Reliability of a phased-mission system with a storage component

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1. Introduction

Many practical systems need to perform multiple missions in sequence, and the system fails as long as it fails in any phase. This kind of system is termed as the phased mission system, and has been studied by many researchers. However, none of them have considered the possibility to store the redundant capacity in one phase so that it can be used in a later phase if the system capacity cannot meet the demand. This paper considers the reliability of a phased-mission system with a storage component. A multi-valued decision diagram based approach is proposed to evaluate the system reliability. An illustrative example is proposed to show the application of the approach.
2. The model

2.1. System description

Consider a phased-mission system with \( n \) components and \( M \) phases. The failure time distribution for any component \( i \) is \( F_i(t) \), with an acceleration factor in each phase \( j \) being \( \alpha_j \). The nominal capacity of any component \( i \) in phase \( j \) is \( w_{ij} \). The demand of each phase \( j \) is \( D_j \). The duration of each phase \( j \) is \( T_j \). A storage component stores the redundant capacity whenever the system capacity is bigger than the system demand. The stored capacity can be consumed by the system whenever the system demand cannot be met by the system components. The system fails whenever the nominal capacity of the storage component becomes negative, meaning that the stored capacity is not able to cover the system deficiency.

A practical example may be a power system with several generators, which needs to support the electricity supply of some district, the demand of which changes from phase to phase. In this case, the generators correspond to the system components, the power supplied by each generator per unit time corresponds to the component capacity, and the power required by the district per unit time corresponds to the system demand. In order to save energy, an accumulator is used to store the redundant capacity whenever the combined capacity of the generators is greater than the demand of the district. In case when the demand is bigger than the combined capacity of the generators, the district demand needs to be satisfied by the stored power in the accumulator. Whenever the stored power of the accumulator is used up, the district experiences power deficiency and the system is regarded as failed.

2.2. MDD for the system

A multi-valued decision diagram (Levitin & Xing, 2017; Mo, Xing, Cui, & Si, 2017; Peng, 2018) is used to represent all the possible sequence of failure events that happen in the system. The steps are as below:

Step 1: The MDD for the first event is as follows.

The first event can be no failure, or the failure of any component in any phase. As is shown in Figure 1, the first level for the decision tree shows all the possible scenarios that can happen in the first event. The number of the branches is \( n^M + 1 \).

\( A_{ij} (1 \leq i \leq n, 1 \leq j \leq M) \) represents that the i-th component fails in the j-th phase as the first failure for the system, i.e. \( A_{1,2} \) means the first failure for the system happens in phase 2 and the failure unit is component 1. The leftmost path means the system has no failure in any phase for any component, which means the success of the system. The value of each branch represents maximum nominal stored capacity of the storage component in each phase. For each possibility, the corresponding maximum nominal stored capacity at the end of each phase is calculated.

To obtain the maximum possible stored capacity, each failure that happens at certain phase is regarded as happening as the end of the phase. For example, \( A_{1,1} \) means the first component fails in phase one.

![Figure 1. MDD of the first event.](image)
as the first failure of the system, then the maximum nominal stored capacity at the end of phase 1 is \((w_{1,1} + w_{2,1} + \ldots + w_{n,1} - D_j)T_1\) (the maximum nominal stored capacity happens when the component 1 fails at the end of the phase). Since the component 1 has failed in phase 1, then we should exclude component 1 when calculating the maximum nominal stored capacity for the following phases. So the maximum nominal stored capacity in phase 2 is \((w_{1,1} + w_{2,1} + \ldots + w_{n,1} - D_1)T_1 + (w_{2,2} + \ldots + w_{n,2} - D_2)T_2, \ldots\), the maximum nominal stored capacity in phase \(j(2 \leq j \leq M)\) is \((w_{1,1} + w_{2,1} + \ldots + w_{n,1} - D_1)T_1 + (w_{2,2} + \ldots + w_{n,2} - D_2)T_2 + \ldots + (w_{2j} + \ldots + w_{nj} - D_j)T_j\).

In case that the maximum nominal stored capacity of the storage component is negative for any phase, stop further branching on the path and change the terminal of the path to be ‘1’ indicating system failure.

**Step 2:** Except for the leftmost path and the paths with terminal ‘1’, all other paths need to be further branched.

For such a path, its further branches correspond to different possibilities of the next event, which can be no more failure, or the failure of any remaining component to fail at the latest phase for the previous failures and later phases. For the leftmost branch corresponding to no failure, the terminal of the branch will be the same as the original terminal and this path will stop branching. For any other branch corresponding to failure of component \(i\) in phase \(j\), the maximum nominal stored capacity in phase \(j\) and before is the same as the original terminal, and the maximum nominal stored capacity in phase \(j + 1\) and later will be the original maximum nominal stored capacity minus the sum of product of the nominal capacity of component \(i\) in the phase and the duration of the phase for the phase from \(j + 1\) to the phase of concern. In case that the maximum nominal stored capacity of the storage component is negative for any phase, stop further branching on the path and change the terminal of the path to be ‘1’ indicating system failure.

**Step 3:** The system MDD is obtained when all the paths have stopped branching.

Branching on each path stops either it is the leftmost path corresponding to no more failure or its terminal is ‘1’ corresponding to system failure, or all the components have already failed.

### 2.3. Reliability Calculation

In the system MDD, all the paths except the paths leading to ‘1’ contribute to system reliability. For such a path, if the components have failed in \(k\) different phases and the number of failed components in each of the \(k\) phases is \(n_k\), it can be denoted as \(m_1, v_{m1,1}, \ldots, v_{m1,n1}; m_2, v_{m2,1}, \ldots, v_{m2,n2}; \ldots; m_k, v_{mk,1}, \ldots, v_{mk,nk}\) where \(v_{ij}\) is the index of the \(j\)-th failed component in phase \(i\). The probability that this path leads to system success can be calculated as

\[
P(m_1, v_{m1,1}, \ldots, v_{m1,n1}, \ldots, m_k, v_{mk,1}, \ldots, v_{mk,nk}) = 1 \left(1 - \prod_{i=1}^{k} (1 - F_i(v_{i1} + \ldots + v_{im,1}))\right)
\]

The Equation (1) can be explained explicitly with Figure 2. All the failures happen in \(k(k \leq M)\) phases. Let \(t_{ms,1}, t_{ms,2}, \ldots, t_{ms,n}\) \((1 \leq s \leq k)\) be the sequence of the components failure time in \(ms\)-th phase, then the sequence satisfy \(t_{ms,1} < t_{ms,2} < \ldots < t_{ms,n}\) \(s \leq \sum_{i=1}^{m} T_i\). For example, the first failure appears in the \(m_1\)-th phase and there are \(n_1\) failures in this phase, so the failure time in phase \(m_1\) satisfy \(t_{m_1,1} < t_{m_1,2} < \ldots < t_{m_1,n_1}\) \(\leq \sum_{i=1}^{m_1} T_i\). In order to calculate the probability that the system succeeds, the \(\sum_{i=1}^{k} n_j\) integral functions are conducted as Equation 1.

In Equation 1, \(1(k)\) is the probability that the maximum nominal stored capacity at the end of phase \(k\) is
nonnegative. In particular, we have
\[
1(1) \ldots 1(m_1-1) = 1((w_{1,1} + w_{2,1} + \ldots + w_{n,1} - D_1)T_1 \geq 0) \\
- \ldots - (w_{n,m-1} - D_m)T_m-1 \\
+ (w_{1,m-1} - D_m)T_m-1 \\
+ (w_{1,m1-1} + w_{2,m1-1} + \ldots + w_{n,m1-1} - D_m)T_m-1 \\
+ (w_{1,m1} + w_{2,m1} + \ldots + w_{n,m1} - D_m)T_m1 \\
+ (w_{1,m1} + w_{2,m1} + \ldots + w_{n,m1} - w_{1,m1,1,m1} - D_m)(T_1 + \ldots + T_m - t_{m1,1}) \\
= e^{-1.2},
\]
where \( z = 0, \ldots, m_2 - m_1 - 1, \) and
\[
1(m_1 + z) = 1((w_{1,1} + w_{2,1} + \ldots + w_{n,1} - D_1)T_1 \\
+ \ldots + (w_{1,m1-1} + w_{2,m1-1} + \ldots + w_{n,m1-1} - D_m)T_m-1 \\
+ (w_{1,m1} + w_{2,m1} + \ldots + w_{n,m1} - D_m)(T_1 + \ldots + T_m - t_{m1,1}) \\
= e^{-0.6}(1 - e^{-0.1})
\]
where \( j = 1, \ldots, k, z = 0, \ldots, m_{j+1} - m_j - 1 \) for \( j < k, \) and \( z = 0, \ldots, M - m_k \) for \( j = k. \)

The system reliability can be obtained by summing up the probabilities of all the paths except the paths leading to "1".

### 3. Examples

Consider a system with three phases. The demand for the three phases are \( D_1 = 3, D_2 = 4, D_3 = 5, \) and the durations of the three phases are \( T_1 = T_2 = T_3 = 1. \) The system consists of two parallel and independent components, where \( w_{1,1} = w_{1,2} = w_{1,3} = 2 \) and \( w_{2,1} = w_{2,2} = w_{2,3} = 4. \) Without loss of generality, it is assumed that the failure time of both components observes exponential distribution, and the failure rates in the three phases are respectively 0.1, 0.2, 0.3. This is equivalent to say that the baseline failure time distribution function is \( F(t) = 1 - e^{-0.1t}, \) and the acceleration factors in the three phases are 1, 2, 3 respectively. According to the procedures in Section 2, the multi-valued decision diagram of the system can be constructed, as shown in Figure 3.

It can be seen that there are thirteen paths which do not have leaf node '1'. It indicates that these paths may have some contributions to system success. The probabilities for these paths to lead to system success are

\[
P(\text{no failure}) = e^{-1.2},
\]

\[
P(\text{component 1 fails at phase 1 and no more failure, and system succeeds}) = e^{-0.6}(1 - e^{-0.1})
\]
MDD of the illustrative system 1.

\( p(\text{component 1 fails at phase 1 and component 2 fails at phase 3, and system succeeds}) \)

\[ p(m_1 = 1, v_{1,1} = 1, m_3 = 3, v_{3,2} = 2) \]

\[ = \int_0^2 \int_1^3 1(2t_1 + 4t_2 - 12 > 0) d(1 - e^{-0.1t_1}) \times d(1 - e^{0.3 - 0.3t_2}) \]

\[ = \int_0^1 \int_3^{t_1} d(1 - e^{-0.1t_1})d(1 - e^{0.3 - 0.3t_2}) \]

\[ = \int_0^1 \left[ \int_3^{t_1} [d(1 - e^{0.3 - 0.3t_2})] \right] d(1 - e^{-0.1t_1}) \]

\[ = e^{-0.7}(2e^{0.05} + 1)(e^{0.05} - 1)^2 \]

\( P(\text{component 1 fails at phase 2 and no more failure, and system succeeds}) \)

\[ = e^{-0.6}(e^{-0.1} - e^{-0.3}) \]

\( P(\text{component 1 fails at phase 2 and component 2 fails at phase 2, and system succeeds}) = 0 \)

\( P(\text{component 1 fails at phase 2 and component 2 fails at phase 3, and system succeeds}) \)

\[ = \int_1^2 \int_2^3 1(2t_1 + 4t_2 - 12 > 0) d(1 - e^{0.1 - 0.2t_1}) \times d(1 - e^{0.3 - 0.3t_2}) \]

\[ = 4e^{-0.6}(e^{0.05} - 1) - e^{-0.9}(e^{0.2} - 1) \]

\( P(\text{component 1 fails at phase 3 and no more failure, and system succeeds}) \)

\[ = e^{-0.6}(e^{-0.3} - e^{-0.6}) \]

\( P(\text{both components fail at phase 3, component 1 fails first, and system succeeds}) \)

\[ = 0.5(e^{-0.3} - e^{-0.6})(e^{-0.3} - e^{-0.6}) \]

\( P(\text{component 2 fails at phase 2 and component 1 fails at phase 3, and system succeeds}) = 0 \)

\( P(\text{component 2 fails at phase 2 and no more failure, and system succeeds}) = e^{-0.6}(e^{-0.3} - e^{-0.6}) \),

\( P(\text{component 2 fails at phase 2 and component 1 fails at phase 3, and system succeeds}) = 3e^{-0.6} + e^{-0.8} - 4e^{-13/20} \)

Summing up all the above probabilities, the system reliability can be obtained as

\[ p(\text{no failure}) = e^{1.2} + e^{-0.6}(1 - e^{-0.1}) \]

\[ + e^{-0.6}(e^{-0.1} - e^{-0.3}) + 2e^{-0.6}(e^{-0.3} - e^{-0.6}) \]

\[ + (e^{-0.3} - e^{-0.6})(e^{-0.3} - e^{-0.6}) + e^{-0.6}(e^{-0.2} - e^{-0.3}) \]

\[ + 4e^{-0.6}(e^{1/20} - 1) - e^{-0.9}(e^{-0.2} - 1) \]

\[ + 3e^{-0.6} + e^{-0.8} - 4e^{-13/20} \]

\[ + e^{-0.7}(2e^{1/20} + 1)(e^{1/20} - 1)^2 \]

\[ = 0.7121 \]

For a bigger example, consider a system with four components and three phases. The demands for the three phases are \( D_1 = 8, D_2 = 9, D_3 = 10 \), and the durations of the three phases are \( T_1 = T_2 = T_3 = 1 \). The nominal capacities of the three components in different phases are \( w_{1,1} = w_{1,2} = w_{1,3} = 2, w_{2,1} = w_{2,2} = w_{2,3} = 3, w_{3,1} = w_{3,2} = w_{3,3} = 3, \) and \( w_{4,1} = w_{4,2} = w_{4,3} = 2 \). The baseline failure time distribution of the four components are assumed to be the same as \( F(t) = 1 - e^{-(0.3t)^{1/2}} \). The accelerating factors in the three phases are assumed to be \( \alpha_1 = 1, \alpha_2 = 1.5, \alpha_3 = 2 \). According to the procedures in Section 2, the multi-valued decision diagram of the system can be constructed, as shown in Figure 4.
Figure 4. MDD of the illustrative system 2.
Figure 4. Continued.

It can be seen that there are ninety-seven paths which do not have leaf node ‘1’. It indicates that these paths may have some contributions to system success. The probabilities for these paths to lead to system success are

\[ p(\text{no components fail}) = (e^{-0.1(1+1.5+2)}1.2)^4 = 0.2156 \]

\[ p(\text{component 1 fails at phase 2, no more failure, system succeeds}) = \int_{T_1+T_2}^{T_1+T_2+T_3} 1(1.2)1(3)dF(T_1)
+ \int_{T_1}^{T_1+T_2} 1(3)dF(T_1 + 1.5(t_1 - T_1)) \]
\[ = \int_{T_1}^{T_1+T_2} 1(3)dF(T_1 + 1.5(t_1 - T_1)) \]
\[ (1 - F(T_1 + 1.5T_2 + 2T_3))^3 \]
\[ \int_{1}^{2} 1(2t + 3 \times 3 + 3 \times 3 + 2 \times 3 - 27 > 0)
\]
\[ d(1 - e^{(-0.1(1+1.5)(t-1))1.2}) (e^{-0.451.2})^3 \]
\[ = \int_{1}^{2} d(1 - e^{(-0.15t-0.05)1.2}) (e^{-0.451.2})^3 \]
\[ = 0.0179 \]

\[ p(\text{component 1 fails at phase 2, component 2 fails at phase 3, no more failure, system succeeds}) \]
\[ = \int_{T_1+T_2+T_3}^{T_1+T_2+T_3+T_4} 1(1)1(2)1(3)
\times dF(T_1 + 1.5(t_1 - T_1))dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]
\[ \times [1 - F(T_1 + 1.5T_2 + 2T_3)]^2 \]
\[ = \int_{1.5}^{2} \int_{-1/3}^{3} t_1 d(1 - e^{(-0.1(1+1.5+2(t-1))1.2)}) \]
\[ \times d(1 - e^{(-0.1(1+1.5+2(t-1))1.2}) (e^{-0.451.2})^2 \]
\[ = 6.1679 \times 10^{-4} \]
\[
\begin{align*}
&\quad p(\text{component 1 fails at phase 2 and}
\text{component 2 fails at phase 3,}
\text{no more failure, system succeeds})
\quad + p(\text{component 1 fails at phase 2 and}
\text{component 3 fails at phase 3,}
\text{no more failure, system succeeds})
\quad + p(\text{component 2 fails at phase 3,}
\text{component 4 fails at phase 3,}
\text{no more failure, system succeeds})
\quad = \int_{t_1}^{t_2} \int_{t_1+T_3}^{t_1+T_3+T_2} \int_{t_1+T_3}^{t_1+T_3+T_2} 1(1)123
\quad \times dF(t_1 + 1.5(t_1 - T_1)) dF(t_1 + 1.5T_2 + 2(t_2 - T_1 - T_2))
\quad \times dF(t_1 + 1.5T_2 + 2(t_3 - T_1 - T_2))
\quad \times [1 - F(t_1 + 1.5T_2 + 2T_3)]
\quad = 2.1260 \times 10^{-5}
\end{align*}
\]

\[
\begin{align*}
&\quad p(\text{component 1 fails at phase 2 and}
\text{component 3 fails at phase 3,}
\text{no more failure, system succeeds})
\quad + p(\text{component 1 fails at phase 2 and}
\text{component 4 fails at phase 3,}
\text{no more failure, system succeeds})
\quad + p(\text{component 2 fails at phase 3,}
\text{component 3 fails at phase 3,}
\text{no more failure, system succeeds})
\quad = \int_{t_1}^{t_2} \int_{t_1+T_3}^{t_1+T_3+T_2} \int_{t_1+T_3}^{t_1+T_3+T_2} 1123
\quad \times dF(t_1 + 1.5(t_1 - T_1)) dF(t_1 + 1.5T_2 + 2(t_2 - T_1 - T_2))
\quad \times dF(t_1 + 1.5T_2 + 2(t_3 - T_1 - T_2))
\quad \times [1 - F(t_1 + 1.5T_2 + 2T_3)]
\quad = 1.4111 \times 10^{-5}
\end{align*}
\]

\[
\begin{align*}
&\quad p(\text{component 1 fails at phase 2, component 3 fails}
\text{at phase 3, no more failure, system succeeds})
\quad + p(\text{component 2 fails at phase 3, component 1 fails}
\text{at phase 3, no more failure, system succeeds})
\quad = \int_{t_1}^{t_2} \int_{t_1+T_3}^{t_1+T_3+T_2} \int_{t_1+T_3}^{t_1+T_3+T_2} 1123
\quad \times dF(t_1 + 1.5T_2 + 2(t_1 - T_1 - T_2))
\quad \times dF(t_1 + 1.5T_2 + 2(t_2 - T_1 - T_2))
\quad \times [1 - F(t_1 + 1.5T_2 + 2T_3)]^2
\quad = 0.0065
\end{align*}
\]
\[ p(\text{component 1 fails at phase 2, component 4 fails} \times \text{at phase 3, no more failure, system succeeds}) \]
\[
= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2}^{T_1+T_2+T_3} 1(1)1(2)1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] dF[T_1 + 1.5T_2 + 2T_3]
\]
\[
= \int_{T_1}^{T_1+T_2+T_3} 1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] [1 - F(T_1 + 1.5T_2 + 2T_3)]^3
\]
\[
\times d(1 - e^{-0.1(2T_1 - 1.5)})^3 \left( e^{-0.45T_2} \right)^3
\]
\[
= 0.0462
\]

\[ p(\text{component 1 fails at phase 3, no more failure, system succeeds}) \]
\[
= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2}^{T_1+T_2+T_3} 1(1)1(2)1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] dF[T_1 + 1.5T_2 + 2T_3]
\]
\[
= \int_{T_1}^{T_1+T_2+T_3} 1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] [1 - F(T_1 + 1.5T_2 + 2T_3)]^3
\]
\[
\times d(1 - e^{-0.1(2T_1 - 1.5)})^3 \left( e^{-0.45T_2} \right)^3
\]
\[
= 0.0462
\]

\[ p(\text{component 1 fails at phase 2 and component 4 fails}) \]
\[
\times \text{at phase 3, no more failure, system succeeds}
\]
\[
= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2}^{T_1+T_2+T_3} 1(1)1(2)1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] dF[T_1 + 1.5T_2 + 2T_3]
\]
\[
= \int_{T_1}^{T_1+T_2+T_3} 1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] [1 - F(T_1 + 1.5T_2 + 2T_3)]^3
\]
\[
\times d(1 - e^{-0.1(2T_1 - 1.5)})^3 \left( e^{-0.45T_2} \right)^3
\]
\[
= 0.0462
\]

\[ p(\text{component 1 fails at phase 2 and component 3 fails} \times \text{at phase 3, no more failure, system succeeds}) \]
\[
= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2}^{T_1+T_2+T_3} 1(1)1(2)1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] dF[T_1 + 1.5T_2 + 2T_3]
\]
\[
= \int_{T_1}^{T_1+T_2+T_3} 1(3) dF[T_1 + 1.5T_2 + 2(T_2 - T_1 - T_2)] [1 - F(T_1 + 1.5T_2 + 2T_3)]^3
\]
\[
\times d(1 - e^{-0.1(2T_1 - 1.5)})^3 \left( e^{-0.45T_2} \right)^3
\]
\[
= 0.0462
\]
\[
\begin{align*}
\int_{T_1}^{T_1+T_2} \int_{T_1+T_2}^{T_1+T_2+T_3} dF(T_1 + 1.5(t_1 - T_1)) \\
\times (1 - F(T_1 + 1.5T_2 + 2T_3))^3 \\
= \int_1^2 1(2t + 3 \times 3 + 3 \times 2 + 2 \times 3 - 27 > 0) \\
\times d(1 - e^{[-0.1(1+1.5(t-1))]})(e^{-0.45}_{12})^3 \\
= \int_1^2 d(1 - e^{-(0.15t-0.05)}_{12})(e^{-0.45}_{12})^3 \\
= 0.0179
\end{align*}
\]

\[p(\text{component 1 fails at phase 3 and component 2 fails at phase 3, and component 3 fails at phase 3, component 4 fails at phase 3, system succeeds})
\]

\[= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2+T_3}^{T_1+T_2+T_3+T_4} dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \\
\times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \\
\times dF(T_1 + 1.5T_2 + 2(t_3 - T_1 - T_2)) \\
\times dF(T_1 + 1.5T_2 + 2(t_4 - T_1 - T_2)) \\
= 3.8686 \times 10^{-5}
\]

\[p(\text{component 1 fails at phase 3 and component 2 fails at phase 3, and component 4 fails at phase 3, component 2 fails at phase 3, system succeeds})
\]

\[= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2+T_3}^{T_1+T_2+T_3+T_4} dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \\
\times [1 - F(T_1 + 1.5T_2 + 2T_3)]^2 \\
= \int_1^3 d(1 - e^{[-0.1(1+1.5(t-1))]}_{12}) \\
\times d(1 - e^{-(0.15t-0.05)}_{12})(e^{-0.45}_{12})^3 \\
= 9.3049 \times 10^{-4}
\]

\[p(\text{component 4 fails at phase 2, component 1 fails at phase 3, no more failure, system succeeds})
\]

\[= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2+T_3}^{T_1+T_2+T_3+T_4} dF(T_1 + 1.5(t_1 - T_1)) \\
\times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \\
\times [1 - F(T_1 + 1.5T_2 + 2T_3)]^2 \\
= \int_1^3 d(1 - e^{[-0.1(1+1.5(t-1))]}_{12}) \\
\times d(1 - e^{-(0.15t-0.05)}_{12})(e^{-0.45}_{12})^3 \\
= 6.0557 \times 10^{-4}
\]

\[p(\text{component 4 fails at phase 3, component 1 fails at phase 3, no more failure, system succeeds})
\]

\[= \int_{T_1}^{T_1+T_2+T_3} \int_{T_1+T_2+T_3}^{T_1+T_2+T_3+T_4} dF(T_1 + 1.5(t_1 - T_1)) \\
\times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \\
\times [1 - F(T_1 + 1.5T_2 + 2T_3)]^2 \\
= \int_1^3 d(1 - e^{[-0.1(1+1.5(t-1))]}_{12}) \\
\times d(1 - e^{-(0.15t-0.05)}_{12})(e^{-0.45}_{12})^3 \\
= 6.0557 \times 10^{-4}
\]
\[ p(\text{component 4 fails at phase 2, component 3 fails at phase 3, no more failure, system succeeds}) = \int_{1.5} \int_{4-2/3} t_1^3 d\left(1 - e^{-0.1(1+1.5(t_1-1))^{1.2}}\right) dt_1 
\times d\left(1 - e^{-0.1(1+1.5+2(t_2-1))^{1.2}}\right) \left[e^{-0.45^{1.2}}\right] 
= 6.1679 \times 10^{-4} \]

\[ dF[T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)]dF[T_1 + 1.5T_2 + 2(t_3 - T_1 - T_2)][1 - F(T_1 + 1.5T_2 + 2T_3)] 
= 1.4111 \times 10^{-5} \]
\[ p \left( \text{component 4 fails at phase 2, and component 1 fails at phase 3, and component 3 fails at phase 3, system succeeds} \right) \]

\[ = \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{1(1)1(2)1(3)} 1(1)1(2)1(3) \times dF(T_1 + 1.5(t_1 - T_1)) dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]

\[ = \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{1(1)1(2)1(3)} 1(1)1(2)1(3) \times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]

\[ = 3.6354 \times 10^{-7} \]

\[ p(\text{component 2 fails at phase 3, no more failure, system succeeds}) \]

\[ = \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{1(1)1(2)1(3)} 1(1)1(2)1(3) \times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]

\[ = 0.0048 \]

\[ p(\text{component 3 fails at phase 3, no more failure, system succeeds}) \]

\[ = \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{1(1)1(2)1(3)} 1(1)1(2)1(3) \times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]

\[ = 0.0462 \]

\[ p(\text{component 2 fails at phase 3, component 3 fails at phase 3, no more failure, system succeeds}) \]

\[ + p(\text{component 3 fails at phase 3, component 2 fails at phase 3, system succeeds}) \]

\[ = \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{1(1)1(2)1(3)} 1(1)1(2)1(3) \times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]

\[ = 0.0462 \]

\[ p(\text{component 2 fails at phase 3, component 4 fails at phase 3, no more failure, system succeeds}) \]

\[ + p(\text{component 4 fails at phase 3, no more failure, system succeeds}) \]

\[ = \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{T_1 + T_2 + T_3} \int_{T_1 + T_2}^{1(1)1(2)1(3)} 1(1)1(2)1(3) \times dF(T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)) \]

\[ = 4.8446 \times 10^{-6} \]

\[ p(\text{component 2 fails at phase 3, component 4 fails at phase 3, no more failure, system succeeds}) \]

\[ + p(\text{component 3 fails at phase 3, component 2 fails at phase 3, component 4 fails at phase 3, no more failure, system succeeds}) \]
fails at phase 3, no more failure, system succeeds

\[
\begin{align*}
&= \int_{t_1 + t_2}^{t_1 + t_2 + T_3} 1(1)1(2)1(3) dF[t_1 + 1.5T_2 \\
&+ 2(t_1 - t_1 - T_2)]dF[T_1 + 1.5T_2 + 2(t_2 - T_1 - T_2)] \\
&\times [1 - F(T_1 + 1.5T_2 + 2T_3)]^2 \\
&= \int_{1.5}^{2} \int_{4/2/3} t_1 \ d(1 - e^{[0.1(1+1.5+2(t_2-1))]^{1.2}}) \\
&d(1 - e^{[0.1(1+1.5+2(t_1-1))]^{1.2}} [e^{-0.45^{1.2}}] \\
&= 0.0065
\end{align*}
\]

\( p(\text{component 4 fails at phase 3}, \text{no more failure, system succeeds}) \)

\[
\begin{align*}
&= \int_{t_1 + t_2}^{t_1 + t_2 + T_3} 1(1) \cdot 1(2)1(3) dF[T_1 + 1.5T_2 \\
&+ 2(t_1 - t_1 - T_2)]dF[T_1 + 1.5T_2 + 2(t_1 - T_1 - T_2)] \\
&\times [1 - F(T_1 + 1.5T_2 + 2T_3)]^3 \\
&= \int_{t_1 + t_2}^{t_1 + t_2 + T_3} (3)dF[T_1 + 1.5T_2 + 2(t_1 - T_1 - T_2)] \\
&\times [1 - F(T_1 + 1.5T_2 + 2T_3)]^3 \\
&= \int_{2}^{3} 1(2t + 3 \times 3 + 3 \times 3 + 2 \times 3 - 27 > 0) \\
&\times d(1 - e^{[0.2(1-1.5)]^{1.2}} (e^{-0.45^{1.2}})^3 \\
&= 0.0462
\end{align*}
\]

\( p(\text{component 1 fails at phase 3}, \text{and component 3 fails at phase 3}, \text{no more failure, system succeeds}) \)

\( \begin{align*}
&\begin{cases}
p(\text{component 1 fails at phase 3}, \text{and component 3 fails at phase 3}) \\
p(\text{component 1 fails at phase 3}, \text{and component 4 fails at phase 3}) \\
p(\text{component 3 fails at phase 3}, \text{and component 1 fails at phase 3}) \\
p(\text{component 3 fails at phase 3}, \text{and component 4 fails at phase 3}) \\
p(\text{component 1 fails at phase 3}, \text{no more failure, system succeeds}) \\
p(\text{component 4 fails at phase 3}, \text{no more failure, system succeeds}) \\
+ p(\text{component 3 fails at phase 3}) \\
\end{cases}
\end{align*} \)

\( 7.0557 \times 10^{-4} \)

\( p(\text{component 4 fails at phase 3}, \text{component 3 fails at phase 3}, \text{component 4 fails at phase 3}, \text{no more failure, system succeeds}) \)

\( \begin{align*}
&\begin{cases}
p(\text{component 4 fails at phase 3}, \text{component 3 fails at phase 3}) \\
p(\text{component 4 fails at phase 3}, \text{component 3 fails at phase 3}) \\
p(\text{component 4 fails at phase 3}, \text{component 4 fails at phase 3}) \\
p(\text{component 1 fails at phase 3}) \\
\end{cases}
\end{align*} \)

\( 0.0065 \)

\( p(\text{component 1 fails at phase 3}, \text{component 4 fails at phase 3}, \text{no more failure, system succeeds}) \)

\( \begin{align*}
&\begin{cases}
p(\text{component 1 fails at phase 3}, \text{component 3 fails at phase 3}) \\
p(\text{component 3 fails at phase 3}, \text{no more failure, system succeeds}) \\
p(\text{component 4 fails at phase 3}, \text{no more failure, system succeeds}) \\
\end{cases}
\end{align*} \)

\( 0.0098 \)

Summing up all the above probabilities, the system reliability can be obtained as

\( p(\text{no failure}) = 0.2156 + 0.0079 + 6.1679 \times 10^{-4} \\
+ 2 \times 1.4111 \times 10^{-5} \\
+ 2 \times 3.6354 \times 10^{-7} + 3 \times 2.1260 \times 10^{-5} \\
+ 3 \times 6.1679 \times 10^{-4} + 2 \times 9.3049 \times 10^{-4} \\
+ 2.1260 \times 10^{-5} + 4 \times 0.0462 + 2 \times 0.0065 \\
+ 2 \times 4.8446 \times 10^{-4} \)
$+ 3.8686 \times 10^{-5} + 2 \times 7.0557 \times 10^{-4} + 2 \times 0.0065 + 0.0179 + 0.0048 + 0.0098 = 0.4832$

4. Conclusions

This paper studied a phased mission system with a storage component. The storage component stores the redundant capacity whenever the system capacity is bigger than the demand, and uses its stored capacity to compensate the system deficiency whenever the system capacity is smaller than the demand. A multi-valued decision diagram based approach is adopted to evaluate the system reliability. An illustrative example is proposed to show the application of the procedures. This work can be extended to the case where the storage component has a maximum storage capacity in the future. Moreover, it would be interesting to investigate the case where the stored capacity may reduce with time even without use.

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