ACCUMULATION CHARTS FOR INSTANT-RUNOFF ELECTIONS

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Abstract. We propose a new graphical format for instant-runoff voting election results. We call this proposal an “accumulation chart.” This model, a modification of standard bar charts, is easy to understand, clearly indicates the winner, depicts the instant-runoff algorithm, and summarizes the votes cast. Moreover, it includes the pedigree of each accumulated vote and gives a clear depiction of candidates’ coalitions.

Who won the election? How did it play out? When election results are announced and analyzed, details matter. A standard bar chart gets the job done for a plurality election, but it falls short for instant-runoff voting (IRV) elections, in which candidates are eliminated one-by-one according to voters’ rankings. A single bar chart or table cannot capture the multiple rounds of IRV tallying, thus obscuring and perhaps undermining the public’s trust in IRV itself.

The media frequently report IRV results using multiple tables or bar charts, in a (not altogether successful) attempt to communicate the full story of an election. While IRV has been used for decades in state and local elections in the U.S. [3], the recent adoption of IRV by the state of Maine for a number of state races, including the U.S. Senate, marked the first time that such an alternative to plurality voting would be used in a U.S. congressional election [2]. Figure 1 illustrates a typical way in which the results of the 2018 election for Maine’s 2nd Congressional District were communicated. Unfortunately, this represents the IRV election as a plurality election plus some mysterious extra steps that were invoked when no candidate won a majority in the first round, retaining little connection to the nature and features of IRV.

One of the selling points of IRV is the ability to vote for a hopeless candidate without wasting your vote: the hopeless candidate will be eliminated in an early round and your vote reallocated to your highest-ranked serious contender. The fact that IRV allows voters to rank

| Candidate | Party | Votes | Pct. | Ranked-Choice Results |
|-----------|-------|-------|------|-----------------------|
| ✓ Golden  | Dem.  | 131,954 | 45.6% | 139,231 | 50.5% |
| Poliquin  | Rep.  | 134,061 | 46.3 | 136,326 | 49.5 |
| Bond      | Ind.  | 16,452  | 5.7  | —         | —     |
| Hoar      | Ind.  | 6,865   | 2.4  | —         | —     |

Figure 1. Reproduction of the New York Times [10] graphic showing the 2018 election results from Maine’s 2nd Congressional District.
candidates, and then processes the full list of those rankings, has an indisputable effect on how people vote in an IRV election. The first round tally in IRV reflects an important part of voters’ preferences and candidates’ coalitions, but the results of that first round do not tell the full story of the election — particularly for supporters of a candidate who receives the most votes in the first round but does not ultimately win the majority. For example, as Wright claimed after his loss in the 2009 mayoral election in Burlington, Vermont (conducted under IRV, see Figure 2), “under the plurality system, I would have won tonight” [6]. That first round tally is just one facet of what the voters prefer. Moreover, the voters might not have voted for those top choices if they had not been casting ranked-choice votes at the time. The point of IRV is that important data is contained in the lower-ranked votes as well. Just as the first-lap leader of a race has no claim on being the eventual winner, neither is the leader after the first round of IRV tallying anything more than a temporary front runner.

Besides avoiding this inappropriate emphasis on first round results, we propose that an election graphic — whatever the election framework — should, at a minimum, achieve the following goals (cf. [5]).

**Objectives of an election graphic:**

- Be easy to understand
  
  *Graphics exist to display and convey information, and that information should be readily interpretable to the consumer.*

- Clearly indicate a winner
  
  *The winner’s identity is typically the most important aspect of an election’s results, and should be immediately apparent.*

- Reflect the methodology and integrity of the election procedure
  
  *The graphic should echo the mechanism by which the winner is selected from the votes that are cast.*

- Summarize the votes that were cast
  
  *The story of an election is not simply the answer to “who won?” but is also the record of the votes that were cast.*

The first objective is a precondition for any graphical depiction of data. Likewise, the point of holding an election is to determine a winner, justifying the second objective. Consumers of election results have different priorities, and the last two objectives combine to serve those needs. For example, a voter might be most interested in how the winner was determined from the votes cast, whereas a candidate might be most interested in the coalitions of support that were demonstrated through those votes. These issues are especially pertinent in an IRV election, when not all members of a candidate’s broadest coalition may have ranked that candidate as their top choice.

The third and fourth objectives are nontrivial in the context of IRV elections, where the winner is determined by a multi-step algorithm. As discussed above, while the first round of this algorithm looks deceptively like a simple plurality election, its role in the IRV framework is different. Furthermore, the voters’ expressed preferences are, themselves, complicated and hard to summarize. A list of votes according to frequency quickly becomes unwieldy — for example, there are more than 300 possible ranked-choice votes on five candidates.

In this paper, we propose an **accumulation chart** for illustrating the results of IRV elections. It can be read at a glance, the winner is clearly indicated, it shows the impact of each round of the instant-runoff procedure, and every vote can be traced throughout the tallying.
Moreover, candidates can use the accumulation chart to understand their coalitions of support. These coalitions are displayed up through a candidate’s last round of participation, with a full description of the “pedigree” of all votes that were included in the candidate’s final vote totals.

Our proposal is organized as follows. In Section 1, we define instant-runoff voting and highlight its main features. In Section 2, we discuss how the results of recent IRV elections have been reported in the press, and some of the questions left unanswered by that reporting. In Section 3, we introduce accumulation charts. We demonstrate their value using those same recent IRV elections, and explore their benefits for different constituencies of interested parties. We conclude with Section 4 a call to use accumulation charts more broadly.

1. Ranked-choice and instant-runoff voting

Instant-runoff voting considers not only each voter’s first-choice candidate but, as necessary, her lower-ranked choices as well. Whereas plurality (a.k.a. first-past-the-post or winner-takes-all) elections require each voter to name at most one preferred candidate, IRV involves a more complicated vote.

Definition. A ranked-choice vote is a list of candidates, ordered from top choice to bottom.

Different methods have been proposed for identifying a winner from a collection of ranked-choice votes. Our focus is on the following procedure.

Definition. An instant-runoff tally is a method for tallying ranked-choice votes. It works, iteratively in rounds, as follows.

Step 1: The top-ranked selection of each vote is tallied.
Step 2: If there are two candidates, then the candidate with the most votes is declared the winner.
Step 3: If there are more than two candidates, then the candidate with the fewest votes is eliminated and that candidate’s name is stricken from each vote, with lower-ranked candidates advancing one position in that vote’s ranking. The procedure now begins again with a new round.

We present the following example as a demonstration.

Example. Irvtown is electing a mayor from three candidates: Alf, Bugs, Chester. The election is being conducted under IRV. The 22 votes cast in the election are described in the table below:

| # votes | 1st Choice | 2nd Choice | 3rd Choice |
|---------|------------|------------|------------|
| 1       | Alf        | Bugs       | Chester    |
| 5       | Alf        | Chester    | Bugs       |
| 3       | Bugs       | Alf        | Chester    |
| 2       | Bugs       | Chester    | —          |
| 4       | Bugs       | Chester    | Alf        |
| 3       | Chester    | Alf        | Bugs       |
| 4       | Chester    | Bugs       | Alf        |

For example, two voters ranked Bugs as their top choice, Chester as their second choice, and listed no one as their third choice.
After the first round of IRV tallying, Alf receives six votes, Bugs receives nine votes, and Chester receives seven votes. Alf, with the fewest votes, is eliminated. In the second round, one vote is reallocated from Alf to Bugs, and five votes are reallocated from Alf to Chester. Thus, after the second round of tallying, Bugs has received 10 votes and Chester has received 12 votes. Of these two remaining candidates, Chester is declared the winner.

IRV is sometimes described with “Step 1.5: If any candidate has received a majority of votes, then that candidate is declared the winner.” While including this step might shorten the tallying process, it does not change the eventual outcome of the election. Moreover, truncating the procedure can obscure important information, and muddles what occurred in the intermediate rounds.

Following common usage, we refer to an election consisting of ranked-choice voting followed by an instant-runoff tally as an election carried out under instant-runoff voting (IRV). The terminology used is not consistent in the literature, and several other terms — including ranked-choice voting and alternative voting — are also used to describe this kind of election system [3, 4].

The iterative nature of IRV relies on something that does not exist in plurality elections.

Definition. Suppose that Candidate A accumulates a particular ranked-choice vote in a round of IRV. The pedigree of that accumulated vote is the ranked list of all candidates who appeared on that particular ballot ahead of and including Candidate A.

Our framing of IRV emphasizes two important elements of the procedure:

- what matters in each round is not who has the most votes, but who has the fewest votes, and hence who is eliminated from future rounds; and
- allowing the procedure to continue until it decides between two candidates (instead of utilizing “Step 1.5”) does not change the final outcome, and gives substantially more information to the voters and the candidates.

2. Examples of IRV election graphics

Before introducing accumulation charts, we look at IRV election results in the media. The Maine results shown in Figure 1 give the candidates’ vote tallies after only two rounds, suppressing earlier (including write-in candidates) and intermediate (after Hoar’s elimination) data. The 2009 mayoral election in Burlington, Vermont was also conducted under IRV, and Figure 2 depicts how the results of that election were presented in the Burlington Free Press [6]. The election was summarized in three bar plots and accompanying blurbs, one for each round of the tallying process after elimination of all write-in candidates and the Green Party candidate Simpson.

Faced with either of these examples, a reader might well ask, which chart should I look at? Which table’s winner is the election’s winner? Why did two candidates switch positions between the first and second table? I ranked the candidates A>B>C>D — where is my vote in this picture? And so on. These graphics have suppressed so much information that they barely reflect the IRV process, and they give no sense of what votes were cast in the election. For example, of the 767 votes added to Wright’s tally in Figure 2’s chart for Round 3, there is no way to distinguish between votes that begin with the following rankings:
Round 1
No candidate reached 50% plus one vote in the first IRV round. Smith eliminated.

Round 2
Second-choice votes from supporters of Smith and the Green Party’s James Simpson redistributed to remaining candidates. Still no candidate receives 50% plus one vote. Montroll eliminated.

Round 3
Second-choice votes from Montroll supporters are redistributed to remaining candidates. Bob Kiss surpasses 50% threshold and is re-elected.

Also, these graphics do a weak job of describing candidates’ coalitions. The information gained becomes more vague with each successive round, and there is ambiguity about the coalitions forming support for the candidates who make it to the final round. For example, Wright might want to know the precise makeup of his support in order to craft a subsequent campaign. Kiss, similarly, might want to know what coalition put him over the top, and how best to represent them during his time in office. Neither candidate would be able to find this information from the data as currently depicted.

Another method for displaying IRV results is by a Sankey diagram [9], as used for the 2017 Minnesota mayoral election [8]. These diagrams, which illustrate resource flow (in this case, the redistribution of votes among candidates), do a reasonably good job of describing candidates’ coalitions, at least qualitatively, but are not especially successful at meeting the other objectives.

3. Accumulation charts

Accumulation charts are motivated by the fact that, in IRV, candidates can accumulate votes during each round of the procedure. Illustrating that accumulation makes substantial progress towards achieving the goals outlined at the beginning of this paper. We do this by modifying a standard bar chart, noting that

- bar charts are familiar graphical tools,
- the longest bar can indicate the winner,
- each round of IRV can be depicted by a segment of the candidates’ bars, and
- ranked-choice votes can be expressed by coloring slices within those segments.

We introduce accumulation charts with a demonstration — this paper is about graphical tools, after all. This example, shown in Figure 3, describes the results of the 2018 election for Maine’s 2nd Congressional District (cf. Figure 1). General accumulation charts are outlined below, and we conclude this section with a second demonstration (Figure 4), this time using the 2009 mayoral race in Burlington, Vermont.

### Accumulation charts

- An accumulation chart is a modified bar chart, with one bar per candidate.
- A candidate’s bar consists of multiple segments, one for each IRV round that the candidate participates in before elimination. We use long vertical lines to separate segments (that is, to separate votes by the round in which they are accumulated), and segments for earlier rounds appear further to the right (so the rightmost segment in each candidate’s bar represents the votes in which that candidate was ranked first).
- The length of a candidate’s Round X segment represents the number of (new) votes accumulated by the candidate during Round X.
- The total length of a candidate’s bar represents the number of votes accumulated by that candidate in all rounds, through the candidate’s final round of participation before elimination or victory.
- The pedigree of a vote is indicated by top-to-bottom coloring within the segment in which it gets accumulated by the candidate. In each round, we groups votes by pedigree for ease of interpretation.

For example, using the coloring of Figure 3, the ranked-choice vote

| Candidate | Votes   |
|-----------|---------|
| Golden    | 142440  |
| Poliquin  | 138931  |
| Bond      | 19173   |
| Hoar      | 6875    |

**Figure 3.** Accumulation chart for the 2018 election in Maine’s 2nd Congressional District using data from [1].
would appear in Hoar’s 1st round segment as the slice

in Poliquin’s 2nd round segment as the slice

not in Bond’s third round segment (because Bond was eliminated after Round 2), and in Golden’s fourth round segment (if a fourth round had been necessary in this election) as the slice

The top-to-bottom coloring of a slice describes the ranked-choice vote’s (initial) ranking. All slices in a candidate’s accumulation bar have the same bottom color, because the candidate accumulated each of those votes when he or she was the votes’ highest non-eliminated choice.

We have arranged the segments so that votes from early rounds appear to the right of votes from later rounds. We do this deliberately, to remove any understandable but inappropriate emphasis on Round 1 tallies. While some rounds contribute relatively skinny segments to the accumulation bars and might appear crunched in a printed version of the chart, this is not a concern if the chart is displayed electronically with zooming capability, providing both coarse and granular detail.

Note that the slices comprising a given segment may not all have the same number of colors. For example, in Figure 3, Golden’s Round 3 segment shows two three-colored slices and one two-colored slice. From left to right, these represent the 1362, 3027, and 4835 votes cast that began, respectively,

shows two three-colored slices and one two-colored slice. From left to right, these represent the 1362, 3027, and 4835 votes cast that began, respectively,

1. Hoar
2. Bond
3. Golden

In each case, these votes are allotted to Golden during the third round of IRV tallying (after Hoar’s elimination in Round 1 and Bond’s elimination in Round 2), and so they appear in the third round segment of Golden’s bar in the accumulation chart.

Figure 4 uses an accumulation chart to present the results of the 2009 mayoral election in Burlington, Vermont. Write-in candidates appeared in the ranked-choice votes, but their presence was scant enough that we have made the decision to eliminate all write-in candidates in a single round, for brevity.
Figure 4. Accumulation chart for the 2009 mayoral election in Burlington, Vermont using ballot-level data from [7].

4. Conclusion

Our focus in this article is on general-purpose IRV election graphics — suitable for, say, a newspaper. While currently there is no standard for reporting IRV results, prominent outlets have tended to present a first-round tally followed by the final tally, such as in Figure 1, which we have established is problematic.

While we believe that accumulation charts address the deficiencies of commonly used graphics, one must still be thoughtful about their use and analysis. For example, candidates (and their bars) get eliminated in different rounds, so they are not directly comparable. However, given the correct interpretation — that bar length corresponds to the total number of votes accumulated before elimination — this is not a concern. There are aspects of the election results that might be better displayed through other means (for example, using a Sankey diagram to indicate the flow of votes) but, again, our goal here is for a general-purpose graphic.

We believe that accumulation charts achieve the objectives of an election graphic. They are easy to understand at a glance, with each candidate corresponding to a single bar and bar lengths indicating overall support. Importantly, accumulation charts allow a reader to follow the algorithm as it narrows down the field, and the visible pedigrees give a sense of each candidate’s coalition of support. As future elections unfold, we hope that IRV election results can be reported in such a data-rich way.
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