Further Analysis on $\sigma$-particle Properties

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(Received May 25, 1997)

In a previous work, we have reanalyzed the $I=0$ S-wave $\pi\pi$-scattering phase-shift through a new method of Interfering Breit-Wigner amplitudes, and have shown the existence of a light scalar, $\sigma$ particle, accompanied by a negative “repulsive core”-type background phase-shift, whose origin may have some correspondence to a “compensating” $\pi\pi$ contact interaction. In this work we make further analysis on the phase shift under $KK$ threshold from the same standpoint, to determine precise values of $\sigma$ mass and width, so far as present experimental data are concerned.

§1. Introduction

Whether a light iso-scalar resonance ($J^{PC} = 0^{++}$), called ”$\sigma$”-particle, exists or not is one of the most interesting and important problems in hadron spectroscopy. Phenomenologically, since of its light mass and of its “vacuum” quantum number, it may affect various processes. Theoretically, for example, in the Nambu-Jona-Lasinio-type models, realizing the situation of dynamical breaking of chiral symmetry and believed to be a low-energy effective theory of QCD, existence of $\sigma$-meson is predicted as a chiral partner of the Nambu-Goldstone $\pi$-boson. Correspondingly the extensive experimental investigations in the $\pi\pi$-channel have been made for many years. The $I=0$ S-wave $\pi\pi$ phase shift $\delta_0$ is now well known to rise smoothly to 90° at around 900 MeV, then shows a rapid step-up by 180° near the $KK$ threshold, and reaches only to 270° at $m_{\pi\pi} \sim 1200$ MeV. This behavior of $\delta_0$ was thought, in the 1976-through-1994 editions of PDG, to be due mainly to the narrow $f_0(980)$ and the broad $f_0(1370)$, since there remains no phase shift for light $\sigma$-particle.

In contrast with this interpretation, we have shown in the previous work (to be referred as I), re-analyzing the phase shift $\delta_0$ systematically up to 1300 MeV, a possibility for the existence of ”$\sigma(555)$”-resonance with a rather narrow width of a few hundred MeV in addition to these two resonances.

The reasons which led us to a different result from the conventional one, even

* However in the latest edition, the behavior of $\delta_0$ is understood as due to, in addition to the $f_0(980)$ and $f_0(1370)$, a very broad $f_0(400 \sim 1200)$ or $\sigma$. See also the analyses suggesting the $\sigma$ existence, which introduce a repulsive $\delta_{B.G.}$, similarly as in our case.
with the use of the same data of phase shifts, are twofold: On one hand technically, we have applied a new method of Interfering Breit-Wigner Amplitude (IA method) for the analyses, where the $T$-matrix (instead of $K$-matrix in the conventional treatment) for multiple resonance case is directly represented by the respective Breit-Wigner amplitudes in conformity with unitarity, thus parametrizing the phase shifts directly in terms of physical quantities, such as masses and coupling constants of the relevant resonant particles.

On the other hand physically, we have introduced a “negative background phase” $\delta_{B.G.}$ of hard core type, (with a core radius of about pion size,) making enough room for $\sigma$-resonance. Here it is suggestive to remember that a similar type of $\delta_{B.G.}$ is well-known to exist in the $\alpha$ nucleus-$\alpha$ nucleus scattering and in the nucleon-nucleon scattering. A possible origin of $\delta_{B.G.}$ in the $\pi\pi$ system seems to have some correspondence to the “compensating” repulsive $\lambda\phi^4$ interaction in the NJL-model or the linear $\sigma$ model, which is required from the viewpoint of the current algebra and the PCAC.

In this work, we shall extend the re-analysis of the phase shift in $I$ to determine the mass and coupling constant of $\sigma$ and core radius as precise as possible, from the present experimental data of $\delta_{0}^0$, inspecting especially the data under $K\bar{K}$ threshold.

§2. Applied formulas

For our purpose we shall analyze the $\delta_{0}^0$ between the $\pi\pi$-threshold and an energy slightly below the $K\bar{K}$-threshold (980 MeV), taking into account the effects of two resonances, $\sigma$ and $f_{0}(980)$, and of the $\delta_{B.G.}$. Effects from other higher-mass resonances ($f_{0}(1370), f_{0}(1500)...$) and from other channels than $\pi\pi$ are ignored.

The applied formulas of IA-method (in the case of one channel with two resonances) are as follows.

The relevant partial S-wave $S$ matrix element $S$ in the $2\pi$ system is represented by the phase shift $\delta(s)$ and the amplitude $a(s)$. The $\delta$ is given by the sum of $\delta^{Res.}$ and $\delta^{B.G.}$, respectively, due to the resonance and background. The $\sigma$ and $f_{0}(980)$ resonances contribute additively to $\delta^{Res.}$.

$$S = e^{2i\delta(s)} = 1 + 2ia(s)$$  \hspace{1cm} (1)

$$\delta(s) = \delta^{Res.}(s) + \delta^{B.G.}(s), \quad \delta^{Res.}(s) = (\sigma)\delta^{(s)}(s) + (f_{0})\delta^{(s)}(s).$$  \hspace{1cm} (2)

Correspondingly the total $S$ matrix is given by the product of individual $S$-matrices.

$$S = S^{Res.}S^{B.G.} = (\sigma)(f_{0}) S S^{B.G.}.$$  \hspace{1cm} (3)

The unitarity of the total $S$ matrix is now easily seen to be satisfied by the “unitarity of individual $S$-matrices”. Each of $S^{Res.}$ is given by corresponding amplitudes $^{(R)}a\ 's$, which is taken as the following relativistic Breit-Wigner (BW) form

$$^{(R)}S(s) = e^{2i\delta^{(R)}} = 1 + 2ia^{(R)}(s) \quad : R = \sigma, f_{0}$$  \hspace{1cm} (4)
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\[
\left( a^R \right) (s) = \frac{-\sqrt{s} \Gamma_R(s)}{(s - M_R^2) + i\sqrt{s} \Gamma_R(s)},
\]

\[
\sqrt{s} \Gamma_R(s) \equiv \rho_1 g_R, \quad p_1 = \frac{p_1}{\sqrt{s}}, \quad p_1 \equiv \sqrt{s/4 - m_\pi^2},
\]

where $\Gamma_R(s = M_R^2)$ represents the peak width $\Gamma_R^{(p)}$ of the resonance $R$, $g_R$ is the $\pi\pi$-coupling constant, $\rho_1$ is the $\pi\pi$-state density and $p_1$ is the CM momentum of the pion. Here it is to be noted that the total resonance amplitude $a^{Res.} \equiv (S_{Res.} - 1)/2i$ is represented by the respective amplitudes as

\[
a^{Res.} = a^\sigma + f_0^\sigma, \quad a = a^\sigma + 2i f_0^\sigma,
\]

where the last term, looking like an “interference” of the amplitudes, guarantees our amplitude to satisfy the unitarity constraint.

The $\delta_{B.G.}$ is supposed to be of the repulsive hard core type;

\[
\delta_{B.G.}(s) = -p_1 r_c.
\]

In the actual analysis we have applied, in order to have a global fit in all relevant energy region, the relativistic BW formula (5) with a revised width

\[
\tilde{\Gamma}_R(s) \equiv \Gamma_R(s) F(s),
\]

\[
F(s) = 1 - \frac{s_0}{s + s_0} e^{-s/M_0^2}, \quad (M_0 = 400 \text{ MeV})
\]

instead of $\Gamma_R(s)$ defined in Eq.(6). As a matter of fact there has been no established form of relativistic BW formula, and the concrete form of $F(s)$ is considered to be determined following the dynamics of strong interactions. The conventional BW form (5) ($\tilde{\Gamma}_R(s)$ with $F(s) = 1$ in Eq.(9)) gives, in the case of broad width, an unphysical mass spectrum in the low energy region. The phenomenological form of $F(s)$ with a parameter $s_0$ given in (9) is so designed as to give a good fit in the low-energy region affecting it only in the energy region below 400 MeV. Thus $s_0$ plays a similar role as a scattering length.

§3. Mass and width of $\sigma$ and core radius

The $\delta^\sigma_0$ over $m_{\pi\pi} > 600$ MeV is well determined by the analysis of the CERN-Münich experiment. Among 5 independent analyses performed\[^{[4]}\] in Grayer74, (i) the one originally presented in Hyams73 (b-analysis in Grayer74) is widely accepted, partly because of its agreement to those reported by Protopopescu73\[^{[4]}\]. (ii) Srinivasan75 gave the phase between 350-600 MeV, (iii) Rosselet77 and (iv) Bel’kov79 determined the phase between $\pi\pi$-threshold and 400 MeV. (i)-(iv) will be used as the “standard phase shift $\delta^\sigma_0$” in the present analysis. However, there are several works to report different behaviors of $\delta^\sigma_0$ within $\pm 20^\circ$ ambiguities in the $m_{\pi\pi} = 400$-800 MeV region. We will utilize them to estimate the allowed region of $m_{\sigma}$ and $g_{\sigma}$ in our analysis: (v) The $c$-analysis of Grayer74 and (vi) Cason82 are taken as the upper and lower bounds of $\delta^\sigma_0$, respectively.
Fig. 1. I=0 ππ scattering phase shift. (a) Best fit to the standard δ^0. The respective contributions to δ^0 from σ, f_0(980) and δ_{B.G.} are also given. The dotted line with label of “r_c=0” represents conventional fit without the repulsive background thus far made. (b) $\chi^2$, $M_\sigma$ and $g_\sigma$ versus $r_c$. (c) Best fits to the upper and lower δ^0 (dashed lines), and fittings with 3-sigma deviation to the standard δ^0 corresponding to $r_c=2.6$ and 3.35 GeV$^{-1}$ (dotted lines).

The properties of $f_0(980)$ have already been obtained in I by analyzing systematically the standard δ^0, elasticity, and ππ → KK phase shift together in the range of $m_{\pi\pi}=600 \sim 1300$ MeV, with the 2-channel ($\pi\pi$, $K\bar{K}$) three resonance ($\sigma$, $f_0(980)$, $f_0(1370)$) formula in the IA method. The obtained values are $M_{f_0}=993.2\pm6.5_{st}\pm6.9_{sys}$ MeV and $g_{f_0}=1680\pm91$ MeV, which are used as fixed parameters in the following analysis.

Fig.2(a) shows the result of the best fit (solid line) to standard δ^0 by the formula (2) given above, which includes the sum of three contributions: σ, $f_0(980)$, and δ_{B.G.}. The best-fitted values of parameters are $M_\sigma=585$ MeV, $g_\sigma=3.6$ GeV (corresponding

*) The $f_0(980)$ does not play essential role in the relevant mass region since of its small coupling (width).
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Table I. Obtained values of parameters and their errors. $g^r \equiv g \cdot F^{1/2}(s = M^2_R)$. As for the definitions of $\Gamma^{(p)}$ and $\Gamma^{(d)}$, see the text.

| Parameter | Standard $\delta^0_0$ (i-iv) | Upper bound (v) | Lower bound (vi) |
|-----------|-------------------------------|-----------------|-----------------|
| $M_\sigma$ | 585±20 MeV | 535 MeV | 650 MeV |
| $g^{\sigma \pi \pi}$ | 3600±350 MeV | 3670 MeV | 3465 MeV |
| $\Gamma^{(p)}_{\sigma \pi \pi}$ | 385±70 MeV | 430 MeV | 335 MeV |
| $\Gamma^{(d)}_{\sigma \pi \pi}$ | 340±45 MeV | 370 MeV | 315 MeV |
| $r_c$ | 3.03±0.35 GeV$^{-1}$ | 3.03(fixed) | 3.03(fixed) |
|          | 0.60±0.07 fm | (0.6 fm) | (0.6 fm) |

to decay width of 340 MeV), and $r_c$=3.03 GeV$^{-1}$(0.60 fm) with $\chi^2=23.6$ for 30 degrees of freedom (34 data points with 4 parameters $^{*}$). Note that the best-fitted core radius is nearly the same as the “structural size” (charge radius) of pion~0.7 fm.

In order to justify the preciseness of these values and to estimate their errors it is apparently necessary to take into account of the correlation between $M_\sigma$, $g_\sigma$ and $r_c$. Several fits are performed for various fixed values of $r_c$ between 0 and 4.0 GeV$^{-1}$. In Fig.2(b) $\chi^2$, $M_\sigma$ and $g_\sigma$ are plotted as functions of $r_c$, where $M_\sigma$ and $g_\sigma$ decrease as $r_c$ becomes larger. The $\chi^2$ shows deep parabolic shape, and gives its minimum at 3.03 GeV$^{-1}$, where we obtain the above mentioned “best-fit” values. Note that the fit in the case of $r_c$=0, which corresponds to the results of the conventional analyses without $\delta_{BG}$ thus far made, gives $\chi^2 = 163.4$, about 140 worse than the best fit. In the first row of Table 1 the errors of relevant parameters are quoted which correspond to three sigma deviation in the $\chi^2$ behavior in Fig.2(b). In Fig.2(c) the fits with $r_c$=2.6 and 3.35 GeV$^{-1}$ corresponding to the 3-sigma deviations are shown. The small bump around 750 MeV of the standard $\delta^0_0$ is not reproduced with small $r_c$ (correspondingly with large $g_\sigma$ - width). To estimate the effect due to the ambiguities of experimental $\delta^0_0$ data mentioned above, data (v) and (vi) are analyzed with fixed $r_c$=3.03 GeV$^{-1}$. The results are also shown in Fig.2(c) and the obtained values of parameters are given in the second and the third rows of Table 1.

From all of these studies, we may conclude that the $M_\sigma$ is in the range of 535-650 MeV.

§4. Supplementary discussions

We would add some comments on the results of our analysis:

i) First a possible influence of the tail of $f_0(1370)$ even below $m_{\pi \pi}$=980 MeV should

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$^*$ The significant difference of the value of $M_\sigma$ from that(553.3±0.5 MeV) in I is due to including a consideration on the correlation between $M_\sigma$ and $r_c$ in the present work, and also, regrettably, due to mis-reading of the errors of some data points of (ii) in I.

$^{**}$ In (i), one point at $M_{\pi \pi}$=910 MeV seems to have a too small error and to disturb the continuity to adjacent data points. In fact, this point turned out to occupy large fraction of total $\chi^2$. The values in the text are obtained without this point, while $M_\sigma$=600 MeV, $g^{\sigma \pi \pi}_\sigma$=3750 MeV, $r_c$=2.75 GeV$^{-1}$ (0.54 fm), and $\sqrt{s_0}$=475 MeV with $\chi^2=35.9$ for 31 degrees of freedom, in the case of including this point.
Table II. Pole positions on sheet II in present one-channel analysis. The errors corresponds to the 3-sigma deviation from the best fit to standard $\delta_0$.

| $s_{\text{pole}}$/GeV$^*$ | $\sqrt{s}_{\text{pole}}$/GeV | $p_{\text{pole}}$/GeV |
|---------------------------|-----------------------------|----------------------|
| (0.324±0.020)            | (0.602±0.026)               | (0.271±0.015)        |
| -i(0.236±0.044)          | -i(0.196±0.027)             | -i(0.109±0.014)      |

Would $f_0(1370)$ have a large width, the value $r_c=3.03$ GeV$^{-1}$ in Table 1 should be regarded as including the $f_0(1370)$ contribution. However, we may expect that it does affect very little the values of $M_\sigma$ and $g_\sigma$, since these are determined mainly by the fine structure of the standard $\delta_0$ around 750 MeV.

**ii)** In our treatment the S-matrix is parametrized directly in terms of physically meaningful quantities, the masses and $\pi\pi$-coupling constants of resonances. So it is not necessary to argue about pole positions on analytically continued complex Riemann sheets. However, for convenience to compare with other works, we give these values for the $\sigma$ resonance in Table 2. Our S-matrix has the form of product of respective resonance-S-matrices. Each BW resonance (in our form of Eq.(5)) produces three poles, a virtual state pole, a BW pole and its complex conjugate pole. We are able to derive the approximate relations:

$$Re\sqrt{s}_{\text{pole}} \approx M_R$$ and

$$Im\sqrt{s}_{\text{pole}} \approx \frac{1}{2} \Gamma_p.$$ 

This situation corresponds to the case of one-channel ($\pi\pi$). In the two-channel case ($\pi\pi$ and $K\bar{K}$), one BW resonance induces four poles (except for virtual state pole), $s_P^{III}$, its complex conjugate $s_P^{III*}$, and the two in sheet $II$ or $IV$. In $I$, $g_{\sigma K\bar{K}} \approx 0$ is obtained. This case leads to the four poles with the same $s$-values $s_p$ and $s_p^*$ on sheets $II$ and $III$.

Here the decay width $\Gamma^{(d)}$ should be discriminated from the peak width $\Gamma^{(p)}$. The former is defined by the formula

$$\Gamma^{(d)} = N^{-1} \int ds \Gamma(s)/[(s - M_\sigma^2)^2 + s\Gamma(s)^2];$$

$$N = ds \int 1/[(s - M_\sigma^2)^2 + s\Gamma(s)^2].$$

**iii)** In our analysis we have introduced a parameter $s_0$ in Eq.(9). The obtained value of it is $\sqrt{s}_0 = 365$ MeV, which corresponds to the scattering length $a_0 = 0.23$ in $m^{-1}$ unit. We have introduced no ad hoc prescriptions for Adler-zero condition. However, we have found that our resultant amplitude has a zero at $s = 1.0m_\pi^2$.

**iv)** In the $I = 2 \pi\pi$ system, there is no known/expected resonance, and accordingly it is expected that the repulsive core will show up itself directly. In Fig.2 the experiment-
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Fig. 2. $I=2$ $\pi\pi$ scattering phase shift. Fitting by hard core formula is also shown.

tal data$^{[4]}$ of the $I = 2$ $\pi\pi$-scattering S-wave phase shift $\delta_0^2$ is shown from threshold to $m_{\pi\pi} \approx 1400$ MeV, which is apparently negative, and is fitted well also by the hard core formula $\delta_0^2 = -r_c^{(2)}|p_1|$ with the core radius of $r_c^{(2)} = 0.87$ GeV$^{-1}(0.17$ fm). The core radius is smaller by a factor of 3 than that in the $I = 0$ system, which suggests the importance of quark-pair-annihilation force in the $\pi\pi$ system (see Ref.$^{[17]}$ for possible explanation in the framework of the linear $\sigma$ model).

§5. Concluding remarks

In this work and in I we have shown the strong possibility of existence of $\sigma$-particle with light mass ($\approx 580$ MeV) and comparatively narrow width ($\approx 350$ MeV). The value of $M_\sigma$ is close to $M_\sigma \approx 2m_q$ ($m_q$ being constituent quark mass) expected in the NJL type of models. Recently one of the present authors has argued$^{[10]}$ that our obtained values$^{[4]}$ of $m_\sigma$ and $\Gamma_\sigma$ are consistent with the relation predicted in the linear $\sigma$-model (L$\sigma$M). This fact seems to us to show that our “observed” $\sigma$-particle is really a chiral partner of $\pi$-meson as a Nambu-Goldstone boson.

Here we also note an interesting argument$^{[0,17]}$, that the origin of repulsive core corresponds (at least in the low energy region where the structure of composite hadrons is negligible,) to the “compensating” $\lambda\phi^4$ contact interaction, which is required from the current algebra and the PCAC.$^{[3,4,7]}$

$^*)$ Recently Harada et al.$^{[4]}$ have made a similar analysis of $\pi\pi$-scattering data as I leading to the $\sigma$-existence with similar values of mass and width. However, they start from the viewpoint of Non-linear $\sigma$ model and do not recognize it as a chiral partner of $\pi$ meson.

$^{**)}$ The “effective” $\sigma\pi\pi$ coupling, which includes both effects of an intermediate $\sigma$-production and of the repulsive $\lambda\phi^4$ interaction, becomes of a derivative type, while in the conventional Breit-Wigner formula, a non-derivative coupling of $\sigma$ resonance is supposed. This seems to be a reason why $\sigma$-meson existence has been overlooked for many years.
Historically the existence of $\sigma$-particle, although it was anticipated from various viewpoints, had been rejected for many years. One of the main reasons was that (i) the $\sigma$ has been missing in the $\pi\pi$ phase shift analyses, and the other was due to (ii) the negative results of applications of L$\sigma$M to the low energy $\pi\pi$ scattering and to the $K_{l4}$ decay form factors.

Concerning to (i), the present work may give a possible and clear solution. Recently there are several other phase shift analyses resulting in $\sigma$-particle existence. Also the problem (ii) should be reexamined under the light of these recent progress of phenomenological search for $\sigma$-particle. Finally we should like to note that recently a rather strong evidence for direct $\sigma$ production has been obtained in the central $pp$-collision. As a matter of fact, it was the motivation of our investigation.

References

[1] S. Ishida, M.Y. Ishida, H. Takahashi, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. 95 (1996), 745.
[2] R. Delbourgo and M.D. Scadron, Phys. Rev. Lett. 48 (1982), 379.
[3] T. Hakioglu and M.D. Scadron, Phys. Rev. D42 (1990), 941.
[4] T. Hatsuda and T. Kunihiro, Prog. Theor. Phys. 74 (1985), 765; Phys.Rep.247 (1994), 221.
[5] Y. Kohyama, M. Takizawa et al., Phys. Lett. B208 (1988), 165; Nucl.Phys. A507 (1990), 617.
[6] Review of Particle Properties, Phys. Rev. D54 Part I (1996) 329 and also 355.
[7] R. Kamiński, L. Lesniak and J.-P. Maillet, Phys. Rev. D50 (1994), 3145, private communication.
[8] N.A. Törnqvist and M. Roos, Phys. Rev. Lett. 76 (1996), 1575.
[9] M. Taketani et al., Prog. Theor. Phys. Suppl. No.39(1967); No.42(1968). In particular see Chapter 7 (S. Otsuki, No.42, p.39) and also Chapter 6 (N. Hoshizaki, No.42, p.1).
[10] M.Y. Ishida, Prog. Theor. Phys. 96 (1996), 853.
[11] (i)(v)G. Grayer et al., Nucl. Phys. B75 (1974), 189. B. Hyams et al., Nucl. Phys. B64 (1973), 134. (ii)V. Srinivasan et al., Phys. Rev. D12 (1975), 681. (iii)J. Rosselet et al., Phys. Rev. D15 (1977), 574. (iv)A.A. Bel’kov et al., JETP.Lett.29 (1979), 579. (vi)N.M. Casson et al., Phys. Rev. D28 (1983), 1586.
[12] S.D. Protopopescu et al., Phys. Rev. D7 (1973), 1279.
[13] M. Svec, Phys. Rev. D53 (1996), 2343.
[14] R. Kamiński, L. Leśniak and K. Rybicki, Henryk Niewodniczański Institute of Nuclear Physics Report No.1730/PH(1996).
[15] W. Hoogland et al., Nucl. Phys. B126 (1977), 109; Nucl. Phys. B69 (1974), 266, J.P. Prukop et al., Phys. Rev. D10 (1974), 2055, M.J. Losty et al. Nucl. Phys. B69 (1974), 185, D. Cohen et al., Phys. Rev. D7 (1973), 661, E. Colton et al., Phys. Rev. D3 (1971), 2928, W.D. Walker et al., Phys. Rev. Lett. 18 (1967), 630.
[16] M. Harada, F. Sannino and J. Schechter, Phys. Rev. D54 (1996), 1991.
[17] M.Y. Ishida, Proc. of YITP conference, 1996, Kyoto, Yukawa Hall, to be published in Soryusiron-kenkyuu; Proc. of the Workshop on Particle Physics at K arena with 50-GeV PS, KEK, Tsukuba, Japan Dec 19-20,1996, edited by N. Sasao, S. Sugimoto, Y. Kuno and T. Tomatsuara, Inst. for Nuclear Study, Univ. of Tokyo.
[18] S. Weinberg, Phys. Rev. 166 (1968), 1568, T. Shiozaki, Prog. Theor. Phys. Suppl. Extra Number (44), 1968.
[19] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep.164 (1988), 217.
[20] T. Morozumi, C.S. Lim and A.I. Sanda, Phys. Rev. Lett. 65 (1990), 404, M. Takizawa, T. Inoue and M. Oka, Prog. Theor. Phys. Suppl.120 (1995), 335.
[21] K.L. Au, D. Morgan and M.R. Pennington, Phys. Rev. D41 (1987), 1633.

\( ^{\ast} \text{Ref.26) argues that the L}\sigma\text{M describes successfully both of the mentioned processes.} \)
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[22] J. Gasser and H. Leutwyler, Ann.Phys. 158 (1984), 142.
[23] E.P. Shabalin, Yad.Fiz. 49 (1989), 588 (Sov.J.Nucl.Phys. 49 (1989), 365).
[24] D. Alde et al., Phys. Lett. B397 (350), 1997; T. Ishida, Doctor Thesis of Tokyo University (1996).