NEW AND RECENT TRENDS IN MODERN COSMOLOGY

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Abstract

Non Conventional treatments in modern cosmology, in both Steady State and Big Bang, are given. The motivation behind these treatments is to solve some of the problems of the conventional treatments in cosmology. For this aim, different geometric structures and alternative field theories, used to construct world models, are given. A brief review of Absolute parallelism (AP) geometry and its parameterized version (PAP), as a wider geometry than the Riemannian one, is presented. World models constructed using alternative field theories, constructed in the AP geometry, are discussed and compared. Some points about using topology in the field of cosmology are commented. A new path equation, admitted by the PAP geometry, is used to get the effect of spin-gravity interaction on the cosmological parameters.

1 INTRODUCTION

Cosmology is that branch of science which deals with the Universe as one system. Modern Cosmology started in 1917 when A. Einstein built his world model, in an attempt to understand the large scale structure of the Universe (cf. [1]). This model has been built using his theory of gravity, the "General Theory of Relativity" (GR), which has been constructed using Riemannian Geometry (RG). The first problem faced by this model was that it is not static, while it was generally believed that the Universe is static. The model predicted that the Universe is expanding while observations, of that time, did not support this prediction. Einstein was enforced to modify the field equations of GR by adding a term, called the "Cosmological term", to these equations, in order to stop expansion and to get a static model. When A. Friedmann used the modified equations of GR in 1922, he got again an expanding world model. In 1929 E. Hubble confirmed, by studying the red-shift of distant galaxies, the prediction of GR that the Universe is expanding. Afterwards, Einstein rejected this term as it does not stop expansion.

After Hubble’s discovery, many researches started to build models for the Universe, in the context GR, investigating the consequences of different assumptions about the distribution of matter in the Universe. At that time it was generally agreed, on observational
bases, that the spatial distribution of matter in the universe is isotropic and homogeneous. This fact represents one of the basic assumptions of modern cosmology, the "Cosmological Principle".

Geometers quickly entered the playground and constructed geometric structures that satisfy the cosmological principle [2], [3]. They assumed that a Riemannian space whose metric is given by,

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \]

(1)

where \( g_{\mu\nu} \) is the metric tensor and \( x^\mu \) is the coordinate system used, should satisfy certain conditions (the Killing equations), which are relations between the components of the metric tensor and the generators of certain groups. The solutions of the Killing equations in this case give rise to the well-known Friedmann-Robertson-Walker (FRW) metric,

\[ ds^2 = dt^2 - \frac{R^2(t)}{(1 + \frac{1}{4}kr^2)^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) , \]

(2)

where \( R(t) \) is a function of time, called the "Scale Factor", to be obtained from the solution of the field equations of GR, and \( k(= +1, 0, -1) \) is the curvature constant. The Riemannian structure given by (2) is the basis of any world model assuming the validity of the Cosmological Principle. It is to be considered that GR alone (i.e. without observations) cannot fix a value for the constant \( k \).

1.1 Big Bang Cosmology

Einstein’s Field equations of GR can be written in the form,

\[ G_{\mu\nu} = -\kappa T_{\mu\nu} \]

(3)

where \( G_{\mu\nu} \) is Einstein tensor, \( T_{\mu\nu} \) is the material-energy tensor and \( \kappa \) is Einstein’s constant. Cosmology built in the context of GR, using the FRW-space time (2) and the field equations (3), comprises a class known as "Big Bang Cosmology". This is because all the solutions of (3) are singular at \( t = 0 \), which gives rise to the term Big Bang. The tensor \( T_{\mu\nu} \) is a phenomenological object, and not a part of the geometric structure. Its components are chosen to satisfy the Cosmological Principle. The simplest choice for this tensor is that of a perfect fluid.

The Big Bang scenario has predicted that the Universe has passed through a very hot and dense phase when it was very young. As it expands it cools down until it reaches an estimated temperature of 3\(^9\)K at present. Also, this scenario predicted that about 25\% of the matter contents of the Universe are made of Helium and it was formed in the first few minutes from the Big Bang moment, when suitable conditions for this formation were set up. These theoretical results have been obtained by G.Gamow and his collaborators in the mid 1940’s.

1.2 Steady State Cosmology

Cosmological observations of 1940’s show that, although the Universe is expanding, the cosmic density of the material distribution remains constant! Also, cosmologists using a
value of Hubble’s parameter $H(130 \text{ Kms/sec/Mpc})$ found that the age of the Universe is less than the age of the Earth! This represented a puzzle for cosmologists of that time. To solve this puzzle, H. Bondi and T. Gold in 1948 [4] suggested a different scenario for the Universe, the "Steady State Scenario". In the framework of this scenario, matter in the Universe is continuously created as the Universe expands. This violates laws of conservation on which GR was originally constructed. However, they rejected GR and modified the Cosmological Principle to imply that homogeneity and isotropy are manifested in space and time. This modification is known as the "Perfect Cosmological Principle". They have built their world model without using GR or any alternative gravity theory, as follows. In the context of the Perfect Cosmological Principle, the curvature of space should be constant and since it is proportional to the quantity $\left(\frac{k}{R(t)}\right)$ they took $k = 0$ to switch off the time evolution of this term. From Hubble’s diagram (the red shift-apparent magnitude relation), they concluded that the rate of expansion of the Universe is constant, i.e. $\frac{\dot{R}}{R} = H$ which has the solution,

$$R(t) = e^{Ht}, \quad (4)$$

where $\dot{R}$ is the time derivative of $R$. This solution solved the singularity and the age problems appeared in the Big Bang cosmology. The metric of the space (2) will now be reduced to,

$$ds^2 = dt^2 - e^{2Ht}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \quad (5)$$

In the same year, F. Hoyle [5] modified GR field equations in an attempt account for continuous creation of matter. He dropped conservation by adding a term $(V_{\mu\nu})$, with non-vanishing vectorial divergence, to Einstein’s field equations (3) which he wrote in the modified form:

$$G_{\mu\nu} + V_{\mu\nu} = -\kappa T_{\mu\nu}. \quad (6)$$

$V_{\mu\nu}$ is a second order symmetric tensor called the "Creation Tensor". In 1949 Hoyle constructed a Steady State model[6], as a solution of the modified equations (6), assuming that the creation tensor is derived from a vector field, called the "Creation Field", $V_{\mu}$ by the relation,

$$V_{\mu\nu} \overset{\text{def}}{=} \frac{1}{2}(V_{\mu} ; \nu + V_{\nu} ; \mu). \quad (7)$$

In 1960, Hoyle constructed another Steady State model assuming that the creation tensor is derived from a scalar field which is a function of time only. Hoyle and Narlikar constructed a steady state model[7] as a solution of a new set of field equations derived from an action principle, and studied its properties [8].

Each of the two rival scenarios, given above, has its own problems. For example, the Steady State theory was ruled out in the mid sixties since it has some problems with observations of the cosmic microwave background radiation (CMBR) and the abundance of light elements. On the other hand, standard Big Bang cosmology has its own problems e.g. singularity, horizon, flatness ..... Among the history of these rival theories, scientists tried to overcome these problems by suggesting different solutions, which may be one or more of the following:

(1) By using other alternative theories of gravity, different from GR.
By changing the basic geometry used to construct the theory.

By relaxing one or more of the basic assumptions, used to construct world models.

It is the aim of the present work to give a brief review of alternative, less famous, treatments in theoretical cosmology, which are suggested to overcome one or more of the problems encountered by standard cosmology. In the following sections, we are going to review briefly the consequences of following one or more of the above suggestions. In Section 2, a brief review of the "Absolute Parallelism" (AP)-geometry, as an alternative to Riemannian geometry, is given. In Section 3, two geometric AP-structures, usually used in cosmological applications, are given. In Section 4, we review briefly some field theories, built in AP-geometry, and give some results of their applications in the field of cosmology. Some comments on the possibility of using topology, in place of geometry, are given in Section 5. The work is discussed in Section 6.

2 THE ABSOLUTE PARALLELISM GEOMETRY

This type of geometry, the AP-geometry, was first used by Einstein (from 1928 to 1932) in an attempt to unify gravity with electromagnetism. As it is show, in the following brief review, this type of geometry is more wider than the Riemannian one. Recently, a new version of this geometry, the parameterized Absolute Parallelism (PAP), in which both curvature and torsion are simultaneously non-vanishing, is suggested. In the following sub-sections, we are going to give a brief review on the AP and PAP geometry. For more details the reader is referred to references [9], [10], [11], [12], [13].

2.1 The Conventional AP-Version

The AP-space \((M, \lambda(x))\) is an \(n\)-dimensional space, each point of which is labeled by \(n\)-variables \(x^\mu\). Its structure is defined completely by \(n\) linearly independent contravariant vectors \(\lambda_i^\mu(i = 1, 2, 3, ..., n)\), denotes the vector number and \(\mu = 1, 2, 3, ..., n\) denotes the coordinate component) defined at each point of the manifold \(M\) and are subject to the condition,

\[
\lambda_i^\mu \frac{\partial}{\partial x^\nu} = 0,
\]

where the stroke denotes absolute differentiation to be defined later. Equation (8) is the condition for the absolute parallelism. The covariant components of \(\lambda_i^\mu\) are defined such that,

\[
\lambda_i^\mu \lambda_i^\nu = \delta_\nu^\mu,
\]

and

\[
\lambda_i^\nu \lambda_j^\nu = \delta_{ij}.
\]

Using these vectors, the following second order symmetric tensors are defined:

\[
g^{\mu\nu} \overset{\text{def}}{=} \lambda_i^\mu \lambda_i^\nu,
\]

\[
g_{\mu\nu} \overset{\text{def}}{=} \lambda_i^\mu \lambda_i^\nu.
\]
consequently,
\[ g^{\mu \alpha} g_{\nu \alpha} = \delta^\mu_\nu. \] (13)

These second order tensors can serve as metric tensors of Riemannian space, associated with the AP-space, when needed. This type of geometry admits, at least, four affine connections. The first is a non-symmetric connection given as a direct solution of the AP-condition (8), i.e.
\[ \Gamma^\alpha_{\mu \nu} \overset{\text{def}}{=} \lambda_i^\alpha \lambda_{i \mu \nu}, \] (14)
where the comma denotes ordinary partial differentiation. The second is its dual \( \tilde{\Gamma}^\alpha_{\mu \nu} \) (\( \overset{=}{=} \Gamma^\alpha_{\nu \mu} \)), since (14) is non-symmetric. The third one is the symmetric part of (14), \( \Gamma^{\alpha}_{(\mu \nu)} \).

The fourth is Christoffel symbol defined using (11),(12) (as a consequence of a metricity condition). The torsion tensor is twice the skew symmetric part of the affine connection (14), i.e. [9]
\[ \Lambda^\alpha_{\mu \nu} \overset{\text{def}}{=} \Gamma^\alpha_{\mu \nu} - \Gamma^\alpha_{\nu \mu}. \] (15)

Another third order tensor (contortion) is defined by the expression,
\[ \gamma^\alpha_{\mu \nu} \overset{\text{def}}{=} \lambda_i^\alpha \lambda_{i \mu \nu}, \] (16)
the semicolon is used for covariant differentiation using Christoffel symbol. The two tensors are related by,
\[ \gamma^\alpha_{\mu \nu} = \frac{1}{2} (\Lambda^\alpha_{\mu \nu} - \Lambda^\alpha_{\nu \mu} - \Lambda^\alpha_{\mu \nu}). \] (17)

A basic vector could be obtained by contraction of the above third order tensors,
\[ C_\mu \overset{\text{def}}{=} \Lambda^\alpha_{\mu \alpha} = \gamma^\alpha_{\mu \alpha}. \] (18)

One of the advantages of AP-geometry (for more details see [11]) is that for any world tensor \( T^\alpha_{\beta \gamma} \) defined in the AP-space, one can construct a set of scalars \( T_{(ijk)} \),
\[ T_{(ijk)} \overset{\text{def}}{=} \lambda_i^\alpha \lambda_j^\beta \lambda_k^\gamma T^\alpha_{\beta \gamma}. \] (19)

If \( T^\alpha_{\beta \gamma} \) is the contortion (16) then the corresponding scalars are those known in the literature as Ricci coefficients of rotation [9].

The curvature tensor is defined by,
\[ B^\alpha_{\mu \nu \sigma} \overset{\text{def}}{=} \Gamma^\alpha_{\mu \nu, \sigma} - \Gamma^\alpha_{\mu \sigma, \nu} + \Gamma^\alpha_{\nu \sigma} \Gamma^\epsilon_{\mu \nu} - \Gamma^\alpha_{\epsilon \sigma} \Gamma^\epsilon_{\mu \nu} \equiv 0. \] (20)

This tensor vanishes identically because of (8). From the above tensors, the following second order tensors could be defined in Table 1 [10], [14].
Table 1: Second Order World Tensors [10], [14]

| Skew-Symmetric Tensors | Symmetric Tensors |
|------------------------|-------------------|
| $\xi_{\mu \nu} \overset{\text{def}}{=} \gamma^{\alpha}_{\mu \nu |+} \gamma_{\alpha |}$ | $
abla_{\alpha \beta} \overset{\text{def}}{=} \Delta_{\mu \nu}^{\alpha}$ |
| $\zeta_{\mu \nu} \overset{\text{def}}{=} C_{\alpha} \gamma_{\mu \nu |}$ | $\phi_{\mu \nu} \overset{\text{def}}{=} C_{\alpha} \Delta_{\mu \nu}^{\alpha}$ |
| $\eta_{\mu \nu} \overset{\text{def}}{=} C_{\alpha} \Lambda_{\mu \nu}^{\alpha}$ | $\psi_{\mu \nu} \overset{\text{def}}{=} \Delta_{\mu \nu |}$ |
| $\chi_{\mu \nu} \overset{\text{def}}{=} \Lambda^{\alpha}_{\mu \nu |}$ | $\theta_{\mu \nu} \overset{\text{def}}{=} C_{\mu |}^{\nu} + C_{\nu |}^{\mu}$ |
| $\varepsilon_{\mu \nu} \overset{\text{def}}{=} C_{\mu |}^{\nu} - C_{\nu |}^{\mu}$ | $\omega_{\mu \nu} \overset{\text{def}}{=} \gamma_{\mu \nu |}^{\alpha \beta} \gamma_{\alpha \beta |}$ |
| $\kappa_{\mu \nu} \overset{\text{def}}{=} \gamma_{\mu \nu |}^{\alpha \beta} \gamma_{\alpha \beta |}$ | $\sigma_{\mu \nu} \overset{\text{def}}{=} \gamma_{\mu \nu |}^{\alpha \beta} \gamma_{\alpha \beta |}$ |
| $\chi_{\mu \nu} \overset{\text{def}}{=} \gamma_{\mu \nu |}^{\alpha \beta} \gamma_{\alpha \beta |}$ | $\alpha_{\mu \nu} \overset{\text{def}}{=} C_{\mu \nu}$ |
| $\omega_{\mu \nu} \overset{\text{def}}{=} \gamma_{\mu \nu |}^{\alpha \beta} \gamma_{\alpha \beta |}$ | $R_{\mu \nu} \overset{\text{def}}{=} \frac{1}{2} (\psi_{\mu \nu} - \phi_{\mu \nu} - \theta_{\mu \nu}) + \omega_{\mu \nu}$ |

The autoparallel path equation can be written in the form,

$$\frac{d^2 x^\mu}{dp^2} + \Gamma^\mu_{\alpha \beta} \frac{dx^\alpha}{dp} \frac{dx^\beta}{dp} = 0.$$  \hspace{1cm} (21)

where $p$ is an evolution parameter.

Using the above mentioned affine connections, one can define the following absolute derivatives [11]:

$$A^\mu_{\mu |} = A^\mu_{\mu |} + A^\alpha \Gamma^\mu_{\alpha \nu},$$  \hspace{1cm} (22)

$$A^\mu_{\mu |} = A^\mu_{\mu |} + A^\alpha \Gamma^\mu_{\alpha \nu},$$  \hspace{1cm} (23)

$$A^\mu_{\mu |} = A^\mu_{\mu |} + A^\alpha \Gamma^\mu_{\alpha \nu},$$  \hspace{1cm} (24)

$$A^\mu_{\mu |} = A^\mu_{\mu |} + A^\alpha \left\{ \mu \right\}_{\alpha \nu},$$  \hspace{1cm} (25)

where $A^\mu$ is any arbitrary contravariant vector. Using these derivatives, one can define the following curvature tensors [15], as consequence of non commutation of these absolute derivatives,

$$\lambda^\mu_{\mu | \sigma} - \lambda^\mu_{\mu | \sigma} \overset{\text{def}}{=} \lambda^\alpha D^\mu_{\alpha | \nu},$$  \hspace{1cm} (26)

$$\lambda^\mu_{\mu | \sigma} - \lambda^\mu_{\mu | \sigma} \overset{\text{def}}{=} \lambda^\alpha L^\mu_{\alpha | \nu},$$  \hspace{1cm} (27)
\[
\lambda^\mu_{\nu\sigma} - \lambda^\mu_{\sigma\nu} \overset{\text{def}}{=} \lambda^\alpha_{\alpha\nu\sigma},
\]
(28)

\[
\lambda^\mu_{\nu;\sigma} - \lambda^\mu_{\sigma;\nu} \overset{\text{def}}{=} \lambda^\alpha_{\alpha\nu\sigma}.
\]
(29)

Table 2 gives a brief comparison between the AP and the Riemannian geometries. It shows how wide is the AP-geometry compared to the Riemannian one.

### Table 2: Comparison Between The Riemannian Geometry and AP-Geometry

| Object                      | Riemannian geometry | AP-geometry |
|-----------------------------|---------------------|-------------|
| Building Blocks             | \( g_{\mu\nu} \)    | \( \lambda^\mu_{\nu} \), \( \lambda^\alpha_{\alpha\nu\sigma} \) |
| Affine Connection           | \( \{ \alpha_{\mu\nu} \} \) | \( \{ \alpha_{\mu\nu} \}, \Gamma^\alpha_{\mu\nu}, \tilde{\Gamma}^\alpha_{\mu\nu}, \Gamma^\alpha_{\mu\nu} \) |
| Second Order Symmetric Tensors | two \( (g_{\mu\nu}, R_{\mu\nu}) \) | many, Table 1 |
| Second Order Skew Tensors  | \(--\)              | many, Table 1 |
| Third Order Tensor          | \(--\)              | \( \gamma^\alpha_{\mu\nu}, \Lambda^\alpha_{\mu\nu} \) |
| Vectors                     | \(--\)              | \( C_{\mu} \) |
| Scalars                     | \( R \)             | Many        |
| Curvature                   | \( R^\alpha_{\beta\gamma\delta} \neq 0 \) | \( B^\alpha_{\beta\gamma\delta} \equiv 0 \) |
|                            |                     | \( L^\alpha_{\beta\gamma\delta} \neq 0 \) |
|                            |                     | \( N^\alpha_{\beta\gamma\delta} \neq 0 \) |
|                            |                     | \( R^\alpha_{\beta\gamma\delta} \neq 0 \) |
Because of (20) many authors believe that the AP-space is a flat one. This is not the case, since (27), (28) and (29) are non-vanishing. For more details about this problem the reader is referred to [12].

2.2 The Non-Conventional PAP-Version

There are at least two convincing physical reasons for parameterizing the AP-geometry [11]. To clarify the first, let us examine the structure of the curvature tensor given by (20). As stated before, this tensor vanishes identically because of the AP-condition (8). This tensor can be written in the form,

\[
B^\alpha_{\mu\nu\sigma} \overset{\text{def}}{=} R^\alpha_{\mu\nu\sigma} + Q^\alpha_{\mu\nu\sigma},
\]

where \( R^\alpha_{\mu\nu\sigma} \) is the Riemann-Christoffel curvature tensor, of the associated Riemannian space, given by,

\[
R^\alpha_{\mu\nu\sigma} \overset{\text{def}}{=} \left\{ \frac{\alpha}{\mu\sigma} \right\}_{\nu} - \left\{ \frac{\alpha}{\mu\nu} \right\}_{\sigma} + \left\{ \frac{\beta}{\mu\sigma} \right\}_{\nu} - \left\{ \frac{\beta}{\mu\nu} \right\}_{\sigma},
\]

and

\[
Q^\alpha_{\mu\nu\sigma} \overset{\text{def}}{=} \gamma^{\alpha}_{\mu\nu\sigma} - \gamma^{\alpha}_{\mu\nu\sigma} + \gamma^\beta_{\mu\sigma} \gamma^\alpha_{\beta\nu} - \gamma^\beta_{\mu\nu} \gamma^\alpha_{\beta\sigma},
\]

It is clear from (31) that \( R^\alpha_{\mu\nu\sigma} \) is made of Christoffel symbols only, while from (32) we can see that \( Q^\alpha_{\mu\nu\sigma} \) is made of the contortion (or the torsion via (17)) only. Some authors believe that \( R^\alpha_{\mu\nu\sigma} \) and \( Q^\alpha_{\mu\nu\sigma} \) are equivalent. Others consider \( Q^\alpha_{\mu\nu\sigma} \) as giving an alternative definition of \( R^\alpha_{\mu\nu\sigma} \). Let us examine these two tensors from a different point of view. It is well known that Christoffel symbol is related, in applications, to the gravitational field. So, its existence in (31) indicates that gravity is responsible for the curvature of space-time. In our point of view [11], the identical vanishing of the curvature \( B^\alpha_{\mu\nu\sigma} \) may indicate that there is another physical interaction (anti-gravity, say) which is related to the contortion (or the torsion) and is represented by the tensor \( Q^\alpha_{\mu\nu\sigma} \). This interaction balances the effect of gravity in such a way that the total effect vanishes. If so, it is better to call the tensor \( Q^\alpha_{\mu\nu\sigma} \) The Curvature Inverse of Riemann-Christoffel Tensor. But since gravity is dominant in our observable Universe, which means that \( R^\alpha_{\mu\nu\sigma} \) is more effective than \( Q^\alpha_{\mu\nu\sigma} \), thus one has to parameterize torsion terms in AP-expressions.

The second reason is that the AP-geometry admits paths [16] that are different from those of the Riemannian geometry. The new paths contain a torsion term, together with the Christoffel symbol term. These paths cannot be reduced to the geodesic one, unless the torsion vanishes. It has been shown that the vanishing of the torsion of the AP-space will reduce the space to a flat one [17]. So, what are the physical trajectories of particles that can be represented by these paths? Clearly there are no particles that move along the new paths. The reason is that the effect of the Christoffel symbol term, in these equations, is of the same order of magnitude as the effect of the torsion term. So, for these paths to represent physical trajectories, the torsion term in the path equations should be parameterized, in order to reduce its effect [11].
As it is shown the two reasons for which we parameterize the geometry are the vanishing of the curvature tensor (20) and the problem of the physical meaning of the set of path equations admitted by the AP-geometry. As it is clear, the common factors between these two reasons are the affine connections. So, it is necessary to start parameterizing these connections first.

**Parameterized Connection**: One way to parameterize the AP-geometry is to define a general affine connection by linearly combining the affine connections defined in the geometry. In doing so, we get after some manipulations [18]:

\[
\nabla^{\mu}_{,\alpha\beta} = a_1 \left\{ \frac{\mu}{\alpha\beta} \right\} + (a_2 - a_3) \Gamma^{\mu}_{,\alpha\beta} - (a_3 + a_4) \Lambda^{\mu}_{,\alpha\beta},
\]

where \(a_1, a_2, a_3\) and \(a_4\) are parameters. It can be easily shown that \(\nabla^{\mu}_{,\alpha\beta}\) transforms as an affine connection, under the group of general coordinate transformations provided that a metricity condition is imposed. It is clear that this parameterized connection is non-symmetric.

**Parameterized Absolute derivatives**: If we characterize absolute derivatives, using the connection (33), by a double stroke, then we can define the following derivatives:

\[
A^{\mu}_{+\parallel \nu} \overset{\text{def}}{=} A^{\mu}_{,\nu} + A^{\alpha} \nabla^{\mu}_{,\alpha\nu},
\]

\[
A^{\mu}_{-\parallel \nu} \overset{\text{def}}{=} A^{\mu}_{,\nu} + A^{\alpha} \nabla^{\mu}_{,\alpha},
\]

\[
A^{\mu}_{\parallel \nu} \overset{\text{def}}{=} A^{\mu}_{,\nu} + A^{\alpha} \nabla^{\mu}_{,(\alpha\nu)},
\]

where \(A^{\mu}\) is any arbitrary vector. The metricity, using the parameterized connection, is given by:

\[
g^{\mu}_{+\parallel \nu} = 0,
\]

which gives rise to the condition,

\[
a + b = 1,
\]

where \(a = a_1, b = a_2 + a_4, (a_3 = -a_4)\) are two parameters. In this case the general affine connection (38) can be written in the form:

\[
\nabla^{\alpha}_{,\mu\nu} = \left\{ \frac{\alpha}{\mu\nu} \right\} + b^{\alpha}_{\gamma,\mu\nu}.
\]

It is clear from this equation that we have parameterized the contortion (or equivalently the torsion) term in a general connection of the AP-geometry. Now we will explore the consequences of this parameterization.

**Parameterized Path Equation**: Using the parameterized connection (39) and following the same approach followed before in [16], we can get the following parameterized path equation admitted by the geometry [18],

\[
\frac{dZ^{\mu}}{d\tau} + \left\{ \frac{\mu}{\nu\sigma} \right\} Z^{\nu} Z^{\sigma} = -b \Lambda_{(\nu\sigma),\mu} Z^{\nu} Z^{\sigma},
\]
where \( Z^\mu \left( \equiv \frac{dx^\mu}{d\tau} \right) \) is the tangent to the path and \( \tau \) is the evolution parameter along it.

**Parameterized Curvature Tensors:** There are two methods for defining the curvature tensor. The first is by replacing Christoffel symbols, in the definition of Riemannian-Christoffel curvature tensor, by any affine connection. The second is by using the non-commutation properties of the absolute derivatives as done in Subsection 2.1. The two methods are equivalent in RG only. Using the first method, we can define the following curvature tensor,

\[
\hat{B}^\alpha_{\mu\nu\sigma} \equiv \nabla^\alpha_{\mu\nu\sigma} - \nabla^\alpha_{\mu\sigma\nu} + \nabla^\beta_{\mu\sigma} \nabla^\alpha_{\beta\nu} - \nabla^\beta_{\mu\nu} \nabla^\alpha_{\beta\sigma}.
\]  

Using the definition of \( \nabla^\beta_{\mu\nu} \) given by (39) then we can write,

\[
\hat{B}^\alpha_{\mu\nu\sigma} = R^\alpha_{\mu\nu\sigma} + b\hat{Q}^\alpha_{\mu\nu\sigma},
\]  

where

\[
\hat{Q}^\alpha_{\mu\nu\sigma} \equiv \gamma^\alpha_{\mu\nu,\sigma} - \gamma^\alpha_{\mu\sigma,\nu} + b(\gamma^\beta_{\mu\sigma} \gamma^\alpha_{\beta\nu} - \gamma^\beta_{\mu\nu} \gamma^\alpha_{\beta\sigma}).
\]  

It is clear that the tensor \( \hat{B}^\alpha_{\mu\nu\sigma} \) is a parameterized replacement of the tensor \( B^\alpha_{\mu\nu\sigma} \) given by (20). But here \( \hat{B}^\alpha_{\mu\nu\sigma} \) is, in general, non-vanishing.

Using the second method, for defining curvature tensors we get the following tensors,

\[
\lambda^\mu_{\mu\nu\sigma} = \hat{W}^\mu_{\mu\nu\sigma},
\]  

\[
\lambda^\mu_{\nu\sigma} = \hat{L}^\mu_{\nu\sigma},
\]  

\[
\lambda^\mu_{\nu\sigma} = \hat{N}^\mu_{\nu\sigma}.
\]  

Note that every tensor with a hat is the parameterized replacement of that without a hat. We can show that the tensors given by the second method are more general than those obtained using the first method. For example we can write,

\[
W^\alpha_{\mu\nu\sigma} = \hat{B}^\alpha_{\mu\nu\sigma} - b(b - 1)\gamma^\alpha_{\mu\beta}\Lambda^\beta_{\nu\sigma}.
\]  

An important results is that the PAP-geometry is more general than both RG and AP-geometry. It possesses curvature and torsion which are simultaneously non-vanishing. Furthermore, from the PAP geometry, we can get RG as a special case corresponding to \( b = 0 \) and we can get AP-geometry corresponding to \( b = 1 \).

### 3 AP-STRUCTURES FOR COSMOLOGICAL APPLICATIONS

In the context of gravity theories written in Riemannian geometry, certain geometric structures are needed to construct world models, e.g. FRW-structure (2). Similarly for field theories constructed in AP-geometry, one needs certain AP-structures, satisfying the
cosmological principle, in order to construct world models. Robertson [13] constructed two AP-structures for cosmological applications. The two structures satisfy the cosmological principle. The structure of an AP-space, of 4-dimensions, is given by a tetrad vector field. The following two tetrad vector fields give the complete structure of the two AP-spaces used for cosmological applications, which can be written in spherical polar coordinate [10], respectively as,

The 1st structure

\[
\lambda^\mu_i = \begin{pmatrix}
\sqrt{-1} L^+ \sin \theta \cos \phi & 0 & 0 & 0 \\
0 & (L^- \cos \theta \cos \phi - 4K^\frac{1}{2} r \sin \phi)/4rR & (L^- \sin \phi + 4K^\frac{1}{2} r \cos \phi \cos \phi)/4rR & (L^- \cos \phi - 4K^\frac{1}{2} r \cos \phi \sin \phi)/4rR \\
0 & 0 & (L^- \cos \phi \sin \phi - 4K^\frac{1}{2} r \cos \phi)/4rR & (L^- \cos \phi - 4K^\frac{1}{2} r \cos \phi \sin \phi)/4rR \\
0 & 0 & 0 & (L^+ \cos \phi)/(4rR)
\end{pmatrix}.
\] (48)

The 2nd structure

\[
\lambda^\mu_i = \begin{pmatrix}
\sqrt{-1} L^+ & -K^\frac{1}{2} r r & 0 & 0 \\
0 & (4K^\frac{1}{2} r \sin \theta \cos \phi)/L^+ & L^+ \sin \theta \cos \phi & -L^+ \sin \theta \cos \phi/4RrR \sin \theta \\
0 & 0 & (L^+ \cos \theta \sin \phi)/4R & (L^+ \sin \phi \cos \phi + 4K^\frac{1}{2} r \cos \phi \cos \phi)/4rR \sin \theta \\
0 & 0 & 0 & (L^+ \cos \phi)/(4rR)
\end{pmatrix}.
\] (49)

where \( L^\pm = 4 \pm kr^2 \), and \( R(t) \) is an unknown function of \( t \). It is to be considered that the Riemannian space associated with each one of the structures (48) and (49) is that given by the FRW-metric (2).

4 COSMOLOGICAL APPLICATIONS OF ALTERNATIVE THEORIES

In the present section, we are going to review briefly some alternative theories, different from GR, and world models resulting from the solution of their field equations. The general feature of these field theories is that all of them are constructed in spaces with absolute parallelism. Some of these theories were constructed to overcome one or more of the problems appeared in the applications of GR, especially in the cosmological case. Other theories are constructed to widen the domain of GR. In each of the following subsections, we review briefly a theory, the motivation for constructing its field equations and some features of the world models resulting from its cosmological applications.

4.1 McCrea-Mikhail Treatment of Creation of Matter

Hoyle’s modification of GR [6], to account for continuous creation of matter stimulated many questions. One of these questions was about the role of the skew part of the tensor \( V_{\mu \nu} \). Another important questions is whether it is better to define the creation vector from the geometric structure used. Mikhail [10], McCrea and Mikhail [19] have used the AP-geometry in order to define this vector. They wrote Hoyle’s modified equations (6) in
AP-geometry by using the basic vector given by (18) to play the role of the creation vector. Their field equations are similar to (6). An advantage of this treatment is that the AP-geometry possesses sufficient structure to allow for the creation vector to be represented, as a geometric object. This represents an example for applying Einstein’s philosophy of geometrization of physics. Applying the modified field equations using the geometric structures (48), (49), they have got a Steady State model. It is to be noted that the skew part of the creation tensor as defined by McCrea and Mikhail, in the above mentioned tetrads, vanishes identically. This is another advantage of this treatment.

4.2 A Pure Geometric Approach to the Steady State

Mikhail [20] constructed a Unified field theory using AP-geometry. This theory is pure geometric in the sense that it has no phenomenological objects. Its field equations can be written as,

\[ \Lambda^\alpha_{\mu \nu} + c_{\mu \nu} = 0, \]
\[ c_{\alpha} = 0, \]
\[ c_{\nu} = 0. \]

Applying this set of field equations to the AP-structure given by (48), Mikhail [21] found that the first two equations of (50) are satisfied identically while the last one gives,

\[ \frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} = 0 \]

which has the Steady State solution,

\[ R(t) = Ae^{\frac{ct}{2}}, \]

where \( A, a \) are constants. The Riemannian space associated with (48) is the FRW-structure (2), which can be written in the form

\[ ds^2 = c^2 dt^2 - \frac{e^{\frac{2\pi}{k r^2}}}{(1 + \frac{1}{4}kr^2)^2}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \]

It is to be considered that, here again as in the previous subsection, the skew part corresponding to the creation tensor vanishes identically. This answers, partially, one of the questions raised above.

One of the objections which can be raised against this treatment is that the field equations (50) are twenty two while the field variables are only sixteen (the tetrad components). Another objection is clear from (52) in which \( k \) takes all possible values which may violate the perfect cosmological principle. However, any of the values of this constant could be inserted by hand, since this treatment does not fix it.
4.3 Møller’s Tetrad Theory (MTT) and the Big Bang

In 1978 C. Møller [22] attempted to modify GR in order to remove the inevitable singularities appearing in the solutions of its field equations. He wrote a new gravity theory, in the AP-geometry, whose field equations are derivable from an action principle. The Lagrangian function suggested by Møller is in the form:

\[ \mathcal{L} = \mathcal{L}_g + \mathcal{L}_m \]

where

\[ \mathcal{L}_g \equiv \sqrt{-g}(\alpha_1 c^\mu c^\mu + \alpha_2 \gamma^{\mu\nu\sigma} \gamma^\mu \gamma^\nu \gamma^\sigma + \alpha_3 \gamma^{\mu\nu\sigma} \gamma^\sigma) \] (53)

\[ \mathcal{L}_m \equiv \kappa \sqrt{-g} T^\mu_\nu g_\mu^\nu \] (54)

where \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are parameters to be fixed later, and \( T^\mu_\nu \) is a phenomenological material-energy tensor. Møller imposed the condition that his suggested theory must have a correct Newtonian limit. Using this condition, he was able to reduce the three parameters to only one parameter, \( \chi \), where

\[ \alpha_1 = -1, \quad \alpha_2 = \chi, \quad \alpha_3 = 1 - 2\chi. \]

His field equations can be written in the form,

\[ G^\mu_\nu + H^\mu_\nu = -\kappa T^\mu_\nu \] (55)

\[ f^\mu_\nu = 0 \] (56)

where,

\[ H^\mu_\nu \equiv \chi[\gamma_{\alpha\beta\gamma}^{\mu} \gamma_{\nu}^{\alpha\beta} + \gamma_{\alpha\beta}^{\mu} \gamma_{\nu}^{\alpha\beta} + \gamma_{\alpha\beta}^{\mu} \gamma_{\mu}^{\alpha\beta} + g_\nu^{\mu}(\gamma_{\alpha\beta\gamma}^{\sigma} \gamma_{\sigma}^{\mu} - \frac{1}{2} \gamma_{\alpha\beta}^{\sigma} \gamma_{\sigma}^{\alpha\beta})] \] (57)

and

\[ f^\mu_\nu \equiv \chi[\phi_{\mu,\nu} - \phi_{\nu,\mu} - \phi_{\alpha}^{\mu} \lambda^{\alpha}_{\mu,\nu} + \gamma_{\mu,\alpha}^{\nu}] \] (58)

Saez and De Juan [23] applied Møller’s field equations to construct world models. They have obtained a number of Big Bang models depending on the choice of the material distribution and the value of \( k \).

Saez [24] proposed two generalization for the MTT by introducing a scalar field in addition of the tetrad vector field. His theories compose a class known as "Scalar-Tetradic Theories of Gravity". He examined [25], among other things, the cosmological consequences of the suggested class and found that his result could be reduced to the corresponding results of MTT. It is to be noted that all world models obtained from MTT have the same problems as those obtained from GR.

4.4 Cosmology of the New General Relativity (NGR)

Hayashi and Shirafuji [26] constructed a theory which they called "New General Relativity". The theory is constructed in the AP-geometry and the field equations have been derived using an action principle. The Lagrangian function used is in the form:

\[ \mathcal{L} \equiv \lambda \left( \frac{R}{2k} + d_1 (t^\lambda_\mu_\sigma b^\mu_\sigma) + d_2 c^\mu c^\mu + d_3 a^\mu a_\nu \right) \] (59)
where \( d_1, d_2, d_3 \) are three parameters and \( \lambda \) is the determinant of \( \lambda_{ij} \). The tensor \( t_{\lambda \mu \nu} \) is defined by:

\[
t_{\lambda \mu \nu} \overset{\text{def}}{=} \frac{1}{2}(\Lambda_{\lambda \mu \nu} + \Lambda_{\mu \lambda \nu} - \frac{1}{6}(g_{\nu \lambda}c_{\mu} + g_{\mu \nu}c_{\lambda}) + \frac{1}{3}g_{\lambda \mu}c_{\nu},
\]

and the axial vector \( a_\mu \) is defined by,

\[
a_\mu \overset{\text{def}}{=} \frac{1}{6}\epsilon_{\mu \nu \rho \sigma}\Lambda^{\nu \rho \sigma},
\]

\( \epsilon_{\mu \nu \rho \sigma} \) is the Levi-Civita totally anti-symmetric tensor.

The field equations of NGR can be written in the form:

\[
G_{\mu \nu} + S_{\mu \nu} = \kappa \tilde{T}_{\mu \nu}
\]

where,

\[
S_{\mu \nu} \overset{\text{def}}{=} 2kF_{\mu \nu \lambda}d - 2kC_\mu F_{\mu \nu \lambda} + 2kK_{\mu \nu} - kg_{\mu \nu}L
\]

and,

\[
F_{\mu \nu \lambda} \overset{\text{def}}{=} d_1(t_{\mu \nu \lambda} - t_{\mu \lambda \nu}) - d_2(g_{\mu \nu}c_\lambda - g_{\mu \lambda}c_\nu) - \frac{1}{3}d_3\epsilon_{\mu \nu \lambda \sigma}a_\sigma,
\]

\[
K_{\mu \nu} \overset{\text{def}}{=} \Lambda^{\rho \sigma \mu}F_{\rho \sigma \nu} - \frac{1}{2}\Lambda^{\rho \sigma \nu}F_{\rho \sigma},
\]

and

\[
L \overset{\text{def}}{=} d_1(t_\lambda \mu \nu t_{\lambda \mu \nu}) + d_2c^\mu c_\mu + d_3a^\mu a_\mu.
\]

It is clear that \( \tilde{T}_{\mu \nu} \) in (63) is non-symmetric phenomenological material-energy tensor. Taking the vectorial divergence of both sides of (63), Hayashi and Shirafuji found that,\[
\tilde{T}_{\mu \nu} = 0.
\]

They considered (67) as a generalization of the law of conservation. This will reduce to conservation in orthodox GR when \( \tilde{T}_{\mu \nu} \) is symmetric.

Mikhail et al.[27] applied the NGR field equations (63) together with conservation (67) using the two AP-structures (48), (49). They have assumed that the material distribution is described by a material energy tensor of a perfect fluid, usually used in GR. They have obtained two families of World models corresponding to equations of state for dust and radiation. The models obtained have the same problems of standard Big Bang cosmology, expect that the existence of horizons is conditional in some models.

### 4.5 A Pure Geometric Approach to the Big-Bang

Mikhail and Wanas [28] have constructed a field theory in an attempt to unifying gravity and electromagnetism, in the context of AP-geometry. The theory is a pure geometric one and it is called the ”Generalized Field Theory ” GFT. The field equations of this theory were obtained using variational calculus but without using an action principle. However,
the same field equations could be obtained using an action principle \[29\]. In both cases 
the Lagrangian used can be written in the form:

$$\mathcal{L} = \lambda (\Lambda_{\mu\nu\alpha} \Lambda^{\alpha\mu\nu} - c_{\mu} c^{\mu}).$$ \hspace{1cm} (68)

This Lagrangian has been constructed using certain assumptions generalizing the Scheme 
of GR \[15\]. The field equations obtained from this Lagrangian can be written in the form,

$$E_{\mu\nu} = 0,$$ \hspace{1cm} (69)

where \(E_{\mu\nu}\) is a second order non-symmetric tensor defined in the AP-space. The symmetric 
part of this tensor gives rise to the equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = B_{\mu\nu},$$ \hspace{1cm} (70)

where \(B_{\mu\nu}\) is a geometric material-energy tensor defined by,

$$B_{\mu\nu} \overset{\text{def}}{=} \frac{1}{2} g_{\mu\nu} (\sigma - \omega) + \omega_{\mu\nu} - \sigma_{\mu\nu},$$ \hspace{1cm} (71)

where the tensors on the R.H.S. of this definition are given in Table 1. The skew- 
symmetric part of (69) gives rise to the equations,

$$F_{\mu\nu} = c_{\mu,\nu} - c_{\nu,\mu},$$ \hspace{1cm} (72)

where \(F_{\mu\nu}\) is a second order skew symmetric tensor defined by,

$$F_{\mu\nu} \overset{\text{def}}{=} \zeta_{\mu\nu} - \xi_{\mu\nu} + \eta_{\mu\nu},$$ \hspace{1cm} (73)

also, the tensor on the R.H.S. of this expression are defined in Table 1. It is to be 
considered that \(B_{\mu\nu}\) is subject to a conservation condition as a consequence of (70).

Since the GFT theory is a pure geometric theory, a certain scheme, known as "Type 
Analysis" has been suggested \[12\], \[30\] to attribute some physical meaning to the geo-
metric objects of the AP-space. This scheme enables one to know, off hand, the capabil-
ities of any AP-structure to represent physical systems. Applying this scheme to the two 
structures given by (48), (49), respectively, it is shown \[31\] that the first AP-structure 
(48) can represent a gravitational field within a material distribution, while the structure 
(49) is capable of representing a gravitational field in free space. Thus the structure (48) 
is to be used in order to construct non-empty world model.

The author \[32\] has applied the GFT field equations (69) to the AP-structure (48), 
and got a unique world model. This model is non-empty and has no particle horizons. It 
fixes a value for \(k(= -1)\) i.e. it has no flatness problem, but it still has a singularity at 
t=0. A further advantage of using pure geometric theories \[33\] is that we do not need to 
impose any condition from outside the geometry used (e.g. equation of state) in order to 
solve the field equations.
4.6 Spin-Gravity Interaction and the Cosmological Parameters

Recently, a type of interaction between the quantum spin of a moving particle and the background gravitational field, is suggested [18]. The equation of motion of a spinning particle in a gravitational field is that given by (40). The parameter $b$ is given by,

$$b = \frac{n}{2} \alpha \gamma,$$

(74)

where $n$ is a natural number, $\alpha$ is the fine structure constant and $\gamma$ is a dimensionless parameter of order unity. The use of this equation in the weak field limit, removed the discrepancy from the results of the COW-experiment [34]. Also, it helped in constructing a temporal model for SN1987A [35], which is in good agreement with supernovae mechanism.

The author [36] studied the effect of the new suggested spin-gravity interaction on the cosmological parameters. The results obtained are tabulated in Table 3. Equation (40) indicates that trajectories of massless spinning particles in gravitational fields is spin dependent. The natural number $n$ takes the values 0, 1, 2, 3, ..., for particle with spin 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, ... respectively. The null geodesic equation, which is a special case of (40) corresponding to $n = 0$, is usually used in the context of GR to represent the trajectory of massless spinning particles in gravitational field. This means that, in using null geodesic, we neglect the effect of the spin-gravity interaction. Table 3 summarizes the results of calculations of the effect of this interaction, on the cosmological parameters. The second column gives the conventional values of these parameters, i.e. those values obtained when neglecting the interaction. The third, fourth and fifth columns give the effect of this interaction on values of the parameters.

Table 3: Spin-Dependence of Cosmological Parameters

| Parameter | Spin-0 | Spin-1/2 (neutrino) | Spin-1 (photon) | Spin-2 (graviton) |
|-----------|--------|---------------------|----------------|-----------------|
| Hubble    | $H_o$  | $(1 - \frac{3}{2})H_o$ | $(1 - \alpha)H_o$ | $(1 - 2\alpha)H_o$ |
| Age       | $\tau_o$ | $\frac{\tau_o}{(1-\frac{3}{2})}$ | $\frac{\tau_o}{(1-\alpha)}$ | $\frac{\tau_o}{(1-2\alpha)}$ |
| Acceleration | $A_o$ | $(1 - \frac{3}{2})(A_o - \frac{3}{2}H_o)$ | $(1 - \alpha)(A_o - \alpha H_o)$ | $(1 - 2\alpha)(A_o - 2\alpha H_o)$ |
| Deceleration | $q_o$ | $\frac{(q_o - \frac{3}{2}H_o)}{(1-\frac{3}{2})}$ | $\frac{(q_o - \frac{3}{2}H_o)}{(1-\alpha)}$ | $\frac{(q_o - \frac{3}{2}H_o)}{(1-2\alpha)}$ |
| Matter    | $\Omega_o$ | $\Omega_o$ | $\Omega_o$ | $\Omega_o$ |
It is clear from this Table that, the value of the matter parameter is not affected by this interaction. This is because both the mean cosmic density and the critical density have the same dependence on Hubble’s parameter. It is of interest to note that, if we measure these parameters using different carriers of cosmological information (e.g. photons and neutrino), one would obtain a further confirmation, on the cosmological scale, of the existence of spin-gravity interaction.

5 TOPOLOGY AND COSMOLOGY

All what is given, so far, in the previous sections can be classified under the title "Geometry and Cosmology". In recent years, some articles appeared in periodicals connecting topology to cosmology. Of course, one cannot give a complete review about this topic in such a limited number of pages. But I will focus on a single trend in this class, that is the recent work of El Nashie, which I consider as related to the subject of the present review. El Nashie suggested a special scheme, which has been published in a number of papers, to understand nature. This scheme depends mainly on a type of topology "The Wild Topology" and a type of geometry "The Noncommutative Geometry", both related to 4-dimensional fusion algebra and M.Fredmann 4-dimensional Topological spaces. Using this scheme he was able to obtain physical results in excellent agreement with all micro-physical experiments and some macrophysical observations. For example, he obtained the mass spectrum of quarks [37]; and an acceptable value for the cosmic microwave background radiation (CMBR) temperature [38]. Moreover, on the same bases, he obtained convincing results concerning unification of fundamental interactions [39], a general theory for quantum gravity [40], the dimensions of heterotic string theory [41], and a value for the super-symmetric quantum gravity coupling constant [42]. His main calculations depend on the golden mean \( \phi \equiv \frac{\sqrt{5}-1}{2} \).

El-Naschie’s results are not only more than promising but also, they stimulate many questions and various comments. For instance we could ask the following questions:

(1) First one could ask a formal question about the title of this theory. Why it is \( \varepsilon^{(\infty)} \) theory ? Why it is not called the golden mean field theory.

(2) A More important question is that: Are we really able to interpret any phenomena in the Universe without using evolutionary scenarios ?

El-Naschie results, in the context of his constructed wild topology, depend on the quantity \( \phi \). And since \( \phi \) is constant, then every subsequent result will be constant ! What does this mean ? As clear from the present review, there were two rival scenarios used to interpret the general features of our universe : The Big-Bang scenario, and the Steady State one. In the first, the Universe as well as its constituents evolve. In the Steady State scenario the constituents of the Universe are evolving while the global characteristics of the Universe remain the same. The observation of the CMBR-temperature, in the mid sixties of the 20th century, ruled out the Steady State theory, since there is no place for the CMBR in this theory, as CMBR is a result of evolution of the universe. The most astonishing thing is that the CMBR-temperature is obtained from El-Naschie scheme, while everything is constant in his calculations!

Is El-Naschie working in a Steady State background and consequently giving a new
chance to this theory to revive? Let us try to give an answer to this question. If we accept Mach’s principle, then any property of the constituents of the Universe is a reflection of the large scale material distribution in the Universe. And since, in the context of a Steady State model, this distribution does not change (the material-energy density is constant), then we get constant properties of the constituents including masses of elementary particles and even the CMBR-temperature. Other questions may be raised if we accept this interpretation.

(3) Is El-Naschie theory dealing with stable configurations (in the Universe) only? An answer to this question may throw some light on his way of understanding nature. It is widely accepted that stable systems were not born in this situation (stability situation). It is usually assumed that such systems are born as unstable systems, and gradually arrive to stability, satisfying certain stability conditions. In this case, it seems that there is no escape from evolution, the assumption which does not exist in El-Naschie treatment.

(4) Is it possible to construct a general (or let us say, universal) stability theory, which can transfer unstable configurations to El-Naschie stable systems? If yes, the stability conditions of this theory would be algebraic (equations or inequalities) rather than differential, in order to be consistent with the \( \varepsilon^{(\infty)} \)-theory. If this is done, it would be considered as a complement of El-Naschie theory. Moreover, the golden mean \( \phi \) would be a real root (or the real root) necessary to satisfy such conditions. This would construct an acceptable bridge between existing physics and El-Naschie’s "Topologization of Physics".

6 DISCUSSION AND CONCLUDING REMARKS

This brief review gives alternative treatments of theoretical cosmology. In particular it gives alternative theoretical treatments leading to Big-Bang or Steady State cosmologies. The standard treatment in theoretical cosmology can be summarized in the following steps:

(1) Riemannian geometry is assumed to give a complete representation of the physical World, including space-time.

(2) Certain Riemannian structures (e.g. (2)), satisfying some conditions (e.g. the cosmological principle) are to be used as basic structures for constructing World models.

(3) The equations of GR, written in Riemannian geometry, represent good constraint connecting the material distribution in the Universe and the geometric structure used to describe it.

Problems of standard cosmology motivated investigators to change the conventional treatment, summarized above, in the hope that this change may remove one or more of these problems. The use of a more wider geometry, the AP-geometry, in place of GR represents a change in the first step (this is done in Section 2). Consequently the use of the structures (48) and (49) in place of (2) represents a change in the second step (which is given in Section 3). Finally the use of the equations of alternative field theories, different from GR, is a change of the third step (as presented in Section 4).

In the following we are going to compare the results obtained, from the alternative treatments given in the present work with standard theoretical cosmology. (note: one
can reconstruct GR in AP-geometry by taking Ricci tensor as defined in Table 1, and the geodesic (or null geodesic) is obtained from the AP-path (40) upon taking $n = 0$.

1) In order to compare the Big Bang results of alternative field theories, written in the AP-geometry, with those of GR, we first give a brief comparison of these theories in Table 4.

Table 4: Comparison Between Field Theories Giving Big Bang Models

| Field Theory | Reference | Field Equations | Field Variables | Gravitational Potential | $T_{\mu\nu}$ |
|--------------|-----------|-----------------|-----------------|-------------------------|--------------|
| GR (1916)    | cf.[1]    | $G_{\mu\nu} = -\kappa T_{\mu\nu}$ | $g_{\mu\nu}$ | $g_{\mu\nu}$ | Phenom. |
| GFT (1977)   | [28]      | $G_{\mu\nu} = B_{\mu\nu}$ | $\lambda_\mu$ | $g_{\mu\nu}$ | Geomet. |
|              |           | $F_{\mu\nu} = c_{\mu,\nu} - c_{\nu,\mu}$ |         |             |             |
| MTT (1978)   | [23]      | $G_{\mu\nu} + H_{\mu\nu} = -\kappa T_{\mu\nu}$ | $\lambda_\mu$ | $g_{\mu\nu}$ | Phenom. |
|              |           | $f_{\mu\nu} = 0$ |         |             |             |
| NGR (1979)   | [26]      | $G_{\mu\nu} + S_{\mu\nu} = -\kappa \tilde{T}_{\mu\nu}$ | $\lambda_\mu$ | $g_{\mu\nu}$ | Phenom. |

The last column of this table indicates whether the material-energy tensor is phenomenological or geometric.

Table 5 gives a comparison between the Big Bang cosmology resulting from the alternative theories given in the present work and that resulting from GR. It is of interest to note that the existence of horizon in the NGR is conditional. It is clear from this Table that in case of GFT, the horizon and flatness problems disappeared from the model obtained. Also this model is a unique one and has the advantage that extra conditions (equation of state) is not needed to construct the model, but a relation between the pressure and density is obtained as a result of this model. This is the main advantage of using a pure geometric approach to cosmology [33].
Table 5: Comparison between the resulting Big Bang cosmologies

| Criterion                                | GR, cf[1] | GFT, [32] | MTT, [22] | NGR, [27] |
|------------------------------------------|-----------|-----------|-----------|-----------|
| Space                                    | Riemannian| AP-space  | AP-space  | AP-space  |
| Free parameters                          | No        | No        | One       | Three     |
| Energy-momentum tensor $T^{\mu\nu}$      | Symmetric Phenomological tensor | Symmetric Geometric tensor | Symmetric Phenomological tensor | Non-symmetric Phenomological tensor |
| Basic assumption                         | Homogeneity & Isotropy | Homogeneity & Isotropy | Homogeneity & Isotropy | Homogeneity & Isotropy |
| $k$ for non-static, non-empty models     | +1, 0, −1 | −1        | +1, 0, −1 | +1, 0, −1 |
| Number of models allowed                  | Many      | One       | Many      | Many      |
| Particle Horizons                         | Yes       | No        | Yes       | Conditional |
| Flatness Problem                         | Yes       | No        | Yes       | Yes       |
| Need for extra condition                  | equ. of state | -        | Equ.of state | Equ.of state |
(2) Also to compare Steady State results of alternative theories constructed in AP-geometry with those resulting from the modified equations of GR(6), we establish Table 6.

Table 6: Comparison between the resulting Steady State cosmologies

| Criterion               | Hoyle [6]         | McCrea and Mikhail [19] | Mikhail [21] |
|-------------------------|-------------------|-------------------------|--------------|
| Space                   | Riemannian        | AP-space                | AP-space     |
| Creation tensor         | Phenom.           | Geometric               | ?            |
| Extra conditions        | Equation of state | Equation of state       | -            |
| $k$                     | 0                 | 0                       | +1, 0, -1    |

It is to be considered that all the Steady State results listed in Table 6 are free from singularity and horizon problem. Furthermore, the use of a pure geometric approach (fourth column of the table) indicates that there is no need for an equation of state to construct a world model.

(3) Some authors [23] construct World models without using the AP-structures (48), (49). However, it is more appropriate to use (48) or (49) in order to guarantee homogeneity and isotropy in the general case. The use of the roots of the metric tensor to produce tetrads is not sufficient to obtain the most general AP-structure satisfying the cosmological principle. This is because the ten components of the metric tensor $g_{\mu\nu}$ cannot fix the sixteen tetrad components $\lambda_{\mu}$ uniquely.

(4) As in subsection 2.2, the PAP-geometry is more wider than the conventional AP and the Riemannian geometries. It has sufficient structure for other physical interactions to be represented, beside gravity. For example, its general path equation (40) are used [18] to describe the trajectories of massless spinning particles in a background gravitational field. Its R.H.S. is interpreted as representing a type of interaction between the quantum spin of the moving particle and the gravitational field. Since particle carrying the cosmological information are massless spinning particles, then their trajectories in the cosmic space will be affected by the spin-gravity interaction. This will affect information carried by such particles, and then it will be of interest to know how to free information from this interaction. This is clear from Table 3 which gives the effect of this interaction on the cosmological parameter. This is one of the achievements of using the non-conventional approach given in the present review.

(5) El-Nashie [43] quoted two arguments to support, what I am calling, topologization of physics. The first is the relation between the golden mean and the Fibonacci series. The second is the relation between the Hausdorff dimensions of the Menger sponge and the CMBR-temperature. Let us discuss the consequences of these two arguments.

The Fibonacci series (cf. [44]) (0, 1, 1, 2, 3, 5, 8, ...) is a series in which each term
$t_{n+1}$ is the sum of the preceding two terms ($t_n + t_{n-1}$). The golden mean is obtained as:

$$\phi = \lim_{n \to \infty} \frac{t_n}{t_{n+1}}$$

By this definition, although $\phi$ is not an exact number, it converges to a constant. Consequently, everything in El-Nashie theory would be constant, or converges to a constant. There is no direct time evolution as stated in Section 5, but there is another type of evolution i.e. the Fibonacci gross law.

The second argument gives a further confirmation to the above remark. The CMBR-temperature has a strong relation, via entropy and complexity theory, with the Hausdorff dimensions of the Menger sponge, which is given by $\frac{\log 20}{\log 3} = 2.7268...$. This dimension, although it is not exact, it converges to a constant value without any time evolution. So, if there is any causal relation between the CMBR-temperature and the Hausdorff dimensions of the Menger sponge, then one of El-Nashie important results [38] is a consequence of using a constant! Is there any physical relationship between this argument and the Zeldovich idea that our Universe is similar to a sponge? El Nashie states that there is [43].

The conclusion is that El-Nashie theory may need a general stability theory (may be a generalization of a theory of the type of the KAM theory) in order to take over unstable systems to El- Nashie stable systems. In this case, once again, we believe that El-Nashie is working in a Steady State background.

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