Charge Fractionalization in a Kondo Device

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We study nonequilibrium transport through a charge Kondo device realizing the two-channel Kondo critical point in a recent experiment by Iftikhar et al.3 By computing the current and shot noise at low voltages near the critical point, we obtain a universal Fano factor $e^2/e = 1/2$. We identify elementary transport processes as weak scattering of emergent fermions carrying half-integer charge quantum numbers. This forms an experimental fingerprint for fractionalization in a non-Fermi liquid, which, compared to spin-Kondo devices, could be observed at elevated temperatures.

**Introduction and Results.**—Deconfinement and fractionalization are fascinating phenomena in which particles that are initially found as bound states become independent of each other. Such phenomena emerge in strongly interacting condensed matter systems, for example, in the form of spin-charge separation in Luttinger liquids,2 possible emergence of a spinon Fermi sea in spin liquid13, and the appearance of magnetic monopoles in spin-ice2. Deconfinement is often associated with asymptotic freedom as occurring in gauge theories4 leading to the quark-gluon plasma in high energy physics. Arguably, the simplest strongly interacting model that displays asymptotic freedom is the Kondo effect, describing magnetic impurities in metals, and revived in the 90’s in the realm of quantum dots.21 In this paper we argue that a similar phenomenon may be observed in on-going experiments on a charge Kondo device.2,5

As introduced in 1980, the two-channel Kondo (2CK) model3 describes a single impurity spin $S$ coupled to two electronic channels $\alpha = 1, 2$ via the spin-flip interaction

$$H_K = J \sum_{\alpha = 1, 2} \psi_\alpha^\dagger(0) \psi_\alpha(0) S^+ + H.c.$$  \hspace{1cm} (1)

While a single channel of electrons can completely screen the impurity spin, the presence of two (or more) competing channels turns this model into a paradigmatic example of frustration and non-Fermi liquid (NFL) behavior, with possible broader significance in bulk systems such as heavy fermion materials. Due to the spin-flip process $H_K$, the number of electrons from each channel $\alpha = 1, 2$ and each spin $\sigma = \uparrow, \downarrow$, $N_{\alpha \sigma} = \int dx \psi_{\alpha \sigma}^\dagger(x) \psi_{\alpha \sigma}(x)$, may change but only by unit steps. As a precursor to fractionalization in this model, through the Emery-Kivelson (EK) solution2, one introduces charge, spin, flavor, and spin-flavor quantum numbers,

$$N_{c,s} = \frac{1}{2} (N_{\uparrow} \pm N_{\downarrow} + N_{2\uparrow} \pm N_{2\downarrow}),$$

$$N_{I,sf} = \frac{1}{2} (N_{\uparrow} \pm N_{\downarrow} - N_{2\uparrow} \pm N_{2\downarrow}),$$

and associated new fermions, $\psi_\mu^\dagger(x)$ (here $\mu = c, s, f, sf$), that change only the corresponding $N_\mu$ quantum numbers by a unit step. An exact rewriting of the Kondo interaction is $H_K = J[(\psi_{sf}(0)\psi_{sf}^\dagger(0)) + \langle \psi_{s}(0)\psi_{sf}(0) \rangle]S^+ + H.c.$. Crucially, at weak coupling physical operators such as $H_K$, involve the new fermions in pairs.10–11 This is a necessary constraint to describe Fermi liquid (FL) states, since upon inverting Eq. (2), each single new fermionic particle changes electronic numbers $N_{\alpha \sigma}$ by half-integers10–11, for example $\psi_{sf}^\dagger$ takes $N_{sf} \rightarrow N_{sf} + 1$ or equivalently $\delta(N_{\uparrow}, N_{\downarrow}, N_{2\uparrow}, N_{2\downarrow}) = (1, -1, -1, 1)$. In this sense, the new fermions are “confined” to occur in pairs in physical processes in FLs. However, the non-perturbative Kondo interaction leads to non-Fermi liquid behavior12–14. In light of this, one may wonder - is unpairing of these fermions possible and can it be manifest in a physical system?

In recent years the multichannel Kondo effect was experimentally studied in highly tunable semiconductor quantum dot systems15–18. Our work is primarily motivated by charge 2CK setups, theoretically suggested by Akhmerov et al. and recently realized in the quantum Hall regime. The impurity “spin” is encoded by two nearly degenerate macroscopic charge states of a large quantum dot, see Fig. 1 which is coupled to normal leads via quantum point contacts (QPCs) allowing to flip the “spin” via single electron tunneling. Upon decreasing temperature below the Kondo temperature $T_K$ the conductance reaches half of the conductance quantum $G \rightarrow \frac{1}{\pi} \frac{G}{2}$, corresponding to two perfectly transmitting quantum resistors in series.

We study non-equilibrium transport through such devices and analyze the non-linear current $I(V)$ and shot noise $S(V)$, focusing on the vicinity of the 2CK critical point. Generally, shot noise informs on the charge of the current carrying particles, examples ranging from the fractional quantum Hall effect19 to superconduc-
tor junctions exhibiting Cooper pair tunneling. Applying methods borrowed from Gogolin and Komninos and Schiller and Hershfield, we find interesting universal properties encoded in the current and noise in the non-equilibrium Kondo regime $T \ll eV \ll T_K$ (for simplicity we set $T = 0$ for now). The current $I = \frac{2eV}{\hbar} (1 - eV/T_K) + O(V^3)$ contains a non-linear correction that corresponds to a backscattering current $I_b = \frac{e^2V}{2\pi e T_K}$. While the first term describes noiseless current through two perfectly transmitting QPCs, the backscattering current produces shot noise $S = 2e^2I_b + O(V^3)$, with a Fano-factor $e^2/e^2 = e^2/2$.

This fractional Fano factor can be precisely interpreted in terms of unpairing of a spin-flavor fermion $\psi_{sf}^\dagger$ in physical processes at the NFL state: we identify the elementary backscattering processes consisting of annihilation of this individual fermion which yield half-integer changes in electronic occupation numbers, as directly reflected in this fractionalization.

**Model.**—Our system in Fig. 1 consists of a large metallic quantum dot in the quantum Hall regime with spinless electrons coupled to two normal leads via QPCs and described by the Hamiltonian:

$$H_K = \sum_{\alpha=1,2} \int dx \psi_{\alpha \sigma}^\dagger(x) \psi_{\alpha \sigma}(x) + \frac{i}{\hbar} eV F_{\alpha \sigma} + J a \left( \psi_{\alpha \uparrow}^\dagger(0) \psi_{\alpha \downarrow}(0) S^z + H.c. \right) + \Delta E S^z.$$  

Here, $\sigma = \uparrow$ describes states in the lead and $\sigma = \downarrow$ in the dot; the index $\alpha = 1, 2$ labels the two QPCs. We assume that the dot is sufficiently large, such that its level spacing is small compared to the temperature, as a result of which edge states in the dot near different QPCs are incoherently coupled. We specialize to the large charging energy limit such that only two macroscopic charge states, with $N = N_0$ or $N_0 + 1$ electrons in the dot, are relevant at the experiment’s temperatures $T \ll E_c$, and play the role of the impurity spin $S$. Upon detuning the gate voltage from the degeneracy point, an energy splitting $\Delta E$ is formed between these macroscopic charge states.

The two-channel Kondo state is a critical point occurring at charge degeneracy $\Delta E = 0$ and for left-right symmetry $J_1 = J_2 = J$, which will be assumed. Towards the end we will comment on deviations from these conditions which lead to a crossover at low energies to a FL state. The parameters of the model include the density of states $v$ and a high-energy cutoff $D$, set by the minimum of the band-width and the charging energy, defining through the tunneling amplitude $J$ the Kondo temperature $T_K \sim D e^{-v/J}$.

The model Eq. (3) is an anisotropic XY Kondo Hamiltonian with the $J_z$ term omitted. In our calculations below we will add such a term $H_z = J_z \sum_{\alpha, \sigma, \sigma'} \psi_{\alpha \sigma}^\dagger(0) \psi_{\alpha \sigma}(0) S^z$, keeping in mind that spin anisotropy does not affect the low energy physics.

**Strategy.**—Our primary interest is in the non-equilibrium transport properties and specifically on the shot noise in the vicinity of the 2CK fixed point. As a non-perturbative tool allowing to approach the vicinity of the strong coupling 2CK fixed point, we use the EK solution near the Toulouse point $J_z = 2e\hbar v_F$. The analysis at the Toulouse point gives correctly only the fixed point properties such that the $T = V = 0$ value of the linear conductance $G \to G_0 = \frac{e^2}{2}\hbar$. However this free fermion description misses the leading low energy corrections. It is well known that for multichannel Kondo models these arise from the leading irrelevant operator known from CFT. We clarify that in the 2CK model the leading irrelevant operator is turned on via any small deviation from the Toulouse point. Thus, treating $J_z = 2e\hbar v_F$ as a perturbation, allows us to perform a controlled non-equilibrium calculation in terms of the original electronic degrees of freedom, and capture the leading low energy corrections at the 2CK fixed point.

**Mapping to the Toulouse Hamiltonian.**—Following the standard EK transformation we (i) bosonize the fermionic fields $\psi_{\alpha \sigma}(x) \sim \sum_{\mu} \frac{1}{\sqrt{2\pi a}} e^{i\phi_{\alpha \sigma}(x)}$, with $a$ being a short distance cutoff, (ii) perform the rotation in Eq. (2) to define charge, spin, flavor, and spin-flavor bosons, $\Phi_{\alpha \sigma} \to \Phi_{\alpha \mu}(\mu = c, s, f, sf)$, and finally (iii) referomize these bosons into new fermion operators $\psi_{\mu} \sim \frac{1}{\sqrt{2\pi a}} e^{i\phi_{\mu}}$. The transformed Hamiltonian becomes

$$H_K = i\hbar v_F \sum_{\mu} \int dx \psi_{\mu \uparrow}^\dagger(x) \partial_x \psi_{\mu \uparrow}(x) + iJ \chi_{sf}(0) \hat{b} - \frac{eV}{2} \int dx \left[ \psi_{\sigma \uparrow}^\dagger(x) \psi_{\sigma \downarrow}(x) + \psi_{\sigma \downarrow}^\dagger(x) \psi_{\sigma \uparrow}(x) \right] + iJ_z - 2\pi e\hbar v_F \psi_{\mu \uparrow}^\dagger(0) \psi_{\mu \downarrow}(0) \hat{a} \hat{b},$$

where $\hat{a}, \hat{b}$ are a local Majorana operators associated with the impurity degrees of freedom, $\hat{a} \hat{b} = S^z$, satisfying $\hat{a}^2 = \hat{b}^2 = \frac{1}{2}$, $\chi_{sf}(x) \sim \psi_{sf}^\dagger(x) \psi_{sf}(x) / \sqrt{2}$, and $J = J_1 + J_2$. We included the source-drain voltage $eV$, setting a chemical potential difference in the leads $eV N_{1\uparrow} N_{2\downarrow}$, which after the transformation Eq. (2) simply becomes a chemical potential of the new fermions. The last term accounts for deviations from the Toulouse point. The current operator is given by $\hat{I} = \frac{ie}{\hbar} \left[ \frac{N_{1\uparrow} - N_{2\downarrow}}{2}, H_K \right]$.

Applying this free fermion Hamiltonian at the Toulouse point, as detailed in the appendix, one obtains the current $I(V) = \frac{e^2}{\hbar} V (1 + O(V^2/T_K^2))$ and noise $S(V) = O(V^3/T_K^2)$, where $T_K = \pi e^2 / J_2^2$, valid for $eV \ll T_K$. As noted above, the 2CK fixed point $eV/T_K \to 0$ corresponds to two perfectly transmitting QPCs in series which do not produce any partitioning noise and thus we have $S = 0$. The quadratic voltage corrections in $I(V)$ and the cubic term in $S(V)$ are artifacts of the free fermion resonant level
structure. We shall now obtain the leading universal corrections in $eV/T_K$ that emerge due to deviations from the free fermion point.

Irrelevant operator and shot noise near the critical point. From the CFT solution of the multichannel Kondo effect\(^{21}\), one can identify the leading irrelevant operator, which captures the low energy corrections around the strong coupling fixed point. For the 2CK model this is the dimension 3/2 operator\(^{21}\), which can be written in terms of EK fermions as\(^{21}\)

$$H_{irr} = \frac{1}{\nu^{3/2}} eV\sqrt{\pi T_K} \sum k \psi_k^\dagger(0)\psi_k(0) \hat{\chi}_{sf}(0) \hat{a}. \quad (5)$$

Here, $\nu = \frac{1}{2\pi\hbar \Gamma}$ is the density of states, $T_K$ acts as a high energy scale, and $\hat{\chi}_{sf}(x) = \chi_{sf}(x)\text{sign}(x)$ is a modified spin-flavor Majorana fermion, reflecting the absorption of the local Majorana fermion $\hat{b}$ as detailed in the appendix. How does the source-drain voltage couple to $\hat{\chi}_{sf}$? Answering this question requires one to formulate an approach fully in terms of the original degrees of freedom in Eq. (4), in which the voltage enters in a simple way.

We obtain such a controlled approach by treating the deviations from the Toulouse point $v_1$, in which the voltage enters in a simple way, straightforward though tedious computation of the current and noise. Another approach fully in terms of the original degrees of freedom in Eq. (4), in which the voltage enters in a simple way.

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Thus, by setting the deviation from the anisotropic Toulouse point to $v_1 = -\sqrt{\pi}/\nu$, we generate the leading dimension 3/2 irrelevant operator Eq. (5) up to the 2CK fixed point. As described in detail in the appendix, a straightforward though tedious computation of the current as well as shot noise in the framework of Eq. (4), to infinite order in $\nu$ and to the leading second order in $v_1/\sqrt{\pi T_K}$, gives our main results

$$I = \frac{e^2V}{2h} \left( 1 - \frac{\pi^2|eV|}{8 T_K} + O\left( \frac{eV}{T_K} \right)^2 \right),$$

$$S = \frac{e^2V}{2h} \left( \frac{\pi^2|eV|}{8 T_K} + O\left( \frac{eV}{T_K} \right)^2 \right). \quad (8)$$

Notably, we can define the backscattering current $I_b \equiv G_0 V - I = \frac{e^2V}{h} \frac{\pi^2|eV|}{T_K}$, and write the shot noise as $S = 2e^2I_b$ with $e^* = e/2$.

The same result, supplemented by an intelligible physical picture, can be obtained by a simple calculation based on the Fermi’s golden rule applied directly with respect to the irrelevant operator Eq. (5). Decomposing the operator $\hat{\chi}_{sf}(0) = \frac{1}{\sqrt{8L}} \sum k \psi_k^\dagger \psi_k(0)\hat{\chi}_{sf}(0)\hat{a}$ into normal fermionic modes, we see that it either creates a particle or a hole in the spin-flavor Fermi sea. Eq. (4) shows that the source-drain voltage sets an enhanced chemical potential $eV/2$, and the Fermi sea. Thus annihilation of one spin-flavor particle at $k_\pm$ above the equilibrium Fermi level $0 < \epsilon_{k_\pm} < eV/2$ lowers the energy. Energy conservation is attained via a creation of a particle-hole excitation in the spin sector via the factor $\psi_k^\dagger(0)\psi_k(0) = \frac{1}{L} \sum_{k_\pm} \sum_{\epsilon_{k_\pm}} e_{\epsilon_{k_\pm}} c_{k_\pm} c_{k_\pm}$. Eq. (5) is revealed by the ratio between the coefficients of $eV/T_K$ and $\delta(\epsilon_{k_+} - \epsilon_{k_-} - \epsilon_{k_\pm}) = \pi (eV)^2 / h 167K$. (9)

Crucially, the unit change in $\hat{N}_{sf}$, modifies electronic occupations by half integers. Thus this Poissonian process describes backscattering of charge $e^* = e/2$, and the backscattering current is

$$I_b = -e^* \frac{d \langle \hat{N}_{sf} \rangle}{dt} = \frac{e^2V}{h} \frac{\pi^2|eV|}{167K},$$

with an associated noise $S = 2e^* I_b$, in agreement with Eq. (8).

Finite temperature effects. – The universality of our results Eq. (8) is revealed by the ratio between the coefficients of $eV/T_K$ in the current and noise. Another experimentally testable universal ratio can be obtained from the leading $T$ dependence of the current, which we find to be $I = G_0 V \left[ 1 - \frac{\pi^2}{8} \left( \frac{eV}{T_K} + 2\frac{T}{\pi T_K} \right) \right]$. Similarly, the noise $S(V,T)$ has a temperature dependence where, at $T \gg eV$ it must cross from the shot noise limit Eq. (8) to thermal noise $S = 4k_B T G$ with $G = dI/dV|_{V=0}$. 

![Energy diagram of EK Fermi model](image-url)
Deviations from the critical point.}—We first test the influence of relevant perturbations. The intricate properties of the critical point are destabilized by left-right asymmetry $\Delta J = J_1 - J_2$ or by gate voltage deviations from the charge degeneracy point $\Delta E$. These create an energy scale, $T^* = c_1 T_K (\nu \Delta J)^2 + c_2 (\Delta E)^2 / T_K$, with $c_{1,2}$ coefficients of order unity. Below this energy scale the system crosses over to a FL state, whereby the non-linear conductance gradually decreases below $G_0 = e^2 / 2 h$. Since $T_K$ may become high and approach the charging energy ($\sim 290 mK$) in charge-kondo devices, one may realistically assume $T^* \ll T_K$. This gives a finite voltage window $T^* \ll \epsilon V \ll T_K$ within which our shot noise predictions, dominated by the leading irrelevant operator, hold. Nevertheless, what is the leading influence of finite $T^*$? Including for instance channel asymmetry, and assuming $\epsilon V \ll T_K$, the current and noise Eq. [5] acquire the corrections\(^{25}\)

$$\delta I = -\frac{e}{h} T^* \arctan \frac{\epsilon V}{2 T^*},$$

$$\delta S = \frac{e^2}{h} T^* \int_{-\epsilon V/2 T^*}^{\epsilon V/2 T^*} dy \left( \frac{1}{1 + y^2} \right),$$

(10)

which are valid for any ratio $T^*/\epsilon V$. This current remains a small backscattering correction compared to $G_0 V$ for $\epsilon V \gg T^*$. Under this condition Eq. (10) gives $\delta I = -\frac{e^2}{2 \pi} T^* + \mathcal{O}(\epsilon V^{-1})$, and $\delta S = \frac{e^2}{2 \pi} T^* + \mathcal{O}(\epsilon V^{-1})$, the total current and noise are $I + \delta I$ and $S + \delta S$. Interestingly, to leading order in $T^*$ the fractional Fano factor $e^*/e = (S + \delta S)/(2c(G_0 V - I - \delta I)) = 1/2$ reap-pears. This is expected since, similar to the irrelevant operator Eq. [5], also the dimension 1/2 relevant channel asymmetry operator consists of any unpaired spin-flavor fermion $(\psi_{sf} - \psi_{sf})/\sqrt{2}$, or another $\psi_{sf}$ fermion for other relevant perturbations e.g. $\Delta J$.\(^{33}\) Thus, although relevant operators eventually destabilize the critical point, the $e^*/e = 1/2$ Fano factor is remarkably stable and includes the leading effects of relevant perturbations as well.

We herein validate the stability of our results in presence of generic marginal operators. These are quadratic forms of the original electrons $\psi_{\alpha\sigma} \psi_{\alpha'\sigma'}$, which map into quadratic forms of the new fermions $\psi^{(1)} \psi^{(1)}$. Such operators, if present, lead to corrections to the current due to single electron processes and hence will affect the Fano factor. First, consider $\psi^{(1)} \psi^{(1)} + h.c.$ whereby one electron moves between a lead and the dot. At the free fermion fixed point, this marginal operator changes the charge of the dot, hence, it must involve $S^\pm = (\hat{a} \mp i \hat{b}) / \sqrt{2}$. Using Eq. (9), the latter becomes the unpaired fermion $\chi_{\alpha}$, changing electronic numbers in the leads by half integers. Thus, exactly like the marginal operator describing deviations from the Toulouse point, any marginal operator involving the impurity spin $\hat{S}$ at the free fermion fixed point changes into a dimension 3/2 operator at the critical point. Secondly, consi-

![FIG. 3. Charge fractionalization using a weak probe: one electron tunnels from the weakly coupled lead no. 3 and is equally and simultaneously partitioned into the two leads.](image-url)
processes, while our current result $e^* = e/2$ cannot be accommodated within such a Fermi liquid picture. In contrast to charge-Kondo devices, calculations of non-equilibrium transport in spin-multichannel Kondo devices remain challenging, but expectedly doable for 2CK devices, due to the free fermion effective description. Also, non-equilibrium noise in $N > 2$ multichannel charge Kondo devices whose linear transport properties were addressed recently remains an interesting question for future work. For this theoretical task, connections $e.g.$ to topological Kondo devices and their non-equilibrium properties may become useful.

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Appendix A: Full counting statistics (FCS)

This appendix is devoted to a controllable and detailed derivation of the main results based on the full counting statistics (FCS) method following Gogolin and Komnik. In Sec. A.1 we start by recapitulating the main definitions of the FCS generating function and its relation to Keldysh Green functions (GF); we apply these definitions in Sec. A.2 at the Toulouse point as well as including relevant perturbations generating the energy scale $T^*$; finally we apply the FCS method in Sec. A.3 perturbatively from the definitions from the Toulouse point providing a controlled derivation of our main result Eq. (8).

1. Preliminaries

We define a generating function $\chi(\lambda) = \sum q e^{i q \lambda} P_q$, where $P_q$ is the probability for transfer of charge $q$ through our system within the measurement time $T$. Then the cumulants are given by

$$\langle \delta^n q \rangle = (-i)^n \frac{\partial^n}{\partial \lambda^n} \ln \chi(\lambda) \bigg|_{\lambda=0}. \tag{A1}$$

The generating function is given by the following average on the Keldysh contour $C$ [49],

$$\chi(\lambda) = \left<T_C \exp \left[-\frac{i}{\hbar} \int_C T_\lambda(t) dt \right]\right>, \tag{A2}$$

where $T_C$ is the contour ordering operator, and $\lambda(t)$ is a contour dependent measuring field which is non-zero only during the measurement time. The operator $T_\lambda(t)$ here describes direct tunneling between the left and right leads. It is coupled to the measuring field $\lambda$ with the generic form

$$T_\lambda(t) = e^{i \lambda/2} T_R + e^{-i \lambda/2} T_L, \tag{A3}$$

where $T_R$ and $T_L$ are operators transferring an electron through the system to the right or to the left, respectively. In fact the measuring field enters via a gauge transformation

$$\psi_L \rightarrow \psi_L e^{i \lambda/4}, \quad \psi_R \rightarrow \psi_R e^{-i \lambda/4}, \tag{A4}$$

where $\psi_L/R$ annihilates a particle in the left/right leads. The generating function can be expressed as $\ln \chi = -i T U(\lambda, -\lambda)$, in terms of a field $U$ which satisfies

$$\frac{\partial}{\partial \lambda} U(\lambda_-, \lambda_+) = \left\langle \frac{\partial H(\lambda)}{\partial \lambda_+} \right\rangle_\lambda. \tag{A5}$$

Here, $-$ and $+$ denote the forward and backward parts of the Keldysh contour. Thus, on evaluating Eq. (A5), integrating the result over $\lambda_-$ and finally setting $\lambda_- = -\lambda_+ = \lambda$, one obtains the generating function for the cumulants. The current and noise are then obtained from the first two cumulants,

$$I = \frac{e \langle \delta q \rangle}{T}, \quad S = \frac{2 e^2 \langle \delta^2 q \rangle}{T}. \tag{A6}$$

For a weak current composed of uncorrelated tunneling events, the effective charge $e^*$ is $e^* / e = \langle \delta^2 q \rangle / \langle \delta q \rangle$.

2. FCS at the Toulouse point

Our next step is to incorporate the measuring field $\lambda(t)$ into the Toulouse Hamiltonian [1]. This is done by generalizing the gauge transformation Eq. (A4) to include the dot-states, $\sigma = \downarrow$, which remain gauge invariant,

$$\psi_{1,\uparrow} \rightarrow e^{i \lambda(t)/4} \psi_{1,\uparrow}, \quad \psi_{2,\uparrow} \rightarrow e^{-i \lambda(t)/4} \psi_{2,\uparrow}, \quad \psi_{\alpha,\downarrow} \rightarrow \psi_{\alpha,\downarrow}, \quad (\alpha = 1, 2). \tag{A7}$$

Going through the EK transformation, one can show that this gauge transformation maps to

$$\psi_{c,s} \rightarrow \psi_{c,s}, \quad \psi_{sf} \rightarrow e^{i \lambda(t)/4} \psi_{sf}. \tag{A8}$$

Thus, the tunneling term $J$ in Eq. (4) transforms to

$$H_\lambda = i \mathcal{J} \hat{b} [\chi_{sf}(\lambda(t)/4) + \eta_{sf} \sin(\lambda(t)/4)], \tag{A9}$$

where $\chi_{sf} = \psi_{\uparrow}^{\dagger} \psi_{\downarrow}$ and $\eta_{sf} = \psi_{\uparrow}^{\dagger} - \psi_{\downarrow}^{\dagger}$ are the Majorana fermions associated with the spin-flavor field. We observe that the gauge transformation Eq. (A9) can be identified with a rotation of the Majorana components. Defining a rotated Majorana basis

$$\begin{pmatrix} \chi_{sf}^\lambda \\ \eta_{sf}^\lambda \end{pmatrix} = \begin{pmatrix} \cos(\lambda(t)/4) & \sin(\lambda(t)/4) \\ -\sin(\lambda(t)/4) & \cos(\lambda(t)/4) \end{pmatrix} \begin{pmatrix} \chi_{sf} \\ \eta_{sf} \end{pmatrix}, \tag{A10}$$

the Toulouse Hamiltonian takes the form

$$H_\lambda = i \hbar v_F \sum_{\mu=c,s,f} \int dx \psi_{\mu}^{\dagger} (x) \partial_x \psi_{\mu} (x) \tag{A11}$$

where $\mathcal{J}_- = \frac{\mathcal{J} T}{\sqrt{2} \pi a}$. We first address the symmetric lead couplings setup, where $\mathcal{J}_- = 0$. In this case, note that only $\chi_{sf}^\lambda$ is coupled to the impurity, and the RHS of Eq. (A5) is

$$\left\langle \frac{\partial H_\lambda}{\partial \lambda} \right\rangle_\lambda = \frac{i}{4} \left\langle \hat{b} [\eta_{sf} \cos(\lambda(t)/4) - \chi_{sf} \sin(\lambda(t)/4)] \right\rangle_\lambda. \tag{A12}$$

Expressing the above equation in terms of Keldysh GF’s (for the form of the electronic free GF’s, see Eq. (34) in the paper of Gogolin and Komnik), we replace $V$ →...
calculate quadratic structure of this Hamiltonian, one can exactly
\[ \alpha = D \]
where we define \( \Gamma = T_K = \pi \nu J^2 \), \( \lambda = \lambda^- - \lambda^+ \),
\[ D_{bb}(t) = -i \langle T_C \hat{b}(t) \hat{b}(0) \rangle \] is the full GF of the \( \hat{b} \) op-
erator and \( n_{1,2}(\omega) \) are Fermi-Dirac functions of leads \( \alpha = 1, 2 \) respectively. Taking the advantage of the quadratic structure of this Hamiltonian, one can exactly calculate(Eq. (A12)) for the asymmetry term by replacing \( \Gamma \)
Due to the similar structure of the equation one can write
now like to include the effect of channel asymmetry
Focusing on low energies \( \omega \ll T_K \), we see that \( T(\omega) \to 1 \). The \( \lambda \)-dependence \( e^{\pm i \lambda} \) signifies single electron transport processes. At \( T = 0 \) the generating function becomes \( \ln \chi = T \frac{\lambda \nu V}{2 \hbar} \), such that only the first moment is finite. Consequently, we obtain conductance \( G = \frac{\pi^2}{2} \), to geth-\er, with noise \( S = 0 \). This happens naturally in the limit \( eV, k_BT \ll T_K \) where the system is exactly at the
fixed point.
Before looking at the irrelevant operator, we would now like to include the effect of channel asymmetry \( J_\rightarrow \frac{i - J_\leftarrow}{\sqrt{2\pi}a} \) and observe the resulting current and noise. The change \( \delta \left( \frac{\partial H}{\partial \lambda} \right) \) in Eq. (A11) is
\[ \delta \left( \frac{\partial H}{\partial \lambda} \right) = \frac{i}{4} \langle \hat{a} \left[ \chi_{st} \sin(\lambda(t)/4) + \eta_{st} \cos(\lambda(t)/4) \right] \rangle. \]
Due to the similar structure of the equation one can write
Eq. (A14), only with \( \lambda \to -\lambda \) and \( T(\omega) = \frac{\pi^2}{\omega^2 + i\omega} \). It
is then easy to obtain Eq. (10) at \( T = 0 \). Note that the change in the sign of \( \lambda \) in the form \( e^{-i\lambda} \) indicates a negative contribution to the current.

3. Deviations from the Toulouse point

We now consider the effect of irrelevant operators Eq. (5) emerging at the vicinity of the 2CK fixed point on the generating function and its cumulants. We have clarified that the irrelevant operator is generated by the deviations from the Toulouse point, see Eq. (7). Thus, by evaluating Eq. (A5) in the presence of \( \nu_1 \) as a perturbation we expect to obtain at low energies the same behavior of the dimension 3/2 irrelevant operator.

In order to calculate the generating function in the presence of the coupling Eq. (7) one has to evaluate perturbative corrections to the \( \hat{b} \)-impurity’s GF. To lowest order in \( \nu_1 \), the impurity’s GF is
\[ D_{bb} = D_{bb} + \nu_1^2 D_{bb} \Sigma D_{bb} = D_{bb} + \delta D_{bb}, \]
where \( \Sigma \) is the self energy. Thus, Eq. (A12) acquires the correction
\[ \langle \frac{\partial H}{\partial \lambda} \rangle = \frac{i}{4} \int \frac{d\omega}{2\pi} \left\{ \delta D_{bb}^{-\frac{1}{2}}(\omega)(n_2 - n_1) \right\} \]
\[ + \delta D_{bb}^{+\frac{1}{2}}(\omega) \left[ e^{\frac{i\lambda}{4}(1 - n_2)} - e^{-i\frac{\lambda}{4}(1 - n_1)} \right]. \]
The self-energy takes the form
\[ \Sigma^ij(\omega) = \int \frac{d\omega_1}{2\pi} \frac{\delta D_{aa}}{\omega - \omega_1} \int \frac{d\omega_2}{2\pi} G^ij_\omega(\omega_1 + \omega_2) G^ij_{\omega_2}, \]
where \( \delta D_{aa} = -i \langle T_C \hat{a}(t) \hat{a}(0) \rangle \) and \( G_{\omega} = -i \langle T_C \hat{a}(t) \hat{a}(0) \rangle \) are bare \( \hat{a} \)-GF of the impurity
and the spin fermionic operators, respectively. At \( T = 0 \) these two different GF’s take the form
\[ G^ij_\omega(\omega) = 2\pi \nu \left[ \begin{array}{cc} -\frac{i}{2} \text{sign}(\omega) & i\Theta(\omega) - \frac{i}{2} \text{sign}(\omega) \\ -i\Theta(\omega) & \frac{i}{2} \text{sign}(\omega) \end{array} \right], \]
\[ \delta D_{aa}^i(\omega) = \left[ \begin{array}{c} \frac{1}{\omega} \\ -i\pi\delta(\omega) \end{array} \right]. \]
Evaluating \( \Sigma^ij(\omega) \) at \( T = 0 \), we obtain
\[ \Sigma^ij(\omega) = \nu^2 \omega \left[ \begin{array}{c} \ln |\omega| - 1 - i\pi\Theta(\omega) \\ -i\pi\Theta(\omega) \end{array} \right]. \]
To obtain \( D_{bb}(\omega) \) at low energies, we take the limit \( \omega \ll \Gamma \),
We find that the integral over \( \delta D_{bb} \)
\[
\int \delta D_{bb} \sum \text{such that the only contribution to the generating function}
\]
the value of the effective charge
\[ e \] easily able to calculate the cumulants of Eq. (A25) and
ative correction to the current,
\[ \lambda \]
Not the factor 1
\[ n_1 + n_2 - 1 \]
\[ e^{i\lambda/4 n_1} + e^{-i\lambda/4 n_2} \]
We find that the integral over \( \delta D_{bb} \) in Eq. (A17)
vanishes. Looking at the second term \( \delta D_{bb}^+ = \sum_{i,j} D_{bb}^{ij} \delta D_{bb}^+ \) in detail, we find
\[ D_{bb}^{ij} \Sigma^{-} D_{bb}^{ij} + = - D_{bb}^{ij} \Sigma^{+} D_{bb}^{ij} +, \] (A23)
such that the only contribution to the generating function
comes from the term
\[ D_{bb}^{ij} \Sigma^{+} D_{bb}^{-} = i \pi \nu^2 \omega \Theta(\omega) \left[ \frac{\Gamma(e^{i\lambda/4 n_1} + e^{-i\lambda/4 n_2})}{\text{Det}(g_0^{-1} - \Sigma_0)} \right]^2. \] (A24)
Plugging this term into Eq. (A17), we obtain the correction for the generating function at \( T = 0 \)
\[ \ln \delta \chi = \frac{T}{\hbar} \left( \frac{v_F e V}{4} \right)^2 e^{-i\lambda/2}. \] (A25)
Note the factor 1/2, to be reflected in the fractional Fano factor, and the negative sign of \( \lambda \) which indicates a negative correction to the current,
\[ I = \frac{e^2 V}{2 \hbar} \left[ 1 - \pi \nu^2 e^2 |V|^2 \right] \]. (A26)
Defining the backscattering current \( I_b \equiv I - e^2 V \) we are easily able to calculate the cumulants of Eq. (A23) and the value of the effective charge
\[ e^*/e = \left( \frac{\partial^2 \ln \chi}{\partial \lambda^2} \right) \left( \frac{\partial \ln \delta \chi}{\partial \lambda} \right) \bigg|_{\lambda = 0} = \frac{1}{2}. \] (A27)

**Appendix B: Derivation of Eq. (6)**

This short appendix provides a derivation of Eq. (6) following Refs. [20] and [27]. We examine the exact behavior of the coupled operators \( \chi_{cs}(x) \) and \( b \) in the vicinity of the fixed point as described by Eq. (4) in the absence of the perturbation Eq. (5). Consider the mode expansion
\[ \chi_{cs}(x) = \sum_k \varphi_k(x) \psi_k + H.c \] (B1)
\[ \hat{b} = \sum_k u_k \psi_k + H.c, \]
where \( \psi_k \) are operators in Fock-space satisfying \( \{ \psi_k, \psi_k^\dagger \} = \delta_{k,k'}, \varphi_k(x) \) are wave functions and \( u_k \) are local coefficients. In this basis, the Hamiltonian reads
\[ H' = \sum_k \varepsilon_k \psi_k^\dagger \psi_k \]
such that the wave functions \( \varphi_k(x) \) and \( u_k \) satisfy a set of Schrodinger equations
\[ [H_K, \chi_{cs}(x)] = [H', \chi_{cs}(x)], [H_K, \hat{b}] = [H', \hat{b}] \] yielding
\[ i \lambda \delta(x) u_k + i \hbar v_F \partial_x \varphi_k(x) = \varepsilon_k \varphi_k(x), \] (B2)
\[ - i \lambda \varphi_k(0) \]
Solving these equations, one obtains \( \varphi_k(x) \propto e^{i k x} [\theta(x) \varphi_k^{(+)} - \theta(-x) \varphi_k^{(-)}], \varphi_k(0) = \frac{1}{2} (\varphi_k^{(+)} + \varphi_k^{(-)}), \quad u_k = \frac{i \lambda}{\hbar v_F} \varphi_k(0), \quad \varphi_k^{(+)} / \varphi_k^{(-)} = e^{-\frac{2i \lambda}{\pi v_F} \tan^{-1} \left( \frac{x}{v_F T_K} \right)}. \] Thus, for energies \( \ll T_K \) one finds \( \varphi_k^{(+)} = - \varphi_k^{(-)}. \) Although the wave functions have a discontinuity at \( x = 0, \) the local Majorana \( b \) field can be written as
\[ \hat{b} = \frac{1}{\sqrt{\pi \nu T_K}} \hat{\chi}_{cs}(0) \] (B3)
where \( \hat{\chi}(x) = \chi_{cs}(x) \text{sign}(x) \) is continuous at \( x = 0. \)