Higher Harmonics in Heavy Ion Collisions

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Abstract. As the QGP expands and cools, it carries much information on its creation and evolution imprinted on the patterns of higher harmonic flow. In this proceeding we report on the progress in simulating and understanding the higher harmonics by the McGill group using the 3+1D event-by-event viscous hydrodynamics simulation suite named MUSIC.

1. Introduction
One of the major goals of heavy ion collision phenomenology is to study the creation and the subsequent evolution of the Quark-Gluon Plasma (QGP). Accomplishing this goal requires a remarkable range of theoretical tools, from classical and quantum field theory to Monte-Carlo simulations of hadronic collisions. Currently, one of the most important theoretical tools is the relativistic hydrodynamics.

While the whole of heavy ion collision may take around 15 fm/c, the system spends a bulk of that time in the hydrodynamic phase. Hydrodynamics studies assume that after a brief initial thermalization phase (on the order of 1 fm/c), hydrodynamics begins to apply. The properties of QGP such as the equation of state and the viscosities then critically influence subsequent evolution of the system. Hydrodynamics is essentially the response of a fluid to its local pressure gradient. Therefore, the imprint of the QGP properties needs to be sought in the fluid properties reflected in the final state hadrons. More explicitly, in the global shape of the momentum distribution of hadrons.

Mathematically, the shape is encoded in the coefficients of the Fourier series

$$\frac{dN}{dyd^2p_T} = \frac{1}{2\pi} \frac{dN}{dydp_Tdp_T} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \psi_n)) \right)$$

where $\psi_n$ is the $n$-th order event-plane angle defined as

$$\psi_n = \frac{1}{n} \tan^{-1} \left( \frac{\langle p_T \sin(n\phi) \rangle}{\langle p_T \cos(n\phi) \rangle} \right).$$

It is customary to choose the orientation in such a way that $v_1 = 0$. The second coefficient $v_2$ is usually referred to as the elliptic flow and the third coefficient $v_3$, the triangular flow. The elliptic flow is a measure of how efficiently hydrodynamics translates the spatial anisotropy to the momentum anisotropy. Triangular flow depends more on the spatial distribution of the initial energy density fluctuations. Higher harmonic coefficients encode more detailed information on the initial fluctuations and their propagation through the QGP evolution. Therefore, how well
one understands these flow coefficients is an indication of how well one understands the bulk properties of QGP.

Hydrodynamic studies of heavy ion collisions have a long history [1]. Until recently, most of the study was conducted using the boost-invariant 2+1D ideal hydrodynamics with smooth averaged initial conditions (For reviews, see Refs.[2, 3]). More recent efforts have generalized this to include viscosity [4, 5], 3+1D dynamics [6, 7], and fluctuating initial conditions [8]. However, none of the above mentioned studies has implemented all three improvements over the 2+1D ideal hydrodynamics. Only recently, the first implementation of 3+1D event-by-event viscous hydrodynamics was made by the McGill group [9, 10, 11]. In this proceeding, I report on recent progresses made by the McGill group based mostly on Ref.[11].

2. 3+1D Event-by-Event Viscous Hydrodynamics

Our implementation of hydrodynamics (named MUSIC) uses the hyperbolic $\tau-\eta_s$ coordinate system but without assuming the invariance in $\eta_s$. For the evolution of the viscous tensor $\pi^{\mu\nu}$, we use a variant of the Israel-Stewart formalism [12]

$$\Delta^{\mu\nu}\Delta^{\alpha\beta}u^\alpha \partial_\gamma \pi_{\alpha\beta} = -\frac{1}{\tau_\pi} \left( \pi^{\mu\nu} - 2\eta \nabla^{(\mu} u^{\nu)} + \frac{4}{3} \tau_\pi \pi^{\mu\nu} (\partial_\alpha u^\alpha) \right)$$

(3)

where $u^\mu$ is the flow velocity and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the local 3-metric. For solving conservation equations, we use a recently developed algorithm of Kurganov and Tadmor [13] in the semi-discrete form together with the Heun’s method. This is relatively simple to implement yet very stable. The fluctuating initial condition is implemented by using the Glauber approximation. Namely, the nucleons are first distributed in the projectile and the target nuclei according to the Wood-Saxon distribution function. Then according to their transverse position and the nucleon-nucleon cross-section the colliding pairs are chosen. At each collision point, a Gaussian shaped energy density is assigned. The width of the Gaussian is chosen not to exceed $c\tau_0$ where $\tau_0$ is the initial time when hydrodynamic evolution begins. The energy density is adjusted to reproduce the final state hadron entropy.

For the equation of state, our choice is the lattice based parameterization “s95p-v1” from [14] although other choices are also available. Freeze-out space-time point for each cell at $(x, y, \eta_s)$ is given by

$$\Sigma = (\tau_f(x, y, \eta_s) \cosh \eta_s, x, y, \tau_f(x, y, \eta_s) \sinh (\eta_s))$$

(4)

where $\tau_f$ is the freeze-out time when the local temperature reaches the freeze-out temperature. Cooper-Frye formula then is used with the normal vector given by

$$d^3 \Sigma_\mu = -\epsilon_{\mu\nu\lambda\rho} \frac{\partial \Sigma^\nu}{\partial x} \frac{\partial \Sigma^\rho}{\partial y} \frac{\partial \Sigma^\lambda}{\partial \eta_s} dxdyd\eta_s$$

(5)

to get the final state hadronic spectrum with the full resonance decays adapted from Heinz and Kolb’s 2-D scheme. The viscous correction to the thermal spectrum is given by

$$\delta f = f_0 (1 \pm f_0) \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2sT^3}$$

(6)

where the $\pm$ signs are for Bosons (+) and Fermions (−) and $s$ is the entropy density. Up to $p^2 = O(T)$, this is $O(\eta/s)$ or smaller.

3. Results and Discussion

In our previous publications, we have shown that our implementation of the hydrodynamics reproduces very well the measured elliptic flows at RHIC with the shear viscosity to entropy
ratio of \( \eta/s = 1/4\pi \). We have also predicted a sizable triangular flow due to the initial state fluctuations which has been confirmed by the experiment [16] indicating that at RHIC, the ratio \( \eta/s \) is indeed very small. Our calculations for the LHC are shown in Figures 1 and 2. Shown in Figure 1 are the results for the elliptic flow for the 10–20 \% centrality class and the 30–40 \% centrality class. For more central collisions, it does not make much difference whether the average initial condition is used or the fluctuating initial condition is used. This is because for more central collisions, the nucleon-nucleon collision sites are close together. Therefore a single event does not look much different from the average one. One can also see that it makes little difference whether \( \eta/s = 1/4\pi \) or \( \eta/s = 1/2\pi \). This is due to the fact that the eccentricity is not so large in this centrality class. The short amount time it takes for the system to cease developing anisotropic flow is in effect too short for the difference in the viscosity to manifest.

For the 30–40 \% centrality class, average initial condition and fluctuating initial condition lead to fairly different results. This is due to the fact that in this centrality class, the nucleon-nucleon collision sites are far enough apart that the average eccentricity tends to be smaller in the event-by-event cases. It is also evident from these figures that \( \eta/s \) at the LHC is bigger than that at RHIC by a factor of 2. This is not unexpected since higher temperature at the LHC would imply smaller \( \alpha_S \) which in turn would imply higher \( \eta/s \).

Shown in Figure 2 are our calculations of the integrated \( v_2 \) and \( v_3 \) compared to the ALICE
data. We underestimate $v_2$ and $v_3$ slightly. This may be due to the fact that the Glauber initial condition tends to underestimate the eccentricity. More study is underway.

4. Conclusions and Outlook
Using the first implementation of 3+1D event-by-event viscous hydrodynamics, we have begun to investigate the higher harmonic components of the QGP flow. As the experiments accumulate more data, the higher harmonics can potentially provide a very discriminating probe of the nature of QGP and its evolution throughout relativistic heavy ion collisions. Our investigation with the Glauber initial conditions shows that the viscosity parameter at the LHC is about twice as big as the viscosity parameter at RHIC in accordance with the asymptotic freedom property of QCD. Studies with other initial conditions such as the IP-Sat model are underway. Efforts to make improvements such as the hadronic afterburner and jet-fluid interactions are also underway.

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