SpectralFPL: Online Spectral Learning for Single Topic Models

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Abstract

This paper studies how to efficiently learn an optimal latent variable model online from large streaming data. Latent variable models can explain the observed data in terms of unobserved concepts. They are traditionally studied in the unsupervised learning setting, and learned by iterative methods such as the EM. Very few online learning algorithms for latent variable models have been developed, and the most popular one is online EM. Though online EM is computationally efficient, it typically converges to a local optimum. In this work, we motivate and develop SpectralFPL, a novel algorithm to learn latent variable models online from streaming data. SpectralFPL is computationally efficient, and we prove that it quickly learns the global optimum under a bag-of-words model by deriving an $O(\sqrt{n})$ regret bound. Experiment results also demonstrate a consistent performance improvement of SpectralFPL over online EM: in both synthetic and real-world experiments, SpectralFPL’s performance is similar with or even better than online EM with optimally tuned parameters.

1 Introduction

Latent variable models are classical approaches to explain observed data via unobserved concepts in supervised and unsupervised learning. They have been successfully applied in a wide variety of fields, such as speech recognition, natural language processing, and computer vision (Kabiner 1989, Wallach 2006, Nowozin and Lampert 2011, Bishop 2006). Despite their extensive success, classical latent variable models are limited to the supervised/unsupervised learning setting since they require an available dataset. On the other hand, however, in many practical problems a learning agent needs to learn a latent variable model online while interacting with real-time streaming data with unobserved concepts. For instance, a recommender system would like to learn to cluster its users online based on streaming data recording user impressions and clicks. The goal of this paper is to develop efficient learning algorithms for such online learning problems.

Previous works have proposed several algorithms to learn latent variable models online by extending the classical expectation maximization (EM) algorithm. Those algorithms are known as online EM algorithms, and include the stepwise EM (Cappé and Moulines 2009, Liang and Klein 2009) and the incremental EM (Neal and Hinton 1998). Similarly as the EM algorithm, each iteration of an online EM algorithm also includes an E-step to fill the values of latent variables based on an estimated distribution, and an M-step to update the model parameters. The main difference is that each step of online EM algorithms only uses currently available data, rather than the whole dataset. This ensures that the online EM algorithms are computationally efficient and can be used to learn a latent variable model online. However, like the EM algorithm, online EM algorithms also have one major drawback: they may converge to local optima and hence suffer from a non-diminishing performance loss.

In this paper, we aim to overcome such limitations by developing a new online learning algorithm that learns the globally optimal latent variable model. Specifically, we propose a novel online learning algorithm, referred to as SpectralFPL, by combining ideas from the state-of-the-art spectral method in unsupervised learning and the follow-the-perturbed-leader method in online learning. Spectral method (Anandkumar et al. 2014) is recently proposed to learn the parameters for latent variable models in the unsupervised learning setting, with theoretical guarantee of convergence to a global optimum. Specifically, it learns the model parameters by constructing and decomposing high-order tensors of moments. On the other hand, the follow-the-perturbed leader method is a classical online learning algorithm (Kalai and Vempala 2005). As is standard in online learning, in this paper we measure the algorithm performance based on the notion of regret, which is the difference of the cumulative loss between our online model at each step and the best model learned in hindsight. We prove that our proposed SpectralFPL algorithm achieves $O(\sqrt{n})$ regret, where $n$ is the number of time steps. This shows that SpectralFPL learns the globally optimal latent variable model efficiently.

The contributions of this paper are fourfold. First, to the best of our knowledge, this is the first paper to formulate the online learning of latent variable models as a regret minimization problem. Moreover, we show that by properly choosing the per-step loss function, the regret in this online learning setting is closely related to the loss function of the spectral method in the unsupervised learning setting. Second, we propose SpectralFPL, an online learning variant of the spectral method, for the considered on-
line learning problem. Motivated by follow-the-perturbed-leader (Kalai and Vempala 2005), we introduce random perturbation into SpectralFPL to ensure its worst-case regret is low. SpectralFPL might also exploit reservoir sampling (Vitter 1985) to reduce its computational and memory complexities. Third, as discussed above, we prove an $O(\sqrt{T})$ regret bound for SpectralFPL. Finally, we compare SpectralFPL and the stepwise EM in extensive synthetic and real-world experiments. Experiment results show that the stepwise EM is very sensitive to its hyper-parameter setting; and in all experiments, SpectralFPL performs similarly as or even better than the stepwise EM with optimally tuned hyper-paramters.

2 Related Work

This section reviews related work on (i) spectral method for latent variable models and (ii) online learning methods for latent variable models.

The spectral method by tensor decomposition has been widely applied in different latent variable models such as mixtures of tree graphical models (Anandkumar et al. 2014), mixtures of linear regressions (Chang and Liang 2013), hidden Markov models (HMM) (Anandkumar, Hsu, and Kakade 2012), latent Dirichlet allocation (LDA) (Anandkumar et al. 2012), Indian buffet process (Tung and Smola 2014), and hierarchical Dirichlet process (Tung et al. 2017). These methods first empirically estimate low-order moments of observations, and then apply decomposition methods (e.g., SVD) to recover the model parameters. The solution of the decomposition is usually unique, such that we can compute a globally optimal solution to those latent variable models.

Traditional online learning methods for latent variable models usually extend the traditional iterative methods for learning latent variable model in the batch setting. Batch EM calculates the sufficient statistics based on all the data (Liang and Klein 2009). In online EM, the sufficient statistics can be updated with only recent data in each iteration (Cappé and Moulines 2009; Neal and Hinton 1998; Liang and Klein 2009). Online variational inference is used to learn LDA efficiently (Hoffman, Bach, and Blei 2010). Online spectral learning method has also been developed (Huang et al. 2015), with a focus on improving computational efficiency, by conducting optimization of multilinear operations in SGD and avoiding directly forming the tensors. Online stochastic gradient for tensor decomposition has been analyzed in (Ge et al. 2015) with a different online setting. They do not look at the online problem as regret minimization and the analysis focuses on the convergence to the local minimum.

In contrast, in our paper, we develop an online spectral method with a theoretical guarantee of convergence to global optimum. Besides, we discuss designing robust online spectral method in non-stochastic setting where the topics of documents over time are correlated, a setting that has not been studied in the context of online spectral learning (Huang et al. 2015).

3 Spectral Method for Topic Model

In this section, we introduce the background of spectral method in latent variable models. Spectral method works in a wide range of latent variable models; we describe how spectral method works in the simple bag-of-words model (Anandkumar et al. 2014).

In the bag-of-words model, the goal is to understand the latent topic of the documents, based on the observed words in each document. Assume we have $K$ distinct topics, $L$ observed words in each document and the size of the vocabulary is $d$. This model can be viewed as a mixture model; for the $t$th document, there is a latent variable $Y_t$ representing the topic and the observed $X_t^{(1)}, X_t^{(2)}, \ldots, X_t^{(L)}$ are conditionally i.i.d. given topic $Y_t$. Therefore, the parameters of the model include probability of each topic $j$

$$w_j = P[Y_t = j], \quad j \in [K].$$

and the conditional probability of each word $u_j \in \mathbb{R}^d$ given topic $j$, where

$$[u_j]_i = P[X_t^{(l)} = i | Y_t = j], \quad i \in [d].$$

When number of observed words $L = 3$, it is enough to construct the third order tensor $M_3$ as

$$\mathbb{E}[x_t^{(1)} \otimes x_t^{(2)} \otimes x_t^{(3)}] = \sum_{1 \leq i, j, k \leq d} P[X_t^{(1)} = i, X_t^{(2)} = j, X_t^{(3)} = k | e_i \otimes e_j \otimes e_k],$$

by representing the $L$ words $X_t^{(1)}, X_t^{(2)}, \ldots, X_t^{(L)}$ in the document as $d$ dimensional vector $x_t^{(1)}, x_t^{(2)}, \ldots, x_t^{(L)} \in \mathbb{R}^d$, where $x_t^{(l)} = e_i$ if and only if the $l$-th word in the document is $i, l \in [L]$. Here $e_1, e_2, \ldots, e_d$ is the standard coordinate basis for $\mathbb{R}^d$. When $L \geq 3$, we can construct the $M_3$, for example, by averaging over all $(\frac{L}{3})!$ ordered triples of words in a document with $L$ words (Anandkumar et al. 2014). For simplicity, in rest of this paper we discuss the case when $L = 3$ but our algorithm and analysis generalize when $L > 3$. To recover the parameters $w_j$ and $u_j$, we decompose the third order tensor

$$M_3 = \sum_{i=1}^{K} \omega_i u_i \otimes u_i \otimes u_i.$$  

However, in general, obtaining such decomposition for general symmetric tensors (e.g., $M_3$) is NP-hard. Thus, we do not want to deal with general tensors. Instead, we can efficiently obtain the decomposition for orthogonal decomposable tensor, where the eigenvectors are orthogonal. One way to make the tensor $M_3$ orthogonal decomposable is whitening. We can define whitening matrix as $W = U A^{-1/2}$, where $U \in \mathbb{R}^{d \times k}$ is the matrix of orthonormal eigenvectors of

$$M_2 = \mathbb{E}[X_t^{(1)} \otimes X_t^{(2)}] = \sum_{i=1}^{K} \omega_i u_i \otimes u_i,$$

and $A \in \mathbb{R}^{K \times K}$ is the diagonal matrix of positive eigenvalues of $M_2$. To summarize, the spectral method has the following steps (Anandkumar et al. 2014) in Algorithm 1.
Algorithm 1: Spectral method for latent variable model.

Data: One hot encoding \((x^{(1)}_t)_{t=1}^n, (x^{(2)}_t)_{t=1}^n\) and \((x^{(3)}_t)_{t=1}^n\) for a sequence of data \((X^{(1)}_t)_{t=1}^n, (X^{(2)}_t)_{t=1}^n, (X^{(3)}_t)_{t=1}^n\).

Result: The latent variable model \(\omega_i\) and \(u_i\), where \(i \in [K].\)
1. \(M_2 = \frac{1}{n} \sum_{t=1}^n x^{(1)}_t \otimes x^{(2)}_t.\)
2. \(W = M_2^{-2}.\)
3. \(y^{(1)}_t = W^T x^{(1)}_t, y^{(2)}_t = W^T x^{(2)}_t, y^{(3)}_t = W^T x^{(3)}_t.\)
4. \(T = \frac{1}{K} \sum_{t=1}^n y^{(1)}_t \otimes y^{(2)}_t \otimes y^{(3)}_t.\)
5. Decompose \(T\) by power iteration to get \(\lambda_i\) and \(v_i.\)
6. \(\omega_i = \frac{1}{\sqrt{\lambda_i}}.\)
7. \(u_i = \lambda_i (W^T) + v_i.\)

4 Online Learning for Topic Models

We study the following online learning problem in a single topic model. Fix a sequence of \(n\) document latent topics \((\hat{Y}_t)_{t=1}^n\), one for each document per time step. The words of the document at time \(t\), \(X_t\), are generated i.i.d. conditioned on \(Y_t\). The sampling distribution of the words is identical at all steps \(t \in [n]\). The goal of the learning agent is to predict a sequence of parameters with low cumulative regret, with respect to the best solution in hindsight of knowing \((\hat{Y}_t)_{t=1}^n\) and \((X^{(3)}_t)_{t=1}^n\).

To simplify notation and reduce clutter, we introduce the following notation. We denote by \(T_t\) the tensor constructed from the first \(t\) observations, in line 4 of Algorithm 1 and by \(\hat{T}_t\) its expectation with respect to random observations \((X^{(3)}_t)_{t=1}^n\). We refer to tensor decompositions in line 5 of Algorithm 1 by \(\hat{\theta} = ((\lambda_i)_{i=1}^K, (v_i)_{i=1}^K)\) and to the corresponding estimated tensor by \(\hat{T}(\theta) = \sum_{i=1}^K \lambda_i v_i \otimes v_i \otimes v_i\). When the tensor decomposition is computed from the first \(t\) observations, we denote it by \(\hat{\theta}_t = f(T_t)\), where \(f\) is a mapping from tensor \(T_t\) to \(\hat{\theta}_t\) in line 5 of Algorithm 1.

Given a model \(\theta\), we define the cumulative loss over the first \(t\) steps as

\[
L_t(\theta) = \|\hat{T}_t - T(\theta)\|, \tag{6}
\]

where \(\| \cdot \|\) is the tensor spectral norm in Anandkumar et al. (2014). This loss is the same as the offline batch loss of the spectral method on the data in the first \(t\) steps (Anandkumar et al. 2014). We define the loss at time \(t\) as

\[
\ell_t(\theta) = L_t(\theta) - L_{t-1}(\theta) = \|\hat{T}_t - T(\theta)\| - \|\hat{T}_{t-1} - T(\theta)\|, \tag{7}
\]

and assume that \(\|\hat{T}_0 - T(\theta)\| = 0.\) The reason for the above formulation is that the sum of the first \(n\) losses, which is

\[
\sum_{t=1}^n \ell_t(\theta) = \|\hat{T}_n - T(\theta)\|, \tag{8}
\]

is minimized by Algorithm 1. This definition of the loss function \(\ell_t(\theta)\) is motivated by the work of Kar et al. (2014) on non-decomposable loss functions. Roughly speaking, \(\ell_t(\theta)\) measures the extra loss due to including the observation at time \(t\), \(X_t\), when \(T_{t-1}\) is updated to \(T_t.\)

Our goal is to bound the cumulative regret

\[
R(n) = \sum_{t=1}^n \ell_t(\tilde{\theta}_{t-1}) - \sum_{t=1}^n \ell_t(\theta_n), \tag{9}
\]

where \(\tilde{\theta}_{t-1}\) is the solution of the learning agent at time \(t\) and \(\theta_n\) is the best solution in hindsight.

5 Algorithm SpectralFPL

We propose SpectralFPL, an online learning algorithm for minimizing the regret in a single topic model, which is defined in (9). Our algorithm is a variant of the follow-the-perturbed-leader (FPL) algorithm (Kalai and Vempala 2005). At time \(t\), it chooses solution

\[
\tilde{\theta}_{t-1} = f(T_{t-1} + p_t), \tag{10}
\]

where \(T_{t-1}\) is the tensor constructed from the first \(t - 1\) observations, in line 4 of Algorithm 1 and \(p_t\) is symmetric tensor noise with the magnitude of \(t^{-1/2}.\) The noise \(p_t\) is chosen uniformly at random from \([0, t^{-1/2}]^{K \times K \times K}\).

The key idea in the design of SpectralFPL is to introduce perturbations to the follow-the-leader (FL) algorithm, \(\tilde{\theta}_{t-1} = f(T_{t-1}).\) This algorithm is natural but may suffer high losses on some sequences of observations (Kalai and Vempala 2005). The amount of the perturbation is chosen carefully to balance losses due to bad sequences and adding potentially harmful noise (Section 6).

SpectralFPL is not computationally efficient because its time complexity at time \(t\) depends on time \(t\). In particular, the whitening operation in line 3 of Algorithm 1 depends on \(t\) because all past observations are whitened by a whitening matrix \(W\) that changes with \(t\).

A more memory and computationally efficient algorithm can be designed using reservoir sampling. Originally, \(\tilde{T}_{t-1}\) is calculated based on all observations from the first \(t - 1\) steps. Instead, we maintain a pool of \(R\) observations. When \(t < R\), a new observation is added to the pool. When \(t \geq R\), a new observation replaces a random observation in the pool with probability \(R/t.\) Then we can approximate \(T_{t-1}\) by calculating it only based on the observations in the pool. With reservoir sampling, SpectralFPL has per-step memory and computational cost independent of \(t\).

6 Analysis

In this section, we derive a \(O(\sqrt{n})\) bound on the \(n\)-step regret of SpectralFPL. The proof has three main parts. First, we show that the “cheating” algorithm has low regret. Second, we show that the losses of the “cheating” algorithm increase only a little when it is perturbed. Finally, we argue that the losses of \(\tilde{\theta}_{t-1}\) in (10) and the perturbed “cheating” algorithm are similar.

Recall that \(\tilde{T}_t\) is the tensor constructed from the first \(t\) observations and \(\hat{T}_t\) is its expected value with respect to random words. Let \(E_t = T_t - \hat{T}_t\) and \(\epsilon_t = \| E_t \|\), where
\[ \mathbf{∥ · ∥} \text{ is the tensor spectral norm defined in Section} \]

\[ 6.2 \text{ Technical Lemmas} \]

**Lemma 1** (Be the leader). For any \((T_t)_{t=1}^n\) and \((\bar{T}_t)_{t=1}^n\),

\[
\sum_{t=1}^n \ell_t(\theta_t) \leq \sum_{t=1}^n \ell_t(\theta_n) + c_1 \sum_{t=1}^{n-1} \varepsilon_t \cdot
\]

**Proof.** The claim holds for \( n = 1 \). For \( n \geq 1 \), we get that

\[
\sum_{t=1}^n \ell_t(\theta_t) = \sum_{t=1}^n \ell_t(\theta_t) + \ell_n(\theta_n)
\]

\[
\leq \sum_{t=1}^{n-1} \ell_t(\theta_{n-1}) + c_1 \sum_{t=1}^{n-1} \varepsilon_t + \ell_n(\theta_n)
\]

\[
\leq \sum_{t=1}^{n-1} \ell_t(\theta_n) + c_1 \sum_{t=1}^{n-1} \varepsilon_t + \ell_n(\theta_n)
\]

\[
= \sum_{t=1}^{n-1} \ell_t(\theta_n) + c_1 \sum_{t=1}^{n-1} \varepsilon_t,
\]

where the first inequality is by induction and the second inequality follows from our assumption that \( \theta_{n-1} \) minimizes \( \mathcal{L}_{n-1}(\theta) = \| \bar{T}_{n-1} - T(\theta) \| \) up to an error of \( c_1\varepsilon_{n-1} \).

**Lemma 2** (Perturbation error). Let \( \theta_t = f(T_t) \), \( \theta_t' = f(T_t + p_t') \), and \( p_t' \in [0, t^{-1/2}]^{K \times K \times K} \) be any symmetric tensor noise. Then

\[ \ell_t(\theta_t') - \ell_t(\theta_t) \leq 2c_1(\varepsilon_t + c_2t^{-1/2}) \]

for some \( c_2 \geq 0 \) that does not depend on \( t \).

**Proof.** Recall that the spectral norm is sub-additive. Therefore, \( \| A + B \| \leq \| A \| + \| B \| \) and \( \| A + B \| - \| A \| \leq \| B \| \) for any \( \| A \| \) and \( \| B \| \). We apply these inequalities and get that the loss due to the perturbation at time \( t \) is bounded as

\[
\ell_t(\theta_t') - \ell_t(\theta_t) = \| \bar{T}_t - T(\theta_t') \| - \| \bar{T}_{t-1} - T(\theta_t') \| - \| \bar{T}_t - T(\theta_t) \| + \| \bar{T}_{t-1} - T(\theta_t) \|
\]

\[
\leq \| \bar{T}_t - T(\theta_t') \| - \| \bar{T}_{t-1} - T(\theta_t') \| + \| T(\theta_t) - \bar{T}_{t-1} + \bar{T}_t - T(\theta_t') \|
\]

\[
= 2\| \bar{T}_t - T(\theta_t') \|
\]

\[
\leq 2c_1\| \bar{T}_t - T(\theta_t') \|
\]

\[
\leq 2c_1(\varepsilon_t + c_2t^{-1/2})
\]

where the last step is from the definitions of \( E_t \) and \( p_t' \).

**Lemma 3** (Almost the leader). Let \( \bar{\theta}_{t-1} = f(T_{t-1} + p_t) \), \( \theta_t' = f(T_t + p_t') \), and \( p_t, p_t' \in [0, t^{-1/2}]^{K \times K \times K} \) be uniformly random symmetric tensor noise. Then in expectation over \( p_t \) and \( p_t' \),

\[ \mathbb{E}[\ell_t(\bar{\theta}_{t-1}) - \ell_t(\theta_t')] \leq c_3t^{-1/2} \]

for some \( c_3 \geq 0 \) that does not depend on \( t \).
Proof. By the union bound and from our assumption that \( \max_{i,j,k \in [K]} |T_i(i, j, k) - T_{i-1}(i, j, k)| \leq c_i t^{-1} \), there exists at least \( 1 - 2c_i K^3 t^{-1/2} \) fraction of perturbed tensors \( T_{i-1} + p_t \) such that \( T_{i-1} + p_t = T_t + p_t' \) for some \( p_t' \). Because the matched tensors are identical and equally likely, the difference of losses on these tensors is zero. At most \( 2c_i K^3 t^{-1/2} \) fraction of tensors \( T_{i-1} + p_t \) is not matched with any tensor \( T_t + p_t' \). For any such pair, the maximum loss is finite and independent of \( t \). This concludes our proof.

\[ \sum_{\theta} \text{model} \]

7 Experiments

In this section, we evaluate SpectralFPL and compare it with state-of-the-art baselines in multiple numerical experiments. Specifically, we demonstrate experimental results in synthetic problems with both stochastic and non-stochastic settings, as well as two problems based on large-scale real-world datasets.

| \( X \) | \( P(X | Y = 1) \) | \( P(X | Y = 2) \) | \( P(X | Y = 3) \) |
|---|---|---|---|
| 1 | \( p \) | \( \frac{1-p}{2} \) | \( \frac{1-p}{2} \) |
| 2 | \( \frac{1-p}{2} \) | \( p \) | \( \frac{1-p}{2} \) |
| 3 | \( \frac{1-p}{2} \) | \( \frac{1-p}{2} \) | \( p \) |

Table 1: The conditional distribution of words in the synthetic problems.

Our chosen baseline is stepwise EM (Cappé and Moulines 2009), an online EM algorithm. We choose this baseline as previous work shows that it outperforms other online EM algorithms, such as incremental EM (Liang and Klein 2009). As detailed in (Liang and Klein 2009) Cappé and Moulines 2009, stepwise EM has two key tuning parameters: the stepsize reduction power \( \alpha \) and the mini-batch size \( m \). In particular, the learning rates in stepwise EM decrease with \( \alpha \). In the following experiments, we compared SpectralFPL to stepwise EM with various \( \alpha \) and \( m \).

To have a fair comparison, we define two metrics:

(i) negative predictive log likelihood up to step \( n \), \( \mathcal{L}^{(1)}_n = \sum_{i=2}^n \left( - \log \sum_{Y=i}^K P_{\theta_{i-1}}(Y = i) \prod_{l=1}^i P_{\theta_{l-1}}(X = X^{(l)}_t | Y = i) \right) \)

and (ii) recovery error up to step \( n \), \( \mathcal{L}^{(2)}_n = \sum_{i=2}^n \| M_2(\theta^*) - M_2(\theta_{i-1}) \|_F^2 \), where \( M_2(\theta^*) \) and \( M_2(\theta_{i}) \) are the reconstructed second order moments (reconstructed by (6)) from optimal model \( \theta^* \) and learned model \( \theta_{i} \), respectively. Because metric \( \mathcal{L}^{(2)}_n \) is easy computed for both SpectralFPL and stepwise EM. This metric measures parameter reconstruction error, and therefore is also closely related to the objective of the spectral method. In synthetic problems, we know \( \theta^* \). In real-world problems, we learn \( \theta^* \) by the spectral method because we have all data in advance. Note that the EM in batch setting minimizes the negative log likelihood, but SpectralFPL in batch setting minimizes the recovery error of tensors. In the following experiments, at step \( n \), we report (i) average negative log likelihood up to step \( n \), which is \( \frac{1}{n} \mathcal{L}^{(1)}_n \) and (ii) average recovery error up to step \( n \), which is \( \frac{1}{n} \mathcal{L}^{(2)}_n \). All the reported results are averaged over 10 runs.

![Figure 1: The comparisons between SpectralFPL and stepwise EM on synthetic stochastic problems, when \( m = 1 \). The x-axis is step \( n \). In the first row the y-axis is the average negative log likelihood up to step \( n \), while in the second row the y-axis is the average recovery error up to step \( n \).](image)

7.1 Synthetic Stochastic Setting

We compare our methods with stepwise EM on two synthetic problems in the stochastic setting. In this setting, the topic of the documents at each \( t \) is sampled from a fixed distribution. This setting is to simulate the scenario where there is no correlation between the topics in a sequence of documents. We set number of distinct topics \( K = 3 \), number of observed words \( L = 3 \) in each document, and the size of the vocabulary \( d = 3 \). Considering in practice, some topics are more popular than other topics, we set the following distribution for topic \( Y \). At each step, the topic is randomly sampled from the distribution where \( P(Y = 1) = 0.15 \), \( P(Y = 2) = 0.35 \), and \( P(Y = 3) = 0.5 \). Given each topic, the conditional probability of words is listed in Table I. We evaluate on easy problems where \( p = 0.9 \), and then evaluate on more difficult problems where \( p = 0.7 \). With smaller \( p \), the conditional distribution of words given different topic becomes similar, the difficulty of distinguishing different topics increases. For SpectralFPL, we do not add perturbations to handle the correlated topics, because the topics are i.i.d. We show the impact of adding perturbation later in synthetic non-stochastic setting in Section 7.2. For stepwise EM, the main tunable parameter is \( \alpha \). The smaller the \( \alpha \), the more quickly the old sufficient statistics are forgotten.

We show that stepwise EM is very sensitive to its param-
The best parameters of stepwise EM depend on the individual problem, and a careful grid search in online learning is almost impossible: we can not see all the data in advance, to select best parameters for stepwise EM in the online setting.

### 7.2 Synthetic Non-stochastic Setting

We evaluate different methods on two synthetic problems in the non-stochastic setting, where the topic of the documents at each $t$ is correlated. This setting is the same as stochastic setting, except that topics of the documents are strongly correlated in the streaming data. We look at an extreme case of correlated topics in the streaming data. For each $t$, we do an experiment to show the impact of adding perturbations, considering that adding perturbation should be helpful in the non-stochastic setting. We experiment with adding different symmetric tensor noise $p_t$, in $(10)$ with the magnitude of $c \times t^{-1/2}$, where $c \in \{0, 0.2, \ldots, 0.8\}$. Second, we compare SpectralFPL and stepwise EM in this non-stochastic setting.

The impact of adding perturbations is shown in Figure 3. First, adding noise is helpful in this non-stochastic setting. Compared to no perturbation where $c = 0$, a larger $c$ enables SpectralFPL to achieve lower negative log likelihood as well as the recovery loss. Second, the gap between stepwise EM is again sensitive. Even for the same problem, with different $m$, the best $\alpha$ of stepwise EM is different. As an instance, the best $\alpha = 0.9$ in Figure 1b while the best $\alpha = 0.5$ in Figure 2b.

These results indicate that the best parameters of stepwise EM depend on the individual problem, and a careful grid search of $\alpha$ and $m$ is usually needed to optimize stepwise EM. However, in practice a grid search in online learning is almost impossible: we can not see all the data in advance, to select best parameters for stepwise EM in the online setting.

| Figure 2: The comparisons between SpectralFPL and stepwise EM on synthetic stochastic problems, when $m = 100$. The x-axis is step $n$. In the first row the y-axis is the average negative log likelihood up to step $n$, while in the second row the y-axis is the average recovery error up to step $n$. |
|---|
| Figure 4: The comparisons between SpectralFPL and stepwise EM on synthetic non-stochastic problems, when $m = 1$. The x-axis is step $n$. In the first row the y-axis is the average negative log likelihood up to step $n$, while in the second row the y-axis is the average recovery error up to step $n$. |
Table 2: A summary of the comparison between SpectralFPL and stepwise EM with different m and α in real world problems. P1 is YTimes news articles dataset and P2 is PubMed abstracts dataset. The black font highlights the best algorithm for a fixed m on one dataset, while the underline highlights the worst algorithm.

adding different levels of noise in Figure 3 is much smaller, than the gap in the comparison between SpectralFPL and stepwise EM later discussed in Figure 4. Therefore, in the rest of experiments, we set c = 0 and do not perturb tensors in SpectralFPL.

We show the competitive performance of SpectralFPL in non-stochastic setting in Figure 4. First, for stepwise EM, the α leading to lowest negative log likelihood is 0.5. This result well matches the fact that the smaller the α, the more quickly the old sufficient statistics is forgotten, and more stepwise EM adapts to the non-stochastic setting. Second, in terms of adaptation to correlated topics in non-stochastic setting, SpectralFPL is even better than stepwise with α = 0.5. Note that 0.5 is the smallest valid value of α = 0.5 for stepwise EM [Liang and Klein 2009].

7.3 Real World Problems

In this section, we compare our SpectralFPL to stepwise EM on real world problems. We evaluate on both NYTimes news articles and PubMed abstracts [Lichman 2013], which are popularly used in the evaluation of bag-of-words model [Huang et al. 2015] [Lichman 2013]. As a preprocessing step, we retain the top 500 frequent words in the vocabulary. We set K = 5. We evaluate our online learning experiments on the scale of 100K documents. In order to do this, we filter out the document with less than 50 words. For those documents with more than 50 words, we randomly uniform down sample 50 words for each document. This downsampling is to ensure that in each step (or mini-batch) of online learning, there is the same amount of words.

We compare SpectralFPL to stepwise EM with different α, when mini-batch size is m = 1, 100 and 1000. The setting of m = 1000 is to evaluate the performance of SpectralFPL with reservoir sampling on large scale streaming data with 5M words. We set the window size of reservoir sampling to 10000. We show the competitive results of SpectralFPL on real world datasets with various α and m in Table 2 and Figure 5. We show the recovery error curves of different algorithms when m = 1000 in Figure 5. When m = 1 and 100, actually the curves are similar to the ones when m = 1000. We summarize results for various m in Table 2. For m = 1, we report both of our metrics at n = 1000, and for m = 100 and m = 1000, we report both of our metrics at n = 100.

Figure 5 showed that for recovery error, SpectralFPL performs better than stepwise EM with its best α for both real world datasets. When we downsample data with window size 10000 by reservoir sampling, the model still learns, as shown in Figure 5. Note that at each step n, we process m = 1000 documents. When n > 10, the data starts to be downsampled by reservoir sampling. We observe that the curves of SpectralFPL still decrease when n > 10.

Similar comparative behavior is observed in Table 2. SpectralFPL performs better than stepwise EM with best α and across different m in terms of parameter recovery loss. In terms of negative predictive log-likelihood, SpectralFPL is not as good as the stepwise EM with its best parameter setting. However, directly using SpectralFPL without the effort of tuning any parameters can still provide reasonably good performance under negative log-likelihood. SpectralFPL usually performs much better than the stepwise EM with its worse parameter setting. Thus, even under negative log likelihood metric, SpectralFPL is very useful in practice: we can quickly build reasonable baseline results by SpectralFPL without any parameter tuning.

8 Conclusions

We propose SpectralFPL, a novel online learning algorithm for latent variable models. With an instance of bag-of-words model, we prove that SpectralFPL converges to a global optimum and derived an O(√̃m) cumulative regret bound for SpectralFPL. Experiment results show that SpectralFPL is more robust than online EM. In most cases, it performs similar to or better than an optimally-tuned online EM. In the future work, we would like to extend SpectralFPL to more complicated latent-variable models, such as HMMs and LDA.
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