Different effect of the hyperons Λ and Ξ on the nuclear core

Yu-Hong Tan¹, Ping-Zhi Ning²
1. Theoretical Physics Division, Nankai Institute of Mathematics, Nankai University, Tianjin 300071, P.R.China
2. Department of Physics, Nankai University, Tianjin 300071, P.R.China

Abstract

We demonstrate the different effect of strange impurities (Λ and Ξ) on the static properties of nuclei within the framework of the relativistic mean-field model. Systematic calculations show that the gluelike role of Λ hyperon is universal for all Λ-hypernuclei considered. However, Ξ⁻ hyperon has the gluelike role only for the protons distribution in nuclei, and for the neutrons distribution Ξ⁻ hyperon plays a repulsive role. On the other hand, Ξ⁰ hyperon attracts surrounding neutrons and reveals a repulsive force to the protons. Possible explanations of the above observation are discussed.

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I Introduction

Change of bulk properties of nuclei under the presence of strange impurities, like the lambda hyperon (Λ), is an interesting subject in hypernuclear physics. Since a Λ does not suffer from Pauli blocking, it can locate at the center of a nucleus, then the Λ attracts surrounding nucleons (the gluelike role of Λ) and makes the nucleus shrink. One might expect only a bit change of the size for most of the nuclei. However, significant shrinkage of hypernuclei size could be expected when a Λ is added to loosely bound light nuclei such as \(^{6}\)Li\(^{1,2,3}\). Recently, the experiment KEK-PS E419 has found clear evidence for this shrinkage of hypernucleus \(^{7}\)Λ\(^{4,5}\).

In order to obtain a more profound understanding of the gluelike role of strange impurities in nuclei, it is necessary to consider other strange impurities, like the sigma (Σ⁻) and cascade (Ξ⁻, Ξ⁰). The behavior of these hyperons in the nuclear medium as well as the hyperon-nucleus potential, is of particular importance for this study. However, the Σ-nucleus potential has been still unclear until more recently because experimental information is limited\(^{6}\). For the sake of the improvement of this situation, a new experiment at KEK is carried out to measure the inclusive (\(π^−, K^−\)) spectrum, which is sensitive to the Σ-nucleus potential\(^{10}\). The result shows that a strongly repulsive Σ-nucleus potential is required to reproduce the observed spectrum. So, we have reason to believe that Σ hyperon does not have any gluelike role and can not make the nucleus shrink. Next in mass are the Ξ⁻ and Ξ⁰ hyperons. Experimental evidence suggests that Ξ-nucleus potential is attractive\(^{8}\).

Therefore we may only consider Λ- and Ξ-hypernuclei in this work. Our purposes are: (i) to test the universality of the gluelike role of Λ impurity in a variety of Λ-hypernuclei which may not be loosely bound light nuclei; (ii) to see whether or not the Λ- and Ξ-impurities behave the same, in view of the fact that both Λ- and Ξ-nuclear potentials are attractive; (iii) to predict the properties of the Ξ-impurities in Ξ-hypernuclei.

To accomplish these, a standard approach to the subject is the relativistic mean-field (RMF) model, to which a brief description for the hypernuclei is given in Sec.II. In Sec.III, after testing the validity of force parameters used in the RMF model, systematic calculations are performed for Λ-hypernuclei and the universality of the gluelike role of Λ impurity is revealed. In Sec.IV, we provide the RMF results for Ξ-hypernuclei and different effects of Ξ⁻ and Ξ⁰ on the nucleus are discussed. A brief summary and conclusions are drawn in Sec.V.
II  The RMF Model

The relativistic mean-field model (RMF) has been used to describe nuclear matter, finite nuclei, and hypernuclei successfully. Here, we start from a Lagrangian density of the form

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_Y,$$

where, $Y = \Lambda$, or $\Xi^-$, or $\Xi^0$, $\mathcal{L}_N$ is the standard Lagrangian of the RMF model

$$\mathcal{L}_N = \bar{\psi}_N (i\gamma_\mu \partial^\mu - M_N - g_\sigma N \sigma - g_\omega N \gamma_\mu \omega^\mu$$
$$- \frac{1}{2} g_\rho N \gamma_\mu \vec{p}_N \cdot \vec{p}_\mu - c_\gamma \frac{1}{2} \tau_3 N A^\mu) \psi_N$$
$$+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2)$$
$$- \frac{1}{3} b_\sigma^3 - \frac{1}{4} c_\sigma^4$$
$$- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$
$$- \frac{1}{4} \bar{R}_{\mu\nu} \cdot \bar{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \bar{p}_\mu \cdot \bar{p}^\mu$$
$$- \frac{1}{4} H_{\mu\nu} \cdot H^{\mu\nu}$$

where

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu,$$

$$\bar{R}_{\mu\nu} = \partial_\mu \bar{p}_\nu - \partial_\nu \bar{p}_\mu,$$

$$H_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

It involves nucleons ($\psi_N$), scalar $\sigma$ mesons ($\sigma$), vector $\omega$ mesons ($\omega_\mu$), vector isovector $\rho$ mesons ($\bar{p}_\mu$), and the photon ($A_\mu$). The scalar self-interaction $-\frac{1}{4} b_\sigma^3 - \frac{1}{4} c_\sigma^4$ are included, as well. The parametrization of the nucleonic sector (NL-SH) are adopted from Ref.~[10], the properties of finite nuclei can be well described.

The Lagrangian density $\mathcal{L}_\Lambda$ describes the hyperon $\psi_\Lambda$ and its couplings to mesonic fields includes the $\omega-\Lambda$ tensor coupling term

$$\mathcal{L}_\Lambda = \bar{\psi}_\Lambda (i\gamma_\mu \partial_\mu - m_\Lambda - g_{\sigma \Lambda} \sigma - g_{\omega \Lambda} \gamma_\mu \omega^\mu) \psi_\Lambda$$
$$+ \frac{f_{\omega \Lambda}}{2 m_Y} \bar{\psi}_\Lambda \sigma_{\mu\nu} \gamma^{\mu\nu} \omega^\alpha \psi_\Lambda.$$

Since $\Lambda$ is neutral and isoscalar baryon, it does not couple with the $\rho$ mesons and the photon. We adopt the parametrization of $\Lambda$ sector from Ref.~[10]: $g_{\sigma \Lambda}/g_{\sigma N} = 0.49, g_{\omega \Lambda}/g_{\omega N} = 0.512, f_{\omega \Lambda}/f_{\omega N} = -0.616$. Using these coupling constants, the properties of $\Lambda$-hypernuclei can be well described.

The Lagrangian density $\mathcal{L}_\Xi$ describes the hyperon $\psi_\Xi$ and its couplings to the $\sigma, \omega, \rho$ mesonic fields and the photon field

$$\mathcal{L}_\Xi = \bar{\psi}_\Xi (i\gamma_\mu \partial_\mu - m_\Xi - g_{\sigma \Xi} \sigma - g_{\omega \Xi} \gamma_\mu \omega^\mu$$
$$- \frac{1}{2} g_{\rho \Xi} \gamma_\mu \vec{p}_\mu \cdot \vec{p}^\mu - c_\gamma \frac{1}{2} \tau_3 \Xi A^\mu) \psi_\Xi.$$

We fix the coupling constants of $\Xi$, say the one to the vector fields with the quark model (SU(6) symmetry),

$$g_{\omega \Xi} = \frac{1}{3} g_{\omega N},$$

$$g_{\rho \Xi} = g_{\rho N},$$

and those to the scalar field with the experimental information—the optical potential. It turns out that two coupling constants of $\Xi$, $g_{\sigma \Xi}$ and $g_{\omega \Xi}$, are strongly correlated because they are fixed by the depth of the $\Xi$ potential

$$V_0^\Xi = g_{\sigma \Xi} \sigma^q + g_{\omega \Xi} \omega^q.$$
in the saturation nuclear matter. But the experimental data on $\Xi^-$-hypernuclei are very little. Dover and Gal analyzing old emulsion data on $\Xi^-$ hypernuclei, conclude a nuclear potential well depth of $V_0^\Xi = -21 \sim -24$ MeV. Fukuda et al fit to the very low energy part of $\Xi^-$ hypernuclear spectrum in the $^{12}\text{C}(K^-,K^+)$ reaction in experiments E224 at KEK, estimate the value of $V_0^\Xi$ between $-16$ and $-20$ MeV. Recently, E885 at the AGS indicates a potential depth of $V_0^\Xi = -14$ MeV or less. So the depth $V_0^\Xi$ of $\Xi$ in nuclear matter is not well fixed on.

### III The effect of the $\Lambda$ impurity

We start from calculation of the single-particle energies for $\Lambda$ in $\Lambda$-hypernuclei within the framework of the RMF model with force parameters taken from Ref. [10], and present the results in Fig.1. It can be seen that the results are in good agreement with the experimental data. Very small spin-orbit splitting for $\Lambda$-hypernuclei are also observed. This shows the RMF theory with the parameter set used for the $\Lambda$-hyperonic sector is reliable for studying the effect of the $\Lambda$ impurity, and has a predicting ability.

In order to observe the universality of the gluonlike role of the $\Lambda$ hyperon impurity, an unified RMF calculation is needed and careful tests should be done. Hence in our calculation typical hypernuclei between $^7\Lambda\text{Li}$ and $^{209}_{\Lambda}\text{Pb}$ are selected. Our results are shown in Table 1 in which some results for medium and heavy hypernuclei have been given in our previous work. In the table, $-E/A$ (in MeV) is the binding energies per baryon, $r_{cm}$ is the r.m.s. charge radius, and $r_n$, $r_p$ are the calculated r.m.s. radii (in fm) of the hyperon ($\Lambda$ or $\Xi$), neutron and proton, respectively. Here, hyperon is at its $1s_{1/2}$ configuration. The definition of these quantities can be found in Ref. [10]. For comparison, the results for normal nuclei are also given. From Table 1, it can be seen that for lighter $\Lambda$-hypernucleus, the size of the core nucleus in a hypernucleus is smaller than the core nucleus in free space (i.e., normal nucleus). Although there are only a bit change in the core nucleus due to the presence of the $\Lambda$ impurity. For instance, the r.m.s. radius $r_n$ ($r_p$) of the neutrons (protons) decreases from 2.32 fm (2.37 fm) in $^6\text{Li}$ to 2.25 fm (2.29 fm ) in $^7\Lambda\text{Li}$. We also see from the Table that the change of $r_n$ and $r_p$ gradually decreases with increasing mass number. The above RMF results reveal the universality of the shrinkage effect for $\Lambda$-hypernuclei, but not for $\Xi$-hypernuclei. It is particular interesting to observe a quite different effect caused by $\Xi$ hyperon impurity.

### IV The effect of the $\Xi$ impurity

In order to see whether or not there is the shrinkage effect of $\Xi$-hypernuclei, we have carried out the standard RMF calculations for some $\Xi^-$- and $\Xi^0$-hypernuclei. Due to insufficient experiment information on $\Xi$-hypernuclei, the $\Xi$ potential well depth is relatively uncertain, values appearing in the literature range from about -30 to -10 MeV. Recent experiments with light nuclei suggest that the value lies on the less bound size of this range. However, it may be more deeply bound for heavy nuclei. As a result, a number of values of the $\Xi$ potential well depth $V_0^\Xi$ for each hypernuclei are used to test the sensitivity of the position of the $\Xi$ single-particle energy levels to the potential depth. In Fig.2, we only present results of calculations for the nucleus $\text{Zr}$ with a comparison of the $\Xi^-$ (upper part) and the $\Xi^0$ (lower part) single-particle levels. It can be seen that the change of the potential well depth causes large change in the single-particle energies. As $V_0^\Xi$ deeper, the single-particle energies of hyperon increase significantly, and the spin-orbit splitting has a little larger. We also see that the attractive Coulomb interaction for $\Xi^-$ leads to a considerable stronger binding of $\Xi^-$ in nuclei when compared with $\Xi^0$-hypernuclei. In Fig.3, we give the $\Xi^-$ (upper part) and $\Xi^0$(lower part) binding energies in the nuclei O, Ca, Zr, Pb for $V_0^\Xi = -10$ and $-28$ MeV (only the $1s, 1p, 1d, 1f$ states are given). The solid (dashed) curves are the results for $V_0^\Xi = -10$(-28)MeV.

Next let us go further into the question how the static properties of the $\Xi$-hypernuclei are affected by the potential depth $V_0^\Xi$. Both $\Xi^-$ and $\Xi^0$ are at the 1s state in hypernuclei. Our RMF results are shown in Table 2 with $V_0^\Xi = -10, -18$ and $-28$ MeV, respectively. As seen from the Table 2 with increasing depth $|V_0^\Xi|$ from 10 MeV to 28 MeV, the binding energies per baryon ($-E/A$) become larger. We can see such an uncertainty of $\Xi$ potential well depth gets clearly reflected in an important variation of the $\Xi$ binding energies, as it is shown in Figs. 2 and 3 and also of the binding energies per baryon ($-E/A$), presented in Table 2. Because of that, no firm conclusions can be drawn from the quoted values of $-E/A$. We can also notice that the charge radius and the r.m.s. radii of the $\Xi$ hyperon, neutrons and protons become smaller with increasing potential well depth. Note that the reduction of r.m.s. radius for the neutrons ($r_n$) and protons ($r_p$) is different. In the case of $\Xi^-$-hypernuclei, the reduction of the $r_p$ is faster than that of the $r_n$. While in the case of $\Xi^0$-hypernuclei, the...
reduction of the $r_p$ is slower than that of the $r_n$. Thus the RMF model predicts that the proton and neutron distribution have different response to the potential depth $V_0^\Xi$ for the $\Xi^-$- and $\Xi^0$-hypernuclei.

Now, we study whether the $\Xi$ hyperon impurity has the gluon-like role as the $\Lambda$ does. The results are shown in table 1 in the form $C_+A$, where the central values ($C$) are the results obtained with the -18 MeV $\Xi$ potential well depth, while the extremes of the uncertainty interval $C + A$ and $C + B$ are obtained with $V_0^\Xi = -10$ MeV and -28 MeV, respectively. A similar presentation is used in table 3 (where rho exchange is not considered, i.e., $g_{\rho\Xi} = 0$).

From table 1 we find, by adding a $\Xi^-$ hyperon to the nuclei, the r.m.s. radius of the neutrons becomes a little larger, while r.m.s. radius of the protons becomes much smaller, comparing with that in the normal nuclei. Contrast to the situation of $\Xi^-$-hypernuclei, by adding a $\Xi^0$ hyperon, the r.m.s. radius of the protons become larger and that of the neutrons become smaller. This is different from the situation of adding a $\Lambda$ hyperon. We know that $\Lambda$ has a gluon-like role, both the r.m.s. radii of the protons and neutrons become smaller when adding a $\Lambda$. Note that $\Lambda$, $\Xi^-$ and $\Xi^0$ are different particles from proton and neutron, they are all not constrained by the Pauli exclusion, it is obviously that the common explanation for the shrinkage does not suit the case of $\Xi^-$ and $\Xi^0$. Otherwise, both $\Lambda$ and $\Xi^0$ hyperon are neutral, hence the origin of the above difference can not be attributed to the Coulomb potential. There must be some other source that we don’t recognize.

To reach a better understanding of the different behavior of the $\Lambda$, $\Xi^-$ and $\Xi^0$ impurities in the nucleus, we make an inspection of their isospin. $\Lambda$, $\Xi^-$ and $\Xi^0$ have different third component of isospin, which may be responsible for their different behavior. The different third component of isospin works through the coupling of baryon with the $\rho$ mesons in the RMF model. We may imagine if the $\rho$ mesons couplings for $\Xi^-$ and $\Xi^0$ are omitted from the RMF calculation ($g_{\rho\Xi} = 0$), the above mentioned different behavior of $\Xi^-$ and $\Xi^0$ shall disappear. After eliminating the contribution of the $\rho$ mesons, the RMF results are shown in table 3 from which we find the r.m.s. radii of the protons and neutrons reduce when adding a $\Xi^-$ or $\Xi^0$ hyperon to the normal nuclei, the same as the situation of adding a $\Lambda$ hyperon. We obtain the same nuclear shrinkage by $\Xi^-$ and $\Xi^0$ when ignoring the contribution of the $\rho$ mesons. From the interactive term of nucleons with the $\rho$ mesons, we can find : when adding a $\Xi^-$, the attractive force increases for the protons and the repulsive force increase for the neutrons, the situation is contrary to the above when adding a $\Xi^0$. That explains the above RMF results reasonably. So, we can conclude the $\rho$ mesons play an important role, and the different behavior of the $\Lambda$, $\Xi^-$ and $\Xi^0$ impurities is due to their different isospin. Although the changes are small, the different response of $r_p$ and $r_n$ to $\Xi^-$ and $\Xi^0$ may be interesting to know what kind of properties the two-body $\Xi N$ interaction. Probably the isospin $T = 0$ interaction is attractively large, while the $T = 1$ interaction is repulsive and small. Although the r.m.s. radius is reduced only for one kind of nucleons, but the r.m.s radius of other kind of nucleons become larger, that is seems that the nuclei may even swell somewhat when adding a $\Xi^-$ or $\Xi^0$. That is very different from the nuclear shrinkage by $\Lambda$.

V Summary and conclusion

Within the framework of the RMF theory, the $\Lambda$ single-particle energies was calculated and the results are in good agreement with the experiments for all of the hypernuclei considered. Very small spin-orbit splitting for $\Lambda$-hypernuclei was observed, which is agreement with earlier phenomenological analysis. From the investigation of the effects of $\Lambda$ on the core nucleus, We obtain the shrinkage effect inducing by $\Lambda$ hyperon impurity, otherwise, we find other light and medium $\Lambda$-hypernuclei also have this shrinkage effect, i.e., the gluon-like role of $\Lambda$ impurity is universal.

For $\Xi$-hypernuclei, first, we study the effect of the potential well depth $V_0^\Xi$ on the static properties of $\Xi$-hypernuclei. We can see the uncertainty of $\Xi$ potential well depth gets clearly reflected in an important variation of the $\Xi$ binding energies, because of that, no firm conclusions can be drawn from the quoted $\Xi$ binding energies and values of $-E/A$. In the $\Xi^-$-hypernuclei, the reduction of r.m.s. radius of the protons is larger than the reduction of that of the neutrons, while in the $\Xi^0$-hypernuclei, the reduction of r.m.s. radius of the neutrons are larger than that of the protons with the deeper potential well depth. The strength of the effect of $V_0^\Xi$ on different nucleons is different in $\Xi$-hypernuclei. The effect of $V_0^\Xi$ on the hypernuclei decreases with the increasing of the atomic number.

After that, we study the effect of the adding $\Xi$ hyperon on the nuclear core, we find: by adding a $\Xi^-$ hyperon to the nucleus, the r.m.s. radius of the neutrons become a little larger, while the r.m.s. radius of the protons become smaller, comparing with that in the normal nucleus, and the decrease of the r.m.s. radius of the protons is larger as the $V_0^\Xi$ deeper. While when adding a $\Xi^0$ hyperon, the r.m.s. radius of the protons become a little larger and that of the neutrons become smaller. Although the r.m.s. radius is reduced only for one kind of
nucleons, but the r.m.s radius of other kind of nucleons become larger, that is seems that the nuclei may even swell somewhat when adding a \( \Xi^- \) or \( \Xi^0 \). That is very different from the nuclear shrinkage by \( \Lambda \). And we find the \( \rho \) mesons play an important role, the different effect on the nuclear core by \( \Lambda, \Xi^- \), \( \Xi^0 \) is due to their different isospin. Although the changes are small, the different response of \( r_p \) and \( r_n \) to \( \Xi^- \) and \( \Xi^0 \) may be interesting to know what kind of properties of the two-body \( \Xi N \) interaction. Probably the isospin \( T = 0 \) interaction is attractively large, while the \( T = 1 \) interaction is repulsive and small.

The present work only focuses on the pure \( \Lambda \) and \( \Xi \) hypernuclei, the coupling between \( \Xi N \) and \( \Lambda \Lambda \) channels in \( \Xi \) hypernuclei isn’t taken into consideration. The physics of \( \Lambda \Lambda \) hypernuclei (\( \Lambda \Lambda \) and \( \Xi N \) mix up in a formalism of coupled channel) and \( \Lambda \Xi \) hypernuclei have attracted a lot of attention and are subject of current investigation, because of that, more reliable information on \( \Xi N \) interaction and \( \Xi \)-nucleus are desired.

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Figure 1: A binding energies (in MeV) for some hypernuclei($^{13}_\Lambda$C, $^{16}_\Lambda$O, $^{51}_\Lambda$V, $^{89}_\Lambda$Y, $^{139}_\Lambda$La, $^{208}_\Lambda$Pb). The solid lines are our RMF results with parameters of Ref.[10]. The experimental data are taken from Ref.[16], Ref.[17], and Ref.[18] denoted by $\triangle$, $\circ$, $\star$, respectively.

Figure 2: Dependence of the position of the $\Xi^-$ (upper part) and the $\Xi^0$ (lower part) single-particle levels in Zr for $V_0^{\Xi} = -10, -14, -18, -24, -28$ MeV (only the 1s, 1p, 1d, 1f states are given).
Figure 3: Comparison of the $\Xi^-$ (upper part) and the $\Xi^0$ (lower part) binding energies in O, Ca, Zr, Pb for $V_0^\Xi = -10$ and -28 MeV.

Table 1: Binding energy per baryon, $-E/A$ (in MeV), r.m.s. charge radius $r_{ch}$ (those of the nucleons, in fm), r.m.s. radii of the hyperon ($A, \Xi^-, \Xi^0$), neutron and proton, $r_n$ and $r_p$ (in fm), respectively, including the contribution of the $\rho$ mesons. The configuration of hyperon is $1s_{1/2}$ for all hypernuclei. The meaning of $Z$ in $^A Z$ is the number of protons. The results of $\Xi$-hypernuclei are given in the form $C^+A$, where $C, C + A$ and $C + B$ are the results obtained with $V_0^\Xi = -18, -10$ and -28 MeV, respectively.

| $^A Z$ | $-E/A$ | $r_{ch}$ | $r_n$ | $r_p$ | $^{16}O$ | $r_{ch}$ | $r_n$ | $r_p$ |
|--------|--------|---------|-------|-------|--------|---------|-------|-------|
| $^6$ Li | 5.67   | 2.52    | 2.32  | 2.37  | 8.04   | 2.70    | 2.55  | 2.58  |
| $^7$ Li | 5.47   | 2.43    | 2.58  | 2.25  | 2.29   | 8.27    | 2.70  | 2.55  | 2.58  |
| $^7$ Li | 5.18   | 2.39    | 3.20  | 2.35  | 2.25   | 8.14    | 2.67  | 2.59  | 2.58  | 2.58  |
| $^7$ Li | 4.99   | 2.55    | 3.49  | 2.23  | 2.41   | 7.92    | 2.73  | 2.71  | 2.53  | 2.60  |
| $^8$ Be | 5.42   | 2.48    | 2.30  | 2.34  | 8.52   | 3.46    | 3.31  | 3.36  |
| $^9$ Be | 5.58   | 2.44    | 2.40  | 2.27  | 2.30  | 8.75    | 3.46  | 3.31  | 3.36  |
| $^9$ Be | 5.30   | 2.41    | 2.80  | 2.33  | 2.26  | 8.75    | 3.44  | 3.33  | 3.34  |
| $^9$ Be | 5.09   | 2.50    | 2.90  | 2.25  | 2.30  | 8.56    | 3.47  | 3.30  | 3.37  |
| $^{10}$ C | 7.47  | 2.46    | 2.30  | 2.32  | 7.90  | 5.51    | 5.71  | 5.45  |
| $^{12}$ C | 7.79  | 2.44    | 2.19  | 2.28  | 7.98  | 5.51    | 4.05  | 5.71  | 5.44  |
| $^{14}$ C | 7.55  | 2.42    | 2.43  | 2.32  | 8.01  | 5.56    | 3.68  | 5.72  | 5.44  |
| $^{13}$ C | 7.31  | 2.48    | 2.56  | 2.26  | 7.96  | 5.51    | 4.05  | 5.70  | 5.45  |
Table 2: Binding energy per baryon, \(-E/A\) (in MeV), r.m.s. charge radius \(r_{ch}\) (those of the nucleons, in fm), r.m.s. radii of the hyperon, neutron and proton, \(r_y, r_n\) and \(r_p\) (in fm), respectively, including the contribution of the \(\rho\) mesons. The meaning of \(Z\) in \(^A\Z\) is the number of protons in hypernuclei. The configuration of hyperon is 1s\(_{1/2}\) for all hypernuclei. The results of \(\Xi\) hypernuclei are given for \(V_0^\Xi = -10, -18, -28\) MeV.

| \(^A\Z\) | \(V_0^\Xi\) | \(-E/A\) | \(r_{ch}\) | \(r_y\) | \(r_n\) | \(r_p\) |
|---|---|---|---|---|---|---|
| \(^6\)Li | 5.67 | 2.52 | 2.32 | 2.37 |
| \(^7\)Li | 5.91 | 2.46 | 4.75 | 2.35 | 2.32 |
| \(^9\)Be | 5.76 | 2.31 | 2.30 | 2.15 |
| \(^9\)Be | 5.76 | 2.31 | 2.30 | 2.15 |
| \(^{13}\)C | 7.47 | 2.46 | 2.30 | 2.32 |

Table 3: Binding energy per baryon, \(-E/A\) (in MeV), r.m.s. charge radius \(r_{ch}\) (those of the nucleons, in fm), r.m.s. radii of the hyperon, neutron and proton, \(r_y, r_n\) and \(r_p\) (in fm), respectively, without the contribution of the \(\rho\) mesons. The meaning of \(Z\) in \(^A\Z\) is the number of protons. The configuration of hyperon is 1s\(_{1/2}\) for all hypernuclei. The results of \(\Xi\)-hypernuclei are given in the form \(C^{+A}_{+B}\), where \(C, C + A\) and \(C + B\) are the results obtained with \(V_0^\Xi = -18, -10\) and –28 MeV, respectively.

| \(^A\Z\) | \(-E/A\) | \(r_{ch}\) | \(r_y\) | \(r_n\) | \(r_p\) |
|---|---|---|---|---|---|
| \(^6\)Li | 5.67 | 2.52 | 2.32 | 2.37 |
| \(^7\)Li | 5.91 | 2.46 | 4.75 | 2.35 | 2.32 |
| \(^9\)Be | 5.76 | 2.31 | 2.30 | 2.15 |
| \(^{13}\)C | 7.47 | 2.46 | 2.30 | 2.32 |

| \(\Xi\)-hypernuclei | \(-E/A\) | \(r_{ch}\) | \(r_y\) | \(r_n\) | \(r_p\) |
|---|---|---|---|---|---|
| \(^7\)\(\Xi\)O | 8.19 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{13}\)\(\Xi\)O | 7.94 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{27}\)\(\Xi\)O | 7.85 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{41}\)\(\Xi\)Ca | 8.77 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{63}\)\(\Xi\)Ca | 8.56 | 2.70 | 2.30 | 2.32 | 2.37 |

| \(\Xi\)-hypernuclei | \(-E/A\) | \(r_{ch}\) | \(r_y\) | \(r_n\) | \(r_p\) |
|---|---|---|---|---|---|
| \(^7\)\(\Xi\)O | 8.04 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{13}\)\(\Xi\)O | 7.94 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{27}\)\(\Xi\)O | 7.85 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{41}\)\(\Xi\)Ca | 8.77 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{63}\)\(\Xi\)Ca | 8.56 | 2.70 | 2.30 | 2.32 | 2.37 |

| \(\Xi\)-hypernuclei | \(-E/A\) | \(r_{ch}\) | \(r_y\) | \(r_n\) | \(r_p\) |
|---|---|---|---|---|---|
| \(^7\)\(\Xi\)O | 8.00 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{13}\)\(\Xi\)O | 7.94 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{27}\)\(\Xi\)O | 7.85 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{41}\)\(\Xi\)Ca | 8.77 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{63}\)\(\Xi\)Ca | 8.56 | 2.70 | 2.30 | 2.32 | 2.37 |

| \(\Xi\)-hypernuclei | \(-E/A\) | \(r_{ch}\) | \(r_y\) | \(r_n\) | \(r_p\) |
|---|---|---|---|---|---|
| \(^7\)\(\Xi\)O | 8.00 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{13}\)\(\Xi\)O | 7.94 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{27}\)\(\Xi\)O | 7.85 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{41}\)\(\Xi\)Ca | 8.77 | 2.70 | 2.30 | 2.32 | 2.37 |
| \(^{63}\)\(\Xi\)Ca | 8.56 | 2.70 | 2.30 | 2.32 | 2.37 |