Supporting information for "Silicon channeled spectropolarimeter for on-chip single-detector Stokes spectroscopy"

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1 The principle for generating a channeled spectrum

A schematic of the proposed silicon photonic channeled spectropolarimeter (SiPh-CSP) is shown in Fig. S1. Two orthogonal linearly polarized components of the incoming light, $E_x$ and $E_y$, are split into two separate waveguides by the surface polarization splitter (SPS). They then propagate into the channeled spectrum generator (CSG). The CSG comprises two pairs of differential delay lines that are connected by a $2 \times 2$ multimode interference (MMI) coupler. The differential lengths of the first and second pairs of differential delay lines are $L_1$ and $L_2$, respectively. Before they are combined in a waveguide Y-junction, the electric fields $E_{o1}$ and $E_{o2}$ of the second pair of differential delay lines can be calculated by

$$
\begin{align*}
\begin{pmatrix}
E_{o1}(\sigma) \\
E_{o2}(\sigma)
\end{pmatrix}
= T_2 C T_1
\begin{pmatrix}
E_x(\sigma) \\
E_y(\sigma)
\end{pmatrix},
\end{align*}
$$

(S1)

where $\sigma$ is the wavenumber; $T_1$ and $T_2$ are the transfer matrices of the first and second MZI circuits, respectively; $C$ is the transfer matrix of the MMI coupler. $T_1$ and $T_2$ are given by

$$
T_1 = \begin{pmatrix}
e^{i2\pi n_{eff}(\sigma)\sigma L_1} & 0 \\
0 & 1
\end{pmatrix},
T_2 = \begin{pmatrix}
e^{i2\pi n_{eff}(\sigma)\sigma L_2} & 0 \\
0 & 1
\end{pmatrix},
$$

(S2)

where $i$ represents an imaginary unit; $L_1$ and $L_2$ are the path length differences of the first and second MZI circuits, respectively; and $n_{eff}(\sigma)$ is the effective index of the waveguide at wavenumber $\sigma$. $C$ can be expressed as

$$
C = \begin{pmatrix}
\tau(\sigma) & 0 \\
\kappa(\sigma) e^{-i\pi} & \tau(\sigma)
\end{pmatrix},
$$

(S3)

where $\kappa(\sigma)$ and $\tau(\sigma)$ indicate the cross-coupling and straight-through coefficients of the MMI coupler, respectively. $\tau^2(\sigma) + \kappa^2(\sigma) = 1$. For a 3-dB MMI coupler, $\tau(\sigma) = \kappa(\sigma) = \sqrt{2}/2$.

The Stokes spectrum can be described by the spectrum of the Stokes vector $S(\sigma) = (S_0(\sigma), S_1(\sigma), S_2(\sigma), S_3(\sigma))^T$, and can be calculated by

$$
\begin{align*}
S_0(\sigma) &= |E_x(\sigma)|^2 + |E_y(\sigma)|^2 \\
S_1(\sigma) &= |E_x(\sigma)|^2 - |E_y(\sigma)|^2 \\
S_2(\sigma) &= E_x(\sigma) \cdot E_y^*(\sigma) + E_y(\sigma) \cdot E_x^*(\sigma) \\
S_3(\sigma) &= i [E_x(\sigma) \cdot E_y^*(\sigma) - E_y(\sigma) \cdot E_x^*(\sigma)]
\end{align*}
$$

(S4)

where $E_x^*(\sigma)$ and $E_y^*(\sigma)$ indicate the complex conjugates of $E_x(\sigma)$ and $E_y(\sigma)$, respectively. Based on Eqs. S1-S4, the channeled spectrum $I(\sigma)$ can be obtained by

$$
I(\sigma) = \frac{1}{2} \left| E_{o1}(\sigma) + E_{o2}(\sigma) \right|^2
$$

$$
= \frac{1}{2} \left( S_0(\sigma) + 2\kappa(\sigma) \cdot \tau(\sigma) \cdot S_1(\sigma) \cos \left[ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot L_2 + \frac{\pi}{2} \right] ight.
$$

$$
+ S_2(\sigma) \left\{ \kappa^2(\sigma) \cdot \cos \left[ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot (L_1 - L_2) \right] + \tau^2(\sigma) \cdot \cos \left[ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot (L_1 + L_2) \right] \right\}
$$

$$
- S_3(\sigma) \left\{ \kappa^2(\sigma) \cdot \sin \left[ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot (L_1 - L_2) \right] + \tau^2(\sigma) \cdot \sin \left[ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot (L_1 + L_2) \right] \right\} \right).
$$

(S5)

Equation S5 indicates that $I(\sigma)$ is linear with $S_0(\sigma)$, $S_1(\sigma)$, $S_2(\sigma)$, and $S_3(\sigma)$. $I(\sigma)$ also can be expressed as

$$
I(\sigma) = \frac{1}{2} \left( S_0(\sigma) + 2\kappa(\sigma) \cdot \tau(\sigma) \cdot S_1(\sigma) \cos \left[ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot L_2 + \frac{\pi}{2} \right] ight.
$$

$$
+ \kappa^2(\sigma) \cdot |S_{23}(\sigma)| \cos \left\{ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot (L_1 - L_2) + \arg [S_{23}(\sigma)] \right\}
$$

$$
+ \tau^2(\sigma) \cdot |S_{23}(\sigma)| \cos \left\{ 2\pi n_{eff}(\sigma) \cdot \sigma \cdot (L_1 + L_2) + \arg [S_{23}(\sigma)] \right\} \right),
$$

(S6)

where $S_{23}(\sigma) = S_2(\sigma) + iS_3(\sigma)$, and $\arg [S_{23}(\sigma)]$ means the argument of $S_{23}(\sigma)$. Based on Taylor’s theorem\textsuperscript{1}, $n_{eff}(\sigma) \cdot \sigma$ can
be expressed as

\[
n_{\text{eff}}(\sigma) \cdot \sigma = n_{\text{eff},0} \cdot \sigma_0 + \frac{\partial [n_{\text{eff}}(\sigma_0) \cdot \sigma]}{\partial \sigma} \cdot (\sigma - \sigma_0) + o(\sigma - \sigma_0)
\]

\[
= n_{\text{eff},0} \cdot \sigma_0 + \left[ n_{\text{eff},0} + \sigma_0 \frac{\partial n_{\text{eff}}(\sigma_0)}{\partial \sigma} \right] \cdot (\sigma - \sigma_0) + o(\sigma - \sigma_0)
\]

\[
= n_{\text{eff},0} \cdot \sigma_0 + n_{g,0} \cdot (\sigma - \sigma_0) + o(\sigma - \sigma_0)
\]

\[
= n_{g,0} \cdot \sigma + (n_{\text{eff},0} - n_{g,0}) \cdot \sigma_0 + o(\sigma - \sigma_0),
\]

where \( \sigma_0 \) is the center wavenumber, \( n_{g,0} \) and \( n_{\text{eff},0} \) are the group and effective index of waveguide, respectively, at wavenumber \( \sigma_0 \), \( o(\sigma - \sigma_0) \) represents the Peano’s form of the remainder and can be neglected in our calculation. Based on Eqs. S6 and S7, \( I(\sigma) \) can be rewritten as

\[
I(\sigma) = \frac{1}{2} \left[ S_0(\sigma) + 2 \kappa(\sigma) \cdot \tau(\sigma) \cdot S_1(\sigma) \cdot \cos \left[ 2\pi n_{g,0} \cdot L_2 \cdot \sigma + \phi_2(\sigma) + \frac{\pi}{2} \right] \right. \\
+ \left. \kappa^2(\sigma) \cdot |S_{23}(\sigma)| \cdot \cos \left[ 2\pi n_{g,0} \cdot (L_1 - L_2) \cdot \sigma + \phi_1(\sigma) - \phi_2(\sigma) + \arg [S_{23}(\sigma)] \right] \right] \\
+ \tau^2(\sigma) \cdot |S_{23}(\sigma)| \cdot \cos \left[ 2\pi n_{g,0} \cdot (L_1 + L_2) \cdot \sigma + \phi_1(\sigma) + \phi_2(\sigma) + \arg [S_{23}(\sigma)] \right] \right.
\]

where

\[
\phi_1(\sigma) = 2\pi L_1 \cdot \left[ (n_{\text{eff},0} - n_{g,0}) \cdot \sigma_0 + o(\sigma - \sigma_0) \right],
\]

and

\[
\phi_2(\sigma) = 2\pi L_2 \cdot \left[ (n_{\text{eff},0} - n_{g,0}) \cdot \sigma_0 + o(\sigma - \sigma_0) \right].
\]

**Figure S1.** Schematic of the proposed Si-CSP. The lengths of the waveguides are represented by \( L_a, L_b, L_a + L_1 \) and \( L_a + L_2 \). SPS: surface polarizaton splitter. CSG: channeled spectrum generator. VDMS: vernier dual-microring spectrometer. PD: photodetector.

**2 The principle of the compressive sensing method**

According to Eq. S5, the channeled spectrum can also be expressed as

\[
I(\sigma) = a_0(\sigma) S_0(\sigma) + a_1(\sigma) S_1(\sigma) + a_2(\sigma) S_2(\sigma) + a_3(\sigma) S_3(\sigma),
\]

where \( a_0(\sigma), a_1(\sigma), a_2(\sigma) \) and \( a_3(\sigma) \) are the linear operators that depend on the structure of the device instead of the input Stokes spectrum. The operation from the Stokes spectrum to a channeled spectrum is therefore linear, and the linear operator of the proposed device can be easily obtained by injecting four known Stokes spectra. Using the linear operator, we can generate the virtual channeled spectrum for each estimated Stokes spectrum. By comparing the virtual channeled spectrum with the measured one, we can determine whether the estimated Stokes spectrum is close to the input Stokes spectrum. Finally, after multiple iterations using the optimization algorithm, an estimated Stokes spectrum that is closest to the input Stokes spectrum can be obtained, as shown in Fig. S2. The method described above is what is called the compressive sensing method\(^2\). The details of this method are described as follows.
According to Eq. S11, if we define four diagonal matrices as

\[
\begin{align*}
M_0 &= \text{diag}\{a_0(\sigma_1), a_0(\sigma_2), \ldots, a_0(\sigma_n)\}, \\
M_1 &= \text{diag}\{a_1(\sigma_1), a_1(\sigma_2), \ldots, a_1(\sigma_n)\}, \\
M_2 &= \text{diag}\{a_2(\sigma_1), a_2(\sigma_2), \ldots, a_2(\sigma_n)\}, \\
M_3 &= \text{diag}\{a_3(\sigma_1), a_3(\sigma_2), \ldots, a_3(\sigma_n)\},
\end{align*}
\]

(S12)

an estimated channeled spectrum \( \hat{\mathbf{s}} \) can be calculated by

\[
\hat{\mathbf{s}} = M_0 \cdot \mathbf{s}_0 + M_1 \cdot \mathbf{s}_1 + M_2 \cdot \mathbf{s}_2 + M_3 \cdot \mathbf{s}_3,
\]

(S13)

where \( \mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \) and \( \mathbf{s}_3 \) represent the four components of the estimated Stokes spectrum, and can be expressed as

\[
\begin{align*}
\mathbf{s}_0 &= [S_0(\sigma_1), S_0(\sigma_2), \ldots, S_0(\sigma_n)]^T, \\
\mathbf{s}_1 &= [S_1(\sigma_1), S_1(\sigma_2), \ldots, S_1(\sigma_n)]^T, \\
\mathbf{s}_2 &= [S_2(\sigma_1), S_2(\sigma_2), \ldots, S_2(\sigma_n)]^T, \\
\mathbf{s}_3 &= [S_3(\sigma_1), S_3(\sigma_2), \ldots, S_3(\sigma_n)]^T.
\end{align*}
\]

(S14)

To well perform the optimization algorithm, the Stokes spectrum can be expressed in terms of Legendre polynomials (LP) and discrete cosine transform (DCT) bases, as

\[
\mathbf{s}_j = [M_{\text{lp}} \cdot M_{\text{dct}}] \cdot \begin{bmatrix} \hat{s}_j^\text{lp} \\ \hat{s}_j^\text{dct} \end{bmatrix} = M_{\text{support}} \cdot \hat{s}_j,
\]

(S15)

where \( \hat{s}_j^\text{lp} \in \mathbb{R}^{\gamma \times 1} \) and \( \hat{s}_j^\text{dct} \in \mathbb{R}^{\chi \times 1} \) are the basis coefficients that represent LP and the DCT frames for Stokes parameters \( \mathbf{s}_j \), respectively (\( j = 0, 1, 2, 3 \) from Eq. S14). \( \gamma \) is the number of LP. \( \chi \) is the index related to the carrier frequency. Besides, \( \gamma + \chi \geq n \). The LP matrix, \( M_{\text{lp}} \in \mathbb{R}^{n \times \gamma} \), can be used to model the signals such as linear, quadratic and cubic polynomials. The \( (q, p)^{th} \) element of \( M_{\text{lp}} \) can be calculated by

\[
M_{\text{lp}}(q, p) = 2^p \sum_{k=0}^{p} \binom{2p-n-1}{n-1} k^k \left( \begin{array}{c} p \\ k \end{array} \right) \left( \begin{array}{c} q+p-1 \\ 2p \end{array} \right).
\]

(S16)

\( M_{\text{dct}} \in \mathbb{R}^{n \times \chi} \) is a DCT matrix the helps to capture sinusoidal variations. The \( (q, p)^{th} \) element of \( M_{\text{dct}} \) can be given by

\[
M_{\text{dct}}(q, p) = \begin{cases} 
\sqrt{\frac{1}{n}} \cos \left( \frac{2\pi}{n} (q-1)(2p-1) \right) & \text{if } p \neq 1 \\
\sqrt{\frac{1}{n}} & \text{if } p = 1
\end{cases}
\]

(S17)

Based on Eqs S13 and S15, a likelihood function \( L(\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3) \) and a regularizer function \( R(\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3) \) are defined as

\[
L(\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3) = \| \mathbf{I}_{\text{estimated}} - \mathbf{I}_{\text{measured}} \|_2^2,
\]

\[
R(\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3) = \| \hat{s}_0 \|_1 + \| \hat{s}_1 \|_1 + \| \hat{s}_2 \|_1 + \| \hat{s}_3 \|_1,
\]

(S18)
where $I_{\text{measured}}$ represents the measured channeled spectrum of the fabricated device. To find a Stokes spectrum that is closest to the input, an optimization problem needs to be solved:

$$\min_{\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3} L(\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3) + \beta R(\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3)$$

subject to $0 \leq M_{\text{support}} \cdot \hat{s}_0$

(S19)

where $\beta$ is the regularizer weight. Increasing the parameter $\beta$ can improve robustness against noise as the signal-to-noise ratio (SNR) decreases. Because $M_{\text{hp}}$ and $M_{\text{det}}$ respond to different kinds of variations, we can improve the accuracy of the Stokes spectrum reconstruction via adjusting the values of $\gamma$ and $\chi$. For example, $\chi$ can be set at a large value when the input Stokes spectrum exhibits sinusoidal variation with a high frequency. After solving the optimization problem, the input Stokes spectrum can be calculated according to Eq. S15.

In the main manuscript, we have discussed how to choose the value of $L_1$ for the Fourier transform method. Next, we study how to choose the value of $L_1$ for the compressive sensing method. As shown in Fig. S3, for a given spectral resolution of 83.24 m$^{-1}$, increasing $L_1$, the RMSE firstly decreases rapidly, then maintains at a certain level after $L_1$ reaches the value given by Eq.5 of the main manuscript. A larger $L_1$ does not lead into a higher accuracy but results in a higher waveguide propagation loss. Therefore, the best choice of $L_1$ for the compressive sensing method is same as that for the Fourier transform method.

![Figure S3](image-url)

**Figure S3.** Root mean squared error as a function of $L_1$, calculated using the compressive sensing method to reconstruct the Stokes spectra with $f = 0.003$ mm.

### 3 3-dB Multimode interference coupler

The schematic of the MMI coupler is presented in Fig. S4a. Four trapezoidal tapers are used to connect the input/output waveguides with the multimode region. The width of the taper varies from 0.5 $\mu$m to 1.5 $\mu$m, and the length is 50 $\mu$m. After a series of optimization simulations, we set the width and length of the multimode region to 4.1 $\mu$m and 57.5 $\mu$m, respectively. These structure parameters helped us to obtain a broadband 3-dB MMI coupler with a center wavelength of 1550 nm. The simulated and measured results of the fabricated MMI coupler are shown in S4b. They indicate that the straight-through and cross-coupling coefficients of the fabricated MMI coupler can maintain close to $\sqrt{2}/2$ over a wavelength range of larger than 70 nm.

### 4 Experiment

#### 4.1 Heating power calibration

Figure S5a presents a schematic of the experiment for calibrating the heating powers that are applied to the microring resonators (MRs) of the proposed vernier dual-microring spectrometer (VDMS). The device has two MRs: MR1 and MR2. $HP_1$ and $HP_2$ represent the heating powers applied to MR1 and MR2, respectively. $I_1$ and $I_2$ represent the optical intensities of the through and drop ports of the the cascaded dual-MR system. Adjusting $HP_1$ and allows us to control the resonance wavenumber of the cascaded dual-MR system. In theory, heating power is linear with the resonance wavenumber, so that the heating power of any resonance wavenumber can be calculated according to several other known heating powers. In practice, it is difficult to completely eliminate thermal crosstalk between two MRs. As shown in Fig. S5b, although the heating power is only applied to MR1, the resonance wavenumber of the MR2 also shifts slightly. Therefore, the $HP_1$ and $HP_2$ need to be individually calibrated for each of the 560 resonance wavenumbers over the entire free spectral range (FSR) of the cascaded dual-MR system. To shorten the time required for heating power calibration, Fig. S5c shows our proposed two-step method that first searches the $HP_1$, and then simultaneously sweeps both $HP_1$ and $HP_2$ with $HP_1$ being swept over a small power range:
Figure S4. Performance of the fabricated 3-dB MMI coupler. (a) Schematic of the MMI coupler including the structure’s parameters. (b) The straight-through (red line) and cross-coupling (blue line) coefficients of the fabricated MMI coupler as a function of wavelength. The solid and dotted lines are the experimental and simulated results, respectively.

1) Inject the light with a wavenumber $\sigma$.

2) Set $HP_2$ at 0 mW and sweep $HP_1$ from 10 mW to 130 mW in 0.5 mW increments to obtain the minimum $I_1$. Simultaneously, record the value for $P$ of the corresponding $HP_1$.

3) Simultaneously sweep the $HP_1$ and $HP_2$ to obtain the maximum $I_2$. $HP_2$ is swept from 0 mW to 130 mW in 0.5 mW increments. Note that $HP_1$ is swept from $(P - 6)$ mW to $(P + 6)$ mW in 0.5 mW increments in order to compensate for thermal crosstalk.

4) Record the optimized values $(HP_1, HP_2)$ for the wavenumber $\sigma$.

5) Determine whether the $HP_1$ and $HP_2$ for all the wavenumbers have been calibrated. If not, increase the value for $\sigma$ and start the sequence.

The above steps provided details on how to calibrate $HP_1$ and $HP_2$ for each wavenumber. However, $HP_2$ can also be calibrated before simultaneously optimizing $HP_1$ and $HP_2$.

Figure S5. A schematic of heating power calibration for two MRs. (a) A schematic of the experiment setup for calibrating heating powers. (b) The resonance wavenumber shifts of MR1 and MR2 as a function of $HP_1$ when $HP_2$ is set at 0 mW. (c) A flow chart for calibrating heating power. $\sigma$ represents the wavenumber of light. MR1 and MR2 indicate the first and second microring resonators of the proposed vernier dual-microring spectrometer, respectively. $HP_1$ and $HP_2$ are the heating powers applied to MR1 and MR2, respectively.
4.2 Heating power calibration results for a single VDMS
A schematic of the experimental setup for the heating power calibration of the single VDMS is presented in Fig. S6a. The optical input and output of the single VDMS are located in the same place. 25° fiber arrays are used to couple the light into and out of the silicon photonic (SiP) chip. We used Python code to control the whole system including the tunable laser, photodetector, and heating power controller, for automatic calibration.

The calibration results for the heating power applied to the MR1 and MR2 in the VDMS are presented in Fig. S6b. By individually applying these heating power pairs to the two MRs, the input spectrum could be obtained. The tuning efficiencies for both MR1 and MR2 were ∼30 mW/nm. Due to the Vernier effect, the maximum heating power that was required for either MR1 or MR2 to cover a wavelength range of 56 nm was lower than 120 mW.

Figure S6. (a) A schematic of the experimental setup for the heating power calibration for the proposed spectrometer. (b) The calibrated heating powers (HP) applied to MR1 and MR2 as a function of the wavenumber.

4.3 Heating power calibration results for the VDMS used in SiPh-CSP
A schematic of the experimental setup for the heating power calibration for the VDMS used in SiPh-CSP is shown in Fig. S7a. The optical input and output of the Si-CSP were located in two different places. 0° and 25° fiber arrays are used to couple the light into and out from the SiP chip, respectively.

The calibration results for the heating power applied to the MR1 and MR2 in SiPh-CSP are presented in Fig. S7b. The channeled spectrum can be obtained by individually applying these heating power pairs to two MRs.

Figure S7. (a) A schematic of the experimental setup of the heating power calibration for the spectrometer used in SiPh-CSP. (b) The calibrated heating powers (HPs) applied to MR1 and MR2 as a function of the wavenumber.

4.4 Experiment for characterizing the VDMS
A schematic of the experimental setup for the spectrum measurement using the fabricated VDMS is presented in Fig. S8a. A broadband light source was used as the input spectrum. The input spectrum can be reconstructed by sweeping all the heating power pairs shown in Fig. S6b and simultaneously reading the optical intensity of the drop port of the dual-MR system. In order to provide a detailed demonstration on how the proposed VDMS works, Fig. S8b, presents the optical intensity in the
drop port as a function of the heating powers applied to MR1 and MR2, when a broadband light source is injected into the VDMS. By picking up all the heating power pairs shown in S6b, the spectrum of the light source can be obtained. For example, the circles seen in S8b represent the optical intensities when the resonance wavenumber varies from $6.443758 \times 10^5 \text{ m}^{-1}$ to $6.4329366 \times 10^5 \text{ m}^{-1}$ (following the direction of the long white arrow) with increments $41.62 \text{ m}^{-1}$. The result for the final measurement is presented in the main manuscript.

Figure S8. (a) A schematic of the experimental setup for the spectrum measurement using our spectrometer. PBS: polarization beam splitter. (b) The optical intensity read by the photodetector as a function of the heating powers (HPs) that were applied to MR1 and MR2 when measuring a broadband source. The circles are the points when the resonance wavenumber varied from $6.443758 \times 10^5 \text{ m}^{-1}$ to $6.4329366 \times 10^5 \text{ m}^{-1}$ (following the direction of the white arrow).

4.5 Experiment for characterizing the SiPh-CSP

A schematic of the experimental setup for the Stokes spectrum measurement using the fabricated SiPh-CSP is presented in Fig. S9. The light generated by a light source passes through a collimating lens, a polarizer, a half waveplate (HWP), and a quarter waveplate (QWP). The light is then injected into the surface of the SPS. The HWP and QWP are rotated by the stepper motor rotation stage with an accuracy of $0.14^\circ$. Controlling the angles of the HWP and QWP allowed us to control the input Stokes spectrum.

Figure S9. A schematic of the experimental setup for the Stokes spectrum measurement using the fabricated SiPh-CSP. PBS: polarization beam splitter; HWP: half waveplate; QWP: quarter waveplate; P0: polarizer with transmission axis parallel to x-axis; PD: photodetector.

According to Eq. S11, the values for $a_0(\sigma)$, $a_1(\sigma)$, $a_2(\sigma)$ and $a_3(\sigma)$ are very important for the optimization model method and can be calibrated by four know Stokes spectra. Their calibration results are presented in Fig. S10. According to Eq. S4, $a_0(\sigma)$ should not be modulated into any carrier, but the experimental result is not consistent with this. This is because when the incoming light interacts with the SPS, there is a small variation in its polarization state due to imperfections in the fabrication which induces an additional phase or/and intensity difference between two components of the incoming light. A mueller matrix $M_{mu}(\sigma)$ can be used to connect the Stokes vectors before and after the light interacts with the SPS, calculated as:

$$S'(\sigma) = M_{mu}(\sigma) \cdot S(\sigma),$$  

(S20)
Figure S10. Experimental calibration results of (a) $a_0(\sigma)$, (b) $a_1(\sigma)$, (c) $a_2(\sigma)$ and (c) $a_3(\sigma)$.

where $S'(\sigma)$ represents the Stokes vector injected into the CSG. If we define the linear operators of CSG as $a'_0(\sigma)$, $a'_1(\sigma)$, $a'_2(\sigma)$ and $a'_3(\sigma)$, the channeled spectrum can be calculated by

$$I(\sigma) = \begin{bmatrix} a'_0(\sigma) & a'_1(\sigma) & a'_2(\sigma) & a'_3(\sigma) \end{bmatrix} \cdot S'(\sigma)$$

$$- \begin{bmatrix} a'_0(\sigma) & a'_1(\sigma) & a'_2(\sigma) & a'_3(\sigma) \end{bmatrix} \cdot M_{mul}(\sigma) \cdot S(\sigma).$$

Equation S22 indicate that $a_0(\sigma), a_1(\sigma), a_2(\sigma)$ and $a_3(\sigma)$ are the calibrated results of both SPS and CSG, and it is therefore possible to modulate $a_0(\sigma)$ into some carriers. However, this does not influence the results from the Stokes spectrum reconstruction using the fabricated SiPh-CSP because they can be calibrated.

### 4.6 The measured results for 16 input Stokes spectra

The main manuscript only shows the measured results for three Stokes spectra. Here, we provide more Stokes spectra measurements in order to further demonstrate the capabilities of our device. Figures S11 and S12 show the measured results that were reconstructed using the Fourier transform and compressive sensing methods, respectively.

### References

1. Rudin, W. *Principles of mathematical analysis*, vol. 3 (McGraw-hill New York, 1964).

2. Lee, D. J., LaCasse, C. F. & Craven, J. M. Compressed channeled spectropolarimetry. *Opt. Express* **25**, 32041–32063 (2017).
Figure S11. Reconstructed results obtained using Fourier transform method. $\theta_H$ and $\theta_Q$ represent the angle of the fast-axis of the half and quarter waveplates, respectively.
Figure S12. Reconstructed results obtained using the compressive sensing method. $\theta_H$ and $\theta_Q$ represent the angle of the fast-axis of the half and quarter waveplates, respectively.