A Robust $\mathcal{H}_\infty$ Approach of In-flight Calibration for UAVs with Low-cost IMU

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Abstract. In this paper, the on-line calibration problem for the low-cost IMU of UAV has been studied. A robust $\mathcal{H}_\infty$ estimation scheme is designed such that, in the presence of modelling error, asynchronous sampling issue, and filtering bias, the overall estimation dynamics is exponentially stable in the mean square; at the same time, this scheme is applicable to the on-line calibration with asynchronous sampling time. More specifically, a novel attitude angle matching calibration scheme is proposed. Hereinto, the measurement model is deduced in detail in the scheme, and the influences of the amplitude and frequency on the estimation of the misalignment angle is analyzed in detail. A numerical simulation example has been used to demonstrate the potential of the proposed approach to robust estimation in terms of both performances and computational tractability.

1. Introduction

Unmanned aerial vehicle (UAV) is one of the emerging methods to implement unmanned logistics transport and aerial photography. And thus, how to complete the online calibration of UAV airborne inertial navigation system (INS) quickly and accurately is the key to improve localization accuracy and route planning efficiency. However, INS has the weakness of low observability, especially in the case of static pedestals, which will lead to the least observability [1]. Besides, during the state estimation of the navigation system of UAV, the observability of the INS will affects the convergence speed and convergence accuracy of Kalman-like estimator. Meanwhile, during the calibration process of the moving base UAV, the movement of the pedestal can improve the observability of the airborne INS. Therefore, the accuracy and speed of the INS online calibration can be improved by the purposeful maneuvering of the pedestal under certain conditions [2, 3].

However, the online calibration problem of the UAV airborne INS has the following two characteristics: (i) the flight environment is complex: the navigation process in the urban environment requires rapid response, and the flight environment is intensive, if the calibration time is too long It may reduce the safety of flight due to insufficient navigation accuracy; (ii) UAV usually equipped with low-cost IMU; (iii) the security requirement of UAV needs the low-cost IMU has high accuracy [4-6]. And thus, the traditional velocity matching calibration scheme cannot meet the special requirement of the UAV.

Literature [7] proposed an angular velocity matching scheme, which shows that it can get better results with the swaying wing; however, this scheme is only applicable to the strapdown to strapdown
system. Literature [8] proposes a combination matching scheme with velocity and attitude angle; however, this approach didn’t give the specific Kalman-like filtering model. In this paper, a novel attitude angle matching calibration scheme is proposed. Hereinto, the measurement model is deduced in detail in the scheme, and the influence the amplitude and frequency the wing on the estimation of the misalignment angle is analyzed in detail.

2. Mathematical description of robust $H_{\infty}$ filter

The key feature of $H_{\infty}$ filtering approach is applied the $H_{\infty}$ norm to the filter design, in order to solve the various uncertainties of the target system. Besides, $H_{\infty}$ filtering approach regards the noise and uncertain inputs as the random signals with limited energy, making the $H_{\infty}$ norm of the closed-loop transfer function from system disturb to estimation error less than a given positive value $\gamma$.

In this paper, let the dynamic equation of the generalized controlled object be

$$
\begin{align*}
\dot{x} &= Ax + Bw \\
y &= Cx + Dv \\
z &= Lx
\end{align*}
$$

(1)

where $x$ is the state vector, $y$ is the observation vector, $z$ is the estimated vector, $w$ and $v$ denote the system noise and observation noise, respectively; $A$, $B$, $C$, $D$, and $L$ denote the constant matrices with corresponding dimensions.

The system is supposed to satisfy the following constraints: (i) $(A, B)$ is controllable, while $(A, C)$ is observable; (ii) $DD^T = I$. More specifically, constraints (i) is the prerequisite for the existence of the estimator, while constraints (ii) is equivalent to the performance index in the general quadratic form.

Let $\hat{z}$ be the estimation of $z$, $\hat{z} = L\hat{x}$, and then $\Delta z = z - \hat{z} = Tw$, where $T$ denotes the transfer function matrix from $w$ to $\Delta z$. For a linear steady-state system (1) with finite energy interference $w$, the optimal estimated performance index can be written as

$$
J = \|T\|_\infty = \sup_{0 \neq w \in L_2(0,\infty)} \left\{ \frac{\|z - \hat{z}\|_2}{\|w\|_2} \right\} < \gamma
$$

(2)

In order to make the estimation result of $H_{\infty}$ estimator unbiased and have a predictive correction structure, the state estimator should satisfy the structure of the observer as follows

$$
\begin{align*}
\dot{x} &= A\hat{x} + K(y - C\hat{x}), \\
\dot{z} &= L\hat{x}, \\
\hat{x}(0) &= 0
\end{align*}
$$

(3)

where, denotes the feedback gain matrix of the state estimator. Let $e = x - \hat{x}$, and then

$$
\begin{align*}
\dot{e} &= (A - KC)e + Gw - KDv \\
\Delta z &= z - \hat{z} = Le
\end{align*}
$$

(4)

The transfer function from interference $w$ to estimation error $\Delta z$ can be expressed as follows

$$
T = \begin{bmatrix}
A - KC & G & -KD \\
L & 0 & 0
\end{bmatrix}
$$

For the system model (1), which considers the disturbance effects and satisfies the constraints (i) and (ii), given a positive number $\gamma > 0$, the necessary and sufficient condition for equation (2) to be
established is that, there is a unique positive definite symmetric matrix \( P(P = P^T > 0) \) that satisfies the following algebraic Riccati equation

\[
AP + PA^T + P(y^{-1}L^T L - C^T C)P + BB^T = 0
\]

(5)

where the feedback gain matrix to make the estimator (3) stable is \( K = PC^T \).

An thus, we can conclude from the above analysis that, the existence of the \( H_\infty \) filter depends not only on its own structural parameters, but also on the vector to be estimated, i.e., \( z \). \( \gamma \) is required to be as small as possible in the design of the \( H_\infty \) filter, and thus the value of \( \gamma \) should choose the minimum value of the solution of equation (5).

3. Error Models of online calibration for airborne low-cost IMU

Coordinate frames used in the case study of on-line calibration of the low-cost IMU can be referred in [3,7]; hereinto, in the earth fixed frame (e-frame), we can obtain \( \Omega = [0 \ \omega_e \cos L \ \omega_n \sin L] = [0 \ \Omega_y \ \Omega_z] \), where \( \omega_e \) is the rotational angular velocity of the earth, \( L \) denotes the local latitude. The IMU error differential equations in the navigation frame mechanization are given as follows:

\[
\begin{aligned}
\dot{X} &= AX + \eta \\
Y &= DX + \zeta
\end{aligned}
\]

(6)

where

\[
X = [\phi_e \ \phi_n \ \phi_u \ \delta V_e \ \delta V_n \ \delta \lambda \ \nabla_e \ \nabla_n]^T
\]

hereinto, \( \phi_e, \phi_n, \phi_u \) denote the misalignment errors in east angle, north angle and azimuth angle, respectively, \( \delta V_e, \delta V_n \) denote the errors in east velocity and north velocity, \( \delta \lambda \) denotes latitude error, \( \nabla_e, \nabla_n \) denote the accelerometer bias errors, \( \eta \) denote the system white noise vector, \( E(\eta,\eta) = Q\delta_{ij} \).

\[
\begin{bmatrix}
0 & \omega_e \sin L + V_e \tan L / R_e & -\omega_e \cos L + V_e / R_e & 0 & -1 / R_e & 0 & 0 \\
-\omega_e \sin L - V_e \tan L / R_e & 0 & -V_e / R_e & 1 / R_e & 0 & \omega_e \sin L & 0 \\
\omega_e \cos L + V_e / R_e & V_e / R_e & 0 & \tan L / R_e & 0 & \omega_e \cos L + V_e \sec L / R_e & 0 \\
0 & -f_e & f_e & V_e \tan L & 2 \omega_e \sin L + V_e \tan L / R_e & 2 \omega_e \cos L + V_e \sec L / R_e & 0 \\
f_e & 0 & -f_e & 2 \omega_e \sin L + V_e \tan L / R_e & 0 & -2 \omega_e \cos L + V_e \sec L / R_e & V_e / R_e \\
0 & 0 & 0 & 0 & 1 / R_e & 0 & 0 \\
0 & 0 & 0 & \sec L / R_e & 0 & 0 & 0
\end{bmatrix}
\]

where \( f_e, f_n, f_u \) denote the east, north and celestial component of the specific force, respectively, \( R_e \) denotes the radius of the earth.
\[ \mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 \\ 0 & 0 & 0 & C_{11} & C_{12} \\ 0 & 0 & 0 & C_{21} & C_{22} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

where, \( C_{ij} \) is an element in the attitude transformation matrix from sensor coordinate system to e-frame. The horizontal velocity error from IMU/GPS output is used as observation equation as follows,

\[ Y = V_{\text{INS}} - V_{\text{GPS}} = DX + \zeta \] (7)

and

\[ D = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

where, \( \zeta \) denotes observation white noise vector, \( E(\eta_i\eta_j) = R\delta_{ij} \).

4. Numerical Simulation and Analysis

4.1. Simulation configurations and metrics

In this section, we present a numerical simulation example on the low-cost IMU of the UAV to demonstrate the proposed estimator design method. The primary sensor system used is the low-cost IMU that generates position, velocity and attitude information. Velocity error formed from the difference between the IMU and the GPS output yield observation to the robust \( H_\infty \) estimator. Here, the IMU computations are carried out at 20Hz; while the robust \( H_\infty \) estimator update interval is 1Hz. This test has been carried out for 475s.

In the experimentation, the designed values of matrices for the process noise covariance and the measurement noise covariance are as follows:

\[ Q = \text{diag}\{(50\mu g)^2, (50\mu g)^2, (0.25^\circ/h)^2, (0.25^\circ/h)^2, (0.25^\circ/h)^2, 0, 0, 0, 0, 0\} \]

\[ R = \text{diag}\{(0.01m/s)^2, (0.01m/s)^2\} \]

where the initial horizontal and azimuth misalignment angles are 1\(^\circ\), initial velocity error is 0.1m/s, the constant error of accelerometer is 100\(\mu g\), the constant bias of gyroscope is 0.3\(^\circ\)/h, the random bias is 0.25\(^\circ\)/h, the measurement noise of system velocity is 0.01m/s, and the initial localization error is (10\(^{-4}\))^\(\circ\).

The weighted matrix of the proposed robust \( H_\infty \) estimator is

\[ S = \text{diag}\{(1^\circ/3600)^2, (1^\circ/3600)^2, (1^\circ/3600)^2, (10^{-4})^2, (10^{-4})^2, (10^{-6})^2, (10^{-6})^2, 0, 0, 0, 0, 0\} \]

\[ W = Q / 3, \ V = R / 5, \ \gamma = 3 \]
4.2. Simulation results and analysis

In this subsection, several key properties of the proposed robust $H_{\infty}$ estimation approach are tested on the UAV with the dynamics models in Subsection 4.1. During the simulation evaluation, the motion of UAV is divided into two stages, i.e., (i) uniformly accelerated motion with acceleration value 0.04m/s$^2$, and initial velocity 5m/s, and (ii) uniform circular motion with acceleration value 0.04m/s$^2$.

For comparison, a Kalman-like linear estimator in [7] is chosen to compare with the newly proposed robust $H_{\infty}$ estimation algorithm in our simulation. The estimation results, i.e., misalignment error estimation of UAV and error estimation of UAV velocity, are listed in Figures 1 and 2, respectively.

(a) (b) (c)
Furthermore, in Table 1, we analyse the calibration accuracy of Kalman-like estimator and robust $H_\infty$ estimator in the form of numerical results. It is clearly from the results that the calibration accuracy of robust $H_\infty$ estimator is higher than Kalman-like estimator, especially when the heading angle of the UAV is changing constantly. Besides, the calibration velocity of the proposed $H_\infty$ estimator is faster than the Kalman-like estimator.

Table 1. Simulation comparison between Kalman filter and robust $H_\infty$ filter

| Algorithm                  | $\phi_e \, /''$ | $\phi_n \, /''$ | $\phi_\alpha \, /''$ |
|----------------------------|-----------------|-----------------|---------------------|
| Accelerated motion         |                 |                 |                     |
| Kalman                     | 14.457          | 14.365          | 7.978               |
| $H_\infty$                 | 12.675          | 13.354          | 5.786               |
| Uniform Circular Motion    |                 |                 |                     |
| Kalman                     | 11.564          | 11.567          | 7.464               |
| $H_\infty$                 | 9.786           | 9.452           | 5.243               |

5. Conclusions
In this paper, the on-line calibration problem for the low-cost IMU of UAV has been studied. A robust $H_\infty$ estimation scheme has been designed such that, in the presence of modelling error, asynchronous sampling issue, and filtering bias, the overall estimation dynamics is exponentially stable in the mean square and, at the same time, this scheme is applicable to the on-line calibration with asynchronous sampling time. A numerical simulation example has been used to demonstrate the potential of the proposed approach to robust estimation in terms of both performances and computational tractability.

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