The types of Mott insulator

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There are two classes of Mott insulators in nature, distinguished by their responses to weak doping. With increasing chemical potential, Type I Mott insulators undergo a first order phase transition from the undoped to the doped phase. In the presence of long-range Coulomb interactions, this leads to an inhomogeneous state exhibiting “micro-phase separation.” In contrast, in Type II Mott insulators charges go in continuously above a critical chemical potential. We show that if the insulating state has a broken symmetry, this increases the likelihood that it will be Type I. There exists a close analogy between these two types of Mott insulators and the familiar Type I and Type II superconductors.

The study of Mott insulators and doped Mott insulators constitutes one of the central focuses of condensed matter physics of the past two decades. At a simple level, a Mott insulator is a system in which the strong repulsion between particles impedes their flow; the simplest cartoon of this is a system with a classical ground-state in which there is one particle on each site of a crystalline lattice and such a large repulsion between two particles on the same site that fluctuations involving the motion of a particle from one site to the next are suppressed. The Mott insulator is thus essentially classical in nature (and so accessible to particles of any\textsuperscript{1} statistics.) In contrast a band insulator\textsuperscript{2}, including all familiar semiconductors, is a state produced by subtle quantum interference effects which arise from the fact that electrons are fermions. Nevertheless, one generally considers band insulators to be “simple” because the band theory of solids successfully accounts for their properties; although the physical origin of the insulating character of a Mott insulator is understandable to any child, other properties, especially the response to doping, are still only at best partially understood.

There is even debate concerning the definition of a Mott insulator. In many of the most familiar cases\textsuperscript{3} involving electrons, one can, in theory, continuously vary the strength of the repulsion between electrons, such that in the limit of vanishing strength interactions the system can be well understood on the basis of band theory, while in the strong interaction limit Mott physics better accounts for the insulating properties; if there is no phase transition between these two limits, it is not clear at what interaction strength one should start thinking of the system as a Mott insulator. However, recently an interesting Bose Mott insulator has been produced by trapping bosonic neutral atoms in an optical lattice\textsuperscript{4}; there is no non-interacting limit in which bosons are insulating, so no such ambiguity arises here.

Mott states, in addition to being insulating, can be characterized by the presence or absence of a spontaneously broken symmetry (e.g. spin antiferromagnetism), by the nature of the low energy excitation spectrum (e.g. gapped or gapless)\textsuperscript{5}, and by the presence or absence of topological order and charge fractionalization\textsuperscript{6, 7, 8, 9}. To this list, we add a “type” index that classifies Mott insulators into two categories depending on their response to doping.

Generally speaking, states with charge gaps (this includes both band and Mott insulators) occur in crystalline systems at isolated rational “occupation numbers,” \( \nu = \nu^\ast \), where \( \nu \) is the number of particles per chemical unit cell. By “doping” we mean a process which causes the occupation number to shift away from \( \nu^\ast \). When lattice translation symmetry is not spontaneously broken, \( \nu^\ast \) is typically an integer for bosons, consistent with the simple classical cartoon described above, and an even integer for spin 1/2 fermions (e.g. electrons). The significance of this latter statement is that a necessary condition for an electronic system to be adiabatically connected to a weakly interacting band-insulator is that \( \nu^\ast \) be an even integer. Insulating states can also occur when \( \nu^\ast \) is a fraction (for fermions this includes odd-integer \( \nu^\ast \)). Usually when that happens, translational symmetry is spontaneously broken so that the new unit cell has integral occupation number (or even integral occupation number for fermions)\textsuperscript{10, 11}. For instance, electronic Mott insulators with \( \nu^\ast = 1 \) often exhibit antiferromagnetic Néel long-range order, which doubles the unit cell leading to an effective \( \nu^\ast_{\text{eff}} = 2 \). Nevertheless, we will mention \textsuperscript{6, 7, 8} model bosonic systems for which the Mott state can be shown to have no broken symmetries for \( \nu^\ast = 1/2 \), and fermionic systems in which \( \nu^\ast = 1 \). (Currently, no laboratory system has been found which unambiguously exhibits this exotic behavior.)

The purpose of this paper is to address the response of Mott insulators to light doping (i.e. \( \nu \to \nu^\ast - \delta \)). Our central observation is that there are two types of Mott insulators (henceforth referred to as Type I and Type II). In a Type I Mott insulator an increasing chemical potential induces a first order phase transition from an undoped state to a charge-rich state so that changes occur discontinuously. In other words, there is a range of “forbidden charge density,” and two-phase coexistence. In a Type II Mott insulator, charges go in continuously above a critical chemical potential. Since doping of a band insulator is continuous, a doped Type II Mott insulator may be
adiabatically connected to a band insulator, but a doped Type I Mott insulator is a thermodynamically distinct state of matter.

In the presence of long-range Coulomb forces macroscopic charge inhomogeneity is impossible. However, so long as the long-range forces are not too strong, there can exist a range of low doping in which the doped charges tend to cluster[12]. We still refer to this kind of system as a Type I Mott insulator[13]. For instance, if a low density of doped charges form puddles or stripes[14], we consider this a form of charge clustering, while a Wigner crystal of doped charges is deemed homogeneous.

What is the situation in real materials? That simple semiconductors can be continuously doped, and hence are Type II, is well known, although it is not clear they are profitably thought of as Mott insulators, even when correlation effects significantly renormalize the gap magnitude. There are many materials in which the insulating state is also antiferromagnetically ordered. It has been argued[15, 16, 17] that doping of this class of Mott insulators generically[18] leads to phase separation, i.e. that they are Type I. We will show below that the presence of a broken symmetry always increases the tendency of an insulator to be Type I, although the conjectured strict connection has neither been proven[19] nor disproved.

For the cuprates and manganates, both of which have antiferromagnetic order when undoped, there exists considerable evidence that doping induces spatial inhomogeneity[20, 21, 22, 23, 24]. Conversely, Sr1−xLaxTiO3, which is also an antiferromagnet when undoped[25], is generally believed[26] to be Type II on the basis of the observation of conventional Fermi liquid behavior[27] for doping as low as x = 5%, with an effective mass that shows a tendency to diverge with decreasing x; taken at face value[28], this evidence of a quantum critical precursor to a continuous metal-insulator transition indeed suggests that this material is Type II.

Another purpose of this paper is to elucidate the analogy between Type I and Type II Mott insulators and the familiar types of superconductors. In particular, when the constituent particles are bosons there exists a mathematically precise mapping, the so-called "duality transformation"[10, 29], that relates the zero-temperature properties of the doped Mott insulator in two spatial dimensions, D=2, to the finite-temperature response of a 3D superconductor to a magnetic field. Table I summarizes the correspondence between the two. This same mapping was exploited previously by Balents, Fisher, and Nayak[30] in the context of a theory of a proposed "nodal liquid" Mott insulating state; in this regard, the present paper simply underlines the generality of the analogy.

| T = 0 properties of 2D Bose Mott insulators | T > 0 properties of 3D Superconductors |
|--------------------------------------------|--------------------------------------|
| Doping                                    | Applying magnetic field              |
| Chemical potential μ                      | Applied magnetic field H             |
| Induced particle density ρ                | Magnetic induction B                 |
| World line of doped particles             | Flux tubes                           |
| Quantum delocalization of doped particles  | Thermal meandering of flux tubes     |
| Type I Mott insulator                     | Type I superconductor                |
| Mott gap                                  | Hc                                   |
| Effective attraction between doped particles | Positive N-S interface energy |
| Type II Mott insulator                    | Type II superconductor               |
| Effective repulsion between doped particles | Negative N-S interface energy       |
| Mott gap                                  | Hc1                                 |
| Wigner crystal of doped particles          | Abrikosov flux lattice               |
| Superfluid state                          | Entangled vortex fluid               |
| Critical μ at which Wigner crystal melts   | Hc2                                 |

I. DOPING A TRANSLATIONALLY INVARIANT BOSE MOTT INSULATOR

To begin with, we avoid the interesting complications which arise from spontaneous symmetry breaking or from the fermionic character of electrons by focusing on the simplest kind of Mott insulators formed by spin zero point bosons on a lattice at $\nu = 1$. This is not a purely academic exercise[31], as it applies rather directly to the experiments in Ref.[4]. Consider the following Hamiltonian

$$H = -\frac{t}{2} \sum_{<ij>} (a_i^+ a_j + h.c.) + \frac{U}{2} \sum_i (a_i^+ a_i)^2 (a_i^+ a_i - 1) + \frac{1}{2} \sum_{i,j} V_{ij} a_i^+ a_i a_j^+ a_j - \mu \sum_i a_i^+ a_i,$$ (1)
where \( i, j \) label the lattice sites on a D-dimensional hypercubic lattice, and \( a_j^+ \) creates a boson at site \( i \). The first term of Eq. (1) describes the quantum mechanical “hopping” of bosons from a site \( i \) to its nearest neighbors \( j \), the second and third terms describe the pair-wise interactions between bosons. The \( U \) term is a contact interaction and the \( V_{ij} \) terms describe interaction between bosons separated by \( |\mathbf{r}_i - \mathbf{r}_j| \). This model is known as the extended Bose-Hubbard model, and has been extensively studied from various perspectives[32].

Let us focus on the \( U/t \to \infty \) limit. In this “hardcore” limit, doubly occupied sites cost too much energy, hence are excluded. In this case for large positive \( \mu \) there is an unique ground state

\[
|\text{Mott} > = \prod_j a_j^+ |0 > ,
\]

in which each site is occupied by one and only one boson and hence \( \nu = 1 \). Since this state is separated from all other states with the same particle number by an energy gap of order \( U \), clearly we have an insulator. (For \( \mu \) large and negative, the groundstate is the empty lattice, \( |0 > \).

The behavior of the system upon varying \( \mu \) (i.e. doping) depends on the values of \( V_{ij} \). For \( V_{ij} = 0 \) the doped holes (i.e the empty sites) interact only through the hardcore exclusion. Since the kinetic energy favors a uniform density state with delocalized holes, this limit corresponds to a Type II Mott insulator. Indeed, a small concentration of doped holes, \( \delta \equiv 1 - \nu \ll 1 \), is equivalent to a system of dilute bosons (defined relative to the vacuum state \(|\text{Mott} > \) with short-range interactions, a problem whose solution has long been known[33, 34]. The energy is clearly minimized by delocalizing the holes so the ground state in \( D = 2 \) is a uniform superfluid with superfluid stiffness proportional to \( \delta \nu \). Conversely, if \( V_{ij} \) is attractive (negative), it is clear that the holes will cluster when \( \sum_j \nu_j \gg t \). Thus depending the nature of \( V_{ij} \) Eq. (1) can describe either a Type I or Type II Mott insulator. Note that in this example, the Mott state does not break any symmetry, so any inhomogeneity of the the doped state cannot be attributed to order parameter competition. The Type I behavior discussed here is more general[35] than that which occurs in the bicritical point scenario in the Landau theory of two competing order parameters[36].

To be still more explicit, consider the case in which \( V_{ij} = -V \) for (ij) nearest-neighbor sites, and \( V_{ij} = 0 \) otherwise. Technically in this limit Eq. (1) is equivalent to the \( S=1/2 \) ferromagnetic XXZ model in a z-direction magnetic field

\[
H = -J_{xy} \sum_{<ij>} (S_i^x S_j^x + S_i^y S_j^y) - J_z \sum_{<ij>} S_i^z S_j^z - h_z \sum_i S_i^z .
\]

The mapping between these two models relates \( J_{xy} \) to \( t \), \( J_z \) to \( V \), \( h_z \) to \( \mu + Vc/2 \) (\( c \) = the coordination number). The \( z \) component of the magnetization is related to the boson density according to \( M_z = (\nu - 1/2)N \). (Here \( N \) is the total number of lattice sites.) For \( J_z < J_{xy} \) (i.e. \( V < t \)), this model has XY order in the absence of \( h_z \). In this range of parameters varying \( h_z \) causes the magnetization \( M_z \) to vary continuously, but the ground state is uniform and, so long as \( |M_z| < N/2 \), has a non-zero component of the magnetization which lies in the XY plane and spontaneously breaks the XY symmetry. In terms of bosons this means that the insulating state is Type II, and the doped system is a uniform superfluid for all \( 1 > \nu > 0 \).

Conversely, for \( J_z > J_{xy} \) (i.e. \( V > t \)) the model is effectively an Ising ferromagnet with a fully polarized ground state. In this case \( M_z \) exhibits a discontinuity \( \Delta M_z = N \) as \( h_z \) is varied through zero. If we consider a constrained groundstate, with \( |M_z| < N/2 \), the state exhibits phase separation into two oppositely polarized domains. In terms of the bosons, these two domains are Mott insulating (\( \nu = 1 \)) and empty (\( \nu = 0 \)) respectively. On the boundary between the two behaviors (\( J_z = J_{xy} \)), the system is equivalent to a Heisenberg ferromagnet in a magnetic field. In the absence of \( h_z \) the ground state is \( \nu+1 \) fold degenerate correspond to \( M_z = -N/2, ..., N/2 \). In the presence of \( h_z \) the state with \( M_z \) = \( \pm N/2 \) (\( \pm \) depends on the sign of \( h_z \)) has the lowest energy. For this system there is again a first order transition at \( h_z = 0 \). However, precisely at the transition point the boson inverse compressibility \( \kappa^{-1} \) diverges, and a uniform ground state exists for any \( \nu \); at this critical point, only, the system is neither truly Type I nor Type II.

II. DUALITY TRANSFORMATION FOR THE BOSE MOTT INSULATOR IN D=2

There is a particularly convenient way of thinking about Bose Mott insulators that is specific to \( D = 2 \). It turns out that for the class of model given by Eq. (1), there is an exact mapping, the “duality” transformation, that provides us an alternative view of the physics of Eq. (1) in terms of the vortices of the boson field[10, 29]. It is this mapping that enables us to establish a precise connection between the two types of Mott insulators with Type I and Type II superconductors[10, 29, 30]. In the following we discuss the physical content of the duality transformation without going into its technical details of the transformation[10].

A vortex is a topological defect in the Bose field. When a boson is adiabatically transported around a vortex, the boson wavefunction acquires a phase factor - the Aharonov-Bohm phase \( \theta = 2\pi \). In the dual picture, the role of the vortices and the particles are interchanged: the vortices are the dual particles (they turn out to have Bose statistics as well), and when a dual particle is transported around an original boson, the wave function acquires the same Aharonov-Bohm phase[10, 29] discussed above. The fact that a boson and a vortex acquire a phase when they go around one another implies that bosons and vortices can not Bose condense simultaneously. As
is well known, in the Bose superfluid phase the vortices must be absent or localized, i.e., form an Abrikosov lattice. Conversely, in the dual phase, where the vortices form a superfluid[37], the boson density (and hence the dual magnetic flux) must be frozen; the vortex superfluid phase is the Mott insulating phase of the original bosons[10, 29].

It is important to note that the absence of dynamical boson density fluctuation is a necessary but not a sufficient condition for vortex condensation. A static boson density acts like a background magnetic field to the vortices, which can still frustrate vortex condensation. However when the static boson density corresponds to an integral $\nu$, the vortices see a background magnetic flux corresponding to integral number of flux quanta per plaquette. (The vortices live on the dual lattice, i.e., the centers of the square plaquettes.) This type of flux is “invisible” because it can be “gauged away”. The $\nu = 1$ Bose Mott insulator, discussed in the previous section, corresponds to precisely this situation.

When the boson density is a fraction ($\nu = p/q$) it is also possible for the vortices to condense. This is well known[38] in the context of the classical frustrated XY model (the XY order parameter is the Bose amplitude of the vortices) in $D = 2$. Quantum analogues of this, especially for low order rationals ($q$ a small integer) can readily be imagined. Such a state is most naturally accompanied by spontaneous translation symmetry breaking[10, 11], as discussed above (and as occurs in the classical model), leading to an enlarged unit cell with an effective integer $\nu$. However, it is possible to imagine a more exotic Mott state at $\nu = p/q$ in which the translation symmetry is unbroken. As has been discussed in Ref. [6], this could happen if $q$ elementary vortices form a bound-state, and these composites then condense. Such a composite condensation is unfrustrated by an uniform static boson density with $\nu = p/q$. Moreover, since the condensate vortex consists of $q$ elementary vortices, charge $1/q$ bosonic soliton excitation (viewed by the condensate vortices as a flux quantum) can become a finite energy excitation. Thus there is fractional charge solitons!

In short, a vortex condensate requires the bosons to Mott insulate. Consequently we can view a boson Mott insulator as a vortex superconductor[10, 29]. Doping changes the average background boson density. To the vortices this appears as a change in the background magnetic field. The zero temperature boson Mott insulator is mapped onto a zero-temperature vortex superconductor (with quantum fluctuating two-dimensional electromagnetic fields)[39]. However, the quantum partition function of a (particle-hole symmetric)[37] two dimensional superconductor with a fluctuating gauge field is equivalent to the classical (i.e. thermal) partition function of a three-dimensional superconductor with thermally fluctuating magnetic field[40]. This leads to the final correspondence between the Mott insulator and the classical fluctuating 3D superconductor summarized in Table I.

The properties of classical 3D superconductors in a magnetic field are generally well known, so most of the comparisons in Table I are self-evident. We therefore confine ourselves to commenting on a few of the subtleties of this comparison:

Although a Type I superconductor typically expels any applied field of magnitude $H < H_c$, by choosing an appropriate experimental geometry it is possible to study its behavior at fixed average magnetic induction - which is analogous to studying the Mott insulator at fixed doping concentration. This can be done, for example, by subjecting a flat slab of a Type I superconductor to a perpendicular magnetic field of strength $H < H_c$. It is known that when that is done the “intermediate state” consists of a mixture in particular, among many possible inhomogeneous structures which have been observed[41], one of the most common is a laminar (or stripe) structure.

Much recent attention has been focussed on fluctuation effects in the mixed state of a Type II superconductor. Mean-field theory predicts that the magnetic induction $B$ increases from zero continuously as $H$ is raised above the lower critical field $H_{c1}$. Moreover, the magnetic induction first appears in the form of flux tubes each enclosing a single quantum of magnetic flux which (in the absence of disorder) form a regular (Abrikosov) lattice. However, thermal meandering of the flux tubes can affect the physics dramatically near $H_{c1}$ as well as near the mean-field $H_{c2}$. Near $H_{c1}$, the distance between neighboring flux tubes is much greater than the range of their interaction (the London penetration depth $\lambda$). As a result thermal meandering of the flux tubes will melt the flux lattice to form a flux liquid[42]. At larger magnetic fields the density of flux tubes becomes higher so that their interaction can stabilize the flux lattice. This flux lattice persists until $H \rightarrow H_{c2}$ where the thermal meandering melts the flux lattice again. The corresponding behavior of a Type II Bose Mott insulator is a reentrant series of transitions to a superfluid, an insulating crystal of doped holes, and again a superfluid as a function of increasing doping concentration.

Finally, it is worth recalling that what determines whether a superconductor is Type I or Type II is the ratio between the London penetration depth and the core size of the vortices, or more physically the sign of the interface energy between normal and superconducting regions[41]. Type I superconductors have a positive interface energy while in Type II superconductors it is negative. As a result, the flux tubes effectively attract each other in Type I superconductors and repel each other in Type II. As we have seen in Eq. (1), this is exactly how we turn a Type I Mott insulator into a Type II.

III. DOPING A MOTT INSULATOR WITH AN ORDER PARAMETER

As we discussed at the beginning of this paper a Mott insulating state can be accompanied by transla-
tion (and/or other) symmetry breaking. For example let us consider Eq. (1) with $U/t \to \infty$ and $V_{ij} = +V$ for nearest neighbor $< ij >$ and 0 otherwise. For sufficiently strong $V$ and $\mu = 0$ the ground state breaks translation symmetry and bosons form a checkerboard lattice and Mott insulate. In this two-fold degenerate ground state the unit cell is doubled. Is this Mott insulator Type I or Type II?

With the above specific choice of $U$ and $V_{ij}$, Eq. (1) is equivalent to Eq. (3) with $J_{xy} = -t$, $J_z = V$ and $h = \mu - Vc/2$. For this choice of parameters it is known that as a function of $h$, Eq. (3) exhibits a “spin flop” transition from the antiferromagnetic Ising ($S_z$) ordered phase into the ferromagnetic XY ordered phase. Translate this into the boson language it implies that it is Type I.

There is, in fact, a general reason to expect broken symmetry to increase the tendency of a Mott insulator to be Type I. If there is a broken symmetry in the insulating state, then generally there is a corresponding local order parameter ($\psi$) for the above example $\psi$ is the two-sublattice density wave order parameter. We now consider the effect of this order parameter on the interface energy between the insulating and the metallic (doped) state. Typically, $\psi$ will have a reduced value (perhaps 0) in the doped state. Thus, at the interface between doped and undoped region a spatially varying $\psi$ is necessary. The spatial gradient energy

$$\int d^d x |\nabla \psi|^2$$

will thus make a positive contribution to the interface energy, increasing the tendency of the Mott insulator to be Type I.

IV. AN EXAMPLE OF TRANSLATIONALLY INVARIANT BOSE MOTT INSULATOR AT $\nu = 1/2$

A recently discovered system which exhibits a Bose Mott insulating state at a fractional $\nu$ is the quantum hardcore dimer model on a triangular lattice. The model is defined in terms of bosons (called dimers) which live on the nearest neighbor bonds of a lattice, i.e. $b_{ij}^\dagger$ creates a boson on the bond connecting sites i and j. We impose a hardcore constraint - this can be thought of as deriving from the $U \to \infty$ limit of a contact interaction, which forbids two dimers to occupy the bonds emanating from a single site, i.e. $b_{ij}^\dagger b_{jk}^\dagger b_{kj} b_{jk} = 0$ for any triplet of nearest neighbor sites, $< ij >$. Thus, since each bond touches exactly two sites, it is possible on any regular lattice with one site per unit cell to satisfy this constraint if and only if $\nu \leq 1/2$. The simplest (shortest range) Hamiltonian[8] one can construct for these dimers is

$$H = -t \sum_{(ijkl)} [b_{ij}^\dagger b_{kl}^\dagger b_{il} b_{jk} + \text{h.c.}] + V \sum_{(ijkl)} [b_{ij}^\dagger b_{kl} b_{il} + b_{il}^\dagger b_{kj} b_{ji}^\dagger b_{kj}^\dagger b_{il}]$$

where $(ijkl)$ denotes a “square” of nearest-neighbor sites, such that $< ij >, < jk >, < kl >$, and $< li >$ are all pairs of nearest-neighbors.

On a square lattice, all zero temperature Mott insulating phases with $\nu = 1/2$ derived from this Hamiltonian are believed to break translational symmetry, at least doubling the unit cell size, so that the effective $\nu$ is integral[7]. However, on a triangular lattice, it has now been established[7] that for a range of parameters near $t \sim V$, there is a dimer liquid phase, with exponentially falling correlation functions and no broken symmetries. From the dual viewpoint, this phase can be viewed as a superfluid of vortex pairs[6]. As mentioned previously, there are observable topological consequences of the vortex pairing, including certain predictable ground-state degeneracies on closed surfaces.

This same model can serve as a paradigmatic example of a Mott insulating state of spinfull fermions at an odd integer $\nu = 1$. To see this, we merely need to define fermion creation operators, $c_{\sigma,ij}^\dagger$, for an electron with spin polarization $\sigma$ on bond $< ij >$. In terms of this, we can express the dimer creation operators as

$$b_{ij}^\dagger = c_{\uparrow,ij}^\dagger c_{\downarrow,ij}^\dagger$$

and an additional constraint, $\sum_\sigma c_{\sigma,ij}^\dagger c_{\sigma,ij}^\dagger = 0$ or 2, which can be thought of as a consequence of a strong attractive interaction between two electrons on the same bond.

Is this sort of fractional Mott insulator Type I or Type II? The intensive study of what happens when one dopes such a “spin-liquid” state was initiated by the proposal[43] that doping such a liquid would lead inevitably to high temperature superconductivity. The basis for this proposal was the thought that pairing correlations (e.g. a spin gap[9]) might be present already in the insulating state, and these would evolve smoothly into superconducting pairing upon doping. Central to this proposal is the assumption that the spin-liquid is a Type II Mott insulator. Indeed, an early triumph of this idea was the observation by Ioffe and Larkin[8, 44] that, for at least a range of parameters, doped holes in the quantum dimer model do not phase separate. Moreover, reversing the logic of the previous section, the fact that this state has no broken symmetry increases the likelihood that it is Type II.

V. CONCLUSIONS

To summarize, in this work we introduce the concept that there are two types of Mott insulator. In particular, there exist a whole class of Mott insulators, the type I Mott insulators, that become inhomogeneous upon doping. The type I behavior does not have to originate from order parameter competition. We give a familiar lattice boson example where the Mott state exhibits no symmetry breaking, however depending on the sign
of certain interaction, it can be either type I or type II after doping. Through duality these two types of insulating states have a close tie to Type I and Type II superconducting states.[30] Finally after addressing the “zeroth order” physics of doped Mott insulator, we point out that many important “first order” issues remain unanswered. Among them are “what determines the structure of the inhomogeneity in a doped Type I Mott insulator?” “What is the ultimate low energy behavior of these inhomogeneities?”

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[18] There are, in each case, various additional conditions concerning the nature of the antiferromagnet specified in the various conjectural relations between antiferromagnetism and Type I behavior. Before gleefully constructing proposed counterexamples, such as a doped Ising antiferromagnet with additional strong, short-range repulsive interactions, the reader should consult the original papers.
[19] It was shown by L. Pryadko, D.Hone, and S.A.Kivelson [Phys. Rev. Lett. 80, 5651-5654 (1998)] that a long-range uniformly attractive interaction between static charges is induced by spin-wave fluctuations in a Heisenberg quantum antiferromagnet - these forces are sufficient to prove that phase separation is generic in doped antiferromagnets up to a few nasty caveats.
[20] For a review of stripes in the cuprates see V. J. Emery, S. A. Kivelson, J. M. Tranquada Proc. Natl. Acad. Sci. USA 96, 8814 (1999).
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[28] Although the cited evidence suggests that LaTiO$_3$ is a Type II Mott insulator, and thus a counterexample to the general conjecture that antiferromagnetic insulators are generically Type I, it seems to us that more work needs to be done to establish this fact. The apparent divergence of the effective mass with decreasing $x$ reported in [27], and the good Fermi liquid character of the metallic state even at the smallest reported $x$, certainly argue in favor of a
continuous transition to the insulating state. However, indirect NMR evidence, Y. Furukawa et al., Phys. Rev. B 59, 10550-10558 (1999), apparently implies that there is little or no antiferromagnet character to the magnetic fluctuations in the metallic phase; a continuous transition between a paramagnetic metal and an antiferromagnetically ordered insulator would be expected to exhibit diverging (quantum critical) antiferromagnetic fluctuations as the critical point is approached. The failure to see such fluctuations is most naturally understood if the transition is ultimately first order. The issue is further complicated by the fact that the correlation effects are relatively weaker in this material than in the cuprates or manganates; the charge gap in the insulating state is only about 0.1eV. This might well mean that the range of doping over which the actual physics of the “doped antiferromagnet” is dominant may be rather small, and in fact the experiments seem never to access the important range of doping with $x < 5\%$. It would be very interesting to look more closely at the low doping regime of this and related materials, and especially to study the magnetic structure by neutron scattering in this regime, to unambiguously determine whether they are truly Type II.

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[40] In the correspondence between a quantum superconductor at $T = 0$ and a thermally fluctuating superconductor at finite temperature the magnetic field $(F_{12})$ and electric field $(F_{01}, F_{02})$ in the 2+1 dimensional quantum theory are mapped on to $B_z$ and $(B_y, -B_x)$ respectively.
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