Study on production of exotic $0^+$ meson $D_{sJ}^*(2317)$ in decays of $\psi(4415)$

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Abstract

The newly observed $D_{sJ}^*$ family containing $D_{sJ}^*(2317)$, $D_{sJ}(2460)$ and $D_{sJ}(2632)$ attracts great interests. Determining their structure may be important tasks for both theorists and experimentalists. In this work we use the heavy quark effective theory (HQET) and a non-relativistic model to evaluate the production rate of $D_{sJ}^*(2317)$ from the decays of $\psi(4415)$, and we find that it is sizable and may be observed at BES III and CLEO, if it is a p-wave excited state of $D_s(1968)$. Unfortunately, the other two members of the family cannot be observed through decays of charmonia, because of the constraints from the final state phase space.

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1 Introduction

The recently observed exotic mesons $D_{sJ}^*(2317)$, $D_{sJ}(2460)$ and $D_{sJ}(2632)$ [1] seem to constitute a new family of mesons which are composed of charm and strange flavors. The mesons possess spin-parity structures of $0^+$, $1^+$ and $0^+$ respectively. This new discovery draws great interests of both theorists and experimentalists of high energy physics. Some authors [2] suppose that $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are the chiral partners of the regular $D_s$ and $D_s^*$, while $D_{sJ}(2632)$ may be a radially excited state of $D_{sJ}^*(2317)$. They may also be considered to be p-wave excited states of $D_s$ [3]. Alternatively, many authors suggest that they can possibly be four-quark states or molecular states [4, 5]. The most peculiar phenomenon is that in some experiments the three resonances are observed with clear signals [1], whereas not by other prestigious experimental groups. One would ask if the observed resonances actually exist or the background was misidentified as a signal. It is noted that similar situations exist for pentaquarks [6]. The goal of the research is to help designing experiments which can help clarifying the mist.

The key point is to experimentally explore the resonances and find a convincing explanation why they are observed in certain experiments, but not in others. However, before it,
one needs to design certain experiments to confirm the existence of $D_{sJ}^*(2317)$, $D_{sJ}(2460)$ and $D_{sJ}(2632)$ and determine their hadronic structures. As aforementioned, there are several different postulates. Measurements may tell which one is more realistic.

Because of the constraint of final-state phase space, observing a final state which involves any of the exotic states can only be realized via decays of higher excited states in the $\psi$ family. From the data-booklet [7], we can see that the lowest excited state which can offer sufficient energy to produce $D_{sJ}^*(2317) + D_s(1668)$ is $\psi(4415)$, but still not enough for $D_{sJ}^*(2317) + D_s(1668)$. However, since $D_{sJ}^*(2317)$ is a $0^+$ meson and $D_s(1668)$ is a $0^-$ meson, a careful analysis on the total angular momentum and parity indicates the decay of $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668)$ is forbidden. Moreover, if only considering the central value of $\psi(4415)$, the phase space is not enough for $D_{sJ}^*(2317) + D_s(1668) + \pi$ and $D_{sJ}^*(2317) + \bar{D}_s^*(2112)$ which could be produced via pure strong interaction and the only possible mode is the radiative decay $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668) + \gamma$. That is a decay with a three-body final state and is an electromagnetic process where a p-wave is necessary to conserve the total angular momentum and parity. This observation tells us that the corresponding branching ratio must be very suppressed and is a rare decay. Recently, Barnes et al. [8] suggest to observe $D_{sJ}^*(2317)$ via the process

$$\psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s^*(2112).$$

The advantage is that it is a strong decay with a two-body final state, therefore the amplitude may be large, but meanwhile, by the central values, $m_{D_{sJ}^*(2317)} + m_{D_s^*(2112)} > 4415$ MeV, thus this reaction can only occur via the threshold effect and would suffer from a corresponding suppression. If it is of a larger rate (we will estimate it later in the work), the decays $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668) + \pi$ and $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668) + \gamma$ can also be realized via secondary decays of $D_{sJ}^*(2112) \to D_s(1668) + \pi$ and $D_{sJ}^*(2112) \to D_s(1668) + \gamma$ and these are the dominant modes over the direct decay modes $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668) + \gamma$ which are three-body decays.

The picture is that the charmonium $\psi(4415)$ dissolves into a $c\bar{c}$ pair and both $c$ and $\bar{c}$ are free and on mass shell, and the soft gluons emitted from $c\bar{c}$ can excite the physical vacuum to create a pair of $s\bar{s}$. The process of $s\bar{s}$ pair creation is quantitatively described by the quark-pair-creation model (QPC) [9] [10]. Then the $s\bar{s}$ join the corresponding $\bar{c}c$ and $c$ to compose charmed mesons. Indeed, the creation process is fully governed by the non-perturbative QCD effects, thus the rate is not reliably calculable so far and can only be estimated in terms of models. In this work, we use QPC model [9] [10] to evaluate the rates of $\psi(4415) \to D_{sJ}^*(2317) + D_s^*(2112)$ and the direct decay $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668) + \gamma$ where a photon is emitted during the process.

In this work, we consider the transitions of $\psi(4415) \to D_{sJ}^*(2317) + D_s^*(2112)$ and the subsequent observable modes $\psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s^*(2112) \to D_{sJ}^*(2317) + D_s(1668) + \gamma$ ($D_{sJ}^*(2317) + \bar{D}_s(1668) + \pi$). We also calculate the ratio of the direct radiative decay process $\psi(4415) \to D_{sJ}^*(2317) + D_s(1668) + \gamma$ which is not produced via the resonance $D_s^*(2112)$.

The key point is how to evaluate the hadronic matrix elements. Here we must adopt suitable models to do the job.
$D_{sJ}^{*}(2317)$ and $D_s^{(*)}$ all are heavy mesons, therefore one can expect that the heavy quark effective theory (HQET) applies for evaluating the hadronic matrix elements. For a completeness, we keep the $1/m_c$ corrections in the formulation, however, it is obvious that such corrections are practically negligible in the concerned case, so that we do not really include them in the numerical calculations. As a check, we employ a non-relativistic model to re-evaluate the hadronic matrix elements and compare the results obtained in the two approaches.

To obtain the concerned parameters and testify the applicability of the model, we calculate the branching ratios of $\psi(4040) \to D^{(*)} + \bar{D}^{(*)}$ and $D_s + \bar{D}_s$. By fitting data, we determine the vacuum production rate of the quark-pairs in HQET. Moreover, when using the non-relativistic model, we also need to determine the concerned parameters in the wavefunctions. More concretely, there are several decay channels in $\psi(4040)$ with $c$ and $\bar{c}$ in the final states ($D^0(\bar{sJ})D^{0(\bar{s})}$, $D^{\pm(\bar{s})}\bar{D}^{\pm(\bar{s})}$), and their branching ratios are experimentally measured. Actually, in HQET, the only free parameter is the rate of quark-pair creation from vacuum, i.e. $\gamma_q$, then one mode is enough to fix it. We can check the obtained model and the parameter by applying them to evaluate other modes which have also been experimentally measured. Our numerical results respect the pattern determined by the experiments. For $\psi(4415)$ more channels are available, that is $D_s\bar{D}_s$ (or $D_s^+D_s^-$), etc. We may naively consider that the production of $D_s\bar{D}_s$ in $\psi(4415) \to D_s + \bar{D}_s$ is somehow related to $\psi(4040) \to D^{(*)} + \bar{D}^{(*)}$, then all the parameters obtained from decays of $\psi(4040)$ can be applied to study decays of $\psi(4415)$ while assuming the parameters are not very sensitive to the energy scale.

In both HQET and non-relativistic model, we derive the formulation for the branching ratio of $\psi(4415) \to D_{sJ}^{(*)}(2317) + \bar{D}_s^{(*)}(2112)$ and obtain the final numerical results. We also formulate the direct process $\psi(4415) \to D_{sJ}^{*}(2317) + \bar{D}_s(1986) + \gamma$ which is the only channel allowed by the phase space if neglecting the threshold effects. Even though these results with the aforementioned approximations cannot be very accurate, one expects that the order of magnitude of the calculated result would be right.

If the exotic states $D_{sJ}^{*}(2317)$ is of the 4-quark structure as suggested [4, 5], in the production process at least three pairs of quarks are created from vacuum, and the final state would involve more quarks and anti-quarks, thus the integration over the final-state phase space would greatly suppress the rate. By our rough numerical evaluation, at least 4 orders suppression would be resulted for the decays, if the exotic meson $D_{sJ}^{*}(2317)$ is a four-quark state. Thus by measuring the branching ratio of $\psi(4415) \to D_{sJ}^{*}(2317) + \bar{D}_s^{(*)}(2112)$, we may judge (1) if the exotic meson $D_{sJ}^{*}(2317)$ indeed exists, (2) what quark structure it possesses.

This work is organized as follows, after the introduction, in Sect. 2, we formulate the decay rates of $\psi(4415) \to D_{sJ}^{*}(2317) + \bar{D}_s^{(*)}(2112)$ and direct process $\psi(4415) \to D_{sJ}^{*}(2317) + D_s(1986) + \gamma$. In Sect.3, we present our numerical results along with all the input parameters. Finally, Sect. 4 is devoted to discussion and conclusion. Some detailed expressions are collected in Appendix.
2 Formulation

The QPC model about the process that a pair of quarks with quantum number $J^{PC} = 0^{++}$ is created from vacuum was first proposed by Micu\cite{9} in 1969. In the 1970s, QPC model was developed by Yaouanc et al. \cite{10, 11, 12, 13} and applied to study hadron decays extensively. Recently there are some works \cite{14, 15} to study QPC model and its applications \cite{16}. In the QPC model, the interaction which represents the mechanism of a pair of quarks created from vacuum can be written as \cite{15}

$$S_{\text{vac}} = g_{tq} \int d^4 x \bar{\psi}_q \psi_q, \quad \text{with} \quad L = g_{tq} \bar{\psi}_q \psi_q,$$

(1)

where $g_{tq} = 2 m_q \gamma_q$. $\gamma_q$ is a dimensionless constant which denotes the strength of quark pair creation from vacuum, and can only be obtained by fitting data. $m_q$ ($q = u, d, s$) are the masses of light quarks. In the non-relativistic approximation \cite{13}, the interaction Hamiltonian (1) can be expressed as

$$\mathcal{H}_{\text{vac}} \rightarrow \mathcal{H}_{\text{vac}}^{\text{non}} = \sum_{i,j} \int d^4 p d^4 p' [3 \gamma_q \delta^3(p_q + p_{\bar{q}})] \sum_m \langle 1, 1; m, -m | 0, 0 \rangle$$

$$\times \mathcal{X}_m^1 (p_q - p_{\bar{q}}) (\chi_{m,0}^{(i)} \gamma_0 \psi_0^{(j)}) b_i^+(p_q, s) d_j^+(p_{\bar{q}}, s'),$$

(2)

where $i$ and $j$ are SU(3)-color indices of the created quarks and anti-quarks; $s$ and $s'$ are spin polarizations; $\chi_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $(\omega_0)_{ij} = \delta_{ij}$ for flavor and color singlets respectively; $\chi_m$ is a triplet state of spin, $\mathcal{X}_m^1$ is a solid harmonic polynomial corresponding to the $p$-wave quark pair.

2.1 The transition amplitude of $\psi(4415)$ in QPC model.

In this work, we study the strong decay $\psi(4415) \rightarrow \bar{D}^*_s(2112) + D^*_s(2317)$ and the direct radiative decay $\psi(4415) \rightarrow \gamma + \bar{D}_s(1668) + D^*_s(2317)$. With the QPC model, during these transitions charm-quark and antiquark from $\psi(4415)$ combine with the $ss$ created from vacuum to form final state particles. The Feynman diagrams of these transitions are depicted in Fig. \[\]

The transition matrix element of $\psi(4415) \rightarrow \bar{D}^*_s(2112) + D^*_s(2317)$ is

$$T^{\text{strong}} = \langle \bar{D}^*_s(2112)D^*_s(2317)|\mathcal{H}_{\text{vac}}(x)|\psi(4415) \rangle.$$

(3)

For the direct radiative decay $\psi(4415) \rightarrow \gamma + \bar{D}_s(1668) + D^*_s(2317)$, the transition matrix element reads as

$$T^{\text{dir}} = \langle \bar{D}_s(1668)D^*_s(2317)\gamma|T \int dx dy [\mathcal{L}_{\text{vac}}(x)\mathcal{L}_{\text{em}}(y)]|\psi(4415) \rangle,$$

(4)

where $\mathcal{L}_{\text{em}}(y)$ is the electromagnetic interaction Hamiltonian and have the following form

$$\mathcal{L}_{\text{em}}(y) = \pm \frac{2e}{3} \int d^4 x \bar{\Psi} \gamma_\mu \Psi A^\mu(y).$$

(5)
where the sign ± corresponds to charges of \( c \) and \( \bar{c} \) respectively. Considering the weak binding approximation, \(|\psi(4415)\rangle\) can be expressed as

\[
|\psi(4415)\rangle \rightarrow N \Psi(0) \bar{c} \gamma \epsilon \langle 0 |,
\]

(6) where \( \Psi(0) \) is the wave function at origin and \( \epsilon \) denotes the polarization vector. \( N \) is the normalization constant.

It is also noted that for decays \( \psi(4040) \rightarrow D^{(*)} \bar{D}^{(*)} \), the Feynman diagrams are similar to that in Fig.1 (a).

2.2 Evaluation of the hadronic matrix elements in HQET.

(i) The strong decay \( \psi(4415) \rightarrow D_s^*(2112) + D_{sJ}^*(2317) \).

The diagram in Fig. (a) involves the q-meson-Q vertices. In refs. [17, 18], the effective Lagrangian for these vertices has been constructed based on the heavy quark symmetry and chiral symmetry

\[
\mathcal{L}_{HL} = \bar{h}_v (iv \cdot \partial) h_v - [g \bar{\chi}(\bar{H} + \bar{S}) h_v + H.c.] + g' [Tr(\bar{H}H) + Tr(\bar{S}S)],
\]

(7) where the first term is the kinetic term of heavy quarks with \( \bar{\phi} h_v = h_{\bar{v}} \); \( H \) is the super-field corresponding to the doublet \((0^-, 1^-)\) of negative parity and has an explicit matrix representation: \( H = \frac{1+\gamma_5}{2} (P^*_\mu \gamma^\mu - P^* \gamma_5) \); \( P \) and \( P^* \) are the annihilation operators of pseudoscalar and vector mesons which are normalized as \( \langle 0 | P | M(0^-) \rangle = \sqrt{M_H} \), \( \langle 0 | P^{*\mu} | M(1^-) \rangle = \sqrt{M_H} \epsilon^\mu \); \( S \) is the super-fields related to \((0^+, 1^+)\) and \( S = \frac{1+\gamma_5}{2} (P_{\mu^*}^* \gamma^\mu \gamma_5 - P_0) \); \( \chi = \xi q \) \((q = u, d, s \) is the light quark field and \( \xi = e^\gamma \), here we only take the leading order as \( \xi \approx 1 \)).
The effective Lagrangian in Eq. (7) contains only the leading order for the coupling of meson with quarks. We may also include the $1/m_Q$ corrections in the expressions. The heavy-light quark interaction Lagrangian given in [18] is

$$L_{HL} = \bar{Q} (i \partial - m_Q) Q - \frac{2g^2}{\Lambda^2} \bar{Q} \gamma_\mu \frac{\lambda^A}{2} Q \bar{\psi} \gamma_\mu \frac{\lambda^A}{2} \psi,$$  

where $Q = (b, c)$ and $\psi = (u, d, s)$, $m_Q$ is the heavy quark mass. We can obtain the $1/m_Q$ corrections from two aspects. The first comes from the quark wavefunction [19]

$$Q(x) = e^{-im_Q v \cdot x} \left( 1 + \frac{iD}{2m_Q} + \cdots \right) h_v(x), \quad D_\perp = D^\mu - v^\mu v \cdot D,$$  

the $1/m_Q$ correction is obtained by replacing $h_v$ with $(1 + iD_\perp/2m_Q) h_v$ in (7). Secondly, the superfields $H$ and $S$ in (7) should also receive $1/m_Q$ correction. Falk et al. [20] presented the changes as

$$H \rightarrow H + \frac{1}{2m_Q} \{\gamma^\mu, iD_\mu H\}, \quad S \rightarrow S + \frac{1}{2m_Q} \{\gamma^\mu, iD_\mu S\}.$$  

Then, we can include the $1/m_Q$ corrections in (7).

Now, let us write down the transition amplitude for the decay of $\psi(4415) \rightarrow \bar{D}_s^*(2112) + D_s^*(2317)$ as

$$M(\psi(4415) \rightarrow \bar{D}_s^*(2112) + D_s^*(2317))$$

$$= \frac{\Psi(0)}{6\sqrt{M_A}} \text{Tr} \left[ g \frac{i}{D_\perp} \frac{\sqrt{M_C}}{\bar{p}_C - \bar{p}_A/2 - m_s} g_{sA} \frac{i}{\bar{p}_A - \bar{p}_B - m_s} g \sqrt{M_B} \right]$$

$$\times (M_A + \bar{p}_A) f_A$$

$$= \frac{2\Psi(0)g^2 g_{sA} \sqrt{M_B M_C}}{3\sqrt{M_A}} \left[ M_A m_q^2 - \frac{1}{4} M_A^3 - M_A (p_A \cdot p_C) - M_A M_B^2 \right] (\epsilon_{D_s^*} \cdot \epsilon_A)$$

$$\times \frac{1}{(p_A^2 - p_B^2)^2 - m_s^2} [(p_A^2 - p_B^2)^2 - m_s^2],$$  

where $p_A$, $p_B$ and $p_C$ represent the four momenta of $\psi(4415)$, $D_{sJ}^*(2317)$ and $D_s^*(2112)$; $M_A$ and $\epsilon_A$ are the mass and the polarization vector of $\psi(4415)$; $M_B$ and $M_C$ are the masses of two produced mesons; $g$ is the coupling constant of Q-meson-q vertex which is given in literature [17]. It is noted that by the central values

$$m_{D_{sJ}^*(2317)} + m_{D_s^*(2112)} > 4415 \text{ MeV},$$

thus the process can only occur through the threshold effect. The resonance $\psi(4415)$ has a total width $\Gamma_A$. Considering the distribution, we adopt the typical Gaussian form suggested by the data group [7], and set the lower and upper bound for the integration of final phase space as $M_A - \delta < M < M_A + \delta$ and the delta-function guarantees the energy-momentum conservation.
Finally we obtain the width
\[
\Gamma(\psi(4415) \rightarrow \tilde{D}^*_s(2112) + D^*_s(2317)) = \frac{2}{(1 - \beta)\sqrt{2\pi}} \int_{M_A - \delta}^{M_A + \delta} \left\{ \frac{1}{6M} \int \frac{d^3p_B d^3p_C}{(2\pi)^32E_B(2\pi)^32E_C} |M(\psi(4415) \rightarrow \tilde{D}^*_sD^*_s(2317))|^2 \times (2\pi)^4\delta^4(M - p_B - p_C) \right\} \exp \left[ - \frac{(M - M_A)^2}{2(\Gamma_A/2)^2} \right] dM, \tag{11}\]
where
\[
|p_B| = \sqrt{(M^2 - (M_B + M_C)^2)(M^2 - (M_B - M_C)^2)}, \tag{12}\]
\[
E_B = \sqrt{M_B^2 + p_B^2}, \quad E_C = \sqrt{M_C^2 + p_B^2}; \quad \delta = 1.64 \frac{\Gamma_A}{2}, \quad \beta = 10\%. \tag{13}\]

For the indirect subsequent decays \(\psi(4415) \rightarrow D^*_s(2317) + \tilde{D}_s(1968) + \gamma\) and \(\psi(4415) \rightarrow D^*_s(2317) + \tilde{D}_s(2112) \rightarrow D^*_s(2317) + D_s(1968) + \pi\), the rates are obtained as
\[
\Gamma^{ind}(\psi(4415) \rightarrow D^*_s(2317) + \tilde{D}_s(1968) + \gamma) = \Gamma(\psi(4415) \rightarrow D^*_s(2317) + \tilde{D}_s(2112) \rightarrow D^*_s(2317) + D_s(1968) + \pi), \tag{14}\]
\[
\Gamma^{ind}(\psi(4415) \rightarrow D^*_s(2317) + \tilde{D}_s(2112) \rightarrow D^*_s(2317) + \tilde{D}_s(1968) + \pi). \tag{15}\]

(ii) The direct radiative decay \(\psi(4415) \rightarrow \gamma + \tilde{D}_s(1968) + D^*_s(2317)\).

This process is much more complicated than the strong decay depicted in (i).

By the QPC model a pair of \(ss\) quarks is created from vacuum and the underlying mechanism is the soft gluon exchanges which excite the vacuum sea. The momentum of the light quark pair created from vacuum is small and the photon hardly has possibility to be produced from light quark. Thus we can ignore the contribution of Fig. (d) and (f) for the direct \(\psi(4415) \rightarrow \gamma + \tilde{D}_s(1968) + D^*_s(2317)\) process.

The amplitude for radiative decay \(\psi(4415) \rightarrow \gamma + \tilde{D}_s(1968) + D^*_s(2317)\) includes several pieces. For Fig. (b),
\[
\mathcal{M}_{(b)} = Q_c \left[ \frac{1}{m_c} \int \frac{d^3q}{(2\pi)^3/2} \psi_{s_1s_2}(q) \bar{u}(p_2, s_2) \mathcal{O}_{(b)}(q) u(p_1, s_1) \right], \tag{16}\]
\[
\mathcal{O}_{(b)} = \gamma_5 g \sqrt{M_C} \sqrt{p_C - \hat{p}_A/2 - m_s} \cdot g_{is} \sqrt{\frac{i}{\hat{p}_B + \hat{p}_A/2 - s - m_s}} \cdot g_{ks} \sqrt{M_B} \times \sqrt{\frac{i}{\hat{p}_A/2 - s - m_c}} \hat{f}_k. \]

for Fig. (c), the transition amplitude reads as
\[
\mathcal{M}_{(c)} = -Q_c \left[ \frac{1}{m_c} \int \frac{d^3q}{(2\pi)^3/2} \psi_{s_1s_2}(q) \bar{u}(p_2, s_2) \mathcal{O}_{(c)}(q) u(p_1, s_1) \right], \tag{17}\]
\[ O^{(c)} = \frac{i}{\hbar} \frac{g}{\sqrt{M_C}} \gamma_5 \frac{\sqrt{M_B}}{i \vec{p}_A/2 - m_s} g_{1s} \]

where \( p_1, p_2 \) and \( s_1, s_2 \) are the momenta and spin projections of the charm quark and anti-charm quark, and the following relations hold:

\[ p_1 + p_2 = p_A = (M_A, 0), \quad p_1 - p_2 = 2q = (0, 2q), \quad \sum_{s_1, s_2} d^4q |\psi_{s_1s_2}(q)|^2 = 1. \]

Using the method of \[21\], we have

\[
\frac{1}{m_c} \int \frac{d^4q}{(2\pi)^3/2} \psi_{s_1s_2}(q) \bar{u}(p_2, s_2) O^{(i)} u(p_1, s_1) = \frac{1}{m_c} \int \frac{d^4q}{(2\pi)^3/2} \psi_{s_1s_2}(q) \text{Tr}[M_{A} O^{(i)} + \{ O^{(i)} \}, \gamma_5 + M_A q \cdot \hat{O}^{(i)}] \frac{1 + \gamma_0}{2\sqrt{2}} (-f_A) \]

where \( O^{(i)} = O^{(i)}|_{q=0} \) and \( \hat{O}^{(i)} \equiv \frac{\partial}{\partial q^{\mu}} O^{(i)}|_{q=0} \), and \( i \) denotes \( (b) \) or \( (c) \).

Dissociation of the charmonium into \( c\bar{c} \) can be well described by the non-relativistic model where the wavefunction at origin \( \Psi(0) \) corresponds to the binding effect. \[16\] and \[17\] can be further expressed as

\[
\mathcal{M}_{(b)} = \frac{\Psi(0) Q_c^c}{6\sqrt{M_A}} \text{Tr} \left[ \frac{\gamma_5 \sqrt{M_C}}{\vec{p}_C - \vec{p}_A/2 - m_s} \cdot g_{1s} \cdot \frac{i}{\sqrt{M_B}} \right] \frac{i}{\hat{p}_B + \hat{p}_A/2 - \vec{k} - m_s} \frac{1}{\hat{p}_A/2 - \vec{k} - m_c} \left( \hat{f}_k(M_A + \hat{p}_A) \hat{f}_A \right),
\]

\[
\mathcal{M}_{(c)} = -\frac{\Psi(0) Q_c^c}{6\sqrt{M_A}} \text{Tr} \left[ \frac{\gamma_5 \sqrt{M_C}}{\vec{p}_C - \vec{p}_A/2 - m_s} \cdot g_{1s} \cdot \frac{i}{\sqrt{M_B}} \right] \frac{i}{\hat{p}_B + \hat{p}_A/2 - \vec{k} - m_s} \frac{1}{\hat{p}_A/2 - \vec{k} - m_c} \left( \hat{f}_k(M_A + \hat{p}_A) \hat{f}_A \right),
\]

where \( p_A, p_B, p_C \) and \( k \) correspond to the four momenta of \( \psi(4415), D_s^*(2317), D_s(168) \) and photon respectively; \( \epsilon_k \) is the polarization vector of the emitted photon.

The decay width for \( \psi(4415) \to \gamma + D_s(168) + D_s^*(2317) \) radiative decay is expressed as

\[
\Gamma = \frac{1}{6M_A} \prod_{i} \left( \frac{d^3p_i}{(2\pi)^3/2E_i} \right) (2\pi)^4 \delta^4(M_A - p_B - p_C - k) |\mathcal{M}_{(b)} + \mathcal{M}_{(c)}|^2.
\]

In next section, we carry out the multiple integration to obtain numerical results.
2.3 Evaluation of the hadronic matrix elements in the non-relativistic model.

As discussed above, for a comparison\(^1\) we are going to employ a non-relativistic model i.e. the harmonic oscillator model to repeat the calculations made in terms of HQET. Application of such model should be reasonable in this case.

(i) Strong decay \(\psi(4415) \rightarrow \bar{D}_s^*(2112) + D_{sJ}^*(2317)\).

We calculate the \(\psi(4415) \rightarrow \bar{D}_s^*(2112) + D_{sJ}^*(2317)\) by using QPC model in the non-relativistic approximation. The decay width is

\[
\Gamma(\psi(4415) \rightarrow \bar{D}_s^*(2112) + D_{sJ}^*(2317)) = \frac{2}{(1 - \beta)^2} \frac{\sqrt{2} \pi E_B E_C |k|}{M} \sum_{l,s} |M_{ls}|^2 \exp \left[ -\frac{(M - M_A)^2}{2(\Gamma_A/2)^2} \right] dM, \tag{22}
\]

and the concrete expression of \(\sum_{l,s} |M_{ls}|^2\) is collected in Appendix. The definitions of \(\delta\) and \(\beta\) in the expression are exactly the same as those given in eq.\(^{[13]}\).

(ii) For direct radiative decay \(\psi(4415) \rightarrow \gamma + \bar{D}_s(1968) + D_{sJ}^*(2317)\).

Following the traditional method \(^{[13]}\), the matrix element of radiative decay \(\psi(4415) \rightarrow \gamma + \bar{D}_s(1968) + D_{sJ}^*(2317)\) in nonrelativistic approximation can be written as

\[
\langle \Psi_{D_s}(p_2, s_2'; p_4', s_4'); \Psi_{D_{sJ}}(p_1', s_1'; p_3, s_3) \Delta \gamma (k, \epsilon(k)) | T[\mathcal{H}_\text{em}^\text{non}] \rangle = \frac{\gamma \mathcal{F} \mathcal{C}}{n_1 n_2 n_3 n_4 n_{D_{sJ}} n_{D_{sJ}} n_{D_{sJ}} n_{D_{sJ}}} \int \prod_{d=1}^{4} dp_a \prod_{b=1}^{4} dp_b' \delta^3(p_1 + p_2 - p_0) \delta^3(p_2 + p_4 - p_3) \delta^3(p_3 + p_4 - p_1) \times \langle 1, 1; -n_{D_{sJ}}, n_{D_{sJ}} | 0, 0 \rangle \langle 1, 1; n_{D_{sJ}}, -n_{D_{sJ}} | 0, 0 \rangle \prod_{d=1}^{4} \delta^3(p_3 - p_4) \nonumber \times \varphi_{\psi}(p_1 - \frac{1}{2} p_2, p_2 - \frac{1}{2} p_0) \varphi_{D_{sJ}}(p_1' - \frac{1}{2} p_{D_{sJ}}, p_3 - \frac{1}{2} p_{D_{sJ}}) \nonumber \times \varphi_{D_{s}}(p_4' - \frac{1}{2} p_{D_{sJ}}, p_4' - \frac{1}{2} p_{D_s}) \rangle 0 | b_p_1 d_{p_3} b_{p_4} a_k \int \frac{d^3 x}{3} \frac{2e}{\sqrt{\gamma c}} \Psi_c \gamma^\mu \Psi_c A_\mu(x) \nonumber \times b_{i}' p_{i}' d_{p_3} b_{p_4} b_{p_1} 0 |, \tag{23}
\]

where \(\mathcal{F}\) and \(\mathcal{C}\) correspond to the flavor and color factors in this transition; \(\chi\)'s are the spin wave functions; \(p_\psi, p_{D_s}\) and \(p_{D_{sJ}}\) are three-momenta of \(\psi(4415), D_s(1968)\) and \(D_{sJ}^*(2317)\); \(\varphi_{\psi}, \varphi_{D_{sJ}}\) and \(\varphi_{D_s}\) are the harmonic oscillator wave functions of \(\psi(4415), D_{sJ}^*(2317)\) and \(D_s(1968)\) respectively.

\(^1\)There is another reason to employ the non-relativistic harmonic oscillator model. If \(D_{sJ}^*(2317)\) is of four-quark structure, the HQET no longer applies and the only model we can use for the multi-constituents structure is the harmonic oscillator model. Therefore a comparison of the results in the model with that obtained by HQET is indeed meaningful. Namely, HQET is believed to be applicable in this case, thus consistence of the results obtained in the two approaches can confirm applicability of the harmonic oscillator model. Then we can use it to calculate the production rate if \(D_{sJ}^*(2317)\) is of four-quark structure.
Neglecting some technical details, finally one can derive the decay amplitude of Fig. 1 (b)

\[ M_b(\psi(4415) \rightarrow \gamma + D_s^*(2317) + \bar{D}_s(1968)) \]

\[ = i\gamma S \Psi(0) \left( \frac{4R^2}{\sqrt{35}} \right) \left( \frac{R_B^2}{\pi} \right)^{3/4} \sqrt{2} \frac{R_C^{3/2}}{9 \pi^{1/4}} \left[ - \frac{2e}{3} \left( \frac{1}{2\pi} \right)^{2/3} \frac{1}{\sqrt{2E_k}}(2\pi)^4 \right] \]

\[ \times \int dp_2 \exp \left[ - \frac{1}{8} R_A^2 (2p_2)^2 - \frac{1}{8} R_C^2 (p_C + 2p_B + 2p_2)^2 - \frac{1}{8} R_B^2 (2p_2 + p_B)^2 \right] \]

\[ \times \mathcal{Y}_1^{-n_B}(-2p_2 - p_B) \mathcal{Y}_1^{n_B}(-2p_B - 2p_A + 2p_2) \bar{v}(p_2, s_2) \gamma_{\mu} v(p_C + p_B - p_A + p_2, s_2') e_{s_1}^{S}(k) \]

\[ \times \frac{-105 + 210R_A^2p_2^2 - 84R_A^4p_2^4 + 8R_A^6p_2^6}{12\sqrt{35}}, \] (24)

where \( S \) is a spin factor, \( \Psi(0) \) is the wave function of \( \psi(4415) \) at origin. The indices \( A, B \) and \( C \) are for \( \psi(4415), D_s^*(2317) \) and \( D_s(1968) \) respectively.

With the same treatment, we also obtain the amplitude \( M_c(\psi(4415) \rightarrow \gamma + D_s^*(2317) + \bar{D}_s(1968)) \) of Fig. 1 (c), and for saving space we keep its expression in Appendix.

2.4 A rough estimation of the production rate of \( D_s^*(2317) \) in \( \psi(4415) \) decays, if \( D_s^*(2317) \) is of a four-quark structure.

There have been some works which suggest that \( D_s^*(2317) \) is of a four-quark structure\[4, 5\], the situation would be completely different. We draw a possible Feynman diagram in Fig. 2, and one can notice that three quark-pairs are created from vacuum. As more particles are produced, the final state phase space would greatly reduce the rate.

![Figure 2](image)

Figure 2: The Feynman diagram describing the production of \( D_s^*(2317) \) in \( \psi(4415) \) inclusive decay considering the four quark structure of \( D_s^*(2317) \) in the QPC model, where ellipsis denotes the diagrams for other possible quark combinations.

The inclusive transition matrix element can be written as

\[ \langle \bar{c}, s, q, \bar{q}, D_s^*(2317) | T[H_{\text{vac}}(x_1)H_{\text{vac}}(x_2)H_{\text{vac}}(x_3)] | \psi(4415) \rangle. \] (25)

Thus the decay width reads as

\[ \Gamma(\psi(4415) \rightarrow D_s^*(2317) + \bar{c} + s + q + \bar{q}) \]
\[
= \frac{1}{6M_A} \int \frac{d^3 p_1}{(2\pi)^3 2\omega_1} \prod_{i=1}^4 \int \frac{d^3 k_i}{(2\pi)^3} \frac{m_i}{E_i} (2\pi)^4 \delta^4(M_A - p_1 - \sum k_i) \exp(i p_1 \cdot \vec{x} - i k_i \cdot \vec{p}) \times |M(\psi(4415) \to D_{sJ}^*(2317) + \bar{c} + s + q + \bar{q})|^2.
\]

(26)

Two points are noted: First this amplitude cannot be evaluated in the framework of HQET, but only in the non-relativistic model because of the complicated quark-structure; Secondly Fig.2 depicts an inclusive process \( \psi(4415) \to D_{sJ}^*(2317) + 4 \) free quarks, considering hadronization, observable processes can only be \( \psi(4415) \to D_{sJ}^*(2317) + D_s(1968) + \pi \) and \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s(1968) + \gamma \) where \( q\bar{q} \) annihilate into a photon. As discussed above, such direct processes are much suppressed.

Because the inclusive decay \( \psi(4415) \to D_{sJ}^*(2317) \) is related to multi-body final states, the calculation is very complicated. The multi-integration over the phase space is very difficult, even in terms of the Monte-Carlo method. Generally the rate is proportional to

\[ \alpha \sim (\gamma_q)^3 \left[ \frac{4\pi}{(2\pi)^3} \right]^4, \]

which is a remarkable suppression factor.

Thus if \( D_{sJ}^*(2317) \) is of a four-quark structure, one can expect that the inclusive decay width of \( \psi(4415) \to D_{sJ}^*(2317) \) is at least four orders smaller than the corresponding value if \( D_{sJ}^*(2317) \) is a p-wave excited state of the regular \( D_s \) meson.

3 Numerical results

(1) Determination of the concerned parameters in the two approaches.

(a) The parameters for the transition in HQET.

The coefficient \( G = \gamma g^2 \), which is introduced in the transition amplitude, is obtained by fitting the decay width of \( \psi(4040) \to D \bar{D} \). In Appendix, we present the formula for the decay rate of charmonium into two charmed pseudoscalar mesons in HQET. For calculating \( G \), the value of \( \Psi_{\psi(4040)}(0) \) is obtained by fitting the experimental data of \( \psi(4040) \to e^+e^- \) [22]. We get \( \Psi_{\psi(4040)}(0) = 0.101 \text{ GeV}^{3/2} \) and \( G = 12.3 \text{ GeV}^{-1} \). In ref. [17], the value of \( g^2 \) is obtained as \( g^2 = 4.17 \text{ GeV}^{-1} \), thus we obtain \( \gamma_q = 2.95 \). Yaouanc et al. used to employ the harmonic oscillator to evaluate such processes, and they got \( \gamma_q \approx 3 \) [13], which is very close to the value we obtain with HQET. However, it is also noted that \( \gamma_q \) is purely a phenomenological parameter and its value may vary within a reasonable range, for example, in their later work, Yaouanc et al. took \( \gamma_q \) to be 4 instead, when they fitted data [12].

Since there are not enough data to determine \( \gamma_s \), we adopt the relation [12]

\[ \gamma_s = \gamma_q / \sqrt{3}, \]

for later numerical computations.
(b) The parameters in the non-relativistic model.

In this scenario, the non-relativistic approximation is taken and the expression is no longer Lorentz invariant, the relevant parameters may be somehow different from the values in HQET, especially the value of $\gamma_q$ which corresponds to the vacuum creation of a quark pair. However, as pointed above the values obtained in these two approaches are very close, thus we can use $\gamma_q = 2.95$ for later calculations.

Using the experimental results of $\psi(4040) \rightarrow D\bar{D}$, $\psi(4040) \rightarrow D^*\bar{D}^*$ and $\psi(4040) \rightarrow D_s^+D_s^-$ decays [23], we obtain all the relevant parameters which are needed for later numerical computations. For the readers’ convenience the relevant formulations [12] are collected in Appendix. With all the information, we obtain the values of $R$'s in the harmonic oscillator wave functions as:

$$\psi(\gamma)$$

values in HQET, especially the value of $\gamma$.

It is believed that this approximation is reasonable for estimating the order of magnitude of these transitions.

(2) Our numerical results for the $D_{sJ}^*(2317)$ production.

In the calculations of $\psi(4415) \rightarrow D_{sJ}^*(2317)$ and the direct decay $\psi(4415) \rightarrow \gamma + D_s(1968) + D_{sJ}^*(2317)$ by the two approaches, we employ the following parameters as inputs: $M_{\psi(4415)} = 4.415$ GeV, $M_{D_{sJ}^*} = 1.968$ GeV, $M_{D_s^*} = 2.112$ GeV, $M_{D_{sJ}^*(2317)} = 2.317$ GeV [7]. By fitting the data of $\psi(4415) \rightarrow e^+e^-$ which is available at present, we obtain $\Psi(\psi(4415))(0) = 0.088$ GeV$^{3/2}$.

We now present the numerical results obtained with the two approaches in Table. 2.

|                  | I     | II    |
|------------------|-------|-------|
| $Br(\psi(4415) \rightarrow D_{sJ}^*(2112) + D_{sJ}^*(2317))$ | 9.16% | 9.58% |
| $Br(\psi(4415) \rightarrow \gamma + D_s(1968) + D_{sJ}^*(2317))(\text{ind})$ | 8.63% | 9.03% |
| $Br(\psi(4415) \rightarrow \gamma + D_s(1968) + D_{sJ}^*(2317))(\text{dir})$ | $5.31 \times 10^{-3}$ | $5.56 \times 10^{-3}$ |
| $Br(\psi(4415) \rightarrow \gamma + D_s(1968) + D_{sJ}^*(2317))(\text{dir})$ | $8.46 \times 10^{-5}$ | $2.29 \times 10^{-5}$ |

Table 1: Columns I and II correspond to the numerical results obtained in HQET and the non-relativistic model respectively.

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2 The BES measurements of the inclusive charm cross section at 4.03 GeV are [23]: $\sigma_{D^0} + \sigma_{\bar{D}^0} = 19.9 \pm 0.6 \pm 2.3$ nb, $\sigma_{D^+ + \sigma_{D^-}} = 6.5 \pm 0.2 \pm 0.8$ nb and $\sigma_{D^+_s + \sigma_{D^-_s}} = 0.81 \pm 0.16 \pm 0.27$ nb. Considering the relation [13]: $\Gamma(D^0\bar{D}^0) : \Gamma(D^+\bar{D}^-) : \Gamma(D^0\bar{D}^-) \approx 1 : 7 : 9$, we obtain the following decay widths of $\psi(4040)$: $\Gamma(\psi(4040) \rightarrow D\bar{D}) = 2.97 \pm 0.68$ MeV, $\Gamma(\psi(4040) \rightarrow D^*\bar{D}^*) = 26.73 \pm 6.13$ MeV and $\Gamma(\psi(4040) \rightarrow D_{sJ}^* D_{sJ}^-) = 1.55 \pm 0.69$ MeV.
4 Discussion and conclusion

It is obvious that the newly discovered \( D_{sJ} \) family may be very significant for better understanding of the hadronic structure and low energy QCD. The members of the family, \( D_{sJ}^*(2317), \ D_{sJ}(2460), \ D_{sJ}(2632) \), all have positive parity, so that they cannot fit in an s-wave \( c\bar{s}(\bar{c}s) \) structure. The literatures suggest that they may be p-wave excited states, namely chiral partners of \( D_s, \ D_s^* \) etc. or four-quark states as well as molecular states. It is necessary to look for a more plausible way to determine their configurations, i.e. design an experiment(s) to clarify the picture. At least we would like to find an experiment to judge (1) if such states indeed exist, (2) their quark configuration (p-wave excited states or four-quark states).

To have a larger production rate, it is reasonable to look for \( D_{sJ}^*(2317) \) via strong decays of higher excited states of charmonia. The most possibly available charmonium is \( \psi(4415) \). Since \( D_{sJ}^*(2317) \) mesons have positive parity, the decay mode of \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s(1968) \) is forbidden and if considering the central values of the masses of the concerned particles and constraints from phase space of final states, only \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s(1968) + \gamma \) is allowed. This direct radiative decay must be much suppressed as discussed in the introduction.

Barnes et al. \(^8\) suggested to observe decay \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s^*(2112) \) which can occur via the threshold effects. The consequent decays \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s^*(2112) \to D_{sJ}^*(2317) + \bar{D}_s(1968) + \gamma \) and \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s^*(2112) \to D_{sJ}^*(2317) + \bar{D}_s(1968) + \pi \) can be observed. Even though such processes may only occur via threshold effects and should be suppressed, it is noted that \( m_{D_{sJ}^*(2317)} + m_{D_s^*(2112)} \) is only slightly above 4415 MeV, one can expect that the suppression is not very strong.

In this work, we carefully study the production of \( D_{sJ}^*(2317) \) in the decays of \( \psi(4415) \) and evaluate its production rate. The processes are realized as the charmonium \( \psi(4415) \) dissolves into a \( c\bar{c} \) pair which then combines with \( \bar{s}s \) created from vacuum due to the non-perturbative QCD effects and constitute two mesons. The first step is determined by the wavefunction of \( \psi(4415) \) at origin and the light-quark-pair creation is described by the QPC model \(^11\). For evaluating the hadronic transition matrix elements, we employ two approaches, i.e. HQET and the non-relativistic model. Our final numerical results achieved in the two approaches confirm this allegation as they are reasonably consistent with each other.

To guarantee the plausibility of the results, we obtain all necessary parameters by fitting data. However, it is understood that there must be some errors from both theoretical and experimental aspects, and the parameters should have some uncertainties, especially the vacuum creation rate of the light quark pair. Thus the real rates may be within a range around the values estimated with the input parameters and theoretical approaches, the order of magnitude should be correct and trustworthy.

For a comparison, we have also evaluated the transition rate of the direct radiative decay \( \psi(4415) \to D_{sJ}^*(2317) + \bar{D}_s(1968) + \gamma \) in the same approaches and find that the resultant rate is two orders smaller than that through the intermediate state \( D_s^*(2112) \), even though it is realized via the threshold effects.
We find that even though the threshold effects suppress the production rate of \( \psi(4415) \to D_{s J}^*(2317) + D_s^*(2112) \), it is still sizable if \( D_{s J}^*(2317) \) is a p-wave excited state. If so, it can be observed in the future experiments of BES III, CLEO and maybe at Babar or even LHC-b. However, as our calculations indicate that if \( D_{s J}^*(2317) \) is of a four-quark structure, its production rate is much more suppressed and cannot be observed in decays of charmonia.

Unfortunately, in such decays, one can only expect to observe \( D_{s J}^*(2317) \), but not the two other members of the new family. However, once the existence and structure of \( D_{s J}^*(2317) \) are definitely confirmed, we have reasons to believe existence of the other two. Moreover, we can have more knowledge on the hadronic structure and may design experiments to testify the other two. We are looking forward to new experimental results to clarify this theoretical problem. In a recent work, some authors \[24\] calculate the decay rates of \( D_{s J}^*(2317) \) and \( D_{s J}^*(2460) \). They claim that their results prefer the ordinary \( c \bar{s} / (c \bar{s}) \) quark-structure for the mesons. However, a decisive conclusion must be drawn from a deterministic experiment(s), and \( D_{s J}^*(2317) + D_s^*(2112) \) suggested by Barnes et al. as well as subsequent observable modes \( D_{s J}^*(2317) + D_s(1968) + \gamma, \) \( D_{s J}^*(2317) + D_s(1968) + \pi \) would provide an ideal possibility to make this judgement.

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Appendix

(a) In the Eq. (22), the \( \sum_{l,s} |M_{l s}| \) is

\[
\sum_{l,s} |M_{l s}| = \frac{R_A^2 R_B^2 R_C^5}{147456 \sqrt{35} \pi \eta^{11/2}} \exp \left[ - \frac{k^2 R_A^2 (R_B^2 + R_C^2)}{8 (R_A^2 + R_B^2 + R_C^2)} \right] \left\{ R_A^6 \left[ - 48 \alpha^5 (2 \zeta + 1) \eta^3 k^6 + 8 \sqrt{3} + 3 \right] k^4 - 420 \alpha (2 \zeta + 1) \eta k^2 + 42 \alpha^2 \eta \left( 10 \zeta (\zeta + 1) \eta k^2 + 8 \sqrt{3} + 3 \right) k^2 + 105 (2 \zeta (\zeta + 1) \eta k^2 + 9) \right] - 84 \eta R_A^4 \left[ - 16 \alpha^3 (2 \zeta + 1) \eta^2 k^4 + 4 \alpha^4 \eta^2 (2 \zeta (\zeta + 1) \eta k^2 + 3) k^4 - 40 \alpha (2 \zeta + 1) \eta k^2 - 4 \alpha^2 \eta (2 \zeta (\zeta + 1) \eta k^2 - 4 \sqrt{3} + 21) k^2 + 15 (2 \zeta (\zeta + 1) \eta k^2 + 7) \right] + 1680 \eta^2 R_A^2 \left[ 6 \zeta (\zeta + 1) \eta k^2 - 4 \alpha (2 \zeta + 1) \eta k^2 + 2 \alpha^2 \eta (2 \zeta (\zeta + 1) \eta k^2 + 3) k^2 + 15 \right] - 6720 \eta^3 (2 \zeta (\zeta + 1) \eta k^2 + 3) \right\}, \tag{27}
\]

where

\[
\zeta = \frac{R_A^2}{R_A^2 + R_B^2 + R_C^2}, \quad \eta = \frac{R_A^2 + R_B^2 + R_C^2}{8}, \quad a = 1 + \zeta.
\]
(b) The concrete expression of $\mathcal{M}_{(c)}$ is

$$
\mathcal{M}_{(c)}(\psi(4415) \to \gamma + D^*_s(2317) + \bar{D}_s(1968)) = i\gamma S\Psi(0) \left( \frac{4R^3}{\sqrt{35}} \right) \left( \frac{R^2_B}{\pi} \right)^{3/4} \sqrt{2} \frac{R^5_C}{9 \pi^{1/4}} \left[ - \frac{2e}{3} \left( \frac{1}{2\pi} \right)^{2/3} \frac{1}{\sqrt{2E_k}} (2\pi)^4 \right] 
$$

$$
\times \int dp_1 \exp \left[ -\frac{1}{8} R_A^2 (2p_1)^2 - \frac{1}{8} R_B^2 (2p_1 + 2p_C + 2p) - \frac{1}{8} R_C^2 (2p_1 + p_C)^2 \right] 
$$

$$
\times \mathcal{Y}_1^{-n_B}(p_B + 2p_C + 2p) \mathcal{Y}_n^0(-2p_C - 2p_1) \bar{u}(p_B + p_C + p_1, s_2) \gamma_\mu u(p_1, s_2') \varepsilon^\mu(k) 
$$

- $105 + 210R_A^2 p_1^2 - 84R_A^4 p_1^4 + 8R_A^6 p_1^6 \right] \frac{12\sqrt{35}}{12}.
$$

(c) The amplitude of $\psi(4040)$ decay into two pseudoscalar mesons in HQET is

$$
\mathcal{M}(\psi(4040) \to P + P) = \frac{2ig_{\psi s} s^2 m_q \sqrt{M_A M_B M_C} \Psi(0)}{3(p_A/2 - p_B)^2 - m_q^2} \epsilon_A \cdot (p_B - p_C).
$$

(d) In ref. [12], the authors gave a general expression for calculating decays of $\psi(4040) \to \bar{D}D$, $\psi(4040) \to \bar{D}D^*$ and $\psi(4040) \to D^*\bar{D}^*$

$$
\Gamma(\psi(4040)) = Ck^3 N_2(k^2),
$$

where $C$ means a spin-SU(3) factor corresponding to the particular channel under consideration ($C = 1/3$ for $\bar{D}D$, $C = 4/3$ for $\bar{D}D^* + \bar{D}D^*$ and $C = 7/3$ for $D^*\bar{D}^*$), $k$ is the three-momentum of the final particles in the CM frame of $\psi(4040)$, and $N_2(k^2)$ is a normalization factor and has the following expression

$$
N_2(k^2) = \frac{R^3 \gamma^2 M}{43740 \pi^{3/2}} \left[ L_2^{3/2}(4\xi) \exp(-\xi) \right]^2,
$$

where $L_2^{3/2}$ is a Laguerre polynomial, and $\xi = k^2 R^2/6$.

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