Quantum Pattern Recognition of Classical Signal

Chao-Yang Pang

Sichuan Key Lab of Software, Sichuan Normal University, Chengdu 610068, P.R. of China and
College of Mathematics and Software Science, Sichuan Normal University, Chengdu 610068, P.R. of China

Cong-Bao Ding

College of Physics and Electronic Engineering, Sichuan Normal University, Chengdu 610068, P.R. of China

Ben-Qiong Hu

College of Information Management, Chengdu University of Technology, 610059, P.R. of China

Abstract

It’s the key research topic of signal processing that recognizing genuine targets real time from the disturbed signal which has giant amount of data. A quantum algorithm for pattern recognition of classical signal which has time complexity $O(\sqrt{N})$ is presented in this paper.

Keywords: Pattern recognition, Grover’s algorithm, Rotation on subspace
I. INTRODUCTION

It's the key research topic of signal processing that recognizing genuine targets real time from the disturbed signal which has giant amount of data. A quantum pattern recognition of classical signal is presented. To understand the idea of this paper easily, the example of designing quantum pattern recognition to detect the saturated raid from armada is presented, and all contents are focused on this example.

Saturated raid is the topical raid mean of an armada, which many missiles, many planes, many spurious weapons generated by electromagnetic wave, and et. al. will be appeared at a same. Defender’s first task is to recognize the genuine space targets real time from the signal which is captured by phased array radar. This is a hard problem for classical computer. Peter Shor’s quantum algorithm [1] and Grover’s quantum search algorithm [2] show the high efficiency of quantum computer. Is there fast quantum algorithm to detect the saturated raid from an armada and recognize genuine space targets at real time?

Pang presented a quantum loading scheme (QLS) [3] to make quantum computer compatible with classical memory. The QLS can load all giant signal data captured by phased array radar into quantum registers at a time. Quantum image match algorithms and the method rotation on subspace show that image recognition is possible [4, 5, 6, 7]. Quantum discrete Fourier transform with classical output (QDFT) shows that quantum digital signal processing (especially for radar signal) is possible [8].

Introduction of Quantum Loading Scheme $U_L$ [3]: Suppose an arbitrary record (or data) $record_i$ is stored in classical database with the corresponding index $i$. The function of unitary operation $U_L$ can be described as

$$\frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle |0\rangle) |ancilla\rangle \xrightarrow{U_L} \frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle |record_i\rangle) |ancilla\rangle$$

(1)

, where ancillary state $|ancilla\rangle$ is known.

That is, unitary operation $U_L$ loads all information of records stored in a classical database into quantum state. Unitary operation $U_L$ has time complexity $O(logN)$ (unit time: phase transform and flipping the qubits of registers) [3]. Operator $U_L$ is so fast that its running time can be ignored when analyzing the time complexity of an algorithm.

Introduction of Quantum Search Algorithm with Complex Computation (i.e., the Method of Rotation at Subspace) [4, 5, 6, 7]:

2
Grover’s algorithm can find a database record according to the given index. However, database search is complex in general. E.g., police often hopes to find a mug shot from the database in which many sample photos are stored by the method of matching every sample photo and the photo captured by the vidicon at the entrance of airport at real time. Grover’s algorithm is invalid for this kind of search case because the coupling between search and other computation (e.g., image matching) is required at this case. Pang et.al. presents a quantum method named "rotation at subspace" \[4, 5, 6, 7\] to generalize Grover’s algorithm to the search case with arbitrary complex computation, that is derived from the research of quantum image compression \[4\]. The method of rotation at subspace is described as following briefly:

First, All input datum are stored in classical memory as database records. Assume that total number of records is \(N\). All these records can be loaded into a superposition of state using quantum loading scheme \(U_L\).

Second, construct the general Grover iteration (GGI) \(G_{\text{general}}\) as

\[
G_{\text{general}} = (2|\xiangle\langle\xi| - I)(O_c)^\dagger O_f O_c U_L
\]

where \(O_c\) denotes computation oracle such as image matching, \(f\) denotes the judge function (i.e., if the output of \(O_c\) satisfies some conditions, let \(f = 1\), else \(f = 0\)), \(O_f\) is the oracle of the judge function, and \(|\xi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle\).

Third, similar to Grover’s algorithm, let unitary operation \(G_{\text{general}}\) act on initial state \(\frac{1}{\sqrt{N}}(\sum_{i=0}^{N-1} |i\rangle_{\text{registers1}})|0\rangle_{\text{registers2}} O(\sqrt{N})\) times, we will find the optimal solution.

II. THE PROPOSED ARCHITECTURE OF MULTI-PATTERN RECOGNITION TO DETECT SATURATED RAID

Target pattern set is denoted by

\[P = \{t_i| i = 0, 1, ..., n - 1\}\]

, where \(n\) is the number of elements in \(P\) and \(t_i\) is the information of target pattern, such as image captured by radar.

Spurious pattern set is denoted by

\[S = \{s_j| j = 0, 1, ..., s - 1\}\]
where $s$ is the total number of elements in $S$ and $s_j$ is the information of spurious pattern which is generated by enemy’s electromagnetic wave.

We often have $s + n \neq 2^x$, where $x$ denotes some integer. If $s + n \neq 2^x$, add some virtual patterns $v \in V$ for the condition $|P \cup S \cup V| = 2^x$ to simplify computation.

Let the whole set 

$$I = P \cup S \cup V$$

, and the number of elements of set $I$ is $N$.

We always depend on feature to recognition target pattern in set $P$. Assume that all characteristics are collected as set

$$C = \{c_{-2}, c_{-1}, c_0, c_1, \cdots, c_{m-1}\}$$

, where $m \leq n$ in general.

The computation of the element of set $C$ depends on the computation method of feature. E.g., we can calculate out the affine and projective invariants \cite{9} (or other topology invariants) of bomb carrier or aircraft carrier as the pattern characteristics, which are not changed by disturbing signal in theory.

The mapping between pattern set $I$ and feature $C$ (i.e., $g : I \mapsto C$) is denoted as

$$g(x_i) = \begin{cases} 
  c_{i_k}, & x_i \in P \\
  c_{-1}, & x_i \in S \\
  c_{-2}, & x_i \in V 
\end{cases}$$

, where $0 \leq i_k < m$. The feature of spurious pattern (i.e., $x_i \in S$) is denoted by $c_{-1}$ and the feature of the meaningless virtual pattern (i.e., $x_i \in V$) is denoted by $c_{-2}$.

We often have a sample set (or codebook) about all target patterns previously, which is stored as a database \cite{4, 5, 6, 7, 10}. Assume that the sample set is

$$CB = \{(t_i, c_{ik})|k = 0, 1, ..., m - 1\}$$

, where $c_{ik}$ is the feature of target pattern.

The whole quantum multi-pattern recognition proposed in this paper is described as follows:

First, radar captures the signals and they are stored in memory, denoted as set $P \cup S$. Store all datum of set $I = P \cup S \cup V$ in classical memory with indices as a database.
Second, load all information of data in set $I$ into a state using QLS. Third, for a given feature stored in $CB$, design the following quantum algorithm to judge if the feature is hidden in the state and recognize it. Using the same method to full search the sample set $CB$, all target patterns will be recognized.

**FIG. 1:** The Architecture of Quantum Multi-Pattern Recognition

### III. QUANTUM MULTI-PATTERN RECOGNITION

First, construct the following data structure (DS) and unitary operations:

**DS1.** Save all elements of set $I$ in classical memory, and these records have indices $i = 0, 1, ..., N - 1$.

**DS2.** Construct six registers to denote the information of the element in set $I$. The six registers have data format

$$|\alpha\rangle_{\text{register1}}|c_{i_0}\rangle_{\text{register2}} \otimes |i\rangle_{\text{register3}} \otimes |x_i\rangle_{\text{register4}} \otimes |c_i\rangle_{\text{register5}} \otimes |d(c_i, c_{i_0})\rangle_{\text{register6}}$$

That is, 1st, 2nd, 3rd, 4th, 5th, and 6th register are used to save the input parameter $\alpha$, the pattern characteristics $c_{i_0} \in CB$ which required to be extracted from the superposition of state, the index of the element of $I$, pattern $x_i \in I$, the feature of pattern $x_i$, and the similarity measurement value between $c_{i_0}$ and $c_i$.

Let initial state is

$$|\psi_0\rangle = |\alpha\rangle|c_{i_0}\rangle|0\rangle|0\rangle|0\rangle$$
Hardmard transform $H$ acts on $|\psi_0\rangle$ will generate the following state:

$$|\psi_0 \rangle \xrightarrow{H} |\psi_1\rangle = \frac{1}{\sqrt{N}} |\alpha\rangle |c_{i_0}\rangle (\sum_{i=0}^{N-1} |i\rangle |0\rangle |0\rangle)$$

**DS3.** Construct loading operation $U_L$ as:

$$|\alpha\rangle |c_{i_0}\rangle |i\rangle |0\rangle |0\rangle \xrightarrow{U_L} |\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |0\rangle |0\rangle$$

All information of set $I$ will be loaded into registers by operation $U_L$.

**DS4.** Design oracle $O_c$ to compute the pattern feature, i.e.,

$$|\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |0\rangle |0\rangle \xrightarrow{O_c} |\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |c_i\rangle |0\rangle$$

If the computation of oracle $O_c$ is complex, decompose it as many simple oracles. That is supported by the method of rotation at subspace $[4, 5, 6, 7]$.

**DS5.** Design oracle $O_d$ to compute the similarity measurement value between $c_{i_0}$ and $c_i$, i.e.,

$$|\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |c_{i_0}\rangle |0\rangle |0\rangle \xrightarrow{O_d} |\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |c_{i_0}\rangle |d(c_i, c_{i_0})\rangle$$

**DS6.** Design oracle $O_f$ to mark the target patterns:

$$|\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |c_{i_0}\rangle |d(c_i, c_{i_0})\rangle \xrightarrow{O_f} (-1)^{f(i)}|\alpha\rangle |c_{i_0}\rangle |i\rangle |x_i\rangle |c_{i_0}\rangle |d(c_i, c_{i_0})\rangle$$

where $f(i) = \begin{cases} 1 & \text{if } 0 \leq d(c_i, c_{i_0}) \leq \alpha \\ 0 & \text{otherwise} \end{cases}$.

**DS6.** Construct pattern recognition iteration $G_{pr}$:

According to Eq[2], $G_{pr}$ is

$$G_{pr} = (2|\xi\rangle \langle \xi| - I)(O_dO_cU_L)^\dagger O_fO_dO_cU_L$$

Second, design the following quantum multi-pattern recognition algorithm to find a target pattern for that $0 \leq d(c_i, c_{i_0}) \leq \alpha$.

Multi-Quantum Pattern Recognition Algorithm:

**Step1.** Initialize $m = 1$ and set $\lambda = 6/5$. (Any value of $\lambda$ strictly between 1 and 4/3 would do.)

**Step2.** Choose $j$ uniformly at random among the nonnegative integers smaller than $m$. 

6
Step 3. Apply $j$ iterations of $G_{pr}$ acting on state $|\psi_1\rangle = \frac{1}{\sqrt{N}} |\alpha\rangle |c_{i_0}\rangle (\sum_{i=0}^{N-1} |i\rangle |0\rangle |0\rangle$.

Step 4. Observe the 3rd register: let $i_0$ be the outcome.

Step 5. Calculate value $d(c_i, c_{i_0})$ using classical computation. If $0 \leq d(c_i, c_{i_0}) \leq \alpha$, preserve $i_0$ and the signal $x_{i_0}$ captured by phased array radar is target pattern, and exit.

Step 6. Otherwise, set $m$ to $\min(\lambda m, \sqrt{N})$ and go back to step 2.

The above quantum multi-pattern recognition algorithm is similar to BBHT algorithm [11], which is the improved algorithm of Grover’s algorithm. And the main difference between BBHT algorithm and the presented algorithm in this paper is that Grover iteration is replaced by pattern recognition iteration $G_{pr}$ that realizes the coupling between quantum search and the computation of pattern recognition. The above algorithm has time complexity $O(\sqrt{\frac{N}{M}})$ [11], where $M$ denotes the number of target pattern that satisfy the condition $0 \leq d(c_i, c_{i_0}) \leq \alpha$. The above presented algorithm is also suitable to the case in which $N$ is not a big integer according to Long’s research [12].

IV. CONCLUSION

It’s the key research topic of signal processing that recognizing genuine targets real time from the disturbed signal which has giant amount of data. A quantum algorithm for pattern recognition of classical signal which has time complexity $O(\sqrt{N})$ is presented in this paper. Quantum discrete Fourier transform [8] shows quantum signal processing is possible. This paper shows that quantum pattern recognition (or quantum image recognition) for classical signal is also possible.

Acknowledgments

The first author thanks his teacher prof. G.-C. Guo and the Key Lab. of Quantum Information, USTC because the first author is brought up from the lab. The first author thanks prof. Z. F. Han for that he encourages and helps the first author up till now. The first author thanks assistant prof. Xudong Huang who is at Harvard Uni. for his interest at the author’s research topic of quantum image compression and quantum image recognition. The discussion between prof. Huang and the first author makes the first author decide to
open this paper soon.

[1] P.W. Shor. Algorithms for quantum computation discrete log and factoring. In Proc. of the 35th Annual Symposium on the Foundations of Computer Science, pages 20–24, Los Alamitos, CA, 1994. IEEE Computer Society Press.

[2] Lov K. Grover. A fast quantum mechanical algorithm for database search. In Proc. of 28th Annual ACM Symposium on the Theory of Computing, pages 212–218, Philadelphia, Pennsylvania, 1996. ACM Press.

[3] Chao-Yang Pang. Loading N-dimensional vector into quantum registers from classical memory with O(logN) steps. arXiv:quant-ph/0612061, 2006.

[4] Chao-Yang Pang. Quantum image compression. Postdoctoral report, Key Laboratory of Quantum Information, University of Science and Technology of China (CAS), Hefei, China, Jun 2006.

[5] Pang Chao-Yang, Zhou Zheng-Wei, Chen Ping-Xing, and Guo Guang-Can. Design of quantum vq iteration and quantum vq encoding algorithm taking sqrt(n) steps for data compression. CHIN. PHYS., 15(3):618–623, 2006.

[6] Chao-Yang Pang, Zheng-Wei Zhou, and Guang-Can Guo. Quantum discrete cosine transformation for image compression. arXiv:quant-ph/0601043, 2006.

[7] Pang Chao-Yang, Zhou Zheng-Wei, and Guo Guang-Can. A hybrid quantum encoding algorithm of vector quantization for image compression. CHIN. PHYS., 15(12):3039–3043, 2006.

[8] Chao-Yang Pang and Ben-Qiong Hu. Quantum Discrete Fourier Transform with Classical Output for Signal Processing. arXiv:0706.2451, 2007.

[9] Chong-Shan Luo, Chao-Yang Pang, and Yu-Ping Tian. Higher Geometry. Higher Education Press, Beijin, China, 2006.

[10] Shao-Qi Bian, Xue-Gong Zhang, and et.al.. Pattern Recognition. Tsinghua University Press, Beijin, China, 2000.

[11] M. Boyer, G. Brassard, P. Hoyer, and A. Tap. Tight bounds on quantum searching. arXiv:quant-ph/9605034, 1996.

[12] G. L. Long. Grover algorithm with zero theoretical failure rate. PHYSICAL REVIEW A, 64(2):022307/1–4, 2001.