Reliability Analysis of Supply Chain System with the Multi-Suppliers and Single Demander

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Abstract The time-dependent solution of a kind of supply chain system with the multi-suppliers and single demander is investigated in this paper. By choosing state space and defining operator of system, we transfer model into an abstract Cauchy problem. We are devoted to studying the unique existence of the system solution and its exponential stability by using the theory of $C_0$-semigroup. We prove that the system operator generates $C_0$-semigroup by the theory of cofinal operator and resolvent positive operator. We derive that the system has a unique nonnegative dynamic solution exponentially converging to its steady-state one which is the eigenfunction corresponding eigenvalue 0 of the system operator.

Keywords supply chain; resolvent positive operator; cofinal operator; exponential stability

1 Introduction

Supply chain, as an extended enterprise, is a complex and dynamic system. It’s complexity and dynamism are further intensified by the global economic integrating and market internationalization. The performance of a supply chain is a result of the coordinated operations between companies in the supply chain. Any breakdown on the coordinated operations in the supply chain will largely affect the performance of a supply chain. Hence, effective supply chain management relies on the appraisal and maintenance of the reliability of a supply chain. How to analyze, evaluate, monitor, control and improve the supply chain reliability is gaining more and more attention recently.

Received May 15, 2018, accepted November 8, 2018
Supported by Premium Funding Project for Academic Human Resources Development in Beijing Union University (BPHR2020CZ06)
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Because of the widely application of the supply chain, many researchers paid attention to this issues. Voudouris presented a mathematical programming approach to improve the efficiency and responsiveness of the supply chain of a fine chemical industry\cite{1}. Beamon established a multi-stage supply chain model\cite{2}. Thomas first proposed reliability engineering to supply chain and defined the reliability of supply chain\cite{3}. Wang and Zhang, from the reliability analysis of the single-stage supply chain, conducted the reliability of the multi-stage supply chain based on the Markov process\cite{4}. Chen, Wang and Han presented a reliability analysis of chain model and mesh model of supply chain\cite{5}. In the same year, to simplify the calculation, Mu and Du presented the concept of basic unit of supply chain based on different stages of the supply chain\cite{6}. Zeng and Li gave a quantitative study on the reliability of the basic structure of the supply chain by AHP\cite{7}. Qi and Wang established the overall reliability of the supply chain and a mathematical model between its union scale of soft and selection methods enter prise\cite{8}. Cai and Zeng studied the quantitative analysis of the reliability of the supply chain issues by GO algorithm\cite{9}. Liu introduced G system into the supply chain system, established a corresponding supply chain reliability model, and got some important reliability indices\cite{10}.

However, in these literatures, the authors either did not take into account the impact of random factors, or assumed that the random factors followed exponentially distribution. Obviously, a lot is not the case. In addition, in practical applications, manufacturers always reduce the risk by contracting with several suppliers. Therefore, it is meaningful to research the supply chain of multi-suppliers and single demander. This paper make a exponential stability study of the model of supply chain system with multi-suppliers and the single demander.

In many previous literatures on reliability study, see\cite{11–15}, almost all results are obtained by the method of Laplace and Laplace-Stieltjes transforms of semi-Markov process based on two hypotheses: 1) The system concerned has a unique nonnegative time-dependent solution; 2) The solution is asymptotic stability. Both the hypotheses evidently hold if the repair time follows exponential distribution. However, whether they hold or not if the repair time follows arbitrary distribution is still an open question and should be justified. Moreover, in traditional reliability research, it is well known that the dynamic solution of the system is difficult or even impossible to be obtained. Therefore it is ordinary to substitute the steady-state solution for the dynamic one approximately. Especially due to the importance of the steady-state availability, it is usual to replace the instantaneous one with the steady-state one. However, the replacement is not true in general (see \cite{16, 17}).

It is easy to know that when the repair time follows exponential distribution, the availability is monotonously decreasing, thereby the steady-state one can substitute for the instantaneous one, which is accepted in reliability research but not be certified. While if the repair time follows arbitrary distribution, the replacement does not always hold unless a safety factor is considered. Therefore it is quite necessary to study the unique existence and the expression of the dynamic solution of the system to determine the safety factor at least. Obviously, it is indispensable to discuss the stability of the system likewise. It is important that the exponential stability ensures the system stability not be subject to some factors such as failure rate and repair rate. Hu discussed the exponential stability of a parallel repairable system with warm standby by analyzing the spectra distribution and the quasi-compactness of the system\cite{17}. Zhang dealt
with a cold standby repairable system with repairman extra work, and derived that dynamic solution of the system exponentially converges to its steady-state one by using the theory of resolvent positive operator\cite{18}.

This paper considers the model of supply chain system with multi-suppliers and the single demander. We will mainly discuss the unique existence of the time-dependent solution of the system as well as its exponential stability. We demonstrate that the system of interest has a unique nonnegative dynamic solution and it is exponentially stable by the theory of $C_0$-semigroup. This not only presents the expression of the system solution but also provides strictly theoretical foundation for stability research. We prove that the system operator generates $C_0$-semigroup by cofinal operator and resolvent positive operator. This provides a new method for the stability analysis of repairable systems.

The rest of this paper is organized as follows: Section 2 presents the system model with concerned notations and translates it into an abstract Cauchy problem; Section 3 discusses the unique existence of the system solution; Section 4 studies the exponential stability of the system; Section 5 concludes the paper.

2 System Formulation

The system of interest is a supply chain system with a number of the same type of independent parts suppliers and the single demander. The system transition diagram is shown in Figure 1.

![Figure 1](image)

Figure 1. The state transition diagram for the supply chain system

The following assumptions are associated with this system:

1) The system contains a number of the same type of independent parts suppliers and the single demander.

2) The supply time $X$ of the component suppliers follows exponential distribution $F(t)$, $F(t) = 1 - e^{-\lambda t}$, $t \geq 0$. The repair time $Y$ after supply break follows generally continuous distribution $G(t)$, $G(t) = \int_0^t g(x)dx = 1 - e^{-\int_0^t \mu(x)dx}$ and $\int_0^\infty xG(x)dx = \frac{1}{\mu}$.

3) At the initial time $t = 0$, there are $n$ suppliers who supply the units properly and there is one manager who manages the supply chain. If all the suppliers do their work properly, the manager is idle. Otherwise, (known as failure), the suppliers received the optimized adjustments (known as repairment) in order. If the manager is in a busy state, the supplier who works improperly waits the adjustments in line. After the adjustment, the adjusted suppliers run into their work immediately and do exactly as well as the properly ones. Once the number of the suppliers who fails is $k$, the demander works improperly (i.e., the system fail). At this period, the suppliers without failures work properly. In addition, suppose the time when the suppliers work properly is independent of the time spent in the optimized adjustment.

Define the states of the system:
i: \(i\)th state of the system, means there are \(i\) suppliers failed in the system, \(i = 0, 1, 2, \cdots, k\). And when \(i = k\), the system failed.

Then by probability analysis, the model of the system can be formulated as

\[
\left(\frac{d}{dt} + n\lambda\right)P_0(t) = \int_0^\infty P_1(t, x)\mu(x)dx,
\]

\[
\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n-1)\lambda + \mu(x)\right]P_i(t, x) = 0,
\]

\[
\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n-i)\lambda + \mu(x)\right]P_i(t, x) = (n-i+1)\lambda P_{i-1}(t, x), \quad i = 2, 3, \cdots, k-1,
\]

\[
\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x)\right]P_k(t, x) = (n-k+1)\lambda P_{k-1}(t, x).
\]

The boundary conditions are

\[
P_1(t, 0) = n\lambda P_0(t) + \int_0^\infty P_2(t, x)\mu(x)dx, \quad (5)
\]

\[
P_i(t, 0) = \int_0^\infty P_{i+1}(t, x)\mu(x)dx, \quad i = 2, 3, \cdots, k-1,
\]

\[
P_k(t, 0) = 0. \quad (7)
\]

The initial conditions are

\[
P_0(0) = 1, \text{ the others equal to } 0. \quad (8)
\]

Here \(P_0(t)\) represents the probability that the system is in state 0; \(P_i(t, x)dx\) represents the probability that the system is in state \(i\) with elapsed repair time lying in \([x, x+dx]\), \(i = 1, 2, \cdots, k\). \(\lambda\) represents constant failure rate of one supplier. \(\mu(x)\) represents repair rate with an elapsed repair time of \(x\). By the assumptions, \(\lambda_i = (n-i)\lambda, i = 0, 1, \cdots, k-1\).

Concerning the actual background, we can assume that:

\[
\mu(x) \geq 0; \quad \sup_{x \in [0, \infty)} \mu(x) < \infty;
\]

and for any \(0 < T < \infty\),

\[
\int_0^T \mu(x)dx < \infty, \quad \int_0^\infty \mu(x)dx = \infty.
\]

Furthermore, we know that many repairs are done periodically in practice. For example, the system works in the daytime and is maintained at night. So we can assume that the mean rate of repair exists and is not equal to 0 (see [17,18]). That is

\[
0 < \lim_{x \to \infty} \frac{1}{x} \int_0^x \mu(\tau)d\tau = \hat{\mu} < \infty. \quad (9)
\]

We translate the system (1)~(8) into an abstract Cauchy problem in a Banach space. First, let the state space \(X\) be

\[
X = \left\{ P \in \mathbb{R} \times (L^1_{[0, \infty)})^k \left| ||P|| = |P_0| + \sum_{i=1}^k ||P_i(x)||_{L^1_{[0, \infty)}} \right. \right\}.
\]
Obviously, \((X, ||\cdot||)\) is a Banach Lattice. Next, we will define some operators in \(X\) for simplicity.

\[
A = \text{diag} \left( -n\lambda, -\frac{d}{dx} (n - 1)\lambda - \mu(x), \cdots, -\frac{d}{dx} (n - k + 1)\lambda - \mu(x), -\frac{d}{dx} - \mu(x) \right),
\]

\[
D(A) = \left\{ P \in X \left| \begin{array}{c}
\frac{dP(x)}{dx} \in L^1_{[0,\infty)}, P_i(x) \text{ are absolutely continuous functions,} \\
(i = 1, 2, \cdots, k) \text{ satifying } P_1(0) = n\lambda P_0 + \int_0^\infty P_2(x)\mu(x)dx, \\
P_1(0) = \int_0^\infty P_{i+1}(x)\mu(x)dx, (i = 2, 3, \cdots, k - 1), P_k(0) = 0
\end{array} \right. \right\}
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 \\
0 & (n - 1)\lambda & 0 & \cdots & 0 & 0 \\
0 & 0 & (n - 2)\lambda & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & (n - k + 1)\lambda & 0
\end{bmatrix}_{k+1},
\]

\[
E = \begin{bmatrix}
0 & \int_0^\infty \mu(x) dx & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}_{k+1}
\]

Then the equations (1)~(8) can be written as an abstract Cauchy problem in the Banach space \(X\):

\[
\begin{aligned}
\frac{dP(t, \cdot)}{dt} &= (A + B + E)P(t, \cdot), \quad t \geq 0, \\
P(t, \cdot) &= (P_0(t), P_1(x, t), \cdots, P_k(x, t)), \\
P(0, \cdot) &= P_0 = (1, 0, \cdots, 0).
\end{aligned}
\]

### 3 Unique Existence of the System Solution

Since the system (1)~(8) is formulated as an abstract Cauchy problem, it is necessary to prove the well-posedness of the system. In this section, we will prove the unique existence of the system solution by cofinal operator and resolvent positive operator theory. This not only verifies the first hypothesis used in traditional reliability study mentioned in Section 1 but also presents the expression of the dynamic system solution.

**Definition 1** Let \(X\) be an ordered Banach space whose positive cone \(X_+ \) is generating and normal. An operator \(A\) on \(X\) is called resolvent positive if there exists \(w \in R\) such that \((w, \infty) \subset \rho(A)\) (the resolvent set of \(A\)) and \(R(\lambda, A) := (\lambda I - A)^{-1} \geq 0\) for all \(\lambda > w\).

**Definition 2** A subset \(X_0\) is called cofinal in \(X_+\) if for every \(f \in X_+\) there exists \(g \in X_0\) such that \(f \leq g\).

We note \(s(A) = \inf\{w \in R : (w, \infty) \subset \rho(A)\} \text{ and } R(\lambda, A) \geq 0 \text{ for all } \lambda > w\}. \text{ If } (T(t))_{t \geq 0} \text{ is a positive } C_0\text{-semigroup with generator } A, \text{ then the growth bound } w(A) \text{ is defined by } w(A) =
Thus, there exists constant $c$, such that $P_i = 0$ in $[0, c_i]$, $i = 1, 2, \cdots, k$. According to corollary 2.30 in [20], we know that $L$ is dense in $X$. So we only need to prove that $D(A + B + E)$ is dense in $L$.

For all $P \in L$, there exists $c_i(i = 1, 2, \cdots, k)$ such that $P_i = 0$, $x \in [0, c_i]$. Let $0 < 2s \leq \min\{c_i, i = 1, 2, \cdots, k\}$, $P_i(x) = 0$, $i = 1, 2, \cdots, k$, when $x \in [0, 2s]$. Let

\[
f^*(0) = (f_0^*, f_1^*(0), f_2^*(0), \cdots, f_k^*(0)) = \left(P_0, n\lambda P_0 + \int_0^\infty P_2(x)\mu(x)dx, \int_0^\infty P_3(x)\mu(x)dx, \cdots, \int_0^\infty P_k(x)\mu(x)dx, 0\right),
\]

\[
f^*(x) = (P_0, f_1^*(x), \cdots, f_k^*(x)).
\]

Here,

\[
f_i^*(x) = \begin{cases} f_i^*(0) \left(1 - \frac{x}{s}\right)^2, & x \in [0, s], \\ -u_i(x-s)^2(x-2s)^2, & x \in (s, 2s], \\ P_i(x), & x \in (2s, \infty), \end{cases}
\]

\[
u_i = \frac{f_i^*(0) \int_s^\infty (1 - \frac{x}{s})^2dx}{\int_s^{2s} (x-s)^2(x-2s)^2dx}, \quad i = 1, 2, \cdots, k.
\]

Obviously, such defined $f^*(x) \in D(A + B + E)$. Moreover

\[
||P - f^*|| = \sum_{i=1}^k \int_0^\infty |P_i(x) - f_i^*(x)|dx \rightarrow 0, \quad s \rightarrow 0.
\]

Thus, $D(A + B + E)$ is dense in $L$.

**Lemma 2** The system operator $A + B + E$ is a resolvent positive operator.

**Proof** First, we prove that $A$ is a resolvent positive operator. For any $G \in X$, consider the operator equation $(\gamma I - A)P = G$. That is

\[
(\gamma + n\lambda)P_0 = g_0,
\]

\[
\frac{dP_i(x)}{dx} + [\gamma + (n - i)\lambda + \mu(x)]P_i(x) = g_i(x), \quad i = 1, 2, \cdots, k - 1,
\]

\[
\frac{dP_k(x)}{dx} + [\gamma + \mu(x)]P_k(x) = g_k(x).
\]
Solving the equations (11)∼(13) yields

\[
P_0 = \frac{g_0}{\gamma + n\lambda},
\]

\[
P_i(x) = P_i(0)e^{-\int_0^x \gamma + (n-i)\lambda + \mu(\xi)dx} = \int_0^x g_i(\tau)e^{-\int_\tau^x \gamma + (n-i)\lambda + \mu(\xi)dx}d\tau,
\]

\[i = 1, 2, \ldots, k-1,
\]

\[
P_k(x) = \int_0^x g_k(\tau)e^{-\int_\tau^x \gamma + \mu(\xi)dx}d\tau.
\]

That is,

\[
P_0 = \frac{g_0}{\gamma + n\lambda},
\]

\[
P_i(x) = \left[ n\lambda P_0 + \int_0^\infty P_2(x)\mu(x)dx \right] e^{-\int_0^x \gamma + (n-i)\lambda + \mu(\xi)dx} + \int_0^x g_i(\tau)e^{-\int_\tau^x \gamma + (n-i)\lambda + \mu(\xi)dx}d\tau,
\]

\[i = 2, 3, \ldots, k-1,
\]

\[
P_k(x) = \frac{1}{k!} \left( \frac{\partial}{\partial x} \right)^{k-1} \left[ \frac{1}{\gamma + n\lambda} \right] + \int_0^x g_k(\tau)e^{-\int_\tau^x \gamma + \mu(\xi)dx}d\tau.
\]

We can derive that \((\gamma I - A)^{-1}\) exists for \(Re\gamma > 0\). When \(G\) is nonnegative, \(P = (\gamma I - A)^{-1} G\) is also nonnegative. This implies that \((\gamma I - A)^{-1}\) is a positive operator, that is \(A\) is a resolvent positive operator. From the expression of \(B\) and \(E\), it is not difficult to know that \(B + E\) is positive, and \(\|B + E\| \leq M\). This manifests that \(A + B + E\) is a resolvent positive operator.

**Lemma 3** Let \((A + B + E)^*\) be the adjoint operator of \(A + B + E\), then for any \(Q \in X^*\)

\[
(A + B + E)^* Q = \begin{pmatrix}
-n\lambda Q_0 + n\lambda Q_1(0) \\
\frac{d}{dx} - (n - 1)\lambda - \mu(x) \\
\frac{d}{dx} - (n - i)\lambda - \mu(x) \\
\frac{d}{dx} - \mu(x)
\end{pmatrix}
\begin{pmatrix}
Q_1(x) + \mu(x)Q_0 + (n - 1)n\lambda Q_2(x) \\
Q_i(x) + \mu(x)Q_{i-1}(0) + (n - i)n\lambda Q_i(x),
\end{pmatrix}
\]

\[i = 2, 3, \ldots, k-1,
\]

\[
D((A + B + E)^*) = \left\{ Q \in X^* \left| \frac{dQ_i(x)}{dx} \in L^\infty(R_+), Q_i(x) \in L^\infty(R_+), Q_i(x) \text{ are absolutely continuous, } i = 1, 2, \ldots, k \right. \right\}.
\]
Proof For any \( P \in D(A + B + E) \) and \( Q \in X^* \), we have

\[
\langle (A + B + E)P, Q \rangle
\]
\[
= \left[ -n\lambda P_0 + \int_0^\infty \mu(x)P_1(x)dx \right] Q_0 - \int_0^\infty \left[ \frac{d}{dx} + (n-1)\lambda + \mu(x) \right] P_1(x)Q_1(x)dx \]
\[
- \sum_{i=2}^{k-1} \int_0^\infty \left[ \frac{d}{dx} + (n-i)\lambda + \mu(x) \right] P_i(x)Q_i(x)dx + \sum_{i=2}^{k-1} (n-i+1)\lambda \int_0^\infty P_{i-1}(x)Q_i(x)dx \]
\[
- \int_0^\infty \left[ \frac{d}{dx} + \mu(x) \right] P_k(x)Q_k(x)dx + (n-k+1)\lambda \int_0^\infty P_{k-1}(x)Q_k(x)dx \]
\[
= -n\lambda Q_0 P_0 + \int_0^\infty Q_0 \mu(x)P_1(x)dx + n\lambda P_0 Q_1(0) + \int_0^\infty P_2(x)\mu(x)Q_1(0)dx \]
\[
+ \sum_{i=1}^{k-1} \int_0^\infty \left[ \frac{d}{dx} + (n-i)\lambda + \mu(x) \right] P_i(x)Q_i(x)dx + \sum_{i=2}^{k-1} \int_0^\infty P_{i+1}(x)\mu(x)Q_i(0)dx \]
\[
+ \int_0^\infty \left[ \frac{d}{dx} - \mu(x) \right] P_k(x)Q_k(x)dx + \sum_{i=2}^{k-1} (n-i+1)\lambda \int_0^\infty P_{i-1}(x)Q_i(x)dx \]
\[
= [-n\lambda Q_0 + n\lambda Q_1(0)] P_0 \]
\[
+ \int_0^\infty \left[ \left( \frac{d}{dx} - (n-1)\lambda - \mu(x) \right) Q_1(x) + \mu(x)Q_0 + (n-1)\lambda Q_2(x) \right] P_1(x)dx \]
\[
+ \sum_{i=2}^{k-1} \int_0^\infty \left[ \left( \frac{d}{dx} - (n-i)\lambda - \mu(x) \right) Q_i(x) + \mu(x)Q_{i-1}(0) + (n-i)\lambda Q_i(x) \right] P_i(x)dx \]
\[
+ \int_0^\infty \left[ \left( \frac{d}{dx} - \mu(x) \right) Q_k(x) + \mu(x)Q_{k-1}(0) \right] P_k(x)dx \]
\[
= \langle (A + B + E)^* Q \rangle.
\]

**Theorem 2** The system operator \( A + B + E \) generates a positive \( C_0 \)-semigroup \( T(t) \).

Proof Let \( X^* \) be the adjoint space of \( X \) whose positive cone is \( X^+_+ = X^* \cap \{ Q = (Q_0, Q_1, \cdots, Q_k) | Q_i \geq 0, i = 0, 1, \cdots, k \} \). From Lemma 3, we have \( D((A + B + E)^*)_+ = X^+_+ \cap D((A + B + E)^*) \). For any \( F = (F_0, F_1, \cdots, F_k) \in X^+_+ \), \( \| F \| = \sup \{ \| F_0 \|, \| F_i \|_{L^\infty(R_+), i = 1, 2, \cdots, k} \} \). Since \( 1(x) \in L^\infty(R_+) \) is absolutely continuous and \( \frac{d}{dx} 1(x) \in L^\infty(R_+) \), then \( G = (\| F \|, \| F \|_1(x), \cdots, \| F \|_1(x)) \in D((A + B + E)^*)_+ \), and \( G \geq F \). This implies that \( D((A + B + E)^*)_+ \) is cofinal in \( X^+_+ \). Theorem 1 implies that \( A + B + E \) generates a positive \( C_0 \)-semigroup \( T(t) \).

**Theorem 3** The system (10) has a unique nonnegative time-dependent solution \( P(t, \cdot) \) which satisfies

\[
\| P(t, \cdot) \| = 1, \quad \forall \ t \in [0, \infty).
\]

Proof From Theorem 2 and [21] we can derive that the system (10) has a unique nonnegative solution \( P(t, \cdot) \) and it can be expressed as

\[
P(t, \cdot) = T(t)P_0, \quad \forall t \in [0, \infty). \]
Because $P(t, \cdot)$ satisfies the equations (1)∼(8), it is not difficult to know that
\[
\frac{d \| P(t, \cdot) \|}{dt} = 0.
\]

Therefore
\[
\| P(t, \cdot) \| = \| T(t)P_0 \| = \| P_0 \| = 1, \quad \forall t \in [0, \infty).
\]

This just reflects the physical meaning of $P(t, \cdot)$.

Note Because $P_0 \notin D(A)$, the solution we obtained in (21) is the generalized solution. However, it is not difficult to prove that it is just the classical one when $t > 0$ with pure analysis method.

4 Exponential Stability of the System

In this section, we study the exponential stability of the system by cofinal operator and resolvent positive operator theory. This proves the second hypothesis used in traditional reliability study mentioned in Section 1.

First, we discuss the spectra distribution of the system operator $A + B + E$. By the same method in [16–18] we can have the following two lemmas:

**Lemma 4** \{ $\gamma \in \mathbb{C}$ | $\Re \gamma > 0$ or $\gamma = ia$, $a \in \mathbb{R}\{0\}$ \} belongs to the resolvent set of the system operator $A + B + E$.

**Lemma 5** $0$ is an eigenvalue of the system operator $A + B + E$ with algebraic multiplicity one.

From Section 3 together with Theorem 1, we can deduce that $A$ is the generator of a positive $C_0$-semigroup $S(t)$. Moreover, $s(A) = \omega(A)$. For $\Re \gamma > -\hat{\mu}$, $\gamma \in \rho(A)$. Thus $s(A) \leq -\hat{\mu}$, $\omega(A) = s(A) \leq -\hat{\mu}$. We have the following theorem.

**Theorem 4** For any $0 < \epsilon < \hat{\mu}$, there exists $C \geq 1$, such that $\| S(t) \| \leq Ce^{-\epsilon t}$, $t \geq 0$.

**Theorem 5** For $\epsilon > 0$, there are finite isolated eigenvalues of $A + B + E$ in $D = \{ \gamma \in \mathbb{C} | -\hat{\mu} + \epsilon \leq \Re \gamma < 0 \}$ with algebraic multiplicity finite.

**Proof** Because $B$ is a compact operator, make use of perturbation theorem of semigroup, the result is immediate.

**Theorem 6** The time-dependent solution of the system (1)∼(8) strongly converges to its steady state solution. That is
\[
\lim_{t \to \infty} P(t, \cdot) = \hat{P}.
\]

Furthermore
\[
\| P(t, \cdot) - \hat{P} \| \leq Ce^{-\epsilon t},
\]
where $\hat{P}$ is the eigenfunction corresponding to eigenvalue 0 of the system operator $A + B + E$ satisfying $\| \hat{P} \| = 1$.

**Proof** Recalling Theorem 2.10 (see [22]) together with Theorem 4 and Theorem 5, we can deduce that $C_0$-semigroup $T(t)$ generated by the system operator $A + B + E$ can be decomposed as $T(t) = T_0 + R(t)$ where $T_0$ is the residue corresponding to the eigenvalue 0 and $\| R(t) \| \leq Ce^{-\epsilon t}$ for suitable constant $\epsilon > 0$, $C > 0$. 
However, according to Theorem 3, the nonnegative solution of the system (1)–(8) can be expressed as $P(t, \cdot) = T(t)P_0$, $t \in [0, \infty)$. Then in the light of Theorem 2.3 in [23], we can obtain

$$P(t, \cdot) = T(t)P_0 = ( \overline{P}_0 + R(t))P_0 = (P_0, Q^*)\hat{P} + R(t)P_0 = \hat{P} + R(t)P_0,$$

where $Q^* = (1, 1, \cdots, 1)$ is the eigenvector corresponding to eigenvalue 0 of $(A + B + E)^*$. Then the result is immediate.

5 Conclusion

There are several papers concerning the stability problem of repairable system (see [17, 18] and reference therein). However, none of these papers deal with this problem by the theory of cofinal and resolvent positive operator. In this paper, we considered a supply chain system with the multi-suppliers and single demander. For the importance of the system dynamic solution and its stability both in theory and in practice, we proved that the system has a unique nonnegative dynamic solution by using the theory of resolvent positive operator and presented its expression. Further, we discussed the exponential stability of the system by using the theory of cofinal operator and resolvent positive operator. Thereby, it is rational that the steady-state indices can substitute for the instantaneous ones by considering a safety factor. And the exponential stability ensures the system stability not be subject to some factors such as failure rate and repair rate. The same method can also be applied to other repairable system.

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