BEAM-BEAM EFFECTS UNDER THE INFLUENCE OF EXTERNAL NOISE

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Abstract

Fast external noise, which gives fluctuation into the beam orbit, is discussed in connection with beam–beam effects. Phase noise from crab cavities and detection devices (position monitor) and kicker noise from the bunch by bunch feedback system are the sources. Beam-beam collisions with fast orbit fluctuations with turn by turn or multi-turn correlations, cause emittance growth and luminosity degradation. We discuss the tolerance of the noise amplitude for LHC and HL-LHC.

INTRODUCTION

Beam-beam effects under external noise are studied with the weak-strong model in this paper. The strong beam is regarded as a target with a Gaussian charge distribution. In the model, an external noise is introduced into the transverse position of the strong beam at the collision point.

We first discuss an orbit (transverse position) shift of the strong beam given as:

$$\Delta x_{i+1} = (1 - 1/\tau)\Delta x_i + \delta x \cdot \hat{r}$$  \hspace{1cm} (1)

where $\Delta x_i$ is the orbit shift at the $i$th turn, $\tau$, $\delta x$ and $\hat{r}$ are the damping times, a constant characterizing the random fluctuation amplitude and a Gaussian random number with unit standard deviation. This is known as the Ornstein-Uhlenbeck process. This type of noise is referred to as first type later.

All particles in the weak beam experience the fluctuation of the strong beam, thus a transverse collective motion is induced. The collective motion results in an emittance growth due to filamentation caused by the nonlinear beam–beam force.

The stable amplitude of the fluctuation of the strong beam is given by:

$$\Delta x^2 = \langle \Delta x^2_{n \to \infty} \rangle = \frac{\tau \delta x^2}{2}.$$  \hspace{1cm} (2)

The correlation function between $i^{th}$ and $i^n$-th turns is expressed by the damping time as:

$$\langle \Delta x_i \Delta x_{i+n} \rangle = \Delta x^2 e^{-|n|/\tau}.  \hspace{1cm} (3)$$

The damping time is regarded as the correlation time of the fluctuation. For white noise, which corresponds to $\tau = 1$, the correlation function is expressed as:

$$\langle \Delta x_i \Delta x_{i+n} \rangle = \Delta x^2 \delta_{n0}  \hspace{1cm} (4)$$

where $\delta_{n0}$ is the Kronecker delta.

The beam oscillates with the betatron frequency. We consider a second type of noise as:

$$\Delta x_{i+1} = (1 - 1/\tau)(\Delta x_i \cos \mu_o + \Delta p_i \sin \mu_o) + \delta x \cdot \hat{r} \hspace{1cm} (5)$$

$$\Delta p_{i+1} = (1 - 1/\tau)(-\Delta x_i \sin \mu_o + \Delta p_i \cos \mu_o) + \delta x \cdot \hat{r} \hspace{1cm} (6)$$

where $x$ and $p$ are the coordinate and canonical momentum normalized by the beta function, so that $J = (x^2 + p^2)/2$. $\mu_o = 2\pi \nu_o$ is the betatron tune multiplied by $2\pi$. In collision the offset causes an emittance growth. The stable dipole oscillation amplitude is expressed by the same equation as Eq. (2). The correlation function contains the betatron tune as:

$$\langle \Delta x_i \Delta x_{i+n} \rangle = \Delta x^2 e^{-|n|/\tau} \cos n\mu_o.  \hspace{1cm} (7)$$

We discuss the effect of noise for the cases of the LHC and High Luminosity-LHC. The parameters are listed in Table 1. The phenomena depend on the beam–beam parameter, the noise amplitude normalized by the beam size and the Piwinski angle.

EMITTANCE GROWTH DUE TO THE EXTERNAL NOISE

The emittance growth under an external noise and the non-linear force of the beam–beam interaction is discussed in [1, 2, 3]. Previous work is reviewed in this section.

The beam–beam potential for a bunch population $N_p$ and the transverse size $\sigma_r$ is expressed as:

$$U(x) = \frac{N_p r_p}{\gamma_p} \int_0^{\infty} \frac{1 - e^{-x^2/(2\sigma_r^2 + q)}}{2\sigma_r^2 + q} dq  \hspace{1cm} (8)$$

where $r_p$ and $\gamma_p$ are the classical radius of the proton and the relativistic factor of the (weak) beam, respectively. The potential is expanded as a Fourier series:

$$U(x) = \frac{N_p r_p}{\gamma_p} \sum_{k=0}^{\infty} U_k(a) \cos 2k\psi  \hspace{1cm} \text{(9)}$$

where

$$U_k(a) = \int_0^a \left[\delta_{0k} - (2 - \delta_{0k})(-1)^k e^{-w} I_k(w)\right] \frac{dw}{w}  \hspace{1cm} (10)$$

and $a = \beta^* J/2\sigma_r^2 = J/2\epsilon$. The change of $J$ per revolution is given by the derivative of the beam–beam potential with respect to $\psi$ as:

$$\Delta J = -\frac{\partial U}{\partial \psi} = \frac{N_p r_p}{\gamma} \sum_{k=0}^{\infty} 2kU_k \sin 2k\psi.  \hspace{1cm} (11)$$
This change, which indicates a stable sinusoidal modulation of the betatron amplitude, does not induce emittance growth.

We consider the case in which the strong beam has a small offset (Δx). The beam–beam potential with the offset is expanded for Δx:

\[ U(x + \Delta x) = U(x) + U'(x)\Delta x. \]  

Here Δx is a random variable fluctuating described by Eq. (1) or (6).

The potential with the offset is expanded as a Fourier series:

\[ U'(J, \psi) = \frac{\partial U}{\partial J} \frac{\partial J}{\partial x} + \frac{\partial U}{\partial \psi} \frac{\partial \psi}{\partial x} \]

\[ = \frac{N_p r_p}{2\gamma \sigma_r} \sum_{k=0}^{\infty} G_k(a) \cos(2k + 1)\psi. \]

The Fourier coefficients as a function of a are expressed as:

\[ G_k(a) = \sqrt{a} [U'_k + U''_k] + \frac{1}{\sqrt{a}} [(k + 1)U_{k+1} - kU_k]. \]

where \( U'_k \) is the derivative with respect to \( a \).

The diffusion of \( J^2 \) after N revolutions is given by:

\[ \langle \Delta J^2(N) \rangle = \frac{1}{N} \sum_{\ell=1}^{N} \sum_{n=-\ell+1}^{\ell-1} \frac{\partial U'(\ell)}{\partial \psi} \frac{\partial U'(\ell + n)}{\partial \psi} \langle \Delta x_{\ell} \Delta x_{\ell+n} \rangle \]

For turn-by-turn white noise, the correlation function is replaced by the Kronecker delta, \( \delta_{n,0} \). The diffusion of \( J \) is expressed by:

\[ \langle \Delta J^2 \rangle = \frac{\langle \Delta J^2(N) \rangle}{N} \approx \frac{N^2 \sigma_p^2}{8\gamma^2 \sigma_r^4} \sum_{k=0}^{\infty} (2k + 1)^2 G_k(a)^2. \]

The diffusion of \( J \) per revolution is given for the fluctuation in Eq. (1) by:

\[ \langle \Delta J^2 \rangle \approx \frac{N^2 \sigma_p^2}{8\gamma^2 \sigma_r^4} \]

\[ \sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} (2k + 1)^2 G_k^2 \cos((2k + 1)n\mu_o)e^{-|n|/\tau} \]

\[ \approx \frac{N^2 \sigma_p^2}{8\gamma^2 \sigma_r^4} \sum_{k=0}^{\infty} (2k + 1)^2 G_k(a)^2 \sinh 1/\tau \]

\[ \left[ \frac{1}{\cosh 1/\tau - \cos(2(k + 1)\mu_o)} + \frac{1}{\cosh 1/\tau - \cos(2(k + 1)\mu_o + \delta \mu)} \right] \]

where \( \delta \mu \) is the tune difference between the weak and strong beam oscillations (\( \delta \mu = \mu - \mu_o \)).

Figure 1 shows the diffusion rate of \( J \) as a function of \( J \). The diffusion rate is proportional to the square of the fluctuation amplitude \( \Delta x \) and the square of the beam–beam parameter \( N_p \). The rate is normalized by the factor \( C = (N_p r_p \Delta x/\gamma \sigma_r)^2/8 \) in the figure.
Figure 1 shows that the rate is proportional to $J$ for small $J/2\varepsilon < 2$. The slope of $\langle \Delta J^2 \rangle$ for turn-by-turn noise ($\tau = 1$) is:
\[
\langle \Delta J^2 \rangle \approx \frac{N_p r_p^2}{8 \gamma^2 \sigma_r^2} \Delta x^2 \times 4.4.
\]
(21)

The luminosity degradation rate per collision is estimated by the emittance growth rate as:
\[
\Delta L/L = \left( \frac{\Delta x}{\sigma_r} \right)^2 \times 21.7.
\]
(22)

For two IPs, the formula is corrected by a factor two, i.e. $21.7 \rightarrow 10.8$ and $\xi \rightarrow \xi_{tot}$. The tolerance for the noise amplitude is given for a luminosity life time $\Delta L/L = 10^{-3}$:
\[
\xi_{tot} \frac{\Delta x}{\sigma_r} = 9.8 \times 10^{-6}.
\]
(23)

We now discuss the second type of noise given by Eq. (6). Figure 2 shows the diffusion rates. Figures 2 (a) and (b) are given for the beam-orbit oscillation with the same tune ($\delta \mu = 0$) and a difference of $\delta \mu = \xi = 0.01$, respectively. A strong enhancement of the diffusion is seen at small amplitudes at a large correlation time in shown in Fig. 2 (a). This behavior mainly comes from a contribution at $k = 0$.
\[
\langle \Delta J^2 \rangle \approx \frac{N_p r_p^2}{16 \gamma^2 \sigma_r^2} G_0(a)^2 \tau
\]
(24)

The strong beam modulation with the same tune gives an external force oscillation to the weak beam particles. For colliding beams, the assumption, that beam-orbit oscillations have the same tune, is not obvious. The diffusion rate for $\delta \mu = \xi$ in Fig. 2 (b) may be better to represent the beam–beam system. The diffusion rate, which is saturated at $J/2\varepsilon = 1$, is similar as that of $\tau = 1$ on the whole. Therefore we study the diffusion rate for $\tau = 1$ in simulations.

It may be better that the noise effects are studied in the framework of a strong-strong model, especially for the second type of noise. The noise induces either coherent $\sigma$ or $\pi$ modes or a continuous frequency spectrum. The $\sigma$ mode does not contribute the emittance growth. Emittance growth based on the strong-strong model had been discussed in [3]. The author discussed that 18% of the dipole growth based on the strong-strong model had been discussed in [3]. The author discussed that 18% of the dipole growth based on the strong-strong model had been discussed in [3]. The author discussed that 18% of the dipole growth based on the strong-strong model had been discussed in [3]. The author discussed that 18% of the dipole growth based on the strong-strong model had been discussed in [3].

We now discuss weak-strong simulations taking into account external noise. The weak beam is represented by 131072 macro-particles. The particles are tracked one million turns interacting a strong beam located at two interaction points. The luminosity is calculated turn-by-turn, and averaged every 100 turns. Luminosity degradation is evaluated by fitting its evolution.

Figure 3 shows the emittance growth given by Eq. (25) and by a strong-strong beam–beam simulation [4], where the beam–beam tune shift is $\xi = 0.0034/IP$. The results agree fairly well. The strong-strong simulation suffers from numerical noise related to the statistics of macro-particles. One million macro-particles are used in the simulation, thus 0.1% of the offset noise is induced by the statistics.

**SIMULATION OF EXTERNAL NOISE**

**Study based on LHC**

The analytical theory is based on the near solvable system far from resonances. There is no such limitation in beam–beam simulations, while simulations take considerable computing time to evaluate a slow emittance growth. Simulations considering external noise are straightforward: a modulation is applied to the strong beam with Eq. (1) or (6). Effects of resonances, longitudinal motion and a crossing angle are taken into account in simulations.

We only discuss weak-strong simulations taking into account external noise. The weak beam is represented by 131072 macro-particles. The particles are tracked one million turns interacting a strong beam located at two interaction points. The luminosity is calculated turn-by-turn, and averaged every 100 turns. Luminosity degradation is evaluated by fitting its evolution.

Figure 4 shows the luminosity degradation for collisions without a crossing angle. The degradation is plotted as a
function of the fluctuation amplitude for three total beam–beam parameters, $\xi_{\text{tot}} = 0.02, 0.04$ and 0.05. Three lines given by the analytical formula Eq. (23) are shown in the figure. The simulation results agree with the formula fairly well.

There was no qualitative change for collision without a crossing angle. For $\xi_{\text{tot}} = 0.035$, a degradation due to the crossing angle is seen, but a significant cross-talk is not observed. The degradation of the luminosity due to the fluctuation depends on $\xi_{\text{tot}}$, but hardly on the presence of the crossing angle.

**High Luminosity LHC (HL-LHC)**

For the HL-LHC, a higher luminosity is the target and obtained by increasing the bunch population and squeezing to smaller beta function, while the pile up of collision events sets an upper limit for the luminosity at $L_{\text{coll}} = 2.6 \times 10^{31}$ cm$^{-2}$s$^{-1}$. The luminosity at $\beta = 0.15$ m is expected to be $L_{\text{coll}} = 8.6$ or $18 \times 10^{31}$ cm$^{-2}$s$^{-1}$ for the bunch population of 2.2 or $3.5 \times 10^{11}$, respectively. Therefore, luminosity leveling keeping the luminosity constant at $L_{\text{coll}} = 2.6 \times 10^{31}$ cm$^{-2}$s$^{-1}$ is proposed. The level-

![Figure 3: Emittance growth given by Eq. (25) and by a strong-strong beam–beam simulation.](image)

![Figure 4: Diffusion rate given by a weak-strong simulation using Eq. (23).](image)

![Figure 5: Diffusion rate for crossing collision given by weak-strong simulation.](image)

![Figure 6: Luminosity degradation as a function of the beam–beam parameter with offset noise.](image)
leveling can be done by controlling the crab cavity voltage or the beta function at the interaction point (IP). Leveling with the beta function, the total beam–beam parameter (2IP) is \(0.011 \times 2 = 0.022\) (25ns) or \(0.014 \times 2 = 0.028\) (50ns) at the early stage of the collision, where the beta function is 0.49 m or 1.02 m. The results given in the previous subsection are applied for the parameters:

\[
\frac{\Delta x}{\sigma_x} = 4.5 \times 10^{-4} \text{ or } 3.5 \times 10^{-4}. \tag{26}
\]

for 25 ns and 50 ns, respectively.

Using a levelling with crab cavities, the crab voltage increases to keep the luminosity constant while the beam current decreases. At the early stage of collision, the crab voltage is low and two beams collide with a large Piwinski angle, where \(\phi_c/2\sigma_r = 3.14\) or 2.87 for 25 ns or 50 ns, respectively. We study the effects of noise for collisions with a large Piwinski angle.

Figure 7 shows the luminosity degradation rate as a function of the offset amplitude. The simulation is performed for two IPs. The tune shift is 0.0015 or 0.0050 in the crossing or orthogonal plane for the design bunch population of \(N_p = 2.2 \times 10^{11}\) (25ns). The tune shift is 0.0065 in both planes, due to the combination of the horizontal and vertical crossing. The fluctuation amplitude 0.2% is a tolerable limit for \(\Delta L/L_0 = 10^{-9}\) as shown in the figure. The simple formula Eq. (23) is satisfied for the HL-LHC, 0.0065 \(\times\) 0.002 = \(1.3 \times 10^{-5}\), with 30% difference from the formula.

Using beta function levelling the beam–beam parameter \(\xi = 0.02\) in this paper. The relation of the phase noise and collision offset is given by

\[
\Delta \varphi_{cc} = \frac{\omega_{cc}}{c\phi_c/2} \Delta x,
\]

where \(\Delta \varphi_{cc}\) and \(\omega_{cc}\) are the phase fluctuation and frequency of the crab cavity.

Using beta function levelling the beam–beam parameter is very high, \(\xi_{tot} = 0.022\) or 0.028 for 25 ns or 50 ns, respectively. The tolerance of the noise amplitude is given by Eq. (26). The corresponding phase error is \(\Delta \varphi = 1.6 \times 10^{-4}\) or \(2.3 \times 10^{-4}\) rad.

For the crab cavity levelling, the beam–beam parameter is \(\xi_{tot} = 0.0065\). The tolerance of the noise amplitude is \(\Delta x/\sigma_x = 0.002\) and the corresponding phase error is \(\Delta \varphi = 4 \times 10^{-5}\), where the crab angle is 10% of the crossing angle \(\phi_c = 59\) rad \((L/coll = 2.7 \times 10^{11} \text{ cm}^{-2}\text{s}^{-1})\).

The crab cavity noise was measured at KEKB, \(1.7 \times 10^{-4}\) rad for frequencies above 1 kHz \((\tau < 10)\). The value is critical for beta function levelling, because of the large beam–beam parameter. Using four crab cavities, the noise tolerance is twice as large, while for the crab voltage levelling, the measured phase error is tolerable.

**Incoherent noise due to intra-beam scattering**

Emittance growth rates due to intra-beam scattering (IBS) are 105 h and 63 h for the horizontal and longitudinal planes, respectively, in the nominal LHC [13]. The transverse emittance and bunch population in the nominal are \(5.0 \times 10^{-10}\) and \(1.15 \times 10^{11}\), respectively. The horizontal IBS growth rate is approximately proportional to the particle density in the six dimensional phase space. The growth time is 40 h for \(\xi_{tot} = 0.02\) in this paper \((\varepsilon = 2.7 \times 10^{-10} \text{ and } N_p = 1.63 \times 10^{11})\). The fluctuation is \(\delta x/\sigma_x = 5.5 \times 10^{-3}\) for \(\xi_{tot} = 0.05\). The luminosity degradation is determined by geometrical emittance growth \(\delta L/L_0 = \delta x^2/\sigma_x^2\) for incoherent noise.
COHERENT BEAM–BEAM EFFECTS UNDER EXTERNAL NOISE

Effects of external noise in crab cavity were performed in KEKB during 2008 and 2009 [6]. Sinusoidal noise is applied to the crab cavity RF system. Near the $\sigma$ mode tune, a strong luminosity drop of 80% was seen when suddenly exceeding a threshold excitation amplitude. A smaller luminosity drop ($L = 0.9L_0$) was seen near $\pi$ mode frequency. Strong-strong simulations reproduced these luminosity drops. A systematic study using the strong-strong simulation showed that these characteristic phenomena for coherent nonlinear beam–beam interactions. A similar phenomenon was observed in Ref. [7]. The detailed analysis is published in Ref. [6].

CONCLUSIONS

Fast noise of the collision offset degrades the luminosity performance in hadron colliders. The luminosity degradation depends on the product of the noise amplitude and the beam–beam parameter as shown in Eq. (23), with little dependence on the crossing angle. A tolerance for the crab cavity phase error was obtained for the HL-LHC.

The crab cavity noise was measured at KEKB, $1.7 \times 10^{-4}$ rad above 1 kHz ($\tau < 10$). The value is critical for beta function levelling, because of the high beam–beam parameter. For the crab voltage levelling, the measured phase error is tolerable because of the small beam–beam tune shift.

Further studies related to beam–beam modes should be done using strong-strong models.

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