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COVID-19 and Rumors: A Dynamic Nested Optimal Control Model

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\section*{A B S T R A C T}

Unfortunately, the COVID-19 outbreak has been accompanied by the spread of rumors and depressing news. Herein, we develop a dynamic nested optimal control model of COVID-19 and its rumor outbreaks. The model aims to curb the epidemics by reducing the number of individuals infected with COVID-19 and reducing the number of rumor-spreaders while minimizing the cost associated with the control interventions. We use the modified approximation Karush–Kuhn–Tucker conditions with the Hamiltonian function to simplify the model before solving it using a genetic algorithm. The present model highlights three prevention measures that affect COVID-19 and its rumor outbreaks. One represents the interventions to curb the COVID-19 pandemic. The other two represent interventions to increase awareness, disseminate the correct information, and impose penalties on the spreaders of false rumors. The results emphasize the importance of interventions in curbing the spread of the COVID-19 pandemic and its associated rumor problems alike.

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1. Introduction

Due to closed and isolated government measures, people have been forced to stay at home and spend more time using cell phones and computers to get information from the outside world. Massive data has been generated on social media during the COVID-19 pandemic. Many people and companies have also used social media to spread their concerns about the spread of this virus and the rapid increase in deaths globally [1]. These measures also contributed to widespread rumors on social media [2]. Even though social media and other communication channels have played an essential role in making people aware of the dangers of COVID-19 and facilitating telemedicine, they have also contributed to spreading some false rumors about COVID-19. Some companies have exploited the pandemic to market some of their products as effective treatments for COVID-19 and to apply wrong prescriptions that have caused harm to some people. In addition, rumors have spread that the virus is a political game between enemy countries. Additionally, some rumors have contributed to spreading panic among people about food stocks, resulting in people buying large quantities of supplies to store. Moreover, rumors have caused psychological exhaustion, even causing suicide. In some societies, the virus is considered a social stigma, and the infected individuals bear the responsibility for spreading the virus. There have also been rumors that doctors are using mercy killing because of the large number of deaths due to the virus and many other rumors that have spread during the COVID-19 outbreak. Recently, numerous studies have been published discussing the rumors that abounded during the COVID-19 pandemic. For instance, S Tasnim, and et al. [3] addressed the impact of rumors that accompanied the COVID-19 pandemic and the importance of publishing authentic messages issued by health organizations.

We can consider the spread of rumors similar to the spread of an epidemic, where an epidemic is mainly transmitted through close contact, while rumors are transmitted through social media, especially in modern society. The population can be divided according to their exposure to rumors into several categories: the first category consists of the people who have not been exposed to rumors, and the second category consists of those who carry these rumors without publishing them, similar to asymptomatic carriers. The third category consists of the individuals who spread the rumors the same way infected people spread the disease, and the final category includes those who have recovered from the rumors.
through awareness or experience of the ineffectiveness of the rumors.

Therefore, several studies have addressed the rumors as an epidemiological model, whereby the optimal control model was formulated, and the parameters for controlling this epidemic of rumors were defined. An H. Zhu, J. Ma in [4] introduced an exciting study that addressed the Susceptible–Hesitated–Infected–Removed (SHIR) rumor propagation model in random heterogeneous networks with dynamic friendships. Another approach of rumors models studies deals with the epidemiological model for more than one rumor and, hence, deals with this model like a dual infection model such as the study in [5]. Regardless, most of the control parameters used in the optimal control model address the impact of increasing the community’s awareness and imposing restrictions and penalties for everyone who spreads false rumors, as well as the importance of enabling specialists to publish the correct information that benefits the community.

Numerous recent studies have discussed the epidemiological model of COVID-19. A Novakovic, and AH Marshall [6] proposed a novel methodology based on the hybridization of change point detection and agent-based modeling techniques for modeling COVID-19 infection dynamics and quantifying the effects of non-pharmaceutical interventions on a national level. In the related study [7], the authors discussed the COVID-19 humanitarian response plan for high-priority countries as an optimal distribution model. At the same time, PSO was used to solve the co-infection between COVID-19 and Chikungunya model [8].

Some of these studies have focused on the mathematical analysis of the epidemiological model, and some have focused on predicting the curve of the pandemic and data analysis. Additionally, most of these studies have formulated optimal control models to curb the spread of COVID-19. The impacts of non-pharmaceutical interventions have been discussed, such as lockdown, isolation and quarantine, availability of screening test kits, protection, social distancing, and the wearing of personal protective equipment, as well as the treatment of people with COVID-19. Furthermore, some studies focused on how to detect COVID-19. G. Deshpande et al. [9] reviewed the studies on human audio signals using artificial intelligence techniques to screen, diagnose, monitor, and spread awareness about COVID-19. Another interesting study in [10] used the distance deviation naive Bayesian classification method and PSO to detect COVID-19-infected patients. T.K. Dash et al. [11] proposed a new bio-inspired Cepstral feature set to detect COVID-19 patients. In addition, several studies have discussed the priority of covid-19 vaccine. L. Romeo and E. Frontoni [12] introduced the Hierarchical Priority Classification eXtreme Gradient Boosting for priority classification for the COVID-19 vaccine. In contrast, Hezam et al. [13] used the MCDM approach to determine the priority groups of the COVID-19 vaccine.

On the other hand, nested optimization problems (bi-level programming) are particular optimization problems applied to hierarchical decision structures. A good review of bi-level optimization is presented in [14] for further information, which includes the solving approaches and their application. However, Pontryagin’s maximum principle with an augmented Hamilton function is the most widely used method to solve nested optimization, as in studies [15], while Knauer [16] converted the bi-level to the single-level problem using indirect methods and direct approximation. Additionally, the authors in [17] used evolutionary techniques to solve inverse optimal control problems. However, some works, such as [18], reformulated the bi-level to the single-level optimal control problem using the Karush–Kuhn–Tucker (KKT) condition. In the same way, Hezam [19] proposed a new bilevel model that combined the COVID-19 and unemployment problem. Although nested optimization is considered one of the NP-hard optimization problems, it becomes more complex when included in a dynamic optimal control model and solved by metaheuristic algorithms.

In this work, we design an interactive mathematical model framework that integrates COVID-19 and rumor outbreaks. Then, we formulate a new dynamic nested optimization model where the COVID-19 model represents the outer model, and the rumors model represents the inner model. Two types of objectives will be proposed. The first type aims to minimize the number of individuals infected with COVID-19 and rumor-spreaders. The second type aims to minimize the total cost associated with the intervention strategies. Then, we convert the nested model to a single-level model using the modified approximation KKT condition. Also, we replace the Lagrangian function of the KKT condition via the Hamiltonian function. After that, we use a genetic algorithm (GA) to solve the new model with consideration of the KKT constraints. Moreover, we investigate three control inputs, including interventions to curb the COVID-19 pandemic, measures taken to increase awareness by publishing truthful information directly from specialists, and actions to punish false rumors spreaders. Finally, depending on the sensitivity of the time-varied inputs, we evaluate different strategies to achieve the general goal of controlling the COVID-19 epidemic and stopping the spread of false rumors.

The remainder of this work is organized as follows. Section 2 discusses the interactive mathematical model framework of COVID-19 and the rumors, while section 3 presents a dynamic nested optimal control (DNOC) model. Reformulation of the model is presented in Section 4. The methodology of the solution is introduced in Section 5. In Section 6, we discuss the numerical simulation. Finally, in Section 7, we summarize and conclude the work.

2. Model Formulation

The population will be divided into eight categories, the first four describe the dynamics of COVID-19 infection, and the other four describe the rumors model. The descriptions of all the current categories with associate parameters are reported in Tables 1 and 2, respectively.

The categories of population based on the infection with COVID-19 are $S_c(t), I_c(t), R_c(t),$ and $P_c(t),$ corresponding to the numbers of individuals in the four epidemiological categories at the time $t.$ The total population at the time $t,$ denoted by $N_c(t),$ is given by:

$$N_c(t) = S_c(t) + I_c(t) + R_c(t) + P_c(t)$$

Moreover, the categories of the population based on infection with rumors are $S_r(t), H_r(t), I_r(t),$ and $R_r(t)$; where the total population at the time $t$ is given by:

$$N_r(t) = S_r(t) + H_r(t) + I_r(t) + R_r(t)$$

The following dynamic differential equation system describes both the COVID-19 and the rumor models.

$$\frac{dS_c(t)}{dt} = \Delta N_H + \mu I_r(t) - \left(1 - \frac{u_1(t)}{N_H}\right) S_c(t)$$

$$\frac{dI_c(t)}{dt} = \left(1 - \frac{u_1(t)}{N_H}\right) S_c(t) + \left(\frac{\rho_2 - \rho_1(1 - u_2(t))}{N_I}\right) I_r(t) - (\gamma_1 + \omega_1) I_c(t)$$

$$\frac{dR_c(t)}{dt} = \gamma_1 I_c(t) - \mu I_r(t)$$

$$\frac{dP_c(t)}{dt} = \omega_1 I_c(t)$$
The individuals infected with COVID-19 are defined by Equation (2). This category is increased when the transmission rate \( \alpha_1 \) is increased and when government intervention \( u_1(t) \) is absent. We added a new term related to the impact of rumors on infection with COVID-19, where the infection is increased at rate \( \rho_1 \), and we can reduce this term using the second intervention related to the awareness \( u_2(t) \). Also, leaving this category is accomplished either through recovery, at a rate of \( \gamma_1 \), or through death, at a rate of \( \omega_1 \). The recovered individuals from COVID-19 are determined via Equation (3). The high recovery rate, \( \gamma_1 \), leads to an increase in this category. Also, this category is decreased by infected again with COVID-19 at a rate \( \mu_1 \). Equation (4) describes the individuals who have perished due to COVID-19 at a rate of \( \omega_1 \).

On the other hand, the rumor model is described by the remaining four equations. Individuals who have not been exposed to rumors are described by Equation (5). This category decreases when these individuals are exposed to rumors at a rate of \( \delta \), and there is a possibility that they are attracted to the rumor at a rate \( \sigma \) or are not be attracted to this rumor at a rate \( (1 - \sigma) \). We can control the infection rate by increasing the society awareness \( u_2(t) \).

Hesitant individuals are described by Equation (6), and they are those individuals who have heard about the rumors but are still hesitating in a latent period. Individuals in this category will become spreaders at a rate \( \omega_2 \), or they will be resistant to rumors at a rate \( \alpha_2 (1 - \beta) \).

Infected individuals who actively spread the rumors are described by Equation (7). The spreaders increase as the rate of attractiveness to rumors increases and decreases with the increase
in the rate of recovery \( \gamma_2 \). Here, interventions can play a role in decreasing the number of spreaders by imposing deterrent penalties for each false spreading of rumors, as well as by increasing community awareness. We added a new term related to the effect of the COVID-19 infection rate on the rumored rate where \( \rho_2 \) measures the interaction rate between being infected both with COVID-19 and with rumors. Stiflers are identified by Equation (8), and they are those who have lost their interest in spreading rumors or are immune to false rumors. We are working to increase this state by increasing \( u_3(t) \) community awareness and conveying correct information from reliable sources to all members of society; also, in parallel, we work to curb the spread of rumors and correct misconceptions \( u_3(t) \). It is worth noting that we can classify the parameters in Tables 1 and 2 into three categories. The first one includes values that have been assumed, such as \( Q \). The second one includes values that have been estimated based on the real data obtained from the case study, for instance, \( \Lambda \) value. The third one includes literature-based values as \( \alpha_2 \).

3. Dynamic Nested Optimal Control Model (DNOC)

Nested optimization consists of models that overlap with each other, and because of their hierarchical structure, they are considered to be a type of NP-hard optimization problem. The outer model is called the leader or the upper level, and the inner model is called the follower or the lower level. Therefore, we can formulate simple nested optimization as:

\[
\begin{align*}
\min_{x \in X} & \quad F(x, y) \\
\text{s.t.} & \quad G(x, y) \leq 0 \\
y & \in P(x) \\
\text{where} & \\
P(x) = \arg\min_{y \in Y} f(x, y)
\end{align*}
\]

s.t. \( g(x, y) \leq 0 \)

where \( F(x, y) \) is the outer objective with respect to the decision variable \( x \) and the constrains \( G(x, y) \). In addition, the constraints of the outer problem include the optimal solution of the inner objective \( f(x, y) \) with respect to the decision variable \( y \) and the constrains \( g(x, y) \).

In the present work, the DNOC model contains two models: 1) the COVID-19 model as an outer model; 2) the rumor model as an inner model. The proposed DNOC model can be formulated as follows:

**MODEL I:**

**Outer Level:**

\[
f_1 = \min_{L(t), u_1(t), u_2(t), u_3(t)} \int_{t_0}^{t_f} \left( z_1 L(t) + A(u_1(t))^2 + B(u_2(t))^2 + C(u_3(t))^2 \right) dt
\]

\[
\max_{t} \left( l_i(t) \right) \leq l_i^{\text{peak}}, \quad i = c, r
\]

\[
u_1(t) \in [0, 0.5]
\]

\[
u_2(t) \in [0, 0.5]
\]

\[
u_3(t) \in [0, 0.5]
\]

**Equations (1-4)**

**Inner Level:**

\[
f_2 = \min_{L(t), u_1(t), u_2(t), u_3(t)} \int_{t_0}^{t_f} \left( z_2 L(t) + B(u_2(t))^2 + C(u_3(t))^2 \right) dt
\]

\[
u_2(t) \in [0, 0.5]
\]

\[
u_3(t) \in [0, 0.5]
\]

**Equations (5-8)**

Fig. 1. Schematic diagram of the interaction of the rumors and COVID-19 models.
The nested optimal control model is defined by equations (10)–(17). Eq. (10) represents the objective function of the outer level, while Eq. (15) represents the inner level’s objective function. Each objective function aims to minimize the number of infected individuals with COVID-19 and the rumor-spreaders and to minimize the cost associated with the interventions of control. Eq. (11) defines the maximum peaks of the number of infected individuals with COVID-19 and rumor-spreaders. Eqs. (12)–(14) and (16)–(17) determine the bounds of the time-varying variables that can be dynamically changed to determine the best strategy of control. Eqs (1–4) represent the COVID-19 dynamic constraints, while Eqs. (5–8) represent the dynamic rumor constraints. Herein, $z_1$, $z_2$, $A$, $B$, and $C$ are weight coefficients. The optimal control model is assumed during the full-time horizon $[t_0, t_f]$, where $t_f$ is 260 days.

4. The Reformulation of the DNOC Model

This section presents the reformulation of the DNOC model. First, we will find the Lagrangian function of the inner-level model that will be used to reformulate the proposed model. Second, the inner-level problem is replaced by a modified approximation KKT condition. Finally, we will also replace the Lagrangian function using the Hamiltonian function.

The Lagrangian function of the inner-level model can be formulated as follows:

$$L(X(t), u_i(t), \mathcal{L}, t) = \int_{t_0}^{t_f} \left( L_2(t) + L_1(t) \right) dt$$

where $L_1(t)$, $L_2(t)$, $L_3(t)$, and $L_4(t)$ are functions of time compared to the Lagrange multipliers in a static optimization model. $X(t)$ refers to the rumor states $S(t), H(t), I(t)$, or $R(t)$.

In nested optimization, although the KKT conditions of the inner level can often be used to convert the model to a single-level model, the KKT conditions cannot be used for all models, especially the DNOC, due to non-convexities in the KKT conditions. So, the modified approximation KKT condition is used in this work.

The formulation of the single level that we obtained by the substitution of the inner level with the modified approximation KKT conditions is as follows:

**MODEL II:**

$$f_1 = \min_{L(t), u_i(t), \mathcal{L}, t} \int_{t_0}^{t_f} \left( L_2(t) + L_1(t) \right) dt$$

$$j = 1, 2, 3, i = 1, \ldots, 4$$

Equations (1–4), Equations (5–8),

$$\nabla_{u_i(t), u_j(t)} L(X(t), u_i(t), \mathcal{L}(t), t) \leq \varepsilon$$

$$L_1(t) \left[ \frac{dS(t)}{dt} - \frac{Q}{\mu} - \mu_2 R(t) + \frac{\delta(1 - u_2(t))S(t)}{\mu} + (\delta(1 - \sigma) + u_2(t))S(t) \right] \geq -\varepsilon$$

$$L_2(t) \left[ \frac{dH(t)}{dt} - \delta S(t) - \alpha_2 \beta H(t) + (\alpha_2(1 - \beta) + u_3(t))H(t) \right] \geq -\varepsilon$$

$$L_3(t) \left[ \frac{dI(t)}{dt} - \alpha_2 \beta H(t) - (\rho_1 - \rho_2)(1 - u_2(t))I(t)I(t) + (\gamma_2 + u_3(t))I(t) \right] \geq -\varepsilon$$

$$L_4(t) \left[ \frac{dR(t)}{dt} - (\delta(1 - \sigma) + u_2(t))S(t) - (\alpha_2(1 - \beta) + u_3(t))H(t) - (\gamma_2 + u_3(t))I(t) + \mu_2 R(t) \right] \geq -\varepsilon$$

$$L_i \geq 0, i = 1, \ldots, 4, \varepsilon \leq \varepsilon_0, \varepsilon_0 \equiv 0$$

where $\varepsilon$ is a small number bounded by fixed-parameter $\varepsilon_0$.

Now, the Hamiltonian function can be derived from the Lagrangian function. In Equation (18), partial integration can be employed to rewrite the last term on the right-hand side as:

$$\int_{t_0}^{t_f} \left( L_2(t) \frac{dS(t)}{dt} + L_1(t) \frac{dH(t)}{dt} + L_3(t) \frac{dI(t)}{dt} + L_4(t) \frac{dR(t)}{dt} \right) dt$$

$$= L_1(t)S(t_0) - L_1(t_0)S(t_0) + \int_{t_0}^{t_f} \frac{dL_1(t)}{dt} S(t) dt + L_2(t)H(t_0) - L_2(t_0)H(t_0)$$

$$+ \int_{t_0}^{t_f} \frac{dL_3(t)}{dt} I(t) dt + L_3(t_0)I(t_0) + \int_{t_0}^{t_f} \frac{dL_4(t)}{dt} R(t) dt$$

$$+ L_4(t_0)R(t_0) - L_4(t_0)R(t_0) + \int_{t_0}^{t_f} \frac{dL_4(t)}{dt} R(t) dt$$

26.
Substituting equation (26) into the Lagrangian function (18) we get:

\[
L(X(t), u_i(t), L(t), t) = \int_{t_0}^{t_f} \left( z_2 \dot{L}(t) + B(u_2(t))^2 + C(u_3(t))^2 \right. \\
+ L_1(t) \left[ -Q - \mu_2 R_r(t) + \delta \sigma (1 - u_2(t)) S_r(t) + (\delta (1 - \sigma ) + u_2(t)) S_r(t) \right] \\
+ L_2(t) \left[ -\delta \sigma S_r(t) + \alpha_2 \beta H_r(t) + (\alpha_2 (1 - \beta) + u_2(t)) H_r(t) \right] \\
+ L_3(t) \left[ -\alpha_2 \beta H_r(t) - (\rho_1 - \rho_2)(1 - u_2(t)) L_r(t) + (\gamma_2 + u_3(t)) L_r(t) \right] \\
+ L_4(t) \left[ -\delta (1 - \sigma ) + u_2(t)) S_r(t) - (\alpha_2 (1 - \beta) + u_3(t)) H_r(t) - (\gamma_2 + u_3(t)) L_r(t) + \mu_2 R_r(t) \right] \\
+ \frac{d L_1(t)}{d t} S_r(t) + \frac{d L_2(t)}{d t} H_r(t) + \frac{d L_3(t)}{d t} I_r(t) + \frac{d L_4(t)}{d t} R_r(t) \\
\left. + L_1(t) S_r(t) - L_1(t_0) S_r(t_0) + L_2(t_0) H_r(t_0) - L_3(t_0) H_r(t_0) + L_3(t_0) I_r(t_0) \\
- L_3(t_0) R_r(t_0) + L_4(t_0) R_r(t_0) - L_4(t_0) R_r(t_0) \right) \\
\right) \, dt
\]

(27)

To derive the first-order conditions, we have that the solution of the Lagrangian function is found. Then, any change to \(X(t)\) and \(u_i(t)\) must cause the value of the Lagrangian function to decline. Specifically, the total derivative of the Lagrangian function obeys:

\[
dt L(X(t), u_i(t), L(t), t) = \int_{t_0}^{t_f} \left( \frac{d L_1(t)}{d t} S_r(t_0) + \frac{d L_2(t)}{d t} H_r(t_0) + \frac{d L_3(t)}{d t} I_r(t_0) + \frac{d L_4(t)}{d t} R_r(t_0) \right) \, dt \]

(28)

To get the critical points, we will find the first derivative of the Lagrangian function and equal it to zero.

\[
dt L(X(t), u_i(t), L(t), t) = \int_{t_0}^{t_f} \left( \frac{d L_1(t)}{d t} S_r(t) + \frac{d L_2(t)}{d t} H_r(t) + \frac{d L_3(t)}{d t} I_r(t) + \frac{d L_4(t)}{d t} R_r(t) \right) \, dt
\]

(29)

The \(S_r(t_0), S_r(t_0), H_r(t_0), H_r(t_0), I_r(t_0), I_r(t_0), R_r(t_0)\), and \(R_r(t_0)\) are fixed. So, their derivatives equal zero. Thus, we will obtain the following equations equivalent to Equation (20).

\[
\begin{align*}
2B u_2(t) + (1 - \delta \sigma ) L_1(t) S_r(t) + \rho_2 L_2(t) I_r(t) - \rho_2 L_4(t) S_r(t) = 0 \\
2C u_3(t) + L_3(t) I_r(t) - L_4(t) (H_r(t) + I_r(t)) = 0 \\
L_1(t) \delta \sigma (1 - u_2(t)) + L_2(t) - (L_1(t) - L_4(t))(\delta (1 - \sigma ) + u_2(t)) - \delta \sigma L_2(t) + \frac{d L_1(t)}{d t} = 0 \\
\alpha_2 L_3(t) - \alpha_2 \beta L_3(t) + L_4(t)(-\alpha_2 + \alpha_2 \beta + u_3(t)) + \frac{d L_2(t)}{d t} = 0 \\
z_2 - L_3(t)(\rho_1 - \rho_2)(1 - u_2(t)) L_r(t) + (L_3(t) - L_4(t))(\gamma_2 + u_3(t)) + \frac{d L_3(t)}{d t} = 0 \\
\mu_2 (L_4(t) - L_1(t)) + \frac{d L_4(t)}{d t} = 0
\end{align*}
\]

MODEL II can be rewritten as the following:

MODEL III:

\[
f_1 = \min_{u_i(t), u_i(t) \leq 0} \int_{t_0}^{t_f} z_1 L_1(t) + A(u_1(t))^2 + B(u_2(t))^2 + C(u_3(t))^2 \, dt
\]

(36)

\[
j = 1, 2, 3, 4
\]

Equations (1-4), (11-14)

Equations (5-8), (16-17)

Equations (21-25)

\[
2B u_2(t) + (1 - \delta \sigma ) L_1(t) S_r(t) + \rho_2 L_2(t) I_r(t) - L_4(t) S_r(t) \leq \varepsilon
\]

(37)

\[
2C u_3(t) + L_3(t) I_r(t) - L_4(t) (H_r(t) + I_r(t)) \leq \varepsilon
\]

(38)
\[ L_1(t)\delta \sigma (1 - u_2(t)) + (L_1(t) - L_4(t))(\delta (1 - \sigma) + u_2(t)) \]
\[ -\delta \sigma L_2(t) + \frac{dL_1(t)}{dt} \leq \varepsilon \]  \hspace{1cm} (39)
\[ \alpha_2 L_2(t) - \alpha_2 \beta L_3(t) + L_4(t)(-\alpha_2 + \alpha_2 \beta + u_3(t)) \]
\[ + \frac{dL_2(t)}{dt} \leq \varepsilon \]  \hspace{1cm} (40)
\[ z_2 - L_3(t)(\rho_1 - \rho_2)(1 - u_2(t))l_i(t) \]
\[ + (L_3(t) - L_4(t))(\gamma_2 + u_3(t)) + \frac{dL_3(t)}{dt} \leq \varepsilon \]  \hspace{1cm} (41)
\[ \mu_2 (L_4(t) - L_1(t)) + \frac{dL_4(t)}{dt} \leq \varepsilon \]  \hspace{1cm} (42)
\[ L_i(t) \geq 0, \hspace{0.5cm} i = 1, \ldots, 4. \]  \hspace{1cm} (43)

5. The DNOC Solution Approach

This section presents the solution methodology to solve the DNOC model in this work. After reformulating the nested model to model III, we will solve model III using the GA.

5.1. Overview of the genetic algorithm

The GA is a metaheuristic algorithm proposed by Holland (1975) that mimics natural evolutionary processes like selection, crossover, and mutation. It is widely applied in engineering, computer science, management, mathematical optimization, and applications. The basic components of the GA can be summarized: first, the parameters of the algorithm are set: the initial population \( P(0) \), the population size \( Z \), the crossover probability \( P_c \), the mutation probability \( P_m \), as well as the maximum iterations \( T_{\text{max}} \), and the stopping criteria are also defined. The second element is encoding a chromosome (solution) as either binary or float according to the given problem type. In addition, the solutions are evaluated using the fitness function that will be defined according to the objective function under study. Then, the crossover operator is employed to produce the offspring generation (new chromosomes) from the mating of the parents (old chromosomes) by swapping the genes of the chromosomes. \( P_c \) often is high: 0.8–0.95. After that, the mutation operator generates new chromosomes by flipping some part of the string. \( P_m \) is often low: 0.001–0.05 [21]. So, the crossover operator is used to increase the intensification, while the mutation operator is used to increase the diversification. The selection is also an important element of the algorithm so that the fraction of the chromosomes will be selected based on their fitness and by any selection methods (e.g., the roulette wheel). Finally, the GA is terminated when the stopping criteria are met, e.g., the maximum iteration is reached.

5.2. Description GA for the DNOC model

This section describes the solution procedures. The first phase of the solution procedures is to reduce the nested optimal control to a single-level optimal control using KKT and the Hamiltonian function. Then, the optimal solution of the inner model will be obtained by solving the equations (30)–(35). After that, the optimal solution will be sent to the GA to solve the outer model. So, the GA is used to solve the outer problem, including the inner problem as a constraint.

\[
\begin{array}{cccc}
\text{Solution 1} & \text{Solution 2} & \text{Solution 3} & \text{Solution 4} \\
\hline
\text{Solution 1} & \text{Solution 2} & \text{Solution 3} & \text{Solution 4}
\end{array}
\]

In the steps of the GA, the solution population \( (u_1(t)) \) of the objective function (36) is generated. Here, the solution is represented by a matrix. The matrix dimensions are \( k \times (T_f + 1) \) where \( k \) is the number of decision variables of the objective function in the outer model and \( T_f \) is the final finite horizon time. It should be noted that \( u_2(t) \) and \( u_3(t) \) are critical points of the inner problem. Table 3 illustrates the solution representation.

In this work, the initial population is randomly generated in the feasible range, and it will be evaluated and sorted. Evaluation is scaled according to the fitness function. We define the fitness function as the following:

\[
F = (L_i(t), u_1(t), u_2(t), u_3(t)) = f_1(L_i(t), u_1(t), u_2(t), u_3(t)) - f_1^{\text{min}}
\]

\[
f_1^{\text{min}} = \min_{u(t)} \int_{t_0}^{T_f} z_i L_i(t) + A(u_1(t))^2 + B(u_2(t))^2 + C(u_3(t))^2 dt \]

Then, the roulette wheel method will be used to select higher-fit solutions according to their fitness values. After that, the crossover operator is used to reproduce new solutions by swapping parts of the two solutions in the mating pool. For example, two solutions are chosen for a crossover. Suppose we swap two parts; each portion contains three genes for \( T_f = 5 \). Then, the swapping result of the crossover is shown in Fig. 2.

After that, the mutation operator is performed at a fraction of the populations (with a probability of less than 0.1), where one solution is randomly selected, and some part of the string is flipped to get a new solution. The mutation operator scans the feasible region efficiency and is useful for avoiding trapping at the local optimum, thus increasing diversity. Finally, the algorithm will be terminated when the criteria are met.

We can summarize the procedure of the solution methodology in the following steps:

Step 1: Set up the model parameters and set up the GA parameters: population size \( Z \), probability of crossover \( P_c \), mutation \( P_m \), and the maximal generation of terminating the algorithm \( T_{\text{max}} \); then set the counter of generation \( t = 0 \)
Step 2: Reduce the nested optimal control to the single level using the modified approximation KKT condition
Step 3: Use Pyomo to get the critical points \( u_2 \) and \( u_3 \) from equations (30)–(35) and save these values
Step 4: Send the critical point values to the GA
Step 5: Initialize the initial population of the test solutions randomly
Step 6: Evaluate the population
Step 7: Sort the population and save the best current solution
While the end criterion is not achieved:

Table 3

| Chromosome representation |
|---------------------------|
| \( t_0 \) | \( t_1 \) | \( t_f \) |
| \( u_1(t_0) \) | \( u_1(t_1) \) | \( u_1(t_f) \) |
| \( l_i(t_0) \) | \( l_i(t_1) \) | \( l_i(t_f) \) |

Fig. 2. Crossover example
Select the potential solutions using the roulette wheel method
Apply the crossover to the $S_1$ and $S_2$ and get $S'_1$ and $S'_2$
Evaluate $S'_1$ and $S'_2$
Select a new solution $S_3$ and apply the mutation to obtain $S'_3$
Sort the new populations
Save the best solution of the outer problem
End while.
Output the best current solution.

6. Numerical Simulations

In this section, numerical simulations were performed and were implemented in Python 3.7. First, we set the parameter values listed in Table 2 with the initial categories as reported in Table 1. Additionally, the weights $z_1, z_2, A, B,$ and $C$ were assumed to be equal to one for all simulations. The range of the active control variables was $0.0 \leq u_1(t) \leq 0.5$, $0.0 \leq u_2(t) \leq 0.5$, $0.0 \leq u_3(t) \leq 0.5$, while their values in the inactive cases were equal to zero. In addition, the parameters of the GA were: the population size $Z = 1000$, $P_c = 0.9$, mutation $P_m = 0.01$, and $T_{\text{max}} = 100$.

Fig. 3 shows the dynamic change in the population for each state. The COVID-19 panel illustrates the infected, recovered, and death categories, and it can be seen that the curve of infected people increases until reaching the peak before decreasing with time. By contrast, the curves of the recovered and dead people increase for some time and then become stable.

The dynamic change in the rumor categories can be seen in the rumor panel. The ignorant category decreases with time, while the stifler category increases. The other two rumor categories, the hesitators and the spreaders, increase to the peak of rumor and then decrease with time.

Here, we will investigate the impact of the three control variables i.e., $u_1, u_2$ and $u_3$ where $u_1$ indicates the interventions to curb the COVID-19 pandemic, $u_2$ refers to the measures taken to increase the awareness of fake rumors, and $u_3$ signifies the actions taken against the spreaders of false rumors.

**Strategy 1**: $u_1 \neq 0$, $u_2 \neq 0$, and $u_3 \neq 0$

In this strategy, the government takes strict measures to curb the COVID-19 epidemic and stiffens the penalties against the spreaders of false rumors; this, in return, contributes to spreading community awareness about fake rumors by providing correct information.

**Strategy 2**: $u_1 = 0$, $u_2 = 0$, and $u_3 = 0$

In this strategy, the government does not interfere with any measures to curb the COVID-19 epidemic or against the spreaders of false rumors; also, it does not have an awareness program to refute rumors and spread awareness in the community.

**Strategy 3**: $u_1 = 0$, $u_2 \neq 0$, and $u_3 \neq 0$

In this strategy, we consider that the control variable related to COVID-19 is not active, while the other control variables related to rumor are active. Herein, the government’s priority is to fight the rumors, and in return, no significant measures are taken to curb the COVID-19 pandemic.

**Strategy 4**: $u_1 \neq 0$, $u_2 = 0$, and $u_3 = 0$

In this strategy, one control variable related to COVID-19 is active, while the other two control variables related to rumor are inactive. The government’s priority is to fight the COVID-19 pandemic; in return, the government does not take any interventions to curb the rumors.

The simulation results comparison of all the above strategies is illustrated in Fig. 4. We can see the effect of the control variables’ activity on the current categories of both pandemics.

The numbers of people infected with COVID-19 for the strategies mentioned above are shown in Panel A. The differences in the peaks and the ends of the curves, according to the strategy followed in dealing with the pandemics, can be observed. In strategy 2, when the government does not intervene with any measures either to curb the COVID-19 pandemic or to stop the rumors, the number of infected people is high, while the peak is small in strategy 1 when the government takes measures to stop both the COVID-19 pandemic and the rumors, and the curve, in this case, is more flattened. When comparing strategies 3 and 4, there is a slight preference for strategy 4, in which the government took measures only to curb the COVID-19 pandemic. Likewise, the same order of preference for these strategies can be seen from Panel B on the number of deaths due to COVID-19. The number was highest in strategy 2, followed by strategy 4, then strategy 3, and the lowest number of deaths occurred when strategy one was applied. Panel C compares among the numbers of people recovering from COVID-19 for all the strategies. Clearly, the number of recoveries varies according to the number of infected, but the recovery curve for strategy 2 is at a lower level than the curves for the other strategies.

The other three panels show the categories’ comparison to the rumor model. Panel E illustrates the number of hesitators with rumors for all strategies. We can see a high number of hesitators occur when no measures are taken to curb the course of COVID-19 or the rumors. The second-highest number occurs when strategy four is applied, and intervention is taken only to curb COVID-19. Then comes strategy 3, followed by strategy 1, in which measures were taken to curb both COVID-19 and the rumors problem.
Likewise, the number of spreaders increases when the government does not take measures to stop either COVID-19 or the rumors problem, as shown in Panel (F). Strategy 2 has the highest peak, followed by strategy 4. A low number of spreaders is achieved when the maximum control of COVID-19 and the rumors problem is applied. In contrast, the highest number recovering from the rumors is observed when implementing strategy 1, while the lower number occurs when implementing strategy 4.

On the other hand, although the lowest cost associated with the control strategy was achieved when applying strategy 2, in which no significant measures were taken to curb COVID-19 or the rumors problem, this will have significant economic and health effects.

It can be concluded that some intervention is needed to reduce both the epidemic and the rumors. These measures alleviate the spread of the pandemic, and the imposition of organized rules to control rumors and increase transparency in publishing true information from trusted sources on a timely basis contributes to building a conscious and fortified society. From the previous cases, it can be stated that strategy 1, which is the maximum control, is the best among the strategies.

7. Conclusions

The cross-effect between the COVID-19 pandemic and rumors was investigated in this work using a DNOC model. It was assumed that the COVID-19 model was the outer model, and the rumor model was the inner model. The DNOC model aims to reduce the number of individuals infected with COVID-19 and the number of rumor-spreaders and minimize the costs associated with the various controls. The modified approximation KKT condition with the Hamiltonian function was used to convert the nested model to a single-level model. Hence, the GA was used to solve the proposed model. Three control variables were considered: the first controlling variable related to government measures to curb the COVID-19 pandemic, represented by quarantine, social distancing, and per-
sonal protection, and the other two controlling variables related to government interventions to fight rumors by spreading the correct information and in return imposing penalties for each spreader of false rumors. Four different cases were investigated to verify the effect of the control variables, and the results showed the importance of government measures to combat COVID-19 and government interventions in curbing rumors. As a future suggestion, a robust improvement model could be considered.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

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