Kinematics of Planar Quabody Mechanism with More Kinematics Bifurcation Positions

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Abstract. Based on multibody dynamics, some definitions of mechanism are defined. The kinematics bifurcation position is a cold researching spot for many many years. It is common in planar quabody mechanisms but has not been paid enough attention to. The kinematics bifurcation position refers to the mechanism position with a pressure angle of 90° of the follower, including the stuck position (dead point). In this paper, a planar quabody mechanism is studied. In a motion cycle, there are three kinematics bifurcation positions in the mechanism. The third derivative of angle to time, angular velocity, angular acceleration of the connecting rod in kinematics bifurcation positions is obtained by solving the kinematics parameters of kinematics bifurcation positions according to L'Hôpital's rule. If the driving body swings in sinusoidal law, the motion of the mechanism is continuous. The kinematics characteristics of the mechanism are independent of the simulation convergence and response bifurcation of nonlinear dynamics. The length of the rocker is 1 m, the length of the connecting rod is 0.5 m, the distance between the track of the slider and the fixed point of the rocker is 0.5 m, and the rocker swings according to the sine law with a period of 9 seconds. The computer simulation shows that the speed and acceleration are continuous and so there is no impact in the kinematics bifurcation positions.

1. Introduction
Machinery is the main tool to replace or reduce human labor and increase labor productivity. It is a theoretical model or entity device composed of several parts with definite mutual motion and certain functions. From the motion of one object to get the motion of other objects, and there is a certain relationship between the motions of every two objects. This kind of motion is called mechanical motion. A machine is a movable device that performs mechanical motions. A mechanism is the theoretical model that performs mechanical motions.

Mechanical principle focuses on theory, so its concept should be consistent with theoretical mechanics and multibody dynamics. A body (component) is an independent object moving in a mechanism or machine. The connecting parts between the bodies are called motion pairs. In the mechanism, the body is rigid, which can be abstracted as the object formed by rigid connection of each motion pair element. A body has three freedoms (independent motions) in plane motion, two mutual vertical motions and one rotation. When one body rotates with respect to the other, two freedoms are constrained. This kind of motion pair is the rotating pair and the biconstraint pair (low...
pair). When a body moves relative to another body, which is equivalent to rotating around infinity far, a motion freedom and a rotation freedom are constrained. This kind of motion pair is the translational pair and also the biconstraint pair. When one body rolls and slides with respect to the other, a motion freedom perpendicular to the rolling direction is constrained. The motion pair between the two bodies is the rolling-sliding pair and the monoconstraint pair (high pair). The body with two or more rotating pairs is called a rod (crank, rocker, connecting rod, multipair rod, etc.). The body with one rotating pair is called a block (slider, track rod, swing block, etc.). The body without rotating pair is called a board. Prefix multi, mono, bi and qua mean many, one, two and four respectively.

For the crank rocker mechanism, if the rocker can bypass the fixed body, and if the crank rotates at a constant speed, at the position where the connecting rod and the rocker are collinear, the rocker can rotate in the counterclockwise direction, or in the clockwise direction, which is called the kinematics bifurcation position. If the rocker is an active body, and if it swings according to the sinusoidal law, the position of the collineation between the crank and the connecting rod is also the kinematics bifurcation position. It can be seen that the pressure angle of the follower (the angle between the force direction and the motion direction) is 90 degrees in the kinematics bifurcation position. For the static kinematics bifurcation position, if no matter how large the driving force can not make the mechanism move, the position is also called the stuck position (dead point [1-2]).

The kinematics bifurcation position is common in planar mechanism. This motion characteristic is also called motion bifurcation. It is independent of simulation convergence and response bifurcation of nonlinear dynamics. However, in mechanism kinematics, the kinematics bifurcation position has not been paid enough attention to for many years[3].

The kinematic study[4] is the basis of the dynamic study[5-6]. In the searched single slider quadrabody mechanism, the connecting rod is not the longest rod, and it has three kinematics bifurcation positions. Assuming that the driven slider is affected by a certain flow rate, the motion law of the driven connecting rod and the driven slider is studied.

2. The rocker slider mechanism

In the rocker slider mechanism shown in figure 1, the rocker swings within a certain range. The rotating pair between rocker AB and fixed body is A, and the rotating pair of connecting rod BC and slider is C. The track path of the slider refers to a straight line or curve that passes through another motion pair and is parallel to the motion track. The distance between the track path and point A is half of the length of the rocker.

![Figure 1. Rocker-slider mechanism with kinematic bifurcation positions](image-url)

In this mechanism, the fixed body can be regarded as an infinitely long rod with A pointing to the infinitely far end which is vertical to the slide track, and the slide can be regarded as an infinitely long rod with a rotating pair C pointing to the infinitely far end perpendicular to the track. Point A is taken as the coordinate origin. The horizontal right direction is taken as the positive direction of $x_1$ axis. The vertical up direction is taken as the positive direction of $x_2$ axis. So a rectangular coordinate system is established. $l_1$ and $l_2$ are the length of rocker AB and connecting rod BC respectively, and the diameter vectors of the two bodies are $l_1$ and $l_2$ respectively. $d$ is the vertical distance from A to the slider track. Both $l_2$ and $d$ are half of $l_1$. The diameter vector of point C relative to the coordinate origin is $l_3$. 
According to the geometric relationship between the bodies and the geometric meaning of the vector equation (the vectors on both sides of the equal sign are connected with each other to form two vector chains. If their starting point is the same, their ending point is coincident). The vector equation of the mechanism is as the following\cite{7-8}.

\[ l_1 + l_2 = l_3 \]  

(1)

If \( \theta_1 \) and \( \theta_2 \) are taken as the included angles of \( l_1 \) and \( l_2 \) with respect to the positive direction of \( x_1 \) axis. There is, \( \theta_1 \in [0 \pi] \).

3. Projection equations of mechanism vector equation

By projecting the vector equations (1) onto two coordinate axes respectively, a projection equations set can be obtained as the following.

\[ l_1 \cos \theta_1 + l_2 \cos \theta_2 = x_{1,c}, \quad l_1 \sin \theta_1 + l_2 \sin \theta_2 = d \]  

(2)

If the rocker is an active part, its motion law is known. The unknowns of equations (2) are \( \theta_2 \) and \( x_{1,c} \). \( \theta_2 \) can be obtained from the second equation of equations (2) as the following.

\[ \theta_2 = \arcsin \left( \frac{d - l_1 \sin \theta_1}{l_2} \right), \quad \theta_2 = \pi - \arcsin \left( \frac{d - l_1 \sin \theta_1}{l_2} \right) \]  

(3)

According to \( \theta_2 \) and \( \dot{\theta}_2 \) of the previous position, as well as \( \dot{x}_{1,c}, \) \( \theta_2 \) can be obtained from formula (3). Then, \( x_{1,c} \) can be obtained from the first equation of equations (2).

Calculating the first derivative of time, equations (s) changes to equations (4). It can be reorganized to matrix equation.

\[ \begin{bmatrix} -l_1 \dot{\theta}_1 \sin \theta_1 - l_2 \dot{\theta}_1 \sin \theta_2 = \dot{x}_{1,c} \; \; , \; \; & -l_1 \sin \theta_1 \; \; -1 \; \; & \dot{\theta}_1 \; \; -\sin \theta_1 \; \; \cos \theta_1 \; \; \cos \theta_1 \; \; \cos \theta_1 \; \; \cos \theta_1 \end{bmatrix} = \left[ \begin{array}{c} \dot{x}_{1,c} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{x}_{1,c} \end{array} \right] \]  

(4)

The second equation of equation group (4) has only one unknown variable, so it can be solved in turn without the simplified formula of the second-order matrix inversion to solve matrix equation (4). From the downer equation of equations (4), we can get:

\[ \dot{\theta}_2 = \frac{-l_1 \dot{\theta}_1 \cos \theta_1}{l_2 \cos \theta_2} \]  

(5)

\( \dot{x}_{1,c} \) can be obtained from the upper equation of equations (4). Calculating the first derivative of time, equations (4) changes to equations (6). It can be reorganized to matrix equation (7).

\[ \begin{bmatrix} -l_1 \dot{\theta}_1 \sin \theta_1 - l_2 \dot{\theta}_1 \cos \theta_1 - l_1 \dot{\theta}_1 \sin \theta_2 - l_2 \dot{\theta}_1 \cos \theta_2 = \dot{x}_{1,c} \; \; , \; \; & -l_1 \sin \theta_1 \; \; -1 \; \; & \dot{\theta}_1 \; \; -\sin \theta_1 \; \; \cos \theta_1 \; \; \cos \theta_1 \; \; \cos \theta_1 \; \; \cos \theta_1 \end{bmatrix} = \left[ \begin{array}{c} \dot{x}_{1,c} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{x}_{1,c} \end{array} \right] \]  

(6)

\[ \dot{\theta}_2 = \left( l_2 \dot{\theta}_1^2 \sin \theta_2 - l_1 \dot{\theta}_1 \cos \theta_1 + l_1 \dot{\theta}_1^2 \sin \theta_1 \right) \left( l_2 \cos \theta_2 \right)^{-1} \]  

(8)
Then, \( \dot{x}_{1,c} \) can be obtained from the upper equation of equations (6).

By calculating the first derivative of equations (6) for time, the third derivative of vector equation (1) in \( x_2 \) axial projection can be obtained. Then \( \ddot{\theta}_2 \) can be arranged out.

\[
l_1\ddot{\theta}_1 \cos \theta_1 - 3l_1\dot{\theta}_1 \dot{\theta}_1 \sin \theta_1 - l_1\dddot{\theta}_1 \cos \theta_1 + l_1\dot{\theta}_1^3 \cos \theta_1 + 3l_2\dot{\theta}_2 \dot{\theta}_2 \sin \theta_2 - l_2\dddot{\theta}_2 \cos \theta_2 = 0
\]

(9)

\[
\ddot{\theta}_2 = \left(-l_1\ddot{\theta}_1 \cos \theta_1 + 3l_1\dot{\theta}_1 \dot{\theta}_1 \sin \theta_1 + l_1\dot{\theta}_1^3 \cos \theta_1 + 3l_2\dot{\theta}_2 \dot{\theta}_2 \sin \theta_2 + l_2\dddot{\theta}_2 \cos \theta_2 \right)(l_2 \cos \theta_2)^{-1}
\]

(10)

The third derivative of the vector equation projected on the \( x_1 \)-axis is also obtained. At the same time, \( \dot{x}_{1,c} \) is obtained.

\[
\dot{x}_{1,c} = -l_1\ddot{\theta}_1 \sin \theta_1 - 3l_1\dot{\theta}_1 \dot{\theta}_1 \cos \theta_1 + l_1\dot{\theta}_1^3 \sin \theta_1 - l_2\dddot{\theta}_2 \sin \theta_2 - 3l_2\dot{\theta}_2 \dot{\theta}_2 \cos \theta_2 + l_2\dddot{\theta}_2 \theta_2 \sin \theta_2
\]

(11)

4. Angular velocity and acceleration of the connecting rod in kinematics bifurcation positions

For the mechanism shown in figure 1, there are motion bifurcations in three positions \( \theta_1 = 0 \), \( \theta_1 = \pi / 2 \) and \( \theta_1 = \pi \), which are the kinematics bifurcation positions of the mechanism. In the position of \( \theta_1 = \pi / 2 \), the connecting rod is pulled by the rocker without the feature of stuck position, which belongs to the general kinematics bifurcation position. In the other two positions, because the connecting rod is pressed by the rocker, if the sliding block remains stationary, the mechanism remains stationary no matter how much driving force, so both positions are the stuck positions of the mechanism.

If the rocker swings according to the sinusoidal law, \( T \) is the period, \( \theta_1 \) and its derivatives from first to forth are as the following.

\[
\theta_1 = \frac{\pi - 0}{2} \sin \left( \frac{2\pi}{T} \right) + \frac{\pi + 0}{2}, \quad \dot{\theta}_1 = \frac{\pi^2}{2} \cos \left( \frac{2\pi}{T} \right)
\]

(12)

\[
\ddot{\theta}_1 = -\frac{2\pi^3}{T^3} \sin \left( \frac{2\pi}{T} \right), \quad \dddot{\theta}_1 = -\frac{4\pi^4}{T^3} \cos \left( \frac{2\pi}{T} \right), \quad \ddot{\theta}_2 = \frac{8\pi^5}{T^4} \sin \left( \frac{2\pi}{T} \right)
\]

The connecting rod \( BC \) can rotate all the way. In the position of \( \theta_1 = 0 \), there are,

\[
t = \frac{3}{4} - nT , \quad \frac{2\pi}{T} t = \frac{3}{2} - 2n\pi , \quad \cos \left( \frac{2\pi}{T} \right) = 0, \quad \dot{\theta}_1 = 0, \quad \ddot{\theta}_1 = 0, \quad \theta_1 = \frac{\pi}{2}
\]

(13)

According to the previous research, \( \ddot{\theta}_1 \) of formula (5), \( \dddot{\theta}_2 \) of formula (8) and \( \ddot{\theta}_2 \) of formula (10) are all infinitive of type 0/0. The value of \( \ddot{\theta}_2 \) can be determined according to L’Hopital’s rule as the following[9].

\[
\theta_2 = \left(-\ddot{\theta}_1 \cos \theta_1 + \dddot{\theta}_1 \dot{\theta}_1 \sin \theta_1 \right)(-\dot{\theta}_2 \sin \theta_2) = \left(\ddot{\theta}_1 \dot{\theta}_1 \right) \left(l_2 \dot{\theta}_2 \right) \left(l_2 \dot{\theta}_2 \right)^{-1}, \quad \theta_2 = \overline{\theta} \sqrt{\ddot{\theta}_1 \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_2}
\]

(14)

Where, “\( \overline{\theta} \)” is the same as the \( \ddot{\theta}_2 \) symbol of the previous position, but opposite to the \( \dot{x}_{1,c} \) symbol of the previous position.

If \( \ddot{\theta}_1 = 0 \), the mechanism remains stationary. There are, \( \theta_2 = 0 \), \( \ddot{\theta}_2 = 0 \). From (4), \( \dot{x}_{1,c} \) is:

\[
\dot{x}_{1,c} = -l_2 \dot{\theta}_2
\]

(15)
According to L’Hopital’s rule, $\dot{\theta}_2$ can be obtained from formula (8).

$$\ddot{\theta}_2 = \frac{2l_1 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2 + l_2 \dot{\theta}_2^3 \cos \theta_2 - l_1 \dot{\theta}_1 \cos \theta_1 + 3l_1 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_1 + l_2 \dot{\theta}_2^3 \cos \theta_1}{-l_2 \dot{\theta}_2 \sin \theta_2} = \frac{2l_1 \dot{\theta}_2 \dot{\theta}_2^3 - l_1 \dot{\theta}_1 - l_2 \dot{\theta}_2^3}{-l_2 \dot{\theta}_2}$$

(16)

$$\ddot{\theta}_2 = \frac{l_1 \dot{\theta}_1 + l_2 \dot{\theta}_2^3}{3l_2 \dot{\theta}_2} = 0$$

(17)

According to L’Hopital’s rule, $\ddot{\theta}_2$ can be obtained from formula (10).

$$\ddot{\theta}_2 = -l_1 \dot{\theta}_1 \cos \theta_1 + l_1 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_1 + 3l_1 \left( \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^3 \right) \sin \theta_1 + 3l_1 \dot{\theta}_1 \dot{\theta}_2^3 \cos \theta_1 + 3l_1 \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_2^3 \cos \theta_1$$

$$\ddot{\theta}_2 = -l_1 \dot{\theta}_1 \sin \theta_1 + 3l_1 \left( \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^3 \right) \sin \theta_1 + 3l_1 \dot{\theta}_1 \dot{\theta}_2^3 \cos \theta_1 + 3l_1 \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_2^3 \cos \theta_1 - l_1 \dot{\theta}_1^3 \sin \theta_1$$

$$\ddot{\theta}_2 = (l_1 \dot{\theta}_1 + l_2 \dot{\theta}_2^3) \left( 3l_2 \dot{\theta}_2 \right)^{-1}$$

(18)

In the position of $\theta_1 = \pi$, there are, $t = \frac{T}{4} + nT$, $\frac{2\pi}{T} = \frac{\pi}{2} + 2n\pi$, $\cos \left( \frac{2\pi}{T} \right) = 0$, $\dot{\theta}_1 = 0$, $\dot{\theta}_1 = 0$, $\dot{\theta}_2 = \frac{\pi}{2}$

$\dot{\theta}_2$ and $\ddot{\theta}_2$ are also 0/0 infinitive type. According to L’Hopital’s rule, from formula (5), we can get the value of $\dot{\theta}_2$ as the following.

$$\dot{\theta}_2 = \pm \sqrt{-\dot{\theta}_1 l_2^{-1}}$$

(20)

Where, "$\mp$" is the same as the $\dot{\theta}_2$ symbol of the previous position, but opposite to the $\dot{x}_{1,c}$ symbol of the previous position.

If $\dot{\theta}_1 = 0$, the mechanism remains stationary, there are, $\dot{\theta}_2 = 0$, $\ddot{\theta}_2 = 0$. From equation set (4), formula (15) can be obtained. According to L’Hopital’s rule, $\ddot{\theta}_2$ can be obtained from formula (8).

$$\ddot{\theta}_2 = (l_1 \dot{\theta}_1 + l_2 \dot{\theta}_2^3) \left( 3l_2 \dot{\theta}_2 \right)^{-1} = 0$$

(21)

According to L’Hopital’s rule, $\ddot{\theta}_2$ can be obtained from formula (10).

$$\ddot{\theta}_2 = (l_1 \dot{\theta}_1 + l_2 \dot{\theta}_2^3) \left( 3l_2 \dot{\theta}_2 \right)^{-1}$$

(22)

In the position of $\theta_1 = 0.5\pi$, there are,

$$t = \frac{T}{4} + nT \text{ or } t = \frac{T}{4} + 2n\pi, \frac{2\pi}{T} = \frac{\pi}{2} + 2n\pi, \sin \left( \frac{2\pi}{T} \right) = 0, \dot{\theta}_1 = 0, \dot{\theta}_1 = 0, \dot{\theta}_2 = \frac{3\pi}{2}$$

According to L’Hopital’s rule, $\ddot{\theta}_2$ can be obtained from formula (5) as the following.

$$\ddot{\theta}_2 = (l_1 \cos \theta_1 \dot{\theta}_1 + l_2 \dot{\theta}_2 \sin \theta_1 \dot{\theta}_2) \left( -l_1 \sin \theta_1 \dot{\theta}_1 \dot{\theta}_2 \right)^{-1} = (\dot{\theta}_1 \dot{\theta}_2) \left( l_2 \dot{\theta}_2 \right)^{-1}$$

(23)

$$\ddot{\theta}_2 = \pm \sqrt{\dot{\theta}_1 \dot{\theta}_2 l_2^{-1}}$$

(24)

Where, "$\mp$" is the same symbol as the $\ddot{\theta}_2$ of previous position, otherwise a flexible impact occurs.
If $\dot{\theta}_1 = 0$, then $\dot{\theta}_2 = 0$. $\dot{x}_{1,C}$ can be obtained from equations (4).

$$
\dot{x}_{1,C} = -l_1 \dot{\theta}_1 + l_2 \dot{\theta}_2
$$

(25)

According to L’Hopital’s rule, $\ddot{\theta}_1$ can be obtained from formula (8).

$$
\ddot{\theta}_2 = \frac{2l_1 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2 + l_2 \dot{\theta}_2^3 \cos \theta_2 - l_1 \dot{\theta}_1 \cos \theta_1 + 3l_1 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_1 + l_1 \dot{\theta}_1^3 \cos \theta_1 - 2l_2 \dot{\theta}_2 \sin \theta_2 + 3l_1 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_1}{l_2 \dot{\theta}_2 \sin \theta_2}
$$

$$
\ddot{\theta}_2 = \left( l_1 \dot{\theta}_1 \dot{\theta}_2 \right)^{-1} = \pm \sqrt{l_2 l_1} \ddot{\theta}_1 = 0
$$

(26)

Where, "±" is the same as that of formula (24).

If $\dot{\theta}_1 = 0$, then $\dot{\theta}_2 = 0$. When $\dot{\theta}_1 = 0$ and $\dot{\theta}_2 = 0$ are held at the same time, the mechanism remains stationary. According to L’Hopital’s rule, $\ddot{\theta}_2$ can be obtained from formula (10).

$$
\dddot{\theta}_2 = \frac{+3l_1 \left( \ddot{\theta}_1 \ddot{\theta}_2 + \dot{\theta}_1 \dddot{\theta}_2 \right) + l_2 \dddot{\theta}_2 - 3l_1 \left( \ddot{\theta}_2 \dddot{\theta}_2 + \dot{\theta}_2 \dddot{\theta}_2 \right) + l_2 \dddot{\theta}_2^4}{l_2 \dot{\theta}_2}
$$

(27)

$$
\dddot{\theta}_2 = \frac{+3l_1 \dddot{\theta}_2 + 4l_2 \dddot{\theta}_2 - 3l_2 \left( \dddot{\theta}_2 \dddot{\theta}_2 + \dddot{\theta}_2 \dddot{\theta}_2 \right) + l_2 \dddot{\theta}_2^4}{4l_2 \dddot{\theta}_2}
$$

(28)

5. Computer simulation results

5.1. The mechanism parameters

The lengths of the active rocker and the connecting rod are $l_1 = 1.0\text{m}$ and $l_2 = 0.5\text{m}$, respectively. The distance between the slide track and the rocker fixed point is $d = 0.5\text{m}$. The swing period of the rocker is $T = 9.0\text{s}$.

5.2. Simulation result

The time history of kinematics parameters of each body is shown in figure 2 and figure 3. There is no sudden change of velocity and acceleration. So there is no impact in the kinematics bifurcation position[10-11]. This mechanism has motion continuity[12].

**Figure 2.** The kinematics parameters of rocker and connecting rod

**Figure 3.** The kinematics parameters of slider
6. Conclusion
There are 3 points mainly as the following.

1) In the kinematics bifurcation position, the calculation formula of kinematics parameters is infinitive. If the corresponding calculation formula is not deduced in advance, it will be divided by zero and the calculation fails in the dynamic simulation of the mechanism.

2) For a planar quabody mechanism with three kinematics bifurcation positions in one motion cycle, this paper deduces the calculation formula of kinematics parameters of each body, which provides the research basis for the dynamic programmed computer simulation.

3) The given basic concepts of mechanical principle are in line with theoretical mechanics and multibody dynamics, such as mechanism, machine, monoconstraint pair, biconstraint pair, rod, block and board. Those concepts will make the subject more scientific.

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