Phase-retrieval methods with applications in composite-material tomography

Anna Burvall,1 Daniel H Larsson,1 Ulf Lundström,1 Fredrik Stig,2 Stefan Hallström,2 and Hans M Hertz1

1Biomedical and X-ray Physics, Royal Institute of Technology (KTH), Albanova, SE-106 91 Stockholm, Sweden
2Dept. of Aeronautical and Vehicle Engineering, Royal Institute of Technology (KTH), SE-100 44 Stockholm, Sweden
E-mail: anna.burvall@biox.kth.se

Abstract. In-line phase-contrast x-ray imaging is emerging as a method for observing small details when the contrast in absorption x-ray imaging is low. It gives images with strong edge enhancement, and phase retrieval is necessary to obtain quantitative thickness information. In particular for tomography, clarity can be enhanced by phase retrieval, as here demonstrated on a 3D-weave reinforced composite material. Seven suitable phase-retrieval methods are identified and integrated into a single method, where each version is marked by variations in particular steps. The general method and its variations are outlined and a comparison shows which methods are most suitable in different situations.

1. Introduction
In-line phase-contrast imaging [1] is a method for visualizing phase for hard x-rays used, e.g., in material studies [2]. As illustrated in figure 1, the distance between object and detector allows phase variations to develop into intensity differences. The resulting image shows the position of object details very clearly marked by their edges, but to get quantitative information on e.g. object density distribution, phase-retrieval [3] is needed. This post-processing can be of special interest in tomography, as here demonstrated on composite-material samples. We compare phase-retrieval methods intended for use on tomographic data [4].

Figure 1. Illustration of in-line phase contrast, and of phase retrieval.
An object, as a function of three-dimensional position \( r \), is characterized by its refractive index 
\[ n(r) = 1 - \delta(r) + i \beta(r) \]
where \( \delta(r) \) describes refractive or phase properties, whereas \( \beta(r) \) describes absorption and is related to the absorption coefficient \( \mu \) as 
\[ \mu = 4\pi \beta / \lambda, \]
where \( \lambda \) is the wavelength. For true phase retrieval \( \delta(r) \) and \( \beta(r) \) should be recovered independently, but this cannot be done from a single phase-contrast image as it does not contain enough information. It could be done from two images captured at different distances from the object, but this is inconvenient in x-ray tomography. Instead, a relation between \( \delta(r) \) and \( \beta(r) \) is assumed. Two options are in use: that there is no absorption, i.e. \( \beta(r) = 0 \), or that \( \delta(r) \) and \( \beta(r) \) are proportional to each other. In some cases those assumptions are accurate: for example the second assumption is true if the object is homogeneous, or for light materials at high photon energies. In other cases, such as multi-material samples at lower photon energies, the assumptions are inaccurate but still used for lack of other options.

The scope of this comparison [4] is limited to methods that use a single image. Furthermore, we limit it to analytical methods for speed, and we only consider monochromatic x-rays in order to compare as many methods as possible. Monochromatic theory in this case works surprisingly well for broadband x-rays. We identify seven different phase-retrieval methods in the literature, all summarized in the next section. We use one of them on a 3D-weave reinforced fibre composite sample, and finally outline how phase-retrieval methods are chosen in different situations.

2. Summary of methods

**Table 1.** Methods of phase retrieval and their properties. The functions \( g(I) \), \( H_p(w) \) and \( f(g_F) \) are introduced in Eq. 1. Table from Burvall et al.[4].

| Method                  | \( g(I) \)                                      | \( H_p(u, v) \)                                      | \( f(g_F) \)                                      | Assumptions |
|------------------------|-------------------------------------------------|----------------------------------------------------|--------------------------------------------------|-------------|
| Bronnikov              | \( \frac{I}{I_{in}} - 1 \)                      | \( [2\pi \lambda d |w|^2]^{-1} \)                       | \( g_F \)                                         | \( \beta = 0, F \gg 1 \) |
| Modified Bronnikov     | \( \frac{I}{I_{in}} - 1 \)                      | \( [2\pi \lambda d |w|^2 + \alpha]^{-1} \)         | \( g_F \)                                         | \( \beta \approx 0, F \gg 1 \) |
| Phase-att. duality     | \( \frac{I}{I_{in}} \)                          | \( [2\pi \frac{r_\lambda d}{\sigma K N} |w|^2 + 1]^{-1} \) | \( \frac{\lambda r_\lambda}{\sigma K N} \ln g_F \) | \( \beta \propto \delta, F \gg 1, \text{SVP} \) |
| Single material        | \( \frac{I}{I_{in}} \)                          | \( [4\pi^2 d F_\lambda |w|^2 + 1]^{-1} \)         | \( -\frac{1}{\mu} \ln g_F \)                     | \( \beta \propto \delta, F \gg 1 \) |
| Two materials          | \( \frac{I_{exp[\mu_2 A(r) \lambda]}}{I_{in}} \) | \( [4\pi^2 d F_\lambda |w|^2 + 1]^{-1} \)         | \( -\frac{1}{\mu_1 - \mu_2} \ln g_F \)           | \( \beta \propto \delta, F \gg 1, \text{SVP} \) |
| Fourier method, Born type | \( \frac{1}{2} \left( \frac{I}{I_{in}} - 1 \right) \) | \( [\sin (\pi \lambda d |w|^2)]^{-1} \) | \( g_F \)                                         | \( \beta = 0, \text{Born} \) |
|                        | \( \frac{1}{2} \ln \frac{I}{I_{in}} \)         | \( [\gamma \cos (\pi \lambda d |w|^2) + \sin (\pi \lambda d |w|^2)]^{-1} \) |                                               | \( \beta \propto \delta, \text{Born} \) |
| Fourier method, Rytov type | \( \frac{1}{2} \ln \frac{I}{I_{in}} \)         | \( [\sin (\pi \lambda d |w|^2)]^{-1} \) | \( g_F \)                                         | \( \beta = 0, \text{Rytov} \) |
|                        | \( \frac{1}{2} \ln \frac{I}{I_{in}} \)         | \( [\gamma \cos (\pi \lambda d |w|^2) + \sin (\pi \lambda d |w|^2)]^{-1} \) |                                               | \( \beta \propto \delta, \text{Rytov} \) |

The seven methods included in this comparison are 1) the Bronnikov method [5], 2) the modified Bronnikov algorithm by Grosset al. [6], 3) the phase-attenuation duality algorithm by Wu et al. [7], 4) the method for homogeneous object or single material by Paganin et al. [8], 5) the two-material method by Beltran et al. [9], and the Fourier method in 6) the Born or 7) the
Rytov approximation derived by Gureyev et al. [10]. Number 6, the Fourier method in Born approximation, has also been derived by Zabler [11], Guigay [12], and Turner [13] under slightly different approximations. Modifications to this method have been suggested by Moosmann [14].

All these methods follow the same pattern – the phase $\varphi(r_\perp)$, where $r_\perp$ is the two-dimensional space coordinate in the plane transverse to the direction of propagation, is retrieved from registered intensity $I(r_\perp)$ as

$$
\varphi(r_\perp) = f \left( F^{-1} \left\{ H(w) \cdot F \{ g[I(r_\perp)] \} \right\} \right)
$$

(1)

where $F$ denotes Fourier transform and $w$ spatial frequency. The three functions $g(I)$, $H(w)$, and $f(gF)$ will differ between the different methods, and are summarized in Table 1. For derivation and details on method parameters, see Ref. [4]. All methods use the Fresnel approximation; the other assumptions listed in Table 1 are large Fresnel numbers ($F \gg 1$), slowly varying phase (SVP), a Born-type approximation (Born), and a Rytov-type approximation (Rytov).

3. Examples of phase retrieval

![Figure 2](image_url)

**Figure 2.** Reconstructed slices of a 3D-weave reinforced composite sample a) without phase retrieval, b) using the single-material method for $\delta/\mu = 6.8 \cdot 10^{-9}$ m and c) $\delta/\mu = 1.5 \cdot 10^{-8}$ m.

A 3D-weave reinforced composite sample, consisting of vinyl ester resin surrounding carbon fibre bundles and some air voids, has been imaged using phase-contrast x-ray tomography. This is a multi-material sample, but the single-material phase-retrieval method has still been applied to demonstrate how the parameter $\delta/\mu$ can be adjusted to reconstruct interfaces between different materials. A liquid-metal-jet x-ray source [15] with Galinstan as anode material, operated at 40 W with a 50 kV acceleration voltage, was used. The source-to-object distance was 625 mm and the object-to-detector distance 2375 mm. The detector is a Gadox scintillator-based fiber-coupled CCD detector from Photonic Science with $9 \times 9$ $\mu$m$^2$ pixels and a total area of $36 \times 24$ mm$^2$, and images are binned $4 \times 4$ times before phase retrieval and the subsequent tomographic reconstruction using filtered back-projection (FBP). A slice through the tomographic reconstruction is shown in figures 2 and 3 with and without phase retrieval. For lower values of $\delta/\mu$, as in figures 2b or 3a, the air-vinyl interface is properly reconstructed, while edge amplification is still visible for vinyl-carbon. For higher values of $\delta/\mu$, as in figures 2c or 3b, the vinyl-carbon interface is well reconstructed but the air-vinyl interface smeared. This
shows particularly well around the voids in figure 3, which are quite well reconstructed for a low \( \delta/\mu \) but less visible for a higher value.

Figure 3. Zoom-in of the regions indicated in figure 2 for a) \( \delta/\mu = 6.8 \cdot 10^{-9} \) m and b) \( \delta/\mu = 1.5 \cdot 10^{-8} \) m.

4. Conclusions: choosing your phase-retrieval method

Comparison of the different methods, based mainly on the approximations and assumptions of their derivations but also on testing their noise sensitivity, lead to a simple scheme for choosing a reconstruction method [4], as illustrated in figure 4.

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