Lifetimes of $b$-flavoured hadrons

Fulvia De Fazio†
† Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE, UK

Abstract. I discuss the heavy quark expansion for the inclusive widths of heavy-light hadrons, which predicts quite well the experimental ratios of $B_q$ meson lifetimes. As for $\Lambda_b$, current determinations of $O(m_b^{-3})$ contribution to $\tau(\Lambda_b)$ do not allow to explain the small measured value of $\tau(\Lambda_b)/\tau(B_d)$. As a final topic, I discuss the implications of the measurement of the $B_c$ lifetime.

1. Lifetimes of heavy-light hadrons

Inclusive particle widths describe the decay of the particle into all possible final states with given quantum numbers $f$. For weakly decaying heavy-light $Q\bar{q}$ ($Qqq$) hadrons $H_Q$, the spectator model considers only the heavy quark $Q$ as active in the decay, the light degrees of freedom remaining unaffected. Hence, all the hadrons containing the same heavy quark $Q$ should have the same lifetime; this picture should become accurate in the $m_Q \to \infty$ limit, when the heavy quark decouples from the light degrees of freedom. However, the measurement of beauty hadron lifetime ratios [1]:

\[
\frac{\tau(B^-)}{\tau(B_d)} = 1.066 \pm 0.02, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.99 \pm 0.05, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.794 \pm 0.053 \quad (1)
\]

shows that $\tau(\Lambda_b)/\tau(B_d)$ significantly differs from the spectator model prediction.

A more refined approach consists in computing inclusive decay widths of $H_Q$ hadrons as an expansion in powers of $m_Q^{-1}$ [2]. Invoking the optical theorem, one can write $\Gamma(H_Q \to X_f) = 2i m(H_Q)\langle\hat{T}|H_Q\rangle/2M_{H_Q}$, with $\hat{T} = i \int d^4x T[L_w(x)\mathcal{L}_w^\dagger(0)]$ the transition operator describing the heavy quark $Q$ with the same momentum in the initial and final state, and $\mathcal{L}_w$ the effective lagrangian governing the decay $Q \to X_f$. An operator product expansion of $\hat{T}$ in the inverse mass of the heavy quark allows to write: $\hat{T} = \sum_i C_i \mathcal{O}_i$, with the local operators $\mathcal{O}_i$ ordered by increasing dimension, and the coefficients $C_i$ proportional to increasing powers of $m_Q^{-1}$. As a result, for a beauty hadron $H_b$ the general expression of the width $\Gamma(H_b \to X_f)$ is:

\[
\Gamma(H_b \to X_f) = \Gamma_0 \left[ c_f^i (\bar{b}b)_{H_b} + \frac{c_f^j}{m_b^2} (\bar{b}ig_\ast \sigma \cdot Gb)_{H_b} + \sum_i \frac{c_f^{(i)}}{m_b^3} \langle\mathcal{O}_i\rangle_{H_b} + \mathcal{O}\left(\frac{1}{m_b^4}\right)\right], \quad (2)
\]

with $\langle\mathcal{O}\rangle_{H_b} = \frac{\langle H_b | \mathcal{O} | H_b \rangle}{2M_{H_b}}$, $\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{qb}|^2$ and $V_{qb}$ the relevant CKM matrix element.
The first operator in (4) is $\bar{b}b$, with dimension $D = 3$; the chromomagnetic operator $O_G = \bar{b}^{\mu} b_{\nu} G^{\mu \nu b}$, responsible of the heavy quark-spin symmetry breaking, has $D = 5$; the operators $O^b_i$ have $D = 6$. In the limit $m_b \to \infty$, the heavy quark equation of motion allows to write:

$$\langle \bar{b}b \rangle_{H_b} = 1 + \frac{\langle O_G \rangle_{H_b}}{2m_b^2} - \frac{\langle O_\pi \rangle_{H_b}}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^4}\right), \quad (3)$$

with $O_\pi = \bar{b}(i\partial^2)b$ the heavy quark kinetic energy operator. When combined with (4), the first term in (3) reproduces the spectator model result. $\mathcal{O}(m_b^{-1})$ terms are absent \footnote{\cite{3,4}} since $D = 4$ operators are reducible to $\bar{b}b$ by the equation of motion. Finally, the operators $O_G$ and $O_\pi$ are spectator blind, not sensitive to light flavour. Their matrix elements can be determined from experimental data; as a matter of fact, defining $\mu^2_G(H_b) = \langle O_G \rangle_{H_b}$ and $\mu_\pi^2(H_b) = \langle O_\pi \rangle_{H_b}$, one has: $\mu^2_G(B) = 3(M_{B^-} - M_{B^0})/4$, while $\mu^2_\pi(\Lambda_b) = 0$ since the light degrees of freedom in the $\Lambda_b$ have zero total angular momentum relative to the heavy quark. Moreover, from the mass formula:

$$M_{H_b} = m_b + \bar{\Lambda} + \mu^2_G - \mu_\pi^2 + \mathcal{O}(m_b^{-2}) \quad \text{with} \quad \bar{\Lambda}, \mu^2_G \text{ and } \mu_\pi^2 \text{ independent of } m_b,$$

from the experimental data, one can infer $\mu^2_\pi(B_d) \simeq \mu^2_\pi(\Lambda_b)$, as confirmed by QCD sum rule estimates \footref{1}.

The $\mathcal{O}(m_b^{-3})$ terms in (4) come from four-quark operators, accounting for the presence of the spectator quark in the decay. Their general expression is \footnote{\cite{5}}:

$$O^q_{-A} = (\bar{b}_L \gamma_\mu q_L)(\bar{q}_L \gamma_\mu b_L) \quad \text{and} \quad T^q_{-A} = (\bar{b}_L \gamma_\mu \gamma^a q_L)(\bar{q}_L \gamma_\mu \gamma^a b_L)$$

$$O^q_{-S-P} = (\bar{b}_R q_L)(\bar{q}_L b_R) \quad \text{and} \quad T^q_{-S-P} = (\bar{b}_R \gamma^a q_L)(\bar{q}_L \gamma^a b_R). \quad (4)$$

Their matrix elements over $B_q$ can be parametrized as:

$$\langle \bar{O}^q_{-A} \rangle_{B_q} = \langle \bar{O}^q_{-S-P} \rangle_{B_q} \left(\frac{m_b + m_q}{M_{B_q}}\right) = f_{B_q}^2 \frac{M_{B_q}}{8}, \quad \langle T^q_{-A} \rangle_{B_q} = \langle T^q_{-S-P} \rangle_{B_q} = 0, \quad (5)$$

$f_{B_q}$ being the $B_q$ decay constant. As for $\Lambda_b$, one can write:

$$\langle \bar{O}^q_{-A} \rangle_{\Lambda_b} = f_{B_B}^2 M_B r/48, \quad \langle \bar{O}^q_{-A} \rangle_{\Lambda_b} = -\tilde{B} \langle \bar{O}^q_{-A} \rangle_{\Lambda_b} \quad (6)$$

with $\bar{O}^q_{-A} = (\bar{b}_L \gamma_\mu b_L)(\bar{q}_L \gamma_\mu q_L)$. In the valence quark approximation $\tilde{B} = 1$.

Actually, with the computed values of the Wilson coefficients in (4), only large values of the parameter $r$ in \footnote{\cite{8}} (namely $r \simeq 3 - 4$) could explain the observed difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. This, however, seems not to be the case.

2. $\langle \bar{O}^q_{-A} \rangle_{\Lambda_b}$ from QCD sum rules

The parameter $r$ in (6) can be determined using quark models or lattice QCD \footnote{\cite{7}}. HQET QCD sum rules allow to estimate it from the correlator:

$$\Pi_{CD} = (1 + \gamma_5)_{CD} \Pi(\omega, \omega') = i^2 \int dx dy \, e^{i\omega x - i\omega' y} \langle 0 \left| T[J_C(x) \bar{O}^q_{-A}(0) J_D(y)] \right| 0 \rangle \quad (7)$$

between $\Lambda_b$ interpolating fields $J_{C,D}$ ($C, D$ Dirac indices) \footnote{\cite{8}} and the operator $\bar{O}^q_{-A}$; $\omega$ ($\omega'$) is related to the residual momentum of the incoming (outgoing) current $p^\mu = \bar{O}^q_{-A}$.
m_b v^\mu + k^\mu with k^\mu = \omega v^\mu. The projection of the interpolating fields on the \Lambda_b state is parametrized by \langle 0 | J_C | \Lambda_b (v) \rangle = f_{\Lambda_b} (\psi_v) C (with \psi_v the spinor for a \Lambda_b of velocity v).

Saturating the correlator \Pi(\omega, \omega') with baryonic states and considering the low-lying double-pole contribution in the variables \omega and \omega', one has:

\[ \Pi^{had}(\omega, \omega') = \langle \tilde{O}_q^{V-A} \rangle_{\Lambda_b} \frac{f_{\Lambda_b}^2}{2} \frac{1}{(\Delta_{\Lambda_b} - \omega)(\Delta_{\Lambda_b} - \omega')} + \ldots \]  
(8)

with \Delta_{\Lambda_b} defined by \( M_{\Lambda_b} = m_b + \Delta_{\Lambda_b} \). Besides, for negative values of \omega, \omega', \Pi can be computed in QCD in terms of a perturbative contribution and of vacuum condensates:

\[ \Pi^{QCD}(\omega, \omega') = \int d\sigma d\sigma' \frac{\rho_{11}(\sigma, \sigma')}{(\sigma - \omega)(\sigma' - \omega')} \]  
(9)

with possible subtractions omitted [9]. The sum rule consists in equating \( \Pi^{had} \) and \( \Pi^{QCD} \). Moreover, invoking global duality, the contribution of higher resonances and of continuum to \( \Pi^{had} \) can be modeled as the QCD term in the region \omega \geq \omega_c, \omega' \geq \omega_c, with \( \omega_c \) an effective threshold. Finally, a double Borel transform to \( \Pi^{QCD} \) and \( \Pi^{had} \) in \omega, \omega', with Borel parameter \( E_1, E_2 \), removes the subtraction terms in (9), improves factorially the convergence of the OPE and enhances the contribution of the low-lying resonances in \( \Pi^{had} \). Choosing \( E_1 = E_2 = 2E \), one gets a sum rule the result of which is depicted in figure 1. Considering the variation with \( E \) and the threshold \( \omega_c \), one has an estimate of \( \langle \tilde{O}_q^{V-A} \rangle_{\Lambda_b} \):

\[ \langle \tilde{O}_q^{V-A} \rangle_{\Lambda_b} \simeq (0.4 - 1.20) \times 10^{-3} \, GeV^3, \]  
(10)

corresponding to \( r \simeq 0.1 - 0.3 \) [10]. The same calculation gives \( \tilde{B} \simeq 1 \). This result produces \( \tau(\Lambda_b)/\tau(B_d) \geq 0.94 \), at odds with the experimental result. The discrepancy discloses exciting perspectives both from experimental and theoretical sides [10].

3. \( B_c \) lifetime

A different hadronic system, whose lifetime can be determined by OPE-based methods, is the \( B_c \) meson, observed at Fermilab with mass \( M_{B_c} = 6.40 \pm 0.39 \pm 0.13 \, GeV \) and lifetime \( \tau_{B_c} = 0.46 \pm 0.18 \pm 0.03 \, ps \) [11]. Like quarkonium states, \( B_c \) can be treated in a non relativistic way, but unlike them it can decay only weakly, with the main decay
mechanisms induced by the quark transitions $b \to cW^-$, $\bar{c} \to \bar{s}W^-$ and $\bar{c}b \to W^-$ (annihilation). Predictions for $\tau_{B_c}$ spread in the range $0.4 - 1.2$ ps \cite{12, 13, 14}. In the $m_b, m_c \to \infty$ limit one would have $\Gamma_{B_c} = \Gamma_{b,spec} + \Gamma_{c,spec}$. Corrections to this result can be computed using an OPE organized in powers of the heavy quark velocity \cite{14}. The result is: $\tau_{B_c} \simeq 0.4 - 0.7$ ps, together with the prediction of the dominance of charm transitions; as a matter of fact, $b$-decay dominance would imply a larger lifetime: $\tau_{B_c} = 1.1 - 1.2$ ps \cite{13}. Hence, the measurement of $\tau_{B_c}$ provides us with the first hints on the underlying dynamics in this meson. For this system, it is interesting to investigate the validity of the non relativistic approximation: actually, one estimates $\langle k^2 \rangle/m_c^2 \simeq 0.43$, where $\langle k^2 \rangle$ is the average squared momentum of the charm quark, implying possible deviations from the non relativistic limit \cite{13}.

4. Conclusions

$1/m_Q$ expansion can be used to compute inclusive widths of heavy-light hadrons. A QCD sum rule calculation of the matrix element $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ contributing to $\mathcal{O}(m_b^{-3})$ to the $\Lambda_b$ lifetime gives the result: $\tau(\Lambda_b)/\tau(B_d) \geq 0.94$, thus implying that such a correction does not explain the observed difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. Finally, the measurement of $B_c$ lifetime already enlightens some aspects of the quark dynamics in this meson.

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