FLUXES AND GAUGINGS: \( N = 1 \) EFFECTIVE SUPERPOENTIALS*  

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Abstract

We illustrate the correspondence between the \( N = 1 \) superstring compactifications with fluxes, the \( N = 4 \) gauged supergravities and the superpotential and Kähler potential of the effective \( N = 1 \) supergravity in four dimensions. In particular we derive, in the presence of general fluxes, the effective \( N = 1 \) supergravity theory associated to the type IIA orientifolds with D6 branes, compactified on \( T^6/(Z_2 \times Z_2) \). We construct explicit examples with different features: in particular, new IIA no-scale models, new models with cosmological interest and a model which admits a supersymmetric AdS\(_4\) vacuum with all seven main moduli \((S, T_A, U_A, A = 1, 2, 3)\) stabilized.

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1 Introduction

Superstring and M-theory compactifications admit four-dimensional vacua with some exact or spontaneously broken supersymmetries. The breaking scheme depends on the choice of compactification, including orbifold or orientifold projections, and the phenomenologically most attractive patterns are $N = 8$ or $4 \rightarrow N = 1 \rightarrow N = 0$. Even with broken supersymmetry, the underlying ten-dimensional theory encodes the constraints of $N \geq 4$ supersymmetry, which can then be used to derive information on the structure of the effective $N = 1$ supergravity. This effective four-dimensional theory typically includes moduli fields originating either from the dilaton field $\Phi$ and the internal metric $g_{IJ}$, or from $p$-form potentials $F_p$.

Determining the vacuum expectation values of the moduli fields is essential in order to (i) reduce the number of massless scalars, (ii) induce supersymmetry breaking, (iii) determine the four-dimensional background geometry. This requires to generate a potential for these scalars.

In $N \geq 4$ supergravities, the only available tool for generating a potential is to turn abelian gauge symmetries, naturally associated with vector fields, into non-abelian ones. This procedure of *gauging* introduces in the theory a gauge algebra $G$ acting on the vector fields in the gravitational and/or vector supermultiplets. The important fact is that from the point of view of the “daughter” $N = 1$ supergravity, the gauging modifications only affect the superpotential $W$, whereas the Kähler potential $K$ and hence the kinetic terms remain the same as in the ungauged theory.

At the superstring level, the generation of a superpotential is in general obtained when the background field configuration includes non-trivial fluxes and/or Scherk–Schwarz periodicity conditions [1]. Our main point is to establish the correspondence of these data of the superstring theory, with, at the effective field theory level, the gauging of some algebra allowed by the massless multiplet content of the theory. It is then possible to exhaustively study phenomenological aspects like supersymmetry breaking, generation of masses, four-dimensional space–time geometry, soft breaking terms, condensation phenomena, cosmological behaviour directly in this simple field-theory approach. This method has been developed in Ref. [2] with particular emphasis on type IIA strings, and in more general terms in Ref. [3]. The present contribution is a summary of these works.
To illustrate more concretely the method, consider the case of the $N = 4$ supergravity theory corresponding to heterotic strings on $T^6$, type II strings on orientifold or on $K3 \times T^2$, or any system with 16 supercharges. For all these cases, the $N = 4$ scalar manifold is 

$$
\mathcal{M} = \left( \frac{SU(1,1)}{U(1)} \right) \times \left( \frac{SO(6,6+n)}{SO(6) \times SO(6+n)} \right).
$$

(2.1)

Reducing supersymmetry to $N = 1$ also truncates this scalar manifold and the final structure depends on the specific choice of compactification. We will focus here on a $Z_2 \times Z_2$ orbifold (or Calabi–Yau) projection, which leads to the following Kähler manifold:

$$
\mathcal{K} = \left( \frac{SU(1,1)}{U(1)} \right)_S \times \prod_{A=1}^{3} \left( \frac{SO(2,2+n_A)}{SO(2) \times SO(2+n_A)} \right)_{T_A,U_A,Z_A^I}.
$$

(2.2)

The first factor is the complex scalar field of the $N = 4$ gravitational supermultiplet, which can be parameterized by the solution

$$
\phi_0 - \phi_1 = \frac{1}{(S + \bar{S})^{1/2}}, \quad \phi_0 + \phi_1 = \frac{S}{(S + \bar{S})^{1/2}},
$$

(2.3)

of the $U(1,1)$-invariant constraint

$$
|\phi_0|^2 - |\phi_1|^2 = \frac{1}{2}.
$$

(2.4)

The next three factors are produced by the $Z_2 \times Z_2$ orbifold truncation of the vector multiplet sector. The resulting scalars are the solutions of the $N = 4$ constraints

$$
|\sigma_A^1|^2 + |\sigma_A^2|^2 - |\rho_A^1|^2 - |\rho_A^2|^2 - |\chi_A^I|^2 = \frac{1}{2},
$$

(2.5)

$$
|\sigma_A^1|^2 + (\sigma_A^2)^2 - (\rho_A^1)^2 - (\rho_A^2)^2 - (\chi_A^I)^2 = 0.
$$

Specifically, we choose the solution:

$$
\sigma_A^1 = \frac{1 + T_A U_A - (Z_A^I)^2}{2Y_A^{1/2}}, \quad \sigma_A^2 = i \frac{T_A + U_A}{2Y_A^{1/2}},
$$

$$
\rho_A^1 = \frac{1 - T_A U_A - (Z_A^I)^2}{2Y_A^{1/2}}, \quad \rho_A^2 = i \frac{T_A - U_A}{2Y_A^{1/2}}, \quad \chi_A^I = \frac{i Z_A^I}{Y_A^{1/2}}.
$$

(2.6)

This choice of parameterization of the physical fields is appropriate for string compactifications, since it singles out the geometric moduli and dilaton as $\text{Re}T_A$, $\text{Re}U_A$, $\text{Re}S$. Each
string construction is then characterized by its own complexification of these seven real scalars and by specific couplings to matter fields $Z^I_A$. The index $A$ labels the three complex planes defined by the $Z_2 \times Z_2$ symmetry used for the orbifold projection.

Equipped with this set of physical fields, we can simply extract the Kähler potential and the superpotential of the $N = 1$ theory, by direct inspection of the gravitino mass matrix of the $N = 4$ theory submitted to the $Z_2 \times Z_2$ projection, which reads

\[
e^{K/2}W = (\phi_0 - \phi_1) f_{IJK} \Phi^I \Phi^J \Phi^K + (\phi_0 + \phi_1) \tilde{f}_{IJK} \Phi^I \Phi^J \Phi^K,
\]

with $\Phi^I_A = \{ \sigma^1_A, \sigma^2_A; \rho^1_A, \rho^2_A, \chi^I_A \}$,

where $f_{IJK}$ and $\tilde{f}_{IJK}$ are the structure constants of the gauge algebra. They differ, however, in their dependence with respect to the $S$-field through the $S$-duality phases. With our solutions, the non-holomorphic part defines the Kähler potential,

\[
K = -\ln (S + \bar{S}) - \sum_{A=1}^{3} \ln Y_A,
\]

where

\[
Y_A = (T_A + \bar{T}_A)(U_A + \bar{U}_A) - \sum_I (Z^I_A + \bar{Z}^I_A)^2.
\]

And the holomorphic part defines the superpotential generated by the structure constants $f_{IJK}$ and the $S$–duality phases.

With Eqs. (2.3), the superpotential has two terms. The first contribution involves the structure constants $f_{IJK}$ and does not depend on $S$. The second term involves the second set of structure constants $\tilde{f}_{IJK}$ and is linear in $S$.\(^1\) In the heterotic construction where the geometric moduli are $\text{Re} \ T_A$ and $\text{Re} \ U_A$, these two contributions would respectively correspond to a perturbative “electric gauging” and a non-perturbative “magnetic gauging”.

3 Gauging and fluxes

A gauging with non-zero $f_{IJK}$ generates a superpotential and then, in particular, a potential for the moduli. Furthermore, if the structure constants are such that the fields

$\sigma^1_A, \sigma^2_A; \rho^1_A, \rho^2_A$

\(^1\)Consistency constraints apply on the structure constants: $S$-duality phases should be compatible with the gauge algebra, and, of course, Jacobi identities should be verified.
are involved, the superpotential also induces supersymmetry breaking. These directions correspond to the $N = 4$ graviphotons and the non-abelian gauge algebra also involves the $N = 4$ $R$-symmetry.

In string and M-theory, the non-trivial $f_{IJK}$ and $\tilde{f}_{IJK}$ are generated by non-zero electric and/or magnetic fluxes, R–R and fundamental $p$-form fields:

- three-form fluxes $H_3$, in the heterotic string and in the NS–NS sector of type IIA and type IIB theories;
- fluxes of $p$–forms $F_p$ in M-theory and in the R–R sector of type IIA and type IIB;
- fluxes of gauge two-forms $\tilde{H}_2$, in heterotic $(E_8 \times E_8$ or $SO(32))$ as well as in type I, and spin-connection fluxes $\omega_3$ in all strings and M-theory.

Many particular cases have already been considered in the literature by direct study of the ten- or eleven-dimensional field equations, including specific choices of fluxes. In particular:

- $H_3$ fluxes in heterotic superstrings [5], including spin connection and Yang–Mills fluxes [6];
- fluxes of $\omega_3$, $H_3$ and $\tilde{H}_2$, as generated in exact string solution via freely-acting orbifold (the generalization to superstring theory of the Scherk–Schwarz gauging) and group manifold compactifications [7];
- IIB strings with simultaneous NS–NS ($H_3$) and R–R ($F_3$) fluxes [8].

The fact that such configurations can also be studied in the gauging approach of the effective field theory does not necessarily shed new light on some of these cases.

The simplicity of our approach using gauging of the underlying $N = 4$ supersymmetry algebra allows however to study exhaustively more complex cases, including also spin-connection geometric fluxes. This was the main motivation of Ref. [2], where we concentrated on orientifolds of type IIA strings which offered the broadest structure of allowed fluxes and had been explored to a lesser extent [9]. The results of this study is reported in the next two sections.
4 Fluxes in type IIA orientifold on $T^6/Z^2 \times Z^2$

The $Z_2 \times Z_2$ orbifold projection used in this paper is only compatible with the orientifold admitting $D6$–branes. The possible fluxes in this type IIA orientifold are as follows [9, 10]. Firstly, in the R–R sector, the IIA theory compactified to four dimensions generates all even-form fluxes,

$$F_0, F_2, F_4, F_6$$

(where the subscript gives the degree of the form in the compact directions only). The mass parameter of massive IIA supergravity and the ten-dimensional R–R four-form in space–time only, $\sim \lambda \epsilon_{\mu\nu\rho\sigma}$, induce then $F_0$ and $F_6$, while internal values of the R–R two- and four-forms generate $F_2$ and $F_4$. The NS–NS sector provides then three-form fluxes $H_3$, as well as the geometrical fluxes arising from background values of spin connections $\omega_3$.

To identify the superpotential terms related to the above fluxes, we firstly need to find the appropriate complexification of the seven moduli fields

$$S, T_1, T_2, T_3, U_1, U_2, U_3$$

compatible with $N = 1$ supersymmetry. It is actually simpler to begin with a discussion of the heterotic case, and then to find the non-trivial change of variables relating heterotic and IIA strings. For heterotic strings on $T^6/Z^2 \times Z^2$, the dilaton $\Phi$ is singled out as the gauge coupling, geometric moduli have a natural definition in terms of the internal metric $G_{IJ}$ and complexification involves the metric and the two-form field:

$$\begin{align*}
(G_{IJ})_A &= \frac{t_A}{u_A} \begin{pmatrix} u_A^2 + \nu_A^2 & \nu_A \\ \nu_A & 1 \end{pmatrix}, \\
T_A &= t_A + i(B_{IJ})_A, \\
U_A &= u_A + i\nu_A,
\end{align*}$$

(4.1)

These heterotic definitions of moduli $s, t_A, u_A$ are also called geometrical variables herebelow.

The supersymmetric complexification in the type IIA orientifold with D6-branes is more involved, due to the dilaton rescaling, the presence of R–R fields and also the orientifold projection which in particular eliminates the geometrical modes $\nu_A$ of the metric. Since the NS–NS two-form field $B_2$ is odd in this orientifold, it is clear that $T_B$ moduli in IIA and heterotic are the same:

$$T_{B,\text{IIA}} = T_B \quad (B = 1, 2, 3).$$

(4.2)
Inspection of the complexification of scalar kinetic terms and also of D6-brane gauge kinetic terms indicates that the redefinitions

\[ s_{\text{IIA}} = \sqrt{\frac{s}{u_1u_2u_3}}, \quad u_{1,\text{IIA}} = \sqrt{su_2u_3}, \quad u_{2,\text{IIA}} = \sqrt{su_1u_3}, \quad u_{3,\text{IIA}} = \sqrt{su_1u_2} \quad (4.3) \]

do the real moduli scalars are required so that the \( N = 1 \) complex scalars receive their imaginary part from the four components of the three-form R–R field which survive the orbifold projection:

\[ S_{\text{IIA}} = s_{\text{IIA}} + iA_{6810}, \]
\[ U_{1,\text{IIA}} = u_{1,\text{IIA}} + iA_{679}, \quad U_{2,\text{IIA}} = u_{2,\text{IIA}} + iA_{589}, \quad U_{3,\text{IIA}} = u_{3,\text{IIA}} + iA_{5710}. \quad (4.4) \]

In the next section, we will omit the subscript “IIA”, but it should be understood that the fields defined by Eqs. (4.3) and (4.4) will be used whenever we discuss superpotential contributions.

5 \( N = 1 \) superpotentials versus IIA fluxes

With the definitions (4.3), (4.4) and Eqs. (4.1), it is now easy to translate the effect of IIA fluxes into specific contributions to the \( N = 1 \) superpotential. In this section, we merely enumerate the various contributions and describe some examples with combined fluxes. More detail can be found in Refs. [2] and [3]. Compact dimensions will be labelled 5, 6, \ldots, 10, with 5, 7, 9 odd and 6, 8, 10 even under the orientifold \( Z_2 \).

5.1 Single fluxes and their superpotentials

The R–R sector generates four types of fluxes: \( F_0 \) and \( F_6 \) arise from switching on the four-form field in space–time (Freund–Rubin ansatz) and from the mass parameter of massive IIA supergravity [11]; switching on the two- and four-form fields in internal directions generates \( F_2 \) and \( F_4 \).

- \( F_6 \) flux

Suppose that we switch on an internal R–R six-form \( F_6 \). Using geometrical variables as defined in Eqs. (4.1), a scalar potential of the form \( V = F_6^2 / (s^2 t_1 t_2 t_3) \) in these variables is generated. Changing to IIA variables using (4.3) and (4.4) and comparing
with the \( N = 1 \) supergravity potential leads to the superpotential

\[
W = F_0. \tag{5.1}
\]

- **\( F_0 \) flux**

Similarly, switching on an internal (real) zero-form \( F_0 \) leads to the superpotential

\[
W = -iF_0 T_1 T_2 T_3 \tag{5.2}
\]

in IIA variables. Via complexification, this superpotential introduces terms depending on imaginary parts of \( T_A \), i.e. terms depending on the NS–NS two-form field \( B_2 \). These terms are as predicted by the equations of massive IIA supergravity with mass parameter \( F_0 \).

- **\( F_2 \) fluxes**

The orbifold and orientifold projections allow internal two-form fluxes \( F_{56}, F_{78} \) and \( F_{910} \). The induced superpotential is

\[
W = -F_{56} T_2 T_3 - F_{78} T_3 T_1 - F_{910} T_1 T_2. \tag{5.3}
\]

- **\( F_4 \) fluxes**

Finally, the orbifold and orientifold projections allow internal four-form fluxes \( F_{5678}, F_{78910} \) and \( F_{56910} \). The induced superpotential is

\[
W = iF_{5678} T_3 + iF_{78910} T_1 + iF_{56910} T_2. \tag{5.4}
\]

In the NS–NS sector, we need to consider \( H_3 \) fluxes and geometric fluxes of the spin connection \( \omega_3 \).

- **\( H_3 \) fluxes**

The directions in \( H_3 \) allowed by the orbifold and orientifold projections are \( H_{579}, H_{5810}, H_{6710} \) and \( H_{689} \). These fluxes generate the superpotential

\[
W = iH_{579} S + iH_{5810} U_1 + iH_{6710} U_2 + iH_{689} U_3. \tag{5.5}
\]
• **Geometric $\omega_3$ fluxes**

The projections allow background values of several components of the spin connections. These fluxes produce two categories of superpotential terms. Firstly,

$$W = -\omega_{679} ST_1 - \omega_{895} ST_2 - \omega_{1057} ST_3. \quad (5.6)$$

Both $H_3$ and $\omega_3$ are then sources for the $S$-dependent, “non-perturbative” part of the superpotential. Secondly, geometric fluxes allow to generate all bilinear contributions of the form $T_A U_A$ or $-T_A U_B \ (B \neq A)$. For instance,

$$W = \omega_{6810} T_1 U_1 + \omega_{8106} T_2 U_2 + \omega_{1068} T_3 U_3$$

will be used in the next paragraph.

### 5.2 Type IIA examples with combined fluxes, gauging and moduli stabilization

The set of allowed fluxes in this IIA orientifold is rich enough to provide examples of combined fluxes where some or all moduli are stabilized. We give here only a brief description of some of these cases.

• **Flat gaugings, no-scale models, stabilization of four moduli**

If four moduli are stabilized, the resulting scalar potential is positive definite with the flat directions of a no-scale model [12].

1. **Standard perturbative Scherk–Schwarz superpotential from $\omega_3$ fluxes**

   The superpotential reads:
   $$W = a (T_1 U_1 + T_2 U_2). \quad (5.7)$$

   The scalar potential is semi-positive, $V \geq 0$. At the minimum, the four imaginary parts of $T_1, T_2, U_1$ and $U_2$ vanish and one quadratic condition applies on the real parts: $t_1 u_1 = t_2 u_2 \equiv v^2$. Then, at the minimum, $W = 2av^2$ and the gravitino mass reads:
   $$m_{3/2}^2 = \frac{|a|^2}{32 \operatorname{str}_3 u_3},$$

   which depends on the flat directions $S, T_3, U_3$, but not on the three flat directions left at the minimum in directions $t_1, t_2, u_1$ and $u_2$. This model is a gauging of the two-dimensional euclidean group $E_2$.  

8
2. A "non-pertubative" no-scale example with $\omega_3, F_2, H_3$ and $F_6$ fluxes

The superpotential of this example is

$$W = a(ST_1 + T_2 T_3) + ib(S + T_1 T_2 T_3). \quad (5.8)$$

The potential is again semi-positive, the gravitino mass is

$$m^{2 \ 3/2} = \frac{a^2 + b^2}{32 u_1 u_2 u_3},$$

and there are in addition two "decoupled" flat directions.

3. A no-scale model based on a gauging of the euclidean group $E_3 \times E_3$

This gauging is equivalent to non-zero fluxes of $\omega_3, F_0, F_2$ and $H_3$. Specific choices of these fluxes allow the following superpotential:

$$W = a(ST_1 + ST_2 + ST_3) + a(T_1 T_2 + T_2 T_3 + T_3 T_1) + 3ib(S + T_1 T_2 T_3). \quad (5.9)$$

Four moduli are stabilized, and the resulting no-scale model has gravitino mass given by

$$m^{2 \ 3/2} = \frac{9 \ a^2 + b^2}{32 u_1 u_2 u_3}.$$

- Gaugings with positive definite potential

Examples can be easily found, in which less than four moduli are stabilized and the potential is always strictly positive-definite, leading to runaway solutions (in time).

Superpotentials with a single monomial are of course examples where no modulus gets stabilized. For instance, we can choose the fluxes $F_6$, $F_0$ or $H_3$, leading to

$$W = F_6, \quad W = -iF_0 T_1 T_2 T_3 \quad \text{or} \quad W = iH_3 S. \quad (5.10)$$

This leads to $V = 4 \ m^{2 \ 3/2} > 0$ and the gravitino mass term is of the form

$$m^{2 \ 3/2} = \frac{1}{2^7 s t_1 t_2 t_3 u_1 u_2 u_3} \times \left\{ |F_6|^2, \ |F_0 T_1 T_2 T_3|^2 \text{ or } |H_3 S|^2 \right\}, \quad (5.11)$$

respectively.

9
An example where three moduli are stabilized is obtained by switching on a system of R–R fluxes \((F_0, F_2, F_4, F_6)\), with superpotential

\[
W = A \left( 1 + T_1 T_2 + T_2 T_3 + T_3 T_1 \right) + i B \left( T_1 + T_2 + T_3 + T_1 T_2 T_3 \right).
\] (5.12)

This choice of fluxes and superpotential is actually a gauging of \(SO(1, 3)\). It is immediate to see that, since the superpotential does not depend on four of the seven main moduli (the \(T\)-moduli are stabilized at one), supersymmetry is broken and a positive-definite runaway\(D = 4\) scalar potential is generated,

\[
V = m_{3/2}^2, \quad \text{with} \quad m_{3/2}^2 = \frac{A^2 + B^2}{8 s u_1 u_2 u_3},
\] (5.13)

possibly leading to time-dependent vacua of cosmological interest.

**Gaugings with negative-definite potential**

Situations where more than four moduli are stabilized lead to negative-definite potentials once the stabilized moduli are set to their vacuum values.

We begin with a gauging of \(E_3\) with fluxes \(\omega_3\) (geometric) and \(F_6\) (R–R six-form). The R–R six-form corresponds to the \(SO(3)\) directions in \(E_3\) while \(\omega_3\) corresponds to the translations. The superpotential reads

\[
W = \omega_3 \left( T_1 U_1 + T_2 U_2 + T_3 U_3 \right) - F_6.
\] (5.14)

The six equations for the non-trivial supergravity auxiliary fields are solved at \(\tau_A = \nu_A = 0\) and \(t_1 u_1 = t_2 u_2 = t_3 u_3 = F_6/\omega_3\). At these values, \(W = 2F_6\), and the \(s\)-dependent scalar potential and gravitino mass term read:

\[
V = -2m_{3/2}^2 = -\frac{\omega_3^3}{16 F_6 s}.
\] (5.15)

At the string level, this is the well-known NS five-brane solution plus linear dilaton [13], in the near-horizon limit. The original gauging is \(SU(2)\), combined with translations, which emerge as free actions at the level of the world-sheet conformal field theory. It is remarkable that this \(E_3\) algebra remains visible at the supergravity level. It is also interesting that, if we allow extra fluxes, induced by the presence of fundamental-string sources, we can reach AdS\(_3\) background solutions with stabilization of the dilaton. All moduli are therefore stabilized. This has been studied recently at the string level [14].
• Stabilization of all moduli

Using all fluxes admissible in IIA, $Z_2 \times Z_2$ strings, we can obtain the stabilization of all moduli in AdS$_4$ space–time geometry. Switching on all fluxes ($\omega_3, H_3, F_0, F_2, F_4, F_6$), we can obtain the superpotential

$$ W = A \left[ 2 S (T_1 + T_2 + T_3) - (T_1 T_2 + T_2 T_3 + T_3 T_1) + 6 (T_1 U_1 + T_2 U_2 + T_3 U_3) - 9 \right] $$

$$ + i B \left[ 2 S + 5 T_1 T_2 T_3 + 2 (U_1 + U_2 + U_3) - 3 (T_1 + T_2 + T_3) \right]. $$

(5.16)

This is a consistent $N = 4$ gauging only if

$$ 6 A^2 = 10 B^2, $$

(5.17)

as a consequence of Jacobi identities, as shown in Refs. [2, 3]. Notice that this condition relates the even and odd terms in the superpotential, thus its sign ambiguity is irrelevant. The superpotential (5.16) leads to a supersymmetric vacuum at $S = T_A = U_A = 1$ ($A = 1, 2, 3$). Since at this point $W = 4 (3 A + i B) \neq 0$, implying $V = -3 m_{3/2}^2 < 0$, this vacuum has a stable AdS$_4$ geometry with all seven main moduli frozen.

Notice that the appearance of non-integer flux numbers is actually an artifact of our choice for presenting the model, with $S = T_A = U_A = 1$ at the minimum. One can recover integer flux numbers by rescaling appropriately the moduli. A possible choice (among many others) is the following:

$$ (S, T_A, U_A) \rightarrow b (S, T_A, U_A), \quad b = \frac{B}{A} = \sqrt{\frac{3}{5}}. $$

(5.18)

With that choice

$$ W = N \left[ 2 S (T_1 + T_2 + T_3) - (T_1 T_2 + T_2 T_3 + T_3 T_1) ight. $$

$$ + 6 (T_1 U_1 + T_2 U_2 + T_3 U_3) - 15 \right] $$

$$ + i N \left[ 2 S + 3 T_1 T_2 T_3 + 2 (U_1 + U_2 + U_3) - 3 (T_1 + T_2 + T_3) \right], $$

(5.19)

where $N = (3/5) A$.

6 Conclusion

We should firstly emphasize that our last example is the only known case with complete stabilization of the moduli, reached in IIA by switching on fundamental fluxes. We should
also stress that this cannot happen in the heterotic string, because of the absence of $S$-dependence in the general flux-induced superpotential. Such a dependence could appear with gaugino condensation. In type IIB with D3-branes (and D7), the orientifold projection that accompanies the $Z_2 \times Z_2$ orbifold projection eliminates the $\omega_3$ fluxes, thus the $T$ moduli are not present in the superpotential and cannot be stabilized by fluxes. The case of D9-branes (open string) is similar to the heterotic case.

Our approach of supergravity gauging can be viewed as a bottom-up approach to the problem of generating moduli and matter superpotentials in superstring vacuum configurations. As in all approaches essentially based on effective Lagrangians, it is not in principle expected that low-energy symmetries are powerful enough to completely replace a top-bottom analysis using ten-dimensional string or M-theory equations. Our studies of many examples [2, 3], in heterotic and type II strings, actually shows that the effective supergravity approach based upon $N = 4$ gaugings accurately reproduces the conditions imposed by the full field equations of the ten-dimensional theories. This of course requires to include all necessary brane and orientifold plane contributions to these equations.

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