Strongly gravitational lensed SNe Ia as multi-messengers: Direct test of the Friedmann-Lemaître-Robertson-Walker metric

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We present a new idea of testing the validity of the Friedman-Lemaître-Robertson-Walker metric, through the multiple measurements of galactic-scale strong gravitational lensing systems with type Ia supernovae in the role of sources. Each individual lensing system will provide a model-independent measurement of the spatial curvature parameter referring only to geometrical optics independently of the matter content of the universe. This will create a valuable opportunity to test the FLRW metric directly. Our results show that the LSST would produce robust constraints on the spacical curvature comparable to Planck 2014 results, with 650 strongly lensed SNe Ia observed in the future.

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Introduction. — Friedmann-Lemaître-Robertson-Walker (FLRW) metric commonly used in cosmology stems from the homogeneity and isotropy of the Universe, supported by observations of the large-scale distribution of galaxies and the near-uniformity of the CMB temperature [1,2]. The FLRW metric, provides the context for interpreting the observed accelerated expansion of the Universe – one of the most important issues of modern cosmology since historical observations of type Ia supernovae (SNe Ia) [3,4]. However, there were also suggestions that the failure of the FLRW approximation could potentially explain this late-time accelerated expansion phenomenon, which gave birth to a variety of cosmological models deviating from exact homogeneity and isotropy [5,6].

Growing observational data obtained with increasing precision, enabled testing the robustness of the FLRW metric [7,11]. More recently, it was proposed that combination of SNe Ia and strong lensing data may provide another consistency test of the FLRW metric [12–15] in the framework of the so called sum rule of distances (derived from geometrical optics).

In this letter, we propose a new idea of testing the validity of the FLRW metric, through the multiple measurements of galactic-scale strong gravitational lensing systems with type Ia supernova acting as background sources. Strongly lensed SNe Ia (SGLSNe Ia thereafter), which have long been predicted in the literature [16,17], had not been discovered until very recently. Goobar et al. [18] reported the discovery of a new gravitationally lensed SNe Ia iPTF16geu (SN 2016geu) from the intermediate Palomar Transient Factory (iPTF). Time-delay predictions of this system were discussed in [19]. The advantage of our method is that, it is independent of the matter content of the Universe and its relation to spacetime geometry, and each individual lensing system will provide a model-independent measurement of the cosmic geometrical optics along the line of sight the lensed SNe Ia, without any need to seek for redshift correspondence from other observational data. Therefore, with a sample of measurements of cosmic curvature at different positions on the sky, one could directly test the validity of the FLRW metric. Moreover, if the sum rule is consistent with the observational data, the test would provide a measurement of the spatial curvature of the Universe.

Method. — The deflection of light caused by foreground mass overdensities (galaxies, galaxy clusters) is able to create multiple images of the distant source. Such phenomenon, also known as strong gravitational lensing (SGL), occurs whenever the source, the lens and the observer are so well aligned that the observer-source direction lies inside the so-called Einstein radius of the lens. In this letter, we focus on gravitational lensing caused by a galaxy-sized lens.

For a specific strong lensing system with the lensing galaxy (at redshift $z_l$), angular separation of multiple images of the source (at redshift $z_s$) depends on the ratio of angular-diameter distances between lens and source $D^A_{ls}$ and between observer and source $D^A_s$. Let us denote the dimensionless comoving distances as $d_{ls} = d(z_l, z_s)$, $d_l = d(0, z_l)$ and $d_s = d(0, z_s)$. According to the distance sum rule, these three dimensionless distances satisfy the following relation [12]

$$\frac{d_{ls}}{d_s} = \sqrt{1 + \Omega_k d_l^2} - \frac{d_l}{d_s} \sqrt{1 + \Omega_k d_s^2}. \quad (1)$$

Therefore, the value of $\Omega_k$, which in the framework of FLRW metric would correspond to spatial curvature parameter, could be directly derived from the distance ratio of $d_{ls}/d_s$, provided the other two distances, $d_l$ and $d_s$ are known. In this work, we focus on lensing systems with early-type galaxies acting as lenses and SNe Ia acting as
I. The angular diameter distance ratio can robustly be determined via the measurement of the so-called Einstein radius, which is given by the formula:

$$\theta_E = \left( \frac{4GM_E}{c^2} \frac{D_s^4}{D_A^4} \right)^{1/2}$$

(2)

where $c$ is the speed of light and $M_E$ is the mass enclosed in the cylinder of radius equal to the Einstein radius. According to Treu et al. 20, the mass enclosed in the Einstein radius can be measured to within 1-2%, including all random and systematic uncertainties. We will assume spherically symmetric power-law mass distribution $\rho \sim r^{-\gamma}$, which has been widely used in studies of lensing caused by early-type galaxies 21 22. After solving the spherical Jeans equation 23 based on the assumption that stellar and mass distributions follow the same power-law and velocity anisotropy vanishes, the combination that stellar and mass distributions follow the same spherical Jeans equation 25 based on the assumption that the Fermat potential difference $\Delta \phi_{i,j} = [(\theta_i - \beta)^2/2 - \psi(\theta_i) - (\theta_j - \beta)^2/2 + \psi(\theta_j)]$ depends on the lens mass distribution and the source position $\beta$, $\psi$ is the two-dimensional lensing potential, satisfying the corresponding two-dimensional Poisson Equation: $\nabla^2 \psi = 2\kappa$, where $\kappa$ is the surface (projected) mass density of the deflector in units of the critical density $\Sigma_c = \frac{c^2}{4\pi G D_s^2 L}$. The so called time-delay distance introduced in Eq.(1) can be expressed as

$$D_{\Delta l} = \frac{D_A^4 D_s^4}{D_s^8} = \frac{c}{1+z_l} \Delta \phi_{i,j},$$

(5)

where the first equivalence is its definition and second equality shows how it relates to observable quantities. One can see that measurements of $\Delta t_{i,j}$ and $\Delta \phi_{i,j}$ (from lens model reconstruction) provide the way to measure time-delay distance, which combined with the distance ratio $D_{ls}^4/D_s^4$, provides the comoving distance from the observer to the lens

$$D_l = (1+z_l)D_{\Delta l} \frac{D_{ls}^4}{D_s^4}.$$  

(6)

III. It is commonly believed that SNe Ia can be calibrated as standard candles, and provide the luminosity distance $D^L_s$ through the distance modulus $\mu_D$ as $D^L_s = 10^{\mu_D/5}$ (Mpc). Theoretically, the distance modulus can be assessed as $\mu_D = m_X - M_B - K_{BX}$, where $m_X$ is the peak apparent magnitude of the supernova in the filter $X$, $M_B$ is its rest-frame B-band absolute magnitude, and $K_{BX}$ denotes the cross-filter K-correction 24. In our context, the unlensed SNe flux should be scaled up by a magnification factor $\mu$ due to gravitational lensing. This effect can be taken into account by correcting the peak apparent magnitude of the source SNe Ia $m_X = m_{X,obs} + 2.5 \log \mu$. Therefore, the comoving dis-

FIG. 1: Individual measurements of the cosmic curvature parameter from future observations of strongly lensed SNe Ia: without and with the effect of microlensing on the standardisation of SGLSNe Ia.
tions of the Large Synoptic Survey Telescope (LSST).

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\[ \Omega_k(z_1, z_2) = \frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2d_s^2 - 2d_l^2d_{ls}^2 - 2d_s^2d_{ls}^2}{4d_l^2d_s^2d_{ls}^2} \]  

(8)

The above function is general, i.e., applicable to any spacetime, but in the FLRW spacetime it should be equal to the present value of the spatial curvature parameter \( \Omega_{k,0} \) and thus should give the same result for any pair of source and lens. In the following section, we will apply the above methodology to the simulated future observations of the Large Synoptic Survey Telescope (LSST).

Simulated data and constraints.— Recent analysis of \[28\] revealed that LSST can discover up to 650 multiply imaged SNe Ia in a 10 year \( z \)-band search, more than an order of magnitude improvement over previous estimates \[17\]. Following the approach proposed by \[29\], we have simulated a realistic population of strongly lensed SNe Ia. Our simulated population of lenses is dominated by early-type galaxies with intermediate velocity dispersion \( \sigma_0 \). This assumption, together with the distribution of \( \theta_E \) in the population of lenses, reveals similarity between our simulations and the real data from the SL2S sample \[30\].

1. For a specific galactic-scale SGL system, by applying state-of-the-art lens modeling techniques \[31, 32\] and kinematic modeling methods \[33, 34\] to high-quality imaging observations from HST, the parameters characterizing the lens mass distribution could be inferred with high precision. As pointed out in the recent analysis by \[35\], fractional uncertainty of the observed velocity dispersion and the Einstein radius is respectively at the level of 5% and 1%. Note, that although the line-of-sight contamination might introduce 3% uncertainties in the Einstein radii \[36\], this systematics might be reduced to the level of 1% in future strong lensing surveys, which makes the assumption of 1% accuracy on the Einstein radius measurements reasonable.

II. Three sources of uncertainty are included in our simulation of time-delay measurements: time delay, Fermat potential difference, microlensing effect, and LOS effects. It is well recognized that SNe Ia have many advantages over AGNs and quasars as time delay indicators \[28\]. The typical quasar-galaxy lensing system could provide \( \Delta t \) measurements typically at 3% accuracy \[26, 37–39\] with new curve shifting algorithms \[40\]. Time delays measured through lensed SNe Ia are supposed to be very accurate due to exceptionally well-characterized spectral sequences and considerable variation in light curve morphology \[41, 42\]. In our analysis, the fractional uncertainty of \( \Delta t \) is taken at the level of 1%, which is reasonable with well-measured light curves of lensed SNe Ia. However, it has long been known that microlensing due to field stars in the lens galaxy, may significantly magnify and demagnify distant background supernovae \[43, 44\]. Following recent predictions concerning LSST \[45\], the distribution of absolute time delay error due to microlensing is unbiased at the sub-percent level with color curve observations in the achromatic phase. In our analysis, an additional 1% uncertainty of \( \Delta t \) will be added for SGLSNe Ia in which the microlensing is significant. In a system with the lensed SN Ia image of quality typical to the HST observations and the relevant parameters recovered with the state-of-the-art lens modeling techniques, one can achieve \( \sim 3\% \) precision of the Fermat potential difference for a well-measured time-delay lens system, which is in agreement with the lens modeling precision used for the lensed quasar analysis \[46\]. Finally, the well-known spectral energy distributions of SNe Ia allow one to correct for extinction along the paths of each SNe Ia image. Therefore, in order to characterize the typical effect of LOS contamination, a typical 1% uncertainty will be added to the estimation of lens potential \[47\].

III. Concerning the uncertainty budget, following the strategy described by the WFIRST Science Definition Team (SDT) \[48\], the distance precision per SN is taken as \( \sigma_{\text{dist}} = \sigma_{\text{stat}}^2 + \sigma_{\text{int}}^2 + \sigma_{\text{lens}}^2 \) \[49\], with the mean uncertainty (including both statistical measurement uncertainty and statistical model uncertainty) \( \sigma_{\text{meas}} = 0.08 \) mag, the intrinsic scatter uncertainty \( \sigma_{\text{int}} = 0.09 \) mag, and the lensing uncertainty set to be \( \sigma_{\text{lens}} = 0.07 \times z \) mag \[50, 51\]. Moreover, the total systematic uncertainty is also considered in the corrected SN Ia distances, which is modeled as \( \sigma_{\text{sys}} = 0.01(1+z)/1.8 \) (mag) \[49\]. Then, because they are standardizable candles, strongly lensed SNe Ia can be used to directly determine the lensing magnification factor \( \mu \) \[52\], which can be determined by
solving the lens equation using glafic [52]. In our analysis, we assume a conservative 5% uncertainty of lensing magnification factor, which is consistent with the results concerning gravitationally lensed supernova, iPTF16geu [19]. Finally, according to recent discussion of [54] using a range of plausible strong lens macromodels, only 22% of strongly lensed SNe Ia discovered by LSST will be standardisable due to microlensing effect. This means that an additional uncertainty $\sim 0.70$ mag should be taken into account for the remaining 78% of lensed SNe Ia [53], especially in quadruple image systems, symmetric doubles and small Einstein radii lenses. Table I lists the relative uncertainties of factors contributing to the accuracy of $\Omega_k(z_l, z_s)$ measurements.

Based on the fiducial cosmological model (with relevant cosmological parameters taken after Planck 2014 [56]), we simulated 650 measurements $\Omega_k(z_l, z_s)$ according to the methodology outlined above. Following the analysis of [54], we simulated two sets of realistic lensed SNe Ia with and without effect of microlensing on the standardisation of lensed SNe Ia. The results are shown in Fig. 1. Considering the magnification difference between different image configuration, lenses with asymmetric double images, which dominate the source plane in real observations, are harder to be detected than more symmetric doubles and quads. Note that two-image systems can be well characterized by singular isothermal sphere (SIS) model, which is generally supported by analyses of both individual and statistical lensing systems [21, 57].

Turning to the mock SGLSNe catalogue of [28], only 22% of the full sample of SGLSNe Ia discovered by LSST will be standardisable. Such conclusion is consistent with the predicted relation between the standardisable fraction and Einstein radius assuming the Salpeter IMF [54], which implies that 90% of the source plane with $\theta_E \geq 1''$ on the image plane is standardisable. In our simulated data sets, the median Einstein radius for a standardisable LSST SGLSNe Ia is 1.5'', compared to 0.73'' for SGLSNe Ia unsuitable to be standard candles. Time delay for a standardisable LSST GSN Ia is 71 days, compared to 36 days for SGLSNe Ia unsuitable to be standard candles. Now, the question is: Are these measurements sufficient enough to detect possible deviation from the Friedmann-Lemaître-Robertson-Walker metric? As can be clearly seen from Fig. 1, relatively low precision of individual $\Omega_k(z_l, z_s)$ measurements, especially with sources at lower redshifts ($z < 1$), makes it very difficult to achieve competitive results about this issue. However, it is very likely to find different $\Omega_k(z_l, z_s)$ for any two pairs $(z_l, z_s)$, which may indicate that light propagation on large scales is not described by the FLRW metric. From this point of view, another question arises: Is it possible to achieve a stringent measurement of the present value of the spatial curvature density parameter from a statistical sample of strongly lensed SNe Ia? Fig. 2. displays the probability distribution function (PDF) of cosmic curvature parameter derived from 650 measurements of $\Omega_k$ with different strongly lensed SN Ia pairs. It can be seen that, using only standardizable SN Ia (i.e. 22% of the full sample) our approach is able to constrain the cosmic curvature parameter with the precision of $\Delta \Omega_k = 0.08$. If applied only to the remaining 78% taking into account microlensing effect, the precision will be $\Delta \Omega_k = 0.07$. Finally, the full sample of 650 lensed SNe Ia will improve the constraint to $\Delta \Omega_k = 0.05$, which is comparable to that of the Planck 2014 CMB data [56]. One should remark that there is a chance that our method might perform better. Namely the estimates of standardizable fraction of SN Ia adopted above were based on the Salpeter IMF, while with the Chabrier IMF more lensed SNe Ia can be classified as standard candles: lenses with smaller Einstein radius ($\theta_E \sim 0.4''$) can have a source plane which is 90% standardisable [54]. Actually, such possibility will dramatically increase the preci-
sociated with the large-scale structure formation?\(^\text{1}\) approaches like the back-reaction from inhomogeneities as-
technically possible to confirm or falsify alternative ap-
account. The most striking conclusion of these works is 
universe in which the back-reaction of inhomogeneities 
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mogeneities is taken into account.

\[^{1}\] For instance, Bolejko \(^{58, 59}\) examined the emergence of the negative spatial curvature within the so called silent universe in which the back-reaction of inhomogeneities (coming from the structure formation) was taken into account. The most striking conclusion of these works is the emergence of spatial curvature in the local universe: \(\Omega_k = 0.19^{+0.04}_{-0.03}\) (95% confidence level). It is interesting to see if our method can be used to test this prediction. Fig. 4 shows the precision of the curvature parameter assessment according to our approach as a function of strongly lensed SNe Ia sample size. Apparently, in a framework of the cosmological-model-independent method proposed in this work, 100 strongly lensed SNe Ia can effectively differentiate between the silent universe and the concordance ΛCDM cosmology. This means that the phenomenon of emerging curvature will soon be directly testable with observational data.

Summarizing, one may expect that strongly lensed SNe Ia acting as multi-messengers can become an independent and complementary alternative to current probes, useful for more precise empirical studies of the FLRW metric as well as the phenomenon of the emerging spatial curvature. Such accurate tests of the FLRW metric can become a milestone in precision cosmology. Other possible ways to achieve the same goal can be come from strongly lensed quasars and gravitational wave (GW) events \(^{47, 60}\), which can be observed at much higher redshifts. Therefore, it is expected that combing the strongly lensed SNe Ia data with other astronomical observations such as lensed quasars and GW events, the FLRW metric could be constrained more precisely than currently possible. These will be a subject of future studies.

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| TABLE I: Uncertainties of factors contributing to the measurement of \(\Omega_k(z_l, z_s)\): \(\delta \theta_E\) and \(\delta \sigma_{ap}\) denote Einstein radius and aperture velocity dispersion; \(\delta t\), \(\delta \psi\), \(\delta \text{LOS}\) correspond to time delay, Fermat potential difference and light-of-sight contamination, respectively; \(\Delta \mu_D\) and \(\delta \mu\) correspond to the distance modulus and magnification factor of lensed SNe Ia. Additional uncertainties of \(\delta \Delta t = 1\%\) and \(\Delta \mu_D = 0.70\) mag will be added to 78% lensed SNe Ia due to microlensing effect. |
| --- |
| \(\delta \theta_E\) | \(\delta \sigma_{ap}\) |
| Image configuration | 1% | 5% |
| \(\delta \Delta t\) | \(\delta \Delta t\) (macrolensing) | \(\delta \psi\) | \(\delta \text{LOS}\) |
| Time delay | 1% | 1% | 3% | 1% |
| \(\Delta \mu_D\) | \(\Delta \mu_D\) (macrolensing) | \(\delta \mu\) |
| Lensed SNe Ia | Ref. [49] | 0.70 mag | 5% |

![FIG. 4: Error bars represent 68% confidence intervals and the best fitted values of \(\Omega_k\) as a function of the number of strongly lensed SNe Ia (sample size). For comparison, the gray shaded area is the 95% confidence level of effective curvature parameter predicted by the simulations of so called silent universe in which the back reaction of small-scale inhomogeneities is taken into account.](image-url)
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