Advanced optimal tolerance design of machine elements using teaching-learning-based optimization algorithm

R.V. Rao* and K.C. More

Department of Mechanical Engineering, S.V. National Institute of Technology, Ichchanath, Surat, Gujarat 395 007, India

(Received 25 October 2013; accepted 6 February 2014)

Tolerance design has become a very key issue in product and process development because of an informal compromise between functionality, quality, and manufacturing cost. The problem formulation becomes complex with simultaneous selection of design and manufacturing tolerances and the optimization problem is difficult to solve with the traditional optimization techniques. In this paper, a recently developed optimization algorithm called teaching-learning-based optimization (TLBO) is used for optimal selection of design and manufacturing tolerances with an alternative manufacturing process to obtain the optimal solution which is nearer to global optimal solution. Three problems are considered and these are: overrunning clutch assembly, knuckle joint assembly with three arms, and a helical spring. Out of these three problems, the problems of overrunning clutch assembly and knuckle joint assembly with three arms are multi-objective optimization problems and the helical spring problem is a single-objective problem. The comparison of the proposed algorithm is made with the genetic algorithm (GA), Non-dominated sorting genetic algorithm-II (NSGA-II), and multi-objective particle swarm optimization algorithm (MOPSO). It is found that the TLBO algorithm has produced better results when compared to those obtained by using GA, NSGA-II, and MOPSO algorithms.

Keywords: tolerance design; multi-objective optimization; machine elements; teaching-learning-based optimization algorithm

1. Introduction

Assembly tolerance in an acceptable functioning of a product assembly is ensured by making the assembly response to be included in a specified range. Allocation of this assembly tolerance among the ingredient dimensions while gathering the functionality requirements is known as tolerance design. Design engineers normally tend to specify rigid tolerances for ensuring the performance and functionality of the product assembly; while manufacturing engineers do prefer slack tolerances to produce parts economically. Therefore, design of tolerance plays an essential role in bonding the between designs and manufacturing phases. Selection of optimal tolerances for minimum manufacturing cost and meeting the functional requirements of the product assembly has been an appealing topic of research for some decades (Singh, Jain, & Jain, 2005).

The tolerance design problem becomes more intricate for manufacturing every dimension in the existence of different processes or machines, as shown in Figure 1 (Prabhaharan, Asokan, & Rajendran, 2005). Due to this the manufacturing cost–tolerance
features change from machine to machine and from process to process. The total costs incurred throughout a product’s life cycle can be divided in two main categories: manufacturing cost, which occurs before the product reaches the customer; and quality loss, which occurs after the product is sold. A rigid tolerance (i.e. high manufacturing cost) indicates that the variety of product quality characteristics will be low (i.e. little quality loss). On the other hand, a slack tolerance (i.e. low manufacturing cost) indicates that the variability of product quality characteristics will be high (i.e. high quality loss). Therefore, there is a need to adjust the design tolerance between the quality loss and the manufacturing cost and to reach an economic balance during product tolerance design.

During the last few decades, researchers have focused their attention on obtaining the best tolerance allocation in such a way that the product design not only meets the efficient needs but also minimizes manufacturing cost. In order to solve the tolerance allocation problem, various numerical methods were employed to deal with complicated computations associated with tolerance design models. Forouraghi (2002) introduced an approach to worst-case tolerance design using a genetic algorithm (GA) for the optimal design of a clutch assembly without considering alternative manufacturing process selection. Ye and Salustri (2003) introduced a new method of synthesizing tolerances simultaneously for both manufacturing cost and quality. The authors had constructed a non-linear optimization model and had minimized the quality loss and the manufacturing cost simultaneously in a single-objective function by setting both the process tolerances and design tolerances. Singh, Jain, and Jain (2003, 2004) obtained an optimal solution to a set of tolerance design problems with simple dimension chain involving sets of alternative processes using GAs.

Prabhaharan et al. (2005) used ant colony algorithm as an optimization tool for minimizing the critical dimension deviation and for allocating the cost-based optimal tolerances. Krishna and Rao (2006) used a scatter search method to simultaneously allocate both the design and manufacturing tolerances based on minimum total manufacturing cost. Huang and Shiau (2006) obtained tolerance allocation of a sliding vane rotary compressor and the required reliability, minimum cost, and quality loss were optimized. Huang and Zhong (2007) established the sequential linear optimization models to maximize the 2D-sized, angular, and directional working tolerances of a 3D-machined part based on the process capabilities. This approach releases the working tolerances, reduces manufacturing costs, and enhances the acceptance rate of machined parts.

Singh, Jain, and Jain (2008) introduced GA to obtain an optimal solution to the advanced tolerance synthesis problem by considering a continuous cost function and this
method was used for both single and multiple tolerance stack-ups. Huang and Zhong (2008) implemented a linear programming approach to obtain process tolerances concurrence from an assembly by using the information of process planning. Sivakumar, Kannan, and Jayabalan (2009) used a hybrid algorithm for optimum tolerance allocation in complex assemblies with alternative process selection. For hybridization, the authors had used a tabu search and heuristic algorithm.

Gonzalez and Sanchez (2009) developed a methodology to allocate optimal statistical tolerances to dependent variables and the manufacturing costs were minimized using a statistical approach. Forouraghi (2009) developed a methodology to allow an automatic tolerance allocation capable of minimizing manufacturing costs based on particle swarm optimization (PSO) algorithm. Sivakumar and Stalin (2009) used a Lagrange multiplier method for selection of alternative processes for tolerance allocation and total manufacturing cost minimization. Wu, Jean-Yves, Alain, Ali, and Patrick (2009) adopted Monte Carlo simulation and GA methodology for tolerance allocation of minimizing both manufacturing cost and quality loss.

Muthu, Dhanalakshmi, and Sankaranarayanasamy (2009) used metaheuristic techniques such as GA and PSO to the design and process tolerances simultaneously for minimum combined manufacturing cost and quality loss over the life of the product. Sivakumar et al. (2011a) used Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) and multi-objective particle swarm optimization algorithm (MOPSO) for simultaneous optimum selection of design and manufacturing tolerances with alternative process selection. Sivakumar et al. (2011b) used NSGA-II and MOPSO for tolerance allocation for mechanical assemblies with considering alternative manufacturing process selection. Sivakumar, Balamurugan, and Ramabalan (2012) used evolutionary NSGA-II and multi-objective differential evaluation (MODE) optimization algorithms for the simultaneous selection of optimal process tolerances for a wheel assembly. Guofu, Yuege, Ye, and Hu (2013) proposed a method of multi-objective reliability tolerance design for electronic systems. It improved the performance of output response and reduced the operating stresses on components.

It has been observed from the literature review that some of the researchers had used traditional optimization techniques like scatter search method, sequential linear method, Lagrange multiplier method, Monte Carlo simulation, linear programming method, etc. for optimum tolerance design of machine elements. However, these approaches suffer from some drawbacks as following: (i) these traditional techniques may be relevant only for simple cost functions and may not be relevant for discontinuous cost functions. (ii) the traditional techniques may not give global optimum solution. In order to overcome these drawbacks of traditional optimization techniques, few researchers had used the so-called advanced optimization techniques such as GA, SA, NSGA-II, MODE, and MOPSO algorithms. The advantages of these advanced optimization techniques are: (i) these techniques are relevant for solving all types of complex optimization problems and (ii) these techniques are population-based and hence they may give global optimum solution.

Many researchers, except Sivakumar et al. (2011a, 2011b, 2012) and Guofu et al. (2013), considered only one objective function i.e. minimization of manufacturing cost. But the real-world tolerance design problems have more than one objective function. Some of the other important objective functions are quality loss, stack-up tolerance, stock removal allowance, etc.

The advanced optimization techniques like GA, SA, NSAG-II, MOPSO, MODE, etc. need tuning of algorithm-specific parameters. For example, GA requires crossover
probability, mutation probability (Claudio, Jose, & Nestor, 2003) and NSGA-II requires
crossover probability, mutation probability, real-parameter SBX parameter, and real-
parameter mutation parameter (Daniel, Claudio, & Blas, 2006). SA requires controlling
temperature and Boltzmann constant (Luis & Claudio, 2007).
MOPSO requires mutation probability, inertia weight, and social and cognitive
parameters. MODE requires scaling factor and crossover constant. The proper tuning of
the algorithm-specific parameters is a very essential factor, which affects the perform-
ance of those algorithms. The improper tuning of algorithm-specific parameters either
increases the computational effort or yields the local optimal solution (Rao & Patel,
2013a). Hence to overcome the problem of tuning of algorithm-specific parameters, we
have used a recently developed algorithm-specific parameter-less algorithm known as
teaching-learning-based optimization (TLBO) algorithm (Rao, Savsani, & Vakharia,
2011; Rao, Savsani, & Vakharia, 2012; Rao & Savsani, 2012). It requires only common
controlling parameters like population size and number of generations for its working
(Just like any other population-based optimization algorithms). For more details about
the effects of the common controlling parameters on the performance of TLBO
algorithm, readers may refer to Rao and Patel (2012, 2013b).
John Holland had developed GAs in the 1960s. Most commonly used genetic opera-
tors are reproduction, crossover, and mutation. They are implemented as a computer
simulation in which a population of abstract representations (called chromosomes or the
genotype of the genome) of candidate solutions (called individuals, creatures, or pheno-
types) to an optimization problem evolves toward better solutions (Singh et al., 2005).
NSGA-II algorithm is utilized here as a multi-objective optimization method in order
to find the Pareto frontier using the GA. The number of ways NSGA-II works differs
from Non-Dominated Sorting Genetic Algorithm (NSGA). It uses an elite-preserving
mechanism. Secondly, it uses a fast Non-Dominated Sorting procedure. The solutions
on each pareto-front are pareto-optimal with respect to each other (Sivakumar et al.,
2011a).
The single-objective PSO algorithm extends to handle multi-objective optimization
problems called MOPSO algorithm. The crowding space mechanism together with a
mutation operator maintains the diversity of non-dominated solutions in the outer
archive. The motivation behind this concept is to attain better convergence to the
Pareto-optimal front, while giving sufficient emphasis to the diversity consideration
(Sivakumar et al., 2011b).
The case studies presented by Sivakumar et al. (2011b) and Singh et al. (2005) are
considered in this work to demonstrate the applicability of the TLBO algorithm for the
overrunning clutch assembly, knuckle joint with three arms, and the helical spring
design problem to see if any further improvement can be obtained in the results and in
the computational time. Out of these three optimization problems, the first two problems
are multi-objective problems and the third one is a single-objective problem.
The next section presents the details of the TLBO algorithm.

2. Teaching-learning-based optimization algorithm.

TLBO is a teaching-learning process inspired algorithm proposed recently by Rao et al. (2011, 2012) based on the effect of influence of a teacher on the output of learners in a class. The algorithm mimics the teaching-learning ability of teachers and learners in a classroom. Teacher and learners are the two vital components of the algorithm and describe two basic modes of the learning, through teacher (known as teacher phase) and
interacting with the other learners (known as learner phase) (Rao et al., 2011, 2012; Rao & Savsani, 2012).

The output in TLBO algorithm is considered in terms of results or grades of the learners which depend on the quality of teacher. So, teacher is usually considered as a highly learned person who trains learners so that they can have better results in terms of their marks or grades. Moreover, learners also learn from the interaction among themselves which also helps in improving their results. TLBO is a population-based method and a group of learners is considered as population and different design variables are considered as different subjects offered to the learners and learners’ result is analogous to the ‘fitness’ value of the optimization problem. In the entire population, the best solution is considered as the teacher. The flowchart of TLBO algorithm is shown in Figure 2 (Rao & Kalyankar, 2012). The working of TLBO is divided into two parts, ‘teacher phase’ and ‘learner phase’. Working of both the phases is explained below.

![Flow chart of TLBO algorithm](image-url)
2.1. Teacher phase

It is the first part of the algorithm where learners learn through the teacher. During this phase, a teacher tries to increase the mean result of the classroom from any value \( M_1 \) to his or her level (i.e. \( T_j \)). But practically it is not possible and a teacher can move the mean of the classroom \( M_1 \) to any other value \( M_2 \) which is better than \( M_1 \) depending on his or her capability. Consider \( M_j \) be the mean and \( T_i \) be the teacher at any iteration \( i \).

Now \( T_i \) will try to improve the existing mean \( M_j \) towards him/her so that the new mean will be designated as \( M_{\text{new}} \) and the difference between the existing mean and new mean is given by,

\[
\text{Difference}_i = r_i(M_{\text{new}} - T_j M_j)
\]

where \( T_f \) is the teaching factor which decides the value of mean to be changed, and \( r_i \) is the random number in the range \([0,1]\). Value of \( T_f \) can be either 1 or 2 which is a heuristic step and it is decided randomly with equal probability as,

\[
T_f = \text{round} \left[ 1 + \text{rand}(0,1) \right]
\]

Based on this \( \text{Difference}_i \), the existing solution is updated according to the following expression

\[
X_{\text{new},i} = X_{\text{old},i} + \text{Difference}_i
\]

2.2. Learner phase

It is the second part of the algorithm where learners increase their knowledge by interaction among themselves. A learner interacts randomly with other learners for enhancing his or her knowledge. A learner learns new things if the other learner has more knowledge than him or her. Mathematically, the learning phenomenon of this phase is expressed below.

At any iteration \( i \), considering two different learners \( X_i \) and \( X_j \) where \( i \neq j \)

If \( f(X_i) < f(X_j) \)

\[
X_{\text{new},i} = X_{\text{old},i} + r_i(X_{\text{old},j} - X_{\text{old},i})
\]

If \( f(X_i) > f(X_j) \)

\[
X_{\text{new},j} = X_{\text{old},i} + r_j(X_{\text{old},j} - X_{\text{old},i})
\]

Accept \( X_{\text{new}} \) if it gives better function value.

The Equations (4) and (6) are applicable for minimization problems and the Equations (5) and (7) are applicable for maximization problems.

It is important to mention that in the basic TLBO algorithm, the solution is updated in the teacher phase as well as in the learner phase (Rao & Patel, 2013a). For more details on TLBO algorithm, one may refer to: https://sites.google.com/site/tlborao. In this paper, the TLBO algorithm is applied for the optimum tolerance design of few machine elements like overrunning clutch assembly, knuckle joint with three arms, and helical spring.
The next section describes the role of design and manufacturing tolerances, stock removal allowances, selection of manufacturing processes, manufacturing cost, and quality loss function.

3. **Problem definition**

3.1. **Design and manufacturing tolerances**

In any product design, the proposed functions and assembly needs are distorted into a set of related tolerances and appropriate dimensions. The dimensions are known as the assembly dimensions and the related tolerances are known as assembly tolerances. The assembly tolerance is sensibly distributed among the part dimensions in particular dimension sequence considering the practical aspects. A tolerance specified in the design stage is further refined to suit the requirements of process planning for producing the constituent dimensions (Singh et al., 2009).

3.2. **Stock removal allowances**

The amount of stock removal allowance has very much effect on the manufacturing tolerance selection. The stock removal allowance is the layer of material to be removed from the surface of a work piece in manufacturing to obtain the required profile accuracy and surface quality through different machining processes. The stock removal allowance is very much affecting the quality and the production efficiency of the manufactured features. A disproportionate stock removal allowance will add to the consumption of material, machining time, tool, and power, and hence raise the manufacturing cost. On the other hand, with an insufficient stock removal allowance, the faulty surface layer caused by the previous operation cannot be rectified. Variation in the stock removal is the sum of manufacturing tolerances in the prior and the current operations. An appropriate stock removal allowance is required for each successful manufacturing operation and the concept is shown in Figure 3 (Haq, Sivakumar, Saravanan, & Muthiah, 2005).

3.3. **Selection of machining process**

The selection of machining process is depending on the equipment precision, machining series, set-up mode, and cutting parameters. The selection of machining process is strongly affected by the tolerance of the part to be machined. So it is vital to do a simultaneous selection of the best machining process while allocating the tolerance (Sivakumar et al., 2011a).

3.4. **Manufacturing cost**

The total expenditure for developing a blueprint dimension is called the manufacturing cost of the dimension. The overall manufacturing cost will constitute both the direct cost and the overheads. For a given manufacturing process, the material, set-up, tool, and inspection costs, in addition to overheads, comprise the fixed cost. The variable cost, which usually depends on the value of tolerance assigned, is mostly because of rework and/or rejection. The actual processing (labour) cost also accounts for the variable cost, though it is not predominant.
Manufacturing cost usually increases as the tolerance of quality characteristics in relation to the ideal value is reduced. Due to this, more refined and accurate operations are essential while the suitable ranges of output are reduced. In other hand, large tolerances are having minimum cost to get as they want less specific manufacturing processes; but usually they provide poor result in performance, premature wear, increase in scrap, and part rejection.

The manufacturing cost function for the available manufacturing processes is assumed to be exponential (Equation (16)) (Singh et al., 2005).

$$C = c_0 * e^{-c_1 t} + c_2$$  \hspace{1cm} (8)

where $C$ is manufacturing cost as a function of tolerance; $c_0$, $c_1$, and $c_2$ are cost function values; and $t$ is the design tolerance dimension.

### 3.5. Quality loss function

Variability in the production process is compulsory due to changeability in tool work piece, material, and process parameters. In this study, it is referred as the quality loss function (Feng & Kusiak, 1997). This loss function is a quadratic expression for measuring the cost in terms of financial loss due to product failure in the eyes of the consumers. The quality loss function (QL) is:

$$QL = \frac{A}{T^2} \sum_{i=1}^{J} \sigma_i^2$$ \hspace{1cm} (9)

$$\sigma_i = \frac{t_i}{3}$$ \hspace{1cm} (10)

$$QL = \frac{A}{9T^2} \sum_{i=1}^{J} t_i^2$$ \hspace{1cm} (11)
where $T$ is the tolerance stack-up limit of the dimensional chain, $A$ is the quality loss cost, $i$ is the component tolerance index, $j$ is the process index, $t$ is the design tolerance dimension, and is the standard deviation of dimension $i$.

The next section presents three problems of optimization and the application of the TLBO algorithm for same.

4. Examples

The examples of overrunning clutch assembly, knuckle joint assembly with three arms which were solved previously by Sivakumar et al. (2011b), and the design of helical spring problem, which was solved previously by Singh et al. (2005), are attempted now by using the TLBO algorithm.

4.1. Example 1: overrunning clutch assembly (Sivakumar et al. 2011b)

The overrunning clutch assembly as shown in Figure 4 (Sivakumar et al., 2011b) consists of hub, rollers, and cage. The rollers are supplied by vendor hence their having fixed value tolerance and manufacturing cost. So, only for the hub and the cage the tolerances and the manufacturing cost are optimized. The following are the requirements and information for manufacture of the assembly. These data are adopted from (Singh et al., 2008; Sivakumar et al., 2011b; Feng & Kusiak, 1997). Assembly response function is given by

$$Y = a \cos \left( \frac{X_1 + X_2}{X_3 - X_2} \right)$$

where $X_1$ is hub dimension in mm, $X_2$ is rollers dimension in mm which is supplied by vendor, $X_3$ is cage dimension in mm, $Y$ is the assembly dimension, and $a$ is constant.

Figure 4. Overrunning clutch assembly (Sivakumar et al., 2011b).
Note: ($X_1 = 55.291$ mm; $X_2 = 22.86 \pm 0.0102$ mm; $X_3 = 101.6$ mm and $Y = 7.0124 \pm 10$ [Sivakumar et al., 2011b; Singh et al. 2009]).
The manufacturing processes for the hub are forging, milling, and grinding; and for the cage are forging, turning, and grinding. Also, there are three machining tolerance parameters for manufacture of each of the hub and the cage \( t_{ij} \) \((i = 1 \text{ for the hub, and } 3 \text{ for the cage; } j = 1 \text{ to } 3 \text{ for the three manufacturing operations in the respective process plan})\). For each manufacturing operation, alternative machines are available with the details given in Table 1.

Sivakumar et al. (2011b) used NSGA-II and MOPSO for simultaneous optimum selection of design and manufacturing tolerances with alternative process selection.

The objective functions considered are: minimum tolerances stack-up, minimum total manufacturing cost of the assembly, and minimum quality loss function. The objective functions considered in this paper are same as those considered by Sivakumar et al. (2011b) and these are:

Minimize:

\[
Z_1 = AY = \xi_1 t_{13} x_{13} + \xi_2 t_2 + \xi_3 t_{33} x_{33} \tag{13}
\]

\[
Z_2 = C_{asm} = \sum_{i=1}^{3} \sum_{j=1}^{3} (a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) |_{x_{ij}} \tag{14}
\]

\[
Z_3 = QL = \frac{A}{9T^2} \sum_{i=1}^{j} t_{ij} \tag{15}
\]

where \( Z_1 \) is the minimum tolerances stack-up; \( Z_2 \) is the minimum total manufacturing cost; \( Z_3 \) is the minimum quality loss function; \( C_{asm} \) is the total assembly manufacturing cost; \( \xi_1, \xi_2, \xi_3 \) is the value of sensitivity for component dimensions \( X_1, X_2, X_3 \); \( t_{ij} \) is the tolerance on the dimension \( x_i \) produced by the \( j \)th process; \( a_0, a_1, a_2, a_3, a_4 \) and \( a_5 \) is the parameters of cost function.

Constraints on the design tolerances are formulated based on the assumed stack-up criteria. The approach opted by Krishna and Rao (2006) and Sivakumar et al. (2011b) is followed in the present work. Constraints on the design tolerances are formulated based on the assumed stack-up criteria. The manufacturing tolerance constraints can be formulated as follows:

\[
\delta_{ij} + \delta_{i(j-1)} \leq \Delta A_{ij} \tag{16}
\]

\[
\sqrt{\delta_{ij} + \delta_{i(j-1)}} \leq \Delta A_{ij} \tag{17}
\]

where \( \delta_{ij} \) and \( \delta_{i(j-1)} \) are the manufacturing tolerances obtainable in the \( j \)th and \( j-1 \)th operations, respectively, in production of the \( i \)th dimension. \( \Delta A_{ij} \) is the difference between the nominal and the minimum stock deduction/deformation allowance for manufacturing operation \( j \).

The following equations are the representative design constraints:

\[
\xi_1 t_{13} x_{13} + \xi_2 t_2 + \xi_3 t_{33} x_{33} \leq 0.0349/2 \tag{18}
\]

\[
\xi_1^2 t_{13}^2 x_{13} + \xi_2^2 t_2^2 + \xi_3^2 t_{33}^2 x_{33} \leq (0.0349/2)^2 \tag{19}
\]

The following equations are the representative machining tolerance constraints:

For the hub,
Table 1. Manufacturing process characteristics for the overrunning clutch assembly (Sivakumar et al., 2011b).

| Dimension | Manufacturing operation | Process no. | Parameters of cost function | Minimum tolerance (mm) | Maximum tolerance (mm) |
|-----------|-------------------------|-------------|-----------------------------|------------------------|------------------------|
|           |                         |             | \(a_0\)                     | \(a_1\)                | \(a_2\)                | \(a_3\)                | \(a_4\)                | \(a_5\)                |                         |
| Hub       | Forging                 | 1           | 304                         | -2175                  | 6208                   | -8620                  | 7930                   | -5670                  | .080                   | .500                   |
|           |                         | 1           | 220                         | -1861                  | 5888                   | -7985                  | 2155                   | 3859                   |                         |                        |
|           | Milling                 | 2           | 208                         | -1791                  | 5848                   | -8065                  | 2225                   | 3659                   | .05                    | .350                   |
|           |                         | 3           | 199                         | -1761                  | 5821                   | -8062                  | 2215                   | 3648                   | .010                   | .100                   |
|           | Grinding                | 4           | 252                         | -1952                  | 5938                   | -8005                  | 2315                   | 3648                   | .010                   | .100                   |
|           |                         | 1           | 94.60                       | -2602.75              | 75,010                 | -1,114,190             | 8,219,697              | -23,717,949            |                         |                        |
|           |                         | 2           | 101.22                      | -2717.55              | 75,108                 | -1,108,090             | 8,160,697              | -23,595,949            |                         |                        |
| Cage      | Forging                 | 1           | 304                         | -2175                  | 6208                   | -8620                  | 7930                   | -5670                  | .08                    | .5                     |
|           |                         | 1           | 112.3                       | -1060                  | 5833                   | -15,340                | 18,450                 | -8269                  |                         |                        |
|           | Turning                 | 2           | 106                         | -1020                  | 5480                   | -14,830                | 19,660                 | -10,160                | .05                    | .5                     |
|           |                         | 3           | 110                         | -1035                  | 5475                   | -14,830                | 19,650                 | -10,120                | .010                   | .100                   |
|           | Grinding                | 4           | 500                         | -1035                  | 5475                   | -14,820                | 19,650                 | -10,120                | .010                   | .100                   |
|           |                         | 1           | 111.28                      | -2978.96              | 58,304                 | -543,354               | 2,386,396              | -3,966,766             |                         |                        |
|           |                         | 2           | 120.67                      | -4946.68              | 166,770                | -2,929,392             | 25,171,322             | -83,271,352            |                         |                        |
where \( t_{ij} \) is the manufacturing tolerance on the \( i \)th dimension by \( j \)th process.

Parameter cost functions and tolerance limits for the overrunning clutch assembly are given in Table 1. The combined objective function considered by Sivakumar et al. (2011b) is used as it is in the present research work to make the comparison of results of optimization meaningful. Sivakumar et al. (2011b) had combined the multiple objectives into scalar objective by using weight vector. Hence, the same approach is used in the present work for comparison purpose. For this problem, the combined objective function \( f_c \) by Sivakumar et al. (2011b) is as follows:

\[
\text{Minimize } f_c = W_1/Z_1 + W_2/Z_2 + W_3/Z_3 \tag{24}
\]

The values of \( N_1 = 0.1 \), \( N_2 = 100 \), and \( N_3 = 10 \) are normalizing parameters of the objective functions \( Z_1 \), \( Z_2 \), and \( Z_3 \), and \( W_1 \), \( W_2 \), and \( W_3 \) are the weights given to the objective functions 1, 2, and 3, respectively. \( N_1 = 0.1 \) means that it is the minimum value of \( Z_1 \) if only \( Z_1 \) is considered as a single objective function (by originally all the other objective function). Similarly, \( N_2 = 100 \) and \( N_3 = 10 \) indicate the minimum value of \( Z_2 \) and \( Z_3 \) considering them as single objectives (by ignoring the other objective functions). The designer may give any weight to a particular objective function. But the summation of all weight values should be equal to 1. It means that the total weight should be 100%. Sivakumar et al. (2011b) used the weights of \( W_1 = W_2 = W_3 = 1/3 \) and the same weights are used in the present work for comparison purpose. The calculated values of first, second, and third objective functions using TLBO algorithm are 0.0046, 233.1067, and 3.2549, respectively. So to get the normalized values, the first, second, and third objective functions are divided by their individual average values (0.1, 100, and 10), respectively. The normalized values of first, second, and third objective functions are 0.046, 2.331067, and 0.32549, respectively.

### 4.2. Example 2: Knuckle joint with three arms (Sivakumar et al., 2011b)

There are several dimensions in the Knuckle joint assembly as shown in Figure 5 (Sivakumar et al., 2011b). There are three interrelated dimension chains corresponding to the respective design functions giving rise to three constraints. The permissible variation \( T \) associated with the respective assembly dimension \( Y \) has been assumed to be equal to 0.2. All similar dimensions are manufactured in same machine, hence the design tolerance associated to \( X_{2a} \) and \( X_{2b} \) is \( t_2 \), and that associated to \( X_{3a} \) and \( X_{3b} \) is \( t_3 \). For each manufacturing operation, alternative machines are available with the details given in Table 3.

Design functions:

\[
Y_1 = X_{2b} - X_1 \tag{25}
\]
\[ Y_2 = X_{3b} - (2X_{2a} + X_{2b}) \]  
(26)

\[ Y_3 = X_4 - (2X_{3a} + X_{3b} + X_5) \]  
(27)

where, \( Y_1, Y_2, Y_3 \) is design dimensions; \( X_1, X_{2a}, X_{2b}, X_{3a}, X_{3b}, X_4, X_5 \) are individual dimensions in product assembly.

Sivakumar et al. (2011b) used NSGA-II and MOPSO for simultaneous optimum selection of design and manufacturing tolerances with alternative process selection.

The objective functions considered are: minimum tolerances stack-up, minimum total manufacturing cost of the assembly, and minimum quality loss function. The objective functions considered are same as those considered by Sivakumar et al. (2011b) and these are:

Minimize:

\[ Z_1 = \Delta Y_1 = t_1 + t_2 \]  
(28)

\[ Z_2 = \Delta Y_2 = 3t_2 + t_3 \]  
(29)

\[ Z_3 = \Delta Y_3 = 3t_3 + t_4 + t_5 \]  
(30)

\[ Z_4 = C_{asm} = Cx_1 + 2Cx_{2a} + Cx_{2b} + 2Cx_{3a} + Cx_{3b} + Cx_4 + Cx_5 \]  
(31)

\[ Z_5 = QL = \frac{A}{9T^2} \sum_{i=1}^{T} t_{ij}^2 \]  
(32)

where \( Z_1, Z_2, \) and \( Z_3 \) are the minimum tolerances stack-up; \( Z_4 \) is the minimum total manufacturing cost and \( Z_5 \) is the minimum quality loss function.

The following tolerance stack-up constraints are considered:

Figure 5. Knuckle joint assembly with three arms (Sivakumar et al., 2011b).

Note: \( X_i \) is the individual dimensions in product assembly; \( Y_1, Y_2, Y_3 \) is design dimensions).
 Parameter cost functions and tolerance limits for the knuckle joint assembly with three arms are given in Table 2. Multiple objectives are combined into scalar objective using a weight vector. For this problem, the combined objective function \( f_c \) is defined as follows [18]:

\[
 f_c = W_1 Z_1 + W_2 Z_2 + W_3 Z_3 + W_4 Z_4 + W_5 Z_5
\]

(36)

The values of \( N_1 = N_2 = N_3 = N_5 = 1.0 \) and \( N_4 = 100 \) are the normalizing parameters of the objective functions \( Z_1, Z_2, Z_3, Z_4, \) and \( Z_5 \) and \( W_1, W_2, W_3, W_4, \) and \( W_5 \) are the weights given to the objective functions 1, 2, 3, 4, and 5, respectively. \( N_i = 1.0 \) means that it is the minimum value of \( Z_i \) if only \( Z_i \) is considered as a single objective function (by originally all the other objective function). Similarly, \( N_2 = 1.0, N_3 = 1.0, N_4 = 100, \) and \( N_5 = 1.0 \) indicate the minimum value of \( Z_2, Z_3, Z_4, \) and \( Z_5 \) considering them as single objectives (by ignoring the other objective functions). As mentioned earlier, the designer may give any weight to a particular objective function. But the summation of all weight values should be equal to 1. It means that the total weight should be 100%.

Sivakumar et al. (2011b) used the equal weights of \( W_1 = W_2 = W_3 = W_4 = W_5 = .2 \) and the same weights are used in the present work for comparison purpose. The calculated values of first, second, third, fourth, and fifth objective functions using TLBO algorithm are \( .186008, .191812, .196046, 1008.7, \) and \( .2466 \), respectively. So to get the normalized

| Dimensions | Process | Parameters of cost function | Minimum tolerance | Maximum tolerance |
|------------|---------|-----------------------------|-------------------|------------------|
| \( X_1 \)  | 1       | 311.0 15.8 24.2  .01  .15 |                   |                  |
|            | 2       | 280.0 14.0 19.8  .01  .15 |                   |                  |
|            | 3       | 296.4 19.5 23.82  .01  .15 |                   |                  |
|            | 4       | 331.2 17.64 20.0  .01  .15 |                   |                  |
| \( X_{2a}, X_{2b} \) | 1 | 311.0 15.8 24.2  .01  .15 |                   |                  |
|            | 2       | 280.0 14.0 19.8  .01  .15 |                   |                  |
|            | 3       | 296.4 19.5 23.82  .01  .15 |                   |                  |
|            | 4       | 331.5 17.64 20.1  .01  .15 |                   |                  |
| \( X_{3a}, X_{3b} \) | 1 | 311.0 15.8 24.2  .01  .15 |                   |                  |
|            | 2       | 280.0 14.0 19.8  .01  .15 |                   |                  |
|            | 3       | 296.4 19.5 23.82  .01  .15 |                   |                  |
|            | 4       | 331.5 17.64 20.0  .01  .15 |                   |                  |
| \( X_4 \)  | 1       | 92.84 82.45 32.5  .02  .20 |                   |                  |
|            | 2       | 82.43 16.70 21.0  .02  .20 |                   |                  |
| \( X_5 \)  | 1       | 128.25 82.65 32.5  .01  .10 |                   |                  |
|            | 2       | 160.43 86.70 29.2  .01  .10 |                   |                  |
|            | 3       | 231.42 50.05 28.05  .01  .10 |                   |                  |
|            | 4a      | 134.16 78.82 500.0  .01  .10 |                   |                  |
values, the first, second, third, fourth, and fifth objective functions are divided by their individual average values \( (i.e. 1.0, 1.0, 1.0, 1.0, 1.0) \), respectively. The normalized values of the first, second, third, fourth, and fifth objective functions are \( 0.186008, 0.191812, 0.196046, 1.087, \) and \( 0.2466 \), respectively.

### 4.3. Example 3: Helical spring (Singh et al., 2005)

In this example, only one objective function is considered \( i.e. \) minimizing the total manufacturing cost (Singh et al., 2005). For the helical spring shown in Figure 6 (Singh et al., 2005), there are three decision variables and Singh et al. (2005) used GA for tolerance design considering interrelated dimension chains and process precision limits. The Design functions are:

![Helical spring diagram](image)

**Figure 6.** Helical spring (Singh et al., 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Figure 6.** Helical spring (Singh et al., 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |

---

**Table 3.** Manufacturing process characteristics for the helical spring (Singh et al. 2005).

| Dimensions | Parameter | Minimum tolerance (mm) | Maximum tolerance (mm) |
|------------|-----------|------------------------|------------------------|
| \( d_w \)  | \( C_0 \) | .018                  | .80                    |
|            | \( C_1 \) | .020                  | .82                    |
|            | \( C_2 \) | .022                  | .8                    |
Table 4. Optimization results for the knuckle joint with three arms.

| Technique                  | Dimension | Machine | Tolerance notation | Tolerance value (mm) | \(Z_1\)  | \(Z_2\)  | \(Z_3\)  | \(Z_4\)  | \(Z_5\)  | Combined objective function (\(f_c\)) |
|----------------------------|-----------|---------|--------------------|----------------------|---------|---------|---------|---------|---------|----------------------------------------|
| NSGA-II (Sivakumar et al., 2011b) | \(X_1\)   | 3       | \(tX_1\)           | 0.069665             | 0.119203| 0.194848| 0.179988| 1076.895| 0.124079| 2.2774136                              |
|                            | \(X_{2a}-X_{2b}\) | 3       | \(tX_{2a},tX_{2b}\) | 0.049538             |         |         |         |         |         |                                        |
|                            | \(X_{3a}-X_{3b}\) | 3       | \(tX_{3a},tX_{3b}\) | 0.046234             |         |         |         |         |         |                                        |
|                            | \(X_4\)   | 2       | \(tX_4\)           | 0.020007             |         |         |         |         |         |                                        |
|                            | \(X_5\)   | 2       | \(tX_5\)           | 0.021279             |         |         |         |         |         |                                        |
| MOPSO (Sivakumar et al., 2011b) | \(X_1\)   | 3       | \(tX_1\)           | 0.087427             | 0.137138| 0.197005| 0.195297| 1029.393| 0.154244| 2.1955228                              |
|                            | \(X_{2a}-X_{2b}\) | 3       | \(tX_{2a},tX_{2b}\) | 0.049711             |         |         |         |         |         |                                        |
|                            | \(X_{3a}-X_{3b}\) | 3       | \(tX_{3a},tX_{3b}\) | 0.047872             |         |         |         |         |         |                                        |
|                            | \(X_4\)   | 2       | \(tX_4\)           | 0.027344             |         |         |         |         |         |                                        |
|                            | \(X_5\)   | 2       | \(tX_5\)           | 0.024337             |         |         |         |         |         |                                        |
| TLBO                       | \(X_1\)   | 3       | \(tX_1\)           | 0.137619             | 0.186008| 0.191812| 0.196046| 1008.7  | 0.2466  | **2.1814**                              |
|                            | \(X_{2a}-X_{2b}\) | 3       | \(tX_{2a},tX_{2b}\) | 0.048389             |         |         |         |         |         |                                        |
|                            | \(X_{3a}-X_{3b}\) | 3       | \(tX_{3a},tX_{3b}\) | 0.046645             |         |         |         |         |         |                                        |
|                            | \(X_4\)   | 2       | \(tX_4\)           | 0.030979             |         |         |         |         |         |                                        |
|                            | \(X_5\)   | 2       | \(tX_5\)           | 0.025132             |         |         |         |         |         |                                        |

Note: The number in bold indicate the better values.
Table 5. Optimization results for the overrunning clutch assembly.

| Technique         | Machine | Tolerance notation | Tolerance value (mm) | Machine | Tolerance notation | Tolerance value (mm) | Objective functions | Combined objective function ($f_c$) |
|-------------------|---------|--------------------|---------------------|---------|--------------------|---------------------|---------------------|----------------------------------|
| NSGA-II (Sivakumar et al., 2011b) | 1       | $t_{11}$           | .261619             | 1       | $t_{31}$           | .196208             | .0112              | 288.175 13.201 1.436501          |
|                   | 3       | $t_{12}$           | .165386             | 2       | $t_{32}$           | .094096             | .0127              | 271.952 13.702 1.404177          |
|                   | 2       | $t_{13}$           | .02747              | 2       | $t_{33}$           | .028307             | .0046              | 233.1067 3.2549 .8917            |
| MOPSO (Sivakumar et al., 2011b) | 1       | $t_{11}$           | .203162             | 1       | $t_{31}$           | .22952              | .0127              | 271.952 13.702 1.404177          |
|                   | 3       | $t_{12}$           | .200655             | 2       | $t_{32}$           | .11676              | .0127              | 271.952 13.702 1.404177          |
|                   | 2       | $t_{13}$           | .023635             | 2       | $t_{33}$           | .042379             | .0046              | 233.1067 3.2549 .8917            |
| TLBO              | 1       | $t_{11}$           | .385848             | 1       | $t_{31}$           | .465393             | .0046              | 233.1067 3.2549 .8917            |
|                   | 3       | $t_{12}$           | .24247              | 2       | $t_{32}$           | .121685             | .0046              | 233.1067 3.2549 .8917            |
|                   | 2       | $t_{13}$           | .010973             | 2       | $t_{33}$           | .0165781            | .0046              | 233.1067 3.2549 .8917            |

Note: The number in bold indicate the better values.
\[
k = \frac{Gd_w^4}{8(d_i + d_w)^3N}
\]

(37)

\[
d_o = d_i + 2d_w
\]

(38)

where \(k\) is the spring rate, \(G\) is the modulus of rigidity of the spring wire material, \(d_w\) is the wire diameter of spring, \(d_i\) is the inner diameter of spring, \(d_o\) is the outer diameter of spring, and \(N\) is the number of active coils in spring.

The objective function considered in this paper is same as that considered by Singh et al. (2005) and these are:

Minimize:

\[Z_1 = C_{asm} = C_{d_w} + C_{d_i} + C_n\]

(39)

where \(C_{asm}\) is the total assembly manufacturing cost, \(C_{d_w}\), \(C_{d_i}\), and \(C_n\) are manufacturing costs of wire diameter of spring, inner diameter of spring and number of active coils in spring, respectively.

For each manufacturing operation, alternative machines are available with the details given in Table 3.

The following tolerance stack-up constraints are considered:

Table 6. Optimization results for the helical spring.

| Dimension | Process | Individual tolerance value (mm) | Resultant assembly tolerance | Minimum cost |
|-----------|---------|---------------------------------|-------------------------------|--------------|
| GA (Singh et al., 2005) |
| \(d_w\) | 2 | .02 | \(\Delta K = .04770^*\) | 5.94 |
| \(D_i\) | 1 | .4645 | \(\Delta d_o = .5045\) | |
| \(N\) | 2 | .2054 | | |
| TLBO |
| \(d_w\) | 2 | .02007159 | \(\Delta K = .08105\) | 5.8365 |
| \(D_i\) | 1 | .49925285 | \(\Delta d_0 = .5392\) | |
| \(N\) | 2 | .17866543 | | |

*Corrected value is \(\Delta K = .08561\).
Note: The number in bold indicates the better value.

Table 7. Results obtained by using NSGA-II, MOPSO, and TLBO algorithms for the multi-objective optimization problems.

| Example 1 (Overrunning clutch assembly) | \(Z_1\) | \(Z_2\) | \(Z_3\) | \(Z_4\) | \(Z_5\) | Combined objective function (\(f_c\)) |
|----------------------------------------|---------|---------|---------|---------|---------|---------------------------------------|
| TLBO | .0046 | 233.1067 | 3.2549 | NA | NA | **.8917** |
| NSGA-II | .0112 | 288.175 | 13.201 | NA | NA | 1.436501 |
| MOPSO | .0127 | 271.952 | 13.702 | NA | NA | 1.404177 |

Example 2 (Knuckle joint assembly with three arms)

| Example 2 (Knuckle joint assembly with three arms) | \(Z_1\) | \(Z_2\) | \(Z_3\) | \(Z_4\) | \(Z_5\) | Combined objective function (\(f_c\)) |
|-------------------------------------------------|---------|---------|---------|---------|---------|---------------------------------------|
| TLBO | .186008 | .191812 | .196046 | 1008.7 | .2466 | **2.1814** |
| NSGA-II | .119203 | .194848 | .179988 | 1076.895 | .124079 | 2.2774136 |
| MOPSO | .137138 | .197005 | .195297 | 1029.393 | .154244 | 2.1955228 |

Note: NA – not applicable.
The numbers in bold indicate the better values.
\[
\frac{G(d_w + t_{wj})^4}{8\{(d_i - t_{ij}) + (d_w + t_{wj})\}^3(N - t_{Nj})} - \frac{G(d_w + t_{wj})^4}{8\{(d_i + t_{ij}) + (d_w - t_{wj})\}^3(N + t_{Nj})} \leq .477 \tag{40}
\]

\[
t_{ij} + 2t_{wj} \leq .508 \tag{41}
\]

For each manufacturing operation, alternative machines, parameter cost functions, and tolerance limits for the helical spring are given in Table 3.

5. Results and discussion

Tables 4–6 compare the optimum selection of the manufacturing process (machine), allocated tolerance value, the values of objective functions, and the combined objective function value obtained by using various optimization techniques. For optimal result, overrunning clutch assembly, the knuckle joint assembly with three arms and the helical spring, were used population size of 50 and number of generations of 20, 60, and 40, respectively, for overrunning clutch assembly, the knuckle joint assembly with three arms, and the helical spring. As the evaluation is done in the both the teacher and student phases, the number of function evaluation used in TLBO is calculated as, Number of function evaluation = Population size * number of generations * 2. Thus the function evaluations are 2000, 6000, and 4000, respectively, in the case of for overrunning clutch assembly, the knuckle joint assembly with three arms, and the helical spring. It is observed that TLBO algorithm gives better results than GA, NSGA-II, and MOPSO algorithms. The TLBO algorithm is found superior to GA, NSGA-II, and MOPSO for

![Figure 7. Convergence of combined objective function obtained by TLBO algorithm for Overrunning clutch assembly.](image)
Figure 8. Convergence of combined objective function obtained by TLBO algorithm for Knuckle joint assembly with three arms.

Figure 9. Convergence of combined objective function obtained by TLBO algorithm for Helical spring.
all the three case studies considered, i.e. overrunning clutch assembly, the knuckle joint assembly with three arms, and the helical spring. Table 7 shows that the results obtained using NSGA-II, MOPSO, and TLBO algorithms for the multi-objective optimization problems. Each problem is run 30 times. Standard deviation for clutch assembly is .00156, for knuckle joint assembly is .0246, and for spring design is .0894.

Figures 7–9 show the convergence rate of the TLBO algorithm for the three examples. In the example of overrunning clutch assembly convergence takes place after 7th iteration, in knuckle joint assembly with three arms convergence takes place after 12th iteration, and in design of helical spring the convergence takes place after 11th iteration of the TLBO algorithm. It is noted that the TLBO algorithm gives the best results (i.e. minimum values of $Z_1$, and $Z_3$ for overrunning clutch assembly and minimum values of $Z_1$, $Z_2$, $Z_3$, and $Z_5$ for knuckle joint assembly) and the computational time to find the optimum solutions by TLBO algorithm is less than that of GA, NSGA-II, and MOPSO algorithms which implies that the TLBO algorithm is faster than the GA, NSGA-II, and MOPSO algorithms. Figure 10 shows the optimal solution trade-offs obtained from NSGA-II, MOPSO, and TLBO algorithms for the knuckle joint assembly. Figure 11 shows the optimal solution trade-offs obtained from NSGA-II, MOPSO, and TLBO algorithms for the overrunning clutch assembly.

In overrunning clutch assembly, TLBO gives 38 and 36.5% improvements in the combined objective function as compared to the results given by the NSGA-II and MOPSO algorithms, respectively. In the case of knuckle joint with three arms assembly, TLBO gives 4.2% and .62% better value of the combined objective function than those
given by NSGA-II and MOPSO algorithms, respectively. In helical spring design, the TLBO gives 1.742% reduction in total cost as compared to the result given by the GA. Table 7 shows the values of all the objective functions and the combined objective function for the multi-objective optimization problems.

6. Conclusions
Three different problems are considered in this paper for optimization of design and manufacturing tolerances of three machine elements: (i) overrunning clutch assembly, (ii) knuckle joint assembly with three arms, and (iii) helical spring. For the optimal tolerance allocation and alternative process selection for mechanical assemblies, a recently developed advanced optimization algorithm, called teaching-learning-based optimization (TLBO) algorithm, is considered. This algorithm does not require selection of algorithm-specific parameters. Out of the three problems considered, the first two are the multi-objective problems and the third is a single-objective problem and these problems are used to evaluate the strength of the TLBO algorithm. The same models were earlier attempted by other researchers using GA, SA, NSGA-II, and MOPSO algorithms. The results obtained using TLBO algorithm is compared with those obtained by using the NSGA-II, GA, and MOPSO algorithms. However, the TLBO algorithm has given considerable improvements in the results and in the convergence rate.

The TLBO algorithm has shown its ability in solving multi-objective optimization problems by using normalizing weighting objective function. The convergence behaviour of the TLBO algorithm to a near global solution has been observed to be
more effective than that of GA, NSA-II, and MOPSO algorithms. The results indicate that the TLBO technique gives better results than GA, NSGA-II, and MOPSO algorithms. Hence, the TLBO algorithm is proved better than the other optimization algorithms in terms of results and the convergence. The TLBO algorithm may be conveniently used for the optimal tolerance design of the other machine elements also.

References
Claudio, M. R., Jose, A. M., & Nestor, C. (2003). Robust design using a hybrid-cellular-evolutionary and interval-arithmetic approach: A reliability application. *Reliability Engineering and System Safety, 79*, 149–159.

Daniel, S., Claudio, M. R., & Blas, J. G. (2006). Optimization of constrained multiple-objective reliability problems using evolutionary algorithms. *Reliability Engineering and System Safety, 91*, 1057–1070.

Feng, C.-X., & Kusiak, A. (1997). Robust tolerance design with the integer programming approach. *Transaction of ASME Journal Manufacturing Science Engineering, 119*, 603–610.

Forouraghi, B. (2002). Worst-case tolerance design and quality assurance via genetic algorithms. *Journal of Optimization Theory and Applications, 113*, 251–268.

Forouraghi, B. (2009). Optimal tolerance allocation using a multiojective particle swarm optimizer. *International Journal of Advanced Manufacturing Technology, 44*, 710–724.

Gonzalez, I., & Sanchez, I. (2009). Statistical tolerance synthesis with correlated variables. *Mechanism and Machine Theory, 44*, 1097–1107.

Guofu, Z., Yuege, Z., Ye, X., & Hu, B. (2013). A method of multi-objective reliability tolerance design for electronic circuits. *Chinese Journal of Aeronautics, 26*, 161–170.

Haq, A. N., Sivakumar, K., Saravanan, R., & Muthiah, V. (2005). Tolerance design optimization of machine elements using genetic algorithm. *International Journal of Advanced Manufacturing Technology, 25*, 385–391.

Huang, Y. M., & Shiau, C.-S. (2006). Optimal tolerance allocation for a sliding vane compressor. *Journal of Mechanical Design, 128*, 98–107.

Huang, M. F., & Zhong, Y. R. (2007). Optimized sequential design of two-dimensional tolerances. *International Journal of Advanced Manufacturing Technology, 33*, 579–593.

Huang, M., & Zhong, Y. (2008). Dimensional and geometrical tolerance balancing in concurrent design. *International Journal of Advanced Manufacturing Technology, 35*, 723–735.

Krishna, G., Rao, K. M. (2006). Simultaneous optimal selection of design and manufacturing tolerances with different stack-up conditions using scatter search. *International Journal of Advanced Manufacturing Technology, 30*, 328–333.

Muthu, P., Dhanalakshmi, V., & Sankaranarayanasamy, K. (2009). Optimal tolerance design of assembly for minimum quality loss and manufacturing cost using metaheuristic algorithms. *International Journal of Advanced Manufacturing Technology, 44*, 1154–1164.

Pierluissi, L., & Rocco, C. M. (2007). Optimal design centring through a hybrid approach based on evolutionary algorithms and monte carlo simulation. ICANNGA’07, 8th International Conference Adapt Nat Comput Algo. Warsaw, Polonia, Lecture Notes in Computer Science (LNCS), series by Springer Verlag, 31–38.

Prabhaharan, G., Asokan, P., & Rajendran, S. (2005). Sensitivity-based conceptual design and tolerance allocation using the continuous Ants colony algorithm (CACO). *International Journal of Advanced Manufacturing Technology, 25*, 516–526.

Rao, R. V., & Kalyankar, V. D. (2012). Parameter optimization of machining processes using a new optimization algorithm. *Materials and Manufacturing Processes, 27*, 978–985.

Rao, R. V., & Patel, V. (2012). Comparative performance of an elitist teaching-learning-based optimization algorithm for solving unconstrained optimization problems. *International Journal of Industrial Engineering Computations, 4*, 29–50.

Rao, R. V., & Patel, V. (2013a). Multi-objective optimization of two stage thermo electric cooler using a modified teaching–learning-based optimization algorithm. *Engineering Applications of Artificial Intelligence, 26*, 430–445.

Rao, R. V., & Patel, V. (2013b). An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems. *International Journal of Industrial Engineering Computations, 3*, 535–560.
Rao, R. V., & Savsani, V. J. (2012). Mechanical design optimization using advanced optimization techniques. London: Springer-Verlag.

Rao, R. V., Savsani, V. J., & Vakharia, D. P. (2011). Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems. Computer-Aided Design, 43, 303–315.

Rao, R. V., Savsani, V. J., & Vakharia, D. P. (2012). Teaching–learning-based optimization: An optimization method for continuous non-linear large scale problem. Information Sciences, 3, 1–15.

Rocco, C. M., Moreno, J., & Carrasquero, N. (2003). Robust design using a hybrid-cellular-evolutionary and interval-arithmetic approach: A reliability application. Reliability Engineering and System Safety, 79, 149–159.

Salazar, D., Rocco, C. M., & Galvan, B. (2006). Optimization of constrained multiple-objective reliability problems using evolutionary algorithms. Reliability Engineering and System Safety, 91, 1057–1070.

Singh, P. K., Jain, S. C., & Jain, P. K. (2003). Tolerance allocation with alternative manufacturing processes-suitability of genetic algorithm. International Journal of Modeling and Simulation, 2, 22–34.

Singh, P. K., Jain, S. C., & Jain, P. K. (2004). A GA based solution to optimum tolerance synthesis of mechanical assemblies with alternate manufacturing processes: Focus on complex tolerancing problems. International Journal of Simulation and Modelling, 42, 5185–5215.

Singh, P. K., Jain, P. K., & Jain, S. C. (2005). Advanced optimal tolerance design of mechanical assemblies with interrelated dimension chain and process precision limits. Computers in Industry, 56, 179–194.

Singh, P. K., Jain, P. K., & Jain, S. C. (2008). Optimal tolerance design of mechanical assemblies for economical manufacturing in the presence of alternative machines- a genetic algorithm-based hybrid methodology. Proceeding Institution of Mechanical Engineers Journal of Engineering Manufacture, 222B, 591–604.

Singh, P. K., Jain, P. K., & Jain, S. C. (2009). Important issues in tolerance design of mechanical assemblies. Part-2: Tolerance synthesis. Proceedings IMechE Journal of Engineering Manufacture, 223B, 1249–1287.

Sivakumar, K., Balamurugan, C., & Ramabalan, S. (2011a). Concurrent multi-objective tolerance allocation of mechanical assemblies considering alternative manufacturing process selection. International Journal of Advanced Manufacturing Technology, 53, 711–732.

Sivakumar, K., Balamurugan, C., & Ramabalan, S. (2011b). Simultaneous optimal selection of design and manufacturing tolerances with alternative manufacturing process selection. Computer-Aided Design, 43, 207–218.

Sivakumar, K., Balamurugan, C., & Ramabalan, S. (2012). Evolutionary multi-objective concurrent maximisation of process tolerances. International Journal of Production Research, 50, 3172–3191.

Sivakumar, M., Kannan, S. M., & Jayabalan, V. (2009). A new algorithm for optimum tolerance allocation of complex assemblies with alternative processes selection. International Journal of Advanced Manufacturing Technology, 40, 819–836.

Sivakumar, M., & Stalin, B. (2009). Optimum tolerance synthesis for complex assembly with alternative process selection using Lagrange multiplier method. International Journal of Advanced Manufacturing Technology, 44, 405–411.

Wu, F., Jean-Yves, D., Alain, E., Ali, S., & Patrick, M. (2009). Improved algorithm for tolerance allocation based on Monte Carlo simulation and discrete optimization. Computers and Industrial Engineering, 56, 1402–1413.

Ye, B., & Salustri, F. A. (2003). Simultaneous tolerance synthesis for manufacturing and quality. Research in Engineering Design, 14, 98–106.