Charge shelving and bias spectroscopy for the readout of a charge qubit on the basis of superposition states.

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Charge-based qubits have been proposed as fundamental elements for quantum computers. One commonly proposed readout device is the single-electron transistor (SET). SETs can distinguish between localized charge states, but lack the sensitivity to directly distinguish superposition states, which have greatly enhanced coherence times compared with position states. We propose introducing a third dot, and exploiting energy dependent tunnelling from the qubit into this dot (bias spectroscopy) for pseudo-spin to charge conversion and superposition basis readout. We introduce an adiabatic fast passage-style charge pumping technique which enables efficient and robust readout via charge shelving, avoiding problems due to finite SET measurement time.

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The experimental observation, manipulation and utilization of coherent quantum mechanical properties in solid-state systems are key technological challenges for this century. The importance of incoherent quantum properties has been essential for the development of microelectronics and it is hoped that coherent quantum effects will spawn new technologies including, but not necessarily limited to, quantum computers.

In the development of coherent solid-state systems compatible with quantum computing, superconducting systems have a clear advantage due to the presence of macroscopic quantum states and key milestones have already been reached.4 Coherent transport in semiconductor two-dimensional electron-gas (2DEG) systems has been observed2 and recently a charge qubit has been realized in a GaAs double dot.5 Despite this, there is a strong impetus to develop coherent technologies that are compatible with the semiconductor industry, especially those based on silicon-metal-oxide technology, owing to its mature manufacturing technology and potential scalability advantages.7, 8, 9

Of particular interest are charge-based quantum computers,10 for example in Cooper-pair box arrangements,11 and semiconductor systems,8, 9, 12 because of the relative ease of readout using high sensitivity electrometers. One such electrometer is the radio-frequency single-electron transistor (rf-SET),13 which has been shown to be compatible with quantum computing requirements.14 The relative ease of coupling to a charge-based qubit is, however, also responsible for giving the qubit a short decoherence time, as the charge distribution of the qubit couples readily to the local electrostatic environment. By operating a charge qubit at the degeneracy point, the natural basis is the superposition basis. These states couple less strongly to the electrostatic environment, so the decoherence rate of a superposition basis qubit should be much less than that for a position basis qubit. Conventional electrometers lack the sensitivity to directly distinguish between superposition states. Our scheme provides a robust mechanism to convert information from the superposition basis to an accessible position basis, incorporating charge shelving in a fashion not available with conventional double-dot schemes.

We consider a coherent triple-dot, one-electron system, with a strongly-coupled qubit, and a weakly-coupled ‘probe’ dot, illustrated in Fig. 1. Varying the energy of the third dot, relative to the qubit, achieves a form of bias spectroscopy, reminiscent of the optical Autler-Townes (AT) experiment.15 Such spectroscopy serves to probe the system dynamics. However for single-shot qubit readout we propose an adiabatic fast passage (AFP)16-like process to perform superposition-to-position state pumping. This constitutes a form of charge-shelving and is analogous to a scheme for adiabatic transport in a

![FIG. 1:](image-url)
double-dot system by Brandes and Vorrath\textsuperscript{12}. Although our scheme should be applicable to any three-state, one-electron system, for clarity we focus on the phosphorus-in-silicon system of Hollenberg \textit{et al.}.\textsuperscript{13} We note that subsequent to our initial suggestion, a similar scheme has been realized in a beautiful experiment by Astafiev \textit{et al.}\textsuperscript{14} for a Josephson charge qubit.

Our scheme requires a small increase in the complexity required to perform a charge-qubit measurement over simpler two-donor schemes. Therefore we must identify the circumstances where it will be advantageous.

For any quantum computing scheme there are many important timescales, and the choice of an effective measurement scheme depends on the relative values of each. There are the two environmental dephasing times, $T_1$ and $T_2$ corresponding to population and coherence relaxation respectively. $\tau_{\text{meas}}$ is the time required for a measurement to occur, and because a measurement projects the system into a basis state of the measurement device, this contributes an effective $T_2$. We define $\tau_{\text{osc}}$ as the time for one coherent oscillation to occur and $\tau_{\text{gate}}$ as the maximum rate at which signals can be sent to manipulate the qubit. For a functioning qubit we must have $\tau_{\text{osc}}, \tau_{\text{gate}} \ll T_1, T_2$, and this is assumed in our discussion.

Nondestructive single-shot readout requires $\tau_{\text{meas}} \ll T_1$. $T_1$ is a function of the energy separation between states, and for good charge localization (necessary for SET readout) we must operate far from degeneracy, i.e. where $T_1$ is minimized. Recent experiments and analysis of rf-SETs\textsuperscript{15} suggest that if the induced SET island charge is $\sim 0.01e$, it will take $\tau_{\text{meas}} \sim 2\mu$s to achieve an error rate of $0.1$. This $\tau_{\text{meas}}$ means that although proof of principle experiments using signal averaging will be possible, single shot measurements for readout and error correction in a practical device will be problematic as the population will decay faster than it can be measured. By introducing charge shelving to isolate the charge at the bias position that maximizes $T_1$ during readout, we ameliorate this.

To exploit SET readout and superposition basis operation, we must transfer quantum information from the superposition basis, to a position basis. This may be done either nonadiabatically or adiabatically.

Nonadiabatic operations, which could be performed in either double or triple dot geometries, require $\tau_{\text{gate}} \ll T_1, T_2$ and $\tau_{\text{osc}}$. If sufficient bandwidth is available they constitute the fastest mechanism for transferring population. Nonadiabatic operations are extremely sensitive to noise and gate errors, and hence attention has turned to adiabatic methods.

There are at least two adiabatic timescales, the first being the usual quasi-static case where $\tau_{\text{gate}} \gg T_1, T_2, \tau_{\text{osc}}$. Adiabatic transfer based on quasi-static operations is conceptually easy and applicable to both double and triple dot schemes. By necessity such manipulations are slow, and there may be incompatibility between $\tau_{\text{gate}}$ and $T_1$, meaning such rotations are not suitable for quantum computer readout. Triple-dot systems afford the possibility for a further adiabatic timescale, where adiabatic passage techniques exploiting appropriate parameter modulation, allow decoupling of the adiabatic pathway from the dephasing times, i.e. $\tau_{\text{osc}} \ll \tau_{\text{gate}} \ll T_1, T_2$. It is this second adiabatic timescale that we exploit for AFP charge-shelving. For a more complete discussion of timescales in coherently driven systems see Ref.\textsuperscript{15}.

To summarize, any practical qubit requires $\tau_{\text{osc}}, \tau_{\text{gate}}, \tau_{\text{meas}} \ll T_2, T_1$, and most implementations proposed to date also require $\tau_{\text{gate}} \ll \tau_{\text{osc}}$. Our scheme can function with $\tau_{\text{gate}} > \tau_{\text{osc}}$, to identify the signature of coherent oscillations, and charge-shelving scheme provides a mechanism to increase $T_1$ to ensure that $\tau_{\text{meas}} \ll T_1$. AFP provides the advantages of adiabatic control without restricting gate operations to the timescales for decoherence rates. Even so, it is necessary to explore any given implementation fully to determine if our scheme will provide a tangible advantage.

One further consideration is that access to multiple bases is necessary for state tomography,\textsuperscript{16} which is important for qubit characterization. The extra freedom afforded by the triple-donor system suggests that a hybrid position/superposition readout system may be realizable which would have advantages for tomography and we will investigate this possibility elsewhere.

The three-donor system is shown in Fig. 1 with three ionized phosphorus donors (open circles) sharing a single electron. A strongly coupled qubit is defined by donors $l$ (left) and $r$ (right). The weakly coupled probe is labelled $p$. We follow the gate notation used in Refs.\textsuperscript{15,16,17,9,12}. The energies of each single-electron state are controlled using...
The Hamiltonian is identical to an optically driven three-level atom in the rotating-wave approximation.

The density matrix equations of motion are written

$$\dot{\rho} = -(i/\hbar)[H, \rho] + \mathcal{L},$$

with $\rho$ the density matrix and $\mathcal{L}$ a dephasing operator. $T_\alpha$ processes are modelled by a dephasing rate $\Gamma$ which is assumed affect all coherences, $T_\beta$ processes are described by rates of incoherent population transfer

$$\Gamma_{\alpha\beta} = \chi_{\alpha\beta}\Delta_{\alpha\beta}/[1 - \exp(\Delta_{\alpha\beta}/kT)],$$

where $k$ is Boltzmann’s constant, $T$ the temperature and $\chi_{\alpha\beta}$ a rate from the tunnelling probability between $\alpha$ and $\beta$. $\Gamma_{\alpha\beta}$ is the population transfer rate from $\beta$ to $\alpha$.

The density matrix equations of motion are therefore

$$\dot{\rho}_{ll} = i\Omega_{lr}(\rho_{rl} - \rho_{rl}) - \Gamma_{rl}\rho_{ll} + \Gamma_{rl}\rho_{rr},$$

$$\dot{\rho}_{lr} = i\left[\Delta_{l}\rho_{lr} + \Omega_{lr}(\rho_{rr} - \rho_{ll}) - \Omega_{rp}\rho_{rp}\right] - \Gamma_{lr},$$

$$\dot{\rho}_{lp} = i\left[\Delta_{p}\rho_{lp} + \Omega_{rp}\rho_{rp} - \Omega_{rp}\rho_{pr} + \Omega_{lp}\rho_{lp} - \Omega_{lp}\rho_{pr}\right] - \Gamma_{lp},$$

$$\dot{\rho}_{rp} = i\left[\Delta_{p}\rho_{rp} + \Omega_{rp}\rho_{rp} - \Omega_{rp}\rho_{pr} + \Omega_{rp}\rho_{lp} - \Omega_{lp}\rho_{lp}\right] - \Gamma_{rp},$$

$$\dot{\rho}_{pp} = i\Omega_{rp}(\rho_{rp} - \rho_{rp}) + \Gamma_{rp}\rho_{rr} - \Gamma_{rp}\rho_{pp} - \Gamma_{lp}.$$

$$1 = \rho_{ll} + \rho_{rr} + \rho_{pp}.$$  \hspace{1cm} (1)

We numerically integrated Eqs. (1) with $\rho_{ll}(0) = 1$ to highlight the dynamics. Maximum coherence times require $E_l = E_r$ which is used in our calculations. For the third dot to act as a weak probe, $\Omega_{rp} \ll \Omega_{lr}$. We have therefore chosen $\Gamma = \Omega_{lr}/100$, $\chi_{\alpha\beta} = 0$ and $\Omega_{rp} = \Omega_{lr}/20$. Fig. 2 (a) shows the time dependent populations when $E_p = E_{AS}$. The dominant feature is the coherent population oscillations between $l$ and $r$. There is a steady buildup of population in $p$, which is our measurement signal. Figs. 2 (b)-(d) show $\rho_{ll}$, $\rho_{rr}$, and $\rho_{pp}$ respectively, as a function of time and $\Delta_{pr}$. Again, the dominant behavior is the coherent oscillation between $l$ and $r$, however when $\Delta_{pr} = \pm \Omega_{lr}$, resonant tunnelling into $p$ occurs, yielding a doublet in $\rho_{pp}$ similar to the AT doublet. Measurement of probe population is therefore sensitive to the population in the symmetric and anti-symmetric modes of the $l-r$ system.

Bias spectroscopy should be useful for characterizing qubit properties, albeit incompatible with readout of a practical quantum computer. This is due to (i) coherent oscillations on the $r-p$ transition reducing readout fidelity, (ii) the need to set and accurately maintain $\Delta_{pr}$ over the measurement time, and (iii) small average populations tunnelling into $p$ requiring multiple experiments. To avoid these problems, we propose a form of charge-shielding that adiabatically drives the population into $|p\rangle$.
from one of the superposition states, realized by controlling the tunneling in a fashion related to AFP. The advantages of this are that $\rho_p$ is adiabatically driven to a large value (approaching unity) in a short time (typically a few $\Omega_{tr}^{-1}$), with robustness to gate errors. Because of the energy dependence of $T_1$, it is most useful to perform AFP between state $|p\rangle$ and the more energetic of the two superposition states, i.e. $|AS\rangle$. One must take care in choosing the modulation trajectory for $|p\rangle$ in this case, as there will be some off-resonant interactions. Our results are promising however, and further optimization can be done.

The trajectory taken by state $|p\rangle$ is governed by both $\Delta_{pl}$ and $\Omega_{rp}$ and for the traces in Figs. 3 they were:

$$\Delta_{lp} = 2\Omega_{lr} (1 - t/t_{max}),$$
$$\Omega_{rp} = \Omega_{rp}^{max} \left[ 1 - \cos\left(2\pi t/t_{max}\right) \right]/2,$$

where $\Omega_{rp}^{max} = 0.4\Omega_{lr}$, $t_{min} = 0$ and $t_{max} = 5\pi/\Omega_{lr}$. In order to make this more explicit, Fig. 3(a) shows $\Omega_{rp}(t)$, in keeping with conventional AFP schemes, our scheme is fairly insensitive to the exact form of $\Omega_{rp}$; and (b) is a diagram showing the energy levels as a function of time.

Fig. 3 (c) shows the populations for the qubit being initially prepared in $|AS\rangle$. After AFP most of the population has been driven into $|p\rangle$. Similarly Fig. 3(d) shows the effect of the AFP trajectory on $|S\rangle$, in this case there is minimal population transfer. We are presently performing more detailed numerical experiments to optimize the population transfer and examine tomographic applications of the scheme, as foreshadowed above. One important issue is the potential re-initialization of the qubit after readout. The AFP scheme as presented is entirely time reversible, and therefore one can simply reverse the scheme to pump an electron from the probe state into the anti-symmetric state to reset the qubit.

In summary, we have presented a scheme for performing bias spectroscopy on a qubit, where resonant tunneling from the superposition states to a third dot is read out with a SET. Such a scheme is a solid-state analog of the optical Autler-Townes scheme. Using bias spectroscopy, one can map out the energy-level space and this may prove a useful probe of coherent coupling where conventional electrometers are unable to resolve the dynamics. Because the bias spectroscopy described here should be able to resolve arbitrary energy differences, this idea may be applied to discriminating between the singlet and triplet states of a two-spin system, such as in Ref. 2. This constitutes an alternate approach to spin readout which we will describe elsewhere. For useful readout, we propose an AFP style scheme, where population is driven into the probe state. This has the advantage of high fidelity readout with robustness to gate errors. It also introduces a form of charge-shelving, avoiding problems due to measurement times long compared with $T_1$.

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