Enhancing convolutional neural network generalizability via low-rank weight approximation

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Abstract

Noise is ubiquitous during image acquisition. Sufficient denoising is often an important first step for image processing. In recent decades, deep neural networks (DNNs) have been widely used for image denoising. Most DNN-based image denoising methods require a large-scale dataset or focus on supervised settings, in which single/pairs of clean images or a set of noisy images are required. This poses a significant burden on the image acquisition process. Moreover, denoisers trained on datasets of limited scale may incur over-fitting. To mitigate these issues, we introduce a new self-supervised framework for image denoising based on the Tucker low-rank tensor approximation. With the proposed design, we are able to characterize our denoiser with fewer parameters and train it based on a single image, which considerably improves the model’s generalizability and reduces the cost of data acquisition. Extensive experiments on both synthetic and real-world noisy images have been conducted. Empirical results show that our proposed method outperforms existing non-learning-based methods (e.g., low-pass filter, non-local mean), single-image unsupervised denoisers (e.g., DIP, NN+BM3D) evaluated on both in-sample and out-sample datasets. The proposed method even achieves comparable performances with some supervised methods (e.g., DnCNN).

Keywords: Self-supervised learning; low-rank approximation ; out-sample performance.

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1 Introduction

Noise in images, attributed to various factors such as noise corruption (Zhang and Zuo 2017) and resolution limit (Buban et al. 2010), imposes great challenges in image processing (Buades et al. 2005b). The removal of noise, i.e., denoising, is often a crucial step in advance of further tasks, e.g., image segmentation, recognition, and classification. Among many image denoising methods in the literature (see, e.g., Fan et al. (2019), for a survey), deep learning frameworks, especially the convolutional neural networks (CNNs), stand out as a prominent approach. However, most of these frameworks focus on supervised settings, in which single/pairs clean images, e.g., (Zhang et al., 2017; 2018) or a set of noisy images, e.g., (Lehtinen et al., 2018; Krull et al., 2019) are required for learning the denoising mapping $f_\theta(\cdot)$. But collecting a large number of useful images poses a burden to time and budgets, and sometimes such scrutinized images might not even exist in practice. In addition, leveraging millions of image datasets almost exclusively requires Graphics Processing Units (GPUs), which presents another challenge to run on conventional desktops or specialized computer hardware.

Recently, various denoising methods were proposed that only take single images of interest as input to the deep learning process. Ulyanov et al. (2018) proposed a single-image deep learning model for image recovery using image priors. Soltanayev and Chun (2018) proposed to train the denoisers on single noisy images using Stein’s unbiased risk estimator to deal with Gaussian additive noise, with extensions in Cha and Moon (2019); Zhussip et al. (2019). However, these methods require that the noise levels be known a priori. Wu et al. (2020) introduced the dilated blind-spot network to explicitly estimate the noise levels from unpaired noisy images for better performance. Quan et al. (2020) developed a Self2Self dropout scheme for single-image unsupervised learning, which achieves much bet-
ter denoising performance than the existing self-similarity non-local methods. Zheng et al. (2020) proposed another competitive single-image denoiser that can handle more complex noise distributions by fusing deep learning-based methods with traditional non-learning-based Gaussian denoisers (e.g., BM3D, NLM) within the plug-and-play framework. In theory, the convergence of exploiting plug-and-play priors for fusing two bounded denoisers has been established in Shi et al. (2023); Shi and Liu (2023). More recently, Huang et al. (2021) trained image denoisers using single noisy images by generating training pairs with a random neighbor sub-sampler. This approach avoids heavy dependence on assumptions about the noise distribution. However, those single-image convolutional neural networks often lack the ability for generalization. Since our primary interest is in self-supervised learning methods, it is unlikely they will be exposed to sufficient noise types when implemented in practice. Such iterative algorithms tend to be trapped in a sharp loss surface, which bears the risk of producing a highly unstable prediction. As it is needed, more generalizability of neural networks should be considered.

A number of recent works have investigated the generalizability of deep neural networks and the potential methods for improving it. Apart from image denoising, there have been many works focusing on alleviating over-parameterization in order to improve the generalizability of the neural networks, such as parameter pruning (Han et al., 2015b), weights sharing (Chen et al., 2015b), weights tensorization (Huang et al., 2017), weights binarization (Hubara et al., 2016), knowledge distillation (Hinton et al., 2015), etc. Moving forward, some researchers applied the mentioned methods together to achieve a simpler model representation (Han et al., 2015a; Kozyrski and Phan, 2020). Nonetheless, most of them are subject to model accuracy decrease when a simplified model structure is deployed, even if a fine-tuning process is included afterward. Thus, it still remains a challenging task
to remove the redundant parameters while maintaining model performance, especially in
few-shot or one-shot learning tasks (e.g., single-image denoising).

Meanwhile, tensor decomposition provides an insightful perspective for investigating
the generalizability of neural networks. One recent work of Arora et al. (2018) character-
ized the generalizability of a neural network based on the canonical polyadic decomposition
(CPD) (see Kolda and Bader (2009) for a survey on tensor decomposition), and achieved a
provable generalization error bound. Hereafter, the low-rankness of weight kernels has been
considered as a critical measuring tool when the number of parameters, i.e., generalization
ability, is of interest. Most current applications in the image restoration paradigms are
based on the higher-order tensor decomposition techniques (e.g., CP decomposition (Wu
et al. 2018), tensor train decomposition (Novikov et al. 2015; Phan et al. 2020), tensor
SVD (Zhang et al. 2020)) to exploit the nature of latent low-rankness structures. Although
exact tensor factorization typically tends to be computationally intractable, the optimal so-
lution can be obtained via a “warm-start” based on a proper tensor matricization (Richard
and Montanari, 2014). However, such vanilla tensor decompositions, even paired with a
“warm-start” can be numerically unstable and may not converge to the desired optimum
in the end (Rabanser et al. 2017). In addition, it is worth noting that some weight kernels
among the practical deep learning networks (e.g., the well-trained VGG-16 and WRN-28-
10) do not necessarily admit the low-rank CP representation as demonstrated by Li et al.
(2020).

**Main contributions** To overcome the above limitations, this paper proposes a new
algorithm that can boost the model performance with the compressed parameters. We focus
on low-rank approximation (Lebedev et al. 2014; Kim et al. 2016), primarily for its clean
format and good interpretability. Owing to the nature that the weight kernels in the CNN are usually in high-order (i.e., 4D tensors to be explained later), we specifically resort to Tucker decomposition \cite{Tucker1966} to exploit the low rankness with the aid of variational Bayesian matrix factorization (VBMF) for rank selection. The Tucker decomposition is particularly desirable in our denoising framework since it allows low-rank approximation on a part of tensor modes. Such tensor decomposition has also been exploited to realize the CNN compression schemes in other scenarios \cite{Kimetal2016,Nakajimaetal2011,Bulatetal2019} and the references therein. In addition, we adopt the newly proposed Tucker decomposition-based weight distortion scheme \cite{Leeetal2019} within our single-image denoising context, in which the original model structure is retained and only the weight values are twisted by low-rank approximation after certain times of iterations. Then, such a weight distortion method is incorporated to the similar alternating direction method of multipliers (ADMM) framework as \cite{Zhengetal2020} to increase its generalization ability. The main contributions of this paper are summarized as follows:

1. We apply the low-rank tensor decomposition to adjust the direction of ADMM to enable denoising on single images and reduce the computational complexity. By applying the Tucker low-rank approximation to the weight tensors at each twist stage, we can alleviate stochastic variance induced by the stochastic gradient descent optimizer while preventing the likelihood from trapping in undesirable local minima (to be explained in Section 3.2).

2. The hyperparameter ranks are selected by the data-driven VBMF scheme to bypass any intermittent interventions. The components of the weight tensor with small variation are pruned via VBMF on mode-3 and mode-4, and the resulting refined neural networks can achieve a more robust estimation while preserving the original
training (in-sample) performance.

3. We also conduct extensive numerical experiments on both the benchmark and real-world noisy image datasets that show our method significantly outperforms the existing methods in the literature. It is worth noting that our method can even achieve a higher peak signal-to-noise ratio (PSNR) under many scenarios. Moreover, our proposed framework can also achieve significantly better image denoising performance on other external (out-sample) testing datasets, indicating its good generalization ability. Our implementation code and experimental details will be made publicly available after the acceptance of this manuscript.

2 Preliminaries and Notation

Lowercase letters (e.g., $x, y$), bold lower case letters (e.g., $\mathbf{x}, \mathbf{y}$), and uppercase letters (e.g., $X, Y$) are used to denote scalars, vectors, and matrices, respectively, unless defined otherwise. Let $X_{ij}, X_i, \text{ and } X_j$ be the $(i,j)$th entry, the $i$th row, and the $j$th column of $X$, respectively. We use calligraphic-style letters $\mathcal{X}$ and $\mathcal{Y}$ to denote higher-order tensors with their entries $\mathcal{X}_{ijk}$ and $\mathcal{Y}_{ijk}$, respectively. For a general tensor $\mathcal{X} \in \mathbb{R}^{p_1 \times p_2 \times \cdots \times p_n}$, its mode-1 tensor-matrix product with a matrix $W \in \mathbb{R}^{r_1 \times p_1}$ is

$$\mathcal{X} \times_1 W \in \mathbb{R}^{r_1 \times p_2 \times \cdots \times p_n},$$

$$(\mathcal{X} \times_1 W)_{i_1,\cdots, i_n} = \sum_{j=1}^{p_1} \mathcal{X}_{j,\cdots, i_n} W_{i_1, j}.$$ 

The general mode-$k$ tensor-matrix product is defined similarly. The mode-$k$ matricization of $\mathcal{X}$ is obtained by unfolding it to the matrix $\mathcal{X}_{(k)}$ along the $k$th mode, and the multilinear rank (Tucker rank) is then defined as $(\text{rank}(\mathcal{X}_{(1)}), \cdots, \text{rank}(\mathcal{X}_{(n)}))$. The Tucker
decomposition of an order-$n$ multilinear rank-$(r_1, r_2, \cdots, r_n)$ tensor $\mathbf{X} \in \mathbb{R}^{p_1 \times p_2 \times \cdots \times p_n}$ is

$$
\mathbf{X} = \mathbf{S} \times_1 U_1 \cdots \times_n U_n = [\mathbf{S}; U_1, U_2, \cdots, U_n],
$$

where $\mathbf{S} \in \mathbb{R}^{r_1 \times r_2 \times \cdots \times r_n}$ is a core tensor, and $U_i \in \mathbb{R}^{p_i \times r_i}, i = 1, \cdots, n$ are the factor matrices. Higher-order orthogonal iteration (HOOI) (De Lathauwer et al., 2000; Zhang and Xia, 2018) is a classic algorithm that finds the rank-$(r_1, r_2, \cdots, r_n)$ approximation for $\mathbf{X}$ and we denote the result from HOOI as $\tilde{\mathbf{X}}$. In addition, the Frobenius norm of a matrix $\mathbf{X}$ and a tensor $\mathbf{X}$ is denoted as $\| \mathbf{X} \|_F = \sqrt{\sum_{i_1, i_2} X_{i_1, i_2}^2}$ and $\| \mathbf{X} \|_F = \sqrt{\sum_{i_1, \cdots, i_n} X_{i_1, \cdots, i_n}^2}$, respectively.

The standard 2D convolutional operation of a CNN can be presented in a tensor format: with $\mathbf{X} \in \mathbb{R}^{h \times w \times s}$ as the input, the convolutional operation generates a new tensor $\mathbf{Y} \in \mathbb{R}^{\tilde{h} \times \tilde{w} \times \tilde{s}}$ based on an order-4 weight kernel tensor $\mathbf{W} \in \mathbb{R}^{d \times d \times s \times \tilde{s}}$. Elementwisely, we have

$$
\mathbf{Y}_{\tilde{h}\tilde{w}\tilde{s}} = \sum_{i=1}^{d} \sum_{j=1}^{d} \sum_{k=1}^{s} W_{ijk} \times \mathbf{X}_{h_i w_j k},
$$

where $d$ is the spatial width of the weight kernel $\mathbf{W}$, $h_i = (\tilde{h}-1)\delta+i-p$, $w_j = (\tilde{w}-1)\delta+j-p$, $\delta$ is the stride size, and $p$ is the zero-padding size.

### 3 The Proposed Methods

Our denoising algorithm starts with the following additive model (Buades et al., 2005a; Aharon et al., 2006; Dabov et al., 2007; Gu et al., 2014):

$$
\mathbf{Y} = \mathbf{X}^* + \mathbf{E},
$$

where $\mathbf{Y} \in \mathbb{R}^{h \times w \times s}$ is the noisy image, $\mathbf{X}^* \in \mathbb{R}^{h \times w \times s}$ is the latent clean image, and $\mathbf{E} \in \mathbb{R}^{h \times w \times s}$ is the noise, each with height $h$, width $w$, and number of channels $s$. Below, we
introduce the proposed algorithm to recover $X^*$ step by step. The pseudocode of the overall procedure is summarized in Algorithm 1. The sub-algorithms Variational Bayesian matrix factorization (VBMF) (Nakajima et al., 2013) and Partial high-order orthogonal iteration (PHOOI) (De Lathauwer et al., 2000) are described in the supplementary materials.

3.1 Model Formulation

To construct an initial objective function, we introduce the maximum a posterior (MAP) framework, which aims at maximizing the posterior distribution $p(X | Y)$ as follows:

$$\max_{X} \ln p(X | Y) \propto \max_{X} \{ \ln p(Y | X) + \ln p(X) \}. \quad (3)$$

Here, $p(X)$ is the prior distribution of $X$, representing the information before acquiring the image, and $p(Y | X)$ is the likelihood function. For example, if $p(X) \propto \exp\{-\lambda R(X)\}$ and $Y | X \sim \mathcal{N}(X, \sigma^2 I)$, (3) becomes a loss minimization problem

$$\min_{X} l(Y, X) = \min_{X} \left\{ \frac{1}{2\sigma^2} \|Y - X\|_F^2 + \lambda R(X) \right\}, \quad (4)$$

where $R(X)$ can be seen as a regularization term. In practice, the loss function in (4) is high-dimensional and often non-convex, making direct computation unstable. To resolve the computational challenge, we introduce $\mathcal{M}, \mathcal{A} \in \mathbb{R}^{h \times w \times s}$ and rewrite the loss function of (4) in an augmented Lagrangian format:

$$l(X, Y, \mathcal{M}, \mathcal{A}; \rho, \lambda) = \frac{1}{2\sigma^2} \|Y - X\|_F^2 + \lambda R(\mathcal{M}) + \frac{\rho}{2} (\|X - \mathcal{M} + \mathcal{A}\|_F^2 - \|\mathcal{A}\|_F^2), \quad (5)$$

where $\lambda$ and $\rho$ are two hyperparameters to be further discussed in Section 4. By formulating (4) into (5), the constraints are split into two parts, $\|Y - X\|_F^2$ and $R(\mathcal{M})$. As a result, we only need to minimize the distance between $X$ and $Y$ while controlling the complexity
Algorithm 1 Proposed Optimization Scheme

Input: Single noisy image \( Y \)

Initialization: \( \theta^{(0)} \) by PyTorch, \( \mathcal{M}^{(0)} = Y, \mathcal{A}^{(0)} = 0_{h \times w \times s}, \rho = 10^2, \eta = .5, \mathcal{S} = \{S_D, 2S_D, \cdots\}, S_D = 200 \)

for \( k = 0 \) to \( K \) do
  for \( i = 0, 1, \cdots, m \) do
    \( \theta^{(k),i+1} = \arg \min_{\theta} \left\{ 2\sigma^2 \| Y - f^{(k)}(Y) \|_F^2 + \rho \| f^{(k)}(Y) + A^{(k)} - \mathcal{M}^{(k)} \|_F^2 / 2 \right\} \)
    if \( km + i \in \mathcal{S} \) then
      for \( W_n^{(k)} \in \theta^{(k),i+1} \) and \( n \neq 1 \) do
        \( (r_3, r_4) = \text{VBMP}(W_n^{(k)}, \sigma^2) \)
        \( (\mathcal{G}^{(k)}, U_{r_3}^{(3)}, U_{r_4}^{(4)}) = \text{PHOOI}(W_n^{(k)}, r_3, r_4) \)
        \( \tilde{W}_n^{(k)} = \mathcal{G}^{(k)} \times_3 U_{r_3}^{(3)} \times_4 U_{r_4}^{(4)}, W_n^{(k)} = \tilde{W}_n^{(k)}. \)
      end for
    end if
  end for
  \( \theta^{(k+1)} = \theta^{(k),m}, f^{(k+1)} = f_{\theta^{(k+1)}}, \mathcal{X}^{(k+1)} = f^{(k+1)}(Y) \)
  \( \mathcal{M}^{(k+1)} = \arg \min_{\mathcal{M}} \left\{ R(\mathcal{M}^{(k)}) + \rho \| f^{(k+1)}(Y) - \mathcal{M}^{(k)} + \mathcal{A}^{(k)} \|_F^2 / 2 \right\} \quad \triangleright \lambda \text{ is absorbed in } \rho \)
  \( \mathcal{A}^{(k+1)} = \mathcal{A}^k + \eta \left\{ f^{(k+1)}(Y) - \mathcal{M}^{(k+1)} \right\} \)
end for

Output: \( \hat{X} = f^{(K)}(Y) \)

of the auxiliary variable \( \mathcal{M} \). To this end, we introduce a dual variable \( \mathcal{A} \) to control the proximity between \( \mathcal{M} \) and \( \mathcal{X} \) to promote the constraint \( \mathcal{X} - \mathcal{M} = 0 \).
3.2 Computational Method

We solve (5) by alternating direction method of multipliers (ADMM) (Boyd et al., 2011), which updates the auxiliary and dual variables alternatively and iteratively. The procedure includes the following three sub-problems.

1. Update $X$: We establish a CNN-based mapping $f_{θ}(·)$ from the noisy image $Y$ to the update of $X$. The mapping $f_{θ}(·)$ essentially characterizes the features of $X$ parametrized by $θ$, where $θ$ is the collection of trainable parameters, including a convolution weight tensor in each layer. We note $W_n \in \mathbb{R}^{d \times d \times s \times s}$ as the convolution weight tensor for the $n$th layer. After substituting $X$ with $f_{θ}(Y)$, $θ$ can be iteratively updated through the backpropagation (BP) scheme (Rumelhart et al., 1986). For example, in the $(k+1)$th iteration, we update the parameters by

$$
θ^{(k+1)} = \arg \min_{θ} l\{ f_{θ}(Y), Y, M^{(k)}, A^{(k)}; ρ, λ \},
$$

$$
X^{(k+1)} = f_{θ^{(k+1)}}(Y),
$$

where the subscript $k$ or $k+1$ denotes the number of epochs. When our target image has a large size (e.g., high-resolution micrographs), we can apply mini-batch stochastic gradient descent on image patches to accelerate the computation.

2. Update $M$: We update $M$ by applying a proximal mapping $Υ(·)$. This sub-problem is equivalent to applying a Plug-and-Play technique to $X^{(k+1)} + A^{(k+1)}$, which can be generalized to any existing denoisers, e.g., BM3D (Dabov et al., 2007) and NLM (Buades et al., 2005a) among others (Venkatakrishnan et al., 2013; Zheng et al.)
Specifically,
\[
\mathcal{M}^{(k+1)} = \arg \min_{\mathcal{M}} \left\{ R(\mathcal{M}) + \frac{\rho}{2} \| \mathcal{X}^{(k+1)} - \mathcal{M} + \mathcal{A}^{(k+1)} \|^2_F \right\}
= \Upsilon (\mathcal{X}^{(k+1)} + \mathcal{A}^{(k+1)}).
\]  

(7)

3. Update \( \mathcal{A} \): As the variable \( \mathcal{A} \) only appears in the last term of our augmented loss function, it can be updated by
\[
\mathcal{A}^{(k+1)} = \mathcal{A}^{(k)} + \eta (f_{\theta^{(k+1)}}(\mathcal{Y}) - \mathcal{M}^{(k+1)}),
\]

where \( \eta \) can be seen as the learning rate of ADMM.

In Algorithm 1, a Tucker low-rank approximation distortion is applied to the weight tensors in all CNN layers except the first one for the update of \( \mathcal{X} \) at the pre-specified twist steps, i.e., \( S_D, 2S_D, \cdots \), while retaining the original structures of the approximated layers. This strategy not only boosts the model’s generalizability but also obviates significant damage to model performance. Specifically, the model generalizability corresponds to the flatness of the loss function (4) near the local minimum (Baldassi et al., 2020). If the loss surface is steep, measured by \( \partial l/\partial \theta \), small turbulence of \( \theta \) may lead to a deterioration of performance. Thus it is often more desirable to find another local minimum in which minor weight distortion only makes a slight impact on the value of training loss, i.e., \( \partial l/\partial \theta \) is of small magnitude. Specifically, our training procedure is coupled with such low-rank approximation after every \( S_D \) iterations to achieve stable convergence, where \( S_D \) is a carefully selected hyperparameter that controls the trade-off between model quality and generability. This scheme can even boost the resulting model performance on various occasions. Figure 1 gives a schematic illustration: given the same level of estimation error due to the stochastic variability, represented by \( \Delta \theta \), the weight distortion scheme allows
us to settle down on the blue point, which is more desirable than the red point when no weight adjustment is implemented. In addition, through the low-rank approximation of the

\[ \text{Figure 1: A schematic illustration of weight distortion in the stochastic gradient descent training process} \]

4D weight tensor, the computation time at that step is significantly reduced. For example, the common kernel weight tensors \( W \) typically have small spatial mask size \( d \) but large numbers of input/output channels \( s \) and \( b \). Thus, we apply Tucker decomposition with low-rank structures on modes-3 and 4:

\[ W = G \times_3 U^{(3)}_{r_3} \times_4 U^{(4)}_{r_4}, \]

with \( G \in \mathbb{R}^{d \times d \times r_3 \times r_4}, U^{(3)}_{r_3} \in \mathbb{R}^{s \times r_3}, U^{(4)}_{r_4} \in \mathbb{R}^{b \times r_4}, \)
where $\mathcal{G}$ is the core tensor with loading matrices $U_{r_3}^{(3)}$ and $U_{r_4}^{(4)}$ for mode-3 and mode-4, respectively. Then the original 2D convolution operator in (1) can be rewritten in three consecutive convolutions as follows:

$$
Y_{hwr3}^{(1)} = \sum_{i=1}^{s} U_{r_3,ir_3}^{(3)} X_{hiw},
$$

$$
Y_{hwr4}^{(2)} = \sum_{i=1}^{d} \sum_{j=1}^{d} \sum_{k=1}^{r_3} G_{ijkr4} \times Y_{hwr4}^{(1)},
$$

$$
Y_{hws}^{(3)} = \sum_{i=1}^{r_4} U_{r_4,ris}^{(4)} Y_{hws}^{(2)}.
$$

The compression ratio of computation time of this convolution operation is

$$
d^2 s \tilde{s} / \left( d^2 r_3 r_4 + s r_3 + \tilde{s} r_4 \right), \quad (9)
$$

which is typically greater than 1 for small values of ranks $r_3$ and $r_4$. In summary, the weight tensors are approximated by the VBMF-aided Tucker decomposition to filter out any unnecessary redundancy at each twist stage, and the original 2D convolution operator can be replaced with three consecutive convolutions to speed up the gradient computation. Throughout our training, the original structures of the approximated layers are retained since only the numerical values of the weight tensors are substituted. By applying the Tucker low-rank approximation to the weight tensors at each twist stage, we can alleviate stochastic variance induced by the stochastic gradient descent optimizer while preventing the likelihood of trapping in undesirable local minima. In addition, the weight distortion step does not change the structure of the convolution structure, which avoids significant in-sample quality drop but generalizes better to out-sample datasets.
3.3 Rank Selection

The selection of optimal Tucker rank can be challenging in practice. One popular data-driven approach is cross-validation, which can be computational intensive to apply in the deep learning framework. Other researchers suggest a human-in-the-loop approach for rank tuning, in which the “elbow” points are sought to partition the spectral values of the matricized weight tensors. However, this approach relies on the manual intervention at each iteration and does not allow the framework to be end-to-end. Towards this reason, we deploy the variational Bayesian matrix factorization (VBMF) sub-algorithm into our proposed optimization scheme for choosing \( r_3 \) and \( r_4 \), which achieves global optimality under certain conditions (Nakajima et al., 2013). The detailed VBMF algorithm is illustrated in the supplementary materials.

4 Experiments

We assess the proposed single-image denoiser method on real-world noisy images, synthetic images corrupted by Gaussian noise and synthetic images corrupted by Poisson-Gaussian noise. For simplicity, all experiments are done on gray-scale images; colored image denoising can be done similarly. The deep learning network structure of \( f_{\theta}(\cdot) \) are chosen to be the U-net architecture with a 5-layer encoder and decoder (Ronneberger et al., 2015) (see Figure 2 for a schematic plot); the denoiser \( \Upsilon(\cdot) \) is chosen as the block-matching and 3D filtering (BM3D) (Dabov et al., 2007). The hyper-parameters \( \rho \) and \( \eta \) in the ADMM algorithm are set to be 100 and 0.5, respectively. The noise level \( \sigma \) is estimated following the method in Chen et al. (2015a). Our algorithm is termed as NN+BM3D+T.

In the low-rank approximation step, we select the ranks \( r_3, r_4 \) by VBMF every \( S_D = 200 \)
Figure 2: Illustration of the U-net deep learning framework. The left side (before the Bottleneck) is the encoder and the right side (after the Bottleneck) is the decoder.

4.1 Results

Experiments on real-world noise  We first apply our proposed algorithm to a real-world dataset of the SARS-CoV-2 2P protein produced by cryo-electron microscopy (cryo-em). The image includes $5760 \times 4092$ pixels with a pixel size around $1.058\text{Å}$. The raw image is obtained by running the MotionCor2 algorithm in Zheng et al. (2017) on the collected movie frames, and its image quality is substantially limited by various factors, such as dose fractionation, specimen heterogeneity, and radiation damage (Vulović et al., 2013), which makes the particles almost invisible in Figure 3. Our goal is to apply the proposed method to improve the damped contrast in the cryo-em image and reveal the particles from the background noise. Figure 3 visualizes the denoised images after apply-
ing the proposed algorithms trained for 100 epochs compared with BM3D. Our proposed algorithm can effectively reduce the background noise and reveal particles of interest, outperforming BM3D. Since some signal-dependent noises, e.g., the black ribbons, we apply a classic transformation technique, Variance Stabilizing Transform (VST), during our training process as suggested in Makitalo and Foi (2012); Zheng et al. (2020). After applying VST, such signal-dependent noises are further reduced.

To study the generalizability of the proposed algorithm, we further consider out-sample denoising, i.e., apply the trained denoiser to images outside the training set. We specifically apply the tuned models to other cryo-em images to assess the model out-sample performance. As BM3D is a non-learning-based method that cannot be evaluated on an external image, the out-sample performance is omitted in Figure 3. From Figure 3, we can see our method enhances the contrast of particles and reveals more information from the noisy image. Moreover, our trained model only takes $\sim 300$ seconds for patch-wise denoising on an external micrograph, whereas the BM3D requires $\sim 1500$ seconds per micrograph.

**Synthetic Gaussian noise** In this section, we evaluate the in-sample performance of the proposed procedure on one selected image in the training dataset SET12 (Zhang et al., 2017). The chosen image is corrupted by additive Gaussian noise with various noise standard deviation $\sigma = 20, 25, 30$. We compare our proposed method with two traditional denoising methods, Low-pass filter and Non-local mean (NLM) (Buades et al., 2005a), and three learning-based methods, denoising CNN (DnCNN) (Zhang et al., 2017), deep image prior (DIP) (Ulyanov et al., 2018), and NN+BM3D (Zheng et al., 2020); Specifically, DnCNN is a supervised method trained that requires clean/noisy image pairs; Low-pass and NLM do not provide out-sample denoiser; DIP, NN+BM3D, and our method only
Figure 3: Visualized in-sample (top) and out-sample (bottom) performance comparison of on two raw micrographs of SARS-CoV-2 2P protein microscopy. The hyper-parameters for the ADMM framework is chosen as $\eta = 0.5, \rho = 100$. Recall BM3D is a non-learning-based method and therefore does not provide a out-sample denoiser.

require single noisy images and yield out-sample denoisers. As suggested by Quan et al. (2020), we choose a non-blind version of DIP guided by the estimated noise level for comparison. For a fair comparison, DnCNN is fine-tuned on the pre-trained model given the provided single image and the NN+BM3D method is trained with the same neural network architecture as depicted in Figure 2. All self-supervised learning methods are retrained on the selected single image for $K = 100$ epochs after initialization.

The results are assessed in both peak signal-to-noise ratio (PSNR) and structured similarity index (SSIM) computed by the built-in Python functions in the skimage package.
Table 1: In-sample (top) and out-sample (bottom) performance in terms of PSNR/SSIM for synthetic additive Gaussian noise removal under different noise level

| In-sample | Traditional methods | Supervised method | Self-supervised methods |
|-----------|---------------------|-------------------|------------------------|
| σ         | Noisy               | Low-pass NLM      | DnCNN                  | DIP        | NN+BM3D | NN+BM3D+T |
| 20        | 22.14dB/0.5212      | 29.84dB/0.9023    | 30.06dB/0.8823         | 30.22dB/0.9013 | 30.07dB/0.8932 |
| 25        | 20.19dB/0.4255      | 28.46dB/0.6238    | 29.55dB/0.8729         | 28.62dB/0.8856 | 29.81dB/0.8861 | 30.44dB/0.9092 |
| 30        | 18.72dB/0.3512      | 28.50dB/0.8623    | 29.98dB/0.8765         | 28.23dB/0.8431 | 28.54dB/0.8461 | 29.34dB/0.8823 |

| Out-sample | σ = 20 | Low-pass NLM | DnCNN | DIP | NN+BM3D | NN+BM3D+T |
|------------|--------|--------------|-------|-----|---------|-----------|
| SET12      | 22.14dB/0.5921 | NA           | NA    | 28.57dB/0.8832 | NA | 27.01dB/0.8745 | 27.16dB/0.8689 |
| BSD68      | 22.13dB/0.6014 | NA           | NA    | 28.00dB/0.8577 | NA | 24.32dB/0.8424 | 26.76dB/0.8444 |

| Out-sample | σ = 25 | Low-pass NLM | DnCNN | DIP | NN+BM3D | NN+BM3D+T |
|------------|--------|--------------|-------|-----|---------|-----------|
| SET12      | 20.19dB/0.5102 | NA           | NA    | 28.04dB/0.8721 | NA | 27.58dB/0.8732 | 28.13dB/0.8921 |
| BSD68      | 20.19dB/0.4834 | NA           | NA    | 27.39dB/0.8478 | NA | 26.43dB/0.8421 | 27.06dB/0.8456 |

| Out-sample | σ = 30 | Low-pass NLM | DnCNN | DIP | NN+BM3D | NN+BM3D+T |
|------------|--------|--------------|-------|-----|---------|-----------|
| SET12      | 18.62dB/0.4211 | NA           | NA    | 26.44dB/0.8205 | NA | 26.67dB/0.8432 | 27.06dB/0.8647 |
| BSD68      | 18.61dB/0.4289 | NA           | NA    | 26.00dB/0.8017 | NA | 25.90dB/0.8013 | 26.41dB/0.8332 |

Figure 4 illustrates one example of the in-sample performance in the dataset SET12 when σ = 25; see Table 1 for a comprehensive in-sample comparison under different noise levels. We can see (i) learning-based methods perform better and generalize better to out-samples; (ii) our proposed method outperforms its precedents NN+BM3D in terms of PSNR/SSIM under moderate noise levels. However, when the noise is diminishing in intensity, i.e., σ = 20, our method is subject to a minor quality drop – this is reasonable as the image presents less noise, our low-rank approximation might filter out some useful information.

**Synthetic Poisson-Gaussian noise** We also test the performance of the proposed method on images corrupted by Poisson-Gaussian distributed noise (Foi et al., 2008; Wang et al., 2020). First, we synthesize the noisy images by generating Poisson-Gaussian pixel values based on one selected single image. Next, we apply the proposed method coupled
Figure 4: Visual comparisons of our method against other competing methods in terms of in-sample performance from dataset SET12 coupled with PSNR and SSIM. See more in-sample comparisons in the supplementary materials.

with the VST technique on the synthetic noisy image for comparison. The visual and quantitative evaluation in Figure 5 shows our method works effectively for Poisson-Gaussian noise removal.

Out-sample performance We further compare the out-sample performance of our method to the one without weight distortions (i.e., ablation analysis) (Figure 6a) and a supervised learning-based denoiser DnCNN (Figure 6b) when $\sigma = 25$; see Table 1 for a comprehensive out-sample comparison. Specifically, we train the denoisers based on the single noisy image LENA and evaluate the out-sample performance on the remaining test images in datasets SET12 and BSD68 (Martin et al., 2001) corrupted by additive Gaussian noises under the same noise level as the training image.

In addition, we present the out-sample performance under misaligned noise levels in
Figure 5: A visual comparison of (a) Poisson-Gaussian-noisy image, (b) denoised by BM3D, (c) denoised by NN+BM3D+T, and (d) denoised by NN+BM3D+VST+T ($\eta = 0.5$, $\rho = 100$ and $S_D = 200$).

Table 2, where the performance is evaluated on the test images (e.g., BSD68) with different noise levels from the training image (e.g., one noisy image from SET12). This demonstrates the advantages of weight distortion to the model generalization ability.

The results demonstrate that our proposed network can achieve better results when being evaluated with an external-image tuned parameters on average, is even competitive with the supervised denoiser (e.g., DnCNN) across all considered (aligned or misaligned) noise levels. Such results manifest the benefit of adding a Tucker low-rank approximation for the weight tensor in terms of improving generalization despite a minor quality drops of in-sample performance. See more out-sample experiments in the supplementary material.

4.2 Discussions

Advantages of low-rank approximation To assess the benefits of low-rank tensor approximation on convergence, we present in Figure 7 the loss function $l_{\theta(k),i}(\mathcal{X}, \mathcal{Y})$ over each iteration $i$ within epoch $k$, average changes of the weight kernels in Frobenius...
Table 2: Out-sample performance in terms of PSNR/SSIM for synthetic additive Gaussian noise removal under misaligned noise level

| Training σ | Test σ | DnCNN PSNR/SSIM | NN+BM3D PSNR/SSIM | NN+BM3D+T PSNR/SSIM |
|------------|--------|-----------------|-------------------|---------------------|
| 20         | 20     | **28.00dB/0.8621** | 24.32dB/0.8433 | 26.76dB/0.8421 |
| 25         | 20     | **26.54dB/0.8145** | 23.97dB/0.8123 | 26.07dB/0.8126 |
| 30         | 20     | 24.36dB/0.7234 | 23.57dB/0.7821 | **25.37dB/0.7848** |
| 25         | 20     | **28.14dB/0.8745** | 27.06dB/0.8579 | 27.69dB/0.8723 |
| 25         | 25     | **27.39dB/0.8548** | 26.34dB/0.8421 | 27.06dB/0.8532 |
| 30         | 25     | 26.08dB/0.8011 | 25.54dB/0.8046 | **26.18dB/0.8089** |
| 30         | 30     | **28.00dB/0.8631** | 27.84dB/0.8746 | 27.61dB/0.8639 |
| 25         | 30     | 26.54dB/0.8109 | 26.92dB/0.8411 | **27.07dB/0.8541** |
| 30         | 30     | 26.00dB/0.8002 | 25.90dB/0.8033 | **26.41dB/0.8329** |

norm versus epochs (average difference of weight tensor is measured by $\Delta W/(d^2s\tilde{s})$ for $W \in \mathbb{R}^{d \times d \times s \times \tilde{s}}$), and the average compression ratio (over layers) defined in (9) at each low-rank approximation step for all $\sigma$ levels. One can observe that (i) the loss functions in both models decrease significantly after a few iterations; (ii) changes of weights tend to fluctuate less when coupled with the weight distortion (Figure 7b), and the embedded low-rank Tucker approximation does not noticeably affect the final loss value (Figure 7a); (iii) average compression ratios are significantly reduced after one low-rank approximation step but remain above 1 henceforth under all noise levels (Figure 7c). Supported by the empirical evidence, it can be concluded that applying the low-rank approximation to the weight tensors can be beneficial to streamline the neural network structure, leading to a
Figure 6: Out-sample performance profile of our algorithm on two datasets (SET12 and BSD68) when $\sigma = 25$ compared to (a) the single-image-based denoiser NN+BM3D and (b) the supervised learning-based denoiser DnCNN. Note that the our denoiser is trained on LENA but tested on the other images.

Table 3: Average training time in seconds per epoch (standard deviation in parentheses) and inference time in seconds per image for learning based denoisers

| Methods      | DnCNN  | DIP    | NN+BM3D | NN+BM3D+T |
|--------------|--------|--------|---------|-----------|
| Training time| 283.83 (78.1) | 495.86 (6.8) | 340.23 (9.0) | **332.71 (1.2)** |
| Inference time| 4.99    | 2.52   | 2.28    | 2.32      |

flatter local minima without notable loss increment.

We further consider the computational complexity of training. Our proposed method takes $\sim 330$ seconds per epoch to denoise one image of size $256 \times 256 \times 1$, which requires less variant training time as implied by its smaller standard deviation. See Table 3.
Figure 7: (a) Training loss $l\left(f_{\theta(k)}(\mathcal{X}), \mathcal{Y}\right)$ of NN+BM3D and ours versus number of iterations when $\sigma = 25$; (b) Average changes in weight tensors $\Delta W = \|W^k - W^{k-1}\|_F$ of NN+BM3D and ours versus epochs when $\sigma = 25$; (c) Average compression ratio of parameters at each distortion step in NN+BM3D+T when $\sigma = 20, 25, 30$; (d) Average PSNR of 5 initial starts for training single image LENA over epochs.

**Sensitivity analysis** We also assess the robustness of model performance under different initialization weight values ($\theta^0$) and hyper-parameters ($\rho, \eta$ and $S_D$). As the gradient-based optimization is known to be sensitive to initialization [Bubeck 2014], we first investigate
the model robustness under different weight initializations based on in-sample PSNR stabilizability. Specifically, we re-run the learning-based methods (DIP, DnCNN, NN+BM3D, and ours) for 5 times after 100 epochs on one single image, *LENA*, with the Xavier weight initialization strategy in Glorot and Bengio (2010). Figure 7d displays the average training PSNR values against the epochs, which shows our algorithm reaches stability with the highest average PSNR after 100 epochs and shows its robustness to the randomness of the initialization.

Next, we perform sensitivity analysis on hyper-parameters for our proposed algorithm. We calculate the average PSNR on the dataset *SET12* by fixing $\eta = 0.5, S_D = 200$ and varying $\rho$ from 95 to 125 (Figure 8, left), by fixing $\rho = 100, S_D = 100$ and varying $\eta$ from 0.2 to 1.4 (Figure 8, middle), and by fixing $\rho = 100, \eta = 0.5$ and varying $S_D$ from 100 to 300 (Figure 8, right). The resulting PSNR curves are stable in all scenarios, which shows our model is robust to different choices of hyper-parameters.
5 Conclusion

In this paper, we propose to substitute the weight kernels in the CNN-based network with their approximated low-rank tensors in the original learning structure. By combining this technique with the state-of-the-art single-image denoising methods, the resulting network is equipped with more flexibility and improved generalization ability. In addition, with the aid of VBMF for optimal rank selection, our algorithm is data-driven and automatic as it requires little manual interference and achieves the end-to-end fashion. Low-rank weight approximation for multi-channel images (e.g., RGB-colored images) might require more carefulness in rank selection to deal with the inter-channel correlated noises, where the column-wise independence required by the VBMF algorithm is not satisfied. Empirical evidence based on the synthetic noisy images and real-world noisy images demonstrates the advantages of proposed weight distortion compared with various learning-based and non-learning-based methods. As the column-wise independence required by the VBMF algorithm is not satisfied for multi-channel images (e.g., RGB-colored images), low-rank weight approximation for multi-channel images might require more careful handling of inter-channel correlated noises. An extension to handling rank selections under inter-channel correlated noises is one of our future directions (e.g., incorporating the estimation of noise covariance across channels [Dong et al., 2018]). Further tasks in image processing, such as deblurring, segmentation, and classification, are to be explored in future research as well.

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Supplementary Material for “Self-supervised Denoising via Low-rank Tensor Approximated Convolutional Neural Network”

These supplementary materials collect the implementation details and additional experimental results of the proposed algorithm in the main content.

S1 IMPLEMENTATION DETAILS

S1.1 Details of U-net Architecture

We implement the U-net with a similar architecture to the one in Lehtinen et al. (2018). Different from Lehtinen et al. (2018), we substitute the first two convolutional layers with spatial width 3 by a single convolutional layer with spatial width 11 as suggested in Bepler et al. (2020). All the convolutional layers are implemented with pad='same', i.e., the output has the same shape as the input. Except for the last layer followed by linear activation, the other convolutions are coupled with a ReLU activation function; the pooling layers used in the U-net are the max-pooling downsampling with the stride 2, realized by function nn.MaxPool2d in the PyTorch package. The up-sampling layers are the nearest-neighbor upsampling block, realized by function nn.Upsample in the PyTorch package.

S1.2 Details of Training

In each of our experiments, each image is first normalized by subtracting its individual mean pixel intensity and dividing by its standard deviation. After finishing the denoising procedure described earlier, each denoised image is multiplied by its original standard deviation and added by its original mean.
For denoising of moderate size images (e.g. 256 × 256 × 1), image patches of 32 × 32 × 1 with possible overlapping are randomly sampled from each image. Then, all these patches are randomly rotated by 90, 180, or 270 degrees (to avoid interpolation artifacts) and mirrored as data augmentation. After that, mini-batches of size 128 are formed to fuel the subsequent stochastic gradient descent (SGD) algorithm. There are around \( m = 60 \) iterations in each epoch when denoising images of size 256 × 256 × 1.

For denoising of large size images (e.g. 5760 × 4092 × 1), image patches of 800 × 800 × 1 are randomly selected with proper data augmentations. For the sake of computational capacity, mini-batches of size 4 are formed for the optimizer. There are around \( m = 1000 \) iterations in each epoch when denoising the micrographs of size 5760 × 4092 × 1. In addition, to avoid the edge artifacts, we include a padding of 500 pixels when denoising the image by patches \( \text{(Bepler et al., 2020).} \)

S1.3 Details of Additional Algorithms

Let \( M_k(\cdot) \) be the matricization operator of tensor \( \cdot \) along its \( k \)-th mode (see (Kolda, 2001, Section 2.4) for a formal definition); \( \mathcal{N}_d(\cdot; \mu, \Sigma) \) denotes the density of the \( d \)-dimensional Gaussian random variable with mean \( \mu \) and covariance matrix \( \Sigma \); \( \text{SVD}(A, r) \) is defined as the matrix comprised of the top \( r \) left singular vectors of \( A \). The implementations of VBMF (variational Bayes matrix factorization) and Partial HOOI (partial high-order orthogonal iteration), two key sub-algorithms in our proposed method, are provided in Algorithms S1 and S2, respectively.
Algorithm S1 Rank selection by variational Bayes matrix factorization (VBMF)

**Input:** Weight kernel $\mathcal{W} \in \mathbb{R}^{d \times d \times s \times b}$, $\sigma^2 \in \mathbb{R}$

$M = \mathcal{M}_3(\mathcal{W}) \in \mathbb{R}^{s \times (d^2)}$ and assume: $M = BA^\top$ with $B \in \mathbb{R}^{s \times h}, A \in \mathbb{R}^{(d \times d \times b) \times h}$

$M = \sum_{i=1}^{h} \gamma_i w_{ai} w_{ai}^\top$ with $\gamma_1 \geq \cdots \geq \gamma_h$

▷ Gaussian Priors on $A = (a_1, \cdots, a_h)$ and $B = (b_1, \cdots, b_h)$

$$
\phi(A) \propto \exp \left\{ -\sum_{i=1}^{h} \|a_i\|^2/(2c^2_{ai}) \right\} \quad \text{and} \quad \phi(B) \propto \exp \left\{ -\sum_{i=1}^{h} \|b_i\|^2/(2c^2_{bi}) \right\}
$$

$r(A, B \mid \mathcal{W}) = \prod_{i=1}^{h} \{N_{d \times d \times s}(a_i; \mu_{ai}, \Sigma_{ai}) \times N_s(b_i; \mu_{bi}, \Sigma_{bi})\}$  ▷ Assume probabilistic independence of $A$ and $B$ given $\mathcal{W}$

$F_{VB}(r \mid \mathcal{W}) = F_{VB}\{\mu_{ai}, \mu_{bi}, \Sigma_{ai}, \Sigma_{bi} : i = 1, \cdots, h\}$ ▷ The variational Bayes free energy $F_{VB}$ can be analytically derived

$\{\mu_{ai}^*, \mu_{bi}^*, \Sigma_{ai}^*, \Sigma_{bi}^*\} = \arg\min F_{VB}(r \mid \mathcal{W})$ ▷ The global solution can then be analytically obtained

$$
\alpha_i^* = \|\mu_{ai}\|^2 + \text{tr}(\Sigma_{ai}^*), \beta_i^* = \|\mu_{bi}\|^2 + \text{tr}(\Sigma_{bi}^*)
$$

$$
c^2_{ai} = \frac{\alpha_i^*}{(d \times d \times s)}, c^2_{bi} = \frac{\beta_i^*}{s}
$$

$$
\tilde{\gamma}_i = \sqrt{\frac{(2d+s+\hat{s})\sigma^2}{2} + \frac{\sigma^4}{2c^2_{ai}\tilde{\gamma}_i} + \sqrt{\left(\frac{(2d+s+\hat{s})\sigma^2}{2} + \frac{\sigma^4}{2c^2_{ai}\tilde{\gamma}_i}\right)^2 - 2ds\hat{s}\sigma^4}} \quad \text{Define the cutoff value}
$$

**Output:** Global optimal mode-3 rank $r_3 = \arg\max_i \{\gamma_i > \tilde{\gamma}_i : i = 1, \cdots, h\}$ of $\mathcal{W}$

(optimal mode-4 rank $r_4$ can be deduced analogously)
**Algorithm S2** Partial higher order orthogonal iteration (Partial HOOI) for partial Tucker decomposition

**Input:** Weight kernel $W \in \mathbb{R}^{d \times d \times s \times b}$, mode-3 and mode-4 ranks $(r_3, r_4)$

$U^{(0)}_{r_3} = \text{SVD}\{\mathcal{M}_3(W), r_3\}, U^{(0)}_{r_4} = \text{SVD}\{\mathcal{M}_4(W), r_4\}$ \> Initialization

for $k = 0, 1, \cdots, K - 1$ do

$G^{(k)} = W \times_3 (U^{(k)}_{r_3})^T \times_4 (U^{(k)}_{r_4})^T$

$U^{(k+1)}_{r_3} = \text{SVD}\{\mathcal{M}_3(G^{(k)}), r_3\}$

$U^{(k+1)}_{r_4} = \text{SVD}\{\mathcal{M}_4(G^{(k)}), r_4\}$

end for

$G^{(K)} = W \times_3 (U^{(K)}_{r_3})^T \times_4 (U^{(K)}_{r_4})^T$

**Output:** Decomposed parts $G, U_{r_3}, U_{r_4}$

**S2 ADDITIONAL NUMERICAL EXPERIMENTS**

For completeness, we compare our method with other existing representative self-supervised learning methods in Table S1 in terms of their in-sample performance, i.e., trained on each image individually.

Table S1: In-sample performance in terms of PSNR/SSIM for synthetic additive Gaussian noise removal

|         | N2N [Lehtinen et al., 2018] | N2V [Krull et al., 2019] | Neighbor2Neighbor [Huang et al., 2021] | NN+BM3D | NN+BM3D+T |
|---------|-----------------------------|---------------------------|----------------------------------------|---------|-----------|
| SET12   | 30.66dB/0.95                | 28.84dB/0.80              | 31.09dB/0.86                           | 29.81dB/0.89 | 30.41dB/0.91 |
| BSD68   | 28.86dB/0.82                | 27.72dB/0.79              | 30.79dB/0.87                           | 27.42dB/0.78 | 28.80dB/0.80 |

N2V is trained on unorganized images and N2N is trained on paired noisy images. Intuitively, they are expected to outperform our method since they have more information available during training. However, our method only subject to small PSNR/SSIM drops
and it can achieve comparable state-of-the-art performance as the Neighbor2Neighbor, which requires computational subsampling scheme.

Figures S1, S2, and S3 visualize the out-sample denoising performance of our denoiser trained on LENA on the other images in SET12. Figures S4–S20 provide more experimental results on the generalization ability of our trained model on the external datasets BSD68. In Figure S21 we provide more denoising results for real Cryo-EM images on SARS-CoV-2 2P protein.
Figure S1: Visualization of out-sample denoising results with PSNR/SSIM on the (rest) SET12. Noted that our denoiser is trained based on LENA.
Figure S2: Visualization of out-sample denoising results with PNSR/SSIM on the (rest) SET12. Noted that our denoiser is trained based on LENA.
Figure S3: Visualization of out-sample denoising results with PNSR/SSIM on the (rest) SET12. Noted that our denoiser is trained based on LENA.
Figure S4: Visualization of out-sample denoising results with PSNR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S5: Visualization of out-sample denoising results with PSNR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S6: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S7: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S8: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S9: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S10: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S11: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S12: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S13: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets *BSD68*. Noted that our denoiser is trained based on *LENA*. 

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Figure S14: Visualization of out-sample denoising results with PSNR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S15: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S16: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S17: Visualization of out-sample denoising results with PSNR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S18: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S19: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S20: Visualization of out-sample denoising results with PNSR/SSIM on the external image sets BSD68. Noted that our denoiser is trained based on LENA.
Figure S21: Visualization of out-sample denoising results on the Cryo-EM images of SARS-CoV-2 2P protein with the proposed algorithm coupled with VST.