Possibilities for calculating rubber-cord shells in modern finite-element analyses packages

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Abstract. As flexible inserts of various designs, including pipeline systems, vibration-insulating pipes are used, which act as compensators to reduce the resulting deformations and vibration loads. The main element of the design of the pipe, perceiving the action of internal overpressure, is a rubber-cord sheath. To study the possibility of calculating rubber-cord shells in modern finite element analysis packages in the present work, the cylindrical shell of the vibration-insulating pipe has been simulated and its stress-strain state in the software package has been calculated. Since the RCS is a composite design, the simulation was carried out using the built into modules. The results obtained make it possible to analysis the composite material of the RCS and to determine the criteria for shell destruction.

1. Introduction
Vibration insulating pipes are used as flexible inserts in pipeline systems and operate as compensators to reduce the occurring deformations and vibration loads. The main structural element of the nozzle is the rubber-cord shell (RCS), which perceives the action of internal overpressure. In order to study the possibility of finite element analysis of RCS in modern software packages Ansys, Abaqus, Nastran, the strength characteristics of the shell are determined on its basis, a cylindrical shell of a vibration-insulating pipe is modeled in the work and the stress-strain state (SSS) is calculated under three different load conditions.

2. The problem statement
Modeling and calculation of the SSS of a composite cylindrical shell is required to be performed having the following parameters: the shell radius is \( R = 50 \text{ mm} \); and the shell length is \( L = 1000 \text{ mm} \).

The shell is made of four layers of two-dimensional rubber-cord material with the following characteristics:
- modulus of elasticity in the longitudinal direction (direction of the fibers) is \( E_1 = 140000 \text{ MPa} \);
- modulus of elasticity in the transverse direction (in the direction of the matrix) is \( E_2 = 97000 \text{ MPa} \);
- Poisson's ratio is \( \nu = 0.3 \);
- shear modulus in the plane of the axes of the material is \( G_{12} = 5400 \text{ MPa} \);
- transverse shear modulus is \( G_{13} = 3600 \text{ MPa} \);
- transverse shear modulus is \( G_{23} = 5400 \text{ MPa} \).

The composite structure is:
- the number of layers is 4;
- thickness of each layer is 1 mm;
- the direction of the fibers of the layer (the direction of reinforcement is the circumference) in the first layer is 0°; in the second layer it is 45°; in the third layer it is 90°; in the forth layer it is 135°.

As a calculation model, a straight cylindrical shell with a certain number of layers of rubber-cord material is adopted, fixed between the flanges of the pipe. The model of the vibration isolation pipe is shown in figure 1.

Since vibration isolating nozzles are operated in various load conditions, the article considers three cases in which the simulation is designed for the following loads:
1. The lower section is fixed (movements across all 6 degrees of freedom are limited). Internal pressure $P=1$ MPa is applied to the shell.
2. The lower section is fixed (movements across all 6 degrees of freedom are limited). Internal pressure $P=1$ MPa is applied to the shell.
3. The lower section is fixed (movements across all 6 degrees of freedom are limited). Internal pressure $P=1$ MPa and axial displacement compressing the shell by 10 mm are applied to the shell.
4. The lower section is fixed (movements across all 6 degrees of freedom are limited). A bending force $Q=1200$ N and moment $M_x=2.4 \cdot 10^6$ N·mm is applied to the upper section.

3. Theory
The static equilibrium equation system [1] is applied in the framework of the SSS analysis:

$$[K]\{U\} = \{F\},$$

where $[K]$ is global stiffness matrix;
- $\{U\}$ is nodal displacement vector;
- $\{F\}$ is vector of external forces.

Direct or iterative methods are used [2] to solve the system of equilibrium equations. They include:
- Right-Handed Method;
- Left-Handed Method using the Cholesky method;
- Sparse Method;
- Parallel Method.

Iterative methods include:
- Global method using a full matrix for the structure;
- Elemental method using element matrices.

Isotropic, orthotropic and ply material [1] can be used as layer material in NX LC. The ply material combines the properties of the fibers and matrix materials, and on their basis the equivalent properties of the ply are calculated. The ply material can be: unidirectional, woven, and granular and with randomly oriented short fibers.

When creating a ply material, volume fractions of fiber material ($f$) and matrix ($m$) are introduced:

$$V_f + V_m = 1,$$

where $V_f$ is fiber volume fraction ($0 < V_f < 1$); $V_m$ is volume fraction of the matrix ($0 < V_m < 1$).

Equivalent material properties for unidirectional material are calculated as follows (the resulting material is orthotropic) [3]:

$$E_1 = V_f E_f + V_m E_m,$$
$$E_2 = \frac{E_f E_m E_1}{E_f (V_f E_m + V_m E_f) - V_f V_m (E_f - E_m)^2}.$$
\[ G_{12} = \frac{G_f G_m}{V_f G_m + V_m G_f}, \]
\[ \nu_{12} = V_f \nu_f + V_m \nu_m, \quad \nu_{21} = \frac{\nu_{12} E_2}{E_1}, \]

where \( E_1, E_2 \) are elastic moduli of equivalent material;
\( E_f, E_m \) are elastic moduli of fiber and matrix;
\( G_{12} \) is equivalent material shear modulus;
\( G_f, G_m \) are fiber and matrix shear modules;
\( \nu_{12}, \nu_{21} \) are Poisson's ratios of equivalent material;
\( \nu_f, \nu_m \) are Poisson's ratio of fiber and matrix.

4. Simulating of the rubber-cord shell

A three-dimensional finite-element model of the rubber-cord shell of the vibration-insulating pipe was created (figure 2), an orthotropic material was selected as the shell material, it is composite by type, and the parameters are corresponding to it.

![Figure 2. Finite-element model of RCS.](image-url)

Since the shell is made of composite material, the orientation of the material is very important. The first layer is to be oriented perpendicular to the direction of the generatrix in this model, i.e. circumference and correspond to the model shown in figure 3.

![Figure 3. Checking the orientation of the material.](image-url)
5. Calculation results
In the first design case the shell is rigidly fixed along the upper and lower faces, and is loaded with internal pressure 1 MPa. As a result of calculations, a deformed shell model has been obtained (figure 4).

![Volumetric diagram of the RCS deformed model displacements.](image)

The displacements of the model nodes are increased for clarity and make up 10% of the model size. The calculations showed that the maximum displacements were 0.011 mm.

![Volumetric diagrams: a) strains of the first layer, b) strains of the fourth layer, c) stresses of the second layer; d) stresses of the third layer.](image)
The software package displays strains and stresses for each RCS layer separately, both in volume diagrams and in graphs. For example, strains are shown selectively for the first and fourth layers of the RCS (figure 5, a, b) and stresses for the second and third layers of the RCS (figure 5, c, d).

There is shown the distribution of stresses and strains in the section along the generatrix of the RSC model for each of their four layers of RCS on the graphs (figures 6, 7).

![Figure 6](image1.png)

**Figure 6.** Charts of equivalent stresses layered distribution along the RCS generatrix.

![Figure 7](image2.png)

**Figure 7.** Charts of the layered strain distribution along the RCS generatrix.

When modeling the second design case, the shell is rigidly fixed along the lower flange, loaded with internal pressure 1 MPa and compressed along the axis by 10 mm. In order to compress the shell, it is necessary to set 10 mm displacement of the upper face in the axial direction.

As a result of the calculations, a volumetric diagram of the deformed shell model displacements has been obtained (figure 8).
Figure 8. Volumetric diagram of deformed shell displacements.

For clarity, the movements of the model nodes are increased and make up 10% of the model size. The distribution of stresses and strains in each layer of the shell in graphs is shown in figures 9, 10.

Figure 9. Charts of equivalent stresses layered distribution along the RCS generatrix.

Figure 10. Charts of the layered strain distribution along the RCS generatrix.
There is the third design case considered, in which the shell is rigidly fixed along the lower flange, and bending force and moment are applied to the upper one. The results of calculations in the form of a deformed model are shown in figure 11.

![Figure 11. Volumetric diagram of deformed shell displacements.](image)

The displacements of the model nodes are increased for clarity and make up 10% of the model size. The color gamut distribution shows the amount of displacements. The displacements of the loose end were 16.5 mm.

For the third case of loading, strains and stresses for various shell layers are shown selectively (figure 12).
In this case, the loading is asymmetric; therefore, the distribution of strains and stresses around the circumference in the middle section of the shell will be more interesting. The layered distribution of stresses and strains in the cross section of the shell at distance 500 mm from the lower edge is displayed on the graphs (figures 13, 14).

Figure 12. Volumetric diagrams: a) strains of the first layer, b) strains of the second layer, c) stresses in the third layer; d) stresses in the fourth layer.

Figure 13. Charts of equivalent RCS stresses layered distribution.

Figure 14. Charts of RCS strains layered distribution.
6. Discussion of the results
The results obtained consider the composite material of the RCS and determine the criteria for the destruction of the shell. The distribution of stresses over the composite thickness (figure 15) showed that the most stressed element under number 2783 was in the second design case, in the third layer. Stresses amounted to 1406.45 MPa in this case.

![Figure 15. The distribution of stresses over the RCS composite thickness.](image)

7. Conclusion
In this work, the problem of finite element modeling of the stress-strain state of a vibration-isolating pipe using the software packages of finite element analysis tools has been solved. The problem of modeling a cylindrical shell consisting of four layers of a composite material is considered. Calculations were performed for three cases of shell loading: 1 – under the influence of internal pressure, 2 - under the influence of internal pressure and compressive load, 3 – under the influence of internal pressure, and bending force, and moment.

The calculations of the stress-strain state made it possible to obtain the values of strains and stresses both for the shell as a whole and for each layer separately. It is also possible to calculate the layer strength and interlayer strength according to various criteria.

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