Dynamical breaking of $U(1)_R$ and supersymmetry in a metastable vacuum

Steven Abel, Callum Durnford, Joerg Jaeckel, Valentin V. Khoze

Institute for Particle Physics Phenomenology, Durham University, Durham DH1 3LE, United Kingdom

Abstract

We consider the metastable $\mathcal{N}=1$ QCD model of Intriligator, Seiberg and Shih (ISS), deformed by adding a baryon term to the superpotential. This simple deformation causes the spontaneous breaking of the approximate $R$-symmetry of the metastable vacuum. We then gauge the flavour $SU(5)_f$ and identify it with the parent gauge symmetry of the Standard Model (SM). This implements direct mediation of supersymmetry breaking without the need for an additional messenger sector. A reasonable choice of parameters leads to gaugino masses of the right order. Finally, we speculate that the entire “ISS $\times$ SM” model should be interpreted as a magnetic dual of an (unknown) asymptotically free theory.

The issue of supersymmetry (SUSY) breaking has recently been reinvigorated by Intriligator Seiberg and Shih [1] (ISS). Their observation – that metastable SUSY breaking vacua can arise naturally and dynamically in the low-energy limit of supersymmetric $SU(N)$ gauge theories – has important implications for our understanding of how SUSY is broken in nature. Following this work there has been exploration of both the cosmological consequences [2; 3; 4; 5; 6], and the possibilities for gauge or direct mediation of the SUSY breaking to the visible sector [1; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24; 25; 26; 27; 28]. On the cosmological side, the attractive feature of these models is that the metastable vacua are naturally long lived due to the flatness of the potential. Moreover, at high temperatures the SUSY breaking vacua are dynamically favoured over the SUSY preserving ones because they have more light degrees of freedom, so the early Universe would naturally have been driven into them [1]. On the phenomenological side, attention has focused on a striking aspect of metastability, namely that the models do not have an exact $U(1)_R$-symmetry, and indeed the $U(1)_R$-symmetry is anomalous under the same gauge group that dynamically restores the supersymmetry in the supersymmetric global minima.

In principle this allows one to evade the theorem by Nelson and Seiberg that SUSY breaking requires $R$-symmetry in a generic model (i.e. one that includes all couplings compatible with the symmetries) [32]. $R$-symmetry is unwelcome because it implies that gauginos are massless, so the fact that it can be broken by metastability is an encouraging sign. In a more recent paper [18]

\footnote{An additional cosmological development which is somewhat orthogonal to our discussion is the use of the models of ISS in supergravity to “uplift” supersymmetric models to small non-zero cosmological constant [29; 30; 31]. This possible phenomenon is relevant for larger SUSY breaking than that we will be considering here.}
it was emphasized that the relation between SUSY breaking and \( R \)-symmetry is a continuous one, in the sense that the lifetime of the metastable vacuum decreases in proportion to the size of any explicit \( R \)-symmetry breaking terms that one adds to the theory. This allows one to play the “approximate \( R \)-symmetry” game: add to the superpotential of the effective theory explicit \( R \)-symmetry breaking terms of your choosing, whilst trying to keep the metastable minimum as stable as possible. Such approximate \( R \)-symmetry models can then be motivated by appealing to (for example) higher dimensional operators of the underlying high energy physics.

Clearly there is some tension in this procedure. For example the gauge mediation scenario explored in refs. [12; 17] invokes a messenger sector (denoted by \( f \)). The field \( f \) has to have an explicit \( R \)-breaking mass-term to give gauginos a mass, and consequently a new SUSY restoring direction opens up along which \( f \) gets a vev. One is then performing a rather delicate balancing act: in order to avoid disastrously fast decay of the metastable vacuum, large SUSY breaking scales must be invoked so that the \( R \)-breaking mass can be sufficiently small. It should also be noted that here the \( R \)-symmetry breaking responsible for the globally supersymmetric minima of ISS models plays no direct role in the generation of gaugino masses, and consequently this is expected to be a generic problem for gauge mediation of metastable SUSY breaking. This is also a problem for the models that were constructed to implement direct mediation [14], and again, in those cases certain operators had to be forbidden by hand, making the superpotential non-generic.

To avoid these problems, the next option for generating non-zero gaugino masses would be to use the explicit \( R \)-breaking of the ISS model itself, associated with the metastability and the existence of a global supersymmetric groundstate. This is in fact a more difficult proposition than one might suppose for the following reason. At the metastable minimum there is an unbroken approximate \( R \)-symmetry (which is of course why it is metastable in the first place). The \( R \)-symmetry is explicitly (more precisely anomalously) broken only by the nonperturbative term,

\[
W_{np} \propto \left( \det N_f \Phi \right)^{\frac{1}{\mathfrak{N}}} \sim \Phi^{\frac{N_f}{\mathfrak{N}}},
\]

where \( \Phi \) is the meson field, \( SU(N)_{mg} \) is the gauge group of the magnetic theory, and \( N = N_f - N_c \) with \( SU(N_c) \) being the gauge group of the electric theory [1]. One might hope that this would induce (for example) \( R \)-symmetry breaking mass-terms that contribute to gaugino masses in perturbation theory. However such mass-terms will be typically of order \( \frac{\partial^2 W}{\partial \Phi^2} \sim \Phi^{2N_c - N_f} \). Thus since ISS models are valid in the interval \( N_c + 1 \leq N_f < \frac{3}{2}N_c \), they are exactly zero in the metastable minimum where \( \langle \Phi \rangle = 0 \).

We are led to an alternative – the focus of this paper – which is to spontaneously break the approximate \( R \)-symmetry of the ISS model to generate gaugino masses. The explicit breaking of the model then ensures that any \( R \)-axions get a mass and are made safe. The natural avenue to explore is to gauge (part of) the \( SU(N_f) \) flavour symmetry of the ISS model, identifying it with the Standard Model gauge groups. This would allow the quarks and mesons in the theory to mediate the SUSY breaking directly to the Standard Model, thereby avoiding the need for any messenger sectors which as we have seen are liable to destabilize the metastable vacuum. Once spontaneous \( R \)-breaking has been achieved, there is in principle nothing to prevent it being mediated via these fields to the rest of the model including the gaugino masses\(^2\). (Note that the nonperturbative explicit \( R \)-breaking can also now contribute to gaugino masses since \( \Phi \) will get a vev.)

In this paper we demonstrate that perfectly viable direct mediation of SUSY breaking can indeed be implemented in this way, by making the simplest deformation to the ISS model that one can imagine, namely the addition of a baryon term to the superpotential. This “baryon-

\(^2\)Indeed the very same point was made in Ref. [10] which was presented in the language of retrofitting. There however, successful mediation required a messenger sector which, in general, may lead to new and unstable directions.
gauge symmetry and $N$ to $SU$ indices and the 3...7 indices have a different status and the flavour symmetry is broken explicitly.

The dual theory has a $SU$ are forbidden by R-symmetry.

in the mesons that could arise from lower dimensional irrelevant operators in the electric theory by R-symmetry (as well as the gauge and flavor symmetries discussed above). Terms quadratic the parent $SU$ with the exception of the last term which is a baryon of $SU$ SU with the Standard Model gauge groups) acquires a vev at this point as well; the latter gives $R$-breaking masses to the magnetic quarks which are charged under the SM gauge groups. This enables them to act as messenger fields giving the gauginos masses at one-loop. We stress that all of this happens automatically upon adding a baryon. There is no need for any messenger sector outside the ISS model, and therefore no additional instability is induced. Moreover, we will show that the resulting gaugino masses can be naturally of the right order.

Let us begin by introducing our model which is based on the model of ISS with $SU(N_c)$ gauge symmetry and $N_f$ flavours of quark/anti-quark pairs in the electric theory. The magnetic dual theory has a $SU(N)_{mg}$ gauge symmetry, where $N = N_f - N_c$, $N_f$ flavours of fundamental quark/anti-quark pairs, and is IR free if $N_c + 1 \leq N_f < \frac{3}{2}N_c$. The minimal values consistent with this equation and leading to a non-trivial magnetic gauge group are $N_f = 7$ and $N_c = 5$ giving $SU(2)_{mg}$ in the magnetic dual theory. Now consider the following superpotential:

$$W = \Phi_{ij} \varphi_i \varphi_j - \mu_{ij}^2 \Phi_{ji} + m \varepsilon_{abc} \varepsilon_{rs} \varphi_a^b \varphi_r \varphi_s$$ (2)

where $i, j = 1...7$ are flavour indices, $r, s = 1, 2$ run over the first two flavours only, and $a, b$ are $SU(2)_{mg}$ indices (we set the coupling $\hbar = 1$ for simplicity). This is the superpotential of ISS with the exception of the last term which is a baryon of $SU(2)_{mg}$. Note that the 1,2 flavour indices and the 3...7 indices have a different status and the flavour symmetry is broken explicitly to $SU(2)_f \times SU(5)_f$. The $SU(5)_f$ factor will be gauged separately and will be identified with the parent $SU(5)$ of the SM.

The baryon deformation is the leading order deformation of the ISS model that is allowed by R-symmetry (as well as the gauge and flavor symmetries discussed above). Terms quadratic in the mesons that could arise from lower dimensional irrelevant operators in the electric theory are forbidden by R-symmetry.

Using the $SU(2)_f \times SU(5)_f$ symmetry, the matrix $\mu_{ij}^2$ can be brought to a diagonal form

$$\mu_{ij}^2 = \begin{pmatrix} \mu^2 & 0 \\ 0 & \tilde{\mu}^2 \end{pmatrix}.$$

We will assume that $\mu^2 > \tilde{\mu}^2$. The parameters $\mu^2$, $\tilde{\mu}^2$ and $m$ have an interpretation in terms of the electric theory: $\mu^2 \sim \Lambda m_Q$ and $\tilde{\mu}^2 \sim \Lambda \tilde{m}_Q$ come from the electric quark masses $m_Q$, $\tilde{m}_Q$, where $\Lambda$ is the Landau pole of the theory. The baryon operator can be identified with a corresponding operator in the electric theory. Indeed the mapping from baryons $B_E$ in the electric theory to baryons $B_M$ of the magnetic theory, is $B_M \Lambda^{-N_c} \leftrightarrow B_E \Lambda^{-N_f}$ (we neglect factors of order one). Thus one expects

$$m \sim M \left( \frac{\Lambda}{M} \right)^{2N_c - N_f} = \frac{\Lambda^3}{M^2}.$$ (4)

Where $M$ represents the scale of new physics in the electric theory at which the irrelevant operator $B_M$ is generated. We will think of it as being $M_P$ or $M_{GUT}$ although as we shall see a large range of values can be accommodated.

It is encouraging that this rather minimal choice of parameters allows us to identify $SU(5)_f$ flavour symmetry with the Standard Model gauge group. Thus the magnetic quarks $\varphi, \tilde{\varphi}$

---

1Note that the breaking of $SU(5)$ is assumed to take place or be included explicitly in the SM sector.

2It is also an amusing coincidence that the electric theory has the same gauge groups for colour and flavour, $SU(5)_f \times SU(5)_c$. 

---

3
decompose into 4 singlets (which we will call $\phi$, $\tilde{\phi}$) plus 2 fundamentals of $SU(5)_f$ (which we call $\rho$, $\tilde{\rho}$), while the magnetic mesons $\Phi_{ij}$ decompose into 4 fundamentals of $SU(5)_f$ ($Z$ and $\tilde{Z}$), an adjoint+trace singlet of $SU(5)_f$ ($X$), plus 4 more singlets ($Y$). The charges are as follows:

| $\Phi_{ij}$ | $SU(5)$ | $SU(2)_{mg}$ | $U(1)_R$ |
|-----------|----------|--------------|---------|
| $\Phi_{ij} \equiv \begin{pmatrix} Y & Z \\ \tilde{Z} & X \end{pmatrix}$ | $\begin{pmatrix} 4 \times 1 \square \end{pmatrix}$ | $\begin{pmatrix} 1 \ 1 \\ 1 \ 1 \end{pmatrix}$ | 2 |
| $\varphi \equiv \begin{pmatrix} \phi \\ \rho \end{pmatrix}$ | $\begin{pmatrix} 1 \square \end{pmatrix}$ | $\square$ | 1 |
| $\tilde{\varphi} \equiv \begin{pmatrix} \phi \\ \tilde{\rho} \end{pmatrix}$ | $\begin{pmatrix} 1 \square \end{pmatrix}$ | $\square$ | -1 |

At this point it is worth noting that, thanks to the baryon, the model has $R$-charges that are not 0 or 2. As discussed in Ref. [21] this condition is necessary for Wess-Zumino models to spontaneously break $R$-symmetry. Therefore, our model allows for spontaneous $R$ symmetry breaking and we will see in the following that this does indeed happen.

Now let us consider the potential at tree-level. The $F$-term contribution to the potential at tree-level is

$$V_F = \sum_{ar} |Y_{rs}\tilde{\phi}_s^a + Z_{ri}\tilde{\rho}_i^a + 2m\varepsilon_{ab}\varepsilon_{rs}\phi_b^a|^2$$

$$+ \sum \left[ |\tilde{Z}_{ir}\phi_i^a + X_{ij}\delta_j^a|^2 + \sum |\phi_i^aY_{rs} + \rho_j^a\tilde{Z}_{ik}|^2 + \sum |\phi_i^aZ_{rij} + \rho_j^aX_{ij}|^2 \right]$$

$$+ \sum_{rs} |(\phi_r\phi_s - \mu^2\delta_{rs})|^2 + \sum_{ri} |\phi_r\tilde{\rho}_i|^2 + \sum_{ri} |\rho_i\phi_s|^2 + \sum_{ij} |(\rho_i\tilde{\rho}_j - \hat{\mu}^2\delta_{ij})|^2$$

where $a$, $b$ are $SU(2)_{mg}$ indices. The flavor indices $r$, $s$ and $\hat{i}$, $\hat{j}$ correspond to the $SU(2)_f$ and $SU(5)_f$, respectively. It is straightforward to see that the rank condition works as in ISS; that is the minimum for a given value of $X, Y, Z$ and $\tilde{Z}$ is along $\rho = \tilde{\rho} = 0$ and

$$\langle \phi \rangle = \frac{\mu^2}{\xi} I_2$$

$$\langle \tilde{\phi} \rangle = \xi I_2,$$

where $\xi$ parameterizes a runaway direction that will eventually be stabilized by the Coleman-Weinberg contribution to the potential. This then gives

$$Z = \tilde{Z} = 0$$

but the pseudo-Goldstone modes $X = \chi I_5$ are undetermined. (Note that all the $D$-terms are zero along this direction and the $SU(2)_{mg}$ is Higgsed but $SU(5)_f$ is unbroken.) In addition $Y$ becomes diagonal and real (assuming $m$ is real). Defining $\langle Y_{rs} \rangle = \eta I_2$, the full potential is

$$V = 2 \left| \eta \xi + 2m\frac{\mu^2}{\xi} \right|^2 + 2 \left| \frac{\mu^2}{\xi} \right|^2 + 5\hat{\mu}^4.$$

Using $R$ symmetry we can choose $\xi$ to be real\(^5\). Minimizing in $\eta$ we find

$$\eta = -2m \left( \frac{\xi^2}{\mu^2} + \frac{\mu^2}{\xi^2} \right)^{-1}.$$

\(^5\)The phase of $\xi$ corresponds to the $R$-axion which will be dealt with later.
Substituting $\eta(\xi)$ into Eq. (8) we see that $\xi \to \infty$ is a runaway direction along which

$$V(\xi) = 8m^2\mu^2 \left( \frac{\xi^6}{\mu^6} + \frac{\xi^2}{\mu^2} \right)^{-1} + 5\mu^4.$$  

It is worth emphasizing that even in the limit $\xi \to \infty$, the scalar potential $V$ is non-zero, so we have a runaway to broken SUSY (a ‘pseudo-runaway’ in the language of \[33\]). Proceeding to one loop, the Coleman-Weinberg contribution to the potential is therefore expected to lift and stabilize this direction at the same time as lifting the pseudo-Goldstone modes $\chi$.

Let’s see how this works. Firstly, recall that the Coleman-Weinberg effective potential \[34\] sums up all one-loop quantum corrections into the following form:

$$V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2} \text{STr} M^4 \log \frac{M^2}{\Lambda_{\text{UV}}} \equiv \frac{1}{64\pi^2} \left( \text{Tr} m_0^4 \log \frac{m_0^2}{\Lambda_{\text{UV}}^2} - 2 \text{Tr} m_{1/2}^4 \log \frac{m_{1/2}^2}{\Lambda_{\text{UV}}^2} + 3 \text{Tr} m_1^4 \log \frac{m_1^2}{\Lambda_{\text{UV}}^2} \right)$$

where $\Lambda_{\text{UV}}$ is the UV cutoff\[4\], and the mass matrices are given by \[35\]:

$$m_0^2 = \begin{pmatrix} W_{abc} W_b^c + D^{\alpha a} D^c_\alpha + D^{\alpha a} D^c_\alpha & W_{abc} W_b^c + D^{\alpha a} D^c_\alpha + D^{\alpha a} D^c_\alpha \\ W_{abc} W_b^c + D^{\alpha a} D^c_\alpha + D^{\alpha a} D^c_\alpha & W_{abc} W_b^c + D^{\alpha a} D^c_\alpha + D^{\alpha a} D^c_\alpha \end{pmatrix}$$

$$m_{1/2}^2 = \begin{pmatrix} W_{abc} W_b^c + 2D^{\alpha a} D^c_\alpha & -\sqrt{2}W_{abc} W_b^c \\ -\sqrt{2}W_{abc} W_b^c & 2D^{\alpha a} D^c_\alpha \end{pmatrix}$$  \[12\]

$$m_1^2 = D^{\alpha a} D^c_\alpha + D^{\alpha a} D^c_\alpha.$$  \[13\]

As usual, $W_c \equiv \partial W/\partial \Phi^c$ denotes a derivative of the superpotential with respect to the scalar component of the superfield $\Phi^c$, and $D^\alpha$ are the appropriate $D$-terms, $D^\alpha = g_\alpha \partial_\alpha$. Of course, $D$-terms can be switched off by setting the gauge coupling $g = 0$, which we will do until further notice. All the above mass matrices will generally depend on field expectation values. The effective potential $V_{\text{eff}} = V_F + V_{\text{eff}}^{(1)}$ is the sum of the F-term (tree-level) and the Coleman-Weinberg contributions. To find the vacua of the theory we now have to minimize $V_{\text{eff}}$. The true, stable vacuum will be the global minimum, with other minima being only meta-stable.

It is interesting to note that as the 1-loop corrections are of a supertrace form, they vanish around supersymmetric vacua. If the runaway was to a supersymmetric vacuum at infinity, the Coleman-Weinberg corrections wouldn’t lift it. In our case, we have a runaway to a non-supersymmetric vacuum at infinity, so it is reasonable to expect that these loop corrections will modify the asymptotic behaviour.

Now let’s see how the classical runaway direction is lifted by quantum effects. We parameterize the pseudo-Goldstone and runaway field vacuum expectation values by

$$\langle \phi \rangle = \xi I_2$$

$$\langle Y \rangle = \eta I_2$$

$$\langle X \rangle = \chi I_5.$$  \[14\], \[15\]

These are the most general vevs consistent with the tree level minimization. It can be checked that at one loop order all other field vevs are zero in the lowest perturbative vacuum. By computing the masses of all fluctuations about this valley we can go about constructing the one-loop effective potential from eq. (11). We have done this numerically using Mathematica as well as V$\text{scape}$, a program specifically written to explore the properties of metastable vacua \[36\].

Table 1 shows the result of minimizing the vevs in the one-loop effective potential for some sample values of the parameters. As expected, the vevs in Eqs. (14), (15) are seen to approximately, i.e. up to small Coleman-Weinberg corrections, satisfy the analytic tree-level relations

\[\text{As usual we can “eliminate” } \Lambda_{\text{UV}} \text{ by trading it for a renormalization scale at which the couplings are defined.}\]
Figure 1: This plot demonstrates the stabilization in the $\xi$ direction. The red curve is the tree-level runaway potential. The purple is the Coleman-Weinberg contribution (we have added a constant shift of 5 to it). The blue line depicts the full stabilized potential. (We use $\mu = 4\hat{\mu}$, $m = 2\hat{\mu}$.)

| Model                  | $\xi/\hat{\mu}$ | $\kappa/\hat{\mu}$ | $\eta/\hat{\mu}$ | $\chi/\hat{\mu}$ |
|------------------------|-------------------|----------------------|-------------------|-------------------|
| Vscape Unconstrained   | 22.55451          | 0.709338             | $-0.125660$       | $-1.00041$        |
| Vscape Constrained     | 22.55581          | 0.709352†            | $-0.125671†$      | $-1.00132$        |
| Mathematica            | 22.5559           | 0.70935†             | $-0.12567†$       | $-1.0014$         |
| Gauged $SU(2)_{mg}$, $g = 0.4$ | 22.4385         | 0.71306†             | $-0.12699†$       | $-1.0115$         |

Table 1: Stabilized vevs for different models: $\mu = 4\hat{\mu}$, $m = 2\hat{\mu}$. The values$^{\dagger}$ are obtained from the tree-level constraints Eqs. (6), (9).

\[
\kappa = \frac{\mu^2}{\xi} \quad \eta = -2m \left( \frac{\xi^2}{\mu^2} + \frac{\mu^2}{\xi^2} \right)^{-1}.
\] (16)

We have checked that this is indeed the case for a wide range of input parameters. Hence, in what follows, we impose the condition above and only consider the two independent vevs $\xi$ and $\chi$.

A plot of the potential in the $\xi$ direction, Fig. 1 shows the Coleman-Weinberg terms do indeed stabilize the $\xi \to \infty$ runaway at finite, non-zero values of the fields. A contour plot in the $\xi - \chi$ plane, Fig. 2 reveals that the pseudomodulus $\chi$ is also stabilized at a non-zero value $O(\hat{\mu})$.

Thus, for a natural choice of parameters, all the vevs $\xi$, $\kappa$, $\eta$ and $\chi$ obtain stable, finite $O(\hat{\mu})$ values. Notice that $\Phi$, $\varphi$ and $\tilde{\varphi}$ all carry $R$-charge, so the $R$-symmetry of the model is spontaneously broken in this minimum.

Until now we have neglected the $D$-terms from $SU(2)_{mg}$ but, as we can see from Tab. 1 including them does not significantly alter the vev-structure of the vacuum.

What about the stability of this vacuum? When the gauge fields are turned on, this model has non-zero Witten index, so the global minimum will be supersymmetric. As in the ISS model, this minimum is induced by the non-perturbative contribution to the superpotential,

\[
W_{np} = 2\Lambda^3 \left[ \det \left( \frac{\Phi}{\Lambda} \right) \right]^{\frac{1}{2}}.
\] (17)
Figure 2: This contour plot of the effective potential $V_{\text{eff}}$ shows that the pseudo-modulus $\chi$ is also stabilized at a non-vanishing vev. (We use $\mu = 4\hat{\mu}$, $m = 2\hat{\mu}$.)

Adapting the supersymmetric vacuum solution from the ISS model to our case with $\mu > \hat{\mu}$ we find,

$$\varphi = 0, \quad \tilde{\varphi} = 0, \quad \eta = \hat{\mu}^2 \mu^{\frac{5}{3}} \Lambda^{\frac{1}{3}}, \quad \chi = \mu^{\frac{4}{3}} \Lambda^{\frac{1}{3}}.$$

Note that the supersymmetric minimum lies at $\varphi = \tilde{\varphi} = 0$ and is completely unaffected by the baryon deformation.

So far we have established that supersymmetry is broken dynamically and $R$-symmetry spontaneously in the metastable vacuum of the ISS sector. We now need to transmit both these effects to the Standard Model. The most concise way to do it is to gauge the $SU(5)$ and to identify it with the parent gauge group the Standard Model. Since both supersymmetry and $R$-symmetry are broken, gauginos do acquire a mass.

Gaugino masses are generated at one loop order (cf. Fig. 3). The fields propagating in the loop are fermion and scalar components of the direct mediation ‘messengers’ $\rho$, $\tilde{\rho}$ and $Z$, $\tilde{Z}$. For fermion components,

$$\psi = (\rho_{ia}, Z_{ir})_{\text{ferm}}, \quad \tilde{\psi} = (\tilde{\rho}_{ia}, \tilde{Z}_{ir})_{\text{ferm}},$$

the mass matrix is given by

$$m_f = I_5 \otimes I_2 \otimes \begin{pmatrix} \chi & \xi \\ \mu^2 \xi & 0 \end{pmatrix}.$$  

We assemble the relevant scalars into

$$(\rho_{ia}, Z_{ir}, \tilde{\rho}_{ia}, \tilde{Z}_{ir})_{\text{sc}},$$

7In contrast to the ISS model which only has small anomalous $R$-symmetry breaking our model has in addition a rather large spontaneous $R$-symmetry breaking by the vacuum expectation value $\langle \chi \rangle$. 
and for the corresponding scalar mass-squared matrix we have

\[ m_{sc}^2 = I_5 \otimes I_2 \otimes \begin{pmatrix} |\xi|^2 + |\chi|^2 & \chi |\mu| \xi & -\bar{\mu}^2 & \eta^* |\mu| \xi \\
\chi |\mu| \xi & |\mu|^2 |\xi|^2 & (\xi \eta)^* + 2m |\mu|^2 & 0 \\
-\bar{\mu}^2 & \xi \eta + 2m |\mu|^2 \xi & |\mu|^2 + |\chi|^2 \xi & (\chi \xi)^* \\
\eta |\mu| \xi & 0 & \chi \xi & |\xi|^2 \end{pmatrix}. \] (22)

Gaugino masses arise from the one-loop diagram in Fig. 3. Due to the non-diagonal form of the matrices $I_5$, $I_2$, we find it easiest to evaluate the appropriate expressions numerically. The scale $\bar{\mu}$ is the SUSY-breaking scale. We will keep it fixed, and measure all other dimensionful parameters in units of $\hat{\mu}$. Then for fixed $\bar{\mu} = 1$ there are only two independent input parameters, $\mu$ and $m$, while the VEVs $\xi$, $\kappa$, $\eta$ and $\chi$ are generated from minimizing the effective potential, as above. For the purposes of this paper we will focus on generating the largest possible values for gaugino masses (in units of $\hat{\mu}$). We find that this occurs when $\mu \simeq \bar{\mu}$. For example, for $\mu = 1.1 \bar{\mu}$ and $m = 0.3 \bar{\mu}$ we have

\[ m_{\lambda A} \simeq \frac{g_A^2}{16\pi^2} 0.0089 \bar{\mu}, \] (23)

where $A = 1, 2, 3$ labels the three gauge groups of the Standard Model. Requiring that all the gaugino masses are

\[ m_{\lambda A} \sim (0.1 - 1) \text{ TeV}, \] (24)

we conclude that

\[ \bar{\mu} \sim (10^4 - 10^5) \text{ TeV} \] (25)

in this point in the parameter space of our model.

We would like to compare this numerical evaluation of the gaugino mass with the simple analytical expression one might have anticipated. Assuming that the dominant effect comes from magnetic quarks, $\rho$ and $\tilde{\rho}$, propagating in the loop, as shown in Fig. 3 and working to the leading order in susy breaking, i.e. to order $F_\chi$, gaugino mass goes as

\[ m_{\lambda A}^{\text{naive estimate}} \sim \frac{g_A^2}{16\pi^2} \langle F_\chi \rangle \sim \frac{g_A^2}{16\pi^2} \langle \chi \rangle \sim \frac{g_A^2}{16\pi^2} \bar{\mu}. \] (26)

For the last part of (26) we have assumed that all VEVs and mass parameters are of the same order $\mathcal{O}(\bar{\mu})$. This expression is two orders of magnitude greater than our numerical result and therefore is too simplistic to give a correct estimate. In fact, the correct $\mathcal{O}(F_\chi)$ contribution to $m_{\lambda A}^{(1)} \sim \frac{g_A^2}{16\pi^2} F_\chi (m_f)^{-1}_{11}$ vanishes in our model, and one needs to go to order $F_\chi^3$ to find a non-vanishing contribution. This effect was first pointed out in Ref. [38]: the zero element in the lower right corner of the fermion mass matrix [20] implies that $(m_f)^{-1}_{11} = 0$ and hence $m_{\lambda A}^{(1)} = 0$.

The vanishing of the leading order contribution $m_{\lambda A}^{(1)}$ to the gaugino mass contrasts with the usual gauge mediation argument that the scalar masses should be roughly similar to the gaugino masses $m_{\lambda A} \sim M_{sc}$. Clearly, the $R$-symmetry breaking (together with the structure of the messenger mass matrices) plays a crucial role in suppressing the gaugino masses. Since scalars are not protected by R-symmetry the generation of their masses is less constrained. Hence, we expect the appropriate two-loop diagrams (cf., e.g., [40, 41]) to give something closely approximating the naive estimate for the scalar masses,

\[ M_{sc}^2 \sim \left( \frac{g_A^2}{16\pi^2} \right)^2 \bar{\mu}^2. \] (27)

\[ ^{4}\text{In Ref. [33] we will explore the parameter space of the model in more detail.} \]

\[ ^{5}\text{For example, as long as supersymmetry is broken, we can have scalar masses even when R-symmetry is unbroken.} \]
Figure 3: One-loop contribution to the gaugino masses. The blob on the scalar line indicates an appropriate number of insertions of $\langle F_\chi \rangle$ to make the diagram non-vanishing.

In our model therefore the scalars are always heavier than the gauginos. The phenomenology for this particular type of model is expected to be of the "heavy-scalar" type as reviewed in Ref. [39]. In the region $\hat{\mu} \simeq \mu \sim m$ their masses are only about two orders of magnitude larger than the gaugino masses, and a focus-point type of phenomenology [42] may be possible. (Note that by choosing $\mu$ to be even closer to $\hat{\mu}$ one may get gaugino masses closer to those of the scalars.) Increasing $\mu$ and decreasing $m$ takes us continuously to the split SUSY scenario [43, 44]. A fuller investigation will be carried out in Ref. [37].

Non-perturbative effects due to $W_{np}$ are suppressed by the scale $\Lambda$ of the Landau pole of the ISS sector, which we have not yet constrained. Choosing $\Lambda \gg \hat{\mu}$ (so that the magnetic theory is weakly coupled and the metastable vacuum is long lived) the non-perturbative corrections to our discussion are small.

Our model has a spontaneously broken R-symmetry that is explicitly broken only by the non-perturbative contribution $W_{np}$ to the superpotential. In such a situation we generally expect a pseudo-Goldstone boson – the R-axion $a_R$ (cf., e.g., [32, 45, 46]). If such a particle is light it can have dangerous phenomenological consequences [47, 48, 49]. Since the R-symmetry is an axial symmetry triangle diagrams typically couple the R-axion to gauge fields via a term (see, e.g., [47])

$$\sim \frac{\alpha}{2\pi f_R} a_R F^{\mu\nu} \tilde{F}_{\mu\nu}$$

(28)

where $F^{\mu\nu}$ is a gauge field and $f_R$ is the scale of spontaneous R-symmetry breaking. Particularly dangerous are the couplings of this type to gluons and photons. Moreover, there can exist couplings of the R-axion to matter fields. For small masses $m_{a_R} \lesssim 100 \text{ MeV}$ astrophysical considerations [48, 49] constrain the scale of spontaneous R-symmetry breaking to be

$$f_R \gtrsim \text{few} \times 10^7 \text{ GeV} \quad \text{for} \quad m_{a_R} \lesssim 100 \text{ MeV}.$$  

(29)

Let us now estimate the mass of the R-axion in our model to check whether it is harmless. The R-axion is the phase of the fields that spontaneously break the R-symmetry,

$$\eta = |\eta| \exp \left( \frac{2i}{f_R} a_R \right), \quad \chi = |\chi| \exp \left( \frac{2i}{f_R} a_R \right)$$

(30)

where the 2 arises from the R-charge 2 of the $\Phi$-field. The dominant contribution to spontaneous R-symmetry breaking comes from $\langle \eta \rangle$. This sets the scale

$$f_R \sim \langle \eta \rangle.$$  

(31)

The R-axion mass arises from the explicit breaking due to $W_{np}$.

Another contribution to the R-axion mass may arise from supergravity. A constant term in the superpotential that cancels the cosmological constant also breaks R-symmetry explicitly [46].
account $W_{np}$ contribution to the $F_X$-terms,

$$V_F \supset |F_X|^2 \sim \langle \eta \rangle \langle \chi \rangle \frac{3}{2} \exp \left( \frac{5i a_R}{\langle \eta \rangle} \right) \Lambda^{-\frac{1}{2}} - \hat{\mu}^2 \right|^2 \tag{32}$$

$$= \left[ \langle \eta \rangle^2 \langle \chi \rangle^3 \Lambda^{-1} + \hat{\mu}^4 - 2\hat{\mu}^2 \langle \eta \rangle \langle \chi \rangle^2 \Lambda^{-\frac{1}{2}} \cos \left( \frac{5i a_R}{\langle \eta \rangle} \right) \right].$$

the $R$-axion mass arises from the last term on the right hand side. (For simplicity, we have chosen $\hat{\mu}$ and all the vevs to be real.) Expanding to second order in $a_R$ we find the $R$-axion mass to be,

$$m^2_{a_R} \sim \hat{\mu}^2 \langle \eta \rangle^{-1} \langle \chi \rangle^2 \Lambda^{-\frac{1}{2}} \tag{33}$$

For our values this turns out to be sufficiently heavy to easily avoid the astrophysical constraints for any $\Lambda < M_P$.

We now want to comment on a particular feature of our model, and indeed all direct mediation models based on embedding the Standard Model gauge group into the flavor group of the ISS sector. As already mentioned in refs.\cite{1, 14} this embedding adds a significant number of matter multiplets charged under the SM gauge groups. Above the mass thresholds of these fields this leads to all Standard Model gauge groups being not asymptotically free and therefore to Landau poles in the SM sector. Since the additional fields are in $SU(5)$ multiplets, the beta functions of the SM gauge couplings are modified universally. For example, in our model as

$$b_A = b_A^{(MSSM)} - 9 \tag{34}$$

where the additional contributions are 2 from $\varphi$ and $\tilde{\varphi}$, and 7 from $\Phi$. The SM gauge couplings at a scale $Q > \mu$ in our model are therefore related to the traditional MSSM ones as

$$\alpha_A^{-1} = (\alpha_A^{-1})^{(MSSM)} - \frac{9}{2\pi} \log(Q/\hat{\mu}), \tag{35}$$

where the fields $\varphi$, $\tilde{\varphi}$ and $\Phi$ contribute to the running above the scale $\mu$. The Landau pole $Q \equiv \Lambda^{(MSSM)}$ we will take to be situated where $g_A \sim 4\pi$ which corresponds roughly to

$$\frac{\Lambda^{(MSSM)}}{\hat{\mu}} \sim 10^5. \tag{36}$$

Values of $\hat{\mu} \gtrsim 10^8 TeV$ would be required in order to reach the conventional GUT scale in the MSSM sector before the Landau pole.

We would now like to suggest that the change of sign in the slopes of the Standard Model gauge couplings and the very existence of Landau poles is an interesting feature rather than an insurmountable problem. Presence of Landau poles in all sectors of theory indicates that we should interpret not only the ISS sector as a magnetic dual of an asymptotically free theory, but also apply the same reasoning to the Standard Model itself. In other words, at energy scales above $\hat{\mu}$ the Standard Model sector and the ISS sector are not decoupled from each other and, in general, should be treated as part of the same theory. We already know that the UV completion of the ISS sector is its electric Seiberg dual and we propose the whole theory has such a UV completion. This seems to be a rather symmetric construction. One consequence of this interpretation is, of course, that gauge unification is lost, or at least buried in the unknown details of the dual theory.

In summary we find that direct mediation (i.e. mediation in which there is no separate messenger sector) is relatively simple to implement in ISS-like models by inducing spontaneous breakdown of the approximate $R$-symmetry associated with the metastable minimum which in turn allows us to generate gaugino masses alongside other soft SUSY-breaking terms. We
presented a baryon-deformed ISS model in which this occurs automatically due to the Coleman-Weinberg potential. Once the $R$-symmetry is broken, the magnetic quarks of the ISS sector are able to play the role of messengers, if one identifies an $SU(5)_f$ subset of the flavour symmetry with the SM gauge groups. We have speculated that the entire theory is a magnetic dual of an (unknown) asymptotically free theory.

References

[1] K. Intriligator, N. Seiberg and D. Shih, “Dynamical SUSY breaking in meta-stable vacua,” JHEP 0604 (2006) 021 [hep-th/0602239].

[2] S. A. Abel, C. S. Chu, J. Jaeckel and V. V. Khoze, “SUSY breaking by a metastable ground state: Why the early universe preferred the non-supersymmetric vacuum,” JHEP 0701, 089 (2007) [arXiv:hep-th/0610334].

[3] N. J. Craig, P. J. Fox and J. G. Wacker, “Reheating metastable O’Raifeartaigh models,” arXiv:hep-th/0611006.

[4] W. Fischler, V. Kaplunovsky, C. Krishnan, L. Mannelli and M. Torres, “Meta-stable supersymmetry breaking in a cooling universe,” arXiv:hep-th/0611018.

[5] S. A. Abel, J. Jaeckel and V. V. Khoze, “Why the early universe preferred the non-supersymmetric vacuum. II,” JHEP 0701, 015 (2007) [arXiv:hep-th/0611130].

[6] L. Anguelova, R. Ricci and S. Thomas, “Metastable SUSY breaking and supergravity at finite temperature,” arXiv:hep-th/0702168.

[7] S. Forste, “Gauging flavour in meta-stable SUSY breaking models,” Phys. Lett. B 642, 142 (2006) [arXiv:hep-th/0608036].

[8] A. Amariti, L. Girardello and A. Mariotti, “Non-supersymmetric meta-stable vacua in SU(N) SQCD with adjoint matter,” JHEP 0612 (2006) 058 [arXiv:hep-th/0608063].

[9] M. Dine, J. L. Feng and E. Silverstein, “Retrofitting O’Raifeartaigh models with dynamical scales,” Phys. Rev. D 74, 095012 (2006) [arXiv:hep-th/0608159].

[10] M. Dine and J. Mason, “Gauge mediation in metastable vacua,” arXiv:hep-ph/0611312.

[11] R. Kitano, H. Ooguri and Y. Ookouchi, “Direct mediation of meta-stable supersymmetry breaking,” arXiv:hep-ph/0612139.

[12] H. Murayama and Y. Nomura, “Gauge mediation simplified,” arXiv:hep-ph/0612186.

[13] O. Aharony and N. Seiberg, “Naturalized and simplified gauge mediation,” arXiv:hep-ph/0612308.

[14] C. Csaki, Y. Shirman and J. Terning, “A simple model of low-scale direct gauge mediation,” arXiv:hep-ph/0612241.

[15] S. A. Abel and V. V. Khoze, “Metastable SUSY breaking within the standard model,” arXiv:hep-ph/0701069.

[16] A. Amariti, L. Girardello and A. Mariotti, “On meta-stable SQCD with adjoint matter and gauge mediation,” arXiv:hep-th/0701121.

[17] H. Murayama and Y. Nomura, “Simple scheme for gauge mediation,” arXiv:hep-ph/0701231.
[18] K. Intriligator, N. Seiberg and D. Shih, “Supersymmetry Breaking, R-Symmetry Breaking and Metastable Vacua,” arXiv:hep-th/0703281.

[19] K. Intriligator and N. Seiberg, “Lectures on Supersymmetry Breaking,” arXiv:hep-ph/0702069.

[20] S. A. Abel, J. Jaeckel and V. V. Khoze, “Naturalised supersymmetric grand unification,” arXiv:hep-ph/0703086.

[21] D. Shih, “Spontaneous R-symmetry breaking in O’Raifeartaigh models,” arXiv:hep-th/0703196.

[22] L. Ferretti, “R-symmetry breaking, runaway directions and global symmetries in O’Raifeartaigh models,” arXiv:0705.1959 [hep-th].

[23] H. Ooguri, Y. Ookouchi and C. S. Park, “Metastable Vacua in Perturbed Seiberg-Witten Theories,” arXiv:0704.3613 [hep-th].

[24] A. Katz, Y. Shadmi and T. Volansky, “Comments on the meta-stable vacuum in N(f) = N(c) SQCD and direct mediation,” arXiv:0705.1074 [hep-th].

[25] F. Brummer, “A natural renormalizable model of metastable SUSY breaking,” arXiv:0705.2153 [hep-ph].

[26] E. Dudas, J. Mourad and F. Nitti, “Metastable Vacua in Brane Worlds,” arXiv:0706.1269 [hep-th].

[27] A. Delgado, G. F. Giudice and P. Slavich, “Dynamical mu Term in Gauge Mediation,” arXiv:0706.3873 [hep-ph].

[28] H. Y. Cho and J. C. Park, “Dynamical $U(1)_R$ Breaking in the Metastable Vacua,” arXiv:0707.0716 [hep-ph].

[29] E. Dudas, C. Papineau and S. Pokorski, “Moduli stabilization and uplifting with dynamically generated F-terms,” JHEP 0702, 028 (2007) arXiv:hep-th/0610297.

[30] R. Kallosh and A. Linde, “O’KKLT,” JHEP 0702, 002 (2007) arXiv:hep-th/0611183.

[31] O. Lebedev, V. Lowen, Y. Mambrini, H. P. Nilles and M. Ratz, “Metastable vacua in flux compactifications and their phenomenology,” JHEP 0702, 063 (2007) arXiv:hep-ph/0612035.

[32] A. E. Nelson and N. Seiberg, “R symmetry breaking versus supersymmetry breaking,” Nucl. Phys. B 416, 46 (1994) arXiv:hep-ph/9309299.

[33] R. Essig, K. Sinha and G. Torroba, “Meta-Stable Dynamical Supersymmetry Breaking Near Points of Enhanced Symmetry,” arXiv:0707.0007 [hep-th].

[34] S. R. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.

[35] R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo and C. A. Savoy, “Quartic Mass Matrix And Renormalization Constants In Supersymmetric Yang-Mills Theories,” Phys. Lett. B 115 (1982) 212.

[36] K. van den Broek, “Vscape V1.1.0: An interactive tool for metastable vacua,” arXiv:0705.2019 [hep-ph].

[37] S. A. Abel, C. Durnford, J. Jaeckel and V. V. Khoze, in preparation.
[38] K. I. Izawa, Y. Nomura, K. Tobe and T. Yanagida, “Direct-transmission models of dynamical supersymmetry breaking,” Phys. Rev. D 56 (1997) 2886 [hep-ph/9705228]

[39] J. L. Feng and F. Wilczek, “Advantages and distinguishing features of focus point supersymmetry,” Phys. Lett. B 631, 170 (2005) [hep-ph/0507032]

[40] S. P. Martin, “Generalized messengers of supersymmetry breaking and the sparticle mass spectrum,” Phys. Rev. D 55 (1997) 3177 [hep-ph/9608224]

[41] G. F. Giudice and R. Rattazzi, “Theories with gauge-mediated supersymmetry breaking,” Phys. Rept. 322 (1999) 419 [hep-ph/9801271]

[42] J. L. Feng, K. T. Matchev and T. Moroi, “Focus points and naturalness in supersymmetry,” Phys. Rev. D 61 (2000) 075005 [hep-ph/9909334]

[43] N. Arkani-Hamed and S. Dimopoulos, “Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC,” JHEP 0506 (2005) 073 [hep-th/0405159]

[44] G. F. Giudice and A. Romanino, “Split supersymmetry,” Nucl. Phys. B 699 (2004) 65 [Erratum-ibid. B 706 (2005) 65] [hep-ph/0406088]

[45] M. Dine and A. E. Nelson, “Dynamical supersymmetry breaking at low-energies,” Phys. Rev. D 48 (1993) 1277 [arXiv:hep-ph/9303230].

[46] J. Bagger, E. Poppitz and L. Randall, “The R axion from dynamical supersymmetry breaking,” Nucl. Phys. B 426 (1994) 3 [arXiv:hep-ph/9405345].

[47] J. E. Kim, “Light Pseudoscalars, Particle Physics and Cosmology,” Phys. Rept. 150 (1987) 1.

[48] G. G. Raffelt, Stars As Laboratories For Fundamental Physics: The Astrophysics of Neutrinos, Axions, and other Weakly Interacting Particles, University of Chicago Press, Chicago, 1996.

[49] G. G. Raffelt, “Astrophysical axion bounds,” arXiv:hep-ph/0611350