New method in medical tomography based on vibrating wire: bench-test experiment on laser beam

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Abstract. A new method for fast transverse beam profiling, where a vibrating wire is served as a resonant target, has been developed. The speed of scan up to a few hundred mm/s provides opportunity to make a set of beam profiles at different directions of the scan within a reasonable measurement time. This profile set allows us to reconstruct 2D beam profile by filtered back-projection algorithm. The new method may be applied for proton, X-ray, gamma, and neutron beams, and can also be of interest in tomography including medical applications. The method has been tested experimentally by means of laser beams.

1. Resonant target method

Wire scanners widely used for profile measurements of various types of beams, are based on the detection of secondary particles/radiation generated when the beam particles interact with the wire. To pick out this beam signal from the high level background, we propose using vibrating wire as a target whose oscillation frequency serves as a reference to separate signal from noise in the measurements [1-3]. The principle of operation of the proposed method is described in Fig. 1.

Figure 1.1. The main principle of resonant target method is based on the measurements of scattered/reflected particles/photons in opposite positions of the wire during oscillation process. In
figure: (a) the nonuniform beam flux is represented by vertical lines with different thicknesses (1), vibrating wire is stretch perpendicular to the figure plane and wire center is moving along segment (2), (3) - scattered/reflected particles/photons are detected by detector (4) with synchronism with wire oscillation frequency: (b) plane of unmovable vibrating wire, (1), (3) - positions of the wire at "left" measurements, (2), (4) - at "right" measurements, beam is presented by ellipse with nonuniform distribution; (c) plane of movable vibrating wire, (1), (3), (5), (7) - positions of the wire at "left" measurements, (2), (4), (6), (8) - at "right" measurements. Each step of wire feed at one period is less than oscillation amplitude. All schematics are not to scale.

Serial subtraction of measurements [1] of the photons reflected from the opposite positions of the vibrating wire oscillations (mentioned in Fig. 1 as "left" and "right") eliminates a high level of background noise falling on the photodiode during the photon measurements and also minimizes the noise in the measurement circuits.

In Fig. 1.2 we present the main results of laser beam 1D profiling [3] made by this method for the case where besides the main measurement object (i.e., laser beam), a photodiode also measures a 50 Hz lighting and considerable reflection of the laser beam from the holder of vibrating wire. The proposed measurement method allows reconstructing the laser beam profile by acquiring profile gradients. Speed of scan was about 12 mm/s, the scan of about 3 mm wide laser beam took about 250 ms.

Figure 1.2. Laser beam 1D profiling made by RT method separate the photon reflections on the vibrating wire from the reflections on holder of the monitor and 50 Hz background: 1 - ADC measurements by fast photodiodes in synchronism with wire oscillations, the left peak is originated by the reflections from the wire while the right is reflections from the monitor holder; 2 - differential signal of wire half-period measurements, 3 - reconstructed profile provide only laser beam. All scan from -2 mm to 6 mm lasted about 600 ms.

2. Tomography mathematics
The aim of tomography is the reconstruction of distribution of a 2D object from a set of its 1D projections (line integrals along a finite number of lines of known directions). From many of mathematical methods of such a reconstruction we chose the filtered back projection method described in details in [4-6] and proposed in [2] for recovering of complex proton beam profile by mean of vibrating wire monitors. In the case the vibrating wire accumulate the information of particles penetrating the wire either by measuring of natural frequency shift caused by temperature increase or
measuring of scattered/reflected particles in known wire position. Such measurements give the integral of particles distribution along the wire (one point in 1D profile at given scan with fixed direction of the wire). Cause by thermal type of interaction of the beam with vibrating wire (wire heating at beam impact) the each 1D scan requires tens of seconds. Usage of vibrating wire as a resonant target allows substantially speed up the fulfillment of the 1D scan and solve the tomography problem at reasonable time.

We assume that the beam intersects the investigated object in so-called reconstruction region and work in following geometry: the wire direction remains the same (assumed horizontal) while the investigated object is rotating. In discrete space we define as reconstruction region square \((N_{scr}, N_{scr})\) (numbering of indexes according to Visual Basic mathematics that we would like apply started from 0). By first index of arrays in this space we present the \(x\)-coordinate and by second index \(- y\)-coordinate. By Beam \((N_{scr}, N_{scr})\) we denote the array of initial beam profile at rotation angle \(\varphi = 0\).

Array of beam projections along the \(x\)-axis we denote as \(\text{Pro}_y(N_{\varphi}, N_{scr})\) (first index – angle \(\varphi\) of projection, second index – \(y\)-coordinate). The array of reconstructed beam profile we denoted as \(\text{RezBeam}(N_{scr}, N_{scr})\).

### 2.1. Reconstruction algorithm

When there is no a priori knowledge of the object, we always assume that the intensity of the object is uniform along the ray path. In other words, we distribute the projection intensity evenly among all pixels along the ray path. This process leads to the concept of backprojection.

The projection process and following trivial back projection scheme is presented in Fig. 2.1.

![Figure 2.1](image)

Figure 2.1. "Trivial" reconstruction of one projection deposition (backprojection). Given projection corresponds to \(\varphi = 0\), \(N_{scr} = 10\). Left - the model beam in discrete reconstruction region, middle column - the projection for angle \(\varphi = 0\), right - the uniformly spread of projection value along the corresponding horizontal rows.

Cause to chosen geometry the aim is to reconstruct the beam profile for \(\varphi = 0\). So the procedure of reconstruction is following: for given angle \(\varphi > 0\) we make back-projection then rotate it back on the angle \(- \varphi\) and accumulate into reconstruction array. For this purposes we introduce two auxiliary arrays \(\text{Rec}_\varphi(N_{scr}, N_{scr})\) and \(\text{Rec}_0(N_{scr}, N_{scr})\) - first for reconstructed back-projection for angle \(\varphi > 0\) and the second for back-projection array rotated back on the angle \(- \varphi\).

The whole algorithm for "trivial" back-projection can be presented as following...
Rec_φ(i_x, i_y) = Pro_y(i_φ, i_y)/N_{scr}, for i_x = 0...N_{scr}, i_y = 0...N_{scr}, \hspace{1cm} (1)
Rec_0(N_{scr}, N_{scr}) = Rotate(Rec_φ(N_{scr}, N_{scr}), -φ), \hspace{1cm} (2)
RezBeam(N_{scr}, N_{scr}) => RezBeam(N_{scr}, N_{scr}) + Rec_0(N_{scr}, N_{scr}), \hspace{1cm} (3)

where Rotate(Array(N_{scr}, N_{scr}), φ) procedure returns rotated array RotArray(N_{scr}, N_{scr}) of Array(N_{scr}, N_{scr}).

Algorithm is repeated for each projection angle φ = Δφ * i_φ, where Δφ = 2π/N_φ, i_φ = 0...N_φ. This algorithm however is not mathematically correct and leads to blurring of recovered profile. The double Fourier transformation provides a straightforward solution for tomographic reconstruction, but it presents some problems in actual implementation [4] (e.g. an error produced on a single sample in Fourier space affects the appearance of the entire image). Alternative implementation of the Fourier method is the so-called filtered backprojection (FBP) algorithm. In FBP reconstruction process, each projection is first convoluted with a specific and suitable filtering function [7]. Corresponding convolution function called a convolution kernel, or a filter, or a transfer function [8]. We follow to J.Alonso’s approach [9] based on the transfer function algorithm [10-13], where the specific cell in projection column is spread in reconstruction region not only to the corresponding row but also with some weighs in neighbor rows as presented in Fig. 2.2.

Figure 2.2. Back-projection with filtering on three rows from specific cell of projection column with weighs w_1, w_2, w_3. According to Cho [13] we choose the optimal set should be w_1 = -0.5232, w_2 = 0.1016, w_3 = -0.0531. Procedure will be repeated for all cells of projection. Presented projection corresponds to φ = 0 of a test beam of Fig. 2.1.

The whole filtered back-projection method can be rewritten in form:
Rec_φ(i_x, i_y) = Pro_y(i_φ, i_y)/N_{scr}, for i_x = 0...N_{scr}, i_y = 0...N_{scr}, \hspace{1cm} (4)
Rec_0(N_{scr}, N_{scr}) = Rotate(Rec_φ(N_{scr}, N_{scr}), -φ), \hspace{1cm} (5)
RezBeam(i_x, i_y) => RezBeam(i_x, i_y) + Rec_0(i_x, i_y), \hspace{1cm} (6)
RezBeam(i_x, i_y) => RezBeam(i_x, i_y) + w_1Rec_0(i_x, i_y ± 1), \hspace{1cm} (7)
RezBeam(i_x, i_y) => RezBeam(i_x, i_y) + w_2Rec_0(i_x, i_y ± 2), \hspace{1cm} (8)
RezBeam(i_x, i_y) => RezBeam(i_x, i_y) + w_zRec_0(i_x, i_y ± 3), \hspace{1cm} (9)

for i_x = 0...N_scr, i_y = 0...N_scr (indexes outside of the space are ignored). Same as for (1-3) the procedure should be repeated for each projection angle \( \varphi = \Delta \varphi \ast i_\varphi \), where \( i_\varphi = 0...N_\varphi \). One can see that the method is very similar to "trivial" backprojection except the equation (3) that transforms to set of equations (6-9).

2.2. Rotation algorithms

In filtered back-projection method, huge role plays rotation procedure (function Rotate in (5)). Generally rotation described by the usual rotation matrix

\[
\begin{pmatrix}
    x^* \\
    y^*
\end{pmatrix} =
\begin{pmatrix}
    \cos \varphi & \sin \varphi \\
    -\sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}, \hspace{1cm} (10)
\]

where \( x, y \) are the coordinates of pixel before rotation on angle \( \varphi \) and \( x^*, y^* \) are coordinates after rotation. However, in discrete rotation there is present so called aliasing problem – rotated pattern of pixel does not match to the discrete space. In some pixels of space do not set not a single rotated pixel, and in the some two. In [2] proposed in [9] algorithm for discrete object rotation was used. After rotation each pixel of rotating pattern was rotated according the pixel center backward on global rotation angle to match pixel direction to the main coordinate system of reconstruction region. Usually the center of pixel is not lie on the digital space grid. To generate the discrete pixel system each intersection of re-rotated pixel of rotated pattern with grid is spread to the corresponding pixel of reconstruction region. However this algorithm leads to smearing of the reconstructed view.

In this paper we use another algorithm that preserves each rotating pixel value – so called rotation by three shears. Mathematically it means that instead of rotation matrix (10) we use three shearing matrices (\( y -\text{share on } -\tan(\varphi/2) \), \( x -\text{share on } \sin \varphi \) and \( -y -\text{share on } -\tan(\varphi/2) \)) [7, 8]:

\[
\begin{pmatrix}
    x^* \\
    y^*
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 & \sin \varphi \\
    -\tan(\varphi/2) & 1 & 0
\end{pmatrix}
\begin{pmatrix}
    1 & 0 \\
    -\tan(\varphi/2) & 1
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}. \hspace{1cm} (11)
\]

The process of such rotation on 27 deg is illustrated in Fig. 2.3.

Figure 2.3. Rotation by three shears (to show algorithm we use Paint Image/Stretch/Skew instruments).

To allow vertical shears the discrete space in vertical direction should be more than \( N_{\text{scr}} \). If the rotation angles are limited by \( \pi/2 \) the size of vertical space (temporary value, only for rotation
procedure) can be chosen as $2N_{scr}$. This is a reason when operate with angles $> \pi/2$ transform them to rotation of mirrored images on angles $< \pi/2$.

One can see that the shears along the $y$–axis (first and the third matrices) contains $\tan(\theta/2)$ diverged at $\theta = \pi$. To avoid big values of $\tan(\varphi/2)$ should be better rotation angles limited in range $|\varphi| < \pi/2$. In formal way the projection procedure gives the same result for mirrored along the axis perpendicular to projection axis. This allows us when rotate to angles more than angles $>\pi/2$ used mirrored objects. Thus the rotation of object on the angle $\pi/2 < \varphi < \pi$ by mean of projection gives the same result as rotation of the same object mirrored along $x$-axis and rotated on angle $0 < \pi - \varphi < \pi/2$.

### 2.3. Numerical simulation

Based on the (4-9) a special numerical program was developed that operates either with model test beams or with experimental data. To allow use data of rotation set up to $2\pi$ the corresponding algorithm with mirror and rotation by shears is introduced. Special instruments to prepare test beam profiles are developed. An example of test beam 2D profile is presented in Fig. 2.4.

![Figure 2.4. Test beam profile made by program graphical instruments.](image)

The corresponding for this model set of projections is presented in Fig 2.5a (here horizontal axis present the rotation angle and vertical axis correspond to $y$-axis of reconstruction region). The whole pattern of reconstructed profile is presented in Fig. 2.5b.

![Figure 2.5. Projections (left) and reconstruction (right) of beam of a model presented in Fig. 2.4.](image)
3. Tomography experiment

The aim of the experiment is to reconstruct complicated 2D profile of laser beam. As a scanning mechanism we use VWM mounted on the shaft rotating with uniform angular velocity. The vibrating wire is directed perpendicular to the rotating plane. Radius of the wire trajectory is 91.5 mm, the rotation frequency is about 0.166 Hz, so the linear scanning velocity is about 95.8 mm/s (it is much more than speed about 12 mm/s used in the previous experiment described in Sec. 1. The only small part of the vibrating wire trajectory penetrates the laser beam, so the measurement process starts by signal from opto-interrupter which reacts on the hole of the disk conjugated with the rotating shaft.

Parameters of the laser used in the experiment are following: maximum output power < 200 mW, wavelength 532 nm. To make a nonuniform distribution we used a nozzle with wire along the diameter.

![Figure 3.1. Photograph of laser beam.](image)

The main view of laser beam cross section is presented in Fig. 3.1. The photograph of beam was made by digital CCD camera in manual mode operation after reflection of the beam from one surface of optical orange filter which absorbs all transient parts of laser radiation (by this procedure the flux of laser beam has been reduced so that we can adjust CCD camera range without saturation).

The layout of the experiment is presented in Fig. 3.2.
Figure 3.2. Vibrating wire 1 mounted on the rotating shaft with uniform angular speed axis 2, disk 3 with two contact pileups provide the electrical connection with autogeneration electronics 4, opto interrupter reacts on the hole 5 of the contact disk and generates the measurement cycle at the moment before the wire is submerged into the laser beam 6; laser 7 is mounted on the axis of stepper motor 8, which makes a definite number of steps after end of measurement cycle (electronic unit 9); fast photodiode 10 collects the laser photons reflected from the wire and measurements are done with ADC based electronic unit 11; measurements are done in short time in synchronism with wire oscillations (two measurements in oscillation period); the reflected photon measurement results, vibrating wire frequency and laser rotating stepper motor steps are transferred, visualized and stored in computer 12; the RS232 interface 13.

Photograph of the experimental layout is presented in Fig. 3.3.

Figure 3.3. Layout of experiment: 1- vibrating wire monitor, 2 - laser mounted on the stepper motor 3 rotating shaft, 4 - DC motor with fixed angular speed rotates the VWM, 5 - contact disk with two pileups 6, 7 - opto interrupter, 8 - fast photodiode with front-end electronics.
During the one tomography experiment we made 200 projections in $0 \sim \pi$ range. Each scan needed 120 ms. Some of the projection data are plotted in Fig. 3.4.

![Figure 3.4](image.png)

Figure 3.4. Plots of some projection data. From the original photodiode signals (ranged in counts 0-4096), we calculated the differential signals of each pair of the measurement points).

The complete set of projections is presented in Fig. 3.5.

![Figure 3.5](image.png)

Figure 3.5. The set of projections obtained from the experiment in the same format as in Fig. 2.5a.
The 1D profiles for each rotation angle are obtained by the integration of the corresponding differential signal over the y-coordinate. The final reconstruction of 2D profile of laser beam was made by equations (4-9) and is presented in Fig. 3.6.

![Figure 3.6. Reconstruction of the laser beam by experimental data presented in Fig. 3.5. The achieved pattern approximately coincides with the direct photograph of the laser beam.](image)

4. Conclusion
The main advantage of the proposed method is that it is applicable for beams of different origins - charged particles (electrons, protons, and ions), neutrons and photons in wide range of energies. Compared to the direct method, which is based on the measurements of wire’s temperature increase (or corresponding frequency shifts), we achieved much faster operation speed with the resonant target method, leading to a rapid decrease in scan time. This gives an opportunity to apply the method in different types of tomography including the medical tomography.

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