Classicalization is a phenomenon of redistribution of energy - initially stored in few hard quanta - into the high occupation numbers of the soft modes, described by a final state that is approximately classical. Using an effective Hamiltonian, we first show why the transition amplitudes that increase occupation numbers are exponentially suppressed and how a very special family of classicalizing theories compensates this suppression. This is thanks to a large micro-state entropy generated by the emergent gapless modes around the final classical state. The dressing of the process by the super-soft quanta of these modes compensates the exponential suppression of the transition probability. Hence, an unsuppressed classicalization takes place exclusively into the states of exponentially enhanced memory storage capacity. Next, we describe this phenomenon in the language of a quantum neural network, in which the neurons are represented as interconnected quantum modes with gravity-like negative-energy synaptic connections. We show that upon an injection of energy in form of a hard quantum stimulus, the network reaches the classicalized state of exponentially enhanced memory capacity with order one probability. We construct a simple model in which the transition results into classical states that carry an area-law micro-state entropy. In this language, a non-Wilsonian UV-completion of the Standard Model via classicalization implies that above cutoff energy the theory operates as a brain network that softens the high energy quanta by bringing itself into the state of a maximal memory capacity. It is striking that one and the same underlying mechanism can be responsible for seemingly remote phenomena across the disciplines, such as, the formation of classical black holes in elementary particle collisions, the solution to the Hierarchy Problem via classicalization of the Higgs field, as well as, the transition to a maximal memory state in brain networks.
sion of the transition amplitudes into the states of high occupation numbers, in generic theories. Secondly, we must understand how the classicalizing theories escape this exponential suppression. What is special about such classical states?

An useful qualitative guideline [2, 3, 5] is provided by the well known example of black holes [6–8]. Namely, a simple common sense semi-classical argument indicates that the production of a macroscopic black hole in the collision of high energy particles should exhibit no exponential suppression. In the microscopic language, the absence of the suppression can be attributed to a high entropy of the final state, which provides a required enhancement factor. That is, while a production of each black hole micro-state is exponentially suppressed, the suppression is compensated by their multiplicity, since they all amount to the same classical macro-state.

This tells us that an unsuppressed quantum-to-classical transition must represent a transition into a state of exponentially enhanced memory storage capacity and high complexity.

Recently [9, 10], it has been shown that certain quantum neural networks, with gravity-like synaptic connections, exhibit the states of exponentially-enhanced memory storage capacity, in the way somewhat analogous to a black hole situation [12, 13]. The analogy appears even more intriguing, as some of these neural networks turn out to be isomorphic to a $D$-dimensional quantum field model constructed in [11]. There, the enhanced memory state translates as a critical state with the emergent gapless modes inhabiting an area of a $D - 1$-dimensional sphere. Therefore, the corresponding micro-state entropy obeys an area-law, reminiscent of the Bekenstein entropy of a black hole [17].

Due to the above connections with quantum field theoretic models, such a quantum neural network represents a nice laboratory for testing the idea of classicalization. The gain from such analysis is two-fold. First, the neural networks can teach us about classicalization. Secondly, we learn a possible mechanism by which the brain network can efficiently bring itself into a state of enhanced memory and pattern recognition capacities in response to an external stimulus.

The connection between the quantum neural networks and fields is established by identifying the neural degrees of freedom with the momentum modes of the field, whereas, the synaptic connections between the neurons are mapped on the interactions among the different modes in the Hamiltonian [10].

In general, visualising the quantum field theory systems, such as the Standard Model, as quantum neural network, allows us to get some fresh perspective on things. For example, we can understand a non-Wilsonian UV-completion [1] as the ability of the system to move itself into the state of a maximal memory storage capacity. In this light, the solution of the Hierarchy Problem by classicalization would mean that the Standard Model works as a remarkable quantum brain network, which above cutoff energy swiftly evolves into a maximal memory state.

In addition, the neural network language, can help us to comprehend - at the level of a simple effective Hamiltonian - the essence of the phenomena that are usually blurred by secondary technicalities. For example, since the key focus is on a quantum transition to the classical macro-state of high micro-state entropy, it can teach us a qualitative lesson about an analogous transition from a two-particle state into a classical black hole.

Below we shall proceed as follows. Following [9, 10], we shall first construct a simple Hamiltonian that describes a quantum neural network that exhibits a classical state of macroscopic occupation number $N$ and an exponentially enhanced memory capacity, due to $N$ emergent gapless modes. We than study the question of transitions to such a classical state from an initial quantum state of sufficiently high energy. In very general terms, we first show why in any sensible quantum theory the transition to each particular micro-state is necessarily exponentially suppressed by a factor $e^{-N}$. We then show, how the multiplicity of the micro-states overpowers this suppression. This multiplicity originates from the states of the gapless modes that emerge around the classical state.

We establish the following general bound. For an unsuppressed transition into a given classical macro-state, it is necessary that the number $N_{\text{gapless species}}$ of distinct species of the gapless modes emergent in this state, is not less than the occupation number $N$ of the respective “constituent” soft modes. In other words, the occupation number $N$ of each soft mode, must be accompanied by at least an equal number of the emergent species of the gapless modes:

$$N_{\text{gapless species}} \geq N.$$  \hspace{1cm} (1)

This also implies that the micro-state entropy of the classical state must be higher or equal to the occupation number of the soft modes in it:

$$\text{Entropy} \geq N.$$ \hspace{1cm} (2)

Finally, we briefly review some implications of this phenomenon, for the Standard Model, for Black holes and for quantum brain networks in general.

Note: Since we work with a well-defined quantum Hamiltonian, one can feel free to ignore the term “neural network”. Our results remain valid for an arbitrary
quantum system with a large number of interacting degrees of freedom of the type we consider. They show, why the transition to each high occupation number state is exponentially suppressed, and how this suppression is compensated by the large micro-state entropy factor within a very special family of the classicalizing theories.

Note: While viewing our models as quantum neural networks [14] of Hopfield type [15], we should point out the key differences in our approach. Our emphasis is not on the learning algorithms, but rather on the energy efficiency of the memory storage. The latter we measure by the two criteria:

1) Achieving the largest number of patterns that can fit within the maximally-narrow energy gap;
and
2) Minimizing the energy cost of the response to an external stimulus.

This pushes us to a very different way of the network usage. In our cases, the states of enhanced memory capacity are the states that support a large number \( N \) of the emergent gapless neurons. This boosts the memory efficiency: The number of patterns stored in such states scales as \( e^N \).

The crucial point is that this exponentially large number of patterns occupies a microscopically-narrow (or even zero) energy gap, and moreover, can respond to arbitrarily-soft stimuli. This is very different from a generic quantum Hopfield network [16], where the same number of patterns would occupy the macroscopically large energy gap, scaling as \( N \), and they would be blind to the soft stimuli.

This type of exponential enhancement of the memory storage capacity is precisely what makes the classicalizing theories possible. The system can enjoy the unsuppressed quantum transitions to the states that can store exponentially large number of patterns within maximally narrow energy gaps.

II. A MODEL

Following [9, 10], let us first construct a simple prototype model of a quantum neural network, which possesses a state of an exponentially enhanced memory storage capacity. The model consists of the set of neural degrees of freedom represented by the creation and annihilation operators \( \hat{a}_k^\dagger, \hat{a}_k \), that satisfy the usual oscillator algebra,

\[
[\hat{a}_r, \hat{a}_k^\dagger] = \delta_{rk}, \quad [\hat{a}_r, \hat{a}_k] = [\hat{a}_r^\dagger, \hat{a}_k^\dagger] = 0,
\]

where \( k, r = 0, 1, 2, ..., N \) are the labels. We shall be interested in the situation when \( N \) is large. The excitation of a neuron \( k \) in a given quantum state is described by an eigenvalue (or an expectation value) of the corresponding number operator \( \hat{n}_k = \hat{a}_k^\dagger \hat{a}_k \). Thus, the basic states of a neural network are the Fock states described by the ket-vectors \( |n_0, n_1, ..., n_N \rangle \) labeled by the excitation levels of the different neurons.

A network represented by the following simple Hamiltonian suffices to capture the essence of the emergence of an enhanced memory state,

\[
\hat{H} = \epsilon_0 \hat{n}_0 + \sum_{k \neq 0} \epsilon_k \hat{n}_k - \frac{\epsilon_0}{\Lambda} \hat{n}_0 \sum_{k \neq 0} \epsilon_k \hat{n}_k ,
\]

where \( \Lambda \) is a parameter of dimensionality of energy. The parameters \( \epsilon_k, (k = 0, 1, ..., N) \), are the threshold energies required for the excitation of individual neurons, in the absence of the interactions. The third term describes the interactions, or equivalently the synaptic connections among the neurons. The structure of this term is very important. Here, we have singled out a neuron \( k = 0 \) as the master neuron and have only included the interactions between this neuron and the rest. As long as the interactions are sufficiently weak (as we assume), the rest of the connections play no role in what follows and have not been included explicitly. The same applies to the higher order terms that must be included in order to make the Hamiltonian bounded from below. Nothing in our analysis is affected by the presence of such terms, and therefore, they will be ignored for simplicity.

The interaction among \( \hat{n}_0 \) and \( \hat{n}_k \) is universally proportional to the threshold energy \( \epsilon_k \) and the sign is negative. In this way, the interaction is sort of “gravity-like”. Since, the interesting critical phenomena take place when the occupation numbers reach the inverse connection strength 3 [9 10], from now on we shall set \( \frac{\epsilon_0}{\Lambda} = \frac{1}{N} \). This makes the coupling strength of the master neuron inverse of the number of modes connected to it. The interaction structure of such a network resonates with ’t Hooft’s large-\( N \) limit idea in gauge theories [18]. A network representation of the model is given by Fig.(1).

The above network represents a simplified version of the models introduced in [9 10], and reduces the phenomenon of memory capacity enhancement to its essence. This essence can be described as follows.

The excitation of the master neuron \( \hat{n}_0 \), due to the negative energy synaptic connections, lowers the excitation thresholds of all the \( \hat{n}_k \neq 0 \) neurons connected to it. That is, on a state \( \langle \hat{n}_0 \rangle = n_0 \), the effective excitation threshold of the \( k \)-neuron is reduced as,

\[
\epsilon_k^{(\text{eff})} = \epsilon_k \left( 1 - \frac{n_0}{N} \right).
\]

Thus, for \( n_0 = N \), all the \( k \neq 0 \) neurons become effectively gapless.
It is important to mention the following. In the above model, for \( n_0 = N \), the modes \( \hat{n}_k \neq 0 \) are exactly gapless and therefore the states \( |N, n_1, \ldots, n_N \rangle \) are degenerate for an arbitrary choice of the occupation numbers, \( n_k \neq 0 \). Nevertheless, only the states with small \( n_k \neq 0 \) are indistinguishable classically, and correspondingly only such states count as the representatives of one and the same classical state with \( n_0 = N \). For \( n_k \neq 0 \gg 1 \), the states correspond to different classical states. Of course, such states are equally good for storing the patterns and thus play important role in the memory storage. However, as we shall see, in the classicalizing transition in which the system reaches the classical macro-state of high entropy from some initial low entropy quantum state, only the states with \( n_k \neq 0 \sim 1 \) can be reached with an unsuppressed probability.

**III. QUANTUM-TO-CLASSICAL TRANSITION**

We shall now modify our model in such a way that to allow for a *quantum-to-classical* transition. For this we shall introduce an additional “hard” degree of freedom described by the number operator \( \hat{n}_b \equiv \hat{b}^\dagger \hat{b} \) and with a high excitation threshold energy given by \( \epsilon_b = N \epsilon_0 \). We assume that \( \hat{b}^\dagger, \hat{b} \) operators satisfy the commutation relations analogous to (3) and commute with all the \( \hat{a} \)-modes.

Next, we shall introduce a coupling between \( b \) and \( a \) degrees of freedom allowing a transition \( |1\rangle_b \rightarrow |N\rangle_a \). That is, an input quantum stimulus - carried by a hard neuron \( b \) in its lowest excited state - is converted into a classical state of a macroscopically excited soft neuron \( a \). As we shall see, although the transitions to individual micro-states belonging to this classical macro-state are exponentially suppressed, the suppression can be overpowered, since the classical final state \( \langle \hat{n}_0 \rangle = n_0 = N \) exhibits the exponentially enhanced memory storage capacity.

Thus, the modes emerging around the “master” state \( n_0 = N \), can be classified in three categories, according to the level of their softness. These are:

1) The *hard* \( b \)-mode, with the gap \( N \epsilon_0 \);
2) The *soft* \( a_0 \)-mode, with the gap \( \epsilon_0 \); and
3) The *super-soft* \( a_k \neq 0 \)-modes, which are essentially gapless.

The latter modes play an absolutely crucial role in making the classicalizing transition probable: It is the summation over the final states that are “dressed” by the different occupation numbers of these gapless modes that compensates for the exponential suppression factor. Clearly, in the same time, this goes hand in hand with the enhancement of the memory capacity, since an exponentially large number of patterns are stored in the pattern storage.
states of the gapless modes.

The above-described classicalizing transitions are generated by the following Hamiltonian,

$$\hat{H} = \hat{H}_{\text{numb}} + \hat{H}_{\text{tr}}, \quad (6)$$

where $\hat{H}_{\text{numb}}$ denotes the part that conserves the individual occupation numbers of each mode,

$$\hat{H}_{\text{numb}} = N\epsilon_0\hat{n}_b + \epsilon_0\hat{n}_0 + \sum_{k \neq 0}\epsilon_k \left(1 - \frac{\hat{n}_0}{N}\right)\hat{n}_k, \quad (7)$$

whereas $\hat{H}_{\text{tr}}$ generates the number-non-conserving transitions. It has the following form

$$\hat{H}_{\text{tr}} = \hat{b}^\dagger\hat{a}_0^N \sum_{n_1, n_2, \ldots, n_N} g_{n_1, \ldots, n_N}\hat{a}_1^{n_1}\hat{a}_2^{n_2}\ldots\hat{a}_N^{n_N} + \text{h.c.}, \quad (8)$$

where the summation is taken over all possible sets of integers $n_1, \ldots, n_N$.

Notice, the above interaction must be viewed as an effective Hamiltonian describing the transition. At the level of a fundamental quantum field theory it can emerge as a fundamental vertex or be a result of summation of an infinite set of Feynman diagrams, as, e.g., in \[3\]. The power of the analysis at the level of an effective Hamiltonian is that it uncovers the generic constraints that such interaction must satisfy, regardless of its underlying origin.

We are interested in the time-evolution generated by the above Hamiltonian starting from an initial state $|in\rangle \equiv |1\rangle_b \times |0, 0, \ldots, 0\rangle_a$, in which the hard neutron $b$ is excited to the first level, whereas the $a$-neurons are unexcited. An each member of the sum in $[8]$, after acting on the initial state $|in\rangle$, converts it into the following state,

$$\sqrt{N!n_1!\ldots n_N!}g_{n_1, \ldots, n_N}|N\rangle_{n_1, \ldots, n_N}, \quad (9)$$

where we have introduced a notation, $|N\rangle_{n_1, \ldots, n_N} \equiv |0\rangle_b \times |N, n_1, \ldots, n_N\rangle_a$. This conversion results into the following transition matrix elements,

$$|\langle in | \hat{H}_{\text{tr}} | N\rangle_{n_1, \ldots, n_N}|^2 = N!n_1!\ldots n_N!|g_{n_1, \ldots, n_N}|^2. \quad (10)$$

Correspondingly, the probability of the transition to one of the classical states with $n_0 = N$, can be obtained by summing over the numbers $n_1, \ldots, n_N$.

Here we arrive to a very important point. Naively, one may think that the above sum can be made arbitrarily large at the expense of adjusting the parameters $g_{n_1, \ldots, n_N}$, as well as, by making use of the multiplicity of states. However, things are not so cheap, because the quantities $g_{n_1, \ldots, n_N}$ are subject to the severe consistency constraints.

The point is that in order to have a well-defined transition process, both the initial and the final states must be the approximate eigenstates of the Hamiltonian.

This implies that the off-diagonal (number-non-conserving) terms in the Hamiltonian must be sub-leading to the number conserving part $\hat{H}_{\text{numb}}$ over the entire part of the Hilbert space that can be probed during this evolution of the system. In particular, this must be true for all the states for which the expectation values of the occupation numbers are centered around the initial and final values involved in the transition. Therefore, this requirement must be satisfied on all the coherent states $|C\rangle$ with the relevant values of the mean occupation numbers, since such states represent the Poissonian distributions centered around the mean occupation numbers $n$ with the variances given by $\Delta n^2 \sim n$.

Thus, consider a coherent state $|C\rangle$, with the following mean occupation numbers:

$$\langle C | \hat{n}_b | C \rangle = 1, \quad \langle C | \hat{n}_0 | C \rangle = N, \quad \langle C | \hat{n}_k | C \rangle = n_k. \quad (11)$$

We require that on such a state the expectation values of all off-diagonal term must be less than the expectation values of the diagonal ones. Taking the expectation values and remembering that the coherent state is an eigenstate of all the annihilation operators with the eigenvalues given by the square-roots of the corresponding mean-occupation numbers, we obtain the following constraint

$$|g_{n_1, \ldots, n_N}| \lesssim \lambda(\epsilon_0 N)^{-\frac{N}{2}} n_1^{-\frac{n_2}{2}} \ldots n_N^{-\frac{n_N}{2}}. \quad (12)$$

Here the factor $\epsilon_0 N$ comes from the expectation value of $\hat{H}_{\text{numb}}$ and $\lambda$ is a small dimensionless parameter that controls the relevant strength of off-diagonal expectation values. The precise value is unimportant for our conclusions. For example, taking, $\lambda = \frac{1}{N\sqrt{N}}$ implies that the relative strength is less than $\frac{\epsilon_0}{N^{3/2}}$. Below, for definiteness, we shall make this choice, although it is not unique and has no important meaning at this level.

The equation (12) represents an extremely powerful constraint which reveals a lot of information about the nature of the suppression of quantum transition to the states with high occupation numbers.

In order to see this, let us examine the cases of small and large occupation numbers $n_k \neq 0$ separately. Consider first the transition into a particular micro-state with $n_k \neq 0 \lesssim 1$. From (12) it is clear that the corresponding coefficients are suppressed as $|g_{n_1, \ldots, n_N}| \lesssim \frac{\epsilon_0}{N^{3/2}} N^{-\frac{n_2}{2}}$. Then, evaluating the matrix element (10) we obtain,

$$|\langle in | \hat{H}_{\text{tr}} | N\rangle_{n_1, \ldots, n_N}|^2 \sim \frac{\epsilon_0^2}{N} N! N^{-N}, \quad (13)$$
where we took into account that \( n_{k\neq 0} \lesssim 1 \). Using Stirling formula for large \( N \) we get,

\[ | \langle \text{in} | \hat{\mathcal{H}}_{\text{tr}} | N \rangle_{n_1...n_N} |^2 \sim e^2 \frac{1}{\sqrt{N}} e^{-N}. \]  

(14)

Thus, from very general considerations, we have obtained an anticipated exponential suppression factor of

the matrix element describing a quantum transition into a classical state of some large occupation \( N \). This also represents a nice consistency check of our analysis.

The above suppression factor is due to a large occupation number of \( n_0 \)-mode in the final state. However, in the present model this state is accompanied by the emergence of the \( N \) gapless \( \hat{n}_k \)-modes, which results into an exponential enhancement of the number of micro-states with \( n_0 = N \).

As a result, the above suppression factor is matched by the multiplicity of states with \( n_{k\neq 0} \lesssim 1 \), which is approximately \( e^{N} \). Summing over all such micro-states, we obtain, in the leading order in \( g \)-expansion, the following transition probability to a classical macro-state, per time \( t \),

\[ P_{b \rightarrow N n_0} = \sum_{n_k \neq 0 \lesssim 1} | \langle \text{in} | \hat{\mathcal{H}}_{\text{tr}} | N \rangle_{n_1...n_N} |^2 \sim \frac{t^2 e^2}{\hbar^2} \frac{1}{\sqrt{N}}. \]

(15)

Thus, summarizing: We observe that the transition probability to the master macro-state of enhanced memory capacity is exponentially more probable than a would-be transition into an analogous state in absence of the memory enhancement.

This enhancement is due to a large micro-state entropy provided by the modes \( \hat{n}_k \neq 1\) that are rendered gapless by the high occupation number of the master neuron \( \hat{n}_0 \). Indeed, as it is clear from (14), in the absence of the gapless modes, the transition to a state \( n_0 = N \), would be exponentially unlikely. It is the presence of the gapless modes that provides the necessary compensating factor \( e^{N} \), which makes this transition highly probably.

One may wonder whether we would have gotten an even more enhanced transition rate, had we taken into account also the transitions into the states with large values of \( n_{k \neq 0} \)-s. After all, the multiplicity of states with \( n_{k \neq 0} \gg 1 \) is much higher than the one of the states with \( n_{k \neq 0} \sim 1 \). For example, the number of all possible states with \( n_{k \neq 0} < d \), where \( d \) is some integer, scales as \( \sim d^N \). This may create a false impression that by increasing \( d \) we can make the transition more and more probable.

In reality, this is not the case and the reason is the constraint (12). Evaluating the transition matrix element subject to this constraint, we get:

\[ | \langle \text{in} | \hat{\mathcal{H}}_{\text{tr}} | N \rangle_{n_1...n_N} |^2 \sim N!n_1!...n_N!N^{-N} \hat{n}_1^{-n_1}...\hat{n}_N^{-n_N}, \]

(16)

which after using Stirling formula gives:

\[ | \langle \text{in} | \hat{\mathcal{H}}_{\text{tr}} | N \rangle_{n_1...n_N} |^2 \sim \sqrt{Nn_1...n_N} e^{-(N+n_1+...+n_N)}. \]

(17)

In both expressions we have omitted an unimportant factor \( \lambda e_0 N \). From the expression (17) it is obvious that a high multiplicity of states with large \( n_{k \neq 1} \)-s is unable to compensate the respective exponential suppression. Indeed, the number of such states can at best provide an enhancement factor \( \sim e^{N\ln(d)} \), which for \( d \gg 1 \) is absolutely powerless against the suppression factor in (17), which scales as \( \sim e^{-N(1+d)} \). We thus conclude that the compensation of the exponential suppression factor takes place exceptionally for the transitions into the states with small \( d \).

This result makes perfect physical sense. Indeed, the states with large occupation numbers \( n_{k \neq 0} \), are classically-distinguishable from the master state \( | N \rangle_{0...0} \). So, despite of not being separated by a large energy gap, such states nevertheless correspond to different macro-states. However, unlike the master state \( | N \rangle_{0...0} \), they lack their own “entourages” of the exponentially large number of the quantum micro-states. The reason is that the increase of the occupation numbers \( n_{k \neq 0} \) is not accompanied by an emergence of any new gapless modes. But, such modes are necessary for providing the further exponential increases of the numbers of micro-states by the respective factors \( e^{N} \). Correspondingly, the exponential suppressions of transitions to large-\( n_{k \neq 0} \) states remain uncompensated.

Thus, we observe that in a classicalizing transition the system mainly explores the micro-states that are not distinguishable classically from the master state \( | N \rangle_{0...0} \), i.e., the micro-states that are obtained from the latter state via small quantum excursions. The more distant classical states remain unexplored.

**IV. VARYING NUMBER OF SOFT QUANTA**

We are now prepared for the next step, in which we can allow also the transitions to the classical states in which the number of soft quanta \( n_0 \) can be different from the critical value \( N \). For this, we modify the transition Hamiltonian (8) as,

\[ \hat{\mathcal{H}}_{\text{tr}} = \hat{b}^\dagger \sum_{n_0,n_1,...n_N} g_{n_0} \hat{n}_1...\hat{n}_N \hat{a}^\dagger n_1...\hat{a}^\dagger n_N + \text{h.c.}, \]

(18)

where the parameters obey the respective conditions of the type (12). This leads to processes in which creation of \( n_0 \) soft quanta is accompanied by certain distributions
of the rest, \( n_{k \neq 0} \). As it is clear from [3], for the special case of \( n_0 \approx N \), the rest of the modes become super-soft (i.e., nearly-gapless) and the process can be presented as:

\[
1_{\text{hard}} \rightarrow n_0 + n_{\text{supersoft}},
\]

where \( n_{\text{supersoft}} = \sum_{k \neq 0} n_k \).

Due to the previous discussion, the probability of classicalization for large \( n_0 \) effectively factorizes,

\[
P_{\text{class}} = P_{1 \rightarrow n_0} P_{\text{super-soft}}(n_0, \epsilon),
\]

where \( P_{1 \rightarrow n_0} \sim e^{-n_0} \) represent the transition probability into \( n_0 \) soft quanta, without accompanying super-soft quanta, whereas, \( P_{\text{super-soft}}(n_0, \epsilon) \) takes into the account the “dressing” of the final state by populating it with the super-soft quanta of the gapless modes. It includes summation over all the micro-states \( |n_0, n_1, \ldots, n_N \rangle \) that fit within some fixed energy gap \( \epsilon \). That is, these are the micro-states that satisfy the constraint \( \sum_{k \neq 0} n_k \epsilon_k (1 - \frac{n_0}{N}) < \epsilon \). Due to this, the quantity \( P_{\text{super-soft}}(n_0, \epsilon) \) is a function of both \( n_0 \) and of \( \epsilon \).

The choice of \( \epsilon \) is dictated by how much energy uncertainty one is willing to attribute in defining the classical macro-state \( |n_0 \rangle_{\text{class}, 0} \). An obvious requirement is that this uncertainty must vanish in the exact classical limit, \( n_0 \to 0 \). In our case, the physically justified choice of \( \epsilon \) is to take it to be less or equal to an elementary gap \( \epsilon_0 \). Then, given \( \epsilon_k \geq \epsilon_0 \), we have the following situation.

First, for \( n_0 = N \) all the \( k \neq 0 \) modes are gapless. Since, as explained above, the states with \( n_k \gg 1 \) contribute the exponentially suppressed weights into the probability, the sum is effectively cut off above \( n_k \neq 0 \sim 1 \), so that \( P_{\text{super-soft}}(n_0 = N, \epsilon_0) \sim e^N \).

On the other hand, for \( n_0 \neq N \), the number of the states that can fit within the gap \( \epsilon \) rapidly diminishes as a function of \( |n_0 - N| \). Correspondingly, the function is exponentially sharply peaked at \( n_0 = N \) and rapidly falls off away from it. Thus, for \( \epsilon \lesssim \epsilon_0 \), we can approximate,

\[
P_{\text{super-soft}}(n_0, \epsilon) = \delta_{n_0 N} e^N,
\]

where \( \delta_{n_0 N} = \begin{cases} 0 & \text{for } n_0 \neq N \\ 1 & \text{for } n_0 = N \end{cases} \) is Kronecker delta. Correspondingly, the transition probability is peaked around \( n_0 = N \), where it is given by [15].

V. CLASSICALIZATION FOR ARBITRARY \( N \) AND ENERGY-DEPENDENCE OF ENTROPY

By now, the following is clear from the previous discussion. The necessary condition for an unsuppressed classicalization into a macro-state \( |N\rangle_{a_0 N} \), in which the occupation number of a soft mode \( a_0 N \) is equal to \( N \), is that the micro-state entropy of \( |N\rangle_{a_0 N} \) is larger or equal to \( N \), in accordance with the bound [1]. That is, the number of species of the emergent gapless modes in this state is bounded from below by \( N \), as indicated in [1].

This statement fully generalizes to the case when the classical macro-state represents a distribution over the occupation numbers of several different soft modes:

\[
|\text{class}\rangle = \bigotimes_N |N\rangle_{a_0 N},
\]

where \( \otimes \) stands for a tensor product. In such a case, a macroscopic occupation number \( N \) of each soft mode \( a_0 N \) must be accompanied by \( N \) emergent gapless (i.e., super-soft) modes \( a_k N \), where \( k = 1, 2, \ldots, N \). An each set of the gapless modes will contribute a corresponding factor \( N \) in the micro-state entropy. Thus, the total micro-state entropy of the state \( |\text{class}\rangle \) is equal to a total number of the emergent gapless modes:

\[
\text{Entropy} = \sum_{a_0 N} N. \tag{23}
\]

In the same time, the relation between the micro-state entropy and the energy of the state is not uniquely fixed by the requirement of classicalization.

In order to make the above statements clear, below we give an example of the network that allows for classicalizing transitions to the macro-states with arbitrarily-high occupation number \( N \) and varying energy \( E_N \). For this, we shall endow both the soft master mode as well as the hard modes, with an index \( N \), as we did above. A given master neuron \( a_0 N \), when excited to a corresponding critical level \( N \), renders a set of \( N \) neuron species, \( a_k N \), \( k = 1, 2, \ldots, N \), gapless. In this way, the system delivers an infinite sequence of macro-states, \( |N\rangle_{a_0 N} \), each accompanied by \( N \) emergent gapless modes \( a_k N \) and a respective micro-state entropy equal to \( N \). All such states saturate the bounds [1] and [2].

A simple Hamiltonian that accomplishes this goal has the following form,

\[
\hat{H} = \sum_N N \epsilon_0 \hat{a}_N^\dagger \hat{a}_N + \sum_N \epsilon_{0 N} \hat{a}_0 N^\dagger \hat{a}_0 N + \tag{24}
\]

\[
\sum_N \sum_{k_N = 1}^N \epsilon_{k N} \left( 1 - \frac{\hat{a}_0 N^\dagger \hat{a}_N}{N} \right) \hat{a}_{k N}^\dagger \hat{a}_{k N} + \\
\sum_N \delta_N^N \sum_{n_1, n_2, \ldots, n_N} g_{n_1 \ldots n_N}^{(N)} \hat{a}_{n_1 N}^\dagger \hat{a}_{n_2 N}^\dagger \ldots \hat{a}_{n_N N}^\dagger + \text{h.c.}.
\]

Notice, the different species of the soft master neurons \( a_0 N \) are now allowed to have different energy gaps \( \epsilon_{0 N} \). Correspondingly, we have introduced a spectrum of the hard neuron species, \( b_N \), with their threshold energies
given by $E_N = N \epsilon_{0N}$.

In this system, an input excitation $|1\rangle_{bN}$, of energy $E_N = N \epsilon_{0N}$, and zero entropy, classifies into a macro-state $|N\rangle_{a0N}$ of the same energy and the entropy given by $N$. Obviously, the transition probability is given by $|15\rangle$, where $\epsilon_0$ must be replaced by $\epsilon_{0N}$.

It is clear that the relation between energy and entropy is not uniquely fixed by the requirement of classicalization. It appears that for various relations, the system can classicalize equally well.

For example, consider the following two choices of the soft energy gap $\epsilon_{0N}$ as function of $N$.

In the first case we take, $\epsilon_{0N} = \epsilon_0 = \text{constant}$, whereas in the second case, we take $\epsilon_{0N} = \frac{\Lambda}{\sqrt{N}}$, where $\Lambda$ is some fundamental scale. For both choices, the micro-state entropy of the macro-state $|N\rangle_{a0N}$ is equal to $N$. However, the energy dependences in the two cases are very different. In the first case, the entropy scales proportional to energy, $\text{Entr} = N = \frac{E_N}{\epsilon_0}$. In contrast, in the second case it scales as the square of the energy $N = \frac{E_N^2}{\Lambda^2}$. Notice, the latter scaling is very similar to the relation between the black hole entropy and its energy in $D = 3$ space dimensions.

Despite the above difference, in both cases the classicalizing transition from the state $|1\rangle_{bN}$ to the state $|N\rangle_{a0N}$ takes place without any exponential suppression, since in both cases the number of the emergent gapless species is equal to $N$. This is all that is needed for the compensation of the suppression factor $e^{-N}$ of the transition amplitude.

VI. IMPLICATIONS

By implementing the construction of $|9\rangle, |10\rangle$, we have obtained a simplest quantum neural network which exhibits a state of exponentially enhanced memory capacity due to emergent gapless neurons. We then showed that when a stimulus is injected in the network in form of an elementary excitation of some high-energy neuron $\hat{b}$, the system reaches the state of enhanced memory capacity with order one probability. In this process the energy of the hard stimulus gets converted into a macroscopically high excitation of a soft master neuron $\hat{a}_0$. Schematically, this transition can be described as,

$$|\text{Quant}, n_b = 1\rangle_{\text{Entr}=0} \rightarrow |\text{Class}, n_{a0} = N\rangle_{\text{Entr}=N},$$

(25)

where the subscripts indicates the value of the micro-state entropy of the initial and final states.

Reduced to its bare essentials the phenomenon can be described as follows. A system with many degrees of freedom with negative energy connections can exhibit the enhanced memory state in which the high occupation number of one of the soft modes renders the set of other modes gapless $|9\rangle, |11\rangle$. This enhancement makes the transition to such a high occupation number state maximally probable, even when initially the entire energy of the system is concentrated in a single quantum.

Such a system defies the ordinary intuition about the exponential suppression of the quantum-to-classical transitions, due to the fact that the resulting classical state is accompanied by an exponential enhancement of memory storage capacity and corresponding large micro-state entropy.

Below we shall briefly discuss some implications of our results.

A. Quantum Brain Networks

One obvious implication of our results is for understanding an accessibility of nearly-classical states of enhanced memory storage capacity in quantum brain networks, under the influence of the external quantum stimuli. As we explained, in such networks the existence of energy-efficient memory capacity is linked with the emergence of the gapless neurons $|9\rangle, |10\rangle$. The latter property also ensures the sensitivity to an arbitrarily soft external stimuli. Correspondingly, the property of classicalization is intrinsically linked with the ability of the network to reach the highest possible memory state, in which it will also become sensitive to maximally soft stimuli.

To put it shortly: A hard stimulus prompts the brain network to become responsive to the exponentially large variety of the super-soft input patterns.

B. Classicalization in Black Hole Formation in High Energy Particle Collisions

It is a well-accepted idea $|6\rangle, |8\rangle$ that black holes are expected to form with order one probability in the collision of quantum particles of sufficiently high center of mass energy.

The qualitative argument is rather simple: Such a system should lead to a black hole formation once the initial center of mass energy becomes concentrated within the corresponding gravitational radius. This phenomenon represents an example of classicalization, since it describes a transition from an initial quantum state of small occupation number of hard quanta into a classical final state. However, the precise microscopic mechanism behind this process in still unknown. The purpose of the present note is not to offer one, but rather
to point out a remarkable analogy.

Namely, in its essence, the classicalization process of a black hole formation is strikingly similar to the classicalization process in the neural network described above. Indeed, the black hole formation, say, in two-particle collision, represents a transition from an initial quantum state - in which the entire energy of the system is carried by two hard degrees of freedom - into a classical state (i.e., a macroscopic black hole) of enormous memory storage capacity. This is exactly what is happening in the neural network presented above. In both cases an initial quantum state of low memory storage capacity evolves into a classical state of sharply enhanced memory storage capacity:

\[ |\text{Quant}\rangle_{\text{Low Entropy}} \rightarrow |\text{Class}\rangle_{\text{High Entropy}} \]  

(26)

Of course, it would be too naive to suggest that our model captures all the aspects of gravitational physics. However, it may very well be capturing some key aspects of the classicalization in the black hole formation, by highlighting the role of the memory capacity enhancement in this process.

However, there is more to it. In fact, the computations of [2], [4], may serve as strong supporting evidence for such a connection. In these papers, the relevant quantum gravity process for the black hole formation was identified as \(2 \rightarrow N\) transition, in which a two initial gravitons of high center of mass energy scatter into \(N\) equally soft ones. Especially interesting things happen in the regime when the number \(N\) of the soft gravitons is equal to the inverse strength of their quantum gravitational coupling. As it is easy to see, the same number coincides with the entropy of a black hole of the size given by the de Broglie wavelength of the soft gravitons. Equivalently, \(N\) is the entropy of a black hole with the mass equal to the center of mass energy of the initial two gravitons. This striking coincidence with the qualitative properties of classicalization in the gravity-like neural network considered above, should already ring a bell.

In fact, according to the hypothesis of [13], such a soft \(N\)-graviton state is at the quantum critical point [12] and represents an approximate (in large \(N\)) description of a black hole macro-state. Due to its quantum criticality, there emerges a number \(N\) of nearly-gapless super-soft modes. These were assumed to be responsible for the origin of black hole entropy. In this description, the black hole micro-states represent the states of \(N\)-soft gravitons, accompanied by the distributions of the super-soft ones. The latter gravitons represents the excitations of the gapless modes.

Now, both, the string-theoretic as well as the quantum field-theoretic computations of [3] show that such a scattering process indeed delivers a suppression factor \(\sim e^{-N}\) for the probability of producing a particular micro-state. Notice, this is strikingly similar to the analogous factor in [14]. Moreover, this is precisely the factor that is compensated by the summation over the black hole micro-states, produced by the super-soft modes.

In addition, the computation of Addazi, Bianchi and Veneziano [4] suggests that the summation over micro-states may have a diagrammatic counterpart in form of the “dressing” of the \(2 \rightarrow N\) process by both virtual and real super-soft gravitons.

So, if the above picture is even remotely correct, it follows that a most naive gravity-like quantum network captures the qualitative properties of a classicalization process in black hole creation in particle collisions.

In order to make the analogy with a black hole formation even sharper, let us note that using the recipe of [11], it is rather straightforward to modify the model (24) in such a way that the micro-state entropy of the enhanced memory classical state shall obey an area-law, reminiscent of black hole’s Bekenstein entropy [17]. For this, we need to impose a spherical symmetry on the neural degrees of freedom, by visualizing them as the angular momentum modes of some “parent” field living in \(D\)-dimensions [10]. In particular, the index \(k_N\) in (24) must be understood as the label of spherical harmonics of a level of degeneracy \(N\).

This mapping allows to attribute a well-defined meaning of a local geometry to such an intrinsically non-local system as the neural network. The example of a network exhibiting an area-law entropy (for the case of angular invariance in \(D = 3\)) is given by the following Hamiltonian:

\[
\hat{H} = \sum_{N} N \epsilon_{N} \hat{b}_{N}^{\dagger} \hat{b}_{N} + \sum_{N} \epsilon_{N} \hat{a}_{0,N}^{\dagger} \hat{a}_{0,N} + \\
\sum_{N} \epsilon_{N} \left( 1 - \frac{\hat{a}_{0,N}^{\dagger} \hat{a}_{0,N}}{N} \right) \sum_{m,l} \hat{a}_{slm}^{\dagger} \hat{a}_{slm} + \\
\sum_{N} \hat{b}_{N}^{\dagger} \hat{a}_{0,N} \sum_{n_1, n_2, \ldots, n_N} \sum_{m_1, m_2, \ldots, m_N} g_{n_1 \ldots n_N} (N) \hat{a}_{sl_1 m_1}^{n_1} \hat{a}_{sl_2 m_2}^{n_2} \ldots \hat{a}_{sl_N m_N}^{n_N} + \text{h.c.} 
\]

This Hamiltonian has a structure very similar to (24) except it incorporates the spherical symmetry. Here, \(s = 0, 1, \ldots \infty\) is an integer that labels the level of angular harmonics on a three-sphere. For a given \(s\), the integers \(m, l\) label the usual angular harmonics on \(S_2\) and take the values \(|m| \leq l \leq s\). Thus, for fixed \(s\), the sum over \(l, m\) is the sum over the spherical harmonics of a 2-sphere belonging to the level \(s\) of an angular harmonics of a 3-sphere. The summation is subject to an obvious constraint of the angular momentum conservation. Since, the degeneracy of the level \(s\) is \((s + 1)^2\), we constrain \(N\) to take the values \(N = (s + 1)^2\). The couplings \(g_{n_1 \ldots n_N} (N)\), as before, obey the constraints.
Notice, we do not need to require the full $S_3$-invariance of the Hamiltonian. For the desired count of the gapless modes it suffice to maintain intact the structure of $S_2$ angular harmonics. The lifting of a model to an entire $S_3$ invariance, which can be done along the lines of [10], is unnecessary for the present purposes.

The, above Hamiltonian possess an infinite family of macro-states labeled by the occupation number of the corresponding master modes $n_{0_N} = N$ and the energies $E_N \simeq N \epsilon_0$, where $N = (s + 1)^2$. On each of these states the corresponding modes $a_{slm}$ become gapless. For large $s$, their number is $N \sim s^2$. The entropy of the degenerate micro-states created by these gapless modes therefore scales as $\sim s^2$. This represents an area of a sphere of radius $s \sim \sqrt{N}$ measured in units of some fixed fundamental length.

However, the area scaling of entropy a priori does not specify its energy dependence. This is an additional information encoded in $N$-dependence of the gaps $\epsilon_0$. For example, as explained in the previous chapter, the choice $\epsilon_0 = \frac{\sqrt{N}}{N}$ reproduces the dependence between the black hole entropy and its energy, with $\Lambda$ playing a role of the Planck mass.

Thus, once the neurons are visualized as the angular momentum modes of a field, the above network exhibits a “holographic” behaviour of the type [11]. Here the term holography is manifested in the fact that the emergent gapless modes effectively “inhabit” an area of a lower dimensional sphere. This is intriguing, since holography is usually considered to be characteristic of the gravitational systems [20–24].

Applying the previous analysis to the above Hamiltonian, we reach the following conclusion. Starting from an arbitrary initial state $|in\rangle = |1\rangle_{b_N}$ with a single excited quantum of one of the $b_N$-modes, for arbitrarily large $N$, the Hamiltonian generates an unsuppressed transition into a classical state with $n_{0_N} = N = (s + 1)^2$, with the micro-state entropy scaling as $\sim s^2$.

It is straightforward to further enhance the complexity of the network, and allow for more channels of classicalizing transitions. We shall not enter this model building here, since the presented example suffice for explaining the key ideas of the framework.

C. Hierarchy Problem: Standard Model as Brain Network

Our analysis can provide a guideline for UV-completion of the Standard Model through the mechanism of classicalization of the Higgs field. For accomplishing this goal, the Higgs boson must be endowed by a new interaction that would enable its unsuppressed transition into some multi-particle states [11]. The lesson we have learned is that such states must posses an exponentially enhanced memory storage capacity. This is necessary for compensating the exponential suppression of the transition amplitude.

A quantum neural network of the type [24] represents a toy effective model describing the classicalization of the Higgs field in the Standard Model. In order to see this, we should perform the following identification. First of all, we must identify the neural degrees of freedom with the momentum modes of the quantum fields.

In order to understand who is who, let us recall [1] that the key ingredient of solving the hierarchy problem via classicalization is to allow an unsuppressed transition of the high energy Higgs particles into a classical state composed out of many soft quanta.

Thus, we establish the following dictionary between the classicalizing neural network and the Standard Model. The hard $b$-neurons of energy $N \epsilon_0$ must be mentally replaced by the high momentum modes of the Higgs field with the center of mass energy given by $N \epsilon_0$. For example, $\hat{b}_N \to \hat{k}^\dagger \hat{\bar{k}}^\dagger$, where $\hat{k}$ is a momentum vector, and $\hat{k}^\dagger$ and $\hat{\bar{k}}^\dagger$ denote the creation operators of the Higgs boson and of its antiparticle respectively.

With such a replacement the effective Hamiltonian [27] shall generate classicalizing transitions of high energy Higgs pairs into multi-particle states. Namely, an initial quantum state with Higgs particle-antiparticle pair of energy $E = 2 |k| = N \epsilon_0$, represented by a ket-vector, $|in\rangle = |1\rangle_{b_k} \otimes |1\rangle_{\bar{b}_{-k}} \otimes |0\rangle_a$, gets converted into a classical state, $|final\rangle = |0\rangle_{b_k} \otimes |0\rangle_{\bar{b}_{-k}} \otimes |N\rangle_{a_0} \otimes |n_1\rangle_{a_1} \otimes \ldots \otimes |n_N\rangle_{a_N}$, in which $N$ soft $a_0$-quanta are present. The Feynman diagram corresponding to this process is given in Fig.2. As shown above, the rate of the transition is not exponentially suppressed due to the multiplicity of micro-states.

Notice, the $a$-modes can either correspond to the momentum modes of the Higgs field itself or to some additional Bosonic field introduced in the theory. In other words, the theory could classicialize either due to a new energy-dependent self-interaction of the Higgs field or due to its interactions with the additional fields from beyond the Standard Model. This is a matter of the model building, which is beyond our present goal.
that the Standard Model operates as a brain network that exploits an arbitrarily high energy - pumped in it through the hard quanta - for bringing itself into a classical state of the maximal available memory storage capacity.

VII. OUTLOOK

In this paper we described an effective theory of classicalization [1–4, 25]. While our model represents a fully self-consistent quantum system, in parallel, we gave its interpretation [9, 10] in terms of a quantum neural network. In this description, the momentum modes of particles are identified with the neural degrees of freedom, whereas their interactions are mapped on synaptic connections between the respective neurons.

The key aspect was to highlight the role of the enhancement of micro-state entropy due to the emergence of gapless modes, via the mechanism of [9–11]. There it was shown that, in case of gravity-like connections, a macroscopic excitation of one of the soft modes renders a set of modes, connected to the excited “master” mode, effectively gapless. In a neural network language, this means that the system attains a state in which it acquires a maximal capacity of the memory storage and an ability to respond to super-soft input stimuli. In such states, an exponentially large number of patterns can be stored within an arbitrarily narrow energy gap.

In order to avoid the potential problems with terminology, we gave a careful definition of our criteria and explained why the network that gives rise to the gapless neurons via our mechanism is special as compared to a generic quantum Hopfield network.

Next, we studied the question of a classicalizing transition from an initial quantum state into a “classical” state of $N$ soft quanta,

$$|1\rangle_{\text{quant}} \rightarrow |N\rangle_{\text{class}}.$$  

Using a simple effective Hamiltonian, we first explained why the transitions to the individual $N$-particle states must be exponentially-suppressed by the factors $e^{-N}$.

To the best of our knowledge, the presented way of reasoning showing that the physical amplitudes of the processes that increase occupation numbers must be exponentially suppressed, is new and can be useful on its own right. In particular, this gives yet another non-perturbative argument showing that without introduction of new classicalizing interactions that become strong above few TeV energies, in the Standard Model alone the production of classical states with high occupation number of the Higgs quanta should remain exponentially suppressed (regardless of a possible breakdown of perturbation theory (see, e.g., [26])) and the Hierarchy Problem will remain unsolved.

We then showed, how this suppression is compensated by the multiplicity of the micro-states in cases when the classical $N$-particle state exhibits an exponentially enhanced memory capacity. We showed that a necessary condition (1) is that a high occupation number $N$ of each soft master mode $a_{0N}$ composing the classical state, is matched by the equal number of species $(a_{kN}, k = 1, 2, ...N)$ of the emergent gapless modes.

In such a situation, the production of $N$ soft quanta of $a_{0N}$-mode is accompanied by a number distribution, $n_1, n_2, ..., n_N$, of the super-soft ones $a_{kN}$, coming from the gapless modes. It is the summation over the distributions of these super-soft quanta that provides the desired exponential enhancement of the transition probability.

There is something striking about understanding this phenomenon in terms of quantum neural networks. In
this language, the effect of classicalization is translated as the ability of a network to readily bring itself into a (classical) state of maximal memory storage capacity, in response to an external quantum stimulus of very high energy.

The presented way of looking at the field-theoretic systems, such as, the classicalizing Standard Model Higgs and/or quantum creation of black holes, puts the seemingly-disconnected phenomena in an unified perspective. They all emerge as the brain networks that aspire to attain the states of maximal available memory capacity.

1. NOTE ADDED

Allen Caldwell shared with us the results of his numerical analysis [27], which show that under a random walk evolution the enhanced memory states of the neural network model of [9] are the attractor points to which the evolution converges. The Hamiltonian of [9], used in this analysis, represents a slightly expanded version of [4], with unessential difference. It appears that this convergence, primarily due to its stochastic nature, although seen by a different protocol, captures some aspects of the quantum evolution presented here. This study would be worth pursuing further.

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2. NOTE ADDED

After this paper was finished, we became aware of the work by Cesar Gomez [28] in which he is also discussing the role of infrared physics in classicalization, but from a different angle. Although Gomez in his analysis is touching neither a mechanism of micro-state entropy nor the emergence of the gapless modes, nevertheless, the clear-cut connection point between the two proposals is the crucial role of dressing by the infrared physics in classicalizing theories. In our approach, this is expressed as an accompaniment to a large occupation number of each soft mode, by an equal number of the emergent gapless species. It is the summation over the distributions of the super-soft quanta of these gapless modes that makes an unsuppressed classicalization possible.

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