The NLO Chiral Lagrangian from the meson-baryon interaction in the $S = -1$ sector.

A Feijoo and V K Magas and A Ramos
Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, Martí Franquès 1, E08028 Barcelona, Spain
E-mail: feijoo@fqa.ub.edu

Abstract. The present study consists of establishing the role played by the next-to-leading order (NLO) terms as well as the Born terms of a chiral SU(3) Lagrangian in the $S = -1$ meson-baryon interaction. This necessarily leads to the determination of the low energy constants present in the NLO contribution, which have not been fixed so far. In order to constrain them, we employ scattering data from the $K^- p$ interaction, paying a special attention to $K^- p \rightarrow K \Xi$ reactions and to isospin filtering processes such as $K^- p \rightarrow \eta \Lambda, \eta \Sigma^0$. In addition, the stability and the accuracy of all the parameters involved in the model have been examined by means of the inclusion of high spin hyperonic resonances.

1. Theoretical background and procedure
In the last years, the interest in studying the meson-baryon interaction in the $S = -1$ sector has been sparked by the availability of more precise data coming from the measurement of the energy shift and width of the 1s state in kaonic hydrogen by the SIDDHARTA collaboration [1]. All these works were carried out as a response to the need to extend the approach to higher orders and energies aiming for greater accuracy in data description. Since this scenario requires a non-perturbative resummation of Chiral Perturbation Theory (ChPT), the authors employed a chiral SU(3) Lagrangian up to next-to-leading order (NLO) and implemented unitarization in coupled channels. Unitarized Chiral Perturbation Theory (UChPT) consists in solving the Bethe-Salpeter equation (BSE), which can be converted into a system of algebraic equations using the on shell factorization [2,3]: $T_{ij} = (1 - V_i G_i)^{-1} V_j$, where $T_{ij}$ is a scattering amplitude for a given incoming i-channel and an outgoing j-channel, and $G_i$ is a loop function, which after dimensional regularization introduces the so called subtraction constants ($a_i$) which are normally fitted to the experimental data. Considering the isospin symmetry, in this sector the subtraction constants become 6. On the other hand, $V_{ij}$, the interaction kernel, can be derived from the chiral Lagrangian. The leading order of the chiral Lagrangian at low momenta consists of the famous Weinberg-Tomozawa (WT) interaction kernel and the Born vertices.

$$L_{\phi B}^{(1)} = i\langle \bar{B} \gamma_\mu [D^\mu, B] \rangle - M_0 \langle \bar{B} B \rangle - \frac{1}{2} D \langle \bar{B} \gamma_\mu \gamma_5 \{u^\mu, B\} \rangle - \frac{1}{2} F \langle \bar{B} \gamma_\mu \gamma_5 [u^\mu, B] \rangle,$$

where $M_0$ is the common baryon octet mass in the chiral limit. The constants $D$, $F$ denote the axial vector couplings of the baryons to the mesons. $B$ is the baryon octet field and the symbol $\langle \ldots \rangle$ stands for the trace in flavour space, while the pseudoscalar meson octet field $\phi$ enters in
a more complicated way: $u_\mu = i u^\dagger \partial_\mu U u^\dagger$, where $U(\phi) = u^2(\phi) = \exp \left( \sqrt{2} i \phi / f \right)$ (see [4]). At NLO, the contributions of the Lagrangian to meson-baryon scattering in S wave can be written as:

$$\mathcal{L}_{\phi B}^{(2)} = b_D \langle \bar{B} \{ \chi_+, B \} \rangle + b_F \langle \bar{B} \{ \chi_+, B \} \rangle + b_0 \langle \bar{B} B \rangle \langle \chi_+ \rangle + d_1 \langle \bar{B} u_\mu \{ u^\mu, B \} \rangle + d_2 \langle \bar{B} u_\mu \{ u^\mu, B \} \rangle + d_3 \langle \bar{B} u_\mu \{ u^\mu B \} \rangle + d_4 \langle \bar{B} B \rangle \langle \mu u_\mu \rangle,$$

(2)

where $\chi_+ = 2B_0(u^\dagger M u^\dagger + u M u)$ breaks chiral symmetry explicitly via the quark mass matrix $M = \text{diag}(m_u, m_d, m_s)$ and $B_0 = -\langle 0 | \bar{q} q | 0 \rangle / f^2$ relates to the order parameter of spontaneously broken chiral symmetry, with $f$ being the pseudoscalar decay constant in the chiral limit. The coefficients $b_D$, $b_F$, $b_0$ and $d_i$ ($i = 1, \ldots, 4$) are the low energy constants. In principle, the first three coefficients, involved in terms proportional to the $\chi_+$ field, should fulfill constraints related to the mass splitting of baryons. However, since our study goes beyond tree level and incorporates higher order terms via coupled channel scattering equations, we will relax those constraints and we will fit these $b$-type constants, together with the $d_i$ ones, to experimental data.

Apart from a good reproduction of the experimental data, one of the most challenging aspects of this sort of study is the determination of the non constrained low energy constants involved in the NLO terms of the Chiral Lagrangian. For the sake of extracting information about these coefficients, it would be convenient to explore processes in which the higher order terms of the Lagrangian play a significant role beyond that of fine tuning. With this motivation, in previous studies, we focused on the $K^- p \to K^+ \Xi^-$, $K^0 \Xi^0$ reactions, because they do not proceed via the WT term of the Chiral Lagrangian and, hence, they are expected to be especially sensitive to the higher order terms.

After checking the effects on the low energy constants of the systematic inclusion of terms beyond the WT one, our models demonstrated the sensitivity of the $K^- p \to K \Xi$ reactions to the NLO and Born terms and, therefore, more reliable values of the low energy constants were obtained.

Another interesting finding of [5] is that models that include or do not include the Born terms give very different isospin projections of the $K^- p \to K \Xi$ total cross sections while offering very similar reproduction of the $K^- p \to K \Xi$ scattering data. This fact points towards the idea that processes that filter a single isospin component are an essential requirement to get much more reliable values for the NLO coefficients. This background marks the next natural step: the extra experimental data from the $K^- p \to \eta \Lambda, \eta \Sigma^0$ reactions in the fitting procedure should be included to discriminate possible ambiguities in the isospin distributions since they are pure $I = 0$ and $I = 1$ processes, respectively.

Finally, in accordance with [4], the accommodation to the experimental $K^- p \to K \Xi$ total cross sections was improved by the incorporation of additional contributions to the pure chiral scattering amplitude coming from high-spin and high-mass resonant diagrams. Complementary to this phenomenological reason, they allow us to study the accuracy and stability of the NLO coefficients. Moreover, the inclusion of these resonant contributions implicitly simulates higher-angular-momента contributions involving low lying meson-baryon states of the coupled channel problem. This is due to the fact that the low energy constants absorb these contributions in order to reproduce the experimental data at the expense of taking realistic values. Thus, the resonant contributions permit such parameters to relax avoiding a possible overestimation of their values. Consequently, guided by [4] and after an exhaustive study of the effects on the $K^- p \to K \Xi, \eta \Lambda$ total cross sections of the incorporation of all the possible resonances [6]
ranging in energy from 1800 to 2300 MeV, we conclude that the candidates which reproduce the experimental data better were $\Sigma(2030)$, $\Sigma(2250)$ and $\Lambda(1890)$ with $J^P = 7/2^+, 5/2^−$ and $3/2^+$, respectively. The resonant contributions have been implemented applying the Rarita-Schwinger effective Lagrangians that allow one to couple 1/2 spins with the higher ones \cite{7,8}. Furthermore, an exponential form factor has been included due to the high momentum dependence of the scattering amplitudes as done in \cite{7}. The resonant terms are only taken into account for the amplitudes connecting $K^−p$ states with $K^+\Xi^−$, $K^0\Xi^0$ and $\eta$ ones replacing the scattering amplitudes by:

\[
T_{K^−p→K\Xi}^C \rightarrow T_{Chiral}^{C} + T_{K^−p→K\Xi}^{3/2^+} + T_{K^−p→K\Xi}^{5/2^-} + T_{K^−p→K\Xi}^{7/2^+} \\
T_{K^−p→\eta\Lambda} \rightarrow T_{Chiral}^{C} + T_{K^−p→\eta\Lambda}^{3/2^+} 
\]

where $T_{ij}^{Chiral}$ is the scattering amplitude obtained by means of (BSE), and where $T_{ij}^J$ stands for the corresponding resonant term with $J^P$ quantum numbers.

2. Results and discussion

With the aim of improving the reliability of the NLO parameters and discussing the implications of including resonant terms, two fits have been performed. The Model 1 fit corresponds to a unitarized calculation employing the full chiral Lagrangian, eqs. (1) and (2), which involves 16 fitting parameters: the pseudoscalar decay constant, the 2 axial vector couplings, the 6 subtraction constants, and the 7 NLO low energy constants. While the Model 2 fit improves upon the first one by using the inclusion of resonant terms in the way described before. It must be mentioned that 13 new parameters have been introduced in the last fit that we avoid giving any further information for reasons of simplicity (see \cite{8} for specific details). Both fitting procedures take into account $K^−p$ elastic and inelastic cross section data for all the two body channels of the sector \cite{10-24} and the precise SIDDHARTA value of the energy shift and width of kaonic hydrogen \cite{1}, as well as, branching ratios at threshold \cite{25,26}.

| Table 1. Values of the parameters for a regularization scale $\mu = 1$ GeV and the corresponding $\chi^2_{\text{MINUIT}}$ values, defined in \cite{4}, for both described fits. The error of the parameters are given by the MINUIT minimization procedure. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $a_{KN}$ (10$^{-3}$)          | $a_{\alpha}$ (10$^{-3}$) | $a_{\epsilon\Xi}$ (10$^{-3}$) | $a_{\delta\Xi}$ (10$^{-3}$) | $a_{\epsilon\Xi}$ (10$^{-3}$) | $a_{\alpha\Xi}$ (10$^{-3}$) | $f/f_*$ | $D$ | $F$ |
| Model 1 | 1.27 ± 0.12 | −6.1 ± 12.9 | 0.68 ± 1.43 | −0.67 ± 1.06 | 8.00 ± 3.26 | −2.51 ± 0.99 | 1.20 ± 0.03 | 0.70 ± 0.16 | 0.51 ± 0.11 |
| Model 2 | 1.52 ± 0.21 | −2.6 ± 13.9 | 2.1 ± 1.2 | 0.76 ± 1.21 | 10.1 ± 3.7 | −2.01 ± 0.74 | 1.18 ± 0.03 | 0.70 ± 0.15 | 0.49 ± 0.11 |
| $b_0$ (GeV$^{-1}$) | $b_D$ (GeV$^{-1}$) | $b_T$ (GeV$^{-1}$) | $d_1$ (GeV$^{-1}$) | $d_2$ (GeV$^{-1}$) | $d_3$ (GeV$^{-1}$) | $d_4$ (GeV$^{-1}$) | $\chi^2_{\text{MINUIT}}$ (defined in \cite{5}) |
| Model 1 | 0.13 ± 0.04 | 0.12 ± 0.01 | 0.21 ± 0.02 | 0.15 ± 0.03 | 0.13 ± 0.03 | 0.30 ± 0.02 | 0.25 ± 0.03 | 1.14 |
| Model 2 | −0.07 ± 0.01 | 0.13 ± 0.01 | 0.27 ± 0.02 | 0.14 ± 0.03 | 0.13 ± 0.01 | 0.40 ± 0.02 | 0.02 ± 0.02 | 0.96 |

One of main findings of Model 1, when comparing the fitting parameters of this model to the ones from the models in \cite{4,5}, is the homogeneity and accuracy achieved by the NLO coefficients and the natural sized values for all the subtraction constants obtained. The fitting parameters are displayed in Table 1, from which, one can appreciate that the most remarkable result is the 16% improvement in the goodness of the fit, as is clearly reflected in $\chi^2_{\text{MINUIT}}$ values. It can be noted that the fitting parameters are quite stable; this stability being more marked for the NLO coefficients. The reason stems from the fact that we have employed more observables sensitive to the NLO term. In addition, as a consequence of this stability, we still obtain natural sized subtraction constants. Regarding the accuracy, we obtain similar errors associated to the fitting parameters for both models which is in contrast to what was found in \cite{4}.
Figure 1. Total cross sections of the $K^{-}p \rightarrow \eta\Lambda, \eta\Sigma^{0}, K^{0}\Xi^{0}, K^{+}\Xi^{-}$ reactions obtained from Model 1 (solid line) and Model 2 (dashed line). The experimental data of these total cross sections with their corresponding symmetric error bars are taken from [14–24].

In Figure 1, we only represent those cross sections where the novelty comes, because non significant differences between the models exist for the rest due to the dominance of the WT term. From the solid line (Model 1), one sees that the cross sections of the $\eta$ channels can be properly reproduced, which is not the case with previous models [9], even the resonant structure from $\Lambda(1670)$ seen in $K^{-}p \rightarrow \eta\Lambda$. In the $K\Xi$ channels, one can observe a loss of agreement with the data in order to reproduce the $K^{-}p \rightarrow K^{0}\Xi^{0}$ reaction at low energy. Partly, the use of resonant processes was motivated by this fact and to improve the $K^{-}p \rightarrow \eta\Lambda$ cross section around 1950 MeV. From the dashed lines, one appreciates a clear improvement in reproducing the experimental $K^{-}p \rightarrow \eta\Lambda$ cross section in the energies ranging from 1850 to 2200 MeV, and, at the same time, Model 2 respects the $\Lambda(1670)$ resonant structure. But, the most notable effect is the one observed in the $K^{-}p \rightarrow K^{0}\Xi^{0}$ cross section, which reproduces better the experimental data located just above threshold due to the $\Sigma(1890)$ resonance. Moreover, the combined contribution of the $\Sigma(2030)$ and $\Sigma(2250)$ resonances provides a clear bump structure reaching its maximum at around 2100 MeV. A similar behavior for low energies can be noticed in the $K^{-}p \rightarrow K^{+}\Xi^{-}$ cross section, while, in the vicinity of the energy where the experimental data shows a maximum, the $\Sigma(2030)$ and $\Sigma(2250)$ resonant contributions produce some structure, together with a slight reduction of strength. With respect to the $K^{-}p \rightarrow \eta\Sigma^{0}$ cross section, we do not notice any difference in the reproduction of the experimental data, but for energies around 1800 MeV Model 2 presents a more pronounced slope.
3. Acknowledgements
This work is partly supported by the Spanish Ministerio de Economia y Competitividad (MINECO) under the project MDM-2014-0369 of ICCUB (Unidad de Excelencia ‘María de Maeztu’), and, with additional European FEDER funds, under the contract FIS2014-54762-P, by the Generalitat de Catalunya contract 2014SGR-401.

References
[1] Bazzi M et al. 2011 Phys. Lett. B 704 113
[2] Oset E and Ramos A 1998 Nucl. Phys. A 635 99
[3] Hyodo T and Jido D 2012 Progress in Particle and Nuclear Physics 67 55
[4] Feijoo A, Magas V and Ramos A 2015 Phys. Rev. C 92 015206
[5] Ramos A, Feijoo A and Magas V 2016 Nucl. Phys. A 954 58
[6] Olive K et al. [Particle Data Group Collaboration] 2014 Chin. Phys. C 38 090001
[7] Sharov D, Korotkikh V and Lansky D 2011 Eur. Phys. J. A 47 109
[8] Feijoo A 2017 Meson-Baryon interactions from effective Chiral Lagrangians (Barcelona: UB PhD Thesis)
[9] Feijoo A, Magas V and Ramos A 2017 EPJ Web Conf. 137 05003
[10] Kim J K 1965 Phys. Rev. Lett. 14 89
[11] Mast T S et al. 1976 Phys. Rev. D 14 13
[12] Bangerter R O et al. 1981 Phys. Rev. D 23 1484
[13] Ciborowski J et al. 1982 J. Phys. G 8 13
[14] Burgun G et al. 1968 Nucl. Phys. B 8 447
[15] Carlson J R et al. 1973 Phys. Rev. D 7 2533
[16] Dauber P M et al. 1969 Phys. Rev. 179 1262
[17] Haque M et al. 1966 Phys. Rev. 152 1148
[18] London G W et al. 1966 Phys. Rev. 143 1034
[19] Trippe T G and Schlein P E 1967 Phys. Rev. 158 1334
[20] Trower W P et al. 1968 Phys. Rev. 170 1207
[21] Starostin A et al. [Crystal Ball Collaboration] 2001 Phys. Rev. C 64 055205
[22] Baxter D F et al. 1973 Nucl. Phys. B 67 125
[23] Jones M et al. 1975 Nucl. Phys. B 90 349
[24] Berthon A et al. 1974 Nuovo Cim. A 21 146
[25] Nowak R J et al. 1978 Nucl. Phys. B 139 61
[26] Tovee D N et al. 1971 Nucl. Phys. B 33 493