Higgs mass prediction in the MSSM at three-loop level in a pure \( \overline{\text{DR}} \) context

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The impact of the three-loop effects of order \( \alpha_t \alpha_s^2 \) on the mass of the light CP-even Higgs boson in the MSSM is studied in a pure \( \overline{\text{DR}} \) context. For this purpose, we implement the results of Kant et al. [1] into the C++ module Himalaya and link it to FlexibleSUSY, a Mathematica and C++ package to create spectrum generators for BSM models. The three-loop result is compared to the fixed-order two-loop calculations of the original FlexibleSUSY and of FeynHiggs, as well as to the result based on an EFT approach. Aside from the expected reduction of the renormalization scale dependence with respect to the lower order results, we find that the three-loop contributions significantly reduce the difference from the EFT prediction in the TeV-region of the SUSY scale \( M_S \). Himalaya can be linked also to other two-loop \( \overline{\text{DR}} \) codes, thus allowing for the elevation of these codes to the three-loop level.
1 Introduction

The measurement of the Higgs boson mass at the Large Hadron Collider (LHC) represents a significant constraint on the viability of supersymmetric (SUSY) models. Given a particular SUSY model, the mass of the Standard Model-like Higgs boson is a prediction, which must be in agreement with the measured value of $(125.09 \pm 0.21 \pm 0.11)$ GeV \cite{2}. Noteworthy, the experimental uncertainty on the measured Higgs mass has already reached the per-mille level. Theory predictions in SUSY models, however, struggle to reach the same level of accuracy. The reason is that the Higgs mass receives large higher order corrections, dominated by the top Yukawa and the strong gauge coupling \cite{3–5}. Both of these two couplings are comparatively large, leading to a relatively slow convergence of the perturbative series. Furthermore, the scalar nature of the Higgs implies corrections proportional to the square of the top-quark mass, on top of the top-mass dependence due to the Yukawa coupling, which enters the loop corrections quadratically. On the other hand, corrections from SUSY particles are only logarithmic in the SUSY particle masses due to the assumption of only soft SUSY-breaking terms. If the SUSY particles are not too far above the TeV scale \cite{6, 7}, the SUSY Higgs mass can be obtained from a fixed-order calculation of the relevant one- and two-point functions with external Higgs fields. In this case, higher order corrections up to the three-loop level are known in the Minimal Supersymmetric Standard Model (MSSM) \cite{1, 5, 8–23}.

There are plenty of publicly available computer codes which calculate the Higgs pole mass(es) in the MSSM at higher orders: CPsuperH \cite{24–26}, FeynHiggs \cite{9, 27–31}, FlexibleSUSY \cite{32, 33}, H3m \cite{1, 20}, ISASUSY \cite{34}, MhEFT \cite{35}, SARAH/SPheno \cite{36–42}, SOFTSUSY \cite{43, 44}, SuSpect \cite{45} and SusyHD \cite{46}. FeynHiggs adopts the on-shell scheme for the renormalization of the particle masses, while all other codes express their results in terms of \text{MS}/\text{DR} parameters. All these schemes are formally equivalent up to higher orders in perturbation theory, of course. The numerical difference between the schemes is one of the sources of theoretical uncertainty on the Higgs mass prediction, however. All of these programs take into account one-loop corrections, most of them also leading two-loop corrections. H3m is the only one which includes three-loop corrections of order $\alpha_t \alpha_s^2$, where $\alpha_t$ is the squared top Yukawa and $\alpha_s$ is the strong coupling. It combines these terms with the on-shell two-loop result of FeynHiggs after transforming the $\mathcal{O}(\alpha_t)$ and $\mathcal{O}(\alpha_t \alpha_s)$ terms from there to the \text{DR} scheme.

Here we present an alternative implementation of the $\mathcal{O}(\alpha_t \alpha_s^2)$ contributions of Refs.\cite{1, 20} for the light CP-even Higgs mass in the MSSM into the framework of FlexibleSUSY \cite{32], referring to the combination as FlexibleSUSY+Himalaya in what follows. This allows us to study the effect of the three-loop contributions in a pure \text{DR} environment, i.e. without the trouble of combining the corrections with an on-shell calculation. The three-loop terms are provided in the form of a separate C++ package, named Himalaya, which one should be able to include in any other \text{DR} code without much effort. The Himalaya package and the dedicated version of FlexibleSUSY which incorporates the three-loop contributions from Himalaya, can be downloaded from Refs.\cite{47, 48}, respectively. In this way, we hope to contribute to the on-going effort of improving the precision of the Higgs mass prediction in the MSSM.
In the present paper we study the impact of the three-loop corrections for low and high SUSY scales and compare our results to the two-loop calculations of the public spectrum generators of FlexibleSUSY and FeynHiggs. By quantifying the size of the three-loop corrections, we also provide a measure for the theoretical uncertainty of the DR fixed-order calculation.

As will be shown below, the implementation of the $\alpha_t \alpha_s^2$ corrections also applies to the terms of order $\alpha_b \alpha_s^2$, where $\alpha_b$ is the bottom Yukawa coupling. Therefore, Himalaya will take such terms into account, and we will refer to the sum of top- and bottom-Yukawa induced supersymmetric QCD (SQCD) corrections as $O(\alpha_t \alpha_s^2 + \alpha_b \alpha_s^2)$ in what follows. However, it should be kept in mind that this does not include effects of order $\alpha_s^2 \sqrt{\alpha_t \alpha_b}$, which arise from three-loop Higgs self energies involving both a top/stop and a bottom/sbottom triangle. The results of Himalaya are thus unreliable in the (rather exotic) case where $\alpha_t$ and $\alpha_b$ are comparable in magnitude.

The remainder of this paper is structured as follows. Section 2 describes the form in which the three-loop contributions of order $(\alpha_t + \alpha_b) \alpha_s^2$ are implemented in Himalaya. Its input parameters are to be provided in the DR scheme at the appropriate perturbative order. Section 3 details how this input is prepared in the framework of FlexibleSUSY. It also summarizes all the contributions that enter the final Higgs mass prediction in FlexibleSUSY + Himalaya. Section 4 analyzes the impact of various three-loop contributions on this prediction as well as the residual renormalization scale dependence, and it compares the results obtained with FlexibleSUSY + Himalaya to existing fixed-order and resummed results for the light Higgs mass. In particular, this includes a comparison to the original implementation of the three-loop effects in H3m. Our conclusions are presented in Section 5. Technical details of Himalaya, its link to FlexibleSUSY, and run options are collected in the appendix.

2 Higgs mass prediction at the three-loop level in the MSSM

The results for the three-loop $\alpha_t \alpha_s^2$ corrections to the Higgs mass in the MSSM have been obtained in Refs. [1, 20] by a Feynman diagrammatic calculation of the relevant one- and two-point functions with external Higgs fields in the limit of vanishing external momenta. The dependence of these terms on the squark and gluino masses was approximated through asymptotic expansions, assuming various hierarchies among the masses of the SUSY particles. For details of the calculation we refer to Refs. [1, 20].

2.1 Selection of the hierarchy

A particular set of parameters typically matches several of the hierarchies mentioned above. In order to select the most suitable one, Ref. [1] suggested a pragmatic approach, namely the comparison of the various asymptotic expansions to the exact expression at two-loop level. Himalaya also adopts this approach, but introduces a few refinements in order to further stabilize the hierarchy selection (see also Ref. [49]).

In a first step the Higgs pole mass $M_h$ is calculated at the two-loop level at order $\alpha_t \alpha_s$, using the result of Ref. [12] in the form of the associated FORTRAN code provided
by the authors. We refer to this quantity as $M_{DSZ}^h$ in what follows. Subsequently, for all hierarchies $i$ which fit the given mass spectrum, $M_h$ is calculated again using the expanded expressions of Ref. [1] at the two-loop level, resulting in $M_{h,i}$. In the original approach of Ref. [1], the hierarchy is selected as the value of $i$ for which the difference

$$\delta_{2L}^i = |M_{DSZ}^h - M_{h,i}|$$

(1)

is minimal. However, we found that this criterion alone causes instabilities in the hierarchy selection in regions where several hierarchies lead to similar values of $\delta_{2L}^i$. We therefore refine the selection criterion by taking into account the quality of the convergence in the respective hierarchies, quantified by

$$\delta_{\text{conv}}^i = \sqrt{\sum_{j=1}^{n} (M_{h,i} - M_{h,i}^{(j)})^2}.$$  

(2)

While $M_{h,i}$ includes all available terms of the expansion in mass (and mass difference) ratios, in $M_{h}^{(j)}$ the highest terms of the expansion for the mass (and mass difference) ratio $j$ are dropped. We then define the “best” hierarchy to be the one which minimizes the quadratic mean of Eqs. (1) and (2),

$$\delta_i = \sqrt{(\delta_{2L}^i)^2 + (\delta_{\text{conv}}^i)^2}.$$  

(3)

The relevant analytical expressions for the three-loop terms of order $\alpha_t \alpha_b^2$ to the CP-even Higgs mass matrix in the various mass hierarchies are quite lengthy. However, they are accessible in Mathematica format in the framework of the publicly available program H3m. We have transformed these formulas into C++ format and implemented them into Himalaya.

The hierarchies defined in H3m equally apply to the top and the bottom sector of the MSSM, so that the results of Ref. [1] can also be used to evaluate the corrections of order $\alpha_t \alpha_b^2$ to the Higgs mass matrix. Indeed, Himalaya takes these corrections into account. However, as already pointed out in Section 1, a complete account of the top- and bottom-Yukawa effects to order $\alpha_b^2$ would require to include the contribution of diagrams which involve both top/stop and bottom/sbottom loops at the same time. These were not considered in Ref. [1], and thus the Himalaya result should only be used in cases where such mixed $\sqrt{\alpha_t \alpha_b}$ terms can be neglected.

### 2.2 Modified DR scheme

By default, all the parameters of the calculation are renormalized in the DR scheme. However, in this scheme, one finds artificial “non-decoupling” effects [12], meaning that the two- and three-loop result for the Higgs mass depends quadratically on a SUSY particle mass if this mass gets much larger than the others. Such terms are avoided by transforming the stop masses to a non-minimal scheme, named MDR (modified DR) in Ref. [1], which mimics the virtue of the on-shell scheme of automatically decoupling the heavy particles.
If the user wishes to use this scheme rather than pure $\text{DR}$, *Himalaya* writes the Higgs mass matrix as
\[
\hat{M}(\hat{m}_t) = \hat{M}^{\text{tree}} + \hat{M}^{(\alpha_t)}(\hat{m}_t) + \hat{M}^{(\alpha_t \alpha_s)}(\hat{m}_t) + \hat{M}^{(\alpha_t^2)}(\hat{m}_t) + \cdots
\]
\[
= M^{\text{tree}}(m_t) + M^{(\alpha_t)}(m_t) + M^{(\alpha_t \alpha_s)}(m_t) + \delta M(m_t, \hat{m}_t) + M^{(\alpha_t^2)}(\hat{m}_t) + \cdots, \tag{4}
\]
where $M$ and $\hat{M}$ are the Higgs mass matrices in the $\text{DR}$ and the $\text{MDR}$ scheme, respectively, $M^{\text{tree}} = M^{\text{tree}}$ is the tree-level expression, and the superscript $(x)$ denotes the term of order $x \in \{\alpha_t, \alpha_s, \alpha_t \alpha_s, \ldots\}$. The ellipsis in Eq. (4) symbolizes any terms that involve coupling constants other than $\alpha_t$ or $\alpha_s$, or higher orders of the latter. For brevity we suppress the stop mass indices “1” and “2” here. *Himalaya* provides the numerical results for $M^{(\alpha_t^2)}(\hat{m}_t)$ as well as
\[
\delta M(m_t, \hat{m}_t) \equiv \left( \hat{M}^{(\alpha_t)}(\hat{m}_t) + \hat{M}^{(\alpha_t \alpha_s)}(\hat{m}_t) \right) - \left( M^{(\alpha_t)}(m_t) + M^{(\alpha_t \alpha_s)}(m_t) \right), \tag{5}
\]
where the $\text{MDR}$ stop mass $\hat{m}_t$ is calculated from its $\text{DR}$ value $m_t$ by the conversion formulas through $O(\alpha_t^2)$, provided in Ref. [1]. Note that these conversion formulas depend on the underlying hierarchy, and may be different for $m_{t,1}$ and $m_{t,2}$.

Even if the result is requested in the $\text{MDR}$ scheme, the output of *Himalaya* can thus be directly combined with pure $\text{DR}$ results through $O(\alpha_t \alpha_s)$ according to Eq. (4) in order to arrive at the mass matrix at order $\alpha_t \alpha_s^2$. Of course, one may also request the plain $\text{DR}$ result from *Himalaya*, in which case it will simply return the numerical value for $M^{(\alpha_t \alpha_s^2)}(m_t)$ which can be directly added to any two-loop $\text{DR}$ result.

In any case, the difference between the $\text{DR}$ and $\text{MDR}$ result is expected to be quite small unless the mass splitting between one of the stop masses and other, heavier, strongly interacting SUSY particles becomes very large. As a practical example, in Figure 1 we show the difference of the lightest Higgs mass at the three-loop level calculated in the $\text{DR}$ and $\text{MDR}$ scheme. All $\text{DR}$ soft-breaking mass parameters, the $\mu$ parameter of the MSSM super-potential, and the running CP-odd Higgs mass are set equal to $M_S$ here. The running trilinear couplings, except $A_t$, are chosen such that the sfermions do not mix. The $\text{DR}$ stop mixing parameter $X_t = A_t - \mu / \tan \beta$ is left as a free parameter. For this scenario we find that the difference between the $\text{DR}$ and $\text{MDR}$ scheme is below 100 MeV for different values of the stop mixing parameter.

Note that for all terms in the Higgs mass matrix except $\alpha_t$, $\alpha_t \alpha_s$, and $\alpha_t \alpha_s^2$, it is perturbatively equivalent to use either the $\text{DR}$ or the $\text{MDR}$ stop mass as defined above. Predominantly, this concerns the electroweak contributions as well as the terms of order $\alpha_t^2$. In this paper, we use the $\text{DR}$ stop mass for these contributions.

3 Implementation into FlexibleSUSY

3.1 Determination of the MSSM $\text{DR}$ parameters

FlexibleSUSY determines the running $\text{DR}$ gauge and Yukawa couplings as well as the running vacuum expectation value of the MSSM along the lines of Ref. [50] by setting
Figure 1: Difference between the lightest Higgs pole mass calculated in the DR scheme and the MDR scheme as a function of the SUSY scale $M_S$ for $\tan \beta = 5$. In the left panel the soft-breaking stop and gluino mass parameters are set equal to $M_S$. In the right panel, we use $m_{\tilde{g}} = 2M_S$. We have cut off curves with non-zero $X_t$ around or below the TeV scale, where the DR CP-even Higgs mass becomes tachyonic at the electroweak scale.

The scale to the $Z$-boson pole mass $M_Z$. In this approach, the following Standard Model (SM) input parameters are used:

$$\alpha_{\text{em}}^{\text{SM}(5)}(M_Z), \alpha_s^{\text{SM}(5)}(M_Z), G_F, M_Z,$$

$$M_e, M_\mu, M_\tau, m_{u,d,s}(2 \text{ GeV}), m_c^{\text{SM}(4),\overline{\text{MS}}}(m_c), m_b^{\text{SM}(5),\overline{\text{MS}}}(m_b), M_t,$$

where $\alpha_{\text{em}}^{\text{SM}(5)}(M_Z)$ and $\alpha_s^{\text{SM}(5)}(M_Z)$ denote the electromagnetic and strong coupling constants in the $\overline{\text{MS}}$ scheme in the Standard Model with five active quark flavours, and $G_F$ is the Fermi constant. $M_e, M_\mu, M_\tau,$ and $M_t$ denote the pole masses of the electron, muon, tau lepton, and top quark, respectively. The input masses of the up, down and strange quark are defined in the $\overline{\text{MS}}$ scheme at the scale $2 \text{ GeV}$. The charm and bottom quark masses are defined in the $\overline{\text{MS}}$ scheme at their scale in the Standard Model with four and five active quark flavours, respectively.

The MSSM DR gauge couplings $g_1$, $g_2$ and $g_3$ are given in terms of the DR parameters...
$\alpha_{em}^{\text{MSSM}}(M_Z)$ and $\alpha_{s}^{\text{MSSM}}(M_Z)$ in the MSSM as:

$$g_1(M_Z) = \frac{\sqrt{\frac{5}{3}}}{\frac{\sqrt{4\pi\alpha_{em}^{\text{MSSM}}(M_Z)}}{\cos \theta_w(M_Z)}},$$

$$g_2(M_Z) = \frac{\sqrt{\frac{4\pi\alpha_{em}^{\text{MSSM}}(M_Z)}}}{\sin \theta_w(M_Z)},$$

$$g_3(M_Z) = \frac{\sqrt{\frac{4\pi\alpha_{s}^{\text{MSSM}}(M_Z)}}}{\sin \theta_w(M_Z)}.$$

The couplings $\alpha_{em}^{\text{MSSM}}(M_Z)$ and $\alpha_{s}^{\text{MSSM}}(M_Z)$ are calculated from the corresponding input parameters as

$$\alpha_{em}^{\text{MSSM}}(M_Z) = \frac{\alpha_{em}(5)}{1 - \Delta\alpha_{em}(M_Z)},$$

$$\alpha_{s}^{\text{MSSM}}(M_Z) = \frac{\alpha_{s}(5)}{1 - \Delta\alpha_{s}(M_Z)},$$

where the threshold corrections $\Delta\alpha_i(M_Z)$ have the form

$$\Delta\alpha_{em}(M_Z) = \frac{\alpha_{em}}{2\pi} \left( \frac{1}{3} - \frac{16}{9} \log \frac{m_t}{M_Z} - \frac{4}{9} \sum_{i=1}^{6} \log \frac{m_{\tilde{u}_i}}{M_Z} - \frac{1}{9} \sum_{i=1}^{6} \log \frac{m_{\tilde{d}_i}}{M_Z} ight),$$

$$\Delta\alpha_{s}(M_Z) = \frac{\alpha_{s}}{2\pi} \left[ \frac{1}{2} - 2 \log \frac{m_{\tilde{g}}}{M_Z} - \frac{2}{3} \log \frac{m_t}{M_Z} - \frac{1}{6} \sum_{i=1}^{6} \left( \log \frac{m_{\tilde{q}_i}}{M_Z} + \log \frac{m_{\tilde{l}_i}}{M_Z} \right) \right].$$

The $\overline{\text{DR}}$ weak mixing angle in the MSSM, $\theta_w$, is determined at the scale $M_Z$ from the Fermi constant $G_F$ and the $Z$ pole mass via the relation

$$\sin^2 \theta_w \cos^2 \theta_w = \frac{\pi \alpha_{em}^{\text{MSSM}}}{\sqrt{2} M_Z^2 G_F (1 - \delta_r)},$$

where

$$\delta_r = \frac{\text{Re} \Sigma_{V,T}(0)}{M_W^2} - \frac{\text{Re} \Sigma_{Z,T}(M_Z^2)}{M_Z^2} + \delta_{\text{VB}} + \delta^{(2)}_r,$$

$$\hat{\rho} = \frac{1}{1 - \Delta\hat{\rho}}, \quad \Delta\hat{\rho} = \text{Re} \left[ \frac{\Sigma_{Z,T}(M_Z^2)}{\hat{\rho} M_Z^2} - \frac{\Sigma_{W,T}(M_W^2)}{M_W^2} \right] + \Delta\hat{\rho}^{(2)}.$$
two-loop corrections taken from Refs. [51, 52]. The DR vacuum expectation values of the up- and down-type Higgs doublets are calculated by

\[ v_u(M_Z) = \frac{2m_Z(M_Z) \sin \beta(M_Z)}{\sqrt{3/5g_1^2(M_Z) + g_2^2(M_Z)}}, \]  
\[ v_d(M_Z) = \frac{2m_Z(M_Z) \cos \beta(M_Z)}{\sqrt{3/5g_1^2(M_Z) + g_2^2(M_Z)}}, \]

where \( \tan \beta(M_Z) \) is an input parameter and \( m_Z(M_Z) \) is the \( Z \) boson DR mass in the MSSM, which is calculated from the \( Z \) pole mass at the one-loop level as

\[ m_Z^2(M_Z) = M_Z^2 + \text{Re} \Sigma_{Z,T}(M_Z^2). \]  

In order to calculate the Higgs pole mass in the DR scheme at the three-loop level \( \mathcal{O}(\alpha_t \alpha_s^2 + \alpha_t \alpha_b^2) \), the DR top and bottom Yukawa couplings must be extracted from the input parameters \( M_t \) and \( m_b^{\text{SM}(5),\text{MS}}(m_b) \) at the two-loop level at \( \mathcal{O}(\alpha_s^2) \). In order to achieve that, we make use of the known two-loop SQCD contributions to the top and bottom Yukawa couplings of Refs. [51–54], as described in the following: We calculate the DR Yukawa couplings \( y_t \) at the scale \( M_Z \) from the DR top mass \( m_t \) and the DR up-type VEV \( v_u \) as

\[ y_t(M_Z) = \sqrt{2} \frac{m_t(M_Z)}{v_u(M_Z)}. \]

In our approach, we relate the DR top mass to the top pole mass \( M_t \) at the scale \( M_Z \) as

\[ m_t(M_Z) = M_t + \text{Re} \Sigma_t^S(M_t^2, M_Z) 
+ M_t \left[ \text{Re} \Sigma_t^L(M_t^2, M_Z) + \text{Re} \Sigma_t^R(M_t^2, M_Z) 
+ \Delta m_t^{(1),\text{SQCD}}(M_Z) + \Delta m_t^{(2),\text{SQCD}}(M_Z) \right], \]

where \( \Sigma_t^{S,L,R}(p^2, Q) \) denote the scalar (superscript \( S \)), and the left- and right-handed parts (\( L, R \)) of the DR renormalized one-loop top self-energy without the gluon, stop, and gluino contributions, and \( \Delta m_t^{(1),\text{SQCD}} \) and \( \Delta m_t^{(2),\text{SQCD}} \) are the full one- and two-loop SQCD corrections taken from Refs. [51, 52],

\[ \Delta m_t^{(1),\text{SQCD}} = -\frac{\alpha_s}{4\pi} C_F \left[ \left( \frac{m_{\text{g}}^2 m_{l_1}^2 s_2 \theta_t}{m_t(m_{l_1}^2 - m_g^2)} - \frac{m_{\text{g}}^2 m_{l_2}^2 s_2 \theta_t}{m_t(m_{l_2}^2 - m_g^2)} + \frac{m_{l_1}^4}{2(m_{l_1}^2 - m_g^2)^2} \right) - \frac{m_{l_1}^4}{m_{l_1}^2 - m_g^2} + \frac{m_{l_2}^4}{2(m_{l_2}^2 - m_g^2)^2} - \frac{m_{l_2}^4}{m_{l_2}^2 - m_g^2} + 1 \right] \log \frac{m_g^2}{Q^2} 
+ \left( \frac{m_{\text{g}}^2 m_{l_1}^2 s_2 \theta_t}{m_t(m_{l_1}^2 - m_g^2)} - \frac{m_{l_1}^4}{2(m_{l_1}^2 - m_g^2)^2} + \frac{m_{l_1}^4}{m_{l_1}^2 - m_g^2} \right) \log \frac{m_{l_1}^2}{Q^2}, \]
In Eq. (28), except the bottom quark, the top quark and the

\[
\begin{aligned}
\Delta m_t^{(2),\text{SQCD}} &= \left( \Delta m_t^{(1),\text{SQCD}} \right)^2 - \Delta m_t^{(2),\text{dec}}.
\end{aligned}
\]

In Eq. (22), it is $C_F = 4/3$ and $s_{2\theta_t} = \sin 2\theta_t$, with $\theta_t$ the stop mixing angle. The two-loop term $\Delta m_t^{(2),\text{dec}}$ is given in Ref. [51] for general stop, sbottom, and gluino masses.

The MSSM DR bottom-quark Yukawa coupling $y_b$ is calculated from the DR bottom-quark mass $m_b$ and the down-type VEV at the scale $M_Z$

\[
y_b(M_Z) = \sqrt{2} m_b(M_Z) v_d(M_Z).
\]

We obtain $m_b(M_Z)$ from the input \(\overline{\text{MS}}\) mass $m_b^{\text{SM}(5),\overline{\text{MS}}}(m_b)$ in the Standard Model with five active quark flavours by first evolving $m_b^{\text{SM}(5),\overline{\text{MS}}}(m_b)$ to the scale $M_z$, using the one-loop QED and three-loop QCD renormalization group equations (RGEs). Afterwards, $m_b^{\text{SM}(5),\overline{\text{MS}}}(M_Z)$ is converted to the DR mass $m_b^{\text{SM}(5),\text{DR}}(M_Z)$ by the relation

\[
m_b^{\text{SM}(5),\text{DR}}(M_Z) = m_b^{\text{SM}(5),\overline{\text{MS}}}(M_Z) \left( 1 - \frac{\alpha_s}{3\pi} + \frac{3g_2^2}{128\pi^2} + \frac{13g_Y^2}{1152\pi^2} \right).
\]

Finally, the MSSM DR bottom mass $m_b(M_Z)$ is obtained from $m_b^{\text{SM}(5),\text{DR}}(M_Z)$ via

\[
m_b(M_Z) = \frac{m_b^{\text{SM}(5),\text{DR}}(M_Z)}{1 + \Delta m_b^{(1)} + \Delta m_b^{(2)}},
\]

\[
\Delta m_b^{(1)} = -\text{Re} \Sigma_b^S((m_b^{\text{SM}(5),\overline{\text{MS}}})^2, M_Z)/m_b - \text{Re} \Sigma_b^L((m_b^{\text{SM}(5),\overline{\text{MS}}})^2, M_Z) - \text{Re} \Sigma_b^R((m_b^{\text{SM}(5),\overline{\text{MS}}})^2, M_Z),
\]

\[
\Delta m_b^{(2)} = \Delta m_b^{(2),\text{dec}} - \frac{\alpha_s}{3\pi} \Delta m_b^{(1)},
\]

where $\Sigma_b^{S,L,R}(p^2, Q)$ are the scalar, left- and right-handed parts of the DR renormalized one-loop bottom quark self-energy in the MSSM, in which all Standard Model particles, except the bottom quark, the top quark and the $W$, $Z$, and Higgs bosons, are omitted. In Eq. (28) $\Delta m_b^{(2),\text{dec}}$ denotes the two-loop decoupling relation of order $\mathcal{O}(\alpha_s^2)$ between the \(\overline{\text{MS}}\) bottom mass $m_b^{\text{SM}(5),\overline{\text{MS}}}$ and the DR bottom mass in the MSSM calculated in Refs. [53, 54].

Note that the matching of the SM to the MSSM leads to large logarithmic contributions in the MSSM DR parameters in the case of a heavy SUSY particle spectrum. These contributions can be resummed in a so-called EFT approach [31, 33, 46, 55, 56].
3.2 Calculation of the CP-even Higgs pole masses

FlexibleSUSY calculates the two CP-even Higgs pole masses $M_h$ and $M_H$ by diagonalizing the loop-corrected mass matrix

$$M = M^\text{tree} + M^{1L}(p^2) + M^{2L} + M^{3L}$$

(29)

at the momenta $p^2 = M_h^2$ and $p^2 = M_H^2$, respectively ($M^{2L}$ and $M^{3L}$ are evaluated at $p^2 = 0$). The one-loop correction $M^{1L}(p^2)$ contains the full one-loop MSSM Higgs self energy and tadpole contributions, including electroweak corrections and the momentum dependence. The two-loop correction $M^{2L}$ contains the known corrections of order $\mathcal{O}(\alpha_s^2 + \alpha_b^2)$ [12–16]. The three-loop correction $M^{3L}$ incorporates the terms of order $\mathcal{O}(\alpha_t \alpha_s^2 + \alpha_b \alpha_s^2)$ from the Himalaya package, as described in Section 2. In Eq. (29) all contributions are defined in the DR scheme by default. The renormalization scale is chosen to be $Q = \sqrt{m_t m_t}$ and the DR parameters which enter Eq. (29) are evolved to that scale by using the three-loop RGEs of the MSSM [57, 58]. Since the two CP-even Higgs pole masses are the output of the diagonalization of $M$ but at the same time must be inserted into $M^{1L}(p^2)$, an iteration over the momentum is performed for each mass eigenvalue until a fixed point for the Higgs masses is reached with sufficient precision.

4 Results

4.1 Size of three-loop contributions from different sources

In the DR calculation within FlexibleSUSY+Himalaya, there are three sources of contributions which affect the Higgs pole mass at order $\mathcal{O}(\alpha_t \alpha_s^2 + \alpha_b \alpha_s^2)$: The one-loop threshold correction $\mathcal{O}(\alpha_s)$ to the strong coupling constant, the two-loop threshold correction $\mathcal{O}(\alpha_t)$ to the top and bottom Yukawa couplings, and the genuine three-loop contribution to the Higgs mass matrix. In Figure 2, the impact of these three sources on the Higgs pole mass is shown relative to the two-loop calculation without these three corrections. The left panel shows the impact as a function of the SUSY scale $M_S$, and the right panel as a function of the relative stop mixing parameter $X_t/M_S$ for the scenario defined in Section 2.2.

First, we observe that the inclusion of the one-loop threshold correction to $\alpha_s$, Eq. (13), (blue dashed line) leads to a significant positive shift of the Higgs pole mass of around $+2.5$ GeV for $M_S \approx 1$ TeV. For larger SUSY scales the shift increases logarithmically as is to be expected from the logarithmic terms on the r.h.s. of Eq. (13). The inclusion of the full two-loop SQCD corrections to $y_t$ (green dash-dotted line) leads to a shift of similar magnitude, but in the opposite direction (the effect due to $y_b$ is negligible). Thus, there is a significant cancellation between the three-loop contributions from the one-loop

---

1 We do not distinguish between $\overline{\text{DR}}$ and $\overline{\text{MDR}}$ parameters here, and drop the hat over $\hat{M}$ introduced in Eq. (4) for simplicity.

2 FlexibleSUSY+Himalaya provides a flag to calculate the corrections of order $\mathcal{O}(\alpha_t(1+\alpha_s^2) + \alpha_b(1+\alpha_s^2))$ in the $\overline{\text{MDR}}$ scheme, as described in Section 2.2. See Appendix C for more details.
threshold correction to $\alpha_s$ and the two-loop SQCD corrections to $y_t$. The genuine three-loop contribution to the Higgs pole mass (black dotted line) is again positive and around $+2\text{ GeV}$ for $M_S \approx 1\text{ GeV}$. This is consistent with the findings of Ref. [1], of course. As a result, the sum of these three three-loop effects (red solid line) leads to a net positive shift of the Higgs mass relative to the two-loop result without all these corrections.

The size of the individual three-loop contributions depends on the stop mixing parameter $X_t/M_S$, as can be seen from the r.h.s. of Figure 2: between minimal ($X_t/M_S = 0$) and maximal stop mixing ($X_t/M_S \approx \sqrt{6}$) the size of the individual three-loop contributions changes by 1–2 GeV. For maximal (minimal) mixing, their impact is maximal (minimal). The direction of the shift is independent of $X_t/M_S$.

Note that the nominal two-loop result of the original FlexibleSUSY (i.e., without Himalaya) includes by default the one-loop threshold correction to $\alpha_s$ and the SM QCD two-loop contributions to the top Yukawa coupling [32, 33]. This means that the two-loop Higgs mass as evaluated by the original FlexibleSUSY already incorporates partial three-loop contributions. As a result, the two-loop result of the original FlexibleSUSY does not correspond to the zero-line in Figure 2, but is rather close to the blue dashed line. This implies that, compared to the two-loop result of the original FlexibleSUSY, the effect of the remaining $\alpha_t\alpha_s^2$ contributions in the Higgs mass prediction is negative.

4.2 Scale dependence of the three-loop Higgs pole mass

To estimate the size of the missing higher-order corrections, Figure 3 shows the renormalization scale dependence of the one-, two- and three-loop Higgs pole mass for the
scenario defined in Section 2.2 with \( \tan \beta = 5 \) and \( X_t = 0 \). The one- and two-loop calculations correspond to the original \texttt{FlexibleSUSY}. In the one-loop calculation the threshold corrections to \( \alpha_s \) and \( y_t \) are set to zero, and in the two-loop calculation the one-loop threshold corrections to \( \alpha_s \) and the two-loop QCD corrections to \( y_t \) are taken into account. The three-loop result of \texttt{FlexibleSUSY+Himalaya} includes all three-loop contributions at \((\alpha_t + \alpha_b)\alpha_s^2\) discussed above, i.e. the one-loop threshold correction to \( \alpha_s \), the full two-loop SQCD corrections to \( y_t, b \), and the genuine three-loop correction to the Higgs pole mass from \texttt{Himalaya}. In addition, the Higgs mass predicted at the two-loop level in the pure EFT calculation of \texttt{HSSUSY} is shown as the black dotted line, see Section 4.3. The bands show the corresponding variation of the Higgs pole mass when the renormalization scale is varied using the three-loop renormalization group equations [57–63] for all parameters except for the vacuum expectation values, where the \( \beta \)-functions are known only up to the two-loop level [64, 65]. In \texttt{FlexibleSUSY} and \texttt{FlexibleSUSY+Himalaya}, the renormalization scale is varied in the full MSSM within the interval \([M_S/2, 2M_S]\), while in \texttt{HSSUSY} it is varied in the Standard Model within the interval \([M_t/2, 2M_t]\), keeping the matching scale fixed at \( M_S \). The plot shows that the successive inclusion of higher-order corrections reduces the scale dependence, as expected. In particular, the three-loop corrections to the Higgs mass reduce the scale dependence by around a factor two, compared to the two-loop calculation. The scale dependence of \texttt{HSSUSY} is almost independent of \( M_S \), because scale variation is done within the SM after integrating out all SUSY particles at \( M_S \). Note that the variation of the renormalization scale only serves as an indicator of the theoretical uncertainty due to missing higher order effects.

\[ \text{Figure 3: Variation of the Higgs pole mass when the renormalization scale is varied} \]

\[ \text{by a factor two at which the Higgs pole mass is calculated, for} \tan \beta = 5 \text{ and } X_t = 0. \]
4.3 Comparison with lower order and EFT results

In Figures 4–5, we compare the three-loop calculation of \texttt{FlexibleSUSY+Himalaya} (red) with other MSSM spectrum generators. As input we use $M_t = 173.34$ GeV, $\alpha_{\text{em}}^{\text{SM}(5)}(M_Z) = 1/127.95$, $\alpha_s^{\text{SM}(5)}(M_Z) = 0.1184$ and $G_F = 1.1663787 \cdot 10^{-5}$ GeV$^{-2}$. All $\overline{\text{DR}}$ soft-breaking mass parameters as well as the $\mu$ parameter of the super-potential in the MSSM, and the running CP-odd Higgs mass are set equal to $M_S$. The running trilinear couplings, except for $A_t$, are chosen such that there is no sfermion mixing. The stop mixing parameter $X_t = A_t - \mu / \tan \beta$ is defined in the $\overline{\text{DR}}$ scheme and left as a free parameter. The lightest CP-even Higgs pole mass is calculated at the scale $Q = \sqrt{M_{\tilde{t}_1} M_{\tilde{t}_2}}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.pdf}
\caption{Comparison of Higgs mass predictions between two- and three-loop fixed-order programs and a two-loop EFT calculation as a function of the SUSY scale for $\tan \beta = 5$ and $X_t = 0$. In the left panel the absolute Higgs pole mass and in the right panel the difference w.r.t. the three-loop calculation is shown (FS=\texttt{FlexibleSUSY}, FS+H=\texttt{FlexibleSUSY+Himalaya}, FH=\texttt{FeynHiggs}).}
\end{figure}

\textbf{FlexibleSUSY 1.7.4.} The blue dashed line shows the original two-loop calculation with \texttt{FlexibleSUSY} 1.7.4\cite{42}. Note that, by construction of \texttt{FlexibleSUSY}, this result coincides exactly with the one of \texttt{SOFTSUSY} 3.5.1. As described above, it includes the one-loop threshold corrections to $\alpha_s$ and the two-loop QCD contributions to $y_t$, and it uses the three-loop RGEs of the MSSM \cite{57,58}. \texttt{FlexibleSUSY} 1.7.4 (and \texttt{SOFTSUSY}) use the explicit two-loop Higgs pole mass contribution of order $O(\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b)^2 + \alpha_s^2)$ of Refs.\cite{12–16}.

\textbf{HSSUSY 1.7.4.} The black dotted line has been obtained using the pure two-loop effective field theory (EFT) calculation of \texttt{HSSUSY} \cite{48}. \texttt{HSSUSY} is a spectrum generator from the \texttt{FlexibleSUSY} suite, which implements the two-loop threshold correction for the
quartic Higgs coupling of the Standard Model at $O(\alpha_t(\alpha_t + \alpha_s))$ when integrating out the SUSY particles at a common SUSY scale [46, 55]. Renormalization group running is performed down to the top mass scale using the three-loop RGEs of the Standard Model [59–63] and finally the Higgs mass is calculated at the two-loop level in the Standard Model at order $O(\alpha_t(\alpha_t + \alpha_s))$. In terms of the implemented corrections, HSSUSY is equivalent to SusyHD [46], and resums large logarithms up to NNLL level while neglecting terms of order $v^2/M_S^2$. The $O(v^2/M_S^2)$ corrections calculated in Ref. [66] have not been taken into account here.

**FeynHiggs 2.13.0-beta.** The green dash-dotted line shows the Higgs mass prediction using FeynHiggs 2.13.0-beta without large log resummation [9, 27–31].3 FeynHiggs 2.13.0-beta includes the two-loop contributions of order $O(\alpha_t\alpha_s + \alpha_t\alpha_b + \alpha_s^2 + \alpha_t^2)$. Consider first Figure 4. The left panel shows the Higgs mass prediction as a function of $M_S$ according to three codes discussed above, together with the FlexibleSUSY+Himalaya result (solid red). The stop mixing parameter $X_t$ is set to zero. The right panel shows the difference of these curves to the latter. Note that the resummed result of HSSUSY neglects terms of order $v^2/M_S^2$, and thus forfeits reliability towards lower values of $M_S$. The deviation from the fixed order curves below $M_S \approx 400$ GeV clearly underlines this.

---

3We use the SLHA input interface of FeynHiggs, which performs a conversion of the DR input parameters to the on-shell scheme. Resummation is disabled, as it would lead to an inconsistent result in combination with the DR to on-shell conversion of FeynHiggs [56]. We call FeynHiggs with the flags 4002020110.
contrast, the fixed order results start to suffer from large logarithmic contributions toward large $M_S$, which on the other hand are properly resummed in the HSSUSY approach. From Figure 4, we conclude that the fixed-order $\overline{\text{DR}}$ result loses its applicability once $M_S$ is larger than a few TeV, while the deviation between the non-resummed on-shell result of FeynHiggs and HSSUSY increases more rapidly above $M_S \approx 1$ TeV. Note that the good agreement of FlexibleSUSY with HSSUSY above the few-TeV region is accidental, as shown in Ref. [33].

The effect of the three-loop $\alpha_t \alpha_s^2$ terms on the fixed-order result is negative, as discussed in Section 4.1, and amounts to a few hundred MeV in the region where the fixed-order approach is appropriate. They significantly improve the agreement between the fixed-order and the resummed prediction for $M_h$ in the intermediate region of $M_S$, where both approaches are expected to be reliable. Between $M_S$ of about 500 GeV and 5 TeV, our three-loop curve from FlexibleSUSY+Himalaya deviates from the HSSUSY result by less than 300 MeV. This corroborates the compatibility of the two approaches in the intermediate region. Considering the current estimate of the theoretical uncertainty in the Higgs mass prediction [28, 33, 46, 55, 67], our observation even legitimates a naive switching between the fixed-order and the resummed approach at $M_S \approx 1$ TeV, instead of a more sophisticated matching procedure along the lines of Ref. [31, 56]. Nevertheless, the latter is clearly desirable through order $\alpha_t \alpha_s^2$, in particular in the light of the observations for non-zero stop mixing to be discussed below, but has to be deferred to future work at this point.

Figure 5 shows the three-loop effects as a function of $X_t$, where the value of $M_S = 2$ TeV is chosen to be inside the intermediate region. The figure shows that, for $|X_t| \lesssim 3 M_S$, the qualitative features of the discussion above are largely independent of the mixing parameter, whereupon the quantitative differences between the fixed-order and the resummed results are typically larger for non-zero stop mixing. Figure 6 underlines this by setting $X_t = -\sqrt{6} M_S$ and varying $M_S$. The kink in the three-loop curve originates from a change of the optimal hierarchy chosen by Himalaya. The red band shows the uncertainty $\delta_i$ as defined in Eq. (3), which is used to select the best fitting hierarchy. We find that $\delta_i$ is comparable to the size of the kink, which indicates a reliable treatment of the hierarchy selection criterion.

4.4 Comparison with other three-loop results

The three-loop $O(\alpha_t \alpha_s^2)$ corrections to the light MSSM Higgs mass discussed in this paper were originally implemented in the Mathematica code H3m. We checked that the implementation of the $\alpha_t$ and $\alpha_t \alpha_s$ terms in Himalaya leads to the same numerical results as in H3m, if the same set of $\overline{\text{DR}}$ parameters is used as input. Since the $\alpha_t \alpha_s^2$ terms of Himalaya are derived from their implementation in H3m, it is not surprising that they also result in the same numerical value if the same set of input parameters is given and the same mass hierarchy is selected. But since Himalaya has a slightly more sophisticated way of choosing this hierarchy (see Section 2.1), its numerical $\alpha_t \alpha_s^2$ contribution does occasionally differ slightly from the one of H3m.

In Figure 7 we compare our results to the three-loop calculation presented in Ref. [68],
assuming the input parameters for the “heavy sfermions” scenario defined in detail in the example folder of Ref. [69]. In the left panel the blue circles show the \( H3m \) result, including only the terms of \( \mathcal{O}(\alpha_t + \alpha_t \alpha_s + \alpha_t \alpha_s^2) \), where the MSSM DR top mass is calculated using the “running and decoupling” procedure described in Ref. [68]. The black crosses show the same result, except that the DR top mass at the SUSY scale is taken from the spectrum generator FlexibleSUSY+Himalaya. We can reproduce the latter result with FlexibleSUSY+Himalaya if we take the same terms into account, i.e., \( \mathcal{O}(\alpha_t + \alpha_t \alpha_s + \alpha_t \alpha_s^2) \); see the dotted red line in Figure 7. The small differences between the two results are due to the fact that \( H3m \) works with on-shell electroweak parameters, while FlexibleSUSY+Himalaya uses DR parameters. The inclusion of all one-loop contributions to \( M_h \) and the momentum iteration reduces the Higgs mass by 4–6 GeV, as shown by the red dashed line. Including all two- and three-loop corrections which are available in FlexibleSUSY+Himalaya, i.e., \( \mathcal{O}(\alpha_t + \alpha_t \alpha_s + (\alpha_t + \alpha_b)^2 + \alpha_s^2 + (\alpha_t + \alpha_b)\alpha_s^2) \), further reduces the Higgs mass by up to 2 GeV, as shown by the red solid line.\(^4\) The right panel of Figure 7 shows again our one-, two-, and three-loop predictions obtained with FlexibleSUSY, FlexibleSUSY+Himalaya, as well as the EFT result of HSSUSY. Similar to Figure 4, we observe that the higher-order terms lower the predicted Higgs mass and bring it closer to the resummed result. A detailed comparison of FlexibleSUSY+Himalaya to a result where \( H3m \) is combined with the lower-order results of FeynHiggs is beyond the scope of this paper and left to a future publication.

\(^4\)By default all available two- and three-loop corrections are included in FlexibleSUSY+Himalaya.
Figure 7: Comparison of the lightest Higgs pole mass calculated at the one-, two- and three-loop level with FlexibleSUSY, FlexibleSUSY+Himalaya, H3m and HSSUSY as a function of the SUSY scale for the “heavy sfermions” scenario of Ref.[68]. The horizontal orange band shows the measured Higgs mass $M_h = (125.09 \pm 0.32)$ GeV including its experimental uncertainty.

Figure 8 shows the lightest MSSM Higgs mass as obtained by FlexibleSUSY at one- and two-loop level, the FlexibleSUSY+Himalaya result, as well as the EFT prediction obtained with HSSUSY. The MSSM parameters are defined in the DR scheme and are chosen in the style of Ref.[70]: The soft-breaking mass parameters of the left- and right-handed stops are set equal at the SUSY scale $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, i.e. $m_{\tilde{t}_L}(M_S) = m_{\tilde{t}_R}(M_S)$. All other soft-breaking sfermion mass parameters are set to $m_{\tilde{f}}(M_S) = m_{\tilde{t}_L,R}(M_S) + 1$ TeV. Stop mixing is disabled, $X_t(M_S) = 0$, and the remaining trilinear couplings are set to zero at the scale $M_S$. The gaugino mass parameters, the super-potential $\mu$ parameter and the CP-odd DR Higgs mass are set to $M_1(M_S) = M_2(M_S) = M_3(M_S) = 1.5$ TeV, $\mu(M_S) = 200$ GeV and $m_A(M_S) = M_S$, respectively, and we fix $\tan \beta(M_Z) = 20$. As opposed to the results shown in Fig.1 of Ref.[70], we observe a reduction of $M_h$ towards higher loop orders, thus leading to the opposite conclusion of a heavy SUSY spectrum in this scenario, given the current experimental value for the Higgs mass. Reassuringly, the higher order corrections move the fixed-order result closer to the resummed result, leading to agreement between the two at the level of about 1 GeV even at comparatively large SUSY scales.

5The scenario of Ref.[70] appears to be not fully defined; in particular, $m_A$ and the sfermion mixing parameters other than $X_t$ remain unspecified.

6Note that, in contrast to Ref.[70], we are using a logarithmic scale in Figure 8.
Figure 8: Comparison of the lightest Higgs pole mass calculated at the one-, two- and three-loop level with FlexibleSUSY, FlexibleSUSY+Himalaya and HSSUSY as a function of the lightest stop pole mass for the benchmark point of Fig. 1 of Ref. [70]. The horizontal orange band shows the measured Higgs mass $M_h = (125.09 \pm 0.32) \, \text{GeV}$ including its experimental uncertainty. The bands around the calculated Higgs mass values show the parametric uncertainty from $M_t = (173.34 \pm 0.98) \, \text{GeV}$ and $\alpha_{\text{SM}}(5) = 0.1184 \pm 0.0006$.

5 Conclusions

We have presented the implementation Himalaya of the three-loop $O(\alpha_t \alpha_s^2 + \alpha_b \alpha_s^2)$ terms of Refs. [1, 20] for the light CP-even Higgs mass in the MSSM, and its combination with the DR spectrum generator framework FlexibleSUSY. These three-loop contributions have been available in the public program H3m before, where they were combined with the on-shell calculation of FeynHiggs. With the implementation into FlexibleSUSY presented here, we were able to study the size of the three-loop contributions within a pure DR environment. Despite the fact that the genuine $O(\alpha_t \alpha_s^2)$ corrections are positive [1], the combination with the two-loop decoupling terms in the top Yukawa coupling lead to an overall reduction of the Higgs mass prediction relative to the “original” two-loop FlexibleSUSY result by about 2 GeV, depending on the value of the stop masses and the stop mixing. This moves the fixed-order prediction for the Higgs mass significantly closer to the result obtained from a pure EFT calculation in the region where both approaches are expected to give sensible results. Contributions of order $O(\alpha_b \alpha_s^2)$ are found to be negligible in all scenarios studied here.

To indicate the remaining theory uncertainty due to higher order effects, we have varied the renormalization scale which enters the calculation by a factor two. The results show that the inclusion of the three-loop contributions reduces the scale uncertainty of the Higgs mass by around a factor two, compared to a calculation without the genuine three-
loop effects. We conclude that our implementation leads to an improved CP-even Higgs mass prediction relative to the two-loop results. Our implementation of the three-loop terms should be useful also for other groups that aim at a high-precision determination of the Higgs mass in SUSY models.

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A Installation of Himalaya

\texttt{Himalaya} can be downloaded as compressed package from [47]. After the package has been extracted, \texttt{Himalaya} can be configured and compiled by running

\begin{verbatim}
cd $HIMALAYA_PATH
mkdir build
cd build
make
\end{verbatim}

where \texttt{$HIMALAYA_PATH} is the path to the \texttt{Himalaya} directory. When the compilation has finished, the build directory will contain the \texttt{Himalaya} library \texttt{libHimalaya.a}. For convenience, a library named \texttt{libDSZ.a} is created in addition, which contains the two-loop $O(\alpha_t \alpha_s)$ corrections from Ref.[12].

B Installation of FlexibleSUSY with Himalaya

We provide a dedicated version of \texttt{FlexibleSUSY} 1.7.4, which uses \texttt{Himalaya} to calculate the Higgs pole mass at the three-loop level. This package contains three pre-generated MSSM models:

- **MSSMNoFVHimalaya**: This model represents the MSSM without (s)fermion flavour violation, where $\tan \beta$ is fixed at the scale $M_Z$ and the other SUSY parameters are fixed at a user-defined input scale. The parameters $\mu$ and $B\mu$ are fixed by the electroweak symmetry breaking conditions. The SUSY mass spectrum, including
the Higgs pole masses, is calculated at the scale \( Q = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \), where \( m_{\tilde{t}_i} \) are the two DR stop masses.

- **MSSMNoFVatMGUTHimalaya**: This is the same model as the MSSMNoFVHimalaya, except that the input scale is the GUT scale \( M_X \), defined to be the scale where \( g_1(M_X) = g_2(M_X) \).

- **NUHMSSMNoFVHimalaya**: This is the same model as the MSSMNoFVHimalaya, except that the soft-breaking Higgs mass parameters \( m_{H_u}^2 \) and \( m_{H_d}^2 \) are fixed by the electroweak symmetry breaking conditions.

The package **FlexibleSUSY-1.7.4-Himalaya.tar.gz** can be downloaded from Ref. [48]. To extract the package at the command line, run

```bash
tar -xf FlexibleSUSY-1.7.4-Himalaya.tar.gz
cd FlexibleSUSY-1.7.4-Himalaya/
```

After the extraction, **FlexibleSUSY** must be configured and compiled by running

```bash
./configure
   --with-himalaya-incdir=$HIMALAY_PATH/source/include
   --with-himalaya-libdir=$HIMALAY_PATH/build
make
```

See ./configure --help for more options. One can use make -j<N> to speed-up the compilation if \(<N>\) CPU cores are available. When the compilation has finished, the MSSM spectrum generators can be run from the command line as

```bash
models/MSSMNoFVHimalaya/run_MSSMNoFVHimalaya.x
   --slha-input-file=models/MSSMNoFVHimalaya/LesHouches.in.MSSMNoFVHimalaya
   --slha-output-file=LesHouches.out.MSSMNoFVHimalaya
```

The file `LesHouches.out.MSSMNoFVHimalaya` will then contain the SUSY particle spectrum in SLHA format. Alternatively, the **Mathematica** interface of **FlexibleSUSY** can be used:

```bash
math -run "<< \"models/MSSMNoFVHimalaya/run_MSSMNoFVHimalaya.m\""
```

For each model an example SLHA input file and an example **Mathematica** script can be found in `models/<model>/`.

### C Configuration options to calculate the Higgs mass at three-loop level with **FlexibleSUSY**

To calculate the CP-even Higgs pole masses at order \( \mathcal{O}(\alpha_t^2 \alpha_b^2 + \alpha_t \alpha_b^2) \) at the scale \( Q = M_S \), the top and bottom Yukawa couplings \( y_t(M_S) \) and \( y_b(M_S) \) as well as the strong cou-
pling constant $\alpha_s(M_S)$ must be extracted from the input parameters at the appropriate loop level.

**Strong coupling constant.** To calculate $M_h$ at the three-loop level at $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ correctly, $\alpha_s(M_S)$ must be extracted at the one-loop level from the input value $\alpha_s^{\text{SM}(5)}(M_Z)$ as described in Section 3.1. To achieve that in **FlexibleSUSY**, the global threshold correction loop order (EXTPAR[7]) must be set to 1 (or higher) and the specific threshold correction loop order for $\alpha_s$ (3rd digit from the right in EXTPAR[24]) must be set to 1 (or higher) in the SLHA input file. See the next paragraph for an example.

**Top and bottom Yukawa couplings.** **FlexibleSUSY** by default determines $y_t(M_Z)$ from the top pole mass at the full one-loop level including two-loop Standard Model QCD corrections, see Ref. [32]. The bottom Yukawa coupling $y_b(M_Z)$ is determined at the full one-loop level from the running bottom quark mass in the Standard Model with five active quark flavours, $m_b^{\text{SM}(5),\text{MS}}(m_b)$, where $\tan\beta$-enhanced higher order corrections are resummed. Both calculations are not sufficient for the calculation of $M_h$ at the three-loop level at $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$, because strong two-loop corrections from SUSY particles would be missing. For this reason, the complete two-loop strong corrections to the top and bottom Yukawa couplings of Refs. [51–54] have been implemented into **FlexibleSUSY**. They must be activated by setting the global threshold correction loop order (EXTPAR[7]) order to 2 and by setting the threshold correction loop order for $y_t$ and $y_b$ (7th and 8th digit from the right in EXTPAR[24]) to 2 in the SLHA input file:

```
Block FlexibleSUSY
  7 2 # threshold corrections loop order
  24 122111121 # individual threshold correction loop orders
```

In the **Mathematica** interface of **FlexibleSUSY** these two settings are controlled using the thresholdCorrectionsLoopOrder and thresholdCorrections symbols:

```mathematica
handle = FS<model>OpenHandle[
  fsSettings -> {
    thresholdCorrectionsLoopOrder -> 2,
    thresholdCorrections -> 122111121
  }
];
```

Here, `<model>` is the used **FlexibleSUSY** model from above, i.e. either MSSMNoFVHimalaya, MSSMNoFVatMGUTHimalaya or NUHMSMNoFVHimalaya.

**Three-loop corrections to the CP-even Higgs mass.** To use the three-loop corrections of order $\mathcal{O}(\alpha_t\alpha_s^2 + \alpha_b\alpha_s^2)$ to the light CP-even Higgs mass in the MSSM from Refs. [1, 20], the pole mass and EWSB loop orders must be set to 3 in the SLHA input.
file. In addition, the individual three-loop corrections should be switched on, by setting the flags 26 and 27 to 1. The user can select between the DR and MDR scheme for the three-loop corrections by setting the flag 25 to 0 or 1, respectively:

\begin{verbatim}
Block FlexibleSUSY
  4 3 # pole mass loop order
  5 3 # EWSB loop order
  25 0 # ren. scheme for Higgs 3L corrections (0 = DR, 1 = MDR)
  26 1 # Higgs 3-loop corrections O(alpha_t alpha_s^2)
  27 1 # Higgs 3-loop corrections O(alpha_b alpha_s^2)
\end{verbatim}

In the Mathematica interface of FlexibleSUSY the pole mass and EWSB loop orders are controlled using the poleMassLoopOrder and ewsbLoopOrder symbols, respectively. The individual three-loop corrections can be switched on/off by using the higgs3loopCorrectionAtAsAs and higgs3loopCorrectionAbAsAs symbols. The renormalization scheme is controlled by higgs3loopCorrectionRenScheme. The above shown SLHA input settings read in FlexibleSUSY’s Mathematica interface:

\begin{verbatim}
handle = FS<model>OpenHandle[
  fsSettings -> {
    poleMassLoopOrder -> 3,
    ewsbLoopOrder -> 3,
    higgs3loopCorrectionRenScheme -> 0,
    higgs3loopCorrectionAtAsAs -> 1,
    higgs3loopCorrectionAbAsAs -> 1
  }
];
...]
\end{verbatim}

Three-loop renormalization group equations. Optionally, the known three-loop renormalization group equations can be used to evolve the MSSM DR parameters from $M_Z$ to $M_S$ \cite{57, 58}. To activate the three-loop RGEs, the $\beta$ function loop order must be set to 3 in the SLHA input file:

\begin{verbatim}
Block FlexibleSUSY
  6 3 # beta-functions loop order
\end{verbatim}

In the Mathematica interface of FlexibleSUSY the $\beta$ function loop order is controlled using the betaFunctionLoopOrder symbol:

\begin{verbatim}
handle = FS<model>OpenHandle[
  fsSettings -> {
    betaFunctionLoopOrder -> 3
  }
];
...]
\end{verbatim}
**Recommended configuration options for FlexibleSUSY+Himalaya.** We recommend to run FlexibleSUSY+Himalaya with the following SLHA configuration options:

```
Block FlexibleSUSY
4 3 # pole mass loop order
5 3 # EWSB loop order
6 3 # beta-function loop order
7 2 # threshold corrections loop order
24 122111121 # individual threshold correction loop orders
25 0 # ren. scheme for 3L corrections (0 = DR, 1 = MDR)
26 1 # Higgs 3-loop corrections O(\alpha_t \alpha_s^2)
27 1 # Higgs 3-loop corrections O(\alpha_b \alpha_s^2)
```

At the **Mathematica** level we recommend to use:

```
handle = FS<model>OpenHandle[
  fsSettings -> {
    poleMassLoopOrder -> 3,
    ewsbLoopOrder -> 3,
    betaFunctionLoopOrder -> 3,
    thresholdCorrectionsLoopOrder -> 2,
    thresholdCorrections -> 122111121,
    higgs3loopCorrectionRenScheme -> 0,
    higgs3loopCorrectionAtAsAs -> 1,
    higgs3loopCorrectionAbAsAs -> 1
  }
];
```

**D Himalaya interface**

**Input parameters.** To calculate the three-loop corrections to the light CP-even Higgs pole mass at order \(O(\alpha_t \alpha_s^2 + \alpha_b \alpha_s^2)\) with **Himalaya**, the set of **DR** parameters is needed, which is shown in the following code snippet. The parameters are stored in the `Parameters` struct which contains the following members:

```
typedef Eigen::Matrix<double,2,1> V2;
typedef Eigen::Matrix<double,2,2> RM22;
typedef Eigen::Matrix<double,3,3> RM33;

struct Parameters {
  // DR-bar parameters
  double scale(); // renormalization scale
  double mu();  // mu parameter
  double g3();  // gauge coupling g3 SU(3)
  double vd();  // VEV of down Higgs
  double vu();  // VEV of up Higgs
  RM33 mq2(RM33::Zero()); // soft-breaking squared left-handed squark
};
```
All these parameters are given at the scale stored in the scale variable, which is typically the SUSY scale. The input values of the stop/sbottom masses and their associated mixing angle are optional, so their default value is set to \( \text{nan} \) (\( \text{std::numeric_limits<T>::quiet_NaN() } \)). If no input is provided, the DR stop masses will be calculated by diagonalizing the stop mass matrix

\[
\mathcal{M}_t = \begin{pmatrix}
(m_{\tilde{Q}}^2)_{33} + m_t^2 + g_t M_Z^2 c_2 \beta & X_t \\
X_t^* & (m_{\tilde{u}}^2)_{33} + m_t^2 + Q_t s_W^2 M_Z^2 c_2 \beta
\end{pmatrix}
\]  

(30)

Here, \((m_{\tilde{Q}})_{33}\) is the left third generation scalar quark mass parameter, \(g_t = 1/2 - Q_t s_W^2\), \(X_t = m_t (A_t - \mu \cot \beta)\), \((m_{\tilde{u}})_{33}\) the right scalar top mass parameter, \(Q_t = 2/3\), \(s_W\) the sine of the weak mixing angle and \(c_2 \beta = \cos(2 \beta)\). The sbottom mass matrix is obtained by replacing \(t \rightarrow b\) and \(\tilde{u} \rightarrow \tilde{d}\) in (30) with \(g_b = -(1/2 + Q_b s_W^2)\), \(X_b = m_b (A_b - \mu \tan \beta)\) and \(Q_b = -1/3\).

**Calculation of the three-loop corrections.** All the functions which are required for the calculation of the three-loop corrections are implemented as methods of the class HierarchyCalculator.

In the context of Himalaya, the procedure described in Section 2 is implemented by the member function

```
HierarchyObject ho = HierarchyCalculator::calculateDMh3L(bool isAlphab, int mdrFlag);
```
Here, the integer \( \text{mdrFlag} \) is optional and can be used to switch between the \( \overline{\text{DR}} \)- (0) and the \( \text{MDR} \)-scheme (1). The \( \overline{\text{DR}} \)-scheme is chosen as default. The returned object holds all information of the hierarchy selection process, such as the best fitting hierarchy, or the relative error \( \delta_{i_0}^{2L}/M_h^{\text{DSZ}} \), where \( \delta_{i_0}^{2L} \) is defined in Eq. (1), and \( i_0 \) denotes the “optimal” hierarchy as determined by the procedure of Section 2.1. The latter represents a lower limit on the expected accuracy of the expansion by comparison to the exact two-loop result \( M_h^{\text{DSZ}} \). In addition to that, the \texttt{HierarchyObject} offers a set of member functions which provide access to all intermediate results. These functions are summarized in Table 1. The selection method described in Section 2 is also applied to the (s)bottom contributions by replacing \( t \to b \), so that only terms of order \( \mathcal{O}(\alpha_b \alpha_s) \) are considered in the comparison. By setting the Boolean parameter \( \text{isAlphab} \) to \texttt{false} (true) the \texttt{calculateDMh3L} function returns the \texttt{HierarchyObject} for the loop corrections proportional to \( \alpha_t \) (\( \alpha_b \)).

Example: Function calls for the benchmark point SPS2:

```cpp
#include "HierarchyCalculator.hpp"
#include "HierarchyObject.hpp"

h3m::Parameters setupSPS2 ()
{
    h3m::Parameters pars;
    // Further code...
}
```

### Table 1: Description of the member functions of the \texttt{HierarchyObject} class.

| Function name                  | Returned value                                                                 |
|-------------------------------|-------------------------------------------------------------------------------|
| getIsAlphab()                 | Returns the bool \( \text{isAlphab} \).                                    |
| getSuitableHierarchy()        | Returns the suitable hierarchy as an int.                                   |
| getAbsDiff2L()                | Returns the double \( \delta_{i_0}^{2L} \) for the suitable hierarchy.      |
| getRelDiff2L()                | Returns the double \( \delta_{i_0}^{2L}/M_h^{\text{DSZ}} \) for the suitable hierarchy. |
| getExpUncertainty(int loops)  | Returns the uncertainty of the expansion at the given loop order (cf. Section 2.1). |
| getDMh(int loops)             | Returns the Higgs mass matrix proportional to \( \alpha_t \) or \( \alpha_b \) at the given loop order. Note that at the two-loop level only corrections of order \( \mathcal{O}(\alpha_t \alpha_s) \) are considered. |
| getDRToMDRShift()             | Returns the loop correction to the Higgs mass matrix to convert from the \( \overline{\text{DR}} \) to \( \text{MDR} \) scheme, according to Eq. (5). The \( \text{MDR} \) corrections are of order \( \mathcal{O}(\alpha_s + \alpha_s^2) \) by convention. |
| getMDRMasses()                | Returns the vector of \( \text{MDR} \) masses \( \{\hat{m}_{t,1},\hat{m}_{t,2}\} \) \( \{\hat{m}_{b,1},\hat{m}_{b,2}\} \), if \( \text{isAlphab} \) is \texttt{false} (true). |

...
pars.scale = 1.11090135E+03;
pars.mu = 3.73337018E+02;
pars.g3 = 1.06187116E+00;
pars.vd = 2.51008404E+01;
pars.vu = 2.41869332E+02;
pars.mq2 << 2.36646981E+06, 0, 0, 0, 0, 1.63230152E+06;
pars.md2 << 2.35612778E+06, 0, 0, 0, 0, 2.31917415E+06;
pars.mu2 << 2.35685097E+06, 0, 0, 0, 0, 9.05923409E+05;
pars.Ab = -784.3356416708631;
pars.At = -527.8746242245387;
pars.MA = 1.48446235E+03;
pars.MG = 6.69045022E+02;
pars.MW = 8.04001915E+01;
pars.MZ = 8.97603070E+01;
pars.Mt = 1.47685846E+02;
pars.Mb = 2.38918959E+00;
pars.MSt << 9.57566721E+02, 1.28878643E+03;
pars.MSb << 1.27884964E+03, 1.52314587E+03;
pars.s2t = sin(2*asin(1.13197339E-01));
pars.s2b = sin(2*asin(-9.99883015E-01));
return pars;
}

int main() {
    h3m::HierarchyCalculator hc(setupSPS2());
    // get the HierarchyObject with entries proportional to alpha_t
    // in the DR scheme
    auto hoTop = hc.calculateDMh3L(false);

    // get the 3-loop correction O(alpha_t * alpha_s^2)
    auto DMh_top_3L = hoTop.getDMh(3);
}

Estimation of the uncertainty of the expansion. In addition to the relative error
of the hierarchy choice \(\delta^{2L}_{i_0}/M_{DSZ}^2 \) (see above), we provide a member function which
returns a measure for the quality of convergence of the expansion at a given loop order,
given by \(\delta^{\text{conv}}_{i_0}\) defined in Eq. (2), where again \(i_0\) labels the “optimal” hierarchy. It can be
called with

```cpp
Eigen::Matrix2d HierarchyCalculator::getExpansionUncertainty(
    HierarchyObject ho, const Eigen::Matrix2d& massMatrix
```
int oneLoopFlag, int twoLoopFlag, int threeLoopFlag);

Its arguments are a `HierarchyObject`, the Higgs mass matrix `massMatrix` up to the loop order of interest, and three flags (`oneLoopFlag`, `twoLoopFlag`, `threeLoopFlag`) to define the desired loop orders. Using the member function `calculateDMh`, the returned `HierarchyObject` provides the user with the quantity $\delta^\text{conv}_b$ at two and three loops by default.

**Example:** For the benchmark point SPS2 one could estimate the uncertainty by calling

```cpp
... // get the HierarchyObject with entries proportional to alpha_t // in the DR scheme auto hoTop = hc.calculateDMh3L(false);

// get the expansion uncertainty for the // 3-loop correction $O(\alpha_t \cdot \alpha_s^{-2})$
auto expansionUncertainty3LTop = hoTop.getExpUncertainty(3);

// calculate the expansion uncertainty for the // 1-loop correction $O(\alpha_t)$
auto expansionUncertainty1LTop = hc.getExpansionUncertainty(hoTop, ho.getDMh(0), 1, 0, 0);
```

## References

[1] P. Kant, R. V. Harlander, L. Mihaila and M. Steinhauser, *Light MSSM Higgs boson mass to three-loop accuracy*, JHEP **08** (2010) 104, [1005.5709].

[2] ATLAS, CMS collaboration, G. Aad et al., *Combined measurement of the Higgs boson mass in $pp$ collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS experiments*, Phys. Rev. Lett. **114** (2015) 191803, [1503.07589].

[3] H. E. Haber and R. Hempfling, *Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than $m_Z$?*, Phys. Rev. Lett. **66** (1991) 1815–1818.

[4] J. R. Ellis, G. Ridolfi and F. Zwirner, *Radiative corrections to the masses of supersymmetric Higgs bosons*, Phys. Lett. **B257** (1991) 83–91.

[5] S. Heinemeyer, W. Hollik and G. Weiglein, *QCD corrections to the masses of the neutral CP-even Higgs bosons in the MSSM*, Phys. Rev. **D58** (1998) 091701, [hep-ph/9803277].

[6] The ATLAS collaboration, *Search for a scalar partner of the top quark in the jets+$E_T^{\text{miss}}$ final state at $\sqrt{s} = 13$ TeV with the ATLAS detector*, ATLAS-CONF-2017-020.
The ATLAS collaboration, *Search for direct top squark pair production in events with a Higgs or Z boson, and missing transverse momentum in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector*, ATLAS-CONF-2017-019.

S. Heinemeyer, W. Hollik and G. Weiglein, *Precise prediction for the mass of the lightest Higgs boson in the MSSM*, *Phys. Lett.* **B440** (1998) 296–304, [hep-ph/9807423].

S. Heinemeyer, W. Hollik and G. Weiglein, *The masses of the neutral CP-even Higgs bosons in the MSSM: Accurate analysis at the two loop level*, *Eur. Phys. J.* **C9** (1999) 343–366, [hep-ph/9812472].

R.-J. Zhang, *Two loop effective potential calculation of the lightest CP even Higgs boson mass in the MSSM*, *Phys. Lett.* **B447** (1999) 89–97, [hep-ph/9808299].

J. R. Espinosa and R.-J. Zhang, *MSSM lightest CP even Higgs boson mass to $O(\alpha_s \alpha_t)$: The Effective potential approach*, *JHEP* **03** (2000) 026, [hep-ph/9912236].

A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *On the neutral Higgs boson masses in the MSSM for arbitrary stop mixing*, *Nucl. Phys.* **B611** (2001) 403–422, [hep-ph/0105096].

J. R. Espinosa and R.-J. Zhang, *Complete two loop dominant corrections to the mass of the lightest CP even Higgs boson in the minimal supersymmetric standard model*, *Nucl. Phys.* **B586** (2000) 3–38, [hep-ph/0003246].

S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *High-precision predictions for the MSSM Higgs sector at $O(\alpha_t \alpha_s)$*, *Eur. Phys. J.* **C39** (2005) 465–481, [hep-ph/0411114].

S. P. Martin, *Complete two loop effective potential approximation to the lightest Higgs scalar boson mass in supersymmetry*, *Phys. Rev.* **D67** (2003) 095012, [hep-ph/0211366].
[20] R. V. Harlander, P. Kant, L. Mihaila and M. Steinhauser, *Higgs boson mass in supersymmetry to three loops*, *Phys. Rev. Lett.* **100** (2008) 191602, [0803.0672].

[21] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, *Momentum-dependent two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM*, *Eur. Phys. J.* **C74** (2014) 2994, [1404.7074].

[22] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, *Renormalization scheme dependence of the two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM*, *Eur. Phys. J.* **C75** (2015) 424, [1505.03133].

[23] G. Degrassi, S. Di Vita and P. Slavich, *Two-loop QCD corrections to the MSSM Higgs masses beyond the effective-potential approximation*, *Eur. Phys. J.* **C75** (2015) 61, [1410.3432].

[24] J. S. Lee, A. Pilaftsis, M. Carena, S. Y. Choi, M. Drees, J. R. Ellis et al., *CPsuperH: A Computational tool for Higgs phenomenology in the minimal supersymmetric standard model with explicit CP violation*, *Comput. Phys. Commun.* **156** (2004) 283–317, [hep-ph/0307377].

[25] J. S. Lee, M. Carena, J. Ellis, A. Pilaftsis and C. E. M. Wagner, *CPsuperH2.0: an Improved Computational Tool for Higgs Phenomenology in the MSSM with Explicit CP Violation*, *Comput. Phys. Commun.* **180** (2009) 312–331, [0712.2360].

[26] J. S. Lee, M. Carena, J. Ellis, A. Pilaftsis and C. E. M. Wagner, *CPsuperH2.3: an Updated Tool for Phenomenology in the MSSM with Explicit CP Violation*, *Comput. Phys. Commun.* **184** (2013) 1220–1233, [1208.2212].

[27] S. Heinemeyer, W. Hollik and G. Weiglein, *FeynHiggs: A program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM*, *Comput. Phys. Commun.* **124** (2000) 76–89, [hep-ph/9812320].

[28] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Towards high precision predictions for the MSSM Higgs sector*, *Eur. Phys. J.* **C28** (2003) 133–143, [hep-ph/0212020].

[29] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *The Higgs Boson Masses and Mixings of the Complex MSSM in the Feynman-Diagrammatic Approach*, *JHEP* **02** (2007) 047, [hep-ph/0611326].

[30] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *High-precision predictions for the light CP-even Higgs boson mass of the minimal supersymmetric Standard Model*, *Phys. Rev. Lett.* **112** (2014) 141801, [1312.4937].

[31] H. Bahl and W. Hollik, *Precise prediction for the light MSSM Higgs boson mass combining effective field theory and fixed-order calculations*, *Eur. Phys. J.* **C76** (2016) 499, [1608.01880].
[32] P. Athron, J.-h. Park, D. Stöckinger and A. Voigt, FlexibleSUSY — A spectrum generator generator for supersymmetric models, *Comput. Phys. Commun.* **190** (2015) 139–172, [1406.2319].

[33] P. Athron, J.-h. Park, T. Steudtner, D. Stöckinger and A. Voigt, Precise Higgs mass calculations in (non-)minimal supersymmetry at both high and low scales, *JHEP* **01** (2017) 079, [1609.00371].

[34] H. Baer, F. E. Paige, S. D. Protopopescu and X. Tata, Simulating Supersymmetry with ISAJET 7.0 / ISASUSY 1.0, in *Workshop on Physics at Current Accelerators and the Supercollider Argonne, Illinois, June 2-5, 1993*, pp. 0703–720, 1993. [hep-ph/9305342].

[35] G. Lee and C. E. M. Wagner, Higgs bosons in heavy supersymmetry with an intermediate m_A, *Phys. Rev.* **D92** (2015) 075032, [1508.00576].

[36] W. Porod, SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+ e- colliders, *Comput. Phys. Commun.* **153** (2003) 275–315, [hep-ph/0301101].

[37] F. Staub, From Superpotential to Model Files for FeynArts and CalcHep/CompHep, *Comput. Phys. Commun.* **181** (2010) 1077–1086, [0909.2863].

[38] W. Porod and F. Staub, SPheno 3.1: Extensions including flavour, CP-phases and models beyond the MSSM, *Comput. Phys. Commun.* **183** (2012) 2458–2469, [1104.1573].

[39] F. Staub, Automatic Calculation of supersymmetric Renormalization Group Equations and Self Energies, *Comput. Phys. Commun.* **182** (2011) 808–833, [1002.0840].

[40] F. Staub, SARAH 3.2: Dirac Gauginos, UFO output, and more, *Comput. Phys. Commun.* **184** (2013) 1792–1809, [1207.0906].

[41] F. Staub, SARAH 4 : A tool for (not only SUSY) model builders, *Comput. Phys. Commun.* **185** (2014) 1773–1790, [1309.7223].

[42] F. Staub and W. Porod, Improved predictions for intermediate and heavy Supersymmetry in the MSSM and beyond, [1703.03267].

[43] B. C. Allanach, SOFTSUSY: a program for calculating supersymmetric spectra, *Comput. Phys. Commun.* **143** (2002) 305–331, [hep-ph/0104145].

[44] B. C. Allanach, A. Bednyakov and R. Ruiz de Austri, Higher order corrections and unification in the minimal supersymmetric standard model: SOFTSUSY3.5, *Comput. Phys. Commun.* **189** (2015) 192–206, [1407.6130].
[45] A. Djouadi, J.-L. Kneur and G. Moultaka, *SuSpect: A Fortran code for the supersymmetric and Higgs particle spectrum in the MSSM*, Comput. Phys. Commun. **176** (2007) 426–455, [hep-ph/0211331].

[46] J. Pardo Vega and G. Villadoro, *SusyHD: Higgs mass Determination in Supersymmetry*, JHEP **07** (2015) 159, [1504.05200].

[47] https://github.com/Himalaya-Library/Himalaya or https://www.particle-theory.rwth-aachen.de/cms/Particle-Theory/Forschung/~gmuw/Publikationen/.

[48] https://flexiblesusy.hepforge.org.

[49] A. Pak, M. Steinhauser and N. Zerf, *Supersymmetric next-to-next-to-leading order corrections to Higgs boson production in gluon fusion*, JHEP **09** (2012) 118, [1208.1588].

[50] D. M. Pierce, J. A. Bagger, K. T. Matchev and R.-j. Zhang, *Precision corrections in the minimal supersymmetric standard model*, Nucl. Phys. **B491** (1997) 3–67, [hep-ph/9606211].

[51] A. Bednyakov, A. Onishchenko, V. Velizhanin and O. Veretin, *Two loop O(α_s^2) MSSM corrections to the pole masses of heavy quarks*, Eur. Phys. J. **C29** (2003) 87–101, [hep-ph/0210258].

[52] A. Bednyakov, D. I. Kazakov and A. Sheplyakov, *On the two-loop O(α_s^2) corrections to the pole mass of the t-quark in the MSSM*, Phys. Atom. Nucl. **71** (2008) 343–350, [hep-ph/0507139].

[53] A. V. Bednyakov, *Running mass of the b-quark in QCD and SUSY QCD*, Int. J. Mod. Phys. **A22** (2007) 5245–5277, [0707.0650].

[54] A. Bauer, L. Mihaila and J. Salomon, *Matching coefficients for α_s and m_b to O(α_s^2) in the MSSM*, JHEP **02** (2009) 037, [0810.5101].

[55] E. Bagnaschi, G. F. Giudice, P. Slavich and A. Strumia, *Higgs Mass and Unnatural Supersymmetry*, JHEP **09** (2014) 092, [1407.4081].

[56] H. Bahl, S. Heinemeyer, W. Hollik and G. Weiglein, *Reconciling EFT and hybrid calculations of the light MSSM Higgs-boson mass*, 1706.00346.

[57] I. Jack, D. R. T. Jones and A. F. Kord, *Three loop soft running, benchmark points and semiperturbative unification*, Phys. Lett. **B579** (2004) 180–188, [hep-ph/0308231].

[58] I. Jack, D. R. T. Jones and A. F. Kord, *Snowmass benchmark points and three-loop running*, Annals Phys. **316** (2005) 213–233, [hep-ph/0408128].
[59] L. N. Mihaila, J. Salomon and M. Steinhauser, *Gauge coupling beta functions in the Standard Model to three loops*, *Phys. Rev. Lett.* **108** (2012) 151602, [1201.5868].

[60] A. V. Bednyakov, A. F. Pikelner and V. N. Velizhanin, *Anomalous dimensions of gauge fields and gauge coupling beta-functions in the Standard Model at three loops*, *JHEP* **01** (2013) 017, [1210.6873].

[61] A. V. Bednyakov, A. F. Pikelner and V. N. Velizhanin, *Yukawa coupling beta-functions in the Standard Model at three loops*, *Phys. Lett.* **B722** (2013) 336–340, [1212.6829].

[62] K. G. Chetyrkin and M. F. Zoller, *Three-loop β-functions for top-Yukawa and the Higgs self-interaction in the Standard Model*, *JHEP* **06** (2012) 033, [1205.2892].

[63] A. V. Bednyakov, A. F. Pikelner and V. N. Velizhanin, *Higgs self-coupling beta-function in the Standard Model at three loops*, *Nucl. Phys.* **B875** (2013) 552–565, [1303.4364].

[64] M. Sperling, D. Stöckinger and A. Voigt, *Renormalization of vacuum expectation values in spontaneously broken gauge theories*, *JHEP* **07** (2013) 132, [1305.1548].

[65] M. Sperling, D. Stöckinger and A. Voigt, *Renormalization of vacuum expectation values in spontaneously broken gauge theories: Two-loop results*, *JHEP* **01** (2014) 068, [1310.7629].

[66] E. Bagnaschi, J. Pardo Vega and P. Slavich, *Improved determination of the Higgs mass in the MSSM with heavy superpartners*, *Eur. Phys. J.* **C77** (2017) 334, [1703.08166].

[67] B. C. Allanach, A. Djouadi, J. L. Kneur, W. Porod and P. Slavich, *Precise determination of the neutral Higgs boson masses in the MSSM*, *JHEP* **09** (2004) 044, [hep-ph/0406166].

[68] D. Kunz, L. Mihaila and N. Zerf, *O(α_s^2) corrections to the running top-Yukawa coupling and the mass of the lightest Higgs boson in the MSSM*, *JHEP* **12** (2014) 136, [1409.2297].

[69] https://www.ttp.kit.edu/Progdata/ttp10/ttp10-23/H3m-v1.3/.

[70] J. L. Feng, P. Kant, S. Profumo and D. Sanford, *Three-Loop Corrections to the Higgs Boson Mass and Implications for Supersymmetry at the LHC*, *Phys. Rev. Lett.* **111** (2013) 131802, [1306.2318].