Multiple Access for Transmissions Over Independent Fading Channels

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Abstract

We propose to employ a multilevel detection (MLDT) technique to allow multiple users which respectively transmit messages over independent fading channels to share the same resource, e.g., the same signature sequence in the CDMA (code division multiple access) system. The users are separated by the different channel gains including amplitudes and phases resultant from the independent fading channels. In a CDMA system with a fixed amount of available signature sequences, the number of users can be doubled or tripled by using MLDT although there is the cost of some BER performance degradation.

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I. INTRODUCTION

Multiple access (MA) techniques are essential for multiple users to share the same bandwidth in the mobile communication systems, e.g., code division multiple access (CDMA) for the third generation (3G) mobile telecommunications and orthogonal frequency division multiple access (OFDMA) for the fourth generation (4G) systems. In recent years, many MA techniques have been proposed such as the Interleave division multiple access (IDMA) [1], non-orthogonal multiple access (NOMA) [2], low-density signature CDMA (LDS-CDMA) [3] and sparse code multiple access (SCMA) [4] and many more. Each of these MA works aims to either increase the sum rate of all users or serve more users in a limited bandwidth.

For an MA system, each user is usually assigned with a unique resource so that the receiver can separate out the message of the desired user. In the conventional CDMA system, the unique resource is the unique signature sequence. In the IDMA system, the unique source is the unique interleaver. In this paper, our goal is to find a technique to allow $P$ users to share the same resource, where $P \geq 1$. In this paper, $P = 2$ and $3$ are studied. Such a situation is referred to as user collision. In this way, the number of users in the CDMA system which is supplied with $K$ signature sequences can be multiplied to $KP$. This technique may also be applied to other MA systems for increasing the number of users.

The basic idea of our work is the observation that the channel gains of independent users are likely to be distinct in the uplink transmission of mobile communication system. For NOMA in [2], different users can be separated by distinct power levels. In our work, not only the amplitude of the channel gain (equivalently, the power level) but also the phase of the channel gain will be utilized for separation of users. The superimposition of signals from the $P$ collided users transmitted over independent fading channels can be viewed as a multi-level ($2^P$-level) signal.
Exploiting the feature of the superimposed signal, the detector can recover the message for each of the collided users respectively. Hence, such a technique is referred to here as the multi-level detection (MLDT) technique. We specifically show the formulae of bit error rates (BERs) of uncoded MLDT for $P = 2$ and $3$. The derived upper bounds indicate that at high SNR the BERs for $P = 2$ and $3$ are respectively $50\%$ and $133\%$ higher than that for $P = 1$.

Error-correcting codes (ECC) are usually used to enhance the system reliability. For a $P$-user MLDT system in which the $P$ users employ $P$ identical binary LDPC (low density parity check) \cite{5, 6} codes, the receiver can be implemented by using a MLDT device followed by $P$ binary decoders each of which uses a conventional sum product algorithm (SPA) to recover the message for a user. Alternatively, the receiver can be implemented by using an MLDT device followed by a nonbinary generalized sum product algorithm (GSPA) \cite{7} to recover messages of all the $P$ users. The concept of MLDT is inspired by the work in \cite{7}, of which the physical-layer network coding for the relay node can be considered as a coded MLDT device for two users followed by a GSPA decoder.

The asymptotic performances of coded MLDT can be evaluated by its capacity. Through the capacity performances of two-user MLDT using BPSK transmission over independent Rayleigh fading channels and a single-user using QPSK transmission over a Rayleigh fading channel, we see that the cost of two-user MLDT using BPSK transmission is the slightly reduced rate as compared to the single-user QPSK transmission. In the single-path fading channel environment, using a fixed-rate ECC can achieve very little coding gain. We will see that Raptor coded MLDT \cite{8} has the potential to achieve the coding gain implied by the capacity analysis.

The MLDT technique can be applied to CDMA systems and possibly other multiple access systems. In this paper, we restrict our study to only CDMA systems. In the CDMA system, orthogonal signature sequences, such as Hadamard-Walsh (HW) codes, can be employed to maintain the orthogonality among users. However, after the transmission over multipath fading channels, the sequences carried by different paths may no longer remain orthogonal and hence
severe interference among users may occur. To alleviate the multipath interference, multi-carrier system can be considered. We apply the MLDT scheme to a multi-carrier direct-sequence CDMA (MC-DS-CDMA) system, where the spreading is performed for each subcarrier so that the orthogonality among users can be maintained and the number of users can be multiplied.

The remainder of this paper is organized as follows. Section II introduces the proposed MLDT scheme and analyzes its BER performances. In Section III, capacity analysis together with some coded MLDT designs are provided. In Section IV, the MLDT technique for CDMA systems is explored. We conclude the paper in Section V.

Notation: Vectors are denoted by boldface case. \( \Pr \{ \cdot \} \) and \( \mathbb{E} [ \cdot ] \) stand for the probability and the expected value of a random variable, respectively.

II. MLDT Receiver

Consider a symbol-synchronous communication system in which \( P \) users share the same resource. For user \( p \), let \( \mathbf{b}_p = [b_p(1), ..., b_p(n), ... b_p(N)] \) be the bit sequence to be transmitted, where \( b_p(n) \in \{0,1\} \). Denote the modulated sequence of user \( p \) as \( \mathbf{x}_p = [x_p(1), ..., x_p(n), ... x_p(N)] \), where \( x_p(n) = (-1)^{b_p(n)} \). In this paper, we concentrate on uplink transmissions over the quasi-static Rayleigh channel with BPSK modulation and assume that all users are perfectly aligned in time. The received signal can be represented as

\[
\mathbf{r}(n) = \sum_{p=1}^{P} h_p x_p(n) + \mathbf{w}(n), \quad n = 1, 2, ..., N, \tag{1}
\]

where \( h_p \) is the complex Gaussian channel gain of user \( p \) and \( \mathbf{w}(n) \) is the additive Gaussian noise with variance \( \sigma^2 = \frac{N_0}{2} \) each dimension. Both the amplitude of channel gain \( |h_p| \) and the phase of the channel gain \( \theta_p \) are useful in identifying the user.

A. Multi-level Detection

Let \( P = 2 \) and \( N = 1 \). Suppose that user A and user B share the same resource such as bandwidth, signature sequence and subcarrier allocation. The superimposed signal representing
$b_A$ and $b_B$ transmitted from user $A$ and user $B$ respectively is denoted as $s_{AB}$, which should be one of the four levels given by $\{S_{AB}(0), S_{AB}(1), S_{AB}(2), S_{AB}(3)\}$. With given $h_A$ and $h_B$, the possible levels of $s_{AB}$ are shown in TABLE [I]. Note that a larger table can be established by a similar rule if more than two users share the same resource.

| $i$ | $b_A$ | $b_B$ | $x_A$ | $x_B$ | $S_{AB}(i)$ |
|-----|-------|-------|-------|-------|-------------|
| 0   | 0     | 0     | 1     | 1     | $h_A + h_B$ |
| 1   | 0     | 1     | 1     | -1    | $h_A - h_B$ |
| 2   | 1     | 0     | -1    | 1     | $-h_A + h_B$ |
| 3   | 1     | 1     | -1    | -1    | $-h_A - h_B$ |

At the receiver, the received signal is $r = r_{AB} = [h_A x_A + h_B x_B] + w$. The a posteriori probability (APP) for $s_{AB}$ given the received signal $r_{AB}$ can be calculated by

$$p_i = \Pr \{s_{AB} = S_{AB}(i) | r_{AB} \}$$

$$= \Pr \{r_{AB} | s_{AB} = S_{AB}(i) \} \cdot \frac{\Pr \{s_{AB} = S_{AB}(i) \}}{\Pr \{r_{AB} \}}$$

where

$$\Pr \{r_{AB} | s_{AB} = S_{AB}(i) \} = \frac{1}{2\pi \sigma^2} \exp \left( \frac{-|r_{AB} - S_{AB}(i)|^2}{2\sigma^2} \right) \quad (2)$$

and $\Pr \{s_{AB} = S_{AB}(i) \}$ is the a priori probability for $s_{AB}$. The APP values for $b_A$ and $b_B$ given the received signal $r_{AB}$ are respectively given by

$$\begin{cases} 
\Pr \{b_A = 0 | r_{AB} \} = \Pr \{s_{AB} = S_{AB}(0) | r_{AB} \} + \Pr \{s_{AB} = S_{AB}(1) | r_{AB} \} = p_0 + p_1 \\
\Pr \{b_A = 1 | r_{AB} \} = \Pr \{s_{AB} = S_{AB}(2) | r_{AB} \} + \Pr \{s_{AB} = S_{AB}(3) | r_{AB} \} = p_2 + p_3 
\end{cases} \quad (3)$$
and
\[
\begin{align*}
\Pr\{b_B = 0|r_{AB}\} &= \Pr\{s_{AB} = S_{AB}(0)|r_{AB}\} + \Pr\{s_{AB} = S_{AB}(2)|r_{AB}\} = p_0 + p_2 \\
\Pr\{b_B = 1|r_{AB}\} &= \Pr\{s_{AB} = S_{AB}(1)|r_{AB}\} + \Pr\{s_{AB} = S_{AB}(3)|r_{AB}\} = p_1 + p_3.
\end{align*}
\]
(4)

The corresponding log likelihood ratio (LLR) values of bits \(b_A\) and \(b_B\) are
\[
\begin{align*}
\ell_{LLR}(b_A) &= \ln \frac{\Pr\{b_A = 0|r_{AB}\}}{\Pr\{b_A = 1|r_{AB}\}} = \ln \frac{p_0 + p_1}{p_2 + p_3} \\
\ell_{LLR}(b_B) &= \ln \frac{\Pr\{b_B = 0|r_{AB}\}}{\Pr\{b_B = 1|r_{AB}\}} = \ln \frac{p_0 + p_2}{p_1 + p_3}.
\end{align*}
\]
(5)

These LLR values can be used to estimate \(b_A\) and \(b_B\) by
\[
\hat{b}_p = \begin{cases} 
0, & \text{if } \ell_{LLR}(b_p) \geq 0, \\
1, & \text{otherwise}
\end{cases}, \quad p = A \text{ or } B,
\]
(6)
or can be fed to a decoder if channel coding is considered.

For \(P = 3\), the MLDT can be similarly executed, where the superimposed signal representing \(b_A, b_B\) and \(b_C\) transmitted from users \(A, B\) and \(C\) respectively should be one of the eight levels given by \(\{S_{ABC}(0), S_{ABC}(1), S_{ABC}(2), S_{ABC}(3), S_{ABC}(4), S_{ABC}(5), S_{ABC}(6), S_{ABC}(7)\}\), where \(S_{ABC}(i) = (-1)^{b_A}h_A + (-1)^{b_B}h_B + (-1)^{b_C}h_C\) and \(i = 4b_A + 2b_B + b_C\).

B. BER Analysis

By assuming equally likely a priori probability for each \(s_{AB}\) (or \(s_{ABC}\)), the bounds of BER for MLDT receiver with \(P = 2\) (or 3) respectively over independent and identically distributed (i.i.d.) Rayleigh fading channels can be derived as follows. Let
\[
p_R(r) = \frac{2r}{E[R^2]} \exp \left( -\frac{r^2}{E[R^2]} \right)
\]
(7)
be the probability density function of a Rayleigh distributed random variable \(R\).

Let \(Q(x) = \int_{x}^{\infty} \exp \left( -\frac{u^2}{2} \right) du\) and let \(\gamma = \frac{1}{N_0}E[h_A^2] = \frac{1}{N_0}E[h_B^2] = \frac{1}{N_0}E[h_C^2]\), where \(|h_A|, |h_B|\) and \(|h_C|\) are i.i.d. Rayleigh random variables.
1) \( P = 1 \): It is well known that the average BER for the BPSK modulation over the Rayleigh fading channel [9] is

\[
P_e = \int_0^\infty Q\left(\frac{2r}{\sqrt{2N_0}}\right) p_R(r) dr = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right).
\]  

(8)

2) \( P = 2 \): The BER of user \( A \) is

\[
P_e = \frac{1}{2} \Pr \left\{ \hat{b}_A = 1 \mid b_A = 0 \right\} + \frac{1}{2} \Pr \left\{ \hat{b}_A = 0 \mid b_A = 1 \right\}
= \Pr \left\{ \hat{b}_A = 1 \mid b_A = 0 \right\} \text{ (from symmetry)}
= \frac{1}{2} \Pr \left\{ \hat{b}_A = 1 \mid s_{AB} = S_{AB}(0) \right\} + \frac{1}{2} \Pr \left\{ \hat{b}_A = 1 \mid s_{AB} = S_{AB}(1) \right\}.
\]

(9)

The Euclidean distance between \( S_{AB}(0) \) and \( S_{AB}(i) \) is \( 2|h_A| \) for \( i = 2 \), and is \( 2|h_A + h_B| \) for \( i = 3 \). Moreover, the Euclidean distance between \( S_{AB}(1) \) and \( S_{AB}(i) \) is \( 2|h_A - h_B| \) for \( i = 2 \), and is \( 2|h_A| \) for \( i = 3 \). Hence, an upper bound of \( P_e \) can be derived as

\[
P_e \leq \frac{1}{2} \sum_{i \in \{2,3\}} \Pr \left\{ |r_{AB} - S_{AB}(i)| < |r_{AB} - S_{AB}(0)| \mid S_{AB}(0) \right\}
+ \frac{1}{2} \sum_{i \in \{2,3\}} \Pr \left\{ |r_{AB} - S_{AB}(i)| < |r_{AB} - S_{AB}(1)| \mid S_{AB}(1) \right\}
\]

(10)

Assume that \( |h_A| \) and \( |h_B| \) are independent and identically Rayleigh distributed. Also assume that \( \theta_A \) and \( \theta_B \) are independent and identically uniformly distributed. The average of the righthand side of (10) is

\[
\frac{1}{2} \int_0^\infty \int_0^\infty \int_0^{\pi} \int_0^{\pi} Q\left(\frac{2|h_A|}{\sqrt{2N_0}}\right) + Q\left(\frac{2|h_A + h_B|}{\sqrt{2N_0}}\right) \cdot p_R(|h_B|) p_R(|h_A|) \left(\frac{2}{\pi}\right)^2 \, d|h_B| \, d|h_A| \, d\theta_B \, d\theta_A
+ \frac{1}{2} \int_0^\infty \int_0^\infty \int_0^{\pi} \int_0^{\pi} Q\left(\frac{2|h_A|}{\sqrt{2N_0}}\right) + Q\left(\frac{2|h_A - h_B|}{\sqrt{2N_0}}\right) \cdot p_R(|h_B|) p_R(|h_A|) \left(\frac{2}{\pi}\right)^2 \, d|h_B| \, d|h_A| \, d\theta_B \, d\theta_A.
\]

(11)
Since $|h_A + h_B|$ and $|h_A - h_B|$ are both Rayleigh distributed with variance twice of that for $|h_A|$, the average upper bound of BER provided in (11) can be simplified as

$$\bar{P}_e \leq \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma}{1 + \gamma}} \right) + \frac{1}{2} \left( 1 - \sqrt{\frac{2\gamma}{1 + 2\gamma}} \right)$$

(12)

Since the BER of the collided users cannot be lower than that of the non-collided users, the average BER for the BPSK modulation over Rayleigh fading channels can be regarded as the lower bound of the MLDT receiver. We have

$$\bar{P}_e \geq \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma}{1 + \gamma}} \right).$$

(13)

---

Fig. 1. Bounds of BER and simulated results for the uncoded MLDT receiver over Rayleigh fading channels.
3) \( P = 3 \): Similar to (9), (10), (11) and (12) for \( P = 2 \), upper bounds of \( P_e \) and \( \bar{P}_e \) for \( P = 3 \) can be derived as,

\[
P_e \leq \frac{1}{4} \sum_{i \in \{4,5,6,7\}} \Pr \{ |r_{AB} - S_{AB}(i)| < |r_{AB} - S_{AB}(0)| \}
+ \frac{1}{4} \sum_{i \in \{4,5,6,7\}} \Pr \{ |r_{AB} - S_{AB}(i)| < |r_{AB} - S_{AB}(1)| \}
+ \frac{1}{4} \sum_{i \in \{4,5,6,7\}} \Pr \{ |r_{AB} - S_{AB}(i)| < |r_{AB} - S_{AB}(2)| \}
+ \frac{1}{4} \sum_{i \in \{4,5,6,7\}} \Pr \{ |r_{AB} - S_{AB}(i)| < |r_{AB} - S_{AB}(3)| \}
\] (14)

and

\[
\bar{P}_e \leq \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma}{1 + \gamma}} \right) + \left( 1 - \sqrt{\frac{2\gamma}{1 + 2\gamma}} \right) + \frac{1}{2} \left( 1 - \sqrt{\frac{3\gamma}{1 + 3\gamma}} \right)
\] (15)

4) Numerical Results: For a large \( x \), the term \( 1 - \sqrt{x/(1 + x)} \) can be approximated by \( 1/(2x) \). Hence, for \( P = 1 \), we have \( \bar{P}_e \approx 1/(4\gamma) \); for \( P = 2 \), we have \( \bar{P}_e \approx 3/(8\gamma) \); for \( P = 3 \), we have \( \bar{P}_e \approx 7/(12\gamma) \). Hence, we expect that for high SNR, the penalty for doubling the user number is the increase of BER by about 50% and for tripling the user number is the increase of BER by about 133%. Fig. 1 shows the analytical results obtained in (12) (13) (15) and the simulation results, where \( E_b/N_0 \) denote the average of \( E_b/N_0 \) over the Rayleigh fading channel. We see that simulation results are very close to the analytic prediction.

III. Coded MLDT

Consider a symbol-synchronous coded system in which \( P \) users transmit messages over independent quasi-static Rayleigh channels. For user \( p \), a message block \( d_p \) is encoded by the encoder of an error-correcting code \( V \) into a codeword (or code sequence) \( b_p \). We will first consider the design for which the MLDT receiver is followed by multiple SPA (sum product algorithm) decoders and then consider the design for which the MLDT receiver is followed by
a single GSPA (generalized sum product algorithm) [7] decoder. The transmitters with \( P = 2 \) is illustrated in Fig. 2.

Fig. 2. The transmitters of a coded system with \( P = 2 \).

A. MLDT Receiver using SPA

For the coded system with \( P = 2 \), the MLDT receiver followed by two conventional SPA decoders is illustrated in Fig. 3. The LLR values \( e_{\text{LLR}}(b_A) \) and \( e_{\text{LLR}}(b_B) \) obtained from the MLDT receiver are respectively fed to two binary decoders which use the SPA to obtain the decoded LLR values \( e_{\text{DEC}}(d_A) \) and \( e_{\text{DEC}}(d_B) \) for message bits \( d_A \) and \( d_B \) respectively.

Fig. 3. The MLDT receiver followed by two SPA decoders for a coded system with \( P = 2 \).

B. MLDT Receiver using GSPA

For the coded CDMA system, the MLDT receiver followed by \( P \) SPA decoders can be replaced by an MLDT receiver followed by a single generalized sum product algorithm (GSPA) decoder.
to obtain the decoded LLR values $e_{\text{DEC}}(d_1)$, $e_{\text{DEC}}(d_2)$, ... and $e_{\text{DEC}}(d_P)$ for message bits $d_1$, $d_2$, ... and $d_P$ respectively. Such a receiver with $P = 3$ is depicted in Fig. 4.

![Fig. 4. The MLDT receiver followed by a GSPA decoder for a coded system with $P = 3$.](image)

Here, the ECC code $V$ is a binary one. However, both the codewords represented by code bits $\{b_A\}$, $\{b_B\}$ and $\{b_C\}$ for users $A$, $B$ and $C$ respectively must satisfy the same factor graph of code $V$. A variable node in the conventional SPA decoder of a binary code is represented by the bit $b_k$. Such a variable node in the GSPA decoder is now represented by a three-tuple $(b_A, b_B, b_C)$ for which its likelihood vector is $\mathbf{p} = \{p_0, p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$, where $p_i$ at the input of the GSPA decoder is defined in a way similar to (2) through replacing $r_{AB}$, $s_{AB}$ and $S_{AB}$ by $r_{ABC}$, $s_{ABC}$ and $S_{ABC}$ respectively.

The updated likelihood vector $\mathbf{p}$ of the GSPA in the iterative operation can be obtained as follows. Suppose that there is a degree-3 variable node which takes two likelihood vectors $\mathbf{p} = [p_0 \ p_1 \ p_2 \ p_3 \ p_4 \ p_5 \ p_6 \ p_7]$ and $\mathbf{q} = [q_0 \ q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6 \ q_7]$ as input. The updated likelihood vector will be

$$\text{VAR}(\mathbf{p}, \mathbf{q}) = \beta [p_0 q_0 \ p_1 q_1 \ p_2 q_2 \ p_3 q_3 \ p_4 q_4 \ p_5 q_5 \ p_6 q_6 \ p_7 q_7],$$  \hspace{1cm} (16)$$

where $\beta$ is a normalized factor. Suppose that there is a degree-3 check node which takes two
likelihood vectors \( p \) and \( q \) as input. The updated likelihood vector will be

\[
\begin{pmatrix}
p_0 q_0 + p_1 q_1 + p_2 q_2 + p_3 q_3 + p_4 q_4 + p_5 q_5 + p_6 q_6 + p_7 q_7 \\
p_0 q_1 + p_1 q_0 + p_2 q_3 + p_3 q_2 + p_4 q_5 + p_5 q_4 + p_6 q_7 + p_7 q_6 \\
p_0 q_2 + p_1 q_3 + p_2 q_0 + p_3 q_1 + p_4 q_6 + p_5 q_7 + p_6 q_4 + p_7 q_5 \\
p_0 q_3 + p_1 q_2 + p_2 q_1 + p_3 q_0 + p_4 q_7 + p_5 q_6 + p_6 q_5 + p_7 q_4 \\
p_0 q_4 + p_1 q_5 + p_2 q_6 + p_3 q_7 + p_4 q_0 + p_5 q_1 + p_6 q_2 + p_7 q_3 \\
p_0 q_5 + p_1 q_4 + p_2 q_7 + p_3 q_6 + p_4 q_1 + p_5 q_0 + p_6 q_3 + p_7 q_2 \\
p_0 q_6 + p_1 q_7 + p_2 q_4 + p_3 q_5 + p_4 q_2 + p_5 q_3 + p_6 q_0 + p_7 q_1 \\
p_0 q_7 + p_1 q_6 + p_2 q_5 + p_3 q_4 + p_4 q_3 + p_5 q_2 + p_6 q_1 + p_7 q_0 
\end{pmatrix}_T .
\]

For a node with degree more than 3, the update likelihood vector can be extended by

\[
\text{VAR}(p, q, \cdots) = \text{VAR}(p, \text{VAR}(q), \text{VAR}(\cdot, \cdot)),
\]

and

\[
\text{CHK}(p, q, \cdots) = \text{CHK}(p, \text{CHK}(q), \text{CHK}(\cdot, \cdot)).
\]

After the iterations, we can determine the LLR values of the collided users by

\[
\begin{cases}
e_{\text{DEC}}(d_A) = \ln \frac{p_0 + p_1 + p_2 + p_3}{p_4 + p_5 + p_6 + p_7} \\
e_{\text{DEC}}(d_B) = \ln \frac{p_0 + p_1 + p_4 + p_5}{p_2 + p_3 + p_6 + p_7} \\
e_{\text{DEC}}(d_C) = \ln \frac{p_0 + p_2 + p_4 + p_6}{p_1 + p_3 + p_5 + p_7} .
\end{cases}
\]

The \( P = 1 \) and \( P = 2 \) scenarios are simply the degenerate cases of \( P = 3 \) case.

C. LDPC-Coded System

Fig. 5 shows the BER performances of an LDPC-coded system with \( P = 2 \), where the (1008, 504) (3, 6)-regular LDPC code [10] is used. The single-path Rayleigh fading channel
is considered. The number of iterations for both SPA and GSPA is set to 10. The “LDPC, $P = 2$” curve shows the BER performances of the conventional receiver for two collided users without MLDT. Clearly, such a arrangement has extremely poor performances. With the MLDT structure for $P = 2$, using either the two SPA decoders or the single GSPA decoder can achieve similar performances which are only slightly inferior to the $P = 1$ case.

Compared to the uncoded MLDT system, the LDPC coded MLDT system does not provide noticeable coding gain. Hence, in the following section, we will conduct the capacity analysis to see whether we can achieve benefit by applying ECC to the MLDT system.

![Graph](image)

Fig. 5. LDPC-coded system over single-path Rayleigh fading channels.

**D. Capacity Analysis**

It would be interesting to compare capacity $C_{AB,BPSK,R}$ of two users employing BPSK averaged over independent quasi-static Rayleigh fading channels to the capacity $C_{QPSK,R}$ of a single user employing QPSK averaged over the Rayleigh fading channel. Let $x_A, x_B \in \{+1, -1\}$,
with fixed channel gain of $h_A$ and $h_B$, the superimposed signal without AWGN will be $s_{AB} = x_A h_A + x_B h_B = (x_A|h_A| + x_B|h_B| \cos \theta) + j(x_B|h_B| \sin \theta)$.

Assume equally likely a priori probability for $s_{AB}$. Then, we have

$$C_{AB,BPSK,R} = \sum_{x_A,x_B} \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \int_{0}^{\infty} f(x_A,x_B,u,v,\theta,|h_A|,|h_B|)$$

$$\log \left\{ f(x_A,x_B,u,v,\theta,|h_A|,|h_B|) \left[ \frac{1}{4} \sum_{x_A,x_B} f(x_A,x_B,u,v,\theta,|h_A|,|h_B|) \right]^{-1} \right\}$$

$$\cdot p_R(|h_A|)p_R(|h_B|) \frac{1}{2\pi} dudvd\theta |h_A|d|h_B|$$

(21)

where

$$f(x_A,x_B,u,v,\theta,|h_A|,|h_B|) = \frac{1}{2\pi\sigma^2} \exp \left\{ \frac{-|r_{AB} - s_{AB}|^2}{2\sigma^2} \right\}$$

$$= \frac{1}{2\pi\sigma^2} \exp \left\{ \frac{-((u - (x_A|h_A| + x_B|h_B| \cos \theta))^2 + (v - x_B|h_B| \sin \theta)^2)}{2\sigma^2} \right\},$$

(22)

and $r_{AB} = u + jv$. The capacity $C_{QPSK,R}$ can be obtained by setting $|h_A| = |h_B|$ and $\theta = \pi/2$ in (21) and (22).

Let $E_s/N_0$ denote the average of $E_s/N_0$ over the Rayleigh fading channel. From Fig. 8, we see that under the quasi-static Rayleigh fading scenarios, $C_{AB,BPSK,R}$ is close to $C_{QPSK,R}$, especially in the high SNR regime. The capacity of $C_{QPSK,A}$ over the AWGN channel is also provided in Fig. 8 as a reference for comparison. This result implies that in the coded systems over the Rayleigh fading channels, MLDT for two-user multiple access employing BPSK suffers only a slight loss of average capacity as compared to a single user employing QPSK transmission.

### E. Raptor-Coded System

In Fig. 5 we note that over the single-path Rayleigh fading channel, using LDPC virtually obtains no coding gain as compared to the uncoded system. This is probably due to the fact that a fixed-rate ECC cannot cope with the varying SNR in the deep fading condition. In case
that feedback channel is available, Raptor coding [8] can indefinitely increase its redundancy until the decoding is successful. A Raptor code $V$ can be constructed as the concatenation of a high-rate ECC $V'$ followed by a rateless Luby Transform (LT) code $C$ which can generate limitless output stream $b_p$ until the transmitter receives an "ACK" signal sent by the receiver through the feedback channel. Hence, the length $N$ of $b_p$ is a random variable. The system throughput will be

$$R_V = \frac{k}{E[N]},$$

(23)

where $k$ is the length of each message block $d_p$.

The transmitters of a Raptor coded system with $P = 2$ can also be illustrated in Fig. 2, where the the ECC encoder $V$ is now a Raptor encoder. For Raptor coded MLDT system with $P = 2$, both user $A$ and user $B$ will increase the output stream length $N$ until both $b_A$ and $b_B$ are successfully recovered.

The MLDT receiver with $P = 2$ using multiple SPA decoders illustrated in Fig. 3 must be modified as shown in Fig. 6, where each SPA decoder is used as the decoder for the ECC $V'$ which is usually a binary LDPC code.

Likewise, the MLDT receiver with $P = 2$ using a single GSPA decoder must be modified as shown in Fig. 7 where the single GSPA decoder is used for the decoding of the two binary ECC $V'$. LLR values $e_{LLR}(b_A)$ and $e_{LLR}(b_B)$ obtained from the MLDT receiver and LT decoders will

Fig. 6. The MLDT receiver followed by two LT decoders and two SPA decoders for an Raptor-coded system. $P = 2$. 

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be used to calculate likelihood vector \((p_0, p_1, p_2, p_3)\) according to (24). This likelihood vector \((p_0, p_1, p_2, p_3)\) is then fed to a GSPA decoder.

\[
\begin{align*}
    p_0 &= \beta \left( 1 - \frac{1}{1 + \exp[e_{LLR}(b_A)]} \right) \left( 1 - \frac{1}{1 + \exp[e_{LLR}(b_B)]} \right) \\
    p_1 &= \beta \left( 1 - \frac{1}{1 + \exp[e_{LLR}(b_A)]} \right) \left( \frac{1}{1 + \exp[e_{LLR}(b_B)]} \right) \\
    p_2 &= \beta \left( \frac{1}{1 + \exp[e_{LLR}(b_A)]} \right) \left( 1 - \frac{1}{1 + \exp[e_{LLR}(b_B)]} \right) \\
    p_3 &= \beta \left( \frac{1}{1 + \exp[e_{LLR}(b_A)]} \right) \left( \frac{1}{1 + \exp[e_{LLR}(b_B)]} \right),
\end{align*}
\]

(24)

where \(\beta\) is the normalization factor.

The throughput performances of a Raptor coded MLDT system are shown in Fig. 8, where the Raptor code is designed in [11], of which the ECC \(V'\) is a \((10000,9500)\) binary LDPC code and the LT code has an adaptive degree distribution. In the simulation, each incremental redundancy (IR) contains 400 symbols, the number of iterations for LT decoder is set to 200 and the number of iterations for both SPA and GSPA is set to 100. For each block if Raptor-code rate less than \(\frac{1}{4}\) and a certain user is not able to obtain successful decoding, we stop decoding and the associated \(N\) output bits will contribute to zero message bit in calculating the throughput. Although Raptor-coded systems performance is about 10% to 66% below the
associated capacity, the Raptor coded MLDT system does achieve appreciable coding gains in the quasi-static environment. Raptor code designed for low SNR [12] over the AWGN channel may also help to increase the throughput of MLDT system for low SNR over the independent quasi-static Rayleigh fading channels.

![Graph showing throughput performances of Raptor-coded system and the derived capacities of QPSK and MLDT](image)

**Fig. 8.** Throughput performances of Raptor-coded system and the derived capacities of QPSK and MLDT

### IV. MLDT FOR CDMA SYSTEMS

Consider a symbol-synchronous uncoded CDMA system with $K$ users. For user $(k,p)$, $1 \leq k \leq K, 1 \leq p \leq P$, let $b_{k,p}$ be the transmitted message bit and $s_k = [s_k(1), ..., s_k(j), ..., s_k(J)]$ be the spreading signature with spreading length $J$. Denote the modulated sequence of user $(k,p)$ as $x_{k,p} = [x_{k,p}(1), ..., x_{k,p}(j), ..., x_{k,p}(J)]$, where $x_{k,p}(j) = b_{k,p}s_k(j)$.

We consider the multipath channels which have $L$ paths, where the delay between adjacent
paths is the period of a chip and all the paths have equal average powers. The received signal is
\begin{equation}
    r(j) = \sum_{\ell=0}^{L-1} \sum_{p=1}^{P} \sum_{k=1}^{K} h_{k,p,\ell} x_{k,p}(j-\ell) + w(j), \quad j = 1, \ldots, J + L - 1,
\end{equation}
where \( h_{k,p,\ell} \) is the channel coefficient of \( \ell \)-th path for user \((k,p)\) and \( x_{k,p}(j) \) with \( j \notin \{1, 2, \ldots, J\} \) is modulated from a message bit \( b_{k,p} \). We assume that all the \( h_{k,p,\ell} \) are independently identical and Rayleigh distributed. The uncoded CDMA system with user collisions and MLDT receiver for \( P = 2 \) is depicted in Fig. 9.

\[ r^f_{k,AB} = \frac{1}{J} \sum_{j=1}^{J} r(j + l)s_k(j) \]
\[ = \frac{1}{J} \sum_{j=1}^{J} [h_{k,A,\ell}x_{k,A}(j) + h_{k,B,\ell}x_{k,B}(j)]s_k(j) + w_{ms} + w_{mp} + w_k, \tag{26} \]
where \( l = 0, 1, \ldots, L - 1, w_{ms} = \frac{1}{J} \sum_{j=1}^{J} \sum_{\ell=0}^{L-1} \sum_{k' \neq k} [h_{k',A,\ell}x_{k',A}(j + l - \ell) + h_{k',B,\ell}x_{k',B}(j + l - \ell)]s_{k'}(j) \) is the interference from users employing sequences \( s_{k'} \neq s_k \), \( w_{mp} = \frac{1}{J} \sum_{j=1}^{J} \sum_{\ell \neq k} [h_{k,A,\ell}x_{k,A}(j + l - \ell)]s_{k'}(j) \) is the interference from users employing sequences \( s_{k'} \neq s_k \), \( w_k \) is the noise corruption at the receiver, \( P \) is the number of users, \( K \) is the number of chips per user, \( L \) is the chip duration, \( J \) is the number of chips in the system, \( b_{k,p} \) is the message bit for user \((k,p)\), \( x_{k,p}(j) \) is the chip for user \((k,p)\) at time \( j \), \( w(j) \) is the noise at the receiver, \( h_{k,p,\ell} \) is the channel coefficient of \( \ell \)-th path for user \((k,p)\), and \( r(j) \) is the received signal at the receiver.
$l - \ell + h_{k,B,l}x_{k,B}(j + l - \ell)s_k(j)$ is the interference from other paths, and $w_k = \frac{1}{J} \sum_{j=1}^{J} w(j) s_k(j)$ is the Gaussian distributed noise with zero mean and variance $\sigma^2/J$. The parameters $p_i, e_{LLR}(b_{k,A}), e_{LLR}(b_{k,B})$ should be modified to $p_i', e_{LLR}^l(b_{k,A}), e_{LLR}^l(b_{k,B})$ respectively. Then, we have

$$
\begin{align*}
  e_{LLR}^l(b_{k,A}) &= \ln \frac{\Pr\{b_{k,A} = 0 | r_{k,AB}^l\}}{\Pr\{b_{k,A} = 1 | r_{k,AB}^l\}} = \ln \frac{p_{k,0} + p_{k,1}}{p_{k,2} + p_{k,3}} \\
  e_{LLR}^l(b_{k,B}) &= \ln \frac{\Pr\{b_{k,B} = 0 | r_{k,AB}^l\}}{\Pr\{b_{k,B} = 1 | r_{k,AB}^l\}} = \ln \frac{p_{k,0} + p_{k,2}}{p_{k,1} + p_{k,3}}.
\end{align*}
$$

(27)

The LLR for bit $b_{k,i}$ of user $(k,i)$, $i = A, B$ is

$$
e_{LLR}(b_{k,i}) = \sum_{l=0}^{L-1} \alpha_l e_{LLR}^l(b_{k,i}),
$$

(28)

where

$$
\alpha_l = \frac{|h_{k,p,l}|^2}{\sum_{l=0}^{L-1} |h_{k,p,l}|^2}.
$$

(29)

A. Systems with Hadamard Walsh codes

Hadamard Walsh (HW) codes can be used in the CDMA applications employing orthogonal signature sequences. We set $K = J$. With these codes, the interference from other users can be removed perfectly for $L = 1$. Hence, in case of single-path Rayleigh fading, the BER performances for CDMA using HW codes with $P = 2$ and $3$ are exactly the same as those derived in Section II.B and shown in Fig. [1]. That means that we can double or triple the number of users in the CDMA system using HW codes with only slight BER degradation.

For $L > 1$, the term $w_{mp}$ is significant since for the HW code, the autocorrelation of a sequence with its shift may be significant. Hence, the BER performances will be very poor.

B. Systems with m-sequences

We replace HW codes by m-sequences (maximum length sequences) in multipath channels, where the autocorrelation of a sequence with its shift is small. In Fig. [10] the BER performances of the m-sequence CDMA system, in the two-path Rayleigh environment with $J = 15$, $P = 1$
and 2 respectively are compared, where the two paths have equal average powers. We see that using the MLDT receivers is not able to perform well in the non-orthogonal environment. The multipath interference severely affect the BER performances. Note that if $KL > J$ which implies that the number of m-sequences in use multiplied by the number of paths exceeds the number of available m-sequences, then the BER performances will be extremely poor. To tackle the multipath interference, we consider using the $(1008, 504)$ $(3, 6)$-regular LDPC code [10] and binary SPA decoders following the MLDT at the receiver.

![Graph showing BER performance for different m-sequences and LDPC configurations](image)

**Fig. 10.** CDMA system with m-sequences of length $J = 15$ over a 2-path Rayleigh fading channel. MLDT receiver for $P = 2$.

In Fig. [10], we can see that the BER performances for the 2-path CDMA system can be some-
what improved by using the LDPC code. However, the BER performances are not satisfactory even for only \( P = 2 \). We will resort to multi-carrier CDMA in the following subsection.

C. Multi-Carrier Direct-Sequence CDMA (MC-DS-CDMA) System

To maintain the orthogonality of HW code over multipath channels, the MC-DS-CDMA system is considered.

1) System Model: The transmitter structure of the MC-DS-CDMA system is illustrated in Fig. 11. The input sequence of user \((k, p)\) is \( b_{k,p} = [b_{k,p}^0, ..., b_{k,p}^p, ..., b_{k,p}^{N-1}] \). For \( p = 1, ..., P \), the bit \( b_{k,p}^n \) of user \((k, p)\) is spread by the signature sequence \( s_k = [s_k(1), ..., s_k(J)] \) for the \( n \)th subcarrier. We have \( X_{k,p}^n = [X_{k,p}^n(1), ..., X_{k,p}^n(J)] \), where \( X_{k,p}^n(j) = b_{k,p}^n s_k(j) \), which is then processed by an \( N \)-point IFFT (inverse fast Fourier transform). At the end, the cyclic prefix (CP) is added to cope with the multipath effect.

\[ X_{k,p} = [X_{k,p}^1(1), ..., X_{k,p}^J(J)] \]

\[ R_n(j) = \sum_{k=1}^{K} \sum_{p=1}^{P} H_{k,p}^n X_{k,p}^n(j) + W(j), \quad 1 \leq j \leq J \]
where \( H_{k,p}^n \) is the channel coefficient of user \((k, p)\) on the \(n\)-th subcarrier and \( W(j) \) is the Gaussian noise. For \( P = 2 \), after de-spreading, we have

\[
R_{k,AB}^n = \frac{1}{J} \sum_{j=1}^{J} R_j^n s_k(j).
\]  

(31)

![Fig. 12. The receiver of the MC-DS-CDMA system for user \((k, p)\).](image)

For \( P = 2 \), the corresponding table for the superimposed transmitted signal of the collided user \(A\) and user \(B\) in the frequency domain are slightly modified and are shown in TABLE II. For \( P = 3 \), \( S_{ABC}(i), i = 0, 1, 2, ..., 7 \) can be similarly obtained.

**TABLE II**

| \(i\) | \(b_{k,A}^n\) | \(b_{k,B}^n\) | \(X_{k,A}^n\) | \(X_{k,B}^n\) | \(S_{k,AB}(i)\) |
|-------|-------------|-------------|-------------|-------------|----------------|
| 0     | 0           | 0           | 1           | 1           | \(H_{k,A}^n + H_{k,B}^n\) |
| 1     | 0           | 1           | 1           | -1          | \(H_{k,A}^n - H_{k,B}^n\) |
| 2     | 1           | 0           | -1          | 1           | \(-H_{k,A}^n + H_{k,B}^n\) |
| 3     | 1           | 1           | -1          | -1          | \(-H_{k,A}^n - H_{k,B}^n\) |

2) **Simulation Results:** In the simulation, we consider an LDPC-coded MC-DS-CDMA system in multipath Rayleigh fading channels of which the path length is \(L = 5\) and each path has the
same average power. We set $K = J = 16$. The $(1008, 504)$ LDPC code is used. The FFT size and the CP length are set to 16 and 4, respectively. The simulated BER performances are provided in Fig. 14.

It is interesting to see that using MLDT with a single GSPA decoder for either $P = 2$ or $P = 3$ can obtain BER performances very close to those obtained for only $P = 1$. Hence, the number of users can be doubled from 16 to 32 or tripled from 16 to 48. This is a significant advantage.

Compared to MLDT with a single GSPA decoder, we see that MLDT with multiple SPA decoders can obtain somewhat inferior BER performances for both $P = 2$ and $P = 3$. However, using multiple SPA decoders has the advantage of lower decoding complexity. The BER performances of MLDT with multiple SPA decoders for $P = 2$ can be improved by employing the interchange of LLR values between the two SPA decoders. The modified MLDT receiver followed by two SPA decoders, which is denoted as MLDT with inter SPA, is depicted in Fig. 13. The output of the MLDT receiver is first processed by the SPA decoder of user $A$, which generates updated $e_{LLR}(b_{k,A})$ to update $\Pr\{s_{AB} = S_{AB}(i)\}$. Then, the MLDT is able to generate $e_{LLR}(b_{k,B})$ by (32), which is then processed by the SPA decoder of user $B$. We can repeat these steps to obtain improved LLR values. From Fig. 14, we see that for $P = 2$, the BER performances of MLDT with inter SPA are very close to those of MLDT with a single GSPA.

![Fig. 13. Illustration of MLDT with inter SPA.](image-url)
\[
\begin{align*}
\frac{p_0}{2\pi\sigma^2} & = \beta \exp\left(-\frac{|r_{AB} - S_{AB}(0)|^2}{2\sigma^2}\right) \left(1 - \frac{1}{1 + \exp[e_{LLR}(b_{k,A})]}\right) \\
\frac{p_1}{2\pi\sigma^2} & = \beta \exp\left(-\frac{|r_{AB} - S_{AB}(1)|^2}{2\sigma^2}\right) \left(1 - \frac{1}{1 + \exp[e_{LLR}(b_{k,A})]}\right) \\
\frac{p_2}{2\pi\sigma^2} & = \beta \exp\left(-\frac{|r_{AB} - S_{AB}(2)|^2}{2\sigma^2}\right) \left(1 + \exp[e_{LLR}(b_{k,A})]\right) \\
\frac{p_3}{2\pi\sigma^2} & = \beta \exp\left(-\frac{|r_{AB} - S_{AB}(3)|^2}{2\sigma^2}\right) \left(1 + \exp[e_{LLR}(b_{k,A})]\right)
\end{align*}
\]

\[e_{LLR}(b_{k,B}) = \ln \frac{p_0 + p_2}{p_1 + p_3},\]

where \(\beta\) is a normalization factor.

Fig. 14. (1008, 504) LDPC-coded MC-DS-CDMA system in multipath Rayleigh fading Channels.
V. CONCLUDING REMARKS

We propose to use an MLDT technique to allow multiple users which transmit signals over independent fading channels to share the same resource. Both BER analysis and simulation for the uncoded system show that using MLDT can double or triple the number of users for multiple access with the price of some degradation of BER performances. Capacity analysis is provided to show the possible gains that we can achieve. Designs of Raptor coded systems using MLDT over the single-path quasi-static Rayleigh fading channels and LDPC-coded multi-carrier direct-sequence CDMA systems using MLDT over the multi-path quasi-static Rayleigh fading channels show that both appreciable coding gains and increased number of users can be obtained.

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