On the Infrared Behavior of Landau Gauge Yang-Mills Theory
with a Fundamentally Charged Scalar Field

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Recently it has been shown that infrared singularities of Landau gauge QCD can confine static quarks via a linearly rising potential. We show that the same mechanism can also provide a confining interaction between charged scalar fields in the fundamental representation. This confirms that within this scenario static confinement is a universal property of the gauge sector even though it is formally represented in the functional equations of the matter sector. The simplifications compared to the fermionic case make the scalar system an ideal laboratory for a detailed analysis of the confinement mechanism in numerical studies of the functional equations as well as in gauge-fixed lattice simulations.

I. INTRODUCTION

The confinement of quarks is a remarkable phenomenon that features both static and dynamical aspects. To reveal its underlying mechanism it is on the one hand essential to restrict the analysis by considering certain limits where the system is considerably simpler and particular aspects of the mechanism become transparent. On the other hand one extends the analysis to similar systems, in order to identify the requirements and characteristic properties of the mechanism. An important limit in this respect is the static case and the corresponding confinement of fundamental color sources which presents an analytic result of non-Abelian gauge theory in the strong-coupling limit \cite{11} and which has been confirmed in numerical simulations at realistic coupling. This is reflected by an area law behavior of large Wilson loops corresponding to a linear potential between the sources and presents a genuine property of the gauge dynamics that is independent of the detailed aspects of the sources and merely depends on the representation of the gauge group.

Within the last years a detailed picture of the infrared (IR) sector of Yang-Mills theory and QCD has evolved, mainly due to investigations within either functional approaches as Dyson-Schwinger equations \cite{2,10,12,20} and functional renormalisation group equations \cite{13,21,25}, respectively, or lattice gauge theory \cite{26,28}. It has been shown that the infrared singularities of Landau gauge quantum chromodynamics (QCD) can provide a mechanism for the confinement of static quarks \cite{16}. This mechanism, relying on the IR scaling solution of Yang-Mills theory \cite{4,13}, is driven by a strong kinematic singularity of the quark-gluon vertex that overturns the IR suppression of the gluon propagator and leads to a long-range gluonic interaction. Yet, in this approach this is inherently obtained from the static limit of the solution of the functional equations of the quark sector of the theory. Since the mechanism exhibits a relation between chiral symmetry breaking and confinement, this poses the question whether the Dirac structure of the quarks is essential for this mechanism. In order to answer this question we present here results for a related theory where the quarks are replaced by a fundamentally charged scalar field, for details we refer to \cite{29}. This theory is interesting because of its simplicity owing to the absence of internal spin degrees of freedom compared to the fermionic theory and has been studied before as a model system for the QCD dynamics. On the other hand, this theory involves additional self-interactions between the scalars and thereby could exhibit a rather different dynamical behavior. Most notably it involves a Higgs phase in addition to a confining phase, and it has been shown that in the fundamentally charged scalar model the confined and the Higgs phase are not separated by a phase boundary \cite{30}.

In dynamical QCD a gluonic interaction that rises with distance is only realized over a certain range that varies continuously with the quark mass(es) \cite{17}. A simplified model system to study all these aspects would be highly desirable. Interestingly string breaking signatures have likewise been observed in lattice simulations of the scalar model, cf. e.g. \cite{31}, before corresponding studies were possible in dynamical QCD, cf. e.g. \cite{32}.

II. DYNAMICS OF FUNDAMENTALLY CHARGED SCALAR FIELDS COUPLED TO LANDAU GAUGE YANG-MILLS THEORY

In order to construct a model system for full QCD, where (fermionic) quarks carry a conserved charge and transform according to the fundamental representation of the gauge group, we attribute the same transformation properties also to the (bosonic) scalars. Therefore the scalars will firstly be implemented in the fundamental representation, and secondly the scalar field must be a complex field in order to be able to define a conserved charge. These scalars will be coupled to an $SU(N)$ gauge theory, and as we aim at comparing the system to Landau gauge QCD we also fix to Landau gauge, i.e. we take the limit for the gauge fixing parameter $\zeta \to 0$. Considering only renormalizable
interactions this generally results in the Lagrangian given by

\[ \mathcal{L} = (D_{\mu,ij} \phi_j^a)(D_{\mu,ik} \phi_k) - m^2 \phi_i^a \phi_i - \frac{\lambda}{4!} (\phi_i^a \phi_i)^2 + \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + \frac{1}{2 \kappa} (\partial_{\mu} A_{\mu}^a)^2 + i \bar{\psi} \gamma_{\mu} D_{\mu}^a \psi, \]

with

\[ D_{\mu}^a = g_{\mu}^a \partial_{\mu} + g f_{abc} A_{\mu}^c, \quad D_{\mu,ij}^a = \delta_{ij} \partial_{\mu} - ig \left( \frac{\mu^a}{2} \right)_{ij}^a A_{\mu}^a, \quad F_{\mu \nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a - g f_{abc} A_{\mu}^b A_{\nu}^c, \]

wherein \( D_{\mu,ij} \) denotes the covariant derivative, involving the Gell-Mann matrices \( t^a \), for the complex scalar field \( \phi(x) \) with the associated mass \( m \), \( \lambda \) is the coupling constant for a quartic scalar interaction, \( F_{\mu \nu}^a \) is the field-strength tensor involving the gluons \( A \) and the structure constants \( f_{abc} \), and \( D_{\mu}^a \) is the covariant derivative for the Faddeev-Popov (anti-)ghosts \( \bar{c}c \) in the adjoint representation. Lorentz-indices are written in Greek letters, roman indices starting with \( a \) are color-indices and the fundamental representation is indexed by roman letters starting with \( i \).

In contrast to QCD the tensor structure of the scalar model is strongly simplified. In the quark propagator one has to consider a Dirac-vector as well as a Dirac-scalar component. Instead for scalar bosons, there is only one (scalar) tensor component in the scalar propagator \( S^{ij} \). Similarly the scalar-gluon vertex \( \Gamma^{\mu,ij}_g \) depending on two independent momenta can be decomposed into two tensors, in contrast to 12 independent tensors in the quark-gluon vertex. This simplification becomes even more significant for higher correlation functions. We will choose the parametrization

\[ S^{ij}(p) = -\delta_{ij} \frac{\tilde{S}(p^2)}{p^2}, \quad \Gamma^{\mu,ij}_g(p_s, p_{gl}) = ig \left( \frac{\mu^a}{2} \right)_{ij}^a \left( \tilde{\Gamma}^a_s(p_s^2, p_{gl}^2, p_s \cdot p_{gl}) p_s, \mu + \tilde{\Gamma}^a_{gl}(p_s^2, p_{gl}^2, p_s \cdot p_{gl}) p_{gl}, \mu \right), \]

where the two independent momenta are conveniently chosen as the incoming scalar and the gluon momentum. Note that besides the simplification in the tensor structures two additional subtleties arise that are not present in QCD. Firstly, the classical Lagrangian contains additional 4-particle-interactions involving scalars, whose analogous terms are not present in QCD due to dimensional reasons (renormalizability). Secondly, a scalar field theory gives rise to a possible scalar condensate via the Higgs mechanism.

Here a note is in order. The theory described above develops a more complicated phase structure compared to QCD. As the gauge is already fixed the residual symmetry that is broken must be a global one \[33\]. Note that it is ascertained that these two different "phases" of the system are not separated by a phase boundary \[34\], but rather by a Kertész line, i.e. they are continuously connected. In that sense the terminology of "phases" is not strictly correct, but as it is frequently used in the literature we will adapt this nomenclature. A suitable quantity to investigate the phase transition is an order parameter \[34\] of the form

\[ Q = \left( \int d^4x \ \phi(x) \right) \left( \int d^4x \ \phi^\dagger(x) \right). \]

The relation of screening to confinement has been subject to other investigations \[35\]. In this work we will not discuss the effects due to scalar condensation and concentrate on the confinement properties of the model - leaving a detailed analysis of a condensate for future work.

In the following we will use the functional Dyson-Schwinger equations (DSEs) to describe the non-perturbative dynamics of the theory. The DSE analysis we perform here relies on the framework described in detail in \[10, 13, 15\] and is similar to the analysis in QCD \[16, 17\]. The details on the derivation of the presented results can be found in \[29\]. The DSEs for the theory can be derived algorithmically \[36\]. The corresponding equations in the gauge sector are given in \[15\], which also hold for coupled scalars in the quenched approximation. In the case of a dynamical scalar these equations are extended by unquenching graphs with closed scalar loops analogous to the case of QCD. The leading equations in the scalar sector are shown in fig. \[1\] where 2-loop diagrams arising in these equations are omitted. These equations represent an infinite tower of coupled integral equations and in general would require some truncation. In order to study interesting qualitative aspects, like the confinement of static sources, it is sufficient to study the long-range behavior of the theory. In momentum space this is encoded in the IR regime of correlation functions, and as far as only the IR scaling laws are concerned a solution of the whole tower is actually possible. This has previously been achieved via the help of a skeleton expansion \[10\] and we will also employ this approach in this work. The skeleton expansion presents a loop expansion in terms of dressed Green functions and this way the equations for the primitively divergent Green functions are transformed into a closed system of equations. However, as argued recently the skeleton expansion is only a convenient tool and not mandatory since the IR scaling is strongly restricted and fully determined by the equations for the primitively divergent correlation functions, see e.g. \[13, 17, 18\] and references therein. Since in the scaling solution of Yang-Mills theory \[14\] the ghost dynamics is strongly dominant in the IR regime and there is no direct coupling between scalars and ghosts in the Lagrangian, the first ghost corrections in the scalar sector arise from two loop diagrams in the skeleton expansion. Correspondingly, we have to consider these graphs in our analysis whereas otherwise higher loop graphs in the skeleton expansion must not be more divergent.
than the corresponding lower order graphs. The resulting leading ghost contribution in the scalar-gluon vertex DSE is given in fig. 2. Similar diagrams emerge from the ghost contributions to the scalar-2-gluon vertex equation. Besides masses, QCD has no inherent scales far below the MeV scale. Thus in the IR regime $p \ll \Lambda_{\text{QCD}}$ all Green functions should exhibit a scaling form in terms of the external momenta. In the case that all external momenta vanish uniformly (uniform limit) with a single external scale $p$ power law scaling solutions of the form

$$\Gamma(p) \sim (p^2)^{\chi + \delta}$$

are a natural ansatz for Green functions. Herein the canonical exponent $\chi$ reflects the dimension of the corresponding operator and the anomalous exponent $\delta$ describes the anomalous scaling induced by the dynamics. In the case of the scalar propagator and scalar-gluon vertex defined above the corresponding canonical dimensions are $\chi_s = -1$ and $\chi_{sg} = 1/2$. In the uniform limit there is effectively only a single scale, and the loop integrals that are dominated by the poles of the integrand have to scale with this external scale. The scaling of a given loop graph can then be determined by a power counting analysis. Yet, when there are additional mass scales present the loop integrals can also be dominated by scales of the order of the mass which has to be considered in detail in the power counting analysis [15]. For a massive particle the propagator can alternatively be parametrized by a mass dressing function $M$ which features a massive IR behavior for $m \equiv M(0) > 0$ and contains in this case an additional hard scale

$$\frac{\tilde{S}(p^2)}{p^2} = \frac{1}{p^2 + M^2(p^2)} \sim (p^2)^{-1+\eta}.\]  

Therefore the mass dressing function $M$ yields an additional exponent $\eta$ in the power counting, with $\eta = 1$ for massive particles, and $\eta = 0$ for massless particles, respectively.
Moreover, as discussed in detail below it also provides a mechanism for the confinement of the matter fields \[16\]. Yet, the confinement of the gauge degrees of freedom within the Kugo-Ojima \[41\] and Gribov-Zwanziger \[42, 43\] scenarios.

In this section we discuss the fundamentally charged scalar model in the limit where all external momenta of Green functions vanish uniformly. In order to find such infrared solutions we perform a power counting analysis. The goal of this analysis is to find the scaling behavior of Green functions under the assumption that all Green functions scale with some power of \( p^2 \) in the deep infrared. In this case we can simply count the exponents of the different dynamical contributions in the DSEs and determine a solvable system for the scaling exponent of each Green function.

As \( p \to 0 \) the dominant term which determines the scaling of the corresponding Green function is the one with the most divergent power law and correspondingly the one with the minimal IR exponent. With this procedure and the help of the skeleton expansion we obtain a closed system for the IR exponents of the primitively divergent vertex functions. For an illustrative example of this counting procedure consider the scalar propagator given in fig. I in the top left corner. On the left hand side the power law ansatz gives \( (p^2)^{1-\delta_s} \) since it is the inverse of the propagator that arises in the DSE, giving an additional overall-sign. The right hand side involves four different diagrams, and the dominant term is correspondingly given by a minimum function of the individual exponents. The first diagram is simply the inverse bare propagator giving \( (p^2)^{1-\eta} \). In the second diagram one has to count the momentum dependence of the loop integration \( (p^2)^2 \), a scalar-propagator \( (p^2)^{1+\eta} \), a gluon-propagator \( (p^2)^{-1-\delta_g} \) and a scalar-gluon vertex \( (p^2)^{\frac{1}{2}+\delta_{sg}} \). Performing the analogous power counting for the last two graphs and subtracting the canonical dimensions one obtains an equation for the anomalous dimension of the scalar-propagator

\[
-\delta_s = \min (-\eta, \delta_s + \delta_{gl} + \delta_{sg}, \delta_s, \delta_g).
\]

Applying the same procedure for all skeleton expanded DSEs of the primitively divergent vertex functions of pure gauge theory in \[15\] and the scalar sector in fig. I one obtains a closed system of equations for the IR exponents of the primitively divergent vertex functions. Due to the scalar self-interaction this system of equations for the anomalous IR exponents proves to be rather extensive in the scalar model and is explicitly given in \[29\].

The next task is then to find the leading infrared behavior of all Green functions for the different infrared fixed points as solution of this purely algebraic system. As discussed in detail in \[29\] it turns out that it is a convenient starting point to consider the equations from the gauge sector first, wherein the ghost equation is the most convenient one to start with. Depending on the boundary condition of the DSE that determines if the bare term is present or absent in the corresponding renormalized equation, this equation can, in addition to the trivial perturbative case, have two qualitatively different non-trivial solutions. These solutions are standardly referred to as scaling \[14, 15, 37-39\], see \[26, 27, 44, 46\] and references therein for lattice results. Note that both solutions yield a confining Polyakov-loop potential \[40\].

For each of these two classes of solutions one can subsequently solve the remaining system of coupled equations, starting with the gauge sector and continuing with the matter part. In the scalar sector different solutions are found in the massive and the massless case. The complete fixed point structure is given in table I and presents the main result of our analysis. The mass of the scalar is here taken into account via the parameter \( \eta \), thus these fixed points in principle hold for both, a massless and a massive scalar field, but in the following we will see that there are further subtleties for massive particles.

It is a remarkable result that, as in the case of QCD \[16\] \[17\], the additional scalar dynamics does not change the known IR fixed point structure of the Yang-Mills sector. The latter features two rather different possible IR scenarios of the continuum theory. First, the decoupling solution with a massive gluon propagator and no IR enhanced Green functions. Second, the scaling solution with a strongly IR singular ghost propagator, which induces similar divergences in gluonic vertex functions but an IR-suppressed gluon propagator. The scaling solution provides a mechanism for the confinement of the gauge degrees of freedom within the Kugo-Ojima \[41\] and Gribov-Zwanziger \[42, 43\] scenarios. Moreover, as discussed in detail below it also provides a mechanism for the confinement of the matter fields \[16\]. Yet,
only the first of these solutions has been observed in current (4-dimensional) lattice simulations [26, 27, 44, 46] and it is a challenging question whether the IR behavior they show is indeed the only solution that is realized when the continuum limit is taken, cf. e.g. [13, 28, 44, 50].

Whereas the scalar sector is entirely trivial in the decoupling solution, there are two qualitatively different IR fixed points for the scaling solution, one with trivial vertices and another one with strongly divergent vertices. These two different fixed points are analogous to the case of QCD. In the massive case the divergent solution for the full scalar-gluon vertex including its canonical dimension features in particular precisely the same IR power law $-1/2 - \kappa$ as the quark-gluon vertex in QCD [11]. In both theories this IR divergence of the vertex is completely induced by the gauge sector. The above scaling laws are not altered by the neglected 2-loop diagrams, as is checked explicitly in [29]. Furthermore, the DSEs for higher $n$-point functions with $n > 4$ are linear. Accordingly these equations cannot induce additional non-trivial fixed points and therefore the infrared exponents of these $n$-point functions are purely determined by the lower $n$-point functions. Moreover, we find that in all cases there are leading graphs that do not involve 4-point functions, in complete analogy to the case of QCD where such 4-point functions are not primitive divergent.

In the approximation discussed so far, the scalar 4-point vertices in table I feature in the massive case only rather mild divergences. In particular, the obtained divergence of the one-particle irreducible 4-scalar vertex is less than the divergence of the corresponding one-particle reducible vertex built using a gluon exchange via two scalar-gluon vertices. We will show below that this is a shortcoming of the present restriction to uniform scaling exponents. In general a more diverse IR behavior can be realized in which Green functions have also kinematic divergences if only a subset of the external momenta vanishes whereas the others remain finite. In this case the loop integrals can receive dominant contributions from hard modes even when all external scales are small. Correspondingly, kinematic divergences can alter the uniform power laws. Due to this the present results for the 4-scalar and the scalar-2-gluon vertex from the uniform limit in table I are given in parentheses. They are only correct for massless particles but underestimate the actual divergence in the massive case. We will explain in the next section how the correct exponents are recovered once kinematic divergences are taken into account.

Finally, we want to point out that a solution with a divergent scalar propagator is not possible, due to the self-interaction of the scalar, cf. [18] for a general treatment of this issue.

IV. KINEMATIC SINGULARITIES AND STATIC CONFINEMENT

As mentioned before, the uniform solution discussed so far presents only a special case of the actual possible IR behavior. In the following we will extend this by the inclusion of kinematic divergences of the vertex functions [13, 15, 51]. Such kinematic divergences provide a mechanism for a long-range interaction that can confine quarks in quenched QCD [10]. Therefore we are interested whether this mechanism can also confine static scalar sources, i.e. if the relevant Green functions feature the same IR scaling exponents. A general discussion of kinematic divergences in the case of the scalar theory considered here is quite complicated due to the additional interactions and the many possible kinematic limits of the 4-point vertices, and is therefore beyond the scope of this work. However, in the following we will present the line of argument that shows that the kinematic divergence of the scalar-gluon vertex in the static case is indeed as strong as that of the quark-gluon vertex in QCD. To this end we restrict the discussion in the following to the quenched approximation where the scalar sector does not affect the gauge sector and can be analyzed independently. Since it was found above that the 4-point functions do not alter the IR scaling laws we can restrict our investigation to the equation for the scalar-propagator and the scalar-gluon vertex. We have checked explicitly in [29] that this remains true if taking into account the enhanced scaling of the vertices obtained below.

A comparison with the corresponding QCD equations shows that the equations for the scalar-propagator and the scalar-gluon vertex in fig. 1 are diagrammatically similar to the corresponding equations for the quark-propagator and the quark-gluon vertex, except for additional terms in the scalar theory that stem from the additional 4-point interactions. In order to find the IR exponents a refined power counting analysis has to be employed that takes into account the possibility of momentum and mass scales that stay finite when the IR limit is taken. As shown in detail in [13], the scale separation between these soft and hard scales in the IR limit allows to identically decompose the arising loop integrals into several integrals that depend only on a single external scale which directly determines the scaling of the corresponding contribution. This yields a system for the IR exponents of the scalar propagator $\delta_\sigma$ and the scalar-gluon vertex in the uniform limit $\delta_{sg}^\sigma$ as well as $\delta_{sg}^{\mu I}$ and $\delta_{sg}^{\mu I}$ in the limits that only a scalar respectively gluon momentum vanishes. Strikingly the additional terms from the 4-point interactions in these equations can be shown to be subleading [29] using constraints from the inequivalent towers of DSEs and RGEs [13]. Correspondingly the scalar system effectively reduces to a DSE system that is up to the different canonical dimensions identical to that of quenched QCD. The different canonical dimensions only further suppress terms that are subleading in the case of QCD. Thus the same fixed point structure is realized and the scalar-gluon vertex features the same scaling behavior as the quark-gluon vertex.

The solution is given in table II where the superscripts denote the soft momenta in the specific limit. It features strong kinematic divergences of the scalar-gluon vertex in the limit that only the external gluon momentum vanishes.
which leads after Fourier transformation in the static limit to a linear potential in coordinate space again effectively adding a forth dressed vertex. The corresponding graph with a soft gluon exchange scales therefore on the left of fig. 3 and the dominant kinematic contribution is given by graph (b) on the right hand side - correlator can be IR enhanced. Due to the same mechanism described above the leading contribution arises from the graph on the left of fig. 3 (a). In the heavy quark limit the hard loop simply shrinks to a point and precisely presents a forth dressed vertex which is present from the outset in functional RG equations. This mechanism finally yields the leading IR scaling laws for the 4-point vertices in table I.

Now let us discuss the interaction between static scalar sources which is described by the heavy mass limit of the full 4-scalar DSE as visualized in the left part of fig. 3 yields a 2-loop diagram that looks very similar to the ordinary gluon exchange graph in the 4-scalar DSE with the difference that the bare vertex has now a vertex correction. When the scalars are massive, this vertex correction loop receives contributions from hard loop momenta and scales only due to the kinematic divergence of the dressed vertex, cf. fig. 3 (a). In the heavy quark limit the hard loop simply shrinks to a point and precisely presents a forth dressed vertex which is present from the outset in functional RG equations. This mechanism finally yields the leading IR scaling laws for the 4-point vertices in table I.

Now let us discuss the interaction between static scalar sources which is described by the heavy mass limit of the full 4-scalar DSE. Here the relevant limit is it not the uniform kinematic configuration where momenta of the external scalars are in the IR regime and the scalars would correspondingly be on the light cone in Minkowski space, but the exactly opposite limit where their mass is large and the external momenta are of the order of this large scale. Nevertheless, when the scalars are far spatially separated the exchanged gluon momentum becomes soft and the correlator can be IR enhanced. Due to the same mechanism described above the leading contribution arises from the graph on the left of fig. 3 and the dominant kinematic contribution is given by graph (b) on the right hand side - again effectively adding a forth dressed vertex. The corresponding graph with a soft gluon exchange scales therefore in the IR as

$$\left(p^2\right)^2 \left(p^2-\frac{1}{2}-\kappa\right)^4 \left(p^2-1+2\delta\right)^2 = \left(p^2\right)^{-2}$$

which leads after Fourier transformation in the static limit to a linear potential in coordinate space

$$V(r) \sim \int d^3p \frac{e^{ipr}}{p^2} \sim |r|.$$  

Table II: The anomalous power law exponents of the leading correlation functions of the quenched scalar model when taking into account kinematic divergences.

| $\delta_n$ | $\delta_n^{\alpha}$ | $\delta_n^{\beta}$ | $\delta_n^\gamma$ |
|------------|----------------------|---------------------|-------------------|
| scaling    | $\eta$, $-\eta-\kappa$, $\sqrt{0}$, $-\eta-\kappa$, $\sqrt{0}$ | 0, 0, 0, 0, 0 |
| decoupling | $\eta$, 0, 0, 0, 0 |

Note that this soft-gluon divergence is of the same size as the uniform divergence and that the inclusion of kinematic divergences does not change the uniform scaling of the scalar-gluon vertex. However as in the case of QCD, for higher uniform correlation functions the consideration of kinematic divergences is required in a DSE study even to get the proper scaling in the uniform limit. As explained in [10] this is a peculiarity of the DSEs owing to the fact that they involve a bare vertex in each graph. Thereby in a theory with enhanced vertices IR strength can be missing in the lowest order corrections and is represented dynamically in the contributions from correlation functions with one more external leg. To see this consider the last term in the equations of the 4-scalar vertex that involves a 5-point function. The latter satisfies its own DSE which contains a graph with a simple gluon exchange correction involving only scalar-gluon vertices. Inserting this correction into the corresponding term in the 4-scalar DSE as visualized in the left part of fig. 3 yields a 2-loop diagram that looks very similar to the ordinary gluon exchange graph in the 4-scalar DSE with the difference that the bare vertex has now a vertex correction. When the scalars are massive, this vertex correction loop receives contributions from hard loop momenta and scales only due to the kinematic divergence of the dressed vertex, cf. fig. 3 (a). In the heavy quark limit the hard loop simply shrinks to a point and precisely presents a forth dressed vertex which is present from the outset in functional RG equations. This mechanism finally yields the leading IR scaling laws for the 4-point vertices in table I.

Now let us discuss the interaction between static scalar sources which is described by the heavy mass limit of the full 4-scalar DSE and the involved propagators and vertices. The case of a complex scalar field considered here presents the renormalizable theory of matter fields coupled to a Yang-Mills sector with the most general interactions in four spacetime dimensions. The explicitly considered cases of scalars and Dirac fermions present precisely the two distinct possibilities for the dynamics as far as the topology of possible graphs is concerned. Deviations from the scaling laws obtained within a power counting analysis are only possible if there are identical cancellations of the leading graphs in the DSEs. In the case of the scalar- respectively quark-gluon vertex there is actually only a single leading diagram in the corresponding DSE, cf. figs. 1 and 2 so that cancellations between different graphs are impossible here. This shows that the universality of the long-range interaction between static fundamentally charged sources, which is a natural property in the lattice framework, is indeed realized in the functional framework as well. Finally, we note that although in this work we studied only gauge dependent Green functions, the above 4-point correlator represents the lowest term in a power series representation of the exponential arising in the corresponding gauge invariant correlator where the two quark sources are connected by a Wilson line. Therefore, in case this leading term is not cancelled identically by higher order terms in the series, this gauge independent quantity likewise shows the observed long-range interaction.
FIG. 3: left: An IR leading contribution to the 4-scalar vertex when inserting a corresponding contribution arising in the DSE of the 4-scalar-gluon vertex, right: Kinematic configurations that yield the leading order contributions for the scaling of the vertex in the case that all external momenta vanish uniformly (a) and in the limit when only the momentum transfer between the scalars becomes small (b). The labels denoted that the momenta running through the corresponding propagators are soft $s$ and vanish in the IR limit or that they are hard $h$ of the order of the scalar mass.

V. CONCLUSION

We have studied the IR fixed point structure of a non-Abelian gauge theory coupled to a scalar matter field in the fundamental representation of the gauge group as a schematic model for the QCD dynamics. We find that the IR fixed point structure is indeed identical to the case of QCD and that for one type of solutions a kinematic divergence of the scalar-gluon vertex induces a linear confining interaction between static sources. The qualitatively identical confinement aspects of the scalar model compared to QCD show that this confinement mechanism is indeed universal and does not depend on the particular features of the matter fields. Instead the long-range interaction between fundamental sources is a property of the gauge sector in complete analogy to results of lattice gauge simulations. Therefore, it is merely a technical complication, that within functional Green function methods the coupling of a fundamental color source to the gauge sector has to be obtained from the static limit of the dynamical equations of the corresponding matter fields. In this limit the kinematic divergence of the matter-gauge vertex simply describes the non-trivial dressing of the static color source.

Finally, we want to emphasize that due to the presented results the scalar theory presents an ideal model system to study a potential confinement mechanism in detail. Within functional approaches the minimal system of coupled integral equations for the matter sector decreases from 14 for fermionic fields to 3 in the scalar case. This should strongly simplify the numerical treatment and allow to study quantitative aspects of the mechanism. Even more importantly, the dramatic simplifications of scalars compared to fermions in lattice gauge simulations could allow to test this mechanism in lattice simulations.

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