Nomogram of Effective Length Coefficient of Frame Column

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Abstract. In this paper, the method of making a nomogram of the length calculation coefficient of the frame column is elaborated. There are two types of frame columns: no sideways and sideways. Therefore, there are two types of corresponding calculation lengths. In the common literature, the calculated length factor $\mu$ is made into a table. However, it is difficult to list all the situations, and making a nomogram can overcome this shortcoming. A nomogram can represent all the changes of $\mu$ with the constraint stiffness and the usage is simple and convenient. It only needs to select the point to connect the straight line, and the obtained $\mu$ value precision also meets the requirements of actual engineering calculation.

1. Introduction

In calculating the stability of the frame, the calculated length $l_0$ of the frame column is first determined, which is determined by the coefficient $\mu$. Calculating the length factor can be calculated by determining the elastic stability critical load of the frame column. The tables and their calculation formulas for calculating the length coefficients of the two types are given in Appendix E.0.1 and E.0.2 of China's latest “Steel Structure Design Standard” (GB 50017-2017). In this paper, we will use the approximate algebraic equations with no lateral shift and lateral displacement to calculate the length coefficient to derive and make a nomogram [1].

2. The length coefficient $\mu$ of the frame column without sideways

2.1. Derivation of point equation

The approximate expression of the $\mu$ coefficient without the side-shifting frame column is [2]:

$$
\mu = \frac{3 + 1.4(k_1 + k_2) + 0.64k_1k_2}{3 + 2(k_1 + k_2) + 1.28k_1k_2}
$$

Among them, $k_1 = 1/G_A$, $k_2 = 1/G_B$, $G_A$ and $G_B$ are the line stiffness of the two points of A and B and the ratio of the line stiffness of the beam. Then equation (2.1) can be written as:

$$
\mu = \frac{3G_AG_B + 1.4(G_A + G_B) + 0.64}{3G_AG_B + 2(G_A + G_B) + 1.28}
$$

When $G_A = G_B$, the logarithm of both sides of equation (2.2) can be obtained:

$$
\ln(\mu) = \frac{1}{2}\ln\left(\frac{3G_A^2 + 2.8G_A + 0.64}{3G_A^2 + 4G_A + 1.28}\right) + \frac{1}{2}\ln\left(\frac{3G_B^2 + 2.8G_B + 0.64}{3G_B^2 + 4G_B + 1.28}\right)
$$

$$
= U + V
$$

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From equation (2-3), we can get the following formula:

\[
\begin{align*}
\frac{1}{2} \ln \left( \frac{3G_x^2 + 2.8G_x + 0.64}{3G_x^2 + 4G_x + 1.28} \right) - U &= 0 \\
\frac{1}{2} \ln \left( \frac{3G_y^2 + 2.8G_y + 0.64}{3G_y^2 + 4G_y + 1.28} \right) - V &= 0 \\
- \ln \mu + U + V &= 0
\end{align*}
\]  \tag{2-4}

Thus the original algebraic equation (2-1) is transformed into three point equations in parallel coordinates, and each point equation contains only one variable \(G_x\), \(G_y\) and \(\mu\). Further, the determinant of coefficient can be obtained by the equation (2-4). If the determinant result is zero, this means that the three point equations transformed from the original algebraic equation are collinear, so equation (2-1) can draw a nomogram [3].

2.2. Drawing method of nomogram

The three point equations derived from the original algebraic equation in 2.1 are indeed collinear. In this case, only the images of the three point equations in parallel coordinates are drawn, and the three point equations are represented by scales. Then, the nomogram showing the original algebraic equation is also successfully produced. Therefore, the process of making a nomogram is essentially the process of drawing a point equation image in parallel coordinates. When drawing a nomogram, draw three point equation curves with scales, and use the coordinates in the Cartesian coordinate system to calculate the formula "P"[4]. If there is a point equation in parallel coordinates: \(f + g \cdot u + h \cdot v = 0\), its coordinates at the "P" point in Cartesian coordinates are: \(x = h/k \cdot (g + h)\), \(y = -f / (g + h)\).

When plotting the trajectory of a point equation motion point in parallel coordinates, it is exactly the same as drawing a curve graph in Cartesian coordinates [5]. First, the scales of the \(u\) and \(v\) axes should be determined. Therefore, we assume that the unit length of the \(U\) axis in parallel coordinates is \(m_u\), and the unit length of the \(V\) axis is \(m_v\). When the values of the \(U\) and \(V\) axes are \([u]\) and \([v]\), respectively, their corresponding lengths are: \(u_i = m_u \cdot i\), \(v_i = m_v \cdot i\). In the formula, \(m_u\) and \(m_v\) are called the scale modulus of the scale, and the unit is (mm). Substituting \(u_i\) and \(v_i\) into (2-4) gives the following equation:

\[
\begin{align*}
\frac{1}{2} \frac{m_u}{m_1} \ln \left( \frac{3G_x^2 + 2.8G_x + 0.64}{3G_x^2 + 4G_x + 1.28} \right) - U_i &= 0 \\
\frac{1}{2} \frac{m_v}{m_2} \ln \left( \frac{3G_y^2 + 2.8G_y + 0.64}{3G_y^2 + 4G_y + 1.28} \right) - V_i &= 0 \\
-m \cdot m_v \ln \mu + \frac{1}{2} \frac{m_u}{m_1} \ln \left( \frac{3G_x^2 + 2.8G_x + 0.64}{3G_x^2 + 4G_x + 1.28} \right) + \frac{1}{2} \frac{m_v}{m_2} \ln \left( \frac{3G_y^2 + 2.8G_y + 0.64}{3G_y^2 + 4G_y + 1.28} \right) &= 0
\end{align*}
\]  \tag{2-5}

Then according to the coordinate calculation formula of the "P" point, the following three formulas are obtained:

\[
\begin{align*}
x_i &= 0 \\
y_i &= \frac{1}{2} \frac{m_u}{m_1} \ln \left( \frac{3G_x^2 + 2.8G_x + 0.64}{3G_x^2 + 4G_x + 1.28} \right) \\
x_2 &= k \\
y_2 &= \frac{1}{2} \frac{m_v}{m_2} \ln \left( \frac{3G_y^2 + 2.8G_y + 0.64}{3G_y^2 + 4G_y + 1.28} \right) \\
x_3 &= \frac{m_u \cdot m_v \cdot k}{m_1 + m_2} \\
y_3 &= \frac{m_u \cdot m_v}{m_1 + m_2} \ln \mu
\end{align*}
\]

These three calculation formulas, because they all have the modulus of the scale, can determine the corresponding \((x_i, y_i)\) value at a certain modulus, and this calculation formula is also called the scale function. It can be seen from the three ruler functions: \(x_i = 0\), the \(G_x\) scale coincides with the \(U\) axis; \(x_2 = k\), and the \(G_y\) scale coincides with the \(V\) axis[5]. When the proportional modulus is positive,
there is $x_1 < x_3 < x_2$, so the $\mu$ axis is between $G_A$ and $G_B$. Therefore, the layout of the nomogram is as follows:

![Figure 1. Nomogram layout.](image)

3. There is a length coefficient $\mu$ of the side frame column

3.1. Derivation of point equation

The approximate expression of the $\mu$ coefficient of the side-shift frame column is [2]:

$$\mu = \sqrt{\frac{1.6 + 4(k_1 + k_2) + 7.5k_1k_2}{k_1 + k_2 + 7.5k_1k_2}}$$  \hspace{1cm} (3-1)

Where $k_1 = 1/G_A$, $k_2 = 1/G_B$. Square the equation to obtain the following formula:

$$\mu^2 = \frac{1.6G_A G_B}{G_A + G_B + 7.5} = \frac{1}{2} \left( \frac{4G_A + G_B + 7.5}{G_A + G_B + 7.5} + \frac{1}{2} \left( \frac{4G_A + G_B + 7.5}{G_A + G_B + 7.5} \right) \right)$$  \hspace{1cm} (3-2)

If $G = G_A = G_B$, then equation (3-2) can be written as:

$$\mu^2 = \frac{1.6G^2}{2G + 7.5} = \frac{1}{2} \left( \frac{8G_A + 7.5}{2G_A + 7.5} + \frac{1}{2} \left( \frac{8G_B + 7.5}{2G_B + 7.5} \right) \right)$$  \hspace{1cm} (3-3)

= $U + V$

Then, as with the method without side shift, we can finally get the calculation formula for each ruler as follows:

$$\begin{align*}
\begin{cases}
  x_1 = 0 \\
  y_1 = \frac{1}{2} m_1 \times \frac{8G_A + 7.5}{2G_A + 7.5}
\end{cases} \quad &
\begin{cases}
  x_2 = k \\
  y_2 = \frac{1}{2} m_2 \times \frac{8G_B + 7.5}{2G_B + 7.5}
\end{cases} \quad &
\begin{cases}
  x_3 = \frac{m_1}{m_1 + m_2} \times k \\
  y_3 = \frac{m_1}{m_1 + m_2} \times (\mu^2 - \frac{1.6G}{2G + 7.5})
\end{cases}
\end{align*}$$

4. Data calculation and drawing

In order to facilitate the calculation when drawing, you can use: $m_1 = m_2 = 30$ (mm); wheelbase $k = 10$ (mm); Therefore, the coordinates of each ruler are calculated as follows:

(1) The coordinate calculation formula without side shift:

$$\begin{align*}
\begin{cases}
  x_1 = 0 \\
  y_1 = 15 \ln \left( \frac{3G_A^2 + 2.8G_A + 0.64}{3G_A^2 + 4G_A + 1.28} \right)
\end{cases} &
\begin{cases}
  x_2 = 10 \\
  y_2 = 15 \ln \left( \frac{3G_B^2 + 2.8G_B + 0.64}{3G_B^2 + 4G_B + 1.28} \right)
\end{cases} &
\begin{cases}
  x_3 = 5 \\
  y_3 = 15 \ln \mu
\end{cases}
\end{align*}$$

(2) The coordinate calculation formula with side shift:

$$\begin{align*}
\begin{cases}
  x_1 = 0 \\
  y_1 = 15 \times \frac{8G_A + 7.5}{2G_A + 7.5}
\end{cases} &
\begin{cases}
  x_2 = 10 \\
  y_2 = 15 \times \frac{8G_B + 7.5}{2G_B + 7.5}
\end{cases} &
\begin{cases}
  x_3 = 5 \\
  y_3 = 15 \times (\mu^2 - \frac{1.6G}{2G + 7.5})
\end{cases}
\end{align*}$$
According to the above coordinate calculation formula, the scale value corresponding to each scale can be calculated by Matlab software programming. Then use the Origin software to draw the \( \mu \) coefficient nomogram with no sideways and side-shifted frame columns as follows:

![Figure 2. \( \mu \) coefficient nomogram without side shift and side shift.](image)

5. Calculation examples

Arbitrarily select 10 points of \( G_A \) and \( G_B \), and compare the \( \mu \) value found by the nomogram with the actually calculated \( \mu \) value. The results are shown in the following table:

| \( G_A \) | \( G_B \) | No side shift | Side shift |
|-----------|-----------|--------------|------------|
| \( G_A \) | Calculated | \( \mu \) | \( \mu \) | Error | \( \mu \) | \( \mu \) | Error |
| \( 0 \)   | \( \infty \) | 0.7 | 0.7 | 0.000 | 0 | \( \infty \) | 2 | 2 | 0.000 |
| 0.1       | 50        | 0.736 | 0.74 | 0.005 | 1 | 50 | 2.232 | 2.21 | -0.001 |
| 0.3       | 10        | 0.779 | 0.78 | 0.001 | 2 | 30 | 2.421 | 2.41 | -0.005 |
| 0.6       | 5         | 0.814 | 0.82 | 0.007 | 3 | 20 | 2.532 | 2.54 | -0.003 |
| 0.9       | 1         | 0.771 | 0.77 | -0.001 | 4 | 10 | 2.435 | 2.43 | -0.002 |
| 1         | 0.1       | 0.656 | 0.65 | -0.009 | 5 | 9  | 2.51  | 2.5  | -0.004 |
| 5         | 0.2       | 0.744 | 0.75 | 0.008 | 10 | 8  | 2.853 | 2.85 | -0.001 |
| 10        | 0.3       | 0.779 | 0.78 | 0.001 | 50 | 2  | 2.512 | 2.51 | -0.001 |
| 50        | 0.4       | 0.809 | 0.81 | 0.001 | 100 | 1  | 2.295 | 2.29 | -0.002 |
| \( \infty \) | 0         | 0.7  | 0.7  | 0.000 | \( \infty \) | 0  | 2  | 2  | 0.000 |

It can be seen from the comparison of the data in the table that the \( \mu \) value found by the nomogram is very close to the actually calculated \( \mu \) value, and the difference value does not exceed.
± 0.01, so the nomogram production method described above is correct. In addition, the nomogram produced by this method has better precision and can be used for actual engineering calculation.

6. Conclusions
(1) The nomogram can represent the case where all μ values vary with the constraint stiffness.
(2) The nomogram can quickly find the value of μ, just take a point on each of the axes, and then connect the two points to draw a line to get the μ value in the intermediate axis.
(3) The nomogram value is compared with the actual calculated value, and the error does not exceed ±0.01, so the nomogram can be used to calculate the actual project.
(4) The nomogram production method described in this paper can also be used to make other nomograms that need to look up the table to know the result.

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