SU(4) pure-gauge string tensions

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In response to recently renewed interests in SU(N) pure-gauge dynamics with large \( N \) both from M/string duality and from finite-temperature QCD phase structure, we calculate string tensions acting between the fundamental 4, diquark 6 and other color charges in SU(4) pure-gauge theory at temperatures below the deconfining phase change and above the bulk phase transition. Our results suggest 4 and 6 representations have different string tensions, with a ratio of \( \sigma_6/\sigma_4 \sim 1.3 \). We also found the deconfining phase change is not strong.

There are renewed interests in pure-gauge dynamics with SU(N) gauge groups with \( N > 3 \): New developments in M/string theory like Maldacena’s duality conjecture\textsuperscript{[1]} map super conformal field theory spectra to those in large-\( N \) and strong-coupling pure-gauge theory\textsuperscript{[2]}. Another line of thought is questioning how well the conventional large-\( N \) expansion describes quenched SU(3) deconfining and related phase transitions or changes\textsuperscript{[3]}. And there is a long-standing problem of how hadronic string tension is related to finite-temperature phase structure of hadrons.

From a M/string-provoked viewpoint, an interesting quantity to look at is the ratio of string tensions. In SU(2) and SU(3) pure-gauge theories there cannot be different string tensions: any irreducible representation is screened by gluons (in the adjoint representation) down to either singlet or fundamental representations: in SU(3), \( 6 \otimes 8 = 24 \oplus 15 \oplus 6 \oplus 3 \). This has been confirmed by a numerical Monte-Carlo calculation\textsuperscript{[4]}. However SU(N) gauge groups with \( N > 3 \) can allow different string tensions for different representations. For example in SU(4), the 6- and 10-dimensional diquark representations cannot be screened by the 15 gluon down to the 4 fundamental: \( 6 \otimes 15 = 64 \oplus 10 \oplus \overline{10} \oplus 6 \) and \( 10 \otimes 15 = 70 \oplus 64 \oplus 10 \oplus 6 \). So this theory allows at least two different string tensions, one for 4 and another for 6 representations. For general \( N \) these string tensions are classified by the \( N \)-ality \( k \) in the center \( Z(N) \) group\textsuperscript{[2]}: the fundamental representation has \( N \)-ality \( k = 1 \), the diquark representations \( k = 2 \), and the adjoint \( k = 0 \), etc. One would expect \( \sigma_{k=0} = 0 \) and \( 1 \leq \sigma_{k \neq 0}/\sigma_{k=1} \leq k \). In response to recently renewed interests in SU(4)\textsuperscript{[3]} pure-gauge string tensions, with a ratio of \( \sigma_6/\sigma_4 \sim 1.3 \). We also found the deconfining phase change is not strong.

\textsuperscript{*}SO thanks the hospitality of the RIKEN BNL Research Center where he stayed for a year from September 1997. This research started during this stay.
washed away by a few light quark flavors. On the other hand if large-\(N\) were to give second-order phase transition, as suggested by the disappearance of center symmetry there \([6]\), then it would still serve as a useful guide: in this case the weak first-order phase transition of SU(3) pure-gauge theory is just a peculiarity arising from the cubic effective interaction present only in this case.

Also interesting is the ratio of the deconfining phase transition temperature and the fundamental string tension. In the classic Hagedorn analysis the ratio should be \(\sqrt{\frac{3}{\pi(d-2)}}\) in \(d\) space-time dimensions. As is well known at \(d=4\) and \(N=2\) and \(3\) the actual ratio are smaller than this prediction, and so are in \(d=3\) \([7]\).

In the present work we study pure-gauge SU(4) dynamics with single-plaquette action in the fundamental 4 representation. We use combinations of Metropolis or pseudo-heatbath update and over-relaxation update algorithms. Our lattices are \(4\) or \(6\times(4, 6, 8, 12, \text{ or } 16)^3\) at more than a dozen values of inverse-squared coupling in \(10.10 \leq \beta = 8/g^2 \leq 11.00\) with at least 1000 sweeps and up to 8000 near the deconfinement phase change. We impose periodic boundary condition in all the four axes in order to look at finite-temperature systems.

We calculate the plaquette and Polyakov lines in 4 (fundamental), 6 (anti-symmetric diquark), 10 (symmetric diquark) and 15 (adjoint) representations, the deconfinement fraction derived from them \([8]\) and correlations of Polyakov lines in all the mentioned representations. We use the PPR multihit method \([9]\) to reduce noise.

We confirmed the known bulk phase transition \([11]\) at around \(\beta \sim 10.20\) where fluctuation in both plaquette and deconfinement fraction becomes large. This is a phase transition that separates two different confining phases with different correlation lengths. The bulk nature of this transition is known because its position in \(\beta\) does not move as we change the temperature lattice size from \(L_t=4\) to 6. However with \(L_t=4\) the change of correlation length drives deconfinement. So we choose \(L_t=6\) for the rest of our calculations.

The average Polyakov line is the order parameter of the center \(Z(4)\) symmetry. In the confining phase it vanishes, while in the deconfining phase it acquires a finite value in one of the four \(Z(4)\) (real and imaginary) axes for 4 and 10 representations, and one of the two \(Z(2)\) (real) axes for 6 and 15 which are self-dual representations. At temperatures lower than \(\beta=10.7\), all these Polyakov lines cluster around the origin. Above that temperature they start to fluctuate along the allowed axes and gradually deviates from the origin. At temperatures above \(\beta = 11.0\) they have finite values in one of the allowed axes. The deconfinement fraction behaves in accord: it starts to deviate from zero at \(\beta=10.7\), crosses the 50% mark at about 10.75 and reaches 100% by 11.0 for all threshold angle between 15° and 35° (see Figure 1). From these we conclude that there is a finite-temperature deconfining phase change at around \(\beta=10.75\) for \(L_t=6\). On the other hand we do not find any discontinuity in the average plaquette, and especially in the difference of space-like and timelike plaquettes, in this temperature range. In other words we are not finding any evidence for a first-order phase transition for \(L_t=6\) and volumes up to \(16^3\). This does not exclude first-order deconfining phase transition on larger lattices.
Figure 2. Polyakov line correlation in the fundamental (4, cross) and anti-symmetric diquark (6, diamond) representations at $\beta = 10.65$ on a $6 \times 16^3$ lattice with 800 configurations. The curves are fits to tensions of $\sigma_4 = 0.102$ and $\sigma_6 = 0.128$ volumes, but it would be a weak one at best [5].

The correlations of dual Polyakov lines for irreducible representations $i = 4, 6, 10$ and 15, etc, are related to the free energy $F_i(r)$ of a pair of infinitely heavy dual color charges in the respective representations separated by the distance $r = |\vec{r}|$:

$$C_i(r) = \langle L_i(\vec{x})L_i^*(\vec{x} + \vec{r})\rangle_\beta \propto r^{-1}e^{-F_i(r)/T}.$$ 

Below the deconfining temperature, the free energy consists of a constant-shift, Lüsher-Coulomb, and the string-tension terms, $F_i(r) = \sigma_i r + c_i + \alpha_i/r$, so we should be able to extract the string tension from its long-range part. Indeed even with the current limited statistics we obtain an estimate of $\sigma_4 = 0.102(5)$ for the fundamental 4 representation from our $6 \times 16^3$ lattice at $\beta = 10.65$ (see Figure 2). The correlation in the antisymmetric diquark 6 representation is noisier but still yields an estimate of $\sigma_6 = 0.128(28)$. So there seem two different string tensions. The higher 10 and 15 representations are still too noisy to extract any tension estimates.

Conclusions: For the phase structure, we confirmed with better data that in SU(4) pure-gauge theory the known bulk phase transition is at $\beta_b \sim 10.2$ and does not move with $L_t$, and for $L_t = 6$ the $T \neq 0$ deconfining phase change is separated from the bulk transition and is at $\beta_d \geq 10.75$. A new observation is that this $T \neq 0$ deconfining phase change does not appear stronger than weakly first-order SU(3) deconfining transition [7]. For string tensions we discovered many things: We see string tension signals for 4 and 6, but not yet for 10 or 15. There seem two different string tensions, $\sigma_4 \sim 0.102(5)$ and $\sigma_6 \sim 0.128(28)$ at $\beta = 10.65$. So their ratio is $1 < \sigma_6/\sigma_4 < 2$. And combining the phase structure and tension calculations, $T_d/\sqrt{\sigma_4(T = 0)} < T_d/\sqrt{\sigma_4(T \sim T_d)} \sim 0.53 < \sqrt{3/\pi(d-2)}$. We plan to accumulate more statistics on larger and finer lattices, probably using several motherboards of the QCDSP parallel supercomputer being built at the RIKEN BNL Research Center.

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