Neutron stars structure in the context of massive gravity

S. H. Hendi$^{1,2}$, G. H. Bordbar$^{1,3}$, B. Eslam Panah$^{1,4}$ and S. Panahiyan$^{1,5}$

$^1$ Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran
$^2$ Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P. O. Box 55134-441, Maragha, Iran
$^3$ Center for Excellence in Astronomy and Astrophysics (CEAA-RIAAM)-Maragha, P. O. Box 55134-441, Maragha, Iran
$^4$ ICRANet, Piazza della Repubblica 10, I-65122 Pescara, Italy
$^5$ Physics Department, Shahid Beheshti University, Tehran 19839, Iran

Motivated by the recent interests in spin$-2$ massive gravitons, we study the structure of neutron star in the context of massive gravity. The modifications of TOV equation in the presence of massive gravity are explored in 4 and higher dimensions. Next, by considering the modern equation of state for the neutron star matter (which is extracted by the lowest order constrained variational (LOCV) method with the AV18 potential), different physical properties of the neutron star (such as Le Chatelier’s principle, stability and energy conditions) are investigated. It is shown that consideration of the massive gravity has specific contributions into the structure of neutron star and introduces new prescriptions for the massive astrophysical objects. The mass-radius relation is examined and the effects of massive gravity on the Schwarzschild radius, average density, compactness, gravitational redshift and dynamical stability are studied. Finally, a relation between mass and radius of neutron star versus the Planck mass is extracted.

I. INTRODUCTION

The Einstein theory of gravity has been a pioneering tool for understanding and describing the gravitational systems. Most of the results and validations of this theory have been confirmed by observations done on solar system level. In addition, the results of LIGO proved the existence of gravitational wave which was one of the challenging predictions of general relativity (GR) \cite{1}. It is expected to observe the advantages of GR for beyond the Newtonian theory regimes (high curvature regimes) such as near the compact objects. The existence of compact objects has been confirmed in the Einstein theory. Due to the physical properties of these objects, it is necessary to take the curvature of spacetime into account in order to have reliable predictions. Therefore, we employ the Einstein theory of gravity to study the neutron star.

Einstein theory predicts the existence of massless spin-2 gravitons with two degrees of freedom as intermediate particles for signaling the gravitational interactions. But, there have been several arguments regarding the possibility of existence of massive gravitons. These arguments are supported by studies that are conducted on quantum level of gravity and brane-world gravity. Especially problems such as the hierarchy problem and their brane-world gravity solutions have expressed on the possibility of existence of massive spin-2 gravitons \cite{2, 3}. Therefore, it is natural to relax massless constraint and consider the modifications and generalizations of the general relativity to include massive graviton. In this paper, we generalize the Einstein theory of gravity to include the massive graviton and investigate its effects on the hydrostatic equilibrium equation of a typical neutron star.

The first attempt for constructing the massive gravity was done by Fierz and Pauli \cite{4}. This theory has the specific problem known as vDVZ (van Dam-Veltman-Zakharov) discontinuity which indicates that the propagators of massless and massive in the limit of $m \to 0$, are not the same \cite{5, 6}. One of the resolutions of this problem was Vainshtein mechanism which requires the system to be considered in the nonlinear framework \cite{8} (it is notable that in nonlinear dRGT, there are also vacua that are free from vDVZ discontinuity \cite{11}). Such generalization to nonlinear case introduces a ghost into the theory which is known as Boulware-Deser ghost \cite{10}. There are various ghost free scenarios for considering the massive gravity in the nonlinear framework. One of the interesting ghost free theories of massive gravity is known as dRGT theory which was developed by de Rham, Gabadadze and Tolley \cite{11, 12}. In this theory, a reference metric is employed to build massive terms \cite{11, 13}. These massive terms are inserted in the action to provide massive gravitons. Cosmological results, black hole solutions and their thermodynamical properties in this massive gravity are investigated by many authors \cite{15, 31}. In addition, Katsuragawa et al, studied the neutron stars in the context of dRGT theory and showed that, the massive gravity leads to small deviation from the GR \cite{31}.

It is worthwhile to mention that the reference metric plays a key role for constructing the massive theory of gravity.

* email address: hendi@shirazu.ac.ir
† email address: ghbordbar@shirazu.ac.ir
‡ email address: behzad.eslampanah@gmail.com
§ email address: sh.panahiyan@gmail.com
One of the modifications in reference metric was done by Vegh which introduced a new massive gravity [33]. This new massive theory has specific applications in the gauge/gravity duality especially in lattice physics which motivate one to use it in other frameworks as well. This theory was employed in the context of black holes and it was shown that geometrical and thermodynamical structures of the black holes will be modified and new phenomena were reported for massive black holes [34–42]. Here, we use this massive theory to conduct our studies in the properties of neutron stars.

The structure of stars and their phenomenological properties are described with hydrostatic equilibrium equation (HEE). This equation is based on the fact that a typical star will be in equilibrium when there is a balance between the internal pressure and the gravitational force. Historically speaking, the first HEE equation for GR was introduced and employed by Tolman, Oppenheimer and Volkoff (TOV) [43–45]. After that, a series of studies were dedicated to obtain HEE of neutron star [46–53]. In addition, the compact objects and their TOV equations were investigated in the presence of different modified gravities such as; gravity’s rainbow [54], vector-tensor-Horndeski theory of gravity [55], dilaton gravity [56], $F(R)$ and $F(G)$ gravities [57–61] (see [62–81] for more details).

According to recent studies on the neutron stars and observations of the interesting properties of them, we want to investigate these stars in the context of massive gravity. In other words, our main motivation in this paper is studying the effects of considering the massive gravity on the structure of neutron stars. Previous studies in the context of other astrophysical objects have proven a wide variation in the properties of these objects comparing to the massless graviton case. Therefore, we are expecting to see the specific modifications in the properties of neutron stars as well. Here, we would like to address how the structure of neutron star will be modified in the presence of massive gravity and which contributions this generalization has into properties of these objects. We regard the HEE equation in 4 and higher dimensions with a suitable equation of state (EoS), which satisfies stability, energy conditions and Le Chatelier’s principle. We obtain the maximum mass and corresponding radius, Schwarzschild radius, compactness, gravitational redshift and dynamical stability of the neutron stars. We give details regarding the effects of massive gravity on these properties. These studies provide an insight into the structure of neutron stars and enable one to make a comparison between the massive and massless gravity theories. Remembering that the neutron stars, similar to other massive objects, propagate the gravitational waves, one is urged to study the neutron stars in the presence of massive gravity which is the aim of this paper.

The outline of our paper is as follows. In Sec. II we consider a spherical symmetric metric and obtain the modified TOV in Einstein-massive gravity in four dimensions. Next, we employ the specific many-body EoS and study its properties such as the Le Chatelier’s principle, stability and energy conditions. Then, considering the Einstein-massive gravity, we investigate the neutron star structure and obtain other properties of this star. In next section, we extract mass and radius of this star versus the Planck mass as a fundamental physical constant. Finally, we finish our paper with some closing remarks.

II. MODIFIED TOV EQUATION IN THE MASSIVE GRAVITY

The action of Einstein-massive (EN-massive) gravity with the cosmological constant in $d$-dimensions is given by

$$I = -\frac{1}{16\pi} \int d^d x \sqrt{-g} \left( R - 2\Lambda + m^2 \sum_i^d c_i U_i(g, f) \right) + I_{\text{matter}},$$

where $R$ and $m$ are the Ricci scalar and the massive parameter, $\Lambda$ is the negative cosmological constant, and $f$ and $g$ are a fixed symmetric tensor and metric tensor, respectively. In addition, $c_i$’s are constants and $U_i$’s are symmetric polynomials of the eigenvalues of $d \times d$ matrix $K_{\mu}^{\nu} = \sqrt{g^{\mu\nu}} f_{\mu\nu}$ where they can be written in the following forms

$$U_1 = [K], \quad U_2 = [K]^2 - [K^2],$$

$$U_3 = [K]^3 - 3[K][K^2] + 2[K^3],$$

$$U_4 = [K]^4 - 6[K^2][K^2] + 8[K^3][K] + 3[K]^2 - 6[K^4].$$

By variation of Eq. (1) with respect to the metric tensor $g_{\mu}^{\nu}$, the equation of motion for EN-massive gravity can be written as

$$G^{\nu}_{\mu} + \Lambda g^{\nu}_{\mu} + m^2 \chi^{\nu}_{\mu} = K_d T^{\nu}_{\mu},$$

where $K_d = \frac{8\pi G_d}{c^4}$, $G_d$ is $d$-dimensional gravitational constant, $G_{\mu\nu}$ is the Einstein tensor and $c$ is the speed of light in vacuum. Also, $T^{\nu}_{\mu}$ denotes the energy-momentum tensor which comes from the variation of $I_{\text{matter}}$ and $\chi_{\mu\nu}$ is the...
massive term with the following explicit form

\[
\chi_{\mu\nu} = -\frac{c_1^2}{2} (U_1 g_{\mu\nu} - K_{\mu\nu}) - \frac{c_2}{2} (U_2 g_{\mu\nu} - 2U_1 K_{\mu\nu} + 2K_{\mu\nu}^2) \\
- \frac{c_3}{2} (U_3 g_{\mu\nu} - 3U_2 K_{\mu\nu} + 6U_1 K_{\mu\nu}^2 - 6K_{\mu\nu}^3) \\
- \frac{c_4}{2} (U_4 g_{\mu\nu} - 4U_3 K_{\mu\nu} + 12U_2 K_{\mu\nu}^2 - 24U_1 K_{\mu\nu}^3 + 24K_{\mu\nu}^4). 
\]

(3)

A. Modified TOV equation in (3+1)-dimensions

In this section, the static solutions of EN-massive gravity in (3 + 1)-dimensions are obtained. For this purpose, we consider a spherical symmetric space-time in the following form

\[
g_{\mu\nu} = \text{diag} (f(r), -g(r)^{-1}, -r^2, -r^2 \sin^2 \theta),
\]

(4)

where \( f(r) \) and \( g(r) \) are unknown metric functions. Now, in order to obtain exact solutions, we should consider a suitable reference metric. Obeying the ansatz of Ref. [34], we consider the following relation for the reference metric

\[
f_{\mu\nu} = \text{diag}(0, 0, C^2 r^2, C^2 r^2 \sin^2 \theta),
\]

(5)

in which \( C \) is a positive constant. Considering the metric ansatz (5), we can obtain the explicit forms of nonzero \( U_i \)'s as [34]

\[
U_1 = \frac{2C}{r}, \quad U_2 = \frac{2C^2}{r^2}.
\]

Here, we regard the neutron star as a perfect fluid with the following energy-momentum tensor

\[
T_{\mu\nu} = (c^2 \rho + P) U_{\mu} U_{\nu} - Pg_{\mu\nu},
\]

(6)

where \( P \) and \( \rho \) are the pressure and density of the fluid which are measured by the local observer, respectively, and \( U^\mu \) is the fluid four-velocity. Using Eqs. (2) and (6) with the metric introduced in Eq. (4), it is easy to obtain the components of energy-momentum as

\[
T^0_0 = \rho c^2 \quad & \quad T^1_1 = T^2_2 = T^3_3 = -P.
\]

(7)

In addition, taking into account Eqs. (4) and (7), it is straightforward to achieve the following nonzero components of field equation (2)

\[
Kc^2 r^2 \rho = \Lambda r^2 + (1 - g) - rg^2 - m^2 C (c_1 r + c_2 C),
\]

(8)

\[
Kf r^2 P = -\Lambda r^2 f - (1 - g) f + rg f + m^2 f C (c_1 r + c_2 C),
\]

(9)

\[
4Kf^2 r P = -4\Lambda r f^2 + 2 (gf) f' - r g f'^2 + r [g' f' + 2g f''] f + 2m^2 C c_1 f^2,
\]

(10)

where \( K = \frac{8\pi G}{c^4} \), and \( f, g, \rho \) and \( P \) are functions of \( r \). Also, we note that the prime and double prime denote the first and the second derivatives with respect to \( r \), respectively.

Using Eqs. (8)-(10) and after some calculations, we obtain

\[
\frac{dP}{dr} + \left( \frac{c^2 \rho + P}{2f} \right) f' = 0.
\]

(11)

In addition, for extracting \( g(r) \), we can use Eq. (8) which leads to

\[
g(r) = 1 + \frac{\Lambda}{3} r^2 - m^2 C \left( \frac{c_1 r}{2} + c_2 C \right) - \frac{c^2 KM(r)}{4\pi r},
\]

(12)
in which $M(r) = \int 4\pi r^2 \rho(r) dr$. Now, we obtain $f'$ from Eq. (11) and insert it with Eq. (12) in Eq. (11) to calculate HEE in the Einstein-massive gravity with the following form

\[ \frac{dP}{dr} = \left( \frac{c^2 \rho + P}{r} \right) \left[ \frac{\rho c^2 M(r)}{2} + 2\pi r^3 \left( \frac{2\Lambda}{3} + KP \right) - m^2 \pi r^2 c_1 C \right] \]

which is modified TOV equation due to the presence of massive graviton. As one expects, for $m = 0$, Eq. (13) is reduced to the following TOV equation obtained in Einstein-Λ gravity [102, 103]

\[ \frac{dP}{dr} = \frac{3c^2 GM(r) + r^3 (\Lambda c^4 + 12\pi GP)}{c^2 r [6GM(r) - c^2 r (\Lambda r^2 + 3)]} \left( c^2 \rho + P \right). \]  

In addition, in the absence of both massive term and cosmological constant ($m = \Lambda = 0$), Eq. (13) leads to the usual TOV equation of Einstein gravity (see [43–45] for more details). It is notable that, the generalization to higher dimensions is done in the appendix A.

Before applying the mentioned gravitational framework on the neutron star structure, we should point out some comments for the mentioned massive gravity. As we mentioned before, the massive gravity employed in this paper is essentially a dRGT like [32]. It was shown that all dRGT like theories of the massive gravity in $d$ dimensions with $N$ scalar fields (Stückelberg fields) enjoy at most $\frac{d}{2}(d-3) + N$ number of degrees of freedom. The reference metric employed in this paper is spatial reference metric which in its simpler form in the appropriate orthonormal coordinate, it will be $(0, 0, 1, 1)$. This specific choice in reference metric leads into interesting results which among them one can point out that under certain coordinate transformation, the general covariance is preserved in radial and temporal coordinates while it breaks in spatial dimensions [33]. Therefore, this theory indeed enjoys Lorentz violating property. On the other hand, it was shown that this choice of reference metric and more general ones enjoy the preservation of the Hamiltonian constraint which leads to removing one of the degrees of freedom. In addition, based on the diffeomorphism, another degree of freedom is eliminated. In general, since this is a 4 dimensional theory of massive gravity, there might be up to 6 degrees of freedom. Two of these degrees of freedom were eliminated due to mentioned properties. On the other hand, it was also shown that only two Stückelberg fields exist in the diffeomorphism invariant formulation of this theory [33]. This indicates that two degrees of freedom are absent which leads to absence of Boulware-Deser (BD) ghost.

To summarize, it is notable that depending on the notion of time, the number of degrees of freedom can differ from two to five. Although an observer with arbitrary time has to describe five degrees of freedom, the observers with Stückelberg time function will describe two degrees of freedom such as the situation of massless gravitons in the usual general relativity.

The full details regarding absence of the BD ghost in this massive gravity are given in Refs [33, 104]. Especially in Ref. [104], the stability of massive gravity with singular metric of arbitrary rank was studied and the absence of BD ghost was proven.

Evaluating the second derivatives of the massive action with respect to the background graviton field, one can obtain a mass matrix. In this case, the eigenvalues of the matrix would correspond to the masses of each mode, the so-called, $m_{tt}$, $m_{ij}$, etc. These are the mass fluctuations of the modes which are depending on the non-trivial contributions of the Stückelberg function. In other words, such mass fluctuations are depending on the free parameters of the massive theory such as $c_1$ and $c_2$, etc. So, it would be useful to calculate the physical mass of fluctuations and impose appropriate conditions to avoid tachyon-like instabilities. Taking into account the point of Ref. [32], we can regard that the mass parameter is of the order of the Hubble parameter today, and therefore, such an instability would not be problematic. However, since the tachyon-like instabilities are very important in some gravitational framework, such as black holes, we will address such substantial point in an independent paper.

### III. Structure Properties of Neutron Star

#### A. Equation of state of neutron star matter

The interior region of a typical neutron star is a mixed soup of neutrons, protons, electrons and muons in charge neutrality and beta equilibrium conditions (beta-stable matter) [108]. This balanced mixture is governed by unknown EoS. One of the EoS which could be employed to study the neutron star is the microscopic constrained variational calculations based on the cluster expansion. This EoS has been employed to study the structure of neutron star matter before [50, 106]. Fundamentally, the mentioned model is based on two-nucleon potentials which are the modern Argonne AV18 [107] and charged dependent Reid-93 [108]. It is notable that this method requires no free
parameter, has a good convergence and is more accurate comparing to other semi-empirical parabolic approximation methods. These advantages come from a microscopic computation of asymmetry energy which is carried on for the asymmetric nuclear matter calculations. The necessity of microscopic calculations with the modern nucleon-nucleon potentials which is isospin projection \((T_z)\) dependent was pointed out in Ref. \[109\]. Here, we employ the lowest order constrained variational (LOCV) method with the AV18 potential \[106\] for obtaining the modern EoS for neutron star matter and investigating some physical properties of neutron star structure.

As we mentioned, the energy of the system under study is obtained by the LOCV method which is a fully self-consistent formalism. Through a normalization constraint, this method keeps the higher order terms as small as possible \[110\]. In addition, this method has been employed to calculate the properties of neutron, nuclear and asymmetric nuclear matters at zero and finite temperatures \[110–112\]. The functional minimization procedure represents an enormous computational simplification over the unconstrained methods which attempt to go beyond the lowest order.

A trial many-body wave function is

\[
\psi = F\phi, \tag{15}
\]

where \(\phi\) is the uncorrelated ground-state wave function of \(N\) independent neutrons, and \(F\) is a proper \(N\)-body correlation function. Here, we apply Jastrow approximation \[113\] to replace \(F\) as

\[
F = S \prod_{i>j} f(ij), \tag{16}
\]

where \(S\) and \(f(ij)\) are a symmetrizing operator and the two-body correlation function, respectively. Besides, we consider a cluster expansion of the energy functional up to the two-body term

\[
E([f]) = \frac{1}{N} \left< \frac{\psi|H|\psi}{\psi|\psi} \right> = E_1 + E_2, \tag{17}
\]

in which \(\psi\) and \(H\) are wave function and Hamiltonian system, respectively. In other words, the energy per particle up to the two-body term is

\[
E([f]) = E_1 + E_2, \tag{18}
\]

where \(E_1 = \sum_{i=+,-} \frac{3}{4} \hbar^2 k_f^2(r^2) \rho(i)\) and \(E_2 = \frac{1}{2N} \sum_{i,j} \langle ij|\nu(12)|ij-j\rangle\) are one-body and two-body energy terms, respectively. It is notable that, \(k_f^2 = (6\pi^2 \rho(i))^{1/3}\) is the Fermi momentum of a neutron with spin projection \(i\). The operator \(\nu(12)\) is nuclear potential and it has been given in Ref. \[114\] (see Refs. \[109\] for more details). The behavior of obtained EoS of neutron star matter is shown in Fig. 1. We extract the mathematical forms for the EoS presented in Fig. 1 as

\[
P = \sum_{i=1}^{7} A_i \rho^{7-i}, \tag{19}
\]

in which \(A_i\) are

\[
A_1 = -3.518 \times 10^{-57}, \quad A_2 = 3.946 \times 10^{-41}, \quad A_3 = -1.67 \times 10^{-25}, \\
A_4 = 3.242 \times 10^{-10}, \quad A_5 = -1.458 \times 10^{9}, \quad A_6 = 2.911 \times 10^{19}, \\
A_7 = -9.983 \times 10^{31}.
\]

In order to investigate the properties of such EoS with more details, we study Le Chatelier’s principle condition in the following subsection.

1. **Le Chatelier’s principle**

The matter of star satisfies \(dP/d\rho \geq 0\) which is a necessary condition of a stable body both as a whole and also with respect to the non-equilibrium elementary regions with spontaneous contraction or expansion (Le Chatelier’s principle) \[115\]. As one can see, in Fig. 1 Le Chatelier’s principle is established.

The stability \(0 \leq \nu^2 = \left(\frac{dP}{d\rho}\right) \leq c^2\) and energy conditions for this EoS are investigated in Ref. \[54\], and it is shown that this EoS satisfied these conditions.
FIG. 1: Equation of state of neutron star matter (pressure, $P$ ($10^{35}$ erg/cm$^3$) versus density, $\rho$ ($10^{15}$ g/cm$^3$)).

B. Mass-radius relation and other properties of neutron star in massive gravity

Considering the maximum gravitational mass of a neutron star for dynamical stability against gravitational collapse into a black hole, one is able to make differences between neutron star and black holes. In other words, there is a critical maximum mass for the massive object in which for masses larger than the maximum value, the massive object becomes a black hole [105]. The value of maximum mass originated from the nucleons degeneracy pressure is evidently the possible maximum mass of neutron star. Therefore, obtaining the maximum gravitational mass of neutron star is of a great interest, and important in astrophysics. Unfortunately, the advanced observational technologies for measuring the mass of neutron star by investigating the X-ray pulsars and X-ray bursters were not able to produce accurate results. Nevertheless the measurements that are done with the binary radio pulsars [116–119], provided highly accurate results for the mass of neutron star. In Ref. [49], the Einstein gravity has been investigated, and maximum mass of neutron star has been obtained using the modern equations of state of neutron star matter obtained from the microscopic calculations. It was shown that the maximum mass of neutron star is about $1.68 M_\odot$. In addition, the EoS with dilaton gravity was employed and the properties of neutron star were investigated [56]. The results showed that by increasing the effects of dilaton gravity, the maximum mass of this star decreases ($M_{\text{max}} \leq 1.68 M_\odot$). Here, we intend to obtain the maximum mass of neutron star by considering the obtained TOV equation for Einstein-massive gravity (Eq. (13)) and investigate the properties of neutron star.

Now, by employing the EoS of neutron star matter presented in Fig. 1 and numerical approach for integrating the HEE obtained in Eq. (13), we can calculate the maximum mass and other properties of the neutron star. To do so, one can consider the boundary conditions $P(r = 0) = P_\odot$ and $m(r = 0) = 0$, and integrates Eq. (13) outwards to a radius $r = R$ in which $P$ vanishes for selecting a $\rho_c$. This leads to the neutron star radius $R$ and mass $M = m(R)$. We present the results in different figures and tables (see Figs. 2 and 3, and tables I, II and III for more details).

It is notable that, here, we ignore the effects of cosmological constant on the structure of neutron star. For investigating its effects, we refer the interested reader to Ref. [103], in which, it was shown that, this constant has no effect on the structure of this star when the cosmological constant is about $10^{-52} m^{-2}$. Now, we are in a position to study the properties of neutron star in massive gravity. First, we consider the mass of graviton about $1.78 \times 10^{-55} g$, which was obtained in Ref. [120]. Next, we use obtained results by A. W. Steiner et al. [121] in which an empirical dense matter EoS from a heterogeneous data set of six neutron stars was obtained. Their results showed that the radius of a neutron star must be in the range of $R \leq (11 \sim 14) km$. In the present paper, we consider the maximum radius of neutron star in the range of $R \leq 11 km$ and investigate the maximum mass for neutron star in the massive gravity by employing the modern EoS of neutron star matter derived from microscopic calculations. According to the table [1] considering the spacial values for the parameters of the modified TOV equation, the maximum mass of neutron star is an increasing function of $m^2 c_2$. Calculations show that the maximum mass of neutron star can be more than $3 M_\odot$ ($M_{\text{max}} \approx 3.8 M_\odot$), whereas in the Einstein gravity and by using this EoS, the maximum mass was in the range of $M_{\text{max}} \leq 1.68 M_\odot$. A mass measurement for PSR J1614-2230 [122] showed that the mass for neutron star was about $2 M_\odot$. In other words, our results cover the mass measurement of massive neutron star, and also, predict that the mass of neutron star in massive gravity can be in the range upper than $3 M_\odot$ (see the table [1] for more
TABLE I: Structure properties of neutron star in massive gravity for \( C = 2 \) and \( m^2c_1 = 3.168 \times 10^{-5} \).

| \( m^2c_2 \)     | \( M_{\text{max}} (M_\odot) \) | \( R (\text{km}) \) | \( R_{\text{Sch}} (\text{km}) \) | \( \bar{\rho} \left( 10^{14} \text{g cm}^{-3} \right) \) | \( \sigma(10^{-1}) \) | \( z(10^{-1}) \) |
|-----------------|-------------------------------|-------------------|-------------------------------|-------------------------------|-------------------|-------------------|
| \(-3.168 \times 10^{-5}\) | 1.68                          | 8.42              | 4.95                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-4}\) | 1.68                          | 8.42              | 4.95                          | 13.36                          | 5.88              | 5.57              |
| \(-3.168 \times 10^{-3}\) | 1.71                          | 8.47              | 4.97                          | 13.36                          | 5.87              | 5.57              |
| \(-1.584 \times 10^{-2}\) | 1.84                          | 8.68              | 5.10                          | 13.36                          | 5.87              | 5.57              |
| \(-3.168 \times 10^{-2}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-6.337 \times 10^{-2}\) | 2.36                          | 9.43              | 5.56                          | 13.36                          | 5.89              | 5.59              |
| \(-9.505 \times 10^{-2}\) | 2.73                          | 9.89              | 5.83                          | 13.40                          | 5.89              | 5.61              |
| \(-1.267 \times 10^{-1}\) | 3.11                          | 10.34             | 6.08                          | 13.36                          | 5.88              | 5.58              |
| \(-1.584 \times 10^{-1}\) | 3.52                          | 10.76             | 6.35                          | 13.41                          | 5.90              | 5.62              |
| \(-1.774 \times 10^{-1}\) | 3.76                          | 11.00             | 6.48                          | 13.40                          | 5.89              | 5.60              |

TABLE II: Structure properties of neutron star in massive gravity for \( m^2c_1 = 3.168 \times 10^{-5} \) and \( m^2c_2 = -3.168 \times 10^{-2} \).

| \( C \)          | \( M_{\text{max}} (M_\odot) \) | \( R (\text{km}) \) | \( R_{\text{Sch}} (\text{km}) \) | \( \bar{\rho} \left( 10^{14} \text{g cm}^{-3} \right) \) | \( \sigma(10^{-1}) \) | \( z(10^{-1}) \) |
|-----------------|-------------------------------|-------------------|-------------------------------|-------------------------------|-------------------|-------------------|
| \(0.01\)       | 1.68                          | 8.42              | 4.20                          | 13.36                          | 5.60              | 5.58              |
| \(0.10\)       | 1.68                          | 8.42              | 4.84                          | 13.36                          | 5.75              | 5.58              |
| \(0.50\)       | 1.70                          | 8.45              | 4.96                          | 13.37                          | 5.87              | 5.58              |
| \(1.00\)       | 1.76                          | 8.55              | 5.03                          | 13.37                          | 5.88              | 5.58              |
| \(2.00\)       | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(3.00\)       | 2.45                          | 9.55              | 5.62                          | 13.36                          | 5.88              | 5.58              |
| \(4.00\)       | 3.11                          | 10.34             | 6.08                          | 13.36                          | 5.88              | 5.58              |
| \(4.73\)       | 3.76                          | 11.00             | 6.49                          | 13.40                          | 5.90              | 5.61              |

TABLE III: Structure properties of neutron star in massive gravity for \( C = 2 \) and \( m^2c_2 = -3.168 \times 10^{-2} \).

| \( m^2c_1 \)     | \( M_{\text{max}} (M_\odot) \) | \( R (\text{km}) \) | \( R_{\text{Sch}} (\text{km}) \) | \( \bar{\rho} \left( 10^{14} \text{g cm}^{-3} \right) \) | \( \sigma(10^{-1}) \) | \( z(10^{-1}) \) |
|-----------------|-------------------------------|-------------------|-------------------------------|-------------------------------|-------------------|-------------------|
| \(-3.168 \times 10^{-14}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-13}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-12}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-11}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-10}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-11}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
| \(-3.168 \times 10^{-10}\) | 2.01                          | 8.94              | 5.26                          | 13.36                          | 5.88              | 5.58              |
Central density may exceed a few times as $10^{15} \rho_{\odot}$ reduce to the obtained results of massless Einstein gravity [49].

Not affected. In other words, considering the value of $m$ details). Also, by decreasing the value of $m^2c_2$ less than $(10^{-4})$, the maximum mass and radius of neutron star are not affected. In other words, considering the value of $m^2c_2$ about $-10^{-4}$, the maximum mass and radius of neutron star reduce to the obtained results of massless Einstein gravity [49].

On the other hand, the average density $\rho_c$ of the neutron star calculated in the tables [II and III] shows that the central density may exceed a few times as $10^{15} g cm^{-3}$. In other words, it is larger than the normal nuclear density, $\rho_0 = 2.7 \times 10^{14} g cm^{-3}$ [107].

For further investigation, we plot the mass of neutron star versus the central mass density ($\rho_c$) in left panels of Figs. 2 and 3. As one can see, the maximum mass of this star increases as $m^2c_2$ increases. On the other hand, the variation of maximum mass versus radius is also shown in right panels of Figs. 2 and 3.

Now, we complete our discussion by considering the gravitational mass equal to $1.78 \times 10^{-65} g$, with various values for different parameters of modified TOV equation (see Eq. (13)) and obtain the maximum mass of neutron star in the massive gravity. The results are presented in the tables [I and III]. According to the table [II], the maximum mass of neutron star is an increasing function of $C$. It is notable that considering the values less than 0.1 for $C$, the maximum mass and corresponding radius of neutron star are not affected. In other words, these results reduce to the obtained results of the maximum mass and radius of neutron star in the Einstein gravity [49]. The variation of $m^2c_1$ has very interesting effects. In this case, the maximum mass and radius of this star are constant and by variation of $m^2c_1$, these quantities are not affected (see the table [III]).

For completeness, in the following, we investigate other properties of neutron star in this gravity such as the Schwarzschild radius, average density, compactness, the gravitational redshift and dynamical stability.

1. modified Schwarzschild Radius

It is clear that by applying the massive term to the Einstein gravity, the Schwarzschild radius is modified. Considering Eq. (12) and using the horizon radius constraint ($g(r) = 0$), we can obtain the Schwarzschild radius ($R_{Sch}$) for the EN-massive gravity. After some calculations, the Schwarzschild radius for this gravity without the cosmological constant is obtained as

$$R_{Sch} = \frac{c \left(1 - m^2c_2C^2\right)}{m^2cc_1C} - \sqrt{\frac{c^2 (m^2c_2C^2 - 1)^2 - 4m^2c_1CGM}{m^2cc_1C}}.$$ (20)
FIG. 3: Gravitational mass versus central mass density (radius), $\rho_c \,(10^{15}\text{gr/cm}^3)$, for $m^2c_1 = 3.168 \times 10^{-12}$ and $m^2c_2 = -3.168 \times 10^{-2}$.

Left diagrams: gravitational mass versus central mass density for $C = 1.20$ (bold line), $C = 2.32$ (doted line), $C = 3.08$ (dashed line), $C = 3.65$ (dashed-dotted line) and $C = 3.85$ (continuous line).

Right diagrams: gravitational mass versus radius for $C = 1.20$ (bold line), $C = 2.32$ (doted line), $C = 3.08$ (dashed line), $C = 3.65$ (dashed-dotted line) and $C = 3.85$ (continuous line).

Using the series expansion of $R_{Sch}$ for the limit $m^2 \rightarrow 0$, we find that

$$R_{Sch} \approx \frac{2GM}{c^2} + \frac{2GMC (c^2c_2C + c_1GM)}{c^4} m^2 + O(m^4),$$

where the first term is the Schwarzschild radius in Einstein gravity [123], as expected, and the second term indicates the massive correction.

In order to investigate the effects of various parameters on the modified Schwarzschild radius, one can look at the tables I, II and III. As one can see in tables I and II, by increasing the maximum mass and radius of neutron star, the Schwarzschild radius increases and these stars are out of the Schwarzschild radius. Also, considering the negative value of $m^2c_2$ and increasing $m^2c_2$, the Schwarzschild radius increases (see table I). On the other hand, by increasing $C$, the Schwarzschild radius increases (see table II). In addition, considering the positive (negative) values of $m^2c_1$ and increasing (decreasing) $m^2c_1$, the Schwarzschild radius almost does not change (see table III).

2. Average Density

Now, using the maximum mass and radius obtained in the massive gravity, we can calculate the average density of neutron star in $4-$dimensions as

$$\rho = \frac{3M}{4\pi R^3},$$

where the results for variation of the massive parameters are presented in the tables I, II and III. Considering different parameters introduced in this theory, the average density of this star is almost the same. In other words, by variations of the different parameters, the average density remains fixed.

3. Compactness

The compactness of a spherical object may be defined by the ratio of Schwarzschild radius to radius of that object

$$\sigma = \frac{R_{Sch}}{R}.$$
which may be indicated as the strength of gravity. For the massive gravity, we obtain the values of \( \sigma \) in the tables I, II and III. For different values of \( m^2c_1 \) and \( C \), the results show that the strength of gravity is almost the same (see tables I and II). But, for different values of \( m^2c_1 \), there are two interesting behaviors. A) considering the positive value of \( m^2c_1 \) and increasing \( m^2c_1 \), the strength of gravity do not change. B) considering the negative values of \( m^2c_1 \) and increasing \( m^2c_1 \), the strength of gravity decreases and the strength of gravity is not affected for \( m^2c_1 > -3.168 \times 10^{-13} \) (see the table III).

4. Gravitational redshift

Considering Eq. (12) for vanishing \( \Lambda \) and by using definition of the gravitational redshift, we obtain this quantity in the massive gravity as

\[
z = \frac{1}{\sqrt{1 - m^2c_1 \left( c_1^2 + c_2^2 \right) - \frac{2GM}{c^2r}} - 1},
\]

in which it reduces to the gravitational redshift in the Einstein gravity when \( m^2 = 0 \). The results show that, the gravitational redshift of neutron star is almost independent of different parameters. The gravitational redshift of each compact object depends on its average density, so as one can see, the average density for these stars are almost the same, therefore the gravitational redshift of them must be the same.

5. Dynamical Stability

The dynamical stability of stellar model against infinitesimal radial adiabatic perturbation was introduced by Chandrasekhar in Ref. [124]. This stability condition was developed and applied to astrophysical cases by many authors [125–128]. The adiabatic index (\( \gamma \)) is defined as

\[
\gamma = \frac{\rho c^2 + P}{c^2 P} \frac{dP}{d\rho}.
\]

It is notable that, in order to have the dynamical stability, \( \gamma \) should be more than \( \frac{4}{3} \) (\( \gamma > \frac{4}{3} = 1.33 \)) everywhere within the isotropic star. Therefore, we plot two diagrams related to \( \gamma \) versus radius for different values of \( m^2c_2 \) and \( C \) in Fig. 4. As one can see, these stellar models in massive gravity are stable against the radial adiabatic infinitesimal perturbations.

Also, we plot the density (pressure) versus distance from the center of neutron star. As one can see, the density and pressure are maximum at the center and they decrease monotonically towards the boundary (see Figs 5 and 6).
IV. NEUTRON STAR PROPERTIES VIA PLANCK MASS

Here, our aim is to obtain the mass of neutron star according to the Planck mass. The neutron stars are supported against the gravitational force by degeneracy pressure of nucleons which is mainly related to the strong repulsive inter-nucleons force. It is notable that the nucleon-nucleon interaction is so strong, and it is taken place through the pion exchange. Therefore, we can consider the average density of a neutron star in term of the nucleus density using the following form (see Refs. [54, 129] for more details)

\[ \rho_{nuc} \sim \frac{3m_p}{4\pi\lambda_\pi^3} \]  \hspace{1cm} (26)

where \( m_p \) and \( \lambda_\pi = \frac{\hbar}{m_\pi c} \) are, respectively, the proton mass and Compton wavelength (\( m_\pi \) is the pion mass). Now, we are going to use an analogy for obtaining a relation between the mass of neutron star in massive gravity and the Planck mass. Using the equations (20) and (22) and by considering \( \rho_{nuc} \), one can derive the following corresponding
mass as

\[
\frac{3m_p}{4\pi \lambda^4} \sim \frac{3M}{4\pi R_{S,c}^3},
\]

\[
M \sim \frac{1}{128m^2c_1^2m^2_p m^6 \pi^2} \left\{ \left[ A_1 + 8cm_p m^3 G \right] \sqrt{32m^4c_1^2C^2h^3 \left( \frac{m^4c_1^2C^2h^3}{32} - A_2 \right)} + 24m^4c_1^2C^2h^3 \left[ m^2C^2A_3 - cm_p m^3 G \right] \right\},
\]

where

\[
A_1 = m^2C^2 \left( m^2C^2h^3/3 - 8cc_2m_p m^3 G \right),
\]

\[
A_2 = cGm^3m^3 \left( m^2C^2 - 1 \right),
\]

\[
A_3 = cc_2m_p m^3 G - \frac{m^2C^2h^3}{24}.
\]

Now, we use the relation between the proton (pion) mass and the Planck mass \[129\] to obtain the mass of neutron star with respect to the Planck mass

\[
m_p = \frac{m_{pl}}{\eta_p} \quad \& \quad m_\pi = \frac{m_{pl}}{\eta_\pi},
\]

\[
M \sim \frac{3\eta_p \eta_\pi^3}{16m^2m^8 \pi^2} \left\{ \left[ \frac{m^4c_1^2C^2h^3 \eta_\pi \eta_\pi^2}{24} - B_1 \right] \sqrt{\frac{32m^4c_1^2C^2h^3 \eta_\pi \eta_\pi^2}{32} - 3B_1} \right\} + m^4c_1^2C^2h^3 \left( \frac{m^4c_1^2C^2h^3 \eta_\pi \eta_\pi^2}{24} + 3B_1 \right),
\]

where

\[
B_1 = \frac{cm^4G \left( m^2c_2C^2 - 1 \right)}{3}.
\]

It is notable that in the absence of massive term \((m^2 \rightarrow 0)\), the obtained relation reduces to the usual general relativity case

\[
M \sim \left( \frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_p^2} \left( \eta_\pi \right)^{3/2} \sim m_{pl} \eta_p \left( \frac{\eta_\pi}{2\eta_p} \right)^{3/2}.
\]

Now, we are in a position to obtain a constraint on the neutron star radius. Regarding the fact that the radius of neutron star should be greater than the Schwarzschild radius, one can extract a limitation for the radius of neutron star via the Planck mass as a fundamental physical constant. Using Eq. \[28\] with \(R_{NS} > R_{S,c}\), we obtain

\[
R_{NS} > \frac{1}{8mc_1m^4 \pi^2} \left\{ 8\sqrt{2cGm^4 \left( m^2c_2C^2 - 1 \right)} \left( -m^2c_1 Ch_\pi \left[ m^4c_1^2C^2h^3\eta_\pi \eta_\pi^3 \right] - 24B_1 \right) \right. \times \left. \sqrt{\hbar_\pi \eta_\pi \left[ m^4c_1^2C^2h^3\eta_\pi \eta_\pi^3 \right] - 96B_1 \right) + \left( m^4c_1^2C^2h^3\eta_\pi \eta_\pi^3 \right)^2 - 72m^4c_1^2C^2h^3\eta_\pi \eta_\pi^3B_1 + 32c^2Gm^8 \left( m^2c_2C^2 - 1 \right)^2 \right\}^{1/2}.
\]

As a final comment, we should note that in the absence of the massive term, Eq. \[30\] reduces to

\[
R_{NS} > G \left( \frac{\eta_\pi}{2\eta_p} \right)^{1/2} \left( \frac{\eta_\pi}{cm_{pl}} \right) \left( \frac{\hbar c}{G} \right)^{3/2},
\]

which may indicate a minimum value for neutron star radius in usual general relativity.
V. CLOSING REMARKS

In this paper, we considered the spherically symmetric metric and extracted a modified TOV equation of stars in the Einstein-massive gravity in $d$-dimensions. Then, we showed that for $m \to 0$ limit, the obtained TOV in Einstein-massive gravity reduces to the Einstein-$\Lambda$ gravity. The generalization of modified TOV equation to arbitrary $d$-dimensions was also done (see appendix A). Furthermore, we have considered an EoS, which was derived from microscopic calculations and investigated Le Chatelier’s principle for the mentioned EoS. It was shown that this equation is suitable for investigating the structure of neutron star.

Considering the modified TOV obtained in this paper, the structure of neutron star was investigated. The results showed that, the maximum mass of these stars increases when $m^2c^2$ and $C$ increase (the results represented in various tables numerically). In addition, it was shown that by considering the constant values of $C$ and $m^2c^2$, the maximum mass of neutron star is independent of $m^2c_1$.

Then, we showed that by increasing the maximum mass of neutron star, the radius and the Schwarzschild radius increase as well. It is notable that, by regarding massive graviton, the Schwarzschild radius is modified. In order to conduct more investigations, we plotted some diagrams related to the mass-radius and mass-central mass density. We found that these figures are similar to the diagrams related to the mass-radius and mass-central mass density in usual GR. In addition, these diagrams confirmed the validity of obtained results in massive gravity.

After that the adiabatic index was investigated. It was shown that this star is dynamically stable. It is notable that the density and pressure are maximum at the center of the star and decrease monotonically towards the boundary.

Jacoby et al. [130] and Verbiest et al. [131] used the detection of Shapiro delay to measure the masses of both the neutron star and its binary component. Also, using the same approach, the masses of compact objects were obtained for Vela X-1 (about $1.8M_\odot$) [132], PSR J1614-2230 (about 1.97$M_\odot$) [122], PSR J0348+0432 (about 2.01$M_\odot$) [133], 4U 1700-377 (about 2.4$M_\odot$) [134] and J1748-2021B (about 2.7$M_\odot$) [135]. It is notable that, in this paper, we showed that the obtained maximum mass of neutron star in massive gravity can cover all the measured masses of pulsars and neutron stars. Also, we predicted the existence of possible mass of more than $3M_\odot$.

Briefly, we obtained the quite interesting results from massive gravity for the neutron star such as:

I) Obtaining the modified TOV equation. II) Prediction of maximum mass for neutron star more than $3M_\odot$ ($M_{\text{max}} \approx 3.8M_\odot$), due to the existence of massive gravitons. III) Dynamically stable neutron star in the massive gravity. IV) EoS derived from microscopic calculations satisfied the energy, stability conditions and Le Chatelier’s principle, simultaneously. V) The Schwarzschild radius was modified in the presence of massive gravity. VI) Due to the considering massive graviton, the gravitational redshift was modified. VII) The relations between the mass and the radius of neutron star versus the Planck mass as a fundamental physical constant were extracted. VIII) Our consequences covered previous results and reduce to the Einstein gravity for massless graviton ($m = 0$), as expected.

Finally, it is notable that the investigation of other compact objects such as quark star and white dwarf in the context of massive gravity and its modified TOV equation are interesting subjects. Moreover, it is worth studying the effects of higher dimensions and other equation of states on the structure of compact objects. Also, anisotropic compact objects [131,142], rotating, slowly rotating [143,149], rapidly rotating [150,155] neutron stars and obtain the Buchdahl limit [156,162] in the context of massive gravity are interesting topics. Furthermore, regarding the considerable effects of free parameters on the existence of tachyon-like instabilities, it will be useful to address the mentioned substantial instability. We leave these issues for the future works.

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Appendix: Modified TOV equation in higher dimension

Here, we are interested in obtaining the modified TOV equation in Einstein-massive gravity in higher dimensions. So, we consider a spherical symmetric space-time in higher dimensions as

$$ds^2 = f(r)dt^2 - g^{-1}(r)dr^2 - r^2h_{ij}dx_i dx_j,$$  \hspace{1cm} (32)

where $i, j = 1, 2, 3, \ldots, d - 2$, and also $h_{ij}dx_i dx_j$ is the line element of a ($d - 2$)-dimensional unit sphere

$$h_{ij}dx_i dx_j = d\theta_1^2 + \sum_{i=2}^{d-2} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2.$$  \hspace{1cm} (33)
We also use the following ansatz for the reference metric which is introduced in Ref. 34:

\[ f_{\mu\nu} = \text{diag}(0, 0, C^2 r^2 h_{ij}). \]  

(34)

Using the mentioned information and ansatz, we can find the explicit functional forms of \( U_i \)'s as

\[
U_1 = \frac{d_2 C}{r}, \quad U_2 = \frac{d_2 d_3 C^2}{r^2}, \\
U_3 = \frac{d_2 d_3 d_4 C^3}{r^3}, \quad U_4 = \frac{d_2 d_3 d_4 d_5 C^4}{r^4},
\]

(35)

where we denoted \( d_i = d - i \). We can also obtain the nonzero components of the energy-momentum for \( d \)-dimensional perfect fluid as

\[
T^0_0 = \rho c^2 \quad \text{and} \quad T^i_0 = T^2 = T^3 = \ldots = T^{d-1} = -P.
\]

(36)

Considering the metric \([2] \) with Eq. \((36)\), we can find the components of Eq. \((2)\) are calculated as

\[
\begin{align*}
K_{d} e^{2} r^2 \rho &= \Lambda r^2 + \frac{d_2 d_3}{2} (1 - g) - \frac{d_2}{2} r g' - \frac{m^2 d_3 C}{2 r^{d_4}} \left[ c_1 r^{d_3} + d_3 c_2 C r^{d_4} + d_3 d_4 \left( c_3 C^2 r^{d_5} + d_5 c_4 C^3 r^{d_6} \right) \right], \\
K_{d} f r^2 P &= - \Lambda r^2 f - (1 - g) f + r g f' + \frac{m^2 d_3 C}{2 r^{d_4}} \left[ c_1 r^{d_3} + d_3 c_2 C r^{d_4} + d_3 d_4 \left( c_3 C^2 r^{d_5} + d_5 c_4 C^3 r^{d_6} \right) \right], \\
4 K_{d} f r^2 P &= - 4 \Lambda r f^2 + 2 \left( g f' \right)' - r g f'^2 + r \left[ g f' + 2 g f'' \right] f + \frac{2 m^2 d_3 C f^2}{r^{d_4}} \left[ c_1 r^{d_3} + d_4 c_2 C r^{d_5} + d_4 d_5 \left( c_3 C^2 r^{d_6} + c_4 C^3 r^{d_7} \right) \right].
\end{align*}
\]

(37-39)

Considering Eqs. \((37) - (39)\) and after some calculations, one can find a relation which is the same as Eq. \((11)\). In addition, we can obtain the functional form of \( g(r) \) by using Eq. \((37)\) as

\[
g(r) = 1 + \frac{2 \Lambda}{d_1 d_2} r^2 - \frac{c^2 K_d M(r) \Gamma \left( \frac{d_1}{d_2} \right)}{d_2 \pi^{d_1/2} r^{d_3}} - m^2 \left[ \frac{c_1 C}{d_2} r + c_2 C \frac{d_3 C^3}{r} + d_3 d_4 c_4 C^4 \right],
\]

(40)

where \( M(r) = \int \frac{2 \pi^{d_1/2}}{\Gamma(d_1/2)} r^2 \rho(r) dr \) and \( \Gamma \) is the gamma function, which satisfies some conditions such as \( \Gamma(1/2) = \sqrt{\pi} \), \( \Gamma(1) = 1 \) and \( \Gamma(x + 1) = x \Gamma(x) \).

Now, we obtain \( f' \) from Eq. \((38) \) and insert it with Eq. \((40)\) into Eq. \((11)\) to obtain the following higher dimensional HEE in Einstein-massive gravity

\[
\frac{dP}{dr} = \left( c^2 \rho + P \right) \left\{ \frac{d_1}{2} \left[ g(r) - 1 \right] - \frac{d_2}{r} \left( \Lambda + K_d P \right) + \frac{m^2 C}{2 r^{d_4}} \left[ (c_1 r^{d_3} + d_3 c_2 C r^{d_4}) \right] \right\},
\]

(41)

where \( g(r) \) is presented in Eq. \((40)\).

As a special case, it is notable that for \( m = 0 \), Eq. \((11)\) reduces to the following \( d \)-dimensional TOV equation obtained in Einstein-\( \Lambda \) gravity \([103]\):

\[
\frac{dP}{dr} = \left( c^2 \rho + P \right) \left[ \frac{d-1}{4 \pi^{(d-1)/2}} r^2 K_d M(r) \left[ \Lambda + \frac{d-1}{2} K_d P \right] \right] r^{-d-1} + \left( d - 1 \right) \frac{r^{d-1}}{2 \pi^{(d-1)/2}} c^2 K_d M(r) - \frac{d-2}{2} r^{d-3}.
\]

(42)

Regarding a suitable EoS for higher dimensional spacetime, with the obtained modified \( d \)-dimensional TOV equation, one can investigate the neutron stars in higher dimensional massive gravity. We leave the mentioned problem for the future works.
Appendix: B brief dimensional analysis of massive parameters and its values

Here, we are going to investigate the massive coefficients via dimensional analysis. In general, all terms of Eq. (40), including $m^2c_2$, $m^2c_2C^2$, $m^2d_3c_3C^3$ and $m^2d_3d_4c_4C^4$, must be dimensionless. On the other hand, in dimensional analysis we know that $[m] = M$, $[r] = L$ and $[d_n] = 1$. Therefore, the dimensional interpretation of massive terms are

$[c_1C] = M^{-2}L^{-1}$, \hspace{1cm} (43)

$[c_2C^2] = M^{-2}$, \hspace{1cm} (44)

$[c_3C^3] = M^{-2}L$, \hspace{1cm} (45)

$[c_4C^4] = M^{-2}L^2$. \hspace{1cm} (46)

Using Eqs. (43)-(46), one can show that massive coefficients are, dimensionally,

$[C] = L \quad \& \quad [c_i] = M^{-2}L^{-2}, \quad i = 1, 2, 3, 4$

On the other hand, regarding the dimensionless action (1), we find that dimensional interpretations of all $R$, $2\Lambda$ and $m^2\sum_i c_i u_i(g, f)$ are $L^{-2}$. Remembering that $[m^2c_i] = L^{-2}$, one can conclude that $U_i$'s are dimensionless. In addition, like the cosmological constant, $m^2c_i$ terms could play the role of the pressure in the extended phase space (see Ref. [163] to find details regarding the relation between pressure and the cosmological constant).

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