Statistical data analysis in the DANSS experiment

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Abstract.

DANSS is a one cubic meter highly segmented solid scintillator detector. It is placed under a 3.1 GW industrial reactor at the Kalinin NPP (Russia) on a movable platform. The distance from the reactor core center is varied from 10.7 m to 12.7 m on-line. The inverse beta decay (IBD) process is used to detect antineutrinos. DANSS detects about 5000 IBD events per day. Sterile neutrinos are searched for assuming a 4 neutrino model (3 active and 1 sterile neutrino). The exclusion area in the sterile neutrino parameter plane is obtained using a ratio of positron energy spectra collected at different distances. Therefore, results do not depend on the shape and normalization of the reactor \(\bar{\nu}_e\) spectrum, as well as on the detector efficiency. The excluded area covers a wide range of the sterile neutrino parameters up to \(\sin^2 \theta_{14} < 0.01\) in the most sensitive region. The Reactor Antineutrino Anomaly optimum point is excluded with a confidence level higher than 5\(\sigma\). A Gaussian CL\(_s\) method was used to obtain exclusion areas. CL\(_s\) method includes a point from \(\sin^2 \theta, \Delta m^2\) plane into exclusion area only if the experiment sensitivity to such point is good. Details of this approach are presented.

1. Introduction

The number of active neutrinos is limited to 3 by the measurements of the Z boson decay width [1]. However, existence of additional sterile neutrinos is not excluded. Several effects [2, 3] observed with about 3\(\sigma\) significance level can be explained by active-sterile neutrino oscillations [4].

The survival probability of a reactor \(\bar{\nu}_e\) at short distances in the 4\(\nu\) mixing scenario (3 active and 1 sterile neutrino) is described by a familiar expression

\[
1 - \sin^2 2\theta_{14} \sin^2 \left( \frac{1.27 \Delta m^2_{14} [eV^2] L [m]}{E_{\nu}[MeV]} \right).
\]

The existence of sterile neutrinos would manifest itself in distortions of the \(\bar{\nu}_e\) energy spectrum at short distances. At longer distances these distortions are smeared out and only the rate is reduced by a factor of \(1 - \sin^2(2\theta_{14})/2\). Measurements at only one distance from a reactor core are not sufficient since the theoretical description of the \(\bar{\nu}_e\) energy distribution is considered not to be reliable enough. The most reliable way to observe such distortions is to measure the \(\bar{\nu}_e\) spectrum with the same detector at different distances. In this case, the shape and normalization...
of the $\tilde{\nu}_e$ spectrum as well as the detector efficiency are canceled out. The DANSS experiment
uses this strategy and measures $\tilde{\nu}_e$ spectra at 3 distances from the reactor core centre: 10.7 m,
11.7 m, and 12.7 m to the detector centre. Antineutrinos are detected by means of the Inverse
Beta Decay (IBD) reaction

$$\tilde{\nu}_e + p \rightarrow e^+ + n \quad \text{with} \quad E_{\tilde{\nu}} = E_{e^+} + 1.80 \text{ MeV}. \quad (2)$$

2. The DANSS Detector
The DANSS detector was constructed by the ITEP-JINR collaboration. It is installed under
the core of a 3.1 GW$_\text{th}$ industrial power reactor at the Kalinin Nuclear Power Plant (KNPP).
The reactor materials provide a good shielding equivalent to $\sim 50$ m of water, which removes
the hadronic component of the cosmic background and reduces the cosmic muon flux by a factor
of 6. DANSS consists of 2500 polystyrene-based scintillator strip with a thin Gd-containing
surface coating. The coating serves as a light reflector and a $(n,\gamma)$-converter simultaneously.
The detector is placed inside a composite shielding of copper, borated polyethylene and lead. It
is surrounded on 5 sides (excluding bottom) by double layers scintillator plates to veto cosmic
muons. Light from the strip is collected with three wavelength-shifting (WLS) fibers, glued into
grooves along the strip. The central fiber is read out with a Silicon PhotMultiplier (SiPM).
The side fibers from 50 parallel strips (a module — 10 layers with 5 strips each) are read out
with a compact photomultiplier tube (PMT). So that the whole detector (2500 strips) is a
structure of 50 intercrossing modules. SiPMs (PMTs) register about 18 (20) photo-electrons
(p.e.) per MeV. These numbers were obtained using measurements with cosmic muons and
artificially driven LEDs. So the total number is 38 p.e./MeV. Parameterized strip response non-
uniformities have been incorporated into the GEANT4 (Version 4.10.4) MC simulation of the
detector. The experimental energy resolution for cosmic muon signals in the scintillator strips
is 15% worse than that from the MC calculation. Therefore, the MC estimations are scaled up
by the corresponding factor. More detailed description of detector construction can be found
elsewhere [5]. Data taking and analysis are described in [6].

3. Spectra calculation
For each point in the the $\Delta m^2_{14}$, $\sin^2 2\theta_{14}$ plane the predictions for the positron spectra at the
two positions were calculated. Several factors were taken into account:

- $\tilde{\nu}_e$ spectrum. Here we used Huber [7] and Mueller [3] calculations, but this choice does not
  influence final results, since the ratio of spectra is compared with experiment.
- IBD cross-section [8].
- Reactor and detector size.
- Reactor burning profile (provided by KNPP). Distribution averaged over the campaign was
  used in this analysis. It was verified that this approximation practically does not influence
  the final results.
- Detector resolution obtained from GEANT4 MC simulations.
- Oscillation probability (1).
- Solid angle ($1/R^2$)

Figure 1 shows the simulated DANSS response to a 4.125 MeV positron signal. The visible
positron energy is converted using MC simulations into the deposited energy by taking into
account average losses in the inactive reflective layers of the strips and dead channels. Sometimes
photons from the positron annihilation produce signals in the strips attributed to the positron
cluster. This leads to an increase of the visible energy. Such a shift is also corrected on average
using MC simulations. A typical size of the total correction is $\sim 2\%$. 

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The energy resolution is modest ($\sigma/E = 17\%$ for the Gaussian part of the spectrum). This leads to additional smearing of the oscillation pattern, comparable with the smearing due to the large reactor core size. Figure 2 shows calculated spectra ratios for different assumptions about detector resolution and reactor core size.

![Figure 1. Reconstructed positron energy for 4.125 MeV positrons from MC simulations. The curve represents the fit of the Gaussian part of the spectrum ($\sigma_E = 0.7$ MeV).](image)

![Figure 2. Calculated spectra ratios for different cases. Red dotted — zero reactor size and ideal energy resolution; black dashdotted — real reactor profile and ideal resolution; cyan solid — real conditions.](image)

4. $\chi^2$ statistics
The $\chi^2$ for each hypothesis was constructed using 24 data points $R^{\text{obs}}_i$ in the (1-7) MeV positron energy range

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{R^{\text{obs}}_i - k \times R^{\text{pre}}_i}{\sigma_i^2} \right)^2,$$

(3)

where $R^{\text{obs}}_i$ ($R^{\text{pre}}_i$) is the observed (predicted) ratio of $\bar{\nu}_e$ counting rates at the two detector positions and $\sigma_i$ is the statistical standard deviation of $R^{\text{obs}}_i$, and $k$ is a normalization factor equal to the ratio of the total number of the IBD events per day at the bottom and top detector positions. The total numbers of IBD events per day at the two positions were equal in the calculations of the $R^{\text{pre}}_i$. Thus, the $\chi^2$ does not depend on the integral IBD event rate dependence on the distance from the reactor core. Only differences in the positron energy shapes are considered. This is the most conservative approach. The results do not depend even on the changes of the detector efficiency as long as they do not depend on the positron energy. This approach reduces also the sensitivity of the results to the position of the reactor fuel burning profile center and the reactor power. Figure 3 shows the ratio of positron energy spectra at the bottom and top detector positions.

5. Gaussian CL$_\alpha$ method
The theoretical predictions for 4$\nu$ and 3$\nu$ hypotheses for the ratio of the positron spectra at the two distances at a given point in the $\Delta m^2_{14}, \sin^2 2\theta_{14}$ plane were compared using the difference in $\chi^2$ for the two hypotheses $\Delta \chi^2_{\exp} = \chi^2_{4\nu} - \chi^2_{3\nu}$, where $\chi^2$ is defined in(3). As it was demonstrated in [9] the difference in $\chi^2$ has a Gaussian distribution with the mean value $\mu$ and the standard deviation $\sigma$ calculated using Asimov data set, a data sample with values following exactly
theoretical curve for the corresponding $4\nu$ or $3\nu$ hypotheses and error bars taken from the real experiment. We calculate the corresponding $\Delta\chi^2_{Asimov}$ by putting the experimental points with their statistical errors on the predicted curves for the $4\nu$ or $3\nu$ hypotheses respectively. The obtained $\Delta\chi^2_{Asimov}$ determines the standard deviation of the $\Delta\chi^2_{exp}$: $\sigma = 2\sqrt{\Delta\chi^2_{Asimov}}$. It is the same for the two Asimov data sets while the mean values $\Delta\chi^2_{Asimov}(4\nu)$ and $\Delta\chi^2_{Asimov}(3\nu)$ differ by the sign. Figure 4 shows Asimov dataset for $4\nu$ hypothesis (left), $3\nu$ hypothesis (right) and two corresponding $\Delta\chi^2$ distributions (center). The confidence levels for the $4\nu$ and $3\nu$ hypotheses are calculated as follows:

$$CL_i = \int_{\Delta\chi^2_{exp}}^{\infty} G_i(\mu_i, \sigma_i),$$

where $CL_i$ is $CL_{4\nu}$ or $CL_{3\nu}$, and $G_i(\mu_i, \sigma_i)$ — corresponding Gaussian distribution of $\Delta\chi^2$ ($\sigma_{3\nu} = \sigma_{4\nu}, \mu_{3\nu} = -\mu_{4\nu}$). The $CL_{4\nu}$ and $CL_{3\nu}$ quantify the consistency of the data with the corresponding hypothesis and the $CL_s = CL_{4\nu}/CL_{3\nu}$. The point in the $\Delta m^2_{14}, \sin^2 2\theta_{14}$ plane is excluded at the $1 - \alpha$ confidence level if $CL_s < \alpha$. Therefore the point is excluded only if the $3\nu$ hypothesis fits the data much better than the $4\nu$ hypothesis. Figure 5 shows spectra ratio with $4\nu$ hypothesis (solid line) and two $\Delta\chi^2$ distributions for $4\nu$ (left) and $3\nu$ (right) hypotheses.

Figure 3. Ratio of positron energy spectra measured at the bottom and top detector positions (statistical errors only). The dashed curve is the prediction for $3\nu$ case ($\chi^2 = 35.0$, 24 degrees of freedom). The solid curve corresponds to the best fit in the $4\nu$ mixing scenario ($\chi^2 = 21.9$, $\sin^2 2\theta_{14} = 0.05$, $\Delta m^2_{14} = 1.4$ eV$^2$). The dotted curve is the expectation for the optimum point from the RAA and GA fit [4] ($\chi^2 = 83$, $\sin^2 2\theta_{14} = 0.14$, $\Delta m^2_{14} = 2.3$ eV$^2$).

Figure 4. An example of $\Delta\chi^2$ distributions and Asimov datasets. Shaded areas represent confidence levels for the $4\nu$ ($\chi^2_{4\nu} = 32.8$) and $3\nu$ ($\chi^2_{3\nu} = 35.0$) hypotheses calculated by integration of the two Gaussian distributions ($CL_{3\nu} = 0.948$, $CL_{4\nu} = 0.242$, $CL_s = 0.255$).
6. Systematics

The systematic uncertainties are treated as the nuisance parameters in [9]. The corresponding parameters with their errors are included into the minimization of the $\chi^2$ (see for example Eqn. 5 [9]). We treat the systematic uncertainties differently. We repeat the CL$_s$ analysis without the nuisance parameters for all combinations of the systematic uncertainties taken at their maximal deviations from the nominal values.

A point in the $\Delta m^2_{14}$, $\sin^2 2\theta_{14}$ plane was included into the final excluded area if it appeared in the excluded areas for all tested variations of the parameters. The following variations of the parameters were tested:

- The energy resolution multiplied by the factors 1.1 and 0.9 with respect to the MC predictions;
- A flat background which gives $\pm 0.1\%$ events at the top position of the detector which corresponds to 100% variation of this background;
- A background with the energy distribution identical to the distribution of the background produced by cosmic muons inside the detector. The fraction of such background was $\pm 0.5\%$ of the IBD rate at the top position of the detector which corresponded to $\pm 15\%$ variation of this background;
- The energy scale changed by $\pm 2\%$;
- All possible combinations of changes listed above;
- The reduced range of the energies used in the fit to (1.5-6) MeV.

It was checked that this approach is practically identical (but more transparent) to using the systematic uncertainties as the nuisance parameters with the minimization of the $\chi^2$ over a limited set of deviations of the systematic parameters taken at their maximal values. Since the influence of the systematic uncertainties on the final results is small this approximation of the minimization procedure should be sufficient. Moreover, such approach allows to perform a separate study to check the stability of the results with respect to the change of the energy range used in the analysis.

In addition exclusion contours were also calculated using raster scan method. It considers every particular $\Delta m^2_{14}$ slice separately from the whole $\Delta m^2_{14}, \sin^2 2\theta_{14}$ plane and $\Delta \chi^2$ is determined as follows:

$$\Delta \chi^2_{\Delta m^2, \theta} = \chi^2_{\Delta m^2, \theta} - \chi^2_{\min(\Delta m^2)}.$$ \hspace{1cm} (5)
where $\chi^2_{\text{min}}(\Delta m^2)$ – minimum $\chi^2$ value of particular slice. Point $\Delta m^2_{14}, \sin^2 2\theta_{14}$ is excluded at 90% level if corresponding $\Delta \chi^2 > 2.71$ [1]. Figure 6 shows the comparison between two method. We can conclude that CL$_S$ method is more conservative on average. For some values of $\Delta m^2_{14}$ the obtained limits are more stringent than previous results [10, 11, 12]. It is important to stress that our results are based only on the comparison of the shapes of the positron energy distributions at the two distances from the reactor core measured with the same detector. Therefore the results do not depend on the $\bar{\nu}_e$ spectrum shape and normalization as well as on the detector efficiency. The excluded area covers a large fraction of regions indicated by the GA and RAA. In particular, the most preferred point $\Delta m^2_{14} = 2.3$ eV$^2$, $\sin^2 2\theta_{14} = 0.14$ [4] is excluded at more than 5$\sigma$ CL. In our analysis the point $\Delta m^2_{14} = 1.4$ eV$^2$, $\sin^2 2\theta_{14} = 0.05$ has the smallest $\chi^2 = 21.9$. The difference in $\chi^2$ with the 3$\nu$ case is 13.1. The significance of this difference will be studied taking into account systematic uncertainties after collection of more data.

![Figure 6. 90% CL exclusion area in $\Delta m^2_{14}$, $\sin^2 2\theta_{14}$ parameter space obtained with CL$_S$ and raster scan methods](image)

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