Two-Loop Analysis of Gauge Coupling Unification with Anomalous $U(1)$ Symmetry and Proton Decay

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Abstract

Recently, a new mechanism, which explains why the three gauge coupling constants meet at a certain scale in the minimal supersymmetric standard model, has been proposed in a scenario of grand unified theories with anomalous $U(1)_A$ gauge symmetry. It is a non-trivial result that although there are many superheavy fields whose mass scales are below the unification scale, this mechanism explains this fact by means of one-loop renormalization group equations. Since the unification scale generically becomes below the usual GUT scale, $2 \times 10^{16}$ GeV, and proton decay via dimension 5 operators is suppressed, the scenario predicts that proton decay via dimension 6 operators, $p \rightarrow e\pi^0$, will be observed in the near future. In this paper, we attempt to estimate a reasonable range of values of the lifetime of the proton predicted within this scenario by using a two-loop renormalization group calculation and the ambiguities of $O(1)$ coefficients.

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1 Introduction

Low energy supersymmetry (SUSY), which was originally introduced for stabilization of the weak scale, plays a critical role in explaining the hierarchical gauge couplings of the standard model in the context of $SU(5)$ grand unified theory (GUT). It is a non-trivial result that the three gauge couplings meet at a GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV if a reasonable SUSY breaking scale is assumed. However, the experimental lower bound of the proton lifetime via dimension 5 operators has become so severe that many GUTs that naturally realize coupling unification, for example, the minimal $SU(5)$ GUT, have been rejected.\(^1\) This is a kind of puzzle in the SUSY GUT scenario. Of course, if we consider the coincidence of the three gauge couplings at some scale to be accidental, then this puzzle is not difficult to solve. For example, after suppression of dimension 5 proton decay, the gauge couplings can be caused to meet at a certain scale by tuning the mass scales under the GUT scale. However, if the coincidence of these couplings is not accidental, solving this puzzle is not so simple, although some solutions have been proposed. Most of these solutions aim to show that the minimal SUSY standard model (MSSM) is realized under the GUT scale $\Lambda_G$ with the dimension 5 proton decay suppressed or forbidden. There are several solutions of this type in the context of extra dimensions. One employs parity assignment, which forbids dimension 5 operators.\(^8\)\(^9\) The other employs wave function suppression due to localization of quark and lepton fields.\(^8\)\(^9\) Even in the context of four-dimensional field theory, solving this puzzle is possible by introducing a special vacuum structure of one or two adjoint Higgs fields of $SO(10)$.\(^10\) However, recently, another type of solution has been proposed\(^11\) in the context of GUT with anomalous $U(1)$ gauge symmetry,\(^12\) whose anomaly is cancelled by the Green-Schwarz mechanism.\(^13\) It is surprising that in the GUT scenario with a simple unification group, gauge coupling unification is realized generically, despite the fact that many superheavy fields, whose mass spectrum does not respect the GUT symmetry, become lighter than the unification scale $\Lambda_A$ and there are several gauge symmetry breaking scales. This is because the mass spectrum of superheavy fields and the symmetry breaking scales are determined by the anomalous $U(1)_A$ charges, and most of the charges are cancelled under the conditions of coupling unification. (The unique exception is the charge of the doublet Higgs.) Moreover, the GUT scenario has many interesting features which we now describe.\(^11\)\(^14\)\(^15\)\(^16\)\(^17\)\(^1\) The interaction is generic in the sense that all the interactions that are allowed by the symmetry are introduced. Therefore, once we fix the field content with their quantum numbers (integers), all the interactions are determined, except the coefficients of order 1.\(^2\) It naturally solves the so-called doublet-triplet (DT) splitting problem,\(^18\) using the Dimopoulos-Wilczek (DW) mechanism.\(^19\)\(^20\)\(^2\) It reproduces the realistic structure of the quark and lepton mass matrices, including neutrino bi-large mixing,\(^21\) using the Froggatt-Nielsen (FN) mechanism.\(^22\)\(^4\) The anomalous $U(1)_A$ accounts for the hierarchical structure of the symmetry breaking scales and the masses of heavy particles.\(^5\) All the fields, except those of the minimal SUSY standard model (MSSM), can become heavy.\(^6\) The gauge couplings are unified just below the

\(^1\)Recently, it was pointed out that allowing arbitrary soft masses and fermion and sfermion mixing, there is still only a small portion of parameter space consistent with experimental results in the ‘decoupling’ region.\(^6\)
usual GUT scale, $\Lambda_G \sim 2 \times 10^{16}$ GeV. 7) In spite of the lower unification scale, proton decay via dimension 6 operators, $p \rightarrow e^+\pi^0$, is still within the experimental bound and therefore we expect to observe proton decay in the near future. 8) The cutoff scale is lower than the Planck scale. 9) The $\mu$ problem is also solved.

In the above-mentioned scenario, one of the most interesting predictions regards proton decay. Because the dimension 5 operators are suppressed, the main decay mode of proton decay is due to dimension 6 operators. Therefore, a more accurate estimate of the unification scale or the cutoff scale is important in order to obtain a more accurate prediction of the lifetime of the proton. In this paper, we determine an allowed range of values of the cutoff scale for several GUT models by means of two-loop renormalization group equations (RGEs) and using the freedom of $O(1)$ coefficients.

2 Gauge coupling unification (one-loop renormalization group)

In our scenario, because generic interactions are introduced, the order of every coefficient is determined by anomalous $U(1)_A$ charges. Therefore, the gauge symmetry breaking scales and the mass spectrum of superheavy fields are also determined by these charges. Thus we can examine whether the gauge couplings meet at the GUT scale once we determine all the charges. This is a consistency check of our scenario, and it has been shown that it is realized in a non-trivial way. In this section, we review this point using one-loop RGEs.

First, we note that the symmetry breaking scales are determined by anomalous $U(1)_A$ charges. Generically, the vacuum expectation value (VEV) of the gauge singlet operator $O$ is determined by its charge $o$ as

$$\langle O \rangle \sim \lambda^{-o}. \quad (1)$$

(Here $\lambda$ is the ratio of the cutoff scale $\Lambda$ to the VEV of the Froggatt-Nielsen field $\Theta$. In our scenario, we adopt $\lambda \sim 0.22$. ) Actually, in our scenario, the VEV of the adjoint field $A$ of $SO(10)$ becomes of the Dimopoulos-Wilczek type:

$$\langle A \rangle = i\tau_2 \times \text{diag}(v, v, v, 0, 0). \quad (2)$$

This fact plays an important role in realizing doublet-triplet splitting, and the scale is determined by the charges $a$ as $v \sim \lambda^{-a}\Lambda$. Throughout this paper we denote all superfields by uppercase letters and their anomalous $U(1)_A$ charges by the corresponding lowercase letters. We often use units in which $\Lambda = 1$. The VEV of $A$ breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The VEVs of the spinors $C(16)$ and $\bar{C}(1\overline{6})$, which break $SU(2)_R \times U(1)_{B-L}$ into $U(1)_Y$, are obtained from $\langle \bar{C}C \rangle \sim \lambda^{-(c+\bar{c})}$ and the $D$-flatness condition $| \langle C \rangle | = | \langle \bar{C} \rangle |$ as

$$| \langle C \rangle | = | \langle \bar{C} \rangle | \sim \lambda^{-(c+\bar{c})/2}. \quad (3)$$

Because $| \langle C \rangle | < | \langle A \rangle |$ is required to realize doublet-triplet splitting, at the scale $\Lambda_A \equiv \langle A \rangle \sim \lambda^{-a}$, the $SO(10)$ gauge group is broken into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.
which is broken into the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ at the scale $\Lambda_C \equiv \langle C \rangle \sim \lambda^{-(c+c)/2}$.

Second, we explain how the mass spectrum of superheavy fields is determined by anomalous $U(1)_A$ charges. Using the definitions of the fields $Q(3,2)_{\frac{1}{3}}, U^c(3,1)_{\frac{1}{3}}, D^c(3,1)_{\frac{1}{3}}, L(1,2)_{-\frac{1}{3}}, E^c(1,1)_1, N^c(1,1)_0$ and $X(3,2)_{-\frac{1}{3}}$, along with their conjugate fields, and $G(8,1)_0$ and $W(1,3)_0$ with the standard gauge symmetry, under $SO(10) \supset SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$, the spinor $16$, vector $10$ and adjoint $45$ of $SO(10)$ are decomposed as

\begin{align*}
16 & \rightarrow \underbrace{[Q + U^c + E^c]}_{10} + \underbrace{[D^c + L]}_{5} + \underbrace{N^c}_{1}, \\
10 & \rightarrow \underbrace{[D^c + L]}_{5} + \underbrace{[\bar{D}^c + \bar{L}]}_{\bar{5}}, \\
45 & \rightarrow \underbrace{[G + W + X + \bar{X} + N^c]}_{24} + \underbrace{[Q + U^c + E^c]}_{10} + \underbrace{[\bar{Q} + \bar{U}^c + \bar{E}^c]}_{10} + \underbrace{N^c}_{1}.
\end{align*}

(4) \quad (5) \quad (6)

A straightforward calculation of the mass matrices $\bar{M}_I$ of the superheavy fields $I = Q, U^c, E^c, D^c, L, G, W$ and $X$ shows that

$$\det \bar{M}_I = \lambda^{\sum_i c_i},$$

(7)

where the quantities $c_i$ are the anomalous $U(1)_A$ charges of the superheavy fields.

Third, we carry out an analysis based on the RGEs up to one loop. The conditions of gauge coupling unification are

$$\alpha_3(\Lambda_A) = \alpha_2(\Lambda_A) = \frac{5}{3} \alpha_Y(\Lambda_A) \equiv \alpha_1(\Lambda_A),$$

(8)

where $\alpha_1^{-1}(\mu > \Lambda_C) \equiv \frac{3}{2} \alpha_R^{-1}(\mu > \Lambda_C) + \frac{2}{3} \alpha_B^{-1}(\mu > \Lambda_C)$, Here $\alpha_X = \frac{g_X^2}{4\pi}$, and the parameters $g_X (X = 3, 2, R, B - L, Y)$ are the gauge couplings of $SU(3)_C, SU(2)_L, SU(2)_R$, $U(1)_{B-L}$ and $U(1)_Y$, respectively.

The gauge couplings at the scale $\Lambda_A$ are roughly given by

\begin{align*}
\alpha_1^{-1}(\Lambda_A) &= \alpha_1^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_1 \ln \left( \frac{MSB}{\Lambda_A} \right) + \sum_i \Delta b_{1i} \ln \left( \frac{m_i}{\Lambda_A} \right) - \frac{12}{5} \ln \left( \frac{\Lambda_C}{\Lambda_A} \right) \right), \\
\alpha_2^{-1}(\Lambda_A) &= \alpha_2^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_2 \ln \left( \frac{MSB}{\Lambda_A} \right) + \sum_i \Delta b_{2i} \ln \left( \frac{m_i}{\Lambda_A} \right) \right), \\
\alpha_3^{-1}(\Lambda_A) &= \alpha_3^{-1}(M_{SB}) + \frac{1}{2\pi} \left( b_3 \ln \left( \frac{MSB}{\Lambda_A} \right) + \sum_i \Delta b_{3i} \ln \left( \frac{m_i}{\Lambda_A} \right) \right),
\end{align*}

(9) \quad (10) \quad (11)

where $M_{SB}$ is a SUSY breaking scale, $(b_1, b_2, b_3) = (33/5, 1, -3)$ are the renormalization group coefficients for the minimal SUSY standard model (MSSM), and $\Delta b_{ai}$ ($a = 1, 2, 3$) are the corrections to the coefficients from the massive fields with mass $m_i$. The last term
in Eq. (14) is from the breaking $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ caused by the VEV $\langle C \rangle$. Because the gauge couplings at the SUSY breaking scale $M_{SB}$ are given by

$$
\alpha_i^{-1}(M_{SB}) = \alpha_G^{-1}(\Lambda_G) + \frac{1}{2\pi} \left( b_i \ln \left( \frac{\Lambda_G}{M_{SB}} \right) \right), \quad (i = 1, 2, 3)
$$

(12)

where $\alpha_G^{-1}(\Lambda_G) \sim 25$ and $\Lambda_G \sim 2 \times 10^{16}$ GeV, the above conditions for unification can be rewritten as

$$
b_1 \ln \left( \frac{\Lambda_A}{\Lambda_G} \right) + \Sigma_I \Delta b_{1I} \ln \left( \frac{\Lambda^{\beta I}_A}{\det M_I} \right) - \frac{12}{5} \ln \left( \frac{\Lambda_A}{\Lambda_C} \right)
$$

(13)

$$
b_2 \ln \left( \frac{\Lambda_A}{\Lambda_G} \right) + \Sigma_I \Delta b_{2I} \ln \left( \frac{\Lambda^{\alpha I}_A}{\det M_I} \right)
$$

(14)

$$
b_3 \ln \left( \frac{\Lambda_A}{\Lambda_G} \right) + \Sigma_I \Delta b_{3I} \ln \left( \frac{\Lambda^{\alpha I}_A}{\det M_I} \right)
$$

(15)

Here, $\bar{r}_I$ represents the ranks of the mass matrices of the superheavy fields, $\bar{M}_I$. The corrections to the renormalization coefficients $\Delta b_{aI}$ are given in the following table:

| $I$ | $Q + Q$ | $U^c + U^c$ | $E^c + E^c$ | $D^c + D^c$ | $L + L$ | $G$ | $W$ | $X + X$ |
|-----|---------|-------------|-------------|-------------|--------|-----|-----|--------|
| $\Delta b_{1I}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{5}{8}$ | $0$ | $0$ | $5$ |
| $\Delta b_{2I}$ | $3$ | $0$ | $0$ | $0$ | $1$ | $0$ | $2$ | $3$ |
| $\Delta b_{3I}$ | $2$ | $1$ | $0$ | $1$ | $0$ | $3$ | $0$ | $2$ |

The unification conditions $\alpha_1(\Lambda_A) = \alpha_2(\Lambda_A)$, $\alpha_1(\Lambda_A) = \alpha_3(\Lambda_A)$ and $\alpha_2(\Lambda_A) = \alpha_3(\Lambda_A)$ can be rewritten as

$$
\left( \frac{\Lambda_A}{\Lambda_G} \right)^{14} \left( \frac{\Lambda_G}{\Lambda_A} \right)^6 \left( \frac{\det \bar{M}_L}{\det M_{Dc}} \right) \left( \frac{\det \bar{M}_Q}{\det M_{Uc}} \right)^4 \left( \frac{\det \bar{M}_Q}{\det M_{Ec}} \right)^3 \left( \frac{\det \bar{M}_W}{\det M_X} \right)^5
$$

$$
= \Lambda_A^{-\bar{r}_{Dc}+\bar{r}_L-4\bar{r}_{Uc}-3\bar{r}_{Ec}+7\bar{r}_Q-5\bar{r}_X+5\bar{r}_W},
$$

(16)

$$
\left( \frac{\Lambda_A}{\Lambda_G} \right)^{16} \left( \frac{\Lambda_G}{\Lambda_A} \right)^4 \left( \frac{\det \bar{M}_{Dc}}{\det M_L} \right) \left( \frac{\det \bar{M}_Q}{\det M_{Uc}} \right) \left( \frac{\det \bar{M}_Q}{\det M_{Ec}} \right)^2 \left( \frac{\det \bar{M}_G}{\det M_X} \right)^5
$$

$$
= \Lambda_A^{-\bar{r}_{Dc}+\bar{r}_L+\bar{r}_{Uc}-2\bar{r}_{Ec}+3\bar{r}_Q-5\bar{r}_X+5\bar{r}_G},
$$

(17)

$$
\left( \frac{\Lambda_A}{\Lambda_G} \right)^4 \left( \frac{\det \bar{M}_{Dc}}{\det M_L} \right) \left( \frac{\det \bar{M}_L}{\det M_Q} \right) \left( \frac{\det \bar{M}_G}{\det M_W} \right)^2 \left( \frac{\det \bar{M}_G}{\det M_X} \right)
$$

$$
= \Lambda_A^{-\bar{r}_{Dc}+\bar{r}_L+\bar{r}_{Q}+\bar{r}_U-2\bar{r}_W-\bar{r}_X+3\bar{r}_G}.
$$

(18)

Note that the above conditions are dependent only on the ratio of the determinants of the mass matrices that are included in the same multiplet of $SU(5)$ and on the symmetry breaking scales $\Lambda_A$ and $\Lambda_C$. If all the component fields in a multiplet were superheavy, the above ratios would be of order 1. However, because some of the component fields, for example massless Higgs doublets or Nambu-Goldstone modes, do not appear in the mass matrices generically, the above ratios are dependent only on the charges of these massless
modes. If all the fields other than those in MSSM become superheavy, the above ratios are easily estimated as

$$\frac{\det \bar{M}_L}{\det \bar{M}_{D^c}} \sim \lambda^{-(h_u+h_d)},$$

(19)

$$\frac{\det \bar{M}_Q}{\det \bar{M}_{U^c}} \sim \lambda^{c+\bar{c}-2a},$$

(20)

$$\frac{\det \bar{M}_G}{\det \bar{M}_{X}} \sim \lambda^{-2a},$$

(21)

where $h_u$ and $h_d$ are the anomalous $U(1)_A$ charges of the massless Higgs doublets $H_u$ and $H_d$, respectively. Then the conditions for coupling unification become

$$\Lambda \sim \lambda^{\frac{h_u+h_d}{14}} \Lambda_G,$$

(22)

$$\Lambda \sim \lambda^{\frac{h_u+h_d}{16}} \Lambda_G,$$

(23)

$$\Lambda \sim \lambda^{\frac{h_u+h_d}{4}} \Lambda_G.$$  

(24)

Therefore the unification conditions become $h_u + h_d \sim 0$, and thus the cutoff scale must be taken as $\Lambda \sim \Lambda_G$. It is obvious that if the cutoff scale were some other scale (for example, the Planck scale), then in MSSM the three gauge couplings would meet at that scale. This implies that in this scenario it is not accidental that the three gauge couplings meet at some scale in MSSM. Note that these results are independent of the details of the Higgs sector, because the requirement that all the fields other than those in MSSM become superheavy determines the field content of the massless fields, whose charges are important to examine whether the gauge couplings meet at the unification scale $\Lambda_A$. The above argument can also be applied to the scenario of $E_6$ unification, though instead of the usual doublet Higgs charge $h$, we have to use effective Higgs charges,

$$h_{\text{eff}} \equiv h + \frac{1}{4}(\phi - \bar{\phi}),$$

(25)

where $E_6$ is broken into $SO(10)$ by the VEV $|\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| \sim \lambda^{-\frac{1}{2}(\phi + \bar{\phi})}$.

Note that the condition $h \sim 0$ does not mean that $h = 0$, because there is an ambiguity involving order 1 coefficients, and we have used only one-loop RGEs. However, the above analysis is useful in providing a rough picture of the behavior in which we are interested.

3 Proton decay

In order to understand how proton decay via dimension 5 operators is suppressed in our scenario, we now examine the mass matrix of triplet Higgs. From the interaction

$$W = \lambda^{h+h'+a} H A H' + \lambda^{2h'} H'^2,$$

(1)

the mass matrices of doublet and triplet Higgs are

$$(5, \bar{5}) \left( \begin{array}{cc} 0 & \lambda^{h+h'+a} \langle A \rangle \\ \lambda^{h+h'+a} \langle A \rangle & \lambda^{2h'} \end{array} \right) \left( \begin{array}{c} 5_H \\ \bar{5}_{H'} \end{array} \right).$$

(2)
Here $H$ and $H'$ represent $10$ of $SO(10)$ and their charges $h < 0$ and $h' > 0$. Note that the $H^2$ term is forbidden by the SUSY zero mechanism. The colored Higgsobtain masses of order $\lambda^{h+h'+a} \langle A \rangle \sim \lambda^{h+h'}$. Because in general, $\lambda^{h+h'} > \lambda^{h'}$, proton decay is naturally suppressed. The effective colored Higgs mass is estimated as $m_{\text{eff}} \sim (\lambda^{h+h'})^2 / \lambda^{h'} = \lambda^{h}$, which is larger than the cutoff scale, because $h < 0$. If the cutoff scale is the usual GUT scale, $\Lambda_G = 2 \times 10^{16}$ GeV, the condition $m_{\text{eff}} > 10^{18}$ GeV requires $h \leq -2$.

Strictly speaking, the condition for gauge coupling unification, $h = 0$, is not satisfied. However, we emphasize that the ambiguity of the coefficients of order 1 can naturally allow for the recovery of coupling unification. In order to change the unification condition, we have to consider the terms including the adjoint field $A$ whose VEV $\langle A \rangle$ breaks GUT symmetry. In addition to the mass term $m_X X X$, higher-dimensional operators $m_A X A X$ must be taken into account. In the usual GUT scenario, such a correction is much smaller than the tree level mass term, because of the suppression factor $\langle A \rangle / \Lambda$. Therefore, even if $m \sim m_X < \langle A \rangle$ is realized due to some symmetry, it is not easy to change the unification condition without finetuning or some special assumptions, for example, forbidding the mass term by some symmetry. However, the GUT with anomalous $U(1)_A$ symmetry naturally realizes $m_X \sim m_X(A)$, because $m_X \sim \lambda^{2a}$, $m \sim \lambda^{2a+2a}$ and $\langle A \rangle \sim \lambda^{-a}$. Moreover, in addition to these terms, $X A A X$ also contributes to the mass of the $X$ field after developing the VEV $\langle A \rangle$ if these terms are allowed by the symmetry. In other words, almost none of the coefficients respect GUT symmetry. Therefore, by adjusting such coefficients we can naturally change the unification condition.

Finally, we recall proton decay via dimension 6 operators in our scenario. Because the cutoff scale $\Lambda$ is around the usual GUT scale, $\Lambda_G = 2 \times 10^{16}$ GeV, and the unification scale becomes $\lambda^a$ ($a < 0$), proton decay via dimension 6 operators may be seen in future experiments. If we roughly estimate the lifetime of the proton using the formula in Ref. [4] and a recent result provided by a lattice calculation for the hadron matrix element parameter $\alpha$, we obtain

$$
\tau_p(p \rightarrow e\pi^0) \sim 2.8 \times 10^{33} \left( \frac{\Lambda_A}{5 \times 10^{15} \text{ GeV}} \right)^4 \left( \frac{0.015(\text{GeV})^3}{\alpha} \right)^2 \text{years.} \quad (3)
$$

This value is near the present experimental limit.[23]

4 Two-loop analysis

As mentioned above, in our scenario, the cutoff scale $\Lambda$ is deeply related to the $SO(10)$ breaking scale $\langle A \rangle \sim \lambda^{-a}$, and also to the mass of the massive gauge bosons, which cause proton decay. Therefore, to estimate the lifetime of the proton, we need to know how large the cutoff scale is. In addition, to realize somewhat accurate predictions, a two-loop analysis is necessary, because in our scenario there are many rather light superheavy particles, and therefore the coupling constants become larger than those of MSSM. In

Note that this value is independent of the gauge coupling $g_{10}$ at the unification scale. This is because the ratio $\frac{\Lambda_A}{m_X}$ is independent of $g_{10}$, because the mass of massive gauge bosons $m_X$ is given by $m_X = \sqrt{2} g_{10} v$. 

6
Table 1: Anomalous $U(1)_A$ charges of the non-trivial representational Higgs fields $(A(45, -), A'(45, -), C(16, +), C'(16, -), C'(16, +), H(10, +)$ and $H'(10, -))$ and of the matter fields $(\Psi_1(16, +), \Psi_2(16, +), \Psi_3(16, +)$ and $T(10, +))$ in some models of the $SO(10)$ scenario. Here, the sign $+$ or $-$ represents the $Z_2$ parity, and we assign odd R-parity for the matter fields.

|   | $A$ | $A'$ | $C$ | $C'$ | $H$ | $H'$ | $\Psi_1$ | $\Psi_2$ | $\Psi_3$ | $T$ |
|---|-----|-----|-----|-----|----|----|--------|--------|--------|----|
| i | -1  | 3   | -4  | -1  | 3  | 6  | 6      | 4      | 9/2    | 7/2 |
| ii| -1  | 3   | -3  | 0   | 2  | 5  | -2     | 3      | 4      | 3   |
| iii| -1  | 3   | -4  | -1  | 3  | 6  | -4     | 5      | 5      | 4   |
| iv| -1  | 3   | -7/2| 1/2 | 3/2| 11/2| -3     | 4      | 9/2    | 7/2 |
| v | -1  | 3   | -1  | -2  | 4  | 3  | -6     | 7      | 6      | 5   |
| vi| -1/2| 3/2 | -4  | -1  | 5/2| 11/2| -3     | 7/2    | 7/2    | 3/2 |
| vii| -1/2| 3/2 | -1  | -2  | 7/2| 5/2 | -6     | 13/2   | 6      | 5   |

Table 2: Anomalous $U(1)_A$ charges of the non-trivial representational Higgs fields $(A(78, -), A'(78, -), \Phi(27, +), \Phi(27, +), C(27, +), C(27, +), C'(27, -)$ and $C'(27, -))$ and of the matter fields $(\Psi_1(27, +), \Psi_2(27, +)$ and $\Psi_3(27, +))$ in some models of the $E_6$ scenario. Here, the sign $+$ or $-$ represents the $Z_2$ parity, and we assign odd R-parity for the matter fields. The MSSM doublet Higgs are contained in $\Phi$.

|   | $A$ | $A'$ | $\Phi$ | $\Phi'$ | $C$ | $C'$ | $C'$ | $\Psi_1$ | $\Psi_2$ | $\Psi_3$ |
|---|-----|-----|--------|--------|-----|-----|-----|--------|--------|--------|
| I | -1/2| 5/2 | -3     | 2      | -5  | 1   | -1  | 13/2   | 13/2   | 9/2    |
| II| -1/2| 5/2 | -3     | 1      | -4  | 1   | -1  | 13/2   | 11/2   | 9/2    |
| III| -1  | 4   | -3     | 2      | -6  | -2  | 7   | 8      | 9/2    | 7/2    |

In fact, in the $E_6$ scenario, they sometimes become too strong to rely on the perturbation analysis. However, in the $SO(10)$ scenario, the two-loop effect is still not excessively large, and we can treat it as a small correction to the one-loop analysis.

In this section, for several models, we investigate how large the cutoff scale can be by using the two-loop RGEs and altering $O(1)$ coefficients in the range of $y^{-1}_{\text{max}} \leq y \leq y_{\text{max}}$ as free parameters. The detailed procedure of this analysis is explained in Appendix A.

4.1 Models

There are several models that may work well. Among them we examine some representative models in Tables [1] and [2].

4.1.1 $SO(10)$ scenario

There are four groups of models, characterized by different values of $h$: ii with $h = -2$; i, iv and vi with $h = -3$; iii with $h = -4$; v and vii with $h = -6$. A model with a larger charge
Table 3: $\Lambda_{\text{max}}$ in units of $10^{16}\text{GeV}$ for some models of the $SO(10)$ scenario. Basically, the parameters are set as $y_{\text{max}} = 2$, $\lambda = 0.22$ and $\alpha_s^{-1}(M_Z) = 8.44$, but in each case, one of these values is different from these, as indicated in the first row. The notation “—” means that there is no solution for gauge coupling unification.

|   | $y_{\text{max}} = 2^2$ | $y_{\text{max}} = 2^\frac{3}{2}$ | $\lambda = 0.20$ | $\lambda = 0.25$ | $\alpha_s^{-1} = 8.30$ | $\alpha_s^{-1} = 8.58$ | 1 loop |
|---|---------------------|---------------------|-----------------|-----------------|-------------------|-------------------|--------|
| i | 3.00                | 15.3                | —               | 2.80            | 3.32              | 3.45              | 2.61   | 3.89  |
| ii| 4.17                | 18.3                | 1.88            | 3.97            | 4.43              | 4.85              | 3.63   | 5.47  |
| iii| 1.95                | 10.0                | —               | 1.77            | 2.22              | 2.24              | 1.68   | 2.53  |
| iv| 2.69                | 13.3                | —               | 2.48            | 2.97              | 3.09              | 2.34   | 3.89  |
| v | —                   | 3.78                | —               | —               | —                 | —                 | —      | —     |
| vi| 2.80                | 14.6                | —               | 2.58            | 3.09              | 3.21              | 2.41   | 3.89  |
| vii| —                  | 3.93                | —               | —               | —                 | —                 | —      | —     |

of the Higgs doublet can realize gauge coupling unification more naturally. Therefore, in the models v and vii it is not so easy to realize gauge coupling unification, though these models have the advantage that the FCNC constraint is weaker because $\psi_1 = t$. Half integer charges can play the same role as $Z_2$ parity or R-parity. For example, model i does not require R-parity, model vii does not require $Z_2$ parity, and model vi requires only either of R-parity or $Z_2$ parity. Models vi and vii have a value $-1/2$ for the charge of $A$, and therefore, because the unification scale is given by $\Lambda_A \sim \lambda^{-a}$, these models predict a more stable proton than the other models. Note that the model ii has the possibility that proton decay via dimension 5 operators dominates that via dimension 6 operators.

4.1.2 $E_6$ scenario

The model III is the unique consistent model with $a = -1$. Models I and II are characterized by $a = -1/2$ and the “effective charge” $h_{\text{eff}} = -17/4$ and $-4$, respectively. These models of the $E_6$ scenario require only either R-parity or $Z_2$ parity, because half integer charges play the same role as these parities. Also, they automatically meet the condition for suppressing FCNC.

4.2 Results

For each model, we calculate the maximal value of the cutoff scale $\Lambda_{\text{max}}$, with which gauge coupling unification is realized, setting the parameters $y_{\text{max}}$, $\lambda$ and $\alpha_s(M_Z)$ as in Tables 3 and 4. In the $E_6$ scenario, we additionally adopt the constraint that the gauge coupling constants do not become too large.

4.2.1 $SO(10)$ scenario

Roughly speaking, as $h$ increases by 1, $\Lambda_{\text{max}}$ increases by a factor of approximately 1.5. On the other hand, for a single value of $h$ (models i, iv and vi), the values of $\Lambda_{\text{max}}$ may
Table 4: $\Lambda_{\text{max}}$ in units of $10^{16}$GeV for some models of the $E_6$ scenario. Basically, the parameters are set as $y_{\text{max}} = 2$, $\lambda = 0.22$ and $\alpha_s^{-1}(M_Z) = 8.44$, and $\alpha_i(\Lambda_A) < 1$ is required, but in each case, one of these values is different from these, as indicated in the first row. The notation “—” means that there is no solution for gauge coupling unification with the conditions considered. Most cases with no solution result from the condition $\alpha_i(\Lambda_A) < 1$, which is much different from $SO(10)$ cases.

|   | $y_{\text{max}} = 2^2$ | $y_{\text{max}} = 2\pi$ | $\lambda = 0.20$ | $\lambda = 0.25$ | $\alpha_s^{-1} = 8.30$ | $\alpha_s^{-1} = 8.58$ | 1 loop |
|---|-----------------|-----------------|----------------|----------------|----------------|----------------|--------|
| I | 9.4             | 79.0            | 2.91           | 4.57           | 19.3           | 10.2           | 8.66   | 11.0   |
| II| 11.7            | 99.3            | 3.63           | 5.75           | 21.1           | 12.5           | 10.7   | 11.6   |
| III| —               | 16.4            | —              | —              | 6.10           | —              | —      | 6.68   |

These values differ from each other by only about 10%, and $\Lambda_{\text{max}}$ does not depend strongly on the parameters $\lambda$ and $\alpha_s$, as seen in Table 3. If we set $y_{\text{max}} = 2$, $\Lambda_{\text{max}}$ is not far from the naively expected value $\Lambda \sim 2 \times 10^{16}$ GeV. Therefore, the prediction of the proton lifetime is at most a factor of $\sim 20$ larger than the naively estimated value $3 \times 10^{33}$ years for the models with $a = -1$, and $5 \times 10^{34}$ for the models with $a = -1/2$. Note that this factor of $\sim 20$ represents the largest value allowed by the ambiguity in the value of the $O(1)$ coefficients. This is an unlikely situation, and therefore we believe that the actual value of this factor is smaller.

Unfortunately, the value $\Lambda_{\text{max}}$ is strongly dependent on $y_{\text{max}}$, and unless we fix $y_{\text{max}}$, we cannot precisely predict the lifetime of the proton. For this reason, we analyse more precisely the dependence on $y_{\text{max}}$ in Fig. 11. It is found that the most probable $\Lambda$ becomes smaller than the naively expected value $\Lambda_G \sim 2 \times 10^{16}$ GeV, and this difference becomes larger as $h$ decreases. Furthermore, it is obvious that gauge coupling unification is more difficult to realize for smaller $h$. Therefore, if we take account of the present limit on the proton lifetime provided by experiments, the models v and vii seem to be unrealistic.

Finally, comparing the one- and two-loop results, it is seen that the two-loop effect in fact slightly worsens the situation, causing $\Lambda_{\text{max}}$ to decrease by a factor of $\sim 1.4$, which corresponds to a prediction of the proton lifetime that is shorter by a factor of $\sim 1/4$.

4.2.2 $E_6$ scenario

Since the $E_6$ scenario has a larger Higgs sector, the models in this case have more superheavy fields than in the $SO(10)$ scenario. This means that the $E_6$ models have larger degrees of freedom with regard to the values of the $O(1)$ coefficients than do the $SO(10)$ models. Therefore, it is expected that gauge coupling unification will be easier to realize in the $E_6$ models. Actually, as seen in Table 3, the maximal values of the cutoff $\Lambda_{\text{max}}$ tend to be larger here than in the $SO(10)$ cases. However, since the $E_6$ models have more superheavy fields, the gauge couplings around the unification scale tend to become so strong that the perturbative analysis is not reliable. For this reason, we add an additional constraint at the unification scale $\Lambda_A$,

$$\alpha_X(\Lambda_A) < 1.$$ (1)
Figure 1: $y_{\text{max}}$ vs $\Lambda_{\text{min}}$ and $\Lambda_{\text{max}}$. Here, the four curves corresponding to the four models, i, ii, iii and vii ($h = -3, -2, -4, -6$, respectively), are plotted together. The allowed region is the upper region of the curve for each model. Other parameters are set as $\lambda = 0.22$ and $\alpha_{s}^{-1}(M_{Z}) = 8.44$.

Mainly because of this constraint, the results displayed in Table I behave significantly differently. In order to satisfy this condition that $\alpha_{X}$ be small, for many cases in the $E_{6}$ scenario, most of the degrees of freedom concerning the $O(1)$ coefficients are lost. We can understand this from the $\lambda$ dependence elucidated in Table II. Because the parameter $\lambda$ represents a unit of mass of the superheavy fields, $\alpha_{X}$ around the unification scale depends significantly on this parameter. Actually, as $\lambda$ becomes larger, the gauge couplings at the unification scale become smaller, and therefore more degrees of freedom of the $O(1)$ coefficients remain unfixed and can be freely adjusted for the purpose of realizing gauge coupling unification. Through this effect, the increase of the maximal value of the cutoff $\Lambda_{\text{max}}$ in the $E_{6}$ scenario becomes much larger than in the $SO(10)$ scenario. In particular, in model III, $\alpha_{X}$ becomes larger than in models I and II, and hence more degrees of freedom are lost due to the requirement that $\alpha_{X}$ be small. Therefore it is more difficult to realize gauge coupling unification. Actually, when $y_{\text{max}} = 2$, only the case with $\lambda = 0.25$ can realize gauge coupling unification. If we tighten the constraint and require $\alpha_{X}$ to be smaller than unity at the cutoff scale $\Lambda$, even in models I and II, coupling unification is difficult to realize.

5 Discussion and summary

In the $SO(10)$ scenario, because the allowed range of values of the parameters $\lambda$ and $\alpha_{s}(M_{Z})$ is limited, in this range, the maximal value of the cutoff $\Lambda_{\text{max}}$ does not change greatly. Because $\Lambda_{\text{max}}$ is obtained by tuning the $O(1)$ coefficients to give the largest value, a reasonable value of $\Lambda$ would be less than the maximal value. If we fix $y_{\text{max}} = 2$, the
maximal value of the cutoff does not differ by too much from the naively expected value, \( \Lambda \sim 2 \times 10^{16} \text{ GeV} \), and therefore a realistic prediction for proton decay can be considered to be not greatly different from the naive prediction, which is, for \( a = -1 \),

\[
\tau_p(p \rightarrow e\pi^0) \sim 2.8 \times 10^{33} \left( \frac{\Lambda_A}{5 \times 10^{15} \text{ GeV}} \right)^4 \left( \frac{0.015(\text{GeV})^3}{\alpha} \right)^2 \text{ years,} \tag{1}
\]

and for \( a = -0.5 \),

\[
\tau_p(p \rightarrow e\pi^0) \sim 4.5 \times 10^{34} \left( \frac{\Lambda_A}{1 \times 10^{16} \text{ GeV}} \right)^4 \left( \frac{0.015(\text{GeV})^3}{\alpha} \right)^2 \text{ years.} \tag{2}
\]

Unfortunately this prediction is strongly dependent on the unknown parameter for the \( \mathcal{O}(1) \) coefficients, \( y_{\text{max}} \). If \( y_{\text{max}} = 4 \), the upper bound of the prediction may be beyond the scope of future experiments.

In the \( E_6 \) scenario, on the other hand, because the coupling constants tend to be too large to allow a reliable perturbative analysis, we have added an artificial constraint to preserve the validity of our perturbative analysis. Though the \( E_6 \) models have more degrees of freedom among the \( \mathcal{O}(1) \) coefficients than do the \( SO(10) \) models, most of them are lost to the requirement of suppressing the gauge couplings. If we had some mechanism to suppress the gauge coupling constants other than the degrees of freedom of the \( \mathcal{O}(1) \) coefficients, then the \( E_6 \) models could much more easily realize gauge coupling unification. Actually, typical values of the maximal cutoff \( \Lambda_{\text{max}} \) in the \( E_6 \) cases are larger than in the \( SO(10) \) cases. It may be more natural in the \( E_6 \) scenario to assume that the gauge couplings are in the non-perturbative region. Though the perturbative analysis is not reliable in such cases, we may expect that the actual result is not so different from the perturbative prediction. This follows from the argument that if gauge coupling unification in MSSM is not accidental, it may be natural to realize a situation similar to that in the perturbative region, which can explain gauge coupling unification in MSSM.

Note that there are other effects that can change the running of the gauge couplings that are not taken into account in the analysis presented in this paper. One effect is from SUSY breaking parameters. Generically, the SUSY breaking scale is not one scale, and therefore there are some effects contributing to the condition for gauge coupling unification. Here, we roughly estimate the contribution in the case of model i of \( SO(10) \) GUT. If we set the masses of the colored superpartners to 1TeV and the masses of colorless superpartners to 100 GeV, then \( \Lambda_{\text{max}} \) decreases by a factor of \( \sim 0.4 \). It would seem that, this is probably an overestimation. For example, if we restrict the gaugino masses to satisfy the GUT relations

\[
\frac{M_3}{\alpha_3} = \frac{M_2}{\alpha_2} = \frac{M_1}{\alpha_1}, \tag{3}
\]

the factor by which \( \Lambda_{\text{max}} \) is decreased becomes 0.5-0.6. The contribution of the SUSY breaking parameters tends to decrease \( \Lambda_{\text{max}} \). Therefore, the estimations given in this paper still give conservative limits on the proton lifetime. Another effect is from the lack of our knowledge about the values of the \( \mathcal{O}(1) \) coefficients. We have used 1 as the central value for the \( \mathcal{O}(1) \) coefficients, but this is only an assumption. The result necessarily depends on this central value, especially in \( E_6 \) case, because the mass spectrum of superheavy
fields depends on this value. We have not used the ambiguities of the gauge symmetry breaking scale due to the $O(1)$ coefficients. This effect can change not only the running of the gauge couplings directly but also the spectrum of superheavy fields if $\langle \bar{C}C \rangle > \lambda^{-(c+\epsilon)}$. From the non-renormalizable term $XX\bar{C}C$, by developing a non-vanishing VEV, the field $X$ can acquire a mass that is larger than expected. Though this effect exists only for fields lighter than $\lambda^{c+\epsilon}$, it may suppress the gauge couplings at the unification scale.

Finally, we emphasize that even if the upper bound for the prediction of the proton lifetime is beyond the scope of future experiments, a more likely prediction of our analysis is near the naive prediction, which is not very far from the present experimental bound.

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A Recipe

As mentioned in §2, in our scenario, the mass spectrum of every heavy Higgs is determined within the range allowed by the ambiguity of the $O(1)$ coefficients. Therefore we can easily calculate the second order $\beta$ function of the effective theories appropriate for each scale (see Appendix B). If we use the $\overline{\text{DR}}$ scheme, the naive step function approximation is good for connecting each gauge coupling constant of the neighboring effective theory, including the case in which the symmetries of these effective theories differ.

We adopt $SU(3)_C \times SU(2)_L \times U(1)_Y$ for $\mu < \lambda^{\phi/2} \Lambda$, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ for $\lambda^{\phi/2} \Lambda < \mu < \lambda^{-a} \Lambda$, $SO(10)$ for $\lambda^{-a} \Lambda < \mu < \Lambda$ in the $SO(10)$ scenario and $\lambda^{-a} \Lambda < \mu < \lambda^{-\phi/2} \Lambda$ in the $E_6$ scenario, and $E_6$ for $\lambda^{-\phi/2} \Lambda < \mu < \Lambda$ in the $E_6$ scenario as the symmetries of the effective theories. Here, we approximate the masses of massive gauge bosons as being the same as the symmetry switching scale.

Regarding the Yukawa coupling effect, we consider only that of the interaction directly related to the left- and right-handed top quark in terms of the relevant symmetry, i.e. $U_3^cQ_3H_u$ for $SU(3)_C \times SU(2)_L \times U(1)_Y$, $Q_{R3}Q_{L3}H_D$ for $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, $\Psi_{3(16)}\Psi_{3(16)}H(10)$ for $SO(10)$, and $\Psi_{3(27)}\Psi_{3(27)}\Phi(27)$ for $E_6$. For the scale dependence of the Yukawa coupling, we use one loop RGEs, which are sufficient for the purpose of investigating the flow of the gauge coupling constants.

We start from the central values of the $\overline{\text{MS}}$ gauge coupling constants at $\mu = M_Z$ given in Ref. [26], including $1\sigma$ ambiguity for $\alpha_s(M_Z)$. We set $M_{t^{\overline{\text{MS}}}} = 165$ GeV, $\tan \beta = 5$, and $v = 174$ GeV, which correspond to $y_t = 0.967$. From $M_Z$ to the SUSY breaking scale, we use the RGEs of the standard model, which contain three family fermions and one Higgs doublet. Since we do not specify the SUSY breaking mechanism, we fix the SUSY breaking scale at 1 TeV and adopt a naive step function approximation, except in the transformation from the $\overline{\text{MS}}$ scheme into $\overline{\text{DR}}$, as the SUSY breaking threshold effect. At the $SU(2)_R \times U(1)_{B-L}$ breaking threshold, we must transform $\{\alpha_Y, \alpha_2, \alpha_3\}$ into 12.
\{\alpha_{B-L}, \alpha_R, \alpha_2, \alpha_3\}, in contrast to the one loop analysis. We set \(\alpha_{B-L}^{-1} = \frac{3}{2}(\alpha_Y^{-1} - \alpha_R^{-1})\) at \(\mu = \lambda^{-(c+\bar{c})/2} \Lambda\), and iteratively adjust \(\alpha_R\) so that it is equal to \(\alpha_2\) at the \(SO(10)\) breaking scale \(\Lambda\).

We investigate whether gauge coupling unification can be realized as explained above, by using the ambiguity of the \(O(1)\) coefficients, for some values of the parameters \(\Lambda, y_{\text{max}}, \lambda,\) and \(\alpha_s(M_Z)\). In practice, we adjust all the independent masses of the superheavy Higgs by factors of \(y_{\text{max}}^{-1} y_{\text{max}}\), instead of adjusting the \(O(1)\) coefficients. Then, if all the differences of the values of \(\alpha_X^{-1}\) can be made smaller than 0.05, we regard gauge coupling unification to be possible.

**B Renormalization group equations**

We use two-loop RGEs for the gauge coupling constants and one-loop RGEs for the top Yukawa coupling constant:

\[
\beta_i = \frac{g_i^3}{16\pi^2} b_i + \frac{g_i^3 g_j^2}{(16\pi^2)^2} b_{ij} + \frac{g_i^3 y_t^2}{(16\pi^2)^2} a_i, \\
\beta_t = \frac{g_t^3}{16\pi^2} C + \frac{y_t g_i^2}{16\pi^2} C_i. \quad (1)
\]

Here, \(a_i, b_i, b_{ij}, a_i, C\) and \(C_i\) are some constants.

Considering only the top Yukawa interaction, as explained in Appendix A, \(a_i, C\) and \(C_i\) are determined as follows.\[28\] For \(SU(3)_C \times SU(2)_L \times U(1)_Y\) (non-SUSY) \((i = 1, 2, 3)\),

\[
a_i = \begin{pmatrix} 17/10 \\ 3/2 \\ 2 \end{pmatrix}, \quad C = 9/2, \quad C_i = \begin{pmatrix} 17/20 \\ 9/4 \\ 8 \end{pmatrix}, \quad (3)
\]

for \(SU(3)_C \times SU(2)_L \times U(1)_Y\) \((i = 1, 2, 3)\),

\[
a_i = \begin{pmatrix} 26/5 \\ 6 \\ 4 \end{pmatrix}, \quad C = 6, \quad C_i = \begin{pmatrix} 13/15 \\ 3 \\ 16/3 \end{pmatrix}, \quad (4)
\]

for \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) \((i = B - L, R, 2, 3)\),

\[
a_i = \begin{pmatrix} 2 \\ 12 \\ 12 \\ 8 \end{pmatrix}, \quad C = 7, \quad C_i = \begin{pmatrix} 1/3 \\ 3 \\ 3 \\ 16/3 \end{pmatrix}, \quad (5)
\]

for \(SO(10)\) \((i = SO(10))\),

\[
a_i = 64, \quad C = 40, \quad C_i = 63, \quad (6)
\]

and for \(E_6\) \((i = E_6)\)

\[
a_i = 180, \quad C = 60, \quad C_i = 104. \quad (7)
\]
The constants $b_i$ and $b_{ij}$ are determined by the matter content of the effective theory. For example, for the standard model ($i = 1, 2, 3$), which contains three families of fermions and one Higgs doublet,

$$b_i = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}, \quad (8)$$

and for the MSSM ($i = 1, 2, 3$), which contains three family chiral superfields and two Higgs doublet chiral superfields,

$$b_i = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}. \quad (9)$$

The correction due to each representational chiral superfield is as follows. For $SU(3)_C \times SU(2)_L \times U(1)_Y$ ($i = 1, 2, 3$),

$$\Delta b_i = \begin{pmatrix} 1/5 \\ 3/2 \\ 2 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 1/75 & 3/5 & 16/15 \\ 1/5 & 21 & 16 \\ 2/15 & 6 & 68/3 \end{pmatrix} \quad \text{for } Q + \bar{Q}, \quad (10)$$

$$\Delta b_i = \begin{pmatrix} 8/5 \\ 0 \\ 1 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 128/75 & 0 & 128/15 \\ 0 & 0 & 0 \\ 16/15 & 0 & 34/3 \end{pmatrix} \quad \text{for } U^c + \bar{U}^c, \quad (11)$$

$$\Delta b_i = \begin{pmatrix} 6/5 \\ 0 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 72/25 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{for } E^c + \bar{E}^c, \quad (12)$$

$$\Delta b_i = \begin{pmatrix} 2/5 \\ 0 \\ 1 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 8/75 & 0 & 32/15 \\ 0 & 0 & 0 \\ 4/15 & 0 & 34/3 \end{pmatrix} \quad \text{for } D^c + \bar{D}^c, \quad (13)$$

$$\Delta b_i = \begin{pmatrix} 3/5 \\ 1 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 9/25 & 9/5 & 0 \\ 3/5 & 7 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{for } L + \bar{L}, \quad (14)$$

$$\Delta b_i = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 54 \end{pmatrix} \quad \text{for } G, \quad (15)$$

$$\Delta b_i = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{for } W, \quad (16)$$

$$\Delta b_i = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 25/3 & 15 & 80/3 \\ 5 & 21 & 16 \\ 10/3 & 6 & 68/3 \end{pmatrix} \quad \text{for } X + \bar{X}. \quad (17)$$

For $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, first we define the abbreviation of each representation $(SU(3)_C, SU(2)_L, SU(2)_R, U(1)_{B-L})$ as $Q_L(3, 2, 1)_\pm$, $Q_R(\bar{3}, 1, 2)_\mp$, $L_L(1, 2, 1)_-$, $L_R(1, 2, -1)_-$,
\[ L_R(1, 1, 2)_1, H_T(3, 1, 1)_{-\frac{3}{2}}, U(3, 1, 1)_{\frac{3}{2}}, X_Q(3, 2, 2)_{-\frac{3}{2}} \] and their conjugates, and \( H_D(1, 2, 2)_0, G(8, 1, 1)_0, W_L(1, 3, 1)_0, W_R(1, 1, 3)_0 \) and \( N(1, 1, 1)_0. \] The spinor 16, vector 10 and adjoint 45 of \( SO(10) \) are decomposed as

\[ 16 \rightarrow Q_L + Q_R + L_L + L_R, \quad (18) \]
\[ 10 \rightarrow H_T + H_T + H_D, \quad (19) \]
\[ 45 \rightarrow G + W_L + W_R + N + X_Q + \bar{X}_Q + U + \bar{U}. \quad (20) \]

Then, the corrections due to these representational chiral superfields are \((i = B - L, R, 2, 3)\)

\[
\Delta b_i = \begin{pmatrix} 1/2 \\ 0 \\ 3 \\ 2 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 1/12 & 0 & 3/2 & 8/3 \\ 0 & 0 & 0 & 0 \\ 1/2 & 21 & 16 \\ 1/3 & 6 & 68/3 \end{pmatrix} \quad \text{for } Q_L + Q_L, \quad (21)
\]

\[
\Delta b_i = \begin{pmatrix} 1/2 \\ 3 \\ 0 \\ 2 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 1/12 & 3/2 & 0 & 8/3 \\ 1/2 & 21 & 16 \\ 0 & 0 & 0 & 0 \\ 1/3 & 6 & 68/3 \end{pmatrix} \quad \text{for } Q_R + \bar{Q}_R, \quad (22)
\]

\[
\Delta b_i = \begin{pmatrix} 3/2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 9/4 & 0 & 9/2 & 0 \\ 0 & 0 & 0 & 0 \\ 3/2 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{for } L_L + \bar{L}_L, \quad (23)
\]

\[
\Delta b_i = \begin{pmatrix} 3/2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 9/4 & 9/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3/2 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{for } L_R + \bar{L}_R, \quad (24)
\]

\[
\Delta b_i = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 2/3 & 0 & 0 & 16/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2/3 & 0 & 0 & 34/3 \end{pmatrix} \quad \text{for } H_T + \bar{H}_T, \quad (25)
\]

\[
\Delta b_i = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 0 \\ 0 & 3 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{for } H_D, \quad (26)
\]

\[
\Delta b_i = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 54 \end{pmatrix} \quad \text{for } G, \quad (27)
\]

---

\(^3\)Here we write \( U(1)_{B-L} \) charges with the usual normalization, but in the context of \( SO(10) \) symmetry, they are multiplied by \( \sqrt{3/8} \).
\[
\Delta b_i = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\text{for } W_L, \quad (28)
\]

\[
\Delta b_i = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\text{for } W_R, \quad (29)
\]

\[
\Delta b_i = \begin{pmatrix} 4 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 32/3 & 0 & 0 & 64/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8/3 & 0 & 0 & 34/3 \end{pmatrix}
\text{for } U + \bar{U}, \quad (30)
\]

\[
\Delta b_i = \begin{pmatrix} 4 \\ 6 \\ 6 \\ 4 \end{pmatrix}, \quad \Delta b_{ij} = \begin{pmatrix} 8/3 & 12 & 12 & 64/3 \\ 4 & 42 & 18 & 32 \\ 4 & 18 & 42 & 32 \\ 8/3 & 12 & 12 & 136/3 \end{pmatrix}
\text{for } X_Q + \bar{X}_Q. \quad (31)
\]

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