Anomalous Nernst Effect in the Vortex-Liquid Phase of High-Temperature Superconductors by Layer Decoupling

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Abstract. – Linear diamagnetism is predicted in the vortex-liquid phase of layered superconductors at temperatures just below the mean-field phase transition on the basis of a high-temperature analysis of the corresponding frustrated XY model. The diamagnetic susceptibility, and the Nernst signal by implication, is found to vanish with temperature as $(T_c - T)^3$ in the vicinity of the mean-field transition at $T_c$. Quantitative agreement with recent experimental observations of a diamagnetic signal in the vortex-liquid phase of high-temperature superconductors is obtained.

Introduction. – The Abrikosov vortex lattice melts into an extended vortex-liquid phase in high-temperature superconductors subject to an external magnetic field oriented perpendicular to the conducting copper-oxygen planes that make them up. The large size in temperature and magnetic field of the vortex-liquid phase can be attributed to such layer anisotropy. A cross-over from a vortex-line liquid at temperatures just above the melting point of the Abrikosov vortex lattice to a decoupled vortex liquid at higher temperature that shows negligible correlations of the superconducting order parameter across layers is predicted if the vortex lattice in isolated layers melts through a continuous or a weakly first-order phase transition. Such dimensional cross-over is observed experimentally in electronic transport studies of the vortex-liquid phase in moderately anisotropic high-temperature superconductors. The Abrikosov vortex lattice is predicted to sublimate directly into a decoupled vortex liquid at large enough layer anisotropy, on the other hand, if the vortex lattice in isolated layers melts through a first-order phase transition. Electronic transport studies of the mixed phase in extremely layered high-temperature superconductors are consistent with the last sublimation scenario.

An anomalous Nernst effect is also observed in the vortex-liquid phase of high-temperature superconductors. In particular, a gradient in temperature along the copper-oxygen planes generates an electric field perpendicular to it along the copper-oxygen planes as well. The low-temperature onset of the anomalous Nernst signal coincides with the melting point of the Abrikosov vortex lattice, while the high-temperature onset can lie above the critical temperature of the superconducting state at zero field. The authors of ref. argue that this
effect is principally due to vortex excitations in the mixed phase of high-temperature superconductors. It is then tempting to identify the cross-over between three-dimensional (3D) and two-dimensional (2D) vortex-liquid behavior that is predicted for layered superconductors in certain instances\[6\] with the peak in the Nernst signal. The fact that anomalous Nernst signals are also observed in the vortex-liquid phase of extremely layered high-temperature superconductors that do not show the former dimensional cross-over\[8\][9] rules out that interpretation, however.

The anomalous Nernst effect observed in the vortex-liquid phase of high-temperature superconductors may instead be principally due to vortex excitations in copper-oxygen planes that are virtually isolated from one another\[9\]. In this Letter, the theoretical consequences of that proposal are examined through a duality analysis of the uniformly frustrated $XY$ model for the mixed phase of extremely type-II superconductors\[5\][6]. We find first that weak collective pinning of the vortex lattice results in a melting/decoupling temperature that does not extrapolate to the mean-field transition in zero field. Instead, a relatively big region of vortex liquid that is stabilized by random pinning centers is predicted to exist at temperatures below the mean-field transition. Second, a high-temperature expansion of the uniformly frustrated $XY$ model yields linear diamagnetism at temperatures just below the mean-field transition. The temperature dependence of the predicted equilibrium magnetization is found to agree quantitatively with recent experimental reports of a diamagnetic signal extracted from the vortex-liquid phase of high-temperature superconductors\[10\]. Last, we emphasize that an anomalous Nernst effect is generally expected inside of the vortex liquid phase\[9\], where it tracks the temperature dependence shown by the diamagnetism in the vicinity of the mean-field phase transition.

Vortex-Lattice Melting/Decoupling. – The $XY$ model with uniform frustration is the minimum theoretical description of vortex matter in extremely type-II superconductors. Both fluctuations of the magnetic induction and of the magnitude of the superconducting order parameter are neglected within this approximation. The model hence is valid deep inside the interior of the mixed phase. Its thermodynamics is determined by the superfluid kinetic energy
\[
E^{(3)}_{XY} = - \sum_r \sum_{\mu=x,y,z} J_\mu \cos[\Delta_\mu \phi - A_\mu]|r|
\]
where $\mu$ is a functional of the phase of the superconducting order parameter, $e^{i\phi}$, over the cubic lattice, $r$. Here, $J_z$ and $J_y$ denote the local phase rigidities over nearest-neighbor links within layers. These are equal and constant, except over links in the vicinity of a pinning center. The Josephson coupling across adjacent layers, $J_z$, shall be assumed to be constant and weak. It can be parameterized by $J_z = J_0/\gamma^2$, where $J_0$ is the Gaussian stiffness of the $XY$ model for each layer in isolation, and where $\gamma$ is the model anisotropy parameter. The vector potential $A_\mu = (0, 2\pi f x/a, 0)$ represents the magnetic induction oriented perpendicular to the layers, $B_\perp = \Phi_0 f/a^2$. Here $a$ denotes the square lattice constant, which is of order the coherence length of the Cooper pairs, $\Phi_0$ denotes the flux quantum, and $f$ denotes the concentration of vortices per site.

The thermal/bulk average of the Josephson coupling between adjacent layers is given by the expression\[5\][11]
\[
\langle \cos \phi_{l,l+1} \rangle \equiv y_0 \sum_{l=1} C_l(0, 1) \cdot C_{l+1}^*(0, 1) e^{i[A_z(l+1) - A_z(l)]}
\]
in the decoupled vortex liquid to lowest order in the fugacity $y_0 = J_z/2k_BT$. Here $\phi_{l,l+1}(\vec{r}) = \phi(\vec{r}, l+1) - \phi(\vec{r}, l) - A_z(\vec{r})$ is the gauge-invariant phase difference across adjacent layers $l$ and...
Fig. 1 – Schematic profile of the density of pinned vortices versus the total density of vortices within an isolated layer. Pinning centers are assumed not to crowd together.

\[ l + 1, \text{ and } C_l(1, 2) = \langle e^{-i\phi_0^{(1)}(1)}e^{i\phi_0^{(2)}(1)} \rangle_0 \] is the autocorrelation function of the superconducting order parameter within layer \( l \) in isolation (\( J_z = 0 \)). Short-range correlations on the scale of \( \xi_{2D} \) following \( C_l(1, 2) = \gamma_0 \rho^{-r_{1,2}/\xi_{2D}} e^{-i\phi_0^{(1)}(1)}e^{i\phi_0^{(2)}(2)} \) yields the result \[ \langle \cos \phi_{l,l+1} \rangle \sim g_0^2 \left( J_0/k_B T \right) \left( (\xi_{\phi})^{-1} - 1/\Lambda_0 \right)^2 \] for the inter-layer “cosine” \[ \text{(2)} \]. Here, \( l_\phi \) is a quenched disorder scale for the vortex lattices across adjacent pairs of isolated layers that appears through the autocorrelation

\[ \exp[i \phi^{(0)}_{l,l+1}(1)] \cdot \exp[-i \phi^{(0)}_{l,l+1}(2)] = e^{-r_{1,2}/l_\phi}. \] 

of the quenched inter-layer phase difference, \( \phi^{(0)}_{l,l+1}(\vec{r}) = \phi_0(\vec{r}, l + 1) - \phi_0(\vec{r}, l) - A_\phi(\vec{r}) \). It is set by the density of dislocations quenched into the 2D vortex lattices found in each layer at zero temperature in the present case of uncorrelated pinning centers. Also, above we have \( \xi_{\phi} = \xi_{2D}/2 \) and the Josephson penetration depth \( \Lambda_0 = \gamma/a \).

In the absence of inter-layer coupling, arbitrarily weak random point pins result in a stack of 2D vortex lattices with dislocations quenched in \[ \text{(3)} \]. Let us assume that each 2D vortex lattice is in a hexatic vortex glass state \[ \text{(4)} \], such that dislocations do not arrange themselves into grain boundaries. The quenched disorder scale \( l_\phi \) that renormalizes down the interlayer Josephson coupling is then set by the density of such dislocations \[ \text{(4)} \]. Recent theoretical calculations find that each isolated layer shows a net superfluid density near zero temperature in the collective pinning regime, where the number of dislocations quenched into each 2D vortex lattice is small in comparison to the number of pinned vortices \[ \text{(5)} \]. Application of collective pinning theory to the 2D vortex lattices found in isolated layers yields a density of quenched-in dislocations identical to the density of Larkin domains \[ \text{(6)} \]. Here the critical state is assumed to be limited by plastic creep of Larkin domains by an elementary burgers vector \( \vec{b} \) of the 2D vortex lattice. Consider now the limit of weak pinning centers that
do not crowd together: \( f_p \to 0 \) and \( \pi r_p^2 \cdot n_0 \ll 1 \), respectively, where \( n_0 \) denotes the density of pinning centers per layer, and where \( r_p \) denotes the range of each pinning center. Simple probabilistic considerations then yield the identity \( n_p / n_0 = \pi r_p^2 / a_{x}^2 \) between the fraction of occupied pinning centers and the ratio of the effective area of each pinning center to the area per vortex, \( a_{x}^2 = a^2 / f \). This yields the result \( n_p = (n_0 \cdot \pi r_p^2) n_{vx} \) for the density of pinned vortices\( ^{[10]} \), where \( n_{vx} = 1 / a_{x}^2 \) is the density of vortices per layer. Finally, substitution of the estimate \( n_0 = (\pi / 4) n_{vx} J_0 \) for the shear modulus\( ^{[17]} \) yields the result \( R_c^{-2} \sim (f_p r_p / J_0)^2 n_0 \) for the density of Larkin domains, which is independent of magnetic field. Note, however, that all of the above is valid only in the 2D collective pinning regime that exists at perpendicular magnetic fields above the threshold \( B_{cp}^{(2D)} \sim (f_p / J_0)^2 \Phi_0 \), in which case many vortices are pinned in each layer within a Larkin domain of dimensions \( R_c \times R_c \)\( ^{[11]} \). Single-vortex pinning exists at magnetic field below that threshold, on the other hand, in which case each Larkin domain contains only a single pinned vortex: \( n_p = R_c^{-2} \). Assembling the above suggests the profile for the density of pinned vortices per layer versus the density of vortices that is depicted by fig.\( ^{[1]} \) It implies that the quenched disorder scale \( l_\phi \sim R_c \) is independent of magnetic fields above the threshold \( B_L^{(2D)} \sim \Phi_0 / R_c^2 \).

We are finally in a position to determine the melting/decoupling line of the 3D vortex lattice at temperatures outside of the 2D critical regime, \( \xi_{2D} \sim a_{vx} \), at big enough perpendicular magnetic fields such that Larkin domains can be defined, \( B \perp \gg B_L^{(2D)} \). The identification of the separation between dislocations quenched into each 2D vortex lattice with the 2D Larkin scale\( ^{[15]} \), \( l_\phi \sim R_c \), necessarily yields the inequality \( \xi_\phi \ll l_\phi \) in such case. At temperatures lying inside of the interval \( [T_m^{(2D)} , T_c^{(2D)}] \) bounded by melting of the 2D vortex-lattice and by the Kosterlitz-Thouless transition in isolated layers, yet lying outside of the 2D critical regime, a partial duality analysis of the pristine layered \( XY \) model with uniform frustration\( ^{[1]} \) finds a first-order melting/decoupling transition of the 3D vortex lattice at interlayer Josephson coupling\( ^{[5]} \)

\[
\langle \cos \phi_{l,l+1} \rangle \simeq 1/2.
\] (5)

The first-order nature of this melting/decoupling line and its coincidence with the contour defined above is consistent both with Monte Carlo simulations of the same model\( ^{[11]} \) and with elastic medium descriptions of the vortex lattice in layered superconductors\( ^{[4]} \). Observe now that the criterion \( ^{[4]} \) for first-order melting/decoupling should remain valid in the present regime of weak pinning such that \( l_\phi \gg \xi_\phi \). Substitution of expression \( ^{[4]} \) for the inter-layer “cosine” in the decoupled vortex liquid then yields the melting/decoupling line

\[
B_D \sim \sqrt{\Phi_0 / \Lambda_0^2} \left( \sqrt{B_{0}^{(0)}} - \sqrt{B_L^{(2D)}} \right)^2,
\] (6)

where \( B_{0}^{(0)} \sim g_0^2 (J_0 / k_B T) (\Phi_0 / \Lambda_0^2) \) is the melting/decoupling field in the pristine limit\( ^{[4]} \). These results are summarized by the phase diagram shown in fig.\( ^{[2]} \) The short sections of dashed and solid lines that emanate perpendicularly from the horizontal axis originate respectively from the decoupling cross-over\( ^{[6]} \) and the second-order phase transition shown by the layered \( XY \) model in the absence of uniform frustration (cf. ref.\( ^{[18]} \)). We conclude this section by observing that the melting/decoupling line does not extrapolate to the mean-field critical temperature at zero-field \( [J_0 (T_\phi) = 0] \) due to the presence of dislocations quenched into the weakly pinned vortex lattices found in isolated layers.

**Vortex-Liquid Diamagnetism.** – The phase diagram for the mixed phase of layered superconductors shown by fig.\( ^{[2]} \) implies a large region of vortex liquid in the vicinity of the meanfield transition at zero magnetic field because of the effects of random point pins.
In particular, the equilibrium diamagnetic susceptibility due to the emergence of Cooper pairs is well defined at temperatures inside of the window $[T_c, T_{c0}]$. The former quantity can be obtained from the uniformly frustrated $XY$ model in the vicinity of the mean-field transition via a high-temperature expansion in powers of the fugacity $z_0 = J/2k_BT$. In particular, a duality analysis yields that the corresponding partition function is approximated by

$$Z_{XY} \sim Z_0 + Z_4$$

as $z_0 \to 0$, where

$$Z_0 = \prod_{\langle ij \rangle \in \Box} \left(\frac{2}{\theta} \frac{J_{ij}}{k_BT}\right),$$

and

$$Z_4 = \sum_{\Box} \prod_{\langle ij \rangle \in \Box} t_1(J_{ij}/k_BT).$$

Here $t_1(x) = I_1(x)/I_0(x)$ is the ratio between a first-order and a zero-order modified Bessel function, which is approximately $t_1(x) \approx x/2$ for $|x| \ll 1$. Also, $\langle ij \rangle$ represents nearest-neighbor links within layers, and $\Box$ represents elementary plaquettes within layers. The equilibrium magnetization is given by

$$M_\perp = -\frac{2\pi}{\Phi_0}(J_4/d)(J_4/2k_BT)^3\sin(2\pi f),$$

where $J_4 = \left(\prod_{\langle ij \rangle \in \Box} J_{ij}\right)^{1/4}$, and $d$ denotes the spacing between layers. The magnetization therefore varies linearly with vanishing magnetic field like $M_\perp = \chi H_{\perp}$, with a diamagnetic susceptibility

$$4\pi\chi = -\kappa^{-2}(J_0/2k_BT)^3(J_4/J_0)^4(a/\xi)^2.$$  

Here $\kappa = \lambda_L/\xi$ is the usual ratio of the London penetration depth to the coherence length of the Cooper pairs. The former is related to the Gaussian phase stiffness of each layer by $J_0 = \Phi_0^2d/16\pi^3\lambda_L^2$.

Non-linear diamagnetism is observed experimentally in the vortex liquid phase of the extremely layered high-temperature superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO), at temperatures just above the superconducting transition in zero field. Linear diamagnetism is displayed at yet higher temperature in the same samples, on the other hand. By Eq. (7), the uniformly frustrated $XY$ model predicts such linear diamagnetism, $M_\perp = \chi H_{\perp}$, at
perpendicular magnetic fields that are small compared to the upper-critical scale $\Phi_0/2\pi a^2$, at temperatures just below the mean-field phase transition. The corresponding diamagnetic susceptibility predicted by the $XY$ model is given by Eq. (8). Use of the relation quoted previously between the Gaussian phase rigidity within planes and the London penetration length in conjunction with physical parameters $\kappa = 100$, $\lambda_L(0) = 0.2\,\mu$m, and $d = 1.5\,\text{nm}$ appropriate for BSCCO yields the estimate

$$4\pi\chi/\mu_0 = -[(503/27)(n_s/n)]^3(J_4/J_0)^2(\alpha/\xi)^2\text{A/Tm}$$

for the diamagnetic susceptibility of that material in the vicinity of the mean-field phase transition. Here $n_s(T)/n = \lambda_L^2(0)/\lambda_L^2(T)$ is the superfluid fraction. The mean-field superfluid density expected from a pristine $d$-wave state in 2D is approximately $2/3$ the corresponding $s$-wave result in the vicinity of the meanfield transition at zero field$^{23}$: i.e., $n_s/n \cong (4/3)(1-t)$, where $t = T/T_{c0}$ is the reduced temperature. Equation (9) then implies that the diamagnetic susceptibility vanishes like $(1-t)^3$ with temperature as it approaches the mean-field transition. Figure 3 displays the cube-root of the diamagnetic signal extracted experimentally in ref. $^{10}$ from an underdoped sample of BSCCO with $T_c = 50^\circ\text{K}$, in perpendicular magnetic field $H_\perp = 14\,\text{T}$, as a function of temperature. The solid line is a fit to the linear diamagnetism, $M_\perp = \chi H_\perp$, predicted by the high-temperature expansion of the uniformly frustrated $XY$ model, Eq. (7), with $n_s/n = (4/3)t(1-t)$, $T_{c0} = 158^\circ\text{K}$, and with $XY$ model parameter $a = 0.28(J_0/J_4)^2\xi$. The success of the fit indicates that the onset of the diamagnetic signal observed in the vortex liquid phase of high-temperature superconductors reflects nothing other than the mean-field phase transition at which Cooper pairs emerge. The large suppression of $T_c$ compared to the meanfield transition temperature $T_{c0}$ obtained here can be accounted for by quenched disordering of the superconducting order parameter, which could be generic to under-doped high-temperature superconductors.$^{24}

**Anomalous Nernst Effect and Conclusions.** – A gradient in temperature along the layers in the vortex liquid phase of high-temperature superconductors generates a voltage in the perpendicular direction within the layers.$^9$ In particular, the Nernst signal defined by the
ratio \( e_y = E_y/\partial_x T \) between the electric field that is generated and the gradient in temperature peaks inside of the vortex liquid. Standard transport theory yields the identity [9]

\[ e_y = \rho_x \cdot \alpha_{xy} \]  

(10)

between the Nernst signal and the product of the flux-flow electrical resistivity \( \rho_x \) with the off-diagonal Peltier coefficient \( \alpha_{xy} \). Also, application of Ginzburg-Landau theory for the superconducting order parameter yields the estimate [25]

\[ \alpha_{xy} \approx \beta M_\perp \]  

(11)

for the Peltier coefficient near the mean-field transition, where \( \beta \) is of order \( T^{-1} \). Observe now that the flux-flow resistance increases with temperature in the vortex liquid, while the equilibrium magnetization decreases with temperature there [cf. Eq. 8]. Substitution of the estimate (11) into the identity (10) then yields (i) that the low-temperature onset of the anomalous Nernst signal is given by the melting/decoupling temperature of the vortex lattice. Also, the linear diamagnetism [8] extracted from the high-temperature regime of the frustrated XY model implies (ii) that the anomalous Nernst signal vanishes with temperature at the mean-field transition as \( (T_{c0} - T)^3 \). Where exactly the Nernst signal peaks inside of the vortex-liquid phase depends on how pinning affects the flux-flow resistance [22], which is beyond the scope of the paper.

In conclusion, a high-temperature analysis of the layered XY model with uniform frustration finds that the simultaneous onset of linear diamagnetism and of an anomalous Nernst effect in the normal phase of high-temperature superconductors [10] can be identified with the mean-field transition for Cooper pairing. The low-temperature onset of the anomalous Nernst signal at the melting/decoupling line of the vortex lattice was also found to be depressed substantially by the presence of dislocations quenched into the vortex lattice in isolated layers.

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