Modeling and Simulation Analysis of a Two-degree-of-Freedom Nonlinear Piezoelectric Energy Harvester

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Abstract. In order to improve energy harvesting efficiency, this paper shows the design of a two-degree-of-freedom (2-DOF) nonlinear piezoelectric energy harvester. The proposed harvest is realized by introducing magnetic attraction to a 2-DOF piezoelectric cantilever beam in order to broaden the energy harvesting bandwidth. Electromechanical equations describing this nonlinear system are presented. The simulation results confirmed the advantage of this new 2-DOF nonlinear harvester. Performance comparison shows that the 2-DOF nonlinear harvester has much higher energy harvesting efficiency than a conventional one-degree-of-freedom (1-DOF) nonlinear one.

1. Introduction
Piezoelectric materials are the most common intelligent materials in electromechanical and vibration systems [1]. The previous studies have shown that the energy conversion efficiency of vibration energy harvester is the highest in resonance state [2]. When the operating frequency deviates from the natural frequency of the beam, the output power of energy harvesting structure decreased obviously. Different forms of energy harvesting structures for the broadband and multi-direction vibration excitation need to be studied. Many researches have been conducted on designing energy harvester such as tuning methods [3], frequency-up methods [4], nonlinear methods [5] and so on.

In the design of piezoelectric energy harvester, the nonlinear method is an effective method to improve the performance of the harvester. Erturk et al. introduced the bistable concept into the design of energy harvester to gain much higher voltage [6]. Zhou et al. proposed the theoretical model and experimental investigations of a broadband piezoelectric energy harvester with a magnetic field which generated higher energy output over a wider range of frequency [7]. Zhao et al. designed a new two degree of freedom bistable piezoelectric vibration energy to broaden the working frequency band of the piezoelectric energy harvester [8]. To improve energy harvester efficiency, a new 2-DOF nonlinear piezoelectric energy harvester is proposed in this paper.

In this study, the design principle and the dynamic model of the proposed 2-DOF nonlinear piezoelectric energy harvester is reported, and its dynamic performance is investigated by simulation.

2. Theoretical Modeling
The typical configuration of the 1-DOF nonlinear energy harvester and the position of the piezoelectric element are shown in figure.1(a). The nonlinear magnetic force created by magnet A and magnets (B, C) is more important to the nonlinear characteristics of harvester. Magnet A and magnets (B, C) are
arranged by means of magnetic attraction. $d_1$ is the separation horizontal distance between the magnets (B, C) and the tip magnet A. $d_2$ is the gap distance between magnet B and magnet C. Monostable or bistable type of harvester could be obtained by adjusting the distances $d_1$ and $d_2$. Especially in the bistable state, the harvester exhibits the high-energy interwell motion. Higher voltage is generated and the effective bandwidth increases.

Figure 1. The structure diagram of the nonlinear piezoelectric energy harvester, (a) 1-DOF, (b) 2-DOF.

A 2-DOF cantilever nonlinear energy harvester is designed shown in figure1(b). Beam 1 and mass block D are added between the frame and magnet A. Then the harvester becomes a 2-DOF system and nonlinear phenomena can occur at the first two nature frequencies to broaden the effective frequency bandwidth. According to Newton’s second law and Kirchhoff laws [9], the dynamic equation of the 2-DOF cantilever nonlinear system can be written as

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\Omega_1^2 x_1 - 2\zeta_1 \Omega_1 x_2 + \mu (\Omega_2^2 x_4 + 2\zeta_2 \Omega_2 x_3) + \frac{1}{m_1} (\alpha_{eq} x_3 + F_z) - A \cos(\omega t) \\
\dot{x}_3 &= \frac{\alpha_{eq}}{C_p} x_5 - \frac{1}{C_p R_L} x_3 \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= \Omega_1^2 x_1 + 2\zeta_1 \Omega_1 x_2 - (\mu + 1) (\Omega_2^2 x_4 + 2\zeta_2 \Omega_2 x_3) - \frac{1}{m_2} ((\mu + 1) \alpha_{eq} x_3 - F_z)
\end{align*}
$$

where $z_0 = A \cos(\omega t)$ is the base excitation, $z_1$ is the relative displacement of the beam 1 to the base, $z_2$ is the relative displacement of the beam 2 to the beam 1. $m_1$, $k_1$, and $c_1$ is the equivalent mass, stiffness, damping of the beam 1. $m_2$, $k_2$, and $c_2$ is the equivalent mass, stiffness, damping of the beam 2. $F_z$ is the attractive force. $V_L$ is the voltage across the load resistance $R_L$, $\alpha_{eq}$ is backward electromechanical coupling in the dynamic equation, $C_p$ is the clamped capacitance.

Equation (2) can be solved by MATLAB software.

In the same way, we can obtain 1-DOF nonlinear piezoelectric energy harvester state equation.
3. Magnetic Analysis
The transverse displacement of magnet A is \(z(t)\). The magnetic fields of dipoles B acting on dipole A is given by [10]

\[ B_{BA} = -\frac{\mu_0}{4\pi} \frac{m_B \cdot r_{BA}}{\|r_{BA}\|^3} \]  

(3)

where \(r_{BA}\) is the vector directed from the magnetic moment source of B to that of A. \(\|\cdot\|_2\) denote Euclidean norm and \(\Delta\) vector gradient operator. The flux density generated by the magnet C on magnet A can be evaluated by the similar formula. The potential energy of the total magnetic fields can be shown by

\[ U_m = U_{mBA} + U_{mCA} = -B_{BA} \cdot m_A - B_{CA} \cdot m_A \]  

(4)

where \(m\) is the magnetic moment vector.

The magnetic dipole moment vectors \((m_A, m_B, m_C)\) and the vectors \((r_{AC}, r_{BC})\) can be written as

\[
\begin{align*}
[r_{BA}] &= \left[ -d_1, z - d_2 \right], \quad [r_{CA}] = \left[ -d_1, z + d_2 \right] \\
[m_C] &= \begin{bmatrix} M_C & V_C & 0 \end{bmatrix}, \quad [m_B] = \begin{bmatrix} M_B & V_B & 0 \end{bmatrix} \\
[m_A] &= \begin{bmatrix} M_A & V_A \cos \theta & M_A & V_A \sin \theta \end{bmatrix}
\end{align*}
\]  

(5)

where \(M_A\) and \(V_A\) represent the magnetization intensity and the volume of magnet A. The meaning of the other parameters is the same. The subscript B, C stands for magnet B and magnet C.

Thus the magnetic force can be given by

\[ F_z = \frac{\partial U_m}{\partial z} \]  

(6)

The bistable state can be implemented by adjusting the separation distance \(d_1\) and \(d_2\).

4. Numerical Simulation
The parameters of the 2-DOF nonlinear energy harvester are shown in table 1. The magnetic potential can be obtained by equation (4). The separation distance \(d_1\) is 7 mm and the gap distance \(d_2\) vertically is 8 mm to make sure the harvester is a bistable system. Then the potential energy curve has two stable points and one unstable point.

| Parameter                  | Value       |
|----------------------------|-------------|
| Mass of beam 1 \((m_1)\)  | 25 g        |
| Mass of beam 2 \((m_2)\)  | 4.8 g       |
| Stiffness of beam 1 \((k_1)\) | 604 Nm\(^{-1}\) |
| Stiffness of beam 2 \((k_2)\) | 133 Nm\(^{-1}\) |
| Damping of beam 1 \((c_1)\) | 0.0787 kgs\(^{-1}\) |
| Damping of beam 2 \((c_2)\) | 0.016 kgs\(^{-1}\) |
| Piezoelectric constant \((d_{31})\) | 285 C/N     |
| Capacitance of the PZT \((C_p)\) | 25.7 nF    |
| Magnetization vector \((M_A, M_B, M_C)\) | \(1.2 \times 10^6\) Am\(^{-1}\) |
| Volume of magnet \((V_A, V_B, V_C)\) | \(100\times\pi\) m\(^3\) |
The governing equation (2) and the 1-DOF nonlinear equation are solved by numerical method. The excitation amplitude is 5 m/s², the frequency $\omega$ is changing. The simulation parameters of the system are given in table 1. Figure 2 shows the root mean square RMS voltage responses of 2-DOF and 1-DOF nonlinear harvester at different excitation frequency. The system of 2-DOF has two effective voltage regions. The peak voltage is 1V at frequency 18Hz in the first region and the peak voltage is 0.6V in the second region.

![Figure 2](image)

**Figure 2.** The theoretical voltage responses of 2-DOF and 1-DOF nonlinear harvester.

In the first region, at frequency 18Hz the voltage vs. velocity trajectory, voltage history and phase portrait are showed in figure 3(a), figure 3(b) and figure 3(c). The 2-DOF system shows very large amplitude electromechanical response and the large amplitude periodic orbit is observed. Figure 3(c) shows the system has periodic motions between two equilibrium points. So the voltage is larger. But the amplitude of motion decreases with increasing frequency. At frequency 24Hz the voltage vs. velocity trajectory, voltage history and phase portrait are showed in figure 3(d), figure 3(e) and figure 3(f). The system moves around an equilibrium point and the voltage is reduced to 0.2V. In the second region, compared with the first phase, the voltage has a similar trend. The voltage increases at frequency 30 Hz and with increasing frequency, then the RMS voltage decreases. The effective bandwidth of the first region is 5Hz and the effective bandwidth of the second region is 3Hz. so the total bandwidth is 8 Hz.

![Figure 3](image)

**Figure 3.** The dynamic response at different excitation frequency of 2-DOF nonlinear harvester for $\omega=18$Hz (a) voltage vs. velocity trajectory (b) voltage history (c) phase portrait and $\omega=24$Hz (d) voltage vs. velocity trajectory(e) voltage history (f) phase portrait.
But the 1-DOF nonlinear harvester only has one effective voltage region and the peak voltage is 0.8V. The effective bandwidth is 5Hz. The performance of 2-DOF nonlinear harvester is better than 1-DOF nonlinear harvester under the same conditions.

5. Conclusion
In this paper, a new 2-DOF nonlinear piezoelectric energy harvester with magnetic coupling is developed to increase the frequency bandwidth and improve the efficiency of energy harvesting from low frequency vibrations. The new harvester can be designed to harvest the vibration energy efficiently over a frequency range (16-21Hz) in the first region and effective frequency (30-33Hz) in the second region. The total effective bandwidth is about 1.6 times to that of the conventional 1-DOF nonlinear harvester. The simulation results indicate that compared 1-DOF nonlinear harvester the 2-DOF nonlinear harvester has a good dynamic characteristic.

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