Two-loop effects in the evolution of non-forward distributions.

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Abstract

We study the effects of next-to-leading order corrections on the evolution of the twist-two non-forward parton distribution functions in the flavour non-singlet sector. It is found that the deviation from leading order evolution is small for all values of the parton momentum fraction variable for moderately large values of the scale parameter.

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1. Introduction. Recently there was significant progress in the perturbative QCD approach to deeply virtual Compton scattering — a process which allows to access the so-called non-forward parton distribution functions [1, 2, 3, 4]. The latter possess hybrid properties: in different regions of phase space they share features with ordinary parton densities of deep inelastic scattering and with exclusive distribution amplitudes. They smoothly interpolate between these two limits. At the present stage all the required perturbative inputs are available, i.e. one-loop coefficient functions [5, 6, 7, 8, 9] and two-loop anomalous dimensions of the moments of the non-forward functions [10, 11], which make possible its analysis in next-to-leading order (NLO) of perturbation theory in the flavour singlet channel. In the above list of corrections the latter were derived within a formalism which allows to determine the eigenfunctions of the two-loop generalized Efremov-Radyushkin-Brodsky-Lepage (ER-BL) evolution equations in closed analytical form. This is a crucial result which is not accessible in the direct calculation of NLO kernels.

The leading order solution of the evolution equations, which are governed by the kernels evaluated in Refs. [1, 2, 3, 12, 13, 14, 6, 15], in terms of the conformal partial wave expansion was given in Refs. [3, 6, 16] (see also [17] and for a direct numerical integration of evolution equation Ref. [12]). In two-loop approximation the corrections to the eigenfunctions were derived in [6, 11]. In the present investigation we will use these results for an explicit study of the magnitude of these effects on the evolution of the flavour non-singlet non-forward parton distribution function versus the LO evolution considered by us previously [16].

2. Solution of the two-loop evolution equation. To start with let us first describe the formalism and spell out our conventions. Here we adopt the definition of the non-forward parton distributions introduced by Radyushkin [2, 3] which fulfill the evolution equation

\[ \mu^2 \frac{d}{d\mu^2} \mathcal{O}(x, \zeta) = \int dx' K \left( x, x', \zeta \mid \alpha_s(Q^2) \right) \mathcal{O}(x', \zeta), \]  

where the kernel \( K (x, x', \zeta \mid \alpha_s(Q^2)) \) can be calculated order by order in perturbation theory. Since the leading order evolution equation can be diagonalized with the help of the conformal operators it is convenient to employ this partial conformal wave expansion also beyond leading order although the Gegenbauer polynomials are not the eigenfunctions of the two-loop generalized ER-BL equation. Namely, the solution of the Eq. (1) can be written in the form

\[ \mathcal{O}(x, \zeta, Q^2) = \sum_{j=0}^{\infty} \phi_j \left( x, \zeta \mid \alpha_s(Q^2) \right) \tilde{\mathcal{O}}_j(\zeta, Q^2), \]  

where the multiplicative renormalizable moments evolve as follows

\[ \tilde{\mathcal{O}}_j(\zeta, Q^2) = \exp \left\{ -\frac{1}{2} \int_{Q_0^2}^{Q^2} \frac{d\tau}{\tau} \gamma_j^D (\alpha_s(\tau)) \right\} \tilde{\mathcal{O}}_j(\zeta, Q_0^2), \]
with the forward anomalous dimensions we need at \( \mathcal{O}(\alpha_s^2) \) accuracy \( \gamma_j^D(\alpha_s) = \frac{\alpha_s}{2\pi} \gamma_j^0 + \left( \frac{\alpha_s}{2\pi} \right)^2 \gamma_j^{(1)} + \ldots \), where \( \gamma_j^{(0)} = -C_F \left( 3 + \frac{2}{(j+1)(j+2)} - 4\psi(j+2) + 4\psi(1) \right) \) and \( \gamma_j^{(1)} \) can be found in Ref. [18]. We have chosen the initial condition so that there are no radiative corrections at the low normalization point \( Q_0^2 \) so that the \( \tilde{O}_j(\zeta, Q_0^2) \) are given by ordinary Gegenbauer moments of the non-forward distribution which are related to the matrix elements of the tree level conformal operators by

\[
\tilde{O}_j(\zeta, Q_0^2) = \int dx \ C_j^{3/2} \left( 2 \frac{x}{\zeta} - 1 \right) \mathcal{O}(x, \zeta, Q_0^2) = \frac{1}{\mathcal{Q}} \langle h' | \bar{\psi}(i\partial_+)^j \Gamma C_j^{3/2} \left( \frac{\partial^+}{\partial^+} \right) \psi \rangle \bigg|_{Q_0^2} |h\rangle. \tag{4}
\]

The problem is reduced to finding the correction to the eigenfunction. It was solved in our previous studies [3, 10, 11, 19]

\[
\phi_j(x, \zeta | \alpha_s(Q^2)) = \phi_j(x, \zeta) + \frac{\alpha_s(Q^2)}{2\pi} \sum_{k=j+2}^{\infty} \phi_k(x, \zeta) \Phi_{kj} \left( \alpha_s(Q^2) \right), \tag{5}
\]

where the LO partial conformal waves are defined via the Gegenbauer polynomials

\[
\phi_j(x, \zeta) \equiv \frac{1}{N_j} \frac{x}{\zeta^2} \left( 1 - \frac{x}{\zeta} \right) C_j^{3/2} \left( 2 \frac{x}{\zeta} - 1 \right), \quad \text{with} \quad N_j = \frac{(j+1)(j+2)}{4(2j+3)}. \tag{6}
\]

The function \( \Phi_{jk} \) is [3, 10, 11, 19]

\[
\Phi_{jk} \left( \alpha_s(Q^2) \right) = S_{jk} \left( \alpha_s(Q^2) \right) \left\{ d_{jk} \left( \gamma_k^{(0)} - \beta_0 \right) - g_{jk} \right\}, \tag{7}
\]

with

\[
d_{jk} = -\frac{1}{2} \left[ 1 + (-1)^{j-k} \right] \frac{(2k+3)}{(j-k)(j+k+3)}, \quad g_{jk} = 2C_F \ d_{jk} \left\{ 2A_{jk} + (A_{jk} - \psi(j+2) + \psi(1)) \frac{(j-k)(j+k+3)}{(k+1)(k+2)} \right\}, \tag{8, 9}
\]

\[\text{with} \quad A_{jk} = \psi \left( \frac{j+k+4}{2} \right) - \psi \left( \frac{j-k}{2} \right) + 2\psi(j-k) - \psi(j+2) - \psi(1).\]

The factor \( S_{jk} \) appears as result of the evolution of the coupling constant and reads

\[
S_{jk} \left( \alpha_s(Q^2) \right) = \frac{\gamma_j^{(0)} - \gamma_k^{(0)}}{\gamma_j^{(0)} - \gamma_k^{(0)} + \beta_0} \left( 1 - \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{1+\left( \frac{\gamma_j^{(0)} - \gamma_k^{(0)}}{\alpha_s(Q_0^2)} \right)/\beta_0}, \tag{10}
\]

with \( \beta_0 = \frac{4}{3} T_F N_f - \frac{14}{3} C_A \).

Obviously, taken as they stand the above Eqs. (2, 4) are valid only for the non-forward distributions with support \( 0 \leq x \leq \zeta \) since the Gegenbauer polynomials \( C_j^r(2x-1) \) form a complete set only on the interval \( x \in [0, 1] \). But as has been noted in Ref. [3] that means that we
can understand the above expansion only in a restricted sense. Namely, we have to represented it in the form

$$\left[ \frac{x}{\zeta^2} \left( 1 - \frac{x}{\zeta} \right) \right]^{\nu - 1/2} C_j^\nu \left( 2 \frac{x}{\zeta} - 1 \right) = 2^{1-2\nu} \frac{\Gamma \left( \frac{j}{2} \right) \Gamma (j + 2\nu)}{\Gamma (\nu) \Gamma (j + \nu + \frac{1}{2}) \Gamma (j + 1)} \int_0^1 dt (\zeta t)^{j+\nu-1/2} \delta^{(j)} (\zeta t - x),$$

(11)

and treat the RHS as a mathematical distribution. In order to circumvent this disadvantage we expand the non-forward distribution in a series of polynomials $P_j(x)$ orthogonal in the region $0 \leq x \leq 1$. Then the expansion coefficients will be given as convolution of $P_j(x)$ with the RHS of Eq. (11).

Note that we can exploit any set of orthogonal polynomials $P_j(x)$ available. Namely, the general expansion looks like

$$\mathcal{O}(x, \zeta, Q^2) = \sum_{j=0}^{\infty} \tilde{P}_j(x) M_j^P (\zeta, Q^2),$$

(12)

where the conjugated polynomials $\tilde{P}_j(x)$ are defined such that

$$\int_0^1 dx \tilde{P}_j(x) P_k(x) = \delta_{jk}.$$ 

(13)

And the moments $M_j^P (\zeta, Q^2)$ are given by a finite sum

$$M_j^P (\zeta, Q^2) = \sum_{k=0}^{j} E_{jk}^P (\zeta) \mathcal{O}_k (\zeta, Q^2),$$

(14)

with $\mathcal{O}_j$-moments expressed in terms of the original ones as follows

$$\mathcal{O}_j (\zeta, Q^2) = \tilde{\mathcal{O}}_j (\zeta, Q^2) + \frac{\alpha_s (Q^2)}{2\pi} \sum_{k=0}^{j-2} \Phi_{jk} \left( \alpha_s (Q^2) \right) \tilde{\mathcal{O}}_k (\zeta, Q^2).$$

(15)

Taking into account the above observation, the expansion coefficients are given by the integral

$$E_{jk}^P (\zeta) = \int_0^1 dx \frac{x \bar{x}}{N_k} C_k^{3/2} (2x - 1) P_j (\zeta x).$$

(16)

Let us repeat that we can use any appropriate polynomials for this purpose. The criterion for choosing a specific one is thus the fastest convergence of the series. Below we give the result for the expansion coefficients of the Jacobi polynomials. Namely,

$$P_j (x) = P_j^{(\alpha, \beta)}(2x - 1), \quad \tilde{P}_j (x) = \frac{x^\alpha \bar{x}^\beta}{n_j (\alpha, \beta)} P_j^{(\alpha, \beta)}(2x - 1),$$

with

$$n_j (\alpha, \beta) = \frac{\Gamma (j + \alpha + 1) \Gamma (j + \beta + 1)}{(2j + \alpha + \beta + 1) j! \Gamma (j + \alpha + \beta + 1)}$$

(17)

For this reason we omit the spectral constraint which appears on the LHS as a result of integration.
The expansion coefficients can easily be obtained by the methods developed in Ref. [11] and read

\[ E^{J}_{jk}(\zeta) = (-1)^{j-k} \theta_{jk} \frac{\Gamma(k+2) \Gamma(j+\beta+1) \Gamma(j+k+\alpha+\beta+1)}{\Gamma(2k+3) \Gamma(k+\beta+1) \Gamma(j-k) \Gamma(j+\alpha+\beta+1)} \times 2^{\frac{k}{3}} F_{2} \left( -j+k, j+k+\alpha+\beta+1, k+2 \middle| 2k+4, k+\beta+1 \right). \tag{18} \]

The results for all other classic orthogonal polynomials immediately follow from this expression. For special values of the parameters the Jacobi polynomials coincide \[20\] either with Gegenbauer\[1\], \[P^{(\lambda-\frac{1}{2},\lambda-\frac{1}{2})}_{j}(x) = \frac{\Gamma(j+\lambda+\frac{1}{2})}{\Gamma(j+\lambda+\frac{1}{2})} C_{j}^{\lambda}(x)\], or Legendre\[1\], \[P^{(0,0)}_{j}(x) = P_{j}(x)\], or Chebyshev polynomials of the first, \[P^{(-\frac{1}{2},-\frac{1}{2})}_{j}(x) = \frac{\Gamma(j+\frac{1}{2})}{\sqrt{\pi}j!} T_{j}(x)\] and second kind, \[P^{(\frac{1}{2},\frac{1}{2})}_{j}(x) = \frac{\Gamma(j+\frac{3}{2})}{\sqrt{\pi}j!} U_{j}(x)\].

The solution of the renormalization group equation (3) in two-loop approximation can be written in the form

\[ \tilde{O}_{j}(\zeta, Q^{2}) = \tilde{O}_{j}(\zeta, Q_{0}^{2}) \left( \frac{\alpha_{s}(Q_{0}^{2})}{\alpha_{s}(Q^{2})} \right)^{\gamma_{j}^{(0)} / \beta_{0}} \left( \frac{\beta_{0} + \beta_{1} \alpha_{s}(Q^{2})}{\beta_{0} + \beta_{1} \alpha_{s}(Q_{0}^{2})} \right)^{\gamma_{j}^{(0)/\beta_{0}} - 2 \gamma_{j}^{(1)/\beta_{1}}} \]

\[ = \tilde{O}_{j}(\zeta, Q_{0}^{2}) \left( \frac{\alpha_{s}(Q_{0}^{2})}{\alpha_{s}(Q^{2})} \right)^{\gamma_{j}^{(0)} / \beta_{0}} \left\{ 1 + \left( \frac{\beta_{1} \gamma_{j}^{(0)}}{2 \beta_{0}^{2}} - \frac{\gamma_{j}^{(1)}}{\beta_{0}} \right) \alpha_{s}(Q^{2}) - \alpha_{s}(Q_{0}^{2}) \right\} \tag{19} \]

where the expansion in the second line is done in order to treat two-loop corrections to the evolution on the same footing as one-loop corrections to the Wilson coefficients, which when both are summed in the amplitude allows to minimize the renormalization scheme dependence (see, for instance, the first paper in Ref. [18] and [21]). The coupling constant in NLO of perturbation theory can be approximated by

\[ \alpha_{s}(Q^{2}) = -\frac{4\pi}{\beta_{0} \ln(Q^{2}/\Lambda_{MS}^{2})} \left( 1 + \frac{\beta_{1} \ln(Q^{2}/\Lambda_{MS}^{2})}{\beta_{0} \ln(Q^{2}/\Lambda_{MS}^{2})} \right). \tag{20} \]

where \( \beta_{1} \) is the second coefficient in the expansion of the QCD \( \beta \)-function \( \beta_{g} = \frac{\alpha_{s}}{4\pi} \beta_{0} + \left( \frac{\alpha_{s}}{4\pi} \right)^{2} \beta_{1} + \ldots \)

and it reads \( \beta_{1} = \frac{16}{3} C_{A} N_{f} + 2 C_{F} N_{f} - \frac{34}{3} C_{A}^{2} \).

3. NLO evolution of the model distributions. In this section we will use the results given above for explicit numerical studies of the evolution of the non-forward parton distributions. To this end we need an initial condition for the evolution equation. The most adequate model for the low scale input functions was given by Radyushkin in Ref. [22]. Namely, \( O(x, \zeta) \) is defined in terms of the double distribution function \( F(y, z) \) \[3\] via the following relation

\[ O(x, \zeta, Q_{0}^{2}) = \int_{0}^{1} dy \int_{0}^{1} dz F(y, z, Q_{0}^{2}) \theta(1-y-z) \delta(x-y-\zeta z). \tag{21} \]

\[ ^{3}\text{We have used them in our preliminary study of LO evolution effects [16].} \]

\[ ^{4}\text{This possibility has been discussed in Ref. [17] but the authors did not manage to find an explicit analytical expression for the expansion coefficients.} \]
Here we have omitted the dependence on the \( t \)-channel momentum transferred squared. Although for a massive target the formal limit \( t \to 0 \) is not accessible for nonvanishing \( \zeta \) due to the kinematical restriction, \( m_h^2 \zeta^2 / \bar{\zeta} \leq -t \) \([3, 23]\), this condition does not affect the results of the evolution.

For our study we accept the following model for the double distribution function corresponding to the non-singlet function \( \mathcal{O}(x, \zeta, Q_0^2) \) \([22]\)

\[
F(y, z, Q_0^2) = q(y, Q_0^2)\pi(y, z), \tag{22}
\]

with the plausible profile functions \( \pi(y, z) \)

\[
\pi(y, z) = 6\frac{z}{y^3}(\bar{y} - z). \tag{23}
\]

In Eq. (22) the function \( q(y, Q_0^2) \) is an ordinary forward parton density measured in deep inelastic scattering taken at a low normalization point. We will also consider the asymptotic distribution functions which although phenomenologically probably irrelevant serves as a good probe for the net evolution effects in NLO approximation since its scale dependence is governed by the non-diagonal elements of the anomalous dimension matrix of the conformal operators which appear only beyond leading order. It is given by the first term in the expansion \([2] \,[22, 16]\)

\[
\mathcal{O}_{as}(x, \zeta, Q_0^2) = 6\frac{x}{\zeta^2} \left( 1 - \frac{x}{\zeta} \right) \left( 1 - x \right) \mathcal{O}_0(\zeta, Q_0^2), \tag{24}
\]

(see Fig. 4 (a)) where \( \mathcal{O}_0(\zeta, Q_0^2) \) (see Eq. (4)) is a matrix element of the conserved local (axial-)vector current, \( \mathcal{O}_0(\zeta, Q_0^2) = \langle h' | \overline{\psi}(0) \Gamma \psi(0) | h \rangle \) with \( \Gamma = \gamma_+ + i\gamma_5 \). Therefore, it depends neither on the skewedness parameter nor on the renormalization point \( Q_0^2 \).

Since in present paper we are studying only the non-singlet evolution we consider the combination

\[
q^{NS}(x) = u(x) - d(x), \tag{25}
\]

and take as initial condition the following CTEQ4M parametrizations for the \( u \) and \( d \)-quark distributions \([24]\)

\[
u(x) = 1.344x^{-0.499}(1 - x)^{3.689}(1 + 6.042x^{0.873}), \tag{26}
\]

\[
d(x) = 0.640x^{-0.499}(1 - x)^{4.247}(1 + 2.690x^{0.333}), \tag{27}
\]

at an input scale \( Q_0 = 1.6 \) GeV. Then the above model reads form these and is given in Fig. 2 (a). Note that the factor \( \mathcal{O}_0(\zeta, Q_0^2) \) in the asymptotic distribution is defined by the first moments of \( q^{NS}(x) \), \( \mathcal{O}_0(\zeta, Q_0^2) = \int_0^1 dx \, q^{NS}(x) \). Let us mention that the models we use satisfy the positivity
Figure 1: Evolution of the asymptotic distribution function. The $x - \zeta$-dependence given by Eq. (24) is shown in (a). The input distribution (dashed-dotted line) was evolved in NLO approximation (solid line) up to $Q^2 = 100$ GeV$^2$ for the skewness parameters $\zeta = 0.1$ (b), $\zeta = 0.5$ (c) and $\zeta = 1.0$ (d).

Moreover, it is useful to mention that Eq. (21) with profile defined by (23) saturates the constraint inequality (28) in the region of its validity $x > \zeta$.

Now we are in a position to present the numerical results for the evolution of the models introduced above. The appropriate values for the parameters which we did not mention so far are $N_f = 4$, $\Lambda_{\text{MS}} = 220$ MeV. For the NLO anomalous dimensions we have used the simplified expression derived by Yndurain et al. [18] which works with an accuracy better than 0.2%, namely

$$
\gamma_{j}^{(1)} = \frac{1}{4} \sum_{\ell=0}^{\infty} \frac{A_{\ell} \ln(j+1) + B_{\ell}}{(j+1)^{\ell}},
$$

(29)

Note an extra factor of $1/\sqrt{\zeta}$ in Eq. (28) found in Ref. [22] which was missed in [25].
In the following we will have a closer look on the distribution functions evolved up to the reference scales $Q^2 = 10, 100$ GeV$^2$ for the skewedness parameters set equal to $\zeta = 0.1, 0.5, 1.0$ and compare them with LO results. We exploit for these purposes an expansion in terms of Legendre polynomials. We perform the evolution\footnote{The calculations were done with a code written for MAPLE.} by evaluating 70 moments in the series (12) in the case

\begin{align}
A_0 &= \frac{32}{27} (201 - 9\pi^2 - 10N_f), \quad A_1 = \frac{512}{9}, \quad A_2 = -\frac{256}{9}, \quad A_3 = \frac{1792}{27}, \quad A_4 = -\frac{256}{3}, \\
B_0 &= \frac{16}{9} \left( -\frac{63}{4} - 134\psi(1) + 6\zeta(3) - 7\pi^2 + 6\pi^2\psi(1) \right) + \frac{32}{27} \left( \frac{3}{4} + \pi^2 + 10\psi(1) \right) N_f, \\
B_1 &= \frac{16}{9} \left( 109 - 32\psi(1) - 3\pi^2 - \frac{22}{3} N_f \right), \quad B_2 = \frac{8}{9} \left( \frac{-1015}{3} + 32\psi(1) + 7\pi^2 + \frac{178}{9} N_f \right), \\
B_3 &= \frac{32}{27} \left( 263 - 56\psi(1) - \frac{9}{2} \pi^2 - 18N_f \right), \quad B_4 = -\frac{42692}{135} + \frac{256}{3} \psi(1) + \frac{236}{45} \pi^2 + \frac{1912}{81} N_f.
\end{align}

Figure 2: 3D shape of the Radyushkin’s model (a). Evolution of input distribution (dashed-dotted line) in leading (dashed lines) and next-to-leading order (solid lines) with $\zeta = 0.1$ (b), $\zeta = 0.5$ (c), $\zeta = 1.0$ (d) and $Q^2 = 10$ GeV$^2$ (curve a), $Q^2 = 100$ GeV$^2$ (curve b).
of the Radyushkin’s model distribution. In the particular case of the asymptotic distribution the
calculation was made in a different way. Due to the support properties of the latter we used
the expansion of the non-forward distribution in Gegenbauer polynomials \( C_j^{3/2}(2x/\zeta - 1) \), i.e. we
directly employed Eqs. (2,5,6). Thus there is no need for a double expansion which would restrict
the number of terms one can treat in the expansion. We have used up to 100 polynomials. In
both cases we made a fit to get smooth curves instead of rapidly oscillating ones.

The results are shown in Fig. 1,2. As can be seen there is only a very small difference between
NLO and LO evolved distribution. As expected this deviation grows with increasing \( Q^2 \), but it
remains small even for large \( Q^2 \). Note that for asymptotically large \( Q^2 \) the excitation of higher
harmonics will die out and both distributions take the form \( O_{as}(x, \zeta) \) from Eq. (24).

4. Conclusion. To conclude, in this note we presented our results on the numerical evolution of the
non-forward distribution function in two-loop approximation in the flavour non-singlet channel.
We studied two models: the asymptotic function and Radyushkin’s model \[22\]. We have found
that the net effects of NLO corrections to the evolution kernels are extremely small and do not
exceed the level of a few percent for moderately large \( Q^2 \). These results together with the fact
that the evolution of the \( O_{as}(x, \zeta) \) (24) is governed by off-diagonal elements of the kernel in the
basis of Gegenbauer polynomials suggest that the latter are small as compared to diagonal entries
and, therefore, the kernel is quasi-diagonal in this basis.

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