Orbital Eccentricity Effects on the Stochastic Gravitational Wave Background from Coalescing Binary Neutron Stars

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Abstract

Unresolved coalescing binary neutron stars in the Galaxy and beyond produce a stochastic gravitational wave background within the LISA frequency band, which can be potentially dangerous for possible detection of fundamentally more interesting relic cosmological backgrounds. Here we address the question what effects the unavoidable eccentricity of orbits of these systems should have on the properties of this background. In particularly, we examine starting from which frequency one-year observations with LISA can be secure from contamination by the noise produced by galactic binary neutron stars in eccentric orbits. We come to the conclusion that harmonics from such binaries do not contribute significantly above $\sim 10^{-3}$ Hz.

1 Introduction

It is generally recognized that in the 21st century gravitational wave astronomy will open up a new window to study physical processes in strong gravitational field and in the early Universe (see Grishchuk et al. 2001 for a recent review). Among the potential sources of gravitational waves (GW) for the forthcoming GW detectors, relic stochastic GW backgrounds, coming up from the early Universe, play a special role due to their fundamental importance and potential detectability by the space-born laser interferometer LISA (Grishchuk 1997, Grishchuk et al. 2001).

The existence of relic gravitational waves is a consequence of quite general assumptions. In the framework of the conventional cosmological picture, the strong variable gravitational field of the early Universe amplifies the inevitable zero-point quantum oscillations of the gravitational waves and produce a stochastic background of relic gravitational waves in a wide frequency band from $10^{-18}$ Hz to $10^{10}$ Hz measurable today. The detection of relic gravitational waves is a feasible way to learn about the evolution of the very early Universe, up to the limit of Planck era and Big Bang.

The expected level of this background is model-dependent so its robust estimation is complicated. Nevertheless, if the level of some background turns out to be higher than the detector sensitivity limit in a given frequency band, it can be directly detected using only one interferometer LISA, provided that other backgrounds do not contribute at these frequencies (for more deep discussion of the LISA capabilities see recent papers by Cornish and Larson (2001), Cornish (2001a,b), and Ungarelli and Vecchio (2001)).
Among various astrophysical GW backgrounds, those produced by several classes of merging binary compact stars (white dwarfs, neutron stars, black holes and their combinations) dominate in the LISA frequency band $10^{-5} - 10^{-2}$ Hz (e.g. Hils and Bender 1998, Kosenko and Postnov 1999, Ferrari et al. 2001). A feature of all such backgrounds is that their level (in terms of the characteristic dimensionless strain metric amplitude $h$) is $h \propto \sqrt{R}$, where $R$ is the merging rate of these binaries. The most “dangerous” from the point of view of "contamination” of the LISA frequencies are galactic binary white dwarfs with the highest galactic merging rate $R_{wd} \sim 1/300$ yr$^{-1}$. For one-year observational run they will make up a stochastic GW background up to frequencies around 1 mHz (Hils and Bender 1998, Kosenko and Postnov 1999).

However, if a binary star has a non-circular orbit, it emits GW at a number of harmonics to its orbital (Keplerian) frequency, and above estimates should be modified. This is precisely the case of binary neutron stars, since they generically should reside in highly eccentric orbits due to two supernova explosions and large mass loss in the progenitor binary system. The possible asymmetry of supernova explosion with a natal kick velocity of the newborn NS additionally affects the orbital parameters by increasing orbital eccentricities. Namely, it is not clear from the very beginning up to what frequency one can expect such binaries to form a stochastic background in one year observation (i.e. individual harmonics cannot be resolved within the frequency resolution bin $\Delta \nu \sim 3 \times 10^{-8}$ Hz). The answer to this question is needed to ensure that indeed there are frequency ”windows” inside the LISA frequency band in which a cosmological GW background can be detected.

\section{Gravitational waves in eccentric orbit}

We start with the simple case of two point masses in a circular orbit. The energy losses caused by the quadrupole gravitational wave emission is

$$L_0 = \frac{dE}{dt} (\nu_K) = \frac{32 G^4 m_1^2 m_2^2 (m_1 + m_2)}{5 c^5 a^5}$$

where $G$ is Newton’s gravitational constant, $c$ is the speed of light and $m_1, m_2$ are the masses of stars. All GW radiation in this case is emitted at the second harmonic to the Keplerian frequency.

For a non-circular orbit with eccentricity $e$, the GW luminosity increases:

$$\frac{dE}{dt} = L_0 \times \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^{7/2}}$$

The emission now is widely spread over higher-order harmonics to the Keplerian frequency (Peters & Mathews 1963):

$$\frac{dE}{dt} (\nu = n \nu_k) = L_0 \times g(n, e)$$

$$g(n, e) = \frac{n^4}{32} (|J_{n-2}(ne)| - 2e J_{n-1}(ne) + \frac{2}{n} J_n(ne) + 2e J_{n+1}(ne) - J_{n+2}(ne))^2 + (1 - e^2)|J_{n-2}(ne)| - 2J_n(ne) + J_{n+2}(ne)|^2 + \frac{4}{3n^2} J_n^2(ne))$$
Here \( J_n \) is Bessel functions and \( n \) is the number of harmonics. The harmonic at which the maximum of energy is radiated increases with eccentricity, in accordance with the third Kepler’s law:

\[
   n_{\text{max}} \approx (1 - e^2)^{-3/2}
\]

This number is asymptotically precise for large eccentricities.

3 Merging neutron star binaries

As we noted above, an important quantity is the upper frequency \( \nu_{\text{lim}} \) above which each individual source from a given population of binaries can be resolved during one-year observation time (i.e. within the frequency bin \( \Delta \nu = 3 \times 10^{-8} \text{ Hz} \)). In the case of binary white dwarfs, this limiting frequency is of order of \( \sim 1.2 \times 10^{-3} \text{ Hz} \) for galactic binary WD coalescing rate \( R = 1/300 \text{ yr}^{-1} \). The coalescing rate of binary NS in most optimistic scenarios is about \( R = 10^{-4} \text{ yr}^{-1} \) so the limiting frequency would be \( \nu_{\text{lim}} = 3 \times 10^{-4} \text{ Hz} \) if they were in circular orbits from the very beginning. However, it is not the case.

There is observational evidence for several galactic NS binaries (observed as binary radio-pulsars), all of them reciting in eccentric orbits (see the Table). The total number of NS binaries in the Galaxy can be roughly estimated as \( R_{\text{NS}} T_{\text{gal}} = 10^{-4} \times 10^{10} \sim 10^6 \), where \( T_{\text{gal}} \) is the galactic age. In compact binaries, the gravitational radiation back reaction is the only driving force for orbital evolution. Given the initial distribution of these system over orbital parameters, \( F_{\text{in}}(a, e) \), it is straightforward to calculate their steady-state distribution (see Buitrago et al. 1994 for more detail).

| Binary PSR with NS secondaries |
|-------------------------------|
| PSR  | Period(days) | eccentricity,e | \( t_{\text{coalescing}} \), yr |
| B1913+16 | 0.323 | 0.617 | \( 1.0 \times 10^{8} \) |
| B1534+12 | 0.420 | 0.274 | \( 1.0 \times 10^{9} \) |
| B2721+11c | 0.335 | 0.681 | \( 8.0 \times 10^{7} \) |
| B2303+46 | 12.340 | 0.658 | \( 1.6 \times 10^{12} \) |

4 Delta-function examples

Some features of interest here can be learned already from an (un realistic) delta-function-like initial distribution of systems: \( F_{\text{in}}(a, e) = \delta(a - a_0, e - e_0) \) (hereafter we assume the NS+NS coalescing rate to be \( R_{\text{NS}} = 10^{-4} \text{ yr}^{-1} \)). The stationary stochastic GW background calculated is shown in Fig. 1 and Fig. 2 for \( a_0 = 5R_\odot \) and \( e_0 = 0.5, \ 0.9 \), respectively. The contribution of first three harmonics is shown by separate curves. It is seen that increase in the initial eccentricity strongly affects the shape of the background up to some frequency at which the eccentricities of binaries in the stationary distribution become sufficiently small. Above this frequency only the second harmonics from almost circular binaries contributes to the total signal. The non-monotonic dependence of the background is due to the non-monotonic behavior of the energy emitted at every harmonics with eccentricity.
Figure 1: The GW background from a model stationary population of binary NS with initial delta-function like distribution at $a_0 = 5R_\odot$, $e_0 = 0.9$ and the contribution from the first, second and third harmonics. For comparison, the would-be background from initially circular NS binaries coalescing with the same rate is shown.

Figure 2: The same as in Fig 1 for systems with initial $a_0 = 5R_\odot$, $e_0 = 0.5$. 
5 Stationary stochastic GW noise from coalescing binary NS

Now we turn to calculation of a realistic GW background produced by binary NS. This can be done with the following steps.

(1) Calculate the initial galactic distribution of binary NS stars over orbital semi-major axes and eccentricities $F_{in}(a, e)$ using e.g. binary population synthesis method (Lipunov et al. 1996).

(2) Calculate a stationary distribution function $F_{st}(a, e)$ (Buitrago et al. 1994, Pierro V. and Pinto I.M 1996).

(3) Compute the total stochastic GW background from these sources.

We shall assume for simplicity that all the sources are at one and the same distance $r = 7.9$ kpc away, which is close to the average distance to stars in our Galaxy. This simplifying assumption does not change our general conclusions significantly. Basically, at each frequency $\nu$ we sum up the GW flux from all the harmonics that fall within the chosen frequency bin $\Delta \nu = 3 \times 10^{-8}$ Hz from lower-frequency non-circular systems in the calculated stationary distribution $F_{st}(\nu, e)$.

$$\frac{dE}{dt} = \sum_{n=1}^{\infty} \int_{0}^{\nu} L_0(\frac{\nu}{n}) g(n, e) F_{st}(\frac{\nu}{n}, e) \, de \frac{d\nu}{n}$$

$$\approx \left( \sum_{n=1}^{n_{lim}} \int_{0}^{\nu} L_0(\frac{\nu}{n}) g(n, e) F_{st}(\frac{\nu}{n}, e) \, de \right) \frac{\Delta \nu}{n}$$

(6)

In the last equation we take into account $\Delta \nu \ll \nu$. The amplitude of high-order harmonics rapidly decreases, so we stopped the summation of harmonics for $n > n_{lim}$, $n_{lim} : g(n_{lim}, e) = \epsilon \times \max_n [g(n, e)]$. We assumed $\epsilon = 10^{-4}, 10^{-6}, 10^{-10}$. Decreasing $\epsilon$ increases the total number of harmonics which contribute to the given frequency bin, but practically does not change the number of the most powerful harmonics within it (see Fig. 4).

The resulting noise curve is shown in Fig. 3 in terms of dimensionless amplitude $h$

$$h^2(\nu) = \frac{G}{c^3 r^2 (2\pi \nu)^2} \frac{dE}{dt}$$

(7)

As expected, the NS+NS confusion noise lies below WD+WD curve, mainly due to lower $R_{NS}$. High-order harmonics from non-circular NS binaries mostly contribute at lower frequencies, so starting from $\nu \sim 10^{-4}$ Hz the calculated noise curve practically coincides with that formed by circular NS binaries coalescing with the same rate $R_{NS} = 10^{-4} \text{yr}^{-1}$.

Extragalactic NS binaries would also form an isotropic confusion noise, but the level of extragalactic GW backgrounds is generally an order of magnitude lower than the galactic one (e.g. calculations of Kosenko and Postnov 1998) and is beyond reach by the expected LISA sensitivity (the bottom dashed curve in Fig. 3). The limiting frequency $\nu_{lim}$ for extragalactic NS binaries finds from the condition for circular systems and can be as higher as $\sim 0.3$ Hz (Ungarelli and Vecchio 2000).
6 The limiting frequency

More important in the non-circular case is the increase of the limiting frequency \( \nu_{\text{lim}} \) above which individual sources can be resolved during a one-year observation. Formally, the number of harmonics inside the frequency bin \( \Delta \nu \) is more than one at any frequency, and in this sense the GW background extends up to very high frequency \( \sim 1 \text{KHz} \). But at high frequencies, the main contribution must come from circular binaries (2-nd harmonics), with the total GW power from higher-order harmonics from low-frequency eccentric systems becoming gradually less and less. So the number of harmonics contributing in a given frequency bin \( \Delta \nu = 3 \times 10^{-8} \text{ Hz} \) were counted starting with the strongest one and continuing until 99bin (Eq. (6)) had been included. The number of such harmonics as a function of frequency and the assumed neutron star kick velocity amplitude (which mostly affect the population synthesis results) \( N(\nu, \Delta \nu) \) is shown in Fig. 4. The limiting frequency \( \nu_{\text{lim}} \) is determined from equation \( N(\nu_{\text{lim}}, \Delta \nu) = 1 \). To within uncertainties of our calculations (\( R_{NS}, F_{in}(a, e) \), etc.), \( \nu_{\text{lim}} \approx 10^{-3} \text{ Hz} \), close to the break in the confusion limit from binary WD. For comparison, in the same Fig. we show \( N(\nu, \Delta \nu) \) for stationary circular NS binaries coalescing with the
The same rate $R_{NS} = 10^{-4} \text{ yr}^{-1} (\nu_{lim}(e = 0) \propto 3 \times 10^{-4} \text{ Hz})$. The effect of increasing the chosen level for the GW power from the strongest harmonics in the bin is illustrated in Fig. 5. Increasing this level from 95% to 100% changes the limiting frequency $\nu_{lim}$ by almost an order of magnitude.

This procedure, however, is not complete. Indeed, consider a frequency bin next to thus defined $\nu_{lim}$. We may ask the question which harmonics will be most probably absent inside it. The answer is those which are the least probable at this frequency. The probability to find a harmonics in the bin at a given frequency is determined by the number of the harmonic and the stationary distribution function of sources $F_{st}(\nu, e)$. It happens that this is harmonics number one (mostly due to a steep decrease of the systems’ stationary distribution function). If we remove all 1st harmonics of systems with orbital frequencies falling into the chosen bin from the sum Eq.(6), we are left again with some (smaller) GW power in the bin and may wish to find the limiting frequency exactly in the same way as described above (i.e. by fixing some level and summing up the strongest harmonics up to this level), call it $\nu_{lim,-1}$. Above this new limiting frequency, we can repeat the entire procedure to find $\nu_{lim,-2}$ (this time the 2d harmonics is least probable to be found in the bin next to the new limiting frequency), etc., until the noise level of the detector is reached. The step-like line which continues the spectrum above $\sim 1 \text{ mHz}$ in Fig. 3 illustrates this procedure. This line of course do not represent the ”real” GW noise curve from binary NS and just gives a feeling of how it most probably behaves at $\nu_{lim}$, above which individual harmonics from coalescing binary NS can be singled out.

7 Discussion and conclusions

A stochastic GW background is frequently described by the quantity $\Omega_{GW}(\nu)$, defined as the ratio of the energy density GW in the bandwidth equal to frequency to the total energy density to close the Universe (see critical discussion of this quantity in Grishchuk et al. 2001). In terms of this quantity, any stochastic cosmological GW background with $\Omega_{GW} \simeq 10^{-11} \div (a \text{ few}) \times 10^{-12}$ can be detected by LISA interferometer within the frequency range from $\sim 10^{-3}$ to $\sim 10^{-2} \text{ Hz}$ (see Fig. 6).

The primordial nucleosynthesis arguments give an upper limit for possible cosmological GW density $\Omega_{GW} \leq 10^{-6}$. The so-called ”Standard Inflationary Model” predicts the energy density of relic gravitational waves at an even smaller level $\Omega_{GW} \approx (a \text{ few}) \times 10^{-14}$. So weak a background cannot be certainly detected by LISA. However, more realistic appears the model (Grishchuk et al. 2001) in which the cosmological stochastic gravitational wave background level can be well above the LISA sensitivity limit. Detecting such a background by the LISA interferometer remains one of the most exciting goals of this challenging project.

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Figure 4: Number of harmonics from non-circular galactic NS binaries producing the stochastic noise in the frequency bin $\Delta \nu = 3 \times 10^{-8}$ Hz as a function of the neutron star kick velocity. The curves are labeled with kick velocity amplitudes 100, 200 and 300 km/s assumed in the population synthesis calculations. The straight line is for the circular NS binaries.

Figure 5: Number of the most powerful harmonics from non-circular galactic NS binaries which contribute 95%, 99% and 100% of the total GW energy collected in the bin $\Delta \nu = 3 \times 10^{-8}$ Hz. The effect of increase in the number of harmonics $n_{lim}$ determining the total energy collected in the bin is shown for different $\epsilon$. Kick velocity amplitude 200 km/s.
Figure 6: Levels of equal $\Omega_{GW}$ in comparison with the expected stationary galactic NS+NS stochastic background in the LISA frequency band.

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