Color superconducting 2SC+s quark matter and gapless modes at finite temperatures

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We investigate the phase diagram of color superconducting quark matter with strange quarks (2SC+s quark matter) in beta equilibrium at zero as well as finite temperatures within a Nambu-Jona-Lasinio model. The variational method as used here allows us to investigate simultaneous formation of condensates in quark-antiquark as well as in diquark channels. Color and electric charge neutrality conditions are imposed in the calculation of the thermodynamic potential. Medium dependance of strange quark mass plays a sensitive role in maintaining charge neutrality conditions. At zero temperature the system goes from gapless phase to usual BCS phase through an intermediate normal phase as density is increased. The gapless modes show a smooth behaviour with respect to temperature vanishing above a critical temperature which is larger than the BCS transition temperature. We observe a sharp transition from gapless superconducting phase to the BCS phase as density is increased for the color neutral matter at zero temperature. As temperature is increased this however becomes a smooth transition.

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I. INTRODUCTION

Color superconductivity has become a compelling topic in QCD during the last few years. At sufficiently high baryon densities, when nucleons get converted to quark matter, the resulting quark matter is in one kind or the other of the many different possible color superconducting phases at low enough temperatures [1]. The rich phase structure is essentially due to the fact that the quark quark interaction is not only strong and attractive in many channels but also many degrees of freedom are possible for quarks like color, flavor and spin so that various kinds of BCS pairing are possible. Studying properties of color superconducting phases in heavy ion collision experiments seems unlikely in the present accelerators as one cannot avoid producing large entropy per baryon in heavy ion collisions and hence cannot produce the dense and cold environment that is needed to support the formation of superconducting phases. However, in the future accelerator facility planned at GSI for compressed baryonic matter experiments, one possibly can hope for observing fluctuations signifying precursory phenomena of color superconducting phase [2].

On the other hand, it is natural to expect some color superconducting phase to exist in the core of compact stars where the densities are above nuclear matter densities and temperatures are of the order of tens of keV. However, to consider quark matter for neutron stars, color and electrical charge neutrality conditions need to be imposed for the bulk quark matter. Such an attempt has been made in Ref. 3 as well as in Ref. 4 where the lighter up and down quarks form the two flavor color superconducting (2SC) matter while the strange quarks do not participate in pairing. A model independent analysis was done in Ref. 5 that is valid in the limit $m_s << \mu$ and $\Delta \sim m_s^2/\mu$, where, $\Delta$ is the pairing gap and $\mu$ is the quark chemical potential. It has been shown, based upon comparison of free energies that such a two flavor color superconducting phase would be absent in the core of neutron stars 6. Within

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Nambu Jona-Lasinio (NJL) model in Ref. [4] it has been argued that such conclusions are consistent except for a small window in density range where superconducting phase is possible. There have also been studies to include the possibility of mixed phases [5] of superconducting matter demanding neutral matter on the average. Recently it was observed that imposition of neutrality conditions lead to pairing of quarks with different fermi momenta giving rise to gapless modes [6]. Within a Nambu Jona-Lasinio model the two flavor superconducting quark matter (2SC) was shown to exhibit gapless modes (g2SC) arising due to difference in the fermi momenta of the pairing quarks when charge and color neutrality conditions are imposed. Superconducting quark matter with strange quarks (2SC+s) was shown to exhibit gapless superconductivity (g2SC+s) within a window of about 80 MeV in baryon chemical potential [7]. The number densities of the strange quarks were seen to be as large as about 40% of the number density of e.g. the up quarks in the gapless phase [7]. Temperature effects on the gapless modes were also studied for two flavor quark matter in Ref. [8].

We had applied a different approach to study the problem in Ref. [7, 9]. We considered a variational approach with an explicit assumption for the ground state having both quark–antiquark as well as diquark condensates. The actual calculations are carried out for the NJL model with a vector interaction such that the minimisation of the free energy density determines which condensate will exist at what density. Charge neutrality conditions were introduced through the introduction of appropriate chemical potentials. We note here that possibility of diquark condensates along with quark–antiquark condensates has also been considered in Ref. [10, 11, 12, 13].

In the present work we generalise the variational approach of Ref. [7] to study the color superconductivity as well as the chiral symmetry breaking to include the effects of temperatures. This will be particularly relevant for the physics of proto neutron stars. The gapless modes in the two flavor color superconductivity were investigated at finite temperatures [8]. However, in the present work we take the analysis further in the sense that the diquark and the quark-antiquark condensates are uniquely calculated selfconsistently and elaborate the transition from gapless phase and BCS phase as the density is increased for different temperatures. Further, since the realisation of color and charge neutrality is nontrivial for physically relevant values of strange quark mass, we retain all orders in \( m_s \) in the calculation of the thermodynamic potential. This of course is important to understand the phase structure of quark matter at densities relevant to the interior of neutron stars. The temperature dependence and the nature of the gapless modes as a function of the baryon density shall be particularly relevant for cooling of the neutron stars through neutrino emission [7].

We organize the paper as follows. In the next subsection we discuss the ansatz state with the quark–antiquark as well as the diquark condensates. In section IV we consider the Nambu Jona-Lasinio model Hamiltonian and calculate its expectation value with respect to the ansatz state to compute the thermodynamic potential. We minimise the thermodynamic potential to calculate all the ansatz functions and the resulting mass as well as superconducting gap equations here. In section III we discuss the results of the present investigation. Finally we summarise and conclude in section IV.

### A. An ansatz for the ground state

To make the notations clear, let us note first the quark field operator expansion in momentum space given as

\[
\psi(x) = \frac{1}{(2\pi)^{3/2}} \int \psi(k)e^{ik \cdot x} dk
\]

\[
= \frac{1}{(2\pi)^{3/2}} \int [U_0(k) q^0(k) + V_0(-k)q^0(-k)] e^{ik \cdot x} dk, \tag{1}
\]

where,

\[
U_0(k) = \begin{pmatrix}
\cos(\phi_0^\sigma) \\
\sigma \cdot k \sin(\phi_0^\sigma)
\end{pmatrix}, \quad V_0(-k) = \begin{pmatrix}
-\sigma \cdot k \sin(\phi_0^\sigma) \\
\cos(\phi_0^\sigma)
\end{pmatrix}.
\tag{2}
\]

The operators \( q^0 \) and \( \bar{q}^0 \) are the two component particle annihilation and antiparticle creation operators respectively which annihilate or create quanta acting upon the perturbative or the chiral vacuum \(|0\rangle\). We have suppressed here the color and flavor indices of the quark field operators. The function \( \phi_0(k) \) in the spinors in Eq. (1) are given as \( \cot \phi_0^i = m_i/|k| \), for free massive fermion fields, \( i \) being the flavor index. For massless fields \( \phi_0(|k|) = \pi/2 \).
We now write down the ansatz for the variational state as a squeezed coherent state involving quark antiquark as well as diquark condensates as given by

\[ |\Omega\rangle = U_d |\text{vac}\rangle = U_d U_Q |0\rangle, \]

(3)

Here, \( U_Q \) and \( U_d \) are unitary operators creating quark–antiquark and diquark pairs respectively. Explicitly,

\[ U_Q = \exp \left( \int q^{0i}(k)^\dagger (\mathbf{\sigma} \cdot \mathbf{k}) h_i(k) q^{0i}(k) dk - h.c. \right). \]

(4)

In the above, \( h_i(k) \) is a real function of \(|k|\) which describes vacuum realignment for chiral symmetry breaking for quarks of a given flavor \( i \). We shall take the condensate function \( h(k) \) to be the same for \( u \) and \( d \) quarks and \( h_3(k) \) as the chiral condensate function for the s-quark. Clearly, a non trivial sum over three colors and three flavors is understood in the exponent of \( U_Q \) in Eq. (4). Similarly, the unitary operator \( U_d \) describing diquark condensates is given as

\[ U_d = \exp (B_d^\dagger - B_d) \]

(5)

where, \( B_d^\dagger \) is the pair creation operator as given by

\[ B_d^\dagger = \int \left[ \hat{a}_{ia}^\dagger(k)^\dagger r f^{ia}(k) q_r^{jb}(-k)^\dagger \epsilon_{ij} \epsilon_{3ab} + \hat{a}_{ia}^\dagger(k)r f^{ia}(k) q_r^{jb}(-k) \epsilon_{ij} \epsilon_{3ab} \right] dk. \]

(6)

In the above, \( i, j \) are flavor indices, \( a, b \) are the color indices and \( r(= \pm 1/2) \) is the spin index. The operators \( q(k) \) are related to \( q^0(k) \) through the transformation \( q(k) = U_d q^0(k) U_Q^{-1} \). As noted earlier we shall have the quarks of colors red and green \((a=1,2)\) and flavors \( u,d \) \((i=1,2)\) taking part in diquark condensation. The blue quarks \((a=3)\) do not take part in diquark condensation. We have also introduced here (color, flavor dependent) functions \( f^{ia}(k) \) and \( f^{ia}(k) \) respectively for the diquark and diantiquark channels. As may be noted the state constructed in Eq. (5) is spin singlet and is antisymmetric in both color and flavor. Clearly, by construction \( f^{ia}(k) = f^{jb}(k) \) with \( i \neq j \) and \( a \neq b \). The ansatz for the ground state has been written down keeping quark–antiquark condensates for the three flavors and diquark condensates for two flavors and two colors.

Finally, to include the effects of temperature and density we next write down the state at finite temperature and density \(|\Omega(\beta, \mu)\rangle\) taking a thermal Bogoliubov transformation over the state \(|\Omega\rangle\) using thermofield dynamics (TFD) as described in refs. [17, 18]. We then have,

\[ |\Omega(\beta, \mu)\rangle = U_{\beta, \mu} |\Omega\rangle = U_{\beta, \mu} U_d U_Q |0\rangle. \]

(7)

where \( U_{\beta, \mu} \) is given as

\[ U_{\beta, \mu} = e^{B(\beta, \mu) - B(\beta, \mu)}, \]

(8)

with

\[ B(\beta, \mu) = \int \left[ q_i^0(k)^\dagger \theta_-(k, \beta, \mu) q_i^0(k)^\dagger + q_i^0(k) \theta_+(k, \beta, \mu) q_i^0(k) \right] dk. \]

(9)

In Eq. (9) the ansatz functions \( \theta_\pm(k, \beta, \mu) \) will be related to quark and antiquark distributions and the underlined operators are the operators in the extended Hilbert space associated with thermal doubling in TFD method. In Eq. (9) we have suppressed the color and flavor indices on the quarks as well as the functions \( \theta(k, \beta, \mu) \). All the functions in the ansatz in Eq. (9) are to be obtained by minimising the thermodynamic potential. This will involve an assumption about the effective Hamiltonian. We shall carry out this minimisation in the next section.

II. EVALUATION OF THERMODYNAMIC POTENTIAL AND GAP EQUATIONS

Since we shall be dealing with non–asymptotic densities we can not use the weak coupling method. For this reason we shall be working in a model in which the interaction among the quarks is simplified – while still respecting the
The Hamiltonian is given as

\[ \mathcal{H} = \sum_{i,a} \psi^{ia\dagger}(-i\alpha \cdot \nabla + \gamma^0 m_i)\psi^{ia} - G_s \sum_{A=0}^8 \left[ (\bar{\psi} \lambda^A \psi)^2 - (\bar{\psi} \gamma^5 \lambda^A \psi)^2 \right] - G_D (\bar{\psi} \gamma^5 \epsilon_{cb} \psi^c)(\bar{\psi} \gamma^5 \epsilon_{cb} \psi) \]  

(10)
In a similar manner, the contribution from the diquark interaction from Eq. (10) to the energy density is given as

\[ V_D = -G_D(\Omega(\beta, \mu)|\langle \bar{\psi} \gamma^5 e^b \psi^C \rangle|\bar{\Omega}(\beta, \mu)) = -G_D \left( \sum_{i,j=1,2; a,b=1,2} I^{ia,jb} |e^{ij}| |e^{3ab}| \right)^2 \]  

(17)

with

\[ I^{ia,jb} = \frac{1}{(2\pi)^3} \int d\mathbf{k} S^{ia,jb}(\mathbf{k}) \cos \left( \frac{\phi_i - \phi_j}{2} \right) \]  

(18)

where, \( S^{ia,jb}(\mathbf{k}) \) has been defined as

\[ S^{ij,ab}(\mathbf{k}) = \sin^2 f^{ia}(\mathbf{k}) \cos 2\theta^{ia,jb}(\mathbf{k}, \beta, \mu) + \sin^2 f^{ia}(\mathbf{k}) \cos 2\theta^{ia,jb}(\mathbf{k}, \beta, \mu). \]  

(19)

In the above we have defined \( \cos 2\theta^{ia,jb} = 1 - \sin^2 \theta^a\pm - \sin^2 \theta^b\pm \), with \( i, j = 1, 2 \) being the flavor indices and \( a, b = 1, 2 \) being the color indices and \( i \neq j, a \neq b \).

To calculate the thermodynamic potential we shall have to specify the chemical potentials relevant for the system. Here we shall be interested in the form of quark matter that might be present in compact stars older than few minutes so that chemical equilibration under weak interaction is there. The relevant chemical potentials in this case are the baryon chemical potential \( \mu_B \), the chemical potential \( \mu_E \) associated with electromagnetic charge \( Q = \text{diag}(2/3, -1/3, -1/3) \) in flavor space, and the two color electrostatic chemical potentials \( \mu_3 \) and \( \mu_8 \) corresponding to \( U(1)_3 \times U(1)_8 \) subgroup of the color gauge symmetry generated by cartan subalgebra \( Q_3 = \text{diag}(1/2, -1/2, 0) \) and \( Q_8 = \text{diag}(1, 1/3, -2/3) \) in the color space. Thus the chemical potential is a diagonal matrix in color and flavor space, and is given by

\[ \mu_{ij,ab} = (\mu \delta_{ij} + Q_{ij}(\beta, \mu)) \delta_{ab} + (Q_{3ab} + Q_{8ab}(\beta, \mu)) \delta_{ij}. \]  

(20)

Here, \( i, j \) are flavor indices and \( a, b \) are color indices.

The total thermodynamic potential, including the contribution from the electrons, is then given by

\[ \Omega = T + V_S + V_D - \langle \mu N \rangle - \frac{1}{\beta} s + \Omega_e \]  

(21)

where, we have denoted

\[ \langle \mu N \rangle = \langle \psi^i a | \mu_{ij,ab} \psi^{j b} \rangle = 2 \sum_{i,a} \mu^ia I^i_a \]  

(22)

with \( \mu^ia \) being the chemical potential for the quark of flavor \( i \) and color \( a \), which can be expressed in terms of the chemical potentials \( \mu, \mu_E, \mu_3 \) and \( \mu_8 \) using Eq. (20). Further

\[ I^i_a = \frac{1}{(2\pi)^3} \int d\mathbf{k} (F^i_a - F^{ia}_1) \]  

(23)

is proportional to the number density of quarks of given color and flavor. The thermodynamic potential for electrons is given as

\[ \Omega_e = -\frac{\mu_E^2}{12\pi^2} \left( 1 + \frac{2\pi^2 T^2}{\mu_E^2} \right) \]  

(24)

where we have taken the electron mass to be zero which suffices for the system we are considering.

Finally, for the entropy density for the quarks we have [17]

\[ s = -\frac{2}{(2\pi)^3} \sum_{i,a} \int d\mathbf{k} \left( \sin^2 \theta^i_a \ln \sin^2 \theta^i_a + \cos^2 \theta^i_a \ln \cos^2 \theta^i_a \right. \]

\[ + \sin^2 \theta^i_+ \ln \sin^2 \theta^i_+ + \cos^2 \theta^i_+ \ln \cos^2 \theta^i_+ \left. \right) \]  

(25)
Now functional minimisation the thermodynamic potential $\Omega$ with respect to the chiral condensate function $h_i(\mathbf{k})$ leads to
\[
\cot \phi_i(\mathbf{k}) = \frac{m_i + 8G_s I^i_s}{|\mathbf{k}|} = \frac{M_i}{|\mathbf{k}|}
\]  

where, $M_i = m_i + 8G_s I^i_s$. Substituting this back in Eq. 16 yields the mass gap equation as
\[
M_j = m_j + \frac{8G_s}{(2\pi)^3} \int \frac{M_j}{\epsilon_j} \sum_{a=1,3} (1 - F_{j}^{ia} - F_{1}^{ia}) d\mathbf{k},
\] 

where, $\epsilon_j = \sqrt{k^2 + M_j^2}$ being the energy of the constituent quarks of $j$-th flavor. Clearly, the above includes the effect of diquark condensates as well as temperature and density through the functions $F$ and $F_1$ given in Eqs. 14 and 15 respectively.

As noted earlier, there are only two independent diquark condensate functions $f^{11}$ and $f^{12}$ corresponding to the indices u-red and u-green respectively. This is due to the fact that the function with indices d-green is the same as that with indices u-red and the condensate function with indices d-red is the same as that with indices u-green.

Minimisation of the thermodynamic potential $\Omega$ with respect to the diquark condensate functions $f^{11}(\mathbf{k})$ and $f^{12}(\mathbf{k})$ yields
\[
\tan 2f^{11}(\mathbf{k}) = \frac{\Delta}{\bar{\epsilon} - \bar{\mu}_{11}} \cos(\frac{\phi_1 - \phi_2}{2})
\]  

and
\[
\tan 2f^{12}(\mathbf{k}) = \frac{\Delta}{\bar{\epsilon} - \bar{\mu}_{12}} \cos(\frac{\phi_1 - \phi_2}{2})
\] 

where, we have defined $\Delta = 4G_D(I^{11,22} + I^{12,21})$ with $I^{ia,jb}$ as defined in Eq. 18. Further, in the above $\bar{\epsilon} = (\epsilon_1 + \epsilon_2)/2$, $\bar{\mu}_{11} = (\mu_{11} + \mu_{22})/2$, $\bar{\mu}_{12} = (\mu_{12} + \mu_{21})/2$. It is thus seen that the diquark condensate functions depend upon the average energy and the average chemical potential of the quarks that condense. We also note here that the diquark condensate functions depends upon the masses of the two quarks which condense through the function $\cos \left( (\phi_1 - \phi_2)/2 \right)$. The function $\cos \phi_i = M_i/\epsilon_i$, can be different for $u,d$ quarks, when charge neutrality condition is imposed. Such a normalisation factor is always there when the condensing fermions have different masses as has been noted in Ref. 22 in the context of CFL phase.

In an identical manner the di-antiquark condensate functions are calculated to be
\[
\tan 2f^{11}_d(\mathbf{k}) = \frac{\Delta}{\epsilon + \bar{\mu}_{11}} \cos(\frac{\phi_1 - \phi_2}{2})
\]  

and
\[
\tan 2f^{12}_d(\mathbf{k}) = \frac{\Delta}{\epsilon + \bar{\mu}_{12}} \cos(\frac{\phi_1 - \phi_2}{2}).
\] 

Substituting the solutions for the condensate functions in the expressions for $I^{ia,jb}$ in Eq. 18 we have the gap equation for $\Delta$ given as
\[
\Delta = 4G_D(I^{11,22} + I^{12,21})
\]  

\[
= \frac{4G_D}{(2\pi)^3} \int d\mathbf{k} \Delta \left( \frac{\cos 2\theta^{12,21}_\pm}{\bar{\omega}_-^{12}} + \frac{\cos 2\theta^{11,22}_\pm}{\bar{\omega}_-^{11}} + \frac{\cos 2\theta^{12,21}_\pm}{\bar{\omega}_+^{12}} + \frac{\cos 2\theta^{11,22}_\pm}{\bar{\omega}_+^{11}} \right) \cos^2 \left( \frac{\phi_1 - \phi_2}{2} \right)
\] 

In the above, $\bar{\omega}^{ia}_\pm = \sqrt{\Delta^2 \cos^2((\phi_1 - \phi_2)/2) + \xi_\pm^{ia}}$, $\xi_\pm^{ia} = \bar{\epsilon} \pm \bar{\mu}_{ia}$, and, $\cos 2\theta^{ia,jb}_\pm$ has been defined after Eq. 19. Finally, the minimisation of the thermodynamic potential with respect to the thermal functions $\theta_\pm(\mathbf{k})$ gives
\[
\sin^2 \theta_\pm^{ia} = \frac{1}{\exp(\beta\omega_\pm^{ia}) + 1}
\]
Various $\omega^{\alpha\beta}$'s are given explicitly as follows.

$$\omega^{11}_\pm = \tilde{\omega}^{11}_\pm + \delta_\epsilon \pm \delta_{\mu}^{11}$$

$$\omega^{12}_\pm = \tilde{\omega}^{12}_\pm + \delta_\epsilon \pm \delta_{\mu}^{12}$$

$$\omega^{21}_\pm = \tilde{\omega}^{21}_\pm - \delta_\epsilon \mp \delta_{\mu}^{12}$$

$$\omega^{22}_\pm = \tilde{\omega}^{11}_\pm - \delta_\epsilon \mp \delta_{\mu}^{11}$$

(34)

and, finally, for the noncondensing colors $\omega^{3\alpha}_\pm = \epsilon^3 \pm \mu^3$. Also for the strange quark $(i=3)$ $\omega^{3a}_\pm = \epsilon^3 \pm \mu^3$, with $\epsilon_3 = \sqrt{k^2 + M^2}$. As already mentioned, the first index refers to flavor and the second index refers to color. Here $\delta_\epsilon = (\epsilon_1 - \epsilon_2)/2$ is half the energy difference of the two quarks which condense and e.g. $\delta_{\mu}^{11} = (\mu_{11} - \mu_{22})/2$, is half the difference of the chemical potentials of the two quarks which condense. Further $\tilde{\omega}^{\alpha\beta}_\pm$ are defined earlier after Eq. (32). Note that in the absence of imposing the charge neutrality condition all the four quasi particles will have the same energy $\tilde{\omega}_-$. It is clear from the dispersion relations given in Eq. (34) that it is possible to have zero modes, i.e., $\omega^{ia}_\pm = 0$ depending upon the values of $\delta_\epsilon$ and $\delta_{\mu}$. So, although we shall have nonzero order parameter $\Delta$, there will be fermionic zero modes or the gapless superconducting phase. We shall discuss more about it section III.

Next, let us focus our attention for the specific case of superconducting phase and the chemical potential associated with it. First let us note that the diquark condensate functions depend upon the average of the chemical potentials of the quarks that condense. Since this is independent of $\mu_3$ we can choose $\mu_3$ to be zero. In that case, $\mu^{11} = \mu + 2/3\mu_E + \mu_s/\sqrt{3} = \mu_1^{12}$ and also $\mu^{21} = \mu - 1/3\mu_E + \mu_s/\sqrt{3} = \mu_2^{22}$. Thus the chemical potentials of the light flavors become degenerate for both the colors that take part in condensation. This also means that the average chemical potential of both the condensing quarks are the same i.e. $\bar{\mu}^{12} = \bar{\mu} = \bar{\mu} + \mu_1/6\mu_E + \mu_s/\sqrt{3}$. For the same reason we also have $\delta_{\mu}^{12} = \mu_E/2 = \delta_{\mu}^{11} = \delta_{\mu}$.

With this condition, it is also clear from Eq. (34) that the quasi particle energies for each flavor becomes degenerate for both the colors which take part in condensation. Thus the quasi particle energies now become $\omega_{-1} = \omega_+ + \delta_\epsilon - \delta_{\mu}$ and $\omega_{-2} = \omega_+ - \delta_\epsilon + \delta_{\mu}$, for u and d quarks respectively. Similarly for the antiparticles the energies are given as $\omega_{+1} = \omega_+ + \delta_\epsilon + \delta_{\mu}$ and $\omega_{+2} = \omega_+ + \delta_\epsilon - \delta_{\mu}$, for u and d quarks. The gap equation given by Eq. (32) then reduces to

$$\Delta = \frac{8G_D}{(2\pi)^3} \int d\mathbf{k} \Delta \left[ \frac{1}{\bar{\omega}_-} \left( \cos^2 \theta_1 - \sin^2 \theta_2 \right) + \frac{1}{\bar{\omega}_+} \left( \cos^2 \theta_1 - \sin^2 \theta_2 \right) \right] \cos^2 \left( \frac{\phi_1 - \phi_2}{2} \right),$$

(35)

where, $\bar{\omega}_\pm = \sqrt{\Delta^2 \cos^2((\theta_1 - \theta_2)/2) + (\epsilon + \mu)^2}$.

Now using these dispersion relations, the mass gap equation Eq. (27) and the superconducting gap equation Eq. (35), the thermodynamic potential given in Eq. (21) becomes

$$\Omega = \Omega_{ud} + \Omega_s + \Omega_{\epsilon},$$

(36)

where, $\Omega_{ud}$ is the contribution from the light quarks and is given as

$$\Omega_{ud} = \frac{8}{(2\pi)^3} \int d\mathbf{k} \left[ \sqrt{k^2 + m^2} - \frac{1}{2} \left( \frac{\Delta^2}{2G_D} + \sum_{i=1,2} \frac{(M^i - m)^2}{8G_s} \right) \right]$$

$$+ \frac{4}{(2\pi)^3} \int d\mathbf{k} \left[ \sqrt{k^2 + m^2} - \frac{1}{2} (\epsilon_1 + \epsilon_2) \right]$$

$$- \frac{2}{\beta(2\pi)^3} \sum_{i=1,2} \int d\mathbf{k} \left[ \log(1 + \exp(-\beta(\epsilon_1 - \mu^3))) + \log(1 + \exp(-\beta(\epsilon_2 + \mu^3))) \right].$$

(37)
Similarly the contribution from the strange quarks to the thermodynamic potential, \( \Omega_s \), is given as

\[
\Omega_s = \frac{6}{(2\pi)^3} \int dk \left[ \sqrt{k^2 + m_s^2} - \sqrt{k^2 + M_s^2} \right] - \frac{2}{\beta(2\pi)^3} \sum_{a=1,3} \int dk \left[ \log(1 + \exp(-\beta(\epsilon_3 - \mu_3^a))) + \log(1 + \exp(-\beta(\epsilon_3 + \mu_3^a))) \right] + \frac{(M_s - m_s)^2}{8G_s}.
\]

Finally, the contribution of the electron to the total thermodynamic potential \( \Omega_e \) is as given in Eq. (24). The first three lines in Eq. (37) correspond to the contribution from the quarks taking part in the condensation while the fourth and fifth lines correspond to the contribution from the two light quarks with the blue color.

Using Eqs. (37) and (38), the gap equations for the masses of u,d quarks are given by

\[
M_i - m_i = \frac{8G_s}{(2\pi)^3} \int dk \frac{M_i}{\epsilon_i} \left( 1 + \frac{\xi_+}{\omega_+} \left( \cos^2 \theta^i_- - |\epsilon_{ij}| \sin^2 \theta^j_+ \right) + \frac{\xi_-}{\omega_-} \left( \cos^2 \theta^i_+ - |\epsilon_{ij}| \sin^2 \theta^j_- \right) \right)
- \sin^2 \theta^i_- - \sin^2 \theta^i_+ + |\epsilon_{ij}| \left( \sin^2 \theta^j_+ + \sin^2 \theta^j_- \right) - \sin^2 \theta^i_3 - \sin^2 \theta^i_3 \right),
\]

The mass gap equation for the strange quarks is given as

\[
M_3 - m_3 = \frac{8G_s}{(2\pi)^3} \int \frac{M_3}{\epsilon_3} \sum_{a=1,3} \left( 1 - \sin^2 \theta_{a3}^- \sin^2 \theta_{a3}^+ \right).
\]

It is worthwhile to take the zero temperature limit of Eq. (37) to compare with the results obtained earlier in Ref. [2, 3]. Using the relation \( \lim_{\beta \to \infty} \frac{1}{4} \ln(1 + \exp(-\beta \omega)) = -\omega \theta(-\omega) \), we have the contributions from the u,d quarks beyond chiral symmetry restored case (i.e. \( M_u = 0 = M_d \)) given as

\[
\Omega_{ud}(T = 0, M_{u,d} = 0) = \frac{3\Lambda^4}{2\pi^2} - \sum_{i=1,2} \frac{\mu_i^3}{12\pi^2} \frac{2}{\pi^2} \int k^2 \left( \sqrt{(k - \bar{\mu})^2 + \Delta^2} + \sqrt{(k + \bar{\mu})^2 + \Delta^2} \right) dk
+ \frac{2}{\pi^2} \int_{\mu_-}^{\mu_+} k^2 \left( \sqrt{\Delta^2 + (k - \mu)^2 + \delta^2} \right) dk
\]

where, \( \mu_{\pm} = \bar{\mu} \pm \sqrt{\delta^2 - \Delta^2} \). The limits of the integrations over momentum in the third integral arises from the conditions of nonnegativity of the argument of theta function for nonzero contributions while taking the zero temperature limit. It is reassuring that the expression is identical as in Ref. [2], where the two flavor color superconductivity was investigated.

Thus the thermodynamic potential is a function of three parameters: the two mass gaps and a superconducting gap which need to be minimised subjected to the conditions of electrical and color charge neutrality. The electric and charge neutrality constraints are given respectively as

\[
Q_E = \frac{2}{3} \rho^1 - \frac{1}{3} \rho^2 - \frac{1}{3} \rho^3 - \rho_e = 0,
\]

and,

\[
Q_s = \frac{1}{\sqrt{3}} \sum_i (\rho_i^1 + \rho_i^2 - 2\rho_i^3) = 0.
\]

In the above \( \rho^i = \langle \psi_1^i \psi_1^i \rangle \) (not summed) and \( I_{1a}^a \) is as given in Eq. (28). Further, \( \rho^i = \sum_{a=1,3} \rho^a_i \).

It may be worthwhile to note that the neutrality conditions Eqs. (42) and (43) can be combined to give rise to a simpler equation as

\[
3Q_E - \frac{\Delta^2}{3}Q_s = 0, \text{ i.e.}
\rho_{11}^1 - \rho_{22}^2 + \rho_{33}^3 - \rho_e = 0
\]
At zero temperature, for the BCS phase, the number densities of the condensing u and d quarks are the same i.e., \( \rho^{11} = \rho^{22} \), and Eq. (41) can be solved for the electric chemical potential as \( \mu_E = -\frac{1}{2} \left( \mu - \frac{\rho \Lambda}{\sqrt{2}} \right) \). It can also be shown by substituting this solution in Eq. (45) that \( \mu_8 \) is very small compared to the chemical potentials \( \mu_E \) and \( \mu \) as long as \( \Delta/\mu \) is small. On the other hand, for the case of gapless phase (\( \Delta < \left| \delta_{\mu} \right| \)), one can solve Eq. (44) for the gap at zero temperature as

\[
\Delta = \sqrt{\frac{\mu^2}{4} - \left( \frac{\rho^3 \mu^2}{6\bar{\mu}^2} \right)} \tag{45}
\]

Eq. (36-43) and the superconducting gap equation Eq. (35) constitute the basis of the numerical calculations that we shall discuss in the next section.

### III. RESULTS AND DISCUSSIONS

For numerical calculations we have taken the values of the parameters of NJL model as follows. The cutoff \( \Lambda \) and the scalar coupling \( G_s \) are chosen by fitting the pion decay constant \( f_\pi = 93 \) MeV and the chiral condensate \( \langle \bar{u}u \rangle^{1/3} = -250 \text{ MeV}=\langle \bar{d}d \rangle^{1/3} \). In case of non zero current quark masses, the additional parameters can be fixed from the values of pion and kaon masses. In the following, however, we shall choose the current quark masses for the light quarks as zero. This leads to the coupling constant \( G_s \) and the cut off \( \Lambda \) as \( G_s = 5.0163 \text{ GeV}^{-2} \) and \( \Lambda = 0.6533 \text{ GeV} \). Similar to Ref. [25] we take the current quark mass of strange quarks as \( m_s = 120 \) MeV as a typical value giving “reasonable” vacuum properties. With this choice of parameters, the constituent quark masses at zero temperature and density are given as \( M_1 = 0.313 \text{ GeV}=M_2 \), and for strange quark \( M_3 = 0.514 \text{ GeV} \). These values are similar to those obtained in Ref. [20], where the parameters have been fixed by fitting vacuum masses and decay constants of pseudoscalar mesons.

Let us begin with the discussions of results without imposition of charge neutrality conditions. At zero temperature, the behaviour of the gap parameters as a function of baryon chemical potential (three times the quark chemical potential) is displayed in Fig.1-a. We may point out here that these solutions for the gaps corresponds to the solutions for which the thermodynamic potential is minimised. In fact, in general, for certain values of the chemical potential there can be several solutions of the gap equations, but we have chosen the ones which minimise the thermodynamic potential.

At low chemical potentials \( \mu_B < \mu_1 \approx 0.6 \text{ MeV} \), the diquark gap vanishes and the masses of the quarks stay at their vacuum values. The entire region below \( \mu_B = \mu_1 \) corresponds to vacuum solution and has zero baryon number. At \( \mu_B = \mu_1 \) a first order phase transition takes place and the system is a two flavor color superconductor. The diquark gap jumps from zero to about 80 MeV at this point. At the same point, the masses of u and d quarks drop from their vacuum values to zero.

A first order transition is characterised by the existence of meta stable phases, the equivalent of e.g. oversaturated vapour. The masses corresponding to these metastable phases are the nontrivial (\( M \neq 0 \)) solutions of the mass gap equation but have higher thermodynamic potential as compared to the solution corresponding to the stable phase.

With increasing \( \mu_B \) the superconducting gap increases until it reaches a maximum at \( \mu_B \approx 1.565 \text{ MeV} \). The baryon number density becomes nonzero in the superconducting phase. At \( \mu_B = \mu_1 \), it jumps from zero to 0.34 fm\(^{-3} \) (around twice the nuclear matter density). The strange quark density remains zero till \( \mu_B \approx 1.630 \text{ MeV} \) as strange quark mass remains at its vacuum value in this regime. Just beyond this point, the constituent mass of strange quark drops to about half its vacuum value and then decreases slowly. The density of strange quarks also becomes nonzero beyond \( \mu_B = 1.630 \text{ MeV} \). This will have important consequences regarding charge neutrality conditions as we shall discuss little later. A flavor mixing interaction probably could lead to a reduced value of the strange quark and hence to a lower threshold for nonzero density of strange quarks. At larger chemical potential, the results become cutoff dependent and the diquark condensate decreases with \( \mu_B \). We also note here that contrary to the vector-interactions of Ref. [3], we do not find a window in chemical potential where both chiral and the diquark condensate coexist.

In Fig. 1-b we show the temperature dependence of masses at zero density. Chiral symmetry is restored for light quarks at temperature about 195 MeV. The sharp first order transition of zero temperature becomes a second order transition at zero baryon density as is reflected in the smooth variation of the mass which is proportional to the order parameter \( \langle \bar{\psi} \psi \rangle \).

In Fig. 2 we show the resulting phase diagram for the chiral transition in the temperature and quark chemical potential (1/3\( \mu_B \)) plane in the present model when charge neutrality conditions are not imposed. The middle line is...
FIG. 1: Gap parameters when charge neutrality conditions are not imposed. Fig.1-a shows the gaps at zero temperature as a function of baryon chemical potential. Fig. 1-b shows the gaps at zero baryon density as a function of temperature. Solid curve refers to masses of u and d quarks, the dotted curve refers to mass of strange quarks and the dashed curve corresponds to the superconducting gap.

the critical line and corresponds to the state where the two nontrivial solution of the gap equations have the same thermodynamic potential. Along this line the thermodynamic potential has two minima of equal depth separated by a barrier and the barrier height decreases with temperature. At the tricritical point this barrier vanishes. Beyond this point there is only one solution for the gap equation and the transition is second order. This critical point \((T_c, \mu_c)\) turns out to be \((74.9, 285)\) MeV. The other two lines are the spinodal lines constraining regions of spinodal instability.

We might note here that for this transition the strange quarks do not play a role as their mass is much too large to contribute to the dynamics.

In Fig.3 we display the density dependence of the superconducting gap for various temperatures. Similar to various BCS-type calculations, this transition is of the second order. The critical temperature is about half of the gap value at zero temperature as is in BCS case. This phase transition is a second order phase transition with the transition temperature \(T_c \simeq 0.57\Delta(T = 0)\).

We next extend our discussions to the case where the charge neutrality conditions \((\mu_B = 0 = \mu_E)\) are imposed. In Fig.4 we show the dependence of masses when these conditions are imposed. For low baryon chemical potential, we see that the d quark masses start decreasing earlier than that of u- quark. Charge neutrality conditions force the d quark density to be larger (almost twice) than that of u-quark density making their masses vanishing earlier.

At higher chemical potentials, strange quarks help in maintaining the charge neutrality condition. We might note here that, in general there could be several solutions of the mass gap solutions. A solution which may be free energetically favorable when charge neutrality condition is not imposed can ingeneral be disfavored when charge neutrality condition is imposed. A typical example is shown in Fig.5 for strange quark mass gap for electrically neutral normal \((\Delta = 0)\) u-d-s matter.

In Fig. we display graphical solutions for strange quark mass when electrical charge neutrality condition is imposed. The two figures – Fig.5-a–b correspond to two different values for chemical potentials \(\mu_B\). Each value of electron chemical potential given in x axis then gives the value of the strange quark chemical potential as \(\mu_s = \mu_B - (1/3)\mu_E\). The dotted lines in these figures gives the solutions of the mass gap equation for the strange quark for each \(\mu_s\). As may be seen there can be three different solutions for given baryon chemical potential and a given value of electric chemical potential. We have also plotted here the charge neutrality line i.e. strange quark mass parameter as a function of electric chemical potential such that the total electric charge is zero. Thus, along the solid line the charge neutrality condition is satisfied. Along the dashed line mass gap equation is satisfied. The intersection of
FIG. 2: Phase diagram in the $(\mu, T)$ plane. Middle curve is the critical line and the outer lines are the lower and upper spinodals.

FIG. 3: Superconducting gap as a function of baryon density for different temperatures. Here charge neutrality conditions (i.e., $\mu_E = 0 = \mu_B$) are not imposed.

the charge neutrality line with the curves satisfying the gap equation defines the desired solution satisfying both gap equation and the charge neutrality condition. When charge neutrality condition is not imposed, the one which has minimum free energy is the required solution. However, this need not be the solution when charge neutrality condition is imposed. Of the two cases, shown in Fig. 5, for $\mu_B = 1528$ MeV (Fig. 5-a), the solution with mass about 520 MeV is the one which satisfies the charge neutrality condition. For $\mu_B = 1606$ MeV, (Fig. 5-b) $M_s \simeq 266$ MeV satisfies the charge neutrality condition. We might note here that, when charge neutrality condition is not imposed, the branch
We compute the thermodynamic potential numerically as follows. For a given temperature and quark chemical potential, the thermodynamic potential is minimised with respect to the strange quark constituent mass after solving the superconducting gap equation Eq. (35) selfconsistently. The values of $\mu_E$ and $\mu_8$ are varied so that the charge
neutrality conditions Eq. (12) and Eq. (13) are satisfied. The resulting superconducting gap is shown in Fig. 6 for different temperatures.

As may be seen for smaller values of the baryon chemical potential ($\mu_B < 1470$ MeV) we have smaller values of the gap which reaches a maximum about 80 MeV at $\mu_B = 1390$ MeV at zero temperature. Then it decreases and vanishes at $\mu_B = 1470$ MeV. We have also verified that the difference between the value of the gap as obtained numerically satisfying the charge neutrality condition and the the approximate expression given in Eq. (45) is less than a percent for the gapless phase. The gap remains zero till $\mu_B = \mu_3 = 1530$ MeV. In fact, in the region between $\mu_2$ and $\mu_3$ we do not have any real solution for $\Delta$. At $\mu_3$ the system jumps from the normal phase to the BCS phase with a gap about 132 MeV. The baryon number density jumps from $\rho_B = 0.96 \text{fm}^{-3}$ to $1.54 \text{fm}^{-3}$ i.e. from 6 to 9.5 times the nuclear matter density. The interval in the baryon chemical potential between the gapless and BCS phase within which it is normal quark matter decreases with increase in temperature and then disappears as may be noted in Fig. 6. The sharp transition between gapless phase to the BCS phase as a function of baryon chemical potential at smaller temperature becomes a smooth transition as temperature increases as may be seen from Fig. 6–b. We also would like to note that the strange quark number density at zero temperature is zero in the gapless phase. This is in contrary to observation made in Ref. [7] where a vector vector point interaction was considered. However, as temperature is increased, the number densities of the strange quarks become nonzero. E.g., at a temperature of 50 MeV, strange quarks contribute upto about 15% of that of density of u–quarks.

Let us next discuss now some of the characteristics of the gapless modes which occur for smaller values of the baryon chemical potential ($\mu_B < \mu_2$). In fact, in this region, in general, there are two solutions to gap equations for given $\bar{\mu}$ and $\delta\mu$. One is the usual BCS solution with a larger value of the gap and the other one with a smaller value of the gap. However, it so happens that in this range of the chemical potential, the solution of the gap equation with larger value of the gap (the BCS solution) does not satisfy the charge neutrality condition. The situation here is similar to the case for strange quark mass when charge neutrality condition is imposed.

At zero temperature, gapless modes occur when the gap is less than half the difference of the chemical potential $\delta\mu$ of the two condensing quarks. It is easy to show also that the excitation energy $\omega_2$ of the d-quark vanishes at momenta $\mu_- = \bar{\mu} - \sqrt{\delta\mu^2 - \Delta^2}$ and $\mu_+ = \bar{\mu} + \sqrt{\delta\mu^2 - \Delta^2}$ at zero temperature. In this phase the number densities of the condensing u and d quarks are not the same. Let us note that the number densities of u-red quarks are given as

$$\rho^{11} = \langle \psi^{11\dagger} \psi^{11} \rangle = \frac{2}{(2\pi)^3} \int d\mathbf{k} (F^{11} - F_1^{11}) = \rho^{12}$$

(46)
where

\[ F^{11} = \sin^2 \theta_1 - \frac{1}{2} \left( 1 - \frac{\bar{\xi}_- - \bar{\omega}_-}{\bar{\omega}_-} \right) \left( 1 - \sin^2 \theta_1 - \sin^2 \theta_2 \right) \]  
(47)

for the quarks and

\[ F_{11}^1 = \sin^2 \theta_+^1 - \frac{1}{2} \left( 1 - \frac{\bar{\xi}_+ - \bar{\omega}_+}{\bar{\omega}_+} \right) \left( 1 - \sin^2 \theta_+^1 - \sin^2 \theta_+^2 \right) \]  
(48)

for the antiquarks. Interchanging indices 1 and 2 we shall have the expression for the number density of the d quarks taking part in condensation. At zero temperature, the distribution function \( \sin^2 \theta_1 \) vanishes for the u quarks as corresponding energy \( \omega_1 = \bar{\omega} - \delta \mu \) is always positive (let us remember that \( \delta \mu \) is negative at zero temperature) where as the distribution function for d quarks is nonzero between \( \mu_- \) and \( \mu_+ \). In fact, the number densities at zero temperature are given as

\[ \rho^u = 2 \rho^{11} = \frac{4}{(2\pi)^3} \int \frac{d^3 k}{2} \left[ \left( 1 - \frac{\bar{\xi}_- - \bar{\omega}_-}{\bar{\omega}_-} \right) \left( 1 - \theta(-\omega_2) - \left( 1 - \frac{\bar{\xi}_+ - \bar{\omega}_+}{\bar{\omega}_+} \right) \right) \right] \]  
(49)

for the u quarks. Similarly the density of the d quarks taking part in condensation is given at zero temperature by

\[ \rho^d = 2 \rho^{21} = \frac{4}{(2\pi)^3} \int \frac{d^3 k}{2} \left[ \theta(-\omega_2) + \frac{1}{2} \left( 1 - \frac{\bar{\xi}_- - \bar{\omega}_-}{\bar{\omega}_-} \right) \left( 1 - \theta(-\omega_2) - \left( 1 - \frac{\bar{\xi}_+ - \bar{\omega}_+}{\bar{\omega}_+} \right) \right) \right] \]  
(50)
The integrands of Eq.s (49) and (50) which are the occupation numbers in the momentum space are plotted in Fig. 7. It is easy to see from the expressions in Eq.s (49) and (50) that except in the interval \((\mu^-, \mu^+)\), in the momentum space, the theta function does not contribute and the distribution is like the BCS distribution. The occupation number distribution is a ‘dented’ theta function with the ‘dent’ \(\sim \Delta\) around the average fermi surface. In the region between \(\mu^-\) and \(\mu^+\), the occupation numbers resemble like that of normal matter as \(\rho_u\) becomes almost zero but for the vanishingly small antiparticle contribution. On the other hand, the integrand of \(\rho_d\) becomes unity in the same

FIG. 8: Number densities of u quarks (solid) and d quarks (dot-dashed) participating in superconducting phase at zero (Fig. 8.a) and at finite temperatures (fig. 8.b).

FIG. 9: Excitation energies as a function of chemical potential.
limit for the antiparticle contributions. This is shown in Fig. 7. It follows therefore that the total number of u-quarks and d-quarks (integrated over the momentum) are not the same. The difference in their number densities is in fact given by

$$\delta \rho_{sc} = \rho_2 - \rho_1 = \frac{4}{(2\pi)^3} \int \! dk \left[ (\sin^2 \theta_2^2 - \sin^2 \theta_1^2) + (\sin^2 \theta_2^1 - \sin^2 \theta_1^1) \right]. \quad (51)$$

Thus at finite temperatures the $\delta \rho_{sc}$ will always be nonzero as long as the difference in the chemical potential is nonzero. At zero temperature this difference will be nonzero only for the gapless solution i.e. for gaps less than half the magnitude of the difference in chemical potentials. This is in fact given by $\delta \rho_{sc}(T = 0) = 2/(3\pi^2)(\mu_3^+ - \mu_3^-)$. In Fig. 8, we plot the number densities of u and d quarks taking part in the condensation. At zero temperature, the number densities become the same in the BCS phase. However, because of thermal distributions, the two densities do not become exactly identical even in the BCS phase at finite temperatures.

In Fig. 9 we plot the dispersion relations - the excitation energies of the quasi-particles as a function of momentum for quark chemical potential $\mu_q = (1/3)\mu_B \simeq 379\text{MeV}$ and at zero temperature. Let us note that the dispersion relations are given by $\omega_1 = \bar{\omega} - \delta \mu$ and $\omega_2 = \bar{\omega} + \delta \mu$. The gap in this case turns out to be $\Delta = 59 \text{MeV}$ and the difference of the chemical potential turns out to be $\delta \mu \simeq -66 \text{MeV}$ at zero temperature. Far from the pairing region $\bar{\mu} \simeq 354 \text{MeV}$, the spectrum looks like that of usual BCS type dispersion relations. Of the two excitation energies, $\omega_1$ shows a minimum at the average fermi momentum $\bar{\mu}$ with a value $\omega_1(|k| = \bar{\mu}) = \Delta + \delta \mu \simeq 125 \text{MeV}$. On the other hand, $\omega_2$ becomes gapless ($\omega_2 = 0$) at momenta $\mu_-$ and $\mu_+$. In this ‘breached’ pairing region we have only unpaired down quarks and no up quarks.

In Fig. 10 we show the temperature dependence of the gapless modes. We have taken the quark chemical potential as 379 MeV as earlier. Initially the gap increases and then decreases monotonically and vanishes at $T_C = 48\text{MeV}$ i.e. $T_C = 0.81\Delta(T = 0)$. Clearly, this shows a departure from the BCS behaviour with $T_C/\Delta(T = 0) \neq 0.567$. Further, it is also seen that for temperatures between 28 MeV to 32 MeV, the gap is larger than difference of chemical potential and $\omega_2$ no longer is exactly gapless in between these temperatures. The excitation energies however are much smaller than temperature and are almost similar to the excitation energies between $\mu_-$ and $\mu_+$ at zero temperature. For temperatures beyond 32 MeV, the gap becomes smaller than $|\delta \mu|$ and we have again the gapless modes occurring at corresponding $\mu_-$ and $\mu_+$.

FIG. 10: Temperature dependence of gap parameter in the gapless phase (solid line). $|\delta \mu|$ as a function of temperature is shown as the dotted line.

It may be worthwhile to mention here that gapless modes in superconductivity were known theoretically in the context of superconducting matter with finite momentum as well as in condensed matter systems with magnetic
impurities \cite{27}. Recently it has been investigated for cold fermionic atoms \cite{26,28}. Gapless modes in the context of quark matter has been first proposed in Ref.\cite{24} for color flavor locked matter. However, this corresponded to a meta stable phase. Gapless modes in neutral quark matter were first emphasized for the two flavor color superconductivity in Ref.\cite{6} and for the 2SC+s quark matter in Ref.\cite{7}. Stable gapless modes for color flavor locked matter has been first proposed in Ref.\cite{29} and has been confirmed in Ref. \cite{30} in a more general structure for the gap. The temperature dependence of the CFL gapless modes has been studied in Ref.\cite{31}.

As the chemical potential is increased beyond 1530 MeV at zero temperature the solution for the gap jumps to a higher value of about 130 MeV almost similar to the case when charge neutrality condition is not imposed. This corresponds to the usual BCS solution. In this case, the numbers of u quarks and d quarks participating in condensation are the same. The charge neutrality condition however is maintained by the third color, the electron as well as the strange quarks. One essential effect of including strange quarks is that the electron density starts decreasing for chemical potentials greater than the strange quark mass as the strange quark can carry the negative charge to maintain electric charge neutrality condition. This has the effect that the lowest excitation energy $\omega_2 = \bar{\omega} + \delta_{\mu}$ in the BCS pairing case becomes large due to both the large value of the gap as well as the smaller magnitude of the electron chemical potential.

IV. SUMMARY

We have analysed here in NJL model the structure of vacuum in terms of quark–antiquark as well as diquark pairing at finite temperature. The methodology uses an explicit variational construct of the trial state. The gap function as well as the thermal distribution functions are also determined variationally. To consider neutron star matter, we have imposed the color and electric charge neutrality conditions through the introduction of appropriate chemical potentials. It has been noted and emphasized earlier that projecting out the color singlet state from color neutral state costs negligible free energy for large enough chunk of color neutral matter \cite{3}. We observe that the mass of the strange quark plays a sensitive role in maintaining charge neutrality condition. Further, we observed that solutions for the gap equations which may not be free energetically preferable solutions, can be the preferred solutions when constraints of charge neutrality conditions are imposed. We also find that with increase in chemical potential, the quark matter has a transition from a gapless phase to the BCS phase through an intermediate normal quark matter phase. This interval between the gapless and BCS phase decreases and then disappears with increase in the temperature. We also find that there are no strange quarks in the gapless phase at zero temperature contrary to the observation in the vectorial point interaction model as in Ref.\cite{7}. However, at higher temperatures only the contribution of strange quarks becomes significant even in the gapless mode. We have looked into the gapless to BCS transition as a function of density for various temperatures. The sharp transition at zero temperature becomes a smooth transition at higher temperatures as density is increased.

We have focussed our attention here to the two flavor superconducting phase with strange quarks. The variational method adopted can be directly generalised to include color flavor locked phase and one can then make a free energy comparison regarding the possibility of which phase would be thermodynamically favourable at what density. This will be particularly interesting for cooling of neutron stars with a CFL core. We have considered here homogeneous phase of matter. However, we can also consider mixed phases of matter with matter being neutral on the average. Surface tension between the superconducting phase and the normal quark phase will become an important factor in determining the stability of this mixed phase \cite{32}. Some of these problems are being investigated and will be reported elsewhere \cite{33}.

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