Non-Gaussianity from violation of slow-roll in multiple inflation

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Abstract: Multiple inflation is a model based on N=1 supergravity wherein there are sudden changes in the mass of the inflaton because it couples to ‘flat direction’ scalar fields which undergo symmetry breaking phase transitions as the universe cools. The resulting brief violations of slow-roll evolution generate a non-gaussian signal which we find to be oscillatory and yielding $f_{NL} \sim 5 - 20$. This is potentially detectable by e.g. Planck but would require new bispectrum estimators to do so. We also derive a model-independent result relating the period of oscillations of a phase transition during inflation to the period of oscillations in the primordial curvature perturbations generated by the inflaton.

Keywords: Cosmic Microwave Background Radiation: non-gaussianity, Early Universe: inflation, supersymmetry and cosmology.
1. Introduction

A major goal in modern observational cosmology is to detect any non-gaussianity of the temperature anisotropies in the cosmic microwave background (CMB) \[1, 2, 3\]. All single field, slow roll, inflation models give rise to anisotropies that are gaussian \[4\]. Therefore detection of a significant non-gaussian signal would falsify such models and provide new insights into the dynamics of inflation.

Quantifying such a signal is however not a straightforward task. Whilst gaussianity is a well-defined property, non-gaussianity is not and the anisotropies can, in principle, deviate from gaussianity in many different ways \[3\]. Also, the CMB temperature anisotropies measured by WMAP are very close to gaussian \[5\]. Therefore any measure designed to detect non-gaussianity needs to be sensitive to a very small signal.

One measure of non-gaussianity that is particularly useful is the three point correlation function, or ‘bispectrum’ of the temperature anisotropies. Gaussian random variables (and functionals of gaussian random variables) have the property that all odd power correlation functions are zero. This makes the bispectrum the lowest order statistic for which any non-zero result would indicate a departure from gaussianity. The bispectrum contains much more information than the power spectrum as, in general, it depends on both scale and shape. Therefore, if a non-zero bispectrum is detected it will also be an extremely useful statistic for constraining models of the early universe.

However there are limitations to how much information can be inferred from the bispectrum:

- Modern CMB experiments possess very large numbers of pixels of data. e.g. \(N \simeq 10^6\) for WMAP and \(N \simeq 10^7\) for Planck \[6\]. Considering that each higher order of correlation function will require an additional factor of \(N\) calculations, exact determination of the bispectrum quickly becomes impossible. Therefore various estimators have been constructed \[7\], but each can look only for a specific type of bispectrum and so might miss a signal different from the type being searched for.

- Higher order correlation functions suffer more from cosmic variance because more information is required at each scale to compute them. In this work we will deal with a scale dependent signal, therefore limitations due to cosmic variance are particularly relevant.

These problems are not necessarily insurmountable. Computing power will only continue to get stronger, making bispectrum estimators increasingly powerful with time. For each potential signal, estimators can also be constructed for that specific signal, ensuring processing time is used as efficiently as possible. The estimator can be optimised for detecting primordial non-gaussianity while discriminating against secondary non-gaussianities arising from e.g. unsubtracted point sources or residuals from component separation \[8\]. Finally, in models that have non-trivial scale dependence, a correlation will likely exist between the power spectrum and bispectrum as we illustrate in this paper.
One of the first methods for quantifying non-gaussianity involved rewriting the Newtonian potential $\Phi(x)$ as \[1\],

$$
\Phi(x) = \Phi_L(x) + f_{NL}(\Phi^2_L(x) - \langle \Phi^2_L(x) \rangle),
$$

(1.1)

where $f_{NL}$ is a constant, and $\Phi_L(x)$ is a gaussian variable. However, this parameterisation has a rather specific form and does not capture other possible deviations from gaussianity. A more general parameterisation can be obtained by allowing $f_{NL}$ to depend explicitly on the wavevector, $k$. In terms of the adiabatic curvature perturbation, $\zeta$, the parameterisation becomes \[2\],

$$
\zeta(x) = \zeta_L(x) - \frac{3}{5} f_{NL} \star (\zeta^2_L(x) - \langle \zeta^2_L(x) \rangle).
$$

(1.2)

Here, the $\star$ product is used because $f_{NL}$ has been allowed to depend on scale.\(^1\) The factor of -3/5 comes from the relationship between the adiabatic curvature and the Newtonian potential at matter domination.

For $f_{NL}$ as defined in Eq.(1.1), the WMAP 5-year constraint is $-9 < f_{NL} < 111$.\(^3\) This reinforces the point made already: the primordial temperature anisotropies are very close to gaussian. Given that the amplitude of these anisotropies is $\Delta T/T \sim 10^{-5}$ the non-gaussian contribution to these anisotropies is at most $\sim 10^{-8}$ (or $\sim 0.1\%$ of the overall anisotropy). In general the expectation from inflation is that the anisotropies should be close to gaussian, therefore this result can be considered as a success of the inflationary paradigm.

More precisely the expectation from inflation of non-gaussianity as parameterised by Eq.(1.2) depends on the specific model. It is known that for single field, slow-roll inflation with a canonical kinetic term and a vacuum initial state, the result is $f_{NL} \sim \epsilon \ll 1$ where $\epsilon$ is the usual slow-roll parameter defined later in Eq.(3.6). The best sensitivity expected from the Planck satellite is to $f_{NL}$ of $\mathcal{O}(5)$ while the secondary contribution from post-inflationary evolution is of $\mathcal{O}(1)$.\(^4\) Therefore the prediction of the simplest inflationary toy models\(^2\) is that there should not be a detection of primordial non-gaussianity in the near future.

Conversely a deviation from the simplest toy models can produce a larger value for $f_{NL}$. The study of these effects serves two purposes. Firstly, from the cosmological perspective, a detection of $f_{NL}$ will immediately rule out single field slow-roll inflation and focus attention on determining how inflation actually occurred. Secondly, from the perspective of inflationary model building based on fundamental physics, as the bounds on $f_{NL}$ grow tighter, some interesting models can be ruled out if there is no detection.

Much work has been done on non-gaussianity generated due to multiple scalar fields (e.g. Ref.[11]), non-canonical kinetic terms (e.g. Ref.[12]) or non-vacuum initial states (e.g. Ref.[13]). However, surprisingly little attention has been paid to the possibility of non-gaussianity generated by a violation of slow-roll. Ref.[14] did consider this in a context

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\(^1\)See Eq.(229) in Ref.[2] for how to define this product.

\(^2\)By this we mean a generic fine-tuned potential such as the frequently used $V(\phi) = m^2 \phi^2$ which has not yet been convincingly obtained from a physical theory, especially since inflation occurs in such models at $\phi > M_P$. 

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when multiple fields are present; however, the non-gaussianity itself is generated by the multiple fields, not the violation of slow-roll (see also Ref.\[15\]). Refs.\[16, 17\] do consider the non-gaussianity generated by violation of slow-roll, but for a toy inflationary model. We follow their methods closely but consider a physical model of inflation. In both models the violation of slow-roll generates sharp features in the power spectrum and also generates a ringing in the bispectrum with a characteristic period which we calculate below.

1.1 Features in the Power Spectrum of Primordial Fluctuations

It is an expectation in the simplest toy models of inflation that the power spectrum of temperature anisotropies in the CMB should be nearly scale-invariant \[18\]. However this need not be the case for physical models. If the observed spectrum was indeed perfectly scale-invariant, such models would be ruled out and naively this might seem to be the case. Assuming the “concordance” \(\Lambda\)CDM cosmology, the primordial spectrum is well parameterised as a power law with spectral index \(n_s = 0.960 \pm 0.014\) \[3\]. This indicates that there cannot be any significant scale dependence that affects the spectrum over a large range of scales but it does not preclude the existence of e.g. localised oscillations in the spectrum, or other sharp features. Moreover since the observed anistropies arise from the convolution of the unknown primordial spectrum with the transfer function of the assumed cosmological model whose parameters are being determined, it is obvious that both unknowns cannot be determined simultaneously without further assumptions.

In fact, there are indications that the primordial spectrum might not be a scale-free power law, even assuming the \(\Lambda\)CDM cosmology \[13, 20, 21, 22, 23, 24\]. At large angular scales (\(> 50^\circ\)) there is essentially no power and there are anomalous ‘glitches’, especially in the range of multipoles \(\ell \approx 20 − 40\) \[25, 26\]. The statistical significance of these anomalies is not sufficient to claim a definite detection, however it is strong enough to provide some tension with the fit to a scale-free power-law primordial spectrum which has only a 3% probability of being a good description of the WMAP 1-year data \[25\], although this did improve to \(\sim 7\%\) with the WMAP 3-year data release \[26\]. Future measurements of the \(EE\) and \(TE\) mode polarisations by the Planck and (proposed) CMBPol satellites will throw light on whether these glitches are real or not \[27\].

“\textit{In the absence of an established theoretical framework in which to interpret these glitches (beyond the Gaussian, random phase paradigm), they will likely remain curiosities}” \[26\]. Indeed if there were a model that had predicted, without ambiguity, the position and amplitude of the glitches, this would be seen as very strong evidence for the model. Although no such model exists, the general possibility of generating glitches over a range of scales had in fact been proposed prior to the WMAP observations in the context of ‘multiple inflation’ wherein the mass of the inflaton field undergoes sudden changes during inflation \[28\]. This generates characteristic localized oscillations in the spectrum, as was demonstrated numerically in a toy model of an inflationary potential with a ‘kink’ parameterised as \[29\]:

\[
V(\phi) = \frac{1}{2} m^2 \phi^2 \left[ 1 + c \tanh \left( \frac{\phi - \phi_s}{d} \right) \right]. \quad (1.3)
\]
By tuning the position, amplitude and gradient of the kink, the locations of the glitches can be varied to match the glitches seen in the power spectrum. One can then perform statistical likelihood tests to determine whether the fit is better with the glitches, but with the additional parameters, or with the simple scale-free spectrum. It was found by the WMAP team that the fit to the 1-year data improves significantly (by $\Delta \chi^2 = 10$) for the model parameters $\phi_s = 15.5 \, M_P$, $c = 9.1 \times 10^{-4}$ and $d = 1.4 \times 10^{-2} \, M_P$, where $M_P \equiv (8\pi G_N)^{-1/2} \simeq 2.44 \times 10^{18} \, \text{GeV}$ [30]. This analysis was repeated later using the WMAP 3-year data, with similar results [31].

This seems encouraging, however $m$ in the toy model above is not the mass of the inflaton — in fact in all such monomial ‘chaotic’ inflation models with $V \propto \phi^n$, inflation occurs at field values $\phi_{\text{infl}} > M_P$, hence the leading term in a Taylor expansion of the potential around $\phi_{\text{infl}}$ is always linear in $\phi$ (since this is not a point of symmetry), rather than quadratic as for a mass term [32]. The effect of a change in the inflaton mass can be sensibly modelled only in ‘new’ inflation where inflation occurs at field values $\phi_{\text{infl}} << M_P$ and an effective field theory description of the inflaton potential is possible. The ‘slow-roll’ conditions are violated when the inflaton mass changes due to its (gravitational) coupling to ‘flat direction’ fields which undergo thermal phase transitions as the universe cools during inflation [28]. The resulting effect on the spectrum of the curvature perturbation was found by analytic solution of the governing equations to correspond to a ‘step’ followed by rapidly damped oscillations [28].

The next step should be to predict other observables, having used the power spectrum to constrain all the parameters in the model. In this paper we calculate the bispectrum of the multiple inflation model [28], using its predicted power spectrum [33] and the set of parameters which provide the best reduced $\chi^2$ in the $\Lambda$CDM cosmology [35]. We also examine the effect on the bispectrum of varying these parameters over their full natural range.[4] The non-gaussianity is found to be potentially detectable by the Planck satellite, or perhaps even a reanalysis of the WMAP data.

2. Multiple inflation

The biggest difficulty in inflationary model building is obtaining a potential that is both sufficiently flat to support inflation for $\sim 60$ e-folds and stable towards radiative corrections. $N = 1$ supergravity (SUGRA), the locally realised version of supersymmetry (SUSY), is a natural framework for achieving this (see Ref. [36] for a comprehensive review, especially of the cosmological issues discussed below). In SUSY there are usually many ‘flat directions’ in field space, i.e. scalar degrees of freedom which, while SUSY remains unbroken, have perfectly flat potentials. When SUSY is broken, the flat directions are lifted and usually acquire a mass-squared related to the SUSY breaking scale. It would seem natural to identify one of these flat directions as the inflaton but to achieve a sufficient number of e-folds of inflation, the mass-squared of the inflaton needs to be much smaller than the Hubble

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3 Although a similar phenomenon had been noted earlier for the case where the inflaton potential has a jump in its slope [24], such a discontinuity has no physical interpretation.

4 We use the word ‘natural’ in this context to mean “stable towards radiative corrections”. 
expansion rate during inflation — if it is not, the evolution is too quick and inflation ends after only a few e-folds. However, due to SUSY breaking by the vacuum energy driving inflation, the natural expectation is $m_\phi^2 \sim H^2$. This is commonly known as the $\eta$ problem in supergravity and superstring model building because it relates to the slow-roll parameter $\eta$ (defined in Eq. 3.7) being too large to support inflation.

There has been much work done to attempt to circumvent the $\eta$ problem (see Ref. [37] for a review and Ref. [38, 39] for some recent attempts in the framework of string/brane models). In this work we will assume that the inflaton mass has a symmetry protecting it against SUSY-breaking corrections. This can be done for example by having the inflaton in a ‘hidden sector’ where it interacts with the other sectors only gravitationally [40].

In the simplest inflationary models, scalar fields other than the inflaton itself are usually ignored. This is justified by the argument that, because of their larger masses, they will not contribute to the adiabatic density perturbations produced during inflation. If these fields decay before the end of inflation then their decay products will be diluted by the expansion of the universe and they will also not contribute any isocurvature perturbations. Such fields usually cannot couple to the inflaton either because any such coupling could endanger the flatness of the inflaton potential.

Multiple inflation [28] is based on $N = 1$ SUGRA and assumes that while ‘flat direction’ scalar fields have their natural SUSY-breaking scale masses of order the Hubble parameter, there is just one scalar field (the inflaton) with a mass that is kept anomalously small through a symmetry. The amplitude of the potential during inflation will be the SUSY breaking scale (which need not be the electroweak scale as SUSY breaking can be different during and after inflation). The flat directions couple gravitationally to the inflaton and thus affect its evolution when their own masses change as the universe cools during inflation and they undergo symmetry-breaking phase transitions.

2.1 Evolution of the flat direction fields

It is assumed [28] that before inflation begins the universe is in a thermal state at temperature $T$. At this point the flat directions, $\psi$, are trapped at the origin by a thermal potential $\propto \psi^2 T^2$. Unless protected by a symmetry, the mass $\mu$ of the field will be of the order of the Hubble parameter, $H$, which is itself determined by the scale $V_0 \sim \Delta^4$ of the vacuum energy driving inflation. Henceforth we work in units where the reduced Planck mass $M_P = 1$, therefore $\mu^2 \simeq H^2$ and $H^2 \simeq \Delta^2/3$.

Although the mass-squared is negative, the flat directions will be lifted at large field values by non-renormalisable operators $\propto \psi^n (M_P = 1)$. For small field values, at non-zero temperatures, the full potential for the flat directions is thus [28],

$$V(\psi) = V_0 + \left( -\frac{\mu^2}{2} + \alpha T^2 \right) \psi^2 + \gamma \psi^n. \quad (2.1)$$

Before inflation, when the temperature is much larger than the mass of the field, it is trapped at the origin by a thermal barrier (quantum tunneling through which is negligible). When inflation starts, the temperature drops rapidly and after $\sim \ln(M_P/\Delta) \approx 10$ e-folds of inflationary expansion the thermal barrier disappears. The field can then evolve to
its minimum at $\Sigma = (\mu^2/n\gamma)^{1/n-2}$ and meanwhile there are $\sim \ln(\Sigma M_P/\Delta^2)$ e-folds more of inflation \footnote{The field evolves as $\psi \propto e^{\mu t}$ so the occupation number of thermal states drops as $\sim e^{-\Delta^2}$ and there is no non-gaussianity generated due to the thermal fluctuations while the field is at the origin (c.f. \cite{AMT}) since observable perturbations today leave the horizon much later.}. The field evolves (as a critically damped oscillator) according to the equation of motion:

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{\partial V}{\partial \psi} = \left[\mu^2 - n\gamma\psi^{(n-2)}\right] \psi; (2.2)$$

when it reaches the minimum the oscillations of the field are damped rapidly (within 1–2 cycles) by the $3H\dot{\psi}$ term (see Fig.1 of \cite{AMT}). Damping due to particle creation is far less effective since this is happening in an inflating background \cite{AMT}. Consequently no isocurvature perturbations are generated by such oscillations.

**The relationship of the flat directions to the inflaton**

It was shown in Ref.\cite{AMT} that a term $\kappa\phi^i\phi\psi^2$ in the Kähler potential with $\kappa$ of $O(1)$ is consistent with the underlying symmetries and corresponds to a term $\frac{1}{2}\lambda^i\phi^2\psi^2$ in the scalar potential where $\lambda = \kappa H^2$. The full potential, including the inflaton, is thus:

$$V(\phi, \psi_i) = V_0 - \frac{1}{2}m_\phi^2\phi^2 - \sum_i \left(\frac{1}{2}\mu_i^2\psi_i^2 - \frac{1}{2}\lambda_i^i\phi^2\psi_i^2 - \gamma\psi_i^{n_i}\right). (2.3)$$

There will be a running of the inflaton mass too, however the field value of the inflaton remains small throughout inflation so we can neglect the higher-dimensional operators that would parameterise such running.

Hence as each field, $\psi_i$, rolls to its minimum at $\Sigma_i$, the effective inflaton mass-squared changes by $\lambda_i\Sigma_i^2$. The amplitude of the potential also changes as each field falls into its minimum, however, this change will be very small compared to the dominant term in the potential $V_0$ \cite{AMT}.

During periods of slow-roll, the amplitude of the primordial perturbation spectrum can be expressed in terms of the potential and its derivatives as,

$$P_k = \frac{H_*^2}{8\pi^2\epsilon_*} = \frac{V^3}{12\pi^2\gamma^2}. (2.4)$$

Where the $*$ indicates that the quantities should be evaluated as the relevant scales ‘cross the horizon’, i.e. when $k^2 = 2a^2H^2$. A transition in the mass of the inflaton, though not causing a significant change in $V$, will cause a significant change in $V'$. If $\lambda_i$ is positive, the mass will decrease causing a jump in the amplitude of the spectrum, while if it is negative then the amplitude will fall. As the flat direction oscillates in its minimum, the mass of the coupled inflaton will briefly oscillate as well. If the amplitude of the oscillations is large enough, one would expect a ‘ringing’ in the primordial power spectrum \cite{Felder} for which there is tentative observational indication \cite{FeldervanHove, FelderMoller, FelderMoller2, FelderMoller3}.

The WMAP measurements indicate that, from $\ell \simeq 40$ through to the limit of $\ell \sim 800$ set by signal-to-noise considerations, the inferred primordial spectrum is nearly flat. However, as was noted earlier, there is tentative evidence of a step at very large scales of
order the present Hubble radius \[19, 21, 22\]. It was in fact a prediction of the original paper on multiple inflation \[28\] that there should be of \(\mathcal{O}(1)\) such feature in the \(\sim 10\) e-folds of inflationary history probed by the CMB.

The advantage of the multiple inflation model over the kink potential \([13]\) considered in Ref.\[14, 29\] (or the toy model considered in Ref.\[34\]) is its grounding in a consistent, particle physics framework. While the latter model is able to improve the fit to the glitches in the data \([80, 81]\) the constraints derived on the model parameters do not tell us anything about the relevant physical processes at these high energy scales. By contrast, constraints on multiple inflation are related directly to the masses and couplings of fields potentially present during inflation. For example, the scalar potential of the Minimal Supersymmetric Standard Model (MSSM) is flat along many directions in field space and a catalogue exists of all these directions \([42]\). These would be lifted during inflation due to SUSY-breaking by the large vacuum energy present and conceivably undergo symmetry breaking phase transitions.

3. Method

3.1 Single field justification

We follow closely the method of Ref.\[16\], who use the framework of Ref.\[4\] in calculating the bispectrum. This method uses the comoving gauge where, in single field inflation, the inflaton fluctuations are zero at all orders. When there are other, more massive, fields present this is no longer true because these extra fields will slightly perturb the instantaneous, adiabatic, direction in field space away from the direction of the main inflaton field. In multiple field scenarios a simpler gauge to describe the evolution of perturbations within the horizon is the uniform curvature gauge. In this gauge, neglecting gravitational waves, the slicing of spacetime into equal time hypersurfaces is made such that the spatial curvature on each of these surfaces is zero.

It was shown in Ref.\[44\] (and generalised beyond linear order in Ref.\[45\]) that there exists a gauge-independent, non-perturbative quantity \(\zeta\) which, on sufficiently large scales, coincides with the adiabatic, scalar density perturbation at the perturbative level. Defining the number of e-folds of expansion as \(N = \int H dt\), it was shown in Ref.\[44, 45\] that if we start in the uniform curvature gauge, this quantity can be expressed as \(\zeta = dN\). This allows us (on sufficiently large scales) to express the adiabatic curvature perturbation as \[47\],

\[ \zeta = \sum_I \frac{\partial N}{\partial \phi_I} \delta \phi_I + \sum_{I,J} \frac{1}{2} \frac{\partial^2 N}{\partial \phi_I \partial \phi_J} \delta \phi_I \delta \phi_J + \cdots. \]  

(3.1)

If we set the initial, uniform curvature, hypersurface a few e-folds of expansion after the modes cross the horizon then due to the much smaller mass of the inflaton, we have \(\dot{\phi} \ll \dot{\psi}\), hence \(\partial N/\partial \phi (= H/\dot{\phi}) \gg \partial N/\partial \psi\) for all the flat direction fields. For all fields involved (including the inflaton), the higher order terms in Eq.\,(3.1) are also very small.\(^6\) Under these

\(^{6}\)These terms become significant in models where non-gaussianity is seeded by entropy perturbations, after the perturbations have left the horizon \([46]\).
conditions, the result \( \zeta = \partial N \partial \phi \delta \phi \) holds to a very good approximation. The flat direction fields do not affect the adiabatic curvature perturbation and thus isocurvature perturbations do not arise gravitationally during inflation. Therefore the inflaton fluctuations will be zero in the comoving gauge during inflation and we are free to use the action derived in Ref.[4] using this gauge.

It should be stressed however that the flat directions do play a crucial role in the generation of the non-gaussianities. The point of the preceding argument is that they do this only through their coupling to the inflaton and not directly through their coupling to gravity. What this means for the calculation is that when solving the zeroth order equations of motion for the inflaton we must include the effects of the flat direction fields. However, when computing the perturbative action to quadratic and cubic order in the perturbations, we need consider only the inflaton.

In principle it might have been less confusing to work initially in the uniform curvature gauge, where the contribution from all the fields is unambiguous, and to convert to the comoving gauge once the curvature perturbation has left the horizon. This would have the benefit of less ambiguity in the present case and be a necessity in genuine multi-field models. In future work we intend to generalise the method of Ref.[16], as used in this paper, to inflationary models involving several, dynamically equally important, fields.

3.2 Calculating the bispectrum

We are interested in calculating the bispectrum (three-point correlation function) of the quantum observable \( \zeta \), corresponding to the scalar, adiabatic curvature perturbation. The model we are considering has the standard, purely gaussian vacuum state, and the quadratic action of the field, \( \zeta \), is effectively just the free field action (i.e. a kinetic term and a mass term). A free gaussian field will remain gaussian, therefore in order to probe any non-gaussianity in the fluctuations it is necessary to consider the cubic action. We know from observations of the power spectrum that \( |\zeta| \sim 10^{-5} \) i.e. higher terms in the perturbative expansion of the action will be less important in general.\(^7\) Therefore, we need only consider the action up to cubic order.

We proceed by re-writing the quantum operator \( \zeta(t) \) in the interaction picture:

\[
\zeta_1(t) = e^{i H_f (t-t_0)} \zeta_1(t_0) e^{-i H_f (t-t_0)},
\]

where \( H_f \) is the free field Hamiltonian derivable from the quadratic action and \( \zeta_1(t_0) \) is the initial vacuum state value of the operator. The full quantum operator is then,

\[
\zeta(t) = U_1(t, t_0) \zeta_1(t) U_1(t, t_0),
\]

where,

\[
U_1(t, t_0) = T e^{- i \int_{t_0}^t H_i(t')dt'},
\]

is the time evolution operator for the interaction terms (and \( T \) denotes the time ordering operator).

\(^7\)This need not be true always, e.g. if a symmetry forces the cubic action to be anomalously small then the quartic action will produce the dominant contribution to any non-gaussianity [48].
Putting these terms into the three-point function gives,
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = \langle U_1^\dagger \zeta(t)^3 U_1 \rangle
\]
\[
= -i \int_{t_0}^t \langle [\zeta(t)^3, H_I(t')] \rangle dt'.
\] (3.5)

Where the last line of equality holds to first order in the expansion of the exponentials.

It is usually emphasised at this point that the vacuum with respect to which the expectation value is being taken is the fully interacting vacuum, not the free vacuum. This is important because the \( \zeta_I \) appearing in these equations will be expressed in terms of the ladder operators that annihilate the free vacuum. Fortunately, this is not an issue in this particular case because, similarly to what is done in Minkowski space, one can slightly deform the integral contour into Euclidean space. This projects out the free vacuum, as explicitly shown in Ref.[9].

Up to field redefinitions proportional to the free field equations of motion, the Hamiltonian in Eq.(3.5) can be obtained from the action derived in Ref.[4] (Eq.(3.9) in that paper and Eq.(3.4) in Ref.[16]). This action can be expressed as a sum of terms whose coefficients are functions of the slow-roll parameters, defined as:
\[
\epsilon \equiv \frac{\dot{\phi}^2}{2H^2},
\] (3.6)
\[
\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \frac{2\ddot{\phi}}{\dot{\phi}H} + \frac{\dot{\phi}^2}{H^2},
\] (3.7)
where the approximate equality holds when both \( \epsilon \) and \( \eta \) are small. Note that there are other common definitions for \( \epsilon \) and \( \eta \) in terms of derivatives of the potential: \( \epsilon_V \equiv \frac{1}{2}(V''/V)^2 \) and \( \eta_V = V''/V \); the ones defined above are related through \( \epsilon_V = \epsilon \) and \( \eta_V = -\frac{1}{2} \eta + 2\epsilon \).

These equalities hold in the slow-roll approximation when all four quantities are small.

In terms of these quantities, the cubic action for \( \zeta \) is:
\[
S_3 = \int d^4x \ a^3\epsilon \zeta^2 \dot{\zeta}^2 + a^3\epsilon \zeta (\partial \zeta)^2 - 2a^3\epsilon \dot{\zeta} (\partial \zeta)(\partial \chi) + \frac{a^3\epsilon}{2} \frac{d\eta}{dt} \zeta^2 \dot{\zeta}
\]
\[
+ \frac{a^3\epsilon}{2} (\partial \zeta)(\partial \chi) \partial^2 \chi + \frac{a^3\epsilon}{4} (\partial^2 \zeta)(\partial \chi)^2 + 2f(\zeta) \frac{\delta L}{\delta \zeta} \bigg|_1,
\] (3.8)
where \( \chi = \epsilon \partial^{-2} \zeta \).

The term \( 2f(\zeta) \frac{dL}{\delta \chi} \bigg|_1 \) is proportional to the free field equations of motion, \( \frac{dL}{\delta \chi} \bigg|_1 \), and in Ref.[4] is removed by a field redefinition. This can have an important effect on the bispectrum, however, in the present case it does not, due to its relative size. There is just one term in \( f(\zeta) \) (Eq.(3.10) in Ref.[4]) which does not include a derivative on one of the \( \zeta \) terms. As we can choose to apply the correction due to the redefinition after the relevant scales have left the horizon, only this term will have a non-zero effect — its coefficient is \( \eta/4 \), which is much smaller than our leading term. 

Note that although \( \eta \) does become large during the phase transition in multiple inflation, we can choose to apply the correction after the phase transition is over, at which point \( \eta \) is again \( \ll 1 \). The redefined field will evolve outside the horizon while \( \eta \) is significant so this can be done only if we also calculate the evolution of the redefined bispectrum out to this time, which we have indeed done.
With the field redefinition taken care of, the Hamiltonian in Eq.(3.3) just becomes \( H_1 = -L_3 \). To use this to evaluate Eq.(3.5) we need to solve the commutator in this equation. We are free to use the standard commutation relations for \( \zeta \); to do so we first need to express \( \zeta_I(k, t) \) in terms of the ladder operators of the free field vacuum

\[
\zeta_I(k, t) = u_k(t) a_k + u^*_{-k}(t) a^\dagger_{-k}, \tag{3.9}
\]

with the usual commutation relations, \([a_k, a^\dagger_{k'}] = (2\pi)^3 \delta^3(k-k')\) and mode functions \(u_k(t)\).

The relationship between \( \zeta_I(k) \) and \( \zeta_I(x) \) is the usual

\[
\zeta_I(x) = \int \frac{d^3k}{(2\pi)^3} \zeta_I(k) e^{ikx}. \tag{3.10}
\]

When these definitions are substituted back into Eq.(3.5), the result is proportional to an integral over time of the free field mode functions \(u_k(t)\), evaluated on the interaction Hamiltonian, \( H_1 \) \((3.11)\).

**The dominant term in multiple inflation**

It has been stated already that multiple inflation violates slow-roll, nevertheless \( \epsilon \) remains \( \ll 1 \) throughout multiple inflation for all parameter values considered here.\(^9\) There is only one term in the action \((3.8)\) that does not have a factor of at least \( \epsilon^2 \) in it — this is the term with \( d\eta/dt \) (note that \( \chi \) is of order \( \epsilon \)). However although \( \epsilon \) remains small, due to the transition in the inflaton mass, \( \eta \) and \( d\eta/dt \) can temporarily become large. The size of the latter term is dictated by both the magnitude and the rate of the mass change in the phase transition. Multiple inflation is a small field model, hence \( \epsilon \) is very small \((\lesssim 10^{-10})\) and the \( d\eta/dt \) term dominates over all others because it has one less factor of \( \epsilon \).

**3.2.1 Numerical method for calculating bispectrum**

Upon substituting this leading term into Eq.(3.3), using the definitions of \( \zeta(k) \) and \( \zeta(x) \) and working through the commutation, the result is (in conformal time \( d\tau \equiv dt/a \)):

\[
\langle \zeta(k_1, \tau) \zeta(k_2, \tau) \zeta(k_3, \tau) \rangle = -2 \text{Im} \left\{ u_{k_1}(\tau) u_{k_2}(\tau) u_{k_3}(\tau) \int_{\tau_0}^\tau d\tau' \left[ a^2 \frac{d\eta}{d\tau'} (2\pi)^3 \delta^3 \left( \sum_i k_i \right) \times \left( u_{k_1}(\tau') u_{k_2}(\tau') \frac{du_{k_3}}{d\tau'} + u_{k_1}^*(\tau') u_{k_3}^*(\tau') \frac{du_{k_2}}{d\tau'} + u_{k_2}^*(\tau') u_{k_3}^*(\tau') \frac{du_{k_1}}{d\tau'} \right) \right] \right\}. \tag{3.11}
\]

This should be compared to Eq.(3.21) in Ref.[16].

To obtain the bispectrum for multiple inflation, we must solve this integral. To do this we first need to numerically solve for the free field mode functions \(u_k(\tau)\), the scale factor \(a(\tau)\) and the slow roll parameters \(\epsilon\) and \(d\eta/d\tau\). Once this has been done, the results are put into the above integral and evaluated out to a value of \(\tau\) after the end of inflation. All that is needed after that is a rescaling of the full bispectrum to obtain a scale dependent generalisation of \(f_{NL}\).

\(^9\)If \( \epsilon \) does become large, inflation will stop; this is potentially possible if there is a later inflationary epoch \([28]\) but we do not consider this possibility here.
Fortunately, the slow roll parameters in the above integral are composite variables of other variables more fundamental to the model (e.g. $\epsilon = \dot{\phi}^2/2H^2$). Therefore, we need only solve for these more fundamental variables and then evaluate $\epsilon$ and $d\eta/d\tau$ from them. The full list of variables that need to be solved for, with their equations of motion, are [49]:

$$v''_k + \left(k^2 - \frac{z''}{z}\right)v_k = 0,$$

(3.12)

with $z = a\sqrt{2\epsilon}$, $v_k = zu_k$, and:

$$\phi'' + a^2\frac{dV}{d\phi} + \frac{2a'\phi'}{a} = 0,$$

(3.13)

$$\psi'' + a^2\frac{dV}{d\psi} + \frac{2a'\psi'}{a} = 0,$$

(3.14)

$$a'' + \frac{a}{6} \left(\phi'^2 + \psi'^2\right) - \frac{2a^3V}{3} = 0.$$

(3.15)

In all of the above, the $'$ indicates differentiation with respect to conformal time. These four equations are just the free field equation of motion for the observable $\zeta$, the two zeroth order equations of motion for $\phi$ and $\psi$, and finally the Friedmann equation for the Hubble parameter $\dot{a}/a$. We solve these equations numerically, using the Matlab function ode113 [50]. For initial conditions we take the Bunch-Davies vacuum state [49]. This amounts to:

$$v_k(\tau_0) = \sqrt{\frac{1}{2k}},$$

(3.16)

$$v'_k(\tau_0) = -i\sqrt{\frac{k}{2}}.$$  

(3.17)

The initial value of $\tau = \tau_0$ in the integrand was set such that integration of the mode functions, $v_k$, begins when the following condition is first satisfied:

$$10^4 \frac{z''}{z} > k^2.$$  

At earlier times, the mode functions are highly oscillatory and would cancel in the integral in Eq.(3.11). In fact there is only a small window in which this integral is non-zero — very early on, the mode functions are highly oscillatory and any contribution to this integral will be washed out, while very late on, the mode functions are frozen and thus the $d\eta/d\tau$ term will be zero. Finally, when the phase transition is not occurring, the $d\eta/d\tau$ term will be negligible. It is only those modes that are leaving the horizon during the phase transition that will give a non-zero contribution to the bispectrum.

A regularisation scheme is used in Ref.[16] to counter a sharp cutoff in the integral in Eq.(3.11) (and improved upon in Ref.[17]). We find that a regularisation scheme is useful for calculational efficiency but not necessary for accuracy. It was argued in Ref.[16] that although the integral is technically convergent as $\tau \to -\infty$, this sharp cutoff would add a spurious contribution to the bispectrum, due to the highly oscillatory nature of
the mode functions at early times. We find that this is a problem only when the cutoff also co-incides with the violation of slow-roll. If this occurs, there was indeed a spurious contribution due to the sharp nature of the cutoff; however, if we begin our integration earlier, when the slow-roll contribution to the integral, $\epsilon d\eta/d\tau$ is negligible, no spurious contribution is obtained.

In ordinary slow-roll, to accurately calculate the contribution from the other, dominant terms in Eq.(3.8) by numerical means, a regularisation scheme would be necessary. For multiple inflation (and for the kink model considered in Ref.[14]) these other terms are small enough to be ignored. This is especially true in multiple inflation where $\epsilon$ is exceptionally small. What the regularisation scheme does do is allow the integration to begin later without sacrificing accuracy. Nevertheless, we did not use a regularisation in calculating our results and altered the integration start time when there was a risk of a spurious contribution.

3.2.2 Comparing a bispectrum to $f_{NL}$

We have defined a scale-dependent generalisation of the $f_{NL}$ parameter in Eq.(1.2). Now we discuss how to calculate $f_{NL}$ from a given bispectrum. For a flat bispectrum a convention has been well established, however there is an ambiguity in the present case due to the scale dependence of both the power spectrum and the bispectrum. For a flat spectrum, following Refs.[4, 9], we start by re-expressing the bispectrum as,

$$
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 \delta^3 \left( \sum_i k_i \right) \frac{H^4}{16\epsilon^2} \frac{A}{\Pi_i k_i^3}.
$$

(3.18)

In this notation, we can rewrite $f_{NL}$ in the form $^{10}$

$$
f_{NL} = -\frac{5}{6} \frac{A}{\sum_i k_i^3}.
$$

(3.19)

However, as pointed out in Ref.[10], this definition entangles some of the ringing of the power spectrum in the definition of $f_{NL}$. The factors of $H^2/\epsilon$ in the bispectrum that are divided out to get $A$ are proportional to the power spectrum which is itself oscillatory. This can be overcome by factoring out a number $(\tilde{P}_k)^2$ that is equal to the square of the value of the power spectrum that is measured when assuming scale invariance $^{10}$. This gives

$$
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta^3 \left( \sum_i k_i \right) \left( \frac{\tilde{P}_k}{2} \right)^2 \frac{A}{\Pi_i k_i^2}.
$$

(3.20)

In the case of slow-roll and a flat spectrum this definition is equivalent to our previous one in Eq.(3.18) (with $\tilde{P}_k = H^2/8\pi^2\epsilon$). It is then appropriate to define $f_{NL}$ in the same way as in Eq.(3.19) and it is this that we discuss in the following section. Note that although Refs.[4, 9, 16] define $A$ differently, the definitions for $f_{NL}$ coincide. We have followed the convention of Ref.[9] for $A$.

$^{10}$This follows directly from Eq.(1.2) if we rewrite it in $k$ space and then evaluate the bispectrum.
4. Results

4.1 Reproduction of the kink model

To check the accuracy of our code and calculations we first set out to reproduce the bispectrum of the ‘kink’ model in Eq.(1.3) [16]. Note that $\phi_s$ dictates where the kink occurs, $c$ dictates the magnitude of the total shift in slope and $d$ dictates the rate of the change in slope. For models where the non-gaussianity is generated as the modes cross the horizon, the bispectrum will be peaked over the equilateral shape ($k_1 = k_2 = k_3$). The authors of Ref.[16] consider the statistic $G(k_1, k_2, k_3)/k_1 k_2 k_3$, where $f_{NL} = -10 k_1 k_2 k_3 \sum_k \frac{G(k_1, k_2, k_3)}{k_1 k_2 k_3}$; in the equilateral case this becomes $f_{NL} = -10 G/k^3$. In Fig.1 we show the equilateral bispectrum for the kink model over the full range over which it is non-zero, to be compared with Fig.4 of Ref.[16] which shows this over a narrower range in $k$. Note that although $G/k^3 \sim -f_{NL}$ becomes as large as 10, it does not remain large over an extensive range of $k$. This has important consequences for any attempt to detect the non-gaussianity.

![Figure 1: Full equilateral bispectrum for the ‘kink’ model [16], with $m = 10^{-6}$, $c = 0.0018$, $\phi_s = 14.81$ and $d = 0.022$ [31] ($M_P = 1$). The scale of $k$ is arbitrary.](image)

4.2 Bispectrum of multiple inflation

The most important, and difficult, part of the bispectrum calculation is obtaining the mode functions, $u_k$. Therefore we first demonstrate that we can reproduce the power spectrum for multiple inflation accurately as shown in Fig.2 (to be compared with Fig.2 of Ref.[35]). These spectra were calculated considering only one flat direction field, with a positive coupling ($\lambda > 0$) to the inflaton. The parameters used in the multiple inflation potential (2.3) are $\phi_0 = 0.01$, $m^2 = 0.005 H^2$, $\lambda = H^2$, $\gamma = 1$ and $\mu^2 = 3H^2$ [35]. Note that $V_0$ is fixed through Eq.(2.4) by requiring the amplitude of perturbations to be the observed value ($\simeq 2.5 \times 10^{-9}$) after the phase transition, and is different for each value of $n$, the order of the non-renormalisable operator which lifts the flat direction potential at large values of the field. The time at which the phase transition occurs also varies with $n$. We emphasise that the value $n = 16$ is particularly favoured as it is a well known flat direction in the MSSM scalar potential [42].
We can now confidently use our code to calculate the bispectrum of multiple inflation as shown in Fig.3. Note that as $k \to 10^{-2} \, h \, \text{Mpc}^{-1}$, the bispectrum begins to exhibit noise — this is a consequence of our not applying the regularisation scheme described in Ref. [16] and is only present at these scales for the reasons explained earlier. Secondly, the non-gaussianity, while not as large as in the kink model, is non-zero over a wider range of $k$. This can be traced to the fact that being a physical rather than a toy model, the features in the power spectrum generated by multiple inflation are not as sharp as in the kink model [29]. Because the dominant term in the action is proportional to $d\eta/dt$, if the transition is not sharp, then the contribution to the bispectrum will not be large either. In multiple inflation the $\psi$ field evolves to its minimum analogous to a critically damped oscillator and oscillates for a period determined by the Hubble damping, which means that the bispectrum is non-zero for longer.

Due to cosmic variance (the transition occurs at large scales) and secondary non-gaussianities, it would be difficult to detect this signal even with a perfect estimator. We
have a similar concern about the bispectrum of the kink model [16] because although it has a larger amplitude, it is non-zero over only a decade in $k$ making it very hard to acquire sufficient statistical significance for a convincing detection.

4.2.1 The effect of varying parameters

In Ref.[35], when finding the best-fit for multiple inflation to the WMAP $TT$ and $EE$ data, the only parameters varied were the amplitude of the potential, the location of the phase transition and the power $n$ of the non-renormalisable term lifting the flat direction. All other parameters were held fixed at their most natural values, in order to maximise the predictive power of the model. Unlike the toy kink model [16] or other empirical models [43], the parameters of multiple inflation are not free to range over any set of values because of its grounding in the effective field theory framework. If one simply allowed all parameters to vary and then performed a naive $\chi^2$ test, the physically constrained (and thus predictive) nature of multiple inflation would not be factored in.

We note in passing that there is a tension between the two methods of determining the likelihood of a model being correct. From the fundamental physics perspective one can calculate the parameter values that give the best fit and then estimate how natural they are. From the cosmology model fitting perspective one can determine the goodness of fit, given the number of parameters needed to achieve it. Neither test is fully satisfactory because natural valued parameters could give a bad fit and a good fit could come from an unnatural model. A Bayesian treatment that can quantify the naturalness of a model as a prior probability distribution would solve this problem. The resulting Bayesian likelihood would incorporate both the naturalness of the underlying model and the goodness of the fit to the data.

Irrespective of this issue, if multiple inflation is true, we do not know for certain what the values of the parameters will be. Therefore we have explored the effect of varying the parameters $\lambda$, $\mu^2$ and $\gamma$ in turn and show the results in Figs. 5, 6 and 7. In each plot, we change $V_0$ as we vary the corresponding parameter to ensure that the power spectrum is always normalised to its observed value of $2.5 \times 10^{-9}$. For ease of comparison we also alter the starting point of the phase transition so that the oscillations in each figure begin at the same point (this does not alter the shape of the bispectrum in any way). Each plot has been approximately equated to spatial scales probed by the CMB observations, although we are not here comparing with actual data.

The simplest parameter to consider is $\lambda$ which does not affect the $\psi$ field’s dynamics but changes the effective mass of the inflaton through $-m^2_{\text{eff}} = -m^2 + \lambda \psi^2$, so increasing $\lambda$ increases the magnitude of the oscillations. Therefore we expect that increasing $\lambda$ has no effect on the bispectrum except to increase its amplitude and this is just what we see in Fig. 4. It is clear that a large bispectrum can be generated from natural values of the parameter $\lambda$ (although values above $\sim 5H^2$ can probably be ruled out already from the power spectrum).

4.2.2 The relationship between $T$ and $\ln k_T$

To consider the effects of $\mu^2$ and $\gamma$ on the bispectrum it is necessary first to derive an
Figure 4: Effect of parameter $\lambda$ on the bispectrum ($n = 14$, $\gamma = 1$ and $\mu^2 = 3H^2$ for each curve).

identity relating the period of the oscillations over time during the phase transition to the period of the oscillations over $k$ in the power spectrum and bispectrum. Eq. (3.12) describes the time evolution of the mode functions $u_k = v_k/z$ of the operator $\zeta_I(k, t)$. In the slow-roll approximation, $z''/z = 2(aH)^2$. When this quantity is small, $v_k$ oscillates as $v_k = e^{ik(\tau - \tau_0)}/\sqrt{2k}$, and when this quantity is large we have $v_k = \alpha z$, with $\alpha$ constant. To a good approximation, the value of the constant $\alpha$ is determined by equating the two solutions when $z''/z = k^2$. This approximation will continue to hold even if $z''/z \neq 2(aH)^2$ so long as $z''/z$ continues to grow exponentially.

The quantity $z = a\sqrt{2}\epsilon$ can be rewritten as $z = \phi'/H$. Since $H$ is almost constant we can write $z' = \phi''/H$ and $z'' = \phi'''/H$. If there are oscillations induced on $\phi'$ they will be seen more sensitively by higher derivatives of this quantity, hence $z'' = \phi''/H$ will have a relative amplitude of oscillation much greater than $z' = \phi'/H$. This means that $z''/z$ will oscillate with the same frequency as $\phi'$, which in turn will oscillate with the same frequency as $\psi$ in its minimum.

In the slow-roll limit, $a$ grows as $e^{HT}$ and this holds even during the phase transition since the vacuum energy $V_0$ remains sensibly constant.\(^{11}\) Therefore, if the oscillation repeats itself every $T$ units of time, then it will repeat itself every $(\ln a)_T = HT$ units of $\ln a$. It was established earlier that this means $z''/z$ will also repeat itself every $(\ln a)_T$ units of $\ln a$. We can thus parameterise the deviation of $z''/z$ from $2(aH)^2$ by the equation:

$$\frac{z''}{z} = 2(aH)^2 f(\ln a), \quad (4.1)$$

where $f(x)$ is a function with period $HT$. When we do the matching of the solutions inside and outside the horizon and equate $\frac{z''}{z} = k^2$, we get,

$$\left(\frac{z''}{z}\right)^{\frac{1}{2}} = k = \sqrt{2aHf^{\frac{1}{2}}}. \quad (4.2)$$

\(^{11}\)This will of course not hold precisely, however the relative deviation will be much smaller than the deviation of $z''/z$ from its slow-roll value, due to the $\phi'$ term in $z$. 

If the period of $f(x)$ is $HT$ then the period of $f^{1/2}$ will be $2HT$ because it will repeat itself half as often. If we ask what the difference in $k$ will be in the matching solution when $f^{1/2}$ undergoes one period, from $t = t_2$ to $t = t_1$, with $t_2 - t_1 = 2T$, we get,

$$k_2 - k_1 = \sqrt{2H} \left[ e^{2Ht_2} f(2Ht_2)^{1/2} - e^{2Ht_1} f(2Ht_1)^{1/2} \right].$$

Unfortunatly this result is not independent of the values of $t_2$ and $t_1$ and will vary as the oscillations occur. However if we consider what will happen to $\ln k$ during this same time period we get, with $f_n = f(2Ht_n)$:

$$\ln k_2 - \ln k_1 = 2H(t_2 - t_1) + \frac{1}{2} (\ln f_2 - \ln f_1).$$

Now the only time dependence comes in the difference between the function $\ln f$ at the two repeating points. These points were chosen because they were at the same point of the oscillation in $f^{1/2}$, therefore to a good approximation $\ln f$ will be equal at each point. This approximation will only hold exactly if the oscillations in $z''/z$ are exactly repeating.

It is thus expected that the mode functions $u_k$ will oscillate with a period over $\ln k$ equal to $(\ln k)_T = 2HT$, with a small drift due to the change of the function $f(\ln a)$ during each oscillation. This result is general to any oscillatory phase transition during inflation and is not specific to multiple inflation. It is a simple matter now to calculate the expected period, $T$, of the oscillations in multiple inflation. The minimum of the potential for $\psi$ occurs at the point, $\Sigma = (\mu^2/n\gamma)^{1/(n-2)}$. If we expand the potential around this point, with respect to the variable $\psi_* = \psi - \Sigma$, then the potential becomes,

$$V(\psi_*) = \frac{\mu^2(n-1)}{2} \psi_*^2 + \text{higher order terms.}$$

It is useful to note that $\gamma$ drops out of the quadratic term and only appears in the higher order terms. Seen from this perspective, the equation of motion for the field $\psi$, when in its minimum, is to first approximation a damped harmonic oscillator. The period, $T$, of a damped harmonic oscillator with damping term $3H$ and potential $\mu^2(n-1)\psi_*^2/2$ is simply:

$$HT = \frac{2\pi}{\sqrt{\left(\frac{\mu}{H}\right)^2 (n-1) - \frac{9}{4}}}. $$

Figure 5: Effect of parameter $\mu^2$ on the bispectrum ($n = 16$, $\gamma = 1$ and $\lambda = H^2$ for each curve).
From the results earlier we expect then that the period of oscillations in both the power spectrum and bispectrum should depend only on the ratio \((\mu/H)^2\) and the power, \(n\), of the non-renormalisable term lifting the potential of the flat direction, \(\psi\). A brief check of the period of \(P(\ln k)\) confirms the expectation that \((\ln k)_T = 2HT\) for the mode functions.\(^{12}\) This can be read off Fig.6 for the parameters listed in the figure.

Therefore we expect that the result of increasing \(\mu^2H^2\) will be to decrease the period of oscillations in the bispectrum. Since this means more rapid oscillations, we also expect a magnification in \(\eta'\) during the phase transition, hence a magnification in the amplitude of the bispectrum. This is just what we see in Fig.6 (there is also a small effect on the shape of the bispectrum).

**Figure 6:** Effect of parameter \(\gamma\) on the bispectrum \((n = 16, \mu^2 = 3H^2\) and \(\lambda = H^2\) for each curve).

Based on this calculation we expect that changing \(\gamma\) will have only a secondary effect on the bispectrum. Indeed in Fig.6 the amplitude changes from \(\sim 5\) to \(\sim 2\) when \(\gamma\) is changed by over an order of magnitude. By comparison, when \(\lambda\) is similarly altered, the change in the bispectrum amplitude was from \(\sim 1\) to \(\sim 20\). The increase in the amplitude is because changing \(\gamma\) changes the minimum of the \(\psi\) field, which alters the magnitude of the change in the mass of the inflaton.

### 4.3 The bump model of [35]

Finally, we point out that in Ref. [35] a second example of multiple inflation was considered with two flat directions which have opposite sign couplings to the inflaton and produce a ‘bump’ in the primordial power spectrum. Using this model it was found that the WMAP 3-year data can be fitted by an Einstein de-Sitter cosmology without a cosmological constant.

We have calculated the bispectrum for this model which has \(\mu_1^2 = \mu_2^2 = 3H^2\), \(\gamma_1 = \gamma_2 = 1\), \(\lambda_1 = \lambda_2 = H^2\), \(n_1 = 12\), \(n_2 = 13\), and find the amplitude of the oscillations induced

\(^{12}\)\(P(\ln k)\) should have period \((\ln k)_T/2\) because it involves the square of the mode functions \(u_k\). To a good approximation, the bispectrum will have a period \((\ln k)_T/3\); however the bispectrum is more complicated because the \(u_k\) are the mode functions of the \(\zeta_I\), which are the free field operators. The bispectrum is a correlation function between the full operators \(\zeta(t) = U_I(t,t_0)^\dagger \zeta_I(t) U_I(t,t_0)\) and there will be additional oscillations generated by the \(U_I\) terms.
by the $\eta'$ term to be only $\sim 0.1$. Such a small value will be swamped by secondary non-gaussianities and could never be observed.

5. Conclusions

There is tentative evidence that the primordial power spectrum of scalar perturbations is imprinted with sharp features on large scales [25, 26]. These could have resulted from one or more phase transitions early on in the inflationary epoch as happens in the multiple inflation model [28]. This model is well motivated by fundamental physics ($N = 1$ supergravity) and such spectral features were predicted before WMAP provided observational indications for them.

It was shown [16] that any such departure from slow-roll during inflation should also generate non-gaussianity. We have calculated the non-gaussianity in multiple inflation for the parameter values that best fit the features in the power spectrum [20]. This is on the edge of being observable but a clear detection would require a new type of bispectrum estimator due to its scale dependence and non-factorisability [7]. Significantly larger non-gaussianities can be generated during multiple inflation if the parameters are allowed to range over values which are technically natural (i.e. stable towards radiative corrections).

The form of this non-gaussianity is tightly correlated with the power spectrum — the oscillations in the bispectrum should begin at the same multipole as the oscillations in the power spectrum and have two thirds of the period. There have been a number of attempts, to deconvolve the primordial power spectrum directly from the WMAP data [20, 21, 22] (see also Ref. [51]). It is generally found that there is a suppression of power on the scale of the present Hubble radius, followed by a ‘ringing’ at medium scales. By measuring the period of the oscillations in the power spectrum one can predict the period (and phase) of oscillations in the bispectrum, in a model independent manner.

Forthcoming measurements of CMB polarisation by Planck ought to shed light on whether these features in the power spectrum are systematic errors or genuine evidence of non-trivial dynamics during the inflationary era. The associated non-gaussian signal should also be detectable according to our model calculation and provide insight into the dynamics.

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References

[1] E. Komatsu and D. N. Spergel, Phys. Rev. D 63 (2001) 063002.
[2] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, Phys. Rept. 402 (2004) 103.
[3] X. Chen, M. x. Huang, S. Kachru and G. Shiu, J. Cosmo. Astropart. Phys. 01 (2007) 002.
[4] J. M. Maldacena, J. High Energy Phys. 05 (2003) 013.
[5] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180 (2009) 330.
[6] M. A. J. Ashdown et al. [Planck CTP Collaboration], Astron. Astrophys. 467 (2007) 761.
[7] J. R. Fergusson and E. P. S. Shellard, Phys. Rev. D 80 (2009) 043510.
[8] D. Munshi and A. Heavens, arXiv:0904.4478 [astro-ph.CO].
[9] D. Seery and J. E. Lidsey, J. Cosmo. Astropart. Phys. 06 (2005) 003.
[10] E. Komatsu et al., arXiv:0902.4759 [astro-ph.CO].
[11] M. Sasaki, J. Valiviita and D. Wands, Phys. Rev. D 74 (2006) 103003.
[12] D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. D 78 (2008) 063523.
[13] J. Martin, A. Riazuelo and M. Sakellariadou, Phys. Rev. D 61 (2000) 083518.
[14] C. T. Byrnes and G. Tasinato, J. Cosmo. Astropart. Phys. 08 (2009) 016.
[15] Y. F. Cai and H. Y. Xia, Phys. Lett. B 677 (2009) 226.
[16] X. Chen, R. Easther and E. A. Lim, J. Cosmo. Astropart. Phys. 06 (2007) 023.
[17] X. Chen, R. Easther and E. A. Lim, J. Cosmo. Astropart. Phys. 04 (2008) 010.
[18] D. H. Lyth and A. Riotto, Phys. Rept. 314 (1999) 1.
[19] J. Martin and C. Ringeval, Phys. Rev. D 69 (2004) 083515, Phys. Rev. D 69 (2004) 127303.
[20] N. Kogo, M. Sasaki and J. Yokoyama, Phys. Rev. D 70 (2004) 103001, Prog. Theor. Phys. 114 (2005) 553.
[21] A. Shafieloo and T. Souradeep, Phys. Rev. D 70 (2004) 043523; A. Shafieloo, T. Souradeep, P. Manimaran, P. K. Panigrahi and R. Rangarajan, Phys. Rev. D 75 (2007) 123502.
[22] D. Tocchini-Valentini, M. Douspis and J. Silk, Mon. Not. R. Astron. Soc. 359 (2005) 31.
D. Tocchini-Valentini, Y. Hoffman and J. Silk, Mon. Not. R. Astron. Soc. 367 (2006) 1095.
[23] G. Nicholson and C. R. Contaldi, J. Cosmo. Astropart. Phys. 07 (2009) 011.
[24] K. Ichiki and R. Nagata, Phys. Rev. D 80 (2009) 083002.
[25] D. N. Spergel et al., Astrophys. J. Suppl. 148 (2003) 175.
[26] G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. Suppl. 170 (2007) 288.
[27] M. J. Mortonson, C. Dvorkin, H. V. Peiris and W. Hu, Phys. Rev. D 79 (2009) 103519.
[28] J. A. Adams, G. G. Ross and S. Sarkar, Nucl. Phys. B 503 (1997) 403.
[29] J. Adams, B. Cresswell and R. Easther, Phys. Rev. D 64 (2001) 123514.
[30] H. V. Peiris et al., Astrophys. J. Suppl. 148 (2003) 213.
[31] L. Covi, J. Hamann, A. Melchiorri, A. Slosar and I. Sorbera, Phys. Rev. D 74 (2006) 083509.
J. Hamann, L. Covi, A. Melchiorri and A. Slosar, Phys. Rev. D 76 (2007) 023503.
[32] G. German, G. G. Ross and S. Sarkar, Phys. Lett. B 469 (1999) 46, Nucl. Phys. B 608 (2001) 423.
[33] P. Hunt and S. Sarkar, Phys. Rev. D 70 (2004) 103513.
[34] A. A. Starobinsky, \textit{JETP Lett.} \textbf{55} (1992) 489.

[35] P. Hunt and S. Sarkar, \textit{Phys. Rev.} \textbf{D 76} (2007) 123504.

[36] D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken and L. T. Wang, \textit{Phys. Rept.} \textbf{407} (2005) 1.

[37] L. Randall, \texttt{arXiv:hep-ph/9711471}.

[38] S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, \textit{J. Cosmo. Astropart. Phys.} \textbf{0310} (2003) 013.

[39] D. A. Easson and R. Gregory, \textit{Phys. Rev.} \textbf{D 80} (2009) 083518.

[40] G. G. Ross and S. Sarkar, \textit{Nucl. Phys. B} \textbf{461} (1996) 597; J. A. Adams, G. G. Ross and S. Sarkar, \textit{Phys. Lett.} \textbf{B 391} (1997) 271.

[41] S. Das and S. Mohanty, \textit{Phys. Rev.} \textbf{D 80} (2009) 123537.

[42] T. Gherghetta, C. F. Kolda and S. P. Martin, \textit{Nucl. Phys. B} \textbf{468} (1996) 37.

[43] M. Joy, V. Sahni and A. A. Starobinsky, \textit{Phys. Rev.} \textbf{D 77} (2008) 023514; M. Joy, A. Shafieloo, V. Sahni and A. A. Starobinsky, \textit{J. Cosmo. Astropart. Phys.} \textbf{0906} (2009) 028.

[44] M. Sasaki and E. D. Stewart, \textit{Prog. Theor. Phys.} \textbf{95} (1996) 71.

[45] D. H. Lyth, K. A. Malik and M. Sasaki, \textit{J. Cosmo. Astropart. Phys.} \textbf{05} (2005) 004.

[46] D. H. Lyth, C. Ungarelli and D. Wands, \textit{Phys. Rev.} \textbf{D 67} (2003) 023503.

[47] D. H. Lyth and Y. Rodriguez, \textit{Phys. Rev. Lett.} \textbf{95} (2005) 121302; D. Seery and J. E. Lidsey, \textit{J. Cosmo. Astropart. Phys.} \textbf{0509} (2005) 011.

[48] K. T. Engel, K. S. M. Lee and M. B. Wise, \textit{Phys. Rev.} \textbf{D 79} (2009) 103530.

[49] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, \textit{Phys. Rept.} \textbf{215} (1992) 203.

[50] L. F. Shampine and M. W. Reichelt, \textit{SIAM Journal on Scientific Computing} \textbf{18} (1997) 1.

[51] M. Bridges, A. N. Lasenby and M. P. Hobson, \textit{Mon. Not. R. Astron. Soc.} \textbf{369} (2006) 1123; \textit{Mon. Not. R. Astron. Soc.} \textbf{381} (2007) 68.