Hybrid Inflation in Intersecting Brane Worlds

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Abstract

Non-supersymmetric brane world scenarios in string theory display perturbative instabilities that usually involve run-away potentials for scalar moduli fields. We investigate in the framework of intersecting brane worlds whether the leading order scalar potential for the closed string moduli allows to satisfy the slow-rolling conditions required for applications in inflationary cosmology. Adopting a particular choice of basis in field space and assuming mechanisms to stabilize some of the scalars, we find that slow-rolling conditions can be met very generically. In intersecting brane worlds inflation can end nearly instantaneously like in the hybrid inflation scenario due to the appearance of open string tachyons localized at the intersection of two branes, which signal a corresponding phase transition in the gauge theory via the condensation of a Higgs field.
1. Introduction

Recently, there have been various attempts to realize the inflationary scenario \[1-3\] of cosmology within string theory \[4-12\] (an alternative to inflation within M-theory was proposed in \[13\]). In order to do so, it appears natural to work in a non-supersymmetric string vacuum from the beginning, as such string models provide non-trivial potentials for the scalar fields of the effective theory. In particular non-symmetric (non-BPS) D-brane configurations offer a promising avenue for inflation, as the displacement of D-branes in the extra dimensions results in a non-vanishing vacuum energy which induces an inflationary phase in the three uncompactified dimensions. More concretely, the displacement parameters of the D-branes in the internal space are identified with the vacuum expectation values of some Higgs scalar fields in the effective four-dimensional theory. So far, it remains a major challenge to find explicit constructions that lead to realistic potentials with suitable properties for interpreting one of the scalars as the inflaton field whose vacuum expectation value gives rise to an effective cosmological constant which then drives the inflationary expansion of the early universe. The ultimate goal would be to combine inflation with realistic particle physics as given by the Standard Model or GUT theories.

A class of phenomenologically interesting string models in this respect is given by so-called intersecting brane worlds \[14-32\]. As in common brane world scenarios the gauge fields are confined to topological defects, D-branes, which spread a four-dimensional Minkowski space and extend into the internal compact space of total dimension six. In the present setting, 6+1 dimensional D6-branes wrap 3-cycles of the internal geometry and intersect each other in certain patterns which determine the breaking of supersymmetry as well as the low energy spectrum of chiral fermions \[33,34\]. The absence of gauge and gravitational anomalies is guaranteed by the stringy RR-tadpole cancellation conditions. An important feature is the fact that the gravitational sector in the bulk of space-time still can be supersymmetric at tree level, while it may be broken in the gauge theory on the branes. Non-supersymmetric and supersymmetric models of this kind have been studied in the literature. It has turned out that one can find examples of non-supersymmetric compactifications that come very close to the Standard Model of particle physics. In particular it was possible to construct models which contain precisely the gauge fields and the chiral fermions of the Standard Model, the only difference showing up in the Higgs sector.

But these constructions are plagued by the same problems as all non-supersymmetric string vacua or, respectively, non-BPS brane configurations that have been found so far,
namely, the appearance of instabilities of scalar fields due to uncanceled massless NSNS tadpoles. In [24] the leading order perturbative potential for the closed string moduli fields has been investigated with the result that it usually implies run-away instabilities for some of the scalars that were pushed to a degenerate limit. While in [24,32] strategies to avoid such dangerous behavior were advocated, one may also contemplate to make use of these potentials by reinterpreting them on cosmological scales.

This is the purpose of the present paper: We investigate whether an inflationary scenario can be realized in an intersecting brane world. In doing so, we study the scalar potentials that arise in these models at the leading and next-to-leading order in string perturbation theory and then look for suitable candidates for the inflaton field. The typical inflationary scenario requires a scalar field $\psi$ whose contribution to the overall energy is dominated by its potential term and thus works as an effective cosmological constant roughly given by its vacuum expectation value. A key criterion to decide if any scalar field is appropriate is the issue of slow-rolling. During its evolution the perturbations must be very tiny in order to fit the bounds set by the highly homogeneous CMB data. This is conveniently rephrased in terms of the potential $V(\psi)$, which must obey

$$\epsilon = \frac{M_{Pl}^2}{2} \left( \frac{V'(\psi)}{V(\psi)} \right)^2 \ll 1, \quad \eta = M_{Pl}^2 \frac{V''(\psi)}{V(\psi)} \ll 1.$$  \hspace{1cm} (1.1)$$

These two conditions (see for instance [35,36]) of course need to be complemented by many more checks like the possibility to get 60 e-folding and a realistic fluctuation spectrum. Therefore it only serves as a minimal requirement needed for $\psi$ to be called an inflaton candidate.

Moreover, in inflationary cosmology one needs a mechanism to end inflation “gracefully”. In the context of intersecting brane worlds, a quite natural exit from inflation is provided by the appearance of open string tachyons after some evolution of closed string moduli. This exit fits into the pattern of a hybrid inflationary model [37,38] effectively described by a term

$$V_{YM}(\psi, H) \sim (M(\psi)^2 - \frac{1}{4} H^2)^2$$  \hspace{1cm} (1.2)$$
in the scalar potential of the gauge theory sector. It combines the merits of both chaotic inflation and spontaneous gauge symmetry breaking. The slowly evolving inflaton field $\psi$ affects the mass $M(\psi)^2$ of a second scalar field, the Higgs field $H$. When $M(\psi)^2$ becomes negative, a phase transition occurs and inflation ends immediately. In string theory open string tachyons may take the role of this scalar signaling also a phase transition in string
theory \[33,40\]: the condensation of higher dimensional D-branes into lower dimensional ones, respectively the condensation of two intersecting D-branes into a single one wrapping a non-trivial supersymmetric 3-cycle \[41\]. Intriguingly, these tachyonic scalars are well suited to serve as Higgs fields and drive the gauge theoretic spontaneous symmetry breaking mechanisms \[33,15,20,22\]. This could naturally link the exit from inflation to some phase transition in the gauge sector, possibly even the electroweak phase transition itself.

Coming back to the main point, we want to identify the inflaton $\psi$ with some scalar field corresponding to a geometric modulus of the model. Conceptually, there are two different classes of such scalars. The first class contains the closed string moduli, some of them parameterize the shape and size of the geometric, six-dimensional gravitational background space. In our cases, this space is given by either a six-dimensional torus or a toroidal orbifold. In particular, there are the Kähler moduli for the size of the space, the complex structure moduli for its shape, as well as twisted moduli localized at the singularities (fixed points) of the orbifolds. The second class contains the positions of the D-branes on the internal space and the Wilson lines of gauge fields along the branes. They are open string moduli.

From earlier work \[7,11\] one expects that the open string moduli could satisfy slow-rolling properties if one makes the severe simplification to assume that the closed string moduli are frozen. Geometrically speaking, this means that if the background space is fixed and no backreaction on the presence of the branes takes place, their motion along this space can be very slow for a certain time. After this time, they start approaching each other faster and at a critical distance a tachyon appears to signal their condensation. Since the dynamics of the entire setting is determined by the fastest rolling field, this assumption implies that the closed string moduli have to roll slower than the open string moduli. Otherwise, the space could for instance shrink very quickly and bring the two branes within their critical distance much faster than originally estimated from the simplified analysis with frozen volume. The problem of the simultaneous slow-rolling properties of all scalar fields involved will be addressed in the present paper and solutions which allow slow-rolling closed string fields will be proposed.

The paper is organized as follows. In section 2 we give a very short review of intersecting brane world models focusing on the main conceptional ingredients. For more details we refer the reader to the by now extensive literature.

In section 3 we derive the disc-level scalar potential (disc tadpole) in the effective four dimensional theory arising from intersecting brane worlds with D6-branes. Note that
this potential only depends on closed string moduli. The open string moduli (positions of D-branes, Wilson lines) first appear in the open string one-loop diagrams. The essential assumption of the whole paper is that the ten-dimensional string coupling is small, so that we can trust string perturbation theory. It is one of the major unsolved problems in string theory by which actual mechanism one can stabilize the dilaton. Nevertheless, in all recent works on inflation from string theory this working hypothesis has been made. On the contrary, no assumption about a small curvature in the target space is required as the potential we derive from divergences in one-loop string amplitudes is exact to all orders in $\alpha'$. We will introduce two different sets of coordinates for the four-dimensional parameter space appearing in the open string tree level potential. We call the first set the “gauge coordinates”, as they are proportional to the effective gauge couplings on certain D-branes. Moreover, these coordinates would naturally appear in an $\mathcal{N} = 1$ effective theory in four dimensions. The second set is called the “Planck coordinates”, for one of them is proportional to the four-dimensional Planck scale.

In section 4 we will investigate whether this leading order potential can satisfy the slow-rolling conditions which are essential for an inflationary cosmological model. The result can be summarized as follows. Working with the “gauge coordinates” we get a result very similar to [12]. In fact the potential investigated there is just a very specific example of the intersecting brane world potentials studied in this paper. In fixing some of the “gauge coordinates”, the remaining ones can satisfy the slow-rolling condition. Such a mechanism was assumed to consist in some, albeit unknown, dynamics that generates a potential which then fixes the moduli. However, not even two of them can be slow-rolling at the same time and therefore all scalars except the inflaton would need to be fixed. One generic feature when working with “gauge coordinates” is that the string scale is evolving during the inflationary era as well. The other option is to work with the “Planck coordinates”, assuming some most likely non-perturbative stringy mechanism to stabilize the four-dimensional dilaton. This immediately implies that both the Planck and the string scale are fixed. In this case it turns out that the remaining three complex structure moduli are generically not slow-rolling.

In section 5 we discuss a rather different scenario where the complex structure moduli are frozen at the string scale such that the relevant dynamics affects the Kähler moduli exclusively. A way to realize such a model in the context of intersecting brane worlds consists in using orbifold background spaces [12,13,14,15] for compactifications of intersecting brane worlds of type I string theory [24,25]. Another option would be a model
where the complex structure moduli are dynamically stabilized by the open string tree level scalar potential, which happens in type IIA or type I intersecting brane worlds with some negative wrapping numbers, i.e. with some effective anti-branes present. The leading order contribution to the scalar potential then comes from the one-loop diagrams. By demanding some reasonable conditions to simplify the situation, the only diagram relevant is the annulus, which not only depends on the closed string Kähler moduli but also on open string fields, distances of the branes and Wilson lines. We find that, interestingly, the Kähler moduli are stabilized dynamically by the resulting potential, leading to small radii of the order of the string scale. In this regime all the closed string moduli are frozen, such that the open string moduli appear as inflaton candidates. While for large values of the Kähler moduli slow-rolling in the open string fields is possible as in [7], for small radii at the true minimum of the potential this property is lost. Therefore, a cosmological application of this scenario seems unlikely.

2. Toroidal Intersecting Brane Worlds

The formalism for describing intersecting brane worlds in type I, type II and even type 0′ string theory has been developed in [15-32]. Here, we mostly concentrate on type I strings but also include comments on the modifications for the other models. In order to proceed, we sketch the general idea and collect the formulae we need in the remainder of the discussion.

The starting point is a T-dual version of type I string theory. In toroidal models that we are going to use, one compactifies the type IIA closed string on a six-dimensional torus $T^6$ which, for simplicity, is taken to be of the form $T^6 = \bigotimes_{I=1}^{3} T^2_I$, each $T^2_I$ with coordinates $(X^I, Y^I)$ and radii $R^I_1$ and $R^I_2$. Then the $\Omega R$ orientifold projection is performed with $R$ being the reflection of all three $Y^I$ coordinates. As a toroidal compactification of type I theory down to four dimensions, this model has 16 supercharges and $\mathcal{N} = 4$ supersymmetry in the closed string sector. The orientifold projection introduces O6-planes stretching along the fixed loci of $R$, the $X^I$ directions, in compact space and filling out the entire four-dimensional uncompactified space-time.

In order to obtain the maximally symmetric solution that corresponds to pure type I strings, one cancels the Ramond-Ramond (RR) tadpoles induced in the non-orientable Klein bottle diagram by placing D6-branes on top of the O6-planes. This leads to a gauge group $SO(32)$ with 16 unbroken supercharges. However, the RR tadpole cancellation
conditions, stating that the sum of the homological cycles of all the D6-branes is the same as the homological cycle of the O6-planes

\[ \sum_{a=1}^{K} N_a \Pi_a = \Pi_{O6}, \]  

(2.1)
can also be satisfied by more general configurations of D6-branes which are no longer parallel but intersect on the internal space [13]. A D6\(_a\)-brane belonging to the stack \( a \in \{1, ..., K\} \) and wrapped on a 3-cycle of the \( T^6 \) can be specified by the wrapping numbers \( (n^I_a, m^I_a) \) along the fundamental cycles of the torus. Then (2.1) translates into conditions for these wrapping numbers and the brane multiplicities \( N_a \). The effective low energy theory in four dimensions for such intersecting brane models possesses interesting features which can be summarized as follows:

- **D-branes intersecting at angles generically break supersymmetry, so that at open string tree level supersymmetry is broken at the string scale.**
- **There is a \( U(N_a) \) gauge group on any stack of parallel D-branes.** In a supersymmetric vacuum a massless vector multiplet of \( \mathcal{N} = 4 \) exists in the adjoint of the gauge group, which splits up into massless and massive components when supersymmetry is broken. Interactions will then generate potentials and masses for all fields except for the gauge boson.
- **At an intersection point of two D6-branes there appears a massless chiral fermion in the bifundamental representation of the gauge groups \( U(N_a) \) and \( U(N_b) \) on the two respective branes.** Since two D-branes on a torus may have multiple intersections, the number of such fermions is degenerate and gives rise to the number of families. The tadpole cancellation conditions imply that the effective theory is anomaly free.
- **The mass of the lowest scalar excitation in the open string spectrum of strings stretching between two branes can be phrased in terms of their relative angles \( \varphi^I_{ab} \) on the three tori (given here for \( |\varphi^I_{ab}| \leq \pi/2 \)):**

\[ M^2_{\text{scal}} = \frac{1}{2\pi} \sum_{I=1}^{3} |\varphi^I_{ab}| - \frac{1}{\pi} \max\{|\varphi^I_{ab}| : I = 1, 2, 3\}. \]  

(2.2)

Obviously, depending on the angles at the intersection, a tachyonic scalar with negative mass can appear, which was proposed to have the interpretation of a Higgs field in the effective low energy theory [33,13,20]. The phase transition signaled by this scalar may simultaneously be responsible for a graceful exit from the inflationary expansion.
of the early universe. In fact, (2.2) can be interpreted as a string realization of a field
dependent mass term as in (1.2).

As mentioned, some of the above features are slightly modified in type II or type 0′ theory.
The general phenomenological patterns, however, remain unaltered. Motivated by these
general perspectives, a systematic investigation has revealed that one can indeed find
models with the Standard Model gauge group and three families of appropriate chiral
fermions. A more detailed study of the phenomenological properties of these models was
carried out in [19,20,22].

3. Tree level scalar potential

Based on the earlier work [7] where a simpler but less realistic brane-anti-brane system
was employed for modeling an inflationary universe, the cosmological use of intersecting
brane world models was studied in [11]. In both works, the dynamics of the closed string
moduli has been ignored and only the open string scalars, the positions of the D6-branes
on the internal transverse space were considered. Only in the simplified brane-anti-brane
model this restriction recently has been relaxed [12]. An argument in favor of such a
simplification could be that in such a brane-anti-brane setting the tree level tadpoles which
drive the dynamics of the transverse geometry are proportional to the inverse of the volume
of the space transverse to the branes. In a large extra dimension scenario [15,16], which
may be advisable in order to avoid the pitfall of the hierarchy problem anyway, this is large
and the tadpoles thus suppressed. But this argument may turn out to be misleading, as
the evolution of the transverse volume under consideration can still be fast on cosmological
scales.

In the following we will derive the leading order scalar potential for the closed string
moduli for general toroidal intersecting D6-branes. This potential occurs already at open
string tree level and is exact to all orders in α′. It also represents the potential for the
untwisted moduli of the orbifold models considered as another option for compactifications
in section 5.
3.1. The scalar potential in string frame

In [24] the open string tree-level scalar potential for toroidal intersecting brane worlds has been computed. The result can either be extracted directly from the divergence in the one-loop amplitudes or by integrating the Born-Infeld action for the D6-branes

\[ S_{\text{BI}} = -T_p \int_{\mathcal{M}_{p+1}} d^{p+1}x \ e^{-\phi_{10}} \sqrt{G_{p+1}} \]  

(3.1)

over their compact world volume. For type I intersecting brane worlds the potential is simply

\[ V = M^7_s e^{-\phi_{10}} \left( \sum_a N_a V^{\text{D6}}_a - V^{\text{O6}} \right) \]  

(3.2)

where \( V^{\text{D6}}_a \) denotes the three-dimensional internal volume of the D6\(_a\)-branes with wrapping numbers \( (n^I_a, m^I_a) \)

\[ V^{\text{D6}}_a = \prod_{I=1}^3 \sqrt{\left( n^I_a R^I_1 \right)^2 + \left( m^I_a R^I_2 \right)^2} \]  

(3.3)

and \( V^{\text{O6}} \) denotes the internal volume of the O6-planes stretched along the \( X^I \) axes

\[ V^{\text{O6}} = -16 \prod_{I=1}^3 R^I_1. \]  

(3.4)

For the case of intersecting D-branes in type IIA or even in type 0' string theory, the contribution from the orientifold planes is absent. In type IIA this implies the absence of any net RR-charge due to supersymmetry, but not so in type 0'. In this non-supersymmetric string theory the orientifold planes are rather exotic objects that carry charge but no tension. Defining the complex structure moduli and the four-dimensional dilaton as

\[ U^I = \frac{R^I_1}{R^I_2}, \quad e^{-\phi_4} = M^3_s e^{-\phi_{10}} \prod_I \sqrt{R^I_1 R^I_2}, \]  

(3.5)

one can express the disc level potential entirely in terms of these variables

\[ V_S(\phi_4, U^I) = M^4_s e^{-\phi_4} \left( \sum_{a=1}^K N_a \prod_{I=1}^3 \sqrt{(n^I_a)^2 U^I + (m^I_a)^2 \frac{1}{U^I}} - 16 \prod_{I=1}^3 \sqrt{U^I} \right). \]  

(3.6)

The non-trivial tadpole cancellation conditions phrased in terms of wrapping numbers read

\[ \sum_{a=1}^K N_a \prod_{I=1}^3 n^I_a - 16 = \sum_{a=1}^K N_a n^I_a m^J_a m^K_a = 0, \]  

(3.7)
for any combination of $I \neq J \neq K \neq I$. Furthermore, the brane spectrum is required to be invariant under

$$\Omega R : (n_a^I, m_a^I) \mapsto (n_a^I, -m_a^I).$$

Note that the potential only depends on the imaginary part $U^I$ of the complex structure of the torus, while its real part is frozen to take the value $b^I = 0$. The four-dimensional dilaton and the complex structures $U^I$ appear to be the natural variables for expressing the string frame leading order scalar potential. In the remainder of this paper we call them “Planck coordinates”. Moreover, the potential for the imaginary part $T^I = M_s^2 R_1^I R_2^I$ of the Kähler structures is flat at tree-level and thus can be neglected at this order. We will come back to the Kähler moduli in section 5 when discussing higher order one-loop corrections.

In $N = 1$ supersymmetric effective field theories in four dimensions the particular combinations of scalars

$$s = M_s^3 e^{-\phi_{10}} \prod_I R_1^I = e^{-\phi_4} \prod_I \sqrt{U^I},$$

$$u^I = M_s^3 e^{-\phi_{10}} R_1^I R_2^J R_2^K = e^{-\phi_4} \sqrt{\frac{U^I}{U^J U^K}}$$

appear in chiral superfields such that the effective gauge couplings can be expressed as a linear function of these variables [47]. In terms of these “gauge coordinates” the string frame scalar potential reads

$$V_S(s, u^I) = M_s^4 \sum_{a=1}^{K} N_a \left( \left( n_a^1 n_a^2 n_a^3 \right)^2 s^2 + \sum_{I=1}^{3} \left( m_a^I m_a^J m_a^K \right)^2 \left( u^I \right)^2 \right)$$

$$+ \left( m_a^1 m_a^2 m_a^3 \right)^2 \left( \frac{u^1 u^2 u^3}{s} \right) + \sum_{I=1}^{3} \left( m_a^I n_a^J n_a^K \right)^2 \left( \frac{s u^J u^K}{u^I} \right)^{\frac{1}{2}} - 16 M_s^4 s,$$

where the last term is the contribution from the O6-planes.

3.2. The scalar potential in Einstein frame

In order to discuss the slow rolling conditions we have to transform to the Einstein frame and need to make sure that the kinetic terms for the scalar fields are canonically
normalized. In terms of the “gauge coordinates” the resulting potential is given by

\[ V_E(s, u^I) = M_{pl}^4 \sum_{a=1}^{K} N_a \left( \frac{n_a^1 n_a^2 n_a^3}{u^1 u^2 u^3} \right)^2 + \sum_{I=1}^{3} \left( n_a^I m_a^J n_a^K \right)^2 \left( \frac{1}{s u^I u^K} \right)^2 + \right. \\
\left. \left( m_a^1 m_a^2 m_a^3 \right)^2 \left( \frac{1}{(s)^3 u^1 u^2 u^3} \right) + \sum_{I=1}^{3} \left( m_a^I m_a^J n_a^K \right)^2 \left( \frac{1}{s (u^I)^3 u^J u^K} \right)^{\frac{1}{2}} \right) \\
\left. - 16 M_{pl}^4 \left( \frac{1}{u^1 u^2 u^3} \right)^{\frac{1}{2}} - 16 M_{pl}^4 \left( \frac{1}{u^1 u^2 u^3} \right) . \right] \\
(3.11)

The rescaling to the Einstein frame performed above is simply defined by

\[ V_E(\phi_4, U^I) = \frac{M_{pl}^4}{M_s^4} e^{4\phi_4} V_S(\phi_4, U^I). \]
(3.12)

Since there is only one fundamental scale in string theory, one has the following relation between the string scale \( M_s \) and the Planck scale \( M_{pl} \)

\[ \frac{M_s}{M_{pl}} = e^{\phi_4} = (s u^1 u^2 u^3)^{-1/4} . \]
(3.13)

Obviously, a running of any single one of the four fields \( s, u^I \) at fixed \( M_{pl} \) implies an evolution of the fundamental string scale \( M_s \). After dimensional reduction to four dimensions, the kinetic terms for the scalar fields read

\[ S_{kin} = M_{pl}^2 \int d^4 x \left[ -(\partial^\mu \phi_4)(\partial_\mu \phi_4) - \frac{1}{4} \sum_{I=1}^{3} (\partial^\mu \log U^I)(\partial_\mu \log U^I) \right] , \]
(3.14)

respectively

\[ S_{kin} = M_{pl}^2 \int d^4 x \frac{1}{4} \left[ -(\partial^\mu \log s)(\partial_\mu \log s) - \sum_{I=1}^{3} (\partial^\mu \log u^I)(\partial_\mu \log u^I) \right] . \]
(3.15)

Thus, for the fields \( s, u^I \) with a logarithmic derivative appearing in (3.14) and (3.15), the correctly normalized field are \( \tilde{s}, \tilde{u}^I \) defined via

\[ s = e^{\sqrt{2} \tilde{s}} / M_{pl} , \quad u^I = e^{\sqrt{2} \tilde{u}^I / M_{pl}} . \]
(3.16)

In the following section we will investigate these leading order potentials in intersecting brane world models and look for slowly rolling scalar fields.
4. Complex structure and dilaton inflation

Let us emphasize again, that our main assumption from the very beginning is that we are allowed to work in string perturbation theory. This means that the ten-dimensional string coupling has to be small, i.e. $e^{\phi_{10}} \ll 1$. Next, since the open string tree level potential is exact to all orders in $\alpha'$ we do not need to impose that the internal radii are large compared to the string scale, which is in contrast to [12]. Moreover, we assume that all integer numbers appearing in the intersecting brane world construction, like the numbers $N_a$ of D-branes and the wrapping numbers $(n^I_a, m^I_a)$, are not exceedingly big. This assumption seems realistic, as with very big numbers it would seem impossible to realize a reasonable low energy particle spectrum.

In order for one of the scalars $(\phi_4, U^I)$ or $(s, u^I)$ of the closed string sector to be identified with the cosmological inflaton field, its potential has to satisfy the two slow-rolling conditions:

$$\epsilon = \frac{M_{pl}^2}{2} \left( \frac{V'_E(s, u^I)}{V_E(s, u^I)} \right)^2 \ll 1, \quad \eta = M_{pl}^2 \frac{V''_E(s, u^I)}{V_E(s, u^I)} \ll 1. \quad (4.1)$$

It is crucial that the derivatives of $V(s, u^I)$ are taken with respect to canonically normalized fields of (3.16).

We will investigate these conditions under the assumption that three of the relevant four scalar fields are frozen by some so far unknown mechanism, and then check the slow-rolling condition for the remaining scalar field. Depending on whether we work with the variables $(s, u^I)$ or $(\phi_4, U^I)$, our conclusions will turn out to be very different. Physically, the main difference between these two possibilities is that, due to the relation (3.13) in the first case, the string scale is forced to change during inflation, whereas in the second case it can be made constant by freezing $\phi_4$. In this sense the physical distinction between the respective sets of coordinates only plays a role when particular fields are frozen. If one would not just consider to freeze some of the coordinate fields, but also allow to impose relations among them, the distinction of coordinates would turn irrelevant.

4.1. Slow-rolling in “gauge coordinates”

As explained in [12], the “gauge coordinates” $(s, u^I)$ are the natural coordinates to work with if we assume some $\mathcal{N} = 1$ supersymmetric dynamics at some higher energy scale.
For toroidal type I intersecting brane world models, the breaking of supersymmetry looks spontaneous in the sense that $\text{Str}(\mathcal{M}^2) = 0$ in the open string spectrum. But one needs to be careful in interpreting the breaking mechanism. The potential (3.11) is in general not of the kind which can occur as the scalar potential in a supersymmetric theory. As long as no tachyon condensation in the open string sector takes place, which would induce a discrete change of some of the winding numbers $(n_a^I, m_a^I)$, while respecting charge conservation, this property will be unaffected by the dynamics. Hence, the theory is separated from a supersymmetric vacuum by a phase transition. In particular, for a given set of winding numbers, the theory will be non-supersymmetric at all scales. There is actually a situation, where the potential indeed can be cast into the form of a D-term potential in $\mathcal{N} = 1$ supersymmetric theories which refers to adding an FI term in the effective Lagrangian. At such a point, the potentially tachyonic NS groundstate is just massless, the scalar signalling a marginal deformation of the cycle wrapped by the two branes, which is a continuous deformation of the theory on their world volume. In this special situation, the non-supersymmetric effective theory is in the same phase as the supersymmetric theory which appears near the string scale.

Analogously to [12], we will now investigate the slow-rolling conditions, freezing three of the four “gauge coordinates” by hand. Here, we will merely discuss the generic situation neglecting the logical possibility that for very specific choices of the wrapping numbers or very special regions in parameter space new features might appear.

**s-Inflation**

Under the assumption that all three $u^I$ moduli are frozen, the potential for the field $s$ without the contribution from the orientifold planes is of the form

$$V^D_6 = M_{pl}^4 \sum_a N_a \left[ \alpha_a + \frac{\beta_a}{s^2} + \frac{\gamma_a}{s} + \frac{\delta_a}{s^3} \right]^{\frac{1}{2}} ,$$

(4.2)

where the coefficients can be read off from (3.11) and involve the fixed scalars $u^I$ and some numbers of order one. In particular, $\alpha_a > 0$. In the region $s \gg 1$ one has

$$V^D_6 = M_{pl}^4 \left[ \left( \sum_a N_a \sqrt{\alpha_a} \right) + \frac{1}{2} \left( \sum_a N_a \frac{\gamma_a}{\sqrt{\alpha_a}} \right) \frac{1}{s} + \cdots \right] .$$

(4.3)

1 For the quasi-supersymmetric models discussed in [47], all higher order corrections in $1/s$ are automatically absent at string tree level.
In type I the orientifold planes contribute

$$V_{E}^{O6} = -M_{pl}^4 \frac{16}{\prod_I u^I}. \tag{4.4}$$

If we choose all wrapping numbers $\prod_I n_a^I$ to be positive, then the constant term in (4.3) cancels precisely against the O6-planes contribution (4.4) due to the RR-tadpole cancellation conditions. In this case, one simply gets $V \sim 1/s$, which implies $V' \sim V$, with a constant of proportionality of order one. Note that the derivative has to be taken with respect to the canonically normalized field $\tilde{s}$, see (3.16). Thus, this case does not show slow-rolling behavior. However, if some of the $\prod_I n_a^I$ are negative, which should generically be expected to be the case, then we get a potential of the form

$$V_E = V_{E}^{D6} + V_{E}^{O6} = M_{pl}^4 \left( A + B e^{-\sqrt{2}s/M_{pl}} + \cdots \right), \tag{4.5}$$

using (3.16). The above distinction actually applies to the case of type I models, whereas in type II or type 0' no negative orientifold contribution appears in the potential, so that (4.5) always applies. Anyway, (4.5) is of the same kind as (4.3) in the absence of orientifold planes, i.e. without (4.4). This potential is identical to the one which appeared in the recent analysis in [12]. In fact, their configuration of D9- and D5-branes is only a very specific choice of D9-branes with magnetic flux, actually infinite magnetic flux for a D5-brane, which is just T-dual to the intersecting D6-branes studied in this paper.

For a potential of the form (4.5) the slow-rolling parameters are readily computed to be

$$\epsilon = \frac{B^2}{A^2} \frac{1}{s^2}, \quad \eta = \frac{2B}{A} \frac{1}{s}, \tag{4.6}$$

such that $\eta \ll 1$ directly implies $\epsilon \ll 1$. Inserting the expressions for $\alpha_a$ and $\gamma_a$, we deduce the following expression

$$\eta = \sum_I \zeta_I \frac{u^I u^K}{u^I s} = \sum_I \frac{\zeta_I}{(U^I)^2}, \tag{4.7}$$

where the coefficients $\zeta_I$ are of order one. Thus, in order to have slow-rolling, all complex structure moduli have to satisfy $U^I \gg 1$ with their relative ratios fixed. Note that due to (3.3) this is self-consistent with our assumption $s \gg 1$. We have thus found that $s$ is a successful inflaton candidate under the given assumptions. In the following we therefore discuss the properties of $s$-inflation in some more detail.
Exit from inflation

Since both $A > 0$ and $B > 0$, we have $\eta > 0$ in (4.3) and indeed face a positive cosmological constant. Hence $s$ rolls towards larger values, i.e. deeper into the slow-rolling region. Therefore, inflation can not end when the slow-rolling conditions cease to be satisfied. However, intersecting D6-branes can have open string tachyons localized on their intersection locus. Their appearance depends on the angles of the intersecting branes, which for fixed wrapping numbers depend on the complex structure of the internal torus, as was expressed in (2.2). Therefore, it appears very suggestive that the entire system may evolve from a tachyon free configuration of intersecting D6-branes, then during inflation rolls down to a region in parameter space where tachyons suddenly appear on certain intersections. These then dominate the dynamics and will trigger a decay of the brane configuration to a different, finally to a stable one. This solution to the graceful exit problem is very reminiscent of the hybrid inflation scenario [37,38]. As the tachyon field localized at some brane intersection carries the bifundamental representation of the respective unitary gauge groups on the two stacks of branes, it is well adapted to act as a Higgs field in the effective gauge theory. Thus, the breakdown of inflation by the condensation of D6-branes is interpreted as a phase transition that involves a spontaneous breaking of gauge symmetry.

To investigate this possible exit in some more detail, we assume that we have two intersecting D6-branes. At the starting point of inflation, the three parameters for the angles under which they intersect on the three $T^2_I$ are defined by

$$\theta^I_0 = \frac{1}{\pi} \arctan \left( \frac{\left(\frac{m^I}{n^I_2} + \frac{m^I}{n^I_1}\right)U^I_0}{1 + \left(\frac{m^I}{n^I_1} \frac{m^I}{n^I_2}\right)^2 \left(U^I_0\right)^2} \right), \quad (4.8)$$

indicating the initial values by their index 0. To simplify the situation, they are supposed to satisfy $0 < \theta^I < 1/2$ throughout their evolution. The mass of the lowest bosonic mode is given by

$$M^2_{\text{scal}} = \frac{1}{2} \sum_{I=1}^{3} \theta^I - \max\{\theta^I : I = 1, 2, 3\}. \quad (4.9)$$

Thus, the region in $\theta^I$ space where no tachyons appear is the interior of the cone with edges given by the bold diagonal lines in figure 1. On the faces of the cone the system preserves $N = 1$ supersymmetry and on the edges $N = 2$ supersymmetry. The origin
corresponds to parallel branes which preserve the maximal $\mathcal{N} = 4$ supersymmetry. During $s$-inflation the background geometry is driven towards larger values of all three complex structures $U^I$ but with their ratios fixed. Using (4.8), we conclude that if none of the two D6-branes is parallel to the $X^I$-axis, i.e. $m^I_a \neq 0$ for $a = 1, 2$, then the intersection angle $\theta^I$ is driven to 0. If in fact any one of the two is parallel to the $X^I$-axis, then $\theta^I$ goes to $1/2$. To summarize, up to permutations we have the following four possible endpoints of the flow

\[
(\theta^1, \theta^2, \theta^3) \rightarrow (0, 0, 0), \quad \mathcal{N} = 4 \text{ SUSY, no tachyons} \\
(\theta^1, \theta^2, \theta^3) \rightarrow (0, 0, 1/2), \quad \mathcal{N} = 0 \text{ SUSY, tachyons} \\
(\theta^1, \theta^2, \theta^3) \rightarrow (0, 1/2, 1/2), \quad \mathcal{N} = 2 \text{ SUSY, no tachyons} \\
(\theta^1, \theta^2, \theta^3) \rightarrow (1/2, 1/2, 1/2), \quad \mathcal{N} = 0 \text{ SUSY, no tachyons}. 
\]

This classification actually leaves out the fact that the points that the parameters are driven to cannot be reached within a given set of winding numbers for any finite value of $U^I$. For instance in the first case, two branes may approach vanishing intersection angles very closely, but only if their $(n^I_a, m^I_a)$ were proportional, they could become parallel. Thus, there may occur a situation where a set of branes evolves towards an $\mathcal{N} = 4$ supersymmetric
setting dynamically, approaching it arbitrarily well, but never reaching it without tachyon condensation. In fact, tachyons can then no longer be excluded for such a brane setting of the first type, as with three very small relative angles, the mass of the NS groundstate may still become negative. But one thing clearly can be deduced: Whenever the model contains two intersecting D-branes, where one of the D-branes is parallel to exactly one of the $X^I$-axes, the system evolves to a region where tachyons do appear. Unfortunately, it is difficult to determine in general the precise end-point of inflation, i.e. the point where the model crosses one of the faces in figure 1.

- **Number of e-foldings and spectrum of perturbations**

  For an inflationary model to be successful it not only has to satisfy the slow-rolling conditions, but also has to yield the right number of e-foldings

  \[ N = \int_{s_h}^{s_e} d\tilde{s} \frac{1}{M_{pl}^2} \frac{V_E}{V_E'} \simeq \frac{A}{2B} (s_e - s_h) \simeq 60 - \log \left( \frac{10^{16} \text{ GeV}}{V_{\text{inf}}^{1/4}} \right), \quad (4.11) \]

  where the index $h$ refers to horizon exit and the index $e$ to the end point of inflation, while $V_{\text{inf}}$ refers to the approximately constant value of the potential $V_E$ during inflation. Moreover, the amplitude of primordial density fluctuations [48] in our case is

  \[ \delta_H \sim \frac{1}{5\sqrt{3\pi}} \left( \frac{V_E^{3/2}}{M_{pl}^3 V_E'} \right) \simeq \frac{1}{5\sqrt{6\pi}} \left( \frac{A^{3/2}s_h}{B} \right), \quad (4.12) \]

  which must be tuned to yield the size of the observed temperature fluctuations of the CMB $\delta_H = 1.9 \times 10^{-5}$. The spectral index of the fluctuations is

  \[ n - 1 = -6\epsilon_h + 2\eta_h \simeq 2\eta_h \simeq \frac{4B}{A\varsigma_h}. \quad (4.13) \]

  In the present case of $s$-inflation we have $A, B > 0$ so that $s_e \gg s_h$, which implies $N = \eta_e^{-1}$. Without knowing $s_e$ and $s_h$, we cannot make any more detailed prediction.

- **$u^I$-Inflation**

  If one freezes all “gauge coordinates” except one of the complex structure moduli $u^I$, the story is very similar. Again, one gets a potential of the form (4.3) in a $1/u^I$ expansion. Even simpler, in this case the constant term $A$ never vanishes, not even in type I models, so that the slow-rolling parameter can always be written as

  \[ \eta = \zeta_1 \frac{u^J u^K}{u^I u^I} + \zeta_2 \frac{u^J s}{u^I u^K} + \zeta_3 \frac{u^K s}{u^I u^J} = \frac{\zeta_1}{(U^I)^2} + \zeta_2 (U^J)^2 + \zeta_3 (U^K)^2, \quad (4.14) \]
with the $\zeta_i$ of order one. Thus, slow rolling requires $U^I \gg 1$ and $U^J, U^K \ll 1$ which is self-consistent with the assumption $u^I \gg 1$. The constant $A$ is again positive but now the constant $B$ can in principal become negative in type I, as the orientifold planes also contribute. In case it is negative, the evolution would lead towards smaller values of $u^I$ until the slow-rolling condition is no longer satisfied or open string tachyons do appear. We have $u^I_h \gg u^I_e$ and, using $u^I_h = -2BN/A$, we can express the density fluctuations in terms of just $N$ and $A$

$$\delta_H \simeq \frac{2}{5\sqrt{6\pi}} A^{1/2}N, \quad n - 1 = -\frac{2}{N}.$$  \hfill (4.15)

Note that by fixing only two of the four parameters, it is not possible to satisfy the slow-rolling conditions for any two fields among the $(s,u^I)$ at the same time. Thus, if we do not fix three “gauge coordinates”, the potential is definitely not slow-rolling. Fast rolling scalars then destabilize the background before the slow-rolling of others becomes relevant cosmologically. From the mathematical point of view, this result is almost trivial. Qualitatively, in the “Planck coordinates” the potential looks like a four-dimensional generalization of the potential shown in figure 2. Starting at any point, there always does exist a direction in which the potential does not change, namely, if we move on a line of constant $V$. The only non-trivial fact is that the directions along the “gauge coordinates” are close to such lines of constant $V$ in their respective regions of slow-rolling.

Given that the Planck scale is a fixed constant in nature, $s$- and $u^I$-inflation implies that the string scale must evolve during inflation. Thus, for $B$ being positive, the length of the string inflates as well.

4.2. Slow-rolling in “Planck coordinates”

In this section we investigate the slow-rolling properties, working in the coordinates $(\phi_4, U^I)$. If two D-branes intersect under non-trivial angles on all three $T^2_I$, the annulus amplitude will only depend on the intersection angles and therefore on the complex structure moduli. These parameters are more natural in the string framework. Since $\phi_4$ is apparently not rolling slowly, we have to assume that this parameter is fixed by some non-perturbative string dynamics. Note, that in this case both the Planck-scale and the string scale are fixed, which seems to be automatic from the string point of view.
The only remaining candidate inflaton fields are the complex structure moduli $U^I$. The natural scale to freeze any of these “Planck coordinates” would in fact be the string scale. This can either be achieved by imposing orbifold symmetries or by the tree level potential itself. In \[32\] we have shown that in type $0'$ models such a dynamical stabilization takes place. Note that the same happens in type II and type I intersecting brane world models if some wrapping numbers $\prod_n n_a^I$ are chosen to be negative. This option has been excluded in \[24\] implicitly, where the run-away behaviour of the type I potential was discussed. Assuming that we have fixed two of these complex structures, the leading order potential for $U^I \gg 1$ is of the form

$$V_E(U^I) = M^4_{pl} A \sqrt{U^I}. \quad \text{(4.16)}$$

Only in type I with $\prod_n n_a^I > 0$ for all $a$ one has

$$V_E(U^I) = M^4_{pl} \frac{A}{\sqrt{U^I}}. \quad \text{(4.17)}$$
In any case, this does not satisfy the slow-rolling conditions. A similar analysis for \( U^I \ll 1 \) leads to the same negative result. The region in the parameter space near to local or global minima of the potential is less straightforward to discuss. Clearly, \( \epsilon \ll 1 \) near an extremum, but usually \( \eta \sim 1 \) for all examples we have studied, although we cannot present a general prove for this statement. Thus, we conclude that in “Planck coordinates” \((\phi_4, U^I)\) the disc level scalar potential is not of an inflationary type upon freezing any of these coordinates.

5. Kähler structure inflation

The only option that we have in the “Planck coordinates” to avoid the fast rolling of the complex structure moduli \( U^I \) is to freeze them. As it was shown in [24], this can be achieved in certain orbifold models from the very beginning while preserving some of the intriguing phenomenological properties of intersecting brane world models. Furthermore, for type 0' backgrounds, generically the complex structures are frozen dynamically at values of order one, as it was shown in [32]. The same holds for type I models with some negative wrapping numbers \( \prod_I n_a^I \), as discussed above.

From now on, we assume that the complex structure moduli \( U^I \) are frozen in some way at order one and study the leading order potential in the Kähler structure moduli \( T^I = M_s^2 R_1^I R_2^I \). The Einstein frame open string tree level potential now only depends on the four-dimensional dilaton

\[
V_{E}^{\text{tree}} = M_{pl}^4 C e^{3\phi_4},
\]

\( C \) being a constant of order one. A non-trivial dependence of the scalar potential on the Kähler moduli can arise at the one-loop level. Assuming that the closed string sector preserves supersymmetry, the torus and Klein-bottle amplitude vanish. Moreover, the annulus and Möbius strip amplitude can depend non-trivially on \( T^I \) via possible Kaluza-Klein (KK) and winding modes in non-supersymmetric sectors. Therefore, the interesting sectors are those, where two D-branes are parallel on one or two of the three \( T^I \) in a way that this open string sector still breaks supersymmetry. Beyond the Kähler moduli, the KK and winding spectrum then also depends on open string moduli, the distance \( x \) between the branes and their relative Wilson line \( y \). Thus, the full potential is a function of these three fields and the entire analysis will be more involved. We will restrict to a rather simple case, where we assume that the only contribution to the potential that depends
on the respective $T^I$ comes from a single set of two branes, which are parallel there but not parallel to any of the O6-planes. The only relevant amplitude left is then the annulus diagram of strings stretching between the two branes. We further neglect any dependence of the potential on $T^I$ other than $T^1 = T$, which finally is a function of only $(T, x, y)$. Up to this order in perturbation theory it then explicitly takes the form

$$V_{E}^{1\text{-loop}}(T, x, y) = M_{pl} M_s^3 C_0 - M_s^4 C_1 - A_{12}(T, x, y).$$  

(5.2)

The first two terms summarize contributions independent of $(T, x, y)$. The situation described above can indeed be met in the intersecting brane world orbifold models of [24].

In this sense, we now consider the annulus amplitude for two D-branes which are parallel on the first $T^1_2$ as the only one-loop contribution to the potential that depends on $T$. Then the open string KK and winding spectrum can be written as

$$M_{op}^2 = \Delta \left( \frac{(r + x)^2}{T} + T (s + y)^2 \right),$$

(5.3)

where $\Delta$ is of order one and depends on the wrapping numbers of the D-branes and the fixed complex structure $U^1$. Here, $0 \leq y \leq 1$ denotes the relative transversal distance between the two D-branes and $0 \leq x \leq 1$ the relative Wilson line along the longitudinal direction of the two D-branes on the $T^2_3$. These variables are related to the canonical normalized open string moduli by

$$Y^2 = \frac{1}{M_s^2} \Delta T y^2, \quad X^2 = \frac{1}{M_s^2} \Delta T x^2.$$  

(5.4)

The annulus loop channel amplitude reads

$$A_{12}(T, x, y) = \frac{M_{pl}^4}{(8\pi^2)^2} e^{4\phi_4} N_1 N_2 I_{12} \int_0^\infty \frac{dt}{t^3} \left( \sum_{r,s \in \mathbb{Z}} e^{-2\pi t \Delta [(r+x)^2/T + (s+y)^2]} \right)$$

$$\times \left( \vartheta_{[0]}^2 \vartheta_{[ei]} \vartheta_{[e_2]} - e^{-\pi i(e_1 + e_2)} \vartheta_{[0]}^2 \vartheta_{[e_1]} \vartheta_{[e_2]} - \vartheta_{[\frac{1}{2}]}^2 \vartheta_{[e_1 + \frac{1}{2}]} \vartheta_{[e_2 + \frac{1}{2}]} \right),$$

(5.5)

where $I_{12}$ denotes the intersection number of the two D-branes on $T^4 = T^2_2 \times T^2_3$. The argument of the $\vartheta$-functions is $q = \exp(-2\pi t)$ and $\epsilon^I = \varphi_{12}^I/\pi$ denotes the intersection angles of the two D-branes. The NS ground state energy is (again take $0 < \epsilon^I < 1/2$)

$$M_{scal}^2 = \left( \frac{\Delta x^2}{T} + \Delta T y^2 \right) + \frac{1}{2} (\epsilon^1 + \epsilon^2) - \max\{\epsilon^I : I = 1, 2\}.$$  

(5.6)
which might be tachyonic depending on \((T, x, y)\). Transforming the amplitude (5.5) into tree channel via the modular transformation \(l = 1/(2t)\) one obtains

\[
\tilde{A}_{12}(T, x, y) = \frac{M_{pl}^4}{(8\pi^2)^2} e^{4\phi} N_1 N_2 I_{12} \int_0^\infty dl \left( \frac{1}{\Delta} \sum_{r,s \in \mathbb{Z}} e^{-\pi l \Delta^{-1} [T r^2 + s^2/T]} e^{-2\pi i (r x + s y)} \right) \times \left( \vartheta \left[ 0^0 \right] \vartheta \left[ 0\epsilon \right] \vartheta \left[ 0\epsilon^2 \right] - \vartheta \left[ \frac{1}{2} \epsilon \right] \vartheta \left[ \frac{1}{2} \epsilon^2 \right] \vartheta \left[ \frac{1}{2} \epsilon' \right] \vartheta \left[ \frac{1}{2} \epsilon'^2 \right] - \vartheta \left[ \frac{1}{2} \epsilon' \right] \vartheta \left[ \frac{1}{2} \epsilon \right] \vartheta \left[ \frac{1}{2} \epsilon'^2 \right] \vartheta \left[ \frac{1}{2} \epsilon^2 \right] \right),
\]

where the argument of the \(\vartheta\)-functions is \(\tilde{q} = \exp(-4\pi l)\). Since all the non-supersymmetric vacua suffer from the presence of a NSNS tadpole, the one-loop amplitude contains divergences coming from the exchange of massless modes between the two D-branes. Thus, to continue, we have to regularize the expression (5.7) by subtracting the divergent piece

\[
\tilde{A}^{\text{reg}}_{12}(T, x, y) = \tilde{A}_{12}(T, x, y) - \tilde{K}_{12}
\]

with

\[
\tilde{K}_{12} = \frac{M_{pl}^4}{(8\pi^2)^2} N_1 N_2 I_{12} \int_0^\infty dl \frac{4}{\Delta} \sin^2 \left( \frac{(\epsilon_1 + \epsilon_2)}{2} \right) \sin^2 \left( \frac{(\epsilon_1 - \epsilon_2)}{2} \right) \frac{\sin(\pi \epsilon_1)}{\sin(\pi \epsilon_1)} \frac{\sin(\pi \epsilon_2)}{\sin(\pi \epsilon_2)}. \tag{5.9}
\]

Concerning the dynamics of \((T, x, y)\) the subtraction appears to be unimportant, since it does not depend on these fields. The first point to notice is that the potential (5.2) indeed stabilizes the Kähler modulus \(T\) dynamically, which can be seen as follows. For \(x = y\) the KK and winding sum is invariant under a T-duality, which maps \(T \to 1/T\) and exchanges \(x \leftrightarrow y\). Thus, there is an extremum at the self-dual point \(T = 1\), which fixes the internal radii at values of the order of the string scale. We have numerically evaluated the regularized annulus amplitude for specific choices of the angles and confirmed this expectation.

With \(T\) thus frozen, the open string modulus \(x = y\) could be a candidate inflaton field, if it satisfies the slow-rolling conditions, assuming that \(x = y\) is in fact dynamically stable. For \(T \gg 1\) and fixed a very similar result has in fact been proven in [7] in the neighbourhood of the instable antipodal point. The essential observation there was that the second derivate \(V''\) of the potential at the antipodal point vanishes, so that not only the slow-rolling paramter \(\epsilon\) but also \(\eta\) becomes arbitarily small. The question we now want to address is, whether this behavior still exists, if \(T\) comes close to its true minimum value at \(T = 1\). Then, not only massless modes contribute to the force between the two D6-branes but also massive string excitations.
For analyzing this question we expand the contribution to (5.10) from the \( \vartheta \)-functions in a \( \tilde{q} \)-series. Considering first the \( \tilde{q}^0 \) term and summing over all KK and winding modes, one essentially has to evaluate the integral

\[
\int_0^\infty dl \left[ -1 + \left( 1 + 2 \sum_{r \geq 1} e^{-\pi l \Delta^{-1} T r^2} \cos(2\pi r x) \right) \left( 1 + 2 \sum_{s \geq 1} e^{-\pi l \Delta^{-1} T s^2} \cos(2\pi s y) \right) \right].
\]

(5.10)

This can be done straightforwardly. By expanding the result around the “symmetric antipodal” point, using \( x = 1/2 - \bar{x}, \ y = 1/2 - \bar{y} \), we find that the linear and the quadratic terms in the fluctuations \( \bar{x} \) and \( \bar{y} \) precisely vanish. This computation exactly yields the large distance result of \([7]\). However, the minimum for \( T \) is not at large distances, but at distances of the order the string scale. Now, by taking the \( \tilde{q}^1 \) term into account and performing the same computation, we find that still the linear terms in \( \bar{x}, \bar{y} \) vanish, but that the quadratic terms do not. Thus, taking the exchange of massive string modes into account destroys the slow-rolling property \( \eta \ll 1 \). We have also done a numerical analysis and found this result confirmed.

Summarizing, in intersecting brane world models the leading order potential for the Kähler moduli is well suited to stabilize these dynamically. But at the real minimum of its potential near \( T = 1 \) the well appreciated slow-rolling properties of the open string moduli for large \( T \) are lost. A cosmological application therefore appears problematic.

6. Conclusions

In this paper we have analyzed intersecting brane world models with respect to their ability to give rise to inflation. Our starting point was the open string tree level scalar potential transformed to the Einstein frame. We have investigated this potential for two different scenarios. In the first scenario, as proposed in \([12]\), we have assumed that some of the “gauge coordinates” are frozen by an unknown mechanism. Analogously to \([12]\) we have found that the potential generically satisfies the slow-rolling conditions both for \( s \)-inflation and for \( u^I \)-inflation, if the remaining three moduli are frozen by hand.

In the second scenario we have worked with the “Planck coordinates”. Even after fixing the four-dimensional dilaton by hand, the potential for the complex structures was generically not of slow-rolling type. On the one hand it might look encouraging to find inflation at least in one scenario, on the other hand it is of course disappointing that one
has to freeze rather artificially a very particular choice of scalar fields, without knowing any explicit mechanism which could do this job.

Finally, we also have studied the leading order potential for the Kähler and open string moduli in models where the running of the complex structure moduli was frozen. First of all, we found that the Kähler moduli are generically stabilized dynamically. However, again rather disappointingly, the slow-rolling properties of the open string moduli at antipodal points, which were appreciated in simpler models, get lost for small values of the internal radii at the minimum of the scalar potential.

Apparently, we are still far away from a viable and realistic string theoretic realization of inflation. For all scalar potentials coming from string theory that have been studied so far, one can only achieve inflationary models by freezing some of the moduli by hand. At this time, this seems to be the state of the art and any substantial improvement would have to be considered a real progress in relating string theory to inflationary cosmology.

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