How Spacetime Foam modifies the brick wall.

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We re-examine the brick-wall model in the context of spacetime foam. In particular we consider a foam composed by wormholes of different sizes filling the black hole horizon. The contribution of such wormholes is computed via a scale invariant distribution. We obtain that the brick wall divergence appears to be logarithmic when the cutoff is sent to zero.

I. INTRODUCTION

Gerard 't Hooft \[1\], in 1985 considered the statistical thermodynamics of quantum fields in the Hartle-Hawking state (i.e. having the Hawking temperature $T_H$ at large radii) propagating on a fixed Schwarzschild background of mass $M$. To control divergences, 't Hooft introduced a "brick wall" with radius a little larger than the gravitational radius $2MG$. He found, in addition to the expected volume-dependent thermodynamical quantities describing hot fields in a nearly flat space, additional contributions proportional to the area. These contributions are, however, also proportional to $\alpha^{-2}$, where $\alpha$ is the proper distance from the horizon, and thus diverge in the limit $\alpha \rightarrow 0$. For a specific choice of $\alpha$, he recovered the Bekenstein-Hawking formula

$$ S_{BH} = \frac{1}{4} A/l_p^2. \quad (1) $$

The prescription for assigning a “Bekenstein-Hawking entropy” $S_{BH}$ to a black hole of surface area $A$ was first inferred in the mid-1970s from the formal similarities between black hole dynamics and thermodynamics\[2\], combined with Hawking’s discovery\[3\] that black hole radiate thermally with a characteristic (Hawking) temperature

$$ T_H = \frac{\hbar \kappa_0}{2 \pi}, \quad (2) $$

where $\kappa_0$ is the surface gravity. Since then, many attempts to renormalize or eliminate the “brick wall” have been done. In a series of paper, it has been suggested\[4, 5, 6\] that this divergence could be absorbed in a renormalization of Newton’s constant, while other authors approached the problem of the divergent brick wall using Pauli-Villars regularization\[7, 8, 9\]. In the Pauli-Villars covariant regularization method, one introduces bosonic and fermionic regulator fields to regulate the divergences. What happens is that the free energy of the anti-commuting regulator fields comes with a minus sign with respect to the commuting fields. This leads to a cancellation of the ultraviolet divergence when the 't Hooft brick wall is removed. Recently a proposal coming by the modification of the Heisenberg uncertainty relations has been taken under consideration\[10, 17\]. The modified inequality takes the form\[1\]

$$ \Delta x \Delta p \geq \hbar + \frac{\lambda_p^2}{\hbar} (\Delta p)^2, \quad (3) $$

where $\hbar$ is the Planck constant and $\lambda_p$ is the Planck length. The interesting point regards exactly the modified number of quantum states, which is changed into

$$ \frac{d^3 x d^3 p}{(2\pi \hbar)^3 (1 + \lambda_p^2)^3}. $$

When $\lambda = 0$, the formula reduces to the ordinary counting of quantum states. If Eq. (4) is used for computing the entropy, the brick wall can be removed. Another interesting recent proposal comes from non-commutative geometry which introduces a natural thickness of the horizon replacing the 't Hooft’s brick wall\[18\]. In this paper we wish to repeat the brick wall computation in the context of spacetime foam. It was J. A. Wheeler who first conjectured that spacetime could be subjected to topology fluctuation at the Planck scale\[19\]. These fluctuations appearing at this scale form the “spacetime foam”. An interesting calculation scheme in this context comes by L. Crane and L.
Smolin\cite{20}. They show that in a foamy spacetime, general relativity can be renormalized when a density of virtual black holes coupled to $N$ fermion fields in a $1/N$ expansion is taken under consideration. The idea they propose is that the high-energy behavior of perturbation theory can be modified by the presence of nonperturbative structure in the vacuum at arbitrary small length scales. In their work, they used a distribution of virtual black holes $\rho(w)$ suggested by the uncertainty principle with the assumption that the perturbations of matter and gravitational fields contributing as virtual states in perturbation theory vanish on and inside any apparent horizons present in the background spacetime. Thus they proposed that in the spacelike slice $\Sigma$ of the background geometry, the distribution $\rho(w)\,dw$ per unit available volume (including also the case of an interior of a black hole of larger size) with masses between $w$ and $w + dw$ was of the form

$$\rho(w) = \begin{cases} 0 & w > l_p \\ \frac{C}{w^q} & w < l_p, \end{cases}$$

where $C$ is a dimensionless constant and $q$ is a parameter such that $q \geq 0$. At this point a natural question arises: how can we relate the Crane-Smolin picture of foam with distribution $\rho(w)$ with the brick wall? Motivated by a recent proposal of a model of spacetime foam based on a superposition of wormholes\cite{21}, we explore the possibility that such a non trivial configuration can affect the behavior of a quantum field near the black hole horizon. The main idea is to substitute the superposition of commuting and anti-commuting fields of Refs.\cite{7, 8, 9} renormalizing the entropy with the gravitational field itself and to use the distribution $\rho(b)$ in \eqref{eq:rho} to deal with the modified number of quantum states of Eq.\eqref{eq:conc}. This combination could change the divergent behavior of the entropy when the brick wall is removed. The rest of the paper is structured as follows, in section \ref{sec:brick} we recall the fundamental points that lead to a brick wall, in section \ref{sec:foam} we apply the foam model to the brick wall model. We summarize and conclude in section \ref{sec:conclude} Units in which $\hbar = c = k = 1$ are used throughout the paper.

II. BRICK WALL MODEL

We wish to study the thermodynamics of hot quantum fields in a background geometry of the form\cite{22}

$$ds^2 = -\exp(-2\Lambda(r)) \left(1 - \frac{b(r)}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\Omega^2.$$ \hfill (6)

Usually this form is considered for the description of wormholes. However, it is quite general to include as special cases the Schwarzschild, Reissner-Nordström and de Sitter geometries, or any combination of these. The function $b(r)$ will be referred to as the “shape function”. The shape function may be thought of as specifying the shape of the spatial slices. On the other hand, $\Lambda(r)$ will be referred to as the “redshift function” that describes how far the total gravitational redshift deviates from that implied by the shape function. Without loss of generality we can fix the value of $\Lambda(r)$ at infinity such that $\Lambda(\infty) = 0$. If the equation $b(r_w) = r_w$ is satisfied for some values of $r$, then we say that the points $r_w$ are horizons for the metric. For the outermost horizon one has $\forall r > r_w$ that $b(r) < r$. Consequently $b'(r_w) \leq 1$. We will fix our attention to the $b'(r_w) < 1$ case only. The anomalous case $b'(r_w) = 1$ can be thought as describing extreme black holes where an inner and outer horizons are merged. For a spherically symmetric system the surface gravity is computed via

$$\kappa_w = \lim_{r \to r_w} \frac{1}{2} \frac{\partial_s g_{tt}}{\sqrt{g_{tt}g_{rr}}}$$

and for the metric \eqref{eq:metric}, we get

$$\kappa_w = \lim_{r \to r_w} \frac{1}{2} \left\{ \exp(-\Lambda(r)) \left[-2\Lambda'(r) \left(1 - \frac{b(r)}{r}\right) + \frac{b(r)}{r} - \frac{b'(r)}{r}\right]\right\}.$$ \hfill (8)

By assuming that $\Lambda(r_w)$ and $\Lambda'(r_w)$ are both finite we obtain that

$$\kappa_w = \frac{1}{2r_w} \exp(-\Lambda(r_w)) \left[1 - b'(r_w)\right],$$ \hfill (9)

where, in the proximity of the throat we have approximated $1 - b(r)/r$ with

$$1 - \frac{b(r)}{r} \approx \frac{r - r_w}{r_w} \left[1 - b'(r_w)\right].$$ \hfill (10)
Now that the geometrical framework has been set up, we begin with a real massless scalar field described by the action

$$ I = -\frac{1}{2} \int d^4x \sqrt{-g} [g^\mu\nu \partial_\mu \phi \partial_\nu \phi] $$

in the background geometry of Eq. (6) whose Euler-Lagrange equations are

$$ \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) = 0. $$

If $\phi$ has the separable form

$$ \phi(t, r, \theta, \phi) = \exp(-i\omega t) Y_{l m}(\theta, \phi) f(r), $$

then the equation for $f(r)$ reads

$$ \frac{\exp(\Lambda(r))}{r^2} \partial_r \left[ r^2 \exp(-\Lambda(r)) \left( 1 - \frac{b(r)}{r} \right) \partial_r f_{nl} \right] - \left[ \frac{l(l+1)}{r^2} \right] f_{nl} + \omega_{nl}^2 \frac{\exp(2\Lambda(r))}{1 - \frac{b(r)}{r}} f_{nl} = 0, $$

where $Y_{l m}(\theta, \phi)$ is the usual spherical harmonic function. In order to make our system finite let us suppose that two mirror-like boundaries are placed at $r = r_1$ and $r = R$ with $R \gg r_1$, $r_1 > r_w$ and consider Dirichlet boundary conditions $f_{nl}(r_1) = f_{nl}(R) = 0$. We also assume the set of real functions $\{f_{nl}(r)\}$, defined by Eq. (14), be complete with respect to the space of $L^2$-functions on the interval $r_1 \leq r \leq R$ for each $l$. The positive constant $\omega_{nl}$ is defined as the corresponding eigenvalue. In order to use the WKB approximation, we define an $r$-dependent radial wave number

$$ k^2(r, l, \omega_{nl}) \equiv \frac{1}{1 - \frac{b(r)}{r}} \left[ \frac{\omega_{nl}^2}{\exp(2\Lambda(r))} \frac{\exp(2\Lambda(r))}{1 - \frac{b(r)}{r}} \right]. $$

The number of modes with frequency less than $\omega$ is given approximately by

$$ \tilde{g}(\omega) = \int \nu(l, \omega)(2l+1)dl, $$

where $\nu(l, \omega)$ is the number of nodes in the mode with $(l, \omega)$:

$$ \nu(l, \omega) = \frac{1}{\pi} \int_{r_1}^{R} \sqrt{k^2(r, l, \omega)} dr. $$

Here it is understood that the integration with respect to $r$ and $l$ is taken over those values which satisfy $r_1 \leq r \leq R$ and $k^2(r, l, \omega) \geq 0$. The free energy is given approximately by

$$ F \simeq \frac{1}{\beta_\infty} \int_0^\infty \ln \left( 1 - e^{-\beta_\infty \omega} \right) \frac{d\tilde{g}(\omega)}{d\omega} d\omega = \int_{r_1}^{R} \tilde{F}(r) 4\pi r^2 dr, $$

where the ‘free energy density’ $\tilde{F}(r)$ is defined by

$$ \tilde{F}(r) \equiv \frac{\exp(-\Lambda(r))}{\beta(r)} \int_0^\infty \ln \left( 1 - e^{-\beta(r)p} \right) \frac{4\pi p^2 dp}{(2\pi)^3}. $$

Here the “local inverse temperature” $\beta(r)$ is defined by the Tolman’s law

$$ \beta(r) = \exp(-\Lambda(r)) \sqrt{1 - \frac{b(r)}{r} \beta_\infty}. $$

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2 see also Ref. [22] for a derivation of the brick wall in the Boulware state.
The total entropy is given by the integral
\[
S = 4\pi \int_{r_1}^{R} \frac{s(r)}{\sqrt{1 - \frac{b(r)}{r}}} r^2 dr = \frac{16\pi^3}{90\beta_\infty^3} \int_{r_1}^{R} \frac{\exp(2\Lambda(r))}{(1 - \frac{b(r)}{r})^2} r^2 dr = S_{r_1} + S_R,
\]
where
\[
S_{r_1} = \frac{16\pi^3}{90\beta_\infty^3} \int_{r_1}^{r_1+\epsilon} \frac{\exp(2\Lambda(r))}{(1 - \frac{b(r)}{r})^2} r^2 dr
\]
and
\[
S_R = \frac{16\pi^3}{90\beta_\infty^3} \int_{r_1+\epsilon}^{R} \frac{\exp(2\Lambda(r))}{(1 - \frac{b(r)}{r})^2} r^2 dr
\]
with \( R \gg r_1 \) and \( r_1 \gg \epsilon \gg r_1 - r_w \). For large \( R \), \( S_R \) is dominated by
\[
S \sim \frac{16\pi^3}{90\beta_\infty^3} \frac{R^3}{3},
\]
representing the entropy of a homogeneous quantum gas in flat space at a uniform temperature \( T_\infty \). However, the brick wall divergence is in the integral of Eq. (22). If we set \( r_1 = r_w + h \), then we are led to consider the following integral
\[
S_{\text{brick}} = \frac{16\pi^3}{90\beta_\infty^3} \int_{r_w+h}^{r_w+h+\epsilon} \frac{\exp(2\Lambda(r))}{(1 - \frac{b(r)}{r})^2} r^2 dr = S(\epsilon, h, r_w),
\]
where
\[
S(\epsilon, h, r_w) = \frac{16\pi^3}{90\beta_\infty^3} \frac{\exp(2\Lambda(r_w))}{(1 - b'(r_w))^2} \int_{r_w+h}^{r_w+h+\epsilon} \frac{r^2 dr}{(r-r_w)^2} = \frac{16\pi^3}{90\beta_\infty^3} \frac{1}{4\kappa_w^2} \int_{r_w+h}^{r_w+h+\epsilon} \frac{r^2 dr}{(r-r_w)^2}.
\]
By defining
\[
g(\epsilon, h, r_w) = \int_{r_w+h}^{r_w+h+\epsilon} \frac{r^2 dr}{(r-r_w)^2},
\]
and with the help of Eq. (3), we get
\[
S(\epsilon, h, r_w) = \frac{16\pi^3}{90} \frac{g(\epsilon, h, r_w)}{4\beta_\infty^3 \kappa_w^2}.
\]
The brick wall is obtained by keeping only the leading divergence in Eq. (27) and introducing the proper distance from the throat
\[
\alpha = \int_{r_w}^{r_w+h} \frac{dr}{\sqrt{1 - \frac{b(r)}{r}}} = \frac{2\sqrt{h}}{\sqrt{1 - \frac{b(r_w)}{r_w}}}.
\]
Thus Eq. (28) becomes
\[
S(\epsilon, h, r_w) = \frac{16\pi^3}{90\beta_\infty^3} \frac{r_w^2}{4\kappa_w^3} \frac{\exp(-\Lambda(r_w))}{\alpha^2} = \frac{16\pi^3}{90} \left( \frac{T_\infty}{\kappa_w/2\pi} \right)^3 \frac{r_w^2}{(2\pi)^3} \frac{\exp(-\Lambda(r_w))}{2\alpha^2}
\]

\[3 \text{ Recall that } b(r) \leq r \text{ and } \Lambda(\infty) = 0.\]
\[ \begin{align*}
A &= \frac{T_\infty}{90} \left( \frac{T_\infty}{\kappa_w / 2\pi} \right)^3 \exp \left( -\Lambda \left( r_w \right) \right) \frac{1}{4\pi\alpha^2}. 
\end{align*} \]

The area law is recovered if we make the following identifications

\[ T_\infty = \frac{\kappa_w}{2\pi} \]

and

\[ \exp \left( -\Lambda \left( r_w \right) \right) = \frac{1}{l_p^2}. \]

**Remark 1** In this paper the redshift function is considered practically as a constant. But even at this level the short distance behavior is affected. This means that in a less approximated scheme something could change the short distance cutoff of Eq.\((32)\).

**Remark 2** We have hitherto considered the wormhole practically as a black hole with a horizon located at \( r_w \).

However if one deals with traversable wormholes, the brick wall is softened in a logarithmic divergence, due to the horizon absence.

### III. THE BRICK WALL MODEL AND THE FOAM

In previous section, we have reproduced the ‘t Hooft brick wall result by fixing the background geometry of one wormhole which behaves as a black hole. In this section, we consider the idea that the divergent horizon entropy may be affected by the presence of nonperturbative structure in the vacuum at the Planck length scales. Instead of using the Crane-Smolin virtual black holes, we consider as a natural candidate for such structure, a distribution of virtual wormholes suggested by the uncertainty principle. Thus in the spacelike slice \( \Sigma \), we consider the distribution \( \rho \left( r_w \right) \) with radii between \( r_w \) and \( r_w + dr_w \) expressed by

\[ \rho \left( r_w \right) = \begin{cases} 
0 & r_w > r_h \\
C / \left( 64\pi^2 r_w^4 \right) & l_p \leq r_w \leq r_h,
\end{cases} \]

where \( C \) is a dimensionless constant and where the Crane-Smolin distribution \((5)\) has been restricted only to the value \( q = 0 \) of the exponent corresponding to a scale invariant distribution. The choice of the form of \( \rho \left( r_w \right) \) is also suggested by the behavior of the energy density of spacetime foam described by a collection of non-interacting wormholes\[21\].

\[ \rho \left( E_w \right) \sim -\frac{\Lambda^4}{64\pi^2}. \]

Actually the correct expression found in Ref.\[21\] is

\[ \rho \left( E_w \right) \sim \frac{\Lambda^4}{64\pi^2}. \]

The minus sign appears because the computation has been done looking at flat space as the reference space.

We recall that our main purpose is to see if and how the brick wall divergence is modified by an underlying nontrivial spacetime structure. Since we have assumed that \( \Lambda \left( r_w \right) \) is a constant, without loss of generality and for future purposes we can modify Eq.\((33)\) in the following way

\[ \rho \left( r_w \right) = \begin{cases} 
0 & r_w > r_h \\
C' \exp \left( \Lambda \left( r_w \right) \right) / \left( 64\pi^2 r_w^4 \right) & l_p \leq r_w \leq r_h,
\end{cases} \]

where \( C' \) is a dimensionless constant.

\[ \rho \left( E_w \right) \sim -\frac{\Lambda^4}{64\pi^2}. \]
We proceed to compute the total black hole entropy beginning with the expression (22) obtained in the previous section

\[ S(h, r_h, r_w) = \frac{16\pi^3}{90} g(h, r_h, r_w) \kappa_w^2, \]

where the integration range is now \( l_p \leq r_w \leq r_h \) and

\[ g(h, r_h, r_w) = \int_{r_h}^{r_h+h} \frac{r^2}{(r-r_w)^2} dr. \]

Eq. (37) describes the entropy generated by one wormhole with a throat located at \( r_w \) with respect to the black hole horizon \( r_h \). The main difference between Eq. (38) and Eq. (27) is in the integration limits: indeed in Eq. (27), we have considered the wormhole exactly like a black hole, while in Eq. (38), the black hole is formed by wormholes of smaller radius. It is obvious that the typical brick wall divergence in Eq. (38) is absent. However, this is not the complete and correct expression of the brick wall calculation, because we have to sum over all wormholes contributing the black hole. If we define \( N_w \equiv N_w(A_{r_w}) \) as the number of wormholes filling the area of a two-sphere \( S^2 \) of radius \( r_w \), then the variation of black hole entropy due to a variation in the number of wormholes filling the black hole area is

\[ dS(r_h) = S(h, r_h, r_w) dN_w, \]

we can write

\[ dS(r_h) = S(h, r_h, r_w) \frac{dN_w}{dA_{r_w}} dA_{r_w}, \]

and the total entropy is

\[ S(r_h) = \int_{A_{l_p}}^{A_{r_h}} S(h, r_h, r_w) \frac{dN_w}{dA_{r_w}} dA_{r_w}. \]

If we assume that for every \( r_w \)

\[ T_{\infty, r_w} = \frac{\kappa_w}{2\pi}, \]

then exchanging the integration order, Eq. (41) becomes

\[ \int_{A_{l_p}}^{A_{r_h}} S(h, r_h, r_w) \frac{dN_w}{dA_{r_w}} dA_{r_w} = \frac{\pi}{90} \int_{r_h}^{r_h+h} r^2 \left[ \int_{A_{l_p}}^{A_{r_h}} \frac{\kappa_w}{(r-r_w)^2} dA_{r_w} \right] dr. \]

The last point concerns the number of wormholes per unit area \( dN_w/dA_{r_w} \), representing the maximum number of wormholes that can be stored in an area of radius \( r_w \) beginning with an area of Planckian size \( A_{l_p} \). This number can be computed by means of the distribution \( \rho(r_w) \). This is simply obtained by the following expression

\[ \frac{dN_w}{dA_{r_w}} = \int_{A_{l_p}}^{A_{r_w}} \rho(r_w') dA_{r_w}, \]

which affects the brick wall behavior. Indeed the expression in Eq. (44) establishes the counting of the constituents of the horizon with respect to the area. Unfortunately, a direct comparison with the usual expressions obtained in Refs. [10, 17, 17] is not immediate, because we have worked in terms of radial coordinates and not with the momentum representation. A comparison should be possible if Eq. (44) should be Fourier transformed in terms of the momentum \( p \). However, this goes beyond the purpose of this paper. Thus Eq. (44) can be written as

\[ C' \left[ 1 - b'(r_w) \right] \frac{\pi}{90} \int_{r_h}^{r_h+h} r^2 \left[ \frac{(4\pi)^2}{64e\pi^2} \int_{l_p}^{r_h} \frac{dr_w}{(r-r_w)^2} \frac{1}{r_f} \frac{1}{r_w} \right] dr, \]

where we have used the explicit expression of \( \kappa_w \) given by Eq. (9). In Eq. (45), we have set

\[ \frac{C'}{e} \left[ 1 - b'(r_w) \right] = 1. \]
This is true for Schwarzschild-like wormholes where $b'(r_w)$ vanishes or for wormholes with $b'(r_w) = \text{const} < 1$. When $b'(r_w)$ is a function of $r_w$, the result depends on a case to case. To leading order in $h$, one gets

$$S(r_h) = \frac{\pi}{360} \left[ \frac{r_h^2}{r_p^2} - 1 \right] \ln \left( \frac{1}{h} \right) + \text{finite terms as } h \to 0. \quad (47)$$

If $r_h^2/r_p^2 \gg 1$ then

$$S(r_h) = \frac{A}{1440r_p^2} \ln \left( \frac{1}{h} \right). \quad (48)$$

IV. SUMMARY AND DISCUSSION

In this paper, we have examined the possibility that a complicated structure like a foamy spacetime may affect the ultra-violet behavior of the brick wall. What we have obtained is a softening of the divergence that is turned from a linear to a logarithmic type. Although a certain number of assumptions has been considered in using the wormhole metric (6), the result seems to be quite general. Indeed, the model of spacetime foam picture we have used depends strictly on the constituents, which in our case, are Schwarzschild-like wormholes. However, we have found that Schwarzschild-Anti-de Sitter wormholes could be used as representatives of the foam. If this choice is adopted, in that case the brick wall will exhibit a completely different behavior due to the different form of $b'(r_w)$ involved. Thus, we expect that Eq.(48) can be valid also for more complicated black holes.

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