Flipped $SO(10)$ model

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Abstract

We show that as in the flipped $SU(5)$ models, doublet-triplet splitting is realized by the missing partner mechanism in the flipped $SO(10)$ models. The gauge group $SO(10)_F \times U(1)_{V'}$ includes $SU(2)_E$ gauge symmetry, that plays an important role in solving supersymmetric (SUSY) flavor problem by introducing non-abelian horizontal gauge symmetry and anomalous $U(1)_A$ gauge symmetry. The gauge group can be broken into the standard model gauge group by VEVs of only spinor fields, such models may be easier than $E_6$ models to be derived from the superstring theory.

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1 Introduction

In the previous paper[1], one of the authors shows that the SUSY flavor problem can be solved in $E_6$ unification by using non-abelian horizontal gauge symmetry and anomalous $U(1)_A$ gauge symmetry[2], whose anomaly is cancelled by the Green-Schwarz mechanism[3], even if large neutrino mixing angles are obtained. It is essential that the fundamental representation $27$ of $E_6$ has two $ar{5}$ fields of $SU(5)$. Actually $27$ is decomposed as

$$27 \rightarrow \left[ \begin{array}{c} 10_{(1,1)} + \bar{5}_{(1,-3)} + 1_{(1,5)} \\ \bar{5}_{(2,2)} + 5_{(-2,-2)} \\ 1_{(4,0)} \end{array} \right]$$

under $E_6 \supset SO(10) \times U(1)_V \supset SU(5) \times U(1)_V \times U(1)_V$, where the representation of $SO(10) \times U(1)_V$, $SU(5) \times U(1)_V \times U(1)_V$ are explicitly denoted in the above. If we introduce three $27$ fields $\Psi_i$ ($i = 1, 2, 3$) for three generation quarks and leptons, three of six $\bar{5}$ fields become massive with three $5$ fields after breaking $E_6$ into $SU(5)$, and the remaining fields $(3 \times \bar{5})$ remain massless. In the $6 \times 3$ mass matrix for $\bar{5}$ and $5$ fields, it is natural to expect that the elements for the third generation field $\Psi_3$ become larger to realize larger Yukawa couplings than the first and second generation fields $\Psi_1$ and $\Psi_2$. Therefore, all the three massless modes of $\bar{5}$ come mainly from the first two generation fields $\Psi_1$ and $\Psi_2$.

This structure is interesting because it can explain larger mixing angles of lepton sector than of quark sector as discussed in Ref.[4]. Moreover, if we introduce non-abelian horizontal symmetry $SU(2)_H$ and take the first two generation fields as doublet, then all three generation $\bar{5}$ fields have degenerate sfermion masses, which are very important to suppress flavor changing neutral current (FCNC) processes with large neutrino mixing angles as discussed in Ref.[1].

In the above arguments, $E_6$ gauge group plays an important role. Actually, it is essential that a single field includes two $\bar{5}$ fields to realize large neutrino mixing angles with suppressing FCNC processes. However, in order to break $E_6$ into the standard model (SM) gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, adjoint Higgs fields $78$ are required, which may not be so easily realized in the framework of superstring models. To avoid the adjoint Higgs, it is a simple way to adopt non-simple group as a unification group. Which kinds of non-simple group do not spoil the above interesting features? The answer is simple. In order to satisfy the essential point that two fields with the same quantum number under the SM gauge group are included in a single multiplet, $SU(2)_E$, which is a subgroup of $E_6$ group and rotates $(\bar{5}_{(1,-3)}, \bar{5}_{(-2,2)})$ and $(1_{(1,5)}, 1_{(4,0)})$ as doublets, is sufficient. Therefore, it is interesting to consider the unification group which include $SU(2)_E$. The $SU(3)^3 \subset E_6$ is in the case and we know that realistic $SU(3)^3$ model can be straightforwardly constructed[5], in which doublet-triplet splitting problem is solved and realistic quark and lepton mass matrices are obtained including large neutrino mixing angles. Therefore, if we introduce non-abelian horizontal symmetry in addition to $SU(3)^3$, FCNC processes can be naturally
suppressed with large neutrino mixing angles. In this paper, we consider another non-simple gauge group, $SO(10)_F \times U(1)_{V'F}$, which can include $SU(2)_E$ because of the unusual embedding of the SM gauge group. We show that in this model, doublet-triplet splitting is realized by missing partner mechanism. The original missing partner mechanism was introduced in $SU(5)$ unification group[6], but it requires several large dimensional representation Higgs fields. To avoid the large dimensional Higgs fields, flipped $SU(5)$[7] has been considered. It is known that the gauge group $SU(5)_F \times U(1)_X$ cannot be unified into $SO(10)$ without spoiling the missing partner mechanism, but we show that $SO(10)_F \times U(1)_{V'F} \subset E_6$ can embed the flipped $SU(5)$ without spoiling the missing partner mechanism. As noted in the above, the flipped $SO(10)$ gauge group includes $SU(2)_E$, that is important to solve the SUSY flavor problem by introducing non-abelian horizontal gauge symmetry and anomalous $U(1)_A$ gauge symmetry.

2 Review of flipped $SU(5)$ model

We briefly review the flipped $SU(5)$ model and the reason why the flipped $SU(5)$ model cannot be embedded in $SO(10)$ GUT.

It is well-known that one family standard model fermions $Q(3, 2)_\frac{1}{6}, U^c(\bar{3}, 1)_{-\frac{4}{3}}, D^c(\bar{3}, 1)_{-\frac{4}{3}}, L(1, 2)_{-\frac{1}{2}},$ and $E^c(1, 1)_{-1}$ plus the right-handed neutrino $N^c(1, 1)_0$ under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ are unified into an $SO(10)$-spinorial 16 superfield:

$$\Psi(16) \rightarrow 10_{\Psi}(10_1) + \bar{5}_{\Psi}(\bar{5}_{-3}) + 1_{\Psi}(1_5), \quad (2.1)$$

where the decomposition is specified into $SU(5) \times U(1)_V$. The matter content of the flipped $SU(5)$ models can be obtained from the corresponding assignment of the standard $SU(5)$ GUT model by means of the “flipping” $U^c \leftrightarrow D^c$, $N^c \leftrightarrow E^c$:

$$10_{\Psi} = (Q, D^c, N^c)$$

$$\bar{5}_{\Psi} = (U^c, L)$$

$$1_{\Psi} = E^c. \quad (2.2)$$

It is important that if 101 representation Higgs 10C is introduced, $SU(5) \times U(1)_X$ can be broken into the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ by the vacuum expectation value (VEV) of the component of $N^c$. Here, the hypercharge operator is written

$$Y = \frac{1}{5}(X - Y'), \quad (2.3)$$

where $Y'$ is the generator of $SU(5)_F$ which commutes with $SU(3)_C \times SU(2)_L$. Then the $SO(10)$-vectorial 10 superfield decomposed as

$$H(10) \rightarrow 5_H(5_{-2}) + \bar{5}_H(\bar{5}_2) \quad (2.4)$$
includes the SM doublet Higgs $H_d = L'$ and $H_u = \bar{L}'$ as

\[
\begin{align*}
5_H &= (\bar{D}'c, L') \\
\bar{5}_H &= (D', \bar{L}'),
\end{align*}
\]

where $D'$ and $L'$ have the same quantum number of SM gauge group as $D^c$ and $L$, respectively. If we introduce interactions in the superpotential as

\[
W_{MP} = 10_C10_C5_H + \bar{10}_C\bar{10}_C\bar{5}_H,
\]

only the triplet Higgs $\bar{D}'c$ and $D'^c$ can be superheavy with $D^c$ in $10_C$ and $\bar{D}^c$ in $\bar{10}_C$, respectively, by developing the VEVs of $10_C$ and $\bar{10}_C$, but the doublet Higgs $L'$ and $\bar{L}'$ have no partner and remain massless. This is essential of the missing partner mechanism in the flipped $SU(5)$ model.

Unfortunately, this missing partner mechanism in the flipped $SU(5)$ model cannot be extended to $SO(10)$ unification. In $SO(10)$ unification the interactions (2.6) are included in the $SO(10)$ symmetric interactions $C(16)C(16)H(10)$ and $\bar{C}(\bar{16})\bar{C}(\bar{16})H(10)$, which include also

\[
10_C\bar{5}_C\bar{5}_H + \bar{10}_C5_C5_H.
\]

Through these interactions, the doublet Higgs $(\bar{L}')_H$ and $(L')_H$ become super-heavy with $L_C$ and $(L^*)_C$, respectively, by developing the VEVs of $10_C$ and $\bar{10}_C$. (In this paper, $X^*$ is a component of $16$ of $SO(10)$ and denotes the complex conjugate representation of $X$ which is a component of $16$ of $SO(10)$.) Therefore, doublet-triplet splitting is spoiled by this extension.\(^1\)

3 Flipped $SO(10)$ model

As noted in the introduction, $27$ of $E_6$ is decomposed as

\[
27 \rightarrow \begin{bmatrix}
[10_{(1,1)} + 5_{(1,-3)} + 1_{(1,5)}]_{16} \\
[5_{(-2,2)} + 5_{(-2,-2)}]_{10} \\
[1_{(4,0)}]_{1}
\end{bmatrix}
\]

under $E_6 \supset SO(10) \times U(1)_{V'} \supset SU(5) \times U(1)_{V'} \times U(1)_{V'}$. There are two ways to embed the flipped $SU(5)$ matters $10_\Psi = (Q, D^c, N^e), \bar{5}_\Psi = (U^c, L)$ and $1_\Psi = E^c$ in the above decomposition of $27$ of $E_6$ into $SO(10) \times U(1)_{V'}$. As discussed in the previous section, the usual embedding $SU(5)_F \times U(1)_X$ in $SO(10)$,

\[
\begin{bmatrix}
[10_\Psi + 5_\Psi + 1_\Psi]_{16} \\
[5_H + 5_H]_{10} \\
[1_S]_{14}
\end{bmatrix}
\]

\(^1\)Of course, if we neglect the component fields $5_C$ and $\bar{5}_C$ by hand, such extension becomes possible [8].
where $\mathbf{5}_H = (\bar{D}^c, L')$, $\bar{\mathbf{5}}_H = (D^c, \bar{L}')$ and $\mathbf{1}_S$ is singlet under $SU(5)_F \times U(1)_X$, spoils the missing partner mechanism. The other embedding can be obtained by means of the "flipping" $\bar{\mathbf{5}}_\psi \leftrightarrow \mathbf{5}_H$ and $\mathbf{1}_\psi \leftrightarrow \mathbf{1}_S$:

$$\left[ \mathbf{10}_\psi + \bar{\mathbf{5}}_H + \mathbf{1}_S \right]_{\mathbf{16}_1} + \left[ \mathbf{5}_\psi + \mathbf{5}_H \right]_{\mathbf{10}_-2} + \left[ \mathbf{1}_\psi \right]_{\mathbf{1}_4}.$$  \hspace{1cm} (3.3)

In this embedding, if $\mathbf{1}_S$ component of $\mathbf{16}_1$ field have non-vanishing VEV, $SO(10)_F \times U(1)_{V'_F}$ is broken into $SU(5)_F \times U(1)_X$. Here, the operator $X$ is obtained as

$$X = \frac{1}{4}(5V'_F - V_F),$$

where $V_F$ is the generator of $SO(10)_F$ which commute with $SU(5)_F$. The hypercharge operator is

$$Y = \frac{1}{5}(X - Y') = \frac{1}{20}(5V'_F - V_F - 4Y').$$  \hspace{1cm} (3.4)

Note that the each $SU(2)_E$ doublet $(D^c, D^c)$, $(L', L)$ and $(N^c, S)$, which has the same quantum number of SM gauge group, is included into a single multiplet $\mathbf{16}_1$, $\mathbf{10}_-2$ and $\mathbf{16}_1$, respectively. This means that $SU(2)_E$ is embedded in $SO(10)_F$.

We introduce two pairs of Higgs fields $[\Phi(\mathbf{16}_1), \Phi(\overline{\mathbf{16}}_{-1})]$ and $[C(\mathbf{16}_1), \overline{C}(\overline{\mathbf{16}}_{-1})]$ to break $SO(10)_F \times U(1)_{V'_F}$ into the SM gauge group. Supposing that the VEVs $|\langle \Phi \rangle| = |\langle \overline{\Phi} \rangle|$ breaks $SO(10)_F \times U(1)_{V'_F}$ into $SU(5)_F \times U(1)_X$, the components $\mathbf{10}_\phi$ and $\overline{\mathbf{10}}_{\overline{\phi}}$ are absorbed by the Higgs mechanism. The VEVs $|\langle C \rangle| = |\langle \overline{C} \rangle|$ break $SU(5)_F \times U(1)_X$ into the SM gauge group, and the components $Q$ and $N^c$ are absorbed by the Higgs mechanism. All the remaining components $\bar{\mathbf{5}}_\psi$, $\mathbf{5}_\psi$, $\mathbf{5}_C$, $\bar{\mathbf{5}}_C$, $(D^c)_C$ and $(D^c*)_C$ must be massive except a pair of doublets. For example, through the interactions in the superpotential,

$$W_{SO(10)} = \Phi \overline{C} C + \overline{C} \overline{C} \Phi \Phi,$$  \hspace{1cm} (3.5)

which include the interactions (2.6) after developing the VEVs $|\langle \Phi \rangle| = |\langle \overline{\Phi} \rangle|$, pairs $[(D^c)_\Phi, (D^c)_C]$ and $[(D^c*)_\Phi, (D^c*)_C]$ become massive. If we introduce the mass term for $C$ and $\overline{C}$, then only $(L')_\Phi$ and $(L'^*)_\overline{\phi}$ remain massless, namely, doublet-triplet splitting is realized. There are several interactions which unsta-

bilize the doublet-triplet splitting. For example, the terms $\Phi \Phi F(\overline{C} C, \Phi \Phi)$ give directly the doublet Higgs mass, so they must be forbidden. (We will return to this subject later in a concrete model.)

We assume that three generation matter fields $\Psi_i(\mathbf{27}) = \mathbf{16}_{\psi_i} + \mathbf{10}_{\psi_i} + \mathbf{1}_{\psi_i}$, $(i = 1, 2, 3)$ respect $E_6$ symmetry. It is an easy way to guarantee the cancellation of gauge anomaly. Among the three generation matter fields $\Psi_i$, there are six fields which have the same quantum number under the SM gauge group as
(Dc, L). Only three linear combinations of these fields become quarks and leptons, and other modes become superheavy with the three (Dc, L′) fields through the interactions 16ψi, 10ψjΦ and 16ψi, 10ψjC by developing the VEVs of Φ and C. It is interesting that up-type Yukawa coupling can be obtained from the renormalizable interactions 16ψi, 10ψjΦ, because O(1) top Yukawa coupling can be naturally realized. But Yukawa couplings of down quark sector and of charged lepton sector are obtained from the higher dimensional interactions 16ψi, 16ψj, 10ψi, 1ψj, CΦ, respectively. Because we have six singlets Ni and Si in the matter sector, the mass matrix for right-handed neutrino becomes a 6 × 6 matrix which are obtained from the interactions Ψi, Ψj, 1ψi, 1ψj, φC. Yukawa couplings of Dirac neutrino sector are obtained from the interactions 16ψi, 10ψj, φ. Therefore, the mass terms of all quarks and leptons can be obtained in this scenario.

Unfortunately, as in the flipped SU(5) model, this missing partner mechanism in the flipped SO(10) model cannot be extended to E6 unification. In E6 unification the interactions (3.6) are included in the E6 symmetric interactions Φ(27)Φ(27)C(27)C(27) and Φ(27)Φ(27)C(27)C(27), which also include 16ψi, 10ψj, CΦ, and 16ψi, 10ψj, CΦ, of SO(10)F. After developing the VEVs |⟨Φ⟩| = |⟨Φ⟩|, these interactions give 5ψ5ψ, 5ψ5ψ, 5ψ5ψ, and 5ψ5ψ, 5ψ5ψ, 5ψ5ψ, of SU(5)F, which give mass terms to doublet Higgs by taking non-vanishing VEVs |⟨C⟩| = |⟨C⟩|. Therefore, doublet-triplet splitting is spoiled in this extension.

4 Flipped SO(10) model with anomalous U(1)A

It is important to find a concrete flipped SO(10) model in which doublet-triplet splitting is realized with generic interactions and to examine whether the realistic quark and lepton mass matrices are realized or not. In a series of papers [1, 4, 5, 9, 10, 11], we have pointed out that anomalous U(1)A symmetry plays an important role in solving various problems in SUSY grand unified theory (GUT) with generic interactions. This is mainly because the SUSY zero mechanism (holomorphic zero)\(^2\) can control various terms which must be forbidden.

In this section, we present a concrete flipped SO(10) model with generic interaction by introducing anomalous U(1)A symmetry.

4.1 Higgs sector

The Higgs contents are listed in Table I.

\(^2\)Note that if the total charge of an operator is negative, the U(1)A invariance and analytic property of the superpotential forbids the existence of the operator in the superpotential, since the Froggatt-Nielsen [12] field Θ with negative charge cannot compensate for the negative total charge of the operator (the SUSY zero mechanism).
Table I. The typical values of anomalous $U(1)_A$ charges are listed. ± is $Z_2$-parity and $i = 1, 2$.

|       | non-vanishing VEV | vanishing VEV |
|-------|-------------------|---------------|
| $16_1$ | $\Phi(\phi = 0, -)$ $C(c = -2, +)$ | $\Phi_i'(\phi_i' = 5, -)$ |
| $\overline{10}_{-1}$ | $\overline{\Phi}(\phi = -1, -) \; \overline{C}(\overline{c} = -2, +)$ | $\Phi_i'(\phi_i' = 4, -)$ |
| $1$ | $\Theta(\theta = -1, +) \; \overline{Z}_i(\overline{z}_i = -1, +)$ $Z(z = -4, -)$ | $S'(s' = 8, +)$ |

Following the general discussion on the determination of VEVs of the models with anomalous $U(1)_A$ charges, the only negatively charged fields can have non-vanishing VEVs [4, 9, 10, 11]. The scale of these VEVs are determined by the anomalous $U(1)_A$ charges as

$$\langle \Phi \Phi \rangle \sim \lambda^{-(\phi + \phi')}, \; \langle C C' \rangle \sim \lambda^{-(c + \overline{c})},$$  

(4.1)

where $\lambda$ is the ratio of the VEV of Froggatt-Nielsen field $\Theta$, which is essentially determined by the Fayet-Illiopoulos $D$-term parameter, to the cutoff $\Lambda$. In this paper, we take $\lambda$ as around the Cabbibo angle $\sin \theta_W \sim 0.22$. If the $1_{(1,5)}$ component of $\Phi$ and the $1_{(-1,-5)}$ component of $\overline{\Phi}$ have non-vanishing VEVs, $SO(10)_F \times U(1)_{Y'}$ is broken into $SU(5)_F \times U(1)_X$. The $10_{(1,1)}$ of $\Phi$ and $\overline{10}_{(-1, -1)}$ of $\overline{\Phi}$ are absorbed by the Higgs mechanism at that time. Moreover, if the $10_{(1,1)}$ component of $C$ and the $\overline{10}_{(-1, -1)}$ component of $\overline{C}$ have non-vanishing VEVs, $SU(5)_F \times U(1)_X$ is broken into the SM gauge group. Then the $Q$ component of $10_{(1,1)}$ of $C$ and the $\overline{Q}$ component of $\overline{10}_{(-1, -1)}$ of $\overline{C}$ are absorbed by the Higgs mechanism. Therefore, the remaining negatively charged fields except singlets under the SM gauge group are the $5_{(1, -3)}$ components of $\Phi$ and $C$, the $D^c$ component of $C$, and the mirror components of $\overline{\Phi}$ and $\overline{C}$. Among these negatively charged fields, no mass term appears because of the SUSY zero (holomorphic zero) mechanism. In order to make them massive, we have to take account of the positively charged fields $\Phi_i'$ and $\overline{\Phi}_i'$. Note that in a $16_1$ field, there are two colored Higgs $D^c$ and $D^c'$ because of $SU(2)_E$ symmetry, but only one doublet $\overline{L}'$. Therefore, the colored Higgs mass matrix becomes $7 \times 7$ matrix $M_T$ which is given by

\[
\begin{pmatrix}
\begin{array}{cccccccc}
\bar{D}^c & D^c \\
10_C & 5_C & 5_\phi & 10_{\phi_i} & 10_{\phi_2} & 5_{\phi_i} & 5_{\phi'_2} \\
\hline
\overline{10}_{\bar{C}} & 0 & 0 & 0 & \lambda^{\bar{c} + \phi_i' + \Delta} & \lambda^{\bar{c} + \phi_i' + \Delta} & \lambda^{\bar{c} + \phi_i' - \Delta} & \lambda^{\bar{c} + \phi_i' - \Delta} \\
5_{\overline{\phi}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\overline{10}_{\phi_i'} & 0 & \lambda^{\phi_i' + c - \Delta} & 0 & \lambda^{\phi_i' + c - \Delta} & \lambda^{\phi_i' + c + \Delta} & \lambda^{\phi_i' + c + \Delta} & \lambda^{\phi_i' + c - \Delta} \\
\overline{10}_{\phi_2'} & 0 & \lambda^{\phi_2' + c - \Delta} & 0 & \lambda^{\phi_2' + c - \Delta} & \lambda^{\phi_2' + c + \Delta} & \lambda^{\phi_2' + c + \Delta} & \lambda^{\phi_2' + c - \Delta} \\
5_{\phi_i'} & \lambda^{\phi_i' + c + \Delta} & 0 & \lambda^{\phi_i' + c + \Delta} & \lambda^{\phi_i' + c + \Delta} & \lambda^{\phi_i' + c + \Delta} & \lambda^{\phi_i' + c + \Delta} & \lambda^{\phi_i' + c + \Delta} \\
5_{\phi_2'} & \lambda^{\phi_2' + c + \Delta} & 0 & \lambda^{\phi_2' + c + \Delta} & \lambda^{\phi_2' + c + \Delta} & \lambda^{\phi_2' + c + \Delta} & \lambda^{\phi_2' + c + \Delta} & \lambda^{\phi_2' + c + \Delta} \\
\end{array}
\end{pmatrix}
\]  

(4.2)
where $\Delta \equiv \frac{1}{2}(\bar{\phi} - \phi - \bar{c} + c)$. The rank becomes seven for the charge assignment in Table I. On the other hand, the mass matrix for doublet Higgs becomes $4 \times 4$ matrix. The charges in Table I lead to

\[
M_D = \begin{pmatrix}
(\bar{L}^s) \bar{L} & C & \Phi & \Phi_1 & \Phi_2 \\
\bar{C} & 0 & 0 & 0 & 0 \\
\bar{\Phi} & 0 & \lambda_{\phi}^{\phi_1}c + \lambda_{\phi}^{\phi_2} & \lambda_{\phi}^{\phi_1}c & \lambda_{\phi}^{\phi_2}c \\
\bar{\Phi}_1 & 0 & \lambda_{\phi}^{\phi_1}d & \lambda_{\phi}^{\phi_1}d & \lambda_{\phi}^{\phi_2}d \\
\bar{\Phi}_2 & 0 & \lambda_{\phi}^{\phi_1}d & \lambda_{\phi}^{\phi_1}d & \lambda_{\phi}^{\phi_2}d
\end{pmatrix}.
\]

(4.3)

It is obvious that the rank is reduced to three, and therefore one pair of doublet Higgs appears in this model. The massless modes are written

\[
H_u = (\bar{L})_C, \quad H_d = (\bar{L}^s)_C
\]

(4.4) (4.5)

where $H_u$ and $H_d$ are the doublet Higgs for up-quark sector and for down-quark sector, respectively.

### 4.2 Quark and lepton sector

In this subsection, we use the standard definition of $\bar{5} \equiv (D^c, L)$ field. If we introduce three generation matter fields $\Psi_i(27) = 16_{\Psi_i} + 10_{\Psi_i} + 1_{\Psi_i} (i = 1, 2, 3)$ with their charges $(\psi_1, \psi_2, \psi_3) = (4, 3, 1)$ in addition to the Higgs sector in Table I, the massless modes of $\bar{5}$ fields, where we have used the usual definition for $\bar{5}$, become

\[
\bar{5}_1 = \bar{5}_{\psi_1} + \lambda^{2}\bar{5}_{\psi_2} + \lambda^{1.5}\bar{5}_{\psi_3} \\
\bar{5}_2 = \bar{5}_{\psi_1} + \lambda^{2.5}\bar{5}_2^\prime + \lambda^{1}\bar{5}_{\psi_2} + \lambda^{3}\bar{5}_{\psi_3} \\
\bar{5}_3 = \bar{5}_{\psi_2} + \lambda^{2.5}\bar{5}_{\psi_3} + \lambda^{0.5}\bar{5}_{\psi_2} + \lambda^{2.5}\bar{5}_{\psi_3}
\]

(4.6)

where $\bar{5}^\prime \equiv (D^c, L^\prime)$ and we fix the three bases of the massless modes $(\bar{5}_1, \bar{5}_2, \bar{5}_3)$ to $(\bar{5}_{\psi_1}, \bar{5}_{\psi_1}, \bar{5}_{\psi_1})$. These are obtained from the mass matrix of three $\bar{5}$ fields and six $\bar{5}$ fields which are given from the interactions $\Psi_i\Psi_j\Phi Z$ and $\Psi_i\Psi_j C$ by developing the VEVs of $\Phi, C$ and $Z$. Then we can estimate the Yukawa couplings of quarks and leptons.

The Yukawa couplings of up quark sector are obtained as

\[
Y_u = \begin{pmatrix}
U_{\Psi_1}^c & U_{\Psi_2}^c & U_{\Psi_3}^c \\
Q_{\Psi_1} & Q_{\Psi_2} & Q_{\Psi_3}
\end{pmatrix}
\]

(4.7)
from the interactions $\lambda^{i_1 + i_2 + i_3}16\psi_i 10\psi_j C$. The Yukawa couplings of down quark sector and of charged lepton sector are given as

$$Y_d^T (\sim Y_e) = \begin{pmatrix} Q_1 \langle E_{\psi_1} \rangle & Q_2 \langle E_{\psi_2} \rangle & Q_3 \langle E_{\psi_3} \rangle \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^{5.5} & \lambda^{4.5} & 0 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix} \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{pmatrix}$$ (4.8)

from the higher dimensional interactions $\lambda^{i_1 + i_2 + i_3} 16\psi_i 16\psi_j \bar{C}\bar{C}$ and $\lambda^{i_1 + i_2 + i_3} 10\psi_i 1\psi_j \bar{C}\bar{C}$, respectively. Note that only $\bar{\psi}_i'$ fields can have non-vanishing Yukawa couplings through the interactions. This is because the interactions $16\psi_i 16\psi_j \bar{C}\bar{C}$ and $10\psi_i 1\psi_j \bar{C}\bar{C}$ are forbidden by $Z_2$-parity. The above mass matrices give almost good values for masses and mixings for quark sector and charged lepton sector.

The Yukawa couplings for the Dirac neutrino are given as

$$Y_{nD} = \begin{pmatrix} N_{\psi_1} \bar{N}_{\psi_2} \bar{N}_{\psi_3} S_{\psi_1} S_{\psi_2} S_{\psi_3} \end{pmatrix} \begin{pmatrix} \lambda^{6.5} & \lambda^{5.5} & \lambda^{2.5} \\ \lambda^6 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^4 & \lambda^3 \end{pmatrix} \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{pmatrix}$$ (4.9)

through the interactions $\lambda^{i_1 + i_2 + i_3} 10\psi_i 16\psi_j C$. The vanishing component is caused by SUSY zero (holomorphic zero). The right-handed neutrino mass matrix becomes

$$M_{nR} = \begin{pmatrix} N_{\psi_1}^c N_{\psi_2}^c N_{\psi_3}^c S_{\psi_1} S_{\psi_2} S_{\psi_3} \\ N_{\psi_1}^c \begin{pmatrix} \lambda^8 & \lambda^7 & \lambda^5 \\ \lambda^7 & \lambda^6.5 & 0 \\ \lambda^5 & \lambda^4 & 0 \end{pmatrix} \lambda \\ N_{\psi_2}^c \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^4 \\ \lambda^6.5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda^3 \\ N_{\psi_3}^c \begin{pmatrix} \lambda^5 & \lambda^4 & 0 \\ 0 & 0 & 0 \\ \lambda^4 & \lambda^3 & \lambda \end{pmatrix} \lambda \\ S_{\psi_1} \begin{pmatrix} \lambda^7.5 & \lambda^6.5 & 0 \\ \lambda^6.5 & 0 & 0 \\ 0 & 0 & \lambda^3 \end{pmatrix} \lambda \\ S_{\psi_2} \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^4 & \lambda^3 & \lambda \end{pmatrix} \lambda \\ S_{\psi_3} \begin{pmatrix} \lambda^7.5 & \lambda^6.5 & 0 \\ \lambda^6.5 & 0 & 0 \\ 0 & 0 & \lambda^3 \end{pmatrix} \lambda \\ \lambda & \lambda & \lambda \end{pmatrix} \Lambda$$ (4.10)

through the interactions $16\psi_i 16\psi_j \bar{C}\bar{C}$, $16\psi_i 16\psi_j \bar{C}\Phi Z 16\psi_i 16\psi_j \bar{C}\Phi$. Here vanishing components are caused by the SUSY zero (holomorphic zero) mechanism. Then the neutrino mass matrix is given by

$$M_\nu = Y_{nD} \Lambda^{-1} \nu_{nD} \langle H_u \rangle^2 \eta^2 \sim \lambda^3 \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_u \rangle^2 \eta^2 \Lambda^{-1},$$ (4.11)

where $\eta$ is a renormalization factor. This gives bi-large neutrino mixings but to realize the mass scale for the neutrino, we have to take the cutoff $\Lambda \sim 10^{13}$ GeV if we take $\langle H_u \rangle \eta \sim 200$ GeV. Such a small cutoff scale leads to too short nucleon
life-time via dimension six operators. Therefore, the charge assignment in Table I looks unrealistic.

However, because the neutrino scale is determined by the anomalous $U(1)_A$ charges as

$$M_\nu \sim \lambda^{-5-l} \left( \begin{array}{ccc} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda^{0.5} & 1 \end{array} \right) \langle H_u \rangle^2 \eta^2 \Lambda,$$

there may be other realistic models with other charge assignments. To obtain larger value of $l$, smaller $c$ and/or larger $\bar{c}$ is needed. Because $C$ includes $H_u$, the charge $c$ is determined as

$$c = -2 \psi_3 = -2n$$

and $\bar{c}$ is determined as $\bar{c} = -2 \Psi_3 = -2n$.

To realize bi-large neutrino mixings (i.e., $\bar{5}$ fields in Eq. (4.6)), we must take

$$r \equiv \frac{1}{2} [(c - \bar{c}) - (\phi - \bar{\phi})] \sim -\frac{1}{2}.$$  

(4.14)

Once we fix the mixing structure of $\bar{5}$ fields, the Yukawa couplings for down quarks are proportional to $\lambda^{\psi_i + \psi_j + 2 \epsilon} \langle \bar{C} \rangle \sim \lambda^{\delta_i + \delta_j + \frac{3}{2}(\epsilon - c)}$. Therefore, roughly speaking, $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ is proportional to $\lambda^{\frac{3}{2}(\epsilon - c)}$. Then, smaller $c$ and/or larger $\bar{c}$ lead to smaller $\tan \beta$. For fixed $\tan \beta$, smaller $c$ and $\bar{c}$ lead to larger $l$. However, unless the condition

$$c - 2 \bar{c} \leq 2$$

(4.15)

is satisfied, $(Y_d)_{33}$ vanishes by the SUSY zero mechanism. A compromised charge assignment is $(\phi, \bar{\phi}, c, \bar{c}, \phi', \bar{\phi}', \bar{c}', \bar{\phi}'', \bar{\phi}''') = (-1, -1, -4, -3, 8, 8, -1, -6, 12)$. Then $l$ becomes $-5$, so the cutoff scale can be larger than the $10^{15}$ GeV. Actually, the running gauge couplings of $SU(3)_C$ and $SU(2)_L$, which should meet at the cutoff scale in this flipped $SO(10)$ scenario, meet around the scale in this charge assignment. And the Yukawa coupling of bottom quark becomes $\lambda^{3.5}$ which can be realistic although the large ambiguity of $O(1)$ coefficients is required.

5 Summary

In this paper, we have shown that the missing partner mechanism in flipped $SU(5)$ model can be embedded in flipped $SO(10)$ model whose gauge group is $SO(10)_F \times U(1)_{V_R} \subset E_6$. It is interesting that the gauge group includes $SU(2)_E$, that plays an important role in solving SUSY flavor problem by the horizontal gauge symmetry and anomalous $U(1)_A$ gauge symmetry. As a proof of existence of a concrete model, we build a flipped $SO(10)$ model by introducing anomalous $U(1)_A$ gauge symmetry.
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