The role of meson exchanges in light-by-light scattering

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Abstract

We discuss the role of meson exchange mechanisms in \( \gamma \gamma \rightarrow \gamma \gamma \) scattering. Several pseudoscalar (\( \pi^0, \eta, \eta'(958), \eta_c(1S), \eta_c(2S) \)), scalar (\( f_0(500), f_0(980), a_0(980), f_0(1370), \chi_c(1P) \)) and tensor (\( f_2(1270), a_2(1320), f_2'(1525), f_2(1565), a_2(1700) \)) mesons are taken into account. We consider not only s-channel but also for the first time t- and u-channel meson exchange amplitudes corrected for off-shell effects including vertex form factors. We find that, depending on not well known vertex form factors, the meson exchange amplitudes interfere among themselves and could interfere with fermion-box amplitudes and modify the resulting cross sections. The meson contributions are shown as a function of collision energy as well as angular distributions are presented. Interesting interference effects separately for light pseudoscalar, scalar and tensor meson groups are discussed. The meson exchange contributions may be potentially important in the context of a measurement performed recently in ultraperipheral collisions of heavy ions by the ATLAS collaboration. The light-by-light interactions could be studied in future in electron-positron collisions by the Belle II at SuperKEKB accelerator.
I. INTRODUCTION

In the Standard Model the light-by-light scattering is usually assumed to proceed through lepton, quark and $W$ gauge boson loops \cite{1-4}. The QCD and QED corrections to the fermion-loop contributions were discussed in \cite{4} and their contribution was found to be very small. In \cite{5} our group considered also double fluctuations of photons into light vector mesons and their subsequent soft interactions treated in the Regge theory. The correction turned out to be important at higher center-of-mass energies ($\sqrt{s} > 2 \text{ GeV}$) and very small scattering angles. In \cite{6} we considered in addition two-gluon exchange contribution. This mechanism, in contrast to the VDM-Regge one survives to larger scattering angles.

In the present paper we shall consider several pseudoscalar ($J^{PC} = 0^{-+}$), scalar ($J^{PC} = 0^{++}$), and tensor ($J^{PC} = 2^{++}$) meson exchange contributions for the $\gamma\gamma \rightarrow \gamma\gamma$ elastic scattering. The meson-$\gamma$-$\gamma$ vertex functions depend on the quantum numbers of objects involved and form factors that are in principle a function of four-momenta of both photons and meson. So far the $\gamma$-$\gamma$-meson processes were investigated at $e^+e^-$ colliders for one or both virtual photons and on-shell $\pi^0$ \cite{7,8}, $\eta$, $\eta'$ \cite{9}, $\eta_c$ \cite{10} mesons and also from recent measurements by the Belle collaboration \cite{11} for $f_0(980)$ and $f_2(1270)$ mesons. Recently, in \cite{12} the authors formulated light-by-light scattering sum rules, that lead to relations between the $\gamma^*\gamma$ transition form factors for $C$-even scalar, pseudoscalar, axial-vector and tensor mesons when assuming sum rule saturation.

The situation for $\gamma\gamma \rightarrow \gamma\gamma$ elastic scattering is different as here photons are on mass shell and meson is off-shell. We do not know very precisely the dependence of the form factor for off-shell mesons on their virtuality. For the s-channel we have time-like meson while for t- and u-channels space-like one (see Fig. 1). We shall discuss here how the results depend on the form factors. In the following we shall parametrise it in either exponential or monopole type. We expect that for the $t$ and $u$ diagrams the corresponding form factors can be rather hard (larger cut-off parameter) compared to purely hadronic processes as they describe the coupling of space-like meson to semi point-like objects (photons).

II. MESON EXCHANGE CONTRIBUTIONS

Below we will discuss the expressions relevant to pseudoscalar ($M_{PS}$), scalar ($M_S$), and tensor ($M_T$) as well as spin-4 $f_4(2050)$ meson exchange contributions for the reaction

$$\gamma(p_1, \lambda_1) + \gamma(p_2, \lambda_2) \rightarrow \gamma(p_3, \lambda_3) + \gamma(p_4, \lambda_4)$$

(2.1)

with the four-momenta $p_i$ ($i = 1, ..., 4$) and the photon helicities, $\lambda_i \in \{1, -1\}$, indicated in brackets. Schematic diagrams for the the process (2.1) are given in Fig. 1.

The differential cross section for the angular distribution for the reaction (2.1) is given by

$$\frac{d\sigma}{d\cos \theta} = \frac{1}{32\pi s} \frac{11}{24} \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |M_{\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4}|^2,$$

(2.2)

where the explicit factor $1/2$ takes into account identity of photons. The kinematical
where \( \epsilon \) collaborations. are relatively well known and were measured in recent years by the Belle and BaBar to the Landau-Yang theorem [13]. The two-photon branching fractions for the resonances process (2.1). The contribution of axial-vector mesons vanishes for on-shell photons due

FIG. 1: Diagrams for light-by-light scattering via a time-like (s-channel) and a space-like (t-channel and u-channel) meson exchanges.

variables used in the present paper are

\[
s = (p_1 + p_2)^2 = (p_3 + p_4)^2, \\
t = (p_1 - p_3)^2 = (p_2 - p_4)^2, \\
u = (p_2 - p_3)^2 = (p_1 - p_4)^2, \\
p_s = p_1 + p_2 = p_3 + p_4, \\
p_t = p_2 - p_4 = p_3 - p_1, \\
p_u = p_1 - p_4 = p_3 - p_2, \\
p_2^2 = s, \ p_2^2 = t, \ p_2^2 = u. \tag{2.3}
\]

The amplitude for the reaction (2.1) with the meson exchanges is written as

\[
\mathcal{M}_{\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4} = \sum_{M_{PS}=\pi^0,\eta,\eta'} \mathcal{M}^{(M_{PS})}_{\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4} \\
+ \sum_{M_{S}=f_0(500),f_0(980),a_0(980),f_0(1370),\chi_c(1P)} \mathcal{M}^{(M_{S})}_{\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4} \\
+ \sum_{M_{T}=f_2(1270),a_0(1320),f_0'(1525)} \mathcal{M}^{(M_{T})}_{\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4}. \tag{2.4}
\]

In Table I we have collected possible potential resonances that may contribute to the process (2.1). The contribution of axial-vector mesons vanishes for on-shell photons due to the Landau-Yang theorem [13]. The two-photon branching fractions for the resonances are relatively well known and were measured in recent years by the Belle and BaBar collaborations.

A. Pseudoscalar meson exchanges

The amplitude for the pseudoscalar meson exchange is written as

\[
i \mathcal{M}^{(M_{PS})}_{\lambda_1 \lambda_2 \rightarrow \lambda_3 \lambda_4} = (\epsilon_3^{i_3})^* i \Gamma_{\mu_3 \mu_4}^{(M_{PS})} (p_3, p_4) (\epsilon_4^{i_4})^* i \Delta^{(M_{PS})} (p_3) \epsilon_1^{i_1} i \Gamma_{\mu_1 \mu_2}^{(M_{PS})} (p_1, p_2) \epsilon_2^{i_2} \\
+ (\epsilon_3^{i_3})^* i \Gamma_{\mu_3 \mu_4}^{(M_{PS})} (-p_3, p_1) \epsilon_1^{i_1} i \Delta^{(M_{PS})} (p_1) (\epsilon_4^{i_4})^* i \Gamma_{\mu_4 \mu_5}^{(M_{PS})} (p_4, p_2) \epsilon_2^{i_2} \\
+ (\epsilon_4^{i_4})^* i \Gamma_{\mu_4 \mu_1}^{(M_{PS})} (p_4, p_1) \epsilon_1^{i_1} i \Delta^{(M_{PS})} (p_u) (\epsilon_3^{i_3})^* i \Gamma_{\mu_3 \mu_2}^{(M_{PS})} (-p_3, p_2) \epsilon_2^{i_2}, \tag{2.5}
\]

where \( \epsilon_i^{i_i} \) are the polarisation vectors of the photons with the helicities \( \lambda_i \).
Above meson exchanges

| Meson  | \(m_M\) (MeV) | \(\Gamma_M\) (MeV) | \(B(M \to \gamma\gamma)\) | \(\Gamma(M \to \gamma\gamma)\) (keV) |
|--------|----------------|-------------------|---------------------------|----------------------------------|
| \(\pi^0\) | 134.9766 ± 0.0006 | (1.31 ± 0.05) \times 10^{-3} | (98.823 ± 0.034) \times 10^{-2} | 7.8 ± 10^{-3} |
| \(\eta\)    | 547.862 ± 0.017  | 0.197 ± 0.009      | (2.21 ± 0.08) \times 10^{-2} | 4.354 |
| \(\eta'(958)\) | 957.78 ± 0.06  | 3.18 ± 0.8        | (1.59 ± 0.13) \times 10^{-4} | 5.056 |
| \(\eta_c(1S)\) | 2983.4 ± 0.5    | 11.3 ± 2.9         | (1.9 ± 1.3) \times 10^{-4} | 2.147 |
| \(\eta_c(2S)\) | 3639.2 ± 1.2    | 400 – 700          | 2.05 ± 0.21                | 2.15 |
| \(f_0(500)\) | 400 – 550       | 50 – 100           | 0.32 ± 0.05                | 0.32 |
| \(f_0(980)\) | 990 ± 20        | 200 – 500          | 0.30 ± 0.10                | 0.30 |
| \(a_0(980)\) | 980 ± 20        | 10 – 100           | 4.0 ± 1.9                  | 4.0 |
| \(f_0(1370)\) | 1200 – 1500     | 20 – 100           | 2.651; 3.14 ± 0.20         | 2.651 |
| \(\chi_{c0}(1P)\) | 3414.75 ± 0.31 | 400 – 700          | 1.00 ± 0.06                | 1.00 |
| \(f_2(1270)\) | 1275.5 ± 0.8    | 186.7 ± 2.27       | 0.081 ± 0.009; 0.13 ± 0.03 | 0.081 |
| \(a_2(1320)\) | 1318.3 ± 0.5    | 107 ± 5            | 0.70 ± 0.14                | 0.70 |
| \(f_2(1525)\) | 1525 ± 5        | 73 ± 5             | 0.30 ± 0.05                | 0.30 |
| \(f_2(1565)\) | 1562 ± 13       | 134 ± 8            | 0.70 ± 0.14                | 0.70 |
| \(a_2(1700)\) | 1732 ± 16       | 194 ± 40           | 0.30 ± 0.05                | 0.30 |

The propagator and \(M_{PS}\)\(\gamma\gamma\) vertex function for the pseudoscalar meson with mass \(m_{MPS}\) are:

\[
i\Delta^{(M_{PS})}(k) = \frac{i}{k^2 - m^2_{MPS} + im_{MPS}\Gamma_{MPS}},
\]

\[
i\Gamma^{(M_{PS}\gamma\gamma)}(k_1, k_2) = -ie^2 \epsilon_{\mu\nu\lambda\kappa\ell}\gamma^\lambda F_{\gamma^\ast\gamma^\ast \to M_{PS}}(0, 0) F^{(M_{PS}\gamma\gamma)}(k^2),
\]

where \(k^2 = p^2_s, p^2_t, p^2_u\) for the \(s, t, u\) diagrams, respectively. The transition form factor \(F_{\gamma^\ast\gamma^\ast \to M_{PS}}(0, 0)\) is related to the two-photon decay width as

\[
F_{\gamma^\ast\gamma^\ast \to M_{PS}}(0, 0) = \frac{1}{4\pi^2 f_{M_{PS}}} = \frac{1}{4\pi\alpha_{em}} \sqrt{\frac{64\pi\Gamma(M_{PS} \to \gamma\gamma)}{m^3_{MPS}}}.\]

Above \(\alpha_{em} = e^2/(4\pi)\) and \(f_{M_{PS}}\) is a decay constant of psudoscalar meson. We have, from Eq. (2.8), \(f_{M_{PS}} = 0.092, 0.093, 0.074, 0.375, 0.776\) GeV for \(\pi^0, \eta, \eta', \eta_c(1S), \eta_c(2S)\) mesons, respectively. The \(f_{M_{PS}}\) can be tried to be calculated within quark model. For \(\eta\) and \(\eta'\) this requires inclusion of \(\eta-\eta'\) mixing (see e.g. [20]). The two-photon decay of heavy quarkonium \(\eta_c\) states was discussed, e.g., in [21].

In our vertices (2.7) the photons are on mass shell and the meson is off-shell. Therefore in calculations we use off-shell meson form factors normalized to \(F(\gamma\gamma)(m^2_{M}) = 1\). These form factors can be parametrised, e.g., by the exponential form for the \(t\)- and \(u\)-channel meson exchanges

\[
F(\gamma\gamma)(t/u) = \exp \left( \frac{t/u - m^2_M}{\Lambda^2_{\xi\gamma}} \right),
\]
while for the s-channel meson exchange we assume

\[ F^{(M\gamma\gamma)}(s) = \exp \left( -\frac{(s - m_M^2)^2}{\Lambda_{\text{exp}}^4} \right), \]  

(2.10)

where \( \Lambda_{\text{exp}} \) is, in principle, a free parameter. In our calculation for pseudoscalar mesons we take \( \Lambda_{\text{exp}} = 2 \text{ GeV} \).

### B. Scalar meson exchanges

In our calculations here we consider four light scalar resonances: \( f_0(500) \), \( f_0(980) \), \( a_0(980) \) and \( f_0(1370) \). The first two states have been seen in \( \gamma\gamma \to \pi\pi \) reactions [22], while the \( a_0(980) \) resonance was observed in the \( \gamma\gamma \to \pi^0\eta \) reaction [23]. The situation for light scalar mesons was discussed, e.g., in [24]. While the direct coupling is expected to be rather small, the rescattering of mesons (\( \pi\pi \), \( KK \) or \( \pi\eta \)) leads to two-photon widths of the order of a fraction of keV or even larger. The partial decay widths were estimated, e.g., in [15, 25] and are collected in Table I. Note that the masses and widths of low-mass resonances are known with a large uncertainties [14]; see also analysis in [26] for the \( f_0(1500) \) and \( f_0(1710) \) states. The two-photon decay of heavy quarkonium states (\( \chi_{c0,2} \) and \( \chi_{b0,2} \)) was discussed, e.g., in [27, 28].

For the scalar meson \( M_S \) exchange the amplitude is similar as (2.5) with \( \Delta^{(M_{PS})} \) and \( \Gamma_{\mu\nu}^{(M_{PS}\gamma\gamma)} \) replaced by \( \Delta^{(M_S)} \) and \( \Gamma_{\mu\nu}^{(M_S\gamma\gamma)} \), respectively. The corresponding vertex can be written as:

\[ i\Gamma_{\mu\nu}^{(M_S\gamma\gamma)}(k_1, k_2) = \frac{e_2}{2m_{M_S}} \left( k_1^2 g_{\mu\nu} - 2 k_{1\nu} k_{2\mu} \right) F_{\gamma^*\gamma^*\to M_S}(0, 0) F^{(M_S\gamma\gamma)}(k^2), \]  

(2.11)

where

\[ F_{\gamma^*\gamma^*\to M_S}(0, 0) = \frac{1}{4\pi\alpha_{\text{em}}} \sqrt{\frac{64\pi\Gamma(M_S \to \gamma\gamma)}{m_{M_S}}}. \]  

(2.12)

It is easy to check that

\[ k_{1\mu} \Gamma^{\mu\nu}(k_1, k_2) = 0, \quad k_{2\nu} \Gamma^{\mu\nu}(k_1, k_2) = 0. \]  

(2.13)

Thus our vertices and as a consequence our s-, t- and u-channel amplitudes are gauge invariant.

For light scalar \( f_0(500) \), \( f_0(980) \), \( a_0(980) \) and \( f_0(1370) \) meson exchanges we take a smaller value of \( \Lambda_{\text{exp}} = 1 \text{ GeV} \).\(^1\) Their contribution will be shown in final summary plots.

\(^1\) This is related to the fact that the \( \gamma\gamma \) coupling to the scalar mesons is dominantly not to quark-antiquark system but rather to mesonic loops.
C. Tensor meson exchanges

The tensor meson exchanges are much more complicated. The $f_2(1270)$ and $a_2(1320)$ couplings to two photons were studied in detail, e.g., in [29] (see section 5). Using the formalism from [29] the amplitude for the tensor meson $M_T$ exchange can be written as

$$i\mathcal{M}_{\lambda_1\lambda_2\rightarrow\lambda_3\lambda_4}^{(M_T)} = (e_3^{(t)} * i\Gamma_{\mu_3\mu_4\nu_3\nu_4}^{(M_T\gamma\gamma)}(p_3, p_4) (e_4^{(t)} * i\Delta^{(M_T)}(\nu_3\nu_4\nu_1\nu_2)(p_3) e_1^{(t)} i\Gamma_{\mu_1\mu_2\nu_1\nu_2}^{(M_T\gamma\gamma)}(p_1, p_2) e_2^{(t)}$$

$$+ (e_4^{(t)} * i\Gamma_{\mu_4\mu_1\nu_1\nu_1}^{(M_T\gamma\gamma)}(p_4, p_1) e_1^{(t)} i\Delta^{(M_T)}(\nu_1\nu_2\nu_3\nu_4)(p_4) (e_3^{(t)} * i\Gamma_{\mu_3\mu_2\nu_3\nu_2}^{(M_T\gamma\gamma)}(p_3, p_2) e_2^{(t)}$$

$$+ (e_3^{(t)} * i\Gamma_{\mu_3\mu_1\nu_1\nu_1}^{(M_T\gamma\gamma)}(p_3, p_1) e_1^{(t)} i\Delta^{(M_T)}(\nu_1\nu_2\nu_3\nu_4)(p_1) (e_4^{(t)} * i\Gamma_{\mu_4\mu_2\nu_4\nu_2}^{(M_T\gamma\gamma)}(p_4, p_2) e_2^{(t)}$$

(2.14)

The $f_2(1270)$ $\gamma\gamma$ vertex is given as (see formulae (3.39), (3.40) and section 3.3 of [29])

$$i\Gamma_{\mu\nu\lambda\kappa}^{(f_2\gamma\gamma)}(k_1, k_2) = i \left[ 2af_{2\gamma\gamma}\Gamma_{\mu\nu\lambda\kappa}^{(0)}(k_1, k_2) - b_{f_2\gamma\gamma}\Gamma_{\mu\nu\lambda\kappa}^{(2)}(k_1, k_2) \right] F^{(f_2\gamma\gamma)}(k^2).$$

(2.15)

Here the so-called helicity-0 and helicity-2 $f_2 \rightarrow \gamma\gamma$ amplitudes are parametrised by the $a$ and $b$ constants, respectively. Two rank-four tensor functions, $\Gamma_{\mu\nu\lambda\kappa}^{(0)}$ and $\Gamma_{\mu\nu\lambda\kappa}^{(2)}$, are defined by (3.18) and (3.19) in [29]. In our calculation for the tensor mesons ($q\bar{q}$ states) we take $\Lambda_{xy} = 2$ GeV in form factors given by (2.9) and (2.10). We adopt also the numerical values of the $a_2(1320)$ $\gamma\gamma$ coupling constants discussed in sections 5.4 and 7.2 of [29].

For a better analysis we shall use model for the $f_2$ propagator considered in [29]; see Appendix A there. The Breit-Wigner formula is used here

$$i\Delta^{(f_2)}_{\mu\nu\lambda\alpha\beta}(k) = \frac{i}{k^2 - m_{f_2}^2 + im_{f_2}^\prime \Gamma_{f_2}} \left[ \frac{1}{2} (\hat{g}_{\mu\alpha} \hat{g}_{\lambda\beta} + \hat{g}_{\mu\beta} \hat{g}_{\lambda\alpha}) - \frac{1}{3} \hat{g}_{\lambda\lambda} \hat{g}_{\mu\alpha} \right],$$

(2.16)

where $\hat{g}_{\mu\nu} = -g_{\mu\nu} + k_\mu k_\nu / k^2$.

In our work we consider also the $f_2^\prime(1525)$ meson contribution. We assume the same ratio of the two-photon decay widths of the specific helicity to the total two-photon decay width as for the $f_2(1270)$ meson. We have

$$a_{f_2^\prime(1525)\gamma\gamma} = \pm \frac{e^2}{4\pi} 0.12 \text{ GeV}^{-3}, \quad b_{f_2^\prime(1525)\gamma\gamma} = \pm \frac{e^2}{4\pi} 0.30 \text{ GeV}^{-1}.$$

(2.17)

A contribution of heavier tensor-meson states are also possible, see Table I. Recently in [12] a role of $f_2(1655)$ and $a_2(1700)$ mesons was discussed in the context of transition form factors. Here we also take into account these two states. In order to estimate the strength of couplings we assume only helicity-2 contributions. From Eq. (5.28) of [29] and with the parameters of Table II we get

$$b_{f_2(1655)\gamma\gamma} = \pm \frac{e^2}{4\pi} 0.93 \text{ GeV}^{-1}, \quad b_{a_2(1700)\gamma\gamma} = \pm \frac{e^2}{4\pi} 0.52 \text{ GeV}^{-1}.$$

(2.18)

In calculation we choose the positive values of above coupling constants.

The $t/u$-channel exchanges of spin-2 particles leads to a growing of the cross section as a function of $\sqrt{s}$. In practice the rapid growth appears quickly above the $s$-channel resonance. No clear unitarization procedure is known to as. From phenomenology we know that the exchange of the meson must be replaced by the exchange of the $f_2$ reggeon [29]. In the following we shall therefore omit the $t$- and $u$-channel meson exchange contributions.
D. $f_4(2050)$ meson exchange

The $f_4(2050)$ resonance was identified in $\gamma\gamma \rightarrow \pi\pi$ processes \[19\]. However, the detailed tensorial coupling for $f_4(2050) \rightarrow \gamma\gamma$ is not known, so as a consequence we do not know corresponding angular distributions. We shall therefore neglect $t$- and $u$-channel contributions. In the following we use a simple Breit-Wigner resonance form for a spin-$J$ resonance $R$: \[2.19\]

$$
\sigma_{\gamma\gamma \rightarrow R \rightarrow \gamma\gamma}(s) = \frac{4(2J + 1)}{(2s_1 + 1)(2s_2 + 1)} \frac{4\pi(s/m_R^2)}{(s - m_R^2)^2 + m_R^2\Gamma_R^2} \left(\sqrt{2} \times \Gamma(R \rightarrow \gamma\gamma)\right)^2 \times (F(s))^4,
$$

which represents only $s$-channel contribution to the total cross section. The resonance form reproduces result of our diagrammatic calculation for lower-spin resonances at $\sqrt{s} \approx m_R$. In (2.19) the resonance width $\Gamma_R$ is a constant. An improved description of resonance shapes can be obtained when the width is made $s$-dependent.

III. RESULTS

In this section we present first numerical results for meson exchange contributions to the $\gamma\gamma \rightarrow \gamma\gamma$ scattering. We shall include not only $s$-channel resonant contributions but also $t$- and $u$-channel exchanges, not discussed so far in the literature.

We start with contributions of the pseudoscalar mesons that were observed in two-photon processes in the $e^+e^-$ collisions (Belle, BaBar). In the left panel of Fig. 2 we show, for example, separate contributions of $s$, $t$ and $u$ channels and their coherent sum for $\eta'(958)$ meson. The $t$ and $u$ channels play a role only for $\sqrt{s} > 2$ GeV. In the right panel we show separate contributions of $\pi^0$, $\eta$ and $\eta'$ exchanges as well as their sum. A large interference effects can be seen.

In Fig. 3 we present an example of angular distribution at $\sqrt{s} = 1.3$ GeV for the light pseudoscalar mesons (similar distribution for the same energy will be discussed for tensor meson contributions). The resulting distribution is relatively flat which is caused by the dominance of the $s$-channel exchange at the selected energy (see the previous figure).

In the left panel of Fig. 4 we show separate contributions of tensor meson exchanges ($s$-channel only) as well as their coherent sum. Similarly as for the pseudoscalar meson exchange contributions relatively large interference effects can be seen. In the right panel we show angular distributions for $\sqrt{s} = 1.3$ GeV. The resulting tensor meson distribution shows an enhancement at $\cos \theta \approx \pm 1$ (compare Fig. 3 for pseudoscalar meson contributions). The subdominant (helicity-0) $f_2(1270)$ $\gamma\gamma$ coupling ($\Gamma^{(0)}$ in (2.15)) plays important role for $\cos \theta \approx 0$.

Finally, in Fig. 5 we show energy dependence of all meson exchange contributions integrated over full $z = \cos \theta$ range (left panel) and for $|z| < 0.6$ (right panel). For reference we show also the fermion-box (lepton and quark) contributions separately and

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2 In the general case the initial particles have spins $s_1$ and $s_2$. In our case, for a real photons, a factors $(2s_i + 1)$ are replaced by 2.
FIG. 2: The energy dependence of the pseudoscalar meson contributions to the $\gamma\gamma \to \gamma\gamma$ scattering.

FIG. 3: Differential cross section for the $\gamma\gamma \to \gamma\gamma$ process at $\sqrt{s} = 1.3\text{ GeV}$. We present results for separate pseudoscalar contributions as well as their coherent sum.

added coherently together. The $W$-boson loops contribution plays a role only for $\sqrt{s} > 200\text{ GeV}$ [30]. The mesonic contribution exceed the quark-box one for different intervals of $\sqrt{s}$. Only a few narrow resonances clearly exceed the total fermion continuum. The reader is asked to note a sizeable interference effect between pseudoscalar and tensor meson contributions.

The $\pi^0, \eta, \eta'(958)$ states were already observed in the $e^+e^-$ collisions. It seems rather difficult to observe the heavy quarkonia $\eta_c(1S), \eta_c(2S)$ and $\chi_{c0}$. In heavy ion ultraperipheral collisions at CERN this is because of poor two-photon invariant mass resolution of
FIG. 4: The energy dependence of the $s$-channel tensor-meson resonances (left) and the angular distributions at $\sqrt{s} = 1.3$ GeV (right).

FIG. 5: The energy dependence of the meson exchange contributions compared with the fermion-box ones. Results integrated over full $z$-range (left) and for $|z| < 0.6$ (right) are plotted. The $f_4(2050)$ meson contribution is calculated from (2.19).

the order of 0.5 GeV while in low-energy $e^+e^-$ collisions because of limited phase space and the presence of two-photon bremsstrahlung background. The region of $f_2(1270)$ seems quite interesting as here some enhancement could be potentially identified by the Belle II at SuperKEKB for instance. Imposing a cut $|z| < 0.6$ (see the right panel of Fig. 5) improves the signal (meson exchanges) to background (boxes) ratio.

The meson exchange contributions are limited only to $\sqrt{s} < 4$ GeV, and should not
influence the recent ATLAS experimental result \[31\].

IV. CONCLUSIONS

In the present work we have discussed for the first time the role of \( s-, t-, u\)-channel meson exchanges in elastic \( \gamma \gamma \rightarrow \gamma \gamma \) scattering. We have included a few mesonic states with large two-photon branching fractions, see Table II. Large interference effects for light pseudoscalar and tensor mesons have been found. The interference of \( \pi^0, \eta \) and \( \eta' \) amplitudes increases the cross section for \( \sqrt{s} > m_{\eta'} \) by almost an order of magnitude. The interference of \( f_2(1270), a_2(1320) \) and the other tensor mesons leads to interesting spectral shape for collision energies in the window between 1 – 2 GeV. Could this be measured at SuperKEKB in a future? Not excluded, provided two-photon bremsstrahlung eliminated experimentally or is taken into account in the calculations.

We have included not only the \( s\)-channel diagrams (leading to peaks at \( \sqrt{s} \approx m_M \)) but also the meson exchanges in \( t\) - and \( u\)-channels (leading to broad continua). In general, the \( t \) and \( u \) diagrams contribute above the resonance peaks associated with the \( s\)-channel exchanges which is caused by kinematics of the process and gauge invariance explicitly fulfilled (imposed) in our calculation. This observation is true for the pseudoscalar, scalar and tensor meson exchanges. Only the \( s\)-channel contributions play a role for heavy pseudoscalar (\( \eta_c(1S), \eta_c(2S) \)) and scalar (\( \chi_{c0} \)) mesons and the \( t/u\)-channel contributions may be safely neglected.

The results related to meson exchanges have been compared with the standard, in the context of \( \gamma \gamma \rightarrow \gamma \gamma \) scattering, fermion-box continuum. The mesonic contributions concentrate in the region of \( \sqrt{s} \in (0.1, 4.0) \) GeV and are in general smaller than box contributions, except of some specific regions of the phase space. For instance, at the resonance positions the meson exchange contributions sometimes even strongly exceed the standard box ones but experimental observation may depend on diphoton invariant mass \( (M_{\gamma \gamma}) \) resolution of a given experiment.

The exchanges of light pseudoscalar (\( \pi^0, \eta \) and \( \eta' \)) and tensor mesons is also important in the context of possible interference with the fermion-box contributions. This goes, however, beyond the scope of the present paper and will be studied elsewhere.

The meson exchange contributions discussed in the present paper could potentially influence the \( ^{208}Pb + ^{208}Pb \rightarrow ^{208}Pb + ^{208}Pb + \gamma \gamma \) ultraperipheral collisions measured recently by the ATLAS collaboration \[31\]. Our present analysis suggest, however, that the meson exchange contributions do not play important role for the recent ATLAS measurement where \( M_{\gamma \gamma} > 6 \) GeV.

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[1] G. Jikia and A. Tkabladze, Phys. Lett. B323 (1994) 453.
[2] M. Bohm and R. Schuster, Z. Phys. C63 (1994) 219.
[3] D. Bardin, L. Kalinovskaya and E. Uglov, Phys. Atom. Nucl. 73 (2010) 1878.
[4] Z. Bern, A. De Freitas, L. J. Dixon, A. Ghinculov and H. L. Wong, JHEP 0111 (2001) 031.
[5] M. Klusek-Gawenda, P. Lebiedowicz, and A. Szczurek, Phys. Rev. C93 (2016) 044907.
[6] M. Klusek-Gawenda, W. Schäfer and A. Szczurek, Phys. Lett. B761 (2016) 399.
[7] S. Uehara et al. (Belle Collaboration), Phys. Rev. D86 (2012) 092007.
[8] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D80 (2009) 052002.
[9] P. del Amo Sanchez et al. (BaBar Collaboration), Phys. Rev. D84 (2011) 052001.
[10] J. P. Lees et al. (BaBar Collaboration), Phys. Rev. D81 (2010) 052010.
[11] M. Masuda et al. (Belle Collaboration), Phys. Rev. D93 (2016) 032003.
[12] I. Danilkin and M. Vanderhaeghen, Phys. Rev. D95 (2017) 014019.
[13] L. D. Landau, Dokl. Akad. Nauk. Ser. Fiz. 60 (1948) 207; C.-N. Yang, Phys. Rev. 77 (1950) 242.
[14] C. Patrignani et al. (Particle Data Group), Chin. Phys. C40 (2016) 100001.
[15] L.-Y. Dai and M.R. Pennington, Phys. Rev. D90 (2014) 036004; Phys. Lett. B736 (2014) 11.
[16] C. Amsler, Rev. Mod. Phys. 70 (1998) 1293.
[17] M. R. Pennington, T. Mori, S. Uehara and Y. Watanabe, Eur. Phys. J C56 (2008) 1.
[18] V. A. Shchegelsky, A. V. Sarantsev, V. A. Nikonov, A. V. Anisovich, Eur. Phys. J. A27 (2006) 207.
[19] M. Klusek-Gawenda and A. Szczurek, Phys. Rev. C87 (2013) 054908.
[20] J. F. Donoghue, E. Golowich and B. R. Holstein, “Dynamics of the Standard Model”, Cambridge 1992.
[21] J. P. Lansberg and T. N. Pham, Phys. Rev. D74 (2006) 034001; Phys. Rev. D75 (2007) 017501.
[22] S. Uehara et al. (Belle Collaboration), Phys. Rev. D78 (2008) 052004; Phys. Rev. D79 (2009) 052009.
[23] S. Uehara et al. (Belle Collaboration), Phys. Rev. D80 (2009) 032001.
[24] N. N. Achasov and G. N. Shestakov, Phys. Rev. D81 (2010) 094029; Phys. Usp. 54 (2011) 799, arXiv:0905.2017.
[25] A. V. Anisovich, V. V. Anisovich, D. V. Bugg and V. A. Nikonov, Phys. Lett. B456 (1999) 80.
[26] R. Barate et al. (ALEPH Collaboration), Phys. Lett. B472 (2000) 189.
[27] J. P. Lansberg and T. N. Pham, Phys. Rev. D79 (2009) 094016.
[28] I. Danilkin and M. Vanderhaeghen, arXiv:1705.01179.
[29] C. Everz, M. Maniatis and O. Nachtmann, Annals Phys. 342 (2014) 31.
[30] P. Lebiedowicz, R. Pasechnik and A. Szczurek, Nucl. Phys. B881 (2014) 288; P. Lebiedowicz, M. Łuszczak, R. Pasechnik and A. Szczurek, Phys. Rev. D94 (2016) 015023.
[31] M. Aaboud et al. (ATLAS Collaboration), arXiv:1702.01625.