Fluctuation-induced self-force and violation of action–reaction in a nonequilibrium steady state fluid

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Abstract. We show that the fluctuations of a fluid driven out of equilibrium can induce a net force on a single asymmetric object immersed in it. The force originates in the restriction of the fluid’s fluctuations at the object’s boundaries, as in the Casimir effect. In contrast to the equilibrium situation, its emergence on a single obstacle is not ruled out by the second law of thermodynamics since the fluid is in a nonequilibrium state. We explicitly calculate this self-force on a deformed circle embedded in a fluid whose density fluctuations obey a stochastic reaction–diffusion equation. When two objects are considered, the presence of self-forces can violate the action–reaction principle. We illustrate this by calculating the internal Casimir-type forces between a circle and a plate. Their sum, instead of vanishing, provides the self-force exerting on the circle–plate assembly.

1. Introduction
Since the pioneering work of Casimir [1], it has been known that the microscopic fluctuations of a media can induce net forces between macroscopic objects immersed in it. The paradigmatic example is that of the zero-temperature Casimir force between two metallic plates, induced by the quantum fluctuations of the vacuum electromagnetic field [2, 3]. Many examples of similar forces have since been exhibited. Lifshitz [4] showed that additional contributions to the Casimir force arise when thermal fluctuations of the quantum field are considered. Thermal fluctuations are known to induce forces in critical fluids close to criticality, liquid crystals in the nematic phase, systems with a broken continuous symmetry and other systems [5, 6, 7, 8]. A necessary ingredient for such forces to arise is that the range of the correlations in the media be large enough to explore the macroscopic disturbances provoked by the embedded objects. Fluctuation-induced forces thereby strongly depend on these objects’ shape [9, 10].

Most of the systems considered in this field are usually assumed to be in equilibrium states. Situations out of equilibrium, however, are also known to develop fluctuations that can have long ranges [11]. Examples are shear Couette flows, Rayleigh–Bénard instabilities, but also granular fluids [12], shaken fluids [13], reaction–diffusion fluids [14], or simply systems of the kind mentioned above, that are driven out of equilibrium [15, 16]. One thus expects that similar fluctuation-induced forces can be generated between inclusions in nonequilibrium systems [17, 18]. The characterization of these forces, however, does not rely on general first principles. Indeed, (equilibrium) thermodynamic potentials are generally not available. It is only recently that such forces have been obtained between two planar objects immersed into nonequilibrium driven systems [19, 16], granular fluids [20], or reaction–diffusion systems that break the detailed balance [21].
Various properties of nonequilibrium fluctuation-induced forces can make them very interesting in applications. A recurrent theme in Casimir-like forces is the control of its sign (attraction or repulsion). Thermodynamic equilibrium reduces the number of parameters controlling the system to only a few (such as temperature, the density, etc.). These external parameters, on which the system’s fluctuations depend, are then usually able to control only the magnitude, but not the sign, of the induced forces. (Thermal fluctuation-induced forces, e.g., are proportional to $k_B T$.) Nonequilibrium fluctuations, on the other hand, can be tuned by as many control parameters as one may wish and by modifying their relative strength (as we will see) or their properties one may achieve sign reversal of the forces (see also [22, 19]).

In addition, we will see in this work that the extension of Casimir-like forces to fluctuating fluids driven out of equilibrium allows for two interrelated effects that arise in nontrivial geometries and that are not possible in equilibrium systems. Namely, nonvanishing forces and torques can be induced on single asymmetric obstacles and the action–reaction principle between two intruders can be violated. These phenomena can have significant consequences in experiments, as they lead to directed motion and unevenness in the measurements of the forces between two objects.

Whether at equilibrium or not, accurate experimental measurements of fluctuation-induced forces need to go beyond the idealized geometry of infinitely long plates, predominant in theory for its simplicity. While a long-studied topic, the objects’ shape dependence of Casimir forces is notoriously difficult for nontrivial geometries. The most widely used technique by experimentalists to date relies on the so-called Derjaguin construction (proximity force theorem), which in essence integrates the two-plate expression of the force along the curved surfaces [23, 24]. An unbalance of action and reaction in a nonequilibrium situation would impede the use of the Derjaguin approximation where it would normally be valid at equilibrium. Note that this unbalance does not seem to be systematic: in [16], e.g., the Casimir forces exerting between two plates of different material constitutions seem to satisfy the action-reaction principle. A violation of Newton’s third law, however, has also been noted in depletion forces between identical spherical objects immersed into a flowing fluid [25]. It furthermore prevents the two-body forces from being derived from an effective smooth potential, in striking contrast to equilibrium situations. We will see that this unbalance directly results from the presence of self-forces.

Since nonequilibrium systems are thermodynamically open, it is not entirely unexpected that self-forces can appear. In fact, provided that both the microscopic time-reversibility and space rotation-invariance symmetries are broken, such forces have been implicitly suggested by the occurrence of sustained motions in other nonequilibrium contexts, such as in ratchets [26], Brownian motors [26, 27], molecular motors [28], or the adiabatic piston [29]. However, in these systems the space asymmetry usually lies in an external temperature gradient or another anisotropic field exerting on the object. More recently, the directed motion of an asymmetric object immersed into vibrated granular matter has been exhibited [30, 31]. Here, a direct calculation of the force exerting on single objects is presented from a “Casimir effect” point of view. Knowing these forces allows for a better understanding of the different effects at play in such sustained motions. It also makes possible the evaluation of additional stresses exerting on asymmetrical static structures in microdevices. Jointly with the sign reversal control of the force, it could be used as a powerful tailoring mechanism for the self-assembly of ordered structures.

2. A nonequilibrium fluid model

To illustrate these effects, we introduce here a rather simplified nonequilibrium fluid. We consider a reaction–diffusion system in which two chemical components, $A$ and $B$, diffuse and react according to

$$A + B \xrightarrow{k_1} 2B, \quad B \xrightarrow{k_2} A$$

(1)
with rates $k_1$ and $k_2$, respectively. The average densities $\rho_A$, $\rho_B$ of these species thus satisfy the rate equations

$$\frac{\partial}{\partial t}\rho_A = D \nabla^2 \rho_A - \frac{k_1}{\rho_{tot}} \rho_A \rho_B + k_2 \rho_B,$$

$$\frac{\partial}{\partial t}\rho_B = D \nabla^2 \rho_B + \frac{k_1}{\rho_{tot}} \rho_A \rho_B - k_2 \rho_B,$$

(2)

where $D$ is the diffusion constant and $\rho_{tot}$ is the total density. When $k_1 > k_2$, these equations admit the homogeneous stationary solution

$$\bar{\rho}_A = \frac{k_2}{k_1} \rho_{tot}, \quad \bar{\rho}_B = \rho_{tot} - \bar{\rho}_A.$$

(3)

The $A$ and $B$ density departures from this solution, that are generically denoted in the following by $\Phi$, both satisfy after linearization of (2) and addition of Langevin-type noises, the equation

$$\frac{\partial \Phi}{\partial t} = -\nabla \cdot (-D \nabla \Phi + \xi_c) - \gamma \Phi + \xi_{nc}.$$

(4)

In this equation, $\gamma = k_1 - k_2 > 0$ is the rate at which the system relaxes to local equilibrium. The added conservative ($c$) and nonconservative ($nc$) random noises $\xi_c$ and $\xi_{nc}$ are relevant to the fluid’s fluctuations once it has relaxed to the steady state. The conservative noise corresponds to fluctuations in the diffusive flux whilst the nonconservative noise, to fluctuations in the local density production. These fluctuations are assumed to have zero average and to be delta-correlated in space and time:

$$\langle \xi_{nc}(r,t) \xi_{nc}(r',t') \rangle = \Gamma_{nc} \delta(r-r') \delta(t-t'),$$

$$\langle \xi_c^\mu(r,t) \xi_c^\nu(r',t') \rangle = \Gamma_c \delta_{\mu\nu} \delta(r-r') \delta(t-t'),$$

$$\langle \xi_c^\mu(r,t) \xi_{nc}(r',t') \rangle = 0,$$

(5)

where $\mu, \nu = 1, 2, 3$ denote the cartesian coordinates of $\xi_c$. This model for the fluid is slightly improved compared to [32], where only nonconservative noise is considered. Let us add that other nonequilibrium systems in their steady state can be described by Eqs. (4)–(5) [19, 29].

In the presence of static objects in the fluid, the density fluctuations get constrained as no flow of matter can cross the rigid surfaces. Eq. (4) is thus supplemented by the non-flux condition

$$\mathbf{n} \cdot (-D \nabla \Phi + \xi_c) = 0$$

(6)

at the objects’ surface, where $\mathbf{n}(r)$ is a unit normal vector chosen to point outward from the fluid’s domain. The total force experienced by an immersed object results from integrating the fluid’s average pressure along its surface. In a steady state, one may calculate this pressure $p$ from a local equation of state that relates it to the density $p = p(\rho(r,t))$ [33, 34]. (Such a relation is experimentally measured in a number of cases of interest, e.g., in driven granular media [35].)\footnote{In the reaction–diffusion model, one may consider that $\rho_{tot}$ in (3) is maintained exactly constant so that only the fluctuations $\Phi = \rho_A - \bar{\rho}_A$ of the density of $A$ (for example) need to be considered.} Assuming that this relation is expandable around the homogeneous reference density and that density fluctuations stay small, one has $\langle p \rangle = p_0 + \frac{\alpha_0'}{2} \langle \Phi^2 \rangle$, where $p_0$ and $p_0'$ are constant parameters depending on the equation of state. Since the homogeneous pressure $p_0$ cannot generate a force, the total force $\mathbf{F}_S$ induced on the object $S$ is thus given by

$$\mathbf{F}_S = \frac{p_0'}{2} \int_S d\sigma \mathbf{n} \langle \Phi^2 \rangle.$$

(7)
To evaluate (7), one needs to calculate $\Phi$ as the solution of (4), (6). After a characteristic time of the order of $\gamma^{-1}$ the system relaxes to a stationary state only affected by the random noises. Given a realization of $\xi_c$ and $\xi_{nc}$, the local density deviation then reads

$$\Phi(r,t) = \int \! \! dt' \int \! \! d^2r' G(r,r',Dt-Dt')(-\nabla \cdot \xi_c + \xi_{nc})(r',t')$$

$$+ \int \! \! dt' \int \! \! d^2\sigma G(r,r',Dt-Dt')n(r') \cdot \xi_c(r',t'),$$

(8)

where $G$ is the Green function propagating the effect of a unit source localized at $r'$ at time $t'$ to the point $r$ at time $t$. It is used here to forward both the conservative and nonconservative noises acting in the whole volume $\Omega$ of the fluid and the conservative noise acting on the boundaries $\partial \Omega$ by (6). In Fourier representation with respect to the scaled time $\tau = Dt$, $G$ is solution of

$$(-\nabla^2 + \kappa^2 - i\omega) G(r,r',\omega) = \delta(r-r')$$

(9)

$$n(r) \cdot \nabla G(r,r',\omega)|_{r \in \partial \Omega} = 0 \ \forall r' \in \Omega, \forall \omega$$

(10)

where $\kappa^{-1} \equiv (\gamma/D)^{-1/2}$ is the correlation length of the fluid. Using the properties of $G$ and the correlations (5) of the noises, one can show from (8) (see details in [36]) that the static structure factor is given by

$$\langle \Phi(r,t)\Phi(r',t) \rangle = \frac{\Gamma_c}{2D} \delta(r-r') + \frac{\Gamma}{2D} G(r,r'),$$

(11)

with $\Gamma \equiv \Gamma_{nc} - \gamma \Gamma_c/D$ and $G(r,r') \equiv G(r,r',\omega = 0)$. The fluctuation-induced modification of the local pressure of the fluid at $r$, given by taking the limit $r' \to r$ in the above formula, obviously diverges so that the expression (7) needs to be regularized due to the inaccuracy of the continuous model at microscopic distances.

At thermal equilibrium, the fluctuation intensities $\Gamma_c$ and $\Gamma_{nc}$ are determined by the temperature of the fluid via the fluctuation–dissipation theorem: $\Gamma_c = 2k_B T D$, $\Gamma_{nc} = 2k_B T \gamma$, so that $\Gamma = 0$. Thus, only microscopic (delta) correlations are left. Being independent of the immersed bodies, these correlations cannot induce any force on them and we will simply omit their contribution in (7). In contrast, when the reaction–diffusion fluid reaches a nonequilibrium steady state where the detailed balance condition is not fulfilled [14], $\Gamma$ can be different from zero and correlations of mesoscopic range (of the order of $\kappa^{-1}$) can occur through $G$ in (11). However, two other divergencies emerge: first in taking the limit $r' \to r$ on $G(r,r')$ and then when evaluating $r$ on the surface $S$ in (7). The first divergency turns out to be independent of the immersed objects and can easily be removed by subtracting from $G$ the Green function $G_0$ of the unconstrained fluid. The second divergency, on the other hand, is more difficult to handle as it inherently depends on the object’s shape and is only compensated between different sides through the surface integral (7) [32]. A hard-core cutoff $\epsilon$ is introduced for that purpose and the regularized force reads

$$F_S = \lim_{\epsilon \to 0} F_0 \kappa \int_S \! \! d\sigma \! n [G-G_0](r-\epsilon n, r-\epsilon n),$$

(12)

where $F_0 \equiv \frac{D}{4}\Gamma$ has the dimension of a force. In two dimensions $G_0(r,r') = K_0(\kappa|r-r'|)/2\pi$ ($K_0$ is the modified Bessel function of order zero) and in three dimensions, $G_0(r,r') = \exp[-\kappa|r-r'|]/4\pi |r-r'|$.

Since $F_0$ is proportional to $\Gamma$, one is not only able to turn the fluctuation-induced force (12) on or off by driving the system out of equilibrium or at equilibrium; one can also modify its sign by controlling the balance between $\Gamma_{nc}$ and $\Gamma_c$ through the external parameters characterizing the nonequilibrium state. This important feature can have very appreciable consequences in experiments and applications.
3. Two nonplanar systems

Formula (12) is illustrated in the following on two classes of objects immersed into the fluid. For simplicity, the examples are all two-dimensional, but similar conclusions can be drawn in three-dimensional cases. Here, we only state the main results; explicit calculations will be developed in [36].

3.1. Self-force and self-torque

We first consider the immersion of a single asymmetric object, a deformed circle, into the fluid. In an equilibrium system, no net force would be induced on such an object. Here, however, one is induced due to the nonequilibrium character of the fluid’s fluctuations, that breaks the microscopic time-reversibility. This force is called “self-force” as there are no other rigid bodies in the fluctuating media.

The generic Green function in the presence of a single bounded object at the origin can be written from the differential equation (9) (at $\omega = 0$) in polar coordinates $r = (\rho, \theta)$, $r' = (\rho', \theta')$ as

$$G(r, r') = G_0(r - r') + \sum_{m, n \in \mathbb{Z}} \frac{e^{im\theta + in\theta'}}{2\pi} a_{mn} K_m(\kappa \rho) K_n(\kappa \rho'), \quad (13)$$

where $K_m$ is the modified Bessel function of order $m$. (The two-dimensional bulk Green function $G_0(r - r')$ is expressed in polar coordinates by the addition theorem for $K_0$ [37].) The reality of $G(r, r')$ and the reciprocity relation $G(r, r') = G(r', r)$ have been used and further imply that the coefficients $a_{mn}$ satisfy $a_{mn} = a_{nm} = a_{m, -n}^*$. These coefficients depend on the particular shape of the object. Considering a slightly deformed circle represented by a polar curve $R(\theta) = R + \eta s(\theta)$ (with $\eta \ll R, \kappa^{-1}$), they can be obtained perturbatively in $\eta$ by the enforcement of the boundary condition (10) up to a certain order. The whole force (12) is then expanded in $\eta$ (note that its dependence upon $R(\theta)$ occurs both through $G$ and through the line integral). The first nonvanishing contribution is of second order in $\eta$. The absence of first order can be understood as follows: the only perturbation of the circle exhibiting a preferred direction comes from the dipolar Fourier mode; however, a small dipolar perturbation amounts to a circle that is merely shifted—it stays symmetric. At second order, on the other hand, the dipolar mode couples to another mode, and leads to a finite result [32]. For instance, superposing a dipolar and quadrupolar deformation as in

$$s(\theta) = 2s_1 \cos(\theta) + 2s_2 \cos(2\theta) \quad (14)$$

results in the shapes displayed in Figure 1, and an average force

$$\mathbf{F} = -F_0 s_1 s_2 (\kappa \eta)^2 H_F(\kappa R) \hat{x}, \quad (15)$$

is induced on them (up to $O(\eta^3)$ terms). In (15), $H_F$ is a dimensionless function obtained as the limit as $\epsilon \to 0$ of a nontrivial series of Bessel functions evaluated at either $\kappa R$ or $\kappa(R + \epsilon)$. It can be shown that the series is absolutely convergent provided $\epsilon > 0$ but the limit cannot be taken under the summation sign. Nevertheless, the numeric computation of this function is accurately fitted by $2/\kappa R$ in the whole range $0.1 \leq \kappa R \leq 100$ (see Figure 2). Note that the shapes displayed in Figure 1 experience opposite forces.

Following the same lines, one can calculate the torque induced by the nonequilibrium fluctuations of the fluid on a single asymmetric object. Choosing a deformation

$$s(\theta) = 2s_2 \cos(2\theta) + 2s_4 \sin(4\theta) \quad (16)$$
of the circle (see Figure 3), the average torque up to $O(\eta^4)$ contributions is

$$T = -\frac{F_0}{\kappa} s_2^2 s_4 \left(\kappa \eta\right)^3 H_\tau(\kappa R) \hat{z}.$$  

(17)

The absence of second order $O(\eta^2)$ in (17) is associated to the antisymmetry of the torque under a reflection $s(\theta) \mapsto s(-\theta)$ of the deformation [36]. The dimensionless function $H_\tau$ is a nontrivial series of Bessel functions similar to $H_F$. However, it exhibits a different asymptotic behaviour for $\kappa R \ll 1$, where it approaches $\approx 80/\kappa R$, than for $\kappa R \gg 1$, where it approaches $\approx 50/\kappa R$, as can be seen in Figure 4. The transition between these regimes occurs at $\kappa R \approx 1$, i.e., when the correlation length $\kappa^{-1}$ matches the size $R$ of the deformed circle.

3.2. Two obstacles

When a second object is immersed into the fluid, the fluid’s fluctuations are modified and the force already exerting on the first object will likewise be affected. If $S$ and $S'$ denote these two
objects, the total force $\mathbf{F}_S$ on $S$ can be decomposed as the contribution of the self-force $\mathbf{F}^0_S$, already present in the absence of $S'$, plus a contribution $\mathbf{F}_{S\rightarrow S'} = \mathbf{F}_S - \mathbf{F}^0_S$ due to the additional asymmetry provoked by the insertion of $S'$. From (12), one has

$$\mathbf{F}_{S\rightarrow S'} = F_0 \kappa \int_S \mathbf{n} \left[ G_{SS'} - G^0_S \right] (\mathbf{r}, \mathbf{r}), \quad (18)$$

where $G_{SS'}$ is the Green function in the presence of both $S$ and $S'$, and $G^0_S$ the Green function when only $S$ is immersed into the fluid. In the expression (18), the hard-core cutoff $\epsilon$ has been removed: the divergencies present in both $G_{SS'}$ and $G^0_S$ cancel in the subtraction. The force (18) represents the inherent “two-body” force analogous to the Casimir interaction with zero set at infinitely separated objects. Considering the similar force $\mathbf{F}_{S\rightarrow S'}$ acting on $S'$, one has

$$\mathbf{F}_{S\rightarrow S'} + \mathbf{F}_{S'\rightarrow S} = \mathbf{F}_{SS'} - \mathbf{F}_S - \mathbf{F}^0_{S'}, \quad (19)$$

where $\mathbf{F}_{SS'}$ is the self-force exerting on the assembly $S \cup S'$ considered as a whole. Clearly, in the presence of self-forces, the right hand side of (19) does not necessarily vanish (as fluctuation-induced forces are not additive) and the action-reaction principle between the internal two-body forces can be violated.

To calculate $\mathbf{F}_{S\rightarrow S'}$ in a regime of large separation $d$ between the objects, a multiple scattering approach can be devised [9]. The differential problem (9)–(10) can equivalently be written in integral form as

$$G(\mathbf{r}, \mathbf{r}') = G_0(\mathbf{r}-\mathbf{r}') - \int_S d\sigma_1 \mathbf{G}(\mathbf{r}, \mathbf{r}_1) \mathbf{n}_1 \cdot \nabla_1 G_0(\mathbf{r}_1 - \mathbf{r}'). \quad (20)$$

Recursively iterating this integral equation expands $G$ as a series of multiple scatterings of the bulk green function $G_0$ on the surface $S$: using the abbreviation $G = G_0 + G \star G_0$ for (20), one has $G = G_0 + G_0 \star G_0 + G_0 \star G_0 \star G_0 + \ldots$. In the presence of the two objects $S$ and $S'$, $G_0$ is scattered on both surfaces. When the separation $d$ is much larger than the correlation length $\kappa^{-1}$, the dominant terms in the expansion of $G_{SS'}$ are the ones with the least number of propagations between $S$ and $S'$. Indeed, for $\mathbf{r} \in S$ and $\mathbf{r}' \in S'$, $G_0(\mathbf{r}-\mathbf{r}') = O(e^{-\kappa d})$. However, any number of scatterings from $S$ to $S'$ or from $S'$ to $S$ can be done without affecting the order as $\kappa d \to \infty$. The sum of these scatterings can be recast as $G^0_S$ and $G^0_{S'}$, respectively, and one can then show that

$$(G_{SS'} - G^0_{S'})(\mathbf{r}, \mathbf{r})|_{\mathbf{r} \in S} \xrightarrow{\kappa d \to \infty} G^0_S \star G^0_S + G^0_S \star G^0_{S'} \star G^0_{S'}. \quad (21)$$

The Green function of the composite system can thus be calculated in a large-separation regime from the knowledge of the Green functions associated to the single objects, which are usually much easier to obtain.

As a concrete example, we consider a circle $C$ of radius $R$ in front of a thin, infinitely-long plane $P$. The distance $d$ is taken as their separation at the closest point. An explicit expression for $G^0_C$ is straightforward to obtain from (13), with the result $a_{nm} = -[l'_m(\kappa R)/K'_m(\kappa R)]\delta_{n,-m}$, where $l'_m$ and $K'_m$ are the derivatives of the modified Bessel functions of order $m$. It can be checked that $G^0_C(\mathbf{r}, \mathbf{r}') = G_0(\mathbf{r}-\mathbf{r}') + G_0(\mathbf{r}-\mathbf{r}')$, where $\mathbf{r}'$ is the point symmetric to $\mathbf{r}'$ with respect to the plate. Using (18) and (21), one can then evaluate $\mathbf{F}_{C\rightarrow P}$ and $\mathbf{F}_{P\rightarrow C}$ in a regime where $\kappa^{-1} \ll d$. Note that since both the circle and the plane are symmetric, $\mathbf{F}_C = \mathbf{F}_P^0 = 0$.

We performed such an evaluation with the additional assumption that $R \ll \kappa^{-1}$. From an asymptotic analysis based on small-$\kappa R$ expansions and steepest-descent values of integrals as $\kappa d \to \infty$, we find that these forces are given by

$$F_{C\rightarrow P} \sim -F_0 \sqrt{\pi(\kappa R)^2 e^{-2\kappa d}} \sqrt{\kappa d}, \quad F_{P\rightarrow C} \sim -\frac{3}{2} F_{C\rightarrow P}, \quad R \ll \kappa^{-1} \ll d, \quad (22)$$
on the axis perpendicular to the plate. The action–reaction principle between these forces is clearly violated and the circle-plate assembly experiences the nonzero global force

\[ F_{CP} = F_{C\rightarrow P} + F_{P\rightarrow C} \sim \frac{1}{2} F_0 \sqrt{\frac{\pi}{\kappa d}} e^{-2\kappa d} \sqrt{\kappa d} \]

(23)
in this regime. Note that this violation is not a negligible effect: the force \( F_{CP} \) has the same order of magnitude as \( F_{C\rightarrow P} \) and \( F_{P\rightarrow C} \).

4. Concluding remarks

The extension of Casimir-like forces to fluctuating media driven out of equilibrium opens up the possibility for new effects that might have important consequences in experiments and applications. The contributions to the forces due to the nonequilibrium character of the fluctuations can be tunable by more parameters than in equilibrium and both their strength and sign may be controlled. Furthermore, in nontrivial geometries, nonequilibrium fluctuations can induce forces on single asymmetric rigid bodies and can break the action-reaction principle between two objects noticeably. These two consequences are clearly ruled out at equilibrium by the second law of thermodynamics.

The occurrence of a violation of the action-reaction principle impedes that an effective interaction potential holds in nonequilibrium and could prevent the use of the Derjaguin approximation. The magnitude of this violation can be of the same order as the internal forces, so that special care should be exercised in measurements performed in asymmetric setups.

If the immersed objects are let free to move, the presence of self-forces would put them into directed motion as in the case of ratchets. Jointly with the control of the sign of the force, this could be used as a powerful tool to tailor micro-devices by self-assembling or to construct motor axles with external energy source. The dynamical properties of such devices, however, need a more in-depth analysis as their motion will affect the fluid’s fluctuations and a self-dynamical interaction could take place.

Refining the initial model to describe more realistic fluids would clearly be needed to draw more quantitative predictions for systems such as colloidal solutions, granular fluids, dusty plasmas etc. Nevertheless, the simple model for the nonequilibrium fluid that we presented here allows to deal efficiently with the complexity of nonplanar geometries via a straightforward Green function formalism and multiple-scattering scheme.

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