A Cosmography Approach to Dark Energy Cosmologies: New Constraints Using the Hubble Diagrams of Supernovae, Quasars, and Gamma-Ray Bursts

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Abstract

In the context of a cosmography approach to using the data of the Hubble diagram for supernovae, quasars, and gamma-ray bursts, we study dark energy (DE) parameterizations and the concordance cold dark matter ($\Lambda$CDM) universe. Using different combinations of data samples including (i) supernovae (Pantheon), (ii) Pantheon + quasars, and (iii) Pantheon + quasars + gamma-ray bursts, and applying the minimization of $\chi^2$ function of the distance modulus of data samples in the context of the Markov Chain Monte Carlo method, we obtain constrained values of cosmographic parameters in a model-independent cosmography scenario. We then investigate our analysis, for different concordance $\Lambda$CDM cosmology, $w$CDM, Chevallier–Polarski–Linder, and Pade parameterizations. Comparing the numerical values of the cosmographic parameters obtained for DE scenarios with those of the model-independent method, we show that the concordance $\Lambda$CDM model has serious issues when we involve quasar and gamma-ray burst data in our analysis. While high-redshift quasars and gamma-ray bursts can falsify the concordance model, our results using a cosmography approach indicate that the other DE parameterizations are still consistent with these observations.

Unified Astronomy Thesaurus concepts: Cosmology (343); Cosmological parameters (339); Dark energy (351); Cosmological constant (334)

1. Introduction

Recent advances in observational cosmology have revealed that the current universe has experienced a stage of accelerated expansion. This expansion can be explained by introducing an exotic component with negative pressure, dark energy (DE), which violates the strong energy conditions, $\rho + 3p > 0$. This expansion can also be justified by modifying the standard theory of gravity on extragalactic scales (Riess et al. 1998; Perlmutter et al. 1999; Kowalski et al. 2008). In the framework of general relativity (GR), it appears that approximately 70% of the energy budget of the universe is in the form of dark energy (Bennett et al. 2003; Peiris et al. 2003; Spergel et al. 2003). The cosmological constant $\Lambda$ in which the equation of state (EoS) parameter is equal to $-1$, is the most likely possibility for dark energy. Although by assuming the cosmological constant and cold dark matter (CDM) in the context of standard $\Lambda$CDM cosmology, the accelerated expansion of the universe and the model can be found to be in good agreement with cosmological observations, this approach suffers from the serious problems of cosmic coincidence and fine tuning (Weinberg 1989; Padmanabhan 2003; Copeland et al. 2006).

Also, from an observational point of view, $\Lambda$CDM cosmology is plagued with some significant issues with respect to estimation of some key cosmological parameters. In particular, there is a discrepancy between the amplitude of matter fluctuations from large-scale structure (LSS) data (Macaulay et al. 2013), and the value predicted by CMB experiments based on the $\Lambda$CDM. Second, the $Ly\alpha$ forest measurement of the baryon acoustic oscillations (BAO) reported in Delubac et al. (2015) suggests a smaller value of the matter density parameter ($\Omega_m$) in comparison with the value obtained by CMB data. Furthermore, there is a statistically significant disagreement between the value of the Hubble constant measured by the classical distance ladder and that obtained from the Planck CMB data (Freeman 2017). Quantitatively speaking, the $\Lambda$CDM cosmology deduced from Planck CMB data predicts $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1}/\text{Mpc}^{-1}$ (Aghanim et al. 2018), while from the Cepheid-calibrated SNeIa (Riess et al. 2019) we have $H_0 = 74.03 \pm 1.42 \text{ km s}^{-1}/\text{Mpc}^{-1}$. To solve these problems, various kinds of DE models have been proposed in the literature (Veneziano 1979; Armendariz-Picon et al. 2001; Erickson et al. 2002; Caldwell 2002; Gasperini & Veneziano 2002; Padmanabhan 2002; Thomas 2002; Elizalde et al. 2004; Gomez-Valent & Sola 2015). Comparing with different observations, some of these models have been ruled out and many of them lead to good consistency with data (see also Malekjani et al. 2017, 2018; Rezaei et al. 2017; Rezaei 2019a; Rezaei et al. 2019b; Rezaei et al. 2020; Lusso et al. 2019; Lin et al. 2019). The results of these investigations show that by using the current observations, it is difficult to distinguish different DE models. This confusion about different DE models suggests that a more conservative way to justify the cosmic acceleration, relying on as few model-dependent quantities as possible, is welcome. As a solution, a well known model-independent approach that is commonly used in the literature for testing the fitting capability of models with data, is cosmography (see Section 2). Applying the cosmographic approach to distinguish between different DE models was proposed in Sahni et al. (2003) and Alam et al. (2003). Using cosmographic parameters, Capozziello & Salzano (2009) tried to constrain a cosmological model, $f(R)$-gravity. Because these parameters are model-independent, they lead to natural “priors” to any theory. Using cosmography, Capozziello & Salzano (2009) discussed how $f(R)$ – gravity could be useful to solve the problem of mass profile and dynamics of galaxy clusters. Capozziello et al. (2011) studied the possibility of extracting the model-independent information about the dynamics of the universe by using a cosmography approach. Their results showed with a cosmography approach, our predictions considerably deviate from the $\Lambda$CDM cosmology. Based on the cosmography approach, authors of Capozziello et al. (2019) constrained the late time
evolution of the universe using low-redshift observations. Their results confirmed issues with the ΛCDM model in a low-redshift universe. Lusso et al. (2019) assumed two different cosmographic models consisting of a fourth-order logarithmic polynomial and a fifth-order linear polynomial, and fitted these models with different data sets. Then, by comparing the results with the expectations from the concordance ΛCDM model, they found significant tensions between the best-fit cosmographic parameters and the concordance found with the expectations from the concordance models with different data sets. Then, by comparing the results with the expectations from the concordance ΛCDM model, they found significant tensions between the best-fit cosmographic parameters and the concordance found with the expectations from the concordance models with different data sets. Then, by comparing the results with the expectations from the concordance ΛCDM model, they found significant tensions between the best-fit cosmographic parameters and the concordance found with the expectations from the concordance models with different data sets. Then, by comparing the results with the expectations from the concordance ΛCDM model, they found significant tensions between the best-fit cosmographic parameters and the concordance found with the expectations from the concordance models with different data sets.

The cosmographic approach also is used in Li et al. (2020) to determine the spatial curvature of the universe. They showed that by combining the supernovae (Pantheon sample), the latest released cosmic chronometers, and the BAO measurements, the most favored cosmography model prefers a nonflat universe with Ω_k = 0.21 ± 0.22. Following these works, in this paper we want to study some relevant DE models including the standard ΛCDM, wCDM, Chevallier–Polarski–Linder (CPL), and Pade parameterizations in the context of a cosmography approach. By combining different data sets including the distance modulus of quasars, the Pantheon, and publicly available gamma-ray burst (GRB) data, we try to find the best-fit values of cosmographic parameters using the minimization of χ^2 function based on the Markov Chain Monte Carlo (MCMC) method. Notice that we first obtain the best-fit values of the cosmographic parameters without considering a cosmological model. We then put constraints on the free parameters of the models under study. Using the constrained values and their confidence regions within the 1σ uncertainties of cosmological parameters of the models, we compute the best-fit values of the cosmographic parameters for each model. By comparing the computed cosmographic parameters of the models and those obtained from a model-independent approach, we can examine the cosmological models against observations. The layout of our paper is as follows. In Section 2, we present the cosmographic approach. Then we introduce the observational data that we have used in our analysis. In Section 3, we first briefly introduce the DE models and parameterizations in our study and then present the numerical results. In Section 4 we present discussions based on our numerical results for different models. In Section 5, we present our conclusions.

### 2. Cosmographic Approach

Recently, the cosmographic approach to cosmology has been used in the literature to obtain as much information as possible directly from observations. In this approach, without addressing issues such as which model of DE is required to satisfy the accelerated expansion of the universe, and just by assuming the minimal priors of homogeneity and isotropy, we can study the evolution of the universe. Cosmography provides information about cosmic flow and its evolution derived from measured distances, using Taylor expansions of the basic observables (Demianski et al. 2017b). The distance–redshift relations obtained from these expansions only rely on the assumption of the Friedman–Lemaître–Robertson–Walker (FLRW) metric and are therefore fully model-independent. First, we introduce the cosmographic functions through the first five derivatives of scale factor a(t) as follows (Visser 2004):

\[
H(t) = \frac{1}{a} \frac{da}{dt}, \quad (1)
\]

\[
deceleration function: \quad q(t) = -\frac{1}{aH^2} \frac{d^2a}{dt^2}, \quad (2)
\]

\[
\text{jerk function: } j(t) = \frac{1}{aH^3} \frac{d^3a}{dt^3}, \quad (3)
\]

\[
\text{snap function: } s(t) = \frac{1}{aH^4} \frac{d^4a}{dt^4}, \quad (4)
\]

\[
\text{lerk function: } l(t) = \frac{1}{aH^5} \frac{d^5a}{dt^5}. \quad (5)
\]

The cosmographic parameters \((H_0, d_0, H_0, s_0, l_0)\) correspond to the present values of the above functions. Furthermore, it is easy to find the relation between the derivatives of the Hubble parameter and the cosmographic parameters as follows:

\[
\dot{H} = -H^2(1 + q), \quad (6)
\]

\[
\ddot{H} = H^3(3q + j + 2), \quad (7)
\]

\[
\dddot{H} = H^4(-3q^2 - 12q - 4j + s - 6), \quad (8)
\]

\[
\ddddot{H} = H^5(30q^2 + 60q + 10j + 20j - 5s + l + 24), \quad (9)
\]

where each over-dot denotes a derivative with respect to cosmic time \(t\). One can compute the Taylor Series expansion of the Hubble parameter to the fourth order in redshift \(z\) around its present value \(z = 0\):

\[
H(z) = H|_{z=0} + \frac{dH}{dz} \bigg|_{z=0} \frac{z}{1!} + \frac{d^2H}{dz^2} \bigg|_{z=0} \frac{z^2}{2!} + \frac{d^3H}{dz^3} \bigg|_{z=0} \frac{z^3}{3!} + \frac{d^4H}{dz^4} \bigg|_{z=0} \frac{z^4}{4!}. \quad (10)
\]

The above Taylor series expansion is valid for small redshifts \(z < 1\), whereas much of the most interesting recent observational data sets occur at higher redshifts \(z > 1\). In other words, the radius of convergence of any series expansion in redshift is equal or less than \((z < 1)\), thus any \(z\)-based expansion will break down at \(z > 1\). To solve this problem we use an improved redshift definition that is commonly used in the literature, the \(y\)-redshift definition \(y = \frac{z}{1+z}\) (Capozziello et al. 2011). Although changing the \(z\)-redshift in to the \(y\)-redshift will not change the physics, it can improve the series of convergence. In terms of the \(y\)-redshift, we see that the radius of convergence of a Taylor expansion is \((\leq 1)\), which corresponds to \(z \rightarrow \infty\). Thus, using the \(y\)-redshift definition, we can use the Taylor expansion of the Hubble parameter at higher redshifts in the following form (Capozziello et al. 2011):

\[
H(y) = H|_{y=0} + \frac{dH}{dy} \bigg|_{y=0} \frac{y}{1!} + \frac{d^2H}{dy^2} \bigg|_{y=0} \frac{y^2}{2!} + \frac{d^3H}{dy^3} \bigg|_{y=0} \frac{y^3}{3!} + \frac{d^4H}{dy^4} \bigg|_{y=0} \frac{y^4}{4!}. \quad (11)
\]

We note that there are some other procedures that can solve the convergence problem. Capozziello et al. (2020) compared some of these procedures to find the best approach to explain low- and high-redshift data sets. They expanded the luminosity distance \(d_L\), using the Taylor series and its alternatives, rational polynomials, and auxiliary variables. Their results show that at low redshifts there is no apparent need to adopt the \(y\)-variables or rational polynomials instead of a Taylor series. However, differences appear at high redshifts, where the results of Capozziello et al. (2020) indicate that a (2, 1) polynomial
performs better than the \( y \)-variables. In this work we use different observations in the redshift range up to \( z \sim 7 \). Thus, we cannot use the Taylor expansion and we should apply one of its alternatives. Since we are not using the high-redshift CMB data, we can use an alternative approach that performs well at low and intermediate redshifts. Assuming this condition, and aiming to prevent complexity arising from inserting more additional degrees of freedom, in this work we select the \( y \)-redshift procedure. By changing the time derivatives of Equations (6)–(9) into derivatives with respect to \( y \), inserting the results in Equation (11) and using Equations (1)–(5) we find the following:

\[
E(y) = \frac{H(y)}{H_{y=0}} = 1 + k_1 y^2 + k_2 y^4 + \frac{k_3 y^6}{6} + \frac{k_4 y^8}{24}, \quad (12)
\]

where different \( k_i \) are:

\[
k_1 = 1 + q_0, \quad (13)
\]

\[
k_2 = 2 - q_0^2 + 2 q_0 + j_0, \quad (14)
\]

\[
k_3 = 6 + 3 q_0^3 - 3 q_0^2 + 6 q_0 - 4 q_0 j_0 + 3 j_0 - s_0, \quad (15)
\]

\[
k_4 = -15 q_0^4 + 12 q_0^3 + 25 q_0^2 j_0 + 7 q_0 s_0 - 4 j_0^2 - 16 q_0 j_0 - 12 q_0^2 + j_0 - 4 s_0 + 12 j_0 + 24 q_0 + 24. \quad (16)
\]

In the above equations \( q_0, j_0, s_0, \) and \( l_0 \) are the current values of cosmographic parameters. By knowing the evolution of \( E \) as a function of redshift, we can investigate the evolution of cosmic fluid. In this paper we want to put constraints on the cosmographic parameters using the Hubble diagrams of low-redshift observational data. To do this, we set the current values of cosmographic parameters \((q_0, j_0, s_0, \) and \( l_0)\) as free parameters in an MCMC algorithm. Then, the best-fit values for the free parameters are those that can minimize the \( \chi^2 \) function. Note that the \( \chi^2 \) function is defined based on the distance modulus of observational objects. The Hubble diagram of low-redshift observational data used in this work is as follows:

1. Pantheon sample: this sample, as a set of the latest data points for the apparent magnitude of type Ia supernovae (SNIa) (Scolnic et al. 2018) in the range \( 0.01 < z < 2.26 \), is one of three sample data points we use in this work. This sample includes 279 spectroscopically confirmed SNIa discovered by the Pan-STARRS1 (PS1) Medium Deep Survey (Rest et al. 2014; Scolnic et al. 2018) in the redshift range \( 0.03 < z < 0.68 \). The pantheon sample also includes the SNIa data from the Sloan Digital Sky Survey (SDSS) (Frielan et al. 2008; Sako et al. 2018) and the Supernova Legacy Survey (SNLS) (Conley et al. 2011; Sullivan et al. 2011). This sample is the largest combined sample of SNIa data, consisting of a total of 1048 data points up to redshift \( \sim 2.3 \).

2. Gamma-ray bursts (GRBs): GRBs are the most energetic and powerful explosions in the universe and can be detectable up to very high redshifts. GRBs are mysterious objects in the universe. A mechanism that indicates the high amounts of energy released from a typical GRB emits is not yet completely known. Some investigations show that the GRBs are produced by core-collapse events (Meszaros 2006). Despite these difficulties, the GRBs are astrophysical objects for studying the expansion scenario of the universe at high redshifts. In fact, using the Hubble diagrams of GRBs, one can study the expansion rate of the universe and investigate the observational properties of DE up to higher redshifts. One of the most important aspects of the observational property of GRBs is that they show several correlations between spectral and intensity properties (luminosity, radiated energy). Demianski et al. (2017a) proposed an empirical correlation between the observed photon energy of the peak spectral flux, \( E_{p,i} \), and the isotropic equivalent radiated energy, \( E_{iso} \). This correlation not only provides constraints on the model of the prompt emission, but also naturally suggests that the GRBs can be used as distance indicators. In fact, to use the GRBs as distance indicators, it is necessary to consistently calibrate this correlation. Unfortunately, due to the lack of GRBs at very low redshifts, the calibration of GRBs is more difficult than that of SNIa. In this regard, several calibration procedures have been suggested so far (Dainotti et al. 2008; Demianski & Piedipalumbo 2011; Demianski et al. 2012; Postnikov et al. 2014). Demianski et al. (2017a), by applying a local regression technique and using the SNIa sample, have constructed a new calibration for the GRB Hubble diagram that can be used for cosmological investigations. They showed how the \( E_{p,i} - E_{iso} \) correlation can be calibrated to standardize the long GRBs and to build a GRB Hubble diagram, which we use to investigate the cosmology at very high redshifts (Demianski et al. 2017a). Note that their \( E_{p,i} - E_{iso} \) correlation has no significant redshift dependence. In this work, we use the 162 data points for the distance modulus of GRBs derived and reported in Demianski et al. (2017a). This sample contains the low and high redshifts GRBs in the range of \( 0.03 < z < 6.67 \). More details and discussions about the calibration method and construction of the Hubble diagram of GRBs can be found in the literature (e.g., Amati & della Valle 2013; Demianski et al. 2017a, 2017b).

3. Quasars: quasars are extremely luminous active galactic nuclei (AGNs) in which a supermassive black hole (SMBH) is surrounded by a gaseous accretion disk. As gas in the disk falls toward the SMBH, energy is released, which can be observed across the electromagnetic spectrum. The observed properties of a quasars depend on factors such as the mass of the SMBH and the rate of gas accretion. The spectral energy distribution (SED) of quasars shows significant emission in the optical-UV band \( L_{UV} \), the so-called big blue bump (BBB), with a softening at higher energies (Sanders et al. 1989; Elvis et al. 1994; Trammell et al. 2007; Shang et al. 2011). This emission is thought to originate from an optically thick disk surrounding the SMBH. Also, the X-ray photons, \( L_X \), are generated by inverse Compton scattering of disk UV photons by a hot electron plasma, the so-called X-ray corona. Note that the energy lost through X-ray emission may cool down the electron plasma, if there is no efficient energy transfer mechanism from the disk to the corona. However, the physical nature of such a process is still poorly understood. An important
observational feature concerning the connection between the UV disk and X-ray corona is provided by the nonlinear correlation between the $L_{\text{UV}}$ from the disk and $L_X$ from the corona. The nonlinear relationship between $L_X$ and $L_{\text{UV}}$ as $\log L_X = \gamma L_{\text{UV}} + \beta$ has been obtained in both optical and X-ray AGN samples, with slope parameter $\gamma$ around 0.5–0.7 (Vignali et al. 2003; Strateva et al. 2005; Steffen et al. 2006; Just et al. 2007; Green et al. 2009; Young et al. 2009, 2010; Jin et al. 2012; Lusso & Risaliti 2016a) showing that optically bright AGNs emit relatively less X-rays than optically faint AGNs. It has been shown that such a relation is independent of redshift and it is very tight (Lusso & Risaliti 2016a). This relation has also been used as a distance indicator to estimate cosmological parameters. Using the $L_X - L_{\text{UV}}$ relationship, Risaliti & Lusso (2015) have constructed a complete sample of a quasar Hubble diagram up to $z \sim 6$, which is in excellent agreement with the analogous Hubble diagram for SNIa in the common redshift range (i.e., $z \sim 0.01$–1.4). This capability turns quasars into a new class of standard candles (Lusso & Risaliti 2017). The main quasar sample is composed of 1598 data points in the range 0.04 < $z$ < 5.1. In this work instead of the main sample, we use a binned catalog including 25 data points from the literature (Risaliti & Lusso 2015; Lusso & Risaliti 2016a). All the details of the sample selection, X-ray and UV flux computation, and the analysis of the nonlinear relation, calibration, and a discussion on systematic errors are provided in Risaliti & Lusso (2019).

We combine gamma-ray bursts and quasars with Pantheon because we can then probe a redshift range (0.03 < $z$ < 6.67) better suited for investigating DE than the one covered by the Pantheon sample (0.01 < $z$ < 2.26). Hence, by adding these data samples to the Hubble diagram, we have more observational data at higher redshifts. Using these data sets we calculate the $\chi^2$ function of the distance modulus based on the MCMC algorithm to find the best-fit values of cosmographic parameters in a model-independent cosmology. To run the MCMC algorithm, we select two different sets of the initial values for free parameters. This can guarantee that our results are independent from the initial values of free parameters. For all of the free parameters we choose big $\sigma$ values to ensure that the MCMC can sweep the whole of parameter space. Using these choices, we have removed the risk of finding local best-fit values in parameter space. Note that for both of the initial value sets, we obtained similar posteriors for $q_0$ and $j_0$. But in the case of $s_0$ and $l_0$, the posteriors are slightly different. Thus, we have repeated our analysis by setting an initial value for $s_0$ and $l_0$ between the two best-fit values obtained in previous steps (we presented the initial values of free parameters in Table 1).

### Table 1

| Parameter | Initial Values | Best-fit Values |
|-----------|----------------|-----------------|
| $q_0$     | $j_0$ | $s_0$ | $l_0$ | $q_0$ | $j_0$ | $s_0$ | $l_0$ |
| Set (1)   | -2.0 | 5.0 | -4.0 | -5.0 | $-0.838^{+0.006}_{-0.04}$ | $2.27^{+0.15}_{-0.31}$ | $-3.8^{+0.67}_{-1.0}$ | $-5.2^{+2.2}_{-3.0}$ |
| Set (2)   | 2.0  | -5.0 | 4.0  | 5.0  | $-0.811 \pm 0.090$ | $2.51^{+0.24}_{-0.31}$ | $-0.11^{+0.8}_{-1.7}$ | $0.91^{+2.10}_{-2.37}$ |
| Final Set | -0.8 | 2.5  | -2.0 | -3.0 | $-0.819 \pm 0.065$ | $2.21^{+0.37}_{-0.42}$ | $-3.44^{+0.36}_{-1.3}$ | $-3.8^{+2.3}_{-0.2}$ |

### Table 2

The Best-fit Values of Cosmography Parameters and Their 1σ Uncertainties Obtained for Different Combinations of Data Samples

| Data Sample | $q_0$ | $j_0$ | $s_0$ | $l_0$ |
|-------------|-------|-------|-------|-------|
| Pantheon    | $-0.702 \pm 0.104$ | $1.60 \pm 0.71$ | $-3.54^{+0.38}_{-1.5}$ | $-4.9^{+6.5}_{-5.0}$ |
| Pantheon+GRB| $-0.775 \pm 0.048$ | $2.61^{+0.29}_{-0.19}$ | $2.8 \pm 1.4$ | $-1.3^{+3.0}_{-3.6}$ |
| Pantheon+quasars | $-0.844 \pm 0.048$ | $2.42 \pm 0.25$ | $-2.5^{+1.4}_{-1.2}$ | $-3.2^{+2.5}_{-2.1}$ |
| Pantheon+GRB+quasars | $-0.819 \pm 0.065$ | $2.21^{+0.27}_{-0.42}$ | $-3.44^{+0.46}_{-1.5}$ | $-3.8^{+8.2}_{-6.2}$ |

In order to see the influence of each data sample of quasars and GRB in our analysis, we consider different combinations of data samples as Pantheon, Pantheon+GRB, Pantheon+quasars, and Pantheon+GRB+quasars. For all of these combinations, we seek to find the best-fit values of the free parameters and their 1– $\sigma$ and 2 – $\sigma$ uncertainties. Note that a procedure to choose the proper initial value for each of the free parameters was described above. The results of our analysis are presented in Table 2. For all combinations of data samples, we can see that the deceleration parameter $q_0$ is tightly constrained. The constraints for jerk parameter $j_0$ are approximately tight. However, our analysis cannot put tight constraints on the snap $s_0$ and jerk $l_0$ parameters. We observe that adding the high-redshift observational data of quasars and GRB causes higher values of $q_0$ and $j_0$. Due to the large values of uncertainties for $s_0$ and $l_0$, we cannot reach a clear conclusion when we compare the results of different combinations. Note that the $s_0$ and $l_0$ parameters are appearing in the fourth and fifth terms of Equation (12), as the third- and fourth-order coefficients of redshift, respectively. In these terms, the big error bar of data points leads to very weak constraints on these two parameters.

### 3. DE Models and Parameterizations

In this section we first briefly introduce some DE models and parameterizations that we want to study using cosmography approach. Note that we also consider the standard $\Lambda$CDM cosmology as a concordance model. Then, by using the data samples presented in the previous section and by applying the minimization of $\chi^2$ function based on the MCMC algorithm, we find the best-fit values of the cosmological parameters of DE models. Using the chain obtained for cosmological parameters of each model within the $1-\sigma$ level, we compute the best fit and $1-\sigma$ uncertainty of cosmographic parameters for each model. Finally, we will compare the best-fit cosmographic parameters of each model with those of the
model-independent approach obtained in Table 2. The DE models that we examine in our analysis are as follows:

1. wCDM: the first model is the DE model with a constant equation of state (EoS) parameter \( w_{de} \). The Hubble parameter of the model in a flat FRW universe reads (Mota & Barrow 2004; Barger et al. 2007)

\[
E^2(z) = \Omega_{m,0}(1 + z)^3 + (1 - \Omega_{m,0})(1 + z)^{3(1+w_{de})},
\]

where \( \Omega_{m,0} \) is the energy density of pressureless matter at the present time. Note that we study the model in late time cosmology where the energy density of radiation is negligible. Using the above equation and rewriting Equations (1)–(5) in terms of redshift, we can obtain cosmographic parameters in the context of wCDM cosmology as follows:

\[
q(z) = \frac{\Omega_{m,0}(1 + z)^3 + (1 + 3w_{de})\Omega_{d,0}(1 + z)^{3(1+w_{de})} - 2E^2(z)}{2E^2(z)},
\]

\[
j(z) = 1 + \frac{3w_{de}(1 + z)^{3(1+w_{de})} - 2E^2(z)}{2E^2(z)}.
\]

In order to obtain the best-fit values and the confidence regions of the cosmographic parameters, we need to obtain the best fit and also the confidence regions of the cosmological parameters \( \Omega_{m,0} \) and \( w_{de} \) of the model. Note that in the flat FRW universe, we have \( \Omega_{d,0} = 1 - \Omega_{m,0} \). So using the different combinations of observational data —Pantheon, Pantheon + GRB, Pantheon + quasar, and Pantheon+GRB+quasars—we obtain the best-fit values of \( \Omega_{m,0} \) and \( w_{de} \) as well as their confidence regions in \( 1 - \sigma \) uncertainty. Our results are reported in the left part of Table 3. Using Equations (18) and (19) and the data of \( \Omega_{m,0} \) and \( w_{de} \) in \( 1\sigma \) error, we put constraints on the cosmographic parameters in wCDM cosmology. Results for the best-fit values and \( 1 - \sigma \) confidence regions are presented in the right part of Table 3.

2. Concordance ΛCDM: in fact when we analyze a given DE model, we should redo our analysis for a standard ΛCDM model as a concordance model. So now we study the standard model from the viewpoint of a cosmography approach. In order to obtain the cosmographic parameters for a ΛCDM model, we can easily set \( w_{de} = -1 \) in Equations (17)–(19). Then we follow the procedure implemented for the wCDM model to find the cosmographic parameters in ΛCDM cosmology. Our results are presented in Table 4. Note that in the ΛCDM model, the jerk parameter is exactly equal to one, independent of the values of cosmological parameters.

3. Pade parameterization: as a well known parameterization for the EoS of DE, we consider the Pade parameterization in this work. The Pade Parameterization is the rational approximation of order \((m,n)\) for an arbitrary function \(f(z)\) as follows:

\[
f(x) = \frac{a_0 + a_1x + a_2x^2 + \ldots + a_nx^n}{b_0 + b_1x + b_2x^2 + \ldots + b_mx^m},
\]

where exponents \((m, n)\) are positive and the coefficients \((a_n, b_j)\) are constants (Pade 1892). In this work, we consider the Pade expansion of the EoS parameter \(w_{de}(a)\) up to the order \((1, 1)\) around the variable \((1 - a)\), where \(a\) is scale factor. Previously, this parameterization was studied in Rezaei et al. (2017) in the light of different observational data. But here we investigate this parameterization from the cosmography point of view. The EoS parameter for the Pade \((1, 1)\) parameterization can
easily be written as follows (Rezaei et al. 2017; Rezaei 2019b):
\[
w_d(z) = \frac{w_0 + (w_0 + w_1)z}{1 + z + w_2 z^2}, \tag{21}
\]
Following (Rezaei et al. 2017; Rezaei 2019b), we can find the evolution of dimensionless Hubble parameter of the Pade parameterization, \( E(z) \), as
\[
E^2(z) = \Omega_{m,0}(1 + z)^3 + \left(1 + w_2 - \frac{w_2}{1 + z}\right)^{p_1} \times (1 - \Omega_{m,0})(1 + z)^{p_2}, \tag{22}
\]
where \( p_1 \) and \( p_2 \) are:
\[
p_1 = -3 \left( \frac{w_1 - w_0 w_2}{w_2 (1 + w_2)} \right),
\]
\[
p_2 = 3 \left( 1 + w_0 + w_1 + w_2 \right). \tag{23}
\]
Using Equation (22) in Equations (1)–(3), we can obtain the cosmographic parameters for the Pade parameterization as follows:
\[
q(z) = \frac{3\Omega_{m,0}(1 + z)^3 + (1 + z)(A_1B_1 + C_1D_1)}{2E^2(z)} - 1, \tag{24}
\]
\[
j(z) = 1 + (1 + z)^2 \frac{2A_1C_1 + B_1F_1 + G_1D_1}{2E^2(z)} - (1 + z) \frac{A_1B_1 + C_1D_1}{E^2(z)}, \tag{25}
\]
where constants \( A_1, B_1, C_1, D_1, F_1 \), and \( G_1 \) are, respectively, given by:
\[
A_1 = \frac{w_2 p_1}{(1 + z)^2} \left( 1 + w_2 - \frac{w_2}{1 + z} \right)^{-1 + p_1},
\]
\[
B_1 = \Omega_{d,0}(1 + z)^{p_2},
\]
\[
C_1 = p_2 \Omega_{d,0}(1 + z)^{-1 + p_2},
\]
\[
D_1 = \left( 1 + w_2 - \frac{w_2}{1 + z} \right)^{p_1},
\]
\[
F_1 = p_1 \left( 1 + w_2 - \frac{w_2}{1 + z} \right)^{p_1 - 1} - \frac{2p_1 w_2}{(1 + z)^3} + \frac{(p_2^2 - p_1) w_2^2}{(1 + z)^4} \left( 1 + w_2 - \frac{w_2}{1 + z} \right)^{p_1 - 2},
\]
\[
G_1 = p_2 (p_2 - 1) \Omega_{d,0}(1 + z)^{-2 + p_2}. \tag{26}
\]
In a cosmology based on the Pade parameterization for the EoS parameter of DE, we have four free parameters including \( \Omega_{m,0}, w_0, w_1 \), and \( w_2 \). We redo our analysis for the Pade parameterization following what was done for \( \Lambda \)CDM and \( \Lambda \)CDM cosmologies. We first find the best fit as well as the confidence regions of the parameters within the \( 1 - \sigma \) level. Then, we obtain the best fit and the error bar of the cosmographic parameters for the Pade approximation for different combinations of data samples. Results are presented in Table 5.

4. CPL parameterization: the other parameterization that we study in this work is the well-known Chevallier-Polarski-Linder (CPL) parameterization of DE in which the EoS parameter is simply expanded around \((1 - a)\) by a Taylor approximation up to the first order, e.g., \( w = w_0 + w_1 z / (1 + z) \) (Chevallier & Polarski 2001; Linder 2003). It is easy to see that for a particular value of \( w_2 = 0 \), we can recover the CPL parameterization from the Pade formula. In the CPL parameterization, the Hubble parameter is written as (Chevallier & Polarski 2001; Linder 2003)
\[
E^2(z) = \Omega_{m,0}(1 + z)^3 + (1 - \Omega_{m,0})(1 + z)^3 (1 + w_0 + w_1) \times \exp \left[ -3w_1 - \frac{z}{1 + z} \right]. \tag{27}
\]
Hence, inserting Equation (27) into Equations (1)–(3), the cosmographic parameters in CPL cosmology are obtained as follows:
\[
q(z) = \frac{A_2 [1 + z + 3(1 + z)w_0 + 3zw_1] + (1 + z)B_2}{2(1 + z)}, \tag{28}
\]
\[
j(z) = \frac{A_2 C_2 + 2(1 + z)^2 B_2}{2(1 + z)^2 [A_2 + B_2]}, \tag{29}
\]
where the constants \( A_2, B_2, \) and \( C_2 \) are:
\[
A_2 = \Omega_{d,0}(1 + z)^{3(w_0 + w_1)},
\]
\[
B_2 = \Omega_{d,0} \exp \left[ 3w_1 - \frac{z}{1 + z} \right],
\]
\[
C_2 = 9z^2 w_1^2 + 3w_1 (1 + z)(6w_0 z + 3z + 1) + (1 + z)^2 (9w_0^2 + 9w_0 + 2). \tag{30}
\]
In this case we have three free parameters including \( \Omega_{m,0}, w_0, \) and \( w_1 \). The best-fit values, the \( 1 - \sigma \) confidence region of these parameters and also the best-fit values of the cosmographic parameters of the model are reported in Table 6.

Using these numerical results, in the next section we will compare the cosmographic parameters of the above DE parameterizations with the cosmographic parameters obtained from the model-independent approach presented in Table 2. In Figure 1, we plot the \( 1 - \sigma \) and \( 2 - \sigma \) confidence regions of the cosmographic parameters \( q_0 \) and \( j_0 \) obtained for the model-independent approach. We can easily observe that the \( 2 - \sigma \) confidence region of the jerk parameter \( j_0 \) for different combinations of Pantheon+GRB, Pantheon+quasars, and Pantheon+GRB+quasars is above the critical value \( j_0 = 1 \). The confidence region of \( j_0 \) for the Pantheon sample covers the critical point \( j_0 = 1 \). Hence, as a quick result, we can see that the \( \Lambda \)CDM cosmology has significant tension with the high-redshift GRB and quasar observations.

4. Discussions

In this section we will compare the numerical results our analysis obtained in previous sections. As shown in Table 2 our analysis leads to fairly tight constraints on two of the cosmographic parameters, \( q_0 \) and \( j_0 \), while the other two parameters of cosmography, \( s_0 \) and \( l_0 \), are not tightly constrained. Furthermore, the best-fit values of \( s_0 \) and \( l_0 \) are significantly varying based on the initial conditions of the
The MCMC algorithm. Also, their related confidence regions are also large. Therefore, in order to compare DE models, we focus on our results for \( q_0 \) and \( j_0 \) and ignore the other ones. Note that the impact of \( q_0 \) and \( j_0 \) on the Taylor expansion of the Hubble parameter is much larger than \( s_0 \) and \( l_0 \). Overall, our consideration to compare DE parameterizations based on \( q_0 \) and \( j_0 \) cannot restrict our conclusion. Based on the results of Table 2, when we just use the Pantheon sample, the deceleration parameter \( q_0 \) has the largest value, \( q_0 = -0.702 \), while adding other data samples to Pantheon leads to a smaller \( q_0 \) value. Thus, we can say that the larger value of deceleration parameter \( q_0 \) is favored by low-redshift data points, while using relatively higher-redshift data points causes a smaller \( q_0 \). In contrast, in the case with a jerk parameter, we obtain a smaller value of \( j_0 \) when we use the Pantheon sample and a larger value of \( j_0 \) when we add the other data samples to Pantheon. These results are completely in agreement with those of Lusso et al. (2019), who used one of our combinations of data samples (Pantheon+GRB+quasars) in a different way to constrain cosmographic parameters. Our results also nearly confirm the results of Li et al. (2020), which were obtained using different data samples and a different approach. Now we compare the best-fit values of the cosmographic parameters \( q_0 \) and \( j_0 \) for each DE parameterization obtained in the previous section with those of the model-independent way. Our comparison for different combinations of data samples is as follows.

1. **Pantheon sample:** using the Pantheon sample, the best-fit values of deceleration and jerk parameters within 1\( \sigma \) uncertainty are \( q_0 = -0.702 \pm 0.104 \) and \( j_0 = 1.60 \pm 0.71 \) for a mode-independent approach. Our results for the \( \Lambda \)CDM model (Table 3) show that both \( q_0 \) and \( j_0 \) are in full agreement (at the 1 \( \sigma \) confidence level) with those we obtained from model-independent constrains. In the case of the \( \Lambda \)CDM model, the results are almost different from those for the model-independent case. For \( \Lambda \)CDM we have \( q_0 = -0.572 \pm 0.019 \ (j_0 = 1.0) \), which is in 1.25\( \sigma \) (0.85\( \sigma \)) tension with the result of \( q_0 \) \( j_0 \), which we have obtained for a model-independent case (see Tables 2 and 4). Both Pade and CPL parameterizations have nearly the same results. The results of \( q_0 \) in these two parameterizations are in full agreement with the value of \( q_0 \) that we have found for model-independent case, while the value of \( j_0 \) that we obtained for these parameterizations is in more than 1\( \sigma \) tension with the \( j_0 \) of the model-independent case (see Tables 2, 5, and 6). In summary, our results are presented in the top left panel of Figure 2 in which the contour plot shows the model-independent constraints on the \( j_0 - q_0 \) plan up to the 3 \( \sigma \) confidence level and the error bars show the computed value of cosmographic parameters for different cosmological models up to 1 \( \sigma \). Accordingly, we can say that using solely the Pantheon sample, the constrained parameters \( q_0 \) and \( j_0 \) for \( \omega \)CDM, Pade, and CPL parameterizations are compatible with model-independent constraints in \( \sim 1 \sigma \) error. While the standard \( \Lambda \)CDM model can be falsified by 1\( \sigma \) uncertainty because of its jerk parameter, the \( \Lambda \)CDM model is still consistent with model-independent results at the 2\( \sigma \) level.

2. **Pantheon + GRB data:** using this sample the best-fit values of cosmographic parameters and their 1 \( \sigma \) confidence levels in a model independent approach are \( q_0 = -0.755 \pm 0.048 \) and \( j_0 = 2.61 \pm 0.29 \). We see that adding the GRB to Pantheon data leads to a smaller deceleration parameter and larger jerk parameter compared to solely using the Pantheon sample. Thus, as stated previously, the \( q_0 \) parameter in this model has a \( \sim 3 \sigma \) tension with the model-independent result. But the \( j_0 \) parameter in this model has a 3 \( \sigma \) CD model is in full agreement with the model-independent result. But the \( j_0 \) parameter in this model has a \( \sim 3 \sigma \) tension with the model-independent scenario. Note that here 1\( \sigma \) is the average of the error bars obtained in model-independent \( j_0 \) (see Table 2). The results of the \( \Lambda \)CDM model are disappointing in this regard. The best-fit value of \( q_0 \) in this model leads to a 3.9\( \sigma \) tension with that of the model-independent one and \( j_0 \) is in more than 5\( \sigma \) tension with the model-independent approach. We emphasize that for all comparisons, we define the agreement or tension between DE models and the model-independent approach.

### Table 5

The Best-fit Values of Cosmological Parameters for the Pade Parameterization (Left Part) and the Best Fit of Cosmographic Parameters (Right Part)

| Data            | \( \Omega_{m,0} \) | Best-fit Parameters | Computed Values |
|-----------------|---------------------|---------------------|-----------------|
| Pantheon        | 0.330^{+0.060}_{-0.045} | \( w_0 \) -1.24^{+0.15}_{-0.13} | \( w_1 \) 0.21^{+0.72}_{-0.40} | \( w_2 \) 0.16^{+0.66}_{-0.42} | \( q_0 \) -0.741 \pm 0.097 | \( j_0 \) 2.4 \pm 1.0 |
| Pantheon+GRB    | 0.327^{+0.063}_{-0.045} | -1.22^{+0.15}_{-0.12} | 0.08 \pm 0.59 | 0.21^{+0.64}_{-0.40} | -0.725 \pm 0.094 | 2.22^{+0.91}_{-1.1} |
| Pantheon+quasars| 0.402^{+0.031}_{-0.024} | -1.40^{+0.15}_{-0.12} | -0.098^{+0.58}_{-0.24} | -0.36^{+0.63}_{-0.63} | -0.756^{+0.11}_{-0.08} | 2.05^{+0.91}_{-1.3} |
| Pantheon+GRB+quasars | 0.391^{+0.038}_{-0.026} | -1.41^{+0.15}_{-0.13} | -0.10^{+0.55}_{-0.67} | -0.07^{+0.55}_{-0.64} | -0.78 \pm 0.10 | 2.5^{+1.0}_{-1.3} |

### Table 6

The Best-fit values of Cosmological Parameters for the CPL Parameterization (Left Part) and the Best Fit of Cosmographic Parameters of the Model (Right Part)

| Data            | \( \Omega_{m,0} \) | Best-fit Parameters | Computed Values |
|-----------------|---------------------|---------------------|-----------------|
| Pantheon        | 0.281^{+0.024}_{-0.009} | -1.17 \pm 0.17 | 0.55^{+1.1}_{-0.52} | \( q_0 \) -0.74 \pm 0.10 | \( j_0 \) 2.33^{+1.26}_{-0.91} |
| Pantheon+GRB    | 0.326^{+0.064}_{-0.033} | -1.22 \pm 0.15 | 0.30^{+0.53}_{-0.41} | \( q_0 \) -0.724^{+0.086}_{-0.075} | 2.17^{+0.59}_{-0.74} |
| Pantheon+quasars| 0.382^{+0.035}_{-0.024} | -1.41 \pm 0.14 | 0.08^{+0.69}_{-0.51} | \( q_0 \) -0.798 \pm 0.090 | 2.70 \pm 0.85 |
| Pantheon+GRB+quasars | 0.384^{+0.033}_{-0.022} | -1.41 \pm 0.14 | 0.05 \pm 0.50 | \( q_0 \) -0.801 \pm 0.090 | 2.71^{+0.82}_{-0.91} |
based on the average of the error bar obtained for cosmographic parameters in the model-independent way presented in Table (2). As stated above, the results of the $q_0$ parameter in the Pade and CPL parameterizations are completely compatible with those of the model-independent approach, while $j_0$ is in a $\sim 2\sigma$ tension with the best value of $j_0$ in the model-independent approach. Therefore, we can say that using the combination of GRB and Pantheon data points, Pade and CPL are the best models and $\Lambda$CDM is completely disfavorable. In the upright panel of Figure 2, the contour plot shows the best fit of cosmographic parameters and related confidence levels up to $3 - \sigma$, obtained using Pantheon+GRB data points in the model-independent approach. The best-fit values and their error bars obtained for different DE models also are plotted for comparison. We can see that in the $q_0 - j_0$ plan, $w$CDM, CPL, and Pade parameterizations are located inside the confidence regions while the standard $\Lambda$CDM model is outside.

3. **Pantheon + quasars:** now let us see the effect of adding quasars data to Pantheon sample in our analysis. By combination of quasars and Pantheon data sets, the model independent approach leads to $q_0 = -0.844 \pm 0.048$ and $j_0 = 2.42 \pm 0.25$. Now we compare this result with the best-fit values of $q_0$ and $j_0$ obtained for different DE models. In the case of $w$CDM (see Table 3), we observe that $q_0$ ($j_0$) of the model deviates from model-independent values in the $0.98\sigma$ ($1\sigma$) region. This result for CPL parameterization (see Table 6) is $1\sigma$ and $1.2\sigma$ deviations, respectively, for $q_0$ and $j_0$. In the case of Pade parameterization (see Table 5), we obtain a $1.9\sigma$ ($1.6\sigma$) deviation from model-independent constraints of $q_0$ ($j_0$). Finally, in the case of concordance $\Lambda$CDM (see Table 4), we see large tension between the values of cosmographic parameters of the model and those of model-independent approach. Numerically, this tension is approximately $6\sigma$ for both $q_0$ and $j_0$. In the bottom right panel of Figure 2, the contour plots show the confidence levels of cosmographic parameters up to $3 - \sigma$, obtained using Pantheon+quasar+GRB data points in the model-independent approach. The best-fit values and their error bars of cosmographic parameters obtained for different DE models also are plotted for comparison. We see that the $\Lambda$CDM model is completely outside of the confidence regions, while

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**Figure 1.** The confidence regions in the $q_0 - j_0$ plan obtained in a model-independent approach using different combinations of Pantheon, quasar, and GRB data samples.
wCDM, CPL, and Pade parameterizations are still inside the regions.

4. Pantheon + quasars + GRB: in the last step, we combine all of our data samples and compare the results of the model-independent approach with those of DE parameterizations. As one can see in Table 2, the best-fit values of cosmographic parameters in the model-independent approach are \( q_0 = -0.819 \pm 0.065 \) and \( j_0 = 2.21^{+0.37}_{-0.42} \). Assuming the results obtained for our models (see Tables 3, 4, 6, and 5), we observe that the best-fit values of \( q_0 \) and \( j_0 \) for wCDM model are respectively in \( 0.3 \sigma \) and \( 1 \sigma \) tension with the results of the model-independent approach. These tensions in the case of \( \Lambda \)CDM enhance to \( 3.7 \sigma \) for \( q_0 \) and \( 4 \sigma \) for \( j_0 \). For Pade parameterization the differences are smaller. Here we have tensions of \( 0.6 - \sigma \) for \( q_0 \) and \( 0.8 \sigma \) for \( j_0 \). In the CPL case, the \( q_0 \) parameter has \( 0.3 \sigma \) tension and \( j_0 \) has \( 1.3 \sigma \) tension with the best-fit values in the model-independent analysis. In the bottom right panel of Figure 2, the contour plots show the confidence levels of cosmographic parameters obtained using Pantheon+GRB+quasars data points in a model-independent approach. The best-fit values and their error bars obtained for different DE parameterizations are also plotted for comparison. The \( \Lambda \)CDM cosmology is far from the confidence region in \( q_0 - j_0 \) space, while other models are still not refuted.

Now we examine the DE parameterizations and also the concordance \( \Lambda \)CDM universe by reconstructing the Hubble parameter in the context of a cosmography approach. In Figure 3, we have reconstructed the redshift evolution of the Hubble parameter, \( H(z) \), within the \( 1 - \sigma \) confidence region, using Equations (12)–(16). Note that we consider Equation (12) up to \( \gamma^2 \), which involves \( q_0 \) and \( j_0 \) parameters. So we can use the best-fit values of cosmographic parameters \( q_0 \) and \( j_0 \) for the model-independent approach in Table (Table 2) and also for DE models and parameterizations in Tables 3–6. Each of the panels of Figure 3 is obtained from one of our combinations of data samples. In all cases, we set \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\) (Abbott et al. 2017). The \( 1 - \sigma \) confidence level of \( H(z) \) (green band) is calculated using the upper and lower limits of best-fit values of \( q_0 \) and \( j_0 \) obtained in the model-independent approach from Table 2. In the upper left panel we show the reconstructed \( H(z) \) obtained from the Pantheon sample. The evolution of \( H(z) \) for different DE models and parameterizations is also plotted for comparison. As shown in the figure, the \( H(z) \) curve of \( \Lambda \)CDM deviates from the \( 1 - \sigma \) region at redshifts higher than \( z \sim 0.8 \), while \( H(z) \) for other DE parameterizations evolve within the \( 1 - \sigma \) region even at high redshifts. The results obtained from the Pantheon+GRB sample are presented in the upper right panel. In this plot, the \( H(z) \) curve of \( \Lambda \)CDM has the maximum differences from the best curve among different models. The bottom left panel, which shows the reconstruction of \( H(z) \) for the Pantheon+quasars sample, represents the \( \Lambda \)CDM cosmology as the most incompatible model again. We see that the deviation from the confidence region is large at higher redshifts. Furthermore, in this plot, the Pade parameterization also evolves outside of the \( 1 - \sigma \) region. Finally, in the bottom right panel, we present the results obtained using the Pantheon+GRB+quasars sample. This plot again confirms the results of the previous panels. We see that the reconstructed Hubble parameters of \( \Lambda \)CDM cosmology evolve outside the confidence region at redshifts larger than \( z \sim 0.8 \). So among the cosmological DE scenarios studied in this work, the \( \Lambda \)CDM is the worst one.

5. Conclusions

In this work we first used the data points of low-redshift Hubble diagrams for Pantheons, quasars, and GRBs to put constraints on the present value of cosmographic parameters in
an independent cosmography approach. To do this, we used different combinations of data samples including Pantheon, Pantheon+quasars, Pantheon+GRB, finally Pantheon+quasars+GRB. In the context of a cosmography approach, we obtained the best-fit values of cosmographic parameters as well as their confidence regions up to $3-\sigma$ uncertainties for different combinations of data samples. Our results showed that the best-fit value of the deceleration parameter $q_0$ varies in the range of $-0.844$ to $-0.702$ and the best fit of jerk parameter $j_0$ varies in the range of $1.60$ to $2.61$ for different combinations of data samples.

Note that here we used the Hubble diagrams of quasars and GRBs, respectively, derived in Lusso & Risaliti (2016a) and Demianski et al. (2017a). In the calibration procedure to form the Hubble diagrams of both quasars and GRBs, they have used the SNIa data at low redshifts. Their results for quasars and GRBs samples are consistent with that of the SNIa samples at the low-redshift universe. Hence we adopted their calibrations and used their Hubble diagrams for quasars and GRBs. In the case of concordance $\Lambda$CDM cosmology, our results are also compatible with recent work in Lusso et al. (2019). They confirmed the presence of a tension between $\Lambda$ cosmology and the best-fit cosmographic parameters $\sim 4\sigma$ with SNIa+quasars, at $\sim 2\sigma$ with SNIa+GRBs, and at $4\sigma$ with the whole SNIa+quasars+GRBs data set (Lusso et al. 2019). Furthermore, we studied some relevant DE parameterizations as well as the concordance $\Lambda$CDM cosmology using the Hubble diagrams of Pantheons, quasars, and GRB observations in the context of a cosmography approach. The DE parameterizations studied in our analysis are $\omega$CDM, CPL, and Pade parameterizations. First, using the different combinations of data samples and in the context of the MCMC algorithm, we calculate the $\chi^2$ function of the distance modulus to find the best-fit values and also the $1-\sigma$ uncertainty of cosmological parameters for each DE parameterization. Using the chain of data obtained for cosmological parameters, we found the best-fit values and the $1-\sigma$ confidence region of the cosmographic parameters of DE parameterizations. Comparing the results for DE models with those obtained for the model-independent approach leads us to conclude that the model is in better (worse) agreement with Hubble diagrams of Pantheons, quasars, and GRBs. In the first stage, using the solely Pantheon sample, we found that the $\omega$CDM model is the most compatible model with the result of model-independent constraints and on the other hand the concordance $\Lambda$CDM model is the worst model. In the second stage, by combining the GRB data to the Pantheon sample, we obtained disappointing results for the $\Lambda$CDM model. In this case the $q_0$ parameter of the $\Lambda$CDM has a $3.8-\sigma$ tension with that of the model-independent cosmography approach. Moreover, the $j_0$ parameter of $\Lambda$CDM cosmology has roughly $5-\sigma$ tension with that of the model-independent approach. These results will be more complicated when we see the results of other DE models and parameterizations that we studied in this work. We observed that $\omega$CDM, CPL, and Pade parameterizations are in better agreement with the results of the model-independent cosmography approach rather than the concordance model. In the third and fourth steps, using the combinations Pantheon+quasars and Pantheon+GRB+quasars data points, we obtained the same results again, supporting our results in previous steps. So we conclude that the concordance $\Lambda$CDM cosmology has much tension with observations of quasars and GRBs at higher redshift. Note that the DE parameterizations studied in this work are in better agreement with Hubble diagrams of high-redshift quasars and GRB observations. Finally, we reconstructed the Hubble parameter using the best-fit value of cosmographic parameters for both model-independent approaches, the $\Lambda$CDM model, and DE parameterizations. We observed that for different data sample combinations, the evolution of reconstructed $H(z)$ in a...
The concordance $\Lambda$CDM model has a maximum deviation from the confidence region compared to different DE parameterizations. Upon this result, we can conclude that among the different cosmological models studied in this work, the $\Lambda$CDM has the minimum compatibility with the predictions of a model-independent approach and thus it is falsified by a cosmography approach. The large value of tensions (between $3\sigma$ to $6\sigma$ for different data combinations) that we observed between the cosmographic parameters of $\Lambda$CDM and those we observed in the model-independent approach support this claim again that we should explore other alternatives for standard $\Lambda$CDM cosmology. We observed that other DE parameterizations in this study cannot be refuted in the context of a cosmography approach. Upon this result, we can conclude that among the different data combinations we observed tensions which we observed using a different approach and different data sets. Although in the literature it has been thoroughly affirmed that the $\Lambda$CDM describes the evolution of the universe until recent times, our conclusion confirms the result of Benetti & Capozziello (2019) that show large tensions emerge at higher redshifts for $\Lambda$CDM. Our analysis can be extended by considering other cosmic observations in the context of a cosmography approach.

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11