A New Method for Geophysical Induced Polarization Inversion Based on Stochastic Medium Model and Sample-Compressed Artificial Neural Network

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Abstract. Induced polarization (IP) is a near-surface geophysical exploration method. Inverting the electrical properties of the underground medium from surface apparent IP parameters is a highly nonlinear problem. To further improve the accuracy, the artificial neural network (ANN) algorithm is applied to the two-dimensional (2D) IP data inversion for the first time. We firstly produced smooth geo-electric models based on the stochastic medium theory, and obtained the corresponding theoretical responses through forward modelling. Then, we compressed the responses and models through image compression technology. Finally, the above compressed responses and models were used as input and output samples to train an optimal network system for inversion. We tested the algorithm with synthetic examples. The results show that ANN can improve the longitudinal resolution of the inversion results and make the inversion results more focused.

Keywords. Induced polarization; inversion; stochastic medium model; artificial neural network.

1. Introduction
Induced polarization (IP) method is an effective geophysical method to characterize the complex resistivity structure of the shallow crust (down to -2 km) [1]. It is a highly nonlinear problem to invert the real electrical properties of underground medium based on surface apparent IP parameters [2]. At present, IP inversion mainly adopts two kinds of methods. The first is the quasilinear inversion methods, including Gauss-Newton, conjugate gradient, OCCAM, least squares, etc. [3]. The second is the global optimization algorithm, including particle swarm optimization, simulated annealing, genetic algorithm, etc. [4]. However, most quasilinear algorithms rely on initial models and smooth constraint conditions. When the initial model is far from the real model or the smooth constraint is too powerful, the iteration results will fall into the local extremum. The global optimization algorithms have a large search range in the solution space, resulting in low iteration speed and high computational cost. Therefore, the global optimization algorithm is not widely used in 2D and 3D massive-scale data inversion.
The development of machine learning provides new ideas for IP inversion [5]. Scholars have long proposed a geophysical inversion framework based on machine learning [6]. Considering that the application of machine learning algorithms in IP exploration is still seldom, this paper realized the 2D inversion of IP data based on the artificial neural network for the first time. We also improved the stability and efficiency of the inversion algorithm by constructing stochastic medium models and compressing the data samples. Finally, the inversion algorithm was tested by synthetic data.

2. Method

2.1. Stochastic Medium Model
In order to improve the generalization ability of ANN, this paper uses smooth stochastic models as training samples. The stochastic medium theory is used to generate the electrical model, and then the apparent resistivity and apparent phase responses are obtained by forward modelling. Finally, the apparent responses are used as the input samples and the corresponding electrical models are used as the output samples to train the neural network.

According to the stochastic medium theory [7], we assume that the 2D geo-electric model consists of a large-scale model and some small-scale stochastic disturbances. The stochastic disturbances coefficient is the inverse Fourier transform of the random spectrum, which can be calculated by using the autocorrelation function and random phase. There are many types of autocorrelation function, including exponential function, Gaussian function and power function. The exponential elliptic function is used in this paper, which is represented by two autocorrelation lengths parameters A and B. When A or B approaches infinity, the medium in the corresponding direction becomes a layered model. In electrical exploration, the observation data and inversion results usually vary continuously. To this end, we further process the stochastic medium model. First, we set the edge of the model to a uniform-half space, and then smooth the entire model to convert the stochastic medium model to a continuous resistivity model, which is then used for forward modelling to get apparent resistivity responses.

2.2. Sample-Compressed Artificial Neural Network
Artificial neural network (ANN) inversion requires more than tens of thousands of training samples, and the dimensions of each input sample or output sample are also between thousands and tens of thousands. The large-scale training samples cause the large size and high computational cost of the neural network [8]. In order to improve training efficiency, we use image compression technology based on discrete cosine transform (DCT) to compress the input and output samples, and then we use the compressed data to train the neural network. Finally, we use the inverse discrete cosine transform (IDCT) to reconstruct the new output of the neural network and obtain the complete data.

For a smooth image, the large DCT coefficients are mostly concentrated in the low-frequency part, and the high-frequency coefficients are approximately 0 [9]. The original image can be reconstructed with high precision by using a small number of low-frequency coefficients. In ANN, we use only the low-frequency coefficients of the input and output data to train the network, which can reduce the dimensions of input and output samples by tens of times, thus improving the training speed.

Figure 1 shows the flow chart of the ANN inversion method for IP data. The input data are apparent resistivity and apparent phase responses of the survey points, and the outputs are inverted resistivity and phase of the 2D medium profile.
3. Simulated Data Test

We firstly used simulated data to test the 2D inversion effect of ANN algorithm. We designed a 2000 m survey line of a total of 100 survey points above a 2D geological profile, and then arranged 13 groups of current electrodes in sequence. The current electrode distances AB were respectively: 300 m, 600 m, 900 m, 1200 m, 1500 m, 1800 m, 2100 m, 2400 m, 2700 m, 3000 m, 3300 m, 3600 m, 3900 m, and the number of observed apparent resistivity or phase data points was 100 * 13. The inverted 2D profile was 2500 m * 1000 m, the number of meshed grids was 120 * 50. We first randomly generated 10,000 sets of 2D smooth stochastic medium models, the range of model resistivity was 10~10000 Ωm, and the range of model phase was 0~100 mrad. Then, the corresponding apparent resistivity and apparent phase responses were calculated by using a 3D IP forward code [10]. In order to improve the stability of the neural network, 1%~10% random noises were added to the apparent resistivity and apparent phase respectively. The training samples were expanded to 110, 000 groups. Then all input samples and output samples were subjected to 2D discrete cosine transform, and only the largest 200 low-frequency coefficients were used to train the network system. Before compression, the dimension of each input sample (including both apparent resistivity and apparent phase) was 2600, and the dimension of output sample was 12000; after compression, the dimensions of the input sample and output sample were both reduced to 400.

To analyse the precision loss of data compression, the root-mean-square (RMS) errors between original data and reconstructed data were counted respectively. For these 10000 smooth models, the average errors of apparent resistivity, apparent phase, model resistivity, and model phase were 0.34%, 0.12%, 0.64% and 0.81% respectively. The maximum errors are 0.8%, 0.5%, 1.5%, 2%. Figure 2 shows the data compression processing of one model. The RMS errors between the original and compressed data shown in the figure are: 0.48%, 0.42%, 0.55%, 0.73%, respectively.

The above comparison shows that sample compression can reduce the data dimension by more than 10 times on the premise of keeping the original data information. Then, we designed an ANN system for inversion. The structure included three layers: input layer, hidden layer and output layer. The number of nodes in the hidden layer was set as two times the sample dimension, i.e., 800 nodes. In addition, cross-validation [11] was used to avoid overfitting [12], and the proportion of training, verification and testing was 70%, 15% and 15% respectively. The optimal weights of the neural network were obtained by iterative search with conjugate gradient method [13], and the training took about 24 hours. Then, 2000 groups of testing models were used to test the trained network, and the RMS of relative errors between the real models and inverted models were counted. The average relative error of resistivity and phase were 7.8% and 11.06% respectively. We randomly selected five models from the testing samples, and compared the real model and inverted results, as shown in Figure 3. The ANN inversion results can effectively reveal the distribution of the electrical resistivity and induced polarization phase of the major underground anomalies.
Figure 2. Data compression based on discrete cosine transform; (a1-d1) original data of apparent resistivity, apparent phase, model resistivity and model phase; (a2-d2) discrete cosine transform coefficients; (a3-d3) reconstructed data by using the low-frequency coefficients in the red box.

Figure 3. Resistivity and phases of five real models and inverted results; (a1)-(a5) model resistivity; (b1)-(b5) inverted resistivity; (c1)-(c5) model phase; (d1)-(d5) inverted phase.
To test the anti-interference ability of the ANN algorithm, we chose one model and added 1%, 2%, 3%, 4%, 5%, 6%, 7%, 8%, 9%, 10% Gauss noise interferences to the observed apparent resistivity in turn. Then we inverted the noisy data. Figure 4 shows the inverted results when the observed data contains various levels of noises. Noise interferences do cause inversion distortion, but the major anomalous bodies are still distinguishable. We calculated the RMS of relative error between the inverted result and the real model, which are in order: 6.39%, 7.01%, 6.1%, 5.9%, 8.2%, 8.1%, 7.2%, 6.45%, 9.23%, 8.46%. The analysis shows that the ANN algorithm has a strong anti-noise ability.

Figure. 4. Comparisons of the model resistivity and the inverted results from apparent responses with various noises; sub-figure (a) and (b) show the apparent resistivity and model resistivity; sub-figure (c), (d) to sub-figure (w), (x) are the apparent resistivity with noises of various value and inverted resistivity.

4. Conclusion
In this paper, the artificial neural network is first introduced into geophysical induced polarization inversion. The training samples are generated by stochastic medium model theory, and the dimensions of input and output samples are reduced by data compression to improve the computing speed of ANN. The simulated data show that compared with the quasi-linear inversion method, the artificial neural network can effectively obtain high-precision inversion results while having a strong anti-interference ability. This paper only trains the smooth models. In the future, we will study the inversion of the sharp boundary model.
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References
[1] Gross L, Soueid Ahmed A and Revil A 2021 Induced polarization of volcanic rocks. 4. Large-scale induced polarization imaging Geophysical Journal International 225 (2) 950-967.
[2] Ramazi, H and Jalali, M 2015 Contribution of geophysical inversion theory and geostatistical simulation to determine geoelectrical anomalies Studia Geophysica et Geodaetica 59 (1) 97-112.
[3] Xu Z and Zhdanov M S 2015 Three-dimensional Cole-Cole model inversion of induced polarization data based on regularized conjugate gradient method IEEE Geoscience and Remote Sensing Letters 12 (6) 1180-1184.
[4] Sharifi F, Arab Amiri A R and Kamkar Rouhani A 2019 Using a combination of genetic algorithm and particle swarm optimization algorithm for GEMTIP modeling of spectral-induced polarization data Journal of Mining and Environment 10 (2) 493-505.
[5] Thibaut R, Kremer T, Royen A, Ngun B K, Nguyen F and Hermans T 2021 A new workflow to incorporate prior information in minimum gradient support (MGS) inversion of electrical resistivity and induced polarization data Journal of Applied Geophysics 187 104286.
[6] Kim Y and Nakata N 2018 Geophysical inversion versus machine learning in inverse problems The Leading Edge 37 (12) 894-901.
[7] Jiang Z, Zeng Z, Li J, Liu F and Li W 2013 Simulation and analysis of GPR signal based on stochastic media model with an ellipsoidal autocorrelation function Journal of Applied Geophysics 99 91-97.
[8] Wójcik P I and Kurdziel M 2019 Training neural networks on high-dimensional data using random projection Pattern Analysis and Applications 22 (3) 1221-1231.
[9] Zhang J, Liao Y, Zhu X, Wang H and Ding J 2020 A deep learning approach in the discrete cosine transform domain to median filtering forensics IEEE Signal Processing Letters 27 276-280.
[10] Liu W, Lin P, Lü Q, Chen R, Cai H and Li J 2017 Time domain and frequency domain induced polarization modeling for three-dimensional anisotropic medium Journal of Environmental and Engineering Geophysics 22 (4) 435-439.
[11] Jena R and Pradhan B 2020 Integrated ANN-cross-validation and AHP-TOPSIS model to improve earthquake risk assessment International Journal of Disaster Risk Reduction 50 101723.
[12] Jabbar H and Khan R Z 2015 Methods to avoid over-fitting and under-fitting in supervised machine learning (comparative study) Computer Science, Communication and Instrumentation Devices 163-172.
[13] Khadse C B, Chaudhari M A and Borghate V B 2016 Electromagnetic compatibility estimator using scaled conjugate gradient backpropagation based artificial neural network IEEE Transactions on Industrial Informatics 13 (3) 1036-1045.