Cosmology dependency of halo masses and concentrations in hydrodynamic simulations

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10th May 2023

ABSTRACT

We employ a set of Magneticum cosmological hydrodynamic simulations that span over 15 different cosmologies, and extract masses and concentrations of all well-resolved haloes between \( z = 0 \) – 1 for critical over-densities \( \Delta_{200m}, \Delta_{2500}, \Delta_{2000} \) and mean overdensity \( \Delta_{200m} \). We provide the first mass-concentration (Mc) relation and sparsity relation (i.e. \( M_{\Delta_{2500}} - M_{\Delta_2} \) mass conversion) of hydrodynamic simulations that is modelled by mass, redshift and cosmological parameters \( \Omega_m, \Omega_b, \sigma_8, h_0 \) as a tool for observational studies. We also quantify the impact that the Mc relation scatter and the assumption of NFW density profiles have on the uncertainty of the sparsity relation. We find that converting masses with the aid of a Mc relation carries an additional fractional scatter (\( \approx 4\% \)) originated from deviations from the assumed NFW density profile. For this reason we provide a direct mass-mass conversion relation that depends on redshift and cosmological parameters. We release the package hydro_mc, a python tool that perform all kind of conversions presented in this paper.

Key words: halo - large-scale structure of Universe - cosmological parameters

1 INTRODUCTION

Early studies of numerical N-body simulations of cosmic structures embedded in cosmological volumes (see e.g. Navarro et al. 1997; Kravtsov et al. 1997) showed that dark matter haloes can be described by the so called Navarro-Frank-White (NFW) profile (Navarro et al. 1996). The NFW density profile \( \rho (r) \) is modelled by a characteristic density \( \rho_0 \) and a scale radius \( r_s \) in the following way:

\[
\rho (r) = \frac{\rho_0}{\left( \frac{r}{r_s} \right)^2 \left( 1 + \frac{r}{r_s} \right)}.
\]

The NFW profile proved to match density profiles of dark matter haloes of dark-matter-only (DMO) simulations (see e.g. Bullock et al. 2001; Suto 2003; Prada et al. 2012; Menechetti et al. 2014; Klypin et al. 2016; Gupta et al. 2017; Brainerd 2019) up to the largest and most resolved ones whose analyses trace the route for the next generation of (pre-)Exascale simulations. However, density profiles of hydrodynamic simulations have small deviations from the NFW profile (see e.g. Balmès et al. 2014; Tollet et al. 2016).

Since this kind of density profile does not have a cut-off radius, the radius of a halo is often chosen as the virial radius \( R_{vir} \) (see e.g. Ghigna et al. 1998; Frenk et al. 1999), namely, the radius at which the mean density crosses the one of a theoretical virialised homogeneous top-hat overdensity. Bryan & Norman (1998) showed that the virial overdensity can be written as

\[
\Delta_{vir}(a) = 18\pi^2 + 8\Omega(a) - 39\Omega(a)^3\left(\frac{\Omega_m(a)^2}{\Omega_m} + \frac{\Omega_r(a)}{\Omega_m} + \frac{\Omega_k(a)}{\Omega_m} + \Omega_{\Lambda}\right)^{-1},
\]

where \( a \) is the scale factor and \( \Omega(a) \) is the energy density parameter (see Dodelson 2003, for a review), namely

\[
\Omega(a) = \Omega_m(a)^3 \left( \frac{\Omega_m(a)^2}{\Omega_m} + \frac{\Omega_r(a)}{\Omega_m} + \frac{\Omega_k(a)}{\Omega_m} + \Omega_{\Lambda} \right)^{-1},
\]

where \( \Omega_m, \Omega_r, \Omega_k \) and \( \Omega_{\Lambda} \) are the density fractions of the total matter, radiation, curvature and cosmological constant, respectively. Numerical cosmological simulations, as in this work, typically use negligible radiation and curvature terms (they set \( \Omega_r = \Omega_k = 0 \) in Eq. 3).

Observational studies typically define galaxy cluster (GC) radii as \( R_{200c} \), where \( A \) is an arbitrary overdensity and the “c” suffix indicates that the overdensity is relative to the critical overdensity given by

\[
M(r < R_{200c}) = \Delta \times \frac{4}{3} \pi R_{200c}^3 \rho_c.
\]

X-ray observations typically use overdensities \( \Delta_{500c} \) and \( \Delta_{2500c} \) and...
the corresponding radii $R_{200c}$ and $R_{2500c}$ (see e.g. Bocquet et al. 2019; Umetzu et al. 2019; Mantz 2019; Bulbul et al. 2019), whereas, observational studies that compute dynamical mass distributions usually take $\Delta = \Delta_{200c}$ (see e.g. Biviano et al. 2017; Capasso et al. 2019).

Weak lensing studies on the other hand often utilize radii whose overdensities are proportional to the mean density of the Universe. For instance, works such as Mandelbaum et al. (2008); McClintock et al. (2019) measure halo radii as $R_{200m}$, where the suffix "m" means that the radius is defined so the mean density of the halo in Eq. 4 crosses $\Delta_p$, (in this case $200\rho$) where $\rho$ is the average matter density of the Universe.

The concentration $c_A$ of a halo is defined as, $c_A \equiv R_A/r_s$, where, $r_s$ is the scale radius of Eq. 1 and quantities how large the internal region of the cluster is compared to its radius for a given overdensity (see Okoli 2017, for a review). Both numerical and observational studies analyse the concentration of haloes in the context of the so called mass-concentration (Mc) plane (see Table 4 in Ragagnin et al. 2019, for comprehensive list of recent studies). Within the context of hydrodynamic simulations, one can define the DM mass-concentration plane which can be used by observations that estimate DM profiles (e.g. Merten et al. 2015). On the other hand observations that have only information on the total-matter profile must rely on total-matter mass concentration planes (e.g Raghunathan et al. 2019).

In search of a realistic estimate of halo concentrations, one must consider the various sources that affect this value. The $c$ parameter in both observational and numerical studies is found to have a weak dependence on halo mass and a very large scatter (Bullock et al. 2001; Martinsson et al. 2013; Ludlow et al. 2014; Shan et al. 2017; Shirasaki et al. 2018; Ragagnin et al. 2019). Concentration has been found to depend on a number of factors, as formation time of haloes (Bullock et al. 2001; Rey et al. 2018), accretion histories (see e.g. Ludlow et al. 2013; Fujita et al. 2018a,b), dynamical state (Ludlow et al. 2012), triaxiality (Giocoli et al. 2012, 2014), and halo environment (Corsini et al. 2018; Klypin et al. 2016). The fractional scatter in the Mc plane is larger than $\gtrsim 33\%$ (Heitmann et al. 2016), and observations found outliers with extremely high concentration (Buote & Barth 2019) or very low concentration (Andreon et al. 2019). When all major physical phenomena of galaxy formation are taken into account (cooling, star formation, black hole seeding and their feedback), then concentration parameters are lower than their dark-matter-only counterpart (see e.g. results from NIHAO simulations as in Wang et al. 2015; Tollet et al. 2016).

Halo concentration parameters are also affected by the underlying cosmological model (see e.g. Roos 2003, for a review on cosmological models). The derived Mc relation is found in fact different in Cold Dark Matter (CDM), ΛCDM, wDM and varying dark energy equation of state (Kravtsov et al. 1997; Ludlow et al. 2016; Dolag et al. 2004; De Boni 2013; De Boni et al. 2013). In general, the Mc dependency of DMO simulations on cosmological parameters has been extensively studied in works as the Cosmic Emulator (Maccì et al. 2008; Bhattacharya et al. 2013; Heitmann et al. 2016), works as Ludlow et al. (2014) and Prada et al. (2012).

Mass-concentration relations allow observational works to convert masses between overdensities. For this purpose, Balmès et al. (2014) defined the sparsity parameter $s_{A1, A2}$ as the ratio between masses at over-density $\Delta_1$ and $\Delta_2$. This quantity is a proxy to the total matter profile (Corasaniti et al. 2018) and enables cosmological parameter inference (Corasaniti & Rasia 2019) and testing for some dark energy models without assuming an NFW profile (Balmès et al. 2014). Observations use the sparsity parameter to infer the halo matter profile (Bartalucci et al. 2019), as a potential probe to test $f(R)$ models (Achitouv et al. 2016), a less uncertain measurement of the mass-concentration relation (Fujita et al. 2019), and to find outliers in scaling relations involving integrated quantities with different radial dependences (see conclusions in Andreon et al. 2019).

In this work we use data from the Magneticum suite of simulations (presented in works such as Biffi et al. 2013; Saro et al. 2014; Steinborn et al. 2015; Teklu et al. 2015; Dolag et al. 2015, 2016; Steinborn et al. 2016; Bocquet et al. 2016; Remus et al. 2017) to calibrate the cosmology dependence of the mass-concentration and of the mass-sphericity relation of the total matter component from hydrodynamic simulations with the purpose of facilitating cluster-cosmology oriented studies. These studies typically calibrate the observable-mass relation from stacked weak lensing signal under the assumption that mass-calibration can be correctly recovered from DMO Mc relations (e.g. Rozo et al. 2014; Dietrich et al. 2014; Baxter et al. 2016; Simet et al. 2017; Geach & Peacock 2017; McClintock et al. 2019; Raghunathan et al. 2019 and references therein), an approach that has to be quantified by calibrating the total-mass mass-concentration relations within hydrodynamic simulations (see discussion in Sec. 5.4.1 of McClintock et al. 2019).

This work represents a first necessary step in this direction and it provides mass-concentration and mass-likelihood relations that depends on cosmology and that simultaneously accounts for the presence of baryons. While in fact previous works in the literature studied either the dependency of the concentration on cosmological parameters or on baryon physics, in this analysis we calibrate for the first time the dependency of concentration on cosmological parameters in the context of hydrodynamic simulations that include a full description of the main baryonic physical processes.

In Section 2 we present the numerical set up of the simulations used in this work. In Section 3 we fit the concentration of haloes as a function of mass and scale factor for all our simulations and compare our results with both observations and other theoretical studies. In Section 4 we provide a fit of the concentration as a function of mass, scale factor and cosmology. As uncertainty propagation is a delicate and important matter for cluster cosmology experiments, in Sec. 5 we test sparsity parameter and study the origin of its large uncertainty. In order to facilitate cluster cosmology studies that include mass-observable relations which are calibrated at different radii (e.g. Bocquet et al. 2016, 2019; Mantz 2019; Costanzi et al. 2019), we study how to convert masses at different overdensities (the (sparsity-mass relation). We summarise our findings, including a careful characterisation of the associated intrinsic scatter. in Sec. 5. We draw our conclusions in Section 7.

2 NUMERICAL SIMULATIONS

Magneticum simulations are performed with an extended version of the N-body/SPH code P-Gadget3, which is the successor of the code P-Gadget2 (Springel et al. 2005b; Springel 2005; Boylan-Kolchin et al. 2009), with a space-filling curve aware neighbour search (Ragagnin et al. 2016), an improved Smoothed Particle Hydrodynamics (SPH) solver (Beck et al. 2016); treatment of radiative cooling, heating, ultraviolet (UV) background, star formation and stellar feedback processes as in Springel et al. (2005a) connected to a detailed chemical evolution and enrichment model as in Tornatore et al. (2007), which follows 11 chemical elements (H, He, C, N, O, Ne, Mg, Si, S, Ca, Fe) with the aid of CLOUDY photo-ionisation code (Ferland et al. 1998). Fabjan et al. (2010); Hirschmann et al.
C8 uses the reference cosmology from Komatsu et al. (2011). Magnetism simulations do reproduce realistic haloes\(^1\), thus one can assume that its concentration parameter are realistic and of general applicability for purposes of calibration on observational studies.

Table 1 gives an overview of the cosmological simulations used in this work. They have already been presented in Singh et al. (2019) (see Table 1 in their paper) and labelled as C1–15. Each simulation covers a volume of 896 Mpc/h, gas and DM particle masses respectively equal to \(m_{\text{gas}} = 2.6 \cdot 10^8 M_\odot/h\) and \(m_{\text{DM}} = 1.3 \cdot 10^{14} M_\odot/h\) and a softening \(\epsilon = 10\) in units of comoving kiloparsec. They have different cosmological parameters \(\Omega_m, \Omega_b, h\), and \(\sigma_8\) exception for two simulations with the same setup as C1 and C15 (C1_norad and C15_norad) that have been run without radiative cooling and star formation.

For each simulation we study the haloes at a timeslice with redshifts \(z = 0.00, 0.14, 0.29, 0.47, 0.67\), and \(z = 0.90\). In the following sections we repeat the same analyses for overdensities \(\Delta_{\text{crit}}, \Delta_{200\text{c}}, \Delta_{250\text{c}}, \Delta_{200m}\) performing a corresponding mass-cut (respectively on \(M = M_{\text{crit}}, M_{200\text{c}}, M_{250\text{c}}, M_{200m}\) that ensures that all haloes have at least \(10^4\) particles. This cut is different for each of our simulations. This is opposed to what was used in Singh et al. (2019), where they choose a fixed mass cut for all C1-C15 simulations. The mass range of these haloes is between \(10^{14} - 4 \times 10^{15} M_\odot\), which fits the typical range of galaxy cluster weak-lensing masses (see e.g. Applegate et al. 2014).

In this work we fit the NFW profile (see Eq. 1) over the total matter component (i.e. dark matter and baryons) as opposed to previous works (see Ragagnin et al. 2019) where the NFW profile fit was performed over the dark matter component only. We fit the density profile over 20 logarithmic bins, starting from \(r = 100 kpc\) (similar to the cut applied in observational studies as Dietrich et al. 2019). All fits with a \(\chi^2 > 10^3\) have been excluded from our analyses (which accounts for a few hundred haloes per snapshot) as they correspond to heavily perturbed objects.

Although works on simulations typically present quantities in comoving units of \(h\) (e.g. distances in \(a \cdot kpc/h\)), unless specified, all quantities expressed in this work are in physical units and are not in units of \(h\).

### 3 HALO CONCENTRATIONS

In Appendix A we study the effect of baryons on mass-concentration planes and show how an incorrect treatment of baryons can lead to under-estimation of the concentration up to 20% and how the interplay between dark matter and baryons put the dynamical state of hydrodynamic simulations in a much more complex picture than the one of DMO simulations. This motivates us to study halo masses and concentrations on hydrodynamic simulations, and in particular we focus on their dependency on cosmological parameters. We perform a fit of the concentration as a function of mass and redshift for each simulation at each over-density of Magneticum simulations. The functional form of the concentration is chosen as a power law on mass and scale factor as done in the observational studies (see e.g. Merten et al. 2015) as:

\[
\ln c_A \left( M_A \right) = \ln A + B \ln \left( \frac{M_A}{M_p} \right) + C \ln \left( \frac{a}{d_p} \right). \tag{5}
\]

Here \(A, B, M_p\) are fit parameters, \(a, d_p\) are median of mass and scale.

| Name | \(\Omega_m\) | \(\Omega_b\) | \(\sigma_8\) | \(h_0\) | \(N_{\text{haloes}}\) | \(N_{\text{haloes}}\) |
|------|----------|---------|---------|------|--------|--------|
|      |          |         |         |      | all     | snapshot |
| C1   | 0.153    | 0.0408  | 0.614   | 0.666| 29206  | 9153   |
| C1_norad | 0.153  | 0.0408  | 0.614   | 0.666| 27613  | 9208   |
| C2   | 0.189    | 0.0455  | 0.697   | 0.703| 54094  | 16236  |
| C3   | 0.200    | 0.0415  | 0.850   | 0.730| 107423 | 27225  |
| C4   | 0.204    | 0.0437  | 0.739   | 0.689| 66351  | 19051  |
| C5   | 0.222    | 0.0421  | 0.793   | 0.676| 84087  | 22037  |
| C6   | 0.232    | 0.0413  | 0.687   | 0.670| 47045  | 14930  |
| C7   | 0.268    | 0.0449  | 0.721   | 0.699| 55815  | 17990  |
| C8   | 0.272    | 0.0456  | 0.809   | 0.704| 79417  | 22353  |
| C9   | 0.301    | 0.0460  | 0.824   | 0.707| 96151  | 26473  |
| C10  | 0.304    | 0.0504  | 0.886   | 0.740| 120617 | 32551  |
| C11  | 0.342    | 0.0462  | 0.834   | 0.708| 97392  | 27100  |
| C12  | 0.363    | 0.0490  | 0.884   | 0.729| 118342 | 33571  |
| C13  | 0.400    | 0.0485  | 0.650   | 0.675| 35503  | 14626  |
| C14  | 0.406    | 0.0466  | 0.867   | 0.712| 104266 | 30918  |
| C15  | 0.428    | 0.0492  | 0.830   | 0.732| 92352  | 28348  |
| C15_norad | 0.428 | 0.0492  | 0.830   | 0.732| 79399  | 25270  |

\(^1\) In particular, Magneticum simulations match observations of angular momentum for different morphologies (Teklu et al. 2015, 2016); the mass-size relation (Remus & Dolag 2016; Remus et al. 2017; van de Sande et al. 2019); the dark matter fraction (see Figure 3 in Remus et al. 2017); the baryon conversion efficiency (see Figure 10 in Steinborn et al. 2015); kinematical observations of early-type galaxies (Schulze et al. 2018); the inner slope of the total matter density profile (see Figure 7 in Bellstedt et al. 2018), the ellipticity and velocity over velocity dispersion ratio (van de Sande et al. 2019); and reproduce the high concentration of high luminosity gap of fossil objects (Ragagnin et al. 2019).
factor, respectively and are used as pivot values, and the fit will be performed including a logarithmic scatter $\sigma$.

We maximised the following likelihood $\mathcal{L}^2$ with a uniform prior for all fit parameters:

$$
\ln \mathcal{L} = \frac{-1}{2} \left[ \ln 2\pi \sigma^2 + \frac{1}{\sigma^2} \left( \ln c_\Delta (M_\Delta, A, B, C) - \ln c_\Delta \right)^2 \right].
$$

(6)

Figure 1 shows the mass concentration planes for $\Delta_{210}$ (computed following Eq. 2), for all 15 simulations, together with the concentration from the redshift-mass-concentration (aMc relation) colour coded by $\log_{10} x^2$. Haloes with high $\chi^2$ tend to have lower concentration which qualitatively agrees with other theoretical studies that show how perturbed objects have lower concentrations (see e.g. Balmès et al. 2014; Ludlow et al. 2014; Klypin et al. 2016).

For this reason, in a mass-concentration plane, it is not advisable to weight halo concentrations with $1/\chi^2$, as this would bias the relation towards higher concentrations. Although the concentration is believed to decrease with increasing halo mass, extreme cosmologies such as C1 and C2 (with $\Omega_m < 0.2$) have an overall positive dependency between the mass and concentration. On the other hand, the logarithmic mean slope is low (between -0.03 and 0.08) and its influence in the mass concentration plane is not dominant in our mass regime of interest.

### 3.1 Comparison with other studies

We then compare Magneticum simulations concentrations of haloes with the concentration predicted by the Cosmic Emulator (Heitmann et al. 2016; Bhattacharya et al. 2013). The Cosmic Emulator is a tool to predict the mass-concentration planes for a given $\omega$CDM. We were able to compare only C7, C8 and C9 cosmologies because the other Magneticum simulations had cosmological parameters that were out of the range of the Cosmic Emulator. Note that while the Cosmic Emulator dependency on $\Omega_m$ is encoded in the power spectrum normalisation, our mass-concentration relation dependency on $\Omega_m$ takes into account all physical processes of baryon physics, including star formation and feedback.

The ratio of median concentration $c_{\text{vir}}$ parameters of haloes obtained with our mass-concentration fit and the concentration provided by the Cosmic Emulator is $\approx 1.2$. We notice how the Cosmic Emulator concentrations (retrieved by dark matter only runs) are systematically higher than Magneticum simulations in this mass regime (by a factor of $\approx 10 - 20$%), in agreement with our comparison in Ragagnin et al. (2019). The scatter is constant over mass, redshift and cosmology, to nearly $\sigma \approx 0.38$, in agreement with the value of $\approx 1/3$ presented in the $\omega$CDM dark-matter only model of Kwan et al. (2013).

Figure 2 shows the mass-concentration plane for the full-physics simulations C1–15 against other dark matter only simulations and observations. We compare with the concentration from Omega500 simulations (Shirasaki et al. 2018); CLASH concentrations from Merten et al. (2015), numerical predictions from MUSIC of CLASH (Meneghetti et al. 2014), where a number of simulated haloes have been chosen to make mock CLASH observations. To highlight the high scatter in the mass-concentration relation, we also show high concentration groups from Pratt et al. (2016) and an under-luminous and low-concentration halo studied in Andreon et al. (2019). When analysing this data one must be aware of their selection effects: CLASH data-set underwent some filtering difficult to model, while fossil objects presented in Pratt et al. (2016) by construction lay in the upper part of the Mc plane. There is a general agreement between concentration of Magneticum simulations and these observations.

### 4 COSMOLOGY DEPENDENCE OF CONCENTRATION PARAMETER

The 15 cosmologies we use in this work have different mass-concentration normalisation values and log-slope (see Figure 1). We perform a fit of the concentration as a function of mass, scale factor and cosmological parameters in order to interpolate a mass-concentration plane at a given, arbitrary, cosmology, i.e. a concentration $c_\Delta (M_\Delta, 1/\chi, \Omega_m, \Omega_b, \sigma_8, h_0)$. As the intrinsic scatter is constant (within few percents) we didn’t further parametrise it and assumed it to be independent of mass, redshift and cosmology. The functional form of the fit parameters in Eq. 5, with a dependency on cosmology is as follows:

$$
\ln A = A_0 + \sigma_m \ln \frac{\Omega_m}{\Omega_m^\text{crit}}, + A_p \ln \frac{\Omega_b}{\Omega_b^\text{crit}} + \sigma_8 \ln \frac{\sigma_8}{\sigma_8^\text{crit}}, + \ln L_0.
$$

(7)

The fit has been performed for $\Lambda = \Delta_{210}, \Delta_{200c}, \Delta_{500c}, \Delta_{2500c}$ and $\Lambda_{200m}$ by maximising the Likelihood as in Eq. 6. Table 2 shows the results with cosmological parameter pivots at the reference cosmology C8.

Given the high number of free parameters, in order to not underestimate possible sources of errors in the fit, we decided to evaluate uncertainties as follows in Singh et al. (2019): (1) we first re-performed the fit for each simulation by setting its own cosmological parameter as pivot values; (2) then for each parameter except $A_0$, $B_0$, $C_0$, we considered the standard deviation of the parameter values in the previous fits and set it as uncertainty in Table 2; (3) parameters $A_0$, $B_0$, $C_0$ are presented without uncertainty because the error obtained from the Hessian matrix is negligible compared to the scatter parameter $\sigma$. Being this work first necessary step towards a cosmology-dependent mass-concentration relation, these parameters may be constrained with more precision in future simulation campaigns.

From the above fit we find that the normalisation ($\alpha$ parameters) is mainly affected by $\Omega_m$ and $\sigma_8$. The slope of the mass-concentration plane ($\beta$ parameters) has a weak dependency on cosmology. However, the logarithmic mass slope is pushed towards negative values by an increase in $\Omega_m$ and $h_0$ (i.e. $\beta_m$ and $\beta_h < 0$), while it is pushed towards positive values by an increase in $\Omega_b$ and $\sigma_8$ (since $\beta_B$ and $\beta_\sigma > 0$). This behaviour is also shown in Figure 2. Note that, C1 and C2 have opposite mass-dependency with respect to the other runs. Although the trend can be positive for

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2 we used the python package emcee (Foreman-Mackey et al. 2013)
Figure 1. Each panel shows the mass-concentration plane one full physics Magneticum simulation presented in Table 1. Concentrations are computed at overdensity $\Delta_{\text{vir}}$. Data points represents all selected haloes at redshift $z = 0$, colour-coded by their $\log_{10} \chi^2$. Concentration values are plotted only in the range $c_{\text{vir}} = 1 - 10$, because this range contains vast majority of haloes. Black line corresponds to the mass-concentration relation obtained by the fit in Eq. 5. Gray lines corresponds to the mass-concentration relation obtained for the simulation C8 (which uses the reference cosmology Komatsu et al. (2009)). The different mass-cut on each panel is due to our choice of selecting the smallest mass-cut where all haloes with at least $10^4$ particles. As a consequence, our mass-cuts depend on cosmological parameters.
Figure 2. Mass-concentration plane of our simulations C1–C15 (black solid lines), haloes from the hydrodynamic cosmological simulation Omega500 (Shirasaki et al. 2018) (dashed orange line), observations of fossil groups from Pratt et al. (2016) (green data points), mock observations from Meneghetti et al. (2014) (red data points) and data from CLASH (Meneghetti et al. 2014) (magenta data points) and a low-concentration halo studied in Andreon et al. (2019). Shaded area is the scatter around the C8 relation.

Table 2. Pivots and best fit parameters for the cosmology-redshift-mass-concentration plane and its dependency on cosmology as in Eq. 5 and Eq. 7 for concentration overdensities of $\Delta = \Delta_{\text{vir}}, \Delta_{200c}, \Delta_{500c}, \Delta_{2500c}$ and $\Delta_{2500m}$. The pivots $\Omega_{m,p}, \Omega_{b,p}, \sigma_8$ and $h_0$ in Eq. 7 are the cosmological parameters of C8 as in Table 1 ($\Omega_{m} = 0.272, \Omega_{b} = 0.0456, \sigma_8 = 0.809, h_0 = 0.704$). Pivots $\alpha_p$ and $M_p$ are respectively median of scale factor an mass of all haloes. Errors on $\alpha_0, B_0, C_0$ and $\sigma$ are omitted as they are all < 0.001%. The package hydro_mc contains a script that utilises this relation (http://github.com/aragagnin/hydro_mc/blob/master/examples/sample_mc.py).

| Param | $M_p[M_\odot]$ | vir | 200c | 500c | 2500c | 2200m |
|-------|-----------------|-----|------|------|-------|-------|
|       | $10^{14}$       | $10^{14}$ | $10^{15}$ | $10^{15}$ | $10^{15}$ | $10^{15}$ |
|       | 19.9 $\times 10^{14}$ | 17.4 $\times 10^{14}$ | 13.7 $\times 10^{14}$ | 6.9 $\times 10^{14}$ | 22.4 $\times 10^{14}$ |
| $\alpha_p$ | 0.877 | 0.877 | 0.877 | 0.877 | 0.877 |
| $\alpha_0$ | 1.50 | 1.24 | 0.86 | 0.13 | 1.69 |
| $B_0$ | -0.04 | -0.05 | -0.05 | -0.03 | -0.04 |
| $C_0$ | 0.52 | 0.20 | 0.19 | 0.11 | 0.91 |
| $\sigma_m$ | 0.454 ± 0.041 | 0.632 ± 0.042 | 0.662 ± 0.042 | 0.759 ± 0.055 | 0.227 ± 0.037 |
| $\alpha_p$ | -0.249 ± 0.040 | -0.246 ± 0.038 | -0.235 ± 0.049 | -0.272 ± 0.134 | -0.266 ± 0.035 |
| $\alpha_0$ | 0.554 ± 0.030 | 0.561 ± 0.034 | 0.519 ± 0.047 | 0.422 ± 0.050 | 0.528 ± 0.022 |
| $B_0$ | -0.005 ± 0.030 | -0.026 ± 0.166 | -0.031 ± 0.065 | -0.021 ± 0.167 | 0.016 ± 0.028 |
| $C_0$ | -0.122 ± 0.001 | -0.118 ± 0.001 | -0.112 ± 0.001 | -0.116 ± 0.001 | -0.116 ± 0.001 |
| $\beta_p$ | 0.117 ± 0.005 | 0.112 ± 0.004 | 0.126 ± 0.005 | 0.289 ± 0.007 | 0.115 ± 0.008 |
| $\beta_0$ | 0.051 ± 0.003 | 0.056 ± 0.002 | 0.088 ± 0.004 | 0.103 ± 0.005 | 0.050 ± 0.006 |
| $\beta_0$ | -0.079 ± 0.013 | -0.044 ± 0.009 | -0.156 ± 0.014 | -0.342 ± 0.017 | -0.094 ± 0.027 |
| $\gamma_m$ | 0.240 ± 0.006 | 0.352 ± 0.007 | 0.346 ± 0.009 | 0.384 ± 0.011 | -0.043 ± 0.009 |
| $\gamma_p$ | -0.126 ± 0.034 | -0.039 ± 0.040 | -0.045 ± 0.051 | -0.133 ± 0.062 | -0.063 ± 0.053 |
| $\gamma_0$ | 0.664 ± 0.027 | 0.767 ± 0.026 | 0.856 ± 0.032 | 0.846 ± 0.046 | 0.635 ± 0.039 |
| $\gamma_0$ | -0.030 ± 0.109 | -0.276 ± 0.112 | -0.347 ± 0.136 | 0.003 ± 0.171 | -0.405 ± 0.135 |
| $\sigma$ | 0.388 ± 0.001 | 0.384 ± 0.001 | 0.377 ± 0.001 | 0.383 ± 0.001 | 0.388 ± 0.001 |
some cosmologies (see Table 2 and Figure 1), the slope is always close to zero. The redshift dependency \((y)\) parameters is driven by both \(\sigma_8\) and \(\Omega_m\), while a high baryon fraction can lower the dependency (see parameter \(y_b\)). The scatter is nearly constant for all the overdensities with a value close to 0.38 and even if it is of the same order of the difference between Mc relations of different cosmologies (see shaded area in Fig. 2), in the next sub-section we will show that statistical studies on large samples of galaxy clusters are still affected by the cosmological dependency of Mc relations.

Since the logarithmic slope of the mass has a weak dependency on cosmology, we provide a similar fit as the one in this section without \(B\) having any cosmological dependencies i.e. \(B = B_0\) in Appendix B (see Table B1). In Appendix B (see Table B2) we also provide the same reduced fit parameters with the scale radius computed on the dark matter density profile.

4.1 Impact on inferred weak-lensing masses

There are differences between the Mc relation extracted from our simulations at different cosmologies and the ones from DMO simulations. When Mc relations are used to provide priors and interpret the weak-lensing signal of non-ideal NFW cluster samples, the different Mc relations will ultimately lead to different inferred masses and therefore different cosmological constraints from cluster number counts experiments. In fact, works as Henson et al. (2017) show that it is possible to correctly recover halo masses from mock observations of both DMO and hydro-simulations by using their respective mass-concentration relations. On the other hand, in low signal-to-noise conditions, weak-lensing mass calibration typically constrains the total observed mass using Mc relations derived from DMO simulations (see, e.g., Melchior et al. 2017). In the following, we quantify and discuss this effect on a simplified example.

To this end, we create a simulated projected surface density profile of an NFW model of the RXC J2248.7–4431 cluster (Gruen et al. 2013), at \(z = 0.436\) with mass \(M_{200c} = 1.75 \cdot 10^{15} M_\odot\) (Melchior et al. 2015). The simulated profile is generated using Eq. 41 in Lokas & Mamon (2001) with a concentration \(c_{200}\) from our Mc relation (see Table 2) and with cosmological parameters \(\Omega_m = 0.27, \Omega_b = 0.05, \sigma_8 = 0.8, h_0 = 0.67\). We mimic a simplified observed radial profile sampled with 20 logarithmic equally spaced radial bins from 3 to 30 arcmin. To each data-point we assigned an associated error in order to simulate typical weak-lensing observational conditions \((S/N = 5)\) of a massive clusters in a photometric survey like the Dark Energy Survey (DES, Melchior et al. 2015). We test a simplified mass calibration process by fitting the above described density profile with a gaussian likelihood for each simulated projected density radial bin \(\Sigma_i\):

\[
L = \prod_i P(\Sigma_i | M_{200c}, c_{200}, \Delta \Sigma_i).
\]  

We adopt a flat prior on \(\log M_{200c}\) and test the impact of adopting the following different priors for the concentration:

- Ragagnin20: The Mc relation with a lognormal scatter \(\sigma_{\log M} = 0.38\) as presented in this work.
- Diemer19: The DMO Mc relation proposed in Diemer & Joyce (2019) with a lognormal scatter \(\sigma_{\log M} = 0.39\).

We used the python package https://bdiemer.bitbucket.io/colossus/ (see Diemer 2018).
To show the impact on mass-calibration of Ragagnin20 and the dependency on cosmological relations, we perform the calibration both at the correct input cosmology (here on Cosmo A) and with cosmological parameters randomly extracted from the posterior distribution of the cosmological parameters derived by SPT cluster number counts (Bocquet et al. 2019, \( \Omega_m = 0.26, \Omega_b = 0.04, \sigma_8 = 0.6, h_0 = 0.66 \) from here on Cosmo B).

Figure 3 (top panel) shows the ideal un-perturbed mock profile and best fit realisations of NFW profiles produced using Ragagnin20 (red line) and Diemer19 (blue line). The \( \mathcal{M} \) relation presented in this work has a lower concentration normalisation than Diemer19 (Appendix A), and thus Ragagnin20 produce lower values of surface densities near the centre and higher values on the outskirts with respect to DMO mass-concentration relations. Different prior assumptions on the \( \mathcal{M} \) relation affect the inferred mass, as we see in Figure 3 (middle panel)\(^4\). While the posterior derived assuming the \( \mathcal{M} \) relations of Ragagnin20 and of Diemer19 are in good agreement, the best fit mass recovered with Diemer19 is \( \approx \) 10% higher compared to the one derived with Ragagnin20. This can be better appreciated in the bottom panel of Figure 3 where we instead simulate the mass calibration of a stack of 100 clusters (Melchior et al. 2017). We therefore mimic a \( S/N = 50 \) stacked average profile and decrease by a factor of \( \sqrt{100} \) the intrinsic scatter around the \( \mathcal{M} \) relation in the prior. Assuming the wrong Cosmo B cosmology, we would recover biased results using both the Ragagnin20 and the Diemer19 \( \mathcal{M} \) relation, even if the marginalized posterior on the mass would be almost unbiased for the Ragagnin20 analysis. Furthermore we note that fixing Cosmo B cosmology instead of the correct input Cosmo A cosmology would result in a slightly smaller mass for Ragagnin20 and in a slightly larger mass for Diemer19. While a more sophisticated analysis including a treatment of systematic uncertainties and a self-consistent exploration of the cosmological parameters is beyond the purpose of this work, this simple exercise highlights the importance of a correct modelization of the cosmological dependence of the \( \mathcal{M} \) relation in the the weak-lensing analysis of cluster samples for cosmological purposes.

We stress that in this experiment we wanted to mimic the procedure of most observational works, thus we didn’t model baryon component of DMO simulations.

5 HALO MASSES CONVERSION

In the following subsections, we present and compare different methods of converting masses between overdensities. We also provide a direct fit for converting masses (i.e. SUBFIND masses) from \( \Delta_1 \) to \( \Delta_2 \) (thus without using the \( \mathcal{M} \) relation), in order to study the origin of the scatter coming from the conversions. This kind of conversions is used in computing the sparsity of haloes (i.e. ratio of masses in two overdensities), which itself can probe cosmological parameters (Corasaniti et al. 2018; Corasaniti & Rasera 2019) and dark energy models (Balmés et al. 2014).

5.1 Mass-mass conversion using \( \mathcal{M} \) relation

In this section, we tackle the problem of converting masses via a \( \mathcal{M} \) relation. By combining the definition of mass \( M_\Delta \) (see Eq. 4) and the fact that the matter profile only depends on a proportional parameter \( p_\rho \) and a scale radius \( r_s \), we get

\[
M_\Delta = 4\pi \rho_0 r_s^3 f(c_\Delta) = \frac{4}{3} \pi R^3 \rho_c,
\]

For a NFW profile as in Eq. 1,

\[
f(c_\Delta) = \ln(1 + c_\Delta) - \frac{c_\Delta}{1 + c_\Delta}.
\]

Combining Eq. 9 and 10 gives the following mass conversion formula:

\[
\left\{
\begin{aligned}
M_{\Delta 2} & = M_{\Delta 1} \left( \frac{c_{\Delta 2}}{c_{\Delta 1}} \right)^{\frac{3}{2}} \\
c_{\Delta 2} & = c_{\Delta 1} \left( \frac{\Delta 1}{\Delta 2} \right) \frac{f(c_{\Delta 2})}{f(c_{\Delta 1})} \frac{1}{2}
\end{aligned}
\right.
\]

From the second part of Eq. 11 it is possible to evaluate the concentration \( c_{\Delta 2} \) as a function of only \( c_{\Delta 1} \) (as in Appendix C of Hu & Kravtsov 2003).

Eq. 11 can be used to estimate the theoretical scatter \( \sigma_{\text{theo}} \) obtained in the mass conversion by analytically propagating the uncertainties of the mass-concentration relation, namely:

\[
\sigma_{\text{theo}} = \left| \frac{d M_{\Delta 2}}{d c_{\Delta 1}} \right| \sigma_{c_{\Delta 1}},
\]

where \( M_{\Delta 2} \) is the converted mass, \( c_{\Delta 1} \) the concentration in the original overdensity \( \Delta_1 \) and \( \sigma_{c_{\Delta 1}} \) is the uncertainty in the concentration (in our case it is the scatter in the \( \mathcal{M} \) relation). Appendix C describes how to obtain the theoretical scatter one would expect given a perfectly NFW profile.

There are several sources of error in the mass-mass conversion derived by a mass-concentration relation: (i) the intrinsic scatter of the \( \mathcal{M} \) relation, (ii) the fact that profiles are not perfectly NFW and thus Eq. 10 is not the best choice for this conversion; (iii) the cosmology-redshift-mass-concentration fit (as in Table 2) may not be optimal.

To further study the sources of uncertainties in this conversion, we fit SUBFIND halo masses between two overdensities\(^5\), and compare the two conversion methods.

5.2 \( M_{\Delta 1} \)-\( M_{\Delta 2} \) (M-M) plane

In this subsection we perform a direct fit between halo masses (i.e. SUBFIND masses), as a function of redshift and cosmological parameter. The reason of this fit is twofold: (1) we want to study the uncertainty introduced in the conversion of the previous subsection and (2) we want to provide a way of converting masses without any assumption on their concentration and NFW density profile.

For each pair of overdensities we performed a fit of the mass \( M_{\Delta 2} (M_{\Delta 1}, 1/(1+z), \Omega_m, \Omega_b, \sigma_8, h_0) \) with the following functional form:

\[
\ln M_{\Delta 2} (M_{\Delta 1}, a) = \ln A + \ln \left( \frac{M_{\Delta 1}}{M_p} \right) + C \ln \left( \frac{a}{a_p} \right)
\]

\(^5\) The package hydro_mc contains a sample script to convert masses between two overdensities by using the mass-concentration relation presented in this paper (http://github.com/aragagnin/hydro_mc/blob/master/examples/sample_mm_from_mc Relation.py).
where \( A, B, C \) parameters are parametrised with cosmology as in Eq. 7.

Table 3 show the results of the mass-mass conversion fit between critical overdensities, while Table 4 show the conversion fit parameters between \( \Delta_{200c} \) and \( \Delta_{200m} \). The conversion relation has a strong dependency on \( \sigma_8 \) and a weak dependency on \( h_0 \) (see \( \alpha_m, \beta_m, \gamma_m \) parameters).

5.3 Uncertainties in mass conversions

When converting between masses at different overdensities, we are interested in the following sources of uncertainty:

- \( \sigma_{M-M(M_c)} \): the scatter from the mass-mass conversion obtained with the aid of our Mc relation found in Sec. 5.1.
- \( \sigma_{M-M(c)} \): the scatter obtained from a conversion between the true values of \( M_{M1} \) and \( c_{M1} \) of a given halo to \( M_{M2} \) (i.e. using only Eq. 11). We use this scatter in order to estimate the error coming from non-NFWness (i.e. deviation from perfect NFW density profile).
- \( \sigma_{\text{theo}} \): the scatter obtained by analytically propagating the Mc log-scatter (= 0.38 as in Table 2) with Eq. 12. This value estimates the uncertainty coming from the intrinsic scatter of the Mc relation alone.
- \( \sigma_M \): the scatter that is supposed to be introduced by a non-ideal cosmology-redshift-mass-concentration fitting formula.
- \( \sigma_{M-M} \): the intrinsic scatter of M-M conversion using Table 3 presented in Sec. 5.2.

In a simplistic approach, the quadrature sum of the scatter coming from non-NFWness (\( \sigma_{M-M(M_c)} \)), the theoretical scatter (\( \sigma_{\text{theo}} \)) and the scatter due to a non-ideal Mc fit (\( \sigma_M \)), should add up to the scatter in the mass-mass conversion using a mass-concentration relation:

\[
\sigma_{M-M(M_c)}^2 = \sigma_0^2 + \sigma_{\text{theo}}^2 + \sigma_M^2 .
\] (14)

5.4 Obtaining \( M_{200c} \) from \( M_{500c} \) or \( M_{2500c} \)

In this subsection we test mass conversion to \( M_{200c} \) given \( M_{500c} \) or \( M_{2500c} \). We compare results obtained using the technique described in Sec 5.1 against the mass-mass relation from Eq. 13.

We tested the conversion \( M_{500c} \rightarrow M_{200c} \), in the mass regime of \( M_{200c} = 10^{14} - 10^{15} M_\odot \) and found the following scatter values: \( \sigma_{M-M(M_c)} = 0.09 \), \( \sigma_{M-M(c)} = 0.04 \), and \( \sigma_{\text{theo}} = 0.07 \) by converting masses using Sec. 5.1 and \( \sigma_{M-M} = 0.07 \) by using conversion table in Sec. 5.2. We are confident to have all uncertainty sources under control because the quadrature sum of all scatter coming from conversion described in Sec. 5.2 (i.e. \( \sqrt{\sigma_{M-M(M_c)}^2 + \sigma_{\text{theo}}^2} = 0.09 \)) equals \( \sigma_{M-M(M_c)} \) from Sec. 5.1 as in Eq. 14.

We repeat the same conversion for \( M_{2500c} \rightarrow M_{200c} \) and find the following scatter values: \( \sigma_{M-M(M_c)} = 0.29 \), \( \sigma_{M-M(c)} = 0.07 \), \( \sigma_{\text{theo}} = 0.24 \) by converting masses using Sec. 5.1 and \( \sigma_{M-M} = 0.22 \), by using conversion table in Sec. 5.2. In this conversion, the quadrature sum of the theoretical scatters in Eq. 14 holds only if we attribute an additional source of the uncertainty to a non-ideal \( M_{2500c} \rightarrow M_{200c} \) relation fit \( \sigma_M = 0.14 \).

This means that a direct mass-mass fit is more precise than a conversion that passes through an Mc relation when converting \( M_{2500c} \rightarrow M_{200c} \).

It is interesting to see that in both scenarios the conversion with the lowest scatter is the one performed with the exact knowledge of both mass and concentration (i.e. \( \sigma_{M-M(M_c)} \) is the lowest). On the other hand, in the scenario where one only knows the mass of a halo, then the conversion with the lowest uncertainty is the one that uses relation in Sec. 5.2 (i.e. with a scatter \( \sigma_{M-M} \)).

6 DISCUSSION ON COSMOLOGY DEPENDENCY OF MASSES AND CONCENTRATIONS

The concentration of haloes at fixed mass is a non trivial function of cosmological parameters. We summarise in Figure 4 (upper panel) the variation of \( c_{500c} \) as a function of cosmological parameters for a halo of mass \( M_\Delta = 10^{14} M_\odot \) to provide a more intuitive representation. In general concentration normalisation decreases with baryon fraction \( \Omega_b \). While a small (\( \approx 2\% \)) decrease is expected also on DMO models as Diemer & Joyce (2019), our change in this mass range is likely associated with feedback from AGN, as an increase...
of gas fraction implies more energy released by feedback processes, which is known to lower concentration. The logarithmic mass slope of the Mc (see Table 2) increases with Ω_m, in agreement with SNe feedback being less relevant in massive haloes. For concentration at Δ = 2500 the situation is less clear. Δ = 2500c is closer to the centre of the halo and depends more strongly on physical processes which are not solely regulated by gravity. The qualitative behaviour is nevertheless consistent with Δ = 500c, with the strongest (positive) cosmological dependence on σ_r and Ω_m, a weak (negative) dependency on Ω_b, and a weaker one on h_0. It is not possible to infer the effect of Ω_m on the redshift log-slope as its value is mainly driven by σ_r.

C1 and C2 simulations show a positive correlation between mass and concentration. This is in agreement with Prada et al. (2012), where they found that haloes with low r.m.s. fluctuation amplitude σ have a concentration that increases with mass. In fact C1 and C2 have extremely low values of σ_r (i.e. σ_r < 0.7) which leads to low r.m.s. fluctuation amplitudes.

When converting masses from higher overdensities to lower overdensities the scatter increases as the difference between overdensities increases (see Table 3). Figure 4 (lower panel) shows the variation of sparsity normalisation as a function of cosmological parameters. The log-slope of the mass dependency (β parameters) has almost no dependency on cosmology. One exception is made by σ_{200c}, Ω_m, where normalisation does depend on Ω_m. Note that this relation doesn’t assume any density profile, thus this dependency cannot be caused by a bad NFW fit. This effect is probably due to baryon feedback that at this scale is capable of influencing the total matter density profile.

The redshift dependency (γ parameters) is mostly influenced by Ω_m and σ_r, with a contribution that increases with separation between overdensities, which may indicates a different growth of the internal and external regions of the halo.

7 CONCLUSIONS

We provided mass-concentration relations and mass conversion re-
Table 4. Fit parameters for Eq. 13 and Eq. 7 between overdensities of \( \Delta_{200c} \) to \( \Delta_{200m} \). Errors on \( A_0, B_0, C_0 \) and \( \sigma \) are omitted as they are all < 0.001%. Pivots are as in Table 2. The package hydro_mc contains a script that utilises this relation (http://github.com/aragagnin/hydro_mc/blob/master/examples/sample_mm.py).

| Param From overdensity \( \rightarrow \) to overdensity | \( M_p \) [\( M_\odot \)] | \( \rightarrow \) | \( \rightarrow \) |
|------------------------------------------------|-----------------|-----------------|-----------------|
| \( \alpha_p \)                               | 0.877           | 0.877           |
| \( A_0 \)                                    | 33.11           | 32.71           |
| \( B_0 \)                                    | 0.99            | 1.01            |
| \( C_0 \)                                    | 0.46            | -0.49           |
| \( \alpha_m \)                               | \( -0.288 \pm 0.013 \) | \( 0.318 \pm 0.016 \) |
| \( \alpha_b \)                               | \( -0.017 \pm 0.015 \) | \( 0.014 \pm 0.019 \) |
| \( \alpha_\sigma \)                          | \( -0.053 \pm 0.011 \) | \( 0.088 \pm 0.019 \) |
| \( \alpha_n \)                               | \( 0.078 \pm 0.009 \) | \( -0.103 \pm 0.003 \) |
| \( \beta_m \)                                | \( 0.031 \pm 0.001 \) | \( -0.035 \pm 0.001 \) |
| \( \beta_b \)                                | \( -0.040 \pm 0.003 \) | \( 0.063 \pm 0.003 \) |
| \( \beta_\sigma \)                           | \( 0.029 \pm 0.002 \) | \( -0.050 \pm 0.002 \) |
| \( \beta_n \)                                | \( 0.017 \pm 0.006 \) | \( -0.011 \pm 0.007 \) |
| \( \gamma_m \)                               | \( -0.313 \pm 0.002 \) | \( 0.358 \pm 0.002 \) |
| \( \gamma_b \)                               | \( 0.062 \pm 0.008 \) | \( -0.102 \pm 0.010 \) |
| \( \gamma_\sigma \)                          | \( -0.201 \pm 0.007 \) | \( 0.246 \pm 0.010 \) |
| \( \gamma_n \)                               | \( 0.026 \pm 0.022 \) | \( 0.033 \pm 0.036 \) |
| \( \sigma \)                                 | \( 0.084 \pm 0.001 \) | \( -0.102 \pm 0.001 \) |

ACKNOWLEDGEMENTS

The Magneticum Pathfinder simulations were partially performed at the Leibniz-Rechenzentrum with CPU time assigned to the project ‘pr86re’. AR is supported by the EuroEXA project (grant no. 754337). KD acknowledges support by DAAD contract number 57396842. AR acknowledges support by MIUR-DAAD contract number 34843 „The Universe in a Box“. AS and PS are supported by the ERC-StG ’ClustersXCosmo’ grant agreement 716762. AS is supported by the the FARE-MIUR grant ’ClustersXEuclid’ R16SBKTMA and by INFN InDark Grant. This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy - EXC-2094 - 390783311. We are especially grateful for the support by M. Petkova through the Computational Center for Particle and Astrophysics (C²PAP). Information on the Magneticum Pathfinder project is available at http://www.magneticum.org. We acknowledge the use of Bocquet & Carter (2016) python package to produce the MCMC plots. We thank the referee for the useful comments, which we believe significantly improved the clarity of the manuscript. We also thanks Stefano Andreon for the useful feedback on this paper.

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

References

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APPENDIX A: EFFECTS OF BARYONS

In this appendix section, we show the importance of correctly describing baryonic physics on the estimation of halo concentration. Since all simulations (C1-C15) share the same initial conditions, it is possible to study the evolution of the same halo that evolved differently in different cosmologies.

Figure A1 shows the evolution of both the virial radii and scale radii of haloes in C1 and C1_norad. Figure A1 (upper panel) shows the stacked ratio of concentration, virial radius and the scale radius. On an average, C1 haloes have higher concentration parameters ($\Delta = 10 - 15\%$ higher, up to $\approx 20\%$) and this difference grows with time. Intuitively one may think that the difference in concentration between C1 and C1_norad would be due to a difference in the virial radius. However, the figure shows that it is the scale radius that produce the difference in concentration between the full physics run and the non-radiative one.

Figure A1 (bottom panel) focuses on the evolution of a single halo (bottom left panel shows the evolution of the halo in C1, whereas, the bottom right panel shows the same halo in C1_norad). Simulations without radiative cooling produce haloes with lower concentration with respect to their full physics counterpart (i.e. $C_{\text{vir}} \approx 6$ lowers down to $C_{\text{vir}} \approx 5$). This example shows that in non-radiative simulations, concentration decreases even if the full physics counterpart is characterised by the same accretion history ("jumps" in concentration and $r_s$ values happen at the same scale factor).

Dynamical state is known to be related to halo concentrations (Ludlow et al. 2012) and can be quantified using the virial ratio (Cui et al. 2017), $(2T-E_s)/W$, where $W$ is the total potential energy, $T$ is the total kinetic energy (including gas thermal pressure) and $E_s$ is the energy from surface pressure $(P$ from kinematic and thermal energy) at the halo boundary. As described in Chandrasekhar (1961), $E_s$ is given by,

$$E_s = \frac{1}{2} \int P(r)r \cdot dS. \quad (A1)$$

Cui et al. (2017) showed that baryonic physics can lower the virial ratio up to 10% w.r.t. DMO runs and Zhang et al. (2016) showed that merger timescale is shortened by a factor of up to 3 for merging clusters with gas fractions 0.15, compared to the timescale obtained with no gas.

Figure A2 shows $E_s/W$ $vs.$ $K/W$ for a DMO run (left panel) and a hydrodynamic run (right panel) that shares the same initial conditions$^6$. The two runs display a different behaviour for highly concentrated objects ($c > 4$): DMO ones have low surface pressure and low total kinetic energy, while hydrodynamic ones show a much more complex and noisy relation between $E_s$, $W$ and $c_{200\text{c}}$.

It is well known that concentration does depend on dynamical state. Here we also noted how hydrodynamic simulations compared to DMO runs do show even a more noisy and complex relation between concentration and the virial ratio. However, given that the majority of observational studies that investigate large cluster samples lack data to accurately determine their dynamical state (see e.g. studies presented in Hoekstra et al. 2015, Okabe & Smith 2016, Melchior et al. 2018, Dietrich et al. 2019, Mantz 2019, Bocquet et al. 2019 and references therein) they will benefit from a mass-concentration relation built from hydrodynamic simulations that already averages over all possible dynamical states of a halo, as in this work.

The average concentration of haloes shown in Figure 2 are lower than the concentration computed using the dark-matter density profile presented in a previous work on Magneticum simulations (Ragagnin et al. 2019, which uses the same cosmology as C8). The median concentration for cosmology C8 is $c_{200\text{c}} \approx 3.5$ for the total matter profile, while the dark matter concentration presented in (Ragagnin et al. 2019) has $c_{200\text{c}} \approx 4.3$.

Such discrepancy is due to the fact that dark matter component is more peaked in the central region with respect to the total matter density. Figure A3 shows an example of the matter density profiles of a Magneticum halo. Here the DM halo has a scale radius of 139 kpc/h while the total matter scale radius is 154 kpc/h: collisional particles and stars formed from them (and their associated heating processes as SN and AGN feedback) are capable of lowering a concentration parameter of $\approx 20\%$.

APPENDIX B: COSMOLOGY-MASS-REDSHIFT-CONCENTRATION RELATION LITE

Given the weak dependency of mass from the concentration (at least in the mass range of interests of cluster of galaxies), we provide a cosmology-redshift-mass-concentration fit where, in Eq. 5 we parametrise the dependency of the cosmology only in the normalisation

$\Delta (M, \gamma, \cosmology)$

---

$^6$ We use Magneticum/Box0_mrsimulation, with $2.7Gpc/h$ size and gravitational softening down to $2.6 \times 10^9 kpc/h$, gas and DM mass particles of $2.610^9 M_\odot/h$ and $2.610^9 M_\odot/h$ respectively, as presented in Bocquet et al. (2016).
Figure A1. Evolution of virial and scale radii and concentration of haloes in simulations C1 and C1_norad. Upper panel shows the stacked average over 50 haloes of ratios of $c_{\Delta 2}$ (magenta top line), $1/r_s$ (cyan middle line) and $R_{\text{vir}}$ (bottom green line) between C1 and C1_norad. Lower panel shows the evolution $R_{\text{vir}}$ (blue top line) and $r_s$ (orange bottom line) and $c_{\text{vir}}$ in blue, of the same halo in the simulation C1 (bottom left panels) and C1_norad (bottom right panels).

and in the redshift dependency as the following:

$$A = A_0 + a_m \ln \left( \frac{\Omega_m}{\Omega_{m,p}} \right) + a_b \ln \left( \frac{\Omega_b}{\Omega_{b,p}} \right) + \alpha_m \ln \left( \frac{h_0}{h_{0,p}} \right)$$

(B1)

$$B = B_0$$

$$C = C_0 + \gamma_m \ln \left( \frac{\Omega_m}{\Omega_{m,p}} \right) + \gamma_b \ln \left( \frac{\Omega_b}{\Omega_{b,p}} \right) + \gamma_s \ln \left( \frac{h_0}{h_{0,p}} \right)$$

Table B1 show the results of this fit, with the same procedure as in Section 4, where pivot values are the ones for the reference cosmology C8 and errors are assigned by performing the same fit as in Singh et al. (2019).

Table B2 show the results of the mass-concentration plane where we fit the NFW profile of the dark matter density profile only. The functional form is as in Eq. B1, with the same procedure as the previous one (thus, as in Section 4).

APPENDIX C: THEORETICAL SCATTER OF MASS CONVERSION USING AN MC RELATION

Equation system 11 shows how the concentration in an overdensity $\Delta_2$ is uniquely identified by the concentration in $\Delta_1$ by solving bottom equation in Eq. 11. Although there are four variables in Eq. 11 (namely $M_{\Delta 1}, M_{\Delta 2}, c_{\Delta 1}$ and $c_{\Delta 2}$), since there are two equations the system depends on two of them.

Hu & Kravtsov (2003) provides a fitting formula for $c_{\Delta 2}$ as a function of $c_{\Delta 1}$. On the other hand since $c_{\Delta 2}$ depends monotonically from right side of Eq. 11, in this work we convert the values from $c_{\Delta 1}$ to $c_{\Delta 2}$ using the fixed-point technique derived by solving equation 11 the Banach-Caccioppoli theorem (see e.g. Ciesielski 2007, for a review).

To evaluate $c_{\Delta 2}$ we start with a guess value of $c_{\Delta 1}$ and iterat-
apply it to Eq. 11 in order to get the new value of value of $c_{\Delta_2}$ until it converges, practically we fix $\frac{\Delta_1}{\Delta_2}$ and $c_{\Delta_1}$ rewrite Eq. 11 as

$$\hat{c}(x) \approx c_{\Delta_1} \left( \frac{\Delta_1}{\Delta_2} \frac{\int f(x)}{f(c_{\Delta_1})} \right)^{1/3}$$

(C1)

$$c_{\Delta_2} = \frac{\hat{c}(c_{\Delta_2})}{\hat{c}(c_{\Delta_1})}.$$  

We found that the relative error after 9 iterations is, at the worst, comparable with Hu & Kravtsov (2003) and can go down to $10^{-9}$ for concentration values higher than 20. As a first value we choose $c_{\Delta_1}$, so

$$c_{\Delta_2} \approx \hat{c} \left( \hat{c} \left( \hat{c} \left( \hat{c} \left( \hat{c} \left( \hat{c} \left( \hat{c} \left( \hat{c} (c_{\Delta_1}) \right) \right) \right) \right) \right) \right) \right).$$

(C2)

Figure A2. Kinetic term $K$ vs. energy from the surface pressure $E_s$ scaled by total potential energy $W$ for the same initial condition evolved with baryon physics (left panel) and a DMO run (right panel). Black solid lines show the median $E_s/W$.

Table B1. Pivots and fit parameters for the cosmology dependent redshift-mass-concentration plane as Table 2, here the logarithmic slope of mass is not dependent on cosmology, thus we fit Eq. 5 and Eq. B1, for concentration overdensities of $\Delta = \Delta_{2500c}$, $\Delta_{2500m}$ and $\Delta_{2500c}$ and $\Delta_{2500m}$. The pivots $\Omega_{\text{m},p}, \Omega_{\text{b},p}, \sigma_8$ and $h_0$ in Eq. 7 are the cosmological parameters of C8 as in Table 1 ($\Omega_{\text{m},p} = 0.272, \Omega_b = 0.0456, \sigma_8 = 0.809, h_0 = 0.704$). Errors on $A_0$, $B_0$, $C_0$ and $\sigma$ are omitted as they are all < 0.001%. The package hydro_mc contains a script that utilises this relation (http://github.com/aragagnin/hydro_mc/blob/master/examples/sample_mc_lite.py).

| Overdensity | vir   | 200c  | 500c  | 2500c | 200m |
|-------------|-------|-------|-------|-------|------|
| $M_p [M_\odot]$ | 1.99e+14 | 1.74e+14 | 1.37e+14 | 6.87e+13 | 2.24e+14 |
| $\alpha_p$ | 0.877 | 0.877 | 0.877 | 0.877 | 0.877 |
| $A_0$ | 1.499 | 1.238 | 0.899 | 0.122 | 1.688 |
| $B_0$ | 0.048 | 0.053 | 0.060 | 0.037 | 0.044 |
| $C_0$ | 0.520 | 0.201 | 0.187 | 0.110 | 0.910 |
| $\sigma_m$ | 0.423 ± 0.006 | 0.60 ± 0.01 | 0.63 ± 0.01 | 0.7273 ± 0.0006 | 0.201 ± 0.003 |
| $\alpha_\sigma$ | -0.141 ± 0.006 | -0.152 ± 0.006 | -0.131 ± 0.005 | -0.179 ± 0.004 | -0.186 ± 0.006 |
| $\sigma_\sigma$ | 0.65 ± 0.02 | 0.65 ± 0.02 | 0.61 ± 0.03 | 0.516 ± 0.003 | 0.60 ± 0.02 |
| $\gamma_m$ | 0.19 ± 0.01 | 0.360 ± 0.010 | 0.336 ± 0.009 | 0.36 ± 0.01 | -0.10 ± 0.01 |
| $\gamma_\sigma$ | 0.02 ± 0.06 | -0.15 ± 0.06 | -0.04 ± 0.05 | 0.00 ± 0.07 | 0.00 ± 0.06 |
| $\gamma_{\sigma\sigma}$ | -0.76 ± 0.05 | 0.72 ± 0.04 | 0.89 ± 0.04 | 0.94 ± 0.06 | 0.61 ± 0.05 |
| $\sigma$ | 0.388031 | 0.384516 | 0.376690 | 0.382686 | 0.388477 |

Figure C1 shows the relative error when converting $M_{2500c}$ and $M_{2500m}$ to $M_{200c}$. Both approach have an error smaller than $\approx 0.1\%$, while the iteration proposed here can reach much more precise value and it is easier to implement. Only 9 iterations produce a relative error that in the worst case is comparable with technique in Hu & Kravtsov (2003) and it is capable of going down to $10^{-8}$.

Figure C2 show the conversion from overdensities $\Delta_2 = 2500$ and $\Delta_2 = 500$ to $\Delta_1 = 200$. These relations are nearly linear with a deviation for lower concentrations.

Another interesting property of Eq. 11 is the possibility of knowing $M_{\Delta 2}/M_{\Delta 1}$ only by knowing $c_{\Delta 1}$.

Figure C3 shows such conversions for overdensities $\Delta_{2500c}$ and $\Delta_{500c}$ to $\Delta_{200c}$. This conversion gets flatter and flatter as the
Verticallines correspond respectively to the dark matter profile scalardensity radius of the dark matter density profile, plus the logarithmic slope of mass is not dependent on cosmology. We fit Eq. 5 and Eq. B1, for concentration overdensities of $\Delta = \Delta_{200c}$, $\Delta_{500c}$, $\Delta_{2500c}$ and $\Delta_{200m}$. The pivots $\Omega_m$, $\Omega_b$, $\sigma_8$ and $h_0$ in Eq. 7 are the cosmological parameters of C8 as in Table 1 ($\Omega_m = 0.272$, $\Omega_b = 0.046$, $\sigma_8 = 0.809$, $h_0 = 0.704$). Errors on $A_0$, $B_0$, $C_0$ and $\sigma$ are omitted as they are all $< 0.001%$. The package hydro_mc contains a script that utilises this relation (http://github.com/aragagnin/hydro_mc/blob/master/examples/sample_mc_dm_end Lite.py).

| Parameter | vir | 200c | 500c | 2500c | 200m |
|-----------|-----|------|------|-------|------|
| $A_0$     | 1.499 | 1.238 | 0.979 | 0.213  | 1.798 |
| $B_0$     | $-0.048$ | $-0.053$ | $-0.039$ | $-0.015$ | $-0.034$ |
| $C_0$     | 0.520 | 0.201 | 0.178 | 0.055  | 0.918 |
| $\sigma_m$ | 0.42 $\pm$ 0.05 | 0.60 $\pm$ 0.01 | 0.46 $\pm$ 0.07 | 0.588 $\pm$ 0.001 | 0.008 $\pm$ 0.007 |
| $\sigma_h$ | $-0.14 \pm 0.03$ | $-0.152 \pm 0.006$ | $-0.08 \pm 0.03$ | $-0.204 \pm 0.010$ | $-0.072 \pm 0.006$ |
| $\gamma_m$ | 0.19 $\pm$ 0.04 | 0.360 $\pm$ 0.010 | 0.34 $\pm$ 0.01 | 0.51 $\pm$ 0.03 | $-0.23 \pm 0.01$ |
| $\gamma_h$ | 0.02 $\pm$ 0.06 | $-0.15 \pm 0.06$ | $-0.4 \pm 0.1$ | $-0.7 \pm 0.1$ | $-0.09 \pm 0.06$ |
| $\gamma_r$ | 0.76 $\pm$ 0.06 | 0.72 $\pm$ 0.04 | 0.5 $\pm$ 0.1 | 0.3 $\pm$ 0.1 | 0.45 $\pm$ 0.02 |
| $\sigma_r$ | 0.39 | 0.384516 | 0.51 | 0.484290 | 0.49887 |

The pivots $\Omega_m$, $\Omega_b$, $\gamma_m$, $\gamma_h$, $\gamma_r$, $\sigma_m$, $\sigma_h$ and $\sigma_r$ correspond to the NFW profile (solid lines) for a halo of C1 simulation at $z = 0$. Vertical lines correspond respectively to the dark matter profile scale radius ($139 \text{kpc/h}$) and the total matter profile has a scale radius $r_s = 154 \text{kpc/h}$. A higher concentration increases, implying that the higher the concentration the lower the error one makes in this conversion.

It is possible to estimate this uncertainty analytically. Given the fact that $M_c$ relations are nown with uncertainties, it is interesting to see how to propagate the error analytically when converting from $c_{\Delta 1}$ to $c_{\Delta 2}$, which is proportional to the derivative coming from Eq. 9:

$$\frac{dc_{\Delta 2}}{dc_{\Delta 1}} = \frac{c_{\Delta 2}}{c_{\Delta 1}} \left[\frac{1}{3 \frac{df(c)}{dc}}\right]_{c=c_{\Delta 2}} ^{c=c_{\Delta 1}} - \frac{1}{3 \frac{df(c)}{dc}} \left[\frac{dc_{\Delta 2}}{dc_{\Delta 1}}\right]_{c=c_{\Delta 2}} ,$$

where $f(c)$ is, in case of imposing a NFW profile, given in Eq. 10.

One can rearrange Eq. C3 to isolate the derivative:

$$\frac{dc_{\Delta 2}}{dc_{\Delta 1}} = \frac{1}{3 \frac{df(c)}{dc}} \left[\frac{c_{\Delta 2}}{c_{\Delta 1}} - \frac{1}{\frac{dc_{\Delta 2}}{dc_{\Delta 1}}}\right],$$

(C4)

One can understand how a uncertainty propagates analytically from $M_{\Delta 2}$ ($M_{\Delta 1}$, $c_{\Delta 1}$) in Eq. 11, by computing the derivative

$$\frac{dM_{\Delta 2}}{dc_{\Delta 1}} = \frac{\partial M_{\Delta 2}}{\partial c_{\Delta 1}} + \frac{\partial M_{\Delta 2}}{\partial M_{\Delta 1}} \frac{dM_{\Delta 1}}{dc_{\Delta 1}} ,$$

given the very weak dependency of mass from concentration, we can approximate

$$\frac{dM_{\Delta 1}}{dc_{\Delta 1}} \approx 0.$$

Figure A3. Density profile of both dark matter (dashed black) and total matter (dashed pink) up to the virial radius $R_{\text{vir}} = 930 \text{kpc/h}$ and the corresponding NFW profile (solid lines) for a halo of C1 simulation at $z = 0$. Vertical lines correspond respectively to the dark matter profile scale radius ($139 \text{kpc/h}$) and the total matter profile has a scale radius $r_s = 154 \text{kpc/h}$. Figure C1. Relative error when converting the concentration using Eq. C1 (i.e. Banach-Cacioppoli theorem) or using the method proposed in Hu & Kravtsov (2003).
one gets
\[
\frac{dM_{\Delta}}{dc_{\Delta}} = 3M_{\Delta}\left(\frac{1}{c_{\Delta2}} \frac{dc_{\Delta2}}{dc_{\Delta1}} - \frac{1}{c_{\Delta1}}\right),
\]
where \(dc_{\Delta2}/dc_{\Delta1}\) is evaluated as in Eq. C4.

Figure C4 show the uncertainty variation when converting to \(M_{200c}\) for a scatter in the concentration compatible with the scatter we found in our Mc relation (see Table 2). This is helpful in understanding the actual scatter one find in real case scenarios as Sections 5.2 and 5.1.

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