\( \Upsilon \) and \( J/\Psi \) spectroscopy using clover fermions in the presence of dynamical quarks

S. Collins, R.G. Edwards, U.M. Heller, and J. Sloan

SCRI, Florida State University

We calculate spin average splittings in the \( \bar{b}b \) and \( \bar{c}c \) systems using the clover action. We compare static and kinetic masses of heavy-heavy and heavy-light mesons and discuss the consistency of the results.

1. INTRODUCTION

In this poster presentation, we discuss recent preliminary calculations of spectroscopy in the \( \bar{b}b \) and \( \bar{c}c \) systems, which were carried out using the tadpole improved Clover action and interpreted in the limit of heavy quarks using the Fermilab interpretation of this action \[1\]. In the first half, we discuss our results for the spin-averaged \( 1P - 1S \) and \( 2S - 1S \) splittings, which are commonly used to extract \( \alpha_{MS}(M_Z) \), and compare to those of other groups. In the second half, we investigate the relationship between the static and kinetic masses of mesons, \( M_1 \) and \( M_2 \) respectively, composed of various mixtures of ‘heavy’ and ‘light’ quarks (i.e. heavy-heavy, heavy-light, and light-light). The Fermilab interpretation of the Clover action with temporal and spatial \( \kappa \) values equal (i.e. symmetric \( \kappa \)) identifies the kinetic mass as the physical mass and differences between kinetic and static masses as a shift in the energy zero of the quarks. This interpretation requires

\[
\delta M(\text{HH}) + \delta M(\text{LL}) = 2\delta M(\text{HL}),
\]

where \( \delta M = M_2 - M_1 \), and HH, HL, and LL are different flavor combinations of heavy and light quarks. We find that this is obeyed, at this particular value of \( n_f \) and \( \beta \), for \( M_2(\text{HL}) < 1 \), but is strongly violated for \( M_2(\text{HL}) > 1 \). The implication of this is that, for example, there is no choice of \( \kappa_b \) (for our ensemble) for which \( M_2(T) \) and \( M_2(B) \) simultaneously agree with experiment\[2\].

2. THE SIMULATION AND FITTING

The gauge configurations, smearing procedures, and fitting procedures are discussed in [2]. For \( P \) states, an \( l = 1 \) bound state spatial smearing function was used; one of the quarks was smeared by a gaussian multiplied by \( x + iy \), where \( x \) and \( y \) are the values of the \( x \) and \( y \) coordinates, and the other quark was a point source. For the “local” \( P \)-wave sink we used similar smearings which combined derivatives of a delta function. We used three different smearing radii each for \( \bar{b}b \) and \( \bar{c}c \), and performed both matrix and vector fits with one, two, and three exponentials. All fits were done with \( t_{\text{max}} = 15 \), i.e. omitting the central point of the propagator.

Because onia (especially \( T \)) are so small, our physical box contained several relatively decorrelated volumes. We took advantage of this by superposing the quark propagators from 8 spatial origins, using \( Z(3) \) noise as described in [3]. For \( P \) states, we further doubled our statistics by using \( t = 16 \) (in addition to \( t = 0 \)) as an initial timeslice. This was vital in obtaining a reliable \( 1P-1S \) splitting; the effective mass plots with the original statistics showed a “bump” near the center which led to ambiguous fits.

For our determination of kinetic masses, we fit the ratio of a momentum one state and a zero momentum state to a single exponential, using a plateau of 8-12 in all cases. Looking at effective mass plots of the ratio, we saw that this corresponded to a very conservative value for \( t_{\text{min}} \), so our statistical errors should be much larger than the (systematic) fitting errors.

---

\[1\] This is due to the large dimensionless mass, not quenching effects on the running of the strong coupling
3. SPECTROSCOPY RESULTS

We present the results of a representative sample of our fits in Table 2. The $b\bar{b}$ fits seemed better behaved than the $c\bar{c}$ fits; this can be seen by comparing $h_b$ to $h_c$ results. We see that corresponding $\Upsilon$ and $\eta_b$ fits are very similar; this is also true of the $J/\psi$ and $\eta_c$. We quote preliminary splittings in Table 2; the error bars include our estimate of systematic fitting errors. This leads to inverse lattice spacings of 2.27(13) and 2.17(25) GeV from the bottomonium $1P-1S$ and $2S-1S$ splittings, respectively, and 2.0(2) and 2.3(5) GeV from charmonium. For the respective dimensionless kinetic masses, $aM_{2i}$, of the lowest energy vector states we find 4.45(6) and 1.44(3) (corresponding to dimensionful masses of 10.1(6) GeV for “$\Upsilon$” and 2.93(3) GeV for “$J/\psi$”) when combined with the corresponding $1P-1S$ $a^{-1}$. Hyperfine splittings were estimated by examining effective mass plots of the ratio of vector to pseudo-scalar propagators; rough values of $41(5)$ MeV for bottomonium and $80(20)$ MeV for charmonium were obtained.

Table 2

| Preliminary splittings | 
|------------------------|
| $h_c(1P) - c\bar{c}(1S)$ | 0.23(2) |
| $J/\psi(2S) - J/\psi(1S)$ | 0.26(5) |
| $h_b(1P) - b\bar{b}(1S)$ | 0.199(11) |
| $\Upsilon(2S) - \Upsilon(1S)$ | 0.26(3) |

NRQCD $b\bar{b}$ and Wilson $c\bar{c}$ spectroscopy has been studied on this ensemble by other groups [4, 5]. The NRQCD inverse lattice spacings from the $b\bar{b}$ $1P-1S$ and $2S-1S$ splittings are 2.44(7) and 2.37(10), respectively, roughly 10% higher than our values. This is consistent with experience from quenched calculations, where NRQCD yields splittings about 10% smaller than corresponding clover calculations [5]. Our $c\bar{c}$ $1P-1S$ inverse lattice spacing is consistent with, but larger than that quoted for Wilson fermions in [5]. Because the clover term mainly affects spin-dependent splittings, we expect agreement between Wilson and clover calculations of the spin-averaged spectrum.

4. KINETIC MASSES

QCD is a Lorentz invariant theory. Therefore the dispersion relation of physical states should obey the relation $E(p^2) = \sqrt{p^2 + M^2}$. In [1] it was pointed out that, for non-relativistic particles, a discretization which subtracts off a zero of energy for each (heavy) quark captures the essential physics:

$$E(p^2) - \sum_i \delta M_i = \sqrt{p^2 + M^2} - \sum_i \delta M_i,$$  \hspace{1cm} (2)

where the lhs is typically called the static mass, $M_i$, and $\delta M_i$ is the shift in the zero of energy of the $i$th quark in the state. The kinetic mass is defined as $M_2 = \left(2\frac{dE}{dp^2}\right)|_{p^2=0}^{-1}$, and is equal to $M$ for the dispersion relation (2). A useful definition of $M_2$ given the splitting $\delta E = E(p^2) - E(0)$ is obtained by solving Eq. (2) for $M$; one obtains $M_2 = (p^2/2\delta E) - \delta E/2$.

The sum over energy shifts in Eq. (2) can be calculated in simulations; it is just the discrepancy between kinetic and static masses $\delta M_{\text{state}} = M_2(\text{state}) - M_1(\text{state})$. Because the energy shifts in Eq. (2) depend only on the quark content of the state, states with the same quark content should have the same $\delta M$; the energy shift associated with a $B$ meson should be the average of the $\Upsilon$ and $\pi$ shifts, i.e. half the energy shift of the $\Upsilon$. This leads us to the relation $2\delta M(\text{HL}) = \delta M(\text{HH}) + \delta M(\text{LL})$ as a non-perturbative test of the consistency of the idea of attributing discrepancies between $M_1$ and $M_2$ to a shift in the zero of energy.

The energy shifts $\delta M_1$ have been calculated for free quarks in [4]; this leads to the relation

$$M_{2,\text{pert}}(M_1) = \frac{e^{M_1}}{\sinh(M_1)} + 1 \hspace{1cm} (3)$$

In Fig. 4, we have plotted our results for $M_2(PS)$ vs. $M_1(PS)$ for both mesons composed of degenerate quarks (HH) and those of mixed mass (HL). It is striking that both HH and HL mesons appear to lie on a universal curve for this range of mass values. We also include the ‘perturbative’ curve for HH mesons $M_2 = 2M_{2,\text{pert}}(M_1/2)$, which seems to describe the curve fairly well. Indications of this behavior were first seen in [4].
Table 1
Selected \( \bar{b}b \) and \( c\bar{c} \) fit results.

| State | type Smears | \( N_{cosh} \) | \( T_{min} \) | \( T_{max} \) | \( Q \) | \( E_1 \) | \( E_2 \) | \( E_3 \) |
|-------|-------------|----------------|--------------|--------------|--------|--------|--------|--------|
| \( \eta_b(nS) \) | Vec a,b,c | 3 | 5 | 15 | .06 | 2.4539(8) | 2.709(15) | 2.94(3) |
| | Vec a,b | 2 | 10 | 15 | .37 | 2.4532(9) | 2.763(17) | 2.96(4) |
| | Vec b,c | 3 | 2 | 15 | .03 | 2.4539(7) | 2.690(7) | 3.19(6) |
| | Mat a,b | 3 | 3 | 15 | .11 | 2.4543(7) | 2.70(2) | 2.96(4) |
| | Mat a,b | 2 | 8 | 15 | .18 | 2.4545(6) | 2.751(7) | 2.96(4) |
| | Mat a,b | 2 | 11 | 15 | .72 | 2.4533(6) | 2.701(5) | 2.97(4) |
| \( \Upsilon(nS) \) | Vec a,b,c | 3 | 5 | 15 | .07 | 2.4707(9) | 2.714(15) | 2.95(3) |
| | Vec a,b | 3 | 2 | 15 | .29 | 2.4691(13) | 2.76(3) | 2.97(4) |
| | Mat a,b | 3 | 3 | 15 | .13 | 2.4710(8) | 2.71(2) | 2.97(4) |
| \( h_b(nP) \) | Vec a,b,c | 3 | 1 | 15 | .30 | 2.666(7) | 2.87(2) | 3.083(19) |
| | Vec a,b,c | 2 | 7 | 15 | .41 | 2.668(9) | 2.95(3) | 3.083(19) |
| | Mat a,b,c | 2 | 9 | 15 | .34 | 2.669(8) | 2.94(4) | 3.083(19) |
| \( \eta_c(nS) \) | Vec a,b,c | 3 | 6 | 15 | .24 | 1.2156(17) | 1.48(8) | 1.78(15) |
| | Vec a,b,c | 2 | 9 | 15 | .56 | 1.2150(13) | 1.53(3) | 1.86(15) |
| | Vec a,b | 2 | 8 | 15 | .55 | 1.2157(13) | 1.51(4) | 1.86(15) |
| | Mat a,b | 3 | 3 | 15 | .14 | 1.2138(13) | 1.49(3) | 1.86(15) |
| | Mat a,b | 2 | 9 | 15 | .45 | 1.2138(13) | 1.49(3) | 1.86(15) |
| \( J/\psi(nS) \) | Vec a,b,c | 3 | 4 | 15 | .20 | 1.2547(15) | 1.53(2) | 1.96(4) |
| | Vec a,b | 3 | 4 | 15 | .16 | 1.2541(2) | 1.51(4) | 1.86(3) |
| \( h_c(nP) \) | Vec a,b,c | 3 | 2 | 15 | .66 | 1.481(8) | 1.86(3) | 2.52(2) |
| | Vec a,b,c | 2 | 7 | 15 | .63 | 1.459(18) | 1.79(7) | 2.52(2) |
| | Vec a,b | 2 | 2 | 15 | .52 | 1.490(5) | 1.863(10) | 2.52(2) |
| | Mat a,b | 2 | 5 | 15 | .27 | 1.484(9) | 1.79(4) | 2.52(2) |
| | Mat a,c | 2 | 6 | 15 | .14 | 1.469(8) | 1.84(2) | 2.52(2) |

albeit with large error bars.

In Fig. 2 we plot the relative inconsistency

\[
I(H, L) = \frac{\delta M(HL) - (\delta M(HH) + \delta M(LL))/2}{M_2(HL)}
\]

as a function of \( M_2(HL) \) for many different heavy-light mesons. For \( M_2(HL) < 1 \), we see that Eq. (3) is consistent; for \( M_2(HL) > 1 \), however the relative discrepancy in energy shifts appears to be growing linearly in \( M_2(HL) \) for fixed light quark mass. Since the \( D \) meson should have a dimensionless mass of about .7 on this ensemble, the energy shift ansatz is consistent for charmed mesons. The \( B \) meson, however, should have a dimensionless mass of about 2.5; if we tune \( \kappa_b \) by matching \( M_2(\Upsilon) \) to experiment (as we did), then we expect \( M_2(B) \) to be much too light.

Recall that we found \( M_2(\Upsilon) = 10.1(6)GeV \) for our choice of \( \kappa_b \) (i.e. we used a bare quark mass slightly too heavy). For the same choice of \( \kappa_b \) we find \( M_2(B) = 4.4(3)GeV \) - much lighter than the physical \( B \) meson!

5. CONCLUSIONS

Our results for spin-averaged splittings of the \( b\bar{b} \) and \( c\bar{c} \) systems showed no surprises. All gave inverse lattice spacings between 2.0 and 2.3GeV, with the smallest inverse lattice spacings coming from the softest splittings. This is consistent with a partial removal of quenching effects by the two flavors of dynamical fermions. This is consistent with experience in quenched calculations.

A problem arises, however, when we examine the relationship between static and kinetic masses. We find that, after tuning \( \kappa_b \) so that
the kinetic mass of the $\Upsilon$ agrees with experiment, the kinetic mass of the $B$ meson is roughly 20\% too small. This will obviously cause difficulties with trying to calculate $f_B$ directly at the $B$ meson mass on a 2$\text{GeV}$ lattice, especially since $f_B$ is strongly dependent upon $M_B$. This problem does not appear to be present in NRQCD; Eq. (3) has been checked for the $B_c$ system [6]. Another area where this inconsistency may cause problems is in low-$\beta$ calculations: $aM_\rho \approx 1.5$ when the inverse lattice spacing is 500$\text{MeV}$. With this in mind, the dependence of the inconsistency threshold upon lattice spacing should be checked.

Recently, another group has postulated a particular form of the lattice dispersion relation for mesons [8], which leads them to similar conclusions to those presented here. We stress that we calculate both $M_2$ and $M_1$ directly from the simulation, rather than relating them through a particular ansatz for the dispersion relation.

Finally, we would like to stress that our results do not prove that $B$ physics cannot be done on a 2$\text{GeV}$ lattice. First, we know that the clover action does not include retardation effects correctly for very heavy quarks; it is possible that adding improvement terms to correct this would reduce the inconsistency. A more promising idea is to explicitly break the hypercubic symmetry of the clover action by using different $\kappa$’s in the temporal and spatial directions [1]; the asymmetry could then be tuned to require that $M_1$ agree with $M_2$, obviating the need for energy shifts.

ACKNOWLEDGEMENTS

We thank Andreas Kronfeld for useful conversations. The computations were performed on the CM-2 at SCRI. This research was supported by DOE contracts DE-FG05-85ER250000 and DE-FG05-92ER40742.

REFERENCES

1. A. Kronfeld, Nucl. Phys. B (Proc. Suppl.) 30 (1993) 445.
2. S. Collins et al., these proceedings.
3. F. Butler et al., Nucl. Phys. B421, (1994) 217, and references therein.
4. P. McCallum et al., these proceedings.
5. M. Wingate et al., Phys. Rev. D52 (1995) 307.
6. J. Sloan, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 171.
7. S. Collins, Nucl. Phys. B (proc. Suppl.) 34 (1994) 465.
8. C. T. H. Davies, these proceedings.
9. T Bhattacharya and R. Gupta, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 935.