Yukawa Corrections to Charged Higgs Boson Production in Association with a Top Quark at Hadron Colliders

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ABSTRACT

We calculate the Yukawa corrections of order $O(\alpha_{ew}m_{(b)}^2/m_W^2)$ to charged Higgs boson production in association with a top quark at the Tevatron and the LHC. The corrections are not very sensitive to the mass of the charged Higgs boson and can exceed $-20\%$ for low values of $\tan \beta$, where the contribution of the top quark is large, and high values of $\tan \beta$ where the contribution of the bottom quark becomes large. These Yukawa corrections could be significant for charged Higgs boson discovery searches based on this production process, particularly at the LHC where the cross section is relatively large.

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1. Introduction

There has been a great deal of interest in the charged Higgs bosons appearing in the two-Higgs-doublet models (THDM)\cite{1}, particularly the minimal supersymmetric standard model (MSSM)\cite{2}, which predicts the existence of three neutral and two charge Higgs bosons $h_0, H, A,$ and $H^\pm$. The lightest neutral Higgs boson may be difficult to distinguish from the neutral Higgs boson of the standard model (SM), but charged Higgs bosons carry a distinctive signature of the Higgs sector in the THDM and MSSM. Therefore, the search for charged Higgs bosons is very important for probing the Higgs sector of the THDM and MSSM and, therefore, will be one of the prime objectives of the CERN Large Hadron Collider (LHC). At the LHC the integrated luminosity is expected to reach $L = 100 fb^{-1}$ per year. Recently, several studies of charged Higgs boson production at hadron colliders have appeared in the literature\cite{3,4,5}. For a relatively light charged Higgs boson, $m_{H^\pm} < m_t - m_b$, the dominate production processes at the LHC are $gg \rightarrow t\bar{t}$ and $q\bar{q} \rightarrow t\bar{t}$ followed by the decay sequence $t \rightarrow bH^\pm \rightarrow b\tau^\pm \nu_\tau$\cite{6}. For a heavier charged Higgs boson the dominate production process is $gb \rightarrow tH^-$\cite{7,8,9}. Previous studies showed that the search for heavy charged Higgs bosons with $m_{H^+} > m_t + m_b$ at a hadron collider is seriously complicated by QCD backgrounds due to processes such as $gb \rightarrow t\bar{t}b, g\bar{b} \rightarrow t\bar{t}b,$ and $gg \rightarrow t\bar{t}bb,$ as well as others process\cite{8}. However, recent analyses\cite{10,11} indicate that the decay mode $H^+ \rightarrow \tau^+ \nu$ provides an excellent signature for a heavy charged Higgs boson in searches at the LHC. The discovery region for $H^\pm$ is far greater than had been thought for a large range of the $(m_{H^\pm}, \tan \beta)$ parameter space, extending beyond $m_{H^\pm} \sim 1 TeV$ and down to at least $\tan \beta \sim 3$, and potentially to $\tan \beta \sim 1.5$, assuming the latest results for the SM parameters and parton distribution functions as well as using kinematic selection techniques and the tau polarization analysis\cite{11}.

The one-loop radiative corrections to $H^-t$ associated production have not been calculated, although this production process has been studied extensively at tree-level\cite{7,8,9}. In this paper we present the calculations of the Yukawa corrections to this associated $H^-t$ production process at both the Fermilab Tevatron and the LHC.
in the THDM. These corrections arise from the virtual effects of the third family (top and bottom) quarks, the charged and neutral Higgs bosons, as well as the Goldstone bosons. The one-loop QCD corrections are probably more important, but are also more difficult to calculate, and we will present these calculations in a future publication[12].

2. Calculations

The tree-level amplitude for \( gb \rightarrow tH^- \) is

\[
M_0 = M_0^{(s)} + M_0^{(t)},
\]

where \( M_0^{(s)} \) and \( M_0^{(t)} \) represent the amplitudes arising from diagrams in Fig.1(a) and Fig.1(b), respectively. Explicitly,

\[
M_0^{(s)} = \frac{i g s}{\sqrt{2} m_W} \pi(p_t) [2m_t \cot \beta p^\mu_b P_L + 2m_b \tan \beta p^\mu_b P_R - m_t \cot \beta \gamma^\mu k P_L \\
-m_b \tan \beta \gamma^\mu k P_R] u(p_b) \varepsilon_\mu(k) T_{ij}^a,
\]

and

\[
M_0^{(t)} = \frac{i g s}{\sqrt{2} m_W} \pi(p_t) [2m_t \cot \beta p^\mu_b P_L + 2m_b \tan \beta p^\mu_b P_R - m_t \cot \beta \gamma^\mu k P_L \\
-m_b \tan \beta \gamma^\mu k P_R] u(p_b) \varepsilon_\mu(k) T_{ij}^a,
\]

where \( T^a \) are the \( SU(3) \) color matrices and \( \hat{s} \) and \( \hat{t} \) are the subprocess Mandelstam variables defined by

\[
\hat{s} = (p_b + k)^2 = (p_t + p_{H^-})^2,
\]

and

\[
\hat{t} = (p_t - k)^2 = (p_{H^-} - p_b)^2.
\]

Here the Cabbibo-Kobayashi-Maskawa matrix element \( V_{CKM}[bt] \) has been taken to be unity.
The Yukawa corrections of order $O(\alpha_{ew}m_{t(b)}^2/m_W^2)$ to the process $gb \to H^- t$ arise from the Feynman diagrams shown in Figs.1(c)-1(v) and Fig.2. We carried out the calculation in the t’Hooft-Feynman gauge and used dimensional regularization to regulate all the ultraviolet divergences in the virtual loop corrections using the on-mass-shell renormalization scheme[13], in which the fine-structure constant $\alpha_{ew}$ and physical masses are chosen to be the renormalized parameters, and finite parts of the counterterms are fixed by the renormalization conditions. The coupling constant $g$ is related to the input parameters $e, m_W, \text{and } m_Z$ by $g^2 = \frac{e^2}{s_w^2}$ and $s_w^2 = 1 - m_w^2/m_Z^2$. The parameter $\beta$ in the THDM we are considering must also be renormalized. Following the analysis of ref.[14], this renormalization constant was fixed by the requirement that the on-mass-shell $H^+\ell\nu_\ell$ coupling remains of the same form as in Eq.(2) of ref.[14] to all orders of perturbation theory. Taking into account the $O(\alpha_{ew}m_{t(b)}^2/m_W^2)$ Yukawa corrections, the renormalized amplitude for the process $gb \to tH^-$ can be written as

$$M_{ren} = M_0^{(s)} + M_0^{(t)} + \delta M^{V_1(s)} + \delta M^{V_1(t)} + \delta M^{s(s)} + \delta M^{s(t)} + \delta M^{V_2(s)}$$

$$+ \delta M^{V_2(t)} + \delta M^{b(s)} + \delta M^{b(t)} = M_0^{(s)} + M_0^{(t)} + \sum \delta M^l,$$

(4)

where $\delta M^{V_1(s)}, \delta M^{V_1(t)}, \delta M^{s(s)}, \delta M^{s(t)}, \delta M^{V_2(s)}, \delta M^{V_2(t)}, \delta M^{b(s)}, \text{and } \delta M^{b(t)}$ represent the corrections to the tree diagrams arising, respectively, from the $gbb$ vertex diagram Fig.1(c), the $gtt$ vertex diagram Fig.1(e), the bottom quark self-energy diagram Fig.1(g), the top quark self-energy diagram Fig.1(i), the $btH^-$ vertex diagrams Figs.1(k)-1(m) and Figs.1(o)-1(q), including their corresponding counterterms Fig.1(d), Fig.1(f), Fig.1(h), Fig.1(j), Fig.1(n), and Fig.1(r), and the box diagrams Figs.1(s) - 1(v). $\sum \delta M^l$ then represents the sum of the contributions to the Yukawa corrections from all the diagrams in Figs.1(c)-1(v). The explicit form of $\delta M^l$ can be expressed as

$$\delta M^l = -\frac{ig^3g_s}{4\sqrt{2} \times 16\pi^2 m_W} C^l \pi(p_t) \{ f_1^l \gamma^\mu P_L + f_2^l \gamma^\mu P_R + f_3^l P_\mu^\rho P_L + f_4^l P_\mu^\rho P_R + f_5^l P_\mu^\rho P_L$$

$$+ f_6^l P_R + f_7^l \gamma^\mu k P_L + f_8^l \gamma^\mu k P_R + f_9^l P_\mu^\rho k P_L + f_{10}^l P_\mu^\rho k P_R + f_{11}^l P_\mu^\rho k P_L$$

$$+ f_{12}^l P_\mu^\rho k P_R \} u(p_b) \varepsilon_\mu(k)T^a_{ij},$$

(5)

where the $C^l$ are coefficients that depend on $s, \bar{t}$, and the masses, and the $f_i^l$ are
form factors; both the coefficients $C^l$ and the form factors $f_i^l$ are given explicitly in Appendix A. The corresponding amplitude squared is

$$\sum |M_{\text{ren}}|^2 = \sum |M_0^{(s)} + M_0^{(t)}|^2 + 2Re \sum [(\sum \delta M^l)(M_0^{(s)} + M_0^{(t)})\dagger], \quad (6)$$

where

$$\sum |M_0^{(s)} + M_0^{(t)}|^2 = \frac{g^2 g_s^2}{2N_C m_W^2} \left\{ \frac{1}{(\hat{s} - m_0^2)^2} \left[ (m_0^2 \cot^2 \beta + m_0^2 \tan^2 \beta)(p_b \cdot k - m_0^2 p_t \cdot k + 2m_0^2 m_0^2(p_b \cdot k - m_0^2 p_t \cdot k)ight]ight. + \frac{1}{(t - m_t^2)^2} \left[ (m_t^2 \cot^2 \beta + m_t^2 \tan^2 \beta)(p_b \cdot k + m_t^2 p_b \cdot k - m_0^2 p_t \cdot k + 2m_0^2 m_t^2(p_t \cdot k - m_0^2 p_b \cdot k)) \right.ight.$$

$$\left. \times \left[ (m_0^2 \cot^2 \beta + m_0^2 \tan^2 \beta)(2p_b \cdot k p_t + k + 2m_0^2 k p_b \cdot k - 2(p_b \cdot k)^2 - m_0^2 p_t \cdot k) + m_0^2 m_0^2(p_t \cdot k - m_0^2 p_b \cdot k) + 2m_0^2 m_0^2(p_t \cdot k - m_0^2 p_b \cdot k - 2p_b \cdot p_t) \right] \right\}, \quad (7)$$

$$\sum \delta M^l(M_0^{(s)})\dagger = -\frac{g^2 g_s}{64N_C \times 16\pi^2 m_W^2(\hat{s} - m_0^2)} C^l \sum_{i=1}^{12} h_i^{(s)} f_i^l, \quad (8)$$

and

$$\sum \delta M^l(M_0^{(t)})\dagger = -\frac{g^4 g_s}{64N_C \times 16\pi^2 m_W^2(t - m_t^2)} C^l \sum_{i=1}^{12} h_i^{(t)} f_i^l. \quad (9)$$

Here the color factor $N_C = 3$ and $h_i^{(s)}$ and $h_i^{(t)}$ are scalar functions whose explicit expressions are given in Appendix B.

The cross section for the process $gb \rightarrow tH^-$ is

$$\hat{\sigma} = \int_{t_{\text{min}}}^{t_{\text{max}}} \frac{1}{16\pi s^2} \sum |M_{\text{ren}}|^2 dt \quad (10)$$

with

$$t_{\text{min}} = \frac{m_t^2 + m_{H^-}^2 - \hat{s}}{2} - \sqrt{(\hat{s} - (m_t + m_{H^-})^2)(\hat{s} - (m_t - m_{H^-})^2)/2},$$

and

$$t_{\text{max}} = \frac{m_t^2 + m_{H^-}^2 - \hat{s}}{2} + \sqrt{(\hat{s} - (m_t + m_{H^-})^2)(\hat{s} - (m_t - m_{H^-})^2)/2}.$$
The total hadronic cross section for \( pp \to gb \to tH^- \) can be obtained by folding the subprocess cross section \( \hat{\sigma} \) with the parton luminosity:

\[
\sigma(s) = \int_{(m_t+m_{H^+})/\sqrt{s}}^{1} \frac{dL}{dz} \hat{\sigma}(gb \to tH^- \text{ at } \hat{s} = z^2 s).
\] (11)

Here \( \sqrt{s} \) and \( \sqrt{\hat{s}} \) are the CM energies of the \( pp \) and \( gb \) states, respectively, and \( dL/dz \) is the parton luminosity, defined as

\[
\frac{dL}{dz} = 2z \left[ \int_{z^2}^{1} \frac{dx}{x} f_{q/P}(x, \mu) f_{g/P}(z^2/x, \mu) \right],
\] (12)

where \( f_{q/P}(x, \mu) \) and \( f_{g/P}(z^2/x, \mu) \) are the quark and gluon parton distribution functions.

3. Numerical results and conclusion

In the following we present some numerical results for charged Higgs boson production in association with a top quark at both the Tevatron and the LHC. In our numerical calculations the SM parameters were taken to be \( m_W = 80.33 \text{GeV} \), \( m_Z = 91.187 \text{GeV} \), \( m_t = 176 \text{GeV} \), \( m_b = 4.5 \text{GeV} \), \( \alpha_s = 0.118 \), and \( \alpha_{ew} = \frac{1}{128} \). For simplicity, we also used the relations from the MSSM between the Higgs boson masses \( m_{h_0, H,A,H^\pm} \) and the parameters \( \alpha \) and \( \beta \), and chose \( m_{H^\pm} \) and \( \tan \beta \) as the two independent input parameters. And we used the CTEQ5M[15] parton distributions throughout the calculations.

Figures 3(a) and 4(a) show the tree-level total cross sections as a function of the charged Higgs boson mass for three representative values of \( \tan \beta \). For \( m_{H^\pm} = 200 \text{GeV} \) the total cross sections at the Tevatron are at most only a few fb for \( \tan \beta = 2, 10, \) and 30, and for \( m_{H^\pm} = 300 \text{GeV} \) the total cross sections are smaller than 1 fb for all three values of \( \tan \beta \). However, at the LHC the total cross sections are much larger: the order of thousands of fb for \( m_{H^\pm} \) in the range 100 to 300GeV and \( \tan \beta = 2 \) and 30; and they are hundreds of fb for the intermediate value \( \tan \beta = 10 \). For low \( \tan \beta \) the top quark contribution is enhanced while for high \( \tan \beta \) the bottom quark contribution becomes large. These results agree with ref.[8,9] and, it should be noted, are larger than the \( W^\pm H^\pm \) associated production cross section at the LHC[4].
In Figs. 3(b) and 4(b) we show the corrections to the total cross sections relative to the tree-level values as a function of $m_{H^\pm}$ for $\tan \beta = 2, 10,$ and 30. These corrections decrease the total cross sections significantly for a wide range of the charged Higgs boson mass, especially for the smaller values of $\tan \beta$ where the top quark contribution is greatly enhanced. In particular, for $\tan \beta = 2$ the corrections exceed $-20\%$ for $m_{H^\pm}$ below 300GeV and reach more than $-25\%$ for $m_{H^\pm}$ below 200GeV at both the Tevatron and the LHC.

In conclusion, we have calculated the Yukawa corrections of order $O(\alpha_{ew}m_{t(b)}^2/m_W^2)$ to the cross section for charged Higgs boson production in association with a top quark at the Tevatron and the LHC. These corrections decrease the cross section and are not very sensitive to the mass of the charged Higgs boson, but depend more strongly on $\tan \beta$. At low $\tan \beta$ the top quark contribution is enhanced while at high $\tan \beta$ the bottom quark contribution becomes large. For $m_{H^\pm}$ in the range 100 to 300 GeV the Yukawa corrections are as large as $-30\%$ for $\tan \beta = 2$, then become smaller for the intermediate value $\tan \beta = 10$, but increase to nearly $-20\%$ for $\tan \beta = 30$.

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Appendix A

The coefficients $C^i$ and form factors $f_i^l$ are the following:

$$
C^{V_1(s)} = \frac{m_b^2}{m_W^2(s - m_b^2)}, \quad C^{V_1(t)} = \frac{m_t^2}{m_W^2(t - m_t^2)}, \quad C^{s(s)} = \frac{m_b^2}{m_W^2(s - m_b^2)^2},
$$

$$
C^{s(t)} = \frac{m_t^2}{m_W^2(t - m_t^2)^2}, \quad C^{V_2(s)} = \frac{1}{s - m_b^2}, \quad C^{V_2(t)} = \frac{1}{t - m_t^2},
$$

$$
C^{b(s)} = C^{b(t)} = \frac{1}{m_W},
$$

$$
\begin{align*}
 f_1^{V_1(s)} &= \eta^{(1)} [m_b (g_2^{V_1(s)} - g_3^{V_1(s)}) - 2p_b \cdot k \times g_6^{V_1(s)}], \\
 f_2^{V_1(s)} &= \eta^{(2)} [m_b (g_3^{V_1(s)} - g_2^{V_1(s)}) - 2p_b \cdot k \times g_7^{V_1(s)}], \\
 f_3^{V_1(s)} &= \eta^{(2)} [2(g_1^{V_1(s)} + g_2^{V_1(s)}) + m_b (g_4^{V_1(s)} + g_5^{V_1(s)}) + 2p_b \cdot k \times g_8^{V_1(s)}], \\
 f_4^{V_1(s)} &= \eta^{(1)} [2(g_1^{V_1(s)} + g_3^{V_1(s)}) + m_b (g_4^{V_1(s)} + g_5^{V_1(s)}) + 2p_b \cdot k \times g_9^{V_1(s)}], \\
 f_5^{V_1(s)} &= \eta^{(2)} [-(g_1^{V_1(s)} + g_2^{V_1(s)}) + m_b (g_6^{V_1(s)} + g_7^{V_1(s)})], \\
 f_6^{V_1(s)} &= \eta^{(1)} [-(g_1^{V_1(s)} + g_3^{V_1(s)}) + m_b (g_6^{V_1(s)} + g_7^{V_1(s)})], \\
 f_7^{V_1(s)} &= \eta^{(1)} [g_4^{V_1(s)} + 2g_6^{V_1(s)} + m_b (g_8^{V_1(s)} - g_9^{V_1(s)})], \\
 f_8^{V_1(s)} &= \eta^{(2)} [g_5^{V_1(s)} + 2g_7^{V_1(s)} + m_b (g_9^{V_1(s)} - g_8^{V_1(s)})], \\
 f_9^{V_1(s)} &= 2p_b \cdot k g_3^{V_2(s)}, \quad f_2^{V_2(s)} = 2p_b \cdot k g_4^{V_2(s)}, \\
 f_3^{V_2(s)} &= 2g_1^{V_2(s)} + 2m_t \cot \beta \Delta_{L}^{bH^-} - 2m_t g_3^{V_2(s)} + 2m_b g_4^{V_2(s)}, \\
 f_4^{V_2(s)} &= 2g_2^{V_2(s)} + 2m_b \tan \beta \Delta_{R}^{bH^-} + 2m_t g_3^{V_2(s)} - 2m_b g_4^{V_2(s)}, \\
 f_5^{V_2(s)} &= \frac{1}{2} f_3^{V_2(s)}, \quad f_8^{V_2(s)} = -\frac{1}{2} f_4^{V_2(s)}, \\
 f_6^{V_2(t)} &= 2p_t \cdot k g_3^{V_2(t)}, \quad f_2^{V_2(t)} = 2p_t \cdot k g_4^{V_2(t)}, \\
 f_3^{V_2(t)} &= 2g_1^{V_2(t)} + 2m_t \cot \beta \Delta_{L}^{bH^-} - 2m_t g_3^{V_2(t)} + 2m_b g_4^{V_2(t)}, \\
 f_4^{V_2(t)} &= 2g_2^{V_2(t)} + 2m_b \tan \beta \Delta_{R}^{bH^-} + 2m_t g_3^{V_2(t)} - 2m_b g_4^{V_2(t)}, \\
 f_5^{V_2(t)} &= \frac{1}{2} f_3^{V_2(t)}, \quad f_8^{V_2(t)} = -\frac{1}{2} f_4^{V_2(t)}, \\
 f_1^{s(s)} &= 2\eta^{(1)} p_b \cdot k [g_1^{s(s)} + m_b (g_2^{s(s)} + g_3^{s(s)})], \\
 f_2^{s(s)} &= 2\eta^{(2)} p_b \cdot k [g_1^{s(s)} + m_b (g_2^{s(s)} + g_4^{s(s)})],
\end{align*}
$$
\[
f_3^{s(s)} = 2\eta^{(2)}[2m_b g_{1s(s)}^{(s)} + 2(m_b^2 + p_b \cdot k)g_{2s(s)}^{(s)} + (m_b^2 + 2p_b \cdot k)g_{3s(s)}^{(s)} + m_b^2 g_{4s(s)}^{(s)}],
\]
\[
f_4^{s(s)} = 2\eta^{(1)}[2m_b g_{1s(s)}^{(s)} + 2(m_b^2 + p_b \cdot k)g_{2s(s)}^{(s)} + m_b^2 g_{3s(s)}^{(s)} + (m_b^2 + 2p_b \cdot k)g_{4s(s)}^{(s)}],
\]
\[
f_7^{s(s)} = -\frac{1}{2} f_3^{s(s)},
\]
\[
f_8^{s(s)} = -\frac{1}{2} f_4^{s(s)},
\]
\[
f_1^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)}[2D_{27} - m_b^2(2D_{11} + D_{21}) - m_b^2 D_{23} - 2p_b \cdot k(D_{12} + D_{24})
+ 2p_t \cdot k(D_{13} + D_{26}) + 2p_b \cdot p_t(D_{13} + D_{25})](-p_b, -k, p_t, m_b, m_b, m_j)
+ \frac{m_t m_b}{m_W} \sum_{i=H^b, h^b, C^b, A^b} \xi_{(i)}^{(3)} \{\eta^{(2)}[2m_b(-3D_{312} + (1 - \zeta_i)D_{27}) + m_b^3(D_0 + D_{12} - D_{22} - D_{32}) - m_b^2 m_b(D_{23} + 2D_{39}) - 2m_b p_b \cdot k(2D_{36} + D_{24} + \zeta_i(D_0 + D_{12}))
+ 2m_b p_t \cdot k(D_{25} + D_{310}) + 2m_b m_b \cdot p_t(D_{26} + 2D_{38})] + \eta^{(1)}[2m_t(-3D_{313})
+ (1 + \zeta_i)D_{27} - m_b^3(D_{33} + (1 + \zeta_i)D_{23}) + m_b^2 m_t(D_{13} - 2D_{38} + (1 + \zeta_i)(D_0 - D_{22})) + 2m_t p_b \cdot k(D_{13} - D_{310} - (1 + \zeta_i)(D_{12} + D_{24})) + 2m_t p_t \cdot k(2D_{37}
+ (1 + \zeta_i)D_{25}) + 2m_t p_b \cdot p_t(2D_{39} + (1 + \zeta_i)D_{26})]\}(k, -p_b, p_t, m_b, m_b, m_i, m_t),
\]
\[
f_2^{b(s)} = f_1^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),
\]
\[
f_3^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)}[\{\eta_{(i,j)}^{(2)}2m_b[D_{11} + D_{21} + (1 + \zeta_i)(D_0 + D_{11})] - \eta_{(i,j)}^{(1)}2m_t(D_{13} + D_{25})\}
(\eta^{(2)}[2m_t(-4D_{27} + 2m_b^2(D_{22} - D_0 - (1 - \zeta_i)(D_{12} + D_{22}) + 2m_t^2(D_{23} - (1 + \zeta_i)D_{26}) + 4p_t \cdot k(D_{26} - D_{25})] + \eta^{(2)}2m_t m_b(1 + \zeta_i)(D_{22} - D_{12} - D_{26}))\}(k, -p_b, p_t, m_b, m_b, m_i, m_t),
\]
\[
f_4^{b(s)} = f_3^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),
\]
\[
f_5^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)}[\{\eta_{(i,j)}^{(2)}(2m_b)[D_{25} + (1 + \zeta_i)D_{13}] + \eta_{(i,j)}^{(1)}2m_t D_{23}\}
(\eta^{(2)}[2m_t m_b(1 + \zeta_i)(D_{13} + D_{23} - D_{26})]\}(k, -p_b, p_t, m_b, m_b, m_i, m_t),
\]
\[
f_6^{b(s)} = f_5^{b(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),
\]
\[
f_7^{b(s)} = \sum_{(i,j)} \xi_{(i,j)}^{(1)}[\{\eta_{(i,j)}^{(2)}(2m_b)[D_{11} + (1 + \zeta_i)D_0] + \eta_{(i,j)}^{(1)}m_tD_{13}\}
(\eta^{(2)}\eta_{(i,j)}^{(1)} m_t D_{13})\]
\(-p_b, -k, p_t, m_i, m_b, m_b, m_j\) + \frac{m_t m_b}{m_W} \sum_{i = H^0, b^{0,+}_0, A^0} \xi^{(3)}_i \{\eta^{(1)}(D_{27} - D_{311}) + m_t^2(D_{11} - 2D_{12} - 2D_{22} - 2D_{36} + (1 + \zeta)(D_9 + D_{12})) - m_t^2(2D_{23} + 2D_{37} + (1 + \zeta)(D_{13}) - 2p_b \cdot k(D_{12} + 2D_{24} + 2D_{34}) + 2p_t \cdot k(D_{13} + 2D_{25} + 2D_{35}) + 2p_t \cdot p_b(D_{13} + 2D_{26} + D_{310})) + \eta^{(2)}m_t m_b(1 + \zeta)(D_{12} - D_{13} - D_0)\} (-k, -p_b, p_t, m_b, m_b, m_i, m_t),

f_{8}^{b(s)} = f_{7}^{b(s)}(\eta^{(1)}(\leftrightarrow \eta^{(2)}), \eta^{(1)(i,j)}(\leftrightarrow \eta^{(2)(i,j)})),

f_{9}^{b(s)} = \sum_{(i,j)} \xi^{(1)}_{(i,j)}[\eta^{(1)}_{(i,j)}][2(D_{12} + D_{24})](-p_b, -k, p_t, m_i, m_b, m_b, m_j)
+ \frac{m_t m_b}{m_W} \sum_{i = H^0, b^{0,+}_0, A^0} \xi^{(3)}_i \{\eta^{(1)}(D_{23} - D_{26} - 1 + \zeta)(D_{12} + D_{24})]
+ \eta^{(2)}m_b[-D_{22} + D_{24} + \zeta((D_9 + 2D_{12} + D_{24}))(\leftrightarrow (k, -p_b, p_t, m_b, m_b, m_i, m_t),

f_{10}^{b(s)} = f_{9}^{b(s)}(\eta^{(1)}(\leftrightarrow \eta^{(2)}), \eta^{(1)(i,j)}(\leftrightarrow \eta^{(2)(i,j)})),

f_{11}^{b(s)} = \sum_{(i,j)} \xi^{(1)}_{(i,j)}[-\eta^{(1)}_{(i,j)}][2(D_{13} + D_{26})](-p_b, -k, p_t, m_i, m_b, m_b, m_j)
+ \frac{m_t m_b}{m_W} \sum_{i = H^0, b^{0,+}_0, A^0} \xi^{(3)}_i \{\eta^{(1)}(D_{23} - (1 + \zeta)D_{25})]
- \eta^{(2)}m_b[(D_{26} + D_{25} + \zeta(D_{13} + D_{25}))(\leftrightarrow (k, -p_b, p_t, m_b, m_i, m_t),

f_{12}^{b(s)} = f_{11}^{b(s)}(\eta^{(1)}(\leftrightarrow \eta^{(2)}), \eta^{(1)(i,j)}(\leftrightarrow \eta^{(2)(i,j)})),

f_{1}^{V_i(t)} = f_{1}^{V_i(s)}(U),\quad f_{2}^{V_i(t)} = f_{2}^{V_i(s)}(U),\quad f_{5}^{V_i(t)} = f_{4}^{V_i(s)}(U),\quad f_{6}^{V_i(t)} = f_{3}^{V_i(s)}(U),

f_{7}^{V_i(t)} = f_{8}^{V_i(s)}(U),\quad f_{8}^{V_i(t)} = f_{7}^{V_i(s)}(U),\quad f_{11}^{V_i(t)} = f_{10}^{V_i(s)}(U),\quad f_{12}^{V_i(t)} = f_{11}^{V_i(s)}(U),

f_{1}^{s(t)} = f_{1}^{s(s)}(U),\quad f_{2}^{s(t)} = f_{2}^{s(s)}(U),\quad f_{5}^{s(t)} = f_{4}^{s(s)}(U),\quad f_{6}^{s(t)} = f_{3}^{s(s)}(U),

f_{7}^{s(t)} = f_{8}^{s(s)}(U),\quad f_{8}^{s(t)} = f_{7}^{s(s)}(U),\quad f_{11}^{s(t)} = f_{10}^{s(s)}(U),\quad f_{12}^{s(t)} = f_{11}^{s(s)}(U),

f_{3}^{b(t)} = f_{1}^{b(s)}(U),\quad f_{4}^{b(t)} = f_{2}^{b(s)}(U),\quad f_{5}^{b(t)} = f_{4}^{b(s)}(U),\quad f_{6}^{b(t)} = f_{3}^{b(s)}(U),

f_{7}^{b(t)} = f_{8}^{b(s)}(U),\quad f_{8}^{b(t)} = f_{7}^{b(s)}(U),\quad f_{9}^{b(t)} = f_{8}^{b(s)}(U),\quad f_{10}^{b(t)} = f_{9}^{b(s)}(U),

f_{11}^{b(t)} = -f_{11}^{b(s)}(U),\quad f_{12}^{b(t)} = -f_{12}^{b(s)}(U),

Here the sums over \((i, j)\) run over \((H^0, H^-), (h^0, H^-), (H^0, G^-), (h^0, G^-)\) and \(A^0, G^-\) and \(U\) is a transformation defined by

\[p_b \to p_t,\quad \hat{s} \to \hat{t},\quad k \to -k,\quad \xi^{(1)}_i \to \xi^{(2)}_i,\quad \xi^{(3)}_i \to \xi^{(4)}_i,\]
\[ m_t \leftrightarrow m_b, \quad \eta^{(1)} \leftrightarrow \eta^{(2)}, \quad \eta^{(1)}_{(i,j)} \leftrightarrow \eta^{(2)}_{(i,j)} \]

and \(D_0, D_{ij}, D_{ijk}\) are the four-point Feynman integrals [16]. All other form factors \(f_i^l\) not listed above vanish. In the above expressions we have used the following definitions:

\[
\begin{align*}
\eta^{(1)}_{(H^0, H^-)} &= \eta^{(1)}_{(h^0, H^-)} = \eta^{(1)} = m_b \tan \beta, \\
\eta^{(2)}_{(H^0, H^-)} &= \eta^{(2)}_{(h^0, H^-)} = \eta^{(2)} = m_t \cot \beta, \\
\xi^{(1)}_{H^0} &= \frac{\cos^2 \alpha}{\cos^2 \beta}, \\
\xi^{(1)}_{h^0} &= \frac{\sin^2 \alpha}{\cos^2 \beta}, \\
\xi^{(2)}_{H^0} &= \frac{\sin^2 \alpha}{\sin^2 \beta}, \\
\xi^{(2)}_{h^0} &= \frac{\cos^2 \alpha}{\sin^2 \beta}, \\
\xi^{(3)}_{H^0} &= -\xi^{(3)}_{h^0} = \frac{\sin \alpha \cos \alpha}{\sin \beta \cos \beta}, \\
\xi^{(3)}_{G^0} &= -\xi^{(3)}_{A^0} = 1, \\
\xi^{(1)}_{H^-} &= \frac{m_t^2}{m_b^2} \cot^2 \beta, \\
\xi^{(1)}_{G^-} &= \frac{m_t^2}{m_b^2}, \\
\xi^{(2)}_{H^-} &= \frac{m_b^2}{m_t^2} \tan^2 \beta, \\
\xi^{(2)}_{G^-} &= \frac{m_t^2}{m_b^2}, \\
\xi^{(3)}_{(H^0, H^-)} &= \frac{2 m_b \cos \alpha}{\cos \beta} \left[ m_W \cos(\beta - \alpha) - \frac{m_Z}{2 \cos \theta_W} \cos 2\beta \cos(\beta + \alpha) \right], \\
\xi^{(3)}_{(h^0, H^-)} &= -2 m_b \sin \alpha \left[ m_W \sin(\beta - \alpha) + \frac{m_Z}{2 \cos \theta_W} \cos 2\beta \sin(\beta + \alpha) \right], \\
\xi^{(1)}_{(H^0, G^-)} &= \frac{m_b \cos \alpha}{\cos \beta} \left[ m_W \sin(\beta - \alpha) - \frac{m_Z}{\cos \theta_W} \sin 2\beta \cos(\beta + \alpha) \right], \\
\xi^{(1)}_{(h^0, G^-)} &= \frac{m_b \sin \alpha}{\cos \beta} \left[ m_W \cos(\beta - \alpha) - \frac{m_Z}{\cos \theta_W} \sin 2\beta \sin(\beta + \alpha) \right], \\
\xi^{(1)}_{(A^0, G^-)} &= m_b m_W \tan \beta, \\
\xi^{(2)}_{(H^0, H^-)} &= \frac{2 m_t \sin \alpha}{\sin \beta} \left[ m_W \cos(\beta - \alpha) - \frac{m_Z}{2 \cos \theta_W} \cos 2\beta \cos(\beta + \alpha) \right], \\
\xi^{(2)}_{(h^0, H^-)} &= 2 m_t \cos \alpha \left[ m_W \sin(\beta - \alpha) + \frac{m_Z}{2 \cos \theta_W} \cos 2\beta \sin(\beta + \alpha) \right], \\
\xi^{(2)}_{(H^0, G^-)} &= \frac{m_t \sin \alpha}{\sin \beta} \left[ m_W \sin(\beta - \alpha) - \frac{m_Z}{\cos \theta_W} \sin 2\beta \cos(\beta + \alpha) \right], \\
\xi^{(2)}_{(h^0, G^-)} &= -m_t \cos \alpha \left[ m_W \cos(\beta - \alpha) - \frac{m_Z}{\cos \theta_W} \sin 2\beta \sin(\beta + \alpha) \right], \\
\xi^{(2)}_{(A^0, G^-)} &= m_t m_W \cot \beta, \\
\zeta_{H^0} &= \zeta_{h^0} = \zeta_{H^-} = -\zeta_{A^0} = -\zeta_{G^0} = -\zeta_{G^-} = 1,
\end{align*}
\]
\[
g_1^{V(s)} = \sum_{i = H^0, H^0 G^0, A^0} \xi_i^{(1)} \left\{ \left[ \frac{1}{2} - 2C_{24} + m_b^2 \right] \left\{ -2C_{11} + C_{12} - C_{21} + C_{23} \right\} - \hat{s} \left( C_{12} + C_{23} \right) \right\}
\]
\[
\times \left\{ -p_b, -k, m_i, m_b \right\} + \left\{ -F_0 + F_1 + 2m_b^2 G_1 - \left( 1 + \xi_i \right) 2m_b^2 G_0 \right\} \left\{ m_b^2, m_i, m_b \right\},
\]
\[
g_2^{V(s)} = \sum_{i = H^-, G^-} 2 \left\{ \xi_i^{(1)} \left[ \frac{1}{2} - 2C_{24} + m_i^2 C_0 + m_b^2 \right] \left\{ -C_{11} - C_{12} - C_{21} + C_{23} \right\} \right\}
\]
\[
- \hat{s} \left( C_{12} + C_{23} \right) \left\{ -p_b, -k, m_i, m_t, m_t \right\} + \left\{ \xi_i^{(1)} \left( -F_0 + F_1 \right) - 2m_i^2 \xi_i, G_0 \right\}
\]
\[
+ m_b^2 \left( \xi_i^{(1)} + \xi_i^{(3)} \right) \left( G_1 - \xi_i G_0 \right) \left\{ m_b^2, m_i, m_t \right\},
\]
\[
g_3^{V(s)} = g_2^{V(s)} \left( \xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^- \right),
\]
\[
g_4^{V(s)} = \sum_{i = H^0, H^0 G^0, A^0} \xi_i^{(1)} 2m_b \left\{ C_0 + 2C_{11} + C_{21} + \xi_i \left( C_0 + C_{11} \right) \left\{ -p_b, -k, m_i, m_b, m_b \right\} \right\}
\]
\[
+ \sum_{i = H^-, G^-} 4m_b \left\{ \xi_i^{(3)} \left( C_0 + 2C_{11} + C_{21} \right) + \frac{m_i^2}{m_b^2} \xi_i \left( C_0 + C_{11} \right) \left\{ -p_b, -k, m_i, m_t, m_t \right\} \right\},
\]
\[
g_5^{V(s)} = g_4^{V(s)} \left( \xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^- \right),
\]
\[
g_6^{V(s)} = - \sum_{i = H^0, H^0 G^0, A^0} \xi_i^{(1)} m_b \left( C_0 + C_{11} + \xi_i \left( C_0 \right) \right) \left\{ -p_b, -k, m_i, m_b, m_b \right\}
\]
\[
- \sum_{i = H^-, G^-} 2m_b \left\{ \xi_i^{(3)} \left( C_0 + C_{11} \right) + \frac{m_i^2}{m_b^2} \xi_i \left( C_0 \right) \left\{ -p_b, -k, m_i, m_t, m_t \right\} \right\},
\]
\[
g_7^{V(s)} = g_6^{V(s)} \left( \xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^- \right),
\]
\[
g_8^{V(s)} = \sum_{i = H^0, H^0 G^0, A^0} \xi_i^{(1)} 2 \left( C_{12} + C_{23} \right) \left\{ -p_b, -k, m_i, m_b, m_b \right\}
\]
\[
+ \sum_{i = H^-, G^-} 4 \xi_i^{(1)} \left( C_{12} + C_{24} \right) \left\{ -p_b, -k, m_i, m_t, m_t \right\},
\]
\[
g_9^{V(s)} = g_8^{V(s)} \left( \xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H^-, G^- \right),
\]
\[
g_1^{V_2(s)} = \sum_{i = H^0, H^0 G^0, A^0} \frac{m_i m_b}{m_W^2} \xi_i^{(3)} \left\{ \eta_{(i,j)}^{(1)} \left[ -\frac{1}{2} + 4C_{24} + m_i^2 \left( C_0 + 2C_{11} + \xi_i \left( C_0 + C_{11} \right) + C_{21} - C_{12} - C_{23} \right) + m_H^2 \left( C_{22} - C_{23} \right) + \hat{s} \left( C_{12} + C_{23} \right) \right] \right\}
\]
\[
\times \left\{ -p_t, -p_{H^-}, m_i, m_t, m_b \right\} + \sum_{(i,j)} \frac{1}{m_W} \left\{ \xi_{(i,j)}^{(2)} \eta_{(i,j)}^{(2)} \left( m_t \left[ 1 + \xi_i \right] \left( C_0 + C_{11} \right) \right) \right\}
\]
\[
\left\{ -p_{H^-}, -p_t, m_j, m_i, m_t \right\} + \xi_{(i,j)}^{(1)} \left[ \eta_{(i,j)}^{(1)} m_t \left( C_0 + C_{12} \right) + \eta_{(i,j)}^{(2)} m_b \xi_i C_0 \right]
\]
\[
\left\{ -p_{H^-}, -p_t, m_i, m_j, m_b \right\},
\]
\[
g_2^{V_2(s)} = g_1^{V_2(s)} \left( \eta^{(1)} \leftrightarrow \eta_i^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)} \right),
\]
\[
g_3^{V_2(s)} = \sum_{i = H^0, H^0 G^0, A^0} \frac{m_i m_b}{m_W^2} \xi_i^{(3)} \left\{ \eta_{(i,j)}^{(1)} m_t \left[ C_0 + C_{11} + \xi_i \left( C_0 + C_{12} \right) \right] + \eta_{(i,j)}^{(2)} \xi_i m_b C_{12} \right\}
\[- p_1, - p_{H-}, m_i, m_t, m_b) + \sum_{(i,j)} \frac{1}{m_W}[\xi_{(i,j)}^{(2)}\eta_{(i,j)}(C_0 + C_{11})

- [p_{H-}, - p_t, m_j, m_i, m_t] + \xi_{(i,j)}^{(1)}[\eta_{(i,j)}(C_0 + C_{11})(-p_{H-}, - p_t, m_i, m_j, m_b)]\],

\[g_4^{V_2(s)} = g_3^{V_1(s)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),\]

\[g_4^{V_2(t)} = \sum_{i = H_0, a_0, A_0} \frac{m_t m_b}{m_W^2} \xi_{(i,j)}^{(3)} \left\{ \eta_{(i,j)}^{(1)} m_t [(1 + \xi_i)C_0 + C_{12}](-p_{H-}, p_b, m_j, m_i, m_t)

+ \eta_{(i,j)}^{(2)} \xi_i m_b[(C_0 + C_{12}) - p_{H-}, p_b, m_i, m_j, m_t]\right\},\]

\[g_2^{V_2(t)} = \sum_{i = H_0, a_0, A_0} \frac{m_t m_b}{m_W^2} \xi_{(i,j)}^{(3)} \left\{ \eta_{(i,j)}^{(1)} m_t [(1 + \xi_i)C_0 + C_{11}] + \xi_i m_t C_{12}\right\}

(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),\]

\[g_4^{V_2(t)} = g_3^{V_1(t)}(\eta^{(1)} \leftrightarrow \eta^{(2)}, \eta_{(i,j)}^{(1)} \leftrightarrow \eta_{(i,j)}^{(2)}),\]

\[g_1^{s(s)} = \sum_{i = H_0, a_0, A_0} m_b \xi_i^{(1)} \{- \xi_i F_0(p_b + k, m_i, m_b) + [\xi_i F_0 - 2m_b^2(1 + \xi_i)G_0

+ 2m_b^2 G_1](m_b^2, m_b, m_b)\} + \sum_{i = H_0, A_0} 2m_b\{ - \frac{m_t^2}{m_b^2} \xi_i F_0(p_b + k, m_i, m_t)

+ [-2m_b^2 \xi_i G_0 + m_b^2(\xi_i^{(1)} + \xi_i^{(3)})(G_1 - \xi_i G_0) + \xi_i \frac{m_t^2}{m_b^2} F_0](m_b^2, m_i, m_t)\},\]

\[g_2^{s(s)} = \sum_{i = H_0, a_0, A_0} \xi_i^{(1)} [-F_0 + F_1](p_b + k, m_i, m_b),\]

\[g_3^{s(s)} = \sum_{i = H_0, a_0, A_0} \xi_i^{(1)} [F_0 - F_1 - 2m_b^2 G_1 + 2(1 + \xi_i)m_b^2 G_0](m_b^2, m_b, m_b)

+ \sum_{i = H_0, A_0} 2\{\xi_i^{(1)} [-F_0 + F_1](p_b + k, m_i, m_t) - [\xi_i^{(1)} (-F_0 + F_1)

- 2\xi_i m_b^2 G_0 + m_b^2(\xi_i^{(1)} + \xi_i^{(3)})(G_1 - \xi_i G_0)](m_b^2, m_i, m_t)\},\]

\[g_4^{s(s)} = g_3^{s(s)}(\xi_i^{(1)} \leftrightarrow \xi_i^{(3)}; i = H_0, A_0),\]

\[\delta \Lambda_{L}^{mH^-} = \frac{4N_C}{3m_W^3} (1 - \cot^2 \theta_W)[2m_b^2(\ln \frac{m_b^2}{\mu^2} - 1) + m_t^2 + m_t^2 - \frac{5}{6} m_W^2 + m_b^2 F_0\]
\( \delta \Lambda_{R}^{H-} = \sum_{i=H^0,b^0,G^0,A^0} \frac{1}{2m_W^2} \{ \left( \frac{2m_W^2}{m_i^2} \right) [F_0 + F_1 - 2m_i^2(1 + \zeta_i)G_0 + 2m_i^2G_1][m_i^2, m_i, m_b] \}

Here \( C_0, C_{ij} \) are the three-point Feynman integrals[16] and \( C_{24} \equiv -\frac{1}{4} \Delta + C_{24} \), while

\[
F_n(q, m_1, m_2) = \int_{0}^{1} dy y^n \ln \left[ \frac{-q^2 y (1 - y) + m_1^2 (1 - y) + m_2^2 y}{\mu^2} \right],
\]

\[
G_n(q, m_1, m_2) = -\int_{0}^{1} dy y^{n+1} \frac{y^2 y (1 - y) + m_1^2 (1 - y) + m_2^2 y}{\mu^2},
\]

and

\[
g_V^t = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W, \quad g_A^t = \frac{1}{2}, \quad g_V^b = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W, \quad g_A^b = -\frac{1}{2},
\]

which are the SM couplings of the top and bottom quarks to the Z boson.
Appendix B

\begin{align*}
\text{(B.1)} & \quad h_1^{(s)} = 4m_t^2 \cot \beta (2p_b \cdot k - m_b^2) - 4m_b^2 \tan \beta (p_b \cdot p_t + p_t \cdot k), \\
\text{(B.2)} & \quad h_2^{(s)} = -4m_b m_t \cot \beta (p_b \cdot p_t + p_t \cdot k) + 4m_b m_t \tan \beta (2p_b \cdot k - m_b^2), \\
\text{(B.3)} & \quad h_3^{(s)} = 2m_t \cot \beta (2p_b \cdot kp_b \cdot p_t - m_b^2 p_t \cdot k - 2m_b^2 p_b \cdot p_t) + 2m_b^2 m_t \tan \beta (p_b \cdot k - m_b^2), \\
\text{(B.4)} & \quad h_4^{(s)} = 2m_t^2 m_b \cot \beta (p_b \cdot k - 2m_b^2) + 2m_b \tan \beta (2p_b \cdot kp_b \cdot p_t - m_b^2 p_t \cdot k - 2m_b^2 p_b \cdot p_t), \\
\text{(B.5)} & \quad h_5^{(s)} = 2m_t \cot \beta (m_t^2 p_b \cdot k - 2(m_t^2 p_t \cdot k - 2p_b \cdot p_t)), \\
\text{(B.6)} & \quad h_6^{(s)} = 2m_t^2 m_b \cot \beta (p_t \cdot k - 2p_b \cdot p_t) + 2m_b \tan \beta (m_t^2 p_b \cdot k - 2(p_b \cdot p_t)^2), \\
\text{(B.7)} & \quad h_7^{(s)} = 4m_t \cot \beta (m_t^2 p_t \cdot k - 2p_b \cdot kp_b \cdot p_t - 2p_b \cdot kp_t \cdot k) - 4m_b^2 m_t \tan \beta p_b \cdot k, \\
\text{(B.8)} & \quad h_8^{(s)} = -4m_t^3 m_t \cot \beta p_b \cdot k + 4m_b m_t \tan \beta p_b \cdot k(p_b \cdot k - m_b^2), \\
\text{(B.9)} & \quad h_9^{(s)} = 4m_t^2 \cot \beta p_b \cdot k(p_b \cdot k - m_b^2) - 4m_b^4 \tan \beta p_t \cdot k, \\
\text{(B.10)} & \quad h_{10}^{(s)} = -4m_b^3 m_t \cot \beta p_t \cdot k + 4m_b m_t \tan \beta p_b \cdot k(p_b \cdot k - m_b^2), \\
\text{(B.11)} & \quad h_{11}^{(s)} = 4m_t^2 \cot \beta p_b \cdot k(p_t \cdot k - p_b \cdot p_t) - 4m_b^2 \tan \beta p_t \cdot kp_b \cdot p_t, \\
\text{(B.12)} & \quad h_{12}^{(s)} = -4m_b m_t \cot \beta p_t \cdot kp_b \cdot p_t + 4m_b m_t \tan \beta p_b \cdot k(p_t \cdot k - p_b \cdot p_t), \\
\text{(B.13)} & \quad h_1^{(t)} = 4m_t^2 \cot \beta (2p_b \cdot k - p_b \cdot p_t) = p_t) - 4m_b^2 \tan \beta (m_t^2 + p_t \cdot k), \\
\text{(B.14)} & \quad h_2^{(t)} = -4m_b m_t \cot \beta (m_t^2 + p_t \cdot k) + 4m_b m_t \tan \beta (2p_b \cdot k - p_b \cdot p_t), \\
\text{(B.15)} & \quad h_3^{(t)} = 2m_t \cot \beta (2p_b \cdot kp_b \cdot p_t - m_b^2 p_t \cdot k - 2(p_b \cdot p_t)^2) + 2m_b^2 m_t \tan \beta (p_b \cdot k - 2p_b \cdot p_t), \\
\text{(B.16)} & \quad h_4^{(t)} = 2m_t^2 m_b \cot \beta (p_b \cdot k - 2p_b \cdot p_t) + 2m_b \tan \beta (2p_b \cdot kp_b \cdot p_t - m_b^2 p_t \cdot k - 2(p_b \cdot p_t)^2), \\
\text{(B.17)} & \quad h_5^{(t)} = 2m_t^3 \cot \beta (p_b \cdot k - 2p_b \cdot p_t) + 2m_b^2 m_t \tan \beta (p_t \cdot k - 2m_b^2), \\
\text{(B.18)} & \quad h_6^{(t)} = 2m_t^2 m_b \cot \beta (p_t \cdot k - 2m_b^2) + 2m_b^2 \tan \beta (p_b \cdot k - 2m_b^2), \\
\text{(B.19)} & \quad h_7^{(t)} = -4m_t \cot \beta (m_t^2 p_b \cdot k + 2p_b \cdot kp_t \cdot k) - 4m_b^2 m_t \tan \beta p_t \cdot k \\
\text{(B.20)} & \quad h_8^{(t)} = -4m_t^3 m_b \cot \beta p_t \cdot k - 4m_b m_t \tan \beta (m_t^2 p_b \cdot k + 2p_b \cdot kp_t \cdot k), \\
\text{(B.21)} & \quad h_9^{(t)} = 4m_t^2 \cot \beta p_b \cdot k(p_b \cdot k - p_b \cdot p_t) - 4m_b^2 \tan \beta p_t \cdot p_t p_t \cdot k, \\
\text{(B.22)} & \quad h_{10}^{(t)} = -4m_b m_t \cot \beta p_b \cdot p_t p_t \cdot k + 4m_b m_t \tan \beta p_b \cdot k(p_b \cdot k - p_b \cdot p_t), \\
\text{(B.23)} & \quad h_{11}^{(t)} = 4m_t^2 \cot \beta p_b \cdot k(p_t \cdot k - m_t^2) - 4m_b^2 m_t^2 \tan \beta p_t \cdot k, \\
\text{(B.24)} & \quad h_{12}^{(t)} = -4m_b m_t^3 \cot \beta p_t \cdot k + 4m_b m_t \tan \beta p_b \cdot k(p_t \cdot k - m_t^2). 
\end{align*}
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Figure 1: Feynman diagrams contributing to the $O(\alpha_{\text{ew}} m_t^2/m_W^2)$ Yukawa corrections to $gb \to tH^-$: (a) and (b) are the tree level diagrams; (c) and (e) are $gqq(q=b,t)$ vertex diagrams; (g) and (i) are self-energy diagrams; (k)-(m) and (o)-(q) are $gbH^-$ vertex; (s)-(v) are box diagrams; (d), (f), (h), (j), (n) and (r) are counterterm diagrams. The dashed lines represent $H, h, A, H^\pm, G^0, G^\pm$ for diagrams (c), (e), (g) and (i), and $H, h, A, G^0$ for diagrams (k), (o), (t) and (v). For diagrams (l), (m), (p), (q), (s) and (u), the dashed line (2) represents $H$ and $h$ when the dashed line (1) is $H^-$, and $H, h$ and $A$ when the line (1) is $G^-$. 

(a) \hspace{1cm} (b) \hspace{1cm} (c) 

(d) \hspace{1cm} (e)
Figure 2: Self-energy Feynman diagrams contributing to renormalization constants. The dashed line represents $H, h, A, H^\pm, G^0, G^\pm$ in (a).

Figure 3: The tree-level total cross sections (a) and relative one-loop Yukawa corrections (b) versus $m_{H^\pm}$ at the Tevatron with $\sqrt{s} = 2$ TeV. The solid, dashed and dotted lines correspond to $\tan \beta = 2, 10$ and 30, respectively.
Figure 4: The tree-level total cross sections (a) and relative one-loop Yukawa corrections (b) versus $m_{H^\pm}$ at the LHC with $\sqrt{s} = 14$ TeV. The solid, dashed and dotted lines correspond to $\tan \beta = 2, 10$ and 30, respectively.