Updating constraints on inflationary features in the primordial power spectrum with the Planck data.

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We present new constraints on possible features in the primordial inflationary density perturbations power spectrum in light of the recent Cosmic Microwave Background Anisotropies measurements from the Planck satellite. We found that the Planck data hints for the presence of features in two different ranges of angular scales, corresponding to multipoles $10 < \ell < 60$ and $150 < \ell < 300$, with a decrease in the best fit $\chi^2$ value with respect to the featureless “vanilla” $\Lambda$CDM model of $\Delta\chi^2 \approx 9$ in both cases.

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I. INTRODUCTION

The recent results from the Planck satellite on the cosmic microwave background (CMB, hereafter) angular power spectrum are in very good agreement with the theoretical expectations of the simplest inflationary model based on a single, minimally coupled, scalar field [1].

However, as already discussed in [2], some interesting hints for deviations from scale invariance are present in the Planck data and are certainly worthwhile of further investigation.

In this brief paper, we present new constraints on an inflationary model with step-like features as proposed by [3, 4].

Step-like features in the inflationary potential are expected in theories with multiple interacting scalar fields as supergravity-inspired models, where supersymmetry-breaking phase transitions occur during inflation. At the same time, step-like features are able to produce localized oscillations in the CMB angular power spectra and, in particular, as we have already shown in [5], to provide a better fit with respect to the featureless case in case of the WMAP data. It is therefore timely to analyze the new Planck data, that covers a larger multipole range respect to WMAP, and to quantify the compatibility of the features with this new dataset.

A first analysis has already been provided by the Planck collaboration in [2]. However, as we will discuss in the next section, this analysis assumed an analytical and, therefore, approximate formula for the features and investigated a range of angular scales different from the one analyzed in [5]. In particular, as we will discuss below, the analysis presented in [2] for step-like feature did not cover the range of low multipoles. Moreover, the remaining cosmological parameters were not let to vary freely but fixed at their best fit values, therefore neglecting possible correlations.

Here, on the contrary, we assume the same parameter range of [5] and we integrate the set of differential equations to accurately compute the oscillations in the CMB angular spectrum, given a step-like feature in the inflationary potential (again, see [5]). Moreover, we let all the parameters to vary freely, taking into account possible correlations between the parameters. For comparison, we also use the analytical model adopted in [2].

The paper is organized as follows: in Sec.II we briefly explain the analysis method adopted; in Sec.III we present the results of our analysis and in Sec IV we summarize our conclusions.

II. MODEL AND ANALYSIS METHOD

Following the work of Adams et al. [3], we consider a model with a step-like feature added to a chaotic potential $V(\phi) = m^2 \phi^2 / 2$, for the inflaton field $\phi$, of the form:

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \left[ 1 + c \tanh \left( \frac{\phi - b}{d} \right) \right], \quad (1)$$

where $b$ is the value of the field where the step is located, $c$ is the height of the step and $d$ its slope. In order to evaluate the density perturbation spectrum we numerically evolve the relevant equations that, for brevity, we do not report here and we refer the reader to [5, 14].

Moreover, we also adopt an analytical parameterization for the scalar primordial power spectrum given by [15, 16]:

$$P_R(k) = \exp[\ln P_0(k) + \frac{A_f}{3} \frac{k\eta_f}{\sinh (\frac{k\eta_f}{x_d})} W'(k\eta_f)], \quad (2a)$$

$$W'(x) = -3 + \frac{9}{x^2} \cos 2x + \left( 15 - \frac{9}{x^2} \right) \sin 2x \frac{2x}{2x} \quad (2b)$$

where $P_0(k) = A_s(k^{n_s-1})$ is the smooth spectrum with the standard power law form, $A_f$ is the kinetic energy perturbation of the step, $\eta_f$ is the step crossing time in units of Mpc and $x_d$ the dimensionless damping
scale. Using this method, by placing the features directly on the density power spectrum, we do not integrate the system of differential equations, with a significantly smaller computing time. This is the same approach that has been used in [2].

We therefore consider a “vanilla” theoretical model with the addition of features in the primordial spectrum, parametrized in both cases (numerical and analytical) by three parameters. Together with these parameter we vary the usual cosmological parameters as the baryon density, \( \omega_b \), the cold dark matter density, \( \omega_c \), the ratio between the sound horizon and the angular diameter distance at decoupling, \( \theta \), the optical depth, \( \tau \), the primordial scalar amplitude, \( A_s \), and, finally the primordial spectral index \( n_s \). We also vary the nuisance foregrounds parameters [17], we consider purely adiabatic initial conditions, fix the sum of neutrino masses to 0.06 eV, and we limit the analysis to scalar perturbations.

We then perform a Monte Carlo Markov Chain analysis via the publicly available package CosmoMC [18]. We use a modified version of the CAMB [19] code, needed to compute the CMB anisotropies spectrum for given values of the parameters describing this type of inflationary model. The Gelman and Rubin criteria is used to evaluate the convergence of the chains, demanding that \( R - 1 \leq 0.02 \). By default CosmoMC uses a simple Metropolis-Hastings algorithm that needs to evaluate the model likelihood at each point traversed by the chains. It is designed to draw samples from the posterior distribution and not to find the best-fit model. Thus in the analysis we use the Bound Optimization BY Quadratic Approximation (BOBYQA) algorithm, developed by Powell [20], that is an optimized methods for minimizing functions of more variables and is implemented in CosmoMC. The quoted results in this paper for the best-fit values of the parameters, as well as for the value of the \( \chi^2 \) itself, are obtained using Powell’s routines.

The dataset considered in this work, available from the ESA website [1] are:

- high-\( \ell \) Planck temperature (\( 50 < \ell < 2500 \), derived from the CamSpec likelihood by combining spectra in the frequency range \( 100 - 217 \) GHz [17]),
- low-\( \ell \) Planck temperature (\( 2 < \ell < 49 \), derived from a component-separation algorithm, Commander, applied to maps in the frequency range \( 30 - 353 \) GHz [21]),
- low-\( \ell \) WMAP-9year polarization [22].

The likelihood code is provided by the Planck collaboration [17].
The pivot wave-number selected is \( k_\ast = k_0 = 0.05 \text{ Mpc}^{-1} \), which is the same value chosen by the Planck collaboration for this type of study. This parameter is degenerate with the value of the position of the step in \( \phi \), e.g., changing \( k_0 \) from 0.05 to 0.002 \( \text{Mpc}^{-1} \) shifts the step value \( b \) by \( \sim 0.5 \) towards lower values (see [2]).

### III. RESULTS AND DISCUSSION

We essentially consider three types of analysis with the results reported in Table 1. The first analysis assumes a simple \( \Lambda \)CDM model with a featureless spectrum. For the second analysis we considered the step-like model in the inflationary potential, numerically integrating the relevant equations and assuming the following priors on the corresponding parameters: \( 14.2 \leq b \leq 15.5, -4 \leq \log c \leq -1, -2.5 \leq \log d \leq -0.5 \). These results are reported in fourth and fifth columns of Table 1. The comparison between the results presented in the two tables are useful in order to identify the impact of primordial features on the constraints on the standard \( \Lambda \)CDM parameters.

In the third analysis we used the analytical formula presented in [2] with the same choice of priors and given by: \( 0 \leq A_f \leq 0.2, 0 \leq \ln(\eta_f/\text{Mpc}) \leq 12, -1 \leq \ln x_d \leq 5 \). These values are reported in the last two columns of Table 1.

As we can see, introducing oscillations in the primordial spectrum either by numerical integration the relevant equation or by using the above mentioned analytical formula, reduces the \( \chi^2 \) of the best fit model by \( \Delta \chi^2 \sim 9 \). However, the feature parameters are poorly constrained, as also shown in Fig. 1 where we report the posterior probabilities for the numerical integrating analysis. Moreover, the introduction of features has little effect on the constraints on the remaining, nuisance, cosmological parameters.

In Fig. 1 we can also note that the posteriors are better defined respect to those present in our previous work of [3], although they are significantly different from a gaussian distribution. In particular, we see that the use of the Planck data eliminates a bimodal form in the posterior probability for the \( b \) parameter, present in the WMAP9 data. As we see in Table I, both the analytical and the numerical method provide the same reduction in the \( \chi^2 \) value. In particular, the results for the analytical method, are fully consistent with those reported in [2].

However the effects on the CMB angular spectra are drastically different. The best-fit model obtained from a numerical integration provide significantly different oscillations respect to the best fit model obtained in the case of the analytical approximation.

We can clearly see this in Fig. 2, where we plot the primordial power spectra for the best fit models obtained in the case of numerical integration (red line) and for the case of analytical approximation (blue line) used in the Planck analysis.

In Fig. 3 we compare the best fit CMB angular spectra obtained in the two cases. As we can see, the numerical integration method identifies the oscillations on large angular scales (\( 10 < \ell < 60 \)) while the analytical method provides a better fit by producing oscillations around the first doppler peak.

This difference is essentially due to the different choice of priors on the feature parameters assumed in the two analyses. To check this, we changed the priors for the analysis based on the analytical formula to \( 0.8 \leq A_f \leq 1 \), \( 7 \leq \ln(\eta_f/\text{Mpc}) \leq 8, 0 \leq \ln x_d \leq 0.5 \), obtaining the best fit values reported in Table I, sixth column. As we can see, the best fit has now \( A_f \sim 0.9 \), a value that
was excluded by the choice of priors used in [2]. The corresponding primordial spectrum is reported in Fig. 2 as a black line and, as we can see, is in full agreement with the best fit spectra obtained from the analysis made assuming the numerical integration method.

We can therefore conclude that one needs to be extremely cautious in the choice of priors when looking for features in the CMB spectra since probability distributions for the parameters are highly multimodal.

IV. CONCLUDING REMARKS

We have presented updated constraints on an inflationary model with a step-like feature in the inflaton potential, using WMAP9 low-\(\ell\) polarization data and the recent temperature data released by the Planck experiment. Such a feature would induce oscillations in the anisotropy power spectrum with magnitude, extent and position depending on three step parameters.

We have considered two different methods. The first uses a numerical routine to accurately calculate the primordial density spectrum corresponding to a given inflaton potential. The second employs an approximate form of the power spectrum, reproducing the features caused by a step-like inflaton potential step-like. For the latter analysis, we have also studied the impact of different prior ranges, corresponding to features in the low-\(\ell\) and mid-\(\ell\) ranges.

The analysis done performing the exact integration of the mode equations shows a minimum \(\chi^2\) value with \(\Delta \chi^2 \approx 9\) with respect to the featureless \(\Lambda\)CDM model, at the cost of three new parameters. This improvement is due to the presence of oscillations in the multipole range 10 < \(\ell\) < 60. These results can be matched using instead the analytical approach, by choosing a suitable prior range for the parameters, different from the one used in [2].

On the other hand, the results for the analytical method with the same prior range as [2], corresponding instead to oscillations in the range 150 < \(\ell\) < 300, are fully consistent with those reported there. The improvement in the goodness-of-fit is still \(\Delta \chi^2 \approx 9\), although it is caused by oscillations in a completely different range of scales.

Finally, the constraints on the step parameters are improved with respect to our previous work [5].

Future polarization data, as discussed in [5] will clearly further improve the constraints presented here.

Our results are in reasonable agreement, given the different method of analysis adopted, also with those recently presented in [23].

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