Nonlinear charge reduction effect in strongly coupled plasmas

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Received 13 April 2006
Accepted for publication 16 May 2006
Published 19 July 2006
Online at stacks.iop.org/PhysScr/74/288

Abstract

The charge reduction effect, produced by the nonlinear Debye screening of high-Z charges occurring in strongly coupled plasmas, is investigated. An analytic asymptotic expression is obtained for the charge reduction factor \((f_c)\) which determines the Debye–Hückel potential generated by a charged test particle. Its relevant parametric dependencies are analysed and shown to predict a strong charge reduction effect in strongly coupled plasmas.

PACS numbers: 51.50+v, 52.20−j, 52.27.Gr

1. Introduction

In this work we wish to investigate the phenomenon of charge reduction which affects the so-called Debye–Hückel–Yukawa (DHY) potential in strongly coupled locally Maxwellian plasmas. In particular, we intend to analyse the role of the local plasma sheath, i.e., the region of a plasma surrounding an arbitrary charged particle (test particle) where the plasma must be treated as a discrete system of charged particles (for simplicity this will be identified with a vacuum region). For definiteness, we shall assume that the test particles are high-Z, spherically symmetric charged particles which are assumed to be ‘small-sized’, i.e., such that the radius of each particle \((\rho_p)\) is much smaller than the radius of the plasma sheath \((\rho_o)\). In particular we intend to prove that for strongly coupled plasmas, the DHY effective charge can be estimated asymptotically by matching the DHY potential and the internal asymptotic solution of the Debye–Poisson equation, to be evaluated suitably close to the plasma sheath. As an application, the dependencies of the effective charge with respect to the relevant parameters are displayed. It is found that the effective charge which characterizes the DHY potential results reduced as a consequence of the nonlinear screening phenomenon produced by the background plasma outside the local plasma sheath.

2. Debye screening

The phenomenology of so-called Debye shielding is well known. It consists of the property of plasmas (or electrolytes [1]), either quasi-neutral or non-neutral, to shield the electrostatic field produced by charged particles (to be denoted as test particles) immersed in the same system. This result has fundamental consequences on the interactions occurring in plasmas, since it actually limits the range of static Coulomb field inside the Debye sphere, i.e., at a distance \(\rho \leq \lambda_D\) from the test particle, \(\lambda_D\) being the Debye length. As usual here \(\lambda_D \equiv (\sum_s \lambda_D^s)^{-1}\), where the sum is carried out on all plasma species and \(\lambda_D^s = \sqrt{T_s / 4\pi Z^2 s e^2 N_{os}}\), \(T_s\) and \(N_{os}\) being respectively the \(s\)-species temperature and number density (the latter defined in the absence of test particles). In fact, when both the particles and the plasma are assumed to be non-relativistic, small-amplitude, stationary (or slowly time- and space-varying), electrostatic perturbations generated by isolated test particles, are effectively shielded in the external domain, i.e., at distances larger than the Debye length \(\lambda_D\).

The renewed interest in this problem is particularly related to dusty plasmas or colloidal suspensions [2, 3] which are characterized by the presence of a large fraction of highly charged particles (grains), i.e., having an electric charge \(Z_d e\) with \(|Z_d| \gg 1\). It is well established that the phenomenon of Debye shielding of the electrostatic potential generated...
by a slowly moving or stationary test particle, when occurring in strongly coupled ionized gases, manifests peculiar properties. These are produced, as a consequence of the nonlinear plasma response occurring at distances smaller than the Debye length $\lambda_D$, when the electrostatic potential ($\Phi$) results locally such that $\Phi = \rho_0 / \lambda_D \sim 1$ (or even $\gg 1$), $\Phi$ being the normalized electrostatic potential evaluated at a suitable characteristic distance $\rho_0$, from the position of the test particle $r(t)$ and $T_b = T_0$. The distance $\rho_0$ is defined as the radius of the local plasma sheath around a test particle. For spherically symmetric test particles, such a domain can be identified with the spherical subset of the configuration space, of radius $\rho_0 = (3/4\pi N_\omega)^{1/3}$ (average interparticle distance), $N_\omega = N_{oi}$ being the density of the ion species, in which each test particle can be considered as isolated. Ionized gases (or electrolytes) can be classified according to the characteristic dimensionless parameters $x_0 = \rho_0 / \lambda_D \equiv \rho_0^{1/3}$ and $\gamma = \beta / x_0$ (where $\beta = Z_d e^2 / 4\pi T_0 \lambda_D$ is denoted as the dimensionless electric charge of the test particle), $g$ and $\Gamma$ being respectively so-called plasma and Coulomb coupling parameters. In particular, for plasmas the high-density requirement $x_0^2 = g \ll 1$ is satisfied by assumption and can be interpreted as the condition that the total number of mutually interacting particles in a Debye sphere (expressed by $1 / g$) is $\gg 1$. Instead, the ordering of the parameter $\Gamma$ remains in principle arbitrary. Thus, the orderings $\Gamma \ll 1$ and $\Gamma \sim 1$ (or $\gamma \gg 1$) correspond respectively to so-called weakly and strongly coupled plasmas.

Dusty and colloidal plasmas may be characterized by $Z_d$ up to $10^3$ with plasma temperature and ion density $T_0 \sim 1$ eV, $N_\omega \sim 10^5$–$10^{10}$ cm$^{-3}$. In this case, the Coulomb parameter for a negatively charged grain in the presence of the plasma sheath produced by hydrogen ions can be as large as $\Gamma \equiv (10^{-2} - 10^{-3}) Z_d$, while the dimensionless radius of the ion plasma sheath can be estimated $x_0 \approx 0.01$–0.05. Therefore, dusty plasmas are typically strongly coupled if $Z_d \gtrsim 1$. For these plasmas, it is important to be able to evaluate the electrostatic potential generated by dust grains which is expected to be strongly influenced by Debye shielding.

It is well-known that, in general, the Debye effect occurs provided suitable physical assumptions are introduced. In particular, the plasma must be assumed to be appropriately close to kinetic Maxwellian equilibrium, in which each particle species is described by a Maxwellian kinetic distribution function carrying finite fluid fields (defined respectively by the number density, temperature and flow velocity $(N, T, \mathbf{V})$). In the absence of test particles, these fluid fields must be assumed to be slowly varying in a suitable sense, or constant, both with respect to position ($r$) and time ($t$). In this regard, it is important to remark that the appropriate treatment of the plasma sheath surrounding each test particle is essential also for the validity of the mathematical model for the Debye screening problem (DSP), i.e., for the existence of classical solutions of the Debye screening problem, which do not exist when letting $x_0 = 0$ [4].

Another significant aspect concerns the issue of the absorption of plasma particles by the test particle, which effectively modifies the local charge density of the background plasma species [5–7]. Since the particle capture mechanism is a manifestly charge-dependent and velocity-dependent phenomenon (in particular it depends on the angular momentum of the incoming particle), it is obvious that in principle, it can produce deviations from local Maxwellian equilibrium [8, 9]. However, this phenomenon is expected to become relevant only if the radii of the test particle and of the surrounding plasma sheath are comparable (i.e., $p_0 / \rho_0$ is of the order of unity). Instead, it is negligible when $p_0 / \rho_0 \ll 1$. Since dusty and colloidal plasmas are characterized by typical grain size $p_0$ smaller than $10^{-5}$–$10^{-3}$ cm and radius of plasma sheath $\rho_0$ of the order of $10^{-2}$, these effects will be considered negligible.

3. The modified DSP

In this section, we adopt the modified DSP, previously introduced in [4]. According to such a model inside the plasma sheath (i.e., the domain of vacuum surrounding the test particle), it is assumed that no charge density is present, while in the external domain all plasma species (ions, electrons and dusty species) are assumed to be described by spatially homogeneous, local Maxwellian distributions. In such a case, one can immediately obtain the Debye–Poisson equation for the dimensionless electrostatic potential $\hat{\Phi}_x(x)$ generated by a test particle (dust grain) of charge $Z_d e < 0$. In particular, in the domain $x \geq x_0$, it can be proven to satisfy the integral Debye–Poisson equation

$$\hat{\Phi}_x(x) = \frac{\beta}{x} - \left[ \frac{1}{x} \int_{x_0}^{x} \mathrm{d}x' x'^2 + \int_{x_0}^{\infty} \mathrm{d}x' x' \right] \hat{S}(x', x_0).$$

This is obtained by imposing that $\hat{\Phi}_x(x)$ satisfies the boundary conditions

$$\left| \frac{\mathrm{d}\hat{\Phi}_x(x)}{\mathrm{d}x} \right|_{x = x_0} = -\Gamma \frac{\hat{S}(x, x_0)}{x_0},$$

$$\lim_{x \to \infty} \hat{\Phi}_x(x) = 0.$$  

In particular equation (1) yields for $x = x_0$, the constraint equation

$$\hat{\Phi}_0 = \Gamma - \int_{x_0}^{\infty} \mathrm{d}x' x' \hat{S}(x', x_0).$$

Here, $\hat{\Phi}_0 \equiv \hat{\Phi}_x(x_0)$, while in the case of a negatively charged test particle in a quasi-neutral three species (electron, ion and dust) plasma the source term $\hat{S}(x, x_0)$ reads

$$\hat{S}(x, x_0) = \Theta(x - x_0) (\xi_\varepsilon \exp[|Z_l| \hat{\Phi}_x(x)] - \xi_\varepsilon \exp[-|Z_l| \hat{\Phi}_x(x)] - \xi_\varepsilon \exp[|Z_d| \hat{\Phi}_x(x)]),$$

$$\Theta(x - x_0) - \varepsilon_\varepsilon \exp[-|Z_l| \hat{\Phi}_x(x)] - \xi_\varepsilon \exp[-|Z_d| \hat{\Phi}_x(x)] \sinh \hat{\Phi}_x(x).$$

Another useful representation of equation (1) can also be obtained by means of the transformation

$$\hat{\Phi}_x(x) = \hat{\Phi}_0 \exp[\gamma(x, x_0)].$$

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and there results, by consistency with the integral
Debye–Poisson equation (1), \( y(x_o, x_o) = 0 \) and \( y'(x, x_o) |_{x=x_o} = -\alpha \), with \( \alpha = \Gamma/x_o \Phi_o \). This delivers for \( y(x, x_o) \) the
integral equation
\[
y(x, x_o) = y_o(x, x_o) + F(x, x_o), \quad (8)
\]
\[
y_o(x, x_o) = -\alpha (x - x_o), \quad (9)
\]
where \( F(x, x_o) \) is the solution of the integral equation
\[
F(x, x_o) = \int_{x_o}^x dx' (x - x') \times \left[ S(x', x_o) - y(x', x_o)^2 \right] - 2 \times y(x', x_o), \quad (10)
\]
with \( S(x', x_o) \) expressed in terms of \( y(x', x_o) \) by means of
equation (5). On the other hand, for finite \( x_o \), the
DHY asymptotic approximation for the electrostatic potential
can be proven to hold when measuring the potential at
a position \( r \) sufficiently far from the position of the test
particle \( r(t) \). In fact this involves imposing that the so-called
weak-field condition \( \Phi_o(x) \ll 1 \) be locally satisfied. As a
consequence, the normalized electrostatic potential
generated by a spherically symmetric point particle of charge \( Ze \)
\[\Phi_s(x) \equiv \Phi_{s,0}(x) = q/4\pi \rho \equiv \text{DH potential.} \]
\[\Phi_{s,0}(x) \equiv \Phi_{s,0}^{\text{ext}}(x) \equiv \frac{q}{4\pi \rho} e^{-\Delta x}, \quad (11)\]
where \( \Delta x \equiv x - x_o \), \( q = q(x_o, \beta) \) is a suitable dimensionless
effective electric charge and \( \Phi_{s,0}^{\text{ext}}(x) \) is usually known as DH
potential. The latter identifies the external asymptotic solution
which holds in the subset of the external domain, outside the
Debye sphere, in which the weak-field condition is satisfied.
The remaining notation is standard, thus \( \rho = |r - r(t)| \) is the
distance from the point charge and \( x \equiv \rho/\lambda_D \in [x_o, \infty) \) is the
Corresponding normalized distance. The weak-field condition
is manifestly locally satisfied both for strongly and weakly
coupled plasmas, provided \( x > 1 \), with \( x \) suitably large (i.e.,
actually \( x \gg 1 \)). However, in the case in which either \( x_o \ll 1 \)
and/or \( \Gamma \gg 1 \), i.e., for strongly coupled plasmas, the electro-
static potential may actually decay on a scale much shorter
than the Debye length \( \lambda_D \) and precisely on a scale suitably
close to the boundary of the plasma sheath, i.e., for \( x_o \lesssim x \ll 1 \).
This conclusion is consistent both with previous analytic
estimates based on the exact solution of the one-dimensional
(1D) DSP [10] and direct numerical simulations of the 3D
DSP [11–14]. It implies, however, that the DH approximation

typically holds also at distances suitably larger than the
boundary of the plasma sheath \( x_o \) and which extend also to
values comparable to the Debye length, i.e., for \( x_o \ll x \lesssim 1 \).
Finally, as a further consequence, the effective electric charge
carried by the DH potential \( q \) may appear significantly
reduced with respect to the case of weakly coupled plasmas
[11, 13–17]. To display this effect, it is convenient to
represent the normalized effective charge of the test particle
in the form \( q = \beta f_c \), \( f_c \) being the charge reduction factor.

By dimensional analysis, it follows that \( f_c \) must be of
the form \( f_c \equiv f_c(x_o, \beta, \xi_o) \), i.e., \( f_c \) is a function, to be
determined, of the only dimensionless parameters of the prob-
lem, namely \( x_o \) and \( \Gamma \) (or \( \beta \)) and moreover the ratios \( \xi_o \) (with
\( s = e, i, d \)). It is well-known that for weakly coupled plasmas
\( q \) can be approximated by the normalized electric charge of
the isolated test particle, i.e., \( q \equiv \beta \), which implies \( f_c \equiv 1 \).
Instead, for strongly coupled plasmas, the value of the charge
reduction factor, determined either via 1D analytic estimates
[10] or by means of a variety of numerical simulation methods
(see, for example, [11–14]), has been found to be notably
smaller than unity, thus suggesting the existence of a possible
significant charge reduction effect. Indeed the investigation
of the effective interactions characterizing high-Z grains
in plasmas has attracted interest in recent years especially for
their role in dusty plasmas [16, 17]. However, for strongly
coupled plasmas the precise form of the function \( f_c \) is
still unknown and, in particular, an analytic estimate of the
effective charge characterizing the DH potential in strongly

coupled plasmas is not yet available.

Besides being of obvious interest as a still unsolved
mathematical problem, it is especially important from
the physical standpoint to estimate \( f_c \) as a function of
the parameters \( x_o \) and \( \Gamma \) for strongly coupled plasmas
characterized either by a high density and/or by the presence
of a large fraction of highly charged particles (grains), such as
dusty plasmas and colloidal suspensions [2, 3].

In a previous work [4], the DSP has been formulated
for an electron–hydrogen plasma in order to analyse the
asymptotic properties of its solutions near the plasma sheath
(i.e., for \( x \) suitably close to \( x_o \)). For this purpose, the case
of a strongly coupled plasma was investigated in which the
parameters \( x_o \) and \( \Gamma \) satisfy the asymptotic conditions, to be
denoted as strong-coupling ordering,
\[\Gamma \sim \frac{1}{O(\delta^2)} \gg 1 \quad (12)\]
\( \text{with } x_o \sim O(\delta^k), \quad k = 0, 1 \text{ and } \delta \text{ an infinitesimal. The similar } \)
case of a three-species plasma can be obtained directly. As
a result, invoking for \( \Phi_s(x) \) the representation (5), together
with equations (9) and (10) and imposing the appropriate
boundary conditions for \( \Phi_s(x) \), it follows that the integral
on the right-hand side of equation (4) can be estimated
asymptotically to yield an asymptotic equation for the initial
condition \( \Phi_{s,0} \), i.e.,
\[\Phi_{s,0}(x_o, \beta) \equiv \frac{1}{|Z_i|} \ln \left\{ \frac{|Z_i| \Gamma}{\xi_i x_o^2 s \xi_o (\Gamma - \Phi_{s,0})} \right\}. \quad (13)\]
In particular, for an electron–hydrogen plasma in which
the contribution to \( \lambda_D \) due to the dusty species is negligible,
there results \( |Z_i| = 1 \) and \( \xi_i = 1/2 \) and this reduces (13)
to the expression given in [4]. In addition, neglecting higher-
order infinitesimals of order \( O(\delta^4) \) (or \( r > 0 \) being a suitable real
number), the following inequality can immediately be proven
\[f_c \leq c^{(a)}(x_o, \beta) \equiv x_o \Phi_{s,0}(x_o, \beta), \quad (14)\]
\[c^{(a)}(x_o, \beta) \equiv x_o \Phi_{s,0}(x_o, \beta), \quad (15)\]
4. Asymptotic approximation for the effective charge

In this paper, we propose a more accurate asymptotic approximation for \( f_c \), potentially useful to analyse the charge reduction effect of high-\( Z \) test particles taking place in strongly coupled plasmas. In particular, we intend to prove that, in validity of the previous strong-coupling ordering, the charge reduction factor can be approximated in the form

\[
f_c(x_o, \beta) \equiv c^{(b)}(x_o, \beta) \equiv x_o e^{\Delta x_c} \Phi^{(\text{int})}_c(x_c) / \beta.
\]  
(16)

where, \( x_c \) is defined so that \( x_c > x_o \) and is provided by the root of the equation

\[
y'(x_c, x_o) = -1 - \frac{1}{x_c}.
\]  
(17)

Here, \( \Phi^{(\text{int})}_c(x) \) denotes the leading-order contribution to the internal asymptotic solution of the Debye–Poisson equation (1), which can be obtained from the representation (5), (9), (10) and the integral equation (6) and holds in an appropriate neighbourhood of the plasma sheath \( (x_c) \). There results

\[
\Phi^{(\text{int})}_c(x) \equiv \Phi^{(\text{ext})}_c(x),
\]  
(20)

while \( y'(x, x_o) \) reads

\[
y'(x, x_o) \equiv -\alpha - \alpha^2 \Delta x + 2\alpha \log \frac{x}{x_o} + \frac{\xi_1}{\Phi_o} \int_x x \exp[-\alpha \Delta x'] + \alpha \Delta x'] + \exp \left\{ 2\alpha^2 \Delta x'^2 - 2\alpha \left[ x' \log \frac{x'}{x_o} - \Delta x' \right] \right\},
\]  
(19)

where \( \Delta x' \equiv x' - x_o \). Equations (16) and (17) uniquely follow by imposing suitable matching conditions between the internal and external asymptotic solutions, \( \Phi^{(\text{ext})}_c(x) \) and \( \Phi^{(\text{int})}_c(x) \), precisely

\[
\frac{\partial}{\partial x} \Phi^{(\text{ext})}_c(x) \bigg|_{x=x_c} = \frac{\partial}{\partial x} \Phi^{(\text{int})}_c(x) \bigg|_{x=x_c}.
\]  
(21)

One can immediately prove that equation (21) is equivalent to the stationary condition for the function \( f_c^{(x)}(x, x_o, \beta) \)

\[
\frac{\partial}{\partial x} f_c^{(x)}(x, x_o, \beta) \bigg|_{x=x_c} = 0.
\]  
(22)

Therefore equation (22) also furnishes the same asymptotic approximation for the charge reduction factor given by equation (16). The accuracy of the asymptotic approximation \( c^{(b)}(x_o, \beta) \) and of the upper bound \( c^{(a)}(x_o, \beta) \) can be tested by comparing them with the estimate of the charge reduction factor obtained from direct numerical simulations of the 3D DSP. For this purpose the function

\[
g(x, x_o, \beta) \equiv x e^{x-x_c} \Phi^{(\text{int})}_c(x)/\beta
\]  
(23)

has to be evaluated numerically by solving the Debye–Poisson equation. The charge reduction factor is defined as the limit

\[
f_c(x_o, \beta) = \lim_{x \to +\infty} x e^{x-x_c} \Phi^{(\text{int})}_c(x)/\beta.
\]  
(24)

Figure 1. Comparison between the asymptotic approximation of the charge reduction factor \( c^{(b)}(x_o, \beta) \), the numerical estimate \( f_c(x_o, \beta) \) and the upper bound estimate \( c^{(a)}(x_o, \beta) \). Case (a) corresponds to \( x_o = 0.08 \) and \( \beta = 2.5 \) for which there results \( x_c \approx 0.098, c^{(b)}(x_o, \beta) \approx 0.211, f_c(x_o, \beta) \approx 0.246 \) and \( c^{(a)}(x_o, \beta) \approx 0.33 \). Case (b) corresponds to \( x_o = 0.15 \) and \( \beta = 10 \) for which there results \( x_c \approx 0.171, c^{(b)}(x_o, \beta) \approx 0.106, f_c(x_o, \beta) \approx 0.119 \) and \( c^{(a)}(x_o, \beta) \approx 0.19 \).

In practice, for numerical estimates it can be approximated by \( f_c(x_o, \beta) \approx g(x_f, x_o, \beta) \) with \( x_f \) to be suitably defined. It is important to stress that, in the limit \( \delta \to 0 \), one expects \( x_c \to 0, x_f \to 0 \) and moreover

\[
\lim_{\delta \to 0} c^{(b)}(x_o, \beta) = \lim_{\delta \to 0} f_c(x_o, \beta).
\]  
(25)

In addition, it must also result that

\[
\lim_{\delta \to 0} c^{(b)}(x_o, \beta) = \lim_{\delta \to 0} c^{(a)}(x_o, \beta).
\]  
(26)

Therefore, for \( \delta \) small enough (i.e., for \( \Gamma \gg 1 \) and/or \( x_o \ll 1 \)), it is actually possible to approximate \( f_c(x_o, \beta) \) both in terms of \( c^{(b)}(x_o, \beta) \) and \( c^{(a)}(x_o, \beta) \). Numerical simulations indicate, however, that the asymptotic estimate \( c^{(b)}(x_o, \beta) \) actually converges rapidly to \( f_c(x_o, \beta) \) and furnishes therefore a more accurate approximation than the upper bound \( c^{(a)}(x_o, \beta) \). Nevertheless, particularly thanks to equation (26), \( c^{(b)}(x_o, \beta) \) can still be used for order-of-magnitude estimates.

In figure 1, the result of two numerical simulations are reported, for the case of an electron–hydrogen plasma and for \( \xi_1 = 1/2 \), which correspond respectively to \( (x_o, \beta) = (0.08, 2.5) \) and \( (x_o, \beta) = (0.15, 10) \). In both cases the matching point \( x_c \) is found to be close to the boundary of the plasma sheath, which implies that the DH potential actually approximately applies for \( x \gg x_c \). The upper bound value
\[ c^{\text{f}}(x_o, \beta) \] can be used for order-of-magnitude estimates of the charge reduction effect. Finally, the parametric dependencies of \( c^{\text{f}}(x_o, \beta) \) with respect to \( \beta \) and \( x_o \) are displayed for selected values of the parameters. Figure 2 demonstrates that for typical values of the Coulomb parameter \( \Gamma \) and the radius of the plasma sheath \( x_o \), the effective electric charge of the DH potentially appears dramatically reduced.

5. Conclusions

The present theory is relevant for the investigation of strongly coupled plasmas, such as dusty plasmas, in which a significant fraction of high-Z charged particles occur in a plasma suitably close to kinetic equilibrium. The charge reduction effect has fundamental consequences on the Debye screening phenomenon, since it actually limits the magnitude of the Coulomb interactions produced by these particles. The estimate of the charge reduction factor \( f_c \), here obtained, is a necessary prerequisite for the description of a variety of kinetic processes (for example, condensation \[18\], collisional processes \[19, 20\], etc), as well as electrostatic or electromagnetic instabilities in dusty plasmas (see for example \[21, 22\]).

However, the question arises of its relevance for dusty plasmas in which the background species (ions and electrons) deviate significantly from local Maxwellian equilibria. Our conclusions indicate that the charge reduction effect is largely independent of the shape of the kinetic distribution function of the dust-particle species. Moreover, since the effect is mostly produced by the nonlinear interaction with the oppositely charged background plasma species (i.e., the ion species in the case of negatively charged dust particles), the electron distribution function remains also largely arbitrary. Thus, the critical requirement for the existence of the effect ultimately involves only the ion distribution function, assumed to be described by a local Maxwellian distribution, slowly varying in a suitable sense. Since deviations from local Maxwellian equilibrium for the ion species can arise, in particular due to ion capture by the test particle when \( \nu_i/\rho_i \sim O(1) \), this requires also that \( \nu_i/\rho_i \ll 1 \). Provided such conditions are satisfied, the charge reduction factor for a test particle immersed in a strongly coupled plasma can be estimated by means of the asymptotic approximation \(24\).

Acknowledgments

Work supported by PRIN Research Program ‘Programma Cofin 2004: Modelli della teoria cinetica matematica nello studio dei sistemi complessi nelle scienze applicate’ (MIUR Italian Ministry), the ICTP/TRIL Program (ICTP, Trieste, Italy), and the Consortium for Magnetofluid Dynamics, University of Trieste, Italy.

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