Spectrum-Efficient Triple-Layer Hybrid Optical OFDM for IM/DD-Based Optical Wireless Communications

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This work was supported in part by the Foundation for Excellent Young Talents of Jilin Province under Grant 20190103010JH, in part by the 13th Five-Year Science and Technology Research Project of Education Department of Jilin Province under Grant JJKH20190277KJ, in part by the Fundamental Research Funds for the Central Universities under Grant 2412018QD002, in part by the National Natural Science Foundation of China under Grant 11905028, in part by the Science and Technology Program of Quanzhou under Grant 2019C010R, in part by the International Science and Technology Cooperation Program of Fujian Province under Grant 201700020, and in part by the Scientific Research Instrument and Equipment CAS under Grant YJKYYQ20170052.

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ABSTRACT In this paper, a triple-layer hybrid optical orthogonal frequency division multiplexing (THO-OFDM) for intensity modulation with direct detection (IM/DD) systems with a high spectral efficiency is proposed. We combine N-point asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM), N/2-point ACO-OFDM, and N/2-point pulse amplitude modulated discrete multitone (PAM-DMT) in a single frame for simultaneous transmission. The time- and frequency-domain demodulation methods are introduced by fully exploiting the special structure of the proposed THO-OFDM. Theoretical analysis show that, the proposed THO-OFDM can reach the spectral efficiency limit of the conventional layered ACO-OFDM (LACO-OFDM). Simulation results demonstrate that, the time-domain receiver offers improved bit error rate (BER) performance compared with the frequency-domain with ∼40% reduced computation complexity when using 512 subcarriers. Furthermore, we show a 3 dB improvement in the peak-to-average power ratio (PAPR) compared with LACO-OFDM for the same three layers.

INDEX TERMS LACO-OFDM, PAM-DMT, spectral efficiency, computation complexity, PAPR.

I. INTRODUCTION

In the last decades, the increasing requirement for mobile devices and access to high-speed networks has put additional pressure on the radio-frequency (RF) spectrum usage [1], [2]. To address this problem, optical wireless communications (OWC) including the light-emitting diodes (LEDs)-based visible light communications (VLC) has been investigated to offer high-speed data links in certain key applications [2]–[5]. Due to its advantages of huge spectrum resource, lower power consumption, higher transmission data rates \( R_b \) and the immunity to RF electromagnetic interference, VLC is seen as a promising complementary wireless technology to the current RF in indoor (mostly), outdoor and underwater applications [6]–[11].

However, the first major issue in VLC is the low modulation bandwidth \( B_{mod} \) of commercial LEDs (< 5 MHz), which limits the transmission capacity [12], [13]. Advanced equalization techniques and spectrum-efficient modulation techniques were thereafter proposed to improve \( B_{mod} \) and therefore \( R_b \) [14]–[17]. In addition, multi-carrier modulation scheme of orthogonal frequency division multiplexing (OFDM) has been investigated to increase \( R_b \) because of their higher spectral efficiency \( \eta_{se} \) compared with the classical most widely used on-off keying (OOK) [18]. Since the OFDM signal needs to be both real and positive in intensity modulation with direct detection (IM/DD) optical communications, asymmetrically clipped optical
OFDM (ACO-OFDM), DC-biased optical OFDM (DCO-OFDM) and pulse amplitude modulated discrete multitone (PAM-DMT) have been proposed [19], [20]. In DCO-OFDM, high DC-bias level is required to ensure a unipolar positive OFDM signal at the cost of reduced power efficiency. Although both ACO-OFDM and PAM-DMT do not need high DC-bias, $\eta_{\text{se}}$ is still not fully exploited because of the lower subcarrier utilization and the one-dimension PAM mapping, respectively. Therefore, a hybrid QAM and PAM can be used to improve $\eta_{\text{se}}$ [21].

To further improve $\eta_{\text{se}}$ of OFDM and DMT, several spectrum-efficient schemes were proposed recently [22]–[26]. In [22] and [23], a layered optical OFDM technique was proposed for IM/DD optical systems using anti-periodic OFDM signals for simultaneous transmission. In [24], a layered ACO-OFDM (LACO-OFDM) scheme was proposed by combining L-layer ACO-OFDM signals with different effective subcarriers in the time domain (TD) to improve $\eta_{\text{se}}$ of conventional ACO-OFDM. A similar method termed augmented spectral efficiency discrete multitone (ASE-DMT) was proposed to improve $\eta_{\text{se}}$ of conventional PAM-DMT [25], [26]. Note that, although layered-OFDM schemes can remove the spectral efficiency gap between unipolar OFDM and DCO-OFDM, their efficiency limits will require infinite layers to superimpose, which is not possible in real applications. Moreover, the combination and distortion cancellation of too many layers in the time or frequency domain would lead to an increased system complexity (i.e., hardware implementation).

In this paper, we propose a novel spectrum-efficient triple-layer hybrid optical OFDM (THO-OFDM), which offers a trade-off between $\eta_{\text{se}}$ and complexity compared with DCO-OFDM. We show that, the proposed THO-OFDM can reach $\eta_{\text{se}}$ limit of LACO-OFDM using only 3-layer and including $N$-point ACO-OFDM, $N/2$-point ACO-OFDM and $N/2$-point PAM-DMT in a single TD frame. Analysis and simulation results show that, the proposed THO-OFDM outperforms conventional LACO-OFDM in terms of computation complexity and PAPR.

The remainder of this paper is organized as follows. In Section II, the conventional LACO-OFDM is briefly described, while in Section III the proposed THO-OFDM and its time/frequency-domain (FD) transceiver are described in detail. Theoretical analysis of spectral efficiency, computation complexity and BER performance are given in Section IV. Simulation results and performance comparisons of the proposed THO-OFDM are presented in Section V. Finally, conclusions are drawn in Section VI.

II. CONVENTIONAL LACO-OFDM SYSTEM

In classical ACO-OFDM, the input bits are converted into complex symbols of quadrature amplitude modulation (QAM). Following Hermitian symmetry, only the odd subcarriers are used prior to inverse fast Fourier transform (IFFT). Therefore, for ACO-OFDM the input FD vector is given by:

$$X = \{0, X_1, 0, X_3, \ldots, X_{N/2-1}, 0, X_{N/2}, 0, \ldots, X_{N-1}^*, 0, X_1^*\}.$$  \hspace{1cm} (1)

where $N$ is the number of subcarriers and * denotes the conjugate symmetric operation. Following IFFT, the TD signal is expressed as:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) \exp(\frac{2\pi}{N} k n),$$  \hspace{1cm} (2)

where $n = 0, 1, \ldots, N - 1$, and $x(n)$ is a bipolar signal, which satisfies the antisymmetric property given by:

$$x(n) = -x(n+N/2), \quad n = 0, 1, \ldots, N/2 - 1.$$  \hspace{1cm} (3)

The ACO-OFDM signal is obtained by a negative clipping without losing any information. Based on this feature, L-layer ($L \geq 2$) LACO-OFDM is constructed by superposition of $\lfloor N, N/2, \ldots, N/2^{L-1} \rfloor$ points ACO-OFDM signals in the TD [24]. Following a redundant replication process, each layer with the same ACO-OFDM signal length are combined in each layer. Here, we define $\eta_{\text{se}}$ of conventional ACO-OFDM. A similar method termed augmented spectral efficiency discrete multitone (ASE-DMT) was proposed to improve $\eta_{\text{se}}$ of conventional PAM-DMT [25], [26]. Note that, the negative clipping distortion of ACO-OFDM corresponds to the lower layer should be removed before correctly demodulating the higher layer in order to determine the optimal bit error rate (BER) performance. However, the optimal BER performance is achieved at the cost of further increased system complexity, hardware implementation and latency.

III. PROPOSED THO-OFDM SYSTEM

From Section II we can see that, the data capacity of LACO-OFDM increases with the layer number but at the cost of increased system complexity. Therefore, we propose a novel THO-OFDM scheme including double layers ACO-OFDM and single layer PAM-DMT signals to reach the $\eta_{\text{se}}$ limit of LACO-OFDM with a much simpler transmitter (Tx) structure as shown in Fig. 1.

A. TRANSMITTER

The detailed Tx structure of the proposed THO-OFDM is shown in Fig. 2, where serial-to-parallel conversion, Hermitian symmetry and cyclic prefix (CP) insertion are omitted. The arbitrary binary bit sequence is first allocated to the 3-layer based on the modulation orders and IFFT points in each layer. Here, we define $X_{\text{ACO}}^{(1)}$, $X_{\text{ACO}}^{(2)}$ and $X_{\text{PAM}}^{(3)}$ as
FIGURE 1. Comparisons of the proposed THO-OFDM and LACO-OFDM.

![Diagram](https://example.com/diagram1.png)

FIGURE 2. Block diagram of the proposed THO-OFDM Tx.

![Diagram](https://example.com/diagram2.png)

$X^{(3)}_{\text{PAM}} = j [0, 0, P_2, 0, P_4, \ldots, P_{N/4-2}, 0, 0, 0, -P_{N/4-2}, 0, \ldots, -P_4, 0, -P_2, 0], \quad (6)$

where $j = \sqrt{-1}$ and $P_k (k = 2, 4, \ldots, N/4 - 2)$ is the PAM symbols. Although the effective ACO-OFDM samples in the 2nd layer are half of the 1st layer in THO-OFDM, the length of effective PAM-DMT samples in the 3rd layer is the same as ACO-OFDM in the 2nd layer. By doing this, the spectral efficiency of THO-OFDM is increased significantly compared with conventional LACO-OFDM. Note that, in THO-OFDM Hermitian symmetry is still required to ensure the anti-symmetric property for the first two layers of ACO-OFDM and the periodic property for the 3rd layer of PAM-DMT.

Following the IFFT operation at each layer, we will have the bipolar TD signals of $X^{(1)}_{\text{ACO}}, X^{(2)}_{\text{ACO}}$ and $X^{(3)}_{\text{PAM}}$. Negative clipping is applied to these signals to ensure all positive and real signals prior to applying the 2-time repeat operation to the 2nd and 3rd layers to compensating for the length difference. The constructed unipolar signals with the same length are defined as $X^{(1)}_{\text{ACO},n}, X^{(2)}_{\text{ACO},n}$ and $X^{(3)}_{\text{PAM},n}$, where $n = 0, 1, \ldots, N - 1$. Therefore, the TD signals in different layers have the different anti-symmetric and a periodic property, which can be concluded as below (7)–(9), as shown at the bottom of the next page.

Finally, the combined triple-layer unipolar signal is given as:

$$X_{\text{THO}} = X_{\text{THO}}^{(1)} + X_{\text{THO}}^{(2)} + X_{\text{THO}}^{(3)}.$$

B. RECEIVER

In this part, we make full use of the special structure of THO-OFDM to represent the two different layered demodulation methods in the TD and FD, respectively.
The block diagram of the TD-based Rx for THO-OFDM is shown in Fig. 3. The received optical signal is first converted into an electrical signal using an optical Rx (ORx) prior to removing the CP. The noise due to optical and electrical parts as well as the ambient lights is modeled as an additive white Gaussian noise (AWGN) [27]–[31]. Thus, the received THO-OFDM signal is given as:

\[ r^{(1,2,3)}_{\text{THO}} = R_{\text{THO}} \otimes h(n) + w_n, \]  

(11)

where \( R \) is the photodiode responsivity, \( n = 0, 1, 2, \ldots, N-1 \), \( h(n) \) is the channel impulse response (CIR), \( w_n \) denotes the discrete samples of AWGN and \( \otimes \) represents convolution operation [32]. To simplify the derivation processes, \( w_n \) is omitted in the following equations.

As for the TD-based Rx, following removal of the CP the signal \( r^{(1,2,3)}_{\text{THO}} \) is split into two, which are given as:

\[ r^{(1,2,3)}_{\text{THO, left}} = \left[ x^{(1)}_{\text{ACO, n}} \right]_c + \left[ x^{(2)}_{\text{ACO, n}} \right]_c + \left[ x^{(3)}_{\text{PAM, n}} \right]_c, \]  

(12)

\[ r^{(1,2,3)}_{\text{THO, right}} = \left[ x^{(1)}_{\text{ACO, n}} + N/2 \right]_c + \left[ x^{(2)}_{\text{ACO, n}} + N/2 \right]_c + \left[ x^{(3)}_{\text{PAM, n}} \right]_c. \]  

(13)

where \( n = 0, 1, \ldots, N/2-1 \). According to (7)-(9), estimated ACO-OFDM in the 1st layer is given by:

\[ \tilde{x}^{(1)}_{\text{ACO, left}} = r^{(1,2,3)}_{\text{THO, left}} - r^{(1,2,3)}_{\text{THO, right}} = \left[ \tilde{x}^{(1)}_{\text{ACO, n}} \right]_c - \left[ \tilde{x}^{(1)}_{\text{ACO, n}} + N/2 \right]_c. \]  

(14)

where \( \tilde{x}^{(1)}_{\text{ACO, left}} \) represents the left half part of the estimated 1st layer bipolar signal \( x^{(1)}_{\text{ACO}} \). The N-point \( x^{(1)}_{\text{ACO}} \) is reconstructed by employing the anti-symmetric property of \( x^{(1)}_{\text{ACO}} \). Following N-point FFT operation, the transmitted bits for the 1st layer is obtained by demodulation of QAM using the odd subcarriers in the FD.

As the superimposed N-point signal in the 2nd and 3rd layers in the TD contains half the repeated signal, the
$N/2$-point effective signal can be utilized to realize the sub-optimal demodulation with reduced complexity. Note, the clipping signal of $\tilde{x}_{ACO, left}^{(3)}$ is removed from $r_{THO, left}^{(1, 2, 3)}$ to obtain the $N/2$-point superimposed signal of the 2nd and 3rd layers, which can be expressed as:

$$r_{THO}^{(2, 3)} = r_{THO, left}^{(1, 2, 3)} - \left[\tilde{x}_{ACO, left}^{(1)}\right]_c.$$  

(15)

Since clipping interference due to the 2nd and 3rd layers only effect the even subcarriers of $r_{THO}$, the ACO-OFDM signal of the 2nd layer is first recovered by means of the $N/2$-point FFT operation. Prior to recovering PAM-DMT in the 3rd layer, the clipping interference from ACO-OFDM of the 2nd layer needs to be removed either in the frequency or time domain [30], [33]. Here, we have adopted the latter, which is simpler to implement and effective in reducing the clipping noise. First, $r_{THO}$ is split into left and right $N/4$-point as $r_{THO, left}^{(2, 3)}$ and $r_{THO, right}^{(2, 3)}$ respectively. The left half part of the 2nd layer bipolar ACO-OFDM signal is estimated as:

$$\tilde{x}_{ACO, left}^{(2)} = r_{THO, left}^{(2, 3)} - \left[\tilde{x}_{ACO, left}^{(3)}\right]_c + \left[\tilde{x}_{PAM, n}\right]_c$$

$$- \left[\tilde{x}_{ACO, n+N/4}\right]_c + \left[\tilde{x}_{PAM, n+N/4}\right]_c$$

$$= \left[\tilde{x}_{ACO, n}\right]_c - \left[\tilde{x}_{ACO, n+N/4}\right]_c,$$  

(16)

where $n = 0, 1, ..., N/4 - 1$. Thus, the left half part of PAM-DMT in the 3rd layer can be estimated as:

$$\left[\tilde{x}_{PAM, n}\right]_c = r_{THO, left}^{(2, 3)} - \left[\tilde{x}_{ACO, left}^{(2)}\right]_c$$

$$= \left[\tilde{x}_{ACO, n}\right]_c + \left[\tilde{x}_{PAM, n}\right]_c - \left[\tilde{x}_{ACO, left}^{(2)}\right]_c.$$  

(17)

According to (9), pairwise clipping can be utilized to reduce the noise by almost half and estimate the error for $\left[\tilde{x}_{PAM, n}\right]_c$ for further improvement of the BER performance of PAM-DMT [28], [34], which is expressed by:

$$\left[\tilde{x}_{PAM, left}^{(3)}\right]_c = \left[\tilde{x}_{PAM, n}\right]_c I \left\{\tilde{x}_{PAM, N/4-1-n}\right\}_c \leq \left[\tilde{x}_{PAM, n}\right]_c,$$  

(18)

where $n = 0, 1, ..., N/8 - 1$ and $I (A)$ is an indicator function with $I (A) = 1$ if even $A$ is true and $I (A) = 0$ otherwise. In addition, $\left[\tilde{x}_{PAM, 0}\right]_c$, $\left[\tilde{x}_{PAM, N/8}\right]_c$, $\left[\tilde{x}_{PAM, N/4}\right]_c$ and $\left[\tilde{x}_{PAM, 3N/8}\right]_c$ are also set to zero according to (9).

Next, $\left[\tilde{x}_{PAM, n}\right]_c$ can be reconstructed based on the periodic property. Finally, the transmitted bits for PAM-DMT in the 3rd layer can be demodulated from the imaginary parts of even subcarriers following the $N/2$-point FFT operation.

As for the FD-based Rx, the subcarriers distribution for the signal and clipping distortion are shown in Fig. 4. The block diagram of the FD-based Rx for THO-OFDM is shown in Fig. 5. As shown in Fig. 4, there is no overlapping of clipping distortion with the data-carrying odd subcarriers for the Layer 1 with the index $K_1 = \{1, 3, ..., N - 1\}$. Therefore, the data can be estimated directly from $K_1$ using the standard demodulation of ACO-OFDM. For the Layer 1, following demodulation, the distortion level is estimated using the clipping noise regeneration process as in [24], [30]. Given that the data-carrying subcarriers of Layer 2 with the index $K_2 = \{2, 6, ..., N - 2\}$ is only affected by the clipping distortion of Layer 1, the Layer 2 can be demodulated by substituting the corresponding distortion components of the Layer 1. Unlike the FD structure of LACO-OFDM, for the Layer 3 of THO-OFDM the index of data-carrying subcarriers is $K_3 = \{4, 8, ..., N - 4\}$. The clipping distortion of PAM-DMT falls on the real part of corresponding even subcarriers in Layer 3, while the imaginary part of corresponding even subcarriers is independent of the PAM-DMT induced distortion. Therefore, the subcarriers of PAM can be extracted by substituting the corresponding distortion components of Layers 1 and 2.

### IV. THEORY ANALYSIS OF THE THO-OFDM SYSTEM

#### A. SPECTRAL EFFICIENCY

The spectral efficiency of the proposed THO-OFDM is determined by the constellation combinations of $N$-point QAM, $N/2$-point QAM and $N/2$-point PAM symbols. The spectral efficiency for the standard ACO-OFDM and PAM-DMT are given as [26], [35]:

$$\eta_{ACO} = \frac{N \log_2 M_{ACO}}{4(N + N_{CP})} \text{(bit/s/Hz)},$$  

(19)

$$\eta_{PAM} = \frac{(N - 2) \log_2 M_{PAM}}{2(N + N_{CP})} \text{(bit/s/Hz)},$$  

(20)

where $M_{ACO}$ and $M_{PAM}$ denote the constellation size of QAM and PAM symbols, respectively. Since only the even subcarriers of PAM-DMT are used to carry data information in the proposed scheme, the total spectral efficiency of the proposed THO-OFDM is given by:

$$\eta_{THO} = \frac{1}{N + N_{CP}} \left(\sum_{i=1}^{2} \frac{N \log_2 M_{ACO}^{(i)}}{2^{i+1}} + \frac{(N - 2) \log_2 M_{PAM}^{(3)}}{8} \right)$$

$$= \frac{N \log_2 M_{ACO}^{(1)} + \frac{N}{8} \log_2 M_{ACO}^{(2)} + \frac{N - 2}{8} \log_2 M_{PAM}}{N + N_{CP}} \text{(bit/s/Hz)},$$  

(21)
where $M_{\text{THO}}^{(l)} (l = 1, 2)$ and $M_{\text{PAM}}$ are the constellation size of QAM and PAM symbols, respectively. For comparisons, the spectral efficiency of $L$-layer LACO-OFDM is given by:

$$\eta_{\text{LACO}} = \frac{1}{N + N_{\text{CP}}} \sum_{l=1}^{L} N \log_2 M_{\text{ACO}}^{(l)} \text{ (bit/s/Hz)}.$$ (22)

Based on (21)-(22), it can be concluded that $\eta_{\text{LACO}} \approx \eta_{\text{THO}}$ for $L \to \infty$, ignoring the effect of $N_{\text{CP}}$ and for the same constellation size for LACO-OFDM and the proposed THO-OFDM. Therefore, THO-OFDM could theoretically achieve the spectral efficiency limit of LACO-OFDM with only three layers and the same constellation size.

**B. COMPUTATION COMPLEXITY**

The computation complexity in this work is defined as the number of complex multiplications in FFT/IFFT. In ACO-OFDM, the computation complexity of an $N$-point complex- and real-valued IFFT/FFT operations, respectively are defined as $O (N \log_2 N)$ and $O (N / 2 \log_2 N)$ accordingly. Given that, in PAM-DMT the data is in the imaginary parts, the computation complexity of $N$-point IFFT/FFT operation is $O (N / 2 \log_2 N)$ [26]. For comparisons, the computation complexities of the TD and FD-based Rxs for THO-OFDM and conventional LACO-OFDM [24] are both given in the following.

At the Tx, THO-OFDM requires one $N$-point and $N / 2$-point complex-valued IFFT and IFFT operations for $x_{\text{THO}}^{(1)}$ and for $x_{\text{ACO}}^{(2)}$ and $x_{\text{PAM}}^{(3)}$ respectively. Therefore, the computation complexity of THO-OFDM can be expressed as:

$$O (\text{THO})_{\text{Tx}} = O (N \log_2 N) + O (N / 2 \log_2 (N / 2)) + O (N / 4 \log_2 (N / 2)) = O (N (\log_2 N + 3/4 \log_2 (N / 2))).$$ (23)

Similarly, the computation complexity of $L$-layer conventional LACO-OFDM at the Tx can be given by (24):

$$O (\text{LACO})_{\text{Tx}} = O \left( \frac{1}{N + N_{\text{CP}}} \sum_{l=1}^{L} N \log_2 M_{\text{ACO}}^{(l)} \right) = O \left( 2N \left( 1 - 2^{-L} \right) \log_2 N - 2N \left( 1 - 2^{-L} \right) \right) + \left( L - 1 \right) N / 2 \log_2 N$$

$$= O \left( N \log_2 N + 3N / 2 \log_2 N \right) - N + N / 4 \log_2 (N / 2) + 2^{1-L} N \left( L + 1 - \log_2 N \right) = O (\text{THO})_{\text{TX}} + F (N, L).$$ (24)

Note, $F (N, L) = (N / 4 \log_2 (N / 2) - N + 2^{1-L} N \left( L + 1 - \log_2 N \right)$, which satisfies $F (N, L) > 0$ if $L \geq 4$, $N > 32$ and $F (N, L) < 0$ if $2 \leq L < 4$.

At the Rx, the computation complexity of the proposed the TD and the FD based Rxs for THO-OFDM are respectively given by (25) and (26):

$$O (\text{THO})_{\text{Rx-TD}} = O \left( N / 2 \log_2 N + N / 4 \log_2 (N / 2) \right) + N / 4 \log_2 (N / 2)$$

$$= O \left( N / 2 \log_2 N + \log_2 (N / 2) \right) = O \left( N (\log_2 N - 1 / 2) \right).$$ (25)

$$O (\text{THO})_{\text{Rx-FD}} = O \left( N / 2 \log_2 N + N / 2 \log_2 N + N / 2 \log_2 (N / 2) \right) + N / 4 \log_2 (N / 2)$$

$$= O \left( N (2 \log_2 N + 3 / 4 \log_2 (N / 2)) \right).$$ (26)

Whereas, the computation complexity of the $L$-layer conventional LACO-OFDM at the Rx is given by (27):

$$O (\text{LACO})_{\text{Rx}} = O \left( \frac{1}{N + N_{\text{CP}}} \sum_{l=1}^{L} N \log_2 M_{\text{ACO}}^{(l)} \right) > O (\text{THO})_{\text{Rx-TD}}.$$ (27)
The total computation complexity of the proposed THO-OFDM and the conventional LACO-OFDM are given by (28):

\[
\begin{align*}
O(\text{THO})_{\text{TD}} &= O(\text{THO})_{\text{Tx}} + O(\text{THO})_{\text{Rx-TD}} \\
O(\text{THO})_{\text{FD}} &= O(\text{THO})_{\text{Tx}} + O(\text{THO})_{\text{Rx-FD}} \\
O(\text{LACO}) &= O(\text{LACO})_{\text{Tx}} + O(\text{LACO})_{\text{Rx}}.
\end{align*}
\]

Based on (25)-(26), we can conclude that the computation complexity of the TD-based Rx is lower compared with the FD-based Rx in THO-OFDM. In order to measure the computation complexity, here we define the computation complexity reduction ratio (CCRR) as in (29):

\[
\begin{align*}
\text{CCRR}_1 &= 1 - \frac{O(\text{THO})_{\text{TD}}}{O(\text{LACO})} \\
\text{CCRR}_2 &= 1 - \frac{O(\text{THO})_{\text{FD}}}{O(\text{LACO})}.
\end{align*}
\]

For \( N = 512 \) and the maximum number of layer (i.e., \( L_{\text{max}} = 8 \)) we have \( \text{CCRR}_1 \approx 59\% \) and \( \text{CCRR}_2 \approx 32\% \), which shows reduced computation complexity for the proposed THO-OFDM with TD and FD-based Rxs compared with the conventional LACO-OFDM Rx as in [24]. Finally, \( \eta_{\text{se}} \) and computation complexity of the proposed THO-OFDM and conventional LACO-OFDM for range of layers are summarized in Table 1. Note, the effect of \( N_{\text{CP}} \) is not considered, \( N \) and \( M_{\text{ACO}} = M_{\text{PAM}} \) are set to 512 and 4, respectively.

| THO-OFDM with TD | CC | TX | Rx |
|------------------|----|----|----|
| THO-OFDM with FD | SE | 0.999 | 7680 | 4352 |
| L=2 | 0.750 | 6656 | 13824 | 41.3 |
| L=3 | 0.875 | 7552 | 19720 | 52.8 |
| L=4 | 0.938 | 7936 | 19712 | 56.5 |
| L=5 | 0.969 | 8096 | 20480 | 57.9 |
| L=6 | 0.984 | 8160 | 20800 | 58.5 |
| L=7 | 0.992 | 8184 | 20928 | 58.7 |
| L=8 | 0.996 | 8192 | 20976 | 58.8 |

The BER performance for ACO-OFDM with \( M_{\text{ACO}} \)-ary square QAM and PAM-DMT with \( M_{\text{PAM}} \)-ary PAM are given by [23], [36], [37]:

\[
P_{b,\text{THO}} \approx \frac{4}{\sqrt{M_{\text{ACO}}}} Q\left( \frac{3 \log_2 M_{\text{ACO}} E_b}{M_{\text{ACO}} - 1} N_0 \right),
\]

\[
P_{b,\text{PAM}} \approx \frac{2}{M_{\text{PAM}}} Q\left( \frac{6 \log_2 M_{\text{PAM}} E_b}{M_{\text{PAM}}^2 - 1} N_0 \right),
\]

where \( E_b \) is the bit energy, \( N_0 \) is the noise spectral density and \( Q(\cdot) \) is the tail probability of the standard normal distribution given by \( Q(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\xi}^{\infty} \exp \left( -\frac{x^2}{2} \right) dx \).

Based on (30)-(31), the average \( P_b \) for 3-layer THO-OFDM including \( N \)-point ACO-OFDM with \( M_{\text{ACO}}^{(1)} \)-ary square QAM, \( N/2 \)-point ACO-OFDM with \( M_{\text{ACO}}^{(2)} \)-ary square QAM and \( N/2 \)-point PAM-DMT with \( M_{\text{PAM}} \)-ary PAM is given by (32), as shown at the bottom of this page. Here, \( P_{b,l}(l = 1, 2, 3) \) denotes the BER of the \( l \)th layer of THO-OFDM. Since the BER performance of \( M_{\text{ACO}} \)-ary square QAM is the same as \( M_{\text{PAM}} \)-ary PAM based on the above formulations, then we have \( M_{\text{ACO}}^{(1)} = M_{\text{ACO}}^{(2)} = \sqrt{M_{\text{PAM}}} = M \) for a fair comparison for LACO-OFDM with the same \( \eta_{\text{se}} \).

In order to ensure that different layers have similar BER performance at a given signal to noise ratio (SNR), the energy/bit for different layers should be kept the same. The theoretical analysis show that, the inter-layer BERs approximately satisfy \( P_{b,1} = P_{b,2} = P_{b,3} \) in the FD demodulation provided \( M_{\text{ACO}}^{(1)} = M_{\text{ACO}}^{(2)} = \sqrt{M_{\text{PAM}}} = M \). In the TD demodulation, see Fig. 3, however, the subtracting process following signal splitting will discard half of the noise component, which is totally different from the FD demodulation adopted in THO-OFDM or LACO-OFDM. Therefore, the single layer BER performance will degrade with the number of layers decreasing. Note that, BERs varies in the inter-layer, which can be exploited to improve the final BER, as was demonstrated in the TD iterative-based Rx of the HACO-OFDM and LACO-OFDM [28], [34].

V. NUMERICAL RESULTS

In this section, we outline the comprehensive BER, \( \eta_{\text{se}} \), computation complexity and CCDF analysis obtained using Monte-Carlo simulations in the MATLAB R2016a. To simplify simulations, we consider the AWGN channel and the maximum number of subcarriers \( N \) of 512. Fig. 6 shows the BER as a function of SNR for the proposed THO-OFDM with TD/FD demodulation and conventional LACO-OFDM for 4-, 16-, and 64-QAM, 2-, 4-, and 8-PAM, and a range of \( \eta_{\text{se}} \).

Since PAM only uses a one-dimension mapping to carry data bits compared with the two-dimension QAM constellations, the modulation orders in Fig. 6 should satisfy \( M_{\text{ACO}} = \sqrt{M_{\text{PAM}}} \) for a fair comparison. From Fig. 6, we can see that,
FIGURE 6. BER performance against SNR for the proposed THO-OFDM with TD and FD demodulation and conventional LACO-OFDM for different $\eta_{se}$ (bit/s/Hz).

the proposed THO-OFDM with two demodulation methods can achieve the same $\eta_{se}$ as LACO-OFDM with 3-layer. More specifically, $\eta_{se}$ of 0.875/1.75/2.625 bit/s/Hz and M$_{ACO}$ = 4, 16, 64. Moreover, at a BER of $10^{-4}$ THO-OFDM with TD demodulation offer the SNR gains of 0.6, 0.8, and 1.0 dB compared with FD demodulation for M$_{ACO}$ = 4, 16 and 64, which marginally better than LACO-OFDM with FD demodulation.

To better illustrate the inter-layer BERs comparison of THO-OFDM with two demodulation methods, the BER plots for 1-3 layers in THO-OFDM with two demodulation methods for 4-QAM, 4-QAM and 2PAM are depicted in Fig. 7. Note, the BER plots are obtained for layer-by-layer from low to high. As shown in Fig. 7, the inter-layer BERs plots for TD demodulation approximately approach at a given SNR value while the inter-layer BERs of TD demodulation turn better with the layer number increases. This is consistent with the theoretical analysis for the BER given in Section IV.

The detailed comparisons of $\eta_{se}$ and computation complexity for the proposed THO-OFDM with two demodulation methods and the conventional LACO-OFDM are drawn in Fig. 8 for M$_{ACO}$ = M$_{PAM}$ = 4. Obviously, the proposed THO-OFDM with TD or FD achieve the spectral efficiency limit of LACO-OFDM with the computation complexity reduced significantly, which is also consistent with the previous analysis. E.g., for $L = 3$, we can further verify the advantages of the proposed THO-OFDM in terms of $\eta_{se}$ of 0.124 bit/s/Hz and CCRR$_1$ $\approx$ 53% in TD and CCRR$_2$ $\approx$ 22% in FD compared with the conventional LACO-OFDM. Meanwhile, $\sim$40% computation complexity reduction is achieved in TD demodulation compared with FD demodulation for THO-OFDM.

As the transfer characteristic of the commercial LEDs is nonlinear, the PAPR becomes another key factor to evaluate the performance of the optical OFDM [29], [38]–[40]. The PAPR of discrete optical OFDM signal can be generally defined as the ratio of the maximum power to the average power, which is given by:

$$PAPR = 10 \log_{10} \frac{\max |x(n)|^2}{E[|x(n)|^2]} \text{ (dB),}$$  \hspace{1cm} (33)

where $E[.]$ denotes the statistical expectation. The complementary cumulative distribution function (CCDF) is further employed to illustrate the PAPR performance comparisons between the proposed THO-OFDM and the conventional LACO-OFDM. It denotes the probability that, PAPR of an optical OFDM signal exceeds a certain threshold $PAPR_0$ as given by:

$$CCDF = Pr(PAPR > PAPR_0).$$  \hspace{1cm} (34)

Fig. 9 shows the CCDF against the threshold $PAPR_0$ for the proposed THO-OFDM and LACO-OFDM for a range.
of layers. It is shown that, at the CCDF of $10^{-4}$, the PAPR requirements are $>19$, $>18$, $>17.5$, $>17$ and $>16$ dB for LACO-OFDM for $L = 2, 3, 4, 5$ and $6$, respectively compared with $15$ dB for the proposed THO-OFDM. It is worth noting that, for $L \geq 6$ there is a tendency that, the PAPR requirement for the proposed THO-OFDM is still lower than LACO-OFDM. For LACO-OFDM with more layers, fewer zeros would be found in the TD signal, due to the superposition of more layers. Meanwhile, the average power of the signal increases faster than the peak power as more layers are utilized [29]. Therefore, the LACO-OFDM signal with more layers tends to exhibit lower PAPR, which can also be verified from Fig. 9. However, at the CCDF of $10^{-4}$, the proposed scheme offers lower $\text{PAPR}_0$ by about $3$ dB compared with LACO-OFDM for the same number of layers and the modulation format.

VI. CONCLUSION

In this paper, a spectrum-efficient triple-layer hybrid optical orthogonal frequency division multiplexing was presented and studied. We showed that, the proposed THO-OFDM reached the spectral efficiency limit of the classical LACO-OFDM with only three layers. In addition, theoretical analysis results demonstrated that, THO-OFDM with the TD-based Rx did attain reduced computation complexity by 40% compared with the conventional successive interference cancellation (SIC) demodulation scheme employed in the frequency domain with a marginally improved BER. In addition, CCDF simulation results demonstrated that, a $3$ dB PAPR improvement for THO-OFDM compared with the classical LACO-OFDM for the same number of layers, thus demonstrating its potential applications in IM/DD based optical wireless communications.

ACKNOWLEDGMENT

The authors would like to thank Dr. Qi Wang graduated from Tsinghua University for the fruitful discussions on LACO-OFDM and generously helps on simulation codes.

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