Strange electromagnetic form factors of the nucleon with $N_f = 2 + 1 \mathcal{O}(a)$-improved Wilson fermions

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We present results for the strange contribution to the electromagnetic form factors of the nucleon computed on the coordinated lattice simulation ensembles with $N_f = 2 + 1$ flavors of $\mathcal{O}(a)$-improved Wilson fermions and an $\mathcal{O}(a)$-improved vector current. Several source-sink separations are investigated in order to estimate the excited-state contamination. We calculate the form factors on six ensembles with lattice spacings in the range of $a = 0.049 - 0.086$ fm and pion masses in the range of $m_\pi = 200 - 360$ MeV, which allows for a controlled chiral and continuum extrapolation. In the computation of the quark-disconnected contributions, we employ hierarchical probing as a variance-reduction technique.

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The contributions of strange sea quarks to the nucleon electromagnetic form factors, which characterize the charge and current distribution in the nucleon, have been of high interest in the last decades. Experimentally, strange electromagnetic form factors can be measured through the parity-violating asymmetry, arising from the interference of the electromagnetic and neutral weak interactions, in the elastic scattering of polarized electrons on unpolarized protons. The first measurement by the SAMPLE experiment, at backward angles and low $Q^2$, yielded a result for $G_{1s}^s$, which is consistent with zero [1]. The G0 collaboration combined measurements at forward and backward angles and low $Q^2$, yielding a result for $G_{1s}^s$ which is consistent with zero [1]. A recent nonzero measurement has been obtained by the A4 experiment at MAMI with a four momentum transfer squared of $Q^2 = 0.22$ GeV$^2$, where $G_{1s}^s = 0.050 \pm 0.038 \pm 0.019$ and $G_{1M}^s = -0.14 \pm 0.11 \pm 0.11$ [2]. A recent measurement from the HAPPEX collaboration at $Q^2 = 0.624$ GeV$^2$ found a value for the combination of the strange electromagnetic form factors consistent with zero $G_{1s}^s + 0.517G_{1M}^s = 0.003 \pm 0.010 \pm 0.004 \pm 0.009$ [3], confirming a previous measurement at $Q^2 = 0.48$ GeV$^2$, where a value consistent with zero was found as well [6]. For a recent review of the experimental status of the strange electromagnetic form factors, see [7]. On the theoretical side, lattice QCD simulations allow for a nonperturbative determination of the strange nucleon form factors. This is a challenging calculation, due to the appearance of quark-disconnected diagrams, which are notoriously difficult to evaluate. The most expensive part of the pertinent simulation is the calculation of the trace of an all-to-all propagator. In order to obtain a good signal, the application of variance-reduction techniques, such as hierarchical probing [8], are crucial. A prominent example to illustrate the importance of a precise knowledge of the strange nucleon form factors is the weak charge of the proton. At tree level and without radiative corrections, the weak charge is connected to the weak mixing angle through $Q_W(p) = 1 - 4 \sin^2 \theta_W$. Hence, through measurements of $Q_W(p)$, one can determine a fundamental parameter of the Standard Model. The experiment proceeds by measuring the parity-violating asymmetry, from which $Q_W(p)$ can be isolated, provided that the required nucleon form factors to describe the hadronic contribution are known [7, 9]. Here the strange electromagnetic form factors $G_{1s}^s$ and $G_{1M}^s$, as well as the strange axial form factor $G_A^s$, play a crucial role, as they constitute the leading uncertainty. In this Letter, we closely follow the strategy outlined in [10].

We make use of the coordinated lattice simulation (CLS) $N_f = 2 + 1 \mathcal{O}(a)$-improved Wilson fermion ensembles with the tree-level-improved Lüscher-Weisz gauge action [11]. The fermion fields have open boundary conditions in time in order to prevent topological freezing [12]. Simulations have been performed such that the sum of the bare quark masses is constant, which implies a constant $\mathcal{O}(a)$-improved coupling [13]. See Table 1 for a list of ensembles used in this Letter. We obtain the strange electromagnetic form factors of the nucleon by calculating the disconnected three-point function with a vector current insertion in the strange quark loop. The relevant diagram and our chosen momentum setup is depicted in Fig. 1. The disconnected three-point function factorizes into separate traces for the strange quark loop and the nucleon two-point function

$$C_{3,V_{\mu}}^{s,s}(q, z_0; p', y_0; x; \Gamma_{\nu}) = \left\langle e^{-iqe L_{V_{\mu}}(q, z_0)} \cdot C_2(p', y_0, x; \Gamma_{\nu}) \right\rangle_G,$$

where $L^s$ and $C_2$ denote the strange loop, given in Eq. 4, and the nucleon two-point function respectively. The calculation of nucleon two-point functions $C_2$ pro-

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ceeds via the standard nucleon interpolator

\[ N_\alpha(x) = \epsilon_{abc} \left( u_3^c(x) \left( C \gamma_5 \right)_{\beta \gamma} \bar{d}_\gamma(x) \right) u_\alpha^c(x), \]

and \( \Gamma_0 = \frac{1}{2} (1 + \gamma_0) \), which ensures the correct parity of the nucleon at zero momentum. Wuppertal smearing [10] is applied at the source and the sink for all quark propagators. We increase the statistics of the nucleon two-point function using the truncated solver method [17, 18]. Traces over the strange quark loops can be stochastically estimated using four-dimensional noise vectors \( \eta \). For a local current

\[ V^s = \bar{s}(x) \Gamma s(x), \]

the trace over the strange quark loop then reads

\[ \langle \mathcal{L}^s_\Gamma(q, z_0) \rangle_{G} = - \sum_{z \in \Lambda} e^{i q \cdot z} \left\langle \operatorname{tr} [S^s(z; z) \Gamma] \right\rangle_{G} \]

\[ = - \sum_{z \in \Lambda} e^{i q \cdot z} \left\langle \eta^s(z) \Gamma \psi(z) \right\rangle_{G, \eta}, \]

with

\[ D^s \psi = \eta, \]

where \( D^s \) denotes the Dirac operator for the strange quark, and the sum is taken over the spatial volume \( \Lambda \).

Instead of a local current we consider the \( O(a) \)-improved conserved vector current in this Letter

\[ V_\mu(z)^{\text{Imp.}} = \frac{1}{2} \left( \bar{s}(z + \hat{\mu} a)(1 + \gamma_\mu)U_\mu(z) s(z) - \bar{s}(z)(1 - \gamma_\mu)U_\mu(z) s(z + \hat{\mu} a) \right) + ac_\nu \partial_\nu (\bar{s}(z) \sigma_{\mu \nu} s(z)), \]

with the improvement coefficient \( c_\nu \) taken from [19]. Furthermore, we use hierarchical probing [8], which replaces the sequence of noise vectors by one noise vector multiplied with a sequence of Hadamard vectors. We find that the statistical error of the strange quark loop is reduced by a factor of 5 when using 512 Hadamard vectors, compared to the estimate based on 512 U(1) noise vectors, for nearly the same cost. The quark loops in this study were obtained by averaging two independent noise vectors with 512 Hadamard vectors each. To extract the strange contribution to the electromagnetic form factors of the nucleon, we consider the ratios (see [20, 22])

\[ R_{V_\mu}^s(z_0, q; y_0, p'; \Gamma_{\nu}) = \frac{C_{\nu, V_\mu}^s(q, z_0; p', y_0; \Gamma_{\nu})}{C_{2}^s(p', y_0)} \times \sqrt{\frac{C_2(p', y_0)C_2(p', z_0)C_2(p', y_0 - z_0)}{C_2(p' - q, y_0)C_2(p' - q, z_0)C_2(p', y_0 - z_0)}}. \]

Performing the spectral decomposition and only taking the ground state into account, these ratios read

\[ \text{TABLE I. Gauge ensembles used in this Letter, where } N_{cfg} \text{ denotes the number of gauge configurations and the last column corresponds to the total number of measurements for the ratio in Eq. (7). The values for the lattice spacing and pion and kaon masses are taken from [14], while the nucleon masses are estimated using the two-point function in this work. For the ensemble marked with an asterisk, the pion and kaon masses have been obtained from dedicated runs in connection with [15].} \]

| \( \beta \) | \( a \) [fm] | \( N_\alpha^2 \times N_\nu \) | \( m_{\pi} \) [MeV] | \( m_{K} \) [MeV] | \( m_{N} \) [MeV] | \( m_{KL} \) | \( N_{cfg} \) | \( N_{\text{meas}} \) |
|---|---|---|---|---|---|---|---|---|
| H105 | 3.40 0.08636 | 32^3 \times 96 | 278 | 460 | 1037 | 6.44 | 1020 | 391680 |
| N401 * | 3.46 0.07634 | 48^3 \times 128 | 289 | 462 | 1042 | 8.59 | 701 | 314048 |
| N203 | 3.55 0.06426 | 48^3 \times 128 | 345 | 441 | 1111 | 6.90 | 772 | 345856 |
| N200 | 3.55 0.06426 | 48^3 \times 128 | 283 | 463 | 1061 | 7.23 | 856 | 383488 |
| D200 | 3.55 0.06426 | 64^3 \times 128 | 200 | 480 | 989 | 10.01 | 278 | 124544 |
| N302 * | 3.70 0.04981 | 48^3 \times 128 | 354 | 458 | 1120 | 5.55 | 1177 | 527296 |
we will use the summation method data as our standard dataset, since they are less affected by excited-state contamination.
contamination, compared to the plateau fits. Nevertheless, we include the analysis of the plateau data, for a conservative choice of source-sink separation of 1 fm using 5 points around the midpoint, as an estimate for the uncertainty coming from excited states. In order to further analyze the kaon mass and lattice spacing dependence, we use model-independent \(z\)-expansion fits \([30, 31]\) to fifth order to extract the radii and magnetic moment. (We have explicitly checked that going to a maximum order of 10 does not change the fit results.) The form factors can be expanded as

\[
G_{E/M}(Q^2) = \sum_{k=1/0}^{5} a_k^{E/M} z(Q^2)^k, \tag{15}
\]

\[
z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}.
\]

Since the physical \(\omega\) and \(\phi\) mesons are narrow resonances and because one cannot easily establish whether or not they are unstable particles on the analyzed ensembles, we use \(4m_K^2\) for the value of the cut in the \(z\)-expansion, where we use the ensemble kaon mass for \(m_K\) (see Table I). We stabilize the fits using Gaussian priors centered around zero for all coefficients with \(k > 1\). To this end, we first determine the coefficients \(a_{0,1}\) from a fit without priors and subsequently use the maximum of these coefficients to estimate the width of the priors, i.e.,

\[
a_{k>1} = 0 \pm c \times \max\{|a_0|,|a_1|\}.
\]

We find that for \(c = 5\) the extraction of the radii and the magnetic moment are stable and lead to consistent results even after applying a cut of \(Q^2 < 0.5\) \(\text{GeV}^2\). Finally, we estimate the effect of this choice on the final observables by repeating the analysis with the prior width doubled. From the \(z\)-expansion fits, we can extract the strange magnetic moment \(\mu_s\), as well as the electric and magnetic charge radii \(\left(r_{E/M}^2\right)\):

\[
\mu_s = a_0^M, \tag{16} \\
\left(r_{E/M}^2\right) = -\frac{3}{2t_{\text{cut}}} \phi_{E/M}^L. \tag{17}
\]

We have repeated the analysis in several variations in order to assess systematic errors and subsequently perform chiral and continuum extrapolations. Since the radii and magnetic moments are defined at \(Q^2 = 0\), we perform the fits applying a cut of \(Q^2 < 0.5\) \(\text{GeV}^2\) and treat the difference to fitting all of the data as a systematic uncertainty. This cut also ensures that all ensembles contribute over the whole range in \(Q^2\). In total we thus have four sets of values for the radii and magnetic moments for every ensemble, for which we analyze the lattice spacing and kaon mass dependence.

The analyzed set of ensembles allow for a controlled chiral and continuum extrapolation of the strange electromagnetic form factors. In the following, we will investigate the kaon mass dependence using

\[
\left(r_{E/M}^2\right) = c_1 + c_2 \log(m_K) + \tilde{c}_1 a^2 + c_1^L \sqrt{t_{\text{cut}} e^{-m_K L}}, \\
\mu_s = c_3 + c_4 m_K + \tilde{c}_2 a^2 + c_2^L \left(m_K - \frac{2}{L}\right) e^{-m_K L}, \\
\left(r_{M}^2\right) = \frac{c_5}{m_K} + c_6 + \tilde{c}_3 a^2 + c_3^L \sqrt{t_{\text{cut}} e^{-m_K L}}, \tag{18}
\]

which is derived from \(\text{SU}(3)\) heavy baryon chiral perturbation theory (HBChPT) \([32]\), supplemented by terms describing the dependence on the lattice spacing \(a\) and the finite volume. (Note that the CLS ensembles follow the \(\text{tr} \bar{M}_q = \text{constant trajectory}, \) and so the kaon mass and the pion mass are therefore not varied independently.) Since the finite-volume dependence originates exclusively from kaon loops, we substitute the pion mass in the relevant expression for the magnetic moment \([33]\) by the mass of the kaon. For a detailed discussion of the finite-volume dependence, we refer to the Supplemental Material \([28]\). For the radii, we use the model-dependent ansatz of \([34, 35]\), assuming the finite-volume dependence to be the same as for the pion form factor calculated in \([39]\), again replacing the pion with the kaon mass. Since our data for the magnetic radius do not show the divergent behavior expected from HBChPT (see Fig. 3), we amend the expressions from \([32]\) by the term \(c_6\). While this cancellation of higher order terms was already found in Ref. \([37]\), we note that the convergence of HBChPT, the rate of which strongly depends on the observable, is, in general, not easily established.

For each of the variations of the \(z\)-expansion fit in the previous section, we analyze the chiral behavior separately. The chirally extrapolated values for the standard fit procedure and the variations of the \(z\)-expansion fits performed to assess systematic uncertainties are given in Table I. We treat the difference of the central values for
the variations as an estimate for a (symmetric) systematic error. In addition, we perform a fit including lattice artifacts or a fit including finite-volume dependence to the standard \( z \)-expansion fit. A simultaneous fit of the lattice spacing and finite-volume dependence amounts to the determination of four parameters from six data points for which the AIC\(_c\) value is not defined. Therefore, we choose to perform separate extrapolations in our analysis. The AIC\(_c\) values, i.e., the Akaike information criterion \([38]\) adjusted for small sample size \([39, 40]\), for the fits including lattice spacing or finite-volume effects, are larger by at least 24 in absolute value compared to the fits including lattice spacing or finite-volume effects, respectively.

\[
\begin{align*}
\text{Fit} & & (r_E^2)^* \text{[fm}^2\text{]} & & \phi & & (r_M^2)^* \text{[fm}^2\text{]} & & \chi^2/d.o.f. \\
\text{Standard} & & -0.0046(12) & & -0.020(5) & & -0.010(6) & & 2.04(12) \\
\text{Prior width} & & -0.0053(15) & & -0.020(6) & & -0.012(8) & & 1.47(12) \\
\text{Plateau} & & -0.0045(14) & & -0.022(8) & & -0.014(8) & & 1.62(12) \\
\mathcal{O}(a^2) & & -0.0036(16) & & -0.009(7) & & -0.003(8) & & 1.91(9) \\
\mathcal{O}(\exp[-m_K L]) & & -0.0049(12) & & -0.021(5) & & -0.010(6) & & 1.12(9) \\
\text{No cut in } Q^2 & & -0.0051(9) & & -0.017(5) & & -0.008(5) & & 3.14(12) \\
\end{align*}
\]

TABLE II. Fit results for the standard fit and variations thereof.

The AIC\(_c\) values, we use the maximum likelihood estimator for the sample variance; i.e., the fits omitting \( \mathcal{O}(a^2), \mathcal{O}(\exp[-m_K L]) \) are favored. We therefore quote the fit without lattice artifacts and finite-volume effects as our best value, using the difference in the central value for the respective procedures as a systematic error from finite lattice spacing and finite-volume corrections. At the physical point, we find

\[
(r_E^2)^\text{phys} = -0.0046(12)(7)(1)(9)(3)(6) \text{ fm}^2, \\
\mu^* = -0.020(5)(0)(2)(11)(1)(3), \\
(r_M^2)^\text{phys} = -0.010(6)(2)(5)(7)(0)(2) \text{ fm}^2,
\]

as our final estimate, where the first error is statistical and the remaining errors come from the variations in the fitting procedure given in Table II.

For the radii, our values are in good agreement with other lattice determinations \([34, 35, 42, 43]\). Our value for the magnetic moment is again in good agreement with \([42, 43]\). The magnetic moment from \([34, 35]\) disagrees with our estimate and with \([42, 43]\) by more than 2 standard deviations, see Fig. 4. Our best estimate of the radii and magnetic moment compare favorably to the available experimental data, as can be seen from Fig. 4.

In summary, we have reported on our calculation of the strange contribution to the electromagnetic form factors obtained on six CLS \( N_f = 2+1 \mathcal{O}(a) \)-improved Wilson fermion ensembles. For the calculation of the disconnected contributions, we use the method of hierarchical probing, which significantly reduces the statistical error. To deal with excited-state contamination, we employ the summation method. We find agreement with plateau estimates for large enough source-sink separations. The strange charge radii and the strange magnetic moment are obtained on each ensemble through model independent \( z \)-expansion fits and later extrapolated to the physical point. See the Supplemental Material \([28]\) for a summary of the extracted form factors and \( z \)-expansion fits. Our results are compatible with other lattice QCD studies and in good agreement to experimental data. With the current set of ensembles, the physical values for the strange charge radii and the strange magnetic moment still have large relative statistical errors. We aim to improve this by enlarging the number of ensembles.

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Supplemental Material

For convenience we attach the supplemental material to the published Letter in the following sections.

Finite-Volume Dependence

In this section we derive the finite-volume dependence of the strange magnetic moment \( \mu_s \) of the nucleon in HBChPT to order \( O(q^3) \). We will show that the form of the finite-volume correction is the same as in the SU(2) case for the isovector magnetic moment \[33\] after substituting the kaon for the pion mass. To this end we analyze the relevant diagram in HBChPT \[32\]. Only one diagram contributes to the magnetic moment at one loop to order \( O(q^3) \), see Fig. 6.

![Fig. 6. One-loop contribution to the strange magnetic moment.](image)

The relevant meson-baryon Lagrangian is \[29\]

\[
\mathcal{L} = D \langle \overline{B} S^a [u_\mu, B] \rangle + F \langle \overline{B} S^a [u_\mu, B] \rangle. \tag{22}
\]

Expanding the Lagrangian in terms of the meson fields we obtain

\[
\mathcal{L} = \frac{1}{2F_\phi} D \langle \overline{B} c \lambda_c S^a i \partial_\mu \phi_a [\lambda_a, \lambda_b] B_b \rangle + \frac{1}{2F_\phi} F \langle \overline{B} c \lambda_c S^a i \partial_\mu \phi_a [\lambda_a, \lambda_b] B_b \rangle + \ldots
\]

\[
= 2D \phi^{abc} \overline{B} c S^a i \partial_\mu \phi_a B_b + 2F f^{abc} \overline{B} c S^a i \partial_\mu \phi_a B_b + \ldots \tag{23}
\]

where we only show the terms necessary for the discussion of the finite-volume effects. The \( \lambda_i \) are the Gell-Mann matrices and the \( d \) and \( f \) are the usual SU(3) structure functions. This leads to the Feynman rule

\[
\frac{2i p \cdot S (D \phi^{abc} + iF f^{abc})}{F_\phi}, \tag{24}
\]

for the meson-baryon interaction, where \( p \) is the incoming momentum of the meson with isospin index \( a \), and \( b, c \) are the isospin indices of the incoming and outgoing baryon, respectively. The baryon propagator is given by

\[
\frac{i}{\nu \cdot p}. \tag{25}
\]
The covariant derivative of the mesonic Lagrangian is defined as

\[ D_\mu A = \partial_\mu A - ir_\mu A + iA l_\mu, \tag{26} \]

\[ r_\mu = l_\mu = \lambda_8 v_\mu^{(8)}, \tag{27} \]

where for the magnetic moment only the octet current contributes at the one-loop level. Again expanding the Lagrangian in terms of meson fields and only keeping the relevant terms gives

\[ \mathcal{L} = \frac{F^2}{4} \langle D_\mu U(D^\mu U)^\dagger \rangle + \ldots \]

\[ = \frac{i}{2} \langle \partial_\mu \phi(\phi, \lambda_8^{(8)}) v^{(8)} \rangle + \ldots \]

\[ = -2\partial_\mu \phi_\alpha \phi_\beta f^{ab8} v_\mu^{(8)} + \ldots \tag{28} \]

The Feynman rule for the electromagnetic interaction of the meson reads

\[ -2f^{ab8}(p_\mu + p'_\mu) \tag{29} \]

where \( p, a \) and \( p', b \) are the momenta and isospin indices of the incoming and outgoing meson, respectively. Since the structure functions \( f^{ab8} \) only give non-vanishing contributions for \( a, b = 4, 5 \) and \( a, b = 6, 7 \), only kaons contribute to the loop diagram for the strange magnetic moment of the nucleon. The matrix element of the current (in the Breit frame) is parametrized as

\[ J_\mu = \frac{1}{N_i N_f} \bar{u}(p') P^+ \left[ G_E v_\mu + \frac{1}{m} G_M [S_\mu, S_\nu] q_\nu \right] P^+ u(p) \tag{30} \]

where

\[ P^+ = \frac{1 + \gamma^0}{2}, \quad q = p' - p, \quad N_i = \sqrt{E_i + m \over 2m}, \tag{31} \]

\[ v_\mu = \{1, 0, 0, 0\}, \quad p'^\mu = m v^\mu + r'^\mu, \quad p^\mu = m v^\mu + r^\mu. \tag{32} \]

In the Breit frame the kinematic vectors read

\[ r'^\mu = \{E - m, -q_2\}, \tag{33} \]

\[ r'^\mu = \{E - m, -q_2\}, \tag{34} \]

\[ q^\mu = r'^\mu - r^\mu = \{0, q\}, \tag{35} \]

\[ v^2 = 1, \tag{36} \]

\[ v \cdot q = 0. \tag{37} \]

Using the explicit representation of \( S \)

\[ S_\mu = \frac{i}{2} \gamma_5 g^{\mu\nu} v_\nu, \tag{38} \]

we find that the part of a diagram proportional to \( \gamma_\mu \) corresponds to the magnetic moment. The one-loop diagram of Fig. reads

\[ D = \frac{8m_0}{F_8^2} \frac{1}{i} \int \frac{d^D k}{(2\pi)^D} \frac{k^\mu k}{(v \cdot k - \omega)(k^2 - m_K^2)((k + q)^2 - m_K^2)} + \ldots, \tag{39} \]

where we only display terms proportional to \( \gamma_\mu \), i.e. contributing to the magnetic moment. We have collected the isospin-dependent part in \( \Theta \),

\[ \Theta_{ba} = -i(Dd_{ca}^e + iFf_{ca}^e)f^{cd8}(Dd_{de}^{eb} + iFf_{de}^{eb}), \tag{40} \]
with \( a, b \) the isospin index of the incoming, outgoing nucleon, respectively, and \( \omega = v \cdot p \). We parametrize the tensor integral
\[
\frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{k^\mu k^\nu}{(v \cdot k - \omega)(k^2 - m_K^2)((k + q)^2 - m_K^2)} = g^{\mu\nu} c_1 + g^\mu q^\nu c_2 + (v^\mu q^\nu + v^\nu q^\mu) c_3 + v^\mu v^\nu c_4. \tag{41}
\]
For the subsequent discussion we only need \( c_1 \) which for the case \( q^2 = 0 \) and \( v \cdot q = 0 \) reads\(^1\)
\[
c_1 = \frac{1}{D - 1} \left[ (m_K^2 - \omega^2) K_0 + J_0 - \omega I_0 \right], \tag{42}
\]
with
\[
K_0 = \frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(v \cdot k - \omega)(k^2 - m_K^2)((k + q)^2 - m_K^2)}, \tag{43}
\]
\[
J_0 = \frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(v \cdot k - \omega)(k^2 - m_K^2)}, \tag{44}
\]
\[
I_0 = \frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2 - m_K^2)((k + q)^2 - m_K^2)}. \tag{45}
\]
The magnetic moment to one loop reads
\[
G_{M(8),\text{loop}}^{(s)}(0, b, a) = \frac{8m\Theta_{ba}}{F_\phi^2} \frac{1}{D - 1} \left[ (m_K^2 - \omega^2) K_0 + J_0 - \omega I_0 \right], \tag{46}
\]
with
\[
A_0 = \frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2 - m_K^2)}. \tag{47}
\]
The isospin factor for the nucleon is
\[
\Theta_{44} - i\Theta_{54} = \frac{-5D^2 - 6DF + 9F^2}{4\sqrt{3}}. \tag{48}
\]
Inserting the explicit expression for the loop integrals, e.g. Appendix B in Ref. \[29\], we obtain
\[
G_{M(8),\text{loop}}^{(s)}(0) = -\frac{mm_K}{8\sqrt{3}F_\phi^2} \left( 5D^2 - 6DF + 9F^2 \right), \tag{49}
\]
which is the same result as in Ref. \[32\]. Thus we have shown that the magnetic moment is proportional to the derivative of the self-energy with respect to \( m_K^2 \). Furthermore, we can rewrite \( \Sigma \)
\[
\left[ (m_K^2 - \omega^2) J_0 - \omega A_0 \right] \frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{(m_K^2 - \omega^2) - \omega(v \cdot k - \omega)}{(v \cdot k - \omega)(k^2 - m_K^2)},
\]
\[
= \frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{k^2 - (v \cdot k)^2}{(v \cdot k - \omega)(k^2 - m_K^2)},
\]
\[
= -\frac{1}{i} \int \frac{d^Dk}{(2\pi)^D} \frac{k^2}{(v \cdot k - \omega)(k^2 - m_K^2)}. \tag{50}
\]
This expression coincides with the integral of Eq. (8) from Ref. \[33\] (up to an irrelevant factor), with the kaon mass substituted for the pion mass. Thus the finite-volume corrections for the strange magnetic moment of the nucleon are of the same form as in \[33\], after substituting the kaon for the pion mass.

\(^1\) Note that \( D \) here refers to space-time dimensions.
Effective Mass

For convenience we show the effective mass of the nucleon for ensemble N200 at zero momentum in Fig. 7.

![Graph showing effective mass of the nucleon](image)

FIG. 7. Effective mass of the nucleon at zero momentum for ensemble N200.

Tables

In this section we give the extracted form factors $G_{E/M}^s$ as well as the z-expansion fits for the final result quoted in the main text.

| $a_k$ | $G_{E}^s$ Summation Method | Plateau Fit | $G_{M}^s$ Summation Method | Plateau Fit |
|------|----------------------------|-------------|----------------------------|-------------|
| 0    | -                          | -           | -0.02047 (0.00437)         | -0.01233 (0.00599) |
| 1    | 0.06397 (0.01385)          | 0.06823 (0.01405) | 0.12329 (0.06527)          | 0.04532 (0.07789) |
| 2    | 0.02234 (0.14166)          | -0.02693 (0.14610) | -0.00992 (0.28592)         | 0.00175 (0.23002) |
| 3    | 0.00627 (0.15825)          | -0.00133 (0.16380) | -0.00232 (0.28851)         | 0.00026 (0.23533) |
| 4    | 0.00108 (0.15816)          | 0.00019 (0.16458)  | -0.00038 (0.28453)         | 0.00002 (0.23424) |
| 5    | 0.00015 (0.15918)          | 0.00006 (0.16606)  | -0.00005 (0.28363)         | -0.00000 (0.23054) |
| $\chi^2$/dof | 0.44508  | 1.64311       | 1.46231                    | 0.48937       |

TABLE III. Fit of the z-expansion to the strange electromagnetic form factors on ensemble H105 with a transferred four-momentum cut of $Q^2 < 0.5 \text{GeV}^2$. 
| \(a_k\) | \(G^x_E\) Summation Method | \(G^y_E\) Plateau Fit | \(G^x_M\) Summation Method | \(G^y_M\) Plateau Fit |
|---|---|---|---|---|
| 0 | - | -0.02529 (0.00425) | -0.02095 (0.00435) |
| 1 | 0.09512 (0.01429) | 0.10383 (0.01443) | 0.13998 (0.062712) | 0.14331 (0.08529) |
| 2 | -0.27623 (0.16770) | -0.30477 (0.16933) | 0.1803 (0.67666) | -0.01013 (0.49372) |
| 3 | -0.00203 (0.25734) | -0.03529 (0.26595) | 0.0121 (0.68362) | -0.00025 (0.49920) |
| 4 | -0.00082 (0.25992) | -0.03031 (0.26699) | 0.00121 (0.67480) | -0.00055 (0.49920) |
| 5 | 0.00007 (0.25824) | -0.00025 (0.26746) | 0.00002 (0.67480) | -0.00012 (0.49920) |
| \(\chi^2/\text{dof}\) | 1.73239 | 1.16637 | 1.47959 | 0.96105 |

TABLE IV. Fit of the \(z\)-expansion to the strange electromagnetic form factors on ensemble N401 with a transferred four-momentum cut of \(Q^2 < 0.5\) GeV$^2$.

| \(a_k\) | \(G^x_E\) Summation Method | \(G^y_E\) Plateau Fit | \(G^x_M\) Summation Method | \(G^y_M\) Plateau Fit |
|---|---|---|---|---|
| 0 | - | -0.01899 (0.00345) | -0.01435 (0.00458) |
| 1 | 0.07188 (0.00981) | 0.06983 (0.01178) | 0.06979 (0.06544) | 0.02585 (0.06573) |
| 2 | -0.24568 (0.09677) | -0.22658 (0.11057) | 0.00194 (0.34785) | -0.03506 (0.28291) |
| 3 | -0.03558 (0.15444) | -0.03115 (0.14710) | -0.00407 (0.36546) | -0.00927 (0.28600) |
| 4 | -0.00422 (0.15360) | -0.00345 (0.14939) | -0.00127 (0.37107) | -0.00165 (0.28557) |
| 5 | -0.00049 (0.15529) | -0.00037 (0.14689) | -0.00025 (0.37221) | -0.00025 (0.28763) |
| \(\chi^2/\text{dof}\) | 1.89473 | 1.84721 | 1.71021 | 1.18630 |

TABLE V. Fit of the \(z\)-expansion to the strange electromagnetic form factors on ensemble N203 with a transferred four-momentum cut of \(Q^2 < 0.5\) GeV$^2$.

| \(a_k\) | \(G^x_E\) Summation Method | \(G^y_E\) Plateau Fit | \(G^x_M\) Summation Method | \(G^y_M\) Plateau Fit |
|---|---|---|---|---|
| 0 | - | -0.02586 (0.00341) | -0.01435 (0.00458) |
| 1 | 0.08026 (0.01134) | 0.06574 (0.01291) | 0.20748 (0.06544) | 0.23425 (0.06965) |
| 2 | -0.34096 (0.11673) | -0.19624 (0.13298) | -0.16836 (0.34266) | -0.04257 (0.23831) |
| 3 | -0.06001 (0.15726) | -0.03532 (0.16129) | -0.04070 (0.35692) | -0.01019 (0.24036) |
| 4 | -0.00834 (0.15976) | -0.00498 (0.16238) | -0.00670 (0.35371) | -0.00166 (0.23809) |
| 5 | -0.00106 (0.15581) | -0.00064 (0.16369) | -0.00094 (0.35217) | -0.00023 (0.33128) |
| \(\chi^2/\text{dof}\) | 1.69598 | 0.96513 | 0.96210 | 1.88794 |

TABLE VI. Fit of the \(z\)-expansion to the strange electromagnetic form factors on ensemble N200 with a transferred four-momentum cut of \(Q^2 < 0.5\) GeV$^2$.

| \(a_k\) | \(G^x_E\) Summation Method | \(G^y_E\) Plateau Fit | \(G^x_M\) Summation Method | \(G^y_M\) Plateau Fit |
|---|---|---|---|---|
| 0 | - | -0.01544 (0.00470) | -0.02656 (0.00517) |
| 1 | 0.06857 (0.02031) | 0.06464 (0.02218) | 0.10160 (0.07439) | 0.09163 (0.13288) |
| 2 | -0.01483 (0.22268) | -0.14348 (0.24059) | -0.06996 (0.37373) | -0.00272 (0.63158) |
| 3 | 0.00768 (0.31908) | -0.01647 (0.28613) | -0.01276 (0.36976) | 0.00061 (0.65262) |
| 4 | 0.00191 (0.30663) | -0.00143 (0.28985) | 0.00165 (0.36929) | 0.00019 (0.65950) |
| 5 | -0.00001 (0.30945) | -0.00010 (0.28545) | -0.00019 (0.37093) | 0.00003 (0.65906) |
| \(\chi^2/\text{dof}\) | 1.05330 | 1.14854 | 1.77733 | 0.59909 |

TABLE VII. Fit of the \(z\)-expansion to the strange electromagnetic form factors on ensemble D200 with a transferred four-momentum cut of \(Q^2 < 0.5\) GeV$^2$. 
TABLE VIII. Fit of the $z$-expansion to the strange electromagnetic form factors on ensemble N302 with a transferred four-momentum cut of $Q^2 < 0.5 \text{GeV}^2$.

| $Q^2$ (GeV$^2$) | Summation Method | Plateau Fit | Summation Method | Plateau Fit |
|-----------------|------------------|-------------|------------------|-------------|
| 0.00000         | 0.000054 (0.00107) | 0.00135 (0.00106) | 0.002275 (0.000903) | -0.00662 (0.001681) |
| 0.14300         | 0.000327 (0.00155) | 0.00195 (0.00233) | -0.01549 (0.00658) | -0.00950 (0.00191) |
| 0.19479         | 0.000304 (0.00156) | 0.00585 (0.00197) | -0.01174 (0.00267) | -0.00795 (0.00374) |
| 0.19268         | 0.000316 (0.00080) | 0.00378 (0.00077) | -0.02176 (0.00266) | -0.00875 (0.00374) |
| 0.19397         | 0.000350 (0.00067) | 0.00383 (0.00069) | -0.02289 (0.00514) | -0.00677 (0.00393) |
| 0.19487         | 0.000408 (0.00093) | 0.00557 (0.00113) | -0.00947 (0.00324) | -0.01279 (0.00545) |
| 0.30649         | 0.000399 (0.00154) | 0.00532 (0.00211) | -0.02289 (0.00514) | -0.02235 (0.00393) |
| 0.31545         | 0.000406 (0.00191) | 0.00576 (0.00275) | -0.01645 (0.00510) | -0.02176 (0.01025) |
| 0.37669         | 0.000544 (0.00065) | 0.00423 (0.00081) | -0.00832 (0.00166) | -0.00677 (0.00251) |
| 0.37505         | 0.000537 (0.00093) | 0.00592 (0.00124) | -0.01291 (0.00269) | -0.01148 (0.00406) |
| 0.37833         | 0.000529 (0.00115) | 0.00605 (0.00188) | -0.01197 (0.00295) | -0.00765 (0.00611) |
| 0.40252         | 0.000644 (0.000641) | 0.00666 (0.00076) | -0.00987 (0.00148) | -0.00880 (0.00213) |
| 0.45865         | 0.000495 (0.000340) | 0.00744 (0.00375) | -0.01160 (0.00850) | -0.01750 (0.01360) |
| 0.53690         | 0.000486 (0.00170) | 0.00337 (0.00243) | -0.00795 (0.00313) | -0.01109 (0.00592) |
| 0.55227         | 0.000488 (0.00157) | 0.00524 (0.00189) | -0.00937 (0.00308) | -0.00146 (0.00521) |
| 0.59650         | 0.000496 (0.00083) | 0.00458 (0.00094) | 0.00896 (0.00172) | 0.00855 (0.00219) |
| 0.69338         | 0.000541 (0.00262) | 0.00305 (0.00435) | 0.00433 (0.00486) | 0.01567 (0.00764) |
| 0.70727         | 0.000323 (0.00238) | -0.00021 (0.00309) | 0.00360 (0.00459) | -0.00018 (0.00773) |
| 0.71798         | 0.000484 (0.000308) | 0.00288 (0.00441) | -0.00341 (0.00528) | 0.00410 (0.01151) |
| 0.80515         | 0.000473 (0.00126) | 0.00472 (0.00158) | 0.00471 (0.00199) | 0.00449 (0.00308) |
| 0.84184         | 0.000525 (0.00178) | 0.00582 (0.00258) | 0.00120 (0.00339) | -0.00045 (0.00599) |
| 0.86127         | 0.000340 (0.00207) | 0.00559 (0.00252) | -0.00265 (0.00363) | -0.00752 (0.00603) |
| 0.94815         | 0.000594 (0.00132) | 0.00857 (0.00204) | -0.00682 (0.00227) | -0.00067 (0.00427) |
| 0.95489         | 0.000439 (0.00117) | 0.00710 (0.00166) | -0.00245 (0.00190) | -0.00146 (0.00349) |
| 0.98315         | 0.000440 (0.00073) | 0.00405 (0.00415) | -0.00748 (0.00583) | -0.00590 (0.00838) |
| 0.99902         | 0.000440 (0.00073) | 0.00557 (0.00086) | -0.00336 (0.00108) | -0.00403 (0.00162) |
| 1.00001         | 0.000411 (0.00093) | 0.00390 (0.00127) | -0.00285 (0.00143) | 0.00055 (0.00260) |
| 1.10979         | 0.000439 (0.00182) | 0.00403 (0.00232) | -0.00636 (0.00258) | -0.00900 (0.00425) |
| 1.12050         | 0.000200 (0.00270) | 0.00614 (0.00378) | -0.01003 (0.00381) | -0.00675 (0.00683) |
| 1.18020         | 0.000310 (0.00135) | 0.00278 (0.00207) | -0.00531 (0.00177) | -0.00186 (0.00353) |
| 1.18347         | 0.000606 (0.00169) | 0.00209 (0.00272) | -0.00696 (0.00232) | 0.00083 (0.00451) |
| 1.20767         | 0.000472 (0.00096) | 0.00397 (0.00134) | -0.00422 (0.00133) | -0.00342 (0.00189) |

TABLE IX. Results for the strange electromagnetic form factors from the summation method and plateau fit at 1 fm on ensemble H105.
| \(Q^2 \text{ [GeV}^2\) | \(G_{\Delta}^s\) Summation Method | \(G_{\Delta}^s\) Plateau Fit | \(G_{\Delta}^s\) Summation Method | \(G_{\Delta}^s\) Plateau Fit |
|----|----------------|----------------|----------------|----------------|
| 0.00000 | 0.00116 (0.00115) | 0.00176 (0.00112) | - | - |
| 0.09297 | 0.00256 (0.00135) | 0.00255 (0.00128) | 0.00571 (0.00949) | -0.01468 (0.01318) |
| 0.09455 | 0.00238 (0.00126) | 0.00284 (0.00123) | -0.02148 (0.00782) | -0.01459 (0.00972) |
| 0.11160 | 0.00353 (0.00072) | 0.00371 (0.00072) | -0.01811 (0.00394) | -0.02243 (0.00448) |
| 0.11190 | 0.00404 (0.00066) | 0.00366 (0.00063) | -0.02018 (0.00337) | -0.01543 (0.00396) |
| 0.11209 | 0.00399 (0.00082) | 0.00418 (0.00087) | -0.01899 (0.00413) | -0.01054 (0.00517) |
| 0.19208 | 0.00273 (0.00118) | 0.00442 (0.00109) | -0.01577 (0.00571) | -0.01013 (0.00684) |
| 0.19485 | 0.00321 (0.00122) | 0.00517 (0.00125) | -0.01864 (0.00481) | -0.01690 (0.00558) |
| 0.21804 | 0.00444 (0.00061) | 0.00543 (0.00059) | -0.02050 (0.00219) | -0.01470 (0.00280) |
| 0.21904 | 0.00349 (0.00081) | 0.00563 (0.00080) | -0.01587 (0.00311) | -0.01153 (0.00379) |
| 0.21983 | 0.00278 (0.00085) | 0.00412 (0.00091) | -0.01489 (0.00374) | -0.00720 (0.00490) |
| 0.22895 | 0.00407 (0.00061) | 0.00495 (0.00059) | -0.01420 (0.00214) | -0.01249 (0.00234) |
| 0.28782 | 0.00237 (0.00200) | 0.00585 (0.00184) | -0.02224 (0.00656) | -0.01464 (0.00727) |
| 0.31993 | 0.00327 (0.00138) | 0.00644 (0.00127) | -0.01616 (0.00366) | -0.01296 (0.00478) |
| 0.32360 | 0.00350 (0.00128) | 0.00694 (0.00118) | -0.01782 (0.00372) | -0.00876 (0.00433) |
| 0.34084 | 0.00539 (0.00084) | 0.00640 (0.00079) | -0.01255 (0.00228) | -0.00853 (0.00258) |
| 0.41775 | 0.00395 (0.00200) | 0.00430 (0.00184) | -0.00471 (0.00469) | -0.01755 (0.00620) |
| 0.42102 | 0.00312 (0.00200) | 0.00509 (0.00184) | -0.00454 (0.00446) | -0.00760 (0.00546) |
| 0.42380 | 0.00313 (0.00218) | 0.00520 (0.00212) | -0.00826 (0.00454) | -0.01159 (0.00641) |
| 0.45789 | 0.00629 (0.00118) | 0.00684 (0.00114) | -0.00927 (0.00283) | -0.00928 (0.00338) |
| 0.51211 | 0.00633 (0.00122) | 0.00475 (0.00110) | -0.00534 (0.00317) | -0.00501 (0.00336) |
| 0.51687 | 0.00663 (0.00129) | 0.00522 (0.00116) | -0.00912 (0.00287) | -0.00603 (0.00326) |
| 0.55086 | 0.00599 (0.00102) | 0.00509 (0.00096) | -0.00268 (0.00249) | -0.00478 (0.00300) |
| 0.55255 | 0.00631 (0.00087) | 0.00620 (0.00086) | -0.00577 (0.00193) | -0.00728 (0.00233) |
| 0.56979 | 0.00614 (0.00059) | 0.00550 (0.00062) | -0.00607 (0.00147) | -0.00421 (0.00169) |
| 0.56999 | 0.00677 (0.00074) | 0.00662 (0.00076) | -0.00809 (0.00164) | -0.00626 (0.00201) |
| 0.60319 | 0.00327 (0.00176) | 0.00315 (0.00167) | -0.00668 (0.00370) | -0.00463 (0.00425) |
| 0.64997 | 0.00378 (0.00122) | 0.00377 (0.00117) | -0.00705 (0.00261) | -0.00771 (0.00304) |
| 0.65275 | 0.00278 (0.00154) | 0.00343 (0.00152) | -0.01197 (0.00331) | -0.00859 (0.00380) |
| 0.67693 | 0.00438 (0.00102) | 0.00455 (0.00098) | -0.00664 (0.00194) | -0.00372 (0.00247) |
| 0.67772 | 0.00494 (0.00122) | 0.00396 (0.00131) | -0.00835 (0.00248) | -0.00954 (0.00295) |
| 0.68694 | 0.00512 (0.00083) | 0.00558 (0.00082) | -0.00497 (0.00157) | -0.00342 (0.00184) |

TABLE X. Results for the strange electromagnetic form factors from the summation method and plateau fit at 1 fm on ensemble N401.
| \( Q^2 \) (GeV\(^2\)) | \( G_E \) Summation Method | \( G_M \) Summation Method | \( G_E \) Plateau Fit | \( G_M \) Plateau Fit |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.00000         | -0.00003 (0.00090) | -0.00037 (0.000112) | -0.00476 (0.00624) | -0.01342 (0.01149) |
| 0.12555         | 0.00308 (0.00102) | 0.00534 (0.00123) | -0.00750 (0.00343) | -0.00039 (0.000845) |
| 0.12876         | 0.00289 (0.00096) | 0.00532 (0.00121) | -0.00772 (0.00258) | -0.01591 (0.00487) |
| 0.15658         | 0.00455 (0.00052) | 0.00515 (0.00069) | -0.00772 (0.00258) | -0.00507 (0.00625) |
| 0.15718         | 0.00425 (0.00047) | 0.00481 (0.00064) | -0.00750 (0.01204) | -0.01747 (0.00295) |
| 0.15758         | 0.00409 (0.00062) | 0.00502 (0.00080) | -0.00167 (0.00285) | -0.01592 (0.00403) |
| 0.26184         | 0.00222 (0.00088) | 0.00105 (0.00118) | -0.00891 (0.00390) | -0.00484 (0.00568) |
| 0.26716         | 0.00169 (0.00094) | 0.00067 (0.00140) | -0.00402 (0.00331) | -0.01198 (0.00512) |
| 0.30442         | 0.00357 (0.00041) | 0.00337 (0.00062) | -0.00435 (0.00139) | -0.01117 (0.00233) |
| 0.30633         | 0.00324 (0.00059) | 0.00231 (0.00079) | -0.00329 (0.00197) | -0.01184 (0.00295) |
| 0.30794         | 0.00329 (0.00063) | 0.00271 (0.00089) | -0.00129 (0.00233) | -0.01139 (0.00356) |
| 0.32310         | 0.00390 (0.00043) | 0.00321 (0.00057) | -0.00124 (0.00121) | -0.01160 (0.00188) |
| 0.39291         | 0.00613 (0.00172) | 0.00528 (0.00219) | -0.00256 (0.00419) | 0.00367 (0.00728) |
| 0.44463         | 0.00597 (0.00102) | 0.00611 (0.00127) | -0.01222 (0.00238) | -0.01587 (0.00502) |
| 0.45196         | 0.00499 (0.00103) | 0.00509 (0.00128) | -0.00788 (0.00224) | -0.00743 (0.00363) |
| 0.48029         | 0.00502 (0.00053) | 0.00530 (0.00070) | -0.01001 (0.00134) | -0.01118 (0.00212) |
| 0.57851         | 0.00171 (0.00141) | 0.00175 (0.00165) | -0.01045 (0.00336) | -0.01087 (0.00506) |
| 0.58494         | 0.00135 (0.00137) | 0.00270 (0.00165) | -0.00732 (0.00319) | -0.00271 (0.00468) |
| 0.59036         | 0.00110 (0.00159) | 0.00381 (0.00209) | -0.00874 (0.00341) | -0.00602 (0.00521) |
| 0.64631         | 0.00410 (0.00077) | 0.00375 (0.00100) | -0.00662 (0.00169) | -0.00475 (0.00264) |
| 0.70667         | 0.00527 (0.00093) | 0.00570 (0.00117) | -0.00508 (0.00198) | -0.00662 (0.00344) |
| 0.71601         | 0.00516 (0.00098) | 0.00564 (0.00138) | -0.00762 (0.00214) | -0.01061 (0.00351) |
| 0.77155         | 0.00552 (0.00073) | 0.00506 (0.00099) | -0.00312 (0.00159) | -0.00774 (0.00256) |
| 0.77507         | 0.00540 (0.00063) | 0.00481 (0.00094) | -0.00526 (0.00130) | -0.00642 (0.00214) |
| 0.80349         | 0.00538 (0.00041) | 0.00513 (0.00058) | -0.00574 (0.00086) | -0.00660 (0.00139) |
| 0.80389         | 0.00528 (0.00052) | 0.00488 (0.00076) | -0.00573 (0.00102) | -0.00721 (0.00167) |
| 0.82990         | 0.00616 (0.00157) | 0.00483 (0.00182) | -0.00194 (0.00270) | -0.00748 (0.00448) |
| 0.90814         | 0.00444 (0.00092) | 0.00313 (0.00121) | -0.00523 (0.00175) | -0.00332 (0.00258) |
| 0.91347         | 0.00509 (0.00127) | 0.00514 (0.00177) | -0.00354 (0.00226) | 0.00169 (0.00355) |
| 0.95264         | 0.00493 (0.00076) | 0.00517 (0.00097) | -0.00426 (0.00117) | -0.00516 (0.00192) |
| 0.95424         | 0.00397 (0.00083) | 0.00330 (0.00116) | -0.00509 (0.00151) | -0.00626 (0.00235) |
| 0.96941         | 0.00425 (0.00055) | 0.00417 (0.00072) | -0.00515 (0.00090) | -0.00531 (0.00147) |

TABLE XI. Results for the strange electromagnetic form factors from the summation method and plateau fit at 1 fm on ensemble N203.
| $Q^2$ (GeV$^2$) | $G^s_5$ Summation Method | $G^s_5$ Plateau Fit | $G^s_5$ Summation Method | $G^s_5$ Plateau Fit |
|----------------|-------------------------|---------------------|-------------------------|---------------------|
| 0.00000        | -0.00093 (0.000106)     | -0.00055 (0.000139) | -0.01729 (0.00681)     | -0.02589 (0.01542)  |
| 0.12293        | 0.00313 (0.00116)       | 0.00204 (0.00161)   | -0.01656 (0.00600)     | 0.00015 (0.01105)   |
| 0.12665        | 0.00380 (0.00114)       | 0.00154 (0.00163)   | -0.02071 (0.00244)     | -0.02093 (0.00519)  |
| 0.15618        | 0.00331 (0.00057)       | 0.00255 (0.00079)   | -0.01831 (0.00221)     | -0.02155 (0.00364)  |
| 0.15678        | 0.00343 (0.00054)       | 0.00320 (0.00073)   | -0.02179 (0.00303)     | -0.02532 (0.00509)  |
| 0.15728        | 0.00371 (0.00068)       | 0.00297 (0.00097)   | 0.01230 (0.00460)      | 0.01697 (0.00803)   |
| 0.25772        | 0.00447 (0.00103)       | 0.00183 (0.00147)   | -0.00973 (0.00144)     | -0.00647 (0.00297)  |
| 0.26375        | 0.00407 (0.00110)       | 0.00205 (0.00175)   | -0.00858 (0.00259)     | -0.00497 (0.00401)  |
| 0.30282        | 0.00451 (0.00047)       | 0.00403 (0.00063)   | -0.01039 (0.00279)     | -0.01177 (0.00512)  |
| 0.30693        | 0.00483 (0.00073)       | 0.00300 (0.00122)   | -0.00893 (0.00149)     | 0.00491 (0.00246)   |
| 0.32310        | 0.00462 (0.00047)       | 0.00445 (0.00063)   | -0.00973 (0.00144)     | -0.00647 (0.00297)  |
| 0.38688        | 0.00871 (0.00207)       | 0.00502 (0.00265)   | -0.00705 (0.00280)     | 0.00277 (0.00558)   |
| 0.44152        | 0.00677 (0.00127)       | 0.00548 (0.00154)   | -0.00347 (0.00281)     | -0.00705 (0.00280)  |
| 0.44985        | 0.00721 (0.00120)       | 0.00418 (0.00160)   | -0.00628 (0.00161)     | 0.00084 (0.00495)   |
| 0.47998        | 0.00410 (0.00064)       | 0.00359 (0.00090)   | -0.00628 (0.00161)     | 0.00084 (0.00495)   |
| 0.57339        | 0.00040 (0.00175)       | 0.00546 (0.00223)   | -0.00395 (0.00415)     | -0.00191 (0.00270)  |
| 0.58082        | 0.00504 (0.00167)       | 0.00810 (0.00215)   | -0.00824 (0.00379)     | -0.00594 (0.00637)  |
| 0.58695        | 0.00590 (0.00191)       | 0.00857 (0.00279)   | -0.00484 (0.00420)     | -0.00907 (0.00725)  |
| 0.64631        | 0.00444 (0.00090)       | 0.00471 (0.00120)   | -0.00543 (0.00189)     | -0.007710 (0.00335) |
| 0.69934        | 0.00560 (0.00123)       | 0.00744 (0.00153)   | -0.00483 (0.00234)     | -0.00775 (0.00467)  |
| 0.71008        | 0.00702 (0.00127)       | 0.00810 (0.00172)   | -0.00672 (0.00268)     | -0.00389 (0.00479)  |
| 0.76924        | 0.00663 (0.00093)       | 0.00643 (0.00129)   | -0.00345 (0.00192)     | -0.00493 (0.00360)  |
| 0.77296        | 0.00686 (0.00081)       | 0.00734 (0.00116)   | -0.00603 (0.00164)     | -0.00675 (0.00297)  |
| 0.80309        | 0.00538 (0.00051)       | 0.00581 (0.00074)   | -0.00520 (0.00100)     | -0.00521 (0.00187)  |
| 0.80359        | 0.00580 (0.00065)       | 0.00610 (0.00098)   | -0.00561 (0.00123)     | -0.00451 (0.00226)  |
| 0.82016        | 0.00558 (0.00183)       | 0.00182 (0.00251)   | -0.00303 (0.00299)     | -0.00511 (0.00552)  |
| 0.90403        | 0.00412 (0.00110)       | 0.00443 (0.00154)   | -0.00377 (0.00190)     | -0.00580 (0.00341)  |
| 0.91005        | 0.00503 (0.00154)       | 0.00296 (0.00233)   | -0.00446 (0.00257)     | -0.00640 (0.00468)  |
| 0.95133        | 0.00393 (0.00086)       | 0.00253 (0.00126)   | -0.00525 (0.00139)     | -0.00716 (0.00247)  |
| 0.95324        | 0.00275 (0.00106)       | 0.00381 (0.00157)   | -0.00352 (0.00180)     | -0.01072 (0.00386)  |
| 0.96941        | 0.00418 (0.00063)       | 0.00349 (0.00094)   | -0.00456 (0.00108)     | -0.00367 (0.00185)  |

TABLE XII. Results for the strange electromagnetic form factors from the summation method and plateau fit at 1 fm on ensemble N200.
| $Q^2$ (GeV$^2$) | Summation Method | Plateau Fit | Summation Method | Plateau Fit |
|---------|------------------|-------------|------------------|-------------|
| 0.00000 | 0.00868 (0.00245) | 0.00279 (0.00314) | - | - |
| 0.07543 | -0.01185 (0.00223) | 0.00058 (0.00316) | -0.02945 (0.01863) | -0.02786 (0.03715) |
| 0.07653 | -0.01177 (0.00205) | 0.00065 (0.00278) | -0.01595 (0.01504) | -0.01260 (0.02745) |
| 0.08889 | 0.00109 (0.00132) | 0.00313 (0.00165) | -0.04089 (0.00895) | -0.01793 (0.01775) |
| 0.08899 | 0.00204 (0.00113) | 0.00281 (0.00140) | -0.01097 (0.00774) | -0.01581 (0.01209) |
| 0.08919 | 0.00157 (0.00137) | 0.00168 (0.00191) | -0.01755 (0.00956) | -0.01169 (0.01477) |
| 0.15527 | 0.00327 (0.00198) | 0.00306 (0.00254) | -0.03115 (0.01122) | -0.02594 (0.01966) |
| 0.15708 | 0.00298 (0.00199) | 0.00419 (0.00258) | -0.08910 (0.00922) | -0.06992 (0.01470) |
| 0.17406 | 0.00319 (0.00110) | 0.00288 (0.00132) | -0.07060 (0.00460) | 0.00191 (0.00896) |
| 0.17466 | 0.00421 (0.00143) | 0.00558 (0.00179) | -0.04042 (0.00758) | 0.00724 (0.01104) |
| 0.17516 | 0.00381 (0.00156) | 0.00458 (0.00206) | -0.02756 (0.00786) | -0.02037 (0.01349) |
| 0.18179 | 0.00421 (0.00111) | 0.00455 (0.00129) | -0.01730 (0.00493) | -0.00671 (0.00716) |
| 0.23251 | -0.00073 (0.00312) | 0.00008 (0.00429) | 0.00279 (0.01129) | -0.00205 (0.01903) |
| 0.25591 | 0.00279 (0.00238) | 0.00499 (0.00335) | -0.00007 (0.00698) | -0.00774 (0.01103) |
| 0.25832 | 0.00143 (0.00205) | -0.00023 (0.00281) | 0.00077 (0.00821) | -0.02576 (0.01537) |
| 0.27078 | 0.00365 (0.00134) | 0.00386 (0.00203) | 0.00440 (0.00497) | -0.01025 (0.00781) |
| 0.33485 | -0.00071 (0.00318) | 0.00230 (0.00447) | -0.00616 (0.00991) | 0.00424 (0.01819) |
| 0.33696 | -0.00027 (0.00306) | 0.00356 (0.00401) | -0.00806 (0.00987) | 0.00327 (0.01494) |
| 0.33877 | -0.00012 (0.00322) | 0.00295 (0.00441) | -0.00677 (0.00981) | -0.00817 (0.01694) |
| 0.36358 | 0.00131 (0.00199) | 0.00179 (0.00264) | -0.00295 (0.00648) | -0.00256 (0.00979) |
| 0.41229 | 0.00664 (0.00220) | 0.00348 (0.00334) | -0.01137 (0.00504) | 0.00449 (0.01013) |
| 0.41430 | 0.00626 (0.00208) | 0.00286 (0.00291) | -0.00628 (0.00610) | 0.00222 (0.01017) |
| 0.43901 | 0.00609 (0.00176) | 0.00149 (0.00250) | 0.00627 (0.00490) | -0.00887 (0.00809) |
| 0.44001 | 0.00696 (0.00142) | 0.00384 (0.00199) | -0.00716 (0.00403) | 0.00015 (0.00635) |
| 0.45257 | 0.00690 (0.00110) | 0.00290 (0.00159) | -0.00840 (0.00306) | -0.00460 (0.00514) |
| 0.45267 | 0.00664 (0.00132) | 0.00448 (0.00199) | -0.01535 (0.00388) | -0.00104 (0.00621) |
| 0.48521 | 0.00828 (0.00274) | 0.01279 (0.00343) | 0.00182 (0.00625) | 0.00848 (0.01126) |
| 0.51875 | 0.00518 (0.00197) | 0.01002 (0.00237) | -0.00284 (0.00512) | 0.00437 (0.00758) |
| 0.52056 | 0.00720 (0.00208) | 0.00733 (0.00290) | 0.00961 (0.00570) | 0.02815 (0.00982) |
| 0.53814 | 0.00770 (0.00164) | 0.01075 (0.00218) | -0.00167 (0.00408) | 0.01878 (0.00710) |
| 0.53874 | 0.00379 (0.00197) | 0.01119 (0.00243) | -0.00565 (0.00533) | -0.00729 (0.00865) |
| 0.54527 | 0.00674 (0.00136) | 0.00909 (0.00167) | -0.00219 (0.00337) | 0.00405 (0.00543) |

TABLE XIII. Results for the strange electromagnetic form factors from the summation method and plateau fit at 1 fm on ensemble D200.
| $Q^2$ (GeV$^2$) | $G^s_{1N}$ Summation Method | $G^s_{1N}$ Plateau Fit | $G^s_{1M}$ Summation Method | $G^s_{1M}$ Plateau Fit |
|----------------|-----------------------------|------------------------|-----------------------------|------------------------|
| 0.00000        | 0.00000 (0.00000)           | 0.00000 (0.00000)      | -                           | -                      |
| 0.18358        | 0.00316 (0.00099)           | 0.00582 (0.00153)      | -0.01073 (0.00529)          | -0.01557 (0.01293)     |
| 0.19423        | 0.00389 (0.00096)           | 0.00541 (0.00149)      | -0.00649 (0.00469)          | -0.01503 (0.00961)     |
| 0.25587        | 0.00492 (0.00048)           | 0.00460 (0.00060)      | -0.01281 (0.00146)          | -0.01527 (0.00723)     |
| 0.25800        | 0.00451 (0.00044)           | 0.00433 (0.00051)      | -0.00725 (0.00148)          | -0.00468 (0.00223)     |
| 0.25955        | 0.00407 (0.00057)           | 0.00425 (0.00081)      | -0.00570 (0.000209)         | -0.00661 (0.00381)     |
| 0.39513        | 0.00320 (0.00102)           | 0.00322 (0.00141)      | -0.00884 (0.00350)          | 0.00210 (0.00771)      |
| 0.41206        | 0.00453 (0.00130)           | 0.00395 (0.00192)      | -0.00889 (0.00346)          | -0.00603 (0.00786)     |
| 0.49006        | 0.00439 (0.00041)           | 0.00363 (0.00052)      | -0.00899 (0.00084)          | -0.00878 (0.00167)     |
| 0.49732        | 0.00484 (0.00058)           | 0.00400 (0.00078)      | -0.00973 (0.00155)          | -0.00706 (0.00284)     |
| 0.50264        | 0.00386 (0.00071)           | 0.00247 (0.00107)      | -0.00983 (0.00176)          | -0.00365 (0.00384)     |
| 0.53787        | 0.00444 (0.00041)           | 0.00428 (0.00049)      | -0.00747 (0.00090)          | -0.00889 (0.00143)     |
| 0.59593        | 0.00675 (0.00216)           | 0.00583 (0.00305)      | 0.00616 (0.00541)           | -0.00440 (0.01034)     |
| 0.70713        | 0.00613 (0.00097)           | 0.00583 (0.00125)      | -0.00791 (0.00189)          | -0.01123 (0.00383)     |
| 0.73200        | 0.00547 (0.00096)           | 0.00735 (0.00146)      | -0.00604 (0.00195)          | -0.00920 (0.00419)     |
| 0.79587        | 0.00423 (0.00048)           | 0.00471 (0.00065)      | -0.00583 (0.00102)          | -0.00616 (0.00164)     |
| 0.90155        | 0.00491 (0.00159)           | 0.00683 (0.00207)      | -0.00609 (0.00272)          | -0.00274 (0.00540)     |
| 0.93290        | 0.00550 (0.00147)           | 0.00451 (0.00119)      | -0.00165 (0.00271)          | 0.00012 (0.00500)      |
| 0.94984        | 0.00551 (0.00119)           | 0.00179 (0.00320)      | -0.00200 (0.00323)          | 0.00658 (0.00784)      |
| 1.07564        | 0.00449 (0.00072)           | 0.00409 (0.00094)      | -0.00340 (0.00118)          | -0.00387 (0.00201)     |
| 1.10255        | 0.00445 (0.00119)           | 0.00464 (0.00151)      | -0.00378 (0.00184)          | -0.00325 (0.00371)     |
| 1.13318        | 0.00455 (0.00138)           | 0.00466 (0.00202)      | -0.00100 (0.00243)          | 0.00017 (0.00494)      |
| 1.25922        | 0.00347 (0.00081)           | 0.00439 (0.00119)      | -0.00292 (0.00142)          | 0.00069 (0.00290)      |
| 1.26987        | 0.00344 (0.00072)           | 0.00409 (0.00118)      | -0.00077 (0.00117)          | 0.00009 (0.00262)      |
| 1.28477        | 0.00608 (0.00222)           | 0.00171 (0.00280)      | 0.00267 (0.00330)           | 0.00046 (0.00590)      |
| 1.33664        | 0.00443 (0.00039)           | 0.00437 (0.00055)      | -0.00344 (0.00062)          | -0.00381 (0.00109)     |
| 1.33519        | 0.00417 (0.00053)           | 0.00478 (0.00084)      | -0.00288 (0.00083)          | -0.00004 (0.00174)     |
| 1.47077        | 0.00273 (0.00105)           | 0.00411 (0.00151)      | -0.00292 (0.00149)          | -0.00302 (0.00283)     |
| 1.48771        | -0.00011 (0.00179)          | -0.00035 (0.00295)     | -0.00368 (0.00239)          | 0.00510 (0.00520)      |
| 1.57297        | 0.00232 (0.00077)           | 0.00360 (0.00105)      | -0.00110 (0.00091)          | 0.00059 (0.00197)      |
| 1.57829        | 0.00319 (0.00000)           | 0.00326 (0.00155)      | -0.00203 (0.00125)          | -0.00076 (0.00287)     |
| 1.61351        | 0.00231 (0.00048)           | 0.00331 (0.00070)      | -0.00276 (0.00064)          | -0.00264 (0.00120)     |

TABLE XIV. Results for the strange electromagnetic form factors from the summation method and plateau fit at 1 fm on ensemble N302.