Anomalous magnetic moment of the positronium ion

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Abstract
We determine the gyromagnetic factor of the positronium ion, a three-body system consisting of two electrons and a positron, including first relativistic corrections. We find that the $g$-factor is modified by a term $-0.51 (1) \alpha^2$, exceeding 15 times the $\alpha^2$ correction for a free electron. We compare this effect with analogous results found previously in atomic positronium and in hydrogen-like ions.

Keywords: anomalous magnetic moment, positron, relativistic effects

1. Introduction
The positronium ion $\text{Ps}^-$ is a bound state of two electrons and one positron. Discovered in 1981 [1], it is now being precisely studied with the goal of determining its lifetime [2, 3], the binding energy, and the photodetachment cross section [4]. These observables have been precisely predicted [5–11]. The recent progress has occurred thanks to the developments of an intense positron source [12] and an efficient $\text{Ps}^-$ source [13–16] on the experimental side, and by improved variational calculations of the three-body wave function and incorporation of relativistic and some radiative effects on the theory side.

In this paper we focus on the magnetic moment of this three-body system. In its ground state, the two electrons are in a spatially symmetric wave function forming a spin singlet to make their total wave function antisymmetric. Thus, the whole magnetic moment is due to the

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positron and, if we neglect the bound-state effects, it is given by \( g^2 \frac{\hbar}{2m} \) where \( g \) is the gyromagnetic ratio of a free positron (or electron), \( g = 2 + \frac{\alpha}{\pi} + \ldots \), and \( \alpha \approx 1/137 \) is the fine-structure constant. The free-particle \( g \) factor has become known recently to the astonishing five-loop order, \( O\left((\alpha/\pi)^5\right) \) [17].

The purpose of this paper is to determine to what extent the interaction of the positron with the two electrons modifies the magnetic moment of the ion. This effect is expected to be analogous to that in hydrogen-like atoms and ions, where the nuclear electric field modifies the \( g \) factor of an electron [18], and thus be a correction of order \( \alpha^2 \), enhanced relative to the free-particle effects in this order in the coupling constant. Effects of this origin have been studied with high precision in hydrogen-like ions [19–21]. Combined with measurements with a five-fold ionized carbon [22–24], they are the basis of the most precise determination of the electron mass.

2. Hamiltonian

We are interested in the lowest-order relativistic corrections, or effects \( O(1/c^2) \) (equivalently \( \alpha^2 \)). To this order, the Hamiltonian describing the two electrons (labels 1 and 2) and the positron (label 3) consists of the kinetic energy \( H_0 \), the spin–orbit interaction \( H_3 \), the spin-other orbit term \( H_4 \), and the magnetic moment interaction \( H_5 \). We number the terms in the Hamiltonian in a way consistent with previously published results [25]. The expressions are simplified, since all particles have equal masses, \( m_1 = m_2 = m_3 \equiv m \),

\[
H_0 = \frac{\Pi_1^2}{2m} + \frac{\Pi_2^2}{2m} + \frac{\Pi_3^2}{2m} + \frac{e^2}{r_{12}} - \frac{e^2}{r_{13}} - \frac{e^2}{r_{23}} \tag{2.1}
\]

\[
H_3 = -\frac{e^2}{2m^2 c^2} \mathbf{s}_3 \cdot \frac{\mathbf{r}_{23} \times \Pi_3}{r_{23}^3} - \frac{e^2}{2m^2 c^2} \mathbf{s}_3 \cdot \frac{\mathbf{r}_{13} \times \Pi_3}{r_{13}^3} \tag{2.2}
\]

\[
H_4 = \frac{e^2}{m^2 c^2} \mathbf{s}_3 \cdot \frac{\mathbf{r}_{13} \times \Pi_1}{r_{13}^3} + \frac{e^2}{m^2 c^2} \mathbf{s}_3 \cdot \frac{\mathbf{r}_{23} \times \Pi_2}{r_{23}^3} \tag{2.3}
\]

\[
H_5 = -\frac{e}{mc} \mathbf{s}_3 \cdot \mathbf{B} \left( 1 - \frac{\Pi_3^2}{2m^2 c^2} \right) \tag{2.4}
\]

where \( r_{ij} \equiv r_i - r_j \). We only retain the terms that can contribute to the magnetic moment in the desired order \( \alpha^2 \). The terms proportional to the electron spins \( \mathbf{s}_1 \) and \( \mathbf{s}_2 \) are symmetric in the particle indices 1 and 2. However, the \( Ps \) wave function is antisymmetric in 1 and 2. Therefore, the expectation values of these terms are zero, and they have been omitted. Note that in the expression for \( H_5 \) in [25], there is a factor \( mc^2 \) missing in the denominator of the term corresponding to the second term in the bracket of (2.4).
3. Center of mass coordinates

Expressions (2.1)–(2.4) refer to particle coordinates and momenta in the LAB frame. On the other hand, we determine the wave function in the center of mass (CM) system of the ion. In order to calculate the magnetic moment, we need the Hamiltonian expressed in the CM variables. This can be achieved using the Krajcik–Foldy (KF) relations between the CM and LAB variables [26]. It turns out, however, that most of the terms of those relations do not contribute to the $O(\alpha^2)$ correction to the $g$ factor and we only need

$$r_i = \rho_i + \sum_j \frac{\sigma_j \times \pi_j}{2mMc^2},$$

$$p_i = \pi_i,$$

$$s_i = \sigma_i,$$  

(3.1)

where $r_i$, $p_i$, and $s_i$ are the LAB variables of the $i$th particle, and $\rho_i$, $\pi_i$, and $\sigma_i$ are the corresponding CM variables. $M$ is the total mass of the system. We choose the center of mass as the origin, $R = 0$. None of the terms dependent on the total momentum of the system were found to contribute to the magnetic moment to order $\alpha^2$, so we also set $P = 0$.

4. $g$ factor in two-body atoms

Before we consider the three-body ion, we show how the known corrections for simple one-electron atoms can be reproduced.

4.1. Positronium

Positronium is a two-body system with the symmetry due to equal masses, so the Hamiltonian simplifies. Among the parts of the Hamiltonian shown in equations (2.1)–(2.4), only $H_{0,3,4,5}$ contribute to the order $\alpha^2$. The Ps atom contains only the electron $i = 1$ and the positron $i = 3$, so all terms where the label $i = 2$ appears can be neglected. On the other hand, in $H_{3,4,5}$, we have to account for the spin of the electron (not included in (2.2)–(2.4) in anticipation of cancellations in Ps, due to the symmetry of its wave function). This is achieved by replacing $s_1 \to s_3 - s_1$.

We set $e_1 = -e_1 = e$ and $\pi_3 = -\pi_1 = \pi$. Neglecting terms containing $R$ and $P$, we find that in the transformation LAB $\to$ CM, equation (3.1), the only term relevant for the Ps atom is

$$r_i \to \rho_i + \sum_j \frac{\sigma_j \times \pi_j}{2mMc^2} = \rho_i + \frac{(\sigma_1 - \sigma_3) \times \pi}{4m^2c^2},$$

(4.1)

while the momentum and spin transform trivially, $p_i \to \pi_i$ and $s_i \to \sigma_i$.

Since the transformation (4.1) adds a term suppressed by $1/c^2$, we only need to apply it to the lowest order term $H_0$, where it affects the vector potential in the kinetic term. The resulting contribution to the magnetic moment is (here and below we average over the directions of position and momentum, since we are interested in the S-wave ground state),
\[ \Pi_1^2 = \left( p_i - \frac{e_i}{c} A_i \right)^2 \rightarrow -\left\{ \left[ \mathbf{\pi}, \frac{e}{2c} \mathbf{B} \times \frac{(\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \times \mathbf{\pi}}{4m^2c^3} \right] \right\} \rightarrow \frac{e}{6m^2c^3} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \mathbf{B} \mathbf{\pi}^2. \]  

(4.2)

The same effect arises from the kinetic energy of the positron. In total,

\[ \frac{\Pi_1^2 + \Pi_3^2}{2m} \rightarrow \frac{e}{6m^2c^3} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \mathbf{B} \mathbf{\pi}^2. \]  

(4.3)

The next corrections are expressed by position operators of \( e^\pm \). We have, after the transformation to CM, \( r_i \rightarrow \rho_i \equiv -\frac{r_i}{2}, r_3 \rightarrow \rho_3 \equiv +\frac{r_3}{2}, \) and \( r_3 \rightarrow -r \). The sum of terms 3 and 4 in the Hamiltonian, equations (2.2)–(2.3), gives the magnetic interaction

\[ H_3 + H_4 \rightarrow \frac{e^2}{2m^2c^2} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \frac{\mathbf{r} \times \left( \frac{e}{2} \mathbf{B} \times \mathbf{r} \right)}{r^3} \rightarrow \frac{e^3}{12m^2c^2r} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \mathbf{B}. \]  

(4.4)

Finally, \( H_5 \) gives

\[ H_5 \rightarrow -\frac{e}{mc} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \mathbf{B} \left( 1 - \frac{\pi^2}{2m^2c^2} \right). \]  

(4.5)

The total magnetic moment interaction is the sum of (4.3)–(4.5). Its expectation value with the ground state spatial part of the wave function gives

\[ -\frac{e}{mc} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \mathbf{B} \left( 1 - \frac{\pi^2}{6m^2c^2} - \frac{\pi^2}{2m^2c^2} - \frac{e^2}{12mc} \right) \]

\[ = -\frac{e}{mc} (\mathbf{\sigma}_3 - \mathbf{\sigma}_1) \cdot \mathbf{B} \left( 1 - \frac{5\alpha^2}{24} \right), \]  

(4.6)

confirming the well known result [27–29]. The resulting interaction does not have diagonal elements neither in spin singlet nor triplet states of Ps. However, it mixes the \( m = 0 \) state of the triplet with the singlet. Measurements of the resulting splitting among the oPs states determine the hyperfine splitting of positronium.

### 4.2. Hydrogen

In hydrogen, there are further simplifications, since the spin-other orbit term \( H_4 \) does not contribute in the leading order, due to the suppression by the proton mass. Also, there is no difference between the LAB and the CM frames in the leading order in \( 1/M \). Thus, only \( H_5 \) and the spin–orbit term \( H_3 \) contribute (we replace \( s_3 \rightarrow s \) and \( r_3 \rightarrow r \)),

\[ H_3 \rightarrow \frac{e^2}{2m^2c^2} s \cdot \frac{\mathbf{r} \times \left( \frac{e}{2c} \mathbf{B} \times \mathbf{r} \right)}{r^3} \rightarrow \frac{e^3}{6m^2c^2r} s \cdot \mathbf{B} \]

\[ H_5 \rightarrow \frac{e}{mc} s \cdot \mathbf{B} \left( 1 - \frac{\pi^2}{2m^2c^2} \right) \]


and the total magnetic moment interaction in the ground state of H becomes
\[
\frac{e}{mc} s \cdot B \left( 1 - \frac{\pi^2}{2m^2c^2} + \frac{e^2}{6mc^2} \right) = \frac{e}{mc} s \cdot B \left( 1 - \frac{\alpha^2}{3} \right), \tag{4.7}
\]
in agreement with the classic result by Breit [18].

4.3. Hydrogen-like ions, including recoil effects

Now we consider an ion consisting of a nucleus with charge Ze and a single electron with −e. Among the systems, for which binding effects on the g factors have been evaluated, this is the closest one to the positronium ion, which is also charged and in which recoil effects are not suppressed, since there is no heavy nucleus.

Since we have already established which terms are relevant to the order we need, we set \( c = 1 \) from now on. The relevant terms of the KF transformation become, using \( \equiv r_e - r_p, m \) for the mass of the electron and, only in this section, \( M \) for the mass of the nucleus, for easier comparison with reference [27]
\[
r_e \rightarrow R + \frac{M}{M + m} r + \frac{s_e \times p_e}{2m(M + m)},
\]
\[
r_p \rightarrow R - \frac{m}{M + m} r + \frac{s_e \times p_e}{2m(M + m)}. \tag{4.8}
\]
This introduces the spin interaction into the kinetic energy term \( H_0 \),
\[
H_0 = \frac{\Pi_e^2}{2m} + \frac{\Pi_p^2}{2M} \rightarrow - \frac{es \cdot B}{6(M + m)} \left( \frac{1}{m^2} + \frac{Z}{mm} \right) \langle \pi^2 \rangle, \tag{4.9}
\]
and in the ground state \( \langle \pi^2 \rangle = Z^2\alpha^2\mu^2 \) where \( \mu = \frac{mm}{M + m} \) is the reduced mass.

If the nuclear mass is taken as finite, the spin–orbit and spin-other orbit terms become
\[
H_3 + H_4 \rightarrow \frac{\alpha}{2m^2r_e} s \cdot r \times \Pi_e - \frac{\alpha}{mm_e} s \cdot r \times \Pi_p
\]
\[
\rightarrow \frac{eZ^2\alpha^2\mu}{6m^2M(M + m)} s \cdot B \left( M^2 - 2Zm^2 \right). \tag{4.10}
\]
Finally, the last correction comes from \( H_5 \),
\[
H_5 \rightarrow \frac{es \cdot B}{m} \left( 1 - \frac{Z^2\alpha^2\mu^2}{2m^2} \right). \tag{4.11}
\]
The sum of (4.9), (4.10), (4.11) gives the total magnetic moment interaction in the ion,
\[
\frac{es \cdot B}{m} \left( 1 - Z^2\alpha^2M^2(3m + 2M) + Zm^2(3M + 2m) \right), \tag{4.12}
\]
in agreement with equation (43) in [27]. We note that the correction is symmetric with respect to the exchange of the electron and nucleus mass and charge, \( M \leftrightarrow m, Z \leftrightarrow 1 \); in the limit \( M \gg m \) reproduces our non-recoil result (4.7); and in the limit \( Z \rightarrow 1, M \rightarrow m \) agrees with the correction in the positronium atom (4.6).
5. Positronium ion

For the positronium ion, the correction arises in a way similar to the Ps atom. Setting \( c = 1 \), we find

\[
\rho = - \rho_1 - \rho_2 + \rho_3 + \rho_4,
\]

where the first two terms arise from \( H_5 \), the third from \( H_6 \), and the last one from \( H_3 + H_4 \). We use the notation \( \rho = \rho_{ij} \eta_{ij} \eta \), \( \pi = \pi_{ij} \eta_{ij} \eta \).

For the expectation value, we use the wave function found using the variational calculation as described in [8] (see appendix) and find

\[
\Delta = -0.51 (1) \alpha^2.
\]

Here \( \Delta = 2 \left[ 1 + \frac{\alpha}{2} - 0.328 \left( \frac{\alpha}{2} \right)^2 + \ldots \right] \) is the \( g \)-factor of a free electron [17]. The error in \( \Delta \) arises primarily from higher-order binding corrections, beyond the scope of this paper. Note that the binding correction \( \Delta \) exceeds the same order effect, \( O(\alpha^2) \), in \( g_{\text{free}} \), about 15 times. Our final prediction for the gyromagnetic factor of the positronium ion is

\[
g_{\text{Ps}} = 2.00461 (1).
\]

We see that the correction \( \Delta \) is smaller in magnitude than in hydrogen, equation (4.7), where it is \( -0.67 \alpha^2 \), but larger than in the positronium atom, equation (4.7), \( -0.42 \alpha^2 \). Indeed, this confirms the naive expectation that the value should be in between these two and closer to positronium. The entire magnetic moment of the three-body ion can be thought of as being due to the magnetic moment of the positron, whose gyromagnetic ratio \( g \) is modified by the binding to the two electrons. If the two electrons are considered as a kind of a nucleus in whose field the \( g \) factor of the positron is modified, it is heavier than in the positronium atom, but much lighter than in hydrogen.

Can this quantity be measured? The main challenge is the very short lifetime of the ion, only four times longer than that of the atomic parapositronium, or about half a nanosecond. With an intense beam and a strong external magnetic field, a possible scenario of a measurement could be as follows. An ion with a known initial polarization could be subjected to the magnetic field, where its polarization (the direction of the positron spin) would precess. The annihilation process occurs predominantly within a spin-singlet electron-positron pair, so that the total spin direction of the ion is preserved by the surviving electron, and can be detected.

Production of polarized positronium atoms has recently been demonstrated [30]. In particular, that study has shown that the initial polarization of positrons produced in a \( \beta^+ \) decay can be preserved in a storage trap. Thus, it appears that it will be feasible to also produce polarized positronium ions. (As an aside, we mention the discussion of the feasibility of accelerating \( \text{Ps}^+ \) to relativistic velocities in [31], where the connection of its physics with quantum chromodynamics systems is also discussed. Analogies between QED bound states and hadrons are also explored in [32].)
As the first step towards the measurement of the magnetic moment of Ps\(^-\) one could attempt to observe the polarization of the daughter electron remaining after the decay. As we have shown, the g factor of Ps\(^-\) is very close to the Dirac value 2, so that the spin and velocity of the ion precess in electromagnetic fields with very similar frequencies. The polarization of the initial positron is degraded mainly by spin–orbit and spin–spin interactions in the production target. Thus, a measurement of the daughter electron polarization without a magnetic field may provide information about the dynamics of the ion formation.

What could motivate a measurement of the magnetic moment of the ion? The influence of various ‘new physics’ scenarios on positronium properties has been reviewed in [33]. However, we believe that Ps\(^-\) is more suitable for testing our understanding of bound states within QED, whose value should not be underestimated. Only recently have solutions been found for the puzzles of the positronium decay [34–39], the helium fine structure [40, 41] and, possibly, the positronium hyperfine splitting [42–47]. A special feature of the positronium ion is that its magnetic moment is more sensitive to recoil corrections than is the case in hydrogen-like systems. We hope that in the future its measurement will complement studies of the Ps\(^-\) lifetime and photodetachment reactions as a probe of this exotic system.

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Appendix A. Optimization and expectation values of operators

Here we briefly describe how the operators in equation (5.1) are evaluated using the variational method. We expand the trial wave function in an explicitly correlated Gaussian basis, following the steps described in a study of the di-positronium molecule [48],

\[
\phi = \sum_{i=1}^{N} c_i \exp \left[- \sum_{a<b} w_{ab} \rho_{ab}^2 \right]
\]  

(A.1)

where \(\rho_{ab}\) are the three inter-particle separations and \(N\) is the size of the basis; we use \(N = 200\).

The parameters \(w_{ab}\) are optimized using the non-relativistic Coulomb Hamiltonian

\[
H_C = \sum_{a=1}^{3} \frac{p_a^2}{2m_a} + \sum_{a<b} e_a e_b / \rho_{ab}.
\]  

(A.2)

The inter-particle vectors are related \((\rho_{12}^2 + \rho_{23}^2 - \rho_{13}^2 = 0)\), so one of them can be eliminated in the evaluation of expectation values. The resulting integrands have an exponential whose argument is of second order in two of the inter-particle distances.

The new operator that has to be evaluated is the second term in (5.1). We rewrite it as

\[
\frac{\rho_{13} \cdot \rho_{12}}{\rho_{13}^2} = \frac{1}{2} \left( \frac{1}{\rho_{13}} - \frac{\rho_{23}^2 - \rho_{12}^2}{\rho_{13}^2} \right),
\]  

(A.3)
and find, using $w_{ab}^{ij} \equiv w_{ab}^i + w_{ab}^j$, 
\[
\mathcal{A}_{ij} = \left< \phi_i \left| \frac{1}{\rho_{13}} \right| \phi_j \right> = \frac{2\pi^{5/2}}{\left( \sum_{a \neq b < c} w_{ab}^i w_{ac}^j \right)^{1/2}} \sqrt{w_{12}^i + w_{23}^j}
\]
\[
\mathcal{A}_{ij} \left< \phi_i \left| \frac{\rho_{23}^2 - \rho_{12}^2}{\rho_{13}^2} \right| \phi_j \right> = \left[ \frac{d}{dw_{12}^y} - \frac{d}{dw_{23}^y} \right] \left< \phi_i \left| \frac{1}{\rho_{13}} \right| \phi_j \right>
\]
\[
= \int d^3x \, d^3y \, \frac{1}{y^2} \left[ \frac{d}{dw_{12}^y} - \frac{d}{dw_{23}^y} \right] \exp \left\{ -\alpha_i x^2 - \alpha_j y^2 \right\} \quad (A.4)
\]
\[
= -\left< \phi_i \left| \frac{1}{\rho_{13}} \right| \phi_j \right> \left[ \frac{d}{dw_{12}^y} - \frac{d}{dw_{23}^y} \right] \alpha_y \quad (A.5)
\]
\[
= \frac{2\pi^{5/2}}{\left( \sum_{a \neq b < c} w_{ab}^i w_{ac}^j \right)^{1/2}} \left( w_{12}^i + w_{23}^j \right)^{3/2} \quad (A.6)
\]
where $\alpha_i \equiv w_{12}^i + w_{23}^j$ and $\alpha_i \alpha_j \equiv \sum_{a \neq b < c} w_{ab}^i w_{ac}^j$. In going from line (A.4) to (A.5) we use 
\[
\left[ \frac{d}{dw_{12}^y} - \frac{d}{dw_{23}^y} \right] \exp \left\{ -\alpha_i x^2 \right\} = 0.
\]

References

[1] Mills A P Jr 1981 Phys. Rev. Lett. 46 717
[2] Fleischer F et al 2006 Phys. Rev. Lett. 96 063401
[3] Ceeh H et al 2011 Phys. Rev. A 84 062508
[4] Michishio K et al 2011 Phys. Rev. Lett. 106 153401
[5] Frolov A M 2009 Phys. Rev. A 80 014502
[6] Frolov A M 1999 Phys. Rev. A 60 2834
[7] Drake G W F and Grigorescu M 2005 J. Phys. B: At. Mol. Opt. Phys. 38 3377
[8] Puchalski M, Czarnecki A and Karshenboim S G 2007 Phys. Rev. Lett. 99 203401
[9] Bhatia A K and Drachman R J 1985 Phys. Rev. A 32 3745
[10] Ward S J, Humberston J W and McDowell M R C 1987 J. Phys. B: At. Mol. Phys. 20 127
[11] Igarashi A, Shimamura I and Toshima N 2000 New J. Phys. 2 17
[12] Hugenschmidt C et al 2004 Nucl. Instrum. Methods Phys. Res., Sect. B 221 160
[13] Nagashima Y, Hakodate T, Miyamoto A and Michishio K 2008 New J. Phys. 10 123029
[14] Terabe H, Michishio K, Tachibana T and Nagashima Y 2012 New J. Phys. 14 015003
[15] Hyodo T et al 2011 J. Phys: Conf. Ser. 262 012026
[16] Nagashima Y et al 2012 J. Phys.: Conf. Ser. 388 012021
[17] Aoyama T, Hayakawa M, Kinoshita T and Nio M 2012 Phys. Rev. Lett. 109 111807
[18] Breit G 1928 Nature 122 649
[19] Pachucki K, Jentschura U D and Yerokhin V A 2004 Phys. Rev. Lett. 93 150401
[20] Pachucki K, Jentschura U D and Yerokhin V A 2005 Phys. Rev. Lett. 94 229902 (erratum)
[21] Czarnecki A, Jentschura U D, Pachucki K and Yerokhin V A 2006 Can. J. Phys. 84 453
[22] Sturm S et al 2014 Nature **506** 467
[23] Häffner H et al 2000 Phys. Rev. Lett. **85** 5308
[24] Beier T et al 2002 Phys. Rev. Lett. **88** 011603
[25] Anthony J M and Sebastian K J 1994 Phys. Rev. A **49** 192
[26] Krajcik R and Foldy L 1974 Phys. Rev. D **10** 1777
[27] Groth H and Hegstrom R A 1971 Phys. Rev. A **4** 59
[28] Close F E and Osborn H 1971 Phys. Lett. **B34** 400
[29] Faustov R N 1970 Nuovo Cim. **A69** 37
[30] Cassidy D B, Meligne V E and Mills A P 2010 Phys. Rev. Lett. **104** 173401
[31] Uggerhøoj U I 2006 Phys. Rev. A **73** 052705
[32] Hoyer P 2014 arXiv:1402.5005 (unpublished)
[33] Rubbia A 2004 Int. J. Mod. Phys. A **19** 3961
[34] Vallery R, Zitzewitz P and Gidley D 2003 Phys. Rev. Lett. **90** 203402
[35] Adkins G S, Fell R N and Sapirstein J 2000 Phys. Rev. Lett. **84** 5086
[36] Kniehl B A and Penin A A 2000 Phys. Rev. Lett. **85** 1210
Kniehl B A and Penin A A 2000 Phys. Rev. Lett. **85** 3065 (erratum)
[37] Hill R and Lepage G P 2000 Phys. Rev. **D62** 111301
[38] Melnikov K and Yelkhovsky A 2000 Phys. Rev. D **62** 116003
[39] Czarnecki A, Melnikov K and Yelkhovsky A 1999 Phys. Rev. Lett. **83** 1135
Czarnecki A, Melnikov K and Yelkhovsky A 2000 Phys. Rev. Lett. **85** 2221 (erratum)
[40] Pachucki K and Yerokhin V A 2010 Phys. Rev. Lett. **104** 070403
[41] Borbely J S et al 2009 Phys. Rev. A **79** 060503
[42] Ishida A et al 2014 arXiv:1310.6923 (unpublished)
[43] Baker M et al 2014 Phys. Rev. Lett. **112** 120407
[44] Kniehl B A and Penin A A 2000 Phys. Rev. Lett. **85** 5094
[45] Melnikov K and Yelkhovsky A 2001 Phys. Rev. Lett. **86** 1498
[46] Hill R J 2001 Phys. Rev. Lett. **86** 3280
[47] Czarnecki A, Melnikov K and Yelkhovsky A 1999 Phys. Rev. Lett. **82** 311
[48] Puchałański M and Czarnecki A 2008 Phys. Rev. Lett. **101** 183001