Memory-based ISMC design of DFIG-based wind turbine model via T-S fuzzy approach

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Abstract
This paper investigates the memory-based integral sliding mode control (SMC) design for nonlinear doubly fed induction generator model. The proposed nonlinear doubly fed induction generator model is equivalent to linear sub-systems via fuzzy membership functions by utilising the Takagi–Sugeno fuzzy approach. Then, a memory-based sliding surface is intended, which is different from the conventional sliding surface. Based on suitable Lyapunov functionals and slack matrices, sufficient conditions are obtained, which guarantee the memory-based Takagi–Sugeno fuzzy system to be globally asymptotically stable under the designed memory-based sliding mode control through linear matrix inequality. Further, the desired memory-based fuzzy SMC control gain is obtained by solving the concerned linear matrix inequality.

1 INTRODUCTION

The Takagi–Sugeno (T-S) fuzzy models have received special consideration in the area of non-linear modeling, due to their capability to processes into a weighted sum of simple linear sub-systems. The performances of the T-S fuzzy system depend on its complexity, on the type of membership functions [1–3], and [4]. Thus, the fuzzy control based on non-linear models have become a hot topic among the researchers. It has revealed that many control systems, such as the Lorenz system [5] and Chua’s system [6]. For instance, in [7], the authors have been studied the problem of non-linear singular systems via T-S fuzzy approach, which was illustrated by the non-linear mass-spring-damping system. Also, the authors have been investigated in [8] the T-S fuzzy descriptor systems via an average dwell-time approach, which was shown by a single-link robot arm model. In addition, the stability analysis for T-S fuzzy systems has been studied in [9] by mode-dependent average dwell-time switching approach. Moreover, the stabilisation problem of the non-linear model with communication delay has been studied in [10] by applying the set-theoretic description approach.

In the literature, various control techniques are implemented for analysing the dynamic behavior of non-linear systems such as sampled-data control [11, 12], event-triggered controller [13], dissipative control [14], and fault-tolerant control [15]. Moreover, the SMC is an efficacious robust control approach [16, 17]. It has a lot of advantage such as fast response, good transient performance. Therefore, many researchers have been focused on the stability and stabilisation issues of the non-linear systems under SMC [18] and [19]. For example, the authors have been studied in [20] the problem of active suspension systems via dynamic SMC by the T-S fuzzy approach. Moreover, adaptive SMC problem of uncertain nonlinear systems has been investigated in [21] via interval type-2 T-S fuzzy model, which is illustrated by Rossler’s system. The main advantage of integral SMC is the reaching phase desired in a conventional SMC approach is reduced, and sliding mode motion can be achieved from the very beginning of the control action while preserving the order of the original model. In [22] and [7], the problem of T-S fuzzy singular systems with integral SMC has been investigated. Also, reliable integral SMC for T-S fuzzy descriptor systems with semi-Markov parameters have been analyzed.
in [8]. Recently, observer-based event-triggered fuzzy ISMC for the generalised T-S fuzzy system has been investigated [23] with non-linear chaotic permanent magnet synchronous generator model.

The DFIG-based wind turbine model is a complex system with various parameters. It is valuable for the model-based methods to make exact mathematical models and unambiguous numeric expressions to amplify the finding accuracy. For example, in [24], the problem of \( H_\infty \) controller design for wind generation systems with DFIG has been investigated. In recent times, remarkable studies concerning the stability and stabilisation problems of the DFIG system with various control approach have been carried out. Also, the grid connected DFIG model has been investigated by using SMC approach with space vector modulation [25]. Moreover, rotor-tied DFIG System has investigated with sliding-mode observer control in [26]. The stability and stabilisation criterion for DFIG systems has been studied via harmonically distorted grid voltage with SMC [27]. The problem of fault-tolerant control of non-linear DFIG model has been studied in [28] by utilising an augmented observer approach. Further, the damping controller for DFIG system has been investigated in [29] by using bacterial foraging technique. Moreover, field-oriented control for a DFIG-based wind turbine has been analyzed in [30] based on an extended Kalman filter. Recently, fault-tolerant control problem for the DFIG-based wind turbine model has been investigated [31] with stochastic actuator faults by incorporating a T-S fuzzy technique. To the best of authors knowledge, the problem of memory-based switching surface and stabilisation of the non-linear DFIG-based system under sliding motion switching surface. The stabilisation of the control problem is achieved through the suitable control gain matrices which are determined by solving the given LMI.

• By investigating the memory-based switching surface and sliding mode dynamics, a fuzzy integral SMC law is synthesised to ensure that the system performance satisfies the reaching condition.

• From the application point of view, the obtained theoretical result is validated with the non-linear state space model of the DFIG-based wind turbine.

The remaining of this paper is structured as follows: The modelling of wind energy systems with DFIG is presented in Section 2. The stability and stabilisation conditions are derived in Section 3. Numerical simulation of the proposed model is given in Section 4. Finally, Section 5 gives a conclusion and further research work.

Notations: \( \mathbb{R}^{m \times n} \) stands for the set of all \( m \times n \) real matrices. \( \mathbb{R}^n \) is the Euclidean \( n \)-dimensional space. \( A > 0 \) means that the matrix \( A \) is a real symmetric positive definite (semi-positive definite). The terms induced by symmetry are denoted by \( \| z \| \). The superscripts \( T \) and \( -1 \) stand for the matrix transpose and inverse, respectively. A block diagonal matrix is denoted by \( \text{diag}(M_1, M_2, \ldots, M_r) \) with diagonal matrices \( M_1, M_2, \ldots, M_r \).

2 | WIND ENERGY SYSTEMS WITH DFIG

2.1 | Wind turbine system

For a given wind speed \( v \), the aerodynamic power \( P_m \) created by the rotor is as follows [32]:

\[
P_m = \frac{1}{2} \rho \pi R^2 C_p(\lambda, \beta) v^3, \tag{1}
\]

where \( \rho \) denotes the density of the air in \( kg/m^3 \); \( \pi R^2 \) denotes the area swept by blade in \( m^2 \); and \( R \) represents the radius of the blade in \( m \); the aerodynamic model of a wind turbine is obtained by the \( C_p(\lambda, \beta) \); \( \lambda \) denotes the power coefficient; Also, tip speed ratio is denoted by \( \lambda \), and the blade pitch angle is represented by \( \beta \); Further, the tip speed ratio is described as follows:

\[
\lambda = \frac{R \Psi}{v},
\]

where \( \Psi \) denotes rotational speed of the wind.

Then the aerodynamic torque is expressed as follows:

\[
P_m = \Psi_m T_m.
\]

The aerodynamic torque \( T_m \) is also given by

\[
T_m = \frac{1}{2} \rho \pi R^2 C_s(\lambda, \beta) v^2,
\]

where \( C_s = \frac{C_p(\lambda, \beta)}{\lambda} \) is the torque coefficient.

The turbine is joined to the generator shaft through a gearbox. The transmission losses are ignored, the wind turbine torque and speed of the wind turbine, which are denoted by:

\[
\begin{align*}
T_e &= \frac{T_m}{G} \\
\Psi_e &= \frac{\Psi_m}{G},
\end{align*}
\]
where $\tau_g$ is the driving torque of the generator and $\psi_m$ is the generator shaft speed, respectively. The power coefficient $C_p$ is defined as follows:

\[
\begin{align*}
C_p &= k_1 \left( \frac{k_2}{\lambda_i} - k_3 \beta_1 - k_4 \right) + k_5 \lambda_i \\
\lambda_i &= \frac{1}{\lambda + 0.08\beta} - 0.035 + 1 + \beta^3
\end{align*}
\]

where $k_1, k_2, k_3, k_4, k_5, k_6$ are constant coefficients, $k_1 = 0.5176$, $k_2 = 116$, $k_3 = 0.4, k_4 = 5, k_5 = 21, k_6 = 0.0068$, and $\lambda_i$ relates to $\lambda$ and $\beta$. When $\lambda = \lambda_{opt} = 8.1$ and $\beta = 0^\circ$, the wind turbine should be operated at maximum power coefficient point $C_{\text{p max}} = 0.48$. The maximum power captured by the equation (1) from the wind turbine is obtained as in the following:

\[
p_{opt} = \frac{1}{2} \rho \pi R^5 C_{\text{p max}} \psi_m^3.
\]

### 2.2 DFIG wind turbine model

The voltage equations of the DFIG in the arbitrary $d-q$ reference frame are (stator in the generator convention and rotor in the motor convention) expressed as follows [33]:

\[
\begin{align*}
v_{dq} &= R_{dc} i_{dq} + \frac{d\Omega_{dq}}{dt} - k_i \Omega_{dq} \\
v_{qf} &= R_{qf} i_{qf} + \frac{d\Omega_{qf}}{dt} - k_i \Omega_{qf} \\
v_{dr} &= R_{dr} i_{dr} + \frac{d\Omega_{dr}}{dt} - k_i \Omega_{dr} \\
v_{qf} &= R_{qf} i_{qf} + \frac{d\Omega_{qf}}{dt} - k_i \Omega_{qf}
\end{align*}
\]

with

\[
\begin{align*}
\Omega_{dq} &= L_{dc} i_{dq} + L_{mf} i_{dq} \\
\Omega_{qf} &= L_{qf} i_{qf} + L_{mf} i_{qf} \\
\Omega_{dr} &= L_{dr} i_{dr} + L_{mf} i_{dr} \\
\Omega_{qf} &= L_{qf} i_{qf} + L_{mf} i_{qf}
\end{align*}
\]

The rotor and stator angular velocities are demonstrated as in the following connection:

\[k_r = k_i - p \psi_m^m.\]

**Remark 1.** The non-linear behavior of the induction generator is due to the back electromotive force that based on the rotation speed of the generator [34]. By utilising the clarke transformation to change the rotor currents and voltages of the stator into the $p-q$ frame $(\alpha-\beta)$ method orthogonal coordinates [35], which is expressed in $p-q$ frame are provided as in the following:

\[
\begin{bmatrix} \mathcal{M}_{\beta} \\ \mathcal{N}_{\beta} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \mathcal{M}_{\alpha} \\ \mathcal{N}_{\alpha} \end{bmatrix},
\]

where voltage of stator is denoted by $\mathcal{M}_{\alpha}$ and current of the rotor is represented by $\mathcal{N}_{\beta}$.

### 2.3 Non-linear model of a DFIG

The following assumptions will be used for construct the non-linear DFIG model:

A1: Generator’s stator and rotor windings are uniformly fed. A2: The minor energy losses are ignored such as inductance saturation, iron losses, skin effect, and bearing friction. A3: Let the winding resistance be a constant.

Based on the above, the model transformation the DFIG in the rotational $(d-q)$ reference frame is obtained by

\[
\dot{x}(t) = A(x(t))x(t) + Bu(t),
\]

with

\[
\begin{align*}
x(t) &= \left[\begin{array}{c} v_{dq}(t) \\ v_{qf}(t) \\ v_{dr}(t) \\ v_{qf}(t) \end{array}\right] \\
u(t) &= \left[\begin{array}{c} i_{dq}(t) \\ i_{qf}(t) \end{array}\right]
\end{align*}
\]

Here, the stator current and rotor current of the non-linear DFIG model is denoted by the state vector $x(t)$. Also, stator voltage and rotor voltage of the non-linear DFIG model is represented by the control input vector $u(t)$.
Here, the stator resistance is denoted by \( R_s \), rotor resistance is represented by \( R_r \). Also, the stator inductance is denoted by \( L_s \), the rotor inductance is represented by \( L_r \), the magnetisation inductance is denoted by \( L_m \), \( t \) represents current, and the synchronous speed is denoted by \( \omega_s \).

The non-linear system (4) can be expressed as in the following T-S fuzzy model:

**Model Rule 1:** IF \( \theta_1(t) \) is \( M_{i1} \) and \( \ldots \theta_p(t) \) is \( M_{ip} \), THEN

\[
\dot{x}(t) = A_i x(t) + B_i u(t),
\]

where \( x(t) \in \mathbb{R}^n \) denote the state vector; \( u(t) \in \mathbb{R}^n \) denote the input vector; the premise variables are denoted by \( \theta_1, \ldots, \theta_p \); \( M_{i1}, \ldots, M_{ip} \) denote the fuzzy sets, \( r \) represents the number of T-S fuzzy rules; \( A_i, B_i \) denote the system matrices. Then by applying fuzzy rule, the non-linear systems (4) is described by as follows:

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + B_i u(t)],
\]

where \( \theta(t) = [\theta_1(t), \ldots, \theta_p(t)]^T \), \( h_i(\theta(t)) \) is the membership function given by

\[
h_i(\theta(t)) = \frac{\prod_{j=1}^{p} M_{ij}(\theta(t))}{\sum_{j=1}^{r} \prod_{j=1}^{p} M_{ij}(\theta(t))},
\]

for all \( t > 0 \). \( M_{ij}(\theta(t)) \) is the grade membership of \( \theta_j(t) \) in \( M_{ij} \). The membership functions \( h_i(\theta(t)) \) represent the weights of the sub-systems in the global system which satisfies \( h_i(\theta(t)) \geq 0 \) and

\[
\sum_{i=1}^{r} h_i(\theta(t)) = 1, \quad \forall \ t > 0.
\]

From the considered premise variable \( \theta(t) \) is bounded as:

\[
\theta_{\text{min}} < \theta(t) < \theta_{\text{max}}.
\]

The minimum and maximum values of the considered non-linear term are obtained as follows:

\[
\begin{align*}
\theta_{\text{min}} &= \min(\theta(t)) = \frac{G\lambda_{d\omega l}}{R} r_{\text{min}}, \\
\theta_{\text{max}} &= \max(\theta(t)) = \frac{G\lambda_{d\omega l}}{R} r_{\text{max}}.
\end{align*}
\]

From the above Equation (7), the non-linear DFIG model can be denoted by the maximum and minimum values of membership functions, that is, \( \dot{\theta}(t) = Z_1 \theta_{\text{min}} + Z_2 \theta_{\text{max}} \), where \( Z_1 + Z_2 = 1 \).

Therefore the membership functions can be gained as in the following:

\[
\begin{aligned}
\theta_1(\theta(t)) &= Z_1 = \frac{\theta - \theta_{\text{max}}}{\theta_{\text{max}} - \theta_{\text{min}}} \\
\theta_2(\theta(t)) &= Z_2 = \frac{\theta_{\text{min}}}{\theta_{\text{max}} - \theta_{\text{min}}}.
\end{aligned}
\]

Then the non-linear DFIG model is composed of \( 2 = 2^1 \) linear subsystems, which are acquired by replacing with the bounded values in non-linear state-space model of the DFIG (4). Now, we get the following T-S fuzzy model:

**Model Rule 1:** IF \( \theta_1 \) is \( Z_1 \) THEN

\[
\dot{x}(t) = A_1 x(t) + B_1 u(t).
\]

**Model Rule 2:** IF \( \theta_2 \) is \( Z_2 \) THEN

\[
\dot{x}(t) = A_2 x(t) + B_2 u(t).
\]

In order to get the two possible combinations, \( A_i (i = 1, 2) \) are obtained by utilising the scalars \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) where \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) are the bounds of \( \theta(t) \) which is obtained by using relation (6) when \( \lambda_{d\omega l} = 0.81 \).

Further more, it can be expressed as in the following, which is obtained by the relation (6):

\[
\begin{align*}
A_1 &= \begin{bmatrix}
\frac{-R_s}{\sigma L_r} & \frac{L_{\omega l}^2 \theta_{\text{min}}}{\sigma L_r L_r} \\
\frac{-R_r}{\sigma L_r} & \frac{L_{\omega l}^2 \theta_{\text{min}}}{\sigma L_r L_r} \\
\frac{-L_{d} \theta_{\text{min}}}{\sigma L_r} & \frac{L_{\omega l}^2 \theta_{\text{min}}}{\sigma L_r L_r} \\
\end{bmatrix} \\
A_2 &= \begin{bmatrix}
\frac{R_s L_{\omega l}}{\sigma L_r} & \frac{L_{\omega l}^2 \theta_{\text{min}}}{\sigma L_r L_r} \ \\
\frac{-R_r}{\sigma L_r} & \frac{L_{\omega l}^2 \theta_{\text{min}}}{\sigma L_r L_r} \\
\frac{R_r L_{\omega l}^2}{\sigma L_r} & \frac{L_{\omega l}^2 \theta_{\text{min}}}{\sigma L_r L_r} \\
\end{bmatrix}.
\end{align*}
\]
A_2 = \begin{bmatrix} -R_j \sigma_{I_{r}} & \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}} & (\omega_j + \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}}) \\ -\left(\omega_j + \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}}\right) & -R_j \sigma_{I_{r}} & \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}} \\ \frac{R_j I_{mg}}{\sigma_{I_{r}I_{r}}} & \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}} & (\omega_j - \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}}) \\ -\left(\omega_j - \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}}\right) & -R_j \sigma_{I_{r}} & \frac{I_{mg}^2 \theta_{\max}}{\sigma_{I_{r}I_{r}}} \\ \end{bmatrix}

B_1 = B_2 = \begin{bmatrix} \frac{1}{\sigma_{I_{r}}} & 0 & -\frac{I_{mg}}{\sigma_{I_{r}I_{r}}} & 0 \\ 0 & \frac{1}{\sigma_{I_{r}}} & 0 & -\frac{I_{mg}}{\sigma_{I_{r}I_{r}}} \\ -\frac{I_{mg}}{\sigma_{I_{r}I_{r}}} & 0 & \frac{1}{\sigma_{I_{r}}} & 0 \\ 0 & -\frac{I_{mg}}{\sigma_{I_{r}I_{r}}} & 0 & \frac{1}{\sigma_{I_{r}}} \\ \end{bmatrix}

where 0 \leq \tau_1 < \tau_2, \text{ here } \tau_1, \tau_2, \text{ and } \mu \text{ are known positive constants.}

As stated by SMC theory, if \( \dot{z}(t) = 0 \), then we get

\[
\dot{z}(t) = G\dot{x}(t) - \sum_{j=1}^{r} b_j(\theta(t)) \sum_{j=1}^{r} b_j(\theta(t)) \times [(A_j + B_j K_j) x(t) + B_j K_j x(t - \tau(t))]
\]

Substituting (5) in the above equation. The equivalent controller can be obtained as

\[
n_{sg}(t) = \sum_{j=1}^{r} b_j(\theta(t)) [K_j x(t) + K_j x(t - \tau(t))].
\]

By substituting (12) in (5), the T-S fuzzy sliding-mode dynamics can be obtained as follows:

\[
\dot{x}(t) = \sum_{j=1}^{r} b_j(\theta(t)) \sum_{j=1}^{r} b_j(\theta(t)) [(A_j + B_j K_j) x(t) + B_j K_j x(t - \tau(t))].
\]  

Remark 2. The state-space model of the DFIG-based wind turbine system is provided in (4), which is time-variant and nonlinear. In consequence, to approximate the non-linear DFIG-based wind turbine system, the T-S fuzzy approach is utilised. In order to investigate the stabilisation of the nonlinear DFIG model, we have assumed one premise variable, that is, \( \theta(t) = \Psi_w(t) \). When \( v < v_{\text{min}} \), that is, the wind speed is lower than the cut-in speed \( v_{\text{min}} \), no power can be produced such that \( \Psi_w = \frac{C_{\text{opt}}}{R} v \). The blade pitch will be regularly reserved a constant while the generator torque is the controlling variable [36], if \( v_{\text{min}} < v < v_{\text{max}} \).

The main objective of this paper is to investigate the stability and stabilisation criterion for non-linear DFIG model with memory-based SMC via T-S fuzzy approach, which can be established as follows:

Problem 1. Given all the system matrices in (5), obtain the matrices \( K_j \) and \( K_{pj} \) in the switching surface manifold of (9) such that the T-S fuzzy sliding mode dynamics (13) is globally asymptotically stable under the memory-based fuzzy ISMC scheme (12).

3.2 Stability and stabilisation of the sliding-mode dynamics

The following theorem provides the stability and stabilisation criterion for global asymptotic stability of memory-based nonlinear DFIG sliding mode dynamics as a solution to problem 1:

\[
0 \leq \tau_1 \leq \tau(t) \leq \tau_2 \text{ and } \dot{\tau}(t) \leq \mu < 1,
\]
**Theorem 1.** For given positive scalars $\tau_1$, $\tau_2$, and $\mu$, the memory-based non-linear DFIG sliding mode dynamics (13) is globally asymptotically stable, if there exist positive-definite matrices $P$, $Q$, $R$, $S$, and any appropriately dimensioned matrices $M_1$ and $M_2$, such that

\[
\Xi_{ii} < 0, \quad i = 1, 2, \ldots, r
\]

\[
\Xi_{ij} + \Xi_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, r,
\]

where

\[
\Xi_{ij} = \begin{bmatrix}
\Xi_{11} & \Xi_{12}/j & 0 & 0 & \Xi_{15}j \\
* & \Xi_{22}/j & 0 & 0 & \Xi_{25}/j \\
* & * & \Xi_{33} & 0 & 0 \\
* & * & * & \Xi_{44} & 0 \\
* & * & * & * & \Xi_{55}
\end{bmatrix},
\]

\[
\Xi_{11} = -M_2^T - M_2, \quad \Xi_{12}/j = P + M_1 + M_2^T A_j + M_2^T B_j K_j,
\]

\[
\Xi_{15}j = M_2^T B_j K_j,
\]

\[
\Xi_{22}/j = (\tau_2 - \tau_1) S - M_2^T A_j - A_j^T M_1 - M_2^T B_j K_j - K_j^T B_j^T M_1,
\]

\[
\Xi_{25}/j = -M_2^T B_j K_j, \quad \Xi_{33} = -\tau_1 Q + R,
\]

\[
\Xi_{44} = -(\tau_2 - \tau_1) S, \quad \Xi_{55} = -(1 - \mu) R.
\]

**Proof.** Consider the Lyapunov-Krasovskii functional $V(t)$ for the T-S fuzzy sliding mode dynamics (13) to get the stability condition.

\[
V(t) = x^T(t) P x(t) + \tau_1 \int_{t-\tau_1}^{t} x^T(s) Q x(s) ds
\]

\[
+ \int_{t-\tau_2}^{t-\tau_1} x^T(s) R x(s) ds + (\tau_2 - \tau_1) \int_{t-\tau_2}^{t} x^T(s) S x(s) ds.
\]

Take the time derivative of $V(t)$ along the state trajectory of the nonlinear DFIG sliding mode dynamics (13) yields

\[
\dot{V}(t) = 2x^T(t) P \dot{x}(t) + \tau_1 x^T(t) Q \dot{x}(t) - \tau_1 x^T(t-\tau_1) Q \dot{x}(t-\tau_1)
\]

\[
+ x^T(t - \tau_1) R \dot{x}(t-\tau_1) - (1 - \mu) x^T(t - \tau_1) S \dot{x}(t-\tau_1)
\]

\[
- (\tau_2 - \tau_1) x^T(t-\tau_2) S \dot{x}(t-\tau_2).
\]

For any matrices $M_1$ and $M_2$ with appropriate dimension, we get the following equation:

\[
-2 \sum_{i=1}^{r} b_i(\theta(t)) \sum_{j=1}^{r} b_j(\theta(t))[x^T(t) M_1^T + \dot{x}^T(t) M_2^T]
\]

\[
\times \{A_i + B_j K_j \dot{\varsigma}(t) + B_i K_j \dot{\varsigma}(t - \tau(t)) - \dot{\varsigma}(t)\} = 0. \quad (17)
\]

Combining (15) and (16), we obtain the following:

\[
\dot{V}(t) \leq \sum_{i=1}^{r} b_i(\theta(t)) \sum_{j=1}^{r} b_j(\theta(t)) \xi^T(t) \Xi_{ij}(t) \xi(t)
\]

\[
= \sum_{i,j} b_i(\theta(t)) b_j(\theta(t)) \xi^T(t) \Xi_{ij}(t) \xi(t)
\]

\[
+ \sum_{i,j} b_i(\theta(t)) b_j(\theta(t)) \xi^T(t) (\Xi_{ij} + \Xi_{ji}) \xi(t),
\]

where

\[
\xi^T(t) = [x^T(t) x^T(t - \tau_1) x^T(t - \tau_2) x^T(t - \tau(t))].
\]

By Schur complement lemma and from (14) and (15) that, one can obtain that $\dot{V}(t) < 0$, it gives that under the memory-based fuzzy sliding mode controller scheme the non-linear DFIG sliding mode dynamics (13) is asymptotically stable.

**Remark 3.** Theorem 1 provides sufficient condition of asymptotic stability criterion for the memory-based T-S fuzzy sliding mode dynamics (13) in the form of nonlinear matrix inequalities. In the derived sufficient condition of the above theorem, the memory-based SMC gain matrices $K_j$ and $K_{ej}$ are unknown and this makes the LMIs (14) and (15) are not solvable. Based on the derived result in Theorem 1, the desired memory-based SMC gain matrices $K_j$ and $K_{ej}$ in (9) can be achieved by the following theorem.

**Problem 2.** Given all the system matrices in (5) and the matrices $G$, $K_j$, and $K_{ej}$ in the switching surface manifold of (9), determine what condition the T-S fuzzy sliding mode dynamics (13) is global asymptotically stable under the memory-based fuzzy ISMC scheme (12).

We propose the following theorem as a solution to problem 2:

**Theorem 2.** For given positive scalars $\tau_1$, $\tau_2$, $\mu$, and $\alpha > 0$, the T-S fuzzy sliding mode dynamics (13) is asymptotically stable under fuzzy ISMC controller (12), if there exist positive-definite matrices $P$, $Q$, $R$, $S$, and...
and any appropriately dimensioned matrices $M_1$ and $M_2$, such that
\[
\dot{x}_i < 0, i = 1, 2, \ldots, r,
\]
\[
\dot{x}_{ij} + \dot{x}_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, r,
\]
where
\[
\begin{bmatrix}
\dot{x}_{11} & \ldots & \dot{x}_{1j}
\end{bmatrix} = 0,
\]
\[
\begin{bmatrix}
\dot{x}_{2j}
\end{bmatrix} = 0,
\]
\[
\begin{bmatrix}
\dot{x}_{3j}
\end{bmatrix} = 0,
\]
\[
\begin{bmatrix}
\dot{x}_{4j}
\end{bmatrix} = 0,
\]
\[
\begin{bmatrix}
\dot{x}_{5j}
\end{bmatrix} = 0.
\]

Furthermore, the gain matrices $K_j$ and $K_{c,j}$ involved in fuzzy memory-based sliding switching manifold in (9) are computed by $K_j = X_j^rM_1^{-1}$ and $K_{c,j} = Y_j^rM_1^{-1}$.

Proof. Let us consider $M_1 = M_1^{-1}$, $M_2 = \alpha M_1^{-1}$, $\tilde{P} = M_1^T P M_1$, $\hat{Q} = M_1^T Q M_1$, $\tilde{R} = M_1^T R M_1$, $\hat{S} = M_1^T S M_1$. Then, the inequalities (14) and (15) are pre- and post multiplied by $\text{diag}(M_1^T, M_1^T, M_1^T, M_1^T, M_1^T)$ and its transpose, respectively, we get (18) and (19). Based on the proof of the same procedure in Theorem 1, we conclude that T-S fuzzy sliding motion dynamics (13) is asymptotically stable.

### 3.3 Synthesis of memory-based fuzzy integral SMC law

In this subsection, we synthesize a memory-based fuzzy integral SMC law by using the idea of parallel distributed compensation, in which the state trajectories of the T-S fuzzy sliding motion dynamics (13) can be driven onto the pre-specified sliding manifold in a finite time and then are maintained there for all subsequent time.

**Problem 3.** Synthesize a SMC law to globally drive the system state trajectories onto the pre-defined surface manifold $s(t) = 0$ in a finite time.

The fuzzy SMC law of rule $j$ is given for the nonlinear sliding dynamics (13) as follows:

**Model Rule i:** IF $\theta_j(t)$ is $M_{i1}$ and ... $\theta_j(t)$ is $M_{ir}$, THEN
\[
u(t) = K_j x(t) + K_{c,j} x(t - \tau(t)) - \rho (GB_s)^{-1} \text{sign}(s(t)).
\]

Therefore, memory-based fuzzy SMC law is constructed as
\[
u(t) = \sum_{j=1}^{r} b_j(\theta(t))[K_j x(t) + K_{c,j} x(t - \tau(t))]
\]
\[- \rho (GB_s)^{-1} \frac{s(t)}{||s(t)||},
\]
where $\rho$ is a constant.

**Theorem 3.** Consider the fuzzy integral sliding motion dynamics (13). Suppose that the fuzzy integral sliding manifold is given in (9), matrices $K_j$ and $K_{c,j}$ being solved by $K_j = X_j^rM_1^{-1}$ and $K_{c,j} = Y_j^rM_1^{-1}$, and the memory-based fuzzy integral SMC is constructed as (20). Then, the state trajectories of fuzzy sliding mode dynamics (13) shall be determined onto the established in advance fuzzy sliding switching surface $s(t) = 0$.

**Proof.** Consider the Lyapunov function:
\[
V(t) = \frac{1}{2} s^T(t)s(t),
\]
From (9), we get the following:
\[
\dot{V}(t) = G \sum_{j=1}^{r} b_j(\theta(t)) \sum_{j=1}^{r} b_j(\theta(t))
\]
\[\times [-B_j K_j x(t) - B_j K_{c,j} x(t - \tau(t)) + B_j u(t)].
\]
From the above, we get the following:
\[
\dot{V}(t) = s^T(t)\dot{s}(t)
\]
\[= s^T(t)G \sum_{j=1}^{r} b_j(\theta(t)) \sum_{j=1}^{r} b_j(\theta(t))
\]
\[\times [-B_j K_j x(t) - B_j K_{c,j} x(t - \tau(t)) + B_j u(t)].
\]
We get the following inequality from (20) into (22),
\[
\dot{V}(t) = -\rho ||s(t)|| \leq 0,
\]
which gives $\dot{V}(t) \leq 0, \forall s(t) \neq 0$. This implies the state trajectories of nonlinear DFIG-based sliding motion dynamics (13) can reach onto the predefined memory-based switching manifold $s(t)$ in a finite time. This completes the proof.
| Parameter               | Value               |
|------------------------|---------------------|
| Stator resistance Rs   | 0.005 Ω             |
| Rotor resistance Rr    | 0.0089 Ω            |
| Stator leakage inductance Ls | 0.407 mH       |
| Rotor leakage inductance Lr | 0.2990 mH         |
| Mutual inductance Lm   | 0.016 mH            |
| Air density ρ          | 1.225 kg/m²         |
| Pairs of pole number   | ρ = 2               |
| Gearbox ratio G        | G = 12              |
| Blade length R          | R = 62.8            |

### 4 | SIMULATION EXAMPLE

This section provides a numerical example to demonstrate the superiority of the derived sufficient condition. To do this, a nonlinear DFIG model (13) is utilised. The parameters value of the DFIG is given in Table 1. Besides that, a simplified diagram of the DFIG-based wind turbine is illustrated in Figure 1. Induction generators can be controlled by utilising different control approaches such as field-oriented control and direct torque control [37]. Away from these control approaches, the field-oriented control is extensively applied in the engineering field due to its simplicity of inception and its superior recital [38]. Besides that, memory-based SMC included not only the current state but also the delayed state, which employed for the stabilisation criteria for non-linear systems. The stator winding of DFIG is directly joined to the grid, while the rotor is joined by way of a back-to-back converter. It has adjoined connecting the rotor and the grid of the DFIG to allow power commute amid the grid and the generator. As a consequence, the grid side converter controls the DC-link voltage and reactive power, and the rotor side converter operates to control the DFIG in a way that is free from outside control of active and reactive powers [39].

In this regard, the membership functions are chosen as follows:

\[
b_1(\theta(t)) = \frac{\theta - \theta_{\max}}{\theta_{\max} - \theta_{\min}} \quad b_2(\theta(t)) = 1 - b_1(\theta(t)).
\]

The concerned sliding surface in (9) is obtained as:

\[
s(t) = Gx(t) - \int_0^t \left[ \sum_{i=1}^{2} b_i(\theta(\tau)) \sum_{j=1}^{2} \frac{1}{b_j(\theta(\tau))} \left( A_i + B_i K_j \right) x(\tau) + B_i K_j x(\tau - \tau(\tau)) \right] d\tau.
\]

The constant matrix \( G \) satisfies \( GB_x \) as a non-singular matrix such that

\[
G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

Further, memory-based fuzzy SMC controller in (20) is obtained by

\[
u(t) = \sum_{j=1}^{2} b_j(\theta(t)) \left[ K_j x(t) + K_j x(t - \tau(t)) \right] - 0.6(GB_x)^{-1} \frac{s(t)}{||x(t)||},
\]

where

\[
GB_x^{-1} = \begin{bmatrix} -227.6140 & 0 & -215.9950 & 0 \\ 0 & -227.6140 & 0 & -215.9950 \\ -215.9950 & 0 & -227.6140 & 0 \\ 0 & -215.9950 & 0 & -227.6140 \end{bmatrix}
\]

Moreover, memory parameter is assumed by \( \tau(t) = 0.6 + 0.4 \sin(t) \). that is \( \tau_1 = 0.2 \) and \( \tau_2 = 1.0 \), which implies that...
0.2 \leq \tau(t) \leq 1.0 \text{ and } \dot{\tau}(t) = 0.4 \cos(t) \text{ that is } \dot{\tau}(t) \leq \mu = 0.4 < 1. \text{ By solving the LMI conditions (18) and (19) in Theorem 2 with the scalar } \alpha = 0.5 \text{ through MATLAB-LMI control toolbox, we can obtain the corresponding memory-based control gain matrices as follows:}

\[
K_1 = 10^3 \times \begin{bmatrix}
3.5479 & 0.8637 & 3.5631 & 0.8199 \\
-0.8617 & 3.5557 & -0.8180 & 3.5707 \\
3.5620 & 0.9501 & 3.5972 & 1.0009 \\
-0.9481 & 3.5698 & -0.9989 & 3.6049
\end{bmatrix},
\]

\[
K_2 = 10^3 \times \begin{bmatrix}
3.5478 & 0.8636 & 3.5631 & 0.8198 \\
-0.8618 & 3.5544 & -0.8180 & 3.5696 \\
3.5618 & 0.9500 & 3.5971 & 1.0008 \\
-0.9482 & 3.5684 & -0.9990 & 3.6037
\end{bmatrix},
\]

\[
K_{\tau 1} = 10^3 \times \begin{bmatrix}
-0.0006 & 0.0025 & -0.0003 & 0.0019 \\
-0.0007 & -0.0006 & -0.0008 & -0.0003 \\
-0.0004 & 0.0020 & -0.0002 & 0.0015 \\
-0.0004 & -0.0008 & -0.0005 & -0.0004
\end{bmatrix}.
\]
The solutions of the proposed DFIG-based wind turbine model is illustrated with the help of pictorial representations. Given the initial condition, and the adjustable parameters \( \rho = 0.5 \), the simulation results are given in Figures 2–9. As a consequence (Figures 2–5), one can easily see that the state responses of nonlinear DFIG-based wind turbine model to be controlled for the desired switching manifold \( s(t) = 0 \) in a limited time, and it is asymptotically stable toward the origin. Hence, the stabilisation of the memory-based T-S fuzzy system is well established. One can obtain that from these Figures 2–5 that the system can ensure asymptotically stable under the memory-based fuzzy SMC law. Figure 6 shows the response of control input. Figure 7 depicts the response of sliding surface. Figures 8 and 9 conclude that the considered non-linear DFIG-based wind turbine system with chaotic solutions are stabilising via the proposed memory-based fuzzy integral. Finally, through the simulation tests, we can conclude that the proposed controller gave better system performance based on derived conditions.
FIGURE 8  The dynamic behavior of non-linear DFIG-based wind turbine system (13) without control in (a) $i_{ds}i_{qs}$ plane, (b) $i_{ds}i_{rq}$ plane, (c) $i_{dr}i_{qr}$ plane under initial condition $x(0) = [10 \ 0 \ 10]^T$

FIGURE 9  The dynamic behavior of non-linear DFIG-based wind turbine system (13) without control in (a) $i_{qs}i_{dr}$ plane, (b) $i_{qs}i_{qr}$ plane, (c) $i_{dq}i_{dp}$ plane under initial condition $x(0) = [10 \ 0 \ 10]^T$

5  CONCLUSION

In this paper, the problem of memory-based SMC design for the non-linear systems has been investigated. In addition to that the particular industrial problem, the non-linear DIG model has been adapted for the numerical evaluations, which is one of the objectives of the present work. As the considered model is non-linear, we have designed the fuzzy membership grades which approximates the non-linear DFIG model into linear sub-models and have utilised the derived stability and stabilisation conditions. A novel memory-based fuzzy integral switching surface manifold has constructed, and then, based on suitable Lyapunov functional and few slack matrices, the sufficient conditions have acquired in the form of LMI which provided the globally asymptotic stability of the considered T-S fuzzy sliding mode dynamics. By solving the derived stabilisation condition, we can get the proper gain matrices that are stabilised the state trajectories of the concerned system. To show the effectiveness of the proposed control design, a physical experimental problem on DFIG model has been adapted and demonstrated via the derived theoretical results. Due to the complex environment, the non-linear model with stochastic disturbance has paid more attention to the researchers in recent time. In this regard, we will extend the present study with stabilisation criteria for interconnected systems, and power systems under stochastic disturbance.

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