I. INTRODUCTION

Our knowledge of the proton structure builds on the accumulated world data from the deep-inelastic scattering (DIS) experiments, which cover a broad kinematic range in terms of the scaling variable $x$ and the momentum $Q^2$ transferred to the proton [1]. These data have been gathered in a variety of different scattering experiments, either on fixed targets or through colliding beams, and in the past two decades, especially the HERA electron-proton collider has contributed significantly with very accurate measurements spanning a wide range in $x$ and $Q^2$. Thus, DIS world data form the backbone for the determination of the parton distribution functions (PDFs) in the QCD improved parton model.

Modern PDFs, however, are expected to provide an accurate description of the parton content of the proton not only in a kinematic region for $x$ and $Q^2$ as wide as possible, but to deliver also information on the flavor composition of the proton as well as on other nonperturbative parameters associated to the observables under consideration, such as the strong coupling constant $\alpha_s$ and the heavy-quark masses. In the theoretical predictions the values for these quantities are often correlated with the PDFs and, therefore, have to be determined simultaneously in a fit.

A comprehensive picture of a composite object such as the proton does not emerge without the need for additional assumptions by relying, e.g., on DIS data from the HERA collider alone. Therefore, global PDF fits have to include larger sets of precision data for different processes, which have to be compatible, though. The release of the new data for so-called standard candle processes, i.e., precisely measured and theoretically well-understood Standard Model (SM) scattering reactions, initiates three steps in the analysis:

(i) check of compatibility of the new data set with the available world data
(ii) study of potential constraints due to the addition of the new data set to the fit
(iii) perform a high precision determination of the nonperturbative parameters: PDFs, $\alpha_s(M_Z)$ and heavy-quark masses.

Of course, at every step QCD precision analyses have to provide a detailed account of the systematic errors and have to incorporate all known theoretical corrections. At the Large Hadron Collider (LHC) PDFs are an indispensable ingredient in almost every experimental analysis and the publication of data for $W^\pm$- or Z-boson, top-quark pair or jet production from the runs at $\sqrt{s} = 7$ and $8$ TeV center-of-mass (c.m.s.) energy motivates the investigation of potential constraints on SM parameters anew.

Precision data, of course, has to be confronted to high precision theory descriptions. In a hadron collider environment, the reduction of the theoretical uncertainty below $O(1\%)$ cannot be achieved without recourse to predictions at next-to-next-to-leading order (NNLO) in QCD [2,3] which has thus become the standard paradigm of QCD precision analyses of the proton’s parton content [4]. The PDF fits ABKMO9 [5] and, subsequently, ABM11 [6] on which the current analysis is building, have been performed
precisely in this spirit. At the same time, the NNLO paradigm has motivated continuous improvements in the theory description of processes where only next-to-leading order (NLO) corrections are available, such as the hadron-production of jets.

In the current article, we are, for the first time, tuning the Alekhin-Blumlein-Moch (ABM) PDFs to the available LHC data for a number of standard candle processes including $W^\pm$ and Z-boson production as well as $t\bar{t}$ production. We are demonstrating overall very good consistency of the ABM11 PDFs with the available LHC data. Particular aspects of these findings have been reported previously [7–11]. Subsequently, we perform a global fit to obtain a new ABM12 PDF set and we discuss in detail the obtained results for the PDFs, $\alpha_s(M_Z)$ and the quark masses along with their correlations and the goodness of fit.

The outline of the article is as follows. We recall in Sec. II the footing of our fit and present the basic improvements in the theory description and the new data sets included. These encompass the charm-production and high-$Q$ neutral-current HERA data discussed in Secs. II A and II B, the $W^\pm$- and Z-boson production data from the LHC investigated in Sec. II C and, likewise, in Sec. II D data for the total cross section of $t\bar{t}$ production. The results for ABM12 PDFs are discussed in Sec. III in a detailed comparison with the ABM11 fit in Sec. III A and with emphasis on the strong coupling constant and the charm quark mass, cf. Sec. III B. Finally, in Sec. III C we provide cross section predictions of the ABM12 PDFs for a number of standard candle processes and the dominant SM Higgs production channel. The Appendix describes a fast algorithm for dealing with those iterated theoretical computations in the PDF fit, which are very time consuming.

II. NEW DATA INCLUDED AND THE THEORY UPDATE

The present analysis is an extension of the earlier ABM11 fit [6] based on the DIS and Drell-Yan (DY) data and performed in the NNLO accuracy. The improvements are related to adding recently published data relevant for the PDF determination:

(i) Semi-inclusive charm DIS production data obtained by combination of the H1 and ZEUS results [12]. This data set provides an improved constraint on the low-$x$ gluon and sea-quark distribution and allows amended validation of the $c$-quark production mechanism in the DIS.

(ii) The neutral-current DIS inclusive data with the momentum transfer $Q^2 > 1000 \text{ GeV}^2$ obtained by the HERA experiments [13]. These data allow us to check the 3-flavor scheme used in our analysis up to very high momentum transfers and, besides, to improve somewhat the determination of the quark distributions at $x \sim 0.1$.

(iii) The DY data obtained by the LHC experiments [14–17] improve the determination of the quark distribution at $x \sim 0.1$, and in particular provide a constraint on the $d$-quark distribution, which is not sensitive to the correction on the nuclear effects in deuteron.

(iv) The total top-quark pair-production cross section data from LHC [18–22] and the Tevatron combination [23] provide the possibility for a consistent determination of the top-quark mass with full account of the correlations with the gluon PDF and the strong coupling $\alpha_s$.

The theoretical framework of the analysis is properly improved as compared to the ABM11 fit in accordance with the new data included. In this section we describe details of these improvements related to each of the processes and the data sets involved, check agreement of the new data with the ABM11 fit, and discuss their impact and the goodness of fit.

A. The HERA charm data

The HERA data on the $c$-quark DIS production [12] are obtained by combination of the earlier H1 and ZEUS results. The combined data span the region of $Q^2 = 2.5 \div 2000 \text{ GeV}^2$ and $x = 3 \times 10^{-5} \div 0.05$. The dominating channel of the $c$-quark production at this kinematics is the photon-gluon fusion. Therefore it provides an additional constraint on the small-$x$ gluon distribution. Our theoretical description of the HERA data on charm production is based on the fixed-flavor number (FFN) factorization scheme with 3 light quarks in the initial state and the heavy quarks appearing in the final state. The 3-flavor Wilson coefficients for the heavy-quark electroproduction are calculated in NLO [24,25] and approximate NNLO corrections have been also derived recently [26]. The latter are obtained as a combination of the threshold resummation calculation [27] and the high-energy asymptotic [28] with the available Mellin moments of the massive operator matrix elements [29–32], which provide matching of these two. Two options of the NNLO Wilson coefficient’s shape, A and B, given in Ref. [26] encode the remaining uncertainty due to higher Mellin moments than given in [31]. In the present analysis, the NNLO corrections are modeled as a linear combination of the option A and B of Ref. [26] with the interpolation parameter $d_N$ with the values of $d_N = 0$, 1 for the options A and B, respectively. The interpolation parameter is fitted to the data simultaneously with other fit parameters and the shape of the massive NNLO correction preferred by the data is found to be close to option A with the best fit value of $d_N = -0.10 \pm 0.15$. The same approach was also used in our earlier determination of the $c$-quark mass from the DIS data including the HERA charm-production ones [33] with a similar value of $d_N$ obtained. In our analysis we also employ the running-mass definition for the DIS structure.
functions [34]. For comparison, the ABM11 fit is based on the massive NNLO corrections stemming from the threshold resummation only [27] and their uncertainty is not considered.

The description of the HERA charm data within the ABM12 framework is quite good with the value of $\chi^2$/NDP = 62/52, where NDP stands for the number of data points. The pulls for this data set also do not demonstrate any statistically significant trend with respect to either $x$ or $Q^2$, cf. Fig. 1. In particular, this gives an argument in favor of using the 3-flavor scheme over the full range of existing DIS data kinematics.

B. The high-$Q$ neutral-current HERA data

The HERA data for $Q^2 > 1000$ GeV$^2$ newly added to our analysis are part of the combined inclusive sample produced using the H1 and ZEUS statistics collected during Run-I of the HERA operation [13]. Due to kinematic constraints of DIS these data are localized at relatively large values of $x$, where they have limited statistical potential for the PDF constraint as compared to the fixed-target DIS data used in our analysis. For this reason this piece was not used in the ABM11 fit. In the present analysis we fill this gap for the purpose of completeness. At large $Q^2$ the DIS cross section gets non-negligible contributions due to the $Z$ exchange, in addition to the photon-exchange term sufficient for the accurate description of the data at $Q^2 \ll M_Z^2$, where $M_Z$ is the $Z$-boson mass. The $Z$-boson contribution is taken into account using the formalism [35,36] with account of the correction to the massless Wilson coefficients up to NNLO [37]. In accordance with [35] the contribution due to the photon-$Z$ interference term dominates over the one for the pure $Z$ exchange at HERA kinematics.\(^1\) The values of $\chi^2$/NDP obtained in our analysis for the whole inclusive HERA data set and for its neutral-current subset are 694/608 and 629/540, respectively. The data demonstrate no statistically significant trend with respect to the fit up to the highest values of $Q^2$ covered by the data. This is illustrated in Fig. 2 with the example of the neutral-current $e^+p$ HERA data sample, which contains the most accurate HERA measurements at large $Q^2$. For the $e^-p$ sample the picture is similar and the total value of $\chi^2$/NDP obtained for the newly added neutral-current data with $Q^2 > 1000$ GeV$^2$ is 147/142. For comparison, with the cuts of $Q^2 > 100$ GeV$^2$ and $Q^2 > 10$ GeV$^2$ we get for the same sample the values of $\chi^2$/NDP = 311/344 and 486/469, respectively. In particular this says that the FFN scheme used in our analysis is quite sufficient for the description of the existing HERA data in the whole kinematical range (cf. [39,40] for more details).

C. The LHC Drell-Yan data

Data on the DY process provide a valuable constraint on the PDFs extracted from a global PDF fit allowing us to disentangle the sea and valence quark distributions. At the LHC these data are now available in the form of the rapidity distributions of charged leptons produced in the decays of the $W$-bosons and/or charged-lepton pairs from the $Z$-boson decays [14–17]. Due to limited detector acceptance and the $W/Z$ event selection criteria the LHC data are commonly obtained in a restricted phase space with a cut on the lepton transverse momentum $p_T$ imposed. Therefore, taking advantage of these data to constrain the PDFs requires fully exclusive calculations of the Drell-Yan process. These are implemented up to NNLO in two publicly available codes, DYNNLO [41] and FEWZ [42]. Benchmarking these codes we found good mutual agreement for the LHC kinematics. We note that with the version 1.3 of DYNNLO the numerical convergence is achieved faster than for version 3.1 of FEWZ, although even in the former case a typical CPU time required for computing rapidity distribution with the accuracy better than 1% is 200 hours for the Intel model P9700/2.80 GHz. However, FEWZ (version 3.1) provides a convenient capability to estimate uncertainties in the cross sections due to the PDFs. Therefore we use in our analysis the benefits of both codes combining the central values of DYNNLO (version 1.3) and the PDF uncertainties of FEWZ (version 3.1).

\(^1\)The version 1.6 of the OPENQC DRAD code used in our analysis to compute the DIS structure functions including the contribution due to the $Z$ exchange is publicly available online [38].
FIG. 2 (color online). The same as in Fig. 1 for the pulls of the HERA inclusive combined data [13] binned in Bjorken $x$ versus momentum transfer $Q^2$.

FIG. 3 (color online). The ATLAS data [14] on the rapidity distribution $d\sigma/d\eta_l$ of charged leptons produced in the decays of $W^-$ and $W^+$-boson (left and central panel, respectively) and charged-lepton pairs from the decays of Z-boson (right panel) in comparison with the NNLO calculations based on the ABM11 PDFs (solid curves) taking into account the uncertainties due to PDFs (grey area). The dashed curves display the ABM12 predictions. The cuts on the lepton transverse momentum $P_T$ and the transverse mass $M_T$ imposed to select a particular process signal are given in the corresponding panels.
The predictions obtained in such a way with the ABM11 PDFs [6] are compared to the LHC DY data [14–17] in Figs. 3, 4 and 5. The predictions systematically overshoot the ATLAS data [14]. However the offset is within the experimental uncertainty, which is dominated by the one of 3.5% due to the luminosity, cf. Fig 3. On the other hand, a good agreement is observed for the $Z$-boson data by LHCb [17] in the region overlapping with the ATLAS kinematics, cf. Fig 5. This signals some discrepancy between these two sets of data, which is most likely related to the general experimental normalization. In any case the normalization offset cancels in the ratio and the ATLAS data on the charged-lepton asymmetry are in a good agreement with our predictions [14]. This is in some contrast to the CMS results where a few data points go lower than the ABM11 predictions, cf. Fig 5.

Agreement between the LHC data and the ABM11 predictions is quantified by the following $\chi^2$ functional:

$$\chi^2 = \sum_{i,j} (y_i - t_i^{(0)}) (C^{-1})_{ij} (y_j - t_j^{(0)}),$$

(2.1)

where $y_i$ and $t_i^{(0)}$ stand for the measurements and predictions, respectively, and $C_{ij}$ is the covariance matrix with the indices $i, j$ running over the points in the data set. The covariance matrix is constructed as follows:

$$C_{ij} = C_{ij}^{\text{exp}} + \sum_{k=1}^{N_{\text{unc}}} \Delta t_i^{(k)} \Delta t_j^{(k)},$$

(2.2)

where the first term contains information about the experimental errors and their correlations and the second term

![FIG. 4 (color online). The same as in Fig. 3 for the charged muons rapidity distributions obtained by LHCb [15].](image1)

![FIG. 5 (color online). The same as in Fig. 3 for the LHCb data [17] on the rapidity distribution of the $e^+e^-$ pairs produced in the $Z$-boson decays (left panel) and the CMS data [16] on the charge asymmetry of electrons produced in the $W^\pm$-boson decays (right panel).](image2)
comprises the PDF uncertainties in predictions. The latter are quantified as shifts in the predictions due to the variation between the central PDF value and the ones encoding the PDF uncertainties. For ABM11 the latter appear primarily due to the variation of the fitted PDF parameters and, besides, due to the uncertainty in the nuclear correction applied to the deuteron DIS data. Therefore, the total number of PDF uncertainty members is $N_{\text{unc}} = N_p + 1$, where $N_p = 27$ is the number of eigenvectors in the space of fitted PDF parameters (cf. the Appendix for more details).

The experimental covariance matrix for the ATLAS data [14] is computed by

$$C_{ij}^{\text{exp}} = \delta_{ij}\sigma_i^2 + f_i^j(f_j^0)^{*}\sum_{k=1}^{31}s_i^k s_j^k,$$

where $\sigma_i$ are the statistical errors in the data combined in quadrature with the uncorrelated errors. Here $s_i^j$ are the correlated systematic uncertainties representing 31 independent sources including the normalization, and $\delta_{ij}$ stands for the Kronecker symbol. In view of the small background for the $W$- and $Z$-production signal all systematic errors are considered as multiplicative. Therefore, they are weighted with the theoretical predictions $f_i^j$. The experimental covariance matrices for the CMS and LHCb data of Refs. [15–17] are employed directly as published in Eq. (2.2) after reweighting them by the theoretical predictions similarly to Eq. (2.3) and with the normalization uncertainty taken into account in the same way as for the ATLAS data.

The values of $\chi^2$ computed according to Eq. (2.1) for each of the LHC DY data sets obtained with the ABM11 PDFs are given in Table I. The description quality is somewhat worse for the ATLAS and LHCb muon data, however, in general the agreement between the data and predictions is still good. The values of $\chi^2$/NDP are comparable to 1 within the statistical fluctuations in $\chi^2$. Therefore, the data can be easily accommodated in the ABM fit. Furthermore, in this case the PDF variation is expected to be within the ABM11 PDF uncertainties. This allows us to optimize the computation of the involved NNLO Drell-Yan corrections in the fit by extrapolation of the grid with the precalculated predictions for the ABM11 eigenvector basis (cf. Appendix A for the details on the implementation of this approach). The values of $\chi^2$ obtained for the LHC DY data sets in the ABM12 fit are quoted in Table I. In this case the PDF uncertainties are irrelevant since the PDFs have been tuned to the data. Therefore, they are not included into the second term in the covariance matrix Eq. (2.2). Despite the difference in the definition, the ABM12 values of $\chi^2$ for the LHC DY data are in a good agreement with the ABM11 ones giving

![Figure 6](color online) The LO, NLO and NNLO QCD predictions for the $t\bar{t}$ total cross section $\sigma_{t\bar{t}}$ at the LHC ($\sqrt{s} = 8$ TeV) as a function of the top-quark mass in the MS scheme $m_t(m_t)$ at the scale of $\mu = m_t(m_t)$ (left) and in the on-shell scheme $m_t(pole)$ at the scale of $\mu = m_t(pole)$ (right) with the ABM12 PDFs.
additional evidence for the compatibility of these data with the ABM11 PDFs.

D. The data for $t\bar{t}$ production in the ABM12 fit

The $t\bar{t}$-pair production at the LHC proceeds predominantly through the initial gluon-gluon scattering. Thus, the total $t\bar{t}$ cross section is sensitive to the gluon distribution at effective $x$ values of $x \approx 2m_t/\sqrt{s} \approx 0.04...0.05$ for the runs at $\sqrt{s} = 7$ and 8 TeV c.m.s. energy, a region in $x$ which is well constrained by data from the HERA collider, though.

The available data for the total $t\bar{t}$ cross section from ATLAS and CMS at $\sqrt{s} = 7$ TeV [18,19] and at $\sqrt{s} = 8$ TeV [20–22] c.m.s. energy display good consistency, although, for the data sets at $\sqrt{s} = 7$ TeV only within the combined uncertainties. Generally, the systematic and luminosity uncertainties dominate over the small statistical uncertainty and the CMS data [19,21,22] as well as the result from the Tevatron combination [23] are accurate to $O(5\%)$ while the ATLAS measurements [18,20] have an error slightly larger than $O(10\%)$.

The QCD corrections for inclusive $t\bar{t}$-pair production are complete to NNLO [43–46], so that these data can be consistently added to the ABM11 PDF fit at NNLO. The theory predictions are available for the top-quark mass in the $\overline{\text{MS}}$ scheme with $m_t(\mu_r)$ being the running mass [47] as well as for the pole mass $m_t(\text{pole})$ in the on-shell renormalization scheme [43–46]. The distinction is important, because the theory predictions as a function of the running mass $m_t(\mu_r)$ display much improved convergence and better scale stability of the perturbative expansion [47]. This is illustrated in Figs. 6 and 7 for the total $t\bar{t}$ cross section computed with the program Hathor (version 1.5) [48]. In Fig. 6 we show the size of the higher order perturbative corrections from LO to NNLO taking the PDFs order independent, i.e., the ABM11 set at NNLO, as a function of the top-quark mass for the LHC at $\sqrt{s} = 8$ TeV c.m.s. energy. Likewise, Fig. 7 illustrates the scale stability for two representative top-quark masses, $m_t(m_t) = 162$ GeV and $m_t(\text{pole}) = 171$ GeV. Figures 6 and 7 imply a small residual theoretical uncertainty for the $t\bar{t}$ cross section predictions if expressed in terms of the running mass.

We have performed a variant of the ABM12 fit, adding the combined $t\bar{t}$ cross section data from LHC and Tevatron [18–23] to test the impact of these data on the gluon PDF, on the strong coupling $\alpha_s$, and on the value and scheme choice for the top-quark mass. It is strictly necessary to consider these three parameters together, since they are strongly correlated in theory predictions for the $t\bar{t}$ cross section at the LHC. In Figs. 8 and 9 we present the $\chi^2$ profile versus the top-quark mass for the variants of the ABM12 fit with the $t\bar{t}$ cross section data included and for the two different top-quark mass definitions, i.e., the $\overline{\text{MS}}$ mass $m_t(m_t)$ and the pole mass $m_t(\text{pole})$. Figure 8 displays a steeper $\chi^2$ profile for the pole-mass definition. This implies a bigger impact of the $t\bar{t}$ cross section data in the fit and, as a consequence, greater sensitivity to the theoretical uncertainty at NNLO and uncalculated higher order corrections to the cross section beyond NNLO. In

FIG. 7 (color online). The scale dependence of the LO, NLO and NNLO QCD predictions for the $t\bar{t}$ total cross section at the LHC ($\sqrt{s} = 8$ TeV) for the top-quark mass $m_t(m_t) = 162$ GeV in the $\overline{\text{MS}}$ scheme (left) and $m_t(\text{pole}) = 171$ GeV in the on-shell scheme (right) with the ABM12 PDFs and the choice $\mu = \mu_r = \mu_f$. The vertical bars indicate the size of the scale variation in the standard range $\mu/m_t(\text{pole}) \in [1/2, 2]$ and $\mu/m_t(m_t) \in [1/2, 2]$, respectively.

FIG. 8. The $\chi^2$ profile versus the $t$-quark mass for the variants of ABM12 fit with the $t\bar{t}$ cross section data included and different $t$-quark mass definitions: running mass (left) and pole mass (right).
contrast, the $\chi^2$ profile for the $\bar{\text{MS}}$ mass is markedly flatter. Figure 9 shows the $\chi^2$ profile for the subset of the $t\bar{t}$ cross section data with NDP = 5 and nicely demonstrates that a top-quark mass determination from the fit is feasible.

If one requires a $\chi^2 = 1$, the value for the $\bar{\text{MS}}$ mass is obtained at NNLO

$$m_t(m_t) = 162.3 \pm 2.3 \text{ GeV}, \quad (2.4)$$

where we define the error in $m_t(m_t)$ due the experimental data, the PDFs and the value of $\alpha_s(M_Z)$ as the difference between the value for $m_t(m_t)$ at $\chi^2 = 1$ and the minimum of the $\chi^2$ profile in Fig. 9. The additional theoretical uncertainty from the variation of the factorization and renormalization scales in the usual range is small, $\Delta m_t(m_t) = \pm 0.7$ GeV, see Fig. 7 and [49]. Equation (2.4) is equivalent to the top-quark pole-mass value of

$$m_t(pole) = 171.2 \pm 2.4 \text{ GeV}, \quad (2.5)$$

using the known perturbative conversion relations [50–52]. Equation (2.5) can be compared to the value of $m_t(pole) = 169.6 \pm 2.7$ GeV read off from Fig. 9. This indicates good consistency of the procedure and also with the top-quark mass values obtained from other determinations.\footnote{The values in Eqs. (2.4) and (2.5) supersede the top-quark mass determination in [49], because full account of the correlations among all nonperturbative parameters is kept in the present analysis.}

Having established the sensitivity to the value of the top-quark mass, we have performed further variants of the ABM12 fit by fixing $m_t(m_t)$ and $m_t(pole)$ in order to quantify the impact on the gluon PDF and on $\alpha_s$. The values for $\alpha_s(M_Z)$ which are obtained in these variants span the range $\alpha_s(M_Z) = 0.1133\ldots 0.1142$ for $m_t(m_t) = 161\ldots 163$ GeV and $\alpha_s(M_Z) = 0.1144\ldots 0.1154$ for the range of $m_t(pole) = 171\ldots 173.3$ GeV.

The $m_t$-mass range for the case of $\bar{\text{MS}}$ definition is motivated by the value preferred by the data in our analysis, Eq. (2.4). The pole-mass range is selected in order to touch the average of the experimental determinations of $m_t = 173.2$ GeV at the LHC and Tevatron [1]. The corresponding changes in the gluon PDF due to adding the $t\bar{t}$ cross section data and fixing $m_t(m_t)$ and $m_t(pole)$ at different values within these ranges are illustrated in Fig. 10. For the running-mass definition the changes in the gluon PDF are within the uncertainties of the nominal ABM12 fit and for the pole-mass case they are bigger [cf. the variants of the fit with $m_t(m_t) = 162$ GeV and $m_t(pole) = 171$ GeV related by the conversion of Refs. [50–52]]. We also find a marginal change in the gluon PDF and $\alpha_s(M_Z) = 0.1139(10)$ for a variant of the ABM12 fit with $m_t(m_t) = 162$ GeV fixed and with the CMS [19,21,22] and the Tevatron [23] data included, i.e., leaving out the ATLAS data due to the larger experimental uncertainties. This is to be compared with $\alpha_s(M_Z) = 0.1134(11)$ for the ABM11 PDF fit and, again, demonstrates nicely the stability of the analysis, provided all correlations are accounted for.

We briefly comment here on related studies, that have appeared in the literature. The impact of $t\bar{t}$ cross section data on the gluon density has first been studied in [53] by applying a reweighting procedure to given PDF sets, thereby disregarding correlations.

Reference [54] determines the strong coupling constant from a fit to $t\bar{t}$ cross section data and obtains the value of $\alpha_s(M_Z) = 0.1185(28)$ for the ABM11 PDFs with a fixed $m_t(pole) = 173.2$ GeV. Reference [54] has used version
1.3 of Hathor [48], though, which returns a slightly different central value \[O(1\%)\] change for the cross section compared to version 1.5. The sensitivity to \(\alpha_s\) is determined from fits to sets of PDFs for varying values of \(\alpha_s(M_Z)\), i.e., using the ABM11 set at NNLO (abm11_5n_as_nlo.LHgrid in the LHAPDF library [55,56]) which covers the range \(\alpha_s = 0.105 \ldots 0.12\). The analysis of Ref. [54] assumes that the relative size of the PDF uncertainties for the PDF sets with varying values of \(\alpha_s(M_Z)\) to be constant, i.e., to be determined by the PDF set with the nominal value of \(\alpha_s\). However, as one caveat, it yet misses variation of the gluon PDF central value with the value of \(m_t\) and the mass definition discussed above.

Reference [57] explores the constraints on the gluon PDF from the same set of LHC and Tevatron \(t\bar{t}\) cross section data [18–23] considered here. The analysis of Ref. [57] uses fixed values for \(\alpha_s\) and the pole mass \(m_t(\text{pole})\) and, thereby, disregards the correlation of these parameters with the gluon PDF. As illustrated in Fig. 10 this introduces a significant bias so that the fit results of Ref. [57] are a direct consequence of those assumptions. Reference [57] also compares the ABM11 PDFs [6] to those data [18–23] and quotes a value of \(\chi^2 = 40.2\) for NDP = 5 (Table 7 in Ref. [57]). Unfortunately, this computation of the \(\chi^2\) value is incomplete, since it is obtained by neglecting the PDF uncertainties, the uncertainty in the value of \(m_t(\text{pole})\) as well as other uncertainties, which may have an impact on the \(\chi^2\) value such as the uncertainty in the beam energy, currently estimated to be 1%. The \(\chi^2\) profile in Fig. 9 shows that a faithful account of the uncertainties and their correlation leads to a very good description of the \(t\bar{t}\) cross section data.

### III. THE ABM12 PDF RESULTS

In this section the results of the ABM12 fit are discussed in detail and compared specifically with the previous ABM11 PDFs. Regarding the strong coupling constant \(\alpha_s(M_Z)\) we also review the current situation for \(\alpha_s\) determinations from other processes, where the NNLO accuracy in QCD has been achieved. Finally, we apply the new ABM12 PDF grids in the format for the LHAPDF library [55,56] to compute a number of benchmark cross sections at the LHC.
A. Comparison with ABM11 and other PDFs

The PDFs obtained in the present analysis are basically in agreement with the ABM11 ones obtained in the earlier version of our fit[6] within the uncertainties, cf. Fig.11. The strange quark distribution is particularly stable since in our analysis it is constrained by the neutrino-induced dimuon production that was not updated neither from the experimental nor from the theoretical side. It is still significantly suppressed as compared to the nonstrange sea and this contrasts with the strangeness enhancement found in the ATLAS PDF analysis based on the collider data only[14].

The change in the gluon distribution happens in particular due to impact of the HERA charm data and improvements in the heavy-quark electroproduction description, cf. Ref.[33] for details. At the same time the ABM12 quark distributions differ from the ABM11 ones at most due to the LHC DY data. This input contributes to a better separation of the nonstrange sea and the valence quark distributions. As a result, at the factorization scale $\mu = 3$ GeV and $x \sim 0.2$ the nonstrange sea goes down by somewhat 15%, while the total $d$-quark distribution goes up by some 2%, cf. Fig. 12. In turn, this improvement allows for a better accuracy of both, the sea and the valence distributions, in particular, of the $d$-quark one. This improvement is particularly valuable since the accuracy of the latter is limited in the case of DIS data due to the uncertainty in the nuclear correction employed to describe the deuterium-target data. The LHCb data on $W^+$ and $W^-$ production [15] provide the biggest impact on the PDFs as compared to other LHC data, cf. Fig. 13, due to the forward kinematics probed in this experiment. It is also worth noting that the gluon distribution is also sensitive to the existing LHC DY data and in the ABM fit they pull it somewhat up (down) at small (large) $x$. However, in general, the changes are within the PDF uncertainties. This justifies our approach of using the set of PDF uncertainties to precalculate the NNLO DY cross section grid and then to compute those cross sections by grid interpolation in minimal time. To provide the best accuracy of this algorithm the ABM12 PDFs are produced taking the DY cross section grid calculated for the PDFs obtained in the variant of ABM12, which differs from the nominal ABM12 one by inclusion of the LHC data only. Furthermore, to check explicitly the stability of the algorithm we perform a second iteration of the fit based on the DY cross section grids

![Figure 12](color online) The same as in Fig. 11 for the 1σ band obtained in the variant of the ABM12 fit without the LHC DY data included (shaded area) and the relative change in the ABM12 PDFs due to the LHC DY data obtained with one (solid line) and two (dashes) iterations of the fast algorithm used to take into account the DY NNLO corrections. The dotted lines display 1σ band for the ABM12 PDFs obtained with one iteration of the algorithm.
prepared with the PDFs obtained in the first iteration. The iterations demonstrate nice convergence and the first iteration suffices to obtain an accurate result, cf. Fig. 12.

The NNLO PDFs obtained in this analysis are compared to the results of other groups in Fig. 14. Our PDFs are in reasonable agreement with the newly released CT10 PDFs [61]. The most striking difference is observed for the large-$x$ gluon distribution, which is constrained by the Tevatron jet data in the CT10 analysis. It is worth noting that this constraint is obtained for CT10 using the NLO corrections only, while the NNLO corrections may be as big as 15%–25% [62]. Therefore, the discrepancy between CT10 and our result should decrease once the NNLO corrections to the jet production are taken into account. Comparison of the ABM12 PDFs with the ones obtained by other groups demonstrate the trend similar to the ABM11 case [6]. The most essential difference appears in the large-$x$ gluon distribution. It is also constrained by the Tevatron jet data for MSTW08 [59] and NN23 [60], with the NNLO corrections due to the threshold resummation taken into account in this case. However, the threshold resummation terms used in Refs. [59,60] introduce additional theoretical uncertainties [63]. Therefore, a conclusive comparison with our results is still impractical. The spread in the small-$x$ gluon distribution obtained by different groups can be consolidated with the help of the H1 data on the structure function $F_L$ [64] being sensitive in this region. Similarly, differences in the estimates of the nonstrange sea distribution at $x \sim 0.2$ can be eliminated using the LHC DY data considered in our analysis. At the same time the observed spread in the results for the strange sea shape cannot be explained by a particular data selection or difference in the theoretical accuracy of the analyses since all the groups use the CCFR and NuTeV data on the neutrino-induced dimuon production [65] as a strange sea constraint and take into account the NLO corrections to this process [66,67]. The very recent precise data on the neutrino-induced dimuon production by NOMAD [68] are still not included in the present analysis. However, they demonstrate good agreement with the ABM11 prediction and may help to consolidate different estimates of the shape of the strange sea.

**B. The strong coupling constant and the charm quark mass**

The strong coupling constant $\alpha_s(M_Z)$ is measured together with the parameters of the PDFs, the heavy-quark
mass $m_c$ and the higher twist parameters within the analysis. The present accuracies of the scaling violations of the deep-inelastic world data make the use of NNLO QCD corrections mandatory. At NLO the scale uncertainties typically amount to $O(5\%)$, cf. [69], and, therefore, are simply too large.

The value of $\alpha_s(M_Z)$ obtained in the present analysis is

$$\alpha_s^{\text{NNLO}}(M_Z) = 0.1132 \pm 0.0011. \quad (3.1)$$

This result is in excellent agreement with those given by other groups and by us in Refs. [5, 6, 58, 70–72], see Table II. As has been shown in [6] in detail the $\alpha_s$ values obtained upon analyzing the partial data sets from BCDMS [73, 74], NMC [75, 76], SLAC [77–82], HERA [13], and the Drell-Yan data [83, 84] both at NLO and NNLO compare very well to each other and to the central value within the experimental errors.

Fits including jet data have been carried out before both by JR [85] and ABM [6, 86], along with other groups, performing systematic studies including both jet data from the Tevatron and in [6] also from LHC.\textsuperscript{3} We would like to note that it is very problematic to call present NNLO fits of the world DIS data including jet-data NNLO analyses, since the corresponding jet scattering cross sections are available at NLO only. The complete NNLO results for the corresponding jet cross sections have to be used in later analyses, since threshold resummations are not expected to deliver a sufficient description [63].\textsuperscript{4}

The $\alpha_s(M_Z)$ values for some PDF groups are illustrated in Fig. 15. In Table II a general overview on the values of $\alpha_s(M_Z)$ at NNLO is given, with a few determinations effectively at NNNLO in the valence analyses [70, 71], and the hadronic $Z$ decay [90]. The BBG, BB, GRS, ABKM, JR, ABM11, CTEQ analyses and the present analysis find lower values of $\alpha_s(M_Z)$ with errors at the 1%–2% level, while NN21 and MSTW08 find larger values analyzing the deep-inelastic world data.

\textsuperscript{3}Contrary statements given in Refs. [87, 88] are incorrect; see Ref. [6] for further details.

\textsuperscript{4}Partial NNLO results on the hadronic dijet cross section are available [62].

FIG. 14 (color online). The $1\sigma$ band for the 4-flavor NNLO ABM12 PDFs at the scale of $\mu = 2$ GeV versus $x$ obtained in this analysis (shaded area) compared with the ones obtained by other groups (solid lines: JR09 [58], dashed dots: MSTW [59], dashes: NN23 [60], dots: CT10 [61]).
**Alekhin-Blumlein-Moch Parton Distributions …**

![FIG. 15 (color online). The values of \( \alpha_s(M_Z) \) at NNLO obtained in the PDF fits of ABM (solid bars: this analysis, dashed bars: ABM11 [6]) in comparison with the CT [61], JR [58], MSTW [59] and NNPDF [89] results.](image)

**TABLE II.** Summary of recent NNLO and NNNLO QCD analyses of the DIS world data, supplemented by related measurements using a series of other processes and lattice determinations. In case that jet data from hadron colliders are used in the analysis the values of \( \alpha_s(M_Z) \) cannot be considered NNLO values.

| \( \alpha_s(M_Z) \) | \( \text{BBG} \) | \( \text{BB} \) | \( \text{GRS} \) | \( \text{ABKM} \) | \( \text{ABKM} \) | \( \text{JR} \) | \( \text{JR} \) | \( \text{ABM11} \) | \( \text{ABM12} \) | \( \text{MSTW} \) | \( \text{MSTW} \) | \( \text{NN21} \) | \( \text{CTEQ} \) | \( \text{CTEQ} \) | \( \text{BBG} \) | \( \text{Z decay} \) | \( \text{\tau decay} \) | \( \text{\tau decay} \) | \( \text{\tau decay} \) | \( \text{Lattice} \) | \( \text{Lattice} \) | \( \text{Lattice} \) | \( \text{Lattice} \) | \( \text{Lattice} \) | \( \text{Lattice} \) | \( \text{World} \) | \( \text{average} \) |
|------------------|-----------------|-----------|----------|----------|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|                  | 0.1134^{+0.0019}_{-0.0021} | 0.1132 \pm 0.0022 | 0.112 | 0.1135 \pm 0.0014 | 0.1129 \pm 0.0014 | 0.1128 \pm 0.0010 | 0.1140 \pm 0.0006 | 0.1134 \pm 0.0011 | 0.1132 \pm 0.0011 | 0.1171 \pm 0.0014 | 0.1155 \pm 0.1175 | 0.1173 \pm 0.0007 | 0.1159...1162 | 0.1140 | 0.1141^{+0.0020}_{-0.0022} | 0.1137 \pm 0.0022 | 0.1140 \pm 0.0015 | 0.1131^{+0.0028}_{-0.0022} | 0.1175 \pm 0.0025 | 0.1189 \pm 0.0026 | 0.1212 \pm 0.0019 | 0.1204 \pm 0.0016 | 0.1191 \pm 0.0022 | 0.1205 \pm 0.0010 | 0.1184 \pm 0.0006 | 0.1200 \pm 0.0014 | 0.1156 \pm 0.0022 | 0.1181 \pm 0.0014 | 0.1184 \pm 0.0007 |

The value of \( \alpha_s(M_Z) \) has also been determined in different lattice simulations to high accuracy. The NNLO values for \( \alpha_s(M_Z) \) in the valence analyses [70,71] yield slightly larger values than at NNLO. They are fully consistent with the NNLO values within errors. The corresponding shift can be taken as a measure for the remaining theoretical uncertainty in the nonsinglet case, see Table II.

Finally we would like to comment on recent determinations of \( \alpha_s(M_Z) \) at NLO using the jet data [93,94]. The ATLAS and CMS jet data span a wider kinematic range than those of Tevatron and will allow very soon even more accurate measurements. In the analysis [94] \( \alpha_s(M_Z) \) is determined scanning grids generated at different values of the strong coupling constants by the different PDF-fitting groups. These are used to find a minimum for the jet data. Including the scale uncertainties the following NLO values are obtained for the 3/2 jet ratio by CMS [94]:

\[ \alpha_s(M_Z) = 0.1134^{+0.0019}_{-0.0021}, \]

\[ \alpha_s(M_Z) = 0.1132 \pm 0.0022, \]

\[ \alpha_s(M_Z) = 0.1112, \]

\[ \alpha_s(M_Z) = 0.1135 \pm 0.0014, \]

\[ \alpha_s(M_Z) = 0.1129 \pm 0.0014, \]

\[ \alpha_s(M_Z) = 0.1128 \pm 0.0010, \]

\[ \alpha_s(M_Z) = 0.1140 \pm 0.0006, \]

\[ \text{ABM11} \]

\[ \text{ABM12} \]

\[ \text{MSTW} \]

\[ \text{MSTW} \]

\[ \text{NN21} \]

\[ \text{CTEQ} \]

\[ \text{CTEQ} \]

\[ \text{BB} \]

\[ \text{BBG} \]

\[ \text{Z decay} \]

\[ \text{\tau decay} \]

\[ \text{\tau decay} \]

\[ \text{\tau decay} \]

\[ \text{Lattice} \]

\[ \text{Lattice} \]

\[ \text{Lattice} \]

\[ \text{Lattice} \]

\[ \text{Lattice} \]

\[ \text{Lattice} \]

\[ \text{World} \]

\[ \text{average} \]

Ve

5Very recently MSTW [91] reported lower values for \( \alpha_v(M_Z) \) also related to the LHC data.
\[ \alpha_s(M_Z) = 0.1148 \pm 0.0014(\text{exp.}) \pm 0.0018(\text{PDF})^{+0.0050}_{-0.0055} \text{ (scale)} \text{NNPDF21}, \]  
(3.2)

\[ \alpha_s(M_Z) = 0.1135 \pm 0.0018(\text{exp.})[0.180(\text{favored value})] \text{ CT10}, \]  
(3.3)

\[ \alpha_s(M_Z) = 0.1141 \pm 0.0022(\text{exp.})[0.1202^{+0.0012}_{-0.0015}] \text{ MSTW08}, \]  
(3.4)

based on the NNPDF21 [95], the CT10 [61] and the MSTW08 [96] PDF sets, respectively.

A comparable NLO value has been reported using ATLAS jet data [93]

\[ \alpha_s(M_Z) = 0.1151 \pm 0.0050(\text{exp.})^{+0.0080}_{-0.0073} \text{ (th.)}. \]  
(3.5)

Interestingly, rather low values are obtained already at NLO. In parenthesis we quote the NLO values for \( \alpha_s(M_Z) \) in Eqs. (3.2)–(3.4) which are obtained by the fitting groups at minimal \( \chi^2 \). Obviously the values found in the jet-data analysis do not correspond to these values. Yet, the NLO scale uncertainty in this analysis is large. Recently the jet energy-scale error has been improved by CMS [94], leading to a significant reduction of the experimental error. The gluonic NNLO \( K \) factor is positive; as shown in Fig. 2 of Ref. [62] the scale dependence for \( \mu = \mu_F = \mu_R \) behaves flat over a wide range of scales. It is therefore expected that also the error due to scale variation will turn out to be very small in the NNLO analysis. It will be important to repeat this analysis and to fit the LHC jet data together with the world deep-inelastic data, which will be also instrumental for the determination of the gluon distribution at large scales.

The present DIS world data together with the \( F_2^Z(x, Q^2) \) data, are competitive in the determination of the charm quark mass in a correlated fit with the PDF parameters and \( \alpha_s(M_Z) \). For the \( M_S \) mass the value of

\[ m_c(m_c) = 1.24 \pm 0.03(\text{exp.})^{+0.00}_{-0.07} \text{ (th.) GeV} \]  
(3.6)

is obtained at NNLO, see also [33]. At present this analysis is the only one, in which all known higher order heavy-flavor corrections to deep-inelastic scattering have been considered. This value still should be quoted as of approximate NNLO, since the NNLO corrections are only modeled [26] combining small-\( x \) and threshold resummation effects with information of the 3-loop moments of the heavy-flavor Wilson coefficients [31] at high values of \( Q^2 \). Two scenarios have been considered in [26] to parametrize the Wilson coefficients accounting for an estimated error. Here the fit favors a region of the parameter \( d_N \in [-0.1, 0.5] \), cf. [26], on which the theoretical error is based.\(^6\) The value in Eq. (3.6) compares well to the present world average of \( m_c(m_c) = 1.275 \pm 0.025 \text{ GeV} \) [1].

It is needless to say that the determination of a fundamental parameter of the SM, such as \( m_c \), has to follow a thorough quantum field-theoretic prescription, see Refs. [31,113] for details.

C. Standard candle cross sections

In this section we quantify the impact of the new PDF set on the predictions for benchmark cross sections at the LHC for various c.m.s. energies. To that end, we confine ourselves to (mostly) inclusive cross sections which are known to NNLO in QCD, see [6,7] for previous benchmark numbers, since the NNLO accuracy is actually the first instance, where meaningful statements about the residual theoretical uncertainty are possible given the precision of present collider data and the generally large residual variation of the renormalization and factorization scale at NLO.

In detail, we consider the following set of inclusive observables at NNLO in QCD: hadronic \( W \)- and \( Z \)-boson production [114,115], the cross section for Higgs boson production in gluon-gluon fusion [115–119], and the cross section for top-quark pair production [43–47]. We have used the LHAPDF library [55,56] for the cross section computations to interface to our PDFs provided in the form of data grids for \( n_f = 3, 4 \) and 5 flavors accessible with the LHAPDF library,\(^7\)

\[ \text{abml2hc}_3\text{nnlo.LHgrid(0 + 28)}, \]  
\[ \text{abml2hc}_4\text{nnlo.LHgrid(0 + 28)}, \]  
\[ \text{abml2hc}_5\text{nnlo.LHgrid(0 + 28)}, \]

which contains the central fit and 28 additional sets for the combined symmetric uncertainty on the PDFs, on \( \alpha_s \) and on the heavy-quark masses. All PDF uncertainties quoted here are calculated in the standard manners, i.e., as the \( \pm 1\sigma \) variation.

\(^6\)The calculation of the exact NNLO heavy-flavor Wilson coefficients is underway [32,110–112].

\(^7\)The LHAPDF library can be obtained from http://projects.hepforge.org/lhapdf together with installation instructions.
TABLE III. The total cross sections [pb] for gauge boson production at the LHC with $\sqrt{s} = 7$ TeV for the $n_f = 5$ flavor PDF sets ABM11 and ABM12 at NNLO accuracy. The errors shown are the scale uncertainty based on the shifts $\mu = M_{W/Z}/2$ and $\mu = 2M_{W/Z}$ and, respectively, the 1σ PDF uncertainty.

| LHC7 | $W^+$ | $W^-$ | $W^\mp$ | $Z$ |
|------|-------|-------|---------|-----|
| ABM11 | $59.53^{+0.33}_{-0.32}$ | $39.97^{+0.26}_{-0.29}$ | $99.51^{+0.60}_{-0.41}$ | $29.23^{+0.18}_{-0.19}$ |
| ABM12 | $58.40^{+0.34}_{-0.32}$ | $39.63^{+0.26}_{-0.28}$ | $98.03^{+0.67}_{-0.41}$ | $28.79^{+0.17}_{-0.19}$ |

TABLE IV. The same as Table III for the LHC with $\sqrt{s} = 8$ TeV.

| LHC8 | $W^+$ | $W^-$ | $W^\mp$ | $Z$ |
|------|-------|-------|---------|-----|
| ABM11 | $68.30^{+0.48}_{-0.29}$ | $46.27^{+0.35}_{-0.33}$ | $114.97^{+0.82}_{-0.51}$ | $33.97^{+0.23}_{-0.20}$ |
| ABM12 | $67.03^{+0.30}_{-0.29}$ | $46.27^{+0.35}_{-0.33}$ | $113.29^{+0.84}_{-0.52}$ | $33.49^{+0.22}_{-0.20}$ |

TABLE V. The same as Table III for the LHC with $\sqrt{s} = 13$ TeV.

| LHC13 | $W^+$ | $W^-$ | $W^\mp$ | $Z$ |
|-------|-------|-------|---------|-----|
| ABM11 | $110.77^{+0.97}_{-0.61}$ | $80.02^{+0.72}_{-0.47}$ | $190.79^{+1.68}_{-1.09}$ | $57.62^{+0.48}_{-0.29}$ |
| ABM12 | $108.86^{+0.97}_{-0.61}$ | $79.33^{+0.73}_{-0.48}$ | $188.19^{+1.68}_{-1.09}$ | $56.88^{+0.48}_{-0.29}$ |

TABLE VI. The same as Table III for the LHC with $\sqrt{s} = 14$ TeV.

| LHC14 | $W^+$ | $W^-$ | $W^\mp$ | $Z$ |
|-------|-------|-------|---------|-----|
| ABM11 | $119.03^{+1.07}_{-0.68}$ | $86.63^{+0.80}_{-0.53}$ | $205.66^{+1.87}_{-1.20}$ | $62.31^{+0.53}_{-0.32}$ |
| ABM12 | $116.99^{+1.07}_{-0.69}$ | $85.89^{+0.80}_{-0.53}$ | $202.88^{+1.87}_{-1.22}$ | $61.52^{+0.53}_{-0.33}$ |

1. W- and Z-boson production

We start by presenting results for W- and Z-boson production at the LHC. For the electroweak parameters, we follow [6,7] and choose the scheme based on the set ($G_F, M_W, M_Z$). According to [1], we have $G_F = 1.16638 \times 10^{-5}$ GeV$^{-2}$, $M_W = 80.385 \pm 0.015$ GeV, $M_Z = 91.1876 \pm 0.0021$ GeV and the corresponding widths $\Gamma(W^\pm) = 2.085 \pm 0.042$ GeV and $\Gamma(Z) = 2.4952 \pm 0.0023$ GeV. The weak mixing angle is then a dependent quantity, with

$$\sin^2 \theta_w = 1 - \frac{M_W^2}{\hat{\rho} M_Z^2} = 0.23098 \pm 0.00041,$$  (3.7)

and $\hat{\rho} = 1.01051 \pm 0.00011$. The Cabibbo angle $\theta_c$ yields the value of $\sin^2 \theta_c = 0.05085$.

The change in the predictions between ABM11 and ABM12 is small and for the current theoretical accuracy, the uncertainty due to the scale variation is already significantly smaller compared to the PDF error, see Tables III–VI. This indicates the very good stability of the PDF fit and the consistency of the previous ABM11 PDFs with the new variant including LHC data. An additional source of theoretical uncertainty for W- and Z-boson production, namely the choice of PDF sets with $n_f = 4$ or with $n_f = 5$ flavors (as in Tables III–VI) has already been discussed and quantified in [6]. Generally, those differences are less than 1σ in the PDF uncertainty and become successively smaller as perturbative corrections of higher order are included.

2. Higgs boson production

Let us now discuss the cross section for the SM Higgs boson production in the gluon-gluon fusion channel, which is predominantly driven by the gluon PDF and the value of $\alpha_s(M_Z)$ from the effective vertex. The known NNLO QCD corrections [115–119] still lead to a sizable increase in the cross section at nominal values of the scale, i.e. $\mu = m_H$, and it is well established that a further stabilization beyond NNLO may be achieved on the basis of soft gluon resummation, see e.g., [121]. At NNLO accuracy in QCD the theoretical uncertainty from the scale variation is dominating by far over the PDF uncertainty. Using a Higgs boson mass $m_H = 125$ GeV in Table VII we observe again only rather small changes between the ABM11 and the ABM12 predictions. This demonstrates that the gluon PDF is well constrained from existing data and that the ABM11 results are consistent with the new fit based on including selected LHC ones.
It is therefore interesting to compare the ABM predictions in Table VII to the cross section values recommended for use in the ongoing ATLAS and CMS Higgs analyses [120], cf., Table VIII. The central values of the ABM predictions are significantly lower by some 11%–14%. Only a small fraction of this difference can be attributed to the inclusion of soft gluon resummation beyond NNLO, which typically does reduce the scale dependence, though, as is obvious from Table VIII, and to the inclusion of other quantum corrections in [120], e.g., the electroweak ones. Much larger sensitivity of the Higgs cross section predictions arise from theory assumptions made in the analyses, e.g., for constraints from higher orders in QCD due to the treatment of fixed-target DIS data, see [92]. The most interesting aspect is the fact, that the PDFs + $\alpha_s$ error in [120] is inflated roughly by a factor of 4 in comparison to our predictions in Table VII, where we quote the 1σ PDF (and $\alpha_s$ of course) error entirely determined from the correlated experimental uncertainties in the fitted data.

8See also https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageAt7TeV for details.

In summary, the cross section predictions [120] used in the current Higgs analyses at the LHC are based on specific theory assumptions made in PDF and $\alpha_s$ fits supplied by the largely overestimated uncertainties due to the nonperturbative input. This issue can be consolidated in the future by checking correlations between experimental data for different scattering processes at the LHC and their sensitivity to PDFs along the lines of Sec. II (cf. also Ref. [122]).

3. Top-quark pair production

Finally, we present predictions for the total cross section for $t\bar{t}$-pair hadroproduction in Tables IX and X. Using the program Hathor (version 1.5) [48] which incorporates the recently completed QCD corrections at NNLO [43–46], we give numbers for two representative top-quark masses, that is the running mass $m_t(m_t) = 162$ GeV in the $\overline{\text{MS}}$ scheme and the pole mass $m_t(pole) = 171$ GeV in the on-shell scheme.

At NNLO accuracy in QCD, the PDF uncertainties given in Tables IX and X are dominating in comparison to the theory uncertainties based on scale variation. As discussed
at length in Sec. II D the LHC data for $t\bar{t}$-pair production included in the ABM12 fit predominantly constrains the top-quark mass and has little impact on the gluon PDF and on the value of the strong coupling constant $\alpha_s(M_Z)$. Therefore the cross section predictions of the ABM11 and ABM12 PDFs largely coincide.

IV. CONCLUSIONS

We have presented the PDF set ABM12, which results from a global analysis of DIS and hadron collider data including, for the first time, the available LHC data for the standard candle processes such as $W^\pm$ and $Z$-boson and $t\bar{t}$ production. The analysis has been performed at NNLO in QCD and along with the new data included also progress in theoretical predictions has been reflected accordingly. The new ABM12 analysis demonstrates very good consistency with the previous PDF sets (ABM11, ABKM09) regarding the parameter values for PDFs as well as the strong coupling constant $\alpha_s(M_Z)$ and the quark masses. Continuous checks for the compatibility of the data sets along with a detailed account of the systematic errors and of the correlations among the fit parameters have been of paramount importance in this respect.

In detail, we have considered new HERA data sets on semi-inclusive charm production in DIS in Sec. II A which have allowed us to validate the $c$-quark production mechanism in the FFN scheme relying on 3 light flavors in the initial state and leading to a precise determination of the running $c$-quark mass. As another new DIS data set, the neutral-current inclusive data at high $Q^2$ from HERA has been included, which exhibits sensitivity to the exchange of photons, $Z$-bosons as well as to $\gamma-Z$ interference. Our analysis in Sec. II B corroborates again the fact, that even at high scales the FFN scheme is sufficient for description of the DIS data.

The fit of LHC precision data on $W^\pm$- and $Z$-boson production improves the determination on the quark distributions at $x \sim 0.1$ and constrains especially the $d$-quark distribution. The fit shows good consistency and a further reduction of the experimental systematic uncertainties would certainly strengthen the impact of the LHC DY data in global fits. On the technical side, we remark that the fit of DY data has been based on the exact NNLO differential cross section predictions, expanded over the set of eigenfunctions spanning the basis for the ABM PDF uncertainties. This has served as a starting point for a rapidly converging fit including the LHC DY data with account of all correlations.

Also data for the total $t\bar{t}$-cross section can be smoothly accommodated into the fit, although they are still not used in tuning the ABM12 PDFs published. A proper treatment of the correlation between the gluon PDF, the strong coupling constant $\alpha_s(M_Z)$ and the top-quark mass has been crucial here. Moreover, the running-mass definition for the top-quark provides a better description of data as compared to the pole-mass case, the latter showing still sizable sensitivity to perturbative QCD corrections beyond NNLO accuracy. Our analysis in Sec. II D yields a precise value with an uncertainty of roughly 1.5% for the MS mass $m_t(m_t)$ which has been used to extract $m_t(pole)$ at NNLO.

In summary, the new ABM12 fit demonstrates, that a smooth extension of the ABM global PDF analysis to incorporate LHC data is feasible and does not lead to large changes in the fit results. As we have shown in Sec. III A differences with respect to other PDFs sets remain. However, these differences are based either on a different treatment of the data sets or on different theoretical descriptions of the underlying physical processes and we have commented on the correctness of some of those procedures. In particular, the value of strong coupling constant $\alpha_s(M_Z)$ in our analysis remains largely unchanged as documented in Sec. III B and the theoretical predictions for benchmark cross section at the LHC are very stable. This particularly applies to the cross section for Higgs production in the gluon-gluon fusion shown in Sec. III C. We commented on the implications for the ongoing Higgs analyses at the LHC.

The precision of the currently available experimental data make global analyses at NNLO accuracy in QCD mandatory. This offers the great opportunity for high precision determinations of the nonperturbative parameters relevant in theory predictions of hadron collider cross section. At the same time, the great sensitivity to the underlying theory allows us to test and to scrutinize remaining model prescriptions and, eventually, to reject wrong assumptions.

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Note added.—Recently, a new combination of measurements of the top-quark pair production cross section from the Tevatron appeared [123], which carries a combined experimental uncertainty of 5.4%. This measurement yields $\sigma_{pp\rightarrow t\bar{t}} = 7.82 \pm 0.42$ pb for a value of $m_t(pole) = 171$ GeV for the top-quark pole mass, which is consistent with the NNLO cross section prediction $\sigma_{pp\rightarrow t\bar{t}} = 7.17^{+0.22}_{-0.31} \pm 0.16$ pb based on the ABM12 PDFs at NNLO.
within the uncertainties. We also have been aware of the change in the ATLAS data luminosity monitor that led to an upwards shift by 1.9% in all samples of the 2010 data including the Drell-Yan ATLAS data [14] used in our analysis [126]. Once the data of Ref. [14] go somewhat lower than our fit, cf. Fig. 3, this improves the agreement of the ATLAS Drell-Yan sample with the ABM12 PDFs. As a result the ATLAS value of $\chi^2$ in Table I decreases by one unit. Hence, the impact of this improvement on the ABM12 PDFs is marginal.

APPENDIX: A FAST ALGORITHM FOR INVOLVED COMPUTATIONS IN PDF FITS

The accommodation of the different data sets for the PDF fit demands very involved computations of the QCD corrections to the Wilson coefficients. In particular this applies to the calculation of the rapidity distribution of the $W^\pm$ and $Z$-boson decay products produced in hadronic collisions, which are based on the fully exclusive NNLO codes DYNNLO [41] and FEWZ [42]. The typical CPU runtime needed to achieve a calculation accuracy of much better than the uncertainty of the present data using the codes [41,42] amounts to $O(100)$ hours. Therefore an iterative use of the available fully exclusive DY codes in the QCD fit is widely impossible. Instead, these codes are commonly run in advance for the variety of PDF sets, covering the foreseeable spread in the PDF variations, the results of which are stored in grids. Afterwards the cross section values for a given PDF set can be computed in a fast manner using linear grid interpolations. For the first time this approach was implemented in the code fastNLO [124] for the NLO corrections to the jet productions cross sections. A similar approach is also used in the code AppleGrid [125] which provides a tool for generating the cross section grids of different processes, including the DY process. Since fastNLO and AppleGrid are tools of general purpose, the PDF basis used to generate those grids needs to be sufficiently wide to cover the differences between the existing PDF sets. Meanwhile the possible variations of the PDFs in a particular fit are not very large, i.e. if a new fit is aimed to accommodate a new data set being in sufficient agreement with those used in earlier versions of the fit, one may expect variations of the PDFs being comparable to their uncertainties. In this case the PDF basis used to generate the grids for the cross section can be reliably selected as a PDF bunch, which encodes the uncertainties in a given PDF set. For the PDF uncertainties estimated with the Hessian method this bunch is provided by the PDF set members corresponding to the $1\sigma$ variation in the fitted parameters. This allows us to minimize the size of the precalculated cross section grids and reduces the CPU time necessary to generate these grids correspondingly. Moreover, the structure of the calculation algorithm in using these grids for the PDF fit turns out to be simple. In this appendix we describe how this approach is implemented in the present analysis.

Firstly, we remind the basics of the PDF uncertainty handling, see Ref. [6] for details. Let $\vec{q}(P_i)$ be the vector of parton distributions encoding the gluon and quark species. It depends on the PDF parameters $P_i$ with the index $i \in [1,N_p]$ and $N_p$ the number of parameters. $P_i^0$ denote the parameter values obtained in the PDF fit and $\Delta P_i$ are their standard deviations. In general the errors in the parameters are correlated, which is expressed by a nondiagonal covariance matrix $C_{ij}$. However, it is diagonal in the basis of the covariance matrix eigenvectors which makes this basis particular convenient for the computation of the PDF error. The vector of the parameters $P_i$ transformed into the eigenvector basis reads

$$\vec{P}_i = \sum_{k=1}^{N_p} (\sqrt{C})_{ik}^{-1} P_k,$$

where

$$\sqrt{C}_{ij} = \sum_{k=1}^{N_p} A_{ik} \sqrt{D_{kj}}.$$

Here $A_{ik}$ denotes the matrix with the columns given by the orthonormal eigenvectors of $C_{ij}$. $\sqrt{D_{kj}}$ are the eigenvalues of $C_{ij}$ and $\delta_{jk}$ is the Kronecker symbol. The PDF uncertainties are commonly presented as the shifts in $\vec{q}$ due to variation of the parameters $\vec{P}_i$ by their standard deviation. Since the latter are equal to one the shifts are given by

$$\frac{d\vec{q}}{d\vec{P}_i} = \sum_{k=1}^{N_p} \frac{d\vec{q}}{dP_k} (\sqrt{C})_{ik}.$$  

Moreover, the parameters $\vec{P}_i$ are uncorrelated. Therefore the shifts in Eq. (A3) can be combined in quadrature to obtain the total PDF uncertainty. In a similar way the uncertainty in a theoretical prediction $t(\vec{q})$ due to the PDFs can be obtained assuming its linear dependence on the PDFs as a combination of the variations

$$\Delta t^{(k)} = t\left[\vec{q}(P_i^0) + \frac{d\vec{q}}{d\vec{P}_i} \right] - t[\vec{q}(P_i^0)].$$

in quadrature.

Now we show how new data on the hadronic hard-scattering process can be consistently accommodated into the PDF fit avoiding involved cross section computations. Let $P_i^{\text{fit}}$ be the current values of the PDF parameters in the fit with the new data set included and $\Delta P_i = P_i^{\text{fit}} - P_i^0$, where $P_i^0$ stands for the PDF parameter values obtained in the earlier version of the fit performed without the new data set. The current PDF value can be expressed in terms of $\Delta P_i$ and the PDF variation in the eigenvector basis as follows:

$$\vec{q}^{\text{fit}} = \vec{q}(P_i^0) + \frac{d\vec{q}}{d\vec{P}_i} \Delta \vec{P}_i.$$
where

\[ \delta \tilde{P}_i = \sum_{k=1}^{N_p} (\sqrt{C})_{ik}^{-1} \delta P_k. \]  

(A6)

A shift in the hard-scattering cross section corresponding to the variation of the \( A \) th PDF parameter in the fit reads

\[ \delta t^{(k)} = t \left[ \tilde{q}(P^0_k) + \frac{d\tilde{q}}{dP_k} \delta P_k \right] - t[\tilde{q}(P^0_k)] \]

\[ \approx \Delta t^{(k)} \sum_{i=1}^{N_p} (\sqrt{C})_{ik}^{-1} \delta P_i \]  

(A7)

and the total change in \( t \) is the sum of terms in Eq. (A7) over all parameters being fitted. The approximation Eq. (A7) allows fast calculations of the cross section for the new data added to the PDF fit since the values of \( \sigma[\tilde{q}(P^0_k)] \) and \( \Delta t_i \) can be prepared in advance. This approach is justified if the variation of the parameters in the new fit is localized within their uncertainties obtained in the previous fit or in case of sufficient linearity of the PDFs with respect to the fitted parameters and the cross sections depending on the PDFs. Furthermore, if the algorithm does not seem to guarantee sufficient accuracy, it can be applied iteratively, with the update of the \( \sigma[\tilde{q}(P^0_k)] \) and \( \Delta t_i \) values at each iteration.
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