Fractional spin through quantum strange superalgebra $\tilde{P}_Q(n)$.

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Abstract

The purposes of this paper is to investigate the properties of the quantum extended strange superalgebra $\tilde{P}_Q(n)$ when his deformation parameter $Q$ goes to a root of unity.

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1 Introduction

In recent years, much interest has been made in the study of the Lie superalgebras [1, 2, 3, 4, 5, 6]. These structures can be obtained through consistent realization involving deformed Bose and Fermi operators [7, 8].

In another vein, the geometric interpretation of fractional supersymmetry have gaining increased attention, particularly in the works [9, 10, 11, 12, 13, 14, 15], where the authors show that the one-dimensional superspace is equivalent to the braided line when the deformation parameter goes to a root of unity $Q \rightarrow q$, and the braided line is generated by a generalized odd variable and a (classical)ordinary even variable. In the work [16], R. S. Dunne, using $Q$-oscillator realization, proved that the $U_Q(sl(2))$ is similar to a direct product of the finite classical algebra $U(sl(2))$ and the $q$-deformed one $U_q(sl(2))$ (where $q$ is a root of unity).

Since there exist $Q$-oscillator realization of all deformed algebras and deformed super algebras $U_Q(g)$, it is convenable to explore the splitting of these (super) algebras when $Q \rightarrow q$. In this context, the property of splitting of some particular quantum (Super)-algebras was examined in [18]. The decomposition of the quantum (super) Virasoro algebras is described in [19]. The case of quantum affine algebras with vanishing central charge is developed in [20] and the case quantum algebras $A_n, B_n, C_n$ and $D_n$ and quantum superalgebra $A(m,n), B(m,n), C(n+1)$ and $D(n,m)$ in the $Q \rightarrow q$ limit is investigated in [21].

The Lie superalgebras of classical type are one of the two following classes[22]: basic Lie superalgebras or strange ones. The basic Lie superalgebras have proprieties like as simple Lie algebras. They have an invariant non-degenerate bilinear form, but strange Lie superalgebras $P(n)$ and $Q(n)$ have not.

The strange Lie superalgebra $P(n)$ have attracted a particularly attention. In [23], Dynkin-like diagrams of the strange superalgebra $P(n)$ was examined by Frappat, Sciarrino and Sorba. In [24], polynomial representations of strange Lie superalgebras are investigated. The oscillator realization of the strange superalgebras $P(n)$ has been constructed by Frappat, Sciarrino and Sorba in [25]. A deformation $U_Q(\tilde{P}(n)) = \tilde{P}_Q(n)$ of the extended non-contragredient (strange) superalgebra $\tilde{P}(n)$ is given in [26].

The purpose of this paper is to expore the property of decomposition of the quantum extended non-contragredient (strange) superalgebra $U_Q(\tilde{P}(n)) = \tilde{P}_Q(n)$ in the $Q \rightarrow q$ limit. In next section (section 2) we review some results concerning k-fermions, decomposition property of $Q$-boson in the $Q \rightarrow q$ limit and the equivalence between $Q$-deformed fermions and classical ones. Using these results detailed in [19, 20], we analyse the $Q \rightarrow q$ limit of the quantum $U_Q(sl(n))$ algebra of the $sl(n)$ algebra (The bosonic part of $P(n)$) (section 3) and the quantum extended non-contragredient (strange) superalgebra of $U_Q(\tilde{P}(n))$ (section 4). In the last section (section 5) we shall give some concluding remarks.
2 Preliminaries

In this section we recall some basic facts about $k$-fermions\cite{17}, decomposition property of $Q$-boson in the $Q \to q$ limit and the equivalence between $Q$-deformed fermions and ordinary ones (see \cite{19, 20} for more details).

Let us began by giving the definition of the $Q$-bosonic algebra noted $(\Xi_Q^i)$, generated by a number operator $N_{A_i}$, a creation operator $A_i^+$ and an annihilation operator $A_i^-$, satisfying the relations:

\begin{align}
A_i^- A_i^+ - Q^\pm A_i^+ A_i^- &= Q^\mp N_{A_i} \\
Q^{N_{A_i}} A_i^+ Q^{-N_{A_i}} &= Q^\pm A_i^+ \\
Q^{N_{A_i}} A_i^- Q^{-N_{A_i}} &= Q^{-N_{A_i}} Q^{N_{A_i}} = 1
\end{align}

(1)

then if we put the following operators as given in \cite{16}:

\begin{align}
a_i^- &= \lim_{Q \to q} \frac{Q^{\frac{k N_{A_i}}{2}}}{([k]!/2)} (A_i^-)^k, \\
a_i^+ &= \lim_{Q \to q} \frac{Q^{\frac{k N_{A_i}}{2}}}{([k]!/2)} (A_i^+)^k Q^{-\frac{k N_{A_i}}{2}},
\end{align}

(2)

we can easily show that the above operators (2) gratifies the relations of an ordinary boson algebra noted $\Xi_0$, defined by:

\begin{align}
[a_i^-, a_i^+] &= 1, \\
[N_{a_i}, a_i^\pm] &= \pm a_i^\pm
\end{align}

(3)

The number operator of this new algebra is defined by $N_{a_i} = a_i^+ a_i^-$. 

In order to discuss the splitting of $Q$-deformed boson in the limit $Q \to q$, we introduce the new operators:

\begin{align}
\chi_i^- &= A_i^- q^{-\frac{k N_{A_i}}{2}}, \\
\chi_i^+ &= A_i^+ q^{-\frac{k N_{A_i}}{2}}, \\
N_{\chi_i} &= N_{A_i} - k N_{a_i},
\end{align}

(4)

which satisfies the relations of a $k$-fermionic algebra noted $(\Sigma_Q^i)$ defined by

\begin{align}
[\chi_i^+, \chi_i^-]_q &= q^{N_{\chi_i}}, \\
[\chi^-, \chi^+]_q &= q^{-N_{\chi_i}} \\
[N_{\chi_i}, \chi_i^\pm] &= \pm \chi_i^\pm.
\end{align}

(5)

where the deformation parameter $q = e^{\frac{2i\pi}{r}}$, $r \in N - \{0, 1\}$, is a root of unity.

It straightforward to check that the two algebras generated respectively by the set of operators $\{a_i^\pm, \chi_i^-, N_{a_i}\}$ and $\{\chi_i^+, \chi_i^-, N_{\chi_i}\}$ are mutually commutative. We conclude that in the limit $Q \to q$, the $Q$-deformed bosonic algebra oscillator $\Xi_Q^i$ decomposes into two independent algebras, an ordinary boson algebra $\Xi_0^i$ and $k$-fermionic algebra $\Sigma_Q^i$, formally one can write:

$$
\lim_{Q \to q} \Xi_Q^i = \Xi_0^i \otimes \Sigma_Q^i
$$
We define also the $Q$–deformed fermionic algebra noted $\Omega_Q$ generated by the generators $\Phi_i^-, \Phi_i^+$ and $Q^M_{\Phi_i}, Q^{-M}_{\Phi_i}$ satisfying the following relations

\[
Q^M_{\Phi_i}Q^{-M}_{\Phi_i} = Q^{-M}_{\Phi_i}Q^M_{\Phi_i} = 1 \\
Q^M_{\Phi_i}Q^M_{\Phi_j} = Q^M_{\Phi_j}Q^M_{\Phi_i} \\
Q^M_{\Phi_i}\Phi_i^+ Q^{-M}_{\Phi_i} = Q^{\pm}\Phi_i^+ \\
\Phi_i^+ \Phi_i^- + Q^{\pm}\Phi_i^+ \Phi_i^- = Q^{\pm} e_i^\pm \\
\{\Phi_i^+, \Phi_i^-\} = 0; \\
(\Phi_i^+)^2 = 0, (\Phi_i^-)^2 = 0 \\
\{\Phi_i^+, \Phi_j^-\} = 0 \text{ for } i \neq j
\]

then if put the new generators

\[
\phi_i^- = Q^{-M}_{\Phi_i} \Phi_i^- \\
\phi_i^+ = \Phi_i^+ Q^{-M}_{\Phi_i}
\]

we see that the $Q$–deformed fermion reproduces an ordinary fermion algebra defined by the new operators (7) and the following relations

\[
(\phi_i^-)^2 = 0 \\
(\phi_i^+)^2 = 0 \\
\{\phi_i^-, \phi_j^+\} = \delta_{ij}
\]

3 The quantum algebra $U_Q(sl(n))$

Let $C = [a_{ij}](1 \leq i, j \leq n)$ be a symmetrisable generalized Cartan matrix and let $d_i(1 \leq i \leq n)$ be the non integers such that $d_i a_{ij} = a_{ij} d_i$. Let $Q \neq 0$ be a complex number. For $Q$ generic the quantum enveloping algebra corresponding to $[a_{ij}]$ is a Hopf algebra with 1 and generators $\{E_i, F_i, K_i^+, K_i^-= Q^{d_i a_{ij}}, 1 \leq i \leq n\}$ satisfying the following relations :

\[
[E_i, F_j] = \delta_{ij} \frac{K_i - K_i^{-1}}{Q^{d_i a_{ij}} - Q^{-d_i a_{ij}}} \\
K_i E_j K_i^{-1} = Q^{d_i a_{ij}} E_j \\
K_i F_j K_i^{-1} = Q^{-d_i a_{ij}} F_j \\
K_i^+ K_i^- = K_i^- K_i^+; K_i K_j = K_j K_i
\]

with Serre relations,

\[
\sum_{0 \leq t \leq n} (-1)^t [n]_Q (E_i)^t E_j (E_i)^n-t = 0, \quad i \neq j \\
\sum_{0 \leq t \leq n} (-1)^t [n]_Q (F_i)^t F_j (F_i)^n-t = 0, \quad i \neq j
\]

where $a_{ij}$ is Cartan matrix, $n = 1 - a_{ij}$ and

\[
[n]_Q = \frac{[n]Q!}{[t]Q!(n-t)Q}, \quad [t]_Q! = [t]Q[t-1]Q...[1]Q, \quad [t]_Q = \frac{Q^t - Q^{-t}}{Q - Q^{-1}}
\]

The Cartan matrices of classical type $A_n, B_n, C_n$ and $D_n$ and the corresponding non zero integers are given in [27]. A discussion of the Q-boson and Q-fermion representation was given by Hayashi [28].

We focus here on the algebra $U_Q(sl(n))$ (where $sl(n)$ is the bosonic part of $P(n+1)$) and we
assume that the deformation parameter $Q$ is generic. The explicit expressions for corresponding
generators as linear and bilinear in $Q$-deformed bosonic operators are given by:

$$
E_i = A_i^- A_{i+1}^+ \\
F_i = A_i^+ A_{i+1}^- \\
H_i = -N_{A_i} + N_{A_{i+1}}
$$  \hspace{1cm} (12)

Now we can explore the limit $Q \to q$ of the quantum algebra $U_Q(sl(n))$. The key tool to discuss
this limit is the $Q$-bosonic decomposition presented in detail in [19, 20] when the deformation
parameter $Q$ goes to a root of unity $q$. So, the $n$ $Q$-bosons reproduce $n$ ordinary bosons and $n$
$k$-fermions $\{\chi_i^+; \chi_i^-; N_{\chi_i}\}$ with $1 \leq i \leq n - 1$. The $n$ classical bosons are defined by

$$
a_n^- = \lim_{Q \to q} \frac{Q^\pm \frac{kN_{A_i}}{2}}{(\pm |k|)!^\frac{1}{2}} (A_i^-)^k, \quad a_n^+ = \lim_{Q \to q} \frac{(A_i^+)^k Q^\pm \frac{kN_{A_i}}{2}}{(\pm |k|)!^\frac{1}{2}},
$$  \hspace{1cm} (13)

where their number operators are given by $N_{a_i} = a_i^+ a_i^-$, for $i = 1, 2, ..., n$.

Then, using operators (13), we can construct the classical $U(sl(n))$ algebra (with $1 \leq i \leq n - 1$):

$$
e_i = a_i^- a_{i+1}^+ \\
f_i = a_i^+ a_{i+1}^- \\
h_i = -N_{A_i} + N_{A_{i+1}}
$$  \hspace{1cm} (14)

From the remaining operators $\{\chi_i^+; \chi_i^-; N_{\chi_i}\}$ with $1 \leq i \leq n - 1$ we construct the new generators

$$
E_i = \chi_i^+ \chi_{i+1}^- \\
F_i = \chi_i^- \chi_{i+1}^+ \\
H_i = -N_{\chi_i} + N_{\chi_{i+1}}
$$  \hspace{1cm} (15)

which realize the $U_q(sl(n))$ algebra; where $U_q(sl(n))$ is the same version of $U_Q(sl(n))$ obtained
by simply setting $Q = q$ rather than by taking the limit as above. The elements of $U_q(sl(n))$
and $U(sl(n))$ algebras are mutually commutative.

Then, in the $Q \to q$ limit, the quantum algebra $U_Q(sl(n))$ is a direct product of the form

$$
\lim_{Q \to q} U_Q(sl(n)) = U_q(sl(n)) \otimes U(sl(n))
$$  \hspace{1cm} (16)

Note that, the above direct product $\lim_{Q \to q} U_Q(g) = U_q(g) \otimes U(g)$ valid for quantum algebras
does not appear of a quantum superalgebra $U_Q(g)$. In fact the explicit expressions of the
generators of the quantum superalgebra $U_Q(g)$ are presented as linear and bilinears in $Q$-
deformed bosonic and fermionic oscillator operators, and using the fact that the $Q$-deformed
bosonic operators $\{A_i^+, A_i^-, N_{A_i}\}$ with $1 \leq i \leq n$ decomposes into two independent oscillators
algebras: classical bosons $\{a_i^+, a_i^-, N_{a_i}\}$ and $k$-fermion operators $\{\chi_i^+, \chi_i^-; N_{\chi_i}\}$, and $Q$-fermions
become $q$-fermions which are object equivalent to conventional fermions $\{\phi^+_i, \phi^-_i, M_{\phi_i}\}$. Then
from the classical bosons $\{a_i^-, a_i^+, N_{a_i}\}$ and classical fermions $\{\phi^-_i, \phi^+_i, M_{\phi_i}\}$, one can realize the
nondeformed superalgebra $U(g)$ but from the remaining operators $\{\chi_i^+, \chi_i^-; N_{\chi_i}\}$ we construct
the generators of a different quantum $q$-algebra (see [21] for more details).

4 The quantum extended strange superalgebra $\tilde{P}_Q(n)$

Let $G = G_0 \oplus G_1$ be a $\mathbb{Z}_2$-graded vector space with dim $G_0 = j$ and dim $G_1 = i$. Then there exists a natural superalgebra structure on the algebra $End G$ defined by:

$$
End G = End_0 G \oplus End_1 G \quad \text{where} \quad End_k G = \{ \phi \in End G \mid \phi(G_i) \subset G_{k+1} \}$$
The superalgebra $\text{End} \ G$ supplied with the Lie superbracket is the Lie superalgebra noted $\ell(i, j)$. The elements $M$ of $\ell(i, j)$ have the form

$$M = \begin{pmatrix} N & Q \\ R & S \end{pmatrix}$$

where $N$ and $S$ are $gl(j)$ and $gl(i)$ matrices, $Q$ and $R$ are $j \times i$ and $i \times j$ rectangular matrices.

The superalgebra of matrices $M \in \ell(n, n)$ satisfying the following equalities

$$N^t = -S, \quad Q^t = Q, \quad R^t = -R, \quad \text{tr}(N) = 0$$

is nothing other than the (non contragredient) strange superalgebra $P(n)$.

An oscillator realization of the generators of $P(n)$ given in [2, 25]. In the Chevalley basis, the (non contragredient) strange superalgebra $P(n)$ is spanned by the generators $\{X_i, Y_i, T_i, X_n\}$ with $(1 \leq i \leq n - 1)$ satisfying the following commutation relations:

$$[X_i, Y_j] = \delta_{ij} T_i$$
$$[X_i, T_j] = -a_{ij} X_i$$
$$[Y_i, T_j] = a_{ij} Y_i$$
$$[T_i, X_n] = a_{in} X_n$$
$$[T_i, T_j] = 0;$$

where $(a_{ij})_{1 \leq i, j \leq n}$ is the Cartan matrix of $su(n)$ and $a_{in} = 0$ for $1 \leq i \leq n - 2$ and $a_{n-1,n} = -2$.

Let us to precise that the notions of Dynkin diagram and Cartan matrix are not determined for the non-contragredient Lie superalgebra $P(n)$. However, if we extend the superalgebra of $P(n)$ by an appropriate diagonal matrices, one can obtain a non-null bilinear form on the Cartan subalgebra of this extension and therefore get in this case a generalized form of the notions of Cartan matrix and Dynkin diagram [2].

The extended strange superalgebra $\tilde{P}(n)$ is defined in this basis by $3(n-1)$ bosonic generators $X_i, Y_i, T_i$, with $i = 1, ..., n-1$, and a fermionic generator $X_n$ and a diagonal generator $D$ such that

$$[X_i, Y_j] = \delta_{ij} T_i$$
$$[X_i, T_j] = -a_{ij} X_i$$
$$[Y_i, T_j] = a_{ij} Y_i;$$
$$[T_i, X_n] = a_{in} X_n;$$
$$[T_i, T_j] = 0;$$
$$[D, X_i] = [D, Y_i] = [D, T_i] = 0$$
$$[D, X_n] = X_n$$

The Cartan matrix of the extended strange superalgebra $(a_{ik})_{\tilde{P}(n)}$ of $\tilde{P}(n)$ with $1 \leq i \leq n - 1$. 


and $1 \leq k \leq n$ is given by:

$$(a_{ik})_{\tilde{P}(n)} = \begin{pmatrix}
2 & -1 & 0 & \cdots & \cdots & 0 & 0 \\
-1 & 2 & -1 & \cdots & \cdots & 0 & 0 \\
0 & -1 & \ddots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 2 & -1 & 0 \\
0 & 0 & \cdots & 0 & -1 & 2 & -2
\end{pmatrix}$$

The Cartan matrix is well defined to get the Serre relations for the extended non-contragredient Lie superalgebra $\tilde{P}(n)$ in the quantum case and permits to define a quantum superalgebra on the $Q$-deformed version of the extended non-contragredient (strange) superalgebra $\tilde{P}(n)$.

For $Q$ generic, a quantum deformation $U_Q(\tilde{P}(n)) = \tilde{P}_Q(n)$ of the extended non-contragredient (strange) superalgebra $\tilde{P}(n)$ is proposed in [26] as follows:

$$
\begin{align*}
[\hat{X}_i, \hat{Y}_j] &= \delta_{ij} [\hat{T}_i]_Q \\
[\hat{T}_i, \hat{X}_n] &= a_{in} \hat{X}_i \\
[\hat{Y}_i, \hat{T}_j] &= a_{ij} \hat{Y}_i \\
[\hat{T}_i, \hat{T}_j] &= 0 \\
[\hat{D}, \hat{X}_i] &= [\hat{D}, \hat{Y}_i] = [\hat{D}, \hat{\check{T}}_i] = 0 \\
[\hat{D}, \hat{X}_n] &= X_n \\
[\hat{X}_i, \hat{T}_j] &= -a_{ij} \hat{X}_i
\end{align*}
$$

(19)

and the quantum Serre relations described by the expressions:

$$
\begin{align*}
\sum_{0 \leq t \leq 1 - a_{ik}} (-1)^t t 
&\begin{pmatrix}
1 - a_{ik} \\
t
\end{pmatrix}
\hat{X}_i^{1-a_{ik}-t} \hat{X}_k \hat{X}_i^t = 0 \\
\sum_{0 \leq t \leq 1 - a_{ij}} (-1)^t t 
&\begin{pmatrix}
1 - a_{ij} \\
t
\end{pmatrix}
\hat{Y}_i^{1-a_{ij}-t} \hat{Y}_j \hat{Y}_i^t = 0
\end{align*}
$$

(20)

A possible realization of the generators of $\tilde{P}_Q(n)$ in terms of the $Q$-deformed oscillators $\{\Phi_i^-, \Phi_i^+, M_{\Phi_i}\}$ and $\{A_i^+, A_i^-, N_{A_i}\}$ with $(1 \leq i \leq n)$ is given by:

$$
\begin{align*}
\hat{X}_i &= A_i^+ A_{i+1}^+ Q^{\frac{(M_{\Phi_i} - M_{\Phi_{i+1}})}{2}} + \Phi_i^+ \Phi_{i+1}^- Q^{-\frac{(N_{A_i} - N_{A_{i+1}})}{2}} \\
\hat{X}_n &= A_n^+ \Phi_n^+ Q^{-\frac{1}{2} \sum_{i=1}^{n-1} N_{A_i} - \frac{1}{2} \sum_{i=1}^{n-1} M_{\Phi_i}} \\
\hat{Y}_i &= A_i^+ A_i^- Q^{\frac{(M_{\Phi_i} - M_{\Phi_{i+1}})}{2}} + \Phi_i^+ \Phi_i^- Q^{-\frac{(N_{A_i} - N_{A_{i+1}})}{2}} \\
\hat{T}_i &= N_{A_i} - N_{A_{i+1}} + M_{\Phi_i} - M_{\Phi_{i+1}} \\
\hat{D} &= \frac{1}{2} \sum_{i=1}^{n} N_{A_i} + \frac{1}{2} \sum_{i=1}^{n} M_{\Phi_i}
\end{align*}
$$

(21)
where $A_i^+$ and $A_i^-$ are the $Q$-deformed bosonic algebra operators and $\Phi_i^+$ and $\Phi_i^-$ are the $Q$-deformed fermionic ones.

Using the fact that in $Q \to q$ limit, the $Q$-deformed bosons algebras $\{A_i^+, A_i^-, N_a\}$ reproduces classical bosons algebras $\{a_i^+, a_i^-, N_a\}$ and a $q$-deformed k-fermions generators $\{\chi_i^+, \chi_i^-, N_{\chi}\}$, and in this limit the $Q$-fermions become $q$-fermions which are object equivalent to classical ones $\{\phi_i^+, \phi_i^-, M_{\phi}\}$, then from the classical bosons $\{a_i^-, a_i^+, N_a\}$ and classical fermions $\{\phi_i^-, \phi_i^+, M_{\phi}\}$, one can construct the classical extended strange superalgebra $\tilde{P}(n)$:

\[
\begin{align*}
X_i &= a_i^+ a_{i+1}^+ + \phi_i^+ \phi_{i+1}^-
X_n &= a_n^+ \phi_n^+
Y_i &= a_i^+ a_i^- + \phi_i^+ \phi_i^-
T_i &= N_{a_i} - N_{a_{i+1}} + M_{\phi_i} - M_{\phi_{i+1}}
D &= \frac{1}{2} \sum_{i=1}^{n} N_{a_i} + \frac{1}{2} \sum_{i=1}^{n} M_{\phi_i}
\end{align*}
\] (22)

From the remaining operators $\{\chi_i^-, \chi_i^+, N_{\chi}\}$ we realize the following:

\[
\begin{align*}
E_i &= \chi_i^- \chi_{i+1}^+, \quad 1 \leq i \leq n - 1
F_i &= \chi_i^+ \chi_{i+1}^-, \quad 1 \leq i \leq n - 1
K_i &= q^{-N_{\chi_i} + N_{\chi_{i+1}}}, \quad 1 \leq i \leq n - 1
\end{align*}
\] (23)

which generates the $q$-deformed algebra $U_q(sl(n))$. It is easy to show that $U_q(sl(n))$ and $\tilde{P}(n)$ are mutually commutative. As results, we obtain the following decomposition of the quantum strange superalgebra $\tilde{P}(n)$ in the $Q \to q$ limit:

\[
\lim_{Q \to q} \tilde{P}(n) = U_q(sl(n)) \otimes \tilde{P}(n).
\] (24)

5 Conclusion

It is important to note that we have established this decomposition of the quantum strange superalgebra $\tilde{P}(n)$ only for a particular realization, i.e, the $Q$-oscillator realization and although the quantum extended strange superalgebra $\tilde{P}(n)$ does not have direct product form, we establish, for this realization and the corresponding highest weight representations the decomposition of $\tilde{P}(n)$ into the direct product of undeformed $\tilde{P}(n)$ and $U_q(sl(n))$ (the naive version of $U_Q(sl(n))$ at $Q = q$ obtained by simply setting $Q = q$). The labels of the highest weight representations of the quantum strange superalgebra $\tilde{P}(n)$ and the choice of the basis in which the decomposition (24) is clearly manifested will be investigated elsewhere.
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