Generalized Attitude Model Identification of a Quadrotor

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Abstract. This paper studies the generalized attitude model (GAM) identification of a controlled quadrotor by using flight experimental data. The GAM describes the dynamics of the quadrotor and its driving motors. Its mechanism analysis will be first carried out for guiding the parametrization of its estimation model. Before the prediction error method (PEM) is applied on the GAM identification, the involved closed-loop identifiability problem is addressed. Finally, the accuracy of the estimated GAM is validated by comparisons of Bode diagrams and controller performances achieved by a redesigned robust controller.

1. Introduction
There are numerous researches on the advanced control technology of quadrotors, but most of the advanced controller designs are inseparable from their mathematical models [1]. Therefore, the research and identification of quadrotor attitude models can not only provide a model basis for various modern advanced controller designs, but it can also greatly improve the control performance and quality of the quadrotor control system.

Based on the established aircraft mechanism model, Greenber uses the least-squares identification algorithm to process flight experiment data and proposed a method to determine aircraft aerodynamic parameters and mathematical models [2]. Hashimoto et al. selected three models of autoregressive, Box-Jenkins, and output error, used frequency domain identification methods to identify each channel of the quadrotor [3]. GRASP laboratory has proposed two control algorithms for autonomous flight of quadrotors based on visual feedback, and completed the experiment on the quadrotor experimental platform [4]. Hong gives a variety of nonlinear relationships between regression vectors and output observation signals under a nonlinear structure [5]. Since 2005, in the field of system identification, there have been more and more researches on state space models. For example, Katayama and Pintelon used the subspace identification method for the state space model to obtain the system matrix [6-10].

The main contribution of this paper is to determine the attitude model of the quadrotor based on the collected flight data. After obtaining the prior information of the attitude model, the closed-loop identification framework is analyzed, and the flight data is collected and processed through the designed experiment. Finally, the PEM is used to estimate the quadrotor attitude model and determine the mechanism model of the attitude model.
2. Problem Formulation
This paper uses the conventional four-propeller symmetrically arranged "X" small quadrotor as shown in Figure 1 as the research object.

In order to obtain the mechanism model of the quadrotor, it is necessary to establish the body coordinate system of the quadrotor as shown in Figure 2 [11]. The main forces and moments acting on the quadrotor are generated by the four propellers. Two adjacent propellers in the system rotate in opposite directions to keep the entire flight system in balance. $\phi$, $\theta$ and $\psi$ represent the Euler angles of $x_B$, $y_B$ and $z_B$ body axis respectively.

According to the relationship between the Euler angle and the angular velocity of the quadrotor. When the quadrotor is flying in hover, one can get three attitude angles [12]. The rotational torque relationship between the yaw body axis and its associated equivalent driving motor can be described as

$$ J_z \dot{\psi} = -J_{\omega} \dot{\psi} $$

in which, $\omega_{\psi}$ is the equivalent angular velocity from the four driving motors to the yaw axis of the quadrotor and $J_{\omega}$ its associated equivalent moment of inertia; $J_z$ is the equivalent moment of inertia related to the yaw axis of the body.

The identification of the quadrotor attitude model is the process of using flight experiment data to reverse the mathematical model. Under the premise of a reasonable choice of identification method and model structure, determining identification data is an important factor to ensure the accuracy of attitude model identification.

The signal generated by the remote controller is sent to the receiver of the quadrotor. The microcontroller will output 4 PWM signal waves through the electronic governor to drive the brushless DC motor. Complete the roll, pitch and yaw movement of the quadrotor [13]. According to the "X" flight mode shown in Figure 1 and Figure 2, the coupling relationship of the four-channel control variables is shown in Table 1.

| Motor  | Thr($U$) | Roll($U_{\phi}$) | Pitch($U_{\theta}$) | Yaw($U_{\psi}$) |
|--------|----------|------------------|-------------------|----------------|
| Motor 1| +        | -                | +                 | +              |
| Motor 2| +        | +                | +                 | -              |
| Motor 3| +        | +                | -                 | +              |
| Motor 4| +        | -                | -                 | -              |
In the table, "+" means the addition of the control variables of each channel, and "-" means the subtraction of the control variables of each channel. Then the angular velocity of 4 motors is calculated as

\[
\begin{align*}
\text{PWM}_1 &= U - U_\phi + U_\theta + U_\psi \\
\text{PWM}_2 &= U + U_\phi + U_\theta - U_\psi \\
\text{PWM}_3 &= U + U_\phi - U_\theta + U_\psi \\
\text{PWM}_4 &= U - U_\phi - U_\theta - U_\psi
\end{align*}
\]

(2)

From the above formula, the control input of the three attitude channels of the quadrotor under decoupling control is

\[
\begin{align*}
U_\phi &= -\frac{\text{PWM}_2 + \text{PWM}_3 + \text{PWM}_1 - \text{PWM}_4}{4} \\
U_\theta &= \frac{\text{PWM}_2 + \text{PWM}_1 - \text{PWM}_3 - \text{PWM}_4}{4} \\
U_\psi &= \frac{\text{PWM}_1 - \text{PWM}_2 + \text{PWM}_3 - \text{PWM}_4}{4}
\end{align*}
\]

(3)

The PWM signals of the four driving motors are recorded in the memory card through the internal storage and transmission module. According to the above formula, three attitude angle control variables under decoupling control can be obtained, which are represented as input signals \( U_\psi \) in Figure 3. It is used as the input data for direct closed-loop identification, and the actual attitude angle \( \psi \) is used as the output data for closed-loop identification.

### 3. Generalized attitude model identification

#### 3.1. Closed-loop identifiability

For the identification attitude model of the quadrotor, the research takes into account the position of the input and output data collected during the actual flight and constructs a closed-loop identification framework as shown in Figure 3.

![Figure 3. Closed-loop identification framework for the quadrotor](image_url)

In the figure, \( P(s) \) is the GAM of the quadrotor. \( C(z) \) is a controller that can stabilize \( P(s) \). According to the actual data and the quadrotor manual, it can be known that the proportional relationship between the collected PWM signal and the motor speed is \( k_i \) [14]. \( A(s) \) is the actuator, including electronic governor model and the motor model. Its input and output are the output signal \( U_\psi \) of the controller and the torque \( T_w \) generated by the rotation of the motor, and the expression is \( A(s) = k_i J_s s \). The input data used in the identification study is \( U_\psi \) in the figure. \( U_\psi \) is the control value of the three attitude channels output by the controller, which is obtained by four-three conversion of four-channel PWM. \( T \) is the torque of the three attitude channels. \( T_w \) is the disturbance torque generated by random wind. As shown in the figure, the generalized attitude control framework is very clear. The paper conducts identification research on the input control quantity \( U_\psi \) and output attitude \( \psi \), estimates the mechanism model and identification model of the quadrotor.

Combing the dynamics from \( U_\psi \) to \( \psi \) in the forward path in Fig.1. Yields the yaw-axis mechanical GAM as follows
\[ P_v(s) = -k_1 \frac{J_s}{J_s} = -\frac{k_2}{s} \]  \hspace{1cm} (4)

with

\[ k_2 = \frac{k_1 J_w}{J_z} \]  \hspace{1cm} (5)

As a result, the parameter to be identified is \( k_2 \). The main purpose of this paper is to use the large amount of flight data collected by the quadrotor to identify the mechanism model parameter \( k_2 \) of the GAM of the quadrotor. At the same time, according to the mechanism model of the attitude model, the estimation models of the three GAMs of the quadrotor are determined.

The input control signal and output attitude of the quadrotor's attitude model are measurable. Under certain conditions, these data can be used to directly identify the model of the forward channel, without first identifying the entire system model, and then calculating the forward direction from it. The model of the channel. The identifiable condition of the closed-loop direct identification method is that the order of the feedback channel controller model is greater than or equal to the order of the forward channel attitude model [15]. And when there is a noise signal on the feedback channel, and the input control data and output attitude on the forward channel are measurable. At this time, various closed-loop identification methods can be directly used to obtain the estimation model of the forward channel. In addition, the perturbation signals from the forward channel and the feedback channel are helpful for identification, and can also explain the feasibility of the closed-loop direct identification method.

3.2. Prediction error identification method

The PEM provides both a system model and a noise model. It is a special case of the maximum likelihood method. There is no need to know the prior knowledge of the probability distribution of the data in advance, and the prediction error criterion is directly minimized. The PEM can be regarded as a problem of the optimal algorithm for minimizing parameters [16]. The form of GAM suitable for PEM identification is

\[ \varphi(k) = P_v(z)U_v(k - nk) + e(k) \]  \hspace{1cm} (6)

with

\[ P_v(z) = \frac{B(z^{-1})}{A(z^{-1})F(z^{-1})} \]  \hspace{1cm} (7)

\( e(k) \) is an equivalent disturbing noise which is caused by \( T_w \) as well as process and measurement existing noises the closed-loop system in Fig 1. This disturbing noise can be expressed as

\[ e(k) \sim N(z)n(k) \]  \hspace{1cm} (8)

with

\[ N(z) = \frac{D(z^{-1})}{A(z^{-1})C(z^{-1})} \]  \hspace{1cm} (9)

The involved model orders in (7) and (8) are defined as follows

\begin{align*}
A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na} \\
B(z^{-1}) &= b_1 + b_2 z^{-1} + \cdots + b_{nb} z^{-nb+1} \\
F(z^{-1}) &= 1 + f_1 z^{-1} + f_2 z^{-2} + \cdots + f_{nf} z^{-nf} \\
C(z^{-1}) &= 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{nc} z^{-nc} \\
D(z^{-1}) &= 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{nd} z^{-nd}
\end{align*}  \hspace{1cm} (10)
where $na$, $nb$, $nf$, $nc$, $nd$ and $nk$ are the associated model orders.

4. Identification procedure of generalized attitude model

The research on the identification of the quadrotor attitude model can be summarized as the following steps.

**Step 1** Design the flight experiment and collect flight data based on the prior information of the quadrotor system.

**Step 2** Process the collected actual flight data of the quadrotor to obtain the input control data and output attitude data for identification.

**Step 3** PEM identification is used for the collected input control data and output attitude data to obtain a discrete quadrotor estimated attitude model.

**Step 4** Use the bilinear transformation method to process the obtained discrete attitude model to obtain a continuous-time attitude model.

**Step 5** Verify the accuracy of the identified quadrotor attitude model by redesigning the controller. If the model does not meet the accuracy requirements, go back to step 1.

5. Identification results and model validation

In order to improve the accuracy of the model, the Hankel model reduction method is used for the identified model, and the attitude model will be 1st order. Since there is a lot of noise in actual flight that cannot be actually measured, after many calculations and consideration of the actual situation, it is more reasonable to set the noise model as 2nd order. Regarding the delay time, the impulse response method can be used to determine the value of delay time with a confidence interval of 3 standard deviations. It is easy to get the value of delay time from Figure 4. The filled light-blue region shows the confidence interval for the delay time estimation. There is a clear indication that after 19 samples, the impulse response is always 0, and such pulses do not exceed the shadowed area. This means that the delay time is 0 for delay time. Figure 5 shows the input control data and output attitude data of the attitude model used for identification. Therefore, the formula is as follows

$$
\begin{align*}
na &= 0; nb = 3; nc = 2; \\
nd &= 2; nf = 2; nk = 0.
\end{align*}
$$

(11)

**Figure 4.** Impulse response estimation

**Figure 5.** Input and output data for identification

According to section 3.1, one uses the direct identification for the PEM. The identification process is implemented to obtain the discrete GAM, then the continuous GAM and its associated noise model are obtained through a bilinear transformation as follows
Since the parameter $k_2$ of the mechanism model is unknown, the value of $k_2$ on the Bode diagram is only the translation of the upper and lower positions. The fitting of the Bode diagram in Fig. 6 is proof of the specific values of the attitude parameters $k_2$. It can be found by comparing the estimated attitude model and the mechanism attitude model, in a relatively broad frequency interval intersecting with 0dB axis, both the log-magnitude-frequency curves and the log phase-frequency curves for the estimated attitude model and mechanism attitude models have good fitting effects. This also shows that the direct identification method of prediction error is suitable for the identification of the quadrotor attitude model.

\[ \begin{align*} 
\hat{P}_\nu(s) &= \frac{9.48 \times 10^3 s - 0.6181}{s - 1.49 \times 10^{-7}} \\
\hat{N}(s) &= \frac{0.2223 s^2 + 10.3 s + 114.4}{s^2 + 0.4495 s + 0.2869} 
\end{align*} \] (12)

\[ H(s) = \frac{-7.243 s - 4.091}{s^2 + 3.293 s} \] (13)

The control effects of unit step signal and sine signal are shown in Figures 7 and 8 respectively.
By comparing the control effects, it can be seen that both the estimated attitude model and the mechanism attitude model of the quadrotor have a good fitting effect regardless of the unit step signal or the sine signal.

6. Conclusion

In this paper, the PEM is used to study the identification and model of the quadrotor attitude model. Firstly, the mechanism model of the quadrotor was established and based on the prior knowledge, the closed-loop identification framework of the quadrotor was constructed. Then, the issue of identifiability was discussed, the flight experiment was designed, and the experimental data was collected for processing and analysis to obtain the system control data and attitude data. Finally, the PEM is used to estimate the attitude model of the quadrotor. The estimation model was verified by redesigning the robust controller. The results show that the estimated attitude model has high accuracy.

7. References

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