Adjustment of processing parameters error of planar enveloping hourglass worms based on the deviations of feature lines

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Abstract
Planar enveloping hourglass worm (TP worm) with better load capacity and transmission efficiency is a type of transmission superior to traditional cylindrical worm. It could be used to lightweight design. As the flank of the worm is a complex toroidal surface, higher demands are needed for the manufacture and the assembly. This paper proposes a new principle to adjust the processing precision of the worm. The principle is based on the research on the influence of the parameter errors on the deviations of the feature lines on the flank. With different types of parameter errors, the shape and the value of the deviation curves of the feature lines are different. At last, two cases are introduced to verify that the principle could be used to improve the precision of the TP worm.

Keywords: Error adjustment, Planar enveloping hourglass worm, Feature line, Tooth profile model, Processing error

1. Introduction

In 1865, Hindley first proposed the idea of the hourglass worm (Zhou, 2005). In the next century, this kind of worm developed diversely (Zhao et al, 2010; Oiwa et al, 1990; Sakai et al, 1978; Deng et al, 2012; Chen and Tsay, 2009; Maki et al, 1995; Okamoto, 1996). In 1971, Chinese engineers invented the planar double-enveloping hourglass worm gear transmission (Li and Shen, 1981). It is theoretical proved to be a transmission with superior performance. Compared with cylindrical worms, TP worms with complex surfaces have 2 to 3 times capacity, 5%-10% higher transmission efficiency and the 30 years of service life (GB-T16444, 2008). However, it is always a problem to manufacture high precision TP worm to reach the superior performance.

Since the surface of the TP worm is very complex, multi-axis linkage system is needed to complete the forming. Therefore, the structure of the machine tool is also complicated and its precision is hard to be guaranteed. There are two main methods to process the worm, traditional two-axis linkage grinding and four-axis linkage grinding with a virtual center (Zhang and Li, 2007). The former uses a machine tool with complicated structure that is hard to be adjusted and the process range is limited by the diameter of the rotational table. Although the latter method is easy to operate, it uses multi-axis linkage system that means the demand for the precision of the machine tool is higher. It is inevitable to produce processing error for both methods. The traditional way is to improve the precision of the machine tool to reduce the error. If the demand for the precision of the flank is much higher, it is impossible to use the traditional way to make it. With the development of automatic control technique, the measuring instrument for TP worms is developed (Shi et al, 2016). The instrument could measure the helix deviation, profile deviation and tooth thickness deviation. It is of significance to improve the quality of the worm flanks. During the gear processing, the deviations of flanks come from the error of the movement of the machine. The deviations could be separated into each degree of freedom of the machine and each degree of freedom is corresponding to each axis of the machine. According to compensating the error of each axis, the deviation of the flank could be adjusted (Shih and Chen, 2012; Tang et al, 2014; Jiang and Fang,
2015). This method could be transplanted to the TP worm manufacture. Some scholars have set out for this research. Wenjun Luo regards the minimum error between the theoretical helix and the measured helix as objective function and uses the genetic algorithm to get the error of each parameter (Luo et al, 2015). Datong Qin proposes a method to diagnose the adjustment value of the machine by the measured result of profile (Qin et al, 1996). These adjustment methods reduce the demand for the precision of the machine and ensure the quality and stability of the production.

However, the researches above take the helix deviation or the profile deviation into account to achieve limited improvement of the precision of the flank. A more comprehensive method is needed to improve the processing precision of the TP worms. This paper analyzes the influence of errors of different parameters (the center distance, the transmission ratio, the slope angle of the forming plane, the diameter of the base circle and the axial position) on flank deviation to present two feature lines, helix on the pitch circle and the throat axial profile on the symmetry shaft section. A new method based on the two feature lines are proposed to adjust the errors of the parameters. This is important to improve the processing precision of the TP worms.

2. Model of TP worm flank

In order to research the flank deviation, the model of the theoretical flank and that with errors should be built first.

2.1 Theoretical model

The modeling is consistent to the forming process (as shown in Fig. 1). Both of them regard a plane as the forming plane. The plane rotates around the axis of the base circle and the worm rotates around the axis of itself. They rotate at a constant transmission ratio. Finally, the flank of the worm could be acquired. Actually, the whole process is to copy the contact lines from the forming plane to the surface of the flank.

![Fig. 1. Forming principle of TP worms (Shi et al, 2016)](image)

2.1.1 Coordinate systems

Based on the forming process of the TP worm, the coordinates system could be built as Fig. 2. $S_j(O_j - X_jY_jZ_j)$ and $S_n(O_n - X_nY_nZ_n)$ are the static coordinate systems of the worm and worm gear, respectively. $Z_j$ and $Z_n$ coincide with the axes of the worm and the worm gear. $S_i(O_i - X_iY_iZ_i)$ and $S_0(O_0 - X_0Y_0Z_0)$ are the moving coordinate systems fixed to the worm and worm gear. $Z_i$ and $Z_0$ coincide with the axes of the worm and the worm gear. $S_3(UVZ_3)$ is the moving coordinate system fixed to the forming plane and $U$ is tangent to the base circle at point $O_i$. 

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The shaft angle of the worm and worm gear is $90^\circ$. The distance between the axes is center distance $a_i$. The radius of the base circle is $r_b$. The slope angle of the forming plane is $\beta$.

### 2.1.2 Model of the theoretical flank

According to forming process mentioned above, the contact line on the forming plane could be determined based on meshing principle (Litvin, 1994). Then, the contact line is transfer from $S_i(O_i - UVZ_i)$ to $S_i(O_i - X_iY_iZ_i)$. The model of the contact line on the worm flank could be acquired as Eq. (1).

$$
\begin{align*}
    & y = u_{i0} \cos \beta \sin \beta (u \cos \varphi_0 + r_b \sin \varphi_0 - a_i) \\
    & x_0 = u \\
    & y_0 = v \sin \beta - r_b \\
    & z_0 = v \cos \beta \\
    & x_1 = -\cos \varphi_0 \cos \varphi_1 x_0 + \sin \varphi_0 \cos \varphi_1 y_0 - \sin \varphi_1 y_0 + a_i \cos \varphi_1 \\
    & y_1 = \cos \varphi_0 \sin \varphi_1 x_0 - \sin \varphi_0 \sin \varphi_1 y_0 - \cos \varphi_0 - a_i \sin \varphi_1 \\
    & z_1 = -\sin \varphi_0 x_0 - \cos \varphi_0 y_0
\end{align*}
$$

The rotational angle of the forming plane is $\varphi_0 \in [\alpha - \varphi_x, \alpha - \varphi_x + z_k \cdot \tau]$. The parameter $u$ is with in $[R_{1i}^2 - r_b^2, \sqrt{R_{1i}^2 - r_b^2}]$. The rotational angle of the worm $\varphi_i$ equals to $\varphi_{i0}$. Transmission ratio $i_{i0}$ equals to $1/i_{i0}$. $\alpha$ is pressure angle. $z_k$ is the number of involved teeth. $\varphi_x$ is half of the working angle. $\tau$ is the pitch angle. $R_{1i}$ is the radius of the worm root circle and $R_{ai}$ is that of the worm tip circle. For different $\varphi_i$, contact lines in different positions could be determined. The flank of the worm could be constructed by combining the contact lines together.

### 2.2 Model of the flank with errors

The center distance, transmission ration, slope angle of the forming plane, axial position of the worm and diameter of the base circle are main parameters of to process a worm. During the manufacturing, it is inevitable that errors of these parameter will appear. The errors of them are represented as $\Delta a_i$, $\Delta i_{i0}$, $\Delta \beta$, $\Delta z$ and $\Delta d_0$. The parameters are changed to $a_i = a_i + \Delta a_i$, $i_{i0} = i_{i0} + \Delta i_{i0}$, $\beta' = \beta + \Delta \beta$ and $r'_b = r_b + \Delta d_0/2$. The model of the flank with errors is as following.
\[
\begin{align*}
\varphi' &= u'i'_{u} \cos \beta' + \sin \beta'(u' \cos \phi_{0}' + r_{z}' \sin \phi_{0}' - a_{i}') \\
x_{u}' &= u' \\
y_{0}' &= v \sin \beta' - r_{z}' \\
z_{0}' &= v' \cos \beta' + \Delta z \\
x_{i}' &= -\cos \phi_{0}' \cos \phi_{0}' x_{0}' + \sin \phi_{0}' \cos \phi_{0}' y_{0}' - \sin \phi_{0}' y_{0}' - a_{i}' \cos \phi_{0}' \\
y_{i}' &= \cos \phi_{0}' \sin \phi_{0}' x_{0}' - \sin \phi_{0}' \sin \phi_{0}' y_{0}' - \cos \phi_{0}' y_{0}' - a_{i}' \sin \phi_{0}' \\
z_{i}' &= -\sin \phi_{0}' x_{0}' - \cos \phi_{0}' y_{0}' \\
\end{align*}
\]

(2)

3. Analysis of the flank deviation

3.1. Definition of the normal deviation of the flank

We choose intersection of the throat plane and the pitch circle as the reference point which is regarded as the zero point. The theoretical model and the model with errors are relocated to make their zero points coincided each other. \( P(x_{i}, y_{i}, z_{i}) \) is a point on the theoretical flank. The normal line of the theoretical flank crosses the flank with error at point \( P'(x_{i}', y_{i}', z_{i}') \), as shown in Fig. 3. The distance of the two points is as Eq. (3).

\[
PP' = \sqrt{(x_{i}' - x_{i})^2 + (y_{i}' - y_{i})^2 + (z_{i}' - z_{i})^2}
\]

(3)

When the error flank is higher than the theoretical one, the normal deviation \( \Delta \) is \( PP' \), otherwise it is \( -PP' \).

Fig. 3. Normal deviation \( \Delta \) of the flank

3.2. Selecting the feature lines of flanks

The changing rule of the normal deviations of the flank is analyzed using the worm with the parameters in Table 1.

| Table 1 Parameters of the TP worm |
|----------------------------------|
| Parameter                        | value |
| Center distance \( a \) (mm)     | 125   |
| Tooth \( Z' \)                   | 1     |
| Transmission ratio \( i_{0} \)   | 33    |
| Diameter of the pitch circle in the throat plane \( d_{t} \) (mm) | 53    |
| Diameter of the basic circle \( d_{b} \) (mm) | 80    |
| Slope angle of the generating plane \( \beta \) (°) | 9     |
| Working length \( l \) (mm)      | 61.77 |

The feature lines are those could reflect the quality of the worm. The helix and the profile of the worm are usually...
chosen as the feature lines. The helix could represent the geometry precision in the axial direction and the profile show the geometry precision in the tooth high direction.

As the flank of the TP worm is complicated helical surface, the helices with different radii are not same. The flank could be separated into the tip circle helix, pitch circle helix and root circle helix. The profiles are also different from meshing-in part to meshing-out part because of the varied tooth thickness characteristic. This paper picks up the right flank as research object. The meshing-in part and the meshing-out part are shown in Fig.4. When \( \phi_h = \alpha - \phi_n + z_i / 2 \cdot \tau \), the contact line intersects the pitch circle at point M. A plane crossing the point M and axis of the worm is the symmetrical section. The whole worm could be separated into the meshing-in part, the throat part and the meshing-out part in the axial direction.

- Fig. 4. Axial tooth profiles of the worm in the symmetrical section

The results of the helices of the tip circle, pitch circle and root circle and the axial tooth profiles from meshing-in part to meshing-out part could represent the whole flank. The deviation curves could be drawn with the Z axis as the horizontal axis and the normal deviation as the vertical axis.

When the center distance error is 0.1mm, the curves are as shown in Fig. 5. The deviation curves of the helixes and axial profiles are similar to straight lines. The deviations of the helixes are negative near the meshing-in part and positive near the meshing-out part. The tip has bigger deviation than the root on the same axial profile.

- Fig. 5. Deviations of the tooth caused by the center distance error

When the transmission ratio error is 0.1, the curves are as shown in Fig. 6. The deviation curves of the helixes become a straight line in the opposite direction. The deviations of the helixes are positive near the meshing-in part and negative near the meshing-out part. The tip has bigger deviation than the root on the same axial profile near meshing-in part but that the axial profile near the meshing-out part shows the contrary rule.
Fig. 6. Deviations of the tooth caused by the transmission ratio error

The next simulation is deviation of the flank with the slope angle error of the forming plane 0.1°, the curves are as shown in Fig. 7. Only the deviation curves of axial profiles are similar to straight lines. The deviations of the helixes increase first and then decrease from the meshing-in part to the meshing-out part. The tip has bigger deviation than the root on the same axial profile.

Fig. 7. Deviations of the tooth caused by the slope angle error of the forming plane

The axial position error 0.1mm is considered and the results are as shown in Fig. 8. The deviation curves of axial profiles are similar to straight lines. The deviations of the helixes are positive near the meshing-in part and decrease when going to the meshing-out part. The tip has a bigger deviation than the root on the same axial profile.

Fig. 8. Deviations of the tooth caused by the axial position error

When the diameter error of base circle is 0.1mm, the curves are as shown in Fig. 9. The deviation curves of axial profiles are still similar to straight lines. The deviations of the helixes near the meshing-in part are bigger than other parts. The magnitude of the deviations is smaller than that caused by other parameter errors. The tip has a bigger deviation than the root on the same axial profile.
From Fig. 5 to Fig. 9, we could find the changing rule of the deviation curve of the pitch circle is in accordance with those of the tip circle and the root circle. Except the transmission ratio error, the deviation curve of the axial profile near the throat part is in accordance with those of the meshing-in part and the meshing-out part. Therefore, the pitch circle helix and the axial profile near the throat in the symmetrical section could be selected as the feature lines of the TP worm to research the influence of the parameter errors on the deviation of the whole flank.

4. Influence of the parameter errors on the deviations of feature lines

The feature lines could be used to express the geometric characteristic of the TP worm. In order to learn more about the influence of the parameter errors on the deviation of the flank, this section will study on the impact of each parameter error on the deviations of the feature lines. In Fig. 10 ~ Fig. 14, the red lines mean the parameter errors are positive and the blue ones mean the errors are negative. The vertical axis represents normal deviation of the flank.

4.1. Center distance error

Figure 10 is the impact of the center distance error on the deviations of the pitch circle helix and the axial profile near the throat in the symmetrical section. In Fig. 10a, when there is center distance error, the curves of helix deviation with a quite big radius of curvature are approximate as straight lines. When the center distance error is positive, the deviation of the meshing-in part is negative and that of meshing-out part is positive. The slope of the deviation curve is negative. When the error is negative, the result is opposite and the slope is positive. The slope of the curve decreases with the increase of the center distance error. In Fig. 10b, the curves of the axial profile deviation are more like straight lines. The deviation of the tip is obviously bigger than that of the root. The slope of the curves of the axial profile deviation increases with the center distance error. Based on the curves of the axial curve deviation, we could find that the positive center distance error will lead to larger tooth thickness which is helpful to enhance the endurance bending strength of the tooth. This improves the service life of the worm.

4.2. Transmission ratio error
Figure 11 shows the influence of the transmission ratio error. In Fig. 11a, the transmission ratio error also makes the curves of helix deviation similar to straight lines. What is different from the impact of the center distance error is that when the transmission ratio error is positive, the deviation of the meshing-in part is positive and that of meshing-out part is negative. The slope of the deviation curve is positive. The negative error leads to negative slope. The slope of the curve increases with the transmission ratio error. In Fig. 11b, the curves of the axial profile deviation look like straight lines as well. The deviations of the tip and root are almost the same. The axial profile deviation decreases when the transmission ratio error increases. Based on the curves of the axial curve deviation, we could find that the negative transmission ratio error will lead to larger tooth thickness.

![Fig. 11. Impact of the transmission ratio error](image)

4.3. Slope angle error of the forming plane

Figure 12 is result of the slope angle error of the forming plane. In Fig. 12a, the curves of the helix deviation are not straight lines. The absolute value of the deviation of the meshing-in part is the biggest. When it goes to the throat part, the absolute value decreases. The deviations of the flank between the throat part and the meshing-out part are very small. The deviation of the flank decreases when the slope angle error of the forming plane increases. In Fig. 12b, the curves of the axial profile deviation are almost straight lines. When the error is positive, the deviation of the root is positive and that of tip is negative. The slope of the deviation curve is negative. When the error is negative, the result is opposite and the slope is positive. The slope of the curve decreases with the increase of the slope angle error. Based on the curves of the helix deviation, we could find that the positive slope angle error will lead to negative deviation near the meshing-in part which is useful for reducing the meshing-in impact.

![Fig. 12. Impact of the slope angle error of the forming plane](image)

4.4. Axial position error

Figure 13 is the impact of the axial position error between the rotational center of the forming plane and the throat of the worm on the helix deviation and the axial profile deviation. In Fig. 13a, the rule of the curves of helix deviation caused by the axial position error is different from that caused by the slope angle error. The absolute value of the deviation of the meshing-out part is the biggest. When it goes to the throat part, the absolute value decreases. The deviations of the flank between the throat part and the meshing-in part change slowly and seem like a parabola. In Fig.
13b, it is opposite to the deviation caused by the slope angle error. When the error is positive, the deviation of the root is negative and that of the tip is positive. The slope of the deviation curve is positive. When the error is negative, the result is opposite and the slope is negative. The slope of the curve decreases with the increase of the axial position error.

![Graphs showing the impact of axial position error on helix deviation and axial profile deviation.](image)

4.5. Diameter error of the base circle

Figure 14 is the impact of the diameter error of the base circle on the helix deviation and the axial profile deviation. The shape of the helix deviation caused by the diameter error of the base circle is similar to that caused by the slope angle error of the forming plane. The sign is opposite. The impact of them on the axial profile deviation is almost the same.

![Graphs showing the impact of diameter error on helix deviation and axial profile deviation.](image)

4.6. Analysis of the influence on the deviations of feature lines

With each parameter error changing from positive to negative, the changing rules of the deviations of the pitch circle helix and the axial profile near the throat in the symmetrical section are shown in Table 2. According to the analysis above, when the parameter errors change, the slope of the curves will change as well. This is a rotation process. Another characteristic is that there is an intersection point for the deviation curves caused by the same parameter error and for different parameter errors, the positions of the intersection points are not the same.

| Errors                  | Helix deviation curve | Axial profile deviation curve |
|-------------------------|-----------------------|-------------------------------|
|                         | Rotation direction    | Intersection point            | Rotation direction | Intersection point |
| Center distance error   | Anticlockwise         | Zero point                    | Clockwise          | Not on the flank   |
| Transmission ratio error| Clockwise             | Zero point                    | Clockwise          | Not on the flank   |
The deviation curves of the pitch circle helix and the axial profile near the throat in the symmetrical section caused by the five parameter errors are quite different. These two feature lines could represent the whole flank to evaluate the precision of it. Therefore, according to comparing the measurement results of the feature lines with Fig.10 ~ Fig. 14, the type and the sign of the parameter error could be determined. After more comparison, the value of the error could be got. It could guide the adjustment of the parameters.

5. Adjustment of parameter errors based on the deviations of feature lines

5.1. Principle of adjustment of parameter errors

Based on the rules acquired above, the parameter errors could be adjusted to reduce the deviation of the flank. The principle of the adjustment is as following.

1) The adjustment should make the helix deviation meet the requirement. Then, the axial profile deviation should meet the design demand.

2) If the curve of the helix deviation is linear, the center distance or the transmission ratio needs to be adjusted. According to the axial profile deviation, it is determined to adjust the center distance or the transmission ratio.

3) If the curve of the helix deviation is no-linear, the slope angle of the forming plane, the axial position or the diameter of the base circle should be adjusted. When the absolute value of the deviation near meshing-in part is larger, the axial position should be adjusted. Otherwise, the slope angle or the diameter of the base circle should be adjusted. If the sign of the deviation of the root is same with that near the meshing-in part, the diameter of the base circle is required to be adjusted. In contrast, the slope angle is needed to be adjusted.

5.2. Experiment equipment

5.2.1 Machine tool

The machine tool consists of a double-decked working table, a spindle, a rotary table and a grinding head as shown in Fig. 15. The grinding head is equipped on the rotary table which is on the working table. The working table could carry out the circle interpolation motion and the rotary table rotates. These motions form the rotational motion of the forming plane. At the same time, the worm rotates around C axis. Before processing, the parameters of the TP worm should be put into the CNC system.
5.2.2 Measuring instrument

The measuring machine consists of translations in X axis, Y axis and Z axis and a rotation in $\Theta$ axis, as shown in Fig. 16a. X axis and Y axis use linear guide rails whose strokes are 150mm and 70mm respectively, and Z axis uses a V-plane rail with a stroke of 850mm. The radial runout of $\Theta$ axis is 1 $\mu$m and the face runout is 1.5 $\mu$m. X axis, Z axis and $\Theta$ axis driven by servo motors achieve the three-axis linkage with control system, and Y axis is manually controlled. The resolutions of linear encoders are 0.2 $\mu$m and that of the rotary encoder with two readheads is 0.45°. The one-dimensional inductance probe is equipped on Y axis with the stroke $\pm$0.6mm and resolution of 0.15 $\mu$m, as shown in Fig. 16b.

![a) Measuring machine](image1)
![b) Measuring process](image2)

Fig. 16. Measuring machine (a) and measuring process (b)

5.3. Experiment cases

In the following section, two cases are conducted using the principle of adjustment. The parameters of the TP worm is in Table 1.

5.3.1 Case 1

Worm 1 is chosen from a batch of product. The pitch circle helix and the axial profile near the throat in the symmetrical section of it are measured on the measuring instrument. The results are shown in Fig. 17 (blue lines). On the basis of the principle proposed in section 5.1, the error source of Worm 1 could be determined. According to the first item, the shape of the curve of helix deviation will be considered. In Fig. 17a, the deviation curve before adjustment is nonlinear, so we could go to the third item directly. As the deviation near the meshing-in part is larger than the other part, the error may come from slope angle of the forming plane or the diameter of the base circle. The shape of the curve of the helix deviation is quite similar to those in Fig. 12a and Fig. 14a. In order to determine the source, comparing Fig. 17b with Fig. 12b and Fig. 14b, it could be judged that the axial profile deviation corresponds to that caused by the slope angle error of the forming plane. The value of the error could be acquired by comparing the measured deviation curves with the theoretical curves. After comparison, the error of the slope angle could be approximately determined as 0.35°. The machine tool is adjusted based on the error of the slope angle to reprocess the TP worm. After adjustment, the deviations are measured again and the results are shown in Fig. 17 as red lines. In Fig. 17a, the helix deviation reduced obviously is less than 28 $\mu$m. It could meet GB class 6 precision (GB/T16445, 1996). The deviation of the meshing-in part is still large. The reason is that the meshing-in part is modified to make sure the meshing-in process is smooth. The axial profile deviation is reduced as well. In Fig. 17b, it could be seen that there are other kinds of errors. While the worm could meet the requirements, there is no need for more adjustment.
5.3.2 Case 2

Worm 2 is chosen from another batch of product. The measured results are shown in Fig. 18 (blue lines). On the basis of the principle proposed in section 5.1, the error source of Worm 2 could be determined. According to the first item, the shape of the curve of helix deviation will be considered. In Fig. 18a, the deviation curve before adjustment is linear, so we could go to the second item which tells us the error may come from the center distance or the transmission ratio. Then, comparing Fig. 18b with Fig. 10b and Fig. 11b, it could be judged that the axial profile deviation corresponds to that caused by the center distance error. After more comparison between Fig. 18 and Fig. 10, the center distance error could be approximately determined as -0.2mm. The machine tool is adjusted based on the center distance error to reprocess the TP worm. After adjustment, the deviations are measured again and the results are shown in Fig. 18 as red lines. In Fig. 18a, the helix deviation reduced obviously is less than 20μm. It could meet GB class 6 precision. Different from Fig. 17a, this worm is processed without the meshing-in part modification. In Fig. 18b, the axial profile deviation is reduced to less than 10μm. According to the results, the adjusted worm could meet the requirements.

6. Conclusions

(1) By analyzing the deviation curves caused by the errors of the center distance, the transmission ratio, the slope angle of the forming plane, the diameter of the base circle and the axial position, the pitch circle helix and the axial profile near the throat in the symmetrical section are selected as the feature lines of the TP worm to illustrate the deviation in the tooth longitudinal direction and tooth profile direction.

(2) According to the rules of the deviations of the feature lines changing with each parameter error, a new principle of adjustment is proposed to reduce the processing error caused by the five parameter errors. Two experiments are conducted to verify the feasibility of the principle. The precision of the adjusted flank could reach GB class 6.

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