Transforming task representations to allow deep learning models to perform novel tasks

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An important aspect of intelligence is the ability to adapt to a novel task without any direct experience (zero-shot), based on its relationship to previous tasks. Humans can exhibit this cognitive flexibility. By contrast, deep-learning models that achieve superhuman performance in specific tasks generally fail to adapt to even slight task alterations. To address this, we propose a general computational framework for adapting to novel tasks based on their relationship to prior tasks. We begin by learning vector representations of tasks. To adapt to new tasks, we propose \textit{meta-mappings}, higher-order tasks that transform basic task representations. We demonstrate this framework across a wide variety of tasks and computational paradigms, ranging from regression to image classification and reinforcement learning. We compare to both human adaptability, and language-based approaches to zero-shot learning. Across these domains, meta-mapping is successful, often achieving 80-90\% performance, without any data, on a novel task that directly contradicts its prior experience. We further show that using meta-mapping as a starting point can dramatically accelerate later learning on a new task, and reduce learning time and cumulative error substantially. Our results provide insight into a possible computational basis of intelligent adaptability, and offer a possible framework for modeling cognitive flexibility and building more flexible artificial intelligence.

Intelligent systems should be able to transform their behavior on a task, in accordance with a change in goals, and humans often exhibit this ability [1]. For example, if we are told to try to lose at poker, we can perform quite well on our first try, even if we have always tried to win previously. If we are shown an object, and told to find the same object in a new color or texture, we can do so. By contrast, this type of adaptation is quite difficult for deep-learning models [1][2]. How could deep-learning models reuse knowledge more flexibly? How could they adapt to a new task without any data, even when the new task contradicts prior experience?

We suggest that this ability to adapt can arise from exploiting the relationship between the adapted version of the task and the original. In this work, we propose a computational model of adaptation based on task relationships, and demonstrate its success across a variety of domains, ranging from regression to classification to reinforcement learning. Our approach could provide insights into the flexibility of human cognition, and allow for more flexible artificial intelligence systems.

Our model incorporates several key cognitive insights. First, when performing a task (such as playing poker), it is useful for the system to constrain its behavior by an internal task representation. We draw inspiration from machine learning and cognitive science, and construct task representations from examples via meta-learning (e.g. [3][4]), or from a natural language instruction [5][6]. The model then uses this task representation to respond in a task-appropriate way to its inputs.

Crucially, we allow the model to accommodate task alterations by transforming its representation of a task. We refer to these transformations of tasks as \textit{meta-mappings}. That is, meta-mappings are higher-order functions over tasks — functions that take a task as input and output an adapted version of that task. Meta-mappings allow the model to adapt to a new task \textit{zero-shot} (i.e. without requiring any data from that new task), based on the relationship between the new task and prior tasks. We suggest that meta-mapping can offer a useful inductive bias that complements, and in some cases exceeds, other zero-shot learning approaches, by allowing more effective use of knowledge about the prior task.

Meta-mappings can be cued either by examples of the meta-mapping applied to other tasks, or by an instruction, just as basic tasks can be inferred from examples or instructions. Concretely, our model is able to switch to losing at poker on its first try. To do so, it constructs a representation of poker from experience with trying to win the game. It then infers a “try-to-lose” meta-mapping, either from language or from examples of winning and losing at other games, such as blackjack. It then applies this meta-mapping to transform its representation of poker, thereby yielding a representation for losing at poker. This adapted task representation can then be used to perform the task of trying to lose at poker \textit{zero-shot}.

Our main contributions are to propose meta-mapping as a computational framework for zero-shot adaptation to novel tasks, and to propose a parsimonious implementation of this framework in systems we call homoiconic meta-mapping architectures (see below). We demonstrate the success of meta-mapping across a variety of task domains, ranging from visual classification to reinforcement learning. We further show that meta-mapping is a useful starting point for later learning. To our knowledge, this is the first work that proposes transforming a task representation in order to adapt to task alterations \textit{zero-shot}. We consider related work, and implications for cognitive science and artificial intelligence, in the discussion.

1. Task transformation via meta-mappings

Basic tasks are input-output mappings: We take as a starting point the construal of basic tasks as mappings (functions) from inputs to outputs. For example, poker can be seen as a mapping from hands to bets (Fig. 1a), chess as a mapping of board positions to moves, and object recognition as a mapping from images to labels. This perspective is common in machine learning approaches, which generally try to infer a task mapping from many input/output examples, or meta-learn how to infer it from fewer examples. In our work, we infer these mappings in three different ways: from examples,
from natural language instructions, or by transforming a prior task mapping. We view all three approaches as important; the third is the most novel component of our work.

_Tasks can be transformed via meta-mappings:_ We propose meta-mappings as a computational approach to the problem of transforming a basic task mapping. A meta-mapping is a higher-order task, which takes a task representation as input, and outputs a representation of the transformed task. For example, we might have a “lose” meta-mapping (Fig. 1d). If given poker as an input, the lose meta-mapping would output a losing variation of poker. If we have such a meta-mapping, we can use it to transform a task in order to perform the transformed version of the task. This allows a model to adapt to a transformed task without having any data on it, just as humans are able to easily switch to trying to lose at a game they have only tried to win in the past.

How can a meta-mapping be performed? We exploit an analogy between meta-mappings and basic task mappings – they are both simply functions from inputs to outputs. Thus to perform a meta-mapping we use approaches analogous to those we use to perform basic tasks. In particular, we infer a meta-mapping from examples (e.g. winning and losing at a set of example games), or natural language (e.g. “try to switch to losing”). We can then apply this meta-mapping to other basic tasks, in order to infer losing variations of those tasks. Importantly, the system is able to generalize to new meta-mappings, just as it can generalize to new basic tasks. For example, if it experienced meta-mappings which altered the rank of some cards (e.g. “ace is high rather than low”), it could generalize to switching the rank of other cards, either from examples or an instruction.

2. Homoiconic meta-mapping architectures

We propose homoiconic meta-mapping (HoMM) architectures as an implementation of a model that can both perform basic tasks and adapt to task alterations via meta-mappings. These architectures exploit the analogy between basic-tasks and meta-mappings (that both are functions from inputs to outputs). In this section, we describe the general features of HoMM architectures and their training. See the Methods and SI A.3 for details, including all hyperparameter values (Table 2), depiction of inference and gradient flow (Fig. 7), and links to repositories containing all experiment and analysis code.

_Homoiconicity:_ HoMM models exploit the analogy between basic tasks and meta-mappings by using the same architectural components to perform both. This approach is in keeping with the idea that we, as humans, have a single mind that implements computations of all types. Our approach is also inspired by the computational notion of _homoiconicity._ A homoiconic programming language is one in which programs can be manipulated just as data can. Our task representations are like programs that perform tasks, and our implementation is therefore homoiconic in the sense that it operates on data and tasks in the same way. We refer to architectures with the proposed characteristics as homoiconic meta-mapping (HoMM) architectures.

The HoMM approach is parsimonious, in that it does not require adding new networks for each new type of computation. Furthermore, in many cases, functions have some common structure with the entities they act over. For example, both numbers and functions have inverses. For another example, the set of linear maps over a vector-space is itself a (higher-dimensional) vector space. If the different levels of abstraction share structural features, sharing computation should improve generalization. HoMM could also support the ability to build abstractions recursively on top of prior abstractions, as humans do in mathematical cognition (9–11). We suggest that homoiconic approaches will be beneficial.

_Constructing a task representation (Fig. 1b):_ When humans perform a task, we need to know what the task is. In HoMM, we specify the task using a task representation. HoMM can support several different ways of cueing a task, such as instructions, examples of appropriate behavior (e.g. input, target) tuples), or a transformation of a representation for a known prior task (meta-mapping). To construct a task representation from language, we process the language through a deep recurrent network (LSTM). This is similar to techniques used in other work (e.g. 7 8 12). To construct a task representation from examples, we process each example to construct appropriate representation of it, and then aggregate across these representations by taking an element-wise maximum, to combine examples in a nonlinear but dataset-order-invariant way. This aggregated representation then receives further processing to produce the task representation. This aspect of our approach is related to other meta-learning approaches (e.g. 13).

_Performing a task from its representation (Fig. 1c):_ Once we have a task representation, we use it to perform the task. We allow a large part of the input processing (perception) and output processing (action) to be shared across the tasks within each domain we consider so that the task-specific computations can be relatively simple and abstract. For example, if a human is playing cards, much of the perception of the cards will be identical whether the game is poker or bridge, and the task-specific computations will be performed over abstract features such as suit and rank relationships. We thus allow the system to learn a general basis of perceptual features over all tasks within a domain. This idea is related to the long-standing notion that deep networks (both artificial and biological) construct disentangled representations of the task relevant features in deeper layers (14 15), and is often used in meta-learning (3 e.g.).

The system then uses these features in a task-specific way in order to perform task-appropriate behavior. Specifically, the model uses a HyperNetwork (16 17) which takes as input the representation of a task. The hyper network adapts the values of learned “default” connection weights, to make the network task-sensitive (Fig. 1c detail). The adapted network transforms the perceptual features into task-appropriate output features, which can then be decoded to outputs via a shared output processing (action) network. The whole model (including the construction of the task representations) can be trained end-to-end, just as a standard meta-learning system would be. (Our approach outperforms other possible task-network architectures, e.g. feed-forward networks with a task representation as another input, Figs. 12 19).

_Transforming task representations via meta-mappings (Fig. 1c)._ Above, we defined a meta-mapping to be a higher-order task, which takes as input a task representation, and outputs a transformed task representation. Thus, to perform a meta-mapping, we only need a way of transform-
Performing and transforming tasks with the HoMM architecture. (a) Basic tasks are mappings from inputs to outputs, which can be generalized from examples. (b) Meta-mappings are mappings from tasks to other tasks, which can be generalized from examples. (c) The task representation is used to alter the parameters of a task network (see detail) which executes the appropriate task mapping. (d) The meta-mapping representation is used to parameterize the task network to transform a task representation. The transformed representation can then be used to perform the new task zero-shot (see detail). The HoMM architecture exploits a deep analogy between basic tasks and meta-mappings — both can be seen as mappings of inputs to outputs. This analogy is reflected in the parallels between the top and bottom rows of the figure.

**Training & evaluating the model:** To train the system to perform the basic tasks, we compute a task-appropriate loss at the output of the action network, and then minimize this loss with respect to the parameters in all networks. This includes the networks used to construct the task representation, and even the representations of the examples or language that they receive as input. That is, we train the system end-to-end to perform the basic tasks. When constructing a task representation from examples, we do not allow the example network to see every example, in order to force the system to generalize, in a standard meta-learning fashion. For example, if the basic task is poker, the system will have to construct a task representation from some hands that will be useful for playing other hands. This approach encourages the task representations to capture the structure of the task, rather than just memorizing the provided examples.

To train the system to perform meta-mappings, we first instantiate the meta-mapping representation using either examples or language, and use that representation to parameterize the task network. We then process a set of tasks representations (that were not used as examples) through the task network, and try to match the output task representations to known target task embeddings. Specifically, we minimize an \( \ell_2 \) loss on the difference between the output embedding and the embedding constructed when performing the target task.

**Classifying task representations:** In all domains except RL, we also trained the HoMM model to classify task representations by relevant attributes, again using the same architectural components. This may help the model learn the structure of the task space, and thereby improve generalization (but is not essential, see Fig. 13).

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\(^1\) See Sect. 4 for proof that a simpler vector-analogy approach to meta-mapping is inadequate.
then input the task representation for winning blackjack, and try to match the output to the task representation for losing blackjack. On the next training step, we might use hearts and blackjack as examples, and train the meta-mapping to generalize to rummy. After training, we can test the generalization by passing in the representation for a task like poker, that has never been used for any training on this meta-mapping (either as an example or for generalization). We take the output task representation produced by the meta-mapping, and actually perform the task of losing poker with it. This is how we evaluate meta-mapping performance.

In meta-mapping, generalization is possible at different levels of abstraction. The paragraph above refers to basic generalization — applying a meta-mapping seen during training to a basic task that meta-mapping has not been applied to during training, in order to perform a held-out basic task zero-shot. However, if the system has experienced sufficiently many meta-mappings during training, we can also test its ability to generalize to held-out meta-mappings. For example, if the system has been trained to switch various pairs of colors in a classification task (red for blue, green for yellow, etc.), it should be able to generalize to switching held-out pairs (red for yellow, green for blue, etc.) from an appropriate cue (examples or instructions). We view this as an important part of intelligent adaptability — the system should not only be able to adapt to tasks via meta-mappings that it understands well, but also to infer and use new meta-mappings based on specific instructions or examples. We demonstrate this ability in the subset of our experimental domains where we can instantiate sufficiently many meta-mappings.

Comparing to language-based generalization: Natural language instructions are key to human adaptation to new tasks, and prior work on zero-shot performance has often assumed a description of the task as input (e.g.). For example, a system that has learned to behave in accordance with instructions like “win at poker,” “win at blackjack,” and “lose at blackjack,” should be able to generalize to “lose at blackjack,” given sufficiently many training tasks. This, too, does not require data on the novel task. However, transforming the task representation via a meta-mapping may be a useful inductive bias that allows the system to better use prior task knowledge. To verify this, we compare our meta-mapping approach to an approach that simply constructs task representations from language. We show below that meta-mapping results in better performance on the new tasks, especially when the space of tasks is sparsely sampled or generalization is challenging.

3. Experiments
Meta-mapping is an extremely general framework. Because the assumptions are simply that the basic tasks are mappings from inputs to outputs, and that meta-mappings transform basic tasks, the approach can be applied to most paradigms of machine learning with minor modifications. We demonstrate our results in four settings, ranging from regression to classification and reinforcement learning. We summarize the contributions of the experiments in Table 1.

### A. Polynomials
As a proof of concept, and to demonstrate the benefits of a homoiconic approach, we first apply HoMM to polynomial regression (see Fig. 2a). We construct basic tasks that consist of regressing polynomial functions (of degree ≤ 2) in four variables (i.e. from \( \mathbb{R}^4 \rightarrow \mathbb{R} \)). These polynomials can be inferred from (input, output) examples, where the input is a point in \( \mathbb{R}^4 \) and the output is the evaluation of that polynomial at that point. This is a simple meta-learning regression problem, which the system performs well (Fig. 6).

These tasks/polynomials can then be transformed by various meta-mappings — we considered squaring a polynomial, permuting its variables, or adding or multiplying by a constant. We trained the model on 100 basic polynomials, and we train mapped versions of 60 of these for each meta-mapping. We can evaluate the performance of that meta-mapping on the remaining 40 target tasks (corresponding to the 40 other basic polynomials) that the model has never experienced before. We also held out some of these meta-mappings to evaluate the ability of our method to generalize at the meta-mapping level (see above). For example, we trained the model to adapt to a subset of the input variable permutations, and then evaluated its ability to adapt to a held-out permutation based on examples of that permutation. In total, we trained on 20 meta-mappings, and held-out 16, corresponding to 2260 (\( 100 \times 4 \times 36 \)) trained basic tasks, and 1440 held-out for evaluation.

We trained the system in epochs, during which it received one training step on each trained basic task and one training step on each trained meta-mapping, interleaved in a random order. For one step of training on a basic task, we used 1024 evaluations of the polynomial — we present the model with 50 example evaluations from which to generate a task representation, and make one gradient update that improves the predictions on the remaining evaluations (as well as the example ones). This approach encourages the model to generate an accurate representation of the polynomial from seeing a (relatively) small set of evaluations.

For one step of meta-mapping training, we take representations for each of the 60 basic tasks (and corresponding target tasks) on which the meta-mapping is trained, where each basic task representation is computed from examples as above. We randomly chose half of these (input task, output task) pairs to provide as examples of the meta-mapping from which to generate a representation, and train the system to match the output embeddings from the meta-mapping to the targets for all 60 examples. This approach encourages the model to generate a representation of the meta-mapping that will generalize.

To evaluate HoMM on a trained meta-mapping, we parameterize the mapping using all 60 input-output function embedding pairs that were used to train the meta-mapping, and evaluate the performance resulting from applying that mapping to the other 40 basic tasks to perform the 40 corresponding held-out target tasks, which were never experienced during training. Similarly, to evaluate a held-out meta-mapping, we instan-

### Table 1. The contributions of our four sets of experiments. Our results span various computational paradigms and data types.

| Experiment | Held-out MMs | Lang. Comp. | Type | Input | Output |
|------------|--------------|--------------|------|-------|--------|
| Polynomials | ✓             | Regression   | Vector (\( \mathbb{R}^4 \)) | Scalar (\( \mathbb{R} \)) |
| Cards      | ✓             | Regression   | Several-hot Bet values (\( \mathbb{R}^4 \)) |
| Visual concepts | ✓         | Classification 50 x 50 Label (\( \{0, 1\} \)) image |
| RL         | ✓             | RL           | 91 x 91 Action Q-values (\( \mathbb{R}^4 \)) image |

Then, we evaluate meta-mapping performance.

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Results of applying our HoMM architecture in this setting. We plot zero-shot performance (normalized, see text) on new tasks via meta-mappings. The system not only generalizes trained meta-mappings to examples it has never seen before (purple), but also generalizes to held-out meta-mappings from examples (orange), and does both substantially better than a baseline model which does not adapt (dotted lines). Thus our approach is able to flexibly adapt to a new polynomial without any data from that polynomial, based on that polynomial’s relationship to polynomials it has experienced.

In Fig. 2b we show the success of our meta-mapping approach in this setting. We plot a normalized performance measure, which is 0% if the system performs perfectly. Specifically, we measure performance as \(1 - \frac{\text{loss}}{\text{loss}_0}\), where \(\text{loss}_0\) is the loss for a model which always outputs zero.\(^2\) We also show performance of a baseline model which just performs the original task without adaptation. HoMM is able to achieve 89.0% performance on average (bootstrap 95%-CI across runs [88.3, 89.8]) on a polynomial never experienced during training, based on a trained meta-mapping, and 85.5% performance (bootstrap 95%-CI [85.1, 85.9]) based on a held-out meta-mapping. By comparison, not adapting yields only 4.3% and 19.3% performance, respectively. That is, HoMM is able to achieve good performance on a new task without any data, based only on its relationship to prior tasks.

This success is consistent across all the meta-mapping types we evaluated, see Fig. 10. The task and meta-mapping representations are also systematically organized (Figs. 14 and 15). Finally, HoMM significantly outperforms a non-homoiconic baseline (Fig. 11), showing that exploiting the shared structure between basic tasks and meta-mappings improves generalization. Homoiconicity is beneficial.

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\(^2\)This measure is closely related to the variance explained, except that the square meta-mapping shows the mean of the output polynomials slightly away from zero.
However, there is substantial inter-subject variability in base task performance and adaptation. The evaluation hands were sampled in a stratified way in each phase, so this variability in adaptation is either due to randomness in participants’ behavior (e.g. because they are probability-matching rather than optimizing bets), or in their adaptation approaches. (See Fig. [17] for more detail on human performance.)

Both the language-based model and HoMM perform nearly optimally on the trained task. However, the language based model was not able to generalize well to the losing variation from the given dataset (mean performance on losing variation 2%, bootstrap 95%-CI [−12, 16]) [79]. Intriguingly, this corresponds to behaving approximately randomly; performance would be worse if it did not adapt at all. By contrast, the HoMM model adapts quite well (mean 85%, 95%-CI [79, 90]).

C. Visual concepts. We next applied HoMM to visual concepts, a long-standing cognitive paradigm [18] e.g.). Past work has focused almost entirely on learning a concept from examples. However, adult humans can also understand some novel concepts without any examples at all. If you learn that “blickets” are red triangles from examples, and then are told that “zips” are blue blickets,” you will instantly be able to recognize a zipf without ever having seen an example. This zero-shot performance can be understood as applying a “switch-red-to-blue” meta-mapping to the “blicket” classification function (Fig. [1a]). To capture this ability, we applied HoMM.

We constructed stimuli by selecting from 8 shapes, 8 colors, and 3 sizes. We rendered each item at a random position and rotation within a 50 x 50 pixel image. We defined the basic concepts (basic tasks) as binary classifications of the images (i.e. functions from images to \{0, 1\}). We gave the system all the unidimensional concepts as training examples (i.e. one-vs.-all classification of each shape, color, and size), so that it would be able to recognize all the basic attributes. We also constructed composite tasks based on conjunctions, disjunctions, and exclusive-disjunctions (XOR) of these basic attributes. For each concept, we chose the example stimuli so that the datasets were balanced (that is, there was a 50% chance that each item was a member of the category), both during training and evaluation. We only included negative examples that were one alteration away from being a category member. These careful contrasts may be beneficial – recent work has shown contrasting examples encourage neural networks to extract more general concepts [19]. They also make the task more challenging, and the evaluation more informative.

In this domain we constructed representations for both the basic tasks and meta-mappings from language rather than examples (see Fig. [4b]), to show that HoMM can also perform well with this human-relevant cue.

We trained the system on meta-mappings that switched one shape for another, or switched one color for another. We included 6 training example pairs of each mapping (one for each combination of rule type and other attribute). We also included 6 other pairs for evaluation, where the source concept was trained, but the target was held-out for evaluation. (Note that our selection criteria mean that each held-out example will have a closely matched trained one.) That is, the number of basic concepts the system encounters during training is roughly 18 trained per meta-mapping (roughly because it can be reduced if the meta-mappings have overlapping examples), and the number of evaluation concepts is roughly 6 per meta-mapping. For example, the system might be trained on mappings like “switch-red-to-blue,” with corresponding examples like \text{AND(red, triangle)} \rightarrow \text{AND(blue, triangle)}. It would then be evaluated on closely matched examples like \text{AND(red, circle)} \rightarrow \text{AND(blue, circle)}, where the latter is untrained.

We evaluated the system on its ability to apply these meta-mappings to trained source concepts in order to recognize the
Fig. 4. Applying HoMM to visual concepts [a]. Results are shown after training the model on various numbers of training meta-mappings (or the equivalent set of training concepts). HoMM and language generalization are well above chance (50% with our balanced evaluation sets). [b] HoMM is able to generalize trained meta-mappings to perform new tasks zero-shot, and does so slightly better than language when the datasets are of moderate size, though both systems perform well. [c] HoMM is also able to generalize to adaptation based on held-out meta-mappings, once it experiences sufficiently many training meta-mappings. (Results are from 10 runs of each model with each training set size. Errorbars are bootstrap 95%-CIs across runs.)

D. Reinforcement learning. We next apply our approach to reinforcement learning (RL). RL-like computations relate to neural activity [20, 21], and RL has driven recent AI achievements in complex tasks like Go and StarCraft [22, 23]. Furthermore, there is a rich vein of research on language-based generalization in RL (e.g. [7], [8], [24]). Finally, RL requires more sophisticated adaptation, since actions have lasting consequences. Thus, RL is an important testing domain for meta-mapping.

We created a set of RL tasks in the form of simple 2D games (Fig. 5a). The tasks take place in a 6 x 6 room with an additional impassable barrier of 1 square on each side. The squares are upsampled at a resolution of 7 pixels per square to provide the raw visual input to the agent. (The agent receives egocentric input, i.e. its view is always centered on its position — this improves generalization [5].) The agent has four actions available to it, corresponding to moving in the four cardinal directions. If it makes an invalid action, such as trying to move past the edge of the board, the state does not change.

Furthermore, once the model has experienced sufficiently many meta-mappings, it is able to generalize quite well to held-out meta-mappings from their language description (Fig. 5c). Although the average performance from held-out meta-mappings is not perfect even at 32 training meta-mappings, it is perfect in 40% of the runs (Fig 20). We find these results impressive, given that the model experiences at most 32 training meta-mappings — performing held-out meta-mappings from a description is at least as complicated as performing a held-out task from language, and language generalization is often at chance with 30 examples (e.g. with 36 cards tasks above).
There are two types of tasks, a “pick-up” task, and a “push-off” task. In the pick-up task, the agent is rewarded for picking up the good-colored objects by moving to their grid locations, and is negatively rewarded for picking up the bad-colored objects. In the push-off task, the agent is able to push an adjacent object by moving toward it, if there is no other object behind it. The agent is rewarded for pushing the good-colored objects off the edges of the board, and negatively rewarded for pushing the bad colored objects off. The two types of tasks (“pick-up” and “push-off”) are visually distinguishable, because the shape of the objects used for them are different. However, which color is good or bad is not visually discernable, and must be inferred from the example (state, (action, reward)) tuples used to construct the task representation.

There are in total (2 task types) × (5 color pairs) × (binary switching of good and bad colors) = 20 tasks. We trained the system on 18 tasks, holding out the switched color combinations of (red, blue) in both task types. That is, during training the agent is always positively rewarded for interacting with red objects and negatively rewarded for interacting with blue objects. We train the system on the “switch-good-and-bad-colors” meta-mapping using the remaining four color pairs in both task types, and then evaluate its ability to perform the held-out tasks zero-shot based on this mapping. This is a difficult challenge for a model-free system, since any rewards it receives during training on these tasks contradict these evaluation rewards. (See discussion for a comment on model-based methods.) Perhaps because of this difficulty, we found that two minor model modifications helped stabilize learning: persistent task representations and weight normalization (see SI).

Fig. 5. The reinforcement learning tasks and results. (a) Illustrative state transitions from each task. In the pick-up task example (left), the agent moves downward and picks up the green object. In the push-off task example (right), the agent moves left and pushes the red object. These images are precisely the visual input the agent would receive in these states. Note the agent is always in the center of its view (egocentric perspective). (b) Performance on the held-out tasks via a meta-mapping and via language generalization. Despite the challenging nature of the adaptation (as evidenced by the language generalization performance), HoMM is performing quite well. (c) Correlation of performance on the two hold-out tasks, across runs and time points where performance on the training tasks is high. The correlation is much stronger in the HoMM model than in the language-generalization model, that is, the HoMM model is behaving more systematically in the sense that it is generalizing similarly on both tasks. (Results from 5 runs, error-bars in panel (b) are bootstrap 95%-CIs across runs.)

 bad colors in a pair are switched.

There are two types of tasks, a “pick-up” task, and a “push-off” task. In the pick-up task, the agent is rewarded for picking up the good-colored objects by moving to their grid locations, and is negatively rewarded for picking up the bad-colored objects. In the push-off task, the agent is able to push an adjacent object by moving toward it, if there is no other object behind it. The agent is rewarded for pushing the good-colored objects off the edges of the board, and negatively rewarded for pushing the bad colored objects off. The two types of tasks (“pick-up” and “push-off”) are visually distinguishable, because the shape of the objects used for them are different. However, which color is good or bad is not visually discernable, and must be inferred from the example (state, (action, reward)) tuples used to construct the task representation.

There are in total (2 task types) × (5 color pairs) × (binary switching of good and bad colors) = 20 tasks. We trained the system on 18 tasks, holding out the switched color combinations of (red, blue) in both task types. That is, during training the agent is always positively rewarded for interacting with red objects and negatively rewarded for interacting with blue objects. We train the system on the “switch-good-and-bad-colors” meta-mapping using the remaining four color pairs in both task types, and then evaluate its ability to perform the held-out tasks zero-shot based on this mapping. This is a difficult challenge for a model-free system, since any rewards it receives during training on these tasks contradict these evaluation rewards. (See discussion for a comment on model-based methods.) Perhaps because of this difficulty, we found that two minor model modifications helped stabilize learning: persistent task representations and weight normalization (see SI).

Fig. 5. The reinforcement learning tasks and results. (a) Illustrative state transitions from each task. In the pick-up task example (left), the agent moves downward and picks up the green object. In the push-off task example (right), the agent moves left and pushes the red object. These images are precisely the visual input the agent would receive in these states. Note the agent is always in the center of its view (egocentric perspective). (b) Performance on the held-out tasks via a meta-mapping and via language generalization. Despite the challenging nature of the adaptation (as evidenced by the language generalization performance), HoMM is performing quite well. (c) Correlation of performance on the two hold-out tasks, across runs and time points where performance on the training tasks is high. The correlation is much stronger in the HoMM model than in the language-generalization model, that is, the HoMM model is behaving more systematically in the sense that it is generalizing similarly on both tasks. (Results from 5 runs, error-bars in panel (b) are bootstrap 95%-CIs across runs.)

### Results

**HoMM adapts well, achieving 88.0% of optimal rewards** (mean, bootstrap 95%-CI [75.0-99.0]) on the held-out pick-up task, and **71.7%** (mean, bootstrap 95%-CI [42.0, 94.6]) on the held-out push-off task. By contrast, the language model is showing very little adaptation, with respective performance of **-92.8%** (mean, bootstrap 95%-CI [-96.3, -88.4]) and **-79.7%** (mean, bootstrap 95%-CI [-92.8, -59.1]) on the two tasks. This difference between the models is significant ($t(20.6) = -19.515$, $p < 1 \times 10^{-14}$) in a mixed linear regression controlling for task type and a random effect of run.

Furthermore, HoMM exhibits significantly stronger correlations between its performance on the two tasks, both within runs at different time-points and across runs (Fig. 5c, Fig. 24). HoMM has a correlation of $r = 0.82$ between performance on the two tasks, while the language model only has a correlation

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We reinstate our trained models, and consider how the model may be more like what would be expected from human cognition. (Intriguingly, HoMM also takes longer to complete generalization episodes, see Fig. 26, which might also reflect human behavioral uncertainty in novel situations.)

In SI B.5, we show that HoMM is able to extrapolate meta-mappings beyond the dimensions it has been trained on, to transform new dimensions. Specifically, we show that when trained with the switch-good-and-bad meta-mapping applied to colors, it can generalize to switching shapes. This is further evidence for HoMM’s flexibility and intriguing systematicity.

E. Meta-mapping as a starting point for later learning. Why is zero-shot task adaptation useful? One reason is that it provides an excellent starting point for later learning. Although meta-mapping may result in only an approximation of the correct behavior, this is much better than starting from scratch.

We return to the polynomials domain to demonstrate this. We reinstate our trained models, and consider how the model could learn on the held-out tasks once it encounters them. To do so, we optimize the representations of those tasks in order to improve performance on those tasks, without allowing any of the network weights to change. This approach can improve performance on the new tasks without even the possibility of interfering with prior knowledge (c.f. 6, and below). HoMM outperformed or equaled this approach in every domain we considered. This supports our suggestion that transforming the prior task offers a useful inductive bias, that can effectively exploit prior knowledge.

However, the gap in performance between HoMM and the language model varied considerably. In the RL tasks, the language model generalized much worse than chance (once the training tasks were learned). By contrast, in the visual concepts setting, it performed competitively with the meta-mapping approach, with only slight deficits at moderate sample sizes. What factors are responsible for these variations?

There are a few possibilities. First, because of the structure of the task spaces, there are many more training visual concepts than RL training tasks. Thus, while language-based generalization can be effective, HoMM may be especially useful when there are relatively few training tasks — that is, it may be more sample efficient. However, another factor may be even more critical. The RL training tasks more directly contradict the evaluation tasks. By contrast, in the visual concepts setting, it performed competitively with the meta-mapping approach, with only slight deficits at moderate sample sizes. What factors are responsible for these variations?

We have proposed meta-mappings as a computational account of the human ability to perform a novel task zero-shot (without any data), based on the relationship between the novel task and prior tasks. We have shown that our HoMM approach performs well across a wide range of settings, often achieving 80-90% performance on a new task with no data on that task at all. With enough experience, as in the visual classification settings with enough training tasks, it is able to adapt perfectly. It is also able to adapt using novel relationships (held-out meta-mappings) that it has never encountered during training.

We compared HoMM to a standard zero-shot learning paradigm — constructing a task representation from language (e.g. 6, also see below). HoMM outperformed or equaled this approach in every domain we considered. This supports our suggestion that transforming the prior task offers a useful inductive bias, that can effectively exploit prior knowledge.

4. Discussion

We have proposed meta-mappings as a computational account of the human ability to perform a novel task zero-shot (without any data), based on the relationship between the novel task and prior tasks. We have shown that our HoMM approach performs well across a wide range of settings, often achieving 80-90% performance on a new task with no data on that task at all. With enough experience, as in the visual classification settings with enough training tasks, it is able to adapt perfectly. It is also able to adapt using novel relationships (held-out meta-mappings) that it has never encountered during training.

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There are a few possibilities. First, because of the structure of the task spaces, there are many more training visual concepts than RL training tasks. Thus, while language-based generalization can be effective, HoMM may be especially useful when there are relatively few training tasks — that is, it may be more sample efficient. However, another factor may be even more critical. The RL training tasks more directly contradict the evaluation tasks. By contrast, in the visual concepts domain our task sampling guarantees that each held-out concept will have a “nearby” training concept, one with the same relation type and same other attribute (see above). With less structured visual concept sampling, HoMM’s advantage is slightly more clear (Fig. 21). These results suggest that HoMM may
generalize better to tasks farther outside its training experience than the language model does. Relatedly, HoMM seems to exhibit more systematic generalization than the language model in the RL domain. HoMM exhibits more strongly correlated performance on the held-out tasks, and extrapolates better from switching color to switching shape (SI B.3). Future meta-mapping work, and deep learning more broadly, would benefit from systematic exploration of extrapolation.

These results should not be interpreted as suggesting that language is not important or useful. Instead, language and meta-mapping should be seen as complementary. One example of this is the application of meta-mapping to task representations constructed from language, as in the visual concepts setting. More broadly, meta-mapping and language may be applicable in different domains, and could potentially be mutually supporting (see future directions). Indeed, we see intelligence as multi-faceted, and any single model is a simplification. Our results should not be taken as a suggestion that meta-mapping is the only possible mechanism for adaptation. Instead, our results demonstrate an approach that may be useful as one tool for building models with greater flexibility.

We also highlight the results showing that meta-mapping provides a useful starting point for later learning. While meta-learning approaches can construct a good starting point for learning new tasks, they do not use task relationships to offer a uniquely appropriate starting point for each novel task. Our results show that using a task relationship to adapt a prior task can substantially reduce the errors made along the way to mastering the new task. This could make deep learning more efficient. It could also be useful in settings like robotics, where errors can be extremely costly [27].

HoMM increased the computational flexibility of deep learning (though see limitations below). HoMM can perform tasks from examples, from natural language, and from meta-mappings, which we have shown are an effective way to adapt zero-shot. Thus our work has many potential applications in cognitive science, artificial intelligence, and machine learning.

A. Related work in machine learning. As mentioned above, there is a large body of prior work on zero-shot performance based on natural-language. Larochelle et al. (6) considered the general problem of behaving in accordance with language instructions as simply asking a model to adapt its response when conditioned on different “instruction” inputs. Later work explored zero-shot classification based on only a natural language description of the target class (28-30), or of a novel task in a language-conditioned RL system (7, 8). Some of this work has even exploited relationships between tasks as a learning signal (12). Other work has considered how similarity between tasks can be useful for generating representations for a new task (31), but without transforming task representations to do so. Furthermore, similarity is less specific than a meta-mapping, since it does not specify along which dimensions two tasks are similar. To our knowledge none of the prior work has proposed using meta-mapping approaches to adapt to new tasks by transforming task representations, nor has the prior work proposed a model that can perform these mappings.

Our work is also related to the rapidly-growing literature on meta-learning (e.g. 3, 5, 32). Our architecture builds off of HyperNetworks (16) and other recent applications thereof (e.g. 33, 34). It is also related to work on different time-scales of model adaptation (35-39), and its recent applications in meta-learning (e.g. 40, 42). Some work in visual question answering has explored building a classifier conditioned on a question (65), which is related to our visual-categorization approach.

Work in model-based reinforcement learning has partly addressed how to transfer knowledge between different reward functions (e.g. 93), but our approach is more general. Rather than needing a reward function for a new task to be given, meta-mapping provides a principled way to infer a new reward estimator by transforming a prior one. Meta-mapping could also be used to transform a transition function used in the planning model in response to environmental changes. Meta-mapping could therefore complement model-based approaches. This provides an exciting direction for future work.

There has also been other recent interest in task embeddings. Achille et al. (40) proposed computing embeddings for visual tasks from the Fisher information of a task-tuned model. They show that this captures some interesting properties of the tasks, including some types of semantic relationships, and can help identify models that can perform well on a task. Rusu and colleagues recently suggested a similar meta-learning framework where latent codes are computed for a task which can be decoded to a distribution over parameters (44). Other recent work has tried to learn representations for skills (e.g. 11) or tasks (42 e.g.) for exploration and representation learning, but without exploring zero-shot transformation of these skills.

B. Related work in cognitive science. Our work is inspired by several streams of research in cognitive science as well. One long line of research has suggested that analogical transfer between structurally isomorphic domains may be a key component of “what makes us smart” (43). Transfer has been demonstrated across various cognitive domains (e.g. 18, 44). Yet there has been relatively little exploration of adaptation without any examples of the new task at all.

Our work also touches on complex issues of compositionality, productivity, and systematicity. Fodor and others have advocated that cognition must use strictly compositional representations in order to exhibit systematic and productive generalization (e.g. 45-47). We see our work as following in the tradition of exploring how systematic, structured generalization can instead emerge from the structure of learning experience, without needing to be built into the model (18, 19). Without building in compositional representations of tasks, our model can learn to exploit the shared structure in the concept of “losing” across a few card games to achieve 85% performance in losing a game it has never tried to lose before.

There are a number of potential benefits to letting the compositional structure emerge. First, the structure does not need to be hand-engineered specially for each domain. Our system required no special knowledge about the domains beyond the basic tasks and the existence relationships between them. The fact that some of these relationships correspond to e.g. permutations of variables in the polynomial domain did not need to be hard-coded; instead, the model was able to discover it from the data (presumably, since it was able to generalize well to held-out permutations). Emergence may also allow for novel decompositions at test time. The ability of HoMM to perform well on held-out meta-mappings supports this possibility, but further work will be needed to verify it.

We also believe that HoMM captures some of the recursive processing that Fodor and others have considered to be important (e.g. 48). We found particular inspiration in An-
nette Karmiloff-Smith’s work on re-representing knowledge [50]. It would be interesting to explore whether HoMM could model the phenomena she considered. Could a HoMM-like model infer meta-mappings without instruction, based on unsupervised learning over its internal representations?

We have also been influenced by ideas in mathematical cognition about how concepts can be built on top of more basic concepts [9,11]. This recursive construction reflects the way that HoMM learns meta-mappings as transformations of basic tasks – complex transformations are built upon simpler ones. To the extent that humans can handle arbitrary recursion depths of abstraction, they must use a shared space for representing concepts at different levels, to avoid needing arbitrarily many systems. Relatedly, our shared workspace for data points, tasks, and meta-mappings connects to ideas like the Global Workspace Theory of consciousness [52]. Exploring these connections would be an exciting future direction.

Our work also relates to Fodor’s ideas about the modularity of the mind. Two examples are his view of mental processes as “transforming internal representations,” and his argument that what is accessible about the stimulus is only “what is given in [...] its proximal representations” (53, pp. 200-201). Indeed, our division of the architecture into input and output systems, with the flexible, task-specific computations in the middle, may seem very reminiscent of the modularity that he advocated [51]. However, we chose this implementation to simplify the model — we believe that in reality processes such as perception are not completely task-independent, but involve the interaction of top-down and bottom-up constraints [53].

Reciprocally, we believe that higher-level computations are influenced and constrained by the modalities in which they are supported. This computational feature can emerge in our model, as despite the fact that different types of data and tasks are embedded in a shared latent space, the model generally learns to organize distinct types of inputs into somewhat distinct regions of this space. This means that the task-specific processing can potentially usefully exploit domain-specific features of the input, as for example humans do when they use gestures to think and learn in spatial contexts like mathematical reasoning [50].

At the same time, the shared space can allow a graded overlap in the structure that is shared across different input domains. With this approach, “modularity may not be built in [but] may result from the relationship among representations” [57].

Finally, although it is not our primary focus, our architecture and approach may have interesting connections to cognitive control. A failure to meta-map perfectly could capture some failures of control, as could contamination of task examples with examples from other tasks. Furthermore, the “default” task-network weights could be used to model more automatic processing. This processing can be overridden by more task-specific constraints set by the HyperNetwork, when conditioned on an appropriate task representation. We show some simple default processing experiments in SI [3,7].

C. Limitations & future directions. Although we find our results exciting, the present work has limitations. First, we have demonstrated HoMM within relatively simple, small domains. The model adapts quite well, but has not achieved perfect fidelity of adaptation. One factor that may contribute is the relatively limited range of experience of the model — our models lack the rich lifetime of experience that our human participants have. Furthermore, recent work shows that more realistic and embodied environments can improve generalization [9]. Thus, evaluating our approach in richer, more realistic settings, will be an important future direction. In addition, it would be useful for future work to explore human cognitive flexibility in greater detail, to evaluate the circumstances under which humans can effectively adapt.

A second important limitation is that our approach requires identifying relationships between tasks. This identification will be a challenge in extending to more complex environments. However, we suggest that identification of task relationships will be useful for building more flexible artificial intelligence, and is an important part of human experience and education.

Our work suggests many other exciting future directions. For simplicity we contrasted using language, examples, and meta-mapping to infer task representations in this work. However, it would likely be beneficial to use a combination of these — inferring tasks and meta-mappings from both language and examples, and using multiple constraints to adapt a task representation. Furthermore, we considered language as input, but producing language as output (in the form of explanations) can improve understanding and generalization in both humans [58] and neural networks [59]. While our meta-classifications may provide a small part of this benefit, adding language output would likely improve performance, and better capture the structure and systematicity of human behavior.

A number of architectural choices could also be altered. We used HyperNetworks to parameterize our task network, but it would also be reasonable to have a single feed-forward task network which simply receives the task representation as an additional input. This latter approach performed worse in the polynomial and RL domains (Fig. 12), and hindered later learning (Figs. 29 and 30), but perhaps it would be useful in other cases. Furthermore, cognitive tasks often require more complex processing than our model, and replacing the feed-forward task network with a recurrent network — or a network with external memory (e.g. [60]) — would increase the ability of the model to perform and adapt in such cases. Finally, in principle meta-mapping could be applied within any meta-learning approach that uses task representations (e.g. [31]).

D. Conclusions. An intelligent system should be able to adapt to novel tasks zero-shot, based on the relationship between the novel task and prior tasks. We have proposed a computational implementation of this ability. Our approach is based on constructing task representations, and learning to transform those task representations through meta-mappings. We have also proposed a homogeneous implementation that reuses the same architectures for both basic tasks and meta-mappings. We see our proposal as a logical development from the fundamental idea of meta-learning — that tasks themselves can be seen as data points in a higher-order task of learning-to-learn. This insight leads to the reciprocal idea of transforming task representations just like we transform data.

Meta-mapping is an extremely general approach — we have shown that it performs well across a wide variety of domains and computational paradigms, ranging from polynomial regression to visual classification and reinforcement learning, with task representations constructed from either examples or language. In many cases meta-mapping is able to produce 80-90% performance on a new task without any data at all, even when the new task directly contradicts prior learning.
It is generally able to adapt more drastically after experiencing fewer tasks than language-generalization approaches, and intriguingly sometimes seems to exhibit more systematic behavior. Because of this effective adaptation, meta-mapping provides a valuable starting point for later learning, one that can substantially reduce time to learn a new task and cumulative errors made in learning. Our results thus represent a contribution to understanding a possible mechanism of cognitive adaptability, and the role it may play in future learning. We hope our work will lead to a better understanding of human cognitive flexibility, and the development of artificial intelligence systems that can learn and adapt more flexibly.

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5. Supporting Information (SI)

The Supporting Information is organized as follows: in Section A we describe the details of the model, hyperparameters, and methods for all experiments, as well as providing links to the repositories containing the code for all experiments and analyses. In Section B we show supplemental analyses, and in Section C we provide a proof that a simpler vector-analogy approach is insufficient for meta-mapping.

A. Methods.

A.1. Source repositories. The full code for the experiments and analyses can be found on github:

- HoMM library: https://github.com/lampinen/HoMM
- Polynomials: https://github.com/lampinen/HoMM_polynomial_analysis
- Cards (models): https://github.com/lampinen/HoMM_cards
- Cards (human experiment): https://github.com/lampinen/cards_for_humans
- Concepts: https://github.com/lampinen/categorization_HoMM
- RL: https://github.com/lampinen/HoMM_grids
- Stroop results (below): https://github.com/lampinen/stroop

A.2. Cards experiment. At present, the cards experiment (as seen by the participants) can be experienced at: http://web.stanford.edu/~lampinen/mturk/cards/web/pilot.html

![Diagram](https://example.com/diagram.png)

**Fig. 7.** Schematic of architecture, showing inference and gradient flow through the model on a training step. Thin black lines moving rightward represent inference, thick red lines moving leftward represent gradients. (a) Inference and gradients for the basic tasks. (b) Inference and gradients for meta-mappings. The gradients end at the examples of the meta-mapping, rather than propagating through to alter how those representations are constructed, due to GPU memory constraints. In the future, it might be useful to explore whether allowing further propagation would improve results for both basic tasks and meta-mappings. (These figures depict the inference/gradient flow when performing tasks and meta-mappings from examples, performing from language is similar, except that the example inputs and example network are replaced with language inputs and the language processing network.)

A.3. Model details & hyper-parameters. In Fig. 7 we show the flow of inference (forward) and gradients (backward) through the HoMM architecture on basic task and meta-mapping training steps.

See table 2 for detailed architectural description and hyperparameters for each experiment. Hyperparameters were generally found by a heuristic search, where mostly only the optimizer, learning rate annealing schedule, and number of training epochs were varied. Some of the parameters take the values they do for fairly arbitrary reasons, e.g. the polynomial experiments were run earlier, before 1-layer
## Table 2. Detailed hyperparameter specification for different experiments. A “-” indicates a parameter that does not apply to that experiment. Where only one value is given, it applied to all the experiments discussed. We denote the shared representational space by $Z$. Input encoder: $I : \text{input} \rightarrow Z$. Action decoder $A : Z \rightarrow \text{output}$. Target encoder $T : \text{targets} \rightarrow Z$. Meta-network $E : \{(Z,Z),\ldots\} \rightarrow Z$ maps examples to a task representation. Hyper-network $H : Z \rightarrow \text{parameters}$. Task network $F : Z \rightarrow Z$, parameterized by $H$. Language encoder: $L : \text{natural language} \rightarrow Z$. 

| Z-dimension | Polynomials | Cards | Visual | RL |
|-------------|-------------|-------|--------|----|
| 512         | 512         | 512   | 512    | 512|
| $I$ num. layers | 2           |       |        |     |
| $I$ num. hidden units | 128         |       |        |     |
| $I$ conv. layers. (num filters, size, all strides are 2) | -           | (64, 5), (128, 4), (256, 4), (512, 2), (512, 2), (64, 7), (64, 4), (64, 3)| |
| $\mathcal{L}$ architecture | 2-layer LSTM + 2 fully-connected |   |        |     |
| $\mathcal{L}$ num. hidden units | -           | 512   |        |     |
| $T$ num. layers | 1           | 3     | 1      | 3  |
| $T$ num. hidden units | 128         |       | 128    |    |
| $\mathcal{E}$ architecture | 2 layers per-datum, max pool across, 2 layers | | | |
| Task, MM representations from | Examples | Language | Examples | |
| $\mathcal{H}$ architecture | 4 layers | | 1024 | |
| $\mathcal{H}$ num. hidden units | 512 | | | |
| $\mathcal{F}$ num. layers | 3 | 1 | HoMM: 1, Lang: 3 | 3 |
| $\mathcal{F}$ num. hidden units | 64 | | 128 | |
| $\mathcal{F}$ init. scale | 1 | 1 | 30 | 10 |
| $\mathcal{F}$ weight norm. [61] | No | | Yes | |
| $\mathcal{A}$ num. layers | 1 | 2 | 1 | |
| $\mathcal{A}$ num. hidden units | - | 128 | - | |
| Nonlinearities | Leaky ReLU in most places, except no non-linearity at final layer of networks outputting to the latent space $Z$, and (where applicable) sigmoid for classification outputs, and softmax over actions. | | | |
| Base task loss | $\ell_2$ | $\ell_2$ (masked) | Cross-entropy | $\ell_2$ (masked) |
| Meta-mapping loss | $\ell_2$ | | | |
| Partially-persistent task embeddings | No | Yes | | |
| Persistent embedding match loss weight | | 0.2 | | |
| Optimizer | Adam | RMSProp | | |
| Learning rate (base) | $3 \cdot 10^{-5}$ | $1 \cdot 10^{-5}$ | $3 \cdot 10^{-5}$ | $1 \cdot 10^{-4}$ |
| Learning rate (meta) | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-4}$ |
| L.R. decay rate (base) | $0.85$ | $0.85$ | $0.85$ | $0.85$ |
| L.R. decay rate (meta) | $0.85$ | $0.9$ | $0.85$ | $0.95$ |
| L.R. min (base) | $3 \cdot 10^{-8}$ | $1 \cdot 10^{-8}$ | $3 \cdot 10^{-8}$ | |
| L.R. min (meta) | $3 \cdot 10^{-8}$ | $1 \cdot 10^{-8}$ | $3 \cdot 10^{-8}$ | |
| L.R. decays every | 100 epochs | 200 epochs | 400 epochs | 10000 |
| Num. training epochs | 5000 | 100000 (optimally stopped) | 10000 for 4 train mappings, 7500 for 8, 5000 for others | 300000 (optimally stopped) |
| Num. runs | 5 | 5 | 10 | 5 |
| Num. base tasks (training) | 1300 ($= 60 + 60 \times 20 + 40$) | 36 | Varies | 18 |
| Num. base tasks (held out for meta-mapping eval) | 800 ($= 40 \times 20$) | 4 | Varies | 2 |
| Num. meta classifications | 6 | 8 | 8 | - |
| Num. train meta-mappings | 20 | 3 | Varies | 1 |
| Num. held-out meta-mappings | 16 | 0 | 2 | 0 |
| Base dataset size | 1024 | 1024 | 336 | 64 |
| Base examples size | 50 | 768 | - | 32 |
| Meta dataset size (train) | 60 | 36 | Varies | 18 |
| Meta examples size (train) | Half of train dataset | - | Half of train dataset | |
| Meta examples size (eval) | All of train dataset | - | All of train dataset | |
| Base datasets refreshed | Every 50 epochs | Every 20 | Every 1500 | |
| Target network updated | - | - | Every 10000 epochs | |
| RL discount | - | - | 0.85 | |
| RL exploration probability ($\epsilon$) | - | - | Initial: 1., decay: -0.03 when LR decays. | |
| Action softmax inv. temp. ($\beta$) | - | - | 8 | 8 |
task networks were found to be useful in some settings. While it would be ideal to fully search the space of parameters for all models, unfortunately our computational resource limitations prohibited it. Thus the results in the paper should be interpreted as a lower bound on what would be possible.

Each epoch consisted of a separate learning step on each task (both base and meta), in a random order. In each task, the meta-learner would receive only a subset (the “batch size” above) of the examples to generate a function embedding, and would have to generalize to the remainder of the examples in the dataset. The embeddings of the basic tasks used for meta-mappings were computed and cached once per epoch, so as the network learned over the course of the epoch, these task-embeddings would get “stale,” but this did not seem to be too detrimental. In the case of the RL tasks, where there were persistent task embeddings (see below), they were used instead.

The results reported in the figures in this paper are averages across multiple runs, with different trained and held-out tasks (in the polynomial and visual concepts cases) and different network initializations and training orders each epoch (in all cases), to ensure the robustness of the findings.

A.4. RL model modifications. In the RL domain, we made two changes to improve the stability of learning. The model maintained persistent representations for each trained task, and performed the task with a random convex combination of the persistent task representation and one generated from the present examples, while also trying to match the two via an $\ell_2$ loss. The persistence helped the model overcome conflicting signals from switched-color tasks. We also incorporated weight normalization (61) in the task network, which reparameterizes the weights so that their magnitude and direction are estimated separately. It is likely that neither modification was strictly necessary, but they made training the model easier and faster.

A.5. Polynomials. We randomly sampled the train and test polynomials as follows:

1. Sample the number of relevant variables ($k$) uniformly at random from $0$ (i.e. a constant) to the total number of variables.
2. Sample the subset of $k$ variables that are relevant from all the variables.
3. For each term combining the relevant variables (including the intercept), include the term with probability 0.5. If so give it a random coefficient drawn from $\mathcal{N}(0,2.5)$.

The data points on which these polynomials were evaluated were sampled uniformly from $[-1,1]$ independently for each variable, and an independent set was sampled for each polynomial. The datasets were resampled every 50 epochs of training.

Meta-mappings: We trained on 20 meta-mapping tasks, and held out 16 related meta-mappings.

- Squaring polynomials (where applicable).
- Adding a constant (trained: -3, -1, 1, 3, held-out: 2, -2).
- Multiplying by a constant (trained: -3, -1, 3, held-out: 2, -2).
- Permuting inputs (trained on 12, held-out 12, randomly chosen on each run).

Meta-classifications: We also trained the network on 6 task-embedding classification tasks:

- Classifying polynomials as constant/non-constant.
- Classifying polynomials as zero/non-zero intercept.

For each variable, identifying whether that variable was relevant to the polynomial.

A.6. Card games. Our card games were played with two suits, and 4 values per suit. In our setup, each hand in a game has a win probability (proportional to how it ranks against all other possible hands). The agent is dealt a hand, and then has to choose to bet 0, 1, or 2 (the three actions it has available). We considered a variety of games which depend on different features of the hand:

- **Straight flush**: Most valuable is adjacent numbers in same suit, i.e. 4 and 3 in most valuable suit (royal flush) wins against every other hand. This is the game on which we tested adaptation in the models and human participants.
- **High card**: Highest card wins.
- **Pairs**: Same as high card, except pairs are more valuable, and same suit pairs are even more valuable.
- **Match**: The hand with cards that differ least in value (suit counts as 0.5 pt difference) wins.
- **Blackjack**: The hand’s value increases with the sum of the cards until it crosses 5, at which point the player “goes bust,” and the value becomes negative.

We also considered three binary attributes that could be altered to produce variants of these games:

- **Losers**: Try to lose instead of winning! Reverses the ranking of hands. This is the mapping we evaluated in the models and human participants.
- **Suits rule**: Instead of suits being less important than values, they are more important (essentially flipping the role of suit and value in most games).
- **Switch suit**: Switches which of the suits is more valuable.

Any combination of these options can be applied to any of the 5 games, yielding 40 possible games. We held out all losing variations of the Straight Flush game for evaluation.

Meta-mappings: We trained the network on meta-mappings that toggled each of the binary attributes, but evaluated primarily on switching to losing the Straight Flush game (since that corresponded to the human experiment).

Meta-classifications: For meta-tasks, we gave the network 8 task-embedding classification tasks (one-vs-all classification of each of the 5 game types, and of each of the 3 attributes)

Language: We encoded the tasks in language by sequences of the form

```
[“game”, “<game_type>”, “losers”, “<losers-value>”, “suits rule”, “<suits-rule-value>”, “switch suit”, “<switch-suit-value>”].
```

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A.7. Visual concepts. In Fig. 8 we show all shapes (triangle, square, plus, circle, tee, inverseplus, empty-square, empty-triangle), colors (blue, pink, purple, yellow, ocean, green, cyan, red), and sizes (16, 24, and 32 pixels) that we used in our experiments. All stimuli were rendered at random positions within a 50 x 50 image (constrained so that the full shape remained within the frame), and at random angles within ±20° of their canonical orientation.

Meta-classifications: In addition to the meta-mappings mentioned in the main text, we trained the system on 9 meta-classifications: classifying whether the task was a basic-level rule on any of the three basic dimensions, classifying whether each dimension was relevant (regardless of whether the task was basic or composite), and classifying the type of composite (if the task was not basic).

Language: We encoded the tasks in language by sequences from the following grammar:

- Basic rules: encoded as [['attribute-name', '=', 'attribute-value']], for example ['shape', '=', 'triangle']
- Composite rules: encoded as [['composite-type', '(', '(', '(', 'basic-rule', ')', ')', 'AND', '(', '(', 'basic-rule', ')', ')', ')'], where the 'composite-type' is one of "AND", "OR", or "XOR", and each 'basic-rule' is substituted with a sequence as above.
- Meta-mappings: encoded as ['switch', 'attribute-name', 'old-attribute-value', 'new-attribute-value']
- Meta-classifications: encoded as ['is' 'composite-type'] or ['is', 'basic', 'rule', 'attribute-name'] or ['is', 'relevant', 'attribute-name'], depending on the type of classification.

A.8. RL. The RL tasks were implemented using the open-source Pycolab library [https://github.com/deepmind/pycolab](https://github.com/deepmind/pycolab). Each episode ended after either 150 timesteps elapsed, or the agent had picked up 4 of the 8 objects (regardless of whether they were good or bad) in the pick-up task, or pushed off 4 of the 8 in the push-off task. The agent received a reward of +1 for picking up or pushing off the good-colored objects, and −1 for the bad-colored objects. We used both ε-greedy exploration and chose actions from a softmax over Q-values — while ε-greedy exploration was turned off during evaluation, the softmax choice was left on. Without the softmax over actions, the model generalized somewhat worse, presumably because its Q-values are not adapting perfectly and it could easily get stuck in a loop of incorrect actions.

Meta-classifications: We did not train any meta-classifications in this setting.

Language: We encoded the tasks in language by sequences of the following form:

- Basic task: encoded as [[game-type], [color1], [color2], [switch-colors]], where <game-type> was either “pusher” or “pickup”, colors were strings of a color pair, and <switch-colors> was “TRUE” or “FALSE.”
B. Supplemental analyses & figures.

B.1. Polynomials. In Fig. 9, we show that the basic meta-learning is working well in the polynomials domain. That is, we show that after the example network is presented with a set of example input, output pairs, the system is generalizing well to other points from that polynomial. At the end of training, the mean loss on trained polynomials is 0.025 (bootstrap 95%-CI [0.02, 0.03]), and for held-out polynomials it is 0.58 (bootstrap 95%-CI [0.45, 0.70]). Since the baseline which (outputs all zeros) produces a loss of 11.76 for the trained polynomials, and 11.10 for the held-out, by our measure this performance corresponds to about 99.8% of optimal on the trained polynomials, and 94.8% on the held-out.

In Fig. 10, we show the meta-mapping results in the polynomials domain, broken down by the type of mapping. The system performs well across all mapping types.

We next consider some architecture lesions. In Fig. 11, we compare HoMM to a nonhomoiconic architecture – i.e. one in which there are separate example networks (E_base, E_meta) and hyper networks (H_base, H_meta) for the base tasks and meta-mappings. The nonhomoiconic approach performs substantially worse. Specifically, on trained meta-mappings the HoMM model is achieving a normalized performance of 88.99% (bootstrap 95%-CI [88.20, 89.98]), while the non-homoiconic achieving a normalized performance of 83.2% (bootstrap 95%-CI [81.9, 84.9]). On new meta-mappings the HoMM model is achieving a normalized performance of 85.54% (bootstrap 95%-CI [85.14, 85.94]), while the non-homoiconic model is achieving a normalized performance of 81.3% (bootstrap 95%-CI [80.3, 82.2]).

In Fig. 12a, we show that a simpler task network, which just takes a task representation as another input to feed-forward processing, performs perhaps slightly worse than the HyperNetwork-based approach. Specifically, in the simpler architecture, there is a fixed feed-forward task network, and rather than using the task representation to alter the weights of this network, the task-representation is simply concatenated to the input representation and then propagated through the fixed network.

In Fig. 12a, we show that the meta-classification training is not beneficial in the polynomials domain. Specifically, on trained meta-mappings the HoMM model is achieving a normalized performance of 88.99% (bootstrap 95%-CI [88.20, 89.98]), while without meta-classification it is achieving a normalized performance of 85.54% (bootstrap 95%-CI [85.14, 85.94]), while without meta-classification it is achieving a normalized performance of 81.3% (bootstrap 95%-CI [80.3, 82.2]).

In order to understand the model better, we performed principal components analysis on the task and meta-mapping representations in the HoMM model after training (Fig. 14). This analysis reveals strikingly similar organization of the representation space across different training runs, with constant polynomials pushed to the outside in a semi-circle, and more complex polynomials stretching toward the center, where meta-mappings and meta-classifications are located. This may be due to the learning dynamics — the distance of the task representations from the center appears to be roughly inversely proportional to the complexity of the task, which might imply that the constant polynomials have the largest-magnitude representations because they are easiest to learn, and so their representations receive more consistent updates starting from earlier in the learning process.

To analyze this further, in Fig. 15 we plot the representations for only the constant polynomials, colored by their value (square-root compressed for clarity). This shows that the representations of the constant polynomials are consistently arrayed angularly from lowest to highest value.

Finally, we examined the meta-mapping representations more closely (Fig. 16). This analysis shows that the mappings have a consistent organization across runs, with permutations and addition grouping tightly, but multiplication and squaring, which more drastically alter the polynomials, more dispersed. In particular, multiplying by negative numbers and squaring, which can change polynomials signs and therefore cause a more drastic adaptation, are more separated from the remaining meta-mappings. It is also interesting to note that the addition meta-mappings appear to be organized more by absolute value than sign in at least some runs. There is some interesting structure in higher principle components as well, for example the addition mappings appear to be organized linearly by absolute value in...
principle components 3 and 4 (not shown). The organization of the permutation mappings is more chaotic — while mappings that have similar representations appear more likely to differ by only a transposition, because the relationships among the permutations have a much higher-dimensional group structure, they do not project cleanly into two-dimensional plots.

Fig. 10. Meta-mapping performance in the polynomials domain, broken down by meta-mapping type. We plot a normalized performance measure, as in the main text. The system is performing well across all meta-mapping types, although there is some variability. Triangles show performance of a baseline model that does not adapt — note that some meta-mappings are relatively easier for such a model, while in other cases such a model results in worse performance than outputting all zeros.

Fig. 11. HoMM outperforms or equals a non-homoiconic baseline in the polynomials and cards domains. This figure compares the meta-mapping performance of HoMM with a nonhomoiconic model that instantiates separate copies of the example network \( E_{\text{base}}, E_{\text{meta}} \) and hyper network \( H_{\text{base}}, H_{\text{meta}} \) for the basic tasks and the meta-mappings. In the polynomials domain (a), HoMM significantly outperforms the nonhomoiconic approach, while in the cards domain (b) HoMM is not significantly better. These results suggest that there is sufficient shared structure between the basic tasks and the meta-mappings for the homoiconic approach to improve generalization, at least in the polynomials case, and supports our use of homoiconic architectures.
The HyperNetwork-based architecture we propose in the main text performs as well or better on meta-mappings than an architecture that simply concatenates a task representation to the input before passing it through a fixed MLP, at least on the subset of our domains on which we ran a comparison. Note that the task-concatenated architecture performs just as well at the trained basic tasks (not shown), it is adapting via meta-mappings that proves challenging for it. (See Fig. 19 for a similar comparison for the language generalization baseline.)

Fig. 12. The polynomial domain, compare to Fig. 2b.

The meta-classifications we trained the model with do not appear to be substantially beneficial — a model trained without them performs slightly better in the polynomials domain, while the model trained with them performs marginally better in the cards domain. This difference may be due to the fact that the model is trained on many more basic tasks in the polynomials domain, perhaps obviating the need for meta-classification to shape the representations.

Fig. 13. The cards domain, compare to Fig. 3b.
Fig. 14. Principal components of task and meta-mapping representations of HoMM after training on the polynomials domain. The representation space is organized relatively consistently across runs, with constant polynomials pushed to the outside, and meta-mappings and meta-classifications more centrally located.
Fig. 15. Principal components of constant polynomial representations, showing systematic organization by value. Intriguingly, this relationship appears to be systematically non-linear across runs. (PCs computed across all task representations, color scale of values is compressed with a square-root transformation.)
Fig. 16. Principal components of meta-mapping representations in the polynomial domain, showing systematic organization by type. Permutation mappings cluster tightly, as do addition, while multiplication and squaring are more dispersed. The addition and multiplication mappings are partially organized by absolute value.
**B.2. Cards.** In Fig. [17] we show details of human participants performance on the card game tasks. In Fig. [19] we show that the poor language generalization is not simply due to the HyperNetwork architecture, by comparing to a task-concatenated architecture, as we did for HoMM in Fig. [12].

In Figure [11b] we show that non-homoiconic architectures may perform slightly worse in the cards domain, but the difference is not significant. Specifically, the HoMM model is achieving an average expected reward of 85.38% (bootstrap 95%-CI [79.49, 90.32]), while non-homoiconic meta-mapping is achieving an average expected reward of 79.49% (bootstrap 95%-CI [69.50, 87.34]).

In Fig. [18] we show that the basic meta-learning is working well in the cards domain. That is, we show that after the example network is presented with a set of example (hand, bet, reward) tuples, the system is generalizing well to other hands of that game. At the end of training, the mean reward on trained games is 99.20% of optimal (bootstrap 95%-CI [98.90, 99.40]), and for held-out games it is 83.82% (bootstrap 95%-CI [80.50, 86.00]).

In Figure 13b we show that meta-classification may be slightly beneficial in the cards domain, but the difference is small. Specifically, the HoMM model is achieving an average expected reward of 85.38% (bootstrap 95%-CI [79.49, 90.32]), while without meta-classification it is achieving an average expected reward of 78.68% (bootstrap 95%-CI [71.01, 85.97]). Because the meta-classifications appear to be more useful in this domain than in the polynomials domain, it is possible that they are particularly useful for understanding the structure of the task distribution when there are fewer basic training tasks. However, further work would be needed to verify this.

![Basic game: Bet density by expected value.](image1)

![Basic game: Probability of non-zero bet by expected value. The red dashed line is the optimal threshold, the grey curves are the individual subject fits.](image2)

![Losing variation: Bet density by expected value.](image3)

![Losing variation: Probability of non-zero bet by expected value. The red dashed line is the optimal threshold, the grey curves are the individual subject fits.](image4)

**Fig. 17.** Human performance on the card game task. Top row is basic game evaluation (before being told to lose), bottom is after being told to lose. While participants are performing well above chance, they are far from optimal. They make intermediate value bets, and do not switch optimally between betting and not betting depending on hand value. There is also substantial inter-subject variability.

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Fig. 18. Basic meta-learning performance in the cards domain over learning. The system is generalizing at the meta-learning level. That is, this graph shows that, after the example network receives a set of (hand, bet, reward) example tuples from a game, it is generating a sufficiently good representation of that game to play held-out hands. This is true both for games it was trained with (green), and for games that are held-out and never encountered during training (pink). (Thick dark curves are averages over 5 runs, shown as light curves.)

Fig. 19. Language generalization is similar in the cards domain with either the HyperNetwork architecture used by HoMM, or a simpler task-concatenated architecture. See Fig. 12 above for a similar comparison for HoMM itself.
**B.3. Visual concepts.** In Fig. 20 we show the proportion of runs in which each model achieved > 99% performance. In Fig. 22 we show that the language generalization is better with a more complex architecture (deeper & nonlinear) than we used for the HoMM approach. In Fig. 23 we show learning curves for all runs of the HoMM model on these tasks.

![Graph](image1.png)

**Fig. 20.** In the visual concepts domain, the proportion of runs in which each model attained > 99% accuracy on the transformed concepts. (a) Trained meta-mappings. The HoMM model more frequently shows extremely systematic generalization at moderate sample sizes. (b) Held-out meta-mappings. At the largest sample sizes we considered, the HoMM model is able to adapt near-perfectly to new meta-mappings on many runs. Note that even at this largest sample size, the system is generalizing from only 32 trained meta-mappings.

![Graph](image2.png)

**Fig. 21.** Trained meta-mapping results in the visual concepts domain with 150 randomly sampled training concepts, rather than the structured sampling used in the main text. This task sampling scheme means that some evaluation tasks will be farther from the trained tasks. HoMM has correspondingly more of an advantage here. However, the tasks are still likely to be closer to a trained task than in e.g. the RL setting where the evaluation tasks directly contradict the trained ones, and the language model is performing correspondingly better here than on the RL tasks.
Fig. 22. Comparing language generalization on the visual concepts tasks between a linear task network architecture and a deep, nonlinear one. Although the linear task network worked best for the meta-mapping approaches (comparison not shown, but results reported in text are from linear version), the nonlinear task network generalized better to new language instructions (again, results in main text are from better version).
Fig. 23. Meta-mapping performance in the visual concepts domain broken down by number of training meta-mappings (rows), and by run (columns). The green lines are performance when the transformed task was encountered during training, the pink lines are performance on transformed tasks that were never encountered during training. Panel (a) shows the results for trained meta-mappings, and panel (b) shows the results for held-out meta-mappings. With more training meta-mappings, HoMM both generalizes better when applying the trained meta-mappings to held-out examples (a), and when applying held-out meta-mappings (b). However, even with smaller sample sizes, HoMM is achieving perfect generalization on the trained meta-mappings on many runs.
B.4. RL. In Fig. 24 we show the correlation of performance on the RL tasks broken down by run. In Fig. 25 we show average learning curves for the language generalization model, showing transient (though imperfect) early generalization that decays to a failure to adapt. In Fig. 26 we show intriguing behavioral uncertainty in generalization, where the model exhibits more uncertainty (takes longer to solve the task) even if it does well. Selected recordings of behavior can be found at: https://github.com/lampinen/homm_grids/tree/master/recordings.

In Fig. 12b we also show that the HyperNetwork-based architecture performs better in this domain as well.

Fig. 24. Correlation of performance on the two RL tasks, broken down by run. The correlation is higher in the HoMM model, both within and across runs.

Fig. 25. Average performance of the language generalization model over training on the RL tasks. The model exhibits intriguing but transient generalization early in learning, before it has understood the full structure of the tasks (especially the more difficult and sequential push-off task). However, this quickly decays to below-chance generalization as the model masters the training tasks. This early generalization is not included in the main results since the train accuracy at this time is below the threshold of having adequately learned the tasks.
Fig. 26. The HoMM model exhibits behavioral uncertainty in meta-mapping generalization on the RL tasks, measured by the steps taken to complete each episode. (a) The HoMM model takes more steps to complete episodes from the held-out tasks via a meta-mapping than to complete episodes from tasks used to train the meta-mapping. That is, it appears to be more uncertain about its behavior on the generalization tasks. (b) The behavioral uncertainty effect is not solely driven by the model performing more poorly overall; even on the runs where it performs well, it is almost always taking longer to complete the episodes from the tasks it has never seen before. To show this, we plot the difference in average steps vs. difference in average rewards between train and eval. Note that the step difference is almost always positive (evaluation tasks are slower), even where rewards are comparable. (Panel a: means and bootstrap 95%-CIs across 5 runs. Panel b: each point is one game type within one run.)
B.5. Generalizing from color to shape in RL. We next evaluated the generalization capabilities of HoMM in a more challenging RL experiment. In this experiment, we trained HoMM on tasks similar to those in the main text experiments, but where the good and bad objects could be discriminated by either color (with shape matched) or shape (with color) matched. We trained good-and-bad-switched variations of all color tasks, but did not train any switched variations of the shape-discrimination tasks. Specifically, we used 8 colors, of which we used 4 for the pick-up tasks and 4 for the push-off tasks (so the task type would still be superficially distinguishable. We trained color-discrimination between two pairs of colors in each type, when presented with either both colors appearing on square shapes, or both appearing on diamond shapes. We also trained switched-good-and-bad variations of all those color discrimination tasks. We then trained four shape discrimination tasks for each game type, one in each of that game type’s four associated colors. In the shape discrimination tasks, the tee-shaped objects were always good, and triangular objects were always bad.

We trained the “switch-good-and-bad” meta-mapping on the color discrimination tasks, and evaluated whether HoMM was able to correctly generalize this meta-mapping from switching colors to switching shapes, in order to infer that the triangular objects, which had always been negatively rewarded before, were now beneficial. We found it was useful to increase the initial meta-mapping learning rate to 3 \cdot 10^{-4}, but otherwise used the same hyperparameters as the main text experiments. See Fig. 25 for the results. We found that HoMM was indeed able to perform well above chance at this generalization (average returns across pick-up and pusher 64.3% percent of optimal, 95%-CI [55.1, 72.6]). These experiments show that meta-mapping is able to successfully extrapolate well beyond the training examples of the mapping, to transform behavior along new dimensions.

Intriguingly, the language model performed less poorly at these experiments than at the main text experiments, although it was not statistically different from chance (average returns 17.8% of optimal, 95%-CI [−4.0, 37.4]). The difference in performance between HoMM and the language model was significant in a mixed model controlling for game-type (and its interaction with model) and the random effect of run (t(76.01) = −4.60, p = 1.7 \cdot 10^{-5}), while neither the effect of game type on generalization in the HoMM model, nor the interaction of game-type with model type were significant (respectively, t(76.0) = −0.98, p = 0.33 and t(76.0) = 1.48, p = 0.14).

![Fig. 27. HoMM can generalize switching good and bad objects from the color dimension to the shape dimension. In this experiment, we trained HoMM on tasks similar to those in the main text experiments, but where the good and bad objects could be discriminated by either color (with shape matched) or shape (with color) matched. We trained good-and-bad-switched variations of all color tasks, but did not train any switched variations of the shape-discrimination tasks, to evaluate whether HoMM was able to infer how to transfer a mapping from switching colors to switching shapes. Indeed, HoMM performs well above chance at this task, though not quite as well as on the simpler generalization in the main text. Intriguingly, the language model also appears to be performing somewhat better in this setting, though it is not statistically above chance. (Results from 5 runs, see the text for further details of the experimental setup."

B.6. Meta-mapping as a starting point. In Fig. 25 we show that meta-mapping provides a good starting point for learning in the visual concepts domain as well. In this setting the small random initialization is more competitive, but meta-mapping still yields lower cumulative error over learning than random initialization, and much lower than the centroid (which was better in the polynomials domain). Specifically, initializing with a meta-mapping output results in a mean cumulative error of 0.33 (bootstrap 95%-CI [0.10, 0.57]), while a small random initialization results in a mean cumulative error of 9.62 (bootstrap 95%-CI [6.63, 13.59]). This difference is significant in a mixed linear model (t(4) = 4.628, p = 0.01).

We have compared our hyper-network-based HoMM architecture to the simpler alternative of concatenating a task representation to an input embedding before passing it through a fixed network, in various supplemental analyses (Figs. 12 and 19). The hyper network approach generally performs at least as well as, and sometimes substantially better than, the simpler approach. Hyper networks may also be particularly beneficial for continual learning (22). Furthermore, they may also make it easier to optimize the task representation, by giving it more direct control over the computations of the network. Thus, it seems useful to compare these two architectures in this setting.

We therefore performed the polynomial domain experiments, reported in the main text in the meta-mapping as a starting point section, with the simpler task-network architecture as well. In Fig. 26 we show the learning curves for both architectures for the two best initializations (meta-mapping output, and centroid of the trained task representations). The hyper-network architecture learns much more rapidly than the simpler architecture. The initial meta-mapping outputs do not differ so substantially — most of this effect is due to the slower improvement of the loss when optimizing the task representation in the non-hyper architecture. Indeed, optimization in the non-hyper network architecture appears to be plateauing at a much higher loss value than in the hyper-network architecture.

As before, we quantify this by plotting the cumulative loss on the novel tasks in Fig. 29. The simpler non-hyper architecture resulted in about five times greater cumulative loss than the hyper network architecture when starting from the meta-mapping output (mean = 133.81, bootstrap 95%-CI [102.65, 171.10]), and similarly from the centroid of the trained task representations (mean = 1139.35, bootstrap 95%-CI [943.60, 1344.52]). We therefore conclude that hyper-network-based architectures may be particularly conducive to this perspective on continual learning.
Fig. 28. Meta-mapping provides a good starting point for later learning in the visual concepts domain. This figure is the visual concepts analog of Fig. 6 in the main text, with 16 training meta-mappings. Using meta-mapping as a starting point offers much lower initial loss, and faster learning than other initializations. (Thick curves are averages over 5 individual runs, shown as light curves.)

Fig. 29. Comparing the learning curves of the hyper network architecture and a simpler architecture when optimizing the task representations for new polynomials. The simpler architecture improves much more slowly, and appears to plateau at a higher loss. (Note that the y-axis is log-scale. Results are from 5 runs, individual runs are shown as light curves.)
Fig. 30. Comparing the cumulative losses of the hyper-network architecture and a simpler architecture when optimizing the task representations for new polynomials. The simpler architecture results in substantially more cumulative loss. (Results from 5 runs, errorbars are bootstrap 95%-CIs.)
Measuring the default behavior of the HoMM architecture on a Stroop-like task. We plot the bias of the model towards word or color responses, when given an all-zeros task representation, at different proportions of training on words or colors, and different stages of training. When the model has just mastered the less frequent task, it exhibits a default bias towards the more frequent task. However, later in training, when it has mastered both tasks, it exhibits a paradoxical bias towards the less frequent task.

B.7. Default processing & cognitive control. The HoMM architecture could be of interest to researchers in cognitive control, even beyond the idea of meta-mapping as adaptation. The system can perform different tasks based on task examples or language inputs, which is fundamentally the same problems human face when we must adapt our behavior. There are a number of features of the model that offer the opportunity for intriguing investigations based on this idea. For example, the task network in our architecture has a default set of bias weights that are modulated by the HyperNetwork. These can be thought of as the “automatic” or “default” processing habits of the system, whereas the weight alterations the HyperNetwork imposes can be thought of as the exertion of cognitive control to modulate behavior.

To explore this, we trained our architecture on a very simple stroop task taken from Cohen et al. (63). The model receives two sets of two inputs, that can be thought of as corresponding to “word” and “color” domains. One input in each domain is turned on, representing a color word written in a color. The model’s task is to report either the color or the word, depending on context.

The context we give the model is in the form of examples of the task as (input, output) pairs. These are used to construct a task representation, which is then used to modulate the parameters in the task network, via the HyperNetwork. We trained the model repeatedly with different proportions of training on the word task vs. the color task, in order to investigate the default vs. controlled behavior in different training regimes. Specifically, we compared training the model to the point that it barely mastered the less frequent task (when it first achieves 100% performance and cross-entropy loss < 0.3 on both tasks) to the point that it had mastered both tasks (100% performance and cross-entropy loss < 0.01 on both). We then tested the model’s default behavior by giving it an all-zeros task representation, and seeing whether its performance was more aligned with the “word” or “color” task.

In Fig. 31, we show the results. We plot the bias as $2 \times (\text{word accuracy} - \text{color accuracy})$, which is –1 if the model is responding only to color, 1 if the model is responding perfectly to word, and 0 if it is responding equally to each (or otherwise responding randomly). When the model has just barely mastered the less-frequent task, it exhibits a default bias towards the more frequent task. However, once we train it to full master of both tasks, it exhibits a surprising paradoxical bias towards the task that was mastered more recently. This may relate to observations that switching from a less-practiced task back to a more practiced one is difficult (64), possibly because performing the less-practiced task requires strong suppression of the default behavior. It’s possible that in the course of achieving full mastery on the less-practiced task, the more practiced task must be so suppressed that it fades away from being the default. These phenomena provide possible inspiration for future investigations in cognitive control.

For this experiment, we used similar hyperparameters to the polynomials experiments, except we used a much smaller model — a single-layer task network, a $Z$-dimensionality of 8, and $H, E$ had 64 hidden units per layer. We optimized the model via stochastic gradient descent with a learning rate of 0.01 to follow more closely the approach taken by Cohen et al., although results are similar with other optimizers.
C. Proofs.

C.1. Inadequacy of vector analogies for meta-mapping polynomials. One possible implementation of meta-mapping would be to just construct an analogy vector and use that for the mapping. This is motivated by work showing that word vector representations often support vector analogical reasoning, for example if we denote the vector for the word king as $\vec{v}_{\text{king}}$, relationships like $\vec{v}_{\text{queen}} \approx \vec{v}_{\text{king}} + (\vec{v}_{\text{man}} - \vec{v}_{\text{woman}})$ often hold \cite{mikolov2013distributed}. Thus, a plausible approach to meta-mapping would be to take a similar approach, for example in the polynomials domain, the meta-mapping “Permute ($u, z, x, y$)” could be estimated by taking the vector differences between the representations of inputs and targets, computing an average difference vector, and adding that to the held-out examples to produce an output for each one. In this section, we prove that such an approach cannot accurately represent all the meta-mappings in the polynomials domain. Furthermore, we sketch a proof by construction that the linear task network (i.e. an affine transformation, matrix multiplication plus a bias vector) we used in this domain suffices, if it is parameterized separately for each meta-mapping.

Proof that vector analogies are inadequate: In essence, the proof is simply that many of our meta-mappings are non-commutative, while vector addition is commutative. Consider the mappings for adding 1 to a polynomial, and multiplying by 2. Assume there were vector representations for these mappings, respectively $\vec{m}_{+1}$ and $\vec{m}_{\times 2}$. Let $\vec{f}_x$ be the representation for the polynomial $f(w, x, y, z) = x$. Then $\vec{f}_x + \vec{m}_{+1} = \vec{f}_{x+1}, \vec{f}_x + \vec{m}_{\times 2} = \vec{f}_{2x}$. But then:

$$\vec{f}_{2(x+1)} = (\vec{f}_x + \vec{m}_{+1}) + \vec{m}_{\times 2} = \vec{f}_x + \vec{m}_{+1} + \vec{m}_{\times 2} = (\vec{f}_x + \vec{m}_{\times 2}) + \vec{m}_{+1} = \vec{f}_{2x+1}$$

Thus such a representation would result in contradictions, such as $2x + 1 = 2x + 2$. Similar issues occur for input permutation and other non-commutative mappings.

Proof sketch that affine transformations in an appropriate vector space suffice: Suppose that we have a vector representation for the polynomials, where there is a basis dimension corresponding to each monomial, so that the polynomial can be represented as a vector of its coefficients. (This is the standard vector-space representation for polynomials.) Then permutation corresponds to permuting these monomials, i.e. a permutation of the basis dimensions, which is a linear transformation. Adding a constant corresponds to adding to one dimension, which requires only the vector addition part of the affine transformation. Multiplying by a constant requires multiplying each dimension, i.e. a block-diagonal linear transformation.

Squaring polynomials is slightly more complex, and requires augmenting the vector space with components whose values are the product of the coefficients of each pair of monomials. In this case, squaring corresponds to a simple linear transformation. However, this augmentation makes the other meta-mappings more complex. The most difficult case is adding a constant, which requires shifting each pair term containing a constant by the product of the constant and the coefficient of the other monomial, but this again reduces to simply an appropriately parameterized affine transformation — each pair term containing a constant term simply needs the added constant as a weight times the component for the other monomial. Thus affine transformations suffice in this setting.

Of course, with a sufficiently complex, deep, recurrent, and non-linear task network, any meta-mapping could be computed in principle, since a sufficiently large such network is Turing-complete \cite{dumitrescu2019neural}. Thus, our approach to meta-mapping is fully general, conditioned on a sufficiently complex task network, while simpler approaches may not be.