An Efficient Algorithm for Fuzzy Linear Fractional Programming Problems via Ranking Function

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Abstract:
In many applications such as production, planning, the decision maker is important in optimizing an objective function that has fuzzy ratio two functions which can be handed using fuzzy fractional programming problem technique. A special class of optimization technique named fuzzy fractional programming problem is considered in this work when the coefficients of objective function are fuzzy. New ranking function is proposed and used to convert the data of the fuzzy fractional programming problem from fuzzy number to crisp number so that the shortcoming when treating the original fuzzy problem can be avoided. Here a novel ranking function approach of ordinary fuzzy numbers is adopted for ranking of triangular fuzzy numbers with simpler and easier calculations as well as shortening in the procedures. The fuzzy fractional programming problem is the first reduced to a fractional programming problem and then solved with the technique to obtain the optimal solution. It has a power to give a best solution for supporting the solution theory proposed in this work, some numerical fuzzy fractional programming problem are included to ensure the advantage, efficiency and accuracy of the suggested algorithm. In addition, this research paper describes a comparison between our optimal solutions with other existing solutions for inequalities constrains fuzzy fractional program.

Keywords: Fuzzy fractional programming, Fuzzy set, Generalized triangular fuzzy number, Membership function, Triangular fuzzy number.

Introduction:
Fractional programming problem (FPP) is a special kind of non-linear programming problems in which the objective function is a ratio two functions with the constraints. Fractional model arises in decision making such as construction planning, economic, and commercial planning, health care and hospital planning.

Recently, Charnes and Kooper established a method for transforming the fractional programming (FP) to an equivalent model program. Effati and Pakdaman introduced a technique obtain the solution of the interval valued fractional programming (FP). Tantawy proposed an iterative method for treating fraction programming problem with sensitivity analysis. The fractional programming (FP) problem with interval coefficients in the objective function was invented by Borza.

Many researches proposed some techniques depending on integer solution for FP Problem while multi-objective integer FP problems are treated with the help of an improved method by Mehdi et al.

The fuzzy fractional programming problem is interested and has applications in many important fields.

Some researchers suggested some algorithms to obtain the approximate solution for fuzzy programming (FP) problem such as proposed technique introduced by Kabiraj et al. A new method presented by Malathi et al. to solve special fuzzy programming (FP) problem.

Another way to solve the fuzzy programming (FP) problem is established by using ranking function to convert the fuzzy programming (FP) problem into an equivalent crisp programming problem.
Furthermore, the ranking function is utilized to solve fuzzy fractional programming (FFP) problem by fully fuzzy multi-objective programming problem (FFMOPP), fuzzy rough fractional programming (FRFP) problem, fuzzy fractional transportation problem and multi-objective fractional programming (MOFP) are also solved using different methods.

Here, an efficient ranking method is suggested to obtain an optimal solution for FFP problem by reducing it into a crisp programming. The presented technique is explained through illustrations. This article is organized as follows: some important preliminaries concerning triangular fuzzy number and ranking function are listed is Section 2. In Section 3, a ranking function is adopted while its application to solve FFP problem is derived in Section 4. Two examples of FFP problem are included in Section 5. In order to demonstrate the efficiency of the proposed approach, a comparison with other obtains result is also listed in this section. Section 6 describes the conclusion of our obtain results.

Preliminaries
Some necessary background and definitions are recalled throughout this section.

Definition 1
Let X be a collection of objects denoted by x. A fuzzy set Ā in X can be defined as a set of ordered pairs:

\[ Ā = \{ (x, M_Ā(X)) : x \in X \} \]

where \( M_Ā(X) \) is named the membership function of x in a set Ā. The function \( M_Ā(X) \) maps each element of a set X to a membership grade (or membership value) between 0 and 1.

Definition 2
A fuzzy number Ā is a convex normalized fuzzy set on \( R \) such that:

(i) A least one \( x_0 \) exists in R where \( M_Ā(x_0) = 1 \),
(ii) \( M_Ā(X) \) is piecewise continuous.

Definition 3
A fuzzy number Ā = (a, b, c), \( a \leq b \leq c \) \((a, b, c \geq 0)\) is called a triangular fuzzy number if \( M_Ā(X) \) is defined by

\[ M_Ā(X) = \begin{cases} \frac{w(x-a)}{b-a} & a \leq x \leq b \\ \frac{w(c-x)}{c-b} & b \leq x \leq c \end{cases} \]

In general, the fuzzy number Ā = (a, b, c; w) can be defined to be a generalized triangular fuzzy number if \( M_Ā(X) \) is defined by

\[ M_Ā(X) = \begin{cases} \frac{w(x-a)}{b-a} & a \leq x \leq b \\ \frac{w(c-x)}{c-b} & b \leq x \leq c \end{cases} \]

The Suggested New Ranking Function
In fact, an efficient scheme for ordering the elements is to give a ranking function \( R: D(R) \rightarrow R \) which maps for each fuzzy number into R, and exists a natural order. The orders on is defined by:

\[ Ā \leq Ą \text{ if and only if } R(Ā) \leq R(Ȃ) \]

where \( Ā \) and \( Ą \) are in \( D(R) \)

The proposed ranking of fuzzy numbers can be defined as

Consider the generalised triangular membership function

\[ M_Ā(X) = \begin{cases} \frac{w(x-a)}{b-a} & a \leq x \leq b \\ \frac{w(c-x)}{c-b} & b \leq x \leq c \end{cases} \]

Using the \( \alpha \)-cut when \( 0 \leq \alpha \leq 1 \), yields

\[ w(x-a) = \alpha \rightarrow x = a + \frac{\alpha}{w} (b-a) \]

\[ w(c-x) = \alpha \rightarrow x = c - \frac{\alpha}{w} (c-b) \]

Now applying the following ranking function

\[ R(Ā) : R(Ā) = \frac{1}{2} \int_0^\infty a^\alpha \left[ \inf (Ā(α)) + \sup (Ā(α)) \right] \frac{da}{a^\alpha} \]

then \( R(Ā) \) becomes the equation:

\[ R(Ā) = \frac{1}{2} \int_0^\infty a^\alpha \left[ a + \frac{\alpha}{w} (b-a) + c - \frac{\alpha}{w} (c-b) \right] \frac{da}{a^\alpha} \]

\[ R(Ā) = \left[ \frac{a^\alpha}{w} \right] + \frac{\alpha}{w} (b-a) + c - \frac{\alpha}{w} (c-b) \]

\[ R(Ā) = \left[ \frac{a^\alpha}{w} \right] + \frac{\alpha}{w} (b-a) + c - \frac{\alpha}{w} (c-b) \]

\[ R(Ā) = \left[ \frac{a^\alpha}{w} \right] + \frac{\alpha}{w} (b-a) + c - \frac{\alpha}{w} (c-b) \]
Application of the Proposed Ranking Function to Solve Fuzzy Fractional Programming (FFP) Problems

The Technical is suggested to solve a problem of fuzzy fractional programming utilizing fuzzy programming technique where the objective function coefficients are represented by triangular fuzzy numbers but the values of the right hand sides and the left hand sides variables are real numbers. Ranking approach is used in fuzzy fractional programming (FFP) problem to convert it in fuzzy programming (FP) problem. The approach of ranking algorithm for fuzzy fractional programming (FFP) problem is summarized as below:

Consider fuzzy fractional programming (FFP) problems is:

\[
\text{Maximize } Z(X) = \frac{cX + a}{dX + \beta}
\]

subject to,
\[
AX \leq b
\]
\[
x \geq 0.
\]

where \( A = (A1, A2, \ldots, An) \) is an \( m \) by \( n \) matrix;
\( c, d \text{ and } x \in \mathbb{R}^n, b, y \text{ and } \beta \) are scalars.

The proposed algorithm can be summarized as below:

1. Convert the fuzzy fractional programming problem into the following FP problem via new ranking function of triangular fuzzy number.

2. Transform the obtained FP problem into a model problem by using Charnes-Cooper transformation method.

Maximize \( Z(x) = \tilde{c}y + \alpha t \)
subject to,
\[
\tilde{d}y + \beta t = 1
\]
\[
Ay - bt \leq 0
\]
\[
y \geq 0, t \geq 0.
\]

3. Find the optimal solution \( y \) in step 2.

4. Obtain the optimal solution \( x \) using the value \( y \) in step 2.

5. The above algorithm is illustrated by numerical examples given in the next section a mathematical programming will utilize to get the optimal solution.

6. Compare a new ranking function with other ranking function type triangular fuzzy numbers to obtain the optimal solution of problems.

Numerical Examples

Two numerical fuzzy fractional programming problems are solved in this section with triangular fuzzy numbers, with the help of the suggested ranking functions. The FFP problems are transformed into a crisp programming model.

Example 1:- Consider the fuzzy fractional programming (FFP) problem

\[
\text{Max } Z = \frac{(3.6, 4, 5.6)x_1 + (7.6, 8, 9.6)x_2}{(0.6, 1, 2.6)x_1 + (1.6, 2, 3.6)x_2 + (2.6, 3, 4.6)}
\]

subject to
\[
2x_1 + x_2 \leq 10
\]
\[
3x_1 + 4x_2 \leq 26
\]
\[
x_1, x_2 \geq 0.
\]

Applying the proposed algorithm, Convert fuzzy programming problem into the following FFP problem using new ranking function of triangular fuzzy number

\[
1. R(\tilde{A}) = \frac{a + 3a + b + c}{16}
\]

We first substitute the ranking function of each fuzzy number in objective function, which leads to an equivalent FP problem

\[
\text{Max } Z = \frac{4.0750x_1 + 8.0750x_2}{1.0750x_1 + 2.0750x_2 + 3.0750}
\]

subject to
\[
2x_1 + x_2 \leq 10
\]
\[
3x_1 + 4x_2 \leq 26
\]
\[
x_1, x_2 \geq 0.
\]

Now, transformed this fractional programming problem into model programming problem via transformation of charnes cooper, express this model programming form as

\[
\text{Max } Z = 4.0750y_1 + 8.0750y_2
\]

subject to
\[
1.0750y_1 + 2.0750y_2 + 3.0750 t = 1
\]
\[
2y_1 + y_2 - 10t \leq 0
\]
\[
3y_1 + 4y_2 - 26t + y_5 = 0
\]
\[
y_1, y_2 \geq 0
\]

The standard form of programming problem as follows:

\[
\text{Max } Z = 4.0750y_1 + 8.0750y_2
\]

subject to
\[
1.0750y_1 + 2.0750y_2 + 3.0750 t = 1
\]
\[
2y_1 + y_2 - 10t + y_4 = 0
\]
\[
3y_1 + 4y_2 - 26t + y_5 = 0
\]
\[
y_1, y_2 \geq 0
\]

Thus using step 4, the optimized solution here is

\[
y_1 = 0, y_2 = 0.3925, t = 0.0604
\]
\[
x_1 = \frac{y_1}{t} = 0, x_2 = \frac{y_2}{t} = 6.4983
\]

In step 5, the objective function value is \( z = 3.1691 \).

In addition, using the ranking function (26) for a comparison between the above methods is made to show that the suggested method gives satisfactory
better results. A comparison can be also made using
the following ranking function
\[ R(\tilde{A}) = \frac{a+6b+c}{8} \]
The ranking function of each fuzzy number in
objective function, which leads to an equivalent FP
problem
\[ \text{Max } Z = \frac{4 \cdot 1500x_1 + 8 \cdot 1500x_2}{1.1500x_1 + 2.1500x_2 + 3.1500} \]
subject to
\[ 2x_1 + x_2 \leq 10 \]
\[ 3x_1 + 4x_2 \leq 26 \]
\[ x_1, \ x_2 \geq 0 \]
Now the FP problem is reduced to model
programming problem via transformation of
charsnes cooper, express this model programming
form as
\[ \text{Max } Z = 4 \cdot 1500y_1 + 8 \cdot 1500y_2 \]
subject to
\[ 1.1500y_1 + 2.1500y_2 + 3.1500t = 1 \]

**Example 2:** Consider the fuzzy fractional
programming (FFP) problem
\[ \text{Max } Z = \frac{(3.3,4,5.2)x_1 + (5.3,6,7.2)x_2}{(4.3,5,6.2)x_1 + (3.3,4,5.2)x_2 + (0.3,1,2,2)} \]
subject to
\[ 2x_1 + x_2 \leq 10 \]
\[ 3x_1 + 4x_2 \leq 26 \]
\[ x_1, \ x_2 \geq 0 \]
Applying the proposed algorithm, Convert fuzzy
programming problem into the following FFP
problem using new ranking function of triangular
fuzzy number
2. \[ R(\tilde{A}) = \frac{a+4b+c}{16} \]
The first step is to substitute the ranking function of
each fuzzy number in objective function, which
leads to an equivalent FP problem
\[ \text{Max } Z = \frac{4.0313x_1 + 6.0313x_2}{5.0313x_1 + 4.0313x_2 + 1.0313} \]
subject to
\[ 2x_1 + 4x_2 \leq 12 \]
\[ x_1 + 4x_2 \leq 9 \]
\[ x_1, \ x_2 \geq 0 \]
Now, transformed this FP problem into model
programming problem via transformation of
charsnes cooper, express this model programming
form as
\[ \text{Max } Z = 4.0313y_1 + 8.0313y_2 \]
subject to
\[ 5.0313y_1 + 4.0313y_2 + 1.0313t = 1 \]
\[ 2y_1 + 4y_2 - 12t \leq 0 \]
\[ y_1 + 4y_2 - 9t \leq 0 \]
\[ y_1, \ y_2 \geq 0 \]
The standard form of model programming problem
as follows:
\[ \text{Max } Z = 4.0313y_1 + 8.0313y_2 \]
subject to
\[ 5.0313y_1 + 4.0313y_2 + 1.0313t = 1 \]
\[ 2y_1 + 4y_2 - 12t + y_4 = 0 \]
\[ y_1 + 4y_2 - 9t + y_5 = 0 \]
\[ y_1, \ y_2 \geq 0 \]
In step 5, the objective function value \( z = 3.0934 \).
Afterwards, comparing optimized solution in Table
1.

| Value of the function | The solution of new ranking function | Exact error | 26 | Exact error |
|-----------------------|--------------------------------------|-------------|---|------------|
| 3.2501                | 3.1691                               | 0.0810      | 3.0934 | 0.1567 |

Note that, the new ranking function is more
optimized than the ranking function\(^{25}\).

The ranking function of each fuzzy number in
objective function, which leads to an equivalent FP
problem
\[ \text{Max } Z = \frac{4.0313x_1 + 6.0313x_2}{5.0313x_1 + 4.0313x_2 + 1.0313} \]
subject to
\[ 2x_1 + 4x_2 \leq 12 \]
\[ x_1 + 4x_2 \leq 9 \]
\[ x_1, \ x_2 \geq 0 \]
Now, transformed this FP problem into model
programming problem via transformation of
charsnes cooper, express this model programming
form as
\[ \text{Max } Z = 4.0625x_1 + 6.0625x_2 \]
subject to
\[ 5.0625x_1 + 4.0625x_2 + 1.0625 \]
subject to
\[2x_1 + 4x_2 \leq 12 \\
x_1 + 4x_2 \leq 9 \\
x_1, x_2 \geq 0 \]
Now, transform this FP problem into model programming problem via transformation of charnes cooper. Express this model programming form as
\[\text{Max } Z = 4.0625y_1 + 6.0625y_2 \]
subject to
\[5.0625y_1 + 4.0625y_2 + 1.0625t = 1 \\
2y_1 + 4y_2 - 12t \leq 0 \\
y_1 + 4y_2 - 9t \leq 0 \\
y_1, \ y_2 \geq 0 \]

The standard form of model programming problem as follows:
\[\text{Max } Z = 4.0625y_1 + 6.0625y_2 \]
subject to
\[5.0625y_1 + 4.0625y_2 + 1.0625t = 1 \\
2y_1 + 4y_2 - 12t + y_4 = 0 \\
y_1 + 4y_2 - 9t + y_5 = 0 \\
y_1, \ y_2 \geq 0 \]

Thus using step 4, the optimized solution is obtained
\[y_1 = 0, \ y_2 = 2.205, \ t = 0.0980, \]
\[x_1 = \frac{y_1}{t} = 0, x_2 = \frac{y_2}{t} = 2.2500 \]

In step 5, the objective function value \( z = 1.3369 \)

Afterwards, comparing optimized solution in Table 2.

| Value of the function | The solution of new ranking function | Exact error | 26 | Exact error |
|-----------------------|--------------------------------------|-------------|----|-------------|
| 1.3500                | 1.3434                               | 0.0066      | 1.3369 | 0.0131      |

Note that, the new ranking function is more optimized than the suggested ranking function in 26.

Conclusion

An efficient algorithm is proposed to solve the fuzzy fractional programming problem together with triangular fuzzy numbers. In this paper a novel ranking function has been used to convert fuzzy fractional programming problems to crisp fractional programming problems with simpler and easier calculations.

Author’s declaration:

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Technology.

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خوارزمية فعالة لمشكلات البرمجة الكسرية الخطية الضبابية عبر الدالة الرتبية

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الخلاصة:
في العديد من التطبيقات مثل الإنتاج، يعد التخطيط لصيانة القرار أمرًا مهمًا في تحسين دالة الهدف الضبابية للمسألة حيث تحتوي على نسبة ذاتين ضابطين، والتي يمكن أن تستخدم تقنية مسألة البرمجة الكسرية الضبابية. يتم النظر في فئة خاصة من تقنية التحسين تعني مسألة البرمجة الكسرية الضبابية في هذا العمل عندما تكون معاملات دالة الهدف للمسألة ضبابية. تم اقتراح دالة الترتيب الجديدة واستخدامها لتحويل بيانات مسألة البرمجة الكسرية الضبابية من رقم عام إلى رقم واضح بحيث يمكن تجنب التعقيد عند معالجة المسألة الضبابية الأصلية. هذا يتم من خلال وظيفة الترتيب الجديدة للأرقام الضبابية للتأثير على الأرقام الضبابية المثلثية مع حسابات أبسط وأسهل بالإضافة إلى تيسيرها في الإجراءات. يتم تحليل مشكلة البرمجة الكسرية الضبابية أولاً إلى مسألة البرمجة الكسرية ثم حلها باستخدام التقنية للحصول على الحل الأمثل. لديها القدرة على إعطاء أفضل حل لدعم نظرية الحل المفترضة في هذا العمل، يتم تضمين بعض مسائل التحليل مع الحلول الأخرى القائمة لدعم المساواة للقيود في مسائل البرمجة الكسرية الضبابية.

الكلمات المفتاحية: البرمجة الكسرية الضبابية، المجموعة الضبابية، العدد الضبابي الثلاثي الممتع، دالة الانتقاء، العدد الضبابي الثلاثي.