Inverted Sparticle Hierarchies from Natural Particle Hierarchies

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ABSTRACT

A possible resolution of the flavor puzzle is that the fermion mass hierarchy can be dynamically generated through the coupling of the first two generation fields to a strongly coupled sector, which is approximately conformally invariant and leads to large anomalous dimensions for the first two generation fields over a large range of energies. We investigate the possibility of using the same sector to also break supersymmetry. We show that this automatically gives an “inverted hierarchy” in which the first two generation squarks and sleptons are much heavier than the other superpartners. Implementing this construction generically requires some fine-tuning in order to satisfy the constraints on flavor-changing neutral currents at the same time as solving the hierarchy problem. We show that this fine-tuning can be reduced to be milder than the percent level by making some technically natural assumptions about the form of the strongly coupled sector and its couplings to the standard model.

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1 Introduction

The standard model (SM), when embedded into a theory valid at energy scales higher than a TeV, contains two types of unnaturally small couplings: the mass-squared of the Higgs field (the hierarchy problem), and many of the Yukawa couplings and mixing angles in both the quark and lepton sectors (the flavor puzzle). Both issues can be addressed in various ways, and some of the most elegant solutions involve adding to the theory additional sectors that decouple at some high energy scale.

A natural solution to the hierarchy problem is supersymmetry (SUSY), which to avoid fine-tuning must be broken softly at energies not much higher than the weak scale (see [1] for a review). This breaking cannot happen within the simplest supersymmetric generalization of the SM itself, and so an additional sector breaking supersymmetry is usually required, as well as some mechanism for coupling the two sectors such that SUSY breaking is transmitted to the SM.

The flavor puzzle can be addressed by introducing approximate horizontal flavor symmetries which suppress the small Yukawa couplings [2]. Another way to solve the flavor puzzle is to consider a strongly coupled and approximately conformal sector [3]. The quarks and leptons of the first two generations can obtain large anomalous dimensions through direct couplings to this “conformal field theory (CFT) sector”. If the conformal regime persists for a large range of energy scales, renormalization group (RG) evolution can lead to realistic hierarchical Yukawa couplings at low energies, starting from an unstructured (anarchical) UV theory. The resulting Yukawa structures are similar to those obtained using horizontal symmetries.

In principle the above solutions can be combined by making the “flavor CFT sector” supersymmetric, and some implications of this were discussed in [7, 8]. However, it is natural to wonder whether it may be possible to use the flavor CFT sector also for supersymmetry breaking. This could allow for the construction of more economical models with fewer additional sectors that have not yet been observed. Traditionally models with less unobserved sectors have been preferred, following Occam’s razor, though in the context of a complete theory of nature like string theory it is not clear if generic vacua, agreeing with all observations made so far, contain a small or a large number of sectors. In this paper we will simply assume that the same sector plays both roles described above – it generates the flavor hierarchy through large anomalous dimensions and it also breaks supersymmetry.

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2 We will refer to the sector responsible for generating the flavor hierarchy as the CFT sector, even though its dynamics is only approximately conformal in some range of energies.

3 As discussed in [7], if one assumes that the SUSY breaking is transmitted to the flavor CFT sector above the scale where this sector is superconformal, the RG flow of the CFT sector naturally suppresses also SUSY-breaking flavor-violating terms, so it helps in solving the supersymmetric flavor problem.
and we will analyze the phenomenological implications of this assumption.

Our assumption, however, cannot be taken at face-value since it immediately leads to a severe tension between an appropriate suppression of flavor changing neutral currents (FCNCs) and naturalness of the Higgs sector. On the one hand, the dynamical scale of our CFT sector determines the scale of SUSY breaking, and this cannot be too large without introducing fine-tuning of the Higgs mass (for instance, the supersymmetric partner of the top quark should not be much heavier than a TeV or so [10, 11, 12]). On the other hand, and unlike models with flavor-universal SUSY breaking, in our models the first two generations must directly couple to the CFT sector in a flavor non-universal way in order to generate the flavor hierarchy. Generic non-universal flavor dynamics leads to large FCNCs unless it decouples from the standard model fields at a rather heavy scale; the experimental constraints on FCNCs place lower bounds on the scale of flavor-violating new physics that are of the order of tens of thousands of TeVs [13, 14]. In general, an even more serious problem would arise from the experimental bounds on CP violation [13, 14]. In our analysis in most of this paper we consider the bounds from CP-conserving processes; in the final section we discuss the implications of the stronger bounds from CP violation.

The above fine-tuning problem does not have to be faced in its most severe form, since in our models there is always some built-in hierarchy between the scale of the stop mass and the scale of flavor-violating effects involving the first two generations, to which the strongest FCNC bounds apply. The superpartners of the first two generations couple directly to the CFT sector and obtain masses of the order of the SUSY breaking scale. The existence of these direct couplings implies that the CFT sector must have a symmetry group (which is a global symmetry group from the point of view of the CFT sector) that contains the SM gauge group as a subgroup. As we do not allow direct couplings for them (which would destroy the flavor hierarchy), the third generation scalars, gauginos and Higgs sector fields will obtain SUSY breaking masses mostly through gauge-mediated couplings to the CFT sector [13, 14]. These masses will then be suppressed by an extra small factor from SM gauge couplings. Such inverted hierarchy models [14, 15, 16], where the first two generation scalars are relatively heavy, have been considered before as a possible resolution of the supersymmetric flavor problem, from a variety of different perspectives [21, 22, 23, 24, 25, 26, 27]. In our models this feature is an automatic outcome of our assumptions.

Unfortunately, the natural inverted hierarchy obtained from the above considerations is still not large enough to reduce the fine-tuning to acceptable levels. One reason for this is that our CFT sector generally contains many fields charged under the SM gauge group, so that in the language of gauge mediation we have a large effective number of messengers [28], reducing the suppression of the gauge-mediated superpartner masses compared to
the directly-mediated masses. Another issue to address is a generic problem in inverted hierarchy models, where the two-loop RG flow often leads to negative masses-squared for some of the third generation sfermions, which must be avoided [29, 30, 31].

We are thus led to make several additional assumptions. We first require that the CFT sector does not directly couple the first generation and the second generation fields (“separable CFT sectors”). This can be accomplished either by having a “decomposable CFT sector”, in which the fields coupled to the first generation do not couple to the second and vice versa (implying a factorization of the CFT sector gauge group), or by introducing some horizontal-type symmetry which suppresses couplings between the first two generations. The assumption of separable CFT sectors plays two important roles:

- It helps to suppress FCNCs. Naively FCNCs are completely absent in such a setup, but one must keep in mind that the decoupling between the generations discussed here is in the interaction basis, so in the mass basis some flavor-non-diagonal couplings suppressed by powers of the mixing angles between the first two generations are always present. (For quarks, these are typically of order the Cabibbo angle $\sim 0.22$ or smaller.) Thus, separability helps alleviate the fine-tuning problem, but does not solve it completely.

- It helps to solve the “graceful exit” problem in models of dynamically-generated flavor hierarchies as in [3, 4]. It was emphasized there that the dynamics responsible for the breaking of the conformal symmetry and the superconformal $U(1)_R$ symmetry of the CFT sector should not generate a large mixing in the kinetic terms of the first two generation fields, since this would wash out the hierarchy generated between their Yukawa couplings in the conformal range of energies. The assumption of separable CFT sectors automatically solves this problem, as it implies that there is no additional source of mixing between the first two generation fields.

Even if the CFT sector is separable, we are still left with a rather large amount of fine-tuning, so we then consider two additional ways to reduce this in our models. First, we assume the scale of SUSY breaking $\sqrt{F}$ to be much smaller than the scale $M$ at which the SM fields decouple from the CFT sector. This hierarchy can be achieved – as in traditional models of gauge mediation [32, 33] – if the fields in the CFT sector coupling to the standard model fields have a mass which is larger than that of other fields in the CFT sector involved in the SUSY breaking, or by some other mechanism which suppresses the SUSY breaking scale [34]. This allows us to raise the scale $M$ at which generic flavor-violating operators are generated, though the fact that in these models the first two generation sfermions are much lighter than the scale $M$ implies that one must separately worry about flavor-violating effects involving sfermion loops [35, 36]. Second, we assume that only some of
the first two generation fields directly couple to the CFT sector, while others do not; in particular we consider models where only the left-handed superfields couple directly to the CFT sector, or only the right-handed superfields, or only the superfields which are in the \textbf{10} representation of $SU(5)$ (in the language of $SU(5)$ grand unified (GUT) models). Such partial couplings are natural in brane realizations of the standard model. We are then able to suppress the strongest flavor-violating effects coming from chirality mixing operators in the down sector, and still solve the flavor puzzle by generating large anomalous dimensions for those fields which do couple directly to the CFT sector.

After making all the assumptions mentioned above, we find that only a small fine-tuning (milder than the percent level) is necessary in order to avoid FCNCs and solve the hierarchy problem, taking all couplings and correlation functions of the CFT sector to be otherwise generic. While the CFT sector must obey many requirements, all of these are technically natural and could arise in the context of some complete theory of high-energy physics. Finding an explicit model satisfying all these conditions is complicated, and we do not consider it here, but there is no apparent obstruction to such constructions.\footnote{See \cite{37} for recent examples of superconformal models with dynamical SUSY breaking, and \cite{38} in the context of metastable SUSY breaking.}

The outline of this paper is the following. We begin in Section 2 by reviewing how flavor hierarchies can be generated by a CFT, as well as the experimental constraints on FCNCs. In Section 3 we present a general discussion of our models in which all of the first two generation fields are coupled to the CFT sector, and we introduce the notion of a separable CFT sector. We briefly consider one-scale models, $F \sim M^2$, and then move to a more detailed analysis of two-scale models, $F \ll M^2$. In Section 4 we discuss partially coupled scenarios, which result in less fine-tuning, and present some examples of the spectra of superpartners arising from models of this type. We end in Section 5 with a summary of our results and conclusions.

\section{Review}

In this section we review the two main tools of our constructions. Section 2.1 reviews the Nelson-Strassler mechanism, namely how a CFT sector can explain the intergenerational mass hierarchy. In Section 2.2 we outline the main experimental FCNC bounds derived from general dimension-six flavor-violating operators and from diagrams with squark-gluino boxes. These will be basic constraints on all of our models.
2.1 CFTs and flavor hierarchies

2.1.1 The Nelson-Strassler Mechanism

In [6], Nelson and Strassler considered a mechanism that dynamically generates the IR hierarchical structure of the minimal supersymmetric standard model (MSSM) Yukawa couplings starting from a flavor anarchical UV theory. In these models, short-distance Yukawa couplings are arbitrary order one matrices, and the pattern of masses and mixing angles at low energies arises as the result of renormalization group flow.

The basic idea is the following. Unlike in asymptotically free theories in the weak coupling regime, where dimensionless couplings have logarithmic running, in a nearly scale-invariant theory such couplings can have power law running. Furthermore, if the theory is not weakly coupled, the anomalous dimensions of such couplings can be significant. Following an initial idea put forward in [5], the Nelson-Strassler mechanism is the application of this idea to the Yukawa couplings. When light quarks and leptons have direct couplings to a strongly coupled conformal sector, they can acquire substantial anomalous dimensions. As a result, a large Yukawa hierarchy can be generated if the conformal regime persists over a wide enough range of energy scales, and if the anomalous dimensions of these fields are appreciable.

We will assume below that the fixed point theory, near which the RG flow occurs, has $\mathcal{N} = 1$ SUSY. In this case the anomalous dimensions, and hence the IR Yukawa couplings, are determined by the $U(1)_R$ charges of the fixed point theory [39]. This $U(1)_R$ is approximate and may be accidental from the UV point of view. The resulting masses and mixing angles resemble those predicted in models with horizontal Abelian symmetries, but they differ from these models in that the R-symmetry which sets the charges arises dynamically and accidentally. Furthermore, the characteristic ratio between different Yukawas is not set by a vacuum expectation value (VEV) of a field, but by the number of energy decades spent near the fixed point.

To be more concrete and to set the notation, we consider an $\mathcal{N} = 1$ supersymmetric theory with gauge group $\tilde{G}$, charged matter $Q$, neutral matter $X$ and a superpotential $W(X, Q)$, and we will assume that it flows to a conformal theory in the infrared. Here $Q$ is not to be confused with the SM quarks, which are CFT sector singlets and are included in the $X$ fields.

By unitarity all gauge-invariant operators (except the identity operator) have scaling dimensions $\dim[O(Q, X)] \geq 1$ [40], with a strict inequality when $O$ is not a free field. As a consequence, $Q$ may have a negative anomalous dimension but $X$, being gauge-invariant,

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5We assume for simplicity that the SUSY generalization of the standard model contains only the MSSM. Our considerations are easily generalized when additional fields are present.
always has anomalous dimension \( \gamma_X = 2[\dim[X] - 1] \geq 0 \). Consider now superpotential terms of the form \( c_{k,\mathcal{O}}X^k\mathcal{O}(Q) \), where \( \mathcal{O}(Q) \) is a non-trivial \( \tilde{G} \)-invariant operator built from the charged fields \( Q \), which therefore has \( \dim[\mathcal{O}(Q)] > 1 \). At or near the superconformal fixed point, most of these terms are irrelevant since \( \dim[X^k\mathcal{O}(Q)] > k + 1 \), and thus if \( k \geq 2 \) the coupling \( c_{k,\mathcal{O}} \) will flow to zero. Conversely, relevant superpotential terms can drive the theory to a fixed point in which fields of type \( X \) possess positive anomalous dimensions.

We will be interested in terms of the form \( X\mathcal{O}(Q) \) (for \( \dim[\mathcal{O}(Q)] < 2 \)), which can exist at a non-trivial fixed point when \( 0 < \gamma_X < 2 \).

If the superconformal sector has a global symmetry \( G \) – in which the standard model gauge group \( G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \) can be embedded – one can consider the following scenario. The effective theory has gauge group \( \tilde{G} \times \hat{G} \), with \( \tilde{G} \) strongly coupled and an appropriate subgroup \( \hat{G} \) of \( G \), which contains \( G_{SM} \), weakly gauged. As far as \( \tilde{G} \) is concerned the SM quarks and leptons \( L_i, R_j (i, j = 1, 2) \) of the first two generations are of type \( X \) above – they are charged under \( \hat{G} \) but neutral under \( \tilde{G} \), and they can directly couple in the superpotential to charged matter of \( \tilde{G} \) by couplings of the form \( W = X\mathcal{O} \).

In this case, since they are \( X \)-type fields, the conformal dynamics associated to \( \tilde{G} \) may cause them to develop large positive anomalous dimensions. These will in turn cause their standard model Yukawa couplings \( W = y_{ij}L_iHR_j \) (where we canonically normalize the fields) to run as

\[
y_{ij}(\mu) = y_{ij}(\mu_0) \left( \frac{\mu}{\mu_0} \right) \frac{1}{2[\gamma(L^i) + \gamma(R^j)]}.
\]

We can now suppose that all Yukawa couplings are unstructured and of order one at some high scale, say the Planck or string scale \( M_0 \). If the gauge couplings of \( \tilde{G} \) and the direct couplings become conformal near some scale \( \mu_0 = M_\geq \) and remain so until some lower scale \( \mu = M_\leq \), with \( \epsilon \equiv M_\leq/M_\geq \ll 1 \), then the standard model Yukawa couplings will run down to

\[
y_{ij}(M_\leq) \sim \left( \frac{M_\leq}{M_\geq} \right)^{\frac{1}{2}(\gamma_{L_i} + \gamma_{R_j})} = \epsilon^{\frac{1}{2}(\gamma_{L_i} + \gamma_{R_j})} \equiv \epsilon_{L_i}\epsilon_{R_j}.
\]

We assume here (this will later be slightly modified) that the CFT sector has a mass gap of order \( M_\leq \), so that below this scale we are left only with the weakly coupled MSSM fields, with no remaining mixing to the CFT sector.

The above factorized texture of Yukawa couplings, with each fermion coming with its own suppression factor, leads to the following structure of mixing angles:

\[
(V^q_L)_{ij} \sim \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \sim (V^q_L)_{ji}, \quad (V^q_R)_{ij} \sim \frac{\epsilon_{R_i}}{\epsilon_{R_j}} \sim (V^q_R)_{ji} \quad (i < j), \quad q = u, d, \ell,
\]

\(6\)This is the running during the range of energies where the CFT sector is conformal. In addition, \((2.1)\) will obtain order one corrections from the entrance to and exit from this regime.
and fermion mass relations
\[
\frac{m_i}{m_j} \sim \frac{\epsilon L_i \epsilon R_i}{\epsilon L_j \epsilon R_j},
\]
where we assume without loss of generality \(\gamma_{L1} \geq \gamma_{L2}, \gamma_{R1} \geq \gamma_{R2}\). Combining the two above equations in the case of the quarks one obtains
\[
(V^q_{L,R})_{ii} \sim 1, \quad (V^q_{L})_{ij} \sim |V_{ij}|, \quad (V^q_{R})_{ij} \sim \frac{m_{qi}}{|V_{ij}|} \quad (i < j), \quad q = u, d,
\]
where \(V_{ij}\) is the CKM matrix \(V = V^u_L V^d_L\).

It is essential for our purposes that the measured Yukawa hierarchy can then be explained by coupling only the first two generations to the CFT. To this end, suppose a CFT has a spectrum of chiral operators \(O_{1,2}\) with distinct scaling dimensions \(d_{1,2}\) related to their R-charges \(r_{1,2}\) as \(d_{1,2} = \frac{3}{2}r_{1,2}\), and with \(r_1 < r_2 < 4/3\). If some linear combination of MSSM superfields couples to \(O_1\) and this coupling flows to a non-trivial fixed point, then at the fixed point this linear combination (which we can call the first generation and denote by \(\Phi_1\)) will have \(r_{\Phi_1} = 2 - r_1\) and \(d_{\Phi_1} = 3 - d_1\). If some other linear combination of MSSM superfields couples to \(O_2\), then this other linear combination will contain a component along the \(\Phi_1\) direction in flavor space whose coupling will be irrelevant at the fixed point, and a component orthogonal to \(\Phi_1\) which we can denote by \(\Phi_2\) (and which will be the second generation). Assuming that the \(O_2\Phi_2\) coupling also flows to a fixed point we will have at that fixed point \(r_{\Phi_2} = 2 - r_2\) and \(d_{\Phi_2} = 3 - d_2\). By construction there are no R-symmetry violating operators of the form \(O_1\Phi_2\). The R-symmetry violating operators \(O_2\Phi_1\) are irrelevant, so after the RG flow they will be suppressed by factors of order \(\epsilon^{1\gamma(\Phi_1) - \gamma(\Phi_2)}\). One can then start with a generic gauge-invariant Kähler potential and superpotential in the UV, and the conformal dynamics will guarantee that in the effective low-energy theory at the scale \(M_\prec\) the Kähler potential and superpotential are approximately flavor diagonal in the CFT basis, and exhibit a hierarchy in the Yukawa couplings.

2.1.2 Some further assumptions and requirements

After the conformal dynamics has done its job, some relevant deformation must drive the theory away from the fixed point. For example, the escape from the conformal window can be the result of small masses for some CFT sector particles, or of confinement of the CFT sector gauge group. It is important, however, that threshold effects at \(M_\prec\) do not spoil the hierarchy of the Yukawa couplings built during the conformal regime. In particular, flavor

\footnote{We assume that this new fixed point is stable, namely that it does not have any relevant operators that can appear in the Kähler potential. In our examples we will typically know that the original CFT before coupling to the MSSM fields is stable, and it seems reasonable to assume that this remains true after this coupling, but we do not prove this.}
off-diagonal wavefunction renormalizations $Z_{ij}$ should be small. This is the concept of a graceful exit \[6, 7\]. In our constructions this will be guaranteed through the notion of separable CFT sectors. In such scenarios, described in Section 3.1.2 below, intergenerational couplings are absent in the interaction (CFT) basis, up to terms similar to the previously discussed $O_2\Phi_1$. Such a term can induce off-diagonal wavefunction renormalizations of order $\epsilon_1/\epsilon_2$ (where $\epsilon_1$ and $\epsilon_2$ are typical suppression factors for the operators of the first two generations), which are small enough such that the Yukawa hierarchy in the canonical basis is not offset. Note that this implies that in our models we expect to have flavor violating terms at least of order $\epsilon_1/\epsilon_2$.

We should also worry about proton decay. As emphasized in \[6, 7\], if the CFT sector contains couplings violating baryon number, the graceful exit must take place above a scale $M_\prec \sim 10^{15}$ GeV in order to appropriately suppress proton decay from dimension six operators. Dimension five operators pose additional constraints. It is argued there, that even assuming that these terms are not generated by the CFT, they can already be present at $M_{pl}$ (R-parity can be imposed to forbid dimension four couplings, but it does not preclude the presence of dimension five operators). In that case the natural suppression factor $1/M_{pl}$ has to be strengthened by small coefficients in the range $(10^{-6} - 10^{-7})$ \[6, 11\]. In many of the scenarios considered by Nelson-Strassler (at small tan$\beta$) this is achieved by the same suppression factors $\epsilon_i$ in \[2.2\] responsible for the Yukawa hierarchy. We will assume in our constructions that the CFT sector preserves baryon number and that the appropriate suppression of dimension five operators at $M_{pl}$ proceeds along the same lines as just discussed. This allows for low $M_\prec$ and no proton decay.

To illustrate part of the above discussion, we now review two examples presented in \[3\]. We note that none of these models fully satisfies the requirements discussed above that need to be imposed on our CFT sectors. Apart from the fact that they do not break SUSY, the first example is separable but baryon-violating, whereas the second respects baryon number but is not separable. In this paper we are interested in a qualitative discussion on general classes of possible models, and are not concerned with concrete model building. We assume that a model satisfying all our requirements can be built.

### 2.1.3 An $SU(3)^3$ example

We first present an example which is separable but can induce proton decay. As mentioned, the scale $M_\prec$ must then be above $\sim 10^{15}$ GeV in order to suppress baryon number violating dimension six operators in the effective Kähler potential.

In this model the full symmetry group of the CFT sector is $SU(3)^3 \times Z_3 \times SU(5) \times SU(4)$. The group $SU(5) \times SU(4)$ is a gauge symmetry of the CFT sector under which all MSSM fields are neutral. These groups become strongly coupled at scales $\Lambda_5$ and $\Lambda_4$ respectively,
Table 1: Quantum numbers and scaling dimensions at the fixed point of the chiral superfields in the $SU(3)^3$ model.

|        | $SU(3)^3$                        | $SU(4)$ | $SU(5)$ | dimension |
|--------|----------------------------------|---------|---------|-----------|
| $27_1$, $27_2$, $27_3$ | $(3,3,1) + (1,3,3) + (3,1,3)$ | 1       | 1       | $\frac{5}{3}, \frac{4}{3}, 1$ |
| $27_H$, $27'_H$    | $(3,\bar{3},1) + (1,\bar{3},3) + (3,1,3)$ | 1       | 1       | 1, 1     |
| $27_H$, $27'_H$    | $(3,3,1) + (1,3,3) + (3,1,3)$ | 1       | 1       | 1, 1     |
| $Q$            | $(3,1,1) + (1,3,1) + (1,1,3)$   | 4       | 1       | $\frac{5}{3}$ |
| $\bar{Q}$        | $(\bar{3},1,1) + (1,\bar{3},1) + (1,1,3)$ | 4       | 1       | $\frac{5}{3}$ |
| $Q'$           | $(3,1,1) + (1,3,1) + (1,1,3)$   | 1       | 5       | $\frac{5}{3}$ |
| $\bar{Q}'$       | $(\bar{3},1,1) + (1,\bar{3},1) + (1,1,3)$ | 1       | $\bar{5}$ | $\frac{5}{3}$ |

and the couplings $g_5$ and $g_4$ eventually reach a fixed point. $SU(3)^3$ is a global symmetry of the CFT sector. After weakly gauging an appropriate subgroup, the first $SU(3)$ factor is identified with the color gauge group, the electroweak $SU(2)_L$ group resides in the second $SU(3)$, and hypercharge is embedded in the second and third $SU(3)$’s. $SU(3)^3$ can be seen as a subgroup of the GUT group $E_6$ [42], and the MSSM quarks and leptons are contained in three copies of $27 \equiv (3,\bar{3},1) + (\bar{3},1,3) + (1,3,3)$. In addition to the (grand unified generalization of the) usual chiral superfield content of the MSSM, this example contains chiral superfields $Q$ ($\bar{Q}$) and $Q'$ ($\bar{Q}'$) which transform in the fundamental (antifundamental) representations of $SU(4)$ and $SU(5)$, respectively. Including appropriate Higgs multiplets, the field content of the model is summarized in Table 1.

Apart from gauge and Yukawa interactions this model contains the following gauge and $\mathbb{Z}_3$-invariant superpotential:

$$W = \sum_{i=1,2} \lambda_i \bar{Q}Q 27_i + \lambda' \bar{Q}'Q'27_1.$$  \hspace{1cm} (2.6)

Here we used flavor rotations between the three generations to ensure that only the first generation couples to $\bar{Q}'Q'$ and only the first two to $\bar{Q}Q$, as described above; the third generation then does not couple directly to the CFT sector. With this convention the MSSM fields acquire the anomalous dimensions listed in Table 1 as we will now describe.

Going down in energies, the dynamics of this theory can be analyzed in terms of scales $\Lambda_5$ and $\Lambda_4$ (we take $\Lambda_5 > \Lambda_4$). The $SU(5)$ group has nine charged flavors, which drive the theory to a Seiberg conformal fixed point (since $\frac{3}{2}N_c < N_f < 3N_c$) [43]. At this fixed point the dimensions of gauge-invariant chiral operators are related to their R-charges as $d = \frac{3}{2}r$ through the superconformal algebra of the fixed point theory. The R-charges can in turn be read from the microscopic theory and result in $d_{Q'} = d_{Q''} = \frac{3}{2} \frac{N_f - N_c}{N_f} = \frac{2}{3}$. Therefore
the fields $\bar{Q}'$ and $Q'$ acquire negative anomalous dimensions of $\gamma_{\bar{Q}',Q'} = -2/3$ at the fixed point, making the coupling $\lambda'$ relevant and driving the theory to a new fixed point where the last term in the superpotential (2.6) is marginal. Assuming the existence of this fixed point, and given the anomalous dimensions of $\bar{Q}', Q'$, the field $27_1$ must acquire a positive anomalous dimension of $\gamma_{27_1} = 4/3$. This in turn causes $\lambda_1$ and the coupling of the $27_1$ to the Higgs field to become irrelevant and highly suppressed at low energies.

As we continue flowing, the $SU(4)$ theory becomes strongly coupled at the scale $\Lambda_4$, and its dynamics is also superconformal below this energy scale. In a similar fashion to the primed fields, $\bar{Q}$ and $Q$ acquire negative anomalous dimensions of $\gamma_{\bar{Q},Q} = -1/3$, causing the coupling $\lambda_2$ to become relevant, while $\lambda_1$ remains irrelevant. The coupling $\lambda_2$ drives the theory to a fixed point where the $\lambda_2$ term in the superpotential (2.6) is marginal. The anomalous dimension of the $27_2$ field at the fixed point is $\gamma_{27_2} = 2/3$, causing its Yukawa coupling to the Higgs fields to be suppressed at low energies.

The theory can naturally exit the conformal regime, for example, through confinement of an additional strong group $H$ with no extra matter charged under it. If we have non-renormalizable couplings, generated at some high scale $M_* \geq M_>$, of the form

$$\int d^2 \theta \frac{1}{M_*^2} \text{tr}(W^2_\alpha) \left( \bar{Q}Q + \bar{Q}'Q' \right)$$

(2.7)

where $W_\alpha$ are the gauge superfields of $H$, then this gives small masses to the $Q, \bar{Q}, Q', \bar{Q}'$ fields after condensation of the $H$ gauginos contained in $W^2_\alpha$, and leads $SU(5) \times SU(4)$ to a confining phase.\footnote{The term (2.7) is slightly enhanced by the negative anomalous dimension of the quark bilinears, but it can still easily give small masses.}

Since the anomalous dimension of $27_1$ is twice that of $27_2$, this model makes the generic prediction that $\epsilon_1 \sim \epsilon_2^2$ (assuming for simplicity that $\Lambda_5 \sim \Lambda_4$ and that both strong groups exit the conformal window around the same scale). However, since in this model suppression factors are universal within a generation, it predicts $m_u/m_t \sim m_4/m_6$, a prediction that is off by around two orders of magnitude. For more details on this model, see \cite{6}.

\subsection*{2.1.4 A “10-centered” example}

In this model the CFT sector does not induce proton decay and the exit scale $M_<$ could in principle be as low as 10 TeV. The gauge groups and matter content are listed in Table 3, classifying fields according to their representations in $SU(5)$ GUTs.

The usual MSSM superfields are $T_{1,2,3}, \tilde{F}_{1,2,3}, \tilde{H}, H$. The gauge-invariant superpotential of the model is schematically given by

$$W = T_1QL + \sum_{i=1,2} T_iQM + A^5 + (JK)(JK) + A^3LM + (MJ)(MK).$$

(2.8)
Table 2: Quantum numbers and scaling dimensions of chiral superfields in the $10$-centered model.

Note that in this model only the standard model particles in the $10$ representation of $SU(5)$ couple directly to the CFT sector. Relevant interactions such as $A^3$ and $A^4$ must be forbidden by discrete symmetries, or be initially very small for some reason. In this model the $Sp(8)$ group flows to a fixed point where the above superpotential is marginal, while the $Sp(8)'$ group confines at low energies.

From the anomalous dimensions of the fields $T_1$ and $T_2$ one can see that this model makes the prediction $\epsilon_{101}^0 = \epsilon_{102}^{34/19}$ which is in quite good agreement with predictions from $SU(5)$ models (see [6]).

### 2.2 Flavor Changing Neutral Currents

Generic models of new flavor physics introduce flavor violating terms beyond the SM. Such terms are highly constrained by experimental FCNC measurements. The most stringent constraints in our models will arise from either the $K^0 - \bar{K}^0$ system [35, 36, 13] or the $D^0 - \bar{D}^0$ system [44, 45]. Lepton number violating processes will generically give much milder constraints, so we confine this general discussion to $\Delta S = 2$ and $\Delta C = 2$ processes.

The effective Hamiltonians for $\Delta F = 2$ processes at the scale of new flavor physics $\Lambda_{NP}$ can be written as [36]

$$H_{Eff}^{\Delta S=2} = \frac{1}{\Lambda_{NP}^2} \left( \sum_{I=1}^{5} z_{I}^{K} Q_{I}^{sd} + \sum_{I=1}^{3} \tilde{z}_{I}^{K} \tilde{Q}_{I}^{sd} \right),$$

$$H_{Eff}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \left( \sum_{I=1}^{5} z_{I}^{D} Q_{I}^{cu} + \sum_{I=1}^{3} \tilde{z}_{I}^{D} \tilde{Q}_{I}^{cu} \right),$$

(2.9)
where the 4-fermi operators are
\[ \mathcal{Q}_1^{q_1 q_2} = i \bar{q}_L \gamma_\mu q_R \gamma^\mu q_L, \quad \mathcal{Q}_2^{q_1 q_2} = i \bar{q}_L \gamma_\mu q_R \gamma^\mu q_L, \quad \mathcal{Q}_3^{q_1 q_2} = \bar{q}_R \gamma_\mu \gamma_\nu q_L \gamma^\nu q_R, \quad \mathcal{Q}_4^{q_1 q_2} = \bar{q}_R \gamma_\mu \gamma_\nu q_L \gamma^\nu q_R, \]
and where \( \mathcal{Q}_{1,2,3} \) are obtained from \( \mathcal{Q}_{1,2,3} \) through \( L \leftrightarrow R \). The dimensionless coefficients \( z_I^{K,D}, z_I^{K,D} \) encode the details of the new physics generating the above operators at \( \Lambda_{NP} \).

In order to compare to experiment, the above effective Hamiltonians have to be evolved from the scale of new physics \( \Lambda_{NP} \) down to the \( m_K \) and \( m_D \) scales. Taking into account QCD running and operator mixing one obtains (we do not write the tilded part of the effective Hamiltonian, but it is implicit in our discussion)
\[ \langle K^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle_I = \frac{1}{\Lambda_{NP}^2} \sum_{j=1}^5 \sum_{r=1}^5 \left( b_j^{(r,I)} \right) \eta^{(K)}_j z_I^K \langle \overline{K^0} | Q^{sd} | K^0 \rangle, \]
\[ \langle D^0 | \mathcal{H}_{\text{eff}}^{\Delta C=2} | D^0 \rangle_I = \frac{1}{\Lambda_{NP}^2} \sum_{j=1}^5 \sum_{r=1}^5 \left( b_j^{(r,D)} \right) \eta^{(D)}_j z_I^D \langle \overline{D^0} | Q^{cu} | D^0 \rangle, \]
where \( \eta = \alpha_3(\Lambda_{NP})/\alpha_3(m_t) \), and the other relevant inputs can be found in \[10\] for the \( K^0 - \overline{K^0} \) system and in \[13\] for the \( D^0 - \overline{D^0} \) system. Throughout this paper we only use the bounds coming from CP-conserving processes, see the discussion in the final section.

Imposing that the new physics contribution is not larger than the measured values of \( \Delta m_K = 2 \text{Re}\langle K^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle \simeq 3.483 \times 10^{-12} \text{ MeV} \) and \( \Delta m_D = 2 \text{Re}\langle D^0 | \mathcal{H}_{\text{eff}}^{\Delta C=2} | D^0 \rangle \simeq 1.89 \times 10^{-11} \text{ MeV} \[13, 17\], the following constraints are obtained:
\[ \Lambda_{NP} \gtrsim \sqrt{|z_K^I|} 1.0 \times 10^3 \text{ TeV}, \quad \Lambda_{NP} \gtrsim \sqrt{|z_K^I|} 7.3 \times 10^3 \text{ TeV}, \quad \Lambda_{NP} \gtrsim \sqrt{|z_K^I|} 4.1 \times 10^3 \text{ TeV}, \]
\[ \Lambda_{NP} \gtrsim \sqrt{|z_K^I|} 17 \times 10^3 \text{ TeV}, \quad \Lambda_{NP} \gtrsim \sqrt{|z_K^I|} 10 \times 10^3 \text{ TeV}, \]
and
\[ \Lambda_{NP} \gtrsim \sqrt{|z_D^I|} 1.2 \times 10^3 \text{ TeV}, \quad \Lambda_{NP} \gtrsim \sqrt{|z_D^I|} 3.1 \times 10^3 \text{ TeV}, \quad \Lambda_{NP} \gtrsim \sqrt{|z_D^I|} 1.6 \times 10^3 \text{ TeV}, \]
\[ \Lambda_{NP} \gtrsim \sqrt{|z_D^I|} 6.2 \times 10^3 \text{ TeV}, \quad \Lambda_{NP} \gtrsim \sqrt{|z_D^I|} 3.5 \times 10^3 \text{ TeV}. \]

The above bounds are obtained after running \( \alpha_3 \) up to \( \Lambda_{NP} \) assuming that only SM fields contribute to the beta-function up to that scale. In our models we will always have additional fields below the scale \( \Lambda_{NP} \), which will change the running contribution to these bounds, but since the bounds are up to unknown order one coefficients in any case, we ignore this issue here. For a strongly coupled theory with generic flavor structure one
Flavor violation is encoded in the mass insertion parameters expectations where $f$ gauge couplings. Neglecting $\tilde{\phi}$ directly translate into lower bounds on $\Lambda_{NP}$ exchange \cite{35, 36}, and in this case the coefficients will also be generated through box diagrams with squark-gluino exchange. In the above expression the average squark mass is taken to be $\tilde{m}_q^2/\tilde{m}_{\tilde{q}}^2$. Flavor violation is encoded in the mass insertion parameters

$$\frac{\alpha^2}{216} \left(24xf_6(x) + 66\tilde{f}_6(x)\right) (\delta_{12}^{d,u})^2_{LL},$$

$$\frac{\alpha^2}{216} \left(24xf_6(x) + 66\tilde{f}_6(x)\right) (\delta_{12}^{d,u})^2_{RR},$$

$$\frac{\alpha^2}{216} \left(504xf_6(x) - 72\tilde{f}_6(x)\right) (\delta_{12}^{d,u})_{LL}(\delta_{12}^{d,u})_{RR},$$

$$\frac{\alpha^2}{216} \left(24xf_6(x) + 120\tilde{f}_6(x)\right) (\delta_{12}^{d,u})_{LL}(\delta_{12}^{d,u})_{RR},$$

(2.14)

where $f_6(x)$ and $\tilde{f}_6(x)$ are kinematical functions whose expressions can be found in \cite{36} and $x$ is the ratio of the gluino mass squared to the squark mass squared, $x = \tilde{m}_q^2/\tilde{m}_{\tilde{q}}^2$.

In supersymmetric extensions of the SM with perturbative quark-squark-gluino interactions, the operators (2.10) will also be generated through box diagrams with squark-gluino exchange \cite{35, 36}, and in this case the coefficients will be suppressed by SM gauge couplings. Neglecting $\tilde{q}_L - \tilde{q}_R$ mixing, they can be written as

$$(\delta_{12})_{NN} \equiv (K_{21}^q)(K_{22}^q)(\tilde{m}_{qN_2} - \tilde{m}_{\tilde{q}N_1})/\tilde{m}_{\tilde{q}}, \quad (q = u, d, \quad N = L, R)$$

(2.15)

In the above expression the average squark mass is taken to be $\tilde{m}_q \equiv (\tilde{m}_{q_1} + \tilde{m}_{q_2})/2$ \cite{49} and $K_N^q = V_N^q\tilde{V}_N^q$ ($N = L, R$), where $V_N^q$ and $\tilde{V}_N^q$ are hermitian matrices that diagonalize the quark and squark mass matrices (again, neglecting $\tilde{q}_L - \tilde{q}_R$ mixing), respectively, as

$$\text{diag}(m_{q_1}, m_{q_2}, m_{q_3}) = V_L^q M_q V_R^q, \quad \text{diag}(\tilde{m}_{q_{N_1}}, \tilde{m}_{q_{N_2}}, \tilde{m}_{q_{N_3}}) = \tilde{V}_N^q \tilde{M}_{qN} \tilde{V}_N^q.$$  

(2.16)

With this nomenclature, non-degenerate masses correspond to $(\tilde{m}_{q_{N_2}} - \tilde{m}_{\tilde{q}N_1}) \sim \tilde{m}_{\tilde{q}}$ and the amount of alignment can be estimated from the size of the mixing angles $(K_{21}^q)(K_{22}^q)$. Generic squark mass matrices for the first two generations with no degeneracy and no alignment lead to $(\delta_{12})_{NN} \sim 1$. To assess the amount of flavor violation introduced by SUSY breaking in our subsequent (two-scale) models, we will use $\alpha_3(1 \text{ TeV}) \simeq 0.089$ in (2.14), set $\Lambda_{NP} \sim \tilde{m}_{\tilde{q}}$ in (2.11) with $z_1^{KD}$, $z_1^{KD}$ given by (2.14) and require that the total sum of the contributions proportional to a single $\delta$ does not exceed the experimental values of $\Delta m_{K,D}$.

## 3 Fully coupled models

In this paper we wish to explore the possibility that the same dynamics generating the weak-scale flavor hierarchy is also responsible for SUSY breaking, and to work out the
qualitative form of the resulting spectrum. Our basic framework is an implementation of the Nelson-Strassler scenario as reviewed in Section 2.1 leading to a suppression of the Yukawa couplings through coupling of the first two generations to a strongly coupled CFT sector as in (2.2), but with SUSY dynamically broken at a low scale by the CFT sector itself. This implies that the first two generation superfields feel strong direct SUSY breaking effects; note that this is very different from the high-scale SUSY breaking discussed in [7, 9].

In our scenario the standard model gauge group $G_{SM}$ necessarily couples (weakly) also to the CFT sector, so gauge-mediated soft terms naturally also arise.

The above couplings are summarized in the following interaction Lagrangian:

$$\mathcal{L}_{int} = \left( \int d^2 \theta \left( \lambda_1 \mathcal{O}_1 \Phi_1 + \lambda_2 \mathcal{O}_2 \Phi_2 + W(\mathcal{O}_1, \mathcal{O}_2) \right) + \text{h.c.} \right) + \sum_{A=1}^{3} 2g_A \int d^4 \theta \mathcal{J}^A \mathcal{V}^A, \quad (3.1)$$

where the chiral operators $\mathcal{O}_{1,2}$ have different and definite R-charges, $\Phi_{1,2}$ denote generic first two generation MSSM matter superfields, the couplings $\lambda_{1,2} \sim 1$ and we disregard irrelevant couplings following the discussion in Section 2.1. $\mathcal{V}^A$ are MSSM vector superfields and $\mathcal{J}^A$ are the CFT sector global symmetry current superfields for $G_{SM}$ [16]. Note that, in contrast to models of general gauge mediation, taking $\alpha_A \to 0$ in our extended scenario does not fully decouple the MSSM from the CFT sector, since the direct couplings $\lambda_i$ are dynamically set by the superconformal theory.

We define $M$ as the mass of the degrees of freedom of the CFT sector which couple directly to the first two generations. We assume $M$ to be close to the bottom of the conformal window $M \sim M_\prec$, although this assumption is easily relaxed. The scale $M$ may or may not be equal to the scale of the mass gap for all CFT sector fields. We distinguish models according to the number of scales involved:

- **One-scale models**: models in which the SUSY breaking scale is also given by $M$.
- **Two-scale models**: models where the SUSY breaking scale $F$ is parametrically smaller than the scale of decoupling and conformal symmetry breaking, $F \ll M^2$.

In the class of models that we are discussing, the Nelson-Strassler mechanism necessitates that the CFT sector and SUSY breaking are not flavor blind. This is simply because the first two generations couple differently to the CFT sector. The main qualitative constraint on our models therefore comes from the tension between requiring small FCNCs, which pushes the mass of the CFT sector and the first two generation sfermions upwards,

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9In these works SUSY breaking occurs at a high scale well above the bottom of the conformal window, and so the dynamics of the conformal regime leads to the suppression of many soft terms. There must then be a long enough RG flow so that gauginos can drive the soft masses back up to acceptable values, and so the scale $M_\prec$ is compelled to be high.
and requiring small fine-tuning for electroweak symmetry breaking, pushing the mass scales of the theory downwards.

Our main result will be a class of models which are technically natural and – despite direct couplings between the first two generations and the strongly coupled sector – accommodate the FCNC constraints without too much fine-tuning. The best models do so as a result of being separable rather than non-separable, two-scale models rather than one-scale models, and partially coupled rather than fully coupled.

In this section we consider fully coupled models and their limitations. In Section 3.1 we discuss the dynamics associated with the scale $M$, in particular the constraints coming from four-Fermi terms generated at the threshold scale $M$. Through constraints on FCNCs we obtain a lower bound on $M$ assuming the theory is generic (non-separable). We then show how a technically natural structural requirement on the theory, which we call separability, reduces the bound on $M$. Section 3.2 discusses the soft terms. In Section 3.3 we address one-scale models, where the same scale $M$ sets the soft masses in the theory, leading to large fine-tuning. In Section 3.4 we discuss two-scale models, in which the mass of the first two generation sfermions is reduced compared to one-scale models. Although reducing the fine-tuning, this also introduces new sources of FCNCs which arise due to the running of the first two generation sfermions in loops [35, 36]. Therefore the fine-tuning issue is not fully resolved, but it is brought to a much lower level than the single scale case.

### 3.1 Flavor restrictions on $M$

#### 3.1.1 Non-separable vs. separable models

We are interested in low-scale SUSY breaking, leading to a spectrum compatible with a light enough Higgs without fine-tuning. However, generically, exiting the conformal and SUSY-preserving phase through some strong coupling dynamics can be accompanied by troublesome flavor-violating wave-function renormalizations (the graceful exit problem) and four-Fermi operators. If generic four-Fermi terms of the first two generations are generated at $M \sim M_\ast$, then the bound on $M$ is the largest of the bounds in Section 2.2, $M \gtrsim 17 \cdot 10^3$ TeV. This leads to a very heavy sparticle spectrum, resulting in large fine-tuning in the Higgs sector. A concrete mechanism suppressing such effects must be provided.

A possible resolution is the suppression of direct flavor-mixing terms involving the first two generations in the CFT basis. One way to achieve this is by using horizontal symmetries under which the CFT sector fields are charged. Another way to accomplish this is by assuming a “decomposable” CFT sector. In such a setup, the CFT sector decomposes into two separate sectors, each of which couples to a specific linear combination of the three
MSSM generations. In particular, the gauge group $\tilde{G}$ decomposes into the product of two groups, and the fields charged under one couple to those charged under the other only via SM interactions. In both constructions terms similar to $O_2\Phi_1$ can still exist but, as discussed in Section 2.1, they are negligible and do not affect the Yukawa structure. We will henceforth omit these flavor mixing, R-symmetry violating terms from the discussion; their effect on the flavor structure of the soft terms will be at most comparable to the effects we discuss. We shall refer to CFT sectors which are either decomposable or have horizontal symmetries as “separable”, and restrict ourselves to such scenarios from now on.

Our separable CFT sectors give diagonal wave-function renormalizations in the interaction basis, which realize the graceful exit in a natural way. Nevertheless, even in the case of separable models there can still be dangerous FCNC effects. These come about from four-Fermi operators which appear flavor-diagonal in the interaction basis, but are identified as flavor-changing when switching to the mass basis. (These effects are generically much larger than the couplings induced by the SM interactions between the two sectors.) Constraints from such operators, suppressed by powers of $M$ as well as mixing angles, are still in tension with low SUSY-breaking masses, as discussed in the following subsections.

### 3.1.2 Separable CFT sectors

We now discuss the constraints on separable CFT sectors. In the following we focus on decomposable CFT sectors; the case of horizontal symmetries gives similar results.

We couple the first two generations of the MSSM to a decomposable superconformal sector $H$ with group $G \times \tilde{G}_1 \times \tilde{G}_2$. The gauge subgroups $\tilde{G}_1$ and $\tilde{G}_2$ become strongly coupled at scales $\Lambda_1$ and $\Lambda_2$, respectively. The interaction Lagrangian (3.1) becomes

$$\mathcal{L}_{\text{int}} = \left( \int d^2 \theta \ (\lambda_1 O_1 \Phi_1 + \lambda_2 O_2 \Phi_2 + W_1(O_1) + W_2(O_2)) + \text{h.c.} \right) + \sum_{A=1}^3 2g_A \int d^4 \theta \ J^A V^A,$$

(3.2)

where now the fundamental fields forming the composite operators $O_1, O_2$ are not charged under $\tilde{G}_2, \tilde{G}_1$, respectively, and there are no direct couplings between fields charged under $\tilde{G}_1$ and $\tilde{G}_2$. We collectively denote the CFT sector fields charged under $\tilde{G}_{1,2}$ by $H_{1,2}$. An example of this type of models (although not SUSY breaking itself) is the example reviewed in Section 2.1.3 [3].

In these models each part of the CFT sector is perturbed away from the fixed point by separate relevant deformations, and no new intergenerational couplings are introduced. We then naturally obtain a flavor diagonal exit from the conformal regime. SUSY breaking can occur in either sector separately, or in both. We will assume SUSY is broken in both
sectors, and for simplicity we take the mass scales of $H_{1,2}$ to be of similar order of magnitude $M$.

The leading non-renormalizable interactions of the SM fields are given by dimension six four-Fermi operators as in (2.9). Schematically,

$$\mathcal{L}_{4-\text{fermi}} = \frac{1}{\Lambda_{NP}^2} \left( \sum_{I=1}^{5} z_I Q_I^{q_{k_{q_I}}} + \sum_{I=1}^{3} \tilde{z}_I \tilde{Q}_{I}^{q_{k_{q_I}}} \right),$$

(3.3)

and the constants $z_I$ and $\tilde{z}_I$ depend on the details of the strong dynamics. Operators involving the first two generations are suppressed by $\Lambda_{NP} \sim M$. In the absence of any accidental cancelations in the coefficients, the first two generations’ terms have $z_I, \tilde{z}_I \sim \mathcal{O}(1)$, while terms involving the third generation are further suppressed by powers of SM gauge couplings. Since in separable models the two parts of the CFT sector talk to each other only via SM gauge interactions, in the CFT basis all first two generation four-Fermi operators are flavor-diagonal at leading order. Dangerous flavor violating terms are generated, however, when rotating to the mass basis (2.10):

$$\bar{q}_M \tilde{q}_N \tilde{q}_P \tilde{q}_S \supset \left( \bar{q}_{M_1} q_{N_2} \tilde{q}_P q_{S_2} \right) \left( V_{i_1}^M V_{i_2}^N \tilde{V}_{i_2}^P V_{i_1}^S \right) + \left( \bar{q}_{M_2} q_{N_1} \tilde{q}_P q_{S_1} \right) \left( V_{i_1}^M \tilde{V}_{i_2}^N V_{i_2}^P \tilde{V}_{i_1}^S \right),$$

(3.4)

where in the above $q = u, d; i = 1, 2; M, N, P, S = R, L$; the hatted states on the left-hand side denote quarks in the CFT basis, while the unhatted states on the right-hand side denote quarks in the mass basis.

A bound on $M$ can now be obtained by combining (3.4) with (2.5), (2.10), (2.12) and (2.13). We find that the strongest constraint comes from $Q_{sd}^4$ in the $K^0 - \bar{K}^0$ system, which picks up a factor of $\sqrt{m_d/m_s} \sim 0.22$ compared to the general CFT sector estimate $M \gtrsim 17 \cdot 10^3$ TeV, namely (up to order one numbers coming from the coefficients $z_I$ and $\tilde{z}_I$ in (3.3))

$$M \gtrsim 3.7 \cdot 10^3 \text{ TeV.}$$

(3.5)

In models with partial couplings to the CFT sector the FCNC constraints can be relaxed. Such models will be discussed in Section 4.

### 3.2 The soft terms in our models

In our models there are two mechanisms for transmitting SUSY breaking to the standard model fields. The first two generation superfields couple directly to the CFT sector and obtain soft terms directly through these strong couplings. All other fields of the standard model couple to the CFT sector only indirectly, mostly through their interactions with the standard model gauge fields. These couple directly, though weakly, to the CFT sector, since the SM gauge group $G_{SM}$ is a subgroup of the CFT sector global symmetry group.
There are also very small contributions from the Yukawa couplings of these fields to the first two generations, which we will ignore here. The mechanism for generating these soft terms is thus gauge mediation through coupling to a strongly coupled theory.

In a theory of gauge mediation, the leading order (in standard model gauge couplings) soft terms may be expressed in terms of correlators of the $G_{SM}$ currents in the CFT sector theory [16]. Since our CFT sector is strongly coupled, we assume that these correlation functions are all of order one, up to an overall factor of $N_{\text{eff}}^{(A)}$ which captures the effective number of degrees of freedom in the CFT sector that are charged under the $A$'th factor in the standard model gauge group and participate in SUSY breaking. In a weakly coupled CFT sector this would simply be the number of messenger fields, while in strongly coupled theories it does not have to take integer values. We denote by $\mu_S$ the scale at which the SUSY breaking dynamics is integrated out, and by $\Lambda_S$ the effective SUSY breaking scale transmitted to the MSSM (in one-scale models this is just the scale $M$, but we will later analyze models where this scale is different). Our assumptions then imply that at the scale $\mu_S$ the gaugino masses, sfermion masses-squared (for multiplets which do not couple directly to the CFT sector) and $A$-terms are

$$M_A = N_{\text{eff}}^{(A)} \frac{\alpha_A}{4\pi} \Lambda_S, \quad \tilde{m}_j^2 \sim \sum_{A=1}^{3} N_{\text{eff}}^{(A)} \left( \frac{\alpha_A}{4\pi} \right)^2 \Lambda_S^2, \quad A^{u,d,\ell}_{ij} \sim y^{u,d,\ell}_{ij} \sum_{A=1}^{3} N_{\text{eff}}^{(A)} \left( \frac{\alpha_A}{4\pi} \right)^2 \Lambda_S,$$

where $i, j$ are generation indices, $y^{u,d,\ell}$ are Yukawa couplings, and the sum over $A$ should include only the SM gauge groups that the specific fields in question are charged under. Note that we define $N_{\text{eff}}^{(A)}$ in terms of the corresponding gaugino mass.

In the equations above we ignored possible contributions from a D-term of the hypercharge current coming from the CFT sector [50, 51]. In gauge mediation this D-term comes from a vacuum expectation value for the lowest component of the $U(1)_Y$ current superfield, $J_Y$ [16], and at tree-level it gives additional possible contributions to the sfermion masses going as $\tilde{m}_j^2 = g_Y^2 Y_f \langle J_Y \rangle$, where $Y_f$ is the hypercharge of the sfermions. Naively we expect $\langle J_Y \rangle \sim \pm N_{\text{eff}}^{(1)} \Lambda_S^2/(4\pi)^2$, yielding contributions to sfermion masses of order $\tilde{m}_j^2 \sim \pm Y_f N_{\text{eff}}^{(1)} \frac{\alpha_3}{4\pi} \Lambda_S^2$. Such terms are larger than the other contributions to third generation sfermion masses, and since they always take both positive and negative values, they are problematic.

Thus, we will always assume that $U(1)_Y$ is embedded in a non-Abelian group in the global symmetry $G$ of the CFT sector. This means that at leading order we actually have $\langle J_Y \rangle = 0$, and the dominant contribution to $\langle J_Y \rangle$ comes from the leading effects breaking this non-Abelian symmetry, which we expect to be one-loop effects of order $\alpha_3/4\pi$ [51] (for instance, if it is a GUT symmetry). The D-term contributions to the sfermion masses are then expected to be of order $\tilde{m}_j^2 \sim \pm Y_f N_{\text{eff}}^{(1)} \frac{\alpha_3}{(4\pi)^2} \Lambda_S^2$, which is already of the same order as
the other gauge-mediated contributions (3.6) described above. We will analyze the effect of these terms in each scenario separately. In two-scale models, additional contributions to the D-term from the first two generation sfermions can arise via one-loop RG evolution, the effect of which will be discussed in Section 3.4.

Having outlined some generalities of the soft terms structure, we move on to discuss in detail separable one-scale and two-scale models.

### 3.3 Separable one-scale models

In these models the scale of SUSY breaking is the same as that of conformal symmetry breaking, and therefore all dimensionful quantities are given in terms of powers of $M$. Specifically, the soft terms for the first two generations are dominantly given by

$$
(\tilde{M}^2_{u,d,\ell})_{NNi} \sim M^2, \quad A_{ii}^{u,d,\ell} \sim y_{ii}^{u,d,\ell} M, \quad i = 1, 2, \quad N = R, L,
$$

and off-diagonal terms are not generated at leading order due to separability of the CFT sector. Since their mass is of order $M$, the first two generation sfermions can be viewed as part of the CFT sector and are integrated out when writing the effective action below $M$.

The effective Lagrangian below the scale $M$ thus takes the form:

$$
L_{\text{eff}} = \sum_{i=1,2} i q_i D_\mu \lambda^i + \int d^4 \theta \Phi_3 e^V \Phi_3^\dagger + \int d^2 \theta W_{\text{MSSM}}^{\tilde{q}_1,2=0} + L_{\text{soft}}^{\tilde{q}_1,2=0} + L_{\geq 4},
$$

where $q_i$ are the first two generation fermions, $\tilde{q}_i$ are their superpartners, and the MSSM superpotential $W_{\text{MSSM}}^{\tilde{q}_1,2=0}$ and the soft Lagrangian $L_{\text{soft}}^{\tilde{q}_1,2=0}$ do not include the first and second generation sfermions. As we are discussing separable models, wave-function renormalizations below $M$ are diagonal. The soft Lagrangian is given by

$$
L_{\text{soft}}^{\tilde{q}_1,2=0} = \frac{1}{2} [M_A \lambda A \lambda^A + c.c.] + \left( (\tilde{M}^2_{q})_{NN33} \tilde{q}_N \tilde{q}_N + A_{33}^{q} H_u d \tilde{q}_L R + c.c. \right) + \left[ m_{H_u}^2 H_u H_u + m_{H_d}^2 H_d H_d + (B_{\mu} H_u H_d + c.c.) \right],
$$

where $N = R, L$ and $q = u, d, \ell$. The gaugino masses $M_A$, third generation soft masses $(\tilde{M}^2_{q})_{33}$, soft trilinear terms $A_{33}^{q}$ and soft Higgs terms are all dominantly generated through gauge mediation (3.6) with an effective SUSY breaking scale $\Lambda_S \sim M$.

Substituting the lowest value of $M$ compatible with the four-Fermi operator bound (3.5) into the general formulas (3.6) and (3.7), one finds that the spectrum of such one-scale models is very heavy. For instance, the gluino mass is of order $2^{2N}_{\text{eff}} \text{TeV}$ at the scale $M$, and grows as it evolves down to the weak scale. Clearly, a spectrum of such massive gauginos and third generation sparticles is unacceptable since it implies severe fine-tuning of the weak scale [10, 11, 12]. For this reason we discard further discussion of these one-scale models, and turn to models involving two scales.
3.4 Separable two-scale models

We now study models in which the SUSY breaking scale $F$ is parameterically suppressed, $F \ll M^2$. For simplicity, we discuss models in which SUSY breaking occurs in both $\mathcal{H}_1$ and $\mathcal{H}_2$, assuming $F_1 \sim F_2 \sim F$.

3.4.1 Soft terms

The soft terms for the first two generation sfermions can be determined to leading order in $F/M^2$ by dimensional analysis and by requiring these leading contributions to vanish in the supersymmetric limit $F \to 0$, as well as in the limit of no direct coupling $M \to \infty$. Chirality-preserving diagonal soft masses are dominantly given by

$$\left(\tilde{M}^2_{u,d,\ell}\right)_{NN_i} \sim \left(\frac{F}{M}\right)^2, \quad i = 1, 2, \quad N = R, L. \quad (3.10)$$

Off-diagonal soft masses-squared for the first two generations are not generated from direct couplings, and are zero at leading order in gauge mediation. The first two generation diagonal $A$-terms are given by

$$A_{ii}^{u,d,\ell} \sim y_{i3}^{u,d,\ell} \frac{F}{M}, \quad i = 1, 2, \quad i = 1, 2, 3, \quad N = R, L, \quad (3.11)$$

while off-diagonal first two generation $A$-terms are much smaller (see below). In this scenario the first two generation sfermions are much lighter than the CFT sector fields, so the parametric suppression of the SUSY breaking order parameter $F/M^2 \ll 1$ can account for lighter masses $\tilde{m}_{1,2}$ even for a large CFT sector scale $M$.

The third generation and gaugino soft masses are generated through gauge mediation, as are additional soft trilinear couplings. The precise definition of $\Lambda_S$, the effective scale of SUSY breaking transmitted to the visible sector, in terms of $F$ and $M$ depends on the specific class of models under consideration, as we will discuss shortly. At this point, the only requirement is that $\Lambda_S \to 0$ as $F \to 0$. For example, in weakly coupled messenger models of gauge mediation $\Lambda_S$ corresponds to $F/M$, with $M$ the messenger scale and $F$ the vacuum energy squared. In terms of this scale $\Lambda_S$, we then have at the scale $\mu_S$:

$$M_A = N_{e_{eff}}^{(A)} \frac{\alpha_A}{4\pi} \Lambda_S,$$

$$\left(\tilde{M}^2_{u,d,\ell}\right)_{NN_{33}} \sim N_{e_{eff}}^{(A)} \left(\frac{\alpha_A}{4\pi}\right)^2 \Lambda_S^2 \pm Y_{u,d,\ell}N_{NN_{33}}^{(1)} \frac{\alpha_1 \alpha_3}{4\pi^2} \Lambda_S^2, \quad N = R, L, \quad (3.12)$$

$$A_{ij}^{u,d,\ell} \sim y_{ij}^{u,d,\ell} N_{e_{eff}}^{(A)} \left(\frac{\alpha_A}{4\pi}\right)^2 \Lambda_S, \quad i \neq j \quad \text{or} \quad i = j = 3, \quad i, j = 1, 2, 3,$$

where off-diagonal gauge-mediated soft masses-squared are negligible, $Y_{u,d,\ell_{L,R}}$ denotes the hypercharge of the appropriate sfermion, and in the above we take the largest contribution when several SM gauge factors are possible.
In these models we need to consider the RG contribution of the first two generation sparticles to the D-term \[19\]. The soft masses-squared in (3.10) are more accurately given by
\[
(\tilde{M}_{u,d,\ell}^2)_{NN,i} \sim \left(\frac{F}{M}\right)^2 \pm \frac{\alpha_3}{4\pi} \left(\frac{F}{M}\right)^2, \quad i = 1, 2, \quad N = R, L.
\]
(3.13)
The coefficient of the first term is assumed to be invariant under \(G\). The second term encodes the leading violation of the \(G\) symmetry by the standard model. In the simplest case that \(G\) is a unified version of \(G_{SM}\), the strongest such effect is proportional to \(\alpha_3\). Generally, the masses (3.13) contribute to the D-term of \(U(1)_Y\) and to the soft masses of lighter particles. For some other light sfermion of hypercharge \(Y\), the D-term contributes
\[
Y \frac{\alpha_1}{4\pi} \text{Tr}(Y_f \tilde{m}_f^2)
\]
(3.14)
to its soft mass-squared beta function. Depending on the sign of \(Y\), some of the third generation scalar masses may then be driven tachyonic. Recall, however, that we assume that \(U(1)_Y\) is embedded in a non-Abelian factor in the global symmetry group \(G\) of the CFT sector which includes the first two generations (for example an \(SU(5)\) GUT). As a result, the first contribution from the heavy first two generation masses in (3.13) yields a vanishing hypercharge trace. Since the (say) GUT symmetry is broken by SM gauge couplings, the second term in (3.13) will induce a non-zero D-term, leading to contributions to light sfermion mass-squared beta functions of order
\[
Y \frac{\alpha_1}{4\pi} \text{Tr}(Y_f \tilde{m}_f^2) \sim Y \frac{\alpha_1 \alpha_3}{(4\pi)^2} \left(\frac{F}{M}\right)^2.
\]
(3.15)
In the scenarios considered in the rest of the paper the integral of (3.15) is always smaller than the first contribution to the sfermion masses-squared in (3.12), and will henceforth be neglected. The second term in the sfermion masses-squared (3.12) can be negative and therefore potentially dangerous, so it may require mild tuning. We will discuss the effect of this D-term in further detail for partially coupled models in Section 4.

Operators of dimension \(d \geq 4\) can also be generated. Since dimension four sfermion operators can only come from SUSY breaking, they are parametrically suppressed via \(F/M^2 \ll 1\), and are thus not a concern. In dimension five fermion-sfermion operators the SUSY preserving effects dominate over the SUSY breaking ones. The four-Fermi operators of dimension six are dominated by the SUSY preserving effects, and are down by \(1/M^2\). They dictate the bound (3.5) on \(M\).

Equation (3.10) presents the pleasant feature of two-scale models: the first two generation soft terms and the higher-dimensional operator coefficients can simultaneously be suppressed by taking an enhanced value of \(M\) for fixed \(F\), relaxing the contributions to FCNCs while lowering the scale of the scalar masses.

We consider the following two classes of models:
Class (a)

In these models the CFT sector is assumed to have a mass gap of order $M$, and all of
the CFT sector physics is integrated out at the scale $M \sim \mu_S$. In the effective theory
obtained at scale $M$ supersymmetry is dynamically broken, with an accidentally small
SUSY breaking parameter $F \ll M^2$. As a result $\Lambda_S \sim F/M$. In the limit $M \to \infty$ the
CFT sector decouples and SUSY is not broken.

Class (b)

In these models the physics of the CFT sector coupling to the first two generations is
integrated out at the scale $M$ (namely, the CFT sector fields that couple directly to the
first two generations have a mass of order $M$). The rest of the CFT sector is integrated
out at a lower scale $\sqrt{F} \ll M$, at which SUSY is then naturally broken in the effective
theory. Here the mass gap in the CFT sector is the same as the SUSY breaking F-term.
As a result $\mu_S \sim \Lambda_S \sim \sqrt{F}$, while the soft terms for the first two generations are still given
by (3.10). In this class of models the limit $M \to \infty$ corresponds to a gauge mediation
scenario with SUSY breaking scale $\sqrt{F}$.

Since in models of Class (b) part of the CFT sector dynamics is integrated out at $M$,
we expect the SUSY breaking dynamics in this class to yield smaller values of $N_{\text{eff}}$ than
in Class (a). Given the direct couplings and the MSSM quantum numbers of the operators
$\mathcal{O}_{1,2}$ we expect $N_{\text{eff}}$ to be at least twice the MSSM contribution (in two generations) for
Class (a) models. In Class (b) models we can have lower $N_{\text{eff}}$, though requiring dynamical
SUSY breaking in the CFT sector typically means that it cannot be smaller than $\sim 5$.

3.4.2 Further FCNC bounds on scales

As described above, in both classes of two-scale models generic four-Fermi operators gen-
ergated by the superconformal dynamics are suppressed by $1/M^2$, and bound $M$ according
to (3.3). The non-zero soft terms involving the first two generations are set by $F/M$ and
are of the form (3.10), and chirality mixing terms are negligible. At leading order, sfermion
mass-squared matrices for the first two generations are then already diagonalized in the
CFT basis at the scale $\mu_S$.

In these models the sfermions are much lighter than $M$ and so additional FCNC con-
straints are present. The most stringent bound on $F/M$ is obtained from the $K^0 - \overline{K^0}$
FCNC box diagrams involving squarks and gluino exchange. This process constrains the
first two generation (down sector) masses at the weak scale, obtained by RG evolution
(RGE) from the boundary conditions (3.11) at $\mu_S$ down to $m_Z$. We can neglect RG effects

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on the first two generation mass-squared matrices due to the short range of scales involved, and use (3.10) at the weak scale as well. In light of the above, in the squark sector we have

\[(\tilde{V}_{L,R})_{ij} \sim \delta_{ij}, \quad i,j = 1,2, \quad q = u,d.\]  

(3.16)

The quark sector diagonalizing matrices have the structure (2.5), and so the mixing matrices in (2.13) are of the form

\[(K_L^d)_{ij} \sim (V_L^q)_{ij} \sim |V_{ij}|, \quad (K_R^d)_{ij} \sim (V_R^q)_{ij} \sim \frac{m_{q_i}/m_{q_j}}{|V_{ij}|} \quad (i < j), \quad i,j = 1,2, \quad q = u,d.\]  

(3.17)

Note that \((K_{L}^{d})_{12} \sim |V_{12}| \sim (K_{R}^{d})_{12} \sim \frac{m_{d_i}/m_{d_j}}{|V_{12}|} \sim 0.22\) are all of order a Cabibbo factor. In terms of the \(\delta_{12}^d\) parameters of Section 2.2, the separable CFT sector scenario with no degeneracy then corresponds to taking all \(\delta_{12}^d\) of order a Cabibbo factor \(\sim 0.22\). Additionally, (3.10) and (3.12) imply that the ratio \(x \equiv \frac{m_{d_i}^2}{m_{d_j}^2} \sim |V_{12}|\) appearing in (2.14) is small yet depends on \(N_n^{(3)}\). In models of Class (a), this ratio is independent of \(F/M^2\) and is given by

\[x(a) = \left(\frac{\alpha_3}{4\pi N_n^{(3)}}\right)^2\]  

(3.18)

while in models of Class (b) we have

\[x(b) = \left(\frac{\alpha_3}{4\pi N_n^{(3)}}\right)^2 \frac{M^2}{F}.\]  

(3.19)

The most stringent bounds come from the mixed \((\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\) terms in (2.14). We present plots of these bounds as a function of \(N_n^{(3)}\) for Class (a) and Class (b) models in Figure 1. Representative bounds are

\[\text{Class (a)}: \quad \frac{F}{M} \gtrsim 81 \text{ TeV} \quad (N_n^{(3)} = 27),\]  

\[\text{Class (b)}: \quad \frac{F}{M} \gtrsim 87 \text{ TeV} \quad (N_n^{(3)} = 5).\]  

(3.20)

The values for \(N_n^{(3)}\) are chosen in agreement with the two-loop effect that will be discussed in Section 4. Flavor changing chirality-mixing terms \((\delta_{12}^d)_{LR}\) give much less stringent constraints on \(F/M\) due to the relative smallness of the \(A\)-terms (3.11).

Given some amount of degeneracy in the first two generation down sector, the bound is weakened and lower values of \(F/M\) are accessible. For example, allowing additional \(\sim 0.5\) degeneracy the bounds in both classes are weakened to \(F/M \gtrsim 41 \text{ TeV}\) and \(F/M \gtrsim 43 \text{ TeV}\) respectively, for the same values of \(N_n^{(3)}\) as in (3.20).

\textsuperscript{10}\textsuperscript{Recall that all the bounds we write are up to unknown order one constants coming from correlators in the CFT sector.}
Figure 1: The relevant $K^0 - \overline{K^0}$ system bounds on $F/M$ as a function of $N_{eff}^{(3)}$ for fully coupled two-scale models with a separable CFT sector ($M = 3.7 \cdot 10^3$ TeV). The bounds depicted here come from $(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}$ in Class (a) (left) and Class (b) (right) models.

Using the bounds on $M$ and $F/M$ given in (3.5) and (3.20), we compute a bound for $\sqrt{F}$ relevant for models of Class (b), yielding

$$\sqrt{F} \gtrsim 570 \text{ TeV.}$$

(3.21)

It is clear from (3.10), (3.12), (3.20) and (3.21) that in the two-scale models described above the generic sfermion soft mass spectrum at the scale $\mu_S$ is still given by an inverted hierarchy, provided $N_{eff}$ is not too large. In such models, avoiding third generation tachyons from heavy first two generation two-loop contributions [29, 30] puts a lower bound on initial third generation masses, which in our models translates into a lower bound on $N_{eff}^{(A)}$. This two-loop effect will be discussed in greater detail in Section 4. In the current context, this effect along with the initial soft terms (3.12) imply unnaturally large initial soft masses for the third generation sfermions and the gauginos. For instance, gluino masses at the scale $\mu_S$ are of order $\sim 12$ TeV in Class (a), and $\sim 18$ TeV in Class (b). These fully coupled separable models thus still require large fine-tuning in the electroweak sector.
4 Partially coupled models

In the models analyzed in the previous section all MSSM fields of the first two generations coupled directly to the CFT sector. This resulted in relatively high scales $M$ and $F/M$, which led to large fine-tuning of the weak scale $m_Z$. In this section we explore models with partial couplings, in which the strongest constraints on the scales can be relaxed and the fine-tuning can be alleviated.

When fully coupling all first two generation fields to the CFT sector, the main sources of flavor violation are operators made out of both left- and right-handed fields in the down sector. We can thus allow for a lower scale $M$ by coupling only some of these fields (in particular only some of the down quarks) directly to the CFT sector. Here we will consider several such models – models in which only the left-handed (LH) fields in the first two generations couple directly to the CFT sector, similar models with right-handed (RH) direct couplings, and a $10$-centered model, in which only the $10$’s of $SU(5)$ couple to the CFT sector, thus coupling both up sector chiralities but suppressing mixed chirality-operators in the down sector. We consider here only two-scale models, of both Class (a) and (b).

A few comments concerning D-terms are in order. When coupling only a single chirality to the CFT sector, unification is lost. In order to suppress the tree-level gauge mediation and one-loop beta-function D-terms as in the GUT case (see (3.12) and (3.15), respectively), one is forced to embed $U(1)_Y$ in some non-Abelian group under which left and right chirality superfields transform separately [19, 51]. In this case one should require that the parameter $\zeta$ measuring the magnitude of the breaking of the non-Abelian symmetry should be such that the one-loop gauge-mediated masses-squared

$$\tilde{m}_f^2 \sim \sum_{A=1}^{3} N_{\epsilon f}^{(1)} \left( \frac{\alpha_A}{4\pi} \right)^2 \Lambda_S^2 \pm Y_f N_{\epsilon f}^{(1)} \frac{\alpha_1 \zeta}{(4\pi)^2} \Lambda_S^2$$

(4.1)
do not lead to tachyonic sfermions. As discussed in Section 3.2, the D-term contribution to the one-loop beta function for the sfermion masses-squared (3.15) (where $\alpha_3$ is replaced by a general $\zeta$) is negligible in comparison to the bare D-term and can be neglected from subsequent discussion.\[11\]

We will analyze the effect of the remaining gauge mediation D-term initial masses-squared in (4.1) for the third generation sfermions case by case.

\[11\]We implicitly assume here that $\zeta$ is not larger (at the scale $M$) than GUT breaking parameters in the standard model such as $\alpha_3$, and that $N_{\epsilon f}^{(1)}$ is not much larger than the other $N_{\epsilon f}$’s.
4.1 The two-loop effect

As in fully coupled models, all the constructions discussed here will exhibit a (partial) inverted hierarchy in the sfermion sector. Inverted hierarchy models are known to be susceptible to two-loop effects in the RGE, where the heavy first two generations can render light third-generation sparticles tachyonic [29, 30].

Positive physical masses-squared then require heavy initial soft masses for the third generation, such that a fair amount of fine-tuning may be necessary to stabilize the weak scale. This concern, however, will turn out to be unjustified – some fine-tuning is needed, but for a reasonable range of scales and parameters it can be milder than, say, the percent level.

The beta function for the third generation (and all non-directly coupled) sfermion masses-squared up to two-loops can be written as [52]

\[
\frac{d}{dt} \tilde{m}_f^2 = \frac{1}{16\pi^2} \beta_{\tilde{m}_f^2}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{\tilde{m}_f^2}^{(2)} \quad t \equiv \log (\mu/\mu_0). \tag{4.2}
\]

In our analysis we will neglect Yukawa couplings and A-terms throughout in the RGE, as well as gaugino contributions at the two-loop level. With these approximations the beta-functions can be written as [52]

\[
\beta_{\tilde{m}_f^2}^{(1)} \simeq -8 \sum_A g_A^2 C_A(R_f) |M_A|^2,
\]

\[
\beta_{\tilde{m}_f^2}^{(2)} \simeq +4 \sum_A g_A^2 C_A(R_f) \sigma_A + \frac{12}{5} g_1^2 Y_f S', \tag{4.3}
\]

where the sum over \(A\) runs over the three SM gauge groups, \(C_A(R_f)\) denotes the quadratic Casimir invariant of the representation \(R_f\) of the gauge group \(A\), \(Y_f\) is the hypercharge of the sfermion \(f\), and (ignoring small contributions from soft Higgs masses) one has

\[
\sigma_1 \simeq \frac{1}{5} g_1^2 \text{Tr} \left( \tilde{M}_{QQ}^2 + 3 \tilde{M}_{LL}^2 + 8 \tilde{M}_{ur}^2 + 2 \tilde{M}_{dR}^2 + 6 \tilde{M}_{eR}^2 \right),
\]

\[
\sigma_2 \simeq g_2^2 \text{Tr} \left( 3 \tilde{M}_{Q_L}^2 + \tilde{M}_{L_L}^2 \right),
\]

\[
\sigma_3 \simeq g_3^2 \text{Tr} \left( 2 \tilde{M}_{Q_L}^2 + \tilde{M}_{uR}^2 + \tilde{M}_{dR}^2 \right), \tag{4.4}
\]

and

\[
S' \simeq \frac{8}{3} g_3^2 \text{Tr} \left( \tilde{M}_{QL}^2 - 2 \tilde{M}_{ur}^2 + \tilde{M}_{dR}^2 \right) + \frac{3}{2} g_2^2 \text{Tr} \left( \tilde{M}_{Q_L}^2 - \tilde{M}_{LL}^2 \right)
\]

\[
+ g_1^2 \text{Tr} \left( -\frac{3}{10} \tilde{M}_{LL}^2 + \frac{1}{30} \tilde{M}_{Q_L}^2 - \frac{16}{15} \tilde{M}_{uR}^2 + \frac{2}{15} \tilde{M}_{dR}^2 + \frac{6}{5} \tilde{M}_{eR}^2 \right). \tag{4.5}
\]

In the above equations we write the sfermion mass-squared matrices in the standard SU(2) notation. In all the subsequent estimates made in this section, \(4.2\) is integrated in the
leading log approximation for the appropriate light sparticles, where the two-loop contribution runs down to the approximate decoupling scale for the heavy first two generation scalars $\sim F/M$, and the one-loop term flows down to a scale $\mu_0 \sim 1$ TeV. Neglecting chirality-mixing, we then impose that for the lightest sparticle

$$m_f^2(\mu_0) \simeq m_f^2(\mu_S) + \frac{1}{16\pi^2} \beta_f^{(1)}(\mu_S) \log \left( \frac{\mu_S}{\mu_0} \right) + \frac{1}{(16\pi^2)^2} \beta_f^{(2)}(\mu_S) \log \left( \frac{\mu_S}{F/M} \right) \gtrsim 0. \quad (4.6)$$

Taking $N_{\text{eff}}^{(A)} \equiv N$ for simplicity in all initial sfermion and gaugino masses, a rough lower bound on $N$ may then be found in each scenario, affecting the initial soft terms and thus the physical spectrum. A full analysis will of course introduce corrections to the obtained bounds, but the order of magnitude of $N$ is captured in this approximation. The sample spectra that we present in Section 4.3 use the full two-loop RG evolution, and are consistent with this estimate.

### 4.2 Models

The ground is now set to explore some partially coupled models. We begin with chiral models in which only the MSSM fermion-sfermion fields of one chirality are coupled to the superconformal sector. We then address an example of non-chiral partial couplings – a unified 10-centered scenario.

#### 4.2.1 Chiral models

We assume that only the left-handed (right-handed) MSSM fields are coupled to the CFT sector. Therefore, in the quark and lepton sectors, there are only left-handed (right-handed) suppression factors generated by the superconformal dynamics, and the factorizable Yukawa structure of (2.2) contains a single $\epsilon_L$ ($\epsilon_R$) factor. These models can still lead to acceptable flavor structures, but producing appropriate suppression for given anomalous dimensions requires increasing the ratio $M_\geq/M_\leq$. There are, however, no \textit{a priori} constraints on this ratio.\footnote{We need to require that $M_\geq$ is below the Planck scale, but this is easy to satisfy. Additional constraints from Landau poles will be discussed in the next section.} To be more concrete, in chiral left-handed models, the suppression pattern predicts that the mixing angles are of order the mass ratios, and so some additional suppression is needed in order to be in full agreement with measurement. Right-handed models suffer from a difficulty to parametrically produce the CKM mixing angles. When discussing right-handed models in the following, we assume a Cabibbo factor has been generated in estimating the right-handed mixing matrices (2.5); the bounds on the scales obtained in this way are conservative.
A comment is in order. When only right-handed superfields are coupled to the CFT sector, the minimal and more natural assumption is that the CFT sector is uncharged under $SU(2)_L \subset G_{SM}$. In such a scenario the Wino gauge-mediated mass will not be generated at the high scale $\mu_S$, leading to unacceptably light neutralinos and charginos at the weak scale. Therefore, from here on when discussing chiral right-handed couplings we assume that the CFT sector is charged under $SU(2)_L$. This also leaves open the possibility of directly coupling the Higgs sector to the CFT sector.

Among the four-Fermi operators (3.3) bounding $M$, only the chiral operators $Q_1, \tilde{Q}_1$ are now generated at $M$. In light of (2.5), the bound (3.5) in separable models can then be relaxed here to

$$M \lesssim 264 \text{ TeV} \quad \text{(from the } D^0 - \bar{D}^0 \text{ system)},$$

$$M \lesssim 220 \text{ TeV} \quad \text{(from the } K^0 - \bar{K}^0 \text{ system}).$$

(4.7)

In the sfermion sector, soft mass-squared terms for the left-handed (right-handed) first two generation sfermions are given by direct coupling as in (3.10):

$$\tilde{M}_{u,d,\ell}^{2,LL(LL)_{ii}} \sim \left( \frac{F}{M} \right)^2, \quad i = 1, 2.$$  (4.8)

All other soft terms are gauge-mediated similarly to (3.12):

$$M_A = N^{(A)}_{eff} \frac{\alpha_A}{4\pi} \Lambda_S,$$

$$(\tilde{M}_{u,d,\ell}^{2,LL(LL)_{ii}})_{LL,RR} \sim N^{(A)}_{eff} \left( \frac{\alpha_A}{4\pi} \right)^2 \Lambda_S^2 \pm N^{(1)}_{eff} Y_{u,d,\ell,L,R} \frac{\alpha_1}{4\pi}^2 \Lambda_S^2,$$

$$(\tilde{M}_{u,d,\ell}^{2,RR(LL)_{ii}})_{LL,RR} \sim N^{(A)}_{eff} \left( \frac{\alpha_A}{4\pi} \right)^2 \Lambda_S^2 \pm N^{(1)}_{eff} Y_{u,d,\ell,L,R} \frac{\alpha_1}{4\pi}^2 \Lambda_S^2, \quad i = 1, 2,$$

$$A_{u,d,\ell}^{LL,RR} \sim y_{u,d,\ell}^{LL,RR} N^{(A)}_{eff} \left( \frac{\alpha_A}{4\pi} \right)^2 \Lambda_S, \quad i, j = 1, 2, 3.$$  (4.9)

Avoiding initial tachyonic right-handed sleptons when the sign of the D-term is negative requires $\zeta \sim \alpha_3/5$, implying a mild tuning of the D-term generated by the CFT sector in this case. In Class (b) models we can achieve suppression of the D-term naturally by assuming a messenger parity [51] symmetry (taking $J_Y \rightarrow -J_Y$) below the scale $M$. In such a case $<J_Y>_{1-loop} = 0$ [10, 51] and so

$$<J_Y> = O\left( \frac{\zeta^2}{16\pi^2} \right),$$

(4.10)

i.e., the D-term mass-squared acquires an extra factor of $\zeta/(4\pi)$.

In the chiral coupling scenarios discussed here $N^{(A)}_{eff}$ could be smaller than in the case of direct coupling of both left- and right-handed fields to the superconformal sector, schematically by a factor of a half.
Figure 2: Relevant $D^0 - \bar{D}^0$ system (dashed) and $K^0 - \bar{K}^0$ system (bold) bounds on $F/M$ as a function of $N_{\text{eff}}^{(3)}$ for chiral left-handed models ($M = 264$ TeV). The left graph refers to Class (a) models and the right graph refers to Class (b) models.

In chiral left-handed models the most stringent bounds on the masses of the heavy first two generations, $F/M$, are set by processes involving the $(\delta^u_{12})_{LL}$ term in $D^0 - \bar{D}^0$ box diagrams. The $(\delta^d_{12})_{LL}$ term in $K^0 - \bar{K}^0$ box diagrams gives comparable yet milder constraints. Since the right-handed soft masses-squared are given by gauge mediation there is near degeneracy in the first two generations of right-handed squarks, and so $(\delta^{u,d}_{12})_{RR} \approx 0$ for left-handed models.

On the other hand, in chiral right-handed models, the strongest constraints on the heavy scale $F/M$ are obtained from processes involving $(\delta^d_{12})_{RR}$ terms in $K^0 - \bar{K}^0$ box diagrams. Processes involving $(\delta^u_{12})_{RR}$ terms in $D^0 - \bar{D}^0$ box diagrams are comparatively suppressed by the relative smallness of the right mixing angles in the up sector (see (2.5)). Since the left-handed soft masses-squared are given by gauge mediation there is near degeneracy in the first two generations of left-handed squarks, and so $(\delta^{u,d}_{12})_{LL} \approx 0$ for right-handed models.

Plots of the bounds on $F/M$ as a function of $N_{\text{eff}}^{(3)}$ are given in Figure 4 for chiral left-handed models, and in Figure 3 for chiral right-handed models.
Figure 3: Relevant $K^0 - \bar{K}^0$ system bounds on $F/M$ as a function of $N_{\text{eff}}^{(3)}$ for chiral right-handed models ($M = 220$ TeV). The left graph refers to Class (a) models and the right graph refers to Class (b) models.

Representative bounds in chiral left-handed coupled models are

\[
\text{LH Class (a)} : \quad \frac{F}{M} \gtrsim 5 \text{ TeV} \quad (N_{\text{eff}}^{(3)} = 62), \tag{4.11}
\]

\[
\text{LH Class (b)} : \quad \frac{F}{M} \gtrsim 6 \text{ TeV} \quad (N_{\text{eff}}^{(3)} = 5),
\]

while in chiral right-handed coupled models they are

\[
\text{RH Class (a)} : \quad \frac{F}{M} \gtrsim 6 \text{ TeV} \quad (N_{\text{eff}}^{(3)} = 15), \tag{4.12}
\]

\[
\text{RH Class (b)} : \quad \frac{F}{M} \gtrsim 5 \text{ TeV} \quad (N_{\text{eff}}^{(3)} = 5).
\]

Combining the bounds on $M$ and $F/M$ of (4.7), (4.11) and (4.12) we obtain for models of Class (b):

\[
\text{LH couplings} : \quad \sqrt{F} \gtrsim 40 \text{ TeV},
\]

\[
\text{RH couplings} : \quad \sqrt{F} \gtrsim 33 \text{ TeV}. \tag{4.13}
\]

As explained in Section 4.1, two-loop contributions to third generation masses put a lower bound on $N \equiv N_{\text{eff}}^{(A)}$. For concreteness we obtain the bounds for the point in parameter space where all order-one numbers in (4.8) are equal to one. The analysis
for other points in parameter space is qualitatively the same. Using (4.8) and (4.9) for left-handed models, at this point in parameter space one has

\[ S' \simeq \left[ \frac{16}{3} g_3^2 - \frac{8}{15} g_1^2 \right] \left( \frac{F}{M} \right)^2, \]

\[ \sigma_1 \simeq \frac{8}{3} g_1^2 \left( \frac{F}{M} \right)^2, \quad \sigma_2 \simeq 8g_2^2 \left( \frac{F}{M} \right)^2, \quad \sigma_3 \simeq 4g_3^2 \left( \frac{F}{M} \right)^2. \] (4.14)

For Class (a) the strongest bound on \( N \) comes from the right-handed charged sleptons, for which (4.13) reads

\[ \tilde{m}^2_{e_{Ri}}(\text{TeV}) \sim \frac{\alpha_1 \Lambda_S^2}{(4\pi)^2} \left[ \alpha_1 N + \frac{12}{5} \left( -\frac{16}{3} \alpha_3 - \frac{16}{15} \alpha_1 \right) \left( \frac{F/M}{\Lambda_S} \right)^2 \log \left( \frac{\mu_S}{F/M} \right) \right. \]

\[ + \frac{24}{5} \frac{\alpha_1^2}{4\pi} N_i \log \left( \frac{\mu_S}{\Lambda_S} \right) \left. \right] \gtrsim 0, \quad i = 1, 2, 3. \] (4.15)

For Class (b) the strongest bound comes from \( \tilde{u}_{R_i} \), although other sparticles give comparable bounds.\footnote{Here and in the following, since we neglect Yukawa couplings in the running, all generations of the non-directly coupled fields have similar RG evolution. Including the effects of Yukawas, the strongest constraints will come specifically from third generation sparticles within the relevant sector.}

By solving the relevant inequalities we obtain the bounds:

\[ \text{LH Class (a)} : \quad N \gtrsim 62 \quad \text{for} \quad \Lambda_S \sim F/M = 5 \text{ TeV}, \quad \mu_S \sim M = 264 \text{ TeV}, \]

\[ \text{LH Class (b)} : \quad N \gtrsim 2 \quad \text{for} \quad \Lambda_S \sim \mu_S \sim \sqrt{F} = 40 \text{ TeV} \quad (F/M = 6 \text{ TeV}). \] (4.16)

The analysis is analogous for right-handed models. For Class (a) models all sparticles, except right-handed charged sleptons which do not become tachyonic, give similar bounds, while in Class (b) models all sparticles typically give comparable constraints. We obtain:

\[ \text{RH Class (a)} : \quad N \gtrsim 15 \quad \text{for} \quad \Lambda_S \sim F/M = 6 \text{ TeV}, \quad \mu_S \sim M = 220 \text{ TeV}, \]

\[ \text{RH Class (b)} : \quad N \gtrsim 2 \quad \text{for} \quad \Lambda_S \sim \mu_S \sim \sqrt{F} = 33 \text{ TeV} \quad (F/M = 5 \text{ TeV}). \] (4.17)

The values of \( N^{(3)}_{eff} \) in (4.11) and (4.12) are chosen in agreement with the above.

### 4.2.2 10-centered models

As an alternative scenario, one could contemplate evading the strongest \( K^0 - \overline{K^0} \) mixing bounds by disallowing non-chiral couplings in the down sector alone. In SU(5)-based GUT models, where the \( \mathbf{5} \) contains \( L_L, d_R \) and the \( \mathbf{10} \) contains \( Q_L, u_R \) and \( e_R \), coupling only the \( \mathbf{10}_{1,2} \) to the CFT sector can accomplish this, while still generating the Yukawa hierarchies in the up, down and lepton sectors and guaranteeing a vanishing hypercharge \( D \)-term at...
leading order. In this scenario, suppression factors for both chiralities are generated in the up quark sector. In the down quark and lepton sectors only fields of one chirality, left and right respectively, acquire large anomalous dimensions. A detailed discussion of this fermion flavor structure can be found in [6].

In this setup the strongest bound on \( M \) comes from \( Q_2 \) of (3.3) in the \( D^0 - \overline{D^0} \) system [13] and reads

\[
M \gtrsim 682 \text{ TeV}. \tag{4.18}
\]

First two generation up squarks feel direct SUSY breaking, as do left-handed down squarks and right-handed sleptons, while all other sparticles have gauge mediated soft masses and masses-squared. Diagonal first two generation soft trilinear couplings in the up sector have direct SUSY breaking in them and similarly to (3.7) are proportional to the appropriate Yukawa couplings and are not suppressed by SM gauge factors. All other \( A \)-terms are dominantly given by gauge mediation. The expressions in (4.8) and (4.9) are appropriately modified.

As in chiral models, avoiding initial tachyonic right-handed sleptons requires a tuning of order \( 1/5 \) in the \( D \)-term for one of the possible signs of its contribution to the soft masses-squared. In Class (b) models, this tuning is not necessary if we assume messenger parity below the scale \( M \).

The strongest constraints on the heavy scale are now typically set by processes involving \((\delta^u_{12})_{LL}\) terms in \( D^0 - \overline{D^0} \) box diagrams. At relatively high \( N_{\text{eff}}^{(3)} \), processes involving \((\delta^u_{12})_{LL}(\delta^u_{12})_{RR}\) become dominant. Plots of the bounds on \( F/M \) as a function of \( N_{\text{eff}}^{(3)} \) are given in Figure 4.

Representative bounds can be taken to be

\[
\begin{align*}
\text{Class (a)} : \quad & \frac{F}{M} \gtrsim 7 \text{ TeV} \quad (N_{\text{eff}}^{(3)} = 28), \\
\text{Class (b)} : \quad & \frac{F}{M} \gtrsim 7 \text{ TeV} \quad (N_{\text{eff}}^{(3)} = 5),
\end{align*}
\tag{4.19}
\]

and the bound on the effective scale of SUSY breaking for models of Class (b) is then

\[
\sqrt{F} \gtrsim 70 \text{ TeV}. \tag{4.20}
\]

Two-loop contributions then impose the following bounds on \( N \equiv N_{\text{eff}}^{(A)} \) at, say, the point in parameter space where all order one numbers in (4.3) are equal to one:

\[
\begin{align*}
\text{Class (a)} : \quad & N \gtrsim 28 \quad \text{for} \quad \Lambda_S \sim F/M = 7 \text{ TeV}, \; \mu_S \sim M = 682 \text{ TeV}, \\
\text{Class (b)} : \quad & N \gtrsim 1 \quad \text{for} \quad \Lambda_S \sim \mu_S \sim \sqrt{F} = 70 \text{ TeV} \; (F/M = 7 \text{ TeV}).
\end{align*}
\tag{4.21}
\]

At this point in parameter space, Class (a) bounds come typically from left-handed sleptons, whereas in Class (b) all light sparticles (except right-handed charged sleptons, which do not become tachyonic) give comparable bounds.
Figure 4: Relevant $D^0 - \bar{D}^0$ system bounds on $F/M$ as a function of $N_{eff}^{(3)}$ from $(\delta_{12}^u)_{LL}$ (dashed) and $(\delta_{12}^u)_{RR}$ (dotted) processes, for 10-centered models ($M = 682$ TeV). The left graph refers to Class (a) models and the right graph refers to Class (b) models.

4.3 Spectra

We now present some sample spectra for the various partially coupled models discussed above. The results are obtained using the program SuSpect [53], sampling the parameter space using various effective messenger numbers, specific choices of order one coefficients coming from the unknown correlation functions in the CFT sector, and various values of $\tan \beta$. For simplicity, all CFT sector $N_{eff}^{(4)}$ factors are taken to be equal to $N$ and we always set the scale $M$ to its lower bound. In Class (a) models, at the classical level, the scale $M$ can be taken to be much higher than its lower bound, as long as $F/M$ is kept fixed, but this will modify the RG effects (4.6). Additionally, we set the gauge-mediated D-term to zero, since the qualitative behavior of the spectra is not modified.

In the simulations presented in Tables 4, 5, 6, 7 and 8 at the end of the paper, the $O(1)$ coefficients coming from the CFT sector in direct-mediated soft terms (for instance, multiplying the first two generation sfermion masses-squared in (4.8)) have all been taken to be one for simplicity. This implies that all the heavy sparticles are degenerate, which is certainly not the case generically in our models. (Models with separable CFT sectors involving two different scales of SUSY breaking also seem perfectly viable, though we do not discuss them here.) Otherwise, the spectra we present are typical. In Table 3 we show the inputs for the simulations and references to the corresponding tables containing
Table 3: Inputs and some results of simulations of partially coupled models. In the above we include references to the corresponding tables containing the low-energy spectra. For each simulation, the identity of the NLSP, its mass and the amount of fine-tuning for \( \tan \beta = 10 \) are presented.

representative sparticles of the low-energy spectra. In the tables, the mass scale of the heavy sparticles is of order \( F/M \). For completeness, we also include in Table 3 the identity of the NLSP, its mass and the amount of fine-tuning (defined below) in each type and class of viable model when \( \tan \beta = 10 \). A few examples of our full spectra are depicted in Figure 4.

All spectra obtained in these partially coupled models exhibit inverted hierarchies. The sectors in which the first two generation sfermions are heavy differ amongst the models depending on the coupling scenario. Fields that are not directly coupled to the CFT sector present a mass pattern similar to models of general gauge mediation. In our setups, the gluino typically interpolates between the heavy and light scales. As in models of gauge mediation, the LSP is always the gravitino. The NLSP can vary between sneutrino, stau, neutralino and chargino identities (see Tables 4 through 8 for samples of this, and [54, 55, 56] for recent NLSP parameter space discussions in general gauge mediation). Collider studies involving NLSPs of stau, sneutrino, neutralino and chargino character can be found in [15] (and references therein) and in [57, 58, 59].

In the simulations, the values of \( \mu \) and \( B_\mu \) are determined in order to reproduce the correct pattern of electroweak symmetry breaking (namely, the correct VEVs for the two Higgs fields). A rough order of magnitude estimate of the fine-tuning of the weak scale in these models is then given by \( \sim 2\mu^2/m_Z^2 \). In chiral left-handed models and 10-centered models \( \mu \) is \( \mathcal{O}(600 – 800) \) GeV and so these models typically present fine-tuning at the 1% level, as expected from the relatively large gluino and stop masses. Chiral right-handed couplings have lower values of \( \mu \), \( \mathcal{O}(300) \) GeV, and so these models can present fine-tunings milder than 1%. A more accurate quantification of the fine-tuning with respect to a parameter \( \lambda_i \) can be given by the Barbieri-Giudice parameter \( \Delta(m_Z^2; \lambda_i) \) [10]. In this language, \( \Delta(m_Z^2; \lambda_i) \lesssim 100 \) corresponds to the appropriate fine tuning being milder than

| Model       | Class | \( \Lambda_S \) [TeV] | \( F/M \) [TeV] | \( \mu_S \) [TeV] | \( N \) | Table | NLSP | \( m_{NLSP} \) [GeV] | Fine-tuning |
|-------------|-------|------------------------|-----------------|-----------------|--------|-------|------|-----------------|-------------|
| Left-handed | (b)   | 45                     | 6.5             | 45              | 10     | 1     | \( \tilde{\tau}_1 \) | 215           | 99          |
| Right-handed| (a)   | 9                      | 9               | 220             | 20     | 2     | \( \tilde{\nu} \) | 120           | 16          |
| Right-handed| (b)   | 35                     | 5               | 35              | 5      | 3     | \( \tilde{\tau}_1 \) | 153           | 30          |
| 10-centered | (b)   | 70                     | 7               | 70              | 5      | 7     | \( \tilde{\tau}_1 \) | 280           | 106         |
| Right-handed| (a)   | 9                      | 9               | 220             | 15     | 5     | \( \tilde{\chi}^0 \) | 168           | 16          |
the percent level. In the tables, \( \Delta \) denotes the strongest fine-tuning between \( \mu^2 \) and \( B_\mu \), calculated by SuSpect [3], which in all cases considered is \( \Delta(m_Z^2; \mu^2) \). Fine-tunings with respect to other parameters, e.g. the scale \( \Lambda_S \), are expected in our models to be at most comparable to the ones presented [60].

Note that in the spectra in Table 3 left-handed and 10-centered Class (a) models are not presented. This is because these models require large values of \( N_\text{eff}^{(A)} \), and so run into Landau poles. In fact, many (if not all) of our models exhibit Landau poles for the standard model gauge couplings below the GUT scale, and many of the models do not exhibit gauge coupling unification. We view our models as effective theories valid below the scale \( M > \) , so we do not worry about UV completions above this scale (except for needing to suppress baryon-violating operators by a higher scale \( M_{pl} \)). Models without large numbers of degrees of freedom in the CFT sector charged under the standard model group can be safe from Landau poles within the conformal window. However, our models that do have large numbers of degrees of freedom in the CFT sector charged under the standard model group, say \( N_\text{eff}^{(A)} \gtrsim 20 \), could have Landau poles already below the scale \( M > \), and then these models are not really valid as effective field theories (at least not as analyzed above). This means that some of our Class (a) models are not really self-consistent.

To derive a rough estimate of the upper bound on \( N_\text{eff}^{(A)} \), we impose that the conformal window is such that a \( 10^{-5} \) hierarchy in the up sector can be generated, and demand that the strong coupling \( \alpha_3 \) does not blow up in this window. Combining this, we find the order of magnitude constraint \( -3 + \frac{1}{2} N_{\text{add}} \lesssim 7 \gamma \), where \( N_{\text{add}} \) stands for additional non-MSSM degrees of freedom charged under \( SU(3) \) in the conformal window, and \( \gamma \) is the sum of the relevant anomalous dimensions in the up sector, \( \gamma = \frac{1}{2} (\gamma_Q + \gamma_u) \). Under the definition of \( N_\text{eff}^{(A)} \) through the gaugino masses (3.6) (relating it in weakly coupled theories to, say, the number of pairs of superfields in the fundamental and anti-fundamental representations), we obtain for Class (a) models \( N_{\text{eff},(a)}^{(3)} \approx \frac{1}{2} (N_{\text{add}} + 8) \) for fully coupled models (where the second term here stems from the fact that our definition of \( N_\text{eff}^{(A)} \) contains the first two generations of the MSSM, directly coupled to the CFT), yielding \( N_{\text{eff},(a)}^{(3)} \lesssim 7 \gamma + 7 \). Similarly, in chiral models this gives \( N_{\text{eff},(a)}^{(3)} \lesssim 7 \gamma + 5 \), and in 10-centered models \( N_{\text{eff},(a)}^{(3)} \lesssim 7 \gamma + 6 \). In Class (b) models, the relation between \( N_{\text{add}} \) and \( N_{\text{eff}}^{(A)} \) is less direct, since \( N_{\text{eff}}^{(A)} \) only contains the fields that survive to the lower scale \( \sqrt{F} \); clearly \( N_{\text{eff},(b)}^{(A)} < \frac{1}{2} N_{\text{add}} \). In all our Class (b) models \( N_{\text{eff},(b)}^{(A)} \) is small and consistent with this, and it seems that it should always be possible to add few enough degrees of freedom at

\footnote{We use here equations for the beta functions that ignore the anomalous dimensions; we know that some of our fields always have positive anomalous dimensions and others negative, and we expect some overall correction coming from this issue, but we ignore it here since our bounds are up to order one numbers anyway.}
Figure 5: The full sparticle spectrum for the models described in Tables 4, 5 and 7, for \( \tan \beta = 10 \).

the higher scale to be compatible with the bound on \( N_{\text{add}} \) above. In the above simulations we have presented only models that can be consistent with these bounds. This issue seems to favor Class (b) models, although some Class (a) models can also be consistent.

Note that our superconformal sector has an (accidental) \( U(1)_R \) symmetry that is broken at the exit from the conformal window. If this breaking is spontaneous, one may worry that we would have a light R-axion \([61]\) as the corresponding Nambu-Goldstone boson. However, the R-symmetry in our models is violated by Yukawa couplings and by terms like \( \mathcal{O}_2 \Phi_1 \), and despite the small coefficient of these terms, this is enough to raise the R-axion mass to a level where it does not pose any problems for phenomenology or for cosmology. In any case, we assume that the R-symmetry is broken at the exit by a large amount, so that it does not affect the spectrum of soft masses; it may be interesting to also investigate
models where this breaking is small.

5 Discussion and conclusions

The Nelson-Strassler mechanism is an elegant means of generating the pattern of the Yukawa couplings ex-nihilo – a sector with a strongly coupled fixed point with a finite basin of attraction naturally suppresses the Yukawa couplings via RG effects. In this paper we explored the extension of the Nelson-Strassler mechanism to a CFT sector which not only determines the Yukawa couplings, but also triggers SUSY breaking in the MSSM. A coarse look suggests that “here be dragons” – using this CFT sector to set the scale of SUSY breaking implies that its scale cannot be very high, and then, since the CFT sector is flavor-dependent (as much as conceivably possible), the Nelson-Strassler mechanism can run into problems with FCNC constraints if precautions are not taken when SUSY is broken.

In this work we analyzed these models in detail, focusing on the constraints from FCNCs, as well as on other problems that are typical in inverted hierarchy models. An inverted hierarchy is inevitable in our case, where the strongly coupled SUSY breaking CFT sector couples directly to (some of) the first two generation fields, giving rise to large first two generations sfermion masses, whereas the third generation masses arise from gauge mediation. We show that despite the apparent difficulties one can construct models with a viable level of fine-tuning.

In order to achieve low fine-tuning, we had to make several assumptions about the CFT sector:

1. The CFT sector conserves baryon number.

2. The standard model \( U(1)_Y \) is embedded in a non-Abelian group which is a subgroup of the global symmetry group of the CFT sector dynamics. This is certainly true in GUT models (such as \( SU(5) \) models, on which our 10-centered model is based), but it can be implemented in other cases as well.

3. In order to suppress flavor-violating effects, including those coming from mixings at the exit from the conformal window (accomplishing a graceful exit), we require two features of the CFT sector, both compatible with the Nelson-Strassler construction:

   (a) The CFT sector is separable (for instance by coupling each of the first two generations to a separate CFT sector), meaning that the CFT sector does not directly produce any flavor changing in the interaction basis.
(b) The models are partially coupled – only a subset of the fields in the first two
generations couple directly to the CFT sector.

4. The CFT sector has two scales, namely the scale of SUSY breaking $\sqrt{\mathcal{F}}$ is much
smaller than the scale $M$ where the first two generations decouple from the CFT
sector.

In this paper we did not attempt to construct a model that obeys all of these require-
ments (as well as a Nelson-Strassler mechanism and dynamical SUSY breaking). It seems
that such a model should be quite complicated, but we do not see any obstruction in prin-
ciple to the construction of such models (see [37, 38] for related, though \textit{a priori} not directly
applicable, examples in this direction). Our discussion assumed a strongly coupled CFT
sector, but otherwise it is completely general; in particular it applies to models [22] where
this sector has a weakly coupled description in higher dimensions. In Class (b) models
it is possible that the CFT sector below the scale $M$, and in particular at the scale of
SUSY breaking, could become weakly coupled, but we do not discuss this possibility here.
As discussed above, our models generally exhibit Landau poles below the GUT scale, but
suitable UV completions can perhaps be found, \textit{e.g.} via Seiberg duality.

We believe that even without building explicit models, our constructions should have
various distinctive features that could perhaps be tested at the LHC. It would be interesting
to perform a general analysis of the phenomenology of these models, but this is beyond
the scope of the present paper. Let us just mention here the general form of the spectrum
that the assumptions above imply. We always have a partial inverted hierarchy for the
sparticles, where the masses of the scalar components of the superfields coupled directly
to the CFT sector are around 10 TeV. The gluinos are lighter, with a mass of a few TeV,
and the rest of the superpartners are all around 2 TeV or below. The NLSP can be a
stau, sneutrino, neutralino or chargino, as in general gauge mediation. In comparison to
gauge mediation, our spectra differ by the heaviness of some of the first two generation
fields. The partial inverted hierarchy that we presented here could be even more partial
in separable models with only one SUSY-breaking component, but we do not analyze this
case here.

In comparison to other models with an inverted hierarchy, here the light sparticles have
a mass spectrum dictated specifically by gauge mediation. Additionally, in our partially-
coupled models only some chiralities exhibit an inverted hierarchy (see [38] for an example
of a previous model with related characteristics). We also have rather heavy gluinos.
Recently, a suggestion for joining a different model of flavor physics with supersymmetry
breaking [24] was realized in [26, 27]; there are many similarities of these models to what
we find here, but also some differences. Note in particular that in our models the difference
between the three generations of the standard model is generated purely dynamically, while
in the models of [24, 26, 27] one must put in by hand that only the third generation fields are elementary at high energies. Recent LHC-oriented phenomenological studies of inverted hierarchy models have appeared in [24].

In this paper we have thus far ignored the bounds on new physics coming from CP-violating processes. Even if the hidden sector conserves CP, this is not really justified, since the order one CP-violating phase in the CKM matrix generically means that the CP-violating flavor-changing processes in our model will be of the same order as the CP-conserving ones. Thus, in the general case the bounds used for CP-conserving processes would be replaced by the bounds including CP-violating processes. The fine-tuning required in our models would then be enhanced by a factor of order 25, depending on the precise model, leading to unacceptably large fine-tuning. The level of fine-tuning can be reduced (with or without the CP-violating operators) by making some additional assumptions about the structure of our models. For instance, one could assume that at high energies there is an $SU(3) \times SU(3)$ global symmetry that guarantees that the up-Yukawa couplings (or the down-Yukawa couplings) are proportional to the identity matrix (this symmetry is then broken by the couplings to the hidden sector). Or, one could assume that there is some sort of alignment that guarantees that the up-Yukawa couplings (or the down-Yukawa couplings) are diagonal in the same basis as the couplings to the hidden sector (this can naturally arise in extra dimensional scenarios, where different fields are located at different positions in the extra dimensions). In both cases, most of the discussion in our paper is not modified, but flavor-changing processes involving either up or down quarks are suppressed. Since in most of our models one of the two types of processes gives much stronger bounds than the other, this allows us to reduce the amount of fine-tuning by a factor of 8 or so. Note that in all cases the bounds coming from flavor-conserving CP-violating processes, such as electric dipole moments [65, 66] (see also e.g. [67]), are less constraining or at most comparable. We leave a detailed discussion of the implications of these additional assumptions, and of the construction of such models, to future work.

Finally, we should emphasize that in this paper we did not attempt to address the $\mu/B_\mu$ problem, which is generic in models of gauge mediation. Presumably, recent solutions to this problem (such as [68]) can be applied to our models as well. Note that in the natural scenario in which the Higgs fields do not couple directly to the CFT sector, $B_\mu = 0$ at leading order at the scale $\mu_S$, and it was recently claimed [20] that such a scenario is not impossible if $\tan \beta$ is large enough. We leave a detailed analysis of the Higgs sector and its couplings to future work.

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Table 4: Example of a spectrum for Class (b) models with \( N = 10 \), when coupling only left-handed first two generation fields to a decomposable CFT sector. We present representative sparticles of the low energy spectrum. The appropriate heavy sparticle masses are of order \( F/M \). The results are obtained using \( F/M = 6.5 \) TeV, \( \mu_S = \Lambda_S = 45 \) TeV for two different values of \( \tan \beta \), choosing all unknown coefficients coming from the CFT sector to equal one. The entries for \( \tilde{u}_1, \tilde{d}_1, \tilde{e}_1 \) refer to \( \tilde{c}_1, \tilde{s}_1, \tilde{\mu}_1 \) as well. \( \Delta \) is the fine-tuning of \( m_Z^2 \) with respect to \( \mu^2 \). All dimensionful quantities are given in GeV.

| \( \tan \beta \) | \( m_h \) | \( m_{\chi_1^0} \) | \( m_{\tilde{\chi}_1^0} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( M_3 \) | \( \mu \) | \( \Delta \) |
|-----------------|--------|----------------|----------------|--------|--------|--------|--------|--------|--------|--------|--------|
| 3               | 108    | 754            | 766            | 2150   | 2274   | 215    | 2281   | 2275   | 216    | 4446   | 761    | 218    |
| 10              | 119    | 631            | 636            | 2165   | 2272   | 215    | 2281   | 2275   | 217    | 4445   | 629    | 99     |

Table 5: The same for Class (a) models with \( N = 20 \), when coupling only the right-handed first two generation fields to a decomposable CFT sector, using \( F/M = 9 \) TeV, \( \mu_S = 220 \) TeV and \( \Lambda_S = 9 \) TeV.

| \( \tan \beta \) | \( m_h \) | \( m_{\chi_1^0} \) | \( m_{\tilde{\chi}_1^0} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( m_{\tilde{\nu}_1} \) | \( M_3 \) | \( \mu \) | \( \Delta \) |
|-----------------|--------|----------------|----------------|--------|--------|--------|--------|--------|--------|--------|--------|
| 3               | 98     | 218            | 277            | 567    | 697    | 141    | 722    | 726    | 142    | 123    | 1533   | 295    | 34     |
| 10              | 111    | 206            | 238            | 580    | 696    | 140    | 722    | 727    | 143    | 120    | 1533   | 246    | 16     |
Table 6: The same for Class (b) models with $N = 5$, when coupling only the right-handed first two generations to a decomposable CFT sector, using $F/M = 5$ TeV and $\mu_S = \Lambda_S = 35$ TeV.

Table 7: The same for Class (b) models with $N = 5$ in the 10-centered coupling scenario with a decomposable CFT sector, using $F/M = 7$ TeV and $\mu_S = \Lambda_S = 70$ TeV.

Table 8: Example of a spectrum for Class (a) models with $N = 15$, in the chiral right-handed coupling scenario with a decomposable CFT sector. The results are obtained using $F/M = 9$ TeV, $\mu_S = 220$ TeV and $\Lambda_S = 9$ TeV, choosing the $O(1)$ numbers equal to 1 for direct-mediated soft terms and 3 for gauge-mediated ones.
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