Pionic transitions from $X(3872)$ to $\chi_{cJ}$

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Abstract

We consider transitions from the resonance $X(3872)$ to the $\chi_{cJ}$ states of charmonium with emission of one or two pions as a means of studying the structure of the $X$ resonance. We find that the relative rates for these transitions to the final states with different $J$ significantly depend on whether the initial state is a pure charmonium state or a four-quark/molecular state.
1 Introduction

The narrow resonance $X(3872)$ $[1, 2, 3, 4]$ is an extremely interesting hadronic object: its mass is tantalizingly close to the $D^0 D^{*0}$ threshold, $w = M(D^0 D^{*0}) - M(X) = 0.6 \pm 0.6 \text{ MeV}$ $[5]$, and the isotopic symmetry is unusually strongly violated in its decays as follows from the co-existence of the decay $X(3872) \rightarrow \pi^+\pi^- J/\psi$ on one hand and the decays $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$ and $X(3872) \rightarrow \gamma J/\psi$ $[6, 7]$ on the other. The suggested interpretations of this resonance include a dominantly molecular state $[8, 9, 10, 11]$ of charmed mesons ($D^0 D^{*0} + D^{*0} D^0$), possibly of the type discussed in the literature long time ago $[12, 13]$, with an admixture of pure charmonium $[14, 15]$, a dominantly $2^3P_1$ state of charmonium $[16, 17]$, and a virtual threshold state of $D D^*$ $[18, 19, 20, 21]$.

A quantitative understanding of the internal structure of this peculiar state is a challenging task for experimental studies. As has been previously discussed in the literature $[11, 22, 16]$ a measurement of the rates and of the photon spectrum in the decays $X(3872) \rightarrow D D^* \gamma$ can provide a useful insight into certain features of the structure of $X$. In particular, it has been suggested $[16, 17]$ that the observed properties of $X(3872)$ can be, to an extent, mimicked by a $2^3P_1$ state of charmonium, so that any ‘molecular’ admixture would be viewed as a secondary effect due to the coupling to the $D D^*$ states. In this picture the main available indicator of a significant isospin violation in $X(3872)$, the approximately equal rate of the decays $X(3872) \rightarrow \rho J/\psi \rightarrow \pi^+\pi^- J/\psi$ and $X(3872) \rightarrow \omega J/\psi \rightarrow \pi^+\pi^-\pi^0 J/\psi$ is explained by the kinematical suppression of the isospin-allowed transition $X(3872) \rightarrow \omega J/\psi$.

In this paper we discuss the transitions from $X(3872)$ to the $\chi_{cJ}$ charmonium states with emission of one or two pions, which can be studied in addition to the observed processes $X(3872) \rightarrow \pi^+\pi^- J/\psi$ and $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$, and which may be instrumental in further exploration of the $X$ resonance. The discussed transitions may be accessible for experimental observation and may hold the clue to understanding the isotopic structure of the $X(3872)$ and of the prominence of the four-quark component in its internal dynamics. We argue that the characteristics of such transitions are generally completely different between the possible charmonium and molecular components of the resonance $X(3872)$. The rate of the one-pion transition relative to the process with two pions is sensitive to the $I = 1$ four-quark component of the $X(3872)$, while the isoscalar four-quark component should give rise to relative rates of two-pion transitions to the $\chi_{cJ}$ states with different $J$, which are very likely at variance from those expected for transitions between charmonium levels.

In Sect. 2 we consider the one- and two-pion transitions for the case where $X(3872)$ is
treated as a $2^3P_1$ state of charmonium. In this consideration we use the standard approach based on the multipole expansion in QCD \cite{23} and the chiral properties of soft pions.\footnote{A recent discussion of this method in some detail can be found in Ref. \cite{24}.} We find that of the isospin-conserving two-pion transitions by far the strongest should be the decay $X(3872) \rightarrow \pi\pi\chi_{c1}$ with the transition to $\chi_{c2}$ being heavily suppressed and the transition to $\chi_{c0}$ being forbidden at all in the leading order of the chiral expansion. Given that, the rate of the strongest transition in absolute terms is expected to be quite small $\Gamma(X(3872) \rightarrow \pi\pi\chi_{c1}) \sim 1$ keV. The isospin-breaking transitions $X(3872) \rightarrow \pi^0\chi_{cJ}$ proceeding due to the isotopic symmetry breaking by the $u$ and $d$ quark masses are estimated to be still weaker than the two-pion processes. Additionally, the amplitude of the transition $X(3872) \rightarrow \pi^0\chi_{c0}$ turns out to be zero within the considered approach.

A significantly different set of one- and two-pion decay rates is expected in the case where $X(3872)$ has a substantial four-quark component, so that the emission of the pions can be considered as a ‘shake off’ of light degrees of freedom, as discussed in Sect. 3. In this case no strong suppression of a single pion transition should be expected, and in fact such transitions, proceeding due to the $I = 1$ part of the wave function of $X(3872)$, are likely to be dominant over the kinematically suppressed emission of two pions. Moreover, all three $\chi_{cJ}$ resonances are allowed in the final state, including the $\chi_{c0}$, which is additionally favored by the phase space factor $p^3_{\pi}$.

At present we cannot reliably predict the transition rates from a four-quark state. However a very approximate estimate \cite{14}, based on the general understanding of the hadronic parameters, indicates that the single pion transitions $X(3872) \rightarrow \pi^0\chi_{cJ}$ should not be very strongly suppressed relative to the observed decay $X(3872) \rightarrow \pi^+\pi^-J/\pi$. Thus one may hope that if $X(3872)$ indeed has a significant $I = 1$ four-quark/molecular component, the discussed single-pion transitions may be accessible for experimental studies.

\section{Charmonium}

In this section we treat $X(3872)$ as a charmonium $2^3P_1$ state, so that the hadronic transitions from this resonance can be studied using the standard approach based on the multipole expansion. The two-pion transitions arise \cite{23} in the second order in the $E1$ interaction with the chromoelectric gluon field $\mathbf{E}_a$ described by the Hamiltonian

\begin{equation}
\mathcal{H}_E = \frac{G_F}{\sqrt{2}} \int d^4x \left( \bar{c}(\mathbf{D}) c \mathbf{E}_a + \frac{1}{2} \epsilon_{abc} \partial^a \bar{c} \gamma^b c \mathbf{E}_c \right),
\end{equation}
\[ H_{E1} = -\frac{1}{2} \xi^a r_i E^a_i(0), \]  

where \( \xi^a = t^a_1 - t^a_2 \) is the difference of the color generators acting on the quark and antiquark, and \( \vec{r} \) is the vector of the relative position of the quark and antiquark. The convention used throughout this paper is that the QCD coupling \( g \) is included in normalization of the gluon field operators.

The one pion transitions \( ^3P_1 \rightarrow ^3P_J \pi \) are induced by the interference of the E1 interaction (1) and \( M^2 \) term containing the chromomagnetic field \( \vec{B}^a \) and described by the Hamiltonian [25]

\[ H_{M2} = -\frac{1}{4m_Q} \xi^a S_k r_j (D_j B_k(0))^a, \]  

where \( D \) is the QCD covariant derivative, \( m_Q \) is the heavy quark mass, and \( \vec{S} = (\vec{\sigma}_1 + \vec{\sigma}_2) / 2 \) is the operator of the total spin of the quark-antiquark pair.

Generally, one also needs to consider the contribution to the one pion transition coming from the Hamiltonian of the form:

\[ H^{(L)}_{M2} = -\frac{\xi^a}{48 m_Q} [L_k r_j (D_j B_k)^a + (D_j B_k)^a r_j L_k], \]  

where \( \vec{L} \) is the operator of the orbital momentum of the quark-antiquark pair. However we will argue later that this contribution cancels due to the specific form of the gluonic matrix element.

Using the expressions (1) and (2) one can find the one- and two- pion transition amplitudes in the form

\[ A_{\pi\pi}^{(J)} \equiv A \left( ^3P_1 \rightarrow \pi\pi^3P_J \right) = \langle \pi\pi | E^a_i E^a_j | 0 \rangle A_{ij}^{(J)}, \]  

\[ A_{\pi}^{(J)} \equiv A \left( ^3P_1 \rightarrow \pi^3P_J \right) = m_Q^{-1} \langle \pi | E^a_i (D_j B_k)^a + (D_j B_k)^a E^a_i | 0 \rangle A_{ijk}^{(J)}, \]  

where the heavy quarkonium amplitudes \( A_{ij}^{(J)} \) and \( A_{ijk}^{(J)} \) are defined as follows

\[ A_{ij}^{(J)} = \frac{1}{32} \langle 1^3P_J | \xi^a r_i \mathcal{G} r_j \xi^a | 2^3P_1 \rangle, \]

\[ A_{ijk}^{(J)} = \frac{1}{128} \langle 1^3P_J | \xi^a r_i \mathcal{G} r_j \xi^a S_k + r_j \xi^a \mathcal{G} \xi^a r_i S_k | 2^3P_1 \rangle \]

with \( \mathcal{G} \) being the Green’s function of the heavy quark pair in a color octet state. Also in the expressions (6) and (7) we have used the fact that in the matrix elements between color singlet states the color factor \( \xi^a \ldots \xi^b \) can be replaced by \( (\delta^a_b / 8) \xi^c \ldots \xi^c \)
In the leading nonrelativistic limit the spin of the heavy quark pair decouples from the coordinate degrees of freedom. Thus, it is convenient to write the amplitude in a form with explicitly factorized spin and orbital components. For this purpose we denote $\zeta_i$ and $\eta_i$ the spin polarization amplitudes of the initial $^3P_1$ and the final $^3P_J$ states. We also introduce the polarization amplitudes $\phi_i$, $\psi_i$ and $\psi_{ij}$ of the initial state and the final state with $J = 1$ and $J = 2$ correspondingly. The tensor $\psi_{ij}$ is symmetric and traceless, as appropriate for the $J = 2$ state. All these amplitudes are assumed to be normalized in a standard way, so the sums over the polarization states are defined as

$$\sum_{\text{pol}} \zeta_i^* \zeta_j = \sum_{\text{pol}} \eta_i^* \eta_j = \sum_{\text{pol}} \phi_i^* \phi_j = \sum_{\text{pol}} \psi_{ij}^* \psi_j = \delta_{ij},$$

$$\sum_{\text{pol}} \psi_{ij}^* \psi_{kl} = \frac{1}{2} \left( \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right).$$

Further we define the initial and the final states

$$^3P_1 = \epsilon_{ijk} \zeta_i \alpha_j \phi_k / \sqrt{6}, \quad ^3P_J = P_{ij}^{(J)} \eta_i \beta_j,$$

where $\alpha$ and $\beta$ stand for the $P$-wave coordinate parts of the wave functions and the projection matrices $P_{ij}^{(J)}$ for the definite total spin $J$ are:

$$P_{ij}^{(0)} = \delta_{ij} / \sqrt{3}, \quad P_{ij}^{(1)} = \epsilon_{ijk} \phi_k / \sqrt{2}, \quad P_{ij}^{(2)} = \psi_{ij}.$$

Proceeding to the explicit calculation of the heavy quarkonium matrix elements we assume as in the heavy quark limit that the spin of the heavy quark pair is conserved, i.e. $\langle \eta_i \zeta_j \rangle = \delta_{ij}$. Generally the intermediate states contributing to the Green’s function in equations (6) and (7) could be those corresponding to the $S$- and $D$- wave (the $P$-wave is forbidden by the parity), so that two different structures are allowed: $\langle b_m \xi^a r_i \mathcal{G} r_j \xi^a a_n \rangle = S \delta_{im} \delta_{jn} + D (\delta_{ij} \delta_{mn} + \delta_{in} \delta_{jm} - 2/3 \delta_{im} \delta_{jn})$. However, assuming that the relevant intermediate states are separated by a large energy gap $\Delta$ from the discussed $P$-wave states, one can approximate the Green’s function $\mathcal{G}$ as being proportional to the unit operator: $\mathcal{G} \simeq 1 / \Delta$. In this case the coefficient in front of the $S$ wave is related to that of the $D$ wave: $S = 5/3D$. Thus within the introduced notations the heavy quarkonium amplitudes can be written in terms of a scalar quantity $\mathcal{A}$

$$A_{ij}^{(J)} = \mathcal{A} (\delta_{im} \delta_{ip} + \delta_{ip} \delta_{jn} + \delta_{ij} \delta_{np}) \epsilon_{mpr} P_{mn}^{(J)} \phi_r,$$
\[ A_{ijk}^{(j)} = \frac{i\mathcal{A}}{2} (\delta_{in}\delta_{jp} + \delta_{ip}\delta_{jn} + \delta_{ij}\delta_{np}) \epsilon_{qmk} \epsilon_{qpr} P_{mn}^{(j)} \phi_r. \] (12)

The quantity \( \mathcal{A} \) depends on details of the heavy quarkonium dynamics and at present is highly model dependent, however, it is clear that this quantity cancels in the considered ratios of the rates of the single pion and two-pion transitions.

The gluonic matrix element for the one pion transition in the Eq. (4) can be written using the form described in [26]

\[ i \langle \pi | E_a^a (D_j B_k)^a + (D_j B_k)^a E_i^a | 0 \rangle = \frac{X}{15} (3p_j \delta_{ik} - p_i \delta_{jk}), \] (13)

with the form factor \( X \) related to the well known expression [27]

\[ X = \left( \pi \left| G^a \tilde{G}^a \right| 0 \right) = 8\pi^2 \sqrt{2} \frac{m_d - m_u}{m_d + m_u} f_\pi m_\pi^2, \] (14)

where \( m_u \) and \( m_d \) are the masses of the \( u \) and \( d \) quarks and \( f_\pi \approx 130 \text{ MeV} \). It should be noted that the specific form of the gluonic matrix element Eq. (3), namely the absence of the term proportional to \( \delta_{ij} \), makes the contribution of the orbital chromomagnetic term (3) being proportional to the structure \( (\vec{L} \cdot \vec{r}) \) which is obviously equal to zero.

Using the expressions (13), (14) and (9)-(12) one readily finds the squares of the transition amplitudes summed over the polarizations of the final \( 3P_J \) state:

\[ |A_{\pi}^{(2)}|^2 = \frac{X^2 p_\pi^2}{15 m_Q^2} |\mathcal{A}|^2, \quad |A_{\pi}^{(1)}|^2 = \frac{X^2 p_\pi^2}{9 m_Q^2} |\mathcal{A}|^2, \quad |A_{\pi}^{(0)}|^2 = 0, \] (15)

where \( p_\pi = |\vec{p}| \) is a pion momentum. We note here that the vanishing of the amplitude \( A_{\pi}^{(0)} \) is a result of our approximation for the Green’s function as being proportional to the unit operator. The ratio of the amplitudes (15) then gives the ratio of the decay rates:

\[ \Gamma_2 : \Gamma_1 : \Gamma_0 = 3p_{\pi (2)}^3 : 5p_{\pi (1)}^3 : 0 \approx 1 : 2.70 : 0, \] (16)

where \( p_{\pi (1)} \approx 334 \text{ MeV} \) and \( p_{\pi (2)} \approx 285 \text{ MeV} \) are the pion momenta in the transitions to the \( 3P_1 \) and \( 3P_2 \) states correspondingly.

Let us proceed now to evaluating the two pion transition rates. Using general arguments based on the isospin and parity considerations one can see that the pion pair in the transitions \( 3P_1 \rightarrow 3P_J \pi\pi \) could only be produced in the even partial waves. From the properties of the chiral algebra it follows that the expansion of the \( S- \) and \( D- \)wave amplitudes starts from the terms quadratic in the pion momentum (and mass) and therefore kinematically these amplitudes are both of the same order in the soft pion limit.
Considering the transition $1^{++} \rightarrow 0^{++}$ one can easily see that the pion pair besides the $D$-wave motion in its center of mass frame should also be involved in a $D$-wave motion as a whole relatively to the $0^{++}$ state, thus the discussed amplitude has to be proportional to the fourth power of the pion momentum in the chiral expansion. Taking into account the small energy release in the transitions $2^3P_1 \rightarrow 1^3P_J \pi \pi$ we should thus expect a strong suppression of the transition to the $\chi_{c0}$ state in comparison with the transitions to the $\chi_{c1}$ and $\chi_{c2}$.

The amplitudes of the two pion transitions to the $\chi_{c1}$ and $\chi_{c2}$ can readily be evaluated using the general form [25, 28] of the amplitude of the dipion creation by the chromoelectric field (11)

$$
\left| \langle \pi^+ \pi^- | E_i^a E_j^a | 0 \rangle \right| = \frac{8\pi^2}{3b} \left( q^2 + m^2_\pi \right) \delta_{ij} + \frac{12\pi^2}{b} \kappa \left[ p_{1i} p_{2j} + p_{1j} p_{2i} - (\varepsilon_1 \varepsilon_2 + \vec{p}_1 \cdot \vec{p}_2) \delta_{ij} \right],
$$

(17)

where $\varepsilon_{1,2}$ and $\vec{p}_{1,2}$ are the energy and momentum of each pion, $q = p_1 + p_2$ is the total 4-momentum of the pion pair; $b = 9$ is the first coefficient in the beta function for QCD with three quark flavors, and $\kappa$ is a parameter introduced in [28]. The experimental value of the parameter $\kappa \approx 0.2$ [29] agrees with the original theoretical estimate [28].

The first term in the Eq. (17) arising from the conformal anomaly [24] describes the $S$-wave production of the two pions in their c.m. frame while the term proportional to $\kappa$ corresponds to both $S$- and $D$-waves. For an approximate estimate of the amplitude of the transition $2^3P_1 \rightarrow 1^3P_1 \pi \pi$ we neglect the contribution of the $\kappa$-term since this process is dominated by the large $S$-wave contribution arising from the first term in Eq. (17). On the other hand for the transition $2^3P_1 \rightarrow 1^3P_2 \pi \pi$ there is no $S$-wave pion pair emission and only the $D$-wave part of the $\kappa$-term contributes to the amplitude. In this way one estimates

$$
\left| A^{(1)}_{\pi\pi} \right|^2 = 150 \left( \frac{8\pi^2}{3b} \right)^2 \left( q^2 + m^2_\pi \right)^2 \left| A \right|^2,
$$

(18)

$$
\left| A^{(2)}_{\pi\pi} \right|^2 = 12 \left( \frac{12\pi^2}{b} \kappa \right)^2 \left[ \vec{p}_1^2 \vec{p}_2^2 + \frac{1}{3} (\vec{p}_1 \cdot \vec{p}_2)^2 \right] \left| A \right|^2 \rightarrow \frac{40}{3} \left( \frac{12\pi^2}{b} \kappa \right)^2 \vec{p}_1^2 \vec{p}_2^2 \left| A \right|^2,
$$

(19)

where in the last transition in the Eq. (19) the averaging on the relative angle between the momenta was made.

For estimates of the rates of $\pi \pi$ transitions one needs to evaluate the corresponding phase space integrals at the energy release $\Delta^{(J)} = M(2^3P_1) - M(1^3P_J)$

$$
W^{(J)}_{\pi\pi} = \int \frac{d^3p_1}{(2\pi)^3 2\varepsilon_1} \frac{d^3p_2}{(2\pi)^3 2\varepsilon_2} \delta \left( \Delta^{(J)} - \varepsilon_1 - \varepsilon_2 \right) \frac{d^3p_1}{(2\pi)^3 2\varepsilon_1} \frac{d^3p_2}{(2\pi)^3 2\varepsilon_2}.
$$

(20)
A numerical integration using the expressions (18) and (19) then yields
\[ \frac{\Gamma (2^3P_1 \rightarrow \chi_{c1} \pi \pi)}{\Gamma (2^3P_1 \rightarrow \chi_{c2} \pi \pi)} \approx 10^4, \] (21)
so that the transition \( 2^3P_1 \rightarrow \chi_{c2} \pi \pi \) is quite small in comparison to the \( 2^3P_1 \rightarrow \chi_{c1} \pi \pi \) one. Clearly, the main factor responsible for such strong dominance of the transition to the \( \chi_{c1} \) state is the enhancement by the conformal anomaly of the \( S \) wave production of the pions in the amplitude described by Eq. (17).

Using the expression (15) and the value of the charmed quark mass \( m_c \approx 1.4 \text{ GeV} \), one can also find the ratio of one- and two-pion decay rates
\[ \frac{\Gamma (2^3P_1 \rightarrow \chi_{c1} \pi^0)}{\Gamma (2^3P_1 \rightarrow \chi_{c1} \pi^+ \pi^-)} \approx 0.04. \] (22)
One can see from this estimate that for the case where \( X(3872) \) is considered to be a charmonium \( 2^3P_1 \) state the one-pion transition is significantly suppressed relative to the two pion one as a result of small isospin violation.

The absolute value of the discussed decay rates can be very approximately estimated by using the known rate of the decay \( \psi (2S) \rightarrow J/\psi (1S) \pi^0 \). Within the similar description the gluonic matrix element of the latter decay is found as
\[ \frac{1}{64} \langle \eta_m \psi_m | \xi^a r_i \xi^a S_k | \zeta_n \phi_n \rangle / 3 = \frac{5i}{\sqrt{6}} A_s \delta_{ij} \epsilon_{kmn} \psi_m^* \phi_n, \] (23)
so that the rate of the decay is given by
\[ \Gamma = \frac{4 X^2 p_{\pi}^3}{27 m_Q^2 2\pi} |A_s|^2, \] (24)
where the quantity \( A_s \) is an analog, relevant for the \( 2^3S_1 \rightarrow 1^3S_1 \) transition, of the amplitude \( A \) introduced in Eqs. (11) and (12). Although \( A_s \) and \( A \) arise from overlap integrals between different pairs of charmonium levels, we consider them as being of the same order, \( A \sim A_s \), in lieu of a more reliable understanding. Proceeding in this way, we estimate
\[ \Gamma (2^3P_1 \rightarrow \chi_{c1} \pi^0) \sim \frac{3}{4} \left( \frac{p_{\pi (1)}}{p_\pi} \right)^3 \Gamma (\psi' \rightarrow J/\psi \pi^0) \approx 0.06 \text{ keV}, \] (25)
where \( p_\pi \approx 574 \text{ MeV} \) is the pion momentum in the \( \psi (2S) \) decay. Even with a large uncertainty in this estimate one can conclude that the rate of the strongest of the discussed transitions from \( X(3872) \) treated as charmonium \( 2^3P_1 \) state, \( 2^3P_1 \rightarrow 1^3P_1 \pi \pi \), is expected to be very small.
3 Molecular state

Another possible scenario is to consider the $X(3872)$ as being a four quark state made from the two heavy and two light quarks. In this case it appears reasonable to treat the discussed transitions $X \rightarrow \pi\chi_{cJ}$ and $X \rightarrow \pi\pi\chi_{cJ}$ as a ‘shake off’ of the light quarks. In particular, the spin dependent ‘heavy-light’ quark interaction is proportional to the inverse power of the heavy quark mass $m_Q^{-1}$, so that any exchange of the polarization between the light and heavy degrees of freedom is expected to be suppressed. Neglecting such exchange the total wave function can be written as a product of the two wave functions describing the heavy and light degrees of freedom separately. For this purpose we define the polarization amplitudes $h^{(1,2)}_i$ and $l^{(1,2)}_i$ associated with the heavy and the light quark pairs in the initial and the final states. For the states with the definite $J$ we introduce the polarization amplitudes $\phi_i$ and $\psi_i$ corresponding to the initial and final states with $J = 1$ and the symmetrical and traceless tensor amplitude $\psi_{ij}$ for the final $3P_2$ state. All these amplitudes are assumed to be normalized in the standard way and sums over polarizations are the same as in Eq.(8).

Finally, introducing the operators $L^{(1,2)}_i$ and $H^{(1,2)}_i$ acting on the polarization amplitudes $l_i$ and $h_j$: $\langle h^{(2)}_i | H^{(2)}_s H^{(1)}_s | h^{(1)}_j \rangle \sim \delta_{ij}$ and $\langle l^{(2)}_i | L^{(2)}_m L^{(1)}_n | l^{(1)}_j \rangle \sim \delta_{im}\delta_{jn}$ one can write the amplitude for the one pion transition

$$A^{(J)}_\pi = \langle 3P_J | p_1 \epsilon_{lmn} L^{(2)}_m L^{(1)}_n H^{(2)}_s H^{(1)}_s | 3P_1 \rangle$$

with the initial and the final states

$$3P_1 = \epsilon_{ijk} h^{(1)}_j j^{(1)}_k \phi_i / \sqrt{6}, \quad 3P_J = P^{(J)}_{ij} h^{(2)}_i l^{(2)}_j,$$

where the projection matrices $P^{(J)}_{ij}$ are the same as defined in Eq.(10). Using these relations one can readily find the amplitudes of the one pion transition to the states with the different total spin:

$$A^{(0)}_\pi = 2 p_i \phi_i A_1 / \sqrt{3}, \quad A^{(1)}_\pi = \epsilon_{ijk} \phi_i \psi^*_j p_k A_1 / \sqrt{2}, \quad A^{(2)}_\pi = - \phi_i \psi^*_i p_j A_1.$$  (28)

The quantity $A_1$ accounts for the internal dynamics and at present can only be discussed on a model dependent basis. Taking the square of these expressions and summing over the polarization amplitudes of the final states complete the routine procedure of finding the square of the matrix element

$$|A^{(0)}_\pi|^2 = \frac{4}{3} p_\pi^2 |A_1|^2, \quad |A^{(1)}_\pi|^2 = p_\pi^2 |A_1|^2, \quad |A^{(2)}_\pi|^2 = \frac{5}{3} p_\pi^2 |A_1|^2.$$  (29)
This relation readily gives us the ratio of the partial decay widths for the one pion transition

\[ \Gamma_0 : \Gamma_1 : \Gamma_2 = 4p_\pi^3(0) : 3p_\pi^3(1) : 5p_\pi^3(2) \approx 2.88 : 0.97 : 1, \]  

(30)

where \( p_\pi(0) \approx 436 \text{ MeV} \) is the pion momentum in the transition to the \( \chi_{c0} \) state.

For the two-pion transition from a four-quark state essentially the only guidance is provided by general chiral properties, which require that the expansion of the amplitude in the pion 4-momenta starts with a bilinear term. In this order only \( S \) and \( D \) waves for the dipion are possible, and one can write the amplitude in terms of the explicit partial wave decomposition:

\[ A^{(J)}_{\pi\pi} = \langle 3P_J | L_m^{(2)} L_n^{(1)} (\delta_{mn} S + \Phi_{mn} D) H_s^{(2)} H_s^{(1)} | 3P_1 \rangle, \]  

(31)

where the traceless \( D \)-wave tensor \( \Phi_{ij} \) is defined in a standard way: \( \Phi_{ij} = p_i p_j + p_j p_i - 2/3 \delta_{ij} (\vec{p}_1 \cdot \vec{p}_2) \). In the leading order of the chiral expansion the \( S \) wave amplitude can generally be written as \( S = a q^2 + b \varepsilon_1 \varepsilon_2 + c m_s^2 \) with \( a, b \) and \( c \) being coefficients generally of order one, so that there is in fact no kinematical suppression of the \( D \)-wave in comparison with the \( S \)-wave, and both these terms enter the amplitude on equal footing. It can be also noted, once again, that no amplitude for transition to the \( 1^3P_0 \) state arises in this order of expansion.

One finds from the expressions (27) and (31) the amplitudes of the two pion transitions in the form

\[ A_{\chi_{c0}} = 0, \quad A_{\chi_{c1}} = (2\delta_{ij} S - \Phi_{ij} D) \phi_i \psi_j^* A_2/\sqrt{2}, \quad A_{\chi_{c2}} = D \varepsilon_{ijk} \phi_i \psi_j^* \Phi_{lk} A_2, \]  

(32)

where the quantity \( A_2 \) describes the internal properties of the transition and at present is model dependent. Using these expressions and following the standard prescription it is straightforward to find the averaged squares of the transition amplitudes

\[ |A_{\pi\pi}^{(1)}|^2 = |A_2|^2 \left\{ 2 |S|^2 + \frac{1}{3} \left[ \vec{p}_1^2 \vec{p}_2^2 + \frac{1}{3} (\vec{p}_1 \cdot \vec{p}_2)^2 \right] |D|^2 \right\}, \]  

(33)

\[ |A_{\pi\pi}^{(2)}|^2 = |A_2|^2 \left[ \vec{p}_1^2 \vec{p}_2^2 + \frac{1}{3} (\vec{p}_1 \cdot \vec{p}_2)^2 \right] |D|^2. \]  

(34)

As previously discussed, in this case there is generally no reason to expect an enhancement of the \( S \) wave over the \( D \) wave, so that the ratio of the two decay rates may be not as dramatic as in Eq.(21).
4 Summary

The described estimates of the rates of the pionic transitions from $X(3872)$ to the $\chi_{cJ}$ states of charmonium lead us to conclude that these rates exhibit significantly different patterns depending on the internal structure of the $X(3872)$. Namely, if this resonance is dominantly a charmonium $^3P_1$ state, the pionic transitions are quite weak with the process $X(3872) \rightarrow \pi\pi\chi_{c1}$ being dominant. The single pion transitions, proceeding through the isospin breaking by the light quark masses, are even more suppressed with the amplitude of the decay $X(3872) \rightarrow \chi_{c0}$ vanishing altogether as a result of a special form of the matrix element in Eq. (13). If however $X(3872)$ is a four-quark/molecular state with a significant isovector part in its wave function, the considered single pion transitions should be greatly enhanced and exhibit comparable rates for all three $\chi_{cJ}$ resonances in the final state, as described by Eq. (30). One can notice that the yield of the $\chi_{c0}$ in the final state in this case is the highest, while in the pure charmonium case this yield should be strongly suppressed. For the transitions with emission of two pions in the case of dominantly molecular $X(3872)$ we find that generally there is no reason to expect a very strong dominance of the production of the $\chi_{c1}$ resonance in the final state as compared to $\chi_{c2}$. The production of the $J = 0$ state, $\chi_{cJ}$, is expected to be strongly suppressed in either case due to the general properties of the soft-pion expansion. The relative rate of the single and double pion emission is practically impossible to estimate quantitatively at present for a molecular $X(3872)$. At a qualitative level, once the isospin suppression of the single pion processes is removed by the four-quark structure of $X(3872)$, one might expect a larger rate for the one pion transitions than for the two-pion processes, given the small energy release in the considered decays.

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