Finite size corrections to the blackbody radiation laws

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We investigate the radiation of a blackbody in a cavity of finite size. For a given geometry, we use semiclassical techniques to obtain explicit expressions of the modified Planck’s and Stefan-Boltzmann’s blackbody radiation laws as a function of the size and shape of the cavity. We determine the range of parameters (temperature, size and shape of the cavity) for which these effects are accessible to experimental verification. Finally we discuss potential applications of our findings in the physics of the cosmic microwave background and solonluminescence.

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To a good approximation a blackbody can be represented by a cavity heated to a temperature $T$ and connected to the outside by a small aperture. A useful quantity to describe the radiation properties of a blackbody is the density of energy per unit of frequency,

$$u(\nu) = \frac{\rho(\nu)}{V} c(\nu)$$

where $V$ is the volume of the blackbody cavity, $\rho(\nu) d\nu$ is the number of stationary electromagnetic modes with frequencies between $\nu$ and $\nu + d\nu$ and $c(\nu)$ is the average energy per mode. In the $V \to \infty$ limit,

$$\rho(\nu) = \rho_\nu(\nu) = V \frac{8\pi \nu^2}{c^3}$$

where is the speed of light. Classically the average energy per mode does not depend on the frequency $\nu$ so, $u(\nu) \propto \nu^2$. However experimental results indicate $u(\nu) \rightarrow 0$, $\nu \rightarrow \infty$. Planck noted that agreement with experiments could be achieved if the energy was considered a discrete variable,

$$c(\nu) = \frac{h\nu}{(e^{h\nu/k_B T} - 1)}$$

($k_B$ and $h$ are the Boltzmann and Planck constants respectively). Combining these two results, Planck’s radiation law follows immediately, $u_\nu(\nu) = \frac{\rho_\nu(\nu)}{V} \frac{h\nu}{e^{h\nu/k_B T} - 1}$. Another useful quantity to characterize the blackbody radiation is the total energy emitted per unit of time and area,

$$R(T) = \frac{c}{4\pi} \int_0^\infty d\nu u(\nu).$$

In the $V \to \infty$ limit it is given by the Stefan-Boltzmann law,

$$R(T) = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8} \text{Js}^{-1} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$. The blackbody radiation, besides its role as a precursor of the quantum theory, has been an invaluable tool in a variety of applications. For example, it opened the way to the thermal remote sensing of bodies that allows to diagnose the properties of materials or to measure the surface temperature of celestial objects. The international scale of temperature, in the range above the freezing point of Silver, is based on Eq. (1).

Finite size corrections to these results are expected since $\rho_\nu(\nu)$ is only the leading term of a full expansion in powers of a typical length $L$ of the cavity. A comprehensive theoretical study of $\rho(\nu)$ was carried out in [1, 2] by using the Green function formalism previously developed for scalar waves in Refs. [3, 4]. In the special case of 3d cavities with smooth surfaces and $L \gg c/\nu$,

$$\rho(\nu) = \sum_i \delta(\nu - \nu_i) = \rho_\nu(\nu) + \rho_C + \tilde{\rho}(\nu),$$

where $\nu_i$ stands for the $i$th natural mode of the cavity, $\rho_C = \frac{2\pi \nu^3}{3\pi} C L^3$ and $C \propto L$ is the mean curvature ($C = 4\pi r_0$ for a sphere $r_0$ of radius $r_0$), namely, the mean of the two principal radii of curvature of the cavity [5] integrated over solid angle. This oscillatory part is given by,

$$\tilde{\rho}(\nu) = \sum_p A_p(\nu, L) \cos(\nu l_p/c + \mu_p)$$

where the sum runs over the periodic orbits, $p$, of length $l_p$ of the classical counterpart. For electromagnetic waves [1, 4] periodic orbits are the trajectories of the light rays inside the cavity as dictated by geometrical optics. The amplitude $A_p(\nu, L)$ and the Maslov index $\mu_p$ can be evaluated explicitly from the knowledge of the classical periodic orbits (see Appendix in Ref. [5]). In 3d cavities [1],

$$A_p(\nu, L) \propto (L/c)^{1+n/2} \nu^{n/2}$$

where $n$ is the number of axes of symmetry of the cavity ($n = 3$ for a sphere). Polarization effects also modify this amplitude [1]: To leading order only periodic orbits with an even number of reflections on the boundary of the cavity contribute to the amplitude. Moreover the amplitude picks up an additional factor 2 ($2\cos\alpha$) for planar (non planar) orbits due to polarization effects. For
a given periodic orbit characterized by \( m \) reflections,

\[
2 \cos \alpha = \vec{w} \cdot \vec{w}_m + \vec{w}' \cdot \vec{w}'_m
\]

where \( \vec{w}, \vec{w}' \) are two orthogonal unit vectors transverse to the direction of propagation of the light ray transforming into \( \vec{w}_m, \vec{w}'_m \) after \( m \) reflections. The angle \( \alpha \) is usually referred to as the polarization angle.

We note that in this approach \( \rho(\nu) \) is effectively written as an expansion in power of the parameter \( \frac{\lambda}{\nu} \) with \( \lambda = \frac{\pi}{\nu} \). The limits of applicability of this formalism are thus restricted to a range of sizes \( L \) and frequencies \( \nu \) such that \( \frac{\lambda}{\nu} \ll 1 \).

The main goals of this paper are: a) to provide a detailed account of the impact of finite size effects on the blackbody cavity before escaping through the aperture, b) to determine the range of parameters for which such corrections are accessible to experimental verification, c) to propose potential applications of these results.

CORRECTIONS TO PLANCK’S AND STEFAN-BOLTZMANN’S LAWS.

Finite deviations from the Planck’s law Eq.(3) are only due to \( \rho_c = \frac{1}{\rho} \). As a consequence, the modified Planck’s law can be written as,

\[
\frac{u(\nu)}{u^V} = 1 + a_1 \frac{c^2}{L^2 \nu^2} + \frac{c^{2-n/2}}{L^2 \nu^{2-n/2}} \sum_p \tilde{A}_p \cos(l_p \nu/c + \mu_p),
\]

where \( a_1 \propto \frac{\zeta}{\nu^2} \) is a dimensionless coefficient that depends on the mean curvature \( \zeta \propto L \) of the cavity, \( \tilde{A}_p = A_p c/L \) is the dimensionless amplitude corresponding to a single periodic orbit of length \( l_p \). In the case of symmetric cavities like spheres \( (n = 3) \) and rectangles \( (n = 2) \) these coefficients can be obtained easily (see below and [3]). For chaotic cavities \( (n = 0) \) it is much harder to get explicit analytical expressions for \( A_p \) except in the case of the shortest periodic orbits [4].

In order to fully determine \( u(\nu) \) we have yet to set a cutoff in the sum above. From a physical point of view it is evident that periodic orbits longer than the length \( l_{\text{esc}} \) associated to the typical time that a ray stays in the blackbody cavity before escaping through the aperture cannot contribute significantly to \( u(\nu) \). The explicit expression of the cutoff function depends strongly on the symmetries of the cavity. For chaotic cavities \( (n = 0) \), the contribution to Eq.(3) of a light ray of length \( l_p \) is weighted by the probability that this ray does not escape through the aperture, \( P_{\text{esc}}(l_p) \approx e^{-l_p/l_{\text{esc}}} \) with \( l_{\text{esc}} \sim \frac{A_{\text{ape}}}{V} \gg l_{\text{min}} \), where \( V \) is the volume of the blackbody, \( l_{\text{min}} \) is the length of the shortest periodic orbit and \( A_{\text{ape}} \) is the surface of the aperture [3]. By contrast for symmetric cavities \( (n = 2, 3) \) the cut-off function has power-law tails [8].

The Stefan-Boltzmann’s \( R(T) = \sigma T^4 \) law is also modified by finite size effects. After integrating \( u(\nu) \),

\[
R(T) = \tilde{R}(T) + R_c(T) + \sigma T^4
\]

where

\[
R_c(T) = \frac{C}{T^2 \pi L^3 h}(Tk_B)^2 \approx 1/L^2
\]

comes from the curvature term \( \rho_c \) in the spectral density. The contribution coming from the fluctuating part of the spectral density \( \tilde{R}(T) \) can be written exactly in terms of polygamma functions. This expression is rather cumbersome but in the limit \( l_{\text{min}} > \frac{\hbar c}{\kappa B^T} \) (relevant to experiments) simplifies considerably,

\[
\tilde{R}(T) \approx b_1 \frac{c \hbar T}{L^3} + b_0 \frac{\hbar c^2}{L^4}
\]

where the dimensionless coefficients \( b_0 \) and \( b_1 \) depend on the number, \( n \), of symmetry axes of the cavity. The case \( n = 3 \), the sphere, will be discussed in detail later on. For \( n = 0, n = 2 \),

\[
b_0 = -\frac{n+2}{4} \left(\frac{2\pi L}{c}\right)^{1+n/2} \sum_{p,r} A_{p,r}(L f(\mu_p)) (\tau_p r)^{2(n/2)}
\]

with \( f(\mu_p) = \cos(\mu_p) + (\sin(\mu_p) - \cos(\mu_p)) n/2 \),

\[
b_1 = \left(\frac{2\pi L}{c}\right)^{n/2} \sum_{p,r} A_{p,r}(L g(\mu_p)) (\tau_p r)^{1+n/2}
\]

with \( g(\mu_p) = -\sin(\mu_p) - (\sin(\mu_p) - \cos(\mu_p)) n/2 \). Similar expressions are found for \( n = 1 \). The amplitude \( A_{p,r} \) describes the contribution of a single periodic orbit of period \( \tau_p = l_p / c \) repeated \( r \) times. We note that: a) this sum is convergent even without including a cutoff function \( P_{\text{esc}}(l_p) \) due to the aperture, b) the largest contribution to the sum comes from the shortest periodic orbits (light rays), c) the Maslov indexes \( \mu_p \) and amplitudes \( A_{p,r} \) for the shortest periodic orbits can be computed analytically [8], d) for \( T \to 0 \) and fixed \( L \) our semiclassical formalism fails since the maximum of \( u(\nu) \) is in the region \( \lambda / L > 1 \).

FLUCTUATIONS IN CAVITIES WITH NO SYMMETRY AXES.

An explicit analytical determination of \( \tilde{R}(T) \) or the oscillating part of \( u(\nu) \) involves the knowledge of periodic orbits (light rays) of different lengths. This determination is only straightforward in highly symmetric cavities with several symmetry axis. However semiclassical techniques based on that fact that the classical dynamics is ergodic [6] permit an estimation of these deviations in the case of chaotic cavities with no symmetries. We first
study \( \langle \hat{R}^2 \rangle \propto \sum_{p',r,r'} A_{p',r} A_{p,r'} \) where the average is over cavities characterized by the same typical length \( L \) and with no axes of symmetry. To leading order, the double sum \([5]\) above is given by the diagonal \((p = p')\) term,
\[
\langle \hat{R}^2 \rangle \approx \frac{\hbar^2 c^4}{4L^6} \int_{\tau_{\text{min}}}^{\infty} d\tau K(\tau) \frac{\tau^4}{4} + \frac{c^2 k_B T^2}{L^6} \int_{\tau_{\text{min}}}^{\infty} d\tau \frac{K(\tau)}{\tau^2} \quad \text{(6)}
\]
where \( K(\tau) = \sum_p A_{p,r}^2 \delta(\tau - \tau_p) \). The lower limit of integration corresponds to the period of the shortest periodic orbit \( \tau_{\text{min}} = l_{\text{min}}/c \). In the region around \( \tau_{\text{min}} \) the function \( K(\tau) \) is a collection of well separated peaks located whose exact positions depends on the form of the cavity \([6]\). However for longer periods the number of orbits (light rays) with similar periods increases dramatically (in cavities with no symmetries) and \( K(\tau) = 2\tau \) for all chaotic \((n = 0)\) cavities \([6]\). Finally for \( \tau > \tau_H \) \((\tau_H = \nu(\nu)\) is the Heisenberg time\), \( K(\tau) = \tau \) \([6]\). Since our purpose is only to estimate the magnitude of the fluctuations we follow \([6]\) and assume that the non universal part of \( K(\tau) \) is fully described by the contribution of the shortest periodic orbit. Within this approximation it is straightforward to show that,
\[
\langle \hat{R}^2 \rangle \approx \frac{\hbar^2 c^4}{4L^6 l_{\text{esc}}^2} + \frac{c^2 k_B T^2}{L^6} \log(\tau_H/\tau_{\text{min}}). \quad \text{(7)}
\]
We note this result is valid provided that \( l_{\text{esc}} \gg l_{\text{min}} \). Otherwise one would have to multiply the contribution of each periodic orbit by its probability to stay inside the cavity.

A similar calculation of the fluctuations \( \delta u(\nu) \) of the density of energy \( u(\nu) \) with respect to \( u(V(\nu)) \) in a cavity with no symmetries shows that,
\[
\frac{\langle \delta u(\nu) \rangle^2}{u_V^2} = \frac{c^2}{\nu^2 l_{\text{esc}}^2}.
\]

The dependence of the fluctuations with the cutoff distance \( l_{\text{esc}} \) is expected since the sum over orbits in \( u(\nu) \) Eq. \([4]\), unlike \( R(T) \) Eq. \([4]\), does not really converge. Without a cutoff \( u(\nu) \) is a series of isolated peaks at the natural frequencies of the cavity.

**AN EXAMPLE: A SPHERICAL BLACKBODY.**

The case of a spherical blackbody \((n = 3)\) is of special interest since it can be solved exactly. In addition finite size corrections are stronger due to the high degree of symmetry of the sphere (the study of a cubic blackbody was carried out in Ref.\([10]\), the non oscillatory corrections in other simple geometries were investigated in \([11]\)). Small multipolar corrections to the spherical shape \([12]\) are described with the same periodic orbits that in the spherical case but adding an additional cutoff in term of Fresnel integral that smoothly modulates the amplitude and phase of the fluctuations. The effect of small, non overlapping bumps Ref.\([13]\) is only to suppress periodic orbits of the sphere longer than a certain cut-off related to the typical size of the bump.

**FIG. 1: Density of energy \( u(\nu) \) in \( J/m^3 \) of a blackbody versus frequency. The solid line is the Planck’s law. The dashed line is the analytical prediction for a spherical blackbody Eq.\([3]\) and Eq. \([4]\). Finite size effects are clearly visible. The parameters chosen \((r_0 = 2c, T = 5K)\) fall within the region accessible to experiments (see text). In order to reproduce FIRAS resolution only rays shorter than \( l_{\text{cut}} = 2 \times 10^7 c/\nu \) are included in Eq. \([4]\). In the inset we plot \( R_{\text{esc}}(x) = R(T)/\sigma T^4 \) with \( x = T r_0 k_B/\hbar c \).

For this geometry the density \( \rho(\nu) \) \([4]\) is known explicitly. The closed periodic orbits are given by planar regular polygons with a even number of vertexes along a plane containing the diameter. No polarization angle appears in our expressions due to the planarity of the orbits. The length \( L_{p,t} \) of the trajectories is given by \( L_{p,t} = 2pr_0 \sin(\phi) \) where \( r_0 \) is the radius of the sphere, \( p \) is the number of vertexes of the polygon and \( \phi = \pi t/p \) with \( t \) being the number of turns around the origin of a specific periodic orbit. With these definitions,
\[
\rho(\nu) = \rho_V(\nu) + \rho_C + \tilde{\rho}(\nu)
\]

with
\[
\frac{\tilde{\rho}(\nu)}{V} = -\frac{2\pi}{c} \sum_{t=1}^{3} \frac{3\pi \nu}{cr_0 \nu} \sin(8\nu tr_0/c) + \quad \text{(9)}
\]
\[
+ \frac{6\pi}{c} \sum_{t, p > 2t} (-1)^{t} \left( \frac{2\nu}{c} \right)^{3/2} \sin(2\phi) \times
\]
\[
\sqrt{\frac{\sin(\phi)}{r_0 p^4}} \sin((p + 3/2)\pi/2 + 2\nu pr_0 \sin(\phi)/c)
\]
and \( \rho_C = \frac{8\pi}{3c}. \) The sum above will be effectively cutoff due to the finite aperture that make the cavity open.
We postpone its study to the next section in which we address the experimental detection of these effects.

Corrections to the Stefan-Boltzmann's law can be evaluated explicitly by integration over $\nu$. In the limit $l_T = \frac{hc}{k_B T} < l_{min}$,

$$R(T) = \sigma T^4 + \frac{1}{12\pi^2} (Tk_B)^2 + b_1 k_B T c/r_0^3 - b_0 hc^2/r_0^4$$

where $b_1 \approx 0$ (due to the fact that only orbits with an even number of vertexes contribute to the sum), $b_0 \approx \frac{1}{4\pi^2} \sum_l \frac{n_l}{n_l^2} \approx 2 \times 10^{-4}$ . The second term corresponds to the curvature contribution $\rho_c$. The last two terms come from the oscillating part of the spectral density $\tilde{\rho} (\nu)$. The sum is dominated by the first terms corresponding to the contribution of the shortest periodic orbits and consequently it is quite insensitive to the cutoff $l_{esc} \gg l_{min}$.

In order to examine the importance of these finite size effects we rescale $R(T)$, $R_{sca}(x) = \frac{R(T)}{\sigma T^4} = 1 + \frac{5}{\pi^3 x^2} + \frac{60b_0}{x^2}$

with $x = r_0Tk_B/\hbar c$. This is an expansion in $1/x$ valid for $x \gg 1$. The most interesting region (see inset Fig.1.) to detect finite size corrections is thus $x \geq 1$. We note that since we go up to fourth order the expansion is still valid for $T \gg 0.5 cm$ and $T \geq 2K$. We note that a) The dependence of $R(T)$ with $T$ and $L$ for a spherical cavity Eq. (10) agrees with the general results obtained earlier Eq. (5). Only the dimensionless coefficients in front of Eq. (5) are cavity dependent, b) a similar expansion of $R_{sca}$ valid for $x < 1$ can also be carried out. However we could not find a region of parameters $(T, r_0$ with $r_0 > c/\nu$) accessible to experimental verification.

**EXPERIMENTAL DETECTION.**

We determine the range of parameters in which is feasible to build a spherical blackbody and examine whether finite size corrections in this region could be detected with the current technical capabilities.

We first ask about the optimal radius for experimental verification. In general, the larger the radius the more difficult it is to keep constant and stable the temperature. This constraint excludes cavities with radius $r_0 \gg 0.2 m$. On the other hand in cavities with $r_0 \ll 1 cm$ it is hard to control the geometrical shape. Furthermore, the flux of energy through the aperture is too small to be measured with the accuracy needed. The optimal radius can be estimated to be $r_0 \approx 0.5 - 7 cm$. We look at wavelengths $\lambda = c/\nu$ ($r_0 \approx 8 - 30 \lambda$) such that the semiclassical formalism is accurate, namely, $\frac{A}{r_0} \ll 1$. Finally the optimal temperature is such that for the range of wavelengths of interest, $\lambda \sim 0.5 - 2 mm$, the density of energy $u(\nu)$ is a maximum. These conditions are satisfied for $T \sim 2.5 - 6K$ which is accessible by cooling the cavity with Helium liquid. Thus we propose that the optimal experimental setting is a spherical blackbody with $r_0 \approx 0.5 - 7 cm, T \leq 5K$, and $\nu = c/\lambda \sim 1.5 - 6 \times 10^{11} Hz^{-1}$.

In addition in order to observe the effect of the periodic orbits ($l_{esc} \gg l_{min}$) the area of the aperture must verify $A_{ape} \leq \frac{\pi^2 r_0^2}{3}$.

In order to proceed we must choose an apparatus of measurement. For our purpose it is of paramount importance that the radiation of the blackbody at $T \sim 5K$ can be measured with the highest precision. We also request that the resulting flux of energy Eq. (5) through the aperture can be measured with a precision much larger than the strength of the predicted finite size corrections. Finally we require the apparatus to be precise enough to discern comparatively close wavelength. In order to observe fluctuations in $u(\nu)$ it is required that the apparatus can discern frequencies $\Delta(\lambda)$ comparable with the contribution to $u(\nu)$ of the shortest periodic orbit. In our case this corresponds to $\Delta(\lambda)/\lambda \leq 0.05$.

After a careful research, we conclude that instruments designed to measure the cosmic microwave background are the best suited for our purpose. We focus our analysis on FIRAS [13] apparatus on board of COBE satellite though other differential devices to measure the cosmic microwave background could be used as well. By differential it is meant that an internal calibrator nulls the external signal coming either from the sky or from the external calibrator. This mechanism is responsible for the FIRAS ability to detect small deviations from a blackbody source (ICAL in FIRAS). The FIRAS spectrometer is designed to measure deviations from a blackbody spectrum in the region $\lambda = 0.1 - 10 mm$. The intrinsic frequency resolution is $\Delta(\nu)/\nu \sim 5 \times 10^{-3}$. This implies that the contribution of periodic orbits (light rays) $l_{p} \gg l_{cut} = 2 \times 10^5 c/\nu$ will not be detected by the apparatus. In the range of frequencies of interest the equivalent noise power is $< 4 \times 10^{-15} W/\sqrt{Hz}$ is less than 1% of the measurement. The blackbody temperature can be set in the range $T = 2 - 10 K$ with a precision of a few $mK$.

In Fig.1 we plot $u(\nu)$ for a spherical cavity for a set of parameters ($r_0 = 2 cm, T = 5K$ and $l_{cut} = 2 \times 10^2 c/\nu$) accessible to experimental verification. From the figure it is clear that an apparatus with FIRAS specifications is capable to detect finite size corrections in spherical blackbody cavities. Similar results will be obtained for other symmetric cavities such as cylinders or cubes.

We note that a condition for the experimental observation of these effects is that the radiation is coupled out of the cavity in such a way the original modes of the cavity are not seriously affected. Previously we have shown that the aperture of the blackbody acts just as a natural cutoff for long periodic orbits. Therefore this is not a
problem for the experimental verification of our results. Indeed in recent experiments \cite{7} it has been possible to measure the natural modes of a cavity with a microwave source with a precision enough to even test quantitatively semiclassical estimation for the number of modes.

**APPLICATIONS: SONOLUMINESCENCE AND THE COSMIC MICROWAVE BACKGROUND.**

Differential apparatus as FIRAS compare the sky signal with some calibrator on board which is considered a ‘perfect’ blackbody. These calibrators are effective blackbody cavities which may be affected by the finite volume corrections reported in this letter. These corrections may very well be confused with those coming from the sky. Thus on board calibrators (or cavities used as reference sources in the measurement, for instance, of the spectral emissivity of solid samples \cite{11}) should be carefully designed to minimize these effects.

We speculate that finite size corrections may also play a role in cosmological problems. The CMB observed today had its origin in photons from the last scattering surface. For angular separations larger than 1.5 degree, temperature fluctuations in the CMB comes from regions that have never been in thermal contact. The origin of these fluctuations is supposed to date back to the inflation time. We speculate that finite size effects may play some role in the generation of these fluctuations. In order to determine the true relevance of this effect it would be necessary to: a) understand the physical mechanisms such as decoherence that may lead to a suppression of these effects, b) study these deviations in a non Euclidean cavity resembling the global geometry of the universe. It is encouraging that in Euclidean cavities it can be shown that the shape of the finite size corrections to the blackbody’s laws are not modified as universe expands.

Sonoluminescence, the transduction of sound into light, \cite{15, 16}, occurs when the pulsations of an almost spherical bubble of gas in water (or other substances \cite{14}) produced by a standing sound wave attains sufficient amplitude so as to emit periodic picosecond flashes of light. In a certain range of frequencies the spectrum is consistent with a blackbody \cite{14, 15} of typical size \( r_0 \sim 1 \mu m \) and temperatures \( T \sim 10^4 K \) (for bubbles of Xenon in sulphuric acid \( r_0 \sim 3.9 \mu m \) and \( T \sim 7000 K \)). In this range of parameters finite size effects should be observable. However we note in this case the blackbody is not the perfecting conducting cavity with an aperture studied previously but rather a material of refraction index \( n_1 \) immersed in a medium with refraction index \( n_2 \). A natural question to ask is whether the analytical results of the previous section are also applicable in this situation. The answer is affirmative. The only difference is the way in which the cutoff of long periodic orbits is defined. For \( n_1 > n_2 \) the blackbody is effectively an open cavity. A given periodic orbit (light rays) inside the material of refraction index \( n_1 \) will contribute to the spectral density provided that the angle of incidence \( \theta > \theta_c \) (measured from the normal) with \( \theta_c = \arcsin(n_2/n_1) \). Periodic orbits such that \( \theta < \theta_c \) do not contribute since the light ray escapes to the medium with \( n_2 \) \cite{17}.

In conclusion, by using semiclassical techniques, we have derived explicit expressions for the finite size corrections of the blackbody radiation’s laws as a function of the temperature, size and shape of the cavity and area of the aperture. We have also shown that the experimental detection of these finite size corrections is within the reach of current experimental capabilities and may be of relevance in sonoluminescence.

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