Modeling the stress state of a rotating flexible thread (cable) to optimize its speed

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Abstract. The aim of this work was to develop and verify the operability of the method to determine the maximum permissible rotation frequency of a flexible thread (cable), at which a centrifugal inertia force arises, causing stresses and strains not exceeding the permissible limits. Based on the selected assumptions, the research methodology provides for the development of a method of analytical strength calculation of a rotating cable. The resulting system of expressions is the basis for the development of a computer program to be checked for operability by the accuracy of calculations. The calculation method, reflected in the form of a flowchart and implemented as a computer program, allows determining the maximum possible frequency of rotation of a cable that does not allow its limit state to occur, which will simplify the search for the optimal solution and reduce the time spent. Solving the nonlinear programming problem of determining the frequency of rotation of a cable taking into account the development zone of plastic strains in the material at the given maximum permissible stresses and strains by reducing it to the problem of unconditional optimization using Lagrange multipliers allows us to calculate with an error of not more than 4\% compared with the FEM. The calculation results make it possible to use the proposed method both when there is a restriction on the strength condition (allowable stresses) of the material of the cable, and on the allowable deflection in the middle of the span of the cable.

1. Introduction
In nature and human activity, mechanical bodies are widespread, their structure containing elements that subjected to tensile loads, but being very flexible when bent [1]. Similar structural elements are included in the equipment both in industry [2] and energy [3,4], and in transport [5] and science (including space objects [6]). Cables of various designs are used for lifting and loading operations [4,7], where both wind loads and impacts are possible. Energy is transmitted over long distances via wires suspended on supports [8], where vibrational loads and deformations occur under the influence of wind. The limiting elements of the roadway (barriers) at bends also use flexible elements in the form of cables [9]. In this case, a transverse shock load is threatened [10], sometimes leading to the breaking of barrier cables. Flexible threads (cables) are also used in agriculture, for example, in the design of a honey extractor when taking honey from honeycombs located on a frame with reinforcing threads [11]. Flexible elements are also the basis of various fabrics made from threads [12]. During weaving operations of their production, rotational movements and sometimes shock loads act on the threads.

These flexible ropes can work and be calculated for zones of their working capacity, both for loads in the zone of elastic deformations [1, 5], and leave this zone when residual deformations appear [2,
Flexible elements can be both monolithic in cross-section [13, 14], and can consist of a combination of many threads [1, 9].

Existing functional models for calculating flexible elements have both an experimental basis with regression processing [15], traditional force analysis [1, 14], as well as computer numerical models based on the finite element method [4]. The latter option requires large financial costs for the acquisition of complex modern computer systems and powerful specialized settlement systems. A prerequisite is the calibration of the model with experimental data. The classical calculation approach allows you to implement the calculation on the basis of simpler computer programs, however, it requires the development of these specialized methods and methods for calculating specific design parameters and their operating modes, as well as specialized computer programs for similar calculations.

Taking into account the widespread use of winding flexible elements in the electrical and textile industries, the development of software products allowing the calculation of the initial critical conditions for designing a rotor drive system for winding threads in the textile industry and engineering is important.

In software systems implementing the finite element method, modeling the behavior of any structural element including a cable begins with the assignment of the stiffness characteristics of the cross-sections, boundary conditions, as well as the loads acting on the simulated mechanical system. In this case, the determining load is the centrifugal force, which depends on the frequency of rotation of the rotor of the drive. It is known that with increasing speed, the tension of the cable also increases. In this case, upon reaching a certain value, there occurs a limit state, after which the thread breaks or unacceptable deformations occur that impede further operation.

The aim of this work was to develop a method for determining the maximum permissible rotation frequency of a flexible thread, at which a centrifugal inertia force arises, causing stresses and strains not exceeding the permissible limits.

2. Research methods

Based on the selected assumptions, the research methodology provided for the development of a method of analytical strength calculation of a rotating cable. The resulting system of expressions was the basis for the development of a computer program to be tested for operability by the accuracy of calculations.

We considered a rotating flexible thread (cable), initially having a straight line, located at an angle to the axis of rotation at a given distance, and mounted on elastically flexible supports. The design scheme is presented in Figure 1.

When calculating, the following assumptions were made: the kinetic energy of the rotational motion completely goes into work on changing the position of the flexible thread, including elastoplastic deformation and kinematic movement; gravity and air resistance can be neglected; only one centrifugal inertia force acts on a flexible thread during rotation. When solving the objective, as the objective function we use the longitudinal force, the value of which should be maximized.

The initial formula for determining the longitudinal force is [13]:

$$T(H,q,x) = \left[ H^2 + (Q(q,x) + H\cdot\tan(\beta))^2 \right]^{1/2}$$ (1)

where $H$ – thrust, N; $q$ – transverse uniformly distributed load, which is the equivalent of centrifugal inertia, N/m; $x$ – current abscissa ($0 \leq x \leq l$), m; $l$ - distance between the supports of the thread along the axis of rotation, span, m; $Q(q,x)$ – function of the transverse force in an equivalent pivotally supported beam with a span $l$ depending on the action of centrifugal inertia, N; $\beta$ – chord angle AB (Figure 1), degrees.

The centrifugal inertia force during the rotational movement of the thread changes, since the radius vector of the flexible thread in the rotating coordinate system changes. This is caused by the appearance of a deflection of the final equilibrium line. In this regard, the transverse uniformly
distributed load and thrust are included in the function parameters, since they are not known at the time of determining the longitudinal force.

Figure 1. Flexible thread design.

Flexible thread design:

- initial state of the line of equilibrium;
- final state of the line of equilibrium

3. Research result
The necessary restrictions without which a flexible thread (cable) cannot exist are:

1. Strength condition [10]:

\[
T(H,q,x_1) \leq [\sigma] \cdot A
\]

where \(T(H,q,x_1)\) – the value of the longitudinal force function in the section with a given abscissa \(x_1\), N; \([\sigma]\) – the maximum permissible material stress, Pa; \(A\) – the cross-sectional area, \(m^2\).

2. The condition of deformation continuity [14]:

\[
L_0 + \Delta L(H,q) = L_1(H,q)
\]

where \(L_0\) – the initial length, m; \(\Delta L(H,q)\) – the elastoplastic deformation, m; \(L_1(H,q)\) – the final length, m.

3. Law of energy conservation. The kinetic energy of a rotating flexible thread (cable) is completely transferred to the work of changing its position:

\[
\int \left(\frac{n \cdot q^2}{2} \cdot J(H,q) \cdot x \cdot \tan(\beta) + w(H,q,x)\right) dx
\]

where \(n\) – the rotation frequency, \(sec^{-1}\); \(J(H,q)\) – the function of moment of inertia about the axis of rotation, \(kg \cdot m^2\); \(w(H,q,x)\) – the deflection function, including elastoplastic deformation and kinematic movement, m; \(l\) – the span, m.

An optional, but possible restriction on the behavior of the structure under the action of the centrifugal inertia force resulting from rotation is a condition for not exceeding the maximum
allowable deflection of the final equilibrium line of the flexible thread in cross-section with a given abscissa $x_2$:

$$w(H,q,x_2) \leq [w]$$  \hspace{1cm} (5)

where $w(H,q,x_2)$ – the value of the deflection function in a section with a given abscissa $x_2$, m; $[w]$ – the maximum permissible deflection, m.

Limitations resulting from maintaining the engineering meaning of the task:

$$H>0$$  \hspace{1cm} (6)

$$q>0$$  \hspace{1cm} (7)

$$n>0$$  \hspace{1cm} (8)

The internal forces arising from bending in a single-span beam under the action of a uniformly distributed load were recorded mathematically:

$$M(q,x) = x \left( \int_0^l qdx - \int_0^1 qdx \right) - \frac{x}{2} \int_0^1 x \cdot qdx + \int_0^l x \cdot qdx$$  \hspace{1cm} (9)

$$Q(q,x) = \int_0^l qdx - \frac{1}{2} \int_0^1 x \cdot qdx - \frac{1}{2} \int_0^1 x \cdot qdx$$  \hspace{1cm} (10)

The deflection is a function of the abscissa and is the difference between the ordinates of the initial and final equilibrium lines of the cable in the considered section:

$$w(H,q,x) = y(H,q,x) - y_0(x)$$  \hspace{1cm} (11)

where $y(H,q,x)$ – the function of the line of equilibrium depending on the action of centrifugal inertia, m; $y_0(x)$ – the function of the equilibrium line of the initial outline of the cable, m.

When determining the deformed state of a cable, the rules for constructing a bending-moment curve for a beam are applied. The equilibrium line of the cable under the load coincides with the diagram of bending moments of an equivalent articulated beam with the same span under the same load as the flexible thread; while the ordinates of the plot of the moments are reduced by dividing by the magnitude of the spread and are postponed from the chord AB connecting the suspension points of the cable. Mathematically, this can be written as [14]:

$$y(H,q,x) = M(q,x)H^{-1} + x \cdot tg\beta$$  \hspace{1cm} (12)

where $M(q,x)$ – the function of the bending moment in an equivalent pivotally supported beam with a span $l$ depending on the action of centrifugal inertia, N⋅m.

A flexible thread, initially straightforward, having an initial length not exceeding the span and working to perceive the transverse load, is a string. In this case, the equation of the equilibrium line will have the form [10]:

$$y_0(x) = x \cdot tg\beta$$  \hspace{1cm} (13)

The initial length of the flexible thread is determined by the expression [14]:

$$L_0 = \int_0^l \left[ 1 + \left( \frac{d}{dx} y_0(x) \right)^2 \right]^{1/2} dx$$  \hspace{1cm} (14)

As a calculated state diagram of the material, which determines the relationship between stresses and relative deformations, we take a two-segmented strain diagram presented in [14], which, depending on the angle of inclination of the second segment, can act as a stress diagram with linear hardening and without a yield point or in the form of a Prandtl diagram (elastoplastic body). In this case, the main parametric points of the diagram must be indicated, which are the maximum stresses and the corresponding relative deformations, boundary values, etc. [13].

With the adopted deformation diagram, the elongation of the flexible thread is determined by the formula [13]:

\[ \text{...} \]
\[ \Delta L(H,q) = \begin{cases} \sigma(H,q)H_0E^{-1}, & \sigma(H,q) \leq \sigma_1 \\ L_0 \left( [\sigma(H,q) - \sigma_1]^2 + \varepsilon_1 \right), & \sigma_1 < \sigma(H,q) \end{cases} \]  \tag{15}

where \(\sigma(H,q)\) – the tensile stresses, Pa; \(E_1\) – the hardening module on the 2nd (next) segment of the deformation diagram, Pa.

Tensile stresses are determined from the expression \([11]\):

\[ \sigma(H,q) = H(A \cdot L_0)^{-1} \int_0^1 1 + \left( Q(q,x)H^{-1} + \tan \beta \right)^2 \, dx, \]  \tag{16}

The material hardening modulus in the 2nd (next) segment of the deformation diagram can be determined from the equation \([14]\):

\[ E_1 = \left( \sigma_2 - \sigma_1 \right) \left( \varepsilon_2 - \varepsilon_1 \right)^{-1} \]  \tag{17}

The length of the cable corresponding to the final shape under the action of centrifugal inertia is written in the form \([13]\):

\[ L_1(H,q) = \int_0^{\frac{H}{2}} \left[ 1 + \left( \frac{Q(q,x)}{H} + \tan \beta \right)^2 \right]^{1/2} \, dx \]  \tag{18}

where \(u\) – stiffness of elastically flexible supports, N/m.

During the rotational motion around the axis, the moment of inertia of the cable changes due to its deformation, therefore, the moment of inertia should be written as a function of deflection:

\[ J(H,q) = m \int_0^{\frac{H}{2}} (R + x \cdot \tan \beta + w(H,q,x))^2 \, dx \]  \tag{19}

where \(m\) – linear mass, kg/m; \(R\) – the distance to the axis of rotation, m.

The solution that provides the maximum is made using the objective function which is determined by the desired parameters. We write the objective function in the form:

\[ f(H,q) = T(H,q,0) \rightarrow \max \]  \tag{20}

Moreover, the optimization problem contains a number of constraints. These constraints are given by the combination of the above equations, which are equalities and not equalities. The constraints in the form of equalities come from the condition of continuity of deformations and the law of conservation of energy.

The first limitation is:

\[ g_1(H,q) = L_0 + \Delta L(H,q) - L_1(H,q) = 0 \]  \tag{21}

The second limitation is expressed by the dependency:

\[ g_2(H,q,n) = \left( \frac{n \cdot \pi}{30} \right)^2 \cdot J(H,q) \cdot 2^{1/2} \cdot \int_0^1 (R + x \cdot \tan \beta + w(H,q,x)) = 0. \]  \tag{22}

The constraints in the form of inequalities are reduced to the restrictions in the form of equalities. To do this, we introduce additional variables \(z1\) and \(z2\), and since they are not known at the time of solving the task, we will add them to the available parameters of the specified restriction functions. In this case, the restriction on the permissible deflection will take the form:

\[ g_3(H,q,z_1) = [w] - w(H,q,x_2) + z_1^2 = 0 \]  \tag{23}

In turn, the strength condition is written as follows:

\[ g_4(H,q,z_2) = [\sigma] \cdot A - T(H,q,x_1) + z_2^2 = 0. \]  \tag{24}

\[ \Delta L(H,q) = \begin{cases} \sigma(H,q)H_0E^{-1}, & \sigma(H,q) \leq \sigma_1 \\ L_0 \left( [\sigma(H,q) - \sigma_1]^2 + \varepsilon_1 \right), & \sigma_1 < \sigma(H,q) \end{cases} \]  \tag{15}
In order to reduce the considered nonlinear programming task to the unconditional optimization problem, we introduce the Lagrange multipliers and write the function:

\[
L(H,q,n,\lambda_1,\lambda_2,\lambda_3,\lambda_4,z_1,z_2) = f(H,q) + \lambda_1 g_1(H,q) + \lambda_2 g_2(H,q,n) + \lambda_3 g_3(H,q,z_1) + \lambda_4 g_4(H,q,z_2)
\]  (25)

where \(\lambda_1\), \(\lambda_2\), \(\lambda_3\) and \(\lambda_4\) – Lagrange multipliers.

Obviously, whatever the values of the Lagrange multipliers are, when constraints (21)-(24) are satisfied, the value of the function (25) coincides with the value of the function (20), therefore, the necessary condition for the maximum of the function is that its gradient is equal to zero:

\[
\nabla_{H,q,n,\lambda_1,\lambda_2,\lambda_3,\lambda_4,z_1,z_2} L(H,q,n,\lambda_1,\lambda_2,\lambda_3,\lambda_4,z_1,z_2) = 0
\]  (26)

The condition that the gradient of the Lagrange function is equal to zero can be represented as a system of equations:

\[
\begin{align*}
\frac{d}{dh} f(H,q) + \lambda_1 \frac{d}{dh} g_1(H,q) + \lambda_2 \frac{d}{dh} g_2(H,q,n) + \lambda_3 \frac{d}{dh} g_3(H,q,z_1) + \lambda_4 \frac{d}{dh} g_4(H,q,z_2) &= 0 \\
\frac{d}{dq} f(H,q) + \lambda_1 \frac{d}{dq} g_1(H,q) + \lambda_2 \frac{d}{dq} g_2(H,q,n) + \lambda_3 \frac{d}{dq} g_3(H,q,z_1) + \lambda_4 \frac{d}{dq} g_4(H,q,z_2) &= 0 \\
\frac{d}{dn} f(H,q) + \lambda_1 \frac{d}{dn} g_1(H,q) + \lambda_2 \frac{d}{dn} g_2(H,q,n) + \lambda_3 \frac{d}{dn} g_3(H,q,z_1) + \lambda_4 \frac{d}{dn} g_4(H,q,z_2) &= 0 \\
\lambda_1 \frac{d}{dz_1} g_1(H,q) + \lambda_2 \frac{d}{dz_1} g_2(H,q,n) + \lambda_3 \frac{d}{dz_1} g_3(H,q,z_1) + \lambda_4 \frac{d}{dz_1} g_4(H,q,z_2) &= 0 \\
\lambda_1 \frac{d}{dz_2} g_1(H,q) + \lambda_2 \frac{d}{dz_2} g_2(H,q,n) + \lambda_3 \frac{d}{dz_2} g_3(H,q,z_1) + \lambda_4 \frac{d}{dz_2} g_4(H,q,z_2) &= 0
\end{align*}
\]  (27)

To find the stationary point of the Lagrange function, it is necessary to solve a system of nine equations with nine unknowns. This can be done by one of the numerical methods for solving systems of nonlinear equations, for example, by the conjugate gradient method.

The general sequence of actions to optimize the parameters of the objective function is presented in the form of a flowchart in Figure 2. The proposed method is implemented in the Python 3 programming language using numerical solution schemes in the form of a problem-oriented software package [16].

To assess the adequacy of the results obtained using the proposed method, a comparative analysis of the main parameters of the stress-strain state of the thread, as well as the rotational speed at which the centrifugal inertia occurs, causing the found stress and strain in the flexible thread, is carried out. For this, a direct and then inverse problem are solved. The direct problem of determining the stress-strain state of a flexible thread at a given rotation frequency is solved using the well-known finite element method. The inverse problem of determining the frequency of rotation of a flexible thread is solved using a program [16] that implements the proposed method, equating to the allowable stresses and strains in given sections of the flexible thread the corresponding parameter values found in solving the direct problem by the finite element method.

As an object of study, we consider a rotating flexible thread, the design scheme of which is shown in Figure 3, with the span l=1 m, the section A=7.0686·10^{-6} m^2 (diameter D=3 mm) with linear mass m=0.0555 kg/m (density \(R_0 = 0.07701\) MN/m^3), initially rectilinear \(f_0=0\) m with fixed supports \(u\rightarrow\infty\) N/m, located at the same level \(\beta=0\) deg, and the distance to the axis of rotation \(R=0.5\) m. Taking into account the physically non-linear behavior of the material is modeled by a piecewise-linear relationship between stresses and strains. Parameters of the law of nonlinear deformation shown in Figure 3, are expressed in stresses \(\sigma_1=600\) MPa, \(\sigma_2=800\) MPa and the corresponding relative strains \(\varepsilon_1=0.0029, \varepsilon_2=0.024\).
To conduct a comparative analysis, we solve a number of tasks, at a rotation speed of \( n=1000 \) rpm and \( n=2400 \) rpm, to obtain stresses and strains on each segment of a given deformation diagram. In this case, the transverse uniformly distributed load \( q \), which is the equivalent of the centrifugal inertia force, will be \( q=304.31 \) N/m and \( q=1752.84 \) N/m, respectively.

When conducting a comparative analysis for the main criteria, we take the maximum longitudinal force in the flexible thread (cable) arising in the sections located on the supports \( T(H,q,0) = T(H,q,l) \), the deflection in the middle of the span \( w(H,q,l/2) \), the rotation speed \( n \) and transverse uniformly distributed load \( q \).
The finite element calculation was performed in the Russian-language software package LIRA-SAPR version 2015 release R4. A flexible thread was modeled by universal spatial bar finite elements taking into account physical and geometric nonlinearity (type 410), the breakdown was carried out on 100 finite elements. The calculation was carried out by a non-linear step processor designed to solve physically and geometrically non-linear, as well as contact problems.

As a result of mathematical modeling of the stress-strain state of a flexible thread with specified physical and geometric characteristics, the proposed method and the finite element method (FEM), the corresponding values of the accepted evaluation criteria are obtained. The values of the calculated data are summarized in Table 1.

As an example, in Figure 4 are the results of modeling the stress-strain state of a flexible thread using FEM, in Figure 5, the simulation results using the program [16] that implements the proposed method at a rotational speed \( n = 1000 \text{ min}^{-1} \), which corresponds to 1 data load.

From the analysis of the data presented in Table 1 it can be seen that for the case of loading 1 when determining the rotation speed, the determining factor was the constraint on the strength condition (permissible stresses) of the material of the flexible thread. For the case of loading 2, the constraint on the permissible deflection in the middle of the span of the flexible thread became decisive. Moreover, the differences in the values of the adopted criteria for assessing the adequacy of the results obtained using the proposed method do not exceed 5%, which is a good indicator for engineering calculations.

![Figure 4](image1.png)

**Figure 4.** The results of modeling the stress-strain state of a flexible thread using the FEM: (a) - the diagram of displacements; (b) - the diagram of longitudinal force.
Figure 5. The results of modeling a flexible thread using the proposed method.

Table 1. Comparison of the calculation results.

| Load number | Evaluation criterion | FEM     | Proposed method | Error, % |
|-------------|----------------------|---------|-----------------|----------|
| 1           | n, min⁻¹             | 1000    | 984.53          | 1.547    |
|             | q, N/m               | 304.31  | 303.4           | 0.3      |
|             | T(H,q,0), N          | 1790.56 | 1790.5          | =0       |
|             | w(H,q,l/2), m        | 0.02193 | 0.0212          | 3.329    |
| 2           | n, min⁻¹             | 2400    | 2329.19         | 2.95     |
|             | q, N/m               | 1752.84 | 1761.7          | 0.503    |
|             | T(H,q,0), N          | 4560.6  | 4516.7          | 0.963    |
|             | w(H,q,l/2), m        | 0.04971 | 0.0497          | =0       |

4. Conclusion
The developed calculation method, reflected in the form of a flowchart, allows determining the maximum possible frequency of rotation of a flexible thread (cable) that does not allow its limit state to occur, which will simplify the search for the optimal solution and reduce the time spent. Solving the nonlinear programming problem of determining the frequency of rotation of a flexible thread taking into account the development zone of plastic strains in the material at the given maximum permissible stresses and strains by reducing it to the problem of unconditional optimization using Lagrange multipliers allows calculating with an error of not more than 4% compared with the FEM. The calculation results make it possible to use the proposed method both when there is a restriction on the strength condition (allowable stresses) of the flexible thread, and on the allowable deflection in the middle of the span of the flexible thread.

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