QCD Corrections to Neutralino–Nucleon Scattering

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Abstract

We calculate the dominant loop corrections from both standard and supersymmetric QCD to the effective coupling of neutralinos to nucleons. The potentially largest corrections come from gluino–squark loop contributions to the Higgs boson couplings to quarks; these corrections also affect the leading spin–independent squark exchange contribution. Ordinary QCD corrections to the effective coupling of CP–even Higgs bosons to two gluons are also sizable. For large tan\(\beta\) values, i.e. in the region of parameter space probed by current and near–future direct Dark–Matter search experiments, the total corrections can exceed a factor of three.
1. Introduction

There is convincing evidence that dark (non–luminous) matter (DM) contributes between 20 and 90% of the critical density of the Universe, with values around 30 to 40% currently being favored [1]. Studies of Big Bang nucleosynthesis show that this DM cannot be formed by baryons [2], unless they are bound in primordial black holes. Light neutrino DM is strongly disfavored by analyses of galaxy formation [3]. One thus needs physics beyond the Standard Model (SM) to explain the dark matter in the Universe. The probably best motivated, and certainly most frequently studied, particle physics candidate is the lightest neutralino $\tilde{\chi}$ predicted by supersymmetric extensions of the SM. In the Minimal Supersymmetric extension of the Standard Model (MSSM) [5], its stability against decay into ordinary particles can be guaranteed by a symmetry called $R$–parity [6], and for wide regions of the supersymmetric parameter space one predicts a relic density in the desired range.

These arguments in favor of neutralino DM have motivated several experimental groups around the world to search for ambient relic neutralinos. The strength of the expected signal in the two most promising search strategies is directly proportional to the neutralino–nucleon scattering cross section $\sigma_{\tilde{\chi}N}$; these are the search for high–energy neutrinos originating from the annihilation of neutralinos in the center of the Sun or Earth (the so–called “indirect detection”) [7], and the search of the elastic scattering of ambient neutralinos off a nucleus in a laboratory detector (“direct search”) [8]. An accurate calculation of $\sigma_{\tilde{\chi}N}$ for given model parameters is thus essential for the interpretation of the results of these searches.

The matrix element for $\tilde{\chi}N$ scattering, mediated by Higgs bosons, $Z$ boson and squark exchange diagrams, receives both spin–dependent and spin–independent contributions. The former are important for neutralino capture in the Sun, but are irrelevant for capture in the Earth, and play a subdominant role in most direct search experiments, which employ fairly heavy nuclei (Si, Ge, I, Na). The spin–independent contribution in turn is usually dominated by Higgs exchange diagrams, where the Higgs bosons couple either directly to light ($u, d, s$) quarks in the nucleon, or couple to two gluons through a loop of heavy ($c, b, t$) quarks or squarks. In some cases squark exchange contributions can also be important.

In this note we point out that these spin–independent contributions receive large QCD corrections. The effective Higgs couplings to gluons receive standard QCD corrections with large positive coefficients, related to the QCD $\beta$–function. In addition, gluino–squark loop corrections to the quark masses (or to the Yukawa couplings, for fixed physical quark masses) can become very large for down–type quarks, and for the $s$ quark in particular. We compute both kinds of corrections, and argue that corrections to other $\tilde{\chi}N$ scattering diagrams should be small. The required formulae are given in Section 2, while Section 3 contains some numerical examples. In Section 4 we briefly summarize our results and draw some conclusions.
2. Formalism

The spin–independent $\tilde{\chi} N$ scattering amplitude receives contributions from Higgs bosons and squark exchange diagrams. Models with unified gaugino masses typically have squark masses much larger than the neutralino mass, $m_{\tilde{q}} \gtrsim 5 m_{\tilde{\chi}}$, in which case the dominant contribution usually comes from Higgs boson exchange. Note that only scalar Higgs couplings to neutralinos contribute in the non–relativistic limit. In the absence of significant CP–violation in the Higgs sector, one therefore only has to include contributions of the two neutral CP–even Higgs bosons. The contribution of the heavier Higgs boson often dominates, since its couplings to down–type quarks are enhanced if the ratio of vacuum expectation values (vev’s) is large, $\tan \beta \gg 1$.

CP–even Higgs bosons can couple to nucleons in two different ways: through an effective coupling to two gluons mediated by loops of heavy quarks or squarks, or directly to light quarks in the nucleon. The leading contribution to the $H^0 gg$ couplings comes from heavy quark triangle diagrams. To leading order, they can be described by the effective Lagrangian

$$L^{(Q)}_{H^0 gg} = -H^0 F_{\mu \nu} F^{\mu \nu} \frac{\alpha_s}{12\pi} \sum_{Q=c,b,t} \frac{c_i Q}{M_W},$$

where $H^0$ is one of the neutral CP–even Higgs bosons $h$ and $H$ and $F_{\mu \nu}$ is the gluon field strength tensor, $a$ being a color index. Note that the dimensionless coefficients $c_i Q$ are independent of $m_Q$; the factor $m_Q$ in the $H^0 \bar{Q} Q$ coupling is canceled by a factor $1/m_Q$ from the loop integral. Explicit expressions for these coefficients can e.g. be found in Ref. [11]. The effective Lagrangian eq. (1) gives rise to $H^0 \bar{N} N$ couplings, $N$ being a nucleon, through hadronic matrix elements [11, 12]

$$\frac{\alpha_s}{4\pi} \langle N | F_{\mu \nu} F^{\mu \nu} | N \rangle = -\frac{2}{9} m_N \left( 1 - \sum_{q=u,d,s} f_{Tq} \right),$$

where we have introduced

$$f_{Tq} = \frac{m_q}{m_N} \langle N | \bar{q} q | N \rangle.$$

The first point we wish to make is that the effective interaction (1) is subject to large corrections from ordinary QCD. These can most easily be computed using low–energy theorems [13]. The effective Higgs–gluon interaction can then be rewritten as

$$L^{(Q)}_{H^0 gg} = -\frac{1}{4} \frac{\beta_Q(\alpha_s)}{1 + \gamma_Q(\alpha_s)} F_{\mu \nu} F^{\mu \nu} \sum_{Q} \frac{c_i Q}{M_W}.$$

Here $\beta_Q$ is the contribution of heavy quark $Q$ to the QCD $\beta$–function and $\gamma_Q$ is the anomalous dimension of $m_Q$. Eq.(4) only describes interactions where all three external legs couple to the heavy quark line; but is valid to all orders in perturbation theory. Currently results up to order $\alpha_s^3$ are known [14]:

$$L^{(Q)}_{H^0 gg} = -\frac{1}{4} \frac{\alpha_s}{12\pi} F_{\mu \nu} F^{\mu \nu} \sum_{Q} \frac{c_i Q}{M_W} \left[ 1 + \frac{11}{4} \frac{\alpha_s(m_Q)}{\pi} + \frac{2777 - 201 N_f}{288} \frac{\alpha_s^2(m_Q)}{\pi^2} \right].$$
The scale of the overall factor $\alpha_s$ need not be specified, since the matrix element (2) is scale–independent. However, the size of the higher order corrections in the square brackets are determined by $\alpha_s$ taken at the scale $m_Q$, which is the only energy scale in the problem (note that we are interested in configurations with essentially vanishing momentum flow through the diagram). These corrections are therefore larger for $Q = c$ than for $Q = t$. $N_f$ counts the number of active flavors at scale $m_Q$; we take $N_f = 3, 4, 5$ for $Q = c, b, t$.

The general result eq. (3) can also be used for squark loop contributions to the Higgs–gluon coupling. Here results up to $O(\alpha_s^2)$ are known, giving a correction factor $1 + \frac{25}{6} \frac{\alpha_s(m_Q)}{\pi}$ in the effective Lagrangian, relative to the leading order result listed in Ref. [11]. Note that the correction factor is even larger than for quark loops; however, the overall contributions of squark loops to the effective $H^0_{i\bar{q}q}$ couplings at vanishing external momenta are always much smaller [11] than the quark loop contributions.

Potentially even larger corrections come from gluino–squark loop contributions to the $H^0_{\bar{q}qq}$ couplings, which are closely related to $\tilde{g} - \tilde{q}$ loop corrections to quark masses [13]. These loop contributions induce a coupling of quarks to the “wrong” Higgs current eigenstate. In particular, the masses of down–type quarks will now receive contributions from the vev of the Higgs field with positive hypercharge. Parametrically, these corrections are therefore of the order $\delta m_d/m_d \sim \frac{\alpha_s}{\pi} \tan \beta$. These corrections can thus become $O(1)$ for $\tan \beta \sim 50$, which is within the allowed range.

The large size of these corrections raises concerns regarding the reliability of perturbation theory. However, it has recently been shown [16, 17] that the one–loop corrections can be written in such a way that all higher–order corrections of the form $(\alpha_s/\pi \tan \beta)^n$ are re–summed. Note that the corresponding corrections to the couplings of up–type quarks are much smaller for $\tan \beta > 1$; given the uncertainties of the hadronic matrix elements appearing in the $\tilde{\chi}N$ scattering amplitude, corrections of order 10% or less can safely be ignored. We are thus only interested in the couplings of down–type quarks $D$ to CP–even Higgs bosons $h, H$, which can be written as [17]

\begin{align}
    s_h^D(M_S) &= \frac{g}{2M_W} \left[ -m_{SM}^D(M_S) \cos(\alpha - \beta) + m_{MSSM}^D(M_S) \sin(\alpha - \beta) \tan \beta \right] \\
    s_h^D(M_S) &= \frac{g}{2M_W} \left[ m_{SM}^D(M_S) \sin(\alpha - \beta) + m_{MSSM}^D(M_S) \cos(\alpha - \beta) \tan \beta \right]
\end{align}

(6)

(7)

where $g$ is the $SU(2)$ gauge coupling, and $\alpha$ is the Higgs mixing angle in the notation of Ref. [18]. In eqs. (7) $m_{SM}^D(M_S)$ stands for the running quark masses, where only standard QCD corrections are included; their numerical values are known (with some uncertainties) from experiment. $m_{MSSM}^D(M_S)$ is just the product of the $D$ Yukawa coupling with the vev of the Higgs field with negative hypercharge; it differs from $m_{SM}^D$ by sparticle loop corrections:

$$m_{MSSM}^D(M_S) = m_{SM}^D(M_S) - \frac{\alpha_s(M_S)}{3\pi} m_\tilde{q} \sin(2\theta_D) \left[ B_0(m_D^2, m_{\tilde{q}}, m_{\tilde{D}_1}) - B_0(m_D^2, m_{\tilde{q}}, m_{\tilde{D}_2}) \right].$$

(8)
Here \( \theta_D \) is the \( \bar{D}_L - \bar{D}_R \) mixing angle, \( m_{\bar{D}_1} \) and \( m_{\bar{D}_2} \) are the smaller and larger eigenvalue of the \( \bar{D} \) squark mass matrix, \( m_{\tilde{g}} \) is the gluino mass, and \( M_S \) is a scale of the order of the squark or gluino mass; we will take the value \( M_S = \left( m_{\tilde{g}}^2 m_{\bar{D}_1} m_{\bar{D}_2} \right)^{1/4} \). The first argument of the Passarino–Veltman two–point function \( B_0 \) can, to excellent approximation, be set to zero. In the case of the bottom quark (which contributes to the \( H_i^0 qg \) couplings) we add an additional correction from chargino–stop loops, which comes from the large top Yukawa coupling:

\[
\delta m_b^{\text{MSSM}}(M_S) = \frac{g^2}{64\pi^2} \frac{m_{b}^{\text{MSSM}}(M_S)m_t(M_S)}{M_W^2 \sin(2\beta)} \mu \sin(2\theta_t) \left[ B_0(m_b^2, |\mu|, |m_t|) - B_0(m_b^2, |\mu|, |m_{\tilde{t}_1}|) \right].
\]

Note that for \( \tan \beta \gg 1 \), \( \sin(2\theta_D) \simeq 2m_D\mu \tan\beta/(m_{\bar{D}_2}^2 - m_{\bar{D}_1}^2) \), and \( 1/\sin(2\beta) \simeq \tan\beta/2 \); the corrections eqs. (8) and (9) are therefore enhanced for large \( \tan \beta \), as advertised. Of course, if these corrections are set to zero, eqs. (7) reduce to the standard tree–level \( H_i^0 qq \) couplings [18].

It is well known that Yukawa couplings of quarks receive large QCD corrections, i.e. they show a strong scale dependence. Here we need these couplings at a relatively low scale, of the order of the quark mass itself, or in case of light quarks, at a scale not far from 1 GeV. In contrast, eqs. (8) should be interpreted as boundary conditions of the couplings of the effective theory that emerges when squarks and gluinos are integrated out; these boundary conditions are thus valid at a scale \( M_S \). Below this scale these couplings run according to standard renormalization group equations (RGE) [19]:

\[
s_{H,h}^D(Q_1) = s_{H,h}^D(Q_2) \cdot \left[ \frac{\alpha_s(Q_1)}{\alpha_s(Q_2)} \right]^{12/(33-2N_f)},
\]

where \( N_f \) is again the number of active flavors, and \( Q_1 \) and \( Q_2 \) are two energy scales. Since this number changes whenever we cross a heavy quark threshold, the total effect of the running has to be computed by applying eq. (10) repeatedly; e.g.,

\[
s_{H,h}^s(m_c) = s_{H,h}^s(M_S) \cdot \left[ \frac{\alpha_s(M_S)}{\alpha_s(m_t)} \right]^{12/21} \cdot \left[ \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right]^{12/23} \cdot \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{12/25}.
\]

We have seen in eq. (6) that the size of the \( \tilde{g} - \tilde{q} \) loop corrections to \( m_D \) is proportional to the amount of \( \bar{D}_L - \bar{D}_R \) mixing, which in turn is proportional to \( m_D \). This raises the question which \( m_D \) to use when computing \( \sin(2\theta_D) \). Since the corrections eqs. (8) and (9) are to be applied at the high scale \( M_S \) it is fairly clear that one should use a running mass \( m_D \) at this high scale. At the one–loop order we cannot distinguish between the choices \( m_D^{\text{SM}} \) and \( m_D^{\text{MSSM}} \) here. However, in Ref. [10] it has been shown that loop corrections to Higgs boson decay widths into quarks and squarks will be small only if the tree–level squark mass matrices are written in terms of \( m_D^{\text{MSSM}} \), i.e. if \( m_D^{\text{MSSM}} \) is used in the calculation of \( \sin(2\theta_D) \). This requires an iteration, which however usually converges.
quickly. This iteration is equivalent to the re–summation of higher orders advocated in Ref. [17]. We will show below that the difference between using \( m_D^{\text{SM}} \) and \( m_D^{\text{MSSM}} \) in the calculation of \( \sin(2\theta_D) \) can indeed be numerically significant.

The squark–gluino loop corrections to the mass of \( d \)--type quarks also affect the leading \( \mathcal{O}(m_q^{-2}) \) spin–independent contributions from \( d \)--type squark exchange. These contributions are proportional to the quark masses \([10, 11]\), either through the interference of gauge and Yukawa contributions to the neutralino–quark–squark couplings (which requires gaugino–higgsino mixing in the neutralino sector), or through \( \tilde{q}_L - \tilde{q}_R \) mixing. These corrections can again most easily be understood in terms of an effective \( f_q \bar{q}q\tilde{\chi}\tilde{\chi} \) interaction, where the coefficient \( f_q \) is determined by matching to the full theory at a scale \( Q \approx m_{\tilde{q}} \) \([11]\). Since both the \( \tilde{q}_L - \tilde{q}_R \) mixing angle and the quark Yukawa coupling are determined by \( m_q^{\text{MSSM}} \) rather than by \( m_q^{\text{SM}} \), the gluino–squark loop correction to the effective \( f_q \bar{q}q\tilde{\chi}\tilde{\chi} \) interaction can simply be obtained by multiplying the usual tree–level result \([10, 11]\) with \( m_q^{\text{MSSM}}(M_S)/m_q^{\text{SM}}(M_S) \). Of course, we need these couplings at a low, hadronic scale. However, the scale dependence of these couplings is identical to that of the \( H\bar{q}q \) couplings, i.e. is described by eq. \((10)\). This amounts to a purely multiplicative renormalization, which leaves the relative size of the \( \tilde{g} - \tilde{q} \) loop corrections unchanged. Altogether we thus have

\[
f^{(D)}_{D}\big|_{\text{improved}} = f^{(D)}_{D}\big|_{\text{tree}} \cdot \frac{m_D^{\text{MSSM}}(M_S)}{m_D^{\text{SM}}(M_S)}, \tag{12}
\]

where the superscript \((\tilde{D})\) signifies that this expression only applies to the \( \tilde{D} \) squark exchange contribution to the effective \( \tilde{D}D\tilde{\chi}\tilde{\chi} \) interaction\(^1\).

The remaining potentially important contributions to the \( \tilde{\chi}N \) scattering amplitude are due to gauge interactions, at least in the more appealing scenario where the LSP is Bino– or photino–like\(^2\). These include spin–dependent contributions due to \( Z \) and squark exchange, as well as spin–independent \( \mathcal{O}(m_{\tilde{q}}^{-4}) \) contributions \([11]\). Since to one–loop order electroweak gauge couplings are not renormalized by strong interactions, we do not expect significant QCD corrections to these contributions. However, as already stated in the Introduction, these two contributions are often subdominant.

### 3. Results

We are now ready to present some numerical results. In order to illustrate the importance of the corrections described in the previous Section, we show examples for the ratio \( R \) of the neutralino scattering rate on \(^{76}\text{Ge}\) with and without these corrections. If the small

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\(^1\)In fact, in the “decoupling limit”, where \( \cos(\alpha - \beta) \to 0 \), the total \( \tilde{g} - \tilde{q} \) loop correction to the \( H \) exchange contribution takes the same simple form as in eq. \((12)\), while the \( h \) exchange contribution is not affected by these corrections.

\(^2\)We consider this to be more appealing since one then can obtain an interesting relic density with sparticle masses in the “natural” domain of a few hundred GeV. Higgsino–like LSPs need masses around a TeV or more to contribute significantly to the mass density of the Universe.
The difference between the $\tilde{\chi}_n$ and $\tilde{\chi}_p$ scattering amplitudes is neglected, $R$ is simply the ratio of the corrected and uncorrected $\tilde{\chi}_N$ scattering cross sections.

In Fig. (1a,b) we show results for a scenario where all soft SUSY–breaking contributions to sfermion masses at the weak scale have a common value $m_0 = 600$ GeV. Similarly, we took a common value $A_0 = 1.2$ TeV for all trilinear soft SUSY–breaking parameters at the weak scale. We chose a relatively small mass of the CP–odd Higgs bosons, $m_A = 240$ GeV $\simeq 1.6m_{\tilde{\chi}}$, in order to obtain LSP detection rates of interest for present experiments, at least in the region of large tan$\beta$. We assumed the usual unification conditions for gaugino masses, with an $SU(2)$ gaugino mass $M_2 = 300$ GeV at the weak scale; note that now also the gluino mass becomes relevant, see eq. (8). We present results for $|\mu| = M_2$ (Fig. 1a) and $|\mu| = 2M_2$ (1b), as a function of tan$\beta$. In both cases we show results for both positive (dashed) and negative (solid) $\mu$. Finally, the upper (lower) curve with a given pattern has been obtained using $m_q^{\text{MSSM}}$ ($m_q^{\text{SM}}$) when calculating $\sin(2\theta_q)$.

\begin{align*}
M_2 &= 300 \text{ GeV}, \quad m_0 = 2.5m_A = A_0/2 = 600 \text{ GeV} \\
\text{(a) } |\mu| &= M_2 \\
\text{(b) } |\mu| &= 2M_2
\end{align*}

Figure 1: The ratio $R$ of corrected to uncorrected $\tilde{\chi}_N$ scattering cross section in a model with universal soft breaking parameters at the weak scale. The upper (lower) curve of a given pattern uses the quark mass with (without) sparticle loop corrections when computing the squark $L$–$R$ mixing angle.
For small values of $\tan \beta$ the total corrections are dominated by the standard QCD corrections (3) to the $H^0 gg$ couplings. They increase this contribution to the total $\tilde{\chi}_N$ scattering amplitude by $\sim 25\%$, which leads to an increase of the counting rate by $\sim 15\%$.

As anticipated the $\tilde{g} - \tilde{q}$ loop corrections can be much larger, if $\tan \beta \gg 1$. Our sign convention for $\mu$ coincides with that of Refs. [11, 18]. For $\tan \beta \gg 1$ the $L - R$ mixing of $d$–type squarks is essentially proportional to $\mu$; we then find a negative $\tilde{g} - \tilde{q}$ loop correction to $m_D$, and hence a positive correction to the Yukawa coupling for fixed physical quark mass, if $\mu < 0$. Since it is the Yukawa coupling, rather than the quark mass, which appears in most spin–independent contributions to the scattering amplitude, $\mu < 0$ therefore also leads to positive corrections to the scattering cross section\(^3\). Not surprisingly, these corrections are considerably larger for larger $|\mu|$, as can be seen by comparing Fig. 1a with Fig. 1b.

Note that we have (somewhat arbitrarily) terminated the curves when the bottom Yukawa coupling $h_b(M_S) > 1.2$, which roughly corresponds to its “fixed point” value. An even larger $h_b(M_S)$ would lead to a Landau pole at higher energies, i.e. the Yukawa sector of the theory would become non–perturbative at energies only slightly above $M_S$. Since for a fixed value of $\tan \beta$ the $\tilde{g} - \tilde{q}$ corrections increase the Yukawa coupling if $\mu < 0$, for this sign of $\mu$ the upper bound on $\tan \beta$ becomes significantly stronger; conversely, for $\mu > 0$ larger values of $\tan \beta$ become accessible once the corrections are included. Since the predicted counting rate increases with increasing $\tan \beta$, the maximal possible $\tilde{\chi}_N$ scattering cross section for fixed values of the dimensionful input parameters, which occurs for the biggest allowed value of $\tan \beta$, would thus not be affected by the corrections if the relative size of these corrections were the same for $d$, $s$ and $b$ quarks.

Even though we have assumed flavor–independent soft breaking parameters, this condition is not satisfied. First, while $L–R$ mixing does not change the masses of the $\tilde{d}$ and $\tilde{s}$ squarks very much\(^4\), it does affect the $\tilde{b}$ masses significantly. The corrections (8) are therefore larger, as fraction of the tree–level mass, for $D = b$ than for $D = d, s$. Moreover, the chargino–stop loop correction (9) only contributes to the $b$–quark mass. Note that it has the opposite sign as the $\tilde{g} - \tilde{q}$ loop correction. For the parameters of Fig. 1 this is the dominant effect. In fact, in Fig. 1a this top Yukawa correction is nearly as large as the SUSY–QCD correction. The reason is that $\tilde{t}_L - \tilde{t}_R$ mixing, which is relevant in eq. (13), is proportional to $A_t (= A_0$ in our case) which is four times bigger than $|\mu|$ in this example. As a result, for $\mu < 0$ the loop corrections increase the predicted counting rate at the largest allowed value of $\tan \beta$ by a factor of 1.9 (1.4) in Fig. 1a (1b). On the other hand, the increase of the maximal allowed value of $\tan \beta$ for $\mu > 0$ implies that for this sign of $\mu$ the maximal rate only decreases by a factor 0.79 (0.74) in Fig. 1a (1b); of course, one also has to keep in mind that the corrections (3) to the $H^0 gg$ couplings are always positive.

\(^3\)The cross–over of the two sets of curves at small $\tan \beta$ in Fig. 1a occurs because at small $\tan \beta$ and $\mu > 0$ the most important contribution comes from the $h$ boson exchange, whereas for $\mu < 0$ the $H$ boson exchange is always dominant.

\(^4\)The difference between the squark masses is in these cases mostly determined by the different $D$–term contributions to the masses of $SU(2)$ singlet and doublet squarks.

7
Note that using $m_{q}^{\text{MSSM}}$ rather than $m_{q}^{\text{SM}}$ in the calculation of $\sin(2\theta_{q})$ always increases the prediction of the loop–corrected $\tilde{\chi}N$ scattering rate. Recall that a negative correction to the quark mass implies an increase of the scattering cross section. Since in this case $m_{q}^{\text{MSSM}} > m_{q}^{\text{SM}}$, using $m_{q}^{\text{MSSM}}$ increases $L–R$ mixing and hence gives a larger, positive correction to the predicted rate. Conversely, if the corrections to the quark masses for fixed Yukawa couplings are positive, the correction to the $\tilde{\chi}N$ scattering amplitude is negative. Since now $m_{q}^{\text{MSSM}} < m_{q}^{\text{SM}}$, the use of $m_{q}^{\text{MSSM}}$ reduces $L–R$ mixing, and hence lowers the absolute size of the negative corrections to the scattering rate.

The corrections to $h_{b}$ for fixed physical $m_{b}$ can have even larger effects if one assumes universality of soft breaking parameters at some very high scale, as in the popular mSUGRA models with radiative $SU(2) \times U(1)_{Y}$ symmetry breaking [5]. Here one assumes a common squared soft breaking mass $m_{0}^{2}$ not only for sfermions, but also for Higgs bosons; however, this universality only holds at the scale of Grand Unification, $M_{X} \simeq 2 \cdot 10^{16}$ GeV. The squared soft breaking mass of one of the Higgs bosons is then driven to negative values by the top Yukawa contribution to the relevant RGE. One can show [20] that the physical mass of the CP–odd neutral Higgs boson is then given by:

$$m_{A}^{2} = \frac{m_{\tilde{\nu}}^{2} + \vert \mu \vert^{2}}{\sin^{2}\beta} + \mathcal{O}(h_{b}^{2}),$$

(13)

where the corrections from the bottom Yukawa coupling $h_{b}$ are negative. In fact, the upper bound on $\tan\beta$ often (but not always) comes from the requirement that $m_{A}$ should be sufficiently large. Note that the relevant quantity here is again the bottom Yukawa coupling, the size of which depends on the size and sign of the $\tilde{g} – \tilde{q}$ and $\tilde{t} – \tilde{\chi}^{\pm}$ loop corrections. The value of $m_{A}$ is in turn strongly correlated with the mass $m_{H}$ of the heavier neutral CP–even Higgs boson, the exchange of which usually dominates the spin–independent $\tilde{\chi}N$ scattering amplitude for $\tan\beta \gg 1$. In this kind of model one thus expects a much stronger dependence of the predicted $\tilde{\chi}N$ scattering rate on these corrections than in models where all soft breaking parameters are fixed at the weak scale, independently of $\tan\beta$.

This is illustrated in Fig. 2, where we took a weak–scale $SU(2)$ gaugino mass of 200 GeV, and GUT–scale boundary conditions $m_{0} = A_{0} = 250$ GeV. Note that in this model the absolute size of $\mu$ is a derived quantity, but its sign can still be chosen freely; dashed (solid) lines are again for positive (negative) $\mu$. The two inner curves have been obtained by using $m_{q}^{\text{SM}}$ when calculating $L–R$ squark mixing, as well as $h_{b}$ as it appears in the RGE. In this case the correction “only” amounts to a factor of 2 at most, similar to the case shown in Fig. 1a.

The outer curves in Fig. 2 have been obtained by using $m_{q}^{\text{MSSM}}$ both in the calculation of $\sin(2\theta_{q})$ and in the calculation of the bottom quark Yukawa coupling in the RGE. Mostly due to the change of $m_{A}$ discussed above, we now find correction factors as large as 100 for fixed values of $\tan\beta$. Note that in the given example for $\mu < 0$ the upper bound

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5We ignore small weak–scale threshold corrections.
Figure 2: The ratio $R$ of corrected to uncorrected $\tilde{\chi}N$ scattering cross section in an mSUGRA model. The outer (inner) pair of curves uses the quark mass with (without) sparticle loop corrections when computing the squark $L-R$ mixing angle as well as the $b$–quark Yukawa contribution to the RGE.

6 Note that we always include the $\tilde{g}-\tilde{q}$ loop corrections to the mass matrix of neutral Higgs bosons $[2]$. The dominant contribution here usually comes from $\tilde{t}-\tilde{g}$ loop corrections to $m_t$, but for large $\tan\beta$ and small $m_A$ values, $\tilde{b}-\tilde{g}$ loop corrections to $m_b$ can also have a significant effect on the Higgs mixing angle $\alpha$. On the other hand, for $\mu > 0$ the upper bound on $\tan\beta$ after the inclusion of sparticle loop corrections comes from the requirement that the lighter $\tilde{\tau}$ mass eigenstate should not be lighter than the lightest neutralino. In this case the corrections therefore reduce the maximal possible $b$ (and $s$ and $d$) Yukawa couplings, leading to a reduction of the predicted $\tilde{\chi}N$ scattering rate at the maximal allowed value of $\tan\beta$ by a factor close to 100.

on $\tan\beta$ is determined by the lower bound on $m_A$; we have required $m_A + m_h > 180$ (150) GeV if $\cos^2(\alpha - \beta) \simeq 1 \geq 0.2$, to cope with the experimental constraints. As a result the predicted counting rate at the maximal allowed value of $\tan\beta$ changes very little when the loop corrections are included. On the other hand, for $\mu > 0$ the upper bound on $\tan\beta$ after the inclusion of sparticle loop corrections comes from the requirement that the lighter $\tilde{\tau}$ mass eigenstate should not be lighter than the lightest neutralino. In this case the corrections therefore reduce the maximal possible $b$ (and $s$ and $d$) Yukawa couplings, leading to a reduction of the predicted $\tilde{\chi}N$ scattering rate at the maximal allowed value of $\tan\beta$ by a factor close to 100.

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4. Summary and Conclusions

In this paper we discussed two kinds of (potentially) large QCD corrections to the $\tilde{\chi}N$ scattering amplitude. Standard QCD corrections increase the effective coupling of CP–even Higgs bosons to two gluons, which proceeds through a loop of heavy quarks, by $\sim 25\%$. Supersymmetric QCD corrections to the quark masses, or to their Yukawa couplings for fixed physical quark masses, can have even larger effect if $\tan\beta \gg 1$: we found that an enhancement or suppression of the predicted $\tilde{\chi}N$ scattering cross section by a factor of 2 is quite easily possible for a fixed set of soft breaking parameters and fixed $\tan\beta$. Since the correction can also change the maximal allowed value of $\tan\beta$, the change of the maximal possible counting rate, which occurs at the largest allowed value of $\tan\beta$, is usually smaller, but can still be significant.

Much bigger effects from $\tilde{g} – \tilde{q}$ loop corrections are possible if the size of the $b$ Yukawa coupling affects the particle spectrum at the weak scale, as in mSUGRA models. The dominant effect here comes from the anti–correlation between the $b$ Yukawa coupling and the masses of the heavier Higgs bosons. Since for large $\tan\beta$ the scattering cross section roughly scales like the inverse fourth power of the mass of the heavier neutral CP–even Higgs boson, the sparticle loop–induced change of the $b$ Yukawa coupling can change the predicted neutralino detection rate by up to two orders of magnitude, if $\tan\beta$ is large.\[\text{Note that } \tan\beta \gg 1 \text{ is required in this model to obtain scattering rates that are sufficiently large to be probed in present or near–future experiments.}\]

One consequence of these sparticle loop corrections is a strong dependence of the predicted $\tilde{\chi}N$ scattering rate on the sign of $\mu$ even for $\tan\beta \gg 1$. It is well known that for small and moderate $\tan\beta$, positive values of $\mu$ lead to more gaugino–higgsino mixing in the neutralino eigenstate, and hence to larger couplings of the neutralino to Higgs bosons, than do negative values of $\mu$. This tree–level dependence of the scattering rate on the sign of $\mu$ disappears at large $\tan\beta$. However, there the $\tilde{g} – \tilde{q}$ loop corrections become important. Note that now $\mu < 0$ gives larger counting rate (for fixed $|\mu|$) than $\mu > 0$. These corrections can thus qualitatively change the dependence of the predicted relic neutralino detection rate on the parameters of the supersymmetric model.

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