Communication loophole in a Bell-EPR-Bohm experiment: standard no-signaling may not always be enough to exclude local realism.

David Rodríguez

1Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain

(Dated: April 28, 2013)

Assuming perfect detection efficiency, we present an (indeterministic) model for an EPR-Bohm experiment which reproduces the singlet correlations, without contradicting Bell’s original locality condition. In this model we allow the probability distribution \( \rho_\lambda \) of the state \( \lambda \) at the source to depend parametrically on the orientation \( \xi \) of one of the measuring devices: \( \rho_\lambda(\lambda, \xi) \). In a Bell experiment, no-signaling between the source and each one of the devices would seem clearly sufficient to rule such an influence; however, not even schemes where the choice of observables takes place during the on-flight time of the particles can prevent, in some situations, a model of this type from violating the local bounds. In particular, a random shift \( \rho_\lambda(\lambda, \xi_1) \rightarrow \rho_\lambda(\lambda, \xi_2) \rightarrow \ldots \rightarrow \rho_\lambda(\lambda, \xi_n) \) allows the model to perform a “subensemble selection” for each of the terms involved in the inequality (analogous to what goes on with the efficiency loophole), whenever some correlation of those \( \rho_\lambda \)-shifts with the sequence of measurement choices is allowed. That correlation does not necessarily imply signaling during the photon on-flight time.

PACS numbers:

Previous note: We have been made aware (much thanks to Dr. M. Hall for that) of some recent developments on the subject [22-24]. So far to our knowledge, results regarding our model for the singlet correlations can be understood as a particular case of what Dr. Hall refers to as “measurement dependence”, and in general as a particular case within the broader class of models studied in most of those references (though the formal approach is quite different). Nevertheless, the intentions of this paper being less ambitious than most of the former, we still think it may retain some interest as a simple, hopefully easy to grasp counterexample to some widespread beliefs about the rigidity of the frontier between quantum and classical behaviours. On the other hand, it may also be useful to attract some more attention on the subject. As a general remark, we are not so interested in “measurement dependence” as a mathematical resource as we are in the fact that we may find a feasible and intuitive physical counterpart for that phenomena, for instance thinking in the action of field lines generated by the measurement devices on the source. Besides, our approach to indeterminism (see also additional notes) and its relation with Clauser and Horne’s factorability, as little sophisticated as it may perhaps seem, can nevertheless be, in our opinion, clarifying.

Following John Bell [2], the results of the two distant measurements \( A, B \in \{\pm 1\} \) performed in a bipartite experiment (for instance, of the EPR-Bohm type [1]) can be expressed as

\[
A = A(a, \lambda),
\]

\[
B = B(b, \lambda),
\]

where \( a, b \) are the orientations of each device, respectively, and where \( \lambda \in \Lambda \), with a probability density function \( \rho_\lambda(\lambda) \), summarizes the state of the pair of particles at the source. Bell’s locality condition stands, therefore, simply for the fact that \( A(a, \lambda) \) does not depend on \( b \), nor does \( B(b, \lambda) \) depend on \( a \), either. This (deterministic) Bell locality condition is the main hypothesis not only behind the original Bell inequality [2], but also (to our knowledge) for any other inequality [22].

However, a more realistic view demands a generalization of the former expressions to the so-called indeterministic case [22]:

\[
A = A(a, \lambda, \omega_a),
\]

\[
B = B(b, \lambda, \omega_b),
\]

where \( \omega_a \in \Omega_a, \omega_b \in \Omega_b \) are other hidden variables, with density functions \( \rho_{a}(\omega_a, a), \rho_{b}(\omega_b, b) \), representing the state of the measuring device (and therefore parametrically dependent on the corresponding orientation \( a, b \)). Now, for any function \( f_j(A, B) \) (subindex standing for ”joint”) we have

\[
\langle f_j(A, B) \rangle(a, b) = \int_{\Lambda \oplus \Omega_a \oplus \Omega_b} d\lambda d\omega_a d\omega_b \rho_{j}(\lambda, \omega_a, \omega_b) \cdot f_j(A(a, \lambda, \omega_a), B(b, \lambda, \omega_b)).
\]

(5)

where \( \rho_{j}(\lambda, \omega_a, \omega_b) \) is a joint density function; just as we did before, we simply need to invoke the intuitive idea of locality to see that we must demand that \( \lambda, \omega_a, \omega_b \) are statistically independent, i.e. their joint probability density is factorizable: \( \rho_{j}(\lambda, \omega_a, \omega_b) = \rho_{j}(\lambda)\rho_{a}(\omega_a, a)\rho_{b}(\omega_b, b) \).

This said, in experiments testing Bell inequalities of, for instance and again, the EPR-Bohm type [1], a “loophole” appears (there is room for a local model reproducing the observed results) when the two devices are not sufficiently far apart from one another, and a signal

*Electronic address: drodriguez@us.es*
can be thus (causally) transmitted between them, in the time that elapses between the two almost-simultaneous measurements. This is also known as the "locality" or "communication" loophole, and it would in principle be excluded by the standard "no-signaling" condition (the time between the measurements is not enough to allow for any sub-luminal transmission of information between the two locations). In any case, a violation of a Bell inequality is only meaningful if neither that no-signaling nor any of the other possible additional hypothesis involved is violated too: in the following, we will provide a model that works as a counterexample on any of the former (bipartite) inequalities (a local model capable of reproducing the singlet correlations), without necessarily violating the usual no-signaling condition. This will allow us to show that, in the first place, standard no-signaling requirement between the two observers, as it is post-selected by our hypothesis after all, parametrical dependence of $\rho_1$ on the direction $b$ of one of the measuring devices. Moreover, let us also consider that the state at the source $A$ follows a probabilistic distribution governed by the following density function

$$\rho_1(\lambda, b) = \frac{1}{2} [ \delta(\lambda - b) + \delta(\lambda - b + \pi) ],$$

where the Dirac deltas introduce a strong, but well allowed by our hypothesis after all, parametrical dependence of $\rho_1$ on the direction $b$ of one of the measuring devices. Using the last three expressions in (9) is enough to get to what we were looking for:

$$(A \cdot B)(a, b) = - \cos(a - b),$$

which is precisely the quantum correlation for the singlet state. Obviously, $\rho_1$ is not rotationally invariant, but it will be again capable of producing (apparent) rotationally invariant correlations from the point of view of the experimenter [30]. Besides, this model meets the standard no-signaling requirement between the two observers, but, surprisingly enough, still remains completely local. A first consequence of this is that we need to include no-signaling also between each of the pairs (observer, source), if we want to discriminate between local and non-local models. In a highly ideal experiment where the whole correlation spectrum is analyzed, this may be enough, but perhaps not in a real one, as we will show here.

But first, what if the parametrical dependence is for instance on $\xi \neq b$? It is a matter of algebra to see that for

$$\rho_1(\lambda, \xi) = \frac{1}{2} [ \delta(\lambda - \xi) + \delta(\lambda - \xi + \pi) ],$$

we obtain

$$(A \cdot B)(a, b) = - \cos(a - b) + \sin(a - \xi) \sin(b - \xi).$$

We want to show to what extent a model of this kind could work in an actual experiment; for instance, let us consider the case where the CHSH [8] inequality is tested, and let $a, a'$ and $b, b'$ be the two pairs of alternative orientations of the devices at each side. Moreover, let us adopt the procedures of Aspect’s more restrictive experiment [8], working with photons (and therefore subjected to the efficiency loophole, that we will ignore), but still interesting for us (see [33]) because it uses (uncorrelated) post-selection of observables for both particles, as it is described, for instance, in [20].
Now, we will suppose that $\rho_{\lambda}(\lambda, \xi)$ obeys (14), but is randomly shifting from one to another within a set of four possible density functions given by $\rho_{\lambda}(\lambda, a), \rho_{\lambda}(\lambda, a'), \rho_{\lambda}(\lambda, b)$ and $\rho_{\lambda}(\lambda, b')$ (therefore, $\xi \in \{a, b, a', b'\}$), the four possibilities with equal probability. These shifts are random but not completely uncorrelated with the choices of observable at the devices. This correlation does not necessarily need signaling (between device and source) during photon’s on-flight time, regardless of whether we use observable post-selection or not: however, there must exist some device-source communication, though this can present some delay as well.

We will measure that correlation by a certain parameter $\Gamma$. For instance, if we are going to measure (polarizations, projections of spin) $a, b$, then there is a probability $\xi$ that $a = a'$ or $\xi = b'$ (in principle let us suppose with equal probability), which means that other contributions appear, modifying the overall observed correlation. Now, using (15) we obtain that, on average over all events,

$$\langle A \cdot B \rangle (a, b) = -\Gamma \cos(a - b)$$

$$-\frac{1}{2} (1 - \Gamma) [\cos(a - b) - \sin(a - a') \sin(b - a')]$$

$$-\frac{1}{2} (1 - \Gamma) [\cos(a - b) - \sin(a - b') \sin(b - b')]$$

$$= -\cos(a - b) + \frac{1}{2} (1 - \Gamma) [\sin(a - a') \sin(b - a')$$

$$+ \sin(a - b') \sin(b - b')]$$.

Analogous expressions for $\langle A \cdot B \rangle (a, b'), \langle A \cdot B \rangle (a', b)$ and $\langle A \cdot B \rangle (a', b')$ (see [31]) finally lead us to

$$\beta_m(\Gamma) = \beta_q + \frac{1}{2} (1 - \Gamma) \times$$

$$[\sin(a - b') \sin(b - b') + \sin(a - b) \sin(b' - b)]$$

$$+ \sin(a' - b') \sin(b - b') - \sin(a' - b) \sin(b' - b')]$$.

(17)

with $\beta_q = -\cos(a - b) -\cos(a - b') -\cos(a' - b) +\cos(a' - b')$ the well known quantum mechanical prediction.

Now, for $\Gamma = 1$, shifts in $\rho_{\lambda}$ and the choices of observables $\phi_A \in \{a, a'\}$ and $\phi_B \in \{b, b'\}$ are completely correlated, and we obtain the quantum value; for $1 > \Gamma > \frac{1}{2}$, $\rho_{\lambda}$ and $\phi_A, \phi_B$ only bear some correlation but the model is still capable of producing a value that defies the inequality, and for $\Gamma = 0.5$, $\rho_{\lambda}$ and $\phi_A, \phi_B$ are completely uncorrelated, as a result of which the model cannot violate the inequality.

We have seen that allowing for some dependence of $\rho_{\lambda}$ on certain parameters of the experiment $(a, b, a', b')$, together with some correlation (given by $\Gamma$) of that parametric dependence on the choice of observables, makes the model still capable of producing a value going (considerably) beyond the local bound. Neither that dependence, nor that also necessary correlation, are completely implausible [34], even assuming no-signaling during photon on-flight time, unless we design an experiment where an exhaustive evaluation of the whole spectrum of correlations $(E : a, b \rightarrow E(a, b)$ for all $a, b \in [0, 2\pi]$) is performed, with a random, post-selected choice of measurements for each event.

But, from the mathematical point of view... what is really happening when $\Gamma > 0.5$? The answer is simply that, through the shift in the parametrical dependence of $\rho_{\lambda}$, we are allowing the model to perform what is known as “subensemble selection” [18, 33]: in this case the ensembles are $\{\Lambda_i, \rho_{\lambda}(\Lambda_i, \xi_i)\}$. Some correlation between the choice of observables and shifts in $\rho_{\lambda}$ is needed, nevertheless, to make it possible: with $\Gamma > 0.5$ the full ensemble $\Lambda = \cup_i \Lambda_i$ is unavoidably fairly sampled. This is the same mechanism that lays underneath the efficiency loophole, for experiments based on data rejection (see for instance [4, 12, 23, 24]).

Nevertheless, it is still clear that the more exhaustive and restrictive the conditions of the experiment, the less the model can get close to the quantum prediction, so margins in the actual observed violations still play a key role. Taking this into account, perhaps the situation with the Clauser-Horne inequality should be studied more in detail elsewhere: there, violations are usually close to the local bound (in part due to efficiency constraints).

Some of the first precedents of this work are [3] and [11]. In particular, we use the same “Malus cosine law” as Scully [3] for our probabilities (15), (16) of detection (in Scully’s work, probabilities of passage through an Stern-Gerlach device). A complete analogy with the quantum case is not achieved there, as a difference with Barut and

![Figure 1: A maximum violation of the CHSH inequality by the quantum prediction can be achieved, for instance, with $a = 29, b = \theta, a' = 0, b' = 30$. We have represented, for $\theta \in [\pi, 2\pi]$: (i) $\beta_\theta = \beta_\theta(\Gamma = 1)$: * (red), (ii) $\beta_n(\Gamma = 0.8)$: o (green), (iii) $\beta_n(\Gamma = 0.50)$: x (blue), and (iv) the value we would obtain for a uniform distribution $\rho_{\lambda}$ (non-parametric): $\beta_{\lambda-unif} = \frac{1}{2} \beta_q$ + (yellow). This last satisfies the inequality, adding a factor of $\frac{1}{2}$ to the quantum value [32].](image)
Meystre’s work [11], where the bridge is indeed built, although it also needs (besides some sophisticated mathematics) the introduction of an additional (and a bit obscure) condition (some projector acting only for one of the devices). We also need a sort of additional assumption to make our model’s and the quantum prediction meet, but in our case its interpretation comes up as perfectly clear: we are talking of a causal influence of one of the measuring devices on the source of the state. This influence (taking place during the particle on-flight time or not) can find, without much imagination, a physical, quite plausible counterpart: the effect of far field lines generated by the device (an Stern-Gerlach, a polarizer or whatever it is).

The author thanks Dr. R. Risco-Delgado for his comments, and acknowledges support from the Departments of Applied Physics (II and III) at Sevilla Univ.

[1] D. Bohm, Y. Aharonov. Phys. Rev. 108, 1070–1076 (1957).
[2] J.S. Bell, Physics (Long Island City, N.Y.) 1, 195 (1964).
[3] J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt. Phys. Rev. Lett. 23, 880 (1969).
[4] P.M. Pearle. Phys. Rev. D 2, 1418 (1970).
[5] J.F. Clauser, M. A. Horne. Phys. Rev. D 10, 526 (1974).
[6] F. Selleri, G. Tarozzi, Lett. al Nuovo Cimento 29, 16 (1980).
[7] A. Aspect, P. Grangier, G.Roger. Phys Rev. Lett. 49 91 (1982).
[8] A. Aspect, J. Dalibard, G.Roger. Phys Rev. Lett. 49 1804 (1982).
[9] Marlan O. Scully, Phys. Rev. D 28, 2477 (1983).
[10] A.O. Barut, P. Meystre, Phys. Rev. Lett. 53, 10.1103 (1984).
[11] A.O. Barut, P. Meystre, Phys. Lett. A 105, 458 (1984).
[12] A. Garg and N.D. Mermin. Phys. Rev. D 35, 3831 (1987).
[13] Quantum mechanics and local realism, F. Selleri, ed. Planum, p.433 (1988).
[14] S.L. Braunstein and C.M. Caves, in Bell’s Theorem, Quantum Theory, and Conceptions of the Universe, edited by M. Kafatos (Kluwer, Dordrecht, Holland, 1989).
[15] D. M. Greenberger, M. A. Horne, A. Zeilinger, in Bell’s Theorem, Quantum Theory, Conceptions of the Universe, edited by M. Kafatos (Kluwer Academic, Dordrecht, Holland, 1989).
[16] N. D. Mermin. Phys. Rev. Lett. 65, 3373 (1990). Phys. Rev. Lett. 65, 3373-3376 (1990).
[17] R. Risco-Delgado, PhD Thesis. Universidad de Sevilla (1997).
[18] J.-A. Larsson, Phys. Rev. A 57, 3304 (1998).
[19] R. Risco-Delgado, arXiv:quant-ph/0202099v1 (2002).
[20] A. Aspect, in "Quantum (Un)speakables - From Bell to Quantum Information", ed. by R.A. Bertlmann and A. Zeilinger, Springer (2002).
[21] M.A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W.M. Itano, C. Monroe, and D.J. Wineland, Nature (London) 409, 791 (2001).
[22] D.N. Matsukevich, P. Maunz, D. L. Moehring, S. Olm- schen, and C. Monroe, Phys. Rev. Lett. 100, 150404 (2008).
[23] A. Cabello, D. Rodríguez, I. Villanueva, Phys. Rev. Lett. 101, 120402 (2008).
[24] A. Cabello, J. -Á. Larsson, D. Rodríguez. Phys. Rev. A 79, 062109 (2009).
[25] For instance the Clauser-Horne-Shimony-Holt’s one (CHSH) [26], or some of its main generalizations: the Mermin inequality (N parties, 2 observables per party, [16], based on the GHZ states [15]) or that of Braunstein and Caves (two parties, N observables per party, [14]).

[26] Indeterminism was first explicitly addressed in [5], although it remains still perfectly compatible with the CHSH inequality [3] and its generalizations. A necessary remark is that it is also possible to give a definition of deterministic local realism in terms of probabilities, such as $P(A = \mu_b) = f_A(\lambda, a)$, $P(B = \mu_b) = f_B(\lambda, b)$ for $\mu_a, \mu_b \in \{\pm1\}$ (or, equivalently, $P(A = \mu_a | B = \mu_b, \lambda, a, b) = P(A = \mu_a | \lambda, a)$ for the same for $B$). That definition is, however, always reducible to one of the kind $A = A(\lambda, a), B = B(\lambda', b')$, where the hidden variable has been now redefined from $\lambda$ to $\lambda' = \lambda \oplus \mu$, and where $\mu$ is an extra set of variables that also describe the state of the source. A proof of it goes like this: let us suppose that for a given value $\lambda$ of the hidden variable and a given set $\mathcal{M}$ of simultaneous (compatible) measurements, $n$ different sets of results $\mathcal{R}_i$ of those measurements can occur, each one with probability $p_i$. Then, we can define a new random variable $\mu$ taking values $\mu = \mu_i$ for each of the former sets of results, so, naturally, $P(\mu = \mu_i | \lambda) = p_i$. If we now build a new hidden variable $\lambda' = \lambda \oplus \mu$, we can obtain a completely deterministic description of the system, but this time over $\lambda'$. On the other hand, a hidden variable model built on $\lambda'$ is always factorizable in the sense of Clauser and Horne [3], while the one defined over $\lambda$ may certainly not be so. Proof of this last fact is trivial, given that for $\lambda'$, the probabilities involved $- P(A|a, \lambda'), P(B|b, \lambda')$ - are now either unity or zero.

Therefore, the use of probabilities to define local realism does not necessarily imply indeterminism. This indeterminism can, on the other hand, be understood in the sense of some "loss of information in the measurement process", and described with the introduction of new variables describing the state of the apparatus and any other factor that may locally affect the result of the measurement. If we name this new (set of) variables $\omega_a, \omega_b$, then we will finally have $A = A(\lambda', a, \omega_a), B = B(\lambda', b, \omega_b)$.

[27] The model that we propose satisfies the factorability condition assumed in Clauser-Horne’s inequality [3]: $P(A = \mu_a, B = \mu_b | a, b, \lambda) = P(A = \mu_a | \lambda) \cdot P(B = \mu_b | b, \lambda)$, for any $\mu = \pm1$. Though there has been some controversy on this hypothesis [3, 14], we will consider the problem is solved already: see [20].

[28] We begin our proof by choosing, in eq. (4), $f_s(x) = 1$ for
When we assume (12), the rotational invariance is apparent.

We need to guarantee $0 < \tau_{shift} < \tau_{flight}$ and $\tau_{shift} > \tau_{flight}$, but, although the scope of our idea can perhaps be broadened to include those other schemes, we will leave it aside for the moment, or for elsewhere. What we would rather do is illustrate, in some general situation, how standard procedures do not rule out models like the one we have given.

To add some plausibility to our model, we can say that, let $\tau_{flight}$ be the photon on-flight time from the source to the device, and $\tau_{shift}$ the mean time between shifts of observable at one of the devices, either one of the following two conditions are, for instance, sufficient to make $\Gamma > 0.5$ possible: (a) $\tau_{shift} > \tau_{flight}$, (b) the probability of a shift in $\phi_A, \phi_B$ in an interval $\tau_{flight}$ is less than $\frac{1}{2}$.

By “change of ensemble” or “subensemble selection” we refer to the fact that a Bell inequality is usually only valid when all the evaluated correlations (or probabilities) are significant of the same ensemble (or set) of particles (or states), i.e., they are fairly sampled on the total ensemble $A$. Local models can exploit the lack of detection efficiency to introduce unfair sampling (efficiency loophole), and now we have seen that they can also do the same by exploiting a random shift in the parametrical dependence of $f_{\lambda}$. See also http://lanl.arxiv.org/abs/quant-ph/0507120. A model for the singlet correlations equivalent to ours is given in Sec. II C (Theorem 4), with, for instance, $\rho_{\lambda} = \rho_{\lambda}(\lambda | b)$.

When we assume (12), the rotational invariance is apparent because the parameter $\xi$ in $\rho_{\lambda}(\xi, \lambda)$ follows ($\xi = b$) the orientation of one of the devices throughout its rotation.

Using (15) we also obtain

$$\langle A B \rangle(a, b)$$

$$= -\cos(a - b') + \frac{1}{2}(1 - \Gamma)[ \sin(a - a') \sin(b' - a') $$

$$+ \sin(a - b) \sin(b' - b) ],$$

$$\langle A B \rangle(a', b)$$

$$= -\cos(a' - b) + \frac{1}{2}(1 - \Gamma)[ \sin(a' - a) \sin(b - a) $$

$$+ \sin(a' - b') \sin(b' - b) ],$$

$$\langle A B \rangle(a', b')$$

$$= -\cos(a' - b') + \frac{1}{2}(1 - \Gamma)[ \sin(a' - a) \sin(b' - a) $$

$$+ \sin(a' - b) \sin(b' - b) ],$$

With $\langle AB \rangle(a, b) = -\frac{1}{2} \cos(a - b)$ for $\rho_{\lambda}$ uniform, it is trivial that we get a $\beta_{(\lambda - \mu)} = \frac{1}{2} \beta_{\mu}$.