Cuprates possess a large pseudogap that spans much of their phase diagram. The origin of this pseudogap is as debated as the mechanism for high-temperature superconductivity. In one class of theories, the pseudogap arises from some instability not related to pairing, typically charge, spin or orbital current ordering. Evidence of this has come from a variety of measurements indicating symmetry breaking. On the other side are theories where the pseudogap is associated with pairing. This ranges from preformed pairs to resonating valence bond theories where spin singlets become charge coherent. Here, we study pairing in the cuprates by constructing the pair vertex using spectral functions derived from angle-resolved photoemission data. Assuming that the pseudogap is not due to pairing, we find that the superconducting instability is strongly suppressed, in stark contrast to what is actually observed. We trace this suppression to the destruction of the BCS logarithmic singularity from a combination of the pseudogap and lifetime broadening. Our findings strongly support those theories of the cuprates where the pseudogap is instead due to pairing.

To construct the pair vertex, we must first know the single-particle Green’s function. An issue is that angle-resolved photoemission spectroscopy (ARPES) measures only occupied part of the single-particle Green’s function. An issue is that angle-resolved photoemission data. Assuming that the pseudogap is not due to pairing, we find that the superconducting instability is strongly suppressed, in stark contrast to what is actually observed. We trace this suppression to the destruction of the BCS logarithmic singularity from a combination of the pseudogap and lifetime broadening. Our findings strongly support those theories of the cuprates where the pseudogap is instead due to pairing.

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Using this $\chi$, the resulting electron–electron interaction is

$$V(k,\Omega) = \bar{U}^2 \left[ \frac{3}{2} \chi(k,\Omega) - \frac{1}{2} \chi_0(k,\Omega) \right]$$

(4)

where $\bar{U}$ can differ from $U$ because of vertex corrections. To set $\bar{U}$, we will require that the renormalized Fermi velocity at the $d$-wave node (the Fermi surface along the $(0,0) - (\pi,\pi)$ direction) matches that determined from the ARPES dispersion (1.6 eV Å) assuming a bare velocity of 3 eV Å from band theory. The renormalization factor (3/1.6) can be obtained as

$$Z = \left[ 1 - \frac{\partial \Sigma}{\partial \omega} \right]_{\omega = 0}$$

(5)

where $\Sigma$ is the real part of the fermion self-energy, and we assume $Z$ arises from the same interaction $V$ as above:

$$\Sigma(k,i\omega_n) = T \sum_{q,m} V(k-q,i\omega_n-i\omega_m) G_m(q,i\omega_m)$$

(6)

where $G_m$ is the bare fermion Green’s function

$$G_m^{-1}(k,i\omega_n) = i\omega_n - \xi_k$$

(7)

and $\xi_k$ is the bare dispersion (obtained from a tight-binding fit to the ARPES dispersion by multiplying by the renormalization factor 3/1.6 mentioned above). The real part of $\Sigma$ used to obtain $Z$ is obtained from the real frequency version of equation (6). For the case shown in Fig. 1a, $\bar{U}$ per changes out to be the same as $U$. However, for the case shown in Fig. 1b, we must increase $\bar{U}$ to 928 meV to obtain the same $Z$ for the first case.

We now turn to the pairing problem. The anomalous (pairing) self-energy in the singlet channel is

$$-\frac{T}{N} \sum_{k',\omega_n} V(k-k',\omega_n-i\omega_m) \mathcal{P}_0(k',\omega_n) \Phi(k',\omega_n) = \Phi(k,\omega_n)$$

(8)

with the pairing kernel $\mathcal{P}_0$

$$\mathcal{P}_0(k',\omega_n) = G(k',\omega_n) G(-k',-i\omega_m)$$

(9)

and $G$ is the fully dressed Green’s function, which is formally determined by including the self-energy correction equation (6) in a completely self-consistent approach. Instead, we obtain $G$ from the experimental spectral functions. This is related to the approach of ref. 16 where INS data were used instead.

**Figure 1** Dynamic spin susceptibility. a, b, Imaginary part of the susceptibility $\chi''$ constructed from experimental Green’s functions versus energy along the nodal direction for $U = 860$ meV (a) and $U = 800$ meV (b), where $\bar{U}$ is an effective screened Hubbard interaction.

**Figure 2** No superconductivity in the presence of a non-pairing pseudogap. The leading $d$-wave eigenvalue $\lambda_{\text{max}}$ as a function of temperature, using experimental spectral functions. FSR (thin curves) are from Fermi-surface-restricted calculations obtained from equation (10); FBZ (thick curves) are from full Brillouin zone calculations from equation (8). The interaction parameters $U$ and $\bar{U}$ are indicated in units of millielelectrovolts. Note that $\lambda_{\text{max}}$ is less than unity (that is, no solution to the superconducting gap equation).
temperature. We have verified that at even higher temperatures, $\Delta_{\text{max}}$ reaches a maximum, and then begins to fall, with the more realistic second case ($U = 800$ meV) always remaining below unity. Similar behaviour for the $T$ dependence of $\lambda$ was reported in ref. 17 where a Mott gap is present.

To understand this behaviour, we now turn to some analytic calculations. To a good approximation, we can approximate $V$ for the $d$-wave case in the weak-coupling BCS limit as

$$V(\phi, \phi') = V \cos(2\phi) \cos(2\phi')$$  \hspace{1cm} (12)

and assume an isotropic density of states $N_0$ over the Fermi surface coming from $\xi$. The weak-coupling equation for $T_\text{c}$ is

$$T \sum n \int_0^{2\pi} \frac{d\phi}{2\pi} V \cos^2(2\phi) P_n(\phi, i\omega_n) = 1$$  \hspace{1cm} (13)

For $G$ we use a phenomenological form that is a good representation of ARPES data\cite{note2}

$$G(k, i\omega_n) = \frac{\omega_n + i\Gamma \text{sgn}(\omega_n) + \xi}{(\omega_n + i\Gamma \text{sgn}(\omega_n))^2 + \xi^2 + \Delta_c^2}$$  \hspace{1cm} (14)

Here $\Gamma$ is the broadening and $\Delta_c$ is the anisotropic pseudogap, which, consistent with ARPES, is assumed to have a $d$-wave anisotropy. On the Fermi surface, this can be approximated as $\Delta_c \cos(2\phi)$. The pairing kernel can now be analytically derived

$$P_\phi(\phi, i\omega_n) = \pi N_0 \left[ \frac{1}{\sqrt{\omega_n^2 + \Delta_c^2}} - \frac{\Delta_c^2}{2(\omega_n^2 + \Delta_c^2)^{3/2}} \right]$$  \hspace{1cm} (15)

Here $\omega_n$ is $\omega_n + \text{sgn}(\omega_n) \Gamma$. To obtain an analytic approximation, we replace the sum $T \sum n$ by an integral $\int d\omega/2\pi$, using the Euler–Maclaurin formula\cite{note2} for low temperatures in equation (13) and rewrite the condition for $T_\text{c}$ as

$$1 = N_0 V \int_0^\infty \frac{d\omega}{\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} \cos^2 2\phi \left[ \frac{1}{\sqrt{\omega^2 + \Delta_c^2}} - \frac{\Delta_c^2}{2(\omega^2 + \Delta_c^2)^{3/2}} \right]$$  \hspace{1cm} (16)

The integral over $\omega$ can be carried out analytically. The second term is convergent, so we can integrate it to $\infty$. For the first term, we use a BCS cutoff energy $\omega_c$ and we assume $\omega_n \gg T_c$, $\Delta_c$, $\Gamma$ and in the low-$T$ limit we get

$$1 \simeq N_0 V \int_0^{2\pi} \frac{d\phi}{2\pi} \cos^2 2\phi \left[ \log \sqrt{\frac{\omega}{\gamma + \pi T} + \sqrt{\left(\Gamma + \pi T\right)^2 + \Delta_c^2}} \right]$$

By examining equation (17), we can clearly see that the logarithmic divergence of the first term is cutoff by both $\Gamma$ and $\Delta_c$, so a solution is no longer guaranteed. We can estimate the critical values of the inverse lifetime and pseudogap to kill superconductivity at $T = 0$ for limiting cases. In the clean limit with $\Gamma = 0$, $\Delta_c = 2\pi e^{-(\gamma + 1)} T_c$, where $\gamma$ is Euler’s constant and $T_c$ is $T_c$. For $\Delta_c = 0$, $\Gamma = 0$. Without a pseudogap, we find a critical inverse lifetime $T_{\text{c}}$ of $\pi e^\gamma T_\text{c}$ (Abrikosov–Gor’kov\cite{note2}). Figure 3 shows the numerically evaluated left-hand side of equation (13) (denoted as $i_{\text{num}}$) as a function of temperature for various $\Delta_c$, with the variation of $T_c$ with $\Delta_c$ or $\Gamma$ shown in the inset. One clearly sees that the logarithmic divergence is cut off as $\Delta_c$ increases, leading to a maximum in $\lambda$ at a particular temperature. Once this maximum falls below unity, no superconducting solution exists.

To show that our findings are general and not limited to weak-coupling assumptions, we consider a calculation based on a $V$ derived from a phenomenological form for $\chi$ (refs 21,22):

$$V(k, \Omega) = \frac{3}{2} \frac{g_s^2}{\xi_{\text{AF}}^2} + 2 + \cos k_x + \cos k_y = \frac{g_s^2}{\xi_{\text{AF}}^2}$$  \hspace{1cm} (18)

where $g_s$ is the coupling between fermions and spin fluctuations, $\xi_{\text{AF}}$ is the antiferromagnetic coherence length, $\Omega_{\text{c}}$ is the characteristic spin fluctuation energy scale, and $\chi_{\text{c}}$ is the static susceptibility at the commensurate vector $Q = (\pi, \pi)$. For illustrative purposes, we take $g_s^2/\xi_{\text{AF}}^2 = 0.27$ eV, $\xi_{\text{AF}} = 10$, $\Omega_{\text{c}} = 0.4$ eV with a cutoff energy $\chi_{\text{c}} = 0.4$ eV, although we have studied a variety of parameter sets (particularly variation of $\xi_{\text{AF}}$). In general, these parameters are temperature dependent, but for simplicity we ignore this. We use the same model $G$ from above that was used to study the weak-coupling limit. Figure 4 shows the variation of $T_c$ with the pseudogap for different values of $\Gamma$. As in the weak coupling case, $\Delta_c$ and $\Gamma$ suppress $T_c$. As expected, the size of $\Delta_c$ needed to destroy superconductivity is of order $T_{\text{c}}$. In that context, it should be remarked for the experimental data used in Fig. 2, the ratio of $\Delta_c$ to $T_{\text{c}}$ is about 6.4. The behaviour of $\lambda$ with temperature is similar to the weak-coupling case, as illustrated in Fig. 4. Again, a solution fails to appear once the temperature maximum of $\lambda$ falls below unity. The same behaviour was found in the Fermi-surface-restricted results presented in Fig. 2. In turn, use of our phenomenological $G$ and $\chi$ in the full Brillouin zone formalism leads to similar behaviour to Fig. 2 as well, with weakly temperature dependent $\lambda$ having values much less than unity (see Supplementary Information).

Over much of the doping–temperature phase diagram of the cuprates, ARPES reveals strongly lifetime-broadened features with
Figure 4 | Suppression of $T_c$ by a non-pairing pseudogap (strong coupling). Inset: $d$-wave transition temperature as a function of the pseudogap $\Delta_0$ for various values of the inverse lifetime $\Gamma$. The energy scales are normalized to the value of $T_{c0}$ (4.8 meV) for $\Delta_0, \Gamma = 0$. The dashed curve is the variation of the temperature maximum in $\lambda$ for the $\Gamma = 0$ case. Once the $T_c$ curve intersects this, no solution to the superconducting gap equation exists. This is evident from the main panel, where the temperature dependence of $\Lambda_{\text{max}}$ is plotted for various $\Delta_0$ from 0 to 1.5$T_{c0}$. For the dashed curve and below, no solution exists.

a large pseudogap above $T_c$. Despite this, $T_c$ is large except under extreme underdoping conditions. The work presented above indicates that for such a large pseudogap, there should be no superconducting solution. In our phenomenological study, this can be partially mitigated by using model Green’s functions\(^{18}\) that have Fermi surfaces in the pseudogap phase (as occurs with charge ordering, spin ordering, or more phenomenological considerations such as those of ref. 23). On the other hand, the fact that we find this same behaviour using experimental Green’s functions indicates that this is a general issue, not specific to any particular model.

There is a way out of this dilemma. If the pseudogap were due to pairing, then all of the above conclusions would be invalidated. In this case, the mean field $T_c$ would actually be the temperature at which $\Delta_0$ becomes non-zero (that is, $T^*_c$), with the true $T_c$ suppressed from this owing to fluctuations. In a preformed pairs picture, $T_c$ would be controlled by the phase stiffness of the pairs\(^2\), whereas in resonating valence bond theory, it would be controlled by the coherence temperature of the doped holes\(^4\). Regardless, our results are in strong support for such models. ARPES (refs 24,25) and scanning tunneling microscopy\(^{26,27}\) results are consistent with a pairing pseudogap, because the observed spectra associated with the antinodal region of the zone have a minimum at zero bias as would be expected if the gap were due to pairing (local or otherwise). This does not mean that charge and/or spin ordering does not occur in the pseudogap phase, it is just that our results are consistent with these phenomena not being responsible for the pseudogap itself.

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Author contributions

V.M. and M.R.N. conceived the project, carried out the work, and wrote the paper. U.C. and J.C.C. provided the data as well as ideas for the data analysis.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.R.N.

Competing financial interests

The authors declare no competing financial interests.