Cosmic histories of star formation and reionization: An analysis with a power-law approximation

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Abstract. With a simple power-law approximation of high-redshift ($\gtrsim 3.5$) star formation history, i.e., $\dot{\rho}_*(z) \propto [(1 + z)/4.5]^{-\alpha}$, we investigate the reionization of intergalactic medium (IGM) and the consequent Thomson scattering optical depth for cosmic microwave background (CMB) photons. A constraint on the evolution index $\alpha$ is derived from the CMB optical depth measured by the Wilkinson Microwave Anisotropy Probe (WMAP) experiment, which reads $\alpha \approx 2.18 \lg N_\gamma - 3.89$, where the free parameter $N_\gamma$ is the number of the escaped ionizing ultraviolet photons per baryon. At the same time, the redshift $z_f$ at which the IGM is fully ionized can also be expressed as a function of $\alpha$ as well as $N_\gamma$. By further taking into account the implication of the Gunn-Peterson trough observations to quasars for the full reionization redshift, i.e., $6 \lesssim z_f \lesssim 7$, we obtain $0.3 \lesssim \alpha \lesssim 1.3$ and $80 \lesssim N_\gamma \lesssim 230$. For a typical number of $\sim 4000$ of ionizing photons released per baryon of normal stars, the fraction of these photons escaping from the stars, $f_{\text{esc}}$, can be constrained to within the range of $(2.0 - 5.8)$%.

Keywords: Star formation, Reionization
1 Introduction

The cosmic history of star formation (especially at high redshifts) is one of long outstanding problems in astrophysical cosmology, which is being probed by a growing number of observations such as through Lyman break galaxies (LBG) [1, 2], Ly-α emitter galaxies [3], and gamma-ray bursts (GRBs) [4–8]. One of the most important consequences of star formation is the ionization of intergalactic medium (IGM) that has been recombined at redshift $z \sim 1100$, although some other sources (e.g., accreting massive black holes) may also provide contributions to the reionization. The answer to the question of how and when IGM was reionized must hold many clues to the nature of the first generation of light sources and also to the onset of structure formation in cold dark matter cosmologies. During reionization, the liberated free electrons can scatter cosmic microwave background (CMB) photons, resulting in an increase in the degree of the anisotropies of both CMB temperature and polarization [9].

Therefore, since the launch of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite, it has become of great interest to derive constraints on the cosmic histories of star formation and reionization from CMB observations. As one of the most direct results, the WMAP-7 observation infers a Thomson scattering optical depth of CMB photons $\tau = 0.088 \pm 0.015$ [10, 11]. Then, the redshift for full reionization can easily be derived to $z_f = 10.5 \pm 1.2$ with the assumption that the transition from a fully neutral to a fully ionized IGM is abrupt. However, deeper implications of the optical depth always invoke an elaborate description for the star formation history, either by theoretical modelings in the hierarchical formation model [12, 13] or by observational determinations from some specific observations [14]. In this paper, we alternatively take a power-law assumption for the high-redshift star formation history as an effective description, where all of the relevant physical details are packed into a free parameter (i.e., the power law index). The power-law behavior could be regarded as the first order approximation of the realistic history, which in principle allows us to get some general and substantial inferences.

2 Model

First of all, all analyses throughout the paper is based on the assumption that cosmic reionization is dominated by the early generation of stars. The rate of ionizing ultraviolet photons escaping from stars into IGM reads

$$\dot{n}_\gamma(z) = \frac{\dot{\rho}_* (z)}{m_B} N_{\gamma f_{esc}},$$  \hspace{1cm} (2.1)
where \( \dot{\rho}_e(z) \) is the star formation rate (SFR), \( m_B \) is the mass of a baryon, \( N_\gamma \) is the number of ionizing ultraviolet photons released per baryon of the stars, and \( f_{\text{esc}} \) is the fraction of these photons escaping from the stars. In literature, the value of \( N_\gamma \) is usually taken to be \( \sim 4000 \) for a Salpeter stellar initial mass function and a metallicity \( \sim 0.05 Z_\odot \) (e.g., \cite{15}). For the value of \( f_{\text{esc}} \), a wild range from a few percent to \( \sim 20 \) percent was suggested by some observations \cite{16–19}. Similar values were also predicted by some theoretical modelings and simulations. For example, Razoumov & Sommer-Larsen \cite{20} found that \( f_{\text{esc}} \) evolves from \( \sim 1 \)–2 percent at \( z = 2.39 \) to \( \sim 6 \)–10 percent at \( z = 3.6 \). In this paper, we would use the product of \( N_\gamma \) and \( f_{\text{esc}} \) as a single parameter denoted by \( \mathcal{M}_\gamma = N_\gamma f_{\text{esc}} \), which is considered to be completely free. Accompanying with star formation, IGM was reionized. The density of the liberated electrons \( n_e \) can be connected to the hydrogen density \( n_H \) by

\[
n_e = (1 + y)n_H x,
\]

where \( x = n_{\text{II}}/n_H \) represents the fraction of ionized hydrogen and \( y \) is introduced by considering the ionization of helium. By assuming that the helium was only once ionized, we have \( y = Y/(4X) \approx 0.08 \), where \( X = 0.7523 \) and \( Y = 0.2477 \) are the mass fractions of the hydrogen and helium, respectively, at the reionization era \cite{21}. Following ref. \cite{15}, the evolution of \( x \) can be determined by

\[
\frac{dx}{dz} = \left[ \frac{\dot{n}_\gamma}{(1 + y)n_H^0} - \alpha_B C(1 + z)^3(1 + y)n_H^0 x \right] \frac{dt}{dz}, \tag{2.2}
\]

where \( n_H^0 = 1.9 \times 10^{-7} \text{ cm}^{-3} \) is the present number density of hydrogen, \( \alpha_B = 2.6 \times 10^{-13} \text{ cm}^3 \text{s}^{-1} \) is the recombination coefficient of netral hydrogen for electron temperature of \( \sim 10^4 \text{ K} \), \( C = \langle n_{\text{II}}^2 \rangle/(n_{\text{II}})^2 \) is the clumping factor of ionized gas, and \( dt/dz = -1/[H(z)(1 + z)] \) with \( H(z) = H_0 \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda} \), \( H_0 = 71 \text{ km s}^{-1}\text{Mpc}^{-1} \), \( \Omega_m = 0.267 \), and \( \Omega_\Lambda = 0.734 \) \cite{10}.

For a given reionization history \( x(z) \), the CMB optical depth can be calculated by integrating the electron density times the Thomson cross section along proper length

\[
\tau = \int \sigma_T n_e dl = -(1 + y)\sigma_T n_H^0 c \int_0^{z_h} (1 + z)^3 x(z) \frac{dt}{dz} dz. \tag{2.3}
\]

The WMAP-7 observation gave \( \tau = 0.088 \pm 0.015 \), which hence provides a robust constraint on the reionization history. There seems a trouble in the precise definition of the upper limit of the above integral, because of the present poor knowledge of the onset of star formation. However, due to the rapid decease of the degree of ionization with increasing redshifts (see the results presented in the next section), the CMB optical depth could be mainly contributed by the electrons at relatively low redshifts \( z < z_h \). More importantly, since the WMAP experiment is probably insensitive to too high redshifts \cite{10,28}, the value of \( z_h \) actually can not be taken to extremely high. Therefore, when we solve eq. \( (2.2) \) from high to low redshifts, we somewhat arbitrarily take the initial redshift \( z_h \) to be a moderate value of 20 at which the transition from Population (Pop) III to 1/II stars\footnote{The Pop III-I/II transition is usually considered to take place when the cosmic metallicity is enriched to \( \sim 10^{-3.5} Z_\odot \). However, the recently discovered Caffau star showed that the critical metallicity could be much smaller due to dust cooling, i.e., \( \sim 10^{-7} Z_\odot \) \cite{22}. So the transition could proceed more rapidly than previously considered.} may take place \cite{23–26}. Correspondingly, the value of \( x \) at \( z_h \) is in principle determined by both the residual electrons left from the

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Figure 1. Star formation history with 0.3 \( \leq \alpha \leq 1.3 \) (shaded band), which is constrained by both the WMAP-7 CMB optical depth (i.e., \( \tau = 0.088 \)) and the Gunn-Peterson trough observations (i.e., \( 6 \lesssim z_{\text{full}} \lesssim 7 \)). Some observationally-determined SFRs are also presented for comparisons.

recombination era \([27]\) and, in particular, the electrons ionized by possible Pop III stars at \( z > z_h \). For a generally consideration, a wide range of \( 0 \leq x(z_h) \leq 0.1 \) would be taken into account in our calculations. Here all of the uncertain physics above redshift \( z_h \) is packed into the free parameter \( x(z_h) \). Finally, it should be noticed that the contribution to the CMB optical depth by the residual electrons was already accounted for when deriving \( \tau \) from the CMB data. Therefore, in principle, the fraction of these electrons needs to be deducted from the total \( x(z) \) when we calculate the optical depth by eq. (2.3). Nevertheless, in view of the actually small fraction of the residual electrons on the order of \( 10^{-3} \) \([27]\), such a deduction would be ignored in our calculations.

Solving eqs. (2.2) and (2.3) depends on two crucial but controversial astrophysical inputs, i.e., the star formation rate \( \dot{\rho}_*(z) \) and the clumping factor \( C(z) \). Following a series of measurements of SFRs, especially the complication of Hopkins & Beacom \([29]\) which is shown in figure 1 (solid circles), a consensus on the history of star formation now emerges up to redshift \( z \sim 3.5 \). The behavior includes a steady increase of star formation from \( z = 0 \) to \( z = 1 \), and a following plateau up to \( z \sim 3.5 \), which can be empirically expressed by

\[
\dot{\rho}_*(z) \propto \begin{cases} 
(1 + z)^{3.44}, & \text{for } z \leq 0.97, \\
(1 + z)^0, & \text{for } 0.97 \leq z \lesssim 3.5,
\end{cases}
\]  \hspace{1cm} (2.4)

with a local rate \( \dot{\rho}_*(0) = 0.02 \, M_\odot \, \text{yr}^{-1} \, \text{Mpc}^{-3} \). For \( z \gtrsim 3.5 \), a clue to a decrease in the SFRs was seemingly implied by the Hopkins & Beacom’s data, which was further confirmed by the observations of LBGs and GRBs. Anyway, in view of the obvious insufficiency of the high-redshift observations, the tendency of the star formation history to high redshifts is in fact still ambiguous. Therefore, all of the cases including those in which the history continues to plateau, drops off, or even increases were taken into account in previous literature (e.g.,
Instead of a theoretical modeling or observational determination, we here introduce a free parameter $\alpha$ to parameterize the high-redshift history as a power law

$$\dot{\rho}_* (z) = 0.2 \left( \frac{1 + z}{4.5} \right)^{-\alpha} \text{M}_\odot \text{yr}^{-1} \text{Gpc}^{-3}, \text{ for } z \gtrsim 3.5.$$

(2.5)

Such a simple power-law behavior would be regarded as the first order approximation of the realistic history. Finally, for the clumping factor, its value can be estimated by numerical simulations (e.g., [31, 32]) and by semianalytical models (e.g., [33–36]). All of these calculations showed that the clumping factor decreases with increasing redshifts, which can approximately be described by a power law. Following a recent simulation given by ref. [32], here we adopt

$$C(z) = 2.9 \left( \frac{1 + z}{6} \right)^{-1.1}, \text{ for } z \gtrsim 5.$$

(2.6)

which has a fixed $C(z < 5) = 2.9.$

### 3 Results

Substituting eqs. (2.4–2.6) into eq. (2.2), a cosmic reionization history, $x(z)$, can be solved for a set of values of the parameters $N_\gamma$ and $\alpha$. By equating the CMB optical depth for this reionization to the WMAP-7 one (i.e., $\tau = 0.088$), we get a nearly linear relationship between the parameters $\log N_\gamma$ and $\alpha$, as presented in the top panel of figure 2, which can be fitted by

$$\alpha = a \log N_\gamma - b$$

(3.1)

with $a = 2.18 \pm 0.05$ and $b = 3.89 \pm 0.01$. Here the error bars of the coefficients are resulted from the uncertainty of the parameter $x(z_h)$. Moreover, the small values of the error bars indicate that the $N_\gamma - \alpha$ relationship is actually insensitive to the value of $x(z_h)$, which demonstrates that it is acceptable to ignore the deduction of the fraction of residual electrons. With the above relationship, we present the solutions to eq. (2.2) in figure 3 for different values of $\alpha$, where the redshift at which the IGM is fully ionized (denoted by $z_f$) can be found. Subsequently, we can obtain the relationship between $z_f$ and $N_\gamma$, as shown in the bottom panel of figure 2.

As also shown by figure 3, the fittings to the reionization histories for $z_f < z \lesssim 10$ exhibit a power-law behavior as

$$x \approx \left( \frac{1 + z}{1 + z_f} \right)^{-(\alpha+1.7)}, \text{ for } z \geq z_f.$$

(3.2)

Substituting the above approximation into eq. (2.3), an analytical expression for the optical depth can be obtained

$$\tau \approx \frac{2\sigma T n_e^2 c (1 + y)}{3H_0 \Omega_{m}^{1/2}} \left[ (1 + z_f)^{3/2} \left( 1 + \frac{15\eta}{2 + 10\alpha} \right) - \frac{1}{\Omega_{m}^{1/2}} \right],$$

(3.3)

where $\eta = 1 - [(1 + z_h)/(1 + z_f)]^{\alpha-1/5}$ and the parameter $\Omega_\Lambda$ is omitted due to $\Omega_\Lambda \ll \Omega_m (1 + z_f)^3$. For the WMAP-7 optical depth $\tau = 0.088$, the redshift for full reionization as a function of $\alpha$ can be expressed

$$z_f \approx 11.7 \left( 1 + \frac{15\eta}{2 + 10\alpha} \right)^{-2/3} - 1.$$

(3.4)
Figure 2. Relationships required by the WMAP-7 CMB optical depth ($\tau = 0.088$) between the parameters $\mathcal{M}_\gamma$ and $\alpha$ (top panel) or $\mathcal{M}_\gamma$ and $z_{\text{full}}$ (bottom panel), as shown by the solid bands. The width of the bands is determined by the adopted range of the initial degree of ionization as $0 \leq \chi(z_h) \leq 0.1$. The possible range of $z_f$ denoted by the hatched band is inferred from the Gunn-Peterson trough observations, and consequently the dashed arrows represent the constraints on the parameters of $\alpha$ and $\mathcal{M}_\gamma$.

Obviously, for $\alpha \to \infty$ and thus $\eta \to 1$ which is the case considered in the WMAP analyses, the value of $z_f$ approaches to its upper limit 10.7. In contrast, for a more realistic value of $\alpha$ on the order of unity, eq. (3.4) clearly shows that the redshift for full reionization could be much lower than its upper limit.

4 Discussions

The results presented above demonstrate that the effective power-law approximation for the high-redshift star formation history can dramatically simplify the calculations of reionization and CMB optical depth. More usefully, the connections among the star formation, reionization, and CMB optical depth could be approximately analytically addressed. These results make it convenient to test the theoretical models of star formation and reionization quickly
Figure 3. Numerically calculated evolutions of the ionization degree of hydrogen with redshift for different $\alpha$ (solid lines), where the values of $\mathcal{N}_\gamma$ are taken according to eq. (3.1) with $x(z_h) = 0$. The dashed lines represent power-law fittings to the numerical results.

and effectively. At the same time, our results also provide an observational base to some relevant astrophysical studies, e.g., the estimation of the event rate of high-redshift GRBs.

As implied by the probes to the Gunn-Peterson trough (Ly$\alpha$ absorption) toward high-redshift quasars and galaxies, the full reionization of hydrogens could take place not far beyond $z_{GP} \sim 6\,[37–40]$. Of course, by considering that the Gunn-Peterson test is actually sensitive to small neutral fractions, the value of $z_f$ could be somewhat higher. For example, detections of Ly$\alpha$-emitting galaxies at $z = 6.6$ \cite{41, 42} suggested that the redshift for full reionization might be as high as $z \approx 7$. Therefore, for an independent and supplementary consideration, if we take $6 \lesssim z_f \lesssim 7$, a constraint on the star formation history as well as on the parameter $\mathcal{N}_\gamma$ could be derived from the $\mathcal{N}_\gamma - \alpha$ and $\alpha - z_f$ relationships:

$$0.3 \pm 0.2 \lesssim \alpha \lesssim 1.3 \pm 0.3,$$

and

$$80 \pm 12 \lesssim \mathcal{N}_\gamma \lesssim 230 \pm 60.$$

On one hand, the range of $\alpha$ is well consistent with the one constrained by the observed near infrared excess in the cosmic infrared background spectrum \cite{26}. On the other hand, for a typical value of 4000 of $N_\gamma$ for normal stars, the range of $\mathcal{N}_\gamma$ indicates that the escaping fraction $f_{\text{esc}}$ is within the range from $2.0 \pm 0.3\%$ to $5.8 \pm 1.5\%$. This is well consistent with the previous observations and calculations.

The possible star formation history with $0.3 \lesssim \alpha \lesssim 1.3$ (shaded band) is presented in figure 1 in comparison with some observations. Then the comparison shows that, in general, the SFRs inferred from both GRBs and LBGs are too low to reionize the universe. To be specific, the slight shortage of the GRB-inferred SFRs may suggest that the calibration
between the GRBs and stars needs to be investigated more carefully. For example, some selection effects due to redshift measurements [43] should be taken into account, which are missed by ref. [4]. On the other hand, as also argued by some previous works (e.g., [4, 44, 45]), the significant discrepancy between the LBG-inferred SFRs and the required levels indicates that most of the high-redshift star formation could take place in very small halos hosting dwarf galaxies, which are too faint to be observed in deep LBG surveys.

The result present above robustly exhibits the importance and advantage of the GRB observations in the SFR determination. In some previous works (e.g., [46]), the SFRs inferred from high-redshift GRBs were considered to be too high in contrast to the relatively low LBG-inferred SFRs. Therefore, Cheng et al. [46] proposed that these GRB-inferred SFRs may be overestimated due to that some high-redshift GRBs do not originate from the collapse of massive stars, but could be produced by the bursts of superconducting cosmic strings. However, the result addressed in this paper indicates an opposite situation, i.e., that the GRB-inferred SFRs actually are not overestimated but somewhat underestimated. Therefore, the expected number of the bursts of superconducting cosmic strings could be much lower than the one estimated in ref. [46].

Finally, it is worth noticing that the values of the cosmological parameters we adopted above actually are obtained by fitting the CMB power spectra with a hyperbolic-tangent-function reionization. Therefore, as pointed out by some previous works (e.g., [47, 48]), it seems necessary to revisit the fitting to the CMB power spectra with a more realistic reionization history, e.g., the power-law behavior as presented in eq. (3.2). For a preliminary attempt, with the CosmoMC code [49] (updated CAMB), we plot the probability distributions of several cosmological parameters in figure 4, both for the cases of tangent-function and power-law reionizations. In comparison, the modification of the parameters due to the new reionization module is basically apparent and, specifically, the peak values of the parameters shift about few percent. Nevertheless, such shifts would not significantly influence

Figure 4. The probability distributions of some cosmological parameters resulting from a fitting to the CMB power spectra with a hyperbolic-tangent-function reionization (dashed lines) or a power-law reionization as eq. (3.2) (solid lines).
the results obtained in this paper.

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