1. Introduction

Recently, increasing interest has been paid on solidification of alloys by controlling magnetically driven flow to get products with high quality.1–2) There are two typical examples of application of this technique: single crystal growth of semiconductor materials and continuous casting of steels. In the former case, the subject of theoretical investigations has mainly been focused on the control of convection in the system through application of rotating magnetic fields so as for a more homogeneous temperature, composition distribution and a lower density of inclusions.3–5) However, the detailed mathematical formulations about the effect of rotating magnetic fields on the evolution of solidification structure have not yet been seen in the literatures.

While in the latter case, electromagnetic stirring (EMS), as one of the most effective techniques for grain refinement and alleviation of casting defects, has been practiced in almost all the stages of continuous casting processes.7–9) Furthermore, this technique has also been adopted to produce semi-solid billets for thixo-casting. This slurry was expected to produce near net shape products without defects, such as micro-porosity and shrinkage.10) Nevertheless, few quantitative studies have been carried out to present the effect of electromagnetic stirring on the evolution of solidification grain structures in spite of their indisputable experimental results.

Numerical modeling techniques have made it possible to analyze heat/mass transport phenomena and microscopic phenomena for solidification processes, leading to the development of various kinds of nucleation and growth models for the prediction of microstructure evolution in solidification of metals.11–14) Although these models were developed based on simple heat conduction mechanism, some works have still attained satisfactory success in predicting the temperature distribution and microstructure formation in foundry casting processes. In practice, flow behavior of molten melt in casting processes has been regarded as an unavoidable phenomenon during solidification, and fluid flow has a pronounced effect on solidification structures.15,16) A few study has so far been done to develop computational model to simultaneously predict the evolution of solidification grain structures and the dynamic evolution of fluid flow with heat transfer under electromagnetic stirring because of a complex process involving both macroscopic transport phenomena such as melt and heat flow and microscopic solidification phenomena like nucleation and grain growth.17) A coupled cellular automaton-continuum model, by which the evolution of solidification structures and the deflection behavior of the columnar dendritic grains solidified in a flowing melt of Al alloys were simulated successfully, has been developed in our group in the past few years.15)

The purpose of the present study is to develop a numerical model coupling a macroscopic transport model for heat
and fluid flow analysis with a microscopic microstructure model under the electromagnetic stirring, in which the influence of magnetically driven flow on the formation of solidification grain structures was included. The effects of casting process variables, such as pouring temperature, mold temperature and stirring intensity, on the evolution of solidification grain structures were investigated experimentally and compared with simulations.

2. Experimental Procedure

Figure 1 shows a schematic drawing of the experimental assembly. An Al–5.0wt%Cu alloy prepared from a commercial pure Al (99.9 wt%) and an Al–50wt%Cu master alloy, was melted in a graphite crucible under Ar gas atmosphere with a superheating of 150°C, and then was cast into a square mold made of austenitic stainless steel with an external size of 80×80 mm, an internal size of 50×50 mm and a height of 150 mm. The bottom and top of a mold were insulated using Kaowool. An electromagnetic stirrer for the generation of a rotating magnetic field was used. The regions composed of a mold and the melt are subjected to a rotating magnetic field produced by an electromagnetic stirrer system. The rotating rate of the magnetic field was 1440 rpm, and the intensity of the magnetic field was changed by the adjustment of the AC voltage input employed in a range of 0–250 V. The stirrer was placed in an austenitic steel container in which cooling water was circulated. Two stainless steel sheathed K-type thermocouples, one in the center and the other near the surface, with a distance of 20 mm and a height of 60 mm from the bottom of the mold were embedded in the bottom lid of the mold for the measurement of cooling curves. The solidification grain structures were observed across the transverse section at the central position of ingots after etching with Keller solution.

3. Mathematical Formulation

For the sake of simplicity, we consider a two-dimensional transient fully developed laminar flow of viscous incompressible electrically conducting fluid driven only by an electromagnetic stirring force. In the present study, the electromagnetic stirring intensity is not very large, flow velocity of molten metal is not big, and the Reynold’s number of fluid flow in the casting system. Fluid flow can occur not only in the bulk liquid, but also in the mushy zone. This paper is concerned primarily with bulk flow in the liquid. In addition, the solid phase is assumed to be rigid and attached to the mold wall. The present model does not account for the movement of equiaxed crystals in the melt. All fluid properties are held constant except for the viscosity of the fluid, which is taken to be temperature dependent.

3.1. Macro-scale Transport Equations

In terms of the above assumptions, the governing equations of macroscopic transport consist of the Navier-Stokes equations for fluid flow and the thermal balance equation for heat transfer as follows

**Equation of continuity:**
\[ \nabla \cdot \vec{u} = 0 \] ...............................(1)

**Navier–Stokes equation:**
\[ \rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \cdot \nabla \vec{u} = -\nabla p + \nabla \cdot (\mu \nabla \vec{u}) + \vec{f}_L \] ...............................(2)

where \( \vec{u} \) is a velocity vector, \( \rho \) is the density, \( p \) is the hydrostatic pressure, and \( \vec{f}_L \) is the Lorentz force. \( \mu \) is the dynamic viscosity, which is chosen to vary exponentially with temperature in the form
\[ \mu = \mu_0 e^{-(T - T_0)} \] ...............................(3)

where \( \mu_0 \) is the coefficient of viscosity, \( T_0 \) is the mean nucleation temperature, and \( A \) which is an empirical constant determined numerically, is chosen to be 0.5 in the present study. The viscosity of liquid is considered to be \( \mu_0 \) when its temperature is higher than \( T_0 \). This method can help stabilize the numerical calculation to deal with the phase interaction effects on fluid flow.

**Energy balance in the system is governed by**
\[ \rho c_p \frac{\partial T}{\partial t} + \rho c_p \vec{u} \cdot \nabla T = \nabla \cdot (\lambda \nabla T) + \rho L \frac{df_s}{\partial t} \] ...............................(4)

where \( T \) is the temperature, \( c_p \) is the specific heat, \( \lambda \) is the thermal conductivity, \( L \) is the latent heat of fusion, and \( f_s \) is the fraction of solid.

The continuum formulation is adopted due to the particular geometry of the casting system, which enables us to freely use the fixed single domain to compute fluid flow and temperature.

No-slip boundary condition is exerted along the solid and liquid interface for the fluid flow calculation. The Newton’s law of cooling was used at the melt/mold interface and at the mold/air interface.

3.2. Micro-scale Solidification Kinetics

Nucleation is the dominant stage of microstructural evolution in solidification, which leads to the establishment of final grain population. Therefore, nucleation conditions are of great importance in determining the characteristics of
microstructure. There are two significant methods for the evaluation of heterogeneous nucleation: the instantaneous and continuous nucleation model. The instantaneous nucleation model assumes nucleation site saturation, that is, all nuclei are generated at the nucleation temperature, indicating that the total density of grains is not affected by the cooling condition and the grain size will be uniform. On the other hand, the continuous nucleation model assumes a continuous dependency of nucleation density on temperature. Actually, in most cases of alloy solidification, more than one type of nucleation site or foreign particles, such as inoculant particles, are generally considered to exist, and each of these sites has its own critical undercoolings for nucleation. Thus the number of grains and the grain sizes vary according to the solidification condition. In both cases an empirical relationship between the resultant number of nuclei and the cooling rate or the amount of undercooling must be provided to evaluate microstructural evolution. The basic equations and related parameters must be experimentally evaluated or assumed.

In the present study the continuous nucleation model was adopted in which two different Gaussian distributions characterized by the mean nucleation undercooling $\Delta T_{\text{mean}}$, the standard deviation undercooling $\Delta T_{\sigma}$ and the maximum density of nuclei $n_{\text{max}}$ were considered for treating heterogeneous nucleation both on the mold wall and in the bulk liquid. The evolution of grain structures in solidification of Al alloys were presented successfully by mean of the continuous nucleation model, and more detailed information was provided in the literatures. In the presence of electromagnetic stirring, the temperature field was greatly distorted by the convection in the bulk liquid, resulting in a rapid removal of bulk liquid superheat, instantaneous appearance of nucleation undercooling in the entire liquid and subsequent fine solidification structures. Compared with the case without electromagnetic stirring, more potent heterogeneous nucleation sites with their own critical undercoolings for nucleation, which may be occupied by grain growth, will be activated simultaneously in the presence of electromagnetic stirring. As a result, $n_{\text{max}}$ was dealt differently for the cases with and without electromagnetic stirring, although $\Delta T_{\text{mean}}$ and $\Delta T_{\sigma}$ were assumed to be identical in terms of an alloy nature. The nucleation parameters under various conditions were showed in Table 1. Several theories such as the free chill crystal theory, the secondary dendrite remelting theory, and the dendrite fragmentation theory et al., have been proposed to explain the grain refinement under electromagnetic stirring. From the variety of explanations proposed from the range of experiments performed it would seem probable that different mechanisms operate under different conditions. In the present work, the cooling rate in the casting system is very fast and the stirring intensity is not very strong, and thus there is no enough time or energy for the electromagnetic force to break dendrites. Hence, the dendrite fragmentation was neglected in the present study. Maybe there inevitable exists a small portion of nuclei from the convective transport of dendritic debris, but it has a little effect on the simulation results.

Characteristics of dendritic growth can be classified into two models, the LKT (Lipton–Kurz–Trivedi) and the KGT (Kurz–Giovanola–Trivedi). The LKT model describes a free dendritic growth into the undercooled melt, which is usually used in rapid solidification. On the other hand, the KGT model is used for a constrained dendritic growth with a positive temperature gradient in the liquid. Hence, the KGT model was adopted to evaluate the growth velocity of a dendritic tip at a given undercooling in the melt. The detailed growth algorithm and growth kinetics parameters could be found elsewhere.

### 3.3. Electromagnetic Simulation

The finite-element code ANSYS 5.7 was used for the simulation of electromagnetic field. This code allows solving the transient three-dimensional Maxwell equations for any geometric arrangement and any materials. Far-field elements are provided to solve the problem about infinite boundary. Once the magnetic field $B$ and the induction current $j$ are known, the time-averaged Lorentz force $f_L$ induced in the liquid metal can be calculated by evaluating the relation

$$ f_L = \text{Re}(j \times B) $$

Figure 2 shows the Lorentz force distribution in the melt generated by electromagnetic stirring. Maximum strength and frequency conditions of the rotating magnetic field are 0.05 T and 60 Hz, respectively.

### 4. Numerical Methods

The electromagnetic problem can be treated separately from the fluid flow in the low magnetic Reynolds number approximation. Hence, our numerical simulations consist of two steps. First, we calculate the magnetic field by solving the Maxwell equations in harmonic state based on the finite.

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### Table 1. Nucleation parameters used in the study.

| $\gamma$ | $n_{\text{max}}$ | $\Delta T_{\text{mean}}$ | $\Delta T_{\sigma}$ | $n_{\text{max}}$ | $\Delta T_{\text{mean}}$ | $\Delta T_{\sigma}$ |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Fig. 4, 6(b), 7 | $2.0 \times 10^3$ | 0.5 | 0.1 | $1.0 \times 10^3$ | 5.0 | 0.5 |
| Fig. 5, 6(b), 6(c), 8, 9 | $4.0 \times 10^3$ | 0.5 | 0.1 | $6.0 \times 10^3$ | 5.0 | 0.5 |

Note: the relations between the nuclei density for 3D, $n_{\text{max}}$ (m$^{-3}$), and the ones for 2D, are given in the literature. The subscript $s$ and $b$ indicate the values at the surface and in the bulk, respectively.

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Fig. 2. Distribution of Lorentz force generated by a rotary magnetic field with a frequency of 60 Hz and a maximum strength of 0.05 T.
element method to obtain the resulting distribution of the elemental Lorentz forces acting on the liquid metal. Secondly, continuum equations were solved iteratively at each macro-scale time step using the control volume-based finite difference method for macro-scale transport solutions. The SIMPLE method was applied to solve the momentum and continuity equations, into which the elemental Lorentz forces were directed as a source term. A line-by-line solver based on the tri-diagonal matrix algorithm (TDMA) was used to obtain the solutions for macro-scale transport equations, and the upwind scheme was adopted to evaluate the contribution from both the convection and diffusion terms. For each macro-scale time step, the momentum equations were solved first in the iteration process using the most updated velocity and viscosity. Based on the calculation of velocity field, the temperature distribution can then be obtained in a similar manner. The temperatures in the macro-scale cells were linearly interpolated into the temperatures in the CA cells, upon which nucleation and grain growth were calculated using the CA model.

Considering the difference between the macro-scale and micro-scale time steps, a scheme composed of the following two time-steps, first proposed by Thevoz et al.\textsuperscript{23} and modified by others,\textsuperscript{24,25} was adopted. First, an appropriate macro-scale time step is chosen for the fluid flow and temperature calculations. Within each macro-scale time step, the maximum micro-scale time step is indirectly determined by the maximum growth velocity obtained by scanning the growth velocities of all solidification interface liquid cells, and then the calculation of CA model will be carried on. For each micro-scale time step, the latent heat released by the solidifying CA cells was estimated by a modified temperature recovery method,\textsuperscript{26} by which the latent heat contributions of the growing cells can be fed back to the energy equation for correcting the temperature field. The micro-scale process is repeated until the accumulation of each micro-scale time step is equal or larger than the current macro-scale time step. This series of calculation proceeds in this manner until the end of solidification. The flowchart of the present calculation is shown in Fig. 3. The thermal and physical properties used in the calculation are shown in Table 2.

5. Results and Discussion

The present model was applied to predict the evolution of both macro-scale transport phenomena and micro-scale solidification grain structures solidified under magnetically driven flow. For the macroscopic simulation, the whole computational domain with a square of 80 × 80 mm$^2$, consisting of the casting and mold regions, was first divided into 160 × 160 control volumes (CV) for the calculation of velocity and temperature fields. Then, each macro-scale cell in the casting region was further divided into 50 × 50 CA cells to give a grid resolution of 10 μm for simulating the formation of solidification grain structures.

5.1. Effect of Electromagnetic Stirring

Figure 4 shows the stages of the temperature variation and the corresponding macrostructure evolution with a pouring temperature of 730°C and a mold temperature of 30°C in the absence of electromagnetic stirring. It is noted from Fig. 4 that nucleation first occurs along the mold wall and the dendrites originating from the surface nuclei with favorable orientation grow in the direction roughly perpendicular to the mold wall, leading to the onset of columnar growth. The columnar crystals, progressing in the direction perpendicular to the temperature contours, gradually grow into the liquid as the temperature drops. Finally, heterogeneous nucleation occurs in the bulk liquid ahead of the growth

Table 2. Thermal and physical properties used in the calculation.

| Property          | Value |
|-------------------|-------|
| $\rho$ (Density)  | 2780 (Al-5.0 wt% Cu alloy) |
|                   | 8900 (Stainless steel mold) |
| $\lambda$ (Thermal conductivity) | 192.5 (Al-5.0 wt% Cu alloy) |
|                   | 14.5 (Stainless steel mold) |
| $c_p$ (Specific heat) | 1086 (Al-5.0 wt% Cu alloy) |
|                   | 580 (Stainless steel mold) |
| $\mu_s$ (Viscosity coefficient) | 1.3 × 10$^{-3}$ |
| $T_n$ (Mean nucleation temperature) | 638.2 |
| $h_{m/m}$ (Metal/mold interfacial heat transfer coefficient) | 1000 |
| $h_{m/a}$ (Mold/air interfacial heat transfer coefficient) | 50 |
front of columnar crystals, and the columnar to equiaxed transition appears during solidification process.

Figure 5 shows the stages of the temperature, velocity variation and the corresponding macrostructure evolution with electromagnetic stirring with a voltage input of 220 V. It shows that electromagnetic stirring has a pronounced effect on the evolution of solidification structures. A rotary flow with counter-clockwise is first induced by electromagnetic stirring, and the temperature field is greatly changed by the convection in the bulk liquid. Although both nucleation and grain growth develop in a similar way, grain sizes become much finer with electromagnetic stirring.

5.2. Effect of Stirring Intensity

In order to investigate the effect of stirring intensity on solidification grain structures, macrostructures for various stirring intensities with a pouring temperature of 730°C and a mold temperature of 30°C are shown in Fig. 6. The stirring voltage inputs for (a), (b) and (c) are 0 V, 140 V, 220 V, respectively. Stirring intensity here can be clearly reflected by adjusting the voltage input to the stirrer. It is noted from Fig. 6 that the simulation results have a good agreement with the experimental ones. Without electromagnetic stirring, both the columnar and equiaxed grains are coarse. With an increase of stirring voltage input, grains become finer, which implies that abundant nuclei are induced in the liquid by exerting electromagnetic stirring. Under the present electromagnetic conditions, the columnar grains still appear in spite of successful grain refinement in the center of ingots. Additionally, it was observed in the experiments that only central liquid was stirred under electromagnetic stirring, and outer part of liquid seemed solidified too fast to be rotated. Thus an attempt was made to eliminate the columnar region by varying pouring temperature and mold temperature for producing fine solidification grain structures.

5.3. Effect of Pouring Temperature

Figure 7 shows the effect of pouring temperature on macrostructures without electromagnetic stirring with a mold temperature of 30°C. Figure 8 indicates the effect of pouring temperature on macrostructures with a stirring voltage input of 220 V. It is noted from Fig. 7 that in the absence of electromagnetic stirring, grains are generally coarse and become coarser as the pouring temperature increases. It is worthy to be mentioned that, although the nucleation conditions for the simulation are identical for the above three pouring temperatures, the simulated solidification structures, as the experimental ones indicated, show some difference due to the effect of pouring temperatures on nucleation conditions. Interestingly, almost the same fine solidification structures, as shown in Fig. 8, are obtained in the presence of electromagnetic stirring regardless of quite different pouring temperatures, indicating that the effect of electromagnetic stirring on solidification conditions is dominant. It was also noted that the region of columnar grains decreased a little as the pouring temperature increased from 700 to 750°C. However it did not change any more in spite of the higher pouring temperature of 770°C, which is unrealistic in practice. So, it is supposed that the liquid near the
Fig. 5. Evolution of flow field, temperature distribution and macrostructure of an Al–5.0wt%Cu alloy with a pouring temperature of 730°C and a mold temperature of 30°C in the presence of electromagnetic stirring with a voltage input of 220 V for various times after pouring: (a) 5.90 (s), (b) 14.05 (s) and (c) 22.05 (s).

Fig. 6. Simulated and experimental macrostructures of an Al–5.0wt%Cu alloy with a pouring temperature of 730°C and a mold temperature of 30°C under electromagnetic stirring for various voltage inputs: (a) 0 (V), (b) 140 (V) and (c) 220 (V).
Fig. 7. Simulated and experimental macrostructures of an Al–5.0wt%Cu alloy with a mold temperature of 30°C in the absence of electromagnetic stirring for various pouring temperatures: (a) 700°C, (b) 750°C and (c) 770°C.

Fig. 8. Simulated and experimental macrostructures of an Al–5.0wt%Cu alloy with a mold temperature of 30°C under electromagnetic stirring with a voltage input of 220 V for various pouring temperatures: (a) 700°C, (b) 750°C and (c) 770°C.
mold wall must be solidified so fast that there is no enough time for the liquid to be stirred efficiently like the liquid in the central region. Thus, a relatively large temperature gradient at the solidification front occurs, leading to promote the growth of columnar grains.

5.4. Effect of Mold Temperature

Figure 9 shows the simulated and experimental macrostructures of an Al–5.0wt%Cu alloy with a pouring temperature of 730°C under electromagnetic stirring with a voltage input of 220 V for various mold temperatures of 100, 300 and 500°C. It is noted from Fig. 9 that when the mold temperature is lower than 300°C, there is no particular change in solidification grain structures. However, when the mold temperature is higher than 300°C, the columnar region decreases apparently. As the mold temperature increases up to 500°C, the solidification structure shows fully fine equiaxed grains.

Figure 10 indicates the cooling curves during solidification of an Al–5.0wt%Cu alloy with a pouring temperature of 730°C and a mold temperature of 30°C with and without electromagnetic stirring. It is well known that lots of nuclei usually form on the mold wall due to the chilly metal mold. In the absence of electromagnetic stirring, temperature decreases continuously from the liquid to the solid, and a positive temperature gradient always exists ahead of the solidification front, where considerable latent heat is released. In addition, the latent heat of freezing decreases the cooling rate of the liquid, resulting in the formation of coarse solidification grain structures. However, in case of electromagnetic stirring, strong stirring flow quickly dissipates the latent heat released from the solidification front to the bulk liquid, leading to a relatively uniform temperature distribution in the liquid, which is considered to be necessary for the simultaneous nucleation. Under this situation, copious potential nuclei, mainly originating from heterogeneous nucleation, are induced to appear in the bulk liquid, which is virtually attributed to grain refinement. Actually, there also exists a small portion of nuclei from the convective transport of dendritic debris sheared by fluid flow, although the convective transport of the dendritic debris is not considered in the present study because of a lack of information on detailed kinetics models for the dendrite shearing by fluid flow.

In this study, thermal resistance in the system mainly exists at the interfere between the mold and the melt as a bottleneck due to a thick mold wall, and the liquid near the mold wall solidifies quite rapidly no matter whether electromagnetic stirring is imposed. It seems that a little increase of pouring temperature can’t effectively prevent rapid solidification near the mold wall. However, a large increase of mold temperature can retard the rapid solidification, which benefits to efficient mixture of liquid in the entire liquid region and subsequent instantaneous heterogeneous nucleation, resulting in fine equiaxed grains.
6. Concluding Remarks

A two-dimensional computational model has been developed to investigate the effect of electromagnetic stirring on the evolution of solidification grain structures. The model consists of a macroscopic heat and fluid flow analysis and a cellular automaton model under electromagnetic stirring. The dynamic evolution of macrostructures and transport phenomena in solidification of Al–5.0wt%Cu alloy has been presented successfully by means of the present model. Both the simulation and experimental results showed that grain refinement was improved remarkably in the presence of electromagnetic stirring, and that solidification macrostructures fully composed of fine equiaxed grains were obtained if the casting process variables such as pouring temperature, mold temperature and stirring intensity fitted together. The simulation results are also in good agreement with those obtained experimentally.

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