On Taylor-Couette flow: Inferences of near-wall effect

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Abstract: Flow behavior in the near-wall region, the variation of natural wavelength (a pair of counter rotating vortices) and its influence on the normalized torque in Taylor vortex flow (TVF), transition phase of Taylor vortex flow into wavy vortex and wavy vortex flow (WVF) have been investigated using Direct Numerical Simulation (DNS) in Taylor-Couette flow for radius ratio 0.5. Near wall region is defined by the viscous layer (VL) along the axial direction. VL is found to be thinner in the jet impingement and thicker towards the flow separation region. A distinct drop in the VL thickness with two peak values of VL across the drop is observed in the flow separation region, and the axial and radial spatial behavior of VL thickness in the drop region in TVF, transition of TVF into WVF and WVF provide a possibility of the origin of the non-axisymmetric disturbances and its propagation is in the flow separation region of outer wall (inflow region) which results in the formation of WVF. The study of flow structure reveals that in the transition of TVF into WVF, whole inflow region becomes unstable and this results in the appearance of periodic secondary flow in between two counter rotating vortices. This periodic secondary flow plays most dominating role in the appearance of strong azimuthal wave in the inflow region. Study finds that flow structure remains axisymmetric up to Re 425 beyond which WVF appears. Besides, the natural wavelength varies for TVF and WVF with the change in Reynolds number. The natural wavelength is found to be maximum in the transition phase of WVF with comparatively lower normalized torque. The VL thickness and normalized torque is strongly influenced by the natural wavelength, where decrease in natural wavelength results in the decrease in the thickness of VL and increase in normalized torque and vice versa.

Keywords: Taylor-Vortex flow, wavy vortex flow, viscous layer, non-axisymmetric flow structure, axial wavelength, Flow separation, jet impingement.

Introduction:

The study of the vortex structure concentrated between the walls in Taylor Couette flow has gained a lot of attention from researchers due to its contribution of sudden boost in wall shear stress. The linear stability analysis for small annular gap (annular gap ≪ mean radius) conducted by Taylor (1923) proved that the cause of the sudden increase of wall shear stress is a result of a toroidal vortex structure and the additional energy requirement for its sustenance. Synge (1938), Chandrashekar (1953; 1958), Donnelly (1958), Donnelly & Simon (1960) and Donnelly & Fultz (1960) carried out experimental and theoretical analysis to investigate the onset of instability of Taylor-vortex for wide gap problems. Their studies were limited to the torque measurement and onset of instability for a small range of Reynolds number. The linearized stability model developed by Taylor (1923), Synge (1938), Chandrashekar (1958), Donnelly (1958), Donnelly & Simon (1960) and Donnelly & Fultz (1960) provided an idea whether the amplitude of periodic disturbances amplifies or damps out over time, thereby established the criteria for marginal stability. For a fixed radius ratio, the periodic disturbance is found to be dependent on both Reynolds number and frequency. It will be amplified or diminished depending on whether...
Reynolds number is higher or lower than the critical value. The linearized stability model also illustrates that non-neutral perturbation displays an exponential relationship with time. However, these studies (Taylor, 1923; Synge, 1938; Chandrasekhar, 1953; 1958; Donnelly, 1958) observed the disturbances are of finite amplitudes over time instead of a continuous magnification in equilibrium condition (Davey, 1962). Subsequent studies by Stuart (1957; 1960), Watson (1960) and Davey (1962) show that the linear stability theory cannot satisfy the propagation of nonlinear disturbances. The physics of these nonlinear disturbances was later investigated broadly with the development of nonlinear stability analysis by Stuart (1957; 1960), Watson (1960) and Davey (1962).

Using a nonlinear method of expansion of energy balance, Stuart (1957) investigated Taylor-Couette flow for small gap (radius ratio 0.95) problems to analyze torque and amplitude of wavy vortex regime. His analysis includes non-symmetric disturbances but ignoring the periodic axisymmetric disturbances. The torque determined from Stuart’s theory showed good agreement with Taylor’s experimental results. Later, Stuart (1960) realized that his earlier developed theory did not provide a good approximation for wide gap problems because he ignored the effect of harmonic disturbances. Along the same line of investigation, Davey (1962) also developed a nonlinear model with the expansion of perturbation for arbitrary radius ratios and Reynolds numbers. His analysis considered only periodic axisymmetric disturbances and ignored the non-axisymmetric disturbances. The main focus of Davey’s (1962) theory was to investigate the flow regime related to azimuthal periodic wave and the results showed a fair comparison with the experimental data of narrow gap (radius ratio 0.95) problems but could not provide a good relationship for wide gap problems due to omission of the non-axisymmetric disturbances (Davey, 1962). Thus, it is concluded that the wavy vortex flow is primarily due to the nonlinear and non-axisymmetric growth of disturbances which cannot be predicted by the linear stability analysis. In addition to this, Davey (1962) had also observed a radial jet at the outflow region and claimed that the occurrence of the radial jet was the cause of a strong centrifugal force in the outflow region and it results in the appearance of wavy vortex flow.

Experimentally, Cole (1965) observed the wavy and turbulent Taylor vortex in Taylor-Couette flow experiment for the finite cylinder (aspect ratio 27.9) with radius ratio of 0.875. His result further illustrates that as Reynolds number reaches the first critical value, a periodic wave along the axial direction appears in the form of Taylor vortex. Another periodic wave in the azimuthal direction was noted as the Reynolds number reaches the second critical value and it is denoted as the wavy vortex regime. As the Reynolds number is increased beyond the second critical value, a two dimensional doubly periodic waves or modulated wave appeared along with the previously noted axial wave, and is followed by the transformation into a turbulent Taylor vortex flow at higher Reynolds number. In addition to this, non-uniqueness in axial wavelength and azimuthal wavenumber was identified, which was claimed to be the result of the ends effect of finite cylinders (Davey, Prima & Stuart, 1968). Snyder & Lambert (1965) also studied the end effect of finite cylinder and concluded that the end effect cannot cause the non-uniqueness in axial wave length and azimuthal wave number if the aspect ratio is greater than 10. They suggested that the non-uniqueness observed in Cole’s experiment could be a universal character of the flow in which the axial wave length is influenced by the wave form of the vortex flow. Moreover, they had reported that radial and azimuthal velocity behaves like a strong jet at the outflow region whereas the axial velocity behaves like a shock wave. Their study also mentioned that the jet impingement onto the wall causes the pressure to rise at the stagnation point. Kataoka, Doi & Komai (1977) studied
Taylor vortex flow under the application of axial flow and observed appearance of pair of secondary vortices in the inflow region of inner wall at the end of the Taylor vortex regime and a pair of secondary vortices during the transition between modulated wavy vortex flow into turbulent Taylor vortex flow in the inflow region of outer wall due to the flow separation. In a later study, Gorman and Swinney (1979) observed the frequency related to azimuthal traveling wave and the modulation of the wave, and claimed that the nonlinear and non-axisymmetric growth of disturbances was the cause of the propagation and modulation of these waves.

Although numerous experimental and theoretical analysis have been carried out to explain the physical phenomenon exhibited in the wavy vortex regime, the mechanism of the propagation of instability is yet to be comprehensively understood. Several studies were carried out for the investigation of the wavy regime using Direct Numerical Simulation involving Navier-Stokes equations. Fazel & Booz (1983) conducted a numerical study involving the solving of Navier-Stokes Equation using Direct Numerical Simulation for Taylor-Couette flow configuration with an infinite cylinder and wide gap (radius ratio 0.5) configuration and reported that the onset of the wavy vortex has a Reynolds number 21 times higher than the critical value. Moreover, they observed a strong radial and azimuthal jet at the outflow region of the Taylor vortex structure. Marcus (1984) claimed that a secondary form of instability of the strong radial jet at the outflow region could be the primary reason behind the appearance of wavy vortex region. He further pointed out that the origin of the transition of Taylor-Vortex to wavy vortex regime is at the region where the speed of azimuthal waves and vortex cores matches each other. Sobolik, Benabes & Cognet (1995) considered source and sink in the outflow and inflow region respectively for outer wall. In their study, azimuthal velocity gradient along the radial direction is observed maximum in the source and minimum in the sink. In similar studies, Dumont et al. (2002) and Akonur & Lueptow (2003) observed an azimuthal jet at the outflow boundary and attributed the generation of wavy vortex flow to the distribution of axial shear stress along the axial direction of the inner and outer wall. Their study also indicated that the maximum shear stress was at the outflow region due to the jet impingement. It was suggested in their study that the minimum shear stress occurs at the inflow region of outer wall which is attributed to the appearance of pair of secondary vortices.

A more recent study by Kristiawan, Jirout & Sobolik (2011) claimed that flow separation at the inner wall of the outflow and the outer wall of inflow region leads to the generation of a pair of secondary vortices. They consider this to be the reason for the transition from Taylor-vortex to wavy vortex flow. The study also claimed that the appearance of radial jet flow in the outer wall of outflow region is the reason for the maximum shear stress. Sobolik et al. (2011) suggested that the variation of shear stress appearing at the outflow boundary of the outer wall is a result of the jet impingement effect.

Study of Lim & Tan (2004) have reported that the axial wavelength of Taylor vortex is nearly uniform. For the case of wavy vortex flow and modulated wavy vortex flow, axial wavelength increases with the increase in Reynolds number. Martínez et al (2014) have studied the influence of axial wavelength on torque in Turbulent Taylor vortex flow where they have observed that increase in the axial wavelength results in the comparatively lower normalized torque.

From the above studies, it is apparent that the non-axisymmetric and non-periodic disturbances along the azimuthal and axial direction are the principal cause of the appearance of the wavy vortex
Previous studies showed that flow phenomena in the inflow (flow separation region in outer wall and jet impingement into inner wall) and outflow (flow separation region in inner wall and jet impingement into outer wall) regions of Taylor vortex play a significant role in the transition of Taylor vortex into wavy vortex flow. This provides a possible understanding about the origin and propagation of non-axisymmetric and non-periodic disturbances along azimuthal and axial direction may be directly related to the flow physics in the near wall of inflow and outflow regions. In addition, it was indicated in the earlier studies that the distribution of azimuthal shear stress is maximum in the jet impingement (inflow region of inner wall and outflow region of outer wall) and minimum in the flow separation (outer wall of inflow region and inner wall of outflow region). Thus, the study of near wall flow behavior in the inflow and outflow region may provide significant relationship between near wall region and the distribution of azimuthal shear stress along the axial direction in the jet impingement and flow separation region. Study of Lim & Tan (2004) have reported that the axial wavelength of Taylor vortex is nearly uniform. For the case of wavy vortex flow and modulated wavy vortex flow, natural wavelength increases with the increase in Reynolds number (Lim & Tan, 2004). Martínez et al (2014) have studied the influence of natural wavelength on torque in Turbulent Taylor vortex flow where they have observed that increase in the axial wavelength results in the comparatively lower normalized torque. The study of near wall region may provide some important understanding about influence of near wall flow behavior on the variation of natural wavelength and torque in the Taylor vortex flow, transition of Taylor vortex into wavy vortex flow and wavy vortex flow. Hence, in an attempt to better understand the near-wall physical phenomena, its dependency on Reynolds number and its influence on the origin and propagation of non-axisymmetric disturbances, axial wavelength, normalized torque and resultant change in shear stress distribution in Taylor Vortex flow, transition phase of Taylor vortex into wavy vortex flow and wavy vortex flow, we are motivated to study the behavior of near wall region along axial direction of Taylor Couette flow. Although, Huisman, et al. (2013) and Rodolfo et al. (2016) have studied the near wall region of the turbulent Taylor vortex flow at high Re, none of the above studies have focused on the near wall region of the axisymmetric Taylor vortex flow and wavy vortex flow.

This study considers a viscous layer (VL) in between vortex structure and walls along the axial direction for better understanding of flow behavior in the near wall region of axisymmetric Taylor vortex, transition phase of axisymmetric Taylor vortex into periodic non-axisymmetric Taylor Vortex (wavy vortex flow) and wavy vortex flow. The change in the thickness of VL along the axial direction will explain flow behavior in the near wall region of flow separation and jet impingement, its relationship with the natural wavelength and the dependency on Reynolds number in Taylor vortex flow, transition phase of Taylor vortex into wavy vortex flow and wavy vortex flow. The correlation between the distribution of VL along axial direction and local wall shear stress at jet impingement and flow separation will also be studied.

**Flow configuration:** The schematic diagram of Taylor-Couette flow set up is illustrated in Fig. 1 and unless otherwise stated, cylindrical coordinates are adopted with the z-axis coinciding with the axis of symmetry of the cylinders. The radii of the inner and outer cylinders are \( R_i \) and \( R_o \), respectively, the annular gap is expressed as \( d = R_o - R_i \) and the axial height of the cylinders or fluid column is represented as \( h \). In this study, the inner cylinder is in rotation with a constant angular velocity \( \Omega \) while the outer one is stationary. The fluid in-between the two concentric cylinders is considered to be viscous and incompressible.
**Numerical Methods:** Taylor-Couette flow is one of the characteristic nonlinear problems in fluid dynamics. Numerous studies were focused towards the use of Direct Numerical Simulation (DNS) to better understand the phenomenon behind the nonlinearity exhibited in the Taylor-Couette flow. It is considered to be the most acclaimed numerical tool for providing accurate quantitative and qualitative results in comparison to the experimental study of Taylor-Couette flow (Fasel & Booz, 1983; Marcus, 1984; Prima & Swinney, 1985; Bilson & Bremhorst, 2006; Dong, 2007; Gao, Kong, & Vigil, 2016). In this study, Direct Numerical Simulation (DNS) has been carried out for the radius ratio of 0.5, single and four axial wavelengths with the assumption of an infinite length of the cylinder. Fazel & Booz (1983) have carried out DNS for 0.5 radius ratio and single axial wavelength and infinite length of cylinder with the assumption of an axisymmetric flow structure and Reynolds number ranging from 60 to 650. However, in this study, DNS has been carried out without the above assumption for the same range of Reynolds number.

In this study, DNS has been carried out for the two different heights of fluid column namely single wavelength (Two cells of Taylor vortex) and four wavelengths (Eight cells of Taylor vortex). Single wavelength is carried out for the validation of numerical tools against study of Fazel & Booz (1983). For the case of four wavelengths of fluid column, it has been used to study the variation of natural wavelength and VL along axial direction, dependency of normalized torque or wall shear stress on natural axial wavelength and VL and their behavior with the change in Reynolds number in Taylor vortex flow, transition phase and wavy vortex flow. Periodic boundary condition is considered at the both ends of cylinder for single and four wavelengths to satisfy the infinite length of cylinder (Fazel & Booz, 1983). The whole range of Reynolds number has been divided into four regimes to carry out the grid independence test (see Fig. 2) and a number of grid points along radial, azimuthal and axial direction is summarized in Table.1. The normalize torque presented in Fig. 2, is obtained with the following equation.

\[ \text{Normalized Torque} = \frac{T_{DNS}}{\nu \rho \Omega R_i^2 L} \]

where \( T_{DNS} \) is torque obtained from DNS, \( \nu \) is kinematic viscosity, \( \rho \) is density of fluid, \( \Omega \) is angular velocity of the inner cylinder, \( R_i \) is the radius of the inner cylinder and \( L \) is the height of fluid the column.

The torque deviation between inner and outer cylinder is observed to be less than 0.4% which indicates the finer scale of mesh along the radial direction. A structured mesh (Hexahedral) is
generated using Fluent Mesher, and the growth rate of prism layer from the wall is set as 5. This provides a non-uniform distribution of grid size along the radial direction with the region near the wall having a finer grid size and the other regions having a relatively larger grid size.

Fig. 2: Grid convergence test Reynolds number ranges between 60 to 650 (a) Grid indepdency for Re 500 to 650 (b) Grid independency for Re 300 to 475 (c) Grid independencies for Re 200 to 275 (d) Grid independencies for Re 60 to 175.

| Re      | Radial direction | For each wavelength | Azimuthal direction |
|---------|------------------|--------------------|---------------------|
| 60≤Re≤175 | 70               | 125                | 250                 |
| 175≤Re≤275 | 85               | 150                | 300                 |
| 275≤Re≤475 | 100              | 175                | 350                 |
| 475≤Re≤650 | 115              | 200                | 400                 |

DNS is conducted by solving the three dimensional Navier-Stokes equation using icoFoam solver in OpenFOAM 5. As the problem involves incompressible flow, the pressure based solver has been used. Interpolation scheme without non-orthogonal correction has been used to compute the cell face pressure. The second order discretization scheme without non-orthogonal correction has been applied for the convective, diffusive fluxes and velocity derivatives. A second order implicit method (Backward) has been used for the transient formulation. The Pressure Implicit with
Splitting of Operators (PISO) has been used to obtain pressure correction. Generalized geometric-algebraic multi-grid (GAMG) solver along with Smoother Gauss-Seidel has been used to solve pressure. Preconditioned (bi-) conjugate gradient (PBiCG) linear solver with Diagonal incomplete-LU (asymmetric) (DILU) has been used to solve velocity. From the stability criteria Courant–Friedrichs–Lewy (CFL) < 0.5 is used for the whole range of Reynolds number. The mean torque coefficient approaches to a steady state condition within 4 to 5s. Thus, the time duration of the simulation in this study has been chosen as 5 sec.

![Graph](image)

**Fig. 3:** Comparison of present DNS against the study of Fazel & Booz (1983).

DNS result presented in Fig. 3 for single wavelength shows excellent agreement with the DNS study of Fazel & Booz (1983) up to Re 425 with a maximum of 0.12% error where the flow structure is axisymmetric (see Fig. 4 (a)-(f)). The critical Reynolds number obtained in this study is 68 which is same as the study of Fazel & Booz (1983). However, as Reynolds number increases beyond 425, the error between present DNS of single wavelength and the study of Fazel & Booz (1983) increases (see Fig. 3). In the study of Fazel & Booz (1983), flow structure is assumed to be axisymmetric and considered the Reynolds number at which non-axisymmetric flow structure appears is nearly 10 times greater than critical Reynolds number (Re 680). Conversely, this DNS study does not assume any axis-symmetry flow structure. Flow structure is observed to be axisymmetric for Reynolds number ranging from 60 to 425 (see Fig. (a)-(f)) and beyond which partial non-axisymmetric flow structure is observed (see Fig. 4 (g)-(j)). This explains the increase in the error between present DNS for single wavelength and the study of Fazel & Booz (1983) as Reynolds number increases beyond 425. The deviation also provides idea about the Reynolds...
number at which non-axis symmetric flow structure appears. A DNS for single wavelength case with the axisymmetric assumption has been carried out for Reynolds number 650 and it was found that the present DNS of single wavelength result is very close to the study of Fazel & Booz (1983) with an error of 0.39%. Thus, numerical tools used in this present DNS study for single wavelength is fairly validated against the study of Fazel & Booz (1983). Exactly same numerical and mesh configuration is used in the DNS for four wavelength case.

Fig. 4: Streamline patterns associated with flow structure variation in Taylor Couette flow for radius ratio 0.5, single wavelength and infinite aspect ratio (a)-(f) axisymmetric (g)-(j) non-axisymmetric flow structure.

**Result:** The base line of this present DNS is the study of Fazel & Booz (1983), where radius ratio 0.5, single and four wavelength and infinite length of cylinder is used. Chandrashekar (1958) and Donnelly (1958) studied Taylor Vortex flow for the wide gap problem (radius ratio 0.5) using linearize stability theory and their study found axisymmetric flow structure even upto 680 Reynolds number. Donnelly & Simon (1960) and Donnelly & Fultz (1960) conducted experiment for the radius ratio 0.5 and aspect ratio 5. Their study have shown good agreement with the earlier mentioned study. As flow structure reported axisymmetric in the earlier studies (Chandrashekar, 1958; Donnelly, 1958; Donnelly & Simon, 1960; Donnelly & Fultz, 1960), Fazel & Booz (1983) carried out DNS study with the assumption of axisymmetric flow structure up to Re 650. As far as we are aware, only Fazel & Booz (1983) carried out details DNS study of Taylor Couette flow Reynolds number ranging between 60 to 650 for the radius ratio of 0.5. In this present DNS study
of Taylor Couette flow with the same radius ratio, single and four wavelengths of fluid column, we have observed that axisymmetric flow structure up to Re 425 (see Fig. 3, Fig. 4, Fig. 5, Fig. 6 and Fig. 7) and beyond which periodic non-axisymmetric flow structure is observed (see Fig. 3, Fig. 4, Fig. 5, Fig. 6 and Fig. 7). This provides a partial contradiction about the ranging of axisymmetric flow structure with the study of Chandrashekar (1958), Donnelly (1958), Donnelly & Simon (1960), Donnelly & Fultz (1960) and Fazel & Booz (1983). Stuart (1960) and Davey (1962) proved that linearized theory cannot capture the periodic non-axisymmetric disturbance because of its nonlinear growth. Thus, linearized stability theory of Chandrashekar (1958) and Donnelly (1958) is only valid for axisymmetric flow structure and this could be a possible reason about the appearance of non-axisymmetric flow structure at a delayed Reynolds number. Although, Donnelly & Simon (1960) and Donnelly & Fultz have reported good agreement between experiment and the linearized stability theory of Chandrashekar (1958) and Donnelly (1958), the appearance of non-axisymmetric flow structure in their study could be delayed as they have used aspect ratio smaller than 8 (Coles, 1965). Hence, the appearance of periodic non-axisymmetric flow structure (see Fig. 4, Fig. 5, Fig. 6 and Fig. 7) observed at an earlier Reynolds in this present DNS study is justified by the study of Stuart (1960), Davey (1962) and Coles (1965). In addition, in the study of Fazel & Booz (1983), a single wavelength (twice of gap size) is used as the height of fluid column. This provides the axial wavelength of Taylor vortex to be fixed artificially for the all Reynolds numbers. In this present study where the height of fluid column is equivalent to four wavelengths (Eight times of the gap size), we have observed that the natural wavelength varies with the Reynolds number. This has provided significant information about the variation of natural wavelength (two cells of Taylor vortex) along axial direction and its behavior with the change in Reynolds number which is ignored in the study of Fazel & Booz (1983).

Table 2: The summarized DNS result for single wavelength and four wavelength and the variation of natural wavelength.

| Reynolds number | Aspect ratio 2 (DNS RESULT) (Single wavelength=Gapx2) | Aspect ratio 8 (DNS RESULT) (4 wavelengths=Gapx8) | % Difference in wavelength | % Difference in Torque |
|-----------------|------------------------------------------------------|-------------------------------------------------|---------------------------|------------------------|
| Normalized Torque | Normalized Wavelength | Number of Vortex cell | Normalized Torque | Normalized Wavelength | Number of Vortex cell |                              |                          |
| 125              | 24.322 | 0.25 | 2 | 24.766 | 0.199 | 10 | -20.40% | 1.83% |
| 175              | 27.928 | 0.25 | 2 | 28.759 | 0.199 | 10 | -20.40% | 2.58% |
| 225              | 30.827 | 0.25 | 2 | 31.695 | 0.198 | 10 | -20.80% | 2.82% |
| 275              | 33.398 | 0.25 | 2 | 33.398 | 0.25 | 8 | 0.00% | 0.00% |
| 325              | 35.762 | 0.25 | 2 | 36.936 | 0.200 | 10 | -20.00% | 3.00% |
| 375              | 37.958 | 0.25 | 2 | 37.949 | 0.248 | 8 | -0.80% | -0.02% |
| 425              | 39.989 | 0.25 | 2 | 39.991 | 0.248 | 8 | -0.80% | 0.01% |
| 475              | 41.848 | 0.25 | 2 | 37.199 | 0.335 | 6 | 34.00% | -11.01% |
| 525              | 43.144 | 0.25 | 2 | 38.478 | 0.333 | 6 | 33.20% | -10.81% |
| 575              | 44.474 | 0.25 | 2 | 44.932 | 0.25 | 8 | 0.00% | 1.03% |
| 600              | 45.166 | 0.25 | 2 | 45.274 | 0.25 | 8 | 0.00% | 0.24% |
| 650              | 46.594 | 0.25 | 2 | 46.905 | 0.25 | 8 | 0.00% | 0.67% |

Result obtained in the study of DNS with four wavelengths of fluid column is presented in Fig. 3. Study shows that the number of Taylor vortex cells in four wavelengths is 10 for Reynolds number.
125, 175, 225 and 325 which in turn provides normalized natural wavelength to be $\approx 0.2$ (see Fig. 3, Fig. 5, Fig. 6 and Table 2). For the case of Reynolds number 275, 375, 425, 575, 600 and 650, a total of 8 cells of Taylor vortex appears in four wavelengths fluid column and this results in the normalized wavelength as $\approx 0.25$ (see Fig. 3, Fig. 5 and Fig. 6 and Table 2). Total number of Taylor vortex for Reynolds number 475 and 525 is observed as 6 and this provides normalized natural wavelength as $\approx 0.33$ (see Fig. 3, Fig. 5 and Fig. 6 and Table 2). It was found that when natural wavelength for four wavelength of fluid column exactly matched with that of single wavelength fluid column, the normalized torque shows excellent agreement with the study of Fazel & Booz (1983) (see Fig. 3, Fig. 4, Fig. 5 and Table 2). As the number of Taylor vortex cell increases, the normalized natural wavelength decreases (see Fig. 5 and Fig. 6). This results in the increase of normalized torque for the four wavelength of fluid column than that of single wavelength (see Fig. 3, Fig. 4, Fig. 5 and Table 2). In addition, study finds that number of Taylor vortex cells in the transition phase of Taylor vortex flow into non-axisymmetric periodic flow (wavy vortex flow) (see Fig. 6 b and Fig. 6 c) drops to 6 which provides natural wavelength to be 33% greater than that of study of Fazel & Booz (1983) (see Fig. 3, Fig. 5 and Table 2). As there is an increase in the natural wavelength is observed, the normalized torque in this region ($425 < R_e < 575$) is found nearly 11% smaller than single wavelength fluid column (see Fig. 3, Fig. 4 and Table 2). The study of Lim & Tan (2004) have shown that the number of Taylor vortices in wavy and modulated wavy vortex flow is less and it’s corresponding normalized natural wavelength is higher than that of axisymmetric Taylor vortex for radius ratio 0.893. In addition, their study has also indicated that the total number of Taylor vortices decreases, and normalized natural wavelength increases in wavy and modulated wavy vortex flow with the increase in Reynolds number. In the present DNS study, the total number of Taylor vortices is found to be less and it’s corresponding normalized natural wavelength is greater in the transition phase of axisymmetric Taylor vortex into periodic non-axisymmetric Taylor vortex than that of axisymmetric Taylor vortex which is aligned with the study of Lim & Tan (2004). Despite this finding is aligned with the study of Lim & Tan (2004), such kind of change in the natural wavelength in the transition phase of Taylor vortex into wavy vortex has not been reported in the earlier studies for the radius ratio of 0.5.

The study of vorticity contour and the distribution of wall shear stress along the azimuthal direction indicates that as soon as axisymmetric flow structure transforms into periodic non-axisymmetric flow structure, a periodic wave in the inflow and outflow region along the azimuthal direction appears (see Fig. 6 and Fig. 7). Though both inflow and outflow region exhibit periodic wave along azimuthal direction, the amplitude of wave in the inflow region is significantly greater than that of outflow region (see Fig. 6 b-f and Fig. 7). In the beginning of the periodic non-axisymmetric flow structure, the azimuthal wavenumbers (number of wavelengths along azimuthal direction) is nearly 2, normalized natural wavelength is 0.33 (6 cells of Taylor vortex) and normalized torque is nearly 11% smaller than that of single wavelength (see Table 2, Fig. 6 b-f and Fig. 7). As Reynolds number increased, the azimuthal wavenumber approaches towards 1 and normalized natural wavelength decreases to 0.25 which is same as the single wavelength fluid column (see Fig. 6 b-f and Fig. 7). In this flow regime, the normalized torque obtained for the four wavelengths fluid column closely matches with the normalized torque obtained for single wavelength (see Table 2, Fig. 6 b-f and Fig. 7). Thus, the most dominating factor which influence the normalize torque could be reported as natural wavelength which is align with the study of Martínez et al.(2014). In addition, this provides a significant relationship between normalized torque and azimuthal wavenumber and natural wavelength.
From this study, it is found that total of three flow regimes namely axisymmetric Taylor vortex flow, transition between Taylor vortex into wavy vortex flow and wavy vortex flow (see Fig. 4, Fig. 5 and Fig. 6). It was also found that the natural wavelength varies with the Reynolds number. In addition, it is observed that as natural wavelength increases, normalized torque decreases and vice versa. Most important finding could be reported as the sudden increase in natural wavelength and the appearance of azimuthal wave and its influence on the comparatively lower normalized torque during the transition of axisymmetric flow structure into periodic non-axisymmetric flow structure (see Fig. 5, Fig. 6 and Fig. 7). Previous studies indicated that that wavy vortex flow appears due to the non linear growth of the non-axisymmetric disturbances. Hence, drastic change in natural wavelength in the transition phase of Taylor vortex into wavy vortex and its influence in the normalized torque may directly related to the appearance of periodic non-axisymmetric disturbances. It was reported in the earlier studies that inflow and outflow region of Taylor vortex play significant role in the appearance of periodic non-axisymetric or wavy vortex flow (Sobolik, Benabes & Cognet, 1995; Dumont et al., 2002; Akonur & Lueptow, 2003; Kristiawan, Jirout & Sobolík, 2011; Sobolík et al., 2011). In this present DNS study, we have observed that the azimuthal wave in the inflow region is significantly dominant that of outflow region in the periodic non-axisymmetric flow structure (see Fig. 5, Fig. 6 and Fig. 7). This, provides very important understanding that the flow behavior in the inflow region may strongly influence in the occurrence of peridic disturbances in the transition phase of axissymetric Taylor vortex into periodic non-axisymmetric flow structure.

This study strongly believe that the flow behavior in the near wall region of inflow and outflow region may have strong influence in Taylor vortex flow, transition of Talor vortex into wavy vortex flow and wavy vortex flow. It is also believed that study of near wall region may provide some important understanding about the behavior of natural wavelength in Taylor vortex flow, transition of Taylor vortex into wavy vortex flow and wavy vortex flow. Further more, study of the behavior of near wall region may provide important understanding about the mechanism of the generation of non-axisymmetric disturbances and its propagation with the change in Reynolds number. To better understand above mentioned issues, near wall region is studied with the aid of viscous layer (The region where azimuthal velocities varies linearly with wall distance).

In this study, first of all, the distribution of near wall region along axial direction will be carried out for Taylor vortex flow. Next to this, the behavior of near wall region in the jet impingement and flow separation will be investigated. Following that, dependency of near wall region on Reynolds number and its relation with the natural wavelength will be studied. In addition, the behavior of near wall region during transition of Taylor vortex into wavy vortex will be given concentration to get a better understanding about the mechanism of the origination of non-axisymmetric disturbances and its propagation over time. Finally, the correlation between wall shear stress and near wall region in jet impingement and flow separation will be carried out.
Fig. 5: Streamline patterns associated with flow structure variation in Taylor Couette flow for radius ratio 0.5, four wavelength and infinite aspect ratio (a)-(d) axisymmetric flow structure (e) transition of axisymmetric into periodic non-axisymmetric flow structure (wavy), (f) periodic non-axisymmetric flow structure.
Fig. 6: The vorticity countour around 360 degree at radial position 0.00075m from inner wall (a) vorticity countour of axissymmetric flow structure (b) –(f) periodic non-axissymmetric flow structure.

Fig. 7: The distribution of normalized wall shear stress along azimuthal direction at a fixed axial position (a) the distribution of normalized wall shear at inflow region (b) distribution of normalized wall shear at outflow region.

Near wall region:

The near wall region is defined as region where azimuthal velocity varies linearly with the wall distance. The azimuthal velocity profile (Fig. 8) at different axial locations of a single vortex cell at Re 425(Fig. 9) provides a clear idea about the influence of vortex structure on the azimuthal
velocity distribution along the radial direction. As shown in Fig. 8, azimuthal velocity profile in the inflow and outflow region along the radial direction exhibits approximately two linear regions, one from the inner wall to a certain radial position and another one from the outer wall to another distinct radial position where the viscous force plays a major role, but the influence of vortex structure is minimum. The azimuthal velocity profile for the region between the inflow to outflow region along the radial direction is divided into three regions. In the first and third region, the azimuthal velocity changes linearly with the change in radial position where viscous force is dominant, but in the second region, the velocity profile exhibits a nearly uniform distribution (Fig. 8) and vortex structure plays a major role here. The azimuthal velocity profile presented in Fig. 10 for 8 cells of Taylor vortex shows similar behavior as mentioned in Fig. 8 where linear region appears from inner and outer wall up to certain radial position and nearly uniform region away from walls.

In this present study, the near wall region along the radial direction at different axial locations of a single vortex cell, \( u^+ \) and \( y^+ \) in the inner and outer wall has been computed using the following equations.

\[
\begin{align*}
  u^+_l &= \frac{v_\theta(R_l)}{u_{\tau l}} - \frac{v_\theta(r)}{u_{\tau l}}, \quad y^+_l = \frac{(r-R_l)u_{\tau l}}{\nu}, \\
  u^+_o &= \frac{v_\theta(r)}{u_{\tau o}} - \frac{(R_o-r)u_{\tau o}}{\nu}, \quad \text{where, Friction velocity at inner wall, } u_{\tau l} = \sqrt{\tau_{wal l l}\frac{\rho}{\nu}}, \text{ Friction velocity at outer wall the, } u_{\tau o} = \sqrt{\tau_{wall o}\frac{\rho}{\nu}}, \text{ Azimuthal Velocity=}\ V_\theta, \text{ Radius of inner wall}=R_l, \\
  \text{Radius of outer wall}=R_o, \text{ Wall shear at inner wall }=\tau_{wall l}, \text{ Wall shear at outer wall }=\tau_{wall o}, \\
  \text{Kinematic viscosity of fluid }=\nu, \text{ Density of fluid }=\rho.
\end{align*}
\]

![Fig. 8: The azimuthal velocity profile along radial direction at the different axial location of a single vortex cell at Re 425.](image1)

![Fig. 9: Single Taylor Vortex cell (Streamline) at Re 425.](image2)
Fig. 10: The azimuthal velocity profile along radial direction at the different axial location of a four-wavelength at Re 425.

Fig. 11: Variation of $u^+$ and $y^+$ for single vortex cell at the outer wall at Re 425

Fig. 12: Variation of $u^+$ and $y^+$ for single vortex cell at the inner wall at Re 425
The variation of $u^+$ vs $y^+$ for inner and outer wall along the radial direction at different axial locations of a single vortex (Fig. 9) at Re 425 is shown in Fig. 11 and Fig. 12 respectively. It can be observed that $u^+ \approx y^+$ up to a certain radial position from both inner and outer wall (Fig. 11 and Fig. 12). The similar behavior is observed for the case of four wavelength (Fig. 13 and Fig. 14).

It is clearly understood from Fig. 11, Fig. 12, Fig. 13 and Fig. 14 that the wall distance up to which $u^+ \approx y^+$ is satisfied, varies at different axial location of a single vortex cell. The peak value of $y^+$ within the region where $u^+ \approx y^+$, provides the near wall region. The thickness of normalized viscous layer at inner and outer wall has been computed from the peak value of $y^+$ and equations are shown in the following.

Normalized thickness of VL at inner wall, $(\frac{\delta}{d})_i = \frac{(y^+_{peak})_i}{u_{\tau i}d}$

Normalized thickness of VL at outer wall, $(\frac{\delta}{d})_o = \frac{(y^+_{peak})_o}{u_{\tau o}d}$

Where, Friction velocity at inner wall, $u_{\tau i} = \sqrt{\tau_{wall i}} / \rho$, Friction velocity at outer wall, $u_{\tau o} = \sqrt{\tau_{wall o}} / \rho$,

Wall shear at inner wall = $\tau_{wall i}$, Wall shear at outer wall = $\tau_{wall o}$, Kinematic viscosity of fluid = $\nu$,

Density of fluid = $\rho$ and annuals gap = $d$.

The distribution of normalized VL at inner and outer wall along the axial direction for four-natural wavelengths (8 cells of Taylor vortex) has been illustrated in Fig. 15. The study finds that the thickness of normalized viscous layer along the axial direction is strongly influenced by the vortex structure where the VL thickness is minimum in the inflow region of inner wall and outflow region of outer wall (see Fig. 15). For the case of inner wall, the thickness of VL increases from inflow towards outflow region and reaches to its maximum value in the vicinity of outflow region (see Fig. 15 a). As it approaches to the outflow region, a sudden decrease in the VL thickness is observed (see Fig. 15 a). The study of the relationship between VL thickness and wall shear stress provides an inverse like relationship from inflow towards the vicinity of outflow region and proportional like relation is noted within the outflow region of inner wall (see Fig. 16 a and Fig. 17 a). For the case of outer wall, the thickness of VL increases from outflow towards inflow region.
and reaches to its maximum value in the vicinity of inflow region (see Fig. 15 c). As it reaches to the adjacent of inflow region, a sudden decrease in the VL thickness is appeared (see Fig. 15 c). The study of the relationship between VL thickness and wall shear stress illustrates an inverse like relationship from outflow region towards the vicinity of inflow region of outer wall and proportional like relation is observed within the inflow region (see Fig. 16 c and Fig. 17 b). Thus, in the region of inverse like relationship between VL thickness and wall shear stress, when VL thickness is minimum, wall shear stress is observed as maximum in the inflow region of inner wall and outflow region of outer wall (see Fig. 16), as VL thickness is maximum, wall shear stress is observed very close to its minimum value in the vicinity of inner wall of outflow region and inflow region of outer wall(see Fig. 16). The minimum wall shear stress is found exactly in the inflow region of outer wall and outflow region of inner wall. The distribution of wall shear stress along axial direction shows similar agreement with earlier studies (Dumont et al., 2002; Akonur & Lueptow, 2003; Kristiawan, Jirout & Sobolík, 2011; and Sobolik et al., 2011). The relationship between wall shear stress and the thickness of VL observed in the inflow region towards the vicinity of outflow region of inner wall and outflow towards the vicinity of inflow region for outer wall is aligned with the earlier studies of open boundary flow and internal flow (Pope, 2000). However, the proportional relation between VL thickness and wall shear stress observed in the outer wall of inflow region and inner wall of outflow region has not been reported in the earlier studies. Thus, this study considers the behavior of VL thickness in the inflow region of outer wall and outflow region of inner wall as new findings. This study considers the behavior of VL thickness in the inflow region of outer wall and outflow region of inner wall as a drop region (see Fig. 15 a and Fig. 15 c).

In the study of Dumont et al. (2002), Akonur & Lueptow (2003), Kristiawan, Jirout & Sobolík (2011) and Sobolik et al. (2011), a weak jet impingement in the inflow region of inner wall and a strong jet impingement in the outflow region of outer wall is considered. Those study also assumed a flow separation region in the inflow region of outer wall and outflow region of inner wall. This provides an aligned agreement with the occurrence of thinner VL thickness in the jet impingement region (Arabnejad, 2014). Present study finds a distinct drop in the thickness of VL in the outer wall of inflow region and inner wall of outflow region. This indicates that the drop observed in the VL thickness may directly related to the occurrence of flow separation. Akonur & Lueptow (2003) suggested about occurrence of a pair of secondary vortices in the flow separation region. This also may play a significant role behind the appearance of drop in the VL thickness. The drop observed in the VL thickness provides significant information about the exact location of flow separation and its axial and radial range. In the earlier studies, it was mentioned that flow behavior in the inflow and outflow region contributes a major role in the generation of non-axisymmetric disturbances in Talor Couette flow (Sobolik, Benabes & Cognet, 1995; Dumont et al., 2002; Akonur & Lueptow, 2003; Kristiawan, Jirout & Sobolík, 2011; Sobolik et al., 2011). Present DNS study finds that as axisymmetric flow structure transforms into periodic non-axisymmetric flow structure, distinct wave in the inflow and outflow region along the azimuthal direction is observed (see Fig. 6 and Fig. 7). Hence, the study of the behavior of drop in the flow separation region of inflow and outflow region may provide important information about the origin and growth of the non-axisymmetric disturbances. Furthermore, study of the inter relation between VL thickness and variation of natural wavelength may lead to get understanding about the influence of natural wavelength on the VL and normalized torque. To address above mentioned issues, the behavior
of VL thickness, magnitude of drop region, the location of flow separation and radial and axial location with the change in Reynolds number will be studied in the following sections. In addition, the variation of VL thickness and the drop in the Taylor Vortex flow, transition of Taylor vortex into wavy vortex flow and wavy vortex flow will be studied.

![Fig. 15: The distribution of viscous layer thickness along axial direction for inner and outer wall for Re 425 (a) viscous layer distribution at inner wall (b) The vortex structure for four wavelength fluid column (c) viscous layer distribution at outer wall.](image)

![Fig. 16: The distribution of VL thickness along axial direction and its relationship with the wall shear stress (a) Distribution of wall shear stress along axial direction (b) Flow structure at Re 425 (c) Distribution of VL thickness along axial direction](image)
Behavior of near wall region with the change in Reynolds number and its dependency on flow structure:

In this section, the dependency of near wall region on the Reynolds number has been investigated. This section is divided into four regions based on the number of vortex cells and natural wavelength. For axisymmetric flow structure, at Re 175, 225 and 325, a total of 10 vortex cells are observed with the natural wavelength of 0.20 (see Fig. 5 a and Fig. 5 c). For Re 275, 375 and 425, a total of 8 vortex cells appears which provides natural wavelength 0.25 (see Fig. 5 b and Fig. 5 d). For non-axisymmetric flow structure, it is divided into two regimes. In the first regime, it ranges between $415 < Re < 575$ with 6 cells of Taylor vortex (natural wavelength 0.33) (see Fig. 5 e). The second region is classified as Re > 475 where 8 cells of Taylor vortex appear (0.25 natural wavelength) (see Fig. 5 f).

**First regime of axisymmetric flow structure (10 vortex cells and natural normalized wavelength 0.20):**

The distribution of viscous layer (VL) in the inner and outer walls for the first region of axisymmetric flow structure (Re 175, 225 and 325 with 10 cells of Taylor vortex) is illustrated in the Fig. 18. For the case of inner wall, it is found that the difference in the thickness of VL at the jet impingement and flow separation is not dominant for the smaller Reynolds number (Re 175) (see Fig. 18 a). As Reynolds number increases, the difference in the VL thickness at jet impingement and flow separation becomes dominant (see Fig. 18 a). For the case of outer wall, this difference in the VL thickness is dominant for all the Reynolds number (see Fig. 18 b). For both walls, study finds that the increase in Reynolds number results in the appearance of thinner VL at the jet impingement and flow separation region (Fig. 18, Fig. 19 and Fig. 20) where the decreasing trend is more sensitive to the change in Reynolds number for the outer wall than inner wall. This indicates that the radial range of flow separation in both walls is dampens with the increase in Reynolds number, the response in the outer wall is more sensitive than inner wall. The magnitude of average VL thickness in the drop region increases gradually for both walls where
the increasing trend is observed to be greater in the outer wall than that of inner wall (see Fig. 21). The axial range of flow separation is observed as decreased with the increase in Reynolds number where it is found to be smaller in the inflow region of outer wall than that of outflow region of inner wall (see Fig. 22). This indicates that the flow separation region dampens along the axial direction. In addition, it shows that the flow separation region is stronger in the inner wall than that of outer wall. Study finds that the axial distance between flow separation and jet impingement and flow separation increases with the increase in Reynolds number (see Fig. 23). The axial distance between jet impingement and flow separation is found to be smaller for the outer wall than inner wall (see Fig. 23) which illustrates vortex structure is expanded in the inner wall and compressed in the outer wall (see Fig. 5).

Fig. 18: Distribution of normalized viscous layer for 10 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column (a) normalized viscous layer at inner wall (b) normalized viscous layer at outer wall.

Fig. 19: The variation of VL thickness in the flow separation region (a) VL thickness in inner wall (b) VL thickness in outer wall.
Fig. 20: The variation of VL thickness in the jet impingment region (a) VL thickness in inner wall (b) VL thickness in outer wall.

Fig. 21: The variation of average drop in the flow separation region (a) average drop in inner wall (b) average drop in the outer wall.

Fig. 22: The variation of axial range of flow separation.

Fig. 23: The variation of axial distance between jet impingment and flow separation region.
Second regime of axisymmetric flow structure (8 vortex cells and natural normalized wavelength 0.25):

The behavior of VL thickness with the change in Reynolds number for the second region of axisymmetric flow structure (Re 275, Re 375 and Re 425 with 8 cells of Taylor vortex and natural normalized wavelength 0.25) has been illustrated in the Fig. 24. The behavior of VL in the jet impingement and flow separation is same as described in the first regime of axisymmetric flow structure for both walls except the position of vorticial structure (see Fig. 24). For Re 275, the flow structure exhibits jet impingement in the inner wall and flow separation region in the outer wall at the beginning of natural wavelength in the axial direction (Fig. 24). However, for the case of higher Reynolds number (Re 375 and 425), in the inner wall, flow separation region appears first and then jet impingement region while for the outer wall, jet impingement comes first and then flow separation (see Fig. 24). This explains the reason behind the distinct shift of VL thickness along the axial direction for Re 275 than that of Re 375 and 425 (Fig. 24).

The thickness of VL in the inner wall of the flow separation region continues a steady decrease from the first region of axisymmetric flow structure (Re 175) to the end of the second region of axisymmetric flow structure (Re 425) (see Fig. 19 a). For the case of outer wall, a sudden increase in the VL thickness is observed at the beginning of the second region of axisymmetric flow structure (Re 275) following which, the VL thickness decreases continuously till the end of second region (Re 425) (Fig. 19 b). As similar to the first region of axisymmetric flow structure, the VL thickness decreases rapidly respect to the Reynolds number in outer wall than that of inner wall (Fig. 19). The thickness of VL in the jet impingement keeps continuous decreasing from the first regime of axisymmetric flow structure to the end of the second regime of axisymmetric flow structure for both walls (Fig. 20). Similar to the behavior of VL thickness with the change in Reynolds number in the flow separation region, the decreasing rate of VL thickness in the jet impingement region with the change in Reynolds number is sharp in the outer wall than inner wall (see Fig. 20). The thickness of VL in the drop region continues a steady increase with the increase in Reynolds number for the inner wall from first region of axisymmetric flow structure to the end of second region (see Fig. 21 a). For the case of outer wall, the VL thickness in the drop region increases from the first regime of axisymmetric flow structure to the beginning of second regime (Re 275) beyond which, VL thickness start to decrease till the end of second regime of axisymmetric flow structure (see Fig. 21 b). The axial range of flow separation follows the decreasing trend as it was observed in the first region of axisymmetric flow structure in the inner wall (see Fig. 22). For the case of outer wall, the axial range of flow separation keeps decreasing from first regime of axisymmetric flow structure to the beginning of second regime (Re 275). Following this, the axial range of flow separation start increasing up to Re 375 and it reaches nearly steady region at Re 425 (see Fig. 22). The axial distance between jet impingement and flow separation region increases from first regime till the beginning of second region for both walls (Fig. 22). As Re is increased beyond 275, a rapid decrease in the axial distance between flow separation and jet impingement is observed for the both walls (Fig. 22). Similar to the first regime of axisymmetric flow structure, the axial distance between flow separation and jet impingement is found as smaller in the outer wall than that of inner wall (Fig. 22).
The study of the behavior of the thickness of VL and axial range of flow separation with the change in Reynolds number indicates that the behavior of VL in the inner wall for the first (normalized natural wavelength 0.2) and second (normalized natural wavelength 0.25) regime of axisymmetric flow structure, no significant change in the VL is observed (see Fig. 19a, Fig. 20a, Fig. 21a and Fig. 22). This provides an understanding that the change in natural wavelength of Taylor vortex does not influence the VL thickness in the inner wall. In the outer wall, the behavior of VL thickness in the jet impingement with the change in Reynolds number maintains steady decrease (see Fig. 20b). However, for the case of VL thickness in the flow separation and drop region of the outer wall, there is significant sudden change in the VL thickness and axial range of flow separation is observed at the beginning of the second region of axisymmetric flow structure (Re 275) (see Fig. 19b, Fig. 21b and Fig. 22). This indicates that as Reynolds number increased, the flow separation region in the outer wall (inflow region) may play major role in the transformation of vorticial structure with 10 cells (natural wavelength 0.2) into 8 cells (natural wavelength 0.25) of Taylor vortex. From the study of the behavior of VL thickness with the change in the axisymmetric flow structure, it is understood that the flow separation region in the outer wall (inflow region) undergoes through drastic change which may indicate that a secondary form of instability may directly related to the transition of natural wavelength.

Fig. 24: Distribution of normaized viscous layer for 8 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column (a) normalized viscous layer at inner wall (b) normalized viscous layer at outer wall.
First regime of non-axisymmetric flow structure or transition of axisymmetric flow structure into non-axisymmetric flow structure (6 vortex cells and natural Normalized wavelength 0.33): 

As Reynolds number increased beyond 425, the inflow and outflow regions exhibit a wave along azimuthal direction. The study of flow structure indicates that each pair of counter rotating vortices interacts with each other by a secondary flow (see Fig. 25 b). The most fundamental difference between the axisymmetric flow structure and this periodic non-axisymmetric flow structure is the appearance of azimuthal wave in the inflow and outflow region with 6 vortex cells and normalized wavelength 0.33.

In this section, the study of the behavior of VL thickness in the first region of periodic non-axisymmetric flow structure is investigated (6 vortex cells and normalized wavelength 0.33). The study finds that distinct drop in the VL thickness is observed in the flow separation region with the clear difference as two peaks value of the VL across the flow separation region for both walls (see Fig. 25, Fig. 26 and Fig. 27) which has not been observed in the axisymmetric flow structure. The greater peak is classified as the global maximum and lower peak is named as the local maximum (see Fig. 26). It was mentioned in the previous section that as asymmetric flow structure transforms into periodic non-axisymmetric flow structure, a secondary flow appears in between two counter rotating vortices (see Fig. 26 b). The study of flow structure and the distribution of VL thickness indicates that the global maximum of VL lies in the adjacent to the secondary flow and local maximum is related to the vorticial structure where there is no secondary flow (see Fig. 26). This provides an understanding that presence of secondary flow influences the VL thickness to reach to its global maximum in the flow separation region (see Fig. 26). For the case of inner wall, as it is approached from outflow region towards inflow region, VL thickness becomes thinner and following that, a thicker VL appears in the vicinity of the inflow region and VL thickness again drops in the inflow region (see Fig. 26 a). The study of the relationship between VL thickness and normalized wall shear indicates that a total of three regions are observed (see Fig. 26 a). In the first region, wall shear stress and normalized VL vary linearly from the minimum VL thickness of drop region to the global maximum point of VL thickness and then wall shear stress and VL thickness are inversely like related from global maximum to the inflow region of inner wall (outside the drop region) where wall shear stress and VL thickness are inversely related (see Fig. 26 a). The second region varies between minimum VL thickness (outside flow separation) to the minimum thickness of VL of drop region (see Fig. 26 a and Fig. 28a). Normalized wall shear and normalized VL thickness are inversely like related in the region of minimum VL thickness to the local maximum value of VL and proportional like relation is observed in between local maximum to the minimum VL thickness of drop region (see Fig. 26 a, Fig. 28 a, and Fig. 29 a). As Reynolds number is increased, first and third region remains same except the second region (see Fig. 27 a). The second region exhibits two types of relationship, in one side, it shows linear relationship between wall shear and VL and other side the relationship is inverse (see Fig. 27 a, Fig. 30 a, Fig. 31 a).
In the study of axisymmetric flow structure, it was shown that the inflow region of inner wall experiences jet impingement which in turns results in the appearance of a thinner VL in the inflow region (see Fig. 15). However, in this first section of non-axisymmetric flow structure, inflow region which is classified as second region, the identified linear relationship between VL thickness and wall shear stress indicates that inflow region behaves very similar to the flow separation region (see Fig. 27 a, Fig. 28, Fig. 29 a, Fig. 30 a, and Fig. 31 a). Despite inflow region behaves like flow separation region, the behavior of wall shear stress in the second region indicates that this could be reported as a weak flow separation (see Fig. 28, Fig. 29 and Fig. 31 a). It was shown for the first and third regions that the wall shear stress and VL thickness varies linearly in the outflow region of inner wall (flow separation) and inverse relation is observed along the axial direction in between global maximum to the minimum VL thickness (outside flow separation) (see Fig. 25 a, Fig. 26 a, Fig. 30 a, and Fig. 31 a). This could be reported as the direct influence of the strength of vortex and the position of vortex core.

For the case of the outer wall, the drop in the VL of the inflow region (flow separation), maximum thickness of VL in the vicinity of flow separation and the thinner VL in the jet impingement region are observed which is same as the distribution of VL in the axisymmetric flow structure (Fig. 26 b). Similar to the inner wall, there is certain difference in the peak value of VL across the drop region (inflow region) is observed (Fig. 26 c). Though there is no clear jet impingement is observed for the case of inner wall, outer wall experiences both jet impingement and flow separation regions (see Fig. 26 c). The distribution of VL thickness for the single natural wavelength is divided into two sections (see Fig. 26 c and Fig. 32). The first section lies between the jet impingement to the minimum VL thickness region of the drop through the local maximum of VL and second one ranges between minimum VL thickness of drop region to the jet impingement region through the global maximum of VL (see Fig. 26 c and Fig. 32). As similar to the axisymmetric flow structure, the VL thickness and wall shear stress exhibits linear relationship in the drop region and inverse relationship between global maximum point to the minimum VL thickness of jet impingement (see Fig. 29 b). The distribution of VL thickness against wall shear stress is nearly inversely like related from jet impingement to the local maximum (see Fig. 29 b). In the flow separation region, the second region exhibits more dominant characteristics than that of first region (Fig. 29 b) due to the strong influence of secondary flow.

As shown in Fig. 33, the variation of the peak values of VL thickness across the flow separation of each pair of vortices (each two vortex cells) is nearly uniform for the axisymmetric flow structure. As soon as flow structure transforms into non-axisymmetric flow structure, there is a certain difference in the peak values of VL across the drop region is observed (see Fig. 33). In this first regime of non-axisymmetric flow structure, the variation of VL thickness across the drop region in a pair of vortices is found as maximum at Re 475 and beyond which it decreases for the both walls (see Fig. 33 a and Fig. 33 b). Furthermore, as Reynolds number increased, the difference between global and local maximum VL thickness across the drop region decreases for both walls (see Fig. 33 a and Fig. 33 b). These indicates that the strength of secondary flow magnifies with the increase in Reynolds number which in turns results in the increment of the difference between global and local maximum across the drop region.
The variation of VL thickness in the flow separation region with the change in Reynolds number provides a sudden boost in the thickness of VL for inner (see Fig. 19 a) and outer wall (see Fig. 19 b) where the change in VL thickness in the outer wall is more dominant than that of inner wall (see Fig. 17). The thickness of VL in the jet impingement for inner wall decreases steadily from axisymmetric flow structure till Re 575 (see Fig. 20 a). Nonetheless, for the case of outer wall, the thickness of VL in the jet impingement increases till 475 and then again decreases (Fig. 20 b). A certain boost in the thickness of VL in the drop region is observed for both inner and outer wall (Fig. 21). However, the change in the VL in the drop region is more significant for the outer wall than inner wall (Fig. 21 b). The axial range of flow separation decreases from axisymmetric flow structure to the beginning of first regime of non-axisymmetric flow structure for inner wall (Fig. 22). As flow structure reaches to the first regime of non-axisymmetric, axial range of flow separation increases and again it decreases with the increase in Reynolds number (see Fig. 22). For the case of outer wall, the axial range of flow separation decreases steadily in the first regime of axisymmetric flow structure, then increases in the second regime up to Re 375 and remains steady from Re 375 to 425 (Fig. 22). In the beginning of non-axisymmetric flow structure, the axial range of flow separation increases up to Re 475 and then decreases. In the whole range of Re, the axial range of flow separation is always smaller in the inner wall than that of outer wall (Fig. 22). This indicates that the flow separation region in the outer wall is always stronger than that of inner wall. The behavior of axial distance between flow separation and jet impingement for inner and outer wall follows similar trend where the axial distance between flow separation and jet impingement for outer wall is always greater than the inner wall (Fig. 23). The axial range of flow separation and jet impingement increases from second regime of axisymmetric flow structure to the beginning of non-axisymmetric flow structure. In the first phase of non-axisymmetric flow structure, the axial range of flow separation and jet impingement starts decreasing for both inner and outer wall (Fig. 23).

The study finds that in the beginning of the non-axisymmetric flow structure, there is a distinct change observed in the thickness of VL in the flow separation (drop region) (see Fig. 21) and jet impingement (see Fig. 20) for the both walls. However, the variation of VL thickness in the flow separation region goes through drastic changes as compared to the jet impingement region (see Fig. 20 and Fig. 21). This indicates that the variation of VL thickness in the flow separation region plays significant role in the beginning of non-axisymmetric flow structure. In addition, there is sudden change observed in the axial range of flow separation (see Fig. 22) and axial distance between jet impingement and flow separation (see Fig. 23) in the beginning of non-axisymmetric flow structure which could be also related to the appearance of non-axisymmetric flow structure. The thickness of VL in the jet impingement for the outer wall suddenly increases up to Re 475 and beyond which VL thickness decreases again (see Fig. 20 b). For the case of inner wall, the thickness of VL in the jet impingement is nearly steady for the first regime of non-axisymmetric flow structure (425 < Re < 525) following which it again decreases. The study of flow structure reveals that the inflow region of inner wall which was classified as jet impingement region behaves like a weak flow separation region. This may indicate that the steady behavior of VL thickness in the inner wall of inflow region is related to the replacement of jet impingement region with a weak flow separation region in the early phase of non-axisymmetric flow structure.
The source of the non-axisymmetric disturbances and its propagation:

The study of the flow structure indicates that as Reynolds number increased beyond 425, a strong azimuthal wave in the inflow region and comparatively weak azimuthal wave in the outflow region is observed (Fig. 6 b and Fig. 6 c). The streamline obtained in YZ plane for Re 475 and 525 indicates that there is a shift of inflow region along the axial direction (see Fig. 5 e) which is the result of the appearance of azimuthal wave. Besides, as non-axisymmetric flow structure appears, there is a secondary flow observed in between two counter rotating vortices. In this section, the source of the appearance of non-axisymmetric flow structure and some qualitative understanding about the propagation of non-axisymmetric disturbance will be studied here.

The study conducted by Davey (1962), Marcus (1984), Dumont et al. (2002) and Akonur & Lueptow (2003) provided an idea that the appearance of radial and azimuthal jets in the outflow region are the principle cause of the occurrence of periodic non-axisymmetric disturbances. A more recent study by Kristiawan, Jirout & Sobolík (2011) claimed that flow separation at the inner wall of the outflow and the outer wall of inflow region leads to the generation of a pair of secondary vortices. They consider this to be the reason for the transition from Taylor-vortex to wavy vortex flow. The study of VL thickness in this present DNS study indicates that there is a drastic change observed in the thickness of VL in the flow separation region (inflow region of outer wall and outflow region of inner wall) (see Fig. 21). Despite VL thickness exhibits drastic change both in inner and outer wall, the change in the VL thickness in the flow separation region (inflow region) of outer wall is significantly dominant than that of inner wall (Fig. 21 b). In addition, the study of vorticity contour in Fig. 6 and the distribution of wall shear stress along azimuthal direction (Fig. 7) indicates that strong azimuthal wave is observed in the inflow region. Thus, the result obtained from the study of the distribution of VL thickness and the analysis of vorticity contour and wall shear stress along azimuthal direction provides clear understanding that the flow separation region (inflow region) of outer wall play the most dominating role in the appearance of non-axisymmetric flow structure which is partially aligned with the study of Kristiawan, Jirout & Sobolík (2011). This also suggests that the source of the non-axisymmetric disturbances could be the the inflow region of outer wall.

The study of axial range of flow separation decreases steadily with increase in Reynolds number for inner wall in the first and second regime of axisymmetric flow structure (see Fig. 22). This indicates that as Reynolds number increases, the flow separation region weakens in the outflow region of inner wall. For the case of outer wall, the axial range of flow separation decreases with the increase in Reynolds number in the first regime of axisymmetric flow structure. In the second regime of axisymmetric flow structure, the axial range of flow separation increases with the increase in Reynolds number up to Re 375 and remains steady till Re 425 (see Fig. 22). In the first regime of non-axisymmetric flow structure, the axial range of flow separation increases up to Re 475 and then starts decreasing again (see Fig. 22). This indicates that as Reynolds number increases, in the first regime of axisymmetric flow structure, the flow separation region in the inflow region of outer wall becomes weaker with the increase in Reynolds number. In the second regime of axisymmetric flow structure, the flow separation regimes become stronger with the increase in Reynolds number. As Reynolds number is increased beyond 425, the flow separation region becomes so strong that whole inflow region becomes unstable which in turns results in the replacement of jet impingement region by flow separation region (see Fig. 15). The instability observed in the inflow region magnifies over time and this may result in the appearance of periodic
secondary flow in the inflow region which may directly related to the origination of periodic non-axisymmetric disturbances. The strength of periodic secondary flow magnifies over time and this finally results in the appearance of a wave along the azimuthal direction in the inflow region (see Fig. 5 and Fig. 6).

The flow structure transforms into periodic non-axisymmetric due to the appearance of azimuthal wave with 6 cells of vortices and normalized natural wavelength 0.33 (Fig. 5 e, Fig. 6 b and Fig. 6 c). The most fundamental difference between axisymmetric flow structure and this periodic non-axisymmetric flow structure is the azimuthal wave (Fig. 5 d and Fig. 5 e). Despite azimuthal wave appears in both inflow and outflow regions, the inflow region exhibits strong azimuthal wave. The study of the distribution of VL thickness along axial direction in the axisymmetric flow structure indicates that there is distinct drop in the VL is observed in the inflow region of outer wall which is also classified as flow separation region (see Fig. 15 b). Though inflow and outflow region shift along the azimuthal direction, the shift in the inflow region more dominant than that of outflow region. Study also finds that each pair of counter rotating vortex interact with each other by a secondary flow in the outflow region. The source of this secondary flow could be at the inflow region.

![Diagram of flow structure and normalized viscous layer](image)

**Fig. 25**: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure.
Fig. 26: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column at Re \(475\) (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure

Fig. 27: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column at Re \(525\) (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure
Fig. 28: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column at Re 475 (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure

![Image of normalized viscous layer and streamlines]

Fig. 29: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column at Re 475 (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure

![Image of normalized viscous layer and streamlines]
Fig. 30: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column at Re 525 (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure

Fig. 31: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column (a) normalized viscous layer at inner wall (b) the streamlines associated with flow structure
Fig. 32: Distribution of normalized viscous layer for 6 cells of Taylor vortex in four wavelength (8 times of gap) of fluid column (a) normalized viscous layer at outer wall for Re 475 (b) the streamlines associated with flow structure (a) normalized viscous layer at outer wall for Re 525.

Fig. 33: The variation of global and local maximum value of VL thickness across the drop region of a counter rotating vortex along axial direction (a) VL thickness at the inner wall (b) VL thickness at outer wall.
Fig. 34: The variation of VL thickness at the jet impingment of a counter rotating vortex along axial direction (a) VL thickness at the jet impingment of inner wall (b) VL thickness at the jet impingment of outer wall.

Fig. 35: The variation of global and local maximum value of VL thickness across the drop region of a counter rotating vortex along axial direction (a) VL thickness at the inner wall (b) VL thickness at outer wall.
Second regime of non-axisymmetric flow structure (8 vortex cells and natural Normalized wavelength 0.25):

In the first regime of non-axisymmetric flow structure, a total of six cells of vortex appears which in turns results in the normalized natural wavelength to be 0.33 and this flow structure ranges between Re 425 to Re 575. In this flow regimes, a clear azimuthal wave appears in the inflow and out flow region. Besides, the inflow region of inner wall behaves like flow separation region instead of jet impingement as oppose to the axisymmetric flow structure. As Reynolds number increased beyond 575, a total of 8 vortex cells appears with the natural wavelength to be 0.25. In this flow structure, as similar to the first regime of non-axisymmetric flow structure, the inflow region exhibits strong azimuthal wave. Additionally, the inflow region exhibits jet impingement region as similar to the axisymmetric flow structure. In this section, the behavior of VL thickness with the change in Reynolds number and its dependency on the second regime of non-axisymmetric flow structure will be investigated.

The behavior of VL thickness for the second regime of non-axisymmetric flow structure has been illustrated in the Fig. 36. As similar to the first regime of non-axisymmetric flow structure, there is distinct difference in the VL thickness across the drop region is observed for the both walls with two peak value of VL thickness (see Fig. 33 and Fig. 36). Among these two peak values, the greater one is classified as global maximum and lower one is named as local maximum (see Fig. 33 and Fig. 36). The VL thickness of a normalized wavelength is divided in to two regions (see Fig. 36). For the inner wall, the first region the VL thickness lies between minimum VL thickness of drop region to the jet impingement region through the global maximum of VL and second region ranges between jet impingement to the minimum thickness of VL in the flow separation region through local maximum (see Fig. 36 a). In the outer wall, the first region starts from jet impingement to the minimum thickness of VL in the drop region through the local maximum and second region lies between minimum thickness of VL in the drop region to the jet impingement through the global maximum point (see Fig. 36 c).

The behavior of VL thickness with respect to normalized wall shear stress indicates that in the flow separation region, wall shear stress and VL thickness are linearly-like related and in the jet impingement till the vicinity of flow separation, wall shear stress and VL thickness are related with inversely like relation for both walls (see Fig. 37). There is clear difference is observed in the relationship between VL thickness and wall shear stress for 1st and 2nd region of single natural wavelength in both inner and outer wall (see Fig. 37). For the inner wall, the relationship between VL thickness is more dominant in the 2nd region as compared to the 1st region (see Fig. 37 a). In the outer wall, the VL thickness and wall shear stress varies significantly for the 2nd region than that of first region (see Fig. 37 b). The study of flow structure reveals that for each single wavelength of flow structure, secondary flow appears within a pair of vortices. As shown in Fig. 36 b, in the left section of YZ plane, secondary flow moves towards outer wall in the outflow region and then it moves towards negative axial direction as attached flow with outer wall and from inflow region, it moves towards inner wall and then again it flows towards negative axial direction as attached flow with inner wall (see Fig. 36 b). Study finds that the global maximum point of VL thickness lies within secondary flow of the outflow region of inner wall and inflow
region of outer wall. The local maximum point of VL thickness belongs to the vortex structure where there is no secondary flow in the inflow region of outer wall and outflow region of inner wall (see Fig. 36). Hence, it is understood that the secondary flow results in the appearance of global maximum point of VL thickness in the flow separation region in both walls. It is also understood that the occurrence of different VL thickness across the flow separation region is due to the reaction of secondary flow. The difference in the relationship between VL thickness and wall shear stress observed in the inner and outer wall could be also due to the influence of the secondary flow.

Fig. 36: The behavior of viscous layer for second regime of non-axisymmetric flow structure (a) the variation of VL thickness in inner wall (b) variation of VL thickness at outer wall.
The behavior of VL thickness with the change in Reynolds number for the second regime of non-axisymmetric flow structure has been investigated in this section. As shown in Fig. 36, the average thickness of VL in the second region of inner wall (Fig. 38 a) and first and second region of outer wall (Fig. 38 b), the average VL thickness decreases with the increase in Reynolds number. For the case of first region of inner wall, the global maximum value of VL thickness increases with the increase in Reynolds number which in turns results in the increase of the average VL thickness in the first region of inner wall (Fig. 33 a and Fig. 38 a). The global maximum value of VL in the outer wall decreases with the increase in Reynolds number which is completely opposite to the behavior of global maximum value of VL in the inner wall. As shown in the earlier section, the global maximum point is related to the appearance of secondary flow in the outflow region of inner wall and inflow region of outer wall. The most fundamental reason behind the different kind of behavior observed between global maximum of inner wall and outer wall could be explained such way that as Reynolds number increases, the strength of secondary flow in the outflow region of inner wall gets stronger and the secondary flow in the inflow region of outer wall becomes weaker. The stronger secondary flow may lead the global maximum value of VL thickness to be thicker and weaker one is responsible for the local maximum to be thinner in the inflow region of outer wall.

As shown in Fig. 33, in the flow separation region, there is certain difference is observed in the VL thickness across the drop region (see Fig. 33). However, the average VL thickness of each pair of vortices in the flow separation region remains nearly same along axial direction of a fixed Reynolds number. As Reynolds number increases, the magnitude of average global maximum value of VL thickness increases drastically but local maximum value decreases steadily for the case of inner wall (see Fig. 33 a and Fig. 38 a). For the case of outer wall, the magnitude of global maximum and local maximum decrease with the increase in Reynolds number. The average VL thickness in the flow separation region of inner wall increases drastically as the Reynolds number approached to 575(see Fig. 19 a) as compared to the first regime of non-axisymmetric flow structure. However, the VL thickness in the flow separation region of outer wall keeps continuous decreasing from the first regime of non-axisymmetric flow structure to the Re 650(see Fig. 19 b). As shown in the earlier section, the principle cause of the different kind of behavior of VL thickness in the flow separation region of inner and outer wall could be related to the influence of secondary flow where strength of secondary flow increases with the increase in Reynolds number in the inner wall and the strength of secondary flow decreases with the increase in Reynolds number for the outer wall. The VL thickness in the jet impingement of each pair of vortices is nearly uniform along axial direction for a fixed Reynolds number. As Reynolds number increases, the magnitude to VL thickness decreases steadily for both inner and outer walls (see Fig. 20). The magnitude of VL thickness in the drop region of inner wall shows a sudden increase with the increase in Reynolds number in this second regime of non-axisymmetric flow structure. In the outer wall, the magnitude of VL thickness keeps continuous decreasing from end of first regime of non-axisymmetric flow structure to the Re 650. Difference in the behavior of the VL thickness in the drop region of inner and outer wall could be reported due to the influence of secondary flow.
The axial range of flow separation of a fixed Reynolds number is observed as nearly unchanged for each pair of counter rotating vortex pair (see Fig. 35). In the first regime of non-axisymmetric flow structure, the axial range of flow separation decreases with the increase in Reynolds number. However, for the second regime of non-axisymmetric flow structure, the axial range of flow separation increases sharply with the increase in Reynolds number. For the case of inner wall, in the first regime of non-axisymmetric flow structure, axial range of flow separation initially increases, then it decreases and in the end of the first regime of non-axisymmetric flow structure, it again increases. In the beginning of the second regime of non-axisymmetric flow structure, the axial range of flow separation first increases and then it decreases slowly in the end of this flow regime. In the of end first regime of non-axisymmetric flow structure, the axial distance between jet impingement and flow separation decreases rapidly with the increase in Reynolds number for both inner and outer walls. In the second regime of non-axisymmetric flow structure, the axial distance between jet impingement and flow separation increases very slowly for the outer wall. In the inner wall, the distance between jet impingement and flow separation keeps decreasing, but the decreasing gradient is smaller than the decreasing trend observed in the first regime of non-axisymmetric flow structure.

Fig. 38: The variation of VL thickness in the second regime of non-axisymmetric flow structure (a) the variation of VL thickness with the change in Reynolds number in inner wall (b) the variation of VL thickness with the change in Reynolds number for outer wall.
Conclusion: Direct numerical simulation (DNS) of Taylor-Couette flow with radius ratio 0.5 and infinite cylinder length has been conducted to study the near-wall region of Taylor vortex flow, its behavior during the transition of axis-symmetric Taylor vortex into wavy vortex flow, its variation with Reynolds number and natural wavelength and the dependency of wall shear stress on the near wall region. The analysis is focused on the viscous layer (VL) distribution along axial direction. The thickness of VL is defined as the near wall region where azimuthal velocity changes linearly with the change in radial distance. It is found that the thickness of VL is minimum in the jet impingement region and increases to its maximum value near to the flow separation region and a certain drop with two peak values of VL is observed within the flow separation region. The radial and axial behavior of the VL thickness in the drop indicates that the peak values of VL across the drop region is nearly uniform for axisymmetric flow structure (up to Re 425) at a fixed Reynolds number. As Reynolds number is increased beyond 425, there is a clear difference among the peak values of VL is observed. Study shows that the axial range of flow separation in the inflow region of outer wall magnifies with the increase in Reynolds number which provides an idea that inflow region of outer wall may undergo through instability with the increases in Reynolds number and this results in the formation of periodic secondary flow in between two counter rotating vortices. It is found that as Reynolds number is increased beyond 425, a strong azimuthal wave is observed in the inflow region which could be due to the periodic secondary flow. This provides some understanding about the origin of non-axisymmetric disturbances could be in the inflow region of outer wall. Moreover, this also provides some important understanding about the propagation mechanism of non-axisymmetric disturbances through the appearance of periodic secondary flow. It is found that the flow structure is axisymmetric up to re 425 and beyond which flow structure transforms in to wavy vortex flow which has not been observed in the study of Fazel & Booz (1983). Besides, the natural wavelength varies with the change in Reynolds number for axisymmetric flow structure and non-axisymmetric flow structure which been ignored in the study of Fazel & Booz (1983). Study shows that, in the transition phase of periodic non-axisymmetric flow structure, the normalized natural wavelength reaches to its maximum value which in turns results in the comparatively lower normalized torque. The distribution of azimuthal shear stress and the thickness of VL shows about inversely proportional relationship from jet impingement to just before the flow separation region where thickness of VL is minimum in the jet impingement region, azimuthal shear stress is maximum and just before flow separation region, thickness of VL is maximum but wall shear stress approaches towards its minimum value. In the flow separation region, wall shear stress and thickness of VL shows a nearly linear relationship.

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