Unity fidelity multiple teleportation using partially entangled states

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Received 12 July 2009, in final form 15 October 2009
Published 23 November 2009
Online at stacks.iop.org/JPhysB/42/235504

Abstract

We show that the multiple teleportation protocol (MTP) given in reference (J Modławska and A Grudka 2008 Phys. Rev. Lett. 100 110503) is not restricted to the Knill–Laflamme–Milburn (KLM) framework. Rather, we show that MTP can be implemented using any teleportation scheme. We also present two new MTPs which, under certain situations, are more efficient than the original one, requiring half of the number of its teleportations to achieve at least the same probability of success (\(P_{\text{suc}}\)). One of the protocols, however, uses less entanglement than the others yielding, surprisingly, the greatest \(P_{\text{suc}}\).

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The importance of quantum teleportation [1] is widely recognized today. Not only does it enable the remote transmission of the state describing a quantum system to another one, without ever knowing the state, but it also allows the construction of a new way to perform quantum computation [2, 3]. In the previous and in many other applications of teleportation, it is desirable, if not crucial, that the teleported state arrives at its destination (Bob) exactly as it leaves the preparation station (Alice). In other words, we want a unity fidelity output state, which is always achieved if Alice and Bob share a maximally entangled state (MES) [1]. However, it might happen that our parties do not share a MES or, in addition, intermediate teleportations to other parties must be done before the state reaches Bob. This limitation can be overcome by distilling out of an ensemble of partially entangled states (PESs) maximally entangled ones [4]. But this approach requires a large number of PESs to succeed and is ineffective when just a few copies are available. Another way to achieve unity fidelity teleportation with limited resources is based on the probabilistic quantum teleportation (PQT) protocols of [5–7].

Recently, in an interesting work, Modławska and Grudka [8] presented yet another way of achieving probabilistically unity fidelity teleportation. Their strategy was developed in the framework of the KLM scheme [3] for linear optical teleportation. The main idea behind their approach was the recognition that multiple (successive) teleportations using the same PES increased the chances of getting a perfect teleported qubit. We can also see the ideas of [8], as generalized here, as a way to extend the usefulness of quantum relays [9] whenever non-MESs are at stake and entanglement concentration is not practical (only a few copies of entangled states are available).

In this contribution we show that the features of the MTP of [8] are not restricted to the KLM teleportation scheme. In order to show that we build in section 2 a similar protocol (protocol 1) without relying on the intricacies of the KLM scheme. Actually, we use the same language of the original Bennett et al proposal [1], which allows us to express \(P_{\text{suc}}\), the total probability of getting unity fidelity outcomes, as a function of the number of teleportations and of the shared entanglement between Alice and Bob. We then present two new protocols (protocols 2 and 3, see figure 1), both of which are more efficient than the previous one. An important feature of these protocols is that they give \(P_{\text{suc}} > 1/2\) for a huge class of PESs. This is particularly useful when we have a few copies of the qubit to be teleported, since after a few runs of the MTP the overall \(P_{\text{suc}} \rightarrow 1\). On top of that, protocol 2 possesses the same efficiency of the first one but needs only half the number of teleportations to achieve the same \(P_{\text{suc}}\).

We also show that this protocol is connected to the PQT of [5, 6]. Protocol 3, on the other hand, in addition to requiring just half the number of teleportations of protocol 1 also achieves the highest \(P_{\text{suc}}\). Actually, we show that for some set of PESs \(P_{\text{suc}} \approx 1\) after just a few teleportations within a
single run of the MTP. Moreover, and surprisingly, at each successive teleportation this last protocol requires less and less entanglement to properly work. In section 3 we compare the efficiencies of all the three protocols presented here with a different strategy to achieve unity fidelity teleportation based on entanglement swapping [10]. In particular, we compare our results with those obtained for multiple entanglement swapping as presented in [11]. We show that, under certain conditions, we can achieve a better performance using the protocols presented here.

2. Multiple teleportation protocols

Protocol 1. Let us assume that we have \( j = N \) PESs described by \( \{|\Phi_1^n\rangle\} = f_n|00\rangle + g_n|11\rangle \), with \( f_n = 1/\sqrt{1 + n^2} \) and \( g_n = n/\sqrt{1 + n^2} \). (See panel (a) of figure 1.) We assume that the first PES is shared between Alice and Bob while the remaining \( N - 1 \) are with Bob. Without loss of generality we set \( 0 < n_1 < 1 \) [6] and for this protocol also that \( n_j = n \), \( j = 1, \ldots, N \) [8], i.e same entanglement at each teleportation. We can also build a generalized Bell basis as follows:

\[
\begin{align*}
|\Psi_{m_1}\rangle &= fm_1|00\rangle + gm_1|11\rangle, \\
|\Psi_{m_2}\rangle &= fm_2|01\rangle + gm_2|10\rangle, \\
|\Psi_{m_3}\rangle &= gm_1|00\rangle - fm_1|11\rangle, \\
|\Psi_{m_4}\rangle &= gm_2|01\rangle - fm_2|10\rangle,
\end{align*}
\]

with \( m_1 \) being the original Bell basis and the choice for protocol 1. Alice wants to teleport the qubit \( |\phi^A\rangle = \alpha|0\rangle + \beta|1\rangle \) and at each step \( j \) a Bell measurement (BM) is implemented whose result is known to Bob (see figure 1). This information allows him to correct the final state applying the proper unitary operations conditioned on the results of each BM [1], i.e \( I \) if the BM yields \( |\Phi^+\rangle \), \( \sigma_x \) for \( |\Phi^-\rangle \), \( \sigma_z \) for \( |\Psi^-\rangle \) and \( \sigma_y \) for \( |\Psi^+\rangle \), where \( I \) is the identity and \( \sigma_{x,z} \) the standard Pauli matrices.

Before the first teleportation the state describing all qubits are \( |\Phi\rangle = |\phi^A\rangle \otimes \prod_{m=1}^{N} |\Phi_{m_i}\rangle \), which can be written as:

\[
|\Phi\rangle = |\Phi^+\rangle (f_1|f_1\alpha\rangle|g_1\beta\rangle + g_1|f_1\alpha\rangle|g_1\beta\rangle) + |\Phi^-\rangle (\sigma_z (f_1|f_1\alpha\rangle|g_1\beta\rangle + f_1|g_1\beta\rangle|f_1\alpha\rangle)) + |\Psi^-\rangle (\sigma_y (f_1|f_1\alpha\rangle|g_1\beta\rangle + |g_1\beta\rangle|f_1\alpha\rangle)) + |\Psi^+\rangle (\sigma_x (f_1|f_1\alpha\rangle|g_1\beta\rangle + |g_1\beta\rangle|f_1\alpha\rangle)) \].

Figure 1. Pictorial view of all MTPs after \( q \) teleportations. Note that all but the first PES is shared between Alice and Bob. All the others are at Bob’s. (a) Protocol 1: boxes denote standard BMs \( m = 1 \) and at each teleportation the quantum channel is the same state \( |\Phi_{m_i}\rangle \). (b) Protocol 2: boxes now denote GBMs with \( m = m < n_1 \) with the same state \( |\Phi_{m_i}\rangle \) at each stage. (c) Protocol 3: boxes denote standard BMs \( m = 1 \) but the quantum channel’s entanglement is successively reduced after the second teleportation according to the following rule: \( |\Phi_{m_i}\rangle \rightarrow |\Phi_{m_{i-1}}\rangle \).

Figure 2. For protocol 1: from bottom to top the curves represent \( P^{\text{ suc}}_q \) after \( q = 2, 4, 6, 8, 10 \) and 12 successive teleportations. For protocol 2: from bottom to top the curves show \( P^{\text{ suc}}_q \) after \( q = 1, 2, 3, 4, 5 \) and 6 successive teleportations. The dashed curve shows the optimal probability (1/2) using the PQT protocol. All quantities are dimensionless.

\[
\begin{align*}
\text{Total Probability of Success} & \quad \text{Total Probability of Success} \\
0.2 & \quad 0.4 \\
0.6 & \quad 0.8 \\
1 & \quad 0.2
\end{align*}
\]
the total probability of success after the $q$th teleportation, $P_{\text{succ}} = \sum_{j=1}^{n} P_{\text{succ}}^{(j)}$, as a function of $n$ (the greater $n$ the greater the entanglement). Note that here and in the remaining of this section $P_{\text{succ}}$ is given by the sum of the probabilities of all previous successful teleportations since the $N - 1$ PESs are with Bob. In section 3 we also study other scenarios, in particular the one in which Bob possesses just one PES. Looking at figure 2 we see that after each teleportation $P_{\text{succ}}$ increases at a lower rate. Also, after the 10th teleportation we are already close to the maximal value of $P_{\text{succ}}$ for whatever value of $n$. We should remark that we are considering successful only unity fidelity teleportations. That is why $P_{\text{succ}}$ does not tend to 1 as $n \to 1$. Indeed, no matter how close $n$ is to 1 we are always discarding the sequences of BMs where do not get a balanced set of measurements involving the Bell states $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$.

**Protocol 2.** Before we assume that one has $j = N$ PESs described by $|\Phi^+_n\rangle$, with $0 < n_j < 1$ and $n_j = n$, $j = 1, \ldots, N$. (see panel (b) of figure 1.) However, differently from protocol 1, we now assume $m_j = m = n$, any $j$. The state to be teleported is $|\phi^+_j\rangle = |\alpha \rangle [0] + |\beta \rangle [1]$ and at each step $j$ one implements a generalised Bell measurement (GBM) [5, 6]. A GBM is a projective measurement of two qubits onto one of the four generalised Bell states given above (see [12] for ways of implementing a GBM). The result of each GBM is known to Bob who uses this information to apply the right unitary operations on his qubit as described in the first protocol. The rest of the present protocol is nearly the same as before and is inspired by the PQT of [5, 6].

Before any teleportation the state describing all qubits can be written as $|\Psi\rangle = \left|\Phi^+_n\right\rangle \left|\Psi^+_m\right\rangle$, where $|\Phi^+_n\rangle$ and $|\Psi^+_m\rangle$ are the states described by $|\Phi^+_n\rangle$ and $|\Psi^+_m\rangle$, respectively. $|\Psi\rangle$ is a result of a GBM: (2) The GBM yields $|\Phi^+_n\rangle$ or $|\Psi^+_m\rangle$ we get $|\alpha_i, \beta_i\rangle \rightarrow n(\alpha_i, \beta_i)$; (3) if we measure $|\Psi\rangle$ the qubit goes to $(n\alpha_i, \beta_i)$. Therefore, when we have an equal number of $|\Phi^+_n\rangle$ and $|\Psi^+_m\rangle$, $m = n$, in a sequence of MBs we get unity fidelity. The $n^2\beta_j$ coming from the measurement of $|\Phi^+_n\rangle$ is compensated by $n^2\alpha_j$ coming from another GBM giving $|\Psi^+_m\rangle$. Note that the states $|\Phi^+_n\rangle$ and $|\Psi^+_m\rangle$ are ‘neutral’, giving an overall $n$ that can be ignored for the determination of the successful cases. For example, after the second teleportation we have two possible GBM outcomes where we have a unity fidelity teleportation, namely $|\Phi^+_n\rangle|\Psi^+_m\rangle$ and $|\Psi^+_m\rangle|\Phi^+_n\rangle$ with $P_{\text{succ}}^{(2)} = 2n^2/(1 + n^2)^2$. And after the third teleportation the successful cases are four: $|\Phi^+_n\rangle|\Phi^+_m\rangle|\Psi^+_n\rangle$, $|\Phi^+_n\rangle|\Psi^+_n\rangle|\Phi^+_m\rangle$, $|\Psi^+_m\rangle|\Phi^+_n\rangle|\Phi^+_m\rangle$ and $|\Psi^+_m\rangle|\Phi^+_m\rangle|\Phi^+_m\rangle$, with $P_{\text{succ}}^{(3)} = 4n^6/(1 + n^2)^6$. In general, after the $q$th teleportation we have

$$P_{\text{succ}} = B(q)n^{2q}/[(1 + n^2)^{2q}],$$

(2)

where $B(q)$ is the number of all possible combinations of $q$ GBMs where we have an equal number of $|\Phi^+_n\rangle$ and $|\Psi^+_m\rangle$, excluding, as we did in protocol 1, the cases where we already got an equal number of those two states in the previous teleportations. For the first six teleportations we have $B(1) = 2, B(2) = 2, B(3) = 4, B(4) = 10, B(5) = 28$ and $B(6) = 84$.

Noting that $A(2q)/2^{2q} = B(q)$ we immediately see that equations (1) and (2) are the same. However, in protocol 2, we just need half of the number of teleportations to achieve the same efficiency, which is a quite remarkable economy on entanglement resources. Also, the need for less teleportations reduces other possible errors introduced by imperfect projective measurements. Furthermore, this result connects the PQT of [5, 6] to protocol 1. This is true because two successive teleportations using that protocol are equivalent to one using protocol 2, the latter being an extension of the PQT.

**Protocol 3.** Like protocol 1, here we do not need GBMs. (see panel (c) of figure 1.) The projective measurements are made using the standard Bell basis, i.e. $m_j = 1$, any $j$. However, and differently from the previous protocols, we assume that at each teleportation the entanglement of the quantum channel is reduced according to the following rule: $n_j = n_{j-1}^2$, $j \geq 3$ with $n_1 = n_2 = n < 1$. In other words, the first two teleportations are done spending two entangled states $|\Phi^+_n\rangle$ and after that we start using less and less entanglement. The first two steps of this protocol are identical to the first two of protocol 1 yielding $P_{\text{succ}}^{(1)} = 0$ and $P_{\text{succ}}^{(2)} = 2n^2/(1 + n^2)^2$. After the second teleportation, the unsuccessful cases are described by the state $|\alpha \rangle [0] + n^2|\beta \rangle [1]$, if the BMs resulted in $|\Phi^+_n\rangle|\Phi^+_n\rangle$, or by the state $n^2|\alpha \rangle [0] + |\beta \rangle [1]$, if the two successive BMs yielded $|\Psi^+_n\rangle|\Psi^+_n\rangle$. Since in the third teleportation the entangled state spent is $|\Phi^+_n\rangle$, the previous teleported qubit changes to $(\alpha, \beta) \rightarrow (n\alpha, n\beta)$ if we measure $|\Phi^+_n\rangle$ or to $(\alpha, \beta) \rightarrow (n^2\alpha, \beta)$ if we get $|\Psi^+_n\rangle$. Hence, whenever we get the following sequences of BMs: $|\Phi^+_n\rangle|\Phi^+_n\rangle|\Psi^+_n\rangle$ or $|\Psi^+_n\rangle|\Psi^+_n\rangle|\Phi^+_n\rangle$, we achieve unity fidelity with $P_{\text{succ}}^{(3)} = 2n^2/[1 + n^2]^2/(1 + n^2)$. The unsuccessful cases are given by the following 16 cases: $|\Phi^+_n\rangle|\Phi^+_n\rangle|\Phi^+_n\rangle$ and $|\Psi^+_n\rangle|\Psi^+_n\rangle|\Phi^+_n\rangle$, with the unsuccessful teleported qubits being either $(n\alpha, \beta)$ or $(n\beta, \alpha)$, respectively.

It is now clear why we will use $|\Phi^+_n\rangle$ to implement the fourth teleportation. We are trying to catch up with $n^4$ that multiplies either $\alpha$ or $\beta$. And since the unsuccessful cases after this step will turn to have $n^6$ multiplying either $\alpha$ or $\beta$, we will need $|\Phi^+_n\rangle$ at the fifth teleportation to catch up with it. In general, after the $(q - 1)$th teleportation the unsuccessful cases are those where we got the following sequences of $q - 1$ BMs: $|\Phi^+_n\rangle|\Phi^+_n\rangle$ or $|\Psi^+_n\rangle|\Psi^+_n\rangle$, giving a total of $2 \times 2^{q-1}$ cases with unsuccessful (not normalized) teleported qubits described by $|\alpha \rangle [0] + n^{2q-1}\beta [1]$ or $n^{2q-3}|\alpha \rangle [0] + |\beta \rangle [1]$, respectively. At the $q$th teleportation we succeed if we have either $|\Phi^+_n\rangle$ or $|\Psi^+_n\rangle$ with $P_{\text{succ}}^{(q)} = 2n^{2q}/[1 + n^2]^{2q}$.
teleportations, it uses much less entanglement to achieve those.

This changes, in the above expression for $P_{\text{suc}}$, the term $C_{nj}/2$ to $C_{nj}/C_{nj}/4$. In figure 3 we plot the total probability of success after $q$ teleportations, $P_{\text{suc}} = \sum_{j=1}^{q} P_{\text{suc}}^{(j)}$, as a function of $n$. Comparing figure 2 with figure 3 we see that protocol 3 is far better than the previous two by any aspect we might consider. First, it achieves the greatest $P_{\text{suc}}$. The difference comes from the factor multiplying the product of concurrences. Here, this factor is 2, for the other protocols, they are $A(q)/2^q$ and $B(q)$. In protocol 2 we must also consider the concurrences of the GBM. This changes, in the above expression for $P_{\text{suc}}^{(q)}$, the term $C_{nj}/2$ to $C_{nj}/C_{nj}/4$.

There is another property which is also existent in the previous two protocols. Looking at $P_{\text{suc}}$ as a function of the number of teleportations we see that it achieves its maximal value after a small number of steps. This is more evident the lower the entanglement of the quantum channel. Looking at figure 3 we see that for $n < 0.6$ just three teleportations are enough to achieve the maximal $P_{\text{suc}}$. And for higher values of $n$, a few more give the same feature. This is a practical property of MTP for we do not need to implement a prohibitively large number of teleportations to get the optimal value of $P_{\text{suc}}$. One last remark: we can also look at protocol 3 as a way to correct errors in previous teleportations. If it is discovered that in a previous step of the protocol an error changed the entangled state used in the teleportation process we can correct it by properly choosing the right entangled state for the next teleportation.

3. Comparison with multiple entanglement swapping

So far we have considered a ‘direct approach’ to teleport a qubit using PESs. By direct we mean that we use the PESs as they are offered to us, without any pre-processing. We have also assumed that Bob has access to $N - 1$ PESs out of a total of $N$. But we can change this scenario in at least two ways. On the one hand, we can impose that Bob has access to only one PES. The other $N - 2$ states lie between Alice and Bob. See the bottom of figure 4. On the other hand, we can first try to extract a maximally entangled state out of those $N$ PESs and only then implement the usual, single-shot, teleportation protocol. See the top-left of figure 4, for example. Our goal in this section is to compare the efficiencies (probabilities of success) for the present direct protocols with the ones achieved using the multiple entanglement swapping protocol (‘swapping approach’) of [11], whose goal is to obtain out of $N$ PESs linking Alice and Bob (bottom of figure 4, for example) one maximally entangled state (a Bell state). In this
\(P_{\text{swap}} = 1 - \left( f_n^2 - g_n^2 \right) \sum_{j=0}^{[N/2]} f_n^{2j} g_n^{2j} \binom{2j}{j}, \)

with \( [N/2] \) denoting the integer part of \( N/2 \), \( \binom{2j}{j} \) meaning the binomial coefficient, \( f_n = 1/\sqrt{1 + n^2} \), and \( g_n = n/\sqrt{1 + n^2} \).

In our first analysis we consider for protocols 2 and 3 that the \( N - 1 \) PESs are with Bob. For protocol 2 they all have the same entanglement while for protocol 3 the entanglement decreases as explained in the previous section. (See top-right of figure 4.) Note that for protocol 2 we have generalized Bell measurements. For the swapping approach, we consider the configuration given at the bottom of figure 4. The results for this scenario are illustrated in figure 5, where we plot the probabilities of success for \( N = 6 \) PESs. Note that in this situation, protocols 2 and 3 are superior for almost all the range of the parameter \( n \).

We now compare the swapping approach as given by configuration (1) of figure 4, the optimal way for a swapping-based protocol, with protocol 3 as given by configuration (2) of figure 4. Since we have three chances (three pairs of PESs) for succeeding, we get for the swapping protocol \( P_{\text{suc}} = S_1 + (1 - S_1)S_2 + (1 - S_1)(1 - S_2)S_3 \). Here \( S_j \), \( j = 1, 2, 3 \), gives the optimal probability to obtain a maximally entangled state out of two pairs of PESs. One can show that \([11] S_j = n_j^2/(1 + n_j^2) \), with \( n_1 = n, n_2 = n^4 \) and \( n_3 = n^{16} \).

Looking at figure 6 we see that in this case the swapping protocol is slightly superior for \( n \gtrsim 0.6 \) while for small \( n \) they both give the same efficiencies. We should also mention that if all the six pairs of PESs are shared between Alice and Bob, a complete different scenario from the ones depicted in figure 4, entanglement concentration/filtering techniques applied individually to all the six pairs [13] give a better performance. This is true because the optimal probability to locally concentrate a maximally entangled state from a non-maximally pure one is \( P_{\text{on}} = S_j [13] \). However, entanglement concentration can only be applied if Alice and Bob initially do share entangled states. In the majority of the situations studied here, though, Alice and Bob do not initially share any entangled state and we have no choice but to rely on the multiple teleportation or on the multiple swapping techniques.

We end this section comparing both approaches at the same configuration, namely configuration (4) of figure 4. For the direct approach we employ protocol 1. In this scenario \( P_{\text{suc}} \) for the swapping approach is given by equation D3 of [11], where we assume all PESs to be described by the state \( |\Phi^+\rangle \). For protocol 1 \( P_{\text{suc}} \) is calculated considering only those instances in which the qubit arrives with unity fidelity at its final destination. This always happens whenever the Bell measurements after the six teleportations yield a balanced number of \( |\Phi^+\rangle \) and \( |\Psi^\pm\rangle \). A simple numerical count gives 1280 possible ways that this can happen with the total probability being \( P_{\text{suc}} = 20n^6/(1 + n^4)^6 \). Figure 7 shows \( P_{\text{suc}} \) for both approaches when we have six PESs. It is interesting to note that for \( n < 0.557 \) the direct approach is the best choice. We have numerically checked that the lower the number of PESs the greater the value of \( n \) below which the direct approach is the best choice. For more than ten PESs the swapping protocol can be considered the best choice. Finally, we have
compared the efficiency of protocol 1 in configuration (4) of figure 4 against protocol 3 in configuration (3). We always obtained better results for protocol 1 in this case.

4. Conclusion

We have shown that the properties of the multiple teleportation protocol (MTP) are a general feature of successive teleportations, not being restricted to the Knill–Laflamme–Milburn (KLM) scheme. We have also connected one formulation of MTP to the probabilistic quantum teleportation (PQT), another approach that aims to achieve unity fidelity teleportation via partially entangled states (PESs). Moreover, we have presented two new MTPs that are more efficient than the original one. Indeed, in those two new MTPs we just need half the number of teleportations of the original MTP to achieve at least the same probability of success (unity fidelity teleportation). On top of that, we have shown that the protocol furnishing the highest probability of success (protocol 3) is the one requiring, surprisingly, the least amount of entanglement for its full implementation. On the one hand, this result may have important practical applications, since it is known that entanglement is a difficult resource to produce experimentally and, on the other hand, it suggests that whenever PESs are at stake, perhaps the best strategy to achieve a certain goal is not the one that uses the greatest amount of entanglement. Finally, we have compared the three MTPs here developed with the multiple entanglement swapping approach developed in [11]. We have checked that either one or the other approach furnished a better performance, depending on the amount of entanglement available and on the way the PESs are distributed between Alice and Bob.

Acknowledgment

The author thanks the Brazilian agency Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) for funding this research.

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