Signatures of anisotropic sources in the squeezed-limit bispectrum of the cosmic microwave background

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Abstract. The bispectrum of primordial curvature perturbations in the squeezed configuration, in which one wavenumber, $k_3$, is much smaller than the other two, $k_3 \ll k_1 \approx k_2$, plays a special role in constraining the physics of inflation. In this paper we study a new phenomenological signature in the squeezed-limit bispectrum: namely, the amplitude of the squeezed-limit bispectrum depends on an angle between $k_1$ and $k_3$ such that $B_\zeta(k_1, k_2, k_3) \rightarrow 2 \sum L c_L P_L(k_1 \cdot k_3) P_\zeta(k_1) P_\zeta(k_3)$, where $P_L$ are the Legendre polynomials. While $c_0$ is related to the usual local-form $f_{NL}$ parameter as $c_0 = 6f_{NL}/5$, the higher-multipole coefficients, $c_1$, $c_2$, etc., have not been constrained by the data. Primordial curvature perturbations sourced...
by large-scale magnetic fields generate non-vanishing $c_0$, $c_1$, and $c_2$. Inflation models whose action contains a term like $I(\phi)^2F^2$ generate $c_2 = c_0/2$. A recently proposed “solid inflation” model generates $c_2 \gg c_0$. A cosmic-variance-limited experiment measuring temperature anisotropy of the cosmic microwave background up to $\ell_{\text{max}} = 2000$ is able to measure these coefficients down to $\delta c_0 = 4.4$, $\delta c_1 = 61$, and $\delta c_2 = 13$ (68% CL). We also find that $c_0$ and $c_1$, and $c_0$ and $c_2$, are nearly uncorrelated. Measurements of these coefficients will open up a new window into the physics of inflation such as the existence of vector fields during inflation or non-trivial symmetry structure of inflaton fields. Finally, we show that the original form of the Suyama-Yamaguchi inequality does not apply to the case involving higher-spin fields, but a generalized form does.

**Keywords:** inflation, non-gaussianity, primordial magnetic fields, cosmological perturbation theory

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1 Introduction

Convincing detection of the so-called “local-form” three-point correlation function (bispectrum) of primordial curvature perturbations from inflation would rule out all single-field inflation models [1, 2], provided that an initial quantum state of the curvature perturbation is in a preferred state called the Bunch-Davies state [3–5] and that the curvature perturbation does not evolve outside the horizon due to a non-attractor solution [6, 7].

The curvature perturbation, ζ, is defined as a trace part of space-space components of the metric perturbation, \( \delta g_{ij} = a^2(t)e^{2\xi} \delta_{ij} \), in a uniform density gauge. The bispectrum of ζ is defined as

\[
\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = \frac{1}{(2\pi)^3} \delta^{(3)}(k_1 + k_2 + k_3) B_{\zeta}(k_1, k_2, k_3),
\]

and the local-form bispectrum is defined as

\[
B_{\zeta} \rightarrow \frac{5}{12} f_{NL} P_{\zeta}(k_1) P_{\zeta}(k_3),
\]

where \( P_{\zeta}(k) \propto k^{n_s-4} \) is the power spectrum of the primordial curvature perturbation with \( n_s = 0.96 \pm 0.01 \) [9–11]. This means that the local-form bispectrum has the largest amplitude in the so-called squeezed configuration, in which the smallest wavenumber, \( k_3 \), is much smaller than the other two, i.e., \( k_3 \ll k_1 \approx k_2 \) [12]. In this limit the local-form bispectrum is given by

\[
B_{\zeta} \rightarrow \frac{5}{12} f_{NL} P_{\zeta}(k_1) P_{\zeta}(k_3),
\]

and all attractor single-field inflation models with a Bunch-Davies initial state give \( f_{NL} = \frac{5}{12}(1 - n_s) \) [1, 13].

The current best limit on \( f_{NL} \) is \( f_{NL} = 37 \pm 20 \) (68% CL), which was obtained from the Wilkinson Microwave Anisotropy Probe (WMAP) 9-year data with the expected ISW-lensing bias removed [14]. The forthcoming Planck data are expected to reduce the error bar by a factor of four [8].

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1. Also see workshop summaries of “Critical Tests of Inflation Using Non-Gaussianity” in http://www.mpg-garching.mpg.de/~komatsu/meetings/ng2012/.
Figure 1. Absolute values of the shape function of $L = 0$, $(k_1 k_2 k_3)^2 S_0$ (top left panel), that of $L = 1$, $(k_1 k_2 k_3)^2 S_1$ (top right panel), and that of $L = 2$, $(k_1 k_2 k_3)^2 S_2$ (bottom panel). We restrict the plot range to $k_3 \leq k_2 \leq k_1$ and $|k_1 - k_2| \leq k_3 \leq k_1 + k_2$ for symmetry and the triangular condition. The shape of $L = 2$ peaks at the squeezed configuration, $k_3/k_1 \ll 1$ and $k_2/k_1 \approx 1$, in the same way as that of $L = 0$ whereas the shape of $L = 1$ is suppressed at the squeezed configuration. While the shape function of $L = 0$ has positive values for all $k_2/k_1$ and $k_3/k_1$, those of $L = 1$ and 2 have negative values except in the flattened configurations, $k_2/k_1 + k_3/k_1 \approx 1$.

If the Planck collaboration finds evidence for $f_{NL}$, or the lack thereof, what is next? Measuring the local-form four-point function (trispectrum) [15–17] to check the so-called Suyama-Yamaguchi inequality between the amplitude of the local-form trispectrum and $f_{NL}$, i.e., $\tau_{NL} \geq (6 f_{NL}/5)^2$ [18–25], would be an important next step to understand the nature of sources of non-Gaussianity (or the absence thereof). We shall discuss the Suyama-Yamaguchi inequality within the context of higher-spin fields in section 4.

Can we learn more about sources of non-Gaussianity by further scrutinizing the behavior of the bispectrum in the squeezed configuration? The answer is yes, and this is the main goal of this paper. Namely, in this paper, we shall investigate phenomenological consequences of the following new parametrization of the bispectrum of primordial curvature perturbations:

$$B_\zeta(k_1, k_2, k_3) = \sum_L c_L P_L(\hat{k}_1 \cdot \hat{k}_2) P_\zeta(k_1) P_\zeta(k_2) + \text{(2 perm)},$$

where $P_L(\mu)$ is the usual Legendre polynomials, i.e., $P_0(\mu) = 1$, $P_1(\mu) = \mu$, and $P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$. Here, $c_0$ is equal to $6 f_{NL}/5$.\footnote{Note that, due to symmetry, the $c_1$ term as well as any odd $L$ terms vanish in the exact squeezed limit,}
Why consider $c_L$ with $L \geq 1$? These coefficients appear to be sensitive to the existence of vector fields. For example, curvature perturbations sourced by primordial magnetic fields produce non-zero $c_1$ and $c_2$ [26, 27]. Curvature perturbations sourced by a $\frac{1}{2} I(\phi)^2 F^2$ term in Lagrangian produce $c_2 = c_0/2$ [28, 29]. These coefficients are also sensitive to the existence of a non-trivial realization of SO(3) rotational symmetry during inflation: a recently proposed “solid inflation” model produces $c_2 \gg c_0$ [30]. While the second-order effects in General Relativity also induce non-trivial angular dependence in the bispectrum, it disappears in the squeezed limit [31], and thus will not be considered in this paper.\(^3\)

While we assume that the coefficients, $c_L$, do not depend on wavenumbers, it is entirely possible that they do. There are various ways in which $c_0 = 6 f_{\text{NL}}/5$ depends on wavenumbers [3, 4, 35–40]. Particularly interesting possibilities are strongly infrared-divergent $c_1$ and $c_2$, which can naturally give rise to dipolar (i.e., hemispherical) and quadrupolar modulations, respectively, of the observed power spectrum in our sky [41].

In section 2, we shall briefly review these three scenarios to motivate our choice of parametrization given in eq. (1.1). Specifically, we review non-Gaussianities generated from:

1. large-scale magnetic fields after inflation in section 2.1;
2. a vector field coupled to the inflaton field, $\phi$, through a dilaton-like coupling $I(\phi)F^2$ in section 2.2; and
3. solid inflation, which goes as $\cos(2\phi)$ in the flat-sky approximation where $\cos \phi \equiv \ell_1 \cdot \ell_3$ [33, 34]. We ignore this secondary effect in this paper.

\(^3\)A correlation between the integrated Sachs-Wolfe effect and the gravitational lensing of CMB produces an angle-dependent squeezed-limit bispectrum of the CMB temperature anisotropy [32], which goes as $\cos(2 \phi)$ in the flat-sky approximation where $\cos \phi \equiv \ell_1 \cdot \ell_3$ [33, 34]. We ignore this secondary effect in this paper.
The bispectrum for $L = 1$ peaks when $k_2 + k_3 = k_1$. This signature may also be seen in the “flattened bispectrum template” defined by eq. (5.1) of ref. [42]. Nevertheless, the correlation coefficient between these bispectra is 0.196; these are only weakly correlated because the flattened template does not change sign while $S_1$ is positive in the flattened configurations and is negative otherwise.

The rest of this paper is organized as follows. In section 3, we derive both the full-sky and flat-sky formulae of the bispectrum of temperature anisotropies of the cosmic microwave background (CMB) induced by the angle-dependent bispectrum given in eq. (1.1), and analyze their behaviors. We also estimate the error bars of $c_L$ for $L = 0$, 1, and 2, expected for a cosmic-variance-limited CMB experiment measuring temperature anisotropy up to $\ell_{\text{max}} = 2000$. In section 4, we revisit the Suyama-Yamaguchi inequality within the context of higher-spin fields such as those discussed in this paper. We conclude in section 5. In appendix A, we discuss the precision of the flat-sky approximation. In appendix B, we derive the CMB bispectrum in the Sachs-Wolfe limit (in which the temperature anisotropy is given by $\delta T/T = -\zeta/5$). In appendix C, we present the full Fisher matrix for $c_0$, $c_1$, and $c_2$.

Throughout this paper, we adopt the following convention for Fourier transformation of an arbitrary function, $f(x)$: $f(x) = \int \frac{d^3k}{(2\pi)^3} f(k) e^{i k \cdot x}$.

2 Theoretical motivation

2.1 Helical and non-helical magnetic fields

Astrophysical observations suggest the existence of magnetic fields on the order of $10^{-6}$ G in galaxies and cluster of galaxies [43–46]. There is also indirect evidence for the existence of magnetic fields on the order of $10^{-20} - 10^{-14}$ G in the inter-galactic medium (IGM) [47–50].

There is yet no compelling model for how these vector fields can be generated during inflation, as the existing models suffer from strong backreaction or strong coupling problems [28, 51–55] (see the more detailed discussion in the next subsection). Here we simply assume that a magnetic field has been generated, and study its impact on the primordial perturbations during the radiation era and recombination. In the next subsection, we discuss the additional signatures that take place if vector fields are coupled to the inflaton field.

A large amount of literature exist in the studies of effects of vector fields on CMB anisotropies and the large-scale structure of the universe. See, e.g., refs. [58–71] for effects on the two-point correlation functions, and refs. [26, 27, 72–78] for those on higher-order correlation functions.

Let us assume that super-horizon vector perturbations were produced during inflation, and they generated large-scale magnetic fields. The anisotropic stress of this magnetic field sources the growth of curvature perturbation via Einstein’s field equations during the radiation era. However, after the decoupling of neutrinos at a few MeV, the magnetic anisotropic

\footnote{We thank Christian Byrnes for suggesting to compute this correlation.}

\footnote{See e.g., refs. [56, 57] for attempts to avoid such problems.}

\footnote{While we use the term “magnetic field” and “electric field” here and in the next subsection, these fields are not necessarily the usual electromagnetic fields. For the discussion in this subsection, it is sufficient to have some vector field whose anisotropic stress decays as $T_i^j - \frac{1}{3} \delta_i^j T_k^k \propto a^{-4}$ on super-horizon scales. On the other hand, the anisotropic stress on super-horizon scales is constant during inflation (disregarding slow-roll corrections) for the case we discuss in the next subsection. The important feature of these models is that the anisotropic stress scales with $a$ in the same way as the isotropic pressure dominating the universe: for the former case, it scales in the same way the radiation pressure does, and for the latter it scales in the same way the inflaton pressure does.}
stress is compensated by the neutrino anisotropic stress, and the curvature perturbation on super-horizon scales becomes a constant. This constant curvature perturbation survives till the recombination epoch, and yields additional CMB anisotropies. The solution of curvature perturbations on super-horizon scales is determined by the traceless projection of the magnetic anisotropic stress as $\tau_{\gamma,0}$.

$$\zeta_k \approx 0.9 \ln \left( \frac{\tau_{\nu}}{\tau_B} \right) \left[ \hat{k} \cdot \dot{k}_i - \frac{1}{3} \delta_{ii} \right] \frac{1}{4\pi \rho_{\gamma,0}} \int \frac{d^3k'}{(2\pi)^3} B^i(k') B_j(k - k'),$$  \hspace{1cm} (2.1)

where $\tau_B$ and $\tau_{\nu}$ denote the conformal time of the generation of magnetic fields and that of the decoupling of neutrinos, respectively, and $\rho_{\gamma,0}$ is the present-day value of the photon energy density. This equation shows that, even under the assumption that the magnetic field itself is a Gaussian variable, the curvature perturbations become highly non-Gaussian.

Angular dependence of the power spectrum and bispectrum arises due to the spin-1 nature of magnetic fields. The magnetic field vector is transverse, $k^i B_i = 0$, and thus it is expanded using the spin-1 polarization vector, $\epsilon^{(\sigma)}_j(k)$: $B_i(x) = (2\pi)^{-3} \int d^3k B_i(k) e^{ik \cdot x} = (2\pi)^{-3} \int d^3k \sum_{\sigma = \pm 1} B^\sigma(k) \epsilon^{(\sigma)}_i(k) e^{ik \cdot x}$, where $\sigma$ denotes two circular polarization states. Then, the power spectrum of magnetic fields can be decomposed into the “non-helical,” $P_B(k)$, and “helical,” $P_B(k)$, components as $[79]$

$$\langle B_i(k) B_j(k') \rangle = \frac{(2\pi)^3}{2} \delta^{(3)}(k + k') \sum_{\sigma = \pm 1} [P_B(k) - \sigma P_B(k)] \epsilon^{(\sigma)}_i(k) \epsilon^{(-\sigma)}_j(k')$$

$$= \frac{(2\pi)^3}{2} \delta^{(3)}(k + k') \left[ \delta_{ij} - \hat{k}_i \hat{k}_j \right] P_B(k) + i \epsilon_{ijl} \hat{k}_l P_B(k),$$  \hspace{1cm} (2.2)

where $\epsilon_{ijl}$ is the antisymmetric tensor normalized as $\epsilon_{123} = 1$. These power spectra are defined as

$$- \langle B^+(k) B^+(k') \rangle - \langle B^-(k) B^-(k') \rangle = (2\pi)^3 P_B(k) \delta^{(3)}(k + k'),$$  \hspace{1cm} (2.3)

$$\langle B^+(k) B^+(k') \rangle - \langle B^-(k) B^-(k') \rangle = (2\pi)^3 P_B(k) \delta^{(3)}(k + k').$$  \hspace{1cm} (2.4)

Note that the overall sign of the definition of these spectra depends on the choice of the polarization vector.

Using these power spectra, one can write the angle dependence of the bispectrum of curvature perturbations as $[27]$

$$B_\zeta(k_1, k_2, k_3) \propto P_B(k_\ast) P_B(k_1) P_B(k_2) \left( \frac{1}{3} \mu_{12}^2 + \mu_{23}^2 + \mu_{31}^2 - \frac{2}{3} - \mu_{12} \mu_{23} \mu_{31} \right)$$

$$- P_B(k_\ast) P_B(k_1) P_B(k_2) \left( \mu_{23} \mu_{31} - \frac{1}{3} \mu_{12} \right)$$

$$+ (1 \rightarrow 3, \ 2 \rightarrow 1, \ 3 \rightarrow 2) + (1 \rightarrow 2, \ 2 \rightarrow 3, \ 3 \rightarrow 1),$$  \hspace{1cm} (2.5)

where $\mu_{ab} = \hat{k}_a \cdot \hat{k}_b$ and $k_\ast$ denotes some pivot wavenumber.$^7$

eq. (2.5) clearly shows that helical and non-helical magnetic fields generate $L = 1$ and $L = 2$ angular dependence in the bispectrum of curvature perturbations. If the magnetic

$^7$Note that parity-odd terms, which are proportional to $P_B^2 P_B$ and $P_B^2$, do not appear in eq. (2.5), as $\zeta$ is a scalar. On the other hand, the bispectrum involving vector or tensor perturbations may contain parity-odd terms, which yield the CMB temperature auto-bispectrum with $\ell_1 + \ell_2 + \ell_3 = \text{odd}$ $[27, 80]$. 

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field was generated at a GUT scale, i.e., $\tau_\nu/\tau_B \approx 10^{17}$, with nearly scale-invariant spectra of $P_B$ and $P_\nu$, the Legendre coefficients in eq. (1.1) are related to the amplitudes of non-helical and helical magnetic fields smoothed on 1 Mpc as

$$c_0 \approx -2 \times 10^{-4} \left( \frac{B_{1\text{Mpc}}}{\text{nG}} \right)^6, \quad c_1 \approx -0.9 \left( \frac{B_{1\text{Mpc}}}{\text{nG}} \right)^2 \left( \frac{B_{1\text{Mpc}}}{\text{nG}} \right)^4, \quad c_2 \approx 14c_0.$$ (2.6)

Therefore, if inflation creates $B_{1\text{Mpc}} \sim 3\text{nG}$ and $B_{1\text{Mpc}} \sim 1\text{nG}$, which are consistent with the current observational limits, we may have negative and non-vanishing $c_1$ and $c_2$; namely, $c_1 \sim -8$ and $c_2 \sim -2$.

### 2.2 $I^2(\phi)F^2$ model

Vector fields with the standard Maxwell $-\frac{F^2}{4}$ kinetic term are not produced by the expansion of the universe and, if generated by some other source, they are rapidly diluted away. This poses a challenge to models of primordial magnetogenesis and of vector fields during inflation. Vector fields during inflation can result in broken statistical isotropy of the primordial perturbations, which will be probed by the forthcoming Planck data [81, 82].

Vector fields with a kinetic term given by

$$L = -\frac{I^2(\phi)}{4} F^2,$$ (2.7)

can instead be produced during inflation if $I(t)$ has an appropriate time dependence [83]. It is convenient to define the “electric” and “magnetic” components

$$E_i = -\frac{I}{a^2} A_i', \quad B_i = \frac{I}{a^2} \epsilon_{ijk} \partial_j A_k,$$ (2.8)

where the primes denote derivatives with respect to the conformal time, and $a$ is the scale factor of the universe. In terms of these components, the physical energy density in the vector field assumes the conventional expression, $\rho_A = |E|^2 + |B|^2$.

If $I \propto a^n$, with $n = +2$ or $n = -3$, the magnetic modes are generated during inflation with a scale invariant and frozen spectrum outside the horizon. Rather than assuming that $I$ is an external function, one can obtain the required time dependence by assuming that $I$ is a function of the inflaton $\phi$, with a functional form related to the inflaton potential by [83, 84]

$$I = I_0 \exp \left[ -\int \frac{n d\phi}{\sqrt{2\epsilon(\phi)M_p}} \right] \Rightarrow \langle I \rangle \propto a^n,$$ (2.9)

where $\epsilon$ is the usual slow-roll parameter, $\epsilon \equiv \frac{M_p^2}{2} \left( \frac{1}{V} \frac{dV}{d\phi} \right)^2$, with $M_p \equiv 1/\sqrt{8\pi G}$ denoting the reduced Planck mass.

Some recent work studied whether this coupling can result in visible cross-correlations between primordial perturbations and large-scale magnetic fields [69–71, 85–88]. This is not trivial to realize, as the $n = -3$ choice results in too large an energy density in the electric modes [51, 52], while $n = +2$ leads to too large an electromagnetic coupling constant during inflation [28, 52].

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8These problems persist also for a general evolution of $I$ beyond the $a^n$ scaling [55].
Indeed the field measured by local observers. This is customary used, for instance, in the Affleck-Dine context that this variance contributes to the theoretical expectation value of the scalar addition incoherently to the inflaton vacuum modes, and are highly non-Gaussian.

Ref. [89] computed the resulting bispectrum, $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$, in the equilateral configuration, i.e., $k_1 = k_2 = k_3$. The full bispectrum was computed in refs. [28, 29]. The computations of refs. [28, 29] are restricted to $n = 2$ and $n = -2$ which produce, respectively, scale invariant “magnetic” and “electric” perturbations. The model enjoys a symmetry $f \leftrightarrow \frac{1}{f}$, or $n \leftrightarrow -n$, under which $|E| \leftrightarrow |B|$. Both $\pm n$ result in the same equation for $\zeta$. For brevity of exposition, we only refer to the $n = -2$ case in the reminder of this subsection. In this case one obtains, at the leading order in slow-roll,

$$E_k \simeq \frac{3H^2}{\sqrt{2}k^{3/2}} , \quad B_k \simeq \frac{H}{\sqrt{2}k^{1/2}a} , \quad k \ll aH \quad (2.10)$$

for the mode functions of each of the two polarizations of the “electric” and the “magnetic” fields in the super-horizon regime [29]. We note that the power in the “electric” field is frozen outside the horizon and scale invariant, whereas the power in the “magnetic” field decreases to negligible values.

Let us assume that, at the beginning of inflation, say at the time $t = 0$, the “electric” field has a classical homogeneous value, $E_0^0$, all across the universe with negligible perturbations. For $n = -2$, the classical equations of motion for the vector field are solved by a constant, $E = E^0$. This quantity is, however, not the classical “electric” field that would be measured by a local observer at $t > 0$, which we denote by $E_{cl}$. In fact, the modes given by eq. (2.10) become classical after they leave the horizon (we denote them as infra-red (IR) modes), and they add up with $E^0$ to give $E_{cl}$. A given IR mode of wavelength $\lambda$ averages to zero on regions of size $L \gg \lambda$, but it is constant in each region of the size $L \ll \lambda$, and it adds up stochastically with $E^0$ and with all the other modes with $\lambda \gg L$ generated during inflation to determine the value $E_{cl}$ in that region. An observer at time $t > 0$ during inflation can only experience the value of $E_{cl}$ in its local Hubble patch. The average measured by this observer is drawn from a Gaussian distribution with the mean $E^0$ and the variance given by

$$\langle \mathbf{E} \cdot \mathbf{E} \rangle = \frac{2 \times 4\pi}{(2\pi)^3} \int_{H_{a(t=0)}}^{H_{a(t)}} \frac{dk}{k} k^3 E_k^2 = \frac{9H^4}{2\pi^2} N, \quad (2.11)$$

where $N$ is the number of e-folds from the start of inflation to the time $t$. The lower limit in the integral corresponds to the modes that left the horizon at the start of inflation (larger modes are not generated), while the upper limit corresponds to the modes that left the horizon at the time $t$ (larger-momentum modes are still in the quantum regime and do not contribute to the classical average, $E_{cl}$, in the Hubble patch of length $\frac{1}{aH}$).

The situation is completely identical to what happens to the so-called “stochastic inflation [90],” in which the variance of a massless scalar field, $\chi$, is determined by the stochastic addition of the IR modes, and grows as $\langle \chi^2 \rangle \propto H^2 N$ during inflation. It is well established in that context that this variance contributes to the theoretical expectation value of the scalar field measure by local observers. This is customary used, for instance, in the Affleck-Dine

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\[9\] We continue to adopt the “electromagnetic” decomposition (2.8) for notational convenience.
model of baryogenesis [91] or in the curvaton field [92]. The fact that the vector field has spin 1 does not make any difference for these considerations, which simply follow from eq. (2.10).

Let us consider a mode, \( \zeta_k \), of a given comoving momentum, \( k \). This mode leaves the horizon \( N_k \) e-folds before the end of inflation. We are interested in the modes that affect the CMB anisotropies. Such modes leave the horizon \( N_k \simeq N_{\text{CMB}} \simeq 60 \) e-folds before the end of inflation. The Hubble patch that they exit is the one that eventually becomes our Hubble patch. When the mode \( \zeta_k \) leaves the horizon, the classical average of the “electric” field, \( E_{\text{cl}} \), in this Hubble patch is drawn from a Gaussian distribution with the mean \( E^0 \) and the variance given by eq. (2.11), with \( N = N_{\text{tot}} - N_k \), where \( N_{\text{tot}} \) is the total number of e-folds of inflation.

In the presence of this mean, the kinetic term given in eq. (2.7) results in the coupling of \( \mathcal{L}_{\text{int}} \simeq 4a^4 E_{\text{cl}} \cdot \delta E \zeta \). This is the dominant operator for the part of \( \zeta \) sourced by the vector field [29]. The power spectrum of \( \zeta \) generated by this mechanism is given by

\[
P_\zeta(k) = P_\zeta^{(0)}(k) \left[ 1 + g_*(k) \cos^2 \theta_{\hat{k},E_{\text{cl}}} \right], \quad g_*(k) \simeq -\frac{24 E_{\text{cl}}^2 N_k^2}{\epsilon V(\phi)}, \quad (2.12)
\]

where \( P_\zeta^{(0)} \equiv \frac{2 \pi^2}{k^3} \frac{H^2}{8 \pi^2 E M_p^2} \) is the square amplitude of the standard vacuum modes. The second term in eq. (2.12) is the contribution of the sourced part of \( \zeta \), which (as phenomenologically required) we have assumed to be subdominant.\(^{10}\) It follows from the \( N_k^2 \) proportionality that this term continues to grow in the super-horizon regime. This power spectrum was previously obtained in refs. [93–95] in the context of the anisotropic inflationary model [96], where however \( E_{\text{cl}} \) was identified with \( E^0 \), missing the IR contribution.

The \( \delta E \zeta \) mixing results in the following bispectrum in the squeezed limit, \( k_3 \ll k_1 \approx k_2 [29]\)

\[
B_\zeta(k_1, k_2, k_3) \simeq 24 P_\zeta^{(0)}(k_1) P_\zeta^{(0)}(k_3) |g_*(k_1)| N_{k_3}
\times \left[ 1 - \cos^2 \theta_{k_1, E_{\text{cl}}} - \cos^2 \theta_{k_3, E_{\text{cl}}} + \cos \theta_{k_1, E_{\text{cl}}}, \cos \theta_{k_3, E_{\text{cl}}} \cos \theta_{k_1, k_3} \right]. \quad (2.13)
\]

The predicted power spectrum (eq. 2.12) and bispectrum (eq. 2.13) break statistical isotropy, as \( E_{\text{cl}} \) picks out a preferred direction. However, the prediction for an isotropic measurement is obtained by averaging eq. (2.13) over all directions of \( E_{\text{cl}} \).\(^{12}\) We find

\[
B_\zeta(k_1, k_2, k_3) \bigg|_{\text{isotropic measurement}} \simeq 8 P_\zeta^{(0)}(k_1) P_\zeta^{(0)}(k_3) |g_*(k_1)| N_{k_3} \left( 1 + \mu_{13}^2 \right), \quad (2.14)
\]

where \( \mu_{13} \equiv \hat{k}_1 \cdot \hat{k}_3 \). From this we obtain the Legendre coefficients in eq. (1.1) as

\[
c_0 = 32 \frac{|g_*(k_1)| N_{k_3}}{0.1 \, 60}, \quad c_2 = \frac{c_0}{2}. \quad (2.15)
\]

Due to simplicity of the model, eq. (2.15) is a very predictive result, relating the bispectrum coefficients to the amount of statistical anisotropy of the power spectrum, i.e., \( g_* \).

\(^{10}\)Non-detection of statistical anisotropy in the WMAP 9-year data after the correction of non-circular beam effects [14] would imply a conservative upper bound of \( g_* \lesssim 0.1 \).

\(^{11}\)See ref. [29] for the full expression; due to different conventions, the bispectrum of ref. [29] is the one given here divided by \( (2\pi)^{3/2} \).

\(^{12}\)For studies of the bispectrum without averaging over preferred directions, see refs. [97, 98].
This result holds for all models characterized by eq. (2.7) and scale invariant “magnetic” or “electric” modes, including many analyses of the magnetogenesis mechanism [83] and of anisotropic inflation [96] (for which the departure from scale invariance is negligibly small; we note that in this work $E^0$ evolves on an attractor solution, but this has no consequence for the accumulation of the IR modes). An analogous result will also hold for the model of ref. [99] and for the mechanism of ref. [100], for which the scalar perturbations have been studied in ref. [101].

The smallness of $g_*$ limits the level of non-Gaussianity. However, a larger bispectrum, for a given value of $g_*$, can be obtained if the model is more complicated. For instance, one can arrange for a triplet of U(1) vectors, and assume that they have classical vacuum expectation values which are orthogonal to one another and of equal magnitudes [102]. In this case the power spectrum is statistically isotropic ($g_* = 0$). This requires to assume that the IR sum is subdominant, as there is no reason to assume that the IR modes of the three vectors add up to orthonormal values. A larger bispectrum can also be obtained if there are additional fields and additional couplings, as in the waterfall mechanism of ref. [103], in which a vector field of the kinetic term given in eq. (2.7) is also coupled to the field that determines the end of hybrid inflation (see ref. [29] for more detailed discussion).

2.3 Solid inflation

Ref. [30] studied a rather unusual model, in which inflation is driven by a system which has a field-theoretical description of a solid. An equivalent version of the model was proposed by ref. [104], under the name of “elastic inflation.”

Each volume element of the solid is characterized by a comoving label, $\phi^i$ (for instance, it can be the position of that element at the initial time $t = 0$). The functions, $\phi^i(t, x)$, specify which volume element is located at a given position, $x$, at a given time, $t$. A solid at rest in comoving coordinates then obeys

$$\langle \phi^1 \rangle = x, \quad \langle \phi^2 \rangle = y, \quad \langle \phi^3 \rangle = z \quad (2.16)$$

or, in short, $\langle \phi^i \rangle = x^i$. Even if the vacuum expectation value of each field is $x$-dependent, a homogeneous and isotropic Friedmann-Robertson-Walker solution can still be obtained by requiring that the Lagrangian that controls the solid be invariant under rigid translations, $\phi^i \rightarrow \phi^i + a^i$, and SO(3) rotations, $\phi^i \rightarrow O^i_j \phi^j$.

At the lowest order in a derivative expansion, the translational invariance is guaranteed by considering functions

$$B^{ij} = g^{\mu \nu} \partial_\mu \phi^i \partial_\nu \phi^j, \quad (2.17)$$

and isotropy is obtained by requiring that the Lagrangian is a function of SO(3) invariants built from $B^{ij}$. Only three independent invariants exist, and ref. [30] chose

$$S_{\text{solid}} = \int d^4x \sqrt{-g} F[X, Y, Z], \quad X \equiv [B], \quad Y \equiv \frac{[B^2]}{[B]^2}, \quad Z \equiv \frac{[B^3]}{[B]^3}, \quad (2.18)$$

where the square parenthesis denotes the trace of the corresponding matrix, e.g., $[B] \equiv \sum_i B^{ii}$.

The system has the energy-momentum tensor, $T^\mu_\mu = \text{diag} (-\rho, p, p, p)$, with [30]

$$\rho = -F, \quad p = F - \frac{2}{a^2} F_X, \quad (2.19)$$
where the subscript denotes a partial derivative, and $F$ and $F_X$ are evaluated on the background solutions given by $X = \frac{3}{\pi\mu}$, $Y = \frac{1}{3}$, and $Z = \frac{1}{3}$. These invariants are chosen in such a way that $X$ is the only one affected by the overall physical volume expansion; this immediately explains as to why only the derivative of $F$ with respect to $X$ enters into the expression for the pressure.

Inflation is possible only if $F$ is only mildly affected by the physical expansion, or, equivalently, only if $F_X$ is sufficiently small. Specifically, we require

$$\epsilon \equiv \frac{\dot{H}}{H^2} = \frac{X F_X}{F} \ll 1. \quad (2.20)$$

One also requires $F_{XX}$ to be small, so that $\eta \equiv \frac{\dot{\epsilon}}{\epsilon} \ll 1$ [30].

Let us now discuss cosmological perturbations in this system. It is convenient to work in a spatially flat gauge, where the dynamical scalar and vector perturbations are all encoded in the perturbations of the scalar fields:

$$\delta \phi^i = \pi^i(t, \mathbf{x}) = \frac{\partial_i}{\sqrt{-\nabla^2}} \pi_L + \pi_T^i, \quad (2.21)$$

where the vector components are transverse, $\partial_i \pi_T^i = 0$. The scalar and vector modes are decoupled from each other at the linearized level.\footnote{As always in cosmology, the sectors of scalar and vector perturbations also include non-dynamical modes which are, in the spatially flat gauge, encoded in the $\delta g_{ij}$ metric components. These fields can be integrated out as explained in ref. [30]. This affects the action for the dynamical modes, $\pi_L$ and $\pi_T^i$, at long wavelengths. Finally, there are also tensor perturbations - the gravity waves - encoded in the $\delta g_{ij}$ metric components, and which we do not discuss here.} At the lowest order in the slow roll parameters, and in the deep sub-horizon regime, the sound speeds of the scalar (longitudinal) modes, $c_L$, and vector (transverse) modes, $c_T$, are given, respectively, by [30]

$$c_L^2 \simeq \frac{1}{3} \left[ 1 + \frac{8 F_Y + F_Z}{9 X F_X} \right], \quad c_T^2 \simeq \frac{3}{4} \left( 1 + c_L^2 \right), \quad (2.22)$$

so that the propagation is subluminal and non-tachyonic ($0 < c_L^2 < 1$ and $0 < c_T^2 < 1$) for $0 < F_Y + F_Z < \frac{9}{8} \epsilon |F|$ [30]. Namely, the requirement of an accelerated expansion forces $F_X$ to be small, while subluminality also requires that the combination $F_Y + F_Z$ be small. Finally, the theory involves derivative interactions of the “phonon” fields, $\pi^i$, which necessarily become strong at some scale $\Lambda$. A detailed study in ref. [30] gives an estimate of $\Lambda \gg H$ (so that the linearized theory is also valid in the sub-horizon regime, up to $\Lambda$), provided that $\epsilon c_L^3 \gg \left( \frac{H}{\mu} \right)^{2/3}$, which can always be obtained for a sufficiently small $H$.

In a conformally flat gauge, the gauge-invariant scalar perturbation, $\zeta$, evaluates to $\zeta = -H \frac{\delta \phi}{\epsilon} = \frac{1}{3} \partial \pi$. Its solution exhibits two features that are peculiar in scalar-field inflation models, but that were nevertheless already seen in the models studied in the previous subsection [28]. The first one is the fact that $\zeta$ is not conserved on super-horizon scales, due to the anisotropic stress that does not vanish on super-horizon scales [30]. Indeed, following ref. [30], we obtain

$$\delta T_{ij, \text{scalar}} = a^2 M_p^2 \dot{H} \zeta \left[ 2 (3 - 2\epsilon + \eta) \delta_{ij} - (3 + 3 c_L^2 - 2 \epsilon + \eta) \left( 3 k_i k_j - \delta_{ij} \right) \right]. \quad (2.23)$$

Let us recall that, also for the model described by eq. (2.7), the anisotropic component of the stress-energy tensor, $\propto E_{cl,ij} \delta E_j$, does not vanish outside the horizon, and thus the anisotropic term in eq. (2.12) grows outside the horizon.
The second feature is that, analogously to the model described by eq. (2.7), the bispectrum of solid inflation is largest in the squeezed configurations, and exhibits a nontrivial dependence on the angle between the modes in the squeezed limit. The dominant contribution to the bispectrum is given by the interactions of $\pi$ encoded in the scalar field Lagrangian, while the metric perturbations provide a negligible contribution. The dominant interaction in a slow roll expansion is given by

$$L \supset M_p^2 a^3 H^2 \frac{F_Y}{F} \left[ \frac{7}{81} (\partial \pi)^2 - \frac{1}{9} \partial \pi \partial_j \pi^k \partial_k \pi^j - \frac{4}{9} \partial \pi \partial_j \pi^k \partial_j \pi^k + \frac{2}{3} \partial_j \pi^i \partial_j \pi^k \partial_k \pi^l \right].$$

(2.24)

Also in this respect, the situation is analogous to the model discussed in the previous subsection, where the dominant contribution to the bispectrum is obtained from eq. (2.7), disregarding metric perturbations. When written in terms of $\zeta$, this interaction exhibits nontrivial dependence on the direction of the modes which does not vanish in the squeezed limit, imprinting the nontrivial angular dependence in the bispectrum.

At the leading order, the Legendre coefficients in eq. (1.1) are given by

$$c_0 \simeq 0, \quad c_2 = \mathcal{O}(1) \frac{F_Y}{F} \frac{1}{\epsilon c_s^2}.$$  

(2.25)

Namely, in the squeezed limit, the dominant contribution is given by the quadrupole term, whereas the monopole term is negligible. The dominant quadrupole term is essentially proportional to a free combination of parameters (we recall that avoiding superluminality and strong coupling at $p \ll H$ imposes restrictions on the combination $F_Y + F_Z$ but not on $F_Y$ or $F_Z$ individually).

3 Signatures in the cosmic microwave background

In this section, we shall derive the flat-sky (section 3.1) and full-sky (section 3.2) formulae for the bispectrum of CMB temperature anisotropy from the bispectrum of curvature perturbations given in eq. (1.1). We then calculate, in section 3.3, the error bars of $c_0$, $c_1$, and $c_2$ expected for a cosmic-variance-limited experiment measuring temperature anisotropy up to $\ell = 2000$.

3.1 Flat-sky formula

While the full-sky formula is eventually needed for the analysis of full-sky temperature maps, let us derive first the flat-sky formula, as the flat-sky formula is usually simpler and more intuitively understandable.

Under the flat-sky approximation, which is valid only on sufficiently small angular scales, $\ell \gg 1$, CMB fluctuations on the sky are expanded using the two-dimensional Fourier transform, instead of the spherical harmonics. The Fourier coefficients of CMB anisotropy, $a(\ell)$ are related to the three-dimensional coefficients of the curvature perturbation, $\zeta(k)$, as

$$a(\ell) = \int_0^{\tau_0} d\tau \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} S_I \left( k = \sqrt{k_z^2 + (\ell/D)^2}, \tau \right) \frac{1}{D^2} e^{-ik_z D},$$

(3.1)

where $k \equiv (k_x, k_y)$, $D = \tau_0 - \tau$ is the conformal distance out to a given epoch $\tau$, $\tau_0$ is the present-day conformal time, and $S_I$ is the so-called source function.\footnote{\footnotesize The source function is related to the radiation transfer function defined in eq. (3.9) as $T_\ell(k) = \int_0^{\tau_0} d\tau \frac{S_I(k, \tau)}{j_\ell(k D)}$.} This relation simply
tells us that $a(\ell)$ measures the $\zeta$ modes that are perpendicular to the line-of-sight direction (i.e., the modes on the sky), and the line-of-sight modes are washed out by integration. It is straightforward to compute the bispectrum of $a(\ell)$ following, e.g., ref. [105]:

$$\langle a(\ell_1)a(\ell_2)a(\ell_3) \rangle = (2\pi)^2\delta^{(2)}(\ell_1+\ell_2+\ell_3) \sum_L c_L b_L(\ell_1,\ell_2,\ell_3),$$

where

$$b_L(\ell_1,\ell_2,\ell_3) = \int_{-\infty}^{\infty} r^2 dr \left[ \prod_{n=1}^{3} \int_0^{\infty} d\tau_n \int_{\ell_n/D_n}^{\infty} \frac{dk_n}{2\pi} G(\ell_n,k_n,\tau_n,r) \right] \times \left[ \sum_{n=0}^{L} (\hat{\ell}_1 \cdot \hat{\ell}_2)^n F_L^{(n)} \right] P_\zeta(k_1)P_\zeta(k_2) + (2\ \text{perm}),$$

with

$$G(\ell,k,\tau,r) \equiv \left[ 1 - \left( \frac{\ell}{kD} \right)^2 \right]^{-1/2} S_1(k,\tau) \frac{2}{D^2} \cos \left[ \sqrt{1 - \left( \frac{\ell}{kD} \right)^2} k(r-D) \right].$$

The other kernel functions, $F_L^{(n)}$, for $L \leq 2$ are given by $F_L^{(0)} = 1$, $F_L^{(1)} = \prod_{n=1}^{2} \frac{\ell_n}{\kappa_n D_n}$, $F_L^{(2)} = \frac{3}{2} \prod_{n=1}^{2} \left( \frac{\ell_n}{\kappa_n D_n} \right)^2$, and

$$F_L^{(0)} = -2 \prod_{n=1}^{2} \sqrt{1 - \left( \frac{\ell_n}{\kappa_n D_n} \right)^2} \tan \left[ \sqrt{1 - \left( \frac{\ell_n}{\kappa_n D_n} \right)^2} k_n(r-D_n) \right],$$

$$F_L^{(1)} = -2 \prod_{n=1}^{2} \sqrt{1 - \left( \frac{\ell_n}{\kappa_n D_n} \right)^2} \tan \left[ \sqrt{1 - \left( \frac{\ell_n}{\kappa_n D_n} \right)^2} k_n(r-D_n) \right],$$

$$F_L^{(2)} = -3 \prod_{n=1}^{2} \frac{\ell_n}{\kappa_n D_n} \sqrt{1 - \left( \frac{\ell_n}{\kappa_n D_n} \right)^2} \tan \left[ \sqrt{1 - \left( \frac{\ell_n}{\kappa_n D_n} \right)^2} k_n(r-D_n) \right].$$

While these formulae are still complicated, one can read off the leading behaviours of these expressions in the small-scale limit, in which the dominant contributions in the $k$ integration come from the modes with $k \approx \ell/D$. In this limit the kernel functions become $F_L^{(0)} \to 0$, $F_L^{(1)} \to 0$, $F_L^{(1)} \to 1$, $F_L^{(2)} \to -1/2$, and $F_L^{(2)} \to 3/2$. We thus find

$$b_L(\ell_1,\ell_2,\ell_3) \to \int_{-\infty}^{\infty} r^2 dr \left[ \prod_{n=1}^{3} \int_0^{\infty} d\tau_n \int_{\ell_n/D_n}^{\infty} \frac{dk_n}{2\pi} G(\ell_n,k_n,\tau_n,r) \right] \times P_L(\hat{\ell}_1 \cdot \hat{\ell}_2)P_\zeta(k_1)P_\zeta(k_2) + (2\ \text{perm}).$$

This result shows that the CMB bispectrum is proportional to $P_L(\hat{\ell}_1 \cdot \hat{\ell}_2)$ (and its permutations), which is expected from $P_L(k_1 \cdot k_2)$ (and its permutations) in the three-dimensional bispectrum of the primordial curvature perturbation.

While the approximation of $\ell \approx kD$ gives a simple and transparent result given in eq. (3.8), it is less precise than the original form given by eq. (3.3). In appendix A, we discuss the precision of eqs. (3.3) and (3.8) with respect to the full-sky result given in the next subsection.
3.2 Full-sky formula

Encouraged by the flat-sky results, we now move onto the full-sky case. The CMB temperature anisotropy on the celestial sphere is expanded by the spherical harmonic function as \( \delta T(\hat{n})/T = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\hat{n}) \), where \( \hat{n} \) is a three-dimensional unit vector pointing toward a given direction on the sky. The spherical harmonics coefficients, \( a_{\ell m} \), are related to the primordial curvature perturbation as

\[
a_{\ell m} = 4\pi (-i)^\ell \int \frac{d^3k}{(2\pi)^3} T_\ell(k) \zeta_{\ell m}(\hat{k})
= 4\pi (-i)^\ell \int \frac{k^2 dk}{(2\pi)^3} T_\ell(k) \zeta_{\ell m}(k),
\]

where \( T_\ell(k) \) is the so-called radiation transfer function, and we have defined the curvature perturbation expanded in spherical harmonics as \( \zeta_{\ell m}(k) \equiv \int d^2\hat{k} \, \zeta^*_{\ell m}(\hat{k}) \). Then, the bispectrum of \( a_{\ell m} \) can be straightforwardly calculated as

\[
\left\langle \prod_{n=1}^{3} a_{\ell_n m_n} \right\rangle = \left[ \prod_{n=1}^{3} 4\pi (-i)^{\ell_n} \int \frac{k_n^2 dk_n}{(2\pi)^3} T_{\ell_n}(k_n) \right] \left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle,
\]

with the bispectrum of \( \zeta_{\ell m} \) related to \( B_\zeta(k_1, k_2, k_3) \) as

\[
\left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle = \left[ \prod_{n=1}^{3} \int d^2k_n \zeta^*_{\ell_n m_n}(\hat{k}_n) \right] (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) B_\zeta(k_1, k_2, k_3).
\]

Using the bispectrum of \( \zeta \) given in eq. (1.1), \( B_\zeta(k_1, k_2, k_3) \) can be expanded as

\[
B_\zeta(k_1, k_2, k_3) = P_\zeta(k_1) P_\zeta(k_2) \sum_L c_L \frac{4\pi}{2L + 1} \sum_M Y^*_{LM}(\hat{k}_1) Y_{LM}(\hat{k}_2) + (2 \text{ perm}).
\]

Using the definition of the delta function, \( \delta^{(3)}(k) = (2\pi)^{-3} \int d^3x \, e^{ik \cdot x} \), we also expand the delta function as

\[
\delta^{(3)}(k_1 + k_2 + k_3) = 8 \int_0^\infty r^2 dr \left[ \prod_{n=1}^{3} \sum_{L_n M_n} (-1)^{L_n} j_{L_n}(k_n r) Y^*_{L_n M_n}(\hat{k}_n) \right] \times \left( \begin{array}{ccc} L_1 & L_2 & L_3 \\ M_1 & M_2 & M_3 \end{array} \right) I_{L_1 L_2 L_3},
\]

where the \( 2 \times 3 \) matrix denotes the Wigner-3j symbol, and the \( I \) symbol is defined by

\[
I_{l_1 l_2 l_3} \equiv \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right).
\]

Now, performing the integrals of the spherical harmonics over \( \hat{k}_1, \hat{k}_2, \) and \( \hat{k}_3 \), and performing the summations over \( M_1, M_2, M_3, \) and \( M \) as described in ref. [106], we obtain

\[
\left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle = (2\pi)^3 B_{\zeta, \ell_1 \ell_2 \ell_3}(k_1, k_2, k_3) \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right),
\]

\[\text{– 13 –}\]
where

\[
B_{\ell_1\ell_2\ell_3}(k_1, k_2, k_3) = 8 \int_0^{\infty} r^2 dr \left[ \prod_{n=1}^{3} \sum_{L_n} (-1)^{L_n} \frac{L_n}{2} j_{L_n}(k_n r) \right] I_{L_1 L_2 L_3} \times \sum_{L} c_L \frac{4\pi}{2L + 1} I_{\ell_1 L_1 L}I_{\ell_2 L_2 L}(-1)^{\ell_2+L_1}\delta_{L_3 \ell_3} \times \left\{ \begin{array}{c} \ell_1 \\ L_2 \\ L_1 \end{array} \right\} P_{\zeta}(k_1)P_{\zeta}(k_2) + (2 \text{ perm}) .
\]

(3.16)

Here, the \(2 \times 3\) matrix enclosed by curly brackets denotes the Wigner-6 symbol. As the primordial bispectrum given by eq. (1.1) is rotationally invariant, the bispectrum expanded in spherical harmonics must also be rotationally invariant. This means that the dependence of the bispectrum on \(m_1, m_2\) and \(m_3\) must be given by the Wigner-3 symbol, as shown in eq. (3.15). This property ensures rotational invariance of the CMB bispectrum.

Substituting eqs. (3.15) and (3.16) into eq. (3.10), we finally obtain the full-sky formula for the CMB bispectrum:

\[
\left\langle \prod_{n=1}^{3} a_{\ell_n m_n} \right\rangle = \left( \begin{array}{c} \ell_1 \\ m_1 \\ m_2 \end{array} \right) B_{\ell_1 \ell_2 \ell_3} = \left( \begin{array}{c} \ell_1 \\ m_1 \\ m_2 \end{array} \right) \sum_{L} c_L B_{\ell_1 \ell_2 \ell_3}^L ,
\]

(3.17)

where

\[
B_{\ell_1 \ell_2 \ell_3}^L = \int_0^{\infty} r^2 dr \left[ \prod_{n=1}^{3} \sum_{L_n} (-1)^{L_n} \frac{L_n}{2} j_{L_n}(k_n r) \right] I_{L_1 L_2 L_3} \beta_{\ell_1 L_1 L}(r)\beta_{\ell_2 L_2 L}(r)\alpha_{\ell_3}(r) \times \frac{4\pi}{2L + 1} I_{\ell_1 L_1 L}I_{\ell_2 L_2 L}(-1)^{\ell_2+L_1}\delta_{L_3 \ell_3} \left\{ \begin{array}{c} \ell_1 \\ L_2 \\ L_1 \end{array} \right\} + (2 \text{ perm}) ,
\]

(3.18)

and

\[
\alpha_{\ell}(r) = \frac{2}{\pi} \int_0^{\infty} k^2 dk \ T_\ell(k) j_\ell(kr) ,
\]

(3.19)

\[
\beta_{\ell L}(r) = \frac{2}{\pi} \int_0^{\infty} k^2 dk \ P_{\zeta}(k)T_\ell(k)j_L(kr) .
\]

(3.20)

Owing to the selection rules of the Wigner symbols, \(\ell_1, \ell_2\) and \(\ell_3\) are constrained by parity invariance and the triangular condition:

\[
\ell_1 + \ell_2 + \ell_3 = \text{even} , \quad |\ell_1 - \ell_2| \leq \ell_3 \leq \ell_1 + \ell_2 . \tag{3.21}
\]

The former constraint is a consequence of the bispectrum of curvature perturbations given by eq. (1.1) being parity-even. The summation ranges of \(L_1\) and \(L_2\) are also restricted to

\[
L_n = |\ell_n - L|, |\ell_n - L| + 2, \cdots, \ell_n + L - 2, \ell_n + L . \tag{3.22}
\]

In the full-sky formula given by eq. (3.18), the angle dependence for \(L > 0\) induces a coupling among \(\ell_1, \ell_2\) and \(\ell_3\) via the Wigner-6 symbol. As a result, eq. (3.18) is not separable (or at least not obviously separable) with respect to \(\ell\)'s unlike the usual local-form CMB bispectrum without angle dependence, i.e., \(L = 0\).
Figures 2 and 3 show the absolute values of the full-sky reduced bispectra, $b_{\ell_1\ell_2\ell_3}^L \equiv B_{\ell_1\ell_2\ell_3}^L (I_{\ell_1} I_{\ell_2} I_{\ell_3})^{-1}$, for $L = 0$, 1, and 2. Note that the full-sky reduced bispectrum reduces to the flat-sky bispectrum, $b(\ell_1, \ell_2, \ell_3)$, that we discussed in the previous subsection, in the small-sky limit [8].

Figure 2 shows the equilateral triangles with $\ell_1 = \ell_2 = \ell_3$, while figure 3 shows triangles with $\ell_1 = \ell_2 = 200$, which become squeezed triangles for $\ell_3 \ll 200$. We find that the amplitudes of the equilateral triangles monotonically decrease as $L$ increases. We can understand this by using the flat-sky formula given in eq. (3.8): the Legendre polynomials give the ratio of $L = 0, 1, 2$ terms as

$$b_{\ell_1, \ell_2, \ell_3}^{L=0} : b_{\ell_1, \ell_2, \ell_3}^{L=1} : b_{\ell_1, \ell_2, \ell_3}^{L=2} = 1 : -\frac{1}{2} : -\frac{1}{8},$$

(3.23)

for $\hat{\ell}_i \cdot \hat{\ell}_j = -\frac{1}{2}$ ($i \neq j$).

Figure 3 shows the squeezed triangles with $\ell_3 \ll \ell_1 = \ell_2 = 200$. In the squeezed limit, the CMB bispectrum of $L = 1$ is highly suppressed compared with those of $L = 0$ and 2. This is simply due to symmetry: the $L = 1$ term vanishes in the exact squeezed limit. Again, the flat-sky formula given in eq. (3.8) gives the ratio of $L = 0, 1, 2$ terms as

$$b_{\ell_1, \ell_1, \ell_3}^{L=0} : b_{\ell_1, \ell_1, \ell_3}^{L=1} : b_{\ell_1, \ell_1, \ell_3}^{L=2} = 1 : 0 : -\frac{1}{2},$$

(3.24)
Figure 3. Same as figure 3, but for the squeezed triangles, $|b_{l_1 l_2 l_3}^L|/\sqrt{2\pi}\times 10^{-6}$, with $l_1 = l_2 = 200$, as a function of $l_3$.

for $\hat{l}_1 \cdot \hat{l}_3 = 0 = \hat{l}_2 \cdot \hat{l}_3$.

As these calculations are quite involved, we provide the simplified analytical formula in the Sachs-Wolfe limit in appendix B. This test validates our numerical results shown in figures 2 and 3.

3.3 Expected uncertainties on $c_1$ and $c_2$

In this subsection, we calculate the 1-$\sigma$ error bars of $c_0$, $c_1$ and $c_2$, i.e., $\delta c_0$, $\delta c_1$, and $\delta c_2$, expected for a cosmic-variance-limited experiment measuring temperature anisotropy. Here, we shall focus on a simultaneous estimation of a pair of parameters: $(c_0, c_1)$ and $(c_0, c_2)$. We give the full constraint varying all three parameters simultaneously in appendix C.

Following ref. [8], we calculate the Fisher matrix, $F_{LL'}$, from

$$F_{LL'} \equiv \sum_{2 \leq l_1 \leq l_2 \leq l_3 \leq l_{\text{max}}} \frac{B_{l_1 l_2 l_3}^L B_{l_1' l_2' l_3'}^{L'}}{\sigma_{l_1 l_2 l_3}^2},$$

(3.25)

where the variance of the CMB bispectrum, $\sigma_{l_1 l_2 l_3}^2$, is given by

$$\sigma_{l_1 l_2 l_3}^2 = C_{l_1} C_{l_2} C_{l_3} \left[ (-1)^{l_1 + l_2 + l_3} (1 + 2\delta_{l_1, l_2} \delta_{l_2, l_3}) + \delta_{l_1, l_2} + \delta_{l_2, l_3} + \delta_{l_3, l_1} \right],$$

(3.26)
with $C_\ell$ being the power spectrum of temperature fluctuations. As we consider a cosmic-variance-limited experiment, we ignore instrumental noise here.

As we show in appendix $C$, $c_0$ and $c_1$ are nearly uncorrelated, so are $c_0$ and $c_2$; however, $c_1$ and $c_2$ are highly correlated. Therefore, in this subsection, we shall consider submatrices of $F_{LL'}$ involving only either $(c_0, c_1)$ or $(c_0, c_2)$, and study the full matrix in appendix $C$.

We define the submatrix (a $2 \times 2$ matrix) as

$$ (2) F_{ij} = \begin{pmatrix} F_{00} & F_{0L} \\ F_{L0} & F_{LL} \end{pmatrix}, $$

(3.27)

where $L$ takes on either 1 or 2. The 1$\sigma$ marginalized error bars are then given by the matrix inverse as $(\delta c_0, \delta c_L) = \left( \sqrt{(2) F_{11}^{-1}}, \sqrt{(2) F_{22}^{-1}} \right)$.

In figure 4, we show the ratios of error bars, $\delta c_1/\delta c_0$ and $\delta c_2/\delta c_0$, as a function of the maximum multipole in the sum, $\ell_{\text{max}}$. The solid and short-dashed lines show the exact results for $L = 1$ and $L = 2$, respectively, while the long-dashed and dotted lines show the corresponding Sachs-Wolfe approximations for $L = 1$ and $L = 2$, respectively. We find that the Sachs-Wolfe approximations trace the overall behavior of the exact calculations well.
The error bar on \( c_1 \) is an order of magnitude larger than that on \( c_0 \) for \( \ell_{\text{max}} \gtrsim 100 \), as the \( L = 1 \) bispectrum has a vanishing amplitude in the squeezed limit. On the other hand, the error bar on \( c_2 \) is comparable to that on \( c_0 \); the Sachs-Wolfe approximation gives an asymptotic relation of \( \delta c_2 = 4\delta c_0 \). The exact calculation gives \( \delta c_2 \approx 3\delta c_0 \) for \( \ell_{\text{max}} = 2000 \).

Finally, the 1-\( \sigma \) error bars expected for a cosmic-variance-limited experiment measuring temperature anisotropy up to \( \ell_{\text{max}} = 2000 \) are given by

\[
(\delta c_0, \delta c_1) = (4.4, 61),
\]

\[
(\delta c_0, \delta c_2) = (4.4, 13).
\]

See eq. (C.1) for the full Fisher matrix.

## 4 Consistency relations with higher spin fields

Primordial correlation functions in the limit that some combinations of external momenta go to zero — soft limits — play a special role in constraining the physics of inflation. Significant squeezed non-Gaussianity is associated with the presence of extra light degrees of freedom during inflation, hence soft limits can be understood as probing the spectrum of light fields in the early universe. Moreover, soft limits are observationally relevant and are subject to a number of interesting theoretical consistency relations. The first example of such a consistency relation was noted in ref. [13] and established under much more general conditions in ref. [1]:

\[
\lim_{k_3 \to 0} B_2(k_1, k_2, k_3) = (1 - n_s) P_\zeta(k_1) P_\zeta(k_3).
\]

This holds independently of the inflationary Lagrangian, under the assumption that there is only a single field, an attractor solution has been reached [6, 7], and the initial state is in a Bunch-Davies state [3–5].

Our new parametrization given by eq. (1.1) represents a non-trivial modification of this consistency relation:

\[
\lim_{k_3 \to 0} B_2(k_1, k_2, k_3) = \left( 2 \sum_L c_L P_L(\hat{k}_1 \cdot \hat{k}_3) \right) P_\zeta(k_1) P_\zeta(k_3).
\]

The possibility that some of the \( c_L \) coefficients can be \( \gg |n_s - 1| \sim 10^{-2} \) indicates extra light fields in the early universe, while the non-trivial angular dependence is associated with anisotropic sources (such as higher spin fields). We also expect non-trivial soft limits for higher-order correlation functions. To explore such effects, we introduce the quantities

\[
f_{\text{NL}}^\text{eff}(k_i) \equiv \lim_{k_3 \to 0} \frac{5}{12} \frac{B_2(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_3)},
\]

\[
T_{\text{NL}}^\text{eff}(k_i) \equiv \lim_{k_1 + k_2 \to 0} \frac{1}{4} \frac{T_4(|k_1 + k_2|) P_\zeta(k_1) P_\zeta(k_3)},
\]

where \( \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = (2\pi)^3 \delta^{(3)}(\sum_i k_i) T_4(k_1, k_2, k_3, k_4) \). For the local-type non-Gaussianity the quantities \( f_{\text{NL}}^\text{eff} \) and \( T_{\text{NL}}^\text{eff} \) become the standard (momentum independent) non-linearity parameters. In this case there is an interesting consistency relation:

\[
T_{\text{NL}} \geq \left( \frac{6}{5} f_{\text{NL}} \right)^2,
\]

where we dropped the superscript “eff” to emphasize that this relation is understood in the case where eqs. (4.2) and (4.3) are independent of momenta. This relation was first noted by Suyama and Yamaguchi in ref. [18] and further explored in refs. [19–25].
In this subsection, we explore the non-Gaussianity consistency relations in the context of the $I^2(\phi)F^2$ model given in eq. (2.7). In general this model breaks statistical isotropy, as discussed in section 2.2. From eq. (2.13) we obtain

$$f_{NL}^\text{eff} \simeq 10N_k^2 N_{k_3}^2 \frac{24E_{cl}^2}{\epsilon V(\phi)} \left[1 - \cos^2 \theta_{k_1, \hat{E}_{cl}} - \cos^2 \theta_{k_3, \hat{E}_{cl}} + \cos \theta_{k_1, \hat{E}_{cl}} \cos \theta_{k_3, \hat{E}_{cl}} \cos \theta_{k_1, k_3} \right],$$

(4.5)

where we recall that $|g_s(N_{k_i})| = N_{k_i}^2 \frac{24E_{cl}^2}{\epsilon V(\phi)} \ll 1$ is assumed.

We compute the trispectrum for the first time in the model given by eq. (2.7). The computation follows very closely the analogous one of the bispectrum performed in ref. [29], and so we omit the technical details. We obtain

$$\tau_{NL}^\text{eff} \simeq 144N_{k_1}^2 N_{k_3}^2 \frac{24E_{cl}^2}{\epsilon V(\phi)} \left[1 - \cos^2 \theta_{k_1, \hat{E}_{cl}} - \cos^2 \theta_{k_1, \hat{E}_{cl}} - \cos^2 \theta_{k_3, \hat{E}_{cl}} + \cos \theta_{k_1, \hat{E}_{cl}} \cos \theta_{k_3, \hat{E}_{cl}} \cos \theta_{k_1, k_3} \right] + \cos \theta_{k_1, \hat{E}_{cl}} \cos \theta_{k_3, \hat{E}_{cl}} \cos \theta_{k_1, k_3} \cos \theta_{k_1, k_3} - \cos \theta_{k_1, \hat{E}_{cl}} \cos \theta_{k_3, \hat{E}_{cl}} \cos \theta_{k_1, k_3} \cos \theta_{k_3, k_3} \right],$$

(4.6)

where $k_{12}$ is the unit vector in the direction of $k_1 + k_2$. We note that the unit-vector $k_{12}$ enters in this expression, even though the corresponding vector $k_1 + k_2$ vanishes in the squeezed limit (analogously to the $k_3$-dependence of the bispectrum).

We observe that, in the $I^2(\phi)F^2$ model, both $f_{NL}^\text{eff}$ and $\tau_{NL}^\text{eff}$ exhibit highly non-trivial momentum dependence; thus, it is not sensible to compare different configurations. One can readily see that $\tau_{NL}^\text{eff}$ vanishes for several configurations. This happens, for example, if one of $k_1, k_{12}, k_3$ is parallel to $\hat{E}_{cl}$, while the other two vectors are perpendicular to $\hat{E}_{cl}$. We do not interpret this as a violation of the Suyama-Yamaguchi inequality; the expression given by eq. (4.4) requires either that the non-linearity parameters are momentum-independent, or else that $\tau_{NL}^\text{eff}$ and $(f_{NL}^\text{eff})^2$ have the same momentum dependence so that one can factor out an amplitude which obeys eq. (4.4). The original form of the Suyama-Yamaguchi inequality is simply not applicable to the model given by eq. (2.7).\[15\]

In ref. [25] Assassi et al derived a general inequality relating the soft limits of the bispectrum and trispectrum:

$$\int d^3q_1 d^3q_2 \tau_{NL}^\text{eff} (q_1, k - q_1, q_2, -q_2 - k) P_\zeta(q_1) P_\zeta(q_2) \geq \left[ \int d^3q \frac{6}{5} f_{NL}^\text{eff} (q, -q - k, k) P_\zeta(q) \right]^2,$$

(4.7)

where the $k \to 0$ limit is understood. This inequality reduces to eq. (4.4) in the local case, but is completely general and should be respected by any model. We can easily verify that,

---

\[15\] Ref. [107] presents a detailed discussion on how to generalize the Suyama-Yamaguchi inequality to momentum-dependent non-linearity parameters, with a particular emphasis on the case of broken statistical isotropy. The fact that we have shown that there exist some configurations with vanishing $\tau_{NL}^\text{eff}$ implies that, at least in principle, one may obtain observational evidence for a smaller squeezed trispectrum than the one obtained from scalar fields, provided that one can construct an observable quantity which is sensitive to those configurations. It remains to be seen whether such a measurement is feasible.
although the Suyama-Yamaguchi inequality is not meaningful here, eq. (4.7) is still respected. After evaluating the angular integral, the left hand side of eq. (4.7) becomes
\[
\text{LHS} \simeq 1024\pi^2 \frac{24E^2_{cl}}{eV(\phi)} \sin^2 \theta_{\hat{k}, \hat{E}_{cl}} \left( \int dq q^2 N_q P_\eta(q) \right)^2,
\]
and, proceeding analogously, the right hand side becomes
\[
\text{RHS} \simeq 1024\pi^2 N_k^2 \left[ \frac{24E^2_{cl}}{eV(\phi)} \right]^2 \sin^4 \theta_{\hat{k}, \hat{E}_{cl}} \left( \int dq q^2 N_q P_\eta(q) \right)^2.
\]
Therefore, we find
\[
\frac{\text{LHS}}{\text{RHS}} \simeq \frac{1}{|g_\ast(N_k)| \sin^2 \theta_{\hat{k}, \hat{E}_{cl}}} > 1.
\]
Note that $|g_\ast| \ll 1$ has been assumed (as also required by phenomenology) throughout this subsection. We therefore conclude that the integrated inequality given by eq. (4.7) is satisfied for any orientations of $\hat{k}$ relative to the classical vector field background.

5 Conclusion

The angle dependence of the bispectrum of primordial curvature perturbations in the squeezed configuration is sensitive to the presence of vector fields and non-trivial symmetry during inflation. In this paper, we have explored phenomenological consequences of the new parametrization of the bispectrum given by eq. (1.1): $B_\eta = \sum_L c_L P_L(\hat{k}_1 \cdot \hat{k}_2) P_\eta(k_1) P_\eta(k_2) + (2 \text{ perm})$. This form is physically well motivated, and we have given three examples in section 2: the curvature perturbation sourced by the anisotropic stress of magnetic fields; that sourced by an interaction with a vector field of the form $I_2(\phi) F^2$; and solid inflation.

We find that a cosmic-variance-limited CMB experiment measuring temperature anisotropy up to $\ell_{\text{max}} = 2000$, such as the Planck satellite, can measure $c_1$ and $c_2$ down to $\delta c_1 = 61$ and $\delta c_2 = 13$ (68% CL). The latter error bar is comparable to (and only a factor of three larger than) the error bar of $c_0 = 6f_{NL}/5$; thus, if the forthcoming Planck data reveal evidence for $c_0$, one should also measure $c_2$ to understand the nature of sources of non-Gaussianity. Moreover, even if the Planck data do not reveal evidence for $c_0$, one should still measure $c_2$, as solid inflation can generate large $c_2$ without generating detectable $c_0$. Sensitivity to $c_1$ is an order of magnitude worse than that to $c_0$ or $c_2$ because the term proportional to $c_1$ vanishes in the squeezed limit due to symmetry.

The angle-dependent bispectrum in the squeezed configuration is a natural consequence of broken statistical isotropy. Broken isotropy also leads to a non-trivial modification of the inequality between the local-form trispectrum amplitude, $\tau_{\text{NL}}$, and $f_{\text{NL}}^3$. We find that the original form of the Suyama-Yamaguchi inequality, $\tau_{\text{NL}} \geq (6f_{\text{NL}}/5)^2$, does not apply to the current model, due to the momentum- and shape-dependence of $\tau_{\text{NL}}$ and $f_{\text{NL}}$. For example, we find some squeezed configurations in which $\tau_{\text{NL}}$ vanishes. It remains to be seen how sensitive the forthcoming tests of the Suyama-Yamaguchi inequality using the Planck or the large-scale structure data are to the decrease of the trispectrum amplitudes for these particular shapes. We also find that a general inequality of ref. [25] is satisfied in this model.

What is next? Phenomenological consequences of eq. (1.1) for large-scale structure of the universe such as the dark matter halo bias [108], bispectrum, and trispectrum should certainly be explored. For instance, a consistency relation [109, 110] between the squeezed
bispectrum and the power spectrum of dark matter density fluctuations has been proved at the full nonlinear level, and with an initial isotropic non-Gaussianity, in ref. [109]. It may be interesting to study a signature of anisotropic initial non-Gaussianity in that context. Also, it is quite possible that the coefficients $c_L$ depend on wavenumbers, which would be particularly interesting for dipolar and quadrupolar modulations of the observed power spectrum in our sky [41]. Indeed, in the case of the $I^2(\phi)F^2$ model discussed in section 2.2, the $c_L$ coefficients exhibit a logarithmic running with wavenumber. It would be interesting to study possible effects of such wavenumber dependence.

**Note added.** After our paper was submitted, the Planck collaboration reported constraints on non-Gaussianity parameters [111]. They also evaluated the coefficients $c_1$ and $c_2$ as follows: they first constrain coefficients of the basis functions of the “modal expansion,” from which they construct templates for the shapes corresponding to $L = 1$ and 2 of eq. (1.1). This results in a value of $11.0 \pm 113$ (68% CL) for the coefficient $c_1$, and of $3.8 \pm 27.8$ (68% CL) for the coefficient $c_2$. As also remarked in ref. [111], the template used in their analysis is only 60% correlated with $L = 2$, suggesting that the estimators constructed from the modal expansion can provide an estimate for these shapes, but that they might not be optimal. Our reported forecast for the 1-σ uncertainties, $\delta c_L$, assumes a full sky, cosmic-variance-limited experiment measuring temperature anisotropy up to $\ell = 2000$. However, the Planck data are noise dominated for $\ell \gtrsim 1500$ and the analysis presented in ref. [111] uses 73% of the sky. Rescaling our estimates by $1/\sqrt{0.73}$ but still assuming a cosmic-variance-limited experiment, we find $\delta c_0 = 5.1$, $\delta c_1 = 71$, and $\delta c_2 = 15$. On the other hand, the Planck collaboration finds $\delta c_0 = 7.0$, $\delta c_1 = 113$, and $\delta c_2 = 28$ (recall that $c_0$ is equal to $6f_{NL}/5$). As the Planck collaboration uses the optimal estimator to find a limit on $\delta c_0$, we estimate the effect of noise in the Planck data by rescaling the error bars by the ratio of 7.0/5.1. We find $\delta c_1 = 97$ and $\delta c_2 = 21$. These estimates for $\delta c_1$ and $\delta c_2$ are 16% and 33% lower than the error bars that the Planck collaboration finds. The latter can be understood from the fact that the template for $L = 2$ used by the Planck collaboration is 60% correlated with the true shape. Therefore, it appears that there is still some room for improvement in the limits on these parameters, especially $c_2$, using optimal estimators.

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A Precision of the flat-sky approximation

How precise are the flat-sky formulae given by eqs. (3.3) and (3.8)? In figure 5, we compare the full-sky results with the flat-sky results. We find that, for $L = 0$ and 1, the simplified flat-sky formula given by eq. (3.8) yields the bispectra in the equilateral and squeezed configurations which are in good agreement with the full-sky results at $\ell \gtrsim 100$. However, we find that, for $L = 2$, the simplified formula systematically underestimate the magnitude of the bispectra in both configurations. The equilateral result suggests that the simplified formula provides an adequate result only at $\ell \gtrsim 800$.

While these results appear to suggest that the precision of the simplified formula degrades as $L$ increases, this is not the case: the flat-sky results in the Sachs-Wolfe limit (which are not shown in this paper) show that the simplified formula overestimates the magnitudes of the bispectra of $L = 1$ and 2 in the Sachs-Wolfe limit by a similar amount in both equilateral and squeezed configurations. Therefore, we conclude that the simplified formula given by eq. (3.8) should only be used for quantitative calculations of $L = 0$ or for qualitative calculations of $L = 1$ and 2, and the original formula given by eq. (3.3) should be used for quantitative calculations of $L = 1$ and 2. Needless to say, the full-sky formula should always be used for the calculations involving multipoles of $\ell \lesssim 100$.

B Analysis in the Sachs-Wolfe limit

As the calculations presented in section 3.2 are quite involved, some appropriate approximations would be useful for understanding the analytical structures of the basic results.

The Sachs-Wolfe limit, in which the radiation transfer function is given by $T_\ell(k) \to -\frac{1}{5} j_\ell(kr_*), provides such a convenient approximation. With this transfer function, $\alpha_\ell(r) \to -\frac{1}{5} \delta(r-r_*)$, where $r_* \equiv \tau_0 - \tau_*$ is the conformal distance to the last scattering surface. Similarly, for a scale-invariant spectrum of $\zeta$, $P_\zeta(k) = \frac{2\pi^2}{k^3} A_S$, $\beta_\ell(r)$ (eq. 3.20) becomes

$$\beta_{\ell L}(r_*) \to -\frac{\pi^2}{10} A_S \frac{\Gamma \left(\frac{\ell+L}{2}\right)}{\Gamma \left(\frac{-\ell+L+3}{2}\right)} \Gamma \left(\frac{\ell+L+4}{2}\right),$$

where $\Gamma(x)$ is the Gamma function. Using these $\alpha_\ell$ and $\beta_\ell$ in eq. (3.18), one finds the Sachs-Wolfe approximation of the CMB bispectrum as

$$B^L_{\ell_1 \ell_2 \ell_3} \to \frac{1}{5} \left[ \prod_{n=1}^3 (-1)^{\ell_n+L_n} \right] I_{L_1 L_2 L_3} \beta_{\ell_1 L_1}(r_*) \beta_{\ell_2 L_2}(r_*) \times \frac{4\pi}{2L+1} I_{L_1 L_3} I_{L_2 L_3} (-1)^{L_1+L_3} \delta_{L_3} \left\{ \frac{\ell_1}{L_1} \frac{\ell_2}{L_2} \frac{\ell_3}{L} \right\} + (2 \text{ perm}).$$

Figure 6 shows the reduced CMB temperature bispectra in the Sachs-Wolfe limit. The basic behaviors, such as the monotonic decrease of the equilateral amplitudes as a function of $L$ and the suppression of the $L = 1$ term in the squeezed limit, are all reproduced by the simple Sachs-Wolfe limit calculations. These results may be compared with figures 2 and 3.
Figure 5. Absolute values of the CMB temperature reduced bispectra. The solid, long-dashed and short-dashed lines show the full-sky results for $L = 0$, 1, and 2, respectively, while the plus, cross, and star symbols show the simplified flat-sky results from eq. (3.8) for $L = 0$, 1, and 2, respectively. The square symbols show the original form of the flat-sky result for $L = 2$ from eq. (3.3) before further approximation. (Top panel) Equilateral triangles, $|b^L_{\ell \ell \ell}|$. (Bottom panel) Squeezed triangles, $|b^L_{\ell_1 \ell_2 \ell_3}|$, with $\ell_1 = \ell_2 = 200$, as a function of $\ell_3$.

C Full Fisher matrix

In section 3.3, we have presented the 1-$\sigma$ marginalized constraints on $c_0$, $c_1$, and $c_2$, assuming that only $c_0$ and one of $c_1$ and $c_2$ are varied simultaneously. In this appendix, we provide the
Figure 6. Absolute values of the CMB temperature reduced bispectra for \( L = 0 \) (solid), 1 (long-dashed), and 2 (short-dashed), in the Sachs-Wolfe limit. (Left panel) Equilateral triangles, \( |b^{L}_{\ell_1 \ell_2 \ell_3}| \). (Right panel) Squeezed triangles, \( |b'^{L}_{\ell_1 \ell_3 \ell_3}| \), with \( \ell_1 = \ell_2 = 200 \), as a function of \( \ell_3 \).

full Fisher matrix, \( F_{LL'} \), involving all of \( c_0 \), \( c_1 \), and \( c_2 \), calculated up to \( \ell_{\text{max}} = 2000 \):

\[
F_{LL'} = \begin{pmatrix}
5232 & 16.94 & -5.986 \\
16.94 & 26.53 & 66.85 \\
-5.986 & 66.85 & 618.1
\end{pmatrix} \times 10^{-5} . \tag{C.1}
\]

From this matrix, one can compute the cross-correlation coefficients, \( r_{LL'} \equiv F_{LL'}/\sqrt{F_{LL}F_{L'L'}} \). We find \( r_{01} = 0.045 \) and \( r_{02} = -0.003 \), indicating that \( c_0 \) and \( c_1 \) are nearly uncorrelated, so are \( c_0 \) and \( c_2 \). However, there is a high degree of correlation between \( c_1 \) and \( c_2 \): \( r_{12} = 0.522 \). As a result, the marginalized error bars increase slightly to

\[
(\delta c_0, \delta c_1, \delta c_2) = \left( \sqrt{F_{00}^{-1}}, \sqrt{F_{11}^{-1}}, \sqrt{F_{22}^{-1}} \right) = (4.4, 72, 15) . \tag{C.2}
\]

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