High-precision nonlocal temporal correlation identification
of entangled photon pairs for quantum clock synchronization

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Abstract: High-precision nonlocal temporal correlation identification in the entangled photon pairs is critical to measure the time difference between independent remote time scales for many quantum information applications. The first nonlocal correlation identification was reported in 2009, which extracts the time difference via the algorithm of iterative fast Fourier transformations (FFTs) and their inverse. The least resolution cannot be selected arbitrarily, which is restricted by the peak identification threshold of the algorithm, and the calculation precision accordingly. In this paper, an improved algorithm is proposed both in the resolution and time difference precision, which realizes well-performed coincidence identification with a flexible resolution down to 1 ps. With a common time scale reference, the measured time offset between two timing signals has achieved a minimum precision of 0.72 ps. We believe that this algorithm would be a dramatic step towards the field implementation of entanglement-based high-precision clock synchronization, ranging system and quantum communications.

Key Words: nonlocal temporal correlation identification; entangled photons; quantum clock synchronization

Introduction

Quantum correlated photon pairs generated from spontaneously parametric down conversion (SPDC) have been proven to behave superior performance. Owing to their strong temporal correlation (about a few hundred femtoseconds) in correlated photon pairs, the temporal coincidence identification has become an important facility in applications of
quantum spectroscopy [1-4], quantum communication [5-8], and quantum clock synchronization [9-13], etc. Especially in case of clock synchronization applications utilizing energy-time entangled photon pair sources, an accurate time-offset identification measurement between remote sites is the prerequisite for high-performance protocols. Typically, the time-offset identification can be acquired by temporal correlation measurements, which are usually implemented by a local coincidence device, with the fact that the two signals need to be sent to a local device. Such requirement can be satisfied with no difficulty in laboratory experiments. In field protocols, the coincidence resolution, together with the accuracy of the time offset, would suffer from deterioration under local coincidence measurements. Therefore, a nonlocal time coincidence identification is essential to high-accuracy applications.

In 2009, Ho et al. proposed a first nonlocal time correlation identification algorithm [14]. In the algorithm, the remotely departed entangled photon pairs are individually detected and their time arrivals are nonlocally recorded by independent event timers. To identify the temporal correlation, the recorded time sequences are firstly binned for subsequent fast Fourier transformations (FFTs). By implementing direct product on the Fourier transformed bins and applying the inverse FFTs on the outputs, the time difference between the remote photon pairs can be extracted. As the time offset can be identified exactly only when the cross-correlation peak exceeds a peak threshold, the ultimate calculation resolution and the consequent precision are restricted.

Here we present an updated algorithm for the nonlocal coincidence identification. Instead of iteratively applying time binning onto discrete time sequences and subsequent FFTs, the coincidences are acquired by counting the subtractions of the time sequences that are within a certain time window, which corresponds to the identification resolution. To acquire the coincidence distribution of correlated photon pairs, a coarse extraction is firstly launched onto a segment of the time sequences to determine the rough peak position of the time difference. Referenced by this identified coarse time difference, the coincidence distribution is then constructed by dealing with the counts of the subtractions that are within a much finer time window than the correlation width of the photon pairs after propagation. Based on our algorithm, a much higher time-offset precision is achieved with a flexible resolution down to 1 ps. An analysis of the attainable precision of the algorithm has also been stated by using two
commercial event timers that are referenced to a common time scale, which shows a precision of 0.72 ps in a measurement time of 5 s and pair rate of 0.62 kcps. In the following, the algorithm is firstly addressed.

**Algorithm description**

According to quantum theory [15], the joint detection probability of the time-correlated photon pair that are separately distributed to the space-time points \((r_1, t_1)\) and \((r_2, t_2)\) is proportional to the Glauber second-order correlation function \([16, 17]\),

\[
G^{(2)}(r_1, t_1, r_2, t_2) = \langle E^-(r_1, t_1)E^-(r_2, t_2)E^+(r_2, t_2)E^+(r_1, t_1) \rangle,
\]

where \(E^-(\cdot)\) and \(E^+(\cdot)\) are the negative- and positive-frequency part of the electric field operators at space-time point \((r_j, t_j), j = 1, 2\). In the stationary case, the second-order correlation function \(G^{(2)}\) only depends on \(t_1 - t_2\). Without loss of generality, a pair of ideally frequency-anticorrelated photons generated from spontaneous parametric down-conversion (SPDC) process can be considered. The two-photon joint spectral amplitude for degenerate type-II SPDC is given by \(F(\Omega)\), where \(\Omega\) is the deviation from the center frequency \(\omega_0\) of the SPDC photons. Assume that the photons propagate through dispersive media of length \(l_1\) and \(l_2\) to the space points \(r_1\) and \(r_2\), the wave number of the photon can then be expressed as \(k_i(\omega_0 + \Omega) = k_{i0} + \alpha_i \Omega + \beta_i \Omega^2 (i = 1, 2)\). Here \(\alpha\) and \(\beta\) are the first-order and the second-order dispersion which are responsible for the wave packet delay and the wave packet broadening, respectively. In the far-field approximation, \(G^{(2)}(t_1 - t_2)\) can be approximated as [18]

\[
G^{(2)}(t_1 - t_2) \sim \left| F(\Omega = \frac{t_1 - t_2 + \bar{\tau}}{\beta_1 l_1 + \beta_2 l_2}) \right|^2.
\]

where \(\bar{\tau} = \alpha_2 l_2 - \alpha_1 l_1\) is the overall time delay between the signal and idler photons. Thus by determining the peak position of the \(G^{(2)}\) function, the time difference between the two space-time points is obtained. Assume the FWHM width of \(F(\Omega)\) is \(\Delta \Omega\), the FWHM width of \(G^{(2)}(t_1 - t_2)\) in the far-field approximation will be \(\Delta \tau = (\beta_1 l_1 + \beta_2 l_2) \Delta \Omega\).

In practical experiment, such joint distribution of time detections cannot be directly measured, instead the coincidence counting rate \(R_c\) within a certain time window is measured, which can be expressed as [18]

\[
R_c \sim \int_0^T dt_1 dt_2 S(t_1 - t_2 - t_0) G^{(2)}(t_1 - t_2),
\]

where \(S(t)\) is the single photon detector efficiency.
where $T$ represents the data acquisition time. $S(t_1 - t_2 - t_0)$ is the coincidence window function centered at $t_0$, and can be given by a rectangular function

$$S(t_{1,i} - t_{2,j} - t_0) = \begin{cases} 1, & |t_{1,i} - t_{2,j} - t_0| \leq \tau_{BW}/2 \\ 0, & |t_{1,i} - t_{2,j} - t_0| > \tau_{BW}/2' \end{cases}$$

(4)

where $\tau_{BW}$ denotes the resolution of the coincidence measurement. When $\tau_{BW}$ is chosen to be so small that $S(t_1 - t_2 - t_0)$ is equivalent to a delta function, the probability of $G^{(2)}(t_1 - t_2)$ at $t_0$ is obtained. As the detected time events are actually discrete, the coincidence counting rate $R_c$ should be rewritten as

$$R_c(t_1 - t_2 = t_0) = \sum_{i=1}^n \sum_{j=1}^m S(t_{1,i} - t_{2,j} - t_0) G^{(2)}(t_{1,i} - t_{2,j}).$$

(5)

Where $n$ and $m$ are the numbers of the time sequences recorded for the signal and idler photons within the acquisition time of $T$, denoted as $\{t_{1,i}\}, i = 1, \cdots, n$, and $\{t_{2,j}\}, j = 1, \cdots, m$, respectively. According to Eq. (5), when $t_0 = \bar{\tau}$, the maximum coincidence rate is acquired. Therefore, the issue is turned to search for the maximum coincidences from the two detected time sequences by varying $t_0$ with a step of $\tau_{BW}$.

By first setting the searching resolution about three times of $\Delta\tau$, i.e. $\tau_{BW,C} \approx 3\Delta\tau$, a quick identification procedure is applied to seek for the coarse position of $t_{0c,max}$ at which the coincidence rate reaches maximum. During this procedure, only a short segment of time sequences is used with a length of $M \ll n, m$. Assume the time references at the two remote positions A and B are the same, i.e., $t_{1,1} - t_{1,M} \approx t_{2,1} - t_{2,M}$. The value of $t_{0c}$ is then varied from $t_{0c,ini} = t_{1,1} - t_{2,1}$ to $t_{0c,fin} = t_{0,ini} + N_c \tau_{BW,C}$, with the integer $N_c \approx \frac{t_{1,1} - t_{1,M}}{\tau_{BW,C}} \approx \frac{t_{2,1} - t_{2,M}}{\tau_{BW,C}}$. Thus by successively applying the coincidence window function $S(t_{1,i} - t_{2,j} - t_{0,k})$, where $t_{0c,k} = t_{0,ini} + k\tau_{BW,C}$, to each $t_{1,i} - t_{2,j}$ with $i, j \in M$, a coarse coincidence identification as a function of $t_{0,k}$ can be obtained. By finding the maximum coincidence counts, a coarse time offset $t_{0c,max}$ can be determined.

Centered at the coarsely identified time offset $t_{0c,max}$, a fine coincidence identification is subsequently executed. In this procedure, the variation of $t_{0f}$ is within the range of $\bar{t}_{0c,max} \pm \tau_{BW,C}/2$ with a step of $\tau_{BW,f} = \tau_{BW,C}/N_f$, where $N_f$ is an integer and can be flexibly chosen as long as it is larger than 100. Then applying the similar procedure with the fine resolutions $\tau_{BW,f}$ leads to a final coincidence distribution as a function of $t_{0f} = t_{0c,max} - \tau_{BW,C}/2 + k'\tau_{BW,f}$, $k' \in N_f$. Via a Gaussian fitting of the coincidence distribution, the time offset $\bar{\tau}$
corresponding to the maximum coincidences is therefore derived.

**Experimental implementation**

To demonstrate this nonlocal temporal correlation identification algorithm, a frequency-anticorrelated photon pair source was utilized. Fig. 1 shows the schematic diagram of the setup. The frequency anti-correlated biphoton source was generated by using a continuous wave 780-nm laser, which was produced by the cavity-based frequency doubling of a 10-kHz linewidth 1560nm fiber laser, to pump a 10 mm-long type-II PPKTP crystal with a poling period of 46.25 μm. The details of the experimental system can be found in Ref. [19]. After filtering out the residual pump, the orthogonally polarized signal and idler photons were departed by a fiber polarization beam splitter. The signal photons were transmitted directly to the single photon detector D1 [20, 21] while the idler photons were detected by another single photon detector D2 after propagation through a 10 km fiber. According to Eq. (2) and the experimental parameters, the FWHM coincidence width of the entangled photons after the dispersive broadening is estimated to be 460 ps.

![Fig. 1 Schematic experimental setup](image)

The arrival times of the signal and idler photons to the detectors were recorded independently by two commercial event timers, ET A and ET B (Eventech Ltd, A033-ET), as time tag sequences \( \{t_{1,i}\} \) and \( \{t_{2,j}\} \). Due to the data rate limitation of the ETs, the single photon rates at the inputs of the two event timers were controlled to be 12 kcps. Both ETs were referenced to a common timescale based on the laboratory’s own H-maser frequency standard. Based on the above algorithm, the time difference between the signal and idler photons at D1 and D2 are extracted from the registered time tag sequences \( \{t_{1,i}\} \) and \( \{t_{2,j}\} \).

Fig. shows a coarse coincidence distribution of the entangled photon events acquired in \( T_a = 0.05 \) s. When the coarse resolution is set to be \( \tau_{BW,C} = 1.25 \) ns, the coarse position at
which the coincidence rate reaches maximum was found to be $t_{0c,max} = 434409100$ ps, with the peak has a height about 18, which is significantly higher than that of the average background correlations of the uncorrelated photon noise (normally less than 2).

Fig. 2 The extracted coarse coincidence distribution from the time tagged sequences acquired in 0.05 s, the resolution is set as 1.25 ns.

Centered at the coarsely identified time offset $t_{0c,max}$, a fine coincidence identification is subsequently executed. By setting the fine resolution as $\tau_{BW,f} = 20$ ps, Fig. (a) depicts the fine coincidence distribution of entangled photons acquired in a time of $T_a = 5$ s, which shows a FWHM width of 464.67 ps and agrees well with the theoretical prediction. The time offset $\bar{\tau}$ is found to be at 434409078 ps by the Gaussian fitting. Fig. 3 (b) shows the extracted time offset results by repeating the measurement for 50 times, a standard deviation of 4.59 ps is achieved.

Fig. 3 (a) The constructed coincidence distribution (in black squares) and corresponding Gauss fitting (in red solid line) of entangled photons with a resolution of 20 ps; (b) the extracted time offset results distribution.

In order to evaluate the performance of our time offset extraction program, we further implement the measurement without the 10 km fiber in the setup. In this case, the measured coincidences with an integrated coincidence device (PicoHarp 300) gives a FWHM width of
\(\Delta t_{\text{jitter}} \approx 70\) ps, which is determined by the timing jitter of single photon detectors. The constructed coincidence distributions are shown in Fig. 4 (a) by choosing the fine resolution \(t_{\text{of}}\) successively as 1 ps (black squares), 3 ps (in red circles), 5 ps (in blue up-triangles), 7 ps (in magenta down-triangles), 9 ps (in olive diamonds), 15 ps (in dark cyan stars), 25 ps (in navy left-triangles), 35 ps (in violet right-triangles) and 55 ps (in purple hexagons). From Fig. 4 (a) we see that, the fitted FWHMs for \(\tau_{BW,f}=1\) ps, 3 ps, 5 ps, 7 ps, and 9 ps are 70.49 ps, 70.67 ps, 70.93 ps, 70.83 ps, and 71.03 ps, respectively, which shows very good consistency. Therefore, the very good agreement has been achieved between the results obtained by Picoharp 300 and our program based on independent ETs. It should be noted that the achievable least resolution of our algorithm is determined by the Least Significant Bit (LSB) resolution of the time-tagging device. Based on the above fine resolutions and corresponding Gaussian fittings, the time offset can be achieved. By repeating the measurements for 50 times, the averaged values and relevant precisions of the extracted time offsets in terms of standard deviation (SD) are given in Fig. 4 (b) by blue squares and black dots respectively. One can see that, the achieved precision is approximately independent of the resolution applied to the fine time offset identification when it is smaller than \(\Delta t_{\text{jitter}}\), which results from the timing jitter of the detector setup. When the resolution is above \(\Delta t_{\text{jitter}}\), the precision tends to be worse dramatically. It infers that the timing jitter of the detector setup determines the least precision of the time offset measurement.

In contrast, the average time offsets \(\bar{t}_{\text{of}}\) tend to linearly increase with the resolution \(\tau_{BW,f}\), which can be readily understood by the algorithm shown by Eq. (5). Therefore, besides the requirement of \(\tau_{BW,f} < \Delta t_{\text{jitter}}\), a finer \(\tau_{BW,f}\) is favorable for both constructing the coincidence distribution and optimizing the time offset identifications. Furthermore, for the application of high-precision quantum metrologies, a more accurate time offset measurement determines the ultimate performance of the system, and in order to improve the time-offset accuracy, a better coincidence resolution is demanded.
Fig. 4 (a) The constructed coincidence distributions of entangled photons acquired in 4.5s, with different resolutions of 1 ps (in black squares), 3 ps (in red circles), 5 ps (in blue up-triangles), 7 ps (in magenta down-triangles), 9 ps (in olive diamonds), 15 ps (in dark cyan stars), 25 ps (in navy left-triangles), 35 ps (in violet right-triangles) and 55 ps (in purple hexagons); and (b) relationship between precisions (in black circles), accuracy (in blue squares) of the measured time offsets and the resolution applied to the fine time offset identification.

By fixing the resolution as $\tau_{BW,f}=7$ ps, the precision of the measured time offset was further investigated as a function of the acquisition time $T_a$ and the results are shown in Fig. 4 by black squares. It can be seen that, the precision is enhanced with the increase of $T_a$. According to Ref. [22], its dependence on $T_a$ can be given by

$$\delta t_{SD} = \frac{\Delta \tau'}{\sqrt{2R_c T_a}}, \quad (6)$$

where $\Delta \tau'$ denotes the correlation width of the entangled photons together with the contribution from timing jitter of the detector setup, i.e., $\Delta \tau' = \sqrt{\Delta \tau^2 + \Delta \tau_{jitter}^2}$. In the experiment, $\Delta \tau' \approx \Delta \tau_{jitter}$. $R_c$ is the acquired coincidence count rate, which is about 1140 cps. Based on Eq. (6), the theoretical simulation is made and plotted in Fig. 5 (red solid line) as well. The agreement between theory and experimental results is shown at an acquisition time shorter than 1 s. Afterwards, the dropping of the precision curve starts to deviate from the $1/\sqrt{2R_c T_a}$ slope, and a minimum value of 0.72 ps is finally achieved at $T_a=4.5$ s.

Fig. 5 The acquired precision of the measured time offsets versus the data acquisition time (in black...
squares), and the corresponding simulation (in red solid line).

Discussion

Above we have demonstrated the working principle and properties of our algorithm for nonlocally identifying the time correlation between the entangled photon pairs. To verify the superiority of our method, we also applied the algorithm proposed by Ho et al. [14] to the same time sequences used for the above correlation identification process. Fig. 6 (a) and (b) depict the extracted time correlation results by utilizing Ho’s algorithm with the resolution set as 1.2 ns and 0.8 ns, respectively. According to Ho et al., the correlation peak can be identified accurately when the acquired maximum correlation $S_{\text{max}}$ exceeds the threshold $S_p = \sqrt{N/R_1 R_2}$, with time bin number $N$, single photon rates $R_1$ and $R_2$, which is calculated to be 210 in our case. One can see that, when the resolution is chosen as 1.2 ns, $S_{\text{max}} \geq S_p$ is satisfied, the correlation peak can thus be identified as 434409.8 ns. However, such requirement cannot be reached when the resolution is less than 1 ns (see Fig. (b) for example), indicating that this algorithm cannot be used for high-precision time offset identification. In comparison, when the resolution is set to be 1 ps, the time offset information can also be extracted accordingly.

Fig. 6 The cross correlation by applying Ho’s algorithm to the same time sequences that was used for the above correlation identification, with the calculating resolution of (a) 1.2 ns and (b) 0.8 ns, where $S$ is the statistical significance of the time correlation results, and $k$ represents the correlation index.

Furthermore, as shown by Fig. 4 (a), the constructed coincidence distribution of the entangled photon pairs based on our algorithm is perfectly fitted with that obtained with a dedicated coincidence hardware (e.g. Picoharp 300). In comparison with the dedicated coincidence hardware, the coincidence measurement can be achieved without deteriorating the resolution, meanwhile there is no restriction on the locations of the two correlated photons after
propagation. Utilizing our algorithm, we have successfully demonstrated experimental realizations of the femtosecond-level quantum clock synchronization (QCS) [23] and the quantum nonlocality test based on nonlocal dispersion cancellation (NDC) [24] in laboratory.

Conclusion

In summary, we have presented an improved nonlocal time offset identification algorithm of entangled photons based on event timers. With this algorithm, a minimum precision of 0.72 ps has been demonstrated. We also provided a well-performed coincidence identification method for entangled photons between remote sites with a tunable resolution down to 1 ps. Benefitted from the algorithm, high-precision quantum clock synchronization and nonlocal dispersion cancellation have been achieved in experiment. Through the work, further progress can be expected in applications of practical and field quantum metrologies and quantum communications and so on.

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