Baryon Acoustic Oscillations in 2D II: Redshift-space halo clustering in N-body simulations

Takahiro Nishimichi, Atsushi Taruya

1 Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, Chiba 277-8568, Japan and 2 Research Center for the Early Universe, School of Science, The University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan

(Dated: June 24, 2011)

We measure the halo power spectrum in redshift space from cosmological N-body simulations, and test the analytical models of redshift distortions particularly focusing on the scales of baryon acoustic oscillations (BAOs). Remarkably, the measured halo power spectrum in redshift space exhibits a large-scale enhancement in amplitude relative to the real-space clustering, and the effect becomes significant for the massive or highly biased halo samples. These findings cannot be simply explained by the so-called streaming model frequently used in the literature. By contrast, a physically-motivated perturbation theory model developed in the previous paper reproduces the halo power spectrum very well, and the model combining a simple linear scale-dependent bias can accurately characterize the clustering anisotropies of halos in two dimensions, i.e., line-of-sight and its perpendicular directions. The results highlight the significance of non-linear coupling between density and velocity fields associated with two competing effects of redshift distortions, i.e., Kaiser and Finger-of-God effects, and a proper account of this effect would be important in accurately characterizing the BAOs in two dimensions.

I. INTRODUCTION

Redshift distortions caused by the systematic effect of peculiar velocity of galaxies induce the anisotropies in the galaxy clustering patterns (e.g., [1, 2]). These are now recognized as a powerful tool to test theory of gravity on cosmological scales with a great interest (e.g., [3–7]). On large scales, the magnitude of redshift distortions is simply characterized by the growth-rate parameter \( f \), defined as \( f = d \ln D_+ / d \ln a \), where the quantities \( D_+ \) and \( a \) respectively denote the linear growth factor and the scale factor of the Universe [1, 8, 9]. Since the modification of gravity from the general relativity generally alters not only the cosmic expansion but also the structure formation, the measurement of growth-rate parameter provides a complementary way to distinguish between various models of gravity.

Notice that distortions of the galaxy clustering pattern also arise from apparent mismatch of the underlying cosmology when we convert the redshift and angular position for each galaxy to the comoving radial and transverse distances. This is known as the Alcock-Paczynski effect [10], and with the baryon acoustic oscillations (BAOs) as a standard ruler, it can be utilized for a measurement of both the Hubble parameter \( H(z) \) and angular diameter distance \( D_A(z) \) of distant galaxies at redshift \( z \) (e.g., [11, 13], see [16, 21] for observational status). Hence, a measurement of the anisotropic galaxy clustering serves as a dual cosmological probe to simultaneously constrain both the cosmic expansion and structure growth (e.g., [27]). Planned and ongoing galaxy redshift surveys such as the Baryon Oscillation Spectroscopic Survey (BOSS) [26], Hobby-Eberly Telescope Dark Energy Experiment (HETDEX) [27], Subaru Measurement of Imaging and Redshift equipped with Prime Focus Spectrograph (SuMIRe-PFS) [28], and EUCLID/JDEM [29, 30] aim at precisely measuring the anisotropic power spectrum and/or two-point correlation function in redshift space, from which we can simultaneously determine \( D_A, H \) and \( f \) in a very accurate way.

To get tight and robust cosmological constraints, an accurate theoretical template for the anisotropic power spectrum is highly demanding, taking a proper account of various systematic effects. In particular, in redshift space, in addition to the non-linear gravitational clustering, the clustering statistics generally suffer from two competing effects of redshift distortions, i.e., enhancement and suppression of the clustering amplitude, referred to as the Kaiser and Finger-of-God effects, respectively [8, 31]. While the Kaiser effect comes from the coherent motion of matter (or galaxies) and the magnitude of this effect is simply described by the growth-rate parameter \( f \), the Finger-of-God effect is mainly attributed to the virialized random motion of the mass residing at a halo, and the significance of this is rather sensitive to the properties of the small-scale clustering. In the weakly non-linear regime, a tight correlation between the velocity and the density fields still remains, and a mixture of Kaiser and Finger-of-God effects is expected to be significant. In this sense, a careful treatment is needed to model the anisotropic power spectrum accurately, otherwise one might derive a biased estimate of the growth-rate parameter as shown by, e.g., [32].

In a previous paper [33], based on the analytical treatment with perturbation theory, we have presented an improved prescription for redshift-space power spectrum relevant for the scales of BAOs. The model properly takes account of both the non-linear gravitational clustering and redshift distortions. Contrary to the so-called streaming model (e.g., [2, 34]), which has been phe-
nomenonologically introduced but frequently used in the literature, the new model includes the corrections coming from the non-linear coupling between velocity and density fields, which gives rise to a slight increase in the amplitude of the redshift-space power spectrum. Combining the improved treatment of perturbation theory to accurately predict non-linear clustering [32, 37], the model reproduces the monopole and quadrupole moments of the matter power spectrum measured from N-body simulations quite well, and a percent level precision is achieved over the scales of interest for BAOs [33]. However, the comparison with N-body simulations has been only made with lower multipoles of the matter power spectrum, i.e., monopole and quadrupole spectra.

In this paper, as a natural extension of the previous study, we further test the analytical model of redshift distortions. Using a large set of N-body simulations, we measure the redshift-space power spectrum in two dimensions, characterized as a function of line-of-sight wavenumber \( k \parallel \) and its normal one \( k \perp \). In particular, we examine the halo power spectrum in detail, and investigate the extent to which the perturbation theory description combining a simple halo bias scheme can describe the halo clustering properties in redshift space. We find that the measured halo power spectrum in redshift space at relatively large scales \( k \lesssim 0.2 \, \text{h} \, \text{Mpc}^{-1} \) exhibits a significant enhancement in amplitude relative to the real-space clustering, which cannot be explained by the streaming model. On the other hand, the new model including the corrections can describe the enhancement fairly well. The results indicate that the halo bias can illuminate the non-linear coupling between density and velocity fields, and a proper account of the non-linear velocity-density coupling seems important in accurately modeling the galaxy/halo power spectrum in redshift space.

This paper is organized as follows. We begin with a brief review of the models based on perturbation theory in redshift space in Sec. II. Sec. III describes the setup of N-body simulations and discusses the power spectrum analysis. The results for a detailed comparison between models and simulations are presented in Sec. IV, particularly focusing on the halo and matter power spectrum in two dimensions. We discuss about the impact of the non-linear correction terms on some typical on-going/planned galaxy redshift surveys in Sec. V. Finally, Sec. VI is devoted to the summary of this paper.

II. THE MODELS

In principle, the clustering statistics in redshift space can be mapped from those in real space through the coordinate transformation between the positions in real and redshift spaces, \( \mathbf{r} \) and \( \mathbf{s} \) [66]:

\[
\mathbf{s} = \mathbf{r} + \frac{1 + z}{H(z)} v_z(\mathbf{r}) \hat{z},
\]

where the quantity \( H \) is the Hubble parameter at redshift \( z \), the unit vector \( \hat{z} \) indicates the line-of-sight direction, and the quantity \( v_z \) represents the line-of-sight component of the peculiar velocity field. Although the expression (1) is very simple, the statistical relationship between the real- and redshift-space quantities is rather complicated and is actually difficult to treat without approximations. One reason for the difficulty comes from the anisotropies induced by the velocity field which is very sensitive to the small-scale structure. The other important aspect of the redshift-space clustering is the coupling between velocity and density fields, which produces an apparent structure growth by the coherent motion.

Qualitatively, the clustering properties in redshift space are explained by the Kaiser and Finger-of-God effects. The so-called streaming model is a phenomenological model that accounts for these two effects separately [2, 34]. Here, among various streaming models, we consider the following form of the redshift-space power spectrum (e.g., 37, 58):

\[
P(k, \mu) = D_I(k \mu f \sigma_v) \times \left[ P_{\delta \delta}(k) + 2 f \mu^2 P_{\delta \theta}(k) + f^2 \mu^4 P_{\theta \theta}(k) \right], \tag{2}
\]

where \( f \) is the growth rate parameter, the quantity \( \sigma_v \) is the one-dimensional velocity dispersion, and \( \mu \) is the directional cosine of the angle between line-of-sight \( \hat{z} \) and the Fourier mode \( \mathbf{k} \). In the above, while the function \( D_I \) represents a damping function mimicking the Finger-of-God effect, the term in the bracket indicates an improved prescription of the Kaiser effect which takes account of the non-linear gravitational evolution [37]. The spectra \( P_{\delta \delta}, P_{\delta \theta}, \) and \( P_{\theta \theta} \) respectively denote the auto-power spectra of density and velocity divergence, and their cross-power spectrum, with the velocity divergence \( \theta \) defined by \( \theta = -(1 + z)/(Hf) \nabla v \). The explicit functional form of the damping function will be specified later.

Notice that the streaming models have been originally introduced and frequently used in the literature to explain the observed power spectrum on small scales. On large scales of our interest, non-linearity of the gravitational clustering is rather mild, but a tight correlation between density and velocity fields still remains. In previous paper [33], starting with the rigorous expression of redshift-space power spectrum, we partially applied a low-\( k \) expansion, and found that the model [2] misses some important terms. The missing terms naturally arise from the next-to-leading order corrections of the low-\( k \) expansion, and they represent the nonlinear coupling between velocity and density fields. The model is given by

\[
P(k, \mu) = D_I(k \mu f \sigma_v) \times \left[ P_{\delta \delta}(k) + 2 f \mu^2 P_{\delta \theta}(k) + f^2 \mu^4 P_{\theta \theta}(k) \right. \]

\[
\left. + A(k, \mu; f) + B(k, \mu; f) \right]. \tag{3}
\]
The newly derived terms $A$ and $B$ are written as

$$A(k, \mu; f) = (k \mu f) \int \frac{d^3p}{(2\pi)^3} \frac{p_z}{p^2} \times \{B_\sigma(p, k - p, -k) - B_\sigma(p, k, -k - p)\}, \quad (4)$$

$$B(k, \mu; f) = (k \mu f)^2 \int \frac{d^3p}{(2\pi)^3} F(p)f(k - p); \quad (5)$$

$$F(p) = \frac{p_z}{p^2} \left\{ P_{\delta\theta}(p) + \frac{f^2}{p^2} P_{\theta\theta}(p) \right\},$$

where the function $B_\sigma$ is the cross bispectra defined by

$$\left\{ \delta(k_1) \right\} \left\{ \delta(k_2) + f \frac{k_2^3}{k_1^3} \theta(k_2) \right\} \left\{ \delta(k_3) + f \frac{k_3^3}{k_1^3} \theta(k_3) \right\} = (2\pi)^3 \delta_D(k_1 + k_2 + k_3) B_\sigma(k_1, k_2, k_3). \quad (6)$$

Within the range of the validity of the low-$k$ expansion, these corrections should be small, and we apply the standard PT treatment to calculate the $A$ and $B$ terms. To compute the corrections, we use the expressions (A3) and (B4) presented in Ref. [33], which have been derived for the matter power spectrum. As shown in Sec. IV[A] however, the scale-dependence of the halo bias calibrated from the N-body simulations turned out to be very weak over the scales of our interest. Thus, our treatment of the model (7) with scale-dependent bias would be validated, and does not change the final conclusion.

### III. Analysis

#### A. N-body simulations

To assess the validity and accuracy of the analytic models against the redshift-space halo clustering, we run a set of N-body simulations and identified halos. All the simulations are performed with a publicly available tree PM code GADGET2. We adopt $N = 1, 280^3$ particles in boxes with a side length of $1.14472 h^{-1}$ Mpc [i.e., the volume is $1.5 (h^{-1}Gpc)^3$], and set the softening length to 5\% of the inter-particle distance. We generate the initial conditions at $z = 99$ by second-order Lagrangian perturbation theory (e.g., \cite{43, 44}) starting from particles put on the regular lattices. The matter transfer function is calculated by a publicly available Boltzmann solver CAMB, assuming the best-fit LCDM cosmological model determined by the five-year WMAP. The output data of 15 independent realizations are stored at $z = 0.35$. Table I summarizes the settings of the simulations.

#### B. Halo catalogs

Given the data set of dark matter clustering, we identify the halos by a Friends-of-Friends finder with linking length 0.2 times the mean inter-particle distance. For each realization, we construct nine halo catalogs with different minimum and maximum masses. Allowing a slight overlap of the mass range, each mass bin is determined so that the signal-to-noise ratio of halo power spectrum measured at $k = 0.0975h$Mpc$^{-1}$ becomes roughly comparable among the nine halo catalogs [see Eq. (12) for
the definition of signal-to-noise ratio]. Detailed properties of the nine halo catalogs, including the mass range and the number density of halos, are summarized in Table I. In what follows, we especially call the least and most massive halo catalogs (bin 1 and 9) as light and heavy, and discuss their clustering properties in detail. Note that the heavy catalog has roughly the same values of the bias parameter, number density, and survey volume as observed in the luminous red galaxies (LRG) \cite{48} of the seventh data release of Sloan Digital Sky Survey (SDSS DR7) \cite{49}.

C. Power spectrum

We measure the dark matter and halo power spectra from the N-body simulations using the standard method based on the fast Fourier transform. To be specific, we first assign the dark matter or halo particles onto a $1.024^3$ grid by cloud-in-cells (CIC) interpolation scheme \cite{50} to obtain a density field on a lattice. We then transform it to Fourier space and divide it by the CIC kernel in order to eliminate the window effect. Finally, we take the average of the power over the modes in each $k$ bin in real space, and in $(k, \mu)$ bin in redshift space. The size of each Fourier bin is set to $\Delta k = 0.005 h^{-1}\text{Mpc}$ and $\Delta \mu = 0.05$ for the wavenumber and the directional cosine, respectively.

Our primary focus is to test the analytic models of redshift distortions against the halo clustering in N-body simulations. To do this, we need to incorporate the halo bias properties characterized by the function $b(k)$ into the model prediction \cite{7}. While, in practice, the quantity $b(k)$ should be parametrized by a simple function, and must be determined by fitting the model prediction to the observed power spectrum, in order to make a transparent test, we directly measure the halo bias parameter $b(k)$ from the N-body simulation, and use it to compute the analytic models of redshift-space power spectrum. Note that this treatment still includes several non-trivial aspects. Indeed, the linear bias prescription seems rather simplistic, and it might not capture the real clustering nature of halos over the BAO scales. Further, in redshift space, the presence of correction terms in the TNS model implies that even the simple linear bias can lead to a non-trivial modulation of clustering amplitude, which might drastically alter the acoustic structure in redshift space.

Let us discuss how to characterize and measure the halo bias properties from N-body simulations. According to the treatment by Ref. \cite{51}, we decompose the halo density contrast defined in real space into two pieces:

$$\delta_h(k) = b(k)\delta_m(k) + \epsilon(k). \quad (9)$$

Here, the quantity $\epsilon$, which satisfies $\langle \delta_m\epsilon \rangle = 0$, can be regarded as the residual noise contribution, and is related to the stochasticity of the halo sampling process. Notice that the above decomposition does not rely on the assumption of linear bias. The other additional contributions, which cannot be simply written as the term linearly proportional to $\delta_m$, are all incorporated into the second term. Given the auto and cross power spectra of matter and halos measured in real space, $P_m$ and $P_{hm}$, the decomposition of the signal linearly proportional to the matter density field, and noise can be uniquely done, and the halo bias parameter can be measured in the following way:

$$b(k) = \frac{P_{hm}(k)}{P_m(k)}, \quad (10)$$
$$P_{\epsilon}(k) = P_h(k) - b^2(k)P_m(k), \quad (11)$$

where the quantity $P_{\epsilon}$ is the residual noise power spectrum defined by $\langle \epsilon(k)\epsilon(k') \rangle = (2\pi)^3\delta_D(k + k')P_{\epsilon}(k)$. In deriving the above relations, we have used the property $\langle \delta_m\epsilon \rangle = 0$. Note that the halo subsamples listed in Table I are constructed so as to take the same signal-to-noise ratio among the nine catalogs. The signal-to-noise ratio is defined by

$$\left( \frac{S}{N} \right)_k = \frac{b^2(k)P_m(k)}{P_{\epsilon}(k)}. \quad (12)$$

At $k = 0.0975 h^{-1}\text{Mpc}$, the signal-to-noise ratio roughly reaches $(S/N)_k \approx 4$ for all of our subsamples.

In testing the analytic models of redshift-space power spectrum, we subtract the residual noise contribution from the measured halo spectrum, and then compare the results with analytic prediction in the $(k, \mu)$ space. Note that apart from the bias $b(k)$, the models \cite{2} and \cite{4} involve a single parameter $\sigma_v$, which has to be determined by fitting the model power spectrum to the measured power spectrum. To derive the best-fit value of $\sigma_v$, we assume the Gaussianity, and use the statistical error of the redshift-space power spectrum given by \cite{52}

$$[\Delta P(k, \mu)]^2 = \frac{1}{N_{\text{mode}}} \left[ b^2(k)P_m(k, \mu) + P_{\epsilon}(k) \right]^2 \quad (13)$$

where $N_{\text{mode}}$ denotes the total number of independent Fourier modes which fall into the $(k, \mu)$ bin over the 15 realizations. Note that in estimating the best-fit $\sigma_v$ for dark matter, we simply set $b(k) = 1$ and $P_{\epsilon} = 0$.\hfill
TABLE II: Summary of the halo catalogs. The minimum, maximum and mean mass \((M_{\text{min}}, M_{\text{max}} \text{ and } M_{h})\) are in units of \(\hbar^{-1}M_\odot\), while the halo number density \(n_h\) is in \(h^3\text{Mpc}^{-3}\). The bias parameter, \(b_0\), is defined in Eq. (15). See Sec. [LVA] for more detail.

| Sample | bin 1 (light) | bin 2 | bin 3 | bin 4 | bin 5 | bin 6 | bin 7 | bin 8 | bin 9 (heavy) |
|--------|---------------|-------|-------|-------|-------|-------|-------|-------|---------------|
| \(M_{\text{min}}\) | \(1.77 \times 10^{12}\) | \(2.49 \times 10^{12}\) | \(3.54 \times 10^{12}\) | \(4.98 \times 10^{12}\) | \(7.09 \times 10^{12}\) | \(1.00 \times 10^{13}\) | \(1.42 \times 10^{13}\) | \(2.01 \times 10^{13}\) | \(2.84 \times 10^{13}\) |
| \(M_{\text{max}}\) | \(5.54 \times 10^{12}\) | \(1.02 \times 10^{13}\) | \(1.74 \times 10^{13}\) | \(2.66 \times 10^{13}\) | \(4.04 \times 10^{13}\) | \(6.76 \times 10^{13}\) | \(1.19 \times 10^{14}\) | \(2.08 \times 10^{14}\) | - |
| \(M_h\) | \(2.96 \times 10^{12}\) | \(4.65 \times 10^{12}\) | \(7.08 \times 10^{12}\) | \(9.37 \times 10^{12}\) | \(1.47 \times 10^{13}\) | \(2.18 \times 10^{13}\) | \(3.21 \times 10^{13}\) | \(4.63 \times 10^{13}\) | \(7.03 \times 10^{13}\) |
| \(n_h\) | \(1.57 \times 10^{-3}\) | \(1.26 \times 10^{-3}\) | \(9.46 \times 10^{-4}\) | \(6.87 \times 10^{-4}\) | \(4.87 \times 10^{-4}\) | \(3.47 \times 10^{-4}\) | \(2.43 \times 10^{-4}\) | \(1.64 \times 10^{-4}\) | \(1.09 \times 10^{-4}\) |
| \(b_0\) | 1.08 | 1.16 | 1.25 | 1.35 | 1.47 | 1.62 | 1.80 | 1.99 | 2.26 |

Finally, in discussing the goodness of fit between the streaming and TNS models [i.e., Eq. (7) with and without corrections], it is convenient to introduce the reduced chi-squared statistic:

\[
\chi^2_{\text{red}} = \frac{1}{\nu} \sum_{i,j} \frac{(P_{\text{N-body}}(k_i, \mu_j) - P_{\text{model}}(k_i, \mu_j))^2}{\sigma^2 (P(k_i, \mu_j))},
\]

where the power spectra \(P_{\text{N-body}}\) and \(P_{\text{model}}\) are respectively obtained from the N-body simulations and the analytic models with the best-fit value of \(\sigma_\nu\). The quantity \(\nu\) is the number of degrees of freedom, and depending on the range of fitting, wet set \(\nu = 204, 364, 524, \text{and } 684\) for the maximum wavenumber used to fit, \(k_{\text{max}} = 0.08, 0.12, 0.16, \text{and } 0.2 h\text{Mpc}^{-1}\), respectively.

IV. RESULTS

In this section, the results of the power spectrum measurement are presented, and a detailed comparison between N-body simulations with analytic models is made. We first address the real-space clustering for dark matter and halos, and measure the halo bias in Sec. IV A. We then move to the discussion on the redshift-space power spectrum, and the clustering anisotropies caused by redshift distortions are shown in two-dimensional plane in Sec. IV B. In Sec. IV C, applying the multipole expansion to the anisotropic power spectra, the lowest three multipole spectra, i.e., monopole, quadrupole, and hexadecapole spectra, are quantified and compared with the analytic models. While we mainly analyze the dark matter, and light and heavy halo catalogs, we also examine the other halo catalogs in Sec. IV D and study how well the analytic models can reproduce the N-body results when we vary the halo mass and/or fitting range.

A. Real-space clustering

Since the validity and precision of analytic models given in Eq. (14) heavily rely on the accuracy of the prediction in real space, it is important to first check the PT treatment, and to compare the PT prediction with N-body simulations. Top panel of Fig. 1 shows the ratio of the matter power spectrum to the smooth reference spectrum, \(P_m(k)/P_{\text{nw}}(k)\), where the reference spectrum \(P_{\text{nw}}(k)\) is calculated from the linear transfer function of no-wiggle approximation by Ref. [32]. While the dashed line represents the linear theory prediction, the solid line is the result based on the improved PT calculation by Refs. [35, 36], including the next-to-leading order non-linear corrections up to the second-order Born approximation. At \(z = 0.35\), the development of non-linear gravitational clustering is bit significant, and even the BAO

![FIG. 1: Ratios of the real-space power spectra for dark matter and halos. Top: matter power spectrum divided by the no-wiggle linear power spectrum in [32]. The symbols are N-body data, while the lines are analytical predictions (solid: Closure approx. (2nd Born), dashed: linear theory). Middle: bias parameter defined by the ratio of the halo-matter cross power spectrum to the matter auto power spectrum [symbols: N-body data, lines: fitted results of Eq. (15)]. The results for the halo samples of bin 1, 2, ..., and 9 are shown from bottom to top. Bottom: residual noise power spectrum, \(P_n(k)\), divided by the Poisson noise (bin 1, 2, ..., and bin 9 from top to bottom).](image)
scales at $k \lesssim 0.2 \, h\text{Mpc}^{-1}$ exhibit a prominent enhancement in the power spectrum amplitude. Nevertheless, the improved PT prediction reproduces the N-body result fairly well, and the agreement between simulations and PT prediction is mostly within the errorbars estimated from the 15 realizations of N-body simulations. This indicates that the improved PT by Refs. [32, 33] can be reliably applied to the modeling of redshift-space power spectrum at $k \lesssim 0.2 \, h\text{Mpc}^{-1}$, where the acoustic signature is still visible.

In addition to the PT treatment, the halo clustering bias measured in real space is also an important building block to precisely model the redshift-space power spectrum of halos. Middle and bottom panels of Fig. 1 quantify the halo bias properties based on the expressions (10) and (11). Middle panel plots the halo bias parameter $b(k)$, while bottom panel shows the residual noise spectrum divided by the Poisson noise $P_{\text{Pois}} = 1/n_h$, where $n_h$ is the mean number density of halos in each subsample (see Table II). The halo mass dependence is clearly seen not only in the halo bias but also in the residual noise spectrum. As increasing the halo mass, while the bias parameter increases, the residual noise relative to the Poisson noise decreases and manifests a non-Poissonian feature, i.e., $P_r/P_{\text{Pois}} \neq 1$. Although the mass dependence of the bias parameter is qualitatively and even quantitatively well-understood based on the halo model prescription (see [51] for a review), the non-Poissonian feature of the residual noise indicates that the origin of the residual noise might not be simply explained by the halo sampling process. Recently, an attempt to reproduce the non-Poissonian feature of the residual noise has been made, and the mass dependence of the residual noise is shown to be explained by the halo model [52]. Though we do not discuss at all a quantitative aspect of this, the non-Poissonian feature of the residual noise potentially affects the power spectrum estimation, and the understanding of it is practically important to extract a pure clustering signal.

Apart from the origin of the residual noise, it is worth noting that the scale-dependence of the halo bias as well as the noise power spectrum is quite moderate over the scales of our interest. This partly validates our treatment in computing the power spectrum from the TNS model (see Sec. II). The weak scale-dependence of the halo bias can be well-described by a simple fitting function used in the literature [17]:

$$b^2(k) = b_0^2 1 + Q k^2 1 + Ak.$$  

(15)

As a reference, the fitted results of Eq. (15) are shown in solid lines in Fig. 1 and the best-fit values of the parameter $b_0$, which represents the clustering bias in the large-scale limit, are listed in Table II [67]. The resultant best-fit value $b_0$ of the heavy halo sample is quite close to the observed value of the luminous red galaxies in the SDSS DR7.

B. Redshift-space clustering in two dimensions

We are in position to discuss the clustering properties of dark matter and halos in redshift space, and to examine the validity of the analytic model prescription. In this subsection, we particularly focus on the redshift-space power spectrum in two dimensions, and compare the model predictions with N-body simulations.

Let us first show Fig. 2 in which we plot the ratio of the power spectra defined by

$$\frac{P(k, \mu)}{b(k) + f \mu^2} P_{\text{nw}}(k)$$  

(16)

for the dark matter (top), light (middle), and heavy (bottom) halos. The results are shown in the $(k_\perp, k_\parallel)$ plane, where $k_\perp$ and $k_\parallel$ are the wavenumber perpendicular and parallel to the line-of-sight direction, and are related to the quantities $k$ and $\mu$ through $k_\perp = k (1 - \mu^2)^{1/2}$ and $k_\parallel = \mu k$. Note again that the quantities $P_{\text{nw}}$ and $f$ mean the smooth reference linear spectrum based on the no-wiggle approximation by Ref. [53], and growth-rate parameter defined by $f = d \ln D_{\perp}/d \ln a$, respectively.

In each panel of Fig. 2 the results obtained from the N-body simulations, the analytic models [7] with and without corrections (indicated by ‘TNS’ and ‘streaming’, respectively) are presented from left to right. Here, the analytic model predictions are plotted adopting the best-fit values of $\sigma_8$, which are derived based on the Lorentzian form of the Finger-of-God damping. In all of the panels, a ring-like structure originated from the BAOs is clearly manifest, and the acoustic feature is rather prominent along the $k_\perp$ axis, where redshift distortions have no effect. On the other hand, in the line-of-sight direction, the appreciable reduction of the amplitude is found in the dark matter power spectrum, while no such effect is apparent in the halo power spectra. Rather, the heavy halo subsample exhibits a bit large enhancement at $k_\perp \gtrsim 0.1 \, h\text{Mpc}^{-1}$, and the effect becomes significant as increasing the wavenumber. Note that with the ratio defined by Eq. (16), the linear Kaiser effect, which also enhances the clustering amplitude, is effectively eliminated in Fig 2. While a damping seen in the dark matter clustering would be due to the Finger-of-God effect, the absence of damping or a large enhancement in the halo clustering indicates that the Finger-of-God damping seems ineffective, and/or the new effect of redshift distortions additionally arises and it may compensate for the Finger-of-God damping. Since the clustering amplitude is rather sensitive to the halo subsamples, this new effect seems to have a strong dependence on the clustering bias.

Now, looking at the analytic model predictions [7] with and without the corrections, there appears a noticeable difference between the two models in the result of heavy halo subsample. Remarkably, the TNS model [i.e., Eq. (7) including the corrections] reproduces a large enhancement seen in the N-body simulations quite well. Since the main distinction between the two models comes
from the presence of additional terms given in Eqs. (4) and (5), the enhancement found in the N-body simulations can be interpreted as a non-linear effect of redshift distortions, characterized by the correction terms. Indeed, the corrections originated from the $A$ and $B$ terms exhibit a strong dependence on the halo bias parameter $b$, and this fact naturally explains the sensitive dependence of the clustering enhancement on the halo subsamples. These findings are basically consistent with the recent claims by Ref. [56, 57] (see also Ref. [58]), and the present paper further reveals an impact of the non-linear redshift distortions on the power spectrum in two dimensions.

In Fig. 3, for a more quantitative comparison of the analytic models with N-body simulations, we fix the wavenumber $k$, and plot the ratio (16) as a function of directional cosine $\mu$; dark matter, light and heavy halo subsample, from top to bottom. In each panel, the N-body data are shown in symbols, and the results of
best-fit analytic models with and without corrections, indicated by ‘TNS’ and ‘streaming’, are depicted as the solid and dashed lines. For reference, the fitted results of velocity dispersion \( \sigma_v \) and the goodness of fit \( \chi^2_{\text{red}} \) are also shown. Compared to the dark matter clustering, a large enhancement seen in the heavy subsample is statistically significant, and the discrepancy between simulations and the streaming model is clearly evident. Even in the absence of Finger-of-God damping (i.e., \( \sigma_v = 0 \)), the streaming model cannot reproduce the N-body trend. By contrast, the model including the corrections (i.e., the TNS model) successfully explains the N-body results. Although no appreciable difference of the results between the two models is found by eye in the dark matter and light subsamples, the resultant goodness of fit for the TNS model is better than that of the streaming model, statistically indicating that the model including the corrections successfully describes a real physical effect of redshift distortions seen in the N-body simulations. This point will be further investigated in detail in subsequent subsections.

C. Multipole expansion

The multipole expansion is an alternative technique for quantifying the clustering anisotropies, and it has been frequently applied in the literature to extract the cosmological information from the redshift-space power spectrum \( P_\ell(k) \). Especially, the lower-multipole spectra are shown to be powerful to measure the characteristic scales of BAOs (e.g., [5, 15, 25]). It is also helpful for our purpose to understand visually how well the analytic models can accurately reproduce the simulation results. Given the redshift-space power spectrum in two dimensions, \( P(k, \mu) \), the multipole power spectrum, \( P_\ell(k) \), is defined as

\[
P(k, \mu) = \sum_{\ell=\text{even}} P_\ell(k) P_\ell(\mu),
\]

\[
P_\ell(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu P_\ell(\mu) P(k, \mu),
\]

where \( P_\ell(\mu) \) denotes the Legendre polynomial.

Fig. 4 shows the monopole (\( \ell = 0 \)), quadrupole (\( \ell = 2 \)), and hexadecapole (\( \ell = 4 \)) power spectra (from top to bottom). The results are shown for dark matter, light, and heavy halo subsamples (from left to right), divided by the smooth reference power spectrum, \( b^2(k) P_{\text{nw}}(k) \), where the power spectrum \( P_\ell_{\text{nw}}(k) \) is computed from \( P_{\text{nw}}(k) \) taking the linear Kaiser effect into account:

\[
P_\ell_{\text{nw}}(k) = \begin{cases} 
(1 + \frac{3}{2} \beta + \frac{1}{2} \beta^2) P_{\text{nw}}(k) & ; \ell = 0 \\
(4 \beta + 4 \beta^2) P_{\text{nw}}(k) & ; \ell = 2 \\
\frac{8}{35} \beta^2 P_{\text{nw}}(k) & ; \ell = 4
\end{cases}
\]

with the quantity \( \beta \) defined by \( \beta(k) = f/b(k) \).

Apart from the noisy hexadecapole power spectrum, which is largely due to the finite sampling of the Fourier modes along the \( \mu \) direction, both of the streaming and TNS models are broadly consistent with the N-body results for the dark matter and light halo subsamples. A closer look at the acoustic feature reveals a slight discrepancy between the simulations and streaming model (dashed). Further, for the heavy subsample, the streaming model fails to reproduce the enhancement of clustering amplitude on small scales. These are all what we found in Figs. 2 and 3 and are consistent with Ref. [33]. The notable point is that these discrepancies are visually evident even for the dark matter and light halos. That is, while the streaming model predicts a rather clear BAO signal, the actual acoustic structure seen in the N-body simulations seems somewhat degraded, and because of this, the streaming model slightly overshoots the N-body results at low-\( k \), and eventually turn to underestimate at high-\( k \). By contrast, the model including the corrections reproduces the N-body results fairly well, and quantitatively explains a slightly smeared BAOs in the cases of dark matter and light halos, as well as a large enhancement in the heavy subsample.

Carefully looking at the results of the TNS model, however, the oscillatory behavior seen in the quadrupole spectrum seems to be rather over-smeared, and the acoustic structure becomes featureless. Because of that, the visual impression for the agreement with N-body simulations is somewhat degraded. As it has been discussed in Ref. [33], this is presumably due to our heterogeneous samples, the non-linearity of gravitational clustering is strong, and the application of standard PT might be subtle. Nevertheless, as we will show later, the goodness of fit inferred from \( \chi^2_{\text{red}} \) favors the model including the corrections, and the model broadly gives a good agreement with N-body simulations in all halo subsamples. In this respect, the model \([7]\) captures an important aspect of redshift-space clustering, and the role of the non-linear coupling described by \( A \) and \( B \) terms is quite essential.

D. Dependence on halo mass and maximum wavenumber

So far, we have focused on the specific halo subsamples, light and heavy, as well as dark matter, and explored the sample dependent properties of the redshift-space power spectrum. Here, we examine all the halo subsamples, and investigate the halo mass dependence on the validity of analytic models in a more quantitative
FIG. 3: Power spectra in redshift space normalized by $[b(k) + f \mu^2] P_{nw}(k)$ as a function of directional cosine $\mu$ for fixed wavenumbers, $k = 0.1225, 0.1475, 0.1725$ and $0.1975 \, h$ Mpc$^{-1}$ (from left to right). We adopt the function $b(k)$ in the normalization factor for halos directly measured in real space by Eq. (10), while we set $b(k) = 1$ for dark matter. The symbols are measurements from N-body simulations with errorbars estimated by Eq. (13), while the lines are fits by the formula of Eq. (7) with (solid, labeled by 'TNS(L)') and without (dashed, labeled by 'streaming(L)') correction terms. Top: dark matter. Middle: light halo subsample. Bottom: heavy halo subsample. Note that we adopt the Lorentzian form of the Finger-of-God factor for the model predictions, and the velocity dispersion $\sigma_v$ is determined by fitting to the corresponding N-body data. The best-fit values of $\sigma_v$ as well as the resultant reduced chi-squared defined by Eq. (14) for each model are shown in the panels.

way, by evaluating the goodness of fit, $\chi^2_{\text{red}}$. Fig. 5 summarizes the comparison between analytic
models and N-body simulations. In each panel, varying the maximum wavenumber $k_{\text{max}}$ for the range of the fitting, the resultant best-fit value of $\sigma_v$ divided by the linear theory prediction $\sigma^2_{v,\text{lin}} = \int dq P_{\text{lin}}(q)/(6\pi^2)$ (top) as well as the reduced chi-squared $\chi^2_{\text{red}}$ (bottom) are shown in each analytic model. Here, in computing the analytic models, we examine both the Lorentzian and Gaussian forms of the Finger-of-God damping (indicated by filled and open symbols, and labeled by ‘L’ and ‘G’ in the figure legend, respectively). The results are all plotted as a function of the averaged halo mass $M_h$ for each subsample.

We first notice that the best-fit value $\sigma_v$ and $\chi^2_{\text{red}}$ are irrelevant for the choice of the Finger-of-God damping function, consistent with the result found in Ref. [33]. This is presumably because the maximum wavenumber $k_{\text{max}}$ for the range of fitting is limited, $k_{\text{max}} \lesssim 0.2 \, h \, \text{Mpc}^{-1}$, and no appreciable effect of the small-scale clustering appears on the scales of our interest. On the other hand, the best-fit $\sigma_v$ shows a strong model dependence. While the resultant numerical values decrease with the halo mass in the streaming model, the fitted results of $\sigma_v$ mostly remain unchanged in the TNS model, and interestingly are very close to the linear theory prediction $\sigma^2_{v,\text{lin}}$. Further, the best-fit value in the streaming model is sensitively affected by the range of fitting, and for $k_{\text{max}} \gtrsim 0.1 \, h \, \text{Mpc}^{-1}$, it significantly deviates from the linear theory prediction, and eventually becomes zero for massive halo subsamples. This peculiar behavior is somewhat counter-intuitive, and indicates the breakdown of the model prescription.

Indeed, for the streaming model, the goodness of fit indicated by $\chi^2_{\text{red}}$ becomes worse as increasing the halo mass and maximum wavenumber. Apparently, even at the best-fit values $\sigma_v = 0$, the reduced chi-squared $\chi^2_{\text{red}}$ tends to keep the similar values to those obtained in the TNS model, and the streaming model illusively seems to reproduce the N-body results as well. Note, however, that the goodness of fit indicated by $\chi^2_{\text{red}}$ is merely a statistical measure for the discrepancy between model and data, and cannot be used to judge which model is physically plausible or not. As it has been demonstrated in Ref. [33] (see also Ref. [29]), the parameter estimation based on the streaming model can cause a large systematic bias in the best-fit values of the growth-rate parameter $f$, while the model including the corrections correctly reproduces the fiducial values fairly well within the statistical error at 1-$\sigma$ level. Although this demonstration has been made in the dark matter case, the same would be true for the halo subsamples. Since the velocity dispersion directly measured from N-body simulations is known to be nearly independent of the scales and halo mass (e.g., [61, 62]), we conclude that the streaming model fails to capture some aspects of redshift-space clustering, and the model including the corrections can provide a better prescription which also gives a physically reasonable estimate of the halo velocity statistics.

V. IMPLICATIONS

The goal of accurately modeling redshift distortions is to provide a practically useful theoretical template, and with a precision power spectrum measurement, we can tightly constrain and/or measure the growth history of large-scale structure as well as the cosmological distances. For on-going and/or future galaxy surveys, the precision of the power spectrum measurement is expected to reach at a percent-level, and even a slightly improper modeling of the redshift-space power spectrum can lead to a serious systematic bias in the cosmological parameter estimations. Then, an important question is how seriously the tiny deficit seen in the streaming model can lead to a biased estimation of cosmological parameters.
Here, assuming that the model including the corrections $A$ and $B$, given by Eq. (7), is correct, we estimate the size of systematic biases caused by the incorrect model of redshift distortions, particularly focusing on the growth-rate parameter. To do this, we use the Fisher matrix formalism presented in Ref. [25, 33]. Provided the survey parameters that characterize the galaxy surveys such as the observed redshift $z_c$, survey volume $V$, and number density of galaxies $n_g$, the Fisher matrix analysis enables us to estimate not only the statistical uncertainty but also the systematic bias for the best-fit value of cosmological parameters around the fiducial cosmological model. Following the Fisher matrix calculations given by Ref. [25, 33], we specifically consider the on-going and up-coming galaxy surveys, BOSS [26], SuMIRe-PFS [28], and HETDEX [27], and derive the expected constraints.
on the growth-rate parameter. Table I summarizes the
properties of the three galaxy surveys used in the Fisher
matrix calculations.

For illustrative purpose, we assume a prior knowledge
of the real-space power spectra, $P_{\delta_0}$, $P_{\delta_0}$, and $P_{\theta_0}$. Then,
adopting the streaming model as a wrong prior assumption
for the template spectrum, we try to estimate the
growth-rate parameter $f$ from the redshift-space power
spectrum in two dimensions. Notice that in addition to
redshift distortions, the imperfect knowledge of the late-
time cosmic expansion leads to another appreciable effect
of clustering anisotropies, known as the Alcock-Paczynski
effect. Here, we marginalize over the Alcock-Paczynski
effect characterized by the distance scales, $D_A(z)/D_{A,fid}$
and $H(z)/H_{fid}$ (e.g., [23, 11, 42]), where the subscript
$\text{fid}$ indicates the fiducial value. As a result, the total
number of free parameters assumed in the parameter es-
timation is five, i.e., $f$, $D_A(z)/D_{A,fid}$, $H(z)/H_{fid}$, $b$, and
$\sigma_{\gamma}$. For each galaxy sample obtained from the three sur-
veys, we assume the scale-independent linear bias listed
in Table I and adopt the linear theory estimate, as a fiducial value of the velocity dispersion, $\sigma_{\gamma}$.

Fig. 6 summarizes the forecast result for the Fisher ma-
trix analysis based on the power spectrum measurements
in two dimensions. The fiducial growth-rate parameter is
depicted as solid line, and the errorbar around each sym-
metric analysis based on the power spectrum measurements
in two dimensions. The fiducial growth-rate parameter is
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metric analysis based on the power spectrum measurements
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depicted as solid line, and the errorbar around each sym-

As it is clearly seen in Fig. 6, the estimated best-fit values
of $f$ are systematically lower than the fiducial values. Ap-
parently, the deviation from the fiducial value is not
so large at each point, and the biased estimates of the
best-fit value are sometimes inside the 1-$\sigma$ error around
the fiducial value. However, the combined result with all
the constraints strongly disfavors the fiducial model. For
instance, if we parametrize the growth-rate parameter by
(e.g., [2, 63])

$$f(z) = \left[\Omega_m(z)\right]^\gamma,$$  (20)

we obtain $\gamma = 0.77 \pm 0.04$, which significantly deviates
from $\gamma = 0.55$ expected from the fiducial model.

The reason why the theoretical template neglecting the
corrections leads to the underestimate of growth-
rate parameter is basically explained by the mismatch
of the overall shape of the power spectra. As shown in
Sec. IV.C on large scales, the best-fit streaming model
slightly overtakes the amplitude of each multipole spec-
trum in both the TNS model and N-body simulations,
and then the model eventually turns to underestimate at high-$k$.
Note that these are the outcome of the single-
parameter fit. In order to closely match the TNS model
and/or N-body simulations, a simple way is to further
adjust the parameters other than $\sigma_{\gamma}$. Among the remain-
ing four parameters, the bias parameter $b$, and distance
scales $D_A$ and $H$ are indeed well-constrained, and their
contribution to the systematic bias is likely to be rather
small. In this respect, a slight change of the growth-
rate parameter, which controls the strength of Kaiser ef-
effect, is the only possible way to match the result of TNS
model and N-body simulations on large scales. Note that
whether the streaming model overestimates or underesti-
mates the parameter $f(z)$ heavily depends on what scales
one weighs in the parameter estimation. In our specific
example in Fig. 6 we choose rather conservative values of
$k_{\text{max}}$. Thus, the net effect of the systematic bias appears
as an underestimation of $f$. A great emphasis is that
even the tiny discrepancy seen at low-$k$ (see Fig. 6)
can lead to a serious systematic bias in the on-going and/or
up-coming power spectrum measurements.

Recently, the WiggleZ dark energy survey has provided
a large data set of redshift-space clustering around
$0.1 < z < 0.9$ [64], from which tight constraints on
the growth-rate parameter have been put based on the
streaming model. After an extensive test for various
models of redshift distortions, the authors of Ref. [64]
conclude that both the streaming model combining the
fitting formula by [63] and the TNS model provide a rea-
sonable fit to the observed clustering data, and can be
used to derive the constraints on the growth-rate param-
eter. Indeed, the clustering bias of this galaxy sample
is shown to be rather small $b \sim 1$, and no appreciable
distinction between the TNS and streaming models is
manifest within the statistical errors. In this sense, a rea-
sonable goodness of fit in both the streaming and TNS
models sounds rather consistent with our findings, and
the derived results on the growth-rate parameter would
be unbiased. However, galaxy samples with a large clus-
tering bias $b > 1$ will certainly exhibit a non-negligible
effect of non-linear redshift distortions, and a tiny deficit
in the theoretical template can lead to a large systematic
bias on $f$, as shown in Fig. 6. Hence, the suitable choice
of the theoretical template is very crucial for future pre-
cision measurements of the power spectrum.

VI. SUMMARY

We have presented the redshift-space power spectra of
dark matter and halos measured from a large set of cos-
ological N-body simulations, and tested the analytical
models of redshift distortions against the N-body results.
With a particular focus on the redshift distortion effects
of halos, we have created the nine halo catalogs over a
wide mass range. The resultant volume and signal-to-
noise ratio of each catalog are roughly comparable to
those in the SDSS DR7 LRG.

We found that the measured halo power spectrum in
redshift space exhibits a large enhancement in amplitude
on large scales, and the effect becomes significant as in-
creasing the halo mass. This enhancement cannot be
simply explained by the Kaiser effect, and the popular model of redshift distortions called streaming model fails to reproduce the N-body results. The detailed comparison with N-body simulations further reveals that even for the less massive halos with $M_h \gtrsim 10^{13} h^{-1} M_\odot$, a small but non-negligible discrepancy is manifest on large scales, and the best-fit values of the velocity dispersion $\sigma_v$ is rather sensitive to the range of fitting and halo subsamples. By contrast, the model based on the perturbation theory description (i.e., the TNS model), which includes non-trivial corrections to the streaming model, gives a better agreement with N-body simulations for every halo subsample. In particular, the model quantitatively explains a large enhancement of the power spectrum amplitude in heavy subsample fairly well. These results indicates that the non-linear coupling between density and velocity fields induces the new effect of redshift distortions, which seems to have sensitive dependence on the clustering bias. The corrections in the TNS model \cite{1802.00109} can describe the observed feature of the redshift-space halo clustering very well.

To investigate how the slightly improper modeling of redshift distortions affects the cosmological parameter estimation, we have also estimated the size of systematic bias based on the Fisher matrix formalism. Especially focusing on the growth rate parameter, we consider the on-going and upcoming power spectrum measurements from BOSS, SuMIRe-PFS and HETDEX, and found that even the small deficit in the theoretical template of the power spectrum can produce a large systematic bias in the growth rate parameter, and the fiducial model would be erroneously ruled out. This is true even if we choose a conservative value of $k_{\text{max}}$. Hence, the accurate description of the redshift-space power spectrum is quite essential, and the model including the corrections can be used as a reliable theoretical template of the power spectrum.

Finally, we leave the following issue as a future work. Throughout the analysis, we have calibrated the halo bias relationship directly from the N-body simulations in real space, and the measured result of the bias parameter has been used to compute the analytic model of the redshift-space power spectrum, after subtracting the non-Poissonian residual noise. Of course, this treatment is infeasible in real observations, and we must combine other data set in order to get the information of the clustering bias. One plausible approach is to combine the weak lensing measurement. Since the weak lensing measurement directly probes the matter distribution, the cross correlation between weak lensing and galaxy clustering data can give a powerful way to simultaneously characterize the clustering bias and the residual noise contributions. Synergy with the weak lensing measurement is worth considering, and the methodology to extract a pure clustering signal should be further exploited.

\section*{Acknowledgments}

This work was initiated while staying in Institut d’Astrophysique de Paris and Institut de Physique Théorique. We thank Thierry Sousbie, Sebastien Peirani, Stephane Colombi, Francis Bernardeau and Patrick Valageas for their hospitality and discussions during our stay. We also thank Masahiro Takada, Issha Kayo and Eiichiro Komatsu for their fruitful comments and discussions. T. N. is supported by a Grant-in-Aid for Japan
Society for the Promotion of Science (JSPS) Fellows (PD: 22-181) and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. A. T. is supported by a Grant-in-Aid for Scientific Research from JSPS (No. 21740168). Numerical calculations for the present work have been in part carried out under the "Interdisciplinary Computational Science Program" in Center for Computational Sciences, University of Tsukuba, and also on Cray XT4 at Center for Computational Astrophysics, CfCA, of National Astronomical Observatory of Japan. This work was supported in part by Grant-in-Aid for Scientific Research on Priority Areas No. 467 “Probing the Dark Energy through an Extremely Wide and Deep Survey with Subaru Telescope”, and JSPS Core-to-Core Program “International Research Network for Dark Energy”.

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Throughout the paper, we work with the distant-observer approximation, and neglect the angular dependence of the line-of-sight direction, relevant for the high-redshift galaxy surveys.

Note that in fitting Eq. (15) to the measured bias parameter, we allow to vary $A$ as well as $b_0$ and $Q$, although the model (15) has been originally introduced adopting the fixed value, $A = 1.4$. 