CMB LENSOING POWER SPECTRUM BIASES FROM GALAXIES AND CLUSTERS USING HIGH-ANGULAR RESOLUTION TEMPERATURE MAPS

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ABSTRACT

The lensing power spectrum from cosmic microwave background (CMB) temperature maps will be measured with unprecedented precision with upcoming experiments, including upgrades to the Atacama Cosmology Telescope and the South Pole Telescope. Achieving significant improvements in cosmological parameter constraints, such as percent level errors on $\sigma_8$ and an uncertainty on the total neutrino mass of $\sim 50$ meV, requires percent level measurements of the CMB lensing power. This necessitates tight control of systematic biases. We study several types of biases to the temperature-based lensing reconstruction signal from foreground sources such as radio and infrared galaxies and the thermal Sunyaev–Zel’dovich effect from galaxy clusters. These foregrounds bias the CMB lensing signal due to their non-Gaussian nature. Using simulations as well as some analytical models we find that these sources can substantially impact the measured signal if left untreated. However, these biases can be brought to the percent level if one masks galaxies with fluxes at 150 GHz above 1 mJy and galaxy clusters with masses above $M_{200} = 10^{14} M_{\odot}$. To achieve such percent level bias, we find that only modes up to a maximum multipole of $l_{\text{max}} \sim 2500$ should be included in the lensing reconstruction. We also discuss ways to minimize additional bias induced by such aggressive foreground masking by, for example, exploring a two-step masking and inpainting algorithm.

Key words: cosmic background radiation – gravitational lensing: weak – infrared: galaxies – large-scale structure of universe

Online-only material: color figures

1. INTRODUCTION

Gravitational lensing of the cosmic microwave background (CMB) by intervening large-scale structure has long been recognized as a powerful probe of cosmology (see Lewis & Challinor 2006 for a theoretical review). Only recently, however, have millimeter-wave experiments reached the sensitivity to detect this promising signal.

There are two main methods for detecting CMB lensing. One is to measure the smoothing of the acoustic peaks on small angular scales induced by lensing in the CMB power spectrum (Blanchard & Schneider 1987; Seljak 1996; Challinor & Lewis 2005). This has been detected using data from the Arcminute Cosmology Bolometer Array Receiver (Reichardt et al. 2009), the Atacama Cosmology Telescope (ACT; Dunkley et al. 2011; Das et al. 2013), the South Pole Telescope (SPT; Keisler et al. 2011; Story et al. 2013), and the Planck satellite (Planck Collaboration 2013a).

The second method is to measure the distinctive mode-coupling that lensing generates in the CMB. Optimal filters exploiting this mode coupling can be applied to millimeter-wave maps to generate reconstructed maps of the matter distribution responsible for the lensing (Bernardeau 1997; Seljak & Zaldarriaga 1999; Zaldarriaga & Seljak 1999; Hu 2001). The power spectra of these reconstructed maps measure the matter power spectrum integrated along the line of sight. This method has the advantages of ultimately allowing one to measure the lensing power spectrum directly as a function of scale, and additionally of providing a higher significance detection. Reconstructions performed by the ACT (Das et al. 2011, 2013) and SPT (van Engelen et al. 2012) Collaborations yielded lensing detection significances of $4\sigma$ and $6.3\sigma$ respectively. More recently the Planck Collaboration (2013b) has achieved a detection significance of $25\sigma$.

To date, all reconstructed lensing maps obtained from data have employed the optimal quadratic estimation technique of Hu (2001) and Hu & Okamoto (2002). This technique is derived theoretically under idealized experimental conditions, in which the CMB is taken to be a nearly perfect Gaussian field with only a small mode-coupling induced by gravitational lensing, and with no significant mode-coupling from instrumental or foreground effects. Since then, the realities of data have demanded the investigation of a number of additional sources of mode coupling, including finite sky coverage (Perotto et al. 2010; van Engelen et al. 2012; Namikawa et al. 2013; Benoit-Lévy et al. 2013), nonstationary noise statistics (Hanson et al. 2009), primordial non-Gaussianity (Lesgourgues et al. 2005; Merkel & Schäfer 2013), and higher-order mode couplings induced by lensing itself (Kesden et al. 2003; Hanson et al. 2011).

In addition, astrophysical foregrounds in CMB maps intrinsically have significant mode coupling due to their non-Gaussian nature. Amblard et al. (2004) estimated levels of contamination based on simulations of large-scale structure, finding biases on the reconstructed lensing power spectrum ranging between tens of percents to factors of two due to the presence of thermal and kinetic Sunyaev–Zel’dovich (tSZ and kSZ) effects. In their detections of the CMB lensing power spectrum, Das et al. (2011), van Engelen et al. (2012), and Das et al. (2013) argued that the effects of astrophysical foregrounds could be neglected at the sensitivity levels presented in those works. This was based partly on
an improved understanding of the properties of the millimeter-wave sky as measured by ACT and SPT since the publication of Amblard et al. (2004). These improvements include more accurate measurements of the power spectrum amplitudes of point sources and the tSZ and kSZ effects (Hall et al. 2010; Dunkley et al. 2011; Shirokoff et al. 2011; Reichardt et al. 2012). The biasing signals from astrophysical sources were also minimized in the SPT analysis by filtering out the smallest CMB scales, which minimized the non-Gaussian foreground structure in the maps. In the recent strong detection of lensing by the Planck Collaboration, the lensing signal was measured using even larger CMB scales, due to the resolution of the instrument, such that biases were negligible apart from a small term associated with rare bright sources. This was a ∼2% positive bias that was subtracted off from power spectrum estimates (Planck Collaboration 2013b).

Measures of the lensing power spectrum are expected to continue to improve in signal-to-noise ratio. The lensing analysis of the full SPT temperature survey, which consists of about four times the survey area considered in van Engelen et al. (2012), is ongoing. In addition, ACT and SPT have both been upgraded with polarization-sensitive receivers (Niemack et al. 2010; Austermann et al. 2012), and the new instruments (ACTpol and SPTpol) are currently taking data. ACTpol, in particular, will survey much wider areas than achieved to date with ground-based CMB instruments, and is expected to yield lensing detections in temperature maps that will roughly double the signal-to-noise ratio. This will allow statistical uncertainty in the lensing power spectrum to reach the ∼2% level, improving current errors by a factor of two (Niemack et al. 2010). Achieving this will require biases to the lensing power spectrum from astrophysical foregrounds to be constrained at the percent level.

In this paper, we quantify the astrophysical biases with an eye toward these upcoming data sets. We consider the contributions from millimeter-wave emission by infrared galaxies and radio galaxies, as well as the contribution from galaxy clusters via the tSZ effect. We focus this work on biases to lensing reconstruction from temperature maps, deferring the study of foreground bias in polarization-based reconstructions to future work. In all that follows, we work in the flat-sky approximation, where harmonic transforms on the sphere are approximated using Fourier methods.

2. CMB LENSING RECONSTRUCTION

In the absence of lensing and other sources of CMB mode-coupling, the CMB is a globally Gaussian field. On a given scale \( l \), it is fully described by its power spectrum according to

\[
\langle T^U(l_1)T^U(l_2) \rangle = (2\pi)^2 \delta(l_1 + l_2) C^U_{l_1}.
\]

Here, \( T^U(l) \) is the Fourier transform of the unlensed CMB map, and \( C^U_{l} \) is the unlensed power spectrum. Lensing shifts the unlensed CMB temperature at position \( \hat{n} \), to a new position, \( \hat{n} + \alpha \), where the deflection angle \( \alpha \) is given by the gradient of a scalar field \( \phi(\hat{n}) \). Thus

\[
T(\hat{n}) = T^U(\hat{n} + \nabla \phi(\hat{n})).
\]

This scalar field is the weighted line-of-sight projection of the three-dimensional gravitational potential of matter between the observer and the CMB, and thus probes the evolution of cosmic structure through time.

Since the deflection angle represents a small perturbation about \( \hat{n} \), we can Taylor expand, which to first order gives

\[
T(l) = T^U(l) + (\nabla T^U \cdot \nabla \phi)(l),
\]

where pixel-by-pixel multiplication becomes convolution in Fourier space. Since \( \nabla T(l) = i T(l) \), then for \( L = l_1 + l_2 \) and \( L \neq 0 \), keeping only terms linear in \( \phi \) yields

\[
\langle (T(l_1)T(l_2))_{\text{CMB}} \rangle = \langle (l_1 + l_2) \cdot (l_1 C^U_{l_1} + l_2 C^U_{l_2}) \phi(l_1 + l_2) \rangle \equiv f(l_1, l_2) \phi(L).
\]

We choose to focus on the \( L \neq 0 \) terms because the \( L = 0 \) terms led to the power spectrum. The subscript “CMB” on the left-hand side indicates that we consider an ensemble average of CMB realizations, while holding the \( \phi \) realization fixed. Equation (4) describes the mode coupling lensing generates. If there was no lensing, i.e., \( \phi = 0 \), then this would be zero.

The optimal quadratic estimator for the lensing field \( \phi \) is formulated to take advantage of this mode coupling (Hu 2000, 2001). This is done by filtering the maps with a filtering function, \( F(l_1, l_2) \), that downweights the noisy modes, and selects for the mode-coupling of Equation (4):

\[
F(l_1, l_2) = \frac{f(l_1, l_2)}{2C^U_{l_1}C^U_{l_2}}.
\]

Here, \( C_f \) is the assumed total power in the map, including CMB, noise, and foreground power. For the assumed CMB power, we use a fiducial cosmological model with scalar fluctuation amplitude \( A_s = 2.45 \times 10^{-9} \), scalar spectral index \( n_s = 0.96 \), physical baryon density \( \Omega_b h^2 = 0.224 \), physical cold dark matter density \( \Omega_c h^2 = 0.109 \), Hubble parameter \( h = 0.709 \), and optical depth to recombination \( \tau = 0.0878 \). These are obtained from a best-fit cosmology using CMB data from WMAP7 together with high-z experiments. 6

Thus, we can estimate the lensing deflection field, \( d \), which is related to the lensing potential via \( d(L) = L \phi(L) \), using

\[
\hat{d}(L) \equiv L \hat{\phi}(L) = \frac{A_L}{L} \int \frac{d^2 l_1}{(2\pi)^2} F(l_1, L - l_1) T(l_1) T(L - l_1).
\]

In this reconstruction of the deflection field, the normalization \( A_L \) is chosen to ensure that the resulting lensing map is unbiased, with \( \langle \hat{d}(L) \rangle_{\text{CMB}} = L \hat{\phi}(L) \).

An estimation of the lensing power spectrum can be obtained from \( \hat{d}(L) \) by computing

\[
\langle \hat{d}(l_1)\hat{d}(l_2) \rangle = \frac{A_{L_1} A_{L_2}}{L_1 L_2} \int \frac{d^2 l_1}{(2\pi)^2} \int \frac{d^2 l_2}{(2\pi)^2} \times F(l_1, L_1 - l_1) F(l_2, L_2 - l_2) \times L (l_1) T(l_1) T(l_2) \langle (l_2) \rangle_{\text{CMB}}.
\]

The estimated lensing power spectrum is thus sensitive to the four-point product of CMB modes in the Fourier domain. 7 This four-point product can be decomposed into several terms:

\[
\langle T(l_1)T(l_2)T(l_3)T(l_4) \rangle = ((T(l_1)T(l_2))(T(l_3)T(l_4)) + 2 \text{ perm.}) + \langle T(l_1)T(l_2)T(l_3)T(l_4) \rangle_{\text{CMB}}.
\]

\[
\text{http://lambda.gsfc.nasa.gov/product/map/dr4/parameters.cfm.}
\]

\[
\text{7 The angled brackets ( ) without the “CMB” subscript indicates that this ensemble average is taken over realizations of both the T(l) and } \phi(l) \text{ fields; see Kesden et al. (2003) for a more detailed discussion. We are neglecting the correlation between the two fields, induced by the integrated Sachs–Wolfe effect.}
\]
The first two lines describe unconnected terms, which exist even for a globally Gaussian field with no lensing. We have used the Wick theorem to write this unconnected piece as the sum of three sets of two-point products. The (+2 perm.) refers to the terms obtained by replacing the pairings $(1, 2)(3, 4)$ with the pairings $(1, 3)(2, 4)$ and $(1, 4)(2, 3)$. These unconnected terms are given in terms of the power spectrum via the analogue of Equation (1) using the total power $C_{l}^{\gamma\gamma}$.

The first unconnected term is proportional to a delta function at $L = 0$, giving no contribution at $L \neq 0$. However, the two terms represented by the (+2 perm.) lead to an effective noise bias of

$$N_{l}^{(0)} = \frac{A_{l}^{2}}{L^{2}} \int \frac{d^{3}l_{1}}{(2\pi)^{3}} \frac{d^{3}l_{2}}{(2\pi)^{3}} f(l_{1}, L - l_{1}) F(l_{1}, L - l_{1}) \times \left[ \frac{\langle C_{l_{1}}^{\gamma\gamma} \rangle C_{l_{1}}^{\gamma\gamma} - C_{l_{1}}^{\gamma\gamma} C_{l_{1}}^{\gamma\gamma}}{C_{l_{1}}^{\gamma\gamma} C_{l_{1}}^{\gamma\gamma}} \right].$$  \hspace{1cm} (9)

Here, $\langle C_{l_{1}}^{\gamma\gamma} \rangle$ represents the total power spectrum in the field, as opposed to the power assumed in the denominator of the optimal lensing filter, $C_{l_{1}}^{\gamma\gamma}$ in Equation (5).

The $N_{l}^{(0)}$ bias thus originates from the nonzero CMB and noise power in the map. The superscript (0) refers to the terms being zeroth order in lensing, since it is present if $\phi = 0$. If the map contains the same power as that assumed in the filter, so that $\langle C_{l_{1}}^{\gamma\gamma} \rangle = C_{l_{1}}^{\gamma\gamma}$, then $N_{l}^{(0)} = A_{l}$ (e.g., Hu & Okamoto 2002).

In general, there will be some scatter in $N_{l}^{(0)}$ from realization to realization due to the sample variance in the observed CMB and noise fluctuations. This bias can be characterized by directly evaluating Equation (9), taking $C_{l_{1}}^{\gamma\gamma}$ to be the estimated power spectrum from the given map realization (Dvorkin & Smith 2009; Namikawa et al. 2013).\footnote{Alternatively, the noise bias $N_{l}^{(0)}$ can be avoided altogether by splitting the Fourier domain into disjoint regions such that $I_{1}$ and $I_{2}$ do not share modes in common with $I_{1}$ and $I_{2}$ (Sherwin & Das 2010; van Engelen et al. 2012). We do not consider this further because the characterization of the noise bias directly from the data has been shown to be reliable (Namikawa et al. 2013; Planck Collaboration 2013b).}

The connected terms in Equation (8) give the trispectrum,

$$T(T_{l_{1}} T_{l_{2}} T_{l_{3}} T_{l_{4}})_{\text{conn}} = (2\pi)^{3} \delta^{(3)}(l_{1} + l_{2} + l_{3} + l_{4}) T(l_{1}, l_{2}, l_{3}, l_{4}).$$  \hspace{1cm} (10)

At first order in the lensing potential power spectrum, $C_{l_{1}}^{\phi\phi}$, the trispectrum from the CMB is (Kesden et al. 2003):

$$T_{l_{1}, l_{2}, l_{3}, l_{4}} = C_{l_{1}}^{\phi\phi} f(l_{1}, I_{1}) f(l_{1}, I_{4}) + 2 \text{ perm.}$$  \hspace{1cm} (11)

The first of these terms leads to the direct measure of the CMB lensing power spectrum. The (+2 perm.) terms lead to the $N_{l}^{(1)}$ bias, which are of the same order in the lensing potential power spectrum. This bias, and those at higher orders in $C_{l_{1}}^{\phi\phi}$ have been studied elsewhere (Kesden et al. 2003; Hanson et al. 2011).

Other sources of nonzero trispectrum will also lead to signals which can bias the resulting lensing power spectrum estimates. In particular, a field with a given trispectrum $T'(l_{1}, l_{2}, l_{3}, l_{4})$ will lead to a response in the lensing power spectrum given by

$$\langle C_{l_{1}}^{\gamma\gamma} \rangle' = \frac{A_{l}^{2}}{L^{2}} \int \frac{d^{3}l_{1}}{(2\pi)^{3}} \frac{d^{3}l_{2}}{(2\pi)^{3}} \frac{d^{3}l_{3}}{(2\pi)^{3}} \frac{d^{3}l_{4}}{(2\pi)^{3}} F(l_{1}, L - l_{1}) F(l_{2}, L - l_{2}) \times T'(l_{1}, l_{2}, l_{3}, l_{4}).$$  \hspace{1cm} (12)

In this work, we quantify these biases from known astrophysical sources using simulations and some analytic models. For the majority of the work we assume map noise levels of $18 \mu K$ arcmin and foreground power of $9.1 \mu K$ arcmin, which we assume to be independent of $l$. These values are roughly consistent with those of the completed SPT-SZ survey, and with those expected for a wide survey with ACTpol. We also impose a maximum temperature multipole used in the lens reconstruction of $l_{\text{max}} = 3000$. In Section 6, we relax these assumptions and study the dependence of foreground biases on the map noise level and filtering choice.

3. SIMULATIONS OF THE MICROWAVE SKY

To quantify biases on the lensing power spectrum due to foregrounds we use two independent sets of simulations. The first is described in S. Bhattacharya et al. (in preparation; B14 hereafter) and summarized below. The second is from Sehgal et al. (2010, S10 hereafter), with some modifications which we also describe below.

The B14 simulations use the Coyote N-body simulation performed using the publicly available Gadget-2 code (Heitmann et al. 2010) to describe the dark matter distribution. The simulation has a box size of $1300$ Mpc and $1024^{3}$ particles. A standard LCDM cosmology is adopted with $\Omega_{m} h^{2} = 0.1296$, $\Omega_{b} h^{2} = 0.0224$, $n_{s} = 0.97$, $\sigma_{8} = 0.8$, $h = 0.72$, and $\Omega_{\Lambda} = 0$, consistent with the best-fit cosmological model from WMAP7 (Komatsu et al. 2011). Ten simulation outputs between $z = 0$ and 4 were generated, equally spaced in the scale factor, to create a lightcone filling one octant of sky. Ray tracing through the lightcone was done to create the lensing convergence field. The lack of matter fluctuation snapshots above $z = 4$ was treated by neglecting the effects of nonlinear growth at these high redshifts. A theoretical convergence power spectrum for mass fluctuations between $z = 4$ and $z = z_{\text{CMB}}$ was calculated using linear theory, and a Gaussian field with this power spectrum was then generated and added to the convergence from the lower redshifts generated from the simulations. This led to agreement between the simulated convergence and the theoretical prediction for all redshifts, including the effects of nonlinear growth (we use the publicly available CAMB software; Lewis et al. 2000). The SZ effect was added to halos identified in the N-body simulation by using a semi-analytic model for gas physics (Shaw et al. 2010). Infrared galaxies were also added to the halos using a semi-analytic approach.

The flux distribution of the infrared galaxies, which constitute the cosmic infrared background (CIB), in the B14 simulations was allowed to have mass and redshift dependence in addition to frequency dependence. To model this galaxy distribution, the dark matter halos were first modeled as Navarro–Frenk–White profiles (Navarro et al. 1997). Flux from infrared galaxies was then added to these profiles following

$$\frac{dI}{dM dz}(M, z) = \rho_{\text{DM}}(M, z) A \left( \frac{M}{M_{\text{pivot}}} \right)^{a} (1 + z)^{b},$$  \hspace{1cm} (13)

where $M$ and $z$ are the halo mass and redshift. The halos were populated with flux using $A$, $a$, and $b$ values with dark matter halo mass $M \geq 2.5 \times 10^{13} h^{-1} M_{\odot}$. We choose $M_{\text{pivot}} = 2 \times 10^{13} h^{-1} M_{\odot}$. The smallest halo in this simulation is thus resolved by 50 particles. We consider the minimum number of parameters needed to fit the CIB clustered power spectrum (Reichardt et al. 2013; Sievers et al. 2013) and the CIB bispectrum (Crawford et al. 2014). The advantage with this
approach is that there are few parameters with which to explore the parameter space. The disadvantage is that one cannot predict how different galaxy populations contribute to the infrared background power. There is also significant choice in which halos receive infrared flux. Two such choices are made, yielding models constructed to give the same amount of clustered power such that the models match the observed clustered power. We then compute the CIB bispectra for these two models (shown in Figure 2), but do not attempt to modify the parameters to fit the bispectrum. For the CIB analysis described below, we additionally scale the CIB maps from the two models by factors of 1.21 and 1.12, to exactly match the amplitude of the clustered CIB at 150 GHz found by Reichardt et al. (2012) at \( l = 3000 \).

The second set of simulations we use is the S10 simulations, which are described in Sehgal et al. (2010) and are publicly available. We make two modifications to these simulations that differ from the original S10 version. The first is that the SZ gas model that is implemented is instead the gas model described in Bode et al. (2012). The second is that the fluxes of all the infrared galaxies in this simulation have been scaled down by 25%. This reduction in flux makes the infrared galaxy model of S10 match the amplitude of the total infrared background power spectrum measured by ACT and SPT at 150 GHz at \( l = 3000 \) (Reichardt et al. 2013; Sievers et al. 2013). This infrared model also matches the source counts measured by SCUBA at 350 GHz (Coppin et al. 2005), the total infrared background intensity measured by FIRAS (Fixsen et al. 1998), and the bispectrum measured by both SPT and Planck (Crawford et al. 2014; Planck Collaboration 2013e). The S10 simulations including these two new modifications are publicly available at http://www.slac.stanford.edu/~sehgal/simsv2.0/.

The power spectra for the two CIB simulations we consider, including the flux rescalings, are shown in the upper-left panel of Figure 1. Here, sources above 5 mJy have been removed to approximately match the 6.4 mJy cut of the SPT analysis.\(^9\) We also estimate the bispectrum for our CIB models, using the tools described in detail in Crawford et al. (2014), including masks to 5 mJy. These are shown in Figure 2, together with the measurements from SPT on scales \( l > 1500 \) (Crawford et al. 2014), and from Planck on scales \( l < 800 \) (Planck Collaboration 2013e). For both experiments, we scale the more significant measurement at 220 GHz to 150 GHz with a flux factor given by the ratio of the Poisson CIB bispectrum measurement at 220 to that at 150 GHz, which is 6.03 ± 2.23 (Crawford et al. 2014). We divide the 220 GHz clustered CIB measurements by this factor to obtain the measurement at 150 GHz. The final error bar includes uncertainty in the Poisson power at 150 and 220 GHz, and the uncertainty in clustered power at 220 GHz. The S10 model and the first B14 model are consistent with the SPT-obtained bispectrum, within approximately two sigma. The S10 model is also consistent with the bispectrum recently measured by Planck at low multipoles \( l \lesssim 800 \). The second B14 model has a larger bispectrum than that seen by SPT by a factor of \(~10\), because the distribution of flux in this model is concentrated toward more massive halos. Lensing biases obtained from this model may thus be overestimated.

\(^9\) Note that the S10 infrared galaxy model also predicts an abundance of infrared galaxies in massive clusters that is only 30 times larger than the field, contrary to the claims in Luckey et al. (2010) and Hall et al. (2010). This abundance of infrared galaxies in massive clusters is completely consistent with the measurements of Bae et al. (2007).\(^10\) ACT removed sources above 15 mJy, which results in a negligible difference in power compared to the 6.4 mJy cut.
4. BIAS ON LENSING RECONSTRUCTION FROM GALAXIES

Emission from active galactic nuclei and dusty star-forming galaxies represent the dominant sources of fluctuation in millimeter-wave maps on small angular scales, and can significantly bias lensing estimates. Below we study two different types of bias from galaxies, one due to the intrinsic galaxy four-point function and one due to the correlation between galaxies and the lensing field.

4.1. Bias from the Galaxy Four-point Function

4.1.1. Poisson-distributed Galaxies

A field of uncorrelated, Poisson-distributed point sources will possess a trispectrum. Galaxies, in fact, do not follow a Poisson distribution since they are inherently correlated, residing in clustered dark matter halos. However, we present the scenario as an instructive toy model. In the limit of a few bright sources, the distribution of galaxies can approach Poisson.

If uncorrelated sources per area element on the sky are sufficiently numerous, the central limit theorem will apply and the field will approach a Gaussian field with white noise. In lensing power spectrum reconstruction, it will then form a portion of the $N^0$ bias and will be removed. If this limit does not apply, the uncorrelated sources will generate a trispectrum that will appear as a source of bias in the inferred lensing power spectrum. With high-resolution maps from experiments such as ACT and SPT, bright, rare sources have been cleaned to spectrum. With high-resolution maps from experiments such as ACT and SPT, bright, rare sources have been cleaned to spectrum. With high-resolution maps from experiments such as ACT and SPT, bright, rare sources have been cleaned to spectrum. With high-resolution maps from experiments such as ACT and SPT, bright, rare sources have been cleaned to spectrum.

A galaxy at position $\hat{n}$ with flux density $S$ will contribute a temperature fluctuation of $T(\hat{n}) = G_vS\delta_D(\hat{n})$ to the CMB map, where $G_v$ is a factor to convert between flux density and temperature units. This factor is given by $G_v = dB_v(T)/dT$, evaluated at the CMB temperature $T = 2.73$ K, where $B_v$ is the Planck function at frequency $v$.

In the Fourier domain, the contribution from such a galaxy corresponds to a temperature field of

$$T(\mathbf{l}) = G_vS\epsilon^{-i\mathbf{l}\cdot\hat{n}}.$$  \hspace{1cm} (14)

The power spectrum from this is constant in $\mathbf{l}$.

For two sources, with flux densities $S_1$ and $S_2$, at uncorrelated locations, the power spectrum is given by

$$\langle T_1^*(\mathbf{l})T_2(\mathbf{l}) \rangle = (G_vS_1)^2 + (G_vS_2)^2;$$  \hspace{1cm} (15)

the cross terms cancel due to the uncorrelated nature of the sources. This result generalizes to $N$ sources, leading to a power spectrum for Poisson sources, $C_l^P$, of

$$C_l^P = G_v^2\int_{S_{\text{min}}}^{S_{\text{max}}} dS S^2 \frac{dN}{dS}.$$  \hspace{1cm} (16)

The factor $(dN/dS)$ is the number density of sources per steradian, per unit flux interval. Following similar arguments, the trispectrum is also constant in $\mathbf{l}$ and is given by

$$T(l_1, l_2, l_3, l_4) = G_v^2\int_{S_{\text{min}}}^{S_{\text{max}}} dSS'\frac{dN}{dS}.$$  \hspace{1cm} (17)

The power spectrum and trispectrum of sources both lead to signatures in reconstructed CMB lensing power spectra.

The power spectrum and trispectrum of sources both lead to signatures in reconstructed CMB lensing power spectra.
lensing reconstruction ($l_{\text{max}} \simeq 1800$), reducing the relative impact of foregrounds. However, the higher flux cut increases the bias substantially. The Planck team thus found a bias of amplitude $\sim 2\%$ on the lensing power spectrum from unresolved bright sources whose distribution approaches Poisson. Since this corresponds to one-half of their 4% statistical error, this term needed to be treated explicitly in their analysis. This was done by estimating the amplitude for the trispectrum of these roughly uncorrelated sources directly from the maps, evaluating the associated bias on the lensing power spectrum using the curved-sky generalization of Equation (12), and subtracting the resulting template from the estimated lensing power spectra.

We note, however, that infrared galaxies do not need to show a Poisson-distributed component given the maximum multipoles measured by current experiments. If measurements are made assuming they contain two separate components, clustered and Poisson, then there is a possibility that the amplitude of the Poisson component will be a function of the maximum multipole of the measurement (as is the case for the S10 model). We discuss more realistic, clustered sources below.

4.1.2. Clustered Galaxies

The distribution of sources on the sky exhibits clustering, which affects the higher-order statistics. Clustering in the CIB at millimeter-wave frequencies was first detected in the CIB power spectrum by Hall et al. (2010), Dunkley et al. (2011) and the Planck Collaboration (2011), and has since been measured with increasing precision (Shirokoff et al. 2011; Reichardt et al. 2013; Sievers et al. 2013; Planck Collaboration 2013e). The clustering in the bispectrum at millimeter-wave frequencies has also been recently detected (Crawford et al. 2014; Planck Collaboration 2013e). Although some analytic prescriptions exist for higher-order moments of the galaxy population (e.g., Argüeso et al. 2003; Lacasa et al. 2012), we use the S10 and B14 simulations to estimate lensing biases from clustered CIB sources.

To study the impact of source masking, we mask pixels above different maximal flux values ranging between 0.5 and 20 mJy, replacing masked sources with the median of the map at each step. Although the B14 simulations do not contain individual sources, we nevertheless perform masking on the brightest pixels, using the same conversion of temperature to flux density per pixel as that applied for the S10 simulations. We downsample the S10 simulations from Healpix resolution $N_{\text{side}} = 8192$ to $N_{\text{side}} = 4096$; the B14 simulations are at resolution $N_{\text{side}} = 4096$ natively. We then extract 42 non-overlapping flat-sky maps of 100 deg$^2$ each using the oblique Lambert equal-area azimuthal projection (Snyder 1987) from these $N_{\text{side}} = 4096$ maps. To finely sample the Healpix maps on the flat-sky grid we use a resolution of 0.25, and then downgrade to 1’, which resolution is set by that needed for lensing reconstruction, by averaging contiguous pixels. We account for the effects of this downsampleing process by generating Healpix maps of simulated white noise (with power spectrum $C_l \equiv 1$), passing these maps through the same downsample and reprojection steps, and estimating the power spectra of the resulting maps. This leads to an estimate of the effective transfer function associated with the downsample, which we deconvolve from our flat-sky CIB maps.

The top-left panel of Figure 1 shows the power spectra, estimated on the flat sky, of both sets of CIB simulations when applying a 5 mJy flux cut. The model used to generate the B14 simulations assumes that infrared galaxies have both a Poisson and clustered component, and only describes the clustered piece. This leads to a lower overall power spectrum amplitude at $l = 3000$ compared to the S10 simulations. Also shown at $l = 3000$ are the measurements of the clustered and total CIB power found by Reichardt et al. (2013) and Sievers et al. (2013), the latter of which is a fit to the power spectra found by Das et al. (2013). In the case of the Reichardt et al. (2013) measurement, we also show an approximate systematic error bar associated with the scatter between the five models of the clustered CIB studied in that work.

We perform lensing reconstructions on the CIB fields using the estimator in Equation (6). The power spectra of the reconstructed fields are shown in the bottom-left panel of Figure 1. In each case, we subtract an estimation of the noise bias $N_{l}^{(0)}$ using Equation (9) and the power spectrum ($C_l^{(0)}$) estimated directly from the simulations. We call the remaining bias the “CIB four-point bias” due to its dependence on the four-point function of the CIB.

We show this bias, evaluated at $L = 500$, as a function of flux threshold in the left panel of Figure 4. Without any masking, the biases can be several percent, with the second model of B14 showing the largest bias, followed by the S10 model and the first model of B14. These trends follow those of the bispectrum shown in Figure 2. Masking to $\sim 2$ mJy can reduce them to below one percent for all three simulations tested. Since a 5σ flux cut at 150 GHz corresponds to $\sim 5$ mJy for an ACT/SPT experiment, it may be necessary to use information from higher frequencies to reliably extract sources to this level.

4.2. The Galaxy–Lensing Correlation

As was shown observationally by Holder et al. (2013), Planck Collaboration (2013c), and Hanson et al. (2013), the CIB is strongly correlated with the CMB lensing field, at the $\sim 80\%$ level (Song et al. 2003). This is because both fields are tracers of nearly the same dark matter fluctuations, owing to the very similar redshift responses. The two sets of simulations that we

$^{11}$ The uncertainty shown for the “obs. total” CIB points is obtained by combining in quadrature the uncertainties on the measured amplitudes of CIB sources fit to a model of infrared galaxies with separate clustered and Poisson components. We neglect the covariance between these measured amplitudes, slightly overestimating the uncertainty on the total.
use include maps of the lensing convergence, $\kappa$, which is related to the lensing potential according to $\kappa(\mathbf{n}) = (1/2)\nabla^2 \phi(\mathbf{n})$. The $\kappa$ field is proportional to the projected matter overdensity along the line of sight. In the top-right panel of Figure 1, we show the $\kappa$-CIB cross-power spectra in the simulations. Also shown is the cross-correlation coefficient of $\gtrsim 0.8$ at 143 GHz found by the Planck team (Planck Collaboration 2013c). We estimate the uncertainty on this cross-correlation amplitude by summing, in quadrature, the inverse of the stated fractional bandpower uncertainties taken from Table 2 of Planck Collaboration (2013c). The inverse of the square root of this sum gives a fractional uncertainty of 22%.\footnote{Note that this uncertainty is much smaller at the higher Planck frequencies, where emission from dust dominates over the CMB.} The cross power spectra in the simulations are lower than those found observationally, since dark matter halos are only identified up to a given redshift in the simulations ($z = 4$ for B14, $z = 3$ for S10). As a result, CIB sources do not populate halos at higher redshifts than these in the simulations.

We find that CIB fields that are correlated with the lensing maps lead to a bias on the lensing power spectrum (Smith et al. 2007; van Engelen et al. 2012; Bleem et al. 2012). While trispectra based on these correlations, assuming the CIB is a Gaussian field, have been derived (Cooray & Kesden 2003), analysis of non-Gaussian CIB simulations has indicated that a much larger bias can exist. This bias is negative on the scales of interest ($L < 2000$), and becomes positive at higher $L$. To isolate this effect, we compute the cross power of the lensing reconstructions of CIB fields with the input $\kappa$ maps. This bias is a multiplicative bias, describing the imperfect $\kappa$ reconstruction using maps with correlated non-Gaussian foregrounds. This is in contrast to the additive bias of intrinsic four-point functions from foregrounds (as discussed in the previous section). We multiply our result by a factor of two, since we are considering biases for lensing auto-power spectra. This factor of two would be absent in an analysis of cross-correlations of CMB lensing biases for lensing auto-power spectra. This factor of two would multiply our result by a factor of two, since we are considering from foregrounds (as discussed in the previous section). We consider the tSZ effect, which involves photons scattering off hot electrons in the deep potential wells of galaxy clusters. The kSZ effect, which originates from photons scattering off electrons possessing bulk motions along the line of sight, is expected to be a much smaller contaminant for CMB lensing studies (van Engelen et al. 2012). This is due to its lower fluctuation amplitude and its smoothness compared to the tSZ effect. In addition, masking of tSZ clusters (discussed below) will reduce the kSZ-induced lensing contamination from those clusters (Amblard et al. 2004).

The non-Gaussianity of the tSZ field is significant; the three-point function, or bispectrum, has recently been detected by the ACT (Wilson et al. 2012), SPT (Crawford et al. 2014), and Planck (Planck Collaboration 2013d) teams. For lensing power spectrum estimation the tSZ trispectrum is an important potential contaminant. The tSZ trispectrum has been considered previously as a source of non-Gaussian variance in CMB power spectrum estimation (Cooray 2001; Shaw et al. 2009). Due to its scaling with the fourth power of the temperature decrement, it is dominated by the most massive clusters in the Universe. In the current era of dedicated SZ surveys with low noise levels, namely SPT (Vanderlinde et al. 2010; Benson et al. 2013) and ACT (Sehgal et al. 2011; Hasselfield et al. 2013), many of these massive clusters can be detected on an individual basis. However, the trispectrum from clusters just below the detection threshold can in principle cause a concern for lensing studies.

The power spectrum of the tSZ effect has been extensively modeled (e.g., Komatsu & Seljak 2002; Sehgal et al. 2010; Shaw et al. 2010; Battaglia et al. 2010), and observations of the power spectrum of the millimeter-wave sky have been used to fit these models (most recently, Reichardt et al. 2012; Sievers et al. 2013; Planck Collaboration 2013d). The power spectrum is dominated by clusters with lower masses than those which can be detected individually in current surveys (Shaw et al. 2009; Trac et al. 2011). The bispectrum of the tSZ effect originates from clusters with masses between those dominating the power spectrum and trispectrum, and can provide a useful cosmic probe (Bhattacharya et al. 2012; Hill & Sherwin 2013).

Here, we compute the impact of the tSZ trispectrum on lensing power spectrum estimation using the same halo model approach as that taken in the predictions for the SZ power spectrum and bispectrum. On the scales of interest, the power spectrum and bispectrum have been found to be dominated by the term in which all multipole arguments reside within the same dark matter halo (Komatsu & Kitayama 1999; Bhattacharya et al. 2012); this is known as the “one-halo” term. We expect the one-halo term to dominate here as well, due to the sensitivity of the trispectrum to the brightest objects.

The tSZ trispectrum is then given by (Cooray 2001; Komatsu & Seljak 2002; Bhattacharya et al. 2012)

$$T(l_1, l_2, l_3, l_4) = f_c^4 \int dz \frac{dV}{dz} \int d\ln M \frac{dn(M, z)}{d\ln M} \times y(\ell_1, M, z)y(\ell_2, M, z)y(\ell_3, M, z)y(\ell_4, M, z).$$

Here, $V(z)$ is the comoving volume element per steradian, $f_c = x_c(\cosh(x_v) - 4)$ is the frequency-dependent SZ scaling factor, $x_v = h\nu/k_BT_{\text{CMB}}$, $n(M, z)$ is the halo mass function, which we take to be that of Tinker et al. (2008), and $y(\ell, M, z)$ is the Fourier transform of the projected SZ profile for a cluster mass $M$ and redshift $z$. This last term is given by (e.g., Komatsu...}
The amplitude of the matter power spectrum today, $\sigma_8$, has a strong impact on the moments of the tSZ field, with the power spectrum scaling in proportion to $\sigma_8^{-2}$, and the bispectrum scaling in proportion to $\sigma_8^{-12}$ (Bhattacharya et al. 2012; Hill & Sherwin 2013). In Figure 5 we show the dependence of the power spectrum and the four point-induced lensing bias on both of these parameters. As expected, the fractional changes in the four-point signature are roughly twice those of the power spectrum, leading to a large uncertainty.

Since variation in these parameters can lead to a significant change in the theoretical tSZ four-point induced lensing bias, we compute this feature on a grid of $10 \times 10$ points in the plane formed by $\sigma_8$ and $A_c$. We assume a range of parameter values of $0.70 < \sigma_8 < 0.85$ and $0.3 < A_c < 2.0$, and impose prior information from measurements of the tSZ power spectrum on the angular scales of interest. Specifically, we use the SPT measure from Reichardt et al. (2012) of $D_{l,3000}^{\text{SZ}} = 3.65 \pm 0.69 \mu K^2$, where $D_l \equiv l(l+1)C_l/2\pi$. This measurement is obtained from fitting the amplitudes of multiple templates to power spectra at three frequencies, and is somewhat sensitive to choices of these templates, particularly the properties of the CIB. To be conservative we have chosen the tSZ template fit amplitude from Reichardt et al. (2012) with the widest error bar. We form a simple chi-square-like function defined by

$$\chi^2(\sigma_8, A_c) = \frac{C_l^{\text{SZ, data}}}{\sigma(C_l^{\text{SZ, theory}})} - \frac{C_l^{\text{SZ, theory}}}{\sigma(C_l^{\text{SZ, data}})}$$

(20)

The theory power spectra, $C_l^{\text{SZ, theory}}$, are calculated without any clusters masked, i.e., over all clusters in the range of $2 \times 10^{13} M_\odot \leq M_{\text{vir}} \leq 5 \times 10^{15} M_\odot$, and redshifts in the range $0 < z < 3$. The resulting features in the lensing power spectrum space, shown in Figure 5, have very similar shapes as those for Poisson-distributed point sources, but the spatially extended nature of clusters reduces the amplitude of this template. In Figure 5, we also show the theoretical tSZ power spectrum, computed using the analogue of Equation (18) with two factors of $y(l, M, z)$ (e.g., Komatsu & Seljak 2002).

This analytic approach enables us to evaluate the SZ four-point bias as a function of both the cluster physics parameters and the cosmological parameters. Combinations of these parameters are constrained by measurements of the tSZ power spectrum (Reichardt et al. 2012; Dunkley et al. 2011; Planck Collaboration 2013d). Perturbing the cluster physics parameters described in Bhattacharya et al. (2012) and evaluating the lensing bias, we find that the parameter with the largest impact is the normalization of the concentration-mass relation, $A_c$. This parameter controls the effective radius of the clusters. The amplitude of the matter power spectrum today, $\sigma_8$, also has a strong impact on the moments of the tSZ field, with the power spectrum scaling in proportion to $\sigma_8^{-2}$, and the bispectrum scaling in proportion to $\sigma_8^{-12}$ (Bhattacharya et al. 2012; Hill & Sherwin 2013). In Figure 5 we show the dependence of

**Figure 5.** Impact on tSZ power spectrum (top) and tSZ four-point lensing bias (bottom) from varying the parameters describing the amplitude of fluctuations $\sigma_8$ (red, dashed) and the normalization of the concentration-mass relation $A_c$ (blue, dotted). The green dash-dotted line is the lensing power spectrum.

(A color version of this figure is available in the online journal.)
Figure 6. Bias from the four-point function of the thermal SZ effect, obtained theoretically for two mass thresholds. The error bands indicate the impact of marginalizing over $\sigma_8$ and $A_s$, while forcing the theoretical power spectrum to agree with measurements from SPT (Reichardt et al. 2012), within uncertainties. The black curve shows the lensing power spectrum multiplied by 0.05.

(A color version of this figure is available in the online journal.)

Figure 7. Biases on CMB lensing power spectra from tSZ clusters. Top left: power spectra of the tSZ simulations, including those of B14 (blue) and S10 (red). Fainter colors correspond to more aggressive cluster mass cuts. Bottom left: CMB lensing bias from the four-point function of these reconstructions as a function of lensing multipole, $L$. Error bars denote the error on the mean, based on the scatter from 42 patches of 100 deg$^2$ each. Top right: cross-correlation between the tSZ and the lensing field $\kappa$ in the simulations. Bottom right: absolute value of the bias induced from correlation between the square of the tSZ and the lensing field. The black curves show the lensing power spectrum multiplied by 0.05.

(A color version of this figure is available in the online journal.)

$\sigma_8 = 0.8$ and $A_s = 1.4$; this is well-motivated by estimates of the concentration-mass relations in the simulations.

The left panel of Figure 8 shows the fractional bias on the lensing power spectrum at $L = 500$ for the two tSZ simulations.

In order for the four-point bias to be reduced to sub-percent levels we find that it is necessary to mask to $M_{\text{vir}} \lesssim 5 \times 10^{14} M_\odot$.

5.2. The Galaxy Cluster–Lensing Correlation

As with the CIB, the tSZ sky is correlated with the CMB lensing field (Hill & Spergel 2014). Since both sets of large-scale structure-based simulations contain lensing fields which are obtained from the same dark matter that is used to populate the halos, both will contain a nonzero cross-correlation. As shown in the top-right panel of Figure 7, the S10 simulations (with the updated tSZ model described in Section 3) yield a tSZ–$\kappa$ cross-correlation coefficient of 40%, and the B14 model yields a cross-correlation coefficient of 20%. These values are for effectively no cluster masking; the reduction in the cross-correlation when masking is performed is also shown. The factor of two difference between the tSZ–$\kappa$ cross-correlation obtained from the S10 and B14 simulations is likely due to differences in the modeling of the tSZ effect from the intergalactic medium and at high redshifts. Performing lensing reconstructions on the tSZ fields and cross-correlating with the input lensing maps leads to a bias which is negative at $L < 2000$, and positive at higher $L$, as with the CIB. In the bottom-right panel of Figure 7, we show the bias from this correlation, for the two sets of simulations, and for three levels of cluster masking. The values of the bias at $L = 500$ are also shown as a function of mask level in the right panel of Figure 8. Without cluster masking, this is a $\sim 10\%$ bias for both simulations. Masking to $M_{\text{vir}} = 5 \times 10^{14} M_\odot$ reduces the bias to $\sim 2\%$, and more aggressive masking reduces this bias to a sub-percent level.

5.3. Dependence of SZ Bias on Mask Radius

Much of the tSZ four-point bias originates from large, relatively nearby halos. An insufficiently large mask leaves wings around each large projected cluster, which can become the dominant source of non-lensing fluctuation in reconstructed lensing maps. In Figure 9, we show the bias on the lensing power spectrum at $L = 500$ as a function of maximal cluster mass, for
various mask sizes. The plateaus seen at low mass correspond to incomplete masking of large halos, and it is clear that a mask of at least 5' radius is necessary for massive clusters, for percent-level accuracy on the lensing power. Thus multiple mask sizes may be needed, with larger masks for nearby clusters.

6. DEPENDENCE OF LENSING BIASES ON MAXIMAL TEMPERATURE MULTIPLE

The optimal filter for isolating lensing effects in quadratic CMB lensing reconstruction, Equation (5), naturally down-weights the modes in the observed sky with the largest variance. This can be seen by the presence in the denominator of the filter of the total power spectrum, $C^\ell_{\phi\phi}$, which consists of the CMB, foreground, and noise power. Treating the CMB as being beam-deconvolved, the noise power spectrum $C^\text{noise}_{\phi\phi}$ increases exponentially at the beam scale, leading to a natural cutoff for the modes included in the lens reconstruction. However, for experiments with high angular resolution this weighting can introduce the effects of foreground trispectra which become large at high multipoles, $l \gtrsim 2000$, where the CMB becomes relatively faint due to diffusion damping. For the analysis in all preceding sections we have thus imposed a multipole limit of $l_{\text{max}} = 3000$. In this section we study the dependence of the foreground biases on this choice.

In Figure 10, we show the statistical detection significance for quadratic lensing reconstruction when assuming only statistical errors, neglecting foregrounds, for both an experiment with white noise of amplitude 18 $\mu$K arcmin and a 1' beam, and a no-noise experiment. The statistical errors are calculated following

$$L^2 (\Delta(C^\phi_{L}))^2 = \frac{1}{L^2 f_{\text{sky}} L_{\text{max}}^2} (L^2 C^\phi_{L}^0 + N^0_{L}(l_{\text{max}})^2).$$

(21)

where $\Delta_L$ is the binning size and $f_{\text{sky}}$ the fraction of sky observed. The statistical significance for a lensing detection is then estimated using $S/N = (\sum_L (C^\phi_{L}^0 / \Delta(C^\phi_{L}^0)))^{1/2}$. For the 18 $\mu$K arcmin experiment, increasing the maximum multipole from $l_{\text{max}} = 3000$ to 4000 leads to an increase in signal-to-noise ratio of only 11%, while for the no-noise experiment the gain is a factor of 1.35. We also show the total bias determined for our simulations.

In Figure 11, we show all the simulation-derived lensing biases as a function of $l_{\text{max}}$, for an experiment with a noise level of 18 $\mu$K arcmin. Here clusters are masked to $M_{\text{vir}} = 5 \times 10^{14} M_\odot$ and sources masked to 5 mJy. All biases can be seen to increase quickly with $l_{\text{max}}$. Also shown is the totals band (solid gray), bounded by the spread of the sum of the biases for each model.

7. REDUCING BIAS BY AGGRESSIVE SOURCE MASKING

As we have shown, one way to reduce astrophysical biases on the lensing power spectrum is to mask sources and clusters aggressively, particularly, to lower detection thresholds than the very strongly detected sources which would likely be masked in a standard analysis. This process may introduce new biases, and in this section we use simulations of the lensed CMB to determine the response to aggressive masking on the estimation of the lensing power spectrum. We note that this is the only section in which we perform infilling. In other sections, we analyzed the foreground fields on their own, without any CMB or noise fluctuations, and simply replaced bright sources and clusters with the median of the pre-masked map.

Several approaches have been put forward for dealing with source masks in practice. One approach is to combine source masks with the mask from the edges of the field and any region...
of bright Galactic emission. This can be treated as an additional source of statistical anisotropy, for which one can apply the bias-hardened estimator of Namikawa et al. (2013). With this technique, the effects of lensing and the mask multiplication are both treated as separate sources of statistical anisotropy, and optimized quadratic estimators for each are formulated. An unbiased lensing estimate, which is valid to first order in the masking, can then be formed with a suitable linear combination of the two reconstructed fields. This approach is studied in more detail in Osborne et al. (2014).

Another, more complete approach is to perform lensing reconstruction taking into account the full pixel covariances in the CMB maps. Here the pixels which are to be masked can be assigned infinite variance, effectively projecting them out of the analysis. For large CMB maps this approach is naively very computationally challenging, but can be sped up with appropriate preconditioning (Smith et al. 2007, 2009).

A third method for treating sources is to restore the continuity of the CMB map at the source locations with in-painting techniques. This was studied by Perotto et al. (2010) and Benoit-Lévy et al. (2013), using constrained Gaussian realizations to match the CMB fluctuations near the holes (Hoffman & Ribak 1991). Benoit-Lévy et al. (2013) found that for masks of up to 2% of the sky, a straightforward rescaling of the $N_L^{(0)}$-subtracted reconstructions returns the input lensing power spectrum to good accuracy.

As can be seen in Figure 12, the total area of sky masked increases rapidly with aggressive masking, such that, depending on the mask radius, masking sources to 1 mJy and clusters to $M_{\text{vir}} = 10^{14} M_\odot$ corresponds to 10%–30% of the sky.

We perform a constrained realization of the structure in the masked regions using the routine described in Bucher & Louis (2012). This routine takes as input the power spectrum of the sky, which is assumed known; in a real analysis one would estimate the power spectrum directly from the valid data. Performing lensing reconstruction and removing the $N_L^{(0)}$ bias, we find that for holes of radius 2.5 arcmin and small fractions of the sky masked, the lensing power is reduced approximately in proportion to the amount of sky masked. However, as seen in Figure 13, there is a significant deviation from this for some larger masked area fractions.

A straightforward approach for correcting for this bias would be to estimate it from Monte Carlo simulations. We studied an alternative approach which can in principle use the data directly: we in-paint a second time, this time on the reconstructed lensing deflection maps, at the same hole centers using hole sizes of different radii.

(A color version of this figure is available in the online journal.)
which is determined by dividing by the lensing power spectrum in the fiducial cosmology. In each case the maps of lensed CMB and noise are masked with holes of $2.5$ radius. The reconstructed lensing fields are then in-painted again for a variety of mask sizes. Since this second in-painting is restoring lensing power to the masked regions, one might expect that the bias should be significantly reduced. However, we find that this is the case only when increasing the size of the holes to $4$ in this step. Thus there is some significant nontrivial structure in the lensing maps in the vicinity outside each original point source mask. Figure 13 shows that the residual bias can be greatly reduced using this two-step in-painting procedure, as in the case of using $4$ holes to in-paint the deflection map. This leaves a smaller remaining bias that still needs to be evaluated using Monte Carlo simulations. However, other techniques may still be devised to deal with this bias in a cleaner fashion.

8. DISCUSSION

In this paper we have studied several sources of bias on reconstructed CMB lensing power spectra that originate from known sources of non-Gaussianity in the millimeter-wave sky on small angular scales.

Lensing analyses using high-angular resolution maps, such as those from ACT and SPT, yield much stronger lensing detections per unit sky area than analyses of maps from a lower-resolution experiment such as Planck. Intuitively this is due to the increased number of CMB mode pairs over which to average, out to the higher effective maximum multipole $l_{\text{max}}$. However, at high $l_{\text{max}}$, foreground fluctuations also become increasingly important, to the point that they dominate the observed power spectrum at $l \gtrsim 3000$. To date, the analysis of temperature maps from ACT (Das et al. 2011, 2013) and SPT (van Engelen et al. 2012) has yielded lensing detections at low enough significance that these biases could be neglected, with the smallest uncertainty on the lensing amplitude to date being the $16\%$ of van Engelen et al. (2012). However, current and upcoming analyses will map sky areas which are larger by factors of several than these, and possibly with lower noise levels (in the case of a wide survey with ACTPol). With statistical uncertainties of a few percent on the lensing amplitude, systematic effects need to be understood and controlled, ideally to a percent or better.

Point sources can be detected to the relatively low flux levels of several mJy in maps such as those from ACT and SPT, particularly with the inclusion of data at multiple wavelengths. If point sources are uncorrelated, a nonzero trispectrum impacts the inferred lensing amplitude, but this bias is sub-percent after applying standard masking thresholds. In addition, the fact that the trispectrum is constant in multipole space for these sources means that this bias can be treated with other approaches, such as projecting it out of the reconstructed map (Namikawa & Takahashi 2014).

To treat other types of non-Gaussian foregrounds, particularly those with a different shape in multipole space, we analyzed two independent, realistic sets of simulations (S10; B14). For the CIB portions of these simulations, we first rescaled the amplitudes of the maps to match the observed power spectra. We then estimated the bispectra for these simulations, finding reasonable agreement with recent measures from SPT (Crawford et al. 2014) and Planck (Planck Collaboration 2013e). Performing lensing reconstructions on these fields, we isolated two types of bias: the first originates from the connected four-point function of the CIB, and the second originates from the correlation of the squared CIB with the lensing field. Since these biases are of opposite sign there is some degree of cancellation.

We found that both sources of bias can impact the lensing amplitude at the level of several percent, with the latter type of bias being larger. If masking is chosen as the method to treat this bias, we find that masking to $1$ mJy achieves percent-level biases.

Fluctuations from the tSZ effect can also lead to substantial biases, even when masking objects that are confidently detected. We computed the biases from the tSZ simulations of S10 and B14, both of which contain updated gas models designed to match the recent measurements of the power spectrum of tSZ fluctuations. Again we found that for standard masking levels, biases of a few percent can remain, though there is some cancellation between the two types.

We explored the uncertainty in the tSZ trispectrum, originating from its dependence on the details of the cluster gas profiles and the cosmological model. Using an analytical model of the tSZ trispectrum on the scales of relevance for CMB lensing, we calculated the four point-induced bias. We then perturbed in the space of cosmological and cluster-physics parameters, the parameters which most affect the inferred lensing bias, leading to a large uncertainty. It thus seems necessary to use either aggressive cluster masking, input from other frequencies, or an estimation of the tSZ trispectrum from the map itself to reduce this bias to percent levels.

Figure 14 summarizes our simulation-derived findings, with bands indicating the spread between the mean lensing biases for the various models. The left panel corresponds to “standard” masking, with clusters and sources masked to their approximate $5\sigma$ thresholds of $S_{\text{max}} = 5$ mJy and $M_{\text{vir}} = 5 \times 10^{14} M_{\odot}$. The right panel corresponds to an “aggressive” masking level of $S_{\text{max}} = 1$ mJy and $M_{\text{vir}} = 1 \times 10^{15} M_{\odot}$. In order to reduce the total bias on the lensing power spectrum to be less than one percent, we have additionally found it necessary to reduce the maximal temperature multipole in the lens reconstruction from $l_{\text{max}} = 3000$ to $l_{\text{max}} = 2500$ for this case. In this figure we have also allowed for the anticorrelation between the CIB and tSZ at 150 GHz (Addison et al. 2012), which reduces the overall biases by $\sim 20\%$.

Given the strong dependence of the biases on source masking levels, we also studied the feasibility of masking very aggressively. We used an in-painting routine to fill in the lensed CMB and noise fluctuations at the locations of masked sources and clusters, finding a negative bias on the reconstructed lensing power that roughly scales with the fraction of sky masked, for sufficiently small masked sky fractions. Attempting to reduce this bias by in-painting in the reconstructed lensing fields reduces this bias in a way that depends sensitively on the size of the in-painted region. For an experimental analysis, it may be necessary to include this in-painting bias as a transferring effect on the reconstructed lensing power using detailed simulations, as we have done here.

The next few years promise to be an exciting time for CMB lensing. Here we have shown that although the biases from foregrounds are not trivial, they do not present an insurmountable obstacle to using CMB lensing as a new, clean probe for physics that affects the growth of structure.

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Figure 14. Summary of fractional CMB lensing bias levels from our simulations, as a function of lensing multipole, $L$. Each colored region indicates the range of mean biases from the simulations we used. Green spans the thermal SZ four-point biases, red spans the CIB four-point bias, blue spans the bias from the tSZ–κ correlation, and pink spans the bias from the CIB–κ correlation. The range of total biases is bounded by the gray region, including the tSZ–CIB correlation found in the simulations, which slightly reduces the total bias. A 1% error band is indicated by the dotted lines. The left panel corresponds to masking sources above 5 mJy and clusters above $M_{vir}$ = $5 \times 10^{14} M_{\odot}$. The right panel corresponds to aggressive masking, with sources masked above 1 mJy and clusters above $M_{vir}$ = $10^{14} M_{\odot}$. In the right panel, we also reduce the maximum temperature multipole used in the reconstruction to $l_{max} = 2500$. The region where the total is within 1%, $L < 1400$, accounts for more than 99.9% of the total squared lensing signal-to-noise ratio.

(A color version of this figure is available in the online journal.)
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