Analysis of Power Enhancement for a Row of Wind Turbines Using the Actuator Line Technique

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Abstract. The effect of wake interaction for a row of three wind turbines in a wind farm is analysed using the actuator line technique. Both full wake and half wake situations are considered with the aim of deriving the optimal pitch setting of the foremost turbine, with respect to the total power from the row. The mutual distance between the turbines is 5 diameters and the turbines are considered to operate in a wind shear with an exponent of 0.15, with the rotor centre located at 1.4 radii from the ground. The main findings reveal clear effects of reducing the loading on the foremost turbine towards increased production of turbine 2 and 3 in a row.

Key words: Wind turbines wakes, wake interaction, actuator line, pitch control

1. Introduction

Wind turbines clustered in parks or on a single row experience severe wake interactions in wind sectors aligned with the row. Today turbines are controlled individually to optimize power and life time independent of the other turbines. As a consequence downstream turbines produce around 40-60\% of the power compared to the foremost turbine operating in undisturbed inflow in these wind sectors. The objective with this work is to investigate how a control strategy for the foremost turbine in a row may increase the power for the row in total. A similar approach was suggested by Corten and Schaak[1]. Since simple one-dimensional momentum theory predicts that the loading on the foremost turbine should be reduced in order to increase total power, the objective is to find the optimal operating point for the first turbine, letting the remaining turbines operate individually. Modelling of wake interaction between turbines have been approached by various means, like Frandsen et al.[2] who derived analytical links between multiple merging wakes from momentum consideration, as well as for the infinitely large wind farm. A wake meandering model was proposed by Thomsen et al.[3] combining actuator disc computation using Navier-Stokes, with a method where the ambient turbulence meanders the wake. Full CFD approaches, modelling the entire flow field between turbines, has been pursued by Troldborg et al.[4], revealing that CFD has matured, although still computational expensive, to a level where the detailed unsteady behaviour of interacting wakes is captured. Results for three turbines in a row operating at constant pitch and rotational speed, have shown that the model predicts significantly reduced power produced by turbines 2 and 3 with fatigue loads increasing drastically for the same turbines. In order to limit the present investigation, the full wake and half wake situations are considered for a row of three turbines in a park with a mutual distance of 5 diameters. A single wind speed is considered including an imposed wind shear which is modelled using a method by Mikkelsen et al.[5].

2. Approach

2.1 The actuator line method combined with FLEX5

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The aerodynamic loads from the wind turbines is handled by the FLEX5 [6] code combined with the actuator line technique [7-8] and the 3D general purpose flow solver, EllipSys3D [9] developed at DTU (Technical University of Denmark) and Risø National Laboratory. EllipSys3D solves the discretized incompressible Navier-Stokes equations in general curvilinear coordinates using a block structured finite volume approach. The FLEX5 supports 28 flexible DOF but in the present investigation, all turbines are considered to be stiff. Furthermore, FLEX5 supports control algorithms for the generator and the yaw and pitch systems; however, the focus of this study is on static manoeuvring of the pitch for the first turbine. By systematically altering the pitch setting of the first turbine the performance of a system of a number of turbines on a row is evaluated. In order to limit the computations in the present investigation, remaining operating parameters are chosen to be fixed i.e. one wind speed including wind shear, constant rotational speed and zero yaw. The gross flow field around the rotors is computed with the EllipSys3D code employing Large Eddy Simulation (LES) technique, with the rotor blades represented by body forces (actuator lines).

2.2 Computational domain and boundary conditions
The computational domain is a rectangular Cartesian box measuring 12 radii in height, 10 radii in width and 32 radii in length (flow direction). In order to limit the number of computational nodes, cells are concentrated around the rotors and in the developing wake, referred to as the near domain. The near domain grid is chosen to have an equidistant structure between neighbouring cells, thereby maximising the ability of the grid to preserve vortical structures, mainly tip and root vorticities shed from the actuator lines. The rotor blades are resolved with 16 nodes (32 nodes for the rotor) and the equidistantly distributed near domain extends about 1.3 radii away from the hub. Away from the near domain the grid is stretched towards the outer boundaries. In total 64 cells is used in the horizontal and vertical direction and 384 cells in the streamwise direction. The boundary conditions are the usual in and outflow conditions in the main flow direction, no-slip at the ground, far-field velocity at the top, and cyclic conditions on the sides, thus simulating infinitely many columns with three turbines in each column. The present computation has not yet been subject to a thorough grid sensitivity study and it is expected that with the current resolution, some grid dependence is unavoidable. The chosen resolution is a compromise between the ability to resolve vorticities and computational cost, and the current grid resolution is believed to be near the lower limit of the acceptable using the actuator line technique. A more detailed analysis of grid dependence was carried out by Troldborg et al.[4].

2.3 Wind shear modelling and turbulence
Aerodynamic and aeroelastic computations on wind turbines often approximate effects of wind shear using simple power law profiles, however, actual wind shear profiles are often far more varying than what can be handled by most engineering codes. An actual atmospheric wind shear profile furthermore also experiences changes in wind direction up through the boundary layer. Actual wind shear profiles depend on numerous factors like stratification, temperature, Coriolis force, etc. and the aim of the present modelling is to account for all these factors artificially by use of momentum sources combined with CFD. The method was presented recently by Mikkelsen et al.[5] where initial computation are used to impose a desired wind shear profile which may include directional variation with height. In most cases wind direction effects are neglected in aerodynamic computations and in the in the present investigation, directional variations are not considered. A non-uniform inflow profile imposed at an upstream boundary using CFD, is generally not preserved through the computational domain until impact with the wind turbine rotor. Therefore momentum sources needs to be applied everywhere in the domain in order to impose an arbitrary steady atmospheric wind shear profile. The magnitude of the momentum sources were reported to be 3-4 orders of magnitude smaller than the momentum sources related to the actuator line. Free stream turbulence also has a major impact on the behaviour and development of the rotors wake. A method developed recently by Troldborg et al.[10] used concentrated momentum sources at a plane extending beyond the area of the rotors, but not necessarily
covering the entire inlet plane. The plane is located slightly upstream the first rotor (about 1 radii) and not at the inlet boundary, which typically is located 6-10 radii away from the first rotor. Turbulence is then generated from data files at the plane and then convected by the flow solver downstream until impact with the rotor. The present work does not include turbulence, only wind shear is included, although it is recognized that turbulence is an important parameter for accurate predictions of wake interaction.

2.4 Wind turbine and flow parameters
The computations are carried out using turbine data from the Tjæreborg wind turbine. The blades measure 30.54m in radius and it rotates with 22.1 rpm corresponding to a tip speed of 70.7m/s. The technical details can be found in Øye[11,12]. A wind velocity at hub height of 10m/s is used in all computation equivalent to a tip speed ratio $\lambda=7.07$. Wind shear is applied according to

$$V(y) = V_{hub} \left( \frac{y}{h} \right)^{\alpha}$$

where the exponent is set to $\alpha=0.15$ and the hub height set equal to $h=1.4$ radii, which is less than the actual Tjæreborg turbine. An effective Reynolds number equal to $Re=50,000$ based on blade radius and wind velocity is applied. This level is well below the actual field Reynolds number, but as discussed by Sørensen et al.[7] the influence of the Reynolds number on the rotor loading an wake is minor provided it is above a certain minimum.

2.5 Limitations of the modelling
Although the present approach seeks to model the wake interaction from first principle methods combining a number of complex techniques, important limitations are still inherent in the modelling. From an energy conversion point of view, it is not considered to be that important that the flexible DOFs are disabled. The imposed wind shear also seems reasonable, however, neglecting turbulence is considered to be an important limitation together with the limited numerical resolution. Turbulence is expected to have a major effect on how the wake brakes up and the exchange of momentum with wind outside the wake region, see Troldborg [10], for all downstream turbines. In the current setup, however, turbulence has not been included.

3. Results
3.1 Flow behaviour
With the above parameters, computations have been carried out considering the full wake and half wake situation. The pitch angle is altered according to table 1 below.

| Turbine\Pitch | Full wake       | Half wake       |
|---------------|----------------|----------------|
| 1             | $0^\circ$, $1^\circ$, $2^\circ$ | $0^\circ$, $0.5^\circ$, $1^\circ$, $1.5^\circ$ |
| 2             | $0^\circ$       | $0^\circ$       |
| 3             | $0^\circ$       | $0^\circ$       |
Detailed profiles of spanwise distribution on the blade and on the development of the wake deficit are not included here, but may be found in the papers by Troldborg et al.[4,10], only integrated values are shown. Since the wake develops as the computations progress, simulations are continued until the transient behavior settles at some mean level. Figure 1 shows a side view of the shed vorticity contours and the velocity vectors of the computed flow field through the three turbines.

![Figure 1 Side view of the shed vorticity contours displaying the undisturbed wake from the foremost turbine and the full wake interaction for turbines 2 and 3. Velocity vectors indicate the undisturbed incoming wind shear profile, the steady deficit behind the foremost turbine and the unsteady developing deficit behind the second and third turbine.](image)

Figures 2-5 displays views of fully developed unsteady wakes seen from above and the side for the full wake and the half wake. The wake behind the first turbine is steady and dominated by tip vorticities which are smeared out into a continuous vortex sheet. Root vorticies are also visible as shed contours from the centre of the rotor. On impact with the second turbine the first wake is lifted outside the shed wake of the second turbine which brakes up into smaller turbulent eddies about 1 diameter downstream. The third turbine experiences a highly turbulent inflow resulting in a corresponding highly unsteady loading and production.

![Figure 2 Full wake interaction, view from above, vorticity and vectors](image)

Looking at the half wake situation, figures 3 and 4, the shed wake from the first turbine is not surprisingly similar to that of the full wake, however, skewed at an angle of 5.7°.
Figure 3 Half wake interaction, side view, vorticity and vectors
The second and third turbines are affected by the upstream wake correspondingly with a third turbine being exposed to a more turbulent field on the affected part as compared to the second turbine.

Figure 4 Half wake interactions, view from above, vorticity and vectors
Figure 5 shows the computed wake one radius behind turbine 3 for the full and the half wake. Whereas the full wake results in a nearly homogeneous distribution of vorticies located in the periphery, the half wake case results in a preserved vorticity sheet in the unaffected part. The wake patterns of the different simulation, was found to be similar for the considered case stated in table 1. Looking at the loadings, the transient development of the thrust coefficients $C_T$ for the half wake cases is shown in figure 6. The effect of increasing the pitch is reflected clearly in the reduction of the thrust for the first turbine. The second turbine experiences a fully organized wake from the foremost turbine resulting in a periodic load pattern, figure 6 (right).
Figure 5 Fully developed wakes one radii behind the third turbine, full wake (left) half wake (right), vorticity. The reduction in load of the first turbine does results in a gain in thrust for the second turbine which, however, is somewhat less than the reduction of the first turbines when looking at the fully developed stages. The third turbine also experience a periodic load pattern with some unsteady turbulent contributions due to the separated wake of the second turbine, at a mean level which is slightly higher than that of the second turbine.

Figure 6 Thrust coefficients $C_T$ for turbines in half wake. Full transient (left) and close up (right) The corresponding power coefficients $C_P$ are depicted in figure 7 revealing comparable trends as found for $C_T$.

Figure 7 Power coefficients $C_P$ for turbine 1 and 3 in half wake. Full transient (left) and close up (right) The $C_P$ for full wake cases are depicted in figure 8 (left) showing steady production of turbines 1 and 2 and the more chaotic for the third turbine. The accumulated production for the row is given in figure 8 (right), from which it is difficult to conclude what might be the best of the considered pitch settings.
Figure 8 Power coefficients $C_P$ for the three turbines in full wake. Individual $C_P$ (left) and accumulated power from the row (right)

An averaging procedure has been applied (equivalent to about 5 revolutions) to the accumulated power shown in figure 9 for both the half and full wake. In the initial phase the power is reduced with increasing pitch as expected, which develops to, that the curves are nearing each other shifting position to be the most producing during the considered transient. The transient as such appears somewhat too short to draw more solid conclusions on what may be the most favourable pitch setting of the turbines. Presently, computations have not been carried out for a longer period as they are very time consuming. On the other hand, it seems fair to conclude that the reduced loading and power on the first turbine to some degree is regained by the two downstream turbines. However, longer transients are needed in order to give more solid ground to conclude on. The pitch was altered between $0^\circ$ and $2^\circ$ which resulted in a reduction of about 10-15% on $C_T$, which is a quite narrow variation of the pitch. Further investigation should seek to expand the pitch changes causing larger changes in produced power for the row.

Figure 9 Averaged $C_P$ for the half wake (left) and full wake
4. Conclusions
The objective of a wind farm is to extract as much energy as possible from the incoming wind field. To accomplish this, a global control strategy for the wind farm may be needed. As a first step the implications of changing the operating conditions of the foremost turbine in a row, has been investigated by systematically changing the pitch setting. The actuator line technique appear very feasible to investigate effects of wake interaction between turbines, although computational expensive. The naturally developing wakes interact in a in a fully unsteady nonlinear manner. Regarding optimal pitch settings the present work remains inconclusive with the simulations carried out so far. Future studies should seek to include free stream turbulence and longer transients’ needs to be computed. A detailed validation and comparison with measured data and simpler wake methods is of key importance in view getting confidence to the presented method. Presently however, these important validation step remains to be carried out in full. The potential of the developed method appears promising with the possibility of simulating wind farms in combination with global control strategies of the operating settings of all turbines.

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