The formalization of cellular structures based on optical holographic studies according to the statistical methods

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Abstract
The article considers the issues related to the principles of building information models from laser method. Optical images of the speckle structures are recorded and subjected to statistical processing based on the analysis of the intensity of illumination of discrete points of the studied surfaces. The information thus obtained, saved and interpreted in the article as "phase portrait" of the system under study at a given time. The set thus obtained data is presented in the article as "phase space" of the system under study. On this basis to create an information model that is used in the form of "snapshots" of the state of (ID) in the studied points in time. The parameters of the investigated textures are recorded as statistical data and further formalized.

1. Information portrait.
The article shows the possibility of representing the state of the "pore structures" as "patterns" implemented regression equations. These models allow to create an information system about the state of the object, in the form of "information portrait" at any discrete point in time. The transition of the system from the initial state to the final state is formalized by the recursive algorithm with deferred calculation of texture parameters of the object. The texture parameters recorded in the form of visually observable statistical data. They were identified as the geometric shape of the texture of the material.

The main purpose of this work is to develop an information model of the process of transition from one phase state to another phase state with optical-holographic and statistical methods. The model of the target system state is represented by a system of regression equations of state for phase structures. The process of formation of structures is non-stationary and is connected with the phenomena of redistribution of phases, and their aggregation, fragmentation, interfacial dynamic interactions.

The information model was developed based on the study of images in the transition system from the initial phase state - "concrete-mix" in the end phase condition, - "artificial stone". To achieve this purpose the study used the concept of "texture". The term "texture" means the following: - texture, this is a real image of the object at a given point in time. In some ways this term is similar to the concept of speckle-pattern. Geometric shape, number of elements and their sizes are analyzed at the statistical methods. Textures used in the work obtained by direct interferometry of the process of curing of cellular concrete. Holographic interferometry provides the possibility of the interference study of objects with rough (diffusely scattering) surface. However, for a good visualization of holographic
interferogram (interference pattern) is continuously necessary to fulfill two conditions: 1) during exposure of the hologram, the object must not move; 2) surface microstructure should not change.

The object of research is dynamical systems. In the article the term "dynamic system" refers to any object or process from given set (array) property values in the initial time (the initial coordinates) and the law describing the evolution of the coordinates of the initial state of the object (textures) over time. Representation of dynamic structures by the differential equations shown in Figure 1.1 (a). That is, the dynamic system is a set of elements for which you set the functional relationship between time and position in phase space of each element of the system.

\[
\begin{align*}
    x_1' &= f_1(x_1, \ldots, x_n) \\
    x_2' &= f_2(x_1, \ldots, x_n) \\
    x_n' &= f_n(x_1, \ldots, x_n)
\end{align*}
\]  

(1.1)

Variables \(x_1, x_2, \ldots, x_n\) that determine the state of the system at specific points in time, Figure 1.1 (b).

Thus, the mathematical model of dynamic structures it is necessary to write so it was provided: 1) input parameters (coordinates) characterizing the state of the system at different points in time and 2) the operator, allowing to set the location change status over time:

\[
F[x, y(x), y'(x), \ldots, y^{r}(x)] \rightarrow y'(x) = F[x, y(x), y'(x), y^{r'(x)}, \ldots, y^{r-1}(x)]
\]  

(1.2)

The solution of system (1.2) can be represented in the form of solutions of the Cauchy problem that consists in finding a solution that satisfies the initial (r) conditions. The formulation of the Cauchy problem for multiple parameters of the pore structures is defined as follows:

\[
\begin{align*}
    \frac{dy_1}{dt} &= (a - b \cdot y_2) \cdot y_1 \\
    \frac{dy_2}{dt} &= (c - d \cdot y_1) \cdot y_2 \\
    y_1 &= 3; \ y_2 = 1
\end{align*}
\]  

(1.3)

Investigated in the article cellular porous structure of the present artificial stome materials. They consist of binders \((C)\) and different in shape and size of cells \((P)\). Usually, binders \((C)\) are mixed with fillers \((A)\). Fillers \((A)\) inherently have different properties. An important task in the formation of the pore structure is the need to ensure uniform distribution of pore sizes throughout the volume of artificial stone. Many types of such materials differ from each other according to the method of
forming a porous structure and the type of binder. Their differences lie in the methods of strength development, as well as under the conditions of molding (i.e. production technology (PT)). From production technology (PT), in turn, depend on the physico-mechanical characteristics (PMC) obtained in the final product. Thus produced materials with different properties and dimensions of cells. Pores, in their internal structure, can form a closed, communicating and open porosity of the final product.

A number of properties of cellular structures obey generalized laws. For example, the coefficient of thermal conductivity ($\lambda$) depends on the volume weight ($\gamma$) of the final product, but it is not affected by the kind of binder, and conditions of hardening. This is because the material of the walls, which consist of pores, is a kind of calcium silicate or a material similar in properties to the material, which consists of cement stone. However, in this case, the porosity ($P$) and volume weight ($\gamma$) significantly affect the conductivity.

All of the above clearly demonstrates the complex structure of the cellular structures, a large variety of factors affecting their performance characteristics and of the difficulties of formalization. However, it can be assumed that the consumer quality (CQ) cellular structures are a function of the properties and quantities of the starting components, and also technologies of their forming and fabrication. All this can be presented as an unordered set (aggregate) $\{CQ\}$:

$$CQ = \{C, A, PT, PMC, P, \lambda, Y, \ldots\} \quad (1.4)$$

Reflexivity listed in (1.4) properties is that each of these properties can enter into binary and other relationships with each other. Thus is formed the combinatorial properties of the final product. Designating by “R” the corresponding binary and other properties relate to each other, our statement can be formalized as follows:

$$(CQ) \ R \ (CQ); \ (C) \ R \ (A); \ldots \ (PMC) \ R \ (PT); \ldots \ and \ so \ on \ ... \quad (1.5)$$

Some way possible to organize the transfer and conservation properties of the original components in cellular structures in time (Information model). How is it possible to create a phase portrait of the state of the system at specific points in time? The transfer of the initial properties over time can be organized through the optical fixation of the studied dynamic system using speckle patterns. On the basis of recorded in a strictly defined points in time of the speckle patterns, you can create a so-called phase portrait of the system at a given time. The texture of these information models in specified times, a priori, will correspond to their inherent properties. Thus fixed, the texture will represent a corresponding information state model of the system under study in the given time.

2. Speckle-pattern. Reveal the concept of "laser speckle structure".
When the observer examines in laser light diffusely reflecting or transmissive object, the image seems grainy to him. The impression that the object surface is covered with numerous small, randomly distributed bright and dark spots called speckles. If the observer focus his eyes or optical instrument to a point located before the object or behind it, the speckle structure remains visible. If the observer moves, then it seems that speckle-pattern flickers and shifts relative to the object. The phenomenon of speckles associated with the use of highly coherent light. As one of the major advantages of holographic interferometry is that it provides the opportunity to study diffuse objects, it is useful to know some basic properties of laser speckle-structures.

Using phase portraits (Figure 2.1 (a)) it is possible to assess the impact of combinations of properties of the initial components of the mixture on consumer properties of the pore structures of (1.4).
Figure 2.1 (a) Fixed the intermediate state of the sequential transformation of the original mixture in the artificial stone. (b) The typical picture of laser speckles. (c) Diagram of laser micro-relief. (d) The addition of random complex amplitudes

Introduce the concept of "lightness", "contrast" and "characteristic size" spots, which are shown in Figure 2.1 (b) in texture, resulting in scattering of the reflected laser light from a rough surface. Quantitative characteristics of these concepts will be a statistical estimation of the structure of the image obtained and directly used in this article, when modeling volumetric models of the studied textures.

Under a rough surface refers to a surface having microscopic random variation of the terrain, the scale of which exceeds the wavelength of light. Almost all surfaces are rough. Thus, using light with different characteristics it is possible to get different speckle structure of the systems studied in the form of their respective textures.

The physical nature of the spots is simple. Each point of the object scatters light in the direction of the observer. Due to the high coherence of laser light, the scattered light from one of the points of the object are mixed with the light scattered in any other point of the object. This creates interference of scattered light waves. The detector, in the form of photographic film or retina of the human eye, as an observer, placed in the light field, can register a picture of a chaotic interference. The result is a characteristic speckles, shown in Figure 2.1 (d). An accident caused by surface roughness, since the phase of the scattered light varies randomly from point to point, following the fluctuations of the height of terrain in this place.

Instead of considering changes in luminance from point to point in this plane, it is convenient to consider ensemble average for a single point on the surface of the detector. In other words, we assume that the detector remains stationary in the position \( (x, y, z) \), and will measure the illumination at such points from each set (ensemble) of diffusers, which are macroscopically identical to each other, but microscopic are different from each other. Surfaces of all diffusers have the same statistical characteristics, e.g. the same mean-square surface elevation. We assume that the incident light is monochromatic and linearly polarized, and that polarization in the scattering is not changed. Imagine the light falling on the detector, the complex amplitude:

\[
U(x, y, z) = a(x, y, z) \cdot e^{-i \varphi(x, y, z)}
\]

(2.1)

and present it as a vector in the complex plane, as shown in Figure 2.1 (d). This complex amplitude actually is the sum of a large number of \( U \) components corresponding to the light incoming to the detector from the neighborhood of each point of a diffuse surface. Denoting the \( k \)-th component through \([U] \) \( (x, y, z) \), we get:

\[
U(x, y, z) = \frac{\sum_{k=1}^{N} U(x, y, z)}{\sqrt{N}} = \frac{1}{\sqrt{N}} \cdot \sum_{k=1}^{N} a(x, y, z) \cdot e^{-i \varphi(x, y, z)}
\]

(2.2)
In the article calculated the main statistical properties of the speckle-structure light field. This analysis is essentially equivalent to the classical problem of two-dimensional random walk and is performed in the complex plane. Figure 2.1 (d). It turns out that in this case the complex amplitude obeys the statistical law of Gauss. In particular, the density function of the joint probability of the real and imaginary parts of the function \( U \) has the form:

\[
P_{r,i}(U^r, U^i) = \frac{1}{2\pi \sigma^2} e^{-\frac{(U^r)^2 + (U^i)^2}{2\sigma^2}}
\]

where

\[
\sigma = \lim_{N \to \infty} \left( \frac{1}{N} \sum_{k=1}^{N} \frac{1}{2} \langle |U_k|^2 \rangle \right)
\]

We are most interested in the illumination intensity \( I = UU^* \), because that is the value measured in practice. As is known, the expression (2.3) can be transformed to obtain the probability density function for light:

\[
P(I) = \frac{1}{\langle I \rangle} e^{-\frac{I}{\langle I \rangle}}
\]

Thus, the illuminance distribution in the image speckles obeys the statistical law with a negative exponent of the exponential. The most probable value of the illumination speckle is zero, which indicates a black speckle.

The amount of contrast of speckle structure will serve ratio \( C = \frac{\sigma_I}{\langle I \rangle} \) where \( \sigma_I \) is the standard deviation of the illuminance from the mean. For the distribution described by expression (2.5), the contrast of the speckle structure is equal to one.

Analytical dependences between the different elements of the set (1.4) and relations (1.5) can be obtained as a result of numerical calculation and statistical analysis of speckle patterns. Recorded optical speckle pattern is analytically formalized in a dynamic process of the system under study in time, i.e. as a state of a dynamic system with inherent specific properties at specific points in time. This will allow strictly to identify (index) relevant texture of the speckle patterns at specific points in time during their technological processing and production. The process of optical recording and indexing of textures in the form of the corresponding speckle system will greatly improve product quality and to optimize the technological process of manufacturing products.

3. “Phase space”.

For the study of cellular structures and the visual representation of dynamic processes occurring in the article used the term "phase space".

Statistical processing of the speckle structure is presented in Figure 3.1 (a, b, c, d). Construct phase portraits studied the structural shown in Figure 3.1(d).

Part of the results of the statistical analysis of the investigated speckle structures is shown in figures 3.1(a, b, c). In figure 3.1 (a) shows the texture study of the speckle structure. In the figures 3.1 (b, d) shows the statistical characteristics of the distribution of intensity of illumination. In the figure 3.1 (c) plot a graph of the horizontal profile of the studied texture based analysis of the characteristics of the intensity of illumination of corresponding points of speckle structures. Schedule specified in the axes "distance between speckles" - "the brightness of illumination of the respective speckles". We assume that therefore, the speckle structure is indexed and presented on figure 3.1 (e). With the help of figure 3.1 (e) we can point to regularities in the form of so-called specific structures, called us in the future: - "saddle", "center" and "separates". Shown in figure 3.1 (e) the structure is comparable with the well-known concept of "singular structures". With this comparison in this article, we model the corresponding "phase portraits" of the status of the structures studied in the given time. Take the singular simulated structure for the point of beginning samples of the respective phase equilibria or transient processes study of speckle structures and with their help, we will carry out forecasting of the behavior of the studied textures in time.
4. The formalization of cellular structures based on optical holographic studies

The process of formalization of structures based on a holographic optical studies is shown in figure 4.1 (a, b). The structure is shown in figure 4.1 (a) will present with lines of identical level (isolines), which transform into the figure of "equipotential" surfaces 4.1 (b).

Shown in figure 4.1 (a) contour lines to represent the corresponding singular structures, which, themselves, can be stable or unstable systems.

So, if the dynamic system is in the area of sustainable equilibrium, small perturbations do not violate the stable operation of the system. If the structure of the equilibrium is not stable, the violation will progress which can lead to the destruction of the system as a whole.

This provision is also used in the article to predict the behavior of the investigated speckle structure in time. We apply the method of phase space with respect to the studied dynamic system of second order sets (1.4 and 1.5):

$$\frac{dx}{dt} = F_1(x, y) ; \quad \frac{dy}{dt} = F_2(x, y) \quad (4.1, 4.2)$$
Here, \( F_1(x, y) \) and \( F_2(x, y) \) function of its arguments, the sets (1.4, 1.5). To obtain the phase trajectories eliminate time from (4.1, 4.2). Divide the second equation (4.2) on the first (4.1)

\[
\frac{dy}{dx} = \frac{F_2(x,y)}{F_1(x,y)} \tag{4.3}
\]

At the point of equilibrium the time derivatives become zero and then we get the ratio of uncertainties:

\[
\frac{dy}{dx} = \frac{F_2(x,y)}{F_1(x,y)} = 0 \tag{4.4}
\]

The point at which there are uncertainties, are called singular points. At specific points in the phase plane of solutions can bifurcate that actually observed in Figure 4.1 (a, b).

5. Conclusion.

Comparing the obtained model with the identifier (ID), shown in figure 4.1, we can assume the following:

The speckles in the coordinates on the X-axis from 0 to 270 and Y from 0 to 500, - visually fit the image of an unstable system. The speckles in the coordinates on the X-axis from 400 to 500 and Y is from 0 to 500 is also visually consistent with the concept of a stable system.

Thus, to create identical to the set structures - you can use the speckle structure. Statistical analysis of speckle structures allows us to obtain information model the phase state of the system under study at a given time. Thus obtained models allow to create the corresponding phase portraits. Based on this, the production process can be optimized using the identification of the texture at a given point in time the phase portrait of the system.

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