QUANTIZATION OF GRAVITY: YET ANOTHER WAY

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Abstract

Recently proposed quantization in field theory based on an analogue of Hamiltonian formulation which treats space and time on equal footing (the so-called De Donder-Weyl theory) is applied to General Relativity in metric variables. We formulate a covariant analogue of the Schrödinger equation for the wave function of space-time and metric variables and a supplementary “bootstrap condition” which enables us to incorporate classical metric geometry as an approximate notion - a result of quantum averaging - in the self-consistent with the underlying quantum dynamics way. In this sense an independence of an arbitrarily chosen metric background is ensured.

1. Introduction

Among the conceptual problems of quantum gravity the so-called “Issue of Time” (see, e.g., \cite{1}) is perhaps the most quintessential one. It originates in difficulties with reconciling the basic principles of quantum theory with those of General Relativity. One of these is the fact that, in contradistinction with the space-time unity in relativity, in quantum theory the time dimension is singled out by the probabilistic interpretation of the formalism, the procedure of canonical quantization, and the formulations of quantum evolution laws. It is not unexpected then that the gravitation, by very its general relativistic nature, resists to getting quantized in the way discriminating between space

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and time dimensions. Besides, the inherent nonlinearity of gravitation on the classical level makes the applicability of traditional perturbative techniques of quantum field theory rather doubtful, as it is also demonstrated by the non-renormalizability problem. It would be more natural, therefore, to apply a nonperturbative quantization which would treat space and time variables on equal footing. Thought this idea is partially incorporated in the path integral quantization, the latter essentially serves as a basis of perturbative construction and still implicitly singles out the time parameter on the level of interpretation, for example, by referring to the *time-ordered* Green functions produced by the generating functional.

It is well known that the preferred role of the time dimension appears already in the classical Hamiltonian formalism which underlies canonical quantization. It leads to the picture of fields as infinite dimensional Hamiltonian systems evolving in time. The approach of the present paper stems from the (not yet commonly acknowledged in physics) fact that another generalization of the Hamiltonian formulation from mechanics to field theory is possible, which treats space and time variables on equal footing as analogues of the single time parameter in mechanics. The simplest realization of this idea is the so-called De Donder-Weyl (DW) formulation of the field equations [2]: given a Lagrangian density \( L = L(y^a, y^a_{\mu}, x^\nu) \), a function of field variables \( y^a \), their space-time derivatives (first jets) \( y^a_{\mu} \) and space-time variables \( x^\mu \), we can introduce new Hamiltonian-like variables

\[
p^a_{\mu} := \frac{\partial L}{\partial (\partial_{\mu} y^a)} \quad \text{and} \quad H = H(y^a, p^a_{\mu}, x^\nu) := \partial_{\mu} y^a p^a_{\mu} - L,
\]

(1.1)

to be referred to as *polymomenta* and the *DW Hamiltonian function*, respectively, and write the field equations in the DW canonical form

\[
\partial_{\mu} y^a = \frac{\partial H}{\partial p^a_{\mu}}, \quad \partial_{\mu} p^a_{\mu} = -\frac{\partial H}{\partial y^a}.
\]

(1.2)

In formulation (1.2) fields are treated as “multi-time” generalized (=DW) Hamiltonian systems *varying* in space-and-time (rather than evolving in time). No distinction between space and time dimensions, i.e. no topological restriction to globally hyperbolic space-times, is implied. Moreover, another potential advantage of the formulation (1.2) is that it manages to describe the dynamics of a field, usually viewed as an infinite dimensional mechanical system, in a finite dimensional analogue of the phase space, the space of variables \( (y^a, p^a_{\mu}, x^\nu) \). This, of course, does not mean a loss in the number of degrees of freedom: just instead of the notion of a degree of freedom per space point, which is inspired by the conventional Hamiltonian treatment, here the number of components of the field over space-time is more relevant. The equivalence of (1.2) to the Euler-Lagrange equations, which is only restricted by the regularity of DW Legendre transform: \( y^a \to p^a_{\nu}, L \to H \), proves *de facto* that no degrees of freedom get lost.

It is natural to inquire whether the above DW Hamiltonian formulation can be a basis of quantization of field theory in general and of General Relativity in particular. The structures of canonical formalism which are used in various quantization procedures have been generalized to DW formulation of field theory in [3,4]. This generalization leads to Poisson brackets defined on differential forms playing the role of dynamical
variables and to a graded analogue of a Poisson algebra: the so-called Gerstenhaber algebra and its generalizations. Elements of the corresponding field quantization have been considered in [5–7]. In short, in the \( y \)-representation one is lead to the realization of polymomenta as operators
\[
p_a^\mu = -i\hbar \kappa \gamma^\mu \frac{\partial}{\partial y^a},
\]
which act on spinor (or the space-time Clifford algebra valued) wave functions on the configuration space of field and space-time variables: \( \Psi = \Psi(y^a, x^\mu) \); \( \gamma^\mu \)'s denote the imaginary units of the space-time Clifford algebra. The constant \( \kappa \) of the dimension \( [\text{length}]^{-(n-1)} \) ensures the dimensional consistency of (1.3) and is interpreted as a quantity of the ultra-violet cutoff or the fundamental length scale \([5, 7]\). Note that in this picture all the space-time dependence is transferred from operators to the wave function corresponding to what could be called the “ultra-Schrödinger” picture. The dynamical law for the wave function was proposed to have the form of a generalized covariant Schrödinger equation
\[
i\hbar \kappa \gamma^\mu \partial_\mu \Psi = \hat{H} \Psi
\]
where \( \hat{H} \) is the operator form of the DW Hamiltonian function. For example, for scalar fields \( y^a \) with \( L = \frac{1}{2} \partial_\mu y^a \partial^\mu y^a - V(y) \) we obtain \( \hat{H} = \frac{1}{2} p_\mu^a p_\mu^a + V(y) \) where \( p_\mu^a = \eta^{\mu\nu} \partial_\nu y^a \), and then \( \hat{H} = -\frac{1}{2} \hbar^2 \kappa^2 \partial_\alpha \partial_\alpha + V(y) \). In \([5, 7]\) we argued that the above generalization of the Schrödinger equation fulfills some natural requirements of the correspondence principle. In particular, it leads to an analogue of the Ehrenfest theorem and can be reduced to the field theoretic DW Hamilton-Jacobi equation (with some additional conditions) in the classical limit. It should be noticed, however, that in spite of this formal success the description of quantized fields provided by the present framework is very different from that in the conventional quantum field theory and its physical significance remains to be investigated.

The purpose of this paper is to discuss quantization of General Relativity from the point of view of the above approach (see [8] for a somewhat more detailed discussion).

2. Quantizing General Relativity

2.1 Extension to curved space-time and application to General Relativity.

The application to General Relativity requires an extension of the approach to curved space time with the metric \( g_{\mu\nu}(x) \). We introduce \( x \)-dependent curved space-time Dirac matrices \( \gamma_\mu(x) \) which fulfill \( \gamma_\mu(x) \gamma_\nu(x) + \gamma_\nu(x) \gamma_\mu(x) = 2g_{\mu\nu}(x) \) and can be expressed in terms of the Minkowski space Dirac matrices \( \gamma^A : \gamma^A \gamma^B + \gamma^B \gamma^A = 2\eta^{AB} \), and the vielbein coefficients \( e_\mu^A(x) : e_\mu^A(x) e_\mu^B(x) \eta_{AB} := g_{\mu\nu}(x) \). Then the curved space-time version of (1.4) assumes the form
\[
i\hbar \kappa \gamma^\mu(x) \nabla_\mu \Psi = \hat{H} \Psi
\]
where \( \hat{H} \) is the operator of DW Hamiltonian function and \( \nabla_\mu \) is the covariant derivative. If \( \Psi \) is a spinor wave function \( \nabla_\mu = \partial_\mu + \theta_\mu \) is the spinor covariant derivative with the
spinor connection $\theta_\mu = \frac{1}{4} \theta_{A B \mu} \gamma^{A B}$, where $\gamma^{A B} := \frac{1}{2}(\gamma^A \gamma^B - \gamma^B \gamma^A)$ and

$$
\theta^A_{\cdot B \mu} = e^A_{\alpha} e^\nu_{B} \Gamma^\alpha_{\mu \nu} - e^\nu_{B} \partial_\mu e^A_{\nu}.
$$

In General Relativity treated as the field theory of the metric field the DW configuration space is the bundle of symmetric rank-two (metric) tensors $g^{\mu \nu}$ over space-time and, respectively, the wave function is a function on this space. With field variables taken to be the components of the metric the polymomenta will be some combinations of the connection coefficients and, therefore, according to (1.3), the corresponding operators will involve differentiations with respect to the metric variables. Hence, a generalized Schrödinger equation for gravity will assume the symbolic form (c.f. (2.1))

$$
i \hbar \kappa \hat{\nabla} \Psi = \hat{H} \Psi,
$$

where $\hat{H}$ is the operator form of DW Hamiltonian density of gravity, $\hat{H} := e \hat{H}$, $e := |\det(e^A_\mu)|$, and $\hat{\nabla}$ denotes the quantized Dirac operator in the sense that the corresponding connection coefficients are replaced by appropriate differential operators. A more specific form of this equation will be derived below. Note that quantum gravity possesses an intrinsic fundamental length scale, the Planck length $\ell$, suggesting an identification of the parameter $\kappa$ with the Planck scale quantity: $\kappa \sim \ell^{-(n-1)}$.

It should be noticed that if the wave function in (2.3) is spinor valued the Dirac operator involves the spinor connection which classically depends on the first derivatives of vielbeins (c.f. eq. (2.2)). To quantize this object we need to Legendre transform first derivatives of vielbeins to corresponding polymomenta and then quantize the latter. This procedure is only possible within the vielbein formulation of General Relativity, which would be, therefore, the most suitable framework for discussing the application of the present approach to gravity. However, so far there is no appropriate DW-like formulation of General Relativity in vielbein variables in our disposal. For this reason the following discussion, as a first step, is based on the metric formulation, which, nevertheless, also appears to be useful.

2.2 DW-like formulation of General Relativity. The most straightforward way to represent the Einstein equations in DW form (1.1) is to start from their first order form: symbolically,

$$
\partial \Gamma + \Gamma = 0,
$$

$$
\partial g + \Gamma = 0,
$$

where $\Gamma$'s denote different appropriate linear combinations of the Christoffel symbols and $g$ denotes a combination of metric variables, and to find the combinations such that the Einstein equations assume the form

$$
\partial h^\beta_\gamma = \partial \mathcal{H} / \partial Q^\alpha_{\beta \gamma}, \quad \partial Q^\alpha_{\beta \gamma} = -\partial \mathcal{H} / \partial h^\beta_\gamma.
$$

In this formulation the field variables are $h^{\alpha \beta} = \sqrt{g^{\alpha \beta}}$, where $g := |\det(g_{\mu \nu})|$, the polymomenta are

$$
Q^\alpha_{\beta \gamma} := \frac{1}{8 \pi G} (\delta^\alpha_{\beta} \Gamma^\delta_\gamma - \Gamma^\alpha_{\beta \gamma}),
$$

and the DW Hamiltonian density takes the form

$$
\mathcal{H}(h^{\alpha \beta}, Q^\alpha_{\beta \gamma}) := 8 \pi G h^{\alpha \gamma} \left( Q^\delta_{\alpha \beta} Q^\gamma_\delta + \frac{1}{1 - n} Q^\beta_{\alpha \beta} Q^\delta_\gamma \right) + (n - 2) \Lambda \sqrt{g}.
$$
Note that the above formulation of the Einstein equations has a deeper foundation in the theory of Lepagean equivalents in the calculus of variations [10].

2.3 Local quantization. By formally following the curved space-time version of our scheme we obtain the operator form of polymomenta

\[ \hat{Q}_\beta^\alpha = -i\hbar\kappa^\alpha \left\{ \sqrt{g} \frac{\partial}{\partial h^\beta_\gamma} \right\}_{ord} \] (2.7)

which is determined up to an ordering of the expression inside the curly brackets. Then, plugging (2.7) into (2.6) and assuming in the intermediate calculation the standard ordering of operators we obtain the operator form of the DW Hamiltonian density

\[ \hat{H} = -8\pi G \hbar^2 \kappa^\alpha \left\{ \sqrt{g} \hbar^\gamma_{\alpha \beta} \frac{\partial}{\partial h^\beta_\gamma} \frac{\partial}{\partial h^\gamma_\delta} \right\}_{ord} + (n-2)\Lambda \sqrt{g} \] (2.8)

which is also ordering dependent.

It should be noted, however, that the right hand sides of Eqs. (2.7) and (2.8) are tensorial while the left hand sides are not: polymomenta \( Q_\beta^\gamma \) transform as the connection coefficients and \( H \), being essentially the truncated Einstein-Hilbert Lagrangian density, is not a diffeomorphism scalar. Therefore, operator realizations (2.7) and (2.8) can be valid only locally in a specific coordinate system. This is, however, in accordance with the fact that the Poisson brackets underlying the rule of quantization (1.3) (see, e.g., [4]) can be interpreted as, in a sense, “equal-point”. This is the Schrödinger equation (2.3) which is supposed to tell us how to go from one point of space-time to another. This equation, however, contains the spinor connection coefficients which classically have the form (2.2). They obviously cannot be fully quantized within the metric formulation for, the second term in (2.2) cannot be expressed in terms of the polymomenta of the metric formulation.

To quantize the connection coefficients let us note that classically (c.f. (2.5))

\[ \Gamma^\alpha_\beta_\gamma = 8\pi G \left( \frac{2}{n-1} \delta^\alpha_\beta Q^\delta_\gamma - Q^\alpha_\beta_\gamma \right). \] (2.9)

Using (2.7) we can write for the operator \( \hat{\Gamma}^\alpha_\beta_\gamma \) an ordering dependent expression

\[ \hat{\Gamma}^\alpha_\beta_\gamma = -8\pi i\hbar \kappa \sqrt{g} \left( \frac{2}{n-1} \delta^\alpha_\beta \gamma^\sigma \frac{\partial}{\partial h^\gamma_\sigma} - \gamma^\alpha \frac{\partial}{\partial h^\beta_\gamma} \right) + \hat{\Gamma}^\alpha_\beta_\gamma(x) \] (2.10)

where the auxiliary connection \( \hat{\Gamma}^\alpha_\beta_\gamma(x) \) is introduced to make the operator \( \hat{\Gamma}^\alpha_\beta_\gamma \) transforming like a connection. We see now that (2.7) and (2.8) can be viewed as valid locally (in the vicinity of a point \( x \)) in the geodesic coordinate system in which \( \hat{\Gamma}^\alpha_\beta_\gamma |_x = 0 \). In the same coordinate system the locally valid Schrödinger equation can be written in the form (c.f. (2.3))

\[ i\hbar \kappa \sqrt{g} \gamma^\mu (\partial_\mu + \hat{\theta}_\mu) \Psi = \hat{H} \Psi \] (2.11)
where the operator form of the spinor connection coefficients is given by (c.f. (2.2))

\[
\hat{\theta}^A_{B\mu} = -8\pi i\hbar \kappa \sqrt{g} e^A_{\alpha B} \left( \frac{2}{n-1} \delta_{\mu}^\alpha \gamma^\sigma \frac{\partial}{\partial h^{\mu\nu}} \gamma^\nu \frac{\partial}{\partial h^{\nu\sigma}} \right) + \tilde{\theta}^A_{B\mu}|_x
\]

and involves the ordering dependent operator part \((\theta^A_{B\mu})^{op}\) and a non-vanishing (even though \(\tilde{\Gamma}_{\beta\gamma}|_x = 0\)) auxiliary spinor connection part \(\tilde{\theta}^A_{B\mu}|_x\).

2.4 Covariant Schrödinger equation and the “bootstrap condition.” To formulate a diffeomorphism covariant version of (2.11) we notice that vielbeins do not enter the present DW Hamiltonian formulation of General Relativity and, therefore, within the present consideration may (and can only) be treated as non-quantized classical \(x\)-dependent quantities. The reference vielbein field, \(\tilde{e}^A_{\mu}(x)\), however, is not quite arbitrary. The correspondence principle requires it to be consistent with the mean value of the metric tensor in the sense that

\[
\tilde{e}^A_{\mu}(x)\tilde{e}^B_{\nu}(x)\eta^{AB} = \langle g^{\mu\nu}(x) \rangle.
\]

The latter is given by averaging over the space of the metric components by means of the wave function \(\Psi(g^{\mu\nu}, x^\mu)\):

\[
\langle g^{\mu\nu}(x) \rangle = \int [dg^{\alpha\beta}] \bar{\Psi}(g, x)g^{\mu\nu}\Psi(g, x),
\]

where the invariant integration measure in \(\frac{1}{2}n(n+1)\)-dimensional space of metric components reads (c.f. [11])

\[
[dg^{\alpha\beta}] = \sqrt{\tilde{g}}^{(n+1)} \prod_{\alpha \leq \beta} dg^{\alpha\beta}.
\]

Therefore, the metric geometry explicitly appears as a result of quantum averaging of the metric operator \(g^{\mu\nu}\). The local orientation of vielbeins remains unquantized and is supposed to be exclusively due to a choice of a reference vielbein field (local reference frames) which, however, is restricted by the consistency with the averaged metric according to the “bootstrap condition” (2.13).

Now, a generally covariant version of (2.11) takes the form

\[
i\hbar \kappa \tilde{e}^A_{\mu}(x)\gamma^A(\partial_{\mu} + \tilde{\theta}_{\mu}(x))\Psi + i\hbar \kappa (\sqrt{g} \gamma^\mu \theta^{op}_\mu)\Psi = \hat{H}\Psi,
\]

where the term related to the quantized part \(\theta^{op}_\mu\) of the total spinor connection \(\theta_\mu = \tilde{\theta}_{\mu}(x) + \theta^{op}_\mu\) has, up to an ordering of operators, the form

\[
(\sqrt{g} \gamma^\mu \theta^{op}_\mu) = \left\{ \sqrt{g} h^{\mu\nu} \frac{\partial}{\partial h^{\mu\nu}} \right\}_{ord}
\]

and the \(x\)-dependent reference spinor connection term \(\tilde{\theta}_{\mu}(x)\) in (2.12) can be calculated using the reference vielbein field \(\tilde{e}^A_{\mu}(x)\) consistent with (2.13) and the classical expression

\[
\tilde{\theta}^{AB}_{\mu}(x) = e^{\alpha[A} \left( 2\partial_{\mu} \tilde{e}^{B]}_{\alpha]a} + e^{B]a}_{\beta} \partial_{\beta} \tilde{e}^{C}_{\alpha} \right)
\]
which is equivalent to (2.2).

To complete the description, a gauge-type condition should be imposed in order to distinguish the physically relevant information. For example, the De Donder-Fock harmonic gauge

$$\partial_\mu \langle \sqrt{g} g^{\mu \nu} \rangle (x) = 0$$  \hspace{1cm} (2.19)$$
can be chosen. Notice, that this is a gauge condition on the wave function $\Psi(g^{\mu \nu}, x^\nu)$ rather than on the metric field.

Let us note that eq. (2.16) can be written in a more compact form

$$i\hbar \kappa \tilde{\nabla} \Psi + i\hbar \kappa (\sqrt{g} \gamma^\mu \theta_\mu)^{\operatorname{op}} \Psi = \hat{\mathcal{H}} \Psi,$$

(2.20)$$
where $\tilde{\nabla}$ denotes the “averaged” Dirac operator (multiplied by the density $e$) in which the vielbein fields and the corresponding spinor connection are assumed to be consistent with the “bootstrap condition” (2.13). Obviously, the resulting equation for quantized gravity is integro-differential and nonlinear in essence. The meaning of this description is that the metric of the space-time on which the wave function propagates is self-consistent with the quantum dynamics of the latter. The space-time metric geometry arises as an approximate notion – a result of quantum averaging – and is used as such in the left hand side of (2.20) to describe the wave function propagation. In this sense the formulation is independent of an arbitrarily chosen metric background. At the same time, no explicit description of what could be thought to be an underlying “quantum pre-geometry” has been used. A possible speculation could be that the appearance of the averaged Dirac operator in the left hand side of (2.20) may imply an approximate, not ultimately quantum, character of the description achieved here, which may necessitate a further step in which the space-time itself, not only the metric structure, would be treated quantum theoretically. This is, in fact, a rather common expectation in quantum gravity. Our eq. (2.20) implies that the idea requires a generalized notion of the Dirac operator beyond the framework of classical manifolds. For example, non-commutative geometry \cite{12} provides a framework for treating the Dirac operator in the left hand side of (2.3) or (2.20) and the underlying geometry in a “non-commutative” fashion, very much in the spirit of quantum theory. String/M-theory also enables us to consider quantized space-time coordinates (see, e.g., \cite{13}). An analysis of these perspectives could be a subject of a subsequent research.

3 Conclusion

We have argued that within the quantization scheme based on DW theory quantum General Relativity may be described by Eq. (2.16), with operators $\hat{\mathcal{H}}$ and $(\sqrt{g} \gamma^\mu \theta_\mu)^{\operatorname{op}}$ given, respectively, by Eq. (2.8) and Eq. (2.17), and the bootstrap condition (2.13) ensuring self-consistency and, effectively, background independence. These equations define the wave function $\Psi(g^{\mu \nu}, x^\nu)$ which may be interpreted as the probability amplitude to find the values of the components of the metric in the interval $[g^{\mu \nu} - (g^{\mu \nu} + dg^{\mu \nu})]$ in an infinitesimal vicinity of the point $x^\nu$. Obviously, this description is very different
from the conventional quantum field theoretic one and its physical implications remain to be analysed. Note, however, that it opens an intriguing possibility to approximate the “wave function of the Universe” by a solution of equation (2.16) which expands from an initially localized wave packet making the observation of space-time points beyond the primary “probability clot” more and more probable, in this sense attributing a meaning to the process of a “genesis of space-time”. Obviously, an essential role in this process is played by the self-consistency imposed by the bootstrap condition (2.13).

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