Ant Colony Optimization-based Design of Multiple-target Active Debris Removal Mission*

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The paper considers the ant colony optimization (ACO) methodology for designing an active debris removal mission. The goal is to optimize a sequence of transfers using an orbital transfer vehicle to rendezvous with multiple pieces of debris for the purpose of removal. The methodology consists of two phases: the first phase obtains an optimal removal sequence, and the second phase is related to transfer trajectory optimization. During the sequence planning process, a refined approximation is proposed to estimate the transfer times and necessary costs of individual transfers. The problem can then be mapped into a variant of the traveling salesman problem (TSP). To solve it, an enhanced ACO and the inverse-over algorithm are proposed. The effectiveness of the ACO heuristic is proved over a set of instances with different sizes ranging from 100 to 2000. In the second phase, each transfer leg in the optimal sequence is verified using the continuous ACO proposed. The computational results show that the methodology proposed can optimally select targets from a debris archive of considerable size (i.e., up to 2000 debris pieces). Additionally, the mitigation of 13–20 objects, with total ∆V below 1 km/s, is feasible in less than a year.

Key Words: Active Debris Removal, Traveling Salesman Problem, Ant Colony Optimization, Trajectory Optimization

Nomenclature

- μ: Earth’s gravitational parameter
- a: semi-major axis
- e: eccentricity
- i: inclination
- Ω: right ascension of the ascending node
- ω: the argument of perigee
- M: mean anomaly
- n: mean motion
- J2: Earth’s second zonal harmonic
- s: debris
- S: debris set
- σ: chosen debris set
- xij: removal sequence optimization variables
- pr: debris removal profit
- t: time
- τ*: theoretical favorable transfer opportunity
- T: mission time horizon
- Re: Earth’s equatorial radius
- τ: pheromone
- η: heuristic information
- ρ: pheromone evaporation ratio
- α, β, λ1, λ2: ACO parameters
- N: the size of ant colony
- p: transfer optimization variables
- K: arbitrary integer
- V: velocity

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1. Introduction

Space debris raises an increasing threat to operational orbits and human missions. According to Kessler et al., the debris density in Low-Earth Orbit (LEO) has already reached a critical state, creating a cascading effect that may result in an exponential increase in debris even if no further launches are made.1−3) In order to stabilize the debris population, an active debris removal (ADR) concept has recently received increasing attention.4) The best economy will be achieved when a number of well-chosen pieces of debris (in terms of removal cost and profit) can be removed during a single mission.

To remove LEO space debris, this paper presents a reasonable concept for designing an orbital transfer vehicle (OTV) that is capable of rendezvousing with a target, and attaching a kit that modifies the debris orbit, casing it to fall out of orbit. The debris would so be moved to a reentry or an unused orbit (i.e. The orbit poses no risk to human space flights.). Then the same OTV would travel to the debris target. It has become increasingly clear that the design of such a space mission at a preliminary stage leads to a complex embedded optimization problem as it involves a vast combinatorial part as well as a continuous part that is high dimensional. The typical combinatorial optimization problem is related to choosing the order of the debris among a population, and the continuous part is designing the optimum spacecraft fuel for rendezvous trajectories. The continuous choices obviously affect searching for the combinatorial part in space. According to Izzo et al.,4) the design of an ADR mission can be mapped into the dynamic city selection traveling salesman problem (TSP-DCS). The TSP is one of the most studied combinatorial optimization problems. It aims at finding the
shortest tour that visits the nodes in a given weighted complete graph exactly once and returns to the origin node. In the TSP-DCS, debris is moving and the transfer cost thus depends on the exact epochs of rendezvous. Since the TSP-DCS is an NP-hard problem, exact algorithms can merely find an optimal solution for very limited-size instances due to computational efficiency. Some researchers focused on converting the dynamic problem to a static case so as to reduce complexity and find a satisfactory solution within reasonable time. This simplified assumption is not sufficient for actual ADR missions. In addition, Berend and Olive, preprocessed the dynamic problem with the discretization of the transfer duration. Indeed, a fine-grained discretization step is beneficial with regards to the quality of the solution, but highly detrimental to the computation time. With the purpose of achieving a good balance between computation time and quality of the solution, Casalino approximated the accurate transfer times and the corresponding costs based on the use of the J2 effect only to nullify the right ascension of the ascending node (RAAN) differences between any debris pair. Cerf and Izzo et al. considered the cost and duration for each transfer as a linear function to make the dynamic problem static. The methods proposed are then exemplified, respectively, on a moderate size instance (about 100) and debris clouds of considerable size (approximately 2000).

There is no doubt that only using the J2 effect to nullify the RAAN difference has an advantage in terms of fuel savings, but it is not time efficient in the cases where either the RAAN precession rate difference is rather small or the RAAN difference is relatively large. In order to remove as many as debris targets as possible in limited time, it may be necessary to correct the RAAN difference by actively using engine thrust actively in addition to the J2 effect. Despite the advancements in previous works, most of the approximation methods proposed have focused on only one manner of RAAN correction.

Motivated to refine the approximator for a fuel-optimal ADR mission, this paper presents a fast but accurate technique to determine the favorable rendezvous opportunities adapted to various manners of RAAN correction (i.e., using engine thrust, natural perturbation of the J2 effect or a hybrid manner). The approximation of transfer cost between any debris pair, in terms of ΔV, suited to a high thrust vehicle is attached to the debris, it is responsible for the debris’ removal. The OTV then moves and waits for a favorable opportunity to rendezvous with the next piece of debris. The ADR mission is considered finished when either: 1) the reserved fuel (i.e., the total given ΔV) is exhausted, 2) the mission time has expired, 3) the last piece of debris selected for the mission has been removed.

We denote the OTV by $s_0$ and the set of potential piece of debris by $S = \{s_1, s_2, \ldots, s_n\}$. The OTV $s_0$ performs $m \leq n$ orbital transfers to visit $m$ of the $n$ potential candidates. We use a set $\sigma = \{\sigma_i, \ldots, \sigma_m\}$ to denote a sequence of $m$ debris to be removed in an ADR mission. Each transfer takes the OTV $s_0$ to a target belonging to the set $\sigma$. Clearly, $\sigma_i \subseteq S$ represents the $i$-th piece of debris visited by the OTV $s_0$ during the $i$-th maneuver, and the OTV visits the $m$ debris in the same order as appears in the sequence $\sigma$. Once the disposal of a piece of debris $s_i$ is finished, the corresponding profit $pr_i$ will be earned. Since the largest debris has the most hazardous risk probability to the orbital environment, the more important it is to remove it. Accordingly, the profit is directly related to the radar cross-section area. Note that, a sequence can be formed by choosing and permuting $m$ debris out of the $n$ candidates under domain constraints.

The ADR mission can be represented by a graph $G = (S, E)$. The nodes in the graph are defined by the debris objects, each of which is assigned a profit value $pr_i$. In this paper, the profit $pr$ of each debris object is equal to its radar cross-section area. The directed edge connecting debris node

2. Problem Statement

2.1. ADR mission description

Three separate debris clouds in the LEO region, the 2009 Iridium 33, Cosmos 2251 collision and 2007 Fengyun-1C, which were derived from past unfortunate collision events and contain the resulting fragmented debris, are used as test-instances in this study. The orbital elements in the two-line element (TLE) format and radar cross-section areas (RCS) [m²] of these debris are both extracted from the North American Aerospace Defense Command (NORAD) orbital objects catalogue. Their orbits are predicted in a SGP4 orbital propagator until a reference time, set as 21/11/2016 here. In Fig. 1, we show the distribution of the orbital parameters for the three debris clouds as of 21/11/2016.

In order to make the LEO belt safer, removing the maximum number of LEO debris targets possible in one launch is preferred. However, the required ΔV budgets are intuitively too high to be implemented in any practical space mission. Instead, the current paper focuses on selecting of the most dangerous debris in debris clouds for removal and guidance to those selected using chemical propulsion. With respect to fuel expenditure and overall mission time, the multiple rendezvous maneuvers need to have an optimal design so that the cost of the mission at a given time is minimized.

The mission starts when an OTV is released at the same position as the first debris to be removed. The cost and time consumed to remove the first target are therefore not included in the mission cost and duration. After a de-orbit kit has been attached to the debris, it is responsible for the debris’ removal. The OTV then moves and waits for a favorable opportunity to rendezvous with the next piece of debris. The ADR mission is considered finished when either: 1) the reserved fuel (i.e., the total given ΔV) is exhausted, 2) the mission time has expired, 3) the last piece of debris selected for the mission has been removed.

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with \( s_i \) and \( s_j \) corresponds to the orbital maneuver between them, and is characterized by two cost functions: the \( \Delta v_{ij} \) budgets consumed and the transfer duration \( \Delta t_{ij} \). Note that we assume that there is not any mass exchange happening between the OTV and the debris during a rendezvous process.

For instance, Fig. 2 depicts a graph for a specific ADR mission containing five pieces of debris, all of which are to be removed by the OTV. The edges indicate two different types of transfer: 1) the edges between the OTV \( s_0 \) and the debris \( s_i (i = 1, 4, 5) \) correspond to the transfers from the initial position of the OTV to the piece of debris selected, and 2) the other edges are associated with the orbital transfers between debris pairs. In addition, the directed edges represent all possible transfers that the OTV can make, while the full-line path \( \{s_1, s_5, s_4, s_2, s_3\} \) describes a feasible tour of the OTV for visiting all debris.

2.2. Global optimization problem

Global optimization of the ADR mission defined above involves a complex embedded optimization problem:

- Removal sequence optimization: a combinatorial optimization problem consisting of selecting and permuting debris among the population.
- Transfer trajectory optimization: several continuous optimization problems, each of which is responsible for finding the optimal fuel trajectory for the OTV when moving between debris pairs.

In this study, we focus on the problem of maximizing the total profit gained by removing a selected set of debris objects from orbit. Both the total mission duration and the allowed total \( \Delta V \) add constraints for practical reasons. To this end, the \( \Delta V \) consumed, as well as the time required for each individual transfer should also be minimized. Ideally, the minimum-fuel, minimum-time trajectories between all debris pairs should be provided as the inputs for the removal sequence optimization problem. There could be tens of thousands of possible combinations to consider, and the number of possible trajectories between two specific debris is infinite. Thus, trying to obtain all trajectories would be unpractical. A feasible option is to approximate the transfer cost of any debris pair in advance, and then optimize the removal sequence.

For the sake of clarity, we present an integer programming formulation for the removal sequence optimization problem. The decision variables are comprised of a binary variable \( x_{ij}, i \in [0, \ldots, n], j \in [1, \ldots, n] \) in order to determine which piece of debris to remove and in what order, the transfer duration \( \Delta t_{ij}, i \in [0, \ldots, n], j \in [1, \ldots, n] \) for traveling from debris \( s_i \) to \( s_j \) and the time \( t_i, i \in [1, \ldots, n] \) at which the debris \( s_i \) is visited. The binary decision variable \( x_{ij} = 1 \) if and only if the OTV proceeds from debris \( s_i \) to debris \( s_j \). Otherwise, the decision variable \( x_{ij} = 0 \). Hence, the removal sequence optimization problem can be stated as follows:
maximize $A = \sum_{s_i \in S} \sum_{s_j \in S} x_{ij} p_{R_{ij}}$ (1)

subject to $\sum_{s_j \in S} x_{ij} = 1, \; \forall s_j \in \sigma$ (2)

$\sum_{s_i \in S} x_{ij} = 1, \; \forall s_i \in \sigma \cup s_0$ (3)

$t_{ij} + \Delta t_{ij} = t_j, \; \text{if} \; x_{ij} = 1, \; \forall s_i, s_j \in \sigma$ (4)

$\sum_{s_i \in S} x_{ij} \Delta v_{ij} \leq \Delta V_{\text{max}}$ (5)

$x_{ij} t_f \leq T, \; \forall s_i \in s_0 \cup S, \; \forall s_j \in S$ (6)

$\Delta v_{ij} \geq 0, \; \forall s_i \in s_0 \cup S, \; \forall s_j \in S$ (7)

$\Delta t_{ij} \geq 0, \; \forall s_i \in s_0 \cup S, \; \forall s_j \in S$ (8)

$x_{ij} \in \{0, 1\}, \; \forall s_i \in s_0 \cup S, \; \forall s_j \in S$ (9)

where $\sigma, \sigma \subseteq S$, denotes the debris set selected. Formulation (1) states the objective function. Constraints (2) and (3) ensure that a selected piece of debris cannot be removed more than once and that the OTV can initiate only one transfer at a time. From the graph theory point of view, constraints (2) and (3) make the selected edges form a path without loops. Constraint (4) assumes that the OTV initiates a transfer immediately after completing the preceding one (if any) (i.e., neglecting the time needed for the piece of debris to leave the LEO). Constraints (5) and (6) signify that all of the transfers should be finished within the given mission horizon $T$, during which the total $\Delta V$ incurred cannot exceed the upper limit $\Delta V_{\text{max}}$. Eventually, constraints (7)–(9) impose restrictions on the decision variables.

3. Dynamic model

The ADR mission in this study focuses on uncontrolled pieces of debris traveling in a LEO. Their orbits, subject to perturbations (i.e., the gravitational effect of Earth, Sun and Moon, atmospheric drag), are generally predicted using the SGP4 propagation model.\(^{15}\) Since the prediction accuracy of this dynamic model degrades over time, hence a constant model is employed instead.

Considering the main orbital perturbation to be caused by the Earth’s oblateness ($J2$ effect) due to the Earth flattening, the simplified dynamic model neglects the perturbation terms that have a lesser effect. In addition, we describe the body orbit by means of osculating orbital elements. The $J2$ perturbation causes no secular change on the semi-major axis $a$, eccentricity $e$ or inclination $i$, which it makes the RAAN $\Omega$, argument of the perigee $\omega$ and mean anomaly $M$ vary linearly over time. Given the initial orbital elements $a_0, e_0, i_0, \Omega_0, \omega_0$ and $M_0$ (i.e., $t_0 = 0$), the perturbed values can be calculated at any time $t$ using the following equations.

$$a = a_0$$ (10)

$$e = e_0$$ (11)

$$i = i_0$$ (12)

$$\Omega = \Omega_0 + \Omega \cdot t$$ (13)

$$\omega = \omega_0 + \omega \cdot t$$ (14)

$$M = M_0 + M \cdot t$$ (15)

with

$$\hat{\Omega} = -C_{J2} \cdot a^{-\frac{3}{2}} \cdot \frac{\cos i}{(1 - e^2)^{\frac{3}{2}}}$$ (16)

$$\hat{\omega} = -\frac{1}{2} \cdot C_{J2} \cdot a^{-\frac{3}{2}} \cdot \frac{1 - 5 \cos^2 i}{(1 - e^2)^{\frac{3}{2}}}$$ (17)

$$\hat{M} = n + \frac{1}{2} \cdot C_{J2} \cdot a^{-\frac{3}{2}} \cdot \frac{3 \cos^2 i - 1}{(1 - e^2)^{\frac{3}{2}}}$$ (18)

where,

$$C_{J2} = \frac{3}{2} J_2 \sqrt{\mu R_E^2},$$

$R_E$ is the Earth’s equatorial radius, $\mu$ is the Earth’s gravitational constant, $J_2$ is the second zonal harmonics of Earth’s gravitational potential and $n = \sqrt{\mu/a^3}$ is the mean motion. Clearly, the RAAN precession rate $\hat{\Omega}$, depending on $a, e, i$, is therefore constant.

3.1. Transformation from TSP-DCS to LLTC-TSP

Removal sequence optimization is analogous to the TSP-DCS, where the traveling salesman proceeds along the time-varying ADR mission graph (defined in Section 2.1), searching for a feasible open tour to maximize the cumulative values of the cities covered (i.e., pieces of debris). An open tour is called feasible if it is embedded in a Hamilton path of the ADR mission graph, and the length cannot violate neither the total mission time nor the total transfer cost constraints. When an OTV with an unlimited fuel budget is allowed to perform an ADR mission while having infinite time, the TSP-DCS is thus reduced to a classic dynamic TSP.

3.2. Removal Sequence Optimization with Optimal Rendezvous Times

Removal sequence optimization is analogous to the TSP-DCS, where the traveling salesman proceeds along the time-varying ADR mission graph (defined in Section 2.1), searching for a feasible open tour to maximize the cumulative values of the cities covered (i.e., pieces of debris). An open tour is called feasible if it is embedded in a Hamilton path of the ADR mission graph, and the length cannot violate neither the total mission time nor the total transfer cost constraints. When an OTV with an unlimited fuel budget is allowed to perform an ADR mission while having infinite time, the TSP-DCS is thus reduced to a classic dynamic TSP.
Obviously, the transfer duration is typically set as two days which is generally enough for a multiple-revolution transfer in a LEO orbit.

After both of the favorable rendezvous times and transfer durations have been obtained, they no longer play a role in determining the \( \Delta V \) for the transfers. To this end, we can consider that \( \Delta v_{i} = \Delta v_{i} \left( t_{i}, t_{j}, s_{i}, s_{j} \right) \) \( \equiv \Delta v_{i j} \), where \( s_{i}, s_{j} \in S \). In order to simplify analysis at a preliminary mission design phase and obtain an assessment of transfer cost, we also assume that the OTV in an ADR mission is equipped with a high-thrust engine, and the maneuver durations are considered to be negligible. Consequently, the optimal \( \Delta V \) for an individual transfer can be uniquely approximated.

As assumed above, debris can be removed only when the best opportunity occurs (i.e., on the favorable rendezvous dates). Therefore, a large contribution of the \( \Delta V \) for plane change is derived from the inclination difference. The angle \( \gamma \) between the two orbital planes can be computed as:

\[
\cos \gamma = \cos i_{1} \cos i_{2} + \sin i_{1} \sin i_{2} \cos \Delta \Omega
\]

The \( \Delta V \) for a plane change \( \Delta v_{\text{inter}} \) of \( \gamma \) is

\[
\Delta v_{\text{inter}} = 2v_{\text{cir}} \sin (\gamma/2)
\]

Let \( a_{\text{mean}} \) be the mean value of the semi-major axis among the two debris orbits, and \( v_{\text{cir}} \) is the corresponding local circular speed at the beginning of a maneuver. To approximate the in-plane transfer cost, we follow the idea introduced in \(^9\), that the effects of phasing can be neglected here since a sufficient transfer time is allowed. Thus, the Hohmann-based approximator is adopted for the problem of optimal fuel in-plane transfer. The \( \Delta V \) for in-plane change is defined in \(^9\) as

\[
\Delta v_{\text{intra}} = 0.5v_{\text{cir}} \sqrt{(\Delta a/a_{\text{mean}})^2 + (\Delta e)^2}
\]

This formula contains information on both the debris semi-major axis difference \( \Delta a \) and the relative eccentricity \( \Delta e \). The underlying idea is that orbits of similar shape help to minimize the \( \Delta V \) for in-plane transfer. For all of debris pairs in the three debris clouds, Fig. 3 shows the distribution of approximate \( \Delta V \) along with the amount of planar change and that required for in-plane transfer in the form of box-plots, where the estimated favorable rendezvous dates are used. As shown in Fig. 3, the plan change \( \Delta V \) medians for the individual transfers are relatively small due to the \( J_2 \) effect. Since a large number of costly transfers exist (i.e., shown as outliers with red plus sign), it is necessary to remove excessively expensive transfers (i.e., more than \( 300 \text{ m/s} \) trans-
fers to reduce the computation time.

Based on the above approximation, the $\Delta V$ required for transfer between debris pairs is fixed with the optimal transfer strategy for favorable opportunity. Therefore, the dynamic mission graph (as addressed in Section 2.1) becomes static. Motivated by the removal sequence optimization problem with the favorable rendezvous dates described above, a new variant of the TSP arises when imposing the two additional constraints: 1) a feasible tour is allowed to visit only a subset of the nodes, and 2) nodes can only be visited at favorable opportunities. Owing to these specialized constraints, we named this new variant of the problem Limited Length TSP with Time Constraint (LLTC-TSP). In order to obtain the optimal removal sequence, the static mission graph is searched for a feasible open tour having maximal cumulative node values; doing so applying visit time and tour length constraints. Since the duration of each individual travel between two nodes is positive, the successive pieces of debris selected are visited at monotone increasing times.

### 3.2. ACO heuristics proposed for LLTC-TSP

ACO\(^{18}\) is a metaheuristic framework for solving static combinatorial optimization problems. It takes inspiration from the following behavior of ant species: ants are able to find the shortest path from their home to a food source over a period of time.

ACO represents an optimization problem by a construction graph and uses $N$ artificial ants to walk on the graph where $N$ is the size of the ant colony. Each ant constructs a solution iteratively and its behavior is guided by pheromone and heuristic information. The construction graph for the sequence optimization problem is the static ADR mission graph, in which the nodes are debris and the edges are solution components. The ACO metaheuristic is shown in Algorithm 1.

#### Algorithm 1. ACO for LLTC-TSP

1. Set parameters, initialize pheromone trails
2. while The stopping criterion is not met do
3. for all ants $k = 1, \ldots, N$ do
4. choose a city $i$ randomly from the city set as the start city
5. remove the forbidden cities according to both the time constraints and tour length limitations
6. while $Can(\Pi^k) \neq \emptyset$ do
7. choose a city $j \in Can(\Pi^k)$ with probability $p_{ij}^k$
8. add the chosen city to the partial solution $\Pi^k$
9. end while
10. employ the 2-opt, insertion and swap operators to improve $\Pi^k$
11. end for
12. for all ants $k = 1, \ldots, N$ do
13. evaluate the solution $f(\Pi^k)$
14. update the best-so-far solution $\Pi^*$
15. end for
16. update pheromones on the best-so-far path
17. end while
18. return $\Pi^*$

Its main iterative procedure consists of three steps. At first, each ant acts in the same manner: starting from an empty solution $\Pi^* = \emptyset$, and the partial solution $\Pi^k$ is extended incrementally by adding a solution component from available candidates $Can(\Pi^k) \subset S$ according to the biased probabilistic rule:

$$p_{ij}^k = \begin{cases} \frac{\tau_{ij}^k \cdot \eta_{ij}^k}{\sum_{l \in Can(\Pi^k)} \tau_{il}^k \cdot \eta_{il}^k}, & \text{if solution component } l \in Can(\Pi^k) \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (24)

where $Can(\Pi^k)$ is defined as the set of debris objects that can be added to the partial sequence by the $k$-th ant without violating neither the favorable rendezvous time constraint nor the limitation of the sequence length. $\tau_{ij}$ is the pheromone of edge $(i, j)$ corresponding to the transfer from debris $s_i$ to $s_j$. The heuristic information is chosen as
The iterative process terminates when a stopping criterion is satisfied. In order to improve the exploitation capacity of ACO, we use a modified version of the MAX-MIN ant system with respect to the classic one. In our algorithm, we propose an enhanced local search strategy by utilizing 2-opt, insertion and swap operators in turn. When all of these operators cannot find a better solution, the local search stops. Otherwise, it continues until the stopping condition is met. An illustration of these operators is shown in Fig. 4.

4. Transfer Trajectory Optimization

In order to verify the approximation of rendezvous times and transfer costs, the optimal debris removal sequence is selected to enable an accurate analysis. Moreover, a more precise evaluation of mission costs is beneficial for assessing mission feasibility in a practical design.

Based on the optimal removal sequence with favorable rendezvous dates, the trajectory optimization problem is divided into a series of minimum-fuel, multiple-impulse trajectory optimization subproblems corresponding to individual transfers. Furthermore, the trajectory of each transfer leg is composed of ballistic arcs and impulses are applied at the connecting point. The problem is made complex due to the following reasons: 1) both the number of impulses and maneuvers are unknown, 2) the number of revolutions between impulses is also unknown, and 3) an orbit phase change between the OTV and target debris must be taken into account. With a trade-off between computation time and solution quality, up to three impulses are considered for each transfer leg. Due to the J2 effect, an initial coasting arc should be included prior to the first impulse to obtain a reduce the transfer cost. The final impulse is used to match the OTV velocity with the target debris velocity to complete the rendezvous.

The orbital transfer from debris \( s_i \) to \( s_j \) starts at time \( t_i \), and the complete maneuver strategy of \( i \)-th transfer leg is determined entirely by a set of six variables:

\[
P = (p_1, p_2, p_3, p_4, p_5, p_6)
\]

where \( p_4, p_5, p_6 \) define the times at which impulses are applied. Note that \( t_j \) is allowed a minor adjustment around the theoretical favorable rendezvous date \( t^*_j \), \( p_1 \) is the characteristic velocity of the first impulse, and its direction is determined by \( p_2 \) and \( p_3 \). Clearly, the first impulse vector is

\[
\Delta v_1 = p_1 (\cos(p_2) \cos(p_3), \cos(p_2) \sin(p_3), \sin(p_2))
\]

As described in Section 2.3, the orbital trajectory between impulses can be determined according to the dynamic model. Given both the specified transfer time and orbital boundary conditions of the last arc from impulse 2 to impulse 3, the velocity changes required \( \Delta v_2 \) and \( \Delta v_3 \) are obtained by solving a perturbed Lambert’s problem with an iterated scheme to correct solutions of the unperturbed Lambert’s problem. The transfer trajectory optimization problem aims to minimize the total characteristic velocity (i.e., equivalent to propellant cost) in each leg:

\[
J = \Delta v_1 + \Delta v_2 + \Delta v_3
\]

To resolve the trajectory optimization problem of each transfer leg, a metaheuristic that extends ACO to continuous domains (ACO\(_R\)) is adopted\(^{19}\). ACO\(_R\) has proved to be a competitive approach compared to other continuous optimization methods. Due to its higher efficiency and robustness, ACO\(_R\) is increasingly adopted to tackle many complex real-world problems in continuous domains; however, it has not yet been used for aerospace trajectory design. ACO\(_R\) follows the incremental construction characteristics of the ACO: an ant samples the problem-based continuous probability density function for each continuous variable. Then, unlike ACO, the pheromone table is modified based on the components of good solutions to keep a history of its search process implicit, ACO\(_R\) updates the solution archive as a way of maintaining an explicit memory of the search history. The details and the parameter settings of the ACO\(_R\) can be found in Socha and Dorigo\(^{19}\).

5. Case Studies

In order to test the approach proposed here, we use the
three debris clouds introduced (Iridium 33, Cosmos 2251 and Fengyun 1C) to create ten problem instances with different levels of complexity. The cosmos and iridium 33 cases were generated by downloading orbital parameters of the whole debris clouds from the NORAD catalog, while the $K$ cases were generated considering only the largest $K$ objects. Objects are identified using their NORAD catalog numbers in the following.

5.1. Optimal removal sequences obtained using the ACO heuristics proposed

This section discusses the determination of the optimal removal sequence using the ACO heuristics proposed, during which the transfer cost estimated applying the favorable rendezvous date is maintained. Since the inver-over algorithm has proven to be the current state-of-the-art method for ADR sequence optimization problems, as stated in Izzo et al., a comparison between the ACO heuristics proposed and the inver-over algorithm is considered here. The two metaheuristics were coded in C++ and tested on a computer equipped with a 3.1 GHz Intel Core i5 CPU and 4 GB of RAM running on the Windows 7 operating system. For each instance, the metaheuristics were executed 10 times independently, and only the best results are reported.

All parameter values for ACO were set empirically after preliminary experiments using the instances generated in this study. The number of ants $N$ was set to 10, and the relative importance of pheromone and heuristic factors $\alpha, \beta$ were set to 1 and 0.8, respectively. The pheromone evaporation ratio $\rho$ was 0.95. The bias parameters $\Lambda_1, \Lambda_2$ used for initializing heuristic information were set to 0.2 and 0.125. The maximum number of iterations was set to 1000. We implemented the inver-over algorithm using the characteristics and parameters available in Izzo et al.

A mission to remove the most valuable debris within 1000 a total m/s $\Delta V$ in 365 days is consider here. The goal is to earn maximal cumulative removal profits. Notice that solving the binary integer programming problem addressed in Section 2.2 yields not only the optimal removal sequence, but also the theoretical favorable time schedule for visiting the pieces of debris selected. Table 1 reports both the properties of the problem instances from columns 2–3 and the best results of these algorithms from columns 4–11. The best values are shown in bold. As seen from Table 1, ACO obtains the best results for seven instances. Therefore, in terms of the number of best solutions, ACO outperforms inver-over algorithm. The performance gap becomes larger as the problem scale increases.

5.2. Optimal trajectories obtained applying the ACO$_R$ heuristics proposed

In order to obtain a reliable cost assessment, the exact characteristics (i.e., transfer cost and rendezvous date) of each transfer leg in the removal sequence obtained were verified using the ACO$_R$ developed by Socha and Dorigo. The debris order, as well as the initial mission date, were fixed. However, for each leg, the rendezvous date was re-optimized near the estimated value in order to obtain a better transfer cost $\Delta V$. The maximum number of objective function evaluations was set to 1000 for this analysis, leading to a computational time of approximately 2 minutes.

Taking the optimal removal sequence in the case of Fengyun.2000 for example, the mission planning after the re-optimization of dates and maneuvers is detailed in Table 2. Due to the stochastic behavior of ACO$_R$, ten optimization runs were performed for each leg, and the best result is presented.

The transfer legs using the purely drift RAAN correction strategy are marked with bold values. Estimations of the favorable rendezvous dates using this strategy are often extremely accurate, and usually lower $\Delta V$ is obtained compared to the estimated value; especially for cases requiring larger $\Delta V$. The seven underlined transfer legs indicate that the purely drift RAAN correction strategies are performed. During such transfers, the respective $\Delta \Omega$ between the OTV and the debris are quite small, leading to a rather lower $\Delta V$ requirement for orbital plane change. A slightly larger cost is usually found during the transfer leg when using the hybrid RAAN correction method due to the fact that the rendezvous is later or sooner than the time at which the RAAN coincides, and thus additional RAAN adjustments are introduced. RAAN evolution and inclination are shown in Fig. 5, where the intermediate rendezvous points are highlighted, while the specific times at which the impulses are applied are marked by black square. The plane change maneuvering is better understood by RAAN and inclination. It can be clearly observed from Fig. 5(a) that most of the change in RAAN required is nullified with the effect of $J_2$ perturbation, and only minor adjustments are obtained with the impulse.

| Name      | Debris cloud | Size | A, m$^2$ | $\Delta V$, m/s | $T$, days | #debris | A, m$^2$ | $\Delta V$, m/s | $T$, days | #debris |
|-----------|--------------|------|----------|-----------------|-----------|---------|----------|-----------------|-----------|---------|
| iridium33.100 | Iridium 33 | 100  | 4.7118   | 971.34          | 316.34    | 13      | 4.7118   | 971.28          | 352.69    | 13      |
| iridium33.200 | Iridium 33 | 200  | 4.7381   | 977.94          | 352.69    | 16      | 4.7381   | 977.94          | 352.69    | 16      |
| iridium33 | Iridium 33 | 324  | 5.1856   | 974.75          | 351.98    | 14      | 4.7951   | 979.42          | 352.69    | 18      |
| cosmos.300  | Cosmos 2251 | 300  | 2.9417   | 990.25          | 361.41    | 13      | 2.9417   | 977.06          | 354.47    | 13      |
| cosmos.500  | Cosmos 2251 | 500  | 2.9456   | 999.04          | 330.48    | 14      | 2.9417   | 977.06          | 354.47    | 13      |
| cosmos     | Cosmos 2251 | 1040 | 3.0480   | 975.35          | 354.47    | 16      | 2.9773   | 989.01          | 354.30    | 17      |
| fengyun.500 | Fengyun 1C | 500  | 2.3671   | 995.39          | 364.83    | 20      | 2.1779   | 993.05          | 355.56    | 19      |
| fengyun.1000 | Fengyun 1C | 1000 | 2.4201   | 997.54          | 362.83    | 23      | 2.2119   | 999.05          | 364.75    | 19      |
| fengyun.1500 | Fengyun 1C | 1500 | 2.4256   | 997.45          | 364.83    | 23      | 2.2256   | 997.62          | 364.85    | 17      |
| fengyun.2000 | Fengyun 1C | 2000 | 2.4825   | 994.83          | 351.78    | 22      | 2.2392   | 976.45          | 362.83    | 17      |
instead. Moreover, the pieces of debris selected for the optimal sequence have a similar inclination, and it is gradually adjusted to match the target value.

6. Conclusions

In this paper, we discuss the planning of an object removal mission for multiple pieces of debris in space. Our proposal involves a two-staged procedure. First, a global search is performed to obtain the optimal removal sequence. After the transfer costs and times are estimated using a refined approximator, the sequence optimization problem is formulated as a LLTC-TSP. The problem is then resolved using an enhanced ACO. Using instances containing hundreds of pieces of debris, a fair comparison was made between applying our enhanced ACO and the evolutionary inver-over method, which was the most competitive algorithm used to solve similar problems. According to computational results, the enhanced ACO achieves very promising results. Especially, it can find new best solutions for seven out of ten chosen instances. Furthermore, all transfer legs in the optimal sequence selected are precisely optimized with a continuous ACO. Our results open the possibility of constructing more efficient and reliable searches using ACO for the preliminary design of debris removal missions.

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