Axigluon on like-sign charge asymmetry $A_{s\ell}^b$, FCNCs and CP asymmetries in $B$ decays

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Abstract

A non-universal axigluon in generalized chiral color models leads to flavor changing neutral currents (FCNCs) at tree level. We analyze phenomenologically the new contributions to $B_q$ ($q=\text{d, s}$) mixing and the related CP asymmetries (CPAs) that are generated by axigluon exchange. We find that although $\Delta m_{B_q}$ can give a strict constraint on the parameters of $b \to q$ transition, the precise measurement of $\sin 2\beta_{J/\Psi K^0}$ can further exclude the parameter space of $b \to d$ transition. The axigluon-mediated effects can enhance the like-sign dimuon charge asymmetry $A_{s\ell}^b$ by one order of magnitude larger than the standard model prediction. Accordingly, large CPA $\sin 2\beta_{J/\Psi K^0}$ and CPA difference $\sin 2\beta_{J/\Psi K^0} - \sin 2\beta_{\phi K^0}$ are achieved.

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I. INTRODUCTION

In the standard model (SM), with three families of quarks, the unique CP violating phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix can explain some of the observed CP violating phenomena in $K$ and $B$ systems. However, the failure of the KM phase in explaining the matter-antimatter asymmetry and some recent measurements of CP violating observations in $B$ meson mixings and decays motivates the search for new source of CP violation (CPV). Therefore, it is an important issue to explore and to find new CP violating effects in various systems, such as cosmos, Large Hadron Collider (LHC), Tevatron, $B$ factories etc.

Recently, several hints for the existence of new CP violating sources are revealed in experiments. The first hint is observed in the CP asymmetries (CPAs) of $B \rightarrow \pi K$ decays where by naive SM estimation, one expects that $\bar{B}_d \rightarrow \pi^+K^-$ and $B^- \rightarrow \pi^0K^-$ decays have similar CPAs. However, it is surprising that the world average difference between the two CPAs contradicts the expectation as the experimental result is $\Delta A_{CP} = A_{CP}(\pi^+K^-) - A_{CP}(\pi^0K^-) = -(14.8^{+1.3}_{-1.4})\%$, \(1\) whereas the SM prediction is $\Delta A_{CP}(SM) = 0.025 \pm 0.015$ \(2\). The large deviation from the SM prediction indicates a puzzle in the asymmetries and it is introduced in the literature as $B \rightarrow \pi K$ puzzle \(3\). The second hint is observed in the time-dependent CPA of $B_s$ system, where CDF and DØ have shown an unexpected large CP phase in the mixing-induced CPA for $B_s \rightarrow J/\Psi \phi$ and the two possible solutions are given by $\beta_{s}^{J/\Psi\phi} = 2\beta_{s} + 2\phi_{s}^{NP} = -0.75^{+0.32}_{-0.21}$ or $-2.38^{+0.25}_{-0.34}$ \(2\) at 90% confidence level (CL). Here, $\beta_{s} \approx -0.019$ \(4\) is the SM CP violating phase and $\phi_{s}^{NP}$ is the CP violating phase of new physics. The significant deviation from the SM prediction could be speculated by the contributions of new physics.

The third hint is observed in the like-sign charge asymmetry which is defined as $A_{b} = \frac{N_{b}^{++} - N_{b}^{--}}{N_{b}^{++} + N_{b}^{--}}$, \(3\) where $N_{b}^{++(--)}$ denotes the number of events that $b$- and $\bar{b}$-hadron semileptonically decay into two positive (negative) muons. Recently, DØ Collaboration has announced the measurement
on $A_{s\ell}^b$ in the dimuon events [5] with

$$A_{s\ell}^b = (-9.57 \pm 2.51 \text{(stat)} \pm 1.46 \text{(syst)}) \times 10^{-3}. \quad (4)$$

The SM prediction is $A_{s\ell}^b = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [5, 6]. If the semileptonic b-hadron decays do not involve CP violating phase, then the charge asymmetry is directly related to the mixing-induced CPAs in $B_{d,s}$- and $B_s$-meson oscillations. Although the errors of the data are still large, however the 3.2 standard deviations from the SM prediction can be attributed to new CP violating phases in $b \to d$ and $b \to s$ transitions [7–10].

In order to explore the new physics and to avoid the uncontrollable QCD uncertainties, we will concentrate our study on the mixing parameter $\Delta m_{B_q}$, the charge asymmetry $A_{s\ell}^b$ and the time-dependent CPA in $B_q$ oscillation, where QCD effects can be controlled well by Lattice QCD.

In the literature, many extensions of the SM such as chiral color models [11–16], $Z'$ models [17–19], etc have been proposed. The flavor non-universal axigluon in the generalized chiral color models [20, 21] has been studied for solving the anomalous forward-backward asymmetry (FBA) in the top-quark pair production at the Tevatron [22, 23]. Although other models such as $Z'$, diquarks models [24] etc may have significant contributions to the FBA, however, large gauge couplings and flavor changing effects should be introduced in which chiral color model does not need. Inspired by the effects of the axigluon on the top-quark FBA, we study the axigluon-mediated phenomena in $B$-meson system.

A flavor universal axigluon has flavor-conserving effects only. For non-universal axigluon which has different couplings to different quarks, flavor changing neutral currents (FCNCs) can be generated at tree level. This is achieved after transforming the weak eigenstates of the quarks into their physical eigenstates. As a consequence, many phenomena will be affected by these FCNC effects. In this paper, we analyze in detail the non-universal axigluon contributions to the time-dependent CPAs in $B_q$ oscillation after taking into account the constraint from the mixing parameter $\Delta m_{B_q}$.

This paper is organized as follows. In Sec. III, we formulate the interactions of $b \to q$ transitions which are induced by flavor non-universal axigluon exchange. Accordingly, we derive the corresponding effective Hamiltonian for $\Delta B = 1,2$ processes. Furthermore, we discuss the contributions of the axigluon to the charge asymmetry $A_{s\ell}^b$ and the time-dependent CPAs for $B_d \to J/\psi K^0$, $B_d \to \phi K^0$, $B_s \to J/\Psi \phi$ decays. The detailed numerical
II. FORMALISM

In order to study the contributions of the non-universal axigluon to the FCNC processes, we start by writing the interactions of the massive color-octet gauge boson with quarks as

$$\mathcal{L}_A = g_V \bar{q}' \gamma_\mu T^b q' G_A^{b\mu} + g_A \bar{q}' \gamma_\mu \gamma_5 Z T^b q' G_A^{b\mu},$$

(5)

where we have suppressed the flavor and color indices, $g_{V,A}$ are the gauge couplings of the new gauge group $SU(3)_A \times SU(3)_B$, $T^b$ are the Gell-Mann matrices which are normalized by $Tr(T^b T^c) = \delta^{ac}/2$ and $Z$ is a $3 \times 3$ diagonalized matrix with $\text{diag}(Z) = (1, 1, \zeta)$. Here $\zeta = \tilde{g}_A/g_A$ where $\tilde{g}_A$ denotes the gauge coupling of the third-generation quark and its value depends on a specific model, e.g. $\zeta = -1$ in Ref. [21]. For simplicity, we assume that the new exotic quarks which are required for anomaly free are very heavy and their effects are negligible. Hence, we still focus on three flavors for each up and down type quarks. Following the scenario in Refs. [20, 21] for solving the large top-quark FBA, we assume that the axigluon couplings to the third generation are different from their couplings to the first two generations. The left- and right-handed quarks are $SU(2)$ doublet and singlet respectively. Thus, after spontaneous symmetry breaking, the interacting and physical eigenstates can be related by unitary matrices as $q_\chi = V_Q^{\chi'} q'$ with $\chi$ being the chiralities $L$ and $R$ and $Q$ being up or down type quarks. Since $Z$ is not a unit matrix, the FCNCs are arisen from the axial-vector currents and the corresponding Lagrangian is given by

$$\mathcal{L}_{\text{FCNC}} = g_A \bar{q}' \gamma_\mu (V_Q^{\chi'} Z V_Q^{\chi} P_R - V_L^{\chi} Z V_L^{\chi'} P_L) T^b q' G_A^{b\mu},$$

(6)

with $P_{L(R)} = (1 \mp \gamma_5)/2$. Since $V_{\chi}^Q$ are unknown matrices, the FCNCs are associated with left and right-handed currents generally. Nevertheless, if $V_R^{\chi} = V_L^{\chi}$, from Eq. (6) we see that the FCNCs are only associated with axial-vector currents. In terms of the flavor indices, the matrix $V_{\chi}^Q Z V_{\chi}^{Q'}$ can be decomposed as

$$\left(V_{\chi}^Q Z V_{\chi}^{Q'}\right)_{ij} = \delta_{ij} + (V_{\chi}^Q (Z - 1) V_{\chi}^{Q'})_{ij} = \delta_{ij} + (\zeta - 1)(V_{\chi}^Q)_{i3}(V_{\chi}^{Q'})_{j3}. \quad (7)$$

Therefore, the Lagrangian of $b \rightarrow q$ transition can be written as

$$\mathcal{L}_{b \rightarrow q} = g_A \bar{q}' \gamma_\mu (F_{qb}^{QR} P_R - F_{qb}^{QL} P_L) T^b G_A^{Qb\mu}$$

(8)
with $F_{qb}^{Q_3} = (\zeta - 1)(V_Q^R)_{i3}(V_{Q_3}^{Q_3})_{33}$ where $i = (1, 2, 3)$ denotes the family order of the same type $Q$ quark. Based on Eq. (8), we study the impacts of non-universal axigluon exchange on $\Delta B = 2$ processes and the time-dependent CPAs in $B_q$ system.

By Eq. (8), the effective Hamiltonian for $\Delta B = 2$ transitions which is generated by the tree-level axigluon mediation can be written as

$$
H^A_{\Delta B = 2} = \frac{g_2^2}{4m_V^2} \left[ -\frac{1}{N_C} (\bar{q}_i \gamma_\mu (F_{qb}^{DR} P_R + F_{qb}^{DL} P_L) b_i) \right]^2 \\
+ \bar{q}_i \gamma_\mu (F_{qb}^{DR} P_R + F_{qb}^{DL} P_L) b_j \bar{q}_j \gamma_\mu (F_{qb}^{DR} P_R + F_{qb}^{DL} P_L) b_i,
$$

where $N_C$ denotes the number of colors and we have used the identity

$$
T_{ij} T_{kl}^{\bar{b}} = -\frac{1}{2N_C} \delta_{ij} \delta_{kl} + \frac{1}{2} \delta_{il} \delta_{jk}.
$$

In order to calculate the $B_q - \bar{B}_q$ mixing, we write the relevant hadronic matrix elements to be

$$
\langle B_q | \bar{q}_i \gamma_\mu P_{L(R)} b_j | \bar{B}_q \rangle = \frac{1}{3} m_{B_q} f_{B_q}^2 \hat{B}_{1q}^i,
$$

$$
\langle B_q | \bar{q}_i \gamma_\mu P_R b_j \bar{q}_j \gamma_\mu P_L b_j | \bar{B}_q \rangle = -\frac{5}{12} m_{B_q} f_{B_q}^2 \hat{B}_{1q}^{RL},
$$

$$
\langle B_q | \bar{q}_i \gamma_\mu P_L b_j \bar{q}_j \gamma_\mu P_R b_j | \bar{B}_q \rangle = -\frac{7}{12} m_{B_q} f_{B_q}^2 \hat{B}_{2q}^{RL}.
$$

To estimate the new physics effects, we employ the vacuum insertion method to calculate the above matrix elements, i.e. $\hat{B}_{1q} \sim \hat{B}_{1q}^{RL} \sim \hat{B}_{2q}^{RL} \sim 1$ [25, 26]. Additionally, in the heavy quark limit, we take $m_b \sim m_{B_q}$. As a result, the transition matrix element for $B_q - \bar{B}_q$ oscillation mediated by axigluon exchange becomes

$$
M^A_{12,q} = \langle B_q | H^A_{\Delta B = 2} | \bar{B}_q \rangle = \frac{g_2^2}{18m_V^2} m_{B_q} f_{B_q}^2 U_{qb}^D,
$$

$$
U_{qb} = (F_{qb}^{DR})^2 + (F_{qb}^{DL})^2 + 4F_{qb}^{DR} F_{qb}^{DL}.
$$

For reducing the number of free parameters, we will take the approximation $V_{R}^Q \approx V_{L}^Q = V^D$ in our analysis, i.e. $F_{qb}^{DR} \approx F_{qb}^{DL} = F_{qb}^D$, then $U_{qb} = 6(F_{qb}^D)^2$. We note that the approximation $V_{R}^Q \approx V_{L}^Q$ can be realized in hermitian Yukawa matrices [24].

By combining the contributions of SM and axigluon, the transition matrix element for $\Delta B = 2$ can be formulated as

$$
M_{12,q}^{B_q} = |M_{12,q}^{SM,q}| R_{A} e^{2i(\beta_q + \phi_{NP})},
$$

\[ 5 \]
where the new parameters are defined by

\[
R^q_A = \left(1 + (r^q_A)^2 + 2r^q_A \cos 2(\beta^q_{NP} - \beta_q)\right)^{1/2},
\]

\[
2\beta^q_{NP} = \arg(M^A_{12}^q),
\]

\[
r^q_A = \frac{|M^A_{12}^q|}{|M^{SM}_{12}^q|},
\]

\[
\tan 2\phi^q_{NP} = \frac{r^q_A \sin 2(\beta^q_{NP} - \beta_q)}{1 + r^q_A \cos 2(\beta^q_{NP} - \beta_q)},
\]

and \(M^{SM}_{12}^q\) is given by

\[
M^{SM}_{12}^q = \frac{G^2_F m_B q^2 f_{B_q}^2 \hat{M}(V^*_q V_b)^2 S_0(x_t)}{12\pi^2 \eta_B m_B q^2},
\]

with \(S_0(x_t) = 0.784x_t^{0.76}, x_t = (m_t/m_W)^2\) and \(\eta_B \approx 0.55\) is the QCD correction to \(S_0(x_t)\).

Hence, the mass difference between heavy and light \(B_q\) is

\[
\Delta m_B^q = 2|M_{12}^B_q| = \Delta m_{BM}^{SM} R^q_A.
\]

After obtaining \(M_{12}^B_q\), the time-dependent CPA through inclusive semileptonic decays can be defined as

\[
a^q_{s\ell} = \frac{\Gamma(\bar{B}_q(t) \to \ell^+ X) - \Gamma(B_q(t) \to \ell^- X)}{\Gamma(B_q(t) \to \ell^+ X) + \Gamma(B_q(t) \to \ell^- X)},
\]

\[
a^q_{s\ell} = \frac{1 - |\mathbf{q}/p|^4}{1 + |\mathbf{q}/p|^4}
\]

with

\[
\left(\frac{\mathbf{q}}{\mathbf{p}}\right)^2 = \frac{M_{12}^B_q - i\Gamma_{12}^B_q/2}{M_{12}^B_q - i\Gamma_{12}^B_q/2},
\]

where \(\Gamma_{12}^B_q\) denotes the absorptive part of \(B_q \leftrightarrow \bar{B}_q\) transition. Due to \(\Gamma_{12}^B_q \ll M_{12}^B_q\), the wrong-sign charge asymmetry can be simplified as

\[
a^q_{s\ell} = \text{Im} \left(\frac{\Gamma_{12}^B_q}{M_{12}^B_q}\right) \approx \frac{\Delta \Gamma_{SM}^{SM} B_q}{\Delta m_{BM}^q} \sin(2\beta_q + 2\phi^q_{NP} - \theta^q_{\Gamma}).
\]

Here, \(\theta^q_{\Gamma}\) stands for the phase of \(\Gamma_{12}^B_q\). Since the absorptive part is dominated by the SM contribution, we will assume that \(\Gamma_{12}^B_q = \Gamma_{12}^{q,SM}\) in our numerical analysis. A detailed discussions about new physics effects on \(\Gamma_{12}^B_q\) can be found in Refs. [8, 10]. Since \(a^q_{s\ell}\) is associated with the CP phases directly, a non-zero charge asymmetry will be an indication of CP violation. Accordingly, the like-sign charge asymmetry defined in Eq. [3] can be written as

\[
\mathcal{A}_{s\ell}^B = \frac{\Gamma(\bar{b}b \to \ell^+ \ell^+ X) - \Gamma(\bar{b}b \to \ell^- \ell^- X)}{\Gamma(\bar{b}b \to \ell^+ \ell^+ X) + \Gamma(\bar{b}b \to \ell^- \ell^- X)}
\]

\[
= \frac{f_d Z_d a_{s\ell}^q + f_s Z_s a_{s\ell}^s}{f_d Z_d + f_s Z_s},
\]

(19)
where \( f_q \) is the production fraction of \( B_q \) and

\[
Z_q = \frac{1}{1 - y_q^2} - \frac{1}{1 - x_q^2},
\]

\[
y_q = \frac{\Delta \Gamma_{B_q}}{2 \Gamma_{B_q}}, \quad x_q = \frac{\Delta m_{B_q}}{\Gamma_{B_q}}.
\]

(20)

Using \( f_d = 0.323(37) \), \( f_s = 0.118(15) \), \( x_d = 0.774(37) \), \( y_d \sim 0 \), \( x_s = 26.2(5) \) and \( y_s = 0.046(27) \), the asymmetry can be rewritten as

\[
A_{\text{int}}^b = c_d a_{st}^d + c_s a_{st}^s
\]

(21)

with \( c_d = 0.506(43) \) and \( c_s = 0.494(43) \) [5].

Another important time dependent CPA can be defined by [29]

\[
A_{fCP}(t) = \frac{\Gamma(B_q(t) \to f_{CP}) - \Gamma(B_q(t) \to \bar{f}_{CP})}{\Gamma(B_q(t) \to f_{CP}) + \Gamma(B_q(t) \to \bar{f}_{CP})},
\]

\[
S_{fCP} = \frac{2Im\lambda_{fCP}}{1 + |\lambda_{fCP}|^2}, \quad C_{fCP} = \frac{1 - |\lambda_{fCP}|^2}{1 + |\lambda_{fCP}|^2}
\]

(22)

with

\[
\lambda_{fCP} = -\left( \frac{M_{12}^{B} \pm M_{12}^{\bar{B}}}{M_{12}^B} \right)^{1/2} A(\bar{B} \to f_{CP}) = -e^{-2i(\beta_q + \phi_{NP})} \frac{\bar{A}_{fCP}}{A_{fCP}},
\]

(23)

where \( f_{CP} \) denotes the final CP eigenstate, \( S_{fCP} \) and \( C_{fCP} \) are the so-called mixing-induced and direct CPAs, \( A_{fCP} \) and \( \bar{A}_{fCP} \) are the amplitudes of \( B \) and \( \bar{B} \) mesons decaying to \( f_{CP} \) and \( \bar{f}_{CP} \) and \( \bar{A}_{fCP}/A_{fCP} = -\eta_{fCP} A_{fCP}(\theta_W \to -\theta_W)/A_{fCP}(\theta_W) \) with \( \eta_{fCP} \) and \( \theta_W \) are the CP eigenvalue of \( f_{CP} \) and the weak CP phase respectively. Clearly, besides \( \Delta B = 2 \) effects, the mixing-induced CPA is also related to the \( \Delta B = 1 \) process. In this paper, we will concentrate on \( f_{CP} = J/\Psi K_S \) and \( \phi K_S \) for \( q = d \) and on \( f_{CP} = J/\Psi \phi \) for \( q = s \).

To calculate the decay amplitude of \( B(\bar{B}) \to f_{CP} \), we need to discuss the interactions of \( \Delta B = 1 \) processes. With the approximation \( V_R^Q \approx V_L^Q \), the effective Hamiltonian of \( b \to qq'q' \) can be expressed as

\[
\mathcal{H}_{b \to qq'q'} = \frac{g_A}{m_V} F_{V_q}^D q'_{\gamma\mu} q_{\gamma\nu} T^{b} b \sum_{q'=u,d,s,c} q_{\gamma} q'_{\gamma} (g_+ P_R + g_- P_L) T^{b} q'
\]

(24)

with \( g_\pm = g_V \pm g_A \). Using Eq. (10), we can rewrite the last equation as

\[
\mathcal{H}_{b \to qq'q'}^A = \frac{G_F}{\sqrt{2}} V_{ta} V_{tb} \left[ C_{q3} Q_3^o + C_{q4} Q_4^o + C_{q3} Q_3^r + C_{q4} Q_4^r + C_{q5} Q_5^o + C_{q6} Q_6^o + C_{q5} Q_5^l + C_{q6} Q_6^l \right]
\]

(25)
in which the new Wilson coefficients are expressed by

\[ C'_{q3} = \frac{1}{8N_C} \frac{\sqrt{2} F_D}{G_F V_{tb}^* V_{tb}} g_A g^- , \quad C'_{q4} = -N_C C_{q3} , \]
\[ C'_{q5} = -C'_{q3} , \quad C'_{q6} = -N_C C'_{q5} , \]
\[ C'_{q5} = \frac{1}{8N_C} \frac{\sqrt{2} F_D}{G_F V_{tb}^* V_{tb}} g_A g^+ , \quad C'_{q6} = -N_C C'_{q5} , \]
\[ C'_{q3} = -C'_{q5} , \quad C'_{q4} = -N_C C'_{q3} \]

and the associated operators are

\[ O_3^q = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q'} q')_{V-A} , \quad O_4^q = (\bar{q}_a b_\beta)_{V-A} \sum_{q'} (\bar{q'} q'_a)_{V-A} , \]
\[ O_5^q = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q'} q')_{V-A} , \quad O_6^q = (\bar{q}_a b_\beta)_{V-A} \sum_{q'} (\bar{q'} q'_a)_{V-A} , \]
\[ O_3^{qR} = (\bar{q}b)_{V+A} \sum_{q'} (\bar{q'} q')_{V+A} , \quad O_4^{qR} = (\bar{q}_a b_\beta)_{V+A} \sum_{q'} (\bar{q'} q'_a)_{V+A} , \]
\[ O_5^{qL} = (\bar{q}b)_{V+A} \sum_{q'} (\bar{q'} q')_{V-A} , \quad O_6^{qR} = (\bar{q}_a b_\beta)_{V+A} \sum_{q'} (\bar{q'} q'_a)_{V-A} \]

with \((f' f)_{V^\pm A} = f' \gamma_\mu (1 \pm \gamma_5) f\). Besides the new free parameters that are introduced earlier, the non-leptonic \(B\) decays suffer from large uncertain QCD effects such as \(\langle f_{CP} | \mathcal{H}_{b \rightarrow q q'} | B \rangle\).

For estimating the new physics effects, we employ the naive factorization approach (NFA).

Under the NFA, we find that the related effective Wilson coefficients for \(\bar{B}_d \to J/\Psi \bar{K}^0\) and \(\bar{B}_s \to J/\Psi \phi\) are

\[ C'_{s3} + \frac{C'_{s4}}{N_C} + C'_{s5} + \frac{C'_{s6}}{N_C} + C'_{s5} + \frac{C'_{s6}}{N_C} + C'_{s3} + \frac{C'_{s4}}{N_C} \]

With the results in Eq. (26), we clearly see that the influence of axigluon-mediated effects on \(J/\Psi (\bar{K}^0, \phi)\) modes vanishes. In our analysis we neglect the nonfactorizable contributions as they are subleading and difficult to estimate. Now, only \(B \to \phi K \) can display the axigluon-mediated effects. Using NFA and the interactions of Eq. (25), the total decay amplitude of \(B \to \phi K \) is written as

\[ A_{\phi K^0} = \langle \phi K^0 | \mathcal{H}_{b \rightarrow s s \bar{s}} | \bar{B} \rangle , \]
\[ = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb}^* (a_{SM} + a_{s4} + a_{s4}^R) \langle \phi | s \gamma_\mu s | 0 \rangle \langle \bar{K}^0 | s \gamma_\mu b | \bar{B} \rangle \]

where \(\mathcal{H}_{b \rightarrow s s \bar{s}}\) is the sum of the SM and axigluon effective Hamiltonian and \(a_{SM}^R = a_3 + a_4 + a_5\).
with
\[
\begin{align*}
a_3 &= C_3 + \frac{C_4}{N_C}, \quad a_4 = C_4 + \frac{C_5}{N_C}, \quad a_5 = C_5 + \frac{C_6}{N_C}, \\
a_{s4}' &= C_{s4}' + C_{s3}/N_C, \quad a_{s4}^R = C_{s4}^R + C_{s3}/N_C.
\end{align*}
\]

Here, $C_{3-6}$ are the effective Wilson coefficients from the gluon penguin in the SM \cite{28}. We note that the electroweak penguin contributions are very small and thus we neglect them.

Using $V_{ts} = -|V_{ts}|e^{-i\beta_s}$ \cite{29}, we can write
\[
\frac{\tilde{A}_{\phi K^0}}{A_{\phi K^0}} = -e^{2i\beta_s} a_{s4}^{SM} + a_{s4}^R = -e^{2i(\beta_s + \theta_s^{NP})}.
\]

By Eqs. (22) and (23), the mixing-induced CPA via $B_d \to \phi K^0$ decay is obtained as
\[
S_{\phi K^0} \equiv \sin 2\beta_{\phi K^0} = \sin 2(\beta_d + \phi_d^{NP} - \beta_s - \theta_s^{NP}),
\]
while the CPAs through $B_{d,s} \to J/\Psi(K_S, \phi)$ decays are given by
\[
\begin{align*}
S_{J/\Psi K^0} &\equiv \sin 2\beta_{J/\Psi K^0} = \sin 2(\beta_d + \phi_d^{NP}), \\
S_{J/\Psi \phi} &\equiv \sin 2\beta_{J/\Psi \phi} = \sin 2(\beta_s + \phi_s^{NP}).
\end{align*}
\]

Although the measurement of $\sin 2\beta_{J/\Psi K^0}$ has approached to the precision level, however, it might be difficult to tell if there exists new physics by measuring $\sin 2\beta_{J/\Psi K^0}$ only. Nevertheless, one can investigate a new asymmetry defined by \cite{31}
\[
\Delta_{\beta_d} = \sin 2\beta_{J/\Psi K^0} - \sin 2\beta_{\phi K^0},
\]
which is less than 5% in the SM \cite{31}. If a large value of $\Delta_{\beta_d}$ is measured, it will be a strong hint for new physics beyond SM.

## III. NUMERICAL ANALYSIS

So far, we have introduced seven new free parameters in the general chiral color models and they are: two gauge couplings $g_{V,A}$, four parameters in the two complex quantities $F^{D}_{qb}$
and \( m_V \). In order to display the dependence of \( \Delta_{\beta_d} \) on \( m_V \), we use the results in Ref. [21] and take \( g_V = -0.577 g_s \) and \( g_A = -1.155 g_s \) with \( \alpha_s = g_s^2/4\pi = 0.119 \). Thus, the five remaining parameters are \( |F_{q_b}^D|, \beta_q^{NP} \) for \( q=d, s \) and \( m_V \). We list the input values used for numerical calculations in Table I, where the relevant CKM matrix elements \( V_{i_q} = \tilde{V}_{i_q} \exp(-i\beta_q) \) are obtained from the UTfit Collaboration [32], the decay constant of \( B_q \) is referred to the result given by HPQCD Collaboration [33], the CDF and DØ average value of \( \Delta m_{B_s} \) is from Ref. [1] and the SM Wilson coefficients of \( b \to qq'q' \) are obtained from Ref. [28]. Other inputs are obtained from particle data group (PDG) [29].

| TABLE I: Numerical inputs for the parameters in the SM. |
|-----------------------------------|
| \( \tilde{V}_{td} \) | \( \beta_d \) | \( \tilde{V}_{ts} \) | \( \beta_s \) | \( m_{B_d} \) | \( m_{B_s} \) |
| 8.51(22) \times 10^{-3} | (22 \pm 0.8)° | -4.07(22) \times 10^{-2} | -(1.03 \pm 0.06)° | 5.28 GeV | 5.37 GeV |
| \( f_{B_d} \sqrt{B_d} \) [MeV] | \( f_{B_s} \sqrt{B_s} \) [MeV] | \( f_{B_d} \) [MeV] | \( f_{B_s} \) [MeV] | \( S_{J/\Psi K^0}^{NP} \) | \( m_{d}(\tilde{m}_t) \) |
| 216 \pm 15 | 266 \pm 18 | 190 \pm 13 | 231 \pm 15 | 0.655 \pm 0.024 | 163.8 GeV |
| \( (\Delta m_{B_d})^{exp} \) | \( (\Delta m_{B_s})^{exp} \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) |
| 0.507 \pm 0.005 ps\(^{-1} \) | 17.78 \pm 0.12 ps\(^{-1} \) | 0.013 | -0.0335 | 0.0095 | -0.0399 |

After setting up the inputs, we study the contributions of the axigluon to FCNC processes and their associated CPAs that are defined earlier. We start by exploring the allowed parameter space. Since the non-universal axigluon induces FCNCs at tree level, the observed \( B_q - \bar{B}_q \) mixing parameter \( \Delta m_{B_q} \) will give a strict constraint on the parameter space. In Fig. II(a) [(b)], the allowed range for \( \beta_d^{NP} \) and \( |F_{q_b}^D|/m_V \) (in units of \( 10^{-6} \)) is drawn by the down-left hatched lines where we have taken the SM contributions (\( \Delta m_{B_d}^{NP}, \Delta m_{B_s}^{NP} \)) to be (0.506, 17.76) ps\(^{-1} \). Furthermore, since the observed \( S_{J/\Psi K^0}^{exp} \) has been a precise measurement, it is plausible that the current data can further exclude the values of the parameter space which are allowed by \( \Delta m_{B_d} \). Taking 2\( \sigma \) errors of \( S_{J/\Psi K^0}^{exp} \) as the experimental bound, the allowed region for \( \beta_d^{NP} \) and \( |F_{d_b}^D|/m_V \) sketched by down-right hatched lines is plotted in Fig. II(a). Clearly, \( S_{J/\Psi K^0}^{exp} \) gives a strong constraint on the parameters that contribute to \( M_{12}^{B_d} \). From Fig. II we see that, except the two narrow regions correspond to \( |F_{d_b}^D|/m_V > 1 \times 10^{-6} \) GeV\(^{-1} \), the allowed values of \( |F_{d_b}^D|/m_V \) are limited to be \( |F_{d_b}^D|/m_V \leq 0.4 \times 10^{-6} \) GeV\(^{-1} \), whereas the allowed values of \( |F_{s_b}^D|/m_V \) can be one order of magnitude larger than those of \( |F_{d_b}^D|/m_V \). In general, the range of the CP violating phase \( \beta_q^{NP} \) is \([-\pi, \pi] \), for
illustration, we just show the results within $[-\pi, 0]$. The pattern of the constraint in $[0, \pi]$ is similar to that in $[-\pi, 0]$. In order to illustrate the influence of the uncertainties of the SM on the free parameters, in Fig. 2 we plot the allowed values of $|F_{sb}^D|/m_V$ and $\beta_s^{NP}$ by including the errors of $f_{B_s}\sqrt{B_s}$ and $V_{ts}$. Comparing with Fig. 1(b), we see that the allowed range is extended slightly. We note that due to the strict constraint of $S_{J/\Psi K^0}^{\exp}$, the bounds on the parameters for $b \rightarrow d$ transition are not changed significantly, therefore, we don’t show the corresponding diagram for $b \rightarrow d$ transition.

![Diagram](image)

**FIG. 1:** (a)[(b)] Constraints on $\beta_d^{NP}$ and $|F_{db}^D|/m_V$ (in units of $10^{-6}$) obtained from $B_{d[s]} - \bar{B}_{d[s]}$ mixing (down-left hatched lines) and $\sin 2\beta_{J/\Psi K^0}$ (down-right hatched lines).

![Diagram](image)

**FIG. 2:** Legend is the same as Fig. 1(b), but the errors of $\Delta m_{B_s}$ in the SM are included.

According to Eq. (19), if we assume no new CP violating phase in semi-leptonic decays, we will see that the charge asymmetry $A_{s\ell}^b$ depends on two kinds of CP violating phases.
One of the two phases is originated from $B_d - \bar{B}_d$ mixing which is a $b \rightarrow d$ transition, and the other phase is originated from $B_s - \bar{B}_s$ which is associated with $b \rightarrow s$ transition. In other words, we have to consider four parameters $\beta_{(d,s)}^{NP}$ and $|F_{(d,s)b}^D|/m_V$ simultaneously. However, if we consider $b \rightarrow (d, s)$ transitions at the same time, we may induce a large effect on $s \rightarrow d$ because the $\Delta K = 2$ process is associated with $F_{ds}^D = (\zeta - 1)V_{13}^D V_{23}^{D*}$, i.e. $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ and $K^0 - \bar{K}^0$ mixings have strong correlations. In order to avoid inducing a large $K^0 - \bar{K}^0$ mixing, we set a small value for $V_{13}^D$. This is consistent with the results shown in Fig. 1(a) where $\Delta m_{B_d}$ and $S_{J/\Psi K^0}$ strongly constrain $|F_{db}^D|/m_V$. Hence, we assume that $a_{st}^d$ is dominated by the SM contribution where $a_{st}^d(SM) = -4.8 \times 10^{-4}$ [6]. Consequently, the enhanced $|A_{st}^b|$ can be attributed to $b \rightarrow s$ transition. With Eqs. (14), (18) and (21) and the values given in Table I, the contours of $A_{st}^b$ as a function of $\beta_s^{NP}$ and $|F_{sb}^D|/m_V$ are shown in Fig. 3(a) where the values of the contours are in units of $10^{-4}$. As can be seen from the figure, not only the sign of $A_{st}^b$ can fit the data, but also its magnitude can be enhanced by axigluon-mediated effects. By combining with the constraint of $\Delta m_{B_s}$, the region of $\beta_s^{NP}$ for large $|A_{st}^b|$ is limited. In Fig. 3(b), we display $A_{st}^b$ as a function of $\beta_s^{NP}$ where the solid, dashed and dash-dotted line represents $|F_{sb}^D|/m_V = (3, 4, 5) \times 10^{-6}$ GeV$^{-1}$, respectively. As shown in the figure, negative and positive values of $\beta_s^{NP}$ can enhance $A_{st}^b$. It should be noted that, although the axigluon-mediated effect can not enhance the like-sign charge asymmetry to be as large as the central value of DØ data, however, $|A_{st}^b|$ is enhanced by one order of magnitude larger than the SM prediction.

Unlike the case of the charge asymmetry, the time-dependent CPA of $B_s \rightarrow J/\Psi \phi$ decay depends only on the CP phase in $b \rightarrow s$ transition. As a consequence, when the new CP violating effects are small in $M_{12}^{B_s}$, $A_{st}^b$ and $S_{J/\Psi \phi}$ defined in Eq. (22) can have a strong correlation. By using Eq. (32), the contours of $S_{J/\Psi \phi}$ are plotted as a function of $\beta_s^{NP}$ and $|F_{sb}^D|/m_V$ in Fig. 4(a). From the figure, we find that large $S_{J/\Psi \phi}$ can be archived when $A_{st}^b$ is one order of magnitude larger than the SM prediction. Moreover, we also plot $S_{J/\Psi \phi}$ as a function of $\beta_s^{NP}$ in Fig. 4(b), where the solid, dashed and dash-dotted line denotes $|F_{sb}^D|/m_V = (3, 4, 5) \times 10^{-6}$ GeV$^{-1}$, respectively. Clearly, a large $A_{st}^b$ indicates a large $S_{J/\Psi \phi}$. Although the measured values of $A_{st}^b$ and $S_{J/\Psi \phi}$ contain large errors, however, a few sigma deviations from the SM prediction can be considered as a hint for new physics effect.

It is well known that the golden process to measure the angle $\beta_d$ in the SM is $B_d \rightarrow J/\Psi K^0$ which is dominated by tree diagram. Although new physics can also affect this decay mode
FIG. 3: (a) Contours of $A^b_{st}$ as a function of $\beta^{NP}_s$ and $|F^D_{sb}|/m_V$ (in units of $10^{-6}$). (b) $A^b_{st}$ as a function of $\beta^{NP}_s$, where the solid, dashed and dash-dotted line stands for $|F^D_{sb}|/m_V = (3, 4, 5) \times 10^{-6}$, respectively. The values on the plot (a) are $A^b_{st}$ in units of $10^{-4}$.

FIG. 4: (a) Contours of $S_{J/Ψφ}$ as a function of $β^{NP}_s$ and $|F^D_{sb}|/m_V$ (in units of $10^{-6}$). (b) $S_{J/Ψφ}$ as a function of $β^{NP}_s$, where the solid, dashed and dash-dotted line represents $|F^D_{sb}|/m_V = (3, 4, 5) \times 10^{-6}$, respectively.
via $b \to sc\bar{c}$ transition, however, as discussed in Eq. (28), the axigluon contributions to $B_d \to J/\Psi K^0$ vanish. Hence, the source of the time-dependent CPA in $B_d \to J/\Psi K^0$ decay is only originated from the $B_d$ oscillation. Since $\beta_d$ is also a parameter in the SM, a single measurement of $S_{J/\Psi K^0}$ or $\sin 2\beta_{J/\Psi K^0}$ is hard to uncover the new physics. To probe the new physics, the best way is to compare the CPA of $J/\Psi K^0$ with that of $\phi K_S$. Therefore, we do not discuss each of $S_{J/\Psi K^0}$ and $S_{\phi K^0}$ separately. Instead, we focus on the CPA difference $\Delta \beta_d$ which is defined in Eq. (33) and it is only few percent in the SM. By Eqs. (31) and (33), we see that although $\Delta \beta_d$ is insensitive to $F_{D db}$ however it is strongly dependent on $F_{D sb}$. To see the contributions of the axigluon to $\Delta \beta_d$, we present the contours of $\Delta \beta_d$ as a function of $\beta_s^{NP}$ and $|F_{D sb}|/m_V$ in Fig. 5(a)[b], where we have set $|F_{D db}|/m_V = 0$ and figure (a)[b] corresponds to $m_V = 0.5[1]$ TeV. Since the decay amplitude of $B \to \phi K$ depends on $F_{D sb}/m_V^2$ while $\Delta m_{B_s}$ is $(F_{D sb}/m_V)^2$, thus a specific value for $m_V$ has to be given when calculating the contours of $\Delta \beta_d$. For further understanding the $\beta_s^{NP}$-dependence, we display $\Delta \beta_d$ as a function of $\beta_s^{NP}$ in Fig. 6 where figure (a)[b] is for $m_V = 0.5[1]$ TeV and the solid, dashed and dash-dotted line stands for $|F_{D sb}|/m_V = (3, 4, 5) \times 10^{-6}$ GeV$^{-1}$, respectively. It is clear that the axigluon contributions to $\Delta \beta_d$ are larger than that of the SM.

![Contour plots](image)

FIG. 5: Contours of $\Delta \beta_d$ as a function of $\beta_s^{NP}$ and $|F_{D sb}|/m_V$ (in units of $10^{-6}$) with (a) $m_V = 0.5$ TeV and (b) $m_V = 1$ TeV.

In order to comprehend further the correlations among various physical observables under
FIG. 6: $\Delta \beta_d$ as a function of $\beta_{sNP}$ with (a) $m_V = 0.5$ TeV and (b) $m_V = 1$ TeV, where the solid, dashed and dash-dotted line represents $|F_{sb}^D/m_V = (3, 4, 5) \times 10^{-6}$, respectively.

the influence of the axigluon, we display the scatter plots of $A_{s\ell}^b$, $S_{J/Ψφ}$ and $\Delta \beta_d$ with $m_V = 0.5(1)$ TeV versus $\Delta m_{B_s}$ in Fig. 7 where we have chosen the range of $\beta_{sNP}$ to be $[-\pi, 0]$. As an illustration, we also show the scatter plots of ($A_{s\ell}^b$, $S_{J/Ψφ}$) and ($A_{s\ell}^b$, $\Delta \beta_d$) with $m_V = 1$ TeV in Fig. 8 in which the constraint of $\Delta m_{B_s}$ has been included and $\beta_{sNP}$ belongs to $[-\pi, 0]$. By Fig. 8(a), we see that the correlation between $A_{s\ell}^b$ and $S_{J/Ψφ}$ is linear, where this behavior can be understood by the linear dependence between the like-sign charge asymmetry and the mixing-induced CPA of $B_s$. Due to the linearity, we expect that the correlation between $S_{J/Ψφ}$ and $\Delta \beta_d$ should be similar to that between $A_{s\ell}^b$ and $\Delta \beta_d$. Therefore, we just show the latter case in Fig. 8(b).

IV. CONCLUSION

In general, a flavor non-universal axigluon in generalized chiral color models can induce FCNCs at tree level. We study phenomenologically the axigluon-mediated effects on $\Delta B = 2$ FCNC processes and the associated CPAs. We find that although $\Delta m_{B_s}$ strongly constrain the free parameters, the precise measurement of $S_{J/ΨK^0}$ can further exclude the parameter space of $b \to d$ transition. Furthermore, for avoiding inducing large $K^0 - \bar{K}^0$ mixing, the
FIG. 7: Correlations between $\Delta m_{B_s}$ and (a) $A_{s\ell}^b$, (b) $S_{J/\Psi\phi}$, (c) $\Delta \beta_d$ with $m_V = 0.5[1]$ TeV, where the angle $\beta_{sNP}$ belongs to $[-\pi, 0]$.  

FIG. 8: (a) Correlation between $A_{s\ell}^b$ and $S_{J/\Psi\phi}$ and (b) correlation between $A_{s\ell}^b$ and $\Delta \beta_d$ with $m_V = 1$ TeV, where the constraint of $\Delta m_{B_s}$ has been included and the angle $\beta_{sNP}$ belongs to $[-\pi, 0]$.  

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parameter $V_{13}^D$ is chosen to be small so that the like-sign charge asymmetry $A_{s\ell}^b$ and $\Delta_{d\ell}$ are insensitive to the parameters of $b \to d$ transition. As a result, the CP violating observables $A_{s\ell}^b$, $S_{J/\Psi\phi}$ and $\Delta_{d\ell}$ are strongly correlated and are only sensitive to the parameters of $b \to s$ transition.

By the study, we find that the axigluon effects do not only preserve the negative sign in $A_{s\ell}^b$, but also enhance its magnitude by one order of magnitude larger than the SM prediction. Subsequently, the associated values of the parameters can also enhance the CPA $S_{J/\Psi\phi}$ and the CPA difference $\Delta_{d\ell}$ largely although they are only few percent in the SM.

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