Virial theorem analysis of 3D numerical simulations of MHD self-gravitating turbulence

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Abstract. We discuss the virial balance of all members of a cloud ensemble in numerical simulations of self-gravitation MHD turbulence. We first discuss the choice of reference frame for evaluating the terms entering the virial theorem (VT), concluding that the balance of each cloud should be measured in its own reference frame. We then report preliminary results suggesting that a) the clouds are far from virial equilibrium, with the “geometric” (time derivative) terms dominating the VT. b) The surface terms in the VT are as important as the volume ones, and tend to decrease the action of the latter. c) This implies that gravitational binding should be considered including the surface terms in the overall balance.

1. Introduction

The virial theorem (VT) (Chandrasekhar & Fermi 1953) provides a direct way of analyzing the energy balance of a bounded region in a flow, describing the effect of various forces either in driving changes in the structure of a dynamical system or in determining the character of its equilibrium.

The virial theorem can be cast in either Eulerian or Lagrangian form. The latter applies to a fluid parcel following the flow, i.e., the volume $V$ and its bounding surface $S$ will generally be time-dependent. The Eulerian version of the virial theorem (EVT) applies to a fixed volume $V$ rather than a fixed mass (e.g., McKee & Zweibel 1992). This is best suited for application to fixed-grid, Eulerian numerical simulations. However, when considering a large region of the ISM, clouds constitute an ensemble in which each one is moving and morphing (see Vázquez-Semadeni, this volume). Thus, for studying the EVT we have two options:

(a) We can consider all clouds in the simulation, or “lab” frame (Ballesteros-Paredes & Vázquez-Semadeni 1997). However, in this case, the clouds’ bulk motions will appear as internal kinetic energy, and the energy budget of the volume defining the cloud is not exactly the energy budget of the cloud;
(b) We can consider a different frame for each cloud, so that the frame is moving with the cloud’s center of mass velocity with respect to the lab frame, and has its origin at the cloud’s center of mass.

In the present work, we briefly discuss the choice of reference frame, and then present preliminary statistical results of virial balance in two three-dimensional simulations of MHD isothermal turbulence with self-gravity at a resolution of $100^3$ grid points, and with rms Mach number 2.2. One simulation has a box size $L$ equal to the Jeans length $L_J$, and the other has $L = 2L_J$.

2. Virial Theorem in the lab and cloud frames

The transition from the lab frame is not completely straightforward. We first considered the possibility of staying in the lab frame, but taking a volume that moves with the velocity of the cloud’s center of mass, although maintaining a fixed shape in time. However, in this case, the form of the EVT is altered, and several extra terms appear that have no simple interpretation. Due to these difficulties, we finally have chosen to completely move to each cloud’s frame, as described above. In this case, the EVT remains in its usual form, at the expense that plots for the cloud ensemble comparing the various terms in the EVT contain data from many different frames – one for each cloud. The EVT is then

$$\frac{1}{2} \frac{d^2 I_E}{dt^2} = 2(\epsilon_{th} + \epsilon_{kin} - \tau_{th} - \tau_{kin}) + \epsilon_{mag} + \tau_{mag} + w - \frac{1}{2} \frac{d \Phi}{dt},$$

where $I_E = \int_V \rho r^2 dv$ is the moment of inertia of the cloud, $\epsilon_{th} = \frac{3}{2} \int_V \rho v^2 dV$ is the kinetic energy, $\tau_{th} = \frac{1}{2} \oint_S x_i \rho u_i n_j dS$ is the surface pressure term, $\epsilon_{kin} = \frac{1}{2} \int_V \rho u^2 dV$ is the thermal energy; $\tau_{kin} = \frac{1}{2} \oint_S x_i \rho u_i u_j n_j dS$ is the surface kinetic term; $\epsilon_{mag} = \frac{1}{8\pi} \int_V B^2 dV$ is the magnetic energy and $\tau_{mag} = \oint_S x_i T_{ij} n_j dS$ is the surface magnetic term, with $T_{ij}$ being the Maxwell stress tensor; $w = \oint_S \rho x_i \dot{g}_i dV$ is the gravitational term (not equal to the gravitational energy), and $\Phi = \oint_S \rho r^2 u_i n_i dS$ is the flux of moment of inertia through the surface of the cloud (sums over repeated indices are assumed).

We define the clouds in the numerical simulations as connected sets of pixels with densities larger than an arbitrary threshold value. In order to measure only the contribution associated to the fluctuations, we subtract from the velocity the bulk, mass-averaged velocity of the cloud, defined as $V_{cm} = \frac{1}{M} \int_V \rho u dV$, where $M$ is mass of the cloud, and subtract from the position of each pixel the position of the cloud’s center of mass. We consider several density thresholds to enlarge the cloud ensemble, and consider that a cloud maintains its identity as long as it does not split into several components upon increasing the threshold.

3. Preliminary Statistics

We find the following preliminary results, consistent with those of Ballesteros-Paredes & Vázquez-Semadeni (1997): The time derivative terms (i.e., the LHS and the last term in the RHS of equation [1]) are dominant in the overall virial balance, being much larger than the remaining volume and surface terms (fig.1).
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We refer to these terms as “geometrical”, since they correspond to the (time derivatives of the) mass distribution in the cloud and its flux through the cloud’s boundary. This implies that, far from being in quasi-hydrostatic equilibrium, the clouds are continually changing shape and “morphing”. 2. The surface terms, which are often neglected in virial-balance studies, have magnitudes comparable to those of the volumetric ones (fig.2). 3. However, the surface and the volume terms do not cancel out exactly, leaving a net contribution for shaping the clouds and balancing gravity. In fact, we propose that the correct diagnostic for whether a cloud will undergo gravitational collapse is \(|w| > 2(\epsilon_{th} + \epsilon_{kin} - \tau_{th} - \tau_{kin}) + \epsilon_{mag} + \tau_{mag} \); otherwise, the cloud is transient. Indeed, in the simulation with \(L = L_J\), two clouds are observed to be collapsing by the final time. These two clouds satisfy the above criterion (fig.3, left). Note that comparing the absolute value of gravitational term vs. only the volume term would suggest that none of the clouds be collapsing (fig.3, right), contrary to what occurs in the simulation. Thus, we conclude that the correct diagnostic for determining gravitational binding must include the contribution of the surface terms, or else the gravitational term is underestimated.

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References

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Figure 1. The absolute value of RHS vs. the absolute value of the $1/2 d\Phi/dt$ of eq. (1). Their near equality shows that the time-derivative term is the dominant one in the balance for the run with $L = L_J$.

Figure 2. Volume-vs.-surface balance of the thermal-plus-kinetic terms (left) and the magnetic terms (right) for the run with $L = L_J$. The near equality of both kinds of terms indicates that the contribution of the surface terms is comparable to that of the volume terms, implying partial cancellation.
Figure 3. The gravitational term $w$ vs. the RHS of eq. (1) without the $1/2d\Phi/dt$ term for the simulation with $L = 2L_J$. In the plot on the left, we label each cloud by a pair of numbers, one identifying the cloud, and the other giving the threshold density defining the cloud. This plot accurately represents the fact that only two clouds are collapsing in the simulation (i.e., clouds 1 and 2), while the other clouds are not collapsing. Comparing the absolute value of gravitational term vs. only the volume term would suggest that none of the clouds should be collapsing (right), contrary to what occurs in the simulation.