Study of simply connected domain and its geometric properties

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ABSTRACT

By using composition of a differential operator and a subclass of analytic functions, we introduce a new application of a differential operator for starlike and convex functions. Moreover, we are dealing with starlikeness and convexity properties of hypergeometric and related functions.

1. Introduction

The Geometric Function Theory (GFT) deals with the geometric properties of analytic functions of a complex variable. Geometrically, analytic functions are divided into starlike, convex and close to convex functions. The open unit disk plays an important role in the construction of simply connected domains including starlike and convex domains. The fundamental rule of open unit disk is applied in the foundation of Riemann mapping theorem. It is a classical result in GFT, which states that any non-constant analytic function in the open unit disk maps the open unit disk onto a simply connected domain in the complex plane, then there exists one-to-one, onto and holomorphic mapping (implies conformal map and angle-preserving) from onto \( U \) (the open unit disk). The differential operators defined in the open unit disk has attracted the attention of many researchers.

We are motivated by the research works based on mapping properties of hypergeometric functions, convolutions of starlike and convex functions (cf. [1]). This article provides an idea to introduce a new subclass of analytic functions with the help of differential operator given by (2).

The \( p \)-valent functions analytic and univalent in \( U = \{z \in C : |z| < 1\} \) of the form (1) with \( p \in \mathbb{N} \) are said to form the class \( A(p) \).

\[
f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k.
\]

A function \( f \) of the form (1) is called starlike function of order \( \xi \) if

\[
\Re \left( \frac{zf'(z)}{f(z)} \right) > \xi, \quad 0 \leq \xi < p.
\]

We denote the class of starlike functions of order \( \xi \) by \( S^*(\xi, p) \).

A function \( f \) of the form (1) is called convex function of order \( \xi \) if

\[
\Re \left( 1 + \frac{zf''(z)}{f'(z)} \right) > \xi.
\]

We denote the class of convex functions of order \( \xi \) by \( C(\xi, p) \).

The class \( S^*(\xi, p) \) was introduced by Patil and Thakare [2] and \( C(\xi, p) \) was introduced by Owa [3].

The differential operator \( \mathcal{T}_0^\alpha(p, \alpha, \beta, \mu) \) given by (2) has been considered in [4] and [5] and is defined as

\[
\mathcal{T}_0^\alpha(p, \alpha, \beta, \mu) f(z) = z^p + \sum_{k=p+1}^{\infty} \left( \frac{\alpha + (\mu + \lambda)(k-p) + \beta}{\alpha + \beta} \right)^n a_k z^k,
\]

where \( \alpha, \beta, \mu \) and \( \lambda \) having the same constraints as discussed before in [6] and [7]. Further, a straightforward calculation reveals that many differential operators introduced in other papers (see for example [8–19]) are special cases of the differential operator (2).

Now by using (2) if we consider \( n = 0, \beta = 0, \mu = 0, p = 1, \lambda = 1 \) and \( \alpha = 1 \) then \( \mathcal{T}_0^0(1, 1, 0, 0) f(z) = f(z) \) and for \( n = 1, \beta = 0, \mu = 0, p = 1, \lambda = 1 \) and \( \alpha = 1 \), \( \mathcal{T}_1^1(1, 1, 0, 0) f(z) = zf'(z) \), it follows that \( zf''/f \in S^*(\xi) \).

Similarly if we set \( \beta = 0, \mu = 0, \xi = 0, n = 0, p = 1, \lambda = 1 \) and \( \alpha = 1 \) then \( f(z) = z/(1 - z^2) \) belongs...
to $S^*(0)$ and putting $\xi = \frac{1}{2}$, $\beta = 0$, $\mu = 0$, $n = 0$, $p = 1$, $\lambda = 1$ and $\alpha = 1$ then $f(z) = z/(1-z)$ belongs to $S^*(\frac{1}{2})$.

Setting $n = 1$, $p = 1$, $\lambda = 1$, $\alpha = 1$, $\beta = 0$ and $\mu = 0$ then $\gamma_{\lambda}^1(1,1,0)f(z) = z^\prime(z) + z^{2\prime}(z)$ and $\gamma_{\lambda}^1(1,1,0,0)f(z) = z^\prime(z)$ implies $z^\prime(z) + (z^\prime \prime(z))' \in C\rho(\xi)$.

In this case, $g(z) = (1/(2(1-\xi))(((1/(1-z))^{2(1-\xi)} - 1)$ belongs to $C\rho(\xi)$, $0 \leq \xi < 1$ and $f(z) = z/(1-z)$ belongs to $C\rho(0)$.

From (2), we deduce that $\gamma_{\lambda}^1(\alpha,\beta,\mu)f(z) = g(z) = (z^\prime(z) - (1-(\mu+\lambda)/\alpha+\beta))z^{p+1}/((1-z)^2)$ and

$$g(z) = \frac{z^p - \left(\frac{\alpha + \beta - \lambda}{\alpha + \beta}\right) z^{p+1}}{(1-z)^2}$$

$$= (z^p - \left(\frac{\alpha + \beta - \lambda}{\alpha + \beta}\right) z^{p+1}) (1-z)^{-2}$$

$$= z^p + \left(1 + \left(\frac{\mu + \lambda}{\alpha + \beta}\right)\right) z^{p+1}$$

$$+ \left(1 + 2 \left(\frac{\mu + \lambda}{\alpha + \beta}\right)\right) z^{p+2} + \cdots$$

$$= z^p + \sum_{k=p+1}^{\infty} \left(\frac{\alpha + \mu + \lambda(k - p + \beta)}{\alpha + \beta}\right) z^k.$$

Let $\Phi_{p,n}^{a,b}(\xi,\mu)$ denotes the class of all functions of the form given by (1) and satisfying

$$\Re \left\{ \gamma_{\lambda}^n(\alpha,\beta,\mu)f(z) \right\} > \frac{\xi}{p}, \quad 0 \leq \xi < 1. \quad (3)$$

After computation, we notice that if we set $\beta = 0$, $\mu = 0$, $n = 0$, $p = 1$, $\lambda = 1$ and $\alpha = 1$ then $f(z) = z/(1-z)$ belongs to $\Phi_{p,n}^{a,b}(\xi,\mu)$.

Consider $\beta = 0$, $\mu = 0$, $n = 0$, $\xi = 0$, $p = 1$, $\lambda = 1$ and $\alpha = 1$, then $f(z) = z/(1-z)$ belongs to $\Phi_{p,n}^{a,b}(\xi,\mu)$. Moreover, for $\xi = \frac{1}{2}$, $\beta = 0$, $\mu = 0$, $n = 0$, $p = 1$, $\lambda = 1$ and $\alpha = 1$ then $f(z) = z/(1-z)$ belongs to $\Phi_{p,n}^{a,b}(\xi,\mu)$. For further discussion, we refer to [1].

**Lemma 1.1 (20):** Suppose $\omega(z)$ be a nonconstant analytic function in $U$ with $\omega(0) = 0$. If $|\omega(z)|$ attains its maximum value at a point $z_0 \in U$ on the circle $r < 1$, then $z_0 \omega'(0) = \xi \omega(z_0)$, where $\xi \geq 1$ is some real number.

**Theorem 2:** For $n_1, n_2 \geq 0$, if the function $f$ given in (1) satisfy the analytic criterion

$$\frac{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}{\gamma_{\lambda}^n(\alpha,\beta,\mu)f(z)} - 1 \left| \begin{array}{c} \frac{\gamma_{\lambda}^{n+2}(\alpha,\beta,\mu)f(z)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} - 1 \\ \frac{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}{\gamma_{\lambda}^n(\alpha,\beta,\mu)f(z)} - 1 \end{array} \right|_{\xi = \frac{1}{2}}$$

$$\leq \left\{ \frac{(1 - \frac{\xi}{p})^{n_1}}{(1 - \frac{\xi}{p})^{n_2}} + \frac{\mu + \lambda}{2(\alpha + \beta)} \right\}^{n_2} \quad 0 \leq \xi \leq \frac{\xi}{p}$$

then $f$ belongs to $\Phi_{p,n}^{a,b}(\xi,\mu)$.

**Proof:** By using (2)

$$\frac{(\mu + \lambda)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}$$

$$= \frac{(\alpha + \beta)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} + p(\mu + \lambda) \quad (\alpha + \beta) - 1, \quad (4)$$

implies

$$\frac{(\mu + \lambda)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}$$

$$= \frac{(\alpha + \beta)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} + p(\mu + \lambda) \quad (\alpha + \beta) - 1. \quad (5)$$

Subtracting last two equations and after doing some calculations, we get

$$\frac{(\mu + \lambda)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} = \frac{(\alpha + \beta)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} + p(\mu + \lambda) \quad (\alpha + \beta) - 1. \quad (6)$$

Let for $0 \leq \xi \leq p/2$, we define $\omega(z)$ such that

$$p(\mu + \lambda) \quad \gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z) + 1 - \omega(z) \quad (7)$$

Differentiating (7) with respect to $z$ and multiplying the resulting equation by $z$, we obtain

$$\frac{z(\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z))'}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} - \frac{z(\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z))'}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} = \frac{(p - 2\xi)z\omega(z)}{(p - 2\xi)\omega(z)} + \frac{2z\omega(z)}{1 - \omega(z)} \quad (8)$$

Combining (6) and (8), then

$$(\alpha + \beta) \frac{z\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} + (\alpha + \beta) \frac{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}$$

$$= \frac{(p - 2\xi)z\omega(z)}{(p - 2\xi)\omega(z)} + \frac{2z\omega(z)}{1 - \omega(z)} \quad (9)$$

Therefore from (7) and (9), we conclude that

$$\frac{(\mu + \lambda)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)}$$

$$= \frac{(\alpha + \beta)}{\gamma_{\lambda}^{n+1}(\alpha,\beta,\mu)f(z)} + p(\mu + \lambda) \quad (\alpha + \beta) - 1. \quad (10)$$
After simplification
\[
\frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - \frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} = \frac{(\mu + \lambda)(2p - 2\xi)\omega(\xi)}{(\alpha + \beta)(p + (2p - 2\xi)\omega(\xi))(1 - \omega(z))} + \frac{(p - \mu \omega(z) + (2p - 2\xi)\omega(z))}{p(1 - \omega(z))}.
\]
(11)

Or
\[
\frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - 1 = \frac{(\mu + \lambda)(2p - 2\xi)\omega(z)}{(\alpha + \beta)(p + (2p - 2\xi)\omega(z))(1 - \omega(z))} + \frac{(2p - 2\xi)\omega(z)}{p(1 - \omega(z))}.
\]
(12)

Similarly (7) implies
\[
\frac{p\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - p = \frac{(2p - 2\xi)\omega(z)}{1 - \omega(z)}.
\]
(13)

So using (12) and (13) we get
\[
\frac{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n}^{p}(p, \alpha, \beta, \mu)f(z)} - 1 = \frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z) - \gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}\left(1 - \frac{(\mu + \lambda)(2p - 2\xi)\omega(z)}{p(1 - \omega(z))}\right) + \left(1 - \frac{p}{\xi - 1}\right)\omega(z).
\]
(14)

Let there exists a point \(z_0 \in \mathcal{U}\) such that \(\max_{|z| \leq |z_0|} |\omega(z)| = \omega(z_0) \equiv 1\). Then by using Lemma 1.1, we have \(\omega(z_0) = e^{\theta_1}, 0 < \theta_1 \leq 2\pi\) and \(z_0\omega'(z_0) = \zeta(\omega(z_0), \zeta \geq 1)\). Therefore
\[
\frac{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n}^{p}(p, \alpha, \beta, \mu)f(z)} - 1 = \frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z) - \gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}\left(1 - \frac{(\mu + \lambda)(2p - 2\xi)\omega(z)}{p(1 - \omega(z))}\right) + \left(1 - \frac{p}{\xi - 1}\right)\omega(z).
\]
(15)

Using (6) we get
\[
(\alpha + \beta)f(z) = \frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - \frac{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - \frac{(\mu + \lambda)(\mu - 1)\omega(z)}{1 - (\mu - 1)\omega(z)}.
\]
(16)

Now by using (14) and (16), we have
\[
\frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - 1 = \frac{(\mu + \lambda)(\mu - 1)\omega(z)}{1 - (\mu - 1)\omega(z)}.
\]
(17)

After simple calculation, we get
\[
\frac{\gamma_{n+2}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)} - 1 = \frac{(\mu + \lambda)(\mu - 1)\omega(z)}{1 - (\mu - 1)\omega(z)}.
\]
(18)

Similarly
\[
\frac{\gamma_{n+1}^{p}(p, \alpha, \beta, \mu)f(z)}{\gamma_{n}^{p}(p, \alpha, \beta, \mu)f(z)} - 1 = \frac{(\mu + \lambda)(\mu - 1)\omega(z)}{1 - (\mu - 1)\omega(z)}.
\]
(19)
Now by using (18) and (19) and after simplification, we have
\[
\left| \frac{\nabla_{n+1}^\lambda (p, \alpha, \beta, \mu) f(z)}{\nabla_{n}^\lambda (p, \alpha, \beta, \mu) f(z)} - 1 \right|^{n_1} 
\times \left| \frac{\nabla_{n+2}^\lambda (p, \alpha, \beta, \mu) f(z)}{\nabla_{n+1}^\lambda (p, \alpha, \beta, \mu) f(z)} - 1 \right|^{n_2}
\left| \frac{\left( \frac{\rho}{\xi} - 1 \right) \omega(z)}{1 - \left( \frac{\rho}{\xi} - 1 \right) \omega(z)} \right|^{n_1+n_2}
\left| 1 + \frac{(\mu + \lambda) n z \omega(z)}{(\alpha + \beta) |\omega(z)|} \right|^{n_2}
\geq \left( 1 - \frac{\xi}{\rho} \right)^{n_1+n_2} \left( 1 + \frac{(\mu + \lambda) \xi}{(\alpha + \beta)} \right)^{n_2}.
\]
which is contradiction for \( (p/2) - \xi_2 \leq \xi_1 \leq p - \xi_2 \), so our supposition is wrong, therefore \( |\omega(z)| < 1 \) and hence \( f \in \Phi_{p,n}^{\alpha,\beta}(\xi, \mu) \).

Consider \( n_1 = n_2 = 1 \) in Theorem 1.2, then we have the following result.

**Corollary 1.3:** If \( f \in A(p) \) satisfy the following
\[
\left| \frac{\nabla_{n+1}^\lambda (p, \alpha, \beta, \mu) f(z)}{\nabla_{n}^\lambda (p, \alpha, \beta, \mu) f(z)} - 1 \right|^{n_1} 
\times \left| \frac{\nabla_{n+2}^\lambda (p, \alpha, \beta, \mu) f(z)}{\nabla_{n+1}^\lambda (p, \alpha, \beta, \mu) f(z)} - 1 \right|^{n_2}
\left| \frac{\left( \frac{\rho}{\xi} - 1 \right) \omega(z)}{1 - \left( \frac{\rho}{\xi} - 1 \right) \omega(z)} \right|^{n_1+n_2}
\left| 1 + \frac{(\mu + \lambda) n z \omega(z)}{(\alpha + \beta) |\omega(z)|} \right|^{n_2}
\geq \left( 1 - \frac{\xi}{\rho} \right)^{n_1+n_2} \left( 1 + \frac{(\mu + \lambda) \xi}{(\alpha + \beta)} \right)^{n_2}.
\]
then \( f \in \Phi_{p,n}^{\alpha,\beta}(\xi, \mu) \).

Taking \( n_1 = 0, n_2 = 1 \) in Theorem 1.2, then we have the following

**Corollary 1.4:** If \( f \in A(p) \) satisfy the following
\[
\left| \frac{\nabla_{n+2}^\lambda (p, \alpha, \beta, \mu) f(z)}{\nabla_{n+1}^\lambda (p, \alpha, \beta, \mu) f(z)} - 1 \right|^{n_2}
\left| \frac{\left( \frac{\rho}{\xi} - 1 \right) \omega(z)}{1 - \left( \frac{\rho}{\xi} - 1 \right) \omega(z)} \right|^{n_1+n_2}
\left| 1 + \frac{(\mu + \lambda) n z \omega(z)}{(\alpha + \beta) |\omega(z)|} \right|^{n_2}
\geq \left( 1 - \frac{\xi}{\rho} \right)^{n_1+n_2} \left( 1 + \frac{(\mu + \lambda) \xi}{(\alpha + \beta)} \right)^{n_2}.
\]
then \( f \in \Phi_{p,n}^{\alpha,\beta}(\xi, \mu) \).

Substitute \( \beta = l, \mu = 0, \alpha = p \) in Theorem 1.2, we get

**Corollary 1.5:** If \( f \in A(p) \) and satisfy
\[
\left| \frac{\nabla_{n+2}^\lambda (p, \alpha, \beta, \mu) f(z)}{\nabla_{n+1}^\lambda (p, \alpha, \beta, \mu) f(z)} - 1 \right|^{n_2}
\left| \frac{\left( \frac{\rho}{\xi} - 1 \right) \omega(z)}{1 - \left( \frac{\rho}{\xi} - 1 \right) \omega(z)} \right|^{n_1+n_2}
\left| 1 + \frac{(\mu + \lambda) n z \omega(z)}{(\alpha + \beta) |\omega(z)|} \right|^{n_2}
\geq \left( 1 - \frac{\xi}{\rho} \right)^{n_1+n_2} \left( 1 + \frac{(\mu + \lambda) \xi}{(\alpha + \beta)} \right)^{n_2}.
\]
then \( f \in \Phi_{p,n}^{\alpha,\beta}(\xi, \mu) \).

Consider \( \beta = 0, \mu = 0, \alpha = p, \lambda = 1 \) in Theorem 1.2, we get

**Corollary 1.6:** If \( f \in A(p) \) and satisfy
\[
\left| \frac{D_{n+2}^\rho f(z) - D_{n+1}^\rho f(z)}{D_{n}^\rho f(z)} - 1 \right|^{n_2}
\left| \frac{\left( \frac{\rho}{\xi} - 1 \right) \omega(z)}{1 - \left( \frac{\rho}{\xi} - 1 \right) \omega(z)} \right|^{n_1+n_2}
\left| 1 + \frac{(\mu + \lambda) n z \omega(z)}{(\alpha + \beta) |\omega(z)|} \right|^{n_2}
\geq \left( 1 - \frac{\xi}{\rho} \right)^{n_1+n_2} \left( 1 + \frac{(\mu + \lambda) \xi}{(\alpha + \beta)} \right)^{n_2}.
\]
then \( f \in \Phi_{p,n}^{\alpha,\beta}(\xi, \mu) \).

Setting \( \beta = 0, \mu = 0, \alpha = p, \lambda = 1, \eta_1 = 0 \) in Theorem 1.2, we get

**Corollary 1.7:** If \( f \in A(p) \) and satisfy
\[
\left| \frac{D_{n+2}^\rho f(z) - D_{n+1}^\rho f(z)}{D_{n}^\rho f(z)} - 1 \right|^{n_2}
\left| \frac{\left( \frac{\rho}{\xi} - 1 \right) \omega(z)}{1 - \left( \frac{\rho}{\xi} - 1 \right) \omega(z)} \right|^{n_1+n_2}
\left| 1 + \frac{(\mu + \lambda) n z \omega(z)}{(\alpha + \beta) |\omega(z)|} \right|^{n_2}
\geq \left( 1 - \frac{\xi}{\rho} \right)^{n_1+n_2} \left( 1 + \frac{(\mu + \lambda) \xi}{(\alpha + \beta)} \right)^{n_2}.
\]
then \( f \in \Phi_{p,n}^{\alpha,\beta}(\xi, \mu) \).

2. Conclusions

In this paper, a linear differential operator is used to introduce new simply connected domain of analytic
functions in the open unit disk $U$. It has been discussed that new simply connected domain consist of both members of simply connected starlike domain of order zero, simply connected starlike domain of order $\xi$ and simply connected convex domain. Further, an analytic criterion for $p$-valent analytic function to be a member of family of new simply connected domain has been discussed. Moreover, our results have been discussed and compared with the earlier one.

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