Enhancing stability of industrial turbines using adjustable partial arc bearings

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Abstract. The paper presents the principal of operation, the simulation and the characteristics of two partial-arc journal bearings of variable geometry and adjustable/controllable stiffness and damping properties. The proposed journals are supposed to consist of a scheme that enables the periodical variation of bearing properties. Recent achievements of suppressing rotor vibrations using plain circular journal bearings of variable geometry motivate the further extension of the principle to bearings of applicable geometry for industrial turbines. The paper describes the application of a partial-arc journal bearing to enhance stability of high speed industrial turbines. The proposed partial-arc bearings with adjustable/controllable properties enhance stability and they introduce stable margins in speeds much higher than the 1st critical.

1. Introduction
Journal bearing induced instabilities in rotating machinery is a problem that in general is prevented by using journal bearings of multi-lobe elliptical profile or of tilting-pad scheme. The multi-lobe elliptical journal bearings do contribute in locating the instability threshold at higher speeds than the operational speed range by increasing the effective eccentricity (Childs [1]) and tilting-pad bearings do have the advantage of minimizing the cross-coupling stiffness and damping coefficients eliminating in this way the tangential to the whirl fluid film forces that raise instability (Lang and Steinhilper [2]; Glienicke [3]). However, plain cylindrical journal bearings loaded in such extent that would locate the journal at a sufficient eccentricity would not suffer from instability at a specific range of operational speed as found by Hori [4]. This was a notification that raised the power of the slender generator rotors. Although the generators had to increase their length/mass in order to increase power, the heavy loaded journal bearings could set instability thresholds at higher than 3000/3600RPM speeds.

Modern turbine rotors of medium size steam turbines (30-200MW) do not always rotate at synchronous speeds of 50/50Hz but towards more efficient thermodynamics the geared turbines of higher range of operating speeds (>5000RPM) do apply in many power plants. In order to avoid instability issues the tilting-pad bearings are often implemented in such applications. However, the multi-lobe elliptical bearings do offer stable operation and they are quite often implemented in the design having also the advantage of lower price.

The design of a rotor bearing system implemented on industrial turbines for power generation is nowadays a relatively standardized procedure. However, it is not always the case that the real operation meets demands of the supplier/customer and this can be due to many reasons. There can be the case that thermal distortion can deform the turbine case and thus introduce lateral misalignment at
the bearings. There is also the case that bearing power losses appear higher than estimated and this could lead to severe expenses for the supplier to the customer.

Combining the demand for optimum operation, suppressed vibration amplitude and enhanced stability, the concept of adjustable/controllable journal bearing has been developing in the last 4-5 decades. The theoretical proposals are numerous, however, only some would meet a practical application in industrial turbines mainly due to reasons of cost, simplicity, and reliability. A quite complete overview of the experimental and theoretical achievements on mechatronics applied to controllable journal bearings can be found by Santos [5]. Proposed concepts incorporating movable bearing pads, among others the very recent from Chasalevris and Dohnal [6]. The application of external forces in the bearing shell or the journal, for example magnetic (El-Shafei and Dimitri [7]; Dohnal and Markert [8]; Furst and Ulbrich [9]), or piezo-mechanical (Palazzolo et al [10], Przybylowicz [11], Tuma et al. [12]), without changing the bearing’s clearance geometry.

Furthermore, active devices including hydraulic actuator journal bearings, variable impedance bearings (Goodwin et al. [13]), deformable bushes (Kicinski and Materny [14]), and active journal bearings with flexible sleeve (Krodkiewski and Sun [13]) develop high performance of rotating machinery improving the dynamic properties of the rotating system.

Both concepts of journal bearings (JBs) and Active Magnetic Bearings (AMBs) have been proposed under different schemes for controlling/adjusting system properties with the JBs to have the advantage of large load carrying capacity and high damping, and the AMBs to be promoted through controllability and elimination of oil supply. The limitations of JBs stand on the instability mechanism, known as oil whirl and oil whip (El-Shafei [16]; Crandall [17]) while AMBs cannot support high loads without occupying much space in the machine.

The proposed concept of adjustable fluid film journal bearings presented in this paper aims to contribute at the turbine shaft-line dynamics within three means:

a) As a mean to provide enhanced stability margins of industrial turbine rotors at higher speeds (>5000RPM) under the principle of periodic variation of stiffness and damping properties of the fluid film. Recent investigations from Pfau et al. [18] and Dohnal et al. [19] have shown that the bearing instability margins are improved in certain domain of frequency and amplitude of the periodic variation of stiffness and damping properties of the fluid film. This is the actual concept of the paper.

b) As a real time adjustor of the alignment in both horizontal and vertical planes, additionally to the permanent alignment of the pedestals performed on site. This ability is concerned to be a way to achieve an optimum operation of the shaft-line regarding loading if any deformations appear due to thermal distortion of the casing. In long shaft-lines of industrial turbine-generators very small changes in a bearing lateral alignment (<100μm) can lead to severe changes in bearing loading (>1kN) and in this way an optimum Sommerfeld number can be achieved regarding power losses. This is a secondary issue for this paper and the concept is not covered here.

c) As a mean to apply parametric excitation in a turbine rotor bearing system for modal interaction and vibration suppression during normal operation of the system as Dohnal has extensively studied in the recent past in [20-21], and very recently in [22]. The current paper provides an estimation of the variation of stiffness and damping matrices regarding pad displacement and frequency of excitation. With these results, upcoming works aim to investigate thoroughly the modal interaction in a complete turbine-generator shaft-line consisting of three rotors mounted on the proposed adjustable bearings.

2. Principle of operation and simulation of the adjustable bearings

In this paper two types of adjustable partial arc bearings are proposed; the 2-arc bearing consists of two movable cyclic sectors located anti-diametrically at vertical direction, see figure 1 (left) and the 3-arc bearing that consists of three movable cyclic sectors located in the peripheral of the bearing at 120° arc distance to each other, see Figure 1 (right). The movable pads are attached to actuators that are supposed to provide a predefined sinusoidal displacement of certain amplitude and frequency.
The actuation of the bearing moving pads is considered to occur through piezoelectric actuators. The piezoelectric actuators can offer the demanded force for the moving pad displacement under normal operation of the system with full bearing load, combining accuracy in displacement and relatively simple mechanism enclosed in the bearing shell. This will be an open loop control; the feedback of the bearing pad displacement is theoretically not required. However, the development of the proposed bearing and the practical implementation should consider monitoring of the bearing moving pad displacement for functional safety.
Bearings with two small (of short arc 50°-80°) carrying surfaces at bottom and top as in Figure 1 (left) are normally used for relatively low loads and small diameters in industrial turbine-generator shaft-lines. The proposed structural configuration aims also to give the ability of preloading the bearing without the need of lifting up the entire pedestal. Such type of bearings usually has a preload in order to avoid instability issues. The bearing with three small carrying surfaces distributed symmetrically or asymmetrically on the peripheral of the bearing is usually applied for relatively low loads and of non-vertical direction.

The geometric definition of the proposed adjustable 2-arc and 3-arc bearings is given in figure 2. The radius of the rotor is $R$ and of each bearing pad is $R + c$. The fluid film thickness for the bearing pads at a complete circle is defined with respect to the circumferential coordinate $\theta$ as $h_0(\theta) = c - z \cos(\theta) - y \sin(\theta)$. Function $h_0(\theta)$ can be theoretically defined in the entire circumference of the bearing but practically is applied at the value's domain of $\theta_i < \theta < \theta_i'$ at the $i^{th}$ pad. The 2-arc bearing consists of two arcs of equal arc length while the 3-arc bearing consists of two arcs (the two upper) of equal arc length and one (the one lower) of relatively longer arc length as defined in table 1.

**Table 1.** Definition of the geometry of the bearing arcs with respect to figures 1 and 2

| Bearing type | Definition | Arc 1 | Arc 2 | Arc 3 |
|--------------|------------|------|------|------|
| 2-Arc        | Starting angle | $\theta_2 = 60^\circ$ | $\theta_3 = 240^\circ$ | |
|              | Ending angle | $\theta_2' = 120^\circ$ | $\theta_3' = 300^\circ$ | |
|              | Located at | $\theta_{l,2} = 90^\circ$ | $\theta_{l,3} = 270^\circ$ | |
|              | Of arc length | $a_2 = 60^\circ$ | $a_3 = 60^\circ$ | |
|              | In phase of | $\phi_2 = 0\text{ rad}$ | $\phi_3 = \pi\text{ rad}$ | |
| 3-Arc        | Starting angle | $\theta_1 = 0^\circ$ | $\theta_2 = 120^\circ$ | $\theta_3 = 220^\circ$ |
|              | Ending angle | $\theta_1' = 60^\circ$ | $\theta_2' = 180^\circ$ | $\theta_3' = 320^\circ$ |
|              | Located at | $\theta_{l,1} = 30^\circ$ | $\theta_{l,2} = 150^\circ$ | $\theta_{l,3} = 270^\circ$ |
|              | Of arc length | $a_1 = 60^\circ$ | $a_2 = 60^\circ$ | $a_3 = 100^\circ$ |
|              | In phase of | $\phi_1 = \pi / 6\text{ rad}$ | $\phi_2 = 5\pi / 6\text{ rad}$ | $\phi_3 = 9\pi / 6\text{ rad}$ |

The $i^{th}$ pad at both types of adjustable bearing is supposed to be enabled for a variable in time displacement $\delta_i$ through the radial direction of its middle, defined as $\theta_{l,i}$, within an amplitude $\delta_0$, frequency $\Omega_{ex}$, and in phase $\phi_i$ as $\delta_i = \delta_0 \cos(\Omega_{ex} t - \phi_i)$; for the values of $\phi_i$ see table 1. Then the additional fluid film thickness $d_i(\theta)$ at the $i^{th}$ pad, that is introduced due to the $i^{th}$ pad displacement and is added to the $h_0(\theta)$, is defined as in equation (1).

$$d_i(\theta) = \delta_i \cos(\theta - \theta_{l,i}) = \delta_0 \sin(\Omega_{ex} t - \phi_i) \cos(\theta - \theta_{l,i})$$  \hspace{1cm} (1)

The resulting fluid film thickness function at the $i^{th}$ pad is defined as $h_i(\theta) = h_0(\theta) + d_i(\theta)$. Correspondingly, the rate of change of the additional fluid film thickness due to the displacement of each moving pad is defined as in equation (2) and of the resulting fluid film thickness in equation (3).
\[
\dot{h}_i = \Omega \delta_0 \cos(\Omega_t - \varphi_i) \cos(\theta - \theta_{L,i}) \\
\dot{h}_i = -z \cos(\theta) - y \sin(\theta) + \Omega \delta_0 \cos(\Omega_t - \varphi_i) \cos(\theta - \theta_{L,i})
\] (2)

The partial derivative of the resulting fluid film function \( h \) with respect to \( \theta \) is defined in equation (4).
\[
\frac{\partial h}{\partial \theta} = z \sin(\theta) - y \cos(\theta) - \delta_0 \sin\left(\cos(\Omega_t - \varphi_i) \sin(\theta - \theta_{L,i})\right)
\] (3)

For displacements of \( y \) and \( z \) of the journal within the radial clearance of the bearing, see figure 2, that would result to an absolute eccentricity of \( e = \sqrt{z^2 + y^2} \) and a relative eccentricity of \( e = e / c \) at a range of \( 0.15 < e < 0.75 \), the volume of the lubricant flowing in each pad is assumed to flow the following assumptions [4].

1. The lubricant flow is laminar.
2. The gravity and inertia forces acting on the lubricant can be ignored compared with the viscous force.
3. Compressibility of the lubricant is negligible.
4. The lubricant is Newtonian and the coefficient of viscosity is constant.
5. Lubricant pressure does not change across the film thickness.
6. The rate of change of the velocity of the lubricant in the circumferential and the axial direction of the bearing is negligible compared with the rate of change in the radial direction.
7. There is no slip between the fluid and the solid surface.

Then, the lubricant pressure distribution at the \( P_i(x, \theta) \) is supposed to satisfy the Reynolds equation as given in equation (5) for a finite length bearing as these implemented in this paper.
\[
\frac{1}{\mu R^2} \frac{\partial}{\partial \theta} \left(h_i \frac{\partial P_i}{\partial \theta}\right) + \frac{1}{\mu} \frac{\partial}{\partial x} \left(h_i \frac{\partial P_i}{\partial x}\right) = 6 \Omega \frac{\partial h}{\partial \theta} + 12 \dot{h}_i
\] (5)

The right hand side terms of the Reynolds (RHS) can be defined as \( \text{RHS} = 6 \Omega \frac{\partial h}{\partial \theta} + 12 \dot{h}_i \). Using the definitions given before, RHS is defined as in equation (6) and after some math as in equation (7).
\[
\text{RHS} = 6 \Omega \left(z \sin(\theta) - y \cos(\theta) - \delta_0 \sin\left(\cos(\Omega_t - \varphi_i) \sin(\theta - \theta_{L,i})\right)\right) + 12 \left(-z \cos(\theta) - y \sin(\theta) + \Omega \delta_0 \cos(\Omega_t - \varphi_i) \cos(\theta - \theta_{L,i})\right)
\] (6)

\[
\text{RHS} = -6 (2z + \Omega y) \cos(\theta) - 6 (2y - \Omega z) \sin(\theta) - 6 \delta_0 \left(\Omega \sin(\Omega_t - \varphi_i) \sin(\theta - \theta_{L,i}) - 2 \Omega \cos(\Omega_t - \varphi_i) \cos(\theta - \theta_{L,i})\right)
\] (7)

At a given discrete time moment \( t = n \delta t \) and for a given set of values of operating parameters \( \Omega, y, z \), \( \dot{y}, \dot{z} \), the pressure distribution at the \( i^\text{th} \) pad \( P_i(x, \theta) \) can be evaluated by solving equation (5) using the finite difference method (FDM). The lubricant pressure is evaluated at a finite difference grid of \( 10 \times 30 \) intervals at the axial and the circumferential direction correspondingly.

The assumptions for the pressure distribution consider that the pressure is zero at the ends of each bearing pads where the oil feed takes place, see figure 1.

The pressure distribution at the divergent area of each pad is evaluated but it is neglected (assumed zero or ambient) at the evaluation of the fluid film impedance force. Thus, only the positive pressure is integrated in the following equations to obtain the bearing impedance forces. With one axial interval
of $\delta x$, equal at all pads to $\delta x = L / 10$, and three different circumferential intervals defined at the $i^{th}$ pad as $\delta \theta_i = (\theta^* - \theta_i) / 30$, the impedance force of the 2-arc bearing will be given as in equations (8) and (9) at the horizontal and vertical direction correspondingly, see figure 2, while for the 3-arc bearing will be given as in equations (10) and (11). The angle at each point of the defined grid is defined at the $i^{th}$ pad as $\theta_{i,j} = \theta_i + j \cdot \delta \theta_i$.

$$
F_x = -\sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_2(x_j, \theta_{2,j}) R \cos(\theta_{2,j}) \delta x \delta \theta_2 \right) - \sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_3(x_j, \theta_{3,j}) R \cos(\theta_{3,j}) \delta x \delta \theta_3 \right)
$$

(8)

$$
F_y = -\sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_2(x_j, \theta_{2,j}) R \sin(\theta_{2,j}) \delta x \delta \theta_2 \right) - \sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_3(x_j, \theta_{3,j}) R \sin(\theta_{3,j}) \delta x \delta \theta_3 \right)
$$

(9)

$$
F_x = -\sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_1(x_j, \theta_{1,j}) R \cos(\theta_{1,j}) \delta x \delta \theta_1 \right) - \sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_2(x_j, \theta_{2,j}) R \cos(\theta_{2,j}) \delta x \delta \theta_2 \right) - \sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_3(x_j, \theta_{3,j}) R \cos(\theta_{3,j}) \delta x \delta \theta_3 \right)
$$

(10)

$$
F_y = -\sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_1(x_j, \theta_{1,j}) R \sin(\theta_{1,j}) \delta x \delta \theta_1 \right) - \sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_2(x_j, \theta_{2,j}) R \sin(\theta_{2,j}) \delta x \delta \theta_2 \right) - \sum_{i=1}^{30} \sum_{j=1}^{110} \left( P_3(x_j, \theta_{3,j}) R \sin(\theta_{3,j}) \delta x \delta \theta_3 \right)
$$

(11)

When the journal is located at a fixed equilibrium position ($\dot{y} = \dot{z} = 0$) the minimum fluid film thickness coincides with the circumferential location at which the pressure obtains a zero value. Then, for the greater circumferential angles at the pad, cavitation is assumed to occur and the developed negative pressure is assumed to be zero at equations (8) to (11). The definition of the angle where zero pressure is noticed is not used as a boundary condition for the evaluation of the pressure distribution, but it is yielded from the evaluation with the boundary conditions to assume zero pressure at the beginning and the end of the arc. The same boundary conditions are assumed also for the dynamic case ($\dot{y} \neq 0, \dot{z} \neq 0$) within this case the minimum fluid film thickness angle not to coincide with the zero pressure at this case. Again, the negative pressures are not included in the numerical integration in equations (8) to (11). Although the Gumbel/half Sommerfeld cavitation model is supposed to yield a fair approximation to the performance of only narrow bearings and at low eccentricities, its simplicity and compactness of its mathematics constitutes a useful tool in this preliminary analysis of such a lubrication problem that aims to the estimation of the variance of stiffness and damping coefficients under adjustable bearing geometry.

3. Evaluation of the adjustable bearing characteristics

The resulting stiffness and damping coefficients of the fluid film of the 2-arc and the 3-arc adjustable bearings are evaluated in this section. The Sommerfeld number is defined as $S_o = \mu RL \Omega (R / c)^2 / (\pi W)$ (SI units) for geometric and physical parameters that correspond to usual bearing applications of small steam turbine bearings as will be shown for example in table 2. For a set of geometrical and physical properties close to these of table 2, the equilibrium locus is evaluated for both types of bearings for the range of Sommerfeld number $0.05 < S_o < 0.5$ with the changing parameter of operation to be the rotating speed $\Omega$.

For a given external vertical load $W$ to the journal and rotating speed $\Omega$, and for static conditions ($\dot{z} = \dot{y} = 0$), a simple algorithm was programmed to implement the Newton-Raphson method for the
solution of the system of 2 equations describing the equilibrium position of the journal and the 2 unknowns that are the vertical and horizontal displacement of the journal \( y \) and \( z \), see equation (12).

\[
\begin{bmatrix}
  y \\
  z
\end{bmatrix}
_{t+1} =
\begin{bmatrix}
  y \\
  z
\end{bmatrix}
_t - J^{-1} f,
\]

where \( J = \begin{bmatrix}
  \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\
  \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z}
\end{bmatrix} \), and \( f = \begin{bmatrix}
  f_1 \\
  f_2
\end{bmatrix} = \begin{bmatrix}
  F_y - W \\
  F_z
\end{bmatrix} \). (12)

For a given set of initial values \( \{ y \quad z \}_0 = \{-0.01 \quad 0.01\}_0 \) the Jacobian matrix \( J_t \) is evaluated and the system’s equations obtain their initial values \( \{ f_1 \quad f_2 \}_0 \). After only some iterations (<5) the product \( J_t^{-1} \times \{ f_1 \quad f_2 \}_0 \) is very close to zero and the solution is achieved. The partial derivatives presented in the Jacobian matrix are evaluated numerically with a perturbation of the corresponding variable \( 10^{-9} \). For a given bearing configuration about the 2-arc and the 3-arc bearing the equilibrium locus is evaluated and plotted in figure 3 for a variation of Sommerfeld number corresponding from low to high journal eccentricity, as indicated also in figure 3. Three cases are considered regarding geometry: \( L / D = 0.5 \), \( L / D = 0.75 \), and \( L / D = 1.00 \) with the result of equilibrium position locus not to differ much, among the three cases. However, for the same Sommerfeld number the equilibrium position is found at a considerably different location among the three cases, especially in low eccentricities.

**Figure 3.** Equilibrium locus of the 2-arc (left) and the 3-arc (right) bearing for various values of Sommerfeld number.

In the selected points of equilibrium shown in figure 3 a small perturbation is applied in the journal with respect to displacement in order to evaluate the stiffness coefficients, and with respect to velocity in order to evaluate the damping coefficients. The dimensionless stiffness and damping coefficients of the fluid film in both 2-arc and 3-arc bearings are plotted in figure 4 for a selected \( L / D = 0.75 \) that applies most in turbine bearings. The evaluated coefficients are compared in figure 4 with those corresponding to a plain circular \( 360^\circ \) bearing that are evaluated in the same way.
Figure 4. Stiffness coefficients of the 2-arc bearing (left), and the 3-arc bearing (right) as a function of Sommerfeld number $S_o$. Solid line: proposed bearing, dashed line: 360° plain cylindrical bearing.

The cross-coupling stiffness coefficient $K_{ZY}$ appears sensibly lower in the entire range of Sommerfeld number, in both 2-arc and 3-arc bearing configuration compared to the conventional full cylindrical bearing, while the cross-coupling stiffness coefficient $K_{ZZ}$ appears slightly higher in the heavier loading cases ($S_o < 0.5$) and slightly higher in the lighter loading cases ($S_o > 0.5$), at both 2-arc and 3-arc bearing configuration compared to the conventional full cylindrical bearing.

The direct stiffness coefficient in the vertical direction $K_{YY}$ (direction of load) is higher in the 2-arc and 3-arc bearing configuration compared to the conventional full cylindrical bearing, for the entire range of Sommerfeld number. Conversely, in the horizontal direction, the conventional bearing appears to be less stiff compared to the 2-arc and 3-arc bearing, in the entire range of Sommerfeld number, considering the progress of $K_{ZZ}$ in the graph. The divergence of stiffness coefficient $K_{ZZ}$ between conventional and -arc bearing gets slightly higher for lower loads ($S_o \approx 5$).

The ability of the fluid film to provide stiffness and damping to the rotor changes with respect to the operating conditions of the bearing and the fluid film forces obtain different sensitivity to perturbations that can happen to the journal regarding its displacement and velocity in lower or higher eccentricities. Any movement of lubricating surfaces of journal and bearing have different influence in the resulting forces of the fluid film, comparing low and high eccentricity of operation. The sensitivity of the resulting bearing forces to small displacements of the bearing moving pads is investigated in continue and the evaluation of the variation of the stiffness and damping coefficients under a given pad excitation is the scope of the following.

Many operating conditions have been considered regarding Sommerfeld number for the investigation of the proposed partial arc bearings with a selected $L / D = 0.75$. Here, only the case of $S_o = 1$ is presented. The result of the bearing moving pad excitation to the variation of the bearing properties of stiffness is presented in the following figures 5 and 6. Three different cases of $d_o = 5\%c$, $d_o = 10\%c$ and $d_o = 15\%c$ are considered, while the excitation frequency varies at $\Omega_{ex1} = 100\text{rad/s}$, $\Omega_{ex2} = 200\text{rad/s}$, $\Omega_{ex3} = 300\text{rad/s}$ etc. The selected values of excitation frequency are supposed to be applicable for systems that present 1st and 2nd or even 3rd critical speed within a range of 100rad/s – 900rad/s, as it happens in rotor-bearing systems of industrial turbines. As it is seen in figures 5 and 6 the frequency of the pad excitation has significant influence on the variation of the
coefficient regardless the amplitude of excitation. The incorporation of the bearing pad velocity in
Reynolds equation is proved to be beneficial regarding the approximation of the analysis.

Figure 5. Stiffness coefficients of the 2-arc bearing as a function of time, for different amplitude of
pad displacement a) \( d_0 = 5\% c \), b) \( d_0 = 10\% c \), and c) \( d_0 = 15\% c \), variable cases of excitation
frequency \( \Omega_{ex} \), and for \( S_0 = 1 \).

Figure 6. Stiffness coefficients of the 3-arc bearing as a function of time, for different amplitude of
pad displacement a) \( d_0 = 5\% c \), b) \( d_0 = 10\% c \), and c) \( d_0 = 15\% c \), variable cases of excitation
frequency \( \Omega_{ex} \), and for \( S_0 = 1 \).

4. Application to a turbine rotor and investigation of stability characteristics

A representative to an industrial steam turbine rotor-bearing system of basic properties presented in
table 2 and of geometrical outline shown in figure 7 is simulated using the Transient Transfer Matrix
Method (TTMM) [23]. The linear bearing impedance forces are applied on two of the nodes, shown in
figure 7 under the principle of periodic variation of stiffness and damping coefficients, as defined in
equations (13) and (14) where \( \varepsilon_\theta \) is the approximate variation of the coefficient \( K_{ij}, C_{ij} \) according to
the evaluations of section 3; see figures 5 and 6.

\[
\begin{pmatrix}
K_{YY} (\Omega, t) & K_{YZ} (\Omega, t) \\
K_{ZY} (\Omega, t) & K_{ZZ} (\Omega, t)
\end{pmatrix} = \left( 1 + \varepsilon_\theta \sin(\Omega_{ex} t) \right) \begin{pmatrix}
K_{YY} (\Omega) & K_{YZ} (\Omega) \\
K_{ZY} (\Omega) & K_{ZZ} (\Omega)
\end{pmatrix}
\]  \hspace{1cm} (13)

\[
\begin{pmatrix}
C_{YY} (\Omega, t) & C_{YZ} (\Omega, t) \\
C_{ZY} (\Omega, t) & C_{ZZ} (\Omega, t)
\end{pmatrix} = \left( 1 + \varepsilon_\theta \sin(\Omega_{ex} t) \right) \begin{pmatrix}
C_{YY} (\Omega) & C_{YZ} (\Omega) \\
C_{ZY} (\Omega) & C_{ZZ} (\Omega)
\end{pmatrix}
\]  \hspace{1cm} (14)

The two bearing models presented in this paper are implemented with the properties corresponding
to those of an industrial steam turbine of higher speed and two sets of results are produced, one for the
2-arc and a second for the 3-arc bearing. The linear transient response is evaluated for various constant
speed values on the range of \( \Omega / \Omega_i = 0 - 10 \) where \( \Omega_i \) the 1st un-damped rigid support bending natural
frequency defined approximately as \( \Omega_n = (n \pi)^2 \sqrt{EI / M / L} \).
Using plain cylindrical bearings, the system would experience bearing instability at about $\Omega = 2\Omega_1$. Relatively enhanced stability margins are noticed using the proposed bearing configuration but circular bearing profile would hardly achieve stability at a rated speed of $\Omega / \Omega_1 \approx 3$. For this reason the tilting-pad bearings are included mostly in the design of the current machine so as to assure stable bearing operation.

Table 2. Basic geometrical and physical properties of the turbine rotor-bearing system

| Rotor & Blades | Bearing | #1 | #2 |
|----------------|---------|----|----|
| Mass           | $M = 5000kg$ | $L_\eta / D_\eta$ ratio | $\approx 0.75$ | $\approx 0.75$ |
| Slenderness ratio | $L / D \approx 9$ | Radial clearance | $c \approx D_\eta / 1000$ | $c \approx D_\eta / 1000$ |
| Young Modulus  | $E = 210GPa$ | Oil Viscosity | $\mu = 0.03Pa \cdot s$ | $\mu = 0.03Pa \cdot s$ |
| 1st natural freq. | $\Omega_1 \approx 240rad/s$ | Sommerfeld Number | $So = 0.8$ | $So = 0.6$ |
| Operating speed | $\Omega / \Omega_1 \approx 3$ | | |

Figure 7. Representation of the turbine used in this example.

The stability assessment for the linear system of time-periodic characteristics is performed using Floquet theory [21]. The system’s response is evaluated numerically at various given speeds for a number of periods of the system’s periodic variation of stiffness and damping and for different initial conditions, so as to obtain the monodromy matrix of the system. The eigenvalues of the monodromy matrix declare the stable or unstable progress of the response [21]. For the case of the adjustable bearing proposed, the period of variation of system’s properties is defined as $T = 2\pi / \Omega_{ex}$; see also figures 5 and 6. The stability of the system is investigated for various cases of $\Omega / \Omega_1 = 0 \sim 10$ and $\Omega_{ex} / \Omega_1 = 0 \sim 10$ with $\varepsilon_{xy} = 0, 0.1, 0.2, ..., 0.5$ and the rest $\varepsilon_{ij}$ to obtain the corresponding values evaluated in sections 3; the result is mapped on figure 8. Very similar stability margins appear for both 2-arc and 3-arc bearing configurations and figure 8 concerns the 2-arc bearing.

As it is seen in figure 8, the system experiences stable response at rotating speeds around less than $\Omega = 2\Omega_1$, when $\varepsilon_{xy} = 0$, whatever the $\Omega_{ex}$ is; this is expectable. Increasing the rotating speed further, the system enters instability thresholds except if the frequency of stiffness and damping variation is of specific values that introduce parametric antiresonance. The candidate values of $\Omega_{ex}$ that would introduce parametric antiresonance in the system are $\Omega_{ex} = (\Omega_3 - \Omega_1) / n \ (n \in \mathbb{Z^+})$ if the system’s stiffness and damping matrices are symmetrical (current case), or $\Omega_{ex} = (\Omega_2 + \Omega_1) / n$ if the system’s stiffness and damping matrices are not symmetrical. The system’s second natural frequency in bending is defined approximately as $\Omega_2 = (2\pi)^2 \sqrt{EI / M / L^2}$ or $\Omega_2 = 4\Omega_1$.

The values of natural frequencies $\Omega_2$ are very approximating and are used only as an indication for the expectable thresholds of parametric antiresonance. When parametric antiresonance is introduced, leading to stability, e.g. at $2 < \Omega_{ex} / \Omega_1 < 3$, a parametric resonance is introduced also in frequencies
\( \Omega_{ex} / \Omega_1 < 2 \) or \( \Omega_{ex} / \Omega_1 > 3 \) but this does not affect the possibility of performing a run-up of the machine up to \( \Omega / \Omega_1 > 6 \) without entering instability thresholds.

**Figure 8.** Instability thresholds of the system using the proposed adjustable bearings for the various amplitude of a) \( \varepsilon_{yy} = 0 \), b) \( \varepsilon_{yy} = 0.1 \), c) \( \varepsilon_{yy} = 0.2 \), d) \( \varepsilon_{yy} = 0.3 \), e) \( \varepsilon_{yy} = 0.4 \), f) \( \varepsilon_{yy} = 0.5 \)

With the principle of parametric excitation of the system through the proposed adjustable journal bearings it is possible that the stability margins are extented to much higher rotating speeds, theoretically up to \( \Omega / \Omega_1 > 6 \) in the current example. As it is shown in figure 8, the application of
higher amplitude of bearing pad displacement $\delta$ and consequently in higher $\epsilon_{ij}$ yields a wider stable area of combinations and the transition from lower to higher operating speeds can occur without entering instability thresholds.

The influence of parametric excitation on the system’s response is shown additionally in the following figure 9 where the displacement of the rotor at a bearing location is plotted as a function of time for several periods. In figure 9 it is seen that at a selected rotating speed of $\Omega / \Omega_c = 2$ the system would be unstable if no parametric excitation in certain frequencies is provided, see also figure 8. As long as parametric excitation is introduced at a selected frequency of $\omega / \Omega_c = 2.6$, the displacement amplitude of the system decreases in time, indicating stable operation. The excitation frequency $\Omega_c$ could vary according to the operating speed $\Omega$, so both to intersect in a region of figure 8 where stability is indicated. In figure 9 it is seen that the impact of parametric excitation to introduce stability is more intense as the amplitude of bearing properties variation $\epsilon_{yy}$ increases.

5. Conclusions

The proposed partial-arc journal bearings of variable geometry are considered to benefit the operation of turbines on the following manner:

a) Introducing parametric excitation of the system’s properties of stiffness and damping at certain frequencies at which parametric anti-resonance occurs eliminating instability in theoretically very high rotating speed.

b) Adjusting on real time the journal/rotor center (alignment) at the desired position that was not achieved as in design, due to manufacturing or installation reasons or due to thermal distortion of stator/components after long time operation.

c) Achieving the optimum operation of the journal bearing regarding friction coefficient and minimizing the journal bearing power losses by adjusting the optimum clearance and preload in a certain operating condition.

References

[1] Childs, D. (1993), ‘Turbomachinery Rotordynamics: Phenomena, Modelling, and Analysis’, John Wiley & Sons, Inc. NYC, US.

[2] Lang, O. and Steinhilper, W. (1978), ‘Gleitlager – Berechnung und Konstruktion von Gleitlagern mit konstanter und zeitlich veränderlicher Belastung (Konstruktionsbucher Band 31)’, Springer-Verlag.

[3] Glienicke, J., 1987, Stabilitätsprobleme bei Lagerung schnelllaufender Wellen-Berechnung, Konstruktion und Verhalten von Mehrflächen- und Kippsegmentlagern, Technische Akademie Wuppertal, Wuppertal, Germany.

[4] Hori, Y. (2006), ‘Hydrodynamic Lubrication’, Springer-Verlag, Tokyo, Japan

[5] Santos, I., (1995), “On the Adjusting of the Dynamic Coefficients of Tilting-Pad Journal Bearings,” STLE Tribol. Trans., 38(3), pp. 700–706.

[6] Chasalevris, A. and Dohnal, F. (2015), ‘A journal bearing with variable geometry for the suppression of vibrations in rotating shafts: Simulation, design, construction and experiment’,
Mechanical Systems and Signal Processing 52-53, pp 506–528.

[7] El-Shafei, A. and Dimitri, A. S. (2010), ‘Controlling journal bearing instability using active magnetic bearings’, ASME Journal of Engineering for Gas Turbine & Power 132(1), pp 1–9.

[8] Dohnal, F. and Markert, R. (2011), ‘Enhancement of external damping of a flexible rotor in active magnetic bearings by time-periodic stiffness variation’, Journal of System and Dynamics 5(5), pp 856–865.

[9] Furst, S. and Ulbrich, H. (1988), ‘An active support system for rotors with oil-film bearings’, Proceedings of the 4th International Conference on Vibrations in Rotating Machinery of the Institution of Mechanical Engineers, pp. 61–68.

[10] A. B. Palazzolo, R. R. Lin, R. M. Alexander, A. F. Kascak and J. Montague (1991) ASME Journal of Vibration and Acoustics 113, 167}175. Test and theory for piezoelectric actuator for active vibration control of rotating machinery.

[11] Przybylowicz, P. Active stabilisation of a rigid rotor by a piezoelectrically controlled mobile journal bearing system. Australian J of Mech. Eng. 1(2) (2004), 123–128.

[12] Tuma, J., Simek, J., Skuta, J., and Los, J. Active vibrations control of journal bearings with the use of piezoactuators. Mechanical Systems and Signal Processing 36 (2013), 618–629.

[13] M. J. Goodwin, J. E. T. Penny and C. J. Hooke (1984) IMechE, C288/84, 535}541. Variable impedance bearings for turbogenerator rotors.

[14] J. Kicinski and P. Materny 1995 Proceedings of the International Conference on Vibration and Noise, Venice, 120 127. 25-27 April 1995. Non-linear vibrations in multi degree freedom system on the example of turbine 13K215.

[15] J. M. Krodkiewski and L. Sun (1998) Journal of Sound and Vibration 210, 215}229. Modelling of multi-bearing rotor system incorporating an active journal bearing.

[16] El-Shafei, A., (1994), “Insights Into the Static and Dynamic Characteristics of Journal Bearings,” Proceedings of the Fourth IFToMM International Conference on Rotordynamics, pp. 307–315.

[17] Crandall, S. H., (1995), “The Instability Mechanism Responsible for Oil Whirl and Oil Whip,” Proceedings of the Fourth Greek National Congress on Mechanics, Democritus University of Thrace, Xanthi.

[18] Pfau, B., Rieken, M. and Markert, R. (2015), ‘Numerische Untersuchungen eines verstellbaren Gleitlagers zur Unterdruckung von Instabilitaten mittels Parameter-Antiresonanzen. 1st IFToMM D-A-CH Conference.

[19] Dohnal, F., Pfau, B. and Chasalevris, A. (2015), ‘Analytical predictions of a flexible rotor in journal bearings with adjustable geometry to suppress bearing induced instabilities’, 13th International Conference in Dynamical Systems Theory and Applications DSTA 2015

[20] Dohnal, F. (2008), ‘Damping by parametric stiffness excitation: resonance and anti-resonance’, Journal of Vibration and Control 14(5), pp 669–688.

[21] Dohnal, F. (2012), ‘A contribution to the mitigation of transient vibrations, Parametric antiresonance; theory, experiment and interpretation’, Habilitation thesis, Technische Universitat Darmstadt.

[22] Dohnal, F. and Chasalevris, A. (2015), ‘Inducing modal interaction during run-up of a magnetically supported rotor’, 13th International Conference in Dynamical Systems Theory and Applications DSTA 2015.

[23] Liew, A., Feng, N., and Hahn, E. (2004), ‘On using the transfer matrix formulation for transient analysis of nonlinear rotor bearing systems’, International journal of rotating machinery 10(6), pp. 425-431.