Natural extensions of the Connes–Lott Model and comparison with the Marseille–Mainz Model

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Abstract

An extension of the Connes–Lott model is proposed. It is also within the framework of the A.Connes construction based on a generalized Dirac–Yukawa operator and the K–cycle \((H, D)\), with \(H\) a fermionic Hilbert space. The basic algebra \(A\) which may be considered as representing the non–commutative extension, plays a less important role in our approach. This allows a new class of natural extensions. The proposed extension lies in a sense between the Connes–Lott and the Marseille–Mainz model. It leads exactly to the standard model of electroweak interactions.

* Work supported in part by PROCOPE project Mainz University and CPT Marseille–Luminy.
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In this paper we present an extension of the model developed by A. Connes and J. Lott [1–3]. In some sense the proposed extension lies between the Connes–Lott model and the Marseille–Mainz model [4–7] within the noncommutative geometry approach to the standard model in elementary particle physics.

One of the motivations for the proposed generalisation is to avoid some peculiar features of the Connes–Lott model (CL model). These include the presence of a $\gamma^5$ matrix in Yukawa couplings, unusual in the standard model, the assignment of the fermionic additive quantum numbers and the absence of spontaneous symmetry breaking for one generation of fermions. In addition, we propose an alternative construction whose spirit is close to the one of Connes’ and Lott’s construction but avoids some of their predictions [3] which may be in contradiction with experiment. Finally we stress the role of the basic associative algebra $\mathcal{A}$ in the CL model and its phenomenological implications.

The following observation may be considered as the physical motivation for the approach based on noncommutative geometry. As for the electromagnetic interactions the photon related to the Abelian group $U(1)$ leads to the Minkowski space–time, the electroweak interactions, and especially the W and Z particles which are connected with a noncommutative group $SU(2) \times U(1)$, may lead to a space–time where noncommutative geometry plays an important role. One of the main virtues of the noncommutative geometry approach is the explanation for the spontaneous symmetry breaking and the Higgs effect in the standard model.

A common aspect of the above models, inspired by noncommutative geometry, is the construction and importance of a certain graded differential (or derivative) algebra $\Omega^*$ which may be considered as ”noncommutative” generalisation of the de Rham algebra $\Lambda^*(X)$ of differential forms over space–time $X$, and which is the space where the gauge potential $A$ lives. The models differ from each other in the choice of the specific graded differential (or derivative) algebra $\Omega^*$ they use for the construction of the gauge potential (super–connection) $A$ and field strength (super–curvature) $F$. This leads to somewhat different versions of the standard model in the two cases. There is indeed such a difference between the Marseille–Mainz model (MM model) and the CL model: The MM model leads exactly to the standard model with spontaneous symmetry breaking and to the Higgs potential, and gives a natural framework for the discussion of the CKM matrix [8] but it does not determine any of its parameters. The reason for this is explained and discussed in [9]. The CL model, which contains more structure, seems to lead not exactly to the standard model but to a variation of it. This follows from the fact that, with one generation only, this model shows no spontaneous symmetry breaking. It is not clear whether the presence of $\gamma^5$ in the Higgs sector leads to some unusual coupling. The quantum numbers of fermions are not from the beginning equivalent to those of the standard model unless an additional Poincare duality assumption enters the construction [2, 3, 10]. The results of [3, 7] may suggest
that it is at least questionable whether the CL model can fix some parameters of the standard model \[3, 10\], even at the classical level. In what follows, we first give a brief review of the CL and the MM model (section 2 and section 3), and then proceed with the proposed extension of the CL model (sections 4, 5 and 6).

2. Since we would like to extend the CL model, it is necessary to first review some of its aspects. Here this is formulated in much simpler terms than in the original version \([1, 2, 12]\). This is possible by exploring the mathematical results of \([11]\). This was also demonstrated in \([7]\) with a toy model. Here we use for the first time the new formulation in the realistic case. In addition, we discuss the role of the basic algebra \(A\) which constitutes the most essential difference between the CL and the MM models, and which is also the starting point for the proposed generalisation.

We essentially follow the lines of \([7]\) and \([11]\) but choose the associative algebra to be

\[
A = A_M \otimes C^\infty(X)
\]

with

\[
A_M = \mathbb{H} \oplus \mathbb{C}
\]

instead of \(A_M = \mathbb{C} \oplus \mathbb{C}\) in \([7]\). \(\mathbb{H}\) represents the quaternionic numbers and \(C^\infty(X)\) the smooth functions on the space–time \(X\). This algebra leads to a standard model like version.

Given the associative algebra \(A\), one considers first its universal differential envelope \((\Omega^*(A), \delta)\), which is generated by the formal elements ("words") \(A_0 \delta A_1 \cdots \delta A_n \in \Omega^n(A)\) and the operator \(\delta\) obeying the Leibniz rule \(\delta(AB) = (\delta A)B + A(\delta B)\). By means of a K–cycle (Dirac–Kasparov cycle) \((H, D)\) over \(A\), consisting of a Hilbert space \(H\), a Dirac–Yukawa operator \(D\) on \(H\), and a representation of \(A\) on \(H\), the associative algebra \(\Omega^*(A)\) is represented on the space \(L(H)\) of bounded linear operators over \(H\) by

\[
\pi : \Omega^*(A) \longrightarrow L(H) , \ A_0\delta A_1 \cdots \delta A_n \longrightarrow A_0[D, A_1] \cdots [D, A_n].
\]

The Dirac–Yukawa operator has the form

\[
D = i\gamma^\mu \partial_\mu + D_M \quad , \quad D_M = \eta
\]

with \(\eta\) a matrix as specified below (see eq. (8)). \(D_M\) may be understood to be a fermionic mass matrix. Note that such an interpretation was not made in the MM approach because it is unnecessary in that framework. In the original version of Connes’ and Lott’s construction, the gauge potential and the field strength were taken to be elements of \(\pi(\Omega^*(A))\). Since the representation \(\pi\) fails to respect the differential structure of \(\Omega^*(A)\), this leads to the appearance of auxiliary or adynamic fields (fields without kinetic energy) in the Lagrangian which have to be eliminated by minimization.
At this stage a direct comparison with other approaches, such as the MM model, is not possible. In the more recent version of the CL model given in [12] and later in [3], one considers in addition the space $\Omega^*_D(A)$, obtained from $\Omega^*(A)$ by dividing out the ideals $J^k(A) = K + \delta K^{k-1}$, where $K^k := \ker \pi \cap \Omega^k$:

$$\Omega^*_D(A) = \Omega^k(A) / J^k(A) ,$$

or equivalently

$$\Omega^*_D(A) = \pi(\Omega^k(A)) / \pi(J^k(A)) .$$

In contrast to $\pi(\Omega^*(A))$, the space $\Omega^*_D(A)$ is an N–graded differential algebra (like the universal object $\Omega^*(A)$). Therefore $\Omega^*_D(A)$ is the space which should be compared to the space $\Omega^*_M(X)$ in the MM model (see below). The multiplication law is defined by the ordinary multiplication in $L(H)$ and by taking the quotient. We denote it by the symbol $\odot_D$. Similarly the differential $d_D$ on $\Omega^*_D(A)$ is defined by means of commutators with the Dirac–Yukawa operator and by taking the quotient as above. The structure of this algebra may therefore be summarized as follows:

$$( \Omega^*_D(A) , \odot_D , d_D )$$

The explicit construction of the space $\Omega^*_D(A)$ was given in [11]. Since this result is particularly important for the treatment below, we would like to give a short discussion of it [1]. In the case where basic algebra is of the form $A = A_M \otimes C^\infty(X)$ with $A_M$ a block diagonal matrix algebra, the differential algebra

$$( \Omega^*_D(A) , \odot_D , d_D )$$

is isomorphic to the skew tensor product of the de Rham algebra $(\Lambda^*(X), d_C)$ and a specific (quotient space) matrix differential algebra $\mu^*$ which is N–graded and generated from $A_M$ [11]:

$$\Omega^*_D(A) = \mu^*(A) \hat{\otimes} \Lambda^*(X) , \quad \Omega^*_D(A) = \bigoplus_{i=0}^{k} \mu^{k-i} \otimes \Lambda^i(X)$$

The total grade of homogeneous elements $[a] \otimes \alpha$ with $[a] \in \mu^*$ and $\alpha \in \Lambda^*$ is given by

$$\partial([a] \otimes \alpha) = \partial([a]) + \partial(\alpha) .$$

The multiplication law in $\Omega^*_D(A)$ reads

$$([a] \otimes \alpha) \odot_D ([b] \otimes \beta) = (-1)^{\partial([b])\partial(\alpha)}[a][b] \otimes \alpha \beta .$$

The differential $d_D$ is given by

$$d_D([a] \otimes \alpha) = d_\mu[a] \otimes \alpha + (-1)^{\partial([a])} [a] \otimes d_C \alpha .$$

1 It is indeed this result which allows our simplified treatment and the direct comparison between the two models (CL and MM), and it also gives a hint for the extension of the CL model.
It is important to realize that \((\Omega^0_D(A), \circ_D, d_D)\) depends in an essential way on the basic algebra \(A\) (and of course on \(H\) and \(D\)) and is uniquely determined once \(A\) is given. For the case \(A_{\mathbb{M}} = \mathbb{C} \oplus \mathbb{C}\), the calculation was given in [4]. Here we present the explicit result for the case \(A_{\mathbb{M}} = \mathbb{H} \oplus \mathbb{C}\) which corresponds to the CL model [1–3]. We consider only one generation of fermions. For the purpose of physics, we need to know only the spaces \(\pi(\Omega^k)\) for \(k = 0, 1, 2\). Thus we have to determine the projected ideals \(\pi(\mathcal{J}^k)\) for these three values of \(k\). They are found to be (see also [15])

\[
\pi(\mathcal{J}^0) = \{0\}, \quad \pi(\mathcal{J}^1) = \{0\}, \quad \pi(\mathcal{J}^2) = (\mathbb{C}_{2 \times 2} \oplus \mathbb{C}) \otimes \Lambda^0(X) .
\]

Using

\[
M^0 := A_{\mathbb{M}} = \begin{pmatrix} \mathbb{H} & 0 \\ 0 & \mathbb{C} \end{pmatrix}, \quad M^1 := \begin{pmatrix} 0 & 0 & \mathbb{C} \\ 0 & 0 & \mathbb{C} \\ \mathbb{C} & \mathbb{C} & 0 \end{pmatrix}, \quad M^2 := \begin{pmatrix} \mathbb{C}_{2 \times 2} & 0 \\ 0 & \mathbb{C} \end{pmatrix}, \quad (7)
\]

we obtain, in an obvious notation,

\[
\begin{align*}
\Omega^0_D &= A = M^0 \otimes \Lambda^0 , \\
\Omega^1_D &= (M^0 \otimes \Lambda^1) \circ (M^1 \otimes \Lambda^0) , \\
\Omega^2_D &= (M^0 \otimes \Lambda^2) \circ (M^1 \otimes \Lambda^1) .
\end{align*}
\]

The multiplication law can easily be derived for the case \(\Omega^1_D \times \Omega^1_D \rightarrow \Omega^2_D:\)

\[
( (M^0 \otimes \Lambda^1) \circ (M^1 \otimes \Lambda^0) ) \circ_D \left( (M^0 \otimes \Lambda^1) \circ (M^1 \otimes \Lambda^0) \right) = (M^0 \otimes \Lambda^2) \circ (M^1 \otimes \Lambda^1) .
\]

In particular it is important to note that

\[
(M^1 \otimes \Lambda^0) \circ_D (M^1 \otimes \Lambda^0) = 0 .
\]

Similarly we obtain for the differential \(d_D : \Omega^1_D \rightarrow \Omega^2_D:\)

\[
d_D \left( (M^0 \otimes \Lambda^1) \circ (M^1 \otimes \Lambda^0) \right) = (M^0 \otimes d_C \Lambda^1) \circ (-M^1 \otimes d_C \Lambda^0 + d_{A} M^0 \otimes \Lambda^1) \circ (d_M M^1 \otimes \Lambda^0) .
\]

with

\[
d_M \left( M^0 \otimes \Lambda^1 \right) = [\eta, M^0] \otimes \Lambda^1 \quad \text{and} \quad d_M \left( M^1 \otimes \Lambda^0 \right) = \{\eta, M^1\} \otimes \Lambda^0 ,
\]

\([,]\) representing the commutator, \(\{,\}\) representing the anticommutator and

\[
\eta = i \begin{pmatrix} 0 & C \\ C & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} . \quad (8)
\]

Note that because of the multiplication law \(\circ_D\) we obtain \(\{\eta, M^1 \otimes \Lambda^0\} \circ_D = 0\).

The generalized potential (super–connection) \(A\) is a (skew hermitian) element of \(\Omega^1_D(A)\) and reads explicitly:

\[
A = i \begin{pmatrix} A_{SU(2)} & \Phi \\ \Phi^* & B_{U(1)} \end{pmatrix} \quad \text{with} \quad A_{SU(2)} = \frac{1}{2} \tau_i A^i_\mu dx^\mu , \quad B_{U(1)} = B_\mu dx^\mu , \quad (9)
\]

and \(\Phi\) the scalar field \(\Phi = \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix}\).
The structure group is $SU(2) \times U(1)$ given by the unitary part of $A_M$. The field strength (super-curvature) is obtained from the structure equation

$$F_{CL} = dD + A \otimes_D A$$

A straightforward calculation, using the multiplication rule and the differential given above, leads to the result

$$F_{CL} = i \left( \begin{array}{cc} F^A & -D\Phi \\ -D\Phi & F^B \end{array} \right)$$

with $F^A = dC_A + A_2 \wedge A_2$, $F^B = dC_B + A_1 \wedge A_1$, and $D\Phi = dC_\Phi + A_2(\Phi + C) - B_1(\Phi - C)$.

The Lagrangian $L_{CL} = -trF_{CL}^+F_{CL}$ is given by

$$L_{CL} = -\frac{1}{4} trF^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4} F^B_{\mu\nu}F^{B\mu\nu} + 2D\Phi D\Phi$$

From this it is obvious that the Higgs potential in the CL model with one generation of fermions (leptons) is trivial $V_{CL} = 0$.

3. For the benefit of the reader but also in order to facilitate the comparison of the various models, we would like to give a very short review of some essential ingredients of the MM model [4-7] before starting with the extension of the CL model.

The starting point of the MM model is the $\mathbb{Z}_2$-graded algebra $\Omega^*_M(X)$ obtained from the skew tensor product of the matrix algebra $C_{3 \times 3}$ and the algebra $\Lambda^*(X)$ of differential forms over the space–time $X$. The matrix algebra $\Omega^*_M(X) = C_{3 \times 3} \hat{\otimes} \Lambda^*(X)$ is taken $\mathbb{Z}_2$-graded with $\Gamma = diag(1, 1, -1)$ as the grading automorphism.

The matrix multiplication and generalized differential $d'$ is, with an obvious change of notation and with $a$ instead of $[a]$, given formally as in eq. (5) and eq. (6) respectively. The matrix derivative $d_M$ in $C_{3 \times 3}$ is defined by its action on the even and odd part of $a_0$ and $a_1$ respectively [8], with $\eta$ given as in eq. (8):

$$d_M(a) = [\eta, a_0] + i\{\eta, a_1\}$$

The structure of the algebra may be summarized by

$$(\Omega^*_M, \bullet, d')$$

It should be understood that in the MM model no quotient space is present so that the $(\Omega^*_M)$ multiplication $\bullet$ and differential $d'$ are induced straightforwardly from the tensor structure and of course are much simpler than in the CL model. It is important to realize that $(\Omega^*_M, \bullet, d')$ is not a differential algebra since $d'$ is only a derivation and not a differential. It is interesting, however, to note that $\Omega^1_M = \Omega^1_D$ is valid. So we may start with the same gauge potential $A$ as in eq. (9) in both cases. For the field strength we have in the MM model

$$F_M = d'A + A \bullet A$$
The Lagrangian for the bosonic part is given by 

\[ L_{MM} = L_{CL} - V^* \quad \text{with} \quad V^* = 2(\overline{\Phi}C + \overline{C}\Phi + \overline{\Phi})^2. \]

In the MM model, we obtain a non trivial Higgs potential even with one generation of fermions in accordance with the standard model. This is an important difference with the CL model.

4. With the above preparation we are in the position to formulate our new model in a precise and, as we hope, efficient way. The most important difference between the CL and the MM model is the importance of the basic algebra \( \mathcal{A} \). In the CL model, the entire construction relies on the algebra \( \mathcal{A} \). In addition, the algebra \( \mathcal{A} \) has also direct phenomenological implications. Not only the determination of the relevant structure group \( SU(2) \times U(1) \) but also the determination of the relevant fermion representations and in particular the fermion quantum numbers are derived from the associative algebra \( \mathcal{A} \). In the CL model one uses the particular differential algebra \( \Omega^*(\mathcal{A}) \) since one started with \( \mathcal{A} = \mathcal{A}_M \otimes C^\infty(X) \). So it is this particular \( \mathcal{A} \) which fixes the gauge potential \( A \) as an element in \( \Omega^1(I_\mathcal{A}) \). In addition it is this \( \mathcal{A} \) which determines completely the fermionic part of the Lagrangian including all phenomenological implications.

In the MM model, the starting point is a derivative algebra \( \Omega^* \) which generalizes the algebra \( \Lambda^*(X) \). The gauge potential \( A \) is an element of \( \Omega^1 \). We started with \( C_{3\times3} \) for the construction of \( \Omega^*_M \) only because the representation space of the Lie algebra of \( SU(2|1) \) is a \( C^3 \). Here it is the super Lie algebra \( SU(2|1) \) only which leads to the phenomenological consequences. The result is that the MM model, as is well known, even if it has the nice feature, among others, to explain e.g. spontaneous symmetry breaking and the Higgs effect in a geometrical way, has less predictive power in the fermionic sector and gives exactly the standard model. The CL model, because of the fundamental role played by the associative algebra \( \mathcal{A} \), seems to have more predictive power.

At this point, some comments are appropriate. The fundamental role played by an associative algebra is a new aspect of the phenomenology in elementary particle physics. Usually Lie algebras are used for phenomenological implications because they correspond to infinitesimal symmetry transformations. Associative algebras are used mathematically to define the corresponding Lie algebra by the use of commutators. Furthermore, it is well known that in current algebra, the commutator product which determines the Lie algebra is well defined whereas the associative product itself is not well defined (it may be infinite). Therefore it should be stated that the use of an associative algebra for the phenomenology is not at all a priori well justified from the physical point of view. It is an essential ingredient and an important and basic aspect of the CL model which distinguishes this model from all other models within the framework of noncommutative geometry in elementary particle physics.

\(^2\) This aspect is common also to other models within the noncommutative geometry approach.
5. If we relax this fundamental role of the associative algebra $\mathcal{A}$, we obtain a new class of models $\{N\}$ which rely essentially only on the differential algebra $\{\Omega^*_D\}$. We use $\mathcal{A}$ only as an instrument for the mathematical construction of the new differential algebra $\Omega^*_N$ and for nothing more. We use $\Omega^*_N$ to determine the gauge potential $A_N$ and $(\Omega^*_N, \varnothing_N, d_N)$ to derive the field strength $F_N$:

$$F_N = d_N A_N + A_N \varnothing_N A_N .$$

The bosonic part of the Lagrangian is obtained by taking $tr(F^+_N F_N)$. No specific predictions are made for the fermionic part.

A natural extension of the CL model is obtained by taking a new differential algebra

$$( \Omega^*_N, \varnothing_N, d_N )$$

which generalizes the expression eq. (4) for $\Omega^*_D$:

$$\Omega^*_N = N^* \hat{\otimes} \Lambda^*(X)$$

$\Omega^*_N$ is a skew tensor product of a differential matrix algebra $(N^*, d_N)$ and the differential forms $\Lambda^*(X)$. It is formally fixed by the analogous expression which follows eq. (4). Index $D$ is replaced by index $N$ and $\mu^*(\mathcal{A})$ by $N^*$. Since no specific restriction is made for $N^*$ the algebra $\Omega^*_D$ in the CL model is a special case of the differential algebra $\Omega^*_N$.

We choose now another special case of $\Omega^*_N$ which is very near to the spirit of the CL model. This will give the concrete new model we would like to discuss. We denote this special differential algebra by $\Omega^*_P$ and we have in an obvious notation, following eq. (3), eq. (4):

$$( \Omega^*_P, \varnothing_P, d_P ) \quad \text{with} \quad \Omega^*_P = \mathcal{M}^* \hat{\otimes} \Lambda^*(X) .$$

The matrix algebra $\mathcal{M}^*$ depends on $\mathcal{A}_M$, eq. (8), and $D_M$, eq. (8), and is given by

$$\mathcal{M}^k = \Omega^k_{D_M}(\mathcal{A}_M) = \pi(\Omega^k(\mathcal{A}_M))/\pi(\mathcal{J}^k(\mathcal{A}_M)) .$$

We used the notation of eq. (2). It is obvious that in our model the division in $\pi(\Omega^*)$ does not depend on the $C^\infty(X)$ part of $\mathcal{A}$. The division concerns only the matrix space $\mathcal{A}_M$ we started with. We also choose here $\mathcal{A}_M = \mathbb{H} \oplus \mathbb{C}$ in order to obtain the right structure group $SU(2) \times U(1)$. This is the only reason for that choice and we do not fix anything else in the fermionic sector.

Our next step is to determine the space $\Omega^*_P$. For that purpose we proceed in a slightly different way than in the CL model (for the $\Omega^*_P$) and we first determine the space $\mathcal{M}^*$. The construction of $\Omega^*_P$ is obtained in a straightforward manner by the skew tensor product of $\mathcal{M}^*$ and $\Lambda^*$.
Using $D_m$ as in eq. (4), and $C$ as in eq. (5), we obtain
\[
\pi(\Omega^{2k}(A_m)) = \begin{pmatrix} 0 & 0 \\ C & 0 \end{pmatrix} \quad \text{for} \quad k \geq 1, \quad \pi(\Omega^{2k+1}(A_m)) = \begin{pmatrix} 0 & CC \\ CC & 0 \end{pmatrix} \quad \text{for} \quad k \geq 0
\]
and
\[
\pi(\mathcal{J}^0(A_m)) = \{0\}, \quad \pi(\mathcal{J}^1(A_m)) = \{0\}, \quad \pi(\mathcal{J}^2(A_m)) = \begin{pmatrix} iH & 0 \\ 0 & 0 \end{pmatrix},
\]
\[
\pi(\mathcal{J}^k(A_m)) = \pi(\Omega^k(A_m)) \quad \text{for} \quad k \geq 3.
\]
So we obtain from eq. (11)
\[
\mathcal{M}^0 = \begin{pmatrix} iH & 0 \\ 0 & C \end{pmatrix}, \quad \mathcal{M}^1 = \begin{pmatrix} 0 & CC \\ CC & 0 \end{pmatrix}, \quad \mathcal{M}^2 = \begin{pmatrix} iH & 0 \\ 0 & C \end{pmatrix}, \quad \mathcal{M}^k = \{0\} \quad \text{for} \quad k \geq 3.
\]
The multiplication $\cdot_{\mathcal{M}}$ and the differential $d_{\mathcal{M}}$ are given canonically by the quotient of
\[
\pi(\Omega^*(A_m)) \quad \text{by} \quad \pi(\mathcal{J}^*(A_m))
\]
and we have in an obvious notation:
\[
[a] \cdot_{\mathcal{M}} [b] := [ab] \quad \text{and} \quad d_{\mathcal{M}}[a] = [D_M, a].
\]
The space $(\Omega^*_p, \odot_p, d_p)$ is given by
\[
\Omega^j_p = (\mathcal{M}^0 \odot \Lambda^j) \odot (\mathcal{M}^1 \odot \Lambda^{j-1}) \odot (\mathcal{M}^2 \odot \Lambda^{j-2}).
\]
So we have explicitly for $j = 0, 1, 2$:
\[
\Omega^0_p = A, \quad \Omega^1_p = (\mathcal{M}^0 \odot \Lambda^1) \odot (\mathcal{M}^1 \odot \Lambda^0), \quad \Omega^2_p = (\mathcal{M}^0 \odot \Lambda^2) \odot (\mathcal{M}^1 \odot \Lambda^1) \odot (\mathcal{M}^2 \odot \Lambda^0).
\]
It is interesting to note that $\Omega^0_p = \Omega^0_D$ and $\Omega^1_p = \Omega^1_D$. This allows to start with the same super–potential $A$ as in the CL and MM model (see eq. (9)). The multiplication rule $\odot_p$ and the differential $d_p$ are now different. The field strength in this model reads:
\[
F_p = d_p A + A \odot_p A.
\]
Using the results of eq. (12) we easily obtain in the notation of eq. (10)
\[
F_p = i \left( \begin{array}{cc} F^A - (\Phi \overline{C} + \overline{C} \Phi + \Phi \overline{\Phi}) & -D \Phi \\ -D \overline{\Phi} & F^B - (\overline{\Phi} \overline{C} + \overline{C} \Phi + \Phi \overline{\Phi}) \end{array} \right).
\]
This leads to the Lagrangian
\[
L_p = L_{CL} - V^* \quad \text{with} \quad V^* = 2(\overline{\Phi} \overline{C} + \overline{C} \Phi + \Phi \overline{\Phi})^2.
\]
It is obvious that this Lagrangian, in contrast to the $L_{CL}$, leads to spontaneous symmetry breaking even with one generation of fermions. It is also important to realize that
at this level, $L_P$ is similar to the one in the MM model, although there the structure group was the group $SU(2|1)$.

6. We may now summarize our results. We have constructed a new model within the spirit of the Connes–Lott approach which gives precisely the standard model in elementary particle physics. It is directly formulated in Minkowski space–time. It contains none of the unusual aspects of the Connes–Lott model as e.g. the fixing of the gauge group and consequently the problem of the fixing of fermionic quantum numbers, the non–existence of spontaneous symmetry breaking in the case of one generation of fermions and perhaps the presence of some unusual couplings.

Although formulated in the framework of a differential algebra ($d_Pd_P = 0$), it is interesting to note that the bosonic sector of the proposed model is equivalent to the Marseille–Mainz model, which is based on an algebra with derivative only ($d'd' \neq 0$).

We thank R. Häußling, W. Kalau, T. Schücker, M. Walze, J.M. Warzecha for discussions and F. Scheck for reading the manuscript and discussions.

References

[1] A. Connes and J. Lott, Nucl. Physics. B (Proc.Suppl.) 18 B (1990) 29.
[2] D. Kastler, preprint Marseille–Luminy CPT-91/P.2610.
   D. Kastler, preprint Marseille–Luminy CPT-92/P.2814.
   D. Kastler, preprint Marseille–Luminy CPT-92/P.2824.
[3] E. Álvarez, J. M. Gracia–Bondía and C. P. Martín, Phys. Lett. B 323 (1994) 259.
   B. Iochum and T. Schücker, preprints Marseille–Luminy CPT–94/p.3090
   D. Kastler and T. Schücker, Theor. Math. Phys., 92 (1993) 522
   T. Schücker and J.M. Zylinski , J.Geom. Phys., 16 (1994) 1
[4] R. Coquereaux, G. Esposito–Farèse and G. Vaillant, Nucl. Phys. B 353 (1991) 689–706.
   R. Coquereaux, G. Esposito–Farèse and F. Scheck, Intern. J. Mod. Phys. A 7 (1992) 6555–6593.
[5] R. Häußling, N. A. Papadopoulos and F. Scheck, Phys. Lett. B 260 (1991) 125.
[6] R. Häußling, N. A. Papadopoulos and F. Scheck, Phys. Lett. B 303 (1993) 265.
[7] N. A. Papadopoulos, J. Plass and F. Scheck, Phys. Lett. B 324 (1994) 380.
[8] R. Häußling, F. Scheck, Phys. Lett. B 336 (1994) 477.
[9] M. Dubois–Violette, R. Kerner and J. Madore, Phys. Lett. 217B (1989) 485.
   J. Madore, An introduction to Noncommutative Differential Geometry and its Physical applications, Cambridge University Press (1995).
[10] J.M. Gracia–Bondía, preprint, hep–th/9502120, to appear in Phys. Lett. B.
[11] W. Kalau, N. A. Papadopoulos, J. Plass, J.–M. Warzecha, J. Geom. Phys. 16 (1995) 149-167.

[12] A. Connes, Non commutative geometry, Academic Press, Inc. 1994.
    A. Connes, Les Houches Lectures (1992).

[13] A. H. Chamseddine, G. Felder and J. Fröhlich, Phys. Lett. B 296 (1992) 109-116.
[14] J.–M. Warzecha, Diplomarbeit, Mainz 1994
[15] J. Plass, Diplomarbeit, Mainz 1994.