Radiative transitions of heavy quarkonium states

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We study radiative decays of heavy $Q\bar{Q}$ states, both for $Q = c$ and $Q = b$, using an effective Lagrangian approach which exploits spin symmetry for such states. We use existing data on radiative quarkonium transitions to predict some unmeasured decay rates. We also discuss how these modes can be useful to understand the structure of $X(3872)$.

PACS numbers: 13.20.Gd, 14.40.Gx

I. INTRODUCTION

Heavy quarkonium Physics was born in 1974 with the discovery of the $J/\psi$, the first observed bound state of a heavy quark and a heavy antiquark. Since then, quarkonium spectra and decays have been thoroughly studied by means of potential models, lattice QCD, QCD sum rules and effective theories (for recent reviews see [1, 2]).

In particular, the agreement of the observed mass levels and potential model predictions was considered as a success of the latter, at least until 2003, when a series of observations of new states started enriching our knowledge of $c\bar{c}$ and $b\bar{b}$ states and stimulating new investigations. Indeed, several aspects of such new states seem not to be reconciled with predictions. Thus, we have to face two possibilities: either the accuracy of the theoretical approaches has to be questioned, or the newly observed states are not conventional $Q\bar{Q}$ quarkonia [3, 4].

In order to discuss these topics, it is useful to adopt the usual classification of $Q\bar{Q}$ states in terms of the radial quantum number $n$, the orbital angular momentum $L$, the spin $s$ and the total angular momentum $J$. The state identified by $n^{2s+1}L_J$ corresponds to a meson with parity $P = (-1)^{L+1}$ and charge-conjugation $C = (-1)^{L+s}$. In analogy with potential model terminology, states with $L = 0$ are referred to as $S$ wave states, those with $L = 1$ as $P$ wave, $L = 2$ as $D$ wave states, and so on.

In Table 1 we collect quarkonium resonances corresponding to $S$, $P$ and $D$ wave states with $n = 1$, and $S$ and $P$ wave states with $n = 2$, which are the subject of our analysis. In this Table, we include the established charmonium and bottomonium states, together with their masses and widths, when known [5]. Other known charmonium states are $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$, usually identified with the states $3^3S_1$, $2^3D_1$ and $4^3S_1$, respectively, and therefore are not included in the Table. As for bottomonium, the established states not included in the Table are $\Upsilon(3S)$, $\Upsilon(4S)$, $\Upsilon(5S)$, as well as the meson $\Upsilon(11020)$, which is likely to be $\Upsilon(6S)$.

States below the open flavour threshold ($DD$ for charmonium, $BB$ for bottomonium) are narrow, as well as those states above such threshold whose strong decays to open flavour are forbidden by spin-parity conservation. For these states important decay modes are radiative transitions, which can be conveniently studied according to a perturbative expansion of the Hamiltonian inducing the decay. In this way, one recognizes that the most important transitions are electric dipole transitions (named E1) and magnetic dipole transitions (M1). In the former case quark spins are not flipped and the transitions have $\Delta L = \pm 1$, $\Delta s = 0$, while in the latter quark spin is flipped and $\Delta L = 0$. In the framework of potential models, these can be calculated in terms of the wave functions of the involved quarkonium states, the overlap of which is different from zero only for states with the same radial quantum number $n = n'$. This result is modified by the inclusion of relativistic corrections, which induce non-zero transitions among states with $n \neq n'$ [6, 7].

Another framework in which the analogy of quarkonium with an almost non relativistic system is exploited is non relativistic QCD (NRQCD) [8, 9], an effective theory based upon an expansion in the powers of $v$, the relative velocity of $Q$ and $\bar{Q}$ in the bound state. Several predictions have been derived through this approach, the various quantities (production cross sections, decay widths, etc) being written as sums of contributions of several operators ordered according to the velocity scaling rules [10].

From an experimental point of view, there are several possibilities to access quarkonium states. In the case of charm, apart from $p\bar{p}$ production, direct production happens at $e^+e^-$ machines. Examples are CLEO-c at the center of mass energy of $\psi(2S)$ and BES. Radiative decays of $\psi(2S)$ allow to reach other states which cannot be directly produced from $e^+e^-$ annihilation due to conservation of spin-parity, such as the $c\bar{c}$ states. $B$ factories have also revealed an important source of charmonia. In this environment, $c\bar{c}$ states can be produced $i$) through initial state radiation (ISR) when after the emission of a photon from the initial state the effective center of mass energy is suitable for the production of charmonium; $ii$) in the collision of two photons radiated by $e^+e^-$; $iii$) in $B$ decays. As for bottomonium, the same mechanisms hold in principle (except for production in $B$ decays), even...
 TABLE I: Masses and widths of 1S, 1P, 2S, 2P and 1D quarkonium states, taken from Ref. 8. The state $\chi_{c2}(2P)$ is often referred to as $Z(3930)$.

| $n^2 + 1 L_J$ | JPC | Charm mass (MeV) | width (MeV) | Beauty mass (MeV) | width (MeV) |
|----------------|-----|-----------------|-------------|------------------|-------------|
| 1S0            | 0+  | $\eta_c(1S)$   | 2980.3 ± 1.2| $\eta_b(1S)$   | 9300 ± 20 20|
| 1S1            | 1+  | $\eta_c(1S)$   | 3096.916 ± 0.011| $\Upsilon(1S)$ | 9460.30 ± 0.26|
| 1P0            | 0+  | $\chi_{c0}(1P)$| 3414.75 ± 0.31| $\chi_{b0}(1P)$| 9859.44 ± 0.42 31|
| 1P1            | 1+  | $\chi_{c1}(1P)$| 3510.66 ± 0.07| $\chi_{b1}(1P)$| 9892.78 ± 0.26 31|
| 1P2            | 2+  | $\chi_{c2}(1P)$| 3553.20 ± 0.09| $\chi_{b2}(1P)$| 9912.21 ± 0.26 31|
| 1P3            | 3+  | $h_c(1P)$       | 3525.93 ± 0.27| $h_b(1P)$       | < 1         |
| 2S0            | 0+  | $\eta_c(2S)$   | 3637 ± 4    | $\eta_b(2S)$   | 10023.26 ± 0.31|
| 2S1            | 1+  | $\eta_c(2S)$   | 3686.093 ± 0.034| $\Upsilon(2S)$ | (31.98 ± 2.63) ± 31 |
| 2P0            | 0+  | $\chi_{c0}(2P)$| 3929 ± 7   | $\chi_{b0}(2P)$| 10232.5 ± 0.4 50 |
| 2P1            | 1+  | $\chi_{c1}(2P)$| 4025 ± 7   | $\chi_{b1}(2P)$| 10255.46 ± 0.22 50 |
| 2P2            | 2+  | $\chi_{c2}(2P)$| 4068 ± 7   | $\chi_{b2}(2P)$| 10268.65 ± 0.22 50 |
| 2P3            | 3+  | $h_c(2P)$       | 4121 ± 7   | $h_b(2P)$       |             |
| 1D1            | 1−  | $\psi(1D_{1})$ | 3775.2 ± 1.7| $\Upsilon(1D_{1})$| 10161.1 ± 0.6 16 |
| 1D2            | 2−  | $\psi(1D_{2})$ | 3775.2 ± 1.7| $\Upsilon(1D_{2})$| 10161.1 ± 0.6 16 |
| 1D3            | 3−  | $\psi(1D_{3})$ | 3775.2 ± 1.7| $\Upsilon(1D_{3})$| 10161.1 ± 0.6 16 |

In the following, we study radiative decays of heavy QQ states, both for $Q = c$ and $Q = b$, using an effective Lagrangian approach which exploits spin symmetry for heavy QQ states 27. The advantage of this method is represented by the possibility of describing radiative transitions between states belonging to the same nL multiplet to states belonging to another n'L one in terms of a single coupling constant $\alpha_n^L_{n'L'}$, allowing to use data on known transitions to predict the yet unobserved ones. Unlike the heavy-light $Q\bar{q}$ mesons, in heavy quarkonia there is no heavy flavour symmetry 28 because of the infrared divergences developed in diagrams with two static quarks exchanging gluons. Such divergences can be cured taking into account the heavy quark kinetic energy operator, which is $O(1/m_Q)$ and breaks heavy quark flavour symmetry. Because of this, in our approach it is not possible to exploit data on charmonium to obtain quantitative information on bottomonium or vice versa. However, we shall see that at a qualitative level, bottomonium sys-

though the two photon collision has never succeeded until now to produce bottomonia.

Radiative decays of quarkonia will play a role at the LHC. For example, $\chi_{cJ}$ radiative decays to $J/\psi$ will be considered by the ALICE experiment as a source of $J/\psi$ to probe $J/\psi$ suppression in central heavy ion collisions 11.

Thanks to this rich scenario of experimental facilities, several new quarkonium states have been recently discovered. Among these, some have found their proper collocation in the above classification and are included in Table I These are the charmonia $h_c$ 12, $\eta_c(2S)$ 13, $\chi_{c2}(2P)$ (initially denoted by $Z(3930)$) 14 states, and, in the beauty case, the $\eta_b(1S)$ meson 15.

Other states are still awaiting for the right interpretation, since not only their quantum numbers are not well established, but even their QQ structure is questioned. We do not discuss all of them here, but focus only on the state $X(3872)$ to which part of our analysis is devoted. This resonance was discovered by Belle Collaboration as a narrow $J/\psi\pi^+\pi^−$ mass peak in exclusive $B^− \to K^- J/\psi\pi^+\pi^−$ decay 16, and later on confirmed by CDF 17, D0 18 and BaBar 19. The analysis of the $\pi^+\pi^−$ mass distribution shows that the two pions are likely to originate from a $p^0$ decay. The subsequent measurement 20:

$$\frac{B(X \to \pi^+\pi^- p^0 J/\psi)}{B(X \to \pi^+\pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3,$$

showing evidence of G-parity (isospin) violation, has been considered as the argument against the charmonium interpretation for $X$ and in favour of other exotic interpretations, in particular the molecular one 21. However, as pointed out in 22, assuming that the three pion mode originates from the decay $X \to J/\psi \omega$, the experimental ratio reported above is mainly due to the kinematical suppression of the $J/\psi \omega$ mode, and mechanisms can be found to explain the ratio of the decay amplitudes, leaving the $c\bar{c}$ option still open. Several decay modes have been identified which might help discriminating a possible molecular structure of $X$ from the $c\bar{c}$ one, namely, decays to $\chi_{cJ}\pi(\pi)$ 22 and radiative decays to $D^0D^−\gamma$, $D^+D^−\gamma$ 24, even though the role of the latter ones is controversial 25.

If $X(3872)$ is a charmonium state, its possible quantum numbers have been discussed in 20. Among these, considering that the observation of the mode $X(3872) \to J/\psi\gamma$ 21 allows to fix $C = +1$, the most likely ones are the states $1^1D_2$ and $2^3P_1$. 

In the following, we study radiative decays of heavy QQ states, both for $Q = c$ and $Q = b$, using an effective Lagrangian approach which exploits spin symmetry for heavy QQ states 27. The advantage of this method is represented by the possibility of describing radiative transitions between states belonging to the same nL multiplet to states belonging to another n'L one in terms of a single coupling constant $\alpha_n^L_{n'L'}$, allowing to use data on known transitions to predict the yet unobserved ones. Unlike the heavy-light $Q\bar{q}$ mesons, in heavy quarkonia there is no heavy flavour symmetry 28 because of the infrared divergences developed in diagrams with two static quarks exchanging gluons. Such divergences can be cured taking into account the heavy quark kinetic energy operator, which is $O(1/m_Q)$ and breaks heavy quark flavour symmetry. Because of this, in our approach it is not possible to exploit data on charmonium to obtain quantitative information on bottomonium or vice versa. However, we shall see that at a qualitative level, bottomonium sys-
tem can help in understanding charmonium.

Our first purpose in this paper is to exploit existing data on radiative quarkonium decays (we always refer to the states in Table I) to predict unmeasured decay rates. A second purpose is to get insights on the proper identification of states whose identity is still controversial, starting from the analysis of their radiative transitions. In particular, this will be done in the case of $X(3872)$.

Our study concerns both charmonium, both bottomonium; in the latter case, we shall focus on the newly observed $η_b$ meson, the lowest lying pseudoscalar $b\bar{b}$ state, and the elusive $J^{PC} = 1^{-+} \ h_b$ state.

II. EFFECTIVE LAGRANGIAN FOR RADIATIVE TRANSITIONS OF D, P AND S WAVE STATES

Hadrons containing heavy quarks can be conveniently studied in the infinite heavy quark mass limit. It is well known that in such a limit new symmetries show up for systems containing a single heavy quark, i.e. heavy quark spin and flavour symmetries. The effective theory obtained from QCD in the heavy quark (HQ) limit and displaying such symmetries is the heavy quark effective theory (HQET), within which several advances have been obtained in heavy quark Physics [20]. In particular, due to spin symmetry, states which differ only for the orientation of the heavy quark spin with respect to the light degrees of freedom total angular momentum are expected to be degenerate in the HQ limit. Such states can be collected in multiplets being $4 \times 4$ Dirac matrices which, due to flavour symmetry, can describe charmed and beauty states.

Something similar can be done in the case of heavy quarkonia, with the limitation that flavour symmetry can no more be applied, so that each multiplet describes states with a defined heavy quark flavour. The generic expression for a multiplet with relative orbital angular momentum $L$ of the $Q\bar{Q}$ pair reads:

$$J^\mu_1...^\mu_L = \frac{1+\gamma^\mu}{2} \mu_{L+1} L + \frac{1}{\sqrt{L(L+1)}} \sum_{i=1}^{L} \epsilon^{\mu_1...\mu_L}_{\alpha} H_{L}\gamma^\mu_{L+1}...^\mu_{i+1}^\mu_{L}$$

$$+ \frac{1}{\sqrt{L}} \sum_{i=1}^{L} (\gamma^\mu_{L+1} - v^\mu_{L+1}) H_{L-1}^{\mu_1...\mu_{i-1}} H_{L}^{\mu_{i+1}...\mu_{L}}$$

$$- \frac{2}{L \sqrt{(2L-1)(2L+1)}} \sum_{i<j} \mu_{L} \gamma_5^{\mu L} 1 - \frac{\gamma^\mu}{2}$$

where $v^\mu$ is the heavy quark four-velocity and $H_A, K_A$ are the effective fields of the various members of the multiplets with total spin $J = A$. Since we consider in the following $S, P$ and $D$ wave states, it is convenient to write the corresponding multiplets obtained from (2.1):

- $L=2$ multiplet:

$$J^{\mu\nu} = \frac{1+\gamma^\mu}{2} \mu_3 \gamma_\alpha + \frac{1}{\sqrt{6}} \mu_{L} \gamma_\alpha H_{L}^{\mu_1...\mu_2 \alpha} + \frac{1}{\sqrt{6}} \mu_{L} \gamma_\alpha H_{L}^{\mu_1...\mu_2 \alpha}$$

$$+ \frac{1}{2 \sqrt{5}} [(\gamma^\mu - v^\mu) H_{L-1}^{\mu_1...\mu_2 \alpha} + (\gamma^\nu - v^\nu) H_{L-1}^{\mu_1...\mu_2 \alpha}]$$

$$- \frac{1}{2} \mu_{L} \gamma_5^{\mu L} 1 - \frac{\gamma^\mu}{2}$$

- $L=1$ multiplet:

$$J^\mu = \frac{1+\gamma^\mu}{2} \mu_2 \gamma_\alpha + \frac{1}{\sqrt{2}} \mu_{L} \gamma_\alpha H_{L}^{\mu_1...\mu_2 \alpha}$$

$$+ \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) H_0^{\mu} + K_1^{\mu} \gamma_5^{\mu} \frac{1-\gamma^\mu}{2}$$

- $L=0$ multiplet:

$$J = \frac{1+\gamma^\mu}{2} (H_{L}^{\mu_1...\mu_L \alpha} + K_2^{\mu \alpha} \gamma_5^{\mu L} \frac{1-\gamma^\mu}{2}$$

Interactions of $Q\bar{Q}$ states can be described by effective Lagrangians written in terms of the effective fields $H$ and $K$ (for a review see [20]). This can be done for the strong decays with emission of a light meson and for the radiative decays of interest here. One constructs effective Lagrangians imposing Lorentz invariance, as well as invariance under parity, charge conjugation and heavy quark spin symmetry transformations. The corresponding transformations of the multiplets are:

$$J^{\mu_1...\mu_L} \overset{P}{\leftrightarrow} \gamma^0 J^{\mu_1...\mu_L} \gamma^0$$

$$J^{\mu_1...\mu_L} \overset{C}{\leftrightarrow} (-1)^{L+1} C [J^{\mu_1...\mu_L}] T C$$
\begin{align}
J^{μ_1...μ_ℓ} S^{SU(2)_S} \rightarrow S^J \rightarrow J^{μ_1...μ_ℓ} S^{T^\dagger}.
\end{align}

In (2.7), \( S, S' \in SU(2)_S \), \( SU(2)_S \) being the group of heavy quark spin rotations, with the property: \( [S, S'] = [S', S] = 0 \).

The effective Lagrangian describing radiative transitions among members of the \( P \) wave and of the \( S \) wave multiplets has been derived in \([27]\):

\begin{align}
\mathcal{L}_{nP \rightarrow mS} = \delta_Q^{nPmS} Tr [\tilde{J}(mS)\mu_{n}(nP)] \nu_\mu F^{μν} + \text{h.c.},
\end{align}

where \( \delta_Q^{nPmS} \) \((Q = c, b)\) is a coupling constant and \( F^{μν}\) the electromagnetic field strength tensor. The validity of the description is that of the soft exchange approximation regime, when quarks are supposed to exchange gluons of limited momenta. This is expected to work for quarks of mass up to 80 GeV, as discussed in \([27]\).

Eq. (2.8) shows that a single constant \( \delta_Q^{nPmS} \) describes all the transitions among the members of the \( nP \) multiplet and those of the \( mS \) one. Indeed, the following decay widths stem from (2.8) \([27]\):

\begin{align}
\Gamma(n^3P_J \rightarrow m^3S_1) &= \frac{(\delta_Q^{nPmS})^2}{3\pi} k_3^3 M_{S_1} M_{P_J}, \\
\Gamma(m^3S_1 \rightarrow n^3P_J) &= (2J + 1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_3^3 M_{P_J} M_{S_1}, \\
\Gamma(n^1P_1 \rightarrow m^1S_0) &= \frac{(\delta_Q^{nPmS})^2}{3\pi} k_3^3 M_{S_0} M_{P_1}, \\
\Gamma(m^1S_0 \rightarrow n^1P_1) &= \frac{(\delta_Q^{nPmS})^2}{9\pi} k_3^3 M_{P_1} M_{S_0},
\end{align}

where \( k_3 \) is the photon energy.

Following the same guidelines leading to Eq. (2.8), we can construct the effective Lagrangian describing transitions among the members of the \( nD \) and the \( mP \) multiplets. Our result is:

\begin{align}
\mathcal{L}_{nD \rightarrow mP} = \delta_Q^{nPmD} Tr [\tilde{J}_μ(mP)\mu_{n}(nD)] \nu_\mu F^{μν} + \text{h.c.},
\end{align}

which allows to compute the decay widths:

\begin{align}
\Gamma(m^3D_1 \rightarrow n^3P_0) &= \frac{13 - 4\sqrt{3} (\delta_Q^{nPmD})^2}{15} k_3^3 M_{P_D}, \\
\Gamma(m^3D_1 \rightarrow n^3P_1) &= \frac{7 + 4\sqrt{3} (\delta_Q^{nPmD})^2}{20} k_3^3 M_{P_D}, \\
\Gamma(m^3D_1 \rightarrow n^3P_2) &= \frac{7 - 4\sqrt{3} (\delta_Q^{nPmD})^2}{12} k_3^3 M_{P_D},
\end{align}

and

\begin{align}
\Gamma(m^1D_2 \rightarrow n^1P_1) &= \frac{(\delta_Q^{nPmD})^2}{3\pi} k_3^3 M_{P_D}, \\
\Gamma(m^3D_2 \rightarrow n^3P_1) &= \frac{(\delta_Q^{nPmD})^2}{4\pi} k_3^3 M_{P_D}, \\
\Gamma(m^3D_2 \rightarrow n^3P_2) &= \frac{(\delta_Q^{nPmD})^2}{12\pi} k_3^3 M_{P_D},
\end{align}

in terms of a single new coupling constant \( \delta_Q^{nPmD} \). We do not consider the decays of the \( 3D_3 \) state, which proceed in \( D \) wave and therefore are not described by the Lagrangian (2.10).

In the following, we make use of these results to study radiative transitions of charmonia and bottomonia.

### III. RADIATIVE TRANSITIONS OF P AND S WAVE STATES

We exploit the above results to systematically analyse some radiative transitions among the states appearing in Table II. Some of the predictions stemming from the Lagrangian (2.8) have been already obtained in \([27]\), in such cases we have exploited new or more recent data.

#### A. 1P \( \rightarrow 1S \) transitions

The widths of the decay modes \( \chi_{cJ}(1P) \rightarrow J/ψ \gamma \) can be obtained from the first equation in (2.9). Experimental data are available for the three modes and the accuracy of spin-symmetry, which predicts that the transitions \( \chi_{cJ}(1P) \rightarrow J/ψ \gamma \) are all governed by the same coupling constant \( \delta^{1P1S}_c \), can be tested. Using \([3]\):

\begin{align}
B(\chi_{c0}(1P) \rightarrow J/ψ \gamma) &= (1.28 \pm 0.11) \times 10^{-2} \\
B(\chi_{c1}(1P) \rightarrow J/ψ \gamma) &= (36.0 \pm 1.9) \times 10^{-2} \\
B(\chi_{c2}(1P) \rightarrow J/ψ \gamma) &= (20.0 \pm 1.0) \times 10^{-2}
\end{align}

together with the \( \chi_{cJ} \) full widths in Table II we obtain:

\( \delta^{1P1S}_c = 0.227 \pm 0.013 \text{ GeV}^{-1} \), \( \delta^{1P1S}_c = 0.241 \pm 0.009 \text{ GeV}^{-1} \), and \( \delta^{1P1S}_c = 0.233 \pm 0.010 \text{ GeV}^{-1} \), respectively. Therefore, it is correct to describe all these modes in terms of a single constant, the average value of which is:

\begin{align}
\delta^{1P1S}_c = 0.235 \pm 0.006 \text{ GeV}^{-1}.
\end{align}

The same coupling \( \delta^{1P1S}_c \) also governs the decay \( h_c(1P) \rightarrow η_c(1P) \), which has been observed but no measurement of the rate has been determined, yet. Using the result (3.2) and the third equation in (2.9), we predict:

\begin{align}
\Gamma(h_c(1P) \rightarrow η_c(1P) \gamma) = 634 \pm 32 \text{ KeV}.
\end{align}

This result compares favourably with that obtained by Voloshin \([2]\): \( \Gamma(h_c(1P) \rightarrow η_c(1P) \gamma) \approx 0.65 \text{ MeV} \), derived assuming the equality of the radial wave function overlap integrals and exploiting the data on \( \chi_{cJ} \rightarrow J/ψ \gamma \) decays, a procedure similar to the one adopted here. The outcome in (3.3) also agrees with the result in \([31]\). Agreement is also met with lattice QCD \([32]\), whose predicted rate depends on the use of lattice masses or of physical masses: \( \Gamma(h_c(1P) \rightarrow η_c(1P) \gamma) = 663 \pm 132 \text{ KeV} \) or \( \Gamma(h_c(1P) \rightarrow η_c(1P) \gamma) = 601 \pm 55 \text{ KeV} \), respectively. Notice that the estimated uncertainty is sizeably larger than in (3.3).
For the corresponding beauty states \( \chi_{bJ}(1P) \), the available measurements \( [5] \):

\[
B(\chi_{b0}(1P) \rightarrow \Upsilon(1S) \gamma) < 6 \times 10^{-2}
\]
\[
B(\chi_{b1}(1P) \rightarrow \Upsilon(1S) \gamma) = (35 \pm 8) \times 10^{-2} \quad (3.4)
\]
\[
B(\chi_{b2}(1P) \rightarrow \Upsilon(1S) \gamma) = (22 \pm 4) \times 10^{-2}
\]
do not allow us to determine \( \delta_{b}^{1P1S} \) without a measurement of the full width of a \( \chi_{bJ}(1P) \) state. Nevertheless, it is useful to study these processes as functions of the ratio \( r = \frac{\delta_{b}^{1P1S}}{\delta_{c}^{1S1P}} \) of the couplings, with the result plotted in Fig. 1. We expect that the ratio \( r \) is smaller than one, since it includes the ratio of the beauty and the charm quark electric charges: \( \frac{e_{b}}{e_{c}} \), as well as the effect of the inverse heavy quark mass in each coupling \( \delta_{b}^{1S1P} \).

As a reference, we obtain that, at \( r = 0.5 \), \( \Gamma(\chi_{b0}(1P) \rightarrow \Upsilon(1S) \gamma) = 85 \pm 4 \text{ KeV} \), \( \Gamma(\chi_{b1}(1P) \rightarrow \Upsilon(1S) \gamma) = 107 \pm 5 \text{ KeV} \) and \( \Gamma(\chi_{b2}(1P) \rightarrow \Upsilon(1S) \gamma) = 121 \pm 6 \text{ KeV} \). Notice that, once the value of \( r \) in one decay mode has been determined, the prediction for all the others follows.

The same procedure can be applied to the channel \( h_{b}(1P) \rightarrow \eta_{b}(1S) \gamma \), a mode to access the recently discovered \( \eta_{b} \) and to detect the still unseen \( h_{b} \). We fix the \( h_{b} \) mass to the center of gravity of the \( \chi_{bJ} \) states: \( M_{h_{b}} = \frac{M_{\chi_{b0}} + 3M_{\chi_{b1}} + 5M_{\chi_{b2}}}{9} = 9.89989 \text{ GeV} \), an assumption supported by the corresponding measurements in the charm sector; we obtain the result in Fig. 2 which shows that for \( r = 0.5 \) this mode should have a width \( \Gamma(h_{b}(1P) \rightarrow \eta_{b}(1S) \gamma) = 271 \pm 14 \text{ KeV} \).

### B. \( 2S \rightarrow 1P \) transitions

These transitions are described by the second equation in (29). The measured branching fractions:

\[
B(\psi(2S) \rightarrow \chi_{c0}(1P) \gamma) = (9.4 \pm 0.4) \times 10^{-2}
\]
\[
B(\psi(2S) \rightarrow \chi_{c1}(1P) \gamma) = (8.8 \pm 0.4) \times 10^{-2} \quad (3.5)
\]
\[
B(\psi(2S) \rightarrow \chi_{c2}(1P) \gamma) = (8.3 \pm 0.4) \times 10^{-2}
\]

together with the measurement of \( \Gamma(\psi(2S)) \) reported in Table 1 permit to obtain \( \delta_{c}^{2S1P} = 0.215 \pm 0.007 \text{ GeV}^{-1} \), \( \delta_{c}^{2S1P} = 0.223 \pm 0.008 \text{ GeV}^{-1} \), \( \delta_{c}^{2S1P} = 0.258 \pm 0.009 \text{ GeV}^{-1} \), and the average value:

\[
\delta_{c}^{2S1P} = 0.228 \pm 0.005 \text{ GeV}^{-1}. \quad (3.6)
\]

This value is close to that obtained for \( \delta_{c}^{1S1P} \), Eq. (3.2), in analogy to the outcome in (2) for the corresponding radial wave function overlap integrals.

The result (3.6) allows us to predict the decay width and the branching ratio of the mode \( \eta_{c}(2S) \rightarrow h_{c}(1P) \gamma \):

\[
\Gamma(\eta_{c}(2S) \rightarrow h_{c}(1P) \gamma) = 21.1 \pm 0.9 \text{ KeV}
\]
\[
B(\eta_{c}(2S) \rightarrow h_{c}(1P) \gamma) = (0.15 \pm 0.08) \times 10^{-2} \quad (3.7)
\]

This mode is interesting because it represents another channel to study the poorly known state \( h_{c}(1P) \). However, since the branching ratio turns out to be tiny, the observation is challenging.

For the corresponding states with beauty, the following data are available \( [5] \):

\[
B(\Upsilon(2S) \rightarrow \chi_{b0}(1P) \gamma) = (3.8 \pm 0.4) \times 10^{-2}
\]
\[
B(\Upsilon(2S) \rightarrow \chi_{b1}(1P) \gamma) = (6.9 \pm 0.4) \times 10^{-2} \quad (3.8)
\]
\[
B(\Upsilon(2S) \rightarrow \chi_{b2}(1P) \gamma) = (7.15 \pm 0.35) \times 10^{-2}
\]
Using the $\Upsilon(2S)$ width in Table I, we get:
\[ \delta_b^{2S1P} = 0.097 \pm 0.003 \text{ GeV}^{-1}, \]
which can be used to predict the decay width of the process $\eta_b(2S) \to h_b(1P) \gamma$. In Fig. 3 we show the result as a function of the unknown mass of $\eta_b(2S)$, which we varied in a range obtained considering the maximum and minimum value of the theoretical determinations of the mass splitting $M_{\Upsilon(2S)} - M_{\eta_b(2S)}$ [32]. Although the rate turns out to be small, this mode can be considered as a possible channel to detect $h_b$.

![Graph showing $\Gamma(h_b \to \eta_\gamma)$ (KeV) as a function of the ratio of effective couplings $r = \delta_b^{P1S}/\delta_b^{1P1S}$.

![Graph showing $\Gamma(\eta_b(2S) \to h_b \gamma)$ (KeV) versus $M_{\eta_b(2S)}$ (GeV).

C. $2P \to 1S, 2S$ transitions

As mentioned in the introduction, Belle Collaboration has observed a state, $Z(3930)$, in $\gamma \gamma$ collision and decaying to $DD$, which can be most naturally identified with $\chi_{c2}(2P)$ [14]. For this state no radiative mode of the kind considered here has been detected, yet [37]. On the other hand, radiative branching fractions of the $\chi_{bJ}(2P)$ to $\Upsilon(1S) \gamma$ and to $\Upsilon(2S) \gamma$ have been measured [3], although this piece of information is not enough to determine the couplings $\delta_b^{P1S}$ and $\delta_b^{2P2S}$ without the measurement of the $\chi_{bJ}(2P)$ full widths. However, it is interesting to consider the ratios:

\[ R_{ij}^{(b)} = \frac{\Gamma(\chi_{bJ}(2P) \to \Upsilon(2S) \gamma)}{\Gamma(\chi_{bJ}(2P) \to \Upsilon(1S) \gamma)}. \]

We use [3]:

- $B(\chi_{b0}(2P) \to \Upsilon(1S) \gamma) = (9 \pm 6) \times 10^{-3}$
- $B(\chi_{b0}(2P) \to \Upsilon(2S) \gamma) = (4.6 \pm 2.1) \times 10^{-2}$
- $B(\chi_{b1}(2P) \to \Upsilon(1S) \gamma) = (8.5 \pm 1.3) \times 10^{-2}$
- $B(\chi_{b1}(2P) \to \Upsilon(2S) \gamma) = (21 \pm 4) \times 10^{-2}$
- $B(\chi_{b2}(2P) \to \Upsilon(1S) \gamma) = (7.1 \pm 1.0) \times 10^{-2}$
- $B(\chi_{b2}(2P) \to \Upsilon(2S) \gamma) = (16.2 \pm 2.4) \times 10^{-2}$

which in turn provide:

\[ R_0^{(b)} = 5.1 \pm 4.1, \quad R_1^{(b)} = 2.5 \pm 0.6, \quad R_2^{(b)} = 2.3 \pm 0.5. \]

From these ratios we can extract the ratio of the coupling constants $R_5^{(b)} = \frac{\delta_b^{2S1P}}{\delta_b^{2S2S}}$:

\[ R_5^{(b)} = 15 \pm 6 \\
R_5^{(b)} = 9.3 \pm 1.1 \\
R_5^{(b)} = 8.4 \pm 0.9 \]

from $\chi_{b0}(2P)$, $\chi_{b1}(2P)$ and $\chi_{b2}(2P)$ decays, respectively. These results show that also in this case spin symmetry is fulfilled, even though in the case of $\chi_{b0}(2P)$ the error affecting the result is large. The average value is:

\[ R_5^{(b)} = 8.8 \pm 0.7. \]

It is reasonable that, even though the coupling might be different passing from the beauty to the charm sector, the ratios of the couplings stay stable. Adopting such an assumption, one can predict the corresponding ratios for $\chi_{cJ}(2P)$ states using the result [4.14]:

\[ R_2^{(c)} = \frac{\Gamma(\chi_{c2}(2P) \to \psi(2S) \gamma)}{\Gamma(\chi_{c2}(2P) \to \psi(1S) \gamma)} = 2.95 \pm 0.5. \]

This prediction can be tested when new experimental data will be available and can be used to support the identification of $Z(3930)$ with $\chi_{c2}(2P)$.

Interesting considerations stem for the case $J = 1$. Actually, among the canonical interpretations proposed for the puzzling state $X(3872)$, a possible one is the identification with $\chi_{c1}(2P)$. An important piece of experimental information concerning $X(3872)$ is represented by the
two measurements \[34\]

\[
\begin{align*}
B(B^+ \to XK^+, X \to J/\psi \gamma) &= (2.8 \pm 0.8 \pm 0.2) \times 10^{-6} \\
B(B^+ \to XK^+, X \to \psi(2S) \gamma) &= (9.9 \pm 2.9 \pm 0.6) \times 10^{-6} 
\end{align*}
\]  

(3.16)

from which one has:

\[
R_X = \frac{\Gamma(X(3872) \to \psi(2S) \gamma)}{\Gamma(X(3872) \to \psi(1S) \gamma)} = 3.5 \pm 1.4. 
\]  

(3.17)

If \(X(3872)\) is identified as \(\chi_{c1}(2P)\), the above ratio \(R_X\) can be computed in our framework, as done in \[3.15\] in the case of \(Z(3930)\). The result is:

\[
R_1^{(c)} = \frac{\Gamma(\chi_{c1}(2P) \to \psi(2S) \gamma)}{\Gamma(\chi_{c1}(2P) \to \psi(1S) \gamma)} = 1.64 \pm 0.25. 
\]  

(3.18)

In view of the underlying approximation, i.e. the equality of the ratio of the couplings in the beauty and in the charm sector, we find that the experimental value in \[3.17\] and the theoretical prediction obtained in the hypothesis \(X(3872) = \chi_{c1}(2P)\) are close enough to consider this assumption plausible. This should be contrasted to the composite scenarios, in which the mode \(X(3872) \to \psi(2S) \gamma\) turns out to be suppressed compared to \(X(3872) \to \psi(1S) \gamma\) \[3.20\].

IV. RADIATIVE TRANSITIONS OF D WAVE STATES

Experimental data on radiative transitions of \(D\) wave states exist in the case of \(\psi(3770)\), usually identified with the state \(1^3D_1\) \[38\]. The following branching fractions are available:

\[
\begin{align*}
B(\psi(3770) \to \chi_{c0}(1P) \gamma) &= (7.3 \pm 0.9) \times 10^{-3} \\
B(\psi(3770) \to \chi_{c1}(1P) \gamma) &= (2.9 \pm 0.6) \times 10^{-3}. 
\end{align*}
\]  

(4.1)

From these data we can extract the value of the coupling \(\delta_{\psi D1P}\). The average value obtained from the two modes above is:

\[
\delta_{\psi D1P} = 0.32 \pm 0.02 \text{ GeV}^{-1}. 
\]  

(4.2)

This result allows us to predict width and branching ratio of the third available radiative mode for \(\psi(3770)\):

\[
\begin{align*}
\Gamma(\psi(3770) \to \chi_{c2}(1P) \gamma) &= 0.59 \pm 0.06 \text{ KeV} \\
B(\psi(3770) \to \chi_{c2}(1P) \gamma) &= (2.1 \pm 0.2) \times 10^{-5}. 
\end{align*}
\]  

(4.3)

to be compared to the experimental upper bound \(B(\psi(3770) \to \chi_{c2}(1P) \gamma) < 9 \times 10^{-4}\). For comparison, using two variants of the potential model, Barnes et al. \[35\] find \(\Gamma(\psi(3770) \to \chi_{c2}(1P) \gamma) = 3.3 \text{ KeV}\) or \(\Gamma(\psi(3770) \to \chi_{c2}(1P) \gamma) = 4.9 \text{ KeV}\), corresponding to the branching fraction \(B(\psi(3770) \to \chi_{c2}(1P) \gamma) = 1.2 \times 10^{-4}\) or \(B(\psi(3770) \to \chi_{c2}(1P) \gamma) = 1.8 \times 10^{-4}\), respectively, so that the prediction based on spin symmetry is different.

The same coupling governs all the transitions of the members of the \(1D\) multiplet to the members of the \(1P\) one. Allowed decay modes are:

\[
\begin{align*}
1^3D_2 &\to \chi_{c0,1,2}(1P) \gamma \\
1^3D_2 &\to \chi_{c1,2}(1P) \gamma \\
1^3D_2 &\to h_c(1P) \gamma. 
\end{align*}
\]  

(4.4)

In the case of \(1^3D_2\) the decay to \(\chi_{c0}\) is forbidden.

Analysing these modes is interesting not only per se, but also in view of the already mentioned possibility that \(X(3872)\) might be identified with the state \(1^3D_2\). For this purpose, in Fig. 4 we plot \(\Gamma(1^1D_2 \to h_c \gamma)\) versus \(M(1^1D_2)\). If \(X\) coincides with the \(1^1D_2\) state, hence \(M(1^1D_2) = 3872\) MeV, we find that \(\Gamma(1^1D_2 \to h_c \gamma) = 359 \pm 40\) KeV, while, if we use the masses reported in \[32\], i.e. \(\Gamma(1^1D_2) = 3799\) MeV or \(M(1^1D_2) = 3837\) MeV (depending on the variant of the potential model), we find: \(\Gamma(1^1D_2 \to h_c \gamma) = 185 \pm 20\) KeV or \(\Gamma(1^1D_2 \to h_c \gamma) = 267 \pm 30\) KeV, to be compared to the results in \[32\]: \(\Gamma(1^1D_2 \to h_c \gamma) = 339\) KeV or \(\Gamma(1^1D_2 \to h_c \gamma) = 344\) KeV. In all cases, the decay width to \(h_c\) is rather sizeable.

FIG. 4: \(\Gamma(1^1D_2 \to h_c \gamma) \) (KeV) versus the mass of the \(1^1D_2\) state (in GeV).

The same analysis can be carried out in the case of the state \(1^3D_2\), which was initially proposed as a possible identification for \(X(3872)\), but is now ruled out because of the C-parity of this state (opposite to the one fixed for \(X\)). We compute the decay widths to \(\chi_{c1}\) and \(\chi_{c2}\) as a function of \(M(1^3D_2)\), as plotted in Fig. 5 and, in particular, in correspondence to the masses reported in \[32\]: \(M(1^3D_2) = 3800\) MeV or \(M(1^3D_2) = 3838\) MeV, finding: \(\Gamma(1^3D_2 \to \chi_{c1} \gamma) = 163 \pm 18\) KeV or \(\Gamma(1^3D_2 \to \chi_{c1} \gamma) = 230 \pm 25\) KeV and \(\Gamma(1^3D_2 \to \chi_{c2} \gamma) = 34 \pm 4\) KeV or \(\Gamma(1^3D_2 \to \chi_{c2} \gamma) = 51 \pm 6\) KeV. For comparison, in \[35\] the following results are obtained: \(\Gamma(1^3D_2 \to \chi_{c1} \gamma) = \ldots\).
307 KeV or $\Gamma(1^3D_2 \to \chi_{c1}\gamma) = 268$ KeV and $\Gamma(1^3D_2 \to \chi_{c2}\gamma) = 64$ KeV or $\Gamma(1^3D_2 \to \chi_{c2}\gamma) = 66$ KeV.

In the beauty sector, we have considered some modes involving the $\eta_b$ and $h_b$ states, among which only the $\eta_b(1S)$ has been recently discovered. We find that the mode $h_b \to \eta_b\gamma$ could be detectable. As for the production of $h_b$ in $\eta_b(2S)$ radiative decay, we predict a small rate.

V. CONCLUSIONS

We have analysed radiative decays of several $c\bar{c}$ and $b\bar{b}$ states, using an effective Lagrangian approach valid for heavy quarkonia. Exploiting existing data has allowed us to derive model independent predictions on channels related by the heavy quark spin symmetry. When available, experimental data are consistent with the description based on this symmetry.

We have also considered the case of $X(3872)$, finding that the observed radiative modes are compatible with the identification of this state with $\chi_{c1}(2P)$. As for its identification with the $1^1D_2$ state, we have predicted the decay rate of $1^1D_2 \to h_c\gamma$ as a function of $M(1^1D_2)$ and, in particular, for $M(1^1D_2) = 3872$ MeV. The observation of this decay for $X(3872)$ in agreement/disagreement with such a prediction would support/discard this option. This mode is anyway interesting, representing another channel to access the state $h_c$, one of the newly confirmed charmonium states.

FIG. 5: Widths $\Gamma(1^3D_2 \to \chi_{c1}\gamma)$ (KeV) (a) and $\Gamma(1^3D_2 \to \chi_{c2}\gamma)$ (KeV) (b) versus the mass of the $1^3D_2$ state (in GeV).

Acknowledgments
I thank P. Colangelo and T.N. Pham for reading this manuscript and for discussions. This work was supported in part by the EU contract No. MRTN-CT-2006-035482, "FLAVIAnet".
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