Weak radiative hyperon decays and vector meson dominance

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We study the question whether the phenomenologically successful VMD approach to weak radiative hyperon decays can be made consistent with Hara’s theorem and still yield the pattern of asymmetries exhibited by experimental data. It appears that an essential ingredient which governs the pattern of asymmetries is the off-shell behaviour of the input electromagnetic $1/2^0 - 1/2^+ - \gamma$ couplings. Although this behaviour can be chosen in such a way that the experimentally observed pattern is obtained, and yet Hara’s theorem satisfied, at the same time the approach yields a definite prediction for the size of weak meson-nucleon coupling constants. Comparison with experiment reveals then another conflict.

1. INTRODUCTION

Experimental data on weak radiative hyperon decays (WRHD’s) present a challenge to our theoretical understanding. The puzzle manifests itself as a possible conflict between Hara’s theorem and experiment.

Hara’s theorem states that the parity-violating amplitude of the $\Sigma^+ \to \Lambda\gamma$ decay should vanish in the limit of SU(3) flavour symmetry. For expected weak breaking of SU(3) symmetry the parity-violating amplitude in question and, consequently, the $\Sigma^+ \to \Lambda\gamma$ decay asymmetry should be small. However, experiment shows that the asymmetry is large: $\alpha(\Sigma^+ \to \Lambda\gamma) = -0.72 \pm 0.086 \pm 0.045$. This large size is even more difficult to explain when one demands successful simultaneous description of data on three related WRHD’s, namely $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$. From the measured size of the WRHD branching ratios it follows that the single-quark process $s \to d\gamma$ (measured by the $\Xi^- \to \Sigma^-\gamma$ branching ratio) cannot explain the size of the branching ratios for the $\Sigma^+$ and neutral hyperon decays. Consequently, the latter decays must be dominated by (most probably) two-quark processes $su \to ud\gamma$.

Theoretical calculations may be divided into those performed totally at quark level and those carried out at hadron level. For a review see ref. where recent theoretical and experimental situation in the field is presented. It appears that simple quark model calculations violate Hara’s theorem. On the other hand, in pole-model-based hadron-level calculations Hara’s theorem is usually satisfied by construction. The only exception here is the hadron-level vector-meson dominance (VMD) approach of ref. which admits a pole-model interpretation and yet violates the theorem.

The WRHD puzzle is deepened by the fact that so far experimental data seem to agree with the predictions of the VMD model, and not with those of the (Hara’s-theorem-satisfying) standard pole model. Here, we report on an attempt to maintain the phenomenological success of the VMD approach without violating Hara’s theorem.

2. HARA’S THEOREM

The basic assumptions of Hara’s theorem are: (1) gauge-invariance, (2) CP-conservation, and (3) exact U-spin symmetry. Since $\Sigma^+$ and $p$ differ by the $s \leftrightarrow d$ interchange only, the $\Sigma^+$ behaves essentially like a proton. Instead of the $\Sigma^+p\gamma$ coupling, consider therefore the most general parity-violating $pp\gamma$ coupling (see ref. for a more rigorous proof):

$$\bar{\psi}(g_1q^2)\left(\gamma^\mu - \frac{q^\mu d}{q^2}\right)\gamma_5 + g_2(q^2)i\sigma^{\mu\nu}\gamma_5q_\nu|\psi \cdot A_\mu$$

(1)
Since there cannot be a pole at \( q^2 = 0 \) (in fact this is assumption (4)), it follows that \( g_1(0) = 0 \). Furthermore, the \( g_2 \) term violates CP, and therefore we must have \( g_2 = 0 \). Hence, the \( pp\gamma \) coupling vanishes at \( q^2 = 0 \). Since \( \Sigma^+ \) behaves essentially like a proton, the parity-violating \( A(\Sigma^+ \rightarrow p\gamma) \) amplitude must also vanish at \( q^2 = 0 \).

3. MODEL PREDICTIONS AND EXPERIMENT

Comparison of various model predictions with experiment is given in Table 1. Only selected models with the most predictive power are shown.

A closer look at Table 1 reveals that there are two possible patterns of the signs of asymmetries in the four WRHD’s (\( \Sigma^+ \rightarrow p\gamma, \Lambda \rightarrow n\gamma, \Xi^0 \rightarrow \Lambda\gamma, \) and \( \Xi^0 \rightarrow \Sigma^0\gamma \)) dominated by the \( su \) ordering. In Hara’s-theorem-satisfying approaches all these asymmetries are of the same sign (the pole model of ref.\[3\] predicts the pattern \((-,-,-,-))\). On the other hand, Hara’s-theorem-violating approaches yield the pattern \((-,+,+,-)).\) Experiment (and, in particular, the sign of the \( \Xi^0 \rightarrow \Lambda\gamma \) asymmetry [10]) seems to indicate [3] that it is the latter alternative that is realized in Nature. So far the best description of available data has been provided by the VMD approach [3].

4. THE STANDARD AND VMD-BASED POLE MODEL

4.1. Standard approach

In the standard pole model (ref.[3]) the contributions from diagrams (b1) and (b2) are evaluated at the hadron level as follows (consider \( \Sigma^+ \rightarrow p\gamma \) as an example). The contribution from intermediate states is approximated by the contribution from excited \( \frac{1}{2}^- \) baryons \( B^* \). For the (b1) diagram one calculates the weak transition \( \Sigma^+ \rightarrow N^* \) and the electromagnetic emission \( N^* \rightarrow p\gamma \) in the quark model. The results obtained serve to fix the coefficients \( b \) and \( f \) in the relevant hadron-level couplings \( bu_N^* u\Sigma^+ \) and \( f\epsilon_5 \pi u_{\sigma\mu\nu} u_{N^*} q_{\nu} \). When the contribution from the (b2) ordering is added, the two contributions together yield the parity-violating amplitude in the form

\[
\left( \frac{fb}{\Sigma^+-N^*} - \frac{bf}{p-\Sigma^*} \right) \epsilon_\mu \pi_{\mu\sigma\nu} \gamma_5 u_{N^*} q_{\nu} \quad (2)
\]

where the two terms stem from diagrams (b1) and (b2) respectively, and particle names stand for their masses (here \( \Sigma^* \) denotes a strange \( \frac{1}{2}^- \) baryon). In the SU(3) limit when \( \Sigma^+=p \) and \( \Sigma^*=N^* \), the two contributions cancel ensuring that Hara’s theorem is satisfied.

4.2. VMD-based approach

In the VMD approach one calculates first the \( \Delta S=1 \) parity-violating coupling of vector mesons \( (\rho,\omega,\phi) \). This is done along the lines of ref.[11]. The size of all couplings is fixed by symmetry from the size of the weak nonleptonic hyperon decays. The parity-violating couplings of transverse vector mesons to baryons are identified [11] with the hadron-level terms \( \pi_{\mu\sigma\nu} \gamma_5 u_{N^*} V_{\mu} \). In the pole model such terms are obtained using parity-conserving baryon-vector-meson couplings \( BB^* V \) of the form \( g\pi_{\mu\sigma\nu} \gamma_5 u_{N^*} V_{\mu} \) and lead to the following combination of contributions from diagrams (b1) and (b2):

\[
\left( \frac{b\phi}{\Sigma^+-N^*} + \frac{g\phi}{p-\Sigma^*} \right) \pi_{\mu\sigma\nu} \gamma_5 u_{N^*} V_{\mu} \quad (3)
\]

In the SU(3) limit \( (\Sigma^+=p \) and \( N^*=\Sigma^*) \) the contributions from diagrams (b1) and (b2) add...
rather than subtract. Consequently, when the VMD prescription $V \rightarrow \frac{1}{q^2} A$ is used to evaluate the parity-violating photon coupling to baryon, this leads to the appearance of an effective coupling $g_1(0) \bar{p}_\mu \gamma_5 \gamma_5 u_{Y^+} A^\mu$ (4) with a nonvanishing $g_1(0)$. This violates the assumptions upon which Hara’s theorem is based. Expression (4) may be considered a part of a gauge-invariant coupling $g_1(0) \bar{p}_\mu (\gamma_\mu - q q^\mu / q^2) \gamma_5 u_{Y^+} A^\mu$ (for transverse photons $q \cdot A = 0$). The presence of the pole at $q^2 = 0$ may be worrying, however, as no massless hadrons exist.

### Table 1

Branching ratios (in units of $10^{-3}$) and asymmetries (in italics) - comparison of VMD predictions with those of other models and experiment. Input values are underlined.

| process                  | experiment | VMD       | quark model | pole model | QCD sum rules |
|--------------------------|------------|-----------|-------------|------------|---------------|
| $\Sigma^+ \rightarrow p \gamma$ | $1.23 \pm 0.06$ | $1.26, 1.4$ | $1.24$   | $0.92 \pm 0.26$ | $0.8$         |
| $\Lambda \rightarrow n \gamma$ | $-0.76 \pm 0.08$ | $-0.97, -0.95$ | $-0.56$ | $-0.80 \pm 0.32$ | $+1.0$         |
| $\Xi^0 \rightarrow \Lambda \gamma$ | $1.63 \pm 0.14$ | $1.0, 1.7$ | $1.62$ | $0.62$ | $2.1-3.1$ |
| $\Xi^0 \rightarrow \Sigma^0 \gamma$ | $0.16 \pm 0.16$ | $0.9, 1.0$ | $0.50$ | $3.0$ | $1.1$         |
| $\Xi^- \rightarrow \Sigma^- \gamma$ | $0.60 \pm 0.96$ | $0.3, 0.15$ | $0.23$ | $-0.6$ | $0.1-0.2$ |

4.3. Comparison of two approaches

Subtraction, Eq.(2), (respectively addition, Eq.(3)) of pole expressions corresponding to diagrams (b1) and (b2) leads to asymmetry patterns $(-, -, -,-)$ (respectively $(-, +, +,-)$). Thus, the pattern $(-, +, +,-)$ seems to signify the violation of Hara’s theorem.

5. SATISFYING HARA’S THEOREM WITH $(-, +, +,-)$ PATTERN OF ASYMMETRIES

So far the experiment seems to confirm the $(-, +, +,-)$ pattern of asymmetries. In detailed models (VMD, quark model) this pattern signifies violation of Hara’s theorem. The origin of this violation is, however, slightly different in the two models (3). The quark model calculations directly violate the theorem. VMD may be considered more phenomenological. Thus, the question emerges whether the asymmetry pattern ($-, +, +,-$) obtained in the VMD approach could be maintained and yet Hara’s theorem satisfied. This question has been discussed recently in ref. (3).

Let us reconsider the problem of the most general parity-conserving gauge-invariant coupling of a real photon to $1/2^+ - 1/2^-$ baryonic current. Although it seems that there is only one such coupling that does not involve the pole at $q^2 = 0$, i.e. $\overline{\tau}_B i \sigma_{\mu \nu} \gamma_5 q u_B A^\nu$, in fact one also has to consider $h(-i)(p_k + p_l) \gamma_\mu e^{\lambda \mu \nu \rho} \overline{\tau}_B \gamma_\nu u_B A^\rho \equiv h \overline{\tau}_B (q^2 \gamma_\mu - q^\mu \gamma_5 u_B A^\mu + h \overline{\tau}_B (\not{p}_k i \sigma_{\mu \nu} \gamma_\nu q u_B A^\nu$.

For $q^2 = 0$, $q \cdot A = 0$ and on-mass-shell baryons, expression (6) reduces to that of (3) (barring some mass factors). However, such a reduction cannot be effected in pole model calculations of WRHD’s since $B^*$ is not on its mass shell. One has to keep expression (3) in the calculations and only at the end use the fact that external baryons are on their mass shells. Calculation (3) gives then for the parity-violating amplitude in the weak $i \rightarrow f \gamma$ decay the expression

$$h \overline{\tau}_B i \sigma_{\mu \nu} \gamma_5 q u_B A^\nu$$

(7)
From the form of Eq.(6) we see that one can maintain the pattern (−, +, +, −) (in which contributions from (b1) and (b2) add rather than subtract) and yet have Hara’s theorem satisfied (this is ensured by the overall factor of \(m_i - m_f\)). In this case all (and not just \(\Sigma^+ \rightarrow p\gamma\)) parity-violating WRHD amplitudes vanish in the SU(3) limit. In comparison with the Hara’s-theorem-violating approach of refs.[6,3], the difference consists in the presence of an additional \((m_i - m_f)^2\) factor in the amplitudes. Clearly, the existence of such an overall mass factor cannot be experimentally verified since we cannot move the mass of \(\Sigma^+\) to come closer to that of the proton. However, we can look at the \(\Delta S = 0\) processes, i.e. at the weak coupling of mesons to nucleons (such as \(\rho NN\) etc.). If the relative factor of \((m_i - m_f)^2\) is there, the weak \(\rho^+ pn\) coupling would be scaled down with respect to that estimated by symmetry from nonleptonic hyperon decays by a factor of

\[
\left(\frac{m_n - m_p}{m_{\Sigma} - m_p}\right)^2 \approx 10^{-4}
\]  
(8)

Thus, the resulting weak \(\rho NN\) coupling would be totally negligible. The data indicate, however, that the scale of the \(\Delta S = 0\) coupling is the same as that of the \(\Delta S = 1\) couplings [13]. Thus, there is no such mass factor and the observation of the (−, +, +, −) pattern of asymmetries in WRHD’s will signify violation of Hara’s theorem.

6. CONCLUSIONS

Theoretical VMD predictions for the asymmetries of the \(\Xi^0 \rightarrow \Lambda\gamma\) and \(\Xi^0 \rightarrow \Sigma^0\gamma\) decays are very solid (errors cannot exceed 0.2) [3]. There are experimental indications [15] that in the latter case the asymmetry is indeed moderately negative, in agreement with VMD predictions. Although the VMD approach can be made consistent with Hara’s theorem and still yield positive asymmetry parameter in the \(\Xi^0 \rightarrow \Lambda\gamma\) decay, this requires vanishingly small weak meson-nucleon couplings, in gross disagreement with experiment. It is therefore very important to measure the \(\Xi^0 \rightarrow \Lambda\gamma\) asymmetry precisely. This asymmetry is significantly positive (negative) for all models violating (satisfying) Hara’s theorem. If the KTeV experiment [12,13] finds here a negative value, perhaps one can concoct a model which satisfies Hara’s theorem and describes the data. If, on the other hand, the KTeV experiment reports a positive value, Hara’s theorem is violated in Nature. Questions concerning the meaning of this violation may then be more legitimately asked.

REFERENCES

1. Y. Hara, Phys. Rev. Lett. 12, 378 (1964).
2. M. Foucher et al., Phys. Rev. Lett. 68, 3004 (1992).
3. J. Lach and P. Żenczykowski, Int. J. Mod. Phys. A10, 3817 (1995).
4. A. N. Kamal and Riazuddin, Phys.Rev. D28, 2317 (1983).
5. M. B. Gavela, A. Le Yaouanc, L. Oliver, O. Pêne, and J. C. Raynal, Phys. Lett. 101B, 417 (1981).
6. P. Żenczykowski, Phys. Rev. D40, 2290 (1989); ibid. D44, 1485 (1991).
7. P. Żenczykowski, report INP-Kraków No. 1790/PH (March 1998).
8. R. C. Verma and A. Sharma, Phys. Rev. D38, 1443 (1988).
9. V. M. Khatsimovsky, Sov. J. Nucl. Phys. 46, 768 (1987).
10. C. James et al., Phys. Rev. Lett. 64, 843 (1990).
11. B. Desplanques, J. F. Donoghue, and B. Holstein, Ann. Phys. (N.Y.)124, 449 (1980).
12. P. Żenczykowski, Acta Phys. Pol. B27, 3615 (1996).
13. E. G. Adelberger and W. C. Haxton, Ann. Rev. Nucl. Part. Sci. 35, 501 (1985); W. C. Haxton, Science 2775, 1753 (1997); J. F. Donoghue, E. Golowich, and B. Holstein, Phys. Rep. 131, 319 (1986).
14. The published experimental number is corrected here for a missing factor of 3, (E. Ramberg, these proceedings).
15. E. Ramberg, these proceedings.
16. E. Cheu et al., proposal FERMILAB- P-0799, Dec. 1997.