Spiral-arm instability - II: magnetic destabilisation

Shigeki Inoue & Naoki Yoshida

Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS, The University of Tokyo, Chiba 277-8583, Japan
Department of Physics, School of Science, The University of Tokyo, Bunkyo, Tokyo 113-0033, Japan

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ABSTRACT
Fragmentation of spiral arms can drive the formation of giant clumps and induce intense star formation in disc galaxies. Based on the spiral-arm instability analysis of our Paper I, we present linear perturbation theory of dynamical instability of self-gravitating spiral arms of magnetised gas, focusing on the effect of toroidal magnetic fields. Spiral arms can be destabilised by the toroidal fields which cancel Coriolis force. We find that there can be two instability modes: magneto-Jeans instability and oscillating growing mode. The latter is a necessary instability condition. Our analysis can be applied to multi-component systems that consist of gas and stars. To test our analysis, we perform ideal magneto-hydrodynamics simulations of isolated disc galaxies and examine the simulation results. Our analysis can characterise dynamical instability leading arms to fragment and to form clumps in cases of strong and weak magnetic fields. In moderate magnetic fields, however, spiral arms are often stable in non-linear regimes and do not fragment although our linear analysis predicts instability. Using our analytic model, we estimate a typical mass of clumps forming from spiral-arm fragmentation to be consistent with observed giant clumps \( \sim 10^7 - 10^8 \, M_\odot \). Furthermore, we find that, although the magnetic destabilisation can cause low-density spiral arms to fragment, the estimated mass of resultant clumps is almost independent from strength of magnetic fields since marginal instability occurs at long wavelengths which compensate the low densities of magnetically destabilised arms.

Key words: instabilities – methods: numerical – methods: analytical – galaxies: spiral – galaxies: kinematics and dynamics – galaxies: magnetic fields.

1 INTRODUCTION
Dynamical instability in disc galaxies can play an important role in their formation and evolution processes. In the high-redshift Universe, a significant fraction of galaxies have been observed to have clumpy morphologies in which several star-forming ‘giant clumps’ of \( M_{cl} \lesssim 10^8 \, M_\odot \) are hosted in their disc regions; \( \sim 50 \) per cent of galaxies of \( M_{star} \approx 10^9 - 10^{11} \, M_\odot \) at a redshift of \( z \approx 1.5 - 3 \) are observed to be clumpy (Tadaki et al. 2014; Murata et al. 2014; Guo et al. 2015; Shibuya et al. 2016). Although their clumpy morphologies can be attributed to ongoing mergers in some galaxies (e.g., Weiner et al. 2006; Forster Schreiber et al. 2009; Puech 2010; Ribeiro et al. 2017), the clumpy galaxies are observed to have significant rotations indicative of their disc structures (e.g., Genzel et al. 2006, 2008, 2011).

It is has been discussed that the high-redshift clumpy galaxies may evolve to spiral galaxies in the present-day Universe (e.g. Shlosman & Noguchi 1993; Noguchi 1998, 1999) and that giant clumps may be related to formation processes and dynamical properties of galactic structures such as discs, bulges, dark matter haloes and halo objects (e.g., Elmegreen et al. 2005, 2008; Elmegreen & Elmegreen 2006; Bournaud et al. 2007; Elmegreen & Struck 2013; Inoue & Saitoh 2011, 2012, 2014; Nipoti & Binney 2015; Inoue 2013). It has also been proposed, on the other hand, that the giant clumps observed are transient structures and are soon disrupted within an orbital time-scale (e.g., Hopkins et al. 2010; Genel et al. 2012). Cosmological simulations of Buck et al. (2017), Oklopčić et al. (2017) showed that these clumpy structures are bright in star-forming light such as H\(_\alpha\) but not massive enough to be self-gravitating within galactic potential. Clumpy disc galaxies are also observed at low redshifts although their abundance is lower by far than at high redshifts (e.g., Elmegreen et al. 2013; Bassett et al. 2014; Fisher et al. 2014 [2017b], Garland et al. 2015).

The physical mechanism of clump formation is often at-
distributed to local gravitational instability of radial perturbations in galactic discs (e.g. [Noguchi 1999] [Dekel et al. 2009] [Genzel et al. 2011] [Fisher et al. 2017a]), although see [Inoue et al. 2016], i.e. Toomre instability [Safonov 1960] [Toomre 1964]. In our Paper I [Inoue & Yoshida 2018], for the first time, we have proposed spiral-arm instability (SAI) model for the giant clump formation, in which we consider local fragmentation of spiral arms, rather than discs, against azimuthal perturbations. In our Paper I, we argue that our SAI model describes dynamical properties of giant clumps observed in low-redshift galaxies (see section 5 of Paper I).

Thus, fragmentation of spiral arms could be a possible mechanism of giant clump formation. Our linear perturbation analysis for spiral arms presented in our Paper I can characterise fragmentation of spiral arms quite accurately even in two-component models of gas and stars. This means that spiral-arm fragmentation is basically considered to be a linear process that can be described as balance between self-gravity, pressure and Coriolis force.

In this paper, we study instability of spiral arms in the context of ideal magneto-hydro dynamics (MHD) by taking into account effects of toroidal magnetic field in our SAI model. In previous studies, magnetic effects are incorporated in linear perturbation analysis for a self-gravitating local region in a rotating flat disc (e.g. [Lynden-Bell 1966] [Elmegreen 1987, 1994] [Gammie 1996] [Kim & Ostriker 2001]). In these analyses, spiral arms are considered as perturbations propagating on a disc. [Elmegreen 1994] discusses gravitational instability in the crest of a spiral density wave as swinging azimuthal perturbations in a disc. [Kim & Ostriker 2001] perform ideal MHD simulations with their shearing-box model and show that spiral arms can be destabilised and fragment into clumps when azimuthal magnetic fields are strong. Thus, as was shown in the previous works, toroidal magnetic fields can induce fragmentation of spiral arms and may drive formation of giant clumps. Unlike Toomre analysis for disc instability, our SAI analysis is adopted to geometry of a local ring structure (a tightly wound spiral arm), therefore thought to be better at describing fragmentation of spiral arms. We expect that our SAI analysis including the magnetic effect could give us more accurate description of spiral-arm fragmentation induced by azimuthal magnetic fields.

This paper is organised as follows. In Section 2, based on our Paper I, we present our linear perturbation analysis including the effects of magnetic fields and discuss types of unstable modes of perturbations. Furthermore, we develop our theory to a two-component model that consists of gas and stars. In Section 3, we perform ideal MHD simulations with isolated disc galaxy models to test our theory. In Section 4, we adopt our simulation data to our instability analysis and examine our linear analysis. In Section 5, we discuss how masses of giant clumps forming via SAI are affected by strength of magnetic fields and estimate a typical clump mass. In addition, we argue more general effects of magnetic fields on dynamics of galactic discs and whether disc galaxies and their spiral arms are destabilised by magnetic fields in reality. In Section 6, we present our conclusions and summary of this study.

2 LINEAR PERTURBATION ANALYSIS OF IDEAL MHD

Our analysis presented in Paper I basically follows that proposed by [Takahashi et al. 2016] for a single-component non-magnetised gas disc. The local linear perturbation analysis assumes that a pitch angle of a spiral arm to be negligibly small: the tight-winding approximation, where the spiral arm can be locally approximated as a structure resembling a ring. The arm is assumed to have rigid rotation with an angular velocity \( \Omega \) since the arm is expected to be self-gravitating (see Appendix B of Paper I); therefore the Oort’s constant \( B = -\Omega \) in the arm. In the polar coordinates \((R, \phi)\), we consider azimuthal perturbations propagating inside the arm, which are assumed to be proportional to \( \exp[i(ky - \omega t)] \), where \( y = \phi R \). When \( \omega \) has a positive imaginary part, the perturbation is expected to grow exponentially with time and thus dynamically unstable. For the perturbations, if their wavelengths are small enough compared with the radius of the arm, i.e. \( kR \gg 1 \), then the curvature of the spiral arm is negligible.

We adopt a Gaussian distribution to a radial surface density profile of the spiral arm, \( \Sigma(R) = \Sigma_0 \exp(-\xi^2/2w^2) \), where \( \xi \equiv R - R_0 \), \( R_0 \) is the radius of the density peak in the arm, and \( \Sigma_0 \) is the surface density at \( R_0 \). As is done in [Takahashi et al., 2016], and our Paper I, we define the edges of the spiral arm to be the inner and outer radii at which \( \Sigma(R) = 0.3\Sigma_0 \). In this case, the half arm width is \( W \approx 1.55w \). The line-mass of the arm is given as

\[
Y \equiv 2 \int_0^W \Sigma(R) \, d\xi = AW\Sigma_0,
\]

where \( A \approx 1.4 \) for a Gaussian density distribution.

In this Section, we present our linear perturbation analysis for spiral arm taking into account the effect of azimuthal magnetic field. Our models for single- and two-component spiral arms are considered in Section 2.1 and 2.2

2.1 Single-component analysis

We consider a magnetized gas disc. In a reference frame rotating with the disc at an angular velocity \( \Omega \), the equations of continuity, momentum and magnetic conservation are given as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + 2\Omega \times \mathbf{v} - \Omega^2 \mathbf{R} = - \frac{\nabla p}{\rho} - \nabla \Phi - \frac{\mathbf{B} \times (\nabla \times \mathbf{B})}{4\pi \rho},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}).
\]

In equation (1), we do not take into account magnetic diffusion effects, i.e. ideal MHD. In our analysis, we ignore vertical structures of spiral arms and assume the magnetic pressure is uniform within the spiral arm.

In observations, magnetic fields in nearby spiral galaxies are approximately oriented along their spiral arms with the mean strength of a few \( \mu G \) to about 20 \( \mu G \) [Han 2017 and references therein]. Since our analysis assumes spiral arm to be tightly wound, we consider the equilibrium magnetic
field $B_0$ is parallel to the azimuthal direction. Then, perturbed magnetic field is written as $B = \{\delta B_R, B_0 + \delta B_\phi, 0\}$. By considering two-dimensional space, spatial density $\rho$ is replaced with surface density $\Sigma$. Using the aforementioned assumptions and settings, the linearised equations of continuity, $R$- and $\phi$-momenta for the azimuthal perturbations are obtained as
\begin{align}
 \omega \delta Y &= kY \delta v_\phi, \\
 -i \omega \delta v_R &= 2\Omega \delta v_\phi - i k^2 \omega v_A^2 \delta v_R, \\
 -i \omega \delta v_\phi &= -2\Omega \delta v_R - i k^2 \sigma \delta Y - i k \delta \Phi
\end{align}
where the prefix $\delta$ means the perturbation of a physical value following it, and Alfvén velocity $v_A^2 \equiv B_0^2/(\mu_0 \rho)$. For the pressure term in equation (2), a barotropic equation of state is assumed, and $\sigma^2 \equiv \sigma_\phi^2 + \sigma_{\text{mtd}}^2$, where $\sigma_\phi$ and $\sigma_{\text{mtd}}$ are azimuthal dispersion of turbulent velocities and sound velocity of gas.

The Poisson equation for the perturbed potential of a razor-thin ring with a Gaussian density distribution is given as
\[ \delta \Phi = -\pi G \delta \Sigma \sum_{i=0}^{i=k} \bar{K}_i(kW) L_{-i}(kW) + K_i(kW) L_0(kW), \]
where $K_i$ and $L_i$ are modified Bessel and Struve functions of order $i$ (Takahashi et al. 2016). Hereafter, we denote $f(kW) \equiv [\bar{K}_0(kW) L_{-1}(kW) + K_1(kW) L_0(kW)]$. The function $f(kW)$ decreases with $kW$. For $kW \gg 1$ (i.e. short wavelength $\lambda \ll W$), approximately $f(kW) \propto (kW)^{-1}$, therefore the Poisson equation becomes similar to that of a uniform disc. For $kW \ll 1$ (i.e. long wavelength $\lambda \gg W$), the slope of $f(kW)$ becomes shallower, therefore the Poisson equation deviates from that used in Toomre analysis (see fig. 1 of our Paper I).

Combining equations from (5) to (8), one can obtain a dispersion relation for azimuthal perturbations within a magnetized spiral arm,
\[ \omega^2 = \left[ \sigma^2 - \pi G \sum_{i=0}^{i=k} \bar{K}_i(kW) L_{-i}(kW) + K_1(kW) L_0(kW) \right] k^2 + \frac{4\Omega^2 \omega^2}{\omega^2 + k^2 v_A^2}. \]
The last term in the right-hand side represents the contribution of Coriolis force which stabilises large-scale perturbations with small $k$. As can be seen in the dispersion relation (9), the toroidal magnetic field cancels the Coriolis force and can destabilise the arm. However, the contribution of the magnetic field is proportional to $k^2$, whereas that of Coriolis force itself becomes relatively small for large $k$ since the effects of pressure and gravity increase with $k$. Hence, the magnetic destabilising effect can be significant in an intermediate range of $k$. Similar analyses deriving a dispersion relation for magnetised disc have been presented in previous studies (e.g. Lynden-Bell 1966, Elmegreen 1987, 1994, Gammie 1996, Kim & Ostriker 2001). The effect canceling Coriolis force can be intuitively understood as follows. Azimuthal perturbations drive velocity perturbations $\delta v_\phi$ along a spiral arm. Because both $\delta v_\phi$ and the magnetic field are azimuthal, the perturbations are not directly affected by the field. However, Coriolis force exerts perpendicular to $\delta v_\phi$, i.e. the Coriolis force is radial. Since the magnetic field is azimuthal, the magnetic force exerts radial and thus counteracts the Coriolis force.

Toomre’s instability parameter measured in a spiral arm with rigid rotation is given as
\[ Q_{\text{sp}} \equiv \frac{2\sigma \Omega}{\pi G \Sigma} = \frac{2A \sigma \Omega W}{\pi G T}. \]

Influence on gravitational potential by finite thickness of the spiral arm can be taken into account in $Q_{\text{sp}}$, although we assume an infinitesimally thin density distribution for the spiral arm. To make the dispersion relation dimensionless, we introduce normalised frequency and wavelength of perturbations as $s \equiv \omega/(2\Omega)$ and $\lambda \equiv kW$. With $q \equiv \sigma/(2\Omega W)$ and $\beta \equiv \sigma^2/v_A^2$, equation (9) is written as
\[ s^2 = \left[ q^2 - \frac{Aq}{Q_{\text{sp}}} f(x) \right] x^2 + \frac{s^2}{s^2 + \beta^{-1} q^2 x^2}. \]
In addition, we denote
\[ J(x) \equiv q^2 - \frac{Aq}{Q_{\text{sp}}} f(x). \]

Then, equation (11) is reduced to
\[ s^4 + \left[ \beta^{-1} q^2 x^2 - J(x)x^2 - 1 \right] s^2 - J(x)x^4 \beta^{-1} q^2 = 0. \]
Because this dispersion relation is a biquadratic equation of $s$, one can obtain algebraic solutions. In what follows, we describe how to discriminate different types of the solutions.

2.1.1 Solutions of the dispersion relation
For the dispersion relation (13), the two roots are obtained by the quadratic formula as
\[ s^2 = -\frac{1}{2} \left[ \beta^{-1} q^2 x^2 - J(x)x^2 - 1 \right] \pm \frac{1}{2} \sqrt{\left[ \beta^{-1} q^2 x^2 - J(x)x^2 - 1 \right]^2 + 4J(x)x^4 \beta^{-1} q^2}. \]
The roots are a pair of real numbers or complex conjugates. One of the two roots is, however, considered to be unphysical; in the non-magnetic case with $v_A = 0$, the roots become $s_{\text{NM}}^2 = J(x)x^2 + 1$ and $s_{\text{NM}}^2 = 0$. The former corresponds to equation (9) with $v_A = 0$, whereas the latter is physically meaningless. If the roots (equation (14)) are a pair of real numbers, we consider the root closer to the non-magnetic solution of $s_{\text{NM}}^2 = X(x)x^2 + 1$ to be the physical solution, whereas the other root is discarded since it stems from the meaningless solution: $s_{\text{NM}}^2 = 0$. If the roots are complex conjugates, on the other hand, we do not need to choose either (see below).

As we discuss bellow, the physical solution of $s$ can be classified into three types.

Stable oscillation: The first one is a stable solution: $s^2 > 0$. In this case, because the dimensionless frequency $s$ are real numbers, the perturbations are expected to oscillate with time and do not grow.

Previous studies have proposed models of the thickness correction for disc potential (e.g. Goldreich & Lynden-Bell 1965, Romeo 1992, Elmegreen 1994, 2011, Romeo & Wiegert 2011, Behrendt et al. 2015).

This quantity is similar to plasma-$\beta$ but different in terms of taking into account turbulent pressure.
Magneto-Jeans instability: When \( s^2 < 0 \), the solutions of \( s \) are positive and negative imaginary numbers. The positive imaginary solution corresponds to a perturbation that is expected to grow exponentially with time. Unstable solution appears when \( \beta^{-1} q^2 x^2 - J(x)x^2 - 1 > 0 \) and \( 4J(x)x^2 \beta^{-1} q^2 < 0 \), i.e. \( J(x) < 0 \). The former condition is equivalent to
\[
\sigma^2 - \pi G T f(kW) \beta k^2 + 4\Omega^2 < v_A^2 k^2. \tag{15}
\]
Since the left-hand side of the above condition corresponds to the right-hand side of equation (9) with \( v_A = 0 \), the above condition means that magnetic force dominates over the combination of the other three forces, i.e. gravity, thermal plus turbulent pressure and Coriolis force. Meanwhile, the latter condition, \( J(x) < 0 \), corresponds to
\[
\sigma^2 - \pi G T f(kW) < 0, \tag{16}
\]
and is equivalent to the Jeans instability condition in absence of disc rotation and magnetic effect: gravity dominates over thermal and turbulent pressure when \( J(x) < 0 \). Thus, the solution of \( s^2 < 0 \) would correspond to ‘magneto-Jeans instability’ (MJI, Kim & Ostriker 2001, 2002), in which toroidal magnetic field cancels the stabilising effect of Coriolis force, and then perturbations can collapse unless pressure is strong enough to resist gravity.

Oscillating growing mode: When \( [\beta^{-1} q^2 x^2 - J(x)x^2 - 1]^2 + 4J(x)x^2 \beta^{-1} q^2 < 0 \), \( s^2 \) is a complex number; therefore the solutions of \( s \) are also complex. When the complex \( s^2 \) and the solutions \( s \) are written as \( s^2 = C + iD \) and \( s = \pm(c + id) \), the imaginary part \( d \) can be computed as \( d^2 = (-C + \sqrt{C^2 + D^2})/2 > 0 \). Therefore, one of the complex solutions of \( s \) necessarily has a positive imaginary part, which means that the perturbation is expected to grow exponentially with time. Hereafter, we refer to this type of instability as ‘oscillating growing mode’ (OGM). Since this unstable solution also has a real part, the growing mode is expected to oscillate with time. This implies that the OGMs might show chaotic behaviours.

In the above three types of solutions, the dimensionless growth rate of perturbation is generally computed as
\[
s_{\text{grow}} = \sqrt{-\mathfrak{R}(s^2) + \sqrt{\mathfrak{R}^2(s^2) + 3\mathfrak{I}^2(s^2)}} \frac{1}{2}, \tag{17}
\]
where \( \mathfrak{R}(s^2) \) and \( \mathfrak{I}(s^2) \) are real and imaginary parts of \( s^2 \); \( s_{\text{grow}} = 0 \) for the stable solutions, and \( s_{\text{grow}} = \sqrt{-s^2} \) for the MJI.

As seen in equation (17), the dimensionless dispersion relation \( s^2 \) as a function of \( x \) is described with three parameters: \( Q_{sp} \), \( q \) and \( \beta \). Fig. 1 shows the dispersion relations for three example cases. In the left panels where \( Q_{sp} = 0.7 \), \( q = 1 \) and \( \beta = 0.1 \), both unstable modes of OGM and MJI appear. In the top panel, we compare the solution of \( s^2 \) that of non-magnetic case (\( \beta = \infty \)), and the shaded regions with magenta and cyan indicate ranges of \( x \) of the OGM and MJI modes. In the left panels, the non-magnetic solution with \( \beta = \infty \) indicate \( s^2 > 0 \) and \( s_{\text{grow}} = 0 \) for all \( x \), and it is seen that the toroidal magnetic field destabilises the perturbations in the intermediate range of \( x \). In the case of the left panels, the most unstable perturbation that gives the maximum \( s_{\text{grow}} \) takes place at the boundary between the ranges of MJI and OGMs as seen in the bottom panel. However, the two types of unstable solutions do not necessarily appear simultaneously; in the middle panels of Fig. 1 where \( Q_{sp} = 0.7 \), \( q = 0.2 \) and \( \beta = 10 \), there is no MJI, and all unstable solutions are OGMs. In the case of the right

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\[4\] The real part \( c \) is obtained as \( c^2 = (C + \sqrt{C^2 + D^2})/2 > 0 \)
panels of Fig. 1, where $Q_{sp} = 0.5$, $q = 0.2$ and $\beta = 10$, MJI modes appear in the middle of the $x$-range of OGMs, and $s_{\text{grow}}$ has a bump in the range of the MJI modes. This type of dispersion relation can be seen if non-magnetic solution $s^2 < 0$ with $\beta = \infty$. When a spiral arm is subject to strong gravitational instability, the two unstable modes thus arise in an intricate way.

Fig. 2 illustrates $\max(s_{\text{grow}}(x))$ for each unstable solution of MJI (top) and OGM (bottom) for various $\beta$. The coloured regions indicate $\max(s_{\text{grow}}) > 0$ and are expected to be unstable in our analysis for each instability mode. The leftmost panel shows the result of the non-magnetic case $\beta = 0$, and the other panels show those with $\beta = \sigma^2/v_A^2 = 10$, 1 and 0.1, from left to right. The coloured regions are unstable ($\max(s_{\text{grow}}(x)) > 0$) for each instability mode, and the uncoloured regions are stable ($s_{\text{grow}} = 0$ for all $x = kW$). In the top panels, the green solid lines delineate the boundary between the stable and the OGM-unstable states shown in the bottom panels.

### 2.2 Two-component analysis

Next, we extend the above instability analysis to a multi-component model composed of magnetised gas and stars. We consider distinct values of $\Upsilon$, $\sigma$ and $W$ for each of the gas and stellar component. Hereafter, let suffixes ‘g’ and ‘s’ denote gas and stellar values in the two-component analysis. In this study, we assume that angular rotation velocities are the same between the gas and stars, i.e. $\Omega \equiv \Omega_s = \Omega_g$. We adopt the fluid approximation to the stellar component, in which stars are assumed to have the same form of dispersion relation of gas. Although the fluid approximation for stars may not be appropriate because of the collisionless nature of stars (Toomre 1964; Lin & Shu 1966; Binney & Tremaine 2008; Rafikov 2001; Elmegreen 2011), our Paper I has demonstrated that the fluid approximation does not deteriorate accuracy of our SAI analysis and that our linear analysis can characterise fragmentation of arms in $N$-body disc simulations.

For the gas component, by combining equations (5, 6) and (7), a perturbed line-mass of gas is obtained as

$$\delta \Upsilon_g = k^2 \frac{\Upsilon_g}{\omega^2 - \frac{4\pi^2 \omega^2}{\Omega^2} - \sigma^2 k^2} \delta \Phi. \tag{18}$$

On the other hand, stars are not affected by magnetic fields, therefore

$$\delta \Upsilon_s = k^2 \frac{\Upsilon_s}{\omega^2 - \frac{4\pi^2 \omega^2}{\Omega^2} - \sigma_s^2 k^2} \delta \Phi, \tag{19}$$

where $\sigma_s$ is azimuthal component of stellar velocity dispersion. The gas and stars share the same perturbed potential $\delta \Phi$. However, $\Upsilon_s$ is not the same as $\Upsilon_g$.

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5 Although the analysis presented in our Paper I does not assume $\Omega_g = \Omega_s$, only small differences of $\delta \Upsilon$ between gas and stars are found in isolated simulations of Paper I and cosmological simulations of Hou et al. (2016).
that is described as \( \delta \Phi = \delta \Phi_{\text{g}} + \delta \Phi_{\text{st}} \) \cite{log96,rafikov01}, and the Poisson equation \cite{inoue05} connects the two components as

\[
\delta \Phi = -\pi G \left[ \delta T_{\text{g},f}(kW_{\text{g}}) + \delta T_{\text{st},f}(kW_{\text{st}}) \right].
\]

Thus, the two-component dispersion relation is obtained as

\[
\frac{\pi Gk^2T_{\text{g},f}(kW_{\text{g}})}{\sigma_{\text{g}}^2k^2 + \frac{4G^2}{c_s^4} + \frac{\omega^2}{\sigma_{\text{g}}^2k^2}} = \frac{\pi Gk^2T_{\text{st},f}(kW_{\text{st}})}{\sigma_{\text{st}}^2k^2 + \frac{4G^2}{c_s^4} + \frac{\omega^2}{\sigma_{\text{st}}^2k^2}} - 1.\]

(21)

As for equation \cite{inoue05}, we introduce dimensionless quantities for each of gas and stars, then we obtain

\[
\frac{AQ_{\text{g},f}(x_q)q_s}{\sigma_{\text{g}}^2x_q^2 + \frac{4G^2}{c_s^4}} - s^2 + \frac{AQ_{\text{st},f}(x_q)q_s}{\sigma_{\text{st}}^2x_q^2 + 1 - s^2} = 1.\]

(22)

Furthermore, by denoting \( \gamma_s = AQ_{\text{g},f}(x_q)q_s \), \( \gamma = AQ_{\text{g},f}(x_q)q_s \) and \( \alpha^2 = q_s^2x_q^2 - \gamma_s \) and \( \alpha = q_s^2x_q^2 + \gamma \), the dispersion relation \cite{inoue05} can be reduced to a bicubic equation of \( s \):

\[
s^6 + \left( \beta^{-1}q_s^2 \alpha - \alpha_s - \alpha \right) s^4 + \left[ \alpha_s^2 - \beta^{-1}q_s^2 \alpha \alpha_s + \alpha_s \gamma_s \right] s^2 + \left[ \beta^{-1}q_s^2 \alpha \alpha_s - \gamma_s \gamma \right] = 0.
\]

(23)

Since the algebraic solutions of a cubic equation can be derived in a number of different ways such as Cardano’s and Vieta’s methods \cite{vanderwaerden91}, one can compute the three roots of the bicubic equation \cite{inoue05} for each \( k \). One of the roots is positive real numbers for all \( k \) and increases with \( k \), which does not represent an unstable mode for any parameter sets. Hence, this root is physically meaningless. The other two roots behave the same as those of equation \cite{inoue05}, a pair of complex conjugates or real numbers. If they are complex conjugates, both roots result in the same \( s_{\text{growth}} \) from equation \cite{inoue05}. If they are real numbers, we compute the non-magnetic solution of \( s^2 \) with \( \beta = \infty \) (equation \ref{eq:23} in Appendix \ref{app:c}) and then choose the root closer to the non-magnetic solution, and the other one is discarded as an unphysical solution. Then, \( s_{\text{growth}} \) is computed using equation \cite{inoue05}. The values of \( s_{\text{growth}} \) behave similar to those in the single-component case as shown in Figs. \ref{fig:1} and \ref{fig:2}. Complex and negative real solutions of \( s^2 \) correspond to the instability modes of OGM and MJ1 with \( \max(s_{\text{growth}}) > 0 \), respectively.

3 SIMULATIONS

To test our theory proposed in Section \ref{sec:2}, we perform numerical simulations of isolated disc galaxies. We use the moving-mesh MHD/\( N \)-body code \textsc{Arepo} \cite{springel10} to perform simulations of self-gravitating discs that consist of stars and magnetised gas with an isothermal equation of state. The effect of magnetic fields are assumed to be ideal MHD and implemented with the Powell approach \cite{powell99} for keeping \( \nabla \cdot \mathbf{B} \) negligibly small \cite{pakmor}. Our simulations do not take into account gas cooling, star formation, stellar feedback or magnetic diffusion.

3.1 Initial conditions

Our initial conditions of the galactic discs are similar to those used in our Paper I, which are generated with the method proposed by \textsc{Hernquist} \cite{hernquist93}. The initial profile of radial velocity dispersions is assumed to follow that of surface densities described with an exponential function with the scale radius \( R_d = 3 \) kpc, and kinematic coldness is parameterised by a Toomre parameter \( Q_{\text{min}} \) at \( R \approx 2.5 R_d \).

Vertical structures of the discs are constructed with a density function of \( \exp[\mu z/(2a)] \) with a constant scale height \( z = 50 \) pc and a velocity distribution determined from vertical Jeans equilibrium.

For the single-component model, the total mass of the gas disc is \( M_d = 1.24 \times 10^{10} \) M\(_{\odot} \), and we set \( Q_{\text{min}} = 2 \). For the two-component model, the total mass of gas and stellar discs are \( M_{d, \text{g}} = 7.22 \times 10^8 \) M\(_{\odot} \) and \( M_{d, \text{st}} = 3.40 \times 10^{10} \) M\(_{\odot} \), corresponding to the gas fraction \( f_g = 0.175 \), with \( Q_{\text{min}} = 1.5 \). The gas and stars initially share the same density and velocity distributions described above. The gas discs in both models are represented with \( 1 \times 10^6 \) gas cells, and the stellar disc in the two-component model are represented with \( 5 \times 10^5 \) N-body particles. The simulation code operates mesh regulations such as motions of gas cells, refinement and derefinement so that each gas cell keeps its initial mass within a factor of 2. A gravitational softening length of a stellar particle is set to \( \epsilon = 50 \) pc, and that of a gas cell varies as 2.5 times its approximated cell radius with the lower limit of \( \epsilon_{\text{min}} = 50 \) pc; therefore gas contraction is limited to the scale \( \sim \epsilon_{\text{min}} \).

The other settings of our simulations are the same between the single- and two-component models. The whole halo regions of cubic volume of 200 kpc on a side are filled with diffuse gas the density of which is \( n_0 = 10^{-6} \) cm\(^{-3} \); however the halo gas hardly affects our simulation results. Our simulations with the isothermal equation of state keeps the gas temperature at \( 10^4 \) K independent of density. Haloes and bulges are represented with rigid potentials of \textsc{Navarro-Frenk-White} and \textsc{Hernquist} models, respectively \cite{navarro96, hernquist90}. The masses of the halo and the bulge are \( M_h = 1.1 \times 10^{12} \) M\(_{\odot} \) \cite{springel10} and \( M_b = 4.3 \times 10^{10} \) M\(_{\odot} \), and their scale radii are \( r_h = 20.6 \) kpc and \( r_b = 0.3 \) kpc, respectively. Although we do not change the parameters of the halo and the bulge in this paper, our Paper I has demonstrated that our analysis is fairly robust and can characterise \( \text{SA}1 \) independent of background potentials in our non-magnetic hydrodynamics/\( N \)-body simulations.

Strength of magnetic fields is the most important parameter of this study. Using the initial conditions described above, we run the simulations with a parameter of \( \beta_{\text{ini}} \equiv \sqrt{\alpha_{\text{ini}}/\beta_{\text{ini}}^2} \) = 10, 100, 20 and 5 in \cite{inoue05}. The value of \( \beta_{\text{ini}} \) is spatially uniform; since we assume the isothermal equation of state, all gas cells share the same \( \alpha_{\text{ini}} \) in the initial states. The magnetic fields are oriented azimuthally in the initial conditions and evolve self-consistently after starting the runs. Previous

\footnote{In computing \( Q_{\text{min}} \) in the initial conditions, we do not take into account thermal or magnetic pressure but only turbulent velocity dispersion is included.}

\footnote{The value of \( M_h \) is defined to be the mass enclosed within the galactocentric radius \( r_{200} = 206 \) kpc at which \( 3M_h/(4\pi r_{200}^3) = 2.9 \times 10^9 \) M\(_{\odot} \) kpc\(^{-3} \); and is 200 times as large as the present-day cosmic background density.}

\footnote{Note that the definition of \( \beta_{\text{ini}} \) is not identical to that of \( \beta \) in equation \cite{inoue05}; the former does not take into account turbulent pressure of gas, whereas the latter does.}
studies have performed MHD simulations with various initial conditions and methodology such as initially random orientation of magnetic fields in an isolated disc (Khoperskov & Khrapov 2018), vertical orientation in a spherical collapse model for disc formation (Pakmor & Springel 2013) and a uniform seed field in cosmological simulations (Pakmor et al. 2014, 2017, 2018). However, independent of their initial conditions and types of simulations, these studies demonstrated that magnetic fields become nearly toroidal and oriented along spiral arms during galaxy formation and/or emergence of spiral arms.

3.2 Data analysis

Following our Paper I, we employ the same analysis method based on polar maps. First, we apply two-dimensional Gaussian kernels whose full width at half maximum is 0.5 kpc to gas and stars in snapshots. Then, we vertically integrate physical quantities weighted by mass and compute the quantities needed to solve equation (14 or 23): $\Sigma$, $\Sigma_B$, $\sigma_A$, $\sigma_g$, $\Omega$ and $v_A$. Angular velocity is computed from local mean velocity as $\Omega = \pi_0 / R$. In the two-component models, we compute $\Omega$ as the mass-weighted mean between gas and stellar components. The value of $v_A$ in our analysis is computed using azimuthal component $B_\phi$ and local spatial density of gas. Hereafter we denote this ‘azimuthal Alfvén velocity’ as $v_{A,\phi}$. We make polar plots of these quantities as functions of $(R, \phi)$ for each snapshot.

Our analysis also requires to detect spiral arms and measure their half widths $W_g$ and $W_s$. To this end, we use the same method proposed in our Paper I. In what follows, we describe the method briefly. In the polar plot of $\Sigma(R, \phi)$, we perform one-dimensional Gaussian fitting along the radial direction at a given $\phi$. The Gaussian fitting is iteratively adopted in the range from $R - W$ to $R + W$ while changing $W$. By computing a goodness-of-fit $\chi^2$ for each $W$, we search for the half width $W$ that gives the minimum value of $\chi^2$. Thus, we obtain the best-fit $W$ and its minimum $\chi^2$ at each coordinate point $(R, \phi)$. We perform this fitting procedure for each of gas and stellar component. The goodness-of-fit $\chi^2(R, \phi)$ is expected to become significantly lower than unity if there is a crest of a spiral arm at $R$ and if the radial distribution of $\Sigma$ is close to Gaussian. In this paper, we define spiral arms to be regions where $\log \chi^2_s < -0.25$ for the single-component models, and $\log \chi^2_s < -0.1$ for the two-component models. Although this threshold of $\chi^2$ is arbitrary, our instability analysis to compute $s_{gwov}$ is independent of the threshold. Note that our method to measure $W$ assumes a pitch angle $\theta = 0$, therefore the true width is overestimated by a factor of $1 / \cos \theta$.

4 RESULTS

4.1 The single-component runs

In Fig. 3 we show gas surface density distributions in our single-component runs with $\beta_{ini} = \infty$, 100, 20 and 5 at $t = 400$ Myr. In the absence of magnetic field ($\beta_{ini} = \infty$, top left), spiral arms in the simulation are stable and do not fragment. As seen in the figure, the gas density distributions become more prominent and clumpy for stronger magnetic fields with lower $\beta_{ini}$. In the case of $\beta_{ini} = 100$ (top right), spiral arms are still stable at $t = 400$ Myr; however the density contrast between the arms and the inter-arm regions appears somewhat higher than in the non-magnetic run. Magnetic fields in spiral arms can be amplified because of accretion of gas coupled with magnetic fields on arms, transport of magnetic fields (Khoperskov & Khrapov 2018) and/or dynamo mechanisms by differential rotation of a galactic disc (Shukurov et al. 2006) and small-scale turbulence (Schober et al. 2013). Although the run with $\beta_{ini} = 100$ is stable until $t = 400$ Myr, the magnetic fields in the spiral arms become stronger as time elapses, and then the arms finally fragment at $t = 800$–900 Myr. So, in this sense, our simulation is unstable even with $\beta_{ini} = 100$. However, because spiral-arm fragmentation is not seen at all in the early stages of this run, we consider that the spiral arms in $t \lesssim 400$ Myr of this run to be stable.

In the case of $\beta_{ini} = 20$ (bottom left in Fig. 3), gas densities vary significantly along spiral arms, and the arms have knotty structures. Although the arms in this run may be expected to be unstable and to fragment in the snapshot, they actually do not fragment or form massive clumps soon after this snapshot. We consider this run to be marginally unstable; however the density perturbations within the arms evolve non-linearly. We discuss this run in detail in Section 4.1.3 In the case of $\beta_{ini} = 5$ (bottom right), our simulation clearly shows that spiral arms form massive clumps via fragmentation at $t = 400$ Myr. We regard the arms in this run to be unstable.
The spiral arms in the regions A and B have not fragmented yet, the large values of $\max(s_{\text{grow}}) > 0$ indicate that the regions A and B are unstable. Indeed, in the following snapshots at $t = 280$ and $320$ Myr, the arms fragment and form the clumps in the regions A and B. This means that our linear analysis can predict the fragmentation of the spiral arms before the clumps form. Panel d shows $\max(s_{\text{grow}})$ ignoring the effect of magnetic fields, computed from the same linear analysis but assuming $\beta = \infty$. It is worthy to mention that the non-magnetic analysis indicates $s_{\text{grow}} = 0$ for the fragmenting spiral arms, and $s_{\text{grow}} > 0$ can be seen only inside the gas clumps after their collapse. Thus, the linear analysis ignoring magnetic fields cannot characterise the fragmentation correctly and underestimates the instability since it does not taking into account the magnetic destabilisation effect.

Panel e of Fig. 5 indicates types of unstable modes; black regions are stable with $s_{\text{grow}} = 0$, and blue and red ones are unstable with $s_{\text{grow}} > 0$ although blue regions are rarely seen in this run. The red colour means that solutions of dispersion relation (equation 13) only have OGMs for the instability as exemplified in the middle panel of Fig. 1, whereas the blue colour means that the solutions have MJI and OGM as shown in the right and left panels of Fig. 1. Note that the result shown in Panel e is not the types of instability modes of the most unstable perturbations. In this run, the OGM is the major mode of the instability in most of spiral-arm regions. The MJI modes (blue regions) can be seen only in a very small area in the region A at $t = 320$ Myr. Hence, fragmentation of spiral arms in this run is driven by OGMs rather than MJI. Panel f shows strength of toroidal magnetic fields $\beta$ computed using $v_\Lambda$, $\alpha$ in the simulation. In this run, the fragmenting spiral arms have approximately $\log \beta \simeq 0.5$.

4.1.2 The stable case

The spiral arms in our single-component run with $\beta_{\text{ini}} = 100$ are stable and do not fragment until $t \simeq 800–900$ Myr (the top right panel of Fig. 3). Our instability analysis suggests stable states with $s_{\text{grow}} = 0$ for all perturbations inside the spiral arms. Fig. 4 shows the result of our linear analysis for this stable run at $t = 200$ and $400$ Myr. Panels a and b show gas surface densities, where no fragmentation is seen. Panel c indicates distributions of $\max(s_{\text{grow}})$. Low but non-zero values of $\max(s_{\text{grow}}) \lesssim 0.25$ are seen within a few spiral arms. Although $\max(s_{\text{grow}}) > 0$ in our analysis means an unstable state, these low values seen in this run are considered to be practically stable. Low $\max(s_{\text{grow}})$ corresponds to weakly unstable states in linear regime where perturbations can grow slowly. In such cases, effects that are not taken into account in our analysis can play important roles in the dynamics of spiral arms and may prevent the perturbations from growing; for example, since pitch-angles of the arms are not exactly $\theta = 0$, differential rotation of the disc may stretch the arms and decrease the densities within them.

Panel d in Fig. 5 indicates the strength of magnetic fields $\beta$. In this stable run with $\beta_{\text{ini}} = 100$, the magnetic
Figure 5. Polar-map analysis of the single-component run with $\beta_{ini} = 5$ at $t = 140, 210, 280$ and 320 Myr. Panels a and b: gas surface density distributions in the polar and the face-on Cartesian coordinates. Panel c: the maximum values of dimensionless growth rates $\max[s_{grow}(x)]$ computed from our single-component analysis, where $\max(s_{grow}) > 0$ indicates an unstable state in the linear analysis. Panel d: the same as Panel c but ignoring the effect of magnetic fields. Panel e: types of instability modes. Blue regions have both solutions of MJI and OGM as exemplified in the left and right panels of Fig. 1, and red regions only have OGMs as exemplified in the middle panel of Fig. 1, and black regions indicate stable states with $s_{grow} = 0$ for all wavenumbers. Panel f: strength of toroidal magnetic fields, where $\beta \equiv (c_{maj}^2 + \sigma_\phi^2)/c_A^2$. In Panels c, d and e, the inter-arm regions are uncoloured, where our arm-detection method indicates $\log \chi^2 > -0.25$ in the Gaussian fitting for the gas density distribution. In Panel f, regions of field reversals with $B_\phi < 0$ are uncoloured. The dashed ellipses labelled as ‘A’ and ‘B’ in each panel trace the unstable spiral arms we focus on. The regions A and B fragment into gas clumps at $t = 280$ and 320 Myr, and both regions indicate unstable states with $\max(s_{grow}(x)) > 0$ in our linear analysis before the fragmentation (Panel c). On the other hand, the analysis ignoring the magnetic effect cannot capture the fragmentation (Panel d).
fields are significantly weaker than in the unstable run with \( \beta_{ini} = 5 \) shown in Fig. 5 (note the different colour scales for the panels of \( \beta \) between Figs. 4 and 5). The spiral arms in this run have \( \log \beta \simeq 1.0–1.5 \). Because the other properties such as surface densities and widths of the spiral arms are not largely different between the unstable and stable runs, the instability can be attributed to the strong magnetic fields, i.e. the magnetic destabilisation.

4.1.3 The non-linearly stable case

Fig. 7 shows results of our linear analysis applied to the run with the moderate strength of magnetic field \( \beta_{ini} = 20 \). As seen in Panels a and b, the spiral arms in this run do not clearly fragment or form clumps even in later snapshots at \( t \simeq 500–600 \) Myr although the arms may appear clumpy and to be fragmenting at \( t = 400 \) Myr. In Fig. 7, we focus on two spiral arms that are labelled as ‘C’ and ‘D’. In Panel c, these labelled regions indicate \( \text{max}(s_{\text{grow}}) > 0 \) at \( t = 500 \) Myr and are expected to be unstable according to our analysis. Although the region C keeps high values of \( \text{max}(s_{\text{grow}}) \) in these snapshots, the arm does not form clumps, and appears to be long-lived over an orbital time-scale. The region D also indicates \( \text{max}(s_{\text{grow}}) \simeq 0.8 \) at \( t = 500 \) Myr, but actually \( \text{max}(s_{\text{grow}}) \) decreases in the following snapshots. The arm in the region D is finally dissolved at \( t = 610 \) Myr. Thus, in the single-component run with \( \beta_{ini} = 20 \), although the spiral arms are supposed to be unstable in our linear analysis, they are actually stable in non-linear regimes. In Sections 4.1.2 and 5.2 we discuss the non-linear effects and other physics that are not taken into account in our linear analysis. These effects can possibly stabilise and prevent arms from fragmenting when simulations deviate from the linear analysis and break the assumptions in our linear analysis such as the tight-winding approximation, azimuthal perturbations and toroidal magnetic fields.

Panel d in Fig. 4 shows dimensionless growth rates ignoring magnetic fields. We find \( \text{max}(s_{\text{grow}}) = 0 \) except the region D at \( t = 500 \) Myr; the high values of \( \text{max}(s_{\text{grow}}) \) seen in Panel c are due to the magnetic fields. Panel e shows types of instability modes expected from our analysis. We find that OGM is the instability mode expected from our analysis in most of the regions where \( \text{max}(s_{\text{grow}}) > 0 \) except inner radii \( R \lesssim 3 \) kpc. In Panel f, the spiral arms have approximately \( \log \beta \simeq 0.5–1.0 \) showing that the magnetic fields are stronger than in the stable case shown in Fig. 4.

4.2 The two-component runs

In Fig. 6 we show surface density distributions of gas (top set of panels) and stellar (bottom set of panels) components in our two-component simulations at \( t = 300 \) Myr, in which the initial magnetic field strength is \( \beta_{ini} = \infty, 100, 20 \) and 5. Spiral-arm structures can be seen in both gas and stellar distributions, and we adopt our two-component linear analysis described in Section 2.2 to these simulations.

In the non-magnetic run with \( \beta_{ini} = \infty \), the spiral arms are stable at least until \( t = 1 \) Gyr. Similarly to the single-component runs (Fig. 6), the galaxies become clumpy and their arms tend to have knotty structures as \( \beta_{ini} \) decreases. Even in the case of the strong magnetic field with \( \beta_{ini} = 5 \), however, the clumpy structures in gas do not appear to be massive enough to capture stellar particles; stellar clumps are not seen in the surface density maps of the stellar components. In Fig. 6 such low-mass gas clumps could not completely tear the spiral arms. Thus, the magnetic destabilisation may be limited in the two-component runs even if \( \beta_{ini} = 5 \). This would be because magnetic fields only affect the gas component accounting for the small fraction \( f_g = 0.175 \) of the total disc mass, and the stars dominate dynamics within the discs.

In the two-component run with \( \beta_{ini} = 5 \), a few small gas clumps can be seen at \( t = 300 \) Myr in the top right panel of Fig. 8. Fig. 9 shows our polar-map analysis for the snapshots at \( t = 250, 290 \) and 340 Myr, where a fragmenting arm is marked as ‘E’ with dashed ellipses. Since the arm E is not completely torn off and keeps the spiral structure without destructing the arm even after the gas clump formation, we consider that this arm is weakly fragmenting. Eventually, the clump forming in the arm E migrates along the spiral arm to the galactic centre. Panel e shows the dimensionless growth rates \( \text{max}(s_{\text{grow}}) > 0 \) in the fragmenting arm, which indicate instability and are consistent with the result of the formation of the gas clump. We find that, if we ignore magnetic fields in our linear analysis, the arms are predicted to be stable...
with $s_{\text{grow}} = 0$ for all perturbations in these snapshots. We also find that OGM is the major mode of instability in most of the regions with $\max(s_{\text{grow}}) > 0$. In Panel h, we show distributions of magnetic field strength $\beta$, and the fragmenting arm in this run have quite low $\log \beta \simeq -0.3$. The low values $\log \beta \sim 0$ suggest energy equipartition between (thermal and turbulent) pressure and toroidal magnetic field. Despite the strong magnetic fields, the arm is only weakly unstable in this two-component run. In disc galaxies with subdomi-
nant gas components, the magnetic destabilisation appears to be limited even if the energy equipartition is reached.

We consider that the two-component run with $\beta = 100$ (the top right panels in Fig. 8) is stable until $t \sim 500$ Myr although there are density fluctuations in spiral arms after $t \sim 300$ Myr. In Fig. 10, we show our polar-map analysis for the snapshots at $t = 300$ and 400 Myr. Panel e shows dimensionless growth rates in the spiral arms. At $t = 300$ Myr, $s_{\text{grow}} = 0$ in all of the spiral arms, which predicts dynamical stability for the arms. At $t = 400$ Myr, however, our analysis indicates unstable states with $\max(s_{\text{grow}}) \simeq 0.3$–0.4 for the arms although they do not fragment at least during a few orbital time-scales after this snapshot. We consider, therefore, that the spiral arms in this run are stable at $t \sim 400$ Myr and that the arms might be stabilised by non-linear effects and/or physical mechanisms missing in our analysis. Panel f shows strength of the magnetic fields. The spiral arms have $\log \beta \simeq 0.5$–1.0 approximately, and it appears that the values of $\beta$ decrease with time between $t = 300$ and 400 Myr. The magnetic fields are thus amplified in the arms during the run.

In our two-component simulations, we find that stable states with $\max(s_{\text{grow}}) > 0$ are often seen, such as the above case with $\beta = 100$ at $t = 400$ Myr. Spiral arms in the two-component run with $\beta = 20$ also indicate significantly high $\max(s_{\text{grow}}) > 0$ in our analysis, but the arms do not fragment during the run. These stable arms with high $\max(s_{\text{grow}})$ are often stretched out and/or dissolved after a few orbital time-scales. Our linear perturbation analysis described in Section 2 posits on the assumptions such as equilibrium states in toroidal magnetic fields and rigid rotations for azimuthal spiral arms. Deviation from these assumptions could cause various effects that are not considered in our analysis; these non-linear effects may stabilise (and even dissolve) spiral arms. We discuss some of these effects in Section 5.2.

5 DISCUSSION

5.1 Influence on clump mass

Using our linear perturbation theory of SAI presented in Section 2, we discuss a physical property of giant clumps observed in gas-rich star-forming galaxies. In our Paper I, we have demonstrated that our linear analysis can be applied to estimating masses of clumps forming via spiral-arm fragmentation in non-magnetised isolated galaxy simulations. In observations, the large masses of giant clumps, $M_{\text{cl}} \lesssim 1 \times 10^8 M_\odot$, are most noticeable difference from normal star clusters and giant molecular clouds in spiral galaxies. Here, we discuss how toroidal magnetic fields can affect masses of gas clumps forming via SAI. Because clumpy galaxies are generally observed to be highly gas-rich, we can assume that SAI is driven by gas in these galaxies (Paper I). In the following analysis, we therefore consider the single-component analytic model described in Section 2.1. Hereafter, all variables such as $\Upsilon$, $\Sigma$, $\sigma$, and $W$ represent the physical properties of gas in a spiral arm.

We consider that spiral-arm fragmentation occurs in a marginally unstable state with a non-zero but low $\max(s_{\text{grow}})$. In this case, the onset of instability is expected to occur at a certain wavenumber $k_{\text{os}}$, and this perturbation would collapse first. This marginally unstable state corresponds to the boundary between the stable and OGM-unstable states illustrated in the bottom panels of Fig. 2. From the analysis shown in Fig. 2 at given $q \equiv \sigma/(2 \Omega W)$ and $\beta$, we can obtain Toomre’s $Q$ parameters, $Q_{\text{sp}} = 2 \Upsilon \Omega W/(\sigma G \Upsilon)$, of the marginally unstable arm. In addition, by numerically solving the dispersion relation (13), we can also compute dimensionless wavenumber $x_{\text{os}} \equiv k_{\text{os}} W$ for the marginally unstable state.
Spiral-arm instability in MHD

Figure 9. Polar-map analysis of the two-component run with $\beta_{\text{ini}} = 5$ at $t = 250$, 290 and 340 Myr. Panels from a to d: surface density distributions of gas and stars. Panel e: the maximum values of dimensionless growth rates $\max[s_{\text{grow}}(k)]$ computed from our two-component analysis. The inter-arm regions are uncoloured, where our arm-detection method indicates $\log(\chi_g^2 + \chi_s^2) > -0.1$ in the Gaussian fittings. Panel f: strength of toroidal magnetic fields, where the regions with field reversals $B_\phi < 0$ are uncoloured. In Panels a and b, the region labelled as ‘E’ appears to be unstable and form a gas clump at $t = 340$ Myr in the simulation although the arm is not completely torn off. In Panel e, our linear analysis also indicates unstable states in the region E before the collapse.

When the wavelength of the instability onset $\lambda_{\text{os}} = 2\pi/k_{\text{os}}$ is significantly longer than a width of the arm (i.e. $x_{\text{os}} \ll 1$), the unstable perturbation can be expected to collapse along the spiral arm: one-dimensional collapse (large-scale SAI). In this case, the typical mass of a clump forming via SAI is estimated to be

$$M_{\text{cl},1D} \sim \Upsilon \lambda_{\text{os}} = 8A \frac{\Omega^2 W^3}{G} \frac{q}{Q_{sp} x_{\text{os}}},$$

where again $\Upsilon = AW\Sigma$. Thus, $M_{\text{cl},1D} \propto q/(Q_{sp} x_{\text{os}})$ for given $\Omega$ and $W$, and we define a ‘clump-mass module’ as $m_{\text{cl},1D} \equiv q/(Q_{sp} x_{\text{os}})$.

When $\lambda_{\text{os}}$ is significantly shorter than the arm width (i.e. $x_{\text{os}} \gg 1$), the unstable perturbation is deeply embedded within the spiral arm. In this case, we consider that a round region with a radius of $\lambda_{\text{os}}/2$ collapses: two-dimensional collapse (small-scale SAI). In this case, we estimate the typical clump mass as

$$M_{\text{cl},2D} \sim \pi \Sigma \left(\frac{\lambda_{\text{os}}}{2}\right)^2 = 4\pi^2 \frac{\Omega^2 W^3}{G} \frac{q}{Q_{sp} x_{\text{os}}^2},$$

and a clump-mass module is defined as $m_{\text{cl},2D} \equiv q/(Q_{sp} x_{\text{os}}^2)$.

We assume these clump-mass moduli, $m_{\text{cl},1D}$ and $m_{\text{cl},2D}$, to switch at $x_{\text{os}} = 1$ although the boundary between the two regimes is ambiguous in reality.

In the top panel of Fig. 11, we compare the clump-mass moduli $m_{\text{cl}}$ as functions of $q$ between the cases of $\beta = \infty$ and 0.1. The values of $m_{\text{cl}}$ increase only slightly even in the strong magnetic field with $\beta = 0$. When $\beta = 10$ and 1, we find that $m_{\text{cl}}$ hardly changes from the result with $\beta = \infty$. Thus, presence of toroidal magnetic field does not systematically alter masses of clumps forming via SAI if $\Omega$ and $W$ are the same. This result does not mean, however, that magnetic force is ineffective in the clump formation. The bottom panel of Fig. 11 shows that wavenumbers $x_{\text{os}}$ of the marginally unstable perturbations decrease significantly when $\beta = 0.1$. In the presence of an effective magnetic field, a spiral arm can be unstable at a relatively high $Q_{sp}$ (i.e. low $\Sigma$ at given $\Omega$ and $\sigma$) as shown in Fig. 2; however the unstable wavelength $\lambda_{\text{os}}$ becomes long. Thus, the large physical size of the collapsing region compensates the low density within the arm, therefore the resultant clump mass expected from the resulting clump mass can become smaller than $M_{\text{cl},2D}$.

This small-scale SAI reduces to Toomre instability against azimuthal perturbations within a spiral arm (Paper I).

Note that an unstable perturbation would collapse three-dimensionally if $\lambda_{\text{os}}$ is shorter than a vertical height of the spiral arm. In this case, a resultant clump mass can become smaller than $M_{\text{cl},2D}$.
Figure 10. Same as Fig. 9 but for the two-component run with $\beta_{\text{ini}} = 100$ at $t = 300$ and 400 Myr. The spiral arms appear to be practically stable and do not form clumps until $t \approx 500$ Myr although our linear analysis predicts unstable states with $\max(s_{\text{grow}}) > 0$ at $t = 400$ Myr in Panel e.

analysis is insusceptible to strength of a toroidal magnetic field.

Using the above analysis, we estimate a typical mass of a giant clump that is expected to form by the marginal instability in our SAI model. Assuming the typical values of disc galaxies, such as $c_{\phi} = 200$ km s$^{-1}$ and $W = 0.5$ kpc, and clump formation at $R = 5$ kpc, the factor common between equations (24 and 25) becomes $\Omega^2 W^3/G = 4.7 \times 10^7 \, M_\odot$. The other factors of $SA$ in equation (24) and $4\pi^2$ in equation (25) differ only by a factor of $\sim 4$. Since highly turbulent gas discs of clumpy galaxies have been observed to have $c \sim 50$ km s$^{-1}$ (e.g. Green et al. 2010; Swinbank et al. 2011; 2012; Bassett et al. 2014; Fisher et al. 2017a; b; Oliva-Altamirano et al. 2018), $q \equiv \sigma/(2\Omega W) \sim 1$. From the top panel of Fig. 11, the clump-mass moduli are $m_{\text{cl,1D}} \sim 1$ almost independent of magnetic field strength. We thus estimate the typical clump mass to be $M_{\text{cl}} \sim 10^7$–$10^8 \, M_\odot$ from equations (24 and 25), which is approximately consistent with relatively massive clumps observed in high- and low-redshift galaxies (e.g. Förster Schreiber et al. 2011; Adamo et al. 2013; Elmegreen et al. 2013; Wuyts et al. 2014; Guo et al. 2015; 2018; Fisher et al. 2017a). Although giant clumps can significantly decrease their masses and may be disrupted after their formation, clumps with $M_{\text{cl}} \sim 10^8 \, M_\odot$ are thought to survive even if there are feedback effects by supernovae and radiation pressure from massive stars (Mandelker et al. 2017; although see Hopkins et al. 2010; Genel et al. 2012; Buck et al. 2017).

As shown in the bottom panel of Fig. 11, wavenumber $x_{\text{os}}$ of the marginally unstable perturbations decreases as $\beta$ decreases. This implies that SAI occurs at a long wavelength in a strong magnetic field and that the unstable arm tends to collapse one-dimensionally. This large-scale SAI can be expected to destructively disarrange a configuration of the arm, and such large-scale instability may tear off the arm. On the other hand, the wavelength of the marginal instability becomes short in a weak magnetic field. Such small-scale instability induced within an arm — which is expected to collapse two-dimensionally — might not intensely violate the spiral arm. In this case, the arm may keep its configuration although ‘beads on a string’ structures along the arm may develop (e.g. Elmegreen & Elmegreen 1983; Elmegreen et al. 2018). Thus, although strength of toroidal magnetic fields would not significantly affect a typical clump mass, it may be relevant to how violent SAI is for spiral arms.

5.2 Other possible effects of magnetic fields

As we demonstrate using our simulations in Section 4, magnetic destabilisation by strong toroidal fields can lead spiral arms to fragment and form giant clumps. This result is
consistent with our linear perturbation analysis presented in Section 2, although non-linear stability is often seen in the moderate magnetic fields and the two-component models. Our analysis, however, assumes various simplifications, such as toroidal magnetic field without radial gradient of magnetic pressure, azimuthal perturbation (i.e., parallel to the magnetic field) propagating along spiral arm, the tight-winding approximation and rigid rotation within arm. Of course, these assumptions do not necessarily hold in simulations and real galaxies.

Magnetic force can cause complicated effects especially when orientation of magnetic field is not parallel to perturbations. An analytic study of Kim & Ostriker (2000) has, in the context of ideal MHD, discussed various effects in rotating systems. Although their analysis does not take into account self-gravity, they consider general cases for orientations of perturbations and magnetic fields. For example, if there is significant radial gradient of magnetic pressure in a spiral arm, it exerts a force outwards. As seen in our simulation results, the magnetic field strength β varies with radius especially at inner radii although βini is uniform in our initial conditions. Because a pitch-angle of a spiral arm is not exactly zero, the arm can have significant radial gradient of β. Therefore, the magnetic pressure can push a spiral arm in the radial direction.

If there are vertical and/or radial perturbations, gas in a spiral arm can be subject to Parker instability (Parker 1966) and/or toroidal buoyancy (Kim & Ostriker 2000) which are caused by bending of magnetic field lines. It may be expected that these effects could disturb spiral arms and trigger formation of clumpy structures (e.g., Körtgen et al. 2018) although Kim et al. (2002) have demonstrated, using their shearing-box MHD simulations, that Parker instability appears to play only a secondary role in disc instability. Moreover, since a galactic disc generally rotates differentially, magneto-rotational instability can also operate, which stems from radial perturbations.

Although our SAI analysis is focused on azimuthal perturbations within spiral arms, toroidal magnetic fields can also affect radial perturbations within disc regions. In the context of Toomre’s instability analysis for a uniform gas disc, previous studies have presented their linear perturbation analyses taking into account toroidal magnetic fields (e.g., Lynden-Bell 1966; Elmegreen 1987, 1994; Gammie 1996; Kim & Ostriker 2001). Considering a single-component gas disc in a toroidal magnetic field, dispersion relation for radial perturbations in a local disc region is described as

\[\omega^2 = (\sigma_R^2 + v_A^2) \kappa_R^2 - 2\pi G \Sigma_d k_R + \kappa^2, \tag{26}\]

where \(\Sigma_d\) and \(\kappa\) are surface density of disc and epicyclic frequency, and \(k_R\) is radial wavenumber of perturbation. Toomre’s instability parameter \(Q\) is accordingly modified due to the toroidal magnetic field as

\[Q' = \frac{\sqrt{\sigma_R^2 + v_A^2 \kappa}}{\pi G \Sigma_d}. \tag{27}\]

Toroidal magnetic field thus exerts magnetic pressure in radial direction and can stabilise radial perturbations in a local disc region. If we assume that spiral arms start forming in a uniform disc, magnetic fields can suppress growth of radial perturbations and may result in formation of weak spiral arms. Because such weak spiral arms are expected to have large \(Q_{sp}\), in this sense, toroidal magnetic fields in the initial disc can prevent spiral arms from fragmenting and forming giant clumps. We argue, however, that magnetic fields can destabilise spiral arms after their formation. In our simulations, the spiral arms are marginally stable when \(\beta_{ini} = \infty\) (see our Paper I), therefore expected to be susceptible to magnetic fields. In such cases, as our simulations show, toroidal magnetic fields can induce clump formation.

5.3 On spiral-arm fragmentation in real galaxies and giant clump formation via SAI

Formation of giant clumps generally involves active star formation inside them. Guo et al. (2012) have estimated the total star formation rate within giant clumps in a galaxy at a redshift \(z \sim 2\) to be nearly fifty per cent of that within the entire galaxy. Thus, if these clumps are formed via spiral-arm fragmentation, magnetic fields may cause intense star formation in disc galaxies.

In Pakmor et al. (2017), however, their cosmological simulations (the Auriga simulations, Grand et al. 2017) including ideal MHD effects have demonstrated that magnetic effects hardly change star formation histories and global evolution of their simulated galaxies. Their simulations also showed that magnetic energies within the galaxies increase with time but saturate at redshifts \(z = 2–3\), and their kinematic energies of gas significantly dominate over their magnetic energies even at \(z = 0\). None of the galaxies in the Auriga simulations clearly experience clumpy phases driven by disc instability, and it appears that the magnetic destabilisation is only limited in their simulations.

The epoch of the magnetic saturation at \(z = 2–3\) shown in Pakmor et al. (2017), however, coincides with the peak of abundances of clumpy galaxies (Shibuya et al. 2016). Because galaxies are generally gas-rich at the redshifts \(z = 2–3\), magnetic effects may be influential in galactic dynamics. It has been observed that the onset of spiral galaxies in the Universe occurs at \(z \sim 2\) (Law et al. 2012; Elmegreen & Elmegreen 2014). Yuan et al. (2017) report their discovery of the most ancient spiral galaxy at \(z = 2.54\), thanks to magnification effect by gravitational lensing. They argue that spiral-arm structures can not be observationally resolved at this redshift without the aid of gravitational lensing and that spiral galaxies could exist at even higher redshifts. The high-redshift spiral galaxies discovered by Law et al. (2012) and Yuan et al. (2017) are observed to host massive clumps within their spiral arms. Although it is uncertain whether magnetic force is significantly influential in these high-redshift spiral galaxies, we propose SAI to be a possible mechanism to trigger formation of giant clumps in these galaxies (Paper I), besides Toomre instability (e.g., Noguchi 1998, 1999).

13 Note that estimations of clump sizes and masses can significantly depend on observational resolutions (e.g., Dessauges-Zavadsky et al. 2017; Cava et al. 2018). This would also be the case for estimations of star formation rates of clumps.

14 They find the spiral arms in the galaxy from a photometric image obtained from the Hubble space telescope and confirmed its disc rotation from an integral-field spectroscopy by the Gemini North telescope.
6 CONCLUSIONS AND SUMMARY

Our study presents linear perturbation analysis for self-gravitating spiral arms in toroidal magnetic fields. Based on the SAI analysis of our Paper I, we assume ideal MHD, barotropic equation of state for gas, rigid rotation, Gaussian density distribution inside spiral arms and tight-winding approximation. Furthermore, we extend the magnetic SAI analysis to the two-component model that consists of gas and stellar components. As proposed by previous works in the context of Toomre’s instability analysis, toroidal magnetic fields can destabilise spiral arms by canceling Coriolis force and induce fragmentation of arms.

We run our MHD simulations with single- and two-component disc galaxy models in isolation, and then we test our theory by adopting the simulation results to the linear analysis. We find that our analysis can characterise stable and unstable states of spiral arms in cases of significantly weak and strong magnetic fields. In moderate magnetic fields, however, we often find stable states in non-linear regimes, in which spiral arms do not fragment and are practically stable in the simulations although our linear analysis predicts exponential growth of perturbations with finite growth time-scales. If the linear analysis ignores magnetic fields, it erroneously predicts stable states for fragmenting spiral arms. Hence, it is important to take into account the magnetic effect in order to characterise SAI more accurately.

Using our SAI analysis, we estimate a typical mass of giant clumps forming via fragmentation of spiral arms and find that the clump mass is almost independent from strength of toroidal magnetic fields. The estimated mass is approximately $\sim 10^7$–$10^8 M_\odot$ and nearly consistent with relatively massive clumps observed in the high- and low-redshift galaxies. Hence, SAI could be a possible mechanism to form giant clumps in gas-rich galaxies, besides Toomre instability.

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**APPENDIX A: A NON-MAGNETIC TWO-COMPONENT MODEL**

With $\beta^{-1} = 0$, the two-component dispersion relation in the non-magnetic case (see also Paper I) is

$$\frac{Aq_g f(x_g) x_g^2}{Q_{sp,g} (q_g^2 x_g^2 + 1 - s^2)} + \frac{Aq_s f(x_s) x_s^2}{Q_{sp,s} (q_s^2 x_s^2 + 1 - s^2)} = 1.$$  \hspace{1cm} (A1)

Substituting $\gamma_g$, $\gamma_s$, $\alpha'_g \equiv q_g^2 x_g^2 + 1 - \gamma_g$ and $\alpha_s$ in the above equation, the non-magnetic two-component dispersion relation is reduced to

$$s^4 - (\alpha'_g + \alpha_s) s^2 + \alpha'_g \alpha_s - \gamma_g \gamma_s = 0.$$  \hspace{1cm} (A2)

The roots of this biquadratic equation are

$$s^2 = \frac{\alpha'_g + \alpha_s \pm \sqrt{(\alpha'_g - \alpha_s)^2 + 4 \gamma_g \gamma_s}}{2}.$$  \hspace{1cm} (A3)

Because $\gamma_g$ and $\gamma_s$ are positive, the two roots are real numbers. The root taking the plus sign for the second term does not allow an unstable solution of $s^2 < 0$ (see also Dib et al. 2017), therefore can be discarded as an unphysical solution. Hence, the physical solution of equation (A2) is

$$s^2 = \frac{\alpha'_g + \alpha_s - \sqrt{(\alpha'_g - \alpha_s)^2 + 4 \gamma_g \gamma_s}}{2}. $$  \hspace{1cm} (A4)