The calculation of total reaction cross sections induced by intermediate energy $\alpha$-particles with BUU Model

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(Dated: March 30, 2022)

The Boltzmann-Uehling-Uhlenbeck (BUU) Model, which includes the Fermi motion, the mean field, individual nucleon-nucleon (N-N) interactions and the Pauli blocking effect etc., is used to calculate the total reaction cross section $\sigma_R$ induced by $\alpha$-particles on different targets in the incident energy range from 17.4 to 48.1 MeV/u. The calculation result can reproduce the experimental data. The problem of $\sigma_R$ at intermediate energies has been solved now partly by few body Glauber model. For light-ions like p, d, $^3$He, $^{12}$C, $^{16}$O, $^{28}$Si, $^{40}$Ca, $^{58,60}$Ni, $^{112,116,120,124}$Sn, and $^{208}$Pb targets have been measured at energies around several tens MeV/nucleon. It provided an exciting chance to understand the mechanism of nuclear reaction and give a more reliable test of different models. The measurements were performed with a well-collimated and momentum analyzed beam from the Gustaf Werner cyclotron at The Svedberg Laboratory. The beam energy spread was approximated 100 keV (FWHM), and the intensity was typically $2 \times 10^6$ particles per second. A detailed description of the apparatus and experimental technique is given in Refs. [21, 22].

The results induced by light-ions are compared with predictions from microscopic Glauber multile-scattering theory which is based on the individual nucleon-nucleon (N-N) collisions in the overlap volume of the colliding nuclei. [15, 20] This Glauber model calculation is a useful tool to study $\sigma_R$. It considers the Coulomb correction, uses Yukawa interaction with finite range force and distinguishes neutron and proton inside nuclei. Comparisons of $\sigma_R$ at relativistic energy with that at intermediate energies for projectiles heavier than $\alpha$-particle have been done by Ozawa et al. [12, 13] The result calculated by the Glauber model is always underestimated $\sigma_R$ at intermediate energies. This problem has been solved now partly by few body Glauber model. For light-ions like p, d, $^3$He, $\alpha$ etc., Glauber model can fit experimental results, but the fit quality do not so well. Some nuclear transport theories such as the Boltzmann-Uehling-Uhlenbeck (BUU) model and quantum molecular dynamics (QMD) have been applied into the calculation of $\sigma_R$ to resolve this problem also. [2, 4, 8, 9] These models incorporate the Fermi motion, the mean field, individual nucleon-nucleon (N-N) interactions and the Pauli blocking effect etc. in calculation. They should be more suitable for total reaction cross section calculation in intermediate incident energy. Test particles method and the grid method originated from the fluid mechanics had been introduced to revolve the BUU equation by C.Y. Wong et al. More details can be found in Ref. [30]. For medium projectiles, BUU model have been applied to calculate the total cross section successfully. [3] So it is interesting whether BUU calculation can be used in the calculation of total reaction cross section induced by light ions like $\alpha$-particle.

In this letter we used the BUU model to calculate the total reaction cross sections, $\sigma_R$, measured for 17.4 to 48.1 MeV $\alpha$-particles on targets from $^9$Be to $^{208}$Pb. [21, 22] Within the framework of BUU model and according to Poisson Statistics, the average $N-N$ collision number can be obtained as a function of the impact parameter $b$ by assuming a reasonable parameterization of $\sigma_{NN}$. The probability of n times $N-N$ collisions $T_n(b)$ the course of nucleus-nucleus reaction can be easily obtained in BUU calculation. The sum of $T_n(b)$ over $n(n \geq 1)$ represents the total probability of $N-N$ collisions and is related closely to the absorption probability of nuclear reaction. Therefore, the total reaction cross section is given by

$$\sigma_R = 2\pi \int \left[ \sum_{n=1}^{\infty} T_n(b) \right] b db = 2\pi \int \left[ 1 - \exp(N) \right] b db$$

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as the strong absorption model, the total reaction cross section \( \sigma_R \) can be represented in terms of the interaction radius \( R \) and nucleus-nucleus interaction barrier \( B \), as

\[
\sigma_R = 10\pi R^2(1 - B/E_{C.M.})(mb)
\]  

where \( R \) in fm and \( E_{C.M.} \) in MeV. Different parameterized formulas will have different parameterized forms for \( R \) and \( B \). Shen et al. propose a unified parameterized formula for \( \sigma_R \) using:

\[
B = \frac{1.44Z_iZ_p}{r} - b \frac{R_tR_p}{R_t + R_p}(MeV)
\]

\[
R = r_0 \left[ A_t^{1/3} + A_p^{1/3} + 1.85 \frac{A_t^{1/3} A_p^{1/3}}{A_t^{1/3} + A_p^{1/3}} - C(E) \right] + \frac{\alpha (A_t/2Z_t)Z_p}{A_t A_p} + \beta E_{C.M.}^{-1/3} \frac{A_t^{1/3} A_p^{1/3}}{A_t^{1/3} + A_p^{1/3}}
\]  

where \( r = R_t + R_p + 3.2fm, b = 1MeV \cdot fm^{-1}, R_t = 1.12A_t^{1/3} - 0.94A_t^{1/3}, (i = t, p). \alpha = 1 fm, \beta = 0.176MeV^{1/3} fm \) and the experience value around 1.0fm was used normally for \( r_0 \). The nucleon radius parameter \( r_0 \) was also extracted from \( \sigma_R \) calculated by BUU for 48.1MeV incident energy. As seen, this radius decreases with the mass number for light nuclei, but it stays essentially constant at around 1.0 fm for heavy nuclei, as shown in Fig. 2. The open stars denoted the calculations by using the BUU model. The and the circles denoted the calculations by using the Eq.(2-3-4) for 48.1MeV incident energy also. The deduced \( r_0 \) via BUU can fit the

FIG. 1: Total reaction cross section induced by \( \alpha \)-particle for various targets as function of incident energy. The experimental data are indicated by open circles. Solid curves indicated the calculations by using the BUU model.

where \( N \) is the average \( N-N \) collision number. More details can be found in Ref.\[7\]. The BUU equation can be used to extract nuclear equation of state (EOS) and in-medium \( N-N \) cross section \( \sigma_{NN}^{in-medium} \), etc., from the analysis of the experimental excitation function for the total reaction cross section, \( \sigma_R \), if we know the nucleon density distribution for the projectile and target.\[5\]\[8\]\[7\]\[8\] The projectile and target nucleus are stable nucleus here in this paper, so the well-known Cugnon’s parameterized EOS, and square-type density distribution are used in the BUU calculation.\[5\]\[7\]\[8\] Then the Nucleus-Nucleus interaction radius parameter \( r_0 \) can be deduced by fitting experimental data at one incident energy, which is 48.1 MeV in this paper. As shown in Fig. 1, the experimental data are indicated by closed circles. Solid curves indicated the calculated results by using the

BUU model. As seen, the energy dependence changes with the mass number of targets. These experimental data induced by \( \alpha \)-particles are in very well agreement with our calculated results for all targets at all experimental incident energy, where the experimental data at incident energy 48.1MeV were used in fitting procedure to get \( r_0 \). For light targets the total reaction cross section decreases with increasing energy, in agreement with what is expected from the energy dependence of nucleon-nucleon scattering cross section. For medium and heavy targets, however, the total reaction cross section remains more or less independent of energy and geometrical effects seem to dominate. For medium and heavy-targets at low energies, maybe, Coulomb repulsion suppresses the increase of the total reaction cross section with decreasing the incident energy. The experimental results produced by heavier projectiles indicates also that for light target the total reaction cross sections are much more sensitive to the nucleon-nucleon interaction. The total reaction cross sections induced by deuterons and protons shown same tendency either for the experimental results or for BUU calculation.

The dependence of the total reaction cross section on the atomic mass of target and projectile has been parameterized in many forms. According to the strong absorption model, the total reaction cross section \( \sigma_R \) can be represented in terms of the interaction radius \( R \) and nucleus-nucleus interaction barrier \( B \), as
results deduced from the parametrized formula of Shen. It seems that the deduce $r_0$ shows similar trend with the parametrized formula calculations. For $^9$Be, it has two-$\alpha$ plus one neutron structure, so the radium is larger. Our calculation shows this enhance of radium. S.Q. Zhang and J. Meng propose a unified parametrized formula for the nuclear charge radii using Eq(5-6)[32]. Obvious Eq.6 is exactly the isospin-dependent $A^{1/3}$ formula. It is interesting to study the isospin-dependent of the target and the nuclear charge radii in the future.

$$R_c = r_Z Z^{1/3} = r_Z \left( \frac{A}{2} - \frac{N - Z}{2} \right)$$

$$= \frac{r_Z}{2^{1/3}} (1 - \frac{N - Z}{A})^{1/3}$$

$$\approx \frac{r_Z}{2^{1/3}} (1 - \frac{1}{3} \frac{N - Z}{A})$$

The total reaction cross section calculated by BUU is sensitive with the test particles number. An important peculiar characteristic of the test particles simulation is to allow many test particles to represent one nucleon so as to gain a clear insight into the dynamics of nuclear systems in heavy-ion dynamics. In order to assess the test particles simulation as a steady and useful concept, we wish to carry out some numerical calculations and compare them. We examined the dynamics at 48.1MeV. The calculated results, here it is the total reaction cross section, should reach stable saturation with increasing the test particles number. In our calculations, the calculation with different test particles was carried out and the minimum test particle number is obtained. As shown in Fig. 3, we use different numbers test particles to de-
scribe the dynamics. For $^9$Be, in order to get the steady total reaction cross section the test particles are 1000 at least. And for $^{12}$C, 400 test particles are enough. For $^{208}$Pb, 200 test particles are enough even. It shows that we can get steady $\sigma_{total}$ for $\alpha$-particle as projectile and very light target as $^9$Be when the test particles numbers is large enough.

In summary, we have presented the BUU Model calculation for the total reaction cross sections induced by $\alpha$-particles at intermediate energies on targets from $^9$Be to $^{208}$Pb. The BUU calculations can fit the experimental data very well. The nucleus-nucleus interaction radius parameter $r_0$ have been deduced from the BUU's calculation. It is found that $r_0$ becomes constant with increasing the mass number of target and shows similar trend with the parametrized formula calculations. It shows that the total reaction cross sections induced by light-ion like $\alpha$-particle can be calculated by the BUU Model. Although the projectile has few nucleons, the BUU calculation can be used still for the total reaction cross section calculation if large enough test particle number was used. Because test particle method, here in this paper different test particles were used. The minimum test particle number, which is needed for simulating one nucleon in order to get back enough statistics and fit the experimental total reaction cross section, is a function of the target mass number, it decreases with increasing the target mass number.

This work is supported by the Major State Basic Research Development Program in China under contract No. G200077400 and by the National Natural Science Foundation of China under contract 10125521.

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