Nanoscale quantum-dot devices are a formidable tool for probing the inherent quantum-mechanical nature of electrons. Manifestations of quantum electronic properties in these devices include wave interference in Aharonov-Bohm (AB) rings and many-body phenomena such as the Kondo effect (the screening of a localized magnetic moment by conduction electrons) and quantum phase transitions (QPTs). The interplay between quantum interference and the Kondo effect can be studied by inserting a quantum dot in an AB ring, as shown both experimentally and theoretically.

This Letter focuses on a system in which two quantum dots are embedded in the same AB ring. Here, we consider instead a device in which the presence of one, effectively noninteracting level in resonance with the leads, causes the Kondo temperature to range over many orders of magnitude. This two-dot AB device can also realize the conditions necessary for observation of the pseudogap Kondo effect, in which coupling of a magnetic impurity to a power-law-vanishing density of conduction states gives rise to a pair of QPTs between Kondo ($T_K > 0$) and non-Kondo ($T_K = 0$) phases. Pseudogap Kondo physics has previously been predicted to occur in double-quantum-dot devices, but the ring geometry of the present setup allows a deeper exploration of the interplay between coherent quantum interference and the Kondo effect. The conductance and transmission phase shifts reflect a nontrivial interplay between wave interference and interactions, providing clear signatures of quantum phase transitions between Kondo and non-Kondo ground states.

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and ε_{1,0} destroys a spin-σ electron of wave vector k and energy ε_{1,0} in lead ℓ (ℓ = L, R). Each lead is assumed to have a constant density of states ρ(ε) = ρ_0 Θ(D - |ε|), as well as a local (k-independent) coupling to the dots. The gauge degree of freedom allows one to write W_{1L} = V_{1L} e^{i ω/4}, W_{1R} = V_{1R} e^{−i ω/4}, W_{2L} = V_{2L} e^{−i ω/4}, and W_{2R} = V_{2R} e^{i ω/4} where V_{ij} is real. For simplicity, we consider symmetric couplings V_{jR} = V_{jL} ≡ V_j/√2.

At small bias and low temperatures, transmission through an interacting system can be described by a Landauer-like formula \[ G = \frac{2e^2}{h} \int dω \left( \frac{∂f}{∂ω} \right) |t_{LR}(ω)|^2, \]
where \( f(ω, T) \) is the Fermi function at energy ω (measured from the Fermi level) and temperature T, and \( t_{LR}(ω) = 2πρ_0 \sum_{ij} W^*_{ij} G_{ij}(ω) W_{jR} \) is the transmission coefficient. Here, \( G_{ij}(ω) = -i \int_0^∞ dt e^{i ω t} \langle \{ a^d_σ(t), a^a_σ(0) \} \rangle \) is a standard retarded Green’s function.

The dot-1 Green’s function (calculated in the presence of dot 2 and the leads, and taking the U_1 interaction into full account) can formally be written \( G_{11}(ω) = [ω - ε_1 - \Sigma_{11}^*(ω) - \Sigma_{11}^{(0)}(ω)]^{-1} \), where \( \Sigma_{11} \) and \( \Sigma_{11}^{(0)} \) are, respectively, the interacting and noninteracting contributions to the self-energy. Standard equations of motion techniques can be used to express the remaining \( G_{ij} \)'s in terms of \( G_{11} \) and known quantities, and to obtain the result

\[ \Sigma_{11}^{(0)} = \sum_{i,k} |W_{1i}|^2 + \sum_{i,\ell,\ell',k,k'} W_{i\ell} W^*_{\ell'i} \frac{1}{\omega - ε_{k\ell} - \omega + i Δ_2 / 2} W_{i\ell'} W^*_{\ell'i}, \]

where \( Δ_2 = πρ_0 V_1^2 \). The first term in Eq. (4) describes the effect on dot 1 of coupling purely to the leads, while the second term represents an indirect coupling between the dots. In the wide-band limit |ω| ≪ D, these processes combine to yield an energy-dependent hybridization function \( \Delta(ω) = \epsilon_2 \Theta(D - |ω|) \) for \( ε_2 = 0 \) and different values of \( Φ \).

Variation of the Kondo scale.—Figure 2 shows the dot-1 spectral density \( A_{11}(ω) = -π^{-1} \text{Im} G_{11}(ω) \) for several \( Φ \) values at the special point \( ε_1 = -U_1/2, ε_2 = 0 \) where the system exhibits strict particle-hole (p-h) symmetry. For a general flux, \( A_{11}(ω) \) features a Kondo resonance centered on \( ω = 0 \). For \( Φ = nΦ_0 \), however, \( A_{11}(ω) \) vanishes at \( ω = 0 \), signaling suppression of the Kondo effect by the pseudogap in \( ρ_{\text{eff}}(ω) \).

The Kondo resonance width is proportional to the Kondo temperature \( T_K \), which we define in terms of the
impurity susceptibility via the condition $T_K \chi_{\text{imp}}(T_K) = 0.0701$ \[16\]. $T_K$ values varying over three orders of magnitude under an applied magnetic field have been predicted for small AB rings containing one quantum dot \[4\]. The present setup can greatly amplify this variation. For $|\varepsilon_2| \gtrsim \Delta_2$ [see, e.g., Fig. 2(b)], the dip in $\rho_{\text{eff}}(\omega)$ around $\omega = \varepsilon_2$ produces only a weak field-modulation of $T_K$. The range of $T_K$ is much greater for $|\varepsilon_2| \lesssim \Delta_2$.

In the extreme case $\varepsilon_2 = 0$ [Fig. 2(c)], $T_K$ varies from $T_{K0}$ for $\Phi = (n + \frac{1}{2})\Phi_0$ to zero for the pseudogap case $\Phi = n\Phi_0$. Here and below, $T_{K0} = T_K(\varepsilon_1 = \frac{1}{2}U_1, \Phi = \frac{1}{2}\Phi_0) \approx 7 \times 10^{-4}D$ is a characteristic Kondo scale for dot 2 in the absence of dot 1.

Quantum phase transitions.—As noted in the introduction, the presence of a pseudogap in $\rho_{\text{eff}}(\omega)$ gives rise to a pair of QPTs separating Kondo and local-moment phases \[12\, 13\]. These QPTs occur in the double-dot AB setup for $\Phi = n\Phi_0$ and $\varepsilon_2 = 0$ when $\varepsilon_1$ is tuned to one of two critical values $\varepsilon_{c1}^\pm$. The paragraphs below describe how the system can be brought into the vicinity of one of these zero-temperature transitions by measuring the transmission phase shift $\theta_i(\Phi)$ and/or the conductance $g(\Phi)$ at relatively high temperatures of order $T_{K0}$.

The first step in reaching the QPT is to bring the dot-2 level $\varepsilon_2$ to the Fermi energy. We find that this can be most efficiently accomplished by monitoring $\theta_i(\Phi)$. Figure 3(a) plots $\theta_i$ at $T = 0.59T_{K0}$ over the range $0 \leq \Phi < \Phi_0$ for $\varepsilon_1 = -U_1/2$ and various values of $\varepsilon_2$. The most striking feature is the linear variation of $\theta_i$ with $\Phi$, which can be used to identify the target case $\varepsilon_2 = 0$. The origin of this linearity can be seen most readily at $T = 0$, where for $\varepsilon_2 = 0$, $\theta_i = \pi(\Phi/\Phi_0 - \frac{1}{2}) + \theta_i$, with $\theta_i = 0$. Everywhere else, a conventional Kondo ground state forms. The special case $\varepsilon_1 = -U_1/2$ and $\varepsilon_2 = 0$ shown in Fig. 3(a) exhibits an exact $p-h$ symmetry that ensures $\text{Re} G_{11}(0) = 0$ and $\theta_i = 0$ for all $\Phi$.

Figure 3(a) also reveals interesting features away from $\varepsilon_2 = 0$. For large $|\varepsilon_2|$, $\theta_i$ evolves with increasing $\Phi$ to pass through $-\pi$ from above; since phase shifts $\theta_i$ and $\theta_i \pm 2\pi$ are equivalent, any such curve can instead be plotted with a phase jump from $-\pi$ to $\pi$, so that in all cases, $\theta_i(\Phi_0) = \theta_i(0) + \pi$. Around $\varepsilon_2 = \varepsilon_2^{pl}$, “phase lapses” $\Delta \theta_i \simeq \pm \pi$ (not $\pm 2\pi$) appear over narrow ranges of $\Phi$ \[17\]. For $\varepsilon_2 > \varepsilon_2^{pl}$, $\theta_i$ does not pass through $\pm \pi$, but rather varies smoothly between $\theta_i(0)$ and $\theta_i(\Phi_0) = \theta_i(0) + \pi$.

For general $\varepsilon_1$, $\varepsilon_2$, and $T$, $\theta_i = \theta_i - \pi(\Phi/\Phi_0 - \frac{1}{2})$ is small whenever $T < T_K$ and is appreciably nonzero for $T_K \lesssim T$. This is illustrated in Fig. 3(b), which plots the phase shift at $T = 0.59T_{K0}$ for $\varepsilon_2 = 0$ and different values of $\varepsilon_1$. In each case, the Kondo temperature vanishes for $\Phi = n\Phi_0$ and reaches its maximum value $T_{K\text{,max}}$ at $\Phi = (n + \frac{1}{2})\Phi_0$. With increasing $p-h$ asymmetry (increasing $|\varepsilon_1 + U_1/2|$), $T_{K\text{,max}}$ decreases and the points of first noticeable deviation from linearity in $\theta_i$ vs $\Phi$ move closer to $\Phi = n\Phi_0$.

These results suggest an experimental procedure for tuning to the pseudogap: Measure $\theta_i$ vs $\Phi$ for dot-2 plunger gate voltages, holding all other parameters constant, and seek to maximize the range of fluxes around $\Phi = n\Phi_0$ over which the phase shift satisfies $\theta_i = 0$. If one has truly found the dot-2 gate voltage corresponding to $\varepsilon_2 = 0$, it should in general be possible to increase the flux range over which $\theta_i = 0$ by stepping the plunger gate voltage on dot 1 until one achieves $\varepsilon_1 \simeq -U_1/2$.

Once the dot-2 level is locked at the Fermi level, the system can be steered through (or, at any $T > 0$, above) a QPT by further fine-tuning of $\varepsilon_1$, guided by measurements of $g(\Phi)$ and $\theta_i(\Phi)$. We focus on the QPT at $\varepsilon_1 = \varepsilon_1^\pm$, where $-U_1/2 < \varepsilon_1^\pm < 0$, and define $\Delta \varepsilon_1 = \varepsilon_1^+ - \varepsilon_1^-$. (A $p-h$ transformation maps the system from $\varepsilon_{c1}^+$ to the other QPT at $\varepsilon_{c1}^- = -U_1 - \varepsilon_{c1}^\pm$.) As illustrated in Fig. 4, the properties at temperatures of order $T_{K0}$ reveal clear signatures of the $T = 0$ transition between the local-moment ($\Delta \varepsilon_1 < 0$ and $\Phi = n\Phi_0$) and Kondo ($\Delta \varepsilon_1 > 0$ and/or $\Phi \neq n\Phi_0$) phases.

At $\Delta \varepsilon_1 = 0$ and $\Phi = n\Phi_0$, the finite-temperature conductance reaches a near-unitary value $g \simeq g_0$ [Fig. 4(a)] for $n = 0$ while the transmission phase shift $\theta_i = \pi/2$ [Fig. 4(b)]. However, these characteristics may not be reliable experimental locators for the underlying QPT because absolute measurements of $g$ or $\theta_i$ may be complicated by contributions from additional (spurious) channels or by the presence of stray external flux that prevents accurate identification of the point $\Phi = n\Phi_0$.

The derivatives of the transport properties with respect to applied flux provide a superior method for locating the transition. The critical value $\Delta \varepsilon_1 = 0$ is distinguished by two features around the pseudogap loca-
lar behavior (not shown) occurs at small but nonzero temperatures of order the characteristic Kondo scale of quantum of flux, the effective density of states vanishes at the Fermi energy and the setup maps onto a pseudogap regime, where the phase shift and the linear conductance exhibit clear finite-temperature signatures of underlying zero-temperature phase transitions.

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verses the complex plane from quadrant IV to quadrant II, passing via quadrant III for $\varepsilon_2 < \varepsilon_2^{pl}$, via quadrant I for $\varepsilon_2 > \varepsilon_2^{pl}$, and directly through the origin for $\varepsilon_2 = \varepsilon_2^{pl}$. 