Pion as a Longitudinal Axial-Vector Meson $q\bar{q}$ Bound State

T. N. Pham

Centre de Physique Théorique, CNRS, Ecole Polytechnique, 91128 Palaiseau, Cedex, France

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The success of the Adler-Bell-Jackiw (ABJ) chiral anomaly prediction for $\pi^0 \to \gamma\gamma$ decay rate shows that non-anomaly terms would make a negligible contribution to the decay rate, in agreement with the Sutherland-Veltman theorem. Thus the conventional $q\bar{q}$ bound-state description of the pion could not be valid since it would produce a $\pi^0 \to \gamma\gamma$ decay amplitude not suppressed in the soft pion limit, in contradiction with the Sutherland-Veltman theorem. Therefore, if the pion is to be treated as a $q\bar{q}$ bound state, this bound state would be a longitudinal axial-vector meson. In this paper, we consider the pion to be a longitudinal axial-vector meson $q\bar{q}$ bound state with derivative coupling for the pion $q\bar{q}$ Bethe-Salpeter (BS) wave function. We shall show that this BS wave function could produce a suppressed $\pi^0 \to \gamma\gamma$ decay amplitude in the soft pion limit, in agreement with the Sutherland-Veltman theorem. This explains the almost perfect agreement of the anomaly prediction with experiment and the suppression of the virtual one-photon exchange contribution in $\eta \to 3\pi$ decay. The Goldstone boson equivalence theorem used for longitudinal gauge bosons scattering in the electroweak standard model then identifies the longitudinal axial-vector meson $q\bar{q}$ bound state with the pion.

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I. INTRODUCTION

Chiral symmetry, as known from the success of the Goldberger-Treiman relation for the pion-nucleon coupling constant obtained from the PCAC hypothesis, is a good symmetry of strong interactions. The spontaneous breakdown of the $SU(2) \times SU(2)$ chiral symmetry generates a massless Nambu-Goldstone boson which then acquires a small mass through a chiral symmetry breaking quark mass term. PCAC and Adler-Bell-Jackiw chiral anomaly [1–3] then produce the $\pi^0 \to \gamma\gamma$ decay rate in good agreement with experiment. On the other hand, in a conventional bound-state model, a neutral pseudoscalar $q\bar{q} \ 0^{--}$ state, like the $\eta_c$ meson, is usually massive and could decay into two photons like the two-photon decays of positronium and heavy quarkonium. Being massive, they cannot be identified with the neutral pseudoscalar meson of the ground state $SU(3)$ octet like $\pi^0$ and $\eta$ meson, the Nambu-Goldstone bosons of the $SU(3) \times SU(3)$ chiral symmetry. In the traditional non-relativistic and relativistic bound-state calculations, one could
compute the $\pi^0 \to \gamma\gamma$ decay rate using the physical pion mass and obtains some agreement with experiment \[4\], but this particle could not be the pion, since the two-photon decay amplitude for this pseudoscalar $q\bar{q}$ state is not suppressed in the soft pion limit according to the Sutherland-Veltman theorem \[5,6\]. The pion could however be in a longitudinal axial-vector meson $q\bar{q}$ state, if this state could produce a suppressed $\pi^0 \to \gamma\gamma$ decay amplitude in the soft pion limit so that the agreement with experiment for the ABJ anomaly prediction of the $\pi^0$ two-photon decay rates is preserved. In this paper, we shall show that, with the longitudinal axial-vector meson pion BS wave function, the $\pi^0 \to \gamma\gamma$ decay amplitude would be suppressed in the soft pion limit, in agreement with the Sutherland-Veltman theorem. This explains the almost perfect agreement of the anomaly prediction with experiment for $\pi^0 \to \gamma\gamma$ decay and the suppression of the virtual one-photon exchange contribution in $\eta \to 3\pi$ decay which is then given, to leading order in Chiral Perturbation Theory, by the non-electromagnetic isospin breaking current quark mass term of the QCD Lagrangian of the standard model.

II. THE SUTHERLAND-VELTMAN THEOREM

Since the basis of our analysis is the Sutherland-Veltman theorem, for convenience, we reproduce this theorem here. Writing the $\pi^0 \to \gamma\gamma$ amplitude in the original notation \[8\], we have:

$$g \epsilon_{\alpha\beta\gamma\delta} \epsilon_{1\alpha} \epsilon_{2\beta} k_1 \gamma_5 k_2 \delta \int <0| T[j_{1\alpha}(x) j_{2\beta}(0)] \pi_0^0 > \exp (-i k_1 \cdot x) d^4 x. \quad (1)$$

Using PCAC with

$$\partial_\mu A^\mu = f_\pi m_\pi^2 \varphi_\pi,$$ \hspace{1cm} (2)

one finds:

$$\frac{(q^2 - m_\pi^2)}{f_\pi m_\pi^2} \epsilon_{1\alpha} \epsilon_{2\beta} \int <0| T[j_{1\alpha}(x) j_{2\beta}(0) \partial_\mu j_5^\mu(z)] 0 > \exp (-i k_1 \cdot x + i q \cdot z) d^4 x d^4 z$$

$$= \frac{(q^2 - m_\pi^2)}{f_\pi m_\pi^2} \epsilon_{1\alpha} \epsilon_{2\beta} q_\mu \int <0| T[j_{1\alpha}(x) j_{2\beta}(0) j_5^\mu(z)] 0 > \exp (-i k_1 \cdot x + i q \cdot z) d^4 x d^4 z. \quad (3)$$

Since gauge invariance requires that

$$\int <0| T[j_{1\alpha}(x) j_{2\beta}(0) j_5^\mu(z)] 0 > \exp (-i k_1 \cdot x + i q \cdot z) d^4 x d^4 z \propto \epsilon_{\alpha\beta\gamma\delta} k_1 \gamma_5 k_2 \delta q_\mu. \quad (4)$$

for $q^2 = 0$ ($q$ being the pion momentum), $g \to 0$ in the soft pion limit, the amplitude $\pi^0 \to \gamma\gamma$ is $O(q^2)$ and becomes suppressed as $q^2 \to 0$. This theorem is now evaded by the ABJ the chiral anomaly in the quark triangle graph which gives us the well-known chiral anomaly prediction for
$\pi^0 \rightarrow \gamma \gamma$ decay. To see how this comes about, we reproduce here the derivation of the $\pi^0 \rightarrow \gamma \gamma$ amplitude using the modified PCAC equation:

$$\partial_\mu A^\mu = f_\pi m_\pi^2 \phi + S \frac{e^2}{10\pi^2} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

(5)

with $S = 1/2$ in the standard model. Taking the matrix element of both sides of Eq. 5 and separating the $\pi^0$ pole term, we find, in the notation of [14, 15]:

$$N'^{\mu\nu} = \frac{1}{f_\pi} \left( p_\tau \tilde{R}^{\mu\nu\tau}(q, k) - S \frac{e^2}{2\pi^2} Y^{\mu\nu} \right)$$

(6)

with $<\pi^0(p)|T|\gamma^\ast(q)\gamma(k)> = \epsilon_\mu(q)\epsilon_\nu(k)N^{\mu\nu}(q, k)$ and $N^{\mu\nu}(q, k)$ given by:

$$N^{\mu\nu}(q, k) = e^2 F(q, k) Y^{\mu\nu}, \quad Y^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} q_\alpha k_\beta.$$

(7)

where $F(q, k)$ is the transition form factor and $p, k, q$ are respectively, the pion and the two photons momenta. In Eq. $	ilde{R}^{\mu\nu\tau}(q, k)$ is the triangle graph (the direct coupling between the three currents) or the continuum contribution to the axial vector current matrix element $<0|A_\mu|\gamma^\ast\gamma>$ defined as $R^{\mu\nu\tau}(q, k)$:

$$R^{\mu\nu\tau}(q, k) = \tilde{R}^{\mu\nu\tau}(q, k) - f_\pi \frac{p_\tau N^{\mu\nu}(q, k)}{p^2 - m_\pi^2}$$

(8)

Gauge invariance and Bose symmetry tells us that when both photons are real as in the $\pi^0 \rightarrow \gamma \gamma$ decay ($q^2 = 0, k^2 = 0$), the divergence $p_\tau R^{\mu\nu\tau}(q, k)$ is $O(p^2)$ and becomes negligible. One can then apply Eq. 8 to the $\pi^0 \rightarrow \gamma \gamma$ decay amplitude and finds that it is given by the anomaly [1–3]. From the expressions in Ref. [1, 10], the triangle graph contribution to the direct term is then

$$p_\tau \tilde{R}^{\mu\nu\tau}(q, k) = e^2 S \left( 2mP(q, k) + \frac{1}{2\pi^2} \right) Y^{\mu\nu}$$

(9)

with

$$P(q, k) = \frac{m^2}{2\pi^2} \int_0^1 dx \int_0^{1-x} \frac{dy}{[k^2y(1-y) + q^2x(1-x) - (q^2 + k^2 - p^2)xy - m^2]}$$

(10)

When both photons are real ($q^2 = 0, k^2 = 0$), from Eq. 10, we get:

$$2mP(q, k) = \frac{1}{2\pi^2} + O(p^2)$$

(11)

which implies that the r.h.s of Eq. 9 is $O(p^2)$ in agreement with our previous remark that $p_\tau \tilde{R}^{\mu\nu\tau}(q, k) = O(p^2)$ which is precisely the Sutherland result for the axial vector current matrix element in Eq. 11. However the PCAC equation has been modified by the triangle graph anomaly.
which gives rise to the second term in Eq. (6) from which we get the anomaly prediction for \( \pi^0 \rightarrow \gamma\gamma \),

\[
A(\pi^0 \rightarrow \gamma\gamma) = \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \left( -\frac{e^2}{4\pi^2 f_\pi} \right).
\] (12)

This result is exact to all order in \( \alpha \) and the strong coupling constant \( \alpha_s \) and is in almost perfect agreement with experiment, confirming the validity of the soft-pion theorem in the presence of the ABJ anomaly. Other non-anomaly contribution is \( O(p^2) \) and is suppressed in agreement with the Sutherland-Veltman theorem as mentioned above.

It is then quite surprising to find in the literature paper claiming to reproduce exactly the anomaly prediction using the impulse approximation [11] to calculate the width for \( \pi^0 \rightarrow \gamma\gamma \) decay. It is evident that with an approximation, one cannot expect to reproduce exactly the anomaly prediction of Adler [1], since various corrections have to be included, thus invalidating the result of Ref. [11] which appears as pure numerology, in our opinion. Since the anomaly prediction is an exact model-independent soft-pion result, like the low-energy theorem for Compton scattering on any target, the Thomson formula \(- (e^2/m)(e_1 \cdot e_2)\), it cannot be obtained by any model calculation. Any model calculation of \( \pi^0 \rightarrow \gamma\gamma \) decay amplitude in QCD using BS wave function will get \( \alpha_s \) corrections and errors from neglecting other contributions, and in the calculation of [11], there is no physical principle to protect the result from these \( \alpha_s \) corrections and other contributions neglected in the impulse approximation. Moreover the use of a momentum-independent BS wave function for the pion, the \( \gamma_5 \) term, will produce terms not suppressed in the soft-pion limit, for both the non-anomaly term in \( \pi^0 \rightarrow \gamma\gamma \) and the contribution from virtual one-photon exchange electromagnetic interactions in \( \eta \rightarrow 3\pi \) decay amplitude as mentioned below. Therefore the calculation of [11] is incorrect. The fallacy in that work is the use of an approximation for an exact result.

At the same time with the Sutherland-Veltman theorem for \( \pi^0 \rightarrow \gamma\gamma \), Sutherland [8, 12] and Adler [13] also show that the \( \eta \rightarrow 3\pi \) decay is suppressed in the soft-pion limit. This is because the vanishing of the E. T. C. in the soft-pion expression implies that the \( \eta \rightarrow 3\pi \) decay with \( q \rightarrow 0 \) (\( q \) being the \( \pi^0 \) momentum) is suppressed. The reason is that, as shown by Cantor [16], for the virtual one-photon exchange electromagnetic interactions which transform as \( (1, \bar{8}) + (\bar{8}, 1) \) representation of \( SU(3) \times SU(3) \), only derivative coupling is allowed in the \( \eta \rightarrow 3\pi \) decay amplitude. In fact a suppressed \( \eta - \pi^0 \) mixing could be obtained easily from the virtual one-photon exchange electromagnetic interactions if one uses the momentum-dependent BS wave function, in agreement with the Sutherland theorem [12, 13]. One thus needs a new non-electromagnetic isospin violating tadpole term to obtain the unsuppressed \( \eta \rightarrow 3\pi \) decay rate. For this the tadpole term, Cantor
adds a non-electromagnetic isospin breaking term \( u_3 \) to the Gell-Mann-Oakes-Renner \((\bar{3}, \bar{3}) \times SU(3) \times SU(3)\) breaking term \[17\]:

\[
L_1 = a_0 u_0 + a_8 u_8 + a_3 u_3
\]  

The idea that this \( u_3 \) term is essential for \( \eta \to 3\pi \) decay is due to Cabibbo and Wilson as quoted by Cantor \[16\]. The soft-pion theorem for the matrix element \( \langle \eta | u_3 | 3\pi \rangle \) then produces the non-derivative term for \( \eta \to 3\pi \) decay which is not suppressed in the soft-pion limit. This is the origin of the non-electromagnetic isospin breaking current quark mass term of the QCD Lagrangian which gives, at leading order in Chiral Perturbation Theory, the \( \eta \to 3\pi \) decay amplitude.

### III. PION AS A LONGITUDINAL AXIAL-VECTOR MESON

We have seen that without the ABJ anomaly, the \( \pi^0 \to \gamma\gamma \) decay would be suppressed. In any model calculation, for example, in non-relativistic or relativistic calculation, without PCAC and chiral symmetry, the Sutherland-Veltman theorem does not apply and the two-photon decay is not suppressed in the soft pion limit as found in existing bound-state calculations of quarkonium two-photon decays \[4–7\]. The suppression of the virtual photon exchange electromagnetic interactions in \( \eta \to 3\pi \) decay is another chiral symmetry constraint to be imposed on the pion BS wave function according to Sutherland theorem for \( \eta \to 3\pi \) decay.

It follows that the pion could not be described by the usual momentum-independent \( q\bar{q} \) bound-state wave function. Since many properties of hadrons, and in particular, the light mesons and quarkonium systems, are well described by the \( q\bar{q} \) bound-state picture, the problem is how to reconcile this bound-state picture with the Nambu-Goldstone boson character of the pion. The solution of the problem could be found easily by looking at the solution of the relativistic bound-state Bethe-Salpeter(BS) equation \[18\] for a \( q\bar{q} \) system. For a pseudoscalar meson, there are two possible solutions. The solution with the momentum-independent wave function of the form \( P_{\gamma_5} \) and the longitudinal axial-vector momentum-dependent \( \not{p}_{\gamma_5} A \) solutions. This longitudinal solution has been considered by Kummer \[19\]. The longitudinal axial-vector meson wave function for pion is also used by Chernyak and Zhitnitsky for process involving pion at high energy \[20\].

As mentioned above, the \( P_{\gamma_5} \) solution would be in contradiction with the Sutherland-Veltman theorem and therefore could not be the correct pion \( q\bar{q} \) bound-state wave function. The \( \not{p}_{\gamma_5} A \) solutions would be acceptable. In fact, if the pion is a longitudinal axial-vector meson \( q\bar{q} \) bound state, the \( \pi^0 \to \gamma\gamma \) amplitude computed with this wave function, as shown below, would be similar
to the free quark triangle graph contribution to the two-photon matrix element of the axial-vector current divergence $<0|\partial_\mu A_\mu(0)|\gamma\gamma>$ and therefore vanishes in the massless quark limit and thus does not contribute to the $\pi^0 \rightarrow \gamma\gamma$ decay. In the following we present a computation of the $\pi^0 \rightarrow \gamma\gamma$ amplitude using the longitudinal axial-vector meson as the pion BS wave function. Consider now the BS wave function of [21]:

$$\psi(p,q) = \gamma_5 \psi_0 + \gamma_5 \not{p} \psi_1 + \gamma_5 \not{q} \psi_2 + \gamma_5 [\not{p}, \not{q}] \psi_3.$$ (14)

where $p$ and $q$ is the pion and relative momentum of the $q\bar{q}$ system, with the quark and anti-quark momentum $q_1 = q + p/2$, $q_2 = q - p/2$ and $\psi_i, i = 0, ..., 3$ are the scalar functions of $p$ and $q$. The first term $\psi_0$ in Eq. (14) is the momentum-independent wave function, as mentioned above, produce a $\pi^0 \rightarrow \gamma\gamma$ decay in the soft pion limit and is dropped here. The third term $\psi_2$ which is $O(p \cdot q)$ could give a contribution $O(p)$ in the soft pion limit and need not to be considered here. The last term $\psi_3$, does not make a contribution to $\pi^0 \rightarrow \gamma\gamma$ decay by the triangle graph. This leaves us with the $\psi_1$ term as the longitudinal contribution to the $\pi^0 \rightarrow \gamma\gamma$ decay. The BS equation [21] for $\psi(p,q)$ with the gluon propagator $G_{\mu\nu}(k - q)$ reads:

$$\not{q} \psi(p,q)(\not{q} - \not{p}/2) = -i \int \frac{d^4q'}{(2\pi)^4} \gamma_\mu \psi(p,q') \gamma_\nu G_{\mu\nu}(q' - q).$$ (15)

Since, by definition, the BS vertex function $\Gamma(p,q)$ is the BS wave function with the free quark propagator removed [22, 23], Eq. (15) can be used to express $\Gamma(p,q)$ in terms of the BS wave function $\psi(p,q)$. We have:

$$\Gamma(p,q) = -i \int \frac{d^4q'}{(2\pi)^4} \gamma_\mu \psi(p,q') \gamma_\nu G_{\mu\nu}(q' - q).$$ (16)

FIG. 1: Quark loop triangle graphs with BS longitudinal axial-vector meson wave function for $\pi^0 \rightarrow \gamma\gamma$ decay.
In the following, as our purpose is to obtain the soft pion limit for $\pi^0 \rightarrow \gamma\gamma$ decay, we consider only the longitudinal solution for the BS wave function $\gamma_5 \not{p}\psi_1$ given in Eq. (14), and for simplicity, we use the gluon propagator in the Feynman gauge with $G_{\mu\nu}(q' - q) = -g_{\mu\nu}/(q' - q)^2$. The $\pi^0 \rightarrow \gamma\gamma$ decay amplitude is given by the quark loop triangle graph similar to the ABJ chiral anomaly triangle graph, except that the point-like axial-vector current vertex $\gamma_\mu \gamma_5$ is replaced by the BS longitudinal axial-vector meson wave function $\psi(p, q') = \gamma_5 \not{p}\psi_1(p, q')$, and the factor $1/(q' - q)^2$ from the gluon propagator which makes the integration over $q$ convergent and could be carried out by the usual change of variable, assuming the integral over $q'$ convergent. Similar to the calculation of Ref. [1], the $\pi^0 \rightarrow \gamma\gamma$ decay amplitude with the BS vertex function $\Gamma(p, q)$ shown in Fig. 1 after a change of variable $l = q + p/2$, with $l$ one of the quark momentum in the triangle loop and $\Gamma(p, q) = \Gamma(p, l)$ and putting $m = 0$, is given by:

$$M = -ie^2 \int \frac{d^4 l}{(2\pi)^4} \text{Tr} \left( \not{l} \gamma_5 \gamma_5 \gamma_1 \not{k}_1 \not{k}_2 \right) J(p, l) + (\epsilon_1, k_1 \rightarrow \epsilon_2, k_2 \text{ terms})$$

with the scalar part of the BS vertex function $\Gamma(p, l)$ given by:

$$J(p, l) = -2 \int \frac{d^4 l'}{(2\pi)^4} \frac{\psi_1(p, l')}{(l' - l)^2}.$$  \hspace{1cm} (18)

Using the identity [1],

$$\frac{1}{(l - k_2)} \frac{1}{(l + k_1)} \gamma_5 \frac{1}{(l + k_2)} = \frac{1}{(l - k_2)} \gamma_5 + \gamma_5 \frac{1}{(l + k_1)}.$$  \hspace{1cm} (19)

The Dirac $\gamma$ term (the Trace term), is then split into two contributions. The contributions from the 1st and 2nd diagrams in Fig. 1 are respectively then:

$$T_1 = \frac{\not{k}_2(l - k_2) \not{I} \not{k}_1 \not{I} \gamma_5}{(l - k_2)^2 l^2} - \frac{\not{k}_2(l + k_1) \not{I} \not{k}_1 \not{I} \gamma_5}{(l + k_1)^2 l^2}$$

$$T_2 = \frac{\not{k}_1(l - k_1) \not{I} \not{k}_2 \not{I} \gamma_5}{(l - k_1)^2 l^2} - \frac{\not{k}_1(l + k_2) \not{I} \not{k}_2 \not{I} \gamma_5}{(l + k_2)^2 l^2}$$

We see that, provided that the integral over $l$ converges, the $k_2$-terms in Eq. (20) and Eq. (21) would cancel after integration over $l$ by a change of variable $l - k_2 \rightarrow l$ and $l \rightarrow l + k_2$ in the $k_2$-terms of Eq. (20) and similarly for the $k_1$-terms with a change of variable $l - k_1 \rightarrow l$ and $l \rightarrow l + k_1$ in Eq. (21). This is not the case with point-like axial-vector current in the triangle graph since the shift of the integration variable $l - k_2 \rightarrow l$ in Eq. (20), or $l - k_1 \rightarrow l$ in Eq. (21), would induce an anomaly term [24]. This is the well-known anomaly terms for the divergence of the axial-vector current [1]. In our bound-state calculation, the point-like axial-vector current is replaced by the longitudinal axial-vector meson BS vertex function and the $1/l^2$ behavior of the gluon propagator...
at large $l^2$ would make the integrals over $l$ convergent for $k_1$ and $k_2$ terms in the two diagrams. Taking the trace, the total contribution to $\pi^0 \to \gamma\gamma$ decay amplitude is then given by:

$$M = -i e^2 \int \frac{d^4l}{(2\pi)^4} \left( -\frac{4i\epsilon_1\epsilon_2 k_1 l}{(l^2 - k_1^2)^2(l^2 + k_1^2)^2} + \frac{4i\epsilon_1\epsilon_2 k_2 l}{(l^2 - k_2^2)^2(l^2 + k_2^2)^2} \right) J(p, l).$$

(22)

where $\epsilon(\epsilon_1, \epsilon_2, k_1, l)$ and $\epsilon(\epsilon_1, \epsilon_2, k_2, l)$ denote the contraction of $\epsilon_1, \epsilon_2, k_1, l$ and $\epsilon_1, \epsilon_2, k_2, l$ with the anti-symmetric tensor $\epsilon$. Assuming that the integral over $l'$ in $J(p, l)$ is finite, the integration over $l$ in the above expression will produce terms proportional to $\epsilon(\epsilon_1, \epsilon_2, k_1, l)$, $\epsilon(\epsilon_1, \epsilon_2, k_1, l') l' \cdot k_1$ for $k_1$-term and $\epsilon(\epsilon_1, \epsilon_2, k_2, k_2)$, $\epsilon(\epsilon_1, \epsilon_2, k_2, l') l' \cdot k_2$ for $k_2$-term in Eq. [22]. Since $\epsilon(\epsilon_1, \epsilon_2, k_1, k_1) = 0$, $\epsilon(\epsilon_1, \epsilon_2, k_2, k_2) = 0$, only the $l'$ term survives after integration over $l$. After integration over $l'$, only terms proportional to $p \cdot k_1$ and $p \cdot k_2$ survive, but these are $O(p^2)$ and are suppressed in the soft pion limit, in agreement with the Sutherland-Veltman theorem. Provided that the integrals over $l$ and $l'$ are finite, this result does not depend on the detailed form of the BS wave function and the use of the one-gluon exchange kernel in $J(l, p)$. The $\pi^0 \to \gamma\gamma$ decay is then given by the ABJ anomaly which agrees well with experiment. This implies the absence of the $P\gamma_5$ term in the pion BS wave function and the pion thus behaves as a longitudinal axial-vector meson. This momentum-dependent pion BS wave function will produce a suppression of the $\eta \to 3\pi$ decay which is now identified as the quark mass term in the QCD Lagrangian of the standard model.

In the electroweak standard model, the unphysical Goldstone boson becomes the longitudinal gauge boson, in our case, the longitudinal axial-vector meson $q\bar{q}$ bound state appears as a Goldstone boson, the pion, according to the Goldstone boson equivalence theorem for $W_L W_L$ scattering in the standard model [25]. Since only the kinetic term is generated with this BS wave function, pion remains massless. The pion mass term has to be generated by chiral symmetry breaking term as the $\sigma$-term in the $\sigma$ model [26] which is now identified as the quark mass term in the QCD Lagrangian of the standard model.

IV. CONCLUSION

In conclusion, we have derived the Sutherland-Veltman theorem for the $\pi^0 \to \gamma\gamma$ decay considering the pion as a longitudinal axial-vector meson $q\bar{q}$ bound state. With this longitudinal axial vector meson BS wave we have been able to obtain the suppressions of the non-anomaly term in $\pi^0 \to \gamma\gamma$ decay and the virtual one-photon exchange electromagnetic interaction term in $\eta \to 3\pi$ decay. The Goldstone boson equivalence theorem used for $W_L W_L$ scattering in the electroweak standard model then identifies the longitudinal axial-vector meson $q\bar{q}$ bound state with the pion.
This allows us to say that the pion could be a $q\bar{q}$ bound state at the same time a Nambu-Goldstone boson of chiral symmetry with the two-photon decay given by PCAC and the ABJ chiral anomaly. The momentum-dependent BS wave function could then be used to obtain the derivative couplings with hadrons, in agreement with chiral symmetry.

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