Multilayer Formation Control in Constrained Space

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Abstract. This paper addresses the multilayer formation control (MLFC) problem for multi-agent systems in constrained space. A layered distributed finite-time estimator (LDFE) is proposed to acquire the target states for agents in each layer. To avoid collisions with borders, obstacles, as well as the other agents in the constrained space, an artificial potential function is designed based on the Dirac delta function. Based on the LDFE and the proposed potential function, the MLFC algorithm is proposed for multiple Euler-Lagrange systems (MELSs). The semi-global uniform ultimate boundedness of closed-loop errors is guaranteed by Lyapunov stability theory, while the desired formation of each layer can be achieved without collisions occurring in the constrained space. Simulation results are given to show the effectiveness of the proposed approaches.

1. Introduction

Multi-agent system (MAS) control includes interesting topics, such as the consensus problem [1, 2, 3, 4], the tracking problem [5, 6, 7, 8, 9], the formation problem [10, 11, 12, 13, 14, 15], and the containment problem [16, 17].

Recently, the formation-containment problem is defined by combining formation and containment problems [18, 19, 20, 21, 22]. Using a two-layer framework [23], the formation-containment problem is considered with leading following layers. Further, to articulate a more general formation problem, the multilayer formation control problem is proposed and defined in [24], and this framework is more in line with practical missions with a large number of agents. The agents in a layer are required to receive information from the prior layers, interact within the current layer, and transmit information to the subsequent layers. The agents in the first layer need to achieve a specific formation and the agents in the subsequent layers are expected to reach proper positions based on the primary formation shape formed by the prior layers.

In this paper, we extend the authors’ previous result [24] with the consideration of control in constrained space, since a large number of agents should avoid collisions among each other as well as among unexpected obstacles or borders due to the lack of global environmental information. First, an LDFE is designed to estimate the desired state information for agents in each layer. Then, a novel artificial potential function is proposed with the aid of the Dirac delta function. Applying the potential function and the LDFEs, we finally introduce a model-based MLFC law for MELSs and all the closed-loop errors have been proved semi-globally uniformly ultimately bounded (SGUUB).
2. Layered Formation Problem Formulations

Suppose that the multilayer formation consists of \( l \) layers with \( N_n \) agents in the \( n \)th layer, \( n = 1, 2, \ldots, l \). Let \( m = N_1 + N_2 + \ldots + N_l \). The agents are ordered by layers. Let \( V_{Li} \) denote the agent set of the \( n \)th layer as

\[
V_{Li} = \left\{ \sum_{j=1}^{i-1} N_j + 1, \sum_{j=1}^{i-1} N_j + 2, \ldots, \sum_{j=1}^{i-1} N_j + N_i \right\},
\]

with \( i = 1, 2, \ldots, l \), and Define \( V_{LM} = V_{L1} \cup V_{L2} \cup \ldots \cup V_{Li} \).

We assume the internal information flow in the \( n \)th layer, \( n = 2, 3, \ldots, l \), is bidirectional, while the other information flow are all directed. Define layered Laplacian matrix \( L \in \mathbb{R}^{(m+1) \times (m+1)} \),

\[
L = \begin{bmatrix}
0 \otimes I_p & 0_{1 \times N_1} \otimes I_p & 0_{1 \times N_2} \otimes I_p & \cdots & 0_{1 \times N_l} \otimes I_p \\
L_{L1} \otimes I_p & L_{L1} \otimes I_p & L_{L1} \otimes I_p & \cdots & L_{L1} \otimes I_p \\
0_{N_1 \times 1} \otimes I_p & B_{L1} & B_{L1} & \cdots & B_{L1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0_{N_l \times 1} \otimes I_p & B_{L1} & B_{L1} & \cdots & B_{L1}
\end{bmatrix},
\]

where \( L_{L1} \) represents the topology among the agents in the first layer, \( B_{F_v1} \) the topology among the agents in the \( n \)th layer, \( B_{F_v2} \) the topology among the agents in the \( n-1 \)th layer and the agents in the \( n \)th layer.

Define a virtual agent 0, whose position and velocity are equal to the desired formation position and velocity. Then, virtual edges \( a_{0i}, i = 1, 2, \ldots, N_1 \), are given to denote the information flow between the \( i \)th agent in the first layer and agent 0.

Introduce an Laplacian matrix for the first layer and agent 0, where \( L_1 = [l_{ij}] \in \mathbb{R}^{N_1+1 \times N_1+1} \), \( l_{ii} = \sum_{j=1}^{N_1+1} a_{ij} \) and \( l_{ij} = -a_{ij}, i \neq j \). A bearing-based approach is then given to describe the interaction in the \( n \)th layer, \( n = 2, 3, \ldots, l \), and the bearing of agent \( j \) relative to agent \( i \) is given as [25]

\[
g_{ij} = \frac{q_i - q_j}{\|q_i - q_j\|},
\]

(3)

\[
P_{g_{ij}} = I_p - g_{ij}g_{ij}^T, \; i \in V_{Ln}, \; j \in V_{LM},
\]

(4)

where the desired inter-bearing is shown as \( g_{ij}^* \). The Laplacian matrix \( B_{Ln} \), \( n = 2, 3, \ldots, l \), \( v = 2, 3, \ldots, n \), is defined with the \( ij \)th block as

\[
[B_{Ln}]_{ij} = \begin{cases}
-P_{g_{ij}^*}, & i \neq j, \; i \in V_{Ln}, \; j \in V_{LM}, \\
\sum_{k \in \mathcal{N}_n} P_{g_{ik}}, & i = j, \; i \in V_{Ln},
\end{cases}
\]

(5)

where \( P_{g_{ij}^*} = I_d - g_{ij}^* g_{ij}^{*T} \).

The \( i \)th agent is described by the following equation

\[
\dot{q}_i + C_i(q_i, \dot{q}_i) \ddot{q}_i + g_i(q_i) = r_i(t),
\]

(6)

where \( q_i \in \mathbb{R}^p, \; i = 1, 2, \ldots, m \) denotes the position vector of agent \( i \), \( M_i(q_i) \in \mathbb{R}^{p \times p} \) is the symmetric positive definite inertia matrix, \( C_i(q_i, \dot{q}_i) \in \mathbb{R}^{p \times p} \) is the vector of Coriolis and centrifugal matrix, \( g_i(q_i) \in \mathbb{R}^p \) denotes the gravitational force.
3. Layered Distributed Finite-Time Estimator

In this section, an LDFE is designed for agents in each layer. We propose the following LDFE protocol

\begin{align}
\dot{\hat{q}}_i &= \begin{cases}
\dot{v}_i - \beta_{1}\text{sgn} \left[ \sum_{j \in V_{L1}} a_{ij} (\hat{q}_i - \hat{q}_j) + a_{i0} (\hat{q}_i - q_0) \right], & i \in V_{L1}, \\
\dot{\hat{r}}_i, & i \in V_{L1},
\end{cases} \\
\dot{\hat{r}}_i &= -\beta_{2}\text{sgn} \left[ \sum_{j \in V_{L1}} a_{ij} (\hat{r}_i - \hat{r}_j) + a_{i0} (\hat{r}_i - \hat{r}_0) \right], & i \in V_{L1},
\end{align}

where \( \beta_{1}, \beta_{2}, \ldots, \beta_{2n}, n = 2, 3, \ldots, l \) are positive constants. \( \hat{q}_i \) and \( \hat{r}_i \), \( i \in V_{L1} \) are the estimates of the desired formation position and velocity for each agent in the first layer, respectively. Correspondingly, \( \hat{q}_i \) and \( \hat{r}_i \), \( i \in V_{L1} \) are the estimates of bearing-based positions and velocities of the desired formation for each agent in the \( n \)th layer, respectively. \( r_i (t) \) is the auxiliary function decomposed from the desired formation configuration. Let \( q_0 \) and \( \hat{q}_0 \) denote the virtual agent 0’s position and the velocity, respectively.

The following necessary assumptions are made.

**Assumption 1** Assume \( N_1 \geq 2 \), and there exists a directed spanning tree in the first layer with root agent 0.

**Assumption 2** The acceleration of agent 0 is bounded by a constant \( \delta_{L1} \) as \( \| \ddot{q}_0 \| \leq \delta_{L1} < \infty \).

**Assumption 3** The target formation for the \( n \)th layer \( (n = 2, 3, \ldots, l) \) is infinitesimal bearing rigid, respectively.

Define

\begin{align}
Q_{L1} &= [q_1^T, q_2^T, \ldots, q_{N1}^T]^T, \\
C_{L2} &= [c_{N1+1}^T, c_{N1+2}^T, \ldots, c_{N1+N2}^T]^T = -(L_{L11}^{-1} L_{L12} \otimes I_p) Q_{L1}, \\
& \vdots \\
Q_{Ln-1} &= [q_1^T, q_2^T, \ldots, q_{N1+\ldots+N_{n-2}+N_{n-1}}^T]^T, \\
C_{Ln} &= \left[ c_{N1+\ldots+N_{n-1}+1}^T, c_{N1+\ldots+N_{n-1}+2}^T, \ldots, c_{N1+\ldots+N_{n-1}+N_n}^T \right]^T \\
& = -(B_{Ln1}^{-1} [B_{Ln1}, B_{Ln1-1}, \ldots, B_{Ln2})] Q_{Ln-1},
\end{align}

where \( n = 2, 3, \ldots, l \). \( L_{L11}, B_{L21}, \ldots, B_{Ln1} \) are invertible based on Assumptions 1 and 3 [25][26].

**Lemma 1** [24] Let \( \beta_1 > 0 \) and \( \beta_2 > \delta_{L1} \). If Assumptions 1 and 2 hold, under the LDFFEs (7a) and (7b), the agents in the first layer can acquire the precise estimations of the desired formation velocity and position in finite time, respectively, which means

\begin{align}
\hat{v}_i (t) &= \hat{q}_0 (t) + \hat{r}_i (t), \ t \geq T_1, \\
\hat{q}_i (t) &= q_0 (t) + \hat{r}_i (t), \ t \geq T_2, \ i \in V_{L1},
\end{align}

where \( T_1 \) and \( T_2 \) are positive constants.
Therefore, \( \dot{q}_i \) and \( \dot{v}_i \), \( i \in V_{L1} \) can be used to replace the position and velocity of the desired formation when \( t \geq T_2 \).

Next, consider the \( n \)th layer with \( n = 2, 3, \ldots, l \).

Lemma 2 \[24\] Let \( \beta_{2n-1} > 0 \) and \( \beta_{2n} > \frac{\delta_{Ln}}{\lambda_{\min}(B_{Ln1}^{-1})} \). If Assumption 3 holds, under the DFSEs (8a) and (8b), the agents can acquire the precise estimations of the bearing-based velocity and position of the target formation in finite time, respectively, which means

\[
\begin{align*}
\dot{v}_i(t) &= \dot{c}_i(t), \quad t \geq T_{2n-1}, \\
\dot{q}_i(t) &= c_i(t), \quad t \geq T_{2n}, \quad i \in V_{Ln},
\end{align*}
\]

where \( T_{2n-1} \) and \( T_{2n} \) are positive constants.

Then, we can use \( \dot{q}_i \) and \( \dot{v}_i \), \( i \in V_{Ln} \) to substitute the bearing-based weighted average of the positions and velocities of the agents in the \( n - 1 \)th layer when \( t \geq T_{2n} \).

Combining Lemma 1 and Lemma 2, we know that the agents in all layers can obtain desired position and velocity estimation in settling time \( T_M = \max \{ T_2, T_4, \ldots, T_{2n} \} \).

4. Control Design in Constrained Space

A critical issue for MAS control is to avoid the collision among agents and obstacles. Hence, real-time environment modeling techniques are essential to the agents, and they have to avoid collisions with pop-up threats without global environmental information.

To achieve collision avoidance among the agents, we design the potential functions for agent \( i \) as

\[
P_{i,c}(\tilde{q}_{i,j}) = \sum_{j \in H_i} c_{i,j} \ln \frac{r - d_{safe}}{\|\tilde{q}_{i,j}\| - d_{safe}},
\]

where \( \tilde{q}_{i,j} = q_i - q_j \), \( (i,j \in V_{LM}, i \neq j) \), \( c_{i,j} = c_{j,i} \) are positive constants, \( d_{safe} \) is the required safe distance, \( r \) is the danger detection range of each agent, and \( H_i \) represents all agents within the danger range of agent \( i \) as \( H_i = \{ j \in V_{LM}, j \neq i \| \tilde{q}_{i,j} \| < r \} \).

To avoid collisions with obstacles in the constrained space, we define the constrained space as a point set \( 3 = 3_1 + 3_2 + \ldots + 3_R \), where \( R \) is the number of continuous regions. Define \( L_i \) as the edge of region \( 3_i \), \( i = 1, 2, \ldots, R \). The irregular edges of spatial constraints are considered, where we divide an irregular edge into a number of segments. We assume that the edges of the constrained space are smooth. Using Dirac delta function [27, 28] to gather the potential forces from all segments of each edge in the constrained space, we construct the following potential function

\[
P_{i,o}(q_i) = \sum_{x=1}^{R} \int_{q_i^{(x-1)} - r}^{q_i^{(x)} + r} \cdots \int_{q_i^{(p-1)} - r}^{q_i^{(p)} + r} k_{cr} \delta(||q_i - s - kP_x(s)||) \frac{ds_1 \cdots ds_p}{p}
\]

where \( k \) is a real number, \( k_{cr} \) is a positive constant, \( P_x(s) \) represents the perpendicular direction of \( L_x \), and \( s_i \) denotes the \( i \)th element of vector \( s \), \( i = 1, 2, \ldots, p \).

Next, define the following auxiliary variables,

\[
\begin{align*}
z_{1i} &= q_i - \dot{q}_i, \\
z_{2i} &= \dot{q}_i - \alpha_{1i}, \quad i \in V_{LM},
\end{align*}
\]
where $\alpha_{1i}$ is the virtual control to be defined and $K_{1i} = K_{1i}^T > 0$ is the gain matrix.

We design the virtual control as

$$\alpha_{1i} = \hat{v}_i - (\Theta_i)^\dagger \left[ K_{2i}V_{\chi i} + \left( \frac{\partial P_{i,o}}{\partial q_i} \right)^T \hat{v}_i + \left( \frac{\partial P_{i,c}}{\partial q_i} \right)^T (\hat{v}_i - \dot{q}_j) \right],$$

where $\Theta_i = z_{1i}^T + (\partial P_{i,o}/\partial q_i)^T + (\partial P_{i,c}/\partial q_i)^T$, $V_{\chi i} = \frac{1}{2}z_{1i}^T z_{1i} + P_{i,o} + P_{i,c}$, $K_{2i} = K_{2i}^T > 0$, and $(\Theta_i)^\dagger$ is the Moore-Penrose inverse of $\Theta_i$ defined as

$$\Theta_i(\Theta_i)^\dagger = \begin{cases} 0, & \Theta_i = [0, 0, \ldots, 0]^T, \\ 1, & \text{otherwise.} \end{cases}$$

Propose the following MLFC algorithm for each formation layer

$$\tau_i = -z_{1i} - K_{1i} \hat{z}_{2i} + M_i(q_i) \alpha_{1i} + C_i(q_i, \dot{q}_i) \alpha_{1i} + g_i(q_i) - \frac{\partial P_{i,o}}{\partial q_i} - \frac{\partial P_{i,c}}{\partial q_i},$$

where $K_{1i}$ is the control gain matrix. Define the global closed-loop error signals as $z_1 = [z_{11}^T, z_{12}^T, \ldots, z_{1m}^T]^T$ and $z_2 = [z_{21}^T, z_{22}^T, \ldots, z_{2m}^T]^T$.

**Theorem 1** In each layer, under the MLFC algorithm (19), the multilayer formation can be achieved within bounded errors without any collision, and the closed-loop errors $z_1$ and $z_2$ in each formation layer are SGUUB.

**Proof 1** The proof can be given following the process in [11].
5. Simulation
To show the performance of proposed MLFC algorithms, consider a group of 16 networked satellites with 8 satellites labeled as 1, ..., 8 in the first layer, 4 satellites labeled as 9, ..., 12 in the second layer, and 4 satellites labeled as 13, ..., 16 in the third layer. The parameters of the satellites and the orbits are the same as those in [23]. The other parameters for simulation are omitted due to the limited pages.

The trajectories of satellites are given in Figures 1 and 2. In the constrained space, as shown in Figures 1 and 2, the collisions with the border, the obstacles, as well as the agents, are avoided successfully.

6. Conclusion
This paper using a multilayer framework to describe and extend the leader-follower problem with an arbitrary number of layers. A model-based MLFC algorithm is proposed with the consideration of collision avoidance. Under the MLFC law, the layered agents can achieve flexible formation configurations and transform the formation shape smoothly in the constrained space.

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