Decomposition of waves in time series of data related to Covid-19

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Here it is proposed a decomposition in components of the “waves” which appear in the time series of data related to Covid-19 pandemic. The decomposition is based on functions of $\kappa$-statistics; in particular the $\kappa$-Weibull is used. Fitted data are those ranging from August 2020 to April 2021 in the United Kingdom. The second of the two main peaks observed in the time series of daily infection was driven by the Alpha variant of Sars-Cov-2.

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It seems that no formal definition exists for the "wave" of a pandemic, however the term is associated to a rising number of Covid-19 cases, which is characterized by a specific peak and then by a decline. In Ref.1 it is told that “Public health scientists first began using this term [wave] to describe different peaks and valleys of infections during influenza outbreaks in the late 1800s and the 1918-1929 Spanish flu.” The Reference is also stressing that each “wave” has a different feature and can impact different populations, even within the same country. From data such as those given by www.worldometers.info/, we can see that the trend of Covid-19 infection is generally given, from the second wave, by the composition of two or more peaks. For instance, in Italy we had a first wave which was characterized by an isolated peak, but from October 2020, the time series of data related to pandemic were characterized by a composition of some peaks (more than two peaks). In this framework, we can try to analyse data to distinguish peaks in waves - or wavelets in wave train - in order to have an instrument able of determining the onset of a specific component. In this manner, having the onset-time it is possible to identify what caused the surge of further infections. Let us note that, sometimes, the term “deconvolution” is applied to a process of decomposing peaks that overlap with each other.

In the following, the function used for analysis is that proposed in [2], in the framework of $\kappa$-statistics. In particular the $\kappa$-Weibull function will be used. Moreover, we will consider data from time series with a 7-day moving average. Then, as in [2], the statistics is described by:

\[
f_\kappa(t) = \frac{\alpha \beta t^{\alpha-1}}{\sqrt{1+\kappa^2 \beta^2 t^{2\alpha}}} \exp_\kappa (-\beta t^{\alpha}) \quad (1)
\]

where the $\kappa$-exponential is defined in the following manner:

\[
\exp_\kappa(u) = \left(\sqrt{1+\kappa^2 u^2} + \kappa u\right)^{1/\kappa} \quad (2)
\]
Eq. 1 is describing the $\kappa$-Weibull. Parameters $\alpha, \beta$ are the shape and scale indexes of Weibull distribution, whereas $\kappa$ is the index of $\kappa$-distribution, that is the distribution introduced by G. Kaniadakis in [3,4]. In [2], where the first wave of Covid-19 had been analysed for China, Italy, Germany, Spain and United Kingdom, we have seen that (1) is properly fitting the data of time-series. Now, let us consider the case of a time-series which is characterized by the composition of two peaks. Data (courtesy www.worldometers.info) are from the surge of the second wave in the United Kingdom and are shown in the Figure 1. The daily number of cases is divided by the total number of cases, observed in the considered time period (in days).

In (1), we can add also the threshold parameter $T$, which is a feature of Weibull functions. Therefore, we have:

$$f_k(t|\alpha, \beta, T) = \frac{\alpha \beta (t-T)^{\alpha-1}}{\sqrt{1+\kappa^2 \beta^2 (t-T)^2 \alpha}} \exp_k (-\beta (t-T)^{\alpha})$$ \hspace{1cm} (3)

If we consider the case of data given in the Figure 1, let us try to use two functions (3), to fit the time-series, in the following form:

$$f = f_1 + f_2 = \xi f_{\kappa_1}(t|\alpha_1, \beta_1, T_1) + (1-\xi) f_{\kappa_2}(t|\alpha_2, \beta_2, T_2)$$ \hspace{1cm} (4)

Parameter $\xi$, the mixing parameter, ranging from zero to 1, is used to generalize the addition of peaks, as proposed for the Weibull distribution [5]. It is also a rough manner to consider the fact that the set of population, involved by pandemic, changed for sure during the considered time period.

![Figure 1](image.png)

**Figure 1** – Number of daily cases (victims) divided by the total number of cases.

Parameters are $\kappa_1=0.52$, $\kappa_2=0.90$, $\alpha_1=3.50$, $\alpha_2=3.75$, $\beta_1=1.0\times10^{-7}$, $\beta_2=2.6\times10^{-7}$, $T_1=0$, $T_2=94$, $\xi=0.4$. 
The fitting of data given in the Figure 1 shows the threshold time for the second peak at day 94, which is corresponding to the first week of December 2020. In the Figure 2, the same approach has been applied to the daily new infections. The data baseline has been shifted of 1180 cases.

![Figure 2](image)

Figure 2 – Number of daily cases (infections) divided by the total number of cases, observed in the considered time period. Parameters are $\kappa_1 = 0.72$, $\kappa_2 = 1.25$, $\alpha_1 = 3.62$, $\alpha_2 = 3.85$, $\beta_1 = 1.0 \times 10^{-7}$, $\beta_2 = 2.6 \times 10^{-7}$, $T_1 = 0$, $T_2 = 94$, $\xi = 0.4$. The first day was August 20, 2020. The day 94 was November 21, 2020.

Time $T_2$ was November 21, 2020. This is the onset of the second peak in the Figure 2. The origin is in the spread of Alpha variant of Sars-CoV-2.

From the web site Our World in Data, using the following link https://ourworldindata.org/explorers/coronavirus-data-explorer?zoomToSelection=true&time=2020-12-07&facet=none&pikserSort=asc&pickerMetric=location&hideControls=true&Metric=Variants&Interval=New+per+day&Relative+to+Population=true&Align+outbreaks=false&country=GBR, we can see that the percentage of Alpha variant of Sars-CoV-2 was of 13% on December 7, 2020. Using the link https://ourworldindata.org/explorers/coronavirus-data-explorer?zoomToSelection=true&time=2020-11-23&facet=none&pikserSort=asc&pickerMetric=location&hideControls=true&Metric=Variants&Interval=New+per+day&Relative+to+Population=true&Align+outbreaks=false&country=GBR, we can evidence that on November 23, 2020, the percentage was of 7.8% (see also the Figure 3).

Using these data, we can argue that the second largest peak of infections, which started on November 21, 2020, according to the fit in Figure 2, was due to the spread of Alpha variant. This is the main feature of the second component of the wave train.
Figure 3 – Variants in the United Kingdom according to Our World in Data. Many, many thanks to this site and people involved in it.
Here we have propose the use of κ-Weibull to decompose the peaks in the time series linked to Covid-19 pandemic. The work is preliminary. The use of more than two functions is required for the analysis of the pandemic in other countries. Also the mixing parameter could be refined, in the framework of κ-statistics.

References
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