Nuclear Structure for Double Beta Decay

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Abstract. Neutrinoless double-beta decay, if observed, would signal physics beyond the Standard Model that would be discovered at energies significantly lower than those at which the relevant degrees of freedom can be excited. Accurate nuclear structure calculation of the nuclear matrix elements (NME) necessary to analyze the decay rates could be helpful to narrow down the list of competing mechanisms, and to better identify the more exotic properties of the neutrinos. In this paper we present and analyze the status of the NME shell model calculations, and their relevance for discriminating the possible competing mechanisms that may contribute to the neutrinoless double-beta decay process.

1. Introduction
Should the neutrinoless double-beta transitions occur, then the lepton number conservation is violated by two units, and the black-box theorems [1, 2, 3, 4] indicate that the light left-handed neutrinos are Majorana fermions. As such, through black-box theorems alone, it is not possible to disentangle the dominant mechanism contributing to this process. Most of the theoretical effort dedicated to this subject consists of calculations of leptonic phase-space factors and nuclear matrix elements that are computed via several nuclear structure methods and within specific models. One of the most popular models is the left-right symmetric model [5, 6, 7, 8, 9], which is currently investigated at the Large Hadron Collider (LHC) [10]. In two recent papers [11, 12] we have discussed ways to identify some of the possible contributions to the decay rate by studying the angular distribution and the energy distribution of the two outgoing electrons that could be measured. However, there are still many other possible contributions to this process that one cannot yet dismiss. For these reasons, a more general beyond standard model (BSM) effective field theory would be preferable, as it would not be limited to relying on specific models, but rather considering the most general BSM effective field theoretical approach that describes this process. An important outcome of such a theory is the evaluation of the energy scales up to which the effective field operators are not broken, together with limits for the effective low-energy couplings.

The analysis of the $0\nu\beta\beta$ decay process is generally done at three levels. At the lowest level the weak interaction of the quarks and leptons is considered, and the BSM physics is treated within a low-energy effective field theory approach. At the next level the hadronization process to nucleons and exchanging pion is considered. The nucleons are treated in the impulse approximation leading to free space $0\nu\beta\beta$ transition operators. At the third level the nucleon dynamics inside the nuclei is treated using nonperturbative nuclear wave functions, which are further used to obtain nuclear matrix elements (NME) needed to calculate the $0\nu\beta\beta$ observables, such as half-lives and two-electron angular and energy distributions [11].
To accomplish this goal we need reliable NME. Although there seem to exist many NME results to choose from, most of the literature is dedicated calculations for the light left-handed Majorana neutrino exchange. Ref. [13] provides tables and plots that compare the latest results for the light left-handed neutrino exchange and for the heavy right-handed neutrino exchange. We calculate the NME using shell model techniques, which are consistent with previous calculations [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. The reason for choosing shell model NME is our belief that these are better suited and more reliable for $0\nu\beta\beta$ calculations, as they take into account all the correlations around the Fermi surface, respect all symmetries, and take into account consistently the effects of the missing single particle space via many-body perturbation theory (the effects were shown to be small, about 20%, for $^{82}$Se [25]). Furthermore, we have tested the shell model methods and the effective Hamiltonians used by comparing the calculated spectroscopic observables to the experimental data, as presented in Ref. [15, 26, 13]. We do not consider any quenching for the bare $0\nu\beta\beta$ operator in these calculations. Such a choice is different from that for the simple Gamow-Teller operator used in the single beta and $2\nu\beta\beta$ decays where a quenching factor of about 0.7 is necessary [27]. For the PSF we use an effective theory based on the formalism of Ref. [28], but fine-tuned as to take into account the effects of a Coulomb field distorting finite-size proton distribution in the final nucleus. To our knowledge, some of the NME presented here were calculated for the first time using shell model techniques.

2. $0\nu\beta\beta$ decay effective field theory formalism

Throughout the literature, the main mechanism considered to be responsible for the neutrinoless double beta decay is the so called "mass mechanism" that assumes that the neutrinos are Majorana fermions, and requires that the light left-handed neutrinos have mass. However, the possibility that right-handed currents could contribute to the neutrinoless double-beta decay ($0\nu\beta\beta$) has been already considered for some time [29, 28]. Recently, $0\nu\beta\beta$ studies [30, 9] have adopted the left-right symmetric model [31, 7] for the inclusion of right-handed currents. In addition, the $R$-parity violating ($\mathcal{R}_p$) supersymmetric (SUSY) model can also contribute to the neutrinoless double beta decay process [32, 33, 34]. In the framework of the left-right symmetric model and $R$-parity violating SUSY model, the $0\nu\beta\beta$ half-life can be written as a sum of products of PSF, BSM LNV parameters, and their corresponding NME [11]:

$$\left[ T_{1/2}^{0\nu} \right]^{-1} = G_{01} g_A^4 \left[ \eta_{00} M_{00} + \left( \eta_{L}^L + \eta_{R}^R \right) M_{0N} + \eta_{q} M_{q} + \eta_{X} M_{X} + \eta_{\eta} X_{\eta} \right]^2. \quad (1)$$

Here, $G_{01}$ is a phase space factor that can be calculated with good precision for most cases [35, 36, 37, 38], $g_A$ is the axial vector coupling constant, $\eta_{00} = \sqrt{\frac{m_{0\beta}}{m_e}}$, with $m_{0\beta}$ representing the effective Majorana neutrino mass, and $m_e$ the electron mass. $\eta_{L}^L, \eta_{R}^R$ are the heavy neutrino parameters with left-handed and right-handed currents, respectively [20, 9]. $\eta_{q}$, $\eta_{X}$ are $\mathcal{R}_p$ SUSY LNV parameters [39]. $\eta_{\lambda}$, and $\eta_{\eta}$ are parameters for the so-called "$\lambda$–" and "$\eta$–mechanism", respectively [9]. $M_{00}$, $M_{0N}$, are the light and the heavy neutrino exchange NME. $M_{q}$, $M_{X}$ are the $\mathcal{R}_p$ SUSY NME, and $X_{\lambda}$ and $X_{\eta}$ denote combinations of NME and other PSF ($G_{02} - G_{00}$) corresponding to the $\lambda$–mechanism involving right-handed leptonic and right-handed hadronic currents, and the $\eta$–mechanism with right-handed leptonic and left-handed hadronic currents, respectively [11].

A more general approach is based on the effective field theory extension of the Standard Model. The analysis based on the BSM contributions to the effective field theory is more desirable, because it does not rely on specific models, and their parameters could be extracted/constrained by the existing $0\nu\beta\beta$ data, and by data from LHC and other experiments. In fact, the models considered above always lead to a subset of terms in the low-energy ($\sim 200$
MeV) effective field theory Lagrangian. We consider all the terms in the Lagrangian allowed by the symmetries. Some of the couplings will correspond to the model couplings in Eq. (1), but they might have a wider meaning. Others are new, not corresponding to specific models.

We treat the long-range component of the $0\nu\beta\beta$ diagrams [40] as two point-like vertices at the Fermi scale, which exchange a light neutrino. In this case, the dimension 6 Lagrangian can be expressed in terms of effective couplings [41]:

$$\mathcal{L}_6 = \frac{G_F}{\sqrt{2}} \left[ j_{V-A}^\mu j_{V-A}^\mu + \sum_{\alpha,\beta} \epsilon_{\alpha,\beta} \bar{\nu}_\alpha \nu_\beta \right],$$

(2)

where $j_{V-A} = \bar{u}\gamma_\mu d$ and $j_\beta = \epsilon_{\beta} \nu_\beta$. The definitions of the $\mathcal{O}_{\alpha,\beta}$ operators are given in Eq. (3) of Ref. [41]. The LNV parameters are $\epsilon_{\alpha} = \epsilon_{\alpha}^L, \epsilon_{\alpha}^R$, and $\epsilon_{\beta} = \epsilon_{\beta}^L, \epsilon_{\beta}^R$. The "**" symbol indicates that the term with $\alpha = \beta = (V - A)$ is explicitly taken out of the sum. However, the first term in Eq. (2) still entails BSM physics through the dimension-5 operator responsible for the Majorana neutrino mass [42, 11]. Here $G_F = 1.1663787 \times 10^{-5}$ GeV$^{-2}$ denotes the Fermi coupling constant.

In the short-range part of the $0\nu\beta\beta$ diagrams [40] we consider the interaction to be point-like. The general Lorentz-invariant Lagrangian in terms of effective couplings [33] can be written as:

$$\mathcal{L}_9 = \frac{G_F^2}{2m_p} \left[ \epsilon_1 J J + \epsilon_2 J^\mu J_{\mu} + \epsilon_3 J^\mu J_{\mu} + \epsilon_4 J^\mu J_{\mu} + \epsilon_5 J^\mu J_{\mu} \right],$$

(3)

with the hadronic currents of defined chirality $J = \bar{u}(1 + \gamma_5)d$, $J^\mu = \bar{u}\gamma_\mu(1 + \gamma_5)d$, $J^\mu_\pi = \bar{u}\frac{1}{2}[\gamma^\mu, \gamma^\nu](1 + \gamma_5)\gamma^\nu$). The leptonic currents $j = \bar{e}(1 + \gamma_5)e$, $j^\mu = \bar{e}\gamma^\mu(1 + \gamma_5)e$, and $\epsilon_{\beta} = \epsilon_{\beta}^L, \epsilon_{\beta}^R$, are defined as:

$$\{ \epsilon_1, \epsilon_2, \epsilon_3^{LLz(\nu)}, \epsilon_3^{LRz(\nu)}, \epsilon_4, \epsilon_5 \}. $$

These parameters have dependence on the chirality of the hadronic and the leptonic currents involved, with $xyz = L/R, L/L, L/R$. In the case of $\epsilon_3$, one can distinguish between different chiralities, thus we express them separately as $\epsilon_3^{LLz(\nu)}$ and $\epsilon_3^{LRz(\nu)}$.

The contribution of the light left-handed neutrino exchange and that of long-range diagrams to the $0\nu\beta\beta$ decay amplitude is proportional to the time-ordered product of two effective $\mathcal{L}_6$ Lagrangians [41],

$$T(\mathcal{L}_6^{(1)} \mathcal{L}_6^{(2)}) = \frac{G_F^2}{2} T \left[ j_{V-A}^1 J_{V-A}^1 j_{V-A}^1 J_{V-A}^1 + \epsilon_{\alpha} \beta \epsilon_{\alpha} \beta J_{\alpha}^1 J_{\alpha}^1 \right],$$

(4)

while the contribution of the short-range diagram is proportional to $\mathcal{L}_9$.

However, when calculating the $0\nu\beta\beta$ half-life it is necessary to identify the contributions corresponding to different hadronization prescriptions. In addition to these contributions that were also considered in Ref. [41], we also include the long range diagrams [40] that involve pion(s) exchange. These diagrams were considered before as contributing to the $0\nu\beta\beta$ decay rate, but in the context of $\mathcal{K}_p$ SUSY mechanism.

After hadronization, the extra terms in the Lagrangian require the knowledge of 23 individual NME [32, 41, 43, 44, 42, 39]. We can write the half-life in a factorized compact form

$$\left[ T_{1/2}^{0\nu} \right]^{-1} = g_A^4 \left[ \sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \text{Re} \left[ \sum_{i \neq j} \mathcal{E}_i \mathcal{E}_j \mathcal{M}_{ij} \right] \right].$$

(5)

Here, the $\mathcal{E}_i$ contain the neutrino physics parameters, with $\mathcal{E}_1 = \eta_{0\nu}$, representing the exchange of light left-handed neutrinos, $\mathcal{E}_{2-7} = \{ \epsilon_{V-A}^{V-A}, \epsilon_{V-A}^{V-A}, \epsilon_{\pi}^{\pi}^{\pi}, \epsilon_{\pi}^{\pi}^{\pi}, \epsilon_{\pi}^{\pi}^{\pi}, \epsilon_{\pi}^{\pi}^{\pi}, \eta_{0\nu} \}$ are the long-range
LNV parameters, and \( \mathcal{E}_{8-15} = \{ \varepsilon_1, \varepsilon_2, \varepsilon_3^{LLz}, \varepsilon_3^{LRz}, \varepsilon_4, \varepsilon_6, \eta_{1\tau}, \eta_{2\tau} \} \) denote the short-range LNV parameters at the quark level involved in the pion-exchange diagrams [40]. The rational of including the \( \eta_{\pi\nu} \) in the same class with the LNV entering the quark-level long range diagrams is that Ref. [34] indicate that \( \eta_{\pi\nu} \) is proportional to \( \varepsilon_T^{Rl} \). In the same vein, Ref. [45] indicates that \( \varepsilon_1 \) and \( \varepsilon_2 \) are proportional to a combination of \( \eta_{1\tau} \) and \( \eta_{2\tau} \). Therefore the \( \eta_{1\tau} \) and \( \eta_{2\tau} \) were included in the list LNV couplings associated with quark-level short-range diagrams. Denoted with \( M_2^c \) are combinations of NME [43, 44, 41, 39] and integrated PSF [38] (denoted as \( G_{01} - G_{09} \)). Our values of the PSF are taken from Ref [38]. In some cases the interference terms \( E_2 E_3 M_{ij} \) are small [46] and can be neglected, but not all of them. In Ref. [11] we analyzed a subset of terms contributing to the half-life formula, Eq. (1) originating from the left-right symmetric model. In that restrictive case we showed that one can disentangle different contributions to the \( 0\nu\beta\beta \) decay process using two-electron angular and energy distributions as well as half-lives of two selected isotopes. Obviously, this number of observables is not enough to extract all coupling appearing in the effective field theory Lagrangian. However, they can be used to constrain these couplings, thus adding to the information extracted from the Large Hadron Collider and other related experiments.

In the following, we present the individual NME that enter \( M_2^c \) and are needed to analyze the outcome of Eq. (5). These NME are written as products of two-body transition densities (TBTD) and two-body matrix elements (TBME), with summation over all the nucleon states. Their numerical values when calculated within the shell model approach are presented in Table 1 for the light left-handed Majorana neutrino exchange, in Table 2 for the long-range part, and in Table 3 for the short-range component. The general expressions for the NME are (see Refs. [28, 20, 11]):

\[
M_\alpha = \sum_{j_\rho p', j_n j_{n'}} \text{TBTD} \left( j_p j_p', j_n j_{n'}; J^z \right) \times \left\langle j_p j_p'; J^z \right| \tau_\nu \tau_\nu \tau_{1-2} O^{\phi, \phi, P, \theta}_{12} R \left| j_{n} j_{n'}; J^z \right\rangle.
\]

We group the operators that share similar structure into five families.

- Gamow-Teller operator: \( O^{\gamma}_{12} = \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 H_\gamma (r) \),
- Fermi operator: \( O^{\phi}_{12} = H_\phi (r) \),
- Tensor operator: \( O^{\theta}_{12} = [3(\tilde{\sigma}_1 \cdot \hat{r})(\tilde{\sigma}_2 \cdot \hat{r}) - \tilde{\sigma}_1 \cdot \tilde{\sigma}_2] H_\theta (r) \),
- P operator: \( O^P_{12} = (\tilde{\sigma}_1 - \tilde{\sigma}_2) H_P (r) \),
- R operator: \( O^R_{12} = \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 H_R (r) \).

Here, \( \gamma = GT, GT_\omega, GT_q, GT N, GT', GT'' \), \( \phi = F, F_\omega, F_q \), \( FN, F' \), and \( \theta = T, T_q, T', T'' \), \( T_{\pi\nu}, T_1, T_2 \). Detailed expression for all these NME are found in the Appendix of Ref. [40].

3. Results

The NME presented in this section (Eq. (6)) are calculated using shell model approaches. To take into account the two-nucleon short-range correlation (SRC) we multiply the relative wavefunctions by \( f(r) = 1 - ce^{-ar^2}(1 - br^2) \); in the CD-Bonn parametrization used in this work \( a = 1.52 \text{ fm}^{-2}, b = 1.88 \text{ fm}^{-2}, \) and \( c = 0.46 \text{ fm}^{-2} \) [47]. This method is described in greater detail in Refs. [14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 13]

Table 1 presents the \( M_{GT}, M_F, \) and \( M_T \) NME involved in the standard mass mechanism with left-handed currents of Eq. (5). For these NME, an optimal closure energy \( \langle \tilde{E} \rangle \) was used for each effective Hamiltonian [23]: \( \langle \tilde{E} \rangle \) = 0.5MeV for \(^{48}\text{Ca} \) [19] and the GXPF1A Hamiltonian.
[48], \( \langle E \rangle = 3.4 \text{MeV for } ^{76}\text{Ge} [26] \) and \(^{82}\text{Se} [23] \) calculated with the JUN45 Hamiltonian [49], and \( \langle E \rangle = 3.5 \text{MeV for } ^{130}\text{Te} [24] \) and \(^{136}\text{Xe} [17] \) calculated with the SVD Hamiltonian [50].

Table 1. NME values for the exchange of light left-handed Majorana neutrinos corresponding to the "mass mechanism".

|       | \(^{48}\text{Ca}\) | \(^{76}\text{Ge}\) | \(^{82}\text{Se}\) | \(^{130}\text{Te}\) | \(^{136}\text{Xe}\) |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \(M_{GT}\) | 0.807               | 3.206               | 3.005               | 1.662               | 1.505               |
| \(M_{F}\)  | -0.233              | -0.674              | -0.632              | -0.438              | -0.400              |
| \(M_{T}\)  | 0.080               | 0.011               | 0.012               | -0.007              | -0.008              |

The long-range NME \(M_{\alpha}\) (with \(\alpha = GTq, Fq, Tq, GT\omega, F\omega, P, R, GT', F', T', GT'', T''\)) and \(M_{GT\pi\nu}\) and \(M_{T\pi\nu}\) that appear in Eq. (5) are presented in Table 2.

Table 2. NME for the long-range part of the 0\(\nu\beta\beta\) decay rate.

|       | \(^{48}\text{Ca}\) | \(^{76}\text{Ge}\) | \(^{82}\text{Se}\) | \(^{130}\text{Te}\) | \(^{136}\text{Xe}\) |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \(M_{GTq}\) | 0.709               | 3.228               | 3.034               | 1.587               | 1.440               |
| \(M_{GT\omega}\) | 0.930               | 3.501               | 3.287               | 1.855               | 1.682               |
| \(M_{GT'}\)  | 0.841               | 2.699               | 2.567               | 2.120               | 1.935               |
| \(M_{GT''}\)  | 3.581               | 11.982              | 11.490              | 12.210              | 11.202              |
| \(M_{GT\pi\nu}\) | 86.2                | 331.6               | 313.3               | 184.2               | 167.7               |
| \(M_{Fq}\)  | -0.121              | -0.383              | -0.362              | -0.249              | -0.230              |
| \(M_{F\omega}\) | -0.232              | -0.659              | -0.618              | -0.427              | -0.391              |
| \(M_{F'}\)  | -0.258              | -0.812              | -0.772              | -0.635              | -0.581              |
| \(M_{Tq}\)  | -0.173              | -0.059              | -0.058              | -0.013              | -0.012              |
| \(M_{T'}\)  | 0.337               | 0.015               | 0.025               | -0.077              | -0.085              |
| \(M_{T''}\)  | 2.231               | 0.028               | 0.118               | -0.773              | -0.861              |
| \(M_{T\pi\nu}\) | 21.3                | 7.3                 | 6.9                 | 1.2                 | 1.1                 |
| \(M_{R}\)  | 0.395               | -2.466              | -2.332              | -1.729              | -1.617              |
| \(M_{R}\)  | 1.014               | 3.284               | 3.127               | 2.562               | 2.341               |

Shown in Table 3 are the short-range NME \(M_{GTN}, M_{FN},\) and \(M_{\alpha}\) (with \(\alpha = GT1\pi, T1\pi, GT2\pi, T2\pi\)) that appear in Eq. (5).

Table 3. The short-range NME.

|       | \(^{48}\text{Ca}\) | \(^{76}\text{Ge}\) | \(^{82}\text{Se}\) | \(^{130}\text{Te}\) | \(^{136}\text{Xe}\) |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|
| \(M_{GTN}\) | 58.5                | 162.3               | 150.1               | 107.6               | 96.6                |
| \(M_{GT1\pi}\) | -1.354              | -3.559              | -3.282              | -2.421              | -2.171              |
| \(M_{GT2\pi}\) | -0.676              | -1.983              | -1.854              | -1.257              | -1.136              |
| \(M_{FN}\)  | -22.9               | -62.6               | -58.1               | -41.0               | -36.9               |
| \(M_{T1\pi}\) | -0.590              | -0.010              | -0.027              | 0.106               | 0.115               |
| \(M_{T2\pi}\) | -0.227              | -0.010              | -0.015              | 0.038               | 0.040               |

4. Conclusions
In this paper, the shell model values for 23 individual nuclear matrix elements are presented. These NME could be used to disentangle the 15 lepton number violating parameters that may contribute to the 0\(\nu\beta\beta\) decay rate, and for obtaining the energy and angular distributions of the outgoing electrons. To our knowledge the \(M_{GT'}, M_{GT''}, M_{F'}, M_{T'},\) and \(M_{T''}\) NME were calculated for the first time using shell model techniques.
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