Measurement of the $CP$ Asymmetry and Branching Fraction of $B^0 \rightarrow \rho^0 K^0$

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We present a measurement of the branching fraction and time-dependent CP asymmetry of $B^0 \rightarrow \rho^0 K^0$. The results are obtained from a data sample of $227 \times 10^6 \ U(4S) \rightarrow BB$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC. From a time-dependent maximum likelihood fit yielding $111 \pm 19$ signal events we find $B(B^0 \rightarrow \rho^0 K^0) = (4.9 \pm 0.8 \pm 0.9) \times 10^{-6}$, where the first error is statistical and the second systematic. We report the measurement of the CP parameters $S_{\rho^0 K^0} = 0.20 \pm 0.52 \pm 0.24$ and $C_{\rho^0 K^0} = 0.64 \pm 0.41 \pm 0.20$.

Decays of $B^0$ mesons to the $\rho^0 K^0$ final state are expected to be dominated by $b \rightarrow s$ penguin amplitudes. Neglecting Cabibbo-Kobayashi-Maskawa (CKM) suppressed amplitudes, the mixing-induced CP violation parameter $S_{\rho^0 K^0}$ should equal $\sin 2\beta$, which is well measured in $B^0 \rightarrow J/\psi K^0$ decays $[1]$. Within the Standard Model (SM), only small deviations from this prediction are expected $[2]$. In the Standard Model, a single phase in the CKM matrix governs CP violation $[3]$, but if heavy non-SM particles appear in additional penguin diagrams, new CP-violating phases could enter and $S_{\rho^0 K^0}$ would not equal $\sin 2\beta$ $[4]$. Observation of a significant discrepancy would be a clear signal of new physics.

In this Letter we present the first observation of the decay $B^0 \rightarrow \rho^0 K^0$ and a measurement of the CP-violating asymmetries $S_{\rho^0 K^0}$ and $C_{\rho^0 K^0}$ from a time-dependent maximum likelihood analysis. A non-zero value of $S_{\rho^0 K^0}$ indicates CP violation due to the interference between decays with and without mixing. Direct CP violation leads to a non-zero value of $C_{\rho^0 K^0}$. We take a quasi-two-body (Q2B) approach, restricting ourselves to the region of the $B^0 \rightarrow \pi^+\pi^- K^0_S$ Dalitz plot dominated by the $\rho^0$ and treating other $B^0 \rightarrow \pi^+\pi^- K^0_S$ contributions as non-interfering background. The effects of interference with other resonances are estimated and taken as systematic uncertainties.

The data were collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring at SLAC. An integrated luminosity of 205 fb$^{-1}$, corresponding to $227 \times 10^6 BB$ pairs, was collected at the $\Upsilon(4S)$ resonance (center-of-mass (CM) energy $\sqrt{s} = 10.56$ GeV), and 16 fb$^{-1}$ was collected about 40 MeV below the resonance (off-resonance data). The BABAR detector is described in detail elsewhere $[5]$. Charged particles are detected and their momenta measured by the combination of a silicon vertex tracker (SVT), consisting of five layers of double sided detectors, and a 40-layer central drift chamber (DCH), both operating in the 1.5 T magnetic field of a solenoid. Charged-particle identification is provided by the average energy loss in the tracking devices and by an internally reflecting ring-imaging Cherenkov detector (DIRC) covering the central region.

We reconstruct $B^0 \rightarrow \rho^0 K^0_S$ candidates ($B^0_{\text{rec}}$ in the following) from combinations of $\rho^0$ and $K^0_S$ candidates, both reconstructed in their $\pi^+\pi^-$ decay mode. For the $\pi^+\pi^-$ pair from the $\rho^0$ candidate, we remove tracks identified as very likely to be electrons, kaons, or protons. The mass of the $\rho^0$ candidate is restricted to the interval $0.4 < m(\pi^+\pi^-) < 0.9$ GeV/$c^2$. The $K^0_S$ candidate is required to have a mass within 13 MeV/$c^2$ of the nominal $K^0_S$ mass $[6]$ and a decay vertex separated from the $\rho^0$ decay vertex by at least three times the estimated separation measurement uncertainty. In addition, the cosine of the angle in the lab frame between the $K^0_S$ flight direction and the vector between the $\rho^0$ decay vertex and the $K^0_S$ decay vertex must be greater than 0.995. Vetoes against $B^0 \rightarrow D^+\pi^-$ and $B^0 \rightarrow K^*\pi^-(K^* \rightarrow K^0_S\pi^+)$ are imposed by requiring that the invariant masses of both $K^0_S\pi$ combinations are more than 0.055 GeV/$c^2$ and 0.040 GeV/$c^2$ from the $K^*$ and $D^+$ masses $[7]$ respectively. To exclude events with poorly reconstructed vertices we require the estimated error on $\Delta t$ to be less than 2.5 ps and that $|\Delta t|$ must be less than 20 ps, where $\Delta t$ is the proper time difference between the decay of the reconstructed $B$ meson ($B^0_{\text{rec}}$) and its unreconstructed partner ($B^0_{\text{tag}}$), $t_{\text{rec}} - t_{\text{tag}}$. It is determined from the measured relative displacement of the two $B$-decay vertices and the known boost of the $e^+e^-$ system.

Two kinematic variables are used to discriminate between signal and combinatorial background. The first is $\Delta E$, the difference between the measured CM energy of the $B$ candidate and $\sqrt{s}/2$, where $\sqrt{s}$ is the CM beam energy. The second is the beam-energy substituted mass $m_{\text{ES}} = \sqrt{(s/2 + p_i \cdot p_B)/E_i - p_B^2}$, where the $B^0_{\text{rec}}$ momentum $p_B$ and the four-momentum of the initial $\Upsilon(4S)$ state ($E_i, p_i$) are defined in the laboratory frame. We require $|\Delta E| < 0.15$ GeV and $5.25 < m_{\text{ES}} < 5.29$ GeV/$c^2$.

Continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events are the dominant background. To enhance discrimination between signal and continuum, we use a neural network (NN) to combine five variables: the cosine of the angle between the $B^0_{\text{rec}}$ direction and the beam axis in the CM, the cosine of the angle between the thrust axis of the $B^0_{\text{rec}}$ candidate and the beam axis, the sum of momenta transverse to the direction of flight of the $B^0_{\text{rec}}$ and the zeroth and second angular moments $L_{0,2}$ of the energy flow about the $B^0_{\text{rec}}$ thrust axis. The moments are defined by $L_i = \sum_i p_i \times |\cos \theta_i|$, where $p_i$ is its momentum and $\theta_i$ is the angle with respect to the $B^0_{\text{rec}}$ thrust axis of the track or neutral cluster $i$ excluding the tracks that make up the $B^0_{\text{rec}}$ candidate. The NN is trained with off-resonance data and Monte Carlo (MC) $[8]$ simulated signal events.

Selected signal events are reconstructed incorrectly
with low momentum tracks from the other $B$ meson being used to form the $\rho^0$ candidate. In total, 20,073 events pass all selection criteria in the on-resonance sample.

An unbinned extended maximum likelihood fit is used to extract the $\rho^0 K^0_S$ $CP$ asymmetry and branching fraction. There are ten components in the fit: signal, continuum background and eight separate backgrounds from $B$ decays. Large samples of MC-simulated events are used to identify these specific $B$ backgrounds. Where an individual decay mode makes a significant contribution to the dataset (one or more events expected in the data) we include it as a separate contribution to the fit. Probability density functions (PDFs) are taken from simulation and MC efficiencies (Table I). Where only upper limits are available, decay modes are not included in the default fit but are used in alternate fits to evaluate systematics.

Events from $B$ decays that do not come from individually significant channels are collected together into two “bulk” $B$ contributions to the fit ($B^0$ and $B^+$. The assumption is made that $B^0 \rightarrow f_0(600)K^0_S$ can be neglected, with support from $\mathcal{B}$ and MC efficiencies (Table II). Where only upper limits are available, decay modes are not included in the default fit but are used in alternate fits to evaluate systematics.

The events in the data sample have their untagged $B_s$ flavor-tagged as $B^0$ or $\bar{B}^0$ with the method described in [10]. Events are separated into four flavor-tagging categories and an “untagged” category, depending upon the method used to determine the flavor. Each category has a different expected purity and accuracy of tagging. The likelihood function for the $N_k$ candidates in flavor tagging category $k$ is

$$L_k = e^{-N_k} \prod_{i=1}^{N_k} \left[ N_{S^k} \left( 1 - f_{MR}^k \right) P_{i,k}^{S^k} + f_{MR}^k P_{i,k}^{MR} \right] R_{C,k} \sum_{j=1}^{N_{B,j}} N_{B,j} f_{j,k} P_{i,k}^{B_j} \right],$$

(1)

where $N_k$ is the sum of the signal and background yields for events tagged in category $k$, $N_S$ is the number of $\rho^0 K^0_S$ signal events in the sample, $\epsilon_k$ is the fraction of signal events tagged in category $k$, $f_{MR}^k$ is the fraction of mis-reconstructed (MR) signal events in tagging category $k$ and the superscript CR implies correctly reconstructed signal. $N_{C,k}$ is the number of continuum background events that are tagged in category $k$, and $N_{B,j} f_{j,k}$ is the number of $B$-background events of class $j$ that are tagged in category $k$. The $B$-background event yields are fixed in the default fit to values shown in Table II. The values $\epsilon_k$ and $f_k$ are determined from MC for $B$-backgrounds and from a sample of $B$ decays of known flavor for signal. The total likelihood $L$ is the product of the likelihoods for each tagging category.

Each signal and background PDF is defined as $P_k = \mathcal{P}(m_{ES}) \cdot \mathcal{P}(\Delta E) \cdot \mathcal{P}(NN) \cdot \mathcal{P}(\cos \theta_{\pi^+}) \cdot \mathcal{P}(\Delta t) \cdot \mathcal{P}(m_{\pi^+\pi^-})$

Table I: Expected number of events from each $B$ background source.

| Background Mode | $N_{\text{expected}}$ |
|-----------------|-----------------|
| Bulk $B^+$      | 197±98          |
| Bulk $B^0$      | 197±98          |
| $B^0 \rightarrow D^+\pi^-$ | 40±6 |
| $B^0 \rightarrow \eta' K^0_S$ | 34±5 |
| $B^0 \rightarrow f_0(980)K^0_S$ | 22±4 |
| $B^0 \rightarrow K^0_s(1430)^+\pi^-$ | 7±1 |
| $B^0 \rightarrow \rho^0 K^{*0}$ | 3±3 |
| $B^0 \rightarrow (K^0_s\pi^+\pi^-)_{\text{NR}}$ | 2±1 |

where $m_{ES}$, $\Delta E$, $NN$, $m(\pi^+\pi^-)$ are the variables described previously, and $\cos \theta_{\pi^+}$ is the angle between the $K^0_S$ and the $\pi^+$ from the $\rho^0$ meson’s center-of-mass frame.

The $\Delta t$ PDF for signal events is defined as

$$\mathcal{P}(\Delta t) = \frac{e^{-i \Delta t / \tau_B}}{\tau_B} \times \left[ 1 + \Delta_D + q(D) \left( S_{\rho^0 K^0_S} \sin(\Delta m_d \Delta t) - C_{\rho^0 K^0_S} \cos(\Delta m_d \Delta t) \right) \right] \otimes R_{\text{sig}}(\Delta t, \sigma_{\Delta t}),$$

(2)

where $\tau_B$ and $\Delta m_d$ are the average lifetime and eigenstate mass difference of the neutral $B$ meson, $q = 1$ ($-1$) when $B^0_{\text{rec}} = B^0$ ($\bar{B}^0$). $(D), \Delta D$ and the $\Delta t$ resolution function, $R_{\text{sig}}(\Delta t, \sigma_{\Delta t})$, have parameters fixed to values taken from a sample where $B_s$ of known flavor can be reconstructed [10]. “Untagged” events have a $(D)$ of 0, reflecting the lack of tag information.

The $m_{ES}$, $\Delta E$, $NN$, $m(\pi^+\pi^-)$ PDFs for signal and $B$ background are taken from MC simulation. In general they are non-parametric, with the exception of $m_{ES}$ and $\Delta E$ for signal PDFs appear as solid curves in Figure I. The $CP$ parameters for $\eta' K^0_S$ and $f_0 K^0_S$ backgrounds are fixed to $C = 0$ and $S = \sin 2\beta$ (for $\eta' K^0_S$) and $S = -\sin 2\beta$ (for $f_0 K^0_S$), in accordance with SM expectations. For the remaining $B$ backgrounds the parameters $C$ and $S$ are fixed to 0. The PDF parameters describing the continuum background are either allowed to vary freely in the fit or else determined separately from off-resonance data.

There are 16 free parameters in the fit: the yield of signal events, $S_{\rho^0 K^0_S}$ and $C_{\rho^0 K^0_S}$ and 13 that parameterize the continuum background. The continuum parameters are: the yields (5), and those associated with the second order polynomial describing the $\Delta E$ distribution (2), the ARGUS [11] function describing the $m_{ES}$ distribution (1) and the double Gaussian used to model the $\Delta t$ distribution (5).

The fit yields $111 \pm 19$ signal events. We calculate
the branching fraction from the measured signal yield, efficiency (including the $\rho^0 \rightarrow \pi^+\pi^-$, $K^0 \rightarrow K^0_s$ and $K^0_s \rightarrow \pi^+\pi^-$ branching fractions), and the number of $B\overline{B}$ events. The result is $\mathcal{B}(B^0 \rightarrow \rho^0 K^0) = (4.9 \pm 0.8 \pm 0.9) \times 10^{-6}$, where the first error is statistical and the second systematic. The likelihood ratio between the fit result of 111 signal events and the null hypothesis of zero signal shows that this is excluded at the 8.7σ level. When additive systematic effects are included we exclude the null hypothesis at the 5.0σ level. The fit for CP parameters gives $S_{\rho^0 K^0_s} = 0.20 \pm 0.52 \pm 0.24$ and $C_{\rho^0 K^0_s} = 0.64 \pm 0.41 \pm 0.20$.

Figure 1 shows plots of the discriminating variables in the fit. Knowledge of the level of background and our ability to distinguish it from signal can be gained from the errors in these plots. In addition, Fig. 1(f) shows the ratio $L_S/(L_S + L_B)$ for all events, where $L_S$ and $L_B$ are the likelihoods for each event to be signal or background, respectively.

Systematic errors are listed in Table II. We estimate biases due to the fit procedure from fits to a large number of simulated experiments. We vary parameters fixed in the nominal fit by their uncertainty and include the change in result as the corresponding systematic error. The systematic uncertainties arise from sources including the parameterization of the signal $\Delta t$ resolution function, the mistag fractions, and discrepancies between data and the simulation including the effect of alternative models for resonances.

We estimate the systematic uncertainty due to neglecting the interference between $B^0 \rightarrow K^0_s \pi^+\pi^-$ from both parameterized and full simulations that take interference into account. We include contributions from $\rho^0(770)K^0_s$, $f_2(980)K^0_s$, $K^*_S(1430)^+\pi^-$, $K^*_S(892)^+\pi^-$ and $f_2(1270)K^0_s$, as well as two $K^0_s\pi\pi$ non-resonant contributions. We simulate many samples with different relative phases between modes. We also vary the amplitude of each mode within limits based on the best available information. Each simulation is then subjected to the standard selection and fitting procedure. The systematic uncertainty is taken from the width of a Gaussian fitted to the distribution of the results.

In summary, we have established the existence of the decay $B^0 \rightarrow \rho^0 K^0$ and measured its branching fraction with the significance of 5 standard deviations. Our measurement agrees with errors within $\mathcal{B}(B^0 \rightarrow \omega K^0)$ as measured in Ref. 13, as expected if a single penguin amplitude dominates these decays. We have extracted the $CP$ violating parameters $S$ and $C$ for $B^0 \rightarrow \rho^0 K^0_s$ which are consistent with those measured in charmonium channels [1].

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FIG. 2: Plots of $\Delta t$, overlaid with projected signal PDFs, split into (a) $B^0_{u\bar{s}}$ tags, (b) $B^0_{d\bar{s}}$ tags and (c) the asymmetry $(N_{B^0_{u\bar{s}}} - N_{B^0_{d\bar{s}}})/(N_{B^0_{u\bar{s}}} + N_{B^0_{d\bar{s}}})$ as a function of $\Delta t$.

| Mis-reco'd events and fit bias | 0.12 0.09 10 |
|--------------------------------|--------------|
| PDF uncertainties             | 0.13 0.18 2  |
| Tagging parameters            | 0.02 0.01 -  |
| Neglect of interference       | 0.14 0.09 7  |
| $\rho^0$ mass shape           | 0.07 0.05 3  |
| $B$ Background BF             | 0.02 0.10 13 |
| $CP$ of background            | 0.04 0.00 -  |
| Tracking efficiency & $B$ counting | - - 6    |
| Total                          | 0.24 0.20 19 |

TABLE II: Summary of contributions to the systematic error.