Electron-Electron Interactions and the Hall-Insulator

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Abstract

Using the Kubo formula, we show explicitly that a non-interacting electron system cannot behave like a Hall-insulator, i.e., a DC resistivity matrix $\rho_{xx} \to \infty$ and $\rho_{xy} = \text{finite}$ in the zero temperature limit, as has been observed recently in experiment. For a strongly interacting electron system in a magnetic field, we illustrate, by constructing a specific form of correlations between mobile and localized electrons, that the Hall resistivity can approximately equal to its classical value. A Hall-insulator is realized in this model when the density of mobile electrons becomes vanishingly small. It is shown that in non-interacting electron systems, the zero-temperature frequency-dependent conductance generally does not give the DC conductance.

73.40 -c, 73.40. Hm
A two-dimensional electron system (2DES) in a perpendicular magnetic field displays a rich variety of behaviors at low temperatures as a result of a delicate interplay between localization and electron-electron interaction. The most interesting phenomenon for this 2DES is the quantum Hall effect (QHE), which has been a focus of tremendous amount of research effort for more than a decade. Recently, much attention has been directed to the behavior of the Hall resistivity for a 2DES in an insulating phase. Magneto-transport studies of the 2DES have shown, among many other interesting properties, that the Hall resistivity of a 2DES in a perpendicular magnetic field equals approximately the disorder-free value $\rho_{xy} \sim B/nec$ for a wide range of applied magnetic field strength, except near the QHE plateaus, where $B$ is the applied magnetic field, $n$ is electron density of the 2DES, $e$ and $c$ are the electron charge and speed of light, respectively. The fact that a large portion of the electrons may become localized by disorder scattering does not affect the value of $\rho_{xy}$ substantially. This is in sharp contrast to the behavior of the diagonal resistivity, which is found to change from $\rho_{xx} \to 0$ in the QHE phase to $\rho_{xy} \to \infty$ in an insulating phase. This implies, in particular, the existence of a new insulating phase with $\rho_{xx} \to \infty$ and $\rho_{xy} = \text{finite}$, the so-called Hall-insulating phase, which has generated much experimental and theoretical work recently. The goal of this paper is to show that the Hall-insulating behavior is necessarily an interaction effect. It cannot be explained with any independent-particle model.

It is well known that there exists a different kind of insulator characterized by $\rho_{xx} \to \infty$ and $\rho_{xy} \to \infty$ in a magnetic field, which will be called conventional insulators here for convenience. Examples of the conventional insulators are band insulators in semiconductors and the bulk Anderson insulators. The difference between a conventional insulator and a Hall-insulator is determined by the different behaviors in the Hall resistivities in a magnetic field: while $\rho_{xy}$ in a conventional insulator is a measure of density of mobile carriers $n_c$; i.e., $\rho_{xy} \sim B/n_c$, in a Hall-insulator it is not, but rather $\rho_{xy} \sim B/n$. A successful description of the Hall-insulating behavior should explain the origin of the difference in $\rho_{xy}$. So far, much of the theoretical work on the Hall-insulator has been based on calculating the
zero-temperature frequency-dependent conductivity tensor, where a finite value of $\rho_{xy}$ is achieved by finding $\sigma_{xx} \sim i\omega$ and $\sigma_{xy} \sim \omega^2$ in the low frequency $\omega \to 0$ limit. We shall see that this is not a complete description. If one naively takes $\sigma(\omega \to 0)$ to be the DC conductivity, one would conclude that all insulators are Hall-insulators. In fact, we will see that one can not, in general, take the $T \to 0$ limit before taking $\omega \to 0$.

In an insulating phase, the lowest energy extended state is above the Fermi level by a finite energy difference. Low temperature electrical current is carried by electrons thermally activated to the extended states. If the activated carriers are effectively decoupled from the localized electrons in the background, a transport experiment becomes a measurement of the mobile carriers only. As we will show below, the system is then a conventional insulator. Therefore, in order to have a Hall resistivity $\rho_{xy}$ which is not a measure of the mobile carrier density $n_c$, but a measure of the total electron density $n$ as $\rho_{xy} \sim B/n$, there must exist a strong correlation between the activated mobile electrons and the localized electrons in the background. This simple argument directly implies that a non-interacting electron system cannot become a Hall-insulator. We will exploit this idea and show that the Hall-insulating behavior can be realized for electrical conduction by certain kind of correlated excitation.

There is another unsettled question related to the Hall-insulator. It is about its ground state: is it essentially a pinned Wigner crystal or a new type of insulator? The assumption of a Wigner crystal is found to be qualitatively consistent with some results from the transport study [6], radio-frequency measurement [7], and photoluminescence experiment [8]. However, a ground state of disorder-localized electrons rather than a pinned Wigner crystal has been suggested by a recently proposed globe phase diagram [9] and the observation of a similar Hall-insulating phase [3] at the Landau level filling factor $\nu > 2$. Our work reported here does not settle the question of which ground state is correct in a given situation, but rather shows that the Hall resistivity does not necessarily distinguish between these possibilities. We will see, instead, that it is the properties of the excited states that determine the Hall resistivity.

In the following, we will first discuss the non-interacting 2DES and show explicitly that
it can not display Hall-insulating behavior. We will then discuss the interacting 2DES with correlated excitations and explain how the Hall-insulating behavior is realized.

We start with the Kubo formula for conductivity $\sigma_{ij}$

$$\sigma_{ij}(T, \omega) = \frac{i ne^2}{\omega} \frac{\delta_{ij}}{m} + \frac{i}{\omega} \Pi_{ij}(T, \omega).$$

(1)

For non-interacting electrons with an arbitrary disorder, the correlation function $\Pi_{ij}(T, \omega)$ can be evaluated using the complete set of eigenfunctions of the Hamiltonian. It is straightforward to obtain the result

$$\Pi_{ij}(T, \omega) = \frac{1}{A} \sum_{nm} J_{nm}^i J_{mn}^j \frac{f(\epsilon_n) - f(\epsilon_m)}{\epsilon_n - \epsilon_m + \omega + i0},$$

(2)

where $n$ and $m$ label the eigenstates of the Hamiltonian ($H = \sum_i h_i$ and $h|n> = \epsilon_n|n>$), $J_{nm}$ is the matrix element of current operator, and $f(\epsilon)$ is the Fermi distribution function. From the above expression, one can prove the following properties of the conductivity matrix:

1) Im[$\sigma_{ij}(T, \omega)$] is an odd function of $\omega$

$$\text{Im}[\sigma_{ij}(T, \omega)] = O(\omega) + O(\omega^3) + O(\omega^5) + \ldots.$$  

(3)

2) Re[$\sigma_{ij}(T, \omega)$] is an even function of $\omega$

$$\text{Re}[\sigma_{ij}(T, \omega)] = \text{Re}[\sigma_{ij}(T, 0)] + O(\omega^2) + O(\omega^4) + \ldots.$$  

(4)

In the following, we show that if the system is insulating, it must be a conventional insulator. We will reach this conclusion by calculating the finite-temperature DC conductivity and then taking the low temperature limit.

From Eq. (3) and Eq. (4), we can see that $\text{Im}[\sigma(T)] = 0$ for $\omega = 0$. We only need to consider the real parts, which are easily obtained from Eq. (2). The diagonal conductance is

$$\sigma_{xx}(T) \sim \sum_{nm} |J_{nm}^x|^2 \frac{\partial f(\epsilon_n)}{\partial \epsilon_n} \delta(\epsilon_n - \epsilon_m).$$

(5)

For any localized state $|n\rangle$, we have
Inserting the above expression into Eq. (5), we see that only extended states can contribute to $\sigma_{xx}$, because the $\delta$-function requires $\epsilon_n = \epsilon_m$. Now for an insulator, suppose that the lowest extended state has an energy $\Delta$ away from the Fermi level, Eq. (5) gives

$$\sigma_{xx}(T) \sim e^{-\beta \Delta}, \text{ for } T \to 0. \tag{7}$$

The off-diagonal conductance has the form

$$\sigma_{xy}(T) \sim \sum_n f(\epsilon_n) \sum_m \frac{J^x_{nm} J^y_{mn} - J^y_{nm} J^x_{mn}}{(\epsilon_n - \epsilon_m)^2} \tag{8}$$

Any localized state can not contribute to $\sigma_{xy}$. For example, if $< n|$ is localized, then we are allowed to use Eq. (6), and the contribution from the state $< n|$ is,

$$\sum_m \frac{J^x_{nm} J^y_{mn} - J^y_{nm} J^x_{mn}}{(\epsilon_n - \epsilon_m)^2} = \sum_m [x_{nm} y_{mn} - y_{nm} x_{mn}] \tag{9}$$

The summation $\sum_n$ in Eq. (8) may then be restricted to extended states only. If the lowest energy extended state is $\Delta$ above the Fermi energy, Eq. (8) gives

$$\sigma_{xy}(T) \sim e^{-\beta \Delta}, \text{ for } T \to 0. \tag{10}$$

Putting together Eq. (7) and Eq. (10), one finds

$$\rho_{xy}(T) = \frac{\sigma_{xy}(T)}{\sigma_{xx}^2(T) + \sigma_{xy}^2(T)} \sim \frac{e^{-\beta \Delta}}{e^{-2\beta \Delta}}$$

$$= \infty \text{ for } T \to 0. \tag{11}$$

Similarly, one has $\rho_{xx} \to \infty$.

The above result shows that the non-interacting electron insulator is a conventional insulator. We have not made any approximation in our derivation, except the restriction that we consider only non-interacting electrons with a gap between the Fermi level and the lowest extended state. We therefore conclude that non-interacting electrons can not behave like Hall-insulators.
Several authors have described an AC form of the Hall-insulator, by calculating the zero-temperature AC conductivity and then considering the low frequency limit; i.e., they took the limit $T \to 0$ first and the limit $\omega \to 0$ second. This method will give a finite value for $\rho_{xy}$ when $\rho_{xx} \to \infty$. It is easy to see that this does not describe the DC conductivity and could mislead one to conclude that every insulator is a Hall-insulator.

For an insulator at zero temperature, $\text{Re} \left[ \sigma_{ij} (T = 0, \omega = 0) \right] = 0$. From Eq. (3) and Eq. (4), we have:

$$\sigma_{xx}^2 (0, \omega) + \sigma_{xy}^2 (0, \omega) = O(\omega^2) + \text{higher power in } \omega.$$  

$$\text{Re} [\sigma_{xy} (T, \omega)] = O(\omega^2) + \text{higher powers in } \omega.$$  

Then we have

$$\rho_{xy} = \frac{\text{Re} [\sigma_{xy}]}{\sigma_{xx}^2 + \sigma_{xy}^2} \bigg|_{\omega \to 0} \sim \frac{\omega^2}{\omega^2} = \text{constant}. \quad (12)$$

If one associates $\rho_{xy}$ with the DC Hall resistivity of the system, then every insulator would be a Hall-insulator. However, this is not consistent with experiment, suggesting that interchanging the order of $T \to 0$ and $\omega \to 0$ is in general not valid.

We will now argue that if correlations, in the Laughlin-Jastrow sense \cite{11}, are important between conduction electrons and the localized electrons, then the Hall-insulating behavior may be obtained. We consider low temperature activated conduction of a 2D electron gas in an insulating phase. Let $N = N_c + N_L$ and $n = N/A, n_c = N_c/A, n_L = N_L/A$, where $A$ is the system size and $N, N_c, N_L$ are respectively the number of electron in the system, the number of electrons in extended states, and the number of electrons in localized states, with $n, n_c, n_L$ the corresponding densities. $N_c$ is activated, so $N_c \propto e^{-\Delta/\kappa T}$ as $T \to 0$. We consider only the motion of conduction electrons and treat the remaining localized electrons as a scattering source. Although the electrons below $E_F$ are well localized, they are still dynamic and responsive to the motion of the conduction electrons. This dynamic correlation is a very complicated problem. We would like to find a effective Hamiltonian to represent the coupling between the conduction electrons and the localized electrons, which is much simpler and yet it still gives the same effect on the motion of the conducting electrons. This
may be achieved using a Chern-Simons approach \[12\].

The Hamiltonian for the conduction electrons is

\[
H_c = \sum_{i=1}^{N_c} \left[ -\frac{\hbar}{2m} \nabla_i - \frac{e}{c} A^{\text{ext}}(\mathbf{r}_i) \right]^2 + \frac{1}{2} \sum_{ij=1}^{N_c} V(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i=1}^{N_c} U(\mathbf{r}_i) + H_{cL},
\]

(13)

where \( A^{\text{ext}} \) is the vector potential for applied magnetic field \( B^{\text{ext}} \), \( V(\mathbf{r}) \) is the electron-electron interaction between a pair of the conducting electrons, \( U(\mathbf{r}) \) is disorder scattering potential. \( H_{cL} \) is the interaction between the conduction electrons and the remaining localized electrons, which is important in strongly correlated systems and presumably is the term responsible for Hall-insulator behavior. Our next step is to determine a mean-field form of \( H_c \) which characterizes correctly the influence of the localized electrons on the motion of the conduction electrons. It is enlightening to recall an earlier study \[13\] on correlated interstitials in a weakly disordered Wigner crystal, where one finds that energetically favored excitations are described by the wavefunction

\[
\Psi_{\text{corr}}(z_o) = \Psi_{\text{uncorr}} \prod_{i=1}^{N_L} (z_i - z_o)^{m_i},
\]

(14)

where \( z_i = x_i - iy_i \) are the lattice electrons in complex notation, \( z_o \) is the interstitial coordinate, and \( \nu \) is the Landau level filling fraction with \( B^{\text{ext}} = n\phi_o/\nu \). The values of \( m_i \) may be chosen to minimize the energy of the excitation, and we argued in Ref. [13] that this may be accomplished if \( < m_i > = 1/\nu \), where \( <> \) denotes an average over lattice sites. \( \Psi_{\text{uncorr}} \) describes an uncorrelated interstitial, which in a Hartree-Fock approximation would simply be given by a Gaussian orbital at some favorable interstitial site in the lattice. The addition of the Jastrow factor introduces correlations, and it may be shown \[13\] that its introduction converts the excitation into a delocalized state. Antisymmetrization corrections between the interstitial and the lattice electrons have been shown to be small \[13\].

One can clearly see that the physics of such wavefunction is more general than the Wigner crystal context in which it was derived. In particular, one can choose \( \Psi_{\text{uncorr}} \) to be any insulating state of an excited electron, and introducing the Jastrow factor creates an excited electron in an extended state, provided \( < m_i > = 1/\nu \).
The Jastrow factor in Eq. (14) may be thought of as attaching a flux tube of strength $m_i \phi_o$ to the $i$th localized electron [11]. If we consider the longwavelength response of the conduction electron, then an appropriate mean-field Hamiltonian will account for interaction with the localized electron by an additional field $B^{cL} = n_L \phi_o / \nu$, we then have

$$H_{MF}^c = \sum_{i=1}^{N_c} \left[ \frac{-i\hbar}{2m} \nabla_i - \frac{e}{c} A^{net}(r_i) \right]^2 + \frac{1}{2} \sum_{ij=1}^{N_c} V(r_i - r_j) + \sum_{i=1}^{N_c} U(r_i), \quad (15)$$

where $A^{net}$ is the vector potential for the net magnetic field $\nabla \times A^{net} = B^{net} = B^{ext} - B^{cL} = n_c \phi_o / \nu$. DC transport properties of the conduction electrons described by $H_{MF}^c$ of Eq. (15) are easily obtained from the Kubo formula in the memory-function formalism [15]

$$\rho_{ij} = i \Omega_{ij} + \frac{m n_c e^2}{n_c e^2 \Gamma_{ij}}, \quad (16)$$

where $i \Omega_{xx} = i \Omega_{yy} = 0$ and $i \Omega_{xy} = -i \Omega_{yx} = B^{net} / (n_c e c) = B^{ext} / (n c e)$. Since the electrons are in extended states, the disorder scattering can be treated perturbatively. To the lowest order in the disorder potential, $\Gamma_{xy} = \Gamma_{yx} = 0$. Denoting $\Gamma_{xx} = \Gamma_{yy} = 1 / \tau$, we have

$$\rho_{xx} = \frac{m}{n_c e^2 \tau}, \quad \rho_{xy} = \frac{B^{ext}}{n c e}, \quad (17)$$

The above result shows clearly that $\rho_{xy}$ depends only on the external magnetic field and the number of total electrons in 2D system, independent of how many of the electrons are localized.

The idea can be recast in the Drude picture. Suppose there is a current $j_x = n_c e v$ with $\rho_{xx} = 1 / (n_c e \mu)$. A Hall voltage is generated to balance the Lorentz force $\rho_{xy} = E_y / j_x = (vB / c) / (n_c e v) = B / (n_c e c)$. For uncorrelated conduction electrons, $B = B^{ext}$ and $\rho_{xy}$ depends on $n_c$, not the total electron number $n$. One would get $\rho_{xx} \to \infty$ and $\rho_{xy} \to \infty$ when $n_c \to 0$. For strongly correlated systems, we have shown that $B = B^{net} = (n_c / n) B^{ext}$ so that $\rho_{xy} = B / (n c e)$, regardless of the number of electrons which are localized.

We have demonstrated this model of correlation between the activated conduction electrons and the remaining localized electrons does make the system a Hall-Insulator. The
key is that we characterize the interaction between the activated conduction electrons and remaining localized electrons as flux-tube-like, and describe them using a Chern-Simons statistical field. However, we would like to emphasize that the introduction of a Jastrow factor in the trial-wavefunction in the early work \cite{13} on correlated interstitials yields extremely low energies with a microscopically realistic Hamiltonian. We note also that previous work \cite{9} has shown that a non-divergent $\rho_{xy}$ may be obtained using the Chern-Simons approach. However, in that case the correlation were introduced in the ground state rather in the excited states, and while $\rho_{xy} < \infty$, it was not necessarily equal to $B_{ext} / (nec)$. By introducing the correlation in the excited states, it is possible to have localized electrons for which the flux tube strength are site-dependent, leading to the classical Hall resistivity.

To summarize, we have shown rigorously with the Kubo formula that a non-interacting electron systems can not display the Hall-insulating behavior and treatments based on zero-temperature frequency-dependent conductivity are insufficient to explain this behavior. Instead, the Hall-insulating behavior should be considered as an interaction effect. We have constructed an explicit form for the strong correlations between the temperature-activated mobile electrons and the localized electrons in the background and demonstrated that this kind of correlation does lead to the Hall-insulating behavior.

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