Energy separation efficiency of air and helium-xenon mixture flowing in the single Leontiev tube with finned wall

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Abstract. The heat transfer intensity through the separation wall in Leontiev tubes depends on many factors, in particular, the thermal resistance of the separation wall and the external thermal resistances of the near-wall thermal boundary layers. The thermal resistance of the separation wall can be reduced by choosing a highly conductive material and by reducing the wall thickness. External thermal resistances can be reduced by enhancing heat transfer, in particular, by means of finning. In this paper, a theoretical study was carried out on the influence of the thermal resistance of the separation wall, as well as finning on the part of the supersonic and subsonic channels of the Leontiev tube on the efficiency of energy separation.

1. Introduction

There are several physical effects that lead to the separation of heat in gas flows without heat transfer with external heat sources (gas-dynamic temperature stratification). Among them, the method of energy separation in Ranque-Hilsch vortex tube is the most known [1]. The vortex effect is used in aircraft air conditioning systems, vortex refrigeration units [2, 3], etc. Method of gas-dynamic energy separation in the Leontiev tube was developed and theoretically analyzed in [4, 5]. In this method, the energy separation is carried out by transferring heat through a wall streamlined on one side by the supersonic flow, and on the other side – by the subsonic portion of the gas flow flowing from the receiver with a known temperature and stagnation pressure. It is theoretically shown that the energy separation efficiency is determined by the recovery temperature from the part of supersonic flow at its adiabatic outflow from gas reservoir. To reduce this temperature, it is proposed to use gas mixtures with a low Prandtl number (helium-xenon and hydrogen-xenon mixtures) [6]. Experimental studies of heat transfer in a smooth Leontiev tube with a central cylindrical channel, presented in [7, 8, 9], have shown the possibility of energy separation of the air flow, however the efficiency of energy separation of air is significantly lower than that in gas mixtures with a low Prandtl number.

The heat transfer intensity through the separation wall in the Leontiev tube depends on many factors: the Mach number in the supersonic channel and the ratio of Mach numbers in the supersonic and subsonic channels, the adiabatic index, the permeability of the separation wall [10] and, certainly, the thermal resistance of the wall itself, and the external thermal resistances of the near-wall thermal boundary layers. The thermal resistance of the separation wall can be reduced by selecting a highly conductive material and reducing the wall thickness, although the latter may be subject to a restriction...
on the strength of the tube structure. External thermal resistances decrease with the development of the heat transfer surface area that can be achieved by finning.

In this paper, using the method [10], we carried out a theoretical study of the thermal resistance effect of the separation wall, the efficiency of finning from the part of supersonic and/or subsonic channel on the heat flux through the separation wall.

2. Theoretical analysis

At temperature stratification, certain portion of the gas outflowing through the nozzle from the receiver at a constant stagnation pressure $p_0$ and temperature $T_0$ is accelerated to supersonic velocity and directed to one side of the separation wall, while the other portion of the gas is accelerated to a lower velocity and directed to the other side of the wall (Fig. 1). In general, a lower velocity may be supersonic or subsonic. Flow parameters such as pressure and temperature on both sides of the wall are interconnected, at that, the greater the flow rate in the channel, the lower is its temperature and pressure. In the area of the near-wall boundary layer, the gas temperature increases due to the dissipation of the kinetic energy of the gas flow during deceleration and reaches its equilibrium value. The equilibrium wall temperature, in turn, depends on the recovery factor $r$. If the recovery factor is close to zero, the wall temperature is close to the thermodynamic temperature of the gas in this channel. Since the thermodynamic temperature in the channel decreases with increasing flow velocity, under all other conditions being equal, the temperature in the subsonic channel will be greater than that in the supersonic one. Thus, across the separation wall, a heat flux appears from the channel with a lower gas velocity towards the channel with a higher gas velocity. The presence of fins reduces the thermal resistance of the near-wall boundary layers and leads to heat transfer enhancement between the channels. However, finning increases the area of the wall streamlined by the gas, which in turn increases the hydraulic resistance. The search for optimal ratios of the finned area on both sides of the separation wall is an important technical problem when designing efficient energy separation devices.

![Figure 1. Schematic diagram and main flow parameters of separation wall and fins.](image)

We consider just the thermal aspect of the problem on flow around a flat thin wall with a thickness $\delta$ and the thermal conductivity coefficient $\lambda$, by the flow of an ideal gas in the energy separation tube [10]. The wall on both sides is finned. The areas of the wall without finning from the part of subsonic flow and the supersonic flow equal to $S_{w}$, the finned areas are $S_{R1}$ and $S_{R2}$, respectively, so that the total area of the wall from the part of subsonic flow is $S_{s} = S_{w} + S_{R1}$, while from the part of supersonic flow is $S_{s} = S_{w} + S_{R2}$. In general, for the heat flux from the part of supersonic and subsonic channel, respectively, we can write:

$$Q_{s1} = q_{s1}S_{s} = \alpha_{s1}S_{s}(T_{s1} - T_{w}), \quad Q_{s2} = q_{s2}S_{s} = \frac{\lambda}{\delta}S_{s}(T_{s2} - T_{w}), \quad Q_{s3} = q_{s3}S_{s} = \alpha_{s3}S_{s}(T_{s3} - T_{w}),$$  \hspace{1cm} (1)

where $\alpha_{s1}$, $\alpha_{s2}$, $T_{s1}$, $T_{s2}$ and $T_{s3}$, $T_{w}$ are heat transfer coefficients, separation wall temperatures and thermodynamic temperatures in the core of the gas flow in the supersonic and subsonic channel, respectively. We will calculate the heat transfer in the compressible flow by formulas for the incompressible flow, replacing the temperature differences between the wall and the core of the gas flow.
flow by the corresponding principal temperature differences. Multiplying the heat transfer coefficients by the relative functions $\Psi_1^C$ and $\Psi_2^C$, and taking into account the effect of gas compressibility on the heat transfer in the turbulent boundary layer, expression (1) in the general form can be represented as follows:

$$
Q_{u2} = g_{u2} S_2 = \Psi_2^C \alpha_2 S_2 \left( T_{u2} - T_u \right),
Q_{s1} = g_{s1} S_1 = \Psi_1^C \alpha_1 S_1 \left( T_{s1} - T_{s1}^* \right),
$$

(2)

where $\alpha_1$, $\alpha_2$ are the heat transfer coefficients calculated by the formula for incompressible flow, $T_{s1}$, $T_{s1}^*$ are equilibrium temperatures of the flow stagnation on the adiabatic wall, determined by the flow parameters in the supersonic and subsonic channels. From the heat balance and correlation (2), taking into account the definition of $S_1$ and $S_2$ we can obtain the following:

$$
q_w = \frac{Q_w}{S_w} = \frac{1}{1 + \frac{\lambda_1}{\lambda_2} \left( 1 + \frac{S_1}{S_2} \right)} \left[ \Psi_1^C \alpha_1 \frac{T_{u2} - T_{s1}}{1 + \frac{S_1}{S_2}} \right] + \Psi_2^C \alpha_2 \left( 1 + \frac{S_1}{S_2} \right)
$$

(3)

For heat transfer coefficients (from the definition of the Stanton thermal number) it follows:

$$
\alpha = St_r \rho_0 \beta_{c0} \mu^m,
$$

(4)

where $St_r$ is the Stanton number at a given distance from the leading edge of the wall, in which the heat flux or wall temperature is determined; for the supersonic and subsonic part of the flow, $\rho_0$, $u_0$, $c_{p0}$ are density, velocity, and specific heat at constant pressure, respectively, determined by the flow core parameters. In general, for the Stanton thermal number in a zero-pressure-gradient gas flow on an impermeable wall can be written:

$$
St_r = A_f \left( \frac{Re^*}{Pr^* \gamma^*} \right),
$$

(5)

where $Re_x$ is the Reynolds number, determined by the current coordinate; $Pr$ is the gas Prandtl number, $A_f$, $n$, $m$ are constant coefficients, which, depending on the flow regime take the following values: laminar: $A = 0.332$, $n = 0.5$; $m = 2 / 3$; turbulent: $A = 0.0296$, $n = 0.2$, $m = 0.6$. Let us use the definition of the superficial velocity of the compressible gas flow $U = u/\sqrt{2c_{p0}T_{s0}}$ and write its correlation with Mach number:

$$
U^2 = \frac{M^2}{M^2 + 2(\gamma - 1)} \text{ or } M^2 = \frac{2U^2}{(\gamma - 1)(1 - U^2)},
$$

(6)

then for pressure, stagnation temperature, and recovery temperature the following relations can be written:

$$
p_{s0} = p_0 \left( 1 - U_{s0}^2 \right)^{2/3}, \quad T_{s0} = T_0 \left( 1 - U_{s0}^2 \right), \quad T_{r0} = T_{s0} \left( 1 - (1 - r)U_{r0}^2 \right).
$$

(7)

Based on the properties of ideal gas of Dorodnitsyn $\mu_0 = \mu_{c0} \left( 1 - U_{s0}^2 \right)$, $c_p = c_{p0} = c_{p0}$, and ideal gas state equations, we obtain:

$$
\alpha = \frac{A}{Pr^m} \mu_{c0}^n \left( \rho_0 u_0 \right)^{1-n} \left[ \frac{\gamma - 1}{3} \right] \frac{A^2 \left( \mu_{c0}^n \left( 1 - U_{s0}^2 \right) \right)^{1-n} \left( \rho_{s0} \right)^{1-n} T_{s0}^{(1-n)/2}}{Pr^n \mu_{c0}^n \left( 1 - U_{s0}^2 \right)^{(1-n)/2} \left( \rho_{s0} \right)^{1-n} T_{s0}^{(1-n)/2}} \right] U_{s0}^{1-n} \left( 1 - U_{s0}^2 \right)^{\gamma - 2n + 1 - n}.
$$

(8)

The complex in square brackets for any of the channels takes the same value and depends only on the gas properties, the flow stagnation parameters, and the distance from the leading edge of the separation wall. At that, it has the dimension of the heat transfer coefficient W/m²K. While denoting it as $\alpha_{s0}$, equation (8) can be rewritten as:

$$
\alpha = \alpha_{s0} F_0, \text{ and } F_0 = U_{s0}^{1-n} \left( 1 - U_{s0}^2 \right)^{\gamma - 2n + 1 - n}.
$$

(9)
Given (9), formula (3) is converted to the following form:

\[ q_u = \frac{1}{\Psi_{T_i}^0 F_0} \frac{1}{(1 - \delta) + \frac{\delta}{\lambda_i} + \frac{1}{\Psi_{T_i}^0 F_0 \eta_i}}. \]  
(10)

We introduce the finning efficiency factor \( \eta = (1 + k S_k / S_w) \), where \( k \) is the fin efficiency. The finning efficiency factor takes into account the uneven distribution of temperature over the fin surface and can vary from 1 for low fins of highly conductive material to 0 for low conductive thin fins of large height. In the following, we take the fin efficiency equal to 1. Given \( \eta \) formula (10) is converted to the form:

\[ q_u = \frac{\alpha_{00} \left( T_{w2} - T_{w1} \right)}{\Psi_{T_i}^0 F_0 \eta_i + \frac{\delta}{\lambda_i} + \frac{1}{\Psi_{T_i}^0 F_0 \eta_i}}. \]  
(11)

In the case of laminar flow regime, the relative law of heat transfer \( \Psi_{T_i}^0 = 1 \), while in the case of turbulent flow regime:

\[ \Psi_{T_i}^0 = \frac{\left(1 - U_{01}^2\right)^{0.6}}{\left(1 - (1 - r) U_{02}^2\right)^{0.5}}. \]  
(12)

It follows from formulas (9) and (12) that the complex \( \Psi_{T_i}^0 F_0 \) for the laminar flow regime depends only on the superficial velocity, while for the turbulent flow regime it depends on the superficial velocity and the temperature recovery coefficient. Let convert the numerator of expression (11) taking into account the determination of the recovery temperature (7):

\[ q_u = \frac{\alpha_{00} T_{00} (1 - r) \left( U_{01}^2 - U_{02}^2 \right)}{\Psi_{T_i}^0 F_0 \eta_i + \frac{\delta}{\lambda_i} + \frac{1}{\Psi_{T_i}^0 F_0 \eta_i}}. \]  
(13)

The complex \( \alpha_{00} T_{00} \) has a dimension of heat flux density. Introducing a dimensionless energy separation index in the form of \( \sigma_q = Q / S_a \alpha_{00} F_0 \), we obtain:

\[ \sigma_q = \frac{(1 - r) \left( U_{01}^2 - U_{02}^2 \right)}{1 + \frac{\delta}{\lambda_i} + \frac{1}{\Psi_{T_i}^0 F_0 \eta_i}}. \]  
(14)

3. Influence of thermal resistance on energy separation efficiency

We determine the effect of the thermal resistance of the separation wall \( \delta / \lambda_i \) and the efficiency of finning on the part of the supersonic channel \( \eta_i \) and on the part of the subsonic channel \( \eta_1 \) on the energy separation efficiency in the Leontiev tube (14). Consider the turbulent flow regime of air, which is characteristic to this energy separation method [7, 12], and coolant with low Prandtl number [10, 11], namely the helium – xenon mixture with the mass content of helium equal to 7.2%. The properties of air and helium-xenon mixture are presented in Tables 1 and 2, respectively.

| Table 1. Properties of air | Table 2. Properties of He-Xe mixture |
|-----------------------------|--------------------------------------|
| \( \mu \), \( \mu Pa s \) | \( K_{He}, \% \) | \( \mu \), \( \mu Pa s \) | \( c_p \), J/kg\cdot K |
| 18.50 | 25.58 | 527 |
| 1005 | 29.6 | 0.715 |
| \( \lambda \), mW/m\cdot K | \( M \), g/mole | \( \lambda \), mW/m\cdot K | \( M \), g/mole |
| 26.00 | 0.715 | 65.15 | 39.9 |
| 29.6 | 0.206 |
We will take the stagnation parameters, as well as geometry and material of the separation wall of the Leontiev tube to be close to those considered in works [7, 8, 9, 12]: $p_0 = 7.5$ atm, $T_0 = 295$ K, $x = 0.25$ m, $S_0 = 3.14 \times 10.4 \times 250 = 8164$ mm$^2$, $\delta = 3.45$ mm. Copper separation wall had thermal conductivity $\lambda_0 = 387.6$ W/m·K, while ebony wall – $\lambda_0 = 0.16$ W/m·K. In this case, for air $\alpha_0 = 6513$ W/m$^2$K, and for helium-xenon mixture $\alpha_0 = 7522$ W/m$^2$K, thermal resistance of copper wall $\delta/\lambda_0 = 8.9 \times 10^{-6}$ m$^2$K/W, and for ebony wall – $\delta/\lambda_0 = 21.5 \times 10^{-3}$ m$^2$K/W. The recovery coefficient is determined by the formula $r = 0.9 Pr^{0.1}$ [10] and is considered the same at any point of the separation wall. It is known [10] that for an infinite heat-conducting non-finned separation wall, the ratio (14) allows determining the optimal values of Mach numbers in the core of supersonic and subsonic flows, at which the energy separation efficiency $\bar{q}_w$ is maximal. For the Dorodnitsin gas with properties close to those for helium-xenon mixture of noted composition, the optimal Mach number in the core of supersonic flow should be $2...2.5$, while in the core of subsonic flow it ranges within the limits of $0.36...0.5$. Figure 2 represents the calculation results of the energy separation efficiency $\bar{q}_w$ depending on the thermal resistance of the separation wall without finning and with different finning configurations for $M_{ai} = 2$, and $M_{ai} = 0.5$. It can be noted that at all other conditions being equal, the value of $\bar{q}_w$ for helium-xenon mixture is 4-5 times higher than that for air. Finning of the separation wall leads to an increase in the heat transfer rate.

**Figure 2.** The effect of finning and the thermal resistance of separation wall on the energy separation.

**Figure 3.** The effect of Mach number in the subsonic channel on the energy separation in the smooth Leontiev tube and the tube with a finned separation wall.

This effect becomes significant when the internal thermal resistance of the wall is $10^{-3}$ m$^2$K/W and lower, i.e. for highly conductive walls. The greatest increase in the energy separation efficiency is observed for finned separation wall, both from the parts of supersonic flow and subsonic flow. The heat transfer intensity through the separation wall increases in case of its finning for both air and helium-xenon mixture. In addition, the availability of finning makes it possible to achieve the energy separation efficiency in the Leontiev tube operating on the air, equal to or greater than the energy separation efficiency in the Leontiev tube without finning operating on helium-xenon mixture, which is important in terms of reducing the cost of creating and operating such devices. Figure 3 presents the calculation of the heat flux through the separation wall $\bar{q}_w$ at different Mach numbers in the subsonic...
part of the flow. A decrease in the Mach number of the cooled flow (flow rate in the subsonic part of
the flow) leads to a significant decrease in its temperature at the outlet of the Leontiev tube with a
decrease in heat transfer through the separation wall. This is important for building cooling systems
based on the energy separation effect. As can be seen from the graph, within the studied range of
\( M_{\text{in}} = 0.05...1 \), the presence of finning enhances heat transfer through the highly conductive wall. In
real Leontiev tubes this fact can lead to an increase in power while maintaining the overall dimensions
of the energy separation devices, or to an increase in compactness while maintaining power.

Conclusion
The paper presents a theoretical analysis of the finning effect of the separation wall on the heat
transfer rate from the subsonic flow towards the supersonic flow of air or helium-xenon mixture in the
energy separation taking place in Leontyev tube. It is shown that for the known configuration of the
Leontiev tube, the effect of the wall finning on the increase of the energy separation efficiency
depends on the internal thermal resistance of the wall without finning. It is shown that by means of
finning of the separation wall, it is possible to increase the heat flux for tubes operating on air to
values comparable to the heat flux for tubes operating on helium-xenon mixture, which is important
for the practical application of these devices.

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