Numerical Comparison of Transpiration Cooling Designs Using Real Gas Effects and Approximate Gas Model

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Abstract. In the severe high-temperature environment caused by aerodynamic heating, the vibrational excitation, dissociation and ionization of gas may successively occur, which are known as real gas effects. Under the real gas effects, the thermodynamic properties of gas vary drastically and significantly influence the performances of the active thermal protection system of hypersonic vehicles, especially in the case with coolant outflow, for example transpiration cooling. This paper numerically investigates the transpiration cooling performance with the consideration of the interaction between coolant outflow and hypersonic flow under the real gas effects. The mathematical models and coupled numerical strategy are firstly validated by experimental data, then the influences of real gas effects on the transpiration cooling of a wedged leading edge (WLE) are studied under a flight Mach number range from 8 to 12 and a flight height of 40 km. The analysis and discussions of the numerical results reveal some important phenomena and demonstrate the need to consider real gas effects.

Keywords. Transpiration cooling, real gas effects, porous media, hypersonic vehicles, high-temperature environment.

1. Introduction
Hypersonic vehicles are generally designed to fly or cruise in the near space with the Mach numbers higher than 5 [1], and the superiorities of hypersonic vehicles in future military field and civilian traffic have raised many concerns [2]. However, the hypersonic flight will cause a severe aerodynamic heating effect [3], and therefore a series of active thermal protection systems (ATPS) have been researched for the hypersonic vehicles [4].

It is well known that the air around hypersonic vehicles is significantly heated. In high-temperature environment, the phenomena of gaseous vibrational excitation, dissociation and ionization alleged “real gas effects” (RGE) may successively occur. The air composition will change and the thermal properties may deviate from the behavior of ideal gases and traditional approximate models [5]. RGE is one of the important issues in the design of ATPS for hypersonic vehicles, because when the Mach number exceeds 4, strong RGE are not negligible [6]. Using the numerical method, Emelyanov et al. [7] compared the results calculated by RGE and the ideal gas model. Their comparison indicated that using the ideal gas model, the calculated temperatures may exceed the real values.

In the designs of the hypersonic vehicles with long endurance, the development of effective ATPS is very important, because the aerodynamic heat may exceed the limits of current materials [8]. In
recent years, transpiration cooling as a high-efficiency cooling technique attracts more and more attention. Due to the interaction between coolant flow and hypersonic flow, the accurate predictions of transpiration cooling performances need to consider RGE in the flight with a high Mach number. Overall, the comprehensive study of transpiration cooling with the interaction between coolant flow and hypersonic flow under the real gas effects is still lacking. In practice, due to open pores to external, the aerodynamic force and real gas effects of the hypersonic flow inevitably affect the transport characteristics of the coolant within and outside the transpiration cooling structure, which would cause unexpected changes to the temperature and the driving force of the system. Therefore, it is necessary and worthwhile to study the actual transpiration cooling process, which couples the hypersonic flow under the real gas effects.

This paper exhibits a numerical investigation on the transpiration cooling performances of WLE under the conditions of a Mach number range between 8 and 12, and flight height of 40 km, traditional approximate gas models (AGM) and RGE are used respectively, the numerical results are analyzed and compared. This work aims to provide the investigators and designers of the transpiration cooling system of the hypersonic vehicles with a relatively comprehensive reference.

2. Geometry Model
The geometrical model of the WLE used in this work is illustrated in figure 1. The model consists of a porous matrix and an impermeable part with a constant thickness of 2 mm, a total length of 114 mm and an inner cone angle of 14°. The porous matrix has an external radius of 3 mm, an average sphere diameter of 5×10⁻⁴ m, a porosity of 0.3, that correspond to a permeability of 1×10⁻¹³ m². At the altitude of 40 km, the static temperature of the air is 227 K and the static pressure is 1200 Pa. The gaseous coolant is injected from the inner surface, and exchanges heat with the porous matrix adequately. The outflow covers the hot surface of the porous and impermeable regions to prevent them from direct contact with high-temperature airflow.

![Figure 1. Geometry model of the wedge leading edge (WLE).](image)

3. Mathematical Model
3.1. Conservation Equations
The entire computational domain consists of free flow, porous and impermeable regions. In the free flow region, Reynolds-averaged Navier-Stokes (RANS) equations are solved, and SST \( k - \omega \) turbulent model is employed to close RANS equations.

Continuity equation:

\[
\nabla \cdot (\rho \mathbf{u}) = 0
\]
Momentum equation:
\[ \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot \tau + \frac{\partial}{\partial x_j}(-\rho u_i' u_j') \]  
(2)

where \( \tau \) represents shear stress, and the Reynolds stresses \(-\rho u_i' u_j'\) is calculated by SST \( k - \omega \) model.

Energy equation:
\[ \nabla \cdot (\vec{u}(\rho E + p)) = \nabla \cdot (k_f \nabla T + \vec{\tau} \cdot \vec{u}) \]  
(3)

In the porous region, the Darcy–Forchheimer equation is used to describe the coolant flow:

Continuity equation:
\[ \nabla \cdot (\varepsilon \rho \vec{u} \vec{u}) = 0 \]  
(4)

Momentum equation:
\[ \nabla \cdot (\varepsilon \rho \vec{u} \vec{u}) = -\nabla (\varepsilon p) + \nabla \cdot (\varepsilon \vec{\tau}) - \left( \frac{k}{K} \nu \vec{u} + C_2 \frac{1}{2} \rho \vec{u} |\vec{u}| \right) \]  
(5)

where \( K \) and \( C_2 \) respectively represent the permeability of the porous matrix and the inertial resistance factor.

Energy equation
\[ \nabla \cdot \left[ \vec{u}(\rho_f E_f + p) \right] = \nabla \cdot (k_{eff} \nabla T + \vec{\tau} \cdot \vec{u}) \]  
(6)

3.2. Thermodynamic and Transport Properties

In hypersonic flight, the non-equilibrium of chemical reaction in air occurs only when the vehicles fly above an altitude of 40km and at a velocity higher than 4km/s [9]. Therefore, considering the flight conditions, the properties of the free stream and coolant are derived from the equilibrium state of chemical reactions. The differences in the properties between the two calculation methods (AGM in dotted red lines and RGE in solid black lines) are shown in figure 2 especially above Ma 8.

**Figure 2.** Transport properties calculated by approximate model and real effects.
4. Numerical Strategy

4.1. Calculation Grid and Independence of Numerical Results
As shown in figure 3, structured mesh with refined grids near the leading wedge is adopted. Table 1 displays the stagnation temperatures at Mach 12 calculated by the three mesh with different node numbers. The relative deviations are very small, therefore to balance the computational accuracy and load, the mesh with 76814 nodes is chosen in the following simulations.

| Node number | Stagnation temperature/K |
|-------------|--------------------------|
| 49229       | 3752.26                  |
| 76814       | 3754.02                  |
| 109812      | 3754.49                  |

4.2. Numerical Method
To overcome the convergence difficulty due to the coupled relation of the hypersonic flow in the free stream and the low-speed flow in the porous matrix, a three-step numerical solving strategy is applied:

1. The free stream region is calculated, the properties of air are seen as constant, here the entire outer surface of WLE is set as the adiabatic and impermeable wall.
2. The numerical results of constant properties are input as the initial values, the characteristics of the free-flow region are predicted again with AGM and RGE.
3. Enabling the porous and impermeable wall regions, the converged results in step 2 are used as the initial values of the free stream field, the boundaries among the three regions are set as interior and coupled wall.

4.3. Validation of Numerical Strategy Using Experimental Data
Experimental data obtained in the arc heating wind tunnel FD04 of China Academy of Aerospace Aerodynamic by Shen et al. [10] are used to validate the numerical strategy. The experiment was carried out at a Mach number of 4.2 and a total enthalpy of 2300 kJ/kg. Figure 4 indicates that the numerical results of RGE are closer to the experimental data than of AGM, especially at the stagnation point, which means the validity of the numerical strategy.

Figure 3. Structured mesh of computational domain. 
Figure 4. Comparisons of experiment data with numerical simulations.

5. Result and Discussions
Using the validated strategy, a series of numerical simulations are conducted at a flight altitude of 40km and three Mach numbers of 8, 10 and 12. The important design parameters of transpiration cooling systems are analysed and compared between the numerical results obtained by AGM and RGE.
5.1. Aerodynamic Characteristics Analysis
Due to the higher specific heat capacity of RGE, the temperatures around WLE shown in figure 5 are much lower than that of AGM, especially near the leading edge. At the stagnation point, the temperature predicted by RGE is lower than 3800 K, but by AGM achieves 6800 K. Then influenced by the differences in temperature and properties, the shock wave is closer to WLE using RGE. Figure 6 exhibits the pressure distributions predicted by AGM and RGE are quite similar. But more violently compressed after the shock wave, high-pressure region near the stagnation point occupies a larger area.

![Figure 5. Aerodynamic heat environments around WLE without cooling air injection.](image)

![Figure 6. Pressure fields around WLE in two cases.](image)

5.2. Transpiration Cooling Performances
Figure 7 illustrates the cooled wall temperature distributions predicted by AGM and RGE at different Mach numbers when cooling air is injected into the pores with the same velocity 6m/s. The temperatures predicted by REG are lower than that by AGM. This difference can be explained by the lower temperatures around WLE and more cooling air injected into the porous matrix using REG. Figure 8 illustrates the chamber pressure of the cooling air passing through the pores. From figure 8, a significant excess can be observed using REG due to the higher viscosity resistance. With Mach number increasing, these phenomena become more obvious, because of differences in the thermal properties between RGE and AGM are more remarkable, as shown in figure 2.

Figure 9 shows the cooling air mass flow rate distributions over the outer surface of the porous matrix in the two cases, and the total amounts are respectively calculated and listed in table 2. When Mach number increases, one can find the following three trends: 1) More mass flowrate enters the porous matrix in the cases of AGM and RGE. 2) The non-uniformity of the cooling air on the hot surface of the porous matrix is more obvious. 3) In comparison with the case of AGM, the consumption and non-uniformity of the cooling air of RGE increase significantly. These phenomena can be explained by the higher density of the cooling air due to the higher driving pressure, and the more obviously changing of the viscosity leads to a larger non-uniformity of the surface cooling air distribution.
Figure 7. Temperature fields around and within WLE under different Mach numbers in two cases.

Figure 8. Cooling air injection pressure under different Mach numbers in two cases.

Figure 9. Mass flow rate distributions over porous surface in three Mach numbers in two cases.

Table 2. Mass flow rate at different Mach numbers with the same velocity 6 m/s.

| Mach Number | Mass Flow Rate [kg/(m \cdot s)] | AGM | RGE |
|-------------|---------------------------------|-----|-----|
| 8           | 0.04015                          |     |     |
| 10          | 0.05096                          |     |     |
| 12          | 0.06113                          |     |     |

5.3. Comparison of Cooling Air Consumption and Driving Force

At the same cooled stagnation point temperature, the cooling air consumption and the corresponding driving force are compared between the cases of RGE and AGM. In the case of AGM, the velocity of the cooling air is fixed at 6 m/s. Adjusting the velocity of the cooling air using RGE, the same stagnation point temperatures with AGM, corresponding chamber pressures and mass flow rates are listed in table 3. From table 3, the following important phenomena can be found: 1) The injection...
velocity in the case of RGE is smaller than that in the case of AGM, and with an increase in Mach number, the cooling air velocity reduces further. 2) In the case of RGE, the corresponding mass flow rates of the cooling air are always smaller than that in the AGM case. And at the higher Mach number, the consumptions of different cases become closer due to the more obvious non-uniformity of mass flowrate distributions. 3) Although the injection velocities and cooling air consumption are lower in the case of RGE, the driving forces of the cooling air or the chamber pressures required are higher than that in the case of AGM, especially at a high Mach number.

Table 3. Coolant injection velocity and mass flow rate at different Mach numbers.

| Mach number | Stagnation point temperature [K] | AGM Injection velocity [m/s] | Mass flow rate [kg/(m·s)] | Driving pressure [Pa] | RGE Injection velocity [m/s] | Mass flow rate [kg/(m·s)] | Driving pressure [Pa] |
|-------------|---------------------------------|-----------------------------|---------------------------|----------------------|-------------------------------|--------------------------|----------------------|
| 8           | 1090                            | 0.04015                     | 367668                    | 5.0                  | 0.03619                       | 481829                   |                      |
| 10          | 1734                            | 0.05096                     | 507092                    | 4.5                  | 0.05028                       | 765729                   |                      |
| 12          | 2652                            | 0.06113                     | 651907                    | 3.78                 | 0.06071                       | 1089126                  |                      |

6. Conclusion
The thermal properties of gases including freestream and coolant will significantly change in high temperature, and deviate from the traditional approximate gas model (AGM). Therefore RGE should be used in the cooling system designs of hypersonic vehicles. This paper exhibits a numerical analysis of the transpiration cooling system design of a wedge leading edge under three hypersonic conditions, and compares the differences between using RGE and AGM. Through analysis and comparisons, the following conclusions can be drawn:

1. The thermal environment estimated with RGE is much milder, although the aerodynamic pressures on the hot surface show small differences between the two models.
2. At the same cooling air injection velocity, using RGE, the wall temperatures are much lower, but the cooling air mass flow rates and the driving force is much larger than using AGM, which means that to get the same cooled temperature at the stagnation point, the injection velocity required by RGE is always lower, corresponding driving force is still much higher than that by AGM.
3. The distributions of the temperature and cooling air mass flow rate in the stagnation region are more non-uniform, and these trends become more remarkable when the Mach number increases.

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Reference
[1] Vinh N X, Busemann A and Culp R D 1980 Hypersonic and planetary entry flight mechanics NASA STI/Recon Technical Report A 81.
[2] Schmisseur J D 2015 Hypersonics into the 21st century: A perspective on AFOSR-sponsored research in aerothermodynamics Progress in Aerospace Sciences 72 3–16, doi:10.1016/j.paerosci.2014.09.009.
[3] Sziroczak D and Smith H 2016 A review of design issues specific to hypersonic flight vehicles Progress in Aerospace Sciences 84 1–28 doi:10.1016/j.paerosci.2016.04.001.
[4] Zhu Y H, Peng W, Xu R N and Jiang P X 2018 Review on active thermal protection and its heat transfer for airbreathing hypersonic vehicles Chinese Journal of Aeronautics 31 1929–1953, doi:10.1016/j.cja.2018.06.011.
[5] D’Angola A, Colonna G, Gorse C and Capitelli M 2008 Thermodynamic and transport properties in equilibrium air plasmas in a wide pressure and temperature range Eur. Phys. J. D 46 129–150, doi:10.1140/epjd/e2007-00305-4.
[6] Bertin J J and Cummings R M 2006 Critical hypersonic aerothermodynamic phenomena Annual Review of Fluid Mechanics 38 129–157 doi:10.1146/annurev.fluid.38.050304.092041.

[7] Emelyanov V, Karpenko A and Volkov K 2019 Simulation of hypersonic flows with equilibrium chemical reactions on graphics processor units Acta Astronautica S0094576518320435 doi:10.1016/j.actaastro.2019.01.010.

[8] Glass D 2008 Ceramic Matrix Composite (CMC) Thermal Protection Systems (TPS) and Hot Structures for Hypersonic Vehicles In Proceedings of the 15th AIAA International Space Planes and Hypersonic Systems and Technologies Conference (Dayton, Ohio American Institute of Aeronautics and Astronautics).

[9] Bertin J J 1994 Hypersonic Aerothermodynamics American Institute of Aeronautics and Astronautics, Inc.

[10] Shen L, Wang J H, Dong W J, Pu J, Peng J L, Qu D J and Chen L Z 2016 An experimental investigation on transpiration cooling with phase change under supersonic condition Applied Thermal Engineering 105 549–556 doi:10.1016/j.applthermaleng.2016.03.039.