The Trojan Horse Method as an indirect approach for nuclear astrophysics studies

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Abstract. The Trojan Horse method (THM) is a powerful indirect technique that provides a successful alternative path to determine the bare nucleus astrophysical S(E) factor for rearrangement reactions down to astrophysical energies. This is done by measuring the cross section for a suitable three body process in the quasi-free kinematics regime. Prescriptions and basic features will be presented together with some applications to demonstrate how THM works.

1. Introduction

Despite the Coulomb force is extremely weaker than the nuclear one, its influence cannot be always disregarded. For example, it takes considerable energy to force nuclei to fuse, even those of the lightest element, hydrogen. This is because all nuclei have a positive charge, and as like charges repel, nuclei strongly resist being put too close together. This is a critical point in nuclear astrophysics, that deals with explaining the nucleosynthesis in the universe via nuclear reactions. At the relevant temperatures all reactions are strongly inhibited because of the Coulomb repulsion, responsible for the exponential decrease of the cross section $\sigma(E)$ at those temperatures. For this reason, the behavior of $\sigma(E)$ at low energy is usually extrapolated [?] from higher energies (usually E>100 keV) using the definition of the astrophysical S(E)-factor

$$S(E) = E\sigma(E)\exp(2\pi\eta),$$

where $\eta$ is the Coulomb parameter of the colliding nuclei, $\exp(2\pi\eta)$ is the inverse of the Gamow factor, which essentially removes the dominant energy dependence of $\sigma(E)$ due to the barrier penetration. Although the S(E)-factor allows for an easier extrapolation, large uncertainties to $\sigma(E)$ may be introduced, for instance, because of the presence of unexpected resonances.

A second relevant source of uncertainty in extrapolating the S(E)-factor down to zero energy is its enhancement due to the electron screening effect. In the extrapolation of the cross section using Eq.1, it is assumed that the Coulomb potential of the target nucleus and the projectile is that resulting from bare nuclei. However, as regards nuclear reactions studied in the laboratory, the target nuclei and the projectiles are usually in the form of neutral atoms or molecules.
and ions, respectively. Because of the electron clouds surrounding the interacting nuclei, the projectile effectively sees a reduced Coulomb barrier. This, in turn, leads to a higher cross section for screened nuclei, $\sigma_s(E)$, compared to the cross section one would get in the case of bare nuclei $\sigma_b(E)$ [1, 2]. Therefore the enhancement factor, defined by the following relation

$$f_{lab}(E) = \frac{\sigma_s(E)}{\sigma_b(E)} \approx \exp\left(\pi \eta U_e / E\right), \quad (2)$$

where $U_e$ is the so-called “electron screening potential” [1, 2], must be taken into account to determine the bare nucleus cross section.

In the astrophysical environment, the cross section under plasma conditions $\sigma_{pl}(E)$ is related to the bare nucleus cross section by a similar enhancement factor:

$$f_{pl}(E) = \frac{\sigma_{pl}(E)}{\sigma_b(E)} \approx \exp\left(\pi \eta U_{pl} / E\right)$$

that can be calculated from the knowledge of the plasma screening potential $U_{pl}$, depending on important properties of the plasma such as the Debye-Hückel radius. A measurement of $U_e$, which is needed to calculate $\sigma_s(E)$ from Eq. 2, would also help to better understand $U_{pl}$.

However the $U_e$ values deduced in fusion reactions are quite larger in all cases than the upper limit, given by the adiabatic model as the difference between the electron binding energies of the separate atoms in the entrance channel and that of the composite atom [3, 2]. This disagreement in laboratory experiments is not justified yet, and it does not help in understanding the effects under astrophysical conditions.

A week point in the laboratory approach - and thus in the deduced $U_e$ value - is the need for an assumption about the energy dependence of $\sigma_b(E)$ at ultra-low energies. In this framework, the THM overcomes all these difficulties providing a successful alternative path to determine $\sigma_b(E)$.

2. Basic features of the Trojan Horse Method

The THM [4, 5, 6, 7] replaces the astrophysical $A + x \rightarrow c + C$ two-body reaction by a suitable $A + a \rightarrow c + C + s$ three-body process, establishing a relation between the two reactions by using nuclear reaction theories. The three-body process is chosen in such a way that the involved target $a$ (or equivalently the projectile) has a wave function with a large amplitude for a $x - s$ relative motion. This is indeed a different approach to the THM compared to the original idea of ref. [8], where the initial velocity of the projectile $A$ is compensated for by the Fermi motion of $x$. In that framework a quite large momentum of the order of 200 MeV/c or more is needed. But the relative yield of the experimental momentum distribution at such momenta can be very small, in particular for a $l=0$ inter-cluster motion (for example p-n motion inside $^2$H or alpha motion inside $^6$Li). This would make very critical the separation from other competitive reaction mechanisms. Moreover, the theoretical description of the tails of the momentum distribution is a hard task, their shape being very sensitive to it. In our approach to the THM the inter-cluster motion is only needed to fix the accessible astrophysical energy region within a chosen cutoff in momentum distribution, usually of the order of few tens of MeV/c. In this framework, the so called “quasi-free two-body energy” is given by:

$$E_{QF} = \frac{m_x}{m_x + m_A} E_A - B_{x-s}.$$

$$\quad (4)$$
where $E_A$ represents the beam energy, $m_x$ and $m_A$ are the masses of $x$ and $A$ particles respectively, and $B_{x\rightarrow A}$ is the binding energy for the $x-s$ system. Then, a cutoff in the momentum distribution, which is related to the Fermi motion of $s$ inside the Trojan-horse $a$, fixes the range of energies around the "quasi-free two-body energy" accessible in the astrophysical relevant reaction.

In the Impulse Approximation, based essentially on the assumption that the interaction of the spectator with particles $C$ and $c$ is reduced, the three body-cross cross section can be factorized as:

$$
\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto [KF |\varphi_a(p_{sx})|^2] \left( \frac{d\sigma}{d\Omega_{c.m.}} \right)_{HOES}
$$

where $KF$ is a kinematical factor containing the final state phase-space factor. It is a function of the masses, momenta and angles of the outgoing particles [6]; $\varphi_a(p_{sx})$ is proportional to the Fourier transform of the radial wave function $\chi(r)$ for the $x-s$ inter-cluster relative motion; $(d\sigma/d\Omega_{c.m.})_{HOES}$ is the half-off-energy-shell (HOES) differential cross section for the binary reaction at the center of mass energy $E_{c.m.}$ given in post-collision prescription by

$$
E_{c.m.} = E_{cC} - Q_{2b}.
$$

Here, $Q_{2b}$ is the $Q$-value of the binary reaction and $E_{cC}$ is the relative energy of the outgoing particles $c$ and $C$.

The factorization of Eq.5 is strictly valid in Plane Wave or in the so called Modified Plane Wave [9], this changing only its absolute magnitude but not the energy dependence of the two-body cross section.

3. Experimental prescriptions

In a typical THM experiment the decay products ($c$ and $C$) of the virtual two-body reaction of interest are detected and identified by means of telescopes (silicon detector or ionization chamber as $\Delta E$ step and position sensitive detector as $E$ step) placed at the so called quasi free angles. After the selection of the reaction channel, the most critica point is to disentangle the quasi free mechanism from other reaction mechanisms feeding the same particles in the final state, e.g. sequential decay and direct break-up. An observable which turns out to be very sensitive to the reaction mechanism is the shape of the experimental momentum distribution of the spectator. In order to reconstruct the experimental $p_s$ distribution, the energy sharing method [10] is applied for each pair of coincidence QF angles: the quasi-free coincidence yield with a cutoff in relative energy of few hundreds of keV at most is divided by the kinematic factor, providing a quantity which is proportional to the product of the momentum distribution for the neutron with the differential two-body cross section (see Eq.5). The cutoff is chosen in such a way that the differential two-body cross section in this range can be considered almost constant. Thus the quantity defined above represents essentially the momentum distribution for the spectator that has to be compared with the the Fourier transform of the $x-s$ bound state wave function. An example of experimental momentum distribution for the neutron inside $^2H$ is reported in Fig.1 together with the theoretical behaviour (solid line). The points drawn as full and open circles, belong to two different experimental runs. As expected, they have the same trend. Further data analysis is limited to the data lying in the region where the agreement between the two distributions exists. However, usually a window of few tens of MeV/c is chosen not to move too far from $E_{QF}$.

Therefore, it is possible to derive the HOES $((d\sigma/d\Omega_{c.m.})_{HOES}$ from the three-body coincidence yield by simply inverting Eq.5. In a final step, the HOES cross section has to be related to the relevant on-energy-shell (OES) cross section by applying the corresponding corrections. It was demonstrated that there is no Coulomb barrier in the two-body amplitude extracted from
the TH reaction [11] and this is due to the virtuality of particle \( x \). This seems to be the only consequence of off-energy-shell effects as suggested by the agreement between HOES and OES cross-sections for the \(^6\text{Li}(n,\alpha)^3\text{H}\) reaction [12]. If one looks at the angular distributions no correction is needed because once the energy is fixed, it would mean to introduce simply a scaling factor. Thus, the OES data are directly comparable with HOES ones projected onto the emission angle of \( C \) (or \( c \)) in the \( C-c \) center of mass system, \( \theta_{c.m.} \), as given by [13]:

\[
\theta_{c.m.} = \arccos \left( \frac{\mathbf{v}_A - \mathbf{v}_x}{|\mathbf{v}_A| / |\mathbf{v}_x| - \mathbf{v}_C} \right)
\]

where the vectors \( \mathbf{v}_A, \mathbf{v}_C, \mathbf{v}_x \) are the velocities of projectile, transferred particle and outgoing nuclei respectively. These quantities are calculated from their corresponding momenta in the lab-system, where the momentum of the transferred particle is equal and opposite to that of spectator when the quasi-free break-up takes place in the target, otherwise a little different formula has to be used [13]. If HOES data are projected onto the \( E_{Cc} \) axis, Coulomb suppression has to be introduced before comparison with OES data. In a heuristic approach this is done by means of the penetrability factor:

\[
P_l(k_{Ax} R) = \frac{1}{G_l^2(k_{Ax} R) + F_l^2(k_{Ax} R)}
\]

with \( F_l \) and \( G_l \) regular and irregular Coulomb wave functions. This procedure does not allow us to extract the absolute value of the two-body cross section. However, this is not a real problem since the absolute magnitude can be derived from a scaling to the direct data available at higher energies.

4. Applications

The THM was applied several times to rearrangement reactions connected with fundamental astrophysical problems [14]. A list of reactions studied by means of the THM is given in Table 1 together with the relevant references.

Low-energy cross sections for reactions producing or destroying lithium isotopes are fundamental for a number of still not completely solved astrophysical problems, e.g. understanding Big Bang nucleosynthesis and the "Lithium depletion" in the Sun or in other galactic stars. An experimental program was undertaken to study p-capture reactions on \(^6,^7\text{Li}\), main responsible for their destruction [19, 20, 5, 21]. The extracted astrophysical S(E) factors were compared with those from direct measurements and found in fair agreement in the energy range where screening effects are negligible. Since the THM provides the bare S(E) factor, improved information on the screening potential \( U_e \) was obtained from comparison with shielded direct data. The extracted values of \( U_e=(450\pm100)\text{eV} \) for \(^6\text{Li}\) and \( U_e=(330\pm30)\text{eV} \) for \(^7\text{Li}\) confirmed the isotopic independence of \( U_e \), although they were found to be much larger than the adiabatic limit (176 eV). The S(E) factor for the \(^6\text{Li}(p,\alpha)^3\text{He}\) reaction is shown in fig. 2 (black dots). Direct data from [22, 23] are also shown as open symbols. The solid line represents a polynomial fit to the THM data as reported in [5, 12]. The implications of these THM results for the "Lithium problem" have been discussed in [24, 25], showing that \( \sigma_b(E) \) for these reactions is now well determined and that the solution of the problem of light element destruction in stars lies elsewhere. The \(^{15}\text{N}(p,\alpha)^{12}\text{C}, \; ^{18}\text{O}(p,\alpha)^{15}\text{N}\) reactions are involved in the production path of \(^{19}\text{F}\) in AGB stars, whose chemical evolution is strongly influenced by \(^{19}\text{F}\) abundance. Available data from observations provide evidence that fluorine is deeply produced within the interiors of these stars during helium flashes, and its abundance, enhanced by large factors with respect to the solar one, cannot be matched by the models. Among the primary sources of uncertainty in predicting fluorine abundances in AGB stars are the adopted reaction rates [26]. The astrophysical S(E)
Table 1. Two-body reactions studied via the THM

| Direct reaction          | TH reaction          | E\text{inc} (MeV) | Q (MeV) | TH\text{nucl} | ref   |
|-------------------------|----------------------|-------------------|---------|---------------|-------|
| \[1\] \( ^7\text{Li}(p,\alpha)^4\text{He} \) | \( ^7\text{Li}(d,\alpha \alpha)n \) | 19.22 | 15.122 | d = (p\otimes n) | \[20\] |
| \[2\] \( ^6\text{Li}(d,\alpha)^3\text{He} \) | \( ^6\text{Li}(^6\text{Li},\alpha\alpha)^3\text{He} \) | 5     | 22.372 | \( ^6\text{Li} = (\alpha\otimes d) \) | \[4\] |
| \[3\] \( ^6\text{Li}(p,\alpha)^3\text{He} \) | \( ^2\text{H}(^6\text{Li},\alpha^3\text{He})n \) | 14, 25 | 1.795  | d = (p\otimes n) | \[5\] |
| \[4\] \( ^11\text{B}(p,\alpha)^8\text{Be} \) | \( ^2\text{H}(^1\text{B},^8\text{Be} \alpha \alpha)n \) | 27    | 6.36   | d = (p\otimes n) | \[6\] |
| \[5\] \( ^{10}\text{B}(p,\alpha)^7\text{Be} \) | \( ^2\text{H}(^{10}\text{B},^7\text{Be} \alpha \alpha)n \) | 27    | -1.079 | d = (p\otimes n) | \[15\] |
| \[6\] \( ^6\text{Li}(n,\alpha)^3\text{He} \) | \( ^2\text{H}(^6\text{Li},^3\text{He} \alpha \alpha)p \) | 14    | 4.78   | d = (p\otimes n) | \[12\] |
| \[7\] \( ^2\text{H}(^3\text{He},p)^4\text{He} \) | \( ^6\text{Li}(^3\text{He},p \alpha \alpha)^4\text{He} \) | 5, 6  | 16.88  | \( ^3\text{He} = (p\otimes \alpha) \) | \[18\] |
| \[9\] \( ^3\text{H}(d,p)^3\text{He} \) | \( ^2\text{H}(^3\text{Li},t \alpha p)^3\text{He} \) | 14    | 2.59   | \( ^6\text{Li} = (\alpha \otimes d) \) | \[17\] |
| \[10\] \( ^{15}\text{N}(p,\alpha)^{12}\text{C} \) | \( ^2\text{H}(^{15}\text{N},\alpha^{12}\text{C})n \) | 60    | 2.74   | d = (p\otimes n) | \[7\] |
| \[11\] \( ^{18}\text{O}(p,\alpha)^{15}\text{N} \) | \( ^2\text{H}(^{18}\text{O},\alpha^{15}\text{N})n \) | 54    | 1.76   | d = (p\otimes n) | \[7\] |
| \[12\] \( ^1\text{H}(p,p)^1\text{H} \) | \( ^2\text{H}(p,p\alpha)p \) | 5, 6  | 0      | d = (p\otimes n) | \[31, 32\] |
| \[13\] \( ^{17}\text{O}(p,\alpha)^{14}\text{N} \) | \( ^2\text{H}(^{17}\text{O},\alpha^{15}\text{N})n \) | 41    | 1.19   | d = (p\otimes n) | \[33\] |
| \[14\] \( ^2\text{H}(d,d)^3\text{He} \) | \( ^2\text{H}(^2\text{He},t \alpha p)^1\text{H} \) | 17,18 | 4.03   | \( ^3\text{He} = (p\otimes \alpha) \) | \[7\] |
| \[15\] \( ^2\text{H}(d,n)^3\text{He} \) | \( ^2\text{H}(^2\text{He},^3\text{He} \alpha n)^1\text{H} \) | 18    | 3.27   | \( ^3\text{He} = (p\otimes \alpha) \) | \[16\] |
| \[16\] \( ^{19}\text{F}(p,\alpha)^{16}\text{O} \) | \( ^2\text{H}(^{19}\text{F},\alpha^{16}\text{O})n \) | 14    | 8.11   | d = (p\otimes n) | \[16\] |

Figure 1. Experimental momentum distribution for the neutron inside \( ^d \) (full and open circles). The solid line represents the shape given by the square of the \( n - p \) Hulthén bound state wave function in momentum space.

Figure 2. THM bare nucleus S(E) factor for the \( ^6\text{Li}(p,\alpha)^3\text{He} \) (black dots) \[21\] compared with direct data (open symbols) from \[22\]. The solid line is the result of a polynomial fit to the THM data.

factor for the \( ^{15}\text{N}(p,\alpha)^{12}\text{C} \) has been recently measured by means of the THM down to 20 keV (to be compared to 73 keV in direct measurements), thus covering for the first time the whole
Gamow window \((94 \pm 66 \text{ keV for a temperature of } 10^8 \text{ K})\) without the complications due to the electron screening. A calculation called Modified R-matrix approach [7] was applied. It accounts for the virtuality of the proton in the entrance channel of the relevant two body reaction by replacing the entry resonance widths with the form factors. The calculations include the parameters associated with the first three resonances in the \(^{15}\text{N} + p\) system at \(E_{^{15}\text{N}p} = 0.312, 0.9624\) and 1.0014 MeV, and the subthreshold state at \(E_{^{15}\text{N}p} = -2.53 \text{ MeV}\). For details concerning the formal expression of the reaction amplitude in terms of the form factors see [7]. The calculated S(0) value of the astrophysical factor ranges from 65.0 to 70.0 MeVb for a channel radius \(r_0 = 5.0\) to 6.0 fm, in agreement with the estimate derived from a Breit-Wigner extrapolation [27] on the direct data.

As regards the \(^{18}\text{O}(p,\alpha)^{15}\text{N}\) reaction, its reaction rate strongly depends on the strengths of three low-lying resonances corresponding to the 8.014, 8.084 and 8.138 \(^{19}\text{F}\) states (20, 90 and 144 keV in the \(^{18}\text{O}-p\) relative energy axis respectively). While the nuclear parameters for the 8.138 state are quite well known, the strengths of the first two states are very much uncertain. This experiment has provided the strengths of both resonances. The Modified R-matrix approach was applied in order to extract this information, after measuring the relative strengths of the nearby known resonances. Its uncertainty was reduced almost by a factor 10 [28] with respect to that in the previous value adopted in the NACRE compilation [29]. The resulting off-shell excitation function is reported in Fig.3 for \(\theta_{c.m.} \text{ of } 55^\circ\); in the 0-250 keV energy range relevant for astrophysics. A Gaussian fit was used to disentangle the contribution of each peak to the overall coincidence yield. The solid line is the sum of the three Gaussian functions drawn as dashed, dotted, and dashed-dotted lines, and the straight line (dashed-dotted line) is to provide for the nonresonant behaviour. A reaction rate about 35% larger than the Nacre value has been estimated. The astrophysical consequences are being investigated. Recently we have extended

\[ \text{Figure 3. Excitation function for the } ^{18}\text{O}(p,\alpha)^{15}\text{N reaction (black dots) [7] See the text for the meaning of the different lines.} \]

\[ \text{Figure 4. THM two-body cross section (black dots) vs. pp relative energy [31]. Solid line represents the theoretical OES p-p cross section [30]. The dashed-dotted line is the calculated HOES cross section [31].} \]

the application of the THM to scattering processes. In particular, we have address the study of the \(p + p\) elastic scattering through the \(^2\text{H}(p,pp)n\) reaction [31]. The aim of the experiment was to investigate the suppression of the Coulomb amplitude also for scattering. The extracted \(p - p\) HOES cross section is presented in Fig.5 as a function of \(E_{pp}\) (black dots) and compared to the free \(p-p\) cross section (solid line) [30] where the deep minimum due to the interference
between Coulomb and nuclear amplitude dominates the excitation function. The dashed-dotted line represents the calculated HOES $p-p$ cross section [31] where the Coulomb amplitude is strongly suppressed compared to the nuclear one. We observe a striking disagreement between the THM (HOES) and the free $p-p$ (OES) cross sections throughout the region of the interference minimum, which is missing in the THM data. Instead, the calculated HOES $p-p$ nicely fits the THM data. In order to strengthen this result, the THM $p-p$ cross section was compared with the OES $n-n$, $p-n$ and pure nuclear $p-p$ ones [30] as reported in [31]. A good agreement shows up providing compelling evidence of the validity of the THM method for elastic scattering. Moreover, another issue has been readdressed by looking at the energy dependence of the HOES $n-n$ cross section. Surprisingly, it fairly reproduces the behaviour of the $n-n$ and pure nuclear $p-p$ cross sections. This result is in agreement with the one reported in [12] since it excludes the existence of off-energy-shell effects when the Coulomb barrier is absent. These results are extensively explained and displayed in [32]. Thus, through a mechanism different from that of nuclear rearrangement reactions, the present work strongly sustains the THM basic feature, namely the suppression of Coulomb effects in the two-body cross section at sub-Coulomb energies. This appears to be a universal effect whether we consider binary elastic or rearrangement processes. In conclusions, all these applications put on firmer grounds the applicability of the THM in nuclear astrophysics as well as in all physics contexts where it can be important to investigate nuclear effects at low energies.

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