On the Minimal Messenger Model

Francesca M. Borzumati
Department of Particle Physics, Weizmann Institute, Rehovot, Israel

ABSTRACT

We study the Minimal Messenger Model, a minimal version of Gauge Mediated Supersymmetry Breaking models. Boundary conditions equal to zero for trilinear and bilinear soft parameters at the messenger scale make this model free from the supersymmetric CP problem and extremely predictive. These boundary conditions and the vicinity of the messenger scale to the electroweak one, requires a careful implementation of the mechanism of radiative breaking of $SU(2) \times U(1)$. We assess the importance of considering the complete one–loop effective potential and of including a set of logarithmic two–loop corrections to the $B$ parameter for the correct determination of the electroweak minimum. We analyze the resulting low–energy spectrum and give predictions of interest for future experimental searches.
1. Introduction

Models with low–energy breaking of supersymmetry, communicated by gauge interactions to the observable sector, have recently drawn considerable attention as interesting alternatives to models in which this communication is mediated by gravity [1]. The minimal realization of the latter at the electroweak scale is the well known Minimal Supersymmetric Standard Model (MSSM).

The former, the so–called gauge mediated supersymmetry breaking (GMSB) models have, indeed, several attractive features. Most important of these is the fact that, as gauge interactions are flavor blind, squark and slepton masses are universal. Given the low value of the supersymmetry breaking scale, this universality is hardly broken by the evolution of mass parameters to the electroweak scale through Renormalization Group Equations (RGE). (Universality means, in this context, that scalar masses are only functions of gauge quantum numbers, and that $A$–terms are small or proportional to fermion yukawa couplings.) Moreover, they can be more predictive than the MSSM, having a smaller number of free parameters, and may, at least in a minimal variant [2], provide a solution to the supersymmetric CP problem [3].

In this minimal version, dubbed the Minimal Messenger Model (MMM), trilinear and bilinear soft parameters vanish at the messenger scale $X$. Moreover, if $X$ is of $O(\Lambda)$, with $\Lambda$ the ratio of the messenger F-term over $X$, after the radiative breaking of the electroweak gauge symmetry is implemented, this model turns out to be practically a one–parameter model, $\Lambda$.

Although the MMM was already considered in [2,4,5], the mechanism of breaking of the electroweak gauge group was not always correctly implemented. Furthermore, not all experimental constraints on the model were always included. The aim of this paper is to present a comprehensive analysis addressing these issues. After a definition of the model in Sect. 2, we will impose the breaking of $SU(2) \times U(1)$ through minimization of the RGE improved tree–level Higgs potential (Sect. 3.1). In Sect. 3.2, we will demonstrate the importance of considering the full one–loop corrected effective potential for the determination of the electroweak minimum. We will also argue on the need to include a set of additional two–loop corrections to the parameter $B$ of same size than those induced by the one–loop effective potential. Finally, we will discuss the viability of the model for different values of $\Lambda$. In the last section, Sect. 4, we will verify which regions of $\Lambda$ survive the imposing of experimental bounds coming from direct searches of supersymmetric particles and the indirect constraint due to the measurement of $b \rightarrow s\gamma$ and list the main predictions of this model.

2. The model

The messenger sector of the MMM consists of only one pair of chiral superfields, $\Phi, \Phi$, which transforms as a vectorlike representation of the electroweak gauge group. In particular, having in mind an embedding of this model in an underlying Grand Unified Theory (GUT), $\Phi, \Phi$ are chosen to be in a 5, 5 representation of $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$. They couple at the tree–level to a singlet $S$ ($W = \lambda S \Phi \Phi$). For simplicity, we neglect the difference in the evolution of $\lambda$ for the components of the two 5–plets with different $SU(3)_C$ and $SU(2)_L$ quantum numbers.

The scalar component of $S$ acquires a vacuum expectation value (VEV) giving therefore a supersymmetric mass $\lambda <S>$ to fermion and scalar components of both messengers. The auxiliary component of $S$ acquires a VEV as well, and gives rise to a supersymmetry–violating mass term, $\lambda F_S$, which mixes the scalar components of $\Phi$ and $\Phi$. The information of supersymmetry breaking
is then transmitted to the visible sector via gauge interactions. A soft mass is generated for the \(i\)-th gaugino \((i = 1, 2, 3)\) at the one–loop level, with fermion and scalar components of \(\Phi\) and \(\bar{\Phi}\) exchanged as virtual particles. This is:

\[
M_i(\lambda < S>) = \frac{1}{4\pi} \alpha_i(\lambda < S>) \frac{F_S}{\lambda < S>^2} g(x); \quad x \equiv \frac{F_S}{\lambda < S>^2},
\]

(1)

where a GUT normalization for the \(U(1)_Y\) coupling was chosen: \(\alpha_1 = (5/3)\alpha_Y = (5/3)\alpha/\cos^2\theta_W\). If we indicate with \(\Lambda\) the ratio \(F_S/\lambda < S>\) and with \(X\) the supersymmetric mass \(\lambda < S>\), we can rewrite (1) in a more compact form as:

\[
M_i(X) = \bar{\alpha}_i(X) \Lambda g(x); \quad \bar{\alpha}_i(X) \equiv \frac{\alpha_i(X)}{(4\pi)}.
\]

(2)

A tilde on gauge and yukawa couplings indicates hereafter a division over 4\(\pi\). Natural values for \(x\) are of \(\mathcal{O}(1)\) (i.e. \((\lambda < S>)^2 \sim \lambda F_S\), but at \(x = 1\) massless scalar messengers appear in the theory. We choose in the following \(x = 1/2\) and we shall comment on modifications obtained in the limit \(x \to 1\). In the range \(0 - 1\) the function \(g(x)\), given in (3), is monotonically increasing and has values \(g(0) = 1, \ g(1) = 1.386\).

Communication of supersymmetry breaking is passed to the scalars of the observable sector at the two–loop level with scalar and fermion components of the messenger fields, gauge bosons and gauginos exchanged as virtual particles. The masses obtained are:

\[
m^2_i(X) = 2\Lambda^2 \left\{ C_3 \bar{\alpha}_3^2(X) + C_2 \bar{\alpha}_2^2(X) + \frac{3}{5} Y^2 \bar{\alpha}_1^2(X) \right\} f(x),
\]

(3)

where \(C_3 = 4/3, 0\) for triplets and singlets of \(SU(3)_C\), \(C_2 = 3/4, 0\) for doublets and singlets of \(SU(2)_L\); \(Y = Q - T_3\) is the hypercharge and \(i\) runs over all scalars present in the theory. The function \(f(x)\), derived in (4) and (5), is almost always a flat function equal to 1, except for a sharp drop at \(x = 1\), where it has the value 0.7.

Finally, trilinear and bilinear couplings in the soft scalar potential:

\[
V_{soft} \ni -(A_u h_u)^{ij} H_u \tilde{Q}_i \tilde{U}_j^c + (A_d h_d)^{ij} H_d \tilde{Q}_i \tilde{D}_j^c + (A_e h_e)^{ij} H_d \tilde{L}_i \tilde{E}_j^c - B_{\mu} H_d H_u
\]

(4)

(where \(\tilde{Q}, \tilde{U}^c, \tilde{D}^c\) etc., as well as \(H_d\) and \(H_u\) indicate here the scalar components of the corresponding superfields) vanish at the messenger scale:

\[
A_u^{ij}(X) = A_d^{ij}(X) = A_e^{ij}(X) = 0; \quad B(X) = 0.
\]

(5)

The last relation in (5) is the identifying property of this model. (For a discussion on how such a boundary condition can be theoretically implemented, see (6).) Phenomenologically, it renders the model very predictive; technically, it makes the search of the correct electroweak minimum rather complicated.

Low–scale \((M_Z)\) inputs of our analysis are: \(\alpha_3 = 0.120, \ \alpha_2 = 0.0335, \ \alpha_1 = 0.0168\), corresponding to \(\alpha^{-1} = 127.9\) and \(\sin^2\theta_W = 0.2316\), and loosely compatible with a gauge couplings unification (3). As running fermion masses we use \(m_t(M_Z) = 171\text{ GeV}, \ m_b(M_Z) = 3.0\text{ GeV}\) and \(m_{\tau}(M_Z) = 1.75\text{ GeV}\).

\*This relation, the explicit form of the superpotential \(W = h_u^{ij} H_u L_i E_j^c + h_d^{ij} H_d Q_i D_j^c - h_e^{ij} H_u Q_i U_j^c - m^2 H_u H_u\), and the multiplication rule \(H_u \bar{Q} \equiv \epsilon_{ij} H_d \bar{Q}_j; \ \epsilon_{12} = -\epsilon_{21} = 1\), define the sign–conventions used in this analysis.
3. Radiative Breaking of $SU(2)_L \times U(1)_Y$

3.1. RGE improved tree–level Higgs potential

We evolve all the initial parameters $(\overline{2})$, $(\overline{3})$, and $(\overline{3})$, which depend only on $\Lambda$, from $X (= 2\Lambda)$ to a decoupling scale $Q_0$. Henceforth, we shall refer to $X$ as to the “high–scale”, as compared to the “low–scale” $\sim M_Z$. The evolution is performed using the one–loop MSSM RGE. For reference, we report in Appendix A these equations and the two–loop equation for $B$, in the approximation of Kobayashi–Maskawa matrix $K \sim 1$. Those actually used for this analysis, with all intergenerational mixing terms needed for the calculation of $b \to s\gamma$, can be found in $(\overline{3})$.

Given the much more modest evolution of the masses of weakly interacting sparticles, and the fact that they are much lighter than the strongly interacting ones, we take $Q_0^2$ to be the geometrical mean of the high–scale values of $m_Q^2$ and $m_U^2$. These depend on the decoupling scale itself (see $(\overline{3})$) since the high–scale gauge couplings are to be obtained from our inputs at $M_Z$ through a running of the Standard Model (SM) RGE from $M_Z$ to $Q_0$ and the MSSM RGE from $Q_0$ to $X$. A simple iteration allows to find this scale rather quickly. For $\Lambda \sim 100$ TeV, $Q_0$ is typically $\sim 1$ TeV. All squark masses cluster around this value. Corrections for the inadequacy of this scale for the weak mass parameters involved in the breaking mechanism will be provided by the inclusion of the one–loop corrections to the scalar potential, leaving therefore an overall scale ambiguity of the next order. Unless a different value is explicitly mentioned, the choice of $Q_0$ specified above is that made throughout this paper. It will appear obvious later on why this is indeed a good choice.

For the high–scale yukawa couplings required as inputs of the MSSM RGE, we need the value of $\tan\beta$. This parameter, together with $\mu$, is obtained by imposing that the electroweak minimum is a minimum of the neutral higgses $H_u^0, H_d^0$ of the RGE improved tree–level potential

$$V_0(Q_0) = \mu_H^2 \left| H_d^0 \right|^2 + \mu_H^2 \left| H_u^0 \right|^2 - (B\mu) \left( H_d H_u^0 + h.c. \right) + \frac{(g^2 + g_Y^2)}{8} \left( \left| H_d^0 \right|^2 - \left| H_u^0 \right|^2 \right)^2 \quad (6)$$

( $\mu_H^2 \equiv m_{H_u}^2 + \mu^2$ ($i = u, d$) are equal to zero when it is $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$:

$$\left( m_{H_d}^2 + \mu^2 + \frac{1}{2} (g^2 + g_Y^2) (v_u^2 - v_d^2) \right) 2v_d - (B\mu) 2v_u = 0 \quad (7)$$

$$\left( m_{H_u}^2 + \mu^2 - \frac{1}{2} (g^2 + g_Y^2) (v_u^2 - v_d^2) \right) 2v_u - (B\mu) 2v_d = 0 \quad (8)$$

and that the obtained solution is a minimum. This minimization condition has to be imposed at the low–energy scale $M_Z$. Gauge couplings are indeed evolved down to $M_Z$, whereas, as said before, the evolution of mass parameters is stopped at $Q_0$: hence, the $Q_0$–dependence of the potential $V_0$. The equations $(\overline{3})$ are more often cast in the form:

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2; \quad \sin 2\beta = \frac{2B\mu}{m_{H_u}^2 + m_{H_d}^2 + 2\mu^2}. \quad (9)$$

Self–consistent solutions of this problem are obtained through a numerical iteration.

It is a priori not obvious that the previous equations can yield physically meaningful results, due to the little freedom which the model allows and the smallness of the logarithms resummed by the RGE, i.e. of the period of integration $t_{XQ_0} = 2 \ln (X/Q_0)$. The peculiar interplay between
strongly and weakly interacting sectors in this model, as we shall see, makes possible the breaking of the electroweak gauge group; maintains \( B \) small at low–energy, therefore inducing large values of \( \tan \beta \), and has consequences for the viability of the model itself.

The evolution of supersymmetric parameters for which yukawa couplings do not play any role is independent of the breaking mechanism. Low–scale gaugino masses are:

\[
M_i = M_i(X) Z_i = \Lambda \tilde{\alpha}_i(Q_0) g(x) ; \quad Z_i = \frac{\tilde{\alpha}_i(Q_0)}{\tilde{\alpha}_i(X)} = (1 - \tilde{\alpha}_i(Q_0) b_i t_{XQ_0}) ,
\]  

(10)

where it is: \( \tilde{\alpha}_i(Q_0) = \tilde{\alpha}_i/(1 - \tilde{\alpha}_i b_i^{SM} t_{Q_0Z}) \) and \( t_{Q_0Z} \equiv 2 \ln (Q_0/M_Z) \). (If no scale is specified, it is understood that the relevant variables are low–energy variables \( \sim M_Z \), with evolution frozen at \( Q_0 \) for the massive ones.) For \( \Lambda = 100 \, \text{TeV} \), the period of integration \( t_{XQ_0} \) is 10.6 to be compared with the value \( \sim 60 \) in the MSSM for the same decoupling mass and the high–scale \( X \) coinciding with \( M_{GUT} \sim 3 \times 10^{16} \). The values of the three gauge couplings at \( X \) and \( Q_0 \) as obtained for \( \Lambda = 100 \, \text{TeV} \), starting from our input values at \( M_Z \), are:

\[
\begin{align*}
\alpha_1(X) &= 0.0191 ; \quad \alpha_2(X) = 0.0331 ; \quad \alpha_3(X) = 0.0739 , \\
\alpha_1(Q_0) &= 0.0173 ; \quad \alpha_2(Q_0) = 0.0322 ; \quad \alpha_3(Q_0) = 0.0910 .
\end{align*}
\]  

(11)

The coefficients \( Z_i \) are then: 0.90, 0.97, 1.23 \((i = 1, 2, 3)\). 

Gaugino masses, evolution coefficients \( Z_i \) and ratios \( r_{ij} = M_i/M_j = \alpha_i(Q_0)/\alpha_j(Q_0) \) are shown in Fig. (1) as function of \( \Lambda \). Although we analyze this model for all \( \Lambda \)’s from 15 to 150 TeV, we show in this figure only the region from \( \Lambda \sim 62 \, \text{TeV} \), for reasons which will become clear after the inclusion of one–loop corrections to \( V_0 \). For lower \( \Lambda \)’s, the values of \( Z_i \) hardly deviate from those given above. Almost unchanged is also \( r_{12} \), whereas \( r_{32} \) reaches 3.3 at \( \Lambda \sim 20 \, \text{TeV} \). Again, the values of \( Z_i \) are to be compared to those obtained in the MSSM for the same scale \( Q_0 \): \( Z_i^{GU} = 0.44, 0.84, 2.35 \) \((i = 1, 2, 3)\). For each value of \( \Lambda \), the gaugino sector in the MMM can be identified with that in the MSSM for \( M = \Lambda \tilde{\alpha}_i g(x)/Z_i^{GU} \).
The first two generations of scalar masses, whose boundary conditions can be re–expressed in terms of low–scale gaugino masses \([3]\) as:

\[
m_i^2(X) = 2 \left\{ C_3 \left( \frac{M_3}{Z_3} \right)^2 + C_2 \left( \frac{M_2}{Z_2} \right)^2 + \frac{3}{5} Y^2 \left( \frac{M_1}{Z_1} \right)^2 \right\} \frac{f(x)}{(g(x))^2},
\]

(12)
evolve according to \([A^3]\) in Appendix A and get corrected by quantities \(\Delta m_i^2 \equiv m_i^2 - m_i^2(X)\):

\[
\Delta m_i^2 = 2 \left\{ C_3 \left( \frac{M_3}{Z_3} \right)^2 \left( \frac{1-Z_3^2}{b_3} \right) + C_2 \left( \frac{M_2}{Z_2} \right)^2 \left( \frac{1-Z_2^2}{b_2} \right) + \frac{3}{5} Y^2 \left( \frac{M_1}{Z_1} \right)^2 \left( \frac{1-Z_1^2}{b_1} \right) \right\}.
\]

(13)

For our choice of \(x\), it is \(f(x)/(g(x))^2 \sim 1\). The corrections \([13]\) amount to \(\sim 15\%\) in the squark sector and at most \(5\%\) in the case of sleptons, modestly enlarging the already wide gap existing between squark and slepton spectra. Dropping the indices of first and second generation, we have, for \(\Lambda = 100\text{ TeV}\):

\[
m_Q^2 \simeq 16.8 M_Z^2; \quad m_U^2 \simeq m_D^2 \simeq 15.4 M_Z^2; \quad m_L^2 \simeq 1.64 M_Z^2; \quad m_E^2 \simeq 0.40 M_Z^2. \quad (14)
\]

This large gap sharply distinguish the MMM from the MSSM, which in general predicts for the first two generations of squarks and sleptons the familiar relations:

\[
m_Q^2 \simeq m^2 + 6.4 M_Z^2; \quad m_U^2 \simeq m_D^2 \simeq m^2 + 5.9 M_Z^2; \quad m_L^2 \simeq m^2 + 0.68 M_Z^2; \quad m_E^2 \simeq m^2 + 0.22 M_Z^2.
\]

Therefore, whereas the gaugino sector in the two models can be identified through a specific choice of \(M\) for each value of \(\Lambda\), an identification of the scalar mass parameters would require very different values of \(m^2\) for squarks and sleptons (10.4\(M_Z^2\) and 0.96\(M_Z^2\) for the \(SU(2)\) doublets; 9.5\(M_Z^2\) and 0.18\(M_Z^2\) for the singlets).

The interplay between such a light “weak” sector and the heavy “strong” one is such to turn \(m_{H_u}^2\) to large negative values as in the MSSM, making therefore possible the radiative breaking of \(SU(2)_L \times U(1)_Y\). If, for the purpose of illustration, we keep only the first of the logarithms to be resummed when solving \([X^7]\), we get:

\[
m_{H_u}^2 = m_L^2(X) - 3\tilde{\alpha}_t(X) \left( m_Q^2(X) + m_U^2(X) + m_L^2(X) \right) t_{XQ_0}
\]

(15)

with \(\alpha_t(X) = h_t^2(X)/(4\pi)\) and \(h_t(X)\) of \(O(1)\). The factor \(\sim 6\) lost in \(t_{XQ_0}\) with respect to the MSSM is compensated in \([E]\) by the heaviness of the squark spectrum. A similar effect is observed in the evolution of \(m_{H_d}^2\).

Cancellations between weak and a higher–loop strong terms appear also in the determination of the low–energy value of \(B\). Starting from the boundary condition \([3]\), a value different from zero is generated at the one–loop level through gaugino mediated loops (see first diagram in Fig. \([2]\)). The leading contributions are resummed in a series \(\sum_n c_n^w(\tilde{\alpha}(X) t_{XQ_0})^n\) where \(\alpha(X)\) indicates generically \(\alpha_2(X)\) or \(\alpha_1(X)\). In a closed form, it is:

\[
B = -4 \left\{ C_2 \left( \frac{M_2}{Z_2} \right) \left( \frac{1-Z_2^2}{b_2} \right) + \frac{3}{5} \left( \frac{1}{4} \right) \left( \frac{M_1}{Z_1} \right) \left( \frac{1-Z_1^2}{b_1} \right) \right\}.
\]

(16)

Values of \(A_i\) different from zero are similarly obtained from one–loop diagrams as the second one in Fig. \([2]\). Leading logarithms arising from such diagrams, after resummation, give:

\[
A_i = -4 \left\{ C_3 \left( \frac{M_3}{Z_3} \right) \left( \frac{1-Z_3^2}{b_2} \right) + C_2 \left( \frac{M_2}{Z_2} \right) \left( \frac{1-Z_2^2}{b_2} \right) + \frac{3}{5} C_i^Y \left( \frac{M_1}{Z_1} \right) \left( \frac{1-Z_1^2}{b_1} \right) \right\}.
\]

(17)
The coefficients $C^Y_i$ are given in Appendix A and $C_3, C_2$ are equal to 4/3, 3/4 for $A_t$ and $A_b$, and
0, 3/4 for $A_\tau$. When only gaugino–mediated loops are considered, it is then, $A_t \sim A_b \sim -M_2$ and $B \sim A_\tau \sim -0.1M_2$, roughly for any $\Lambda \gtrsim 70$ TeV.

Thus, the leading contributions to $B$ proportional to yukawa couplings, i.e. “$A_t$–induced”, are first obtained at the two–loop level (see upper diagram in Fig. 3) and give rise to the series $\sum_n c_{n+1}^t \tilde{\alpha}_t(X) (\tilde{\alpha}_3(X))^n (t_{XQ_0})^{n+1}$. At the two–loop level are also obtained the leading logarithmic contributions to $A_i$ proportional to yukawa couplings (see lower diagrams in Fig. 3 for $A_t$). Nevertheless, while these diagrams produce indeed numerically small corrections to the values of $A_i$ previously obtained, the same is not true in the case of $B$, for which one has:

$$\frac{c_3^2}{c_1^w} \simeq -3 \left( \frac{16}{9} \right) \left( \frac{M_3}{M_2} \right) \left( \frac{Z_2}{Z_3} \right) \left( \frac{\tilde{\alpha}_3(X)}{\tilde{\alpha}_2(X)} \right) (\tilde{\alpha}_t(X) t_{XQ_0}) ; \quad \left| \frac{c_3^2}{c_1^w} \right| \gtrsim 1. \quad (18)$$

Large cancellations between weak and higher–loop–order strong terms contributing to $B$ take place. They induce a flip of sign for this parameter, which, then, turns out to be small and positive. The positivity of $B$ forces also $\mu$ to be positive; its smallness, relative to the heavy scalar spectrum, pushes $\tan \beta$ to large values.
At the electroweak minimum, after minimization of $V_0$, one obtains $\tan \beta = 46.4$ and the following values of Yukawa couplings at the three relevant scales $X$, $Q_0$, and $M_Z$:

\begin{align*}
  h_t(X) &= 0.8130; \quad h_t(Q_0) = 0.8755; \quad h_t(M_Z) = 0.9799, \\
  h_b(X) &= 0.6324; \quad h_b(Q_0) = 0.7045; \quad h_b(M_Z) = 0.7984. 
\end{align*}

The Higgs mass parameters $m_{H_u}^2$ and $m_{H_d}^2$ are: $m_{H_u}^2 \simeq -(1.50M_2)^2 \simeq -(0.53M_3)^2$, $m_{H_d}^2 \simeq -(0.97M_2)^2$ and $\mu$, given practically by $-m_{H_u}^2$ (the difference between the two numbers determining $M_Z$), is $\mu \simeq 1.48M_2$. The values of $A_t$ and $A_b$ do not deviate much from those obtained according to \cite{17}: $A_t \sim A_b = -0.9M_2$. $A_{\tau}$ is now $-0.055M_2$, and $B$ only $\sim 0.02M_2$.

The results for $\tan \beta - \mu$ obtained from the minimization of $V_0$, as a function of $\Lambda$ are shown in Fig. 4. Some portions of these lines are dotted to indicate that the corresponding points of parameter space are non-physical. Negative squared masses are obtained in the spectra relative to these points, for scalar other than $H_u$ and $H_d$.

The lightest low–energy soft parameter is certainly $m_{\tilde{E}}^2$. For small values of $\Lambda$, however, the inclusion of $D$-terms renders sneutrinos lighter than charged sleptons. In the leptonic sector, $D$–terms are:

\begin{align*}
  D_{\nu} &= \frac{1}{2} \cos 2\beta M_N^2; \quad D_{\nu_L} = -D_{\nu}(1 - 2 \sin^2 \theta_W); \quad D_{\nu_R} = -D_{\nu} 2 \sin^2 \theta_W, \tag{20}
\end{align*}

and numerically they amount to $D_{\nu} \simeq -(64 \text{ GeV})^2$, $D_{\nu_L} \simeq D_{\nu_R} \simeq (45 \text{ GeV})^2$ for all values of $\tan \beta$ obtained here through minimization of $V_0$. Moreover, if $\Lambda$ is small enough, $m_{\nu_i}^2$ may become negative. By using \cite{10}–\cite{13} (i.e. neglecting, for a rough estimate, third generation mass effects on the evolution of soft parameters) and our gauge couplings inputs, it is easy to see that the condition $m_{L}^2 > |D_{\nu}|$, i.e. $m_{\nu}^2 > 0$, is fulfilled for $\Lambda > \frac{7.3(\cos 2\beta)^{1/2}M_Z/\alpha_2}{2} \simeq 20 \text{ TeV}$. This value is not too dissimilar from that obtained with an exact calculation, i.e. the value of $\Lambda$ in Fig. 4 where the initial dotted intervals of the two curves for $\mu$ and $\tan \beta$ turn into solid ones.

In this same lowest range of $\Lambda$, where it is $m_{\tilde{L}}^2 < 0$, $D_{\nu_L}$ and $D_{\nu_R}$ protect the squared masses of charged sleptons from becoming negative. Nevertheless, the mass of the lightest state, $\tilde{\tau}_1$, is smaller than the experimental lower bound of 45 GeV coming from LEP I and it remains so also in the tiny region of $\Lambda$ around 20 TeV, where $\tan \beta$ and $\mu$ are indicated by solid lines.

When $\Lambda$ increases, sneutrinos become heavier, whereas $m_{\tilde{\tau}_1}$ tends initially to decrease. This is due to the fast increase of $\mu \tan \beta$ in the left–right entry of the $\tilde{\tau}$ mass matrix, initially faster than the increase of the diagonal entries (see the $\Lambda$ dependence of $\mu/M_3$ in Fig. 4). By identifying $D_{\nu_L}$ and $D_{\nu_R}$ ($\equiv D_{\nu}$), and neglecting third generation mass effects on the evolution of soft parameters, as well as $A_{\tau}$ in the off–diagonal entry of the $\tilde{\tau}$ mass matrix, one can cast the condition $m_{\tilde{\tau}_1}^2 \leq 0$ in the form: $m_{\tilde{E}}^2 m_\tau^2 + D_{\nu}(m_{\tilde{E}}^2 + m_\tau^2) - (\mu \tan \beta m_{\tau})^2 + D_{\nu}^2 \leq 0$. This quadratic equation in $M_2^2$ admits indeed two solutions which delimit an interval not too dissimilar from that indicated by the dotted portions of lines in Fig. 4 and determined without any of the above approximations. For further increases in $\Lambda$, the diagonal entries in the $\tilde{\tau}$ mass matrix increase far more rapidly than the off–diagonal ones giving eventually a physically acceptable spectrum.

We observe that choices of $x$ closer to 1 (i.e. $X \sim \Lambda$), affect the supersymmetric spectrum through the functions $f(x)$, $g(x)$ and the smaller size of the logarithm $\log Q_0$. We obtain lighter squarks and sleptons as well as lighter values of $|m_{H_u}^2|$: $\mu$ is, then, in general, $\sim M_2$. All gaugino are heavier and therefore weak and strong contributions to $B$ are also larger. In the range of $\Lambda$
physically acceptable ($\Lambda \gtrsim 50\text{ GeV}$), the values of $B$ are smaller than those obtained with our previous choice of $x$ (see (18)): strong gauge couplings as well as top yukawa couplings are now larger at $X$, but not enough to compensate the decrease of $t_XQ_0$ due to smaller $X$’s for fixed $\Lambda$’s. (The values of $Q_0$ are also slightly smaller, but this change has negligible consequences.) Overall, the new $\tan\beta$’s do not differ appreciably from the values previously obtained, for $\Lambda \gtrsim 50\text{ GeV}$.

3.2. Fully one–loop corrected Higgs potential

We have previously argued about the suitability of our choice of decoupling scale $Q_0$, which will be a posteriori justified by the results of the calculations presented in this subsection. It is obvious, however, that a change in this scale affects those parameters where the interplay between weak and strong sectors with different sensitivity to $Q_0$ has the largest effect. Particularly problematic is the parameter $B$. Since its low–energy value turns out to be rather small, changes of $Q_0$ may easily induce oscillations of $B$ around zero, which in turn, require flips of sign in the parameter $\mu$ with non–negligible consequences for the resulting phenomenology. Higher order corrections than those provided by the one–loop RGE as well as adjustments for the inadequacy of one unique decoupling scale $Q_0$ for the widely separated strong and weak sectors, become therefore mandatory.

Given the smallness of the logarithm $t_{Q_0,X}$, finite corrections are in the MMM more important than in the MSSM, when compared to the leading logarithmic corrections supplied by the same order RGE. In principle, they should be also larger than corrections coming from higher order RGE. In practice, this will not be the case, for the parameter $B$.

For a complete set of corrections to the parameters which determine the radiative breaking of $SU(2) \times U(1)$ we will consider the fully one–loop corrected effective potential $V_1(Q_0)$, defined as $V_1(Q_0) \equiv V_0(Q_0) + \Delta V_1(Q_0)$ and we will include contributions to $\Delta V_1(Q_0)$ coming from all sectors of the theory. Because of the wide gap existing between weak and strong mass parameters, this is
crucial in order to warranty results stable under variation of the decoupling scale.

Among other possible finite corrections, we include those to the b–quark mass, which, as known, can be rather large, when \( \tan \beta \) has values as large as those obtained here. These corrections reduce the value of the coupling \( h_b \) since \( h_b(M_Z) = (m_b - \delta m_b)/v_d = (m_b - \tan \beta \mu \Delta)/(v \cos \beta) \) with \( \Delta > 0 \) given in [9,11]. Consequently, they affect the evolution of \( m_{H_d}^2 \) (see (A7)), decrease the low–energy value of \( B \), therefore increasing \( \tan \beta \). This, in turn, increases \( 1/\cos \beta \) and the size of \( \delta m_b \). A retuning of all parameters finally produces the electroweak minimum for \( \tan \beta \)’s in general larger than those obtained when these corrections were not included. The values of \( \mu \) remain practically unchanged, since the parameters \( m_{H_u}^2 \) are unaffected by these corrections. For \( \Lambda = 100 \text{ TeV} \), in particular, it is \( \mu \sim 397.5 \text{ GeV} \), as before; \( h_b \) decreases from 0.798 to 0.786 and correspondingly \( A_t \) and \( A_b \) are slightly more negative. \( B \) adjusts to values indistinguishable from the previous one, but \( \tan \beta \) increases to 52.1 from the initial value of 46.4. We neglect finite corrections to quark masses other than to \( m_b \), as well as threshold effects for supersymmetric parameters other than those induced by \( \Delta V_1(Q_0) \).

We come now to analyze the effects of the inclusion of the one–loop corrections to the effective potential, which have the well known expression:

\[
\Delta V_1(Q_0) = \frac{1}{64\pi^2} \sum_a n_a m_a^4(H) \left[ \ln \left( \frac{m_a^2(H)}{Q_0^2} \right) - \frac{3}{2} \right]
\]

(21)

where \( m_a(H) \) is the field–dependent mass of the \( a^{th} \)-particle, \( n_a \), the corresponding number of degrees of freedom (negative for fermions): \( n_q = 6, n_d = -12, n_t = 2, n_u = -4, n_{\chi^+} = -4, n_{H^\pm} = 2, n_W = 6, n_W^0 = -2, n_{\mu} = 1, \) and \( n_Z = 3 \). The corrected potential \( V_1(Q_0) \) yields minimization equations which retain the form in (2), (3), provided the parameters \( B \mu, m_{H_d}^2, m_{H_u}^2 \) are substituted by hatted parameters \( \hat{B} \mu, \hat{m}_{H_d}^2, \hat{m}_{H_u}^2 \) defined as \( \hat{m}_{H_i}^2 = m_{H_i}^2 + \delta m_{H_i}^2 \), and \( \hat{B} \mu = B \mu + \delta(B \mu) \):

\[
\left( \hat{m}_{H_d}^2 + \mu^2 + \frac{1}{2} (g^2 + g_Y^2) (v_d^2 - v_u^2) \right) 2v_d - (\hat{B} \mu) 2v_u = 0 \quad (22)
\]

\[
\left( \hat{m}_{H_u}^2 + \mu^2 - \frac{1}{2} (g^2 + g_Y^2) (v_d^2 - v_u^2) \right) 2v_u - (\hat{B} \mu) 2v_d = 0. \quad (23)
\]

The shifts \( \delta m_{H_i}^2 \), \( \delta(B \mu) \) are listed in Appendix B. We do not use different symbols to distinguish between \( v_d \) and \( v_u \) as obtained from the minimization of \( V_0 \) and \( V_1 \). In what follows and in Appendix B we refer to the new minimum as to the \( V_1 \)-minimum as opposite to the \( V_0 \)-minimum obtained from (3).

The corrections \( \delta m_{H_i}^2 \) have been only partially included in [4,5], but not those to \( B \mu \), in principle very important and with strong impact on the value of \( \tan \beta \). In a generic GMSB, one can fix the value of \( \tan \beta \) at will using the freedom in the high– and low–energy parameter \( B \). It is precisely this lack of freedom which makes the MMM a model far more difficult to study.

At the \( V_1 \)-minimum, quark/squark and lepton/slepton contributions to \( \delta(B \mu) \) are [4,5]

\[
\frac{1}{32\pi^2} \left[ 3 \left( A_t \mu \hat{h}_t^2 \right) D(m_{\tilde{t}_1, \tilde{t}_2}) + 3 \left( A_b \mu \hat{h}_b^2 \right) D(m_{\tilde{b}_1, \tilde{b}_2}) + \left( A_\tau \mu \hat{h}_\tau^2 \right) D(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \right] ;
\]

(24)

those coming from charged gauge/gaugino, higgs/higgsino modes:

\[
\frac{1}{32\pi^2} \left[ 2 \left( M_2 \mu g^2 \right) D(m_{\tilde{H}_1, \tilde{H}_2}) - \frac{g^2}{4} \left( 2(B \mu) + \frac{g^2}{2} v^2 \sin 2\beta \right) D(m_{H^\pm, G^\pm}) \right],
\]

(25)

\footnote{Although squark mass matrices were evolved down in their \( 6 \times 6 \) form, the approximation of \( 2 \times 2 \) mass matrices for each generation of squarks and sleptons is used here.}
where \( v^2 \equiv v_d^2 + v_u^2 \) and \( G^\pm \) is what would be the charged Goldstone boson at the \( V_0 \)-minimum. Those due to neutral gauge/gaugino, higgs/higgsino mode are:

\[
\frac{1}{32\pi^2} \left[ \sum_{i=1}^{4} F(m_{\tilde{\chi}_i^0}, Q_0) \frac{\partial m_{\tilde{\chi}_i^0}^2}{\partial \sin 2\beta} - \frac{(g^2 + g_Y^2)}{4} \left( \frac{(B\mu)}{4} + \frac{(g^2 + g_Y^2)}{4} v^2 \sin 2\beta \right) D(m_{A^0}, G^0) \right], \tag{26}
\]

with \( A^0 \) the pseudoscalar Higgs, \( G^0 \) the would–be–neutral Goldstone boson. The functions \( D(m_{a,b}) \) and \( F(m_a, Q_0) \) are defined in Appendix B. Corrections to \( B\mu \) coming from other sectors of the theory are identically zero.

A comparison of (24) with the RGE for \( B \) (see (A4)) shows how the quark/squark, lepton/slepton contributions to \( \delta(B\mu) \) are corrections to the parameter \( B \). They improve upon the arbitrariness of the scale \( Q_0 \), linking it more realistically to the actual mass of the scalars virtually exchanged in the corresponding loop diagram. In the top/stop case, for example, in the limit of vanishing left–right mixing terms, it is indeed, \( m_{\tilde{t}_1} \to m_{\tilde{l}} \) and \( m_{\tilde{t}_2} \to m_{\tilde{q}} \), with \( U \), \( Q \) the two scalars in the first diagram of Fig. 3. Moreover, in the limit \( m_{\tilde{t}_1} \to m_{\tilde{t}_2} \equiv m_t \), it is \( D(m_{\tilde{t}_1,2}) \to 2 \log (m_t^2/Q_0^2) \) and the term \( 3/(16\pi^2)A_t h^2 \log (m_t^2/Q_0^2) \), with opposite sign to the corresponding one arising from (A4), would have the effect of trading \( Q_0^2 \) for \( m_t^2 \). In its actual form, the top/stop contribution includes also finite corrections, of type \( c_2^* \tilde{a}_t(X) \tilde{a}_3(X) t_X Q_0 \), i.e. with one logarithm less than the corresponding term provided by the RGE \( c_2^* \tilde{a}_t(X) \tilde{a}_3(X) (t_X Q_0)^2 \).

Similar considerations hold for chargino and neutralino corrections which take care of scale adjustments and finite pieces inclusions, of type \( \sim c_1^w \tilde{a}(X) \), coming from the first diagram of Fig. 2 to be compared to the terms \( c_1^w \tilde{a}(X) t_X Q_0 \) obtained from the one–loop RGE.

The sign of the contributions (24)–(26) depends crucially on the value of \( Q_0 \) with respect to the masses exchanged in the loop. Keeping the choice of \( Q_0 \) made at the tree–level, we have for \( \Lambda = 100 \) TeV, \( m_{\tilde{t}_1} \sim 977 \) GeV, \( m_{\tilde{t}_2} \sim 1099 \) GeV, and \( Q_0 \sim 993 \) GeV. These masses, roughly independent of further adjustments in \( \tan \beta \) and \( \mu \), are shown in Fig. 3 normalized to the gluino mass. Thus, \( D(m_{\tilde{t}_{1,2}}) \) is positive; the same is true for \( D(m_{\tilde{b}_{1,2}}) \), whereas \( D(m_{\tilde{q}_{1,2}}) \) and \( D(m_{\tilde{\chi}_{1,2}}) \) are negative. Quark/squark corrections are therefore small and negative (\( A_t \) and \( A_b \) are both negative), those due to \( \tau \) and \( c_1 \) are small and positive. Chargino corrections are large and negative. Although this cannot be explicitly seen by the formulas displayed in (23) and in Appendix B, negative are also the neutralino contributions to \( \delta(B\mu) \), although not as large as the chargino contributions. The remaining corrections coming from gauge and higgs boson modes are numerically smaller.

Whereas the overall \( Q_0 \)-dependence of \( B\mu \) and \( \delta m_H^2 \) is of order higher than the order of the calculation presented here, the relative size of the individual corrections changes for different choices of \( Q_0 \). For smaller values of \( Q_0 \), the quark/squark contribution tend to dominate and it may happen that approaching \( M_Z \), a negative \( \mu \) is needed to balance corrections too negative and therefore maintain \( B\mu \) positive (\( B\mu \) is indeed related to the fully one–loop corrected pseudoscalar Higgs mass). Since the bulk of supersymmetric particles is much heavier than \( M_Z \), we believe that definite conclusions cannot be drawn on the solutions obtained with such an “unnatural” choice of scale without including higher order corrections to the minimization conditions.

As for the shifts \( \delta m_H^2 \), the two largest contributions are the top/stop correction to \( m_{\tilde{H}_u}^2 \) and the bottom/stopbottom correction to \( m_{\tilde{H}_d}^2 \). They are both negative and add to the already negative parameters \( m_{\tilde{H}_u}^2 \). Contributions from first two generations squarks with different isospin, although not negligible in absolute value, almost completely cancel each other (see formulas in Appendix B). The next largest contributions are the chargino/charged–boson–sector and neutralino/neutral–boson
Figure 5: Solutions obtained for \(\tan\beta - \mu\) with a fully one-loop corrected neutral higgs potential (solid lines). The short-dashed lines correspond to the solutions obtained when only \(m_b\) corrections are included; the dotted lines are the solutions in Fig. 4. See text for the long-dashed lines.

sector, which contribute with different sign. Their algebraic sum, positive, is however still larger than the tau/stau contribution. The hatted parameters \(\hat{m}^2_{H_u}, \hat{m}^2_{H_d}\) are now more negative than \(m^2_{H_d}\) and \(m^2_{H_u}\).

A retuning of \(\mu\) and \(\tan\beta\) is therefore needed to make the left-hand-side of (22) and (23) vanish. After a numerical iteration, solutions are found for values of \(\tan\beta\) and \(\mu\) larger than those obtained at the \(V_0\)-minimum. Considerable is, in particular, the deviation for \(\tan\beta\): for \(\Lambda = 100 \text{ TeV}\), the new value is 59.0, the old, 46.4. This situation is not improved by a change of decoupling scale, which is symptomatic of some incompleteness in the set of corrections included.

A close inspection of the two-loop evolution equation for \(B\) (see the relative equation in Appendix A) shows the presence of strong terms, not \(A_i\)-induced, which yield contributions of type \(d_{\alpha n}^s (\tilde{\alpha}_s(X) \tilde{\alpha}_t(X) t_{XQ_0})^n\). The first of these terms, with \(n = 1\), is of same size of the \(A_i\)-induced finite corrections. It is indeed originated by the same upper diagram in Fig. 3, with the fermionic loop “open” (i.e. not shrunk to reproduce \(A_t\)). We add this first term in our determination of \(B\); it has a positive sign and produces therefore an increase in the value of \(B\) and a decrease of \(\tan\beta\).

In complete analogy with the situation observed in the case of the one-loop RGE, the remaining strong \(A_i\)-induced terms in the two-loop RGE yield corrections of type \(d_{1n}^s (\tilde{\alpha}_s^2(X) \tilde{\alpha}_t(X) t_{XQ_0}^2)^n\) and \(d_{2n}^s (\tilde{\alpha}_s(X) \tilde{\alpha}_t^2(X) t_{XQ_0}^2)^n\), of same size than the corrections \(d_{n}^w (\tilde{\alpha}^2(X) t_{XQ_0})^n\) due to weak terms. The first terms in these series \((n = 1)\), are larger than the corresponding finite one-loop corrections, \(c_{1s}^s \tilde{\alpha}_s(X) \tilde{\alpha}_t(X) t_{XQ_0}, c_{1w}^s \tilde{\alpha}(X)\), as expected. Their addition, therefore, would certainly require the inclusion of finite thresholds effects and corrections due to the two-loop RGE for all supersymmetric parameters. No other anomalously large corrections are induced by terms in the two-loop RGE for \(m_{H_u}^2\) and \(m_{H_d}^2\).
We show in Fig. 5 (solid lines) our estimate of $\mu$, $\tan\beta$ as obtained following the prescription described above. We start our plots from $\Lambda > 62$ TeV; below this value it is $m_{\tilde{\tau}_1}^2 < 0$, and the calculation of one-loop corrections becomes impossible (see terms $\ln(m_{\tilde{\tau}_1}^2/Q_0^2)$ in the tau/stau corrections). We also neglect a very tiny region at small $\Lambda$ which yields unacceptably low masses for $\tilde{\tau}_1$. For comparison, we give in this figure also the results obtained through minimization of $V_0$ (dotted lines); those obtained when only finite corrections to $m_b$ are added (short–dashed lines) and those coming from the minimization of $V_1$, with finite corrections to $m_b$ included (long–dashed lines). In this last case $\tan\beta$ is large enough to keep $m_{\tilde{\tau}_1}^2$ negative up to $\Lambda \sim 80$ TeV. As already mentioned, corrections to $m_b$ only do not produce deviations in the value of $\mu$ as obtained at the $V_0$–minimum (the dotted and short–dashed lines corresponding to $\mu$ coincide). Similarly the inclusion of the non–resummed term in the two–loop RGE lowers $\tan\beta$, but gives rise to changes in $\mu$ indistinguishable on the left frame, and only barely visible in the right one, where the ratios $\mu/M_3$ are given.

The addition of all the above described corrections to the Higgs potential produce a much milder dependence of the $\tan\beta$–$\mu$ solutions on the decoupling scale $Q_0$. We give in the table below the values of $\tan\beta$ and $\mu$ at the $V_0$– and the $V_1$–minimum for $\Lambda = 100$ TeV, and the two different decoupling scales $Q_0 = 993$ GeV and $Q_0 = 331$ GeV.

| $Q_0$ | $V_0$–min | $V_1$–min |
|-------|-----------|-----------|
| 993 GeV | $\tan\beta = 46.4$, $\mu = 397.5$ GeV | $\tan\beta = 49.9$, $\mu = 433.9$ GeV |
| 331 GeV | $\tan\beta = 36.1$, $\mu = 481.8$ GeV | $\tan\beta = 47.3$, $\mu = 446.0$ GeV |

The large variation obtained when minimizing $V_0$ can be traced back rather easily. The lowering of $Q_0$ induces an increase of the value of the strong gauge coupling in the interval $\{Q_0, X\}$. This is due to the fact that the interval $\{M_Z, Q_0\}$ where the faster SM evolution takes place ($b_3 = -7$; $b_3 = -3$) is now smaller. The trilinear couplings $A_t$, $A_b$ are larger, and, as a consequence, also the values of $B$ increase. Heavier squark spectra and larger top yukawa couplings induce also larger $\mu$’s; the increase of the term $B\mu$ produces smaller values of $\tan\beta$. The variation in the $\tan\beta$–$\mu$ solution is much smaller when corrections to $V_0$ are included. Values of $Q_0$ too close to $M_Z$, are however still problematic.

This shows that our choice of $Q_0$ is already close enough to the optimized scale $Q_S$ discussed in [13][14], where the prediction for the two vacuum expectation values $v_1$ and $v_2$ from $V_0$ and $V_1$ coincide. We can therefore rely on the RGE–improved tree–level scalar potential and our choice of $Q_0$ to verify whether dangerous minima breaking charge and/or colour are not also present and possibly deeper than the observed electroweak minimum. An analysis of this type when all one–loop corrections to the scalar potential are added would clearly be prohibitive. We do not find evidence for the existence of such minima, at least using the analytical criteria given in [14][11].

4. Spectrum, Prediction, Constraints

We display in Fig. 6 the spectrum obtained in the allowed range of $\Lambda$ for our estimate of $\mu$, $\tan\beta$ (solid lines of Fig. 5), and we give explicitly in the following table the values of gaugino, chargino, neutralino, and slepton masses (in GeV) at $\Lambda = 100$ TeV.

---

‡This value of $\Lambda$ is larger than the value of $\sim 50$ TeV obtained when minimizing $V_0$, since $\mu$ and $\tan\beta$ are now larger.
We follow here the conventions in [15] ($m_{\tilde{\chi}_i^\pm} > m_{\tilde{\chi}_j^0}$) and we indicate by $m_{\tilde{\nu}_R}$ ($m_{\tilde{\nu}_L}$) the two degenerate eigenvalues for right– and left–handed $\tilde{c}$ and $\tilde{\mu}$. Similarly degenerate are two of the three $\tilde{b}$ states. For the same $\Lambda$, the up– and down–squark masses are given in Appendix C. Due to the rather large values of $\mu$ and $\tan \beta$, the three heavy Higgs states (two neutral, one charged) range between 346 and 360 GeV; the lightest neutral state has mass roughly 125 GeV.

The almost factor of two between the first two generations left–handed and right–handed masses, already present at high–scale [12] is clearly visible in the fourth frame of Fig. 4. The same splitting in the masses of the first two generations squarks is more modest since it is due to weak gaugino loops, whereas the bulk of the masses is produced by gluino loops [12]. The corresponding ratios of the high–scale first two generations left– and right–handed squark masses ($m_{\tilde{q}_i}^Q(X)$, $m_{\tilde{q}_i}^U(X)$) over the low–energy gluino mass, not shown in this figure, are shifted downward, with respect to the solid lines in the two lower frames, by a factor $\sim (1-Z^2_3)/b_3$ (for $f(x)/g(x)^2 \sim 1$) (see [13]). Their geometrical mean, $Q_0/M_3$, is explicitly indicated in both frames by the short–dashed lines. Tiny differences in the shape of the two solid lines in these two frames are due to isospin effects [12, 13] as well as to the presence of different D–terms in the two cases. Those induced by intergenerational as well as chirality mixing terms in the up– and down–squark mass matrices are not visible in these figures.

Right–handed as well as left–handed $\tilde{c}$ and $\tilde{\mu}$ can easily decay to the lightest neutralino $\tilde{\chi}_1^0$ with mass $\sim M_1$. The decrease for increasing $\Lambda$ of the ratios relative to the two heavy neutralinos (mainly neutral higgsino states) and the heavy chargino (mainly a charged higgsino state) in the first two frames of Fig. 4, can be simply explained by the milder growth of $\mu$ already observed in Fig. 3. Their masses actually increase and their states become increasingly more mixed with the gaugino states.

Third generation yukawa effects in the evolution equations (A6) are responsible for the roughly 10% decrease in the third generation squark masses. In addition, the two stop masses are only a little affected by the presence of left–right mixing terms in the up–squark mass matrix ($\sim A_{tt}m_t \ll M_2^2$ compared to the diagonal elements which are $> M_2^2$). This effect is somewhat larger in the down–squark case. Differently than in the MSSM, the twelve squark states cluster quite closely around a common value of mass, $\sim 1$ TeV at $\Lambda = 100$ TeV. We show in Appendix C the diagonalization matrices for up– and down–squarks, at this same value of $\Lambda$, from where one can read off the composition of the relative squark mass eigenstates.

Small is the effect induced by $h_\tau$ in the sneutrino sector, as the third frame of Fig. 3 shows. On the contrary, the effects due to the presence of left–right mixing terms in the charged slepton mass matrix are rather large. The two states $\tilde{\tau}_1$, $\tilde{\tau}_2$, have masses on opposite sides of the left– and right–handed first two generations sleptons. The lighter of the two, $\tilde{\tau}_1$, is indeed the lightest sparticle in the spectrum and exceeds the LEP I bound of 45 GeV only for $\Lambda > 72$ TeV. Once this constraint is imposed, the remaining spectrum is heavy enough to largely exceed any other experimental lower bound on masses. We obtain, in fact, $m_{\tilde{\tau}_1^\pm} > 180$ GeV, $m_{\tilde{\tau}_2^0} > 100$ GeV, $m_{\tilde{\nu}_e} > 240$ GeV, $m_{\tilde{\tau}_1^\pm} > 130$ GeV, $m_{\tilde{\nu}_e} > 735$ GeV, and $m_{\tilde{\tau}_2} > 725$ GeV.

Thus, the main decay for $\tilde{\chi}_1^0$ is the two body decay $\tilde{\chi}_1^0 \rightarrow \tau \tilde{\tau}_1$. It proceeds with full electroweak gauge strength reducing therefore the partial width for the decay $\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}$ ($\tilde{G}$ is here the gravitino)
Figure 6: Ratios of chargino, neutralino, and sfermion masses over weak and strong gaugino masses. For sleptons and squarks, the solid lines are relative to the degenerate first two generations left-handed (upper lines) and right-handed (lower lines) states. The dashed lines refer to the third generation eigenstates. The short-dashed lines in the two lower frames indicate the ratio of our choice for \( Q_0 \) (see text) over the gluino mass. This spectrum corresponds to the solutions (\( \tan\beta, \mu \)) indicated in Fig. 5 by solid lines.
to practically negligible levels in this model. (For a theoretical discussion of this decay mode see [16].) The subsequent decay \( \tilde{\tau}_1 \rightarrow \tau \tilde{G} \) gives \( \tau \tau \) +missing energy as signature for the decaying \( \tilde{\chi}_1^0 \) and a four \( \tau \)'s signal for \( \tilde{\chi}_1^0 \) pair production in an \( e^+e^- \) collider. Given the rather large values of \( m_{\tilde{\chi}_1^0} \) induced by the constraint \( m_{\tilde{\tau}_1} > 45 \text{ GeV} \), such a signal can be observed only at future \( e^+e^- \) colliders. The only signal LEP II is likely to detect is two \( \tau \)'s +missing energy due to a pair production of \( \tilde{\tau}_1 \). The two final \( \tau \)'s are in this case more energetic than in the similar production mechanism in the MSSM, where \( \tilde{\tau}_1 \) decays into \( \tilde{\chi}_1^0 + \tau \) and the neutralino has non–negligible mass.

As already observed, an increase in \( x \) towards the value one, has the effect of decreasing scalar masses, while increasing gaugino masses. The parameter \( \mu \), now much closer to \( M_2 \), gives chargino and neutralino states much more mixed: the curve relative to the heaviest neutralino in Fig. 8 is lowered to the values 1.3–1.1 in the shown range of \( \Lambda \). The ratios of up– and down–squark over gluino masses are now between 0.9 and 1.0. The lightest particle of the spectrum is still \( \tilde{\tau}_1 \), but also \( \tilde{e}_R, \tilde{\mu}_R \) become lighter than the lightest neutralino.

A severe constraint on this model may come from the \( b \rightarrow s \gamma \) test. (This was considered in [8,17] in the context of more general GMSB models than the MMM.) The estimate for the branching ratio \( Br(b \rightarrow s \gamma) \) is obtained here using the one–loop supersymmetric boundary conditions at the electroweak scale given in [8] and including the leading QCD corrections as in [8,18]. Experimental errors of some relevant low–energy variables as well as theoretical uncertainties in the QCD corrections are kept into account as in [13] and [19]. The main source of uncertainty comes from the ambiguity in the scale at which this process has to be evaluated: we let this scale vary between \( m_b/2 \) to \( 2m_b \).

Among the supersymmetric amplitudes, the collection of those relative to chargino exchange deserves a little attention in this model. We rewrite them in the form:

\[
A_{\tilde{\chi}^-} = C^n \sum_{j,k}^{{2,6}} (-2) x_{uk} \left\{ C^{db}_{1jk} \left( F_1 + \frac{2}{3} F_2 \right) (x_{jk}) + \frac{m_{\tilde{\chi}}} {m_b} C^{d*}_{2jk} \left( F_3 + \frac{2}{3} F_4 \right) (x_{jk}) \right\}
\]

(27)

where the function \( F_i \) are given in [8], the symbols \( x_{ij} \) denote ratios of masses: \( x_{uk} \equiv M_{\tilde{k}}^2 / m_{\tilde{u}_k}^2 \), \( x_{jk} \equiv m_{\tilde{\chi}}^2 / m_{\tilde{u}_k}^2 \), and the constant \( C^n \) is \( C^n \equiv G_{F,E}/(\sqrt{32 \pi^2}) \). The coefficient \( C^{db}_{1jk} \) collects the couplings:

\[
C^{db}_{1jk} = C^{j\bar{k}}_{UL} C^{\bar{k}s} UL - (G^{j\bar{k}}_{UL} H^{j\bar{s}} UL + H^{j\bar{k}}_{UR} G^{j\bar{s}} UR) + H^{j\bar{k}}_{UR} H^{j\bar{s}} UL
\]

(28)

of the “pure gaugino”, “mixed gaugino–higgsino” and “pure higgsino” contributions. By using the definitions in [8] and the fact that \( \sin \beta \sim 1, \cos \beta \sim 1/\tan \beta \) in this model, \( C^{db}_{1jk} \) can be re–expressed as:

\[
C^{db}_{1jk} \simeq |V_{j1}|^2 \Gamma^{k3}_{UL} \Gamma^{k2}_{UL} V_{j2}^* - \frac{x_{tw}}{2} V_{j1} V_{j2}^* \left( \Gamma^{k3}_{UL} \Gamma^{k3}_{UR} K_{ts} + \Gamma^{k3}_{UR} \Gamma^{k2}_{UL} K_{tb} \right) + \frac{x_{tw}}{2} |V_{j2}|^2 \Gamma^{k3}_{UR} \Gamma^{k2}_{UL} K_{ts},
\]

where \( K \) is the Kobayashi–Maskawa matrix, \( U \) and \( V \) the matrices needed to diagonalize the chargino mass matrix. The \( 3 \times 6 \) matrices \( \Gamma_{UL} \) and \( \Gamma_{UR} \), juxtaposed, give the diagonalization matrix \( D_U \) of the up–squark mass matrix. Their numerical components as well as those of \( \Gamma_{DL}, \Gamma_{DR} \), as obtained for \( \Lambda = 100 \text{ TeV} \), are given in Appendix C. Similarly, the coefficient \( C^{d*}_{2jk} \) multiplied by \( m_{\tilde{\chi}} / m_b \) is:

\[
\frac{m_{\tilde{\chi}}}{m_b} C^{d*}_{2jk} = \frac{x_{jw}}{2} \tan \beta \left( -U_{j2} V_{j1}^* \Gamma^{k3}_{UL} \Gamma^{k2}_{UL} V_{j2}^* + \sqrt{\frac{x_{tw}}{2}} U_{j2} V_{j2}^* \Gamma^{k3}_{UL} \Gamma^{k2}_{UL} K_{ts} \right).
\]

(30)
Figure 7: Branching ratio for the decay $b \to s\gamma$ obtained in the MMM (solid lines); the dashed lines indicate the SM’s band and the dotted lines the band obtained in a Two-Higgs doublets model with the same mass for the charged higgs as obtained in the MMM. The horizontal solid lines delimit the experimentally allowed values. The results shown correspond to the solutions $\tan\beta - \mu$ indicated in Fig. 5 by solid lines.

The amplitude in (27) with the pure gaugino coupling (first term in (29)) is the supersymmetric counterpart of the SM amplitude:

$$A_{SM} = C \gamma K_{tb} K_{ts}^* 3 x_{tw} \left( \frac{2}{3} F_1 + F_2 \right) \left( x_{tw} \right); \quad (31)$$

those with pure higgsino couplings (the last terms in (29) and (30)), the counterparts of the Higgs–mediated amplitude:

$$A_{H^-} = C \gamma K_{tb}^* K_{th} x_{th} \left\{ \cot^2 \beta \left( \frac{2}{3} F_1 + F_2 \right) (x_{th}) + \left( \frac{2}{3} F_3 + F_4 \right) (x_{th}) \right\}, \quad (32)$$

with obvious meaning of all ratios $x_{ij}$. In particular, the amplitude with the last coupling in (29) corresponds to the Higgs contribution proportional to $\cot^2 \beta$, that with the last coupling in (30) to the $\tan\beta$–independent one. Both couplings have column–index equal to 2 in the diagonalization matrices $U$ and $V$, which, indeed, selects the higgsino component of the chargino exchanged in the loop (see [15], whose notation we follow here). We remark that in this model the chargino mass matrix, $X \sim ((M_2, \sqrt{2}M_W), (0, \mu))$, has all positive entries. It is, therefore, $\det X > 0$. The two diagonalization matrices $U$, $V$ have then both the structure $((\cos \phi_{U,V}, \sin \phi_{U,V}), (-\sin \phi_{U,V}, \cos \phi_{U,V}))$ with angles $\phi_U$, $\phi_V$ determined by $\tan \phi_U \sim (m_{\tilde{\chi}^-_1}^2 - (M_2^2 + 2M_{\tilde{t}_1}^2)) / (\sqrt{2}M_W \mu)$ and $\tan \phi_V \sim (m_{\tilde{\chi}^-_1}^2 - M_2^2) / (\sqrt{2}M_W M_2)$, both $> 0$. (We remind that in our convention, $m_{\tilde{\chi}^-_1}$ is the heavier chargino.)

The two amplitudes with couplings (30) are, in general, those responsible for the growth of the chargino contribution for increasing $\tan\beta$ and should, in principle, be the most relevant ones when $\tan\beta$ is large. The situation is, however, slightly different in this model. As it can be seen
from Appendix C, the tan$\beta$–dependent gaugino–higgsino amplitude,

$$A_{W\tilde{H}} = C^\gamma \sum_{j,k} 2 x_{wk} \frac{\sqrt{x_{jw}}}{2} \tan\beta \left( U_{j2} U_{j1}^* \Gamma_{UL}^{k3} \Gamma_{UL}^{k2} \left( F_3 + \frac{2}{3} F_4 \right)(x_{jk}) \right),$$

gets contributions from the mainly left–handed stop ($k = 2$) and the mainly left–handed sbottom ($k = 6$), which have non–negligible mixings with $\tilde{c}_L$ and $\tilde{t}_L$, respectively. (The index $k$ is ordered according to increasing values of the squark masses.) Sizes and signs of these mixings are determined by the RGE which generate off–diagonal terms in the left–left sector of the up–squark mass matrix, and the requirement of orthogonality for $D$. For our representative value $\Lambda = 100$ TeV, $|\Gamma_{UL}^{k3} \Gamma_{UL}^{k2}| \sim |\Gamma_{UL}^{63}, \Gamma_{UL}^{62}| \sim 0.02$. The two products have opposite sign and differ only at the $10^{-6}$ level; the presence of the functions $(F_3 + 2/3 F_4)(x_{jk}) \sim O(1)$, however, gives rise to a flavour violation still at the $10^{-3}$ level. An additional cancellation is due to the gaugino mixing. After summation in $k$, the sign of the two terms corresponding to the different $j$'s is different, as it appears evident from the previous discussion on the matrices $U$ and $V$. (This result, obviously does not depend on the freedom used in fixing relative phases in the elements of $U$ and $V$ (see [5]).) Again, for $\Lambda = 100$ TeV, it is: $U_{12} V_{11}^* \simeq -0.36$, $U_{22} V_{21}^* \simeq +0.20$, ($m_{\tilde{\chi}^+} = 456$ GeV, $m_{\tilde{\chi}^0} = 254$ GeV). Furthermore, the ratio $x_{wk}$ is a strong factor of suppression, $x_{wk} \sim 10^{-2}$, reducing the “gaugino–higgsino” mixing amplitude (33) to be more than two orders of magnitude smaller than $A_{SM}$ and $A_{H^+}$ and roughly one order of magnitude larger than the pure “gaugino” contribution. Similar cancellations appear also in the other gaugino–higgsino mixing amplitude (with coupling in the second term of (29)), which gives rise to the smallest amplitude in the collection (27).

The tan$\beta$–dependent pure “higgsino” amplitude gets non–negligible contributions from the two stop states ($k = 1$ and $k = 2$). The product of the two elements of $\Gamma_{UL}$, $\Gamma_{UR}$, with their opposite sign for the two $k$’s, leads to a left–right mixing $< 10^{-4}$, overkilling the enhancement due to tan$\beta$. Numerically, once all summations on $k$ and $j$ are performed, this amplitude turns out to be smaller than the tan$\beta$–independent pure “higgsino” amplitude:

$$A_{\tilde{H}\tilde{H}} = C^\gamma \sum_{j,k} 2 x_{wk} \frac{x_{tw}}{2} |V_{j2}|^2 |\Gamma_{UR}|^2 K_{tb} K_{ts}^* \left( F_3 + \frac{2}{3} F_2 \right)(x_{jk}),$$

where it is $(\Gamma_{UR}^{13})^2 \simeq 1, V_{22}^* \simeq 1$ and $x_{tw}/2 \simeq 1$ and where the flavour violation has the same weight as in the SM. The amplitude (34) is the largest one in the collection (27). Its sign is opposite to that of $A_{SM}$ and $A_{H^+}$, but due to the suppression factor $x_{wk}$, it produces only a tiny cancellation of $A_{H^+}$.

Also the gluino amplitude:

$$A_g = C^\gamma \sum_{k} 16 x_{wk} \left( \frac{\alpha_s}{\alpha_w} \right) \left\{ \Gamma_{DL}^{kkb} \Gamma_{DL}^{ksa} F_2(x_{gk}) - \frac{m_{\tilde{g}}}{m_b} \Gamma_{DB}^{kkb} \Gamma_{DL}^{ksa} F_4(x_{gk}) \right\}$$

is known to increase for increasing values of tan$\beta$ [20]. Of the two terms in (35), the larger contribution comes from the second one, with exchange of the mainly right–handed sbottom ($k = 1$), the mainly left–handed sbottom ($k = 4$) and the mainly left–handed s–strange ($k = 6$). Left–right terms in the down–squark mass matrix play a larger role than in the up–squark mass matrix and the corresponding mass eigenstates are more mixed states. The simultaneous left–right and flavour transitions are, however, still $\lesssim 10^{-4}$. The enhancement factors $m_{\tilde{g}}/m_b(m_b)$ and $\alpha_s/\alpha_w$, together with the fact that down– and up–squarks have very similar masses, makes this amplitude comparable to the tan$\beta$–dependent pure “higgsino” one. In our representative case $\Lambda = 100$ TeV,
the SM, Higgs, chargino and gluino amplitudes are respectively: 
\(-1.1 \times 10^{-9}, -5.7 \times 10^{-10}, 7.9 \times 10^{-12}, \) and 
\(1.3 \times 10^{-11}\). The charged Higgs mass, for this value of \(\Lambda\), is 360 GeV. The neutralino amplitude is completely negligible.

The values of \(Br(b \rightarrow s\gamma)\) predicted by this model are in the region delimited by solid lines in Fig. 7. The MMM’s branching ratio deviates little from that obtained in a Two–Higgs–Doublet Model (band delimited by dotted lines) with \(m_{H^\pm}\) as predicted by the MMM. For reference we report also the SM prediction (region within the dashed lines) and the experimentally allowed value (region within the two horizontal solid lines) [21].

Since the requirement of positivity for the the \(\tilde{\tau}_1\)-mass has already selected relatively large values of \(\Lambda\), \(\Lambda > 62\) TeV, and the LEP I constraint \(m_{\tilde{\tau}_1} > 45\) GeV imposes \(\Lambda > 72\) TeV, no further exclusion comes, at the moment, from the measurement of this decay. Improved experimental results will require improvements in the calculation of the supersymmetric branching ratio before any significant conclusion can be drawn. A significant shift of \(x\) towards one, however, would push already now the MMM’s band outside the experimentally allowed range up to \(\Lambda \sim 100\) TeV. For this value of \(\Lambda\), it is: \(m_{H^\pm} = 300\) GeV and the SM, Higgs, chargino and gluino amplitudes are: \(-1.1 \times 10^{-9}, -6.7 \times 10^{-10}, 4.9 \times 10^{-12}, \) and \(1.5 \times 10^{-11}\).

5. Conclusions

In this paper, we studied the MMM in the approximation of a messenger scale of the same order than the supersymmetry–violating messenger scalar mass, and gave our estimate for the solutions \(\tan\beta – \mu\) enforced by the mechanism of radiative breaking of \(SU(2)_L \times U(1)_Y\).

Differently than generic GMSB models, which benefit from the presence of more free parameters at the messenger scale, the MMM requires a careful handling of corrections to \(B\mu, m^2_{H_u}\) and \(m^2_{H_d}\), which determine the electroweak breaking. To this aim, we considered the one–loop corrected effective potential with contributions coming from all massive modes in the model: quark, squark, lepton, slepton, chargino, neutralino, gauge and higgs modes. Finally we included additional logarithmic two–loop corrections to the parameter \(B\) of same type and size than those induced by the one–loop effective potential. All of these corrections turn out to be important to obtain solutions \(\tan\beta – \mu\) stable under variation of the decoupling scale \(Q_0\) around a typical squark mass.

Among the predictions obtained by studying the MMM’s mass spectrum, the most interesting is that \(\tilde{\tau}_1\) is the lightest sparticle. Indeed, the requirement of positivity of \(m^2_{\tilde{\tau}_1}\) as well as that coming from the LEP I lower bounds on supersymmetric masses, exclude already values of \(\Lambda\) up to \(\sim 70\) TeV. This prediction has other important consequences. The MMM, in fact, a) cannot accommodate the \(e^+e^-\gamma\gamma\) CDF event [22]; b) may be detected in \(e^+e^-\) collisions through two \(\tau\)’s +missing energy or four \(\tau\)’s +missing energy signals.

As expected, flavour violation effects as well as effects due to left–right mixing in the squark sector are small in this model. The sparticle contributions to \(b \rightarrow s\gamma\), opposite to the \(W^\pm\) and \(H^\pm\) contributions are, therefore, not very significant. The overall MMM’s prediction closely resembles that of a Two Higgs Doublet Model with same \(H^\pm\) mass. When \(m_{\tilde{\tau}_1}\) is larger than 45 GeV, we obtain rates consistent with the present experimental measurement.
Acknowledgements

I thank many of my colleagues for discussions: M. Bastero-Gil, A. Brignole, D. Comelli, M. Drees, E. Duchovni, M. Kugler, D. Lellouch, Y. Nir, R. Plesser, N. Polonsky, R. Rattazzi, M. Reuter, F. Vissani, G. Wolf, and D. Wyler. I also acknowledge the hospitality of the CERN theory group, where part of this work was carried out.

Note added

After completion of this work we became aware of the existence of the paper hep–ph/9701341 by D. Dicus, B. Dutta, and S. Nandi which studies one of the experimental signals listed in this paper, and of the content of hep–ph/9612464 by R. Rattazzi and U. Sarid with the same subject of the present paper. The solutions $\tan\beta - \mu$ seem in qualitative agreement with ours. Differences appear, however, in the conclusions reached for the implication of the measurement of $b \rightarrow s\gamma$. 
Appendix A.

We list the one-loop RGE to which we refer in the text. For simplicity we give the approximate form valid in the limit of Kobayashi–Maskawa matrix \( K \sim 1 \). The one-loop equations actually used in our analysis, with a realistic \( K \), are given in \( \mathbb{I} \).

- gauge couplings and gaugino masses:

\[
\tilde{\alpha}_i = -b_i \, \alpha_i^2 \quad \tilde{M}_i = -b_i \, \tilde{\alpha}_i M_i
\]  

(A1)

The dot indicates derivative with respect to \( t_{\chi Q} = 2 \ln (X/Q) \) and the tilde a division over 4\( \pi \). The supersymmetric \( \beta \)-function coefficients \( b_i \) are \((-3, 1, 33/5)\). The corresponding SM ones, \( b_i^{SM} \), are \((-7, -19/6, 41/10)\).

- yukawa couplings and soft trilinear couplings:

\[
\begin{align*}
\hat{\alpha}_t &= (GY)_t - 6\tilde{\alpha}_t - \tilde{\alpha}_b \\
\hat{\alpha}_b &= (GY)_b - 6\tilde{\alpha}_b - \tilde{\alpha}_t - \tilde{\alpha}_\tau \\
\hat{\alpha}_\tau &= (GY)_\tau - 4\tilde{\alpha}_\tau - 3\tilde{\alpha}_b \\
\hat{M}_t &= -(GA)_t - 6A_t \tilde{\alpha}_t - A_b \tilde{\alpha}_b \\
\hat{M}_b &= -(GA)_b - 6A_b \tilde{\alpha}_b - A_t \tilde{\alpha}_t - A_\tau \tilde{\alpha}_\tau \\
\hat{M}_\tau &= -(GA)_\tau - 4A_\tau \tilde{\alpha}_\tau - 3A_b \tilde{\alpha}_b
\end{align*}
\]

(A2)

where \( \alpha_t = h_t^2/4\pi \) and after defining \( C_i^Y = (13/36), (7/36), (3/4) \) for \( i = t, b, \tau, GY \) and \( GA \) are:

\[
\begin{align*}
(GY)_i &= 4 \{ C_3 \tilde{\alpha}_3 + C_2 \tilde{\alpha}_2 + \frac{3}{5} C_i^Y \tilde{\alpha}_1 \} \\
(GA)_i &= 4 \{ C_3 \tilde{\alpha}_3 M_3 + C_2 \tilde{\alpha}_2 M_2 + \frac{3}{5} C_i^Y \tilde{\alpha}_1 M_1 \}
\end{align*}
\]

(A3)

- bilinear coupling:

\[
\hat{B} = -4 \{ C_2 \tilde{\alpha}_2 M_2 + \frac{3}{5} \left( \frac{1}{2} \right)^2 \tilde{\alpha}_1 M_1 \} - (A_\tau \tilde{\alpha}_\tau + 3A_b \tilde{\alpha}_b + 3A_t \tilde{\alpha}_t)
\]

(A4)

- first two generations squark and slepton masses

\[
(m^2)_{11,22} = 4 \left\{ C_3 \tilde{\alpha}_3 M_3^2 + \frac{3}{5} Y^2 \tilde{\alpha}_1 M_1^2 \right\}
\]

(A5)

- third generation sfermion masses:

\[
\begin{align*}
(m^2)_{Q,33} &= (m^2)_{Q,11,22} - \tilde{\alpha}_t (SS)_t - \tilde{\alpha}_b (SS)_b \\
(m^2)_{U,33} &= (m^2)_{U,11,22} - 2 \tilde{\alpha}_t (SS)_t \\
(m^2)_{L,33} &= (m^2)_{L,11,22} - \tilde{\alpha}_\tau (SS)_\tau \\
(m^2)_{D,33} &= (m^2)_{D,11,22} - 2 \tilde{\alpha}_b (SS)_b \\
(m^2)_{E,33} &= (m^2)_{E,11,22} - 2 \tilde{\alpha}_\tau (SS)_\tau
\end{align*}
\]

(A6)

- soft Higgs-potential parameters:

\[
\begin{align*}
(m^2)_{H_u} &= (m^2)_{U,11,22} - 3\tilde{\alpha}_b (SS)_b - \tilde{\alpha}_\tau (SS)_\tau \\
(m^2)_{H_d} &= (m^2)_{U,11,22} - 3\tilde{\alpha}_t (SS)_t \\
(m^2)_{H_u} &= (m^2)_{L,11,22} - 3\tilde{\alpha}_t (SS)_t
\end{align*}
\]

(A7)

where

\[
\begin{align*}
(SS)_t &= (m^2_{Q} + m^2_{D})_{33} + m^2_{H_u} + A_t^2 \\
(SS)_b &= (m^2_{Q} + m^2_{D})_{33} + m^2_{H_d} + A_b^2 \\
(SS)_\tau &= (m^2_{L} + m^2_{E})_{33} + m^2_{H_d} + A_\tau^2
\end{align*}
\]

(A8)
− μ parameter:

\[
(\mu^2) = \left[4 \left(C_2 \bar{\alpha}_2 + \frac{3}{5} \left(\frac{1}{2} \bar{\alpha}_1 \right)^2\right) - (3 \bar{\alpha}_t + 3 \bar{\alpha}_b + \bar{\alpha}_r)\right] \mu^2 \tag{A9}
\]

The low–energy parameter \( \mu \) is obtained from the minimization condition of the scalar Higgs potential. This equation may help to trace back the high–energy value of this parameter.

We report also the additional two–loop terms in the RGE for \( B \) to which we refer in the text (see [12]):

\[
\dot{B} = \left\{ \frac{80}{3} (C_2 \bar{\alpha}_2)^2 M_2 + \frac{12}{5} (C_2 \bar{\alpha}_2) (\bar{\alpha}_1) (M_1 + M_2) + \frac{207}{25} (\bar{\alpha}_1)^2 M_1 \right\} \\
+ \frac{12}{5} \left[ C_3 \bar{\alpha}_3 M_3 + \frac{1}{15} \bar{\alpha}_1 M_1 \right] \bar{\alpha}_t + \left[ C_3 \bar{\alpha}_3 M_3 - \frac{1}{30} \bar{\alpha}_1 M_1 \right] \bar{\alpha}_b + \frac{1}{10} \bar{\alpha}_1 M_1 \bar{\alpha}_r \\
- \frac{12}{5} \left[ C_3 \bar{\alpha}_3 + \frac{1}{15} \bar{\alpha}_1 \right] A_t \bar{\alpha}_t + \left[ C_3 \bar{\alpha}_3 - \frac{1}{30} \bar{\alpha}_1 \right] A_b \bar{\alpha}_b + \frac{1}{10} \bar{\alpha}_1 A_r \bar{\alpha}_r \\
+ 6 \left[ A_t \bar{\alpha}_r^2 + (A_t + A_b) \bar{\alpha}_t \bar{\alpha}_b + 3A_t \bar{\alpha}_t^2 + 3A_b \bar{\alpha}_b^2 \right]. \tag{A10}
\]

Appendix B.

It is easy to see that, if we neglect phases other than that in the CKM matrix, the scalar potential \( V \) (here indifferently \( V_0 \) or the fully one–loop corrected potential \( V_1 \)) depend on the real components \( \phi_i, \psi_i \ (i = u, d) \) of the two neutral higgses \( H_d^0 \) and \( H_u^0 \) \( (H_d^0 \equiv \phi_d + i \psi_d, H_u^0 \equiv \phi_u + i \psi_u) \) through the bilinear operators

\[
\epsilon_d \equiv |H_d^0|^2; \quad \epsilon_u \equiv |H_u^0|^2; \quad \epsilon_3 \equiv H_u^0 H_d^0 + h.c.. \tag{B1}
\]

(If extra phases are allowed, one would have to consider slightly more general operators. The following results would remain unchanged up to complex conjugates of soft parameters and/or \( \mu \)).

The derivatives of \( V \) with respect to \( \phi_i \), which determine the minimum conditions, can then be expressed in terms of derivatives with respect to \( \epsilon_d, \epsilon_u \) and \( \epsilon_3 \) as follows:

\[
\begin{align*}
(\partial_{\phi_d} V) &= 2 \epsilon_d (\partial_{\epsilon_d} V) + 2 \epsilon_u (\partial_{\epsilon_u} V) \\
(\partial_{\phi_u} V) &= 2 \epsilon_u (\partial_{\epsilon_u} V) + 2 \epsilon_d (\partial_{\epsilon_d} V).
\end{align*} \tag{B2}
\]

The derivatives \( \partial_{\epsilon_d} V \) are closely related to the entries of the pseudoscalar Higgs mass matrix, if one imposes that \( \langle \psi_d \rangle = \langle \psi_u \rangle = 0 \): \( \partial^2 V / \partial \psi_d^2 = 2 \epsilon_d V; \partial^2 V / \partial \psi_u \partial \psi_d = -2 \epsilon_3 V; \partial^2 V / \partial \psi_u^2 = 2 \epsilon_u V \). Moreover, at the electroweak minimum, i.e. when \( \partial_{\phi_d} V = \partial_{\phi_u} V = 0 \) for \( \langle \phi_d \rangle = v_d, \langle \phi_u \rangle = v_u \) \( (\tan \beta \equiv v_u / v_d) \), it is:

\[
\begin{align*}
\partial_{\epsilon_d} V &= -\partial_{\epsilon_3} V \tan \beta; \\
\partial_{\epsilon_u} V &= -\partial_{\epsilon_3} V \cot \beta.
\end{align*} \tag{B3}
\]

In other words, the correct minimization condition guarantees the existence of one Goldstone boson mode, corresponding to the eigenstate \( (\cos \beta, -\sin \beta) \). The massive mode \( (\sin \beta, \cos \beta) \) has mass \( (2/\sin 2\beta)(-\partial_{\epsilon_3} V) \). In this sense \( -\partial_{\epsilon_3} V \) identifies \( (B\mu) \) when \( V \) is \( V_0 \) and what will be called \( \bar{B}\mu \) in the case of \( V_1 \).

\( ^{5} \)The physical neutral Higgs field \( H^0, h^0, A^0, \) properly normalized, are expressed in terms of \( H_d^0 \) and \( H_u^0 \) as in [23]
The approximation of \(2 \times \) bottom–sbottom contribution mixing terms. All the corrections listed in this appendix are clearly not sensitive to these tiny effects.

The building blocks for these matrices are determined without neglecting intergenerational compactness, can be expressed in terms of \(\hat{\text{F}}\) expressed in terms of "hatted" variables quark/squark, lepton/slepton, chargino/charged gauge and Higgs mode, neutralino/neutral gauge \(\hat{\epsilon}\), with \(\hat{\delta}\).

For each sneutrino species, we have:

\[
\hat{\delta} m^2_{\tilde{H}_d}|_{\tilde{\tau}, \tilde{\tau}} = \frac{3}{32\pi^2} \left[ S(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \left( \frac{g^2 + g_Y^2}{8} \right) + D(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \left( \frac{(g^2 + g_Y^2)}{8} C_t \Delta^2_t \right) \right]
\]

\[
\hat{\delta} m^2_{\tilde{H}_u}|_{\tilde{\tau}, \tilde{\tau}} = \frac{3}{32\pi^2} \left[ S(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \left( \frac{g^2 + g_Y^2}{8} \right) + D(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \left( \frac{(g^2 + g_Y^2)}{8} C_t \Delta^2_t \right) - 2F(m_t, Q_0) \right]
\]

\[
\hat{\delta}(B\mu)|_{\tilde{\tau}, \tilde{\tau}} = \frac{3}{32\pi^2} \left[ D(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \left( A_t \mu h^2_t \right) \right],
\]

where \(C_t\) is \(C_t = (1 - \frac{\theta}{3} \sin^2 \theta_W)\) and \(\Delta^2_t\) defined as \(\Delta^2_t = (m^2_Q - m^2_{U_{333}} + \epsilon_- C_t (g^2 + g_Y^2))/4\).

The functions \(S(m_{\tilde{\tau}_1, \tilde{\tau}_2}), D(m_{\tilde{\tau}_1, \tilde{\tau}_2})\), where the dependence on the scale \(Q_0\) has been suppressed for compactness, can be expressed in terms of \(F(m, Q_0) \equiv 2m^2 \left[ \log(m^2/Q_{0}^2) - 1 \right]\) as:

\[
S(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \equiv F(m_{\tilde{\tau}_1}, Q_0) + F(m_{\tilde{\tau}_2}, Q_0)
\]

\[
D(m_{\tilde{\tau}_1, \tilde{\tau}_2}) \equiv \left( F(m_{\tilde{\tau}_1}, Q_0) - F(m_{\tilde{\tau}_2}, Q_0) \right) / \left( m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2} \right).
\]

The approximation of \(2 \times 2\) mass matrices for each generation of sfermions is used here, in spite of the fact that the building blocks for these matrices are determined without neglecting intergenerational mixing terms. All the corrections listed in this appendix are clearly not sensitive to these tiny effects.

**bottom–sbottom contribution**

\[
\hat{\delta} m^2_{\tilde{H}_d}|_{\tilde{b}, \tilde{b}} = \frac{3}{32\pi^2} \left[ S(m_{\tilde{b}_1, \tilde{b}_2}) \left( \frac{h^2_b - (g^2 + g_Y^2)}{8} \right) + D(m_{\tilde{b}_1, \tilde{b}_2}) \left( \frac{(g^2 + g_Y^2)}{8} C_b \Delta^2_b \right) - 2F(m_b, Q_0) \right]
\]

\[
\hat{\delta} m^2_{\tilde{H}_u}|_{\tilde{b}, \tilde{b}} = \frac{3}{32\pi^2} \left[ S(m_{\tilde{b}_1, \tilde{b}_2}) \left( \frac{g^2 + g_Y^2}{8} \right) + D(m_{\tilde{b}_1, \tilde{b}_2}) \left( \frac{(g^2 + g_Y^2)}{8} C_b \Delta^2_b \right) \right]
\]

\[
\hat{\delta}(B\mu)|_{\tilde{b}, \tilde{b}} = \frac{3}{32\pi^2} \left[ D(m_{\tilde{b}_1, \tilde{b}_2}) \left( A_b \mu h^2_b \right) \right],
\]

with \(C_b \equiv (1 - \frac{\theta}{3} \sin^2 \theta_W)\) and \(\Delta^2_b\) now defined as \(\Delta^2_b \equiv (m^2_Q - m^2_{D_{333}} - \epsilon_- C_b (g^2 + g_Y^2))/4\).

**sneutrino contribution**

For each sneutrino species, we have:

\[
\hat{\delta} m^2_{\tilde{H}_d}|_{\tilde{\nu}} = \frac{1}{32\pi^2} \left[ F(m_{\tilde{\nu}}, Q_0) \left( \frac{g^2 + g_Y^2}{8} \right) \right]
\]
\[
\delta m_{H_u}^2|_\nu = \frac{1}{32\pi^2} \left[ -F(m_{\nu}, Q_0) \left( g^2 + g_\rho^2 \right) \right]
\]
\[
\delta(B\mu)|_{\nu} = 0.
\] (B8)

**tau–stau contribution**

It can be obtained from the bottom-bottom contribution after suppression of the colour factor 3, with obvious replacements, i.e. $h_b \to h_\tau$, $m_b \to m_\tau$, $m_{\tilde{b}_1} \to m_{\tilde{\tau}_1}$, $C_{\tilde{b}} \to C_{\tilde{\tau}} \equiv (1 - 4 \sin^2 \theta_W)$, and $\Delta_{\tilde{b}}^2 \to \Delta_{\tilde{\tau}}^2 \equiv (m_{\tilde{L}}^2 - m_{\tilde{E}}^2)_{33} - \epsilon_{\tau} C_{\tau}(g^2 + g_\rho^2)/4$.

**first–two–generations quark/squark, lepton/slepton contribution**

Since the approximation of massless first–two–generations quarks and leptons is used in this analysis, this contribution can be obtained from [B5][B7] and the tau-stau contribution putting to zero yukawa couplings, quark and lepton masses, and substituting third generation sfermion masses with the corresponding first and second generation ones. This contribution is rather small: there are no corrections coming from this sector to $B\mu$, whereas the corrections to $m_{H_u}^2$ and $m_{H_d}^2$ from the two component of a $SU(2)_L$ doublet supermultiplet, although singularly not small, tend to cancel each other without the diversifying effect of the yukawa couplings.

**chargino/charged boson contribution**

The spin 1/2 modes, charginos, contribute as:

\[
\delta m_{H_d}^2|_{\tilde{\chi}^\pm} = \frac{1}{32\pi^2} g^2 \left[ -S(m_{\tilde{\chi}_1, \tilde{\chi}_2}) - D(m_{\tilde{\chi}_1, \tilde{\chi}_2}) \left( M_2^2 + \mu^2 + g^2 \epsilon_- \right) \right]
\]
\[
\delta m_{H_u}^2|_{\tilde{\chi}^\pm} = \frac{1}{32\pi^2} g^2 \left[ -S(m_{\tilde{\chi}_1, \tilde{\chi}_2}) - D(m_{\tilde{\chi}_1, \tilde{\chi}_2}) \left( M_2^2 + \mu^2 - g^2 \epsilon_- \right) \right]
\]
\[
\delta(B\mu)|_{\tilde{\chi}^\pm} = \frac{1}{32\pi^2} g^2 \left[ D(m_{\tilde{\chi}_1, \tilde{\chi}_2})(2M_2\mu) \right],
\] (B9)

whereas the charged gauge boson mode gives:

\[
\delta m_{H_u}^2|_{W^\pm} = \frac{3}{32\pi^2} \left[ F(M_W, Q_0) \frac{g^2}{2} \right]
\]
\[
\delta m_{H_d}^2|_{W^\pm} = \frac{3}{32\pi^2} \left[ F(M_W, Q_0) \frac{g^2}{2} \right]
\]
\[
\delta(B\mu)|_{W^\pm} = 0,
\] (B10)

with $M_W$ defined as $M_W = \epsilon_+ g^2/2$. There are also two charged Higgs modes $H^\pm, G^\pm$ with “masses” given by

\[
m_{H_{\pm, G_{\pm}}}^2 = \frac{1}{2} \left\{ \mu_{H_d}^2 + \mu_{H_u}^2 + \frac{g^2}{2} \epsilon_\pm \left[ \left( \mu_{H_d}^2 - \mu_{H_u}^2 + \frac{g^2}{2} \epsilon_- \right)^2 + \left( 2(B\mu) + \frac{g^2}{2} \epsilon_3 \right)^2 \right]^2 \right\},
\]

with $\mu_{H_i} \equiv m_{H_i}^2 + \mu^2$. At the $V_0$–minimum, where the relations $\mu_{H_d}^2 + (g^2 + g^2_\rho)(v_d^2 - v_u^2)/4 = (B\mu)\tan\beta$ and $\mu_{H_u}^2 - (g^2 + g^2_\rho)(v_d^2 - v_u^2)/4 = (B\mu)\cot\beta$ hold, $G^\pm$ is the charged Goldstone boson, $H^\pm$ the usual charged Higgs, with mass $m_{H_{\pm}}^2 = \mu_{H_d}^2 + \mu_{H_d}^2 + M_W^2$. Both modes, however, contribute to the determination of the $V_1$–minimum (where the charged Goldstone boson, as well as the neutral
one mentioned below, are not massless anymore). They shift the variables $m_{H_d}^2$, $m_{H_u}^2$ and $B\mu$ by the quantities:

\[ \delta m_{H_d}^2 |_{H^\pm,G^\pm} = \frac{1}{32\pi^2} \left[ S(m_{H^\pm,G^\pm}) \frac{g_2^2}{4} + D(m_{H^\pm,G^\pm}) \frac{g_2^2}{4} \left( m_{H_d}^2 - m_{H_u}^2 + \frac{g^2}{2} \epsilon_- \right) \right] \]

\[ \delta m_{H_u}^2 |_{H^\pm,G^\pm} = \frac{1}{32\pi^2} \left[ S(m_{H^\pm,G^\pm}) \frac{g_2^2}{4} - D(m_{H^\pm,G^\pm}) \frac{g_2^2}{4} \left( m_{H_d}^2 - m_{H_u}^2 + \frac{g^2}{2} \epsilon_- \right) \right] \]

\[ \delta (B\mu) |_{H^\pm,G^\pm} = \frac{1}{32\pi^2} \left[ -D(m_{H^\pm,G^\pm}) \frac{g_2^2}{4} \left( 2(B\mu) + \frac{g^2}{2} \epsilon_3 \right) \right]. \]  

(B11)

neutralino/neutral boson contribution

In this sector, there are: i) one gauge boson mode, with “mass” $M_Z \equiv \epsilon_+ (g^2 + g_Y^2)/2$, which contribute as:

\[ \delta m_{H_d}^2 |_{Z} = \frac{3}{32\pi^2} \left[ F(M_Z, Q_0) \frac{(g^2 + g_Y^2)}{4} \right] \]

\[ \delta m_{H_u}^2 |_{Z} = \frac{3}{32\pi^2} \left[ F(M_Z, Q_0) \frac{(g^2 + g_Y^2)}{4} \right] \]

\[ \delta (B\mu) |_{Z} = 0; \]  

(B12)

ii) two neutral pseudoscalar Higgs modes $A^0, G^0$ with “masses”

\[ m_{A^0,G^0}^2 = \frac{1}{2} \left\{ \mu_{H_d}^2 + \mu_{H_u}^2 \pm \left[ \left( \mu_{H_d}^2 - \mu_{H_u}^2 + \frac{(g^2 + g_Y^2)}{2} \epsilon_- \right)^2 + (2(B\mu))^2 \right] \right\}^{\frac{1}{2}} \]

At the $V_0$-minimum $G^0$ is massless, $A^0$ is the usual pseudoscalar Higgs with mass $m_{A^0}^2 = m_{H_d}^2 + m_{H_u}^2 + 2\mu^2$. They contribute as:

\[ \delta m_{H_d}^2 |_{A^0,G^0} = \frac{1}{32\pi^2} \left( \frac{(g^2 + g_Y^2)}{8} \right) \left[ D(m_{A^0,G^0}) \left( m_{H_d}^2 - m_{H_u}^2 + \frac{(g^2 + g_Y^2)}{2} \epsilon_- \right) \right] \]

\[ \delta m_{H_u}^2 |_{A^0,G^0} = \frac{1}{32\pi^2} \left( \frac{(g^2 + g_Y^2)}{8} \right) \left[ -D(m_{A^0,G^0}) \left( m_{H_d}^2 - m_{H_u}^2 + \frac{(g^2 + g_Y^2)}{2} \epsilon_- \right) \right] \]

\[ \delta (B\mu) |_{A^0,G^0} = 0; \]  

(B13)

iii) two neutral scalar Higgs modes with “masses”

\[ m_{H^0,h^0}^2 = \frac{1}{2} \left\{ \mu_{H_d}^2 + \mu_{H_u}^2 + \left( \frac{(g^2 + g_Y^2)}{2} \right) \epsilon_+ \pm \left[ \left( \mu_{H_d}^2 - \mu_{H_u}^2 + (g^2 + g_Y^2) \epsilon_- \right)^2 + (2(B\mu) + \frac{(g^2 + g_Y^2)}{2} \epsilon_3) \right] \right\}^{\frac{1}{2}} \]

which, at the $V_0$-minimum, reduce to the conventional neutral Higgs masses $m_{H^0,h^0}^2 = \{ \mu_{H_d}^2 + \mu_{H_u}^2 + M_Z^2 \pm [(\mu_{H_d}^2 + \mu_{H_u}^2 - M_Z^2)^2 \cos^2 2\beta + (\mu_{H_d}^2 + \mu_{H_u}^2 + M_Z^2)^2 \sin^2 2\beta]^{1/2}\} / 2$. They produce the shifts:

\[ \delta m_{H_d}^2 |_{H^0,h^0} = \frac{1}{32\pi^2} \left( \frac{(g^2 + g_Y^2)}{4} \right) \left[ \frac{1}{2} S(m_{H^0,h^0}) + D(m_{H^0,h^0}) \left( m_{H_d}^2 - m_{H_u}^2 + (g^2 + g_Y^2) \epsilon_- \right) \right] \]

\[ \delta m_{H_u}^2 |_{H^0,h^0} = \frac{1}{32\pi^2} \left( \frac{(g^2 + g_Y^2)}{4} \right) \left[ \frac{1}{2} S(m_{H^0,h^0}) - D(m_{H^0,h^0}) \left( m_{H_d}^2 - m_{H_u}^2 + (g^2 + g_Y^2) \epsilon_- \right) \right] \]

\[ \delta (B\mu) |_{H^0,h^0} = \frac{1}{32\pi^2} \left( \frac{(g^2 + g_Y^2)}{4} \right) \left[ -D(m_{H^0,h^0}) \left( (B\mu) + \frac{(g^2 + g_Y^2)}{2} \epsilon_3 \right) \right]; \]  

(B14)
iv) four spin 1/2 modes, the neutralinos. We find their contribution following [26,27,28]:

\[
\delta m_H^2 | \chi^0 = \frac{1}{32\pi^2} \left[ -2 \sum_{i=1}^{4} F(m_{\chi_i},Q_0) \partial_{\epsilon_i} m_{\chi_i}^2 \right]
\]

\[
\delta m_{H_u}^2 | \chi^0 = \frac{1}{32\pi^2} \left[ -2 \sum_{i=1}^{4} F(m_{\chi_i},Q_0) \partial_{\epsilon_u} m_{\chi_i}^2 \right]
\]

\[
\delta (B\mu) | \chi^0 = \frac{1}{32\pi^2} \left[ +2 \sum_{i=1}^{4} F(m_{\chi_i},Q_0) \partial_{\epsilon_3} m_{\chi_i}^2 \right]
\]

where in turn, the derivatives of the masses are:

\[
\partial_{\epsilon_i} m_{\chi_i}^2 = -\frac{m_{\chi_i}^6 (\partial_{\epsilon_i} c_2) + m_{\chi_i}^4 (\partial_{\epsilon_i} c_4) + m_{\chi_i}^2 (\partial_{\epsilon_i} c_6) + (\partial_{\epsilon_i} c_8)}{c_6 + 2c_4 m_{\chi_i}^2 + 3c_2 m_{\chi_i}^4 + 4c_0 m_{\chi_i}^6} \epsilon_i = \epsilon_d, \epsilon_u, \epsilon_3.
\]

The coefficients \( c_i \) are the coefficients of the characteristic equation \( c_0 m_{\chi_i}^8 + c_2 m_{\chi_i}^6 + c_4 m_{\chi_i}^4 + c_6 m_{\chi_i}^2 + c_8 = 0 \), for the matrix \( M_{\chi_0} \), with all Higgs dependences reinstated back in \( M_{\chi_0} \).

(The fact that at the true minimum charged Higgses do not develop vacuum expectation values is already imposed, allowing therefore the chargino and neutralino mass matrices to decouple.) They are:

\[
\begin{align*}
    c_0 &= +1 \\
    c_2 &= -2\mu_1^2 - M_1^2 - M_2^2 - (g^2 + g_Y^2)\epsilon_+ \\
    c_4 &= +\mu^2 + 2\mu^2 (M_1^2 + M_2^2) + M_1^2 M_2^2 + [\mu^2 (g^2 + g_Y^2) + (M_2^2 g^2 + M_1^2 g_Y^2)]\epsilon_+ \\
    &\quad - \mu (M_1 g^2 + M_2 g_Y^2) \epsilon_3 + (g^2 + g_Y^2)^2 \epsilon_+^2 / 4 \\
    c_6 &= -\mu^2 (M_1^2 + M_2^2) - 2\mu^2 M_1^2 M_2^2 + \mu [\mu^2 (M_1 g^2 + M_2 g_Y^2) + M_1 M_2 (M_2 g^2 + M_1 g_Y^2)]\epsilon_3 \\
    &\quad - \mu^2 (g^2 + g_Y^2)^2 \epsilon_d \epsilon_u - \mu^2 (M_2^2 g^2 + M_1^2 g_Y^2) \epsilon_+ - (M_2 g^2 + M_1 g_Y^2)^2 \epsilon_+^2 / 4 \\
    c_8 &= +\mu^2 [\mu^2 M_1^2 M_2^2 - \mu M_1 M_2 (M_2 g^2 + M_1 g_Y^2) \epsilon_3 + (M_2 g^2 + M_1 g_Y^2)^2 \epsilon_d \epsilon_u].
\end{align*}
\]

Appendix C.

We collect here the numerical entries of the up- and down-quark mass matrices, their eigenvalues and eigenvectors, as obtained at \( \Lambda = 100 \text{TeV} \). For better readability, we write here “linear” mass matrices: their elements are the square root of the entries in the real mass matrix; the reported signs are those of the corresponding squared elements. The up–quark mass matrix, split in the four left–left, left–right, right–left and right–right submatrices is:

\[
\tilde{M}_U = \begin{pmatrix}
1099.432 & 0.000 & -25.320 & 0.000 & 0.000 & -3.879 \\
0.000 & 1099.432 & 53.712 & 0.000 & 0.000 & 8.228 \\
-25.320 & 53.712 & 1044.930 & 0.000 & 0.000 & 38.766 \\
0.000 & 0.000 & 0.000 & 1054.342 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 1054.342 & 0.000 \\
-3.879 & 8.228 & -38.766 & 0.000 & 0.000 & 976.915
\end{pmatrix}.
\]

As in [1], we work in a quark basis in which the transition from current to mass eigenstates is obtained through the rotation of the left–handed up quarks only. The rotation matrix is the
Kobayashi–Maskawa matrix. The effect of this matrix on both up– and down–squark mass matrices is kept up to $O(\lambda^3)$, where $\lambda$ is the Cabibbo angle.

The diagonalization matrix for $\tilde{M}_u$ is:

$$
D_U = \begin{pmatrix}
.0001 & -.0004 & .0108 & .0000 & .0000 & .9999 \\
.0053 & -.0238 & .9996 & .0000 & .0000 & -.0108 \\
.0000 & .0000 & .0000 & .0369 & .9993 & .0000 \\
.0000 & .0000 & .0000 & -.9993 & .0369 & .0000 \\
-.9762 & -.2169 & .9759 & .0000 & .0000 & .0244 \\
-.2169 & .9759 & .0244 & .0000 & .0000 & .0001
\end{pmatrix}
$$

$$
k = 1 \quad 976.907 \\
2 \quad 1044.902 \\
3 \quad 1054.342 \\
4 \quad 1054.342 \\
5 \quad 1099.432 \\
6 \quad 1099.466
\end{pmatrix}
\right) \quad (C2)

Each row $k$ represents the decomposition of the $k$–eigenvector over the initial basis $(\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)$. The $6 \times 3$ matrices on the left and right of the vertical bar are, then, nothing else but the matrices $\Gamma_{UL}$ and $\Gamma_{UR}$ used in the text. The last column of numbers in (C2) gives for each $k$, the corresponding eigenvalue in GeV.

The mass matrix for the down–squark sector looks like:

$$
\tilde{M}_D = \begin{pmatrix}
1102.331 & 0.000 & -30.548 & .0000 & .0000 & .0000 \\
0.000 & 1102.331 & 64.803 & .0000 & .0000 & .0000 \\
-30.548 & 64.803 & 1033.967 & .0000 & .0000 & -.256.273 \\
0.000 & 0.000 & 0.000 & 1050.701 & 0.000 & .0000 \\
0.000 & 0.000 & 0.000 & 0.000 & 1050.701 & 0.000 \\
0.000 & 0.000 & -256.273 & 0.000 & 0.000 & 999.164
\end{pmatrix}
$$

\right) \quad (C3)

and the corresponding diagonalization matrix is:

$$
D_D = \begin{pmatrix}
.0018 & -.0081 & .5101 & .0000 & .0000 & .8601 \\
.0000 & .0000 & .0000 & .9998 & .0211 & .0000 \\
.0000 & .0000 & .0000 & -.0211 & .9998 & .0000 \\
-.0073 & .0329 & -.8595 & .0000 & .0000 & .5100 \\
-.9762 & -.2169 & .9756 & .0000 & .0000 & .0244 \\
.2168 & -.9756 & -.0332 & .0000 & .0000 & .0101
\end{pmatrix}
$$

$$
k = 1 \quad 979.331 \\
2 \quad 1050.701 \\
3 \quad 1050.701 \\
4 \quad 1052.701 \\
5 \quad 1102.331 \\
6 \quad 1102.397
\end{pmatrix}
\right) \quad (C4)

where the $6 \times 3$ matrices on the left and right of the vertical bar are $\Gamma_{DL}$ and $\Gamma_{DR}$. The differences in the elements (1,1), (2,2), (4,4), (5,5) in $\tilde{M}_U$ and $\tilde{M}_D$ are due to the different D–terms present in the two sectors. The off–diagonal elements (1,3), (2,3) in $\tilde{M}_D$ are induced by the non-diagonal up quark yukawa couplings during the evolution from $X$ to $Q_0$. The same elements in $\tilde{M}_U$ sum also the supersymmetry non–violating terms induced by up quark yukawa couplings.
References

[1] M. Dine, A.E. Nelson, and Y. Shirman, Phys. Rev. D 51 (1995) 1362; M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D 53 (1996) 2658
[2] K.S. Babu, C. Kolda, and F. Wilczek, Phys. Rev. Lett. 77 (1996) 3070
[3] M. Dine, Y. Nir, and Y. Shirman, Phys. Rev. D 55 (1997) 1501
[4] S. Dimopoulos, S. Thomas, and J.D. Wells, hep-ph/9609434
[5] J.A. Bagger, K. Matchev, D.M. Pierce, and R. Zhang, Phys. Rev. D 55 (1997) xxx, hep-ph/9609444
[6] S. Martin, Phys. Rev. D 55 (1997) xxx, hep-ph/9608224
[7] S. Dimopoulos, G.F. Giudice, and A. Pomarol, hep-ph/9607225
[8] N. Polonsky, Phys. Rev. D 54 (1996) 4537
[9] S. Bertolini, F.M. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. B 353 (1991) 591
[10] M. Carena, S. Pokorski, and C. Wagner, Nucl. Phys. B 426 (1994) 269
[11] R. Rattazzi and U. Sarid, Phys. Rev. D 53 (1996) 1553
[12] S. Martin and M. Vaughn, Phys. Rev. D 50 (1994) 2282
[13] G. Gamberini, G. Ridolfi, and F. Zwirner, Nucl. Phys. B 331 (1990) 331
[14] J.A. Casas, A. Lleyda, and C. Munoz, Nucl. Phys. B 471 (1996) 3
[15] H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75
[16] S. Dimopoulos, M. Dine, S. Raby, and S. Thomas, Phys. Rev. Lett. 76 (1996) 3494 S. Ambrosanio, G. Kane, G. Kribs, S. Martin, and S. Mrenna, Phys. Rev. Lett. 76 (1996) 3498
[17] H. Baer, M. Brhlik, C. Chen, and X. Tata, hep-ph #9610358 N.G. Deshpande, B. Dutta, and S. Oh, hep-ph/9611443
[18] A. J. Buras, M. Misiak, M. Münz, and S. Pokorski, Nucl. Phys. B 424 (1994) 374
[19] F.M. Borzumati, M. Drees, and M.M. Nojiri, Phys. Rev. D 51 (1995) 27
[20] F.M. Borzumati, Zeit. für Physik C 63 (1994) 291
[21] M.S. Alam, CLEO Collaboration, Phys. Rev. Lett. 74 (1995) 2885
[22] D. Toback, CDF Collaboration, DPF96 proceedings K. Wyatt Merrit, “Searches for New Physics at the Tevatron”, DPF96, hep-ex #9701009
[23] J.F. Gunion and H.E. Haber, Nucl. Phys. B 272 (1986) 1
[24] J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B 262 (1991) 477
[25] M. Drees and M. Nojiri, Phys. Rev. D 45 (1992) 2482
[26] A. Brignole, J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B 271 (1991) 123
[27] H. Eberl and W. Majerotto, Vienna University preprint, HEPHY-PUB 595/93 (1993), unpublished.
[28] D. Comelli, M. Pietroni, and A. Riotto, Phys. Rev. D 50 (1994) 7703