BIC-solitons in one-dimensional photonic crystal slab

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Abstract. Dynamics of light in the periodically grated waveguide with defocusing Kerr nonlinearity was studied. Dynamically stable bound states in the continuum (BIC) were found in the considered system. Several types of nonlinear localized waves including solitons formed on BIC background were discovered. Stability and dynamics analyzes of the solitons were provided. A protocol of possible experimental observation of dark solitons was suggested.

1. Introduction
Phenomenon of bound states in the continuum (BICs) were discovered in the first half of the twentieth century and recently started to attract much attention again, see [1], [2]. BICs were first predicted by von Neumann in quantum mechanics [3] but later BICs have been identified in many different systems, for example in elastic waves in solid [4], acoustic waves in air [5], water waves [6] and in photonic systems [7]. BIC is a special class of localized solutions of wave equations, the energy of such states are embedded into the spectrum of the delocalized states and therefore they are often referenced as bound states in the continuum. The existence of the BIC is based on a certain symmetry and in many case BIC can be considered as cancellation of the radiative losses due to the destructive interference. If the internal losses equal to zero then the quality factor of a BIC state becomes infinite.

An interesting and practically important example of the nonlinear optical system with BIC is considered in [8]. The system is a periodically grated one-dimensional optical waveguide with focusing Kerr nonlinearity. The electromagnetic field in the waveguide was considered by perturbation method based on the description of the field as a combination of counter propagating waves which is analogous to nearly free electrons approximation in solid state physics. In this approximation the effect of Kerr nonlinearity leads to the effective shift of the propagation constant. In [8] it was shown that such system can provide BIC. However, these uniform states are modulation unstable, although they can be stabilized in systems of finite size. Let us remark here that in the context of the discussed problem by BIC modes we refer the modes localized in the directions transverse to the fiber. In other words the BIC states are the states that do not leak out of the fiber but they are not necessarily localized along the fiber. In particular, the states considered in [8] are spatially uniform along the fiber length.

In the present work we studied the same system as in [8] but with defocusing nonlinearity. Change of nonlinearity sign alters the properties of the homogeneous states drastically and it turned out that BICs can be stable in this case. The aim of our work was to find out if there any nonlinear localized (along the fiber length) solutions based on states with BIC mode.

In our research we have found several types of spatially localized nonlinear waves formed on
BIC-state background, it’s stability and interaction properties. We also suggested the protocol of observation solitons in experiment.

2. Mathematical model

We considered the structure from [8], it is a one-dimensional slab waveguide with grating that changes effective refraction index of waveguide periodically. Schematically considered system is shown in fig.1(a) The medium of the waveguide is crystal with Kerr-type nonlinearity. In two-mode approximation the normally incident pumping excites two coupled modes that propagate in opposite directions. The system of equations describing propagation of this two modes is following:

\[
\begin{align*}
\frac{\partial E_+}{\partial t} + \frac{\partial E_+}{\partial x} - i\delta E_+ + i\alpha(|E_+|^2 + 2|E_-|^2)E_+ + \gamma E_+ + \gamma' E_- - i\phi E_- &= P, \\
\frac{\partial E_-}{\partial t} - \frac{\partial E_-}{\partial x} - i\delta E_- - i\alpha(|E_-|^2 + 2|E_+|^2)E_- + \gamma E_- + \gamma' E_+ - i\phi E_+ &= P.
\end{align*}
\]

where \( E_+ \) and \( E_- \) are amplitudes of the two modes, propagating forward and backward in the grated waveguide respectively, \( \delta \) is the pump detuning, \( q \) is the wave vector of the pump, \( \alpha \) is the coefficient of Kerr-type nonlinearity (\( \alpha = -1 \), so nonlinearity is defocusing), \( \gamma' \) is full losses of both modes, \( \epsilon = \gamma - \gamma' \) is the material losses and \( \phi \) is mode coupling coefficient.

To study BICs it is convenient to introduce symmetric \( (E_s = E_+ + E_-) \) and antisymmetric modes \( (E_a = E_+ - E_-) \). System of equations for such modes is following:

\[
\begin{align*}
\left( \frac{\partial}{\partial t} - i\delta - 3iK + 2\gamma' - i + \epsilon \right) E_s + \left( \frac{\partial}{\partial x} + iq + iM \right) E_a &= 2P, \\
\left( \frac{\partial}{\partial x} + iq + iM \right) E_s + \left( \frac{\partial}{\partial t} - i\delta - 3iK + \epsilon \right) E_a &= 0,
\end{align*}
\]

where \( M = \frac{1}{4}\alpha(|E_a|^2 + |E_s|^2) \) and \( K = \frac{1}{4}\alpha(|E_s|^2 + |E_a|^2) \).
It should be noted, that antisymmetric mode cannot be excited by the external light and does not have any radiation losses. It is BIC (dark) modes and its excitation is possible only because of the nonlinear coupling with symmetric (bright) modes.

3. Homogeneous solutions

All stationary homogeneous states were studied numerically: to find the solutions we used Newton’s method and by pseudo-arc length continuation method the dependency of such solutions intensity on pump intensity $P$ was obtained.

In figs. 1(b) the blue line shows the bifurcation curve of the symmetrical homogeneous solutions and the green line shows mixed states. Mixing of states can be seen from figs 1(c-f): states shown by green line in fig.1(b) are characterized by non-zero intensities of both bright (blue solid line in 1(c-f)) and dark modes (red dashed line in 1(c-f)). States with BIC-component (mixed states) are invariant to field replacement $E_+ \leftrightarrow E_-$. In the case of the absence of material losses ($\epsilon = 0$) the BIC state can exist even with zero pump intensity and such state is purely dark. Because of the inequality of the amplitudes of the counter propagating waves ($E_+ \neq E_-$) the energy flow is not zero for the mixed states.

The dynamically unstable states are shown in fig.1(b) by dashed line. It can be seen, that the mixed modes appear at the point where the symmetric state gets destabilized. It should be noted, that the stability of degenerate states are the same.

4. Spatially localized solutions

Figure 2. (a) Bifurcation curve of peak intensity of bright soliton (black line) with respect to bifurcation curve of homogeneous states (blue line), $\delta = 1.05$; (b) area of existing of dark solitons, it’s backgrounds shown by green bold line and intensity minimum by red bold line; (c) bright soliton, $P = 0.779$: upper panel shows squares of amplitudes of symmetric and asymmetric modes, lower panel shows intensity of soliton; (d) dark soliton, $P = 0.3$.

Now let us discuss the nonlinear localized states that can nestle on the spatially uniform backgrounds considered above.

In the right turning point of bifurcation curve of homogeneous states saddle-center bifurcation takes place, and it was found that bright soliton-solution bifurcates from this point. The bifurcation diagram showing peak intensity $\frac{1}{2}(|E_+(x)|^2 + |E_-(x)|^2)$ of such soliton as a function of the pump (black line) with respect to homogeneous states (blue line) is shown in fig.2(a). Dashed line is showing dynamically unstable solitons. Soliton shown in fig.2(c).

Second type of solitons, the dark ones, were found on the upper mixed state branch. This localized solutions connect two different degenerate homogeneous states, fig.2(d). It’s bifurcation
diagram (red bold line) with respect to homogeneous states (blue line) is shown in fig.2(b). This diagram shows the dependence of soliton intensity dip on pumping rate. Backgrounds of soliton shown by green bold line. Solid line shows dynamically stable solitons. It should be noted that left and right states of shown in fig.2(d) soliton are characterized by $|E_+| > |E_-|$ and $|E_-| > |E_+|$, respectively. So the energy flows on the left and on the right from the soliton.

5. Suggested protocol for experiment
From the physical point of view it is important to find out if the solitary state can be excited in finite systems from realistic initial conditions. In this section we suggest and discuss a finite system with varying losses and the mode coupling strength where the BIC solitons can be observed.

Profiles of spatially modified coupling coefficient, losses and pumping are shown in fig.3(a). Physically, larger $\phi$ at the edges of the system corresponds to deeper grating of the waveguide. We performed direct numerical simulations of the (1) with the initial conditions in the form of weak noise. As it can be seen from fig.3(b) a lot of solitons are forming at time $t < 1000$ and after numerous collisions a group of solitons is formed. Distribution of solitons at the time moment $t = 5000$ is shown in fig.3(c).

![Figure 3](image-url)

Figure 3. (a) Profiles of spatially modified coupling coefficient (solid blue line), pumping (dashed blue line) and losses (red line); (b) time-space diagram of forming of dark solitons with small random initial conditions; (c) several formed dark solitons at moment $t = 5000$

6. Conclusions
We have discovered and studied localized nonlinear waves forming on states with BIC-component in one-dimensional nonlinear grated waveguide. We found different types of solitons, bright ones and dark ones. We have shown that the solitary waves can be dynamically stable. Also, we have suggested a protocol allowing to observe the solitons in real experiment.

References
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