On the relative positions of the 2$\Delta$ peaks in Raman and tunneling spectra of $d$–wave superconductors.

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We study $B_{1g}$ Raman intensity $R(\Omega)$ and the density of states $N(\omega)$ in isotropic 2D $d$–wave superconductors. For an ideal gas, $R(\Omega)$ and $N(\omega)$ have sharp peaks at $\Omega = 2\Delta$ and $\omega = \Delta$, respectively, where $\Delta$ is the maximum value of the gap. We study how the peak positions are affected by the fermionic damping due to impurity scattering. We show that while the damping generally shifts the peak positions to larger frequencies, the peak in $R(\Omega)$ still occurs at almost twice the peak position in $N(\omega)$ and therefore cannot account for the experimentally observed downturn shift of the peak frequency in $R(\Omega)$ in underdoped cuprates compared to twice that in $N(\omega)$. We also discuss how the fermionic damping affects the dynamical spin susceptibility.

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The unusual physical properties of cuprate superconductors have continued to be of high interest to condensed matter physicists for more than a decade. In recent years, a lot of attention was devoted to the study of collective bosonic excitations in the superconducting and pseudogap phases \[ \text{[10,11]} \]. The most notable experimental observation is the discovery of the resonance mode in the dynamical spin susceptibility \[ \text{[12,13]} \]. This mode is centered at the antiferromagnetic momentum $Q = (\pi, \pi)$, and physically reflects the fact that near antiferromagnetic instability, collective spin excitations in a $d$-wave superconductor are undamped, propagating spin waves at energies smaller than 2$\Delta$ \[ \text{[14]} \]. Less attention is devoted to the study of possible resonance bosonic excitations at zero momentum transfer. These excitations are probed by Raman scattering which generally measures the imaginary part of the fully renormalized particle-hole susceptibility at vanishingly small incoming momentum, weighted with Raman form factors which depend on the scattering geometry \[ \text{[15,16]} \]. The experiments relevant to our discussion were performed in $B_{1g}$ geometry where the Raman form factors are the largest for fermionic momenta near $(0, \pi)$ and symmetry related points where the $d_{x^2-y^2}$ gap $\Delta(k)$ is near its maximum $\Delta$ \[ \text{[17,18]} \].

In a BCS theory for a $d$–wave superconductor $B_{1g}$ Raman intensity $R(\Omega)$ logarhythmically diverges at 2$\Delta$ and rapidly, as $\omega^3$, decreases at smaller frequencies \[ [19] \]. Experimental data for overdoped $B/2212$ are qualitatively consistent with this behavior \[ [20] \]. Furthermore, the 2$\Delta$ extracted from $R(\Omega)$ is almost exactly twice the gap extracted from SIN tunneling data, which measure a single particle density of states (DOS) $N(\omega)$ \[ [21] \]. With underdoping, however, the peak frequency in $R(\Omega)$ progressively deviates down from the 2$\Delta$ extracted from the tunneling experiments \[ [22] \].

Blumberg, Morr and one of us (CBM) \[ [13] \] attributed this deviation to a final state interaction between scattered quasiparticles. They argued that the magnetically mediated final state interaction in $B_{1g}$ geometry is attractive and gives rise to a pseudo-resonance in $R(\Omega)$ at a frequency $\Omega_{res}$, which with underdoping progressively deviates down from 2$\Delta$.

An alternative to the resonance mode scenario is one in which the final state interaction is irrelevant, and the discrepancy between Raman and tunneling data is due to fermionic incoherence, which generally shifts the positions of both the Raman peak and the peak in the DOS. In this paper, we show that the shifts in the peak positions of $R(\Omega)$ and $N(\omega)$ due to fermionic damping are correlated such that without final state interaction, the peak in $R(\Omega)$ is still located at almost exactly twice the peak frequency in $N(\omega)$. This result implies that the experimentally observed relative downturn deviation of the peak in the Raman intensity cannot be explained by purely fermionic self-energy effects, and leaves the resonance mode scenario as the most probable one.

We begin with the general expressions for $R(\Omega)$ and $N(\omega)$ in a superconductor. The DOS is the imaginary part of the fully renormalized particle-hole susceptibility at vanishingly small incoming momentum, weighted with Raman form factors which depend on the scattering geometry \[ [10,11] \]. The experiments relevant to our discussion were performed in $B_{1g}$ geometry where the Raman form factors are the largest for fermionic momenta near $(0, \pi)$ and symmetry related points where the $d_{x^2-y^2}$ gap $\Delta(k)$ is near its maximum $\Delta$ \[ [17,18] \].

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In an ideal gas, the fermionic self-energy is absent (i.e., \( \Sigma(k, \omega) = \omega \)). To simplify the discussion, we assume that the Fermi surface is circular, for which both \( R(\Omega) \) and \( N(\omega) \) can be evaluated exactly \[13\]. Substituting the momentum integration by integration over \( d \xi_{k} \), and approximating the \( k \)-dependences of \( V_{B_{1g}}(k) \) and \( \Delta_{k} \) by \( \cos \Theta \), we obtain for \( \Omega \), \( \bar{\omega} < 1 \)

\[
R(\bar{\Omega}) = \frac{3\pi}{16} \bar{\Omega}^{3} F(\frac{1}{2}, \frac{5}{2}, \frac{3}{2}, \bar{\Omega}^{2}), \quad N(\bar{\omega}) = \frac{2}{\pi} \bar{\omega} K(\bar{\omega}^{2}),
\]

and for \( \Omega \), \( \bar{\omega} > 1 \)

\[
R(\Omega) = \frac{3\pi}{16\Omega^{3}} F(\frac{1}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{\Omega^{2}}), \quad N(\omega) = \frac{2}{\pi} K\left(\frac{1}{\omega^{2}}\right),
\]

where \( \bar{\omega} = \omega/\Delta \), \( \bar{\Omega} = \Omega/(2\Delta) \), and \( F(a, b, c, x) \) and \( K(x) \) are hypergeometric and elliptical functions, respectively.

Our goal is to study how the peak positions and, more generally, the functional forms of \( R(\Omega) \) and \( N(\omega) \) are affected by the fermionic self-energy. In general, the fermionic self-energy comes from various sources, and at least part of it, associated with the scattering by the same bosonic excitations which give rise to superconductivity, has to be determined fully self-consistently from the Eliashberg-type equations \[8\]. In this paper, we assume for simplicity that the primary source for the fermionic damping is impurity scattering. We consider the fermionic self-energy in the self-consistent \( T \)-matrix formalism \[17\] and neglect subtle 2D effects beyond \( T \)-matrix approximation \[18\]. Hirshfeld, Wolffe and Einzel \[17\] demonstrated that for a \( d \)-wave superconductor with a particle-hole symmetry and \( k \)-independent scattering potential, \( \Sigma(k, \omega) = 0 = i \gamma_{\omega} \text{sign}(\omega) \), where \( \gamma_{\omega} \) is a solution of the self-consistent equation \[17\] \[16\]

\[
\gamma_{\omega} = \frac{g_{0}(\gamma_{\omega})}{c^{2} + g_{0}(\gamma_{\omega})}.
\]

Here \( \gamma \) is proportional to the impurity concentration, \( c \) is the cotangent of the scattering phase shift, and \( g_{0}(\gamma_{\omega}) = (i/\pi N_{F}) \sum_{k} G_{sc}(k, \omega) \text{sign}(\omega) \), where \( N_{F} \) is the normal state DOS at the Fermi surface. The self-consistency of \[1\] is in the fact that \( G_{sc}(k, \omega) \) given by \[2\] by itself depends on \( \gamma(\omega) \) through \( \Sigma(\omega) \). Physically this means that the fermionic self-energy due to impurity scattering is itself affected by a superconductivity.

For a circular Fermi surface, the momentum integral over \( G(k, \omega) \) can be performed exactly and yields \( g_{0}(\gamma_{\omega}) = (2/\pi) K(\Delta^{2}/\Sigma^{2}(\omega)) \). In the normal state, \( g_{0}(\gamma_{\omega}) = 1 \), and \( \gamma_{\omega} \) reduces to a constant \( \gamma_{\omega} = \gamma/(1 + c^{2}) \). In a superconducting state, however, \( \gamma_{\omega} \) is complex and frequency dependent. Still, one can easily demonstrate that \( \gamma_{\omega} \) remains finite for all frequencies. Below we will need \( \gamma_{0} \) and \( \gamma_{\Delta} \). For unitary scattering \((c=0)\), we solved \[3\] these frequencies and for \( \gamma \ll \Delta \) obtained with logarithmic accuracy \( \gamma_{\Delta} = (2/\pi) \gamma \log(\Delta/\gamma) \) and \( \gamma_{0} = (\pi\gamma \Delta/\log(\Delta/\gamma))^{1/2} \).

We now proceed with the calculations of the DOS and Raman intensity. As the self-energy is \( k \)-independent, we can use the same trick as in earlier studies \[10\] \[22\], and first integrate over momenta in \[1\]. Substituting the momentum integration by the integration over \( \epsilon_{k} \) and evaluating the integrals, we obtain \[13\]

\[
N(\omega) = Im \int_{0}^{\pi/2} d \Theta \frac{\Sigma(\omega)}{D(\omega)}, \quad R(\Omega) = -Re \int_{0}^{\pi/2} d \Theta \cos^{2} \Theta \times \int_{-\infty}^{\infty} d \omega \frac{(\Sigma_{+} - \Sigma_{-})^{2} + (D_{+} - D_{-})^{2}}{4D_{+}D_{-}(D_{+} + D_{-})^{2}}.
\]

Here \( \omega_{\pm} = \omega \pm \Omega/2, \Sigma_{\pm} = \Sigma(\omega_{\pm}), D_{\pm} = D(\omega_{\pm}) \), and \( D(\omega) = \sqrt{\Delta^{2} \cos^{2} \Theta - \Sigma^{2}(\omega)} \).

As a warm up, consider the limit of small frequencies. Substituting \( \Sigma(\omega) = \omega + i \gamma_{0} \text{sign}(\omega) \) into \[1\] and expanding in frequency, we obtained

![FIG. 1. The behavior of the Raman intensity \( R(\Omega) \) and the DOS \( N(\omega) \) in \( d \)-wave superconductors. a), b) Fermi-gas results (\( \gamma = 0 \)). Solid lines - \( d \)-wave results, dashed lines - \( s \)-wave results shown for comparison. The insert shows the behavior of \( R(\Omega) \) close to the threshold frequency \( 2\Delta \). c) The results for \( \gamma = 0.2\Delta \) (Eq. \[3\]). The normalized frequencies are \( \bar{\omega} = \omega/\Delta \) and \( \bar{\Omega} = \Omega/(2\Delta) \). Observe that for \( d \)-wave superconductors, the peak in the Raman intensity is located at larger normalized frequency than the peak in the density of states.](image-url)
\( R(\bar{\Omega}) = -\bar{\Omega} \bar{\gamma}_0^2 \log 1/\gamma_0 + O(\bar{\Omega}^2) \)
\( N(\bar{\omega}) = N(0) + \bar{\omega} \left( 1 - \frac{2}{\pi} \tan^{-1} \frac{\gamma_0}{\bar{\omega}} \right) - \frac{2\gamma_0}{\pi} \log \sqrt{1 + \left( \frac{\bar{\omega}}{\gamma_0} \right)^2} \) \( (7) \)

\begin{align*}
\text{FIG. 2. The behavior of the spin polarization operator in} \\
\text{the Fermi gas (a) and at a finite } \gamma \text{ (b). Solid line} - \text{Re} \Pi_Q(\Omega), \\
\text{dashed line} - \text{Im} \Pi_Q(\Omega). \\
\end{align*}

where \( N(0) = \frac{2}{\pi} \gamma_0 \log 1/\gamma_0 \), and \( \gamma_0 = \gamma_0/\Delta \). At \( \bar{\omega} \ll 1 \), \( N(\bar{\omega}) \approx N(0) + \bar{\omega}^2/\pi \gamma_0 \).

We see that fermionic damping (i) yields a linear frequency dependence of \( R(\Omega) \), and (ii) yields a finite DOS at zero frequency, and the quadratic frequency dependence of \( N(\omega) \) above \( N(0) \). Both of these results agree with earlier studies [17,16,19].

We now consider \( R(\bar{\Omega}) \) and at \( \Omega \approx 2\Delta \) and \( \omega \approx \Delta \) where the Raman intensity and the DOS diverge in a Fermi gas. Simple estimates show that near \( 2\Delta \), the integral in \( R(\Omega) \) is dominated by \( D_{\pm} \approx 1 \) i.e. \( \omega_{\pm} \approx \pm \Delta \).

Substituting \( \Sigma(\omega \approx \Delta) = \omega + i\gamma_\Delta \) sign(\( \omega \)) into (5) and expanding the integrands to first order in \( \Omega - 2\Delta \) and \( \omega - \Delta \) respectively, we obtain after lengthy but straightforward calculations

\begin{align*}
R(\bar{\omega}) = & \frac{\pi}{4\sqrt{2}} \text{Re} \left[ \frac{F \left( \frac{1}{2}, \frac{1}{2}, 2, -\frac{\bar{\omega} - \gamma_\Delta}{\bar{\omega} - 1 + i\gamma_\Delta} \right)}{\sqrt{\bar{\omega} - 1 + i\gamma_\Delta}} \right] ; \\
N(\bar{\omega}) = & \frac{\sqrt{2}}{\pi} \text{Re} \left[ \frac{K \left( -\frac{\bar{\omega}}{\sqrt{\bar{\omega} - 1 + i\gamma_\Delta}} \right)}{\sqrt{\bar{\omega} - 1 + i\gamma_\Delta}} \right] \tag{8} \end{align*}

Here \( \gamma_\Delta = \gamma_\Delta/\Delta \).

We now analyse these results. Near \( \bar{\Omega} = \bar{\omega} = 1 \), we find from (8) \( R(\Omega) \approx (-1/8) \log[(\Omega - 1)^2 + \gamma_\Delta^2] \), \( N(\bar{\omega}) \approx (-1/4\pi) \log[(\bar{\omega} - 1)^2 + \gamma_\Delta^2] \). We see that the logarithmical divergencies are cut by the fermionic damping, but, to a logarithmical accuracy, the peaks remain at the same positions as in a Fermi gas. In other words, fermionic damping gives rise to a broadening of the peaks in the Raman intensity and the DOS, but still, the peak in \( R(\Omega) \) is located at twice the peak frequency of the DOS.

Calculations beyond the logarithmical accuracy show that the peak positions do shift to higher frequencies but the relative shift is opposite to the one detected in the experiments: the peak in \( R(\Omega) \) shifts to higher frequencies for arbitrary \( \gamma_\Delta \) (\( \Omega_{\text{peak}} - 1 \approx \gamma_\Delta^2 \) for \( \gamma_\Delta \ll 1 \), while the peak in \( N(\bar{\omega}) \) shifts to high frequencies only if the damping exceeds a threshold value of \( \gamma_\Delta \approx 0.77 \). Obviously, the magnitude of the shift in \( N(\omega) \) is smaller than that in \( R(\bar{\Omega}) \). The behavior of \( N(\bar{\omega}) \) and \( R(\bar{\Omega}) \) is illustrated in Fig. 5c.

Note in passing that for \( s- \)wave superconductors, the the same calculations which lead to (8) yield

\[
R(x), N(x) \sim \left( \frac{\sqrt{1 + \alpha_x^2} + \alpha_x}{1 + \alpha_x^2 \gamma_\Delta} \right) \frac{1}{2} \tag{9}
\]

where \( \alpha_x = (x - 1)/(\gamma_\Delta) \) and \( x = \bar{\Omega} \) for \( R(x) \) and \( x = \bar{\omega} \) for \( N(x) \). Again, the divergencies at \( \Omega = \bar{\omega} = 1 \) are gone and substituted by the peaks at higher frequencies for which \( \alpha_x = \frac{1}{2\sqrt{\Delta}} \). At the same time, the functional forms of \( R(\bar{\Omega}) \) and \( N(\bar{\omega}) \) are identical. This implies that in a dirty \( s- \)wave superconductor, the peak in the Raman intensity is also located exactly at twice the peak frequency in \( N(\bar{\omega}) \), though both peak positions shift from the Fermi gas values.

For completeness, we also discuss how fermionic damping affects the spin polarization operator at the antiferromagnetic momentum \( Q \). The form of this polarization operator is relevant for the interpretation of neutron scattering and ARPES data [17,18]

The spin polarization operator is related to the dynamical structure factor as \( S_Q(\bar{\Omega}) \propto \text{Im} (\xi^{-2} - \Pi_Q(\bar{\Omega}))^{-1} \) where \( \xi \) is the magnetic correlation length [4]. It is formally given by the same set of particle-hole bubbles made of normal and anomalous Green’s functions as the Raman intensity, but differs from \( R(\Omega) \) in two aspects. First, the antiferromagnetic spin polarization is a finite momentum probe, and the contribution to low frequency \( \text{Im} \Pi_Q(\bar{\Omega}) \) only comes from the momentum range in the Brillouin zone where both particles in the bubble are near the Fermi surface hot spots. Near hot spots, the superconducting gap is finite and close to \( \Delta \). In other words, the regions near the nodes of the \( d_{x^2-y^2} \) gap do not contribute to the dynamical spin susceptibility near \( Q \). Second, the vertices for \( \Pi_Q(\bar{\Omega}) \) contain Pauli matrices. For the anomalous \( FF \) term, the summation over spin projections yields an extra factor \( -1 \) compared to the Raman bubble. Performing the momentum integration in the \( GG \) and \( FF \) bubbles in the same way as before, and using the fact that \( \Delta_k \Delta_{k+Q} = -\Delta^2 \), we obtain [4]
\[ \Pi_Q(\Omega) = i \int_{-\infty}^{\infty} d\omega \frac{\sum_+ \sum_- + D_+ D_- - \Delta^2}{2D_+ D_-} \]  

(10)

The overall factor is chosen such that in the normal state, \( \Pi_Q(\Omega) = i|\omega| \).

For an ideal gas, the frequency integration in (10) yields \( \text{Im} \, \Pi_Q(\Omega) = 0 \) and \( \text{Re} \, \Pi_Q(\Omega) \propto \Omega^2 \) at \( \Omega = \Omega/(2\Delta) < 1 \). For large enough \( \xi \), this behavior of \( \Pi_Q(\Omega) \) gives rise to a resonant peak in \( S_Q(\Omega) \) at a frequency where \( \text{Re} \, \Pi_Q(\Omega) = \xi^{-2} \). At \( \Omega = 1 \), \( \text{Im} \, \Pi_Q(\Omega) \) jumps to a finite value, and \( \text{Re} \, \Pi_Q(\Omega) \) diverges logarithmically. This behavior is shown in Fig. 2a.

Substituting \( \Sigma(|\omega|) = \Delta(\Omega) \) and performing the same calculations as before, we obtain near \( \Omega = 1 \) and to first order in \( \gamma_\Delta \)

\[ \text{Im} \, \Pi_Q(\Omega) = \frac{\Delta \pi}{2} \left( 1 + \frac{2}{\pi} \arcsin \frac{\alpha_Q}{\sqrt{\alpha_Q^2 + 1}} \right) \]

\[ \text{Re} \, \Pi_Q(\Omega) = \Delta \left( \log \frac{1}{\gamma_\Delta} - \Psi(\alpha_Q) \right) \]

(11)

where, as before, \( \alpha_Q = (\Omega - 1)/(\gamma_\Delta) \), and in the limits of small and large \( \alpha \), \( \Psi(\alpha) \) behaves as \( \Psi(\alpha \ll 1) = \alpha^2/2 \), and \( \Psi(|\alpha| \to \infty) = \log |\alpha| \).

We see, similar to what we found for the Raman intensity and the DOS, the inclusion of the fermionic damping eliminates the singularities in the spin polarization operator: \( \text{Im} \, \Pi_Q \) changes continuously through \( \Omega = 1 \), and \( \text{Re} \, \Pi_Q \) is peaked but does not diverge at \( \Omega = 1 \) (to first order in \( \gamma_\Delta \), the peak does not shift to higher frequencies). This behavior is shown in Fig. 2b.

At small frequencies, the expansion in \( \Omega \) in (10) yields

\[ \Pi_Q(\Omega) = \frac{\pi}{2} \Omega^2 + 2i\gamma_0^2 |\Omega| \]

(12)

Again, similar to the result for \( R(\Omega) \), the inclusion of a finite fermionic damping yields a nonzero \( \text{Im} \, \Pi_Q(\Omega) \) down to the lowest frequencies. This result implies that in the presence of impurity scattering, the resonance peak in \( S_Q(\Omega) \) has a finite width. This effect may account for the width of the resonance neutron peak near optimal doping. We, however, do not believe that fermionic damping is responsible for the increase of the peak width with underdoping - this last effect is likely to be caused by the frequency dependence of \( \Delta \) associated with the pseudogap effects, which we do not consider here.

To summarize, we have considered in this paper a simple model form of the electronic damping and analysed how it affects the forms of the Raman intensity, the DOS, and the spin polarization operator at the antiferromagnetic momentum. We found that a finite damping eliminates artificial divergencies found in a Fermi-gas consideration. Still, however, without final state interaction, the peak in \( R(\Omega) \) occurs at or beyond twice the peak frequency for the DOS, in contradiction with the experimental observations. This negative result implies that fermionic damping alone cannot account for the data, and supports the explanation of the downturn shift of the Raman peak with underdoping in terms of a mid-gap pseudo-resonance mode in the Raman intensity. We also found that fermionic damping gives rise to a finite width of the resonance neutron peak.

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Physics, Addison Wesley (1988); A. Bardasis and J.R. Schrieffer, Phys. Rev. 121, 1050 (1961).

[21] H. Monien and F. Zawadowski, Phys. Rev. B 41, 8798 (1990).

[22] A. V. Chubukov and D. K. Morr, Phys. Rev. Lett. 81, 4716 (1998).

[23] Ar. Abanov, A. Chubukov, and A. Finkel'stein, in preparation.