Concentration of higher dimensional entanglement

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Abstract

Enhancement of entanglement enhancement is necessary for most quantum communication protocols many of which are defined in Hilbert spaces larger than two. In this work we present the experimental realization of entanglement concentration of orbital angular momentum entangled photons produced in the spontaneous parametric down-conversion process which have been shown to provide a source for higher dimensional entanglement. We investigate the specific case of three dimensions and the possibility of generating different entangled states out of an initial state. The results presented here are of importance for pure states as well as for mixed states.

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Most of the current applications of entanglement in quantum communication such as quantum teleportation [1] and quantum cryptography [2,3] work best for maximally entangled states. However, in practice one always has to deal with non maximally entangled or mixed states for example because of the uncontrollable interaction of the particles with the environment. Therefore many quantum protocols such as distillation [4,5] purification [6], concentration [7] and error correction [8,9] have been suggested in order to enhance the quality of entanglement. With the exception of quantum error correction which is most important for quantum computation, the common idea in all other protocols is to start with a sample of entangled states having a low quality of entanglement and using local operations and classical communication only to end up in a smaller ensemble with a higher degree of entanglement. A measure for the enhancement of the entanglement could for example be the non local feature of the state represented by violation of Bell’s inequalities.

There are varying terminologies in the literature. Following reference [7] we will use the term concentration for our experimental achievement which was to extract maximally entangled states out of non maximally entangled pure states. This method is also referred to as local filtering or the ”Procrustean method” 1. In contrast to concentration the term distillation is used for the more general case of extracting entanglement out of initially mixed states. Purification denotes the procedure which makes arbitrary initial states more pure, after local operations and classical communication, but not necessarily more entangled. As demonstrated by Horodecki et al. [4], even if the fidelity of the system to be in the desired entangled state is less than 1/2, all inseparable quantum systems can be distilled to singlet states by local filtering and the Bennett et al. distillation protocol [6]. This means that the Procrustean concentration method as experimentally demonstrated here is not only of importance for pure states but also for extracting entanglement out of mixed states.

1After Procrustes, a fabulous Greek giant who stretched or shortened captives to fit one of his iron beds
There exists a steadily growing interest in entanglement in higher dimensions since it allows realization of new types of quantum communication protocols [10–13]. Such states also have been shown to be more efficient and to provide more security in quantum communication applications such as in quantum cryptography [13,12,14]. However in order to overcome the problem of uncontrollable interaction of the quantum states with the environment and to make quantum communication with higher dimensional entangled states feasible it is important to be in full control of the corresponding distillation and concentration techniques. So far the Procrustean method has been experimentally demonstrated only for entangled qubits, i.e. for two-dimensional systems [5]. Given the motivations discussed above we demonstrate for the first time the experimental realization of quantum concentration in higher dimensions using qutrits entangled in orbital angular momentum.

In our experiment the entangled qutrits were produced via type-I spontaneous parametric down-conversion using a BBO crystal (Beta Barium Borate) of 1,5 mm thickness pumped by an Argon ion laser operating at 351nm and having 120 mW of light power. Since we were not interested in polarization entanglement but in the entanglement of the orbital angular momentum we chose type-I down-conversion thus exploiting the greater efficiency of producing entangled photon pairs. We used the energy-degenerated case where both entangled photons had a wavelength of 702nm.

Due to their helical wave fronts the electromagnetic field of photons having an orbital angular momentum has a phase singularity. There the intensity has to vanish resulting in a doughnut-like intensity distribution. These light fields can be described by means of Laguerre Gaussian ($LG_{pl}$) modes with two indices $p$ and $l$. The $p$-index identifies the number of radial nodes observed in the transversal plane and the $l$-index the number of the $2\pi$-phase shifts along a closed path around the beam center. The latter determines the amount of orbital angular momentum in units of $\hbar$ carried by one photon [15–17]. In all our experiments we only considered LG modes with an index $p = 0$ whereas the $l$-index of the entangled photons varied from -2, -1,...to ..1, 2. Since the $LG_{pl}$ modes form an orthogonal basis they can be used to realize discrete higher dimensional entangled systems.
A common technique to produce $LG_{0l}$ modes out of the Gaussian mode is to use computer generated holograms which are usually transmission gratings with dislocations in the center [18,19]. Inversely such a hologram in connection with a single-mode optical fiber can be used to identify a certain $LG_{0l}$ mode [20]. It has also been demonstrated that by displacing such a hologram it is possible create superpositions of the $LG_{00}$ (=Gaussian) and the corresponding $LG_{0l}$ mode with well-defined amplitudes and relative phases. In the present experiment in order to identify the $LG_{01}, LG_{0-1}, LG_{02}$ and $LG_{0-2}$ modes blazed transmission phase gratings with a period of 20 µm were used which had a diffraction efficiency of about 85%.

The experimental demonstration of the concentration was performed in two steps. Besides our earlier confirmation of conservation of orbital angular momentum [20] we also demonstrated entanglement of the $LG_{0-1}, LG_{00}$ and $LG_{01}$ modes by violating a generalized CHSH type Bell inequality [21]. It was therefore reasonable to expect that those results also hold for the extension to $LG_{0-2}$ and $LG_{02}$. Thus first we confirmed this issue. Restricting ourselves to the case of a pump beam having no orbital angular momentum, that is an $LG_{00}$ (Gaussian) mode, it was shown that the entangled down-conversion state was given by $C_{00}|00\rangle + C_{11}|11\rangle + C_{22}|22\rangle$, where the numbers in the kets are equivalent to the absolute value of the $l$-index $|l|$ of photon 1 and photon 2 respectively. Using the same techniques as in earlier experiments [20] the down converted photons on each side were projected onto the respective eigenstates via computer generated holograms. The amplitudes $C_{ij}$ were determined from the coincidence count rates which are a measure for the probabilities. In order to demonstrate the entanglement the state was also measured in a rotated basis, i.e. the down converted photons were projected onto superpositions of $LG$ modes (Fig.3 upper row). As it is discussed in an earlier paper [22] this can be achieved by displaced holograms.

In the second step which we actually demonstrated entanglement concentration. This we showed by converting the initial entangled state having non-equal relative amplitudes into a state with equal amplitudes representing a maximally entangled state.

The experimental setup is shown in Fig.1. The Gaussian ($LG_{00}$) is focused on the BBO crystal where the entangled pairs are produced. These are emitted from the crystal at an
angle of 4° off the pump beam and are coupled into optical fiber couplers via a lens on each side.
FIGURES

FIG. 1. Experimental setup for the entanglement concentration. After type-I parametric down-conversion two lenses are used for spatial mode filtering by which the relative amplitudes between different LG modes are changed. The second part of the setup projects the photons onto the orbital angular momentum eigen states. This is done by using mode detectors consisting of computer generated holograms and single mode optical fibers.

For convenience the mode identification was done using a probabilistic method since the count rates were sufficiently high. However a deterministic mode separator [23,24] would be possible but significantly more complicated. Two non-polarizing beam splitters, the first one having a transmission to reflectivity ratio of 2:1 the second one of 1:1, redirected each photon with probability of 1/3 to each of three mode detectors. As mentioned above the combination of a computer generated hologram and a single mode optical fiber acts as a mode detector. Using the respective holograms and single photon detectors we were able to detect $LG_{0\pm 2}, LG_{0\pm 1}, LG_{00}, LG_{01}$ and $LG_{02}$ modes. The coincidence logic was designed to record the coincidences between an arbitrary pair of detectors each on one side. This gives
9 possible coincidence count rates.

It has been experimentally observed [20] that the emission probability for the higher order entangled modes decreases with the index \( l \). Besides, in order to have a maximum collection efficiency of the down converted photons it is important to adapt their beam parameters to the spatial mode which can be coupled into the single mode fibers [25]. Usually the proper alignment is achieved by overlapping the waist of the down converted beam on the crystal with the waist of an auxiliary beam being sent in a reverse direction through the single mode fiber and the coupling lenses. However the waist size of the \( LG \) modes grows with their \( l \)-index. Therefore there does not exist a common setting of the coupling lenses which can couple the \( LG_{00}, LG_{01} \) and the \( LG_{02} \) modes all with maximum efficiency. In order to measure the initial down-converted state emitted by the crystal we had to proceed in the following way. First the coupling lenses were positioned to have maximal collection efficiency for the \( LG_{00} \) mode. Then the lens positions were changed to maximize the collection efficiencies for the \( LG_{01} \) and the \( LG_{02} \) mode respectively.

Therefore the measured amplitudes depend on the positioning of the coupling lenses. Therefore it is possible by varying their positions to couple in one mode more effectively than the other one. This method can be considered as a kind of filtering because part of the photon state emitted in the modes for which the collection efficiency of the setup is not optimal is lost.

In Fig.2 the filtering effect is illustrated in one of the down-conversion arms for two different \( LG \) modes (e.g. \( LG_{00} \) and \( LG_{01} \)).
FIG. 2. The principle of the mode filtering for two different modes. The two modes $LG_{00}$ and $LG_{0|1|}$ possess the same beam waist at the crystal. However because of their different beam divergences they are not detected with the same efficiency. This effect is exploited to filter the more intense $LG_{00}$ mode.

The refractive index of the mono-mode fibers determines a certain angle of acceptance for incoming light. The position of the lens is chosen such that the LG mode with the lower emission probability (here $LG_{0|1|}$) has a good overlap with the acceptance mode of the fiber. As a result the LG mode with the higher emission probability (here $LG_{00}$) has not an optimal overlap with the acceptance mode of the fiber which causes a filtering of the amplitude of this LG mode. The same arrangement with another distance can be used in the other arm of the down-conversion to achieve a filtering of the $LG_{0|1|}$ mode with respect to the $LG_{0|2|}$ mode.

It is also important to mention that as a consequence of entanglement each lens acts as a non-local filter on both sides. The filtering action of a lens on one side projects also the corresponding modes on the other side. By varying the distance of the two coupling lenses from the crystal just by the same amount one would be able to equalize the amplitudes of two modes only. It is only by choosing asymmetric positions and by exploiting the entanglement

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For simplicity the hologram which would be needed to transform the LG modes with $|l| \neq 0$ into the Gaussian mode in order to make them detectable via mono-mode fibers is left out in Fig.2.
that it is possible to achieve nearly the same coincidence count rates for all three different modes.

In order to identify the state emitted by the crystal we proceeded as described above choosing three different lens positions for collecting the $LG_{00}$, $LG_{0|1}$ and $LG_{0|2}$ respectively each with the same efficiency. Afterwards the amplitude of each mode was taken from that choice of setup having the maximum collection efficiency for the corresponding mode. It is only by this way that the same collection efficiency is ensured for all modes. Using detectors with nearly equal detection efficiency the normalized initial state was found to be

$$\psi_{\text{initial}} = 0.80|00\rangle + 0.44|11\rangle + 0.41|22\rangle$$ (1)

The amplitudes were calculated from the coincidence count rates which represent the probabilities for detecting the photon pair in the corresponding $LG$ mode. For convenience we chose the basis for describing the initial state such as having no relative phases between the components. Such a choice is always possible. By scanning all holograms only horizontally these relative phases remain unchanged.

In order to demonstrate that the initial state is entangled it was also measured in bases rotated in Hilbert space. This was done by displacing the holograms. A displaced $LG_{01} (LG_{02})$-hologram projects an incoming mode onto a certain superposition of the $LG_{00}$ and the $LG_{01} (LG_{02})$ depending on its displacement [22]. Therefore in one beam the $LG_{01}$- and the $LG_{02}$-holograms were displaced while in the other beam the corresponding holograms performed a scan of the mode of the incoming photons. The resulting coincidences are shown in Fig.3 upper row.
FIG. 3. Measurement of the entangled states in a superposition basis. Upper row: before concentration, lower row: after concentration. In both curves the dip is the signature for entanglement. The measurement of a mixed state in the superposition basis would result in equally distributed coincidences forming curves with no or low contrast.

As shown in an earlier work [20] the high visibility ($\sim 80\%$) of these curves in the rotated basis can be viewed as a signature of entanglement. whereas a mixed state would lead to a equally distributed coincidence measurement of the LG modes resulting in curves having low contrast.

As discussed above different lens positions cause different filterings of the initial state (1). That means by choosing various lens positions on both sides different entangled states can be created out of an initial state. After having identified the ”initial state” this was demonstrated experimentally by for 7 different combinations of lens settings in the two arms and detecting the coincidences. Out of these the normalized amplitudes were calculated for the resulting filtered states (Tab.4). Each of these lens configurations can be identified with a certain filter density for the LG modes. The filter density for each LG mode is
defined as $1 - \frac{C_{LC}}{C_{INT}}$ where $C_{LC}$ and $C_{INT}$ denote the coincidence count rate at a certain lens configuration and the coincidence count rates for the initial state respectively. It is a quantitative measure therefore how a lens configuration acts as a filter for an LG mode.

FIG. 4. "Procrustean" filtering method. Each lens configuration in the setup cases a specific filtering of the three modes $LG_{00}$, $LG_{01}$ and $LG_{02}$. In general the filter density is different for each mode and depends on the positions of the two lenses L1 and L2. Specially the lens configuration 5 causes the concentration of the initial state. The concentrated state has nearly equal amplitudes. Typical errors for the amplitudes of the filtered states are about 0.02.

The filtered states are not always necessarily more entangled for any lens configuration, however the filtering action of the lens configuration Nr. 5 in Tab.2 causes a maximal concentration of the initial state emitted by the crystal. The concentrated state is found to be

$$\psi_{\text{concentrated}} = 0.60|00\rangle + 0.56|11\rangle + 0.57|22\rangle$$  \hspace{1cm} (2)

which is very close to the three-dimensional maximally entangled state

$$\psi_{\text{max}} = \frac{1}{\sqrt{3}} [(|00\rangle + |11\rangle + |22\rangle]$$  \hspace{1cm} (3)
Since state (1) was the only "initial" state available to us other lens configurations yielded filtered state which are not necessarily more entangled. However because of the linearity of the filtering process there exist possible initial states for which each of the local filterings in Tab.4 would cause the concentration of entanglement. For example given the states \( |0, 26\rangle \pm |0, 50\rangle \pm |0, 83\rangle \pm |0, 27\rangle \) or \( |0, 73\rangle \pm |0, 63\rangle \pm |0, 27\rangle \) as initial states the filtering action of the lens configurations Nr. 1 and Nr. 7 in Tab.4 would cause the concentration into the maximally entangled state (3) respectively.

After we had experimentally equalized the amplitudes of the initial state (1) we wanted to demonstrate that the resulted filtered state (2) was entangled. Therefore we proceeded in the same way as for demonstrating the entanglement of the initial state. The state (2) was measured in bases rotated in Hilbert space. Again, the two holograms on one side were displaced while the two holograms in the other arm made scans of the modes of the photons.

The measured coincidences are shown in Fig.3 lower row. One can find that in the \( \psi_{\text{concentrated}} \) case which is closer to the maximally entangled state the visibilities are higher. These are defined as \( V = \frac{C_{\text{max}} - C_{\text{min}}}{C_{\text{max}} + C_{\text{min}}} \) where \( C_{\text{o}} \) denotes the coincidence count rates. The corresponding visibilities in Tab.4 are 86, 4\% for the \( LG_{01} \) modes and 81, 5\% for the \( LG_{02} \) in the \( \psi_{\text{initial}} \) case and 94, 4\% for the \( LG_{01} \) modes and 87, 3\% for the \( LG_{02} \) in the \( \psi_{\text{concentrated}} \) case. The asymmetry in Fig.3 lower row right is due to an imperfection of the corresponding hologram.

By exploiting simple experimental techniques theses results clearly show the possibility to extract maximally entangled states out of non maximally entangled ones in higher dimensions. Furthermore the present work demonstrates that using the same technique it is possible to produce and to identify different entangled states. The results presented here are not only of interest for pure states but also for extracting entanglement out of mixed states. Since, as demonstrated by Horodecki et al. [4], all inseparable quantum systems can be distilled to singlet states by local filtering and the Bennett et al. distillation protocol [6]. Therefore it can be expected that the ideas and techniques presented here will be of importance for future quantum communication networks over long distances which necessarily
will need some kind of entanglement enhancement procedure.

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