Kinematics Analysis of Incomplete Gear and Rack Pumping Unit

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Abstract. A pumping unit with incomplete gear and rack reversal was designed for smooth motion and low energy consumption. This paper first describes the working principle of the pumping unit. The kinematical equation of the pumping unit is established on the basis of analyzing the kinematic relationship between the first tooth meshing area, the normal meshing area and the final tooth meshing area of incomplete gear and rack. The kinematic relationship curve of the pumping unit is obtained by ADAMS simulation. The simulation results show that the theoretical model is correct. By comparing the results with the conventional pumping unit, it can be seen that: the pumping unit has a smooth movement of the up-down strokes, with fluctuations of movement only in the process of changing directions. The incomplete gear and rack drive is smooth, which can improve the kinestate of the pumping unit.

Keywords: Pumping unit; Incomplete gear and rack; Suspension point displacement; Kinematics analysis; Analysis of meshing relationship.

1. Introduction

As oil field production decreases faster, the pumping unit with good balance effect and low energy consumption is getting more and more attention¹⁻². Conventional beam pumping units have the advantages of simple structure and mature technology³, but there are problems such as poor balance effect, low motor load rate and high installed power⁴. Gear transmission has the advantages of stable transmission ratio, high transmission accuracy and reliable work⁵. In order to improve the performance of pumping units, scientific and technological workers have been committed to using gear drive in pumping units. Peng Yong et al.⁶ invented a rack and pinion pumping unit, which uses switched reluctance motor to directly realize reciprocating rotation. It has the characteristics of simple structure, small footprint, good balance and high efficiency. Chen Ruize et al.⁷ designed a pinion and rack double-well pumping unit. Under the same working conditions, the net torque fluctuation of the output shaft of the pumping unit is much smaller than that of the conventional pumping unit, which reduces the load ununiformity coefficient. Shi Qiang et al.⁸ designed a double donkey head pumping unit driven by tooth ring, with fixed tooth ring, the pumping unit has higher efficiency than the conventional pumping unit and can make full use of the original parts and components of the pumping unit to save costs.

To sum up, gear drive can make pumping unit drive more stable. This paper presents a pumping unit scheme based on incomplete gear and rack and analyzes its kinematics.

2. Structure and Working Theory

The structure of the pumping unit⁹ is shown in figure 1, which is mainly composed of power part, reversing mechanism and balance part. The power part is composed of motor and reducer. The reverse
mechanism consists of left gear 1, right gear 6, left incomplete gear 2, right incomplete gear 7, left rack 3, right rack 8, spring damper 9, moving frame 10, wherein left incomplete gear 2 can mesh with left rack 3, right incomplete gear 7 can mesh with right rack 8; Balance of pumping unit is achieved by the weight of moving frame 10.

The power of the pumping unit is transmitted to input gear 5 through the reducer, input gear 5 turns clockwise, and both left gear 1 and right gear 6, which are meshes with input gear 5, turn counterclockwise. In the up-stroke, when left incomplete gear 2 disengages from left rack 3, right incomplete gear 7 enters into meshing with right rack 8, and right incomplete gear 7 turns counterclockwise to drive right rack 8 to move downward, moving frame 10 to move downward, and the end connecting rope connected to it moves upward. In the down-stroke, left incomplete gear 2 on left shaft 11 turns counterclockwise, driving left rack 3 to move up, moving frame 10 to move up, and the end connecting rope to move down. Run for one cycle to complete the oil recovery operation.

1. Left gear 2. Left incomplete gear 3. Left rack 4. Positioning guide rail 5. Input gear 6. Right gear 7. Right incomplete gear 8. Right rack 9. Spring damper 10. Moving frame 11. Left axle 12. Right axle

**Figure 1.** Structure diagram of the pumping unit.

3. Analysis of Suspension Point Displacement

Figure 1 shows that right incomplete gear 7 meshes with right rack 8 during the upstroke. In the downstroke, left incomplete gear 2 meshes with left rack 3, that is, the suspension point displacement is composed of the above two parts. Let the suspension point displacement be \( L \) and the incomplete gear turning angle be \( \phi \), as shown in figure 2, with point G as the starting point (bottom dead centre), the upstroke(downstroke) can be subdivided into three motion areas: The first tooth meshing area; The normal meshing area; The final tooth meshing area.

**Figure 2.** Motion diagram of the reversing mechanism.
3.1. Up-stroke
In the up-stroke, the right incomplete gear meshes with the right rack, and the kinematic relationship analysis of the first tooth meshing area, the normal meshing area and the final tooth meshing area is shown below.

3.1.1. First tooth meshing area. As shown in figure 3, after addendum of the right incomplete gear moves to addendum line of the rack, first tooth point D of the right gear meshes with the rack point $C_{10}$ and enters the first tooth meshing area\(^{10-11}\), and line of action is $DB_{21}$. When first tooth of the right incomplete gear turns $\beta$ (rad), the rack downs $DD_{\beta}$, i.e. $L_4$ (mm). The relationship between $L_4$ and $\beta$ can be obtained from the sine theorem:

$$L_4(\beta) = \frac{r_D \sin(\beta + \beta_1)}{\sin \left( \frac{\pi}{2} + E - \beta - \beta_1 \right)}$$

$$\beta_{\text{max}} = E - E_1 - \beta_{\text{max}}$$

Where, $E$, $E_1$, $\beta_{\text{max}}$, $\beta_1$ can be obtained from equations (3), (4), (5) and (6). $r_D$ is the distance between the initial meshing point D on the right incomplete gear and the center of the circle $O_2$; $\beta_1$ is the motion radian of meshing point when first tooth of the right incomplete gear turns $\beta$; $\beta_{\text{max}}$ is maximum radian of the first tooth meshing area.

![Motion diagram of the first tooth meshing area.](image)

$$E = \arctan \left( \frac{y_D}{r_1 - h^*_a m} \right)$$

$$E_1 = \arctan \left( \frac{h^*_a m \tan \alpha}{r_1 - h^*_a m} \right)$$

$$\beta_{\text{max}} = \tan \alpha_D - \alpha_D - \tan \alpha_{\beta_{\text{max}}} + \alpha_{\beta_{\text{max}}}$$

$$\beta_1 = \frac{\theta_\beta - \theta_D}{2} = \tan \alpha_D - \alpha_D - \tan \alpha_{\beta} + \alpha_{\beta}$$

$$r_D = \frac{r_1 - h^*_a m \cos E}{\cos E_1}$$

$$r_\beta = \left[ (y_D - L_1)^2 + (r_1 - h^*_a m)^2 \right]^{1/2}$$

$$r_{\beta_{\text{min}}} = \frac{r_1 - h^*_a m}{\cos E_1}$$

$$\alpha_{\beta_{\text{max}}} = \arccos \left( \frac{r_1 \cos \alpha}{r_{\beta_{\text{min}}}} \right)$$

$$y_D = r_{a1} \sin (\phi_{\text{max}} + K) - s_e$$

$$K = \alpha_{a1} - \alpha$$

Where, $r_1$ is reference radius; $r_{a1}$ is tip radius; $\alpha$ is pressure angle; $\alpha_{a1}$ is tip pressure angle; $s_e$ is space width; $y_D$ is the distance from the initial meshing point D to point O, that is, $DO$ in Figure 3; $r_\beta$ is the distance between the meshing point $D_{\beta}$ on the right incomplete gear and the center of the circle $O_2$; $\theta_\beta$ is the radian corresponding to the arc tooth thickness at the initial meshing point D of the right gear; $\theta_D$ is the radian corresponding to the arc tooth thickness of the meshing point $D_{\beta}$ after the right
incomplete gear turns β; α₀ is the pressure angle at point D of the gear involute; α₁ is the pressure angle of point D₁ of the gear involute[12-14]; ϕ_max can be obtained from equation (16).

3.1.2. **Normal meshing area.** When the right incomplete gear and rack motion to the normal meshing area, the relationship between the radian of the right incomplete gear motion γ(rad) and the rack drop distance L₂(mm) can be expressed as follows:

\[
L_2(γ) = r_1γ
\]

\[
γ_{\text{max}} = π - ϕ_{\text{max}} - β_{\text{max}}
\]

Where, γ_max is the maximum radian in the normal meshing area.

3.1.3. **Final tooth meshing area.** As shown in figure 4, when point C of the right incomplete gear moves to B, addendum of final tooth of the right incomplete gear starts to mesh with the rack and enters the final tooth meshing area. Line of action is \(\overline{BB_{22}}\). The final tooth of the right incomplete gear rotate φ(rad), the rack drops distance \(\overline{BB_{23}}\), i.e. \(L_3\)(mm). The relation between \(L_3\) and φ can be obtained from the sine theorem:

\[
L_3(φ) = \frac{2r_{a1} \sin \left(\frac{φ}{2}\right) \cos \left(\frac{φ}{2} + α_{a1}\right)}{\cos α}
\]

\[
φ_{\text{max}} = \arccos \left(\frac{r_1 - h_{a}m}{r_{a1}}\right) - K
\]

Where: φ_max is the maximum radian of the final tooth meshing area.

![Figure 4. Motion diagram of the final tooth meshing area.](image)

3.2. **Down-stroke**

In the down-stroke, the left incomplete gear meshes with the left rack, and it’s the first tooth meshing area, the normal meshing area and the final tooth meshing area have the same magnitude of the motion relationship, but their directions are opposite. Therefore, the motion relation can be expressed as:

The motion relationship of the first tooth meshing area is:

\[
L_4(β) = -\frac{r_D \sin(β + β_1)}{\sin \left(\frac{π}{2} + E - β - β_1\right)}
\]

The motion relationship of the normal meshing area is:

\[
L_5(γ) = -r_1γ
\]

The motion relation of the final tooth meshing area is:

\[
L_6(φ) = -\frac{2r_{a1} \sin \left(\frac{φ}{2}\right) \cos \left(\frac{φ}{2} + α_{a1}\right)}{\cos α}
\]

3.3. **Suspension Point Displacement**

According to equations (1), (13), (15), (17), (18) and (19), the relationship between suspension point displacement \(L\) and the incomplete gear rotation angle \(φ\)(rad) can be obtained, i.e. \(L = f(φ)\), mm.
Where, $L_{d}$ is the stroke length, and its value is
$$L_{d} = L_{1}(\phi) + L_{2}(\phi - \beta_{\text{max}}) + L_{3}(\phi - \beta_{\text{max}} - \gamma_{\text{max}}) + L_{4}(\phi - \pi) + L_{5}(\phi - \pi - \beta_{\text{max}}) + L_{6}(\phi - \pi - \beta_{\text{max}} - \gamma_{\text{max}}).$$

4. Suspension Point Velocity and Acceleration

The first-order and second-order derivatives of both sides of equation (20) with respect to time obtain the suspension point velocity (mm/s) and acceleration (mm/s$^2$), respectively.

$$v = \frac{df(\phi)}{d\phi} \frac{d\phi}{dt} = \frac{df(\phi)}{d\phi} \omega$$

$$a = \frac{d^2f(\phi)}{d\phi^2} \omega^2 + \frac{df(\phi)}{d\phi} \frac{d\omega}{dt}$$

Where, $\omega$ is angular velocity of the incomplete gear, rad/s.

5. Calculation and Analysis of Example

Taking the pumping unit stroke $n = 4$ min$^{-1}$, i.e. $\omega = \frac{2\pi}{15}$ rad/s as an example, the geometric parameters of incomplete gear (as shown in table 1) and $\omega$ are substituted into equations (20), (21) and (22) to obtain the suspension point displacement, velocity and acceleration, and compare the kinematic relationship of the conventional pumping unit under the same working conditions[15]. Matlab is used to make kinematics curves of two pumping units, as shown in figure 5, 6 and 7.

Table 1. Main geometric parameters of the incomplete gears.

| Parameter                  | Value | Parameter                  | Value |
|----------------------------|-------|----------------------------|-------|
| Number of complete teeth   | 70    | Module (mm)                | 25    |
| Number of actual teeth     | 29    | Addendum coefficient       | 0.85  |

Figure 5. Suspension points displacement curves of the two pumping units.
Figure 6. Suspension points velocity curves of the two pumping units.

Figure 7. Suspension points acceleration curves of the two pumping units.

From figure 5, 6 and 7, it can be seen that suspension point velocity of the pumping unit is constant in the up-down stroke, and the velocity fluctuates during the reversing process. Suspension point acceleration of the is zero in the up-down stroke, can reduce inertia load of the suspension point, only in the reversing process acceleration fluctuations. Compared with the conventional pumping unit, the pumping unit adopts the incomplete gear and rack as the reversing mechanism, with smooth motion in the up-down stroke and motion fluctuations only during the reversing process.

Solidwords is used to build a 3D model of pumping unit, and the simplified model is imported into ADAMS. After setting the kinematic pair of each part, the contact force between incomplete gear and rack is set based on Impact function method \cite{16,17}. Finally, the driving angular velocity function of the left and right incomplete gears is set as step(time;0, 0.1, 0, 0.3, 24d). The simulation step size is 0.01s and the termination time is 16s, and the suspension point motion curve is obtained, as shown in figure 8.

Figure 8. Simulation diagram of the motion curve of the suspension point.

By comparing figure 8 with figure 5, 6, 7, it can be seen that the theoretical analysis results of the suspension point motion of the pumping unit are consistent with the simulation results, indicating the accuracy of the theoretical analysis.
6. Conclusion
The kinematics analysis shows that the meshing motion of the pumping unit is divided into six parts: the first tooth meshing area, normal meshing area, and final tooth meshing area of the left incomplete gear and the first tooth meshing area, normal meshing area, and final tooth meshing area of the right incomplete gear. The theoretical analysis and ADAMS simulation show that the suspension point velocity of the pumping unit only fluctuates in the reversal process; the suspension point acceleration is zero in up-down stroke, and the acceleration only fluctuates in the reversal process. The motion is stable, which is beneficial to reduce energy consumption.

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