A Method to Compensate Interaction between Actuator Dynamics and Control Allocator under Incremental Nonlinear Dynamic Inversion Controller

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Abstract. When designing flight control allocator for an over-actuated flight system, actuator dynamics, generally, are assumed to be ideal since the bandwidths of actuators are much larger than the frequency of aircraft. However, under incremental nonlinear dynamic inversion controller, the interaction between the constrained control allocation frame and actuator dynamics can cause serious consequence. In this work, a method is developed to compensate for actuator dynamics in s-domain analysis. Here, first-order and second-order actuator dynamics are taken into consideration as more complicated actuator dynamics can be combined by these two simplified models. This method solves for gains, which post-process the output of control allocation frame. The gains are terse in form and are easy to be calculated, which means that it can be implemented into typical flight computer. Simulation results show proposed method can significantly compensate the flaw caused by actuator dynamics.

1. Introduction
Incremental nonlinear dynamic inversion (INDI), which is a nonlinear control law, is established on the assumption that actuator can reflect instantly [1]. The basic ideal of INDI is using Taylor expansion to linearize the control system at every sampling time, therefore it is less dependent on system modelling and shows better robustness compared to nonlinear dynamic inversion [2]. The basic structure of INDI controller is shown in Figure 1.

For an advanced aircraft, it is quite common to be designed as an over-actuated system. Compared to traditional flight system, performance, reliability, and reconfigurability can be improve after extra actuators are utilized [3]. Control allocation algorithms are used to compute aircraft control surface deflections which produce a desired set of generalized moment commands [3]. Typical control allocation equation is,

\[ \mathbf{u}_{cmd} = \mathbf{B}^{-1}\mathbf{\tau}_{des} \]  

(1)

Where, \( \mathbf{u}_{cmd} \) is actuator command vector. \( \mathbf{\tau}_{des} \) is desired virtual input vector and \( \mathbf{y}_{des} \) is desired generalized moment vector. Generally, \( \mathbf{\tau}_{des} = \mathbf{y}_{des}. \mathbf{B} \) is effectiveness matrix [3].

For the overall control system, desired goal is that the moments produced by effectors are equal to desired generalized moments, namely,
Where, $u$ is actual actuator respond vector. A number of approaches have been developed to ensure that the commands sent to the effectors are physically realizable [3]. Generally, it is reasonable to ignore actuator dynamics on the condition that bandwidths of actuators are normally much larger than the frequencies of the vehicle’s rigid body model [4]. But, in some cases, actuator dynamics may have serious influence on the performance of control system if the interactions between constrained control allocators and actuator dynamics are ignored [4]. Bolling has shown that the interaction between first order actuator dynamics and constrained control allocation algorithms can be eliminated by overdriving the actuators [5]. Doman, Oppenheimer and Venkataraman proposed the dynamic control allocation method, which takes effector dynamics into account [6]. Doman and Oppenheimer have proposed a compensation method in discrete analysis [4]. Hä rkegå rd posed the control allocation problem as a constrained quadratic program, which provides automatic redistribution of the control effort when one actuator saturates in position [7]. Chen et al. developed another compensation method based on linear matrix inequality approach, which shows good performance on the unconstrained control allocation problem [8].

In this paper, the discrepancy caused by actuator dynamics is presented first under INDI controller. Then compensation gains are deduced for typical two actuator dynamics separately in s-domain analysis. Test system is built to test the performance of proposed compensation method. Simulation results show the proposed method can improve tracking performance.

### 2. Discrepancy caused by actuator dynamics under INDI controller

$$Bu = \tau_{des}$$  

Figure 1. Structure of incremental nonlinear dynamic inversion baseline controller.

**Figure 2. Structure of inner loop of INDI controller**
Figure 2 shows the inner loop of flight system under incremental nonlinear dynamic inversion. Control allocation frame allocates the incremental desired generalized moments to each actuator. The basic equation for control allocation to solve is as below,

$$\Delta \tau_{des,m \times 1} = B_m \Delta u_{cmd,n \times 1}$$  \hspace{1cm} (3)

Where, $\Delta \tau_{des,m \times 1}$ is desired incremental virtual input vector, which is equal to $\Delta \tau_{des}$, namely desired incremental generalized moment vector. $\Delta u_{cmd,n \times 1}$ is incremental actuator command vector. Basically, $n$ is bigger than $m$ in an over-actuated system. Equation 3 is different with Equation 2, due to the special structure of INDI controller.

Compared to unconstrained control allocation, constrained control allocation takes into consideration of both effectors absolute position limits and rate limits [9]. In an INDI controller, actuator commands also need constraining to respect the most restrictive of the rate or position limits in a sampling time. Mathematically, for a single actuator, the constraints are defined as set $\Omega$,

$$\Omega = \{ \Delta u | \Delta u \leq \Delta u \leq \Delta u \}$$  \hspace{1cm} (4)

Where,

$$\Delta u = \max(u_{min} - u, \dot{u}_{min} \Delta t)$$

$$\Delta u = \min(u_{max} - u, \dot{u}_{max} \Delta t)$$  \hspace{1cm} (5)

$u$ is current respond of actuator and $u_{max}$, $u_{min}$ are the maximum rate and minimum rate of actuator, respectively. $u_{max}$, $u_{min}$ are absolute upper and lower bounds of an actuator, respectively. $\Delta u$, $\Delta u$ are the incremental upper bound and incremental lower bound respectively in next sampling time and $\Delta t$ is sampling period of the digital flight control system.

From Figure 2, the control allocation frame of INDI controller does not directly tell effectors to deflect to a desired absolute position, but to deflect a certain degree which is the difference between desired position and current position [1]. Ideal situation is that actuators are infinitely fast, which means $u = u_{cmd}$ or transfer function of actuator is $G_u(s) = 1$. Under this assumption, transfer function from $\Delta u_{cmd}$ to $u$ can be deduced directly,

$$G_u''(s) = \frac{u}{\Delta u_{cmd}} = \frac{1}{\Delta t \cdot s}$$  \hspace{1cm} (6)

In fact, this ideal case can never be achieved due to the existence of actuator dynamics. Here are two figures to present the discrepancy caused by actuator dynamics under INDI controller.

**Figure 3.** Desired generalized moments and actual moments produced by controller – without actuator dynamics.
In Figure 3, actuators are ignored, the desired result is achieved, which means $Bu = \tau_{des}$. However, in Figure 4, when actuator dynamics are taken into account, it is evident that the tracking error in this case is quite large. Notice that, not only actuator dynamics cause the discrepancy of the system, but the positive feedback, which envelops actuator dynamics in INDI controller, also entails this consequence. Thus, it is obliged to propose a method to compensate this discrepancy.

3. Compensation Method

The objective of compensation method is to modify control allocation framework and actuator dynamics such that $G(s) = u/\Delta u_{cmd} = 1/(\Delta t \cdot s)$. Idea of this method is putting s-domain gains, which post-process the output of control allocation frame, so that it can directly augment the incremental actuator command. The basic structure is shown as Figure 5,

$$G_{PT1}(s) = \frac{u}{u_{cmd}} = \frac{1}{Ts + 1}$$ \hspace{1cm} (7)

In Figure 2, transfer function from $\Delta u_{cmd}$ to $u$ can be deduced before adding a s-domain gain $M$,

$$G_{PT1}(s) = \frac{u}{\Delta u_{cmd}} = \frac{1}{Ts}$$ \hspace{1cm} (8)

In Figure 5, transfer function from $\Delta u_{cmd}$ to $u$ can be deduced after adding a s-domain gain $M$,
The objective is to find a gain, $M$, that modifies the output of control allocation algorithm such that,

$$G_{PT1}'(s) = \frac{M}{\Delta t \cdot s}$$  \hspace{1cm} (9)

solving for $M$ yields,

$$M = \frac{T}{\Delta t}$$  \hspace{1cm} (10)

Where, compensation gain $M$ is a number and $M > 1$, as the time constant of actuator is much bigger than sampling time. Since $M$ can be computed from the known quantities $T$ and $\Delta t$, one can compensate for command increment attenuation using Equation 11 above. For a bank of decoupled first-order actuators with nominal bandwidths of $T_i$ and parameter $\alpha_i$ corresponding values of $M_i$ can be computed using $M_i = T_i/\Delta t$.

For second-order actuator dynamics, assume transfer function is below,

$$G_{PT2}(s) = \frac{u}{\omega^2 s^2 + 2s \xi \omega s + \omega^2}$$  \hspace{1cm} (12)

In Figure 2, transfer function from $\Delta u_{cmd}$ to $u$ can be deduced before adding a $s$-domain gain $M$,

$$G_{PT2}'(s) = \frac{u}{\Delta u_{cmd}} = \frac{G_{PT2}(s)}{1 - G_{PT2}(s)} = \frac{\omega^2}{s^2 + 2s \xi \omega s}$$  \hspace{1cm} (13)

In Figure 5, transfer function from $\Delta u_{cmd}$ to $u$ can be deduced after adding a $s$-domain gain $M$,

$$G_{PT2}'(s) = \frac{u}{\Delta u_{cmd}} = M \cdot G_{PT2}(s) = \frac{M \cdot \omega^2}{s^2 + 2s \xi \omega s}$$  \hspace{1cm} (14)

The objective is to find a gain, $M$, that modifies the output of the control allocation algorithm such that,

$$G_{PT2}'(s) = \frac{1}{\Delta t \cdot s}$$  \hspace{1cm} (15)

solving for $M$ yields,

$$M = \frac{s^2 + 2s \xi \omega s}{\Delta t \cdot \omega^2 \cdot s}$$  \hspace{1cm} (16)

Where, compensation gain of this case is more complex than that of first order actuator. $M$ is a transfer function whose order of numerator is bigger than the order of the denominator, which is non-causal and non-realistic. Since the expected behavior is an integration, it is desired for the element before the integrator to have the same steady state gain, by using final value theorem,

$$\lim_{s \to 0} (s \cdot G_{PT2}'(s)) = \lim_{s \to 0} \left( s \cdot \left( \frac{M \cdot \omega^2}{s^2 + 2s \xi \omega s} \right) \right) = \frac{1}{\Delta t}$$  \hspace{1cm} (17)

Solution of the equation gives,

$$M = \lim_{s \to 0} \left( \frac{s^2 + 2s \xi \omega s}{s \cdot \omega^2 \cdot \Delta t} \right) = \frac{2 \xi}{\Delta t \cdot \omega}$$  \hspace{1cm} (18)

Where, gain $M$ is a number can be calculated easily. Since $M$ can be computed from the known quantities $\xi$, $\omega$ and $\Delta t$, one can compensate for command increment attenuation using Equation 20. For a bank of decoupled second-order actuators with nominal natural frequency $\omega_i$ and damping ratio $\xi_i$, corresponding values of $M_i$ can be computed by using $M_i = 2\xi_i/(\Delta t \cdot \omega_i)$.

Referring to Figure 5, for a system with more than one actuator, compensation gain $M$ would be a diagonal matrix with entries along the main diagonal being $M_1, M_2, M_3 \ldots M_n$. Here, $M_i$ would be computed using Equation 11 and Equation 18, corresponding the actuator type and their parameters.

4. Simulation Results

In this section, results from a simulation of a test system displayed in Figure 6 will be shown. Test system is an inner loop of INDI controller for an over-actuated system with 5 actuators. Control
allocation algorithm is minimum error redistributed pseudo-inverse (MERPI) proposed by Jin, which is a linear programming algorithm [10]. In the inner loop of INDI controller, the plant block just considers the process from directive $\tau_{des}$ to produce of actual moment $\tau_p$ but neglects transformation from actual moment to the attitude change of flight. For this reason, some dynamics process happened in the outer loop is neglected. Here, sampling time is $\Delta t = 1/200$. The indicator is tracking error between desired generalized moment vector $\tau_{des}$ to actual moment vector $\tau_p$. Here, overall system goal is to achieve $u$ such that the real moment vector $\tau_p$ produced by deflection of effectors can follow $\tau_{des}$ as,

$$\tau_p = \tau_{des}$$ (19)

subject to,

$$\tau_p = Bu$$ (20)

![Diagram](image)

Figure 6. Structure of test system

$\tau$ is chosen as two dimensions' vector as below,

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 0.4 \sin(0.3 \pi t^2 + \pi t) \\ -0.3 \sin(0.3 \pi t^2 + \pi t) \end{bmatrix}$$ (21)

In this simulation, assume value of control effectiveness matrix $B$ is below:

$$B = \begin{bmatrix} 0.3091 & -0.3346 & -0.6043 & 0.2922 & 0.6753 \\ -0.7023 & -0.6807 & 0.4852 & -0.9145 & -0.6983 \end{bmatrix}$$ (22)

As for the actuators system block, it contains 5 independent mutually actuators in total. Specifically, the dynamics of each actuator are chosen as below,

$$\begin{align*}
\frac{u_1}{u_{cmd}} &= \frac{36}{s^2 + 9.6s + 36} \\
\frac{u_2}{u_{cmd}} &= \frac{36}{s^2 + 9.6s + 36} \\
\frac{u_3}{u_{cmd}} &= \frac{25}{s^2 + 10s + 25} \\
\frac{u_4}{u_{cmd}} &= \frac{5}{5s + 1} \\
\frac{u_5}{u_{cmd}} &= \frac{3}{3s + 1}
\end{align*}$$ (23)

Each actuator is rate and position limited by the following values,

$$\begin{align*}
\underline{u}^T &= [-3 \ -2 \ -4 \ -5 \ -3](\text{degree}) \\
\overline{u}^T &= [+1 \ +5 \ +4 \ +5 \ -3](\text{degree}) \\
\underline{u}_{min}^T &= [-3 \ -5 \ -2 \ -10 \ -4](\text{degree/second}) \\
\overline{u}_{max}^T &= [+3 \ +5 \ +2 \ +10 \ +4](\text{degree/second})
\end{align*}$$ (24)

Proposed method is implemented to test system and comparison are made with the discrete gains in Michael W. Oppenheimer and David B. Doman’s paper [4].
From Figure 7, without augmentation, the original system cannot follow the $\tau_{des}$ definitely and a large error exists between them. Both proposed method and the method in reference [4] can track the reference signal of $\tau_{des}$ closely. Signals of $\tau_p$ under these two compensation methods are almost overlap with signal of $\tau_{des}$. Compared to original system, proposed method significantly improve the tracking performance.

5. Conclusion

Interactions between control allocation algorithm and actuator dynamics degrades the performance of INDI controller. A method is proposed to deal with this problem. Compensation gains are deducted under first-order actuator dynamics and second-order actuator dynamics, separately. Simulation results show that proposed method can reduce the tracking error greatly. There are two advantages of proposed method compared to the compensation method in Reference [4]. First, the form of $M$ is simpler. Especially, for these two typical actuator dynamics, both compensation gains are numbers. Thus, it is quite easy to be calculated. Second, proposed method does not need to know actuators’ state of previous step, which is necessary for method in Reference [4]. Therefore, there is no need to put extra estimator to observe actuator.

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