COMPURER PREDICTION OF TECHNOLOGICAL REGIMES OF RAPID CONE-SHAPED ADSORPTION FILTERS WITH CHEMICAL REGENERATION OF HOMOGENEOUS POROUS LOADS

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Abstract. Mathematical models for predicting technological regimes of filtration (water purification from the present impurities), backwashing, chemical regeneration and direct washing of rapid cone-shaped adsorption filters, taking into account the influence of temperature effects on the internal mass transfer kinetics at constant rates of the appropriate regimes, are formulated. Algorithms for numerical-asymptotic approximations of solutions of the corresponding nonlinear singularly perturbed boundary value problems for a model cone-shaped domain bounded by two equipotential surfaces and a flow surface are obtained. The proposed models in the complex allow computer experiments to be conducted to investigate the change of impurity concentrations in the filtration flow and on the surface of the load adsorbent, temperature of the filtration flow, filtration coefficient and active porosity along the filter height due to adsorption and desorption processes, and on their basis, to predict a good use of adsorbents and increase the protective time of rapid cone-shaped adsorption filters with chemical regeneration of homogenous porous loads.

Keywords: mathematical model, process of water purification, adsorption, rapid cone-shaped filter, chemical regeneration, homogeneous porous load

KOMPUTEROWE PROGNOZOWANIE TRYBÓW TECHNOLOGICZNYCH SZYBKICH STOŻKOWYCH FILTRÓW ADSORPCYJNYCH Z CHEMICZNĄ REGENERACJĄ JEDNORODNYCH POROWATYCH OBCIĄŻEN

Streszczenie. Sformułowano matematyczne modele do prognozowania trybów technologicznych filtry z czynnika zawierającego zanieczyszczenia, wnikania wstecznego, regeneracji chemicznej i bezpośredniego przemywania szybkich stożkowych adsorpcyjnych filtrów z uwzględnieniem wpływu temperatury na kinetykę wewnętrznego przemywania masy przy zachowaniu stałych prędkości odpowiednich trybów. Opracowano algorytmy numerycznie asymptotycznych aproksymacji rozwiązań odpowiadających problemów nieliniowych pojedynczo zaburzonych brzegowych dla domeny modelu o kształcie stożka, ograniczonej dwiema powierzchniami ekvipotencjalnymi i powierzchnią przepływu. Proponowane modele w kompleksie pozwalają na prowadzenie eksperymentów komputerowych w celu zbadania zmiany stężenia zanieczyszczeń w strumieniu filtracyjnym i na powierzchni adsorbentu obciążającego, temperatury przepływu filtracji, współczynnika filtracji oraz porowatości czynnej wzdłuż wysokości filtra ze względu na procesy adsorpcji i desorpcji, na ich podstawie przewidzieć bardziej optymalne zastosowania adsorbentów i wydłużenia czasu ochronnego szybkich stożkowych filtrów adsorpcyjnych z chemiczną regeneracją jednorodnych porowatych obciążen.

Słowa kluczowe: model matematyczny, proces oczyszczania wody, adsorpja, szybki stożkowy filtr, regeneracja chemiczna, jednorodne porowate obciążenie

Introduction

Any water needs to be purified before it can be used for domestic and drinking water supply. The main methods of water purification are clarification, decolorization and disinfection. The final stage is its purification from various impurities, in particular, calcium and magnesium salts, the total content of which determines the hardness of the water, as well as iron removal, in rapid adsorption filters with chemical regeneration of porous loads [4, 6]. They use natural (bentonite, montmorillonite, peat), artificial (activated carbon, artificial zeolites, polysorbs) and synthetic (nanostructured carbon sorbents) materials as adsorbents [17]. The rate of the adsorption process depends on the concentration, nature and structure of the impurities, filtration rate and temperature seepage, and type and properties of the adsorbent [5]. Maintaining a constant set filtration rate is achieved by automatically adjusting the increase in the opening of the valve on the filtrate pipeline as the resistance of the filter load increases due to the accumulation of impurity particles in it. The impulse to increase the opening of the valve on the filtrate pipeline is a change in the water level on the filter (controlled by a float device) or water flow in the filtrate pipeline (controlled by a throttle device and a differential pressure gauge) [11]. When the latch is fully open, the filter is switched off to regenerate the porous load. First, the backwash regime with a high water supply rate (2–3 times higher than the filtration rate), which lasts for 5–20 minutes and allows the filter material of the porous load to loosen and large particles of impurities to be removed. Next, a regime of chemical regeneration is carried out with a high feed rate of a solution of a certain reagent (potassium permanganate KMnO₄ is usually used), which starts the process of chemical restoration of the adsorption capacity of the porous load, and lasts for 10–30 minutes. Impurity particles from the filter material pass into the reagent solution. Finally, a regime of direct rinsing at a high water supply rate, lasting up to 10 minutes, seals the filter material of the porous load and removes residues of impurities and the chemical solution of the reagent.

The increasing needs for purified water in industrial enterprises and the growing cost of filter materials require research, on the one hand, into more optimal use of adsorbents and increasing the duration of filters by choosing their shape, in particular, taking into account the influence of changes in the temperature of the filtration flow along the filter height on the process of adsorption water purification, and on the other hand, into restoration of the filtration properties of porous loads by chemical regeneration for their reuse [4, 6].

1. Literature review

As an analysis of the literature sources shows, in particular [2, 3, 5, 7, 8, 12, 13, 15, 16, 18, 19], a significant contribution to the development of the theoretical foundations of filtering liquids through porous loads has been made by many scientists, both domestic and foreign. Note that mathematical models for predicting the technological processes of filtration and regeneration of porous loads by domestic researchers often use the model of D. M. Mintz [15] with constant rates of the respective processes and temperature, or some modification (improved model). In [10], its spatial generalization is proposed to predict the process of water purification from impurities in rapid cone-shaped filters while maintaining a constant filtration rate. The model proposed in this work is more efficient for theoretical studies aimed at optimizing the filtering process parameters (duration, shape, filter size, layer height, etc.) by introducing additional equations to determine the change in active porosity and filtration coefficient of filter load along its height, taking into account diffusion in the filtration flow and on the surface of the load grains. An urgent task is to generalize the appropriate model for computer prediction of technological regimes of filtration, backwashing, chemical regeneration and direct washing of rapid cone-shaped adsorption filters, taking into account the influence
of temperature effects on the internal mass transfer kinetics at constant rates of the appropriate regimes.

These models in the complex will allow providing computer experiments to predict a better use of adsorbents and increasing the protective time of rapid cone-shaped adsorption filters with chemical regeneration of homogeneous porous loads by taking into account not only the change in the filtration flow rate along the filter height, but also the effect of temperature on the coefficients that characterize the rates of mass transfer during adsorption and desorption, as well as on filter coefficient.

2. Formulation of the problem

Let’s develop a model of technological regimes of filtration, backwash, chemical regeneration and direct washing of rapid cone-shaped adsorption filters with chemical regeneration of a homogeneous porous load. We assume that in the filtration regime, the convective components of mass transfer and adsorption outweigh the contribution of diffusion and desorption, and in the backwash, chemical regeneration and direct washing regimes, the convective components of mass transfer and desorption outweigh the contribution of diffusion and adsorption. In addition, due to changes in the temperature of the filtration flow due to adsorption and desorption processes, the influence of temperature effects on the internal kinetics of mass transfer is taken into account. We assume that the convective components of mass transfer and adsorption outweigh the contribution of diffusion and desorption. In addition, the impact of temperature effects on the internal kinetics of mass transfer is taken into account due to changes in the temperature of the filtration flow due to adsorption and desorption processes. So, for the domain \( G = G_t \times (0, \infty) \), where \( G_t \) is a spatial one-connected domain \((z = (x, y, z))\) bounded by smooth, orthogonal interlocking lines, by two equipotential surfaces \( S_{s} \), \( S' \) and by the flow surface \( S'' \) (Fig. 1), the corresponding spatial model problems for predicting technological regimes of rapid cone-shaped adsorption filters, taking into account the reverse influence of process characteristics (impurity concentration, respectively, in the filtration flow and on the surface of the adsorbent) on the load characteristics (filtration coefficients, porosity, adsorption, desorption) will consist of equations describing the motion of the filtration flow and the equation of continuity:

\[
\vec{v} = k' \cdot \nabla \varphi, \, \nabla \vec{v} = 0.
\]

Next are equations for determining the change in impurity concentrations in the filtration flow and on the surface of the load adsorbent, temperature of the filtration flow, filtration coefficient and active porosity along the filter height, respectively, for the filtration regime:

\[
\begin{align*}
(\sigma' \cdot C)' &= \text{div} (D \cdot \text{grad} \, C) - \varphi' \cdot \text{grad} \, C - \\
&\quad -\alpha' \cdot C + \beta' \cdot U, \\
\left(\sigma' \cdot U\right)' &= \text{div} (D' \cdot \text{grad} \, U) + \alpha' \cdot C - \beta' \cdot U, \\
\left(\sigma' \cdot T\right)' &= \text{div} (D'' \cdot \text{grad} \, T) + \\
&\quad + \varphi' \cdot (\alpha' \cdot C - \beta' \cdot U), \\
\kappa' &= \mu' \cdot U, \text{ and } \sigma' = \lambda' \cdot U,
\end{align*}
\]

backwash, chemical regeneration and direct washing regimes:

\[
\begin{align*}
(\sigma' \cdot C)' &= \text{div} (D \cdot \text{grad} \, C) - \varphi' \cdot \text{grad} \, C + \\
&\quad + \beta' \cdot U - \alpha' \cdot C, \\
(\sigma' \cdot U)' &= \text{div} (D' \cdot \text{grad} \, U) - \beta' \cdot U + \alpha' \cdot C, \\
(\sigma' \cdot T)' &= \text{div} (D'' \cdot \text{grad} \, T) + \\
&\quad + \varphi' \cdot (\beta' \cdot U - \alpha' \cdot C), \\
\kappa' &= \mu' \cdot U, \text{ and } \sigma' = \lambda' \cdot U,
\end{align*}
\]

which are supplemented by the following boundary conditions, respectively, for filtration and direct washing regimes:

\[
\begin{align*}
\left[\varphi\right] &= \varphi_{x}, \left[C\right] = \varphi'_{x}, \left[C\right]_{x} = 0, \\
\left[U\right] &= U_{x}, U'_{x} = 0, \left[U\right]_{x} = 0, \\
\left[T\right] &= T', T'' = 0, \left[T\right]_{x} = 0,
\end{align*}
\]

backwash and chemical regeneration regimes:

\[
\begin{align*}
\left[\varphi\right] &= \varphi_{x}, \left[C\right] = \varphi'_{x}, \left[C\right]_{x} = 0, \\
\left[U\right] &= U_{x}, U'_{x} = 0, \left[U\right]_{x} = 0, \\
\left[T\right] &= T', T'' = 0, \left[T\right]_{x} = 0,
\end{align*}
\]

and initial conditions:

\[
\begin{align*}
C_{I} = c_{I}, C'_{I} = 0, C''_{I} = 0, \\
U_{I} = u_{I}, U'_{I} = 0, U''_{I} = 0, \\
T_{I} = T', T''_{I} = 0, \left[T\right]_{x} = 0,
\end{align*}
\]

where \( \varphi = \varphi(x, y, z), \) and \( \vec{v} = \vec{V}(v, v, v) \) is the potential vector of the filtration, \( 0 < \varphi < \varphi' < 0 < v < [v] = \sqrt{(x^2 + y^2 + z^2) + v^2(x, y, z)} \gg 0, \)

\( k' \) is the initial filtration coefficient, \( k' \geq 0, \) \( n \) is outer normal to the corresponding surface, \( C = C(x, y, z, t) \) and \( U = U(x, y, z, t) \) are the concentrations of impurities, respectively, in the filtration flow and on the surface of the adsorbent, \( T = T(x, y, z, t) \) is the temperature of the filtration flow at point \((x, y, z)\) at time \( t, \) \( \kappa = \kappa(x, y, z, t) \) is the filtration coefficient, \( \sigma = \sigma(x, y, z, t) \) is the active porosity, \( D \) and \( D' \) are the impurity diffusion coefficients, respectively, in the filtration flow and on the surface of the adsorbent, \( D = \epsilon \cdot d_{s}, \) \( d_{s} > 0, \) and \( \alpha \) and \( \beta \) are coefficients that characterize the rate of mass transfer, respectively, in the adsorption of impurities from the filtration flow on the surface of the load adsorbent and the desorption of impurities from the surface of the load adsorbent into the filtration flow, for the model problem of predicting the filtration regimes\( \alpha = \sum_{k=0}^{2} \sum_{h=0}^{4} e^{s_{k}t_{h}} \cdot a_{k, h} \cdot \varphi \cdot T, a_{k, h} \in R \) (\( s_{k} = (0, 2), s_{h} = (0, 2) \) ) and for the model problems of predicting the backwashing, chemical regeneration and direct washing regimes\( \alpha = \epsilon \cdot \sum_{k=0}^{2} \sum_{h=0}^{4} e^{s_{k}t_{h}} \cdot \beta_{k, h} \cdot \varphi \cdot T, \beta_{k, h} \in R \) (\( s_{k} = (0, 2), s_{h} = (0, 2) \) )

(\( x = (x, y, z) \)).
$\sigma_0^0 = \sigma_0^0(x,y,z)$ are quite smooth functions, consistent with each other on the lines of intersection of surfaces $S_1$, $S_2$ and $S^\prime$ of domain $G$ [1].

![Spatial filtering domain $G_s$ with conditional section $\Gamma$ (cone-shaped filter)](image)

3. Materials and methods

The problem is solved in the same way as in [10] by fixing on the surface $S_1$ some point $A$ ($A=B$) and sequential execution of conditional sections $\Gamma_1 = ALMDBC \cap \Gamma_2 = ADDABCCB_1$ along the corresponding surfaces of the flow (we denote for convenience $\Gamma = \Gamma_1 \cup \Gamma_2$). The model problems of forecasting of technological regimes of filtration (1), (2), (4), (5), (8), backwash (1), (3), (6)–(8), chemical regeneration (1), (3), (6)–(8) and direct wash (1), (3), (4), (5), (8) in rapid cone-shaped filter with chemical regeneration of porous load reduced to the solving of the problems in the received one-connected domain $G_s \setminus \Gamma$ that is a curvilinear parallelepiped $ABCDABCCB_1$, bounded by two equipotential surfaces $ABB_2$, $CDD_2$, and four flow surfaces $ABCD = ALMDBC \cap \Gamma_2 = ADDABCCB_1$ (Fig. 1). The surfaces are smooth and orthogonal to each other at angular points and along the edges, with the addition of the impermeability condition $\varphi_0^0| = 0$ along section $\Gamma$:

$$\left\{ \begin{array}{l}
\varphi|_{ABB_2} = \varphi|_{CDD_2} = \varphi^0, \\
\varphi|_{ABCD} = \varphi|_{ADDABCCB_1} = \varphi^0,
\end{array} \right.$$

$$\left\{ \begin{array}{l}
C|_{ABB_2} = C|_{CDD_2} = 0, \\
C|_{ABCD} = C|_{ADDABCCB_1} = 0,
\end{array} \right.$$

$$\left\{ \begin{array}{l}
U|_{ABB_2} = U|_{CDD_2} = 0, \\
U|_{ABCD} = U|_{ADDABCCB_1} = 0,
\end{array} \right.$$

$$\left\{ \begin{array}{l}
T|_{ABB_2} = T|_{CDD_2} = 0, \\
T|_{ABCD} = T|_{ADDABCCB_1} = 0.
\end{array} \right.$$

(9) (10) (11)

backwash, chemical regeneration and direct wash regimes:

$$\varphi|_{ABCD} = \varphi|_{ADDABCCB_1} = \varphi^0,$$

$$C|_{ABCD} = C|_{ADDABCCB_1} = 0,$$

$$U|_{ABCD} = U|_{ADDABCCB_1} = 0,$$

$$T|_{ABCD} = T|_{ADDABCCB_1} = 0,$$

the initial conditions (8) and the conditions of further “gluing” of the banks of conditional section $\Gamma$:

$$\left\{ \begin{array}{l}
\varphi|_{ALMDBC} = \varphi|_{BCCB_1} = \varphi^0|_{ALMDBC}, \\
\varphi|_{ADDABCCB_1} = \varphi|_{BCCB_1} = \varphi^0|_{ADDABCCB_1},
\end{array} \right.$$

and conditions of agreement of values of impurity concentrations in the filtration flow and on the surface of the load adsorbent and values of the filtration flow temperature on the conditional sections of section $\Gamma$:

$$\left\{ \begin{array}{l}
C|_{ALMDBC} = C|_{BCCB_1}, \\
C|_{ADDABCCB_1} = C|_{BCCB_1},
\end{array} \right.$$

$$\left\{ \begin{array}{l}
U|_{ALMDBC} = U|_{BCCB_1}, \\
U|_{ADDABCCB_1} = U|_{BCCB_1},
\end{array} \right.$$
which are supplemented by the following boundary conditions:
\begin{equation}
\left\{
\begin{array}{l}
\left[c_{t}\right]_{\nu_{1}=0} = c_{0}^{
u_{1}} - c_{0}^{
u_{1}} = 0, \\
\left[u\right]_{\nu_{1}=0} = u_{0}^{
u_{1}} - u_{0}^{
u_{1}} = 0, \\
\left[u_{t}\right]_{\nu_{1}=0} = u_{0}^{
u_{1}} - u_{0}^{
u_{1}} = 0,
\end{array}
\right.
\end{equation}
and initial conditions:
\begin{equation}
\left\{
\begin{array}{l}
\left[c_{t}\right]_{\nu_{1}=0} = c_{0}^{
u_{1}} - c_{0}^{
u_{1}} = 0, \\
\left[u\right]_{\nu_{1}=0} = u_{0}^{
u_{1}} - u_{0}^{
u_{1}} = 0, \\
\left[u_{t}\right]_{\nu_{1}=0} = u_{0}^{
u_{1}} - u_{0}^{
u_{1}} = 0,
\end{array}
\right.
\end{equation}
and conditions of consistency of the values of impurity concentrations in the filtration flow and on the surface of the load adsorbent and the values of the filtration flow temperature on the conditional surfaces of the model problem of predicting the filtration regime:
\begin{equation}
\begin{array}{l}
\mathcal{\tilde{\alpha}} = \sum_{i,j=1}^{m} \sum_{h,s}^{n} e^{
u_{1} \cdot i} \cdot \alpha_{h,s}^{
u_{1}} \cdot \tilde{t}_{i} \cdot \tilde{t}_{i} \\
\mathcal{\tilde{\alpha}}_{h,s} \in \mathbb{R} \quad (s_{1} = 0, 2), \\
\tilde{\beta} = \sum_{i,j=1}^{m} \sum_{h,s}^{n} e^{
u_{1} \cdot i} \cdot \tilde{\beta}_{h,s}^{
u_{1}} \cdot \tilde{t}_{i} \\
\tilde{\beta}_{h,s} \in \mathbb{R} \quad (s_{1} = 0, 2, s_{2} = 0, 2 - s_{1}), \\
\tilde{\mu} = \sum_{i,j=1}^{m} \cdot \tilde{\mu}_{i} \cdot \tilde{t}_{i}, \\
\tilde{\mu}_{i} \in \mathbb{R} \quad (s = 0, 2), \\
\tilde{\beta} = \sum_{i,j=1}^{m} \sum_{h,s}^{n} e^{
u_{1} \cdot i} \cdot \beta_{h,s}^{
u_{1}} \cdot \tilde{t}_{i} \\
\beta_{h,s} \in \mathbb{R} \quad (s_{1} = 0, 2, s_{2} = 0, 2 - s_{1})),
\end{array}
\end{equation}
and (16)–(19) with accuracy $O(e^{-\nu_{1}})$ was found in the form of the following series:
\begin{equation}
\begin{array}{l}
c = \sum_{j=0}^{m} c_{j}^{
u_{1}} + \sum_{j=0}^{m} e^{
u_{1}} \cdot \sum_{j=0}^{m} c_{j}^{
u_{1}} \cdot \sum_{j=0}^{m} P_{j}, \quad j = 0, ..., m, \\
u = \sum_{j=0}^{m} e^{
u_{1}} \cdot u_{j}^{
u_{1}} + \sum_{j=0}^{m} e^{
u_{1}} \cdot \sum_{j=0}^{m} u_{j}^{
u_{1}} \cdot \sum_{j=0}^{m} P_{j}, \quad j = 0, ..., m, \\
\tilde{t} = \sum_{j=0}^{m} e^{
u_{1}} \cdot \tilde{t}_{j}, \quad \tilde{t}_{j} \in \mathbb{R}, \\
\tilde{k} = \sum_{j=0}^{m} \cdot \tilde{k}_{j}, \\
\tilde{\sigma} = \sum_{j=0}^{m} e^{
u_{1}} \cdot \tilde{\sigma}_{j}.
\end{array}
\end{equation}

Fig. 2. Spatial domain of complex potential $G_{c}$

Similar to [10], a numerically asymptotic approximation of the solution $(c, u, \tilde{t}, \tilde{\alpha}, \tilde{\sigma})$ of problems (15), (17)–(19)
\[ g_i(\varphi, \psi, \eta, \iota) = \int \frac{g_i(\varphi, \psi, \eta, \iota) - f(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \bar{g}_i(\varphi, \psi, \eta, \iota) = \int \frac{\bar{g}_i(\varphi, \psi, \eta, \iota) - \bar{f}(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \bar{g}_i(\varphi, \psi, \eta, \iota) = \kappa_i \int \frac{\bar{g}_i(\varphi, \psi, \eta, \iota) - \bar{f}(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ g_i(\varphi, \psi, \eta, \iota) = \int \frac{g_i(\varphi, \psi, \eta, \iota) - f(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ g_i(\varphi, \psi, \eta, \iota) = \int \frac{g_i(\varphi, \psi, \eta, \iota) - f(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \varrho = \varrho_0 - \sigma_0. \]

where:

\[ \bar{g}_i(\varphi, \psi, \eta, \iota) = \kappa_i \int \frac{\bar{g}_i(\varphi, \psi, \eta, \iota) - \bar{f}(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \bar{g}_i(\varphi, \psi, \eta, \iota) = \int \frac{\bar{g}_i(\varphi, \psi, \eta, \iota) - \bar{f}(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \bar{g}_i(\varphi, \psi, \eta, \iota) = \int \frac{\bar{g}_i(\varphi, \psi, \eta, \iota) - \bar{f}(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \bar{g}_i(\varphi, \psi, \eta, \iota) = \int \frac{\bar{g}_i(\varphi, \psi, \eta, \iota) - \bar{f}(\varphi, \psi, \eta, \iota)}{\nu^2(\varphi, \psi, \eta)} \, d\nu, \]

\[ \varrho = \varrho_0 - \sigma_0. \]

and to find \( c_i \), \( u_i \), \( \bar{\varrho} \), \( \bar{\sigma}_i \), \( (i = 0, n) \) in problems (16)–(19), the following formulas are obtained:

\[ c_i = \left\{ \begin{array}{ll}
\bar{g}_i(\varphi, \psi, \eta, \iota) + \bar{c}_i(\psi, \eta, \iota - f(\varphi, \eta, \psi)), & \iota \geq \bar{f}, \\
\bar{g}_i(\varphi, \psi, \eta, \iota) + \bar{c}_i(\psi, \eta, \iota - f(\varphi, \eta, \psi)), & \iota < \bar{f}, \\
u_i = \bar{u}_i e^{-\frac{\Delta t_i}{\nu}}, & \iota = \bar{f}, \\
T_{\bar{\rho}} = \left\{ \begin{array}{ll}
\bar{g}_i(\varphi, \psi, \eta, \iota) + \bar{T}_i(\psi, \eta, \iota - f(\varphi, \eta, \psi)), & \iota \geq \bar{f}, \\
\bar{g}_i(\varphi, \psi, \eta, \iota) + \bar{T}_i(\psi, \eta, \iota - f(\varphi, \eta, \psi)), & \iota < \bar{f}, \\
\end{array} \right.
\]
The proposed models in the complex allow computer experiments of technological processes, computer simulation of technological processes, computer techniques and computer technologies, programming. http://orcid.org/0000-0003-3672-8469

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