Selection of Optimal Tire and Design Optimisation of Steering System for a Formula Student Race Car through Tire Data Treatment

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Abstract. Selection of tires plays a crucial role in Vehicle Dynamics, which is important to design the steering and suspension system. An attempt is made to select the most optimum tire for lateral performance by using the tire data acquired from Tire Testing Consortium. Parameters including the tire’s lateral grip, cornering stiffness, cornering stiffness coefficient, camber stiffness, and lateral frictional coefficient have aided this selection. The tire is modelled using the Pacejka Magic Formula where the lateral and aligning coefficients are found, and used to get pneumatic trail, a parameter not provided in the raw data. The novelty of the work is seen when the selected tire is analysed to set the camber, pressure, and caster using the Design of Experiments method, without the need for building and testing a car. Making the best use of the tires’ forces and moments, the steering geometry is designed for a target range of radii by operating the tire slip angles at its optimum value. Further the influence of the change in toe angle on the Ackermann percentage is examined. Steering moment, calculated using the aligning moment curves of the tire, is applied to the steering column to conduct finite element analysis.

1. Introduction

Tires are vital in vehicle dynamics as they are the only connection between the road and the car through which the required forces and moments are transmitted through the four contact patches of the tires. The stiffness of the tire is in series with the suspension spring and hence plays a significant part to the control of roll, pitch and yaw of the car. [1, 2]. The behaviour of the tire reveals information on the handling characteristics of the car. This information is used by the design engineer to make the best use of the tires so that it enhances maneuverability, ride and roll comfort. Unlike commercial vehicles, Formula Student teams select a single tire and use it for their design. The teams have to make an educated selection of racing tires, by comparing the data for different tires given by the Tire Testing Consortium (TTC), tested at Calspan’s tire research facility [3].

The tires are analysed using its force and moment curves. The lateral force \( F_y \) acts at the tire contact patch in a line perpendicular to the tire’s heading and is responsible for the vehicle’s turning ability. The cornering force is considered as capacity of the tire to resist lateral sliding while turning [4]. The cornering force influences tire’s aligning moment \( M_z \), which describes a tire’s tendency to...
steer about a vertical axis through the center of the tire [1]. The instantaneous cornering stiffness is defined as the slope of the lateral force \( F_y \) vs. slip angle curve. Onkar et al. [4] investigated the effect of cornering stiffness of the vehicle for the selection of tires and also design of the steering system. They have compared between R10 and R13 type of tires of similar category. The study revealed that the higher cornering stiffness is always preferred as it will lead to larger lateral forces while cornering.

The angle between the tire’s heading direction and the wheel’s heading direction is known as the slip angle. The cornering stiffness coefficient is the normalized form of instantaneous cornering stiffness at zero slip angle obtained by dividing the cornering stiffness by the normal load [5]. The coefficient of friction is defined as the peak lateral force, neglecting the camber thrust, divided by the vertical load and is an indication of the tire’s lateral grip.

The performance of the tires is very unpredictable; even if the engine has enough power to supply, there is always a possibility of slip. Mainly made out of rubber, the non-linear characteristics of a tire make it hard to analyse. The raw data of various tires is being provided by TTC and can be directly used in the design; however there is a scope for refinement of the raw data to understand the tire completely. Hence there is an imperative need to fit the curve and predict its characteristics. Researchers indicate that the magic formula by Pacekja, a semi-empirical model, provides reliable results to fit the tire data [6, 7].

Tires can be compared based on lateral as well as longitudinal data. This work gives a detailed explanation about selection of tires based on lateral data. Two Hoosier tires i.e. 43075 16x7.5-10 R25B and 43075 16x7.5-10 LCO [3] has been compared for peak lateral force \( F_y \) on the \( F_y \) vs. slip angle \( (SA) \), aligning moment \( (M_z) \), instantaneous cornering stiffness, cornering stiffness coefficient, lateral frictional coefficient or load sensitivity, instantaneous cambering stiffness and tire wear. The selected tire is further analysed for selecting its operating pressure, camber angle and caster angle [1, 5].

The steering system has a large influence on the handling characteristics of the car. The primary objective of the steering system for Formula Society of Automotive Engineers (FSAE) cars is to provide a better response and feedback to the driver inputs. To achieve this, the steering system must be designed to make best use of the tire’s lateral forces and moments. The steering must be able to generate optimal slip angles to achieve peak lateral forces in the tire [5]. The steering system must be structurally safe to withstand the tires aligning moments and steering torques [8].

The steering geometry is a four bar linkage designed to create the required steering angles, which in turn generates the required slip angles [9]. In FSAE cars, most commonly used steering geometries are parallel, pro-Ackerman, and anti-Ackermann. In parallel steer geometries, both the inner and outer wheels turn the same angle. In the pro-Ackerman steering geometry the inner wheels turn a higher angle than the outer wheels, vice versa in the anti-Ackerman setup [1, 10].

An attempt is made to methodically select appropriate tire using the TTC data and design the suspension and steering system to make the optimum usage of tire.

2. Selection of Tire

2.1. Input Data from TTC

FSAE TTC provides testing data in sweeps of slip angle or slip ratio at different conditions of pressure, inclination angle, velocity, and normal loads. During brake test, free rolling cornering test, static test, speed characterization tests and relaxation length test, measurements of forces, moments, positions, velocities, temperature, pressure and so forth are collected. From the tests conducted by TTC, the data is provided both in ASCII and Matlab formats.
2.2. Types of Tires
The tire industry produces a huge range of tires made of different compounds for different purposes. Racing tires are different from normal passenger car tires as they are made up of a softer material providing higher traction, stiffer side wall, and wears out very fast compared to normal tires. Racing tires may be either radial ply or bias ply based on its cord construction. Bias ply tires having stiffer sidewall stiffness are more sensitive to camber, while radial ply tires are more sensitive to pressure, making the suspension designs for both the tires different. True slick tires (bias ply) give better friction making launches much easier, while radial tires which give more in line stability and better response to breakaway. FSAE teams generally have one set of wets (radial or bias), with tire threads, and one set of slicks, without tire threads. The threads on the tires are useful to push out water from the road to prevent aquaplaning when maneuvering over wet track. Slicks provide more grip, but must be used only in dry condition.

2.3. Tire Modeling and Curve Fitting
The raw data from TTC needs to be fit into smooth curves before comparison and analysis to achieve accurate results. Tire models can be either steady state or dynamic (transient) state. The most commonly used models are Brush model, Stretched string model, and Magic Formula. Brush model is an old, simple and physical model [7] which can handle non steady state. It assumes that the tire made out of rubber deflects and is flexible; however the carcass is considered to be rigid. Although it works great for smaller slip angles, performance is rather questionable at higher slip angles [12]. Stretched string model is a physical model which is quite complex. Elasticity of the carcass has been introduced from the previous model [7]. Magic Formula by Pacejka is the most famous and easiest way to model tires. This model is based on semi-empirical methods which use a part of experimental results and approximation to model the tires efficiently, preferred largely by FSAE teams [7]. With many revisions to capture the non-isentropic nature of the tires, the latest version is MF 6.2 which also accounts for pressure and camber effects unlike the previous versions.

The graphs have been generated on Matlab using Polynomial fit and Magic Formula 6.2. In the first step to generate the graphs, the data has been segregated to select the appropriate data points for the limited input conditions given by the user. Polynomial fit of a suitable $n^{th}$ order will ensure a smooth curve with less error. The data points obtained from TTC (SA and force respectively) is fit for a range of SA to calculate the corresponding force.

For the second type, the logic behind the code in Matlab was to give initial input values to the lateral and aligning coefficients and then plot the curve, using Magic Formula 6.2. The sum of the error square, between the given TTC data points and the curve obtained from the Magic Formula was then minimized through a huge number of iterations. Using the function ‘fminval’ available on Matlab, the iteration is terminated, when the sum of error squared is minimum, and is used to plot the curve. This method is very sensitive to the initial inputs and it is advised to use data of an already existing tire, which then get modified for the required tire. The outputs of the Magic Formula include the lateral force curves, aligning moment curves, and pneumatic trail, in which the last entity can only be obtained by the Magic Formula.

2.4. Tire Comparison
For different tires available, different observations can be recorded based on the difference in behaviour of the compounds, tire widths and rim sizes. In this work two sets of tires of different compounds but same size have been compared. The tires 43075 16x7.5-10 R25B and 43075 16x7.5-10 LCO, on 7” rim thickness were selected from Round 8 for comparison. SAE sign convention has been used for the graphs.

Lateral force ($F_y$) is obtained from TTC data and is plotted against slip angle for a particular velocity, inclination angle, pressure and different normal loads. The data points are fit using ‘polyfit’ function (Figure 1). The curves reveal important information on the amount of lateral grip in the tires.
and give the reactiveness of the tire to the driver input. The energy lost and the cornering stiffness of the tire is also determined from the graph.

From the graph below it is observed that with the increase in normal load the peak lateral force increases, and so does the slip angle. Tire 43075 16x7.5-10 R25B shows higher lateral forces for lower slip angles, even though they converge after slip angles of 10°. This would imply that it has more grip.

At lower normal loads \(F_z\) it is seen that these curves are very similar. As the loads increase, Tire 43075 16x7.5-10 R25B will react more quickly than Tire 43075 16x7.5-10 LCO. Tire 43075 16x7.5-10 R25B would lose grip more abruptly than the other tire. It would take a skilled driver to drive reactive tires like these. On the other hand, Tire 43075 16x7.5-10 LCO is more like a passenger car which would provide smoother feedback to the driver without a sharp indication of breakaway.

![Figure 1. Lateral force Diagram.](image)

The cornering stiffness is more for Tire 43075 16x7.5-10 R25B as the slope of the curve in the linear range is more. This is crucial because in normal FSAE cars the slip angles usually lies in the range of 0 – 5 degrees, seldom reaching higher slip angles.

The energy lost by the tire would be proportional to the area under the graph presented in figure 1. From observation we can see that Tire 43075 16x7.5-10 R25B has lesser area under the curve till its peak and hence will have lesser energy losses.

Aligning moments \(M_z\) from the tire is one of the ways a driver gets feedback from the road. This moment arises due to the fact that the tire contact patch is not symmetric and not at the geometrical center of the tire when in driving condition. Hence the lateral force acts at an offset to it; a few centimeters behind the center. This distance between the center point and the point where the lateral force acts is called pneumatic trial. This causes a moment about the center of the tire known as aligning moment, which depends on the tire and its operating conditions. Figure 2 is a curve fit for aligning moment against the slip angle for different normal loads.

It is noted that Tire 43075 16x7.5-10 R25B reaches a peak through a range of 3 – 4.5° slightly before Tire 43075 16x7.5-10 LCO which peaks through a range of 4.5 to 5.5°. This peak value would be important when select the caster for the tire. The slope at which the aligning torque drops after peaking gives information on the handling behavior of the car. A steeper drop would indicate more feedback about the slip in aligning torque \(M_z\) felt by the driver at the steering wheel as a sudden drop in opposing torque to keep the steering wheel steady.
Figure 2. Aligning Moment Diagram.

Instantaneous cornering stiffness of the tire is the slope of the lateral force vs. slip angle curve at every point (Figure 3), and provides information about sensitivity of the steering. The ‘polyfit’ equation from the lateral force diagram is differentiated with respect to slip angle and used to plot instantaneous cornering stiffness graph. The cornering stiffness is highest at zero slip angle and reduces with the increase in slip angle. Tire 43075 16x7.5-10 R25B has higher instantaneous cornering stiffness as observed from the graph and would have a better response to steering input.

Another notable observation is the steepness in which the curves fall. It falls sharply in Tire 43075 16x7.5-10 R25B indicating quicker alignment after cornering. Imagine having a car already steered, operating at a slip angle of 5°. When steering back to neutral position, this tire would give up the lateral grip much faster, making it easier to transfer the lateral grip to its longitudinal grip, giving the driver confidence to accelerate.

Figure 3. Instantaneous Cornering Stiffness and cornering stiffness coefficient.
The cornering stiffness at zero slip angle is divided by the normal load to obtain the cornering stiffness coefficient and plotted against the normal load. The slope of the graph as shown in Figure 3 is observed to provide an insight in the handling characteristics of the tires.

A similar trend is seen in both the graphs as they decrease with normal load. Sharper decrease in slope indicates that the instantaneous cornering stiffness and peak lateral force is increasing more with normal load, which would be desired for race tires.

Figure 4 is a plot of lateral friction coefficient with normal load. It gives an insight about the load sensitivity of the tires and shows the amount of lateral force the tires exhibit for increasing normal loads. The lesser slope the curve has, the lower load sensitivity. Following the same trend, both the tires are almost equally load sensitive [5]. However, Tire 43075 16x7.5-10 R25B would offer more grip.

![Figure 4. Lateral Friction Coefficient Curves.](image)

Instantaneous camber stiffness \((dF_y/dIA)\), where \(IA\) is inclination angle, tells vital information on how the tire behaves with change in inclination angle or camber angle. The camber sensitivity and the effect of camber angle on the tires grip is observed comparing the curves in figure 5. This curve is obtained by differentiating the lateral forces at same normal load with respect to inclination angle to get the instantaneous camber stiffness for one slip angle. The graph below represents Instantaneous camber stiffness for different slip angles.

![Figure 5. Instantaneous Cambering Stiffness Diagram](image)

After a slip angle of \(8^\circ\) both the tires have almost no response to camber change. However Tire 43075 16x7.5-10 R25B is seen to be more sensitive. It would offer 80 newton of lateral force per degree change in inclination angle when operating at zero slip angle with 250 pounds normal load.
The weight loss of Tire 43075 16x7.5-10 R25B is slightly more compared to Tire 43075 16x7.5-10 LCO, as shown in TTC data.

2.5. Optimization of Camber and Pressure

Once the desired tire is selected, it is further studied to set parameters, later used in designing suspension and steering geometry. Camber is defined as the angle between the wheel centre plane and a plane vertical to the road in the front view of the car. It is available as inclination angle in the TTC Data. The operating pressure of the car is an important parameter which influences the grip and lateral friction coefficient of the tire. These two parameters when selected in the right combination can help the designer get the best performance out of the tires. The camber angle and tire pressure are not constant for the tire and varies as the driver operates the car. The plots shown provide an idea of how the lateral force varies with slip angle, pressure, and inclination angle.

![Figure 6. Lateral Force diagram for different Inclination Angles and Pressures](image)

It is noticed that the peak lateral force for all the different inclination angles (same pressure and normal load) are not the same as clearly seen in figure 6, plotted using Magic formula 6.2. The graph reveals how the lateral force increases with slip angle, reaches a threshold, and then slightly reduces with increase in slip angle. Ideally the graph should show a slip angle of zero degrees for zero lateral force for all normal loads. However due to uneven wearing of the tire which causes conicity and plysteer [1] the above need not be true. This will cause camber thrust. As the inclination angle increases there is a decrease in the lateral force that the tire can offer, with the maximum at zero degree camber.

Operating pressure can also be visualized in the same way as the inclination angle. It is noted from figure 6 that the lateral grip provided, decreases with the increase in pressure. The change in lateral
forces for the different conditions make it necessary for the designer to set the inclination angle (camber) and pressure to the most optimum value to make the best use of the tires.

Since more than one input is present to determine its most optimum working condition, the design of experiments method is used to study the effects of inclination angle and pressure. Design of experiments (DOE) is an efficient method to establish the correlation between factors affecting a process and the output of that process. The input parameters are inclination angle and pressure on which we apply the Magic Formula for different fits and peak lateral force is recorded as the output response.

| Table 1. Parameters for DOE. |
|-----------------------------|
| Parameter | Low Level (-) | Medium Level (0) | High Level (+) |
| Inclination Angle (°) | 0 | 2 | 4 |
| Pressure (psi) | 10 | 12 | 14 |

| Table 2. Output Response Tables. |
|---------------------------------|
| Run Order | Inclination Angle | Pressure | Output Response – Peak $F_y$ (N) |
|-----------|-------------------|----------|---------------------------------|
| 1         | -                 | -        | 2109                            |
| 2         | -                 | 0        | 2071                            |
| 3         | -                 | +        | 1937                            |
| 4         | 0                 | -        | 2072                            |
| 5         | 0                 | 0        | 2059                            |
| 6         | 0                 | +        | 1920                            |
| 7         | +                 | -        | 2010                            |
| 8         | +                 | 0        | 2034                            |
| 9         | +                 | +        | 1883                            |

The maximum grip is offered when the tire is operating at 0° inclination angle and 10 psi pressure. When cornering, it would be advantageous if the outer tire which takes most of the normal load stays close to these conditions.

2.6. Optimization of Caster

Caster is the angle with which the steering pivot axis is tilted forward (positive caster) or rearward (negative caster) from the vertical, as viewed from the side [1]. Positive caster is generally preferred by FSAE teams as this geometry straightens out the tires giving in line stability especially at higher speeds. It also creates negative camber gain while cornering. Caster must be kept optimum; too much increases the steering effort and makes tighter turns rather difficult while a little caster is always preferred to aid the aligning of the steering wheel back to its neutral position.

Trail is defined as the distance between the tire contact point and the point of intersection of the ground plane with the steering axis. It has two components; Mechanical trail (or caster trail) due to the geometry of the suspension system and pneumatic trail that is inherent due to the tires. Mechanical trail can be altered by changing caster, king pin inclination or even fork offset. However pneumatic trail only depends on the tires used, and changes with slip angle [1].

Pneumatic Trail is seen as the feedback the driver receives from the road. For race cars this is desirable and hence the mechanical trail should not be too much that it overpowers the pneumatic trail, unlike passenger cars. Also since power steering cannot be introduced in FSAE cars, it is advisable to set a lower mechanical trail.
Pneumatic trail is an output of the Magic Formula MF 6.2 and is varying with the normal load in Figure 7. The pneumatic trail, as seen in the graph, shows negative values after a slip angle of around \(12^\circ\) indicating that the direction of the moment is steering the tire into the turn, thus making it no longer self-aligning. The total torque about the steering axis, due to lateral forces only, is seen as a sum of pneumatic trail and caster trail, multiplied by the lateral force [1].

In a comparison of Caster trail of 10 mm and 20 mm (Figure 8), we see that with increase in mechanical trail the peak slip angle for the aligning moment increases. Increasing this to an angle just before the lateral force slips would be desirable as it gives the driver a good indication before it skids, while allowing it to reach the maximum lateral force. A caster trail of 20 mm is more advantageous for the given tire.
3. Steering Design
The following are the restrictions adopted as per the FSAE rulebook [11] for the design of steering system. The steering design needs to be operated manually. The steering assembly must leave enough space in the cockpit for the template to pass. Maximum free play in the steering wheel is limited to 7°.

3.1. Design of steering geometry
The four-bar linkage of the steering geometry is designed using kinematics software (LOTUS). The basic consideration while designing the geometry is to determine the percentage of Ackerman required [13]. This percentage is influenced by the lateral force [Figure 1 $F_y$ vs. $SA$] characteristics of the tire. The steering geometry must ensure that the tire is operated at optimum slip angles so as to generate maximum lateral forces while cornering. This work concentrates on designing a steering system which ensures optimum use of the tire to achieve peak lateral forces at radius of turn ranging from 8 to 14 meters.

The peak lateral acceleration that can be achieved by the vehicle and the corresponding normal loads due to the lateral load transfer are determined through an iterative process using equations 1, 2, 3, 4, 5[5]. These equations are coded in Matlab to estimate the normal forces.

\begin{align*}
M_{\text{roll}} &= A_y m h \\
\theta_{\text{roll}} &= M_{\text{roll}} / (K_f + K_r) \\
F_{y_r} &= 0.5 \rho m g + (\theta_{\text{roll}} \times K_f) / t_f \\
F_{y_l} &= 0.5 \rho m g - (\theta_{\text{roll}} \times K_f) / t_f \\
A_y &= (F_{y_r} + F_{y_l}) / \rho m
\end{align*}

Where,
- $m$ is the mass of the car including driver
- $\rho$ is the front weight distribution
- $g$ is the acceleration due to gravity
- $h$ is the distance between the center of gravity and the roll axis
- $K_f$ and $K_r$ are the front and rear roll rates respectively
- $t_f$ is the front track width
- $A_y$ is the lateral acceleration
- $\theta_{\text{roll}}$ is the roll angle
- $M_{\text{roll}}$ is the roll moment
- $F_{y_r}$ and $F_{y_l}$ are the right and left tire loads
- $F_{y_r}$ and $F_{y_l}$ are the right and left tire lateral forces

Figure 9 is a plot of the selected tire’s (Hoosier 16x7.5-10 R25B, 7") variation of peak lateral force and corresponding slip angle against normal load and is derived from Figure 1 ($F_y$ vs. $SA$). From Figure 9, the slip angle increases with increase in normal load. This gives an indication that during cornering, the heavily loaded outside tire must be operated at a larger slip angle than the inside tire. In other words the outside tire steering angle must be larger than inside tire steering angle, indicating that the tire operates best with anti-Ackermann geometry.
Figure 9. Variation of peak lateral force and slip angle with normal load.

The calculated normal loads are used to determine ideal slip angles from Figure 9. The geometric slip angles are approximated using the equations 6 and 7 [5]. Calculation of body slip angle is of no interest as the difference in steering angles (Toe angles) is required.

\[ \alpha_r^G = \frac{1}{R + t/2} + \beta \]  
\[ \alpha_l^G = \frac{1}{R - t/2} + \beta \]  

Where,
\( \alpha_r^G \) and \( \alpha_l^G \) are the right and left geometric slip angles
\( R \) is the turn radius
\( t \) is the track width
\( \beta \) is the body slip angle

\[ \delta_r = \alpha_r^G - \alpha_r^I \]  
\[ \delta_l = \alpha_l^G - \alpha_l^I \]  

\( \alpha_r^I \) and \( \alpha_l^I \) are the ideal right and left slip angles
\( \delta_r \) and \( \delta_l \) are the required right and left toe angles

\text{Ackerman angle} = \frac{\text{Wheelbase}}{\text{(Radius of turn)}} \quad \text{(10)}

Equations (6),(7),(8),(9) coded in Matlab, is employed for different turning radius to obtain a plot of difference in steering angle (Toe angles) against Ackerman angle (Figure 10).

Table 3. Steering data from LOTUS.

| Toe of outside tire (°) | Toe of inside tire (°) | Difference in Toe (°) | Turning Radius (m) |
|-----------------------|-----------------------|----------------------|-------------------|
| 14.03                 | 9.18                  | 4.85                 | 7.818             |
| 13.43                 | 8.66                  | 4.77                 | 8.254             |
| 12.82                 | 8.14                  | 4.68                 | 8.73              |
| 12.23                 | 7.61                  | 4.62                 | 9.28              |
| 11.63                 | 7.01                  | 4.62                 | 9.89              |
| 11.05                 | 6.56                  | 4.49                 | 10.6              |
| 10.46                 | 6.04                  | 4.42                 | 11.41             |
| 9.88                  | 5.51                  | 4.37                 | 12.36             |
| 9.33                  | 4.18                  | 5.15                 | 13.49             |
Table 3 indicates the difference in toe angles (Steering angle) at different turning radius for the designed steering geometry at static toe of 2°, using LOTUS. Figure 10 is a plot of difference in steering angle (Toe angle) with Ackermann angle, as required by the tires and as designed using kinematic linkage software (LOTUS). The graph indicates that with increase in Ackerman angle there is an increase in the difference of angle between the inner and outer tire. Iterations are done in lotus such that the designed geometry is able to generate the difference in toe angles that match with the tire’s requirements.

The peak lateral acceleration of 21.312 m/s² is obtained using (1),(2),(3),(4),(5). It must be noted that the peak lateral acceleration obtained here is as tested for the conditions tested in Calspan’s tire testing facility. These conditions are unachievable for normal road conditions, and the forces and moments must be scaled down. These scaling coefficients can be obtained only through actual road testing. However steering geometry has been designed for the maximum acceleration itself. Figure 10 is plotted for a peak lateral acceleration of 21.312 m/s² for different turning radius. Figure 10 indicates without a static toe the steering geometry will not be able to generate the required steering angles to operate the tire at its optimum slip angles. A static toe in of 2° with the same Ackermann percentage would help the steering geometry meet the tires slip angle requirements.

![Figure 10. Plot of difference in Toe angle of inside and outside tire vs. Ackerman angle as desired by the tire and as designed in LOTUS.](image)

3.2. Kinematics
The steering axis is inclined both in front and side view. The choice of axis determines the caster angle and mechanical trail which combine with the tire’s aligning torque as major contributors to steering wheel torque as shown in Figure 8 in section 2.6. As discussed in section 2.6 the tire operates best with a mechanical trail of 20 mm, which corresponds to a caster angle of 5.07° when designed in LOTUS. There will be an increase in camber gains while steering due to large caster angle. Hence a compromise is made to obtain a caster angle of 2.87° and a mechanical trail of 9.8 mm.

There must be minimum variation of the toe angle with roll and bump. Iterations are made to obtain the bump steer and roll steer characteristics as shown in Figure 11. Bump steer is minimized by designing the tie rods such that they are parallel to suspension control arms (A - Arms).
3.3. Steering torque and structural analysis

The total steering torque as explained in section 3.2 is a consequence of the moments caused at the tire due to both the mechanical and pneumatic trail. With positive caster both the caster torque (due to mechanical trail) and the aligning moment (due to pneumatic trail) will be in the same direction. This will enable the steering axis to self-align to its neutral position.

During cornering the driver has to apply effort to maintain the vehicle in the required turning radius so as to oppose the steering torque. In the skid pad event [11] the steering torque will be high; hence the driver’s effort to oppose this torque will also be high. The resistance to the steering torque is felt as a reaction force in the steering tie rods. This tie rod force will cause a torque in the steering column and universal joint.

The peak lateral acceleration that can be achieved by the vehicle before tire slip is $A_y$ using (1),(2),(3),(4),(5) as discussed in section 3.1. The corresponding normal loads and peak lateral forces on the tire is found out using (1),(2),(3),(4),(5) in section 3.1. Calculated values of normal and lateral forces through MATLAB code are as follows.

![Figure 11. Roll and Bump steer characteristics.](image)

### Table 4. Calculation of normal and lateral forces.

| Parameter                                      | Value       |
|------------------------------------------------|-------------|
| Tire radius ($r$)                              | 0.2 m       |
| Caster angle ($\theta$)                        | 2.87 °      |
| Maximum lateral acceleration ($A_y$)           | 21.315 m/s²|
| Right tire (Outer) normal load ($F_{ZR}$)      | 1046 N      |
| Left tire (Inner) normal load ($F_{ZL}$)       | 106.52 N    |
| Right tire (Outer) lateral force ($F_{YR}$)    | 2258 N      |
| Left tire (Inner) lateral force ($F_{YL}$)     | 245.3 N     |

The aligning moments at different normal loads are calculated through linear interpolation from Figure 2 in section 2.4. The aligning moment increases with increase in normal load.

### Table 5. Calculation of Steering moment.

| Caster angle moment | $\varphi$ | Value       |
|---------------------|-----------|-------------|
| King Pin Inclination angle | 8.75 °   |             |
| Right tire moment   | $M_{LR} = F_{YR} \times r \times \tan(\theta)$ | 22.64 N m  |
| Left tire moment    | $M_{LL} = F_{YL} \times r \times \tan(\theta)$ | 2.4595 N m |
Aligning moment Right Tire aligning moment at the TCP \( M_{ZR} \)
Left Tire aligning moment at the TCP \( M_{ZL} \)

Aligning moment Right Tire aligning moment at the steering axis \( M_{SR} = M_{ZR} \times \cos\left(\frac{\phi^2 + \vartheta^2}{2}\right)^{1/2} \)
Left Tire aligning moment at the steering axis \( M_{SL} = M_{ZL} \times \cos\left(\frac{\phi^2 + \vartheta^2}{2}\right)^{1/2} \)

| Total Moment | Right tire moment \( M_R = 22.64 + 50.796 \) | 76.390 N m |
| Prime | Left tire moment \( M_L = 2.495 + 3.260 \) | 5.755 N m |

The position of the tie rods for a rack travel of 10 mm i.e. for a radius of turn of 8.5 m is modeled in Solidworks. A moment balance about the steering axis is done for both the tires individually and the reaction force at the tie rod joints is calculated as shown in Figure 12.

**Table 6. Calculation of Tie rod forces.**

| Description | Value |
|-------------|-------|
| Right Tire Moment balance \( F_R \times 0.05531 = 73.436 \) | 1327.71 N |
| Reaction Force at right tie rod \( F_R \) | |
| Left Tire Moment balance \( F_L \times 0.05531 = 5.755 \) | 104.049 N |
| Reaction Force at left tie rod \( F_L \) | |

Component of this force along the tie rod is calculated by taking the direction cosines of the force reaction for both the left and right tie rods. The force along the tie rod causes a torque in the steering column. The components of tie rod reaction force vector perpendicular to the tie rod axis causes bending in the tie rods.

**Figure 12.** Top view about the steering axis of the right tire. Magnitude of force vector is scaled down by 10th factor.
Table 7. Calculation of Steering Torque.

| Parameter                        | Value     |
|----------------------------------|-----------|
| Right Tie rod force $F_{RY}$     | 1302.1 N  |
| Left Tie rod force $F_{LY}$      | 89.83 N   |
| Total tie rod force $F_T = F_{RY} + F_{LY}$ | 1391.93 N |
| Distance between tie rod and steering column, $d$ | 0.01727 m |
| Torque at the steering column; $T = F_T \cdot d$ | 24.038 N m |

The structural analysis of the universal joint of the steering column made of mild steel is carried out using ANSYS Workbench 18.1. The steering moment (torque) calculated is applied to one end of the universal joint and a fixed support is given at the other end. Figure 13 is the equivalent Von-Misses stress distribution and shear stress distribution in the universal joint. The yield stress of mild steel is 245 MPa, thus factor of safety is found to be 1.96 for equivalent stress and 1.80 for shear stress.

![Figure 13. Equivalent stress distribution and shear stress distribution.](image)

4. Conclusion

Selection of tire has been carried out based on its most influencing parameters, like lateral force, cornering stiffness and frictional coefficient. The Tire Hoosier 43075 16x7.5-10 R25B was found more favourable than Hoosier 43075 16x7.5-10 LCO. It was observed that tire Hoosier 43075 16x7.5-10 R25B had higher lateral grip, cornering stiffness, and lateral friction coefficient. It behaved more like a racing tire as it showed lesser energy loss and faster reaction to steering input. This tire when investigated further, displayed best possible performance when operated at a pressure of 10 psi and inclination angle of 0°. When optimizing the caster it is seen that the caster influences the manner in which steering moment peaks and has been set such that it peaks just before lateral force, to alert the
driver before losing grip. A caster trail of 9.8 mm was chosen after considering the effect of camber gains.

While designing the steering geometry for the highest achievable acceleration of 21.312 m/s², it was seen that the tire is to be operated at a higher difference in slip angle, which was achieved by giving a 2° toe in. The graphs obtained from TTC data and LOTUS was juxtaposed and anti-Ackermann geometry was chosen optimum for turning radii of 8-14 m. The steering moment about the steering column was calculated using the aligning moment graphs of the selected tire and caster moment. The torque was found to be 24.953 Nm and was used for the structural analysis of the universal joint.

The results obtained above are implemented practically while building a real prototype of the race car to set the parameters obtained above. Before testing the car, the engineer has an idea of what influences the cars behaviour, can easily solve problems if the car is not behaving as expected, and enhance the drivers feel. Validation of the results obtained on this basis is of high priority to further optimize the design.

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Acknowledgement
The authors would like to thank “Formula SAE Tire Test Consortium (FSAE TTC)” and the “Calspan Tire Research Facility (TIRF)” for supporting us and providing the requisite information.