Vortex Nucleation and Transition to Turbulence in two-component Bose–Einstein condensates

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Abstract. We theoretically study the instability of countersuperflow, i.e., two counter-propagating miscible superflows, in uniform two-component Bose–Einstein condensates. When the relative velocity of the counterflow exceeds a critical value, the instability causes the nucleation and expansion of vortex rings. A lot of vortex reconnections are caused and lead to binary quantum turbulence, where both components become turbulent. Then we introduce the unique velocity in two-component Bose–Einstein condensates and investigate the probability distribution of the velocity in the binary quantum turbulence to obtain the probability distribution whose tail in the high-velocity region is considerably suppressed compared to single-component quantum turbulence.

1. Introduction
Hydrodynamic instability in quantum fluids, such as superfluid ⁴He, ³He and cold atomic Bose–Einstein condensates (BECs), causes characteristic patterns or quantum turbulence. In atomic BECs, there are growing interests in the study of the counterparts of hydrodynamic phenomena in classical fluid dynamics, such as Kelvin–Helmholtz instability [1, 2], Rayleigh–Taylor instability [3, 4], the Bénard–von Kármán vortex street [5], and turbulence [6, 7, 8]. In this work we study countersuperflow, i.e., two counter-propagating miscible superflows, in uniform two-component BECs. Law et al. showed theoretically that countersuperflow is unstable when the relative velocity exceeds some critical value by linear analysis [9]. Very recently Hamner et al. study experimentally and numerically dark-bright solitons in counterflowing BECs [10]. We have reported that the instability of countersuperflow induced vortex nucleation and tangles of the vortices, which is binary quantum turbulence [11, 12]. In this paper, we discuss the nonlinear dynamics and the probability distribution of the velocity in binary quantum turbulence.

2. Vortex nucleation and transition to turbulence
We consider two counter-propagating superfluids consisting of two distinguishable particles in an isolated uniform system at zero temperature. Then we consider a binary mixture of BECs described by the condensate wave functions \( \Psi_j(r, t) = \sqrt{n_j(r, t)}e^{i\phi_j(r, t)} \) in a mean-field approximation, where the index \( j \) refers to each component (\( j = 1, 2 \)). The wave functions are
governed by the coupled Gross–Pitaevskii (GP) equations [13]

\[
\frac{i\hbar}{\partial t} \Psi_j = \left( -\frac{\hbar^2}{2m_j} \nabla^2 + \sum_{k=1}^{2} g_{jk} |\Psi_k|^2 \right) \Psi_j, \tag{1}
\]

where \(m_j\) is the mass of the \(j\)th component. The coefficient \(g_{jk} = 2\pi \hbar^2 a_{jk}/m_{jk}\) represents the atomic interaction with \(m_{jk}^{-1} = m_j^{-1} + m_k^{-1}\) and the s-wave scattering length \(a_{jk}\) between the \(j\)th and \(k\)th components. Our system satisfies the conditions \(g_{11} g_{22} > g_{12}^2\) and \(g_{jj} > 0\) that the two miscible condensates are stable [13]. The wave functions \(\Psi_j \equiv \Psi_j^0\) in a stationary state are written as

\[
\Psi_j^0 = \sqrt{\mu_j} e^{i(m_j V_j \cdot r - \mu_j t)/\hbar} \tag{2}
\]

with the velocity \(V_j\) and the chemical potential \(\mu_j\) of the \(j\)th component. The countersuperflow is realized with \(V_1 \neq V_2\).

We investigated the nonlinear dynamics caused by countersuperflow instability by numerically solving the coupled GP equations. The numerical simulations were done in a three-dimensional system, being subject to a periodic boundary condition. The initial state is the stationary state of Eq. (2) with small white noise to trigger the instability, and we set the parameters as \(m_1 = m_2 = m\), \(g_{11} = g_{22} = g\), \(g_{12} = 0.9g\), \(n_1 = n_2 = n\), \(V_1 = -V_2\), \(|V_1| = |V_2| = 2.36c\) with \(c = \sqrt{gn/m}\), and \(\mu_1 = \mu_2 = mV_R^2/8 + (g + g_{12})n\) with \(V_R = |V_R| = |V_1 - V_2|\). Because the parameters of two components are symmetric, both components follow similar scenarios in the nonlinear dynamics. First, the instability causes disk-shaped low-density regions that face the direction parallel to the initial relative velocity \(V_R \parallel \hat{x}\) along the \(x\) axis (b). A tiny vortex ring appears inside the low-density disk (c) and the disk then immediately transforms into a torus like a usual vortex ring (d). The nucleated vortex rings gradually expand due to momentum exchange between the two components. The vortices then start to reconnect with each other (e). Frequent reconnections eventually lead to vortex tangles (f). Then the both component becomes turbulent state, which is binary quantum turbulence. The size and expansion rate of the nucleated vortex rings strongly depend on the initial relative velocity and interaction between two components respectively. As a result, we can control the vortex line density and configuration of the vortices in the turbulent state.

Figure 1. (Color online) Nonlinear dynamics of countersuperflow instability. The curves show the vortex cores of the 1st component [black (red) curve]. The isosurface \(|\Psi_1|^2/n = 0.1\) of the density is also plotted with [grey (green)] surfaces. The time \(t\) is normalized as \(t = t'\tau\) with \(\tau = \hbar/\pi g\). The box size is \(V = (8\xi)^3\) with \(\xi = \hbar/\sqrt{mg\pi}\).

3. Probability distribution of velocity field in binary quantum turbulence

The binary quantum turbulence induced by the countersuperflow instability can be realized experimentally by using the Zeeman shift to induce the relative motion [10]. As described, we
can control experimentally the vortex line density in binary quantum turbulence by changing the magnetic field gradient. Binary quantum turbulence has some interesting features not shared by single quantum turbulence. The core of the vortices in repulsive two-component BECs is generally filled with the other component and the total density \( \rho = \rho_1 + \rho_2 \) with \( \rho_j = n_j m_j \) is smoothed, which corresponds to spin textures referred to as a skyrmion [15]. Additionally, the velocity field around the vortex in repulsive two-component BECs is interesting. The velocity field around a vortex in single component BECs decreases in inverse proportion to the distance from the core and then the velocity field diverges at the center of the vortex core. However, when we consider the total superfluid mass-current velocity, \( \mathbf{v} = (\rho_1 \mathbf{v}_1 + \rho_2 \mathbf{v}_2)/(\rho_1 + \rho_2) \) with \( \mathbf{v}_j = \hbar/m_j \nabla \phi_j \), the velocity field of the vortices does not diverge at the center due to the smoothed density. The profile of the velocity closely resembles that of the Rankine vortex, which is typical in a classical fluid [Fig. 2]. Then, we may expect that the probability distribution of the velocity in binary quantum turbulence gets closer to that of classical turbulence than single quantum turbulence. In uniform isotropic classical turbulence, it is known that the probability density function (PDF) of the velocity field obeys a gaussian distribution [16]. On the other hand, in single quantum turbulence it obeys a gaussian distribution in the low-velocity region.
but a power-law distribution $v^{-3}$ in the high-velocity region because of the $r^{-1}$ decay of the velocity around a vortex [8, 17].

We investigate the PDF of the total superfluid mass-current velocity in binary quantum turbulence. We choose the turbulent state where the vortex line density is maximum and the vortex tangles get isotropic for the above setting parameters. Figure 3 shows double logarithmic plotted PDFs of the velocity in the direction parallel to the initial relative velocity of the first component [grey (red) points] and the total superfluid mass-current velocity [black (blue) points]. The PDF of velocity of the first component obeys a gaussian distribution in the low-velocity region and a power-law distribution $v^{-3}$ in the high-velocity region as shown in previous studies [17]. On the other hand, the tail of the PDF of the total superfluid mass-current velocity is considerably suppressed because the decay of the velocity around the maximum of the velocity is more gradual than the $r^{-1}$ decay of the velocity of a vortex in single component BECs. The PDF of velocity of binary quantum turbulence does not obey a gaussian distribution in the whole velocity region as the classical turbulence but the tail of PDF in the high-velocity region is closer to that of classical turbulence than that of single quantum turbulence.

4. Summary

We theoretically studied the instability of two counter-propagating miscible superflows in uniform miscible two-component BECs. The instability nucleates quantized vortex rings and eventually results in binary quantum turbulence, where both components become turbulent. In two-component BECs, the cores of vortices are filled with the other component and then the total superfluid mass-current velocity of the vortex is similar to a Rankine vortex. Then we investigated the statistical nature of the binary quantum turbulence. The PDF of velocity of binary quantum turbulence does not perfectly obey the same distribution as a classical turbulence in the whole velocity region but the tail of PDF in the high-velocity region is considerably suppressed. In the future work, we will study the details of the probability distribution of the velocity and other statistical natures, e.g. energy spectrum.

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