Non-conformal higher spin supercurrents

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Abstract

In four spacetime dimensions there exist two off-shell formulations for the massless multiplet of superspin \((s+\frac{1}{2})\), where \(s = 2, 3, \ldots\). These supersymmetric higher spin gauge theories, known as longitudinal and transverse, are dual to each other and describe two massless fields of spin \((s+\frac{1}{2})\) and \((s+1)\) upon elimination of the auxiliary fields. They respectively reduce, in the limiting case of \(s = 1\), to the linearised actions for the old minimal and the \(n = -1\) non-minimal \(\mathcal{N} = 1\) supergravity theories. Associated with these gauge massless theories are non-conformal higher spin supercurrent multiplets which we describe. We demonstrate that the longitudinal higher spin supercurrents are realised in the model for a massive chiral scalar superfield only if \(s\) is odd, \(s = 2n + 1\), with \(n = 1, 2, \ldots\).
1 Introduction

Supercurrent [1] is one of the fundamental concepts in supersymmetric field theory, for it contains the energy-momentum tensor and the supersymmetry current(s). In the case of superconformal field theories, the supercurrent is unique. In particular, the \( \mathcal{N} = 1 \) conformal supercurrent in four dimensions is a real vector superfield \( J_{\dot{a}a} \) subject to the conservation equation [1]

\[
\bar{D}^{\dot{a}} J_{a\dot{a}} = 0 \quad \iff \quad D^a J_{a\dot{a}} = 0 .
\]

(1.1)

As an example, we consider the superconformal model for a massless chiral scalar \( \Phi \), \( \bar{D}_{\dot{a}} \Phi = 0 \), with action

\[
S_{\text{massless}} = \int d^4x d^2 \theta d^2 \bar{\theta} \Phi \Phi .
\]

(1.2)

It is characterised by the supercurrent [1]

\[
J_{\dot{a}a} = D_a \Phi \bar{D}_{\dot{a}} \Phi + 2i(\Phi \partial_{\dot{a}} \Phi - \partial_a \Phi \bar{\Phi}) .
\]

(1.3)

The conservation equation (1.1) follows if one makes use of the equations of motion \( D^2 \Phi = 0 \) and \( \bar{D}^2 \bar{\Phi} = 0 \).

In the non-superconformal case, however, the conservation equation (1.1) is replaced by a deformed one. Such a deformation is triggered by a trace supermultiplet containing the trace of the energy-momentum tensor and the \( \gamma \)-trace of the supersymmetry current(s). In general there exist several consistent deformations, and therefore several inequivalent supercurrents. This means that the problem of classifying inequivalent supercurrent multiplets needs to be addressed. A simple approach to achieve this is to make use of the observation that consistent supercurrents are automatically associated with linearised off-shell supergravity actions [1]. Given a linearised off-shell action for \( \mathcal{N} = 1 \) supergravity, the supercurrent conservation equation is obtained by coupling the supergravity prepotentials to external sources and then demanding the resulting action to be invariant under the linearised supergravity gauge transformations. Since the linearised off-shell \( \mathcal{N} = 1 \) supergravity actions have been classified [5], all minimal consistent supercurrents are readily derivable [2]. Reducible supercurrents, such as the \( S \)-multiplet of [6], can be obtained by combining some of the minimal ones.

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1 This approach is explained in detail in [2] [3] for the cases of \( \mathcal{N} = 1 \) and \( \mathcal{N} = 2 \) supersymmetric theories in four dimensions, and in [4] for \( \mathcal{N} = 2 \) supersymmetric theories in three dimensions.
The Ferrara-Zumino supercurrent [1] is described by the conservation equation
\[ \bar{D} \dot{\alpha} J_{\dot{\alpha}\dot{\alpha}} = D_{\alpha} T , \quad \bar{D}_\dot{\alpha} T = 0 . \] (1.4)
It corresponds to the old minimal formulation [7, 8, 9] for \( \mathcal{N} = 1 \) supergravity. An example of a supersymmetric theory in which this supercurrent is realised is the massive chiral model
\[ S_{\text{massive}} = \int d^4 x d^2 \theta d^2 \bar{\theta} \Phi \bar{\Phi} + \left\{ \frac{m}{2} \int d^4 x d^2 \theta \Phi^2 + \text{c.c.} \right\} . \] (1.5)
For this model, \( J_{\dot{\alpha}\dot{\alpha}} \) can be chosen to have the same functional form as in the massless case, eq. (1.3). The trace multiplet is then given by
\[ T = m \Phi^2 . \] (1.6)
The higher spin extension of the conformal supercurrent (1.1) was given in [11]. It is
\[ D_{\beta} J_{\dot{\alpha}_1...\dot{\alpha}_{s+1} \dot{\alpha}_{s+2}...\dot{\alpha}_{s+1}} = 0 \iff \bar{D}_{\dot{\beta}} J_{\dot{\alpha}_1...\dot{\alpha}_{s} \dot{\alpha}_{s+1} \dot{\alpha}_{s+2}...\dot{\alpha}_{s+1}} = 0 , \] (1.7)
where \( J_{\alpha(s)\dot{\alpha}(s)} = J_{\dot{\alpha}_1...\dot{\alpha}_s} = J_{\dot{\alpha}_1...\dot{\alpha}_s}(\dot{\alpha}_1...\dot{\alpha}_s) \) is a real superfield. This conservation equation is superconformal provided the supercurrent \( J_{\alpha(s)\dot{\alpha}(s)} \) is superconformal primary of weight \( 1 + \frac{s}{2}, 1 + \frac{s}{2} \) [12]. The higher spin extension of the massless supercurrent (1.3) was given in [12]. It is
\[ J_{\alpha(s)\dot{\alpha}(s)} = (2i)^{s-1} \sum_{k=0}^{s} (-1)^k \binom{s}{k} \times \left\{ \left( \binom{s}{k+1} \right) \partial_{(\dot{\alpha}_1...\dot{\alpha}_k)D_{\dot{\alpha}_{k+1}} \Phi \bar{D}_{\dot{\alpha}_{k+1}} \partial_{\dot{\alpha}_{k+2}...\dot{\alpha}_s} \bar{\Phi}} + 2i \left( \binom{s}{k} \right) \partial_{(\dot{\alpha}_1...\dot{\alpha}_k) \Phi \partial_{\dot{\alpha}_{k+1}...\dot{\alpha}_s} \bar{\Phi} \right\} . \] (1.8)
This supercurrent was also re-derived in a revised version (v2, 10 Oct) of [13].

To obtain higher spin extensions of non-conformal supercurrents, one can make use of the known gauge off-shell formulations for massless higher spin supermultiplets. Such formulations were developed in the early 1990s in Minkowski superspace [14, 15] and anti-de Sitter superspace [16] (see [17] for a pedagogical review of the results of [14, 15]). In section 2 we briefly review the two formulations for each massless multiplet of half-integer superspin \( s + 1/2 \), with \( s = 2, 3, ... \). In section 3 we present the non-conformal higher spin supercurrent multiplets associated with these gauge theories. As an application, we derive all higher spin supercurrents for the massive model (1.5).

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\( ^2 \)This follows from the fact that the gravitational superfield does not couple to the superpotential [10].
2 Massless half-integer superspin multiplets

For a massless multiplet of half-integer superspin $s + 1/2$, with $s = 2, 3, \ldots$, there exist two off-shell formulations \[14\] which are referred to as transverse and longitudinal. They are described in terms of the following dynamical variables:

\[\mathcal{V}_{s+1/2}^\perp = \left\{ H_{\alpha(s)\bar{\alpha}(s)} ; \Gamma_{\alpha(s-1)\dot{\alpha}(s-1)} ; \bar{\Gamma}_{\alpha(s-1)\dot{\alpha}(s-1)} \right\} , \quad (2.1a)\]
\[\mathcal{V}_{s+1/2}^\parallel = \left\{ H_{\alpha(s)\bar{\alpha}(s)} ; G_{\alpha(s-1)\dot{\alpha}(s-1)} ; \bar{G}_{\alpha(s-1)\dot{\alpha}(s-1)} \right\} . \quad (2.1b)\]

Here $H_{\alpha(s)\bar{\alpha}(s)}$ is a real unconstrained superfield. The complex superfields $\Gamma_{\alpha(s-1)\dot{\alpha}(s-1)}$ and $G_{\alpha(s-1)\dot{\alpha}(s-1)}$ are transverse linear and longitudinal linear in the sense that they obey the constraints\[\]

\[D^\dot{\beta} \Gamma_{\alpha(s-1)\dot{\alpha}(s-2)} = 0 \implies D^2 \Gamma_{\alpha(s-1)\dot{\alpha}(s-1)} = 0 , \quad (2.2a)\]
\[\bar{D}_{\dot{\alpha}_1} G_{\alpha(s-1)\dot{\alpha}_2...\dot{\alpha}_{s-1}} = 0 \implies \bar{D}^2 G_{\alpha(s-1)\dot{\alpha}(s-1)} = 0 , \quad (2.2b)\]

These constraints can be solved in terms of unconstrained prepotentials as follows:

\[\Gamma_{\alpha(s-1)\dot{\alpha}(s-1)} = D^\dot{\beta} \Phi_{\alpha(s-1)}(\dot{\beta}\dot{\alpha}_1...\dot{\alpha}_{s-1}) , \quad (2.3a)\]
\[G_{\alpha(s-1)\dot{\alpha}(s-1)} = \bar{D}_{\dot{\alpha}_1} \Psi_{\alpha(s-1)}(\dot{\alpha}_1\dot{\alpha}_2...\dot{\alpha}_{s-1}) . \quad (2.3b)\]

The prepotentials are defined modulo gauge transformations of the form:

\[\delta_{\xi} \Phi_{\alpha(s-1)\dot{\alpha}(s)} = D^\dot{\beta} \xi_{\alpha(s-1)}(\dot{\beta}\dot{\alpha}_1...\dot{\alpha}_s) , \quad (2.4a)\]
\[\delta_{\xi} \Psi_{\alpha(s-1)\dot{\alpha}(s-2)} = \bar{D}_{(\dot{\alpha}_1} \xi_{\alpha(s-1)}(\dot{\alpha}_2...\dot{\alpha}_{s-2}) \) , \quad (2.4b)\]

with the gauge parameters $\xi_{\alpha(s-1)\dot{\alpha}(s+1)}$ and $\xi_{\alpha(s-1)\dot{\alpha}(s-3)}$ being unconstrained.\[3\]

The gauge transformations of the superfields $H$, $\Gamma$ and $G$ are

\[\delta_{\Lambda} H_{\alpha_1...\alpha_{s+1}\dot{\alpha}_1...\dot{\alpha}_s} = \bar{D}_{(\dot{\alpha}_1} \Lambda_{\alpha_1...\alpha_s\dot{\alpha}_2...\dot{\alpha}_s) \dot{\alpha}_1...\dot{\alpha}_s , \quad (2.5a)\]
\[\delta_{\Lambda} \Gamma_{\alpha_1...\alpha_{s-1}\dot{\alpha}_1...\dot{\alpha}_{s-1}} = -\frac{1}{4} \bar{D}^\beta D^\dot{\beta} \Lambda_{\alpha_1...\alpha_{s-1}\dot{\alpha}_1...\dot{\alpha}_{s-1}} , \quad (2.5b)\]
\[\delta_{\Lambda} G_{\alpha_1...\alpha_{s-1}\dot{\alpha}_1...\dot{\alpha}_{s-1}} = -\frac{1}{2} \bar{D}_{(\dot{\alpha}_1} \bar{D}^{(\beta]} D^\beta \Lambda_{\beta\alpha_1...\alpha_{s-1}\dot{\alpha}_2...\dot{\alpha}_{s-1})\dot{\beta}} \]

\[3\]More generally, complex tensor superfields $\Gamma_{(\alpha(r)\dot{\alpha}(t)}$ and $G_{(\alpha(r)\dot{\alpha}(t)}$ are called transverse linear and longitudinal linear, respectively, if the constraints $\bar{D}^\beta \Gamma_{(\alpha(r)\dot{\alpha}(t-1)} = 0$ and $\bar{D}_{(\dot{\beta}} G_{(\alpha(r)\dot{\alpha}(t))} = 0$ are satisfied. The former constraint is defined for $t \neq 0$; it has to be replaced with the standard linear constraint, $\bar{D}^2 \Gamma_{(\alpha(r)} = 0$, for $t = 0$. The latter constraint for $t = 0$ is the chirality condition $\bar{D}_{(\dot{\beta}} G_{(\alpha(r)} = 0$.

\[4\]For $s = 2$ the gauge transformation law \[2.4b\] has to be replaced with $\delta \Psi = \zeta$, with the gauge parameter $\zeta$ being chiral, $\bar{D}_{\dot{\alpha}} \zeta = 0$. 

3
transformation of $\Gamma$ is

$$+i(s-1)\bar{D}_{(\alpha_1}\bar{D}^{\beta]}\Lambda_{\alpha_1\ldots \alpha_{s-1}\hat{\alpha}_2\ldots \hat{\alpha}_{s-1})\beta} .$$

(2.5c)

Here the gauge parameter $\Lambda_{\alpha_1\ldots \alpha_s\hat{\alpha}_1\ldots \hat{\alpha}_{s-1}} = \Lambda_{(\alpha_1\ldots \alpha_s)(\hat{\alpha}_1\ldots \hat{\alpha}_{s-1})}$ is unconstrained. The symmetrisation in (2.5c) is extended only to the indices $\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_{s-1}$. It follows from (2.5b) and (2.5c) that the transformation laws of the prepotentials $\Phi_{\alpha(s-1)\hat{\alpha}(s)}$ and $\Psi_{\alpha(s-1)\hat{\alpha}(s-2)}$ are

$$\delta_{\Lambda} \Phi_{\alpha_1\ldots \alpha_s\hat{\alpha}_1\ldots \hat{\alpha}_s} = -\frac{1}{4} D^2 \Lambda_{\alpha_1\ldots \alpha_s\hat{\alpha}_1\ldots \hat{\alpha}_s} ,$$

(2.6a)

$$\delta_{\Lambda} \Psi_{\alpha_1\ldots \alpha_{s-1}\hat{\alpha}_1\ldots \hat{\alpha}_{s-2}} = -\frac{1}{2} \left( \bar{D}^{\beta} D^\beta - 2i(s-1)\partial^\beta \right) \Lambda_{\beta\alpha_1\ldots \alpha_{s-1}\beta\hat{\alpha}_1\ldots \hat{\alpha}_{s-2}} .$$

(2.6b)

In the transverse formulation, the action invariant under the gauge transformations (2.5a) and (2.5b) is

$$S^+_{s+1/2}[H, \Gamma, \bar{\Gamma}] = \left( -\frac{1}{2} \right)^s \int d^4xd^2\theta d^2\bar{\theta} \left\{ \frac{1}{8} H^{\alpha(s)\hat{\alpha}(s)} D^\beta D^\beta H_{\alpha(s)\hat{\alpha}(s)}
\right.$$  

$$+ H^{\alpha(s)\hat{\alpha}(s)} \left( D_{\alpha_s} \bar{D}_{\hat{\alpha}_s} \Gamma_{\alpha(s-1)\hat{\alpha}(s-1)} - \bar{D}_{\hat{\alpha}_s} D_{\alpha_s} \bar{\Gamma}_{\alpha(s-1)\hat{\alpha}(s-1)} \right)
\left. + \left( \bar{\Gamma} \cdot \Gamma + \frac{s+1}{s} \bar{\Gamma} \cdot \Gamma + \text{c.c.} \right) \right\} .$$

(2.7)

In the longitudinal formulation, the action invariant under the gauge transformations (2.5a) and (2.5c) is

$$S^\|_{s+1/2}[H, G, \bar{G}] = \left( -\frac{1}{2} \right)^s \int d^4xd^2\theta d^2\bar{\theta} \left\{ \frac{1}{8} H^{\alpha(s)\hat{\alpha}(s)} D^\beta D^\beta H_{\alpha(s)\hat{\alpha}(s)}
\right.$$  

$$- \frac{1}{8} \frac{s}{2s+1} \left( \left[ D_{\gamma}, \bar{D}_{\bar{\gamma}} \right] H^{\gamma\alpha(s-1)\hat{\gamma}\hat{\alpha}(s-1)} \right) \left[ D^\beta, \bar{D}^{\bar{\beta}} \right] H^{-\beta\alpha(s-1)\hat{\beta}\hat{\alpha}(s-1)}
\right.$$  

$$+ \frac{s}{2} \left( \partial_\gamma H^{\gamma\alpha(s-1)\hat{\gamma}\hat{\alpha}(s-1)} \right) \partial^{\bar{\beta}\bar{\gamma}} H^{-\beta\alpha(s-1)\hat{\beta}\hat{\alpha}(s-1)}
\right.$$  

$$+ 2i \frac{s}{2s+1} \partial_{\gamma\bar{\gamma}} H^{\gamma\alpha(s-1)\hat{\gamma}\hat{\alpha}(s-1)} \left( G_{\alpha(s-1)\hat{\alpha}(s-1)} - \bar{G}_{\alpha(s-1)\hat{\alpha}(s-1)} \right)
\right.$$  

$$+ \frac{1}{2s+1} \left( \bar{G} \cdot G - \frac{s+1}{s} \bar{G} \cdot G + \text{c.c.} \right) \right\} .$$

(2.8)

The models (2.7) and (2.8) are dually equivalent [14].

We now briefly comment on the limiting $s = 1$ case which should correspond to supergravity. The transverse linear constraint (2.2a) cannot be used for $s = 1$, however its corollary $\bar{D}^2 \Gamma_{\alpha(s-1)\hat{\alpha}(s-1)} = 0$ can be used,

$$\bar{D}^2 \Gamma = 0 .$$

(2.9)

This constraint defines a complex linear superfield. In accordance with (2.5b), the gauge transformation of $\Gamma$ is

$$\delta_{\Lambda} \Gamma = \frac{1}{4} \bar{D}_{\beta} \bar{D}^2 \Lambda^\beta .$$

(2.10)
The action (2.7) for $s = 1$ coincides with the linearised action for the $n = -1$ non-minimal supergravity, see [5, 17] for reviews.

The longitudinal linear constraint (2.2b) is the chirality condition for $s = 1$,

$$\bar{D}_\alpha G = 0 \ .$$

(2.11)

The gauge transformation law (2.5c) cannot directly be used for $s = 1$. Nevertheless, it can be rewritten in the form

$$\delta_A G_{\alpha_1 \ldots \alpha_{s-1} \dot{\alpha}_1 \ldots \dot{\alpha}_{s-1}} = -\frac{1}{4} \bar{D}^2 D^\beta \Lambda_{\beta \alpha_1 \ldots \alpha_{s-1} \dot{\alpha}_1 \ldots \dot{\alpha}_{s-1}}$$

$$+ i(s - 1) \partial^\beta \bar{D}_{(\dot{\alpha}_1 \Lambda_{\beta \alpha_1 \ldots \alpha_{s-1} \dot{\alpha}_2 \ldots \dot{\alpha}_{s-1}) \dot{\beta}} \ ,$$

(2.12)

which is well defined for $s = 1$:

$$\delta_A G = -\frac{1}{4} D^2 D^\beta \Lambda_\beta \ .$$

(2.13)

The action (2.8) for $s = 1$ coincides with the linearised action for the old minimal supergravity, see [5, 17] for reviews. There are actually three different realisations for $G$ in terms of unconstrained superfields (see also [18, 19] for recent discussions). The standard realisation is

$$G = -\frac{1}{4} D^2 U \ ,$$

(2.14)

where the prepotential $U$ is an unconstrained complex superfield, with the gauge transformation law $\delta_A U = D^\beta \Lambda_\beta$. It is this realisation which corresponds to the old minimal supergravity. The second realisation is to make use of a three-form multiplet [20]

$$G = -\frac{1}{4} D^2 P \ , \quad \bar{P} = P \ ,$$

(2.15)

where $P$ is a real but otherwise unconstrained prepotential, with the gauge transformation law $\delta_A P = D^\beta \Lambda_\beta + D_\beta \bar{\Lambda}^\bar{\beta}$. This realisation corresponds to the so-called three-form supergravity [21]. Finally, $G$ can be chosen to be a complex three-form multiplet [21]

$$G = -\frac{1}{4} D^2 D^\beta \Upsilon_\beta \ ,$$

(2.16)

where $\Upsilon_\beta$ is an unconstrained complex spinor prepotential, with the gauge transformation $\delta_A \Upsilon_\beta = \Lambda_\beta$. This realisation corresponds to the complex three-form supergravity [3].
3 Higher spin supercurrents

In this section we first describe the general structure of non-conformal higher spin supercurrents \[^{[22]}\].

In the framework of the longitudinal formulation, let us couple the prepotentials \(H_{\alpha(s)}\hat{\alpha}(s),\Psi_{\alpha(s-1)}\hat{\alpha}(s-2)\), and \(\bar{\Psi}_{\alpha(s-2)}\hat{\alpha}(s-1),\) to external sources

\[
S^{(s+\frac{1}{2})}_{\text{source}} = \int d^4xd^2\theta d^2\bar{\theta} \left\{ H^{\alpha(s)}\hat{\alpha}(s)J_{\alpha(s)}\hat{\alpha}(s) + \Psi^{\alpha(s-1)}\hat{\alpha}(s-2)T_{\alpha(s-1)}\hat{\alpha}(s-2) + \bar{\Psi}_{\alpha(s-2)}\hat{\alpha}(s-1)T^{\alpha(s-2)}\hat{\alpha}(s-1) \right\}. \tag{3.1}
\]

Requiring \(S^{(s+\frac{1}{2})}_{\text{source}}\) to be invariant under \((2.4b)\) gives

\[
\bar{D}^\beta T_{\alpha(s-1)}\hat{\alpha}_1...\hat{\alpha}_{s-3} = 0, \tag{3.2a}
\]

and therefore \(T_{\alpha(s-1)}\hat{\alpha}(s-2)\) is a transverse linear superfield. Requiring \(S^{(s+\frac{1}{2})}_{\text{source}}\) to be invariant under the gauge transformations \((2.5a)\) and \((2.6b)\) gives the following conservation equation:

\[
\bar{D}^\beta J_{\alpha_1...\alpha_s\hat{\alpha}_1...\hat{\alpha}_{s-1}} + \frac{1}{2} \left( D_{(\alpha_1} \bar{D}_{\alpha_1)} - 2i(s-1)\partial_{(\alpha_1}\hat{\alpha}_1)T_{\alpha_2...\alpha_s)\hat{\alpha}_2...\hat{\alpha}_{s-1}) = 0. \tag{3.2b}
\]

For completeness, we also give the conjugate equation

\[
D^\beta J_{\beta\alpha_1...\alpha_s-1\hat{\alpha}_1...\hat{\alpha}_s} - \frac{1}{2} \left( \bar{D}_{(\hat{\alpha}_1} D_{\hat{\alpha}_1)} - 2i(s-1)\partial_{(\hat{\alpha}_1}\hat{\alpha}_1)T_{\hat{\alpha}_2...\hat{\alpha}_{s-1})\hat{\alpha}_2...\hat{\alpha}_s) = 0. \tag{3.2c}
\]

Similar considerations for the transverse formulation lead to the following non-conformal supercurrent multiplet

\[
\bar{D}^\beta J_{\alpha_1...\alpha_s\hat{\alpha}_1...\hat{\alpha}_{s-1}} - \frac{1}{4}\bar{D}^2 F_{\alpha_1...\alpha_s\hat{\alpha}_1...\hat{\alpha}_{s-1}} = 0, \tag{3.3a}
\]

\[
D_{(\alpha_1} F_{\alpha_2...\alpha_{s+1})\hat{\alpha}_1...\hat{\alpha}_{s-1}) = 0. \tag{3.3b}
\]

Thus the trace multiplet \(F_{\alpha(s-1)}\hat{\alpha}(s)\) is longitudinal linear.

In the remainder of this section we are going to show that it is the longitudinal higher spin supercurrents \((3.2)\) which naturally arise in the massive chiral model \(^{(1.5)}\). As in \(^{[23]}\), it is useful to introduce auxiliary complex variables \(\zeta^\alpha \in \mathbb{C}^2\) and their conjugates \(\bar{\zeta}^\hat{\alpha}\). Given a tensor superfield \(U_{\alpha(p)}\hat{\alpha}(q),\) we associate with it the following field on \(\mathbb{C}^2\)

\[
U_{(p,q)}(\zeta,\bar{\zeta}) := \zeta^{\alpha_1} ... \zeta^{\alpha_p} \bar{\zeta}^{\hat{\alpha}_1} ... \bar{\zeta}^{\hat{\alpha}_q} U_{\alpha_1...\alpha_p\hat{\alpha}_1...\hat{\alpha}_q}, \tag{3.4}
\]

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which is homogeneous of degree \((p, q)\) in the variables \(\zeta^\alpha\) and \(\bar{\zeta}^{\dot{\alpha}}\). We introduce operators that increase the degree of homogeneity in the variables \(\zeta^\alpha\) and \(\bar{\zeta}^{\dot{\alpha}}\),

\[
D_{(1,0)} := \zeta^\alpha D_\alpha, \quad \bar{D}_{(0,1)} := \bar{\zeta}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}}, \quad \partial_{(1,1)} := 2i\zeta^\alpha \bar{\zeta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}},
\]

and their descendants

\[
A_{(1,1)} := -D_{(1,0)} \bar{D}_{(0,1)} + (s - 1) \partial_{(1,1)}, \quad \bar{A}_{(1,1)} := \bar{D}_{(0,1)} D_{(1,0)} - (s - 1) \partial_{(1,1)}.
\]

The fermionic operators \(D_{(1,0)}\) and \(\bar{D}_{(0,1)}\) are nilpotent, \(D_{(1,0)}^2 = 0\) and \(\bar{D}_{(0,1)}^2 = 0\). We also introduce two nilpotent operators that decrease the degree of homogeneity in the variables \(\zeta^\alpha\) and \(\bar{\zeta}^{\dot{\alpha}}\), specifically

\[
D_{(-1,0)} := D^\alpha \frac{\partial}{\partial \zeta^\alpha}, \quad D_{(-1,0)}^2 = 0, \quad \bar{D}_{(0,-1)} := \bar{D}^{\dot{\alpha}} \frac{\partial}{\partial \bar{\zeta}^{\dot{\alpha}}}, \quad \bar{D}_{(0,-1)}^2 = 0.
\]

Making use of the notation introduced, the transverse linear condition (3.2a) and its conjugate become

\[
\bar{D}_{(0,-1)} T_{(s-1,s-2)} = 0, \quad D_{(-1,0)} T_{(s-2,s-1)} = 0.
\]

The conservation equations (3.2b) and (3.2c) turn into

\[
\frac{1}{s} \bar{D}_{(0,-1)} J_{(s,s)} - \frac{1}{2} A_{(1,1)} T_{(s-1,s-2)} = 0, \quad \frac{1}{s} D_{(-1,0)} J_{(s,s)} - \frac{1}{2} \bar{A}_{(1,1)} \bar{T}_{(s-2,s-1)} = 0.
\]

Since the operator \(\bar{D}_{(0,-1)} J_{(s,s)}\) is nilpotent, the conservation equation (3.9a) is consistent provided

\[
\bar{D}_{(0,-1)} A_{(1,1)} T_{(s-1,s-2)} = 0.
\]

This is indeed true, as a consequence of the transverse linear condition (3.8a).

Using the notation introduced, the massless higher spin supercurrent (1.8) becomes

\[
J_{(s,s)} = \sum_{k=0}^{s} (-1)^k \binom{s}{k} \left\{ \left( \binom{s}{k+1} \right) \partial_{(1,1)}^k D_{(1,0)} \Phi \partial_{(1,1)}^{s-k-1} \bar{D}_{(0,1)} \bar{\Phi} \right. \\
\left. + \binom{s}{k} \partial_{(1,1)}^k \Phi \partial_{(1,1)}^{s-k} \bar{\Phi} \right\}
\]

(3.11)
We now turn to constructing non-conformal higher spin supercurrents arising in the massive model (1.5). Guided by the structure of the Ferrara-Zumino supercurrent for the model (1.5), we assume that $J_{(s,s)}$ has the same functional form as in the massless case, eq. (3.11). Making use of the massive equation of motion,

$$\frac{1}{4} \bar{D}^2 \Phi + m \Phi = 0 ,$$

we obtain

$$\bar{D}_{(0,-1)} J_{(s,s)} = F_{(s,s-1)} ,$$

where we have denoted

$$F_{(s,s-1)} = 2m(s+1) \sum_{k=0}^{s} (-1)^{s-1+k} \left( \begin{array}{c} s \\ k \end{array} \right) \left( \begin{array}{c} s \\ k+1 \end{array} \right)$$

$$\times \left\{ 1 + (-1)^{s} \frac{k+1}{s-k+1} \right\} \partial_{(1,1)}^{k} \partial_{(1,1)}^{s-k-1} D_{(1,0)} \Phi .$$

(3.13b)

Keeping in mind eq. (3.9a), we now look for a superfield $T_{(s-1,s-2)}$ such that (i) it obeys the transverse linear constraint (3.8a); and (ii) it satisfies the equation

$$F_{(s,s-1)} = \frac{s}{2} A_{(1,1)} T_{(s-1,s-2)} .$$

(3.14)

We consider a general ansatz

$$T_{(s-1,s-2)} = (-1)^{s} m \sum_{k=0}^{s-2} c_{k} \partial_{(1,1)}^{k} \partial_{(1,1)}^{s-k-2} D_{(1,0)} \Phi .$$

(3.15)

For $k = 1, 2, \ldots s - 2$, condition (i) implies that the coefficients $c_{k}$ must satisfy

$$kc_{k} = (s - k - 1)c_{s-k-1} ,$$

(3.16a)

while (ii) gives the following equation

$$c_{s-k-1} + sc_{k} + (s - 1)c_{k-1} = -4(-1)^{s} \frac{1}{s} \left( \begin{array}{c} s \\ k \end{array} \right) \left( \begin{array}{c} s \\ k+1 \end{array} \right)$$

$$\times \left\{ 1 + (-1)^{s} \frac{k+1}{s-k+1} \right\} .$$

(3.16b)

Condition (ii) also implies that

$$(s - 1)c_{s-2} + c_{0} = 4(-1)^{s}(s+1) \left\{ 1 + (-1)^{s} \frac{s}{2} \right\} .$$

(3.16c)
\[ c_0 = -\frac{4}{s}(s + 1 + (-1)^s) . \] (3.16d)

It turns out that the equations (3.16) lead to a unique expression for \( c_k \) given by

\[
c_k = -\frac{4(s + 1)(s - k - 1)}{s(s - 1)} \sum_{l=0}^{k} \frac{(-1)^k}{s-l} \binom{s}{l} \binom{s}{l+1} \left\{ 1 + (-1)^s \frac{l+1}{s-l+1} \right\} ,
\]

\[ k = 1, 2, \ldots s - 2 . \] (3.17)

If the parameter \( s \) is odd, \( s = 2n + 1 \), with \( n = 1, 2, \ldots \), one can check that the equations (3.16a)–(3.16c) are identically satisfied. However, if the parameter \( s \) is even, \( s = 2n \), with \( n = 1, 2, \ldots \), there appears an inconsistency: the right-hand side of (3.16c) is positive, while the left-hand side is negative, \( (s - 1)c_{s-2} + c_0 < 0 \). Therefore, our solution (3.17) is only consistent for \( s = 2n + 1, n = 1, 2, \ldots \).

Relations (3.11), (3.15), (3.16d) and (3.17) determine the non-conformal higher spin supercurrents in the massive chiral model (1.5), with the trace multiplet \( T_{(s-1,s-2)} \) being the higher spin extension of (1.6). Unlike the conformal higher spin supercurrents (1.8), the non-conformal ones exist only for the odd values of \( s \), \( s = 2n + 1 \), with \( n = 1, 2, \ldots \).

4 Concluding comments

The non-conformal higher spin supercurrent multiplets (3.2) and (3.3) are automatically consistent, since they are associated with the gauge-invariant models (2.8) and (2.7), respectively. An interesting open question is to classify all non-conformal deformations of the higher spin supercurrents (1.7), along the lines of the recent analysis of non-conformal \( \mathcal{N} = (1,0) \) supercurrents in six dimensions [24]. Our results provide the setup required for developing a program to derive higher spin supersymmetric models from quantum correlation functions, as an extension of the non-supersymmetric approaches pursued, e.g., in [25, 26, 27]. Our results also have a natural extension to the case of \( \mathcal{N} = 2 \) supersymmetry in three dimensions, where the off-shell higher spin supermultiplets have recently been constructed in [28].

Shortly before posting this work to the arXiv, there appeared another revised version (v3, 26 Oct) of Ref. [13] containing a new section devoted to the higher spin supercurrents in the massive chiral model (1.5). These authors also observed that the higher spin supercurrents \( J_{\alpha(s)\dot{\alpha}(s)} \) in the massive chiral model (1.5) exist only for the odd values of \( s \), \( s = 2n + 1 \), with \( n = 1, 2, \ldots \).
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References
[1] S. Ferrara and B. Zumino, “Transformation properties of the supercurrent,” Nucl. Phys. B 87, 207 (1975).
[2] S. M. Kuzenko, “Variant supercurrent multiplets,” JHEP 1004, 022 (2010) [arXiv:1002.4932 [hep-th]].
[3] D. Butter and S. M. Kuzenko, “N=2 supergravity and supercurrents,” JHEP 1012, 080 (2010) [arXiv:1011.0339 [hep-th]].
[4] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Three-dimensional N=2 (AdS) supergravity and associated supercurrents,” JHEP 1112, 052 (2011) [arXiv:1109.0496 [hep-th]].
[5] S. J. Gates Jr., S. M. Kuzenko and J. Phillips, “The off-shell (3/2,2) supermultiplets revisited,” Phys. Lett. B 576, 97 (2003) [arXiv:hep-th/0306288].
[6] Z. Komargodski and N. Seiberg, “Comments on supercurrent multiplets, supersymmetric field theories and supergravity,” [arXiv:1002.2228 [hep-th]].
[7] J. Wess and B. Zumino, “Superfield Lagrangian for supergravity,” Phys. Lett. B 74, 51 (1978).
[8] K. S. Stelle and P. C. West, “Minimal auxiliary fields for supergravity,” Phys. Lett. B 74, 330 (1978).
[9] S. Ferrara and P. van Nieuwenhuizen, “The auxiliary fields of supergravity,” Phys. Lett. B 74, 333 (1978).
[10] W. Siegel, “Solution to constraints in Wess-Zumino supergravity formalism,” Nucl. Phys. B 142, 301 (1978).
[11] P. S. Howe, K. S. Stelle and P. K. Townsend, “Supercurrents,” Nucl. Phys. B 192, 332 (1981).
[12] S. M. Kuzenko, R. Manvelyan and S. Theisen, “Off-shell superconformal higher spin multiplets in four dimensions,” JHEP 1707, 034 (2017) [arXiv:1701.00682 [hep-th]].
[13] I. L. Buchbinder, S. J. Gates and K. Koutrolikos, “Higher spin superfield interactions with the chiral supermultiplet: Conserved supercurrents and cubic vertices,” [arXiv:1708.06262 [hep-th]].
[14] S. M. Kuzenko, V. V. Postnikov and A. G. Sibiryakov, “Massless gauge superfields of higher half-integer superspins,” JETP Lett. 57, 534 (1993) [Pisma Zh. Eksp. Teor. Fiz. 57, 521 (1993)].
[15] S. M. Kuzenko and A. G. Sibiryakov, “Massless gauge superfields of higher integer superspins,” JETP Lett. 57, 539 (1993) [Pisma Zh. Eksp. Teor. Fiz. 57, 526 (1993)].
[16] S. M. Kuzenko and A. G. Sibiryakov, “Free massless higher-superspin superfields on the anti-de Sitter superspace” Phys. Atom. Nucl. 57, 1257 (1994) [Yad. Fiz. 57, 1326 (1994)] \[arXiv:1112.4612 [hep-th]].

[17] I. L. Buchbinder and S. M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity, Or a Walk Through Superspace, IOP, Bristol, 1995 (Revised Edition 1998).

[18] F. Farakos, S. Lanza, L. Martucci and D. Sorokin, “Three-forms in supergravity and flux compactifications,” Eur. Phys. J. C 77, no. 9, 602 (2017) \[arXiv:1706.09422 [hep-th]].

[19] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Complex three-form supergravity and membranes,” \[arXiv:1710.00535 [hep-th]].

[20] S. J. Gates Jr., “Super p-form gauge superfields,” Nucl. Phys. B 184, 381 (1981).

[21] S. J. Gates Jr. and W. Siegel, “Variant superfield representations,” Nucl. Phys. B 187, 389 (1981).

[22] S. M. Kuzenko and A. G. Sibiryakov, 1993, unpublished.

[23] S. J. Gates, Jr. and S. M. Kuzenko, “4D, N = 1 higher spin gauge superfields and quantized twistors,” JHEP 0510, 008 (2005) \[hep-th/0506255\].

[24] S. M. Kuzenko, J. Novak and S. Theisen, “Non-conformal supercurrents in six dimensions,” \[arXiv:1709.09792 [hep-th]].

[25] L. Bonora, M. Cvitan, P. Dominis Prester, B. Lima de Souza and I. Smolić, “Massive fermion model in 3d and higher spin currents,” JHEP 1605, 072 (2016) \[arXiv:1602.07178 [hep-th]].

[26] L. Bonora, M. Cvitan, P. Dominis Prester, S. Giaccari, B. Lima de Souza and T. Štemberga, “One-loop effective actions and higher spins,” JHEP 1612, 084 (2016) \[arXiv:1609.02088 [hep-th]].

[27] L. Bonora, M. Cvitan, P. Dominis Prester, S. Giaccari and T. Stemberga, “One-loop effective actions and higher spins. II,” \[arXiv:1709.01738 [hep-th]].

[28] S. M. Kuzenko and D. X. Ogburn, “Off-shell higher spin N=2 supermultiplets in three dimensions,” Phys. Rev. D 94, no. 10, 106010 (2016) \[arXiv:1603.04668 [hep-th]].