Conductance Increase by Electron-Phonon Interaction in Quantum Wires

Tobias Brandes, Arisato Kawabata

Department of Physics, Gakushuin University, 1–5–1 Mejiro, Toshima-ku, Tokyo 171, Japan

Abstract

We investigate the influence of electron-phonon interactions on the DC-conductance $\Gamma$ of a quantum wire in the limit of one occupied subband. At zero temperature, a Tomonaga-Luttinger-like renormalization of $\Gamma$ to a value slightly larger than $2e^2/h$ is calculated for a realistic quantum wire model.

PACS: 72.10 Di, 72.15 Nj
I. INTRODUCTION

In contrast to electron-electron scattering, electron-phonon (e-p) scattering in clean quantum wires at low temperatures is believed to be of minor importance for the DC-conductance change $\Delta \Gamma$ from the ballistic value $\Gamma = 2e^2/h$. The extension of the Landauer-Büttiker formula [1,2] to the interacting case is of theoretical interest in view of Fermi- and/or Luttinger-liquid behavior [3,4] in quasi one-dimensional systems. Experimentally, $\Delta \Gamma \sim T^\alpha$ has been found recently [5] with an exponent $\alpha$ in fair agreement with predictions from a Tomonaga-Luttinger model, while no consent was reached for the prefactor determining the $T = 0$ value of the conductance. It was argued [6,7] that the usual reduction of $\Gamma$ due to electron-electron interactions [8] is not observed in realistic wires which are always coupled to leads where the interaction is screened. Thus, the electrons are free outside and interact only over a finite region within the wire which, upon using proper boundary conditions, finally should reestablish the free value $\Gamma = 2e^2/h$.

However, in a recent paper [9] it has been shown that this argumentation might not be complete. The reason is that the conductivity always has to be understood as the current response to the macroscopic (total) electric field and not to the external field (electric displacement), which in fact was pointed out by Izuyama [10] more than 30 years ago shortly after Kubo and Nakano had presented their theory of linear response. From the technical point of view, in an interacting system this difference in the fields exactly can be taken into account by not including 'improper' diagrams simply connected by single Coulomb lines. Since the renormalization of $\Gamma$ in the Tomonaga-Luttinger approach [8] is effectively equivalent to the RPA ('bubble-series' approximation) [9], within this model the Coulomb interaction has no effect on the (properly defined) conductivity and hence the conductance itself which explains the absence of a possible deviation from the ballistic value observed in the experiment.

Independent of the question if or if not Coulomb interactions are important for the conductance at low $T$, it is natural to ask which other processes can lead to deviations
of $\Gamma$ from $2e^2/h$ in ballistic wires. Recently, interest has grown in the effect of electron-phonon coupling on Luttinger liquids [11–17]. The electron-phonon scattering was shown to be a candidate for changes $\Delta \Gamma$ in previous works [14,15], where at zero temperature the conductance turned out to be increased by the e-p coupling in regular Luttinger liquids. In contrast to this, the DC-conductance of chiral Luttinger liquids, describing the edge of quantum Hall systems at filling factor $\nu$, turned out [17] to be insensitive to phonons.

In this paper, we present an alternative derivation of $\Delta \Gamma$ through perturbation theory in the e-p coupling and evaluate the conductance change for a realistic quantum wire at zero magnetic field. Summing up lowest order perturbation terms, we find a renormalization of $\Gamma$,

$$\Gamma = \frac{2}{(1 - \gamma)^{1/2}} \frac{e^2}{h}. \quad (1)$$

Note that since $\gamma > 0$, the conductance is increased and not decreased, the e-p coupling thus acting like an effective attractive interaction between the electrons. We evaluate the numerical value of $\gamma$ for a wire embedded in a GaAs-AlGaAs heterostructure. Though $\gamma$ is of the order of $10^{-4}$ and the conductance change therefore very small, it at least demonstrates that the 'quantization' of the ballistic conductance is never complete in a realistic system. Furthermore, our calculation also shows that the comparatively simple perturbative result Eq. (1) reproduces the calculation for the regular Luttinger liquid model [14,15]. This could indicate the limitations of the latter when applied to realistic quasi-1d systems, where for strong e-p coupling the perturbation theory breaks down.

**II. MODEL**

In our model we start from the Hamiltonian

$$H = H_0 + H_p + H_{ep}$$

$$H_0 = \sum_\alpha \varepsilon_\alpha c_\alpha^\dagger c_\alpha$$
\[ H_p = \sum_Q \omega_Q a_Q^+ a_Q \]
\[ H_{ep} = \sum_{\alpha\beta Q} M_{\alpha\beta}^Q c_\alpha^+ c_\beta(a_Q + a_Q^+) \]
(2)

where \( \alpha \) refers to exact electronic eigenstates with energy \( \varepsilon_\alpha \) of \( H_0 \), \( a_Q \), \( a_Q^+ \) are phonon annihilation and creation operators, and \( M_{\alpha\beta}^Q \) is the coupling matrix element. To obtain transport quantities, Zubarev correlation functions \( \langle\langle c_\alpha^+ c_\beta; c_\gamma^+ c_\delta \rangle\rangle_z \) are defined as
\[ \langle\langle A; B \rangle\rangle_z = -i \int_0^\infty dt e^{izt} \langle [A(t), B(0)] \rangle_0, \]  
(3)

where the expectation value \( \langle \rangle_0 \) refers to the equilibrium density operator \( \exp(-\beta H)/Z \), \( \beta \) is the inverse temperature, and \( Z = Tr \exp(-\beta H) \). The Zubarev functions can be determined by using the equation of motions in \( z \)-space, \( z \langle\langle A; B \rangle\rangle_z = \langle\langle [A, H]; B \rangle\rangle_z = \langle [A, B] \rangle_0 \). To second order in the coupling, the result consists of a term due to backscattering of electrons with a momentum transfer \( \approx 2k_F \), and a forward scattering term. For real processes, in the limit of \( T \to 0 \), the backscattering term freezes out, and only the forward scattering survives. We do not consider virtual backward scattering processes \(^{18}\) which corresponds to the absence of backscattering \((g_1)\)-processes in the Tomonaga-Luttinger model. The analytical expression
\[ \langle\langle c_\alpha^+ c_\beta; c_\gamma^+ c_\delta \rangle\rangle_z = \delta_{\alpha\delta} \delta_{\beta\gamma} \frac{f_\alpha - f_\beta}{z_{\alpha\beta}} + \frac{1}{z_{\alpha\beta}} \frac{f_\delta - f_\gamma}{z_{\beta\gamma}} \times \]
\[ \times \sum_Q \left[ f_\alpha M_{\beta\alpha}^Q M_{\delta\gamma}^Q - f_\beta M_{\alpha\beta}^Q M_{\gamma\delta}^Q \right] \left[ \frac{1}{z - \omega_Q} - \frac{1}{z + \omega_Q} \right] \]
(4)
corresponds to the first term in a 'bubble' series in diagrammatic language. Here, we introduced the abbreviations \( z_{\alpha\beta} := z + \varepsilon_\alpha - \varepsilon_\beta \) and \( f_\alpha := [\exp(\beta(\varepsilon_\alpha - \mu)) + 1]^{-1} \), where \( \mu \) denotes the chemical potential in the Fermi distribution \( f \). The conductance is obtained from the density-density correlation function
\[ \chi(q, z) = i \int_0^\infty dt e^{izt} \langle \rho_q(t), \rho_{-q}(0) \rangle_0, \quad \rho_q := \frac{1}{\sqrt{L_s}} \sum_{k\sigma} c_{k\sigma}^+ c_{k+q\sigma}, \]
(5)

where the quantum number \( k \) refers to plane waves in \( x \)-direction
\[ \langle \mathbf{r}|k \rangle = \frac{1}{\sqrt{L_s}} e^{ikx} \phi(y) \chi(z), \]
(6)
where \( L_s \) is the length of the wire \( (L_s \to \infty \) in the thermodynamic limit), \( \phi(y)\chi(z) \) is the part of the wave function perpendicular to the wire, and \( \sigma \) denotes the electron spin. The relation of \( \chi(q, z) \) to the conductivity \( \sigma(q, z) \) is given through charge and current conservation, 
\[
\sigma(q, z) = -ie^2(z/q^2)\chi(q, z).
\]
With \( \gamma \) including the spin. In the expression for \( \chi(q, z) \), \( \gamma \) is the density-density correlation function for noninteracting electrons in one dimension.

The appearance of the electron-phonon coupling term in Eq. (8) suggests that the perturbative result Eq. (8) is the second term in a geometric series
\[
\chi(q, z) = \chi_0(q, z) \sum_{n=0}^{\infty} \left[ -\chi_0(q, z) \frac{\pi v_F}{2} \gamma_q(z) \right]^n = \frac{\chi_0(q, z)}{1 + \chi_0(q, z) \frac{\pi v_F}{2} \gamma_q(z)}.
\]
Indeed, Eq. (11) is the standard random-phase approximation for the density-density correlation function, which is obtained diagrammatically by summing up the bubble diagrams.
Moreover, Eq. (10) is the exact (non-perturbative) result in a model that starts from the beginning by describing the electronic system in terms of density operators, i.e. in the sense of a Tomonaga-Luttinger liquid description \[14,15\] with a linearized dispersion relation $\varepsilon_k = v_F |k|$. The conductance $\Gamma(z)$ is defined \[19\] as the spatial average of the conductivity in real space over an interval of the length $L$,

$$\Gamma(z) = \langle \sigma(q, z) \rangle_L, \quad \langle A \rangle_L = \frac{1}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} dx dx' \int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{iq(x-x')} A(q)$$  \hspace{1cm} (12)

with the notation $\langle A \rangle_L$ for a function $A(q)$. The DC-limit $z \to 0$ is determined by the $q \to 0$-behavior of $\chi(q, z)$, Eq. (10). Using the expansion $\langle (z + v_k q)^{-1} \rangle_L = -\left( i/|v_k| \right) \left[ 1/2 + izL/(6|v_k|) + O(z^2) \right]$, one finds the conductance $\Gamma_0(z)$ in the non-interacting case as

$$\Gamma_0(z) := \langle \sigma_0(q, z) \rangle_L = 2 \frac{e^2}{h} \left( 1 + \frac{1}{3} \frac{izL}{v_F} + O(z^2) \right),$$  \hspace{1cm} (13)

which is the well-known result \[1,2,20\] for the conductance of a ballistic one-dimensional channel. With the notation

$$\gamma := -\gamma_{q=0}(z = 0) := \frac{2L_s}{\pi h^2 v_F} \sum_Q |M_{k,k}^Q|^2 \frac{2}{\omega_Q},$$  \hspace{1cm} (14)

we obtain from Eq. (10) and Eq. (12) the result Eq. (1), which agrees with the calculation with a bosonized electronic Hamiltonian \[14\]. We can compare to the result by Apel and Rice \[8\], $\Gamma = g^2 e^2 / h$, where the interaction parameter $g$ is larger than unity for attractive and smaller than unity for repulsive electron-electron interaction. At $T = 0$, the coupling to the phonons thus leads to an effectively attractive interaction between the electrons which increases the conductance. Here, we will not discuss the implications of the singularity \[12\] for $\gamma = 1$ for strong e-p coupling since this regime is not accessible within our perturbative calculation. On the contrary, one has to assure that $\gamma \ll 1$ for consistency. We now show that this is indeed the case for a realistic wire, taking into account the full three dimensional phonon system with its coupling to the one dimensional wire for the evaluation of the factor $\gamma$, Eq. (14).
III. ELECTRON-PHONON-COUPLING

For the electron-phonon scattering in GaAs heterolayers, apart from the deformation potential interaction, the piezoelectric interaction is known to be important especially at low temperatures. We follow Price [21] who has shown how to incorporate both contributions and to take into account the important effect of the screening by the 2DEG as well. The fundamental coupling parameters are $\Xi$, the deformation potential, and $eh_{14}$ for the piezoelectric coupling, where $h_{14}$ is the piezoelectric coupling constant as the only non-vanishing component of the piezoelectric tensor for GaAs (zinc-blende structure). For the three dimensional phonon vector $Q$ we use the notation $Q = (Q_\parallel, Q_z)$, $Q_\parallel = (Q_x, Q_y)$, $Q = |Q|$, and $Q_\parallel = |Q_\parallel|$. The vector $Q_\parallel$ lies in the $x - y$ plane of the 2DEG. With the longitudinal and transversal sound velocities denoted by $c_L$ and $c_T$, respectively, the e-p potential $|V_Q|^2$ in the matrix element $|M_{kk}|^2 = |V_Q|^2|\langle k | e^{iQr}|k \rangle|^2$ is given by

$$\frac{1}{\omega_Q}|V_Q|^2 = \frac{\Xi^2 h}{2\rho c_L^2 \Omega} \left\{ 1 + \frac{1}{Q^2} \left( \frac{eh_{14}}{\Xi} \right)^2 \left[ A_L(Q) + \left( \frac{c_L}{c_T} \right)^2 A_T(Q) \right] S(Q_\parallel) \right\}. \quad (15)$$

The phonon frequency $\omega_Q$ is $c_L Q$ or $c_P Q$ for the longitudinal and transversal phonons, respectively. Furthermore, $\Omega$ is the crystal volume with mass density $\rho$, $A_L(Q) = 9Q_\parallel^2 Q_z^4/(2Q^6)$ and $A_L(Q) + 2A_T(Q) = (8Q_\parallel^2 + Q_z^2)Q_\parallel^2/(2Q^4)$, and $S(Q_\parallel)$ is the screening factor due to the mobile electrons in the $x - y$ plane. We have included the latter only for the piezoelectric coupling which diverges $\sim 1/Q$ for small wave vectors, and not for the deformation potential coupling. The latter is regular for $Q \to 0$ and screening effects are already taken into account in the value of the deformation potential $\Xi$ itself. The screening function $S(Q_\parallel)$ is given by

$$S(Q_\parallel) = \frac{Q_\parallel}{Q_\parallel + PH(Q_\parallel)}, \quad H(Q_\parallel) = \int \int dzdz' \chi(z')^2 \chi(z + z')^2 \exp(-Q_\parallel|z|), \quad (16)$$

where $H(Q_\parallel)$ depends on the quantum well wave function $\chi(z)$ and $P$ is the screening constant [21]. The matrix element $|\langle k | e^{iQr}|k \rangle|^2$ still contains the free electron wave function Eq. (6) of which the plane wave component in $x$ direction gives rise to a Kronecker $\delta_{Q_x,0}$. 


This means that only phonons perpendicular to the wire contribute. On the other hand, the 'formfactor' due to the part of the wave function in direction perpendicular to the wire gives rise to a cutoff of phonons with wave vectors \( Q_z \geq l_z^{-1} \) and \( Q_y \geq l_y^{-1} \). Here, \( l_z \) and \( l_y \) are the thickness of the quantum layer and the quantum wire, respectively. We use a parabolic quantum well and a parabolic quantum wire confinement potential such that \( |\langle k|e^{i\mathbf{Q}\mathbf{r}}|k\rangle|^2 = \delta_{Q_x,0} \exp(-l_z^2Q_z^2 - l_y^2Q_y^2) \). The evaluation of the renormalization factor Eq. (14) is performed by introducing polar coordinates in the \( y-z \) plane, \( Q_z = q \sin \varphi \), \( Q_y = q \cos \varphi \), where \( q = (Q_y^2 + Q_z^2)^{1/2} \). Notice that the sum Eq. (14) is two-dimensional because of the Kronecker \( \delta_{Q_x,0} \), furthermore the factor \( L_s/\Omega \) is the area of the \( y-z \) plane so that the transformation from summation to integration can be performed properly. We assume the thickness of the quantum well to be very small compared to the wire width, \( l_z \ll l_y \), which allows us to evaluate the integrals analytically. The result is

\[
\gamma = \frac{2\Xi^2}{(2\pi)^2hv_Fc_T^2\rho} \left\{ \frac{1}{l_y l_z} + \frac{9}{16} + \frac{(c_L/c_T)^2}{32} g(P_Ly) \left( \frac{e\hbar}{\Xi} \right)^2 \right\}, \quad g(y) := \int_0^\infty dx \frac{e^{-x^2y^2}}{x + H(Px)}.
\]

(17)

The e-p parameters we used to obtain the numerical factor for \( \gamma \) (table) are standard values for GaAs. It turns out that the contribution from the deformation potential coupling is of the same order as the piezoelectric contribution. Since \( P_Ly \gg 1 \), the value of \( g(y) \approx \sqrt{\pi}/2y \) is due to \( x \approx 0 \) and by \( H(0) = 1 \) therefore nearly independent of the exact form of the quantum well wave function. We obtain

\[
\gamma \approx 3 \cdot 10^{-4} \times \frac{1}{v_F[10^4\text{ms}^{-1}]}.
\]

(18)

**IV. CONCLUSION**

The result \( \gamma \ll 1 \) is consistent with our perturbative approach. Note, however, that \( \gamma \sim 1/v_F \) and therefore can in principle be made arbitrarily large by tuning the Fermi velocity to very small values (near the bottom of the subband), corresponding to very high
density of states. This region, however, can never be achieved for realistic values of $v_F$. The case $\gamma \approx 1$ where the theory breaks down corresponds to such small values as $v_F \approx 3ms^{-1}$, where the conductance plateau in any case does not exist any longer. Still, from the technical point of view, the divergence of $\Gamma$ in this case indicates that in principle other higher order perturbation terms have to be considered. On the other hand, for Fermi energies well above the subband edge, our perturbation theory works well, $\gamma$ is small and leads to the increase of the conductance, Eq. (1). The question remains if this increase really is observable in an experiment. Realistic quantum wires are never completely ballistic due to boundary roughness and other impurity effects. These, however, should in any case lead to a reduction and not to an increase of the conduction. Furthermore, the electron-electron interaction which originally [8] was predicted to reduce $\Gamma$, turned out [6,7,9] to be irrelevant for the $T = 0$ value of $\Gamma$. The electron-phonon interaction, on the other hand, neither gives rise to the difference between macroscopic and external electric field in the sense of [8,10], nor is it screened in the leads and locally unscreened in the wire as is the electron-electron interaction according to Ref. [6,7]. It therefore should remain the only interaction mechanism to give a contribution to $\Delta \Gamma$ in the limit of zero temperature. From this point of view, we believe that it is worthwhile to investigate the prediction $\Gamma > 2e^2/h$ in some more detail experimentally.

One of the authors (T.B.) would like to acknowledge support by the EU STF9 fellowship program in Japan.
REFERENCES

[1] R. Landauer, Philos. Mag. 21, 863 (1970).

[2] M. Büttiker, Phys. Rev. Lett. 57, 1761 (1986).

[3] B. Hu and S. D. Sarma, Phys. Rev. B 48, 5469 (1993).

[4] See, e.g: J. Voit, Rep. Progr. Phys. 58, 977 (1995).

[5] S. Tarucha, T. Honda, and T. Saku, Solid State Comm. 94, 413 (1995).

[6] V. V. Ponomarenko, Phys. Rev. B 52, R8666 (1995).

[7] I. Safi and H. J. Schulz, Phys. Rev. B 52, R17040 (1995).

[8] W. Apel and T. M. Rice, Phys. Rev. B 26, 7063 (1982).

[9] A. Kawabata, J. Phys. Soc. Japan 65, 30 (1996).

[10] T. Izuyama, Progr. Theor. Phys. 25, 964 (1961).

[11] V. Meden, K. Schönhammer, and O. Gunnarson, Phys. Rev. B 50, 11179 (1994).

[12] D. Loss and T. Martin, Phys. Rev. B 50, 12160 (1994).

[13] D. Loss and T. Martin, Int. J. Mod. Phys. 9, 495 (1995).

[14] T. Brandes and B. Kramer, in 22nd International Conference on the Physics of Semiconductors, edited by D. J. Lockwood (World Scientific, New York, 1994).

[15] T. Martin, cond-mat 9408088, CNLS 94 Conference ‘Quantum Complexity in Mesoscopic Systems’, Los Alamos (1994).

[16] P. Kopietz, cond-mat 9508021, to appear in Z. Phys. B (1995).

[17] O. Heinonen and S. Eggert, cond-mat 9512089 (1995).

[18] We thank H. Fukuyama and Y. Ono for drawing our attention to this point.
[19] D. S. Fisher and P. A. Lee, Phys. Rev. B 23, 6851 (1981).

[20] B. Kramer and J. Mašek, Z. Phys. B 76, 457 (1989), J. Mašek and B. Kramer, Z. Phys. B 75, 37 (1989).

[21] P. J. Price, Jour. Appl. Phys. 53, 6863 (1982).

[22] H. Bruus, K. Flensberg, and H. Smith, Phys. Rev. B 48, 11144 (1993).
TABLE I. Electron-phonon parameters for evaluating $\gamma$ in GaAs. The first five parameters are taken from Ref. [22].

| Parameter                      | Symbol | Value          |
|--------------------------------|--------|----------------|
| Mass density                   | $\rho$ | $5300 \text{ kg m}^{-3}$ |
| Longitudinal speed of sound    | $c_L$  | $5200 \text{ m s}^{-1}$ |
| Transversal speed of sound     | $c_T$  | $3000 \text{ m s}^{-1}$ |
| Deformation Potential          | $\Xi$  | $2.2 \times 10^{-18} \text{ J}$ |
| Piezoelectric constant         | $eh_{14}$ | $1.38 \times 10^9 \text{ eV m}^{-1}$ |
| Screening constant [21]        | $P$    | $2 \times 10^8 \text{ m}^{-1}$ |
| Fermi velocity                 | $v_F$  | $10^4 \text{ m s}^{-1}$ |
| Quantum well width             | $l_z$  | $10^{-8} \text{ m}$ |
| Quantum wire width             | $l_y$  | $10^{-6} \text{ m}$ |