Non-destructive testing of dielectric layers with defects

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Abstract. A microwave imaging method for non-destructive testing of dielectric surfaces beyond a layered media is presented. The method is based on the analytical continuation of the measured data to the surface under test through a special representation of the scattered field in terms of Fourier transform and Taylor expansion. Then the problem is reduced to the solution of a coupled system of non-linear integral equations which is solved iteratively via the Newton method with regularization in the least square sense. Numerical simulations show that defects having sizes in the order of $\lambda/200$ can be successfully recovered through the presented algorithm.

1. Introduction

Non-destructive testing (NDT) of materials is an important subject in the inverse scattering theory due to its wide range of practical applications such as but not limited to, automotive industry, medicine, aerospace engineering, construction etc. In a typical NDT problem the structure under test is excited by a certain type of field or wave and the reaction is measured on a region (usually non-contact to the structure) to extract the desired properties of the material under test. According to the physical configuration, number of methodologies have been developed such as magnetic particle method, eddy current method, ultrasonics, visual-optical methods, infrared thermography, acoustics, electromagnetics etc [1-5]. In electromagnetic applications microwave signals are capable of penetrating inside the dielectric media, allowing the inspection of the surfaces which are not reachable or not tend to be destructed to test. These applications are very important especially in the areas of detection of the mechanical damages, irregularities or cracks on coated surfaces of vehicles or on dielectric surfaces beyond layered media.

In this study, a method to determine the location and the shape of damages, irregularities, cracks, etc. on a dielectric surface separating two lossy dielectric media beyond a dielectric half space is presented. For the sake of simplicity, we consider surfaces having variation only in one space dimension. A single illumination of plane electromagnetic wave with a fixed frequency is used for excitation and the scattered field measurements are performed on a line parallel to the boundary of the upper half space. The method is based on a special representation of the scattered field in each region where the Fourier transform and Taylor expansion are used together. Then the problem is reduced to the solution of a non-linear algebraic equation which is solved iteratively by the use of classical Newton Method. The presented method is tested by some numerical simulations and satisfactory results are obtained.
2. NDT Method
Consider the problem illustrated in Figure 1.

In this configuration $\Gamma_0$ is the destructed surface which is lying between two layers with electromagnetic parameters $\varepsilon_1, \mu_1, \sigma_1$ and $\varepsilon_2, \mu_2, \sigma_2$, respectively. The upper half space is assumed to be free space. The surface under test can be a flat or a rough one which is represented by a single-valued and continuous function $x_2 = f(x_1)$. $\Gamma_0$ is assumed to be locally rough, i.e.: $f(x_1)$ differs from zero over a finite interval which has a length of $L_0$. The main aim of the non-destructive testing problem considered here is to reconstruct the possible defects $D_1, D_2, \ldots$ on the surface through a set of scattered electromagnetic field measurements performed in the accessible domain $x_2 > h$, where $h$ represents the boundary between the layers above the surface under test. The measurements have also been performed in the same region. For the sake of simplicity, it will be assumed that the incident field is a TM polarized time-harmonic plane wave whose electric field vector is given by $\vec{E}^i = (0, 0, u^i(x_1, x_2))$ with $u^i(x_1, x_2) = e^{-i\omega(x_1 \cos \phi_0 + x_2 \sin \phi_0)}$ where $\phi_0$ is the incident angle while $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$. Due to the homogeneity in the $x_3$ direction, the total and the scattered field vectors will have only $x_3$ components and the problem is reduced to a scalar one in terms of the total field function $u(x)$ which satisfies the Helmholtz equation

$$\Delta u + k^2 u = 0$$

where,

$$k^2(x) = \begin{cases} k_1^2, & x > f(x_1) \\ k_2^2, & x > f(x_1) \end{cases}$$

(2.2)
In order to formulate the problem in an appropriate way we first decompose the total field as

\[
 u(x) = \begin{cases} 
 u_1^s(x) + u'(x), & x_2 > h \\
 u_2^s(x), & h > x_2 > f(x_1) \\
 u_3^s(x), & x_2 < f(x_1) 
\end{cases}
\]

(2.3)

where the functions \( u_1^s \), \( u_2^s \) and \( u_3^s \) are the contributions of the defects and/or the roughness of the surface to the total field in the regions \( x_2 > h \), \( x_2 \in (f(x_1), h) \) and \( x_2 < f(x_1) \) respectively.

The boundary conditions imposed on the total field yield,

\[
 u_1^s + u' = u_2^s \quad \text{on} \quad x_2 = h 
\]

(2.4)

\[
 \frac{\partial (u_1^s + u')}{\partial x_2} = \frac{\partial u_2^s}{\partial x_2} \quad \text{on} \quad x_2 = h 
\]

(2.5)

\[
 u_2^s = u_3^s \quad \text{on} \quad x_2 = f(x_1) 
\]

(2.6)

\[
 \frac{\partial u_3^s}{\partial x_2} = \frac{\partial u_2^s}{\partial x_2} \quad \text{on} \quad x_2 = f(x_1) 
\]

(2.7)

under the appropriate radiation condition for \( x_2 \to \infty \).

In the following we will give a special representation of the scattered field by the use of Fourier Transform and Taylor expansion. To this aim let us first define the Fourier Transform of \( u_1^s \) as:

\[
 \tilde{u}_1^s(\nu, x_2) = \int_{-\infty}^{\infty} u'(x_1, x_2) e^{-i\nu x_1} dx_1, \quad x_2 > h 
\]

(2.8)

The Fourier transform of the reduced wave equation for \( u_1^s \) yields

\[
 \frac{d^2 \tilde{u}_1^s}{dx_2^2} - \gamma_0^2 \tilde{u}_1^s = 0, \quad x_2 > h 
\]

(2.9)

where \( \gamma_0(\nu) = \sqrt{\nu^2 - k_0^2} \) is the square root function defined in the complex cut \( \nu \)-plane as \( \gamma_0(0) = -ik_0 \). The solution of (2.9) can be given as

\[
 \tilde{u}_1^s(\nu, x_2) = A(\nu)e^{-\gamma_0 x_2}, \quad x_2 > h 
\]

(2.10)

by taking the radiation condition into account. Here \( A(\nu) \) is the unknown spectral coefficient. Then by applying the inverse Fourier Transform one can express \( u_1^s \) as
\[ u_i^*(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(v) e^{i\nu x_1 - \gamma_1(v) x_2} dv, \quad x_2 > h \]  

(2.11)

The same procedure can be easily applied for the scattered fields in the regions \( x_2 \in (\beta, h) \) and \( x_2 < \alpha \), in which \( \beta \geq \max(f(x_1)) \) and \( \alpha \leq \min(f(x_1)) \) (see Figure 1) where there is no discontinuity in the \( x_1 \)-direction. Thus,

\[ u_i^*(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( B(v) e^{-\gamma_1(v) x_2} + C(v) e^{\gamma_1(v) x_2} \right) e^{i\nu x_1} dv, \quad \beta < x_2 < h \]  

(2.12)

\[ u_i^*(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} D(v) e^{i\nu x_1 + \gamma_2(v) x_2} dv, \quad x_2 < \alpha \]  

(2.13)

where \( \gamma_1(v) = \sqrt{v^2 - k_1^2} \) and \( \gamma_2(v) = \sqrt{v^2 - k_2^2} \) with \( \gamma_1(0) = -ik_1 \) and \( \gamma_2(0) = -ik_2 \) while \( B(v), C(v) \) and \( D(v) \) are the spectral coefficients to be determined.

Now assume that the scattered field is measured on a line, which is parallel to the layers, in the upper half space i.e.; \( u_i^*(x_1, l) \), \( l > h \) is known for all \( x_1 \in \mathbb{R} \). Inserting \( x_2 = l \) into (2.11) we observe that, in agreement with (2.10) the spectral coefficient \( A(v) \) can be determined from the Fourier transform via

\[ A(v) = \hat{u}_i^*(v, \ell) e^{i\nu(v) \ell}. \]  

(2.14)

Since now \( A(v) \) is known, using boundary conditions (2.4) and (2.5) on \( x_2 = h \), one can obtain the unknown spectral coefficients \( B(v) \) and \( C(v) \) related to the region \( x_2 \in (f(x_1), h) \) from the following equations

\[ u_i(x_1, h) + u_i^*(x_1, h) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( B(v) e^{-\gamma_1(v) h} + C(v) e^{\gamma_1(v) h} \right) e^{i\nu x_1} dv \]  

(2.15)

\[ \frac{\partial u_i^*(x_1, x_2)}{\partial x_2} \bigg|_{x_2 = h} + \frac{\partial u_i(x_1, x_2)}{\partial x_2} \bigg|_{x_2 = h} = \frac{1}{2\pi} \left( B(v)(-\gamma_1(v)) e^{-\gamma_1(v) h} + C(v)(\gamma_1(v)) e^{\gamma_1(v) h} \right) e^{i\nu x_1} dv. \]  

(2.16)

The regularized solution of \( B(v) \) and \( C(v) \) in the sense of Tikhonov are obtained by discretizing the equations (2.15) and (2.16).

To be able to find approximate expressions for the scattered field in the regions \( x_2 \in (f(x_1), \beta) \) and \( x_2 \in (\alpha, f(x_1)) \), we use the Taylor expansions of the scattered field:

\[ u_i^*(x) = \sum_{m=0}^{M} \frac{1}{m!} \frac{\partial^m u_i^*(x_1, \beta)}{\partial x_2^m} (x_2 - \beta)^m + R_m(x), \quad f(x_1) < x_2 \leq \beta \]  

(2.17)
where the remainder terms are:

\[ R_M(x) = \frac{1}{M!} \beta \int \frac{\partial^{M+1} u_i^i(x, \xi)}{\partial \xi^{M+1}} d\xi \]

\[ Q_N(x) = \frac{1}{N!} \alpha \int \frac{\partial^{N+1} u_i^i(x, \xi)}{\partial \xi^{N+1}} d\xi \]

The \( m \) th order derivatives appearing in (2.17) and (2.18) can be obtained in the form of

\[
\frac{\partial^m u_i^i(x, \beta)}{\partial \xi^m} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \left(-\gamma_1(\nu) \right)^m B(\nu) e^{-\gamma_1(\nu)\beta} + \left[\gamma_1(\nu) \right]^m C(\nu) e^{\gamma_1(\nu)\beta} \right) e^{i\nu x} d\nu
\]

\[
\frac{\partial^m u_i^i(x, \alpha)}{\partial \xi^m} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \left[\gamma_2(\nu) \right]^m D(\nu) e^{\gamma_2(\nu)\beta} \right) e^{i\nu x} d\nu
\]

This representation of the scattered field together with the boundary conditions (2.6) and (2.7) reduces the problem to a system of nonlinear equations which comprises the spectral coefficient \( D(\nu) \) related to the scattered field \( u_i^i \) and the variation of the rough surface \( f(x_i) \).

Now we proceed by substituting the pairs (2.17) and (2.18) into the boundary conditions (2.6) and (2.7) and by neglecting the remainder terms to obtain

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} D(\nu) \Phi_1(f)(x_i, \nu) e^{i\nu x} d\nu = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( B(\nu) \Phi_1(f)(x_i, \nu) + C(\nu) \Phi_2(f)(x_i, \nu) \right) e^{i\nu x} d\nu
\]

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} D(\nu) \Psi_1(f)(x_i, \nu) e^{i\nu x} d\nu = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( B(\nu) \Psi_1(f)(x_i, \nu) + C(\nu) \Psi_2(f)(x_i, \nu) \right) e^{i\nu x} d\nu
\]

where

\[
\Phi_1(f)(x_i, \nu) = e^{-\gamma_1(\nu)\beta} \sum_{m=0}^{M} \left[\gamma_1(\nu) \right]^m m! \left( f(x_i) - \beta \right)^m
\]

\[
\Psi_1(f)(x_i, \nu) = e^{-\gamma_1(\nu)\beta} \sum_{m=1}^{M} \left[\gamma_1(\nu) \right]^m (m-1)! \left( f(x_i) - \beta \right)^{m-1}
\]

\[
\Phi_2(f)(x_i, \nu) = e^{\gamma_1(\nu)\beta} \sum_{m=0}^{M} \left[\gamma_1(\nu) \right]^m m! \left( f(x_i) - \beta \right)^m
\]

\[
\Psi_2(f)(x_i, \nu) = e^{\gamma_1(\nu)\beta} \sum_{m=1}^{M} \left[\gamma_1(\nu) \right]^m (m-1)! \left( f(x_i) - \beta \right)^{m-1}
\]

\[
\Phi_3(f)(x_i, \nu) = e^{\gamma_2(\nu)\beta} \sum_{m=0}^{M} \left[\gamma_2(\nu) \right]^m m! \left( f(x_i) - \alpha \right)^m
\]

\[
\Psi_3(f)(x_i, \nu) = e^{\gamma_2(\nu)\beta} \sum_{m=1}^{M} \left[\gamma_2(\nu) \right]^m (m-1)! \left( f(x_i) - \alpha \right)^{m-1}
\]
\[ \Psi_3 (f)(x_1, \nu) = e^{c_2(\nu)\mu} \sum_{m=1}^{M} \frac{\gamma_2(\nu)^m}{(m-1)!} (f(x_1) - \alpha)^{m-1} . \] (2.30)

A more compact expression for the system above is given by the following operator equations:

\[ K_1 (D(\nu), f) = g_1 (f) \] (2.31)
\[ K_2 (D(\nu), f) = g_2 (f) \] (2.32)

In (2.31) and (2.32) \( K_1 \) and \( K_2 \) are non-linear operators with respect to \( f(x_1) \) while they are linear with respect to \( D(\nu) \).

Thus now the problem is reduced to the solution of this non-linear system which can be treated by iterative techniques. In the application of the iterative scheme we first choose an initial guess for the unknown surface variation \( f \). Using this initial guess it is now easy to solve one of the equations mentioned above to obtain the spectral coefficient \( D(\nu) \). Note that since both integral equations given by (2.31) and (2.32) are of the first kind one has to apply some regularization techniques. Here again Tikhonov regularization is applied. Once we have obtained the unknown spectral coefficient from one of the equations say the first one, surface variation \( f \) can be obtained by solving the other non-linear equation, which can also be written in an operator form,

\[ F_M (f) = K_2 (D, f) - g_2 (f) = 0. \] (2.33)

The latter one can be solved iteratively via Newton method. To this aim, for an initial guess \( f_0 \), the nonlinear equation is replaced by the linearized one

\[ F_M (f_0) + F_M' (f_0) \Delta f = 0 \] (2.34)

that needs to be solved for \( \Delta f = f - f_0 \) in order to improve an approximate boundary \( \Gamma_0 \) given by the function \( f_0 \) into a new approximation with surface function \( f_0 + \Delta f \). In (2.34), \( F_M' \) denotes the Frechet derivative of the operator \( F \) with respect to \( f \). It can be shown that \( F_M' \) reduces the ordinary derivative of \( F_M \) with respect to \( f \). The Newton method consists in iterating this procedure, i.e.: in solving

\[ F_M' (f_0) \Delta f = -F_M (f_0), \quad i = 0, 1, 2, 3, \ldots \text{ for } \Delta f_{i+1} = f_i + \Delta f_{i+1}. \] (2.35)

It is obvious that this solution will be sensitive to errors in the derivative of \( F_M \) in the vicinity of zeros. To obtain a more stable solution, the unknown \( \Delta f \) is expressed in terms of some basis functions \( \phi_n (x_i), n = 1, \ldots, N \), as a linear combination

\[ \Delta f (x_i) = \sum_{n=1}^{N} a_n \phi_n (x_i). \] (2.36)
A possible choice of basis functions consists of trigonometric polynomials. Then (2.35) is satisfied in the least squares sense, that is, the coefficients \(a_1, ..., a_N\) in (2.36) are determined such that for a set of grid points \(x_1^j, ..., x_J^j\) the sum of squares
\[
\sum_{j=1}^J \left| \sum_{n=-N}^N a_n \phi_n(x_j^j) + F_M(f(x_j^j)) \right|^2
\]
is minimized. The number of basis functions \(N\) in (2.36) can also be considered as a kind of regularization parameter. Choosing \(N\) too big leads to instabilities due to the ill-posedness of the underlying inverse problem; accordingly too small values of \(N\) results in poor approximation quality. Hence one has to compromise between stability and accuracy and in this sense \(N\) serves as a regularization parameter.

3. Numerical Results
In this section some numerical results which demonstrate the validity and effectiveness of the method will be presented. In all the examples the upper space where the sources and observation points are located is assumed to be free-space and the operating frequency is chosen as 12 GHz. 1% random noise is added to the simulated data for the scattered field. In the application of least squares solution the basis functions are chosen as combinations of \(\cos(\frac{2\pi nx_i}{L_0})\) and \(\sin(\frac{2\pi nx_i}{L_0})\), \(n = 0, \pm 1, ..., \pm N\), and the number \(N\) is determined by trial and error.

In the first example the dielectric surface is located above a non-magnetic painting material having electromagnetic parameters \(\varepsilon_2 = 7\varepsilon_0\), \(\sigma_2 = 10^{-4}\) and below a non-magnetic painting material having electromagnetic parameters \(\varepsilon_1 = 4\varepsilon_0\), \(\sigma_1 = 10^{-4}\). The reconstruction of the circular defects on a planar surface shown in Figure 2 is obtained for the truncation number \(M = 5\) in the Taylor expansion for 2 iterations. The method determines the locations and the shapes of the defects having depths in the order of \(\frac{\lambda}{200}\) very accurately. The results given in the Figure 3 shows that the method can be effectively used for reconstruction of the defects on curved surfaces. It shows the reconstruction obtained for the truncation numbers \(M=5\) with 3 iterations.

![Figure 2. Reconstruction of defects on planar surface](image-url)
4. Conclusion
The method presented in [6] is extended to the non-destructive evaluation of the dielectric surfaces beyond a layered media. Although the method is developed for only two layers it can be extended easily to the multilayered cases. The method is very effective for defects having sizes in order of $\lambda/200$ for an operating frequency of 12 GHz. Future studies are devoted to extend the method for 2D surfaces.

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