constant rate of post-injection seismicity [55]. The proposed model assumes that elevated pore pressure remains trapped within the fault after shut-in at the reservoir level, due to the extremely low permeability of the shale formations. These trapped fluids are therefore thought to be responsible for a nearly steady propagation of aseismic slip that could explain the constant rate of seismicity [55]. Although their proposed model slightly differs from our model (in which the fault permeability is homogeneous), this case study seems to provide relevant evidence for the mechanisms discussed here. Moreover, as suggested by Cornet [52], another case in which aseismic slip might have triggered post-injection seismicity is the Basel earthquakes in 2006 in Switzerland, a hypothesis that has been recently considered in combination with other triggering mechanisms via plane-strain geomechanical modelling [56].

Further and possibly more conclusive evidence for post-injection aseismic slip may come from carefully designed laboratory and/or in situ experiments of fluid injection. Laboratory experiments where a finite rupture grows along a pre-existing interface [57–59] are particularly promising to explore the post-injection stage. On the other hand, in situ experiments where the fault deformation is monitored simultaneously at the injection point and at another point away from it [60], may also provide the opportunity to investigate this mechanism when combined with hydro-mechanical modelling that includes the depressurization stage [61].

(f) Model limitations: solution as an upper bound

Our model contains the minimal physical ingredients to reproduce post-injection aseismic slip in three-dimensional media. As such, the effects of a number of additional effects remain to be investigated. In particular, we have assumed that fluid flow induces mechanical deformation but not vice versa. It has been however suggested for a long time that variations of effective normal stress may induce permeability changes in fractures [62] and faults [63]. Also, it is known that frictional slip may be accompanied by dilatant (or contracting) fracture/fault-gouge behaviour that would inevitably induce changes in fluid flow and thus in the propagation of fault slip (see [64] for example). Poroelastic effects in the surrounding medium around the fault may be of first order in some cases (see, for instance, [65]). Accounting for a permeable host rock may notably speed up the depressurization of pore-fluid within a fault zone after shut-in due to the leak-off of fluid. Consequently, the arrest time and maximum run-out distance of the rupture pulses would likely decrease. In this regard, our model, in which the leak-off of fluid into the surrounding medium is neglected, would represent an upper bound for the arrest time and maximum run-out distance similarly to the case of the arrest of mode I hydraulic fractures [66].

In relation to friction, we consider the simplest description that allows us to produce unconditionally stable fault slip, namely, a constant friction coefficient. However, laboratory-derived friction laws [67,68] are widely used in the geophysics community to reproduce the entire spectrum of slip velocities of natural earthquakes [69]. Unconditionally stable fault slip
may be obtained notably in some regimes of so-called rate-strengthening faults when subjected to continuous fluid sources [27,28]. For the post-injection problem, a more complex constitutive friction law would add a finite amount of fracture energy as well as frictional healing. In this regard, our model represents a situation in which the fracture energy spent during propagation is zero and there is no possibility for the friction coefficient to heal with time. The dissipation of a finite amount of fracture energy at the rupture front would resist propagation of the aseismic pulses. Similarly the healing of the friction coefficient would further contribute to the re-locking process of the fault thus accelerating the propagation of the locking front. Both ingredients are therefore expected to shorten the normalized arrest time and maximum run-out distance of the rupture pulses. Our model thus represents also an upper bound with regard to these two mechanisms.

7. Concluding remarks

We have provided an in-depth investigation of how post-injection aseismic slip propagates and ultimately arrests in three-dimensional media. Our results provide for the first time a conceptual and quantitative framework that may help to understand various observations and applied problems in geomechanics and geophysics associated with slow ruptures driven by the motion of fluids. Among them, a particularly relevant problem for geo-energy applications is the phenomenon of post-injection seismicity. It has been long recognized that aseismic slip may alter the stress state of large rock volumes during borehole fluid injections [3]. Based on our findings, we suggest that aseismic slip may continue stressing even larger and more distant regions after shut-in, during time scales that could span even months for fluid injections of only a few days, if the reactivated discontinuity is critically stressed as quantified by the value of the stress-injection parameter $T$. Our physical model shows quantitative agreement with field observations of documented cases of post-injection-induced seismicity [48,53], thus providing support for the mechanisms presented here. Further and possibly more conclusive evidence may potentially come from revisiting more case studies via geomechanical modelling, notably from laboratory and/or in situ experiments that can either monitor or infer the propagation of slow ruptures.

Current efforts to manage the seismic risk associated with subsurface fluid injections such as the so-called traffic light systems might be subjected to important limitations in their effectiveness in light of our results. Traffic light systems work under the tacit assumption that operational measures will become shortly effective in preventing the occurrence of events of a larger magnitude than some pre-defined threshold [33]. Our results suggest that instead, the stressing of increasingly larger rock volumes perturbed by aseismic slip may be persistent, if the chosen operational measure is shutting in the well. Future works should therefore focus on developing physics-based strategies to mitigate the seismic risk associated with post-injection aseismic slip.

Data accessibility. The data are provided in the electronic supplementary material [70]. The seismic catalogue used in figure 8 is available on the CDGP web services (https://cdgp.u-strasbg.fr/). All raw data of numerical results and computational codes that produce every figure of the paper can be found in https://zenodo.org/record/7859517#.ZEebti8RrFd.

Authors’ contributions. A.S.: conceptualization, formal analysis, funding acquisition, investigation, methodology, software, validation, visualization, writing—original draft, writing—review and editing; B.L.: conceptualization, funding acquisition, methodology, software, supervision, writing—review and editing.

Both authors gave final approval for publication and agreed to be held accountable for the work performed therein.

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