RESONANCE TYPE INSTABILITIES
IN THE GASEOUS DISKS
OF THE FLAT GALAXIES
II. The stability of solitary vortex sheet
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Linear stability analysis of the axisymmetric interface of velocity and density discontinuity in rotating gaseous disk has been performed numerically and analytically. Physical mechanisms leading to development of centrifugal and Kelvin–Helmholtz instability at the kink has been analysed in detail. In the incompressible limit it has been shown in the first time such areas in the parameter space that Kelvin–Helmholtz instability is stabilized by the density kink. This effect is caused by both specific angular momentum conservation and buoyancy. The possibility of application of obtained results to the stability analysis of the gaseous disks of the real flat galaxies is discussed.

Introduction.

The interest to a problem of stability of gaseous galactic disks is caused by the hydrodynamic concept of an origin of spiral structure of galaxies proposed in 1972 by A.M. Fridman (Fridman 1978). According to this concept the galactic spiral arms represent waves of density, arising and then reaching nonlinear stage as result of hydrodynamic (non-Jeans) instabilities development in a gaseous disk.

The most probable applicants for a role of the spiral structure generator are thus Kelvin–Helmholtz and centrifugal instability. The development of these instabilities on a shift layer in rotating cylindrical and disk configurations of gas was already investigated many times (see Morozov et al. 1975, 1976a, 1976b, Morozov 1977, Landau 1944). In accordance to Morozov (1979) and Torgashin (1986) development of hydrodynamic instability in model with smooth VJ-like rotation curve (i.e. having significant velocity fall between solid-body rotating and plato parts) was considered, and the jump of angular speed was concurrent to jump of density.

However, because of impossibility of analytical study the solution was found numerically, and from multiparametricity of a problem it was not possible to trace dependence of instability properties (growth rate) from parameters of model in details. Nevertheless we note that Morozov (1989) reported on instability weakening with growth of density contrast between inner and outer disk areas and about minority of influence of sound speed contrast. In all the other quoted works the analytical asymptotic solutions were received in uniform density models¹.

We continue study of sharp speed jump stability, concurrent on radius with density jump, and promote it in discontinuous non-homentopic model.

In the present part of the article we shall use the references to the formulae from a Part I, adding in this case symbol “1.” before number of formula.

¹ It is impossible to bypass work of Fridman (1989), where the deep analysis of gradient instabilities in a gaseous disk in the application to the hydrodynamic concept of spiral structure origin is spent. Unfortunately in the paragraph devoted to stability of velocity jump concurrent with density jump, faulty boundary condition (then corrected by Friman & Khoruzhii (1993)) was used, but investigation of growth rates of instability was not carried out.
1. Model and dispersion equation.

The most simple and convenient for the analysis model with density profile discontinuity cannot be polytropic (Morozov & Mustsevoy 1989). It is connected that at \( p(r) = \frac{K\sigma}{r^\gamma} \), where \( K = \text{const} \), the density jump existence is equivalent to jump of pressure. The same takes place for three-dimensional quantities too. By virtue of a condition of radial forces balance:

\[
\frac{V_\varphi^2}{r} = \frac{1}{\rho} \frac{dP}{dr} + \frac{d\Psi}{dr},
\]

where \( V_\varphi \) is gas rotation linear speed, \( \Psi \) is gravitational potential, the pressure jump with necessity entails jump of gravitational potential (the potential of double layer).

The data on distributions of galactic clouds velocities dispersion are practically absent; we shall use non-homentopic model with density jump but uniform pressure, that imposes relation on three-dimensional parameters of gas at both sides from jump:

\[
\rho_{ex}/\rho_{in} = \frac{c_{s_{in}}^2}{c_{s_{ex}}^2},
\]

where \( c_{s_{in}} \) is gas sound speed at inner side of jump and \( c_{s_{ex}} \) with outer one.

Considering told we research dynamics of disturbances of small amplitude on a background of thin three-dimensional non-selfgravitating non-homentopic disk unlimited on radius being in external stationary axisymmetic potential and consisting of ideal non-viscous gas. Following a standard linearization procedure we present all describing disk magnitudes as follows:

\[
f(r, \varphi, z, t) = f_0(r, z) + \tilde{f}(r, \varphi, z, t), \quad \text{where } |\tilde{f}| \ll f_0.
\]

The equilibrium axisymmetric stationary condition described by the radial balance equation (1) is characterized by the following parameters distributions:

\[
\begin{align*}
\rho_0(r, z) &= \rho_{in}(z) + (\rho_{ex}(z) - \rho_{in}(z)) \theta(r - R), \\
c_s(r, z) &= c_{s_{in}}(z) \left[ 1 + \left( \sqrt{\frac{\rho_{in}(z)}{\rho_{ex}(z)}} - 1 \right) \theta(r - R) \right], \\
\Omega(r, z) &= \Omega_{in}(z) + (\Omega_{ex}(z) - \Omega_{in}(z)) \theta(r - R), \\
P_0(r, z) &= c_s^2(r, z)\rho_0(r, z)/\gamma = \text{const}(z),
\end{align*}
\]

where \( \theta(x) \) is centered Heaviside function, \( \theta(x) = 0 \) at \( x < 0 \), \( \theta(x) = 1 \) at \( x > 0 \) and \( \theta(0) = 1/2 \), and \( \gamma \) is adiabatic index.

We do not use the Poisson’s equation determining gravitational potential because dealing with model problem we shall hereafter implicitly set necessary for us form of \( \Psi_0(r, z) \) believing condition (1) for given distributions (3) and condition of hydrostatic balance along \( z \)-coordinate holding.

Because of uniformity of model parameters on azimuthal angle \( \varphi \) and time \( t \) the solution of gas dynamics linearized equations system is searched by decomposition on normal modes. Thus separate Fourier-harmonics of a kind \( \tilde{f} \propto f(r, z) \exp(i\omega \varphi - i\omega t) \) evolution is further investigated. As Morozov (1989) show, for non-homentopic in a disk plane model \( (P_0(r, z)/\rho_0^2(r, z) \neq F(z)) \) it is described by the system of equations:

\[
\frac{dp}{dr} = \left( \frac{2m\Omega}{r\hat{\omega}} + \frac{P'_0}{\rho_0 c_s^2} \right) p + \rho_0 \left[ \hat{\omega}^2 - \kappa^2 + \frac{P'_0}{\rho_0} \left( \frac{P'_0}{c_s^2 \rho_0} - \frac{\rho'_0}{\rho_0} \right) \right] \xi,
\]
\[
\frac{d\xi}{dr} = \left( \frac{m^2 c_s^2}{r^2 \omega^2} - 1 \right) \frac{P}{c_s^2 \rho_0} - \left( \frac{2m\Omega}{r\omega} + \frac{1}{r} + \frac{P_0'}{c_s^2 \rho_0} \right) \xi.
\]  
(5)

Here designations of a Part I are kept, i.e. \( p \) is disturbance of pressure, \( \xi \) is perturbed radial Lagrange displacement, correlated to disturbance of a radial speed component by relation: 
\[
\frac{d\xi}{dt} = -i(\omega - m\Omega)\xi = -i\hat{\omega}\xi = \tilde{v}_r, \quad \hat{\omega} \text{ is Doppler frequency, } \kappa^2 = 2\Omega(2\Omega + rd\Omega/dr)
\]
is squared epicyclic frequency, \( c_s \) is sound speed, the prime stands for differentiation on radius.

The system (4)–(5) adequately describes dynamics of small disturbances in such objects as:

- a differentially rotating cylindrical configuration of non-dissipative non-selfgravitating ideal gas with the equation of state \( P_0 = \rho_0 c_s^2 / \gamma \) (\( P_0 \) and \( \rho_0 \) are homogeneous along cylinder forming equilibrium pressure and density respectively, \( c_s \) is adiabatic sound speed, \( \gamma \) is adiabatic index); the description is adequate for disturbances with \( \nu_z \equiv 0; \)

- a thin non-dissipative non-selfgravitating disk of polytropic gas, being in stationary external (star) gravitational potential of a kind \( \Psi(r, z) = \Phi(r) + \chi(z) \) (see Appendix I in the work of Gorkavy & Fridman (1994)); pinch type disturbances on \( z \)-coordinate are in this case considered only, \( v_r \) and \( v_\varphi \) with necessity designates their average on \( z \) values, and \( P_0 \) and \( \rho_0 \) are surface pressure and density accordingly; the two-dimensional adiabatic index \( \gamma_s \) is associated to a volumetric by relation: \( \gamma_s = 3 - 4/(\gamma+1) \) (Churilov & Shukhman 1981).

Note that the realization of the last case is necessary for construction of step-homogeneous two-dimension model of thin disk because for potential of a more general form \( \Psi = \Psi(r, z) \) either rotation velocity averaging \( V_\varphi = V_\varphi(r, z) \) on \( z \)-coordinate will not lead to discontinuous distribution of \( V_\varphi(r) \) or there will be the radial inhomogeneities of pressure, which are necessary for fulfilment of equilibrium forces balance. A really specified form of potential means that the disk is quite thin for applicability of potential decomposition in a series on small parameter \( z/r \) with neglecting of all terms more senior than second-order.

As in each of areas divided by jump all equilibrium parameters are homogeneous, the system (4)–(5) becomes identical to system (1.1)–(1.2) and is reduced to modified Bessel equation on \( p \), the general solution of which is a linear combination of modified Bessel functions \( I_m(k_ir) \) and \( K_m(k_ir) \) where a designation is introduced

\[
k_{i;\in;ex}^2 = \frac{4\Omega^2_{\in;ex} - (\omega - m\Omega_{\in;ex})^2}{c_s^2_{\in;ex}}.
\]  
(6)

Note that completely analogous equation is also valid for quantities \( \rho(r), \eta(r) = \rho(r)/\rho_0, \) \( \mathcal{P} (r) = c_s^2 \eta(r) \). Assuming hereafter for uniqueness of the solution \( \Re \, \kappa_{i;ex} > 0 \), from a natural condition of its limitation in a disk centre and on infinity follows:

\[
p(r) = \begin{cases} 
   AI_m(k_{i;\in}r), & r < R, \\
   BK_m(k_{ex}r), & r > R.
\end{cases}
\]  
(7)

Boundary conditions for the solutions on jump we determine by integrating the equation (4), (5) on radius from \( R - \varepsilon \) up to \( R + \varepsilon \) with account of peculiarity of chosen model — \( \partial P_0/\partial r = 0 \) and then tending \( \varepsilon \to 0 \) (see Fridman & Horuzhii 1993 for details):

\[
\xi(R + \varepsilon) - \xi(R - 0) = 0,
\]  
(8)
\[ p(R + 0) - p(R - 0) = \frac{\rho_{in} + \rho_{ex}}{2} \xi(R) R(\Omega_{in}^2 - \Omega_{ex}^2). \] (9)

Coupling of the solutions at \( r = R \) results in the system of equations on factors \( A \) and \( B \). The condition of simultaneity of this system represents the dispersion equation allowing to determine frequency eigenvalues:

\[
\begin{vmatrix}
\beta_{in} & \frac{Q}{\mu} k_{in}^2 R^2 \alpha_{ex} \\
1 & k_{ex}^2 R^2 - \frac{M^2}{2\mu^2} (1 + Q)(1 - q^2) \alpha_{ex}
\end{vmatrix} = 0. \quad (10)
\]

Here the following designations are made: \( M = R \Omega_{in}/c_{sin} \) is Mach number, \( q = \Omega_{ex}/\Omega_{in} \), \( Q = \rho_{in}/\rho_{ex} \), \( \mu = c_{sex}/c_{sin} \), \( \alpha_i \) and \( \beta_i \) are defined by (1.8) and (1.9) in which the index \( i \) still takes values “\( in \)” and “\( ex \)”, and the top index is omitted because of coincidence of radii of speed and density jumps. Besides let us introduce dimensionless frequency \( x = \omega/\Omega_{in} \). The existence of positive \( \text{Im} \omega \) (growth rate) means instability.

Note the original dispersion equation was satisfied identically for neutral values \( (\text{Im} \omega = 0) \): \( \omega = (m \pm 2)\Omega_{in} \), \( \omega = (m \pm 2)\Omega_{ex} \), that is for gyroscopic modes of fluctuations of a homogeneous solid-body rotating disk. It is natural result, as the disturbances with such frequencies propagate exclusively in an azimuthal direction \( (k_{in} = 0, \text{ or } k_{ex} = 0) \) and are not “sensitive” to radial inhomogeneities. Accordingly the given modes do not influence system stability and deriving (10) we supposed that the frequency does not take such values.

2. The stability analysis.

Let us obtain the approximated solutions of the dispersion equation (10) for limiting cases of large and small compressibility of medium.

\( a) \) The case of large compressibility \( (M \gg 1) \)

By applying representation of arguments of Bessel functions through dimensionless parameters note the module of argument is dictated first of all by Mach number \( M \). Assuming therefore \( |k_{in} R| \gg m, \ |k_{ex} R| \gg m \), from (10) we find:

\[
x \simeq \frac{1}{\mu + Q} \left\{ m(Q + q\mu) + \frac{i}{2} M(1 + Q)(1 - q^2) \right\}. \quad (11)
\]

See \( |x| \sim M, \ |k_{ex} R| \sim M^2, \ |k_{in} R| \sim M^2 \), therefore the assumptions made earlier are justified backdating.

For a case of constant density from (11) we get result obtained by Morozov (1977):

\[ \omega \simeq \frac{\Omega_{in}}{2} \left[ m(1 + q) + iM(1 - q^2) \right]. \quad (12) \]

Taking into account that expression \( \mu = \sqrt{Q} \) takes place in model without self-gravitation let us write down growth rate (11) as:

\[
\gamma \simeq \frac{\Omega_{in}}{2} M \frac{1 + Q^{-1}}{1 + Q^{-1/2}} (1 - q^2). \quad (13)
\]

Remarkable peculiarity of obtained result is extremely weak dependence of growth rate on magnitude of density jump at \( \rho_{in} \gg \rho_{ex} \), and on the contrary proportionality to \( Q^{-1/2} \) at
\( \rho_{in} \ll \rho_{ex} \). Applying similar reasons we determine that for wave pattern angular rotation phase speed \( \Omega_p = Re \omega/m \) the following estimations \( \Omega_p \simeq \Omega_{in} \) at \( \rho_{in} \gg \rho_{ex} \) and \( \Omega_p \simeq \Omega_{ex} \) at \( \rho_{in} \ll \rho_{ex} \) are valid. Hence the wave pattern rotation speed comes nearer to rotation speed of more dense medium at significant density differences.

Note that in absence of speed shift (\( q \equiv 1 \)) neutral solution \( \omega \simeq m \Omega_{in} \) respecting to non-growing disturbances “frozen” in gas follows from (11). Thus density kink alone cannot cause instability.

To shed some light on physics of instability it is convenient to pass from (11) to the dimensional form:

\[
\omega \simeq M \left( \rho_{in} c_{sin} \Omega_{in} + \rho_{ex} c_{sex} \Omega_{ex} \right) + \frac{i}{2} \left( \rho_{in} + \rho_{ex} \right) R \left( \Omega_{in}^2 - \Omega_{ex}^2 \right) / c_{sin} \rho_{in} + c_{sex} \rho_{ex}.
\]

(14)

From (14) follows that the growth rate of considered instability in the main order is equal to the ratio of difference of density of centrifugal force, acting on a liquid particle fluctuating in small vicinity of jump, to the sum of wave resistance (characteristic impedances) of media on both sides from jump. Thus this instability must be called centrifugal one, just same as in case of disk of uniform density, which Morozov (1977) had consider\(^2\).

\( b \) An incompressible limit (\( M \ll 1 \)).

Taking into account \( |k_{in;ex}R| \propto M \) we make following assumption: \( |k_{in}R| \ll 1 \), \( |k_{ex}R| \ll 1 \). Applied asymptotic representation of modified Bessel functions at small values of argument (Handbook of Mathematical Functions... 1964), we find the solution (10) in the dimensionless and dimensional forms:

\[
x \simeq \frac{1}{1 + Q} \left\{ m(q + Q) + (q - Q) \pm \right.
\]

\[
\pm \left[ (q - Q)^2 - \frac{1}{2} m (1 - Q)^2 (1 - q^2) - m^2 Q (1 - q^2) \right]^{1/2} \right\},
\]

(15)

\[
\omega \simeq \frac{1}{\rho_{in} + \rho_{ex}} \left\{ m \left( \Omega_{in} \rho_{in} + \Omega_{ex} \rho_{ex} \right) + \left( \Omega_{ex} \rho_{ex} - \Omega_{in} \rho_{in} \right) \pm \right.
\]

\[
\pm \left[ \left( \Omega_{ex} \rho_{ex} - \Omega_{in} \rho_{in} \right)^2 - \frac{1}{2} m (\rho_{ex} - \rho_{in})^2 (\Omega_{in}^2 - \Omega_{ex}^2) - 
\]

\[
-m^2 \rho_{in} \rho_{ex} \left( \Omega_{in} - \Omega_{ex} \right)^2 \right\]^{1/2} \right\}.
\]

(16)

The negativity of expression under square root sign in (15) and (16) means instability.

In a case \( Q = 1 \) (uniform on density at \( z = const \) disk) the result received by Morozov (1977) follows from (15):

\[
\omega \simeq \frac{\Omega_{in}}{2} \left\{ m(q + 1) + (q - 1) + i \left| 1 - q \right| \sqrt{m^2 - 1} \right\}.
\]

(17)

\(^2\) Term “centrifugal” with reference to considered instability was introduced later (Morozov et al. 1984, 1985a, 1985b).
In absence of velocity difference on jump \( q \equiv 1 \) neutral solutions follow from (15) as well as in the case of large compressibility, but in this case they are the pair: \[ \omega \approx \Omega_{in} \left( m + 2 \frac{1 - Q}{1 + Q} \right). \] (18)

At significant density contrast (18) governs gyroscopic modes: \[ \omega \approx (m + 2)\Omega_{in} \text{ at } Q \ll 1 \] and \[ \omega \approx (m - 2)\Omega_{in} \text{ at } Q \gg 1. \] As one can see at \( M \ll 1 \) the density jump alone cannot cause instability too.

Let us analyse a physical nature of terms in (15) and (16) which stabilize and destabilize the system.

For the disturbances with shortest azimuthal wavelength \( (m \gg 1) \)
\[ \omega \approx m \Omega_{ex} \rho_{ex} + \Omega_{in} \rho_{in} \rho_{in} + \Omega_{ex} + \Omega_{in} |V_{in} - V_{ex}|. \] (19)

follows from (16).

Introducing azimuthal wave number on jump \( k_{\varphi} = m/R \) and linear speeds of rotation inside and outside in immediate jump proximity \( V_{in} = R\Omega_{in} \) and \( V_{ex} = R\Omega_{ex} \) accordingly, it is easy to bring (19) to the form:
\[ \omega \approx k_{\varphi} \rho_{ex} V_{ex} + \rho_{in} V_{in} \rho_{in} + \rho_{ex} + i k_{\varphi} \sqrt{\rho_{in} \rho_{ex}} |V_{in} - V_{ex}|. \] (20)

The last expression is identical to classical expression describing Kelvin–Helmholtz instability (KHI) developing on vortex sheet in plainparallel current of an incompressible fluid in absence of gravitation force \( (g \equiv 0) \):
\[ \omega = k_{\parallel} \frac{\rho_{1} V_{1} + \rho_{2} V_{2}}{\rho_{1} + \rho_{2}} \pm \sqrt{k_{\parallel} g \frac{\rho_{2} - \rho_{1}}{\rho_{1} + \rho_{2}} - k_{\parallel}^{2} \frac{\rho_{1} \rho_{2}(V_{1} - V_{2})^{2}}{(\rho_{1} + \rho_{2})^{2}}. \] (21)

Here the index 1 marks magnitudes concerning to the top layer, 2 to the bottom one respectively, \( k_{\parallel} \) is wave number of disturbances along vortex sheet\(^3\). Thus the nature of the last term under square root sign in (16) always destabilizing jump surface is perfectly transparent: it is caused by Bernoulli effect. This result is quite expected as for disturbances with short on \( \varphi \) wavelength effects of curvature and rotation are the least essential.

Second term under square root sign in (16) can be rewritten as follows:
\[ -k_{\varphi} \frac{(\rho_{ex} - \rho_{in})^{2} R\Omega_{in}^{2} - R\Omega_{ex}^{2}}{(\rho_{ex} + \rho_{in})^{2}}. \] (22)

This term is directly proportional to centrifugal acceleration difference on jump, renders stabilizing influence at \( \Omega_{ex} > \Omega_{in} \) and destabilizing at \( \Omega_{ex} < \Omega_{in} \). So the effects of centrifugal stabilization and destabilization (see, for example, Nezlin & Snezhkin 1990)

\(^3\) Inasmuch as our model is quasi-two-dimensional \( (\bar{v}_{z} \equiv 0) \), equation (21) is written out in the assumption of concurrence of directions \( V_{1}, V_{2} \) and \( k_{\parallel} \).
take place, which are well known for a compressed fluid. However, for an incompressible liquid they are shown for the first time. It is also interesting that these effects act only in a case \( \rho_{in} \neq \rho_{ex} \). It has resulted to such fact that they were not found by Morozov (1977). The latter, on our sight, is a direct consequence of incompes sibility and is explained as follows.

The incompressible fluid is characterized by significant (infinite as an ideal) elasticity, because of that centrifugal effects cannot result instability in the same way, as in compressible fluid, where liquid particle releasing from vortex sheet by non-compensated centrifugal or gravitational force is displaced on radius “tightening” particles being ahead it. In given case the radial rearrangements of liquid particles can come true only by their mutual replacement against each other, as it occurs at development of Rayleigh–Taylor instability, when particle of heavy fluid “sinks” on a place of an emerging particle of light fluid. Not casually therefore similarity of a considered combination (22) with first term under square root sign in (21), just governing the development of Rayleigh–Taylor instability on liquids interface in plainparallel flow. This similarity becomes especially transparent in the case of significant density contrast: assuming in (21) \( \rho_1 \gg \rho_2 \) we get \( k_{∥}g \) and in the case \( \rho_1 \ll \rho_2 \) this term gives \( -k_{∥}g \). From (22) as at \( \rho_{in} \gg \rho_{ex} \) and at \( \rho_{in} \ll \rho_{ex} \) follows

\[
-k_{ϕ} \frac{RΩ_{in}^2 - RΩ_{ex}^2}{2}. \tag{23}
\]

Thus in both cases we have longitudinal wave number and mass force acceleration on vortex sheet production. Told leads to unexpected conclusion that for considered axisymmetric vortex sheet a combination of parameters

\[
\frac{\rho_{in} - \rho_{ex}}{\rho_{in} + \rho_{ex}} \frac{RΩ_{in}^2 - RΩ_{ex}^2}{2} \tag{24}
\]

is equivalent \( g \) in a plainparallel case.

Generalizing told we conclude that (22) describes buoyancy effects in considered model and it is caused by buoyancy forces disbalance.

At last the first term under square root sign in (16) is easily represented as

\[
\frac{\rho_{ex} R^2 Ω_{ex} - \rho_{in} R^2 Ω_{in}}{\rho_{ex} R^2 + \rho_{in} R^2}, \tag{25}
\]

which is possible to interprete as the ratio of a half-difference of density of an angular momentum on both jump sides to specific (per volume unit) momentum of inertia of gas on jump. Tracing genesis of this term in any case rendering stabilizing influence upon jump it is possible to note that it is caused by action of Corio lis’ forces and reflects the tendency to preserve an angular momentum of system.

Rather interesting result is appearence of stability area on parameter \( q \) in a vicinity of \( q = 1 \) at \( Q \neq 1 \). This area wider as stronger \( Q \) differs from unit and less as the less is mode number \( m \). The borders of stability constructed on the asymptotic formula (15) for \( m = 2, 3, 4 \) are shown on Fig. 2.1.

On Fig. 2.2–2.5 results of the numerical solution of the dispersion equation (11) by iterative Newton–Rafson method are drawn. From them follows that the approximated formula (15) will be good coordinated with dispersion curves in an incompressible limit.
The same good consent of results with (11) takes place in an inverse case of essentially supersonic speed difference on jump, if both $M^2(1 - q^2) \gg m$ and $M^2(1 - q^2)/\mu \gg m$ simultaneously are valid.

On our sight Fig.2.5 is rather interesting, on which stabilization of centrifugal mode with growth $q$ is shown. Though it is occurs formally at $M \gg 1$ the point of stabilization and branching on two neutral modes corresponds to subsonic speed difference on jump $M(1 - q)/\mu \simeq 0.3 \ll 1$, therefore the mechanism of stabilization is similar to described above.

Thus comparative analysis of results of numerical and asymptotic analytical researches of the equation (10) allows to conclude that all conclusions made in this section are valid.

3. Conclusions.

Let us summarize the basic conclusions concerning to stability of non-homentopic angular speed and density discontinuity profiles of finite thickness ($\xi \ll \Lambda_\Omega, \Lambda_\rho$, where $\Lambda_\Omega, \Lambda_\rho$ are characteristic radial scales of angular speed and density jumps) in model of uniform on pressure at $z = \text{const}$ gaseous disk.

1. At essentially supersonic speed difference on jump centrifugal instability is raised, which growth rate is equal to the ratio of difference of density of centrifugal force acting on liquid particles in vicinity of jump, to the sum of media wave resistances at both sides of jump. Thus the angular rotation speed of a spiral pattern in case of significant density drop is close to more dense medium rotation speed.

2. At small compressibility of both media the Kelvin–Helmholtz instability develops. It is caused by Bernoulli effect and at presence of density difference also by buoyancy effects, for which the role of external mass force density is played by the following combination of parameters:

$$g_{eff} = \rho_{in} - \rho_{ex} \frac{R\Omega_{in}^2 - R\Omega_{ex}^2}{\rho_{in} + \rho_{ex}}.$$ 

Depending on sign of $g_{eff}$, an additional centrifugal destabilization or stabilization of KHI takes place. Besides, at comparison with KHI on plain parallel vortex sheet there are the effects caused by angular momentum preservation. This effects exert in any case a stabilizing influence, the more essential, the larger is disturbance scale (and the less number $m$) and the more considerable density drop.

3. Density drop alone in absence of speed difference is not capable to result instability of considered model both in supersonic and subsonic case.

It is necessary to note that as the KHI does not develop for disturbances with wavelength comparable to or exceeding characteristic scale $\Lambda_\Omega$ (that is modes with $m \gg 1$ will appear neutral in view of finite jump thickness) and the large-scale disturbances are stabilized by virtue of conclusion 2, for smooth subsonic speed jumps there is the area of stability on parameters $q, Q$. The latter is necessary to take into account at stability analysis of real objects.

The stabilization of Kelvin–Helmholtz instability for long wavelength (with small $m$) disturbances of subsonic non-homentopic rotation speed and density jump in view of suppression of this instability for short-wave disturbances because of inevitably present finite radial “smearing” of jump means an opportunity of rather long-lasting existence of such jump. Thus the rotation speed on radius of jump can be even supersonic. Such jumps are observed in central areas of some flat galaxies (Afanesiev et al. 1988a, 1988b, 1988c, 1988d).
and can play a role of an internal reflecting surface for waves amplifying on a corotation resonance (Fridman et al. 1994). Our analysis allows to assert that in case of significant density contrast on jump the life-time of such surface will be sufficient for development of resonant instability of the specified waveguide layer. Otherwise the jump will be smeared out because of “faster” surface mode development earlier than resonant disturbances will grow to nonlinear amplitudes.

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Fig. 2.1. Border of marginal stability of azimuthal modes $m = 2, 3, 4$ (accordingly, continuous line, long dash, short dash) according to the asymptotic formula (15).
Fig. 2.2. Lines of a level of dimensionless growth rate for \( m = 2 \): (a) on the data of the numerical solution \((10)\), (b) according to asymptotic \((15)\). The breaks of isolines on the top drawing are probably artefact of the numerical solution.
Fig. 2.3. Dependence of dimensionless frequency $Re \omega/\Omega_{in}$ (a), (c) and dimensionless growth rate $Im \omega/\Omega_{in}$ (b), (d) from Mach number on the data of the numerical solution (10) (continuous lines) and asymptotics (11) (dashed lines) for various sets of parameters $m$, $q$, $Q$. 
Fig. 2.4. Dependence of dimensionless frequency (a), (c) and dimensionless growth rate (b), (d) from relative density difference on jump on the data of the numerical solution (10) (continuous lines) and asymptotics (11) (dashed lines) for various sets of parameters: $m, q, M$. 
Fig. 2.5. Dependence of dimensionless frequency (a), (c) and dimensionless growth rate (b), (d) from value of speed difference on jump on the data of the numerical solution (10) for non-axisymmetric ($m = 2$) and axisymmetric ($m = 0$) modes.
