Influence of conjugate convective heat transfer on temperature fields in thin walls that organize liquid layers of various orientations

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Abstract. Numerical studies of unsteady conjugate natural convective heat transfer in the model of a thin-walled fuel tank are carried out in the conjugate formulation. The fields of temperature in liquid and in solid walls of the tank are calculated. The evolution of convective flows and temperature fields after a sudden supply of heat to the base of the tank is investigated. Flat layers of liquid enclosed between flat walls of different orientations with inclination angles of 0, 30, 45, 60 and 90 degrees are considered. With the angle of inclination of 0 degrees the outer surface of the side walls is insulated. The outer surface of the lower wall heats up suddenly, the outer surface of the upper wall is isothermally cold. The system of equations of thermogravitation convection in the Boussinesq approximation in dimensionless form, written in terms of temperature, vector potential of the velocity field and velocity vortex, is solved. It is shown that an inhomogeneous temperature field is formed inside the solid walls of the finite thermal conductivity. The thermal conductivity of walls and conditions at the upper boundary of the liquid layer affect significantly the spatial form of convective fuel flows in fuel tanks and laws of conjugated natural convective heat exchange.

1. Introduction

The thermal state of the thin-walled structure, for example, aircraft during takeoff and landing, at the initial stages of entering the cruising speed depends significantly on the processes of unsteady conjugate convective heat transfer in fuel tanks and in air layers of the fuselage. When flying at supersonic speeds, the processes of heating the aircraft skin are added. With the development of aviation technology, requirements for the quality of calculations of thermal stresses in non-isothermal thin-walled elements of aircraft structures are growing significantly [1]. The distribution of temperature, temperature gradients and thermal stresses in walls of the fuel tanks depends on the conjugated convective heat exchange. Similar problems are typical of many technical devices in the modes of heating or cooling switching on and off. In a non-uniformly heated volume of liquid in the gravity field, the natural convective flow, accompanied by stratification of the fluid temperature, is developed. When heat is supplied to walls of the tank, the heated liquid floats and accumulates from above. The spatial form of convective flows has a significant influence on laws of conjugate heat transfer. In turn, the shape of convective flows depends largely on the configuration of cavity and location of the heated and cooled walls and their fragments [1-4]. For adequate estimates of the thermal stress fields in structures it is necessary to know the local features of hydrodynamics and the
features of local conjugate heat transfer generated by them and, as a consequence, the regularities of
the dependence of temperature fields on time in thin walls.

Reliable knowledge of laws of conjugate natural convective heat transfer with unsteady conditions
on external and internal surfaces is important in evaluation and accurate calculation of thermal stresses
and analysis of the general stress-strain state of the structure. The result of unsteady complex-
conjugate heat transfer is the distribution of thermal stresses in thin-walled structural elements. This
work is the development of series of research aimed at studying the effect of conjugated natural
convective heat transfer on the temperature distribution in thin walls performed at the Kutateladze
Institute of Thermophysics SB RAS [2-4]. Dependencies of temperature distributions on a thin vertical
wall when hot liquid flows on it from the heated opposite vertical wall are experimentally investigated.
Unsteady temperature fields are measured by a thermal imager and thermocouples. Numerical studies
of non-stationary conjugate natural convective heat transfer in the model of a thin-walled fuel tank are
Carried out in the conjugate formulation. The temperature fields were calculated in liquid and in solid
walls of the tank. Evolution of convective flows and temperature fields is studied.

2. Model
Numerical simulation is carried out in dimensionless form in two-dimensional conjugate formulation
in Cartesian coordinates. The study area is a two-dimensional rectangular cavity with a ratio of the
area width to the height of 5:1 is filled with liquid and is bounded from all sides by thin walls with the
thickness of 0.0133 layer height. The outer surface of the upper wall is maintained at the constant
temperature equal to the initial temperature of liquid filling the area. The outer surface of the side
walls is adiabatic. The outer surface of the lower wall is suddenly heated and maintained at the
constant set temperature. Numerical simulation was carried out at the following angles of inclination
of the region: 0⁰, 30⁰, 45⁰, 70⁰, 90⁰. Two types of walls with thermal conductivity equal to the thermal
conductivity of liquid and with thermal conductivity 1041.3 times higher than the thermal conductivity
of liquid are considered.

The layer height H is selected as scaling of the geometric dimensions. The scale of temperature ΔT = T₁ - T₂,
where T₁ and T₂ are the temperatures on the outer surface of the lower and upper walls,
respectively. The velocity scale is v/L, where v is the kinematic viscosity of liquid and the time scale is
H²/v. Convective heat transfer in liquid is described by a dimensionless system of Navier-Stokes
equations in the Boussinesq approximation, written in terms of temperature, vortex, and current
function:

\[
\begin{align*}
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} &= \frac{1}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= -\omega \\
\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \omega}{\partial x} \frac{\partial \psi}{\partial y} &= \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + Gr \frac{\partial T}{\partial x}
\end{align*}
\]

Here Gr = g β H⁻¹ ΔT ν² is the Grasgof number, where g is the acceleration of gravity, β is the
volume expansion coefficient of liquid, Pr = ν/α is the Prandtl number, α is the thermal diffusivity of
liquid, T is the dimensionless temperature, ω is the dimensionless vortex, ψ is the dimensionless
current function.

Conductive heat transfer in solid walls is described by the heat equation:

\[
\frac{\partial T}{\partial t} + \alpha (\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) = 0
\]
where $\alpha_s$ is the thermal diffusivity of the solid wall material.

Numerical simulation is performed for the following parameters: $\rho_f = 819$ [kg/m$^3$] is the fluid density; $\beta_f = 8.3 \times 10^{-4}$ [1/K] is the coefficient of volume expansion of fluid; $\eta_f = 1.49 \times 10^{-3}$ [kg/(m⋅s)] is the dynamic viscosity of fluid; $\lambda_f = 0.1162$ [W/(m⋅K)] is the fluid thermal conductivity; $C_p = 2001.29$ [j/(kg⋅K)] is heat capacity of fluid; $\rho_s = 2780$ [kg/m$^3$] is the density of the material of the walls; $\lambda_s = 121$ [W/(m⋅K)] is the solid thermal conductivity; $C_p = 921$ [j/(kg⋅K)] is heat capacity of the material of the walls. In the simulation when walls thermal conductivity equal to liquid thermal conductivity values of $\rho_s$ and $\lambda_s$, substituted values $\rho_f$ and $\lambda_f$, respectively.

The calculations were carried out by the finite element method on an irregular triangular grid with thickening to solid walls. Linear basis functions on triangles were used. In calculations for all angles of inclination, a unified grid with 23569 nodes was used; the coordinates of the nodes of this grid were rotated by the required angle using an affine transformation before calculations.

3. Results and discussion

Calculations are carried out with $Pr = 25.66$ and $Gr = 1000$ for two kinds of materials of solid walls with the ratio of the thermal conductivities of materials of walls to the thermal conductivity of fluid $\lambda_s/\lambda_f = 1$ and $\lambda_s/\lambda_f = 1041.3$ at angles 0$^\circ$, 30$^\circ$, 45$^\circ$, 70$^\circ$, 90$^\circ$.

Figures 1 and 2 show isotherms at different angles of inclination and ratios $\lambda_s/\lambda_f = 1$ and $\lambda_s/\lambda_f = 1041.3$, respectively. For ease of perception, the results in the computational domain are given to a total slope angle of 0$^\circ$, using the inverse affine transformation.
It is seen that in both cases, when tilted at $0^\circ$, cell convection of Rayleigh-Benard type develops with the same number of convective rolls, 8 pieces. However, the direction of fluid movement in the rolls differs. At $\lambda_s/\lambda_f = 1$, downward flows are formed on the inner surface of the side (end) walls, whereas at $\lambda_s/\lambda_f = 1041.3$, the upward flows are formed. This is due to the fact that under the action of conductive heat exchange, the end walls are heated almost linearly in height (figure 4), as a result of which the liquid near the end wall is heated and forced to rise. Whereas at $\lambda_s/\lambda_f = 1$, the relative role of conductive heat exchange in formation of the temperature field inside the wall is significantly inferior to the role of convective heat exchange, resulting in the wall uneven heating along the height under the action of a cold downward fluid flow (figure 3). It is also noticeable that if at $\lambda_s/\lambda_f = 1$ rolls of relatively equal size are formed, then at $\lambda_s/\lambda_f = 1041.3$ the roll sizes differ significantly.

With a slope angle of $30^\circ$ and $\lambda_s/\lambda_f = 1$, a flow with one convective cell is formed. This is due to the fact that under the action of the buoyancy force, the heated liquid rises along the inclined lower wall and flows to the right end wall, along which it subsequently rises to the cold upper wall. Then the flow goes down along the inclined upper wall, cools and reaches the left end wall, then falls along it to the hot lower wall. Thus, the global advective stream is formed.

When the angle of inclination is $30^\circ$ and $\lambda_s/\lambda_f = 1041.3$, the result of the influence of conjugate heat transfer and resulting lateral heating of fluid on end walls of the spatial form of convective flow are substantially changed. Three convective cells of different sizes are formed. Due to the rising along the inclined hot lower wall at the right end wall, the largest convective cell is formed, which occupies most of the region. A downward flow is formed in the first third of the region. Also, the downward flow of liquid is formed on the left end wall. Between two downstream flows there is an upward flow from the lower hot wall, thus forming two additional convective cells. Moreover, the intensity of convective flows in additional cells is much higher than in the main cell.

At an angle of inclination of $45^\circ$, a single-vortex convective flow with the counterclockwise direction of liquid motion at $\lambda_s/\lambda_f = 1$ and $\lambda_s/\lambda_f = 1041.3$ is formed. The direction of movement of liquid is completely determined by the slope of the area. In this case, similar temperature and fluid velocity distributions are formed, which differ mainly near the vertical walls. A similar pattern is observed at angles of $70^\circ$ and $90^\circ$.

However, it should be noted that depending on the ratio of thermal conductivity of the wall material and liquid, the intensity of convective flow in the main cell may vary slightly. In figures 5-6 it
is noticeable that when the angle of inclination is 30°, the intensity of the flow at $\lambda_s/\lambda_f = 1041.3$ is lower than the intensity of the flow at $\lambda_s/\lambda_f = 1$.

With an increase in the angle of inclination to 45° at $\lambda_s/\lambda_f = 1041.3$, the flow intensity increases markedly faster than at $\lambda_s/\lambda_f = 1$. Thus, it turns out that the most intense flow occurs at $\lambda_s/\lambda_f = 1041.3$, and not at $\lambda_s/\lambda_f = 1$.

Figure 5. Longitudinal velocity component profiles in the cross-section $x = 2.5$ at $Pr = 25.66$, $Gr = 10^3$ and $\lambda_s/\lambda_f = 1$ with angle: 1 – 0°, 2 – 30°, 3 – 45°, 4 – 60°, 5 – 90°.

Figure 6. Longitudinal velocity component profiles in the cross-section $x = 2.5$ at $Pr = 25.66$, $Gr = 10^3$ and $\lambda_s/\lambda_f = 1041.3$ with angle: 1 – 0°, 2 – 30°, 3 – 45°, 4 – 60°, 5 – 90°.

With an increase in the angle to 70° at both ratios of thermal conductivity, the intensity of convective flows increases approximately to the equal values.

When the tilt angle is 90° for all relations of thermal conductivity, the longitudinal velocity is maximal.

Figures 7 and 8 show temperature distributions on the inner surface of the lower wall at different thermal conductivity ratios. It is noticeable that in both cases, the lower wall is heated unevenly, but the amplitude of temperature change at $\lambda_s/\lambda_f = 1041.3$ is orders of magnitude lower than at $\lambda_s/\lambda_f = 1$, due to the fact that thermal conductivity of the wall material is orders of magnitude higher.

Figure 7. Temperature profiles at the level of $y = 0.0133$ at $Pr = 25.66$, $Gr = 10^3$ and $\lambda_s/\lambda_f = 1$ with angle: 1 – 0°, 2 – 30°, 3 – 45°, 4 – 60°, 5 – 90°.

Figure 8. Temperature profiles at the level of $y = 0.0133$ at $Pr = 25.66$, $Gr = 10^3$ and $\lambda_s/\lambda_f = 1041.3$ with angle: 1 – 0°, 2 – 30°, 3 – 45°, 4 – 60°, 5 – 90°.
It is noticeable that at an angle of inclination of $0^\circ$ in both cases there is a periodic temperature fluctuation on the inner surface of the lower wall. At $\lambda_s/\lambda_f = 1$ near the vertical walls, the temperature gradients are noticeably lower than in the center of the region, whereas at $\lambda_s/\lambda_f = 1041.3$ such an effect is not observed.

As the angle of inclination increases, the amplitude of temperature changes on the outer surface of the lower wall increases also. Moreover, at $\lambda_s/\lambda_f = 1$ the amplitude changes more significantly than at $\lambda_s/\lambda_f = 1041.3$. When the angle of inclination is $45^\circ$ for both ratios of thermal conductivity, the same asymmetry of heating of the outer surface of the lower wall is observed, with a pronounced minimum in the left corner, in the absence of a similar maximum in the right corner.

**Conclusion**

Numerical studies of non-stationary conjugate natural convective heat transfer in the model of a thin-walled fuel tank are carried out in the conjugate formulation. The fields of temperature were calculated in liquid and in solid walls of the tank. The evolution of the convective flows and temperature fields after a sudden supply of heat to the base of the tank was studied. Flat layers of liquid enclosed between flat walls of different orientations with inclination angles of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ and $90^\circ$ degrees are considered.

It is shown that conjugate heat transfer has a significant effect on the temperature field distribution, shape and intensity of convective flows in the entire region in the angle range of $0^\circ$-$30^\circ$. Changing of the angle from $45^\circ$ to $90^\circ$ and the changing of the ratio of walls material thermal conductivity and fluid thermal conductivity little effect on convective flows shape and intensity in the entire region.

For all the considered angles of inclination of the region at $\lambda_s/\lambda_f = 1041.3$, the lower and upper walls have an almost uniform temperature distribution, end walls have a temperature distribution close to the linear one. At $\lambda_s/\lambda_f = 1$, convective heat transfer significantly affects the temperature field inside thin solid walls, making it significantly inhomogeneous.

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