An Adaptive Robust Attitude Tracking Control of Quadrotor UAV with the Modified Rodrigues Parameter

Jingxin Dou1 and Bangchun Wen2

Abstract
In this paper, a novel adaptive attitude tracking control method is investigated to enhance tracking performance and robustness for a quadrotor unmanned aerial vehicle (UAV) against the modeling uncertainties and external disturbances. First, the attitude dynamics of a quadrotor UAV based on the modified Rodrigues parameter (MRP) is derived. Then, an adaptive dynamic surface technique with robust integral of the sign of the error (RISE) control approach are designed to improve the tracking performance and robustness of the quadrotor attitude control system suffering from the uncertainties and disturbances. A prescribed performance function is employed to improve the tracking performance of the quadrotor UAV by restraining the tracking errors range. The stability analysis proves the presented control scheme can guarantee all the close-loop control system signals are bounded and control system is uniformly ultimately bounded. Simulation results are carried out to demonstrate the superior and effective performance of the presented control method.

Keywords
Quadrotor, attitude tracking control, modified Rodrigues parameter, prescribed performance, dynamic surface, RISE

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Introduction
Quadrotor UAV has four rotors with symmetrical distribution and simple mechanical structure. Quadrotor UAV has the characteristics of vertical take-off and landing, fixed hover and super maneuvering flight, and it has been applied in various fields of military and civilian. However, the quadrotor UAV is a typical underactuated system, and it is susceptible to disturbances in flight.1,2 For high flight performance, a variety of control schemes have been designed and developed for the quadrotor UAV.3–5 The tracking problems of quadrotor UAV were widely drawn attention from lots of researchers by designing various control schemes including proportional-integral-derivative (PID),6,7 liner quadratic regulator,8 feedback linearization,9 and so on.10–12

The control performances of linear controller is poor when the quadrotor UAV is suffering from aggressive maneuvers operation or unknown disturbances.13,14 Sliding mode control, which is able to reject and eliminate the disturbances, was developed for quadrotor UAV.15,16 In Zheng et al.,17 a second order sliding mode controller was designed to track the position and attitude of a quadrotor.17 In Reinoso et al.,18 a sliding mode control was employed to hold maintaining stability of a quadrotor under different operating scenarios.18 In Wang and Wan,19 a sliding mode attitude controller based on proportional integral observer was designed for a quadrotor UAV to improve the robustness.19 In Xuan-Mung and Hong,20 a robust backstepping saturated controller based on an extended state observer was carried out for a quadrotor UAV to solving the trajectory tracking problem.20

For the robustness of the control system subjecting to modeling uncertainties and external disturbances, some adaptive control schemes have been attracted a great interests of researchers to deal with modeling uncertainties and external disturbances for the quadrotor UAV.21–23 In Moussa and Mohamed,24 an adaptive nonsingular fast terminal sliding mode tracking controller was designed for a quadrotor UAV.24 In

1School of Mechanical Electronic & Information Engineering, Putian University, Putian, PR China
2School of Mechanical Engineering and Automation, Northeastern University, Shenyang, PR China

Corresponding author:
Jingxin Dou, School of Mechanical Electronic & Information Engineering, Putian University, No. 1133, Xueyuan Middle Street, Chengxiang District, Putian, 351100, PR China.
Email: doujingxin@163.com

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Barghandan et al.,\textsuperscript{25} an adaptive fuzzy sliding mode controller was carried out for attitude stabilization of the quadrotor UAV.\textsuperscript{25} In fact, the problems of the non-parametric and parametric uncertainties need to be taken into account simultaneously in the trajectory tracking of the quadrotor UAV.\textsuperscript{26,27} For this purpose, an adaptive global fuzzy sliding mode controller was developed for a quadrotor UAV.\textsuperscript{28} In Wang et al.,\textsuperscript{29} an adaptive robust backstepping controller and a global sliding mode controller was developed respectively for attitude stabilization and position tracking of the quadrotor UAV.\textsuperscript{29} An adaptive backstepping horizontal controller was designed to solve trajectory tracking problem for a quadrotor UAV under uncertain parameters.\textsuperscript{30}

Based on the above literatures and existed researches, the backstepping control and sliding mode control are the popular and common control arithmetic for quadrotor UAV.\textsuperscript{31,32} The backstepping control is highly effective to deal with the cascaded structure of the quadrotor UAV, and the sliding mode control is adept at rejecting the disturbances.\textsuperscript{33–35} To take advantage of their strengths, lots of researches were carried out by combining backstepping control with sliding mode control, and the robustness of the proposed control approaches was obvious improved.\textsuperscript{36–38} In Moussa and Mohamed,\textsuperscript{39} an adaptive backstepping fast terminal sliding mode controller was developed for a quadrotor UAV under the parametric uncertainties and external disturbances.\textsuperscript{39} In Basri,\textsuperscript{40} an adaptive backstepping sliding mode controller was designed for stabilizing and tracking of quadrotor UAV.\textsuperscript{40}

According to the above literatures, it is obtained that the tracking problems of the quadrotor UAV under the disturbances were improved. Nevertheless, the backstepping control and sliding mode control have obvious disadvantages. The backstepping control has the problem of item number expansion during the derivation of the virtual controller, and the sliding mode control has chattering phenomenon with switching function. The dynamic surface control can avoid the number expansion problem of the backstepping control by using first-order filter which calculates the virtual controller. The continuous robust integral of the sign of the error control approach can remarkably accommodate for smooth disturbances and realize asymptotic stability.

Motivated by the above discussions, the dynamic surface control (DSC) approach combine with the robust integral of the sign of the error (RISE) are designed for a quadrotor UAV attitude control system under the disturbances to improve the tracking performance in this paper. Compared with the aforementioned works, the principal theory contributions of this paper are summarized as follows:

1. Compared to pre-existing works, the purpose of this paper aims to improve the problem of the robust tracking performance for a quadrotor UAV attitude control system under modeling uncertainties and external disturbances. To deal with the problem, an adaptive dynamic surface robust integral of the sign of the error controller with the prescribed performance is designed to achieve high attitude tracking performance for the quadrotor UAV.

2. For reaching the high tracking performance of the quadrotor attitude control system, the transient performances are significance for holding the stabilization. A prescribed performance function is designed to hold the transient performance of control system in this paper. With the prescribed performance, the control system can quickly return from the transient state to the stable state, and the track errors can be limited in the predetermined ranges, and the precision of tracking performance can be hold.

3. For highly effective to deal with the cascaded structure of the quadrotor UAV, the dynamic surface control approach is used to avoid the problem of the expansion of the differential terms of the backstepping control. For rejecting the uncertainties and external disturbances and avoiding the chattering phenomenon, the robust integral of the sign of the error control scheme is combined to achieve high tracking performance, which can outstandingly reject the smooth disturbances and achieve asymptotic stability. An adaptive function is carried out to compensate the errors between the RISE term and the disturbances, which can reduce the efforts of the feedback gains of the RISE.

The paper is organized as follows. The quadrotor attitude dynamic based on the MRP is presented in Section 2. In the Section 3, the designed procedure and the stability analyze of the present controller are presented. The numerical simulation results are carried out in the Section 4. Finally, a conclusion is drawn in the Section 5.

The attitude dynamics of the quadrotor UAV based on the MRP

The quaternion often is used to describe the attitude dynamics of the aircraft, and it has applied to the practical projects. The quaternion has much advantages such as high computing precision, overcoming the singularity in the calculation and so on. The quaternion can be described as follows:

\[ q = [q_1 \ q_v]^T = \begin{bmatrix} \cos(\frac{\xi}{2}) & \sin(\frac{\xi}{2}) n^T \end{bmatrix}^T \]  \hspace{1cm} (1)

where \( q_1 \) is the quaternion scaler, \( q_v = [q_{i2} \ q_{i3} \ q_{i4}]^T \) is the quaternion vector, \( \sigma \) is the rotation angle, \( n = [n_1 \ n_2 \ n_3]^T \) is the directional vector of the rotation axis.
The lacks of the quaternion which is used to describe attitude dynamics are that the four elements of the quaternion have constraints, and the elements are not independent of each other. Only three independent Euler angles are needed to determine the position of the three-dimensional space, hence, there is redundancy in attitude calculate by using the quaternion. For achieving three independent parameters to describe the attitude calculate by using the quaternion. For achieving three independent parameters to describe the attitude, the modified Rodrigues parameter is presented and defined as follows:

\[
\eta = \frac{q_v}{1 + q_1} = \frac{\sin(\frac{\gamma}{2})n}{1 + \cos(\frac{\gamma}{2})} = \tan\left(\frac{\gamma}{4}\right)n
\] (2)

where \( \eta = [\eta_1 \ \eta_2 \ \eta_3]^T \).

The attitude dynamics of the quadrotor UAV based on the modified Rodrigues parameter is presented as follows:

\[
\dot{\eta} = \frac{1}{4} \left[ (1 - \eta^T \eta) J_3 + 2 \eta^\times + 2 \eta \eta^T \right] \omega
\]

\[
J_\omega = - \omega^\times J_\omega + U + D
\] (3)

where \( I_3 \in \mathbb{R}^{3 \times 3} \) denotes the unit matrix, \( \omega = [\omega_1 \ \omega_2 \ \omega_3]^T \) denotes the attitude angular velocity of the quadrotor UAV in body coordinate, \( J \in \mathbb{R}^{3 \times 3} \) denotes the inertia matrix of the quadrotor, \( D = [D_1 \ D_2 \ D_3]^T \) denotes the disturbances include the parameters perturbation, unmodeled dynamics and external disturbances, \( U = [U_1 \ U_2 \ U_3]^T \) denotes the control input torque, \( \eta^\times \) and \( \omega^\times \) denote the skew symmetric matrix of \( \eta \) and \( \omega \) as follows respectively:

\[
\eta^\times = \begin{bmatrix}
0 & -\eta_3 & \eta_2 \\
\eta_3 & 0 & -\eta_1 \\
-\eta_2 & \eta_1 & 0
\end{bmatrix}, \quad \omega^\times = \begin{bmatrix}
0 & -\omega_3 & \omega_2 \\
\omega_3 & 0 & -\omega_1 \\
-\omega_2 & \omega_1 & 0
\end{bmatrix}
\] (4)

Define the auxiliary function \( P(\eta) = \frac{1}{4} [(1 - \eta^T \eta) I_3 + 2 \eta^\times + 2 \eta \eta^T] \), and \( P(\eta) \) is bounded.\(^{41,42}\) The first equation of (3) can be rewritten as follows:

\[
\dot{\eta} = P(\eta) \omega
\] (5)

The time derivative of (5) is given as follows:

\[
\dot{\omega} = \dot{P}^{-1} \dot{\eta} + P^{-1} \dot{\eta}
\] (6)

Multiply both sides of the second equation of (3) by \( P(\eta) J_\omega \),

\[
P \omega = - PJ^{-1} \omega \times J_\omega + PJ^{-1} U + PJ^{-1} D
\] (7)

Based on (5) and (6), the (7) can be rewritten as follows:

\[
\ddot{\eta} = \left[-PJ^{-1}(P^{-1} \eta)^\times JP^{-1} - \dot{P}^{-1}\right] \eta + PJ^{-1} U + PJ^{-1} D
\] (8)

where \( P(\eta) \) and \( \omega \) are bounded, hence, \( \dot{\eta} \) is also bounded. Under the assumptions that the control input torque \( U \) and the disturbances \( D \) are bounded, the right side of the formula (8) is bounded, and then it is easy to obtain that \( \ddot{\eta} \) is bounded.

Defining \( x_1 = \eta \) and \( x_2 = \dot{\eta} \), the attitude dynamics of the quadrotor UAV based on the MRP can be rewritten as follows:

\[
\dot{x}_1 = x_2
\]

\[
\dot{x}_2 = \left[-PJ^{-1}(P^{-1}x_2)^\times JP^{-1} - \dot{P}^{-1}\right] x_2 + PJ^{-1} U + PJ^{-1} D
\] (9)

where \( x_1 \) is the output of the control system (9).

**Control design**

*Prescribed Performance transformation*

Defining the tracking reference value \( x_{1id} \), and supposing that the first derivative and second derivative of the \( x_{1id} \) exist, the tracking error of the control system can be obtained as follows:

\[
e_1 = x_1 - x_{1id}
\] (10)

For ensuring that the tracking error \( e_1 \) is astricted within a prescribed performance bound, the constraint condition of the specified prescribed performance can be designed as follows:

\[-\rho e < e_1 < \bar{e} \]

where \( 0 < \rho \leq 1, \ \bar{e} \) is a monotone decreasing performance function \( \bar{e}(t) = (e_0 - \bar{e}_0)e^{-\theta t} + \bar{e}_0 \) such that \( 0 < \lim_{t \to 0} \bar{e} = \bar{e}_0 < e_0, \ \theta > 0, \ e_0 \geq |e_1(0)| \).

There is constraint on the prescribed performance condition (11) that make the calculation complicated. To avoid the problem, the attitude tracking error of the quadrotor can be designed by using an equivalent unconstrained prescribed performance condition as follows:

\[
e_1 = \bar{e} S(z_1)
\] (12)

where \( z_1 \) is transform error that represents the conversion from the constrained prescribed performance condition to the unconstrained prescribed performance condition, \( S(z_1) \) is a strictly increasing smooth function, and is presented as follows:
The detailed design processes are presented as follows:

\[
\begin{align*}
- \rho &< S(z_1) < 1 \\
S(0) & = 0 \\
\lim_{t \to \infty} S(z_1) & = -\rho, \lim_{t \to \infty} S(z_1) < 1
\end{align*}
\]

Based on (12) and (13), then transformed error \( z_1 \) can be obtained as follows:

\[
z_1 = \frac{1}{2} \ln \left( \frac{e_1 + \rho \hat{e}}{e - e_1} \right)
\]

The time derivative of (14) can be calculated as follows:

\[
\begin{align*}
\dot{z}_1 &= \frac{1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( \dot{e}_1 - \frac{e_1 \hat{e}}{\hat{e}} \right) \\
&= \frac{1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right) \\
&= \frac{1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\end{align*}
\]

The attitude control system (9) can be rewritten by replacing the first equation of (9) with corresponding equation (15) as follows:

\[
\dot{z}_1 = \frac{1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

\[
x_2 = \left[ -PJ^{-1}(P^Tz_2)^TJP^{-1} - PP^{-1} \right] x_2 + PJ^{-1}U + PJ^{-1}D
\]

**Tracking control design based on the RISE-DSC**

In this section, an adaptive robust attitude tracking control approach based on RISE-DSC is proposed. The detail design processes are presented as follows:

**Step 1:** Defining the tracking errors of the system (16),

\[
Z_1 = z_1
\]

Differentiating \( Z_1 \), the following equation can be obtained,

\[
\dot{Z}_1 = \dot{z}_1 = \frac{1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

Defining the Lyapunov function \( V_1 \) as follows:

\[
V_1 = \frac{1}{2} Z_1^T Z_1
\]

The time derivative of (19) can be calculated as follows:

\[
\dot{V}_1 = Z_1 \dot{Z}_1
\]

\[
= \frac{1}{2} Z_1 \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

For guaranteeing the Lyapunov function \( \dot{V}_1 \leq 0 \), designing the virtual control law as follows:

\[
x_2 = -c_1Z_1 + \dot{x}_1d + \frac{e_1 \hat{e}}{\hat{e}}
\]

where \( c_1 \) is the design constant.

The dynamic equation of the low pass first-order filter with the output \( \beta_2 \) and the input \( x_2 \) is presented as follows:

\[
\tau_2 \dot{\beta}_2 + \beta_2 = x_2, \quad \beta_2(0) = \dot{x}_2(0)
\]

where \( \tau_2 \) is the design constant.

The filter error can be obtained as follows:

\[
y_2 = \beta_2 - \dot{x}_2
\]

\[
y_2 = \beta_2 - \dot{x}_2 = \beta_2 + c_1Z_1 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}}
\]

Differentiating (23), the equation can be obtained as follows:

\[
y_2' = \dot{\beta}_2 - \dot{x}_2
\]

\[
y_2' = \dot{\beta}_2 - \dot{x}_2 = \frac{y_2}{\tau_2} + c_1 \frac{Z_1}{\tau_2} - \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \left( \dot{x}_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

\[
y_2' = \dot{\beta}_2 - \dot{x}_2 = \frac{y_2}{\tau_2} + \frac{c_1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

\[
y_2' = \dot{\beta}_2 - \dot{x}_2 = \frac{y_2}{\tau_2} + c_1 \frac{1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

\[
y_2' = \dot{\beta}_2 - \dot{x}_2 = \frac{y_2}{\tau_2} + \frac{c_1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]

\[
y_2' = \dot{\beta}_2 - \dot{x}_2 = \frac{y_2}{\tau_2} + \frac{c_1}{2} \left( \frac{1}{e_1 + \rho \hat{e} - e_1 - \hat{e}} \right) \left( x_2 - \dot{x}_1d - \frac{e_1 \hat{e}}{\hat{e}} \right)
\]
Designing the auxiliary function as follows:

\[ L_2(Z_1, Z_2, y_2, e_1, e_2, e_1, e_2, \hat{x}_1) = \frac{c_1}{2} \left( \frac{1}{e_1 + \rho e} - \frac{1}{e_1 + e} \right) (Z_2 + y_2 - e_1 Z_1) \]  

(25)

\[ \dot{e}_1 \hat{e} - \frac{e_1 \hat{e}}{e} + \frac{e_1 e}{e^2} - \hat{x}_1 \]

where \( L_2 \) is a continuous function.

Based on the (25), the equation (24) can be rewritten as follows:

\[ \left| y_2 + \frac{y_2}{\tau_2} \right| \leq L_2 \]  

(26)

**Step 2:** Designing the tracking error of the second equation of the control system (16) as follows:

\[ Z_2 = x_2 - \beta_2 \]  

(27)

Differentiating (27)

\[ \dot{Z}_2 = x_3 - \beta_2 \]

\[ = \left[ -PJ^{-1}(P^{-1}x_2)^T JP^{-1} - P \hat{P}^{-1} \right] x_2 \]

\[ + PJ^{-1} U + PJ^{-1} D - \beta_2 \]

\[ = \left[ -PJ^{-1}(P^{-1}x_2)^T JP^{-1} - P \hat{P}^{-1} \right] x_2 \]

\[ + PJ^{-1} U + PJ^{-1} F - \beta_2 + \mu \]  

(28)

where \( F \) is the RISE control term utilized to reject the effect of the disturbances as follows:

\[ F = k_1(Z_2 - Z_2(0)) - \int_0^t (k_1 Z_2 + k_2 \text{sign}(Z_2)) d\tau \]  

(29)

where \( k_1, k_2 \) are design constants respectively. \( \mu \) denotes the error between the RISE term and the disturbances, and \( \mu \) is bounded.

Defining the Lyapunov function \( V_2 \) as follows:

\[ V_2 = \frac{1}{2} Z_2^T Z_2 \]  

(30)

Differentiating (30)

\[ \dot{V}_2 = Z_2 \dot{Z}_2 = Z_2 \left[ -PJ^{-1}(P^{-1}x_2)^T JP^{-1} - P \hat{P}^{-1} \right] x_2 + PJ^{-1} U + PJ^{-1} F - \beta_2 + \mu \]  

(31)

For guaranteeing the Lyapunov function \( \dot{V}_2 \leq 0 \), designing the control law \( U \) as follows:

\[ U = PJ^{-1} \left[ -PJ^{-1}(P^{-1}x_2)^T JP^{-1} - P \hat{P}^{-1} \right] x_2 - PJ^{-1} F + \beta_2 - c_2 Z_2 - \mu \]  

(32)

where \( \mu \) denotes the estimate value of \( \mu \), and adaptive update law of \( \mu \) is

\[ \dot{\mu} = k_3 Z_2 \]  

(33)

For showing the design process clearly, the structure chart of the presented control approach strategy is depicted in Figure 1.

**Stability analysis**

Defining the bound closed sets as follows:

\[ \Omega_1 = \{ (x_{1d}, \dot{x}_{1d}, \ddot{x}_{1d}) : x_{1d}^2 + \dot{x}_{1d}^2 + \ddot{x}_{1d}^2 \leq \xi_1 \} \]

\[ \Omega_2 = \{ (Z_1, Z_2, y_2) : \| Z_1 \|^2 + \| Z_2 \|^2 + \| y_2 \|^2 \leq \xi_2 \} \]  

(34)

where \( \xi_1, \xi_2 \) represent given positive contents respectively.

**Theorem 1:** Consider the control system (16) under the compounded disturbance, the virtual controller and the control law are chosen with (21) and (32). With all initial bound conditions, there exist \( c_i(i = 1, 2) \) and \( \tau_2 \), then all the close-loop control system signals are bounded and control system is uniformly ultimately bounded. Moreover, the tracking error remains within the prescribed performance bound if \( c_i(i = 1, 2) \) and \( \tau_2 \) satisfy

\[ \begin{align*}
  c_1^2 &\geq s^3 + \gamma s^2 - 1 + 2 \gamma \\
  c_2^2 &\geq s^3 + \frac{1}{2} + \gamma
\end{align*} \]

where \( \gamma \) is the design constant, and \( s = \frac{1}{n - \rho e} - \frac{1}{n - \tau} \).

**Proof:** From the tacking errors \( Z_1, Z_2 \), virtual control \( x_2 \) and filter error \( y_2 \), and estimate error \( \hat{\eta} \), designing the Lyapunov function as follows:
The following equation can be obtained as follows:

\[ V = \frac{1}{2} Z_1^T Z_1 + \frac{1}{2} Z_2^T Z_2 + \frac{1}{2} y_2^2 + \frac{1}{2} \mu^T k_{31} \dot{\mu} \]

(35)

where \( \dot{\mu} = \mu - \mu_0 \).

When \( V = \xi_2 \), \( L_2 \) is bounded, and \( |L_2| \) exists the maximum value \( L_{2_{\text{max}}} \) on the set \( \Omega_1 \times \Omega_2 \).

For clarity and simplify. Differentiating (28), the following equation can be obtained as follows:

\[
\dot{V} = Z_1^T \dot{Z}_1 + Z_2^T \dot{Z}_2 + y_2^2 \dot{y}_2 - \mu^T k_{31} \mu
\]

\[ = \frac{1}{2} Z_1^T \dot{x}_1 \left( x_2 - x_{1_d} - \frac{e_i \dot{c}}{2} \right) + Z_2^T \dot{x}_2 - \beta_1
\]

\[ + y_2^2 \left( -\frac{y_2}{\tau_2} + L_2 \right) + \mu^T Z_2
\]

\[ = \frac{1}{2} Z_1^T \dot{x}_1 (Z_2 + y_2 - c_1 Z_1) + Z_2^T (-c_2 Z_2 + \mu)
\]

\[ + y_2^2 \left( -\frac{y_2}{\tau_2} + L_2 \right) + \mu^T Z_2
\]

(36)

The follow inequation can be obtained by Young's inequality

\[
\dot{V} \leq \frac{3}{4} \| Z_1 \|^2 + \frac{1}{2} s_1^2 \| Z_2 \|^2 + \frac{1}{2} s_2^2 \| y_2 \|^2 - \frac{1}{4} s_3^2 \| Z_1 \|^2
\]

\[ - \frac{3}{4} \| Z_2 \|^2 - \frac{1}{2} s_2^2 \| Z_2 \|^2 - \frac{1}{4} s_3^2 \| y_2 \|^2 + \frac{1}{2} \| L_2 \|^2
\]

\[ \leq \frac{3}{4} \left( \frac{1}{2} s_1^2 - \frac{1}{2} \right) \| Z_1 \|^2
\]

\[ + \left( \frac{1}{4} s_3^2 - \frac{1}{2} \right) \| y_2 \|^2 + \frac{1}{2} \| L_{2_{\text{max}}} \|^2
\]

\[ \leq \frac{3}{4} \left( \frac{1}{2} s_1^2 - \frac{1}{2} \right) \| Z_1 \|^2
\]

\[ + \left( \frac{1}{4} s_3^2 - \frac{1}{2} \right) \| y_2 \|^2 + \frac{1}{2} \| L_{2_{\text{max}}} \|^2
\]

(37)

Substituting (37) into (36), the follow inequation can be obtained:

\[
\dot{V} \leq -\gamma \| Z_1 \|^2 - \gamma \| Z_2 \|^2 - \gamma \| y_2 \|^2 + \frac{1}{2} \| L_{2_{\text{max}}} \|^2
\]

\[ = -2 \gamma V' + \frac{1}{2} \| L_{2_{\text{max}}} \|^2
\]

(39)

where \( V' = \| Z_1 \|^2 + \| Z_2 \|^2 + \| y_2 \|^2 \)

When \( V' = \xi_1 \), defining \( \gamma \geq \frac{1}{L_{2_{\text{max}}}^2} \), it can be obtained that \( V \leq -\gamma \xi_2 + \frac{1}{2} \| L_{2_{\text{max}}} \|^2 \). It also can be obtained that \( V(t) \leq \xi_2, \lim_{t \to \infty} V(t), t > 0 \). The follow inequation can be obtained:

\[
0 \leq V \leq \frac{1}{2} \gamma \| L_{2_{\text{max}}} \|^2 + \left( V(0) - \frac{1}{2} \gamma \| L_{2_{\text{max}}} \|^2 \right) e^{-2 \gamma t}
\]

(40)

Thus, all close-loop system signals are bounded, and tracking error \( Z_1 \) can converge to any small value by adjusting \( c_i(i = 1, 2) \) and \( \tau_2 \), and control system under the proposed control scheme is stability. It is obtained that the tracking error \( e_1 = x_1 - x_{1_d} \) always remains within the prescribed performance bound, when \( c_i(i = 1, 2) \) and \( \tau_2 \) satisfy the design conditions.

This proof is completed.

**Simulation and analysis**

For verifying the effectiveness and superiority and the proposed control scheme, some number simulation results are carried out. The desired trajectory of the modified Rodrigues parameters of the quadrotor UAV attitude is
chosen as: $x_{1d} = \begin{bmatrix} 0.8\sin(t) + 0.3\cos(t), & -0.8\sin(t) - 0.3 \\ -0.5\sin(t) \cos(t) \end{bmatrix}^T$. The initial values of the attitude angles and the angles velocities are given respectively as: $x_1(0) = [0.4, -0.4, -0.3]^T$ and $x_2(0) = [0, 0, 0]^T$. Considering the disturbances that the quadrotor UAV is subjected to during flight are complex, the disturbances $D$ is selected as $D = \begin{bmatrix} 2\sin(2t) + 4\cos(t), & 2\sin(2t) - 3\cos(t), & 10\sin(2t)\cos(2t) \end{bmatrix}^T$.

The performance functions are designed respectively as $(0.5 - 0.01)\times e^{-0.1t} + 0.01, -(0.5 - 0.01)\times e^{-0.1t} - 0.01, (0.7 - 0.05)\times e^{-0.1t} + 0.05$. In order to achieve asymptotic and smooth attitude tracking performance, the design parameters of the proposed control scheme must be tuned to reach the requirement of the tracking performance, and also satisfy the conditions of the inequality (34) for achieving the control objective. Hence, the optimization toolbox in the MATLAB is used to adjust the optimal parameter of the controller, and the model parameters of the quadrotor UAV are kept same as the Ref. 43. The main gains of the proposed controller are chosen as: $c_1 = [0.1, 0.15, 0.12]^T$, $c_2 = [7.5, 7.2, 17]^T$. The results of numerical simulation for the quadrotor UAV (16) and the proposed controller (32) are shown as follows.

In Figures 2 to 5, the simulation results are carried out by using the proposed controller. In Figure 1, the tracking results of the modified Rodrigues parameters of the quadrotor attitude are presented by using the proposed controller. As shown from the figures, the three output values of the quadrotor attitude control system can quickly track the desired trajectory. In Figure 3, the tracking errors of the modified Rodrigues parameters are presented. With the presented controller, the tracking errors can be limited between the performance function $\rho e$ and $\dot{e}$. The tracking errors $e_{11}$ fluctuates in a range of -0.03 to 0.015, and the tracking errors $e_{12}$ fluctuates in a range of -0.025 to 0.01, and the tracking errors $e_{13}$ fluctuates in a range of -0.1 to 0.15. The tracking errors results indicated that the tracking performance of quadrotor attitude control system with the proposed controller could achieve the design goal, when the desired trajectories are periodic time varying functions. In Figure 4, three angular velocities of the quadrotor attitude are presented. As shown from figures, the values of angular velocities can achieve the zero around after 1s. The angular velocities can be held around zero with the proposed control scheme during the smooth steadily increases or decreases of the disturbances, and the control system has been held in the stable state. The control inputs of the quadrotor attitude control system are presented in Figure 5.

In addition, for showing the superiority of the tracking and robustness performances of the proposed control scheme, comparisons with a traditional dynamic surface controller (DSC), a traditional sliding mode controller (SMC) and a DSC combining with SMC for the quadrotor attitude control system are carried out. The same disturbances $D$ are applied to the control system under different control schemes. In Figures 6 to 8, the tracking results of the modified Rodrigues parameters of the quadrotor attitude by using the traditional DSC, traditional SMC, and SMC-DSC are presented respectively. As shown from the figures, the
modified Rodrigues parameters of the quadrotor attitude can roughly track the variation trend of the design trajectory when the quadrotor attitude control system is suffering from the disturbances $D$. In Figure 9, the tracking errors under the different control schemes are presented. The tracking errors $e_{11}$ fluctuates in a range of -0.3 to 0.32 with the traditional DSC, and in a range of -0.16 to 0.12 with the traditional SMC, and in a range of -0.11 to 0.33 with the SMC-DSC. The tracking errors $e_{12}$ fluctuates in a range of -0.4 to 0.4 with the
traditional DSC, and in a range of -0.06 to 0.38 with the traditional SMC, and in a range of -0.3 to 0.12 with the SMC-DSC. The tracking errors $e_{13}$ fluctuates in a range of -0.1 to 0.33 with the traditional DSC, in a range of 0.04 to 0.13 with the traditional SMC, and in a range of -0.1 to 0.1 with the SMC-DSC. From the changing trend of the error curves in the figures and the tracking errors, it is obtained that the proposed

Figure 5. The control input of the quadrotor attitude: (a) $U_1$, (b) $U_2$, and (c) $U_3$.

Figure 6. The tracking results of the modified Rodrigues parameters with traditional DSC: (a) the tracking result of $\eta_1$, (b) the tracking result of $\eta_2$, and (c) the tracking result of $\eta_3$. 
control scheme has better tracking performance and robustness than the traditional DSC, and traditional SMC, and the SMC-DSC for quadrotor attitude control under the disturbances obviously.
Conclusion

In this paper, an adaptive dynamic surface robust integral of the sign of the error control approach with the prescribed performance function is presented to improve the tracking performance of a quadrotor attitude control system surfing from the modeling uncertainties and external disturbances. The dynamic surface control law combined with the RISE are designed to improve the tracking performance and robustness for attitude control system of quadrotor UAV, and an adaptive function are designed to eliminate the errors between the RISE term and the disturbances, and a prescribed performance function are employed to improve the tracking performance of the control system by constraining the attitude tracking errors. The stability analysis and the simulation results have verified to the surprise and effective performance of the presented control strategy.

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ORCID iD

Jingxin Dou https://orcid.org/0000-0003-3597-3606

References

1. Michieletto G, Cenedese A, Zaccarian L, et al. Hierarchical nonlinear control for multi-rotor asymptotic stabilization based on zero-moment direction. *Automatica* 2020; 117: 108991.
2. Moussa L and Mohamed C. Robust integral terminal sliding mode control for quadrotor UAV with external disturbances. *Int J Aerosp Eng* 2019; 2019: 2016416.
3. Xuan-Mung N, Hong SK, Nguyen NP, et al. Autonomous quadcopter precision landing onto a heaving platform: new method and experiment. *IEEE Access* 2020; 8: 167192–167202.
4. Xu QZ, Wang ZS and Zhen ZY. Information fusion estimation-based path following control of quadrotor UAVs subjected to Gaussian random disturbance. *ISA Trans* 2020; 99: 84–94.
5. Ananth S, Bharath G and Inderjit C. A scalability study of the multirotor biplane tailsitter using conceptual sizing. *J Am Helicopter Soc* 2020; 65(1): 012009.
6. Xuan-Mung N and Hong SK. Improved altitude control algorithm for quadcopter unmanned aerial vehicles. *Appl Sci* 2019; 9(10): 2122.
7. Miranda-Colorado R and Aguilar LT. Robust PID control of quadrotors with power reduction analysis. *ISA Trans* 2020; 98: 47–62.

8. Cowling ID, Yakimenko OA, Whidborne JF, et al. Direct method based control system for an autonomous quadrotor. *J Intell Robot Syst* 2010; 60(2): 285–316.

9. Aboudonia A, El-Badawy A and Rashad R. Disturbance observer-based linearization control of an unmanned quadrotor helicopter. *Proc IMechE, Part I: J Syst Control Engineering* 2016; 230(9): 877–891.

10. Chen CC and Chen YT. Feedback linearized optimal control design for quadrotor with multi-performances. *IEEE Access* 2021; 9: 26674–26695.

11. Saetti U, Horn JF, Lakhmani S, et al. Design of dynamic inversion and explicit model following control laws for quadrotor UAS. *J Am Helicopter Soc* 2020; 65(3): 032006.

12. Bolandi H, Rezaei M, Mohsenipour R, et al. Attitude control of a quadrotor with optimized PID controller. *Intell Control Auton* 2013; 4(3): 335–342.

13. Ding WC, Gao WL, Wang KX, et al. An efficient B-spline-based kinematic replanning framework for quadrotors. *IEEE Trans Robot* 2019; 35(6): 1287–1306.

14. Wang JY, Wen GH and Duan ZS. Stochastic consensus control integrated with performance improvement: a consensus region-based approach. *IEEE Trans Ind Electron* 2020; 67(4): 3000–3012.

15. Antonio Cortajarena J, Barambones O, Alkorta P, et al. Sliding mode control of an active power filter with photovoltaic maximum power tracking. *Int J Electr Power Energy Syst* 2011; 33: 747–758.

16. Moussa L and Mohamed C. Novel robust super twisting observer-based disturbance observer for uncertain quadrotor UAV subjected to disturbances. *ISA Trans* 2020; 99: 290–304.

25. Barghandan S, Badamchizadeh MA and Jahed-Motlagh MR. Improved adaptive fuzzy sliding mode controller for robust fault tolerant of a quadrotor. *Int J Control Autom Syst* 2017; 15(1): 427–441.

26. Zou Y and Zhu BY. Adaptive trajectory tracking control for quadrotor systems subject to parametric uncertainties. *J Franklin Inst* 2017; 354(15): 6724–6746.

27. Bouadi H and Mora-Camino F. Modeling and adaptive flight control for quadrotor trajectory tracking. *J Airer* 2018; 55(2): 666–681.

28. Zhang JQ, Ren ZH, Deng C, et al. Adaptive fuzzy global sliding mode control for trajectory tracking of quadrotor UAVs. *Nonlinear Dyn* 2019; 97: 609–617.

29. Wang C, Song B, Huang P, et al. Trajectory tracking control for quadrotor robot subject to payload variation and wind gust disturbance. *J Intell Robot Syst* 2016; 83(2): 315–333.

30. Nguyen AT, Xuan-Mung N and Hong SK. Quadcopter adaptive trajectory tracking control: a new approach via backstepping technique. *Appl Sci* 2019; 9(18): 3873.

31. Hossam Eddine G, Latifa A, Abdelghani C, et al. Optimal model-free backstepping control for a quadrotor helicopter. *Nonlinear Dyn* 2020; 100(4): 3449–3468.

32. Ibarra-Jimenez E, Castillo P and Abaunza H. Nonlinear control with integral sliding properties for circular aerial robot trajectory tracking: real-time validation. *Int J Robust Nonlinear Control* 2020; 30(2): 609–635.

33. Chen ZM, Niu K and Li L. Research on adaptive trajectory tracking algorithm for a quadrotor based on backstepping and the sigma-pi neural network. *Int J Aeros Eng* 2019; 2019: 151034.

34. Silva AL and Santos DA. Fast nonsingular terminal sliding mode flight control for multirotor aerial vehicles. *IEEE Trans Aerosp Electron Syst* 2020; 56(6): 4288–4299.

35. Abaunza H and Castillo P. Quadrotor aggressive deployment, using a quaternion-based spherical chattering-free sliding-mode controller. *IEEE Trans Aerosp Electron Syst* 2020; 56(3): 1979–1991.

36. Jia Z, Yu J, Mei Y, et al. Integral backstepping sliding mode control for quadrotor helicopter under external uncertain disturbances. *Aerosci Tech 2017*; 68: 299–307.

37. Almakhles DJ. Robust backstepping sliding mode control for a quadrotor trajectory tracking application. *IEEE Access* 2020; 8: 5515–5525.

38. Xu LX, Ma HJ, Guo D, et al. Backstepping sliding-mode and cascade active disturbance rejection control for a quadrotor UAV. *IEEE-Asme Trans Mechatron* 2020; 25(6): 2743–2753.

39. Moussa L and Mohamed C. Robust adaptive backstepping fast terminal sliding mode controller for uncertain quadrotor UAV. *Aerosci Tech 2019*; 93: 105306.

40. Basri MAM. Design and application of an adaptive backstepping sliding mode controller for a six-DOF quadrotor aerial robot. *Robotica* 2018; 36(11): 1701–1727.

41. Akella MR, Halbert JT and Kotamaraju GR. Rigid body attitude control with inclinometer and low-cost gyro measurements. *Syst Control Lett* 2003; 49(2): 151–159.
42. Ahmed R, Gu DW and Postlethwaite I. A case study on spacecraft attitude control. In Proceedings of the 48th IEEE conference on decision and control, 2009 held jointly with the 2009 28th Chinese control conference, China, 15–18 December 2009, pp.7345–7350. New York: IEEE.

43. Dou JX, Kong XX, Chen XZ, et al. Output feedback observer-based dynamic surface controller for quadrotor UAV using quaternion representation. Proc IMechE, Part G: J Aerospace Engineering 2017; 231(14): 2537–2548.