Prospective mathematics teachers making sense of multiplication algorithm

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Abstract. Multiplication algorithm is broadly used in Indonesian classroom as a technique to multiply two-digits or more numbers. Traditionally, it is introduced as a set of memorized procedures. Hence, despite of its popularity, multiplication algorithm is hardly understood by the students. To understand the root of problem in teaching multiplication, this study aims at elaborating the abilities of prospective mathematics teachers in making sense the multiplication algorithm. To gather the data, we conducted a survey using written test. The participants were 50 students of mathematics education study program in a state university in Bali, Indonesia. The data were analysed using descriptive quantitative method. From the findings, it is found that 18% of participants were able to explain the logic behind the procedures. Furthermore, the analysis shows the prospective mathematics teachers encountered difficulty in explaining multiplication algorithm due to their inability to making sense the process.

1. Introduction
Multiplication is one of basic operations in mathematics which is beneficial for the students’ foundation before entering higher school’s grade. It can be defined as repeated addition of a group with equal size [1]. Despite of the important of it, learning multiplication is not easy for many students [2–4]. It can be seen for consistent errors occurred when students’ solve multiplication problems [5]. It becomes harder when the numbers grow bigger to two-digits and up. Traditionally, the students will be taught to use algorithm in order to count the multiplication result. Even though algorithm were firstly invented to support people attempt in organize the counting and finding the final result, too much focus on the procedures without providing students with logical reason behind each step has contributes to the number of errors.

According to Booth [6], there are four types of major errors when students applying algorithm multiplication, which is incorrectly multiply by zero (i.e. \(14 \times 0 = 14\)), multiplying tens and unit separately (i.e. \(39 \times 30 = 90\)), place value error (i.e. \(13 \times 12 = 39\)) and confused algorithm (i.e. \(14 \times 20 = 840\)). Some studies argue that algorithm is hardly understood since it has no relation with the students’ intuitive approach [7,8]. Only advance students who able to reflect during the process while others seem to used it blindly [9].

To reduce errors and help students to grasp with the conceptual understanding of how multiplication is generally be carried, teachers need to facilitate the process. In this paper we will not argue which strategy will provide better computational fluency result for the students in solving
multiplication, but the chosen strategy should be taught meaningfully [10]. Nevertheless, students’ strategies development in doing multiplication is influenced by the provided learning resources [11]. Learning multiplication should not be only focused on memorizing but how the numbers were connected through the operation [12].

The teachers’ proficiency to support the students’ learning process in certain topic by employing certain method, strategies, techniques or aids is defined as Pedagogical Content Knowledge (PCK) source. PCK is one of the essential knowledges for teachers, besides the proficiency of the taught subject, general pedagogical knowledge to manage classroom, curriculum knowledge and knowledge of students, educational context and goals [13]. A good PCK is needed to organize learning path for the students. Therefore, this skill is made from the combination of understanding in knowledge of content and characteristics of students [14].

One of the skills in PCK is to realize the students’ misconception or errors in learning [15]. To understand a concept is mistakenly understood by the students, the teachers should know which one is correct. Also, mathematical CK contributes to the teachers’ decision to choose the learning path that will accommodate the students’ prior knowledge and the present learning goals [16]. In other words, the teacher should have a good conceptual understanding in the subject they taught. Furthermore, a mastery in Content Knowledge (CK) does not guarantee a good PCK of teachers but a lack CK will contribute to the struggle in managing PCK in the classroom [17].

Reflect to the aforementioned reasons, this study will figure out how the prospective mathematics teachers making sense the process of multiplication algorithm. By understanding their way of thinking in this specific mathematical topic, we can provide a useful recommendation of how the lesson should be conducted to optimize the students’ mastery in multiplication.

2. Method
The present study is a part of larger study aims at examining Pedagogical Content Knowledge (PCK) in prospective mathematics teachers. To gather the data, 50 students of mathematics education study program from a state university in Bali who already experienced teaching internship were assigned to answer a set of PCK test. The form of the test was essay consists of 9 questions in various mathematical topics. The data from students’ written work were supported by a selected interview with the participants. Afterwards, the data were analysed quantitatively and qualitatively using descriptive method.

The current discussion will only focus on the question related to multiplication algorithm. In the test, an illustration of short algorithm of multiplication which is largely used in Indonesia classroom were given (see Figure 1). The participants were asked to provide their explanation if the students ask why the results of the multiplication in the second row is moved a little bit to the left, not exactly in the same line as the first row.

3. Results and Discussion
The question addressed in the present study is related to multiplication, one of the basic operations in mathematics. We chose basic knowledge as it since the insufficient understanding and mastery of the topic contributes to the difficulty to learn more advance mathematical topics [18].

To answer the given problem, the participants should understand how the algorithm work. Therefore, even though the question were initially designed to evaluate the prospective mathematics teachers’ Pedagogical Content Knowledge (PCK), it also gave insight of how their Content Knowledge (CK) in multiplication and how they use it to making sense the algorithm.

Usually, the multiplication algorithm is introduced in the third grade in elementary school as a method to solve two-digits and more multiplication. Even though the students learn it early in their school time, the multiplication algorithm is not easy for them [19–21]. Nevertheless, the drill and
practice traditional teaching method to enable the students in using algorithm fluently is still preserved since, according to the teachers, it is the most important goal in learning multiplication [22].

In this study we tried to figure out how the prospective mathematics teacher making sense the algorithm process. According to the study of [23], the teachers’ understanding of how the algorithm work contributes to the students’ learning path. Only by reflecting from that, we can provide recommendation of how multiplication with two-digits numbers or more should be taught in the classroom. From the gathered data, we found that most of the participants were not able to clearly explain the reason behind the result position in multiplication algorithm. See the following Figure 1 to provide the details about the participants’ responses.

![Figure 1. Classification of prospective mathematics teachers’ responses](image)

As the participants were a group of last year mathematics education students, we can confirm that they were able in applying the algorithm to solve mathematical related or daily life problems. However, 8% of them were not providing any answer toward the problem. During the short interview right after the test, the participants acknowledged they know how to use it but not all were able to explain why it works as it is.

The majority of the students provided explanation, but we classify it as unclear since they merely re-write the process. The example of unclear responses can be seen in Figure 2.

![Figure 2. An example of unclear explanation](image)

Translation:
We start the multiplication algorithm by the last digit, after that the result should be written parallel to it. Followed by the multiplication of the number in front of it and the result is written parallel to it.

The type of responses as in Figure 2 showed how the prospective teachers were not aware the logic behind the algorithm. Instead of elaborating why the numbers were put in certain position, they focus on the memorizing the steps (see the first and the second lines translation of the explanation). This type of teaching method in multi-digits multiplication will lead to mastery of procedural proficiency [24]. The study of Russell [25] found that impartial understanding towards the steps in algorithm, as
happened in the participants of this study, lead to inability to see the connection between mathematical objects and ideas involved.

In contrary, the participants who make a logical reasoning behind the algorithm provided a concise explanation of how it works. In other words, the answers will be classified as clear if the participant attempted to make sense of the procedures of the multiplication algorithm. See the example of responses which classified as a clear explanation in Figure 3.

Figure 3. An example of clear explanation

Figure 3 showed the explanation using narration to describe the distributive properties in two-digits multiplication. The answer also provided an understanding of why the position were moved to the left because the last space is served for the zero. Similar way of thinking also provided in the answer in Figure 4. However, instead of using narration, the participant directly explained the distributive properties in multiplication and place value in a number.

Figure 4. The illustration of multiplication
In general, most participants responded to the problem by explaining it in words. The comparison of general method used by the participants can be observed in Figure 5. Furthermore, most participants who making sense the algorithm procedure, completed their explanation with illustration using the number scheme to represent the number partition or reconstruction.

![Figure 5. The comparison of explanation method](image)

Another remarkable finding was the participants who clearly explain the procedures behind multiplication algorithm tried to emphasize their reason using distributive properties of multiplication. Surprisingly, all of the prospective teachers who applied distributive properties tend to only restructure one number into the addition of ten and other numbers (i.e. $12 \times 11 = 12 \times (10 + 1)$ or $12 \times 11 = (10 + 2) \times 11$) instead of restructure both numbers (i.e. $12 \times 11 = (10 + 2) \times (10 + 1)$).

![Figure 6. Showing the distributive properties in multiplication](image)

In a real classroom, the strategy provided in Figure 6 may lead to another difficulty when the number fall into non-friendly numbers. For example, $13 \times 17 = 13 \times (10 + 7) = (10 + 3) \times 17$. The multiplication of 13 and 7 or 3 and 17 will not be easy and leads to another number restructuration.
During the interview after the test, the prospective mathematics teachers realized this condition and propose to restructure the multiplier and multiplicand together from the very beginning. Furthermore, they stand for their first reasoning that the initial number should be restructured in the combination of numbers involving 10 since the multiplication with 10 is considered as easy or usually be called as friendly number [12,26]. The participants’ argument was correct because in many places multiplication strategies were taught to develop mental calculation and knowing the result of any number multiplied by 10 is one of the milestone in the lesson [27,28].

On the other hand, since most of the participants of the study hardly explained the process of algorithm and the previous studies (see [6,29]) found that students also have trouble in mastery it, we need also consider different techniques to teach two-digits number multiplication. One of the recommendation is by using manipulatives [30].

After all, as can be seen in the aforementioned findings, the prospective mathematics teachers still have difficulties in making sense the algorithm and connect it with the properties and concepts in multiplication. Therefore, supporting the previous researches, this study confirms that the lack of understanding in mathematical content or in general we refer to Content Knowledge (CK) leads to the lack of creating meaningful lesson nor providing meaningful explanation related to mathematical concepts or Pedagogical Content Knowledge (PCK) [17,31].

Consequently, the teacher training program institution should provide more chances for the prospective teachers to critically asking the question of any mathematical objects, principals and concepts to build their own comprehensive understanding. Only by having a strong foundation in mathematics that a mathematics teacher can organize a rich and engaging lessons [32,33] which impact to the enhancement of students’ learning outcome, self-directed learning and well-being [34], [35,36].

For instance, to illustrate the distributive properties in multiplication with two-digits numbers, the prospective teachers can employ area model [37]. It will help the students to get insight of the connection between numbers and the properties employed in numbers’ operations [38]. Furthermore, the use of an area model may support the students’ attempt in developing number sense ability that will be beneficial for their understanding in multiplication [39,40]. The example of the use of area model to support students in learning multiplication can be seen in following Figure 7.

![Figure 7](image_url)

**Figure 7.** The example of an area model

Area model is one of powerful progressive models from Realistic Mathematics Education (RME) approach. Some previous studies showed the impact of area model to enhance students’ ability in solving multiplication problems. Not only for arithmetic operation, area model is also useful for teaching fraction multiplication and division [41,42] and algebra multiplication and factorization [43,44]. Hence, teaching the model from the beginning of learning algebra will support to students to be familiar with it in future studies.

Teaching multiplication using RME takes different learning path with the traditional teaching method by introducing and focusing on multiplication algorithm that usually learnt in Indonesian classroom. For instance, there will be a realistic problem to start the class in which the area model will occur naturally. Not only that, the students should be familiarized first with the number restructuring in which they can flexibly work with numbers. The students should also grasp the idea of “easy
number” that will be useful to be added and to be multiplied. From the number of previous studies it was found that RME provided better result in students’ mastery on the mathematical concepts [45–48].

4. Conclusion

From the data analysis and discussion, it can be seen that the prospective mathematics teachers were hardly making sense the multiplication algorithm. Since they cannot understand it by themselves, they cannot explain it to the students. Therefore, it can be concluded that the prospective teachers’ inability to making sense the process of multiplication algorithm lead to the lack of clarity to explain how it works. Also, the majority of the prospective teachers tend to use narration only compared to illustrate the process using scheme.

Reflect to the findings, it is necessary to the teacher training institution to provide more chance for the prospective teachers to critically think about the mathematical content knowledge (CK) that will be a strong foundation for them to set the appropriate pedagogical approach to teach certain topic (PCK). The prospective teachers should also be familiarized with the types of model that can be helpful to explain mathematical topics especially for earlier grades in elementary school. One of the recommendations to teach multiplication is to employ the area model from Realistic Mathematics Education (RME) approach.

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