Applicability of perturbative QCD to $\Lambda_b \to \Lambda_c$ decays

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Abstract

We develop perturbative QCD factorization theorem for the semileptonic heavy baryon decay $\Lambda_b \to \Lambda_c l\bar{\nu}$, whose form factors are expressed as the convolutions of hard $b$ quark decay amplitudes with universal $\Lambda_b$ and $\Lambda_c$ baryon wave functions. Large logarithmic corrections are organized to all orders by the Sudakov resummation, which renders perturbative expansions more reliable. It is observed that perturbative QCD is applicable to $\Lambda_b \to \Lambda_c$ decays for velocity transfer greater than 1.2. Under requirement of heavy quark symmetry, we predict the branching ratio $B(\Lambda_b \to \Lambda_c l\bar{\nu}) \sim 2\%$, and determine the $\Lambda_b$ and $\Lambda_c$ baryon wave functions.

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I. INTRODUCTION

Analyses of exclusive heavy hadron decays are a challenging subject because of their complicated QCD dynamics. Recently, we have proposed a rigorous theory for these processes based on perturbative QCD (PQCD) factorization theorems [1,2]. In this approach heavy hadron decay rates are expressed as convolutions of hard heavy quark decay amplitudes with heavy hadron wave functions. The former are calculable in perturbation theory, if processes involve large momentum transfer. The latter, absorbing nonperturbative dynamics of processes, must be obtained by means outside the PQCD regime. Since wave functions are universal, they can be determined once for all, and then employed to make predictions for other processes containing the same hadrons. With this prescription for nonperturbative wave functions, PQCD factorization theorems possess a predictive power.

For semileptonic decays, the PQCD approach complements heavy quark symmetry in studies of heavy hadron transition form factors [3]. Heavy quark symmetry determines the normalization of transition form factors at zero recoil of final-state heavy hadrons, up to power corrections in $1/M$, $M$ being the heavy quark mass, and up to perturbative corrections in the coupling constant $\alpha_s$. While PQCD is appropriate for fast recoil, the region with large energy release, and gives a dependence of transition form factors on velocity transfer. For nonleptonic decays, PQCD is a more systematic approach compared with the phenomenological Bauer-Stech-Wirbel (BSW) model [4]. In PQCD factorization theorems contributions to nonleptonic decay rates characterized by different scales are carefully absorbed into different subprocesses, among which renormalization-group (RG) evolutions are constructed [2], leading to a scale and scheme independent, gauge invariant and infrared finite theory [5]. Not only factorizable but nonfactorizable contributions can be evaluated [6]. The BSW model considers only factorizable contributions: two fitting parameters $a_1$ and $a_2$ are associated with external and internal $W$-emission form factors, respectively. Non-factorizable contributions must be included as additional parameters [7].

The above PQCD formalism has been applied to heavy meson decays successfully. It is
then natural to extend the formalism to more complicated heavy baryon decays. In [8] we
have developed factorization theorem for the semileptonic decay $\Lambda_b \to pl\bar{\nu}$, in which Sudakov
resummation of double logarithmic corrections to the $\Lambda_b$ baryon wave function was included,
and a full set of diagrams for the hard $b$ quark decay amplitudes was calculated. This is an
analysis more complete than the work in the literature [9]. On the other hand, $b$-baryons
have been observed in experiments at LEP and at the Tevatron. Masses and decay widths
of the lightest $b$-baryons, as compared with theoretical predictions, have stimulated many
interesting discussions and investigations [10–14]. When Run II of the Tevatron comes up
with a vertex trigger employed, it will be expected to collect millions of $b$-baryon events.
Therefore, an intensive study of exclusive heavy baryon decays is urgent.

Exclusive heavy baryon decays are dominated by $b \to c$ modes. In this paper we shall de-
velop factorization theorem for the semileptonic decay $\Lambda_b \to \Lambda_c\bar{l}\bar{\nu}$, and locate the kinematic
region where PQCD is applicable. It will be shown that PQCD predictions for the involved
transition form factors are reliable at fast recoil of the $\Lambda_c$ baryon with velocity transfer
greater than 1.2. Under requirement of heavy quark symmetry, we predict the branching
ratio $B(\Lambda_b \to \Lambda_c\bar{l}\bar{\nu}) \sim 2\%$. We shall also determine the unknown parameters in the $\Lambda_b$ and
$\Lambda_c$ baryon wave functions, which can be employed to study nonleptonic $\Lambda_b$ baryon decays
because of the universality.

In Sec. II we develop factorization theorem for the semileptonic decay $\Lambda_b \to \Lambda_c\bar{l}\bar{\nu}$. Su-
dakov resummation of double logarithmic corrections to the process is performed. The fac-
torization formulas for the involved heavy baryon transition form factors and their numerical
results are presented in Sec. III and in Sec. IV, respectively. Section V is the conclusion.

II. FACTORIZATION THEOREM

The amplitude for the semileptonic decay $\Lambda_b \to \Lambda_c\bar{l}\bar{\nu}$ is written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb}\bar{l}\gamma^\mu(1 - \gamma_5)\nu_L(\Lambda_c(p'))\bar{c}\gamma^\mu(1 - \gamma_5)b|\Lambda_b(p)\rangle, \quad (1)$$
where $G_F$ is the Fermi coupling constant, $V_{cb}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $p$ and $p'$ are the $\Lambda_b$ and $\Lambda_c$ baryon momenta, respectively. All QCD dynamics is contained in the hadronic matrix element

$$M_\mu \equiv \langle \Lambda_c(p')|\bar c\gamma_\mu(1 - \gamma_5)b|\Lambda_b(p)\rangle = \bar \Lambda_c(p')[f_1(q^2)\gamma^\mu - if_2(q^2)\sigma^{\mu\nu}q_\nu + f_3(q^2)q^\mu]\Lambda_b(p)\nonumber$$

$$+\bar \Lambda_c(p')[g_1(q^2)\gamma^\mu\gamma_5 - ig_2(q^2)\sigma^{\mu\nu}\gamma_5 q_\nu + g_3(q^2)\gamma_5 q^\mu]\Lambda_b(p). \quad (2)$$

In the second expression $M_\mu$ has been expressed in terms of six form factors $f_i$ and $g_i$, where $\Lambda_b(p)$ and $\Lambda_c(p')$ are the $\Lambda_b$ and $\Lambda_c$ baryon spinors, respectively, and the variable $q$ denotes $q = p - p'$. In the case of massless leptons with $q_\mu \bar l\gamma^\mu(1 - \gamma_5)\nu = 0$, the form factors $f_3$ and $g_3$ do not contribute. Since the contributions from $f_2$ and $g_2$ are small, we shall concentrate on $f_1$ and $g_1$ in the present work.

The idea of PQCD factorization theorems is to sort out nonperturbative dynamics involved in QCD processes and factorize it into hadron wave functions. Nonperturbative dynamics is reflected by infrared divergences in radiative corrections to quark-level amplitudes in perturbation theory. The construction of factorization theorem for the decay $\Lambda_b \to \Lambda_c l\bar \nu$ is basically similar to that for the decay $\Lambda_b \to pl\bar \nu$ in [3]. The lowest-order diagrams for $b \to c$ decays are shown in Fig. 1, where two hard gluons attach the three incoming and outgoing quarks in all possible ways. We then investigate infrared divergences from radiative corrections to these diagrams. Small transverse momenta $k_T$ are associated with the valence quarks, such that they are off mass shell a bit. The transverse momenta $k_T$ serve as a factorization scale, below which dynamics is regarded as being nonperturbative, and absorbed into $\Lambda_b$ and $\Lambda_c$ baryon wave functions, and above which perturbation theory is reliable, and radiative corrections are absorbed into hard $b \to c$ decay amplitudes.

Infrared divergences from radiative corrections are collinear, when loop momenta are parallel to an energetic light quark, and soft, when loop momenta are much smaller than
the $\Lambda_b$ baryon mass $M_{\Lambda_b}$. Collinear and soft enhancements may overlap to give double logarithms. Three-particle reducible corrections on the $\Lambda_b$ baryon side are absorbed into the $\Lambda_b$ baryon wave function. If the light valence quarks move slowly, collinear divergences associated with these quarks will not be pinched [1], and soft divergences are important. However, there is probability, though small, of finding the light quarks in the $\Lambda_b$ baryon with longitudinal momenta of order $M_{\Lambda_b}$. Therefore, reducible corrections on the $\Lambda_b$ baryon side are dominated by soft dynamics, but contain weak double logarithms with collinear ones suppressed. Similarly, three-particle reducible corrections on the $\Lambda_c$ baryon side are absorbed into the $\Lambda_c$ baryon wave function. In the fast recoil region collinear divergences become stronger, and double logarithms associated with the $\Lambda_c$ baryon wave function are more important. The remaining part of radiative corrections, with all collinear and soft divergences subtracted, are characterized by a scale of order $M_{\Lambda_b}$, and absorbed into the hard $b$ quark decay amplitudes. Irreducible corrections, with a gluon attaching a quark in the $\Lambda_b$ baryon and a quark in the $\Lambda_c$ baryon, are infrared finite in the large recoil region [15], and also absorbed into the hard decay amplitudes.

The kinematic variables are defined as follows. The $\Lambda_b$ baryon is assumed to be at rest with the momentum

$$p \equiv (p^+, p^-, k_T) = \frac{M_{\Lambda_b}}{\sqrt{2}} (1, 1, 0) .$$

(4)

The valence quark momenta in the $\Lambda_b$ baryon are parametrized as

$$k_1 = (p^+, x_1 p^-, k_{1T}) , \quad k_2 = (0, x_2 p^-, k_{2T}) , \quad k_3 = (0, x_3 p^-, k_{3T}) ,$$

(5)

where $k_1$ is associated with the $b$ quark. The momentum fractions and the transverse momenta obey the conservation laws,

$$x_1 + x_2 + x_3 = 1 , \quad k_{1T} + k_{2T} + k_{3T} = 0 .$$

(6)

The $\Lambda_c$ baryon momentum is chosen as $p' \equiv (p'^+, p'^-, 0)$ with $p'^+ \gg p'^-$ at fast recoil. We define the velocity transfer $\rho$,
\[ \rho = \frac{p \cdot p'}{M_{\Lambda_b} M_{\Lambda_c}}, \quad 1 < \rho < \frac{M_{\Lambda_b}^2 + M_{\Lambda_c}^2}{2 M_{\Lambda_b} M_{\Lambda_c}}, \]  

(7)

where \( M_{\Lambda_c} \) is the \( \Lambda_c \) baryon mass. Using the on-shell condition \( p'^2 = M_{\Lambda_c}^2 \), the plus and minus components of \( p' \) are written as

\[ p'^+ = \rho_+ p^+ , \quad p'^- = \rho_- p^- , \]  

(8)

with

\[ \rho_+ = (\rho + \sqrt{\rho^2 - 1}) r , \quad \rho_- = (\rho - \sqrt{\rho^2 - 1}) r , \]  

(9)

and \( r = M_{\Lambda_c}/M_{\Lambda_b} \). The valence quark momenta in the \( \Lambda_c \) baryon are parametrized as

\[ k'_1 = (x'_1 p'^+, p'^-, k'_{1T}) , \quad k'_2 = (x'_2 p'^+, 0, k'_{2T}) , \quad k'_3 = (x'_3 p'^+, 0, k'_{3T}) , \]  

(10)

where \( k'_1 \) is associated with the \( c \) quark. The primed variables obey similar relations to Eq. (6).

According to factorization theorem, the hadronic matrix element is expressed as

\[ \mathcal{M}_\mu = \int_0^1 \left[ dx \right] \left[ dx' \right] \int \left[ d^2 k_T \right] \left[ d^2 k'_T \right] \Psi_{\Lambda_c}^{\alpha' \beta' \gamma'}(k'_1, \mu) H^\alpha' \beta' \gamma' \alpha \beta \gamma(k'_1, k_i, \rho, M_{\Lambda_b}, \mu) \times \Psi_{\Lambda_b}^{\alpha \beta \gamma}(k_i, \mu) , \]  

(11)

with the notations

\[ [dx] = dx_1 dx_2 dx_3 \delta \left( 1 - \sum_{i=1}^3 x_i \right) , \quad [d^2 k_T] = d^2 k_{1T} d^2 k_{2T} d^2 k_{3T} \delta^2 \left( \sum_{i=1}^3 k_{iT} \right) . \]  

(12)

\([dx']\) and \([d^2 k'_T]\) associated with the \( \Lambda_c \) baryon are defined in a similar way. The hard amplitude \( H_\mu \) will be computed in Sec. III. The dependence on the factorization (renormalization) scale \( \mu \) will disappear after performing a RG analysis.

The structure of the \( \Lambda_b \) baryon distribution amplitude \( \Psi_{\Lambda_b}^{\alpha \beta \gamma} \) is simplified under the assumptions that the spin and orbital degrees of freedom of the light quark system are decoupled, and that the \( \Lambda_b \) baryon is in the ground state (s-wave). The distribution amplitude is then expressed as \[ \Psi_{\Lambda_b}^{\alpha \beta \gamma} \]
\[ \Psi_{\Lambda_b \alpha \beta \gamma}(k_i, \mu) = \frac{1}{2\sqrt{2N_c}} \int \frac{d y_i}{(2\pi)^3} e^{i k_i \cdot y_i} e^{a b c} \langle 0 | T[\{a_\alpha(y_1) u_\beta(y_2) d_\gamma(0)\}] | \Lambda_b(p) \rangle \]

\[ = \frac{f_{\Lambda_b}}{8\sqrt{2N_c}} [ (\phi + M_{\Lambda_b}) \gamma_5 C ]_{\beta \gamma} [\Lambda_b(p) | \alpha \Phi(k_i, \mu) , \]

(13)

where \( N_c = 3 \) is number of colors, \( b, u, \text{ and } d \) are quark fields, \( a, b, \text{ and } c \) are color indices, \( \alpha, \beta, \text{ and } \gamma \) are spinor indices, \( f_{\Lambda_b} \) is a normalization constant, \( C \) is the charge conjugation matrix, and \( \Phi \) is the \( \Lambda_b \) baryon wave function. Under similar assumptions, the \( \Lambda_c \) baryon distribution amplitude \( \Psi_{\Lambda_c \alpha \beta \gamma} \) is written as

\[ \Psi_{\Lambda_c \alpha \beta \gamma}(k'_i, \mu) = \frac{1}{2\sqrt{2N_c}} \int \frac{d y'_i}{(2\pi)^3} e^{i k'_i \cdot y'_i} e^{a b c} \langle 0 | T[\{c_\alpha(y'_1) u_\beta(y'_2) d_\gamma(0)\}] | \Lambda_c(p') \rangle \]

\[ = \frac{f_{\Lambda_c}}{8\sqrt{2N_c}} [ (p' + M_{\Lambda_c}) \gamma_5 C ]_{\beta \gamma} [\Lambda_c(p') | \alpha \Pi(k'_i, \mu) , \]

(14)

where the normalization constant \( f_{\Lambda_c} \) and the wave function \( \Pi \) are associated with the \( \Lambda_c \) baryon.

Because of the inclusion of parton transverse momenta, Sudakov resummation for a hadron wave function should be performed in the impact parameter \( b \) space with \( b \) conjugate to \( k_T \) \[16\]. The result is \[8\]

\[ \Phi(k^-_i, b_i, \mu) = \exp \left[ - \sum_{i=2}^{3} \int_{b_i}^{b_i^-} d \mu \gamma_q(\mu) \right] \phi(x_i) , \]

(15)

where \( \gamma_q = -\alpha_s/\pi \) is the quark anomalous dimension, and the factorization scale \( w \) is chosen as

\[ w = \min \left( \frac{1}{b_1}, \frac{1}{b_2}, \frac{1}{b_3} \right) , \]

(16)

with \( b_3 = |b_1 - b_2| \). The explicit expression of the Sudakov exponent \( s \) is given by \[17\]

\[ s(w, Q) = \int_{w}^{Q} \frac{d p}{p} \left[ \ln \left( \frac{Q}{p} \right) A(\alpha_s(p)) + B(\alpha_s(p)) \right] , \]

(17)

where the anomalous dimensions \( A \) to two loops and \( B \) to one loop are

\[ A = C_F \frac{\alpha_s}{\pi} \left( \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_0 \ln \left( \frac{e^{\gamma_E}}{2} \right) \right) \left( \frac{\alpha_s}{\pi} \right)^2 , \]

\[ B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln \left( \frac{e^{2\gamma_E-1}}{2} \right) , \]

(18)
$C_F = 4/3$ being a color factor, $n_f = 4$ the flavor number, and $\gamma_E$ the Euler constant. The one-loop running coupling constant,

$$\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda_{QCD}^2)},$$  

(19)

with the coefficient $\beta_0 = (33 - 2n_f)/12$ and the QCD scale $\Lambda_{QCD}$, will be substituted into Eq. (17). The initial condition $\phi$ of the Sudakov evolution absorbs nonperturbative dynamics below the factorization scale $w$.

Following the derivation in [3,18], we obtain the Sudakov resummation for the $\Lambda_c$ baryon distribution amplitude,

$$\Pi(k_1^+, b_i, \mu) = \exp \left[ -\sum_{i=1}^{3} s(w, k_i^+) - 3 \int_{w}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right] \pi(x'_{1i}).$$  

(20)

We have included the Sudakov exponent $s$ associated with the $c$ quark, which carries large longitudinal momentum in the fast recoil region. Notice the same transverse extents $b_i$ as those for the $\Lambda_b$ baryon. This is the consequence of neglecting the transverse momenta which flow through the virtual quark lines in $H_\mu$ [18].

The RG analysis of $H_\mu$ leads to

$$H_\mu(k_1^+, k_1^-, b_i, \rho, M_{\Lambda_b}, \mu) = \exp \left[ -3 \sum_{l=1}^{2} \int_{t_l}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right] H_\mu(x'_i, x_i, b_i, \rho, M_{\Lambda_b}, t_1, t_2),$$  

(21)

where the superscripts $\alpha', \beta', \cdots$, have been suppressed. Since large logarithms have been collected by the exponential, the initial condition $H_\mu$ of the RG evolution on the right-hand side of the above expression can be computed reliably in perturbation theory. To simplify the formalism, we shall make the approximations $M_b \approx M_{\Lambda_b}$ and $M_c \approx M_{\Lambda_c}$, and neglect the transverse momentum dependence of the virtual quark propagators as mentioned before. The two arguments $t_1$ and $t_2$ of $H_\mu$, which will be specified in the next section, imply that each running coupling constant $\alpha_s$ is evaluated at the mass scale of the corresponding hard gluon. Substituting Eqs. (13)-(21) into Eq. (11), we derive the factorization formula for the semileptonic decay $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$, where the $\mu$ dependence has disappeared as stated before.

For the $\Lambda_b$ baryon wave function $\phi(x_1, x_2, x_3)$, we adopt the model proposed in [19],
\[ \phi(\zeta, \eta) = N \eta^2 \zeta (1 - \eta) (1 - \zeta) \exp \left[ -\frac{M_b^2}{2\beta^2 (1 - \eta)} - \frac{m_l^2}{2\beta^2 \eta \zeta (1 - \zeta)} \right], \]  

(22)

with \( N \) being a normalization constant, \( \beta \) a shape parameter, \( m_l \) the mass of light degrees of freedom in the \( \Lambda_b \) baryon. The new variables \( \zeta \) and \( \eta \) are defined by

\[ \zeta = \frac{x_2}{x_2 + x_3}, \quad \eta = x_2 + x_3. \]  

(23)

In terms of \( \zeta \) and \( \eta \), the normalization of \( \phi(\zeta, \eta) \) is given by

\[ \int d\zeta d\eta \phi(\zeta, \eta) = 1, \]  

(24)

which determines the constant \( N \), once the parameters \( \beta \) and \( m_l \) are fixed. The above wave function with the factor \( \eta^2 \zeta (1 - \eta) (1 - \zeta) = x_1 x_2 x_3 \) suppresses contributions from the end points of momentum fractions. The exponents proportional to \( M_b^2/(1 - \eta) = M_b^2/x_1 \) and to \( m_l^2/[\eta \zeta (1 - \zeta)] = m_l^2/x_2 + m_l^2/x_3 \) with \( M_b \gg m_l \) indicate that \( \phi \) has a maximum at large \( x_1 \) and at small \( x_2 \) and \( x_3 \), and that the \( b \) quark momentum \( k_1^2 \) is roughly equal to \( M_b^2 \). For \( \phi(x_3, x_1, x_2) \) which will appear in the factorization formulas presented in Sec. III, the above expression is transformed into

\[ \phi(\zeta, \eta) = N \eta^2 \zeta (1 - \eta) (1 - \zeta) \exp \left[ -\frac{M_b^2}{2\beta^2 \eta (1 - \eta)} - \frac{m_l^2 (1 - \eta + \eta \zeta)}{2\beta^2 \eta \zeta (1 - \eta)} \right]. \]  

(25)

For convenience, we assume that the \( \Lambda_c \) wave function \( \pi(\zeta', \eta') \) possesses the same functional form and the same parameters \( \beta \) and \( m_l \) as of \( \phi(\zeta, \eta) \), but with the \( b \) quark mass \( M_b \) replaced by the \( c \) quark mass \( M_c \). The wave function \( \pi(x'_1, x'_2, x'_3) \) also has a maximum at large \( x'_1 \), such that the \( c \) quark momentum \( k_1^2 \) is roughly equal to \( M_c^2 \).

III. TRANSITION FORM FACTORS

In this section we present the factorization formulas for the form factors \( f_1 \) and \( g_1 \), which are associated with the spin structures \( \bar{\Lambda}_c \gamma_\mu \Lambda_b \) and \( \bar{\Lambda}_c \gamma_\mu \gamma_5 \Lambda_b \) in \( M_\mu \), respectively. Working out the contraction of \( \bar{\Psi}_{\Lambda_c} \gamma^\alpha \bar{\gamma}^\beta \gamma_\mu H^\alpha_\mu \bar{\gamma}^\gamma \gamma^\beta \gamma_\alpha \bar{\Psi}_{\Lambda_b} \) in momentum space, we extract the hard part \( H \). Employing a series of permutations of the valence quark kinematic variables as in
the summation over the leading diagrams in Fig. 1 reduces to two terms for each form factor. The factorization formula for the form factors \( f_1(\rho) \) and \( g_1(\rho) \) are written as

\[
\begin{align*}
  f_1(\rho) &= \frac{4\pi}{27} \int_0^1 [dx'][dx] \int_0^\infty b_1 db_1 b_2 db_2 \int_0^{2\pi} d\theta f_{\lambda_\alpha} f_{\lambda_\beta} \\
  &\times \sum_{j=1}^2 H_j(x', x, b_i, \rho, M_{\lambda_\beta}, t_{ji}) \mathcal{F}_j(x', x, \rho) \exp[-S(x', x, w, \rho, M_{\lambda_\beta}, t_{ji})], \tag{26}

  g_1(\rho) &= \frac{4\pi}{27} \int_0^1 [dx'][dx] \int_0^\infty b_1 db_1 b_2 db_2 \int_0^{2\pi} d\theta f_{\lambda_\alpha} f_{\lambda_\beta} \\
  &\times \sum_{j=1}^2 H_j(x', x, b_i, \rho, M_{\lambda_\beta}, t_{ji}) \mathcal{G}_j(x', x, \rho) \exp[-S(x', x, w, \rho, M_{\lambda_\beta}, t_{ji})], \tag{27}
\end{align*}
\]

where \( \theta \) is the angle between \( b_1 \) and \( b_2 \).

The functions \( \mathcal{F}_j \) and \( \mathcal{G}_j \), which group together the products of the initial and final baryon wave functions, are, in terms of the notations,

\[
\phi_{123} \equiv \phi(x_1, x_2, x_3), \quad \pi_{123} \equiv \pi(x'_1, x'_2, x'_3), \tag{28}
\]

given by

\[
\begin{align*}
  \frac{\mathcal{F}_1}{\phi_{123} \pi_{123}} &= \frac{r^2}{[(1-x'_1-\rho_-)\rho_+ + r^2](1-x'_1) x_2 x_2 \rho_+} \left[ 2 \left( 2\sqrt{\rho^2 - 1} - 1 \right) (1 - x'_1) \\
  &+ (2(1+\rho) - 4r - 1)x_2 + (2(2\rho - 1) + (2\rho - 3)\rho_1)x_2 x'_1 \right] \\
  &+ \frac{r^2}{[(1-x'_1-\rho_-)\rho_+ + r^2](1-x'_1) x_2 x_2 \rho_+} \left[ \left( \rho_1 + 2r\sqrt{\rho^2 - 1} + 3 + 4r - \rho \right) (1 - x_1) \\
  &- (\rho_1 + 2r\sqrt{\rho^2 - 1} + 3 + 4r - \rho) \right] (1 - x_1) \\
  &+ \frac{r}{(1-x_1)^2 x_2 x_2 \rho_+^2} \left[ 2r \left( 2\sqrt{\rho^2 - 1} - 1 + 2\rho \right) (1 - x_1) + 2 \left( \sqrt{\rho^2 - 1} + 2 + \rho \right) x_2' \\
  &- r ((2 - \rho)\rho_1 + 1 + 2\rho) (1 - x_1) x_2' \right] \\
  &+ \frac{r}{(1-x_1)^2 x_2 x_2 \rho_+^2} \left[ 2r \left( 2\sqrt{\rho^2 - 1} - 1 + 2\rho \right) (1 - x_1) + 2 \left( \sqrt{\rho^2 - 1} + 2 + \rho \right) \right] \right] - x_2 \\
  &+ 2r \left( 2(\rho - 1)\sqrt{\rho^2 - 1} - 1 + \rho \right) x_2' \right]. \tag{29}
\end{align*}
\]

\[
\begin{align*}
  \frac{\mathcal{F}_2}{\phi_{312} \pi_{312}} &= \frac{r}{[(1-x'_3-\rho_-)\rho_+ + r^2](1-x_3) x_3 x'_3 \rho_+} \left[ 2r \rho_1 (1 - x'_3) + 4r(1 + \rho) (1 - x_1) \\
  &+ 2r^2 \left( \sqrt{\rho^2 - 1} \right) x_1 - 2r \left( (\rho - 1)\sqrt{\rho^2 - 1} - \rho^2 \right) x_2' \\
  &- (1 + \rho_1)(r(\rho - 1)x_1 + x'_2) (1 - x'_3) \right] \\
  &+ \frac{2r}{[(x'_2 - \rho_-)(1 - x_1)\rho_+ + r^2][1 - (1 - x_1\rho_+)(1 - x'_2)]}
\end{align*}
\]
\[ \times \left[ r \left( \rho + \sqrt{\rho^2 - 1} \right) (x_1 x_2' - \rho_1 (x_1 + x_2')) + \rho_1 \left( 2r x_1 + x_2' + 2r \sqrt{\rho^2 - 1} \right) \right] \\
+ \frac{r}{1 - (1 - x_2 \rho_1)(1 - x_3')(1 - x_3) \rho_1} \left[ 2r \rho_1 (1 - x_3) + 4(1 + \rho)(1 - x_1') \right] \\
- 2 \left( \sqrt{\rho^2 - 1} - 3 \right) x_1' - 2r \left( (\rho - 1)(\rho + \sqrt{\rho^2 - 1}) + 1 \right) x_2 \\
- r(1 + \rho_2) \left( rx_2 + \sqrt{\rho^2 - 1} x_1' \right)(1 - x_3) \right] . \]

\[ G_1 = \frac{r^2}{(1 - x_1' - \rho_+ + r^2)(1 - x_1)x_3 \rho_+} \left[ (2\rho - 3 + (2\rho - 1)\rho_2)x_2(1 - x_1') \right] \\
+ 2 \left( 2 - \rho - \sqrt{\rho^2 - 1} \right) x_2 - 2 \left( 2\rho - 1 + 2\sqrt{\rho^2 - 1} \right)(1 - x_1') \right] \\
+ \frac{r^2}{(1 - x_1' - \rho_+ + r^2)(1 - x_1)x_3 \rho_+} \left[ 2 \left( 2\rho - 1 \right)(\sqrt{\rho^2 - 1} + \rho - 1 \right) x_2' \\
+ 2r \left( \sqrt{\rho^2 - 1} - 1 \right) \left( 1 - x_1 \right) - (3\rho_2 + 1)(1 - x_1)(1 - x_1') \right] , \]

\[ G_2 = \frac{r}{(1 - x_3' - \rho_+ + r^2)(1 - x_3)x_1 \rho_+} \left[ -4r^2(1 + \rho) + r \rho_2(4 - \rho - \rho^2)x_1 \\
+ r(\rho - 1)(\rho_2 - 1)x_3' x_1 + 2r(1 - x_3') + 2r \left( (\rho - 1)(\sqrt{\rho^2 - 1} + \rho) + 1 \right) x_2' \\
- (\rho_2 + 1)x_2'(1 - x_3') \right] \\
+ \frac{2r}{(x_2' - \rho_+ - 1 - x_1') \rho_+ + r^2(1 - x_1 \rho_+)(1 - x_2')} \left[ r \left( \sqrt{\rho^2 - 1} - 2 - \rho \right) \\
+ r^2 x_1 + x_2' - 2r \left( \rho + \sqrt{\rho^2 - 1} \right)(1 - x_1)(1 - x_2') \right] \\
+ \frac{r}{1 - (1 - x_2 \rho_1)(1 - x_3')(1 - x_3) \rho_+} \left[ -4(1 + \rho) + 2r(1 - x_3) \right] \\
- 2 \left( \sqrt{\rho^2 - 1} + 2\rho - 1 \right)x_1' + 2r \left( (\rho - 1)(\sqrt{\rho^2 - 1} + \rho) + 1 \right) x_2 \\
- r(\rho_2 + 1)(rx_2 + (\rho - 1)x_1')(1 - x_3) \right] , \]

with \( \rho_1 = \sqrt{(\rho + 1)/(\rho - 1)} \) and \( \rho_2 = 1/\rho_1 \).

The hard parts are given by

\[ H_1 = \alpha_s(t_{11}) \alpha_s(t_{12}) K_0 \left( \sqrt{(1 - x_1')(1 - x_1') \rho_+ M_{\Lambda_b} b_1} \right) K_0 \left( \sqrt{x_2 x_2' \rho_+ M_{\Lambda_b} b_2} \right) , \]
\[ H_2 = \alpha_s(t_{21})\alpha_s(t_{22}) K_0 \left( \sqrt{x_1 x_1' \rho + M_{\Lambda_b} b_1} \right) K_0 \left( \sqrt{x_2 x_2' \rho + M_{\Lambda_b} b_2} \right), \]  

(34)

with \( K_0 \) being the modified Bessel function of order zero. The complete Sudakov exponent \( S \) is written as

\[ S(x'_i, x_i, w, \rho, M_{\Lambda_b}, t_{jl}) = S_d(x'_i, x_i, w, \rho, M_{\Lambda_b}) + S_s(w, t_{jl}), \]  

(35)

with

\[ S_d = \sum_{l=2}^{3} s(w, x_l p^-) + \sum_{l=1}^{3} s(w, x'_l p'^+) , \]  

(36)

\[ S_s = 3 \int_{w}^{t_{11}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) + 3 \int_{w}^{t_{21}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})). \]  

(37)

The hard scales \( t_{jl} \) are chosen as

\[ t_{11} = \max \left[ \sqrt{(1-x_1)(1-x'_1)\rho + M_{\Lambda_b}, 1/b_1} \right], \]

\[ t_{21} = \max \left[ \sqrt{x_1 x_1' \rho + M_{\Lambda_b}, 1/b_1} \right], \]

\[ t_{21} = t_{22} = \max \left[ \sqrt{x_2 x_2' \rho + M_{\Lambda_b}, 1/b_2} \right], \]  

(38)

which are always greater than \( w \). It is possible that the hard scales \( t_{jl} \) are small and the running coupling constants become large as \( b_i \) are close to \( 1/\Lambda_{QCD} \). These nonperturbative enhancements are, however, suppressed by the Sudakov exponential \( \exp(-S_d) \), which decreases quickly in the large \( b_i \) region and vanishes as \( b_i \geq 1/\Lambda_{QCD} \). The exponential \( \exp(-S_d) \) approaches unity; that is, there is no Sudakov suppression from the all-order summation of infrared logarithmic corrections at small \( b_i \). In these short-distance regions higher-order corrections are regarded as being hard and should be absorbed into \( H \) [20]. Another exponential \( \exp(-S_s) \), as a consequence of single-logarithm summation, describes the RG evolution from the factorization scale \( w \) to the hard scales \( t_{jl} \).

For the case with massless leptons, it is easy to show that the differential decay rate in the rest frame of the \( \Lambda_b \) baryon is given by

\[
\frac{d\Gamma}{d\rho} = \frac{M^5_{\Lambda_b} r^3}{24\pi} G_F^2 |V_{cb}|^2 \sqrt{\rho^2 - 1} \left\{ |f_1|^2 (\rho - 1)[3 + 3r^2 - 2(2\rho - 1)r] + |g_1|^2 (\rho + 1)[3 + 3r^2 - 2(2\rho + 1)r] \right\},
\]  

(39)
where only the contributions from the form factors $f_1$ and $g_1$ are considered. It is straightforward to obtain the total decay rate

$$\Gamma \equiv \int d\rho \frac{d\Gamma}{d\rho}$$

from Eq. (33) and thus the branching ratio $B(\Lambda_b \to \Lambda_c \ell \bar{\nu})$, if the form factors $f_1(\rho)$ and $g_1(\rho)$ in the whole range of $\rho$ are known.

IV. RESULTS

In order to reduce the number of unknown parameters, we make an approximation. Consider the baryonic decay constant $\tilde{f}_\Lambda$ defined, in heavy quark effective theory, by

$$\langle 0 | \bar{\psi} \tilde{j}^v | \Lambda_Q \rangle = \tilde{f}_\Lambda \Lambda_Q,$$

in terms of the $\Lambda$-baryonic current [21,22]

$$\tilde{j}^v = \epsilon^{abc} (u^a C \gamma_5 d^b) h^c_v,$$

where $\Lambda_Q$ is the heavy baryon spinor, $h_v$ the heavy quark field, and $a, b, c$ denote the color indices. We contract a Dirac tensor $(C \gamma_5)_{\beta \gamma}$ with a heavy $\Lambda$-baryon distribution amplitude such as $\Psi_{\Lambda_Q \alpha \beta \gamma}$ in Eq. (13) and integrate out the valence quark momenta $k_i$. Compared with Eq. (41), we extract the baryonic decay constant

$$\tilde{f}_\Lambda = f_{\Lambda_Q} M_{\Lambda_Q}. \tag{43}$$

It implies that in the heavy quark limit the normalization constants $f_{\Lambda_b}$ and $f_{\Lambda_c}$ are related by

$$f_{\Lambda_b} M_{\Lambda_b} = f_{\Lambda_c} M_{\Lambda_c}. \tag{44}$$

Therefore, $f_{\Lambda_c}$ associated with the $\Lambda_c$ baryon will not be treated as a free parameter in the numerical analysis below.
We are now ready to compute the form factors $f_1(\rho)$ and $g_1(\rho)$ from Eqs. (26) and (27), adopting the CKM matrix element $V_{cb} = 0.04$, the masses $M_{\Lambda_b} = 5.624$ GeV and $M_{\Lambda_c} = 2.285$ GeV, and the QCD scale $\Lambda_{QCD} = 0.2$ GeV. We examine the self-consistency of our calculation by considering the percentage of the full contribution to the form factor $f_1$, that arises from the short-distance region with all $\alpha_s(t_{jl})/\pi < 0.5$. The percentages for different $\beta$ with $m_l$ fixed at 0.3 GeV are listed in Table I. It is observed that the perturbative contributions become dominant gradually as $\rho$ and $\beta$ increase: a larger $\rho$ corresponds to larger momentum transfer involved in decay processes, and a larger $\beta$ corresponds to heavy baryon wave functions which are less sharp at the high ends of the momentum fractions $x_1$ and $x_1'$. We conclude that the PQCD analysis of the transition form factors is self-consistent for $\beta > 1.0$ GeV and $\rho > 1.2$, viewing the perturbative percentage of about 80%. Compared to the corresponding meson decay $B \to Dl\bar{\nu}$, a perturbative expansion is less reliable in the baryon case, because partons in a baryon are softer, such that Sudakov suppression is weaker.

To obtain the total decay rate, we need the information of $f_1$ and $g_1$ in the whole range of $\rho$. Since the perturbative analysis is reliable only in the fast recoil region, we extrapolate the PQCD predictions at large $\rho$ to small $\rho$. Hinted by [23], we propose the following parametrization for the form factors:

$$f_1(\rho) = \frac{c_f}{\rho^{\alpha_f}}, \quad g_1(\rho) = \frac{c_g}{\rho^{\alpha_g}}, \quad (45)$$

where the constants $c_f$ and $c_g$, and the powers $\alpha_f$ and $\alpha_g$ are determined by the PQCD results at large $\rho$. The constants $c_f$ and $c_g$, equal to the values of the form factors at zero recoil ($\rho = 1$), should be close to unity according to heavy quark symmetry. We fit Eq. (45) to the PQCD results in the range with $\rho > 1.3$ for $\beta = 1.0$, where perturbative contribution has exceeded 80%. The powers $\alpha_f = 5.18$ and $\alpha_g = 5.14$, close to $\alpha_f \sim 4.6$ at large $\rho$ from the method of wave function overlap integrals [24], are obtained. These values are larger than 1.8 extracted from the transition form factors associated with the corresponding meson decay $B \to Dl\bar{\nu}$ [3]. This is expected, because perturbative baryon decays involve more hard
On the experimental side, there exist only the data of the semileptonic branching ratio $B(\Lambda_b \to Xl\bar{\nu}) \sim 10\%$ [25], where the final-state particles $X$ are dominated by the charm baryons. The data of the $B$ meson semileptonic decays show $B(B \to D^*l\bar{\nu}) \sim 3B(B \to Dl\bar{\nu})$, indicating that each of the three polarization states of the $D^*$ meson contributes the same amount of branching ratio as the $D$ meson does. It is possible that this observation applies to dominant modes in the $\Lambda_b \to Xl\bar{\nu}$ decays with the excited charm baryons $\Lambda_c(2593)$ of spin $J = 1/2$ and $\Lambda_c(2625)$ of $J = 3/2$. That is, the branching ratio $B(\Lambda_b \to \Lambda_c l\bar{\nu})$ is about $1/4$ of $B(\Lambda_b \to Xl\bar{\nu})$, i.e., about $2 \sim 3\%$. This estimation is consistent with the experimental upper bound of the branching ratio from the data $B(\Lambda_b \to \Lambda_c l\bar{\nu} + X) = (8.27 \pm 3.38\%)$ [23].

We substitute Eq. (45) for the form factors $f_1$ and $g_1$ into the decay rate $\Gamma$ in Eq. (40), and adjust the normalization constant $f_{\Lambda_b}$ such that our predictions for the branching ratio are located in the range of $2 \sim 3\%$. The $\Lambda_c$ baryon normalization constant $f_{\Lambda_c}$ changes according to Eq. (44). We adopt the $\Lambda_b$ baryon lifetime $\tau = (1.24 \pm 0.08) \times 10^{-12}\,s$ [25]. The value of $f_{\Lambda_b}$ determines the parameters $c_f$ and $c_g$. It is then found that $f_{\Lambda_b} = 2.71 \times 10^{-3}\,\text{GeV}^2$, corresponding to

$$f_1(\rho) = \frac{1.32}{\rho^{5.18}}, \quad g_1(\rho) = \frac{-1.19}{\rho^{5.14}},$$

(46)

gives the branching ratio $2\%$, and $f_{\Lambda_b} = 3.0 \times 10^{-3}\,\text{GeV}^2$, corresponding to

$$f_1(\rho) = \frac{1.62}{\rho^{5.18}}, \quad g_1(\rho) = \frac{-1.46}{\rho^{5.14}},$$

(47)

gives the branching ratio $3\%$. Since the values of the form factors at zero recoil should be close to unity as stated above, we prefer Eq. (46) with $f_1(1) = 1.32$ and $g_1(1) = -1.19$, which are also consistent with the conclusion in [24]. The corresponding normalization constant $f_{\Lambda_b} = 2.71 \times 10^{-3}\,\text{GeV}^2$, of the same order as $f_P = (5.2 \pm 0.3) \times 10^{-3}\,\text{GeV}^2$ for the proton [26], is reasonable. The PQCD predictions and the corresponding extrapolations are displayed in Fig. 2, which deviate from each other at small $\rho$. If applying the PQCD formalism to the zero recoil region, we shall obtain divergent form factors as shown in Fig. 2, which imply
the failure of PQCD. Note that our results of the form factors exhibit slopes larger than the dipole behavior assumed in [23].

We then examine the sensitivity of our predictions for the branching ratio $B(\Lambda_b \to \Lambda_c l\bar{\nu})$ to the variation of the parameter $\beta$. Choosing $\beta = 2.0$ GeV and $\beta = 4.0$ GeV, and normalizing the corresponding form factors in the way that they have similar values to those for $\beta = 1.0$ GeV in Eq. (46), we obtain the form factors

$$f_1(\rho) = \frac{1.34}{\rho^{5.04}}, \quad g_1(\rho) = \frac{-1.17}{\rho^{4.92}}, \quad (48)$$

and

$$f_1(\rho) = \frac{1.34}{\rho^{4.94}}, \quad g_1(\rho) = \frac{-1.18}{\rho^{4.79}}, \quad (49)$$

respectively. Equations (48) and (49) lead to increases of the branching ratio by 4% and 8%, respectively. That is, our predictions for the branching ratio are not sensitive to the choice of baryon wave functions. This observation is attributed to the fact that the PQCD results of the transition form factors at large recoil are insensitive to the variation of baryon wave functions.

We present in Fig. 3 the differential decay rate $d\Gamma/d\rho$ derived from the form factors in Eq. (46), which can be compared with experimental data in the future. The $\Lambda_b$ and $\Lambda_c$ baryon wave functions determined in this work are given by

$$\phi(\zeta, \eta) = 6.67 \times 10^{12} \eta^2 \zeta (1-\eta)(1-\zeta) \times \exp \left[ -\frac{M_b^2}{2(1.0 \text{ GeV})^2(1-\eta)} - \frac{m_l^2}{2(1.0 \text{ GeV})^2\eta\zeta(1-\zeta)} \right], \quad (50)$$

$$\pi(\zeta, \eta) = 6.94 \times 10^4 \eta^2 \zeta (1-\eta)(1-\zeta) \times \exp \left[ -\frac{M_c^2}{2(1.0 \text{ GeV})^2(1-\eta)} - \frac{m_l^2}{2(1.0 \text{ GeV})^2\eta\zeta(1-\zeta)} \right]. \quad (51)$$

At last, we compare our predictions with those derived from other approaches in the literature. The $\Lambda_b \to \Lambda_c$ transition form factors have been evaluated by means of overlap integrals of infinite-momentum-frame (IMF) wave functions, nonrelativistic and relativistic
quark models, and QCD sum rules. For a review, refer to [27]. Basically, they are nonperturbative methods without involving hard gluons. QCD dynamics is completely parametrized into IMF wave functions in the overlap-integral approach [24,28], and into baryon-three-quark vertex form factors in the relativistic quark model [29]. Information of the above bound-state quantities can be obtained by solving Bethe-Salpeter equations [30]. Most of the analyses, including QCD sum rules [22,31,32], led to the branching ratios about or below 6%. The prediction $B(\Lambda_b \to \Lambda_c l \bar{\nu}) \sim 9\%$ in [28] is a bit higher compared to the data of $B(\Lambda_b \to \Lambda_c l \bar{\nu} + X)$. Our result is close to $(3.4 \pm 0.6)\%$ derived in [31].

V. CONCLUSION

In this paper we have developed PQCD factorization theorem for the semileptonic heavy baryon decay $\Lambda_b \to \Lambda_c l \bar{\nu}$, whose form factors are expressed as the convolutions of hard $b$ quark decay amplitudes with universal $\Lambda_b$ and $\Lambda_c$ baryon wave functions. It is observed that the PQCD formalism with Sudakov suppression in the long-distance region is applicable to $\Lambda_b \to \Lambda_c$ decays for the velocity transfer greater than 1.2. This observation indicates that PQCD is an appropriate approach to analyses of two-body exclusive nonleptonic $\Lambda_b$ baryon decays. Requiring that the normalizations of the form factors at zero recoil are consistent with heavy quark symmetry, we have predicted the branching ratio $B(\Lambda_b \to \Lambda_c l \bar{\nu}) \sim 2\%$. We have also determined the $\Lambda_b$ and $\Lambda_c$ baryon wave functions shown in Eqs. (50) and (51), respectively. These wave functions, because of their universality, will be employed to study nonleptonic $\Lambda_b$ baryon decays in the future.

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FIGURE CAPTIONS

**FIG. 1** Lowest order diagrams for the $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ decay.

**FIG. 2** Dependence of $f_1$ and $|g_1|$ on $\rho$ for $\beta = 1.0$ and $m_l = 0.3$ obtained from PQCD (solid lines) and from the extrapolation in Eq. (46) (dashed lines). The upper (lower) set of curves represents the form factor $f_1 (|g_1|)$.

**FIG. 3** Dependence of $d\Gamma/d\rho$ on $\rho$ obtained from Eq. (46) in units of $10^{-13}$ GeV.
Table I. Percentages of perturbative contributions for various $\beta$ and $\rho$.

| Percentage | $\rho = 1.2$ | $\rho = 1.3$ | $\rho = 1.4$ |
|------------|--------------|--------------|--------------|
| $\beta = 1.0$ GeV | 77.7% | 83.6% | 85.2% |
| $\beta = 2.0$ GeV | 79.3% | 83.0% | 85.7% |
| $\beta = 4.0$ GeV | 82.3% | 84.7% | 86.3% |
FIG. 1
Fig. 2

The graph shows the relationship between $\rho$ and $f_1(\left| g_1 \right|)$. The functions $f_1$ and $\left| g_1 \right|$ are plotted against $\rho$ over the range $1.00$ to $1.40$. The graph indicates a decreasing trend as $\rho$ increases.
Fig. 3