Quantitative Models of Fungi Interaction --based on Logistic models

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Abstract. As the key medium for decomposing wood fibers, fungi play a vital role in promoting the carbon cycle. The purpose of this paper is to establish mathematic models describing the process of fungi decomposing litter and wood fiber. The paper comprehensively consider the effects of temperature, moisture, location and other factors on fungal traits, and introduced the experimental data of the US Forest Service Center for Forest Mycology Research (CFMR). Based on the model of competition among populations and the modern coexistence theory, this paper establishes the competitive model between two fungi, then generalizes it to a variety of fungi and predicts the short-term and long-term effects.

1 Introduction

The carbon cycle is an important part of life on earth. Part of it includes the decomposition of compounds, allowing carbon to be renewed and used in other forms. The decomposition of plant materials and wood fibers is the key to this process. Some of the key factors that decompose wood fibers are fungi. A recent article pointed out that slow-growing fungal strains tend to survive and grow better in the presence of environmental changes in humidity and temperature, while faster-growing strains are less resistant to the same changes.

The paper tries to use mathematical methods rather than experimental methods to combine the interaction of multiple fungi.

2 literature Review

Yu Shuling (2007) discussed the succession and general growth and decline of saprophytic fungi, and obtained the rule of fungi: the evolution process from Zygomycetes to ascomycetes and finally to basidiomycetes.

Daniel s. Maynard (2017) proved that competitive network determines the development direction of diversification function relationship.

Daniel s. Maynard (2019) and others believe that competition can profoundly affect biodiversity patterns by determining whether similar species are likely to coexist. By demonstrating that species richness can serve as a self-reinforcing buffer against competitive exclusion, these results help us understand how biodiversity is maintained in natural systems.

Yang Peng (2019) and others carried out the simulation of the birth and death process model of growth retardation. The results showed that the growth rate of species gradually slowed down with the increase of population biomass after adding the retardation term. A two-dimensional two-state biological mathematical model was proposed.

3 Assumptions and Justification

The composition of litter only considers cellulose; The decomposition of litter is only related to the aerobic respiration of microorganisms, without considering the influence of factors such as soil animals; The intermediate process of litter decomposition is regarded as the whole process, because only this process is closely related to microorganisms;

4 Notations

| Symbols | Description |
|---------|-------------|
| t       | Time        |
| x(t)    | Number of fungal population |
| r       | Inherent growth rate |
| n       | Maximum capacity |

5 Research on the Relationship between Fungi

In the natural environment, various fungi exist on the same land. For the two types of fungi, the relationship between the two can be divided into competition, interdependence, and predation relationship. fungi are mostly in competition with each other, so based on the model of competition among populations, modern coexistence theory combined with the data given in the topic, a competition model between two fungi is established. On this basis, it was promoted, and the interactions between a variety of fungi were obtained, and
the short-term and long-term kinetic effects of this interaction were further analyzed and predicted.

5.1 Fungal Interaction Model

Based on the experimental data of Daniel S. Maynard wood-decaying basidiomycetes grown on nutrient-rich agar medium, we selected 15 main fungi from them by searching for data, and analyzed their characteristics respectively, and finally got Fungal competition model.

5.1.1 Basic Principles of Competitive Model

1. Calculate the retardation coefficient between each pair of fungi.

This paper assumes that when each type of fungus survives alone, the number changes obey the Logistic law, that is, the initial stage is roughly exponential growth; then as it becomes saturated, the increase slows down; finally, the increase stops when it reaches maturity. The mathematical formula is as follows:

\[
\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i}{n_i}\right) \quad (1)
\]

\[
\frac{dx_j}{dt} = r_j x_j \left(1 - \frac{x_j}{n_j}\right) \quad (2)
\]

When multiple populations live together, they compete for limited resources such as food and space. There will be mutual blocking effects between the i-th and j-th populations. Among them, the retarding effect of the i-th population on the growth of the j-th population is proportional to the number of the i-th population. In the same way, the retarding effect of the j-th population on the growth of the i-th population is proportional to the number of the j-th population, so it can be improved the above model into the following form:

\[
\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i}{n_i} - s_i \frac{x_j}{n_j}\right) \quad (3)
\]

\[
\frac{dx_j}{dt} = r_j x_j \left(1 - \frac{x_j}{n_j} - s_j \frac{x_i}{n_i}\right) \quad (4)
\]

The meaning of \(s_i\) is that for the resources that support the i species, the consumption of a unit quantity of j is \(s_i\) times the consumption of a unit quantity of i. In the same way, the meaning of \(s_j\) is that the consumption of a unit quantity of i is \(s_j\) times the consumption of unit quantity j for the resources that support the j species.

5.1.2 Competitive Model of Two Fungi

1. Parameter Determination

The data used in this model comes from the experimental data of Daniel S. Maynard et on wood-rotting basidiomycetes grown on nutrient-rich agar medium.

Inherent growth rate r: fungi belong to the category of microbes, which are quite different from the growth and reproduction methods of animals and plants. The degree of improvement makes the model more in line with the requirements of the problem. The efficiency of fungi in decomposing plant matter and wood fiber is closely related to mycelial elongation and mycelial density, so the data of mycelial elongation under suitable temperature and humidity to fit the growth rate is used.

Maximum capacity n: In the experiment of Daniel S. Maynard and others, the hyphae were collected on cellophane, and related protection work was done to prevent the hyphae from drilling into the agar, and then the maximum hyphae density was calculated. The paper processes the obtained data and calculates the maximum capacity of various fungi based on model assumptions.

Retardation coefficient s: According to the definition of retardation coefficient, it is easy to know that the retardation coefficient is related to the decomposition efficiency of fungi.

2. Model Solving

For the relevant data of the 15 fungi are selected and analyzed in this article one by one to obtain 105 groups of fungal relationships. Next, 105 sets of data are screened and classified 105 sets of data, that is, eliminated those combinations that have no direct interaction between species and merged combinations that have similar interactions. In the end, four representative fungal relationships are shown in the figure below.

a: The abilities of the two fungi are equal, and when they become stable, the two groups reach a coexistence state.

b: The two kinds of fungi first grow simultaneously, and then one of them has the advantage, while the other tends to die.

c: At first, the resources are abundant, and the two kinds of fungi can grow naturally, but the growth rate of the dominant species is obviously greater than that of the disadvantaged species. In the end, one side tends to be stable, and the other becomes extinct.

![Fig 1. Diagram of Interaction between Two Fungi - TYPE a](image)
d: There is a big difference between the advantages and disadvantages of the two fungi. In a stable state, one side reaches the maximum value, and the other side becomes extinct.

5.1.3 Interaction of Multiple Fungi

On the premise that the selected environment is not disturbed by the outside world and the decomposition products are limited, and the interaction between all fungi can be directly accumulated, based on the results obtained from the above model, we can extend the interaction of the two fungi to the interaction of multiple fungi. Under certain temperature and humidity conditions, different fungi have different viability. When the resources were initially abundant, most fungi could grow naturally according to the exponential model, and a small number of fungi were eliminated due to their own disadvantages. Then, when there are more fungi in the environment, the advantages and disadvantages between fungi gradually appear, and the dominant bacteria will eliminate the inferior bacteria. Then a small number of fungi coexisted and tended to a stable state. Finally, when the decomposable substances in the environment are completely decomposed, all fungi enter a dormant state.

5.2 Fungal Long- and Short-Term Dynamic Analysis

Based on the conclusions of the above models, four representative combinations of phase trajectories have been drowned. Then get the quantitative relationship of different populations of fungi living in the same environment, as shown in the figure below.
Eventually, when all the decomposable materials in the environment have been decomposed, all the fungi enter a dormant state.

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