1. Introduction

By all that we know nature evolves in a self-organized critical state.\textsuperscript{1} One of the most fundamental characteristics of a system in a self-organized critical state (SOC) is to exhibit a stationary state with a long-range power law decay of both spatial and temporal correlations.\textsuperscript{2} Power law is a very abundant behavior appearing either in natural phenomena, such as the light emitted from quasars, the earthquakes intensities, the water level of the Nile river, or as a direct result of human activities like the distribution of cities by size, the repetition of words in the Bible and in traffic jams.

Self-organized critical systems evolve to the complex critical state without the interference of any external agent — there is no tuning parameter. The prototypical example of SOC is a pile of sand.\textsuperscript{2} Usually, the self-organized state is attained only after a very long period of transient. Last but not least, a minor change in the system can cause colossal instabilities called avalanches. Intermittent bursts of activity separating long periods of quiescence is called punctuated equilibrium.
Gould and Eldredge conjectured that the biological evolution in our planet is under the auspices of this kind of mechanism.\(^3\)

The evolution of the living beings is basically governed by the theory of natural selection. One model specially tailored to represent the co-evolutionary activities of the species is the Bak–Sneppen model (BS). In this model,\(^4\) each species occupies a site \(i\) of a lattice and has an associated fitness value \(\lambda_i\) between 0 and 1 (randomly drawn from an uniform distribution). At each time step, the species with the smallest fitness as well as its nearest neighbors are selected to replace their fitness with new random numbers. In one dimension, after a long transient time, almost all species have fitness larger than the critical value 0.67.\(^4\)

Recently, inspired by natural processes, some heuristic optimization techniques have been proposed: genetics algorithms,\(^5\) simulated annealing\(^6\) and extremal optimization.\(^7\) The latter, the extremal optimization method (EO), is the most efficient since it brings the system faster and closer to its ground state. In brief words, this method consists of the following rules: (1) a fitness \(\lambda_i\) with values between 0 and 1 (randomly chosen from an uniform distribution) is associated with each site \(i\) of a lattice with \(N\) points; (2) all the lattice sites are increasingly ranked according to their fitness (the site with the worst fitness is of rank 1); (3) a site of rank \(k\) \((1 \leq k \leq N)\) is selected with probability \(P(k) \sim k^{-\tau}\) \((\tau\) is an arbitrary real positive number) and its corresponding variable \(\lambda_i\) is changed to \(\lambda_i'\); (4) repeat at step (2) as long as desired.

We observe that, differently from what happens with the Bak–Sneppen dynamics, the EO dynamics has neither a co-evolutionary feature (the extinction of one species has no influence on its neighbors) nor has the exact (Darwinian) characteristic of the elimination of the worst adapted species. In this sense, we can say that the BS algorithm is a coarse grained description of the biological evolution adopted by nature while the EO algorithm represents an optimized dynamics created by man. In this paper, we compare the efficiencies (measured by their mean fitness in the steady state) of the EO and the BS dynamics. For the EO dynamics, we show that the spatial distribution is constant while the distribution of avalanches has an exponential decay. Using a discrete form of the BS model we argue that variability of the species is an essential prerequisite to keep self-organized criticality.

2. Simulations

To compare both dynamics, we simulated the BS and EO algorithms up to \(1.1 \times 10^9\) runs on a one-dimensional ring with \(N = 4001\) sites. To guarantee that the stationary regime has been achieved, we discarded the first \(1.0 \times 10^8\) runs as the transient time. Time averages were then taken over the remaining steps. Figure 1 shows the average frequency of the fitness \(\lambda\). Clearly, for \(\tau = 0.05\) the EO behaves like a random walk having an almost uniform and constant fitness distribution. At \(\tau = 0.5\) \((1.0)\) the distribution is an increasing linear (exponential) function of \(\lambda\).

For the BS dynamics, however, the distribution has the form of a step function.
There is no such a point. A critical self-organized state has been developed. For the EO there is no such a point. For regular geometries or exponential networks,\textsuperscript{8,10} the BS algorithm is the first sign that a critical point in the BS algorithm is the one and perfect species survives. This limit corresponds to the simplified toy model proposed by K. K. Yee.\textsuperscript{12}

To investigate the main differences between the evolutionary (EO) and co-evolutionary (BS) dynamics, we studied their spatial correlation dependence. Let us consider the form of a step function with a discontinuity at the critical point $\lambda_c \sim 0.67$. The dotted lines are the EO algorithm with $\tau = 0.05, 0.50$ and 1.00. The points represent the discrete BSD algorithm with only ten possible discrete fitness values (see the text).

Another important difference between the two dynamics is the complete absence of the punctuated equilibrium. Instead of a power law, Fig. 3, it is clear that the EO dynamics does not show a discontinuity at the critical point $\lambda_c \sim 0.67$. This critical point exists in all regular geometries or exponential networks,\textsuperscript{8,10} but not in scale-free networks.\textsuperscript{11} The presence of this critical point in the BS algorithm is the first sign that a critical self-organized state has been developed. For the EO there is no such a point.

To measure the algorithm’s efficiencies, we plotted in Fig. 2 the mean fitness obtained in the steady state regime. For the BS dynamics the mean fitness is 0.83.
This mean fitness corresponds to an EO with $\tau = 0.86$. At $\tau = 1.5$ the mean fitness of the EO dynamics is approximately 0.99. This means that for higher values of $\tau$, the EO algorithm leads to an utopian society where only one, and a perfect, species survives. This limit corresponds to the simplified toy model proposed by K. K. Yee in the context of law’s evolution in the judicial system. For $\tau$ in the interval $[0, 0.86]$, the BS surpasses EO.

As we pointed out before, while BS is a co-evolutionary dynamics the EO dynamics is only evolutionary. The species in the EO do not interact. A comparison between co-evolutionary and evolutionary performances have already been done in the context of cellular automata. To investigate the main differences between the evolutionary (EO) and co-evolutionary (BS) dynamics, we studied their spatial correlation dependence. Let $D(x)$ be the probability distribution of the distance $x$ between two subsequently extinct (or mutated) species. From Fig. 3, it is clear that the EO dynamics does not show a critical self-organized behavior. Instead of a power law, its spatial correlation is of infinite range — the probability distribution is constant no matter what is the distance between two subsequently modified species. For the BS dynamics, we find the well known power law dependence $D(x) \sim x^{-3.23\pm0.02}$.14

Another important difference between the two dynamics is the complete absence of the punctuated equilibrium in the EO algorithm. One way to check out the existence of the punctuated equilibrium is to measure the probability distribution $P(A)$ of the avalanches with size $A$. The size $A$ of an avalanche is defined as being the number of subsequent time steps with at least one fitness...
value below a critical threshold $\lambda_c$. This critical point does not exist for the EO (Fig. 1). For the BS algorithm the distribution decays as $P(A) \sim A^{-1.07 \pm 0.01}$ at $\lambda_c = 0.67$.\(^\text{14}\) In the EO algorithm, on the other hand, the decay is exponential with a characteristic avalanche size $A_c(\lambda_c, \tau)$ depending on the choices made for $\lambda_c$ and $\tau$.

3. Conclusions

We conclude that although the efficiency of the EO algorithm may exceed, under certain circumstances (if $\tau > 0.86$), that of the BS dynamics, it is accompanied by three undesirable characteristics: the spatial correlation between the species is constant, i.e., it is independent of their distances, there is an external free parameter $\tau$ to be adjusted by hand and the punctuated equilibrium mechanism is lost. The punctuated equilibrium seems to be a very productive form found by nature to innovate species without the intervening of climatic changes or meteors destruction.

We have learned that the EO dynamics does not conduct the system to a critical self-organized state. However, we would like to point out that even the BS can loose its SOC characteristics and, amazingly, in a very easy and quick manner. Suppose that, instead a continuous and uniform fitness distribution in the interval $[0, 1]$, only some discrete values are now possible. To simplify, assume that the fitness can only have $Q$ equally-spaced values, i.e., $\lambda = m/Q$, with $m = 1, 2, \ldots, Q$. Practically this means, that for some reason, the system’s biodiversity has decreased. Due to the discreteness, there will be an enormous number of species carrying the same (worst) fitness value. Which species should then we choose? The simplest solution is to put all those species in a list and to draw one of them. We will call this dynamics as Bak-Sneppen with draw (BSD).

In Fig. 1 we plotted the case $Q = 10$ and observe that, like in the EO dynamics, there is not a critical threshold $\lambda_c$. The mean fitness is 0.86 (Fig. 2), a value which is a little bit greater than that of the standard BS. The curve of the spatial probability distribution $D(x)$ (see Fig. 3) is even more interesting. It shows that the BSD dynamics is of a mixed kind: it behaves like the BS for small distances and like the EO for large distances. So, the BSD dynamics does not retain the self-organized criticality characteristic. Just like in nature, biodiversity plays a fundamental role in the evolutionary theoretical models: without it self-organized criticality is not possible. For higher plants and animals the conventional explanations of biodiversity are habitat heterogeneity, predation pressure and niche differentiation. For microscopic organisms, however, the high biodiversity found (even in uniform environments) is not completely understood and it is called “the paradox of the plankton”. Theoretically, such difficulties can be surmounted by incorporating a noise $\eta$ into the fitness $\lambda = m/Q \pm (1 - 2r)\eta$ (where $r$ is a random number in the interval $[0, 1]$ and generated from a uniform distribution). Even for $\eta$ as small as $10^{-12}$ the SOC characteristic is preserved.\(^\text{15}\) The noise can be interpreted as the presence of sub-species.
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