Single Image Super Resolution Enhancement Method Based on Fractal Interpolation of Iteration Function System

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Abstract. Based on the similarity and transitivity of local features of image data, self similarity and scale invariance of fractal features, fractal method of Iterated Function System is used to construct fractal interpolation function, and variable vertical scale factor is used to realize super-resolution enhancement of two-dimensional image data. The experimental results show that fractal interpolation method of iterated function system can keep the image details better, and the reconstruction error is smaller, which is better than bilinear interpolation and bicubic interpolation methods.

1. Introduction

Due to the limitation of imaging conditions, there are some phenomena such as limited resolution and poor quality in the process of image acquisition and transmission, which are difficult to meet the requirements of image processing and analysis. Traditional bilinear interpolation, bicubic interpolation and other methods can not effectively use the external information, resulting in blurred edges and loss of high-frequency information. In recent years, with the rapid development of image super-resolution enhancement technology, some new methods are emerging. At present, there are two kinds of image super-resolution enhancement methods. One is the super-resolution enhancement of a single image [1,2], which mainly uses the prior knowledge, or searches the details matching with the image, that are processed through feature registration from a given image database to improve the resolution. The other is the super-resolution enhancement of multiple images or image sequences [3-5], which extracts the features of multiple low resolution images. The overlapped information is estimated to form a high-resolution image. Because the super-resolution enhancement method of single image has the advantages of less information and less computation, it has become a research hotspot [6,7]. However, in the super-resolution enhancement of single image, the method based on prior knowledge can only be applied to the image that meets the specific assumptions, and the universality of the method is difficult to guarantee. The method based on image database feature registration requires that the image database and the image to be processed have a good correlation, which limits the scope of use [8,9].

The image super-resolution enhancement method based on fractal interpolation of Iteration Function System takes advantage of the similarity and transmissibility of the local features of a single image, the self similarity and scale invariance of the fractal features, and uses fractal interpolation of Iteration Function System to reconstruct the high-resolution image from the low resolution image and complete the purpose of super-resolution enhancement.
2. Related Work

IFS (Iterative Function System) was first proposed by Hutchinson in 1981, and has become an important research content in fractal geometry [10]. Based on affine transformation, IFS is generated iteratively according to the self similar structure between the whole and the part of the geometric object. Fractal interpolation function can be constructed by IFS fractal method, which can deal with some irregular one-dimensional curves or two-dimensional surfaces [11,12]. They are generally nondifferentiable and not random.

2.1. Mathematical basis of IFS fractal interpolation

Suppose that \( I \) represents an interval \([x_0, x_N]\), where \( N \) is a positive integer and \( x_j \) satisfies the following conditions:

\[
x_0 < x_1 < x_2 < \cdots < x_N
\]

For the point \( \{P_j\}, j = 0,1,2,\cdots, N \), \( P_j = (x_j, y_j) \), by constructing the interpolation function \( f : I \rightarrow \mathbb{R} \), we make:

\[
f(x_j) = y_j, i = 0,1,2,\cdots, N
\]  

(1)

The existence of a compact set \( K \) in \( I \times \mathbb{R} \) and a set of continuous transformations \( w_j : K \rightarrow K \) are determined so that the attractor of IFS \( G = \{ (x, f(x)) : x \in I \} \) is unique. Such a function \( f \) is called fractal interpolation function. Generally, \( G \) is fractal and \( f \) is nondifferentiable.

Suppose there are two points \((c_1, d_1), (c_2, d_2) \in K\), whose distance is defined as:

\[
d((c_1, d_1), (c_2, d_2)) = \max \|c_1 - c_2\|, |d_1 - d_2|
\]

Suppose \( I_j = [x_{j-1}, x_j] \), let \( L_j : I \rightarrow I_j, j \in \{1,2,\cdots, N\} \), where \( L_j \) is compressed:

\[
L_j(x_0) = x_{j-1}, L_j(x_N) = x_j
\]

For every \( c_1, c_2 \in I \), there are:

\[
|L_j(c_1) - L_j(c_2)| \leq \beta |c_1 - c_2|
\]  

(2)

The \( \beta \) in the formula are satisfied \( 0 \leq \beta < 1 \). Let \( F_j : K \rightarrow [a,b] \) be continuous, \(-\infty < a < b < +\infty \). There are \( \gamma \) satisfied \( 0 \leq \gamma < 1 \), there are:

\[
F_j(x_0, y_0) = y_{j-1}, \quad F_j(x_N, y_N) = y_j
\]

\[
|F_j(c, d_1) - F_j(c, d_2)| \leq \gamma |d_1 - d_2|
\]

Where \( c \in I, d_1, d_2 \in [a,b], j \in \{1,2,\cdots, N\} \).

Now \( w_j \) can be defined as:

\[
w_j(x, y) = (L_j(x), F_j(x, y)), \quad j \in \{1,2,\cdots, N\}
\]  

(3)

So a fractal interpolation function based on IFS is defined.

2.2. IFS fractal interpolation formula

For the same data set, different fractal interpolation functions can be obtained by taking different \( L_j(x) \) and \( F_j(x, y) \). Here we take the following affine transformation:

\[
L_j(x) = x_{j-1} + \frac{x_j - x_{j-1}}{x_N - x_0} (x - x_0)
\]  

(4)

\[
F_j(x, y) = b_j x + a_j y + k_j
\]  

(5)
Where \( j \in \{1, 2, \cdots, N\} \), \( L_j \) satisfies the requirements of fractal interpolation function, and the constant expressions are as follows:

\[
b_j = \frac{y_j - y_{j-1} - a_j(y_N - y_0)}{x_N - x_0} \\
k_j = y_{j-1} - a_jy_0 - b_jx_0
\]

Where \( j \in \{1, 2, \cdots, N\} \), \( a_j \in (-1, 1) \).

3. Method

For regular grid data such as image, the set of image data points is \( I \). The compression transformation is done in the directions of \( x, y \) and \( z \). The point set is \((x_n, y_m, z_{n,m})\), \( n = 0, 1, \cdots, N \), \( m = 0, 1, \cdots, M \), and \( a \leq x_n \leq b, c \leq y_m \leq d \), the mesh is generated in steps of \( \Delta x \) and \( \Delta y \).

\[
a = x_0 < x_1 < \cdots < x_N = b \\
c = y_0 < y_1 < \cdots < y_M = d
\]

The compression transformation of the three directions is as follows:

\[
\begin{align*}
\varphi_n(x) &= x_{n-1} + \frac{(x_n - x_{n-1})(x - x_0)}{x_N - x_0} \\
\varphi_m(y) &= y_{m-1} + \frac{(y_m - y_{m-1})(y - y_0)}{y_M - y_0} \\
F_{n,m}(x, y, z) &= e_{n,m}x + f_{n,m}y + g_{n,m}xy + \alpha_{n,m}z + k_{n,m}
\end{align*}
\]

(6)

\[
\begin{align*}
g_{n,m} &= z_{n-1,m-1} - z_{n-1,m} - z_{n,m-1} + z_{n,m} - \alpha_{n,m}(z_{0,0} - z_{N,0} - z_{0,M} + z_{N,M}) \\
e_{n,m} &= z_{n-1,m-1} - z_{n-1,m} - \alpha_{n,m}(z_{0,0} - z_{N,0}) - g_{n,m}(x_0y_0 - x_Ny_0 + x_0y_M - x_Ny_M) \\
f_{n,m} &= z_{n-1,m-1} - z_{n-1,m} - \alpha_{n,m}(z_{0,0} - z_{0,M}) - g_{n,m}(x_0y_0 - x_0y_M) \\
k_{n,m} &= z_{n,m} - e_{n,m}x_0 - f_{n,m}y_0 - \alpha_{n,m}z_{N,0} - g_{n,m}x_Ny_M
\end{align*}
\]

(7)

the compression transformation in the direction of \( x \) and \( y \) is determined, and the compression transformation in the direction \( z \) adds the vertical scale factor \( \alpha \), \( 0 \leq \alpha < 1 \). Its function is to control the surface roughness of the image. \( \alpha \) is larger, the image surface is rougher, \( \alpha \) is smaller, the image surface is smoother. According to the principle of compression mapping, we can get a fixed point, which becomes the attractor of the IFS, and the image of the attractor is the final super-resolution enhanced image.

When the dynamic range of the image data is large, the gray value of the image in a certain area changes greatly, there are gentle and sudden changes. If only one value \( \alpha \) is used to reflect the gray characteristics of the whole area, it is small regions, and then the values \( \alpha \) of each small region are calculated respectively, the gray distribution is closer to the real image.

The vertical scale factor \( \alpha \) represents the compression ratio of the gray change feature of the whole region to the gray feature of the small region. The vertical scale factor of the local small region is set as \( \omega \). For the grid data \((x_i, y_j, z_k)\) on the local region, the first-order trend surface equation of the region is fitted by the least square method.
The fitting gray value of each grid node is \( \hat{z}_{i}(x_{i}, y_{i}) \), which is recorded as the trend gray value of each point in the region.

\[
\hat{z} = b_0 + b_1 x + b_2 y
\]

\( \hat{z} \) is recorded as the overall average trend gray of the region. It is also known that the real gray value of each grid node.

\[
e_i = z_i - \hat{z}_i
\]

\[
\delta = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}}
\]

\( \delta \) is recorded as the similar standard deviation of the gray value of the grid node in the region. \( \omega \) is recorded as the vertical scale factor.

\[
\omega = \delta / \overline{z}
\]

Compared with the selection method only considering the deviation value, the selection method of vertical scale factor is related to the similar standard deviation and mean value of the gray value of the region. It not only takes into account the characteristics of regular grid data, but also takes into account the trend change of the gray value of the region, and can reflect the gray change characteristics of the region more truly.

4. Experiments

Select the test images Lena and camera, the measured images missile, fighter and warship. At first downsampling the original image with a factor of 2, and then using bilinear interpolation, bicubic interpolation and IFS fractal interpolation to enhance the images, as shown in Figure 1.
The results of three super-resolution enhancement methods are compared with the original image, and the statistics on the root mean square error are shown in Table 1.

| Image    | Bilinear Interpolation | Bicubic Interpolation | IFS Fractal Interpolation |
|----------|------------------------|------------------------|---------------------------|
| Lena     | 8.4535                 | 6.7717                 | 3.0143                    |
| Camera   | 7.7461                 | 5.6436                 | 3.2769                    |
| Missile  | 8.0059                 | 6.0860                 | 4.0536                    |
| Fighter  | 6.1239                 | 4.6505                 | 3.7831                    |
| Warship  | 8.1239                 | 7.7505                 | 4.4866                    |

It shows the results in Figure 1 and Table 1 that compared with bilinear interpolation and bicubic interpolation, the IFS fractal interpolation based super-resolution enhancement method can not only keep the details of the image better, but also ensure a smaller reconstruction error.

5. Conclusion
The image super-resolution enhancement method based on IFS fractal interpolation makes use of the self-similarity and scale invariance of fractal set, which can realize the enhancement of low resolution image in a single image. Compared with bilinear interpolation and bicubic interpolation, it can keep the image details better, and the root mean square error with the original image is smaller. Image super-resolution enhancement can provide a better foundation for target detection and recognition, and its research prospect is very broad.
References

[1] Li, X. (2011) Image Recovery via Hybrid Sparse Representation: a Deterministic Annealing Approach. IEEE J. of Selected Topics in Signal Processing, special issue on adaptive sparse representation, 5: 953–962.

[2] Hemalatha, V., Ranjan, A. (2012) Application of Super-Resolution Reconstruction of Low-Resolution Image Sequences in Spatial Domain. International Journal of Advanced Research in Computer Science, 3: 528–530.

[3] Zeyde, R., Elad, M., Protter, M. (2010) On Single Image Scale-Up Using Sparse-Representations. In: Proceedings of the 7th International Conference on Curves and Surfaces. Avignon, France. pp. 711-730.

[4] Yang, S.Y., Wang M., Chen, Y.G., Sun, Y.X. (2012) Single-Image Super-Resolution Reconstruction via Learned Geometric Dictionaries and Clustered Sparse Coding. IEEE Transactions on Image Processing, 21: 4016-4028.

[5] Chaurasia, V., Somkuwar A. (2010) Review of a Novel Technique: Fractal Image Compression. International Journal on Emerging Technologies, 1: 53-56.

[6] Kang, L., Wu, L., Yang, Y.H. (2013) A Novel Unsupervised Approach for Multilevel Image Clustering from Unordered Image Collection. Frontiers of Computer Science, 7: 69-82.

[7] Zhang, Y., Fan, Q., Bao, F., Zhang C. (2018) Single-Image Super-Resolution Based on Rational Fractal Interpolation. IEEE Transactions on Image Processing, 27: 3782-3797.

[8] Huang, J.B., Singh, A., Ahuja, N. (2015) Single Image Super-Resolution from Transformed Self-Exemplars. In: Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition. Boston, MA, USA. pp. 5197-5206.

[9] Fang, L., Zhuo, H., Li, S. (2018) Super-Resolution of Hyperspectral Image via Superpixel-Based Sparse Representation. Neurocomputing, 273: 171-177.

[10] Wee, Y., Shin, H. (2010) A Novel Fast Fractal Super Resolution Technique. IEEE Transactions on Consumer Electronics, 56: 1537-1541.

[11] Yu, L., Xu, Y., Xu, H., Yang, X. (2013) Self-Example Based Super-Resolution with Fractal-Based Gradient Enhancement. In: Proceedings of the IEEE International Conference on Multimedia and Expo Workshops. San Jose, CA, USA. pp. 1-6.

[12] Yao, X., Wu, Q., Zhang, P., Bao, F. (2019) Adaptive Rational Fractal Interpolation Function for Image Super-Resolution via Local Fractal Analysis. Image and Vision Computing, 82: 39-49.