We study the effects of non-standard interactions on the oscillation pattern of atmospheric neutrinos. We use neutrino oscillograms as our main tool to infer the role of non-standard interactions (NSI) parameters at the probability level in the energy range, $E \in [1, 20]$ GeV and zenith angle range, $\cos \theta \in [-1, 0]$. We compute the event rates for atmospheric neutrino events in presence of NSI parameters in the energy range $E \in [1, 10]$ GeV for two different detector configurations - a magnetized iron calorimeter and an unmagnetized liquid Argon time projection chamber which have different sensitivities to NSI parameters due to their complementary characteristics. As an application, we discuss how NSI parameter, $\epsilon_{\mu\tau}$ impacts the determination of the correct octant of $\theta_{23}$. 
1 Introduction

With the immense progress over the past few decades in establishing neutrino masses and mixings, it is fair to say that neutrino physics has entered an era of precision measurements. The first confirmation came in 1998 courtesy the pioneering experiment, Super Kamiokande (SK) [1]. With more data as well as with the aid of numerous other experiments, we have steadily garnered more and more precise information about the neutrino mixing parameters. As a result, the long list of unanswered questions in the standard scenario has become shorter (for recent global analyses of all neutrino oscillation data, see [2–4]). The focus of the ongoing and future neutrino experiments is on resolving the issue of neutrino mass hierarchy i.e., sign \((\delta m^2_{31})\) measuring the CP phase \((\delta)\) and determining the correct octant of the mixing angle \(\theta_{23}\).

The minimal theoretical scenario needed to describe oscillations requires the existence of neutrino masses. The simplest way is to add right handed neutrino fields to the Standard Model (SM) particle content (something that the originators of the SM would, no doubt, have trivially done were nonzero neutrino masses known then) and generate a Dirac mass term for neutrinos. However it is hard to explain the smallness of the neutrino mass terms via this mechanism. To overcome this, an attractive way is to add dimension-five non-renormalizable terms consistent with the symmetries and particle content of the SM, which naturally leads to desired tiny Majorana masses for the left-handed neutrinos. However in the minimal scenario of this extension, the dominant neutrino interactions involving the light fields are still assumed to be described by weak interactions within the SM in which flavour changes are strongly suppressed.

Once we invoke new physics in order to explain the non-zero neutrino masses, it seems rather unnatural to exclude the possibility of non standard interactions (NSI) which can, in principle, allow for flavour changing interactions. Simultaneously, these are new sources of CP violation which can affect production, detection and propagation of neutrinos [5]. Some of the early attempts discussing new sources of lepton flavour violation (for instance, \(R\)-parity violating supersymmetry) were geared towards providing an alternate explanation for the observed deficit of neutrinos in the limiting case of zero neutrino masses and the absence of vacuum mixing [6,7]. In recent years, the emphasis has shifted towards understanding the interplay between the standard electroweak interactions (SI) and NSI and whether future oscillation experiments can test such NSI apart from determining the standard oscillation parameters precisely. This has led to an upsurge in research activity in this area (see the references in [5]). This is also interesting from the point of view of complementarity with the collider searches for new physics. There are other motivations for NSI as well such as (electroweak) leptogenesis [8], neutrino magnetic moments [9–12], neutrino condensate as dark energy [13,14].

Neutrino oscillation experiments can probe NSI by exploiting the interference with the

\[ 1 \delta m^2_{31} = m^2_2 - m^2_1. \]

\[ 2 \text{Terms such as } \bar{\nu}_R^i \nu^j_R \text{ are gauge invariant too and phenomenologically unconstrained. While they break lepton number, the latter is only an accidental symmetry in the SM. Thus, such terms, in conjunction with the usual Dirac mass terms, would generate tiny observable neutrino masses through the see-saw mechanism. It can be readily seen that, the aforementioned dimension-five term (} L^c \Phi \Phi \text{} \text{essentially mimics this mechanism in the low energy effective theory.} \]
Standard Model amplitude. In view of the excellent agreement of data with standard flavour conversion via oscillations, we would like to explore the extent to which NSI (incorporated into the Lagrangian phenomenologically via small parameters) is empirically viable, with specific focus on atmospheric neutrino signals in future detectors. NSI in the context of atmospheric neutrinos has been studied by various authors [15, 18] while other new physics scenarios using atmospheric neutrinos such as CPT violation have been studied in [19, 20].

Finally, as an application, we discuss how NSI impacts the determination of the correct octant for $\theta_{23}$. Typically, Earth matter effects have been exploited to break the degeneracy associated with this parameter [21–23]. Here we discuss, via an example, how a particular NSI parameter $\epsilon_{\mu\tau}$ interferes with the determination of the correct octant for atmospheric neutrinos that is nominally sought to be effected through the study of the $\nu_\mu \to \nu_\mu$ channel. A detailed study of the octant determination in presence of NSI parameters for the case of atmospheric neutrinos is currently under progress [24].

The plan of the article is as follows. We first briefly outline the NSI framework in Sec. 2 and subsequently discuss the neutrino oscillation probabilities in presence of NSI using the perturbation theory approach (in Sec. 3). We describe the features of the neutrino oscillograms in Sec. 4. We give the details of our analysis in Sec. 5 and the discussion on events generated for the two detector types in Sec. 6. Finally, we conclude in Sec. 7.

2 Neutrino NSI Framework: relevant parameters and present constraints

As in the case of standard weak interactions, a wide class of “new physics scenarios” can be conveniently parameterised in a model independent way at low energies ($E \ll M_{EW}$, where $M_{EW}$ is the electroweak scale) by using effective four-fermion interactions. In general, NSI can impact the neutrino oscillation signals via two kinds of interactions : (a) charged current (CC) interactions (b) neutral current (NC) interactions. However, CC interactions affect processes only at the source or the detector and these are are clearly discernible at near detectors (see for example, [25, 26]). On the other hand, the NC interactions affect the propagation of neutrinos which can be studied only at far detectors. Due to this decoupling, the two can be treated in isolation. Usually, it is assumed that the CC NSI terms (e.g., of the type $(\bar{\nu}_\beta \gamma^\mu P_L l_\alpha)(f_L \gamma^\mu P_C f'_L)$ with $f, f'$ being the components of a weak doublet) are more tightly constrained than the NC terms and, hence, are not considered. It turns out, though, that, in specific models, the two can be of comparable strengths [27]. However, since we are interested in NSI that alter the propagation of neutrinos, we shall consider the NC type of interactions alone.

The effective Lagrangian describing the NC type neutrino NSI of the type $(V - A)(V \pm A)$ is given by

$$\mathcal{L}_{NSI} = -2\sqrt{2} G_F \epsilon_{\alpha\beta}^{fC} [\bar{l}_\alpha \gamma^\mu P_L l_\beta] [\bar{f}_\gamma^\mu P_C f'_L] ,$$

where $G_F$ is the Fermi constant, $\nu_\alpha, \nu_\beta$ are neutrinos of different flavours, and $f$ is a first

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3One could think that other Dirac structures generated by intermediate scalar ($S$), pseudoscalar ($P$) or tensor ($T$) fields may also be there. However, these would only give rise to subdominant effects.
generation SM fermion \((e, u, d)\) \[^{[3]}\]. The chiral projection operators are given by \(P_L = (1 - \gamma_5)/2\) and \(P_C = (1 + \gamma_5)/2\). If the NSI arises at scale \(M_{NP} \gg M_{EW}\) from some higher dimensional operators (of order six or higher), it would imply a suppression of at least \(\epsilon_{\alpha\beta}^{LC} \simeq (M_{EW}/M_{NP})^2\) (for \(M_{NP} \sim 1\ TeV\), we have \(\epsilon_{\alpha\beta}^{LC} \simeq 10^{-2}\)). However, such a naive dimensional analysis argument breaks down if the new physics sector is strongly interacting as can happen in a variety of models. We shall, hence, admit even larger \(\epsilon_{\alpha\beta}^{LC}\) as long as these are consistent with all current observations. In general, NSI terms can be complex. Naively, \(SU(2)\) invariance would dictate that operators involving \(\nu_L\) must be accompanied by ones containing the corresponding charged lepton field, thereby leading to additional CC interactions. This, however, can be avoided by applying to \(SU(2)\) breaking and/or invoking multiple fields and interactions in the heavy (or hidden) sector. Rather than speculate about the origin of any such mechanism, we assume here (as in much of the literature) that no such CC terms exist.

The new NC interaction terms can affect the neutrino oscillation physics either by causing the flavour of neutrino to change \((\nu_\alpha + f \rightarrow \nu_\beta + f)\) i.e., flavour changing (FC) interaction or, by having a non-universal scattering amplitude of NC for different neutrino flavours i.e., flavour preserving (FP) interaction. At the level of the underlying Lagrangian, NSI coupling of the neutrino can be to \(e, u, d\). However, from a phenomenological point of view, only the sum (incoherent) of all these individual contributions (from different scatterers) contributes to the coherent forward scattering of neutrinos on matter. If we normalize \[^{[5]}\] to \(n_e\), the effective NSI parameter for neutral Earth matter \[^{[6]}\] is

\[
\epsilon_{\alpha\beta} = \sum_{f=e,u,d} \frac{n_f f}{n_e \epsilon_{\alpha\beta}^f} = \epsilon_{\alpha\beta}^e + 2 \epsilon_{\alpha\beta}^u + \epsilon_{\alpha\beta}^d + \frac{n_\alpha}{n_e} (2 \epsilon_{\alpha\beta}^d + \epsilon_{\alpha\beta}^u) = \epsilon_{\alpha\beta}^e + 3 \epsilon_{\alpha\beta}^u + 3 \epsilon_{\alpha\beta}^d ,
\]

where \(n_f\) is the density of fermion \(f\) in medium crossed by the neutrino and \(n\) refers to neutrons. Also, \(\epsilon_{\alpha\beta}^f = \epsilon_{\alpha\beta}^{fL} + \epsilon_{\alpha\beta}^{fR}\) which encodes the fact that NC type NSI matter effects are sensitive to the vector sum of NSI couplings.

Let us, now, discuss the constraints on the NC type NSI parameters. As mentioned above, the combination that enters oscillation physics is given by Eq. (2). The individual NSI terms such as \(\epsilon_{\alpha\beta}^{fL}\) or \(\epsilon_{\alpha\beta}^{fR}\) are constrained in any experiment (keeping only one of them non-zero at a time) and moreover the coupling is either to \(e, u, d\) individually \[^{[28]}\]. In view of this, it is not so straightforward to interpret those bounds in terms of an effective \(\epsilon_{\alpha\beta}\). There are two ways: (a) One could take a conservative approach and use the most stringent constraint in the individual NSI terms (say, use \(|\epsilon_{\mu\tau}^u|\)) to constrain the effective term (say, \(|\epsilon_{\mu\tau}|\)) in Eq. (2) and that leads to

\[
|\epsilon_{\alpha\beta}| < \begin{pmatrix} 0.06 & 0.05 & 0.27 \\ 0.05 & 0.003 & 0.05 \\ 0.27 & 0.05 & 0.16 \end{pmatrix}.
\]

\[^{[4]}\]Coherence requires that the flavour of the background fermion \((f)\) is preserved in the interaction. Second or third generation fermions do not affect oscillation experiments since matter does not contain them.

\[^{[3]}\]If we normalize to either up or down quark abundance (assume isoscalar composition of matter) instead, there is a relative factor of 3 which will need to be incorporated accordingly.

\[^{[6]}\]For neutral Earth matter, there are 2 nucleons (one proton and one neutron) per electron. For neutral solar matter, there is one proton for one electron, and \(\epsilon_{\alpha\beta} = \epsilon_{\alpha\beta}^e + 2 \epsilon_{\alpha\beta}^u + 2 \epsilon_{\alpha\beta}^d\).
The constraints involving muon neutrinos are at least an order of magnitude stronger (courtesy the NuTeV and CHARM scattering experiments) than those involving electron and tau neutrino [29]. (b) With the assumption that the errors on individual NSI terms are uncorrelated, the authors in Ref. [27] deduce model-independent bounds on effective NC NSI terms

\[ \epsilon_{\alpha\beta} \lesssim \left\{ \sum_{C=L,R} \left[ (\epsilon_{\alpha\beta}^{eC})^2 + (3\epsilon_{\alpha\beta}^{uC})^2 + (3\epsilon_{\alpha\beta}^{dC})^2 \right] \right\}^{1/2}, \]

which, for neutral Earth matter, leads to

\[ |\epsilon_{\alpha\beta}| < \begin{pmatrix} 4.2 & 0.33 & 3.0 \\ 0.33 & 0.068 & 0.33 \\ 3.0 & 0.33 & 21 \end{pmatrix}. \]

Note that the values mentioned in Eq. (5) are larger by one or two orders of magnitude than the overly restrictive bounds of Eq. (3), which, of course, need not be applicable.

Apart from the model independent theoretical bounds, two experiments have used the neutrino data to constrain NSI parameters. The SK NSI search in atmospheric neutrinos crossing the Earth found no evidence in favour of NSI and the study led to upper bounds on NSI parameters [30] given by $|\epsilon_{\mu\tau}| < 0.033, |\epsilon_{\tau\tau} - \epsilon_{\mu\mu}| < 0.147$ (at 90% CL) in a two flavour hybrid model [5]. The off-diagonal NSI parameter $\epsilon_{\mu\tau}$ is constrained $-0.20 < \epsilon_{\mu\tau} < 0.07$ (at 90% CL) from MINOS data in the framework of two flavour neutrino oscillations [31,32]. However the bounds are still rather uncertain [5] and hence we choose to use less restrictive values than the ones mentioned above. Moreover, we note that the existing experimental bounds depend upon various assumptions such as the two flavour approximation. Additionally, the allowed ranges of NSI parameters have been recently extracted using global analysis of neutrino data in Ref. [33]. Following the other studies on neutrino NSI in propagation [34], we will use a value of $|\epsilon_{\alpha\beta}| = 0.15$ for the parameters $\epsilon_{\mu\tau}, \epsilon_{\mu\mu}$ and $\epsilon_{\tau\tau}$ appearing in the present work. This value is eminently in agreement with Eq. (5).

### 3 Neutrino oscillation probability in matter with NSI

The purpose of the analytic expressions presented here is to understand the features in the probability in the presence of NSI. All the plots presented in this paper are obtained numerically by solving the full three flavour neutrino propagation equations using the PREM density profile of the Earth, and the latest values of the neutrino parameters as obtained from global fits (see Table 1).

The analytic computation of probability expressions in presence of SI [35–41] as well as NSI [34,42–46] has been carried out for different experimental settings by various authors. Note that, for atmospheric neutrinos, one can safely neglect the smaller mass squared difference $\delta m_{21}^2$ in comparison to $\delta m_{31}^2$ since $\delta m_{21}^2 L/4E \ll 1$ for a large range of values of $L$.

---

\[ \text{Note that the experimental uncertainties (statistical and systematic) are substantial for the NSI parameters.} \]
Oscillation Parameter | Best-fit value | 3σ range | Precision (%) |
|----------------------|--------------|----------|---------------|
| \(\sin^2 \theta_{12}/10^{-1}\) | 3.23 | 2.78 - 3.75 | 14.85 |
| \(\sin^2 \theta_{23}/10^{-1}\) (NH) | 5.67 (4.67)<sup>a</sup> | 3.92 - 6.43 | 24.25 |
| \(\sin^2 \theta_{23}/10^{-1}\) (IH) | 5.73 | 4.03 - 6.40 | 22.72 |
| \(\sin^2 \theta_{13}/10^{-2}\) (NH) | 2.34 | 1.77 - 2.94 | 24.84 |
| \(\sin^2 \theta_{13}/10^{-2}\) (IH) | 2.40 | 1.83 - 2.97 | 23.75 |
| \(\delta m^2_{21} \times 10^{-5} \text{ eV}^2\) | 7.60 | 7.11 - 8.18 | 7.00 |
| \(|\delta m^2_{31}| \times 10^{-3} \text{ eV}^2\) (NH) | 2.48 | 2.30 - 2.65 | 7.07 |
| \(|\delta m^2_{31}| \times 10^{-3} \text{ eV}^2\) (IH) | 2.38 | 2.30 - 2.54 | 5.00 |
| \(\delta/\pi\) (NH) | 1.34 | 0.0 - 2.0 | - |
| \(\delta/\pi\) (IH) | 1.48 | 0.0 - 2.0 | - |

<sup>a</sup>This is a local minimum in the first octant of \(\theta_{23}\) with \(\Delta \chi^2 = 0.28\) with respect to the global minimum.

**Table 1:** Best-fit values and the 3σ ranges for the oscillation parameters used in our analysis [4]. Also given is the precision which is defined as ratio (in percentage) of the difference of extreme values to the sum of extreme values of parameters in the 3σ range. Here NH (IH) refer to normal (inverted) hierarchy.

and \(E\) (especially above a GeV). This “one mass scale dominant” (OMSD) approximation allows for a relatively simple exact analytic formula for the probability (as a function of only three parameters \(\theta_{23}, \theta_{13}\) and \(\delta m^2_{21}\)) for the case of constant density matter [40] with no approximation on \(s_{13}\), and it works quite well<sup>9</sup>. In order to systematically take into account the effect of small parameters, the perturbation theory approach is used. We review the necessary formulation for calculation of probabilities that affect the atmospheric neutrino propagation using the perturbation theory approach [34].

In the ultra-relativistic limit, the neutrino propagation is governed by a Schrödinger-type equation (see [47]) with an effective Hamiltonian

\[
\mathcal{H} = \mathcal{H}_{\text{vac}} + \mathcal{H}_{\text{SI}} + \mathcal{H}_{\text{NSI}},
\]

where \(\mathcal{H}_{\text{vac}}\) is the vacuum Hamiltonian and \(\mathcal{H}_{\text{SI}}, \mathcal{H}_{\text{NSI}}\) are the effective Hamiltonians in presence of SI alone and NSI respectively. Thus,

\[
\mathcal{H} = \frac{1}{2E} \begin{pmatrix}
0 & \delta m^2_{21} \\
\delta m^2_{31} & 0
\end{pmatrix} \mathcal{U}^\dagger + A(x) \begin{pmatrix}
1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\
\epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\
\epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau}
\end{pmatrix},
\]

where \(A(x) = \sqrt{2} G_F n_e(x)\) is the standard CC potential due to the coherent forward scattering of neutrinos and \(n_e\) is the electron number density. The three flavour neutrino mixing

<sup>9</sup>This approximation breaks down if the value of \(\theta_{13}\) is small since the terms containing \(\delta m^2_{21}\) can be dropped only if they are small compared to the leading order term which contain \(\theta_{13}\). After the precise measurement of the value of \(\theta_{13}\) by reactor experiments, this approximation is well justified. For multi-GeV neutrinos, this condition \((L/E \ll 10^4 \text{ km/GeV})\) is violated for only a small fraction of events with \(E \simeq 1 \text{ GeV}\) and \(L \geq 10^4 \text{ km}\).
matrix $\mathcal{U} \equiv U_{23} \mathcal{W}_{13} U_{12}$ with $\mathcal{W}_{13} = U_\theta \mathcal{U}_{13} U_\delta^\dagger$ and $U_\delta = \text{diag}\{1, 1, \exp(i\delta)\}$ is characterized by three angles and a single (Dirac) phase and, in the standard PMNS parameterisation, we have

$$
\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1 \end{pmatrix},
$$

(8)

where $s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}$. While, in addition, two Majorana phases are also possible, these are ignored as they play no role in neutrino oscillations. This particular parameterisation along with the fact of $\mathcal{H}_{SI}$ commuting with $U_{23}$, allows for a simplification. Going over to the basis, $\tilde{\nu} = U_{23}^\dagger \nu$, we have $\tilde{\mathcal{H}} = U_{23}^\dagger \mathcal{H} U_{23}$ and [44]

$$
\tilde{\mathcal{H}} = \lambda \left[ \begin{pmatrix} r_A + s_{13}^2 & 0 & c_{13} s_{13} e^{-i\delta} \\
0 & 0 & 0 \\
c_{13} s_{13} e^{i\delta} & 0 & c_{13}^2 \end{pmatrix} + r_\lambda \begin{pmatrix} s_{12}^2 c_{13} & c_{12} s_{12} c_{13} & -s_{12}^2 c_{13} s_{13} e^{-i\delta} \\
c_{12} s_{12} c_{13} & c_{12}^2 & -c_{12} s_{12}^2 s_{13} e^{-i\delta} \\
s_{12}^2 c_{13} s_{13} e^{i\delta} & -c_{12} s_{12}^2 s_{13} e^{i\delta} & s_{12}^2 c_{13}^2 \end{pmatrix} \right] + r_A \begin{pmatrix} \tilde{\epsilon}_{ee} & \tilde{\epsilon}_{em} & \tilde{\epsilon}_{et} \\
\tilde{\epsilon}_{em}^* & \tilde{\epsilon}_{mm} & \tilde{\epsilon}_{mt} \end{pmatrix},
$$

(9)

where we have defined dimensionless ratios

$$
\lambda \equiv \frac{\delta m_{31}^2}{2E}; \quad r_\lambda \equiv \frac{\delta m_{21}^2}{\delta m_{31}^2}; \quad r_A \equiv \frac{A(x)}{\delta m_{31}^2}.
$$

(10)

Once again, $\tilde{\mathcal{H}}_{NSI} = U_{23}^\dagger \mathcal{H}_{NSI} U_{23}$ and the last term in Eq. (9) is

$$
\lambda r_A \begin{pmatrix} \epsilon_{ee} & c_{23} \epsilon_{em} - s_{23} \epsilon_{et} \\
c_{23} \epsilon_{em} - s_{23} \epsilon_{et} & \epsilon_{mm} c_{23}^2 + \epsilon_{mt} s_{23}^2 - (\epsilon_{mm} + \epsilon_{mt}) c_{23} s_{23} \\
s_{23} \epsilon_{em} - c_{23} \epsilon_{et} & c_{23}^2 \epsilon_{et}^* - s_{23} \epsilon_{et}^* - c_{23} s_{23} \epsilon_{et} \end{pmatrix} + \epsilon_{\alpha\beta} \begin{pmatrix} s_{23} \epsilon_{em} - c_{23} \epsilon_{et} \\
s_{23} \epsilon_{em} - c_{23} \epsilon_{et} \end{pmatrix}
$$

where $\epsilon_{\alpha\beta} \equiv |\epsilon_{\alpha\beta}| e^{i\phi_{\alpha\beta}}$ are complex. For atmospheric and long baseline neutrinos, $\lambda L \approx \mathcal{O}(1)$ holds and $r_A L \sim \mathcal{O}(1)$ for a large range of the $E$ and $L$ values considered here. The small quantities are $r_\lambda \approx 0.03$ and $\tilde{\epsilon}_{\alpha\beta}$. We decompose $\tilde{\mathcal{H}}$ into two parts: $\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_0 + \tilde{\mathcal{H}}_I$ such that the zeroth order term $\tilde{\mathcal{H}}_0$ provides the effective two flavour limit with $r_A \neq 0$ and $s_{13} \neq 0$ but $r_\lambda = 0$, i.e.,

$$
\tilde{\mathcal{H}}_0 = \lambda \begin{pmatrix} r_A(x) + s_{13}^2 & 0 & c_{13} s_{13} e^{-i\delta} \\
0 & 0 & 0 \\
c_{13} s_{13} e^{i\delta} & 0 & c_{13}^2 \end{pmatrix},
$$

(11)

while $\tilde{\mathcal{H}}_I$ contains the other two terms (on the RHS of Eq. (9)) which represent corrections due to non-zero $r_\lambda$ and the non-zero NSI parameters $\tilde{\epsilon}_{\alpha\beta}$ respectively. Upon neglecting terms like $r_\lambda s_{13}, r_\lambda s_{13}^2$, we get an approximate form for $\tilde{\mathcal{H}}_I$, viz.,

$$
\tilde{\mathcal{H}}_I \approx \lambda \left[ r_\lambda \begin{pmatrix} s_{12}^2 & c_{12} s_{12} & 0 \\
c_{12} s_{12} & s_{12}^2 & 0 \\
0 & 0 & 0 \end{pmatrix} + r_A \begin{pmatrix} \tilde{\epsilon}_{ee} & \tilde{\epsilon}_{em} & \tilde{\epsilon}_{et} \\
\tilde{\epsilon}_{em}^* & \tilde{\epsilon}_{mm} & \tilde{\epsilon}_{mt} \end{pmatrix} \right].
$$

(12)
In what follows, we use the perturbation method described in \cite{41} to compute the oscillation probabilities. The exact oscillation probability is given by

\[ P_{\alpha\beta} = |S_{\beta\alpha}(x, x_0)|^2 , \]  
\[ (13) \]

where \( S(x, x_0) \) is the evolution matrix defined through \( |\nu(x)\rangle = S(x, x_0) |\nu(x_0)\rangle \) with \( S(x_0, x_0) = \mathbb{I} \) and satisfying the same Schrödinger equation as \( |\nu(x)\rangle \). It can, trivially, be seen to be given by \( S(x, x_0) = U_{23} \tilde{S}(x, x_0) U_{23}^\dagger \) where \( \tilde{S}(x, x_0) \) is independent of \( \theta_{23} \). We first evaluate \( \tilde{S}(x, x_0) \) using

\[ \tilde{S}(x, x_0) = \tilde{S}_0(x, x_0) \tilde{S}_1(x, x_0) . \]
\[ (14) \]

Here, \( \tilde{S}_0(x, x_0) \) and \( \tilde{S}_1(x, x_0) \) satisfy

\[ i \frac{d}{dx} \tilde{S}_0(x, x_0) = \tilde{\mathcal{H}}_0(x) \tilde{S}_0(x, x_0) ; \quad \tilde{S}_0(x_0, x_0) = \mathbb{I} , \]
\[ i \frac{d}{dx} \tilde{S}_1(x, x_0) = [\tilde{S}_0(x, x_0)]^{-1} \tilde{\mathcal{H}}_1(x) \tilde{S}_0(x, x_0) \tilde{S}_1(x, x_0) ; \quad \tilde{S}_1(x_0, x_0) = \mathbb{I} . \]
\[ (15) \]

where \( \tilde{\mathcal{H}}_i \) is given by Eq. \[12\]. To the first order in the expansion parameter, we have

\[ \tilde{S}(x_0, x) \simeq \tilde{S}_0(x_0, x) - i\tilde{S}_0(x_0, x) \int_{x_0}^x \tilde{[S}_0(x', x_0)]^{-1} \tilde{\mathcal{H}}_1(x') \tilde{S}_0(x', x_0) dx' . \]
\[ (16) \]

Finally, the full evolution matrix \( S(x, x_0) \) can be obtained by going back to the original basis from the tilde basis using \( S(x, x_0) = U_{23} \tilde{S}(x, x_0) U_{23}^\dagger \). The oscillation probability for \( \nu_e \to \nu_\mu \) can be obtained as

\[ P_{\nu e\nu_\mu}^{NSI} \simeq 4 s_{13}^2 s_{23}^2 \left[ \frac{\sin^2 (1 - r_A) \lambda L/2}{(1 - r_A)^2} \right] \]
\[ + 8 s_{13} s_{23} c_{23} (|\epsilon_{e\mu}| c_{23} c_\chi - |\epsilon_{e\tau}| s_{23} c_\omega) r_A \left[ \frac{\sin r_A \lambda L/2}{r_A} \frac{\sin (1 - r_A) \lambda L/2}{(1 - r_A)} \cos \frac{\lambda L}{2} \right] \]
\[ + 8 s_{13} s_{23} c_{23} (|\epsilon_{e\mu}| c_{23} c_\chi - |\epsilon_{e\tau}| s_{23} s_\omega) r_A \left[ \frac{\sin r_A \lambda L/2}{r_A} \frac{\sin (1 - r_A) \lambda L/2}{(1 - r_A)} \sin \frac{\lambda L}{2} \right] \]
\[ + 8 s_{13}^2 s_{23}^2 (|\epsilon_{e\mu}| s_{23} c_\chi + |\epsilon_{e\tau}| c_{23} c_\omega) r_A \left[ \frac{\sin^2 (1 - r_A) \lambda L/2}{(1 - r_A)^2} \right] , \]  
\[ (17) \]

where we have used \( s_{13} \equiv \sin \tilde{\theta}_{13} = s_{13}/(1 - r_A) \) to the leading order in \( s_{13} \), and \( \chi = \phi_{e\mu} + \delta \), \( \omega = \phi_{e\tau} + \delta \). Only the parameters \( \epsilon_{e\mu} \) and \( \epsilon_{e\tau} \) enter in the leading order expression \[42\], as terms such as \( r_A \epsilon_{e\alpha\beta} \) have been neglected. Let us discuss the two limiting cases, \( r_A \to 0 \) and \( r_A \to 1 \). When \( r_A \to 0 \), we recover the vacuum limit (given by the first term on the RHS of Eq. \[17\]). When \( r_A \to 1 \), we are close to the resonance condition \( (r_A = \cos 2\tilde{\theta}_{13} \text{ since } \tilde{\theta}_{13} \text{ is small}) \) and the probability remains finite due to the \( (1 - r_A) \) and \( (1 - r_A)^2 \) terms in the denominator of Eq. \[17\].

The survival probability for \( \nu_\mu \to \nu_\mu \) is given by

\[ P_{\nu_\mu\nu_\mu}^{NSI} \simeq 1 - s_{23}^2 \left[ \frac{\sin^2 \lambda L}{2} \right] \]
\[ (18) \]
are given by

\[ s_{2x23}^2 \equiv \sin 2\theta_{23} \quad \text{and} \quad c_{2x23} \equiv \cos 2\theta_{23}. \]

Within the SM, for a given hierarchy (NH or IH) and best-fit values of the oscillation parameters:

\[ \Delta P_{\mu\beta} = P_{\mu\beta}^{NI} - P_{\mu\beta}^{FSI}, \]

where the NSI parameters involving the electron sector do not enter this channel and the survival probability depends only on the three parameters \( \epsilon_{\mu\mu}, \epsilon_{\mu\tau}, \epsilon_{\tau\tau} \). Once again, the vacuum limit is recovered for \( r_A \rightarrow 0 \). Of these three NSI parameters, \( \epsilon_{\mu\mu} \) is subject to the most stringent constraint (Eq. 5). If we look at \( P_{\mu\mu}^{FSI} \), the phase factor results in minima of probability for \( \lambda L/2 = (2p + 1)\pi/2 \) (vacuum dip) and maxima for \( \lambda L/2 = p\pi \) (vacuum peak) where \( p \) is any integer. The oscillation length for the NSI terms, though, is different, and this changes the positions of the peaks and dips.

In order to quantify the impact of NSI, it is useful to define a difference\(^\text{10}\)

\[ \Delta P_{\mu\beta} = P_{\mu\beta}^{FSI} - P_{\mu\beta}^{NI}, \]

where \( P_{\mu\beta}^{FSI} \) is probability of transition assuming standard interactions (i.e., with \( \epsilon_{\alpha\beta} \) being set to zero in Eqs. (17) and (18)) and \( P_{\mu\beta}^{NI} \) is the transition probability in presence of NSI parameters. For the different channels that are relevant to our study, the quantities \( \Delta P_{\mu\beta} \) are given by

\[ \Delta P_{\mu\mu} \simeq \epsilon_{\mu\tau} |\sin \phi_{\mu\tau} s_{2x23}^{2} + \epsilon_{\mu\mu} | c_{2x23} c_{\lambda} - |\epsilon_{\tau\tau} | s_{23} c_{\omega} | r_A | \left[ \frac{\sin r_A \lambda L/2}{r_A} \frac{\sin (1 - r_A) \lambda L/2}{(1 - r_A)} \frac{\cos \lambda L}{2} \right] \]

\[ - 8 s_{13} s_{23} c_{23} | \epsilon_{\mu\mu} | c_{23} c_{\lambda} - |\epsilon_{\tau\tau} | s_{23} c_{\omega} | r_A | \left[ \frac{\sin r_A \lambda L/2}{r_A} \frac{\sin (1 - r_A) \lambda L/2}{(1 - r_A)} \frac{\lambda L}{2} \right] \]

\[ - 8 s_{13} s_{23} c_{23} | \epsilon_{\mu\mu} | c_{23} c_{\lambda} + |\epsilon_{\tau\tau} | c_{23} c_{\omega} | r_A | \left[ \frac{\sin^2 (1 - r_A) \lambda L/2}{(1 - r_A)^2} \right] . \]

\[ \Delta P_{\mu\tau} \simeq \epsilon_{\mu\tau} |\sin \phi_{\mu\tau} s_{2x23}^{2} + \epsilon_{\mu\mu} | c_{2x23} c_{\lambda} - |\epsilon_{\tau\tau} | s_{23} c_{\omega} | r_A | \left[ \frac{\sin r_A \lambda L/2}{r_A} \frac{\sin (1 - r_A) \lambda L/2}{(1 - r_A)} \frac{\lambda L}{2} \right] \]

\[ - 8 s_{13} s_{23} c_{23} | \epsilon_{\mu\mu} | c_{23} c_{\lambda} - |\epsilon_{\tau\tau} | s_{23} c_{\omega} | r_A | \left[ \frac{\sin r_A \lambda L/2}{r_A} \frac{\sin (1 - r_A) \lambda L/2}{(1 - r_A)} \frac{\lambda L}{2} \right] . \]

For the case of anti-neutrinos, \( A \rightarrow -A \) (which implies that \( r_A \rightarrow -r_A \)) while \( \lambda \rightarrow \lambda, r_\lambda \rightarrow r_\lambda \). Similarly for IH, \( \lambda \rightarrow -\lambda, r_\lambda \rightarrow -r_\lambda, r_A \rightarrow -r_A \).

In the present work, for the sake of simplicity, the NSI parameters are taken to be real \((\epsilon_{\alpha\beta} = \epsilon_{\alpha\beta}^*)\) and also \( \delta = 0 \).

### 4 Neutrino oscillograms in presence of NSI

Within the SM, for a given hierarchy (NH or IH) and best-fit values of the oscillation parameters (as given in Table 1), the oscillation probability depends on only two quantities...
\textbf{Figure 1:} Oscillograms of $P_{\mu\mu}$ for NH and IH with SI alone.

The neutrino energy $E$ and the zenith angle of the direction of the neutrino, namely $\theta$, with the vertically downward direction corresponding to $\theta = 0$. The oscillation pattern can, then, be fully described by contours of equal oscillation probabilities in the $E - \cos \theta$ plane. We use these neutrino oscillograms of Earth to discuss the effect of neutrino–matter interactions on the atmospheric neutrinos passing through the Earth (see Refs. \cite{48, 49} for a more detailed discussion of the general features of the SI oscillograms).

$\nu_\mu \rightarrow \nu_\mu$ disappearance channel:

In Fig. 1 we reproduce the neutrino oscillograms in the $\nu_\mu \rightarrow \nu_\mu$ channel for the case of NH (left panel) and IH (right panel) in the $E\cos(\theta)$ plane. As expected, the muon neutrino disappearance probability experiences matter effects (MSW effects as well as parametric resonances) for the case of NH but not for the case of IH where it is essentially given by the vacuum oscillation probability (which depends on $\theta_{23}, \Delta m^2_{31}$). For SI in the $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ channel, $A \rightarrow -A$ and the plots for NH and IH get interchanged \cite{40}. In vacuum, the positions of the peaks ($P_{\mu\mu} \simeq 1$) and dips ($P_{\mu\mu} \simeq 0$) can be calculated from the first line on RHS in Eq. (18) as

$$ (L/E)^{\text{dip}} \simeq \frac{(2p-1)\pi}{1.27 \times 2 \times \delta m^2_{31}} \text{ km/GeV} ; \quad (L/E)^{\text{peak}} \simeq \frac{k\pi}{1.27 \times \delta m^2_{31}} \text{ km/GeV} \quad (22) $$

where $p, k \in \mathbb{Z}^+$. The first dip and peak, then, are at

$$ (L/E)^{\text{dip}} \simeq 499 \text{ km/GeV} ; \quad (L/E)^{\text{peak}} \simeq 998 \text{ km/GeV} \quad (23) $$

which means that for a given $L$ (say $L = 7000$ km or $\cos \theta = -0.549$), we can predict the values of peak energy $E^{\text{peak}} \sim 7$ GeV and dip energy $E^{\text{dip}} \sim 14$ GeV. This can be seen clearly from the right panel of Fig. 1 which corresponds to the IH as the probability in this case is dominated by vacuum oscillations.

The MSW matter effect can occur both in the mantle region as well as the core \cite{50, 51}.
| $L$ (km) | $\cos \theta$ | $\rho_{\text{avg}}$ (g/cc) | $E_{\text{peak}}$ (GeV) | $E_{\text{dip}}$ (GeV) | $E_R$ (GeV) |
|----------|----------------|---------------------------|-------------------------|-------------------------|-------------|
| 3000     | $-0.235$      | 3.33                      | 3.01                    | 6.01                    | 9.00        |
| 5000     | $-0.392$      | 3.68                      | 5.01                    | 10.02                   | 8.14        |
| 7000     | $-0.549$      | 4.19                      | 7.01                    | 14.03                   | 7.15        |
| 9000     | $-0.706$      | 4.56                      | 9.02                    | 18.04                   | 6.57        |
| 11000    | $-0.863$      | 6.15                      | 11.02                   | 22.04                   | 4.87        |

Table 2: Values of $E_{\text{peak}}$ and $E_{\text{dip}}$ in vacuum and $E_R$ for $P_{\mu\mu}$ as a function of $L, \cos \theta$ (for the choice of integers $p, k$ mentioned in the text).

The energy at which the MSW resonance takes place in the 13 sector is

$$\rho E_R \simeq \frac{\delta m_{31}^2}{0.76 \times 10^{-4}} \times \cos 2\theta_{13} \text{ GeV g/cc}.$$ (24)

Using the values of $\delta m_{31}^2$ and $\theta_{13}$ from Table 1, we get $E_R \sim 7.15$ GeV for $\rho \simeq 4.19$ g/cc which is the average density for a neutrino traversing $\sim 7000$ km through the earth to reach the detector. As the neutrino path nears the core, the energy at which the MSW resonance effects occur decreases (see Table 2). As discussed in Ref. [40], when $E_R$ coincides with $E_{\text{peak}}$ or $E_{\text{dip}}$, one expects a large change in the probability. We see this feature in the left plot of Fig. 1 around $E_R \sim E_{\text{peak}} \simeq 7$ GeV where the probability is reduced from the peak value by almost 40%. (Note that $L = 7000$ km ($\cos \theta \simeq -0.549$) implies that the neutrino has passed only through the crust and the mantle regions, without penetrating the core.). Also the pattern in the left oscillogram changes abruptly at a value of $\cos \theta_{\nu} = -0.84$ demarcating two regions: for $\cos \theta < -0.84$, the neutrinos pass through both mantle and core which allows for parametric effects while for $\cos \theta > -0.84$, the neutrinos cross only the mantle region where only the usual MSW effects operate. On the other hand, the parametric resonance occurs when neutrinos traversing the Earth pass through layers of alternating density (mantle-core-mantle) [48,49].

Having described the case of SI, let us now address the impact of NSI on neutrinos and antineutrinos traversing the Earth. To best illustrate the features, we consider only one NSI parameter to be nonzero. In the leading order expression only two combinations of the three NSI parameters ($\epsilon_{\mu\tau}, \epsilon_{\mu\mu}, \epsilon_{\tau\tau}$) appear. Let us discuss these in turn.

(a) $\epsilon_{\mu\tau} \neq 0; \epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 0$ : In Fig. 2, we show the corresponding $P^{\mu\mu}_N$ for the case of NH (top row) and IH (bottom row) and two specific values of the NSI parameter $\epsilon_{\mu\tau}$ consistent with the current bounds. Note that the case of NH and $\epsilon_{\mu\tau} > 0$ is grossly similar to the case of IH and $\epsilon_{\mu\tau} < 0$ (and, similarly, for NH and $\epsilon_{\mu\tau} < 0$ vs. IH and $\epsilon_{\mu\tau} > 0$). From Eq. (18), we see that there are two terms proportional to $\epsilon_{\mu\tau}$, one where the oscillating function is $\sin \lambda L$ with the other being $\sin^2 \lambda L/2$. Thus, the first term can be positive or

\[\text{Note, though, that neutrinos of such energies but travelling a smaller path through the earth would also hit regions with } \rho \simeq 4.19 \text{ g/cc and, thus, suffer resonant conversion.} \]
negative depending upon the value of the phase, while the second term is always positive. It is the interplay of these two terms that leads to the features in these plots. The mass hierarchy dependence comes from the first term since we have $r_A \lambda L \sin(\lambda L)$ which changes sign when we go from NH to IH. As noted earlier, near the vacuum dip $\lambda L = (2p + 1)\pi/2$, this term will be dominant. Consequently, for NH and $\epsilon_{\mu\tau} > 0$, the oscillatory pattern is a modification of the standard one. For IH and $\epsilon_{\mu\tau} > 0$, the term proportional to $|\epsilon_{\mu\tau}|$ will have a negative overall sign and this leads to washout to a certain extent of the oscillation pattern.

The difference between SI and NSI contributions to the probability $\Delta P_{\mu\mu}$ is shown in Fig. 3. $|\Delta P_{\mu\mu}|$ can be as large as 1 for regions in the core and in mantle for some choice of $\epsilon_{\mu\tau}$ and hierarchy. We also note large changes in probability (the regions where the difference is large $\sim \pm 1$) along the diagonal line.

(b) $\epsilon_{\mu\mu} - \epsilon_{\tau\tau} \neq 0; \epsilon_{\mu\tau} = 0$: This case will correspond to diagonal FP NSI parameter, $\epsilon_{\mu\mu} - \epsilon_{\tau\tau}$. As mentioned above, $\epsilon_{\mu\mu}$ is tightly constrained (see Eq. (5)) and the bound on $\epsilon_{\tau\tau}$ is very loose.

(c) Subdominant effects due to $\epsilon_{e\mu}, \epsilon_{e\tau} \neq 0$: For the case of NH, we compare the cases of non-zero $\epsilon_{\mu\tau}, \epsilon_{e\mu}, \epsilon_{e\tau}$ in Fig. 4. From Eq. (5), we see that the bounds for $\epsilon_{e\mu}$ and $\epsilon_{\mu\tau}$ are similar (0.33) while that on $\epsilon_{e\tau}$ is rather loose (3.0). It is seen that the other parameters involving the electron sector play only a sub-dominant role in this channel. This can also be understood from the fact that, in the expression for $P_{\mu\mu}^{NSI}$ (see Eq. (18)), these terms appear only at the second order [44].

$\nu_e \to \nu_\mu$ appearance channel:
In Fig. 3 we have shown the standard neutrino oscillograms in the $\nu_e \rightarrow \nu_\mu$ channel for the case of NH (left panel) and IH (right panel) in the ($E\cdot\cos\theta$) plane. In this case, the probability is negligible in most parts of the parameter space (especially for the case of IH). The $\nu_e \rightarrow \nu_\mu$ appearance probability in matter differs from that in vacuum in the leading order itself and also the position of peaks and dips of the vacuum curves do not, in general, coincide with those in presence of matter (unlike in the case of muon survival probability). In order to analyse the $P_{\epsilon\mu}$ plots, let us look at the OMSD expression [40] (since our analytic
expression is valid to first order in $\theta_{13}$)

$$P_{e\mu}^{\text{OMSD}} = \sin^2 2\tilde{\theta}_{13} \sin^2 \theta_{23} \sin^2 \frac{\delta m^2_{31} L}{4E}$$

(25)

where

$$\sin 2\tilde{\theta}_{13} = \sin 2\theta_{13} \frac{\delta m^2_{31}}{\delta m^2_{31}}$$

$$\delta m^2_{31} \equiv \sqrt{(\delta m^2_{31} \cos 2\theta_{13} - A)^2 - (\delta m^2_{31} \sin 2\theta_{13})^2}$$

(26)

The peak energy in matter will be given by [40],

$$(L/E)^{\text{peak}} \sim \frac{(2p - 1)\pi}{1.27 \times 2 \times \delta m^2_{31}} \text{ km/GeV}$$

(27)

where $p \in \mathbb{Z}^+$. One would expect $P_{e\mu}$ to be large when the matter peak coincides with the resonance energy, which gives $E_R \simeq 7$ GeV. However, the resonance condition which implies that $\sin 2\tilde{\theta}_{13} = 1$ also leads to $\delta m^2_{31}$ taking its minimum value at resonance energy $\simeq \Delta m^2_{31} \sin 2\theta_{13}$. Hence, the probability becomes large when $\delta m^2_{31} \sin 2\theta_{13} L/4E \geq \pi/4$ is satisfied. This gives a value of $L = 10,200$ km for $\sin^2 2\theta_{13} \simeq 0.1$ [40]. Note that the maximum value of $P_{e\mu}$ is given by the value of $\sin^2 \theta_{23} \simeq 0.5$. The range of $E$ and $\cos \theta$ where $P_{e\mu}$ is close to its maximal value due to MSW effect is given by $E \in [5, 7.5]$ GeV and $\cos \theta \in [-0.87, -0.5]$ in the mantle region. In the core region, the MSW peak will occur at smaller energies and the parametric resonance leads to large changes.

Having described the case of SI, let us now address the impact of NSI on neutrinos and antineutrinos traversing the Earth. In the leading order expression for $P_{e\mu}^{\text{NSI}}$ (see Eq. (17)) there are only two NSI parameters ($\epsilon_{e\mu}, \epsilon_{e\tau}$) that appear whereas $\epsilon_{\mu\tau}$ does not appear at all. We discuss them in turn.
Figure 6: Oscillograms of $\Delta P_{e\mu}$ for NSI parameter $\epsilon_{\mu\tau}$.

(a) Subdominant effects due to $\epsilon_{\mu\tau} \neq 0$: In Fig. 6, we show the effect of $\epsilon_{\mu\tau}$ on the oscillograms of $\Delta P_{e\mu}$. Since the parameter $\epsilon_{\mu\tau}$ does not appear at all in the first order expression (Eq. (17)), naturally its impact is expected to be small. Consequently, $|\Delta P_{e\mu}| \neq 0$ only in very tiny regions and can at best be as large as $0.3 - 0.4$.

(b) Comparison of effects due to $\epsilon_{e\mu} \neq 0$, $\epsilon_{e\tau} \neq 0$ and $\epsilon_{\mu\tau} \neq 0$: In Fig. 7, we compare the effects due to the three NSI parameters for the case of NH, allowing only one of them to be non-zero at a time. Since the parameters $\epsilon_{e\mu}, \epsilon_{e\tau}$ appear in the first order expression (Eq. (17)), they naturally have a larger impact as compared to $\epsilon_{\mu\tau}$ and, in the favourable situation, $|\Delta P_{e\mu}|$ can be as large as 0.5. This is to be contrasted with $|\Delta P_{\mu\mu}|$ which could take values as large as 1 under favourable conditions. Also, if we look at Eq. (17), we note that $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ appear on equal footing as far as $P_{e\mu}^{\text{NSI}}$ is concerned.

**An application : $\theta_{23}$ Octant determination in presence of NSI using the muon disappearance channel**

A close examination of the expression for muon neutrino disappearance probability shows that, in vacuum, the two flavour expression depends on $\sin^2 2\theta_{23}$. In matter (in the OMSD approximation) too, the leading order term is proportional to $\sin^2 2\theta_{23}$ [22]. This leads to octant degeneracy which means that $\theta_{23}$ and $\pi/2 - \theta_{23}$ are indistinguishable. For a given value of $\sin^2 2\theta_{23} = X$, the two degenerate solutions for $\theta_{23}$ are,

$$\theta_{23} = \frac{\sin^{-1} \sqrt{X}}{2} \text{ or } \theta_{23} = \frac{\pi}{2} - \frac{\sin^{-1} \sqrt{X}}{2}$$

(28)

Obviously, for maximal 23 mixing, the two coincide and $\theta_{23} = \pi/4$ uniquely. However for
non-maximal ($X \neq 1$) mixing, there is clearly an ambiguity in determining the true value of $\theta_{23}$, since there exist two degenerate solutions: one in the lower octant (LO) and other in the higher octant (HO) (see Eq. (28)). From Table. 1, we note that there is a hint for the value of $\sin^2 \theta_{23}$ to be non-maximal and the preferred value lies in the HO for both the hierarchies (ignoring that there is a local minima in the LO for the case of NH). The data indicates an average value of $\sin^2 \theta_{23} \sim 0.57$ which leads to $\theta_{23} \simeq 49^\circ$ and the corresponding value of $X = 0.98$. Now for $X = 0.98$, one can have $\theta_{23} \simeq 0.43$ ($41^\circ$) and $\theta_{23} \simeq 0.86$ ($49^\circ$) which correspond to $\sin^2 \theta_{23} \simeq 0.43$ and $\sin^2 \theta_{23} \simeq 0.57$ respectively. If we call the true value as $\sin^2 \theta_{23}\text{(true)}$, and the false value as, $\sin^2 \theta_{23}\text{(false)} = 1 - \sin^2 \theta_{23}\text{(true)}$, it leads to degeneracy as $P_{\alpha\beta}(\sin^2 \theta_{23}\text{(true)}) = P_{\alpha\beta}(1 - \sin^2 \theta_{23}\text{(true)})$.

For atmospheric neutrinos, the resolution of this degeneracy relies on the resonant earth matter effects and using the OMSD formulae (which are valid for a large range of $E$ and $L$), one notes that for neutrinos (anti-neutrinos), there is a term $\propto \sin^4 \theta_{23} \sin^2 2\tilde{\theta}_{13}$ which is sensitive to the octant and can be large near MSW resonance for NH (IH). For neutrinos and IH (anti-neutrinos and NH), the vacuum oscillation formula holds which has the dominant term depending on $\sin^2 \theta_{23}$ as well as a sub-dominant $\theta_{13}$ dependent term that can aid in the resolution of degeneracy (via a combination $\sin^2 \theta_{23} \cos 2\theta_{23}$). It is therefore natural to investigate the influence of non-zero NSI parameter $\epsilon_{\mu\tau}$ on resolution of this degeneracy.

In Fig. 8, we show the $P_{\mu\mu}$ as a function of $\sin^2 2\tilde{\theta}_{13}$ within the allowed range (Table. 1) for $\theta_{23}\text{true} = 49^\circ$ and $\theta_{23}\text{false} = 41^\circ$. The thickening of curves into bands take into account the variation of $\delta_{CP}$ in $[-\pi, \pi]$. The vertical solid line depicts the best-fit value of $\theta_{13}$. We compare the effect of non-zero $\epsilon_{\mu\tau}$ (both signs) for NH and IH respectively. For NH, we see that the curves (red and green) for LO and HO are well separated while for IH the curves for LO and HO overlap in the absence of NSI as expected. If we focus on the vertical line, we note that NSI term can shift the probability from $\sim 0.8$ to a lower value $\sim 0.6 - 0.65$ for NH, LO and both signs of $\epsilon_{\mu\tau}$. And, NSI, LO (and HO) (blue and magenta) curves overlap with SI, HO (green) for $\epsilon_{\mu\tau} > 0$ while for $\epsilon_{\mu\tau} < 0$, the curves for NSI, LO (blue) barely separate out from NSI, HO (magenta) but still SI, HO curves (green) overlap with NSI, LO (blue). Similarly, for the IH, the probability changes from $\sim 1$ to a lower value $\sim 0.7 - 0.8$ but the two octants are indistinguishable both in absence and presence of NSI.

**Figure 7:** The effect of $\epsilon_{e\mu}$, $\epsilon_{e\tau}$, $\epsilon_{\mu\tau}$ on the oscillogram of $\Delta P_{e\mu}$. 
Thus, in presence of NSI, the octant determination using matter effects needs to be done more carefully taking into account additional admissible parameters.

5 Simulating an experiment

5.1 Atmospheric events

The neutrino and anti-neutrino CC events are obtained by folding the incident neutrino fluxes with the appropriate probabilities, relevant CC cross sections, the detector efficiency, resolution, mass and the exposure time.

The $\mu^-$ event rate in a specific energy bin of width $dE$ and the angle bin of width $d\Omega$ can be written as

$$\frac{d^2N_\mu}{d\Omega\,dE} = \frac{1}{2\pi} \left[ \left( \frac{d^2\Phi_\mu}{d\cos\theta\,dE} \right) P_{\mu\mu} + \left( \frac{d^2P_e}{d\cos\theta\,dE} \right) P_{e\mu} \right] \sigma_{CC}(\nu_\mu) D_{\text{eff}}(\mu^-). \quad (29)$$

Here $\Phi_{\mu,e}$ are the atmospheric fluxes ($\nu_\mu$ and $\nu_e$), $\sigma_{CC}$ is the total CC cross section and $D_{\text{eff}}$ is the detector efficiency. Similarly, the $\mu^+$ event rate can be obtained using the anti-neutrino flux, probability and cross section, and the efficiency for $\mu^+$ (nominally, the same as for $\mu^-$).

Analogously, the $e^-$ event rates would be given by...
\[
\frac{d^2N_e}{d\Omega\,dE} = \frac{1}{2\pi} \left[ \left( \frac{d^2\Phi_\mu}{d\cos\theta\,dE} \right) P_{\mu e} + \left( \frac{d^2\Phi_e}{d\cos\theta\,dE} \right) P_{ee} \right] \sigma_{CC}(\nu_e) \, D_{\text{eff}}(e^-),
\]
(30)

with the e\(^+\) event rate being expressed in terms of anti-neutrino fluxes, probabilities and cross sections as well \(D_{\text{eff}}(e^+)\).

In a realistic detector, the energy and angular resolution is not infinite, and to mimic this, we consider a Gaussian resolution function, \(R\). For the energy resolution function, we use

\[
R_{\text{EN}}(E_t, E_m) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(E_m - E_t)^2}{2\sigma^2} \right],
\]
(31)

where \(E_m\) and \(E_t\) denote the measured and true values of energy respectively. The smearing width \(\sigma\) is a function of \(E_t\) itself. The functional form of \(\sigma\) for ICAL and LAr detectors are given in Table. 4 and 5. Similarly, the angular smearing function is given by

\[
R_{\theta}(\Omega_t, \Omega_m) = N \exp \left[ -\frac{(\theta_t - \theta_m)^2 + \sin^2\theta_t (\phi_t - \phi_m)^2}{2(\Delta\theta)^2} \right],
\]
(32)

where \(N\) is a normalisation constant.

The experimentally observable \(\nu_\mu\) event rates would, thus, be given by

\[
\frac{d^2N_\mu}{d\Omega_m\,dE_m} = \frac{1}{2\pi} \int \int dE_t\,d\Omega_t \, R_{\text{EN}}(E_t, E_m) \, R_{\theta}(\Omega_t, \Omega_m) \left[ \Phi^d_{\mu,\mu} P_{\mu\mu} + \Phi^d_{e,\mu} P_{\mu\mu} \right] \sigma_{CC} \, D_{\text{eff}},
\]
(33)

and similarly for the \(\nu_e\). Here we have denoted \((d^2\Phi/d\cos\theta\,dE)_{\mu,e} \equiv \Phi^d_{\mu,e}\) etc. We limit the charged lepton phase space to \(E_\ell \in [1, 10]\) GeV and \(\cos\theta \in [-1.0, -0.1]\) which covers the incident atmospheric neutrinos propagating through the earth. For effecting a statistical analysis, we subdivide the energy (cos \(\theta\)) range into 9 (18) equal bins each.

It is worth noting at this stage that even if we incorporate the full detector simulation for detectors such as the Iron Calorimeter (ICAL) at INO or a generic Lithium Argon (LAr) one (such as in Ref. \[52\]), the essential physics points of the present work would not change. Since these studies are not yet available for full-fledged reconstruction of neutrino energy and angle using muons and hadrons, we adopt a simpler approach as mentioned above.

### 5.2 \(\chi^2\) analysis

We quantify the difference between the events with SI and NSI in terms of a \(\chi^2\) function. For a fixed set of parameters, the latter is calculated using the method of pulls, which allows us to take into account the various statistical and systematic uncertainties (such as those on the fluxes, cross sections etc.).

Let \(N^{\text{th}}_{ij}\) (std) be the theoretical event rate for the i-j\(^{th}\) bin, as calculated with the standard values for the inputs. Now, let us allow the k\(^{th}\) input (known with an uncertainty \(\sigma_k\)) to deviate from its standard value by an amount \(\sigma_k \xi_k\). If the relative uncertainties are not very large, the change in \(N^{\text{th}}_{ij}\) can be expressed as a linear function of the pull variables \(\xi_k\). In other words, the value of \(N^{\text{th}}_{ij}\) with the changed inputs is given by

\[
N^{\text{th}}_{ij} = N^{\text{th}}_{ij}\,(\text{std}) + \sum_{k=1}^{n_{\text{pull}}} c^k_{ij} \xi_k,
\]
(34)
where npull is the number of sources of uncertainty, which in our case is 5. The systematic uncertainties are given in Table 3. With these changed inputs, the goodness of fit is quantified in terms of a modified $\chi^2$ function defined as

$$
\chi^2(\xi_k) = \sum_{i,j} \left[ \frac{N_{ij}^{\text{th}}(\text{std})}{N_{ij}^{\text{ex}}} + \sum_{k=1}^{\text{npull}} c_{ij}^k \xi_k - N_{ij}^{\text{ex}} \right]^2 + \sum_{k=1}^{\text{npull}} \xi_k^2 \tag{35}
$$

where the additional term $\xi_k^2$ is the penalty imposed for moving the $k^{\text{th}}$ input away from its standard value by $\sigma_k \xi_k$. The $\chi^2$ with pulls, which includes the effects of all theoretical and systematic uncertainties, is obtained by minimizing $\chi^2(\xi_k)$, given in Eq. (35), with respect to all the pulls $\xi_k$, viz.

$$
\chi^2_{\text{pull}} = \text{Min}_{\xi_k} \left[ \chi^2(\xi_k) \right]. \tag{36}
$$

Note that ICAL magnetised detector will be able to distinguish muon neutrinos and muon anti-neutrinos and hence the effective $\chi^2$ is given by $\chi_{\mu-}^2 + \chi_{\mu+}^2$. On the other hand, the $\chi^2$ for the (unmagnetised) LIAR detector is $\chi^2 = \chi_{\mu-+\mu+}^2 + \chi_{e-e+e+}^2$. Finally, we marginalize the $\chi^2$ over the allowed range of the oscillation parameters as mentioned in Table 1.

### 6 Event spectrum for two detector types

We describe the details used for the two detector types (ICAL and Liquid Argon) used in our analysis:

**ICAL detector**

This is a large magnetised iron detector and is being planned for the INO experiment in South India. It consists of 151 layers of magnetized iron plates interleaved with Resistive Plate Chambers (RPC) as active detector elements with a total mass of about 52 kilotons. Such a detector is capable of detecting muons (especially for GeV energies) and identify their charge by virtue of the magnetization. Additionally, the ICAL can detect hadronic showers. The energy and angular resolution of muons and hadrons for the ICAL have been obtained from the INO simulation code and using that information the initial neutrino energy and angle can be reconstructed. The detailed specifications are given in Table 4.
Energy Resolution ($\sigma(E)$) | $0.1\sqrt{E}$
---|---
Angular Resolution ($\Delta\theta$) | $10^\circ$
Detector efficiency ($\mathcal{E}$) | 85%

**Table 4:** ICAL Detector parameters in the atmospheric neutrino experiment simulation [22]

![N\(_\mu\)(500 kt-yr, ICAL), SI, NH](image-a)

![N\(_\mu\)(500 kt-yr, ICAL), $\epsilon_{\mu\tau}=0.15$, NH](image-b)

**Figure 9:** $\nu_\mu$ events with SI and with NSI for non-zero $\epsilon_{\mu\tau}$ (left). All the data are generated for 500 kT-yr of exposure for magnetized ICAL assuming NH as the true hierarchy.

In Fig. 9 the $\nu_\mu$ events are shown. At low energies, the number of events is around $\sim 100$ for all the zenith angles both for the case of SI and NSI ($\epsilon_{\mu\tau} \neq 0$). The difference with and without NSI of the $\nu_\mu$ events using parameters $\epsilon_{\mu\tau}$ and $\epsilon_{\mu e}$ is shown in Fig. 10. For ICAL, it is evident that $\Delta N_\mu \simeq \pm 10$ in some of the bins for $\epsilon_{\mu\tau} \neq 0$ while for $\epsilon_{\mu e} \neq 0$, $\Delta N_\mu \sim \pm 4$. This was expected since the leading dependence was through $\epsilon_{\mu\tau}$, corroborates the probability level analysis.

**LAr detector**

A LAr detector is capable of detecting not only muons but also electrons, and has a very good angular resolution. Since the detector is unmagnetised, only the total events of a given flavour can be measured. For the proposed Long Baseline Neutrino Experiment (which is designated to operate with a beam and a baseline of 1300 km), a 35 kt unmagnetized LAr detector is to be placed underground to study atmospheric neutrinos along with the beam. The specifications for the LAr detector are given in Table 5. We shall assume here a 10-yr operation period, or, equivalently, an effective fiducial volume of 350 kt-yr.

The difference of the total muon ($\nu_\mu + \bar{\nu}_\mu$) and electron ($\nu_e + \bar{\nu}_e$) neutrino events with and without NSI are shown in Fig. 11. LAr detector is complementary to ICAL as the impact of $\epsilon_{\mu\tau}$ is less compared to $\epsilon_{\mu e}$ for both muon and electron flavours.
$\Delta N_\mu, \epsilon_{\mu\tau} = 0.15$ 

(a) 

$\Delta N_\mu, \epsilon_{\mu\mu} = 0.15$ 

(b) 

Figure 10: The difference with and without NSI of $\nu_\mu$ (only) events for non-zero $\epsilon_{\mu\tau}$ (left) and $\epsilon_{\mu e}$ (right). All the data are generated for 500 kT-yr of exposure for magnetized ICAL assuming NH as the true hierarchy.

| Parameter                  | Value            |
|----------------------------|------------------|
| Rapidity ($y$)             | 0.45 for $\nu$   |
|                            | 0.30 for $\bar{\nu}$ |
| Energy Resolution ($\sigma(E)$) | $\sqrt{(0.01)^2 + (0.15)^2/(yE) + (0.03)^2}$ |
| Angular Resolution ($\Delta\theta$) | 3.2$^\circ$ for $\nu_\mu$ |
|                            | 2.8$^\circ$ for $\nu_e$ |
| Detector efficiency ($\mathcal{E}$) | 85% |

Table 5: The LAr detector parameters used for the atmospheric neutrino experiment simulation [53][54].

7 Conclusion and summary of the constraints on $\epsilon_{\mu\tau}, \epsilon_{\mu e}, \epsilon_{e\tau}$

We have obtained constraints on some of the NSI parameters that enter the neutrino oscillation formalism namely, $\epsilon_{\mu\tau}, \epsilon_{\mu e}$ and $\epsilon_{e\tau}$. As the constraint on $\epsilon_{\mu\mu}$ is already very stringent, it is not expected to improve much. We use two detector types, viz. a magnetised iron one (the specifications being those for the proposed ICAL) and a generic unmagnetized LAr detector and contrast the capabilities of these two detector types for individual parameters.

We show the variation of $\Delta \chi^2$ for ICAL and LAr detectors as a function of $\epsilon_{\mu\tau}, \epsilon_{\mu e}$ and $\epsilon_{e\tau}$ in Fig. [12]. The horizontal black dotted line represents the 3$\sigma$ CL. We discuss, in turn, the case for each parameter holding the others to be zero. As far as $\epsilon_{\mu\tau}$ is concerned, the ICAL detector offers better performance in comparison to LAr detector for both NH and IH (see Figs. [12a] and [12b]). This is primarily due to the fact that the ICAL detector is magnetized (whereas the LAr detector is not), allowing it to distinguish between $\mu^\pm$. Since $\Delta N_{\mu^{-}}$ and $\Delta N_{\mu^{+}}$ behave differently, this separability adds to the $\chi^2$. And even though the LAr detector can detect $\nu_e$ and $\bar{\nu}_e$ events, this advantage cannot offset the lack of charge identification, primarily because $\Delta N_{e^{\pm}}$ is typically less than $\Delta N_{\mu^{\pm}}$ (see Fig. [11]) as $\mu \rightarrow e$.
Δ (N_μ + N_\bar{\mu}) , \varepsilon_{\mu\tau} = 0.15

\begin{align*}
\text{Cos}(\theta) \\
-1 & \quad 2 \\
E(\text{GeV}) & \quad 2 & \quad 4 & \quad 6 & \quad 8
\end{align*}

(a)

Δ (N_e + N_{\bar{\nu}}) , \varepsilon_{\mu\tau} = 0.15

\begin{align*}
\text{Cos}(\theta) \\
-1 & \quad 2 \\
E(\text{GeV}) & \quad 2 & \quad 4 & \quad 6 & \quad 8
\end{align*}

(b)

\begin{align*}
\text{Delta}(N_\mu + N_\bar{\nu}), \varepsilon_{\mu\tau} = 0.15
\end{align*}

\begin{align*}
\text{Delta}(N_e + N_\bar{\nu}), \varepsilon_{\mu\tau} = 0.15
\end{align*}

(c)

\begin{align*}
\text{Delta}(N_e + N_\bar{\nu}), \varepsilon_{\mu\tau} = 0.15
\end{align*}

(d)

**Figure 11:** The difference with and without NSI of the total muon neutrino events (ν_μ + \bar{\nu}_\mu) and total electron neutrino events (ν_e + \bar{\nu}_e). All the data are generated for 350 kT-yr of exposure for unmagnetized LAr detector assuming NH as the true hierarchy.

Conversion rate is small, while the contribution from ν_e survival is small on account of the smallness of the latter’s flux.

As far as \varepsilon_{e\tau} and \varepsilon_{\mu e} are concerned, the LAr detector is expected to be better than the ICAL detector (see Figs. 12c, 12d, 12e and 12f). In order to get precise information on different NSI parameters, the detectors with complementary properties are required.

For the case of IH, the general feature is that ICAL detector has better performance than LAr detector for all the three NSI parameters (Figs. 12b, 12d, 12f). This is primarily due to the fact that the ICAL detector is magnetized, while LAr detector is not. We can see for \varepsilon_{\mu\tau}. If we examine the plots of ΔP_μν and ΔP_{\mu e} (Figs. 3 and 6), we note that there are larger regions in E − cosθ parameter space in case of IH than NH for ΔP_μν. It can be noted that the opposite is seen in case of ΔP_{\mu e} - the regions with large change in probability actually shrink for IH compared to NH. But, since the contribution from P_{\mu e} to N_\mu is suppressed...
Figure 12: $\Delta \chi^2$ vs $\epsilon_{\mu\tau}$, $\epsilon_{e\tau}$ and $\epsilon_{\mu e}$ for NH (left) and IH (right) for the two detector types.

by the electron to muon flux ratio for the atmospheric neutrinos and also the maximum possible change is $\sim \pm 0.5$ (which is much smaller than $\sim \pm 1$ for $\Delta P_{\mu\mu}$), this does not nullify the large changes induced due to $P_{\mu\mu}$. 
These observations are also reflected in Fig. 14 where the constraints on pairs of NSI parameters, $\epsilon_{\mu\tau} - \epsilon_{\mu e}$, $\epsilon_{e\tau} - \epsilon_{\mu e}$ and $\epsilon_{e\tau} - \epsilon_{\mu\tau}$ are shown. The allowed values of pairs of NSI parameters imply that we can demarcate between SI and NSI for those values at a given confidence level. The green solid line corresponds to 90% CL while the dashed red line corresponds to 95% CL.

As an application of our study, Fig. 13 shows how NSI impacts the sensitivity of the ICAL detector for octant determination for the case of NH. The ICAL fiducial volume is taken to be 500 kt-yr. The true value of $\theta_{23}$ is taken to be $\sin^2 \theta_{23} = 0.567$ (Table. 1) and the test value of $\theta_{23}$ is varied in the range 35 – 54 degrees. The black curve is with SI only while the other two curves are with NSI for the opposite signs of $\epsilon_{\mu\tau}$. The octant degeneracy is hard to resolve if we have only SI at 3$\sigma$ confidence level. But upon incorporating NSI terms, the sensitivity increases. As predicted from the probability plot, the effect is stronger for the $\epsilon_{\mu\tau} < 0$ regime. A detailed analysis of the octant determination in presence of NSI is under progress [24].

**Figure 13**: $\chi^2$ plot versus the test value of $\theta_{23}$ to determine the octant using ICAL detector.

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Figure 14: Constraints on pairs of NSI parameters for different detector types. Left panel is for 500 kt-yr of fiducial volume of ICAL and right panel with 350 kt-yr unmagnetized LAr detector. NH is assumed to be true hierarchy.

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