The Solar Neutrino Puzzle: An Oscillation Solution
with Maximal Neutrino Mixing

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Abstract

If, as suggested by the SuperKamiokande results, $\nu_\mu$ and $\nu_\tau$ are maximally and "rapidly" ($\Delta m^2 \approx 2.2 \times 10^{-3} (\text{eV})^2$) mixed, this alone determines the mapping from current to mass eigenstates up to one rotation angle $\theta$ mixing $\nu_e$, "more slowly", with an equal combination of $\nu_\mu$ and $\nu_\tau$. For $\sin^2 \theta = 1$, the resulting minimal number of free parameters, yet maximal mixing, shows agreement between extant observations of solar neutrinos and predictions by the standard solar model with minor modifications.

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When Kajita \textsuperscript{[1]} reported at Neutrino '98 evidence for oscillation of atmospheric neutrinos with $\Delta m^2 \approx 2.2 \times 10^{-3} (\text{eV})^2$ and large mixing, probably between $\mu$ and $\tau$ neutrinos, the conceptual landscape for discussion of neutrino mixing changed dramatically. The simplest interpretation consistent with this result is that there is maximal mixing between $\nu_\mu$ and $\nu_\tau$ and negligible mixing with $\nu_e$. This remarkable conclusion leads to an important application in that other great arena, where neutrino oscillations have long been suspected but have so far eluded definitive proof, solar neutrinos. We do that here by assuming that the one parameter left free by the new result, the amount of mixing of $\nu_e$, also is maximal, and then comparing deductions from that assumption with current observations, as well as predicting consequences for possible future observations.
At the very beginning of particle-physics attacks on the deficit in neutrinos arriving from the sun, as compared with expectations from the standard solar model [SSM], see Ref. [2], it was clear that maximal mixing of $\nu_e$ and $\nu_\mu$ would go a very long way in solving the puzzle. However, before the new SuperKamiokande result, there were strong reasons to be cautious about such a hypothesis: (1) *Phenomenology:* The nearest analogue, the CKM matrix mapping quark electroweak current eigenstates to mass eigenstates shows mixing that is small between adjacent generations and very small between the highest and lowest generations [3]. (2) *Theory:* The widely accepted seesaw mechanism [4] for neutrino masses also suggests small mixing angles [3]. (3) *Superfluity:* The MSW effect (so called after Mikheyev, Smirnov and Wolfenstein) seemed able to give a rigorous explanation for the solar neutrino deficit even with small mixing, provided the relevant values of $\sin^2 \theta$ and $\Delta m^2$ for $\nu_e$ mixing lie in a limited range. (4) *Esthetics:* Once one knew that there were three generations of neutrinos, why should $\nu_e$ be linked strongly with just one other generation? This last objection could be met by the complete three-generation-maximal mixing as discussed by several authors [6], but this scenario suggests too small a reduction. Thus there was neither experimental evidence nor theoretical motivation for large, much less maximal, mixing.

The ideal assumption of maximal mixing between $\nu_\mu$ and $\nu_\tau$ for small values of $L/E$ (earth’s dimensions and GeV energies) has the immediate consequence that by suitable phase convention choices one mass eigenstate $|\nu_3\rangle$ may be written (as illustrated in Fig. 1)

$$|\nu_3\rangle = (|\nu_\mu\rangle + |\nu_\tau\rangle)/\sqrt{2}. \quad (1)$$

The most general form for the two other mass eigenstates then becomes

$$|\nu_1\rangle = \cos \theta |\nu_e\rangle + \sin \theta |\nu'\rangle \quad (2)$$

and

$$|\nu_2\rangle = -\sin \theta |\nu_e\rangle + \cos \theta |\nu'\rangle, \quad (3)$$

with
\[ |\nu'| = (|\nu_\mu| - |\nu_\tau|)/\sqrt{2} \] (4)

and

\[ |m_3^2 - m_2^2| \approx 2.2 \times 10^{-3} eV^2 >> |m_2^2 - m_1^2| . \] (5)

Thus, the issue of \( \nu_e \) mixing becomes a two-state problem, with the only change from what might have been done years ago being that \( \nu' \) takes the place of \( \nu_\mu \) as the mixing partner. (Note that \( \nu' \) is neither a flavor nor a mass eigenstate.) The combination of the atmospheric SuperKamiokande result and the maximal mixing hypothesis for \( \nu_e \) uniquely specifies the mapping from the current eigenstates to the mass eigenstates. Note that because we have been allowed to choose the mapping as completely real, no CP violation arises in the mixing. For that, a necessary requirement would be that each of the three mass eigenstates involves all of the current eigenstates.

It follows from the hypothesis that oscillations of \( \nu_e \leftrightarrow \nu_\mu \) as well as \( \nu_e \leftrightarrow \nu_\tau \) should be negligible for atmospheric neutrinos. This is compatible with present observations by SuperKamiokande (see [1]), but the conclusion depends on the absolute number of atmospheric \( \nu_e \)'s predicted. It will be interesting to see whether the results of calculations which take account of the different paths of pions and muons in the Earth’s magnetic field will affect this conclusion (see Gaisser [3]).

Compared to the expectations from the published Standard Solar Model (SSM) [8], the various detectors for solar neutrinos (Homestake [9], GALLEX [10], SAGE [11], Kamiokande and SuperKamiokande [12]) have shown deficiencies, often interpreted as due to matter-induced resonant oscillations in the sun (the MSW effect), where the electron neutrinos change flavor to a state for which the detectors are insensitive or less sensitive. These oscillations are characterized by a mixing angle \( \theta \) and the difference of squared masses \( \Delta m^2 = m_2^2 - m_1^2 \), where \( m_1 \) and \( m_2 \) refer to mass eigenstates. A mixed state propagates through the vacuum with oscillation length \( L_v \) [2]

\[ L_v = 2.48 \times 10^{-3} \frac{E_\nu(MeV)}{\Delta m^2(eV)^2} km. \] (6)

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Various solutions for the parameters $\theta$ and $\Delta m^2$ are compatible with the data. The MSW effect yields possible central solutions $\Delta m^2 = 5.1 \times 10^{-6} \text{eV}^2$, $\sin^2 2\theta = 8.2 \times 10^{-3}$, and $\Delta m^2 = 1.6 \times 10^{-5} \text{eV}^2$, $\sin^2 2\theta = 0.63$ (see Hata and Langacker [13]). Since matter enhanced effects become unimportant as $\sin 2\theta \to 1$, the MSW mechanism is neither needed nor operative for maximal mixing. The special case of a “just-so” vacuum solution has been discussed by Krastev and Petcov [14]. For a recent review of the entire current solar neutrino situation see e.g. Berezinsky [15].

Let us assume that the neutrino deficiencies found are partially due to oscillations of electron neutrinos to different flavors, and partially due to an overestimate of the last, and probably weakest, link in the main neutrino chain of the SSM, viz. the emission intensity of $^8\text{B}$ neutrinos. The minimum required deficiency in emission is obtained for maximal neutrino mixing. If a detector integrates over a sufficient range of energies and/or a sufficient range of distances, phase averaging leads, after many oscillations, to a reduction of the expected signal by a factor two. Since the number of $^8\text{B}$ neutrinos is found by SuperKamiokande to be less than half of the SSM value [12] the assumed vacuum solution would imply that there is a deficit in emission of $^8\text{B}$ neutrinos, compared with expectations from the SSM.

For the chemical detectors ($^{37}\text{Cl}$ and $^{71}\text{Ga}$) the maximal mixing vacuum solution would lead for phase averaging to a halving of the expected neutrinos detected as the experiments are not sensitive to muon or tau neutrinos. In the water Čerenkov detectors muon or tau neutrinos are both detected at a rate reduced to about 14.7% of the detection rate for electron neutrinos, when averaged over the part of the spectrum detected by SuperKamiokande. Assuming the rate of $^8\text{B}$ neutrinos emitted by the sun to be $(1 - x)$ times the value predicted by the SSM, the ratio $R^{(8\text{B})}$ of electron recoils observed by SuperKamiokande, relative to the expectation from the SSM without oscillations, can be written as

$$R^{(8\text{B})} = \frac{1}{2} \times (1 + 0.147) \times (1 - x) = 0.368 \text{ or } (0.474)$$

(7)

giving a reduction $x \sim 0.36$ or 0.17 for the $^8\text{B}$ neutrinos, when the 1995 [8] or (1998 [16]) version of the Bahcall Pinsonneault SSM is considered.
The reduction of $^8$B from the SSM predictions is shown in Figure 2. This allows us, as explained in the legend, to test the consistency of our model with the results obtained by the $^{37}$Cl and $^{71}$Ga experiments. For BP95 SSM we find a 36% reduction of the $^8$B neutrinos emitted by the sun. This leads to a prediction in agreement with the $^{71}$Ga results but misses the $^{37}$Cl result, overestimating it. The recently revised SSM (BP98) makes use of a $^7$Be ($p, \gamma$) $^8$B cross section reduced by 15% from BP95 and of revised solar dynamics, that reduce the $^8$B neutrino flux to 78% of that predicted by BP95. Our maximal mixing model then calls for only a 17% reduction of $^8$B neutrino flux from BP98. Again our prediction is in agreement with the $^{71}$Ga results, but misses the $^{37}$Cl result similarly, by overestimating it.

The solution of maximal mixing, with a reduction in the emission of $^8$B neutrinos, is consistent with a large range of possible values of $\Delta m^2$. The value of $\Delta m^2$ must be large enough to achieve phase averaging of the oscillations for the various neutrino sources in the sun. At a value of $5 - 9 \times 10^{-11} \ (eV)^2$ there is a “just so” vacuum oscillation solution relying on the oscillation phase [13], corresponding to several ($\sim 2 - 4$) full wave length oscillations on the way from sun to earth (mean distance = $1.49 \times 10^8$ km). The vacuum oscillation formula for survival of an electron neutrino with maximal mixing is [2]

$$P(\nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\pi L}{L_v},$$

where $L$ is the distance from the sun and $L_v$ is given by Eq.(1). For the scattering by electrons of the monoenergetic $^7$Be neutrinos, which BOREXINO intends to observe, the detection rate (normalized to unity for no oscillations) becomes

$$R(^7\text{Be}) = 1 - 0.79 \sin^2 2\theta \sin^2 \frac{\pi \Delta m^2 (eV)^2 L(km)}{(0.862)2.48 \times 10^{-3}},$$

where the muon or tau neutrino scattering relative to electron neutrino scattering at 0.862 MeV is 0.21 [2]. As Krastev and Petcov [14], for example, point out, there is a large change in the $^7$Be electron neutrino flux over the year for the “just-so” vacuum solutions due to the change in phase of the order of $\pi/2$ in a year brought about by the $\pm 1.67\%$ yearly orbital
variation from the mean distance of the sun to the earth. GALLEX, where individual experiments represent averages in neutrino absorption over several weeks, did not observe a seasonal effect [17]. For a value of $\Delta m^2 > \sim 10^{-9}$ (eV)$^2$ the oscillation would go through many complete phases in a year and one would attain the region where our phase averaged vacuum mixing model would hold for the $^{71}$Ga detectors. However Suzuki [12] reports a hint of a distortion of the $^8$B spectrum. If this preliminary result should be confirmed we would have to reconsider some of our conclusions. But for the present, we take our solution to approximately span the mass region $10^{-9} < \Delta m^2 < \sim 0.9 \times 10^{-3}$, using the CHOOZ upper limit [18].

The variation in orbital distance ($5 \times 10^6$ km) may be compared to the average source size of the shell in the sun whence the $^7$Be neutrinos originate ($\sim 10^5$ km) [8]. Since the phase change in a year is $\sim 50$ times the phase averaging due to the source size, it dominates on a yearly average. However, if one had sufficient statistics to measure the $^7$Be intensity on, say, a daily basis, then the change in phase from day to day due to the earth’s orbit would be of the same order of magnitude as the phase variation (averaging) at the source, thus allowing an island of $\Delta m^2$ at $\sim 10^{-8}$ (eV)$^2$ to be explored.

We summarize here some experimental consequences of our solution which can be tested by the existing or soon to be completed neutrino detectors:

(1) There is no distortion of the $^8$B neutrino spectrum of the kind demanded by an MSW effect in the sun. However, see the remark above relating to Suzuki’s report at Neutrino ’98.

(2) The deficit of $\sim 36\%$ ($\sim 17\%$) for $^8$B neutrinos can be tested when neutral current interactions are studied at SNO.

(3) Our value for $R(^7$Be) can be tested at BOREXINO.

(4) There is no seasonal effect for $^7$Be neutrinos, other than the small variation due to the $1/r^2$ effect, where $r$ is the sun-earth distance.

(5) There should be no day-night effect (see [19] arising from matter oscillations in the sun. A different day-night effect where solar electron neutrinos interact with the core of the Earth has recently been proposed by several authors (parametric resonance) [20].
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(While this paper was being completed our attention was drawn to a manuscript posted by V. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant [21], which has some similar considerations.)
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FIGURES

FIG. 1. The figure shows in perspective the three-dimensional principal axis transformation from the current eigenstates to the mass eigenstates. First, the system is rotated $45^\circ$ about the $\nu_e$ direction, thus taking the original $\nu_\tau$ direction into the final $\nu_3$ direction. Secondly, the system is rotated $45^\circ$ about the $\nu_3$ direction, taking the original $\nu_e$ direction into the final $\nu_1$ direction.

FIG. 2. Rates observed by the solar neutrino detectors compared with rates predicted for maximal neutrino mixing as a function of the reduction of the $^8$B neutrino flux in the sun from the predictions of the SSM BP95 (heavy dot-dashed line) and BP98 (faint dot-dashed line) are shown in all three boxes. Note that the vertical scale is logarithmic for all three plots. Heavy horizontal lines represent the experimental values, with dashed lines the errors. Errors shown on the right side for BP95 are similar to those for BP98 (not shown). The $^{71}$Ga data are an average of the GALLEX and SAGE data.
\[ \frac{\phi^8B}{\phi^8B}_{SSM} \]

\( \sin 2\theta = 1 \)