EFFICIENTLY AND GLOBALLY SOLVING JOINT BEAMFORMING AND COMPRESSION PROBLEM IN THE COOPERATIVE CELLULAR NETWORK VIA LAGRANGIAN DUALITY

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ABSTRACT

Consider the joint beamforming and quantization problem in the cooperative cellular network, where multiple relay-like base stations (BSs) connected to the central processor (CP) via rate-limited fronthaul links cooperatively serve the users. This problem can be formulated as the minimization of the total transmit power, subject to all users’ signal-to-interference-plus-noise-ratio (SINR) constraints and all relay-like BSs’ fronthaul rate constraints. In this paper, we first show that there is no duality gap between the considered problem and its Lagrangian dual by showing the tightness of the semidefinite relaxation (SDR) of the considered problem. Then we propose an efficient algorithm based on Lagrangian duality for solving the considered problem. The proposed algorithm judiciously exploits the special structure of the Karush-Kuhn-Tucker (KKT) conditions of the considered problem and finds the solution that satisfies the KKT conditions via two fixed-point iterations. The proposed algorithm is highly efficient (as evaluating the functions in both fixed-point iterations are computationally cheap) and is guaranteed to find the global solution of the problem. Simulation results show the efficiency and the correctness of the proposed algorithm.

Index Terms—Cooperative cellular network, fixed-point iteration, KKT condition, Lagrangian duality, tightness of SDR

1. INTRODUCTION

Lagrangian duality [1,2], a principle that (convex) optimization problems can be viewed from either primal or dual perspective, is a powerful and vital tool in revealing the special structures of the optimization problems arising from engineering and further better solving the problems. Celebrated uplink-downlink duality [3,4] in wireless communications is an engineering interpretation of Lagrangian duality. Usually, the uplink problems, e.g., the transmit power minimization problems subject to quality-of-service (QoS) constraints, can be solved efficiently and globally via the fixed-point iteration algorithm. The uplink-downlink duality result thus enables efficient algorithms for solving the downlink problems via solving the relatively easy uplink problems. In the literature, Lagrangian duality and uplink-downlink duality results have been proved in various ways and applied to solve different downlink problems; see [3–20] and the references therein.

Different from the above works, this paper considers the joint beamforming and quantization problem in the cooperative cellular network, where multiple relay-like base stations (BSs) are connected to the central processor (CP) via rate-limited fronthaul links to cooperatively serve the users for effectively mitigating multiuser intercell interference. Such network includes coordinated multipoint [21], cloud radio access network [22], and cell-free massive multi-input multi-output [23] as special cases. Recently, Refs. [24,25] have established an interesting uplink-downlink duality for such network when relay-like BS compression optimization is considered. Specifically, given the same beamforming vectors in the uplink and downlink, it has been shown that when Wyner-Ziv compression and multivariate compression are adopted in the uplink and downlink, respectively, the transmit power minimization problem in the uplink subject to individual signal-to-interference-plus-noise-ratio (SINR) constraints and fronthaul capacity constraints is equivalent to that in the downlink. Furthermore, [25] has designed an algorithm for solving the joint beamforming and quantization problem based on the established duality result. The algorithm in [25] first solves the uplink problem via fixed-point iteration and then solves the downlink problem with fixed beamformers (which is a convex problem) obtained by solving the uplink problem by calling a solver.

In this paper, we consider the same joint beamforming and quantization problem as in [25] but make further progress in the duality result and the algorithm. The main contributions of this paper are as follows. (1) New Duality Result. We establish the tightness of the semidefinite relaxation (SDR) of the considered problem and thus the equivalence of the two problems. This result further implies that the dual problems of the considered problem and its SDR are the same. Note that the Lagrangian dual of the original problem is studied in this paper, which differs from the Lagrangian dual of the problem with fixed beamformers in [25]. (2) Efficient Fixed-Point Iteration Algorithm. Based on the established duality result, we propose an efficient algorithm for solving the considered problem. The proposed algorithm first solves the dual problem via fixed-point iteration and then solves the primal problem via another fixed-point iteration. The proposed algorithm is highly efficient (as each update of variables in fixed-point iterations is computationally cheap) and is guaranteed to find the global solution of the problem. The proposed algorithm exploits more special structures of the solution of the considered problem than the algorithm in [25] and thus significantly outperforms it in terms of the computational efficiency.

Notations. For any matrix $A$, $A^\dagger$ and $A^\ast$ denote the conjugate transpose and transpose of $A$, respectively; $A^{(m,n)}$ denotes the entry on the $m$-th row and the $n$-th column of $A$; and $A^{(m_1:m_2,n_1:n_2)}$ denotes a submatrix of $A$ defined by

$$
\begin{pmatrix}
A^{(m_1,n_1)} & \cdots & A^{(m_1,n_2)} \\
\vdots & \ddots & \vdots \\
A^{(m_2,n_1)} & \cdots & A^{(m_2,n_2)}
\end{pmatrix}.
$$
For two matrices $A_1$ and $A_2$ of appropriate sizes, $A_1 \bullet A_2$ denotes the trace of $A_1 A_2$. We use $CN(0, Q)$ to denote the complex Gaussian distribution with zero mean and covariance $Q$. Finally, we use $I$ to denote the identity matrix of an appropriate size, $0$ to denote an all-zero matrix of an appropriate size, and $E_{a_k}$ to denote the square all-zero matrix except its $m$-th diagonal entry being one.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

### 2.1. System Model

Consider a cooperative cellular network consisting of one CP and $M$ single-antenna relay-like BSs (will be called relays for short later), which cooperatively serve $K$ single-antenna users. In such network, all users and relays are connected by noisy wireless channels and all relays and the CP are connected by noiseless fronthaul links of finite rate. Let $\mathcal{M}$ and $\mathcal{K}$ denote the sets of the relays and the users, respectively.

We first introduce the compression model from the CP to the relays. The transmitted signal at the CP is $\mathbf{x} = \sum_{k=1}^{K} v_k s_k$, where $v_k = [v_{k,1}, \ldots, v_{k,M}]^T$ is an $M \times 1$ beamforming vector and $s_k \sim CN(0, I)$ is the information signal for user $k$. Because of the limited capacities of the fronthaul links, the signal from the CP to the relays need to be first compressed before transmitted. Let the compression error be $\mathbf{e} = [e_1, \ldots, e_m] \sim CN(0, Q)$, where $Q$ is the covariance matrix to be designed. Then the received signal at relay $m$ is $v_m = \sum_{k=1}^{K} h_{k,m} s_k + e_m$. The channel model from the relays to the users is $y_k = \sum_{m=1}^{M} h_{k,m} v_m + z_k$, where $y_k$ is the signal received by user $k$. $v_m$ is the signal transmitted by relay $m$, $h_{k,m}$ is the channel coefficient from relay $m$ to user $k$, and $\{z_k\}$ are independent and identically distributed (i.i.d.) additive Gaussian noise distributed as $CN(0, \sigma^2)$.

Under the above model, the received signal at user $k$ is

$$y_k = h_k^\dagger \left( \sum_{i=1}^{K} v_i s_i \right) + h_k^\dagger e + z_k,$$  

(1)

where $h_k = [h_{k,1}, \ldots, h_{k,M}]^T$ is the channel vector of user $k$. Then the total transmit power of all the relays is $\sum_{k=1}^{K} \|v_k\|^2 + Q \bullet I$, the SINR of user $k$ is

$$\frac{|h_k^\dagger v_k|^2}{\sum_{j \neq k} |h_j^\dagger v_j|^2 + h_k^\dagger Q h_k + \sigma^2}, \ \forall k \in \mathcal{K};$$  

(2)

and the compression rate of relay $m$ under the multivariate compression strategy [26] is

$$\log_2 \frac{\sum_{k=1}^{K} |v_k|^2}{Q^{(m,M,m,M)}/Q^{(m+1,M,m+1,M)}}, \ \forall m \in \mathcal{M}. \ \ (3)$$

In the above, $Q^{(m,M,m,M)}/Q^{(m+1,M,m+1,M)}$ is the Schur complement of the block $Q^{(m,M,m,M)}$ of $Q^{(m,M,m,M)}$, which is equal to $Q^{(m,m)} - Q^{(m+1,M,m+1,M)} Q^{(m+1,M,m+1,M)}^{-1} Q^{(m+1,M,m+1,M)}$.

### 2.2. Problem Formulation

Now we are ready to present the problem formulation. Given a set of SINR targets $\{\gamma_k\}$ and a set of fronthaul capacities $\{C_m\}$, the interested optimal joint beamforming and compression problem, which minimizes the total transmit power subject to all users’ SINR constraints and all relays’ fronthaul capacity constraints, is as follows [25]:

$$\begin{aligned}
\min_{\{v_k\}, Q} & \sum_{k=1}^{K} \|v_k\|^2 + Q \bullet I \\
\text{s.t.} & \frac{|h_k^\dagger v_k|^2}{\sum_{j \neq k} |h_j^\dagger v_j|^2 + h_k^\dagger Q h_k + \sigma^2} \geq \gamma_k, \ \forall k \in \mathcal{K}, \\
& \log_2 \frac{\sum_{k=1}^{K} |v_k|^2 + Q^{(m,m)}}{Q^{(m+1,M,m+1,M)}} \leq C_m, \ \forall m \in \mathcal{M}, \\
& Q \succeq 0.
\end{aligned}$$

(4)

Let $H_k = h_k h_k^\dagger$ for all $k$ and $\eta_m = 2C_m$ for all $m$. By [25] Proposition 4, problem (4) is equivalent to the following problem:

$$\begin{aligned}
\min_{\{v_k\}, Q} & \sum_{k=1}^{K} \|v_k\|^2 + Q \bullet I \\
\text{s.t.} & v_k^H H_k v_k - \gamma_k \left( \sum_{j \neq k} v_j^H H_k v_j + Q \bullet H_k + \sigma^2 \right) \geq 0, \ \forall k \in \mathcal{K}, \\
& \eta_m \left[ \begin{array}{c} 0 \\ 0 \\ Q^{(m,M,m,M)} \end{array} \right] - E_m^m \left( \sum_{k=1}^{K} v_k v_k^H + Q \right) E_m \succeq 0, \\
& \ \forall m \in \mathcal{M}, \\
& Q \succeq 0.
\end{aligned}$$

(P)

In the following section, we shall focus on problem (P) and design an efficient algorithm for solving it.

## 3. PROPOSED ALGORITHM VIA LAGRANGIAN DUALITY

### 3.1. Tightness of SDR of (P)

Problem (P) is a separable homogeneous quadratically constrained quadratic program. A well-known technique to tackle such problem is the SDR [27]. Applying the SDR technique to (P), we obtain

$$\begin{aligned}
\min_{\{V_k\}, Q} & \sum_{k=1}^{K} V_k \bullet I + Q \bullet I \\
\text{s.t.} & a_k(\{V_k\}, Q) \geq 0, \ \forall k \in \mathcal{K}, \\
& B_m(\{V_k\}, Q) \geq 0, \ \forall m \in \mathcal{M}, \\
& V_k \succeq 0, \ \forall k \in \mathcal{K}, \\
& Q \succeq 0.
\end{aligned}$$

(5)

where

$$a_k(\{V_k\}, Q) = V_k \bullet H_k - \gamma_k \left( \sum_{j \neq k} V_j \bullet H_k + Q \bullet H_k + \sigma^2 \right),$$

$$B_m(\{V_k\}, Q) = \eta_m \left[ \begin{array}{c} 0 \\ 0 \\ Q^{(m,M,m,M)} \end{array} \right] - E_m^m \left( \sum_{k=1}^{K} V_k + Q \right) E_m.$$  

The Lagrangian dual of problem (5) is

$$\begin{aligned}
\max_{\{\lambda_k\}, \{A_m\}} & \sum_{k=1}^{K} (\gamma_k \sigma^2) \beta_k \\
\text{s.t.} & C_k(\{\beta_k\}, \{A_m\}) - \beta_k H_k \succeq 0, \ \forall k \in \mathcal{K}, \\
& D(\{\beta_k\}, \{A_m\}) \succeq 0, \\
& \beta_k \succeq 0, \ \forall k \in \mathcal{K}, \\
& A_m \succeq 0, \ \forall m \in \mathcal{M},
\end{aligned}$$

(6)
where $\beta_k$ is the dual variable associated with the $k$-th SINR constraint in (5), $A_m$ is the dual variable associated with the $m$-th front-end capacity constraint in (5), and

$$C_k(\{\beta_k\}, \{A_m\}) = I + \sum_{m=1}^{M} E_m^j A_m E_m + \sum_{j \neq k} \beta_j^j \gamma_j H_j,$$

$$D(\{\beta_k\}, \{A_m\}) = I - \sum_{m=1}^{M} \eta_m \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_{m,m,M,m,M} \end{bmatrix} + \sum_{k=1}^{K} \beta_k \gamma_k H_k + \sum_{m=1}^{M} E_m^j A_m E_m.$$

An important line of research on the SDR is to study its tightness [27,29]. In the following theorem, we show that the SDR in (5) is tight (if it is feasible), i.e., it always has a rank-one solution. This shows that problem (P) admits a convex reformulation and answers a question in [25, Section IX-B].

**Theorem 1** Suppose that problem (5) is feasible. Then it always has a rank-one solution.

**Proof:** Since the SDR is feasible, there must exist a primal-dual pair $(V_k, Q, \{\beta_k\}, \{A_m\})$ such that the Karush-Kuhn-Tucker (KKT) conditions of problem (5) hold. In particular, the complimentary slackness condition $V_k \cdot (C_k(\{\beta_k\}, \{A_m\}) - \beta_k H_k) = 0$ holds. Since $C_k(\{\beta_k\}, \{A_m\})$ is positive definite and $H_k$ is rank-one and positive semidefinite, it follows that

$$\text{rank}(C_k(\{\beta_k\}, \{A_m\}) - \beta_k H_k) \geq M - 1,$$

which, together with the complimentary slackness condition and the rank inequality, implies that $\text{rank}(V_k) \leq 1$. \qed

### 3.2. Proposed Algorithm

It is well known that the KKT conditions are sufficient and necessary for the global solution of problem (5). By further exploiting the special structure of problem (5) and its KKT conditions, we get the following conditions that the solution of problem (5) must satisfy:

$$D(\{\beta_k\}, \{A_m\}) = 0,$$  \hspace{1cm} (7)

$$\text{rank}(A_m) = 1, A_m \geq 0, \forall m \in M,$$  \hspace{1cm} (8)

$$A_{m,1-m,1-m} = 0, A_{m,m,1-m} = 0, \forall m \in M,$$  \hspace{1cm} (9)

$$\text{rank}(C_k(\{\beta_k\}, \{A_m\}) - \beta_k H_k) = M - 1, \forall m \in M,$$  \hspace{1cm} (10)

$$V_k \cdot (C_k(\{\beta_k\}, \{A_m\}) - \beta_k H_k) = 0, \forall k \in K,$$  \hspace{1cm} (11)

$$V_k \geq 0, \text{rank}(V_k) = 1, \forall k \in K,$$  \hspace{1cm} (12)

$$a_k(V_k, Q) = 0, \forall k \in K,$$  \hspace{1cm} (13)

$$B_m(V_k, Q) \geq 0, \forall m \in M,$$  \hspace{1cm} (14)

$$A_m \cdot B_m(V_k, Q) = 0, \forall m \in M,$$  \hspace{1cm} (15)

$$Q \geq 0.$$  \hspace{1cm} (16)

The above conditions are essentially the KKT conditions of problem (5) except the one in (7), whose proof needs a judicious treatment of the special structure and the KKT conditions of problem (5).

Next, we shall design an algorithm for solving the above equations by further carefully exploiting the special structures in the equations. The idea is to first find $\{\beta_k\}$ and $\{A_m\}$ by solving Eqs. (7)–(10) and then plug $\{\beta_k\}$ and $\{A_m\}$ into Eqs. (11)–(16) and solve for $V_k$ and $Q$.

#### 3.2.1. Solving Eqs. (7)–(10)

Suppose that $\{\beta_k\}$ are given, we first find $\{A_m\}$ that satisfy Eqs. (7) and (8). Define $\Gamma = I + \sum_{k=1}^{K} \beta_k \gamma_k H_k$. Then Eq. (7) is equivalent to

$$\sum_{m=1}^{M} \eta_m \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_{m,m,M,m,M} \end{bmatrix} - \sum_{m=1}^{M} E_m^j A_m E_m = \Gamma.$$

We know from the special properties of $\{A_m\}$ in (8) that only $A_1$ affects the first row and column of matrix $\Gamma$. Therefore, the entries in the first row of $A_1$ should be $\frac{1}{\eta_1} \Gamma(1,1), \frac{1}{\eta_1} \Gamma(1,2:M)$. Since $A_1$ is of rank one, we can further obtain all entries of $A_1$ based on its entries in the first row. After $A_1$ is obtained, we can subtract all terms related to $A_1$ from both sides of (7) and the left-hand side of (17) becomes

$$\sum_{m=2}^{M} \eta_m \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_{m,m,M,m,M} \end{bmatrix} - \sum_{m=2}^{M} E_m^j A_m E_m.$$

Then we can do the same to find $A_2$. Repeat the above procedure until all $\{A_m\}$ are obtained. It can be shown that $\{A_m\}$ that satisfy Eqs. (7)–(8) are unique, and such solution, depending on the given $\{\beta_k\}$, is denoted as $\{A_m(\{\beta_k\})\}$.

To ease the presentation, define $C_k(\{\beta_k\}, \{A_m\})$. Since $C_k \succ 0$ and $H_k \succeq 0$, there exists a unique $\beta_k$ such that one and only one eigenvalue of $C_k - \beta_k H_k$ is equal to zero. Such $\beta_k$ admits the following closed-form solution:

$$\beta_k(\{A_m\}, \{\beta_k\}, j \neq k) = (h_k^j C_k^{-1} h_k)^{-1}.$$  \hspace{1cm} (19)

From the above discussion, we know: if $\{A_m\}$ are known, one can get $\{\beta_k(\{A_m\}, \{\beta_k\}, j \neq k)\}$ such that (9) and (10) hold; if $\{\beta_k\}$ are known, one can get $\{A_m(\{\beta_k\})\}$ such that (7) and (8) hold. If one can find $\{\beta_k\}$ and $\{A_m(\{\beta_k\})\}$ that satisfy

$$\beta_k = I_k(\{\beta_k\}) \succeq \beta_k(\{A_m(\{\beta_k\})\}, \{j \neq k\}), \forall k \in K,$$  \hspace{1cm} (20)

then all Eqs. (7)–(10) are satisfied. Define $\beta = [\beta_1, \ldots, \beta_K]^T$ and $I(\beta) = [I_1(\{\beta_k\}), \ldots, I_K(\{\beta_k\})]^T$, then (19) becomes

$$\beta = I(\beta).$$
(C_k - \beta_k H_k) v_k = C_k v_k - \beta_k h_k h_k^T v_k = 0. Hence, v_k can be solved explicitly as follows:

\[ v_k = \frac{C_k^{-1} h_k}{||C_k^{-1} h_k||}. \]  

(21)

Let U_k = v_k h_k^T. Substituting (21) into (13), one has

\[ p_k U_k \cdot H_k - \gamma_k \left( \sum_{j \neq k} p_j U_j \cdot H_k + Q \cdot H_k + \sigma^2 \right) = 0. \]

Then one can solve for \( p_k \) as follows:

\[ p_k \left( Q, \{ p_j \} \right) = \frac{\gamma_k \left( \sum_{j \neq k} p_j U_j \cdot H_k + Q \cdot H_k + \sigma^2 \right)}{U_k \cdot H_k}. \]  

(22)

Now suppose \( \{ p_k \} \) are known. By Eq. (8), one can decompose \( \Lambda_m \) into: \( \Lambda_m = \lambda_m \lambda_m^T, \) where \( \lambda_m = [0, \ldots, 0, \lambda_{m1}^{(m)}, \ldots, \lambda_{mM}^{(m)}]^T. \)

This decomposition, together with Eqs. (14) and (15), implies

\[ B_m \lambda_m = 0, \quad \forall \ m \in \mathcal{M}. \]  

(23)

Next we solve (23) from \( m = M \) to \( m = 1 \) and can obtain the desired \( Q \). More specifically, when \( m = M \), since \( \lambda_M^{(M)} > 0 \), it follows that

\[ Q^{(M,M)} = \sum_{k=1}^{K} \frac{\gamma_k}{\eta_m - 1} v_m^{(M,M)}. \]  

(24)

When \( m = M - 1 \), we can substitute (24) into (22) to solve for \( Q^{(M-1,M-1)} \) and \( Q^{(M-1,M)} \). In particular, we can obtain \( Q^{(M-1,M)} \) by using the last equation of (23) with \( m = M - 1 \); then we can further obtain \( Q^{(M-1,M-1)} \) by using the second last equation of (23) with \( m = M - 1 \). In fact, each step of the above procedure admits a closed-form solution. We can do the same sequentially to solve problem (23) with \( m = M - 2, M - 3, \ldots, 1 \). Denote the solution to (23) as \( Q(\{ p_k \}) \).

Similarly, we can define a fixed-point iteration to solve Eqs. (11–16) for the desired \( \{ V_k \} \) and \( Q \) and show that the fixed-point iteration is a standard interference function and thus converges to the unique solution. We omit the details due to the space reason.

3.2.3. Proposed Fixed-Point Iteration Algorithm

Now, we present the algorithm for solving problem (P) (equivalent to problem (P)). The algorithm first finds \( \{ \beta_k \} \) and \( \{ \Lambda_m \} \) that satisfy Eqs. (7)–(10); with found \( \{ \beta_k \} \) and \( \{ \Lambda_m \} \) fixed, the algorithm then finds \( \{ V_k \} \) and \( Q \) that satisfy Eqs. (11)–(16). Hence, \( \{ V_k \}, Q, \{ \beta_k \}, \) and \( \{ \Lambda_m \} \) together satisfy Eqs. (7)–(16) and thus is a KKT point of problem (P). Since rank \( \{ V_k \} = 1 \) for all \( k \), we can recover the optimal solution for problem (P). The pseudocodes of the proposed algorithm are given in Algorithm 1.

**Algorithm 1 Proposed Algorithm for Solving Problem (P)**

1. Find \( \{ \beta_k \} \) and \( \{ \Lambda_m \} \) that satisfy Eqs. (7)–(10) by performing the fixed-point iteration in (20) on \( \{ \beta_k \} \) until the desired error bound is met.
2. Find \( \{ V_k \} \) and \( Q \) that satisfy Eqs. (11)–(16) by performing an appropriate fixed-point iteration on \( \{ p_k \} \) until the desired error bound is met.
3. Find \( v_k \) such that \( V_k = v_k v_k^T, \) \( \forall \ k \in K \).
4. Output: \( \{ v_k \} \) and \( Q \).
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