Strongly broken Peccei-Quinn symmetry in the early Universe

To cite this article: Fuminobu Takahashi and Masaki Yamada JCAP10(2015)010

View the article online for updates and enhancements.
Strongly broken Peccei-Quinn symmetry in the early Universe

Fuminobu Takahashi\textsuperscript{a,b} and Masaki Yamada\textsuperscript{b,c}

\textsuperscript{a}Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan
\textsuperscript{b}Kavli IPMU (WPI), TODIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan
\textsuperscript{c}Institute for Cosmic Ray Research, ICRR, The University of Tokyo, Kashiwa, Chiba 277-8582, Japan

E-mail: fumi@tuhep.phys.tohoku.ac.jp, yamadam@icrr.u-tokyo.ac.jp

Received August 4, 2015
Revised September 8, 2015
Accepted September 9, 2015
Published October 6, 2015

Abstract. We consider QCD axion models where the Peccei-Quinn symmetry is badly broken by a larger amount in the past than in the present, in order to avoid the axion isocurvature problem. Specifically we study supersymmetric axion models where the Peccei-Quinn symmetry is dynamically broken by either hidden gauge interactions or the SU(3)\textsubscript{c} strong interactions whose dynamical scales are temporarily enhanced by the dynamics of flat directions. The former scenario predicts a large amount of self-interacting dark radiation as the hidden gauge symmetry is weakly coupled in the present Universe. We also show that the observed amount of baryon asymmetry can be generated by the QCD axion dynamics via spontaneous baryogenesis. We briefly comment on the case in which the PQ symmetry is broken by a non-minimal coupling to gravity.

Keywords: axions, particle physics - cosmology connection, baryon asymmetry, supersymmetry and cosmology

ArXiv ePrint: 1507.06387
1 Introduction

The strong CP phase is constrained to be less than $10^{-10}$ by the neutron EDM experiments [1], and the problem of why it is so small is known as the strong CP problem. One of the solutions is the Peccei-Quinn (PQ) mechanism [2, 3], which renders the strong CP phase a dynamical variable, an axion, which arises as a pseudo Nambu-Goldstone (NG) boson in association with the spontaneous breakdown of the global PQ symmetry [4, 5]. The axion acquires a mass through nonperturbative effects of QCD [6, 7] and its vacuum expectation value (VEV) cancels the undesired CP phase. The dynamical relaxation necessarily induces coherent oscillations of the axion, which can account for dark matter in the Universe [8–10].

The PQ symmetry must be of extremely high quality to solve the strong CP problem, as even a tiny breaking would result in a too large CP phase in contradiction with the experiments [11–17]. The high quality of the PQ symmetry could be due to a non-trivial discrete symmetry of high order $Z_N$ [18] or a higher dimensional gauge symmetry (or its combination with an anomalous $U(1)_A$ symmetry) [19]. Here we would like to emphasize that, from a purely phenomenological point of view, the PQ symmetry could be badly broken in the early Universe, as long as the extra breaking disappears in the present Universe. Interestingly, in the presence of such a large PQ breaking in the early Universe, the axion becomes so heavy that its quantum fluctuations are suppressed, avoiding tight constraints on the axion isocurvature perturbations [25–29].

In this paper we study a possibility that the PQ symmetry is largely broken temporarily in the early Universe, and discuss their cosmological implications based on a couple of models. Specifically, we first discuss supersymmetric axion models in which the PQ symmetry is largely broken by non-Abelian hidden gauge interactions which are strongly coupled during inflation. If the axion acquires a sufficiently heavy mass, its quantum fluctuations would be suppressed. In the present Universe, on the other hand, the hidden gauge interactions must be weakly

\[1\text{See refs. [20–24] for other models of the origin of the PQ symmetry.}\]
\[2\text{The stronger QCD was proposed to suppress the axion abundance [30]. See also ref. [31, 32]. There are various ways to suppress the axion isocurvature perturbations [24, 33–40].}\]
coupled, which is made possible by a dynamics of flat directions in the hidden sector. We shall also see that the axion dynamics after inflation can generate a sizable baryon asymmetry a la spontaneous baryogenesis [41–46]. We also show that the suppression can be realized in a more economical model where the PQ symmetry is broken only by the SU(3)$_c$ strong interactions. The QCD scale can be enhanced if the $H_uH_d$ flat direction acquires a large VEV in the early Universe [26, 30].

The rest of the paper is organized as follows. In section 2, we briefly review observational constraints on the axion isocurvature perturbations. In section 3, we provide supersymmetric axion models where the dynamical scales of the hidden or SU(3)$_c$ gauge interactions are enhanced during and some time after inflation, due to the dynamics of a flat direction. After inflation ends, the axion dynamics generates baryon asymmetry via spontaneous baryogenesis. We also show that the scenario can be naturally realized when the scalar field is identified with the $H_uH_d$ flat direction. The last section is devoted to conclusions. In appendix we briefly discuss the case in which the PQ symmetry is broken by an interaction between the axion and the Ricci scalar.

2 Axion isocurvature problem

The axion is a pseudo NG boson associated with the spontaneous breakdown of the PQ symmetry, which is assumed to be anomalous under QCD [4, 5]. Suppose that the PQ symmetry is spontaneously broken at an intermediate scale $v_a$. Then, the axion acquires a non-zero mass through non-perturbative effects of the QCD instantons as [6, 7]

$$m_a \simeq \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi f_\pi}{f_a},$$

(2.1)

where $f_a (= v_a/N_{DW})$ is the axion decay constant and $N_{DW}$ is the domain wall number. Here, $f_\pi$ is the pion decay constant, and $m_u$, $m_d$, and $m_\pi$ are the masses of up quark, down quark, and pion, respectively.

The axion stays almost massless at high temperatures, and its mass gradually grows as the temperature $T$ decreases down to the QCD scale $\Lambda_{QCD}$. In a thermal plasma with temperature $T \gg \Lambda_{QCD}$, the axion mass is given by [47, 48]

$$m_a^2(T) \simeq 1.68 \times 10^{-7} \frac{\Lambda_{QCD}^4}{f_a^2} \left(\frac{\Lambda_{QCD}}{T}\right)^{-n},$$

(2.2)

where $n \approx 6.68$. When the axion mass becomes comparable to the Hubble parameter, it starts to oscillate about the CP conserving minimum. The energy density of the axion oscillations decreases in proportion to $a^{-3}$, where $a$ is the scale factor, and hence it is a good candidate for cold dark matter (CDM). Neglecting the anharmonic effect [49, 50], the axion abundance is given by [51]

$$\Omega_a h^2 \simeq 0.011 |\theta_{ini}|^2 \left(\frac{f_a}{10^{11} \text{GeV}}\right)^{1.19} \left(\frac{\Lambda_{QCD}}{400 \text{MeV}}\right),$$

(2.3)

where $h$ is the Hubble parameter in units of $100$ km/s/Mpc and $\theta_{ini}$ is the initial misalignment angle. Barring fine-tuned cancellations, $\theta_{ini}$ is considered to be of order unity. The observed DM abundance $\Omega_{DM} h^2 \simeq 0.12$ can be explained if the axion decay constant is given by

$$f_a \simeq 7.4 \times 10^{11} \text{GeV} \times |\theta_{ini}|^{-1.68},$$

(2.4)

where we have substituted $\Lambda_{QCD} = 400 \text{MeV}$.
If the axion mass is smaller than the Hubble parameter during inflation, it acquires quantum fluctuations as

$$ |\delta \theta_{ini}| \equiv \frac{\delta a}{f_a} \simeq \frac{H_{\text{inf}}}{2\pi f_a}, \quad (2.5) $$

where $H_{\text{inf}}$ is the Hubble parameter during inflation. This results in the axion CDM isocurvature perturbations through the dependence of $\Omega_a$ on $\theta_{ini}$ (see eq. (2.3)),

$$ S_{a\gamma} \equiv \frac{\delta \Omega_a}{\Omega_a} \simeq 2 \frac{\delta \theta_{ini}}{\theta_{ini}}, \quad (2.6) $$

where $|\delta \theta_{ini}/\theta_{ini}| \ll 1$ is assumed.

The cosmological data can be explained by purely adiabatic density perturbations, and only a small admixture of isocurvature perturbations is allowed. In fact, the Planck Collaboration derived a tight upper bound on the fraction of the uncorrelated isocurvature perturbation as [52]

$$ \frac{P_{SS}(k_*)}{P_{RR}(k_*) + P_{SS}(k_*)} \lesssim 0.038 \quad \text{(95\% C.L.)}, \quad (2.7) $$

where $P_{RR}$ and $P_{SS}$ are the power spectra of the adiabatic and isocurvature perturbations, respectively, and $k_*$ (= 0.05 Mpc$^{-1}$) is the pivot scale. Thus we obtain an upper bound on the axion isocurvature perturbations as

$$ |S_{a\gamma}| \lesssim 9.1 \times 10^{-6}, \quad (2.8) $$

where we have used $P_{RR} \simeq 2.2 \times 10^{-9}$ [52]. Therefore, the axion isocurvature perturbations tightly constrain the energy scale of inflation as

$$ H_{\text{inf}} \lesssim 0.94 \times 10^7 \text{GeV} \left( \frac{f_a}{10^{11} \text{GeV}} \right)^{0.405}. \quad (2.9) $$

The constraint becomes even severer when the anharmonic effect is relevant around $\theta_{ini} \simeq \pi$ [49, 50]. The upper bound is inconsistent with high or intermediate scale inflation models, such as the $R^2$-inflation model [53], chaotic inflation model [54], and hybrid inflation model [55]. In particular, if the primordial B-mode polarization is detected by the future CMB polarization experiment, it would be in a strong tension with the axion CDM scenario [27, 56, 57].

There is one caveat in the above analysis: the axion is assumed to be massless during inflation. In the rest of this paper, we study axion models with a larger breaking of the PQ symmetry during inflation in order to avoid the constraint of eq. (2.9).

### 3 QCD axion models with dynamically broken PQ symmetry

Now we introduce supersymmetric axion models in which the PQ symmetry is broken by a larger amount in the early Universe either by hidden gauge interactions or by the SU(3)$_c$ strong interactions. We first consider the former case in the following, and we will show that the axion isocurvature perturbations can indeed be suppressed. Then we investigate the dynamics of the axion after inflation and show that a right amount of baryon asymmetry can be generated by spontaneous baryogenesis. In section 3.4, we estimate the amount of axion DM and self-interacting dark radiation made of weakly-coupled hidden gauge fields. Finally, we introduce a more economical model where the PQ symmetry is broken by the stronger QCD and a hidden flat direction is replaced by the $H_uH_d$ flat direction.
3.1 PQ symmetry breaking by hidden gauge interactions

Let us consider a model in which the PQ symmetry is dynamically broken by hidden SU(N)\(_H\) gauge interactions in the early Universe. We introduce a PQ breaking scalar field \(\psi\), a singlet scalar field \(\phi\), and \(N_F(N'_F)\) hidden quarks and anti-quarks \(Q_H, \bar{Q}_H, (Q'_H, \bar{Q}'_H)\) in the fundamental and anti-fundamental representation of SU(N)\(_H\), with (without) PQ charges. The charge assignment is shown in Table 1.\(^3\) For simplicity, we assume that the fields \(Q_H\) and \(\bar{Q}_H\) are also charged under SU(3)\(_c\) in the fundamental and anti-fundamental representation, respectively so that they play a role of the heavy PQ quarks to induce the color anomaly term for the axion [58, 59].\(^4\)

We consider the superpotential of

\[
W = y_{ij} \psi Q_H^i \bar{Q}_H^j + y_{kl} \phi Q_H^k \bar{Q}_H^l + W_{PQ} + W_\phi(\phi),
\]

where \(i, j, k, \) and \(l\) are flavor indices and we omit SU(N)\(_H\) and SU(3)\(_c\) indices which are contracted in a gauge invariant way.\(^5\) The superpotential of \(W_{PQ}\) represents interaction terms for \(\psi\) to develop a nonzero VEV which spontaneously breaks the PQ symmetry. We assume that the axion decay constant (the PQ breaking scale) remains constant throughout the history of the Universe. While we have introduced only a single PQ field \(\psi\) for simplicity, we may introduce another PQ scalar \(\tilde{\psi}\) to write down a superpotential like \(W_{PQ} \supset \kappa S(\psi \tilde{\psi} - v_a^2/4)\), where \(\kappa\) is a parameter and \(S\) is a singlet chiral superfield. Assuming an (approximate) exchange symmetry between \(\psi\) and \(\tilde{\psi}\), they are stabilized at \(|\psi| = |\tilde{\psi}| = v_a/2\). The axion is given by a combination of the phases of \(\psi\) and \(\tilde{\psi}\). As long as the inflation scale is lower than \(v_a\), the PQ breaking scale remains almost constant in this case.

Let us first focus on the axion in the present Universe, assuming that the hidden SU(N)\(_H\) is weakly coupled and \(\phi\) is stabilized at the origin (or its VEV is sufficiently small). For instance, in the non-SUSY limit where the SUSY particles are integrated out, the hidden SU(N)\(_H\) is asymptotic non-free if \(N'_F > 11N/4\). Then, the phase of \(\psi\) (or a combination of the phases of \(\psi\) and \(\tilde{\psi}\) in the above example) becomes the axion, and its anomalous coupling to gluon fields is induced by the diagram with \(Q_H\) and \(\bar{Q}_H\) running in the loop.

---

\(^3\)In general, the high quality of the PQ symmetry is a puzzle because global symmetries are considered to be broken in nature [11–17]. This puzzle is neither worsened or improved compared with the ordinary field-theoretic QCD axion models, because the extra PQ breaking effects we introduced disappear in the present vacuum. The required PQ breaking scale in our model is of order \(10^{13}\) GeV. This implies that \(Z_N\) symmetry with \(N \geq 17\) is necessary to suppress Planck-suppressed PQ breaking operators, if we are to explain the lightness of the QCD axion by a single discrete symmetry [18]. There are also some other mechanisms to explain the origin of the PQ symmetry (see refs. [19–24]).

\(^4\)Instead, one may introduce heavy PQ quarks separately, if \(Q_H\) and \(\bar{Q}_H\) are singlet under SU(3)\(_c\).

\(^5\)In ref. [60], they investigated a similar non-SUSY model. They have considered a scenario that the PQ symmetry is largely broken after inflation due to the strong dynamics of hidden gauge interaction. They assume \(N_F = 1\) so that domain walls decay soon after the phase transition of the hidden gauge symmetry.
The domain wall number $N_{DW}$ is equal to $NN_F$ in this model. The axion is stabilized at the CP conserving minimum, solving the strong CP problem. Note that the PQ symmetry is anomalous under $SU(N)_H$, which however does not affect the axion mass as long as $SU(N)_H$ is weakly coupled.

The situation is different and more complicated if $\phi$ develops a large VEV in the early Universe. Before going into details, let us briefly outline a rough sketch of our scenario. First, as we shall see below, the $U(1)_R$ symmetry is spontaneously broken by a non-zero VEV of $\phi$, and the phase component $\phi$ becomes a pseudo NG boson called the $R$-axion.\footnote{Depending on the $R$ charge of $\psi$, the $R$-axion may be composed of a combination of the phases of $\phi$ and $\psi$. This however does not change our arguments.} The $U(1)_R$ symmetry is not an exact symmetry of nature, and it is explicitly broken by a constant term in the superpotential, $W \supset W_0 = m_{3/2} M_{Pl}^2$, where $m_{3/2}$ is the gravitino mass. It is possible that the inflaton sector also does not respect the $U(1)_R$ symmetry, in which case the $R$-axion can be so heavy that it does not acquire sizable quantum fluctuations at superhorizon scales. In general, the $R$-axion mass is given by \[ m_R^2 = \frac{8}{f_R^2 M_{Pl}^2} |F_i \phi_i - 3W|, \] where the $R$-axion coupling $f_R$ is given by
\[ f_R = \sqrt{2} R_i \phi_i, \]
where $R_i$ is the $R$ charge of field $\phi_i$ and $\phi_i$ represents a field which spontaneously breaks the $U(1)_R$ symmetry. We assume that the $U(1)_R$ symmetry is explicitly broken in the inflaton sector so that the $R$-axion mass is comparable to (or heavier than) the Hubble parameter during inflation. This is the case if $H^2 \sim |F_i|^2 \sim |W|^2 / \phi_i^2$. In other words, the mass of the phase component of $\phi$ is of order the Hubble parameter due to the Hubble-induced $A$-term. Secondly, the hidden quarks $Q_H$ and $\bar{Q}_H$ acquire a heavy mass through its coupling to $\phi$, and they are decoupled from the low-energy physics. As a result, the running of the gauge coupling $g_H$ of $SU(N)_H$ is modified, so that it becomes confined during inflation if the confinement scale is higher than $H_{inf}$. This implies that the PQ symmetry is broken badly by non-perturbative effects of $SU(N)_H$. Thus, the axion mass receives an extra contribution during inflation, suppressing the quantum fluctuations. The axion dynamics just after inflation is involved as we shall study in detail later in this section. Some time after inflation the potential minimum of $\phi$ shifts to the origin, and the $SU(N)_H$ becomes weakly coupled. The axion becomes almost massless and it remains so until the QCD phase transition.

Now let us discuss the dynamics of $\phi$ in a greater detail. We assume that $\phi$ is a flat direction, namely, the potential of $\phi$ is flat in the SUSY limit at the renormalizable level, which can be ensured by assigning a certain $R$ charge or a discrete symmetry. Such flat directions can be lifted by soft SUSY breaking terms as well as non-renormalizable terms. To be concrete we consider the following superpotential for $\phi$:
\[ W_\phi(\phi) = \lambda \frac{\phi^4}{4M_{Pl}^2}, \]
where we assign $R$-charges as $R(\phi) = 1/2$ and $R(Q_H, \bar{Q}_H) = 3/2$, and $M_{Pl} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. During inflation, the potential of $\phi$ is given by
\[ V(\phi) = m_\phi^2 |\phi|^2 - c_H H^2 |\phi|^2 - \left( a_H \lambda H \frac{\phi^4}{4M_{Pl}^2} + c.c. \right) + \lambda^2 |\phi|^6, \]
where $m_{\phi}$ is the soft SUSY breaking mass of $\phi$ and the second term in the r.h.s. is the Hubble-induced mass term [62]. The term in the parenthesis is the Hubble-induced $A$-terms, which generically arises when the inflaton breaks the $U(1)_R$ symmetry. Hereafter, we set $c_H = a_H = 1$ for simplicity.

During inflation, since the flat direction has a mass of order the Hubble parameter, the $\phi$ is stabilized at its potential minimum:

$$\langle|\phi|\rangle(t) \simeq \left(\frac{H(t)M_{Pl}}{\sqrt{3\lambda}}\right)^{1/2}. \quad (3.6)$$

The phase direction of $\phi$ also obtains a mass of order the Hubble parameter due to the Hubble-induced $A$-terms. Therefore, it stays at $\theta = 0$ and it does not acquire any sizable quantum fluctuations at large scales.

After inflation ends, the inflaton (not $\phi$) starts to oscillate around the potential minimum and the energy density of the Universe is dominated by the inflaton oscillation. During the inflaton oscillation dominated era, there is a dilute plasma with temperature,

$$T \simeq \left(\frac{36H(t)\Gamma_I M_{Pl}^2}{g_*(T)\pi^2}\right)^{1/4} \propto a^{-3/8}, \quad (3.7)$$

where $H(t) = 2/(3t)$, and $\Gamma_I$ is the inflaton decay rate. Note that, if the plasma temperature is higher than $\sim y v_a$, the hidden SU($N_H$) gauge fields as well as the hidden quarks are thermalized because $Q_H$ and $\bar{Q}_H$ are charged under both SU($3_c$) and SU($N_H$). At a sufficiently high temperature, the effective relativistic degrees of freedom $g_*$ is given as $g_* = 228.75$ in the MSSM. Hereafter, we use $g_* \simeq 200$ as a reference value. The reheating temperature is given by

$$T_{RH} \simeq \left(\frac{90}{g_*(T_{RH})\pi^2}\right)^{1/4} \sqrt{\Gamma_I M_{Pl}} \simeq 1.0 \times 10^{13}\,\text{GeV} \left(\frac{\Gamma_I}{2 \times 10^8\,\text{GeV}}\right)^{1/2}. \quad (3.8)$$

In a finite temperature, the following thermal log potential is induced via two-loop effects: [63, 64]

$$V_T(\phi) \simeq c_T \alpha_H^2 T^4 \log \left(\frac{\phi^2}{T^2}\right), \quad (3.10)$$

for $\phi \gg T/g_H$, where $\alpha_H = g_H^2/4\pi$ is the fine-structure constant of SU($N_H$) and $c_T (> 0)$ is an $O(1)$ constant determined by the beta function of SU($N_H$) coupling.

The flat direction $\phi$ starts to oscillate at $t = t_{osc}^\phi$ when the Hubble parameter becomes comparable to the soft mass or the curvature of thermal potential. The Hubble parameter at the commencement of oscillations of $\phi$ is given by

$$H_{osc}^\phi \simeq \max \left[m_\phi, \left(\frac{72}{5g_*\pi^2}\right)^{1/2} \left(\frac{c_T \alpha_H^2 T_{RH}^2 M_{Pl}}{\langle\phi\rangle^2 t_{osc}^\phi}\right)^{1/2}\right]. \quad (3.11)$$

7The PQ symmetry remains broken if $y \ll 1$.

8To be precise, we should include the degrees of freedom of hidden fields in $g_*$, which however does not affect our main results qualitatively. In section 3.5, many SM particles are decoupled in the thermal plasma due to a large VEV of the $H_uH_d$ flat direction, so that $g_*$ is smaller.
This implies that \( \phi \) starts to oscillate around the origin before reheating completes, i.e., \( T_{\text{osc}}^{\phi} > T_{\text{RH}} \). In our scenario, we find that \( H_{\text{osc}}^{\phi} \) is determined by the second term for the parameters of our interest. Substituting \( \mathcal{O}(1) \) parameters, we obtain a typical value of \( H_{\text{osc}}^{\phi} \) as

\[
H_{\text{osc}}^{\phi} \simeq 1.4 \times 10^{9} \text{GeV} \left( \frac{T_{\text{RH}}}{10^{13} \text{GeV}} \right)^{2} \left( \frac{\langle |\phi|^{2} \rangle (t_{\text{osc}}^{\phi})}{5 \times 10^{15} \text{GeV}} \right)^{-2},
\]

(3.12)

where we have adopted \( \alpha_{H} \simeq 1/25 \) as a reference value. This implies that \( T_{\text{osc}}^{\phi} \simeq 3.3 \times 10^{13} \text{GeV} \) for the above reference parameters.

### 3.2 Suppression of the axion CDM isocurvature perturbations

When the flat direction \( \phi \) obtains a large VEV as eq. (3.6), its \( F \)-component is given by

\[
\langle F_{\phi} \rangle \simeq \left( \frac{H^{3}(t) M_{\text{Pl}}}{3^{2/3} \lambda} \right)^{1/2},
\]

(3.13)

from eq. (3.4). This implies that the SU\((N)_{H}\) gaugino obtains a soft mass via the gauge mediated SUSY breaking effect:

\[
m_{\lambda} \simeq \frac{N'_{F} g_{H}^{2} F_{\phi}}{16 \pi^{2}} \langle \phi \rangle
\]

\[
\simeq N'_{F} \frac{g_{H}^{2}}{16 \sqrt{3} \pi^{2}} H(t).
\]

(3.14)

(3.15)

We may write the Lagrangian of the SU\((N)_{H}\) gauge field as

\[
\mathcal{L} = \int d^{2} \theta \frac{T_{H}}{2} W^{a} W_{a} + \text{H.c.},
\]

(3.16)

where

\[
\tau_{H} = \frac{1}{2 g_{H}^{2}} - \frac{i \theta_{H}}{16 \pi^{2}} - \frac{\theta^{2} m_{\lambda}}{g_{H}^{2}},
\]

(3.17)

is a chiral spurion superfield that contains the gauge coupling \( g_{H} \), vacuum angle \( \theta_{H} \), and gaugino mass \( m_{\lambda} \) \([65, 66]\). The axion appears in the gauge kinetic function, and it can be taken into account by the replacement of

\[
\theta_{H} \rightarrow \theta_{H} - 3 N_{F} \frac{a}{v_{a}} = \theta_{H} - \frac{3}{N} \frac{a}{f_{a}},
\]

(3.18)

where we have used \( f_{a} = v_{a}/N_{\text{DW}} = v_{a}/(N N_{F}) \) in the second equality.

Next, we calculate the confinement scale of SU\((N)_{H}\) in order to determine the axion mass. First, let us consider the present era, where the VEV of \( \phi \) is absent, \( \langle \phi \rangle = 0 \). In this case, the SU\((N)_{H}\) gauge theory contains \( N'_{F} \) flavors of \( Q'_{H} \) and \( \bar{Q}'_{H} \). For a sufficiently large \( N'_{F} \), SU\((N)_{H}\) remains weakly coupled at present. Above the soft SUSY breaking scale \( m_{\text{SUSY}} \), the running gauge coupling \( g_{H} \) is given by

\[
\frac{1}{g_{H}^{2}(\mu)} - \frac{1}{g_{H}^{2}(m_{\text{SUSY}})} = - \frac{3 N - N_{F}'}{8 \pi^{2}} \ln \left( \frac{m_{\text{SUSY}}}{\mu} \right),
\]

(3.19)
where $\mu$ ($m_{\text{SUSY}} < \mu < y_{\nu}$) is the renormalization scale. Next, suppose that the field $\phi$ has a large VEV as eq. (3.6) in the early Universe. The fields $Q'_H$ and $\bar{Q}'_H$ then obtain heavy masses due to the nonzero VEV of $\phi$, which modify the running of the gauge coupling (see figure 1). The $SU(N)_H$ confines during inflation if the confinement scale $\Lambda_H$ is higher than the Hubble parameter during inflation, $H_{\text{inf}}$. The confinement scale $\Lambda_H$ depends on $\phi$ as

$$\Lambda^3_{H}(\phi) = \det (y'\phi) m_{\text{SUSY}}^{3N - N'_F} e^{-8\pi^2/g_m^2(m_{\text{SUSY}})},$$

(3.20)

because the renormalization group equation changes at the energy scale of $y'\phi$. For later convenience, we define $\tilde{\Lambda}_H$ by

$$\tilde{\Lambda}^3_{H}(\phi) \equiv \det (y'\phi) m_{\text{SUSY}}^{3N - N'_F} e^{-16\pi^2 \tau_H(m_{\text{SUSY}})},$$

(3.21)

where $\tau_H$ is given by eq. (3.17) with the replacement of eq. (3.18).

At the confinement scale $\Lambda_H$, the gaugino condensation gives rise to the effective superpotential given by \[67\]

$$W_{\text{eff}} = N\tilde{\Lambda}^3_{H}(\phi).$$

(3.22)

Therefore, the field $a$ acquires an effective potential such as

$$V = - \int d^2 \theta W_{\text{eff}} + \text{H.c.}$$

(3.23)

$$= \frac{32\pi^2}{g_H^2} m_{\Lambda} \tilde{\Lambda}^3_{H} \cos \left( \frac{\theta_H}{N} - \frac{3a}{N^2 f_a} \right) + \ldots,$$

(3.24)

where we use eqs. (3.13), (3.21), and (3.22). This gives the field $a$ an effective mass of

$$m_a^2(\phi) = c_m H_{\text{inf}} \tilde{\Lambda}^2_{H}(\phi)$$

(3.25)

$$\simeq (1.0 \times 10^{12} \text{GeV})^2 \times c_m \left( \frac{H_{\text{inf}}}{10^{11} \text{GeV}} \right)$$

$$\times \left( \frac{f_a}{5 \times 10^{13} \text{GeV}} \right)^{-2} \left( \frac{\Lambda_H(\phi)}{4 \times 10^{13} \text{GeV}} \right)^3,$$

(3.26)
where
\[ c_m \equiv \frac{6\sqrt{3}N_F'}{N^4}. \] (3.27)

When \( m_a(\langle |\phi|\rangle_{\text{inf}}) > H_{\text{inf}} \) is satisfied, the field \( a \) does not acquire quantum fluctuations during inflation, thus avoiding the isocurvature constraint.

### 3.3 Spontaneous baryogenesis via axion dynamics

Now let us consider the axion dynamics after inflation. We shall show that spontaneous baryogenesis by the QCD axion works for certain parameters as one application of the strongly broken PQ symmetry.

We assume that SU(\( N \)) is deconfined just after inflation. This can be achieved if the inflaton decays into the SU(\( N \)) gauge field as well as standard model particles and the maximum temperature of the Universe (or the hidden sector, if it is decoupled from the standard model sector) is larger than the confinement scale \( \Lambda_H \):
\[ T_{\text{max}} \gtrsim \Lambda_H(\langle |\phi|\rangle_{\text{inf}}). \] (3.28)

Here, the maximal temperature of the Universe after inflation is given by\(^9\)
\[ T_{\text{max}} \simeq \left[ \frac{60}{g_\ast \pi^2} \left(\frac{3}{8}\right)^{8/5} \Gamma_I H_{\text{inf}} M_{\text{Pl}} \right]^{1/4}, \] (3.29)
\[ \simeq 2.9 \times 10^{13} \text{GeV} \left(\frac{\Gamma_I}{0.2 \times 10^{9} \text{GeV}}\right)^{1/4} \left(\frac{H_{\text{inf}}}{10^{11} \text{GeV}}\right)^{1/4}. \] (3.30)

We also require the following condition to avoid the restoration of PQ symmetry by thermal effects:
\[ f_a \gtrsim T_{\text{max}}. \] (3.31)

Now, let us explain the dynamics of the axion and \( \phi \) in our scenario. During inflation, the axion acquires the effective mass of eq. (3.26), which is assumed to be larger than the Hubble parameter to suppress the isocurvature perturbation. After inflation ends, the temperature of the Universe soon reaches the maximal temperature \( T_{\text{max}} \), and then, the axion becomes massless because of \( T_{\text{max}} \gtrsim \Lambda_H(\langle |\phi|\rangle_{\text{inf}}) \). When the temperature decreases to \( T = T_{\text{osc}} \simeq \Lambda_H(\phi) \), the axion again acquires an effective mass due to instanton effects of SU(\( N \)) gauge theory. As we shall see shortly, the minimum could be different from that during inflation, in which case the axion starts to oscillate about the new minimum. Here, the minimum of the axion potential is given by \( \theta_H f_a \), which is determined as
\[ \theta_H = \theta_{H,0} + \arg \left[ \det \left( y(\langle \phi \rangle) \right) \right] + \arg \left[ \det \left( y'(\langle \phi \rangle) \right) \right] + N \arg \left[ m_{\lambda,0} \right], \] (3.32)
where \( \theta_{H,0} \) is the bare theta parameter and \( m_{\lambda,0} \) is the mass of SU(\( N \)) gaugino. Since the minimum of the axion field value is related to \( \theta_H \), it also depends on the phase of U(1)\( _R \)-symmetry breaking term through \( \arg [m_{\lambda,0}] \). As the U(1)\( _R \) symmetry is assumed to be

\(^9\)At such a high temperature, axions can be in the thermal equilibrium via interactions such as \( g+g \leftrightarrow a+g \), where \( g \) represents gluon [68, 69]. Then the SU(\( N \)) hidden gauge fields are also in the thermal equilibrium via similar interactions as \( g_H + g_H \leftrightarrow a + g_H \), where \( g_H \) represents hidden gluon. Also, if the temperature is higher than \( y_v a, Q_H \) and \( \bar{Q_H} \) are thermalized. Thus SU(\( N \)) is in the thermal equilibrium and is deconfined after inflation even if the inflaton decays only into the SM particles.
largely broken in the inflaton sector, the phase of U(1)$_R$-symmetry breaking term generically changes after inflation, which implies that the minimum of the axion potential is shifted by an amount of $\Delta a \simeq O(1) f_a$. Therefore, the axion starts to oscillate around the new minimum at $T = T_{\text{osc}}^a \sim \Lambda_H(\phi)$. This axion dynamics breaks the CPT invariance, which enables the spontaneous baryogenesis [41–46].

Let us focus on the dynamics of axion at the temperature around $T = T_{\text{osc}}^a$ and consider the spontaneous baryogenesis based on ref. [45]. We assume that the axion couples to the electroweak SU(2) gauge fields as

$$\mathcal{L} \supset g_2^2 \frac{a}{f_a} F \tilde{F},$$

where $g_2$ is the SU(2) gauge coupling. Using the anomaly equation of the SU(2) gauge theory, we obtain the following derivative couplings between axion and SM particles:

$$\mathcal{L} = -c_a \frac{a(t)}{f_a} \partial_\mu j^{\mu}_{B+L}$$

$$= c_a \frac{\partial \theta_a}{f_a} j^0_{B+L} + \ldots,$$

where $c_a^0 = 1/3$ and $j^\mu_{B+L}$ is the $B + L$ current. Note that this deformation is valid if the SU(2)$_L$ sphaleron effect is sufficiently efficient [46]. The sphaleron rate is given by $\Gamma_{\text{sphaleron}} \sim 25 \alpha_2^2 T^4$ per unit time and volume [70]. Taking the thermal volume, $1/T^3$, and comparing it to the Hubble rate, the sphaleron decouples above the temperature of order $10^{13}$ GeV. This is of the same order with $T_{\text{osc}}^a$, so that our scenario is marginally consistent. Note that the above sphaleron decoupling temperature is calculated by assuming $\alpha_2 \sim 1/25$. Since we introduce some SU(3)$_c$ charged fields, such as $Q_H$ and $\bar{Q}_H$, the unified gauge coupling constant may be larger than 1/25. This implies that the SU(2)$_L$ coupling constant may also be larger than 1/25 at a high temperature. Therefore, the sphaleron decoupling temperature may be higher than $10^{13}$ GeV in our model and we can safely use eq. (3.35).

One needs baryon or lepton number violating interactions for successful spontaneous baryogenesis. We shall focus on the lepton number violating operator mediated by right-handed neutrinos, and so, let us focus on the lepton asymmetry in the following. From eq. (3.35), the axion dynamics leads to an effective chemical potential for lepton asymmetry as

$$\mu_{\text{eff}} = c_a \frac{\partial \theta_a}{f_a}.$$

At the temperature of $T = T_{\text{osc}}^a$, the axion starts to oscillate around the minimum and the effective chemical potential is given by $\mu_{\text{eff}} \simeq c_a H(T_{\text{osc}}^a) \Delta a / f_a$. The nonzero effective chemical potential results in the following equilibrium number density:

$$n_{L}^{\text{eq}} = \frac{1}{6} \mu_{\text{eff}} T^2.$$

Note that the equilibrium number density is realized only if the baryon/lepton number is explicitly broken and its rate is sufficiently rapid.

\[10\]One needs to add extra matter fields in order to form complete multiplets under SU(5), which would keep the gauge coupling unification intact.
Now we introduce right-handed neutrinos for the seesaw mechanism \cite{71-74} and assume that their masses are close to $10^{15}$ GeV. Thermal leptogenesis \cite{75} does not work for such heavy right-handed neutrinos. Nevertheless, they provide lepton number violating processes, $ll \leftrightarrow HH$ and $lH \leftrightarrow \bar{l}\bar{H}$, where $l$ and $H$ represent left-handed lepton and higgs multiplets, respectively. The effective lepton number violating rate is roughly given by

$$
\Gamma_L \sim \frac{\bar{m}^2 T^3}{16\pi v_{ew}^4},
$$

where $\bar{m}^2$ is the sum of the left-handed neutrino mass squared and is assumed to be of order the atmospheric neutrino mass squared difference, $\Delta m_{atm}^2 \simeq 2.4 \times 10^{-3}$ eV$^2$ \cite{76}. This lepton violating interactions lead to a nonzero lepton asymmetry in the Universe in the presence of the chemical potential of eq. (3.36). The resulting lepton asymmetry just after $T = T_{osc}$ is thus calculated as

$$
n_L |_{T=T_{osc}} \simeq \frac{1}{H_{osc}^2} \Gamma_L n_{eq}^L.
$$

After the axion starts to oscillate, the generated lepton asymmetry is partially washed out due to the inverse processes. This is described by the following Boltzmann equation: \cite{77}

$$
\frac{d}{dt} a^3 n_L(t) \simeq -\Gamma_L a^3 n_L(t).
$$

Using eq. (3.7) before reheating and $T^4 = 90 H^2 M_{Pl}^2/\pi^2 g_*$ after reheating, where $g_*$ is the effective number of degrees of freedom for number density, and matching the solutions at $T = T_{RH}$, we obtain

$$
a^3 n_L(T \to 0) \equiv a^3 n_L(T = T_{osc}^a) \Delta_w,
$$

where

$$
\log \Delta_w \simeq -0.7 \left( \frac{T_{RH}}{10^{13} \text{GeV}} \right).
$$

This implies that the reheating temperature cannot be much larger than $10^{13}$ GeV to avoid the washout effect due to the inverse processes.

Since $T = T_{osc}^a$ occurs before the reheating completes, the baryon asymmetry is calculated as

$$
Y_b \equiv \frac{n_b}{s} \simeq -\frac{8 n_f + 4 n_H}{22 n_f + 13 n_H} T_{RH} \frac{3 n_L}{4 \rho_{tot}} \bigg|_{RH} \frac{1}{\Delta_w} \left( \frac{T_{RH}}{10^{13} \text{GeV}} \right)^{7/2} \left( \frac{H_{osc}^a}{1.4 \times 10^9 \text{GeV}} \right)^{-3/4},
$$

where $n_f = 3$ and $n_H = 2$ in the MSSM. Here, we implicitly assume that $\phi$ and axion oscillations never dominate the Universe, on which we shall comment shortly. We find that the observed amount of baryon asymmetry $Y_b^{obs} \simeq 8.6 \times 10^{-11}$ can be explained by the above mechanism. Since $H_{osc}^a \sim (T_{osc}^a)^4 T_{RH}^{-2} M_{Pl}^{-1}$ and $T_{osc}^a \gtrsim T_{RH}$, there is a lower bound on the reheating temperature to explain the observed amount of baryon asymmetry:

$$
T_{RH}^{min} \simeq 5.0 \times 10^{12} \text{GeV}.
$$
This implies that the confinement scale $\Lambda_H$ during inflation should be higher than $5.0 \times 10^{12}$ GeV.

Our scenario requires the Hubble parameter of inflation to be of order $10^{11}$ GeV. This is because it should be smaller than the mass of the axion during inflation, while the successful baryogenesis requires a high reheating temperature. The energy scale of $10^{11}$ GeV predicts the tensor-to-scalar ratio of order $r = O(10^{-7})$, which is too small to be detected in the future CMB polarization experiments.

Finally let us study the dynamics of $\phi$ to see if the above scenario works successfully. The field $\phi$ starts to oscillate around the origin of the potential at $H = H_{\phi \text{osc}}$, where $H_{\phi \text{osc}}$ is given by eq. (3.11). After the field $\phi$ starts to oscillate, $\Lambda_H(\phi)$ becomes much smaller than the temperature of the Universe, so that the axion becomes massless again. So far we have assumed that

$$T_{\phi \text{osc}} \gtrsim T_{\phi \text{osc}},$$

holds so that axion starts to oscillate before $\phi$ starts to oscillate. Otherwise the dynamical scale becomes much smaller than temperature before axion starts to oscillate to generate the baryon asymmetry. Here, let us check that the energy density of $\phi$ oscillation never dominates that of the Universe. It starts to oscillate around the origin of the potential by the thermal log potential at $H = H_{\phi \text{osc}}$, in which case it is known that Q-balls form after the oscillation [78–84]. The energy density of the flat direction is converted to that of Q-balls. However, in the case of our interest, a typical charge of Q-balls is so small that they completely evaporate via interactions with thermal plasma [85, 86]. Thus, they evaporate soon and dissipate into thermal plasma, and their energy density never dominates the Universe.

### 3.4 Axion dark matter and self-interacting dark radiation

At $T \simeq \Lambda_{\text{QCD}}$, the axion acquires an effective mass through the non-perturbative effect of the SU(3)$_c$ gauge theory. The minimum of the axion potential induced by the QCD instantons is generally different from that by the SU($\mathcal{N}$)$_H$ instantons. Thus, the axion again starts to oscillate around its minimum at $T \simeq \Lambda_{\text{QCD}}$ and we can explain the observed DM abundance by the axion oscillations as eq. (2.3).

Let us mention an interesting prediction of our scenario. To solve the strong CP problem, the SU($\mathcal{N}$)$_H$ gauge theory should not be confined at present. Therefore, our model predicts that there are at least massless hidden gauge bosons, and some of the hidden quarks $Q'_H$ and $\bar{Q}'_H$ may also remain sufficiently light.\textsuperscript{11} Since those light hidden particles are in the thermal equilibrium just after inflation ends, they contribute to the energy density of the Universe as dark radiation [87–90]. Their abundance is commonly expressed by the effective neutrino number and is calculated as [87]

$$\Delta N_{\text{eff}} = \left[ \frac{4}{7} (N^2 - 1) + 2\tilde{N}_F \right] \left( \frac{g_*}{43/4} \right)^{-4/3},$$

\textsuperscript{11}The SU($\mathcal{N}$)$_H$ can be asymptotic non-free at present if there are many massless hidden quarks. If $\phi$ develops a small VEV, hidden quarks $Q'_H$, and $\bar{Q}'_H$ acquire a light mass and some of them may be decoupled at present. Alternatively, if the U(1)$_R$ symmetry is broken down to a discrete $R$ symmetry, some of them acquire a non-zero mass depending on the $R$-charge assignment. The scalar components have a mass of order $m_{\text{SUSY}}$ and they do not contribute to dark radiation.
where \( \tilde{N}_{F}' \) denotes the flavor number of massless (or sufficiently light) hidden quarks. Together with the SM prediction of \( N_{e}^{\text{SM}} = 3.046 \), this is consistent with the present constraint of \( N_{e}^{\text{obs}} = 2.99 \pm 0.39 \) [91, 92]. The ground-based Stage-VI CMB polarization experiment CMB-S4 will measure the effective neutrino number with precisions of \( \Delta N_{\text{eff}} = 0.0156 \), so that it can indirectly test our model [93, 94]. In addition, the dark radiation (i.e., SU(\( N \)) gauge boson and hidden quarks) is self-interacting in our model [89], and so, it has different clustering properties compared to the standard free-streaming one. The clustering properties are represented by its effective sound speed \( c_{\text{eff}}^{2} \) and its viscosity parameter \( c_{\text{vis}}^{2} \). These parameters can be measured by CMB observations [95–101], so that in principle we can distinguish between our model and other models that predicts free-streaming dark radiation.

### 3.5 Application to the \( H_u H_d \) flat direction

In this subsection, we consider a more economical model in which the SU(\( N \)) gauge symmetry and the field \( \phi \) are replaced with QCD and the \( H_u H_d \) flat direction, respectively. In this model, we will introduce extra quark multiplets to obtain a sufficiently large dynamical scale \( \Lambda_{\text{QCD}} \) during inflation [26]. We consider a KSVZ-like axion model [58, 59], where the Higgs fields do not carry PQ charges.

Let us consider the \( H_u H_d \) flat direction with a superpotential of

\[
W = \frac{1}{2} \mu \phi^2 + \frac{\lambda}{4M_{\text{Pl}}} \phi^4,
\]

(3.49)

where we denote \( \phi^2/2 \equiv H_u H_d \). The first term in the r.h.s. is the usual Higgs \( \mu \) term. This superpotential leads to the following potential of the flat direction:

\[
V(\phi) = m_\phi^2 |\phi|^2 + \frac{\lambda \mu}{M_{\text{Pl}}} |\phi|^2 (\phi^2 + \text{c.c.}) - c_H H^2 |\phi|^2 - \left( a_H \lambda H \frac{\phi^4}{4M_{\text{Pl}}} + \text{c.c.} \right) + \lambda^2 \frac{|\phi|^6}{M_{\text{Pl}}^2} + V_T(\phi),
\]

(3.50)

where \( m_\phi \sim \mu \) is the mass of the flat direction. Hereafter, we assume \( c_H = a_H = 1 \) for simplicity. At a finite temperature, the thermal potential \( V_T \) is given as

\[
V_T(\phi) \simeq c_T \alpha_s^2 T^4 \log \left( \frac{|\phi|^2}{T^2} \right),
\]

(3.51)

for \( \phi \gg T/g \). The coefficient is given by \( c_T = 9/8 \) for the \( H_u H_d \) flat direction. Since the flat direction has a tachyonic mass of order the Hubble parameter, it obtains the VEV of eq. (3.6) during inflation and inflaton oscillation dominated era. Then, the flat direction starts to oscillate around the origin of the potential at the time of eq. (3.11).

When the \( H_u H_d \) flat direction has a large VEV during inflation, quark multiplets obtain effective masses much larger than the QCD scale \( \Lambda_{\text{QCD}} \). Since the renormalization group flow of the QCD coupling constant is sensitive to the number of light quark multiplets, the dynamical scale of SU(\( 3 \)) depends on their masses, i.e., the VEV of the flat direction. A large VEV of \( \phi \) can make the effective QCD scale during inflation \( \Lambda_{\text{QCD}} \) much larger than \( \Lambda_{\text{QCD}} \approx 400 \text{ MeV} \). Therefore, the axion mass can be enhanced in the early Universe, and if it is heavier than \( H_{\text{inf}} \) during inflation, the axion quantum fluctuations are suppressed. However, in order to make the dynamical scale as high as \( 10^{13} \text{ GeV} \), we need to introduce extra colored particles. We add \( N_F' \) pairs of \( Q_H \) and \( \bar{Q}_H \), which are charged under SU(\( 5 \)),

\[
\tilde{N}_F' \equiv \frac{\tilde{N}_F + N_F}{2} (\text{massless or sufficiently light}).
\]
with the following interaction,
\[ W_{Q'_H} = \left( M_{Q'_H} + \frac{\phi^2}{M'} \right) Q'_H \bar{Q}'_H. \] (3.52)

Note that the extra quarks do not have any PQ charges. In this case, the effective QCD scale can be as high as [26]
\[ \Lambda^\text{inf}_{\text{QCD}} \simeq 1.3 \times 10^7 \text{ GeV} \left( \frac{M_{\text{GUT}}}{M_{Q'_H}} \right)^{N_f/9}, \] (3.53)

for \( \phi \sim M' \sim M_{\text{GUT}} \). As a result, the axion mass is given by eq. (3.26). If \( m_a \gtrsim H_{\text{inf}} \) during inflation, the isocurvature constraint can be avoided.

Next, we consider the scenario of spontaneous baryogenesis. As in the previous subsections, we assume \( T_{\text{max}} \gtrsim \Lambda^\text{inf}_{\text{QCD}} \) so that SU(3)\(_c\) is deconfined and the axion becomes massless just after inflation. Then, when temperature decreases down to \( \Lambda^\text{inf}_{\text{QCD}} \), the axion again obtains an effective mass due to the non-perturbative effect. Here, the minimum of the axion field is determined by the theta parameter and is given by
\[ \theta_{\text{eff}} = \theta_0 + \text{arg} \left[ \det \left( y_u y_d \right) \right] + 3 \text{arg} \left[ M_g \right] - 3 \text{arg} \left[ \mu B \right], \] (3.54)

where \( \theta_0 \) is the bare theta parameter and \( M_g \) is the mass of SU(3)\(_c\) gaugino. Since the minimum of the axion field value is related to \( \theta_{\text{eff}} \), it also depends on the phase of R-symmetry breaking term via \( \text{arg} \left[ M_g \right] \). In general, the phase of R-symmetry breaking term changes after inflation because the source of R-symmetry changes after inflation. This implies that the minimum of the axion field changes after inflation, so that the axion starts to oscillate at \( T = \Lambda_{\text{QCD}} \). This dynamics can be used to realize the spontaneous baryogenesis.

In contrast to the previous model, the SU(2) gauge symmetry is spontaneously broken by the large VEV of \( H_u H_d \) in the present scenario. This implies that sphalerons are decoupled and one cannot use the anomaly equation to derive the lagrangian of eq. (3.35). So, let us instead introduce a Kähler potential of
\[ K \sim \frac{1}{f_a} (A + A^*) \left[ e_i^a |\psi_i|^2 + \ldots \right], \] (3.55)

where \( A \) represents the axion superfield and \( \psi_i \) represents SM matter superfields. This leads to the following derivative couplings between axion and SM particles:
\[ \mathcal{L} = -e_i^a \frac{a(t)}{f_a} \partial_\mu (\bar{\psi}_i \gamma^\mu \psi_i) \] (3.56)
\[ = e_i^a \frac{\partial_0 a}{f_a} (\bar{\psi}_i \gamma^0 \psi_i) + \ldots. \] (3.57)

This leads to an effective chemical potential for the lepton current as
\[ \mu_{\text{eff}} = \sum_i e_i^a g_i L_i \frac{\partial_0 a}{f_a} = e^a \frac{\partial_0 a}{f_a}, \] (3.58)

\[ ^{12} \text{In our setup, the up and down quarks may be lighter than } \Lambda_{\text{QCD}}^{\text{inf}}. \text{ In this case, we should replace } \Lambda_H \text{ in eq. (3.26) to } \Lambda_H^{\text{inf}} \text{ defined by } (\Lambda_H^{\text{inf}})^{3N} = \Lambda_{H_{\text{inf}}}^{N-2} m_u m_d, \text{ where } m_u \text{ and } m_d \text{ is given by } y_u \phi \text{ and } y_d \phi, \text{ respectively.} \]
where $L_i$ are lepton charges of fields $i$, and $g_i$ are the numbers of spin states, but with an extra factor of 2 for bosons. Thus the axion oscillation can induce a nonzero chemical potential for the lepton current. When we introduce a heavy right-handed neutrinos and realize the seesaw mechanism, it gives us lepton violating interactions. Note that lepton violating processes are efficient via the electron and higgs interactions though the Higgs VEV is much larger than the temperature of the Universe. Thus the lepton asymmetry is approximately given by eq. (3.39). The subsequent calculation and discussion are the same with those explained in the previous section and the final result is given by eq. (3.44).

Some time after inflation, the $H_u H_d$ flat direction starts to oscillate around the origin of the potential. Then, the effective QCD scale becomes equal to $\Lambda_{\text{QCD}}$, and the axion becomes massless again. Finally, the axion starts to oscillate around the QCD phase transition and contributes to CDM. The DM abundance can be explained when the PQ breaking scale satisfies eq. (2.4).

Finally, we comment on another source of baryon asymmetry in this model [102, 103]. When the flat direction starts to oscillate at $H = H_{\text{osc}}$, its phase direction also starts to rotate in the complex plane. This implies that the masses of MSSM particles obtain a time-dependent phase through the Yukawa interactions. Since the time-dependent phase of mass terms can be interpreted as a chemical potential, we obtain the chemical potential of $B + L$ current from the dynamics of the $H_u H_d$ flat direction:

$$\mu_{B+L} = 3\omega \phi,$$

(3.59)

where we define $\omega \phi$ by $\phi = |\phi| e^{i\omega \phi t}$. Let us emphasize that the origin of this chemical potential is completely different from that considered above. Therefore, the baryon and lepton asymmetry may also be generated by the spontaneous baryogenesis via this chemical potential. However, the flat direction starts to oscillate due to the thermal log potential, so that the kick in the phase direction is so small that the rotation frequency $\omega \phi$ is much smaller than the Hubble parameter $H_{\text{osc}}$. This implies that the resulting chemical potential is much smaller than that obtained by eq. (3.36). Therefore, we can neglect this contribution and justify our result of eq. (3.44).

4 Conclusions

The QCD axion is one of the plausible candidates for CDM, which is, however, severely constrained by the isocurvature perturbations. In this paper we have proposed an extension of the QCD axion model to avoid the isocurvature constraint by suppressing quantum fluctuations of the axion. Specifically we have considered a scenario where the PQ symmetry is badly broken by a larger amount in the past than in the present, due to non-perturbative effects of hidden or SU(3)$_c$ gauge interactions. Most importantly, the dynamical scale can be temporarily enhanced during inflation, if the renormalization group flow of the gauge coupling is significantly modified by a flat direction with a large VEV. If the dynamical scale is enhanced so as to make the axion mass heavier than or comparable to the Hubble parameter during inflation, the axion isocurvature perturbations are suppressed.

The dynamics of the axion and flat direction could be slightly involved after inflation. We have focused on the case in which the maximal temperature of the Universe is higher than the dynamical scale so that the axion becomes massless just after inflation. Some time after inflation, the axion becomes massive again and starts to oscillate around the potential minimum when the temperature becomes comparable to the dynamical scale. Interestingly,
if the axion has a coupling to SU(2)$_L$ gauge fields, the axion oscillation induces a nonzero effective chemical potential of the $B+L$ symmetry, which would generate the baryon/lepton asymmetry in the presence of baryon/lepton number violating operators. We have shown that a correct amount of the baryon asymmetry is generated by the QCD axion via spontaneous baryogenesis, by taking account of the $\Delta L = 2$ process mediated by the heavy right-handed Majorana neutrinos. Soon after the baryon asymmetry is generated, the flat direction starts to oscillate around the origin of the potential. In the first model, the hidden gauge interactions then become weakly coupled and it remains so until present. Depending on the flavor number of the hidden quarks, the hidden gauge interactions may become asymptotic non-free. Thus, the axion becomes massless again. Finally, at the QCD phase transition, the axion acquires a tiny mass through non-perturbative effect of QCD instantons and it is stabilized at the CP conserving minimum. The observed DM abundance can be explained by the QCD axion produced by the misalignment mechanism. To realize the above scenario and account for the observed baryon asymmetry and the DM abundance, the Hubble parameter of inflation must be of order $10^{11}$ GeV. We have also shown that the scenario can be similarly realized when the flat direction is identified with the $H_u H_d$ flat direction once we introduce additional colored particles at an intermediate scale.

One of the predictions of our scenario based on the hidden gauge interactions is that there must be a self-interacting dark radiation with $\Delta N_{\text{eff}} = \mathcal{O}(0.01-0.1)$ given by eq. (3.48). The ground-based Stage-VI CMB polarization experiment will be able to detect the dark radiation with this amount and distinguish it from free-streaming dark radiation.

In the appendix we also consider an interaction between axion and Ricci scalar, which results in a heavy axion mass during inflation. We find that the axion abundance as well as isocurvature perturbations are suppressed, so that the axion decay constant can be as large as the GUT scale.

Acknowledgments

This work is supported by Grant-in-Aid for Scientific research on Innovative Areas (No. 23104008 (F.T.)), Scientific Research (A) (No. 26247042 (F.T.)), Scientific Research (B) (No. 26287039 (F.T.)), JSPS Grant-in-Aid for Young Scientists (B) (No. 24740135 (F.T.)), JSPS Research Fellowships for Young Scientists (No. 25.8715 (M.Y.)), World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan (F.T. and M.Y.), and the Program for the Leading Graduate Schools, MEXT, Japan (M.Y.).

A Model with a coupling to the Ricci scalar

In this appendix, we provide another model to suppress the axion isocurvature perturbations. We consider a model where the axion is coupled with the Ricci scalar as

$$\mathcal{L} = c_R^2 R M_{\text{Pl}}^2 \cos \left( \frac{a}{f_a} - \theta_R \right),$$

(A.1)

where $c_R$ and $\theta_R$ are constants. Since $R = -6(\dot{a}/a)^2 + \ddot{a}/a \simeq -12H^2$ during inflation, the axion acquires a mass much larger than the Hubble parameter for $c_R \gtrsim f_a/M_{\text{Pl}}$, and therefore the quantum fluctuations of the axion is suppressed.

Here we simply assume that there are no other PQ breaking terms; in particular, any other Planck-suppressed operators are assumed to be absent or sufficiently suppressed. In
general, the absence of such PQ breaking terms is an issue that has been discussed extensively in the literature [11–17]. Our purpose is not to explain the long-standing problem concerning the high quality of the PQ symmetry, but to study cosmological impacts on such PQ breaking terms that are enhanced in the early Universe.

The peculiarity of the above interaction (A.1) is that it suppresses not only the axion quantum fluctuations, but also the axion DM abundance. To see this, let us estimate the axion abundance. At the time around the QCD phase transition, the energy density of the Universe is dominated by radiation, i.e., $a \propto t^{1/2}$, so that the Ricci scalar is one-loop suppressed as [105, 106]

$$R = -3(1 - 3\omega)H^2$$

(A.2)

$$1 - 3\omega \simeq \frac{162\alpha_s^2}{19\pi^2} + \mathcal{O}(g^3),$$

(A.3)

for the SU(3), gauge theory with three flavors. Thus, it induces the so-called Hubble-induced term for the axion:

$$L = -\frac{1}{2} c_a^2 H^2 (a - \theta_R f_a)^2 + \cdots,$$

(A.4)

$$c_a \simeq 1.6 c_R \alpha_s M_{Pl} f_a,$$

(A.5)

where we have expanded the axion potential around the potential minimum. Since $\alpha_s \sim 1$ at the time around the QCD phase transition, $c_a$ is much larger than unity as long as $c_R \gg f_a/M_{Pl}$. Then, the axion abundance is expected to be suppressed in a similar way as the adiabatic suppression mechanism for the moduli abundance proposed in ref. [107].

In order to estimate axion abundance, let us solve the following approximated equation of motion for the axion:

$$\ddot{a} + 3H \dot{a} \simeq -m_a^2(T)a - c_a^2 H^2(t)(a - \theta_R f_a),$$

(A.6)

where $H = 1/2t$, $m_a(T)$ is given by eq. (2.2), and we have omitted higher order terms.13 Let us explain the behavior of the axion qualitatively. First, in the regime of $H \gg m_a(T)$, the Hubble-induced term dominates the axion potential, so that the axion stays at $a \approx \theta_R f_a$. Then, at the time around $H \sim m_a(T)/c_a$, the potential minimum starts to move as $a_{\text{min}}(t) \equiv \theta_R f_a c_a^2 H^2/(m_a^2 + c_a^2 H^2)$. Here, the typical time scale of the shift of the potential minimum $a_{\text{min}}(t)$ is of order the inverse of the Hubble parameter, while that of the axion oscillations is of order $(c_a H)^{-1} (\ll H^{-1})$. This means that the minimum of the axion potential changes adiabatically, and the axion number density in the comoving volume is the adiabatic invariant, and practically no axion oscillations are induced through this dynamics. Finally, in the regime of $H \ll m_a(T)/c_a$, the Hubble-induced mass becomes negligible and the axion oscillates around the present vacuum, and its energy density decreases as $a^{-3}$.

We can estimate the abundance of the axion oscillations induced by weak violation of the adiabaticity [108]. The resulting axion abundance is roughly given by

$$\rho_a(t) \sim m_a^2(\theta_R f_a)^2 \left( \frac{t}{t_{\text{osc}}} \right)^{-3/2} e^{-2\pi c_a/(4+n)},$$

(A.7)

13This is for ease of comparison with ref. [107]. The axion abundance can be similarly suppressed even if one solve the equation of motion without any approximation.
where \( t_{\text{osc}} \simeq c_a / 2m_a(t_{\text{osc}}) \) for \( c_a \gg 1 \). Therefore, the axion-Ricci scalar coupling suppresses the axion abundance efficiently. The result shows that, for a certain value of \( c_a \), the observed DM abundance can be explained even if the axion decay constant is as large as the GUT scale.

Finally, let us comment on the difference of the present model from the original one in ref. [107], where the moduli mass in the low energy is constant with time. In fact, it was pointed out in refs. [109, 110] that the adiabaticity of the moduli dynamics is necessarily broken by the inflaton dynamics, leading to a non-negligible production of the moduli oscillations. This is because, while the inflaton is lighter than the modulus field which acquires a mass of \( \mathcal{O}(10 – 100)H_{\text{inf}} \) during inflation, the inflaton eventually becomes heavier than the modulus field some time after inflation. When the two masses are comparable, the adiabaticity is necessarily broken, which leads to production of some amount of moduli oscillations. In particular, the moduli abundance is only power suppressed and not exponentially suppressed. In contrast, the axion abundance is exponentially suppressed in our scenario, because the axion mass \( m_a(T) \) is negligible until the QCD phase transition. In the context of moduli problem, our result implies that the adiabatic suppression mechanism works successfully and the moduli abundance is exponentially suppressed, if the moduli potential in the low energy vanishes during and some time after inflation and arises at a sufficiently late time.

References

[1] C.A. Baker et al., An Improved experimental limit on the electric dipole moment of the neutron, Phys. Rev. Lett. 97 (2006) 131801 [hep-ex/0602020] [inSPIRE].
[2] R.D. Peccei and H.R. Quinn, CP Conservation in the Presence of Instantons, Phys. Rev. Lett. 38 (1977) 1440 [inSPIRE].
[3] R.D. Peccei and H.R. Quinn, Constraints Imposed by CP Conservation in the Presence of Instantons, Phys. Rev. D 16 (1977) 1791 [inSPIRE].
[4] S. Weinberg, A New Light Boson?, Phys. Rev. Lett. 40 (1978) 223 [inSPIRE].
[5] F. Wilczek, Problem of Strong p and t Invariance in the Presence of Instantons, Phys. Rev. Lett. 40 (1978) 279 [inSPIRE].
[6] G. ’t Hooft, Symmetry Breaking Through Bell-Jackiw Anomalies, Phys. Rev. Lett. 37 (1976) 8 [inSPIRE].
[7] G. ’t Hooft, Computation of the Quantum Effects Due to a Four-Dimensional Pseudoparticle, Phys. Rev. D 14 (1976) 3432 [Erratum ibid. D 18 (1978) 2199] [inSPIRE].
[8] J. Preskill, M.B. Wise and F. Wilczek, Cosmology of the Invisible Axion, Phys. Lett. B 120 (1983) 127 [inSPIRE].
[9] L.F. Abbott and P. Sikivie, A Cosmological Bound on the Invisible Axion, Phys. Lett. B 120 (1983) 133 [inSPIRE].
[10] M. Dine and W. Fischler, The not so harmless axion, Phys. Lett. B 120 (1983) 137 [inSPIRE].
[11] H.M. Georgi, L.J. Hall and M.B. Wise, Grand Unified Models With an Automatic Peccei-Quinn Symmetry, Nucl. Phys. B 192 (1981) 409 [inSPIRE].
[12] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phenomenology and Cosmology With Superstrings, Phys. Rev. Lett. 56 (1986) 432 [inSPIRE].
[13] M. Dine and N. Seiberg, String Theory and the Strong CP Problem, Nucl. Phys. B 273 (1986) 109 [inSPIRE].
[14] M. Kamionkowski and J. March-Russell, Planck scale physics and the Peccei-Quinn mechanism, Phys. Lett. B 282 (1992) 137 [hep-th/9202003] [inSPIRE].
R. Holman, S.D.H. Hsu, T.W. Kephart, E.W. Kolb, R. Watkins and L.M. Widrow, *Solutions to the strong CP problem in a world with gravity*, Phys. Lett. B 282 (1992) 132 [hep-ph/9203206] [inSPIRE].

S.M. Barr and D. Seckel, *Planck scale corrections to axion models*, Phys. Rev. D 46 (1992) 539 [inSPIRE].

B.A. Dobrescu, *The Strong CP problem versus Planck scale physics*, Phys. Rev. D 55 (1997) 5826 [hep-ph/9609221] [inSPIRE].

M. Dine, R.G. Leigh and D.A. MacIntire, *Of CP and other gauge symmetries in string theory*, Phys. Rev. Lett. 69 (1992) 2030 [hep-th/9205011] [inSPIRE].

P. Svrček and E. Witten, *Axions in string theory*, JHEP 06 (2006) 051 [hep-th/0605206] [inSPIRE].

S.M. Barr and D. Seckel, *The Cosmological constant, false vacua and axions*, Phys. Rev. D 64 (2001) 123513 [hep-ph/0106239] [inSPIRE].

K.I. Izawa, T. Watari and T.T. Yanagida, *Higher dimensional QCD without the strong CP problem*, Phys. Lett. B 534 (2002) 93 [hep-ph/0202171] [inSPIRE].

K.I. Izawa, T. Watari and T.T. Yanagida, *Supersymmetric and CP-symmetric QCD in higher dimensions*, Phys. Lett. B 589 (2004) 141 [hep-ph/0403090] [inSPIRE].

K. Harigaya, M. Ibe, K. Schmitz and T.T. Yanagida, *Peccei-Quinn Symmetry from Dynamical Supersymmetry Breaking*, arXiv:1505.07388 [inSPIRE].

M. Kawasaki, M. Yamada and T.T. Yanagida, *Cosmologically safe QCD axion as a present from extra dimension*, Phys. Lett. B 750 (2015) 12 [arXiv:1506.05214] [inSPIRE].

M. Dine and L. Stephenson-Haskins, *Hybrid Inflation with Planck Scale Fields*, arXiv:1408.0046 [inSPIRE].

K. Choi, E.J. Chun, S.H. Im and K.S. Jeong, *Diluting the inflationary axion fluctuation by a stronger QCD in the early Universe*, Phys. Lett. B 750 (2015) 26 [arXiv:1505.00306] [inSPIRE].

G.R. Dvali, *Removing the cosmological bound on the axion scale*, hep-ph/9505253 [inSPIRE].

K. Choi, H.B. Kim and J.E. Kim, *Axion cosmology with a stronger QCD in the early universe*, Nucl. Phys. B 490 (1997) 349 [hep-ph/9606372] [inSPIRE].

T. Banks and M. Dine, *The Cosmology of string theoretic axions*, Nucl. Phys. B 505 (1997) 445 [hep-th/9608197] [inSPIRE].

A.D. Linde and D.H. Lyth, *Axionic domain wall production during inflation*, Phys. Lett. B 246 (1990) 353 [inSPIRE].

A.D. Linde, *Axions in inflationary cosmology*, Phys. Lett. B 259 (1991) 38 [inSPIRE].

S. Kasuya, M. Kawasaki and T.T. Yanagida, *Cosmological axion problem in chaotic inflationary universe*, Phys. Lett. B 409 (1997) 94 [hep-ph/9608405] [inSPIRE].

S. Kasuya, M. Kawasaki and T.T. Yanagida, *Domain wall problem of axion and isocurvature fluctuations in chaotic inflation models*, Phys. Lett. B 415 (1997) 117 [hep-ph/9709202] [inSPIRE].
[37] S. Folkerts, C. Germani and J. Redondo, Axion Dark Matter and Planck favor non-minimal couplings to gravity, *Phys. Lett. B* 728 (2014) 532 [arXiv:1304.7270] [nSPIRE].

[38] M. Kawasaki, T.T. Yanagida and K. Yoshino, Domain wall and isocurvature perturbation problems in axion models, *JCAP* 11 (2013) 030 [arXiv:1305.5338] [nSPIRE].

[39] K. Nakayama and M. Takimoto, Higgs inflation and suppression of axion isocurvature perturbation, *Phys. Lett. B* 748 (2015) 108 [arXiv:1505.02119] [nSPIRE].

[40] K. Harigaya, M. Ibe, M. Kawasaki and T.T. Yanagida, Dynamics of Peccei-Quinn Breaking Field after Inflation and Axion Isocurvature Perturbations, arXiv:1507.00119 [nSPIRE].

[41] A.G. Cohen and D.B. Kaplan, Thermodynamic Generation of the Baryon Asymmetry, *Phys. Lett. B* 199 (1987) 251 [nSPIRE].

[42] A.G. Cohen and D.B. Kaplan, Spontaneous baryogenesis, *Nucl. Phys. B* 308 (1988) 913 [nSPIRE].

[43] M. Dine, P. Huet, R.L. Singleton Jr. and L. Susskind, Creating the baryon asymmetry at the electroweak phase transition, *Phys. Lett. B* 257 (1991) 351 [nSPIRE].

[44] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Spontaneous baryogenesis at the weak phase transition, *Phys. Lett. B* 263 (1991) 86 [nSPIRE].

[45] A. Kusenko, K. Schmitz and T.T. Yanagida, Leptogenesis via Axion Oscillations after Inflation, *Phys. Rev. Lett.* 115 (2015) 011302 [arXiv:1412.2043] [nSPIRE].

[46] R. Daido, N. Kitajima and F. Takahashi, Axion domain wall baryogenesis, *JCAP* 07 (2015) 046 [arXiv:1504.07917] [nSPIRE].

[47] J.E. Kim and G. Carosi, Axions and the Strong CP Problem, *Rev. Mod. Phys.* 82 (2010) 557 [arXiv:0807.3125] [nSPIRE].

[48] O. Wantz and E.P.S. Shellard, Axion Cosmology Revisited, *Phys. Rev. D* 82 (2010) 123508 [arXiv:0910.1066] [nSPIRE].

[49] D.H. Lyth, Axions and inflation: sitting in the vacuum, *Phys. Rev. D* 45 (1992) 3394 [nSPIRE].

[50] T. Kobayashi, R. Kurematsu and F. Takahashi, Isocurvature Constraints and Anharmonic Effects on QCD Axion Dark Matter, *JCAP* 09 (2013) 032 [arXiv:1304.0922] [nSPIRE].

[51] K.J. Bae, J.-H. Huh and J.E. Kim, Update of axion CDM energy, *JCAP* 09 (2013) 005 [arXiv:0806.0497] [nSPIRE].

[52] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XX. Constraints on inflation, arXiv:1502.02114 [nSPIRE].

[53] A.A. Starobinsky, A New Type of Isotropic Cosmological Models Without Singularity, *Phys. Lett. B* 91 (1980) 99 [nSPIRE].

[54] A.D. Linde, Chaotic Inflation, *Phys. Lett. B* 129 (1983) 177 [nSPIRE].

[55] A.D. Linde, Hybrid inflation, *Phys. Rev. D* 49 (1994) 748 [astro-ph/9307002] [nSPIRE].

[56] D.J.E. Marsh, D. Grin, R. Hlozek and P.G. Ferreira, Tensor Interpretation of BICEP2 Results Severely Constrains Axion Dark Matter, *Phys. Rev. Lett.* 113 (2014) 011801 [arXiv:1403.4216] [nSPIRE].

[57] L. Visinelli and P. Gondolo, Axion cold dark matter in view of BICEP2 results, *Phys. Rev. Lett.* 113 (2014) 011802 [arXiv:1403.4594] [nSPIRE].

[58] J.E. Kim, Weak Interaction Singlet and Strong CP Invariance, *Phys. Rev. Lett.* 43 (1979) 103 [nSPIRE].
[60] S.M. Barr and J.E. Kim, New Confining Force Solution of the QCD Axion Domain-Wall Problem, Phys. Rev. Lett. 113 (2014) 241301 [arXiv:1407.4314] [arXIV].

[61] J. Bagger, E. Poppitz and L. Randall, The R axion from dynamical supersymmetry breaking, Nucl. Phys. B 426 (1994) 3 [hep-ph/9405345] [arXIV].

[62] M. Dine, L. Randall and S.D. Thomas, Baryogenesis from flat directions of the supersymmetric standard model, Nucl. Phys. B 458 (1996) 291 [hep-ph/9507453] [arXIV].

[63] T. Asaka, M. Fujii, K. Hamaguchi and T.T. Yanagida, Affleck-Dine leptogenesis with an ultralight neutrino, Phys. Rev. D 62 (2000) 123514 [hep-ph/0008041] [arXIV].

[64] A. Anisimov and M. Dine, Some issues in flat direction baryogenesis, Nucl. Phys. B 619 (2001) 729 [hep-ph/0008058] [arXIV].

[65] G.F. Giudice and R. Rattazzi, Extracting supersymmetry breaking effects from wave function renormalization, Nucl. Phys. B 511 (1998) 25 [hep-ph/9706540] [arXIV].

[66] N. Arkani-Hamed, G.F. Giudice, M.A. Luty and R. Rattazzi, Supersymmetry breaking loops from analytic continuation into superspace, Phys. Rev. D 58 (1998) 115005 [hep-ph/9803290] [arXIV].

[67] I. Affleck, M. Dine and N. Seiberg, Dynamical Supersymmetry Breaking in Supersymmetric QCD, Nucl. Phys. B 241 (1984) 493 [arXIV].

[68] M.S. Turner, Thermal Production of Not SO Invisible Axions in the Early Universe, Phys. Rev. Lett. 59 (1987) 317 [Erratum ibid. 60 (1988) 1101] [arXIV].

[69] E. Masso, F. Rota and G. Zsembinszki, On axion thermalization in the early universe, Phys. Rev. D 66 (2002) 023004 [hep-ph/0203221] [arXIV].

[70] D. Bödeker, G.D. Moore and K. Rummukainen, Chern-Simons number diffusion and hard thermal loops on the lattice, Phys. Rev. D 61 (2000) 056003 [hep-ph/9907545] [arXIV].

[71] T.T. Yanagida, Horizontal symmetry and masses of neutrinos, Conf. Proc. C 7902131 (1979) 95 [arXIV].

[72] T.T. Yanagida, Horizontal symmetry and masses of neutrinos, Prog. Theor. Phys. 64 (1980) 1103 [arXIV].

[73] M. Gell-Mann, P. Ramond and R. Slansky, Complex spinors and unified theories, Conf. Proc. C 790927 (1979) 315 [arXIV:1306.4669] [arXIV].

[74] P. Minkowski, $\mu \rightarrow e\gamma$ at a Rate of One Out of $10^9$ Muon Decays?, Phys. Lett. B 67 (1977) 421 [arXIV].

[75] M. Fukugita and T.T. Yanagida, Baryogenesis Without Grand Unification, Phys. Lett. B 174 (1986) 45 [arXIV].

[76] Particle Data Group collaboration, K.A. Olive et al., Review of Particle Physics, Chin. Phys. C 38 (2014) 090001 [arXIV].
[81] K. Enqvist and J. McDonald, \textit{B-ball baryogenesis and the baryon to dark matter ratio}, \textit{Nucl. Phys. B} \textit{538} (1999) 321 [hep-ph/9803380] [inSPIRE].

[82] S. Kasuya and M. Kawasaki, \textit{Q-ball formation through Affleck-Dine mechanism}, \textit{Phys. Rev. D} \textit{61} (2000) 041301 [hep-ph/9909509] [inSPIRE].

[83] S. Kasuya and M. Kawasaki, \textit{Q-ball formation in the gravity mediated SUSY breaking scenario}, \textit{Phys. Rev. D} \textit{62} (2000) 023512 [hep-ph/0002285] [inSPIRE].

[84] S. Kasuya and M. Kawasaki, \textit{Q-ball formation: obstacle to Affleck-Dine baryogenesis in the gauge mediated SUSY breaking?}, \textit{Phys. Rev. D} \textit{64} (2001) 123515 [hep-ph/0106119] [inSPIRE].

[85] M. Laine and M.E. Shaposhnikov, \textit{Thermodynamics of nontopological solitons}, \textit{Nucl. Phys. B} \textit{532} (1998) 376 [hep-ph/9804237] [inSPIRE].

[86] R. Banerjee and K. Jedamzik, \textit{On B-ball dark matter and baryogenesis}, \textit{Phys. Lett. B} \textit{484} (2000) 278 [hep-ph/0005031] [inSPIRE].

[87] K. Nakayama, F. Takahashi and T.T. Yanagida, \textit{A theory of extra radiation in the Universe}, \textit{Phys. Lett. B} \textit{697} (2011) 275 [arXiv:1010.5693] [inSPIRE].

[88] S. Weinberg, \textit{Goldstone Bosons as Fractional Cosmic Neutrinos}, \textit{Phys. Rev. Lett.} \textit{110} (2013) 241301 [arXiv:1305.1971] [inSPIRE].

[89] K.S. Jeong and F. Takahashi, \textit{Self-interacting Dark Radiation}, \textit{Phys. Lett. B} \textit{725} (2013) 134 [arXiv:1305.6521] [inSPIRE].

[90] M. Kawasaki, M. Yamada and T.T. Yanagida, \textit{Observable dark radiation from a cosmologically safe QCD axion}, \textit{Phys. Rev. D} \textit{91} (2015) 125018 [arXiv:1504.04126] [inSPIRE].

[91] E. Aver, K.A. Olive, R.L. Porter and E.D. Skillman, \textit{The primordial helium abundance from updated emissivities}, \textit{JCAP} \textit{11} (2013) 017 [arXiv:1309.0047] [inSPIRE].

[92] Planck collaboration, P.A.R. Ade et al., \textit{Planck 2015 results. XIII. Cosmological parameters}, \textit{arXiv:1502.01589} [inSPIRE].

[93] K.N. Abazajian et al., \textit{Neutrino Physics from the Cosmic Microwave Background and Large Scale Structure}, \textit{Astropart. Phys.} \textit{63} (2015) 66 [arXiv:1309.5383] [inSPIRE].

[94] W.L.K. Wu, J. Errard, C. Dvorkin, C.L. Kuo, A.T. Lee, P. McDonald et al., \textit{A Guide to Designing Future Ground-based Cosmic Microwave Background Experiments}, \textit{Astrophys. J.} \textit{788} (2014) 138 [arXiv:1402.4108] [inSPIRE].

[95] N.F. Bell, E. Pierpaoli and K. Sigurdson, \textit{Cosmological signatures of interacting neutrinos}, \textit{Phys. Rev. D} \textit{73} (2006) 063523 [astro-ph/0511410] [inSPIRE].

[96] M. Cirelli and A. Strumia, \textit{Cosmology of neutrinos and extra light particles after WMAP3}, \textit{JCAP} \textit{12} (2006) 013 [astro-ph/0607086] [inSPIRE].

[97] A. Friedland, K.M. Zurek and S. Bashinsky, \textit{Constraining Models of Neutrino Mass and Neutrino Interactions with the Planck Satellite}, \textit{arXiv:0704.3271} [inSPIRE].

[98] T.L. Smith, S. Das and O. Zahn, \textit{Constraints on neutrino and dark radiation interactions using cosmological observations}, \textit{Phys. Rev. D} \textit{85} (2012) 023001 [arXiv:1105.3246] [inSPIRE].

[99] M. Archidiacono, E. Calabrese and A. Melchiorri, \textit{The Case for Dark Radiation}, \textit{Phys. Rev. D} \textit{84} (2011) 123008 [arXiv:1109.2767] [inSPIRE].

[100] R. Diamanti, E. Giussarma, O. Mena, M. Archidiacono and A. Melchiorri, \textit{Dark Radiation and interacting scenarios}, \textit{Phys. Rev. D} \textit{87} (2013) 063509 [arXiv:1212.6007] [inSPIRE].

[101] M. Archidiacono, E. Giussarma, S. Hannestad and O. Mena, \textit{Cosmic dark radiation and neutrinos}, \textit{Adv. High Energy Phys.} \textit{2013} (2013) 191047 [arXiv:1307.0637] [inSPIRE].
[102] T. Chiba, F. Takahashi and M. Yamaguchi, *Baryogenesis in a flat direction with neither baryon nor lepton charge*, Phys. Rev. Lett. 92 (2004) 011301 [Erratum ibid. 114 (2015) 209901] [hep-ph/0304102] [INSPIRE].

[103] F. Takahashi and M. Yamaguchi, *Spontaneous baryogenesis in flat directions*, Phys. Rev. D 69 (2004) 083506 [hep-ph/0308173] [INSPIRE].

[104] F.L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton*, Phys. Lett. B 659 (2008) 703 [arXiv:0710.3755] [INSPIRE].

[105] K. Kajantie, M. Laine, K. Rummukainen and Y. Schröder, *The Pressure of hot QCD up to g6 ln(1/g)*, Phys. Rev. D 67 (2003) 105008 [hep-ph/0211321] [INSPIRE].

[106] H. Davoudiasl, R. Kitano, G.D. Kribs, H. Murayama and P.J. Steinhardt, *Gravitational baryogenesis*, Phys. Rev. Lett. 93 (2004) 201301 [hep-ph/0403019] [INSPIRE].

[107] A.D. Linde, *Relaxing the cosmological moduli problem*, Phys. Rev. D 53 (1996) 4129 [hep-th/9601083] [INSPIRE].

[108] F. Takahashi and M. Yamada, in preparation.

[109] K. Nakayama, F. Takahashi and T.T. Yanagida, *On the adiabatic solution to the Polonyi/moduli problem*, Phys. Rev. D 84 (2011) 123523 [arXiv:1109.2073] [INSPIRE].

[110] K. Nakayama, F. Takahashi and T.T. Yanagida, *Cosmological moduli problem in low cutoff theory*, Phys. Rev. D 86 (2012) 043507 [arXiv:1112.0418] [INSPIRE].