Examination of vehicle impact against stationary roadside objects

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Abstract. In this paper, a model of a car impact against a fixed roadside objects/safety barrier is presented and discussed to solve a mixed task: to determine the pre-impact velocity of the vehicle center of mass taking into consideration the recorded post-impact final rest position. The principle of conservation of mechanical energy was applied, whereas the energy of the rigid barrier deformation was estimated by the finite element method/FEM for an elasto-plastic body. FEM analysis was performed by integrating SolidWorks - SW Simulation software, the output data being the deformation of the safety barrier in the three dimensional space subjected to limited boundary conditions. The post-impact car motion was modelled by a multiscale spatial model, well established in modern accident reconstruction. The findings of the study point out the precision and accuracy of the crash pattern revealed in the accident reconstruction and essential in clarifying the situation with appropriate real facts. Two cases involving real-world head-on accident collisions against a fixed flexible barrier and a tank body are considered, which exemplify the adopted method.

1. Introduction
In statistical analysis reports of traffic accidents, automobiles hitting against semi-rigid barriers and flexible barriers get a particularly large share [1-3, 7-9, 11, 14]. The analysis involves several different stages of computational procedures. On the whole, the accident can be divided into three stages. The first phase is prior to the moment of impact, when, generally, the vehicle moves at a loss of longitudinal stability or the automobile diverges to the metal crash barrier due to other reasons, such as individual differences associated with driving behavior. The second phase is at the point of impact, when the vehicle contacts the crash barrier, resulting in deformation of the car body and deformation of the barrier. At this stage part of the kinetic energy inflicted upon the vehicle is transformed into deformation energy of the car body and deformation energy of the metal crash barrier. The third phase includes the post-impact vehicle motion, which is the result of the residual kinetic energy of the vehicle, continuing lateral motion with relative sliding to its final rest position.

The following study proposes a solution to a traffic road: to define part of the initial data / pre-impact velocity of the center of mass / under certain final conditions / the post-impact final rest position of the vehicle/.
2. Design of a vehicle crash model and dynamic impact against a metal crash barrier and a rigid barrier

The impact phase between the vehicle and the metal flexible barrier is considered to be a deformation phase between two bodies where the transformed kinetic energy before the impact turns into deformation energy and post-impact vehicle kinetic energy. The pre-impact vehicle motion is considered to be a translational motion. According to the impulse-momentum change theorem and the kinetic energy theorem the impact impulse and its momentum to the center of mass imparted to the body make the vehicle lose its longitudinal stability and continue its motion to a final rest position. This motion is assumed to be a motion of a car on one or several inclined planes with different adhesion characteristics. The kinetic energy after impact is determined on the basis of a multiscale mechanics and mathematical model.

\[ \frac{m_1 V_i^2}{2} = \frac{m_1 u_i^2}{2} + \frac{I_1 \omega_i^2}{2} + E_1 + E_2 \]  

where:
- \( m_1 \) – mass of vehicle
- \( I_1 \) – mass inertia for the vehicle around the vertical axis Oz
- \( V_i \) – vehicle velocity of the center of mass at the point of impact
- \( u_i \) – post-impact vehicle velocity of the center of mass
- \( \omega_i \) – post-impact angular velocity of the vehicle body around the vertical axis Oz
- \( E_1 \) – deformation energy of car body
- \( E_2 \) – deformation energy of metal crash barrier

Analysing crash tests with a car against a permanently fixed flexible barrier is a complex dynamic task, where the deformation energy applied to the barrier is usually taken into account. To estimate the necessary kinetic energy, too many components need to be known. The results obtained in the study are based on the implementation of a strategy based on the strain energy method.

3. Post-impact dynamic analysis of the vehicle motion

Undoubtedly, the proposed formula (2) is too general and does not cover the full dynamic analysis of the vehicle post-impact due to the fact that it does not count for the complexity of friction between tires and road surface, elasticity of suspension, damping and so on. Equation (1) clearly shows that not only the deformation energy of the two bodies, but also the post-impact vehicle velocity of the center of mass and the angular speed of the vehicle are exhibited. If we apply the center-of-mass motion theorem, it would be a rather inaccurate approach, for it would not be possible to estimate the kinetic energy for the rotation of the vehicle after the impact [13, 17, 20, 21].

A considerably more precise approach is to use a system of twelve differential equations that take into account friction forces, elasticity and suspension, damping forces, gravity force, air mass resistance, etc. [4, 10, 12, 22].

The automobile is considered to be a multi-mass system (Figure 1) and the initial conditions that meet the differential equations for vehicle motion determine the whole post-impact kinetic energy.
The analysis shows post-impact vehicle velocity of the center of mass "u" and the angular velocity of the vehicle after the impact \( \omega_z \).

The differential equations of motion for each vehicle after impact, considered as a multi-mass spatial mechanical system, have the type

\[
\begin{align*}
4 m \ddot{x}_c &= \sum_{i=1}^{4} \left[ F_{ix} \right] + m \cdot g \cdot \sin \alpha - W_{x} \cdot \sqrt{\dot{x}_c^2 + \dot{y}_c^2}, \\
4 m \ddot{y}_c &= \sum_{i=1}^{4} \left[ F_{iy} \right] + m \cdot g \cdot \sin \beta - W_{y} \cdot \sqrt{\dot{x}_c^2 + \dot{y}_c^2}, \\
4 m \ddot{z}_c &= \sum_{i=1}^{4} \left( N_i - \frac{mg}{\sqrt{1 + tg^2 \alpha + tg^2 \beta}} \right)
\end{align*}
\]

where the corresponding matrices are determined by the expressions

\[
\begin{align*}
\left[ J_{c_1} \right] + \left[ J_{c_3} \right] \cdot [\omega] &= \left[ M_{c_{m}} \right] + \left[ M_{c_{a_1}} \right] + \left[ M_{c_{a_2}} \right] + [M_{c_{a_3}}] + [M_{c_{a_4}}], \\
\left[ J_{y} \right] \cdot [\dot{y}] &= \left[ M_{y} \right], \\
F_{ir} &= \mu_{N_i} \frac{v_{p_i}}{v_{p_i}} \cos(\varphi_{z_1} + \theta_i) + \mu_{N_i} \frac{v_{p_i}}{v_{p_i}} \sin(\varphi_{z_1} + \theta_i),
\end{align*}
\]

where the corresponding matrices are determined by the expressions

\[
\begin{align*}
\left[ M_{c_{a_1}} \right] &= M_{N_{a_1}} + J_{x_{a_1}} \omega_{x_{a_1}} \omega_{y_{a_1}} - J_{y_{a_1}} \omega_{x_{a_1}} \omega_{z_{a_1}} + \left( J_{z_{a_1}} - J_{x_{a_1}} \right) \omega_{y_{a_1}} \omega_{z_{a_1}},
\end{align*}
\]

\[
\begin{align*}
\left[ M_{c_{a_3}} \right] &= M_{N_{a_3}} + J_{x_{a_3}} \omega_{x_{a_3}} \omega_{y_{a_3}} - J_{y_{a_3}} \omega_{x_{a_3}} \omega_{z_{a_3}} + \left( J_{z_{a_3}} - J_{x_{a_3}} \right) \omega_{y_{a_3}} \omega_{z_{a_3}},
\end{align*}
\]

\[
\begin{align*}
\left[ M_{c_{a_4}} \right] &= M_{N_{a_4}} + J_{x_{a_4}} \omega_{x_{a_4}} \omega_{y_{a_4}} - J_{y_{a_4}} \omega_{x_{a_4}} \omega_{z_{a_4}} + \left( J_{z_{a_4}} - J_{x_{a_4}} \right) \omega_{y_{a_4}} \omega_{z_{a_4}},
\end{align*}
\]
Here, $m$ is the total mass of the vehicle; $m_{v,i}/i = 1\ldots4$ - mass of each of the wheels; $m_{mi}/i = 1\ldots2$ - mass of each of the drives; $x_c, y_c, z_c$ - coordinates of the vehicle center of mass in relation to the fixed coordinate system; $\phi_c, \theta_c, \gamma_c$ - Euler angles of body’s corners (unsprung mass); $\bar{\phi}_c$ - average angle of rotation of the steering wheels around their axles; $\bar{\gamma}_c/i = 1\ldots4$ - angles of rotation of the wheels on their own rotary axis; $\bar{F}_i/i = 1\ldots4$ - friction forces in the wheels; $\alpha, \beta$ - road surface angles of the longitudinal and transverse slope; $N_i/i = 1\ldots4$ - normal reactions in the wheels; $w$ -
drag coefficient; $\vec{\omega}$ - angular velocity of the movable coordinate system $C_1'x_1'y_1'z_1'$ permanently connected to the unsprung mass; $[\vec{\omega}] = [\vec{\omega}_1' \ \vec{\omega}_y' \ \vec{\omega}_z']^T$ - matrix column of the derivatives of the projections of angular velocity on the permanently connected to the unsprung mass coordinate axes; $\vec{\omega}_2$ - angular velocity of the sprung mass and the constantly connected to it movable coordinate system $O_2x_2'y_2'z_2'$; $\omega_{knee}$ - angular velocity of the flywheel (if available); $M_{F.Nx}$, $M_{F.Ny}$, $M_{F.Nz}$ - moments of the frictional forces on the wheels and the normal reactions to the permanently connected to the vehicle coordinate axes; $[J_{C_1'}]$ - matrix of the mass inertia of the bodywork related to the coordinate axes, permanently connected to it; $[\vec{\omega}]$ - matrix column of the projections of angular velocity on the same axes determined by Euler’s formula; $I_{k_1y''_1}/I_{k_1z''_1}$ - mass inertia of each wheel relative to its own axis of rotation and its radial axis; $I_{m_1} - I = 1/2$ - intrinsic mass moment of inertia of each of the drives relative to its central axis parallel to $z_2'$; $F_{t1}$ - tangential component of the friction force on the wheel; $\mu$ - coefficient of friction depending on the sliding speed of the contact spot, which is introduced graphically or analytically; $r_1$ - radius of the wheel; $f_i$ - coefficient of rolling friction; $[F]$ - square matrix of coefficients in front of the actual angular accelerations of the drive wheels, depending on the wheel and engine inertia moments; $[\vec{F}]$ - matrix column of its own angular accelerations, of which two or four are propulsive; $M_{dt}, M_{zt}$ - corresponding engine and brake torque applied to each wheel.

Coordinates of the center of mass of the unsprung mass are defined by the expressions

\[
\begin{align*}
x_c1 &= x_c - \frac{m_3}{m_1 + m_2 + m_3}.(a_{11}x_{c1}' + a_{12}y_{c1}' + a_{13}z_{c1}') - \frac{m_2}{m_1 + m_2 + m_3}.
& \cdot [a_{11}x_{c1}' + a_{12}y_{c1}' + a_{13}z_{c1}' + (\cos \varphi_{z1} x_{c2} - \sin \varphi_{z1} y_{c2})] \\
y_c1 &= y_c - \frac{m_3}{m_1 + m_2 + m_3}.(a_{21}x_{c2}' + a_{22}y_{c2}' + a_{23}z_{c2}') - \frac{m_2}{m_1 + m_2 + m_3}.
& \cdot [a_{21}x_{c2}' + a_{22}y_{c2}' + a_{23}z_{c2}' + (\cos \varphi_{z1} x_{c2} - \sin \varphi_{z1} y_{c2})] \\
z_c1 &= z_c - \frac{m_3}{m_1 + m_2 + m_3}.(a_{31}x_{c3}' + a_{32}y_{c3}' + a_{33}z_{c3}') - \frac{m_2}{m_1 + m_3}.z_c2
\end{align*}
\] (4)

where $m_1$ is the unsprung mass; $m_2$ - sprung mass; $m_{knee}$ - mass of flywheel (if available); $a_{ij} / i = 1 \div 3; j = 1 \div 3$ - elements of the matrix formed by the mentioned cosines between the axes of the permanently connected to the sprung mass coordinate system and the fixed coordinate system.

The angle of rotation of the coordinate system $O_2x_2'y_2'z_2'$ relative to the fixed coordinate system is determined by the integral

\[
\varphi_{z1} = \int_{\varphi_{z10}}^{\varphi_{z1f}} \varphi_{z1} dt = \int_{\varphi_{z10}}^{\varphi_{z1f}} [\dot{\varphi}(t) + \cos \varphi(t) \cdot \dot{\varphi}(t)] dt
\] (5)

Coordinates of the center of mass of the sprung mass have the notation

\[
\begin{align*}
x_c2 &= x_c + a_{11}x_{c1}' + a_{12}y_{c1}' + a_{13}z_{c1}' + (\cos \varphi_{z1} x_{c2} - \sin \varphi_{z1} y_{c2}) \\
y_c2 &= y_c + a_{21}x_{c2}' + a_{22}y_{c2}' + a_{23}z_{c2}' + (\sin \varphi_{z1} x_{c2} - \cos \varphi_{z1} y_{c2}) \\
z_c2 &= \text{const}
\end{align*}
\] (6)

where
$m$ - total mass of the vehicle;
$\ddot{a}_c$ - acceleration of the vehicle center of mass in relation to the fixed coordinate system;
\( \bar{F}_i \) – forces acting on the vehicle, including friction, suspension elasticity, damping, air resistance.

4. Examining deformation energy of the car body
To determine the deformation energy of the car body \( E_1 \), indicated in equation (1), it is assumed that the intensity of impact forces results in a linear relationship to plastic deformation [6, 5, 18]. It is determined by the formula:

\[
\frac{dF}{dl} = q = A + B \cdot c
\]  

(7)

where \( F \) is impact force; \( c \) – indentation depth of the plastic deformation; \( A, B \) - energy coefficients taking into account the stiffness characteristics of the frontal crumpledne.

The deformation energy is obtained, as follows:

\[
dE = \left[ \int_0^\delta q_{\text{eq}} \cdot dx + \int_0^c q \cdot dc \right] \cdot dl
\]  

(8)

Equation (8) can be expressed to obtain the following equation:

\[
E = (1 + tg^2 \theta_s) \frac{L}{2} \left[ \frac{c_1 + 2 \cdot c_2}{2} + \frac{c_2^2 + 2 \cdot c_4 + c_6}{6} \right] + \frac{B}{6} \cdot \frac{A^2}{2 \cdot B}
\]  

(9)

5. Investigation of the energy deformation of the metal crash barrier
The deformation energy of the metal crash barrier (Fig. 2) is determined from equation (1) by integrating the finite element method. This method is used as a universal tool for calculating and analyzing the behavior of mechanical structures in power and heat loads as well as for solving problems in fluid mechanics, heat engineering, theory of electric and magnetic fields, acoustics, nuclear physics, etc.

![Figure 2. Models for identification of impact.](image)

6. Design of 3d models in solidworks environment
The object of investigation was a car impact against a metal crash barrier with post-impact manifested residual energy as well as with lack of residual energy. Initial data was the known deformation value of the metal crash barrier and the final rest position of the vehicle after the impact.
The initial conditions of the analysed crash were the deformations of the given metal crash barrier, the shape and the geometrical dimensions of the deformation. Three dimensional patterns were created using SolidWorks software [16, 19] (Fig. 3 and Fig. 4).

The results showed that deformation energy could be obtained, in case of iterative setting of impact crash (distributed load). Thus, a car impact process could be simulated against a given barrier at maximum load.

The exact analogy between the scientific model and the real one can be found in each detail, referring to exact geometric dimensions and material that correspond to real-life characteristics according to the load.

Figure 3. Three-dimensional tank model. Figure 4. Three-dimensional patterns of metal beam crash barrier and road.

The road was modelled in the Oxy plane defining the stationary coordinate system. The longitudinal axis Ox coincides with the lane separation line. The length of the road section is selected from the scope of the accident.

7. Finite elements model

The finite element method provides a good justification for the magnitude of the resultant impact force, its magnitude and direction. In modelling, boundary conditions are introduced, which are known from the real depth deformation measurements. Thus, precision could be achieved by ensuring that the resulting deformation from the numerical experiment was estimated by the real magnitude of the impact force.

Spatial barrier models were analyzed applying Finite Element analysis in SolidWorks - SW Simulation [15] package. Each tetrahedron element of the second order has ten nodes (four angles and six inner ones) and each has three degrees of freedom. The edges and the planes of the second order elements take curvilinear forms after deformation. Therefore, these elements are usually modelled in curvilinear coordinates. In the model, second order elements were analysed, as they have a lower mesh density compared to first order elements.

For each barrier model, boundary conditions were set in accordance with the dynamics of the impact process. The objective was to achieve the most accurate modelling of the shape of the deformation from the preliminary data about its indentation depth.

In a complex of computational procedures, a solution to equation (1) on the vehicle pre-impact kinetic energy was obtained.

8. Analysis of a Solidworks 3D model of metal crash barrier

In the analysis, the elements of the metal crash barrier are modelled as flexible surface-mounted posts. These are concrete bases, which are not subject to complete destruction. The modelling of the dynamics of the impact process includes taking into account incomplete damage of the bollards, as well as their unremoval from the concrete slabs, i.e. there is no separation between post and base. This means that the impact process is in intensity up to a limited boundary condition - without complete damage of posts and bases or metal barrier elements. The deformation on the metal barrier does not include its damage or destruction, the same condition being valid for the posts as well, up to the
separation phase from the concrete base. In the initial conditions, it is assumed that the limit value of the impact force should not be above the critical destructive force of the foundation.

Clamped boundary conditions were defined on the concrete slabs (Figure 5).

![Figure 5. Clamped concrete slabs.](image)

The metal crash barrier model was loaded with a distributed load in a normal and tangential direction over a defined area of the metal beam crash barrier, shown in Figure 6. Specified loads were distributed as pressure applied on the top.

![Figure 6. Applied distributed load on metal crash barrier.](image)

9. Analysis of a solidworks 3d model of tank

Limit conditions were defined for the tightened cylindrical body with threaded fasteners on top and on both sides of the (Figure 7).

The tank model was loaded with a distributed load on the normal of the surface of the tank body (Figure 8). In the context of finite element modelling, the condition to use uniformly distributed loads is to be set by their counterparts and on the surface on which they are applied. These are loads of the so-called nodal pressure type.

![Figure 7. Cylindrical body tightened with threaded joints at the top and on both sides.](image)  ![Figure 8. Applied distributed load on the tank body.](image)
10. Impact results of a car crash against a metal crash barrier with post-impact manifested residual energy

After considering the mass inertial parameters

\[ m = 1680 \text{ kg}; \, J_z = 3190 \text{ kg} \cdot \text{m}^2 \]  

the solution to the post-impact vehicle motion is obtained, shown in Fig. 9. Initial conditions for the velocity of the center of mass and the angular velocity of the vehicle, satisfying the differential equations for vehicle motion, determine the residual energy of post-impact vehicle motion. The graphs in Fig. 14 show the variation of the velocity parameters over time.

**Figure 9.** Post-impact vehicle motion against a metal crash barrier and manifestation of residual energy.

**Figure 10.** Motion graph in idle mode.

**Figure 11.** Rotation angle of front wheels.

**Figure 12.** Change of coordinates of the center of mass.

**Figure 13.** Rotation angle of the vehicle around the Oz axis.
Car body deformation from the impact against a metal crash barrier determines the impact energy by formula (8).

**Table 1. Vehicle collision in a rigid barrier.**

| Parameter                                                                 | Value     |
|---------------------------------------------------------------------------|-----------|
| Vehicle total mass reported                                               | 1680.00   |
| Depth of deformation zone reported C1 in [cm]                            | 0.00      |
| Depth of deformation zone reported C2 in [cm]                            | 25.00     |
| Depth of deformation zone reported C3 in [cm]                            | 48.00     |
| Depth of deformation zone reported C4 in [cm]                            | 62.00     |
| Depth of deformation zone reported C5 in [cm]                            | 75.00     |
| Depth of deformation zone reported C6 in [cm]                            | 86.00     |
| Reported angle between plane normal and impact force                     | 6.00      |
| Reported indentation width L in [m]                                      | 1.12      |
| Coefficient - A energy in [N/cm]                                         | 362.00    |
| Coefficient - B energy in [N/cm²]                                        | 48.30     |
| Deformation energy obtained [N.m]                                        | 108817    |

**Figure 14.** Velocity projections of center of mass.  **Figure 15.** Angular speed of each wheel.

**Figure 16.** Reported depth of deformation zone of the vehicle crash against metal crash barrier.

Fig. 17 and 18 show the results of the analysis. At given loading with uniformly distributed load, the following deformation of the metal crash barrier was achieved.
The pre-impact vehicle velocity of the center of mass against the metal barrier is derived from the dependence following the law of conservation of mechanical energy:

\[ v = \sqrt{\frac{m \cdot u^2 + l_2 \cdot w^2 + 2 \cdot E_1 + 2 \cdot E_2}{m}} = \sqrt{\frac{1680.8 \cdot 3^2 + 3190.2 \cdot 9.5^2 + 2 \cdot 108817 + 2.87024}{1680}} \]

\[ = 17.8 \, \text{m/s} \]  

(11)

The value obtained gives a quantitative estimate of the energy needed to deform the car body, the barrier and to reach final rest position.

11. Impact results in a car crash against a tank body in a lack of residual energy after impact - perfectly plastic collision

Deformation energy of the car body is determined according to (6), shown in Table 2.

| Vehicle total mass reported | 1900,00  |
|----------------------------|---------|
| Depth of deformation zone reported C1 in [cm] | 10,00  |
| Depth of deformation zone reported C2 in [cm] | 7,00  |
| Depth of deformation zone reported C3 in [cm] | 5,00  |
| Depth of deformation zone reported C4 in [cm] | 5,00  |
| Depth of deformation zone reported C5 in [cm] | 5,00  |
| Depth of deformation zone reported C6 in [cm] | 7,00  |
| Reported angle between plane normal and impact force | 0,00  |
| Reported indentation width L in [m] | 1,695  |
| Coefficient - A energy in [N/cm] | 541,00  |
| Coefficient - B energy in [N/cm²] | 93,10  |
| Deformation energy obtained [N.m] | 11346  |

At the given loading with uniformly distributed load, the depth of deformation zone of the tank body was achieved, as follows (Fig. 19 and Fig. 20).
Maximum reported deformation energy of the tank body as a result of impact and according to the adopted approach of finite element analysis is

$$E_1 = 41659.8 \, N \cdot m$$

(12)

The velocity of the mass center of the vehicle at the point of impact is determined by the formula

$$V = \sqrt{\frac{2 \cdot (E_1 + E_2)}{m}} = \sqrt{\frac{2 \cdot (41659.8 + 11346)}{1900}} = 7.47 \, m/s = 26.9 \, km/h$$

(13)

The obtained value gives a quantitative estimate of the energy needed to deform the car body, the barrier in a perfectly plastic collision.

12. Conclusions

1. Solution to a common engineering task necessary for legal analysis and widely used in case-law is provided. It determines the pre-impact speed of a vehicle following roadside objects/barrier impact and the loss of longitudinal stability after the impact. Traditional methods analysing such crashes, which are generally applied to modern impact investigation and accident reconstruction, do not meet accuracy and precision requirements.

2. The proposed FEM approach is a modern innovative means of solving similar engineering tasks in order to identify traffic road accidents. It is based on credibility and justification of facts and evidence.

3. This method for investigating and analysing head-on vehicle impact with a rigid barrier can also be applied to other vehicles and trucking fleets - trailers and semitrailers. The solution to the task requires precise post-impact mechanical and mathematical model for vehicle crashes, which represent the motion of the composition of the vehicle fleet.

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