Transverse scattering with the generalised Kerker effect in high-index nanoparticles

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All-dielectric resonant nanophotonics has attracted a lot of interest due to unique possibilities to control scattering of light in high-index dielectric nanoparticles and metasurfaces. One of the important concepts of dielectric meta-optics and nanophotonics is associated with the Kerker effect that drives the unidirectional scattering of light for nearly transparent Huygens’ metasurfaces. Here we suggest and demonstrate experimentally a novel, so-called transverse Kerker effect with nearly complete simultaneous suppression of both forward and backward scattering. We study this effect analytically and numerically, and also provide the proof-of-principle experimental verification for microwave frequencies. We also discuss the influence of substrate and extend this concept to dielectric metasurfaces that demonstrate zero reflection combined with strong localization of light in the surface plane and the field enhancement for nonlinear and sensing applications.
1. Introduction

Light scattering by a subwavelength particle is closely associated with an optically-induced multipolar response. Co-existence of magnetic and electric dipolar resonances makes it possible to achieve either constructive or destructive interference leading to remarkable scattering properties of subwavelength particles [1–5]. In particular, strong forward-to-backwards asymmetric scattering (often termed as “Kerker effect”) is achieved as a result of interfering electric and magnetic dipole modes [6,7], and/or quadrupolar modes (“generalized Kerker effect”) with appropriate phase relations [8–11]. Overlapping electric and magnetic multipoles of higher orders open a way for the effective shaping of light beams beyond the conventional forward and backward directions [2][10][12]. For example, single-element scatterers such as V-shape plasmonic antennas [13] or trimers [14] have been suggested for achieving side-directed scattering (perpendicular to the incident direction) through the breaking scatterers symmetry. Moreover, in Ref. [15] the conditions for the simultaneous cancellation of both forward and backward scattering are obtained in quasi-static approximation for a special case of radially anisotropic particles. However, to satisfy the power conservation requirements and to suppress the forward scattering at the same time, these particles should be active and possess a gain. We notice, however, that this result doesn’t contradict the optical theorem since an incomplete set of multipoles has been considered.

Some semiconductor materials such as silicon, germanium and gallium arsenide or phosphide possess a relatively high dielectric permittivity in the visible range allowing to excite multipole resonances of different orders in subwavelength particles. Moreover, in contrast to metals, these materials possess rather low losses [12], [13],[16–18], and they are commonly employed as materials for all-dielectric nanoantennas [4], metadevices [19,20], and metasurfaces [21,22].
Here we reveal the existence of a new effect in high-index nanophotonics characterized by the transverse scattering by a subwavelength particle with the simultaneous suppression of both forward and backward scattering. We consider silicon particles of the simplest forms (spheres and cubes) without any additional requirements on anisotropy, gain [15], non-symmetric shape [13] etc. For the first time to our knowledge, we obtain the essential conditions for the multipole contributions and reveal the basic concept for the lateral-only scattering pattern formation depending on the optical properties of a substrate. We present the proof-of-concept experiment in the microwave frequency range and demonstrate a nearly perfect agreement with our analytical (Mie theory [23]) and numerical (COMSOL Multiphysics [24]) calculations. The transverse scattering appears to follow a strong near-field localization inside the particles being similar to the recently found anapole states [25,26]. Both of the effects are of high demand for a variety of linear and nonlinear applications including four-wave mixing [27] and all-optical light modulation [28]. Moreover, metasurfaces consisting of such nanoparticles demonstrate the full transmission of light being of a great importance for antireflection coatings and Huygens’ meta-optics [22].

2. Simultaneous suppression of the forward and backward scattering

Let us start from the Cartesian multipole decomposition of a field scattered by an arbitrary particle being much smaller than the wavelength of an incident light. The surrounding medium is taken to be free space with relative permittivity \( \varepsilon = 1 \). The incident wave is linearly polarized along \( z \)-axis:

\[
E_{inc} = E_0 e^{ikz} x,
\]

the \( e^{-i\omega t} \) term is omitted, and \( k \) - is the wavenumber. The exact pattern of the angular distribution of the total scattered light is defined by the superposition of multipoles’ contributions, and it has the following form (limited to quadrupoles) [29]:

\[
E_{sca}(n) \equiv \frac{k^2}{4\pi\varepsilon_0} e^{ikr} \left( [n \times [p \times n]] + \frac{1}{c} [m \times n] + \frac{ik}{6} [n \times \nabla n] + \frac{ik}{2c} [n \times (\nabla n)] \right) .
\] (1)
where \( \mathbf{n} = \mathbf{r}/r \) is the unit vector from the particle’s center towards an observation point; \( c \) is the speed of light in free space. \( \mathbf{p}(\mathbf{m}), \hat{\mathbf{Q}}(\hat{\mathbf{M}}) \) are the electric (magnetic) dipole, and the electric (magnetic) quadrupole respectively.

From Equation 1 one can derive the conditions for the scattering suppression in both forward and backward directions simultaneously. Hereinafter we will consider them analytically, and compare with the numerical results for a high-index silicon nanosphere in the visible range. We find the angular distribution of the total differential scattering cross-section, here the dipole and quadrupole terms expanded only [30]:

\[
\frac{d\sigma^T}{d\Omega}(\theta) = \left( \frac{\alpha_p}{\varepsilon_0} + \frac{k^2}{4} \frac{\alpha_Q}{3\varepsilon_0} \cos \theta + \frac{1}{\varepsilon_0} \sum_{j \neq p,Q} A_j^E(k,\theta)\alpha_j \right) \\
+ \left( \alpha_m \cos \theta + \frac{k^2}{4} \alpha_M \cos 2\theta + \sum_{j \neq m,M} A_j^H(k,\theta)\alpha_j \right)^2
\]

(2)

where \( \theta \) is the polar angle and the power distribution is considered to be symmetrical in the azimuthal angle plane. \( A_j^E \) and \( A_j^H \) represent the contributions of higher order multipole moments, and the detailed derivation could be found elsewhere [30,31]. The particles are characterized by their electric and magnetic complex dipolar \( \alpha_p \), \( \alpha_m \) and quadrupolar \( \alpha_Q \), \( \alpha_M \) polarizabilities, consequently. We derive the particle polarizabilities by comparing Mie theory expansion expression with that of the scattering from a particle illuminated by plane wave in Equation 1 [31]. The induced multipole moments then could be expressed as [30]:

\[
\mathbf{p} = \alpha_p \mathbf{E}_{inc}; \quad \mathbf{m} = \alpha_m \mathbf{H}_{inc}; \quad \hat{\mathbf{Q}} = \alpha_Q \frac{\nabla \mathbf{E}_{inc} + (\nabla \mathbf{E}_{inc})^T}{2}; \quad \hat{\mathbf{M}} = \alpha_M \frac{\nabla \mathbf{H}_{inc} + (\nabla \mathbf{H}_{inc})^T}{2}.
\]

(3)
Now we proceed with the dipoles’ and quadrupoles’ contributions to the differential scattering cross-section, and neglect $A^E_j$ and $A^H_j$, for the sake of simplicity. Actually, it is quite simple to find a particle, where higher order multipoles are negligible, see, e.g. [2]. The Equation 2 takes the following form:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{k^4}{(4\pi)^2} \left( \left| \frac{\alpha_p}{\epsilon_0} \right|^2 + |\alpha_m| \cos^2 \theta + \frac{k^4}{16} \left| \frac{\alpha_Q}{3\epsilon_0} \right|^2 \cos^2 \theta + \frac{k^4}{16} |\alpha_M| \cos^2 2\theta \\
+ \frac{2}{\epsilon_0} Re\{\alpha_p\alpha_m^*\} \cos \theta + \frac{k^4}{24\epsilon_0} Re\{\alpha_q\alpha_M^*\} \cos \theta \cos 2\theta \\
+ \frac{k^4}{8} Re \left\{ \left( \frac{\alpha_p}{\epsilon_0} + \alpha_m \cos \theta \right) \left( \frac{\alpha_Q}{3\epsilon_0} \cos \theta + \alpha_M \cos 2\theta \right)^* \right\} \right).$$

(4)

Coherent contribution of the electric and magnetic dipoles make the radiation pattern to be directed mainly in forward or backward half-spaces depending on whether $\Re\{\alpha_p\alpha_m^*\}$ is positive or negative, respectively.[7] Note that $\alpha_p$ and $\alpha_m$ are complex values and the equality $\epsilon_0\alpha_m/\alpha_p$ becomes real only if the dipoles are in-phase or anti-phase. To obtain the total suppression of the backward scattering $|\theta| \geq 90^\circ$ (see Figure 1), the dipoles are to be in-phase and have the same polarzabilities amplitudes; for suppression of the forward scattering ($|\theta| \leq 90^\circ$) the dipoles must also have the same polarzabilities amplitudes, however the phase has to be shifted on $\pi$ (anti-phase case). These cases are well known as Kerker and anti-Kerker conditions.[32] We formulate Kerker condition as:

$$|\alpha_p/\epsilon_0| = |\alpha_m|, \ \text{arg}(\alpha_p) = \text{arg}(\alpha_m) + 2\pi n,$$

(5)

where $n$ is an integer number.
Figure 1. (a) Schematics of 3D transverse scattering pattern for cubic nanoparticles. Small forward scattering corresponds to a contribution of higher-order multipoles in accord with the optical theorem. (b). Formation of an ideal transverse scattering pattern. Electric dipole (ED) is in-phase with a magnetic dipole (MD) and an electric quadrupole (EQ) is in-phase with a magnetic quadrupole (MQ), while all coherent dipoles are in anti-phase with the coherent quadrupoles.

For the electric and magnetic quadrupoles contribution only, similar to the aforementioned case of the dipoles, the equality $\alpha_Q/3\varepsilon_0\alpha_M$ becomes real only when the complex magnitudes of $\alpha_Q$ and $3\varepsilon_0\alpha_M$ are either in-phase or anti-phase, and can be called quadrupolar Kerker-like condition:
\[ |\alpha_Q| = |3\varepsilon_0\alpha_M|, \quad \arg(\alpha_Q) = \arg(\alpha_M) + 2\pi n, \] (6)

where \(n\) is an integer number too.

Let us consider the total scattering in the upper and lower half spaces (Equation 2). Obviously, it is governed by the coupling terms \(\Re(\alpha_p\alpha_m^*), \Re(\alpha_Q\alpha_M^*),\) and \(\Re \left( (\varepsilon_0^{-1}\alpha_p + \alpha_m)(3^{-1}\varepsilon_0^{-1}\alpha_Q + \alpha_M) \right)^* \). The simultaneous application of the obtained conditions (5 and 6) for the dipole and quadrupole unidirectional scattering leads to the two possible scattering patterns depending on the phase relation between dipoles and quadrupoles given by:

\[ \arg\left[\frac{\alpha_p}{\varepsilon_0} + \alpha_m\right] = \pm \arg\left[\frac{\alpha_Q}{3\varepsilon_0} + \alpha_M\right]. \] (7)

The first one (with +) corresponds to the constructive interference of the dipoles’ and quadrupoles’ fields scattered forward paving a way for a stronger unidirectional scattering.

Generally, the inter-multipole coupling enhances the directionality and plays a key role for the backward scattering suppression. [8] Actually, we can generalise the conditions (5-7) to the whole set of multipoles, and it could be referred to as the generalized Kerker condition:

\[ \frac{d\sigma^T}{d\Omega}(\theta = 0^o) = 0. \] (8)

The second possible scattering regime (with –) in Equation 7 seems to be even more interesting. It corresponds to the case, when both forward and backward scattering are suppressed because of the destructive interference between the coherent combined dipoles and the coherent combined quadrupoles in the forward half-space. The dipole-quadrupole interaction conditions (5-7) in addition to condition in Equation 8 are also satisfy:

\[ \frac{d\sigma^T}{d\Omega}(\theta = 180^o) \cong 0. \] (9)
Unlike the generalized Kerker condition this kind of interaction leads to the formation of a transverse scattering pattern as it is shown in Figure 1. The side lobes of the quadrupoles form the scattering pattern of a mostly lateral shape. However, for real particles it is impossible to achieve such the pattern, because of the optical theorem connecting the total extinction cross-section of a particle and the forward scattered field. Therefore, the forward scattering cannot be completely eliminated, but significantly minimized with the help of the aforementioned conditions (5-9). Figure 1(b) schematically depicts the result of the interference of the radiation from dipoles and quadrupoles. Note that (Figure 1(b)) these diagrams are obtained artificially for the values of the multipole moments ideally corresponding to the conditions (5-9) and with zero values of other multipol moments.

The differential scattering cross-sections (the energy scattered either in forward or in backward directions) for a silicon spherical particle of 140 nm radius are shown in Figure 2(a). The conditions (6) and (7) are almost satisfied at the wavelength of 685 nm, and, as we have already noticed, the backward intensity drops to zero, while the forward intensity reaches its minimum at the same time. This rather simple approach describes the occurrence of the almost unidirectional scattering or lateral-only scattering pattern.

3. Proof of the concept experiment

Based on Maxwell equations scalability, we verify the previous theoretical discussion by measuring the extinction cross-section and the scattering pattern of a spherical particle in the microwave frequency range. To mimic the scattering properties of the considered silicon nanoparticles at microwaves we use MgO-TiO₂ ceramic spheres characterized by the dielectric constant ε=16 and dielectric loss factor of (0.00112 – 0.00117), measured at 9 - 12 GHz. [33] The spherical particle with the radius of a=7.5 mm is located in a microwave chamber for the measurements in the
Figure 2. (a) The forward ($\theta = 0$) and backward ($\theta = 180$) differential scattering cross-sections calculated for a silicon sphere of 140 nm radius in free space. The inset referring to 685 nm is the 2D presentation of the corresponding transverse scattering patterns: the green (blue) line corresponds to the plane of the incident electric (magnetic) field polarization. The comparison of the extinction cross-section calculated in the frame of the Mie theory (b) and the measurements of the 2D scattering pattern at the frequency $f = 7.85$ GHz, where the green (blue) line corresponds to the plane of the incident electric (magnetic) fields polarization (c). The results are obtained for the ceramic spherical particle with the dielectric permittivity of $16 + 0.001i$ and radius of 7.5 mm, respectively.
frequency range of 4-10GHz. To approximate a plane wave excitation, we employ a rectangular horn antenna (TRIM 0.75-18 GHz; DR) connected to the transmitting port of a vector network analyzer (Agilent E8362C). The ceramic sphere is located in the far-field of the antenna, at a distance of approximately 2.5 m, and the second horn antenna (TRIM 0.75-18 GHz; DR) is used as a receiver to observe the transmitted signal. Forward scattering is obtained from the transmission coefficient. The total extinction cross-section is extracted from the measured complex magnitude of the forward scattered signal by means of the optical theorem. [34] The measured extinction is compared with the theoretically obtained results (Mie theory) in Figure 2(b). To measure the 2D scattering diagram the experimental setup is slightly changed. The transmitting antenna and the ceramic sphere are fixed, whereas the receiving antenna moves around the spherical particle in the xz plane. The scattering cross-section patterns for both theoretical (Mie theory) and measured data in the xz plane are shown in Figure 2(c). We observe the lateral scattering to occur for the broadband off-resonance region from f = 7.6 GHz to f = 7.9 GHz.

4. Non-spherical subwavelength particles

Hereinafter we extend the obtained results and requirements to the more general case of non-spherical particles by the employment of the Cartesian multipole moments formalism [2][35], and prove the requirements on the example of cubic particles.

Under a plane wave illumination, Cartesian multipoles could be reduced to some nonzero components as $p = p_x\hat{x}$, $m = m_y\hat{y}$, $Q = Q_{xz}(\hat{x}\hat{z} + \hat{z}\hat{x})$, and $M = M_{yz}(\hat{z}\hat{y} + \hat{y}\hat{z})$. After the procedure similar to the one described in the previous section, the conditions for the transverse scattering pattern can be written in the form:

$$c p_x / m_y = 1; \ c Q_{xz} / 3 M_{yz} = 1; \ 2 c p_x / i k M_{yz} = -1.$$  

(10)
Figure 3. (a) The amplitudes and phases of the ratios from the Equation 10 for a silicon cubic nanoparticle with the edge of 250 nm. The black vertical line shows the wavelength corresponding to the transverse scattering $\lambda = 788$ nm. 2D (b) and 3D (c) scattering patterns at $\lambda = 788$ nm.

(e,f) Calculated electric field inside the nanoparticle with the transverse scattering pattern, $\lambda = 788$ nm in the xy and xz planes respectively.

As an example, silicon cubic nanoparticles will be considered hereinafter. Figure 3(a) shows the calculated absolute values of the ratios from the Equation 10 and their phases being the phase differences between the involved multipoles. At the target wavelength $\lambda=788$ nm (black vertical
line), one can see that the dipoles are in-phase (red solid line) with the nearly equal amplitudes (blue solid line) indicating the Kerker effect. The quadrupoles, on the other hand, have comparable values and $cQ_{xz} \setminus M_{yz} = 0.94$ (blue dashed line) and are in phase (red dashed line), which explains the generalized Kerker effect under the assumption that higher multipoles are negligible. The phase difference between the coherent dipoles and quadrupoles (red dotted line) is about $0.75\pi$. The corresponding 2D and 3D scattering patterns are shown in the Figures 3(b, c). Hence, there is the almost complete scattering suppression in the forward and backward directions. However, as we have already mentioned, the suppression of the forward scattering is not complete owing to the optical theorem. We demonstrate in Figures 3(e) and (f) the electric near-field in both yz and xz planes inside the cubic nanoparticle at the wavelength of the transverse scattering. The observed near-field enhancement accompanied by the strong scattering suppression is similar to the anapole states, where the dipole radiation is almost cancelled by the radiation of the electric toroidal moment. This effect of the light localization could be of a great interest for nonlinear applications.

5. **Effect of a substrate**

Next, we examine how a substrate influences the forward-backwards scattering suppression. In the presence of a substrate, the total field acting on a particle is the superposition of the incident, and the reflected fields including the fields produced by multipole moments and reflected back. This back action results in the sufficient modification of the multipole moments and reveals the existence of the substrate-induced magneto-electric coupling.[36] So, hereinafter let’s consider a nanoparticle situated on a substrate occupying the $z < 0$ half space and infinitely continuous in lateral directions. Then the total scattered field takes the form:
\[
E_{sca}^T(n) = \left( E_{sca}(\rho; z) + \vec{R}(r)E_{sca}(\rho; -z) \right) u(z)
\]

\[
+ \vec{T}(r)E_{sca}(\rho; -z \sqrt{1 - n_s^2 |\rho|^2 / r^2}) u(-z),
\]

where \( u(z) \) is the usual step function

\[
u(z) = \begin{cases} 
1, & z \geq 0, \\
0, & z < 0,
\end{cases}
\]

\( \vec{R}(r), \vec{T}(r) \) are the reflection and transmission tensors, respectively, defined as in the seminal book of Novotny and Hecht.

\( n = r / r \) is the unit vector from the particle’s center towards an observation point and \( \rho \) is the in-plane vector. The first term describes the field in the upper space (observation point \( r \)) as a superposition of the directly scattered waves, and the scattered waves

**Figure 4.** Scattering pattern evolution for the silicon nanocube with the edge side and the wavelength equal to 250 nm, and 788 nm, respectively. (a) yz plane, (b) xz-plane. The substrate refractive indices: \( n_s = 1 \) (blue line) \( n_s = 1.15 \) (red line) and \( n_s = 1.3 \) (yellow line).

reflected from the substrate; while the second term describes the fields transmitted into the substrate. It should be noted that the reflection and transmission tensors are derived for the fields
scattered by dipoles only and including the higher order multipoles in Equation 11 is an approximate approach for the far-field region. The components $p_x, m_y, Q_{xz}$, and $M_{yz}$ are assumed to be dominant components. Then the condition for the complete backward scattering suppression is satisfied at (see appendix and all details can be found in ref.[9]):

$$p_x + \frac{k}{2ic} M_{yz} = -\frac{1 + r_p e^{ika}}{1 - r_p e^{ika}} \left( \frac{m_y}{c} + \frac{k}{6i} Q_{xz} \right),$$

(13)

where $r_p$ is the parallel-polarization Fresnel coefficient, and the complete suppression of the forward scattering takes place when

$$p_x + \frac{k}{2ic} M_{yz} = \frac{m_y}{c} + \frac{k}{6i} Q_{xz}. $$

(14)

The fact that multipoles in the Equations. 13 and 14 are induced also by the reflected fields imply some limitations on the forward scattering, and the previously observed suppression no longer exists. Even despite that it is theoretically possible to achieve the simultaneous forward-backward suppression for a certain wavelength, the scattered field reflected from a substrate is angle dependent and hence it deforms the scattering pattern. Now let’s examine the scattering pattern of a Si nanocube placed on the substrates with the refractive indices $n_s = 1, n_s = 1.15$, and $n_s = 1.3$ (see Figure 4). One can see that the presence of the substrate causes the scattering enhancement together with bending the side lobes to upper half-space.

6. Metasurface consisting of identical particles with the transverse scattering pattern

Now, let us consider a 2D regular metasurface consisting of the cubic particles, for which it is possible to fulfill the condition (10) for the transverse scattering pattern in the visible range. Figure 5(a) depicts the schematic illustration of a square array of the identical cubic scatterers placed in
free-space and illuminated with a normally incident plane wave (in the Figure 5(a) we show a Gaussian beam instead a plane wave and a substrate only for the sake of better visibility).

In the far-field region, the system’s reflection (for normal incidence) is determined by the effective polarizabilities and multipole moments of a central particle[38] taking into account the interaction with all other particles in the metasurface. However, this interaction appears to be rather weak even for periods less than the wavelength for silicon nanoparticles [39], which allows us to suggest that the obtained conditions (10) will not be ruined. We have performed full-wave numerical simulations [24] for the infinite square arrays of the considered silicon nanocubes with the following periods: 400 nm, 500 nm and 700 nm. Figure 5(b) shows the ratios (10) obtained for a standalone particle, and presented in Figure 3. Since the cube polarizabilities are affected by the inter-particle coupling, the multipole moments of the nanocube within the array still satisfies the transverse scattering pattern conditions (10). In the wavelength range 787-793 nm, the coherent dipoles are nearly in-phase (red solid line), quadrupoles are in-phase too (red dashed line), but they are in anti-phase with each other (red dotted line). With the close to unity amplitude ratios (blue solid and dashed lines), the nanocube array shows a nearly perfect simultaneous forward and backward scattering suppression, just like for the standalone particle (Figure 3). Therefore, in this wavelength range, the metasurface reflection is close to zero (Figure 5(b) background green area), despite the fact that the light interacts with the structure with exciting different multipole moments.

In contrast to the well-known Huygens metasurfaces [22] the presented nanostructure doesn’t scatter anything in the forward direction being nearly invisible. Further we analyze the effect of the array’s period on the optical properties of the metasurface. Figure 5(c) shows that all the resonance change their spectral positions and amplitudes. The most interesting observation is that the zero-reflection region broadens with the increasing the inter-particle distance.
Figure 5. (a) Schematics illustrating a square array of the particles with the transversal scattering pattern (we show a Gaussian beam instead a plane wave and a substrate only for the sake of better visibility). (b) The ratios and phases of condition (10) for the cube with 250 nm edge in the square nanostructure with the period $p = 500 \, \text{nm}$, and the corresponding reflection coefficient (the gray line filled with green). (c) The reflection coefficient from the 2D square lattices of Si cubes with 250 nm edge and periods: $p = 400 \, \text{nm}, 500 \, \text{nm}$ and 700 nm.

7. Conclusions

We have formulated the conditions for the transverse scattering of light from high-index subwavelength nanoparticles of different shapes with the simultaneous suppression of both forward and backward scattering. This unusual behavior takes place when the in-phase electric and magnetic dipoles are out-of-phase with the corresponding in-phase quadrupoles, and all of them have comparable amplitudes. This effect can occur in off-resonance scattering regions of the
spectrum. Employing the Mie theory, we have obtained the conditions for the simultaneous suppression of scattering in both forward and backward directions, and have employed the multipole expansion to generalize the conditions for arbitrary subwavelength particles, with a cubic silicon nanoparticle studied in detail as an example. Experimental measurements in the microwave spectral range proved the concept for the example of a ceramic sphere in a free space with a good agreement between theoretical results and experimental data. We have studied the effect of a substrate on the transverse scattering, and revealed analytically and numerically the electromagnetic inter-multipole coupling deforming the side scattering. Finally, we have studied metasurfaces consisting of nanoparticles with the transverse scattering patterns. We have shown periodicity-dependent zero reflection of such metasurfaces corresponding to the similar conditions for isolated particles. In contrast to Huygens’ metasurfaces, the considered metasurfaces scatter neither forward nor backward being almost invisible for incident light. Our new findings pave the way towards novel designs of nanoantennas and nanostructured coatings circuitry. The discovered effect can be utilized for achieving compact and efficient beams splitting, bending and switching of light owing to controllable formation of scattering patterns. Furthermore, high local field enhancement in the plane of metasurface is valuable for shaping nonlinear interactions and sensing.

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**Keywords:** Kerker effect, all-dielectric nanophotonics, meta-optics, nanoparticle arrays, metasurfaces, multipolar expansion

**Appendix**

**Light reflection and transmission tensors near a substrate**

We list the transmission and reflection tensors that asymptotically describe the far field scattered light from particle near a planar substrate. [37]. The particle placed in the upper half space \((z > 0)\) and light incidence is perpendicular to the interface where \(r = (x, y, z)\) and \(\rho = (x, y)\), then:

\[
\vec{R}(r) = \begin{bmatrix}
-r_p \frac{x^2 z^2}{\rho^2 r^2} + r_s \frac{y^2}{\rho^2} & -r_p \frac{xy z^2}{\rho^2 r^2} - r_s \frac{xy}{\rho^2} & -r_p \frac{xz}{r^2} \\
-r_p \frac{xyz^2}{\rho^2 r^2} - r_s \frac{xy}{\rho^2} & -r_p \frac{x^2 z^2}{\rho^2 r^2} + r_s \frac{x^2}{\rho^2} & -r_p \frac{yz}{r^2} \\
-r_p \frac{xz}{r^2} & -r_p \frac{yz}{r^2} & r_p \left(\frac{x^2 + y^2}{r^2}\right)
\end{bmatrix}
\]  

(A.1)

\[
\vec{T}(r) = n_s \begin{bmatrix}
-t_p \frac{x^2 z^2}{\rho^2 r^2} + t_s f \frac{y^2}{\rho^2} & -t_p \frac{xy z^2}{\rho^2 r^2} - t_s f \frac{xy}{\rho^2} & -t_p n_s f \frac{xz}{r^2} \\
-t_p \frac{xyz^2}{\rho^2 r^2} - t_s f \frac{xy}{\rho^2} & -t_p \frac{y^2 z^2}{\rho^2 r^2} + t_s f \frac{x^2}{\rho^2} & -t_p n_s f \frac{yz}{r^2} \\
t_p \frac{xz}{r^2} & t_p \frac{yz}{r^2} & t_p n_s f \left(\frac{x^2 + y^2}{r^2}\right)
\end{bmatrix}
\]  

(A.2)
with \( f = z/\sqrt{r^2 - n_s^2 \rho^2} \), \( \rho = \sqrt{x^2 + y^2} \),

where \( n_s \) is the substrate refractive index. The Fresnel reflection and transmission coefficients for
\( r_p,r_s,t_p \) and \( t_s \) for p- and s-polarized light under normal incidence, are given as

\[
r_p = -r_s = \frac{n_s - 1}{n_s + 1},
\]

\[
t_p = \frac{2}{n_s + 1}, t_s = \frac{2}{1 + n_s}.
\]

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