Dark energy from gravitoelectromagnetic inflation?

1,2 Federico Agustín Membiela* and 1,2 Mauricio Bellini †

1 Departamento de Física, Facultad de Ciencias Exactas y Naturales,
Universidad Nacional de Mar del Plata,
Funes 3350, (7600) Mar del Plata, Argentina.

2 Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina.

Abstract

Gravitoelectromagnetic Inflation (GI) was introduced to describe in an unified manner, electromagnetic, gravitatory and inflaton fields from a 5D vacuum state. On the other hand, the primordial origin and evolution of dark energy is today unknown. In this letter we show using GI that the zero modes of some redefined vector fields $B_i = A_i/a$ produced during inflation, could be the source of dark energy in the universe.

* E-mail address: membiela@argentina.com
† E-mail address: mbellini@mdp.edu.ar
I. INTRODUCTION

It is believed that our universe has experimented a primordial accelerated expansion called inflation, in which the equation of state was vacuum dominated: \( p/\rho \vert_{\text{infl}} \approx -1 \). This theory explains the origin of large scale structure formation\(^1\) and could also explain the origin of primordial magnetic fields\(^2\). New approaches to inflationary cosmology based on a scalar field formalism have been proposed in the context of higher-dimensional theories of gravity. In particular using ideas of the induced matter theory\(^3\) a new formalism for describing inflation, scalar metric fluctuations and gravitational waves, has been introduced\(^4\). The basic idea of this formalism is the existence of a 5D space-time equipped with a purely kinetic scalar field \( \varphi \), in which our observable 4D universe can be confined to a particular hypersurface by the choice of a particular frame.

The GI theory\(^2\) was developed with the aim to describe, in an unified manner, the inflaton, gravitatory and electromagnetic fields during inflation. Other 4D\(^5\) and 5D\(^6\) formalisms were developed in the last decades. This approach has the advantage that all the 4D sources has a geometrical origin in the sense that they can be geometrically induced when we take a foliation on the spacelike and noncompact fifth coordinate. GI was constructed from a 5D vacuum state on a Riemann-flat metric and can explains the origin primordial magnetic fields on cosmological scales, which are today observed. In this letter we explore, using this theory, the possibility that dark energy can be produced during inflation. Dark energy plays a very important role in the today observed accelerated expansion of the universe. However, the physical explanation of its origin and evolution is still mysterious.

II. FORMALISM

To develop our formalism we shall consider a 5D space-time described by the line element

\[
dS^2 = g_{ab} \, dx^a \, dx^b \\
n\quad dS^2 = e^{2F(\psi)} h_{\mu\nu}(x^\nu) \, dx^\mu \, dx^\nu - d\psi^2, \tag{1}
\]

where \( g_{ab} \) is the 5D covariant tensor metric, \( h_{\mu\nu} \) is the 4D covariant tensor metric, \( a, b \) run from 0 to 4, Greek letters run from 0 to 3 and latin letters run from 1 to 3. Inflation from a warped space was recently studied in\(^7\).
Now we consider the system described by the action

\[(5) S = \int d^5 \sqrt{|g|} \left[ \frac{(5) \mathcal{R}}{16 \pi G} + (5) \mathcal{L}(A_b, A_{c:d}) \right],\]  

for a vector potential with components \(A_a = (A_\mu, \varphi)\), which are nonminimally coupled to gravity

\[(5) \mathcal{L}(A_a, A_{a;b}) = -\frac{1}{4} Q_{bc} Q^{bc} - \frac{(5) \mathcal{R}}{6} A^b A_b,\]  

such that \((5) \mathcal{R}\) is the 5D Ricci scalar and

\[Q_{bc} = F_{bc} + \gamma g_{bc} (A^b)^2, \quad \gamma = \sqrt{\frac{2 \lambda}{5}}, \quad F_{bc} = A_{c;b} - A_{b;c},\]

where \(\cdot\) denotes the covariant derivative. The equations of motion for the vector components \(A^b\) are

\[(\triangle A)^b - \mathcal{R}^b_c A^c + (1 - \lambda) A_{;c}^{c;b} = 0,\]  

where \((\triangle A)^b = -A^{bc}_{;c} + \mathcal{R}^b_c A^c\) denotes the Rham vector wave operator, which is a generalized d’Alambertian for vectors in curved spacetime. We shall consider the Lorentz gauge: \(A^c_{;c} = 0\). Notice that the \(\alpha\) (Greek letters run from 0 to 3) components of \((5)\) with the Lorentz gauge on a Ricci flat metric \((\mathcal{R}^a_a = 0)\), give us the 5D Maxwell equations without sources: \(A^{bc}_{;c} = 0\). One could obtain Maxwell’s equations with sources on an effective 4D space-time by taking a foliation on the fifth coordinate of this 5D flat metric in the particular gauge \(A_4 = 0\). In this case the effective electromagnetic current has only 4 non-zero components which are purely of 5D origin.

### III. 5D KINETIC VECTOR FIELD AND EFFECTIVE 4D VECTOR AND SCALAR FIELDS

We wish to study a period of inflation, where there exist scalars and vector fields that are affected by an extra dimensional vector field \([2]\). During inflation the field that dominates is a neutral scalar field called inflaton. It was shown that also a collection of vector fields may produce also a period of inflation \([8]\). Our study tries to give an unified treatment for both formulations. In principle, we may obtain scalar and vector equations of motion for all of the Kaluza-Klein modes, but only a window of these modes can survive a period of inflation. These remanent modes (more exactly, the zero modes) would carry the energy of the universe.
that after the end of inflation decay to fundamental particles. On the other hand, finite $k$-modes should be responsible for the seeds of magnetic fields and structure formation in the universe once the inflationary period ends. For simplicity, in this letter these modes will be neglected. Only we shall deal only with the zero $k$-modes of the components $A_c$, of the vector potential.

We can define the vector $B_i(t, \vec{r}, \psi) \equiv A_i(t, \vec{r}, \psi)/a(t)$. We work with a Friedmann-Robertson-Walker (FRW) 4D metric $h_{\mu\nu} = \text{diag}[1, -a(t)^2, -a(t)^2, -a(t)^2]$. We are considering the 5D Lorentz gauge, but in the absence of sources we can still use the radiation gauge to eliminate the $A_0$ equation. In this case the 5D Lorentz gauge reduces to

$$e^{2F} \left(4F' + \frac{\partial}{\partial \psi} \right) A_4 + a^{-2} \vec{\partial} \cdot \vec{A} = 0,$$

where $\vec{\partial} \cdot \vec{A} \equiv (A_i)^i = 0$ denotes the 3D divergence of $\vec{A}$.

A. 5D invariant $I$

Taking into account that $I = A^b A_b$ is a 5D scalar, which is invariant under coordinates transformations and characterizes the strength of the vector field on 5D. In particular, on the metric (1) it takes the explicit form

$$I = e^{-2F} [A_0^2 - A^i A_i] - \varphi^2,$$

such that its variation is null: $\delta I = 0$. Taking $A_0 = 0$, one obtains

$$I = - \left[ e^{-2F} A_i A_i + \varphi^2 \right].$$

In particular, for 5D Riemann flat metrics the scalar $I$ in (7) becomes a constant.

B. Spatially homogeneous dynamics of fields

Taking null spatial derivatives we work with homogeneous fields which are given by their 3D expectation values. In this case the gauge reduces to

$$\frac{\partial A_4}{\partial \psi} = 4F' A_4,$$

(9)
and the equations for the vector and scalar reduce to

\[ \langle \ddot{B}_i \rangle + 3H \langle \dot{B}_i \rangle - \left[ 2(4F'^2 + F'')e^{2F} - (2H^2 + \dot{H}) - e^{2F} \left( \frac{\partial^2}{\partial \psi^2} + 2F' \frac{\partial}{\partial \psi} \right) \right] \langle B_i \rangle = 0, \tag{10} \]

\[ \langle \ddot{A}_4 \rangle + 3H \langle \dot{A}_4 \rangle - e^{2F} \left[ \frac{\partial^2}{\partial \psi^2} + 2(6F'^2 + F'') \right] \langle A_4 \rangle = 0, \tag{11} \]

where \( \langle B_i \rangle \) and \( \langle A_4 \rangle \) give us the \((t, \psi)\)-dependent expectation values calculated on the 3D spatial volume

\[ \langle B_i \rangle = B_i(t, \psi), \tag{12} \]

\[ \langle A_4 \rangle = \varphi(t, \psi). \tag{13} \]

In general, the difference between the scalar and vector fields is that (despite their expectation values may be spatially homogeneous), the last ones are essentially anisotropic. Hence, they can serve to explain the existence of primordial cosmological magnetic fields on sub Hubble scales. From another point of view, they can also contribute to an important amount to the energy density of the universe during, but mainly, after inflation. Once we assume the isotropy of \( \langle B_i \rangle \equiv B \), we note that the equation of motion (10) for \( B \), has the same mathematical structure than whole of the 3D expectation value for the inflaton field \( \langle A_4 \rangle = \varphi(t, \psi) \).

**IV. AN EXAMPLE WITH THE PONCE DE LEÓN METRIC**

As an example we can consider the case where \( F(\psi) = \ln(\psi/\psi_0) \), with \( a(t) \sim e^{t/\psi_0} \). This case is very interesting because we obtain the Ponce de León 5D Riemann flat metric

\[ dS^2 = \left( \frac{\psi}{\psi_0} \right)^2 \left[ dt^2 - e^{2t/\psi_0} \, d\vec{r}^2 \right] - d\psi^2, \tag{14} \]

which is 3D spatially flat, isotropic and homogeneous. This means that any invariant defined on the Riemann flat 5D metric (14) will be a constant. If we take a foliation \( \psi = \psi_0 = a/\dot{a} = 1/H \), we obtain the effective 4D de Sitter metric

\[ dS^2 |_{\text{eff}} = dt^2 - e^{2H t} \, d\vec{r}^2, \tag{15} \]

with an effective geometrically induced 4D scalar curvature \( ^{(4)}R = 12H^2 \) (In this particular case the Hubble parameter is a constant of time) [9]. Furthermore, the scalar \( I \) is a constant
on the Ponce de León metric and once we take the particular foliation \( \psi = 1/H \), on comoving coordinates \( U^x = U^y = U^z = 0 \), one obtains

\[
I_{|\psi=1/H} = H^2 \langle A_i \rangle \langle A^i \rangle + \langle \varphi \rangle^2_{|\psi=1/H} = H^2 \langle B_i \rangle \langle B_i \rangle + \langle \varphi \rangle^2_{|\psi=1/H} = \text{const}, \tag{16}
\]

where we assumed the summation over repeated spatial indices. It is well known that when slow rolling conditions are assumed \( \langle \varphi \rangle (t) \) is a constant of time for inflationary models which describe a de Sitter (exponential) expansion \cite{10}. Hence, during inflation \( \langle B_i \rangle \) should be a constant and \( \langle A_i \rangle \sim a \). Notice that \( \langle B_i \rangle \) has nothing to do with magnetic fields. These field components are re-scaled zero-modes of \( A_i \).

V. FINAL REMARKS AND POSSIBLE GENERALIZATIONS

Notice that in the example here developed we have worked an effective de Sitter expansion which becomes after taking a constant foliation \( \psi = 1/H \) on the fifth coordinate. However, one could obtain more general inflationary models where the scale factor grows quasi-exponentially by using a dynamical foliation on the fifth coordinate \( \psi \equiv \psi(t) \). In that case one obtains the more general condition

\[
\frac{d}{dt} \left( H^2 \langle B_i \rangle \langle B_i \rangle \right)_{\psi(t)} = -\frac{d}{dt} \langle \varphi \rangle^2_{\psi(t)} > 0. \tag{17}
\]

In other words, the term \( H^2 \langle B_i \rangle \langle B_i \rangle \) in (17) grows as the vacuum energy density, and the term \( \langle \varphi \rangle^2_{\psi(t)} \), decays. Hence, the increase of \( H^2 \langle B_i \rangle \langle B_i \rangle \) during inflation could be the source of the present day domination of dark energy in the universe. Notice that this term has an electromagnetic origin, but it should be very important on very large (cosmological) scales during inflation.

---

[1] A. H. Guth, Phys. Rev. D23: 347 (1981).
[2] A. Raya, J. E. Madriz Aguilar, M. Bellini, Phys. Lett. B638: 314 (2006); J. E. Madriz Aguilar and M. Bellini, Phys. Lett. B640: 126 (2006); F. A. Membiela and M. Bellini, Power spectrum of large-scale magnetic fields from Gravitoelectromagnetic inflation with a decaying cosmological parameter. E-print: 0712.3032.
[3] P. S. Wesson, Gen. Relativ. Gravit. 16: 193 (1984);
    P. Wesson, Gen. Relativ. Gravit. 22: 707 (1990);
    P. S. Wesson, Phys. Lett. B276: 299 (1992);
    P. S. Wesson and J. Ponce de Leon, J. Math. Phys. 33: 3883 (1992);
    H. Liu and P. S. Wesson, J. Math. Phys. 33: 3888 (1992);
    P. Wesson, H. Liu and P. Lim, Phys. Lett. B298: 69 (1993);
    D. J. McManus, J. Math. Phys. 35: 4889 (1994);
    C. Romero, R. K. Tavakol, R. Zalaletdinov, Gen. Rel. Grav. 28: 365 (1996).
[4] S. P. Gómez Martínez, J. E. Madriz Aguilar, M. Bellini, Phys. Lett. B649: 343 (2007).
[5] K. Bamba, JCAP 0710: 015 (2007);
    K. Bamba, S. Noriri, S. D. Odintsov. Inflationary cosmology and the late-time accelerated
    expansion of the universe in non-minimal Yang-Mills-\(F(R)\) gravity and non-minimal vector
    \(F(R)\) gravity. E-print: 0803.3384;
    L. Campanelli, P. Cea, G. L. Fogli, L. Tedesco, Phys. Rev. D77: 043001 (2008).
[6] J. S. Nodnik, Phys. Rev. Lett. 55, 2519 (1985);
    T. Liko, Phys. Lett. B617, 193 (2005).
[7] J. E. Madriz Aguilar, Eur. Phys. J. C53: 133 (2008).
[8] L. H. Ford, Phys. Rev. D40: 967 (1989);
    A. Golovnev, V. Mukhanov, V. Vanchurin, E-print: arXiv: 0802.2068.
[9] Here, we have used ideas of space-time-matter theory to induce the scalar curvature of the
    effective 4D hypersurface: \((4)R = 12/\psi^2|_{\psi=1/H} = 12H^2\). See for instance, P. S. Wesson.
    *Space-Time-Matter: Modern Kaluza-Klein Theory* (World Scientific: Singapore, 1999).
[10] The reader can see it after the eq. (14) of the paper: A. Membiela and M. Bellini, Phys. Lett.
    B635, 243 (2006).