The Matter Bounce Alternative to Inflationary Cosmology

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(Dated: May 1, 2014)

A bouncing cosmology with an initial matter-dominated phase of contraction during which scales which are currently probed with cosmological observations exit the Hubble radius provides a mechanism alternative to inflation for producing a nearly scale-invariant spectrum of cosmological perturbations. In this review article I first discuss the evolution of cosmological fluctuations in the matter bounce scenario, and then discuss various attempts at realizing such a scenario. Observational signatures which will allow the matter bounce to be distinguished from the inflationary paradigm are also discussed.

PACS numbers: 98.80Cq

I. INTRODUCTION

The idea that instead of originating from a Big Bang singularity, the universe has emerged from a cosmological bounce has a long history (see [1] for a review with an extensive list of references to the original literature). The group of Professor Novello has made a lot of important contributions to the research on this topic. However, it was realized only fairly recently [2, 3] that a bouncing cosmology with a matter-dominated phase of contraction during which scales which are probed today in cosmological observations exit the Hubble radius can provide an alternative to the current inflationary universe paradigm of cosmological structure formation. In this review article, we provide an overview of this “Matter Bounce” scenario of structure formation, and we discuss some recent efforts at obtaining a non-singular bouncing cosmological background (see also [4] for reviews comparing the Matter Bounce with inflation and other alternatives to inflation).

Inflationary cosmology [5] (see also [6–8]) has become the paradigm of early universe cosmology not only because it addresses some of the conceptual problems of Standard Big Bang cosmology such as the horizon, flatness and entropy problems, but because it provided the first causal mechanism for generating the primordial fluctuations which could have developed into the structures we see today on large scales [9] (see also [7, 10, 11]). More specifically, it predicted a roughly scale-invariant spectrum of cosmological fluctuations which in simple models of inflation have a slight red tilt and which are Gaussian and nearly adiabatic. These predictions were spectacularly confirmed in recent precision observations of Cosmic Microwave Background (CMB) anisotropies [12].

On the other hand, it was known since long before the development of inflationary cosmology that any model which produces a roughly scale-invariant and almost adiabatic spectrum of cosmological perturbations will be a good match to observations [13, 14]. Inflationary cosmology is simply the first model based on fundamental physics which yielded such a spectrum. In the mean time, other models have been developed which predict this kind of spectrum, e.g. the Ekpyrotic universe [15], “string gas cosmology” [16], the “Varying Speed of Light” proposal [17], the “Conformal Universe” [18], and the Matter Bounce scenario.

In the following section, I shall show how quantum vacuum fluctuations originating on sub-Hubble scales and exiting the Hubble radius in a matter-dominated phase of contracting develop into a scale-invariant spectrum of curvature perturbations.

As is well known since the pioneering work of Hawking and Penrose (see e.g. [19] for a textbook description)), a cosmological singularity is unavoidable if space-time is described in terms of General Relativity (GR) and if matter obeys certain energy conditions. Thus, in order to obtain a bouncing cosmology it is necessary to either go beyond Einstein gravity, or else to introduce new forms of matter which violate the key energy conditions. In Section 3 of this review I will describe some concrete and recent models which yield a bouncing cosmology (the reader is referred to [1] for an overview of early work on obtaining bouncing universes).

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II. SCALE-INARIANT FLUCTUATIONS FROM A MATTER BOUNCE

A. Background

The space-time background cosmology which we have in mind has time $t$ running from $-\infty$ to $\infty$. The bounce point can be taken to be $t = 0$. For negative times the universe is contracting. In the absence of entropy production at the bounce point it is logical to assume that the contracting phase is the mirror inverse of the expanding cosmology of Standard Big Bang cosmology, i.e. at very early times the universe is dominated by pressure-less matter, and at a time $t = -t_{eq}$ (where $t_{eq}$ is the time of equal matter and radiation in the expanding phase) there is a transition to a radiation-dominated phase. If there is entropy production at the bounce, then the transition from matter to radiation domination will occur closer to the bounce point than $-t_{eq}$.

In Figure 1 we show a space-time sketch of the bouncing cosmology background. The horizontal axis is the comoving spatial coordinate, the vertical is time. The vertical line indicates the wavelength of a fluctuation model. It should be compared with the comoving Hubble radius $H^{-1}$ (labelled as such in the figure). As will be reviewed in the following subsection, the Hubble radius is the maximal scale on which causal and local microphysics can generate fluctuations. In order to have a causal generation mechanism, for fluctuations, it is thus important that wavelengths which are observed today originate on sub-Hubble scales. The first important point to take away from the figure is that fixed comoving scales start out with a wavelength smaller than the Hubble radius and that hence a causal generation mechanism for fluctuations is possible in a bouncing cosmology, like it is in inflationary cosmology. In inflationary cosmology it is the exponential decrease of the comoving Hubble radius during the inflationary phase which allows for a causal generation mechanism. We see that in a contracting universe dominated by regular matter a similar decrease in the comoving Hubble radius occurs.

In this section we discuss how a scale-invariant spectrum of adiabatic curvature fluctuations emerges from initially vacuum perturbations. Whereas in inflationary cosmology there is a symmetry argument (time translation invariance during the inflationary phase) which underlies the scale-invariance of the cosmological fluctuations, in a contracting universe there is no such symmetry. As we will see below, it is only vacuum fluctuations which exit the Hubble radius in a phase of matter domination which end up with a scale-invariant spectrum.

B. Preliminaries

Before studying the generation and evolution of fluctuations in the Matter Bounce model, it is important to discuss to which extent this bouncing cosmology can address the conceptual problems of Standard Big Bang cosmology which the inflationary scenario successfully addresses, namely the “Horizon”, “Flatness”, “Size” and “Entropy” problems.
In the context of Standard Big Bang cosmology, there is no causal explanation for the observed overall isotropy of the cosmic microwave background (CMB) since at the time of last scattering regions in the sky separated by more than a couple of degrees were causally disconnected \[5\]. This is the “Horizon” problem. Inflation solves this problem by providing a mechanism to exponentially increase the causal horizon in the early universe. In a bouncing cosmology, the particle horizon is infinite since time extends to \(-\infty\). Hence, the entire spatial hypersurface at the time of recombination which we observe today in the CMB is within the causal horizon and there is no Horizon problem.

In Standard Big Bang cosmology, spatial flatness is an unstable fixed point under dynamical evolution. Thus, the observed degree of spatial flatness requires extreme fine tuning of the initial conditions. During an inflationary phase, spatial flatness becomes an attractor, thus mitigating the Flatness problem. A bouncing cosmology is “neutral” with respect to the Flatness problem: if we postulate a similar degree of spatial flatness an equal amount of time prior to the bounce point as is observed today a certain time interval after the bounce point, then the observations can be explained.

The Size and Entropy problems of the Standard Big Bang model are based on the idea that the early universe should be described by initial conditions containing no hierarchy of scales: at the Planck temperature the size of space should be Planck size and it should contain a Planckian value of the entropy. With these initial conditions inflation is required to generated the large entropy and size observed today. In a bouncing cosmology the universe starts large and cool. Thus, there are no Size and Entropy problems.

There is another aspect of the Flatness problem which is not addressed in most bouncing cosmologies, namely the anisotropy problem: having a nearly homogeneous and isotropic bounce requires fine tuning of the initial anisotropies. In inflationary cosmology, the anisotropies decrease, and thus the anisotropy problem is addressed. Similarly, in String Gas Cosmology there is no anisotropy problem, as shown in \[20\]. In the Ekpyrotic bouncing cosmology \[15\] anisotropies decrease in importance in the contracting phase. A way of realizing the Matter Bounce in a framework which is safe from the anisotropy problem is to merge ideas from simple Matter Bounce models with ideas from Ekpyrotic cosmology \[21\].

C. Formalism

The fluctuations which we are interested in are inhomogeneities in the curvature of space-time which are induced by perturbations in matter. Fluctuations observed today on large cosmological scales are small in relative amplitude. Hence, we can describe them by linearizing the Einstein equations about the cosmological background solution. The corresponding theory is called the “theory of cosmological perturbations” (see \[22\] for an in-depth survey and \[23\] for an overview). In the following we summarize the essentials.

Fluctuations in the space-time metric in cosmology can be classified according to how they transform under spatial rotations. There are scalar, vector and tensor fluctuations. In total there are ten modes. However, four of the modes correspond to coordinate transformation and are hence not physical. There are only two scalar, two vector and two tensor modes which are physical. The tensor modes correspond to gravitational waves and will not concern us here. Vector modes do not couple at linear order to matter perturbations and hence will not be discussed here. In an expanding universe, vector modes decay, and this yields an additional reason to neglect them. However, in a contracting universe they grow \[24\], and thus neglecting them is only justifiable at linear order. The modes of interest to us are the scalar modes, the ones which at linear order couple of matter inhomogeneities. For simple forms of matter such as scalar fields and perfect fluids there is at linear order no anisotropic stress, and this eliminates one of the scalar modes, leaving one degree of freedom which describes curvature fluctuations. In longitudinal gauge, the metric including the scalar fluctuation mode \(\Phi(t, x)\) can be written as

\[
ds^2 = a^2(\eta)\left[(1 + 2\Phi(\eta, x))d\eta^2 - (1 - 2\Phi(t, x))d\mathbf{x}^2\right],
\]

where \(a(\eta)\) is the cosmological scale factor and we have made use of conformal time \(\eta\) related to physical time \(t\) via \(a(\eta)d\eta = dt\).

By expanding the full action (Einstein action plus action for matter) to second order about the classical background cosmology, one can obtain the action for cosmological perturbations which yields the linearized equations of motion. This action can be canonically quantized. By the logic of the previous paragraph, the resulting action can be written in terms of a single dynamical variable which, in addition, can be canonically normalized. The form of the canonical fluctuation variable \(v\) was derived by Mukhanov \[25\] and Sasaki \[26\] (see also \[27\]). The action for \(v\) takes the following form

\[
S^{(2)} = \frac{1}{2} \int d^4x \left[v'^2 - v_i v_i + \frac{k}{z} v^2\right],
\]
where a prime denotes the derivative with respect to conformal time, the subscript $i$ stands for the derivative with respect to the $i$th spatial comoving coordinate, and the function $z(\eta)$ is a function of the cosmological background. If the equation of state of matter is time-independent, then $z(\eta)$ is proportional to $a(\eta)$.

For scalar field matter $\varphi(t,x)$ the canonical variable is given by

$$v = a[\delta\varphi + \frac{\varphi'}{H}] ; \quad (3)$$

where $\delta\varphi$ is the fluctuation of the matter field, $H = \dot{a}/a$, and

$$z = \frac{a\varphi'}{H}. \quad (4)$$

The canonical variable $v$ is simply related to the variable $R$ which describes the curvature fluctuations in the comoving coordinate system $[28,31]$ and which we are interested in computing at late times:

$$v = zR. \quad (5)$$

The equation of motion which follows from the action (2) is (in momentum space)

$$v'' + k^2v - \frac{z''}{2}v = 0, \quad (6)$$

where $v_k$ is the $k$'th Fourier mode of $v$. We see that each Fourier mode satisfies a harmonic oscillator equation of motion with a time-dependent mass, the time dependence being given by the background cosmology. The mass term in the above equation is in general given by the Hubble expansion rate. Thus, we see that the Hubble radius plays a key role in the evolution of fluctuations. The mode $k$ whose wavelength at time $t$ is equal to the Hubble radius will be denoted by $k_H(t)$. It follows that on small length scales, i.e. for $k > k_H$, the solutions for $v_k$ are constant amplitude oscillations. These oscillations freeze out at Hubble radius crossing, i.e. when $k = k_H$. On longer scales ($k \ll k_H$), there is a mode of $v_k$ which scales as $z$. This mode is the dominant one in an expanding universe, but not in a contracting one.

Canonical quantization of the action for cosmological perturbations corresponds to imposing canonical commutation relations for each Fourier mode $v_k$. If we impose vacuum initial conditions at some time $\eta_i$, this implies

$$v_k(\eta_i) = \frac{1}{\sqrt{2k}}$$

and

$$v'_k(\eta_i) = \frac{\sqrt{k}}{\sqrt{2}} \quad (7)$$

Before applying the above formalism to initial vacuum perturbations in the matter bounce scenario, we will review how a scale-invariant spectrum emerges in inflationary cosmology. The definition of scale-invariance of the curvature power spectrum is

$$P_R = \frac{1}{12\pi}k^3|v_k|^2 \sim k^{n_s-1} \sim \text{const}, \quad (8)$$

i.e. $n_s = 1$, where $n_s$ is called the spectral index of scalar metric fluctuations. Note that the vacuum spectrum, i.e. the spectrum obtained with the values $[7]$ is not scale invariant. Rather, it is blue with $n_s = 3$ (more power on short wavelengths).

In inflationary cosmology the Hubble radius is constant while the wavelength of comoving modes expands exponentially. Thus, provided the period of inflation is sufficiently long, all modes which are currently observed originate on sub-Hubble scales during inflation. A mode with wavenumber $k$ exits the Hubble radius at a time $\eta_H(k)$ given by

$$a^{-1}(\eta_H(k))k = H. \quad (9)$$

In inflationary cosmology, any classical fluctuations existing at the beginning of the period of inflation are red-shifted and (in the absence of a trans-Planckian mechanism which might re-populate the ultraviolet modes - see $[32,33]$ for discussions of this point) leave behind a vacuum state. Thus, it makes sense to start fluctuations on sub-Hubble scales in their vacuum. The fluctuations will oscillate with constant amplitude while on sub-Hubble scales. However, on super-Hubble scales $v_k$ will increase as $a(\eta)$. Since long wavelengths spend more time outside the Hubble radius they experience a bigger growth. Thus, the final spectrum will be less blue. When measured at a fixed final time $\eta$, the increase in the amplitude of $v_k$ will be proportional to $a(\eta_H(k))^{-1}$, which from $[9]$ is proportional to $k^{-1}$. Hence, the slope of the power spectrum changes by $\delta n_s = -2$, converting the vacuum spectrum into a scale-invariant one.

We will now see that a similar mechanism converts a vacuum spectrum into a scale-invariant one in the Matter Bounce scenario.
D. Vacuum Fluctuations in the Contracting Phase

In this section we will be following modes which originate as quantum vacuum fluctuations on sub-Hubble scales at early times and which cross the Hubble radius during the phase of matter dominated contraction. The vacuum spectrum has index \( n_s = 3 \). To convert it into a scale-invariant spectrum we require a mechanism which boosts long wavelengths compared to short wavelengths. Since \( v_k \) grows on super-Hubble scales, such a mechanism naturally arises in a contracting universe. As we see below, in a matter-dominated phase of contraction the boost factor is exactly the right one to turn the vacuum spectrum into a scale-invariant one \[2, 3\].

From the equation of motion (6) it follows that \( v_k \) will oscillate with constant amplitude until the scale exits the Hubble radius. While the equation of state is constant in time, we have \( z(\eta) \sim a(\eta) \). In a matter-dominated phase we have \( a(\eta) \sim \eta^2 \) since \( \eta(t) \sim t^{1/3} \). Hence, it follows by solving (6) in the limit \( k = 0 \) that the dominant mode of \( v_k \) scales as \( \eta^{-1} \). Thus, the power spectrum of curvature fluctuations at some late time \( \eta \) when the modes of interest to us are outside the Hubble radius becomes

\[
P_R(k, \eta) \sim k^3 |v_k(\eta)|^2 a^{-2}(\eta) \sim k^3 |v_k(\eta_H(k))|^2 \left( \frac{\eta_H(k)}{\eta} \right)^2 \sim k^{3-1-2} \sim \text{const},
\]

where in the first line we have used the relation between \( R \) and \( v \), in the second the growth rate of \( v_k \) on super-Hubble scales, and in the third line we have made use of the vacuum amplitude of \( v_k \) at the Hubble crossing time \( \eta_H(k) \) and the Hubble radius crossing condition

\[
\eta_H(k) \sim k^{-1}.
\]

Thus, we have shown that vacuum perturbations which exit the Hubble radius during the matter-dominated phase of contraction acquire a scale-invariant spectrum. After the transition to a radiation-dominated phase of contraction (if such a phase exists) all modes are boosted by the same factor, and hence the scale-invariance of the spectrum persists at least until the bounce phase begins.

E. Matching Conditions

So far we have shown that before the bounce the spectrum of fluctuations of the curvature variable \( v \) is scale-invariant. However, we need to determine the curvature fluctuations in the expanding phase, i.e. after the bounce. Since the curvature fluctuations (in the case of adiabatic perturbations) are constant in time on super-Hubble scales in an expanding universe, it is sufficient to compute the spectrum immediately after the bounce at the onset of the period of Standard Cosmology evolution.

The transfer of curvature fluctuations through a non-singular bouncing phase is a non-trivial topic. In models with only adiabatic fluctuations, the bottom line of many detailed studies is that on length scales which are longer than the time duration of the bounce the spectral index of the power spectrum of \( v \) is not changed \[21, 34–38\] (however, as stressed in \[59\], this is not a completely general result). On the other hand, in the presence of entropy fluctuations changes are to be expected \[40\]. The reason why the analysis is non-trivial is related to the fact that new physics is required in order to obtain a non-singular bounce, and when evolving the fluctuations through the bounce phase the modes associated with the new physics must be taken into account.

There are two ways of following fluctuations through the bouncing phase. The first is my explicit numerical integration. In any given realization of the matter bounce we know what the equations for the fluctuations are, and we can integrate them. However, to obtain a good understanding of the results, it is important to have an analytical method. This method uses matching conditions at the transition surface from one phase to the next. Matching conditions were introduced by Israel \[41\] in the context of matching two solutions of Einstein’s equations across a time-like surface. They were then generalized to the problem of matching across a space-like boundary hypersurface in cosmology by Hwang and Vishniac \[42\] and by Deruelle and Mukhanov \[43\].

The matching conditions state that at the boundary between one phase and another both the induced metric on the boundary hypersurface and the extrinsic curvature must be continuous. Applied to the case of cosmological perturbations, one must first make sure that the background satisfies the matching conditions, a point emphasized in \[40\] (see also the caveats mentioned in \[44\]). Once the matching of the background is successfully achieved, the matching conditions imply that the metric fluctuation variable \( \Phi \) and the extrinsic curvature fluctuation must be continuous across the bounce. If the matching surface is a constant energy density hypersurface, then the continuity of the extrinsic curvature implies the continuity of \( v \) \[40\].
mediate a successful transition of the spectrum across the Ekpyrotic bounce \cite{56} (see also \cite{57,58}).

Ekpyrotic scenario show that modes which from the four space-time dimensional point of view are entropy modes can
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fluctuations across the bounce since the extrinsic curvature of the background changes sign across the S-brane. This
model will be discussed further below. In this context, great care must be taken when matching the cosmological
and thus allows for the violation of the Null Energy Condition which in turn allows for a cosmological bounce. This
radiation phase via an S-brane configuration, a configuration which has vanishing energy density and negative pressure
this case, there are cosmological solutions which correspond to a contracting radiation phase glued to an expanding
example is obtained in the context of the string theory backgrounds with gravitomagnetic fluxes studied in \cite{45}. In

To model a non-singular bounce, we can merge three phases, first the contracting phase obeying the equations of
General Relativity, next the bounce phase during which we can use the parametrization
\[ H(\eta) = \alpha \eta, \]
where \( \alpha \) is some constant. We have normalized the time axis such that the bounce point corresponds to \( \eta = 0 \). We
must now introduce two matching surfaces \cite{34,35}, first the transition surface between the standard contraction
phase and the bounce phase, and second between the bounce phase and the standard expanding phase. At the first
transition surface the background is contracting on both sides, and hence the background matching conditions can
easily be satisfied. Similarly, at the second transition surface the background is expanding on both sides, and hence
the background matching conditions are again satisfied.

If both transition surfaces are constant energy density surfaces (which will automatically be the case if the fluctuations
are adiabatic), then \( v \) is continuous across the surface. On length scales larger than the duration of the bounce
phase the non-trivial evolution during the bounce phase is independent of \( k \) (since the \( k^2 \) term in the equation of
motion is negligible compared to the term which generalizes the \( z''/z \) term in \eqref{6} and hence, although it can change
the amplitude of the spectrum, it will not change the spectral index \( n_s \). We hence conclude that the scale-invariant
pre-bounce spectrum survives the bounce. In Figure 2 we present an example of the numerically obtained evolution of
cosmological perturbations across a non-singular bounce, in the context of the model of \cite{21} which will be discussed
below.

There is also the possibility of realizing the Matter Bounce scenario in the context of a singular bounce. One
example is obtained in the context of the string theory backgrounds with gravitomagnetic fluxes studied in \cite{15}. In
this case, there are cosmological solutions which correspond to a contracting radiation phase glued to an expanding
radiation phase via an S-brane configuration, a configuration which has vanishing energy density and negative pressure
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model will be discussed further below. In this context, great care must be taken when matching the cosmological
fluctuations across the bounce since the extrinsic curvature of the background changes sign across the S-brane. This
issue has been discussed in \cite{40} based on the general analysis of \cite{40}.

Other examples of singular bouncing cosmologies are the Pre-Big-Bang scenario of \cite{47} and the Ekpyrotic scenario
\cite{15}. As stressed in \cite{40}, in these cases the application of the matching conditions is only well defined after careful
specification of the matching surface. This resolved the disagreement between various results obtained in the literature
for the final spectrum of curvature fluctuations in the Ekpyrotic scenario, some of which obtained a scale-invariant
spectrum \cite{48,50}, and others which did not \cite{51,54}. Calculations done in the higher-dimensional framework of the
Ekpyrotic scenario show that modes which from the four space-time dimensional point of view are entropy modes can
mediate a successful transition of the spectrum across the Ekpyrotic bounce \cite{55} (see also \cite{56,58}).
After these general remarks on matching conditions for cosmological fluctuations we will return the main development, and we turn to a discussion of implementations of the Matter Bounce scenario.

III. REALIZING A MATTER BOUNCE

A. Overview

As is well known, in order to obtain a bouncing cosmology it is necessary to either abandon General Relativity as the theory which describes space and time, or else we must introduce matter which violates the Null Energy Condition (NEC). Both options have been used to obtain realizations of a non-singular bouncing cosmology.

There is good reason to consider gravitational actions which differ from that of General Relativity at high energy scales: since General Relativity is not a renormalizable quantum theory, we know that in any approach to quantum gravity there will be terms in the action which contain higher derivatives. In this context the singularity theorems of General Relativity are no longer applicable and it is possible that nonsingular cosmological solutions emerge. On the other hand, since higher derivative terms in the action lead to a larger number of modes, the danger arises that the singularities will in fact be worse.

There are existence proofs of higher derivative actions which have non-singular cosmological solutions. One example is the “non-singular universe” construction of [59] in which the action is constructed in order that space-time approached de Sitter space at a certain limiting curvature. This allows for bouncing cosmological solutions. Another example is the (non-local) action of [60] which was constructed to be ghost-free about Minkowski space-time. Once again, this action leads to bouncing cosmological solutions. Below, I will discuss a recent example of a bouncing cosmology arising from a modified gravitational action, namely the Hořava-Lifshitz bounce [61].

More numerous are examples of bouncing cosmologies which arise from modifying the matter sector. Below, I will discuss a few recent examples.

B. Hořava-Lifshitz Bounce

Hořava-Lifshitz (HL) gravity [62] is a proposal for a power-counting renormalizable quantum theory of gravity in four space-time dimensions. The approach is conservative in that it uses only the usual metric variables, but it is radical in the sense that it discards space-time diffeomorphism and local Lorentz invariance as underlying symmetries of the theory. HL gravity postulates the existence of a preferred time direction. It maintains spatial diffeomorphism invariance, but space-dependent time reparametrizations are no longer a symmetry of the theory. Instead, HL gravity postulates a scaling symmetry which treats space and time anisotropically. If \(x \rightarrow bx\), then there is a scaling symmetry if \(t \rightarrow b^3 t\).

The gravitational action in HL theory is obtained by adding to the Einstein action all terms which are consistent with the residual symmetries and which are power-counting renormalizable with respect to the above-mentioned scaling symmetry. This leads to the possibility of adding higher space-derivative terms to the gravitational action - up to four extra space derivative terms are allowed. These higher space-derivative terms can be moved to the right-hand (matter) side of the Einstein equations. As was realized in [61], in the presence of spatial curvature these terms act as matter with negative effective energy density, thus violating the NEC and allowing for a bouncing cosmology.

Specifically, if we insert the ansatz for a spatially homogeneous and isotropic metric into the Hořava-Lifshitz action, we obtain the following generalization of the Friedmann-Robertson-Walker (FRW) equations:

\[
H^2 = \frac{\ddot{k}}{a^2} - \frac{\Lambda_E}{3} - \frac{2\ddot{k}(\zeta + 3\eta)}{\alpha a^4} + \frac{\rho}{6\alpha},
\]

\[
[\dot{H} + \frac{3}{2} H^2] = -\frac{\ddot{k}}{2a^2} - \frac{\Lambda_E}{2} + \frac{\ddot{k}(\zeta + 3\eta)}{4\alpha a^4} + \frac{p}{4\alpha}.
\]

In the above, \(H\) is the usual Hubble expansion rate, \(t\) is physical time, an overdot indicates the time derivative with respect to \(t\), \(\dot{k}\) is the spatial curvature constant, \(\Lambda_E\) is the cosmological constant, \(\rho\) and \(p\) are energy and pressure densities of matter, respectively, \(\alpha = 2/\kappa^2\) is the coefficient of the standard kinetic term in the gravitational action, and \(\zeta\) and \(\eta\) are coefficients of the higher spatial derivative terms in the potential part of the gravitational action.

Considering the first FRW equation, we recognize all the terms except the third term on the right hand side. This term is due to the higher space derivative terms in the gravitational action. We see that it scales in time like radiation. If the kinetic term in the gravitational action is close to the form it has in General Relativity, then \(\zeta + 3\eta > 0\) and...
thus the new term acts like ghost radiation. Note that this term is absent if the spatial sections are flat, but that the coefficient does not depend on whether the spatial curvature is positive or negative.

The bouncing cosmology now arises as follows. We begin with a matter-dominated phase of contraction. At low curvatures the ghost radiation term is negligible. However, as $a$ decreases, the importance of ghost radiation relative to matter increases. Eventually there will be a time when the magnitude of the ghost radiation term catches up with that of regular matter (for small values of $a$ the first two terms on the right hand side of the FRW equations are negligible). At this time we have $H = 0$. From the second FRW equation it follows that $\dot{H} > 0$ at this time. Thus, a cosmological bounce occurs.

C. Quintom Bounce

Introducing quintom matter [63] yields a way of obtaining a non-singular bouncing cosmology, as discussed in [34]. Quintom matter is a set of two matter fields, one of them regular matter (obeying the weak energy condition), the second a field with opposite sign kinetic term, a field which violates the energy conditions. We can [34] model both matter components with scalar fields. Let us denote the mass of the regular one ($\phi$) by $m$, and by $M$ that of the field $\tilde{\phi}$ with wrong sign kinetic term. We assume that early in the contracting phase both fields are oscillating, but that the amplitude $A$ of $\phi$ greatly exceeds the corresponding amplitude $\tilde{A}$ of $\tilde{\phi}$ such that the energy density is dominated by $\phi$. Both fields will initially be oscillating during the contracting phase, and both amplitudes grow at the same rate. At some point, $A$ will become so large that the oscillations of $\phi$ freeze out [80]. Then, $\tilde{A}$ will grow only slowly, whereas $\tilde{A}$ will continue to increase. Thus, the (negative) energy density in $\tilde{\phi}$ will grow in absolute values relative to that of $\phi$. The total energy density will decrease towards 0. At that point, $H = 0$ by the Friedmann equations.

$$H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\tilde{\phi}}^2 + \frac{1}{2} m^2 \phi^2 - \frac{1}{2} M^2 \tilde{\phi}^2 + V(\phi, \tilde{\phi}) \right],$$

(15)

where $V$ is the potential of the two fields. From the second FRW equation

$$\dot{H} = -4\pi G (\dot{\phi}^2 - \dot{\tilde{\phi}}^2)$$

(16)

it follows that since $\tilde{\phi}$ is oscillating while $\phi$ is only slowly rolling that $\dot{H} > 0$ when $H = 0$. Hence, a non-singular bounce occurs. The Higgs sector of the Lee-Wick model [64] provides a concrete realization of the quintom bounce model, as studied in [35]. Quintom models like all other models with negative sign kinetic terms suffer from an instability problem [65] in the matter sector and are hence problematic. In addition, they are unstable against the addition of radiation (see e.g. [66]) and anisotropic stress.

D. Ghost Condensate Bounce

An improved way of obtaining a non-singular bouncing cosmology using modified matter is by using a ghost condensate field [67] (see also [68] for related work). The ghost condensation mechanism is the analog of the Higgs mechanism in the kinetic sector of the theory. In the Higgs mechanism we take a field $\phi$ whose mass when evaluated at $\phi = 0$ is tachyonic, add higher powers of $\phi^2$ to the potential term in the Lagrangian such that there is a stable fixed point $\phi = v \neq 0$, and thus when expanded about $\phi = v$ the mass term has the “safe” non-tachyonic sign. In the ghost condensate construction we take a field $\phi$ whose kinetic term

$$X \equiv -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

(17)

appears with the wrong sign in the Lagrangian. Then, we add higher powers of $X$ to the kinetic Lagrangian such that there is a stable fixed point $X = c^2$ and such that when expanded about $X = c^2$ the fluctuations have the regular sign of the kinetic term:

$$\mathcal{L} = \frac{1}{8} M^4 (X - c^2)^2 - V(\phi),$$

(18)

where $V(\phi)$ is a usual potential function, $M$ is a characteristic mass scale and the dimensions of $\phi$ are chosen such that $X$ is dimensionless.

In the context of cosmology, the ghost condensate is

$$\phi = ct$$

(19)
and breaks local Lorentz invariance. Now let us expand the homogeneous component of $\phi$ about the ghost condensate:

$$\phi(t) = ct + \pi(t).$$  \hspace{1cm} (20)

If $\dot{\pi} < 0$ then the gravitational energy density is negative, and a non-singular bounce is possible. Thus, in \[67\] we constructed a model in which the ghost condensate field starts at negative values and the potential $V(\phi)$ is negligible. As $\phi$ approaches $\phi = 0$ it encounters a positive potential which slows it down, leading to $\dot{\pi} < 0$ and hence to negative gravitational energy density. Thus, a non-singular bounce can occur. We take the potential to be of the form

$$V(\phi) \sim \phi^{-\alpha},$$  \hspace{1cm} (21)

for $|\phi| \gg M$, where $M$ is the mass scale above which the higher derivative kinetic terms are important. For sufficiently large values of $\alpha$, namely

$$\alpha \geq 6, \hspace{1cm} (22)$$

the energy density in the ghost condensate increases faster than that of radiation and anisotropic stress at the universe contracts. Hence, this bouncing cosmology is stable against the addition of radiation and anisotropic stress.

However, the model is still unstable to the presence of anisotropic stress during the long periods of matter and radiation domination which precede the bounce phase. We now turn to a model which also solves this problem.

### E. Ekpyrotic Bounce

One of the key advantages of the Ekpyrotic bouncing model \[15\] is that anisotropies decay relative to the dominant component of the energy density during the period of contraction \[69\]. The Ekpyrotic scenario is obtained by introducing a scalar field with negative exponential potential. While the scalar field is rolling down the potential towards $\phi = 0$, the equation of state parameter $w \equiv p/\rho$ satisfied

$$w \gg 1,$$

since there is a partial cancellation between the positive kinetic energy density and the negative potential. From the continuity equation it hence follows that the energy density $\rho(\phi)$ increases faster than that in anisotropies.

The spectrum of adiabatic fluctuations in the Ekpyrotic scenario is blue \[51, 52\], and hence additional inputs (e.g. entropy fluctuations) are required to obtain a scale-invariant spectrum of curvature fluctuations.

The Ekpyrotic Bounce model of \[21\] combines the advantages of the “Matter Bounce” scenario in terms of obtaining a scale-invariant spectrum of adiabatic fluctuations with the advantages of the Ekpyrotic model in solving the anisotropy problem of a contracting universe. The model is obtained by introducing a scalar field $\phi$ which combines the ghost condensate or Galileon mechanisms for obtaining a non-singular bounce discussed in the previous subsection with a negative exponential potential which renders $\rho(\phi)$ to be dominant over anisotropies at later stages during the contracting phase.

The model of \[21\] is based on the most general form of single scalar field Lagrangian giving rise to second-order field equations in four-dimensional spacetime \[70, 71\]

$$L = K(\phi, X) + G(\phi, X)g^{\mu\nu}\partial_\mu\partial_\nu\phi + \ldots,$$  \hspace{1cm} (24)

where $K$ and $G$ are functions of a dimensionless scalar field $\phi$ and its canonical kinetic term $X$, and where we have not written down a number of higher order operators which are not important at lower energy scales (we shall construct our bounce to be consistent with the condition that these terms remain negligible).

Following the idea behind the ghost condensate bounce of \[67\] we assume that the term $K$ is given by

$$K(\phi, X) = M_p^2[1 - g(\phi)]X + \beta X^2 - V(\phi),$$  \hspace{1cm} (25)

where we introduce a positive-definite parameter $\beta$ so that the kinetic term is bounded from below at high energy scales. For $g > 1$ a ghost condensate ground state with $X \neq 0$ can arise. This is the first key principle of our model. Note that the first term of $K$ involves $M_p^2$ since in the present paper we adopt the convention that the scalar field $\phi$ is dimensionless.

The second key principle of our model is to introduce a non-trivial potential $V$ for $\phi$ which is chosen such that Ekpyrotic contraction is possible. In the specific model which we will discuss in the following, the scalar field evolves monotonically from a negative large values to a positive large value. The function $g(\phi)$ is chosen such that a phase
FIG. 3: Numerical plot of the Hubble parameter $H$ (vertical axis) as a function of cosmic time (horizontal axis). The main plot shows that a non-singular bounce occurs, and that the time scale of the bounce is short. The inner insert shows a blowup of the Hubble parameter $H$ during the bounce phase. All numerical values are in Planck units $M_p$. The parameters were chosen to be $V_0 = 10^{-7}$, $g_0 = 1.1$, $\beta = 5$, $\gamma = 10^{-3}$, $b_V = 5$, $b_0 = 0.5$, $p = 0.01$, $q = 0.1$.

of ghost condensation only occurs during a short time when $\phi$ approaches $\phi = 0$. This requires the dimensionless function $g$ to be smaller than unity when $|\phi| \gg 1$ but larger than unity when $\phi$ approaches the origin.

We choose $G$ to be a simple function of only $X$:

$$G(X) = \gamma X,$$

where $\gamma$ is a positive number. This term is introduced to stabilize gradients.

The potential is chosen such that during the approach to $\varphi = 0$ from negative values we have Ekpyrotic contraction:

$$V(\phi) = -\frac{2V_0}{e^{-\sqrt{2}\varphi} + e^{b_V\sqrt{2}\varphi}},$$

where $V_0$ is a constant.

Based on the previous discussion, the function $g(\varphi)$ is chosen in the following form

$$g(\varphi) = \frac{2g_0}{e^{-\sqrt{2}\varphi} + e^{b_0\sqrt{2}\varphi}},$$

where $g_0$ is a positive constant greater than 1.

The background equations of motion which follow from the Lagrangian (24) with the choices of the various free functions discussed above can be solved analytically using approximate methods. They can also be solved numerically. The results of the numerical integration are shown in Figure 3. As is evident, a successful non-singular bounce occurs.

We have also studied [21] the evolution of cosmological fluctuations through the bounce phase and verified that on large scales the spectral index does not change (see Figure 2).

F. Stringy S-Brane Bounce

All the models with modified matter we have considered so far are toy models in that they do not arise from a theory which is well behaved in the ultraviolet. We conclude this section by mentioning a model which originates from a theory which is well behaved in the ultraviolet, namely superstring theory.

It has long been realized that string theory has the potential of solving the singularity problem of Standard Big Bang cosmology. As is well known [24], there is a maximal temperature of a gas of weakly coupled strings in thermal equilibrium, the Hagedorn temperature $T_H$. In the context of a toroidal compactification of Heterotic string theory, it was realized [10] that the temperature remains smaller than $T_H$ throughout. However, there are potential instabilities of string theory near the Hagedorn temperature.
Recently [45], ingredients to resolve both the Hagedorn instability as well as the initial curvature singularity of string cosmology were proposed. These ingredients are realized in a class of $\mathcal{N} = (4, 0)$ Type II superstring models where the presence of non-trivial “gravito-magnetic” fluxes modifies the thermal vacuum by adding non-trivial momentum and winding charges, lifting the Hagedorn instabilities. If $R_0$ denotes the radius of the Euclidean time circle, the string partition function $Z$ has a thermal duality symmetry about a critical value $R_c$, which is of the order of the string scale:

$$Z(R_0) = Z(R_c^2/R_0).$$

The inverse temperature $\beta(R_0)$ measured in the string frame is then proportional to $R_0$ or its dual value in the asymptotic regimes:

$$\beta \sim 2\pi R_0 \quad \text{as} \quad R_0 \gg R_c, \quad \beta \sim 2\pi \frac{R_c^2}{R_0} \quad \text{as} \quad R_0 \ll R_c.$$

Thus the temperature acquires a maximal critical value.

In the adiabatic approximation, cosmological dynamics very different from the usual particle physics induced dynamics can be obtained. One possibility is a solution in which $R_0$ runs from $0$ to $\infty$. At sub-critical values of $R_0$, the Universe can be taken to be in a contracting phase. At the critical value of $R_0$, states characterized by non-trivial winding and momentum charges along the Euclidean time circle become massless and give rise to a Euclidean gauge theory. At the level of the four-dimensional low energy effective field theory of dilaton-gravity, these massless states lead to extra terms in the effective action which correspond to an S-brane configuration (see e.g. [73] for early references on S-branes), and the effective action is that of dilaton-gravity coupled to a thermal gas of strings plus the S-brane configuration. An S-brane acts like a topological defect in space-like direction. Its energy density vanishes in the same way that the transverse pressure of a regular defect vanishes. The pressure is negative. Hence, the S-brane provides the violation of the null energy condition (NEC) which leads to a smooth transition from a contracting phase to an expanding phase (when cosmological evolution is viewed in the Einstein frame). Thus, for $R_0 > R_c$ the Universe expands. Thus, the setup of [45] provides a realization of a bouncing cosmology. The metric in this setup is continuous, but the derivative of the metric diverges.

At physical temperatures much lower than the critical value, the partition function of the string gas has the equation of state of radiation as long as supersymmetry is unbroken [81]. However, at low temperatures we expect supersymmetry to be broken, thus generating masses and leading to the emergence of a phase of matter domination at low temperature.

In [40], the evolution of cosmological fluctuations in this bouncing cosmology has been studied in detail. The background cosmology starts with a matter-dominated phase of contraction. At a certain temperature $T_{\text{SUSY}}$ supersymmetry is restored, and a transition to a phase of radiation domination will occur. Once the temperature rises to the dual value, the S-brane will be reached. The S-brane mediates the transition to an expanding radiation phase. The temperature will decrease, and below $T_{\text{SUSY}}$ supersymmetry will break, allowing for the generation of a component of pressureless matter which dominates the late-time dynamics.

This background thus provides the requirements to implement the Matter Bounce scenario: we begin with quantum vacuum perturbations on sub-Hubble scales in the contracting phase. Scales which exit the Hubble radius before the onset of the radiation phase acquire a scale-invariant spectrum. In the absence of entropy fluctuations the perturbations are matched across the S-brane with no change in the spectral index. Hence, at late times in the expanding phase a scale-invariant spectrum of curvature fluctuations results.

**IV. CONCLUSIONS**

The “Matter Bounce” is an alternative to cosmological inflation as a mechanism for generating an almost scale-invariant spectrum of cosmological fluctuations. As in the case of inflationary cosmology, the spectrum has a slight red tilt since smaller scales exit the Hubble radius when the radiation component of matter is more important and the vacuum slope of the spectrum (which corresponds to $n_s = 3$) is rearing its head. In fact, there is a transition to an $n_s = 3$ spectrum on small scales which exit the Hubble radius in the radiation phase of contraction. The current observational constraints on a spectrum with such a transition from $n_s = 1$ on large scales to $n_s = 3$ on small scales were studied in [74], with the result that the power spectrum of quasar absorption lines already mildly constrains a model with no entropy production during the bounce. If there is entropy production during the bounce, then the radiation phase of contraction is shorter than the radiation phase of expansion and the observational constraints recede.

The spectrum of gravitational radiation produced in a matter bounce is also scale-invariant. At first glance, there will be less enhancement of the amplitude during the bounce phase than for scalar metric fluctuations, and hence the
predicted tensor to scalar ratio \( r \) will be consistent with current constraints. However, a detailed study of this issue is needed.

Since the curvature fluctuation \( \mathcal{R} \) increases on super-Hubble scales during the contracting period, terms in the general expression for the bispectrum (three point function of the curvature fluctuation variable) derived in \[75\] will be important, leading to a shape of the bispectrum which is different from the one obtained in simple inflationary models \[76\]. Since the analog of the inflationary slow-roll parameter is of order unity in the case of the Matter Bounce, the predicted amplitude of the bispectrum will be of order unity \[76\]. Thus, amplitude and shape of the bispectrum provide a way to differentiate between the Matter Bounce and simple inflationary models.

Possibly the most serious problem which the Matter Bounce scenario faces is the anisotropy problem (see e.g. \[77\]). The only solution of this problem which we know of now is to postulate a phase of Ekpyrotic contraction following the initial matter contraction phase. In the model of \[21\] this solves the anisotropy problem, whereas in the original New Ekpyrotic scenario the anisotropy problem may re-appear during the bounce phase \[77\].

I have focused on single bounce models. Cyclic bouncing background cosmologies face several challenges. In addition to the “heat death” problem (entropy increasing from cycle to cycle), there is the problem that the dynamics of cosmological fluctuations breaks the cyclicity of the model since fluctuations grow on super-Hubble scales during the periods of contraction and lead to a jump in the index \( n_s \) of the power spectrum by \( \delta n_s = -2 \) from cycle to cycle \[78\]. Thus, the model loses predictability \[52\].

Acknowledgments

I wish to thank my collaborators on whose work I have drawn. In particular, I thank Dr. Yifu Cai for collaboration on many of these topics and for allowing me to make use of the figures. Special thanks to Professor Xinmin Zhang for collaboration over a period of many years and for wonderfully hosting me repeatedly at IHEP in Beijing. This work was supported in part by an NSERC Discovery Grant and by funds from the Canada Research Chair program.

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