QCD improved parton distribution functions of $\gamma^*_L$

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Abstract

QCD corrections to the QED formula for parton distribution functions of the longitudinal virtual photon are derived in the leading-logarithmic approximation. It is shown that the resulting PDF satisfy the same homogeneous evolution equations as those of hadrons, but contrary to the latter are perturbatively calculable. Properties of these distribution functions are discussed and their phenomenological consequences outlined.

1 Introduction

Parton distribution functions (PDF) of the photon can be written as sums of the hadronic (HAD) and pointlike (PL) parts. The former satisfy the same homogeneous evolution equations as those of hadrons, whereas the latter satisfy the inhomogeneous ones. Within a subset of pointlike solutions specified by the value of the scale $M_0$ at which they vanish we can thus write

$$D(x, M_0, M) = D^{\text{HAD}}(x, M_0, M) + D^{\text{PL}}(x, M_0, M), \quad D = q, \bar{q}, G.$$  \hspace{1cm} (1)

Due to the freedom in the choice of $M_0$ the separation of PDF into their pointlike and hadronic parts is, however, ambiguous. In \cite{1} we discussed practical aspects of this ambiguity for the Schuler–Sjöstrand sets of parameterizations \cite{2}. The quark distribution functions of the photon can be written as combinations of singlet and nonsinglet parts

$$\Sigma \equiv \sum_{i=1}^{n_f} (q_i + \bar{q}_i), \quad q_i^{\text{NS}} \equiv (q_i + \bar{q}_i) - \frac{1}{n_f} \Sigma.$$  \hspace{1cm} (2)

Due to the presence of hadronic contributions, individual $q_i^{\text{NS}}$ are in general independent nonsinglet distributions. Their pointlike parts are, however, proportional to the corresponding charges and (neglecting quark mass effects) pointlike parts of $q_i$ can therefore be expressed in terms of one singlet and one nonsinglet quantity

$$q_i^{\text{PL}} = (e_i^2 - \langle e^2 \rangle) q_i^{\text{NS,PL}} + \langle e^2 \rangle q_i^{\text{S,PL}}, \quad \Sigma = 2n_f \langle e^2 \rangle q_s^{\text{S,PL}}.$$  \hspace{1cm} (3)

The pointlike part of $q_i^{\text{NS}}$ is defined via the resummation of multiple gluon emissions off the quark from the primary $\gamma \to q\bar{q}$ splitting. In units of $\alpha/2\pi$ \footnote{Frequently also called anomalous, because they contain the term that is absent in the case of hadrons. I prefer the denomination pointlike, which reflects the fact that they arise from pointlike $\gamma \to q\bar{q}$ coupling.}

$$q_i^{\text{NS,PL}}(x, M_0, M) \equiv k_{\text{NS}}(x) \int_{M_0^2}^{M^2} \frac{d\tau_1}{\tau_1} + \int_x^1 \frac{dy}{y} P_{qq}^{(0)} \left( \frac{x}{y} \right) \int_{M_0^2}^{M^2} \frac{d\tau_2}{\tau_2} \frac{\langle a_a(\tau_2) \rangle}{2\pi} k_{\text{NS}}(y) \int_{M_0^2}^{\tau_2} \frac{d\tau_3}{\tau_3} + \int_x^1 \frac{dy}{y} P_{qq}^{(0)} \left( \frac{y}{w} \right) \int_y^1 \frac{dw}{w} P_{qq}^{(0)} \left( \frac{y}{w} \right) \int_{M_0^2}^{M^2} \frac{d\tau_3}{\tau_3} \frac{\langle a_a(\tau_3) \rangle}{2\pi} k_{\text{NS}}(w) \int_{M_0^2}^{\tau_3} \frac{d\tau_2}{\tau_2} \frac{\langle a_a(\tau_2) \rangle}{2\pi} k_{\text{NS}}(w) \int_{M_0^2}^{\tau_2} \frac{d\tau_1}{\tau_1} + \cdots.$$  \hspace{1cm} (4)
where \( k_{NS} = 3(x^2 + (1-x)^2) \) and \( \alpha_s(M) \) satisfies the standard renormalization group equation

\[
\frac{d\alpha_s(\mu)}{d \ln \mu^2} = \beta(\alpha_s(\mu)) = -\frac{\beta_0}{4\pi} \alpha_s^2(\mu) - \frac{\beta_1}{16\pi^2} \alpha_s^3(\mu) + \cdots, \tag{5}
\]

where, in QCD with \( n_f \) massless quark flavours, the first two coefficients, \( \beta_0 = 11 - 2n_f/3 \) and \( \beta_1 = 102 - 38n_f/3 \), are unique, while all the higher order ones are ambiguous. In this paper only the LO QCD corrections will be considered and therefore only the first term in (5) will be taken into account. In terms of standard moments the series in (4) resums to

\[
q^{NS,PL}(n, M_0, M) = \frac{4\pi}{\alpha_s(M)} \left[ 1 - \left( \frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1 - 2P_{q\bar{q}}^{(0)}(n)/\beta_0} \right] a_{NS}(n), \quad a_{NS}(n) \equiv \frac{k^{(0)}(n)}{\beta_0 - 2P_{q\bar{q}}^{(0)}(n)}. \tag{6}
\]

For transverse virtual photon \( \gamma^*_T \), the above considerations can be repeated with only a minor modification reflecting the fact that its momentum \( P_\gamma \) is off-shell, \( P^2 \equiv -P_\gamma^2 \gg 0 \). The exact meaning of the quantity \( \tau \) denoting in (8) the outgoing parton off-shellness is somewhat ambiguous. I have taken \( \tau \) as \( \tau \equiv -(p^2 - m_q^2)/x \), where \( p \) and \( m_q \) stand for parton momentum and mass respectively. Working out the kinematics of the \( \gamma^* \to q\bar{q} \) vertex in the collinear limit yields \( \tau^{\text{min}} = P^2 + m_q^2/(x(1-x)) \). For massless quarks this implies replacing in (4) \( M_0^2 \) with \( P^2 \).

2 The concept of PDF of \( \gamma^*_L \) in QED

Before discussing PDF of \( \gamma^*_L \) in the framework of QCD, let me illustrate the usefulness of this concept on the QED analysis of DIS of electrons on virtual photons. In the region \( m_q^2 \ll P^2 \ll Q^2 \) experiments measure the following quantity

\[
F^\gamma_{\text{eff}}(x, P^2, Q^2) \equiv \frac{Q^2}{4\pi^2\alpha} (\sigma_{TT} + \sigma_{LT} + \sigma_{TL} + \sigma_{LL}) = \frac{Q^2}{4\pi^2\alpha} \sigma(P^2, Q^2, W^2), \tag{7}
\]

where the cross–sections \( \sigma_{jk} \) are in general functions of \( W^2, P^2 \) and \( Q^2 \) and \( x = Q^2/(W^2+Q^2+P^2) \). The \( \mathcal{O}(\alpha) \) QED expressions for individual contributions to \( F^\gamma_{\text{eff}} \) read \( 8 \)

\[
F^{\text{QED}}_{TT}(x, P^2, Q^2) = 6 \left[ \frac{x^2 + (1-x)^2}{x^2P^2} \ln \frac{Q^2}{x^2P^2} + (2x(1-x) - 1) + (2x(1-x) - 1) \right], \tag{8}
\]

\[
F^{\text{QED}}_{LT}(x, P^2, Q^2) = 6 \left[ 4x(1-x) \right], \tag{9}
\]

\[
F^{\text{QED}}_{TL}(x, P^2, Q^2) = 6 \left[ 4x(1-x) \right], \quad F^{\text{QED}}_{LL}(x, P^2, Q^2) = 0. \tag{10}
\]

The denomination “partonic” refers to the fact that the corresponding term arises from integration over \( \tau \) of the quark from the primary splitting \( \gamma \to q\bar{q} \) close to its lower limit \( \tau^{\text{min}} = P^2 \), i.e. from the region of phase space where the emitted quarks (antiquarks) are almost collinear with the incoming photon. In the case of \( F^{\text{QED}}_{TT} \) part of the integrand is proportional to \( 1/\tau \), which yields the logarithmic term, whereas the “partonic” \( 2x(1-x) - 1 = -(x^2 + (1-x)^2) \) in \( F^{\text{QED}}_{LT} \) and \( 4x(1-x) \) in \( F^{\text{QED}}_{TL} \) arise from integration over \( P^2/\tau^2 \). In both cases the resulting expressions have

\footnote{The first and second indices corresponding to probe and target photon respectively.}
a clear parton model interpretation. On the other hand, the “nonpartonic” contributions result from integration over the whole phase space and cannot be interpreted in terms of PDF. Adding all terms in (11) we get

\[ F_{\text{eff}}^{\text{QED}}(x, P^2, Q^2) = 6 \left( x^2 + (1 - x)^2 \right) \ln \frac{Q^2}{x^2 P^2} + 12x(1 - x) - 2. \]  

(11)

One might argue that there is no reason to distinguish the origins and interpretation of the individual terms that add up to the nonlogarithmic part of (11), and consequently, the effects of target \( \gamma^*_L \) can be included as part of the “constant” term in (11). Although legitimate, I prefer not to adopt this procedure and keep track of the origins and interpretation of individual contributions to the constant part of \( F_{\text{eff}}^{\text{QED}} \) because:

- The “partonic” terms, coming from the vicinity of mass singularities are universal quantities, characterizing the incoming \( \gamma^*_L \), whereas the “nonpartonic” ones are process dependent.

- Virtuality dependence of different terms is different. The above expressions hold for \( m_q^2 \ll P^2 \ll Q^2 \). As \( P^2/m_q^2 \to 0 \) the nonpartonic terms, coming from integration over the whole phase space, remain unchanged, whereas \( F_{\text{eff}}^{\gamma^*_q} \) vanishes by gauge invariance as \( F_{\text{eff}}^{\gamma^*_q} \propto P^2/m_q^2 \).

In the case of \( F_{\text{eff}}^{\gamma^*_q} \) the partonic part of the constant term gets additional contribution from the term \( m_q^2/(m_q^2 + x(1-x)P^2) \to 1 \), absent for \( P^2 \gg m_q^2 \). Taking these facts into account allows us to derive in a physically transparent way the constant term of \( F_{\text{eff}}^{\gamma^*_q}(x, P^2) \) for the real photon: \( [2x(1-x) - 1] + [2x(1-x) - 1] + 4x(1-x) + 1 = 8x(1-x) - 1 \).

The fact that \( F_{\text{eff}}^{\gamma^*_q}(x, P^2) = 24x(1-x) \), which does have a parton model interpretation coincides with \( F_{\text{eff}}^{\gamma^*_q} \), which does not, reflects the symmetry of the total cross section \( \sigma_{\text{TL}}(\gamma^*_q(P^2)\gamma^*_q(P^2)) \) with respect to \( P^2 \) and \( Q^2 \).

3 Pointlike part of \( \gamma^*_L \) in QCD – nonsinglet distribution

The purely QED prediction for the pointlike part \( \gamma^{\text{NS}}_L \) of \( q^{\text{NS}}_L \) depends on the quark mass which provides the scale governing its threshold behavior. In this paper I consider QCD corrections to QED formula for massless quarks (for \( x(1-x)P^2 \gg m_q^2 \)) and in the region \( P^2 \ll M^2 \). For \( \gamma^*_L \) the resummation of diagrams in Fig. 4 in the collinear region takes the form

\[ q^{\text{NS}}_L(x, P^2, M^2) \equiv k_L(x) \int_{P^2}^{M^2} d\tau_1 \frac{P^2}{\tau_1^2} + \int_x^1 \frac{d^4}{y} P^{(0)}_{qq} \left( \frac{x}{y} \right) \int_{P^2}^{M^2} d\tau_2 \frac{\alpha_s(\tau_2)}{2\pi} k_L(y) \int_{P^2}^{\tau_2} d\tau_1 \frac{P^2}{\tau_1^2} + \cdots, \]  

(12)

where \( k_L(x) \equiv 12x(1-x) \) and the first term in (12) defines the QED contribution

\[ q^{\text{QED}}_L(x, P^2, M^2) = k_L(x)P^2 \left( \frac{1}{\tau_{\text{min}}} - \frac{1}{M^2} \right) = k_L(x) \left( \frac{1 - P^2}{M^2} \right) \frac{P^2}{P^2 \to 0} k_L(x). \]  

(13)

Performing in (12) the inner integrals over \( \tau_1 \) we get

\^4In not stated otherwise all expressions for quark and gluon distribution functions mentioned in the following concern their pointlike part and the superscript “PL” will therefore be dropped.
Figure 1: Diagrams defining the pointlike part of nonsinglet quark distribution function of the photon. The resummation involves integration over quark virtualities \( \tau \leq M^2 \).

\[
q_{L}^{\text{NS}}(x, P^2, M) = k_L(x) \left( 1 - \frac{P^2}{M^2} \right) + \int_x^1 \frac{dy}{y} P_{qq}^{(0)} \left( \frac{x}{y} \right) k_L(y) \int_{P^2}^{M^2} \frac{d\tau_2}{\tau_2} \frac{\alpha_s(\tau_2)}{2\pi} \left( 1 - \frac{P^2}{\tau_2} \right) + \cdots \tag{14}
\]

Note that \((1 - P^2/M^2)\) and \((1 - P^2/\tau_2)\) are defined also outside the region \(P^2 \leq M^2\) (or \(P^2 \leq \tau_2\)), but are negative there. In order to avoid this unphysical region, all integrals in (14) will be understood to be multiplied by \(\theta(1 - P^2/M^2)\) (or \(\theta(1 - P^2/\tau_2)\)), guaranteeing thus the vanishing of (14) as \(P^2 \to M^2\). Taking into account in each brackets of (14) only the unity, we get

\[
q_{L}^{\text{NS}}(x, P^2, M^2) = k_L(x) + \frac{2}{\beta_0} \int_x^1 \frac{dy}{y} P_{qq}^{(0)} \left( \frac{x}{y} \right) k_L(y) \ln s + \frac{1}{2} \left( \frac{2}{\beta_0} \right)^2 \int_x^1 \frac{dw}{w} P_{qq}^{(0)} \left( \frac{x}{w} \right) \int_w^1 \frac{dy}{y} P_{qq}^{(0)} \left( \frac{w}{y} \right) k_L(y) \ln^2 s + \cdots, \tag{15}
\]

where \(s \equiv \ln(M^2/\Lambda^2)/\ln(P^2/\Lambda^2)\). In momentum space (15) resums to

\[
q_{L}^{\text{NS}}(n, P^2, M^2) = k_L(n) \left[ \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right]^{-2P_{qq}^{(0)}(n)/\beta_0}, \tag{16}
\]

which satisfies standard hadronic evolution equation in the nonsinglet channel

\[
\frac{dq_{L}^{\text{NS}}(n, P^2, M^2)}{d\ln M^2} = \frac{\alpha_s(M^2)}{2\pi} P_{qq}^{(0)}(n)q_{L}^{\text{NS}}(n, P^2, M^2). \tag{17}
\]

The resummation in (14) resembles the definition of nonsinglet quark distribution function of hadrons, with \(k_L(x)\) playing the role of “bare” quark distribution function and \(P^2\) the initial scale. Note, however, that (16) has been derived under the assumption \(P^2 \ll M^2\). Translating (16) into \(x\)-space by means of inverse Mellin transformation leads to scaling violations illustrated by solid curves in Fig. 2. Multiple gluon emission leads to softening of the distribution with increasing \(M^2\), a feature typical for hadrons.

As emphasized above, the result (16) holds for \(P^2 \ll M^2\) only. For \(P^2 \to M^2\) the expression (14) vanishes by definition, whereas (16) goes to \(k_L\). Anticipating the results of the next paragraph let me approximate the inclusions of the terms neglected in (16) by replacing the first term on the r.h.s. of (13) by \(k_L(x)(1 - P^2/M^2)\). This amounts to replacing (16) by

\[
q_{L}^{\text{NS}}(n, P^2, M^2) = k_L(n) \left[ \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right]^{-2P_{qq}^{(0)}(n)/\beta_0} - \frac{P^2}{M^2}, \tag{18}
\]

and leads to series of dotted curves in Fig. 3. As expected these curves are close to the solid ones for \(P^2 \ll M^2\), but substantially different for \(P^2\) approaching \(M^2\) from below. As emphasized at
where the expression after the arrow follows when in all terms on the r.h.s. of (14) the inner integral in momentum space (21) turns into a contribution of the second part of $O(\alpha^2)$ as given by (19) with the hadronic part of $q_T^{NS}$ evaluated from SAS2D parameterization of $u_T$ according to the formula (23).

In analyzing higher order corrections to (16) I will make use of the following approximation starting from the third term on the r.h.s. of (14) the second term, i.e. $\tau^2 \ll 1$ does not coincide with $\tau^2 = 3$ for $2 \leq x_0 \ll x_M$ and only the first term retained.

Figure 2: Left: Scaling violations for $q_L^{NS}$. Solid curves correspond to (14) for $P^2 = 2 \text{ GeV}^2$ and (from above at large $x$) $M^2 = 3, 10, 19^2, 10^3, 10^4$ $\text{GeV}^2$, the dotted ones to the same values of $M^2$ but the formula (13). For the highest two values of $M^2$ the dotted curves are indistinguishable from the solid ones. Right: Comparison of the pointlike part of $q_L^{NS}$ as given by (16) with the hadronic part of $q_T^{NS}$ evaluated from SAS2D parameterization of $u_T$ according to the formula (23).

In momentum space (21) turns into

$$-\int_x^1 \frac{dy}{y} P_{qq}^{(0)} \left( \frac{x}{y} \right) k_L(y) P^2 \int_{p_2}^M \frac{dt_2}{2\pi} \frac{\alpha_s(t_2)}{\alpha_s(t_1)} = -\frac{\alpha_s(P^2)}{2\pi} k_L \otimes P_{qq}^{(0)}.$$  

where $x \equiv \tau/\Lambda^2$, $x_0 \equiv P^2/\Lambda^2$, $x_M \equiv M^2/\Lambda^2$. This integral first appears in evaluating the contribution of the second part of the $O(\alpha^2)$ term in (14), which has been left out in deriving (14)

$$-\frac{\alpha_s(P^2)}{2\pi} k_L(n) P_{qq}^{(0)}(n) \Rightarrow -\frac{\alpha_s(P^2)}{2\pi} k_L(n) P_{qq}^{(0)}(n) \left[ \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right]^{-2P_{qq}^{(0)}(n)/\beta_0},$$

where the expression after the arrow follows when in all terms on the r.h.s. of (14) the inner integral over $d\tau$ is approximated using (13) as $(\alpha_s(P^2)/P^2 - \alpha_s(t_1)/t_1)$ and only the first term retained. Starting from the third term on the r.h.s. of (14) the second term, i.e. $\alpha_s(t_1)/t_1$ is not negligible as the upper limit $\tau_1$ does not coincide with $M^2$, but spans the whole range $P^2 \leq \tau_1 \leq M^2$. Taking it also into account yields (16) multiplied by a term proportional to $\alpha_s^2(P^2)$. Repeating this procedure at higher orders leads to the general expression

$$q_L^{NS}(n, P^2, M^2) = k_L(n) \left( 1 + \sum_{k=1}^{\infty} c_k(n) \alpha_s^k(P^2) \right) \left[ \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right]^{-2P_{qq}^{(0)}(n)/\beta_0},$$
where \( c_1 = -P_{qG}^{(0)}(n)/2\pi \) etc. Although calculable all the coefficients \( c_k(n), k \geq 1 \) were mentioned only to indicate the origin of the general structure of the results after the integrations in (24), but should be discarded in the leading-log approximation, used in deriving (14).

To estimate the hadronic part of \( q_L^{NS} \), I have assumed that its onset is governed by the same ratio \( P^2/m^2_{TV} \) of the photon virtuality \( P^2 \) and vector meson mass \( m^2_{TV} \) as the decrease of \( q_T^{NS} \) with increasing \( P^2 \):

\[
q_L^{NS,HAD}(x, P^2, M^2) = r(P^2)q_T^{NS,HAD}(x, P^2, M^2), \quad r(P^2) \equiv \left( \frac{P^2}{P^2 + m^2_{TV}} \right). \tag{24}
\]

In Fig. 2b (23) is compared to (24) assuming \( q_T^{NS,HAD} = xu(x)/3e_u = 3xu(x) \) and using the SAS1D parameterization of \( u(x, P^2, M^2) \) [4]. Estimated in such a way, the hadronic part of \( q_L^{NS} \) seems to be safely negligible for \( P^2 \geq 1 - 2 \text{ GeV}^2 \).

4 Pointlike part of \( \gamma^+ \) in QCD – singlet and gluon distributions

The fact that \( q_L^{NS} \) satisfies homogeneous evolution equation suggests the same for the quark singlet and gluon distribution functions. Indeed, following the procedure of previous Section one can resum the series coming from diagrams in Fig. 3 as follows

\[
G_L(n, P^2, M^2) = \frac{2n_f\langle e^2 \rangle k_L(n)P_G^{(0)}(n)}{P_{GG}^{(0)}(n)} \left( \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right)^{-2P_{GG}^{(0)}(n)/\beta_0} - 1 = 2n_f\langle e^2 \rangle G_L^{S}. \tag{25}
\]

Adding multiple gluon emissions on the outgoing quark line modifies (25) as follows

\[
G_L^{S}(n, P^2, M^2) = \frac{k_L(n)P_{Gq}^{(0)}(n)}{P_{GG}^{(0)}(n) - P_{qG}^{(0)}(n)} \left( \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right)^{-2P_{Gq}^{(0)}(n)/\beta_0} - \left( \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right)^{-2P_{qG}^{(0)}(n)/\beta_0}. \tag{26}
\]

Combined with \( q_L^{NS} \) defined in (14) \( G_L^{S}(n, P^2, M^2) \) satisfies the evolution equation

\[
\frac{dG_L^{S}(n, P^2, M^2)}{d\ln M^2} = \frac{\alpha_s(M^2)}{2\pi} \left[ P_{GG}^{(0)}(n)G_L^{S}(n, P^2, M^2) + P_{Gq}^{(0)}(n)q_L^{NS}(n, P^2, M^2) \right]. \tag{27}
\]

As for \( q_L^{NS} \) (26) was derived assuming \( P^2 \ll M^2 \), but from the point of view of mathematics can be considered for arbitrary values of \( P^2 \leq M^2 \). Note that whereas \( q_L^{NS}(n, P^2, P^2) = k_L(n) \), \( G_L^{S}(n, P^2, M^2) \) defined in (24) vanishes for \( P^2 = M^2 \). The equation (27) is not exactly the evolution equation for gluons in hadrons as the latter contains quark singlet instead, but the diagrams

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\(^5\)In this way the hadronic part of \( q_L^{NS} \) is in fact overestimated. For \( P^2 \gtrsim 2 \text{ GeV}^2 \) the suppression factor \( P^2/(P^2 + m^2_{TV}) \) is close to unity and has therefore only small effect.
Figure 4: Scale dependence of $q^S_L$ and $G^S_L$ for $n_f = 4$ and $\Lambda^2 = 0.1 \text{ GeV}^2$. Ordered from below at small $x$ the curves correspond to $M^2 = 3, 10, 10^2, 10^3$ and $10^4 \text{ GeV}^2$ respectively.

Included in the definition (26) do not exhaust all possible ladder diagrams defining $G^S_L$. The above considerations are, however, sufficient to conclude that “dressed” PDF of $\gamma^* L$ are constructed from the “bare” ones in the same way as those of hadrons and therefore satisfy the same evolution equations as the latter, with initial conditions at $M^2 = P^2$ having the form

$$q^S_L(x, P^2, P^2) = k_L(x) = 12x(1-x), \quad G^S_L(x, P^2, P^2) = 0.$$  \hfill (28)

Using the formulae of [4] we can write explicitly

$$q^S_L = k_L(n)\left[ \left( \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right)^{d_+(n)} + a(n) \left( \left( \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right)^{d_-(n)} - \left[ \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right]^{d_+(n)} \right) \right],$$ \hfill (29)

$$G^S_L = k_L(n)\epsilon(n) \left( \left( \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right)^{d_-(n)} - \left[ \frac{\alpha_s(M^2)}{\alpha_s(P^2)} \right]^{d_+(n)} \right),$$ \hfill (30)

$$\kappa(n) = \sqrt{P_{qq}^{(0)}(n) - P_{GG}^{(0)}(n)}^2 + 8n_f P_{qq}^{(0)}(n)P_{GG}^{(0)}(n),$$ \hfill (31)

$$a(n) = \frac{P_{qq}^{(0)}(n) - P_{GG}^{(0)}(n) + \kappa(n)}{2\kappa(n)}, \quad \epsilon(n) = \frac{P_{GG}^{(0)}(n)}{\kappa(n)},$$ \hfill (32)

$$d_\pm(m) = \frac{-P_{qq}^{(0)}(n) - P_{GG}^{(0)}(n) \pm \kappa(n)}{\beta_0}.$$ \hfill (33)

Together with $q^S_{NS}$ as given in (16) this completes the evaluation of PDF of $\gamma^* L$. The singlet charge factor $2n_f\langle e^2 \rangle$ factorizes out of both $\Sigma_L$ and $G_L$ due to vanishing of $G_L$ at the initial scale $P^2$. The nontrivial dependence on $n_f$ remains inside $\kappa(n)$.

Performing numerically the inverse Mellin transformation of (29-30) leads the scale dependence of $q^S_L$ and $G^S_L$ in $x$-space, illustrated in Fig. 4 for $P^2 = 2 \text{ GeV}^2$, $\Lambda^2 = 0.1 \text{ GeV}^2$ and $n_f = 4$. The resulting parameterization of PDF of $\gamma^* L$ based on eqs. (16), (29) and (30) contains $\Lambda$ as a free parameter. The fact that PDF of $\gamma^* L$ satisfy the homogeneous evolution equations implies the validity of standard momentum sum rule. Recall that its violation for $\gamma^* T$ is due to the fact that the “bare” quark distribution function of $\gamma^* T$, $q^{QED}_T \propto \ln M^2$, depends on the factorization scale $M$.

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6Obtainable upon request from chyla@fzu.cz.
Figure 5: Comparison of the contributions $F_{TT}$ and $F_{TL}$ of target $\gamma^*_T$ and $\gamma^*_L$ to $F_{\text{eff}}$ (left) and $D_{\text{eff}}$ (right) for $P^2 = 2, 5$ GeV$^2$ and $M^2 = 100$ GeV$^2$ as given by QED and QCD formulae (13) and (29) for $\gamma^*_L$ and SAS1D parameterization for $\gamma^*_T$.

The QCD induced scale dependence of PDF of $\gamma^*_L$ is formally reminiscent of the idea of “dynamically generated” PDF of hadrons advocated by the GRV group (see [5] and references therein). Note, however, that the initial conditions (28) themselves have no direct physical interpretation as all our formulae were derived under the assumption $P^2 \ll M^2$.

5 Phenomenological consequences

In [6] we discussed the numerical relevance of the contribution of resolved $\gamma^*_L$ for two physical quantities: $F_2^\gamma(x, P^2, M^2)$ and the effective PDF

$$D_{\text{eff}}(x, P^2, M^2) \equiv \sum_{i=1}^{n_f} (q(x, P^2, M^2) + \bar{q}_i(x, P^2, M^2)) + \frac{9}{4} G(x, P^2, M^2)$$

relevant for dijet production in ep and $e^+e^-$ collisions. The contributions of $\gamma^*_L$ plotted in Figs. 2 and 3 of [6] and reproduced by dashed curves in Figs. 5 below were calculated using simple QED formula (13) for the quark content of $\gamma^*_L$. They peaked around $x \approx 0.7$ and were more pronounced for $F_2^\gamma$ than for $D_{\text{eff}}$. QCD corrections to PDF of $\gamma^*_L$, given in eqs. (16), (29) and (30), modify this situation significantly. Compared to purely QED predictions, the QCD effects suppress quark distribution functions of $\gamma^*_L$ at large $x$ and enhance them on the other hand for $x \lesssim 0.02$. This enhancement with respect to the contribution of $\gamma^*_T$ is in fact so large that for $x \lesssim 0.02$ the contribution of $\gamma^*_L$ to $F_2^\gamma$ exceeds that of $\gamma^*_T$. For $D_{\text{eff}}$ the region $x \lesssim 0.4$ is dominated by gluons from both $\gamma^*_T$ and $\gamma^*_L$, and again the contributions of $\gamma^*_L$ is comparable to those of $\gamma^*_T$. The relevance
of $\gamma^*_{L}$ is further enhanced with increasing $P^2$ (but keeping $P^2 \ll M^2$), but decrease with increasing $M^2$. Taking into account experimental conditions at LEP and HERA, Figure 5 suggests that dijet production at HERA offers particularly suitable place to look for the effects of $\gamma^*_{L}$. The quantity $D_{\text{eff}}$ allows us to express the relevant cross sections in a simple but only approximate way. Work on incorporating QCD improved PDF of $\gamma^*_{L}$ within the LO Monte-Carlo event generators as well as NLO parton level calculations is in progress.

References

[1] J. Chýla, M. Taševský, Proc. PHOTON ’99, Freiburg in Breisgau, May 1999, ed. S. Soeldner–Rembold, Nucl. Phys. B. Proc. Sup. 82 (2000), 49, hep-ph/9906552
   J. Chýla, M. Taševský, Proc. Monte–Carlo generators for HERA Physics, Hamburg 1999, p. 239, hep-ph/9905444
   J. Chýla, M. Taševský, hep-ph/9912245

[2] G. Schuler, T. Sjöstrand: Z. Phys. C68 (1995), 607; Phys. Lett. B376 (1996),193

[3] A. Gorski, B.L. Ioffe, A. Yu. Khodjamirian, A. Oganesian, Z. Phys. C 44, 523 (1989)

[4] A. Buras, Rev. Mod. Phys. 52 1980, 199

[5] M. Glück, E. Reya, A. Vogt, Phys. Rev. D 60 (1999), 054019

[6] J. Chýla, M. Taševský, Eur. Phys. J. C, in print, hep-ph/0003300

[7] M. Taševský, PhD Thesis, Prague, 1999, unpublished