The uses of singlets

Bohdan Grzadkowski, José Wudka
1Institute of Theoretical Physics, University of Warsaw, Hoża 69, PL-00-681 Warsaw, Poland.
2Department of Physics, University of California, Riverside CA 92521-0413, USA.
E-mail: bohdan.grzadkowski@fuw.edu.pl
E-mail: jose.wudka@ucr.edu

Abstract. A simple extension of the standard model is proposed that can mitigate some of the difficulties associated with the little hierarchy problem and simultaneously provide a viable dark matter candidate. Possible interactions with the neutrino sector and their consequences are also briefly described.

1. The little hierarchy problem
Despite its enormous experimental success the Standard Model (SM) is not believed to provide a complete description of Nature at all energy scales; the SM then represents then an effective theory valid below a UV cutoff Λ. Being a renormalizable theory, no observable prediction of the SM will depend on Λ. However, one can also examine the naturality of the model and within this context determine to what extent radiative corrections are sensitive to the UV cutoff. The corrections to the Higgs mass play an important role in this approach since they are the only ones depending quadratically on Λ:

\[ \delta^{(SM)} m^2_h = \frac{3\Lambda^2}{8\pi^2 v^2} \left[ 4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2 \right] \]  

(1)
as first calculated by Veltman [1] Consistency and naturality of the model require \( \delta^{(SM)} m^2_h < m_h^2 < \Lambda^2 \) which limits the allowed values of \( m_h \) and \( \Lambda \) to the shaded region in the graph below

In particular a light Higgs (\( m_h < 200 \text{ GeV} \)) is disallowed. For example if we require \( \delta^{(SM)} m^2_h \simeq m_h^2 = (130 \text{ GeV})^2 \) then \( \Lambda \simeq 580 \text{ GeV} \), while the current electroweak precision
limits (mostly from the oblique T-parameter \[2\]) require $\Lambda >$ few TeV. To remedy this situation two approaches have been followed in the literature: (i) cancel the SM ln $m_h$ contributions to T_{oblique} so that heavy Higgs boson is allowed by the experimental constraints; or (ii) cancel SM contributions to $\delta m_h$ (e.g. SUSY). Here we follow the second approach.

The goal of the model to be proposed is to relax the tension between naturality and the EW precision measurements for a UV cutoff $\Lambda \sim <3 − 10$ TeV. Our approach is to cancel the top-quark contributions to $\delta m^2_h$ through the introduction of additional scalars. Since we also want to preserve the wealth of electroweak predictions derived from the SM, we assume such scalars are gauge singlets. We will then consider the SM with an extended scalar sector consisting of one isodoublet and $N_{\varphi}$ scalar singlets $\varphi_i$, and arrange the model parameters so that all naturality problems occur at scales above the $O(10$ TeV) range.

The scalar potential we use is

$$V(H, \varphi_i) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_\varphi^2 \bar{\varphi}^2 + \frac{1}{24} \lambda_\varphi \left( \bar{\varphi}^2 \right)^2 + \lambda_x |H|^2 \bar{\varphi}^2$$

where $\bar{\varphi} = (\varphi_1, ..., \varphi_{N_{\varphi}})$ and we assume $\mu^2_\varphi$, $\lambda_x$, $\lambda_H > 0$ so that the $\bar{\varphi}$ vacuum expectation values (VEV) vanishes $\langle \bar{\varphi} \rangle = 0$. Upon spontaneous symmetry breaking the physical Higgs mass becomes $m^2_H = 2\mu_H^2$ while the singlet mass equals $m^2 = 2\mu^2_\varphi + (\lambda_x / \lambda_H) \mu_H^2$.

We demand that the potential $V$ be symmetric under $\varphi \leftrightarrow -\varphi$. For simplicity we also assume $V$ to have a $O(N_{\varphi})$ symmetry, but this is not essential for the considerations below.

It is now straightforward to calculate the new one-loop corrections to $m^2_h$ generated by $\bar{\varphi}$:

$$\delta^{(\varphi)} m^2_h = -[N_{\varphi} \lambda_x / (8\pi^2)] \left[ \Lambda^2 - m^2 \ln \left( 1 + \Lambda^2 / m^2 \right) \right]$$

and, with this additional contribution it now becomes possible to tame the little hierarchy problem; see Fig. 1. For example, for $130$ GeV < $m_h$ < $230$ GeV we find $|\delta_{tot} m^2_h| \equiv |\delta^{(SM)} m^2_h + \delta^{(\varphi)} m^2_h| \lesssim m^2_h$ when $\Lambda \lesssim 5$ TeV and $\lambda_x \lesssim 1.5$. Similar considerations can be made at the two loop level \[4\]; see Fig. 2.

![Figure 1](image-url)  

**Figure 1.** Values of $m$, $\lambda_x$ that insure $\delta_{tot} m^2_h = 0$ for $N_{\varphi} = 6$, $\Lambda = 8$ TeV (left) and $\Lambda = 12$ TeV (right) and various choices of $m_h$, ranging from 130 GeV (top line) to 230 GeV (bottom line).

2. Dark matter

The existence of a parity symmetry allows us to use the singlets also as dark matter (DM) candidates (for $N_{\varphi} = 1$ see \[3\] ). In this case the relevant averaged cross section $\varphi \varphi \rightarrow$ SM SM is given by

$$\langle \sigma v \rangle \simeq \frac{\lambda_x^2}{8\pi m^2} + \frac{\lambda_x^2 v^2 \Gamma_h(2m)}{8m^5} \simeq \frac{1.73}{8\pi} \frac{\lambda_x^2}{m^2}$$

\[4\]
Figure 2. Regions in the $m_h - \Lambda$ plane for which $m_h^2 < \delta m_h^2$ (white) or $m_h^2 > \delta m_h^2$ (gray), for $0.1 < \lambda_x < 5$ at two loops for the pure SM (left), and the SM with one additional singlet (right). Dark gray regions correspond to parameters that also violate the triviality constraint. These graphs are courtesy of A.A. Drozd.

where the first term represents the $s$-channel $h$ exchange contribution, and the second term the sum of the other channels. For this process the Higgs width is well approximated by $\Gamma(2m) \simeq 1.92 m^2$ TeV, with $m$ in TeV units. The freeze-out temperature and corresponding abundance can now be obtained using standard procedures [5]:

$$\frac{m}{T_{\text{freeze}}} \simeq \ln \left[ 0.038 M_{\text{Pl}} \frac{m^{(\sigma v)}}{(g_\ast x_f)^{1/2}} \right] \Rightarrow \Omega_\varphi h^2 = N_\varphi \frac{1.06 \cdot 10^9 x_f}{(g_\ast)^{1/2} M_{\text{Pl}}^{(\sigma v)} \text{GeV}}$$

(5)

As illustrated in fig. 3 it is quite remarkable that there is a large region in parameter space for which the scalars have the required DM abundance and the little hierarchy problem is postponed to scales $\Lambda \sim 10$ TeV. Note that for a fixed abundance $\Omega_\varphi$ the scalar mass scales as $N_\varphi^{-3/2}$ so one can decrease the $\varphi$ mass by augmenting the number of species.

Figure 3. Allowed $m$ and $\Lambda$ values for which $\delta_{\text{tot}} m_h^2 = 0$ and $\Omega_\varphi h^2 = 0.106 \pm 0.008$ (3$\sigma$) [2], for $N_\varphi = 6$ and $m_h = 130$ GeV (left), 170 GeV (right).
3. Neutrino interactions
In the presence of right-handed neutrinos $\nu_R$ the scalars can have additional couplings:

$$
\mathcal{L}_Y = - \bar{L} Y_e H e_R - \bar{L} Y_e \tilde{H} \nu_R - \frac{1}{2} \nu_R (\nu_R) - \varphi (\nu_R) Y_e \varphi R + \text{H.c.}
$$

(6)

($L$ denotes the left-handed lepton isodoublet and $e_R$ the right-handed charged-lepton isosinglet). For $M \gg \langle H \rangle$ there will be a set of light Majorana neutrinos that we denote by $n$, and a corresponding set of heavy Majorana neutrinos we call $N$. In terms of the $n$ and $N$ fields the mass terms and the neutrino field $\nu$ become

$$
\mathcal{L}_m = - \bar{n} M_n n - \frac{1}{2} \bar{N} M N; \quad M_n = \mu^* P_R + \mu P_L
$$

$$
\nu = \left( n_L + M_D M^{-1} N_L + \cdots \right) + \left( N_R - M^{-1} M_D^T n_R + \cdots \right)
$$

(7)

where $M_D = Y_e \langle H \rangle$ and $\mu = -4 M_D M^{-1} M_D^T$.

The discrete symmetry $\varphi \rightarrow -\varphi$ can be maintained even after the fermions couplings are introduced provided we require $L \rightarrow S_L L$, $e_R \rightarrow S_I e_R$, $\nu_R \rightarrow S_\nu \nu_R$ with (we assume 3 neutrino flavors)

$$
S_\nu = \epsilon \begin{pmatrix} 1 & \cdots & \cdot \\ \cdot & \ddots & \cdot \\ \cdot & \cdot & 1 \end{pmatrix} S_I = \begin{pmatrix} s_1 & \cdots & s_3 \\ \cdot & \ddots & \cdot \\ \cdot & \cdot & s_3 \end{pmatrix} Y_\varphi = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \cdot \cdot \end{pmatrix}
$$

(8)

where $\epsilon = \pm 1$, $|s_i| = 1$, and the $b_i$ are complex numbers. In addition the Yukawa matrix $Y_\nu$ is restricted to a set of 10 possible choices, but requiring (i) consistency with tri-bimaximal mixing [6], (ii) the presence of no more than one massless $n$, and (iii) absence of $\varphi \rightarrow n_i n_j$, there remains a single possibility (up to base permutations): $s_{1,2,3} = \epsilon$, corresponding to $S_I = \epsilon 1$, and

$$
Y_\nu = \begin{pmatrix} a & b \cdot \\ -a/2 & b \cdot \\ -a/2 & b \cdot \end{pmatrix} \mathcal{m}_{m_1} = -6 \langle H \rangle^2 a^2 / M_1
$$

$$
\mathcal{m}_{m_2} = -12 \langle H \rangle^2 b^2 / M_2
$$

$$
\mathcal{m}_{m_3} = 0
$$

(9)

where, for simplicity, $a$ and $b$ are real parameters. The resulting neutrino masses and mixing parameters are consistent with current data (see, e.g. [7]).

For the above choices only $\varphi$ and $\nu_{R,3}$ are odd under the discrete symmetry, while all other fields are even. For example, if we choose $\epsilon = 1$, the only $\varphi - \nu$ coupling is

$$
\mathcal{L}_Y \simeq -2 \varphi \sum_{i=1,2} b_i N_i^3 C_{P_R} N_i + \text{H.c.} + \cdots
$$

(10)

so that the $\varphi$ remain stable if $m < M_3$, so that the singlets can still function as DM candidates even in the presence of this type of neutrino couplings.

4. Comments
The above arguments show that the introducing of a number of gauge singlets $\varphi$ can both ameliorate the little hierarchy problem and introduce a viable DM candidate. In addition, if right-handed neutrinos exist they can have non-trivial couplings to the scalars, though the LHC signatures of such interactions would be difficult to observe (at least within the context of the present model).

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