3D relativistic MHD simulations of bow-shock Pulsar Wind Nebulae with highly asymmetric geometry

Maxim V. Barkov$^{1,2}$, Maxim Lyutikov$^1$ and Dmitry Khangulyan$^3$

$^1$Department of Physics and Astronomy, Purdue University, West Lafayette, IN 47907-2036, USA
$^2$Astrophysical Big Bang Laboratory, RIKEN, 351-0198 Saitama, Japan
$^3$Department of Physics, Rikkyo University, Nishi-Ikebukuro 3-34-1, Toshima-ku, Tokyo 171-8501, Japan

E-mail: mbarkov@purdue.edu

Abstract. Bow-shock pulsar wind nebulae (PWNe) show a variety of morphological shapes. We attribute this diversity to the geometrical factors: relative orientations of the pulsar rotation axis, proper velocity, and the line of sight. Here we report study of morphology in the most complicated geometry, when the pulsar rotation axis and its proper velocity makes an arbitrary angle (in the simulations we adopt a value of $\sim \pi/4$). Using 3D RMHD simulations we obtain the MHD structure of PWN and compute synthetic synchrotron emissivity maps.

1. Introduction

Pulsars produce relativistic magnetized winds that create pulsar wind nebulae (PWNe) ([1] and references therein). A distinct type of PWNe is produced by runaway pulsar, i.e. fast moving pulsars that quickly escape from the supernova remnant (for a recent review see Ref.[2]). Runaway pulsars can have velocities of hundreds kilometers per second, significantly exceeding the typical sound speeds in the interstellar medium (ISM), $c_{s,\text{ISM}} = 10^{-100}$ km s$^{-1}$. The interaction of such pulsars with the ISM produces a bow-shock nebula with an extended tail.

Pulsar proper velocity and rotation axis are often aligned allowing axial symmetric two dimensional treatment of these systems. A number of analytical and numerical models were developed under this assumption (e.g. [3, 4]). Also this configuration was considered in three dimensional (3D) hydrodynamic limit [5] and recently with relativistic magnetohydrodynamics (RMHD) approach [6]. However, the alignment does not always take place, and such not-aligned systems require a 3D MHD simulations in complex geometries.

The structure of magnetic fields in the bow-shock PWNe is, generally, a complicated mixture of the wind-structure produces by the pulsar, and the interaction of the wind with the ISM. The formation of wind produced on the magnetosphere scale, which is much smaller than the termination shock (TS) radius. Thus, the formation of pulsar wind in the bow-shock PWNe is not affected by the processes taking place in the PWN. Thus, one can expect that the properties of pulsar winds in PWNe formed by fast- and slow-moving pulsars are similar. Thus, we utilize the model of the pulsar wind that is used in RMHD modeling of the Crab Nebula. Namely, we assume that on scales larger than the light cylinder radius, the energy flux in the wind (which is mostly Poynting power) changes as $L(\theta_p) \propto \sin^2 \theta_p$, where $\theta_p$ is the angle to the pulsar rotation.
Figure 1. PSR J1135−6055 0.3-10 keV Chandra Imaging from Ref.[7]. The two yellow ellipses highlight two distinct features of the nebular: a single-sided bent jet and an extended tail.

axis [8, 9, 10]. In the equatorial region, occupying a section of the polar angles of the order of the pulsar magnetic inclination angle, the magnetic field is reversing polarity each half a period, while at larger latitudes it is unidirectional [9, 11]. As the wind propagates from the light cylinder, the magnetic field is dissipated in the region of field reversals [12, 13]. Thus, the magnetization \( \sigma \) in the wind is small near the equator and can be significant in the polar region.

In addition to the fairly complected structure of the winds of slow-moving pulsars, for bow-shock pulsars there is an additional complication due to the pulsar motion. This introduces another special direction - along the pulsar velocity - and generally makes the whole structure non-axisymmetric. Thus, the structure of the magnetic field depends on two geometrical factors: (i) the angle between the pulsar rotation axis and the magnetic moment; (ii) the angle between the pulsar rotation axis and the direction of motion. Obviously, the observational appearance of the formed complex 3D structure depends strongly on the line-of-sight direction.

Three distinct geometric cases were considered (see Ref. [6] and in Fig. 2): (i) “Rifle Bullet” geometry – when the rotation axis is aligned with the direction of motion. In this case the whole system has a cylindrical symmetry with concentric areas of toroidal fields of changing polarity. (ii) “Frisbee” geometry – when the rotation axis is perpendicular to the direction of motion and is in the plane of the sky; (iii) “Cart Wheel” geometry – when the rotation axis is perpendicular both to the direction of motion and the plane of the sky. The two physically distinct geometries, rifle-bullet and frisbee – cart-wheel, will have very different magnetic fields in the tail, Fig. 3. Here we consider a mixed case when the angle between the pulsar rotation axis and proper velocity is \( \pi/4 \). We aim at qualitatively explaining the morphology of the PWN formed around PSR J1135−6055 (see in Fig. 1). A detailed consideration of different configurations is presented in Ref. [6].

2. 3D RMHD simulations of bow-shock PWNe

We performed 3D RMHD simulations of the interaction of magnetized pulsar wind with magnetized external medium using a 3D geometry in Cartesian coordinates using the PLUTO code\(^1\) [14]. Spatial parabolic interpolation, a 3rd order Runge-Kutta approximation in time, and an HLL Riemann solver are utilized [15]. PLUTO is a modular Godunov-type code entirely written in C and intended mainly for astrophysical applications and high Mach number flows in multiple spatial dimensions. The simulations were performed on CFCA XC30 cluster of National Astronomical Observatory of Japan (NAOJ). The flow has been approximated as an

\(^1\) Link http://plutocode.ph.unito.it/index.html
ideal, relativistic adiabatic gas, one particle species, and polytropic index of 4/3. The simulations were performed in a domain of size $x \in [-4, 10]$, $y$ and $z \in [-5, 5]$. The initial ISM velocity is directed along the $x$-axis. To achieve a good resolution in the central region and long tail zone, which are the mostly relevant for studying the PWN morphology, we use non uniform resolution stretched grid with the total number of cells $N_X = 468$, and $N_Y = N_Z = 336$, see Table 1 for details.

In this work we use the prescription of pulsar wind similar to what was used by [16]. The pulsar with radius 0.2 is placed at the origin. The pulsar produces supersonic magnetized pulsar wind with a toroidal magnetic field that changes its polarity in northern and southern hemispheres. For the total energy flux density of the wind we adopt the monopole model [8, 9]

$$ f_{\text{tot}}(r, \theta_p) = L_0 \left( \frac{1}{r} \right)^2 \left( \sin^2 \theta_p + g \right). $$

(1)

where we added $g = 0.03$ to avoid vanishing energy flux at the poles.

In the wind the energy is distributed between the magnetic $f_m$ component

$$ f_m(r, \theta_p) = \frac{\sigma(\theta_p) f_{\text{tot}}(r, \theta_p)}{1 + \sigma(\theta_p)} $$

(2)

and kinetic $f_k$ one

$$ f_k(r, \theta_p) = \frac{f_{\text{tot}}(r, \theta_p)}{1 + \sigma(\theta_p)}, $$

(3)

where $\sigma(\theta_p)$ is the wind magnetization, which depends on the co-latitude, $\theta_p$ measured relative to the pulsar spin axis.

---

**Table 1. Parameters of the Grid**

| Coordinates | Left | $N_l$ | Left-center | $N_c$ | Right-center | $N_r$ | Right |
|-------------|------|-------|-------------|-------|--------------|-------|-------|
| $X$         | -4   | 72    | -1          | 144   | 1            | 252   | 10    |
| $Y$         | -5   | 96    | -1          | 144   | 1            | 96    | 5     |
| $Z$         | -5   | 96    | -1          | 144   | 1            | 96    | 5     |

Figure 2. Basic geometries: rifle-bullet (with the spin of the NS aligned with the velocity), frisbee (with the spin of the NS perpendicular to the velocity but lying in the plane of the sky), and cart-wheel (with the spin of the NS perpendicular both to the velocity and the plane of the sky). The central doughnut-like structure indicates the distribution of wind power, $\propto \sin^2 \theta_p$, where $\theta_p$ is the pulsar rotation axis; $\theta$ is a polar angle with respect to the velocity.
Figure 3. Magnetic field in the rifle-bullet (left) and frisbee/cart-wheel (right) geometries. In case of a rifle-bullet geometry (NS spin oriented along the velocity), the pulsar produces axisymmetric current flows and magnetic fields. Far in the tail the electric currents form a triple sequence of concentric regions with oppositely directed current flows (low left). For frisbee, (NS spin is orthogonal to the velocity; Sun is in a frisbee/cart-wheel configuration), the pulsar produces three adjacent regions of opposite current flows (low right).

As one adopts a torroidal geometry of the magnetic field, the numerical stability of the code requires vanishing of the magnetic field close to the polar axis. This is achieved by introducing the following dependence of the pulsar wind magnetization:

$$\sigma_0(\theta_p) = \sigma_0 \min \left(1, \frac{\theta_p^2}{\theta_0^2}, \frac{(\pi - \theta_p)^2}{\theta_0^2} \right)$$

(4)

where $\theta_0$ is a small parameter, which was set to 0.2.

Near the equator the alternating components of magnetic field are assumed to annihilate, leaving a low-magnetized equatorial sector with magnetization varying according to

$$\sigma(\theta_p) = \frac{\sigma_0(\theta_p) \chi_\alpha(\theta_p)}{1 + \sigma_0(\theta_p)(1 - \chi_\alpha(\theta_p))},$$

(5)
Table 2. Parameters of the model

| θ | ψ | σ₀ | α |
|---|---|----|---|
| π/4 | π/4 | 1 | π/4 |

where

\[ \chi_\alpha(\theta_p) = \begin{cases} 
(2\phi_\alpha(\theta_p)/\pi - 1)^2, & |\pi/2 - \theta_p| < \alpha \\
1, & \text{otherwise}
\end{cases} \]

and \( \cos \phi_\alpha(\theta_p) = -\cot(\theta_p)\cot(\alpha) \). The angle \( \alpha \) is an angle between magnetic axis and pulsar rotation axis (see Ref. [17] for detail). The pulsar wind was injected with the initial Lorentz factor, \( \Gamma = 2.9 \).

We start our simulation with a non-equilibrium configuration and evolve it until a quasi-stationary solution is settled. From the left edge (\( X = -4 \)) we inject ISM. To reduce computational expenses we set the initial ISM speed to \( v_{\text{ISM}} = 0.1c \), which corresponds to the Mach number of \( M = 85 \). The density of the ISM was adopted so that in the case of non-magnetized spherical pulsar wind the TS radius equals 1. The adopted initial ISM speed is not realistic, but it does not affect significantly the region inside the contact discontinuity [18].

The ISM flow carries a weak magnetic field with magnetization \( \sigma_{\text{ISM}} = 0.01 \); the ISM magnetic field is directed along the \( z \)-axis.

We choose the intermediate configuration, frisbee – rifle-bullet. The orientation is determined by two angles \( \theta \) (clockwise turn around Y axis) and angle \( \psi \) (clockwise turn around axis Z). The intermediate case has \( \theta = \pi/4 \) and \( \psi = \pi/4 \). The parameters of the models are presented in Table 2. The magnetization of pulsar wind is \( \sigma_0 = 1 \), and pulsar magnetic inclination is \( \alpha = \pi/4 \).

Results are shown in Fig. 4. As it was expected the whole structures is highly asymmetric. This is due to the effects of magnetic hoop stresses near the rotational axis: in the head part the wind is slowed down and efficiently confined by the ISM ram pressure. This allows magnetic stresses to get accumulated and produce larger distortion than in the tailward part. Thus, the front outflow forms a narrow jet-like structure. The tail is formed by the back “jet” and partially by a matter and magnetic field from front “jet” which was turned backwards near the head of the bow shock. Turned back flow has a magnetic field directed differently as compared to the back “jet”. This provides sites suitable for the magnetic field reconnection in the tail. A similar configuration is formed in the pure rifle-bullet configuration (see Ref. [6]). Finally, we note that the shape of pulsar wind TS is complicated, similar to the one revealed with 3D hydrodynamic simulations in Ref. [5] (see also Ref. [6]).

3. Synchrotron synthetic maps

The observed X-ray emission from PWNe is generated via synchrotron radiation by non-thermal particles, which are presumably accelerated at shocks and/or in the reconnection events within the PWNe. Conventional ideal RMHD simulations produce only hydrodynamic quantities – density, thermal pressure, velocity, and magnetic field. Thus, we have no direct information about energy distribution and density of non-thermal particles. To obtain this additional information one needs to perform dedicated simulation of evolution of non-thermal particles (see e.g. [19, 20]). However, if the particle cooling is dominated by adiabatic losses, one can use a simplified approach and reconstruct the spectrum of non-thermal particles based on MHD...
Figure 4. 3D rendering in the mixed rifle-bullet – frisbee configuration. Top row: Plasma density (bottom colour scale, logarithmic) and the velocity field (arrows, upper colour scale) and Pressure/velocity plot at bottom right.

parameters only (see [21]).

In bow-shock PWNe, the strength of the magnetic field might be quite high, exceeding the field inferred in PWNe around slow-moving pulsars. Although the structure of the magnetic field in PWNe is quite complicated, the characteristic magnetic field can be obtained from the pressure, \( \rho_{\text{ISM}} V_{\text{NS}}^2 \), required to support the nebula. Thus, one obtains

\[
B \sim 2 \times 10^{-4} \rho_{\text{ISM},0} V_{\text{NS},7.5} G.
\]

(7)

For the magnetic field of strength \( B' \) and photon energy \( \epsilon' \) (both in flow co-moving frame) the required Lorentz factor of the radiating particles is

\[
\gamma_{\text{syn}} \simeq \sqrt{\frac{2 m_e c \epsilon'}{3 e h B'}} = 7 \times 10^6 \epsilon_{\gamma,1 \text{ keV}}^{1/2} B_t^{-1/2}.
\]

(8)

which should be easily attainable for non-thermal particles in PWNe (e.g. in Crab Nebula one expects particle acceleration to PeV energies, see [19, 22]).

The corresponding synchrotron cooling time is

\[
t_{\text{syn}} \simeq 10^8 \epsilon_{\gamma,1 \text{ keV}}^{-1/2} B_t^{-3/2} \text{ s}.
\]

(9)
This cooling time should be compared to adiabatic cooling time, which can be estimated as the
time required for the flow to cross the characteristic hydrodynamic scale, e.g. \( r_s \):

\[ t_{AD} \approx \frac{r_s}{c} \approx 10^6 n_{\text{ISM},0}^{-1/2} V_{NS,7.5}^{-1} L_{w,36}^{1/2} \text{s}. \]  

The ratio of synchrotron and adiabatic cooling times is

\[ \frac{t_{\text{SYN}}}{t_{AD}} \approx 5 \times 10^2 \epsilon^{-1/2} \gamma^{-1/4} n_{\text{ISM},0}^{-1/4} V_{NS,7.5}^{-1/2} L_{w,36}^{-1/2} \]  

which implies that in a bow-shock PWNe the cooling of particles, responsible for X-ray emission,
proceeds predominately due to adiabatic losses. Thus, one can utilize the simple approach for
computing synchrotron radiation [21, 6].

To compute synthetic synchrotron emission maps, we follow a procedure outlined below.

- Our simulations produce 3D distribution of pressure, density, velocity and magnetic field.
- Using proper \( p \) and \( B' \) we calculate the synchrotron emissivity according to Ref. [6].
- For given local synchrotron emissivity we then choose a given line of sight and integrate the
emissivity, assuming optically thin regime and taking into account the local velocity and
the corresponding Doppler factor.

To make clearly visible the X-ray morphology, we use different quantities to produce the
synthetic maps, depending on the orientation of the line-of-sight. Namely, we found that the
emission intensity maps are more illustrative in the case if the pulsar moves toward the observer.
If the pulsar moves side way, then we plot the square-root of the emission intensity. The latter
quantity is somewhat arbitrary, chosen to allow a better highlighting of faint X-ray features.

In Fig. 5 we show synthetic synchrotron maps for the considered configuration. If such a
system is seen along the axis, then the morphology is similar to the frisbee case [6]. If the PWN
is seen side way, then the PWN appears as an asymmetric bow-shock structure (Fig. 5, right
panel) - the front/up “jet” is brighter and narrower as compared to the down/back “jet”. Thus,
this simulations allow us to reproduce the X-ray structure of PWN formed by PSR J1135−6055:
a single-sided bent jet and an extended tail (see Fig. 1) [7]).

The considered RMHD model also explains the apparent low X-ray efficiency of the bow-
shock PWNe [23, 2]. Bow-shock PWNe are very inefficient in converting spin-down luminosity

---

**Figure 5.** Synchrotron emissivity map projected along X (top) and Y (bottom) axis.
in X-rays, with efficiencies $\sim 10^{-3} - 10^{-5}$ [2]. These values are typically much smaller than for slow-moving pulsars, where the conversion efficiency can be as high as tens of per cents e.g. [1].

Such low effectiveness can be explained by strong adiabatic loses $L_{\text{X-ray}} / E_{\text{sd}} \sim t_{\text{ad}} / t_{\text{syn}} \ll 1/1000$ in the head of PWNe. In bow-shock PWNe the crossing time of the relativistic plasma through the tail is shorter than the synchrotron cooling time. As a results, particles are able to emit only small fraction of the energy that they acquired during acceleration at the reverse shock or in reconnection sites - adiabatic cooling dominates and most of the wind luminosity is spent on $pV$ work inflating the bubble at the large distance from a pulsar.

This model may also explain the apparent disagreement between the estimate of the magnetic field from equipartition arguments (even initially weakly magnetized flow after the shock transition reaches approximate equipartition) and the observed length of the X-ray tail [23] - particles are quickly adiabatically cooled.

4. Conclusion

In this work we present analytical and numerical 3D MHD calculations of the interaction of relativistic wind produced by fast moving pulsars relative to the interstellar medium in highly asymmetric geometry. We capture both the flow dynamics in the head part of the resulting bow-shock PWNe, as well as the evolution of the flow in the tail part. Our results indicate that magnetic fields play the most important role in shaping the morphology of the bow-shock PWNe. We also compute synthetic synchrotron intensity maps for frisbee – rifle-bullet geometry. Our synchrotron emissivity maps can reproduce the key features revealed with Chandra X-ray observation of PSR J1135–6055: a single-sided bent jet and an extended tail. Finally, our model naturally addresses the small X-ray efficiencies seen from many bow-shock PWNe. The rate of adiabatic losses appeared to be significantly shorter than the synchrotron cooling time, thus the bulk of the spin-down losses is spent on mechanical work needed for PWN expansion.

Acknowledgments

We would like to thank Anatoliy Spitkovsky, Joseph Gelfand, Oleg Kargaltsev, Victoria Kaspi, Andrey Bykov and Mallory Roberts for numerous enlightening discussions.

The calculations were carried out in the CFCA cluster of National Astronomical Observatory of Japan. We thank the PLUTO team for the possibility to use the PLUTO code and for technical support. The visualization of the results performed in the VisIt package [24]. This work had been supported by NSF grant AST-1306672, DoE grant DE-SC0016369 and NASA grant 80NSSC17K0757. D.K. acknowledges support by JSPS KAKENHI Grant Number JP18H03722.

References

[1] Kargaltsev O, Cerutti B, Lyubarsky Y and Striani E 2015 SSRv 191 391–439 (Preprint 1507.03662)
[2] Kargaltsev O, Pavlov G G, Klingler N and Rangelov B 2017 Journal of Plasma Physics 83 635830501 (Preprint 1708.00456)
[3] Wilkin F P 1996 ApJ 459 L31
[4] Bucciantini N 2002 A&A 387 1066–1073 (Preprint astro-ph/0203504)
[5] Vigelius M, Melatos A, Chatterjee S, Gaensler B M and Ghavamian P 2007 MNRAS 374 793–808 (Preprint astro-ph/0610454)
[6] Barkov M V and Lutnikov M 2018 ArXiv e-prints (Preprint 1804.07327)
[7] Marelli M 2012 ArXiv e-prints (Preprint 1205.1748)
[8] Michel F C 1973 ApJ 180 207–226
[9] Bogovalov S V 1999 A&A 349 1017–1026 (Preprint arXiv:astro-ph/9907051)
[10] Bogovalov S V and Khangoulian D V 2002 MNRAS 336 L53–L55 (Preprint astro-ph/0209269)
[11] Komissarov S S and Lyubarsky Y E 2004 MNRAS 349 779–792
[12] Coroniti F V 1990 ApJ 349 538–545
[13] Lyubarsky Y E 2003 MNRAS 345 153–160 (Preprint astro-ph/0306435)
[14] Mignone A, Bodo G, Massaglia S, Matsakos T, Tesileanu O, Zanni C and Ferrari A 2007 ApJS 170 228–242 (Preprint arXiv:astro-ph/0701854)
[15] Harten A 1983 Journal of Computational Physics 49 357393 ISSN 0021-9991 URL http://www.sciencedirect.com/science/article/pii/0021999183901365
[16] Porth O, Komissarov S S and Keppens R 2014 MNRAS 438 278–306 (Preprint 1310.2531)
[17] Komissarov S S 2013 MNRAS 428 2459–2466 (Preprint 1207.3192)
[18] Barkov M, Lyutikov M, Klingler N and Bordas P 2018 in preparation
[19] Kennel C F and Coroniti F V 1984 ApJ 283 710–730
[20] Vaidya B, Mignone A, Bodo G, Rossi P and Massaglia S 2018 ApJ 865 144 (Preprint 1808.08960)
[21] Barkov M and Bosch-Ramon V 2018 in preparation
[22] Atoyan A M and Aharonian F A 1996 MNRAS 278 525–541
[23] Kaspi V M, Gotthelf E V, Gaensler B M and Lyutikov M 2001 ApJ 562 L163–L166 (Preprint astro-ph/0110188)
[24] Hank Childs H, Brugger E, Whitlock B and et al 2012 VisIt: An End-User Tool For Visualizing and Analyzing Very Large Data High Performance Visualization—Enabling Extreme-Scale Scientific Insight pp 357–372