Abstract

Ever growing water needs cause the rising of peak demand which influences the sourcing and effluent costs and causes an increased outwear of the water distribution system components. To get closer to the ideal flat-demand case, it is necessary to make consumers use water more rationally and/or shift water uses from high-loading instants. In the case that consumers are equipped with smart meters it is technically possible to apply dynamic water pricing which is determined through an optimization of cumulative costs for the water use profile. These costs are covered by water distribution revenues in a full cost recovery principle.

Keywords: Day-ahead dynamic water pricing; Smart metering; Convex optimization

1. Introduction

Fresh water is considered an essential human right. Since the 19th century the public authorities have been responsible for its sourcing, treatment and distribution to different water consumers [1]. Treating water as a public good has contributed to the increase of water scarcity in the world. An economic regulation of the water demand side management becomes essential for protecting and restoring clean water and ensuring its long-term and sustainable use. Adopted in 2000, the Water Framework Directive (WFD) establishes a legal framework for EU member states action in the field of water policy introducing the economic principles and methods for the management of Europe’s waters [2]. WFD imposes the directive that water services are charged at a price which fully reflects the water utility costs, including the operational and maintenance costs of its supply and treatment, costs invested in infrastructure, as well as environmental and resource costs. It also states that water pricing should create incentives for the efficient water use.

Determination of efficient water pricing policies under the WFD was analysed in literature. In [1] the optimal pricing is determined by maximizing the expected social surplus under the constraint that the total expected water utility profit is non-negative. In [3] an efficient pricing policy is achieved by solving a supply-demand system where marginal price is equalized to marginal cost; however, marginal cost pricing rule has some drawbacks and thus "there

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is a little evidence of its real-world application in water utilities" [4]. Other pricing rules include increasing/decreasing block rates, time-based pricing etc.

Recent developments in smart metering technology have stimulated an increased deployment of the time-based pricing schemes in power systems [5,6]. Among various time-based pricing schemes, recent studies have shown that hourly dynamic pricing is the most efficient in terms of providing incentives for the efficient energy use [5]. A number of papers have analysed the implementation of dynamic pricing in electrical distribution systems [5,7,8]. This paper studies an employment of a day-ahead dynamic pricing system (DPS) in the water distribution system (WDS) by formulating the objective as a convex optimization problem whose solution, if exists, is guaranteed to be the global optimum.

The paper is structured as follows: Section 2 describes the DPS framework. Section 3 describes the water distribution network (WDN) models for evaluating the overall water utility cost and revenue. In Section 4 the principles of constructing and solving the optimization problem are given. Section 5 shows the DPS operation with test results. Finally, Section 6 concludes the paper.

2. Dynamic pricing system framework

In order to apply the DPS within the WDN, some necessary conditions have to be fulfilled: (i) each household in the pricing system is equipped with a smart water meter, and (ii) water demand prediction for a day ahead is available. Fig. 1 shows the envisaged framework of the DPS.

Data management platform (DMP) is required for storing all the data needed for operation of the DPS and other services such as the water demand prediction system (WDPS). It collects the data on an hourly consumption for each household, water demand prediction, water utility parameters, environment data, data needed for the WDPS operation and other data. DPS fetches the data from the DMP and determines the optimal prices for a day ahead which are then presented to consumers (see Fig. 1).

DPS is composed of 3 subsystems. The first one is identification of demand response model which shows how do consumers react on the price change in terms of their water consumption – it is also referred to as price elasticity of demand. Linear demand response model is the most widely used because it is simple and robust [5,7–9]. The second subsystem is constructor of the optimization problem and the third subsystem provides the tools for solving it. The whole DPS is mathematically described in the next section.

![Fig. 1. Reference dynamic water pricing framework](image_url)
3. Cost and revenue models for dynamic pricing of water

The DPS design resides on the representation of WDS given in Fig. 2. The DPS procedure relies on water demand prediction assuming the default (flat) pricing. Its goal is to compute such price profile over time that minimizes the cost of the WDN operation and of its interfacing with the systems leaned to it (water abstraction, wastewater treatment). The standing constraint is the full cost recovery principle, that is, the overall water utility costs are charged:

\[ R \geq C_T + C_I, \]  

(1)

where \( R \in \mathbb{R} \) is a total water utility revenue, \( C_T \in \mathbb{R} \) and \( C_I \in \mathbb{R} \) are total technical and interfacing costs of the water utility matched with monetary valuation of distribution operation and WDS interfacing. The mathematical representation of the model is presented in the following subsections.

3.1. Water use model and price elasticity

We assume that demand prediction is performed on an hour time-scale for a day ahead and that it is obtained under constant price \( p_0 \). The price vector \( p \in \mathbb{R}^{24} \) to be applied within the DPS framework is:

\[ p = p_0 \mathbf{1} + \Delta p, \]  

(2)

where \( \mathbf{1} \in \mathbb{R}^{24} \) is a vector with all components one, and \( \Delta p \in \mathbb{R}^{24} \) is the price change from default price \( p_0 \). As mentioned in Section 2, we assume a linear demand response modelled by the price elasticity matrix (PEM) \( J \in \mathbb{R}^{24 \times 24} \), that is:

\[ w = f + J \Delta p, \]  

(3)

where \( w \in \mathbb{R}^{24} \) is the water use profile, and \( f \in \mathbb{R}^{24} \) is the demand prediction (forecast) in a certain measuring point of the WDN that has users in the dynamic pricing mode attached downstream. It is not necessarily the case that all downstream users are in the dynamic pricing mode – their number of course affects the trace of matrix \( J \).

The diagonal elements of PEM represent the self-elasticities and the off-diagonal elements correspond to the cross-elasticities. Self-elasticities are expected to be non-positive and cross-elasticities to be non-negative, that is, higher price at the specific hour leads to a decreased consumption at that hour and an increased consumption in the neighbouring hours which means that consumers shift their consumption from higher price hour to the lower price hours. In [9] various structures of the PEM for electrical demand are studied and it is stated that this matrix has to be determined by analysing the actual consumers’ response to the price change from its default value. In this paper we assume that PEM is negative semidefinite matrix.

Fig. 2. Water distribution system models for the cost analysis
3.2. Revenue

The overall utility revenue $R$, from the part of the WDN downstream the measuring point where the demand is considered (predicted), can be computed as:

$$R = p^T w = (p_0 1 + \Delta p)^T \cdot (f + J \Delta p) = p_0 1^T f + (p_0 1^T J + f^T) \Delta p + \Delta p^T J \Delta p.$$  \hspace{1cm} (4)

Expression (4) shows that the revenue $R$ is a quadratic function of price change $\Delta p$. Moreover, due to the assumption of negative semidefiniteness of $J$, $R$ is a concave function of $\Delta p$.

If a certain amount of leakage exists in the WDN downstream the point for which the demand prediction is determined, it has to be taken into account. According to (4), leakage is also accounted in revenues, as $w$ does not correspond to the sum of end-users’ consumptions. However, the amount of leakage can then be assigned to technical cost as an expense to balance for that.

3.3. Cost

The overall cost of the WDS for the considered part of the WDN is a sum of technical and interfacing costs (see Fig. 2) and it is assumed that they are composed of fixed and variable costs. Fixed cost can be related to e.g. personnel cost or leakage, while variable cost may refer to energy used for pumping through the WDS, water treatment cost, wastewater cost etc. These costs can be modelled as a linear function of the water use profile $w$ and they reflect the short-term operational cost of the water utility. However, water utility can also consider long-term costs in the cost analysis, e.g. damage frequency and recovery of the WDS components have much larger time constant than the observed 24-hour time horizon. However, if an outwear of the WDS components is affected by the large peaks in water use profile, these can also be included in the cost. For instance, in order to reduce the peak consumption value because it influences a lifetime of the components, we can assign a cost to water consumption exceeding particular consumption tolerance level. For such a case, we can model the overall water utility cost as:

$$C = C_T + C_I = b + c^T w + d \max (w - w_{tol} 1, 0)^T \max (w - w_{tol} 1, 0)$$

$$= b + c^T (f + J \Delta p) + d \max (f + J \Delta p - w_{tol} 1, 0)^T \max (f + J \Delta p - w_{tol} 1, 0),$$  \hspace{1cm} (5)

where $b \in \mathbb{R}$, $c \in \mathbb{R}^{24}$, $d \in \mathbb{R}$ and $w_{tol} \in \mathbb{R}$ are the cost coefficients determined from the water distribution system model and monetary valuation models in Fig. 2, $0 \in \mathbb{R}^{24}$ is a vector with all components zero, and max refers to componentwise maximum function. Here, we assign a quadratic cost to water consumption exceeding the consumption tolerance level $w_{tol}$. The overall water utility cost $C$, defined in (5), is a convex function of price change $\Delta p$.

4. Optimization-based pricing

As mentioned before, the objective of the DPS is to determine a day ahead hourly price profile that minimizes the overall water utility cost while satisfying the financial constraints on the cost recovery and other legal and technical constraints. Therefore, the DPS problem is defined as:

$$\text{minimize} \quad C(\Delta p)$$

subject to

$$R(\Delta p) \geq C(\Delta p)$$
$$p_{\min} 1 \leq p_0 1 + \Delta p \leq p_{\max} 1$$
$$w \geq 0$$
other constraints, \hspace{1cm} (6)

where $\Delta p$ is a decision variable, $p_{\min} \in \mathbb{R}$ and $p_{\max} \in \mathbb{R}$ are legally assessed minimum and maximum price. Apart from price bounds, the pricing should be additionally regulated in order to prevent high prices over the whole day. One possibility is to add constraint which limits the average price, e.g.:

$$1^T \Delta p \leq 0,$$  \hspace{1cm} (7)
that guarantees that the average daily price is lower than default price $p_0$. However, this constraint can cause high prices during daily hours when consumption is high, and low prices during night hours when consumption is low. Better option may be to limit the average price in two consecutive hours:

$$\Delta p(k + 1) + \Delta p(k) \leq 0, \quad k \in \{1, \cdots, 23\},$$

(8)

where $\Delta p(k)$ is the $k$-th element of price change vector $\Delta p$. This constraint introduces the incentive for shifting the consumption from high-consumption hours. Adding this constraint to the DPS problem (6) will not affect the problem convexity. Moreover, it is possible to introduce additional constraints – as long as these constraints are convex, the DPS problem (6) remains convex [10]. Note that sufficient condition for feasibility of the observed optimization problem is that for default price $p_0$ the cost recovery constraint (1) is fulfilled.

5. Testing

The proposed method for determining the optimal dynamic price, in terms of reducing the overall water utility cost under the imposed constraints, is tested in this section. The proposed optimization problem is a quadratically constrained quadratic program (QCQP) and there exists a number of open-source and commercial solvers which can solve it. In this paper optimization procedure is performed using MATLAB® [11] and CVX, a MATLAB®-based modelling system for convex optimization [12].

The test was performed for the typical water demand curve shown in Fig. 3 using the following values of cost parameters: $b = 1000$, $c = 2.2 \cdot 1$, $d = 1$ and $w_{tol} = 140 \text{ m}^3$/h. Default price is $p_0 = 3$ EUR/m$^3$, and minimum and maximum prices are $p_{min} = 1$ EUR/m$^3$ and $p_{max} = 5$ EUR/m$^3$. PEM was chosen to be a tridiagonal matrix with diagonal elements set to $-6$ and off-diagonal elements set to $3$. Fig. 3 shows the calculated optimal price profile and the associated demand curve, while Table 1 shows economic statistics and peak-demand for the default and dynamic pricing scheme.

Results presented in Table 1 show that, under the assumption that consumers’ response to price change is described by the particular PEM, both consumer’s bill and overall water utility cost can be reduced.

![Original and DPS-affected demand with associated price profiles](image)
Table 1. Economic statistics and peak-demand for original and DPS-affected pricing scheme.

| Pricing scheme   | Mean price | Peak-demand value | Revenue   | Variable cost |
|------------------|------------|-------------------|-----------|---------------|
| Default price    | 3.00       | 163.00            | 7149.00   | 6095.60       |
| Dynamic price    | 2.90       | 156.52            | 6773.71   | 5773.71       |
| Relative change  | -3.23%     | -3.98%            | -5.25%    | -5.28%        |

6. Conclusion

This paper analyses application of dynamic pricing within the water distribution system. Dynamic prices are obtained by solving the quadratically constrained quadratic program which is a convex optimization problem. A necessary condition for feasibility of the proposed optimization problem is that for default price \( p_0 \) the full cost recovery constraint is fulfilled. Test results show that shifting the consumption from high-demand hours can lead to reduction of both the consumers’ bill and the overall water utility costs; therefore, the proposed pricing scheme introduces an incentive for the efficient water use.

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