Probabilistic analysis of maneuvering nanosatellites with electrothermal propulsion system

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Abstract. A method is proposed for assessing the results of correction maneuvers for a nanosatellite (NS) with an electrothermal propulsion system (ETPS). Common causes of maneuvering errors associated with technological errors in creation of the propulsion system and the nanosatellite as a whole are revealed by an example of the SamSat-M NS being under development. The analysis of the NS maneuvering process was carried out in a probabilistic setting. A factor analysis is carried out and the contribution of the design parameters of the NS and ETPS to the spread of the projections of the velocity increase vector of the NS motion and the arising angular motion is determined.

1. Introduction
At present, nanoclass spacecraft have completely conquered the market of scientific and educational satellites and are actively exploring the field of practical and scientific application, which was previously occupied by "large" spacecraft. One of the most pressing problems in this case is the creation of miniature propulsion systems that would allow solving a wide range of tasks: elimination of errors in launching the NS into orbit, maintaining orbital parameters, interorbital maneuvering, building NS orbital groupings, inspecting other orbital objects, observing near-earth space, withdrawal of NS into the disposal orbit, etc.

One of the most interesting from a practical point of view are electrothermal propulsion systems since they have relatively high specific impulses and can operate with an inert working fluid (WF). In systems of this type, a preliminary gasification of the WF occurs in the evaporator, and then its final overheating due to onboard energy sources and ejection through a nozzle into the surrounding space takes place. In this regard, many research teams engaged in the development of nanoclass platforms offer various options for ETPS [1-3].

In 2005, Surrey Satellite Technology Ltd (Britain) proposed a xenon ETPS, which allows obtaining a specific impulse of 48 s and a thrust force of 18 mN. In 2008, the University of Southern California proposed an ETPS, which allows obtaining a specific impulse of about 80 s. Water was used as WF. The development of ETPS with freon as WF is actively being carried out. These include ETPS with specific impulses reaching 150 s, proposed in 2015-2017 by the American companies CU Aerospace, VACCO Industries and Busek Co Inc. There are also Russian developments. So, the Omsk State Technical University proposed an ETPS with a realized characteristic speed of more than 60 m / s and ammonia as a WF [4]. Many original variations of the ETPS are proposed, for example, when it is proposed to use solar energy to overheat the WF [5].
By modern trends in the design and assembly of NS at the interuniversity space research department of Samara University, a maneuvering unit for NS of the CubeSat - SamSat-M standard was developed [6]. In the developed maneuvering unit, an ETPS with liquid WF is used, which includes a tank with a displacement RT supply system, a distributor that dispenses and feeds WF into the heater, which evaporates and superheats RT steam to a selected temperature and directs it to the nozzle. A mixture of distilled water and ethyl alcohol was selected as the WF, which ensures environmental safety, which is a requirement for ground testing and the delivery of NS to the ISS for subsequent launch. The characteristics of the ETPS were experimentally determined: the specific impulse is 120 s, the thrust is 0.1 N, the value of one correction impulse for a nanosatellite weighing 4.5 kg is 0.1 m/s, the characteristic velocity margin is not less than 50 m/s. Its appearance is shown in Figure 1.

Figure 1. External view of the maneuvering unit.

To analyze the space missions of the maneuvering NS, it is necessary to form a complex of models. These models should take into account technological manufacturing errors and be probabilistic, as well as to take into account a set of factors that significantly affect the movement of the nanosatellite. To assess the effect of the operation of the maneuvering unit on the nature of the NS movement (both the movement of the center of mass and movement relative to the center of mass), software that makes it possible to carry out numerical simulation of the dynamic system under study was developed. This software allows in a probabilistic setting to evaluate the projection of the speed increment and the torques arising during the operation of the propulsion system.

2. Research methodology
The research consists of several stages. At the first stage, a list of design parameters is determined, which are random factors in the numerical simulation. The parameters of the laws of distribution of random factors are set taking into account the capabilities of the technological process and the features of the functioning of the electrothermal propulsion system. A list of monitored parameters is formed, which includes values that allow evaluating the effectiveness of the corrective maneuver. The controlled parameters include the vector of the increase in speed and momentum of the forces.

In the second stage, a sample of realizations is formed using the Monte Carlo method. For this, a series of simulations of the motion of the NS is carried out on the interval of the ETPS operation. The obtained sample is used to determine the parameters of the distribution laws of the controlled parameters.

Based on the results of statistical modelling, linear regression equations have the form:

\[ y = Gx + b. \]

where \( y \) is the vector of controlled parameters; \( G \) – matrix of coefficients of regression equations; \( x \) – vector of design parameters; \( b \) is the vector of coefficients for dummy variables.
Regression models allow to carry out factor analysis, on the basis of which more significant factors are identified and their contribution to the scatter of controlled parameters is estimated. The percentage of influence of each of the parameters can be calculated by the formula:

\[ P_i(x_j) = 100 \frac{|G_{ij}|}{\sum_{j=1}^{m}|G_{ij}|}, \quad (2) \]

where \( P_i(x_j) \) is the degree of influence of the factor \( x_j \), expressed as a percentage; \( G_{ij} \) – coefficient of the regression equation related to the factor \( x_j \) for the \( j \)-th regression equation; \( i = 1 \ldots n \), where \( n \) is the dimension of the vector of controlled parameters; \( j = 1 \ldots m \), where \( m \) is the number of factors in the regression model.

3. Complex of mathematical models for estimating the efficiency of correction

Rotational motion relative to the center of mass of the spacecraft occurs due to the misalignment of the line of application of the thrust force and the longitudinal axis of the NS passing through the center of mass, as well as under the influence of gravitational and aerodynamic moments.

To describe the angular position of the NS, a reference coordinate system has been introduced, relative to which the angular deviations, angular velocities and accelerations of the coordinate system rigidly connected with the spacecraft are measured.

The \( OXYZ \) orbital coordinate system (OCS) with the origin at the center of mass of the nanosatellite \( O \) was chosen as the reference system. The OCS axes are directed as follows: axis \( OY \) - along the local vertical from the center of the Earth to the center of mass of the apparatus \( O \); \( OX \) axis - perpendicular to the \( OY \) axis in the orbital plane and directed towards the flight; \( OZ \) axis - along the binormal to the orbit to form the right coordinate system [7]. The body axis system (BAS) is selected so that its axes coincide with the main central axes of inertia of the spacecraft. BAS \( OXYZl \) is located at the center of mass of the nanosatellite, the \( OXl \) axis is directed forward along the longitudinal axis of the nanosatellite, \( OYl \) and is directed upward along the normal, the third axis \( OZl \) complements the coordinate system to the right.

The orientation of the spacecraft in space, that is, the position of the BAS \( OXYZl \) relative to the orbital coordinate system \( OXYZ \), is determined by three independent angles: roll angle \( \gamma \), yaw angle \( \psi \) and pitch angle \( \theta \).

The motion of the center of mass of the nanosatellite is considered in the Earth-centered inertial (ECI) coordinate frame \( O_0X_0Y_0Z_0 \), the origin of which coincides with the center of the Earth. The main plane \( O_0X_0Y_0 \) coincides with the equatorial plane, the \( O_0X_0 \) axis is directed to the vernal equinox, the \( O_0Z_0 \) axis is along the Earth's rotation axis, and the \( O_0Y_0 \) axis complements the coordinate system to the right [8].

The transition matrix from ECI to OCS is:

\[
B = \begin{pmatrix}
C_2Z_0' - C_3Y_0' & C_3X_0' - C_1Z_0' & C_1Y_0' - C_2Z_0' \\
Cr & Cr & Cr \\
Cr & Cr & Cr \\
Cr & Cr & Cr \\
C_1 & C_2 & C_3 \\
C & C & C \\
X_0' & Y_0' & Z_0' \\
r & r & r
\end{pmatrix},
\]

where \( C = (C_1; C_2; C_3) \) is the vector constant of the area integral, \( X_0', Y_0', Z_0' \) – coordinates of the nanosatellite center of mass in the ECI, \( r = \sqrt{X_0'^2 + Y_0'^2 + Z_0'^2} \) – radius vector of NS.

The vector constant of the area integral is calculated using the following formulas:
where $V_x, V_y, V_z$ — projections of the velocity vector of the center of mass in the ECI.

A nanosatellite is considered as a solid body that performs rotational-translational motion under the action of forces applied to it. The center of mass moves along a trajectory that changes in inertial space, and at the same time, the rotational motion of the NS occurs due to the misalignment of the line of application of the traction force and the longitudinal axis of the NS passing through the center of mass, as well as under the influence of gravitational and aerodynamic moments.

The motion model has the form [9]:

$$
\begin{align*}
\dot{\omega} &= I^{-1} \left( -\omega \times l \omega + M_\Sigma \right), \\
\dot{\lambda} &= \frac{1}{2} \lambda \cdot \omega, \\
\dot{r} &= V, \\
V &= -\frac{\mu r}{r^3} + a_E - a_a + \frac{F_T}{m},
\end{align*}
$$

(5)

where $\omega$ is the instantaneous angular velocity; $I$ is the NS inertia tensor; $M_\Sigma$ — vector of moments of forces, including gravitational and aerodynamic moments, as well as the disturbing moment arising from the applying of the thrust impulse; $\lambda$ is the quaternion of the BAS orientation in the OCS; $a_E$ — acceleration vector due to the influence of the Earth shape and the uneven distribution of its mass; $a_a$ — braking in the atmosphere; $F_T$ is the vector of the thrust force in the ECI; $m$ is the mass of the NS, taking into account the development of the WF.

The thrust profile has a section of reaching the mode, a section of a steady-state and a section of a decrease in thrust (Figure 2).

![Figure 2. Traction profile.](image)

The thrust profile is described using the following approximate expressions [10]:

$$
\begin{align*}
P_0 &= P_a \left( 1 - e^{-t/T_1} \right), t < T_1, \\
P_0 &= P_a T_1 \leq t < T_1 + T_0, \\
P_0 &= P_a e^{-t/T_2}, t \geq T_1 + T_0,
\end{align*}
$$

(6)
where \( t \) is the current time from the moment of turning on the propulsion system; \( T_0, T_1, T_2 \) – the duration of the steady-state, the duration of reaching the mode and the duration of the thrust decay, respectively.

Taking into account the deviation of the nozzle from the longitudinal axis in the \( X_0OY_1 \) plane (angle \( \alpha \)) and the \( X_0OZ_1 \) plane (angle \( \beta \)), the projection of the thrust force \( \mathbf{P} \) in the BAS can be written in the following form:

\[
\begin{align*}
P_x &= P_0 \cos(\arctan \sqrt{\tan^2 \alpha + \tan^2 \beta}), \\
P_y &= P_0 \sin \alpha, \\
P_z &= P_0 \sin \beta.
\end{align*}
\]

(7)

The thrust projections in the BAS are recalculated in the ECI and are taken into account in the right-hand sides of the differential equations of motion of the center of mass.

The thrust moment is defined as the vector multiplication of the shoulder by the thrust vector in the associated coordinate system:

\[
\mathbf{M}_T = \mathbf{CM}_{\text{noz}} \times \mathbf{P},
\]

(8)

where \( \mathbf{CM}_{\text{noz}} \) is the vector between the nozzle and the center of mass of the NS.

In modelling, the following approximate expressions for the gravitational moment are used [11]:

\[
\begin{align*}
\mathbf{M}_{gs} &= 3 \alpha_0 \left( I_x - I_y \right) \mathbf{M}_{12} \mathbf{M}_{33}, \\
\mathbf{M}_{gy} &= 3 \alpha_0 \left( I_x - I_z \right) \mathbf{M}_{33} \mathbf{M}_{31}, \\
\mathbf{M}_{gz} &= 3 \alpha_0 \left( I_y - I_z \right) \mathbf{M}_{31} \mathbf{M}_{32},
\end{align*}
\]

(9)

where \( \alpha_0 \) – orbital speed; \( M_{11}, M_{32}, M_{33} \) – elements of the direction cosine matrix.

The expressions for the aerodynamic moment are:

\[
\begin{align*}
\mathbf{M}_{av} &= \mathbf{Q} \left( \mathbf{CM}_x \mathbf{M}_{13} - \mathbf{CM}_y \mathbf{M}_{12} \right), \\
\mathbf{M}_{av} &= \mathbf{Q} \left( \mathbf{CM}_z \mathbf{M}_{11} - \mathbf{CM}_y \mathbf{M}_{13} \right), \\
\mathbf{M}_{av} &= \mathbf{Q} \left( \mathbf{CM}_y \mathbf{M}_{12} - \mathbf{CM}_z \mathbf{M}_{11} \right),
\end{align*}
\]

(10)

where \( \mathbf{Q} \) is the modulus of the force of aerodynamic resistance; \( \mathbf{CM}_x, \mathbf{CM}_y, \mathbf{CM}_z \) – coordinates of the center of pressure relative to the center of mass in the BAS; \( M_{11}, M_{32}, M_{33} \) – elements of the direction cosine matrix.

The direction cosine matrix is expressed in terms of the normalized orientation quaternion \( \Lambda = (q_0, q_1, q_2, q_3) \) as follows:

\[
\mathbf{M} = \begin{pmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\
2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\
2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2
\end{pmatrix}.
\]

(11)

Controlled parameters include vectors of speed increase and momentum of forces. The velocity growth vector is calculated as the difference between the nanosatellite velocity vector in the OCS at time \( T \) after the thrust impulse and the velocity vector before the thrust impulse in the OCS:

\[
\Delta \mathbf{v}^{\text{OCS}} = \mathbf{v}^{\text{OCS}}_T - \mathbf{v}^{\text{OCS}}_0,
\]

(12)

where \( \mathbf{v}^{\text{OCS}}_T \) is the vector of the final speed in the CSC, \( \mathbf{v}^{\text{OCS}}_0 \) is the vector of the initial speed in the OCS.

Since the ECI is used in modelling of the motion of the center of mass, the initial velocity vector (before the impulse is applied) in the orbital coordinate system is determined using the transition matrix \( \mathbf{B} \) by the formula

\[
\mathbf{v}^{\text{OCS}}_0 = \mathbf{B} \mathbf{v}^{\text{ECI}}_0,
\]

(13)
where \( B \) is the transition matrix from ECI to OCS; \( \vec{V}^{ECI}_0 \) – vector of initial speed in ECI.

The transition matrix from ECI to OCS \( B \) is determined from the expression (3).

The speed of the nanosatellite in the OCS after the issuance of the pulse \( \vec{V}^{OCS}_t \) is determined in the same way:

\[
\vec{V}^{ECI}_t = B\vec{V}^{ECI}_0,
\]

where \( \vec{V}^{ECI}_t \) is the velocity vector of the nanosatellite in the ECI after the impulse is issued.

The moment of impulse vector is calculated using the expression:

\[
L = \int_0^{T_t} \mathbf{M}_x dt.
\]

The complex of mathematical models described above is used to simulate the process of carrying out corrective maneuvers for a nanosatellite with an ETPS and to evaluate the correction results to identify a group of factors that have the greatest impact on the error in the formation of a corrective pulse. The problem of modelling the motion correction process is considered in a probabilistic setting.

The following design parameters of the NS and ETPS are considered as random: displacement of the center of mass of the NS relative to the nozzle and relative to the geometric center, mass-dimensional characteristics of the WF storage and supply system, nozzle geometry, misalignment of the NS longitudinal axis and the nozzle, WF heating temperature and the duration of the thrust pulse.

The vector of controlled parameters \( \mathbf{K} = (\Delta V_x, \Delta V_y, \Delta V_z, L_x, L_y, L_z) \) includes the projections of the velocity growth vector and momentum of the moments of forces generated by the thrust and leading to the rotation of the NS relative to the center of mass.

The assessment of the degree of influence of technological errors in the design parameters of the NS and ETDU on the spread of the increase in the speed of movement of the center of mass and the occurrence of movement relative to the center of mass has practical interest.

The methodological basis of the research is the method of statistical tests (Monte Carlo method) followed by the use of regression and factor analysis [12].

4. Results of numerical simulation on the example of SamSat-M nanosatellite

The list of the design parameters of the SamSat-M, reflecting the capabilities of the technological process, is shown in Table 1. When calculating the mass-inertial characteristics of the NS, it is assumed that the NS consists of two parts: the storage and supply system of the WF and the rest of the NS. The center of mass of the system for storing and supplying WF is shifted to the BAS in the process of consumption WF, the position of the center of mass of the rest of the body remains constant throughout the simulation interval.

| Design parameter                                      | Value                                      |
|-------------------------------------------------------|--------------------------------------------|
| Offset of the center of mass of the tank relative to the nozzle, mm | (69...71; -1...1; -1...1)                  |
| Offset of the rest of the nanosatellite relative to the nozzle, mm | (209...211; -1...1; -1...1)                |
| Tank length, mm                                        | 104.4 ... 105.2                            |
| Inner radius of the tank, mm                          | 29.95...30.05                              |
| Outer radius of the tank, mm                          | 30.95...31.05                              |
| Displacement piston disc thickness, mm                | 4.95...5.05                                |
| Compressed spring length of the system for storing and supplying, mm | 19.95...20.05                             |
| Nozzle offset from geometric center, mm               | (-150; -1...1; -1...1)                     |
| Angular deviation of the longitudinal axis of the nozzle from the longitudinal axis of the NS in the plane XOY, deg | -0.5...0.5                                |
Angular deviation of the longitudinal axis of the nozzle from the longitudinal axis of the NS in the plane \(X_OZ_l\), deg
-0.5...0.5

Nozzle throat radius, mm
0.15...0.25

Gas temperature at the nozzle inlet, K
850...950

Duration of traction reaching steady-state, s
1...2

The duration of the descent of thrust from the steady state, s
1...2

As a result of modeling, polygons of the distribution frequencies of the projections of the velocity increase vector were obtained, which are shown in Figure 3. To obtain polygons, a series of statistical tests were carried out with a sample size of 40,000, which approximately corresponds to 0.5% accuracy of obtaining probabilistic estimates. The velocity increase vector was determined using the relations (12) – (14).

**Figure 3.** The frequency polygons of the projections of the velocity growth vector in the OCS.

It can be seen that when the acceleration pulse is applied, the increase in speed along the \(OX\) axis does not exceed 30 cm/s, the mathematical expectation is 13 cm/s, and the standard deviation is 5 cm/s. Note that the increase in the velocity along the \(OY\) axis in the OCS (the axis is directed along the local vertical) turned out to be insignificant. This is probably because of the fact that one pulse is not enough to change the shape of the orbit. Therefore, in a circular orbit, the radial velocity was approximately zero and remains the same. The distribution of the increase in velocity \(\Delta V_z\) along the binormal to the orbit is close to normal and has the following characteristics: mathematical expectation -2 cm/s, standard deviation 0.6 cm/s.

As a result of applying a thrust impulse, the NS acquires angular acceleration. Towards the end of the thrust impulse, the angular velocities may vary due to different combinations of design parameters. The frequency polygons of the final angular velocities are shown in Figure 4.
Figure 4. Polygons of frequencies of final angular velocities.

With the ideal alignment of the longitudinal axis of the NS passing through its center of mass and the line of application of the traction force, at zero initial angular velocities, zero final angular velocities will be observed. However, as can be seen from Figure 4, the random distribution of the design parameters leads to scatter in the angular velocities acquired by the NS during the operation of the ETPS. The root-mean-square deviations of the final angular velocities are significant; therefore, they can have a strong influence on the projections of the velocity growth vector, and, consequently, on the efficiency of the orbit correction.

One of the "bad" cases, when the twist reaches 15 degrees per second in yaw, is shown in Figure 5. At time $t = 0$, the NS phase variables are determined by the point (0, 0).

Figure 5. Phase trajectory of motion relative to the center of mass.

Based on Figures 4 and 5, we can say that in the process of applying a thrust pulse, the longitudinal axis of the nanosatellite will most likely rotate by a significant angle in pitch or yaw, which will directly affect the efficiency of orbit correction. Hence, it becomes necessary to set stringent requirements for
the design parameters of the nanosatellite or to introduce additional actuators into the NS design, which allow maintaining the thrust vector in a given position.

When conducting factor analysis, linear regression equations were obtained, with the coefficients presented in Table 2.

| Design parameter                                           | $\Delta V_x$ | $\Delta V_z$ | Angular momentum $OX_i$ | Angular momentum $OY_i$ | Angular momentum $OZ_i$ |
|------------------------------------------------------------|--------------|--------------|-------------------------|-------------------------|-------------------------|
| Fictitious factor coefficient                             | -0.44        | -1.33e-2     | -3.00e-05               | 4.07e-07                | 2.40e-07                |
| Nozzle throat radius                                      | 1.46e3       | 6.13         |                         |                         |                         |
| WF heating temperature                                    | 1.67e-4      | -           |                         |                         |                         |
| Duration of thrust growth                                 | 8.54e-2      | -           |                         |                         |                         |
| Duration of thrust decay                                  | -            | -7.46e-3     |                         |                         |                         |
| Offset tank from the nozzle along the axis $OX_i$         | -            | -           | 4.58e-05                | -                       |                         |
| Offset tank from the nozzle along the axis $OY_i$         | -            | -           |                         | -0.06                   |                         |
| Offset tank from the nozzle along the axis $OZ_i$         | -            | 0.50        | -0.06                   |                         |                         |
| Offset of the rest of the NS from the nozzle $OY_i$       | -            | -           |                         | -0.37                   |                         |
| Offset of the rest of the NS from the nozzle $OZ_i$       | -            | 3.04        | 0.37                    |                         |                         |
| Offset of the nozzle from the geometric center of the NS $OY_i$ | -            | -           |                         | -0.48                   |                         |
| Offset of the nozzle from the geometric center of the NS $OZ_i$ | -            | 3.96        | 0.48                    |                         |                         |
| Angular deviation of the longitudinal axis of the nozzle from the longitudinal axis of the NS in the plane $X_iOY_i$ | -            | -0.55       | -0.09                   |                         |                         |
| Angular deviation of the longitudinal axis of the nozzle from the longitudinal axis of the NS in the plane $X_iOZ_i$ | -            | -           |                         | 0.09                    |                         |
| Compressed spring length of the system for storing and supplying | -            | -           | 1.33e-3                 |                         |                         |
| Fisher's Adequacy Criterion                               | 4.87e05      | 8.35e3      | 6.38                    | 8.89e4                  | 8.88e4                  |

The values from Table 2, taken modulo, allow one to estimate the contribution of each of the parameters. Contribution pie charts are shown in Figures 6 and 7.
The impact of design parameters is shown in the pie charts. It can be seen that the "smearing" of the speed increase along the direction of motion is almost completely determined by the technological error in creating the nozzle throat. The spread of the velocity increase outside the orbital plane is significantly influenced by the nozzle transverse displacement and its angular deviation from the longitudinal axis of the NS. The twist about the transverse axes is caused by the scattering of the same parameters that affect the scatter of the velocity increase outside the orbital plane, which is logical. Rotating about the center of mass, the NS acquires a component of the thrust force vector lying outside the orbital plane.

5. Conclusion
A simulation model of the formation of a correcting pulse for the NS was developed, taking into account the peculiarities of the ETPS, and a probabilistic analysis of the nanosatellite maneuvering process was carried out. Based on the analysis, the factors that have a significant impact on the efficiency of orbit correction have been identified.

Regression models have been built, which made it possible to determine the contribution of specific design parameters of the NS and ETPS to the scatter of the angular velocities of the NS rotation and the projections of the velocity increase vector. As shown by the factor analysis, the "smearing" of the speed increase along the direction of motion is almost completely determined by the technological error in creating the nozzle throat. During the issuance of a thrust impulse, the NS rotates about the center of mass, as a result of which a component of the thrust force vector is acquired, which lies outside the
orbital plane. The rotation relative to the transverse axes is caused by the scattering of the nozzle transverse displacement and its angular deviation from the longitudinal axis of the NS.

To implement an effective orbit correction, it is necessary to formulate requirements for the design parameters of the NS with ETOPS or for auxiliary actuators to maintain orientation during the issuance of the thrust impulse. This task is the reverse of the one discussed in this article. The search for its solution serves as an urgent direction for further research.

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