Crossed channel analysis of quark and gluon generalized parton distributions with helicity flip

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Abstract

Quark and gluon helicity flip generalized parton distributions (GPDs) address the transversity quark and gluon structure of the nucleon. In order to construct a theoretically consistent parametrization of these hadronic matrix elements, we work out the set of combinations of those GPDs suitable for the SO(3) partial wave (PW) expansion in the cross-channel. This universal result will help to build up a flexible parametrization of these important hadronic non-perturbative quantities, using for instance the approaches based on the conformal PW expansion of GPDs such as the Mellin-Barnes integral or the dual parametrization techniques.

PACS numbers: 13.60.-r, 13.60.Fz, 14.20.Dh
1. INTRODUCTION

The transversity quark and gluon structure of the nucleon is a longstanding challenge to theoretical and experimental studies \[1\]. Contrarily to the case of the helicity dependent sector, the quark and gluon parts are well separated within the transversity sector thanks to the chiral-odd property of transversity quark distributions. Notorious experimental difficulties prevented us up to now from accessing directly quark transversity distributions through their golden channel, inclusive dilepton production with transversely polarized beam and target \[2\]. A promising attempt to extract information on this important hadronic sector is currently investigated with the help of transverse momentum dependent distributions (TMDs) (see *e.g.* \[3, 4\] and references therein). An alternative method, which may prove to be fruitful, is the study of exclusive reactions, where transversity dependent generalized parton distributions (GPDs) enter the factorized amplitude in the generalized Bjorken regime \[5, 6\]. Here also, the quark and gluon cases are well separated due to the chiral-oddity of the quark operator. This feature prevents the quark helicity flip GPDs to contribute to photon or meson leptoproduction amplitude at the leading twist \[\] (see however Ref. \[5, 11\] to evade this no-go theorem). On the other hand, the gluon helicity flip GPDs do not suffer from any selection rule and appear at the leading twist level in many amplitudes, for instance in the deeply virtual Compton scattering (DVCS) \(O(\alpha_s)\) contribution to the leptoproduction of a real photon. This contribution can be separated through a harmonic analysis \[11\], as discussed in details in Ref. \[12\].

Whereas quark helicity flip GPDs can be parametrized thanks to a double distribution Ansatz à la Radyushkin \[13\], provided an educated guess of the shape and normalization of the transversity PDF is used, as in \[14\]\(^1\), the absence of a forward limit for helicity flip gluon GPDs makes this procedure impracticable. However, a possible way to get a consistent parametrization of gluon helicity flip GPDs is to take advantage of a partial wave expansion in the crossed channel. This is the goal of the present paper.

The paper is organized as follows. In Sec.\[2\] we specify our set of conventions for both quark and gluon helicity flip GPDs. In Sec. \[3\] we determine the combinations of quark and gluon helicity flip GPDs suitable for the partial wave expansion in the cross channel

\(^1\) Other parametrizations have also been recently proposed in Refs. \[15, 16\].
SO(3) partial waves by applying the method elaborated in Sec. 4.2 of [5]. This analysis provides an independent cross check of the selection rules for the cross channel exchange quantum numbers established in [18, 19] with the help of the general method of X. Ji and R. Lebed [20]. For reader’s convenience we present a short overview of the latter method in App. B. The cross channel SO(3) partial wave (PW) expansion may be used within the advanced model building strategies such as the approach by D. Mueller et al. [21] based on the cross channel partial wave expansion of conformal moments of GPDs. Furthermore, this information is useful within the dual parametrization approach allowing to construct the double partial wave expansion of GPDs (in the conformal and SO(3)) partial waves. In Sec. 4 we consider the alternative method to construct the double partial wave expansion for quark and gluon helicity flip GPDs based on the explicit calculation of the cross channel spin-$J$ resonance exchange contributions. We present the explicit results for the case of $C = -1$ quark helicity flip GPDs. For the case of helicity flip GPDs these kinds of analysis was never presented in the literature, to the best of our knowledge.

2. PRELIMINARIES

Throughout this paper we adopt the set of conventions of Ref. [5]. The definition of quark helicity flip GPDs involves the nucleon matrix element of tensor light-cone operator

$$\hat{O}_T^{\alpha\beta}(-\lambda n/2, \lambda n/2) = \bar{\Psi}(-\lambda n/2) i\sigma^{\alpha\beta} \Psi(\lambda n/2)$$

contracted with the appropriate projector

$$n^\alpha g_\perp^{\beta i} \equiv n^\alpha (g^{\beta i} - n^i \bar{n}^\beta - n^\beta \bar{n}^i),$$

where $n$ and $\bar{n}$ are the light-cone vectors ($n^2 = \bar{n}^2 = 0$, $\bar{n} \cdot n = 1$) and the Latin index $i = 1, 2$ is reserved for the transverse spatial directions.

To the leading twist accuracy the form factor decomposition of the nucleon matrix

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2 We employ the light-cone gauge $A \cdot n = A^+ = 0$, so that the gauge link does not appear in the operator.
element of the operator (1) involves 4 invariant functions [6]:

\[
\frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ixP+\lambda} \langle N(p') | \Psi(-\lambda n/2) i\sigma^{+i} \Psi(\lambda n/2) | N(p) \rangle
\]

\[
= \frac{1}{2P^+} \tilde{U}(p') \left[ H_T^{q^+} P^+ - \tilde{H}_T^{q^+} \frac{P^+ + \Delta^+ P^i}{m^2} + E_T^{q^+} \frac{\gamma^+ P^i - P^+ \gamma^i}{2m} + \tilde{E}_T^{q^+} \frac{\gamma^+ P^i + P^+ \gamma^i}{m} \right] U(p),
\]

where \( m \) denotes the nucleon mass. Throughout this paper we employ the usual kinematical notations for the average momentum \( P \), \( t \)-channel momentum transfer \( \Delta \) and the skewness variable \( \xi \):

\[
P = \frac{1}{2} (p + p'); \quad \Delta = p' - p; \quad \xi = -\frac{(p' - p) \cdot n}{(p' + p) \cdot n} \equiv -\frac{\Delta^+}{2P^+}
\]

defined within the usual DVCS kinematics\(^3\). The convolution with the projecting operator (2) is implied in (3). Each of the four invariant functions \( H_T^q, \tilde{H}_T^q, E_T^q \) and \( \tilde{E}_T^q \) depend on the variable \( x, \) skewness \( \xi, \) momentum transfer squared \( \Delta^2 \equiv t, \) as well as on the factorization scale \( \mu. \) Due to hermiticity and time reversal invariance, the four invariant functions are real valued. Moreover, one may check [6] that \( H_T^q, \tilde{H}_T^q, E_T^q \) are even functions of \( \xi \) while \( \tilde{E}_T^q \) is an odd function of \( \xi. \)

For consistency we provide the relation of the parametrization (3) to that used by Z. Chen and X. Ji [19], which reads

\[
H_T^q|_{\text{eq. (3)}} = H_T^q|_{\text{ref. [19]}}; \quad E_T^q|_{\text{eq. (3)}} = -E_T^q|_{\text{ref. [19]}}; \quad \tilde{H}_T^q|_{\text{eq. (3)}} = -\frac{1}{2} \tilde{H}_T^q|_{\text{ref. [19]}}; \quad \tilde{E}_T^q|_{\text{eq. (3)}} = -\frac{1}{2} \tilde{E}_T^q|_{\text{ref. [19]}}.
\]

In the definition of quark helicity flip GPDs instead of the tensor current operator (1) one can use the pseudotensor current employing the relation [3]\(^4\):

\[
\sigma^{\alpha\beta} \gamma_5 = -\frac{i}{2} \epsilon^{\alpha\beta\gamma\delta} \sigma_{\gamma\delta}.
\]

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\(^3\) We refer to the usual DVCS kinematics \( N(p) + \gamma^*(q) \rightarrow N(p') + \gamma(q') \): \( q^2 \equiv Q^2 \rightarrow \infty \), \( p \cdot q \rightarrow \infty, \) with fixed \( x_{Bj} = \frac{Q^2}{2p_q} \) and small negative \( t. \)

\(^4\) Throughout this paper we use the conventions employed in [3], [5]: \( \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu], \gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \) and the Levi-Civita tensor defined with \( \epsilon_{0123} = +1. \) It worths mentioning that this definition of the Levi-Civita tensor differs from that employed e.g. in [22], where \( \epsilon_{0123} = +1. \)
Therefore, the relation between the tensor and pseudotensor currents reads
\[ \hat{O}^\alpha_\sigma(\bar{\lambda}n/2, \lambda n/2) \equiv \hat{\Psi}(\bar{\lambda}n/2)\sigma^\alpha_\gamma \gamma_5 \hat{\Psi}(\lambda n/2) = \frac{-i}{2} \varepsilon^{\alpha\beta\gamma\delta} \hat{O}^\beta_\delta(\lambda n/2, \bar{\lambda}n/2). \] (7)

As a result, the equivalent parametrization for the quark helicity flip GPDs from the Fourier transform of the pseudotensor operator reads
\[ \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ixP^+\lambda} \langle p' | \hat{\Psi}(\bar{\lambda}n/2)\sigma^+_i \gamma_5 \hat{\Psi}(\lambda n/2) | p \rangle = \frac{-1}{2} \varepsilon^{+i\gamma\rho} \int \frac{d\lambda}{2\pi} e^{ixP^+\lambda} \langle p' | \hat{\Psi}(\bar{\lambda}n/2)i\sigma_\rho \gamma_5 \hat{\Psi}(\lambda n/2) | p \rangle \]
\[ = \frac{-1}{2} \varepsilon^{+i\gamma\rho} \tilde{U}(p') \left[ H^T_2 \sigma^+_i \gamma_5 - \tilde{H}^T_2 \varepsilon^{+i\rho\Delta} \gamma_5 - E^T_\gamma \varepsilon^{+i\rho\gamma_5} \right] U(p), \] (8)
where in the last line we use the abbreviate notations for the contraction of the Levi-Civita tensor with four-vectors.

Let us now turn to gluon helicity flip GPD defined from the nucleon matrix element of the appropriate projection of the gluon light-cone operator:
\[ \hat{O}^\alpha_\rho(\bar{\lambda}n/2, \lambda n/2) = G^{\alpha_\rho}(\bar{\lambda}n/2)G^{\beta_\sigma}(\lambda n/2). \] (9)

The corresponding projection operation reads
\[ S G^{ij}(\bar{\lambda}n/2)\hat{G}^{ij}(\lambda n/2) \equiv \tau^{ij}_\rho \rho \sigma n^\alpha n^\beta G^{\alpha_\rho}(\bar{\lambda}n/2)G^{\beta_\sigma}(\lambda n/2), \] (10)
where the \( S \) symbol stands for the symmetrization in the two transverse spatial indices and removal of the corresponding trace. The explicit expression for the \( S \) operation has the form of the following projecting operator [12]:
\[ \tau^{ij}_\rho \rho \sigma = \frac{1}{2} \left( g_{ij} g_{\rho\sigma} + g_{j\rho} g_{i\sigma} - g_{i\rho} g_{j\sigma} \right), \] (11)
where \( g_{\rho\sigma} = g_{\rho\sigma} - n_{\rho} n_{\sigma} - n_{\sigma} n_{\rho} \).

The parametrization of the nucleon matrix element of the gluon helicity flip operator (10) to the leading twist accuracy involves four invariant functions [3]:
\[ \frac{1}{P^+} \int \frac{d\lambda}{2\pi} e^{ixP^+\lambda} \langle p' | S G^{ij}(\bar{\lambda}n/2)\hat{G}^{ij}(\lambda n/2) | p \rangle \]
\[ = S \frac{1}{2P^+} \left[ P^+ \Delta^j - \Delta^j P^i \right] \tilde{U}(p') \left[ H^T_2 \sigma^+i + \tilde{H}^T_2 \frac{P^+ \Delta^i - \Delta^j P^i}{m^2} + \tilde{E}^T_\gamma \frac{\gamma_5 \Delta^i - \Delta^j \gamma_5}{2m} \right] U(p), \] (12)
with $H_T^g$, $E_T^g$, $\tilde{H}_T^g$ and $\tilde{E}_T^g$ being functions of the usual GPD variables. Similarly to the quark case, from the combination of hermiticity and $T$-invariance they are real valued. $H_T^g$, $E_T^g$, $\tilde{H}_T^g$ and $\tilde{E}_T^g$ are even functions of $\xi$ while $\tilde{E}_T^g$ is an odd function of $\xi$. Moreover, from the $C$-invariance $H_T^g$, $E_T^g$, $\tilde{H}_T^g$ and $\tilde{E}_T^g$ are shown to be even functions of $x$.

Again let us specify the relation of the parametrization (12) to that used by Chen and Ji [19]:

$$H_T^g|_{\text{eq.(12)}} = -2xH_T^g|_{\text{ref.}[19]};$$
$$E_T^g|_{\text{eq.(12)}} = -2xE_T^g|_{\text{ref.}[19]};$$
$$\tilde{H}_T^g|_{\text{eq.(12)}} = -x\tilde{H}_T^g|_{\text{ref.}[19]};$$
$$\tilde{E}_T^g|_{\text{eq.(12)}} = -x\tilde{E}_T^g|_{\text{ref.}[19]}.$$ (13)

Note, that the pioneering papers [23], [12] overlooked the distributions $\tilde{H}_T^g$ and $\tilde{E}_T^g$ (see the discussion in [6] and [19]).

Similarly to the case of quark helicity flip GPDs (which possess two equivalent definitions from the nucleon matrix elements of tensor and pseudotensor quark operators) there is an equivalent definition of gluon helicity flip GPDs involving the dual gluon field strength. Indeed, the gluon non-local operator (9) has the following relation to the dual gluon non-local operator:

$$\hat{O}_T^g\gamma^\mu\gamma^\sigma(-\lambda n/2, \lambda n/2) = \tilde{G}^\mu(-\lambda n/2)G^{\gamma\sigma}(\lambda n/2) = \frac{1}{2}\varepsilon^{\alpha\rho\gamma\tau}G^{\gamma\tau}(-\lambda n/2)G^{\beta\sigma}(\lambda n/2).$$ (14)

This leads to the equivalent parametrization of gluon helicity flip GPDs:

$$\frac{1}{P^+} \int \frac{d\lambda}{2\pi} e^{i\lambda P^+\lambda} \langle p'| S \tilde{G}^+(-\lambda n/2)G^{\gamma+}(\lambda n/2)|p \rangle$$
$$= \frac{1}{2P^+} \frac{P^+ \Delta^+ - \Delta^+ P^i}{2mP^+} U(p') \left[ -H_T^g \varepsilon^{+i\gamma_5} + \tilde{H}_T^g \varepsilon^{+iP\gamma_5} \right] + E_T^g \varepsilon^{+i\gamma_5} + \tilde{E}_T^g \varepsilon^{+iP\gamma_5} \right] U(p).$$ (15)

The presence of the equivalent definition (15) just mirrors the fact that, exactly as the quark helicity flip operator, the gluon helicity flip operator (10) does not possess a definite $P$ parity. We review this issue in Appendix B in which the quantum number selection rules for the cross channel exchanges contributing to the Mellin moments of helicity flip GPDs are considered. Surprisingly, to the best of our knowledge, the definition (15) was never previously discussed in the literature. We take advantage of the existence of two equivalent parametrizations for both quark and gluon helicity flip GPDs in Sec. 4 when considering the contributions of the cross-channel spin-$J$ resonance exchanges of natural ($P = (-1)^J$) and unnatural ($P = (-1)^{J+1}$) parity.
3. SO(3) PARTIAL WAVE DECOMPOSITION OF QUARK AND GLUON GPDS WITH HELICITY FLIP

Building up the phenomenological Ansätze for GPDs in consistency with the fundamental theoretical requirements (such as the polynomiality, analyticity, positivity, Regge theory constraints etc.) is eagerly awaited by the present day phenomenology but represents a considerable theoretical challenge. Historically, the first successful parametrization of GPDs was based on the spectral representation of GPDs in terms of double distributions [24–27], which is the most straightforward way to implement the polynomiality property of GPDs. The alternative way to proceed relies on the expansion of GPDs over the convenient systems of orthogonal polynomials in order to achieve the factorization of functional dependencies of GPDs on their variables. As the first step for such expansion one usually employs the set of eigenfunctions of the leading order (LO) evolution equations which leads to the expansion of GPDs over the basis of the conformal partial waves [28].

One of the ways to proceed with the conformal partial wave expansion of GPDs is to further expand the conformal moments over a basis of suitable orthogonal polynomials carrying the labels of irreducible representations of the cross channel\(^5\) angular momentum SO(3) rotation group [21, 29, 30]. In this way one arrives to a double partial wave expansion of GPDs (both over the conformal basis and in the cross channel partial waves). Different methods were proposed in the literature to handle the double partial wave expansions of GPDs (for the discussion see e.g. [28]).

One of such methods is the framework of the so-called dual parametrization of GPDs [30, 31]. Within this approach the operator matrix elements defining GPDs are seen as infinite sums of the cross channel resonance exchanges of arbitrary high spin \(J\). The double partial wave expansion is first assigned meaning in the cross channel, where it rather represents generalized distribution amplitude (GDA). Then, exploiting the crossing symmetry, it is analytically continued to the direct channel allowing to work out a rigorous expression for GPDs. The term “dual” emphasizes the natural association with the old idea of duality in hadron-hadron low-energy scattering, that for binary scattering can roughly be summarized as the assumption that the infinite sum over only just the cross-

\(^5\) The term “cross channel” refers to the \(t\)-channel of the DVCS reaction: \(\gamma^*(q)+\gamma(-q') \rightarrow N(p')+\bar{N}(-p)\).
channel Regge exchanges may provide the complete description of the process within certain kinematical domain \[32\]. The Ansatz for the SO(3) partial wave amplitudes was also suggested within the Mellin-Barnes transform technique developed in \[21\]. Although it employs rather different mathematical tools, it should be in general equivalent to the dual parametrization approach.

However, even without any respect to a summation method employed to handle the double partial wave expansions, finding out the combinations of GPDs suitable for the \(t\)-channel SO(3) partial waves and the choice of the appropriate basis of the orthogonal polynomials represents an important task. For example, for the case of the unpolarized quark and gluon nucleon GPDs this kind of analysis gives rise to the so-called electric and magnetic combinations of GPDs \[3\]:

\[
H^E \{q,g\} = H \{q,g\} + \tau E \{q,g\}; \quad H^M \{q,g\} = H \{q,g\} + E \{q,g\},
\]

(16)

where

\[
\tau \equiv \frac{\Delta^2}{4m^2}.
\]

(17)

These combinations are to be expanded respectively in terms of \(P_J(\cos \theta)\) and \(P'_J(\cos \theta)\), where \(P_J(\chi)\) stand for the Legendre polynomials and \(\theta\) refers for the \(t\)-channel scattering angle in the \(NN\) center-of-mass frame. The resulting double partial wave expansion for the electric and magnetic combinations of unpolarized and polarized nucleon quark and gluon GPDs within the dual parametrization approach was presented in \[31\] and \[33\].

In this section we address the problem of pointing out the combinations of quark and gluon helicity flip GPDs suitable for the expansion in the \(t\)-channel SO(3) partial waves. We employ the method suggested in Sec. 4.2 of \[3\]. In order to identify the combinations of quark GPDs with helicity flip suitable for the partial wave expansion in the \(t\)-channel partial waves one has to consider the analytically continued to the cross channel form factor decomposition of \(N\)-th Mellin moments of the corresponding operator matrix elements. The analytically continued matrix Mellin moments are then computed in a specific reference frame (\(NN\) center-of-mass) for definite (usual) helicity of nucleons (here denoted as \(\lambda\) and \(\lambda'\)). This allows to specify the rotational functions \(d_{J\lambda,|\lambda-\lambda'|}^1\) governing the polar angle dependence. Also this methods provides a cross check of the
$J^{PC}$ quantum number selection rules worked out in Refs. [18, 19] by the method of X. Ji and R. Lebed [20] (see App. B for a review).

1. **SO(3) partial wave decomposition of quark helicity flip GPDs**

Following the receipt of Sec. 4.2 of Ref. [5], in order to identify the combinations of quark helicity flip GPDs suitable for the partial wave expansion in the $t$-channel partial waves we consider the form factor decomposition of the $N$-th Mellin moments (A1) of quark helicity flip GPDs analytically continued to the cross channel ($t > 0$). Thus we are dealing with the form factor decomposition of $N$-th Mellin moments of quark helicity flip $N\bar{N}$ GDAs.

We establish the following notations for the kinematical quantities (4) analytically continued to the cross channel:

\[
t \equiv \Delta^2 \rightarrow \tilde{s};
\]

\[
\Delta \equiv p'-p \rightarrow \tilde{P} \equiv p' + \tilde{p};
\]

\[
P = \frac{p' + p}{2} \rightarrow \frac{1}{2}\tilde{\Delta} \equiv \frac{p' - \tilde{p}}{2}.
\]  

(18)

The form factor decomposition of the $N$-th Mellin moment of the quark helicity flip $N\bar{N}$ GDA then reads:

\[
\mathcal{S}_{\{\nu_{\mu_1}, \ldots, \nu_{\mu_N}\}} \langle N(p', \lambda')\bar{N}(\tilde{p}, \lambda)|\bar{\Psi}(0)i\sigma^{\mu\nu}(\not{iD}_{\mu_1})\ldots(\not{iD}_{\mu_N})\Psi(0)|0 \rangle
\]

\[
= \mathcal{S}_{\{\nu_{\mu_1}, \ldots, \nu_{\mu_N}\}} \tilde{U}(p', \lambda') \left\{ \sum_{k=0}^{N} \left\{ i\sigma^{\mu\nu} \tilde{P}_{\mu_1} \ldots \tilde{P}_{\mu_k} \frac{1}{2} \tilde{\Delta}^{\mu_{k+1}} \ldots \frac{1}{2} \tilde{\Delta}^{\mu_N} A^q_{T,N+1,k}(\tilde{s}) \right. \right.
\]

\[
+ \frac{1}{2m^2} \tilde{\Delta}_{\mu} \tilde{P}^{\nu} - \frac{1}{2m^2} \tilde{\Delta}_{\nu} \tilde{P}^{\mu} \tilde{P}_{\mu_1} \ldots \tilde{P}_{\mu_k} \frac{1}{2} \tilde{\Delta}^{\mu_{k+1}} \ldots \frac{1}{2} \tilde{\Delta}^{\mu_N} \tilde{A}^q_{T,N+1,k}(\tilde{s})
\]

\[
+ \frac{\gamma_{\mu} \tilde{P}^{\nu} - \gamma_{\nu} \tilde{P}^{\mu}}{m^2} \tilde{P}_{\mu_1} \ldots \tilde{P}_{\mu_k} \frac{1}{2} \tilde{\Delta}^{\mu_{k+1}} \ldots \frac{1}{2} \tilde{\Delta}^{\mu_N} \tilde{B}^q_{T,N+1,k}(\tilde{s}) \right. \}
\]

\[
+ \sum_{k=0}^{N} \frac{\gamma_{\mu} \frac{1}{2} \tilde{\Delta}_{\nu} - \gamma_{\nu} \frac{1}{2} \tilde{\Delta}_{\mu}}{m^2} \tilde{P}_{\mu_1} \ldots \tilde{P}_{\mu_k} \frac{1}{2} \tilde{\Delta}^{\mu_{k+1}} \ldots \frac{1}{2} \tilde{\Delta}^{\mu_N} \tilde{B}^q_{T,N+1,k}(\tilde{s}) \right. \}
\]

\[
\left. \right\} V(\tilde{p}, \lambda),
\]  

(19)

where $U$ and $V$ are the usual Dirac spinors and $\lambda'$ ($\lambda$) denote the corresponding nucleon (antinucleon) helicity. The generalized form factors $A^q_{T,N+1,k}$, $\tilde{A}^q_{T,N+1,k}$, $\tilde{B}^q_{T,N+1,k}$ introduced in (A1) are analytically continued to the cross channel.
Now, in order to find which partial waves can contribute into the matrix elements \( (19) \), we compute the corresponding spin-tensor structures for spinors of definite (usual) helicity in the \( N\bar{N} \) center-off-mass (CMS) frame using the explicit expressions \( (C1) \) for the nucleon spinors with definite ordinary helicity.

For this issue we introduce the following parametrization of the relevant 3-vectors in the \( N\bar{N} \) CMS:

\[
\vec{p}' = \frac{\sqrt{s}}{2} \beta \{ \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \};
\]

\[
\vec{p} = \frac{\sqrt{s}}{2} \beta \{ \sin(\pi - \theta) \cos(\phi + \pi), \sin(\pi - \theta) \sin(\phi + \pi), \cos(\pi - \theta) \},
\]

(20)

where \( \theta \in [0, \pi] \) and \( \phi \in [0, 2\pi] \) are the usual polar and azimuthal angles and \( \beta \) denotes the relativistic velocity \( \beta = \sqrt{1 - \frac{4m^2}{s}} \). The four-vectors \( \tilde{P} \) and \( \tilde{\Delta} \) \( (18) \) in the \( N\bar{N} \) CMS then read:

\[
\tilde{P} \equiv p' + \tilde{p} = (\sqrt{s}, 0, 0, 0);
\]

\[
\tilde{\Delta} \equiv p' - \tilde{p} = \sqrt{s} \beta (0, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta).
\]

(21)

We are now about to compute the matrix elements for the \( N \)-th Mellin moments of \( N\bar{N} \) quark helicity flip GDA

\[
\langle N(p', \lambda')\bar{N}(\tilde{p}, \lambda)|\hat{O}_{q+i,++]^{+|0}
\]

(22)

in the \( \bar{N}N \) CMS for the cases when the helicities of nucleon and antinucleon couple to \( \lambda' - \lambda = 0 \) and \( |\lambda' - \lambda| = 1 \).

We project out the combination of \( (22) \) with definite helicity \( J_3 = \pm 1 \) of the corresponding operator:

\[
\langle N(p', \lambda')\bar{N}(\tilde{p}, \lambda)|\hat{O}_{q+i,++]^{+1,++]^{+|0}} + i \langle N(p', \lambda')\bar{N}(\tilde{p}, \lambda)|\hat{O}_{q+i,++]^{+2,++]^{+|0}}
\]

\[
\equiv \langle N(p', \lambda')\bar{N}(\tilde{p}, \lambda)|\hat{O}_{q+i,++]^{+(1\pm i2),++]^{+|0}}
\]

(23)

Then for \( \lambda = \lambda' \) i.e. the aligned configuration of nucleon and antinucleon helicities\(^6\)

\(^6\) Note, that throughout this section the hadron helicity labeling refers to the \( t \)-channel. Obviously, when crossing back to the direct channel the helicity \( \lambda \) is reversed.
\( (N^\uparrow \bar{N}^\uparrow \text{ or } N^\downarrow \bar{N}^\downarrow) \) we get

\[
\langle N(p', \lambda') \bar{N}(\bar{p}, \lambda) | \hat{O}^{q^+}_{T}^{+(1+i2), \ldots} | 0 \rangle \bigg|_{\lambda = \lambda'} = \eta_{N'N}(\bar{P}^+) N^{+1} \sin \theta \\
\times \sum_{k=0, \text{even}}^{N} \left[ (\beta - 1) A^q_{T,N+1,k}(\tilde{s}) + \beta^2 \frac{\tilde{s}}{2m^2} \bar{A}^q_{T,N+1,k}(\tilde{s}) - B^q_{T,N+1,k}(\tilde{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k}; \quad (24)
\]

and

\[
\langle N(p', \lambda') \bar{N}(\bar{p}, \lambda) | \hat{O}^{q^+}_{T}^{+(1-i2), \ldots} | 0 \rangle \bigg|_{\lambda = \lambda'} = \eta_{\lambda'N}(\bar{P}^+) N^{+1} \sin \theta \\
\times \sum_{k=0, \text{even}}^{N} \left[ - (\beta + 1) A^q_{T,N+1,k}(\tilde{s}) + \beta^2 \frac{\tilde{s}}{2m^2} \bar{A}^q_{T,N+1,k}(\tilde{s}) - B^q_{T,N+1,k}(\tilde{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k}. \quad (25)
\]

Note, that the combinations (23) possess definite phases depending on the azimuthal angle \( \phi \) denoted as \( \eta_{N'N}^{\pm} \). Now one can decompose (24) and (25) in the partial waves with total angular momentum \( J \). The \( \theta \) dependence is governed by the Wigner “small-\( d \)” rotation functions \( d_{J_3,|\lambda'-\lambda|} \). In this case we have \( |\lambda' - \lambda| = 0 \) and \( J_3 = \pm 1 \). Therefore, one has to use

\[
d_{J_3,0}^J(\theta) = (\pm 1) \frac{1}{\sqrt{J(J+1)}} \sin \theta P^J_0(\cos \theta). \quad (26)
\]

After the inverse crossing (18) back to the s-channel, within the DVCS kinematics \( \cos \theta \) up to higher twist corrections becomes

\[
\cos \theta \rightarrow \frac{1}{\xi \beta} + O(1/Q^2). \quad (27)
\]

At this stage we switch to massless hadrons so that we could consider hadron helicities as true quantum numbers thus making simple the crossing relation between the corresponding partial amplitudes (in particular excluding mixing). This implies setting \( \beta = 1 \) (which means systematically neglecting the threshold corrections \( \sim \sqrt{1 - \frac{4m^2}{t}} \)). However, up to the very end we keep the non-zero mass within the Dirac spinors. It would be fair to say that this step is somewhat cumbersome and requires further study. A possible solution to the problem of threshold singularities could be the appropriate resummation of the cross channel partial wave expansion in order to avoid the appearing of the kinematical singularities in the direct channel. Some attempts to follow this program were performed within the dual parametrization approach [34].
Putting aside the mentioned above problem, we conclude that the following combinations of quark helicity flip GPDs are to be expanded in $P_f(1/\xi)$:

\[
\begin{align*}
\tau \tilde{H}^q_T(x, \xi, \Delta^2) &- \frac{1}{2} E^q_T(x, \xi, \Delta^2); \\
-H^q_T(x, \xi, \Delta^2) + \tau \tilde{H}^q_T(x, \xi, \Delta^2) &- \frac{1}{2} E^q_T(x, \xi, \Delta^2).
\end{align*}
\]

(28)

Now we consider the case when $|\lambda' - \lambda| = 1$ (i.e. the opposite helicity configuration of nucleon and antinucleon: $N^\uparrow \bar{N}^\downarrow$ or $N^\downarrow \bar{N}^\uparrow$). For the operator helicity $J_3 = \pm 1$ configurations \((23)\) we get

\[
\begin{align*}
\langle N(p', \lambda') \bar{N}(\bar{p}, \lambda) | \hat{O}^q_{\{1+\text{i}2\}, +} \bar{O}^q_{\{1-\text{i}2\}, +} | 0 \rangle |_{|\lambda' - \lambda| = 1} &= \eta_{\lambda' \lambda}^+ (\bar{p}) N_{N+1}(1 + \cos \theta) \left\{ \sum_{k=0}^{N} \left[ \frac{2m}{\sqrt{s}} A^{q}_{N+1,k}(\bar{s}) + \frac{\sqrt{s}}{2m} B^{q}_{N+1,k}(\bar{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right. \\
&\left. - \sum_{k=0}^{N} \frac{\beta \sqrt{s}}{2m} B^{q}_{N+1,k}(\bar{s}) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right\} \\
&\quad \text{(29)}
\end{align*}
\]

and

\[
\begin{align*}
\langle N(p', \lambda') \bar{N}(\bar{p}, \lambda) | \hat{O}^q_{\{1+\text{i}2\}, +} \bar{O}^q_{\{1-\text{i}2\}, +} | 0 \rangle |_{|\lambda' - \lambda| = 1} &= \eta_{\lambda' \lambda}^- (\bar{p}) N_{N+1}(1 - \cos \theta) \left\{ \sum_{k=0}^{N} \left[ \frac{2m}{\sqrt{s}} A^{q}_{N+1,k}(\bar{s}) + \frac{\sqrt{s}}{2m} B^{q}_{N+1,k}(\bar{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right. \\
&\left. + \sum_{k=0}^{N} \frac{\beta \sqrt{s}}{2m} B^{q}_{N+1,k}(\bar{s}) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right\}, \\
&\quad \text{(30)}
\end{align*}
\]

where $\eta_{\lambda' \lambda}^\pm$ denote the azimuthal angle $\phi$ dependent phases.

The combinations \((29), (30)\) are to be expanded respectively in

\[
\begin{align*}
d_{1,1}^+(\theta) &= \frac{1}{J(J+1)} (1 + \cos \theta) \left[ P^q_f(\cos \theta) + \cos \theta P^q_f(\cos \theta) - P^q_f(\cos \theta) \right] \\
&\quad \text{(31)}
\end{align*}
\]

and

\[
\begin{align*}
d_{-1,1}^-(\theta) &= \frac{1}{J(J+1)} (1 - \cos \theta) \left[ P^q_f(\cos \theta) + \cos \theta P^q_f(\cos \theta) + P^q_f(\cos \theta) \right]. \\
&\quad \text{(32)}
\end{align*}
\]

Performing crossing to the direct channel we conclude that the combinations of quark helicity flip GPDs

\[
H^q_T(x, \xi, \Delta^2) + \tau \tilde{H}^q_T(x, \xi, \Delta^2) \pm \tau \tilde{E}^q_T(x, \xi, \Delta^2)
\]

(33)
are to be expanded in
\[ P'_j(1/\xi) + \frac{1 \mp \xi}{\xi} P''_j(1/\xi). \] (34)

Comparing (24), (25), (29), (30) with the parametrization for the \( N \)-th Mellin moments of quark helicity flip GPDs \([11]\) we work out the set of the selection rules for the \( J^{PC} \) quantum numbers for the \( t \)-channel resonance exchanges contributing into \( N \)-th Mellin moments of quark helicity flip GPDs. Theses selection rules coincide with those worked out with the method of X. Ji and R. Lebed reviewed in Appendix B.

As the example, let us consider the Mellin moments of combinations (28) of quark helicity flip GPDs. Note, that \( T \)-invariance constraints are implemented directly through the requirement that the Mellin moments of \( H^q_T \), \( \tilde{H}^q_T \) and \( E^q_T \) should contain only even powers of \( \xi \), while those of \( \tilde{E}^q_T \) involve only odd powers of \( \xi \).

For definiteness, let us consider the even Mellin moments (\( N = 0, 2, \ldots \)) of (28). Therefore, we are dealing with non-singlet (i.e. \( C = -1 \)) combinations\(^7\). Following our analysis, the matrix elements (24) and (25) are to be expanded respectively in \( d^j_{\pm1,0}(\theta) \).

From \( T \)-invariance requirements the summation over \( k \) in the r.h.s. of (24) and (25) goes only over even \( ks \). Therefore, \( N - k \) is also even. Now we work out the selection rule for \( J \): since \( d^j_{\pm1,0}(\theta) \sim \sin \theta P'_j(\cos \theta) \), only odd \( Js \) are consistent with the \( T \)-invariance. From the covariance condition the highest possible value of \( J \) for given \( N \) is \( J = N + 1 \).

There is no selection in parity \( P \), since the operator in question does not possess definite parity (see App. B). So both \( P = \pm 1 \) exchanges are possible. Thus, e.g. for \( N = 0 \) we recover \( J^{PC} = 1^{--} \) and \( J^{PC} = 1^{++} \) exchanges quoted in second lines of Tables [1] [1] in Appendix B while for \( N = 2 \) \( J^{PC} = 1^{--}, 3^{--} \) and \( J^{PC} = 1^{+-}, 3^{+-} \) are relevant.

The combinations (33) can be considered according to same pattern. However, to get the polynomials of definite parity properties in \( \cos \theta \) in order to work the selection rules in \( J \) one should consider the sum and difference of the two combinations in (33). This analysis completes the set of \( J^{PC} \) exchanges of Tables [1] [1] by including some of the contributions of unnatural parity meson that arise only for combinations involving GPD \( \tilde{E}^q_T \). One can also check that for given \( N \) the number of independent generalized

\(^7\) In particular, the non-singlet combination of \( H^q_T \) in the forward limit \( \xi = 0, \Delta^2 = 0 \) is reduced to \( H^q_T(x, 0, 0) = \delta q(x) - \bar{\delta}q(x) \), where \( \delta q(x) \) is the quark transversity distribution.
FFs (which is controlled by the power of the appropriate polynomials) coincides with that from the last columns of Tables [I, II].

2. SO(3) partial wave decomposition of gluon helicity flip GPDs

In the complete analogy with our previous analysis we now proceed with the SO(3) partial wave decomposition of gluon helicity flip GPDs. The cross channel analytic continuation of the form factor decomposition (A3) of N-th Mellin moments of gluon helicity flip GPDs is given by

\[
{\cal S}_{(\alpha \beta \mu_1...\mu_N)} \langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, \alpha \beta \mu_1...\mu_N} (0) | 0 \rangle = \sum_{k=0}^{N} \left\{ \frac{1}{2} \frac{\hat{A}^\delta \tilde{p}^\sigma}{2m} - \frac{1}{2} \Delta^\sigma \hat{p}^\beta \frac{\hat{p}^\mu_1 \ldots \hat{p}^\mu_k \frac{1}{2} \Delta^\mu_{k+1} \ldots \frac{1}{2} \Delta^N A_T^{g, \mu_1...\mu_N}(0)}{2m} \right\}
\]

We would like to compute the matrix elements of N-th Mellin moment of gluon helicity flip $N\bar{N}$ GDA

\[
\langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+1, j+2, ...+} | 0 \rangle
\]

in $N\bar{N}$ CMS. We find the following combinations with definite operator helicities $J_3$ suitable for PW expansion in the $t$-channel PW:

\[
\langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+1, j+2, ...+} | 0 \rangle - \langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+2, j+2, ...+} | 0 \rangle
\]

\[
2i \langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+1, j+2, ...+} | 0 \rangle = \langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+1, j+2, ...+} | 0 \rangle
\]

\[
-2i \langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+1, j+2, ...+} | 0 \rangle = \langle N(p', \lambda') \tilde N(\bar{p}, \lambda) | \hat{O}_T^{g, i+1, j+2, ...+} | 0 \rangle
\]

We find the following combinations with definite operator helicities $J_3$ suitable for PW expansion in the $t$-channel PW:
and

\[ \langle N(p', \lambda') \bar{N}(\tilde{p}, \lambda) | \hat{O}_T^{g+1, +1, + + \ldots +} | 0 \rangle + \langle N(p', \lambda') \bar{N}(\tilde{p}, \lambda) | \hat{O}_T^{g+2, +2, + + \ldots +} | 0 \rangle \]
\[ \equiv \langle N(p', \lambda') \bar{N}(\tilde{p}, \lambda) | \hat{O}_T^{g+(1+i2), + + \ldots +} | 0 \rangle. \]  

(39)

These combinations correspond to the operator helicities \( J_3 = +2, J_3 = -2 \) and \( J_3 = 0 \) respectively.

Then for \( \lambda = \lambda' \) (i.e. the aligned configuration of nucleon and antinucleon helicities: \( N^\uparrow \bar{N}^\uparrow \) or \( N^\downarrow \bar{N}^\downarrow \)) we get

\[ \langle N(p', \lambda') \bar{N}(\tilde{p}, \lambda) | \hat{O}_T^{g+(1-i2), + + \ldots +} | 0 \rangle \bigg|_{\lambda = \lambda'} \]
\[ = \eta_{\lambda\lambda'}^+(\hat{P}^+)^{N+2} \frac{\beta}{2\sqrt{1-\beta}} \sin^2 \theta \sum_{k=0}^{N} \left[ (1 - \beta) A_{T,N+1,k}^g(\bar{s}) \right] + 2 \left( 1 - \frac{\bar{s}}{4m^2} \right) \tilde{A}_{g,TN+1,k}(s) + B_{g,TN+1,k}(s) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \].  

(40)

\[ \langle N(p', \lambda') \bar{N}(\tilde{p}, \lambda) | \hat{O}_T^{g-(1-i2), + + \ldots +} | 0 \rangle \bigg|_{\lambda = \lambda'} \]
\[ = \eta_{\lambda\lambda'}^-(\hat{P}^+)^{N+2} \frac{\beta}{2\sqrt{1-\beta}} \sin^2 \theta \sum_{k=0}^{N} \left[ (1 + \beta) A_{T,N+1,k}^g(\bar{s}) \right] + 2 \left( 1 - \frac{\bar{s}}{4m^2} \right) \tilde{A}_{g,TN+1,k}(s) + B_{g,TN+1,k}(s) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \].  

(41)

The \( \theta \) dependence of the appropriate partial waves is given by

\[ d_{J \pm 2,0}^J(\theta) = \frac{1}{\sqrt{(J-1)J(J+1)(J+2)}} \sin^2 \theta P_j''(\cos \theta). \]  

(42)

Performing crossing to the direct channel (see discussion in Sec. 3.1) we conclude that the combinations

\( (1 - \tau) \tilde{H}_T^g + \frac{1}{2} E_T^g; \)
\( H_T^g + (1 - \tau) \tilde{H}_T^g + \frac{1}{2} E_T^g \)

(43)

are to be expanded in \( P_j''(1/\xi) \).
For the combination $\langle 39 \rangle$ we find

$$\langle N(p', \lambda') \bar{N} (\bar{p}, \lambda) | \hat{O}_T^{\rho + (1+i2), + (1-i2), + + + + |0} \rangle \big|_{\lambda=\lambda'}$$

$$= \eta^{+-}_{\lambda \lambda'} (\bar{p}^+) N^{+2} \frac{\beta}{4 \sqrt{1 - \beta}} \sin^2 \theta \sum_{k=0}^{N} \beta A_{T+1, k}^0 (\bar{s})$$

$$+ 2 \left( 1 - \frac{\bar{s}}{4m^2} \right) \tilde{A}_{T+1, k}^0 (\bar{s}) + \beta B_{T+1, k}^0 (\bar{s}) \right) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right). \quad (44)$$

It is to be expanded in

$$d^I_{0,0} (\theta) = P_J (\cos \theta). \quad (45)$$

Performing crossing to the direct channel we conclude that the combination

$$\langle \xi^2 - 1 \rangle \left( H_T^0 + 2 (1 - \tau) H_T^2 + E_T^0 \right) \quad (46)$$

is to be expanded in $\xi^2 P_J (1/\xi)$.

Now we repeat the analysis for $|\lambda' - \lambda| = 1 \ (i.e. \ the \ opposite \ helicity \ configuration \ of \ nucleon \ and \ antinucleon: \ N^\uparrow \bar{N}^\uparrow \ or \ N^\downarrow \bar{N}^\downarrow)$. We find out that

$$\langle N(p', \lambda') \bar{N} (\bar{p}, \lambda) | \hat{O}_T^{\rho + (1+i2), + (1-i2), + + + + |0} \rangle \big|_{|\lambda' - \lambda| = 1}$$

$$= \eta^{++}_{\lambda \lambda'} (\bar{p}^+) N^{+2} \frac{1}{2} \beta \sin \theta (1 + \cos \theta) \sum_{k=0}^{N} \left[ A_{T+1, i}^0 (\bar{s})$$

$$+ \frac{\bar{s}}{4m^2} B_{T+1, i}^0 (\bar{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} - \sum_{k=0}^{N} \beta \frac{\bar{s}}{4m^2} B_{T+1, k}^0 (\bar{s}) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right) \bigg); \quad (47)$$

$$\langle N(p', \lambda') \bar{N} (\bar{p}, \lambda) | \hat{O}_T^{\rho + (1+i2), + (1-i2), + + + + |0} \rangle \big|_{|\lambda' - \lambda| = 1}$$

$$= \eta^{--}_{\lambda \lambda'} (\bar{p}^+) N^{+2} \frac{1}{2} \beta \sin \theta (\cos \theta - 1) \sum_{k=0}^{N} \left[ A_{T+1, k}^0 (\bar{s})$$

$$+ \frac{\bar{s}}{4m^2} B_{T+1, k}^0 (\bar{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} + \sum_{k=0}^{N} \frac{\bar{s}}{4m^2} B_{T+1, k}^0 (\bar{s}) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right) \bigg) \quad (48)$$

are to be expanded respectively in

$$d^I_{2,1} (\theta)$$

$$= \sqrt{\frac{1}{(J - 1)(J + 2)} \frac{1}{J(J + 1)} \sin \theta (\cos \theta + 1) \left( 2 P_J'' (\cos \theta) + (\cos \theta - 1) P_J''' (\cos \theta) \right)} \quad (49)$$
and

\[
d_{2,1}^J(\theta) = \sqrt{\frac{1}{(J-1)(J+2)J(J+1)}} \sin \theta (\cos \theta - 1) (2P''_J(\cos \theta) + (\cos \theta + 1)P'''_J(\cos \theta))
\]

(50)

Crossing back to the direct channel we conclude that the combinations

\[
H_T^g + \tau E_T^g \pm \tau \xi \tilde{E}_T^g
\]

(51)

are to be expanded in \(2P''_J(1/\xi) + \frac{1+\xi}{\xi} P'''_J(1/\xi)\).

Finally, the combination

\[
\langle N(p', \lambda') \bar{N}(\bar{p}, \lambda)| \hat{O}^{(1+2),+ (1-2), \ldots +}|0\rangle |_{\lambda'-\lambda}=1
\]

\[
= \eta^{+ \pm}_{N'}(\bar{P}^+) \frac{1}{4} \sin \theta \left\{ \beta \cos \theta \sum_{k=0}^{N} \left[ A_{T,N+1,k}^g(\bar{s}) + \frac{\bar{s}}{4m^2} B_{T,N+1,k}^g(\bar{s}) \right] \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} - \beta^2 \sum_{k=0}^{N} \frac{\bar{s}}{4m^2} \tilde{B}_{T,N+1,i}^g(\bar{s}) \left( \frac{1}{2} \beta \cos \theta \right)^{N-k} \right\}
\]

(52)

is to be expanded in

\[
d_{0,1}^J(\theta) = \frac{1}{\sqrt{J(J+1)}} \sin \theta P_J^f(\cos \theta).
\]

(53)

After crossing back to the direct channel it has to do with the combination

\[
H_T^g + \tau E_T^g + \tau \xi \tilde{E}_T^g
\]

(54)

that is to be expanded in \(P_J^f(1/\xi)\).

Thus, we found out the combinations of gluon helicity flip GPDs suitable for the PW expansion in the cross-channel partial waves. Moreover, similarly to the quark case, from comparing the explicit expressions for the Mellin moments of gluon helicity GDA one may work out the \(J^{PC}\) selection rules for \(t\)-channel exchanges contributing to the Mellin moments of gluon helicity flip GPDs. The results are summarized in Tables III, IV of Appendix E. One may check these selection rules are consistent with those worked out

17
by the method \cite{20} (see Sec. 3.2). Let us emphasize that, due to the fact that gluon is its own antiparticle, gluon helicity flip GPDs are all $C$-even. In other words, gluon helicity flip GPDs are even functions of the variable $x$ \cite{8}. In our Tables III, IV we show the results only for even Mellin moments of gluon helicity flip GPDs.

4. ON THE CROSS CHANNEL MESON EXCHANGE CONTRIBUTIONS TO QUARK AND GLUON HELICITY FLIP GPDS

The alternative method to establish the set of quark and gluon helicity flip GPDs suitable for the partial wave expansion in the cross-channel partial waves consists in the explicit calculation of the cross channel spin-$J$ resonance contributions into corresponding GPD (see \cite{35}). The advantage of this method is that it is fully covariant and allows to determine the net resonance exchange contributions into scalar invariant functions $H^{q,g}_T$, $E^{q,g}_T$, $\tilde{H}^{q,g}_T$, $\tilde{E}^{q,g}_T$. In particular, the transition to the limit of massless hadrons can then be performed for the partial wave expansion of those scalar functions (and not for the operator matrix elements as in the analysis of Sec. 3). This method also allows to work out the explicit expressions for the double partial wave expansion of quark and gluon helicity flip GPDs within the dual parametrization approach. Therefore, we find it useful to present below a short overview of this approach.

\footnote{Therefore, we were a bit puzzled by the consideration of odd Mellin moments of gluon helicity flip GPDs in Ref. 19, that all should be zero.}
FIG. 1: Spin-$J$ resonance exchange contribution into the nucleon matrix element of light-cone operator $\hat{O}$. The upper blob denotes the appropriate distribution amplitude of spin-$J$ meson resonance. The lower blob denotes the minimal (on-shell) effective $R_JNN$ vertex.

The calculation is based on the assumption of the meson pole dominance for the matrix element of the light-cone operator in question. This can be seen as the generalization of the old idea of vector meson dominance for nucleon electromagnetic form factor. In principle, this picture can be justified in the large-$N_c$ limit of QCD in which meson resonances become infinitely narrow. What is more important for us is that, by assuming the most general form of nucleon-meson interaction consistent with a given set of symmetries, we populate the invariant form factors at all possible leading twist spin-tensor structures and recover the usual set of selection rules for the quantum numbers of the cross channel resonance exchanges. Within this approach the nucleon matrix element of the light-cone operator $\hat{O}$ is presented as the infinite sum of contributions depicted on Fig. 1. Symbolically it can be written in the following form:

$$
\langle N(p')| \hat{O} |N(p)\rangle 
\sim \sum_{R_J \text{ polarizations of } R_J} \sum_{\Delta} \frac{1}{\Delta^2 - M_{R_J}^2} \times \frac{\langle N(p') R_J(\Delta) |N(p)\rangle}{V_{R_J NN \text{ effective vertex}}} \otimes \langle 0| \hat{O} | R_J(\Delta) \rangle \ , (55)
$$

where $M_{R_J}$ stand for the resonance masses and $\otimes$ denotes the convolution in the appropriate Lorentz indices. Since we are interested just in the pole contributions to the matrix elements it suffices to use the so-called minimal forms (those which do not change their form when all particles are put on mass shell) of $R_JNN$ effective vertices (see Ref. 19–38).
for the detailed discussion). The resulting on-shell polarization sums for spin-$J$ resonances can be performed with the contracted projectors method (see e.g. Chapter I of [39]).

Now we discuss the application of the approach for the case of quark and gluon helicity flip GPDs, which represents the new result. The effective minimal $R_j\tilde{N}N$ vertices for natural parity ($P = (-1)^J$) mesons can be chosen in the following form [34]:

$$V_{R_j NN} = \tilde{U}(p') \left\{ \frac{g_{1R_j}^R}{M_{R_j}} \gamma^{\mu_1} P^{\mu_2} \ldots P^{\mu_J} + \frac{g_{2R_j}^R}{M_{R_j}} P^{\mu_1} \ldots P^{\mu_J} \right\} U(p) \mathcal{E}^{*}_{\mu_1 \ldots \mu_J}(\Delta, j),$$

(56)

where $g_{1R_j}^R$ and $g_{2R_j}^R$ are dimensionless coupling constants and $\mathcal{E}^{*}_{\mu_1 \ldots \mu_J}(\Delta, j)$ stand for the symmetric traceless tensors describing spin-$J$ resonance polarization vector.

To keep with the requirements of $T$-invariance the effective minimal vertices for $\tilde{R}_j^-NN$ for the unnatural parity ($P = (-1)^{J+1}$) mesons with $P \cdot C = -1$ (i.e. $J^{PC} = 1^-, 2^-, 3-, \ldots$) are chosen in a way that

$$V_{\tilde{R}_j^- NN} = \frac{g_{\tilde{R}_j}^R}{M_{\tilde{R}_j}} \tilde{U}(p') i \sigma^{\mu \nu} \gamma_5 U(p) P^{\mu_1} \ldots P^{\mu_{J-1}} \Delta^\nu \mathcal{E}^{*}_{\mu_1 \ldots \mu_{J-1}}(\Delta, j).$$

(57)

where $g_{\tilde{R}_j}^R$ are dimensionless.

Finally, the effective minimal vertices for $\tilde{R}_j^+ NN$ for the unnatural parity ($P = (-1)^{J+1}$) mesons with $P \cdot C = 1$ (i.e. with $J^{PC} = 1^+, 2^+, 3^+, \ldots$) are chosen as

$$V_{\tilde{R}_j^+ NN} = \frac{g_{\tilde{R}_j}^R}{M_{\tilde{R}_j}} \tilde{U}(p') i \sigma^{\mu \nu} \gamma_5 U(p) P^{\mu_1} \ldots P^{\mu_{J-1}} P^\nu \mathcal{E}^{*}_{\mu_1 \ldots \mu_{J-1}}(\Delta, j).$$

(58)

Other necessary ingredients for the calculation of (55) are the relevant leading twist DAs of mesons. The corresponding parametrization for arbitrary high $J$ can be constructed analogously to vector meson (see e.g. [40]) and tensor meson [11] cases.

Namely, the quark helicity flip DA for the natural parity mesons $J^{PC} = 1^-, 2^+, 3^-, 4^+ \ldots$ will read as

$$\langle 0 | \bar{\Psi}(\lambda n/2) i \sigma^{\mu \nu} \Psi(\lambda n/2) | R_j(\Delta, j) \rangle = n^a g_{\pm}^{i \beta} \otimes f_{T R_j}^q \frac{M_{J-1}}{(\Delta \cdot n)^{J-1}}$$

$$\times n^{\nu_1} \ldots n^{\nu_{J-1}} \left( \mathcal{E}_{\alpha \nu_1 \ldots \nu_{J-1}}(\Delta, j) \mathcal{E}_{\beta \nu_1 \ldots \nu_{J-1}}^{\star}(\Delta, j) \mathcal{E}_{\Delta \alpha} \right) \int_{-1}^{1} dy e^{i y \lambda \cdot a} \Phi_{T R_j}^q(y).$$

(59)

The distribution amplitudes $\Phi_{T R_j}^q(y)$ can be expanded over the basis of Gegenbauer polynomials $C_n^{3/2}(y)$ as

$$\Phi_{T R_j}^q(y) = \langle 0 | \bar{\Psi}(\lambda n/2) \Gamma(J + \frac{1}{2}) \sum_{k=J-1}^{\infty} (a_{T R_j}^q)_{k} C_k^{3/2}(y) \rangle$$

(60)
The expansion (60) runs over even \( k \) for \( C = -1 \) mesons and over odd \( k \) for \( C = +1 \) mesons. The normalization of the DA (60) is chosen such that
\[
\int_{-1}^{1} dy y^{J-1} \Phi_{TR}^q(y) = 1.
\] (61)

The normalization constant \( f_{TR}^q \) has the dimension of energy.

For the case of the unnatural parity mesons \( J^{PC} = 1^\pm, 2^\pm, 3^\pm, 4^\pm, \ldots \) the quark helicity flip DAs take the form
\[
\langle 0 | \bar{\Psi}(-\lambda n/2)^{\sigma+i\gamma_5} \Psi(\lambda n/2) | \bar{R}_J(\Delta, j) \rangle = n^{\alpha}_R g_{\perp}^{i\beta} \otimes i f_{TR}^q \frac{M^{J-1}}{(\Delta \cdot n)^{J-1}}
\times n^{\nu_1} ... n^{\nu_{J-1}} \left( \mathcal{E}_{\alpha\nu_1 ... \nu_{J-1}}(\Delta, j) \mathcal{E}_{\beta\nu_1 ... \nu_{J-1}}(\Delta, j) \right) \int_{-1}^{1} dy e^{iy\cdot\Delta_{\perp}} \Phi_{TR}^q(y).
\] (62)

The Gegenbauer expansion for \( \Phi_{TR}^q(y) \) is the same as (60). Again the sum runs over even \( k \) for \( C = -1 \) mesons and over odd \( k \) for \( C = +1 \) mesons. The normalization of \( \Phi_{TR}^q(y) \) is the same as (61) with \( f_{TR}^q \) also having the dimension of energy.

For the case of the gluon helicity flip operator the relevant DAs for the natural parity even spin mesons \( (J^{PC} = 2^{++}, 4^{++}, \ldots) \) read
\[
\langle 0 | S G^{+i}(-\lambda n/2) G^{+j}(\lambda n/2) | R_J(\Delta, j) \rangle
= n^{\alpha}_R n^{\beta}_{T_{ij};\rho\sigma} \otimes f_{TR}^q M_R^{J-2} \left\{ \frac{1}{4} \Delta^\alpha \Delta^\beta \mathcal{E}^{\rho\sigma\nu_1 ... \nu_{J-2}}(\Delta, j) - \frac{1}{4} \Delta^\rho \Delta^\sigma \mathcal{E}^{\alpha\nu_1 ... \nu_{J-2}}(\Delta, j) \right\}
- \frac{1}{4} \Delta^\rho \Delta^\sigma \mathcal{E}^{\alpha\nu_1 ... \nu_{J-2}}(\Delta, j) + \frac{1}{4} \Delta^\rho \Delta^\sigma \mathcal{E}^{\alpha\nu_1 ... \nu_{J-2}}(\Delta, j) \right\} \int_{-1}^{1} dy e^{iy\cdot\Delta_{\perp}} \Phi_{TR}^q(y),
\] (63)

where the DA \( \Phi_{TR}^q(y) \) is in general expanded over the set of Gegenbauer polynomials \( C_k^{5/2}(y) \):
\[
\Phi_{TR}^q(y) = \frac{3}{2^J J (J + \frac{3}{2}) \Gamma(J + 3)} (1 - y^2)^2 \sum_{k=J-2}^{\infty} (a_{TR}^q)_k C_k^{5/2}(y) \text{ with } (a_{TR}^q)_J = 1.
\] (64)

It is normalized so that
\[
\int_{-1}^{1} dy y^{J-2} \Phi_{TR}^q(y) = 1
\] (65)
and the normalization constant $f^g_{TRJ}$ has the dimension of energy. Note, that the sum in (64) runs over even $k$ since we deal with the $C$-even quantity, because the gluon is its own antiparticle.

Finally, for unnatural parity mesons ($J^{PC} = 2^{-+}, 3^{++}, 4^{-+} \ldots$) the relevant DAs read

$$
\langle 0 | S \tilde{G}^{ij}(\lambda n/2)G^{j+}(\lambda n/2) | R_J(\Delta, j) \rangle = n^{\alpha} n^{\beta} T_{ij; \rho\sigma} \otimes i f^g_{TRJ} M^{J-2}_{TRJ} \left\{ \frac{1}{4} \Delta^{\alpha} \Delta^{\beta} \mathcal{E}^{\rho\sigma\nu_1 \ldots \nu_{J-2}}(\Delta, j) - \frac{1}{4} \Delta^{\rho} \Delta^{\sigma} \mathcal{E}^{\alpha\beta\nu_1 \ldots \nu_{J-2}}(\Delta, j) \right\} n^{\nu_1} \ldots n^{\nu_{J-2}} \left( \frac{2}{(\Delta \cdot n)} \right)^{J-2} \times \int_{-1}^{1} dy e^{iy \Delta \cdot n} \Phi^g_{TRJ}(y),
$$

where for even $J$ the DA $\Phi^g_{TRJ}(y)$ has the same Gegenbauer expansion as in (64). For odd $J$ the Gegenbauer expansion reads

$$
\Phi^g_{TRJ}(y) = \frac{3}{2} 2^{J} \frac{\Gamma(J + \frac{3}{2})}{\Gamma(\frac{3}{2}) \Gamma(J + 3)} (1 - y^2)^2 \sum_{k = J-3}^{\infty} (a^g_{TRJ})_k C^{J-2}_k(y) \text{ with } (a^g_{TRJ})_{J-3} = 1.
$$

It is normalized so that

$$
\int_{-1}^{1} dy y^{J-3} \Phi^g_{TRJ}(y) = 1.
$$

Using the set of effective vertices (56), (57), (58) and the parton helicity flip DAs (59), (62), (63), (66) after a straightforward (though tedious and lengthy) calculation one recovers the same combinations of quark and gluon helicity flip GPDs suitable for the partial wave expansion in the cross channel partial waves as derived in Sec. 3 and may check the selection rules for the $J^{PC}$ quantum numbers.

Moreover, one can work out the double partial wave expansion (both in conformal partial waves and in cross channel SO(3) partial waves\textsuperscript{9}) representing quark and gluon helicity flip GPDs within the dual parametrization approach. As an example we present below the formal series for the non-singlet ($C = -1$) combinations of quark helicity flip GPDs within the dual parametrization approach. For simplicity we quote the results in

\textsuperscript{9} Below we relabel the cross channel angular momentum as $l$ to match the notations of Ref. [30].
the $\beta \to 1$ limit.

\begin{align}
\tau H_T^q (x, \xi, \Delta^2) - \frac{1}{2} E_T^{q-} (x, \xi, \Delta^2) \\
= \sum_{k=0}^{\infty} \sum_{\substack{l=1 \text{ even} \ \text{or} \ \text{odd}}} B_{kl}^{(\overline{\tau}H_T^q - \frac{1}{2} E_T^{q-})} (\Delta^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_k^{3/2} \left( \frac{x}{\xi} \right) \frac{1}{|\xi|} P_l' \left( \frac{1}{\xi} \right) \\
- H_T^q (x, \xi, \Delta^2) + \tau \overline{H}_T^q (x, \xi, \Delta^2) - \frac{1}{2} E_T^{q-} (x, \xi, \Delta^2) \\
= \sum_{k=0}^{\infty} \sum_{\substack{l=1 \text{ even} \ \text{or} \ \text{odd}}} B_{kl}^{(-H_T^q + \tau \overline{H}_T^q - \frac{1}{2} E_T^{q-})} (\Delta^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_k^{3/2} \left( \frac{x}{\xi} \right) \frac{1}{|\xi|} P_l' \left( \frac{1}{\xi} \right) \\
H_T^q (x, \xi, \Delta^2) + \tau \overline{H}_T^q (x, \xi, \Delta^2) \pm \tau \overline{E}_T^{q-} (x, \xi, \Delta^2) \\
= \sum_{k=0}^{\infty} \sum_{\substack{l=1 \text{ even} \ \text{or} \ \text{odd}}} B_{kl}^{(H_T^q + \tau \overline{H}_T^q \pm \tau \overline{E}_T^{q-})} (\Delta^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_k^{3/2} \left( \frac{x}{\xi} \right) \frac{1}{|\xi|} P_l' \left( \frac{1}{\xi} \right) \\
\times \left( P_l' \left( \frac{1}{\xi} \right) + \frac{1}{\xi} P_l'' \left( \frac{1}{\xi} \right) \right). \tag{69}
\end{align}

Here we introduce 4 sets of generalized form factors $B_{kl}(\Delta^2)$ for the 4 combinations in question. Note, that the sum in $l$ for the combinations $H_T^q + \tau \overline{H}_T^q \pm \tau \overline{E}_T^{q-}$ runs over both odd and even $l$. This is consistent with the selection rules summarized in Table II. As a result both even and odd power of $\xi$ will appear when computing even Mellin moments of these combinations from the partial wave expansion (69). This is obviously consistent with $T$-invariance since these combinations explicitly contain the GPD $\overline{E}_T^{q-}$ that produces odd powers of $\xi$ for the Mellin moments (see Appendix A).

The formal series (69) can be handled by means of standard methods developed within the dual parametrization approach [30, 31, 33]. The detailed consideration of these series however lies beyond the scope of this paper and will be considered elsewhere. The gluon case can be considered exactly the same pattern. However, the resulting expressions are bulky and we do not present them explicitly in the present publication.

5. CONCLUSIONS

Much work remains to be done before the helicity flip sector of quark and gluon generalized parton distributions in the nucleon is understood. In this work, we established their crossed channel properties, which are of direct importance for a theoretically consistent model building in the spirit of the double partial wave expansion of GPDs. We
did not address the phenomenological side of this study. Some experimental hints already point to the observability of transversity quark and gluon GPDs. The \((3\phi)\) modulation of the interference contribution to the unpolarized beam-longitudinally polarized target asymmetry seen in the HERMES data \(^{12}\) for deeply virtual Compton scattering on a nucleon call for a significant gluon transversity contribution. The recent transverse target spin asymmetries measured by COMPASS in vector meson exclusive leptoproduction \(^{13}\) have been interpreted \(^{14}\) as a signal for transversity quark contributions. We shall cope with the phenomenology of quark and gluon transversity GPDs in a future publication.

**Acknowledgements**

We are grateful to A. Belitsky, V. Braun, M. Diehl, D. Mueller and M. Vanderhaeghen for very helpful discussions and correspondence. This work is partly supported by the Polish Grant NCN No DEC-2011/01/B/ST2/03915, the Joint Research Activity Study of Strongly Interacting Matter (acronym HadronPhysics3, Grant 283286) under the Seventh Framework Programme of the European Community, by the COPIN-IN2P3 Agreement, by the French grant ANR PARTONS (ANR-12-MONU-0008-01) and by the Tournesol 2014 Wallonia-Brussels-France Cooperation Programme.
A. FORM FACTOR DECOMPOSITION OF THE NUCLEON MATRIX ELEMENTS OF HELICITY FLIP OPERATORS

1. Quark helicity flip operators

The form factor decomposition of the $N$-th Mellin moment of non-local tensor operator (1) is given by eq. (22) of Ref. [18]:

\[
S\{\nu\mu_1...\mu_N\} \langle p'\bar{\Psi}(0)i\sigma^{\mu\nu}(i\not{D}_{\mu_1})...(i\not{D}_{\mu_N})\Psi(0)|p\rangle = S\{\nu\mu_1...\mu_N\} \bar{U}'(p') \left[ \sum_{k=0}^{N} \left\{ i\sigma^{\mu\nu} \Delta^{\mu_1} \ldots \Delta^{\mu_k} p^{\mu_{k+1}} \ldots p^{\mu_N} A^q_{T_{N+1},k}(\Delta^2) \right. \right.
\]
\[
+ \frac{P^\mu \Delta^\nu - P^\nu \Delta^\mu}{m^2} \Delta^{\mu_1} \ldots \Delta^{\mu_k} p^{\mu_{k+1}} \ldots p^{\mu_N} \bar{A}^q_{T_{N+1},k}(\Delta^2) \]
\[
+ \frac{\gamma^\mu \Delta^\nu - \gamma^\nu \Delta^\mu}{m^2} \Delta^{\mu_1} \ldots \Delta^{\mu_k} p^{\mu_{k+1}} \ldots p^{\mu_N} B^q_{T_{N+1},k}(\Delta^2) \right) \]
\[
+ \sum_{k=0}^{N} \frac{\gamma^\mu p^\nu - \gamma^\nu p^\mu}{m^2} \Delta^{\mu_1} \ldots \Delta^{\mu_k} p^{\mu_{k+1}} \ldots p^{\mu_N} \bar{B}^q_{T_{N+1},k}(\Delta^2) \right\} U(p), \quad (A1)
\]

where $S\{\nu\mu_1...\mu_N\}$ stands for the symmetrization and subsequent trace subtraction in the corresponding Lorentz indices. The polynomiality relation for quark helicity flip GPDs then reads:

\[
\int_{-1}^{1} dx x^N H^q_T(x, \xi, \Delta^2) = \sum_{k=0}^{N} (-2\xi)^k A^q_{T_{N+1},k}(\Delta^2);
\]
\[
\int_{-1}^{1} dx x^N E^q_T(x, \xi, \Delta^2) = \sum_{k=0}^{N} (-2\xi)^k B^q_{T_{N+1},k}(\Delta^2);
\]
\[
\int_{-1}^{1} dx x^N \bar{H}^q_T(x, \xi, \Delta^2) = \sum_{k=0}^{N} (-2\xi)^k \bar{A}^q_{T_{N+1},k}(\Delta^2);
\]
\[
\int_{-1}^{1} dx x^N \bar{E}^q_T(x, \xi, \Delta^2) = \sum_{k=0}^{N} (-2\xi)^k \bar{B}^q_{T_{N+1},k}(\Delta^2). \quad (A2)
\]

Note, that in accordance with the requirements of the $T$-invariance the $N = 0$ Mellin moment of GPD $\bar{E}^q_T$ vanishes. This fact is consistent with the counting of number of independent form factors for the $N = 0$ Mellin moment of quark helicity flip GPDs (see Tables II, III).
2. Gluon helicity flip operators

The form factor decomposition of the $N$-th ($N$-even) Mellin moment of non-local gluon tensor operator (9) is given by (c.f. eq. (10) of Ref. [19]):

$$S_{\{\alpha\beta\mu_1...\mu_N\}} \langle p'| G^{\alpha\rho}(0) (\overleftrightarrow{D}_{\mu_1})... (\overleftrightarrow{D}_{\mu_N}) G^{\beta\sigma}(0) | p \rangle$$

$$= S_{\{\alpha\beta\mu_1...\mu_N\}} \bar{U}(p') \left[ \sum_{k=0 \text{ even}}^{N} \left\{ \frac{P^\beta \Delta^\sigma - P^\sigma \Delta^\beta}{2m} i\sigma^{\alpha\rho} \Delta^{\mu_1} \ldots \Delta^{\mu_k} P^{\mu_{k+1}} \ldots P^{\mu_N} A^q_{T,N+1,k}(\Delta^2) ight. ight.$$

$$+ \frac{P^\beta \Delta^\sigma - P^\sigma \Delta^\beta}{2m} \frac{P^\rho \Delta^\alpha}{m^2} \Delta^{\mu_1} \ldots \Delta^{\mu_k} P^{\mu_{k+1}} \ldots P^{\mu_N} \bar{A}^q_{T,N+1,k}(\Delta^2) \left. \right\}$$

$$+ \frac{P^\beta \Delta^\sigma - P^\sigma \Delta^\beta}{2m} \frac{\gamma^\alpha P^\rho - \gamma^\rho P^\alpha}{m^2} \Delta^{\mu_1} \ldots \Delta^{\mu_k} P^{\mu_{k+1}} \ldots P^{\mu_N} \bar{B}^q_{T,N+1,k}(\Delta^2) \right\} \bar{U}(p).$$

(A3)

Applying the projector $\tau_{ij;\rho\sigma}$ and taking the $+$-components in the indices $\alpha$, $\beta$, $\mu_1, \ldots, \mu_N$ we get for even $N$

$$\int_{-1}^{1} dx x^N H^q_T(x, \xi, t) = \sum_{k=0 \text{ even}}^{N} A^q_{T,N+k,1}(\Delta^2) \xi^k;$$

$$\int_{-1}^{1} dx x^N \tilde{H}^q_T(x, \xi, t) = \sum_{k=0 \text{ even}}^{N} \bar{A}^q_{T,N+1,k}(\Delta^2) \xi^i;$$

$$\int_{-1}^{1} dx x^N E^q_T(x, \xi, t) = \sum_{k=0 \text{ even}}^{N} B^q_{T,N+1,k}(\Delta^2) \xi^k;$$

$$\int_{-1}^{1} dx x^N \tilde{E}^q_T(x, \xi, t) = \sum_{k=0 \text{ odd}}^{N} \bar{B}^q_{T,N+1,k}(\Delta^2) \xi^k. \quad \text{(A4)}$$

Again in accordance with the requirements of the $T$-invariance the $N = 0$ Mellin moment of GPD $\tilde{E}^q_T$ vanishes.
B. FORM FACTOR DECOMPOSITION OF TWIST-2 QUARK AND GLUON HELICITY FLIP OPERATORS AND QUANTUM NUMBER SELECTION RULES

Polynomiality property of the Mellin moments of hadronic matrix elements of bilocal twist-2 QCD operators is the direct manifestation of the Lorentz symmetry of the underlying fundamental quantum field theory. The Mellin moments of the hadronic matrix elements of bilocal twist-2 QCD operators give rise to towers of local twist-2 operators. These towers of local operators are parameterized in terms of generalized form factors. The parametrization in question should also properly take into account the consequences of the discrete $P$ and $T$ symmetries, which may reduce the number of independent generalized form factors. Therefore, establishing the correct number of independent form factors with proper implementation of constraints following from the discrete $P$ and $T$ symmetries provides the crucial consistency check. Moreover, this kind of analysis allows one to work out the set of selection rules for $J^{PC}$ quantum numbers of the $t$-channel exchange resonances contributing into the Mellin moments of twist-2 quark and gluon helicity flip GPDs.

The extensive analysis of the tensorial properties and form factor decomposition of the leading twist quark and gluon helicity flip operators was carried in Refs. [17, 18, 20] employing the general method elaborated in [20]. The key idea of the method [20] allowing to implement the constraints from $P$ and $T$ invariance is to switch to the cross channel and match the $J^{PC}$ quantum numbers of the corresponding tower of local operators to those of the nucleon-antinucleon state $\langle N\bar{N}\rangle$. The structure of the form factor expansion and the number of the independent generalized form factors is the same in all channels related by crossing transformation. Therefore, due to the $CPT$ invariance, this kind of $J^{PC}$ matching in the cross channel automatically ensures the $T$ invariance and the correct counting of the independent generalized form factors of the operator matrix element in the direct channel.

In this section we summarize the findings of Refs. [18, 20] and write down the system of $J^{PC}$ selection rules for the $t$-channel exchanges contributing into the Mellin moments of leading of twist quark and gluon helicity flip GPDs. The explicit expressions for
the parametrization of the nucleon matrix elements of the relevant towers of quark and
gluon helicity flip operators rewritten in terms of the notations consistent with the set of
conventions of Sec. 2 are for convenience presented in the Appendix A.

1. Quantum number selection rules: quark case

For the case of quark helicity flip GPDs one has to consider the tower of local operators
\[
\hat{O}^q_{\mu\nu, \mu_1...\mu_N} = S_{\{\nu\mu_1...\mu_N\}} \bar{\Psi}(0)i\sigma^{\mu\nu}(i\vec{D}_{\mu_1})...(i\vec{D}_{\mu_N})\Psi(0).
\] (B1)

As pointed out in [18], the tower of local operators (B1) transforms according to
\[\left(\frac{N+2}{2}, \frac{N}{2}\right) \oplus \left(\frac{N}{2}, \frac{N+2}{2}\right)\] representation of the Lorentz group. Its charge conjugation parity is given by
\[C = (-1)^{N+1}.\]
On the contrary, P parity is not uniquely defined. E.g. consider \[t^{ij} = \bar{\Psi}i\sigma^{ij}\Psi.\] One can check that \[t^i_{0i}\] and \[t^i_{ik}\] (where as usual the Latin indices refer to the spatial directions) have different parity, \[P = -1\] and \[P = +1\] respectively. In particular, this means that \[t^i_i\] has no definite P parity. Therefore, contrary to the case of vector and pseudovector towers of local operators (which have definite P parity), both the natural \((P = (-1)^J)\) and unnatural \((P = (-1)^{J+1})\) parity \(t\)-channel exchanges may contribute to the generalized form factors occurring in the parametrization (A1) of the nucleon matrix element of (B1). This leads to two sequences of possible quantum numbers of resonances [18, 20]:

- Of natural parity \(P = (-1)^J\):
  \[J^{PC} = J^{(-)N+1}(-)^{N+1} = 1^{-(N+1)}, 2^{(-)N+1}, 3^{(-)N+1}, \ldots (N+1)^{(-)N+1}(-)^{N+1}.\] (B2)

- Of unnatural parity \(P = (-1)^{J+1}\):
  \[J^{PC} = J^{(-)N+1(-)^{N+1}} = 1^{+(-)^{N+1}}, 2^{(-)N+1}, 3^{(-)N+1}, \ldots (N+1)^{(-)N+1(-)^{N+1}}.\] (B3)

Now, following the analysis of [20], we list below the possible \(\langle N\bar{N}\rangle\) states with the total angular momentum \(J\). Let \(S\) be the spin \((S = 0, 1)\) and \(L\) be the orbital angular momentum of the \(N\bar{N}\) system: \(J = L + S\). The \(P\) parity of \(\langle N\bar{N}\rangle\) state is given by \(P = (-1)^{L+1}\); The \(C\) parity is given by \(C = (-1)^{L+S}\). The list of allowed \(J^{PC}_L\) of \(\langle N\bar{N}\rangle\)
states then reads \cite{20}:

\[
\begin{align*}
0^{++}, & \quad 0^{-+}, \\
1^{++}, & \quad 1^{+-}, \quad 1^{-}, \quad 1^{--}; \\
2^{++}, & \quad 2^{+-}, \quad 2^{-+}, \quad 2^{--}; \\
3^{++}, & \quad 3^{+-}, \quad 3^{--}, \quad 3^{--}; \\
\ldots
\end{align*}
\]

(B4)

Matching \cite{B2}, \cite{B3} with the set of the allowed \(N\bar{N}\) states (B4) gives \cite{18,20} the two sets of the \(t\)-channel spin \(J\) exchanges contributing into the Mellin moments of quark helicity flip GPDs. They are summarized in Tables I, II.

| \(N\backslash J\) | 1  | 2  | 3  | 4  | \ldots | Number of FFs |
|-----------------|----|----|----|----|---------|---------------|
| 0 | 1^{+} | 0^{+} | 1^{+} | 0^{+} | \ldots | 2            |
| 1 | 2^{+} | 1^{+} | 2^{+} | 1^{+} | \ldots | 2            |
| 2 | 1^{+} | 2^{+} | 3^{+} | 2^{+} | \ldots | 4            |
| 3 | 2^{+} | 2^{+} | 3^{+} | 4^{+} | \ldots | 4            |

TABLE I: Natural parity \(J^{PC}_{L}\) \(t\)-channel exchanges contributing into the Mellin moments of quark helicity flip GPDs.

| \(N\backslash J\) | 1  | 2  | 3  | 4  | \ldots | Number of FFs |
|-----------------|----|----|----|----|---------|---------------|
| 0 | 1^{+} | 0^{+} | 1^{+} | 0^{+} | \ldots | 1            |
| 1 | 2^{+} | 1^{+} | 2^{+} | 1^{+} | \ldots | 2            |
| 2 | 1^{+} | 2^{+} | 3^{+} | 2^{+} | \ldots | 3            |
| 3 | 2^{+} | 2^{+} | 3^{+} | 4^{+} | \ldots | 4            |

TABLE II: Unnatural parity \(J^{PC}_{L}\) \(t\)-channel exchanges contributing into the Mellin moments of quark helicity flip GPDs.

The column “Number of form factors (FFs)” shows the number of the generalized form factors in the parametrization (A1) populated by the corresponding \(J^{PC}_{L}\) \(t\)-channel exchanges for the given \(N\).
2. Quantum number selection rules: gluon case

To consider the case of the gluon helicity flip GPDs one has to introduce the tower of local gluon operators

\[ \hat{O}_{\alpha\rho}^{\beta\sigma} \mu_1...\mu_N = \mathbb{S}_{\alpha\beta\mu_1...\mu_N} G^{\alpha\mu_1}(0)(i\vec{D}^\mu_1)...(i\vec{D}^\mu_N)G^{\beta\sigma}(0). \]  

(B5)

The Mellin moments of gluon helicity flip GPDs are given by the nucleon matrix elements of the operators (B5) convoluted with the usual projection operator

\[ \tau_{ij;\rho\sigma} n^\alpha n^\beta. \]  

(B6)

The tower of the local operators (B5) transforms according to \((N_2, N_2) \oplus (N_2, N_2+2) \oplus (N, N+4)\) representation of the Lorentz group. The explicit expression for the parametrization of the nucleon matrix elements of the operators (B5) is given in (A3). Its charge conjugation parity is given by

\[ C = (-1)^{N+2}. \]  

Note, that as the gluon is its own antiparticle the gluon helicity flip GPDs \(H_T^g, \tilde{H}_T^g, E_T^g, \tilde{E}_T^g\) are even functions of \(x\). Therefore we need to consider only even Mellin moment of the gluon helicity flip GPDs\(^\text{10}\).

Similarly to the quark case, the tower of operators (B5) does not possess the definite \(P\) parity. This leads to two sequences of possible quantum numbers of the \(t\)-channel exchanges:

- Of natural parity \(P = (-1)^J\):
  \[ J^{PC} = J^{(-)J}(-)^{N+2} = 1^{(-)^{N+2}}, 2^{(-)^{N+2}}, 3^{(-)^{N+2}}, \ldots (N+2)^{(-)^{N+2}} (-)^{N+2}. \]  

(B7)

- Of unnatural parity \(P = (-1)^{J+1}\):
  \[ J^{PC} = J^{(-)^{J+1}(-)^{N+2}} = 1^{(-)^{N+2}}, 2^{(-)^{N+2}}, 3^{(-)^{N+2}}, \ldots (N+2)^{(-)^{N+2}} (-)^{N+1}. \]  

(B8)

Now matching these two sequences with the set of the allowed \(\langle N\bar{N}\rangle\) states (B4) one can work out the selection rules \([19]\) for the quantum numbers of the \(t\)-channel exchanges contributing into the Mellin moments of gluon helicity flip GPDs. The results for even \(N\) are summarized in the Tables III, IV.

\(^\text{10}\) At this point we disagree with the analysis of Ref. [19] in which odd Mellin moments of gluon helicity flip GPDs are also considered. See discussion in Sec. 3.2.
| $N \backslash J$ | 1   | 2   | 3   | 4   | 5   | 6   | ... | Number of FFs |
|---------------|-----|-----|-----|-----|-----|-----|-----|---------------|
| 0             | $2_{1,3}^{++}$ |     |     |     |     |     |     | 2             |
| 2             | $2_{1,3}^{++}$ | $4_{3,5}^{++}$ |     |     |     |     |     | 4             |
| 4             | $2_{1,3}^{++}$ | $4_{3,5}^{++}$ | $6_{5,7}^{++}$ |     |     |     |     | 6             |

TABLE III: Natural parity $J_L^{PC}$ $t$-channel exchanges contributing into the even Mellin moments of gluon helicity flip GPDs.

| $N \backslash J$ | 1   | 2   | 3   | 4   | 5   | 6   | ... | Number of FFs |
|---------------|-----|-----|-----|-----|-----|-----|-----|---------------|
| 0             | $2_{2}^{-+}$ |     |     |     |     |     |     | 1             |
| 2             | $2_{2}^{-+}$ | $3_{3}^{++}$ | $4_{4}^{-+}$ |     |     |     |     | 3             |
| 4             | $2_{2}^{-+}$ | $3_{3}^{++}$ | $4_{4}^{-+}$ | $5_{5}^{++}$ | $6_{6}^{-+}$ |     |     | 5             |

TABLE IV: Unnatural parity $J_L^{PC}$ $t$-channel exchanges contributing into the even Mellin moments of gluon helicity flip GPDs.

From Tables III, IV we conclude that we inevitably need the contribution of unnatural parity mesons to obtain the proper number of independent form factors for the $N$-th Mellin moment of gluon helicity GPDs.

C. MISCELLANEOUS

1. Conventions for ordinary helicity spinors

Following App. B of Ref. [5] we use the following conventions for the ordinary helicity spinors:

$$U(p, +) = \left( \begin{array}{c} \sqrt{p^0 + m \chi_+(p)} \\ \sqrt{p^0 - m \chi_+(p)} \end{array} \right) ; \quad U(p, -) = \left( \begin{array}{c} \sqrt{p^0 + m \chi_-(p)} \\ -\sqrt{p^0 - m \chi_-(p)} \end{array} \right) ;

V(p, +) = -\left( \begin{array}{c} \sqrt{p^0 - m \chi_-(p)} \\ -\sqrt{p^0 + m \chi_+(p)} \end{array} \right) ; \quad V(p, -) = -\left( \begin{array}{c} \sqrt{p^0 - m \chi_+(p)} \\ \sqrt{p^0 + m \chi_+(p)} \end{array} \right) , \quad (C1)$$
where the two-spinors read

$$\chi_+(p) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p^3)}} \begin{pmatrix} |\vec{p}| + p^3 \\ p^1 + ip^2 \end{pmatrix};$$

$$\chi_-(p) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p^3)}} \begin{pmatrix} -p^1 + ip^2 \\ |\vec{p}| + p^3 \end{pmatrix}. \quad (C2)$$

Here $p = (p_0, \vec{p}) \equiv (p^0, p^1, p^2, p^3)$ is the corresponding momentum four-vector.

2. Wigner $d$-functions

The expression of the Wigner $d$-functions through the Jacobi polynomials (see eq. (3.72) of [44]) :

$$d^J_{m,m'}(\theta) = \sqrt{(J + m)!/(J + m')!(J - m')!} (\sin \frac{\theta}{2})^{m-m'} (\cos \frac{\theta}{2})^{m+m'} P_{J-m}^{(m-m', m+m')}(\cos \theta). \quad (C3)$$

The relation to the usual Wigner function is given by $d^J_{m,m}(\theta) = D^J_{m,m}(0, \theta, 0)$.

Wigner $d$-functions $d^J_{m,m'}$ are orthogonal for fixed $m$, $m'$:

$$\int_0^{\pi} d\theta \sin \theta d^J_{m,m'}(\theta)d^{J'}_{m',m'}(\theta) = \frac{2}{2J+1} \delta_{J,J'}. \quad (C4)$$

The relation (C3) together with the Rodriguez formula for the Legendre polynomials helps to express the Wigner $d$-functions employed in the present paper through the Legendre polynomials and their derivatives.

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