A possible 3:2 orbital epicyclic resonance in QPOs frequencies of Sgr A*

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Received ...; accepted ...

Abstract. A recent measurement of double peak QPOs frequencies in Sgr A* is consistent with the 3:2 ratio. The same ratio is firmly established by previous observations in all double peak kHz QPOs in microquasars and theoretically explained by orbital epicyclic resonances excited in nearly Keplerian accretion flow in black hole’s strong gravity. If confirmed, the 3:2 ratio of double peak QPOs in Sgr A* will be of a fundamental importance for the black hole accretion theory, by providing another clear argument that the accretion disk oscillations are indeed governed by non-linear, strong-gravity physics.

Key words. black holes – X-ray variability – Sgr A* – observations – theory

1. Double peak QPOs with the 3:2 ratio in Sgr A*?

From the current analysis of stellar orbits within 10-100 light hours of Sgr A*, obtained independently by the MPI Garching group (Schoedel et al. 2002, Schoedel et al. 2003, Eisenhauer et al. 2003) and the UCLA group (Ghez et al. 2003, Ghez et al. 2004) the best estimate of the black hole central mass is $3.6 \pm 0.4 \times 10^6 \, M_\odot$, where the error bars represent both statistical and systematic errors. Earlier lower statistical mass estimates based on proper motions of stars further away gave somewhat lower masses ($2.6 \times 10^6 \, M_\odot$) but in light of new information on stellar distribution and anisotropies these earlier data would now also lead to masses near $3.5 \times 10^6 \, M_\odot$ (see the discussion in Schoedel et al. 2003). This well constrained mass must be contained within a few light hours, i.e. several hundred Schwarzschild radii. The analysis of the spatial distribution of the stellar cusp centered on the BH suggests that most likely no more than $1 \times 10^3 \, M_\odot$ of that is in form of stars or stellar remnants (the latter is less well constrained; Genzel et al. 2003). From the lack of motion of the radio source itself and a theoretical comparison of the stochastic motions of a BH of different masses with surrounding stars, a lower limit of the mass contained within the radius of the radio source (10 light minutes, 20 Schwarzschild radii) is about $\sim 10^5 \, M_\odot$ (Reid et al. 1999, Reid et al. 2003, Backer & Sramek 1999, Schoedel et al. 2003).

From these measurements and discussion, one concludes that the mass of the black hole in Sgr A* is most likely in the interval

$$2.6 \times 10^6 \, M_\odot < M < 4.4 \times 10^6 \, M_\odot,$$

and that a very conservative lower limit is $\sim 10^5 \, M_\odot$.

Genzel et al. 2003 measured a clear periodicity of 17 min (1020 sec) in Sgr A* variability during a flaring event. This period is in the range of Keplerian orbital periods at a few gravitational radii away from a black hole with the mass constrained by (1). More recently, Aschenbach et al. 2004 have reported three other QPOs periodicities, 692 sec, 1130 sec, 2178 sec, roughly in the orbital Keplerian range, and two much shorter periods of 100 sec and 219 sec. The value of 1130 sec differ by 10% from the 1020 sec period found by Genzel et al. 2003 and may correspond to the same periodicity of the source, but a firm conclusion is not possible because of the quality of data. With all reservation and caution that are necessary here, it was noticed (Abramowicz et al. 2004a, Abramowicz et al. 2004b, Aschenbach 2004) that,

$$(1/692) : (1/110) : (1/2178) \approx 3 : 2 : 1,$$

We thank Reinhard Genzel for providing (in summer 2004) this updated discussion on the Sgr A* mass measurements.
i.e. that the “Keplerian” frequencies found in Sgr A* form ratios that are very close to be an exact commensurable sequence, 3:2:1. The commensurability of QPOs frequencies in Sgr A*, if confirmed by a more accurate observations and data analysis, could be of a fundamental importance for a reason that we explain in the next section.

2. Commensurability of QPOs in microquasars: observations and theory

In the case of microquasars, the 3:2 ratio was found in all four sources with double peak QPOs detected. Table II and Figure II summarize the QPOs microquasars data relevant to the present Note. Impressively accurate ratio $\nu_{\text{upp}}/\nu_{\text{down}} = 3/2$ of frequencies was found in all four microquasars that display the double peak QPOs. In three microquasars with known mass $M$, the QPOs frequencies scale as \( \nu_{\text{upp}} = 2.8 \left( \frac{M}{M_\odot} \right)^{-1} \) [kHz].

$$\nu_{\text{upp}} = 2.8 \left( \frac{M}{M_\odot} \right)^{-1} \text{[kHz]}.$$ (3)

### Table 1. Frequencies of twin peak QPOs in microquasars and Galaxy centre black hole

| Source          | $\nu_{\text{upp}}$ [Hz] | $\Delta \nu_{\text{upp}}$ [Hz] | $\nu_{\text{down}}$ [Hz] | $\Delta \nu_{\text{down}}$ [Hz] | $2\nu_{\text{upp}}/3\nu_{\text{down}} - 1$ | Mass $^b$ [M$_\odot$] |
|-----------------|--------------------------|---------------------------------|---------------------------|-----------------------------------|-----------------------------------------------|------------------------|
| GRO 1655–40     | 450 ± 3                  |                                 | 300 ± 5                   |                                   | 0.00000                                      | 6.0 — 6.6              |
| XTE 1550–564    | 276 ± 3                  |                                 | 184 ± 5                   |                                   | 0.00000                                      | 8.4 — 10.8             |
| H 1743–322      | 240 ± 3                  |                                 | 166 ± 8                   |                                   | -0.03614                                     | not measured           |
| GRS 1915+105    | 168 ± 3                  |                                 | 113 ± 5                   |                                   | 0.00885                                      | 10.0 — 18.0            |
| Sgr A*          | 1.445 ± 0.16 mHz         | 0.886 ± 0.04 mHz                |                           |                                   | 0.08728                                      | 2.6 — 4.4 $10^6$      |

$^a$ Twin peak QPOs first reported by Strohmayer 2001, Remillard, Muno, McClintock & Orosz 2002, Homan et al. 2003, Remillard, Muno, McClintock & Orosz 2003, and Aschenbach et al. 2004.

$^b$ See Greene, Bailyn, Orosz 2001, Orosz et al. 2002, Greiner, Cuby, McCaughrean 2001, McClintock & Remillard 2003, and the first part of introduction.

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**Fig. 1.** Left: In all four microquasars where double peak kHz QPOs were detected, the observed frequencies $\nu_{\text{upp}}$ and $\nu_{\text{low}}$ are clearly in 3:2 ratio. Right: The same 3:2 ratio seems to be present in double peak QPOs in Sgr A*. The accuracy is so high that the error bars cannot be shown correctly in this logarithmic plot.

Even before the double peak kHz QPOs have been discovered in microquasars (first by Strohmayer 2001), and the 3:2 ratio pointed out (first by Abramowicz & Kluźniak 2001, Kluźniak & Abramowicz 2000) suggested on theoretical ground that these QPOs should have rational ratios, being due to resonances in oscillations of nearly Keplerian accretion disks. It seems that the resonance hypothesis is now well supported by observations, and that in particular the 3:2 ratio is seen most often in double peak QPOs in LMXB black hole and neutron sources, $2\nu_{\text{upp}} = 3\nu_{\text{down}}$.

According to the resonance hypothesis, the two modes in resonance have eigenfrequencies $\nu_{\text{rad}}$, equal to the radial epicyclic frequency, and $\nu_{\text{v}}$, equal to the vertical orbital frequency $\nu_{\text{vert}}$ or the Keplerian frequency $\nu_{\text{K}}$ (see Kluźniak & Abramowicz 2004).
for recent review). Several resonances of this kind are possible, and have been discussed (see e.g. [Abramowicz et al. 2004b]). Main relations are summarized in Table 2.

Table 2. Relation for observed frequencies for standard ($\nu = \nu_{\text{vert}}$) and “Keplerian” ($\nu = \nu_K$) resonances

| Theory                  | Type of resonance | $n\nu_{\text{rad}} = m\nu$ | Observed frequencies |
|-------------------------|-------------------|-----------------------------|----------------------|
|                         |                   | $n$ | $m$ | $\nu_{\text{vert}}$ | $\nu_{\text{rad}}$ |
| parametric              | 3:1 forced        | 3   | 1   | $\nu_{\text{vert}}$ | $\nu_{\text{vert}} - \nu_{\text{rad}}$ |
|                         | 2:1 forced        | 2   | 1   | $\nu_{\text{vert}} + \nu_{\text{rad}}$ | $\nu_{\text{vert}}$ |
| Keplerian standard      | 3:1 forced        | 3   | 1   | $\nu_K$ | $\nu_{\text{vert}} - \nu_{\text{rad}}$ |
|                         | 2:1 forced        | 2   | 1   | $\nu_K + \nu_{\text{rad}}$ | $\nu_K$ |

Formulae for $\nu_{\text{vert}}$ and $\nu_{\text{rad}}$ in the gravitational field of a rotating Kerr black hole with the mass $M$ and spin $a$ are well known,

$$\nu_{\text{vert}}^2 = \nu_K^2 \left( 1 - 4 ax^{-1/2} + 3a^2 x^{-2} \right), \quad \nu_{\text{rad}}^2 = \nu_K^2 \left( 1 - 6x^{-1} + 8ax^{-3/2} - 3a^2 x^{-2} \right), \quad \nu_K = \frac{1}{2\pi} \left( \frac{GM_0}{r_G^3} \right)^{1/2} \left( x^{3/2} + a \right)^{-1}. \quad (4)$$

Here $x = r/(GM/c^2)$ is the dimensionless radius, expressed in terms of the gravitational radius of the black hole.

For a particular resonance $n:m$, the equation $n\nu_{\text{rad}} = m\nu$ ($\nu = \nu_{\text{vert}}$ or $\nu_K$) determines the dimensionless resonance radius $x_{n:m}$ as a function of $a$.

3. Application to Sgr A*

From the known mass of Sgr A*, the observed $\nu_{\text{down}} = 0.886$ mHz = 1/1130 sec$^{-1}$, and from the equation (4) on may calculate the black hole spin in Sgr A*, consistent with different types of resonances. This procedure was first applied to the microquasar GRO 1655–40 by [Abramowicz & Kloźniak 2001] and more recently for the other two microquasars by [Abramowicz et al. 2004b] and [Török et al. 2005]. These results, together with these calculated in this Note for five representative values of the Sgr A* mass, are summarized in Table 3 and illustrated in Figure 2 for particular resonances.

Table 3. Sgr A* spin estimates from observed 3:2 QPOs, calculated for five representative values of mass outgoing from large spectrum above lower conservative limit include the best mass estimate $3.6 \times 10^{8} M_{\odot}$

| Resonance | Mass [$M_{\odot}$] : |
|-----------|---------------------|
|           | $4.4 \times 10^{5}$ | $0.8 \times 10^{6}$ | $1.8 \times 10^{6}$ | $2.2 \times 10^{6}$ | $2.6 \times 10^{6}$ | $3.6 \times 10^{6}$ | $4.4 \times 10^{6}$ |
| $3:2 [\nu_o, \nu_i]$ parametric | — | 0.22 | 0.90 | 0.98 | — | — | — |
| $2:1 [\nu_o, \nu_i]$ forced | — | — | 0.16 | 0.40 | 0.57 | 0.81 | 0.92 |
| $3:1 [\nu_o, \nu_i]$ forced | — | — | 0.36 | 0.58 | 0.72 | 0.95 (0.99)* | — |
| $3:2 [\nu_K, \nu_i]$ “Keplerian” p. | — | 0.25 | — | — | — | — | — |
| $2:1 [\nu_K, \nu_i]$ “Keplerian” f. | — | — | 0.16 | 0.41 | 0.58 | 0.83 | 0.94 |
| $3:1 [\nu_K, \nu_i]$ “Keplerian” f. | — | — | 0.32 | 0.52 | 0.65 | 0.85 | 0.93 |

* see Figure 2

4. Discussion and conclusions

If commensurability of double peak QPOs frequencies in Sgr A* is confirmed, this together with the already established $1/M$ scaling, would give a very strong support for the suggestion that the double peak QPOs physics, the same in microquasars and in Sgr A*, is due to a non-linear orbital resonance in strong gravity. It would be interesting to see whether other black hole sources, ULXs and AGNs, show the same phenomenon ([Abramowicz et al. 2004a]).
For black hole sources with known mass that display the double peak QPOs, one may measure the black hole spin, but the spin estimate depends on which of the theoretically possible resonance, 2:1, 3:1, or 3:2, is actually excited in the source. At present, neither observations, nor the resonance theory could firmly determine this\(^2\).

Acknowledgements. I thank Marek Abramowicz and Włodek Kluzniak and also Zdeněk Stuchlík and Vladimir Karas for discussion and help. The article was partially written under the Erasmus-Socrates exchange program between Chalmers University and Silesian University at Opava. The final version was completed at Nordita.

References

Abramowicz M.A., Kluzniak W., 2001, A&A 374L, 19A
Abramowicz M.A., Kluzniak W., McClintock & Remillard, 2004a, ApJ 609, L63
Abramowicz M.A., Kluzniak W., Stuchlik Z., Torok G., 2004b, submitted to A&A, astro-ph/0401464
Abramowicz M.A., Kluzniak W., 2004, X-Ray Timing 2003: Rossi and Beyond, ed. P. Kaaret, F. K. Lamb, J. H. Swank (Melville, NY: American Institute of Physics)
Abramowicz M.A., Kluzniak W., Stuchlik Z., Torok G., 2004c, In S. Hledík and Z. Stuchlík, editors, Proceedings of RAGtime 4/5: Workshops on black holes and neutron stars, Opava, 1416/1315 October 2002/03, Opava, 2004. Silesian University at Opava.
Aschenbach B., Grosso N., Porquet D., Predehl P., 2004, accepted by the A&A, astro-ph 040158
Aschenbach B., 2004, accepted by A&A, astro-ph 0406545
Backer D. C., Sramek R. A., 1999, ApJ, 524, 805
Eisenhauer et al. 2003, ApJ 597, L121
Genzel et al. 2003, ApJ 594, 812
Ghez et al. 2001, ApJ, 554, 1290G
Greiner J., Cuby J. G., McCaughrean M. J., 2001, Natur., 414, 522G
Homan J., Miller J.M., Wijnands R., Steeghs D., Belloni T., van der Klis M., Lewin W. H. G., 2003, Atel 16 [http://integral.rssi.ru/atelmirror](http://integral.rssi.ru/atelmirror)
Klužniak W., Abramowicz M.A., 2000, Phys. Rev. Lett. submitted, astro-ph/0105057
Klužniak W., Abramowicz M.A., 2003, 12th Workshop on General Relativity and Gravitation, Tokyo: Tokyo University Press, astro-ph/0304345
Klužniak W., Abramowicz M.A., 2004, in “X-Ray Timing 2003: Rossi and Beyond”, ed. P. Kaaret, F. K. Lamb, J. H. Swank (Melville, NY: American Institute of Physics)
McClintock J.E, Remillard R.A., 2003, astro-ph/0306213 v.2
Orosz J. A., Groot P. J., van der Klis M., McClintock J. E., Garcia M. R., Zhao P., Jain R. K., Bailyn Ch. D., Remillard R. A., 2002, ApJ, 568, 845O

\(^2\) Aschenbach 2004 argues that QPOs data suggests that black holes in three microquasars and Sgr A* listed in Table 1 have nearly the same spin \(a \approx 0.99616\), due to a new relativistic effect that he found (a non monotonic behaviour of orbital velocity with radius for rapidly rotating black holes, see also Stuchlík et al. 2004). Our calculations are based on standard types of non-linear orbital resonances, and do not include the Aschenbach effect.
Remillard, R. A., Muno, M. P., McClintock, J. E., Orosz, J. A. 2002, ApJ, 580, 1030
Remillard, R. A., Muno, M. P., McClintock, J. E., Orosz, J. A. 2003, HEAD....7.3003R
Reid at al. 1999, ApJ, 524, 816
Reid et al. 2003, Astron. Nachr./AN 324, No. S1, 3 9 2003, astro-ph/0304095
Schoedel et al. 2002, Nature 419, 694
Schoedel et al. 2003, ApJ 596, 1015
Stuchlik, Z., Slany, P., Török, G., Abramowicz M.A., 2004, Phys.Rev.D., submitted, gr-qc/0411091
Strohmayer T., 2001, ApJ 552L, 49S
Török G., Abramowicz M.A., Kluźniak W., Stuchlik Z., 2005, submitted to A&A, astro-ph/0401464