Interferometric Approach to Open Quantum Systems and Non-Markovian Dynamics

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We combine the dynamics of open quantum systems with interferometry and interference introducing the concept of open system interferometer. By considering a single photon in a Mach-Zehnder interferometer, where the polarization (open system) and frequency (environment) of the photon interact, we theoretically show how inside the interferometer path-wise polarization dephasing dynamics is Markovian while the joint dynamics displays non-Markovian features. Outside the interferometer and due to interference, the open system displays rich dynamical features with distinct alternatives: Only one path displaying non-Markovian memory effects, both paths individually displaying them, or no memory effects appearing at all. Our results also illustrate that measuring the photon’s path can either create or destroy non-Markovian memory effects depending on whether the measurement takes place in or outside the interferometer. Moreover, the scheme allows to probe the optical path difference inside the interferometer by studying non-Markovianity outside the interferometer. With our framework and interference, it is also possible to introduce dissipative features for the open system dynamics even though the system-environment interaction itself contains only dephasing. In general, the results open so far unexplored avenues to control open system dynamics and for fundamental studies of quantum physics.

\textbf{Introduction}.—Interactions within a multipartite quantum system can destroy the quantum properties of a given subsystem. This leads to disturbance, or decoherence, in the system of interest, i.e., an open quantum system \cite{Breuer2002}. The study of decoherence and open quantum systems in general is important for both practical and fundamental reasons, e.g., to produce feasible quantum devices harnessing the fragile properties threatened by the environment \cite{Nielsen2010}, or to better understand such essentials as quantum to classical transition \cite{Haroche2006} and non-Markovian character of open system evolution \cite{Breuer2002}.

Linear optical systems provide a commonly used practical platform for this open system framework. Here, the system of interest is often the polarization of photon while the environment is the frequency degree of freedom. The system-environment interaction is due to a birefringent medium and the subsequent polarization-frequency coupling \cite{Soykal2007, Duan2000, Schindler2013, Seshadri2014, Mackay2015}. Recent achievements within this framework include, e.g., controlled Markovian to non-Markovian transition \cite{Seshadri2014} and arbitrary control of the dephasing dynamics \cite{Mackay2015}. Sometimes the frequency noise can even turn out to be useful. For example, it has been shown that noise-induced non-Markovianity can be exploited in teleportation \cite{Duan2000, Seshadri2014} and superdense coding \cite{Soykal2007}.

Our current aim is to go beyond the conventional open quantum system framework by combining the system-environment interaction scheme with interferometric studies of quantum optics, i.e., to introduce the concept of \textit{open system interferometer}. We are interested in how noise, appearing in different locations of the interferometer, influences the output of the interferometer. At the same time we describe how the interferometric setup influences the dynamics of open quantum system and the appearance of non-Markovian memory effects. These have been under intensive scrutiny both theoretically and experimentally in the last ten years \cite{Breuer2002, Seshadri2014}, though not yet considered in the interferometer framework to the best of our knowledge.

In addition to the frequency of the photon, the paths of the interferometer introduce another environmental degree of freedom, and allow to apply noise on different paths of the interferometer – both inside and outside. By considering a Mach-Zehnder interferometer, we see how prior noise influences the interference at the output and the subsequent non-Markovian open system dynamics. The framework, due to interference, also allows to mimic dissipative features of open system dynamics. In general and as a result of this “interferometric reservoir engineering”, we obtain non-Markovian memory-effects depending on in which location of the interferometer the state tomography is performed and where the “Heisenberg cut” \cite{Alon2009} between the system and the environment is drawn. It is worth noting that recent studies show both theoretically and experimentally how combining quantum channels in different causal orders allows to improve information transmission for communication purposes, see, e.g., Refs. \cite{Mackay2015, Pirandola2018}. However, our motivation and interest is different. We are interested in the fundamental studies of open quantum systems and non-Markovian features, when combining open quantum system dynamics with interferometry and interference.

\textbf{The scheme of the model}.—We consider a polarization qubit of a single photon experiencing frequency noise on the two paths in and outside a Mach-Zehnder interfer-
the open system is where the coherence terms undergo rotation and decay. In panel (b), the unitaries with possibly different interaction times and refractive indices are applied on the different paths of the interferometer. Throughout this paper, we use labels 0 and 1 for the paths inside the interferometer, and 0' and 1' for the paths outside the interferometer.

Intuitively, same unitaries on paths δ(0) and 1(0) should not alter the open system dynamics from the traditional single-path case. Thus, the main question of the paper is: How do differing unitaries within an interferometric setup affect the open system dynamics, if at all? We will address this question from the point of view of both the total open system and the path-wise states, revealing the intriguing effects related to the which-path-information. Next, we briefly recall the system-environment interaction model.

System-environment interaction and information flow.—Omitting the path qubit for now, the initial polarization-frequency state is

$$|\Psi\rangle = C_H|H\rangle\int d\omega g(\omega) e^{i\theta_H}|\omega\rangle + C_V|V\rangle\int d\omega g(\omega) e^{i\theta_V}|\omega\rangle,$$

where $C_H(V)$ and $g(\omega)$ are the probability amplitudes for the photon to be in the polarization state $|H(V)\rangle$ and the frequency state $|\omega\rangle$, respectively, and $e^{i\theta_H(V)}$ is the complex phase factor corresponding to horizontal (vertical) polarization. Note that in general $\theta_H(V) = \theta_{H(V)}(\omega) \neq H(V)$, constant indicating initial correlations between the polarization and frequency [22, 27, 28]. Here, however, we restrict ourselves to constant initial phase factors and initial product state between the system and environment.

Individually, the action of the dephasing channels on the system is well-known [7, 11, 16, 22, 24, 26, 30]. In the linear optical framework, we have

$$\rho_j(t) = \Phi_j(t)(\rho(0)) = \text{tr}_E[U_j(t)|\Psi\rangle\langle\Psi|U_j(t)^\dagger],$$

where $U_j(t)|\lambda\rangle|\omega\rangle = e^{in_j\lambda\omega}\chi_j(t)|\lambda\rangle|\omega\rangle$ and $\lambda$ labels the polarization components $|H,V\rangle$ while $j$ is the channel label. In the time evolution operator $U_j$ we have $T_j(t) = \int_0^t \chi_j(s)ds$ with $\chi_j(s) = 1$, when $t_{ji} \leq s \leq t_{jf}$, and $\chi_j(s) = 0$ otherwise. Thereby, the polarization and frequency are coupled in a birefringent medium described by the refractive indices $n_j$ from time $t_{ji}$ to $t_{jf}$. Employing a Gaussian frequency distribution [10]

$$|g(\omega)|^2 = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\omega - \omega_0}{\sigma}\right)^2\right],$$

the evolving state of the open system is

$$\rho_j(t) = \begin{pmatrix} |C_H|^2 & C_H C_V^* \kappa_j(t) \\ C_H^* C_V \kappa_j(t)^* & |C_V|^2 \end{pmatrix},$$

where the coherence terms undergo rotation and decay dictated by the decoherence function

$$\kappa_j(t) = \exp\left[i(\theta + \mu \Delta n_j T_j(t)) - \frac{1}{2}(\sigma \Delta n_j T_j(t))^2\right].$$

Here, $\Delta n_j = n_jH - n_jV$ is the birefringence of the medium, and $\theta = \theta_H - \theta_V$.

The flow of information between the system and environment — and its connection to non-Markovian dynamics — is commonly described by the trace distance $D(t)$ between a pair of initially distinguishable states of the system [14] and has been applied in several physical contexts for this purpose in the past, see, e.g., Refs. [18, 34]. The sign of $\frac{d}{dt}D(t)$ tells the direction of the information flow. Positive sign indicates non-Markovian memory effects and information backflow into the open system. For dephasing and choosing the initial state pair to be $|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$, i.e., having the maximum initial coherences, the trace distance has a simple expression $D_j(t) = |\kappa_j(t)|$.

The interferometric setup.—A balanced convex combination of two of the channels can be constructed, e.g., by a Mach-Zehnder type interferometer, inside of which the channels $\Phi_0(t)$ and $\Phi_1(t)$ operate on their own paths. Including the path of the photon — initially in the state $|\tilde{0}\rangle$, not to be confused with the path states $|0\rangle$ and $|0'\rangle$ — in the environment, the overall polarization-frequency-path state inside the interferometer is

$$|MZ(t)\rangle := (U_0(t) \otimes |0\rangle\langle 0| + U_1(t) \otimes |1\rangle\langle 1|)(I \otimes H)|\Psi\rangle|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(U_0(t)|\Psi\rangle|0\rangle + U_1(t)|\Psi\rangle|1\rangle),$$

where, to see how the system-environment interaction affects interference, we have assumed that there is no phase difference between the paths, and $H$ is the Hadamard gate describing a non-polarizing 50/50 beam splitter. From Eq. (4) it is clear that obtaining the which-path-information, i.e., applying $I \otimes |j\rangle\langle j|$ and normalizing the state, results in Markovian dephasing dynamics of the
system when Gaussian frequency distribution is used. However, as long as the path is not measured, we can go beyond Markovian dynamics. The state of the system in the latter case is given by

\[ \rho(t) = \frac{\Phi_0(t)(\rho(0)) + \Phi_1(t)(\rho(0))}{2}. \]  

(5)

Now the question becomes, what kind of open system dynamics we have after the interferometer, both on the individual paths separately and combining them. This time, for simplicity, we have the same unitary coupling \( U(t) \) acting after both exit ports. The total state exiting the interferometer is

\[ |MZ'(t)⟩ := (U(t) \otimes 1)(1 \otimes H)|MZ(t)⟩ \]

\[ = \frac{1}{2} \left[ U(t)(U_0(t) + U_1(t))|\Psi⟩|0'⟩ + U(t)(U_0(t) - U_1(t))|\Psi⟩|1'⟩ \right]. \]  

(6)

The open system state \( \rho'(t) \) is then given by Eq. (5) with the transformation

\[ n_{j'\lambda}T_j(t) \rightarrow n_{j'\lambda}T_j(t) + n_{j'\lambda}T'(t) \]  

applied to it. However, if we now measure the photon's path and obtain the result \( j' \), the state of the system becomes

\[ \rho'_j(t) = \frac{1}{4P_{j'}} \left[ 2\rho'(t) + (-1)^{j'} \left( |C_H|^2 ϕ_H \quad |C_H|C_V\Lambda(t)\right) \left( C_H|C_V|\Lambda(t)^* \right) \right]. \]  

(7)

(8)

where

\[ \kappa_\lambda = 2 \exp \left[ -\frac{1}{2} \sigma^2 (n_{0(\lambda t_0 - n_{1(\lambda t_1)})^2} \cos [\mu(n_{0(\lambda t_0 - n_{1(\lambda t_1)})}] \right. \]  

and

\[ \Lambda(t) = \exp \left\{ i[\theta + \mu(n_{0(\lambda t_0 - n_{1(\lambda t_1)})} + \Delta n'T'(t))] - \frac{1}{2} \sigma^2 (n_{0(\lambda t_0 - n_{1(\lambda t_1)})} + \Delta n'T'(t))^2 \right\} \]

\[ + \exp \left\{ i[\theta + \mu(n_{1(\lambda t_1) - n_{0(\lambda t_0)}} + \Delta n'T'(t))] - \frac{1}{2} \sigma^2 (n_{1(\lambda t_1) - n_{0(\lambda t_0)}} + \Delta n'T'(t))^2 \right\} \]  

(9)

(10)

originate from the cross-terms \( U_0(t)|\Psi⟩⟨\Psi|U_1(t)\) and \( U_1(t)|\Psi⟩⟨\Psi|U_0(t)\) while

\[ P_{j'} = \frac{2 + (-1)^{j'} |C_H|^2 ϕ_H + (-1)^{j'} |C_V|^2 ϕ_V}{4} \]  

(11)

is the probability for the photon to be detected on path \( j' \) outside the interferometer. Note that both the polarization probabilities in the path-wise states [Eq. (8)] and the path probabilities \( P_{j'} \) [Eq. (11)] contain rapidly oscillating terms with the frequency \( \mu \), since they both contain \( \kappa_\lambda \) [Eq. (9)]. An experimental setup which can be used to realize both the path-wise and joint open system dynamics is presented in Fig. 2.

**Dynamical characteristics of the open system interferometer.**—We consider the case where the polarization-specific refractive indices are the same but the interaction times inside the interferometer may differ. Interaction times on the paths outside the interferometer are equal. In terms of notation, \( t \) is the laboratory time, \( t_0 = t_0f - t_{0i} \) (\( t_1 = t_1f - t_{1i} \)) is the duration of the interaction on path 0 (1), and we use \( t_{0i} = t_{1i} = 0 \). The difference in the interaction times inside the interferometer is denoted with \( \Delta t = t_0 - t_1 \). For time scales we use \( \tau = \sigma t \) and in similar manner have \( \tau_0 = \sigma t_0, \tau_1 = \sigma t_1, \) and \( \Delta \tau = \sigma \Delta t \).

We first consider the case where the interaction time difference, \( |\Delta \tau| = 10 \), is so large that the subsequent optical path differences produced inside the interferometer prevent interference at the second beam splitter, BS’. We have plotted the trace distances capturing the non-Markovian features of the dephasing dynamics in Fig. 3 the initial state pair being \( |\pm⟩ \). Taking both paths into consideration yields \( D_\tau \), whereas performing state tomography only on path \( j' \) yields \( D_\tau j' \). Figure 3 (a) shows that inside the interferometer before BS’, the joint open system undergoes non-Markovian dephasing, while the path-wise states behave in a Markovian fashion. As soon as the interaction on path 0 is switched off at \( \tau = 50 \) – while interaction still continues on path 1 – the joint open system dynamics displays oscillatory behaviour of trace distance indicating non-Markovian behaviour. Outside the interferometer, see Fig. 3 (b), the open system displays opposite features. Here, the joint dynamics is Markovian while the path-wise evolution shows non-
Markovianity and backflow of information. Initially on each output path, there are $H$ and $V$ components originating from both paths inside the interferometer. In the subsequent interaction outside the interferometer and on each path, the optical path differences between the $H$ component from one earlier path and the $V$ component from the other earlier path become temporarily equal allowing recoherence and memory effects to arise. The maximum trace distance reached is equal to 0.5 since the other two remaining components have distinct path differences all the times.

It is also interesting to note that $\Lambda(t)$, which gives the path-wise dynamics outside the interferometer, contains information about the system’s entire history – see Eqs. (8) and (10). This has a useful property. When $P_0' = P_1'$ and having no interference at BS’, we can estimate that

$$|t_0 - t_1| = \begin{cases} \frac{\Delta n t_{\text{max}}}{\max\{n_H, n_V\}}, & \text{if } \Delta n > 0 \\ -\frac{\Delta n t_{\text{max}}}{\min\{n_H, n_V\}}, & \text{if } \Delta n < 0 \end{cases}$$

where $t_{\text{max}}$ is the instant of time where $|\Lambda(t)|$ reaches its (observable) maximum, 0.5. Therefore, by studying non-Markovianity outside the interferometer, we can quantitatively estimate what the interaction time difference was inside the interferometer – even though the path probabilities $P_0'$ and $P_1'$ do not carry significant information about this anymore. If interaction times are equal along the two paths and instead indices of refraction are not equal, we can estimate their difference in the same way. Similar calculation also holds for estimating the optical path difference $|t_0 - t_1|$.

Let us now turn to the question on how the increasing amount of interference at BS’ of the interferometer influences the subsequent open system dynamics in the joint and path-wise states. The results are shown in Fig. 4 where from panel (a) to (d) we have $|\Delta \tau| = 2.5$ to $|\Delta \tau| = 0$, respectively. Comparing Fig. 3 (b) having no interference ($|\Delta \tau| = 10$) and Fig. 3 (a) ($|\Delta \tau| = 2.5$) we see that the recoherence peak and the interval of non-Markovianity shifts to smaller times $\tau$ and that the behaviour of the path-wise state dynamics begin to deviate from themselves even though both still display memory effects. Increasing the amount of interference further and having $|\Delta \tau| = 1.5$ in Fig. 4 (b) shows that the dynamics on path 0’ displays information backflow while on path 1’ and joint dynamics behave in Markovian manner. Note also that in the path-wise states, the probabilities $\langle H | 0' | H \rangle$ and $\langle H | 1' | H \rangle$ have changed compared to their initial value 0.5. This means that the interferometric setup also allows to introduce dissipative-type effects for the open system dynamics, due to interference, even though the system-environment interaction consists of only dephasing. This is seen in more significant way in Fig. 4 (c) with $|\Delta \tau| = 0.5$. Here, we have, e.g., on path 0’ $\langle H | 0' | H \rangle \approx 0.183$ and at the same time all the three

![Figure 3](image3.png)

**FIG. 3.** (Color online) Trace distances of the initial state pair $|\pm\rangle$ (a) in and (b) outside the interferometer as functions of the scaled laboratory time $\tau$ when $|\Delta \tau| = 10$. Dashed light blue = path 0; dashed and thick dark red = path 1; dashed dark blue = path 0’; dashed and thick light red = path 1’; solid green = combined paths dynamics. We have fixed $n_H = 1.553, n_V = 1.544, \mu/\sigma = 400, \tau_0 = 50$, and $\tau_1 = 60$. For the dynamics outside the interferometer in panel (b), the interaction times on both output paths start simultaneously at $\tau = 60$ and then run freely.

![Figure 4](image4.png)

**FIG. 4.** (Color online) Trace distance dynamics outside the interferometer for different values of $|\Delta \tau|$. (a) $|\Delta \tau| = 2.5$, (b) $|\Delta \tau| = 1.5$, (c) $|\Delta \tau| = 0.5$, and (d) $|\Delta \tau| = 0$. Other parameters, notation, and units are the same as in Fig. 3 (b), except: (a) $\tau_0 = 57.5$, (b) $\tau_0 = 58.5$, (c) $\tau_0 = 59.5$, and (d) $\tau_0 = 60$. 

different dynamics behave in Markovian way, though distinctly. Finally, Fig. 4(d) ($|\Delta \tau| = 0$) represents the other extreme compared to Fig. 3(b). Here, despite of having noise inside the interferometer, the two previous paths are fully indistinguishable, and due to full interference, the photon always ends up to path 0’ and no memory effects are on display.

Conclusions.—We have gone beyond the traditional view point of open quantum system dynamics by introducing and studying open system interferometer. By considering a single photon in a Mach-Zehnder interferometer, and accounting for polarization-frequency interaction at different stages of the interferometer, we have shown how inside the interferometer path-wise open system (polarization) dephasing dynamics displays Markovian dynamics while the joint dynamics including both of the paths displays non-Markovian memory effects. More importantly and interestingly, at the output of the interferometer, we observe a subtle and rich interplay between the interference and memory effects. Depending on the system-environment interaction times inside the interferometer, the open system dynamics in the output can display non-Markovianity and information backflow only on one path, on both paths individually, or no memory effects at all. It is also important to note that the scheme can be used to estimate the optical path difference inside the interferometer by looking at non-Markovianity at the output – while the path probabilities do not carry this information anymore. Moreover, despite of having system-environment interaction producing dephasing, we have shown how to introduce dissipative elements to the open system dynamics due to the interference effects.

Our results, therefore, open so far unexplored avenues for the control and engineering of open system dynamics. This also includes non-Markovian memory effects, where their source first originates from superposing of two paths having different earlier dynamics and then continuing with ongoing system-environment interaction for the superposed paths. In general, we hope that our results stimulate further work for understanding rich dynamical features of open quantum systems, how to engineer them, and how to explore fundamental aspects of quantum mechanics by combining the concepts of open quantum systems – beyond their traditional use – with other physical frameworks.

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