Design of centralized controller for multivariable process using MOPSO algorithm

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Abstract

Objective: To estimate centralized PID controller parameters for 4 outputs and 5 inputs crude distillation non-square system with RHP zeros process. Methods/Analysis: The Multi-Objective Particle Swarm Optimization (MOPSO) algorithm is applied to determine the PID controller parameters for the considered distillation column process. Findings: The performance of the proposed controller is compared with two centralized controller schemes, Davison’s and Tanttu and Lieslehto methods. The Integral Square Error (ISE), Integral Absolute Error (IAE) and Integral of Time Absolute Error (ITAE) are chosen as performance indices. The simulation results prove that MOPSO tuned centralized controller gives the best performance when compared to other analytical techniques for both set point tracking and in disturbance rejection environment. Novelty: In practice, conventional PID controllers are tuned using classical methods, which require complex numerical calculations. In this paper, an attempt is made to fine tune the PID controller for a MIMO process using Multi Objective optimization technique and obtained challenging results as compared to conventional methods.

Keywords: Nonsquare system; Centralized control; Multi Objective Particle Swarm Optimization; PID controller

1 Introduction

Most of the industrial control processes are multivariable processes. Design of controller for a multivariable process is difficult because of the interaction effect among the loops. The interaction is mainly due to changes in one input effect with respect to several outputs. Distillation column is a major unit of operation in chemical, oil and gas processes. Design of controllers for the distillation column poses a tedious job in the process control instrumentation field. The interaction and location of transmission zeros are important in Multi-Input Multi-Output (MIMO) systems. A system is called non -minimum phase system when it has one or more right half plane (RHP) transmission zeros. These RHP zeros impose limitations on stability and controllability of the system (1,2). Design of controller for the system with positive
zeros is a major task because it will affect both the amplitude and phase angle. Adding extra phase lag by RHP zero causes instability to the process and is also difficult to control. The processes with an unequal number of inputs and outputs are called non-square systems and these systems have two control methods which are called centralized and decentralized controllers. The feedback will be used in the centralized controller and each input is manipulated from all measured outputs. In decentralized controller feedback will be implemented after pairing one output with one input.

Many heuristic algorithms such as Differential Evolution (DE), Simulated Annealing (SA), Particle Swarm Optimization (PSO), Genetic Algorithm (GA) and Bacterial Forage algorithm are powerful methods for solving many non-linear and tough optimization problems. The multi-objective particle swarm optimization (MOPSO) is one of the most promising stochastic search methods among these metaheuristics which originates from the simulation of behavior of bird flocks, because of its easy implementation and high convergence speed.

Liu et al\(^3\) proposed modified Internal Model Control (IMC) method and Smith delay compensator structure based on static decoupling for non-square processes with right half plant zeros and multiple time delays. The performances of the proposed controllers provided improved response when compared with other methods mentioned in the literature. Guo et al\(^4\) suggested Smith Decoupling compensation control method which provides good robustness and less interference performance over model mismatch on the system by model approximation using suboptimal reduction algorithm. The PSO based controller designed for spherical tank system shows improved performances when compared with various other optimization algorithms \((5, 6)\).

Madiouni\(^7\) presented MOPSO algorithm based PID controller for various non-linear problems and proved its performances better than the non-dominated sorting genetic algorithm II in terms of better computation time. Ram and Chidambaram\(^8\) proposed steady state gain matrix (SSGM) based centralized PI controller design for a multivariable process and validated its performance compared with controller design based on gain, time delay and time constant. Zhao et al\(^9\) suggested controller design based on MOPSO for MIMO systems and also proved that the designed controller provided improved result in terms of ISE when compared to other optimal PID controllers. The multi-objective optimization algorithm \((10, 11)\) is applied to determine controller parameters to solve various difficult problems. Fu et al\(^12\) employed multi objective optimization algorithm to estimate PI controller parameters to control superheated steam temperature. The designed controller improved the performance of steam temperature control for both set-point tracking and disturbance rejection. Monica et al\(^13\) designed MOPSO based PID controller for ball and beam system and proved that the proposed controller provided improved performance compared to Skogestad’s Internal Model Control in terms of time response analysis.

Perng et al\(^14\) derived PID controller parameters using MOPSO, Genetic Algorithm, Bee colony optimization algorithm, Reinforcement Learning algorithm to water turbine governor based on the frequency domain sensitivity. The estimated PID controller performances are compared based on rise time \((t_r)\), integral square-error, integral of time-multiplied squared-error, integral absolute error, and integral of time multiplied by absolute error. From the obtained results, they proved that MOPSO tuned PID controller provided improved performances when compared with other optimization algorithms. Gomez et al\(^15\) suggested MOPSO based PID controller tuning for Unmanned aerial vehicles. The selected PID tuning parameters are applied to case study of quadrotor and also proved through the simulation results that MOPSO tuned PID controller provided good performance in terms of overshoot, rise time and root-mean-square error. Oliveira et al\(^16\) proposed PSO based PID controller tuning method for Arduino-based Temperature Control Laboratory and compared its performance with the Grey Wolf Optimization algorithm in terms of integral absolute error and the total variation criteria. Through the obtained result it is observed that the PSO tuned controller provided improved performance than the other optimization algorithm.

Gomez et al\(^17\) introduced MOPSO based controller tuning procedure to control altitude for Px4- based Unmanned aerial vehicles. From the simulation results it is evident that the proposed controller gives good performance in terms of rise time, overshoot and root-mean-square error of step response of the P-PID controllers. Al-Khazraji et al\(^18\) designed MOPSO algorithm to select the best system control parameter for production–inventory system with multivariable input and multivariable output. He tested this algorithm at automatic pipeline, inventory, and order based production control system (APIOBPCS) model and the newly modified two automatic pipeline inventory and order based production control system (2APIOBPCS) model for optimal control of production. The simulation results indicated that 2APIOBPCS model performed better than the APIOBPCS model to achieve optimal performance in terms of balancing the order rate and stock level under different conditions.

In this work, the crude distillation column by Levein was considered for study. The MOPSO based optimization algorithm is used to determine the gain matrix. The controller performance is compared with Davison and Tanttu and Lieslehto control methods in terms of ISE, IAE and ITAE. From the simulation results it is identified that MOPSO optimization tuned controller gives improved response than the other methods.
2 Multi Objective Particle Swarm Optimization

Particle Swarm Optimization (PSO) technique, is an evolutionary-type global optimization algorithm developed by Kennedy and Eberhart\(^{(19)}\) based on social activities in flock of birds and school of fish and is widely applied to solve various engineering problems because of its simplicity and high computational efficiency problems that have more than one objective which is referred to as multi-objective optimization. The multi objective problem is found in various fields such as mathematics, engineering, agriculture, social studies, economics, aviation, etc.

Multi-objective optimization (MOO) problems consist of several objectives that are needed to be handled simultaneously. The most suitable MOO problems are Pareto based approaches, because of its ability to generate multiple solutions in less computation time\(^{(20)}\). Due to the high speed of convergence PSO based algorithms are suitable for multi-objective optimization problems. The flowchart shows Multi Objective Particle Swarm Optimization (MOPSO) and it uses the idea of a repository of particles and a mutation operator to get better solution. In detail, at each iteration, the objective value is calculated for each individual and then used to determine the relationship of non-dominance in the population in order to select a
MOPSO is applied to estimate PID controller parameter for the crude distillation column. Distillation column is a major unit operation in chemical, oil and gas processes. Design of controllers for the distillation column poses a tedious job in the process control instrumentation field. The transfer function of crude distillation unit is shown in equation (4). The process has 4 outputs and five inputs. The controlled variables are naphtha/kerosene cut-point (y1), kerosene/light gas oil (LGO) cut-point (y2), LGO/ heavy gas oil (HGO) cut-point (y3) and measured over flash (y4). Manipulated variables are top temperature (u1), kerosene yield (u2), LGO yield (u3), HGO yield (u4) and heater outlet temperature (u5).

Figure 2 illustrates the basic block diagram of an optimization algorithm based PID controller tuning for a considered distillation process. The MOPSO algorithm based controller design is attempted to study and improve the performance of distillation unit. The controller design process is to select the suitable values for tuning parameters from the search space that minimizes the objective function. The objectives are minimization of ISE, IAE and ITAE values. The accuracy of the heuristic algorithm based controller tuning mainly depends on the objective function, which guides the optimization search to estimate suitable controller parameter values.

![Block diagram of optimization scheme](https://www.indjst.org/)

Fig 2. Block diagram of optimization scheme

### 3 Centralized Controller Design

#### 3.1 Davison’s Method

Centralized multivariable PID controller tuning method as suggested by Davison (1) for square systems is discussed as follows. The proportional and integral gain matrices are given by

\[ K_c = \delta [G(S=0)]^{-1} \] (1)

\[ K_I = \varepsilon [G(S=0)]^{-1} \] (2)

Where \([G(S=0)]^{-1}\) is called the rough tuning matrix, and \(\delta\) and \(\varepsilon\) are the fine-tuning parameters, generally its range from 0 to 1. This method is also applied to non-square system. There exists no inverse for non-square system. The Moore-Penrose pseudo inverse is used. For matrix A, it is given by

\[ A^\dagger = A^H \left( A \times A^H \right)^{-1} \] (3)

Where \(A^\dagger\) is the inverse of non-square matrix and \(A^H\) is the Hermitian matrix of A. The PID controller gains for a non-square system are given by

\[ K_c = \delta [G(S=0)]^\dagger \] (4)
\[ K_I = \varepsilon[G(S = 0)]^\dagger \]  \hspace{2cm} (5)

\[ K_D = [G(S = 0)]^\dagger \]  \hspace{2cm} (6)

### 3.2 Tanttu and Lieslehto method

Based on IMC principles Tanttu and Lieslehto \(^{(1)}\) have devised a multivariable PI controller tuning method. For a first order time delay process,

\[ K_{cij} = \left(2\tau_{ij} + I_{ij}\right) / 2\lambda k_{ij} \]  \hspace{2cm} (7)

\[ \tau_{ij} = \tau_{ij} + 0.5 L_{ij} \]  \hspace{2cm} (8)

\[ K_{lij} = K_{cij} / \tau_{lij} \]  \hspace{2cm} (9)

where \( k_{ij} \) and \( L_{ij} \) are the process gain and dead time of an element in model of the process for the \( i \)th output and \( j \)th input. The proportional gain and integral time constant are \( K_{cij} \) and \( \tau_{lij} \) of the internal model controller of the \( ij \)th loop. Then the multivariable PID controllers are designed by taking the pseudo-inverse for non-square system

\[ K_C = \left[1/K_{cij}\right]^\dagger \]  \hspace{2cm} (10)

\[ K_I = \left[1/K_{lij}\right] \]  \hspace{2cm} (11)

\[ K_D = \left[1/K_{Dij}\right]^\dagger \]  \hspace{2cm} (12)

where \( K_{lij} \) is the integral gain constant of the \( ij \)th loop

### 3.3 MOPSO Algorithm based Tuning method

The MOPSO is applied to estimate a centralized PID controller gain matrix with RHP zero. The Integral Squared Error (ISE), Integral Absolute Error (IAE), and Integral of the Time-Weighted Absolute Error (ITAE) are used as performance indices for minimization of error.

\[ ISE = \int_{0}^{t_0} e^2(t) dt \]  \hspace{2cm} (13)

\[ IAE = \int_{0}^{t_0} |e(t)| dt \]  \hspace{2cm} (14)

\[ ITAE = \int_{0}^{t_0} t|e(t)| dt \]  \hspace{2cm} (15)

Where \( t \) is the simulation time =150 sec
4 Result and Discussion

4.1 Simulation Studies

To analyze the effectiveness of the centralized controller for a non-square process, we considered a crude distillation column with 5-input and 4-output process for simulation. The process transfer function is given by,

\[ G(s) = \begin{bmatrix} \frac{3.8(16s + 1)}{140s^2 + 14s + 1} & \frac{2.9e^{-6s}}{10s + 1} & 0 & 0 \\ \frac{150s^2 + 20s + 1}{3.9(4.5s + 1)} & 6.3 & 0 & 0 \\ \frac{96s^2 + 17s + 1}{16se^{-2s}} & \frac{(5s + 1)(14s + 1)}{3.8(0.8s + 1)} & \frac{6.1(12s + 1)e^{-s}}{337s^2 + 34s + 1} & \frac{3.4e^{-2s}}{6.9s + 1} \\ \frac{23s^2 + 13s + 1}{22se^{-2s}} & \frac{5.1s^2 + 7.1s + 1}{0.32(-9.1s + 1)e^{-s}} & \frac{2s + 1}{12s^2 + 15s + 1} \end{bmatrix} \]

The steady-state gain matrix of the above model and the centralized PID controller matrix is determined by using Davison's, Tanttu and Lieslehto methods as elaborated by Sarma and Chidambaram.(1)

4.2 MOPSO based Centralized Controller method for crude distillation column

The entire operation of the crude distillation unit at Cosmo Oil's Sakai Refinery is discussed by Sarma and Chidambaram.(1) Simulation is carried out 50 iteration for the considered process and the tuning parameter values are obtained by assuming minimum and maximum values with 20 populations to find the global solution in a less competitive time. The PID controller matrix based on MOPSO algorithm is estimated as

\[ G_C = \begin{bmatrix} 0.3519 + \frac{0.2514}{s} + 0.1661s & 0.0974 + \frac{0.1142}{s} + 0.1337s & 0.138 + \frac{0.0115}{s} + 0.117s & 0.0288 + \frac{0.0261}{s} + 0.0279s \\ -0.1380 - \frac{0.1729}{s} - 0.1495s & 0.1804 + \frac{0.1357}{s} + 0.1648s & 0.0099 - \frac{0.0096}{s} - 0.0110s & -0.0254 - \frac{0.0238}{s} - 0.213s \\ -0.0015 - \frac{0.0019}{s} - 0.0014s & -0.1411 - \frac{0.0894}{s} - 0.1859s & 0.1492 + \frac{0.1536}{s} + 0.1663s & -0.00029 + \frac{0.00025}{s} - 0.00022s \\ -0.2890 - \frac{0.2932}{s} - 0.3165s & 0.1965 + \frac{0.292}{s} + 0.2615s & 0.4712 - \frac{0.4215}{s} - 0.2619s & -0.8225 - \frac{1.2905}{s} - 0.1769s \\ 0.0758 - \frac{0.0665}{s} - 0.0953s & 0.0838 + \frac{0.0502}{s} + 0.0352s & 0.0425 + \frac{0.0299}{s} + 0.0318s & 0.0959 + \frac{0.0635}{s} + 0.0819s \end{bmatrix} \]

5 Simulation Results

Crude distillation is a large-scale problem with 4 outputs and 5 inputs. Simulation study is carried out on the Matlab/Simulink platform for both servo and regulatory problems with step input. Simulation is carried out for 150 seconds, keeping a sampling period of 1 second. Results are compared in terms of performance indices such as IAE, ISE and ITAE values for all the three controller methods. The error values are shown in Tables 1, 2 and 3. From the tabulated values it is observed that MOPSO tuned controller provides the lowest error values (ISE, IAE, ITAE) than the other two analytical methods for applying step changes.
in Y1, Y2, Y4 for servo problems. For the given step change at Y3 the Davison controller gives lowest error values than the optimized Tanttu controllers. The Tanttu controller gives four times higher error values than the other two controllers.

A step change is applied in set point y1 and the corresponding output responses are compared which is shown in Figure 3. From Figure 3, it is noted that MOPSO controller quickly follows the changes in the set point with high overshoot but settled in less time period compared to other methods. Settling time of optimization tuned controller decreased around 15% as compared to Davison method; Tanttu and Lieslehto method takes more settling time than the other methods. Similarly set point change is also applied in Y2, Y3 and Y4 and its corresponding output responses are recorded in Figures 4, 5 and 6. From the Figures 4 and 5 it is noted that the optimization algorithm based controller response has greatly improved with regards to settling time and rise time with initial high overshoot. When we apply the step change at Y4 and its corresponding response as shown in Figure 6, it is observed that MOPSO controller provides less undershoot and quickly reaches the set point and also gives reduced settling time compared to Davison and Tanttu methods. The settling time and undershoot of Davison method is more compared to the proposed method and less by Tanttu method.

The designed controller’s performance is also compared for change in load variables. Figures 7, 8, 9, 10 and 11 show comparison of the output variable for unit step change in load variables d1, d2, d3, d4, d5 respectively. From the Figure 7, it is noted that the applied disturbances are eliminated quickly within 60 seconds by the MOPSO controller with less overshoots when compared to other two methods. The Davison method provides more overshoot than optimized controller method and less overshoot than Tanttu method. From all the regulatory responses (Figures 8, 9, 10 and 11 ) the Tanttu controller takes 2-3 times more to eliminate the applied disturbance. The error values for regulatory operations are shown in Tables 4, 5 and 6. From Table 4, it clearly indicates that MOPSO algorithm tuned controller exhibits good response and also provides 2-3 times lower ISE values than other two controller techniques.

When we apply step change in load variable d3, the ISE values of the optimized and the Davison controllers are nearly equal as compared to Tanttu method. From Table 5, it is evident the MOPSO tuned controller for all unit step changes d1, d2, d4, d5 provide 2-3 times lesser IAE values than other analytical control methods. When we apply unit step change at d3, Davison based controller given less IAE values than the proposed and the Tanttu controllers. From Table 6, it is noted that the MOPSO controller provided two times lesser ITAE values than the Davison controller and five times lesser than the Tanttu controller.

**Performance comparison of MOPSO, Davison's method and Tanttu and Lieslehto method for unit step changes**

![Fig 3. Servo problem for (A) y1, (B) y2, (C) y3 and (D) y4 for crude distillation process from y1 step input changes](https://www.indjst.org/)
Fig 4. Servo problem for (A) y1, (B) y2, (C) y3 and (D) y4 for crude distillation process from y2 step input changes

Fig 5. Servo problem for (A) y1, (B) y2, (C) y3 and (D) y4 for crude distillation process from y3 step input changes
Fig 6. Servo problem for (A) y1, (B) y2, (C) y3 and (D) y4 for crude distillation process from y4 step input changes

Fig 7. Regulatory problem for (A) y1, (B) y2, (C) y3 and (D) y4 for crude distillation process from y1 step input changes
Fig 8. Regulatory problem for (A) $y_1$, (B) $y_2$, (C) $y_3$ and (D) $y_4$ for crude distillation process from $y_2$ step input changes.

Fig 9. Regulatory problem for (A) $y_1$, (B) $y_2$, (C) $y_3$ and (D) $y_4$ for crude distillation process from $y_3$ step input changes.
**Fig 10.** Regulatory problem for (A) $y_1$, (B) $y_2$, (C) $y_3$ and (D) $y_4$ for crude distillation process from $y_4$ step input changes.

**Fig 11.** Regulatory problem for (A) $y_1$, (B) $y_2$, (C) $y_3$ and (D) $y_4$ for crude distillation process from $y_5$ step input changes.
**Table 1.** ISE values of the servo problem for centralized controller

| METHOD         | STEP IN | ISE Values | SUM OF ISE |
|----------------|---------|------------|------------|
|                | Y1      | Y2         | Y3         | Y4         |        |
| Davison's      | 2.468518| 0.221825   | 0.604317   | 0.248185   | 3.542845|
|                | 0.620711| 6.528954   | 3.040682   | 0.058512   | 10.24886|
|                | 0.049365| 0.0514     | 4.320502   | 0.191621   | 4.612888|
|                | 0.084841| 0.088282   | 0.295454   | 3.010368   | 3.478943|
| Tanttu & Lieslehto | 11.71437| 1.317035   | 1.515135   | 0.774036   | 15.32058|
|                | 0.149957| 0.250799   | 15.71780   | 1.317035   | 18.90483|
|                | 0.042204| 0.05744    | 0.911667   | 0.911667   | 2.490706|
|                | 0.016358| 0.015908   | 15.93994   | 0.104345   | 3.06087  |
| MOPSO          | 2.011713| 0.895941   | 0.042204   | 0.016358   | 2.951472|
|                | 0.895941| 5.687901   | 0.05744    | 0.016358   | 6.76396  |
|                | 0.042204| 0.05744    | 0.911667   | 0.911667   | 2.490706|
|                | 0.016358| 0.015908   | 15.93994   | 0.104345   | 3.06087  |

**Table 2.** IAE values of the servo problem for centralized controller

| METHOD         | STEP IN | IAE Values | SUM OF IAE |
|----------------|---------|------------|------------|
|                | Y1      | Y2         | Y3         | Y4         |        |
| Davison's      | 7.017111| 3.17635    | 6.096304   | 2.448388   | 18.87944|
|                | 4.464637| 12.75727   | 11.57743   | 0.914591   | 29.71393|
|                | 1.359765| 1.419553   | 7.299364   | 1.357556   | 11.43624|
|                | 1.904284| 1.926667   | 3.583132   | 5.768111   | 13.18219|
| Tanttu & Lieslehto | 29.06583| 4.261004   | 4.626874   | 42.91012   | 65.49724|
|                | 4.261004| 37.13822   | 6.044405   | 1.698667   | 49.14229|
|                | 4.626874| 10.59989   | 10.86113   | 30.37771   | 56.4656  |
|                | 42.91012| 38.79753   | 14.15686   | 3.330265   | 99.19478|
| MOPSO          | 5.201128| 2.284543   | 4.622425   | 2.869113   | 14.97721|
|                | 4.55005 | 10.72069   | 8.394566   | 1.751351   | 25.41666|
|                | 1.159747| 1.412987   | 11.91399   | 3.736781   | 18.22351|
|                | 12.24083| 17.15978   | 18.52329   | 4.07642    | 7.937867|

**Table 3.** ITAE values of the servo problem for centralized controller

| METHOD         | STEP IN | ITAE Values | SUM OF ITAE |
|----------------|---------|------------|-------------|
|                | Y1      | Y2         | Y3         | Y4         |        |
| Davison's      | 1052.567| 497.645    | 914.4456   | 367.2582   | 2831.916|
|                | 669.6956| 1913.591   | 1736.614   | 137.1887   | 4457.09 |
|                | 203.9648| 212.9329   | 1094.905   | 203.6334   | 1715.436|
|                | 3912.632| 3856.545   | 6969.586   | 2166.736   | 1977.329|
| Tanttu & Lieslehto | 4359.875| 1835.827   | 2131.165   | 1497.719   | 9824.586|
|                | 639.1506| 5570.732   | 906.6608   | 254.8      | 7371.344|
|                | 694.031 | 1589.984   | 1629.169   | 4556.656   | 8469.84 |
|                | 6436.518| 5819.629   | 2123.529   | 499.5397   | 14879.22|
| MOPSO          | 780.1693| 342.6814   | 693.3637   | 430.367    | 2246.581|
|                | 682.5074| 1608.104   | 1259.185   | 262.7026   | 3812.499|
|                | 173.962 | 211.948    | 1797.099   | 560.5172   | 2733.526|
|                | 95.62492| 109.0247   | 203.2698   | 782.7607   | 1190.68 |
### Table 4. ISE values of the regulatory problem for centralized controller

| METHOD          | STEP IN | ISE Values |        |        | SUM OF ISE |
|-----------------|---------|------------|--------|--------|------------|
| Davison’s       | d1      | 23.6427778 | 21.90663 | 62.36149 | 8.100033   | 116.0109   |
|                 | d2      | 11.9973427 | 41.79147 | 73.76714 | 4.622125   | 132.1781   |
|                 | d3      | 0.07930666 | 0.081288 | 16.70909 | 2.568755   | 19.43844   |
|                 | d4      | 0.02542171 | 0.026355 | 0.09004  | 0.513724   | 0.65554    |
|                 | d5      | 0.79236529 | 2.166121 | 6.734299 | 0.157927   | 9.868713   |
| Tanttu & Lieslehto | d1   | 182.595704 | 123.2726 | 351.5919 | 61.504     | 718.9642   |
|                 | d2      | 68.3814179 | 396.1179 | 615.2996 | 38.62084   | 1116.42    |
|                 | d3      | 0.2787024  | 0.28236  | 160.3206 | 22.8267    | 183.7082   |
|                 | d4      | 0.0898157  | 0.331235 | 0.307326 | 5.201697   | 5.93075    |
|                 | d5      | 5.4158826  | 3.312883 | 11.7048  | 2.11644    | 22.54928   |
| MOPSO           | d1      | 7.76270852 | 11.89699 | 15.32378 | 1.515756   | 36.49923   |
|                 | d2      | 6.22646555 | 21.72669 | 18.63809 | 1.466382   | 48.05763   |
|                 | d3      | 0.13085538 | 0.161931 | 14.38668 | 4.5622731  | 19.24174   |
|                 | d4      | 0.00431723 | 0.005052 | 0.021837 | 0.2896073  | 0.320813   |
|                 | d5      | 0.49058153 | 1.91373  | 5.091145 | 0.3617842  | 7.857241   |

### Table 5. IAE values of the regulatory problem for centralized controller

| METHOD          | STEP IN | IAE Values |        |        | SUM OF IAE |
|-----------------|---------|------------|--------|--------|------------|
| Davison’s       | d1      | 26.08421   | 25.7103 | 46.46391 | 14.44491   | 112.7033   |
|                 | d2      | 18.24468   | 36.30209 | 52.05609 | 12.16026   | 118.7631   |
|                 | d3      | 1.889074   | 1.911063 | 17.08375 | 5.882622   | 26.76651   |
|                 | d4      | 1.08813    | 1.087037 | 2.056617 | 3.086636   | 7.31842    |
|                 | d5      | 6.317469   | 9.163132 | 16.7351  | 3.044152   | 35.29586   |
| Tanttu & Lieslehto | d1   | 104.3735   | 98.88363 | 195.9327 | 82.20293   | 481.3927   |
|                 | d2      | 71.03731   | 206.2506 | 268.9047 | 65.63882   | 611.8314   |
|                 | d3      | 4.349445   | 5.810961 | 112.7541 | 40.49491   | 163.3639   |
|                 | d4      | 2.761562   | 6.270148 | 6.188787 | 17.89926   | 33.11976   |
|                 | d5      | 24.36947   | 12.68927 | 29.70691 | 16.5995    | 83.36515   |
| MOPSO           | d1      | 12.24083   | 17.15978 | 18.52329 | 4.07642    | 52.00033   |
|                 | d2      | 10.97219   | 23.19806 | 22.97597 | 4.90838    | 62.0546    |
|                 | d3      | 2.024289   | 2.404323 | 19.15299 | 8.770329   | 32.35193   |
|                 | d4      | 0.35452    | 0.422865 | 0.764441 | 1.690606   | 3.232432   |
|                 | d5      | 4.284821   | 8.001342 | 12.14946 | 3.069841   | 27.50546   |

### Table 6. ITAE values of the regulatory problem for centralized controller

| METHOD          | STEP IN | ITAE Values |        |        | SUM OF ITAE |
|-----------------|---------|-------------|--------|--------|-------------|
| Davison’s       | d1      | 3912.632    | 3856.545 | 6969.586 | 2166.736    | 16905.5    |
|                 | d2      | 2736.702    | 5445.313 | 7808.414 | 1824.039    | 17814.47   |
|                 | d3      | 283.3612    | 286.6595 | 2562.563 | 6067.411    | 4014.977   |
|                 | d4      | 163.2195    | 163.0566 | 308.4925 | 462.9954    | 1097.763   |
|                 | d5      | 947.6203    | 1374.47  | 2510.265 | 456.6228    | 5288.978   |
| Tanttu & Lieslehto | d1   | 15656.02    | 14832.55 | 29389.91 | 12330.44    | 72208.91   |
|                 | d2      | 10655.6     | 3093.58  | 4033.57  | 9845.823    | 91774.71   |
|                 | d3      | 652.4167    | 871.6442 | 16913.11 | 6067.411    | 24504.59   |
|                 | d4      | 414.2343    | 940.5222 | 928.318  | 2684.889    | 4967.964   |
|                 | d5      | 3655.42     | 1903.39  | 4456.037 | 2489.925    | 12504.77   |

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6 Conclusion

The MOPSO algorithm based optimization technique is used to design centralized controller for non-square systems. The performances of the controller technique are compared with other simple centralized controller methods such as Davison's method, Tanttu and Lieslehto method. Simulations are carried out for both servo and regulatory operations. The IAE, ISE and ITAE values are tabulated. The MOPSO algorithm based controller method has shown the lowest error values for both servo and regulatory operations and better performance compared to other two methods in terms of settling time. The MOPSO controller reduced about 15% lesser settling time compared to other controllers with higher overshoot. Also, the proposed controller reduces the error two times lesser than Davison controller and 4-5 times lesser than Tanttu controller. A possible direction for future work is that, this algorithm may be combined with other optimization algorithms to improve the convergence. Convergence of MOPSO algorithm seems to be premature for complex large scale systems which limit the searching efficiency for global optimal solution.

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