Application of initial data sequences to the study of Black Hole dynamical trapping horizons

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Abstract. Non-continuous "jumps" of Apparent Horizons occur generically in 3+1 (binary) black hole evolutions. The dynamical trapping horizon framework suggests a spacetime picture in which these "Apparent Horizon jumps" are understood as spatial cuts of a single spacetime hypersurface foliated by (compact) marginally outer trapped surfaces. We present here some work in progress which makes use of uni-parametric sequences of (axisymmetric) binary black hole initial data for exploring the plausibility of this spacetime picture. The modelling of Einstein evolutions by sequences of initial data has proved to be a successful methodological tool in other settings for the understanding of certain qualitative features of evolutions in restricted physical regimes.

Keywords: trapping and dynamical horizons, black hole mergers, numerical relativity
PACS: 04.70.Bw, 04.25.dg, 02.70.Hm

Problem and antecedents. We aim here at gaining some insight into the understanding of non-continuous Apparent Horizon jumps occurring generically in 3+1 evolutions of black hole spacetimes, with a focus on the binary case. We look at this problem from the perspective of a geometric quasi-local approach to black holes, namely the dynamical trapping horizons introduced by Hayward and Ashtekar & Krishnan (cf. [1] for a recent review). This framework suggests a spacetime picture in which these Apparent Horizon jumps actually correspond to different (non-connected) spatial cuts of a unique underlying smooth hypersurface foliated by (compact) marginally outer trapped surfaces, when the spacelike hypersurface Σ is sliced by a given spacelike hypersurface Σ. Our goal here is to assess this qualitative spacetime picture by means of some specific numerical implementations.

The setting of the dynamical trapping horizon framework is the quasi-local characterization of a black hole in terms of a trapped region. The latter is built on the notion of trapped surface, a closed surface on which the expansions associated with outgoing ℓ⁺ and ingoing ℓ⁻ null congruences are negative: θ⁺ < 0 and θ⁻ < 0, respectively. The black hole horizon is then modelled as a worldtube Σ of marginally outer trapped surfaces (MOTS), i.e. θ⁺ = 0. Additional geometric conditions are needed for Σ to fulfill the expected physical properties of the horizon, namely: a) a future condition, θ⁻ < 0, crucial for deriving an area growth law; and b) an outer condition δℓ⁻θ⁺ < 0, meaning that when moving inwards the horizon we enter into the trapped region. Under the null energy condition, horizon Σ is either a spacelike or null hypersurface. From a 3+1 perspective, we consider the intersections Σt of Σ with slices {Σt} in a 3+1 spacetime foliation. A given Σt can multiply intersect Σ giving rise to Apparent Horizon jumps...
in the 3+1 description. In particular, this is the case if the dynamical horizon is actually part of a smooth MOTS-worldtube changing its metric type (signature) along the evolution (i.e. if $\delta_{\ell-\theta_+} > 0$). This is illustrated in Fig. 1 for the spherically symmetric case, where $\pm 45^\circ$ lines correspond to null directions. The suggested picture is that of a single MOTS-worldtube bending in spacetime. Focusing on the binary case, this translates into a scenario where a single smooth MOTS-worldtube would account for every MOTS found in the $\Sigma_t$ slices. If such a picture is correct then, starting from some initial slice with two (non-connected) MOTS $\mathcal{H}_1$ and $\mathcal{H}_2$, once a common Apparent Horizon appears it should immediately split into a growing common outer horizon $\mathcal{H}_{co}$ (the Apparent Horizon) and a shrinking common inner horizon $\mathcal{H}_{ci}$. The outer one $\mathcal{H}_{co}$ should asymptote to the Event Horizon whereas the inner one $\mathcal{H}_{ci}$ should somehow annihilate itself with the original horizons $\mathcal{H}_1$ and $\mathcal{H}_2$. Figure 2, where $\Sigma$ indicates the direction joining both black holes, illustrates this process. Picture on the left represents the moment in which the common horizon appears for the first time, whereas central and right pictures illustrate two possibilities for the merging of $\mathcal{H}_{ci}$ with $\mathcal{H}_1$ and $\mathcal{H}_2$. In the central one, $\mathcal{H}_1$ and $\mathcal{H}_2$ first merge and the resulting MOTS gets annihilated with $\mathcal{H}_{ci}$. In the picture on the right, $\mathcal{H}_{ci}$ first pinches-off into two MOTSs that annihilate respectively with $\mathcal{H}_1$ and $\mathcal{H}_2$. Of course, these or related ideas have already been discussed in the literature. We refer explicitly to recent references [2, 3, 4] where analytical and numerical studies of MOTS-worldtubes are carried out in different scenarios. We also refer to the recent review [1] where the issue of the signature change in $\mathcal{H}$ is discussed. Results in [4] are specially significant in the present context since an overlapping of $\mathcal{H}_1$ and $\mathcal{H}_2$ is found in the dynamical evolution. This explicitly challenges the single hypersurface picture suggested by dynamical horizons. Our main purpose is to ellucidate if a generic
picture for the evolution of the inner horizon exists at all and, in that case, to assess if it corresponds rather to pinchings or crossings of the horizons.

**Methodology.** The considered problem involves an intrinsically evolutive process. We lack a sufficiently accurate evolution code, so we rather follow an alternative methodology by constructing a sequence of data $(\gamma_{ij}, K^{ij})$. We stay in axisymmetry (MOTS are then axisymmetric) and use spectral methods implemented in bispherical coordinates. An excision technique is employed and a negative outer expansion $\theta_+ = \theta_c^o < 0$ is prescribed at the excision surface. Highly accurate snapshots are thus produced.

A criterion must be adopted to define the initial data sequence. We assume that, once the common horizon has formed, the horizon rapidly settles down to stationarity (radiation rapidly falls into the horizon or is radiated away). Our sequences are thus only physically meaningful once the common horizon has already formed and are characterised by the constancy of the $S_{ci}$ area. In practice, we impose the constancy of the ADM mass along the sequence and use the $S_{ci}$ area-constancy as a check.

**Results.** We start by constructing non-boosted Bowen-York data, for two black holes rotating around the axis joining them. We construct a uni-parametric sequence whose elements are labelled by the coordinate distance parameter $D$ between the coordinate centers of the excised surfaces. The sequence starts at the value $D = D_o$ at which the common horizon appears for the first time. Then the sequence is constructed by diminishing $D$. Proper distance between the horizons also diminishes, as checked a posteriori. First, the common horizon splits (smoothly) into outer and inner MOTSs, consistently with the existence of a single MOTS-worldtube. As $D$ gets down, the common inner MOTS $\mathcal{H}_{ci}$ approaches the original horizons $\mathcal{H}_1$ and $\mathcal{H}_2$, producing the kind of pictures found in [3]. However, when pushing down $D$ we observe none of the expected possibilities (cf. Fig. 3). More specifically: i) original horizons $\mathcal{H}_1$ and $\mathcal{H}_2$ do not develop an eight-shape (i.e. they do not seem to merge) and ii) neither a pinching-off of the common inner horizon $\mathcal{H}_{ci}$ is observed (previous experience with Dyson-ring sequences, where a pinching does occur, strongly supports this statement). Moreover, no crossing/overlapping of $\mathcal{H}_1$ and $\mathcal{H}_2$ like in [4] is either observed. However, when trying to assess this last point, an important non-genericity feature of our data becomes apparent: due to the vanishing of $K_{ij}s^is^j$ and $K$, where $s^i$ is the normal to the MOTS, the considered MOTS are actually minimal surfaces, $0 = \theta_+ = D_is^i + K_{ij}s^is^j - K = D_is^i$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Left: splitting of the common horizon in the $J/M^2 = 0.3$ non-boosted sequence. Center: accumulation of $\mathcal{H}_{ci}$ and $\mathcal{H}_{1,2}$ at small $D$. Right: central-part zoom of the $D = 0.07$ sequence element.}
\end{figure}
A maximum principle then applies: if two such minimal surfaces tangentially touch, they must actually coincide. No crossing can then happen. However, this geometric obstruction is just an artifact of the non-generic nature of the constructed data.

In order to test the crossing possibility, we construct head-on boosted Bowen-York data, for which the relevant components of the extrinsic curvature are non-vanishing. The resulting sequence shows a non-trivial intertwining of MOTS and minimal surfaces. However when pushing $D$ down, the qualitative picture does not change with respect to the non-boosted case in Fig. 3: neither pinching nor crossing are observed, but rather the original horizons $\mathcal{H}_1$ and $\mathcal{H}_2$ accumulate against the common inner horizon $\mathcal{H}_{ci}$.

Geometric tests on the sign of $\theta_-$ and $\delta_{\ell}-\theta_+$ (more precisely on $-\delta_{\ell,\theta_+}$) show that the common outer horizon behaves as a dynamical horizon, whereas the common inner one violates the future and outer geometric conditions.

Conclusions and perspectives. Results in [4] challenge the single smooth MOTS-worldtube picture for Black Hole coalescence suggested by the dynamical trapping horizon framework. We have not found numerical evidence for: a) neither interior horizons annihilation (single smooth MOTS-horizon picture) nor b) crossing of the original horizons [4]. As a third possibility, there could exist three distinct never-merging MOTS-worldtubes, namely an exterior one accounting for the two common horizons, and two other worldtubes corresponding to the original black holes. In sum, the existence of a generic picture is seriously challenged. The whole problem could be reformulated and refined as the search of conditions on the 3+1 slicing guaranteeing the existence of a unique underlying smooth MOTS-worldtube $\mathcal{H}$. The main motivation for this is to set geometric flow equations along $\mathcal{H}$ relating quantities before and after the coalescence. Such equations explicitly need the existence of some underlying smooth hypersurface.

There are some important caveats in our discussion. Namely, we are not dealing with an actual evolution, but only with a sequence of snapshots. One would need to assess the error in the evolution Einstein equations. On the other hand, the fact that we have not been able to construct a single initial data showing either pinching or crossing deserves attention. Of course, genericity issues regarding our data must still be addressed.

Regarding future perspectives, the main interest of this research line is geometric, aiming at understanding the dynamics of MOTS-worldtubes. In this sense, numerical relativity simulations can be crucial for establishing generic behaviours. This can provide relevant insights in the gravitational collapse context, e.g. Penrose inequality or more generally cosmic censorship conjecture, as well as in the study of the Hawking radiation process when taking into account corrections to General Relativity.

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