Abstract—Propagations of coherent and partially coherent flat-topped beams through a focusing optical system are formulated. The radiation force on a Rayleigh dielectric sphere induced by focused coherent and partially coherent flat-topped beams is investigated theoretically. It is found that we can increase the transverse trapping range at the planes near the focal plane by increasing the flatness (i.e., beam order) of the flat-topped beam, and we can increase the transverse and longitudinal trapping ranges at the focal plane by decreasing the initial coherence of the flat-topped beam. The trapping stability is also analyzed.

I. INTRODUCTION

In recent years, light beams with flat-topped profiles have attracted more and more attention due to their wide applications in free-space optical communications, inertial confinement fusion, material thermal processing, nonlinear optics and electron acceleration [1-8]. Several theoretical models such as super Gaussian beam, flattened Gaussian beam, flat-topped beam and flat-topped multi-Gaussian beam have been proposed to describe a laser beam with flat-topped beam profile [9-12].

Recently, flat-topped beams were extended to the partially coherent case. Several theoretical models have been proposed to describe a partially coherent beam with flat-topped beam profile [13, 14]. Baykal and Eyyuboğlu studied the scintillations of incoherent flat-topped Gaussian beam in turbulence [15]. More recently, Wang and Cai reported experimental generation of a partially coherent flat-topped beam [16].

The use of radiation pressure for trapping and manipulation of particles is a subject of great interest. In 1970, Ashkin first demonstrated how to capture and manipulate micro-sized particles by using the radiation pressure [17]. Since then, this new technology has found wide applications in manipulating various particles such as micro-sized dielectric particles, neutral atoms, cells, DNA molecules, and living biological cells [18-21]. Up to now, the trapping characteristics of different beams, such as Gaussian beam, Laguerre-Gaussian beam, Hermite-Gaussian beam, evanescent fields, radially polarized beam, Gaussian Schell-model beam and pulsed Gaussian beam have been studied [17, 18, 23-27]. It has been found that the radiation forces produced by a laser beam are closely related to its beam characteristics such as beam profile, coherence and polarization. In this paper, we investigate the radiation force produced by focused coherent and partially coherent flat-topped beams on a Rayleigh particle. Some interesting and useful results are found.

II. THE CHARACTERISTICS OF COHERENT AND PARTIALLY COHERENT FLAT-TOPPED BEAMS

In this paper, we adopt the model for a flat-topped beam proposed by Li [11]. By applying the tensor ABCD law [28, 29], we can express the electric field of a flat-topped beam after passing through a general astigmatic (i.e., nonsymmetrical) optical system as follows [30]

\[
E_{out-N}(p_1, z) = E_{in} \sum_{n=0}^{N} \frac{(-1)^n}{n!} \binom{N}{n} \left[ \text{det}(A + BQ_{in}^n) \right]^{1/2} \exp \left( -\frac{i}{2} \vec{p}^T Q_{in}^{-1} \vec{p} \right)
\]

where det stands for the determinant, \( \vec{r} \) is the position vector in the output plane given by \( \vec{r} = (\rho_1, \rho_2) \). A, B, C and D are the sub-matrices of the general astigmatic optical system, \( Q_{in}^{-1} \) and \( Q_{2n}^{-1} \) are related by following tensor ABCD law

\[
Q_{in}^{-1} = \begin{pmatrix}
-2ni/kw_0 & 0 \\
0 & -2ni/kw_0
\end{pmatrix}
\]

\[
Q_{2n}^{-1} = (C + DQ_{in}^n)^{-1}(A + BQ_{in}^n)
\]

where \( w_0 \) is the waist size of the fundamental Gaussian mode, \( \binom{N}{n} \) denotes a binomial coefficient, and \( N \) is called the beam order of the flat-topped beam, \( E_{0N} \) is a normalization constant. The intensity of a coherent flat-topped beam at the output plane is given by

\[
I_{out-N}(\rho_1, \rho_2, z) = \left| E_{out-N}(\rho_1, \rho_2, z) \right|^2
\]

For the more general case, we need to take the coherence of light into consideration. It is well known that a partially coherent beam can be characterized by the cross-spectral density. The cross-spectral density \( (\tau=0) \) of a partially coherent beam generated by a Schell-model source can be expressed in the following well-known form [31]. We can express the cross-spectral density of a partially coherent flat-topped beam after passing through a general astigmatic optical system as follows
\( \Gamma_{\text{out}}(\rho_1, \rho_2, z) = I_0 \sum_{n=0}^{N} \sum_{m=1}^{N} \frac{(-1)^{n_m}}{N!} (N)_{n}^{m} \left[ \text{det}(\mathbf{X} + \mathbf{B} \mathbf{M}^{-1}_{\text{lum}}) \right] \exp \left[ -\frac{ik}{2} \mathbf{\hat{r}}^{T} \mathbf{M}^{-1}_{\text{2nn}} \mathbf{\hat{r}} \right] \) 

where \( \hat{\mathbf{r}}^{T} = (\rho_1^T, \rho_2^T) \), \( \hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}} \) and \( \hat{\mathbf{D}} \) are defined as follows:

\[
\begin{align*}
\hat{\mathbf{A}} &= \begin{pmatrix} \mathbf{A}I & 0 \\ 0 & \mathbf{A}I \end{pmatrix}, \quad \hat{\mathbf{B}} = \begin{pmatrix} \mathbf{B}I & 0 \\ 0 & -\mathbf{B}I \end{pmatrix}, \\
\hat{\mathbf{C}} &= \begin{pmatrix} \mathbf{C}I & 0 \\ 0 & -\mathbf{C}I \end{pmatrix}, \quad \hat{\mathbf{D}} = \begin{pmatrix} \mathbf{D}I & 0 \\ 0 & \mathbf{D}I \end{pmatrix}.
\end{align*}
\]

\( \mathbf{M}^{-1}_{\text{lum}} \) and \( \mathbf{M}^{-1}_{\text{2nn}} \) are related by following tensor ABCD law

\[
\mathbf{M}^{-1}_{\text{lum}} = \begin{pmatrix} (-2ni/k\omega_0^2 - i/k\sigma_0^2) & i/k\sigma_0^2 \\ i/k\sigma_0^2 & (-2ni/k\omega_0^2 - i/k\sigma_0^2) \end{pmatrix}
\]

\[
\mathbf{M}^{-1}_{\text{2nn}} = \left( \mathbf{C} + \mathbf{D} \mathbf{M}^{-1}_{\text{lum}} \right) / \left( \mathbf{A} + \mathbf{B} \mathbf{M}^{-1}_{\text{lum}} \right)
\]

The intensity of a partially coherent flat-topped beam at the output plane is given by

\[
I_{\text{out}}(\rho_1, \rho_2, z) = \Gamma_{\text{out}}(\rho_1, \rho_2, z).
\]

III. Radiation Force Produced by Focused Coherent and Partially Coherent Flat-Topped Beams on a Rayleigh Particle

Now we study the radiation force produced by focused coherent and partially coherent flat-topped beams on a Rayleigh dielectric sphere, whose radius is much smaller than the wavelength of the laser beam (i.e., \( a << \lambda \)). The schematic for trapping a Rayleigh dielectric sphere with a focused flat-topped beam is same with Fig.1 of Ref [26]. A Rayleigh dielectric sphere with refractive index \( n_p \) is placed near the focus. In this case, the particle is treated as a point dipole. It’s well known that there are two kinds of the radiation force: scattering force and gradient force. The scattering force \( F_{\text{scat}} \) caused by the scattering of light by the sphere is proportional to light intensity and is along the direction of light propagation. The scattering force can be expressed as

\[
\vec{F}_{\text{Scat}}(r, z) = \hat{\vec{\sigma}} n_p \alpha d_{\text{out}} / c
\]

where \( d_{\text{out}} \) is the intensity of the focused beam at the output plane, \( \hat{\vec{\alpha}} \) is a unity vector along the beam propagation, \( \alpha = (128\pi^2 a^6 / 3\lambda^4)((m^2 - 1)/(m^2 + 2))^2 \), \( m = n_p / n_m \) with \( n_m \) being the refractive index of the ambient. The gradient force \( F_{\text{Grad}} \) produced by non-uniform electromagnetic fields is along the gradient of light intensity, and is expressed as

\[
F_{\text{Grad}}(r, z) = 2m n_m \beta \nabla I_{\text{out}} / c
\]

where \( \beta = a^3 (m^2 - 1)/(m^2 + 2) \). By applying the Eqs. (1-3), (4-6), (7) and (8), we can calculate the radiation force induced by focused coherent and partially coherent flat-topped beams on a Rayleigh dielectric sphere. Without loss of generality, we choose the radius of the particles to be \( a = 50 \text{ nm}, f = 10 \text{ mm}, \quad w_0 = 10 \text{ mm}, \quad \lambda = 632.8 \text{ nm} \), the refractive index of the particle to be \( n_p = 1.59 \) (i.e., glass) and the refractive index of the ambient to be \( n_m = 1.33 \) (i.e., water).

We calculate in Fig. 1 (a)-(c) the scattering force (cross line \( y=0 \)) and in Fig. 1 (d)-(f) the transverse gradient force (cross line \( y=0 \)) of a coherent flat-topped beam for four different values of \( N \) at different positions \( z \), and in Fig. 1 (g) and (h) the longitudinal gradient force for two different transverse positions \( x \). The sign of radiation force means the direction of the force. Positive \( F_{\text{scat}} \) means the direction of the scattering force is along \( +z \) direction. Positive \( F_{\text{Grad}} \) or \( F_{\text{Grad}} \) means the direction of the gradient force is along \( +x \) or \( -x \) direction. One finds from Fig. 1 that both scattering force and gradient force decreases as the initial beam order \( N \) increases (i.e., beam profile becomes more flat), and the forward scattering force always is much smaller than the longitudinal gradient force. So the scattering force in this case can be neglected. From Fig. 1(d) and (g), one finds that at the focal plane, there is one stable equilibrium point, and we can use focused flat-topped beam to trap a Rayleigh particle whose refractive index is larger than the ambient at the focus. As the initial beam order \( N \) increases, the trapping stability decreases due to the decrease of radiation force, and both transverse trapping range and longitudinal trapping range becomes smaller (i.e., the positions of peak values approach to the focus as \( N \) increases). So a coherent flat-topped beam \( (N>1) \) at the focal plane doesn’t offer advantage for trapping a Rayleigh particle over a Gaussian beam. From Fig. 1 (e), (f), one finds that we can increase the transverse trapping range by increasing the initial beam order \( N \) suitably at the planes near the focal plane. But the initial beam order \( N \) can’t be very large, because the trapping stability decreases as \( N \) increases. We also note from Fig. 1 (h), because there are two stable equilibrium points, the particle can’t be stably trapped along the \( z \) direction at off-axis points. To solve this problem, we can use two vis-à-vis focused flat-topped beams to construct a true optical potential well or “optical bottle”. To check for stability, we can interrupt one beam for a moment, which causes the particle to be accelerated rapidly in the remaining beam along its propagation direction. When another beam is turned on again, the particle is decelerated slowly and returns to its equilibrium region. So two vis-à-vis focused flat-topped beams can be used to trap a particle stably at on-axis and off-axis points.

Now we study the influence of coherence on radiation force produced by a partially coherent flat-topped beam. We calculate in Fig. 2 (a)-(c) the scattering force (cross line \( y=0 \)) and in Fig. 2 (d)-(f) the transverse gradient force (cross line \( y=0 \)) of a partially coherent flat-topped beam with \( N=3 \) for different values of \( \sigma_0 \) at different positions \( z \), and in Fig. 2 (g) and (h) the longitudinal gradient force for two different
transverse positions $x$. One finds from Fig. 2 that scattering force, transverse and longitudinal gradient forces decrease as the coherence of the initial flat-topped beam decreases. The forward scattering force also is much smaller than the longitudinal gradient force. From Fig. 2 (d) and (g), we find that at the focal plane, as the initial coherence of flat-topped beam decreases, both transverse and longitudinal trapping ranges become larger (i.e., the positions of peak values deviate away from the focus as $\sigma_0$ decreases), while the trapping stability decreases due to the decrease of radiation force. From Fig. 2 (e), (f) and (h), we find at the planes near the focal plane, both trapping range and stability decreases as the initial coherence of flat-topped beam decreases. Similarly, we can use two vis-à-vis focused partially coherent flat-topped beams to trap a particle stably at on-axis and off-axis points.

From above discussions, we can come to the conclusion that by increasing the flatness of the beam profile (i.e., increasing beam order $N$) of the flat-topped beam, we can increase the transverse trapping range at the planes near the focal plane. By decreasing the initial coherence of the flat-topped beam, we can increase the transverse and longitudinal trapping ranges at the focal plane. In both cases, the stability of trapping decreases, so it is necessary to choose suitable beam order $N$ and initial coherence (i.e., $\sigma_0$) in order to increase trapping range. This conclusion is the main results of present paper.

IV. ANALYSIS OF THE TRAPPING STABILITY

In this section, we analyze the trapping stability in more detail by taking the Brownian motion into consideration. We know that the particle usually suffers the Brownian motion due to the thermal fluctuation from the ambient (water). Following the fluctuation-dissipation theorem of Einstein, the magnitude of the Brownian force is expressed as

$$F_B = \frac{3}{2} \frac{\pi \eta a k_B T}{12}$$

where $\eta$ is the viscosity of the ambient (in our case, $\eta = 7.977 \times 10^{-1} \text{Pas}$ at the $T = 300K$), $a$ is the radius of the particle and $k_B$ is the Boltzmann constant. Then we obtain (after calculation) the magnitude of the Brownian force $F_B = 2.5 \times 10^{-3} \text{pN}$. Comparing the radiation forces in Figs. 1 and Fig. 2, we can find that both scattering force and gradient force in our numerical examples are all larger than the Brownian force. So Brownian motion doesn’t influence the main conclusion of present paper. We calculate in Fig. 3 the dependences of the radiation forces $F_{\text{Scat}}^\text{Max}$, $F_{\text{Grad}-x}^\text{Max}$ and $F_{\text{Grad}-z}^\text{Max}$ induced by a flat-topped beam on initial beam order $N$ and initial coherence (i.e., $\sigma_0$) at $z_1 = 0$. For comparison, Brownian force $F_B$ is also shown in Fig. 3. From Fig. 3 (a), one finds that both scattering force and gradient force decrease as $N$ increases, which is consistent with Fig. 1. When $N = 15$, the scattering force equals to Brownian force $F_B$, but the gradient force are still much larger than the scattering force and Brownian force $F_B$, so a flat-topped beam with $N = 15$ can still be used to trap a particle. When $N = 20$, the gradient force nearly equals to the Brownian force $F_B$, so a flat-topped beam with $N \geq 20$ can’t be used for trapping a particle. From Fig. 2, one finds that both scattering force and gradient force decrease as the initial coherence of flat-topped beam decreases, which is consistent with Fig. 3. When $\eta$ is smaller than 0.04, we can’t use a partially coherent flat-topped beam for trapping a particle because the Brownian force is larger than the radiation force. The line Q in Fig. 3 (a) and (b) can be regarded as the critical line. From above discussions, we come to conclusion that the trapping stability decrease as the initial beam order increases or initial coherence decreases. So it is necessary to choose suitable values of $N$ and $\sigma_0$.
increases or as the initial coherence decreases. So it is stability decreases as the flatness of flat-topped beam. We have also found that the trapping focal plane by decreasing the initial coherence of the increase the transverse and longitudinal trapping ranges at the flatness (i.e., beam order) of flat-topped beam, and we can

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V. CONCLUSION

In conclusion, we have studied the focusing properties of coherent and partially coherent beams, and have studied the radiation force on a Rayleigh dielectric sphere induced by focused coherent and partially coherent flat-topped beams. We have found that we can increase the transverse trapping range at the planes near the focal plane by increasing the flatness (i.e., beam order) of flat-topped beam, and we can increase the transverse and longitudinal trapping ranges at the focal plane by decreasing the initial coherence of the flat-topped beam. We have also found that the trapping stability decreases as the flatness of flat-topped beam increases or as the initial coherence decreases. So it is necessary to choose suitable beam order and initial coherence of a flat-topped beam in order to trap a particle. Our results are interesting and useful for particle trapping.

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