Elastic meson production and Compton scattering

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Abstract
I discuss recent progress in the theory of exclusive meson production and Compton scattering, focusing on hard-scattering factorization and on the dipole formalism.

1 Introduction
In this talk I will discuss selected issues in elastic meson production and Compton scattering. Given limited time and space I cannot review all activity in the field since the last Ringberg meeting two years ago [1]; in particular I will say little about heavy-meson production and nothing about odderon physics. My first topic is the dipole description of diffractive processes and its relation with hard-scattering factorization. More on these topics can be found in [2, 3, 4]. A second part is devoted to scaling and its interplay with the helicity structure of hard-scattering processes, a topic where there is now a variety of data. In a third part, I focus on deeply virtual Compton scattering, i.e., the process $\gamma^* p \rightarrow \gamma p$ at large $Q^2$ and small $t$. I will give short summaries at the end of each part.

2 Two pictures
There are two rather different ways of viewing a process like $\gamma^* p \rightarrow \rho p$ or $\gamma^* p \rightarrow \gamma p$. Each comes with its own formalism, region of validity, and its way of organizing the

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scattering amplitude into different building blocks. The dipole picture is a high-energy approximation. Its ingredients, as shown in Fig. 1a, are light-cone wave functions $\psi_\gamma$, $\psi_\rho$ giving the amplitude for the virtual photon or meson to fluctuate into a $q\bar{q}$-pair, and the amplitude $A_{q\bar{q}p\rightarrow q\bar{q}p}$ of this $q\bar{q}$-pair to scatter elastically off the proton. In the high-energy approximation one often retains only the imaginary part of $A$ and replaces it with the total $q\bar{q}p$ cross section $\sigma_{q\bar{q}p}$ via the optical theorem. Hard-scattering factorization, shown in Fig. 1b, is valid in the limit of large photon virtuality $Q$. Its building blocks are a kernel describing the hard scattering between quarks, gluons, and photons, the $q\bar{q}$ distribution amplitude $\varphi_\rho$ of the meson, and a generalized quark or gluon distribution $f_{q,g}$ in the proton.

2.1 Variables and their roles

Let us look in some detail at the variables appearing in the respective building blocks of the two formalisms. The dipole cross section depends on one variable related with the scattering energy. There is an ongoing debate on whether the appropriate choice for this is Bjorken’s scaling variable $x_B$ or rather the $\gamma^*p$ c.m. energy $W^2$. $\sigma_{q\bar{q}p}$ further depends on the transverse size $r$ of the $q\bar{q}$ dipole. Note that $r$ is conserved by the interaction in the high-energy limit, unlike its Fourier conjugate, the relative transverse momentum $k_T$ in the $q\bar{q}$ pair.

Generalized parton distributions depend on two longitudinal momentum fractions, which can be parameterized symmetrically as shown in Fig. 1b. Here $\xi = x_B/(2 - x_B)$ is determined by the external kinematics, whereas $x$ is a loop variable, which needs to be integrated over and thus has no analog in the dipole cross section. Note that generalized parton distributions do not depend on the transverse momentum $k_T$ of the quarks or gluons in the proton. This is because in hard-scattering factorization one makes a collinear approximation, i.e., one sets $k_T = 0$ in the hard-scattering kernel. The loop integral over $k_T$ can then be performed for the lower blob in Fig. 1b alone and gives the
parton distribution. The upper limit of this integral is taken of order $Q$ and provides the factorization scale of $f_{q,g}$.

In the same fashion one makes the collinear approximation for the relative transverse momentum $k_T$ in the $q\bar{q}$ pair. The corresponding loop integral is then performed for the meson wave function alone and gives the distribution amplitude $\varphi_{\rho}$ at a factorization scale of order $Q$. Expressed in transverse position space, this procedure means that one is sensitive only to the meson wave function for $q\bar{q}$ separations $r$ up to order $1/Q$.

Both the dipole cross section and the generalized parton distributions depend on the invariant momentum transfer $t$, which in both pictures may be replaced with the impact parameter $b$ after a Fourier transform in transverse space. The simplest type of ansatz for the $t$ dependence often made in models is that it factorizes, e.g., $\sigma(x_B, r, t) = \sigma(x_B, r) e^{Bt/2}$ or $f(x, \xi, t) = g(x, \xi) F(t)$ with a constant slope parameter $B$ or a general form factor $F(t)$. While this is a convenient first approximation it misses important physics. At small $x_B$ the steepening of the $t$ dependence with energy, i.e., shrinkage, has recently been advocated as deeply connected with confinement [8]. In generalized parton distributions, this phenomenon is reflected by a correlation between the dependence on $t$ and the longitudinal momentum variables $x$ and $\xi$. An intuitive picture can be gained when going to impact parameter $b$, which describes the transverse position of the struck parton in the proton [9]. Representing the parton distribution as the overlap of light-cone wave functions for the initial and final hadron [10], shrinkage is then seen as a nontrivial dependence of the transverse distribution for partons with different longitudinal momentum in the target. Such a picture is not restricted to small $x_B$, and several models for generalized parton distributions at larger $x$ and $\xi$ indicate that a factorizing $t$-dependence is too simple [11, 12], which reflects a nontrivial interplay between transverse and longitudinal degrees of freedom in the hadron.

### 2.2 Regions of validity

The hard-scattering formalism is valid in the limit where the scale $Q$ is large, it gives the amplitude up to corrections in powers of $1/Q$ times logarithms of $Q^2$. This can be proven using general methods as explained in [3, 13]. The hard scale $Q$ may be replaced by a large internal quark mass, for instance in photoproduction $\gamma p \rightarrow J/\Psi p$, although no formal proofs of factorization exist for this case. Hard-scattering factorization can be applied for $x_B$ small or not, and thus provides a link between high-energy phenomena at H1 and ZEUS and physics at HERMES energies. The dipole formalism, on the other hand, requires $x_B$ to be small, but goes beyond the leading power in $1/Q$. Again, one may replace the scale $Q$ from a highly virtual photon with the large quark mass in quarkonium production.

In the joint limit of large $Q$ and small $x_B$, more precisely to leading power $1/Q$ and to double leading logarithm, $\log Q^2$ and $\log(1/x_B)$, both pictures claim validity and their predictions coincide. In this approximation the dipole cross section at small separation $r$ is proportional to the gluon density, $\sigma_{q\bar{q}p} \sim r^2 \overline{\sigma}(\overline{\mathbf{r}}, \overline{O})$, where $\overline{\mathbf{r}}$ is of order $x_B$ and the factorization scale $\overline{O}$ of order $Q$ or $1/r$. In a leading logarithmic approximation we cannot
make the statement “of order” more precise. To the same accuracy we can further not distinguish between the usual gluon density and the generalized one $g(x, \xi; Q)$ at some $\overline{x} \sim \xi$. Numerically however, the difference between the gluon density, say, at $Q$ and at $Q/2$, or at $x_B$ and at $x_B/2$ is quite important in the low-$x_B$ regime, and many attempts have been made to improve on the simple relation $\sigma_{q\bar{q}p} \sim x_B g(x_B, Q)$, making more refined choices for $Q$ and $x_B$, and incorporating the generalized gluon distribution. This is somewhat reminiscent of choosing the scale of $\alpha_s$ in a fixed-order perturbative calculation: while it makes sense from a phenomenological point of view, one should not forget that one is asking a question to which strictly speaking there is no answer at the established accuracy of the calculation.

In contrast to hard-scattering factorization, it is not clear for the dipole formalism up to which parametric accuracy it is valid. It does emerge as a description in the leading $\log(1/x_B)$ approximation, and has been studied in connection with the BFKL pomeron \cite{14, 15}. Whether the dipole picture can provide a systematic description of the scattering amplitude beyond the leading $\log(1/x_B)$ is not known and subject of ongoing studies in the connection with the next-to-leading logarithmic corrections to BKFL \cite{3}. If the picture persists at that accuracy, it will most probably need to be extended in several aspects. Beyond leading $\log(1/x_B)$ the dipole cross section can depend on the longitudinal momentum fraction $z$ of the quark in the $q\bar{q}$ dipole. In general, the transverse dipole size will not be conserved any more \cite{13}. Also, more complicated objects than just a $q\bar{q}$ dipole such as $q\bar{q}g$ states will have to be considered as scattering off the target \cite{2, 16}.

Given the simplicity and versatility of the dipole formalism in the form of Fig. 1a, it is tempting to extend it beyond the strict limits where it has been derived so far. Incorporating the generalized gluon distribution mentioned above goes beyond the leading $\log(1/x_B)$, as does the inclusion of the real part of the dipole scattering amplitude $A_{q\bar{q}p \to q\bar{q}p}$, which is subleading in $\log(1/x_B)$. A perhaps even bolder extension is to try and describe $\sigma_{q\bar{q}p}$ at large dipole size $r$, where the connection between dipole cross section and gluon density becomes unclear, and where perturbation theory ceases to be valid. In the same direction goes the extension in $Q$ down to the photoproduction limit $Q = 0$. The wave function of a real photon is nonperturbative so that at present one has to model it, as one does for meson wave functions. One should however be aware that for a purely soft transition such as $\gamma \to \rho$ the contribution from higher Fock states in the photon is no longer suppressed by a small $\alpha_s$ compared with the $q\bar{q}$ component, and that a Fock state expansion on current quarks and gluons may not be practical at all. At this point one must invoke further arguments, e.g. along the lines of \cite{17} or of \cite{18}.

### 2.3 Applications

There is one particularly interesting phenomenon that can readily be incorporated into the dipole formalism while clearly going beyond the leading-twist physics of hard-scattering factorization. This is the particular correlation of the $r$ and $x_B$ dependence of $\sigma_{q\bar{q}p}$ termed saturation, discussed in more detail in \cite{4}.

One of the strategies pursued is to take a particular model of the dipole cross section,
compare its results to data, and see if one is sensitive to saturation effects in $\sigma_{q\bar{q}p}$. The dipole model has been applied to light and heavy vector meson production in [19], and $J/\Psi$ photoproduction was studied in [20] and [21]. It is interesting to note that the last two studies both achieve agreement with the data, but with different physics mechanisms. Whereas the main ingredient ensuring a good description in [20] were shadowing corrections, the dominant effect in [21] came out to be the choice of factorization scale in the gluon density when expressing the dipole cross section at small $r$. Given our incomplete ability to reliably calculate either of these effects, I conclude that with present theory and with the kinematical range of present data the relevance of saturation cannot be firmly established.

The reverse approach has been followed in [22], where an attempt was made to reconstruct the dipole cross section from $\rho$ production data. Unfortunately, it was found that with the $t$-range of the data no reliable extraction of $\sigma_{q\bar{q}p}$ was possible for small impact parameters $b$, where saturation effects can be expected to set in first.

In conclusion, hard-scattering factorization and the dipole formalism are valid in different kinematical regimes, but these regimes overlap and there the two pictures give complementary descriptions. The dipole picture provides a versatile framework for describing diffractive reactions, and many studies are trying to refine it with quite good phenomenological success. What we are however lacking at this time is a theoretical understanding of what the limits of applicability are for this formalism and whether such refinements can be put on a systematic basis.

3 Scaling and helicity

3.1 Scaling

I focus now on the hard-scattering formalism. To remind us, the factorization theorem, e.g. for $\rho$ production states that the amplitude for $\gamma^*_L p \rightarrow \rho_L p$ scales like $1/Q$ times logarithmic corrections in $Q^2$, in the limit of large $Q^2$ and at fixed $t$ and $x_B$. The latter point is important to remember when looking at plots of the $Q^2$ dependence integrated over a fixed interval not in $x_B$ but in the scattering energy $W$. While this is rather natural from the point of view of the experimentally accessible phase space, it does not allow one to directly check the predicted scaling laws, except for quantities whose dependence on $W$ is rather flat. Notice also that the logarithmic corrections to the predicted power behavior can be large at small $x_B$. They certainly are for the inclusive structure function $F_2$—one would not have discovered Bjorken scaling from the HERA small-$x_B$ data. A fit to preliminary data on the cross section $\sigma_L$ for longitudinal $\rho$ production, giving a $Q^2$ behavior like $(Q^2 + m_p^2)^{-n}$ with $n = 1.89 \pm 0.03$ for $Q^2 > 2$ GeV$^2$ [23] and $n = 2.21 \pm 0.03$ for $Q^2 > 5$ GeV$^2$ [24], is by itself not in contradiction to the scaling prediction that $\sigma_L$ should go like $1/Q^6$ times logarithmic terms.

On the theory side we are far from a systematic understanding of the $1/Q^2$ power corrections in the leading amplitudes such as $\gamma^*_L p \rightarrow \rho_L p$, $\gamma^*_L p \rightarrow \pi p$, $\gamma^*_T p \rightarrow \gamma p$. There
are however estimates based on particular mechanisms. Recently the effects of finite parton $k_T$ in the hard scattering part of these processes were estimated in [25] and found to be rather substantial in the case of meson production. For DVCS, the power corrections due to the hadronic component of the outgoing real photon were estimated in [26] using vector dominance, giving contributions of order 10% to 20% at amplitude level for the kinematics of the H1 and ZEUS data [27].

3.2 Hadron helicity selection

Factorization theorems only tell us how to calculate selected helicity amplitudes for a process, but they do include the statement that all others are suppressed by powers of $1/Q$. Let us review the general ingredients that go into this helicity selection at large scales. The first is the collinear approximation for the $q\bar{q}$ pair forming a meson, described in section 2.1. In this approximation the quark and antiquark leaving the hard scattering process are strictly collinear, so that the spin along their direction of motion is only due to their helicities. Because of angular momentum conservation, the helicity of the produced meson must be the same, i.e., the sum of the $q$ and $\bar{q}$ helicities. This no longer holds at the level of $1/Q$ suppressed terms, where one takes $k_T$ in the hard scattering into account: then the $q\bar{q}$ pair can carry orbital angular momentum. Note that at the same level of accuracy one must also consider the $q\bar{q}g$ Fock state of the meson, where the gluon carries helicity in addition to the quarks. In transverse position space the collinear approximation means that one neglects $1/Q$ compared with the characteristic distance over which the meson wave function varies significantly. For a photon with its pointlike component, one cannot make this approximation. In fact, photons are directly attached to the hard scattering subprocess and one does not set to zero the relative $k_T$ of the $q\bar{q}$ pair they couple to. The hadronic component of the photon, which may contribute at the level of power corrections, can however be treated in a similar way as a meson, with corresponding light-cone wave functions and distribution amplitudes.

The second ingredient for helicity selection is that in a hard-scattering subprocess the chirality along a light quark line is conserved, since the light quark masses are neglected to the accuracy in question. Notice that this is not applicable for heavy quarks, unless $Q$ is much larger than their mass. Chirality breaking due to the axial anomaly does not spoil this type of argument since it does not affect hard-scattering kernels [28].

Consider now $\rho$ electroproduction at small $x_B$, which is dominated by the hard subprocess $\gamma^* g \rightarrow q\bar{q} g$. As shown in Fig. 3 the helicities of the $q\bar{q}$ pair forming the $\rho$ must be antialigned, so that longitudinal $\rho$ production is leading at large $Q^2$. To make a transverse $\rho$ costs a factor of $1/Q$ in the amplitude. Scaling thus predicts that the ratio $R = \sigma_L/\sigma_T$ of longitudinal and transverse cross sections should be linear in $Q^2$ at large $Q^2$, but again only up to logarithmic effects. If we strictly demand $R \propto Q^2$ we are further assuming that these effects, where they are strong in the cross section, should be similar for both polarizations. This need not be the case, and the calculation for small $x_B$ of Martin et al. [29] for instance finds them clearly different, although not much. Data at moderate $x_B$ are interesting in this context, because there one would not expect large logarithmic
Figure 2: Formation of a longitudinal $\rho$ from the hard-scattering process $\gamma^* g \rightarrow q\bar{q} g$.

effects in $Q^2$. Unfortunately, the existing data does not cover a large span in $Q^2$ for values above which one may think of applying hard-scattering arguments. Given the still large statistical errors in the high-energy data [23, 24] I find it difficult to see whether $R$ is approximately linear in $Q^2$ for values above, say, 2 to 3 GeV$^2$. The selection of longitudinal $\rho$ polarization should also hold in the subprocess $\gamma g \rightarrow q\bar{q} g$ at large momentum transfer $t$ between the real photon and the $q\bar{q}$ pair, which is believed to dominate the process $\gamma p \rightarrow \rho Y$ where the proton dissociates into a hadronic system $Y$. The preliminary results from ZEUS [23, 30] are thus puzzling, since transverse $\rho$ clearly dominate even up to $|t|$ of several GeV$^2$. Ivanov et al. [31] have suggested that the reason might be a numerically large power correction due to the hadronic part of the photon. This part has a wave function component where the $q$ and $\bar{q}$ have equal helicities, so that according to our the above arguments they will form a transverse $\rho$ after the hard scattering. It will be interesting to see if this mechanism can explain the $t$-dependence and normalization of the data at large momentum transfer.

3.3 Photon helicity selection

The selection of photon helicity comes about in a slightly different way than for hadrons. Going back to the example of $\rho$ electroproduction, we recall that in the hard-scattering kernel one also makes a collinear approximation for the partons coming from the proton side, and further neglects $t$ compared with $Q^2$. It then follows that the partons from the proton transfer no spin along the direction of the $\gamma^*$ and $\rho$ momenta, except if they change their helicity. Generalized quark and gluon distributions with helicity flip have indeed been studied, as well as the helicity amplitudes in DVCS and meson production where they contribute [32, 33]. Except when such distributions can contribute, one obtains $s$-channel helicity conservation with the above arguments, and since in $\rho$ electroproduction the meson is longitudinal at large $Q$, the photon must be, too. The same holds for the production of a spin-zero meson such as a $\pi$. At the level of power corrections, this selection rule fails since one can have transfer of orbital angular momentum from the transverse momentum of the partons, or of helicity from additional partons exchanged between the proton and the hard-scattering subprocess.

Let me add that the inclusion of parton $k_T$ in the hard scattering does not automatically lead to a change of helicity between the photon and meson. In the high-energy or $k_T$-factorization scheme, for instance, one picks up the leading $\log(1/x_B)$ terms in the amplitude and does include gluon $k_T$ in the hard scattering, but the contribution provid-
ing the leading log(1/x_B) does not transfer any angular momentum. In this case one also retains the transition $\gamma^*_T p \to \rho_T p$ at high energy, although it is subleading in the large-$Q^2$ limit.

Going back to moderate energies, the HERMES data for exclusive $\pi^+$ electroproduction \cite{34,35} on a longitudinally polarized target shows a clear sin $\phi$ signal in the distribution of the angle $\phi$ between the lepton and hadron planes in the target rest frame. The effect is too large to be explained as due to the small transverse spin of the target relative to the exchanged $\gamma^*$, so that at least part of it must come from the interference between amplitudes with transverse and longitudinal polarization of the intermediate $\gamma^*$. We can infer that both amplitudes which are leading and nonleading in the large $Q$ limit are visible in the kinematics of the experiment. Furthermore, not all possible combinations of such interference terms are large. The single-spin asymmetry data shows no indication of a cos $\phi$ modulation in the unpolarized cross section appearing in its denominator. Such a term would go with the real part of an interference term averaged over the initial proton helicity. The observed sin $\phi$ signal involves the imaginary part of the interference term for the difference of the two target helicities. We can at present not decide whether the cos $\phi$ term is small because of the phase between the interfering amplitudes or because of their dependence on the proton spin. Future data, also with transverse target polarization, should bring us closer to understanding the pattern of the different helicity transitions.

While there is at present no theory description of power suppressed helicity amplitudes in pion production, the situation is better for DVCS. The final state photon is transverse, so that the initial one must also be transverse to leading order in 1/Q. Transitions suppressed by just one power in 1/Q involve a $\gamma^*_L$, and we have now a systematic description of these to leading order in $\alpha_s$ \cite{36}, closely following the analog of the spin dependent structure function $g_2$. Broadly speaking, they are due to two types of contributions. One involves the same handbag diagrams that give the leading amplitude with a $\gamma^*_T$, see Fig. 3a, but with one unit of orbital angular momentum transferred by the quarks as described above. Using the equations of motion for the quark fields, the corresponding proton matrix elements can be reduced to the same generalized parton distributions that describe the leading amplitude. This is the contribution retained in the so called Wandzura-Wilczek approximation. The other one, shown in Fig. 3b, involves

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{fig3.png}
\caption{a) Quark handbag diagram for DVCS. b) Diagram with an additional exchanged gluon.}
\end{figure}
the exchange of two quark lines and an additional gluon between the proton and the hard scattering; it comes with corresponding twist-three parton distributions.

Information on the different photon helicity amplitudes is again contained in the $\varphi$ dependence of the $ep \to ep\gamma$ cross section. At sufficiently large $Q^2$, a $\sin \varphi$ distribution of the lepton polarization asymmetry is due to a $\gamma_T^*$ and a $\sin(2\varphi)$ is the signature of a $\gamma_L^*$, up to at least partly calculable corrections in $1/Q$ \cite{37}. The data from HERMES \cite{35,38}, and also from a yet lower energy experiment at Jefferson Lab \cite{39} is consistent with a sole $\sin \varphi$, so that we do not seem to have a signal for longitudinal $\gamma^*$ polarization in these kinematics.

Let me in this context point out a noteworthy difference between Compton scattering and meson production. In the photoproduction limit $Q^2 \to 0$, the transition amplitude with a $\gamma_L^*$ has to vanish like $Q$ because of gauge invariance, so that $\gamma_T^*$ dominates at very small $Q$. In Compton scattering this is also the polarization state dominating at very large $Q$, where the $\gamma_T^* p$ amplitude goes to a constant and the $\gamma_L^* p$ one falls like $1/Q$. The general pattern is thus the one of Fig. 4a, where for simplicity I ignore the proton spin dependence. From general principles we cannot say where the longitudinal amplitude has its maximum and how large it is. There is no reason why it should be small compared with the transverse one there, it might even be bigger and cross the curve for transverse photons twice. This depends on how fast the $\gamma_L^* p$ amplitude rises at small $Q$ and at which $Q$ it starts falling off again. For meson production, say for $\gamma^* p \to \pi p$, the situation is different. Here it is the $\gamma_L^*$ that dominates in the limit of large $Q$, so that the curves for $\gamma_L^*$ and $\gamma_T^*$ have to intersect at some value of $Q$. We do not know where this is for pion production, and whether it is before or after the amplitude for $\gamma_L^*$ starts falling again. In the case of the $\rho$, where we have extensive data (and many more helicity transitions), the transitions $\gamma_T^* p \to \rho_T p$ and $\gamma_L^* p \to \rho_L p$ become equal around $Q^2 = 2 \text{ GeV}^2$.

Summarizing this section, there is an intricate relation between scaling properties and helicity structure in hard-scattering processes. It is a consequence of quite general principles. How well the corresponding helicity selection rules are satisfied at finite $Q^2$ is governed by the interplay between the hard scale in the process and the scale of the trans-
verse distribution of partons in a hadron. It also depends on how important configurations are with additional gluons in the hadron wave functions.

4 Deeply virtual Compton scattering

4.1 Why?

Let me now focus on the exclusive process where theory is most advanced, and where data from both high and low energies have recently come in [27, 35, 38, 39]. Although this reaction has a lower cross section than for instance vector meson production, and although its analysis is more involved due to the competing Bethe-Heitler mechanism, it presents important advantages. The first one is precisely due to the Bethe-Heitler process, since it allows one to study the real and the imaginary parts of the scattering amplitude separately through interference [37, 40].

Secondly, to leading and first subleading power in $1/Q^2$ only the known pointlike component of the real photon is needed, not the hadronic part with its unknown wave function. Let me remark that, whereas hadronic wave functions do present a source of theoretical uncertainty in calculating exclusive meson production, this can be seen as a tradeoff of difficulties. In hard inclusive processes, calculations are done at the parton level, and in many cases one must describe the transition from partons to the final-state hadrons by models with their uncertainties. In exclusive processes the calculation is already at the level of the final-state hadrons, and the physics of hadronization is precisely contained in the meson wave function and similar quantities.

Thirdly, the hard subprocess in Compton scattering involves less external parton lines than in meson production, and as a consequence also less internal ones. As a consequence, the hard external momentum of the process becomes less “diluted” in the hard scattering, and our experience with exclusive processes suggests that the scale $Q^2$ where factorization works will be lower for Compton scattering than for meson production.

4.2 Theory approaches for small $x_B$

Let me now give a brief survey of the theoretical approaches that have been applied to DVCS so far, and begin with the model of Frankfurt et al. [40]. Although sometimes referred to as a “leading order QCD calculation”, it is not the analog of what is commonly called a LO QCD analysis of $F_2$, i.e., the calculation of the quark handbag diagram of Fig. 3a with an ansatz for the quark distribution in the proton. Instead, it is a rather simple way of relating the DVCS amplitude with the measured inclusive structure function $F_2$, which one may represent in three steps:

$$F_2 \propto \Im A(\gamma^* p \rightarrow \gamma^* p) \rightarrow \Im A(\gamma^* p \rightarrow \gamma p)_{t=0} \rightarrow A(\gamma^* p \rightarrow \gamma p)_{t=0} \rightarrow A(\gamma^* p \rightarrow \gamma p).$$

Step 1 is the most involved one. At low scale $Q^2 = Q_0^2$, the ratio $R$ of $\Im A(\gamma^* p \rightarrow \gamma^* p)$ and $\Im A(\gamma^* p \rightarrow \gamma p)_{t=0}$ was estimated to be around 0.5 using an ansatz based on the aligned
jet model. The variation of this ratio with $Q^2$ was then studied using the formalism of parton distributions and their evolution with the hard scale; this is where LO QCD entered the argument. Here the generalized parton distributions were assumed to equal the usual ones at the low factorization scale $Q^2_0$. The authors found little change of $R$ with $Q^2$ and $x_B$. In step 2, the ratio $\eta$ of $\text{Re} A(\gamma^*p \to \gamma p)_{t=0}$ and $\text{Im} A(\gamma^*p \to \gamma p)_{t=0}$ was taken from the derivative of the imaginary part with respect to $\log(1/x_B)$. This is a widely used approximation, also in meson production, based on analyticity and the quantum numbers of two-gluon exchange. It assumes an effective power behavior of the amplitude in $x_B$; analogous prescriptions for more general dependence on $x_B$ also exist. In step 3, an exponential behavior $e^{Bt/2}$ of the amplitude on $t$ was assumed, with a $W^2$ and $Q^2$ dependent slope $B$ estimated from measurements of other diffractive processes.

Each of the above steps reflects a rather fundamental aspect in the physics of DVCS. Step 1 is the transition from a forward to a nonforward process; in the context of factorization it means going from a forward to a nonforward parton distribution with its two independent longitudinal variables. Step 2 concerns the energy behavior at small $x_B$ and key approximations made in the high-energy regime. As discussed in section 2.1, step 3 involves the question of how longitudinal and transverse dynamics interrelate. Since the Compton amplitude can in principle be extracted from data, all three steps can be studied experimentally in DVCS.

There have recently been two applications of the dipole picture to DVCS. The one by Donnachie and Dosch [41] uses a semiclassical approach to model the dipole cross section at large dipole size $r$, and two-gluon exchange for small dipoles. Recently, McDermott et al. [42] compared the predictions of two rather different dipole models for this process. It is interesting to note that both give rather similar results for $\text{Im} A(\gamma^*p \to \gamma p)$ in the kinematics of the HERA measurements, but show more pronounced differences in the real part. Similar findings have been made previously by Frankfurt et al. [43].

### 4.3 Generalized parton distributions

In contrast to the previous approaches, the leading-twist description with generalized parton distributions (GPDs) is amenable to both high and intermediate energies, and thus provides a link between the HERA Collider and the HERMES measurements. Let me sketch how currently predictions are obtained. This proceeds in several steps:

$$q(x; Q^2_0) \overset{1}{\to} q(x; \xi; t; Q^2_0) \overset{2}{\to} q(x; \xi; t; Q^2) \overset{3}{\to} A(\gamma^*p \to \gamma p).$$

(1)

Here $q(x, Q^2)$ denotes the conventional quark density at scale $Q^2$, and $q(\xi, t; Q^2)$ the generalized quark distribution. The procedure is the same for gluons. To make predictions we need $q(\xi, t; Q^2_0)$ at some scale $Q^2_0$, and current strategies use the available parameterizations of $q(x, Q^2_0)$ as a model input. Let me stress that what we can currently do in step 1 is to take model prescriptions, mostly based on symmetry considerations. A widely used strategy is an ansatz based on “double distributions” [44], which was also employed in the recent study [43]. The $t$-dependence is presently modeled by the simple but restricted factorizing ansatz mentioned in section 2.1.
Step 2 is performed using the appropriate evolution equations for GPDs, whose kernels are known to NLO [46]. Notice that, even if one takes a given parameterization of the conventional parton distributions and the same model prescription how to “convert” them into a GPD, the result depends on the scale where this is done. In other words, for the model prescriptions we have, translating $q(x)$ into $q(x, \xi, t)$ at $Q_0$ and then evolving up to $Q_1$ does not give the same as first evolving $q(x)$ from $Q_0$ to $Q_1$ and then translating it into $q(x, \xi, t)$. There are arguments [47] that evolution of the GPDs tends to “wash out” the $\xi$ dependence of the distribution at the starting scale, so that after evolution over a long interval in $Q$ the $\xi$ dependence is essentially generated by evolution itself. This seems to work rather well in the region $x > \xi$ of the GPDs, but not for $x < \xi$, where evolution works very slowly, so that it may be most relevant at small $\xi$ and for observables not sensitive to the region $x < \xi$.

Step 3 above uses the hard scattering kernels, which for DVCS are also known to NLO [48]. The NLO expressions involving a quark box and generalized gluon distributions are however only known for massless quarks. For $Q^2$ comparable to $m_c^2$ this leads to an uncertainty in the results, since there it is neither a good approximation to omit charm in the quark box, nor to treat it as massless. As charm comes with a charge factor $4/9$ in the amplitude compared with $6/9$ for the sum of light quarks, this uncertainty is not negligible, and a calculation of the massive quark box for DVCS would be welcome.

Notice that the imaginary part of the DVCS amplitude is only sensitive to the point $x = \xi$ at LO, and only to the region $x \geq \xi$ in the radiative corrections. The real part probes the entire $x$-region of the GPDs. It can involve important cancellations between the regions $x < \xi$ and $x > \xi$, as is illustrated by the principal value integral $\int dx (x - \xi)^{-1}q(x, \xi, t)$ appearing in the LO expression.

In summary, the theory of Compton scattering belongs to the best-developed branches in the physics of exclusive processes. The phenomenological possibilities of DVCS allow one to study several important questions rather directly in the data. It also has a special status since its theoretical complexity lies in between the totally inclusive DIS structure functions and the much richer (and more complicated) processes of meson production.

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References

References

[1] Teubner T 1999, hep-ph/9910329.
[2] Bartels J, these proceedings.

[3] Collins J C, these proceedings hep-ph/0107252.

[4] Golec-Biernat K, these proceedings.

[5] Golec-Biernat K and Wüsthoff M 1999, Phys. Rev. D 60, 114023 hep-ph/9903358; McDermott M et al. 1999, Z. Phys. C 16, 641 hep-ph/9912547.

[6] Forshaw J R, Kerley G and Shaw G 1999, Phys. Rev. D 60, 074012 (1999) hep-ph/9903341; Schildknecht D et al. 2000, Phys. Lett. B 499, 116 (2001) hep-ph/0010030; Kopeliovich B Z, Nemchik J, Schäfer A and Tarasov A V 2001, hep-ph/0107227.

[7] Müller D et al. 1994, Fortsch. Phys. 42 (1994) 101 hep-ph/9812448; Ji X 1996, Phys. Rev. Lett. 78 (1997) 610 hep-ph/9603249; Radyushkin A V 1997, Phys. Rev. D 56 (1997) hep-ph/9704207; Blümlein J, Geyer B and Robaschik D 1999, Nucl. Phys. B 560, 283 (1999) hep-ph/9903520.

[8] Bartels J and Kowalski H 2000, Eur. Phys. J. C 19, 693 (2001) hep-ph/0010345.

[9] Burkardt M 2000, Phys. Rev. D 62, 071503 (2000) hep-ph/0005108.

[10] Brodsky S J, Diehl M and Hwang D S 2000, Nucl. Phys. B 596, 99 (2001) hep-ph/0009254; Diehl M, Feldmann T, Jakob R and Kroll P 2000, Nucl. Phys. B 596, 33 (2001) hep-ph/0009255.

[11] Penttinen M, Polyakov M V and Goeke K 1999, Phys. Rev. D 62, 014024 (2000) hep-ph/9909489.

[12] Diehl M et al. 1998 Eur. Phys. J. C 8, 409 (1999) hep-ph/9811253; Tiburzi B C and Miller G A 2001, hep-ph/0104198.

[13] Collins J C, Frankfurt L and Strikman M 1996, Phys. Rev. D 56, 2982 (1997) hep-ph/9611433; Collins J C and Freund A 1998, Phys. Rev. D 59, 074009 (1999) hep-ph/9801262.

[14] Nikolaev N N, Zakharov B G and Zoller V R 1993, JETP Lett. 59, 6 (1994) hep-ph/9312268; Mueller A H 1994, Nucl. Phys. B 415, 373 (1994).

[15] Bialas A, Navelet H and Peschanski R 2000, Nucl. Phys. B 593, 438 (2001) hep-ph/0009248.

[16] Bartels J, Gieseke S and Kyrieleis A 2001, hep-ph/0107152.

[17] Nachtmann O 1991, Annals Phys. 209, 436 (1991).
[18] Radyushkin A V 1995, *Acta Phys. Polon. B* 26, 2067 (1995) hep-ph/9511272.
[19] Caldwell A C and Soares M S 2001, hep-ph/0101085.
[20] Gotsman E *et al.* 2001, Phys. Lett. B 503, 277 (2001) hep-ph/0101142.
[21] Frankfurt L, McDermott M and Strikman M 2000, *JHEP* 0103, 045 (2001) hep-ph/0009086.
[22] Munier S, Staśto A M and Mueller A H 2001, *Nucl. Phys. B* 603, 427 (2001) hep-ph/0102291.
[23] Crittenden J, these proceedings.
[24] ZEUS Collaboration 2000, Abstract 880 submitted to the XXXth ICHEP conference, Jul.-Aug. 2000, Osaka, Japan.
[25] Vanderhaeghen M *et al.* 1999, *Phys. Rev. D* 60, 094017 (1999) hep-ph/9905372.
[26] Donnachie A, Gravelis J and Shaw G 2000, *Eur. Phys. J. C* 18, 539 (2001) hep-ph/0009235.
[27] Lobodzińska E, these proceedings hep-ph/0108263; Adloff C *et al.* [H1 Collaboration] 2001, *Phys. Lett. B* 517, 47 (2001) hep-ex/0107003.
[28] Collins J C and Diehl M 1999, *Phys. Rev. D* 61, 114015 (2000) hep-ph/9907498.
[29] Martin A D, Ryskin M G and Teubner T 1999, *Phys. Rev. D* 62, 014022 (2000) hep-ph/9912551.
[30] ZEUS Collaboration 2001, Abstract 556 submitted to the EPS HEP conference, Jul. 2001, Budapest, Hungary.
[31] Ivanov D Y *et al.* 2000, *Phys. Lett. B* 478, 101 (2000) hep-ph/0001254.
[32] Hoodbhoy P and Ji X 1998, *Phys. Rev. D* 58, 054006 (1998) hep-ph/9801369; Diehl M 2001, *Eur. Phys. J. C* 19 (2001) 485 hep-ph/0101335.
[33] Belitsky A V and Müller D 2000, *Phys. Lett. B* 486, 369 (2000) hep-ph/0005028; Mankiewicz L and Kivel N, hep-ph/0106329; Kivel N 2001, hep-ph/0107275.
[34] Thomas E [HERMES Collab.] 2001, talk presented at DIS 2001, Apr.-May 2001, Bologna, Italy.
[35] Bianchi N, these proceedings.
[36] Anikin I V, Pire B and O. V. Teryaev O V 2000, *Phys. Rev. D* **62**, 071501 (2000) [hep-ph/0003203]; Penttinen M *et al.* 2000, *Phys. Lett. B* **491**, 96 (2000) [hep-ph/0006321]; Belitsky A V and Müller D 2000, *Nucl. Phys. B* **589**, 611 (2000) [hep-ph/0007031]; Kivel N *et al.* 2000, *Phys. Lett. B* **497**, 73 (2001) [hep-ph/0007315]; Radyushkin A V and Weiss C 2000, *Phys. Rev. D* **63**, 114012 (2001) [hep-ph/0010296].

[37] Diehl M, Gousset T, Pire B and Ralston J P 1997, *Phys. Lett. B* **411** (1997) 193 [hep-ph/9706344].

[38] Airapetian A *et al.* [HERMES Collaboration] 2001, *Phys. Rev. Lett.* (in press) [hep-ex/0106068].

[39] Stepanyan S *et al.* [CLAS Collaboration] 2001, [hep-ex/0107043].

[40] Frankfurt L L, Freund A and Strikman M 1997, *Phys. Rev. D* **58**, 114001 (1998) [hep-ph/9710356].

[41] Donnachie A and Dosch H G 2000, *Phys. Lett. B* **502**, 74 (2001) [hep-ph/0010227].

[42] McDermott M, Sandapen R and Shaw G 2001, [hep-ph/0107224].

[43] Frankfurt L L, Freund A and Strikman M 1998, *Phys. Lett. B* **460**, 417 (1999) [hep-ph/9806535].

[44] Musatov I V and Radyushkin A V 1999, *Phys. Rev. D* **61**, 074027 (2000) [hep-ph/9905376].

[45] Freund A and McDermott M F 2001, [hep-ph/0106319]; [hep-ph/0106124]; [hep-ph/0106115].

[46] Belitsky A V, Freund A and Müller D 1999, *Nucl. Phys. B* **574**, 347 (2000) [hep-ph/9912379].

[47] L. Frankfurt *et al.* 1997, *Phys. Lett. B* **418**, 345 (1998) [hep-ph/9703449]; K. J. Golec-Biernat *et al.* 1999, *Phys. Lett. B* **456**, 232 (1999) [hep-ph/9903327].

[48] Belitsky A V and Müller D 1997, *Phys. Lett. B* **417**, 129 (1998) [hep-ph/9709379]; L. Mankiewicz *et al.* 1997, *Phys. Lett. B* **425**, 186 (1998) [hep-ph/9712251]; Ji X and Osborne J 1998, *Phys. Rev. D* **58**, 094018 (1998) [hep-ph/9801260]; Belitsky A V *et al.* 1999, *Phys. Lett. B* **474**, 163 (2000) [hep-ph/9908337].