Design and expected performance of a compact and continuous nuclear demagnetization refrigerator for sub-mK applications

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Abstract. Sub-mK temperatures are achievable by a copper nuclear demagnetization refrigerator (NDR). Recently, research demands for such an ultra-low temperature environment are increasing not only in condensed matter physics but also in astrophysics. A standard NDR requires a specially designed room, a high-field superconducting magnet, and a high-power dilution refrigerator (DR). And it is a one-shot cooling apparatus. To reduce these requirements, we are developing a compact and continuous NDR with two PrNi\(_5\) nuclear stages which occupy only a small space next to an appropriate pre-cooling stage such as DR. PrNi\(_5\) has a large magnetic-field enhancement on Pr\(^{3+}\) nuclei due to the strong hyperfine coupling. This enables us to enclose each stage in a miniature superconducting magnet and to locate two such sets in close proximity by surrounding them with high-permeability magnetic shields. The two stages are thermally connected in series to the pre-cooling stage by two Zn superconducting heat switches. A numerical analysis taking account of thermal resistances of all parts and an eddy current heating shows that the lowest sample temperature of 0.8 mK can be maintained continuously under a 10 nW ambient heat leak.

1. Introduction

In general, thermal fluctuations and specific heats of materials are substantially reduced at low temperatures near absolute zero. So one can improve the sensitivity of various particle detectors such as X-ray detector dramatically by cooling. It is also expected that unique ordered states and phase transitions of materials and new physical phenomena appear at lower temperatures. Nowadays, an automatically controllable dilution refrigerator (DR), which can produce a temperature as low as 10 mK continuously, is commercially available. Then many researchers, even who are not familiar with low temperature techniques, can make use of such very low temperatures easily. This contributes to the progress of science in wide areas. However, further low temperature experiments below 1 mK (sub-mK) are still highly demanding for non-specialists to use.

Usually, sub-mK temperatures are achievable with a copper nuclear demagnetization refrigerator (NDR) \cite{1}. The principle of NDR is based on adiabatic demagnetization cooling of nuclear spins, a magnetocaloric effect. This method was established in the 1970s and has been contributed to many discoveries in the research field of quantum liquids and solids. The cooling procedure of NDR is as follows. First we apply a strong magnetic field \(B_{\text{max}}\) (typically...
8 T) to copper nuclear spins \((I = 3/2)\) and pre-cool them down to \(T_i = 10 \sim 15\) mK by a DR. In this process, the entropy of nuclear spin system is reduced by \(\Delta S\) from its maximum value \((= N k_B \ln 4)\). Here, \(N\) is the number of spins. Then we thermally disconnect the spin system from DR by turning a superconducting heat switch (HSW) open and decrease \(B\). During this adiabatic demagnetization process, the temperature of spin system decreases keeping \(B/T\) constant in an ideal condition. Non-idealities come from, for example, heat loads to cool a sample and the copper electron system simultaneously and an eddy current heating of the copper stage. We can keep the lowest temperature, which is determined by the final field and the thermal loads, constant by reducing \(B\) slowly so that an ambient heat leak is just absorbed as an entropy increase of spin system until \(\Delta S\) becomes zero. Thus the cooling capacity of NDR is given by \(\Delta S\), and this is a one-shot cooling method.

In order to obtain a large \(\Delta S\), we need to reduce \(T_i\) and increase \(B_{\text{max}}\) and \(N\) as much as possible. Typical sizes of the copper nuclear stage are more than 30 mm in diameter and 250 mm in length. We thus inevitably need to use a superconducting magnet of large bore diameter and of high magnetic field, and a powerful DR with a cooling power higher than 200 \(\mu\)W at 100 mK. At the same time, the sample space should be well isolated from fringing fields of the demagnetization magnet. As a result, the length of NDR becomes too long (more than 5 m when the DR insert is removed from a dewar) to fit in standard laboratory rooms demanding a specially designed room with a deep pit or a high ceiling. This is one of major barriers for application of the sub-mK environment to wider research fields.

Usage of recently developed cryogen-free DR (dry-DR) as a pre-cooler for NDR [2] can make the system size shorter since usually outer vacuum is separable in dry-DR. However, the large copper nuclear stage and superconducting magnet still limit the space conservation.

In this article, we describe design concept and results of feasibility study of the performance of a new compact demagnetization refrigerator with two nuclear stages of PrNi\(_5\), a hyperfine enhanced nuclear magnet, connected in series to maintain the sub-mK environment continuously and easily.

### 2. Design concept of CNDR

If we have two independent magnetic refrigeration units, it is possible to keep a constant temperature below 1 mK continuously by using one of them as a heat absorber connected to the sample and the other as a shuttle stage to pump an entropy increase of that unit to DR. The two-unit setup is also good for minimizing the apparatus size because we don’t have to maximize the cooling capacity of each unit. Such continuous refrigeration has previously been realized in an adiabatic demagnetization refrigerator for electronic spins (ADR) developed by Shirron et al. [3, 4]. By connecting one CPA and two GLF salt stages in series, they were successful to maintain the temperature of a soft X-ray calorimetric detector at \(T = 50\) mK continuously on a satellite. In principle, it should be possible to extend this technique to the sub-mK region by replacing the electronic spins with nuclear spins.

The new continuous nuclear demagnetization refrigerator (CNDR) we propose here has two nuclear stages of PrNi\(_5\). In this Van Vleck paramagnet, Pr\(^{3+}\) nuclear spins \((I = 5/2)\) feel an about 12 times higher effective magnetic field than the applied one due to the strong hyperfine coupling [1, 5]. This feature is essential to make the apparatus compact, since a sizable amount of \(\Delta S\) can be obtained even in a relatively low field. For example, at \(T_i = 15\) mK, \(\Delta S\) per unit volume of PrNi\(_5\) in \(B_{\text{max}} = 1.2\) T is nearly the same as that of Cu in 8 T. The lowest achievable temperature of the PrNi\(_5\) stage is limited to around 0.5 mK by its nuclear ferromagnetic ordering at 0.4 mK [6]. However, this would not be a serious problem for most realistic applications, and PrNi\(_5\) has eventually been used as an alternative coolant to copper in previous single-stage NDRs [5, 7, 8, 9].

In our CNDR, one of the two nuclear stages will consist of three cylindrical PrNi\(_5\) rods (total
volume ≈ 10 cm$^3$ or 80 g). The size of each rod is 6 mm in diameter and 120 mm in length. Since the size and $B_{\text{max}}$ of our demagnetization magnet are small (bore diameter = 22 mm, length = 140 mm, $B_{\text{max}} = 1.2$ T), the fringing field can easily be suppressed by surrounding it with a high permeability magnetic shield of 40 mm in outer diameter. Such compactness enables us to install the two sets of magnetic units within a small space around the mixing chamber of DR. Since the demagnetization magnet is long enough compared to the PrNi$_5$ rods and the magnetic shielding further improves the field homogeneity, the difference between a root-mean-square field over the whole rods and the field at the magnet center is only 2%.

Although we are still in the process of fine tuning of geometry of the CNDR, the basic concept is to connect the two nuclear stages (NS1 and NS2) in series between the sample and mixing chamber through two HSWs (HSW1 and HSW2). Each HSW consists of 6 zinc foils of 0.25 mm thick contacting to 7 silver foils on both sides. After annealing separately, they will diffusively be bonded by heating to 230 °C. Zinc was chosen because of its low superconducting critical field ($B_c = 5$ mT) and of the resultant low magnetic heat-release ($\propto B_c^2$) on turning open at the lowest temperature. The overall diameter of HSW including a superconducting solenoid and a magnetic shield will be 17 mm.

![Figure 1. Block diagram of CNDR used for the thermodynamic simulations shown in figures 2 and 3.](image)

3. Expected Performance of CNDR
All thermal links connecting each parts will be made of properly annealed pure silver. They will have tiny but non-negligible thermal resistances including contact resistances as shown in figure 1. For instance, $R_{\text{NS1}}$ consists of a resistance of a silver thermal link (15 silver wires of 1 mm in diameter for each PrNi$_5$ rod) and a contact resistance between the wires and rod by Zn soft soldering for NS1. $R_{\text{SW1}}$ is an overall resistance of the assembled HSW1. Based on the thermal block diagram shown in figure 1, we have evaluated the performance of CNDR numerically. In the evaluation, we assumed that $R_{\text{SW1}} = 100 \, \text{n}\Omega$ and $R_{\text{NS1}} = R_{\text{NS2}} = R_{\text{SW2}} = 50 \, \text{n}\Omega$ in the unit of electrical resistance, and thermal resistances were calculated from them through the Wiedemann-Franz law. These values are estimated from results of test annealing and measurements of short pieces [10]. Note that electrical resistance can be measured much more easily than thermal resistance. Various physical properties of PrNi$_5$ were taken from the book [1].

When the HSW1 or the HSW2 is kept open, a heat leak from the mixing chamber at 30 mK, the highest operation temperature, to sample stage will be less than 0.5 nW through it. On
switching HSW2 to the open state, a latent heat of about 250 nJ will be released to the NS2 side. These are realistic estimations based on our previous experiences with a similar Zn HSW [11]. By employing standard thermal isolation techniques at sub-mK temperatures [1], a total heat leak to NS2 would be reduced to 1 nW in several weeks as was obtained in the previous single-stage PrNi5 NDRs [7, 8]. A somewhat larger value (∼5 nW) was reported if dry-DR is used as a pre-cooler presumably due to its larger mechanical vibrations [9].

Figure 2 shows a calculated entropy ($S$) vs. temperature ($T$) diagram of the two PrNi5 stages in the continuous operation. Here we set a constant sample-stage temperature ($T_S$) at 0.8 mK assuming the ambient heat leak to the sample stage ($Q_{\text{leak}}$) to be 10 nW, a much worse value than the ordinary cases. The red closed line in the figure is an entropy pumping cycle for NS1 which is thermally connected either to DR at high $T$ or the sample stage at low $T$. The temperature of NS1 ($T_{\text{NS1}}$) is pre-cooled down to $T_1 = 17$ mK in a magnetic field of 1.2 T by a DR which has a cooling power of 100 µW at $T = 100$ mK and the lowest achievable temperature ($T_0$) of 15 mK. The cooling power of this DR is much lower than that for typical copper NDRs.

The blue upward and downward vertical arrows for NS2 in figure 2 correspond to absorbing $Q_{\text{leak}}$ by slow demagnetizing and releasing an accumulated entropy to NS1 as a heat flow by slow magnetizing, respectively. Note that NS2 is directly connected to the sample stage. The temperature of NS2 ($T_{\text{NS2}}$) is slightly lower than $T_S$ during absorbing $Q_{\text{leak}}$ and is slightly higher than $T_S$ during releasing the entropy to keep $T_S$ constant all the time. Appropriate temperature differences between $T_{\text{NS2}}$ and $T_S$ in the two processes depend on $Q_{\text{leak}}$, $R_{\text{NS1}}$, $R_{\text{NS2}}$, and $R_{\text{SW2}}$. As the resistances increase, the heat (or entropy) transport rate from the sample stage to NS1 or NS2 decreases. This decreases the cooling power of CNDR, and finally we will not be able to keep $T_S = 0.8$ mK in the presence of $Q_{\text{leak}} = 10$ nW. Thus it is essentially important to make the

![Figure 2](image-url)  
**Figure 2.** Simulation result for the entropy vs. temperature diagram for the CNDR designed in this work.

![Figure 3](image-url)  
**Figure 3.** Simulation results for time evolutions of the temperatures, applied magnetic fields, and entropies of NS1 and NS2 for the CNDR designed in this work.
thermal resistances as low as possible for higher cooling power and lower operation temperature for the CNDR.

Figure 3 shows calculated time evolutions of the temperatures, applied magnetic fields and entropies of NS1 and NS2 in the continuous operation. In the CNDR, the cooling capacity divided by the cycle time (= 16210 s here) is more important than the cooling capacity itself of one cycle. Since the cooling power of DR is proportional to $T^2$ and thus the pre-cooling speed decreases near $T_0$, the cycle time becomes too long if we set $T_i$ too low. This degrades the performance of CNDR. On the other hand, if $T_i$ is set too high, we cannot make use of the cooling power of DR effectively, again degrading the performance. This is because the cycle time cannot be shorter than time necessary for demagnetization and magnetization. Therefore, there should be an optimum $T_i$ depending on $T_S$, $\dot{Q}_{\text{leak}}$, thermal resistances of each parts, and other factors. We shall discuss about the fine tuning of $T_i$ elsewhere [10].

From $t = 4830$ to 7240 s in figure 3, NS1 is cooled from 17 to 0.7 mK by adiabatic demagnetization from $B_{NS1} = 1.2$ T to 6 mT. During the demagnetization, $S_{NS1}$ increases by about 4% of $\Delta S$ due to the eddy current heating produced at the silver wires and Zn soldering layers around the PrNi$_5$ rods. An optimum sweep rate should be determined by competition between the eddy current loss and cycle time increase. At $t = 4830$ and 16210 s when HSW1 and HSW2 are turned open, respectively, there must be small entropy losses in NS1 and NS2 due to the magnetic heat release. But they are invisible in the scale of figure 3.

Finally, let us briefly comment on the feasibility to use this CNDR on satellite. Cooling powers of current space-based DR [12] and ADR [4] are only 1–2 $\mu$W at $T = 50$ mK. They are insufficient to pre-cool the PrNi$_5$ stages as is estimated in figure 2. So an additional platform kept around 10 mK consisting of a CMN salt stage [13] should be necessary in between them.

4. Summary

We designed a new compact and continuous nuclear demagnetization refrigerator (CNDR) using two PrNi$_5$ nuclear stages connected in series with two zinc superconducting heat switches. Because of the large hyperfine enhancement of applied magnetic field in PrNi$_5$, it was possible to design all the parts quite compact so that they can be installed in a relatively small space around the mixing chamber of a dilution refrigerator for pre-cooling. The size is substantially smaller than that of standard copper nuclear demagnetization refrigerators, and the cooling power of dilution refrigerator can be relatively low. We evaluated the expected performance of CNDR by numerical calculation based on a simple thermal model assuming realistic thermal resistances of each parts. The simulation results show that the CNDR can maintain an operation temperature of $T = 0.8$ mK continuously with a cooling power of 10 nW by cyclic demagnetization and magnetization of the first nuclear stage between 1.2 T and 6 mT being pre-cooled by a dilution refrigerator with a cooling power of 100 $\mu$W at $T = 100$ mK.

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