Computer simulation of coronas, glories and fogbows in atmospheric clouds and fogs

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Abstract. This paper deals with the development of software that allows one to compute the scattering phase functions of the optical radiation in polydisperse media consisting of spherical particles, based on the Mie scattering theory. The software also enables computer visualization of such optical phenomena as rainbows, glories and coronas. A special technique is used to significantly accelerate the integration over the radius of scattering particles and to increase the calculation rate of the scattering phase functions. Using the software developed, we have performed a detailed analysis of the scattering phase functions for a large number of well-known cloud and fog models, and have created computer images of optical phenomena (rainbows, glories, and coronas) that are inherent to the considered models.

1. Introduction
The software for calculating the phase functions and visualization of optical phenomena, presented in paper [1], was modified, which allowed us to increase the calculation speed by about 10 times while maintaining the accuracy of the calculations with a relatively small calculation error. To increase the calculation speed, an auxiliary array of averaged scattering phase functions is used instead of a large number of monodisperse scattering phase functions with a small step along the drop radius.

2. Algorithm for calculating the phase functions
The phase function $I_\lambda (\theta)$ for the distribution density of the drop radius $p(r)$ and the wavelength $\lambda$ is calculated by the following algorithm.

1. Using W Wiscombe’s algorithm [2], an array of scattering phase functions $G_\lambda (R_l, \theta_j)$ is calculated for monodisperse media with drops of the radius $R_l$, $R_l = il$, $i = 1, ..., N$, given in micrometers, for the scattering angles $\theta_j$, $j = 1, ..., s$.

2. An array of averaged scattering phase functions $g_\lambda (r_i, \theta_j)$, $i = 1, ..., n$, $j = 1, ..., s$, $n = \frac{N}{10}$, $r_i = 10il$, with a larger step of $10h$ along the drop radius is calculated by the following formulas:

$$g_\lambda (r_i, \theta_j) = \frac{1}{m} \sum_{k=K_i}^{K_2} G_\lambda (R_k, \theta_j), \quad l = 10i, \quad R_l = r_i, \quad i = 2, ..., n - 1,$$

$$K_1 = l - \frac{m - 1}{2}, \quad K_2 = l + \frac{m - 1}{2},$$

$$g_\lambda (r_1, \theta_j) = G_\lambda (R_1, \theta_j), \quad g_\lambda (r_n, \theta_j) = G_\lambda (R_N, \theta_j).$$
where \( m \) is an odd number that determines the size of a symmetric window, \( 1 \leq m \leq \frac{n-1}{2} \). For calculations near the boundary values of the interval, the algorithm uses a dynamically changing window size.

3. Using Boole’s rule for numerical integration, the values of the scattering phase function \( I_\lambda(\theta) \) are calculated as follows:

\[
I_\lambda(\theta) = \int_0^{R_{\text{max}}} g_\lambda(r, \theta)p(r) \, dr \approx
\]

\[
\approx \frac{2h}{45} \sum_{i=1,4}^{n-3} (7g_\lambda(r_{i-1}, \theta) + 32g_\lambda(r_i, \theta) + 12g_\lambda(r_{i+1}, \theta) + 32g_\lambda(r_{i+2}, \theta) + 7g_\lambda(r_{i+3}, \theta)),
\]

where \( i = 1, 4 \) means that the index starts from 1 and increases in increments of 4.

Figure 1 shows the values of the relative error

\[
\sigma = \left| \frac{\overline{g}_\lambda(\theta) - \overline{G}_\lambda(\theta)}{\overline{g}_\lambda(\theta)} \right|
\]

depending on the scattering angle \( \mu, \theta = \cos \mu \), for the Cloud C1 model. Here \( \overline{g}_\lambda \) is the result of the numerical integration over the array \( g_\lambda(r) \) and \( \overline{G}_\lambda \) is over the array \( G_\lambda(R) \).

In practice, the algorithm works well with the following parameters: \( h = 0.01, \ m = 51, \ N = 3000 \). For all the considered cloud and fog models, \( R_{\text{max}} = 30 \) is sufficient.

The optical properties of clouds and fogs are defined by the distribution of the water drop radius. The distribution of water drops in clouds and fogs is usually approximated by the modified gamma distribution [3]:

\[
n(r) = NA r^\alpha \exp(-Br^\gamma), \quad B = \frac{\alpha}{\gamma r_{\text{mod}}}, \quad r_{\text{mod}} = \left( \frac{\alpha}{B\gamma} \right)^{\frac{1}{\gamma}}.
\]

where \( N \) is a particle concentration (cm\(^{-3}\)), \( A \) is a normalizing constant, \( \alpha, B, \gamma \) are parameters controlling the shape of the distribution, \( r_{\text{mod}} \) is the modal radius. This function is used as the distribution density of the drop number in one cubic centimeter depending on the drop radius given in micrometers. Without the factor \( N \), this function is the probability density:

\[
\int_0^{+\infty} Ar^\alpha \exp(-Br^\gamma) \, dr = 1.
\]

Thus, \( N \) is the drop number of all sizes in one cubic centimeter.

3. Calculation results

Using the software developed, the scattering phase functions were calculated, and computer visualization of glories, fogbows and coronas was carried out for the well-known cloud and fog models from [4-8]. According to the calculation results, the angular radii of glories and fogbows were estimated (see Table 1).

The calculation results, in particular, show that the Cloud C4 and Radiation Fog (moderate) models have the most pronounced glory and corona, see figure 2.

For all the considered cloud and fog models, a fogbow is observed. For some models, weakly expressed secondary fogbows are observed, separated from the primary fogbows by Alexander’s dark band. Secondary fogbows are most pronounced for the OPAC Cumulus (maritime) and MODTRAN Cumulus models (see figure 3).

We should also note that for the Cloud C2, Cloud C3, Cloud C4, OPAC Stratocumulus (continental, polluted), OPAC Cumulus (continental, clean), OPAC Cumulus (continental, polluted), OPAC Cumulus maritime and MODTRAN Altostratus models, a dark band closer to the center of the primary fogbow is visible (see figure 4). For other models, such a band does not appear.
Table 1. Angular radii (in degrees) of glories and fogbows for cloud and fog models.

| Models of D Deirmendjian [4] | Glory | Primary fogbow | Secondary fogbow | B   | α   | γ   |
|-------------------------------|-------|----------------|------------------|-----|-----|-----|
| Cloud C1                     | 5     | 36             | -                | 1.5 | 6   | 1   |
| Cloud C2                     | 6     | 35             | -                | 0.04166 | 8  | 3   |
| Cloud C3                     | 11    | 33             | -                | 0.33333 | 8  | 3   |
| Cloud C4 (see note below)    | 12    | 35             | 3                | 0.33333 | 8  | 3   |
| OPAC models [5]              |       |                |                  |     |     |     |
| Stratus (continental)        | 5     | 37             | 60               | 0.938 | 5  | 1.05|
| Stratus (maritime)           | 4     | 37             | 58               | 0.193 | 3  | 1.30|
| Stratocumulus (continental, polluted) | 6.5 | 34 | - | 0.247 | 8 | 2.15 |
| Cumulus (continental, clean) | 5     | 36             | -                | 0.0782 | 5  | 2.16|
| Cumulus (continental, polluted) | 6.5 | 35 | - | 0.247 | 8 | 2.15 |
| Cumulus (maritime)           | 2     | 39             | 59               | 0.00713 | 4  | 2.34|
| MODTRAN models [6]           |       |                |                  |     |     |     |
| Stratus                       | 4.5   | 37             | -                | 0.6  | 2   | 1   |
| Stratus / Stratocumulus      | 5     | 37             | -                | 0.75 | 2   | 1   |
| Nimbostratus                 | 3.5   | 38             | 58               | 0.425 | 2  | 1   |
| Altostratus                  | 4     | 37             | 59               | 1.111 | 5  | 1   |
| Cumulus                      | 3.5   | 38             | 59               | 0.5  | 3   | 1   |
| Fog models [7, 8]            |       |                |                  |     |     |     |
| Advection Fog (heavy)        | 2.5   | 40             | 57               | 0.3  | 3   | 1   |
| Advection Fog (moderate)     | 2.5   | 40             | 57               | 0.375 | 3  | 1   |
| Radiation Fog (heavy)        | 5.5   | 36             | 61               | 1.5  | 6   | 1   |
| Radiation Fog (moderate)     | 12    | 33             | -                | 3    | 3   | 6   |

Note. For the Cloud C4 model, the same function \( n(r) \) is used as for the Cloud C3 model, but with a shift to the right by two units: \( n_{C4}(r) = n_{C3}(r - 2), r > 2 \).

In reality, the images of glories and fogbows can be very blurry and become almost invisible due to the multiple scattering of light in clouds. To obtain more realistic images of optical phenomena, we used Monte Carlo simulation of the angular distributions of the radiation scattered by the cloud layer, taking into account multiple scattering (for more details, see [9]). The results of computer visualization of coronas and glories generated by the cloud layers of different optical thicknesses are presented in figures 5 and 6 for the Cloud C4 model.

The software allows user to save simulated images of optical phenomena and computed scattering phase functions. Current versions of the software are freely available on the Internet [10].

![Figure 1](image-url) Figure 1. The relative error in calculating the scattering phase function for the Cloud C1 model for the wavelengths 380 nm (left) and 780 nm (right) during the transition to a larger integration step with preliminary smoothing.
Figure 2. Glories for the models: Cloud C4 (left) and Moderate Radiation Fog (right). The angular radius of the images is 15 degrees.

Figure 3. Primary and secondary fogbows for the models: OPAC Cumulus Maritime (left) and MODTRAN Cumulus (right). The angular radius of the images is 65 degrees.
Figure 4. For the Cloud C2 (left) and OPAC Cumulus maritime (right) models, a dark band is visible to the center of the primary fogbow. The angular radius of the images is 45 degrees.

Figure 5. Computer visualization of coronas for the Cloud C4 model based only on single scattering (left) and multiple scattering in the cloud layer of optical thickness 1 (middle) and 5 (right). The angular radius of the images is 15 degrees.

Figure 6. Computer visualization of glories generated by the Cloud C4 model ignoring multiple scattering (left) and with the account of the multiple scattering by the cloud layer of optical thickness 1 (center) and 5 (right). Alongside the glory in the center, you can see the fogbow with the angular radius of about 40 degrees. The angular radius of the images is 50 degrees.
Acknowledgments
The reported study was funded by RFBR, project numbers 19-31-90104, 18-01-00609.

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