Higher curvature gravity at LHC

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(Dated: March 14, 2014)

We investigate brane world model in different viable $F(R)$ gravity theories where the Lagrangian is an arbitrary function of the curvature scalar. Deriving the warped metric for this model, resembling Randal-Sundrum (RS) like solutions, we determine the graviton KK modes. The recent observations at LHC, constrains the RS graviton KK modes to a mass range greater than 3 TeV, somewhat incompatible to RS model predictions. However the models with $F(R)$ gravity in the bulk is shown to address the issue which in turn constrains the $F(R)$ model itself.

Observable universe being $(3 + 1)$ dimensional keeps room for additional unobserved dimensions. Such extra dimensional models are primary candidates to address the well-known problems like, the hierarchy between electroweak scale and Planck scale and observed smallness of the cosmological constant leading to cosmological fine tuning problem.

In order to address the above issues brane world models were proposed (see [1], [2]) where our universe is a hyper-surface embedded in a higher dimensional bulk. The RS model provides a geometric solution to the hierarchy problem via an exponential warp factor whose magnitude is controlled by separation of two 3-branes and bulk cosmological constant. The RS model constitutes a self-consistent description of our universe with four dimensional standard Friedmann cosmology and Newtonian limit reproduced (see [3], [4]). The brane world model can actually be thought of as some low energy effective theory of some string theory [5].

The RS model line element is given by

$$ds^2_{RS} = e^{-2kr_c \phi} n_{\mu \nu} dx^\mu dx^\nu + r_c^2 d\phi^2 \quad (1)$$

with Greek indices $\mu, \nu, \ldots \, \text{run over } 0, 1, 2, 3$ and they refer to usual observed dimensions, while the co-ordinate $\phi$ signifies the extra dimension which is spacelike of length $r_c$. The constant $k$ is connected to bulk cosmological constant $\Lambda$ such that, $k = \sqrt{-\Lambda/12M^3}$. This is of the order of Planck mass. The quantity $e^{-2kr_c \pi}$ is known as the warp factor. The slices $\phi = 0$ and $\phi = \pi$ represents hidden and visible branes (i.e. our observable universe) respectively. Due to presence of the warp factor a mass scale of the order of Planck scale, $M_{Pl}$ gets warped on the visible brane by an amount,

$$M = M_{Pl} e^{-2kr_c \pi} \quad (2)$$

For $kr_c \approx 12$, we get $M \approx 1 TeV$. Hence in this picture the stability of the Higgs mass against large radiative correction is controlled by the warped geometry of this five dimensional spacetime.

The two brane system can be stabilized by introducing a bulk scalar field $(\theta, \bar{\theta})$. To address the cosmological constant problem various other models have been proposed which includes domain wall scenario (see [8], [9]) and self tuning in large extra dimension (see [10], [11]). However this brane world model has a drawback, it describe the visible brane as our universe but with negative tension, which are intrinsically unstable. Also by the very choice of the metric ansatz it eliminates 3-brane cosmological constant being not consistent with present day observed small value. In order to circumvent these possibilities regarding stability of the 3-brane and non zero cosmological constant (due to negative tension) as well as to generalize the model to include maximally symmetric spaces recently a more general setup were proposed [12]. In this context it would be useful from a phenomenological point of view to use bottom-up approach where for instance we introduce modified Einstein-Hilbert gravity. In this work we consider RS solutions and its generalizations for $F(R)$ gravity theory on the bulk (for recent review on $F(R)$ gravity see [13], see also [14], [15]). Higher derivative gravity and fine tuning problem has been discussed in Ref. [16], while Ref. [17] studies cosmological aspects of $F(R)$ brane world. Some other features like effective Einstein equations and junction conditions for $F(R)$ brane world were studied in [18] and [19].

The search for warp geometry model in LHC is performed through the signal of dilepton or diphoton decay channel of the first graviton KK modes, indicates the lower bound on the masses of lowest graviton KK modes, $>3$ TeV (see for example [20] and [21]). This posed serious problem to the standard RS model, since it cannot explain this large lower bound within its theoretical framework. In this work we will try to introduce a possible generalization of the RS model to address these issues.

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within the framework of warped geometry model.

Recently, there have been a boost in the gravitation research along the line of modifying the gravitational Lagrangian in order to explain the high energy non-renormalizable behavior of gravitation. These modified theories include: $F(R)$ theories, Gauss-Bonnet gravity, Lanczos-Lovelock models etc. In this paper, we shall study the warped geometric model with $F(R)$ Lagrangian on the bulk, and try to find out the graviton KK modes in these warped geometric models.

We start from the following action for $F(R)$ gravity on the bulk,

$$S = \int d^{5}x \sqrt{-G} \left( M^{3}F(R) - \Lambda \right) + \int d^{3}x \sqrt{-g_{i}} V_{i}$$

where $\Lambda$ is the bulk cosmological constant, $R$ is the five dimensional Ricci scalar and $V_{i}$ is the brane tension for $i$th brane. A general warped metric ansatz is taken as,

$$ds^{2} = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + r_{c}^{2} dy^{2}$$

The Einstein equations with constant scalar curvature for this metric are without any higher derivative term and therefore free from ghosts. They are,

$$\begin{cases}
   4R_{\mu\nu} + \frac{e^{-2A}}{r_{c}^{2}} \left( A'' - 4A'^{2} \right) g_{\mu\nu} = \frac{8\Lambda}{M^{3}} e^{-2A} g_{\mu\nu} \\
   -\frac{1}{2} g_{\mu\nu} e^{-2A} F(R) = -\frac{\Lambda}{2M^{3}} e^{-2A} g_{\mu\nu}
\end{cases}$$

where prime denote derivative with respect to $y$ and the five dimensional scalar curvature has the following expression, $R = e^{2A} \left( R + \frac{1}{4} \left( 8A'' - 20A'^{2} \right) \right)$. These equations has to be supplemented by the boundary conditions $[A'(y)]_{i} = \frac{\Omega e^{2A}}{12M^{3}} V_{i}$, where $\epsilon_{i} = 1$. From Eqs. (5) and (6) we are left with an interesting fact that we can actually separate the spacetime part and the extra dimension part which finally leads to,

$$\begin{cases}
   4R_{\mu\nu} = \Omega g_{\mu\nu} \\
   3A'' = \Omega r_{c}^{2} e^{2A}
\end{cases}$$

Note that the first equation could equivalently be written as, $4G_{\mu\nu} = -\Omega g_{\mu\nu}$. Then from Eq. (5) we readily obtain a simplified version of equation satisfied by $A'$ by introducing $F(R) = R + f(R)$, which finally leads to (with rescaling $y \rightarrow r_{c} y$),

$$(A')^{2} \left( 6 - \frac{4}{3} \frac{df}{dR} \right) = -\frac{\Lambda}{2M^{3}} + \frac{f}{2} + 2\Omega e^{2A} \left( 1 - \frac{2}{3} \frac{df}{dR} \right)$$

The above equation has the following solution for the variable $A$,

$$e^{-A} = \omega \cosh \left( \frac{\omega}{c_{1}} + k_{F} y \right)$$

$$k_{F}^{2} = -\frac{1}{6} \left( \frac{\Omega \omega}{c_{1}^{2}} \frac{\omega^{2}}{y^{2}} \right)$$

where $\omega^{2} = -\frac{\Omega \omega}{c_{1}^{2}} > 0$, since for $\text{AdS}$ bulk $\Lambda < 0$ the induced cosmological constant $\Omega$ on the visible brane is negative. Also note that since five dimensional scalar curvature is constant the quantity $k_{F}^{2}$ is actually constant. The respective brane tensions are being given by,

$$V_{\text{vis}} = 12M^{3} k_{F} \left[ \frac{\omega^{2}}{c_{1}^{2}} e^{2k_{F}^{2}r_{c}y} - 1 \right]$$

$$V_{\text{hid}} = 12M^{3} k_{F} \left[ 1 - \frac{\omega^{2}}{c_{1}^{2}} e^{2k_{F}^{2}r_{c}y} \right]$$

The quantity $c_{1}$ could be obtained by normalizing the warp factor to unity at $y = 0$ and leads to $c_{1} = 1 + \sqrt{1 - \frac{\omega^{2}}{c_{1}^{2}}}$. Here also the solution to hierarchy problem can be obtained by calculating the warp factor at $y = \pi r_{c}$. Thus we have obtained the exact form of the warp factor for this generalized RS model. Following the procedure used in [12] we also observe that in this scenario we have both positive and negative cosmological constant on visible 3-brane. Remarkably for anti-de Sitter brane the hierarchy problem can be solved with both the positive tension brane. In all these cases if $k_{F}^{2} > k_{RS}^{2}$ then the brane tensions are greater than that in RS model (see Eq. (11)). However this fine tuning of brane cosmological constant actually generates from fine tuning of modulus $r_{c}$ at inverse Planck scale. Thus if we want to resolve the fine tuning problem without introducing any further hierarchy the brane cosmological constant $\omega$ must be very small.

Now in order to determine the constant $\alpha$ we took the limit to $f(R) \rightarrow R$ and that leads to $\alpha = -2k_{F} r_{c}$. Also in the limit $f(R) \rightarrow 0$ we readily obtain $k_{F}^{2} = k_{RS}^{2} = \frac{\Lambda}{12M^{3}}$. Then requiring $k_{F}^{2} > -\frac{\Lambda}{12M^{3}}$ we arrive at the following criteria,

$$\frac{df}{dR} > \frac{2\Lambda}{3M^{3}}$$

Thus our result is identical to that of RS model except for the fact that it differ in one important aspect, the quantity $k_{F}^{2}$ is different and depends on the $f(R)$ model we are considering. On the visible brane as in the RS case the Higgs mass gets warped by the following form,

$$m = m_{0} e^{-2k_{F} r_{c} \pi}$$
Since \( k_F \) is different from the RS solution \( r_c \), should be modified such that the product \( k_F r_c \) remains \( \approx 11 \) to produce desired warping.

The graviton KK spectrum in these brane world models act as an important phenomenological signature of extra dimensions. This is quiet different from the usual factorizable geometry which results in a distinctive phenomenology. All these KK modes have masses and couplings in the TeV scale, which can be produced on resonance and hence observed in the collider. As in the RS scenario the metric fluctuation can be taken as a linear expansion of the flat metric around Minkowski value, \( \tilde{G}_{\alpha\beta} = e^{-2\sigma}(\eta_{\alpha\beta} + \kappa^* h_{\alpha\beta}) \) leading to graviton KK modes which can be expressed as \( ^{24} \),

\[
h_{\alpha\beta}(x, \phi) = \sum_{n=0}^{\infty} R_{\alpha\beta}^{(n)}(x) \begin{pmatrix} \frac{(n)}{(n)} \phi \\ \sqrt{r_c} \end{pmatrix}
\]

where \( R_{\alpha\beta}^{(n)}(x) \) corresponds to KK modes of the graviton. The equation satisfied by the mode functions \( \chi^{(n)} \) is given by,

\[
\frac{-1}{v_f^2} \frac{d}{d\phi} \left( e^{-4\sigma} \chi^{(n)}(\phi) \right) = m_n^2 e^{-2\sigma} \chi^{(n)}
\]

The functions \( \chi^{(n)} \) satisfies orthonormal condition, \( \int_{-\pi}^{\pi} d\phi e^{-2\sigma} \chi^{(n)}(\phi) \chi^{(m)}(\phi) = \delta_{nm} \). The solutions for \( \chi^{(n)} \) are given by \( ^{24} \),

\[
\chi^{(n)}(\phi) = \frac{e^{2\sigma(\phi)}}{N_n} [J_2(z_n) + \alpha_n Y_2(z_n)]
\]

where \( J_2 \) and \( Y_2 \) are Bessel functions of second order, \( z_n(\phi) = m_n e^{\sigma(\phi)} / k \) and \( N_n \) is the normalization. Then for \( x_+ = z_n(\pi) \) and in the limit \( m_n / k \ll 1 \) and \( e^{kr_c} \pi \gg 1 \), using continuity of the first derivative of \( \chi^{(n)} \) we get \( x_+ \) to be the solution of the equation \( J_1(x_+) = 0 \). The masses of the graviton KK excitations are given by \( m_n = k_F x_+ e^{-kr_c} \).

Having obtained the mass modes, we can now derive the interactions of \( h_{\alpha\beta} \) with the matter fields on the visible 3-brane at \( \phi = \pi \). With these constraint we find the usual form of the interaction Lagrangian such that,

\[
L = \frac{1}{M^{3/2}} T^{\alpha\beta}(x) h_{\alpha\beta}(x, \phi = \pi)
\]

where \( T^{\alpha\beta}(x) \) is the symmetric energy-momentum tensor of the matter fields. Expanding the graviton field into the KK states of Eq. \( ^{14} \) and using the proper normalization for \( \chi_n(\phi) \) we arrive at,

\[
L = \frac{1}{M_{Pl}} R^{\alpha\beta} h_{\alpha\beta}^{(0)} - \frac{1}{\Delta_s} \sum_{n=1}^{\infty} T^{\alpha\beta} h_{\alpha\beta}^{(n)}
\]

Thus as in the RS scenario, the zero mode in \( F(R) \) gravity couples with the 4-dimensional Planck scale. It can also be observed that, all the massive KK states couples to the scale \( \Lambda_{m}^{-1} \), which has the following expression, \( \Lambda_{m} = e^{k_F r_c M_{Pl}} \), which is of order TeV\(^{-1} \) i.e. in the weak scale. Thus even in the \( F(R) \) theory the coupling are the same as in RS model. Hence the 3 TeV experimental bound on first graviton KK modes applies here also.

However as discussed earlier, it is impossible to achieve this in standard RS model. Following our calculation due to inclusion of \( F(R) \) gravity in the bulk then we readily obtain that the graviton masses become modified by the factor \( k_F \) given by Eq. \( ^{10} \). However in an \( F(R) \) model if \( k_F / k_{RS} = p \) then retaining \( k_F r_c \sim 12 \) can set the graviton mass \( p \) times larger than RS model to raise it above the experimental lower bound.

Thus if we let \( k_F^p = -p \Lambda_{12} M^3 \) where \( p > 1 \) we arrive at the following relation,

\[
p \frac{dF}{dR} \frac{\Lambda}{12 M^3} = (p - 1) \Lambda_{12 M^3} + \frac{f}{12}
\]

for the particular choice \( f(R) = \beta R^n \) we find,

\[
\frac{\beta R^{n-1}}{12} \left( \frac{2p \Lambda | \Lambda |}{3 M^3} + R \right) = (p - 1) \Lambda | \Lambda |_{12 M^3}
\]

Thus we get a bound on the scalar curvature given by,

\[
R > \frac{2p \Lambda | \Lambda |}{3 M^3} \quad (\beta > 0, n=odd; \ \beta < 0, n=even)
\]

\[
R < \frac{2p \Lambda | \Lambda |}{3 M^3} \quad (\beta > 0, n=even; \ \beta < 0, n=odd)
\]

Now we would like to know what is the criteria for \( k_F^p \) to be greater than \( k_{RS}^p \). From Eq. \( ^{12} \), with \( f(R) = \beta R^n \) we obtain the bound,

\[
R > \frac{2n \Lambda | \Lambda |}{3 M^3}
\]

If we employ the criteria that \( k_F^p > 0 \) then also we shall retrieve the criteria given by Eq. \( ^{22} \). However the following criteria \( k_F / M < 1 \) (i.e. bulk curvature is less than 5D Planck scale to ensure that the classical solutions of Einstein’s equation can be trusted) leads to an inequality given by

\[
\frac{\Lambda | \Lambda |}{12 M^5} + \frac{\beta R^{n-1}}{2 M^2} \left\{ R + \frac{4nM^2}{3} \right\} < 1
\]

As the bulk is AdS, we infer that five dimensional scalar curvature must be negative. This implies, \( \beta > 0 \) is the valid choice for odd powers of \( R \), while \( \beta < 0 \) is a valid choice for even powers of \( R \). Hence the form of \( F(R) \) should be the following, \( F(R) = R - \alpha R^2 + \)
\[ \beta R^3 - \ldots \] Also we can arrive at the following bound on the bulk curvature,
\[ |R| < \min \left( \frac{2n|A|}{3M^3}, \left[ \frac{3}{2\beta n} \left( 1 - \frac{|A|}{12M^5} \right) \right]^{1/(n-1)} \right) \quad (24) \]

From the above inequality we observe that the magnitude of the bulk curvature should remain less than \( O(\frac{|A|}{12M^3}) \).

Thus in this paper, we have shown that though graviton KK mode bounds by recent LHC experiments can not be explained in standard RS scenario, it can be explained quiet well in \( F(R) \) gravity framework. The solutions on the branes for non flat metric ansatz have been obtained with \( F(R) \) gravity in the bulk, which shows that they have the structure as that of RS scenario, however differ in one important parameter, \( k_F \), which in turn depend on the form of \( F(R) \). From the flat brane limit we recover the RS like solutions in \( F(R) \) gravity on the bulk. The graviton KK modes gets modified by that factor \( k_F \) and they can have values well beyond 3 TeV, without any conflict with recent LHC results. Finally, from various criteria like \( k_F^2 > 0, k_F/M < 1 \) we arrive at a constraint equation on the bulk curvature. By assuming some reasonable properties about the bulk curvature like it should be \( O(|A|/12M^3) \) we observe from Eq. (24) that this holds provided \( F(R) \) has leading order term as \( n = 2 \) with a negative co-efficient.

Hence by introducing \( F(R) \) gravity in the bulk we have achieved a small brane cosmological constant, stability of negative tension brane with non-flat branes. Then we have also derived the graviton KK mass modes and the possibility to have them above 3 TeV has been addressed. Finally some constraint on bulk scalar curvature is obtained.

Acknowledgement

S.C is funded by SPM fellowship from CSIR, Govt. of India.

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