Precision study of $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ and $K^{\pm} \rightarrow \pi^{\pm}\pi^+\pi^-$ Dalitz plot distributions by NA48/2

Evgueni Goudzovski
Scuola Normale Superiore and INFN, Pisa, Italy
E-mail: goudzovs@mail.cern.ch

The NA48/2 experiment at the CERN SPS has collected an unprecedented sample of $K^{\pm} \rightarrow 3\pi$ decays. The high statistics and the good resolution of the detectors allow a unique investigation of the detailed phase space distributions of these decays. The effects of final state pion rescattering observed in the Dalitz plot distribution of the $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ decays turned out to be a powerful tool for extraction of the S-wave $\pi\pi$ scattering lengths. The large statistics also allowed a precise measurement of the Dalitz plot slope parameters for the $K^{\pm} \rightarrow 3\pi^{\pm}$ decays.
Introduction

The primary goal of the NA48/2 experiment at the CERN SPS is the search for direct CP violation in $K^\pm \rightarrow 3\pi$ decays [1]. Data have been collected in 2003–04, providing samples of $\sim 4 \times 10^9$ fully reconstructed $K^\pm \rightarrow 3\pi^\pm$ and $\sim 10^8 K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays. Surprisingly, a study of a partial sample of $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays corresponding to about 25% of the total sample revealed an anomaly in the $\pi^0 \pi^0$ invariant mass ($M_{00}$) distribution in the region around $M_{00} = 2m_+$, where $m_+$ is the charged pion mass [2]. This anomaly, dubbed “cusp effect”, never observed in previous experiments, was theoretically interpreted as an effect due mainly to the final state charge exchange scattering process $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ in $K^\pm \rightarrow 3\pi^\pm$ decay, and was shown to provide a precise determination of $a_0 - a_2$, the difference between the S-wave $\pi\pi$ scattering lengths in the isospin $I = 0$ and $I = 2$ states [3]. A number of theoretical approaches to describe this process are being developed; the original NA48/2 measurement of $a_0 - a_2$ was performed in the framework of the approach [4]. The current paper presents a new step of the analysis, namely a preliminary result of a measurement based on the full NA48/2 data sample within the same theoretical framework.

In addition, a measurement of the Dalitz plot slopes of the $K^\pm \rightarrow 3\pi^\pm$ decay based on a partial NA48/2 data sample is presented.

1. NA48/2 experimental setup

Two simultaneous $K^+$ and $K^-$ beams are produced by 400 GeV protons impinging on a 40 cm long Be target. Particles with a central momentum of 60 GeV/c and a momentum band of $\pm 3.8\%$ produced at zero angle are selected by a system of dipole magnets forming an “achromat” with null total deflection, focusing quadrupoles, muon sweepers and collimators. With $7 \times 10^{11}$ protons per burst of 4.5 s duration incident on the target the positive (negative) beam flux at the entrance of the decay volume is $3.8 \times 10^7$ ($2.5 \times 10^7$) particles per pulse, of which 5.7% (4.9%) are $K^+$ ($K^-$). The decay volume is a 114 m long vacuum tank.

Charged particles from $K^\pm$ decays are measured by a magnetic spectrometer consisting of four drift chambers and a large-aperture dipole magnet located between the second and third chamber. Each chamber has eight planes of sense wires: two horizontal, two vertical and two along each of two orthogonal 45° directions. The spectrometer is located in a tank filled with helium at atmospheric pressure and separated from the decay volume by a thin (0.31%$X_0$) Kevlar window. A 16 cm diameter vacuum tube centered on the beam axis runs through the spectrometer and subsequent detectors. Charged particles are magnetically deflected in the horizontal plane by an angle corresponding to a transverse momentum kick of 120 MeV/c. The momentum resolution of the spectrometer is $\sigma(p)/p = 1.02\% \oplus 0.044\%p$ ($p$ in GeV/c). The spectrometer is followed by a scintillator hodoscope consisting of two planes segmented into horizontal and vertical strips.

A liquid krypton calorimeter is used to reconstruct $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays. It is an almost homogeneous ionization chamber with an active volume of 7 m$^3$ of liquid krypton, segmented transversally into 13248 projective cells of $2 \times 2$ cm$^2$ by a system of Cu-Be ribbon electrodes, and with no longitudinal segmentation. The calorimeter is 27$X_0$ thick and has an energy resolution $\sigma(E)/E = 0.032/\sqrt{E} \oplus 0.09/E \oplus 0.0042$ ($E$ in GeV). Spatial resolution for a single electromagnetic shower is $\sigma_x = \sigma_y = 0.42\%/\sqrt{E} \oplus 0.06$ cm for each transverse coordinate $x, y$. 

A detailed description of the components of the NA48 detector can be found elsewhere [5].

2. Cusp effect and measurement of pion scattering lengths

The reconstructed spectra of $\pi^0\pi^0$ invariant mass $M_{00}$ for 2003 and 2004 data samples (totally $59.6 \times 10^9$ events) are presented in Fig. 1. The change of slope at $\pi^+\pi^-$ threshold is clearly visible. For description of this effect the $K^\pm \rightarrow \pi^\pm \pi^0\pi^0$ amplitude is presented as a sum of two terms:

$$M = M_0 + M_1,$$

(2.1)

where $M_0$ is the “unperturbed” amplitude expressed as a polynomial expansion in terms of the kinematic variables $u = (s_3 - s_0)/m_K^2$ and $v = (s_1 - s_2)/m_i^2$, where $s_i = (P_K - P_i)^2$, $s_0 = (s_1 + s_2 + s_3)/3$, $P_K$ and $P_i$ are 4-momenta of kaon and pions, and $i = 1, 2$ correspond to the two “even” (i.e. identical) pions:

$$M_0(u, v) = M_0(0, 0) \cdot (1 + g_0 u^2 + h' u^2 v^2 / 2 + k' v^2 / 2),$$

(2.2)
Precision study of $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ and $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ Dalitz plot distributions by NA48/2

Figure 2: Deviation of the data spectrum from the fit result with statistical errors (combined 2003+2004 data set): $\Delta = \text{Data/Fit} - 1$. Good quality of the fit and an excess of events in the region of the threshold are demonstrated.

Figure 2: Deviation of the data spectrum from the fit result with statistical errors (combined 2003+2004 data set): $\Delta = \text{Data/Fit} - 1$. Good quality of the fit and an excess of events in the region of the threshold are demonstrated.

and $\mathcal{M}_1$ is a contribution from the $K^\pm \rightarrow 3\pi^\pm$ decay amplitude $\mathcal{M}_+$ through $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ charge exchange, which in particular simplest case of the original Cabibbo theory [5] is given by

$$\mathcal{M}_1 = -2a_3 m_+ \mathcal{M}_+ \sqrt{1 - (M_{00}/2m_+)^2}. \quad (2.3)$$

Here, in the limit of exact isospin symmetry, $a_3 = (a_0 - a_2)/3$. The amplitude $\mathcal{M}_1$ changes from real to imaginary at the threshold $M_{00} = 2m_+$; as a consequence it interferes destructively with $\mathcal{M}_0$ below the threshold (leading to 13% integral depletion in this region), and adds quadratically above the threshold.

The model used for the present measurement is based on the formulation [4], which takes into account all rescattering precesses at the one-loop and two-loop level. In this approach the matrix element of the $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decay includes a number of additional terms depending on five $S$-wave $\pi\pi$ scattering lengths (corresponding to the processes $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$, $\pi^+ \pi^+ \rightarrow \pi^+ \pi^+$, $\pi^+ \pi^- \rightarrow \pi^+ \pi^-$, $\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$ and $\pi^0 \pi^0 \rightarrow \pi^0 \pi^0$) expressed as linear combinations of $a_0$ and $a_2$. In addition to [4], isospin breaking effects are taken into account introducing a single parameter $\epsilon = (m_+^2 - m_0^2)/m_+^2 = 0.065$ [5].

The fit to extract the scattering lengths and Dalitz plot slopes $g_0, h'$ was performed in the $M_{00}$ projection of the data using a full GEANT-based Monte Carlo simulation of the detector response. The used rescattering model does not include radiative corrections, which are particularly important at the threshold $M_{00} = 2m_+$, and contribute to formation of $\pi^+ \pi^-$ atoms (pionium). Thus a group of seven bins near the threshold has been excluded from the fit. The quality of the fit ($\chi^2$/NDF = 164/139 for 2003 analysis, and $\chi^2$/NDF = 119/139 for 2004 analysis) illustrated in Fig. 2 shows an excess of events in this excluded region. This excess, being interpreted as due to pionium formation, yields the rate of pionium formation $R = \Gamma(K^\pm \rightarrow \pi^+ A_{2\pi})/\Gamma(K^\pm \rightarrow 3\pi^\pm) = (1.82 \pm 0.21) \times 10^{-5}$, somewhat higher than a theoretical prediction [7].

Measurement of the quadratic Dalitz plot slope $k'$ was performed using the $v$ projection of the data and fixing the values of $a_0, a_2, g_0$ and $h'$ measured by the above method. Then the fit in $M_{00}$ projection was re-iterated to account for the measured non-zero value of $k'$. 
Systematic uncertainties due to fitting technique, trigger efficiency, description of geometric acceptance and resolution, calorimeter non-linearity, and simulation of showers in the calorimeter have been evaluated. External uncertainties due to limited experimental knowledge of \( \mathcal{M}_+ / \mathcal{M}_0 \) at the \( \pi^+ \pi^- \) threshold have been also considered. Stability checks with respect to decay vertex position, particle separations in the calorimeter front plane, and kaon sign have been performed.

**Results and conclusions**

The original NA48/2 measurement of the of \( \pi \pi \) scattering lengths [2] by exploring the cusp effect in the \( K^\pm \to \pi^\pm \pi^0 \pi^0 \) decay spectrum has been improved: the full NA48/2 data sample has been used, and a more elaborate study of systematic effects performed. The model [4] with isospin breaking corrections has been used. The measured scattering lengths are:

\[
(a_0 - a_2)m_+ = 0.261 \pm 0.006_{\text{stat}} \pm 0.003_{\text{syst}} \pm 0.001_{\text{ext}}.
\]

\[
a_2m_+ = -0.037 \pm 0.013_{\text{stat}} \pm 0.009_{\text{syst}} \pm 0.002_{\text{ext}}.
\]

The external uncertainties are due to the limited knowledge of \( \Gamma(K^\pm \to \pi^\pm \pi^0 \pi^0) / \Gamma(K^\pm \to 3\pi^\pm) \). Moreover, an uncertainty \( m_+ \delta(a_0 - a_2) = 0.013 \) has to be attributed to the result due to precision of the theoretical model. The Dalitz plot slopes corresponding to the used model are found to be:

\[
g_0 = 0.649 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}}.
\]

\[
h' = -0.047 \pm 0.007_{\text{stat}} \pm 0.005_{\text{syst}}.
\]

\[
k' = -0.0097 \pm 0.0003_{\text{stat}} \pm 0.0008_{\text{syst}}.
\]

In addition, a measurement of the Dalitz plot slopes of the PDG parameterization [8] of the \( K^\pm \to 3\pi^\pm \) decay with a sample of \( 4.71 \times 10^8 \) fully reconstructed events yielded the following results:

\[
g = -0.21134 \pm 0.00013_{\text{stat}} \pm 0.00010_{\text{syst}}.
\]

\[
h = 0.01848 \pm 0.00022_{\text{stat}} \pm 0.00033_{\text{syst}}.
\]

\[
k = -0.00463 \pm 0.00007_{\text{stat}} \pm 0.00012_{\text{syst}}.
\]

This measurement is described in detail in [9]. The results are compatible with the world average, and demonstrate the validity of the conventional parameterization at the new level of precision.

**References**

[1] J.R. Batley et al., Phys. Lett. **B634** (2006) 474.

J.R. Batley et al., Phys. Lett. **B638** (2006) 22.

[2] J.R. Batley et al., Phys. Lett. **B633** (2006) 173.

[3] N. Cabibbo, Phys. Rev. Lett. **93** (2004) 121801.

[4] N. Cabibbo and G. Isidori, JHEP **0503** (2005) 021.

[5] V. Fanti et al. (NA48), Nucl. Instrum. Methods **A574** (2007) 433.

[6] K. Maltman and C.E. Wolfe, Phys. Lett. **B393** (1997) 19, erratum-ibid. **B424** (1998) 413.

K. Knecht and R. Urech, Nucl. Phys. **B519** (1998) 329.

[7] Z.K. Silagadze, JETP Lett. **60** (1994) 689.

[8] W.-M. Yao et al. (PDG), J. Phys. **G33** (2006) 1.

[9] J.R. Batley et al., Phys. Lett. **B649** (2007) 349.