Production of two $c\bar{c}$ pairs in gluon-gluon scattering in high energy proton-proton collisions

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Abstract

We calculate cross sections for $gg \rightarrow QQ\bar{Q}Q$ in the high-energy approximation in the mixed (longitudinal momentum fraction, impact parameter) and momentum space representations. Besides the total cross section as a function of subsystem energy also differential distributions (in quark rapidity, transverse momentum, $QQ$, $Q\bar{Q}$ invariant mass) are presented. The elementary cross section is used to calculate production of ($c\bar{c}$) in single-parton scattering (SPS) in proton-proton collisions. We present integrated cross section as a function of proton-proton center of mass energy as well as differential distribution in $M_{(c\bar{c})(c\bar{c})}$. The results are compared with corresponding results for double-parton scattering (DPS) discussed recently in the literature. We find that the considered SPS contribution to ($c\bar{c}$) production is at high energy ($\sqrt{s} > 5$ TeV) much smaller than that for DPS contribution.
I. INTRODUCTION

The cross section for $c\bar{c}$ production in proton-proton or proton-antiproton collisions at high energy is quite large [1, 2] because the gluon-gluon luminosity grows quickly with energy. We have shown recently that the cross section for production of two pairs of $c\bar{c}$ in double-parton scattering (DPS) grows even faster with incident center-of-mass energy [3] and becomes very large at large energies. In order to verify the DPS contribution a single-parton scattering (SPS) contribution has to be evaluated. This was not done so far in the literature as it requires calculation of $2 \rightarrow 4$ subprocesses. At LHC a measurement of the two-pairs of $c\bar{c}$ production should be possible. This could be a good test of methods of calculating higher-order QCD corrections.

It is the aim of the present paper to calculate contribution of single-parton scattering to the inclusive $pp \rightarrow (c\bar{c})(c\bar{c})X$ cross section. In the present paper we shall use high-energy approximation in calculating elementary $gg \rightarrow (c\bar{c})(c\bar{c})$ cross section. At low incident energy and/or low $c\bar{c}c\bar{c}$ invariant mass production a careful treatment of the threshold effects is required. The elementary $2 \rightarrow 4$ cross section is convoluted next with gluon distribution functions. The result is compared with that for DPS presented recently [3]. A prospects how to disentangle SPS and DPS contributions will be discussed in the Result section.

II. THEORETICAL FRAMEWORK

A. $gg \rightarrow Q\bar{Q}Q\bar{Q}$

In the high energy limit of the gluon-gluon subprocess, the $gg \rightarrow Q\bar{Q}Q\bar{Q}$ amplitude will be dominated by the $t$-channel gluon exchange. Furthermore, if the $Q\bar{Q}$-pairs are produced in the respective fragmentation regions of the incoming gluons an intuitively appealing approach based on light-cone perturbation theory is possible. Namely we are looking for the cross section of excitation of the $Q\bar{Q}$-Fock states of the colliding gluons.
We base our calculations on previous works \cite{4, 5}, where the total cross section for the process \( gN \rightarrow Q\bar{Q}X \) and \( gA \rightarrow Q\bar{Q}X \) has been obtained, as well as on \cite{6}, where more detailed kinematical distributions can be found.

Let us briefly recapitulate the derivation of the key formulas. We start by writing out the lightcone Fock-state expansion of the incoming, physical, gluon, taking into account only the heavy-quark-antiquark fluctuation in the two-body sector (see, \cite{5}):

\[
|g^a(b)\rangle = \sqrt{1-n_{Q\bar{Q}}} |g^a_{\text{bare}}\rangle + \int d^2r dz \Psi(z, r) |[Q\bar{Q}]^a_8; z, r\rangle .
\]  

(1)

Here, quark and antiquark in the gluon carry fractions \( z, 1-z \) of the gluon’s large lightcone plus-momentum and are separated by a distance \( r \) in the impact parameter plane.

Notice that the quark-antiquark pair before the interaction is not a color dipole, but carries a color-octet charge. Infrared safety of the relevant cross section is a consequence of the fact, that arbitrarily long-wavelength gluons cannot induce the \( g \rightarrow Q\bar{Q} \) transition and decouple.

The normalized color-states of the quark-antiquark system in the color-octet and color-singlet states are, respectively:

\[
|[Q\bar{Q}]^a_8\rangle = \sqrt{2} (t^a)^i_j |Q^i \bar{Q}^j\rangle ,
\]

\[
|[Q\bar{Q}]_1\rangle = \frac{1}{\sqrt{N_c}} \delta^i_j |Q^i \bar{Q}^j\rangle .
\]  

(2)

A similar Fock-space expansion as (1) can be written for the second gluon participating in the interaction, which will have a large momentum in the light-cone minus-direction.

The interaction between the right- and left moving parton systems is then mediated by the gluon exchange which acts like a helicity conserving potential \cite{7} between partons \( i \) and \( j \).

\[
V(b + b_i - s_j) = (-i)\frac{\alpha_S}{\pi} \int \frac{d^2q}{[q^2 + \mu_G^2]} \exp[i(b + b_i - s_j)q] \sum_k e^{iqs_k} T^b_k |g^a_{\text{bare}}\rangle .
\]  

(3)

Here \( b \) is the impact parameter between the two initial gluons, \( b_i \) and \( s_i \) are the impact parameter distances of \( Q \) and \( \bar{Q} \) relative to their respective parent gluon which are conserved during the interaction. The matrices \( T^b_i \) are the generators of color \( SU(N_c) \) acting on parton \( i \) in the relevant representation. Notice, that the gluon mass parameter \( \mu_G \) is not needed to make the cross section finite, as mentioned above there are no singularities associated with the small-\( q \) behaviour of the gluon propagator. As a physical parameter it enforces a finite propagation radius of gluons in the transverse plane, as is in fact enforced by confinement. In practice, for the problem at hand its precise value is unimportant: as long as \( \mu_G^2 \ll m_Q^2 \) our results are practically independent of \( \mu_G \). The relevant Feynman diagrams for our process are shown in Figs. (1) and (2), and the corresponding scattering amplitude for \( gg \rightarrow (Q\bar{Q})(Q\bar{Q}) \) in the high-energy limit of interest takes the form:

\[
[S(b) - 1]|g^a \otimes g^c; b\rangle = -i\frac{\alpha_S}{\pi} \int \frac{d^2q e^{iqb}}{q^2 + \mu_G^2} \left[ \sum_j e^{-iqs_j} T^b_j |g^c\rangle \right] \otimes \left[ \sum_k e^{iqs_k} T^b_k |g^a\rangle \right].
\]  

(4)
Let us now turn to one of the factors in the square brackets (the so-called “impact-factors”). The quark in the gluon is located at the distance \( b_Q = (1 - z) \vec{r} \), and the antiquark at \( \bar{b}_Q = -z \vec{r} \), where \( \vec{r} = b_Q - b_Q \). In fact we are interested in the respective two-body Fock-state components orthogonal to the physical gluon, and the relevant piece of the amplitude for the excitation \( g \to Q\bar{Q} \) is then given by (see also [3]):

\[
\Phi^b(q, g^a) \equiv \sum_k e^{i q \cdot b_k} T_k^b|G^a\rangle_{\text{excitation}} = \int d^2z d^2r \Psi(z, r) \left\{ e^{i(1-z)q \cdot r} - e^{-izq \cdot r} \right\} \frac{1}{\sqrt{2N_c}} \delta_{ab} \langle [Q\bar{Q}]_{1}; z, r \rangle \\
+ e^{i(1-z)q \cdot r} - e^{-izq \cdot r} \frac{1}{2} d_{abc} \langle [Q\bar{Q}]_{8}; z, r \rangle \\
+ e^{i(1-z)q \cdot r} + e^{-izq \cdot r} - 2i \frac{1}{2} f_{abc} \langle [Q\bar{Q}]_{8}; z, r \rangle \right\} .
\] (5)

Here \( b \) is the color index of the \( t \)-channel gluon which carries the transverse momentum \( q \). Clearly, the impact factor vanishes for \( q \to 0 \). A completely analogous expression can be written for the other factor in square brackets, and the full amplitude is obtained as:

\[
A(g^a g^c \to Q\bar{Q}Q\bar{Q}; b) = -i\frac{\alpha_S}{\pi} \int \frac{d^2q e^{i q \cdot b}}{[q^2 + \mu^2_c]} \Phi^b(q, g^a) \Phi^b(-q, g^c).
\] (6)

The two impact factors correspond to the upper and lower gluons, respectively. The total cross section is then obtained after integrating the squared amplitude over the impact parameter and averaging over initial gluon colors

\[
\sigma_{\text{tot}} = \frac{1}{(N_c^2 - 1)^2} \sum_{a,c} \int d^2b |A(g^a g^c \to Q\bar{Q}Q\bar{Q}; b)|^2 = 4 \alpha_s^2 \int \frac{d^2q I^{bc}(q) I^{bc}(-q)}{[q^2 + \mu^2_c]^2} .
\] (7)

Similar impact factor representations for related QED problems are known for a long time [3]. Introducing the short-hand notation

\[
F(qr) = 2 - \exp(iqr) - \exp(-iqr),
\] (8)
after some calculation, we obtain the impact factors relevant for the total cross section:

\[
I^{bc}(q) \equiv \frac{1}{N_c^2 - 1} \sum_a \Phi^a(q, g^a) \Phi^{\ast c}(q, g^a) = \frac{\delta_{bc}}{2N_c} \frac{N_c^2}{N_c^2 - 1} \int d^2z d^2r |\Psi(z, r)|^2 \left( F((1-z)qr) + F(zqr) - \frac{1}{N_c} F(qr) \right) .
\] (9)

The light-cone wave function for the \( g \to Q\bar{Q} \) transition can be obtained from the well-known case for the photon as [3, 10]:

\[
|\Psi(z, r)|^2 = \frac{\alpha_s(r)}{6\alpha_{em}} |\Psi_{\gamma}(z, r)|^2 = \frac{\alpha_s(r)}{(2\pi)^2} \left[ \left( z^2 + (1 - z)^2 \right) m_{Q}^2 K_1^2(m_{Q}r) + m_{\bar{Q}}^2 K_0^2(m_{\bar{Q}}r) \right],
\] (10)

where \( K_{0,1} \) are generalized Bessel functions, and in the spirit of collinear factorization, we took the gluon to be on-shell.
B. Dipole-dipole cross section

It is now convenient to introduce the total cross section for two color dipoles of sizes \( r, s \), in the two-gluon exchange Born-approximation [3]:

\[
\sigma_{DD}(r, s) = \frac{N_c^2 - 1}{N_c^2} \int \frac{d^2 q \alpha_S^2}{[q^2 + \mu_G^2]^2} F(qr)F(-qs) \\
= 4 \frac{N_c^2 - 1}{N_c^2} \alpha_S^2 \left( \chi(0) - \chi(r) - \chi(s) + \chi(r - s) \right), \tag{11}
\]

where

\[
\chi(r) = \int \frac{d^2 q}{[q^2 + \mu_G^2]^2} \exp(iqr) = \frac{\pi}{\mu_G^2} (\mu_G r) K_1(\mu_G r). \tag{12}
\]

The Born level dipole-dipole cross section now reads

\[
\sigma_{DD}(r, s) = \frac{N_c^2 - 1}{N_c^2} \frac{4 \pi \alpha_S^2}{\mu_G^2} \left[ 1 - \mu_G r K_1(\mu_G r) - \mu_G s K_1(\mu_G s) + \mu_G |r - s| K_1(\mu_G |r - s|) \right]. \tag{13}
\]

Notice, that it is finite for \( \mu_G \to 0 \).

The total cross section for the parton-level process \( gg \to Q\bar{Q}Q\bar{Q} \) can now be written in terms of the dipole-dipole cross section and the light-cone wave-functions for the \( g \to Q\bar{Q} \) transitions as

\[
\sigma_{tot} = \int dz d^2 r d^2 s |\Psi(z, r)|^2 |\Psi(u, s)|^2 \Sigma(z, r; u, s). \tag{14}
\]

where

\[
\Sigma(z, r; u, s) = \left( \frac{N_c^2}{N_c^2 - 1} \right)^2 \cdot \left\{ \sigma_{DD}((1 - z)r, (1 - u)s) + \sigma_{DD}((1 - z)r, us) - \frac{1}{N_c^2} \sigma_{DD}((1 - z)r, s) \right. \\
+ \sigma_{DD}(zr, (1 - u)s) + \sigma_{DD}(zr, us) - \frac{1}{N_c^2} \sigma_{DD}(zr, s) \\
- \frac{1}{N_c^2} \left( \sigma_{DD}(r, (1 - u)s) + \sigma_{DD}(r, us) - \frac{1}{N_c^2} \sigma_{DD}(r, s) \right) \right\}. \tag{15}
\]

C. Momentum distributions

The mixed representation given above could in principle be used to obtain distributions in longitudinal momenta, simply by stripping off the \( dz \) and/or \( du \) integrations.
For the more interesting transverse momentum distributions it is better to start all over in momentum space, where the impact factor reads

\[
\Phi_b(q, g^a) = \int \frac{d^2 k}{(2\pi)^2} \left\{ |\Psi(z, k - (1 - z)q) - \Psi(z, k + zq)| \frac{1}{\sqrt{2N_c}} \delta_{ab} |\bar{Q}Q|_1; z, k \right\} + |\Psi(z, k - (1 - z)q) - \Psi(z, k + zq)| \frac{1}{2} d_{abc} |\bar{Q}Q|^{c^*}; z, k \left\} \right. 
\]

(16)

After squaring, we obtain naturally the same structure as in [6]:

\[
I_{bc}(z, k, q) = \frac{\delta_{bc}}{2N_c} \left\{ \frac{N_c^2}{N_c^2 - 1} \left| \Psi(z, k) - \Psi(z, k + zq) \right|^2 + \left| \Psi(z, k) - \Psi(z, k - (1 - z)q) \right|^2 
- \left| \Psi(z, k - (1 - z)q) - \Psi(z, k + zq) \right|^2 \right\} + \left| \Psi(z, k - (1 - z)q) - \Psi(z, k + zq) \right|^2 \right\}.
\]

(17)

The explicit form for the squares of the light-cone wave functions can been found in [6] and reads:

\[
|\Psi(z, p) - \Psi(z, p + \kappa)|^2 = \alpha_s \left\{ \left[ z^2 + (1 - z)^2 \right] \left( \frac{p}{p^2 + m_Q^2} - \frac{p + \kappa}{(p + \kappa)^2 + m_Q^2} \right)^2 
+ m_Q^2 \left( \frac{1}{p^2 + m_Q^2} - \frac{1}{(p + \kappa)^2 + m_Q^2} \right) \right\}.
\]

(18)

With all of this given, we can put together the differential cross section

\[
d\sigma = \frac{N_c^2 - 1}{N_c^2} \frac{4\pi^2 \alpha_s^2}{|q^2 + \mu_Q^2|^2} I(z, k, q) I(u, l, -q) dz \frac{d^2 k}{(2\pi)^2} du \frac{d^2 l}{(2\pi)^2} \frac{d^2 q}{(2\pi)^2}.
\]

(19)

A brief comment on the kinematics of quarks is in order. The four-momenta of quarks are fully specified by the variables \( z, k, q, u, l \). Incoming gluons carry longitudinal momentum
fractions $x_1, x_2$ of the momenta of the incoming protons. The latter are

$$P_a = P_a^+ n^+ = \sqrt{\frac{s}{2}} n^+, \quad P_b = P_b^- n^- = \sqrt{\frac{s}{2}} n^-,$$

so that $2P_a \cdot P_b = s$. We parametrize four-momenta in light-cone coordinates

$$p = (p_+, p_-, p), \quad 2p_+ p_- - p^2 = m^2. \quad (21)$$

Then, the four momentum, say of the quark/antiquark belonging to parent gluon 1, read

$$p_Q = ((1 - z) x_1 P_a^+ + m_Q^2 P_Q, \frac{p_Q^2 + m_Q^2}{2(1 - z) x_1 P_a^+} P_Q), \quad p_{\bar{Q}} = ((1 - u) x_2 P_b^+ + m_{\bar{Q}}^2 P_{\bar{Q}}, \frac{p_{\bar{Q}}^2 + m_{\bar{Q}}^2}{2(1 - u) x_2 P_b^+} P_{\bar{Q}}). \quad (22)$$

We still need to give the relation between the transverse momenta $p_{Q, \bar{Q}}$ of the (anti-)quark and the momenta $k, q$ used above. These relations read:

$$p_Q = k + z q, \quad p_{\bar{Q}} = -k + (1 - z) q, \quad (23)$$
or, alternatively

$$q = p_Q + p_{\bar{Q}}, \quad k = (1 - z) p_Q - z p_{\bar{Q}}. \quad (24)$$

Notice, that $q$ is the total transverse momentum of the $Q\bar{Q}$-pair, while $k$ is the light-cone relative transverse momentum which was conjugate to the dipole size. Now having the four momenta, we can calculate all sorts of kinematical variables, e.g. the rapidity will be given by

$$y_Q = \frac{1}{2} \log \left( \frac{p_Q^+}{p_Q^-} \right). \quad (25)$$

The kinematics of the “lower” $Q\bar{Q}$-pair is treated analogously. Obviously the transverse momentum of the second pair is just $-q$, because incoming gluons are collinear. When constructing four-momenta, we only need to be careful that the large component is along the lightcone-minus direction

$$p_{Q_2} = \left( \frac{p_Q^2 + m_Q^2}{u x_2 P_b^-}, u x_2 P_b^-, P_Q \right), \quad p_{\bar{Q}_2} = \left( \frac{p_{\bar{Q}}^2 + m_{\bar{Q}}^2}{2(1 - u) x_2 P_b^+}, (1 - u) x_2 P_a^+, P_{\bar{Q}} \right). \quad (26)$$

And here

$$p_Q = l - u q, \quad p_{\bar{Q}} = -l - (1 - u) q. \quad (27)$$

D. On normalization and phase-space

It is important to remember, that the expressions we derived are valid in a high energy limit, in which the invariant masses of $Q\bar{Q}$-pairs are much smaller than the center-of-mass energy squared of the gluon-gluon collision: $M_{12}^2, M_{34}^2 \ll \hat{s}.$
In the practical calculation of the hadron-level cross section, we integrate over all momentum fractions carried by initial state gluons, and hence over the cms-energy of the gluon-gluon subprocess. In practice, we want that our cross section behaves smoothly also at low energies.

Firstly notice, that our states are normalized in such a way, that the formulas are simple in the high energy limit. In particular, the constrained two-body phase space is just \( dz d^2k /(2\pi)^2 \).

Effectively, the four-body phase space in the high-energy limit is just
\[
\frac{d^2q}{(2\pi)^2} dz \frac{d^2k}{(2\pi)^2} du \frac{d^2l}{(2\pi)^2}.
\]

Obviously it doesn’t vanish if \( \hat{s} \) approaches the threshold. To improve upon this, let us first rescale the amplitude, and introduce the Feynman-amplitude \( \mathcal{M}_F \)
\[
\mathcal{M}_F \equiv 2P_+ \sqrt{z(1-z)(2\pi)} \, 2P_- \sqrt{u(1-u)(2\pi)} \cdot A
\]
\[
= \sqrt{z(1-z)2(2\pi)} \sqrt{u(1-u)2(2\pi)} \cdot \hat{s} \cdot A,
\]

which is normalized such, that the fully differential cross section takes the form:
\[
d\sigma = \frac{1}{2\hat{s}} |\mathcal{M}_F|^2 d\Phi(\hat{s}; p_1, p_2, p_3, p_4).
\]

Here, the constrained n-body phase space, with \( P^2_{in} = \hat{s} \) is
\[
d\Phi(\hat{s}; p_1, \ldots, p_n) = (2\pi)^4 \delta(4)(P_{in} - \sum_i p_i) \cdot \prod_{i=1}^n \frac{d^4p_i}{(2\pi)^3} \delta(p^2_i - m^2_i).
\]

We now introduce invariant masses \( M_{12}, M_{34} \), and write the four-body phase-space as
\[
d\Phi(s; p_1, p_2, p_3, p_4) = d\Phi(s; P_{12}, P_{34}) \frac{dM^2_{12}}{2\pi} d\Phi(M^2_{12}; p_1, p_2) \frac{dM^2_{34}}{2\pi} d\Phi(M^2_{34}; p_3, p_4).
\]

Now, going over to light-cone coordinates, we have
\[
\frac{dM^2}{2\pi} d\Phi(M^2; p_1, p_2) \quad = \quad \frac{dM^2}{2\pi} (2\pi)^4 \delta(4)(P_{12} - p_1 - p_2) \prod_{i=1}^2 \frac{dp^+_i dp^-_i d^2p_i}{(2\pi)^3} \delta(2p^+_i p^-_i - p^2_i - m^2_i)
\]
\[
= \quad \frac{dM^2}{2\pi} (2\pi)^4 \delta(p^+_1 + p^+_2 - P^+_{12}) \delta(\frac{p^2_1 + m^2_1}{2p^+_1} + \frac{p^2_2 + m^2_2}{2p^+_2} - P^-_{12})
\]
\[
\cdot \delta^{(2)}(p_1 + p_2 - P_{12}) \prod_{i=1}^2 \frac{dp^+_i d^2p_i}{2p^+_i (2\pi)^3}.
\]

Here we integrated out the \( p^-_i \)-components from the on-shell conditions. Let us now write \( p^+_1 = zP^+__{12} \), then one of the overall delta-functions gives us, that \( p^+_2 = (1-z)P^+_{12} \).
Furthermore, we can use the on-shell condition $P_{12}^2 = M^2$ to write $2P_{12}^+P_{12}^- = M^2 + P_{12}^2$, then we obtain finally

$$\frac{dM^2}{2\pi} d\Phi(M^2;p_1,p_2) = \frac{dz}{z(1-z)} \frac{d^2p_1 d^2p_2}{2(2\pi)^3} \delta^{(2)}(p_1 + p_2 - P_{12}) = \frac{dz}{z(1-z)} \frac{d^2k}{2(2\pi)^3}.$$ (34)

Which agrees, modulo the factors now absorbed in the Feynman amplitude with the result obtained in the high-energy limit. The major simplification in the high-energy limit occurs in the first factor of the four-body phase space (32). Namely we can write the phase space for two clusters $M_{12}^2, M_{34}^2 \ll \hat{s}$, which are separated by a large rapidity distance as

$$d\Phi(s;P_1, P_2) = (2\pi)^4 \delta^{(4)}(P - P_1 - P_2) \prod_{i=1}^{2} \frac{dP_i^+ dP_i^- d^2P_i}{(2\pi)^3} \delta(2P_i^+ P_i^- - P_i^2 - M_i^2).$$ (35)

For brevity, we wrote $P_1 = P_{12}, P_2 = P_{34}$ Now we can neglect the plus-component of $P_2$ and the minus-component of $P_1$ in the overall four-momentum conservation, so that

$$\delta^{(4)}(P - P_1 - P_2) = \delta(P^+ - P_1^+)\delta(P^- - P_2^-)\delta^{(2)}(P_1 + P_2).$$ (36)

Hence all the integrations except for the transverse momentum ones can be done immediately. From the integrals over $\pm$-components, we only get a factor of

$$\frac{1}{2P_1^+ 2P_2^-} = \frac{1}{2\hat{s}},$$ (37)

so that, finally:

$$d\Phi(\hat{s};P_1, P_2) = \frac{1}{2\hat{s}} \frac{d^2P_1 d^2P_2}{(2\pi)^2} \delta^{(2)}(P_1 + P_2) \equiv \frac{1}{2\hat{s}} \frac{d^2q}{(2\pi)^2}.$$ (38)

If we approach the threshold, the exact phase-space goes to zero and our approximation is very bad. Still, we know, that the integrated two-cluster phase-space will be just

$$\int d\Phi(\hat{s};P_1, P_2) = \frac{\sqrt{[\hat{s} - (M_{12} + M_{34})^2][\hat{s} - (M_{12} - M_{34})^2]}}{8\pi \hat{s}}.$$ (39)

In order to improve our calculation, we therefore introduce the correction factor

$$f_{corr} = \sqrt{\left[1 - \frac{(M_{12} + M_{34})^2}{\hat{s}}\right] \left[1 - \frac{(M_{12} - M_{34})^2}{\hat{s}}\right]},$$ (40)

which within the approximations of the high-energy limit of course is exactly unity. A deviation from unity, or as a matter of fact any energy-dependence of the subprocess cross section thus indicates for us how far we are from the high-energy domain.
### E. $pp \rightarrow (Q\bar{Q})(Q\bar{Q})$ inclusive cross section

The cross section for proton-proton (proton-antiproton) can be calculated as usually in the parton model as

$$
\sigma_{pp \rightarrow (Q\bar{Q})(Q\bar{Q})}(W) = \int dx_1 dx_2 \ g(x_1, \mu_F^2) \ g(x_2, \mu_F^2) \ \sigma_{gg \rightarrow (Q\bar{Q})(Q\bar{Q})}(\hat{s}^{1/2}) , \tag{41}
$$

where $\hat{s} = x_1 x_2 W^2$. The last factor is the elementary cross section discussed in the previous sections. In calculating (41) we take into account kinematical constraints and the threshold correction factor (40). The elementary cross section is calculated first on a subsystem energy grid and a simple interpolation is done then when using it in formula (41). We shall use different parton (gluon) distributions from the literature. The factorization scale of the gluon distribution in principle depends on the kinematics of the final state quarks. We use $\mu_F^2 = 4m_Q^2$ when calculating the integral (41). The parton formula (41) is very useful to make differential distribution in invariant mass of the $(Q\bar{Q})(Q\bar{Q})$ system. In principle it will be desirable to obtain results for open-charm mesons, but the inclusion of hadronization goes beyond the scope of the present paper where we wish to present only a first estimation of the cross section for SPS production of $(c\bar{c})(c\bar{c})$.

### III. RESULTS

#### A. $gg \rightarrow (c\bar{c})(c\bar{c})$

To make final calculations we have to fix $\alpha_s$ in formulae (18) and (19). We shall use leading-order running strong coupling constant $\alpha_s(k_t^2 + m_Q^2)$ in impact factors (see Eq.(18)) and $\alpha_s(\mu^2)$ with $\mu^2 = \max(k_t^2 + m_Q^2, q^2)$ for gluon exchange (see Eq.(19)).

In Fig.3 we show total cross section for the $gg \rightarrow (c\bar{c})(c\bar{c})$ as a function of gluon-gluon energy. We show cross section with extra cut $M_{12} + M_{34} < W$ and with extra correction factor (see Eq.(10)). The latter cross section will be used then to calculate corresponding cross section for proton-proton collisions. In the following calculations we have fixed the regularization parameter $\mu_G = 0.5$ GeV. There is only a marginal dependence on the value of the nonperturbative parameter.

Before we go to real observables let us show an auxiliary distributions in $q_t$ and $k_t$ (see Fig.4).

The transverse momentum distributions of quark (antiquark) is shown in Fig.5 for a few selected subsystem energies. The higher subsystem energy the bigger transverse momenta are available kinematically.

Rapidity distributions of quark (antiquark) for different energies are shown in Fig.6. We show distributions for quarks emitted from the upper line (solid line) and from the lower line (dashed line). The higher the energy the better separation of the two contributions can be seen.

The rapidity separation can be better seen in the distributions in rapidity distance between (anti)quark-(anti)quark. The distance between quark-antiquark from the same
FIG. 3: Energy dependence of the total cross section for the $gg \to (c\bar{c})(c\bar{c})$ process. The description of the solid line is given in the main text.

FIG. 4: Auxiliary distributions for the $gg \to (c\bar{c})(c\bar{c})$ process for $W = 20, 50, 100, 200$ GeV (successively growing cross section).

pair is very small compared to the distance between (anti)quark-(anti)quark from dif-
FIG. 5: The quark (antiquark) transverse momentum distribution $d\sigma/dp_t$ in the $gg \rightarrow (c\bar{c})(c\bar{c})$ for $W = 20, 50, 100, 200$ GeV.

Different pairs. In Fig.7 we show an example for subsystem energy $W = 200$ GeV. The distance in rapidity between quarks and antiquarks emitted from different pairs reminds a bit situation in double-parton scattering. There the distance between quarks and antiquarks emitted from two different hard processes can also be large [3].

Finally we close presentation of our results for $gg \rightarrow (c\bar{c})(c\bar{c})$ by showing distributions in quark-antiquark invariant masses. Here there are two distinct classes of subsystems. We shall introduce the notation: $d\sigma/dM_{ij} = d\sigma/dM_{12} \equiv d\sigma/dM_{34}$ (category I) for quark-antiquark emitted in the same pair and
$$d\sigma/dM_{13} = d\sigma/dM_{14} = d\sigma/dM_{23} = d\sigma/dM_{24} \equiv d\sigma/dM_{ij}$$ (category II) for quark-antiquark emitted from different pairs. One can see that the average invariant mass of quark-antiquark from the same pair is smaller than the average invariant mass from the different pairs. At large invariant masses the emission from different pairs dominates over the emission from the same pair. The situation reminds that for double-parton scattering [3].

B. $pp \rightarrow c\bar{c}c\bar{c}X$

Let us come now to proton-proton scattering. In Fig.9 we show distribution in $\xi_1 = \log_{10}(x_1)$ ($\xi_2 = \log_{10}(x_2)$) for $W = 7$ TeV (LHC). We see that typical $x_1$ and $x_2$ are not too small, of the order of $10^{-3} - 10^{-2}$. This is the region where the gluon distributions
are relatively well known. There is no strong dependence of the cross section on the choice of gluon distribution function (GDF).

The distribution in invariant mass of the \((c\bar{c})(c\bar{c})\) system (equal to subsystem energy) is shown in Fig.10. We show distribution for CTEQ6 GDFs [11]. For comparison we show distribution obtained for double-parton scattering (see [3]). While the double parton scattering contribution dominates in the region of small invariant masses, the single parton scattering contribution takes over above \(M_{4c} > 500\) GeV. This is the region where large rapidity gaps between quarks and antiquarks occur.

The energy dependence of the \(pp \rightarrow c\bar{c}c\bar{c}X\) inclusive cross section is shown in Fig.11. One can observe that the inclusive cross section for the \(c\bar{c}c\bar{c}\) final state is much smaller than that for the \(c\bar{c}\) final state but grows somewhat faster at low energies. At higher energies the ratio is almost constant of the order of 1\%. This is in contrast to double parton scattering contribution for the \(c\bar{c}c\bar{c}\) production which grows much faster than the cross section for single \(c\bar{c}\) production [3].

IV. CONCLUSIONS

We have presented for the first time formulae for the production of two pairs of \(Q\bar{Q}\) in single-parton scattering. The elementary \(gg \rightarrow (Q\bar{Q})(Q\bar{Q})\) cross section was given in two
FIG. 7: Distribution in rapidity difference between quark-antiquark from the same pair (dashed line) and quark-quark, quark-antiquark, antiquark-quark, antiquark-antiquark from different pairs in the $gg \rightarrow (c\bar{c})(c\bar{c})$ for the subsystem energy $W = 200$ GeV.

different representations: so-called mixed one (longitudinal momentum fraction, impact parameter) called also dipole representation, and momentum space one within a high (subsystem) energy approximation. While the dipole representation is easy to include energy dependence of the dipole-dipole interaction the momentum representation seems better suited to include threshold effects. We have discussed how to correct the high-energy formulae close to threshold where the phase space is rather limited by energy-momentum conservation.

We have presented energy dependence as well as different differential distributions for the elementary cross section for $gg \rightarrow (c\bar{c})(c\bar{c})$. We have shown that the elementary cross section varies quickly from the kinematical threshold ($W = 4m_c$) up to $W = 100$ GeV where almost a plateau can be observed. The invariant mass distributions $\frac{d\sigma}{dM_{12}} = \frac{d\sigma}{dM_{14}} = \frac{d\sigma}{dM_{35}} = \frac{d\sigma}{dM_{24}}$ are much steeper than that for $\frac{d\sigma}{dM_{13}} = \frac{d\sigma}{dM_{15}} = \frac{d\sigma}{dM_{23}} = \frac{d\sigma}{dM_{24}}$. where 1 and 2 are from the first $c\bar{c}$ pair and 3 and 4 are from the other $c\bar{c}$ pair.

Our high-energy approach does not include processes when a second pair is produced via gluon emitted from quark/antiquark of a first pair and its “subsequent” splitting into $c\bar{c}$. This processes may be important for small rapidity distance between quarks and antiquarks. A good example can be LHCb kinematics when both $c$ quarks (or both $\bar{c}$ antiquarks) are emitted within two units of rapidity. This mechanism should not be, however, important for the case of large rapidity interval between $cc$ or $\bar{c}\bar{c}$ discussed
FIG. 8: Quark-antiquark invariant mass distributions in the $gg \rightarrow (c\bar{c})(c\bar{c})$. The category I (same pair emission) distributions are shown by the dashed line and category II (different pair emission) distributions by the solid line.

recently in the context of double parton scattering \[3\].

The elementary cross section has been convoluted next with gluon distributions in the proton. We have presented inclusive cross section for $pp \rightarrow (c\bar{c})(c\bar{c})X$ as a function of incident energy as well as invariant mass distribution of the $c\bar{c}c\bar{c}$ system. The results have been compared with corresponding contribution of double-parton scattering. We have found that the single-parton scattering contribution is significantly smaller than that for the double-parton scattering. We conclude therefore that a measurement of two pairs of $c\bar{c}$ at LHC would be very useful in testing models of double parton scattering.

An evaluation of distributions for charmed mesons would be very useful in planning and interpreting current measurements at the LHC. The LHCb collaboration started already such an analysis \[12\].

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FIG. 9: Distribution of the proton-proton cross section in $\xi_1$ ($\xi_2$) for the production of $(c\bar{c})(c\bar{c})$ at $W = 7$ TeV. The solid line is for $\mu_F^2 = m_c^2$ and the dashed line is for $\mu_F^2 = 4m_c^2$. In this calculation a simple leading order gluon distribution from [11] was used.

FIG. 10: Distribution of the $pp \to c\bar{c}c\bar{c}X$ cross section in $M_{cc}$ for the production of $(c\bar{c})(c\bar{c})$ at for $W = 7$ TeV. The solid line is for $\mu_F^2 = m_c^2$ and the dashed line is for $\mu_F^2 = 4m_c^2$. In this calculation a simple leading order gluon distribution from [11] was used.
FIG. 11: Energy dependence of the cross section for the $pp \to c\bar{c}c\bar{c}X$ reaction compared with that for the $pp \to c\bar{c}X$ reaction.

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