Violation of Vector Dominance in the Vector Manifestation*

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Abstract

The vector manifestation (VM) is a new pattern for realizing the chiral symmetry in QCD. In the VM, the massless vector meson becomes the chiral partner of pion at the critical point, in contrast with the restoration based on the linear sigma model. Including the intrinsic temperature dependences of the parameters of the HLS Lagrangian determined from the underlying QCD through the Wilsonian matching together with the hadronic thermal corrections, we present a new prediction of the VM on the direct photon-$\pi$-$\pi$ coupling which measures the validity of the vector dominance (VD) of the electromagnetic form factor of the pion. We find that the VD is largely violated at the critical temperature, which indicates that the assumption of the VD made in several analysis on the dilepton spectra in hot matter may need to be weakened for consistently including the effect of the dropping mass of the vector meson.

1 Introduction

Spontaneous chiral symmetry breaking is one of the important features in low-energy QCD. This symmetry breaking is expected to be restored in hot and/or dense QCD and properties of hadrons will be changed near the critical temperature of the chiral symmetry restoration [2, 3, 4, 5, 6, 7]. The CERN Super Proton Synchrotron (SPS) observed an enhancement of dielectron ($e^+e^-$) mass spectra below the $\rho/\omega$ resonance. This can be explained by the dropping mass of the $\rho$ meson (see, e.g., Refs. [9, 4, 7]) following the Brown-Rho scaling proposed in Ref. [10]. Further the Relativistic Heavy Ion Collider (RHIC) has started to measure several physical processes in hot matter which include the dilepton energy spectra. Therefore it is interesting to study the vector meson mass including the all possible thermal effects, which is one of the important quantities in the chiral phase transition.

For studying the vector meson mass in hot matter, it is convenient to use a model including the vector meson in a manner consistent with the chiral symmetry. One of such models is the model based on the hidden local symmetry (HLS) which successfully describes the systems including the pions and vector mesons at zero temperature [11, 12, 13]. In the framework of HLS, the vector meson is introduced as the gauge boson into the system and acquires its mass through the Higgs mechanism. Based on the chiral perturbation theory (ChPT) with HLS, the Wilsonian matching was proposed [14]. This is a manner which determines the parameters of the HLS Lagrangian from the underlying QCD at the matching scale $\Lambda$. Recently in Ref. [15], by using the Wilsonian matching, the vector manifestation

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(VM) was proposed as a novel manifestation of the chiral symmetry in the Wigner realization, in which the chiral symmetry is restored by the massless degenerate pion (and its flavor partners) and vector meson (and its flavor partners) as the chiral partner. (For a review of the ChPT with HLS, the Wilsonian matching and VM, see Ref. [13].)

In Ref. [16], we extended the Wilsonian matching to the one at non-zero temperature and showed that the VM actually occurs at the critical temperature of the chiral symmetry restoration. There, the intrinsic temperature dependences of the parameters of the HLS Lagrangian play the essential roles to realize the chiral symmetry restoration consistently: In the framework of the HLS the equality between the axial-vector and vector current correlators at critical point can be satisfied only if the intrinsic thermal effects are included. Since the VM is a new picture which realizes the dropping mass of the vector meson such as the one predicted by the Brown-Rho scaling [10], it is important to study what the VM predicts on the properties of the pion and vector mesons in hot matter.

In this talk, we shed a light on the validity of the vector dominance (VD) in hot matter. In several analyses such as the one on the dilepton spectra in hot matter done in Ref [7], the VD is assumed to be held even in the high temperature region. However, the analysis in Ref. [17], which shows that the thermal vector meson mass goes up if the VD holds, seems to imply that the assumption of the VD excludes the possibility of the dropping mass of the vector meson from the beginning. In the present analysis, we present a new prediction of the VM in hot matter on the direct photon-π-π coupling which measures the validity of the VD of the electromagnetic form factor of the pion. We find that the VM predicts the large violation of the VD at the critical temperature [1]. This indicates that the assumption of the VD may need to be weakened, at least in some amounts, for consistently including the effect of the dropping mass of the vector meson.

2 Hidden Local Symmetry

In this section, we briefly review the model based on the hidden local symmetry (HLS).

The HLS model is based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = SU(N_f)_L \times SU(N_f)_R$ is the chiral symmetry and $H = SU(N_f)_V$ is the HLS. The basic quantities are the HLS gauge boson $V_\mu$ and two matrix valued variables $\xi_L(x)$ and $\xi_R(x)$ which transform as

$$\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g_{L,R}^\dagger,$$

where $h(x) \in H_{\text{local}}$ and $g_{L,R} \in [SU(N_f)_{L,R}]_{\text{global}}$. These variables are parameterized as

$$\xi_{L,R}(x) = e^{i\sigma(x)/F_\sigma}e^{i\pi(x)/F_\pi},$$

where $\pi = \pi^a T_a$ denotes the pseudoscalar Nambu-Goldstone bosons associated with the spontaneous symmetry breaking of $G_{\text{global}}$ chiral symmetry, and $\sigma = \sigma^a T_a$ denotes the Nambu-Goldstone bosons associated with the spontaneous breaking of $H_{\text{local}}$. This $\sigma$ is absorbed into the HLS gauge boson through the Higgs mechanism, and then the vector meson acquires its mass. $F_\pi$ and $F_\sigma$ are the decay constants of the associated particles. The phenomenologically
important parameter $a$ is defined as
\[ a = \frac{F_\sigma^2}{F_\pi^2}. \] (3)

The covariant derivatives of $\xi_{L,R}$ are given by
\[ D_\mu \xi_L = \partial_\mu \xi_L - i V_\mu \xi_L + i \xi_L \mathcal{L}_\mu, \]
\[ D_\mu \xi_R = \partial_\mu \xi_R - i V_\mu \xi_R + i \xi_R \mathcal{R}_\mu, \] (4)

where $V_\mu$ is the gauge field of $H_{\text{local}}$, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ are the external gauge fields introduced by gauging $G_{\text{global}}$ symmetry.

The HLS Lagrangian with lowest derivative terms at the chiral limit is given by \[ L = F_\pi^2 \text{tr} [\hat{\alpha}_\perp \hat{\alpha}_\perp] + F_\sigma^2 \text{tr} [\hat{\alpha}_\parallel \hat{\alpha}_\parallel] - \frac{1}{2 g^2} \text{tr} [V_{\mu\nu} V^{\mu\nu}], \] (5)

where $g$ is the HLS gauge coupling, $V_{\mu\nu}$ is the field strength of $V_\mu$ and
\[ \hat{\alpha}_\perp = \frac{1}{2i} [D^\mu \xi_R \cdot \xi_R^\dagger - D^\mu \xi_L \cdot \xi_L^\dagger], \]
\[ \hat{\alpha}_\parallel = \frac{1}{2i} [D^\mu \xi_R \cdot \xi_R^\dagger + D^\mu \xi_L \cdot \xi_L^\dagger]. \] (6)

Expanding the Lagrangian (5) in terms of the $\pi$ field with taking the unitary gauge of the HLS ($\sigma = 0$), we find the expressions for the mass of vector meson $M_\rho$, $\rho \pi \pi$ coupling $g_{\rho \pi \pi}$, $\rho$-$\gamma$ mixing strength $g_\rho$ and direct $\gamma \pi \pi$ coupling $g_{\gamma \pi \pi}$. Especially $g_{\gamma \pi \pi}$ coupling constant is expressed as
\[ g_{\gamma \pi \pi} = e \left(1 - \frac{a}{2}\right). \] (7)

By taking the parameter choice $a = 2$, the HLS reproduces the following phenomenological facts: 
- $g_{\rho \pi \pi} = g$ (universality of the $\rho$ coupling) \[18\];
- $M_\rho^2 = 2 g_{\rho \pi \pi}^2 F_\pi^2$ (the KSRF relation, version II) \[19\];
- $g_{\gamma \pi \pi} = 0$ (vector dominance of the electromagnetic form factor of the pion) \[18\]. Note that in framework of the HLS model, the vector dominance (VD) is satisfied by the choice $a = 2$.

In the HLS model it is possible to perform the derivative expansion systematically \[20\] \[21\] \[13\]. In this ChPT with HLS the vector meson mass is considered as small compared with the chiral symmetry breaking scale $\Lambda_\chi$, by assigning $O(p)$ to the HLS gauge coupling \[20\] \[21\]:
\[ g \sim O(p). \] (8)

For details of the ChPT with HLS, see Ref. \[13\].

### 3 Wilsonian Matching in Hot Matter

When we naively extended a result obtained in the low temperature region to the higher temperature region, the axial-vector and vector current correlators do not agree with each
other at the critical temperature. Disagreement between these current correlators is obviously inconsistent with the chiral symmetry restoration in QCD. However the parameters of the HLS Lagrangian should be determined by the underlying QCD. Thus it is natural that these parameters are dependent on temperature. In Ref. [16] the Wilsonian matching, which was originally proposed at $T = 0$ [14], was extended to non-zero temperature and it was shown that the parameters of the HLS Lagrangian have the intrinsic temperature dependences. Further in Ref. [22], it was shown that the effects of Lorentz symmetry violation at bare level are small through the Wilsonian matching at non-zero temperature. As is stressed in Ref. [16], the disagreement between the axial-vector and vector current correlators mentioned above is cured by including the intrinsic temperature dependences of the parameters of the HLS Lagrangian.

The Wilsonian matching proposed in Ref. [14] is done by matching the axial-vector and vector current correlators derived from the HLS with those by the operator product expansion (OPE) in QCD at the matching scale $\Lambda$. In Refs. [16, 22], this matching scheme was extended to the one at non-zero temperature. Since the Lorentz symmetry breaking effect in the bare pion decay constant is small, $F_{\pi,\text{bare}}^t \simeq F_{\pi,\text{bare}}^s$ [22], it is a good approximation that we determine the bare pion decay constant at non-zero temperature through the matching condition at zero temperature with putting possible temperature dependences on the gluonic and quark condensates [16, 22]:

\[
\frac{F_{\pi}(\Lambda; T)}{\Lambda^2} = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \frac{\langle G_{\mu\nu}G^{\mu\nu} \rangle_T}{\Lambda^4} + \pi^3 \frac{1408 \alpha_s \langle \bar{q}q \rangle^2_{T}}{27 \Lambda^6} \right].
\]

Through this condition the temperature dependences of the quark and gluonic condensates determine the intrinsic temperature dependences of the bare parameter $F_{\pi}(\Lambda; T)$, which is then converted into those of the on-shell parameter $F_{\pi}(\mu = 0; T)$ through the Wilsonian RGEs.

Now we consider the Wilsonian matching near the critical temperature with assuming that the quark condensate becomes zero for $T \to T_c$. As was discussed in Ref. [22, 1], we can use the Lorentz invariant Lagrangian at bare level as long as we study the pion decay constant and validity of VD. Then we start from the Lorentz invariant bare Lagrangian even in hot matter. The agreement between the current correlators, which characterizes the chiral symmetry restoration, $G_{A}^{(\text{HLS})} = G_{V}^{(\text{HLS})}$ is satisfied only if the following conditions are met [16]:

\[
\begin{align*}
  g(\Lambda; T) & \to T_c, \\
  a(\Lambda; T) & \to T_c.
\end{align*}
\]

Note that $(g, a) = (0, 1)$ is the fixed point of RGEs for $g$ and $a$ [14]. This implies that the parameter $M_\rho$ goes to zero for $T \to T_c$:

\[
M_\rho \to T_c, 0.
\]

Including the hadronic thermal effects as well as quantum corrections through the RGEs, we obtain the physical quantities. Near the critical temperature, we find that the pole mass
of the vector meson is the form

\[ m_\rho^2(T) \simeq M_\rho^2[1 + \delta_{\text{had}}(T)], \quad (12) \]

where the hadronic correction is obtained as

\[ \delta_{\text{had}}(T) = N_f \frac{15 - a^2}{144 \alpha} \frac{1}{F_\pi^2} T^2 > 0. \quad (13) \]

i.e., the hadronic thermal effect gives a positive correction near \( T_c \). However the pole mass \( m_\rho \) goes to zero by vanishing parameter \( M_\rho \) at the critical temperature and the vector manifestation (VM) of chiral symmetry is realized \[16\].

### 4 Vector Manifestation of Chiral Symmetry

The VM is highly in contrast with the standard chiral symmetry restoration based on the linear sigma model. In order to clarify this difference, we consider the multiplet structure. In the broken phase, the chiral representation does not agree with the mass eigenstate and there exists a mixing. Then the scalar, pseudoscalar, vector and axial-vector mesons belong to the following representations for \( N_f = 3 \) respectively:

\[
|s\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle,
|\pi\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \sin \psi + |(1, 8) \oplus (8, 1)\rangle \cos \psi,
|\rho\rangle = |(1, 8) \oplus (8, 1)\rangle,
|A_1\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \cos \psi - |(1, 8) \oplus (8, 1)\rangle \sin \psi,
\]

(14)

where \( \psi \) denotes the mixing angle and is given by \( \psi \simeq 45^\circ \) \[23, 24\].

Now we consider the chiral symmetry restoration, where it is expected that the above mixing disappears. There are two possibilities of chiral symmetry realization. One possible pattern is the case \( \cos \psi \to 0 \) for \( T \to T_c \). In this case, the pion belongs to \( |(3, 3^*) \oplus (3^*, 3)\rangle \) and becomes the chiral partner of the scalar meson. The vector and axial-vector mesons are in the same multiplet \( |(1, 8) \oplus (8, 1)\rangle \). This is of course the standard scenario of chiral symmetry restoration. Another possibility is the case \( \sin \psi \to 0 \) for \( T \to T_c \) \[15\]. In this case, the pion belongs to purely \( |(1, 8) \oplus (8, 1)\rangle \) and so its chiral partner is the vector meson:

\[
|\pi\rangle = |(1, 8) \oplus (8, 1)\rangle,
|\rho\rangle = |(1, 8) \oplus (8, 1)\rangle \quad \text{for} \quad \sin \psi \to 0.
\]

(15)

The scalar meson joins with the axial-vector meson in the same representation \( |(3, 3^*) \oplus (3^*, 3)\rangle \). This is nothing but the VM of chiral symmetry.

### 5 Violation of the Vector Dominance

As we mentioned in introduction, in Ref. \[1\], we studied the validity of the vector dominance (VD) which describes the phenomena in low energy region very well. At non-zero temperature there exists the hadronic thermal correction to the parameters. Thus it is nontrivial
whether or not the VD is realized in hot matter, especially near the critical temperature. Here we will show that the intrinsic temperature dependences of the parameters of the HLS Lagrangian play essential roles, and then the VD is largely violated near the critical temperature.

We first mention the direct $\gamma\pi\pi$ interaction at zero temperature. At the leading order of the derivative expansion in the HLS, we can read the form of the direct $\gamma\pi\pi$ interaction from Eq. (5) as

$$\Gamma^\mu_{\gamma\pi\pi(\text{tree})} = e(q - k)^\mu (1 - \frac{a}{2}),$$

where $e$ is the electromagnetic coupling constant and $q$ and $k$ denote outgoing momenta of the pions. As we have mentioned in section 2, for the parameter choice $a = 2$ the direct $\gamma\pi\pi$ coupling vanishes, which leads to the vector dominance of the electromagnetic form factor of the pion.

Next we evaluate the $\gamma\pi\pi$ coupling including quantum corrections as well as hadronic thermal effects. We obtained the parameters $a_t(T)$ and $a_s(T)$ which are the extension of the parameter $a$ in hot matter, in low temperature region ($T \ll M_\rho$) as follows:

$$a'(T) \simeq a^s(T) \simeq a(0) \left[ 1 + \frac{N_f}{12} \left( 1 - \frac{a^2}{4a(0)} \right) \frac{T^2}{F^2_\pi(0; T)} \right],$$

where $a$ is the parameter renormalized at the scale $\mu = M_\rho$, while $a(0)$ is the parameter defined from the $\gamma\pi\pi$ interaction at one-loop order. By using $F_\pi(0) = 86.4$ MeV, $a(0) \approx 2.31$ and $a(M_\rho) = 1.38$ obtained through the Wilsonian matching for $(\Lambda_{\text{QCD}}, \Lambda) = (0.4, 1.1)$ GeV and $N_f = 3$ [13], $a'$ and $a^s$ in Eq. (17) are evaluated as

$$a'(T) \simeq a^s(T) \simeq a(0) \left[ 1 + 0.066 \left( \frac{T}{50 \text{ MeV}} \right)^2 \right].$$

This implies that the parameters $a'$ and $a^s$ increase with temperature in the low temperature region. However, since the correction is small, we conclude that the VD is still satisfied in the low temperature region.

When we consider the situation near the critical temperature, the intrinsic thermal effects are important. From the VM conditions in hot matter [10], the parameters $(g, a)$ approach $(0, 1)$ for $T \to T_c$ by the intrinsic temperature dependences. Taking the limit $T \to T_c$, we find that

$$a'(T), a^s(T) \overset{T \to T_c}{\to} 1.$$

This implies that the vector dominance is largely violated near the critical temperature.

6 Summary

In the picture based on the vector manifestation (VM), we presented a new prediction associated with the validity of vector dominance (VD) in hot matter [1]. In the HLS model at zero temperature, the Wilsonian matching predicts $a \simeq 2$ [13] which guarantees the VD
of the electromagnetic form factor of the pion. Even at non-zero temperature, this is valid as long as we consider the thermal effects in the low temperature region, where the intrinsic temperature dependences are negligible. However the situation is changed when we consider the validity of the VD in higher temperature region. We showed that, as a consequence of including the intrinsic effect, the VD is largely violated at the critical temperature. In general, full temperature dependences include both hadronic and intrinsic thermal effects. Then there exist the violations of VD and universality of the ρ-coupling at generic temperature, although at low temperature the VD and universality are approximately satisfied.

In several analyses such as the one on the dilepton spectra in hot matter done in Ref [7], the VD is assumed to be held even in the high temperature region. We should note that the analysis in Ref. [17] shows that, if the VD holds, the thermal vector meson mass goes up. Then the assumption of the VD, from the beginning, seems to exclude the possibility of the dropping mass of the vector meson such as the one predicted by the Brown-Rho scaling [10]. Our result, which is consistent with the result in Ref. [17] in some sense, indicates that the assumption of the VD may need to be weakened, at least in some amounts, for consistently including the effect of the dropping mass of the vector meson into the analysis.

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