Charged rotating black holes as a brane-antibrane system

A. Güijosa\textsuperscript{1}, H. Hernandez\textsuperscript{21} and H. Morales-Técotl\textsuperscript{22}

\textsuperscript{1} Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A.P. 70-543, México D.F. 04510, México
\textsuperscript{2} Departamento de Física, Universidad Autónoma Metropolitana Iztapalapa, A.P. 55-534, México D.F. 09340, México

E-mail: alberto@nucleares.unam.mx, hehe@aei.mpg.de, hugo@xanum.uam.mx

Abstract. In this work we show how, based on a previously proposed brane-antibrane system at finite temperature, we can compute microscopically the Bekenstein-Hawking entropy for a non-extremal seven dimensional rotating and charged black hole, the analogous of a Kerr-Newman black hole. The macroscopic and microscopic entropies match up to a numerical factor close to unity. This talk is based on the paper [1].

1. Introduction
Finding a statistical mechanics interpretation of the Bekenstein-Hawking black hole entropy has been one of the biggest challenges in theoretical physics, to the point of considering this goal as one of the basic features that any theory of Quantum Gravity should be able to describe. Recently the two main approaches to quantum gravity, string theory and loop quantum gravity, have obtained some results that may turn to be the leading point in the establishment of such a theory (the original references are [2, 3]; for reviews, see, e.g., [4, 5]).

The main idea in string theory is to consider higher dimensional analogues of a black hole, the so called black branes. In this spirit, most of the string-theoretic analyses (see for example [2]) have concentrated on black branes that are either extremal or near-extremal. The far-from-extremal regime, which includes in particular the neutral (Schwarzschild) black hole, has proven to be more challenging. There have been, though, some other approaches and several proposals to try to obtain a similar result for those black holes (for a list of references see [1]).

In searching to tackle this problem some time ago it was shown that the entropy of a certain class of extended charged black holes (the threebrane of Type IIB supergravity and the twobrane and fivebrane of eleven-dimensional supergravity) arbitrarily far from extremality (including in particular the Schwarzschild black hole in seven, nine, and six dimensions) can be reproduced microscopically via a model based on branes and antibranes [6, 7]. In the work on which this talk is based [1] we used the above mentioned brane-antibrane model of [6] to describe the rotating charged black threebrane that corresponds to the seven-dimensional Kerr-Newman black hole.

\textsuperscript{1} Present address: Max-Planck-Institut für Gravitationsphysik, Albert Einstein Institut, Am Mühlenberg 1, 14476 Golm, Germany.
\textsuperscript{2} Associate member of ICTP, Trieste, Italy.
We divide this paper as follows: section 2 contains our calculations, starting in 2.1 with a summary of the model of [6] and an analysis of the neutral rotating threebrane, and ending in 2.2 with the generalization to the case with arbitrary charge. Our conclusions are presented in Section 3.

2. Entropy Determination

We review first the brane-antibrane computation obtained in [6] for the entropy and the corresponding matching with the macroscopic Bekenstein-Hawking one.

The goal is to obtain a microscopical counting for the Bekenstein-Hawking entropy of a seven dimensional black hole, hence we describe this black hole macroscopically. As is well known a class of higher dimensional black holes can be obtained via some configuration and intersection of several branes [8].

Particularly in [9] (see also [10]) a type of seven dimensional black hole was constructed, the rotating black threebrane solution of Type IIB supergravity. We take this solution as our starting point with all but one of the rotation parameters set to zero. The metric takes the form

\[
\begin{align*}
    ds^2 &= \frac{1}{\sqrt{f}}(-h dt^2 + d\vec{x}^2) + \sqrt{f} \left[ \frac{dr^2}{h} - \frac{2lr_0^4 \cosh \alpha}{r^4 \Delta f} \sin^2 \theta dt d\phi \\
    &\quad + r^2 (\Delta d\theta^2 + \tilde{\Delta} \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega_3^2) \right],
\end{align*}
\]

where the variables describe the event horizon and the angular momentum.

The macroscopic parameters of mass, angular momentum, charge and the Bekenstein-Hawking entropy for this black hole are

\[
\begin{align*}
    m_{SG} &= \frac{M_{SG}}{V} = \frac{\pi^3}{\kappa^2 r_0^4} \left( \cosh 2\alpha + \frac{3}{2} \right), \\
    j_{SG} &= \frac{J_{SG}}{V} = \frac{\pi^3}{\kappa^2} r_0^4 \cosh \alpha, \\
    Q_{SG} &= \frac{\pi^5/2}{\kappa} r_0^4 \sinh 2\alpha, \\
    s_{SG} &= \frac{A_H/AG_N}{V} = \frac{2\pi^2}{\kappa^2 r_0^4} \sqrt{\frac{1}{4} - l^2/4 + l^2/2 \cosh \alpha}.
\end{align*}
\]

2.1. Neutral case

The higher dimensional Kerr black hole corresponds to the neutral case above, which, as can be seen from (3) corresponds to set\(^4\) \(\alpha = 0\). The entropy is in this case

\[
    s_{SG} = \sqrt{\frac{2^{9/2} \pi^2 m_{SG}^2}{\kappa^2 - 4} + 4\pi^4 j_{SG}^4 - 2\pi^2 j_{SG}^2}.
\]

As was discussed in [6], the microscopic model for this black brane corresponds to a system of \(D3 - \bar{D3}\)-branes\(^5\) (see figure 2.1).

\(^3\) The subscript \(SG\) refers to the macroscopic part, in contrast with the microscopic \(FT\) one.

\(^4\) \(\alpha\) corresponds to a boost parameter, necessary for constructing the rotating brane ([8]).

\(^5\) Recall that the computation of the entropy for, lets say; the extremal black hole constructed in [2] was possible because of the identification of the microscopical degrees of freedom of such system with the degrees of freedom of open strings ending on the several \(D\)-branes, building blocks of the macroscopic black hole. The entropy comes from counting the degeneracy in distributing the energy available between the several modes of the strings (considered as a gas). In the present model something similar happens.
The difference with respect of the extremal or near-extremal case is that the gas of strings ending in different types of branes ($D \rightarrow \bar{D}$) is unstable: as the branes have opposite charge they tend to annihilate. In order to stabilize such system it was proposed in [6] to add temperature (in other words, they condensate the tachyon degrees of freedom present.)

We employ the same model of $D3 - \bar{D}3$ branes at finite temperature of [6] to compute the degeneracy of states arising from the energy partition among the gas of open strings ending on different types of branes ($N$ of each of them). The total mass density of the system is

$$m_{FT} = 2N\tau_3 + e,$$

with $\tau_3 = \sqrt{\pi/\kappa}$ the D3-brane tension and $e$ the total energy available. This microscopic model for the gas is strongly coupled, so we use the AdS/CFT correspondence [11],

$$s_{FT} = 2^{9/4}3^{-3/4}\pi^{1/2}(e/2)^{3/4}\sqrt{N},$$

Equation (6) is then used in (7) to eliminate $e$ in favor of $N$, and the value of $N$ (the number of $D\bar{D}$ pairs) determined by maximizing $s_{FT}$ at fixed $m_{FT}$. This leads to

$$N = \frac{m_{FT}}{5\tau_3} \implies e = \frac{3}{5}m_{FT}. \quad (8)$$

Using (8) and (6) in (7), it was found in [6] that the entropy of the model exactly coincides with that of the supergravity solution except for a factor of $2^{3/4}$. It was also found that this same model could equally well reproduce the entropy of the charged threebrane (where $N \neq \bar{N}$).

Our case under consideration was to test the model with angular momentum $j$. The entropy of the gases for a given energy and charge is [9, 10]

$$s(e,j,N) = \frac{2^{5/4}3^{-3/4}\sqrt{\pi Ne}^{3/4}}{\sqrt{1 + \chi + \sqrt{\chi}}}, \quad \chi = \frac{27\pi^2j^4}{8N^2e^3}. \quad (9)$$

Here we have two gases, one on the D3-branes and the other on the anti-D3-branes. As for the neutral case we have $N = \bar{N}$, we expect $j = j_{FT}/2$. Thus

$$s_{FT} = \frac{2^{9/4}3^{-3/4}\sqrt{\pi Ne}^{3/4}}{\sqrt{1 + \chi + \sqrt{\chi}}}, \quad \chi = \frac{27\pi^2j_{FT}^4}{2^7N^2e^3}. \quad (10)$$

When we maximize (10) with respect to $N$ we found that the equilibrium value of $N$ is again (8). So, with (10) and (8) we obtain a microscopic entropy

$$s_{FT} = s_{SG} \quad (11).$$
2.2. Charged case

With a slight modification we can consider the charged case, which we now analyze.

In the model we now have

\[ Q_{FT} = N - \bar{N} \neq 0, \]  

(12)

From the experience gained from the neutral case we expect that the equilibrium values of \( N, \bar{N} \) will again coincide with those found in [6] for the non-rotating system. So we postulate

\[ N = \frac{\pi^{5/2}}{2\kappa} r_0^4 e^{2\alpha}, \quad \bar{N} = \frac{\pi^{5/2}}{2\kappa} r_0^4 e^{-2\alpha}. \]  

(13)

The final ingredient is the rotation \( j_{FT} = j_{SG} \) that we set:

\[ j_{FT} = j + \bar{j}. \]  

(14)

From (2) it is natural to propose that

\[ j = \frac{\pi^3}{2\kappa^3} lr_0^4 e^\alpha, \quad \bar{j} = \frac{\pi^3}{2\kappa^3} lr_0^4 e^{-\alpha}. \]  

(15)

The entropy is just the sum of both entropies, one for each gas

\[ s_{FT} = \frac{2^{5/4} 3^{-3/4} \sqrt{\pi} N \kappa^{3/4}}{\sqrt{1 + \chi + \sqrt{\chi}}} + \frac{2^{5/4} 3^{-3/4} \sqrt{\pi} \bar{N} \kappa^{3/4}}{\sqrt{1 + \bar{\chi} + \sqrt{\bar{\chi}}}}, \]  

(16)

which, after some simplification, leads to

\[ s_{FT} = 2\pi^4 l^{2} r_0^6 \cosh \alpha \sqrt{r_0^4 + l^4/4 + l^2/2}. \]  

(17)

By inspecting this last expression it is not difficult to see that, for the charged and rotating case we have

\[ s_{FT} = s_{SG}. \]  

(18)

### 3. Conclusions

In the present work we extended the study made in [6] of a system of branes and antibranes at finite temperature to describe microscopically a rotating and charged threebrane, particularly in the calculation of its entropy. This threebrane correspond to a type of seven dimensional Kerr black hole (in the neutral case) and Kerr-Newman black hole (charged).

The key ingredient was the addition of temperature to the unstable system of branes and antibranes, providing a stable configuration which accurately reproduces the Bekenstein-Hawking entropy of the black hole associated. However, there exists a numerical discrepancy between the two: a factor of \( 2^{3/4} \).

So, an immediate extension for this paper would be to try to provide a reasonably explanation of this factor, although following the original paper [1] have been some proposals to try to account for the origin of this discrepancy (see for example [14], where a parallel work was done, or [15] where a possible explanation for this has been given).

Nevertheless the importance of the present work is that it provides a microscopical model where one can obtain statistical-mechanical quantities that provide foundation to the long standing issue of the Bekenstein-Hawking entropy and the formulation of the black hole’s laws. At the very least, then, the agreement found here and in [6] is a rather non-trivial property of the supergravity formulae. Moreover, this property appears to have some degree of universality, for it applies not only to the (rotating or not) Type IIB threebrane, but also to the twobrane and fivebrane of eleven-dimensional supergravity [6], and to other systems as well [14].

Other extensions of the present work, such as computing quantities other than the entropy, have been obtained in recen literature [16].
Acknowledgments
This work was partially supported by Mexico’s National Council of Science and Technology (CONACyT), under grants CONACyT-U-40745-F and CONACyT-NSF E-120-0837. The work of H.H. was supported by CONACyT Scholarship number 113375, who also acknowledges the DAAD-scholarship A/04/21572 and the support of the Max Planck Institute for Gravitational Physics in Golm, where this article was written.

References
[1] Güijosa A Hernández H and Morales-Téoctol H 2004 The entropy of the rotating charged black three-brane from a brane-antibrane system Jour. High En. Phys 0403 069 (Preprint hep-th/0402158).
[2] Strominger A and Vafa C 1996 Microscopic origin of the Bekenstein-Hawking entropy Phys. Lett. B 379 99, (Preprint hep-th/9601029).
[3] Ashtekar A, Baez J, Corichi A and Krasnov K 1998 Quantum geometry and black hole entropy Phys. Rev. Lett. 80, 904 (Preprint gr-qc/9710007).
[4] Das S and Mathur S 2000 The Quantum Physics Of Black Holes: Results From String Theory Ann. Rev. Nucl. Part. Sci. 50 153 (Preprint gr-qc/0105063).
[5] Ashtekar A 2000 Classical and Quantum Physics Of Isolated Horizons: A Brief Overview Lect. Notes Phys. 541 50 (Preprint gr-qc/9910101).
[6] Danielsson U Güijosa A and Kruczenski M 2001 Brane-Antibrane systems at finite temperature and the entropy of black holes Jour. High En. Phys 0109 011 (Preprint hep-th/0106201).
[7] Danielsson U Güijosa A and Kruczenski M 2003 Black brane entropy from brane-antibrane systems Rev. Mex. Fís. 49S2 61 (Preprint hep-th/0204010).
[8] Peet A TASI lectures on black holes in string theory 2000 Preprint hep-th/0008241.
[9] Gubser S 1999 Thermodynamics of spinning D3-branes Nucl. Phys. B 551 667 (Preprint hep-th/9810225).
[10] Kraus P Larsen F and Trivedi S 1999 The Coulomb branch of gauge theory from rotating branes Jour. High En. Phys. 9903 003 (Preprint hep-th/9811120).
[11] Maldacena J 1997 The large N limit of superconformal field theories and supergravity Adv. Theor. Math. Phys. 2 231-252 (Preprint hep-th/9712200).
[12] Gubser S Klebanov I and Peet A 1996 Entropy and Temperature of Black 3-Branes Phys. Rev. D 54 3915 (Preprint hep-th/9602135).
[13] Gubser S Klebanov I and Polyakov A 1998 Gauge theory correlators from non-critical string theory Phys. Lett. B 428 105 (Preprint hep-th/9802109);
Gubser S Klebanov I and Tseytlin A 1998 Coupling constant dependence in the thermodynamics of N = 4 supersymmetric Yang-Mills theory Nucl. Phys. B 534 202 (Preprint hep-th/9805156).
[14] Peet A and Saremi O 2004 Brane-antibrane systems and the thermal life of neutral black holes Phys. Rev. D 70 26008 (Preprint hep-th/0403170).
[15] Siwach S 2005 Note on brane-antibrane description of non-extremal black holes Preprint hep-th/0503164.
[16] Garcia J and Güijosa A 2004 Threebrane absorption and emission from a brane-antibrane system Jour. High En. Phys. 0409 027 (Preprint hep-th/0407075);
Lifschytz G 2004 Black hole thermalization rate from brane-antibrane model Jour. High En. Phys. 0408 059 (Preprint hep-th/0406203);
Kalyana S 2004 A description of multi charged black holes in terms of branes and antibranes Phys. Lett. B 596 221-228 (Preprint hep-th/0405084);
Bergman O 2004 Schwarzschild black branes from unstable D-branes Jour. High En. Phys. 0404 060 (Preprint hep-th/0403189).