Scaling analyses on the critical current density in MgB$_2$/NbN/Si thin film

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Abstract. Scaling analyses are performed on the critical current density $J_c$ in MgB$_2$/NbN/Si thin film. In our previous work on MgB$_2$/SiC/Si film [Mat. Sci. Eng. 502 (2019) 012184], we have shown that $J_c$ data well scale on a single line with the reduced scaling formula more than 10 orders of magnitude. We extend these studies onto MgB$_2$ film with NbN buffer layer in comparison with SiC buffer. Experimental $J_c$ data under perpendicular magnetic field to the film are reduced against transition temperature and critical field, and applicability of the comprehensive scaling formula is examined over 16 orders of magnitude. Our scaling formula is revealed to be well applicable to the NbN-buffered film as well.

1. Introduction
In our previous works [1-3] we studied scaling behaviors of the critical current density $J_c$ in MgB$_2$ thin films with SiC buffers, and showed that single scaling formula without flux creep effect is able to fit $J_c$ in 50 nm and 100 nm films up to 15 orders of magnitude, with various film quality and superconductivity, demonstrating wide applicability of the comprehensive scaling formula.

In order to even further investigate applicability of the scaling formula, we extend our scaling examinations onto MgB$_2$ films with NbN buffer layer. MgB$_2$/NbN/Si thin film studied here is the same as that in our past report [4], where the present reduced scaling formula was not yet treated. We utilize the same experimental data of the upper critical field, critical current density and irreversibility field and normalize them against the critical temperature and critical field, and then compare them with the scaling formula [1-3], which is formulated as shown below.

In the absence of thermal activation of flux pinning, critical current density $J_c$ at temperature $T$ and magnetic field $H$ is expressed as:

$$J_c(H,T) = A\mu_0^{-m-1}H_{c2}^{-m-\gamma}(T)H^{\gamma-1}[1-H/H_{c2}(T)]^{\delta},$$

(1)

where $A$ is a constant, $H_{c2}(T)$ is the upper critical field, and $m, \gamma$ and $\delta$ are parameters depending on the flux pinning mechanism [5]. Employing the scaling parameters $J_f = A\mu_0^{-m-1}H_{c2}^{-m-1}(0)$ and $b_0 = \mu_0 H / \mu_0 H_{c2}(0)$ and empirical temperature dependence of $H_{c2}(T) = H_{c2}(0)(1-t^2)$ with $t = T/T_c$, equation (1) results in the scaling formula as:

$$J_c(b_0,t) = J_f b_0^{\gamma-1}(1-t^2)^{-m\gamma}[1-b_0/(1-t^2)]^{\delta},$$

(2)

which is finally transformed to:
\[ J_C(b_0, t) / (J_C b_0^{-m_1}) = ((1-t^2)/b_0)^{m_2} \left[ 1-b_0/(1-t^2) \right]^{\delta}. \]  

(3)

It should be noted that once the pinning parameters \( m, \gamma \) and \( \delta \) are given the right hand side of this equation (3) contains no adjustable parameter except for \( J_C \).

2. Experimental

MgB\(_2\)/NbN/Si thin film was prepared by sequential evaporation of boron and magnesium followed by in-situ annealing. The magnesium and boron layers were deposited on NbN-buffered Si(100) substrate at the background vacuum of 10\(^{-4}\) Pa. The NbN buffer layer was mainly applied as an effective diffusion barrier. The thickness of the boron layer in the precursor film was adjusted so as to result in 200 nm stoichiometric MgB\(_2\) final film after reaction with the excess Mg top layer. The precursor film was then in-situ heated to 280°C and kept there for 30 min in an Ar atmosphere of 0.06 Pa. Subsequently, the Ar pressure was increased to 16 Pa and the temperature was increased to 700°C and kept there for 10 min. The sample was then cooled down to room temperature. Transmission electron microscopic observations and X-ray diffraction measurements basically indicated characteristics of amorphous-like MgB\(_2\) film.

AC and DC magnetizations were measured using PPMS magnetometer (Quantum Design). The upper critical field \( H_{c2} \) was estimated from AC susceptibility measurements with magnetic fields parallel and perpendicular to the film, while the critical current density \( J_C \) was evaluated from DC magnetization hysteresis under perpendicular magnetic field with the Bean critical state model: \( J_C = 30 \Delta M/r \), where \( \Delta M \) is the height of the magnetization loop and \( r \) is the sample half-width (0.13 cm).

3. Results and discussion

Figure 1 shows temperature dependence of the upper critical field \( H_{c2} \) plotted with the same data as in the past report [4]. Least square fitting (straight line) to the perpendicular data (solid circles) gives estimation of \( H_{c2}^{\perp}(0) = 101 \) kOe, which will be used later for the normalization of the applied fields, i.e. \( b_0 \) parameter in equation (3). The parallel data (open circles) give \( H_{c2}^{\parallel}(0) = 115 \) kOe, and the anisotropy in \( H_{c2} \) is much smaller than that in SiC-buffered films. This probably comes from the amorphous nature of the present 700°C-annealed, NbN-buffered film.

![Figure 1](image1.png)

**Figure 1.** Upper critical field \( H_{c2} \) for 200 nm NbN-buffered film under perpendicular (solid circles) and parallel (open circles) fields. Least square fitting (solid line) to the perpendicular data gives estimation of \( H_{c2}^{\perp}(0) = 101 \) kOe, which is used for field normalization.

![Figure 2](image2.png)

**Figure 2.** Variation of \( J_C \) as a function of \( 1-t^2 \) with \( t = T/T_C \) for NbN-buffered film. Solid lines indicate best linear-fits to respective \( J_C \) at respective values of constant magnetic field by \( J_C(t) = J_C(0)(1-t^2)^{m'} \) scaling equation. Least square fittings provide values of the critical exponents \( m' \).
Figure 2 indicates the critical current density $J_c$ in NbN-buffered 200 nm MgB$_2$ film as a function of $1-t^2$ with $t = T/T_c$, ($T_c = 31.0$ K) at each value of constant magnetic field (0.1~2.2 kOe) perpendicular to the film. It is noted that $J_c$ values exceed 1 MA/cm$^2$ at lower temperature and lower field. The solid straight lines in the figure represent least square fittings with the scaling equation:

$$J_c(t) = J_c(0) (1-t^2)^{m'},$$

estimating values of the critical exponents $m'$ at respective magnetic fields. Although $J_c$ values become a bit scattered at higher magnetic fields ($>3$ kOe), $m'$ values similarly increase with magnetic field up to 15kOe.

Thus estimated $m'$ values are plotted in figure 3 as a function of the applied magnetic field $H$. As discussed previously [1-3], if the applied magnetic field is small, the temperature dependent contribution from $[1-b_0/(1-t^2)]\delta$ term can be neglected and equation (2) reduces to equation (4) with the pinning parameter $m' = m-\gamma$. Thus, the intrinsic value of $m'$ can be deduced from the lower field values of those exponents. Taking average of the lowest two values in figure 3, we estimate the intrinsic critical exponent in equation (4) as $m' = 4.5$. This value is identical with the previous 750°C-annealed, SiC-buffered 50 nm and 100 nm MgB$_2$ films [1,2].

![Figure 3](image1)

**Figure 3.** Critical exponent $m'$ in equation (4) increases with increase in the applied magnetic field. Average of the lowest two values provides the intrinsic critical exponent as $m' = 4.5$.

![Figure 4](image2)

**Figure 4.** Critical current density $J_c$ in 700°C-annealed, NbN-buffered MgB$_2$ film as a function of perpendicular magnetic field $H$. The solid line indicates field dependence as $J_c(H) \sim H^{-0.5}$, comparing with experimental field variation.

The value of $\gamma$, another pinning parameter, can be estimated from magnetic field dependence of $J_c$ at constant temperature low enough to neglect the flux creep effect. Figure 4 represents critical current density $J_c$ in 700°C-annealed, NbN-buffered 200 nm MgB$_2$ film as a function of perpendicular magnetic field $H$. The solid straight line indicates the field dependence as $J_c(H) \sim H^{-0.5}$ which is comparable with experimental field variation of $J_c$ at lower temperatures. Thus, we can regard the field dependence of the critical current density without flux creep as $J_c(H) \sim H^{-0.5}$. Comparing this dependence with equation (2), we obtain $\gamma - 1 = -0.5$ and thus $\gamma$ is estimated to be 0.5, which further results in $m = m' + \gamma = 5.0$. This value of $m$ is also identical with the value in all former SiC-buffered 50 nm and 100 nm films and infers the surface pinning effect as suggested before [2].
The value of the third pinning parameter $\delta$ is assumed to be 2.0 according to the literatures [6,7]. This value of $\delta = 2.0$ corresponds to the presence of saturation effects in the summation problem for the total pinning strength, the assumption being always employed in our scaling analyses [1-3].

Putting values of these pinning parameters in equation (3) as $m = 5.0$, $\gamma = 0.5$ and $\delta = 2.0$, we plot in figure 5 experimental reduced critical current density $y = J_c / (J_c b_0^{4.0})$ as a function of $x = (1-t^2)/b_0$ under perpendicular magnetic field. Here, we employed $H_{c2}^\perp(0) = 101$ kOe according to figure 1 in order to normalize applied field for $b_0$ parameter. As can be seen, wide range of experimental data in temperature from 8 to 22 K and in field from 0.1 to 68 kOe are aligned in a single line. This is the widest range of temperature and field ever examined in our $J_c$ scaling examinations (i. e. over 16 orders of magnitude). Either larger or smaller $m$ than 5.0 deteriorates data alignment from the single line. This also supports appropriateness of the evaluated values of $m = 5.0$ and $\gamma = 0.5$.

\[ y = J_c / (J_c b_0^{4.0}) \]

\[ x = (1-t^2)/b_0 \]

**Figure 5.** Plot of reduced experimental critical current density $y = J_c / (J_c b_0^{4.0})$ in NbN-buffered 200 nm MgB$_2$ film as a function of $x = (1-t^2)/b_0$ with $b_0=H/H_{c2}^\perp(0)$. The solid line represents $y = x^{4.5}(1-1/x)^2$, which fits experimental data well over 10 orders of magnitude in reduced $J_c$ ($y>10^4$). Deviation from the scaling line below $x<10$ suggests flux creep effects at higher temperature and higher magnetic field.

The solid line in figure 5 represents equation (3) as $y = x^{4.5}(1-1/x)^2$ and the line fits experimental data very well over 10 orders of magnitude in reduced $J_c$ ($y>10^4$). On making such fit, the only adjustable parameter is the scaling factor $J_s$ which reflects the pinning strength through the factor $AH_{c2}^{m-1}(0)$ in equation (2), and the value of 0.1 MA/cm$^2$ results in the best agreement between experimental data and the scaling equation (3). This value of $J_s$ is fairly large and comparable to that in
the former 750°C-annealed, SiC-buffered 50 nm film [1]. This infers good film quality and sound superconductivity in both of our films.

On the other hand, reduced $J_c$ deviates from the theoretical scaling line (equation (3)) below $x = 10$. This deviation suggests some effect of thermal flux creep which may happen at higher temperature and higher magnetic field, i.e. at smaller $x$.

Finally, we examine behavior of the irreversibility field $H_{irr}$, which is the criterion of occurrence of flux creep. At higher temperatures $H_{irr}$ follows the well-known scaling relation:

$$H_{irr}(t) = H_{irr}(0) (1-t^2)^n$$

where $n = 2(m-\gamma)/(3-2\gamma)$ [2,6]. Putting values of $m = 5.0$ and $\gamma = 0.5$, we obtain $n = 4.5$ which is 1.5 times larger than $n = 3$ as estimated in our past report (figure 4 in ref. [4]). Such discrepancy was also observed in 750°C-annealed, SiC-buffered 50 nm MgB$_2$ film, and the reason is not yet clarified.

In summary, scaling behaviors of $J_c$ in 700°C-annealed, NbN-buffered 200 nm MgB$_2$ film are examined over 16 orders of magnitude in comparison with our previous analyses in SiC-buffered MgB$_2$ films. The comprehensive scaling formula (equation (3)) has been shown to well explain $J_c$ data in the present NbN-buffered MgB$_2$ film, as well as in the previous SiC-buffered MgB$_2$ films. The commonly estimated values of the pinning parameter $m = 5.0$ in all SiC-buffered 50 nm and 100 nm films and NbN-buffered 200 nm film infer that the surface pinning effect [2] is commonly effective in all these MgB$_2$ thin films. The only failure of fitting experimental $J_c$ with the theoretical equation was in the case of 10 nm SiC-buffered MgB$_2$ film, probably due to nano-granular nature of superconductivity [1,8]. Overall, we have shown good versatility of our scaling formula (3) to predict practical $J_c$ values in various MgB$_2$ films under wide range of temperature and field.

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