Multi-agent Inverse Reinforcement Learning for General-sum Stochastic Games

Xiaomin Lin\(^1\), Stephen C. Adams\(^2\) and Peter A. Beling\(^2\)

\(^1\) MassMutual Financial Group, 470 Atlantic Ave, Boston, MA 02210, USA, xlin63@massmutual.com
\(^2\) University of Virginia, Charlottesville, VA 22903, USA, \{sca2c, pb3a\}@virginia.edu

Abstract. This paper addresses the problem of multi-agent inverse reinforcement learning (MIRL) in a two-player general-sum stochastic game framework. Five variants of MIRL are considered: uCS-MIRL, advE-MIRL, cooE-MIRL, uCE-MIRL, and uNE-MIRL, each distinguished by its solution concept. Problem uCS-MIRL is a cooperative game in which the agents employ cooperative strategies that aim to maximize the total game value. In problem uCE-MIRL, agents are assumed to follow strategies that constitute a correlated equilibrium while maximizing total game value. Problem uNE-MIRL is similar to uCE-MIRL in total game value maximization, but it is assumed that the agents are playing a Nash equilibrium. Problems advE-MIRL and cooE-MIRL assume agents are playing an adversarial equilibrium and a coordination equilibrium, respectively. We propose novel approaches to address these five problems under the assumption that the game observer either knows or is able to accurately estimate the policies and solution concepts for players. For uCS-MIRL, we first develop a characteristic set of solutions ensuring that the observed bi-policy is a uCS and then apply a Bayesian inverse learning method. For uCE-MIRL, we develop a linear programming problem subject to constraints that define necessary and sufficient conditions for the observed policies to be correlated equilibria. The objective is to choose a solution that not only minimizes the total game value difference between the observed bi-policy and a local uCS, but also maximizes the scale of the solution. We apply a similar treatment to the problem of uNE-MIRL. The remaining two problems can be solved efficiently by taking advantage of solution uniqueness and setting up a convex optimization problem. Results are validated on various benchmark grid-world games.

Keywords: inverse reinforcement learning, multi-agent, game theory, general-sum, correlated equilibrium, Nash equilibrium, cooperative games, non-cooperative games

1 Introduction

In the problem of inverse reinforcement learning (IRL), the objective is to estimate the reward function for a Markov decision process (MDP) given observations of the policy followed by an agent assumed to be acting optimally [23].
IRL has been applied to a number of problems, most related to the problems of learning from demonstrations and apprenticeship learning [13,35]. Multi-agent reinforcement learning (MRL) can be approached from an inverse perspective in a manner analogous to that of IRL. A number of algorithms have been developed with theoretically or empirically guarantee to certain equilibriums, such as Nash or correlated equilibriums [18,19,12,8,32,3,7]. Inverse learning problems for MRL, which we term multi-agent inverse reinforcement learning (MIRL), include the problem of estimating the payoffs of games, given only observations of the actions taken by the players. Compared to the IRL problem, MIRL is more challenging in that it is formalized in the context of a stochastic/Markov game (see, e.g., [30,25]) rather than a MDP. Games bring two primary challenges: First, the concept of optimality, central to MDPs, loses its meaning and must be replaced with a more general solution concept, such as the Nash equilibrium (NE). Second, the non-uniqueness of equilibria means that in MIRL, in addition to multiple reasonable solutions for a given inversion model, there may be multiple inversion models that are all equally sensible approaches to solving the problem.

IRL is a special or approximate version of MIRL in the sense that the former treats other agents in the system as part of the environment, ignoring the difference between responsive agents and passive environment. A financial trading example can be used to illustrate the difference. Yang et. al. [34] use the reward functions inferred from IRL as a feature space for the purpose of classifying high-frequency trading algorithms in the stock market. This treatment is reasonable in a typical stock trading market. Usually there are many traders involved and their activities give rise to cancellation effects that make it reasonable for any one trader to model the collective actions of all the other traders as a stochastic system. However, if the market is dominated by a small number of traders—such as is the case currently with crypto-currency trading—the market should not be modeled as a passive system. Rather, each dominant trader should take the other’s possible strategies into account before making decisions. In such a case, a stochastic game framework (used in MIRL) would be more appropriate than a MDP framework (used in IRL).

Recently MIRL has attracted some interest from the machine learning research community. Natarajan et al. address MIRL using an IRL model for multiple agents without dealing with interactions or interference among agents [23]. Waugh et al. [33] contribute to the inverse equilibrium problem, but in the context of simultaneous one-stage games, rather than the sequential stochastic games that are the subject of MIRL. Reddy et al. [29] use the concept of subgame perfect equilibrium (SPE) [20], a refinement of NE used in dynamic games, to address MIRL for general-sum stochastic games that have the property that each player’s rewards do not depend on the actions of the others. Hadfield-Menell et al. [15] introduce a cooperative IRL problem, motivated from an autonomous system design problem, where the robot is required to align its value with those of the humans in its environment in such a way that its actions contribute to the maximization of values for the humans. Their problem is not modeled as a
MIRL problem in a stochastic game context. Lin, Beling, and Cogill [17] develop a Bayesian method but address only two-person zero-sum stochastic games. The approach in [17] is not applicable to the general-sum case because the uniqueness property of the minimax equilibrium in zero-sum games does not carry over to key solution concepts, such as NE, in general-sum games. The non-uniqueness of equilibria, in turn, makes it unclear how to specify the likelihood function for a Bayesian inverse learning algorithm.

In this paper, we consider five special classes of two-person general-sum MIRL problems, uCS-MIRL, advE-MIRL, cooE-MIRL, uCE-MIRL, and uNE-MIRL, each distinguished by its solution concept. The first problem, uCS-MIRL, is a cooperative game in which the agents employ cooperative strategies (CSs) that aim to maximize the sum of their value functions, or the total game value. The second and third problems consider circumstances in which two very special and unique NEs are employed: advE is in general a win-or-lose equilibrium, but not necessarily for a zero-sum game; cooE is such an equilibrium that players maximize their own payoffs by coordinating with others. In the fourth problem, uCE-MIRL, the agents are assumed to follow strategies that constitute a utilitarian correlated equilibrium (uCE), which achieves the maximum total game value among all CEs. In the last problem, uNE-MIRL, players are assumed to follow strategies that constitute a NE that maximizes total game value.

First, advE and cooE come from [19] and uCE comes from [12]. uCS, I do not want to say it is an "equilibrium", I would rather call it a "strategy", because Cooperative Strategy (CS) is for a cooperative game not for a non-cooperative game. Majority of MRL research has been focusing on non-cooperative games. As for uNE, nobody has ever touched it in a forward perspective in any MRL algorithm. Then why did we do it? Because as long as we can do uCE, we can do uNE in a straightforward way without much efforts in theory.

The MIRL problems that we study from an inverse perspective arise in a variety of application contexts. For example, uCS-MIRL, uCE-MIRL, and uNE-MIRL embed solution concepts that correspond to agents trying to achieve a socially efficient outcome (with or without certain constraints) that maximizes the sum of their value functions and is a Pareto optimum, meaning that it is not possible to make one player better off without also making the other player worse off [6]. These equilibria are of particular interest in welfare economics, in which policy makers try to design rules of games to achieve Pareto optimal solutions in social welfare. Also, uCE has been studied from an MRL perspective [12]. Despite the fact the advE and cooE-MIRL equilibria are not guaranteed to exist, they have been studied in MRL [19] and have potential applications. Consider an example in which two power suppliers compete with each other in the local market. Though it can be viewed as a competitive game, the situation is unlikely to evolve to a dominate-or-exit solution typical of zero-sum games. Hence it might be more reasonable to assume the suppliers are playing an advE equilibrium rather than a minimax solution to a zero-sum game. As for cooE, the classic Stag Hunt game (see, e.g.) [31] is representative of a broad range of social cooperation games. In Stag Hunt there are two hunters, each can chose
to hunt hare or stag, with symmetric payoffs. If they both hunt stag(hare), they
both will get a payoff of 2(1); and if their targets are different, the one who hunts
stag will fail to get anything and the other will get a payoff of 1. In this game,
(stag, stag) is a cooE.

We propose novel approaches to address these five problems under the as-
sumption that the game observer knows or is able to accurately estimate the
policies and solution concepts for the players. For uCS/advE/cooE-MIRL, we
first develop a characteristic set of solutions ensuring that the observed bi-policy
is a corresponding strategy/equilibrium and then apply a Bayesian inverse learn-
ing method. For uCE-MIRL, we develop a linear programming problem, subject
to constraints that define necessary and sufficient conditions for the observed
policies to be a correlated equilibrium (CE). For the objective function, we pro-
pose novel heuristics to choose a solution that not only minimizes the total game
value difference between the observed bi-policy and its local uCS, but also max-
imizes the scale of the solution. We apply a similar treatment to the problem of
uNE-MIRL.

The remainder of this paper is structured as follows. Section 2 introduces
notation, terminology and definitions that will be used throughout this paper,
as well as some basic game theory equilibrium concepts through some examples.
Section 3 summarizes several conventional MIRL algorithms. Section 4 provides
the main technical work, developing different approaches for different problems
to learn rewards. Section 5 and Section 6 demonstrate our algorithms through
several benchmark experiments that include comparison with existing MIRL
algorithms. Section 7 offers concluding remarks and a discussion of future work.

2 Preliminaries

This section serves to introduce concepts and notation for MRL that will be
used throughout the paper. It also introduces relevant concepts and formalism
for two-player general-sum games and the equilibria of interest in later sections.

2.1 Stochastic Game

A two-player general-sum discounted stochastic game is a tuple \( \{S, A_i, R_i, P, \gamma\} \),
where \( S \) is the common state space for all players, \( A_i \) and \( R_i \) are the action space
and reward for player \( i \), respectively. \( P \) is the probabilistic function controlling
state transitions, conditioned on the past state and joint actions. The reward
discount factor is \( \gamma \in [0, 1) \). In this paper, we assume that both players share
the same action space. The state and action spaces are both finite, i.e. \( |S| = N \) and \( |A_i| = M \). A stochastic game is a sequence of single-stage games, or
subgames, induced in every state \( s \in S \), such that both players need to deter-
mine an individual strategy \( \pi_i(s) \) or negotiate a bi-strategy \( \pi(s) \) that guides their
actions in every subgame. The collection of all bi-strategies is a bi-policy \( \pi \).
Note that an individual strategy can be a mixed strategy, which is a probability
distribution over all available actions. We define a pure bi-strategy \( a \in A = A_1 \times
A2 as a bi-strategy where both players select deterministic actions. Each player’s reward values are assumed dependent on state and possibly, bi-strategies, but are independent of each other.

### 2.2 MRL

Let \( \tilde{r}_i^\pi (s) \) be the expected reward value received by agent \( i \) at state \( s \) under bi-policy \( \pi \), specifically,

\[
\tilde{r}_i^\pi (s) = \sum_a \pi_1 (a_1 | s) \pi_2 (a_2 | s) R_i (s, a)
\]

\( = [\pi_1 (s)]^T R_i (s) \pi_2 (s), \forall s \in S, \)  

where \( a \) is a pure bi-strategy, \( \pi_i (s) \) is a \( M \times 1 \) vector denoting the probability distribution over actions in state \( s \), \( R_i (s) \) is a \( M \times M \) matrix, each entry of which denotes a pure bi-strategy dependent reward value. Structuring all \( R_i (s, a) \) into a column vector as \( r_i \), we can simplify and represent (1) in a matrix notation as

\[
\tilde{r}_i^\pi = B_\pi r_i.
\]

The linear transformation operator \( B_\pi \) is a \( N \times NM^2 \) matrix constructed from \( \pi \), whose \( k \)th row is:

\[
[\Phi_{1,1}^\pi (k), \Phi_{1,2}^\pi (k), \cdots, \Phi_{M,M}^\pi (k)],
\]

where

\[
\Phi_{i,j}^\pi (k) = \begin{bmatrix}
0, \cdots, 0, \phi_{i,j}^\pi (k), 0, \cdots, 0 \\
\overbrace{0}^{k-1}, \phi_{i,j}^\pi (k), \overbrace{0}^{N-k}
\end{bmatrix},
\]

and

\[
\phi_{i,j}^\pi (k) = \pi_1 (i | k) \pi_2 (j | k).
\]

Player \( i \)’s value function, starting at state \( s \) and under \( \pi \), is defined as

\[
V_i^\pi (s) = \sum_{t=0}^{\infty} \gamma^t E (\tilde{r}_i^\pi (s_t) | s_0 = s),
\]

and its Q-function, upon \( s \) and \( a \), is

\[
Q_i^\pi (s, a) = r_i (s, a) + \gamma \sum_{s'} p (s' | s, a) V_i^\pi (s')
\]

\( = r_i (s, a) + \gamma P_{s,a} V_i^\pi. \)

A major difference between RL/IRL and MRL/MIRL, is the definition of the value function. In MRL/MIRL,

\[
V_i^\pi (s) \in \text{solution concept} (Q_1^\pi (s), Q_2^\pi (s)), \forall s \in S
\]
Players, of course, are free to choose any solution concept. In order to pose an inverse learning problem, though, a particular choice of solution concept has to be known or assumed.

Let \( G_\pi \) denote a transition matrix under bi-policy \( \pi \). Specifically, \( G_\pi \) is the \( N \times N \) matrix with elements

\[
g_\pi (s'|s) = \sum_a \pi_1 (a_1|s) \pi_2 (a_2|s) p(s'|s,a).
\]

Then

\[
V_\pi (s) = r_\pi (s) + \gamma \sum_{s'} g_\pi (s'|s) V_\pi (s').
\]

In addition, \( V_\pi (s) \) can also be expressed in terms of the \( Q \)-function as

\[
V_\pi (s) = [\pi_1 (s)]^T Q_\pi (s) \pi_2 (s),
\]

where \( Q_\pi (s) \) is a \( M \times M \) matrix. We can rewrite (7) in matrix notation as

\[
V_\pi = r_\pi + \gamma G_\pi V_\pi.
\]

Thus

\[
V_\pi = (I - \gamma G_\pi)^{-1} B_\pi r_\pi,
\]

where \((I - \gamma G_\pi)\) is always invertible for \( \gamma \in [0,1) \) since \( G_\pi \) is a transition matrix. Restructuring \( Q_\pi (s,a) \) into a column vector, denoting \( Q_\pi \), we can rewrite equation (4) in matrix notation, over all states and joint actions, as

\[
Q_\pi = r_\pi + \gamma Pv_\pi,
\]

where \( P \) is a \( NM^2 \times N \) matrix with \( p(s'|s,a) \) as its elements. Combining (11) and (10) leads to

\[
Q_\pi = r_\pi + \gamma P (I - \gamma G_\pi)^{-1} B_\pi r_\pi
\]

In addition, (5) can be rewritten more compactly as

\[
V_\pi = B_\pi Q_\pi.
\]

Lastly, we define the total game value of a two-player stochastic game starting at state \( s \), under a bi-policy \( \pi \), \( V_\pi (s) \), as the sum of the value functions of both players, i.e., \( V_\pi (s) = V_1 (s) + V_2 (s) \).

### 2.3 Nash Equilibrium

In non-cooperative game theory, the Nash equilibrium (NE) is one of the most important solution concepts. A NE is an equilibrium where no player will benefit from unilaterally deviating from their current strategy given the other players’
strategies remain unchanged \cite{21, 22}. In a two-player single-stage game (state $s$),
\(\pi(s)\) is a NE if and only if,
\[ R_i(s, \pi(s)) \geq R_i(s, a_i, \pi_{-i}(s)), a_i \in A_i, \]
\[ (15) \]

The most important theorem regarding to the existence of NE, for any \(n\)-player
games, is as follows \cite{22, 25}:

**Theorem 1.** (Nash Existence \cite{22, 25}) Every finite game has at least one mixed
strategy Nash equilibrium.

However, there can exist multiple or even an infinite number of Nash equilibria,
and finding a NE or determining the number of them is a NP-hard problem \cite{9}.
Many algorithms, such as the Lemke-Howson Algorithm, have been developed
to address this problem. See, e.g., \cite{2, 9} for more details on algorithms for finding
NE. A concrete example of the chicken game \cite{28} is shown in Figure 1. In this
game two cars are driving toward each other. Each driver may either chicken-out
by swerving away (action C) or dare by continuing straight (action D), where
there are three NEs, as follows:

1. pure strategy - (C, D), with game values \(R_1 = 2, R_2 = 7\)
2. pure strategy - (C, D), with game values \(R_1 = 7, R_2 = 2\)
3. mixed strategy - \(\pi_1: \left(\frac{2}{3}C, \frac{1}{3}D\right), \pi_2: \left(\frac{2}{3}C, \frac{1}{3}D\right)\),
with game values \(R_1 = R_2 = \frac{14}{3}\)

We can see that different NEs have different game values. This non-uniqueness
property causes the non-convergence issue and is a bottleneck for MRL research.

### 2.4 Correlated Equilibrium

The **correlated equilibrium** (CE) is a more general solution concept in non-
cooperative games. Unlike NE, in which all agents act independently on a selfish
and conservative basis, a CE allows dependencies among agents. A CE is a probability
distribution over the joint space of actions, in which all agents optimize
their payoff with respect to one another’s probabilities, conditioned on their own
probabilities \cite{12}.

To illustrate, consider the chicken game shown in Figure 1. Assume there is
a trusted mediator devising a joint bi-strategy \(\pi\) for Player # 1 and Player # 2
from among these choices:
1. Player #1 takes action $C$ and Player #2 takes action $C$;
2. Player #1 takes action $C$ and Player #2 takes action $D$;
3. Player #1 takes action $D$ and Player #2 takes action $C$;
4. Player #1 takes action $D$ and Player #2 takes action $D$.

The mediator selects one of these four pure bi-strategies by sampling from a distribution with probabilities $\pi_{CC}$, $\pi_{CD}$, $\pi_{DC}$, $\pi_{DD}$, respectively, where $\pi_{CC} + \pi_{CD} + \pi_{DC} + \pi_{DD} = 1$. Once a pure bi-strategy is picked, the mediator will make a recommendation to both players accordingly. However, both players receive the recommendation regarding their own actions without knowing what recommendation the other player receives. Each player has the option to accept the recommendation or not.

A CE is such an equilibrium that each player will accept the recommendation with the belief that the other player will also accept the recommendation. In other words, in the case that the other player takes the recommended action, one will not benefit from deviating from the mediator’s recommendation.

In the Chicken Game example, the mediator’s distribution induces a CE if and only if these inequalities hold:

\begin{align}
6\pi_{CC} + 2\pi_{CD} &\geq 7\pi_{CC} + 0\pi_{CD} \\
6\pi_{CC} + 2\pi_{DC} &\geq 7\pi_{CC} + 0\pi_{DC} \\
7\pi_{DC} + 0\pi_{DD} &\geq 6\pi_{DC} + 2\pi_{DD} \\
6\pi_{CC} + 2\pi_{DC} &\geq 7\pi_{CC} + 0\pi_{DC} \\
7\pi_{CD} + 0\pi_{DD} &\geq 6\pi_{CD} + 2\pi_{DD},
\end{align}

(16)

Armed with some intuition of CE given by the preceding example, we now formally define CE as follows [20]:

**Definition 1.** Let $\Delta (A)$ denote the set of probability distributions over $A$, and $X$ be a random variable taking values in $A = \prod_{i \in I} A_i$ distributed according to $\pi \in \Delta (A)$. Then $\pi$ is a correlated equilibrium if and only if

$$
\sum_{a_{-i} \in A_{-i}} P(X = a|X_i = a_i) [R_i (s, a_i, a_{-i}) - R_i (s, \tilde{a}_i, a_{-i})] \geq 0,
$$

for all $a_i \in A_i$ such that $P(X_i = a_i) > 0$ and all $\tilde{a}_i \in A_i \setminus a_i$.

One real application of CE is the traffic light control shown in Figure 2. A traffic light sends a private message with an action recommendation to each car (“stop” for A and C and “go” for B and D). Each car can decide whether to accept it or not. Rationally, no car will reject the recommendation from the traffic lights given that all other cars obey the rules.

Similar to NE, an important theorem regarding the existence of CE is as follows [14]:

**Theorem 2.** (Correlated Equilibrium Existence [14]) Every finite game has at least one Correlated equilibrium.

In fact, CE is a superset of NE [3], and hence for any general sum game, the number of CEs is larger than or equal to the number of NEs. In contrast to NE, for which no efficient method of computation is known, finding CEs can be done in polynomial time via linear programming.
2.5 Cooperative Strategy

Both NE and CE are equilibria of competitive games. In a cooperative game, by contrast, an agreement on a joint strategy for all players can be called a cooperative strategy (CS). A characteristic function $v$ defines the type of cooperation between players \[10\], and for a two-player single-stage game (state $s$), can be defined as

$$v(s, a) = Val(R_1(s, a), R_2(s, a)), a \in A = A_1 \times A_2.$$  \hspace{1cm} (17)

$Val(\cdot)$ is self-defined, based on the type of cooperation.

3 Conventional MIRL Approaches

Before introducing our algorithms, we review several existing approaches to MIRL and related problems. The first of these is a decentralized MIRL (d-MIRL) algorithm developed by Reddy et al. \[29\]. This algorithm is decentralized in the sense that it infers agents’ rewards one by one, rather than all at once. All agents are assumed to follow a Nash equilibrium at every single game. The key idea is to find a reward that maximize the difference between the $Q$ value of the observed policy and those of pure strategies, which is analogous to the classical approach to single-agent IRL given in \[24\]. Though in the original version of their algorithm reward is assumed dependent only on the state, we can extend it to treat action dependency as well. Using our notation and player 1 as an example, the d-MIRL approach to a two-person general-sum MIRL problem is
to solve the following linear program:

$$\begin{align*}
\text{maximize:} & \quad \sum_{s=1}^{N} \min_{a_1} \left( B_{\pi} - B_{\pi(a_1)} \right) D_{\pi} r_1 - \lambda \|r_1\|_1 \\
\text{subject to:} & \quad \left( B_{\pi(a_1)} - B_{\pi} \right) D_{\pi} r_1 \leq 0,
\end{align*}$$

where $D_{\pi}$ is defined in (25) in the following section, $\lambda$ is an adjustable penalty coefficient for having too many non-zero values in the reward vector.

The key idea of the second approach is to model a two-person general-sum MIRL as an IRL problem. This approach requires us to select one player (e.g., player 1) and treat the other player as part of the passive environment. We extend the Bayesian IRL (BIRL) approach developed in [27], which is only applicable to state-dependent reward recovery, to involve action-dependence cases. Note that the reward to be recovered is $R_1(s, a_1)$ instead of $R_1(s, a_1, a_2)$, as player 2 is not considered adaptive. That is to say, $R_1(s, a_1, a_2) = R_1(s, a_1)$ for all $a_2 \in A_2$.

Using our notation, the algorithm to recover player 1’s reward is:

$$\begin{align*}
\text{minimize:} & \quad \frac{1}{2} (r_1 - \mu r_1)^T \Sigma_{r_1}^{-1} (r_1 - \mu r_1) \\
\text{subject to:} & \quad \left( F_{\pi(a_1)} - C_{a_1} \right) r_1 \geq 0,
\end{align*}$$

subject to:

$$\left( F_{\pi(a_1)} - C_{a_1} \right) r_1 \geq 0,$$

for all $a_1 \in A_1$, where

$$F_{\pi(a_1)} = \left[ \gamma \left( G_{\pi} - G_{\pi(a_1)} \right) (I - \gamma G_{\pi})^{-1} + I \right] C_{\pi},$$

and $C_{\pi}$ is a $N \times NM$ sparse matrix constructed from $\pi_1$, whose $i$th row is

$$\begin{bmatrix}
0, \cdots, \pi_1(i, 1), \cdots, 0, \cdots, \pi_1(i, M), \cdots, 0
\end{bmatrix}_{N \times (M-2)N},$$

and $C_{a_1}$ is conceptually similar to $C_{\pi}$, except for being constructed from a pure strategy $a_1$ for all states.

Strictly speaking, the BIRL approach is not a dedicated algorithm for MIRL problems but rather a way of shoe-horning the multi-agent problem into a single-agent, IRL setting. BIRL will provide a useful point of comparison to quantify the benefits of explicitly modeling the decisions of all players.

The third approach is not applicable to a general MIRL problem but a restricted family: zero-sum games. The algorithm to recover one player’s reward vector (assuming the other player’s reward is the additive inverse) is

$$\begin{align*}
\text{minimize:} & \quad \frac{1}{2} (r - \mu r)^T \Sigma_{r}^{-1} (r - \mu r) \\
\text{subject to:} & \quad \left( B_{\pi(a_1)} - B_{\pi} \right) D_{\pi} r \leq 0 \\
& \quad \left( B_{\pi(a_2)} - B_{\pi} \right) D_{\pi} r \geq 0,
\end{align*}$$

for all $a_1 \in A_1$ and $a_2 \in A_2$. More details can be found in [17].

These three approaches will be revisited as benchmarks in later sections.
4 MIRL Model Development

This section proposes five two-player general-sum MIRL problems and corresponding approaches to them. We first informally define an MIRL problem. Given a bipolicy $\pi$ being played in a two-player, general-sum game with states, actions, dynamics, and discount $\{S, A_i, P, \gamma\}$, the MIRL problem is to find rewards $r_1, r_2$ that best explain the observed policy. Though we will not do so in this paper, MIRL may defined in terms of an set of observed state-action values $O$ rather a bipolicy.

The MRL literature suggests that an agreement over a specific solution concept may be needed to solve a MRL problem. Similarly, in our approaches to MIRL, one basic assumption is required: both players agree on a specific strategy/equilibrium to play and this information is available to us in posing the MIRL problem. We limit attention to the following five solution concepts:

1. **utilitarian Cooperative Strategy** (uCS). In (17), consider $\text{Val}(\cdot) = \sum (\cdot)$. A single-stage game in state $s$ and taking action $a$ is a utilitarian cooperative strategy (uCS) if and only if
   \[ \sum_i R_i(s, a) \geq \sum_i R_i(s, a'), a' \in A = A_1 \times A_2 \setminus a. \] (20)

2. **Adversarial Equilibrium** (advE) An advE is a type of NE. It has another feature that no player is hurt by any change of others [15,19]. That is to say, in a two-player single-stage game (state $s$), $\pi(s)$ is an advE if and only if, in addition to (15),
   \[ R_i(s, \pi_i(s), \pi_{-i}(s)) \leq R_i(s, \pi_i(s), a_{-i}), a_{-i} \in A_{-i}, \] (21)

3. **Coordination Equilibrium** (cooE). A cooE is also a type of NE. It has another feature that all players’ maximum expected payoffs are achieved [15,19]. Mathematically, in a two-player single-stage game (state $s$), $\pi(s)$ is a cooE if and only if, in addition to (15),
   \[ R_i(s, \pi(s)) \geq R_i(s, a), a \in A = A_1 \times A_2. \] (22)

4. **utilitarian Correlated Equilibrium** (uCE). We borrow the concept of utilitarian correlated equilibrium (uCE) from [12] and state that in a two-player single-stage game (state $s$), $\pi(s)$ is a uCE if and only if,
   \[ \Sigma_i R_i(s, \pi(s)) \geq \Sigma_i R_i(s, \hat{\pi}(s)), \hat{\pi}(s) \in \pi_{\text{CE}} \setminus \pi(s). \] (23)

5. **utilitarian Correlated Equilibrium** (uNE). Similar to uCE, in a two-player single-stage game (state $s$), a NE $\pi(s)$ is a utilitarian Nash Equilibrium (uNE) if and only if
   \[ \Sigma_i R_i(s, \pi(s)) \geq \Sigma_i R_i(s, \hat{\pi}(s)), \hat{\pi}(s) \in \pi_{\text{NE}} \setminus \pi(s). \] (24)
Among the above five equilibria, it is easy to show that uCS, uCE and uNE always exist and unique in any games (uCS for cooperative while uCE and uNE for noncooperative). Both advE and cooE are shown to be unique in a noncooperative game, though neither of them is guaranteed to exist \cite{15,19} in any games.

The distinctions between cooE and uCS are worth noting. Intuitively, cooE is a noncooperative game equilibrium, which means that agents are essentially selfish. They are forced to cooperate in order to maximize their individual benefits. When following a uCS, by contrast, agents cooperate actively and may even sacrifice their own benefits to achieve a better overall outcome. Section 5 will help illustrate the differences.

4.1 Extension to stochastic games

Filar and Vrieze \cite{11} show how the $Q$ function links a stochastic game to a single stage game. $Q$ functions at one particular state with different bi-strategy are treated as payoffs for that particular single stage game (note the terms “game” and “state” can be used interchangeably), and the stochastic game is said to be in an equilibrium if and only if all single games (over all states) are in equilibrium. We now extend our definitions of the five strategies/equilibria from a single game to a two-player stochastic game, as follows,

**Definition 2.** A bi-policy $\pi$ is a uCS/advE/cooE/uNE/uCE of a two-player stochastic game $G$ if only if $\pi(s)$ is a uCS/advE/cooE/uNE/uCE of its sub-game $G(s)$, for all $s \in S$.

Correspondingly, we define that a uCS-MIRL/advE-MIRL/cooE-MIRL/uNE-MIRL/uCE-MIRL problem is an MIRL problem in which the players are assumed to employ a uCS/advE/cooE/uNE/uCE.

4.2 uCS-MIRL

A main result characterizing the set of solutions to a two-player uCS-MIRL problem is the following:

**Theorem 3.** Given a two-player stochastic game $\{S,A_i,r_i,P,\gamma\}$, an observed bi-policy $\pi$ is a uCS if and only if

$$ (B_\pi - B_a) D_\pi (r_1 + r_2) \geq 0, \ a \in A = A_1 \times A_2 $$

where $D_\pi = I + \gamma P (I - \gamma G_\pi)^{-1} B_\pi$. $B_a$ is obtained from such a bi-policy that players employ the bi-strategy $a$ in all states.
Proof. According to the definition of uCS, \( \pi \) is a uCS if and only if, for any state \( s \) and pure bi-strategy \( a \in A = A_1 \times A_2 \), we have

\[
\pi (s) \in \arg \max_{a \in A} \sum_{i} Q_i^\pi (s, a)
\]

\[
\iff \sum_{i} Q_i^\pi (s, \pi (s)) \geq \sum_{i} Q_i^\pi (s, a)
\]

\[
\iff r_1 (s, \pi (s)) + r_2 (s, \pi (s)) + \gamma P_s,\pi(s) (V_1^\pi + V_2^\pi) \\
\geq r_1 (s, a) + r_2 (s, a) + \gamma P_s,a (V_1^\pi + V_2^\pi)
\]

\[
\iff B_\pi (r_1 + r_2) + \gamma B_\pi P (I - \gamma G_x)^{-1} B_\pi (r_1 + r_2) \\
\geq B_a (r_1 + r_2) + \gamma B_a P (I - \gamma G_x)^{-1} B_a (r_1 + r_2)
\]

\[
\iff (B_\pi - B_a) (I + \gamma P (I - \gamma G_x)^{-1} B_a) (r_1 + r_2) \geq 0
\]

\[
\iff (B_\pi - B_a) D_a (r_1 + r_2) \geq 0
\]

Since any solution that is consistent with \( r_2 \) ensures a unique uCS, we can borrow the idea introduced in [17] and propose a Bayesian approach. The general idea is to maximize the posterior probability of the inferred rewards, \( p (r_1, r_2 | \pi) \), which can be expressed as

\[
p (r_1, r_2 | \pi) \propto f (r_1, r_2) p (\pi | r_1, r_2),
\]

where \( p (\pi | r_1, r_2) \) is the likelihood of observing \( \pi \) given \( r_1 \) and \( r_2 \) and \( f (r_1, r_2) \) is a joint prior of \( r_1 \) and \( r_2 \) that we need to specify. Recall the assumption that

\[
f (r_1, r_2) = f (r_1) f (r_2),
\]

which allows specification of the prior over \( r_1 \) and \( r_2 \) independently. We adopt a Gaussian prior on both rewards: that is, \( r_i \sim N (\mu_i, \Sigma_{r_i}) \), where \( \mu_i \) is the mean of \( r_i \) and \( \Sigma_{r_i} \) is the covariance. Then the probability density function of \( r_i \) is

\[
f (r_i) = \frac{1}{(2\pi)^{N/2} |\Sigma_{r_i}|^{1/2}} \exp \left( -\frac{1}{2} (r_i - \mu_i)^T \Sigma_{r_i}^{-1} (r_i - \mu_i) \right), \quad i = 1, 2.
\]

To model the likelihood function \( p (\pi | r_1, r_2) \), assume that the bi-policy which the two agents follow is a unique uCS given \( r_1, r_2 \). The likelihood is then a probability mass function given by

\[
p (\pi | r_1, r_2) = \begin{cases} 
1, & \text{if } \pi \text{ is uCS for } r_1, r_2 \\
0, & \text{otherwise}.
\end{cases}
\]

Thus, the optimization problem for uCS-MIRL can be formulated as

\[
\text{maximize: } f (r_1, r_2) \\
\text{subject to: } p (\pi | r_1, r_2) = 1.
\]
Equivalently,

$$\minimize \frac{1}{2} \sum_i (r_i - \mu_{r_i})^T \Sigma_{r_i}^{-1} (r_i - \mu_{r_i})$$

subject to:

$$\left( B_\pi - B_{\pi|a} \right) D_\pi (r_1 + r_2) \geq 0, a \in A = A_1 \times A_2. \tag{32}$$

4.3 advE-MIRL

The main result characterizing the set of solutions to a two-player advE-MIRL problem is the following:

**Theorem 4.** Given a two-player stochastic game \( \{S, A_i, r_i, P, \gamma\} \), the observed bi-policy \( \pi \) is an advE if and only if

\[
\begin{align*}
(B_{\pi|a_1} - B_\pi) D_\pi r_1 &\leq 0, \forall a_1 \in A_1 \\
(B_{\pi|a_2} - B_\pi) D_\pi r_2 &\leq 0, \forall a_2 \in A_2 \\
(B_{\pi|a_1} - B_\pi) D_\pi r_2 &\geq 0, \forall a_1 \in A_1 \\
(B_{\pi|a_2} - B_\pi) D_\pi r_1 &\geq 0, \forall a_2 \in A_2, \tag{33}
\end{align*}
\]

where \( B_{\pi|a} \) is obtained from such a bi-policy that player 2 employs their original policy while player 1 always chooses action \( a_1 \) in any state (game).

**Proof.** Eqs (33) contains four inequalities. In this proof, we will first show that the first and second inequalities constitute a necessary and sufficient condition for \( \pi \) being a NE. Recall that a bi-policy \( \pi \) is a minimax equilibrium for a 2-player zero-sum game if and only if \[Q_\pi(s)]^T \pi_1(s) \geq V_\pi(s) 1_M \]

Similarly, \( \pi \) is a NE if and only if

\[
\begin{align*}
[Q_2^\pi(s)]^T \pi_1(s) &\leq V_2^\pi(s) 1_M \\
Q_1^\pi(s) \pi_2(s) &\leq V_1^\pi(s) 1_M. \tag{35}
\end{align*}
\]

Combining \{14\} and \{35\} leads to

\[
\begin{align*}
B_{\pi|a_2} Q_2^\pi &\leq B_\pi Q_2^\pi, \forall a_2 \in A_2 \\
B_{\pi|a_1} Q_1^\pi &\leq B_\pi Q_1^\pi, \forall a_1 \in A_1. \tag{36}
\end{align*}
\]

Substituting \{12\} into \{35\} and rearranging the two sides of the inequalities yields

\[
\begin{align*}
(B_{\pi|a_1} - B_\pi) D_\pi r_1 &\leq 0, \forall a_1 \in A_1 \\
(B_{\pi|a_2} - B_\pi) D_\pi r_2 &\leq 0, \forall a_2 \in A_2. \tag{37}
\end{align*}
\]
We now turn to the additional feature of advE. Recall (21), it is easy to derive that an advE for a two-player general-sum game if and only if, in addition to
\begin{align*}
\left[Q^\pi_1(s)\right]^T \pi_1(s) & \geq V^\pi_1(s) \mathbf{1}_M \\
Q^\pi_2(s) \pi_2(s) & \geq V^\pi_2(s) \mathbf{1}_M.
\end{align*}
(38)

Following similar steps to those used to derive (36), the additional constraints (38) can be reduced to
\begin{align*}
(B_{\pi|a_1} - B_{\pi}) D_{\pi r_2} & \geq 0, \forall a_1 \in A_1 \\
(B_{\pi|a_2} - B_{\pi}) D_{\pi r_1} & \geq 0, \forall a_2 \in A_2.
\end{align*}
(39)

Since it has been proved that, in a one-stage game, if an advE exists it must be unique [19], an advE for a stochastic game, must also be unique, if it exists. Therefore, we can still use Bayesian approach to solve advE-MIRL problems. The prior (29) is also valid here. But the likelihood would be
\begin{align*}
p(\pi|r_1, r_2) &= \begin{cases} 
1, & \text{if } \pi \text{ is an AdvE for } r_1, r_2 \\
0, & \text{otherwise.}
\end{cases}
(40)
\end{align*}

And the optimization problem for advE-MIRL is
\begin{align*}
\text{minimize: } & \frac{1}{2} \sum_i (r_i - \mu_{r_i})^T \Sigma_{r_i}^{-1} (r_i - \mu_{r_i}) \\
\text{subject to: } & (B_{\pi|a_1} - B_{\pi}) D_{\pi r_1} \leq 0, \forall a_1 \in A_1 \\
& (B_{\pi|a_2} - B_{\pi}) D_{\pi r_2} \leq 0, \forall a_2 \in A_2 \\
& (B_{\pi|a_1} - B_{\pi}) D_{\pi r_2} \geq 0, \forall a_1 \in A_1 \\
& (B_{\pi|a_2} - B_{\pi}) D_{\pi r_1} \geq 0, \forall a_2 \in A_2.
\end{align*}
(41)

In fact, there is a direct link between the minimax equilibrium of a competitive zero-sum game and an advE for a special zero-sum case, as the following proposition,

**Proposition 1.** The minimax equilibrium of a single competitive zero-sum game is an advE, and vice versa.

**Proof.** Let \( r_1 = r = r_2 \). Then (33) reduces to
\begin{align*}
(B_{\pi|a_1} - B_{\pi}) D_{\pi r} & \leq 0, \forall a_1 \in A_1 \\
(B_{\pi|a_2} - B_{\pi}) D_{\pi r} & \geq 0, \forall a_2 \in A_2,
\end{align*}
(42)
which are exactly the constraints of (19), the necessary and sufficient conditions for \( \pi \) being a minimax equilibrium for a zero-sum game, in [17].

From Proposition 1, we can see that advE is a more general concept for general-sum games, whereas the minimax equilibrium is specific to zero-sum games.
4.4 cooE-MIRL

The main result characterizing the set of solutions to a two-player cooE-MIRL problem is the following:

**Theorem 5.** Given a two-player stochastic game \( \{S, A_i, r_i, P, \gamma\} \), the observed bi-policy \( \pi \) is an CooE if and only if

\[
\begin{align*}
(B_{\pi|a_1} - B_{\pi}) D_{\pi} r_1 &\leq 0, \forall a_1 \in A_1 \\
(B_{\pi|a_2} - B_{\pi}) D_{\pi} r_2 &\leq 0, \forall a_2 \in A_2 \\
(B_{\pi} - B_{a}) D_{\pi} r_1 &\geq 0, \forall a \in A = A_1 \times A_2 \\
(B_{\pi} - B_{a}) D_{\pi} r_2 &\geq 0, \forall a \in A = A_1 \times A_2.
\end{align*}
\] (43)

In (43), the first two inequalities, which guarantee \( \pi \) is a NE, has been established in Section 4.3. The latter two inequalities warrant the unique property of cooE, the proof of which is sketched below.

*Proof.* According to the definition of cooE, \( \pi \) is a cooE if and only if, for any state \( s \) and pure bi-strategy \( a \in A = A_1 \times A_2 \),

\[
\begin{align*}
\pi (s) &\in \arg \max_{a \in A} Q^*_i (s, a) \\
\Leftrightarrow Q^*_i (s, \pi (s)) &\geq Q^*_i (s, a) \\
\Leftrightarrow r_i (s, \pi (s)) + \gamma P_{s,\pi(s)} V^*_i &\geq r_i (s, a) + \gamma P_{s,a} V^*_i \\
\Leftrightarrow B_{\pi} r_i + \gamma B_{\pi} P (I - \gamma G_{\pi})^{-1} B_{\pi} r_i &\geq B_{a} r_i + \gamma B_{a} P (I - \gamma G_{\pi})^{-1} B_{\pi} r_i \\
\Leftrightarrow (B_{\pi} - B_{a}) \left(I + \gamma P (I - \gamma G_{\pi})^{-1} B_{\pi}\right) r_i &\geq 0 \\
\Leftrightarrow (B_{\pi} - B_{a}) D_{\pi} r_i &\geq 0.
\end{align*}
\] (44)

We can also develop a similar optimization problem for cooE-MIRL. Using the same reasoning as in the case of advE, it is easy to show that a cooE for a stochastic game is unique, if it exists. As a result, the Bayesian approach is also valid here, with the same prior \( (29) \) but a different likelihood as follows

\[
p (\pi | r_1, r_2) = \begin{cases} 
1, & \text{if } \pi \text{ is an cooE for } r_1, r_2 \\
0, & \text{otherwise.} 
\end{cases}
\] (45)

Hence the optimization problem for cooE-MIRL is

\[
\begin{align*}
\text{minimize:} & \quad \frac{1}{2} \sum_i (r_i - \mu_{r_i})^T \Sigma_{r_i}^{-1} (r_i - \mu_{r_i}) \\
\text{subject to:} & \quad (B_{\pi|a_1} - B_{\pi}) D_{\pi} r_1 \leq 0, \forall a_1 \in A_1 \\
& \quad (B_{\pi|a_2} - B_{\pi}) D_{\pi} r_2 \leq 0, \forall a_2 \in A_2 \\
& \quad (B_{\pi} - B_{a}) D_{\pi} r_1 \geq 0, \forall a \in A = A_1 \times A_2 \\
& \quad (B_{\pi} - B_{a}) D_{\pi} r_2 \geq 0, \forall a \in A = A_1 \times A_2.
\end{align*}
\] (46)
4.5 uCE-MIRL

The result that characterizes the set of solutions to a two-player CE-MIRL problem is as follows:

**Theorem 6.** Given a two-player stochastic game \( \{ \mathcal{S}, \mathcal{A}, r, P, \gamma \} \), the observed bi-policy \( \pi \) is a CE if and only if

\[
\pi^T H (s, a_1) \geq 0, \forall \pi_1, \pi_2 \in \mathcal{A}_1, \ \forall a_i \in \mathcal{A}_i \ \forall \mathcal{A}_1 \ \forall a_i, \ (47)
\]

where \( \pi \) is restructured from \( \pi \) to be a column vector of length \( NM^2 \), and \( H(s, a_i) \) is a linear transformation operator as described in the proof below.

**Proof.** By definition of CE, for a two-player general-sum stochastic game \( \mathcal{G} \), a bi-policy \( \pi \) is a CE if and only if

\[
\sum_{a_2} \pi (a_1, a_2 | s) Q_{1}^\pi (s, a_1, a_2) \geq \sum_{a_2} \pi (a_1, a_2 | s) Q_{1}^\pi (s, \tilde{a}_1, a_2), \forall a_1 \in \mathcal{A}_1, \forall \tilde{a}_1 \in \mathcal{A}_1 \ \forall a_1
\]

\[
\sum_{a_1} \pi (a_1, a_2 | s) Q_{2}^\pi (s, a_1, a_2) \geq \sum_{a_1} \pi (a_1, a_2 | s) Q_{2}^\pi (s, a_1, \tilde{a}_2), \forall a_2 \in \mathcal{A}_2, \forall \tilde{a}_2 \in \mathcal{A}_2 \ \forall a_2,
\]

for all \( s \in \mathcal{S} \). Rearranging (48) yields

\[
\pi (a_1, : | s) \left( [Q_{1}^\pi (s, a_1, :)]^T - [Q_{1}^\pi (s, \tilde{a}_1, :)]^T \right) \geq 0
\]

\[
\pi (:, a_2 | s)]^T (Q_{2}^\pi (s, :, a_2) - Q_{2}^\pi (s, :, \tilde{a}_2)) \geq 0,
\]

where \( \pi (a_1, : | s) \) is a row vectors of \( 1 \times M \), spanning over all \( a_2 \in \mathcal{A}_2 \), and \( \pi (:, a_2 | s) \) is a column vectors of \( M \times 1 \), spanning over all \( a_1 \in \mathcal{A}_1 \). Recall

\[
Q_{1}^\pi (s, a) = R_1 (s, a) + \gamma \sum_{s'} p (s' | s, a) V_{1}^\pi (s'). \quad (50)
\]

So

\[
[Q_{1}^\pi (s, a_1, :)]^T = [R_1 (s, a_1, :) + \gamma p (s | s, a_1, :) V_{1}^\pi - [R_1 (s, \tilde{a}_1, :) + \gamma p (s | s, \tilde{a}_1, :) V_{1}^\pi
\]

\[
Q_{2}^\pi (s, :, a_2) = R_2 (s, :, a_2) + \gamma p (s | s, :, a_2) V_{2}^\pi
\]

Substituting (51) into (49) leads to

\[
\pi (a_1, : | s) \left( [R_1 (s, a_1, :) - [R_1 (s, \tilde{a}_1, :) ]^T + \gamma [p (s | s, a_1, :) - p (s | s, \tilde{a}_1, :) ] V_{1}^\pi \right \}
\]

\[
\geq 0
\]

\[
[\pi (:, a_2 | s)]^T \left( [R_2 (s, :, a_2) - r_2 (s, :, \tilde{a}_2) + \gamma [p (s | s, a_2) - p (s | s, \tilde{a}_2)] V_{2}^\pi \right \}
\]

\[
\geq 0
\]

(52)

The above inequality can be further simplified. First, let \( [R_1 (s, a_1, :) ]^T = H(s, a_1) r_1 \) and \( R_2 (s, :, a_2) = H(s, a_2) r_2 \), where \( H(s, a_i) \) is a sparse \( M \times NM^2 \) matrix. It is also easy to see \( p (s | a_1, :) = H(s, a_1) P \) and \( p (s | s, : a_2) = H(s, a_2) P \). In addition, we can also have \( \pi (a_1, : | s) = [H(s, a_1) \pi ]^T = \pi^T H(s, a_1)^T \),
and \( \pi(\cdot, a_2|s) = H(s, a_2)\pi \). Substituting (10) into (52) and rearranging it, we can get

\[
\pi^T H(s, a_i) \left[ H(s, a_i) - H(s, \tilde{a}_i) \right] \left( I + \gamma P (I - \gamma G_{\pi})^{-1} B_{\pi} \right) r_i \geq 0, \quad i = 1, 2,
\]

Recall

\[
D_{\pi} = I + \gamma P (I - \gamma G_{\pi})^{-1} B_{\pi},
\]

we can express (53) compactly as

\[
\pi^T H(s, a_i) \left[ H(s, a_i) - H(s, \tilde{a}_i) \right] D_{\pi} r_i \geq 0, \quad i = 1, 2, \forall a_i \in A_i, \tilde{a}_i \in \tilde{A}_i \setminus a_i,
\]

Clearly, any sensible point that is consistent with (55) constitutes a CE for the stochastic game. Many points in the convex hull of CE, however, are less meaningful because only the uCE is of interest. Hence, we desire to find some way to choose between solutions satisfying (55). A first idea is to maximize \( \sum_s V_{\pi}^*(s) \). That is not enough though, because reaching a uCE is difficult in practice. Finding a uCS is a much easier problem, by contrast. This fact gives rise to another idea. Before going into details, we introduce four concepts: cooperation gap, local uCS, local improvement and local reduced gap.

**Definition 3.** The cooperation gap \( I_{cg}^\pi(s) \), corresponding to a starting state \( s \) and a bi-policy \( \pi \) in a two-player general-sum stochastic game, is the total game value difference between \( \pi \) and \( \pi^* \), where \( \pi^* \) is any uCS; specifically,

\[
I_{cg}^\pi(s) = V_{\pi^*}(s) - V_{\pi}(s), \quad s \in S.
\]

**Definition 4.** The local uCS, corresponding to a starting state \( s \) and a bi-policy \( \pi \) in a two-player general-sum stochastic game, is a bi-policy for which the two players employ a uCS bi-policy \( \pi^* \) at \( s \) and then employ \( \pi \) afterwards.

**Definition 5.** The local improvement \( I_{imp}^\pi(s) \), corresponding to a starting state \( s \) and a bi-policy \( \pi \) in a two-player general-sum stochastic game, is the total game value gain by employing the local uCS.

**Definition 6.** The local reduced gap \( I_{rg}^\pi(s) \), corresponding to a starting state \( s \) and a bi-policy \( \pi \) in a two-player general-sum stochastic game, is the total game value difference between a uCS and a local uCS; specifically,

\[
I_{rg}^\pi(s) = V_{\pi^*}(s) - Q_{\pi}(s, \pi^*(s)), \quad s \in S,
\]

and the total local improvement for \( \pi \) is

\[
I_{rg}^\pi = \sum_s I_{rg}^\pi(s) = \sum_s V_{\pi^*}(s) - Q_{\pi}(s, \pi^*(s)), \quad s \in S.
\]
An implication from the above definitions is that for a starting state $s$, $I_{rg}^\pi(s) = I_{cg}^\pi(s) - I_{imp}^\pi(s)$, shown in Figure 3.

It is obvious that for a two-player general-sum stochastic game, among all its CEs, the uCE is closest to its uCS in terms of the total game value, as illustrated in Figure 3. In a uCE-MIRL problem, however, all CEs except uCE are unobservable. Therefore, we need to find a way to infer a set of rewards $\{r_1, r_2\}$ such that the observed $\pi$ is most likely the uCE of the game.

![Fig. 3: The game value distance relationship between uCE, uCS and other CEs](image)

By definition, a local uCS improves $V^\pi(s)$ by employing a uCS strategy only at current state $s$, resulting in a local improvement with respect to $\pi$ and a local reduced gap with respect to a uCS. Adding up all those local reduced gaps over all states gives a measure of how close a bi-policy $\pi$ is to a uCS, in terms of the total game value. We now propose an important theorem that captures the relationship between the local reduced gap and uCE, as follows:

**Theorem 7.** Consider a two-person general-sum stochastic game $\Gamma$ with a collection of CEs, $\Pi_{CE}$ and a bi-policy $\pi^*$ that is a uCS. Then $\pi_{CE}^* \in \Pi_{CE}$ is a uCE if and only if its total local reduced gap $I_{rg}^{\pi_{CE}}$ is no greater than that of any other CE, specifically,

$$I_{rg}^{\pi_{CE}} \leq I_{rg}^{\pi_{CE}}, \forall \pi_{CE} \in \Pi_{CE}. \quad (57)$$
Proof. We first show necessity. From (56) and the properties of value function, we have

\[
\pi_{CE}^{*} \text{rg} - \pi_{CE}^{*} = \left[ \sum_s V_{\pi_{CE}}^{*}(s) - Q_{\pi_{CE}}^{*}(s, \pi_{CE}^{*}(s)) \right] - \left[ \sum_s V_{\pi_{CE}}^{*}(s) - Q_{\pi_{CE}}^{*}(s, \pi_{CE}^{*}(s)) \right]
\]

\[
= \sum_s Q_{\pi_{CE}}^{*}(s, \pi_{CE}^{*}(s)) - Q_{\pi_{CE}}^{*}(s, \pi_{CE}^{*}(s)) = \sum_s \left\{ \tilde{r}_1(s, \pi_{CE}^{*}(s)) + \tilde{r}_2(s, \pi_{CE}^{*}(s)) + \gamma P_{s,\pi_{CE}^{*}(s)} V_{\pi_{CE}}^{*} \right\}
\]

\[
= \sum_s \left\{ \tilde{r}_1(s, \pi_{CE}^{*}(s)) + \tilde{r}_2(s, \pi_{CE}^{*}(s)) + \gamma P_{s,\pi_{CE}^{*}(s)} V_{\pi_{CE}}^{*} \right\}.
\]

Since the definition of \(\pi_{CE}^{*}\) implies that \(V_{\pi_{CE}}^{*}(s) \leq V_{\pi_{CE}}^{*}(s)\) for all \(s\), the column vector \(V_{\pi_{CE}}^{*} - V_{\pi_{CE}}^{*}\) is non-positive. Also, \(P_{s,\pi_{CE}^{*}(s)}\) is a non-negative row vector as all its entries are probabilities. Therefore, \(P_{s,\pi_{CE}^{*}(s)}(V_{\pi_{CE}}^{*} - V_{\pi_{CE}}^{*}) \leq 0\) for all \(s \in S\), and consequently, \(I_{\pi_{CE}}^{*} \leq I_{\pi_{CE}}^{*}\).

Next, we show sufficiency by assuming \(I_{\pi_{CE}}^{*} \leq I_{\pi_{CE}}^{*}\), for all \(\pi_{CE} \in \Pi_{CE}\) and that \(\pi_{CE}^{*}\) is not a uCE. Since \(\pi_{CE}^{*}\) is not a uCE, there must exist a uCE, \(\pi_{uCE}\), such that \(V_{\pi_{uCE}}(s) \geq V_{\pi_{CE}}^{*}(s)\), for all \(s \in S\). Then from (58) we can conclude

\[
I_{\pi_{CE}}^{*} - I_{\pi_{uCE}}^{*} = \gamma \sum_s P_{s,\pi_{CE}^{*}(s)}(V_{\pi_{uCE}} - V_{\pi_{CE}}^{*}) > 0,
\]

which contradicts our assumption that \(I_{\pi_{CE}}^{*} \leq I_{\pi_{CE}}^{*}\), for all \(\pi_{CE} \in \Pi_{CE}\).

The intuition behind Theorem 7 is: comparing to any other CE, a uCE is “closer” to the uCS. Its corresponding local uCS is even closer to uCS and as a result, there is less room to further shrink the local reduced gap. Hence the smaller the local reduced gap is, the more likely a CE \(\pi\) is a uCE. Thus, given a CE \(\pi\), a desired pair of \(r_1\) and \(r_2\) satisfies

\[
\text{minimize: } \sum_s y(s) - V_{\pi}(s)
\]

subject to:

\[
Q_{\pi}^{1}(s, a) + Q_{\pi}^{2}(s, a) \leq y(s) \quad V_{\pi}(s) \leq y(s),
\]

for all \(a \in A = A_1 \times A_2\).

However, \(r_1 = r_2 = 0\) is the optimal solution to (59). The reason is that \(V_{\pi}(s)\) needs to be enlarged so that picking a uCE is achievable with higher
We propose the following linear programming problem to find the desired $r_1$ and $r_2$,

$$\begin{align*}
\text{maximize: } & \sum_s V^\pi(s) - \lambda (y(s) - V^\pi(s)) \\
\text{subject to: } & Q_1^\pi(s, a) + Q_2^\pi(s, a) \leq y(s) \\
& V^\pi(s) \leq y(s)
\end{align*}$$

(60)

where $\lambda$ is a regularization parameter. Expressing $V^\pi_i$ and $Q^\pi_i(s, a)$ as functions of $r_i$ and reformulating those inequalities more compactly in matrix notation leads to

$$\begin{align*}
\text{maximize: } & \mathbf{1}_{1 \times N} \times \left[ (1 + \lambda) (I - \gamma G^\pi) \right]^{-1} B^\pi (r_1 + r_2) - \lambda y \\
+ & \text{regularization terms} \\
\text{subject to: } & \pi^T H(s, a_i)^T [H(s, a_1) - H(s, \bar{a}_i)] D_\pi r_i \geq 0, \forall a_1, \bar{a}_i \in \mathcal{A}_i, a_i \in \mathcal{A}_i \setminus a_i \\
& D_\pi (r_1 + r_2) \leq y \cdot \mathbf{1}_{M \times M} \\
& (I - \gamma G^\pi)^{-1} B^\pi (r_1 + r_2) \leq y.
\end{align*}$$

(61)

We now discuss the regularization terms in (61). One challenging issue for MIRL is that there often exists many solutions that are equally sensible so that it is more likely than IRL to recover rewards which are far from actual ones. For example, in [17] the authors emphasize the importance of the structure of rewards. Therefore, some prior knowledge or assumption of the game, as well as the structure of the unknown rewards, is very helpful. For example, it is often assumed that, all other things being equal, an unknown reward vector is sparse [24]. One easy way to incorporate this assumption is to add a penalty term to the objective function to regularize non-sparcity. There might be other problem-specific knowledge/assumption available and taking advantage of it will help infer higher-quality rewards.

4.6 uNE-MIRL

Recall that the necessary and sufficient condition for an observed bi-policy $\pi$ being a NE for a two-player general-sum stochastic game is given by

$$\begin{align*}
(B^\pi_{|a_1} - B^\pi_{a_1}) D_\pi r_1 & \leq 0, \forall a_1 \in \mathcal{A}_1 \\
(B^\pi_{|a_2} - B^\pi_{a_2}) D_\pi r_2 & \leq 0, \forall a_2 \in \mathcal{A}_2.
\end{align*}$$

(62)
Since NE is a subset of CE, we can borrow the idea proposed in Section 4.5 and solve a uNE-MIRL problem by solving the following LP problem:

\[
\begin{align*}
\text{maximize: } & \quad 1 \times N \times \left[ (1 + \lambda) \left( I - \gamma G_{\pi} \right)^{-1} B_{\pi} (r_1 + r_2) \right] \\
& \quad + \text{other problem-specific regularized terms} \\
\text{subject to: } & \quad \left( B_{\pi|a_1} - B_{\pi} \right) D_{a_1} r_1 \leq 0, \forall a_1 \in A_1 \\
& \quad \left( B_{\pi|a_2} - B_{\pi} \right) D_{a_2} r_2 \leq 0, \forall a_2 \in A_2 \\
& \quad D_{\pi} (r_1 + r_2) \leq y \cdot 1_{M \times M} \\
& \quad (I - \gamma G_{\pi})^{-1} B_{\pi} (r_1 + r_2) \leq y.
\end{align*}
\]

\[(63)\]

5 Numerical Examples I: GridWorld

This section describes the behaviour of our algorithms (except advE-MIRL) using two grid games (GGs), shown in Figure 4, namely GG1 for the left and GG2 for the right. These games have been used extensively in many theoretical MRL works [15,19,12]. In both GGs, there are two agents, A and B, and two goals (or homes). The two agents act simultaneously and can move only one step in any of the four compass directions. When adjacent to a wall, choosing a direction into a wall results in a no-op, where the agent remains in the current position. If both agents attempt to move into the same cell, a collision occurs and they are pushed back to their original positions immediately, except for cells in the bottom row. Each agent is rewarded upon reaching its goal. However, since the reward is discounted with time, the earlier to reach the goal, the better. GG1 and GG2 are similar in basic game rules but different in board setup in two aspects. First, in GG1, the two players’ goals are separate while their goals coincide in GG2. Second, in GG2, there are two barriers and if any agent attempts to move downward through the barrier from the top, then with 1/2 probability this move fails and results in a no-op.

![Fig. 4: Grid games. The circle indicates A’s goal and the hexagon indicates B’s goal.](image-url)
We let agents A and B play the go-back-home games together according to either uCS, uCE, uNE or cooE. Our task is to recover their rewards given the equilibrium, the bi-policy, and the state transition dynamics. The basic rewarding rule is: either player receives reward 1 (discounted with time) once reaching home and the game stops immediately, and 0 otherwise. When employing cooE, however, neither player receives reward unless they reach home simultaneously.

Our experiments are conducted as follows. First, we apply the cooE-MRL algorithm by Hu and Wellman [16], and the uCE-MRL algorithm proposed by Green and Hall [12] to obtain cooE and uCE bi-policies, respectively. Then we develop similar Q-learning based iterative algorithms for uCS-MRL and uNE-MRL. The general procedure, namely multi-Q-learning algorithm, is the same for all these four MRL algorithms and described in [Algorithm 1]. It is worth emphasizing that the multi-Q-learning algorithm can be applied to many variants of Q-learning problems as long as the equilibria exists and is unique [15,19,12]. It is easy to show that uCS, uCE, uNE and cooE all meet this requirement.

The second step is to apply our uCS-MIRL, cooE-MIRL uCE-MIRL and uNE-MIRL algorithms accordingly, incorporating basic knowledge and some reasonable assumptions into Gaussian priors for uCS and cooE. For example, one assumption is that both players’ reward vectors are sparse, only depending on reaching home or not. In addition, one agent’s position might have a small effect on the other agent’s reward, or possibly no affect.

For each experiment, we compare recovered rewards of both players $r_{A}^{\text{rec}}$ and $r_{B}^{\text{rec}}$, with the true values $r_{A}$ and $r_{B}$ numerically. We use a normalized root mean squared error (NRMSE) metric, where we first normalize a recovered reward vector $r_{\text{rec}}$ on $[0,1]$, as follows:

$$r_{\text{nrec}} = \frac{r_{\text{rec}} - \min(r_{\text{rec}})}{\max(r_{\text{rec}}) - \min(r_{\text{rec}})},$$

and then compute

$$\text{NRMSE} = \frac{1}{2} \left( \frac{\|r_{A}^{\text{nrec}} - r_{A}\|_2}{\dim(r_{A})} + \frac{\|r_{B}^{\text{nrec}} - r_{B}\|_2}{\dim(r_{B})} \right).$$

We compare our MIRL algorithm with IRL and d-MIRL algorithms. First, we use IRL algorithms to solve the uCS-, cooE-, uCE- and uNE-MIRL problems. Specifically, we focus on B, and try to infer its reward. Obviously, the inferred IRL reward is a function of the state and B’s own action. The IRL algorithm we use is BIRL, proposed in [27]. Note that the reward vector recovered from IRL can be extended to a MIRL reward vector by letting $R(s,a_1,a_2) = R(s,a_2)$ for all $a_1 \in A_1$. Second, we use the d-MIRL algorithm to solve the above four problems. Recall that d-MIRL simply assumes agents employing a Nash equilibrium.

To further evaluate the quality of uCS-MIRL reward, let agents $B_1$, $B_2$ and $B_3$ learn uCS-MIRL, IRL and d-MIRL rewards, respectively, and figure out their own policies $\pi_{B_1}$, $\pi_{B_2}$ and $\pi_{B_3}$. Then let these three agents play with A by using their policies while A still employs $\pi_{A}$ as if it plays with B. We compute their
Algorithm 1 General Multi-Q-learning algorithm

Require: \( f \): uCS, cooE, uCE or uNE; \( \alpha \): learning rate

1: procedure Multi-Q\((f, T, r_1, r_2, \alpha)\)
2: Initialize: \( s, a, Q_1, Q_2, t = 0 \)
3: while \( t < T \) do
4: agents choose bi-strategy \( a \) in state \( s \)
5: observe rewards and next state \( s' \)
6: for \( i = 1 \rightarrow N \) do
7: \( V_i(s') = f_i(Q_1(s'), Q_2(s')) \)
8: \( Q_i(s, a) = (1 - \alpha)Q_i(s, a) + \alpha [(1 - \gamma)r_i(s, a) + \gamma V_i(s')] \)
9: agents choose bi-strategy \( a' \)
10: \( s = s', a = a' \)
11: decay \( \alpha \)
12: \( t = t + 1 \)
13: end
14: end

total game value over all states and compare with true total game values, which are the maximum. Obviously, for any reward, the larger the total game value that can be achieved, the better quality it has.

Numerical results are summarized in Table 1 and Table 2. Table 1 summarizes how close the total game value is to the true total game value, using recovered reward from for each method, by computing
\[
\frac{1}{N} \sum_{s=1}^{N} (V_{\text{true}}(s) - V_{\text{recovered}}(s))^2,
\]
where \( N \) is the number of states, \( V_{\text{true}} \) and \( V_{\text{recovered}} \) are true total game value vector and the one using recovered reward, respectively.

We can easily conclude that our MIRL algorithms generate satisfactory results and performs much better than IRL and d-MIRL algorithms for all the four problems.

| Grid Game #1 | Grid Game #2 |
|--------------|--------------|
| uCS-MIRL     | 1.50 \times 10^{-3} | 2.27 \times 10^{-4} |
| IRL          | 0.122        | 0.121         |
| dMIRL        | 0.104        | 0.106         |
| cooE-MIRL    | 0.026        | 0.026         |
| IRL          | 0.409        | 0.319         |
| dMIRL        | 0.085        | 0.083         |
| uCE-MIRL     | 1.30 \times 10^{-3} | 1.39 \times 10^{-10} |
| IRL          | 0.287        | 0.311         |
| dMIRL        | 0.089        | 0.083         |
| uNE-MIRL     | 0            | 0             |
| IRL          | 0.271        | 0.283         |
| dMIRL        | 0.104        | 0.103         |

Table 1: NRMSE results for reward values comparison
6 Numerical Examples II: Abstract Soccer Game

This section presents a demonstration of the advE-MIRL algorithm on a stylized soccer game. Two-player soccer games of many forms are popular among MRL researchers for algorithm demonstration and comparison purposes [18, 12, 17]. In [17], a zero-sum MIRL algorithm is proposed and performance demonstrated on a game that is similar to that used here. The algorithm in [17], however, requires that the game be zero-sum. In this section, we relax the zero-sum assumption and require only that the two players be foes, which enables us to rely on a weaker assumption that they employ an advE.

The soccer game (see Figure 5) is depicted as follows. Players A and B compete with each other, aiming to score by either bringing or kicking the ball (represented by a circle) into their opponents’ goals (A’s goal are 6 and 11, and B’s goal are 10 and 15). Both players can move simultaneously either in four compass directions, ending in a neighbouring cell or stay unmoved. A ball exchange may occur with some probability in case of a collision in the same cell. A kick action is also available to players. Each one has a perception of how likely she is to making a scoring shot, or the probability of a successful shot (PSS), if kicking the ball at a given position. For simplicity, PSS is assumed to be independent of the opponent’s position. The position based PSS distribution is shown in Table 3.

| PSS = 0.7 PSS = 0.5 PSS = 0.3 PSS = 0.1 PSS = 0 |
|---|---|---|---|---|
| A 1, 7, 12, 16 2, 8, 13, 17 3, 9, 14, 18 4, 10, 15, 19 5, 20 |
| B 5, 9, 14, 20 4, 8, 13, 19 3, 7, 12, 18 2, 6, 11, 17 1, 16 |

Table 3: Original PSS distribution of each player

Note that a player’s PSS at a particular spot is her perceived likelihood of a scoring short, rather than the actual probability of a successful shot. So statistically calculating the successful shot rate from observation data would not help reflect the player’s own beliefs about her shooting skills, which is the reward.

The two players play against each other, employing a minimax equilibrium in a zero-sum game. But this information is not available to us when solving the
MIRL problem. Instead, we are given: (1) the bi-policy of the two players over all states, and; (2) the state transition dynamics (including the ball exchange rate $\beta = 0.6$). In fact, this information can be statistically calculated or estimated with sufficient observations. We simply skip this data pre-processing stage as it is not the emphasis of this paper. We then assume that the two players follow an advE and try to infer their rewards on this basis.

6.1 Prior Specification

As indicated in Section 4.3, one of the key specifications of the advE-MIRL model is the prior, which encodes our beliefs of the unknown rewards. We use two Gaussian priors for the unknowns of A and B with three types of means and two types of covariance matrices, as follows:

- **Weak Mean**: for A, assign 0.5 point in every state where A has possession of the ball and $-0.5$ point in every state where A loses possession of the ball. Same setting for B.

- **Median Mean**: for A, assign 1 point whenever it has the ball and is in one of the corner squares 1, 6, 11 and 16, and $-1$ point to A whenever B has the ball and is in its hypothesized goal area 5, 10, 15 and 20. When A has the ball and takes a shot, it is assigned 0.5 wherever it is; when B has the ball and takes a shot, A is assigned $-0.5$ wherever B is. Otherwise, no points will be assigned. Same setting for B.

- **Strong Mean**: More accurate than Median Mean, in the sense that the hypothesized goal squares for A are 6 and 11, and 10 and 15 for B.

- **Weak Covariance Matrix**: an identity matrix for both A and B, which implies no knowledge or guess about the relationship between any two reward values is available.
– **Strong Covariance Matrix**: a more complex covariance matrix constructed from our following hypothesis of the reward structure, same for both A and B.

1. when one player has the ball and takes a shot, its PSS depends only on its' current position in the field. That means that, take player A as an example, $\rho(r_A(s_1,a_A=\text{shoot},a_B),r_A(s_2,a_A=\text{shoot},a_B)) = 1$, where in $s_1$ and $s_2$, A’s positions are identical;

2. at any state, one’s reward for any non-shoot action is one’s own position dependent. Take player A as an example, for any two states $s_1$ and $s_2$ we assume $\rho(r_A(s_1,a_A\neq\text{shoot},a_B),r_A(s_2,a_A\neq\text{shoot},a_B)) = 1$, as long as A’s positions are identical in these two states.

In addition, for simplicity, the standard deviation of any reward point is assumed 1, so that the covariance matrix is identical to the correlation coefficient matrix.

To make it clear, our prior specifications do not imply a zero-sum relationship between A and B’s reward. As a singularity issue may occur when using strong covariance matrices, we add a small numerical perturbation to the diagonal.

6.2 Monte Carlo Simulation using Recovered Rewards

By solving an advE-MIRL problem, we recover A and B’s reward vectors, over all states and all actions. There are 6 advE-MIRL reward vectors recovered corresponding to the 6 pairs of means and covariance matrices. Since we have seen that evaluating the quality of recovered reward by simply measuring its numerical difference from true value may lead to misleading conclusion, we adopt a Monte Carlo simulation approach to evaluation, as follows:

1. Create agents:
   - C, which uses advE-MIRL reward;
   - D, which uses true reward;
   - E, which uses zero-sum MIRL reward;
   - F, which uses dMIRL reward;
   - G, which uses BIRL reward.

2. Simulate competitive games:
   - C against D;
   - C against E;
   - C against F;
   - C against G;

Note that agent E, F and G use rewards recovered from three conventional MIRL approaches covered in Section 3. Here we let C plays the role of A and others take the place of B (due to symmetry, two parties can switch roles as well). All those games are simulated in three different environmental settings, where the the ball exchange rates $\beta$ are 0, 0.4 and 1. 5000 round games are simulated per case.
The simulation results are presented in Tables 4–7, where WM, MM, SM, WC and SC stand for weak mean, median mean, strong mean, weak covariance matrix and strong covariance matrix, respectively. To interpret the result, take the 2nd row of Table 5 as an example: C uses WM and SC as prior and recovers A’s advE-MIRL reward, while E also use the same prior and learns a zero-sum MIRL reward vector of B. They come up with their own minimax policies according to their learned rewards and environmental settings and play against each other. 32.28/36.40 means C beats E with probability 32.28%, loses with probability 36.40%, and end in a tie with probability 31.32%, when the ball exchange rate is 1. Note that 0/0 shown in these tables means the both parties learn very bad rewards such that no one is able to score a single point even if its opponent is also poorly skilled.

| advE-MIRL Rewards | W/L% ($\beta = 0.4$) | W/L% ($\beta = 1$) | W/L% ($\beta = 0$) |
|-------------------|----------------------|--------------------|--------------------|
| WM & WC           | 0/32.44              | 0/58.00            | 0/49.98            |
| WM & SC           | 20.40/25.46          | 20.50/38.24        | 42.88/50.16        |
| MM & WC           | 4.60/30.12           | 9.36/44.00         | 10.44/49.88        |
| MM & SC           | 24.86/24.94          | 25.10/24.80        | 49.97/50.02        |
| SM & WC           | 14.90/30.52          | 6.80/42.50         | 15.42/50.08        |
| SM & SC           | 25.26/24.80          | 25.00/24.80        | 50.14/49.86        |

Table 4: C vs D

| advE-MIRL Rewards | W/L% ($\beta = 0.4$) | W/L% ($\beta = 1$) | W/L% ($\beta = 0$) |
|-------------------|----------------------|--------------------|--------------------|
| WM & WC           | 0/2.40               | 0/0                | 0/0                |
| WM & SC           | 22.76/28.94          | 32.28/36.40        | 43.14/50.14        |
| MM & WC           | 0/0                  | 9.20/5.60          | 4.12/16.86         |
| MM & SC           | 24.86/25.12          | 25.04/24.96        | 49.54/50.44        |
| SM & WC           | 11.24/10.60          | 8.80/9.18          | 16.10/24.46        |
| SM & SC           | 25.28/25.06          | 24.94/25.12        | 50.13/49.86        |

Table 5: C vs E

The results in Tables 4–7 support the following conclusions:

1. The advE-MIRL algorithm performs, if not better, comparatively with zero-sum MIRL algorithm, though the latter requires a stronger assumption.
2. The advE-MIRL algorithm performs notably better than d-MIRL and BIRL algorithms, particularly when using a strong covariance in prior.
7 Conclusions and Future Work

We present novel and computationally tractable algorithms for five special variants of MIRL problems, as well as demonstrations on several benchmark grid-world examples. The advE-MIRL problem formulation requires weaker assumptions but performs comparably with zero-sum MIRL. However, advE-MIRL outperforms both d-MIRL and BIRL. Both uCS and cooE generate good results if the estimated reward can be scaled correctly. The uCE and uNE problem formulations perform remarkably well in two benchmark grid-world examples by accurately estimating the value of the true reward. There are three reasons why the results are so good. First, these two small GGs are well-defined in the sense that there is no chance of moving in another direction by accident once a certain direction is selected (no noise in action). Second, the bi-policy \( \pi \) we use is exactly the equilibrium of interest because it is generated from a corresponding MRL-Q-learning algorithm. Third, we have incorporated strong prior information about the game, and a good solution can be achieved by tuning the regularized coefficients.

Although this paper is restricted to the two-person case, an extension of the methods to \( n \)-player cases would be straightforward because the equilibria we study are unique in \( n \)-player games, if they exist. In this sense, advE-MIRL has advantages over zero-sum MIRL as how to extend zero-sum MIRL from two-player to \( n \)-player is not yet clear. However, our work is limited in two aspects: (1) we only consider the case where both state and action spaces are discrete and limited, and (2) though it is not explicitly emphasized, we use a strong assumption that a full bi-policy over all states is available. In practice, it might...
not be possible to observe the game long enough to obtain an accurate estimate of the true bi-policy over all states. Two potential directions for future work are worth pursuing. One is to derive continuous versions of the proposed algorithms, and the other is to treat the condition when only a partial bi-policy is available.

References

1. Abbeel, P., Ng, A.Y.: Apprenticeship learning via inverse reinforcement learning. In: Proc. Intl. Conf. Mach. Learning (ICML’04). pp. 1–8 (2004)
2. Abbot, T., Kane, D., Valiant, P.: On algorithms for nash equilibria (2004), http://web.mit.edu/tabbott/Public/final.pdf
3. Abdallah, S., Lesser, V.: A multiagent reinforcement learning algorithm with nonlinear dynamics. Journal of Artificial Intelligence Research 33, 521–549 (2008)
4. Aumann, R.: Subjectivity and correlation in randomized strategies. Journal of Mathematical Economics 1, 67–96 (1974)
5. Baker, C.L., Saxe, R., Tenenbaum, J.B.: Action understanding as inverse planning. Cognition 113(3), 329–349 (2009)
6. Barr, N.: Economics of the Welfare State. Oxford University Press, 5 edn. (2012)
7. Cigler, L., Faltings, B.: Decentralized anti-coordination through multi-agent learning. Journal of Artificial Intelligence Research 47, 441–473 (2013)
8. Conitzer, V., Sandholm, T.: Awesome: A general multiagent learning algorithm that converges in self-play and learns a best response against stationary opponents. Mach. Learning 67(1–2), 23–24 (2007)
9. Daskalakis, C., Goldberg, P.W., Papadimitriou, C.H.: The complexity of computing a nash equilibrium. SIAM Journal on Computing 39(1), 195–259 (2009)
10. Ferguson, T.S.: Game Theory. UCLA (2008)
11. Filar, J., Vrieze, K.: Competitive Markov Decision Processes. Springer-Verlag, New York, NY, 1st edn. (1996)
12. Greenwald, A., Hall, K.: Correlated q-learning. In: Proceedings of the 20th International Conference on Machine Learning, ICML’03, pp. 242–249 (2003)
13. Hadfield-Menell, D., Dragan, A., Abbeel, P., Russell, S.: Cooperative inverse reinforcement learning. In: Proceedings of the 30th Neural Information Processing Systems (NIPS’16) (2016)
14. Hart, S., Schmeidler, D.: Existence of correlated equilibria. Mathematics of Operations Research 14(1), 18–25 (1989)
15. Hu, J., Wellman, M.P.: Multiagent reinforcement learning: Theoretical framework and an algorithm. In: Proc. Intl. Conf. on Mach. Learning (ICML’98), pp. 242–250 (1998)
16. Hu, J., Wellman, M.P.: Nash q-learning for general-sum stochastic games. The Journal of Machine Learning Research 4, 1039–1069 (2003)
17. Lin, X., Beling, P.A., Cogill, R.: Multi-agent inverse reinforcement learning for two-person zero-sum games. IEEE Transactions on Games 10(1), 56–68 (2018)
18. Littman, M.L.: Markov games as a framework for multi-agent reinforcement learning. In: Proc. Intl. Conf. Mach. Learning (ICML’94). pp. 157–163 (1994)
19. Littman, M.L.: Friend-or-foe q-learning in general-sum games. In: Proceedings of the 18th International Conference on Machine Learning, ICML’01. pp. 322–328 (2001)
20. Maskin, E., Tirole, J.A.: Markov perfect equilibrium: I. observable actions. Journal of Economic Theory 100(2), 191–219 (2001)
21. Nash, J.: Equilibrium points in n-person games. Proceedings of the National Academy of Sciences 36(1), 48–49 (1950)
22. Nash, J.: Non-cooperative games. The Annals of Mathematics 54(2), 286–295 (1951)
23. Natarajan, S., Kunapuli, G., Judah, K., Tadepalli, P., Kersting, K., Shavlik, J.W.: Multi-agent inverse reinforcement learning. In: Proc. Intl. Conf. Mach. Learning App. (ICMLA’10). pp. 395–400 (2010)
24. Ng, A.Y., Russell, S.: Algorithms for inverse reinforcement learning. In: Proc. Intl. Conf. Mach. Learning (ICML’00). pp. 663–670 (2000)
25. Owen, G.: Game Theory. W. B. Saunders Company, Philadelphia, PA, 1st edn. (1968)
26. Ozdaglar, A.: Game theory with engineering applications. MIT Open Courseware (2010)
27. Qiao, Q., Beling, P.A.: Inverse reinforcement learning with gaussian process. In: Proc. American Control Conf. (ACC’11). pp. 113–118 (2011)
28. Rapoport, A., Chammah, A.M.: The game of chicken. American Behavioral Scientist 10, 10–28 (1966)
29. Reddy, T.S., Gopikrishna, V., Zaruba, G., Huber, M.: Inverse reinforcement learning for decentralized non-cooperative multiagent systems. In: Proc. IEEE Intl. Conf. Syst., Man, Cybern. (SMC’12) (2012)
30. Shapley, L.S.: Stochastic games. Proc. Nat. Academy Sci., Math. 39, 1095–1100 (1953)
31. Skyrms, B.: The Stag Hunt and the Evolution of Social Structure. Cambridge University Press, Cambridge, UK (2004)
32. Sodomka, E., Hilliard, E., Littman, M., Greenwald, A.: Coco-q: Learning in stochastic games with side payments. In: In proceedings of the 30th International Conference on Machine Learning (ICML’13). pp. 1471–1479 (2013)
33. Waugh, K., Ziebart, B., Bagnell, J.: Computational rationalization: The inverse equilibrium problem. In: Proc. Intl. Conf. Mach. Learning (ICML’11). pp. 1169–1176 (2011)
34. Yang, S.Y., Qiao, Q., Beling, P.A., Scherer, W.T., Kirilenko, A.A.: Gaussian process-based algorithmic trading strategy identification. Quantitative Finance 15(10), 1683–1703 (2015)
35. Ziebart, B.D., Maas, A.L., Bagnell, J.A., Dey, A.K.: Maximum entropy inverse reinforcement learning. In: Proc. Nat. Conf. Artif. Intell (AAAI’08). vol. 3, pp. 1433–1438 (2008)