Scaling of ac susceptibility and nonlinear response in high-temperature superconductors

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Abstract

The magnetic ac susceptibility of high-temperature superconductors is shown to obey some scaling relations. We try to analyse this behavior within the framework of a common nonlinear response function of mixed state. The derived equations for critical current and ac susceptibility ($\chi(T)$) agree with the scaling relations of experimental data.

74.60.Ge, 72.15.Gd, 74.60.Ec
One of the most popular means of investigating vortex dynamics in the high-temperature superconductors (HTS) is the measurement of the response of the vortex system to ac fields [1,2]. Most experiments with high-$T_c$ superconductors (HTS) deal with thin flat sample in a perpendicular magnetic field. The application of a time-dependent field $H(t) = H_0 + h_{ac}e^{-i\omega t}$ to the sample surface results in an electric-field gradient in the sample interior ($h_{ac}$ is the ac-field amplitude and $\omega$ the angular frequency). This gives rise to a shielding current, which in turn exerts a Lorentz force on the vortices in the sample. The measurements of such ac response contain much valuable information about pinning and creep of vortices and turn out to be useful to test models for describing the ac losses which have to be carefully characterized and monitored for many applications.

The controversy over the analysis of the ac response is often noteworthy because of the complicate interplay of hysteretic and eddy-current losses [1,2]. Critical state models like Bean model and its modifications are often used to analyze the results where the hysteretic losses dominate. However some problems with the amplitude dependence of ac responses still remain unsolved. Several common features of the amplitude dependence of the in-phase and out-of-phase susceptibilities have been observed for different kinds of materials [1–6]: a) a parallel shift of the in-phase susceptibility $\chi - T$ curve with increasing $h_{ac}$ toward lower temperatures is observed; b) the onset of diamagnetism and dissipation does not appear to depend on $h_{ac}$ values; c) the out-of-phase peak shifts to lower temperatures with increasing $h_{ac}$ and broadens in the low temperature side; d) the absorption peak increases slightly when $h_{ac}$ increases. In Fig.1 we show the data of high $J_c$ YBCO bulk material [5] for an example.

To describe the behavior of the vortex system on a macroscopic scale, general methods are the Maxwell equations combined with the materials equation of superconductors $J(E, B, T)$. In present work we try to analyze the low-frequency ac susceptibility within the framework of a common nonlinear response function generally valid for all type-II superconductors.
In next section we show some widely observed scaling relations of ac susceptibility experimental data. In section III we introduce the common nonlinear response function of mixed state. The equations of critical current and the ac susceptibility are derived in section IV. A discussion of the connection between the observed scaling relations and the nonlinear response function is given in section V.

II. SCALING BEHAVIOR OF SUSCEPTIBILITY

It is interesting to note that the different measured $\chi''(T)$ curves in Fig.1 can be represented by a single curve in Fig.2 when $\chi''(T) - \chi''(T^*)$ and $T - T_p$ are scaled by $\chi_{peak}'' - \chi''(T^*)$ and $T^* - T_p$ respectively, with $T^*$ the irreversibility point and $\chi''(T_p) \equiv \chi_{peak}''$ the peak value of the out-of-phase susceptibility as

$$\frac{\chi''(T) - \chi''(T^*)}{\chi_{peak}'' - \chi''(T^*)} = f_1\left(\frac{T - T_p}{T^* - T_p}\right)$$ (1)

A rather more amazing fact is that the experimental data of susceptibility from different references [2, 7, 8] at different frequencies up to $26 MHz$ can also be superimposed with the empirical scaling relation

$$\frac{\chi''(T) - \theta(T - T_p)\chi''(T^*)}{\chi_{peak}'' - \theta(T - T_p)\chi''(T^*)} = f_2\left(\frac{T - T_p}{T^* - T_p}\right)$$ (2)

as shown in Fig.3.

The observed $\chi''$ peak position $T_p$ can also be approximately described by an empirical relation

$$[T_p(h_{ac}) - T^*]^\alpha \propto h_{ac}$$ (3)

as illustrated by the inset of Fig.1, where $T^*$ is the limit of $T_p$ as $h_{ac} \to 0$. This power law shift of $T_p$ to lower values by increasing amplitude $h_{ac}$ can be found for various kinds of materials. In Fig.4, we summarized some experimental $h_{ac} \sim T_p$ data from different references in literature.
III. NONLINEAR RESPONSE

The equations that describe the behavior of superconductor on a macroscopic scale are the Maxwell equations combined with the materials equation of superconductor \( J(E, B, T) \). Various models in literature suggested different specific forms of the materials equation. In the case of ideal type-II superconductors with negligible flux pinning, the material can be characterized by the linear equation

\[
E = \rho_f(B, T)J
\]  

(4)

with \( \rho_f \approx \rho_n B/B_c^2 \), the flux-flow resistivity as estimated by Bardeen and Stephen [7]. On the other hand, in nonideal type II superconductors with considerable pinning, the material is described by a set of equations \( E = B \times v, v = v_0 \exp[-U(J)/kT] \) or

\[
E(J) = J\rho_f e^{-U(J,B,T)/kT}
\]  

(5)

The activation barrier \( U \) depends on \( J \) as well as additionally depends on the temperature \( T \) and magnetic field \( B \). Different types of \( U(J) \) have been suggested to approximate the real barrier, for instance, the Anderson-Kim model [8] with \( U(J) = U_c(1-J/J_{co}) \), the logarithmic barrier \( U(J) = U_c \ln(J_{co}/J) \) [9] and the inverse power-law with \( U(J) = U_c[(J_{co}/J)^m - 1] \). [10–12]

We find, if one makes a common modification to the different model barriers \( U(J) \) as

\[
U(J) \rightarrow U(J_p \equiv J - E/\rho_f)
\]  

(6)

then the corresponding modified materials equation

\[
E(J) = J\rho_f e^{-U(J_p)/kT}
\]  

(7)

leads to a common normalized form as

\[
y = x \exp[-\gamma(1 + y - x)^p]
\]  

(8)
with \( x \) and \( y \) the normalized current density and electric field respectively. \( \gamma \) is a parameter characterizing the symmetry breaking of the pinned vortices system and \( p \) is an exponent.

To show the connection of the nonlinear response function Eq. (8) with the critical-state model \( U(J) \), we start from the expression widely used for flux creep with the logarithmic barrier [9],

\[
E(J) = \rho_f J \exp\left[-\frac{U_c}{kT} \ln\left(\frac{J_{c0}}{J}\right)\right]
\] (9)

Substituting \( J_p \equiv J - E(J)/\rho_f \) for the current density \( J \) in the bracket on the right-hand side of Eq. (9), we get

\[
E(J) = \rho_f J \exp\left[-\frac{U_c}{kT} \ln\frac{J_{c0}}{J_p}\right]
\] (10)

The definition of barrier implies \( J_{co} \geq J_p \). Using the approximation

\[
\ln \eta = \sum_{n=1}^{\infty} \frac{1}{n} (1 - \eta^{-1})^n \approx a(1 - \eta^{-1})^p, \ (\eta > \frac{1}{2})
\] (11)

finally we find Eq. (10) in the form

\[
\ln\left(\frac{x}{y}\right) = \gamma (1 + y - x)^p
\] (12)

which is the general normalized form of the materials equation Eq. (8). Here we have

\[
\gamma \equiv a \frac{U_c}{kT}, \quad x \equiv \frac{J}{J_{co}}, \quad y \equiv \frac{E(J)}{\rho_f J_{c0}}
\] (13)

In earlier works, this materials equation for type-II superconductors has also been shown in connection with the Anderson-Kim model and the inverse power-law \( U(J) \) [13,14].

The numerical factor \( a \) in the approximation Eq.(11) should be evaluated with considering the limitation of sample size to the realistic barrier \( U(J) \) as discussed in Refs. [12–14]. Considering this limitation as a cut-off of the series in Eq. (11), we have

\[
a = \sum_{n=1}^{N_c} \frac{1}{n} = C + \ln(N_c)
\]

where \( C \) is the Euler constant and \( N_c \) corresponds to the realistic cut-off of the series in Eq. (11). Usually \( a \) is of the order 2-4. Ignoring this limitation, one gets from Eq. (10) an
even simpler expression

\[ E(J) = \rho_f J \left( \frac{J}{J_c^0} \right)^{U_c/kT} \]

or

\[ y/x = (x - y)^\sigma \]  \hspace{1cm} (14)

with \( \sigma = U_c/kT \), though the latter can not be used to interpret the case with small barrier and thermally assisted flux-flow (TAFF). In Fig. 5, we show the numerical solutions of Eq. (8) and Eq. (14) for comparison.

Therefore, the activation barrier \( U(J, B, T) \) in Eq. (5) can be explicitly expressed as

\[ U(J, B, T) = U_c(B, T) F \left[ J/J_c^0(B, T) \right] \]  \hspace{1cm} (15)

Incorporating it into the commonly observed scaling behavior of magnetic hysteresis \( M(H) \) in superconductors, it can be shown that \( U_c(B, T) \) and \( J_{c0}(B, T) \) in Eq. (15) must take the following forms [15]

\[ U_c(B, T) = \Psi(T) B^n \]
\[ J_{c0}(B, T) = \lambda(T) B^n \]  \hspace{1cm} (16)

IV. CRITICAL CURRENT AND SUSCEPTIBILITY EQUATIONS

The nonlinear response function Eq. (8) gives current-voltage characteristic of the form

\[ E(J) = v_0 B \exp \left[ - \frac{U_c(B, T)}{kT} \left( 1 + \frac{E(J)}{\rho_f J_{c0}(B, T)} - \frac{J}{J_{c0}(B, T)} \right)^{\frac{1}{\beta}} \right] \]  \hspace{1cm} (17)

Where \( v_0 \) is a prefactor with dimension of velocity and \( v_0 B \approx \rho_f J \) as discussed in [13].

Defining the critical current density \( J_c \) by a certain criterion of electric field \( E_c \) as \( E(J_c) \equiv E_c \) one finds from it the expression of the critical surface

\[ J_c(B, T) = J_{c0}(B, T) \left[ \left( 1 - \frac{kT}{U_c(B, T)} \ln \left( \frac{v_0 B}{E_c} \right) \right)^{\frac{1}{\beta}} + \frac{E_c}{\rho_f J_{c0}(B, T)} \right] \]  \hspace{1cm} (18)
commonly used for the engineering calculation in applied superconductivity. In the ac susceptibility measurements we have $E_c = \omega h_{ac}$ and the irreversibility temperature $T^*(B)$ is defined by the condition

$$U_c[B, T^*(B)] = kT^*(B) \ln\left(\frac{v_0 B}{E_c}\right)$$  \hspace{1cm} (19)$$

With this condition, the critical surface equation (18) turns into the ohmic relation of flux-flow regime as

$$J_c[B, T^*(B)] = E_c/\rho_f$$  \hspace{1cm} (20)$$

For $T \leq T^*(B)$, critical current density can be expressed as

$$J_c(B, T) = J_{c0}(B, T)\left[1 - \left(\frac{T}{T^* U_c(B, T^*)}\right)^{\frac{1}{p}} + \frac{E_c}{\rho_f J_{c0}(B, T)}\right]$$  \hspace{1cm} (21)$$

In the case where sample size is much smaller than the wavelength $l$ and the ac amplitude $h_{ac} \ll H_0$ one can neglect the variation of local current density within a period and define two parameters

$$L_P \equiv h_{ac}/J_c, \quad r \equiv L_p/a$$  \hspace{1cm} (22)$$

with $a$ the radius of sample,

Substituting Eq. (16) and Eq. (21) to Eq. (22), and considering the amplitude of electric field induced by $h_{ac} E_c = \omega h_{ac}$, we find the field and temperature dependency

$$r = h_{ac}/a J_c = h_{ac} J_{c0}^{-1}(B, T)\left[1 - \left(\frac{T}{T^* U_c(B, T^*)}\right)^{1/p} + \frac{E_c}{\rho_f J_{c0}(B, T)}\right]^{-1}$$  \hspace{1cm} (23)$$

It has been shown by Clem[3], the in-phase and out-of-phase permeabilities of a type-II superconducting cylinder can be expressed as

$$\mu' = \mu'_{0} g_1(r), \quad \mu'' = \mu''_{0} g_2(r)$$  \hspace{1cm} (24)$$

With the scale function

$$g_1(r) = r(1 - \frac{5}{16} r), 0 \leq r < 1$$

$$= 1 + \frac{2}{\pi} \left[\left(-\frac{1}{2} + \frac{r}{2} - \frac{5r^2}{32}\right)\theta + \left(-\frac{2}{3} + 1 - \frac{7r}{8} + \frac{13r^2}{48}\right)\sin(\theta) + \left(-\frac{1}{4} + \frac{r}{4} - \frac{r^2}{24}\right)\sin 2\theta + \left(-\frac{r}{24} + \frac{r^2}{48}\right)\sin 3\theta + \left(-\frac{r^2}{384}\right)\sin 4\theta, r \geq 1$$
where \( \theta(r) \equiv \sin^{-1}(r^{-\frac{1}{2}}) \) and

\[
g_2(r) = \frac{4}{3\Pi} r(1 - \frac{r}{2}), \quad 0 \leq r < 1
\]

\[
= \frac{4}{3\Pi} \frac{1}{r}(1 - \frac{1}{2r}), \quad r \geq 1
\]

(26)

With a maximum \( g_2^{\text{MAX}} = g_2(r = 1) = 0.21 \).

\( \mu'_0 \) is the dimensionless differential permeability, which increases gradually with decreasing temperature

\[
\mu'_0 \equiv \frac{d B_{eq}(H)}{dH} \bigg|_{H=H_0} = \mu'_0(H_0, T)
\]

(27)

For samples with different geometry we have also the expressions similar to Eq.(25) and Eq.(26) for susceptibilities \( \chi' \) and \( \chi'' \) but with somewhat different specific forms of \( g_1(r) \) and \( g_2(r) \) than Eqs.(25) and (26)[2].

V. DISCUSSION

The scaling behavior of ac susceptibility mentioned in section II can be understood in connection with the nonlinear response function in section III. Denoting the maximum \( g_2^{\text{MAX}}(r) = g_2(r = r_p) \), then from Eqs.(22)-(26) we get the equation for the out-of-phase susceptibility peak position \( T_p(B) \) in the form

\[
r_p = \frac{h_{ac}}{aJ_c(B, T_p(B))}
\]

\[
= \frac{h_{ac}}{aJ^{-1}c(B, T_p(B))[1 - \left( \frac{T_p U_c(B, T^*(B))}{T^*(B)U_c(B, T_p(B))} \right)^{1/p}}
\]

\[
+ \frac{\omega h_{ac}}{\rho fJ_{c0}(B, T_p(B))} \bigg]^{-1}
\]

(28)

where \( U_c \) and \( J_{c0} \) can be expressed as [15]

\[
U_c(B, T) = \Psi(T)B^n \propto [T^*(B) - T]^\beta B^n
\]

\[
J_{c0}(B, T) = \lambda(T)B^m \propto [T^*(B) - T]^\alpha B^m
\]

(29)

Starting from equations (28) and (29) the widely observed scaling relations Eqs.(1),(2), and (3) can be naturally derived.
In the case of low frequency as in Fig.1, the amplitude of electric field $E_c$ induced by the ac magnetic field $h_{ac}$ is negligibly small. Thus from Eqs.(23) and (24) one finds

$$\frac{\chi''(T) - \chi''(T^*)}{\chi_{peak}''} \approx \frac{\chi''}{\chi_{peak}''} = \frac{g_2[r(T)]}{g_2^{MAX}}$$

using equations (28) and (29) we get the form

$$\frac{\chi''(T) - \chi''(T^*)}{\chi_{peak}''} \approx [g_2^{MAX}]^{-1}g_2\{r = r_p \frac{T^*(B) - T_p}{T^*(B) - T}\}$$

$$= [g_2^{MAX}]^{-1}g_2\{r = r_p[1 - \frac{T - T_p}{T^* - T_p}]^{-\alpha}\}$$

which is just the scaling relation Eq.(1)

In the case of radio frequency the ac losses due to flux flow is significant at high temperatures near the irreversibility line. So the last terms in the right hand sides of equations (21),(23) and (28) can no longer be omitted. However, the terms with the Heaviside function $\theta(T - T_p)$ in the empirical scaling relation Eq.(2) properly substract these frequency dependent contributions from the overall critical currents and susceptibilities. Thus, again we see

$$\frac{\chi''(T) - \theta(T - T_p)\chi''(T^*)}{\chi_{peak}'' - \theta(T - T_p)\chi''(T^*)} \approx [g_2^{MAX}]^{-1}g_2\{r = r_p[1 - \frac{T - T_p}{T^* - T_p}]^{-\alpha}\}$$

as the experimental data from different references at different frequencies up to 26MHz are superimposed in Fig.3.

The amplitude effect relation Eq.(3) can also be well understood. Since the contribution to critical current from pinning is dominating at temperature $T = T_p$. Omitting the last term in Eq.(23), one derives from Eqs.(28) and (29) naturally the empirical relation for the peak position of $\chi''$

$$[T_p(h_{ac}) - T^*]^{\alpha} \propto h_{ac}$$

VI. SUMMARY

We find some empirical scaling relations for the ac susceptibility of high temperature superconductors. Based on the analysis of the nonlinear response function of mixed state we
derive the critical current and susceptibility equations which lead naturally to the observed scaling behavior.

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FIGURES

FIG. 1. $\chi'$ and $\chi''$ as functions of temperature for a YBCO sample with high $J_c$ at four values of $h_{ac}$ (A:11.2 Oe;C:2.2 Oe;D:1.1 Oe) with $h//c$ axis. Inset: relation between $h_{ac}$ and the temperature at the peak[5]. ($f = 337Hz$)

FIG. 2. Scaling form of the $\chi''(T)$ curves in Fig.1 with different $h_{ac}$ noted by A,B,C, and D respectively.

FIG. 3. The $\chi''(T)$ curves of high $J_c$ YBCO bulk material, $Y ba_2Cu_3O_7$ single crystal and high quality $Y ba_2Cu_3O_7$ films at different frequencies (337Hz ~ 26Hz), can be superimposed when the susceptibility is scaled as $\frac{\chi''(T)−\chi''(T^*)\theta(T−T_p)}{\chi''(T_p)−\chi''(T^*)\theta(T−T_p)}$ and the temperature is scaled as $\frac{(T−T_p)}{(T^*−T_p)}$.

A denote the $\chi''(T)$ curves in Ref.[5] with $f = 337Hz$; B denote the $\chi''(T)$ curves in Fig.5(b), 6(b), 8(b) of Ref.[4] with $f = 26, 0.1, 9MHz$; C denote the $\chi''(T)$ curves in Fig.2(a), 2(b), 6(a), 6(b) of Ref.[2].

FIG. 4. Relation between $h_{ac}$ and $(1−T_p/T^*)$ in the different experiments. $T_p$ is the temperature at the peak of $\chi''$. □: YBCO bulk sample [5]; ●: Single crystal of $Pr_{1.85}Ce_{0.15}CuO_{4−y}$ at $f = 111Hz$ and $\mu_0H = 1T$ [6]; ■: Single crystal of $Pr_{1.85}Ce_{0.15}CuO_{4−y}$ at $f = 111Hz$ and $\mu_0H = 0.1T$ [6]; ▼: a disk (diameter 1mm) of YBCO film [2]; ◆: a rectangle (2 × 3mm$^2$) of YBCO film [2]; ○: a ring (width 50µm) of film YBCO film [2]; ▲: a ring (width 25µm) of YBCO film [2].

FIG. 5. Numerical solutions of equation (8) (open symbols) and equation (14) (lines) for comparison.