Single-photon transistor based on cavity electromagnetically induced transparency with Rydberg atomic ensemble

Y. M. Hao, G. W. Lin, X. M. Lin, Y. P. Niu & S. Q. Gong

A scheme is presented to realize a single-photon transistor based on cavity quantum electrodynamics (QED) with Rydberg atomic ensemble. By combining the advantages of the cavity-enhanced interaction and Rydberg blockade, we achieve a high gain single-photon transistor. The numerical calculation shows that by using one single gate photon more than one thousand source photons can be switched.

Electromagnetically induced transparency (EIT) induced by the coherent interference effect has many important applications, including optical nonlinearities, quantum storage, observation of parity-time symmetry, and so on. By combining the advantages of the cavity-enhanced interaction and Rydberg blockade, Cavity EIT with Rydberg atomic ensemble becomes a promising platform for the realization of optical nonlinearities. Both theories and experiments have verified strong optical nonlinearities can be realized in this system. In particular, Lin et al. presented a theoretical scheme for strong single-photon nonlinearity with intracavity EIT in blockaded Rydberg ensemble. In this scheme, they showed that the photons in the cavity are in the form of cavity dark-state polaritons, and strong interaction of the polaritons leads to strong blockade effect. In a recent experiment, Jia et al. have observed this strong interaction of the cavity dark-state polaritons, and demonstrated the strong single-photon nonlinearities by measuring the transmission spectrum. By exploiting this strong nonlinearities, the quantum phase gate between a photon and an atomic ensemble or two photons can be realized.

Single-photon transistor is the cornerstone device for quantum information processing. It opens up new perspectives for all-optical information processing and has many potential applications, such as realization of quantum repeaters, nondestructive detection of optical photons, generation of Schrödinger-cat states. Strong single-photon nonlinearities have been made in various systems. Among these systems, cavity quantum electrodynamics (QED) and Rydberg EIT are two significant promising candidates. To obtain a high gain single-photon transistor, the cavity QED scheme and the Rydberg-EIT scheme respectively use the cavity-enhanced interaction and Rydberg blockade to achieve the strong single-photon nonlinearities. Both optical gain of these two systems have been up to several hundreds.

In this paper, we theoretically present a single-photon transistor based on cavity EIT with Rydberg atomic ensemble. In our scheme, a Rydberg atomic ensemble is trapped in an optical cavity. The extremely strong single-photon nonlinearities can be created by combining the advantages of the cavity-enhanced interaction and Rydberg blockade. By means of the strong single-photon nonlinearities, we can implement a single-photon transistor with high gain. We show that the optical gain in our scheme could be boosted above one thousand, which is higher than that in both cavity QED scheme and Rydberg-EIT scheme. Furthermore, even when the scale of atomic ensemble is much larger than the blockade radius, our scheme could still work well.

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Results

As illustrated in Fig. 1, our model consists of a cold ensemble of \( N \) Rydberg atoms trapped inside an optical cavity. Each atom has a stable ground state \(| g \rangle\), two excited states \(| e_1 \rangle\) and \(| e_2 \rangle\), and two Rydberg states \(| r_1 \rangle\) and \(| r_2 \rangle\). The first step for realization of the single-photon transistor is the process of the gate photon. Initially, all atoms are in ground state \(| g \rangle\), i.e.,

\[
| \ldots \rangle = | g \rangle^N
\]

A free-space gate photon is resonant to the transition \(| g \rangle \leftrightarrow | e_1 \rangle\), while a classical control field with Rabi frequency \( \Omega_1 \) drives the transition \(| e_1 \rangle \leftrightarrow | r_1 \rangle\), as shown in Fig. 1(a). These two transitions form EIT configuration. By adiabatically changing the control laser power down, the gate photon can be stored in the Rydberg atomic ensemble\(^4\). As shown in ref. \(^4\), the maximum storage efficiency with cavity EIT could reach

\[
P \approx \frac{\eta}{1 + N \eta}
\]

After this storage process of the gate photon, we apply a weak coherent source beam \( \varepsilon_s \) with frequency \( \omega_s \) to drive the cavity mode \( a \), which is resonant with the transition \(| g \rangle \leftrightarrow | e_2 \rangle\), as seen in Fig. 1(b). Meanwhile another control field with Rabi frequency \( \Omega_2 \) drives the transition \(| e_2 \rangle \leftrightarrow | r_2 \rangle\). The total Hamiltonian of the system can be given by

\[
H_{\text{total}} = \varepsilon_s (a^\dagger e^{-i\Delta s} + a e^{i\Delta s}) + H_I,
\]

where \( \Delta = \omega_s - \omega_a \) being detuning of source photon from the cavity mode, \( a (a^\dagger) \) being the annihilation (creation) operator of the cavity mode, and

\[
H_I = \sum_{j=1}^{N} (g_j | e_{2j} \rangle \langle g_j | a + \Omega_j (| r_{2j} \rangle \langle e_{2j} | + H.c.
\]

\[
+ \sum_{i<j} \Delta_{r_1,r_2} (| r_{2j} \rangle \langle r_{2i} | \otimes | r_{1j} \rangle \langle r_{1i} |)
\]

being the interaction Hamiltonian between atoms and cavity mode. Here \( \Delta_{r_1,r_2} \) is the additional energy shift when two atoms are respectively excited to Rydberg states \(| r_1 \rangle\) and \(| r_2 \rangle\), \( g_j \) is the single-atom cavity coupling coefficient. In Hamiltonian \( H_I \), we have ignored the self-blockade interaction \( \sum_{i<j} \Delta_{r_1,r_2} (| r_{2j} \rangle \langle r_{2i} | \otimes | r_{1j} \rangle \langle r_{1i} |) \) by choosing suitable principal quantum number to satisfy \( \Delta_{r_1,r_2} > \Delta_{r_1,r_2} \) \(^4\).

In the ideal case, each atom has an equal probability-amplitude \( 1/\sqrt{N} \) to be the Rydberg excitation \(| r_i \rangle\) after the gate photon is stored in the atomic ensemble. Without loss of generality, we assume the \( i \)-th atom to be the Rydberg excitation \(| r_i \rangle\) and other atoms remain in their initial ground state \(| g \rangle\). Due to the Rydberg blockade, \( n \)

\[
\text{Storage}
\]

\[
\text{Switching}
\]

Figure 1. Single-photon transistor with an ensemble of \( N \) Rydberg atoms trapped inside an optical cavity. (a) We first stored a gate photon in the medium, which corresponds to a Rydberg excitation to state \(| r_1 \rangle\). (b) This Rydberg excitation blocks the transmission of source photons through the cavity.
atoms within the blockade radius around the \( i \)-th atom have an additional energy shift \( \Delta_{ri} \). The other \( (N-n) \) atoms outside the blockade radius will not be affected by the Rydberg blockade (see Fig. 1(b)). For simplicity, we divide the Hamilton \( H_i \) into two parts: one part is Hamilton \( H_n \) for \( n \) atoms within the Rydberg blockade radius, the other is Hamilton \( H_{N-n} \) for \( (N-n) \) atoms outside the Rydberg blockade radius, then

\[
H_i = H_n + H_{N-n},
\]

with

\[
H_n = \sum_{i=1}^{n} (g_i|e_{2,i}\rangle\langle g_i|a + \Omega_2|r_{2,i}\rangle\langle e_{2,i}| + H.c.) + \Delta_{ri}|r_{2,i}\rangle\langle r_{2,i}|,
\]

\[
H_{N-n} = \sum_{k=1}^{N-n} (g_k|e_{2,k}\rangle\langle g_k|a + \Omega_2|r_{2,k}\rangle\langle e_{2,k}| + H.c.).
\]

Considering photon losses from the cavity as well as the decays associated with the atom, the dynamics of the system governed by the Hamiltonian \( H_{total} \) can be described by quantum Langevin equations\(^{39} \) and the steady-state solution of the cavity mode \( a \) under the mean-field approximation\(^{39} \) is given by (see Methods)

\[
\langle a \rangle = -\frac{E_a}{i\Delta_a + \kappa + \frac{\Omega_2^2}{\Delta_a + \frac{\Delta_{ri}}{2} + \frac{i\gamma}{N-n}} + \frac{\Omega_2^2}{\Delta_a + \frac{\Delta_{ri}}{2} + \frac{i\gamma}{N-n}}},
\]

where \( \kappa \) is the decay rate of the cavity, \( \gamma \) (\( \gamma_2 \)) is the dephasing rate associated with the low excited state (Rydberg state), \( \gamma = \sqrt{\sum_{j=1}^{N} \omega_j^2}/N \) is the effect atom-cavity coupling strength. And the transmitted source field through the cavity is

\[
a_T = \sqrt{\langle a \rangle}.
\]

Figure 2 plots the transmitted intensity \( I_s = a_T^*a_T \) of the source light (normalized by the input source intensity \( I_{in} = |\langle g\rangle| \)) versus the source-cavity detuning \( \Delta_s \) for (a) \( \gamma_2 = 0.01 \kappa \), (b) \( \gamma_2 = 0.1 \kappa \) and (c) \( \gamma_2 = \kappa \). Other parameters are \( \Omega = 20 \text{ MHz}, \kappa = 1 \text{ MHz}, \gamma = 1 \text{ MHz}, N = 3600, n = 400, \gamma_2 = 30 \text{ MHz} \) and \( \epsilon_2 = 5 \text{ MHz} \).
Discussion and Conclusions

Next we quantify the single-photon transistor with optical gain. The transmitted source photon number is
\[ \int_0^{\tau} a^\dagger a dt \]
with \( \tau \) being the source integration time. Then the optical gain
\[ G \]
per stored gate photon can be defined as the gate-photon-induced change in source-light transmission, as
\[ G = \frac{M_{s,\text{out}}^{\text{no gate}} - M_{s,\text{out}}^{\text{with gate}}}{M_{s,\text{out}}^{\text{no gate}}} \]  
where \( M_{s,\text{out}}^{\text{no gate}} \) and \( M_{s,\text{out}}^{\text{with gate}} \) denote the mean numbers of transmitted source photons without and with the storage of the gate photon, respectively. In Fig. 4, we show the transistor gain \( G \) versus \( r = n/N \). From Fig. 4, we see that with the increase of the \( r = n/N \), the optical gain \( G \) increases firstly and then quickly reaches to the maximum value. Even though \( r \ll 1 \), the optical gain \( G \) can exceed \( 10^3 \). Furthermore, when the cavity is in the weak-coupling regime, i.e., the single-atom cooperativity \( \eta = g^2/\kappa^2 \approx 0.1 < 1 \), the single-photon transistor can still work well.

Then we address the experiment feasibility of the proposed scheme. For a potential experimental system, we consider that an optical cavity traps an ensemble of cold \(^{87}\text{Rb} \) atoms with atom number \( N \approx 3600 \). Assuming that \( n \approx 14 \) atoms within a blockade sphere with radius \( r \approx 1.5 \mu \text{m} \) are affected by the Rydberg blockade. For the Rydberg states \( |n_1\rangle = |41s_{1/2}, m = 1/2 \rangle, |n_2\rangle = |40s_{1/2}, m = 1/2 \rangle \), one could achieve strongly asymmetric Rydberg blockade interaction \( \Delta_{n_1 n_2} \approx 56 \text{ MHz} \), which is much larger than \( \Delta_{\eta \gamma} \approx 0.3 \text{ MHz} \). Typically, the relevant cavity parameters are \( \langle \kappa, \gamma \rangle \approx (1.16, 37.6) \text{ MHz} \). We choose the parameters \( \gamma_r = 10^{-2} \kappa, \varepsilon_s = 5 \text{ MHz} \), \( \Omega_s = 20 \text{ MHz} \), \( \tau = 50 \mu \text{s} \) and the single-atom cooperativity \( \eta = g^2/\kappa \gamma \approx 0.1 < 1 \), then we can obtain the optical gain \( G \approx 1125 \) for the single-photon transistor. As the experiment progresses, cavity EIT\(^9\) and multi-wave mixing\(^{50,51}\) for strong nonlinearity has been successfully demonstrated in Rydberg atoms. Therefore, our scheme could be realized in the near future.

In conclusion, we have demonstrated a new scheme to implement a single-photon transistor based on cavity QED and Rydberg-EIT. By combining the advantages of the cavity-enhanced interaction and Rydberg blockade, the optical gain of the single-photon transistor is boosted to over \( 10^3 \), which is higher than both cavity scheme\(^8\) and Rydberg-EIT scheme\(^{34-38}\). Furthermore, even when the scale of atomic ensemble is much larger than the blockade radius\(^44\), our scheme could still work well. Therefore, our work may provide a promising approach for the realization of the single-photon transistor and other all-optical devices.
Methods
For convenience, we define the collective operators
\[ \Lambda_n = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |\Lambda_k\rangle \langle \Lambda_k| \]
(\(\Lambda = e_2, r_2\)) for atoms within the Rydberg blockade radius and outside the Rydberg blockade radius, respectively. In terms of these collective operators, we rewrite the Hamiltonians
\[ H_n = \sum_{\Lambda} (\Lambda_n^\dagger \Lambda_n + \Lambda_n \Lambda_n^\dagger) + \Omega_1 (\Lambda_n^\dagger \Lambda_n n_{e_2} + \Lambda_n \Lambda_n^\dagger n_{e_2} + \Delta_{e_2,r_2} \Lambda_n^\dagger n_{r_2} n_{e_2}), \]
\[ H_{N-n} = \sqrt{N-n} \sum_{\Lambda} (\Lambda_n^\dagger \Lambda_n + \Lambda_n \Lambda_n^\dagger) + \Omega_2 (\Lambda_n^\dagger \Lambda_n N_{e_2} + \Lambda_n \Lambda_n^\dagger N_{e_2} + \Delta_{r_2} \Lambda_n^\dagger n_{r_2} n_{e_2}), \]
where \(\sqrt{N-n} = \frac{\sqrt{\sum_{\Lambda} |\Lambda_{e_2, r_2}|^2}}{\sqrt{\sum_{\Lambda} |\Lambda_{e_2, r_2}|^2}}\) is the effective atom-cavity coupling strength, which is collectively enhanced due to the many-atom interference effect.

Considering photon losses from the cavity as well as the decays associated with the atom, we describe the dynamics of the system governed by the total Hamiltonian
\[ H_{\text{total}} \]
in the rotating frame with the following quantum Langevin equations:
\[ \dot{a} = -(i\Delta_2 + \kappa) a - \sqrt{\kappa} \sqrt{N-n} n_{e_2} - \sqrt{\kappa} \sqrt{N-n} n_{r_2} - i\epsilon, \]
\[ \dot{\Lambda}_{n,e_2} = -\left( i\Delta_2 + i\Delta_{e_2} + \frac{\gamma}{2} \right) \Lambda_{n,e_2} - \sqrt{\gamma} \sqrt{N-n} a - i\Omega_2 \Lambda_{n,r_2}, \]
\[ \dot{\Lambda}_{n,r_2} = -\left( i\Delta_2 + i\Delta_{r_2} + \frac{\gamma}{2} \right) \Lambda_{n,r_2} - \sqrt{\gamma} \sqrt{N-n} a - i\Omega_2 \Lambda_{n,e_2}, \]
\[ \dot{\Lambda}_{N-n,e_2} = -\left( i\Delta_2 + \frac{\gamma}{2} \right) \Lambda_{N-n,e_2} - \sqrt{\gamma} \sqrt{N-n} a - i\Omega_2 \Lambda_{N-n,r_2}, \]
\[ \dot{\Lambda}_{N-n,r_2} = -\left( i\Delta_2 + \frac{\gamma}{2} \right) \Lambda_{N-n,r_2} - i\Omega_2 \Lambda_{N-n,e_2}, \]

Here, we have ignored the decoherence between the ground states. Under the mean-field approximation \(\langle Qc \rangle = \langle Q \rangle \langle c \rangle\), and the mean value equations are given by
\[ \langle a \rangle = -(i\Delta_2 + \kappa) \langle a \rangle - \sqrt{\kappa} \sqrt{N-n} \langle n_{e_2} \rangle - \sqrt{\kappa} \sqrt{N-n} \langle n_{r_2} \rangle - i\epsilon, \]
\[ \langle \Lambda_{n,e_2} \rangle = -\left( i\Delta_2 + i\Delta_{e_2} + \frac{\gamma}{2} \right) \langle \Lambda_{n,e_2} \rangle - \sqrt{\gamma} \sqrt{N-n} \langle a \rangle - i\Omega_2 \langle \Lambda_{n,r_2} \rangle, \]
\[ \langle \Lambda_{n,r_2} \rangle = -\left( i\Delta_2 + i\Delta_{r_2} + \frac{\gamma}{2} \right) \langle \Lambda_{n,r_2} \rangle - \sqrt{\gamma} \sqrt{N-n} \langle a \rangle - i\Omega_2 \langle \Lambda_{n,e_2} \rangle, \]
\[ \langle \Lambda_{N-n,e_2} \rangle = -\left( i\Delta_2 + \frac{\gamma}{2} \right) \langle \Lambda_{N-n,e_2} \rangle - \sqrt{\gamma} \sqrt{N-n} \langle a \rangle - i\Omega_2 \langle \Lambda_{N-n,r_2} \rangle, \]
\[ \langle \Lambda_{N-n,r_2} \rangle = -\left( i\Delta_2 + \frac{\gamma}{2} \right) \langle \Lambda_{N-n,r_2} \rangle - i\Omega_2 \langle \Lambda_{N-n,e_2} \rangle. \]

The steady-state solution of the cavity mode is given by
\[ \langle a \rangle = \frac{-i\epsilon}{i\Delta_2 + \kappa + \frac{\tau^n}{\Delta_2 + \frac{\gamma}{2} + \frac{\Omega_2^2}{\Delta_2 + \frac{\gamma}{2} + \frac{\Omega_2^2}{\Delta_2 + \frac{\gamma}{2}}}} + \frac{\tau^n (N-n)}{(\Delta_2 + \frac{\gamma}{2} + \frac{\Omega_2^2}{\Delta_2 + \frac{\gamma}{2}})^2}. \]

Data Availability
All data generated or analysed during this study are included in this published article (and its Supplementary Information files).

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Additional Information
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