Study on the effects of the light CP-odd Higgs via the leptonic decays of pseudoscalar mesons

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Abstract

To explain the anomalously large decay rate of $\Sigma^+ \to p + \mu^+\mu^-$, it was proposed that a new mechanism where a light CP-odd pseudoscalar boson of $m_{A_1^0} = 214.3$ MeV makes a crucial contribution. Later, some authors have studied the transition $\pi^0 \to e^+e^-$ and $\Upsilon \to \gamma A_1^0$ in terms of the same mechanism and their result indicates that with the suggested mass one cannot fit the data. This discrepancy might be caused by experimental error of $\Sigma^+ \to p + \mu^+\mu^-$ because there were only a few events. Whether the mechanism is a reasonable one motivates us to investigate the transitions $\pi^0 \to e^+e^-$; $\eta(\eta') \to \mu^+\mu^-$; $\eta_c \to \mu^+\mu^-$; $\eta_b \to \tau^+\tau^-$ within the same framework. It is noted that for $\pi^0 \to e^+e^-$, the standard model (SM) prediction is smaller than the data, whereas the experimental central value of $\eta \to \mu^+\mu^-$ is also above the SM prediction. It means that there should be extra contributions from other mechanisms and the contribution of $A_1^0$ may be a possible one. Theoretically calculating the branching ratios of the concerned modes, we would check if we can obtain a universal mass for $A_1^0$ which reconcile the theoretical predictions and data for all the modes. Unfortunately, we find that it is impossible to have such a mass with the same coupling $|g_\ell|$. Therefore we conclude that the phenomenology does not favor such a light $A_1^0$, even though a small window is still open.

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1 Introduction

Searching for new physics beyond the standard model (SM) is the goal of not only the very high energy experiments such as at LHC and even the future ILC, but also the machines of lower energies where new physics signals may be revealed at rare processes. The HyperCP collaboration observed an anomalously large decay rate of $\Sigma \rightarrow p + \mu^+ \mu^-$ which is higher than the prediction of SM by several standard deviations[1]. The discrepancy may be attributed to contributions from new physics. A very possible mechanism is that the extra contribution is due to a light CP-odd pseudoscalar. Indeed, many models suggests its existence. Among all the models the supersymmetric model where there are five Higgs bosons remain after the symmetry breaking is the most favorable one. Namely there are two CP-even scalars $H^0$ and $h^0$, a CP-odd $A^0$ and two charged Higgs bosons $H^\pm$[2].

The search for SM Higgs boson has already spanned for almost half century and covered a rather large regions. Recently, at LHC an excess at 126 GeV is observed and it could be the signal of Higgs, even though firm identification still needs time[3][4][5]. So far, as well known, the SM Higgs must be heavy, but a Higgs demanded by new physics beyond the SM might be light. Phenomenological search for beyond SM Higgs would be an interesting job of theorists and experimentalists. For example, Kao et al investigate the FCNC process $t \rightarrow c\phi$ with the two Higgs Doublet Model at LHC [6] where a heavy scalar or pseudoscalar Higgs $\phi$ of about 130 GeV contributes. On other aspect, it does not exclude the possibility that a light CP-odd pseudoscalar boson might exist, but it definitely is not the SM Higgs. To explain the large decay rate $\Sigma \rightarrow p + \mu^+ \mu^-$ which should be very small if only the SM applies, He, Tandean and Valencia [7] suggested that a light CP-odd $A_1^0$ of mass 214.3 MeV may result in the observed data. Later Chang and Yang applies the same mechanism to evaluate the branching ratios of $\pi^0 \rightarrow e^+ e^-$ while considering a constraint from $B(\Upsilon \rightarrow \gamma A_1^0)$ [8]. They noticed that the SM prediction is lower than the experimental data[9], therefore there should be some extra contributions from the mechanisms which have not been considered yet or are due to new physics beyond SM. Chang and Yang calculated the contribution of the light $A_1^0$, but combining
the constraints from the anomalous magnetic moment of muon and \( B(\Upsilon \rightarrow \gamma A_1^0) \) on \( A_1^0 \), the authors concluded that rigorous constraints on the mass of \( A_1^0 \) and the concerned parameter \( |g_\ell| \) enforced by \( B(\pi^0 \rightarrow e^+e^-) \) and \( B(\Upsilon \rightarrow \gamma A_1^0) \) rule out the mass of 214.3 MeV. Furthermore by fitting data, if the mechanism does make a substantial contribution, one should have \( m_{A_1^0} \sim m_\pi \) and \( |g_\ell| = 0.10 \pm 0.08 \).

Considering that the HyperCP collaboration only recorded a few events for \( \Sigma \rightarrow p + \mu^+\mu^- \), thus relatively large experimental uncertainties could be expected, it is natural to ask if a mass range of \( A_1^0 \) at vicinity of \( m_\pi \) can remarkably enhance the rate of \( \Sigma \rightarrow p + \mu^+\mu^- \)? Moreover, since the new contribution to \( \pi^0 \rightarrow e^+e^- \) is realized via mediating a Higgs-like boson in the s-channel, its coupling to the lepton is proportional to its mass, so that the contribution is suppressed by the electron mass. And due to the phase space restriction, \( \pi^0 \) cannot decay into heavier muons.

In fact, we notice that for similar decay modes, the SM predictions on \( \eta \rightarrow \mu^+\mu^- \) is below the experimental central value, and there are no data available yet for the modes \( \eta' \rightarrow \mu^+\mu^- \), \( \eta_c \rightarrow \mu^+\mu^- \), \( \eta_b \rightarrow \tau^+\tau^- \) etc., and we will show below that they are important for determining if the scenario of \( A_1^0 \) works. The observation may hint that there could exist an unknown mechanism(s) which can make up the gap between the SM prediction and data. Existence of a light \( A_1^0 \) definitely is a reasonable candidate. Thus in this work, we are going to carry out a wider study on the the modes in terms of the theory which involves a light CP-odd boson \( A_1^0 \) originating from the NMSSM theory and the concerned couplings with fermions is:

\[
\mathcal{L}_{A_1^0 q\bar{q}} = \left( \sum_{u\text{-type}} l_u m_u \bar{u}\gamma_5 u + \sum_{d\text{-type}} l_d m_d \bar{d}\gamma_5 d \right) \frac{i A_1^0}{v}, \tag{1}
\]

\[
\mathcal{L}_{A_1^0 \ell\ell} = \frac{i g_\ell m_\ell}{v} \bar{\ell}\gamma_5 \ell A_1^0, \tag{2}
\]

where, \( l_d = -g_\ell = v\delta_-/(\sqrt{2}x) \) and \( l_u = l_d / \tan^2 \beta \).

Our strategy is whether we can find a mass range as well as the parameter \( |g_\ell| \) (see the text for detail), which can tolerate all the observed modes, namely a universal \( A_1^0 \) mass

\[1\text{The experimental error for } B(\eta \rightarrow \mu^+\mu^-) \text{ is large, so that making a definite conclusion needs more precise measurement which will be coming soon.} \]
can make the gaps between the SM predictions and the data.

The paper is arranged as follows. In section 2, we present the necessary theoretical derivations. In section 3, our numerical results are shown in relevant tables and figures. We reserve the last section for our discussion and conclusion.

2 Formalism

To serve our aim of this work, we concentrate ourselves on the application of the light CP-odd pseudoscalar boson $A^0_1$ in the NMSSM. Since the leptonic decays of the pseudoscalar mesons $\pi^0$, $\eta$, $\eta'$, $\eta_c$, $\eta_b$ are less contaminated by the non-perturbative QCD effects, they are ideal for studying the new mechanism. As aforementioned, unlike $\pi^0 \to e^+e^-$, the contributions of $A^0_1$ to the decay modes $\eta(\eta') \to \mu^+\mu^-$, $\eta_c \to \mu^+\mu^-$ and $\eta_b \to \tau^+\tau^-$ do not severely suffer from the mass suppression. Then, pre-assuming the new mechanism, by fitting data we would check if we can obtain a universal mass for $A^0_1$ which reconciles all the modes.

2.1 For leptonic decays of light pseudoscalar mesons

In the SM sector, the dominant contribution to $\eta \to \mu^+\mu^-$ comes from the QED anomaly and the Feynman diagram is shown in Fig.1. For completeness, we re-derive the formulae given in Ref.[8, 11] and show them in this text. The total contribution of the triangle-diagram is written as:

$$
\mathcal{M}_{\gamma\gamma} = i e^2 \int \frac{d^D q}{(2\pi)^D} \frac{L^{\mu\nu} H_{\mu\nu}}{(q^2 + i\varepsilon)((q - p)^2 + i\varepsilon)((q - k_1)^2 - m_\ell + i\varepsilon)},
$$

(3)
where

\[ L^{\mu\nu} = \bar{u}(k_1, s)\gamma^\mu(k_1 - \ell + m_\ell)\gamma^\nu v(k_2, s') \], \tag{4} \\
\[ H^{\mu\nu} = i e^2 \epsilon_{\mu\nu\alpha\beta} q^\alpha (p - q)^\beta f_{\gamma\gamma} F_{\gamma\gamma}(q^2, (p - q)^2) \]. \tag{5} \\

Explicitly, \( H^{\mu\nu} \) is the effective \( \eta\gamma\gamma \) vertex where the Lorentz structure includes a form factor related to the loop integral. The form factor, as usual, can be decomposed into a numerical coupling constant \( f_{\gamma\gamma} = 1/(4\pi^2 f_\eta) \) times a function \( F_{\gamma\gamma}(q^2, (p - q)^2) \).

For the lepton part \( L^{\mu\nu} \), we employ the projection operator technique [11]:

\[
\mathcal{P}(p - k_1, k_1) = \frac{1}{\sqrt{2}} [v(p - k_1, +) \otimes \bar{u}(k_1, -) + v(p - k_1, -) \otimes \bar{u}(k_1, +)] \\
= \frac{1}{2\sqrt{2}p^2} [-2m_\ell p_\mu \gamma^\mu \gamma^5 + \frac{1}{2} \epsilon_{\mu\nu\sigma\tau} (k_1^\sigma (p - k_1)^\tau - (p - k_1)^\sigma p^\tau) \sigma^{\mu\nu} + \frac{p^2}{2} \gamma^5]. \tag{6} \\
\]

Then we have the amplitude[4] for Fig.(1):

\[
\mathcal{M}_{\gamma\gamma}(\eta \to \ell^+ \ell^-) = -2\sqrt{2} \alpha^2_{em} m_\eta f_{\gamma\gamma} A^\ell (m_\eta^2). \tag{7} \\
\]

where \( A^\ell(p^2) \) is the reduced amplitude:

\[
A^\ell(p^2) = \frac{2i}{p^2} \int \frac{d^4q}{\pi^2} \frac{q^2p^2 - (q \cdot p)^2}{(q^2 + i\varepsilon)((q - p)^2 + i\varepsilon)((q - k_1)^2 - m_\ell^2 + i\varepsilon)} F_{\gamma\gamma}(q^2, (p - q)^2). \tag{8} \\
\]

This is the same as the result given in Ref.[11], and in the derivation, we also utilize the same form factor \( F_{\gamma\gamma}(q^2, (p - q)^2) \) therein.

Besides the QED contribution, there exists a tree level contributions induced by exchanging weak interaction gauge boson \( Z^0 \) and a new CP-odd pseudoscalar boson \( A_1^0 \) at s-channel and the Feynman diagrams are shown in Fig.(2).

\[
\mathcal{M}_{Z^0} = \langle \ell^+ \ell^- | \bar{\ell} V_{\ell Z^0} - \frac{i}{p^2 - m_{Z^0}^2 + i\varepsilon} q V_{q Z^0} q | \eta \rangle, \tag{9} \\
\]

where \( q \) stands for the light quarks \( u, d \) and \( s, p = k_1 + k_2, V_{\ell Z^0} \) and \( V_{q Z^0} \) are the interaction vertices of \( \bar{\ell} \ell Z^0 \) and \( qq Z^0 \)[12].

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2Here, we have added the missing minus according the corrections to M. E. Peskin’s QFT book: http://www.slac.stanford.edu/ mpeskin/QFT.html#errors.
It is well known that the physical pseudoscalar particles $\eta$ and $\eta'$ are mixtures of the flavor eigenstates $\eta_q$ and $\eta_s$\cite{13}:

$$\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) ,$$  \hspace{1cm} (10) \\
$$\eta_s = s\bar{s} ,$$  \hspace{1cm} (11)

as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} ,$$  \hspace{1cm} (12)

where $\phi$ is the mixing angle.

Our conventions of the decay constants $f^q_\eta$ and $f^s_\eta$ are taken from Ref.\cite{13, 14}.

$$\langle 0 | J^\mu_{\mu5} | \eta \rangle = i f^j_\eta p_\mu \ (j = q, s) ,$$  \hspace{1cm} (13)

where $q$ stands as the lighter quarks $u$ and $d$, $f^q_\eta = \cos \phi f^q$, $f^s_\eta = -\sin \phi f^s$ and $p_\mu$ is the four-momentum of $\eta$.

After a straightforward calculation, we obtain the contributions of the weak interaction sector to the amplitude:

$$M^u_{Z^0} = \frac{2\sqrt{2}e^2 f^q_\eta m_\ell m_\eta}{(\sin\theta_W \cos\theta_W)^2 p^2 - m_Z^2} ,$$  \hspace{1cm} (14a) \\
$$M^d_{Z^0} = -\frac{2\sqrt{2}e^2 f^s_\eta m_\ell m_\eta}{(\sin\theta_W \cos\theta_W)^2 p^2 - m_Z^2} ,$$  \hspace{1cm} (14b)

where the projection operator for outgoing lepton pair is employed.

In the NMSSM, the light CP-odd pseudoscalar Higgs $A^0_1$ couples to up-, down-type quarks and leptons. Following the general notation of Ref.\cite{7}, one can write the amplitude in terms of the effective couplings Eqs.(11) and (2). As generally suggested in literature...
that by fitting available data, tan β takes a larger value, thus the coupling constant \( l_u \) in Eq. (11) is much suppressed and the contribution of \( u \)-type quarks to the amplitude in the NP part is negligible. Then the extra contribution of for \( d \)-type quarks to the amplitude reads:

\[
M_{A_1}^{d} = -\frac{l_d g_f \eta f_{\eta} m_d m_{\eta}}{2\sqrt{2}v^2} \frac{1}{m_{\eta}^2 - m_{A_1}^2 + i m_{A_1} \Gamma_{A_1}} \bar{u}(k_1)\gamma_5 v(k_2),
\]

where \( \Gamma_{A_1} \) is the total width of \( A_1^0 \). When deriving Eq. (15), we utilize the relation,

\[
\langle 0|\bar{d}\gamma_5 d|\eta \rangle = -i \frac{f_{\eta} m_{\eta}^2}{2\sqrt{2}m_d}, \quad \langle 0|s\gamma_5 s|\eta \rangle = -i \frac{f_{\eta} m_{\eta}^2}{2m_s},
\]

where \( f_{\eta} \) and \( f_{\eta} \) are defined in Eq. (13).

We can further reduce \( M_{A_1}^{d} \) into:

\[
M_{A_1}^{d} = -\frac{l_d g_f \eta f_{\eta} m_d m_{\eta}^3}{2v^2} \frac{1}{m_{\eta}^2 - m_{A_1}^2 + i m_{A_1} \Gamma_{A_1}}.
\]

By the aforementioned notation, one can easily obtain \( M_{A_1}^{s} \) as \( \sqrt{2}M_{A_1}^{d} \frac{f_{\eta}}{f_{\eta}} \).

The total contribution is a sum of all the individual ones:

\[
M_{tot} = M_{\gamma\gamma} + M_{Z_0}^{u,d,s} + e^{i\theta_{NP}} M_{A_1}^{u,d,s},
\]

where \( \theta_{NP} \) represents a possible relative phase between the contributions of SM and NMSSM.

The total decay rate of \( \eta \rightarrow \mu^+\mu^- \) is expressed as:

\[
\Gamma_{tot}(\eta \rightarrow \mu^+\mu^-) = \frac{1}{8\pi m_{\eta}^2} |M_{tot}|^2,
\]

where \( k \) is the three-momentum of one of the leptons in the rest frame of \( \eta \).

### 2.2 For decays of heavy pseudoscalar mesons

Generally, when calculating the anomaly and decay rate of \( \pi^0 \rightarrow \gamma\gamma \), for simplification, one can use an approximation \( q \rightarrow 0 \) [15]. This approximation works well for decays of
light pseudoscalar mesons, but definitely not for heavy pseudoscalar mesons with \( q^2 = M^2 \gg 0 \) where \( M \) stands as the mass of the heavy meson. Thus, we take another approach to take into account the effects induced by the hadronic structure of the decaying heavy meson.

For decay of \( \eta_b \) into lepton pairs, we employ the light-cone distribution amplitude (LCDA) method to calculate the transition amplitude \( \mathcal{M}_{\gamma\gamma} \). The leading-order contributions induced by the photon-Fermion loop are displayed in Fig. 3.

Figure 3: The QED contributions to the \( \eta_b \to \ell^+\ell^- \) decay via box diagrams.

Again, for completeness, we re-derive the QED contribution to \( \eta_b \to \tau^+\tau^- \), and the corresponding formula was obtained for \( \eta_c \to \mu^+\mu^- \) in Ref. [12]. The contribution of the loop diagrams is:

\[
\mathcal{M}_{\gamma\gamma} = \mathcal{M}^A_{\gamma\gamma} + \mathcal{M}^B_{\gamma\gamma} = C_1 \bar{u}(k_1)\gamma_5 v(k_2) + C_2 \bar{u}(k_1)\sigma^{\mu\nu}v(k_2)\varepsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma,
\]

where

\[
\begin{align*}
C_1 &= \frac{G_F^2e^4}{8\pi^2}f_{\eta_b}m_\ell \int_0^1 du \phi(u, \mu) \int_0^1 dx \int_0^{1-x} dy \times \left\{ \frac{2}{D_1} + \frac{1 + (2u - 1)x - y}{D_2} + \frac{1 + (1 - 2u)x - y}{D_3} \right\}, \\
C_2 &= \frac{G_F^2e^4}{8\pi^2}f_{\eta_b}m_\ell \int_0^1 du \phi(u, \mu) \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1 - x - y - z}{D_4^2},
\end{align*}
\]

Here, the notions \( D_{1,2,3,4} \) and the concrete expressions for \( C_1 \) and \( C_2 \) are explicitly presented in Ref. [12]. For \( \eta_b \to \tau^+\tau^- \), the numerical results of \( C_1 \) and \( C_2 \) are displayed in Table [1].
Table 1: The coefficients $C_1$ and $C_2$ in Eq.(20) for $\eta_b \to \tau^+ \tau^-$.  

Our next step would be evaluating the hadronic matrix element $\langle 0 | \bar{b}_\alpha(x) b_\beta(y) | \eta_b \rangle$. Thus we need the wave function for the pseudo-scalars. According to Refs.\[12, 16, 17\], we employ the light-cone distribution amplitude for the calculation:

$$
\langle 0 | \bar{b}(x) b(y) | \eta_b(p) \rangle = -i f_{\eta_b} \int_0^1 du e^{-i(\bar{u} p + \bar{u} p y)} [\gamma_5]_{\beta \alpha} \phi(u, \mu),
$$

(23)

where $\bar{u} = 1 - u$, $\mu$ is the energy scale and $f_{\eta_b}$ is the decay constant of $\eta_b$ defined in Eq.(13). The wave function of $\eta_b$ is adopted as:

$$
\phi(u) = N 4 u (1 - u) e^{-\beta u(1 - u)},
$$

(24)

where $N$ is the normalization factor and the parameter is set as $\beta = 3.8 \pm 0.7$ [12].

3 Numerical Analysis

Firstly, we list some necessary input parameters which are taken from either the PDG book\[18\] or concerned literatures [19, 20, 21, 22]:

$$
\begin{align*}
&f_\eta^q = 1.07 f_\pi, \\
&f_\eta^s = 1.34 f_\pi, \\
&f_\pi = 130 \text{MeV}, \\
&f_{\eta_b} = 0.705 \text{MeV}, \\
&m_{\eta_b} = 9390.9 \pm 2.8 \text{MeV}, \\
&\Gamma_{\eta_b} = 10 \text{MeV}, \\
&\phi = (39.9 \pm 2.6(\exp) \pm 2.3(\text{the}))^o, \\
&\sin^2 \theta_W = 0.23116.
\end{align*}
$$

The reduced amplitude $A^\ell(m_{\eta}^2)$ in Eq.(8) is:

$$
A^\ell(m_{\eta}^2) = 2.61 - 5.21i, \quad A^\ell(m_{\eta'}^2) = 7.59 + 3.41i.
$$

(25)

For the New Physics part, we firstly discuss the relevant input parameters. The electroweak scale $\nu$ is 246 GeV. Furthermore, the coupling constant $|g_\ell|$ in Eq.(2) is stringently constrained by the muon anomalous magnetic moment[7]:

$$
|g_\ell| \lesssim 1.2.
$$

(26)
This bound results in an $A_1^0$ width $\Gamma_{A_1^0} \lesssim 3.7 \times 10^{-7}$ MeV. That implies that the coupling constant $l_d$ is also of order of unity which is consistent with the general estimates for the size of $v\delta_-(\sqrt{2}x)$.\textsuperscript{[23]}

Figure 4: The dependence of $B_{\text{tot}}(\eta \rightarrow \mu^+\mu^-)$ and $B_{\text{tot}}(\eta' \rightarrow \mu^+\mu^-)$ on the NP phase. The horizontal sold lines correspond to the results of the SM, respectively.

Generally, a relative phase between the SM and NP pieces can exist in the Lagrangian, and $\theta_{NP}$ should be determined by fitting data or from a larger symmetry which includes the SM and the concerned NP, in this work we treat it as a free parameter. We illustrate the dependence of $B_{\text{tot}}(\eta(\eta') \rightarrow \mu^+\mu^-)$ on this phase in Fig. 4. From this figure one can observe that, as setting $|g_\ell| = 0.35$, when $\theta_{NP}$ is around 0, $B_{SM}(\eta \rightarrow \mu^+\mu^-)$ is enhanced by the NP effect, whereas if $\theta_{NP}$ is about $\pi$, it decreases. Comparing the result of $\eta \rightarrow \mu^+\mu^-$ with the experimental data

$$B_{\text{Exp}}(\eta \rightarrow \mu^+\mu^-) = (5.8 \pm 0.8) \times 10^{-6}. \quad (27)$$

whose central value is above the SM prediction by about 20%, we are tempted to conclude that as the coupling constant is not very large ($|g_\ell| \lesssim 1.2$) as suggested in the literature, if one expects to substantially enhance theoretical prediction of $B_{SM}(\eta \rightarrow \mu^+\mu^-)$ to the experimental value [18] via the effect induced by $A_1^0$, the phase $\theta_{NP}$ should be around 0. Thus in following calculations we set $\theta_{NP}$ to be 0. By contrast, $B_{SM}(\eta' \rightarrow \mu^+\mu^-)$ decreases near 0 and almost reaches the maximum at $\pi$. Unfortunately, up to now, there is no any experimental data on $\eta' \rightarrow \mu^+\mu^-$, with $\theta_{NP} \approx 0$, its branching ratio is predicted, so that the future experiments may hint us if the scenario works.
Therefore, by fitting the experimental data of $\mathcal{B}_{\text{Exp}}(\eta \rightarrow \mu^+\mu^-) = (5.8 \pm 0.8) \times 10^{-6}$ while taking $|g_\ell| = 0.35$ and $\theta_{NP} = 0$, the proper mass of $A^0_1$ should be 547.5 MeV.

In order to give a better insight, we draw the dependence of $\mathcal{B}_{\text{tot}}(\eta(\eta') \rightarrow \mu^+\mu^-)$ on $|g_\ell|$ and $m_{A^0_1}$ in Fig.(5).

![Figure 5: The dependence of $\mathcal{B}_{\text{tot}}(\eta \rightarrow \mu^+\mu^-)$ and $\mathcal{B}_{\text{tot}}(\eta' \rightarrow \mu^+\mu^-)$ on the coupling constant $|g_\ell|$. For the $\eta$ decay, the dashed line, black line and dot-dashed line correspond to $m_{A^0_1} = 214.3$ MeV, 500 MeV and 1 GeV respectively. For the $\eta'$ decay, the black line, dashed line and dot-dashed line correspond to $m_{A^0_1} = 547.5$ MeV, 1 GeV, and 2 GeV respectively. The phase angle is chosen as $\theta_{NP} = 0$. The abscissas are the results of the SM.

From Fig.(5), we observe that as $|g_\ell|$ is not very large and $\theta_{NP} = 0$, the NP effect for $\eta \rightarrow \mu^+\mu^-$ induced by existence of $A^0_1$ can enhance the branching ratio to the experimental level, but a heavier $A^0_1$ with a mass about 1 GeV or more would not.

Based on these parameters employed in above text, we present all the numerical results and corresponding experimental data for $\eta(\eta') \rightarrow \mu^+\mu^-$ in Table.(2).

$\mathcal{B}(\eta_c \rightarrow \mu^+\mu^-)$ in SM was studied in Ref.[12] and its result is:

$$\mathcal{B}_{\text{SM}}(\eta_c \rightarrow \mu^+\mu^-) = 6.39^{+1.03}_{-0.89} \times 10^{-9}.$$  \hspace{1cm} (28)

With the same method we calculate $\mathcal{B}(\eta_b \rightarrow \tau^+\tau^-)$ in this work. However, since $l_u = l_d / \tan^2 \beta$, for a larger $\tan \beta$ which is usually considered in literature, the effect induced by $A^0_1$ on $\mathcal{B}(\eta_c \rightarrow \mu^+\mu^-)$ is negligible. Indeed, if there were the experimental data for the decay of $\eta_c \rightarrow \mu^+\mu^-$, one could gain more information about $A^0_1$ by comparing it with $\mathcal{B}(\eta_b \rightarrow \tau^+\tau^-(\mu^+\mu^-))$. 

11
Then, using $m_{A^0_1} = 547.5$ MeV, we obtain the branching ratio predicted by the pure SM and NMSSM with this light CP-odd Higgs respectively. In analog to Fig.(4) and Fig.(5), we draw Fig.(6) for the heavy pseudoscalar meson $\eta_b$. Additionally, we give the numerical results in Table 2.

Combining with the upshots of Ref.[8] on $\pi^0 \rightarrow e^+e^-$, we list all the results of the leptonic decays of the pseudoscalar mesons $\pi^0$, $\eta$, $\eta'$, $\eta_c$, $\eta_b$ in Table(2).

| Decay | $B_{Exp}$ | $B_{SM}$ | $B_{tot}^{\theta=0}$ | $|g_\ell|$ | $m_{A^0_1}$(MeV) |
|-------|-----------|----------|-------------------|---------|------------------|
| $\pi^0 \rightarrow e^+e^-$ | $(7.48 \pm 0.38) \times 10^{-8}$ | $(6.25 \pm 0.09) \times 10^{-8}$ | $7.48 \times 10^{-8}$ | 0.35 | 134.95 |
| $\eta \rightarrow \mu^+\mu^-$ | $(5.8 \pm 0.8) \times 10^{-6}$ | $4.94^{+0.44}_{-0.45} \times 10^{-6}$ | $5.80^{+0.44}_{-0.45} \times 10^{-6}$ | 0.35 | 547.5 |
| $\eta' \rightarrow \mu^+\mu^-$ | $-1$ | $(1.27 \pm 0.42) \times 10^{-7}$ | $(1.26 \pm 0.42) \times 10^{-7}$ | 0.35 | 547.5 |
| $\eta_c \rightarrow \mu^+\mu^-$ | $-1$ | $6.39^{+1.03}_{-0.80} \times 10^{-9}$ | $6.39^{+1.03}_{-0.80} \times 10^{-9}$ | 0.35 | 547.5 |
| $\eta_b \rightarrow \tau^+\tau^-$ | $< 8\%$ | $5.56^{+0.44}_{-0.45} \times 10^{-9}$ | $6.67^{+0.44}_{-0.45} \times 10^{-9}$ | 0.35 | 547.5 |

Table 2: The branching ratios of pseudoscalars to a leptonic pair in SM and in NMSSM.
Figure 7: Comparison among the leptonic decays of the pseudoscalar mesons, where we have normalized each branching ratio with its SM prediction. The dot-dash line represents the experimental data and the shadowed region corresponds to the error band, where $|g_e| = 0.35$.

4 Discussion and Conclusion

The starting point of this work is that the SM prediction on $B(\pi^0 \to e^+e^-)$ is smaller than the data by a few standard deviations, and the large rate of $\Sigma \to p + \mu^+\mu^-$ is also beyond the SM prediction, thus there must be something new. But what is it, new physics or a mechanism hidden in the SM but not being taken into account? Definitely, it is not due to the final state interaction because the leptonic and semi-leptonic decays are not contaminated by non-perturbative QCD effects. Thus people are inclined to attribute the discrepancy to new physics effects. But then what new physics could it be?

To explain the anomalously large decay rate of $\Sigma^+ \to p + \mu^+\mu^-$, He, Tandean and Valencia proposed a new mechanism where a light CP-odd scalar boson $m_{A_1^0} = 214.3$ MeV exists[7]. Later, some authors studied the transition $\pi^0 \to e^+e^-$ in terms of the same mechanism and their result indicates that the suggested mass cannot fit the data[8]. This discrepancy might be caused by experimental error of $\Sigma^+ \to p + \mu^+\mu^-$ because there were only a few events. Whether the mechanism is a reasonable one motivates us to investigate the transitions $\pi^0 \to e^+e^-; \eta \to \mu^+\mu^-; \eta' \to \mu^+\mu^-; \eta_c \to \mu^+\mu^-; \eta_b \to \tau^+\tau^-$.
Looking at Fig.(7), one can notice several aspects.

1. Only the mass of $A_0^1$ is close to the mass of the decaying pseudoscalar boson, the branching ratio of the leptonic decay can be remarkably enhanced. That is due to the Breit-Wigner form of the propagator:

$$\frac{1}{q^2 - m_{A_0^1}^2 + i m_{A_0^1} \Gamma_{A_0^1}},$$

and $q^2 = m_{\pi^0, \eta, \eta', \eta_b}$.

2. Unless the mass of the light $A_0^1$ is close to the mass of the decaying pseudoscalar, its contribution to the leptonic decay is not sensitive to the mass of $A_0^1$ at all. On the right panel of Fig.(7), the abscissa corresponds to the decay of $\eta_b \rightarrow \tau^+ \tau^-$, which does not vary with respect to the change of $m_{A_0^1}$, even though the contribution of $A_0^1$ exists.

3. The experimental central value of $B(\eta \rightarrow \mu^+ \mu^-)$ is larger than the SM predictions, so that it implies that there could be additional contributions from some mechanisms which were not taken into account or from new physics beyond SM. On other aspect, the error is large, i.e. within two standard deviations, the SM prediction still coincides with the data. Thus if we take the central values seriously, we should search for new sources of the deviation. To further study, more accurate measurements are necessary. Moreover, so far, there are no data on $\eta' \rightarrow \mu^+ \mu^-$ available, the reason is obvious that $\Gamma(\eta' \rightarrow \mu^+ \mu^-)$ and $\Gamma(\eta \rightarrow \mu^+ \mu^-)$ have the same order of magnitude, but $\Gamma_{\text{tot}}(\eta') \gg \Gamma_{\text{tot}}(\eta)$.

4. The existence of a light $A_0^1$ might be the source, but our numerical results on the various leptonic decays of $\eta, \eta', \eta_b$ as well as $\pi^0$ indicate that we cannot find an universal mass for $A_0^1$ which can make the gaps between SM predictions and data for $\Gamma(\pi^0 \rightarrow e^+ e^-)$ and $\Gamma(\eta \rightarrow \mu^+ \mu^-)$ simultaneously.

5. Just as Chang and Yang indicated, if the mass of $A_0^1$ is close to pion mass, the gap between theoretical prediction and data may be filled out, but another serious problem is raised. Namely, as $\pi^0$ and $A_0^1$ have close masses, they should maximally mix according to the general principle of quantum mechanics, if so, the data on $\pi^0 \rightarrow \gamma \gamma$ would not be explained. Similarly, the argument can be applied to other pseudoscalar mesons. However,
from another aspect, as $\pi^0$ and $A_1^0$ have the same CP behavior and are close in mass, in the two-photon final state, it is hard to distinguish between them.

6. It is noted that the concerned experiments have larger errors, so that one may re-consider if we can reconcile the experimental data and theoretical predictions with the help of $A_1^0$. Since the measurement of $\pi^0 \rightarrow e^+e^-$ has a smaller error, let us assume that the mass of $A_1^0$ obtained by fitting $B(\pi^0 \rightarrow e^+e^-)$ is the right one, then with this value we re-examine $B(\Sigma \rightarrow p + \mu^+\mu^-)$ and we obtain it as $1.16 \sim 1.19 \times 10^{-7}$. Using the measured data $(3.1^{+2.4}_{-1.9} \pm 1.5) \times 10^{-8}$, He, Tandean and Valencia got $m_A^0$ as 214.3 MeV. The estimated branching ratio with a lighter $A_1^0$ is 4 to 5 times larger than what they estimated. As pointed above, there were only a few events, the errors may be large, so that we hope that our experimentalists can strive to obtain more accurate measurements on $B(\Sigma \rightarrow p + \mu^+\mu^-)$ which may provide valuable information about $A_1^0$. Moreover, if $m_A^0$ is close to 140 MeV, as we show in Fig.(7), its contribution to the amplitude of $\eta \rightarrow \mu^+\mu^-$ is almost a constant and negligible because the coupling of $A_1^0$ to $\mu^+\mu^-$ is proportional to $m_{\mu}$ which is small compared to $m_\tau$, meanwhile the measurement on $\eta \rightarrow \mu^+\mu^-$ also possesses a larger error range and within 2 standard deviations the SM prediction is consistent with the data, thus that data cannot exclude an $A_1^0$ of about 140 MeV. There are so far, no data for $\eta^\prime \rightarrow \mu^+\mu^-$, $\eta_c \rightarrow \mu^+\mu^-$ and $\eta_b \rightarrow \tau^+\tau^-$ available, as we show, an $A_1^0$ of about 140 MeV does influence their branching ratios, but remains as a constant. For $\eta_b \rightarrow \tau^+\tau^-$, the result of SM+NP is about 1.2 times larger than the SM prediction. By contrast, as discussed in the introduction, it does not affect $\eta_c \rightarrow \mu^+\mu^-$ at all.

Therefore, much more accurate measurements on $\Sigma \rightarrow p + \mu^+\mu^-$ and the leptonic decays of the pseudoscalar mesons are indeed badly needed.

As a conclusion, the phenomenology seems not to favor the light CP-odd $A_1^0$, even though does not exclude its existence and there exists a narrow window.

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