ASPECTS OF THE STANDARD MODEL AND QUANTUM GRAVITY FROM STRAND SPACETIME

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Abstract. Strands are causal curves in spacetime with no distinct interior points, introduced to describe quantum nonlocality in a spacetime framework. We present a model where the standard model particles are bound states of strands that interact by exchanging strands. In strand spacetime, it is not just the positron whose existence is implied by the Dirac Lagrangian: we show that hidden within this simple Lagrangian are all the quarks, leptons, and gauge bosons, with their correct spin, electric charges, color charges, and, in the electroweak sector, stability. Also encoded in the combinatorics of the Dirac Lagrangian are all the trivalent electroweak interactions (involving both leptons and quarks), electroweak parity violation, as well as 16 independent mass orderings that all agree exactly with experiment. However, the model predicts the existence of massive gluons that are cousins of the $W$ and $Z$ bosons, but no other particles. Using the geometry of strands, we are able to derive many properties of quarks, such confinement, three color charges, and their allowable combinations into baryons and mesons. We also show that CPT invariance holds for all interactions, where C, P, and T each sit in a different connected component of the full Lorentz group. Finally, we introduce a quantum modification to Einstein’s equation by reinterpreting the chiral decomposition of the Dirac Lagrangian.

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1. Introduction

Strand spacetime is obtained from classical general relativity by defining the world-line of a fundamental particle to be a causal curve with no distinct interior points. Such a 1-dimensional ‘smeared-out’ event in spacetime is called a strand. This geometry was recently introduced to give a possible spacetime description of quantum nonlocality [35]. In this article, we consider the following model:

- Strands are ‘circular’ with spin $\frac{1}{2}$ and mass equal to their inverse radius $r$,

$$m = \frac{\hbar}{cr}.$$  \hspace{1cm} (1)

- Bound states of strands interact with each other by exchanging strands according to Newton’s third law of motion, called a splitting.

The mass relation (1) implies that strands satisfy the modified energy-mass relation $E_0 = mc^2u$, where $u$ is their tangential speed (Section 3). This relation in turn implies that the Lagrangian density for strands is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m|\bar{u}^a u_a|^{1/2})\psi,$$  \hspace{1cm} (2)

where $\bar{u}$ and $u$ are projections of the four-velocities of the strands represented by $\bar{\psi}$ and $\psi$ onto a spatial hypersurface (Section 6). From this Lagrangian we obtain a new composite preon model where leptons, quarks, and gauge bosons are bound states of strands, rendering the standard model Lagrangian an effective theory (Section 7). The Lagrangian (2) implies that the strands themselves admit no fundamental interactions other than creation and annihilation in pairs. An example of scattering using bound states of strands is shown in Figure 1.

In our model, we are able to reproduce exactly the leptons, quarks, and electroweak bosons, with their correct spin, electric charge, color charge, and, in the electroweak sector, stability (Table 4). Our model also reproduces exactly the trivalent electroweak Feynman interactions involving both leptons and quarks (Tables 1, 2, and 6). However, our model predicts the existence of neutral and charged massive gluons, and is therefore falsifiable (Table 5). It is particularly surprising that we find much of the standard model – with its various particles and forces – hidden within the Dirac Lagrangian (2).

Furthermore, the combinatorics and geometry of the model have wide explanatory power. In particular, we obtain

- the existence of electric charge and three color charges (Sections 4 and 12.2);
the allowable combinations of quarks into baryons and mesons (Sections 4 and 7.3.2);
• quark confinement (Sections 4 and 7.3.2);
• 16 mass orderings of leptons, quarks, and gauge bosons (Section 8);
• a determination of which electroweak particles are stable (Section 9); and
• neutrino parity violation (Section 10).

In Sections 7 - 10, we introduce the preon model and explore some of its immediate consequences. In Section 11, we present a modification to Einstein’s equation from the Dirac Lagrangian that resolves Bohr’s gedankenexperiment regarding gravitational radiation and which-way information. In Section 12, we show that in the framework of strands, charge conjugation is a Lorentz transformation. Charge conjugation, parity, and time reversal are then found to each sit in a different connected component of the Lorentz group \( O(1,3) \), and their product is the identity transformation. As a consequence, we obtain CPT invariance of standard model interactions. In Section 13, our definition of charge conjugation is used to derive a spin-statistics connection, following Schwinger’s heuristic argument. We conclude with remarks on scattering, and give evidence for why ultraviolet divergences in quantum field theory may not be present in the framework of strands.

Throughout the article, we primarily restrict our attention to a classical analysis of the preon model; quantum aspects of strand spacetime, including the ontology of path and spin quantum states, is considered in the companion article [B5].

We briefly note relations to other work. Our preon model is similar to ’t Hooft’s double line formalism for quarks and gluons [tH]. A well-known preon model related to quantum gravity, and specifically to loop quantum gravity, is the Harari-Shupe (or rishon) model [Har, Shu, HarS], and its topological realization with braids [Bi, BMS, BHKS]. Although the preon model we introduce is different from the Harari-Shupe model, its objectives are similar. It is also possible that the constraints in quantum field theory obtained through strands (Section 14) are related to the discretization of spacetime, and so could potentially be related to loop quantum gravity, Regge calculus, causal dynamical triangulation, or causal set theory [BLMS].

**Notation:** Tensors labeled with upper and lower indices \( a, b, \ldots \) represent covector and vector slots using Penrose’s abstract index notation, and tensors labeled with indices \( \mu, \nu, \ldots \) denote its components with respect to a coordinate basis. We use the signature \((+,-,-,-)\) throughout, and usually use natural units \(\hbar = c = 1\).

## 2. Strand spacetime

In formulating general relativity, Einstein replaced the gravitational force field in Newton’s theory of gravity with the geometry of spacetime. In a similar way, we would like to describe the particles in the standard model not as quantized fields, but
Figure 1. An example of photon entanglement using strands. The corresponding Feynman diagram is shown on the right.

Table 1. The fundamental splittings and their $O(3)$ particle identifications.

| $\gamma$ | $Z$ | $W^-$ | $W^+$ |
|----------|-----|-------|-------|
| $\bar{\psi}_L \psi_R$ | $\bar{\psi}_L \psi = \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$ | $\bar{\psi}^0 \psi := \bar{\psi}^0_L \psi_R + \bar{\psi}^0_R \psi_L$ | $\bar{\psi}^0 \psi := \bar{\psi}^0_L \psi_R + \bar{\psi}^0_R \psi_L$ |
| $\bar{\psi}_L$ | $\bar{\psi}_L$ | $\bar{\psi}_L$ | $\bar{\psi}_L$ |
| $\psi_R = e$ | $\psi_R = e$ | $\psi_R = e$ | $\psi_R = e$ |
| $\nu_\mu = \bar{\psi}_L \otimes \psi_L$ | $\psi_R \otimes \bar{\psi}_R = \bar{\nu}_\mu$ | $\mu = \bar{\psi}_L^0 \otimes \psi_L$ | $\psi_R \otimes \bar{\nu}_R = \bar{\nu}_\mu$ |
| $\nu_\tau = \bar{\psi}_L \otimes \bar{\psi}_R$ | $\psi_R \otimes \psi_L = \bar{\nu}_\tau$ | $\tau = \bar{\psi}_L^0 \otimes \bar{\psi}_R$ | $\psi_R \otimes \psi_L = \bar{\nu}_\tau$ |

as geometric properties of spacetime itself. Quantum field theory would then be an emergent description of particle physics, rather than a fundamental description.

To this end, we define a fundamental particle to be a causal (i.e., timelike or null) curve in spacetime that is deemed to be a single point; that is, it is a curve without
Table 2. The non-fundamental splittings, wherein strands are allowed to be created according to the strand Lagrangian \(^\text{(2)}\): in pairs of opposite sign on a diameter. The only additional rule is that each of the five fields \(\bar{\psi}_{L/R}, \psi_{L/R}, \phi\) is excited in some atom in the splitting. The splitting \(W^± \to ZW^±\) is equivalent to \(Z \to W^±W^±\).

| \(\gamma\) | \(Z\) |
|---|---|
| \(\bar{\psi}_L \psi_R\) | \(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\) |
| \(\mu = \bar{\psi}_L^0 \otimes \psi_L\) | \(\mu = \bar{\psi}_L^0 \otimes \psi_L\) |
| \(\psi_R^0 \otimes \bar{\psi}_R = \bar{\mu}\) | \(\psi_R^0 \otimes \bar{\psi}_R = \bar{\mu}\) |
| \(\tau = \bar{\psi}_L^0 \otimes \bar{\psi}_R\) | \(\tau = \bar{\psi}_L^0 \otimes \bar{\psi}_R\) |
| \(\psi_R^0 \otimes \psi_L = \bar{\tau}\) | \(\psi_R^0 \otimes \psi_L = \bar{\tau}\) |
| \(W^- = \bar{\psi}_L^0 \psi\) | \(W^- = \bar{\psi}_L^0 \psi\) |
| \(\bar{\psi}^0 \psi = W^+\) | \(\bar{\psi}^0 \psi = W^+\) |
| \(\nu_e = \bar{\psi}_L^0\) | \(\nu_e = \bar{\psi}_L^0\) |
| \(\psi_R^0 = \nu_e\) | \(\psi_R^0 = \nu_e\) |

**distinct interior points.** In particular, time does not flow along such a curve, even if it is timelike; this is the source for quantum nonlocality introduced in \([B5]\).

**Definition 2.1.** Let \((\tilde{M}, \tilde{g})\) be a \((3+1)\)-dimensional time-orientable Lorentzian manifold, which we call **emergent spacetime**. Let \(S\) be a collection of causal curves in \(\tilde{M}\), called **strands**. We declare two points \(x, y \in \tilde{M}\) to be equivalent if there is a strand \(\alpha \in S\) that contains both \(x\) and \(y\), and extend the equivalence transitively. We define **spacetime** to be the set of equivalence classes

\[M := \{[x] : x \in \tilde{M}\}.\]

Denote by \(\pi\) the map

\[\pi : \tilde{M} \to M, \quad x \mapsto [x].\]

Each point \(x\) in \(U := \pi(\tilde{M} \setminus \cup_{\alpha \in S} \alpha)\) has a unique preimage \(\pi^{-1}(x)\). Thus, to each point \(x \in U\), we may associate the unique vector space

\[T_x\tilde{M} := T_{\pi^{-1}(x)}\tilde{M}.\]

This allows us to make the following definitions:

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1In the framework of nonnoetherian algebraic geometry introduced in \([B2]\), algebraic varieties with nonnoetherian coordinate rings of finite Krull dimension necessarily contain positive dimensional ‘smeared-out’ points (see \([B3]\) Theorem A for a precise statement). Such a variety may contain, for example, curves that are identified as single points. The original purpose of this framework was to provide a geometric description of the vacuum moduli spaces of certain unstable quiver gauge theories in string theory \([B4]\) (see also \([B1]\)). It was then proposed in \([B6]\) that this geometry could be applied to spacetime itself, with the hope that it could explain, in a suitable sense, quantum nonlocality.
• The exponential map \( \exp : T_x M \to M \) at \( x \in U \) is the composition
\[
T_x M = T_{\pi^{-1}(x)} \tilde{M} \xrightarrow{\exp} \tilde{M} \xrightarrow{\pi} M.
\]

• The metric at \( x \in U \) is the metric at \( \pi^{-1}(x) \),
\[
g_x := \tilde{g}_{\pi^{-1}(x)} : T_x M \times T_x M \to \mathbb{R}.
\]

A strand, then, is a 1-dimensional point of spacetime, and does not possess a tangent space. In a given frame, a strand appears to be a classical particle, which we call a \textit{strand particle}. The difference between a worldline and a strand is that a worldline consists of a continuum of distinct points, whereas a strand is a single point.

2.1. Circular strands. We now define the specific strands that will be used in the preon model.

**Definition 2.2.** Let \( \beta \) be a causal geodesic in \( \tilde{M} \) with affine parameterization. A strand is \textit{circular} if it is a closed segment of a curve of the form
\[
\alpha(t) = \begin{cases} 
\exp_{\beta(t)} (r \cos(\omega t)e_1 + r \sin(\omega t)e_2) & \text{if } \dot{\beta}(t)^2 < 0 \\
\exp_{\beta(t)} (r \cos(\omega t_0)e_1 + r \sin(\omega t_0)e_2) & \text{if } \dot{\beta}(t)^2 = 0
\end{cases}
\]

where \( r > 0 \) and \( \omega \in \mathbb{R} \); \( t_0 \) is fixed; and \( \{e_1, e_2, e_3\} \) is a spacelike orthonormal basis that is parallel transported along \( \beta \),
\[
\dot{\beta}^a \nabla_a e_i = 0.
\]

For the remainder of the article, we will only consider circular strands, and so we will usually omit the prefix ‘circular’.

We define the four-momentum \( p^a \) of the strand particle of \( \alpha \) by the Planck-de Broglie relation in the direction \( \dot{\beta} \) (with \( \hbar = c = 1 \)):
\[
p^a := k^a = |\omega| \dot{\beta}^a.
\]

We say the strand particle is \textit{massive} if \( p^2 = p^a p_a \neq 0 \), and \textit{massless} if \( p^2 = 0 \).

3. The mass-shell condition without Lorentz violation

3.0.1. Massive strands. Suppose \( \alpha \) is a circular strand whose central worldline \( \beta \) is a timelike geodesic. In the inertial frame of \( \beta \), \( \alpha \) is a circular trajectory of radius \( r \), angular frequency \( \omega \), and tangential velocity
\[
u := |\dot{\alpha}| = |\omega|r.
\]

We define the mass \( m \) of \( \alpha \) to be its curvature,
\[
m := \frac{1}{r} = \frac{\hbar}{cr}.
\]
with units restored in the rightmost equality. (Note that \( r = \hbar (\text{cm})^{-1} \) is the reduced Compton wavelength of a particle of mass \( m \).) Consequently, the rest energy \( E_0 \) of \( \alpha \) in the inertial frame of \( \beta \) is

\[
\frac{E_0}{u} = \frac{\hbar|\omega|}{u} = \frac{\hbar}{r} = mc,
\]

that is,

\[
(4) \quad E_0 = mcu.
\]

We thus derive a variant of Einstein’s relation \( E_0 = mc^2 \); Einstein’s relation holds if and only if the tangential velocity \( u \) equals the speed of light \( c \).

3.0.2. Massless strands. A circular strand particle \( \alpha(t) \), with a lightlike central world-line \( \beta \), cannot rotate as it propagates. Indeed, assume to the contrary that \( \alpha(t) \) rotates about \( \beta \), just as a massive strand particle does:

\[
(\alpha(t)^\mu) = (t, r \cos(\omega t), r \sin(\omega t), t).
\]

The tangent vector to \( \alpha \) is then spacelike with length

\[
\dot{\alpha}^2 = -(\omega r)^2.
\]

Thus, \( |\omega|r \) is invariant under Lorentz transformations. But the frequency \( \omega \) varies with boosts in the \( z \)-direction, whereas the radius \( r \) is independent of such boosts. Therefore \( |\omega|r \) cannot be invariant, a contradiction.

The precise time at which two bound states of strands interact will always be uncertain (i.e., indistinguishable) over a nonzero interval of time \([t_0, t_0 + \epsilon]\). Therefore, by the identity of indiscernibles (see [B5]), a circular strand does not actually propagate as a (0-dimensional) particle, but rather as a segment of a helix: for \( s \in [t_0, t_0 + \epsilon] \) and \( t \geq t_0 + \epsilon \), we have

\[
\alpha(s, t) = \begin{cases} 
\exp_{\beta(t)} (r \cos(\omega(t + s))e_1 + r \sin(\omega(t + s))e_2) & \text{if } \beta(t)^2 < 0 \\
\exp_{\beta(t)} (r \cos(\omega(t_0 + s))e_1 + r \sin(\omega(t_0 + s))e_2) & \text{if } \beta(t)^2 = 0
\end{cases}
\]

However, all the interior (0-dimensional) points of the helix in \( \tilde{M} \) are identified as the same point in \( M \). As we have just shown, the helical segment rotates in an inertial frame if and only if the strand is massive. Nevertheless, a massless strand still possesses a frequency \( \omega \), given by the pitch of its propagating helical segment.

For our purposes here, it suffices to regard massive strands as simply (0-dimensional) particles.

3.0.3. The mass-shell condition. Set \( c = 1 \). From (4), the four-momentum

\[
p^\mu = |\omega|\dot{\beta}^\mu = E_0\beta^\mu = mu\beta^\mu
\]

of a massive strand particle satisfies

\[
p^2 = p^\mu p_\mu = E_0^2 = m^2 u^2.
\]
This is a modification of the standard relativistic mass-shell condition $p^2 = m^2$.

In standard quantum field theory, a particle (or field excitation) is said to be on-shell if $p^2 = m^2$, and off-shell, or virtual, if $p^2 \neq m^2$. During a scattering event, most internal particles are off-shell. Under the assumption that $E_0 = m$, such particles violate relativity (hence the name ‘virtual’). However, under the assumption that $E_0 = mu$, i.e., $p^2 = m^2 u^2$, off-shell massive strand particles do not violate relativity; they are simply particles whose tangential velocity is not lightlike.

Consequently, a massive strand particle is

- lightlike ($u = 1$) iff $p^2 = m^2 u^2 = m^2$; and
- timelike ($u \neq 1$) iff $p^2 = m^2 u^2 \neq m^2$.

Furthermore, a massless strand particle is

- lightlike ($u = 1$) iff $\beta$ is lightlike, whence $p^2 = |\omega|^2 \beta^2 = 0$; and
- timelike ($u \neq 1$) iff $\beta$ is timelike, whence $p^2 = |\omega|^2 \beta^2 \neq 0$.

The variability of $u$ thus enables a geometric description of off-shell particles for which $p^2 = |\omega|^2 \beta^2$ always holds (that is, $p^2 = E_0^2$ in the massive case), and therefore relativity is never violated.

Lightlike tangential velocity may be viewed as a geodesic-like property: suppose a circle of radius $r$ is rotating with tangential velocity $u$ measured in an inertial frame. In the accelerated frame of the circle, Ehrenfest observed that the circumference is

$$C = 2\pi r (1 - u^2)^{-1/2} = 2\pi r \gamma(u).$$

Thus, if $u = 1$, then $C$ is infinite. If a circle of infinite circumference is regarded as a straight line, then the particle travels in a ‘straight line’ in its own reference frame if and only if it travels at light speed $u = 1$. Off-shell strand particles are thus unstable, and as such quickly interact with neighboring strands to recover their geodesic states.

3.1. Conservation of angular momentum. Recall that a massive strand particle $\alpha(t) \in \tilde{M}$ has mass $m = r^{-1}$. Thus, the spatial angular momentum $L$ of $\alpha(t)$, in the inertial frame of its central wordline $\beta$, equals its tangential velocity $u = \omega r$,

$$L = m|p| = rmu = u.$$

Conservation of angular momentum therefore implies that the particle’s tangential velocity $u$ is constant.

4. Electric and color charge from the geometry of strands

In this section we will show that both electric charge and color charge, as well as the allowable combinations into mesons and baryons, are novel features of strand spacetime geometry.

The worldline $\alpha$ of a strand particle $\alpha(t)$ is a continuum of distinct points in emergent spacetime $\tilde{M}$, and a single point $\pi(\alpha)$ in spacetime $M$ itself. Thus, there is no tangent vector field ‘along’ the point $\pi(\alpha)$ in $M$. In contrast, the strand particle
has a tangent vector field $\tau$ along its worldline $\alpha$ in $\tilde{M}$, since $\tilde{M}$ is a manifold. But this four-vector is not uniquely determined by the motion of the strand particle in spacetime $M$, because, fundamentally, time does not flow along its worldline. There is therefore an ambiguity in the choice of tangent four-vector, namely

$$\tau = \pm \dot{\alpha}.$$  

We identify $\tau = \dot{\alpha}$ with negative electric charge, and $\tau = -\dot{\alpha}$ with positive electric charge. We thus obtain a new definition of electric charge from the geometry of strands.

Consider a strand $\alpha$ with a timelike central worldline $\beta$. Identify the tangent spaces $T_{\alpha(t)}\tilde{M}$ along $\beta$ via the isomorphism induced by the tetrad $\{e_a\}$,

$$T_{\alpha(t)}\tilde{M} \cong T_{\beta(t')}\tilde{M} \cong \mathbb{R}^{1,3}.$$  

Consider the spatial subspace

$$V := \text{span}_\mathbb{R}\{e_1, e_2, e_3\} \subset T_{\beta(t)}\tilde{M}.$$  

Restricted to $V$, $\alpha$ has circular trajectory

$$\alpha(t) = (r \cos(\omega t), r \sin(\omega t), 0).$$

For ease of notation, we assume that $\alpha$ has unit speed parameterization, $u = \omega r = 1$.

In isolation, or empty space, there is no distinguished direction of space. Thus, to obtain a spatial tangent vector $t \in V$ to $\alpha$ at $\alpha(t)$, we may apply any Lorentz transformation $g \in O(3)$ to the vector $\dot{\alpha}(t) \in V$, with the property that $g$ is invariant under an arbitrary Lorentz change-of-basis $h \in O(3)$:

$$h^{-1}gh = g.$$  

Consequently, $g$ is in the center of $O(3)$,

$$g \in Z(O(3)) = \{w^\pm := \pm \text{diag}(1, 1, 1)\} \cong \mathbb{Z}_2.$$  

The possible tangent vectors to $\alpha(t)$ are therefore

$$t = w^+ \dot{\alpha} = \dot{\alpha} \quad \text{and} \quad t = w^- \dot{\alpha} = -\dot{\alpha}.$$  

We call the choice of $w^+$ or $w^-$ the strand charge of $\alpha$, denoted $q(\alpha)$, and identify these charges with negative and positive electric charges, respectively. We will consider charge conjugation in Section 12.2 below.

Now consider a strand $\alpha$ in a bound state of massive strands that share a common timelike central worldline $\beta$. Consider the worldline Frenet frame $\{t, n, b\}$ of $\alpha$, translated to the origin of $V$. Just as there is an ambiguity in the choice of tangent vector $t$, namely $t = \pm \dot{\alpha}$, there is also an ambiguity in the choice of normal vector $n = \pm \dot{\alpha}$ and binormal vector $b = \pm e_3$, again since time does not flow along the trajectory $\alpha$.

Indeed, the normal line $L = \text{span}_\mathbb{R}\{e_3\} \subset V$ to the plane of rotation

$$P = \text{span}_\mathbb{R}\{e_1, e_2\}$$
is a distinguished direction of space. Thus, to obtain the Frenet frame, we may apply any orthogonal transformation \( g \in O(V) = O(3) \) to

\[
\begin{align*}
\left\{ \hat{\alpha}, \ddot{\alpha}, \frac{\dot{\alpha} \times \ddot{\alpha}}{|\dot{\alpha} \times \ddot{\alpha}|} = \text{sgn}(\omega)e_3 \right\},
\end{align*}
\]

with the property that \( g \) is invariant under an arbitrary orthogonal change-of-basis \( h \) in the subgroup \( O(2) \times O(1) \) of \( O(3) \) specified by \( P \),

\[
h \in O(2) \times O(1) = O(P) \times O(L) \subset O(V).
\]

Consequently, \( g \) is in the center of \( O(2) \times O(1) \),

\[
g \in Z(O(2) \times O(1)) \cong \mathbb{Z}_2 \times \mathbb{Z}_2.
\]

We denote the four elements of \( Z(O(2) \times O(1)) \), with respect to the ordered basis \( \{e_1, e_2, e_3\} \), by

\[
\begin{align*}
\mathcal{w}^\pm := \pm \text{diag}(1, 1, 1) \quad \text{and} \quad \mathcal{c}^\pm := \pm \text{diag}(1, 1, -1).
\end{align*}
\]

These central elements act on \( \{t, n, b\} \) to give the Frenet frame \( \{t, n, b\} \) in \( \tilde{M} \), and we call the choice of central element the strand charge of \( \alpha \), denoted \( q(\alpha) \). Thus, for example, \( \alpha \) has strand charge \( c^- \) if and only if its Frenet frame is

\[
\begin{align*}
t &= c^- \dot{\alpha} = -\dot{\alpha}, \quad n = c^- \ddot{\alpha} = -\ddot{\alpha}, \quad b = c^- \text{sgn}(\omega)e_3 = \text{sgn}(\omega)e_3.
\end{align*}
\]

The possible Frenet frames in \( \tilde{M} \) are therefore

\[
\begin{array}{|c|c|c|}
\hline
\text{charge} & t & n & b \\
\hline
\mathcal{w}^+ & \dot{\alpha} & \ddot{\alpha} & \text{sgn}(\omega)e_3 \\
\mathcal{w}^- & -\dot{\alpha} & -\ddot{\alpha} & -\text{sgn}(\omega)e_3 \\
\mathcal{c}^+ & \dot{\alpha} & \ddot{\alpha} & -\text{sgn}(\omega)e_3 \\
\mathcal{c}^- & -\dot{\alpha} & -\ddot{\alpha} & \text{sgn}(\omega)e_3 \\
\hline
\end{array}
\]

Let \( \mathcal{P} \) be the set of fixed planes \( P \) in a bound state \( \cup \alpha \). The total charge of \( \cup \alpha \) is the \( \mathbb{Z} \)-linear combination

\[
q(\cup \alpha) := \sum_\alpha q(\alpha) = n_w \mathcal{w}^+ + \sum_{P \in \mathcal{P}} n_P \mathcal{c}^+,
\]

where \( n_w, n_P \in \mathbb{Z} \) are integer coefficients. A strand or bound state of strands is able to exist in isolation if and only if it is invariant under \( O(3) \). Therefore, a bound state \( \cup \alpha \) may exist in isolation if and only if

\[
q(\cup \alpha) = m_w \mathcal{w}^+
\]

for some \( m_w \in \mathbb{Z} \). This condition restricts the allowable sets of fixed planes of a bound state that is able to exist in isolation.
The simplest bound state with color charge that may exist in isolation consists of two strands that share the same fixed plane $P$, but have opposite charges $c_P^+$ and $c_P^-$. 

$$c_P^+ + c_P^- = 0w^+.$$

We call such a bound state a mesonic state.

The next simplest bound state with color charge that may exist in isolation consists of three strands $\alpha_1, \alpha_2, \alpha_3$, necessarily with orthogonal binormal lines, say 

$$e_3(\alpha_1) = (1, 0, 0), \quad e_3(\alpha_2) = (0, 1, 0), \quad e_3(\alpha_3) = (0, 0, 1).$$

Their respective possible color charges are then 

$$r^\pm := \pm \text{diag}(-1, 1, 1)$$

$$g^\pm := \pm \text{diag}(1, -1, 1)$$

$$b^\pm := \pm \text{diag}(1, 1, -1)$$

These matrices satisfy the relations

(11) $$r^+ + g^+ + b^+ = w^+$$

and

(12) $$w^+ + w^- = r^+ + r^- = g^+ + g^- = b^+ + b^-.$$

Therefore the strands $\alpha_1, \alpha_2, \alpha_3$ may have color charges 

$$q(\alpha_1) = r^+, \quad q(\alpha_2) = g^+, \quad q(\alpha_3) = b^+$$

or

$$q(\alpha_1) = r^-, \quad q(\alpha_2) = g^-, \quad q(\alpha_3) = b^-.$$

We call such a bound state a baryonic state.

There is a unique configuration of three pairwise orthogonal planes in $\mathbb{R}^3$, up to rotation. Thus there are precisely three orthogonal embeddings of $O(2) \times O(1)$ in $O(3)$, up to rotation. Consequently, every $O(3)$ bound state must be a mesonic state, a baryonic state, a collection of strands each with spatial group $O(3)$, or a union of such states.

The strand charge of a strand $\alpha$ is therefore an element of $\{w^\pm, r^\pm, g^\pm, b^\pm\}$. We make the following identifications between strand charges and electric and color charges:

| strand charge | electric charge | color charge |
|---------------|-----------------|-------------|
| $w^+$         | $-e$ (negative) |             |
| $w^-$         | $+e$ (positive) |             |
| $r^+, g^+, b^+$|                 | red, darkgreen, blue |
| $r^-, g^-, b^-$|                 | anti-red, anti-darkgreen, anti-blue |
We will denote by $c^\pm$ an unspecified color charge $r^\pm, g^\pm, b^\pm$.

The sign of $\alpha$ (or $q(\alpha)$), denoted $\text{sgn}(\alpha)$, is the sign $\pm$ of the superscript of $q(\alpha)$.

**Definition 4.1.** The antiparticle $\bar{\alpha}$ of a strand particle $\alpha$ is obtained by reversing the sign of $\alpha$. The antiparticle $\bar{B}$ of a bound state $B = \cup \alpha$ consisting of a collection of strands is obtained by reversing the sign of each strand in $B$.

**Remark 4.2.** The Stueckelberg interpretation of antiparticles as particles that travel backwards in time [St] is obtained by replacing $t$ with $-t$ in (5), and thus results in the respective tangent vectors

\[ t = \dot{\alpha} \quad \text{and} \quad t = -\bar{\alpha}, \]

in agreement with (9). Indeed, we have

\[ \frac{d}{dt}(\alpha(-t)) = (-1, u \sin(-\omega t), -u \cos(-\omega t), 0) = -\frac{d}{dt}\alpha(t). \]

However, this interpretation does not give color charge. Furthermore, time does not flow along a strand: time does not flow backwards just as it does not flow forwards.

### 5. The Chiral Spinor Representation of Strands

By assumption, circular strands have spin $\frac{1}{2}$. In this section we determine the chirality of a circular strand.

Let $\alpha$ be a strand of radius $r$ centered about a timelike geodesic $\beta(I) \subset \tilde{M}$. Identify the tangent spaces $T_{\beta(t)}\tilde{M}$ along $\beta$ via the isomorphism induced by the tetrad $\{e_a\}$,

\[ T_{\beta(t)}\tilde{M} \cong T_{\beta(t')}\tilde{M} \cong \mathbb{R}^{1,3}. \]

Consider the spatial subspace

\[ V := \text{span}_{\mathbb{R}}\{e_1, e_2, e_3\} \subset T_{\beta(t)}\tilde{M}. \]

Restricted to $V$, $\alpha$ has circular trajectory [5]. Translate the (unnormalized) worldline Frenet frame $\{\dot{\alpha}, \ddot{\alpha}, \dot{\alpha} \times \ddot{\alpha}\}$ to the origin of $V$. Since time does not flow along a strand, a Frenet vector, or a wedge product of Frenet vectors, is a property of $\alpha$ in spacetime $M$ if and only if it is independent of time $t$; we call the set of all such wedge products the $M$-frame of $\alpha$.

**Remark 5.1.** The parameter $t$ is the proper time as measured by a clock alongside the strand particle $\alpha(t)$ in emergent spacetime $\tilde{M}$, but does not represent a time parameter in spacetime $M$ itself.

The wordline tangent and normal vectors, $\dot{\alpha}$ and $\ddot{\alpha}$, vary with time $t \in I$, whereas the binormal vector,

\[ \frac{\dot{\alpha} \times \ddot{\alpha}}{|\dot{\alpha} \times \ddot{\alpha}|} = \frac{\omega^2 r^2}{|\omega^2 r^2|} e_3 = \text{sgn}(\omega) e_3, \]

is independent of $t$. Therefore, since the strand $\alpha$ is a single point in $M$, only the binormal vector $\dot{\alpha} \times \ddot{\alpha}$ of the Frenet frame belongs to the $M$-frame of $\alpha$. 
We may also consider wedge products of worldline Frenet vectors. The two wedge products
\[ \hat{\alpha} \wedge (\hat{\alpha} \times \hat{\alpha}) = -\omega^4 r^3 \sin(\omega t) e_1 \wedge e_3 + \omega^4 r^3 \cos(\omega t) e_2 \wedge e_3 \]
and
\[ \hat{\alpha} \wedge (\hat{\alpha} \times \hat{\alpha}) \]
vary with \( t \), whereas the two wedge products
\[ (14) \quad \hat{\alpha} \wedge \hat{\alpha} = \omega^3 r^2 e_1 \wedge e_2 \]
and
\[ \hat{\alpha} \wedge \hat{\alpha} \wedge (\hat{\alpha} \times \hat{\alpha}) = \omega^6 r^4 e_1 \wedge e_2 \wedge e_3 \]
are independent of \( t \). Thus, the two wedge products (14) also belong to the \( M \)-frame of \( \alpha \).

To determine the possible \( M \)-frames, we act on (13) and (14) by the strand charges \( w^\pm \) and \( c^\pm \) in (8). Set
\[ \chi(\alpha) := \text{sgn}(\alpha) \text{sgn}(\omega). \]
The electric charges \( w^\pm \) act by
\[ \hat{b} = \text{sgn}(\omega) w^\pm e_3 = \chi(\alpha)e_3, \]
(16)
\[ \hat{i} \wedge \hat{n} = \text{sgn}(\omega)(w^\pm e_1) \wedge (w^\pm e_2) = \text{sgn}(\omega) e_1 \wedge e_2, \]
\[ \hat{i} \wedge \hat{n} \wedge \hat{b} = \text{sgn}(\alpha)e_1 \wedge e_2 \wedge e_3. \]

Similarly, the color charges \( c^\pm \) act by
\[ \hat{b} = \text{sgn}(\omega) c^\pm e_3 = \chi(\alpha)e_3, \]
(17)
\[ \hat{i} \wedge \hat{n} = \text{sgn}(\omega)(c^\pm e_1) \wedge (c^\pm e_2) = \text{sgn}(\omega) e_1 \wedge e_2, \]
\[ \hat{i} \wedge \hat{n} \wedge \hat{b} = \text{sgn}(\alpha)e_1 \wedge e_2 \wedge e_3. \]

There are therefore four possible \( M \)-frames, and these are specified by the signs of \( \alpha \) and \( \omega \):
\[ \{ \hat{b}, \hat{i} \wedge \hat{n}, \hat{i} \wedge \hat{n} \wedge \hat{b} \} = \{ \chi(\alpha)e_3, \text{sgn}(\omega)e_1 \wedge e_2, \text{sgn}(\alpha)e_1 \wedge e_2 \wedge e_3 \}. \]

We now briefly recall the chiral decomposition of the Lorentz algebra. The generators of the Lorentz algebra \( \mathfrak{so}(1,3) \), namely the three rotations \( J_i \) and three boosts \( K_i \) admit linear combinations \( A_i := \frac{1}{2}(J_i + iK_i) \) and \( B_i := \frac{1}{2}(J_i - iK_i) \) that satisfy
\[ [A_i, A_j] = i\epsilon_{ijk} A_k, \quad [B_i, B_j] = i\epsilon_{ijk} B_k, \quad [A_i, B_j] = 0. \]
Therefore the complexification \( \mathfrak{so}(1,3)_C := \mathfrak{so}(1,3) \otimes \mathbb{C} \) decomposes as a direct sum
\[ (18) \quad \mathfrak{so}(1,3)_C \cong \mathfrak{su}(2)_C \oplus \mathfrak{su}(2)_C. \]
Since there is a bijection between real representations of a real Lie algebra and complex representations of its complexification, (18) implies that \( \mathfrak{so}(1,3) \) and \( \mathfrak{su}(2) \oplus \mathfrak{su}(2) \)
\[ \text{The generators satisfy} \]
\[ [K_i, K_j] = -i\epsilon_{ijk} J_k, \quad [J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k. \]
have the same irreducible representations. The chiral spinors $\psi_L$ and $\psi_R$ live in the representations of the left and right summands of $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$ respectively.

Again consider the strand $\alpha$. We may represent $\alpha$ by a left or right chiral spinor

$$\psi_L := P_L \psi = \frac{1}{2}(1 - \gamma^5)\psi \quad \text{or} \quad \psi_R := P_R \psi = \frac{1}{2}(1 + \gamma^5)\psi,$$

where $\psi$ is a (4-component) Dirac spinor. The chirality of $\alpha$ is obtained by determining which $\mathfrak{su}(2)_C$ subgroup of the complexified Lorentz algebra $\mathfrak{so}(1,3)_C$ its associated linear combination

$$C_\alpha = \text{sgn}(\omega)J_z + i \text{sgn}(\alpha)K_z$$

belongs. We have

$$C_\alpha \in \mathfrak{su}(2)_C \oplus 0 = \left( \frac{1}{2}, 0 \right) \quad \iff \quad \text{sgn}(\alpha) \text{sgn}(\omega) = +1,$$

$$C_\alpha \in 0 \oplus \mathfrak{su}(2)_C = \left( 0, \frac{1}{2} \right) \quad \iff \quad \text{sgn}(\alpha) \text{sgn}(\omega) = -1.$$

Whence, by (15), the chiral spinor representing $\alpha$ is

$$\begin{cases} 
\psi_L & \text{if } \chi(\alpha) = +1 \\
\psi_R & \text{if } \chi(\alpha) = -1 
\end{cases}$$

The chirality of $\alpha$ is therefore given by $\chi(\alpha)$.

By (16) and (17), the sign of $\alpha$ is the sign of the top wedge product $\hat{t} \wedge \hat{n} \wedge \hat{b}$. It follows that $\text{sgn}(\alpha)$ is also the handedness, left or right, of the ordered basis $\{t, n, b\}$.

We identify the phase of the spinor $\psi_{L,R}$ with the strand particle $\alpha(t) \in M$ itself, by the linear isomorphism from the plane of rotation $P$ in $\partial\bar{M}$ to $\mathbb{C}$,

$$e_1 \mapsto 1, \quad e_2 \mapsto i.$$

Under this isomorphism, $\beta(t)$ is the origin of $\mathbb{C}$.

6. The strand Lagrangian

In Section 2 we defined the mass of a strand $\alpha$ to be equal to its inverse radius $m := r^{-1}$. In Section 3 we showed that this definition implies that $\alpha$ has energy $E_0 = m u$, where $u$ is its tangential speed, rather than $E_0 = m$. Based on this relation, we model strand interactions by the Lagrangian density

$$\mathcal{L}(\beta(t)) := \bar{\psi}(i\partial^\beta - m|\bar{u}^a u_a|^{1/2})\psi,$$

where $\bar{\psi}, \psi$ are Dirac spinors that represent strand particles $\bar{\alpha}, \alpha$ on a circle with central worldline $\beta$; and $\bar{u}^a, u^a$ are the projections in $T_{\beta(t)}\bar{M}$ of the four-velocities (based at the origin of $T_{\beta(t)}\bar{M}$) of $\bar{\alpha}, \alpha$ onto the spatial hypersurface of the inertial frame of $\beta$.

Note that the mass dimensions of the fields are $[\psi] = \frac{3}{2}$ and $[u^a] = 0$, and thus $[m] = 1$ as it should be.
In Section 4 we found that strands are represented by chiral spinors, $\psi_L = P_L \psi$ or $\psi_R = P_R \psi$. Expanding the Lagrangian (19), we have

$$L^{(i)} = i \bar{\psi}_L \gamma^0 \frac{\partial \psi_L}{\partial \psi_L} - m |\bar{u}^a u_a|^{1/2} \bar{\psi}_R \psi_R + (L \leftrightarrow R),$$

and

$$L^{(ii)} = i \bar{\psi}_L \gamma^0 \frac{\partial \psi_L}{\partial \psi_L} - m \phi \bar{\psi}_L \psi_R + (L \leftrightarrow R),$$

where (i) holds since

$$\bar{\psi}_L := (\psi_L)^\dagger \gamma^0 = \bar{\psi}_R$$

and $\bar{\psi}_R = \bar{\psi}_L$. In (ii), $\phi := |\bar{u}^a u_a|^{1/2}$ is viewed as a real scalar field. Each chiral spinor $\bar{\psi}_L, \psi_R, \bar{\psi}_R$ represents a strand $\bar{\alpha}_L, \alpha_R, \bar{\alpha}_R, \alpha_L$ of radius $r$ and angular velocity $\bar{\omega}_L, \omega_R, \bar{\omega}_R, \omega_L$. Since the terms $\bar{\psi}_L \psi_R$ and $\bar{\psi}_R \psi_L$ have the same coupling constant $m\phi$, the respective spatial four-velocities $\bar{u}^a, u^a, \bar{v}^a, v^a$ satisfy

$$|\bar{u}^a u_a|^{1/2} = \phi = |\bar{v}^b v_b|^{1/2}.$$
Thus, the strands have opposite $w^\pm$ charges, and so their bound state $\bar{\psi}_L\psi_R$ has zero charge by (10). We identify these two atoms with photons of opposite circular polarization (for non-circular polarization, see [B5]).

(ii) There is also an atom with $\bar{\omega}_{L/R} = \omega_{R/L}$ consisting of four strands:

$$Z = \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L : \bar{\psi}_L \bar{\psi}_R \psi_L \psi_R$$

This atom also has total charge zero, and so we identify it with the $Z$ boson. The $Z$ atom is not a superposition of two photons because the two diameters are bound together. In particular, there is binding energy that contributes to the atom’s mass; see Section 8.

- Now suppose $\bar{\omega}_{L/R} = -\omega_{R/L}$. Then $\phi = |\cos \theta|$, and in particular the coupling constant $\phi m$ varies in time. From the associativity $(\phi m)\bar{\psi}\psi = \phi (m\bar{\psi}\psi)$, there are two possible representations of such atoms:

  - A coupling representation, where the atom is represented by $(\phi m)\bar{\psi}\psi$. This representation allows to determine certain mass orderings (Section 8), and provides an explicit derivation of electroweak parity violation (Section 10).

  - A field representation, where the atom is represented by $\phi (m\bar{\psi}\psi)$, with $\phi$ an independent real scalar field bound to a strand in the $\gamma$ or $Z$ atom $m\bar{\psi}\psi$. Since $\phi$ varies in time, $\phi$ has a time orientation. Thus $\phi$ has a sign,

  $$\text{sgn}(\phi) = \pm 1,$$

  as described in Section 4. This representation therefore shows that there should be an additional $O(3)$ charge – the charge of the scalar field $\phi$ – in an atom with a time varying coupling constant.

Both representations are useful, as they reveal different aspects of such atoms.

The possible atoms with $\bar{\omega}_{L/R} = -\omega_{R/L}$ are given in Table 3. We assume that the scalar field $\phi$ only binds to the strand of opposite sign in the photon diameter $\bar{\psi}_L\psi_R$ of the $Z$ atom, since the strands in the non-photon diameter $\bar{\psi}_R\psi_L$ may mutually annihilate in a splitting (see Definition 7.1 below).

(i) First consider the atom $B = \phi m\bar{\psi}_L\psi_R$. In the coupling representation

$$B = (\phi m)\bar{\psi}_L\psi_R,$$

$B$ does not have circular polarization since $\bar{\omega}_L = -\omega_R$. In contrast, in the field representation

$$B = \phi (m\bar{\psi}_L\psi_R),$$

$B$ is a bound state of the scalar field $\phi$ with the photon atom $\gamma = \bar{\psi}_L\psi_R$, and thus $B$ has circular polarization. But this implies that $B$ is inconsistent (that is, ill-defined)
Table 3. The possible $O(3)$ symmetric atoms with a time varying coupling constant $\phi m$.

| coupling representation | field representation |
|-------------------------|----------------------|
| inconsistent: $\bar{\psi}_L \psi_R$ | $\neq $ $\bar{\psi}_L^0 = \phi \bar{\psi}_L \psi_R$ |
| $W^-$: $\bar{\psi}_L \psi_R$ | $= $ $\bar{\psi}_L^0 = \phi \bar{\psi}_L \psi_R$ |
| $W^+$: $\bar{\psi}_L \psi_R$ | $= $ $\bar{\psi}_L^0 = \phi \bar{\psi}_L \psi_R$ |

since $(\phi m) \bar{\psi}_L \psi_R = \phi (m \bar{\psi}_L \psi_R)$. We therefore conclude that this atom cannot be physical.

(ii) Now consider the atom $B = \phi m \bar{\psi} \psi$. Since the strands $\bar{a}_L, a_R$ have opposite chirality and $\bar{\omega}_L = -\omega_R$, the chirality relation [15] implies that $\text{sgn}(\bar{a}_L) = \text{sgn}(a_R)$. Similarly $\text{sgn}(a_R) = \text{sgn}(a_L)$. Thus each diameter has charge $2w^+$ or $2w^-$. Therefore, a priori, $B$ has charge

$$q(B) = 4w^\pm + w^\text{sgn}(\phi) \quad \text{or} \quad q(B) = 0 + w^\text{sgn}(\phi),$$

by (10). However, in the field representation $B = \phi (m \bar{\psi} \psi)$, we find that the atom is a bound state of a $Z = \bar{\psi} \psi$ atom, which has zero charge, and the scalar field $\phi$, which has charge $w^+$ or $w^-$. Therefore

$$q(B) = q(Z) + q(\phi) = w^\text{sgn}(\phi).$$

Thus, the two diameters in the coupling representation $B = (\phi m) \bar{\psi} \psi$ must have opposite charge, $2w^+$ and $2w^-$. Consequently, the charge of the atom arises entirely from the particular periodic configuration of the strands. We identify these two symmetric atoms with the $W^-$ and $W^+$ particles.
6.2. The creation and annihilation of strands: apexes. Suppose $\bar{\omega}_{L/R} = \omega_{R/L}$ and $\bar{u}^a u_a = -1$. This configuration describes two strand particles $\bar{\alpha}$, $\alpha$ on a diameter whose spatial tangent vectors $\bar{u}^a$, $u^a$ are parallel. Furthermore, by the chirality relation (15), $\bar{\alpha}$ and $\alpha$ have opposite sign. The configuration therefore describes the collision of two strands of opposite sign:

\[ \bar{\psi} \quad \uparrow \quad \psi \]

Definition 6.2. An apex is a point $x \in \tilde{M}$ where two strands, or two coupling fields, of opposite sign on a diameter are created or annihilated; see Figure 2.

Two strands that meet at an apex and belong to two atoms in a splitting may be transformed into a single strand by reversing the time orientation of one of the atoms; see Remark 7.2.

Energy-momentum is conserved in interactions between bound states of strands. Thus, if an apex appears in such an interaction, then the newly created strands will obtain their energy from other strands involved in the interaction. In particular, unlike particle-antiparticle creation, the strands will not use ‘free energy’ from the vacuum, allowed by the time-energy uncertainty principle, to exist. Therefore, the time-energy uncertainty principle does not constrain the lifetime of strands.

6.3. A remark on the quantization of the Lagrangian. From Dirac’s path integral formulation of quantum theory, Feynman concluded that ‘Nature takes every
Table 4. Particle identifications of the symmetric and split atoms

| atom    | rotational symmetry | ↓ spin | ↓ stability | ↓ strand charge | ↓ electric charge | particle |
|---------|---------------------|-------|------------|----------------|------------------|----------|
| $\psi_L\psi_R$ | $\pi$ 1 | no stable | 0 | $c_1^+ + c_2^-$ | 0 | $\gamma$ |
| $\bar{\psi}\psi = \psi_L\psi_R + \bar{\psi}_R\psi_L$ | $\pi$ 1 | yes unstable | 0 | $c_1^+ + c_2^-$ | 0 | $\tilde{\gamma}$ |
| $(\phi m)\bar{\psi}\psi = \phi(m\bar{\psi}\psi)$ | | | | | | |
| $= \psi_L^0\psi_R + \bar{\psi}_R\psi_L$ | | | | | | |
| $\psi_R$ | $2\pi$ 1/2 | no stable | $w^+$ | -1 | | $W^-$ |
| $\tilde{\psi}_L^0 \otimes \psi_L$ | $2\pi$ 1/2 | yes unstable | $w^+$ | -1 | | $\mu$ |
| $\tilde{\psi}_R^0 \otimes \psi_R$ | $2\pi$ 1/2 | yes unstable | $w^+$ | -1 | | $b$ |
| $\psi_R^0$ | $2\pi$ 1/2 | no stable | 0 | 0 | | $\nu_e$ |
| $\tilde{\psi}_L \otimes \psi_L$ | $2\pi$ 1/2 | no stable | 0 | 0 | | $\nu_\mu$ |
| $\tilde{\psi}_L^0 \otimes \bar{\psi}_R$ | | | | | | |
possible path'. However, in [B5] we introduced the possibility that the geodesic hypothesis of general relativity is also able to describe quantum phenomena, by proposing the following alternative:

*Nature takes a set of all indistinguishable stationary paths.*

*Thus, by the identity of indiscernibles, Nature takes a single stationary path.*

Classical physics and quantum physics would then share the same underlying principle. Furthermore, path superposition would result whenever two paths, that are indistinguishable to all the constituents of the universe, become distinguishable at a collective, emergent scale.

In this framework, it may be that the strand Lagrangian $L$ should *not* be quantized. This would be possible if the classical equations of motion of $L$ correspond to some ‘quantum motion’ determined by the path integral of a different Lagrangian. Indeed, the equations of motion of the Lagrangian

$$L = \bar{\psi}(i\partial - mu)\psi,$$

namely $i\partial\psi = mu\psi$ (that is, $E_0 = mu$), correspond to certain quantum perturbations of the equations of motion of the Dirac Lagrangian,

$$L_{\text{Dirac}} = \bar{\psi}(i\partial - m)\psi,$$

namely $i\partial\psi \neq m\psi$, whenever $u \neq 1$.

One stark difference that would remain, however, is that there are vacuum fluctuations from the path integral, but not from the Lagrangian alone. In our model, then, apexes would only arise so that strands that are off-shell can become on-shell, and would not spontaneously occur without cause.

We leave these speculations for future work.

7. A STRAND MODEL OF LEPTONS, QUARKS, AND GAUGE BOSONS

A primary objective of our model is to provide a spacetime description of quantum nonlocality, and this would not be possible if each elementary particle is simply a different type of strand; see [B5]. We therefore introduce a new preon model of particle physics, where leptons, quarks, and gauge bosons are bound states of strands that interact by exchanging strands.

7.1. Split atoms. We make the following assumption:

(21) *Strands of opposite sign attract, and strands of the same sign repel.*

In the standard model, the physical mechanism that causes the attraction and repulsion between electric charges is the exchange of photons. This will also be the cause of attraction and repulsion between charged strand atoms in our model. However,

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$^3$Vacuum fluctuations account for the Casimir force and the positive cosmological constant, as well as provide a physical mechanism that corrects bare masses to renormalized masses. Further development of our framework is therefore required to address these issues.
we do not know what physical mechanism may cause the attraction and repulsion
between the individual strands within a single atom (though we expect that it may
be related to the microscopic curvature of spacetime). Our model therefore requires
further development to address this question.

Definition 7.1. A symmetric atom may undergo a fundamental splitting into two
split atoms if $\phi = 0$, that is, if the strands in each diameter are no longer bound
together, such that the following holds.

(i) The strands in the non-photon diameter $\bar{\psi}_R \psi_L$ of the symmetric atom may
annihilate each other at an apex.

(ii) Newton’s third law of motion: One strand from each non-annihilated diameter
of the symmetric atom belongs to each split atom.

(iii) By (21), the two strands in a split atom have opposite sign.

A non-fundamental splitting occurs if (i) - (iii) hold, and new strands are created
at apexes. We impose one additional rule for such splittings:

(iv) Each of the five fields $\bar{\psi}_{L/R}, \psi_{L/R}, \phi$ is excited in some atom in the splitting.

The set of all fundamental and non-fundamental splittings are shown in Tables 1 and
2 respectively.

Remark 7.2. An incoming (outgoing) atom in a splitting can be transformed into
an outgoing (resp. incoming) atom by reversing the time orientation, that is, the sign,
of each strand in the atom. In particular, if $B_0 \rightarrow B_1 B_2$ is a splitting of a symmetric
atom $B_0$, then $\bar{B}_1 B_0 \rightarrow B_2$ and $\bar{B}_1 \rightarrow B_0 B_2$ are also allowed interactions (just as is
the case for Feynman interactions).

The vertices in a Feynman diagram represent splittings. Thus, the total four-
momentum is conserved at each splitting,

$$\sum_{\text{incoming}} p_\mu = \sum_{\text{outgoing}} p'_\mu.$$ 

However, the four-momentum along a single strand in a splitting need not be con-
served; momentum of one strand may be transferred to another strand during a
splitting.

With these rules, we reproduce exactly the leptons, quarks, and electroweak bosons,
with their correct spin, electric and color charges, certain mass orderings, and, in
the electroweak sector, stability, as well as the electroweak interactions. Note that
rule (iv) ensures that there are no photon self-interactions, and no photon-neutrino
interactions.

7.2. Spin. Recall that a circular strand has spin $\frac{1}{2}$, by assumption. The spin of a
diameter $\bar{\psi}_{L/R} \psi_{R/L}$ is zero since it is a scalar field. However, the input to the coupling
constant $\phi m$ requires the specification of the plane of rotation $P$, which is determined
by the spatial four-vector $e^a_3$. Thus, although a diameter is a scalar field, symmetric atoms have spin 1.

Split atoms also have a plane of rotation $P$, and so are not scalar fields. In particular, split atoms do not have spin 0. But a split atom can bind with another split atom to form a symmetric atom, which has spin 1. Therefore split atoms must have spin $\frac{1}{2}$.

In the defining representation of $SO(3)$, a vector is returned to its initial position by a rotation of $\theta = 2\pi$, whereas in the spin-$\frac{1}{2}$ representation of $SO(3)$, a spinor is returned to its initial position by a rotation of $4\pi = 2\theta$. The ratio of rotational symmetry between vectors and spinors, namely 2, is precisely the ratio of rotational symmetry between symmetric and split atoms; see Table 1.

7.3. Particle identifications. Based on each atom’s spin, electric charge, and color charge, we make the particle identifications given in Table 4.

**Remark 7.3.** In our model, photons do not a priori interact with all electrically charged atoms; instead, they interact only with those atoms that together satisfy the splitting rules. From these rules, we find that a photon is able to interact with a lepton atom if and only if it has a nonzero electric charge, but this is not the case for quark atoms.

7.3.1. Lepton interactions. There are four $O(3)$ symmetric atoms: the photon $\gamma$, Z-boson, and $W^\pm$-bosons, shown in Table 4. Their splittings into split atoms are shown in Tables 1 and 2. The splittings correspond precisely to the Feynman interactions between leptons and electroweak gauge bosons.

We leave the question of neutrino oscillation (and quark oscillation) for future work.

7.3.2. Quark interactions. The photon diameter $\bar{\psi}_L \psi_R$ specifies an atom’s plane of rotation $P$, since the strands in the photon diameter cannot mutually annihilate. Thus the spatial group of the strands in the photon diameter, $O(2)$ or $O(3)$, is determined by whether $P$ is fixed in a bound state with other atoms (forming a meson or baryon), or whether $P$ is unconstrained. However, the spatial group of the strands in the non-photon diameter $\bar{\psi}_R \psi_L$ is always $O(3)$ because $P$ is fixed entirely by the photon diameter. *Therefore the strands in the photon diameter, namely $\bar{\alpha}_L$ and $\alpha_R$, can carry either $O(2)$ (color) charge or $O(3)$ (electric) charge, whereas the strands in the non-photon diameter, $\bar{\alpha}_R$ and $\alpha_L$, can only carry $O(3)$ charge.*

**Definition 7.4.** If the strands in the photon diameter $\bar{\psi}_L \psi_R$ have $O(2)$ charge, then we say the atom is an $O(2)$ atom; otherwise it is an $O(3)$ atom.

We call the $O(2)$ symmetric atoms *gluons*, though they differ from gluons in quantum chromodynamics. There are three types of gluons:

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*This is not a rigorous argument, but in order to reproduce the correct quark charges, we want only the strands in the photon diameter to be able to carry $O(2)$ charge.*
• Neutral massless gluons $\tilde{\gamma}$, called $\gamma$-gluons, which mediate color charge between $d$, $s$, and $b$ quarks.
• Neutral massive gluons $\tilde{Z}$, called $Z$-gluons, which mediate color charge between all quarks.
• Charged massive gluons $\tilde{W}$, called $W$-gluons, which allow flavor transformations within a generation.

The model therefore predicts the existence of both neutral and charged massive gluons. Their splittings are shown in Tables 5. Note that a $\gamma$-gluon (resp. $Z$-gluon, $W$-gluon) with strands of equal color is simply a photon (resp. $Z$, $W$ boson), by the relations (12); see Table 6.

In our model, quarks do not have fractional electric charge as they do in QCD. Instead, they possess integer combinations of strand charges. Nevertheless, our model gives the correct electric charges for all baryons and mesons:

**Proposition 7.5.** Upon substituting the charges

$$w^\pm \mapsto \mp 1 \quad \text{and} \quad c^\pm \mapsto \mp \frac{1}{3}$$

in Table 4, we obtain the fractional electric charges of the quarks in QCD. Therefore, the strand model and QCD predict the same electric charges for all baryons and mesons.

**Proof.** The second statement follows from the relations (11) and (12). $\Box$

**Remark 7.6.** Our model ‘explains’ two of the prominent features of QCD:

• The reason there are three color charges is because there are three dimensions of space: specifically, there are three pairwise orthogonal embeddings of $O(2) \times O(1)$ into $O(3)$.

• The reason that quarks cannot exist in isolation is because color charge is only possible when there is a distinguished direction of space; in isolation, there is no distinguished direction of space.

However, the different gluon types predicted by the model do not arise in QCD, and this feature may cause the model to fail.

7.3.3. **Gauge boson interactions.** Our model reproduces exactly the standard model trivalent vertices $\gamma W^+W^-$ and $ZW^+W^-$ involving the electroweak gauge bosons; these are shown in Table 2. The four-valent electroweak vertices

$$\gamma \gamma W^+W^-, \quad \gamma ZW^+W^-, \quad ZZW^+W^-, \quad W^+W^-W^+W^-,$$

are obtained by a (simultaneous) composition of two of the splittings

$$\{\gamma \to W^+W^-\} \cong \{W^\pm \to \gamma W^\pm\} \quad \text{or} \quad \{Z \to W^+W^-\} \cong \{W^\pm \to ZW^\pm\}.$$

Furthermore, our model predicts the new photon-gluon vertices, shown in Table 6

$$\gamma \tilde{W}^+\tilde{W}^-, \quad Z\tilde{W}^+\tilde{W}^-, \quad W^\pm \tilde{Z}\tilde{W}^\pm.$$
Table 5. The splittings of the $O(2)$ symmetric atoms (gluons) into $O(2)$ atoms. The strands that do not belong to the original symmetric atom are paired by opposite sign on a diameter.

| $\tilde{\gamma}$ | $\tilde{\beta}$ | $\tilde{W}^-$ |
|------------------|------------------|--------------|
| $\tilde{d} = \tilde{\psi}_L^c$ | $\psi_R^c = d$ | $\tilde{d} = \tilde{\psi}_L^c$ |
| $\tilde{b} = \tilde{\psi}_L^c \otimes \tilde{\psi}_R^c$ | $s = \tilde{\psi}_L^c \otimes \tilde{\psi}_R^c$ |
| $\tilde{W}^- = \tilde{\psi}_L^c \otimes \psi_d$ | $\tilde{b}^c \psi_d^c = \tilde{W}^-$ |

| $\tilde{\psi}_L^c \psi_L^c := \tilde{\psi}_L^c \psi_R^c + \tilde{\psi}_R^c \psi_L^c$ | $\tilde{\psi}_L^c \psi_L^c := \tilde{\psi}_L^c \psi_R^c + \tilde{\psi}_R^c \psi_L^c$ | $\tilde{\psi}_L^c \psi_L^c := \tilde{\psi}_L^c \psi_R^c + \tilde{\psi}_R^c \psi_L^c$ |
|------------------|------------------|--------------|
| $c = \tilde{\psi}_L^c \otimes \psi_L^c$ | $\psi_R^c \otimes \tilde{\psi}_R^c = \tilde{\psi}_R^c \psi_L^c$ |
| $t = \tilde{\psi}_L^c \otimes \tilde{\psi}_R^c$ | $\psi_R^c \otimes \psi_L^c = \tilde{\psi}_R^c \psi_L^c$ |
| $\tilde{W}^- = \tilde{\psi}_L^c \psi_d^c$ | $\tilde{b}^c \psi_d^c = \tilde{W}^-$ |
| $\tilde{b}^c \psi_d^c = \tilde{W}^-$ |

8. Mass orderings

In this section, we show that our model produces 16 independent mass orderings that agree exactly with the standard model particle masses. We consider three sources of mass in a strand atom:

- angular momentum;
- the coupling field $\phi$; and
- binding energy: the number of pairs of strands that do not belong to the same diameter.

In the following, we use these sources, together with Table 4, to derive certain mass orderings of leptons, quarks, and gauge bosons.

8.1. Angular momentum. Consider two atoms $B$, $C$ that are identical except that the diameters of $B$ rotate in opposite directions, clockwise and counter-clockwise, whereas the diameters of $C$ rotate in the same direction. Then the total angular momentum of $B$ will be zero, and that of $C$ will be nonzero,

$$L(B) = 0 \quad \text{and} \quad L(C) > 0,$$

where $L = |\mathbf{L}|$ is the magnitude of the (spatial) angular momentum $\mathbf{L}$. Furthermore, the angular momentum $L$ contributes to the rest energy of the atom. Therefore, the mass of $B$ must be less than the mass of $C$, $m(B) < m(C)$. We find that this agrees with the experimentally determined mass values:
Table 6. The splittings of the $O(3)$ symmetric atoms into $O(2)$ atoms. These splittings all use the relation $w^+ + w^- = c^+ + c^-$ in (12). The strands that do not belong to the original symmetric atom are paired by opposite sign on a diameter. Note that the photon $\gamma$ does not interact with the quarks $u, c, t$, in contrast to the standard model.

Two of these inequalities, for example $m(\mu) < m(\tau)$ and $m(s) < m(b)$, are not retrodictions because we could swap the atoms labeled $\mu$ and $\tau$, and the atoms labeled $s$ and $b$. The other two inequalities, however, are fixed by the $W^\pm$ splittings, and so $m(c) < m(t)$ is an honest retrodiction, and $m(\nu_\mu) < m(\nu_\tau)$ is a prediction.\footnote{We note that, in the standard model, neutrino flavor eigenstates and mass eigenstates do not coincide in order to account for neutrino oscillations, which differs from our model.}

Furthermore, the individual diameters of the $Z$ atom each have nonzero angular momentum, whereas the diameters of the $W$ atom have zero angular momentum.
Therefore we should have

\[(22) \quad m(W) < m(Z),\]

which also agrees with experiment.

8.2. The coupling field $\phi$. In the field representation, the only difference between the $W$ atom and the $Z$ atom is that the $W$ atom contains the scalar coupling field $\phi = |u^a u_a|^{1/2}$. However, the mass of the $W$ atom is less than the mass of the $Z$ atom, by \[22\]. Therefore the field $\phi$ must have negative mass, $m(\phi) < 0$. This presents no problem, however, because $\phi$ cannot exist in isolation, as it is an emergent property of atoms for which $\omega_{L/R} = -\omega_{R/L}$.

Since the field $\phi$ has negative mass, in general adding $\phi$ to an atom which does not contain $\phi$ will decrease the atom’s mass.

However, suppose both strands in a split atom have $O(3)$ charge; then one strand will have charge $w^+$ and the other will have charge $w^-$ by \[21\]. Since $\phi$ also has charge $w^+$ or $w^-$, and ‘like charges repel’, a minimum energy $E_{\phi}$ will be required to keep $\phi$ in the atom. In our model, we assume that this energy is greater than the absolute value of the mass of $\phi$,

\[E_{\phi} > |m(\phi)|.\]

Thus, if both strands have nonzero $O(3)$ charge, then adding the coupling field to the atom will increase its mass, rather than decrease it.

We find exact agreement with experiment:

| At most one diameter with $O(3)$ charge: | Both diameters with $O(3)$ charge: |
|----------------|----------------|
| with $\phi$ | without $\phi$ | without $\phi$ | with $\phi$ |
| $m(\nu_e) < m(e)$ | $m(\nu_\mu) < m(\mu)$ |
| $m(u) < m(d)$ | $m(\nu_\tau) < m(\tau)$ |
| $m(s) < m(c)$ |
| $m(b) < m(t)$ |

8.3. Binding energy. Let $B$ be a strand atom, and let $n(B)$ be the number of pairs of strands of $B$ that do not belong to the same diameter. Each such pair requires energy to bind the strands together in the atom. Thus, if two atoms $B$, $C$ satisfy $n(B) < n(C)$, then $C$ should require more binding energy than $B$. Therefore, if $B$ and $C$ are ‘sufficiently similar’ atoms, then $n(B) < n(C)$ will imply $m(B) < m(C)$.

We find exact agreement with experiment if we consider all $O(3)$ (resp. $O(2)$) atoms containing precisely two charges:
We also find exact agreement with experiment if we consider all $O(3)$ (resp. $O(2)$) atoms containing the coupling field $\phi$:

| $n(B)$ | $n(C)$ |
|--------|--------|
| $m(\gamma)$ | $\min\{m(\nu_\mu), m(\nu_\tau)\}$ |
| $m(\tilde{\gamma})$ | $\min\{m(c), m(t)\}$ * |
| $m(\nu_e)$ | $\min\{m(\nu_\mu), m(\nu_\tau)\}$ |
| $m(u)$ | $\min\{m(c), m(t)\}$ |

The two inequalities marked (*) are predictions for the $\tilde{\gamma}$ and $\tilde{W}$ gluons, as these particles do not belong to the standard model.

9. ELECTROWEAK PARTICLE STABILITY

First consider the electroweak sector. By (21), an $O(3)$ atom is unstable if and only if it contains more than two like charges $w^\pm$. Using Table 4, we find

| stable particles: | electron, electron neutrino, muon neutrino, tau neutrino, photon |
| unstable particles: | muon, tau, $W$ boson, $Z$ boson |

This classification of stability is in exact agreement with experiment.

Now consider the quark sector. The atom consisting of a single strand is an (anti-) electron if it has spatial group $O(3)$, and an (anti-)down quark if it has spatial group $O(2)$. According to the strand Lagrangian (19), both types of strands should have the same radius $r$, given by the coupling constant $m = r^{-1}$. However, electrons and down quarks have different rest energies $E_0 = \omega$, and so the relation

$$E_0 = mu = \frac{u}{r}$$

implies that either the electron satisfies $u \neq 1$, or the down quark satisfies $u \neq 1$. That is, either the electron is off-shell or the down quark is off-shell, and so one of the two particles must be unstable (Section 3). Our model agrees with experiment, since the down quark is indeed unstable and the electron is stable.
However, we cannot explain why electrons and down quarks have different rest energies, or why the only stable quark is the up quark, and so the model requires further development to address stability in the quark sector.

10. Electroweak parity violation

In this section we use the strand model to show that, in the standard model, $e_R$ transforms as an SU(2) singlet, and $(\nu_e)_{e_L}$ transforms as an SU(2) doublet.

Consider a diameter, $\bar{\psi}L\psi_R$ or $\bar{\psi}R\psi_L$, of the atom

$$W^\pm = (\phi m)\bar{\psi}\psi = (\phi m)(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L),$$

in the coupling representation, that is, where the coupling constant $\phi m$ varies in time periodically. As shown in Section 6.1, the two strands on the diameter,

$$\bar{\alpha}(t) = \exp(\beta(t)) (r \cos(\bar{\omega}t)e_1 + r \sin(\bar{\omega}t)e_2),$$
$$\alpha(t) = \exp(\beta(t)) (r \cos(\omega t)e_1 + r \sin(\omega t)e_2),$$

have opposite angular velocity and equal charge,

$$\bar{\omega} = -\omega, \quad q(\bar{\alpha}) = q(\alpha).$$

We may thus regard the lift of the diameter to the tangent space $T_{\beta(t)}\tilde{M}$ as a (classical) superposition of $\bar{\alpha}$ and $\alpha,$

$$(\bar{\alpha}\alpha)(t) := \exp(\beta(t)) (2r \cos(\omega t)e_1),$$

whence as a harmonic oscillator, with charge $q(\bar{\alpha}\alpha) = 2q(\alpha).$ However, in this form it is clear that the sign of $\omega$ is irrelevant to the physical description of the $W$ atom.

We want to determine the signs of the angular velocities $\omega$ of the strands in the split atoms that arise from splitting a $W$ atom. We obtain these signs by the following three steps:

(i) Since the sign of $\omega$ is physically irrelevant in the superposition $\bar{\alpha}\alpha,$ we may take $\omega = |\omega|.$ (This will be the underlying source of the parity violation of neutrinos.) Since $\alpha$ is future-directed, when it splits from the superposition $\bar{\alpha}\alpha$ its angular velocity remains $|\omega|.$

(ii) We now determine the signs of the individual strands in the two split atoms.

Consider an unbarred strand $\alpha$ in one of the split atoms. Recall that the chirality $\chi(\alpha) = \text{sgn}(\omega) \text{sgn}(\alpha)$ of $\alpha$ is determined by which $\mathfrak{su}(2)_C$ subgroup of the complexified Lorentz algebra $\mathfrak{so}(1,3)_C$ its associated element $C_\alpha = \text{sgn}(\omega)J_z + i \text{sgn}(\alpha)K_z$ belongs:

$$\psi_\bullet(\alpha) = \begin{cases} 
\psi_L \text{ if } \chi(\alpha) = +1 \\
\psi_R \text{ if } \chi(\alpha) = -1 
\end{cases}$$

Thus, by step (i), the chirality of $\alpha$ is given by which $\mathfrak{su}(2)_C$ subgroup of $\mathfrak{so}(1,3)_C$ the associated element

$$C_\alpha = \text{sgn}(|\omega|)J_z + i \text{sgn}(\alpha)K_z$$
belongs. Therefore, by the chirality relation (15) we have

\[ \psi \cdot (\alpha) = \psi_L \iff \text{sgn}(\alpha) = +1, \]
\[ \psi \cdot (\alpha) = \psi_R \iff \text{sgn}(\alpha) = -1. \]

Since we are considering the coupling representation, the two strands on a given diameter of the W atom have the same sign. Thus, by (23), both strands from the diameter \( \bar{\psi}_L \psi_R \) have sign \(-1\), and both strands from the diameter \( \bar{\psi}_R \psi_L \) have sign \(+1\). Whence,

\[ \psi \cdot (\bar{\alpha}) = \bar{\psi}_L \iff \text{sgn}(\bar{\alpha}) = -1, \]
\[ \psi \cdot (\bar{\alpha}) = \bar{\psi}_R \iff \text{sgn}(\bar{\alpha}) = +1. \]

(iii) Finally, we use (24) together with the chirality relation (15) to determine the signs of the angular velocities of the barred strands. We find that neutrinos are only able to rotate in one direction, and anti-neutrinos are only able to rotate in the opposite direction:

\[ W^- \rightarrow \bar{\psi}_L \psi_R \bar{\psi}_L \psi_R \bar{\psi}_R \psi_R \bar{\psi}_L \psi_L = \begin{array}{ccccccc}
\chi(B) & +1 & -1 & (+1,+1) & (-1,-1) & (+1,-1) & (-1,+1) \\
\text{sgn}(B) & (-1;+1) & -1 & (-1,+1;+1) & (-1,+1) & (-1,+1;+1) & (-1,+1) \\
\text{sgn}(\omega(B)) & -1 & 1 & (-1,+1) & (+1,-1) & (-1,-1) & (+1,+1) \\
B & \bar{\nu}_e^\dagger & e^\dagger & \mu^\dagger & \bar{\nu}_\mu^\dagger & \tau^\dagger & \bar{\nu}_\tau^\dagger \\
\end{array} \]

\[ W^+ \rightarrow \psi_L \psi_R \psi_L \psi_R \psi_R \psi_L \bar{\psi}_L \bar{\psi}_R = \begin{array}{ccccccc}
\chi(B) & +1 & -1 & (+1,+1) & (-1,-1) & (+1,-1) & (-1,+1) \\
\text{sgn}(B) & (-1;+1) & -1 & (-1,+1;+1) & (-1,+1) & (-1,+1;+1) & (-1,+1) \\
\text{sgn}(\omega(B)) & -1 & 1 & (-1,+1) & (+1,-1) & (-1,-1) & (+1,+1) \\
B & \bar{e}^\dagger & \nu_e^\dagger & \mu^\dagger & \bar{\nu}_\mu^\dagger & \tau^\dagger & \bar{\nu}_\tau^\dagger \\
\end{array} \]

**Remark 10.1.** In the field representation of a split atom, say \( \psi_R^0 \otimes \bar{\psi}_R \) or \( \psi_R^0 \otimes \psi_L \), the coupling field \( \phi \) is bound to the strand \( \alpha \) represented by \( \bar{\psi}_R \). In particular, \( \phi \) has opposite sign to \( \alpha \). However, the signs of the strands in a symmetric atom obtained from the coupling representation are different from the signs obtained in the field representation (though the total charge of the atom is the same in both representations). Thus, the sign of the additional charge that arises from the coupling constant \( \phi m \) in the coupling representation need not be opposite to that of \( \alpha \).

**Remark 10.2.** Our model does not admit sterile neutrinos, that is, right-handed neutrinos and left-handed anti-neutrinos which do not interact with \( W^\pm \) atoms. Indeed, each \( O(3) \) atom \( B \) has
• two degrees of freedom from an overall sign of the tuple
\[ \text{sgn}(\omega(B)) = (\text{sgn}(\omega_\alpha))_{\alpha \in B} \]
of the directions of rotation of its constituent strands \( \alpha \in B \); and
• two degrees of freedom from the sign of the total charge \( q(B) = \sum_{\alpha \in B} q(\alpha) \)
of \( B \) whenever \( q(B) \neq 0 \).

However, neutrino atoms have zero total charge, and so a neutrino atom \( B \) has only two degrees of freedom, namely the overall sign of \( \text{sgn}(\omega(B)) \). Furthermore, as we have just shown, \( \text{sgn}(\omega(B)) \) alone determines whether \( B \) is a neutrino or an anti-neutrino.

We thus obtain a derivation of the parity violation of electroweak interactions. Another derivation of parity violation was recently given in \([F]\), and it would interesting to understand how the two approaches are related. Our model requires further development to address parity violation in the quark sector.

11. A MODIFICATION TO EINSTEIN’S EQUATION FROM THE DIRAC LAGRANGIAN

Suppose a photon passes through a beam splitter. In the framework of strands, the two path eigenstates of the photon are both physically ‘real’ or ontic \([B5]\). We may even suppose that the energy of the two eigenstates are equal, and equal to the initial photon. This does not pose a problem with respect to energy-momentum conservation in the framework of strand spacetime: the two eigenstates are really one and the same photon sitting at the same point in spacetime \( M \) (although at different points of emergent spacetime \( \tilde{M} \)), and so the total photon energy is not doubled when the photon passes through the beam splitter.

However, there is an irreconcilable problem of our two physically real eigenstates of the photon with Einstein’s equation
\[ G_{ab} = 8\pi T_{ab} \]
given by Bohr’s gedankenexperiment: each eigenstate of the photon has energy, and thus produces gravitational radiation as it propagates. But this gravitational radiation transmits which-way information, and so the two eigenstates cannot be in superposition.

To remedy this problem, we propose that spacetime curvature is only sourced at apexes, that is, from the mass terms,
\[ m\bar{\psi}_L\psi_R \quad \text{and} \quad m\bar{\psi}_R\psi_L, \]
of the strand Lagrangian \([20]\). (Recall that in contrast to quantum fluctuations, apexes do not spontaneously occur in a vacuum, but rather only occur to bring off-shell strand particles on-shell.) The chiral spinors \( \psi_L \) and \( \psi_R \) individually have no gravitational mass since there are no terms of the form
\[ m\bar{\psi}_L\psi_L \quad \text{or} \quad m\bar{\psi}_R\psi_R \]
in the strand Lagrangian. The absence of the terms (26) implies that the strand particles represented by \( \psi_L \) and \( \psi_R \) do not source gravitation as they propagate. The Einstein equation on emergent spacetime \( \tilde{M} \) is therefore

\[
G_{ab} = 8\pi 1_A T_{ab},
\]

where \( 1_A : \tilde{M} \to \{0,1\} \) is the indicator function defined by

\[
1_A(x) := \begin{cases} 
1 & \text{if } x \text{ is an apex} \\
0 & \text{otherwise}
\end{cases}
\]

This modification is similar to the semi-classical Einstein equation \( G_{ab} = 8\pi \langle T_{ab} \rangle \).

The problem of Bohr’s gedankenexperiment is resolved by the Dirac Lagrangian in the context of strands. Indeed, the energy density \( T_{ab} v^a v^b \) and momentum density flow \( -T_{ab} v^a \) of an observer with four-velocity \( v^a \) are only defined at points of emergent spacetime \( \tilde{M} \) where an energy-momentum measurement (state reduction) occurs, and these points precisely form the support of \( 1_A \). The classical Einstein equation is thus recovered in the classical limit where energy and momentum are defined at each point of spacetime:

\[
1_A \to 1.
\]

A consequence of (27) is the following. Increasing the thermal energy \( T \) of an object, such as a star, results in an increase in the number of interactions (random collisions) among its constituent strand particles. These interactions in turn result in an increase in the frequency of apexes within the object, and so \( A = A(T) \) is an increasing function of \( T \). Therefore, the hotter an object is, the more it curves spacetime: as the temperature of the object increases, the indicator function \( 1_A \) approaches the identity on the support of the object,

\[
1_A(T) \xrightarrow{T \to 0} 1.
\]

Conversely, if all the particles making up the object ceased interacting, then the object would no longer curve spacetime. Consequently, Einstein’s equation (25) is recovered in the high temperature limit, but (27) differs from (25) at low temperature (although the difference may be extremely small).

With the modification (27), the spacetime curvature \( G_{ab} \) of \( \tilde{M} \) remains classical. In particular, there are no gravitons in this framework. However, there are two possible connections between (27) and other theories of gravity:

- In our strand model, apexes are responsible both for state reduction of path superposition [B5] and gravity. Similar, but different, connections between quantum state reduction and gravity have been developed by Penrose, Diósi, and Oppenheim [P1, P2, D, O]; in these theories, gravity also remains classical-like.
The source of gravitation in (55) is thermal. It is therefore possible that our model is related to the work of Jacobson [J], Padmanabhan [Pa], or Verlinde [V], among others.

Remark 11.1. Our model unifies electromagnetism with gravity, in the sense that if we turn the strength of the electric charge $e$ to zero, so that the positive and negative strands are no longer attracted to each other, then spacetime curvature, and thus gravity, would disappear.

12. CPT INvariance: Charge Conjugation as a Lorentz Transformation

12.1. Preliminary: The action of Lorentz transformations on Dirac spinors. To establish notation, we briefly review the standard derivation of the action of a Lorentz transformation $\Lambda \in SO(1,3)$ on a Dirac spinor $\psi$.

To determine the action of $\Lambda$ on $\psi$, the Dirac projection $(i/\partial - m)$ is required to be invariant under $\Lambda$,

$$\Lambda(i\partial - m)\Lambda^{-1} = i\partial - m.$$  

We denote a spinor representation of $\Lambda$ by $S_\Lambda \in \text{End}(\mathbb{C}^4)$, and a spacetime representation by $\Lambda^\mu_\nu$; whence

$$\Lambda_\psi(x^\mu) = S_\Lambda \psi(\Lambda^\mu_\nu x^\nu).$$

Thus, (28) implies

$$(S_\Lambda \gamma^\nu)(\Lambda^\mu_\nu \partial_\mu) = \gamma^\mu \partial_\mu S_\Lambda = \gamma^\mu S_\Lambda \partial_\mu.$$  

We may therefore obtain $S_\Lambda$ (unique up to a phase) by imposing the constraint

$$S_\Lambda \gamma^\nu \Lambda^\mu_\nu = \gamma^\mu S_\Lambda.$$  

For example, the parity transformation $P = (P^\mu_\nu) = \text{diag}(1, -1, -1, -1)$

acts on $\psi$ by

$$P_\psi(t, x^i) = \gamma^0 \psi(t, -x^i).$$

12.2. Charge Conjugation as a Lorentz Transformation. Recall the strand Lagrangian $L = \bar{\psi}(i\partial - m|\bar{u}^\mu u_\mu|^{1/2})\psi$ from Section 6. In Section 7, we found that the photon atom $\gamma = \bar{\psi}_L \psi_R$ splits into an electron strand and a positron strand, represented by $\psi_R$ and $\psi_L$ respectively. The electron-photon interaction term $e\bar{\psi}A\psi$ in the standard model Lagrangian is therefore absent from the strand Lagrangian $L$. In contrast to the term $e\bar{\psi}A\psi$, the term $m\bar{\psi}_L \psi_R$ in $L$ should not change sign under charge conjugation, since its coupling constant is mass $m$, not electric charge $e$.

Geometrically, $\tilde{M}$ is not fundamentally equipped with a $U(1)$ gauge bundle. Instead, electric charge arises from the tangent bundle of $\tilde{M}$, described in Section 4 and is thus a property of emergent spacetime $\tilde{M}$ itself. An immediate question, then, is how charge conjugation can be realized in this framework. With only spacetime
in hand, charge conjugation must somehow be given by a Lorentz transformation.
Furthermore, this Lorentz transformation must act on spinors by exchanging particle
spinors with anti-particle spinors; a priori, it is not clear whether such a Lorentz
transformation exists, or how it could be derived.

Consider the four positive energy solutions \((E = +\sqrt{|\mathbf{p}|^2 + m^2})\) of the Dirac equa-
tion \((i\partial - m)\psi = 0\) in the Dirac representation:

\[
\begin{align*}
  u_1 &= N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x-ip_y}{E+m} \end{pmatrix}, \\
  u_2 &= N \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}, \\
  v_1 &= N \begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \\
  v_2 &= N \begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix},
\end{align*}
\]

where \(N = \sqrt{E + m}\).

In the standard model, charge conjugation \(C\) acts by reversing the sign of the
electric charge \(e\),

\[(31) \quad C(i\partial - e\mathbf{A} - m)C^{-1} = i\partial + e\mathbf{A} - m.\]

Thus \(C\) acts on \(\psi\) by

\[(32) \quad C\psi = i\gamma^2\psi^*.\]

Consequently, \(C\) exchanges particle spinors with anti-particle spinors,

\[(33) \quad C.u_1 e^{ip_\mu x^\mu} = v_1 e^{-ip_\mu x^\mu}, \quad C.u_2 e^{ip_\mu x^\mu} = v_2 e^{-ip_\mu x^\mu}.\]

Of course, the transformation \((32)\) does not correspond to any Lorentz transforma-
tion.

In the context of strands, however, the difference between a particle strand and an
anti-particle strand lies in the non-uniqueness of tangent vectors (in \(\tilde{M}\)): a particle
strand \(\alpha\) has a future-oriented tangent four-vector \(\tau = \dot{\alpha}\), and an anti-particle strand
has a past-oriented tangent four-vector \(\tau = -\dot{\alpha}\), by Definition 4.1. Therefore, since
\(\dot{\alpha}(t)\) may point in any direction of space, and

\[\tau(t) = \text{sgn}(\alpha)\dot{\alpha}(t),\]

charge conjugation is given by the Lorentz transformation

\[\begin{align*}
  C &= (C^\mu_\nu) = \text{diag}(-1, -1, -1, -1),
\end{align*}\]

In order for our spinor representation of strands to be consistent, this transformation
must exchange particle spinors with anti-particle spinors.

Let us first determine how \(C\) acts on a Dirac spinor \(\psi\). Since \(C\) is now a Lorentz
transformation, we have

\[C(i\partial - m)C^{-1} = i\partial - m.\]
Thus, from (29) we obtain \( S \gamma^\nu C^\mu_{\nu} = \gamma^\mu S \). Whence \( S \gamma^\mu = \gamma^5 \) (times any phase). Therefore

\[
(34) \quad C.\psi(t, x^i) = \gamma^5 \psi(-t, -x^i) = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \psi(-t, -x^i),
\]

where the \( \gamma^5 \) matrix is in the Dirac representation.

We find that \( C \) does indeed exchange particle spinors with anti-particle spinors, similar to (33):

\[
C.\hat{u}_1 e^{ip\mu x^\mu} = (\gamma^5 \hat{u}_1) e^{iC^\mu_{\nu}p^\mu x^\nu} = v_2 e^{-ip\mu x^\mu},
\]

\[
C.\hat{u}_2 e^{ip\mu x^\mu} = (\gamma^5 \hat{u}_2) e^{iC^\mu_{\nu}p^\mu x^\nu} = v_1 e^{-ip\mu x^\mu}.
\]

Our model therefore produces a new definition of charge conjugation based solely on the Lorentz transformation that exchanges a positive strand with a negative strand.

**Remark 12.1.** The two spinor transformations \( S \gamma^\mu C^\mu_{\nu} = i\gamma^2 \) and \( S \gamma^\mu = \gamma^5 \) both exchange particle spinors with anti-particle spinors, although the spinors that are exchanged are different. Specifically, \( i\gamma^2 \) exchanges \( u_1 \) and \( u_2 \) with \( v_1 \) and \( v_2 \) respectively, whereas \( \gamma^5 \) exchanges \( u_1 \) and \( u_2 \) with \( v_2 \) and \( v_1 \) respectively.

**Remark 12.2.** The operator \( \gamma^5 \) cannot act as charge conjugation if the term \( e\bar{\psi}A\psi \) is included in the Lagrangian, since equation (31) does not hold if \( S \gamma^\mu = \gamma^5 \).

12.3. **Time reversal without complex conjugation.** Time reversal is the Lorentz transformation

\[
T = (T^\mu_{\nu}) = \text{diag}(-1, 1, 1, 1),
\]

which we take to act on the tangent space at a point of emergent spacetime \( \tilde{M} \). It is standard to assume that \( T \) acts on the spatial velocity vector \( v \) of a particle by reversing its sign, since

\[
T. v = T. \frac{dx}{dt} = \frac{dx}{d(-t)} = -\frac{dx}{dt} = -v.
\]

Under this assumption, \( T \) acts on the position and momentum operators by

\[
(35) \quad T.\hat{x} T^{-1} = \hat{x} \quad \text{and} \quad T.\hat{p} T^{-1} = -\hat{p}.
\]

Thus, time reversal acts on complex numbers by complex conjugation:

\[
(36) \quad T. i T^{-1} = T. [\hat{x}, \hat{p}] T^{-1} = [T.\hat{x} T^{-1}, T.\hat{p} T^{-1}] = [\hat{x}, -\hat{p}] = -i.
\]

Consequently, \( T \) acts on the Dirac projection by complex conjugation,

\[
(37) \quad T. (i\hat{\phi} - m) T^{-1} = (i\hat{\phi} - m)^*.
\]

From (37), and the fact that \( \gamma^2 \) is pure imaginary, it follows that \( T \) acts on \( \psi \) by

\[
T.\psi = \gamma^1 \gamma^3 \psi^*.
\]
However, we claim that time reversal does not actually reverse the sign of the velocity vector $\mathbf{v}$, and so conjugation of the momentum operator $\hat{p}$ by $T$ does not reverse the sign of $\hat{p}$.

Indeed, consider the worldline $x^\mu = x^\mu(t)$ of a particle. The particle has four-velocity

$$(\dot{x}^\mu) = \gamma(1, \mathbf{v}),$$

where $\gamma = (1 - |\mathbf{v}|^2)^{-1/2}$. Observe that $T$ leaves $\mathbf{v}$ unchanged:

$$(T^\mu_\nu \dot{x}^\nu) = \gamma(-1, \mathbf{v}).$$

In other words, the output of the spatial component of the time derivative $\dot{x}^\mu$ is the spatial vector $\mathbf{v}$, and so the spatial component of the Lorentz transformation $T$ acts on $\mathbf{v}$, not the time component. Therefore the particle’s spatial momentum, $\boldsymbol{p} = \gamma m \mathbf{v}$, is also left unchanged by $T$,

$$(T^\mu_\nu p^\nu) = \gamma m(-1, \mathbf{v}).$$

Consequently, we have

$$T\hat{p}T^{-1} = \hat{p},$$

in contrast to (35). Whence (37) does not hold; instead, the Dirac projection remains invariant under time reversal, as it is a Lorentz transformation:

$$T(i\partial - m)T^{-1} = i\partial - m.$$ 

Therefore, applying (29) we find

(38) $$T\psi(t, x^i) = \gamma^1 \gamma^2 \gamma^3 \psi(-t, x^i).$$

**Remark 12.3.** The Lorentz transformation that does reverse the direction of $\boldsymbol{p}$ is charge conjugation $C$, as we found in Section 12.2. Whence $C\hat{p}C^{-1} = -\hat{p}$. But $C$ also reverses the sign of $\mathbf{x}$, and so $C\hat{x}C^{-1} = -\hat{x}$. Thus $CiC^{-1} = i$ by (36), and therefore $C$ does not induce complex conjugation either.

### 12.4. CPT invariance.

In Section 12.2, we showed that charge conjugation $C$ of a strand is given by the Lorentz transformation

(39) $$C = \text{diag}(-1, -1, -1, -1).$$

Furthermore, in Section 12.3 we showed that time reversal $T$ does not induce complex conjugation.

To summarize, charge conjugation (34), parity (30), and time reversal (38) act on Dirac spinors in the strand preon model by

$$C\psi(t, x^i) = \gamma^5 \psi(-t, -x^i),$$

(40) $$P\psi(t, x^i) = \gamma^0 \psi(t, -x^i),$$

$$T\psi(t, x^i) = \gamma^1 \gamma^2 \gamma^3 \psi(-t, x^i).$$

---

6Note that if we used the signature $(-1, 1, 1, 1)$ instead of $(1, -1, -1, -1)$, then $S_P$ and $S_T$ would be swapped.
Each of these transformations act trivially on the Dirac projection \((i\partial - m)\).

There are two important consequences of \((39)\) and \((40)\):

- The Lorentz group \(O(1,3)\), as a manifold, has four connected components, and so the quotient group \(O(1,3)/SO^+(1,3)\) has four elements. Using \((39)\), this quotient is generated by \(C, P, T\):

\[
O(1,3)/SO^+(1,3) = \langle C, P, T \rangle \cong \mathbb{Z}_2 \times \mathbb{Z}_2.
\]

Each of 1, \(C\), \(P\), and \(T\) belong to a different one of the four connected components of \(O(1,3)\).

- The product \(CPT\), in the spacetime representation, is simply the identity:

\[
(41) \quad C^\mu P^\rho T_\nu = g^\mu_\nu = \delta^\mu_\nu \in SO^+(1,3).
\]

Furthermore, the product \(CPT\), in the spinor representation, is proportional to the identity:

\[
(42) \quad S_CSPST = \gamma^5 \gamma^0 (\gamma^1 \gamma^2 \gamma^3) = -i (\gamma^5)^2 = -i.
\]

Note that \((42)\) is consistent with \((41)\) since \(g^\mu_\nu\) may be taken to act on a Dirac spinor by multiplication by an arbitrary phase \(e^{i\theta}\), by \((29)\).

By \((41)\) and \((42)\), CPT invariance of the strand Lagrangian \(L\) trivially holds.

**Remark 12.4.** The relationships between \(C\), \(P\), and \(T\) that we have obtained cannot occur in the standard model, because in the standard model charge conjugation does not correspond to a Lorentz transformation.

**Remark 12.5.** The relation \((CPT)^\mu_\nu = \delta^\mu_\nu\) in \((41)\) is really just a different incarnation of the chirality relation

\[
\text{sgn}(\alpha) \chi(\alpha) \text{sgn}(\omega) = 1
\]

from \((15)\), resulting from the correspondences

\[
C \sim \text{sgn}(\alpha), \quad P \sim \chi(\alpha), \quad T \sim \text{sgn}(\omega).
\]

The correspondence \(P \sim \chi(\alpha)\) holds because \(\chi(\alpha)\) is the handedness, left or right, of the ordered basis \(\{t, n, b\}\) (shown in Section 5), and parity \(P\) flips the handedness of this basis:

\[
P.\{t, n, b\} = \{-t, -n, -b\}.
\]

Furthermore, the correspondence \(T \sim \text{sgn}(\omega)\) holds because \(t\) and \(\omega\) only appear as the product \((\omega t)\) in the parameterization of circular strands in \((3)\), and \((-\omega)t = \omega(-t)\).

**Remark 12.6.** A rotation by \(2\pi\) in spacetime corresponds to the Lorentz transformation \((PT)^2\). Furthermore, by \((40)\),

\[
(S_P S_T)^2 = (\gamma^0 \gamma^1 \gamma^2 \gamma^3)^2 = -1.
\]

We therefore obtain the elementary fact that a spacetime rotation of a spinor \(\psi\) in \(\mathbb{C}^4\) is trivial if and only if the rotation is a multiple of \(4\pi\).
13. The spin-statistics connection for strands

The following derivation is similar to Schwinger’s heuristic argument for the spin-statistics connection [Sch], with CP in place of T and CPT. However, in the framework of strands, the complications due to time (e.g., spacelike separation of the two particles; rotation to Euclidean spacetime) do not arise.

Consider two strands \( \alpha, \tilde{\alpha} \) of equal radius \( r \). Denote by \( \beta, \tilde{\beta} \) their respective central worldlines in \( \tilde{M} \), and suppose that there are points along \( \beta \) and \( \tilde{\beta} \), say \( \beta(s_0) \) and \( \tilde{\beta}(s_0) \), that are causally connected. Further suppose that the strands have equal chirality, say left-handed, with spinor representations \( \psi_L(\beta(s)) \) and \( \psi_L(\tilde{\beta}(s)) \).

By a possible local change of coordinates, we may suppose that
\[
x^\mu := \tilde{\beta}^\mu(\tilde{s}_0) = -\beta^\mu(s_0).
\]
Furthermore, by the indistinguishability of swapping two identical particles, we have
\[
\text{PT.}(\psi_L(-x^\mu) \otimes \psi_L(x^\mu)) = \psi_L(-x^\mu) \otimes \psi_L(x^\mu).
\]
Thus, using parity and time reversal of strands, we find (suppressing \( \mu \))
\[
\psi_L(-x) \otimes \psi_L(x) \overset{(i)}{=} \text{PT.}(\psi_L(-x) \otimes \psi_L(x)) = (\text{PT.}\psi_L(-x)) \otimes (\text{PT.}\psi_L(x))
\]
\[
\overset{(ii)}{=} (-i\gamma^5)\psi_L(x) \otimes (-i\gamma^5)\psi_L(-x) = i\psi_L(x) \otimes i\psi_L(-x) = -\psi_L(x) \otimes \psi_L(-x).
\]

where (i) holds by \[43\], and (ii) holds by \[40\]. Similarly, for a diameter \( \tilde{\psi}_L\tilde{\psi}_R \) we find
\[
(\tilde{\psi}_L\tilde{\psi}_R)(-x) \otimes (\tilde{\psi}_L\tilde{\psi}_R)(x)
\]
\[
= \text{PT.}((\tilde{\psi}_L\tilde{\psi}_R)(-x) \otimes (\tilde{\psi}_L\tilde{\psi}_R)(x))
\]
\[
= (\tilde{\psi}_L(-i\gamma^5)(-i\gamma^5)\tilde{\psi}_R)(x) \otimes (\tilde{\psi}_L(-i\gamma^5)(-i\gamma^5)\tilde{\psi}_R)(-x) = (\tilde{\psi}_L\tilde{\psi}_R)(x) \otimes (\tilde{\psi}_L\tilde{\psi}_R)(-x).
\]

In field-theoretic terms, an excitation of the field \( \psi_L \) at \( \beta^\mu(s_0) \) is the same field excitation at \( \beta^\mu(s_1) \) since the two points
\[
\alpha^\mu(s_0) \quad \text{and} \quad \alpha^\mu(s_1),
\]
by virtue of being joined by the strand $\alpha$, are the same point in spacetime $M$. Therefore (44) and (45) hold along the entire two central worldlines $\beta$ and $\tilde{\beta}$,

$$\psi_L(\beta) \otimes \psi_L(\tilde{\beta}) = -\psi_L(\tilde{\beta}) \otimes \psi_L(\beta),$$

and similarly for $L \leftrightarrow R$. In particular, the central worldlines of two strands of equal chirality and equal radius cannot intersect: for $x \in \tilde{M}$ we have

$$\psi_L(x) \otimes \psi_L(x) = 0 \quad \text{and} \quad \psi_R(x) \otimes \psi_R(x) = 0.$$

This is the Pauli-exclusion principle for strands. Furthermore, by (46), the central worldlines of diameters are allowed to intersect. In particular, (46) implies that (circular) strands are fermionic, whereas diameters are bosonic. We therefore obtain a spin-statistics connection for strands.

### 14. Future Directions: Scattering with Strands

We assume that the standard Feynman rules hold for propagators and vertices, as an effective field theory, for computing cross sections and decay rates. However, (at least) two new constraints on the path integral arise in our strand preon model.

To describe these constraints, consider a scattering event with fixed incoming and outgoing particles. In this event, all the different configurations of internal lines (strands) that exist in superposition, exist together in spacetime $M$. By the spin-statistics connection for strands (Section 13), the worldlines of strands with the same chirality cannot intersect unless at least one of the strands is paired with another strand in a diameter of an atom, making the pair bosonic. Furthermore, a strand particle cannot simultaneously both terminate at an apex and continue to propagate. We thus expect severe restrictions on both

(i) the possible Feynman diagrams that may exist in superposition; and

(ii) the limits of integration $\int \int d^4x \, d^4p$ of the Lagrangian density.

Scattering amplitudes obtained from strands should therefore differ from those obtained from the full path integral $Z = \int \mathcal{D}\psi e^{i\mathcal{S}[\psi]}$. These constraints could potentially eliminate certain (ultraviolet) divergences in quantum field theory. We leave a formulation of these constraints for future work.

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