Understanding dynamics of Martian winter polar vortex with ”improved” moist-convective shallow water model

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Abstract. We show how the properties of the Mars polar vortex can be understood in the framework of a simple shallow-water type model obtained by vertical averaging of the adiabatic ”primitive” equations, and ”improved” by inclusion of thermal relaxation and convective fluxes due to the phase transitions of $CO_2$, the major constituent of the Martian atmosphere. We perform stability analysis of the vortex, show that corresponding mean zonal flow is unstable, and simulate numerically non-linear saturation of the instability. We show in this way that, while non-linear adiabatic saturation of the instability tends to reorganize the vortex, the diabatic effects prevent this, and thus provide an explanation of the vortex form and longevity.

1. Introduction
A characteristic feature of the general circulation of planetary atmospheres is the presence of polar vortices and/or circumpolar jets. They clearly appear in the data after the flow fluctuations are removed by time and/or zonal (longitudinal) averaging. As such, the polar vortices obey the laws of hydrodynamics, and can be subject to hydrodynamic instabilities according to well-known instability criteria. A surprising property of the polar vortex in the winter hemisphere on Mars is that it keeps its annular form for the long time, while according to the above-mentioned criteria, this structure is unstable. In order to understand this apparent paradox, we employ the simplest atmospheric model which is obtained by vertical averaging of the adiabatic ”primitive” equations of the atmosphere. We, however, ”improve” the model by adding radiative cooling and convective fluxes due to the phase transitions of $CO_2$, which is the major constituent of the Mars atmosphere. The approach we are using was successfully tested in modelling atmospheric vortices [1], [2], and recently allowed to understood the peculiar form of the Saturn’s North pole jet plus vortex system [3]. It consists, first, in analysing stability properties of the vortex configuration with the help of the adiabatic version of the model. Once unstable modes are identified, they are used to initialize numerical simulations with a high-resolution finite-volume code, and compare adiabatic saturation of the instability to the diabatic one, with inclusion of convective fluxes, as explained in [4].

2. The model in adiabatic approximation, and the vortex configuration
The rotating shallow water (RSW) equations are obtained from the adiabatic atmospheric primitive equations by vertical averaging between material surfaces in so-called pseudo-height
pressure coordinates in the absence of forcing and dissipation, and under hypothesis of horizontally constant potential temperature. Although spherical coordinates are natural to use for large-scale atmospheric motions, we will work with cylindrical coordinates on the polar tangent plane. The RSW equations thus read:

\[
\frac{Dv}{Dt} + \left(f + \frac{v}{r}\right)(\hat{z} \times v) = -g \nabla H h, \quad \frac{Dh}{Dt} + h \nabla H \cdot v = 0.
\]  

(1)

Here \((r, \theta)\) are the coordinates in the plane, \(\hat{z}\) is the unit vector normal to the plane, \(v = u\hat{r} + v\hat{\theta}\), \(g\) is gravitational acceleration, \(f\) is the Coriolis parameter, and \(h\) is thickness of the layer. \(D/Dt = \partial_t + \vec{v} \cdot \nabla\) is the material derivative, and \(\nabla H = \hat{r} \partial_r + \hat{\theta} (1/r) \partial_\theta\).

Equations (1) have exact solutions verifying the conditions of cyclo-geostrophic equilibrium, i.e. equilibrium between the pressure, and centrifugal and Coriolis forces, which link azimuthal velocity \(V(r)\) to the thickness \(H(r)\) of the vortex.

We start with the observed mean velocity profile of the Mars’ polar vortex, and make a simple analytic fit of it. We then reconstitute corresponding pressure distribution via cyclo-geostrophic equilibrium, and analyse stability properties of thus obtained solution of (1). The radial distribution of the mean azimuthal velocity obtained by vertical averaging of the MACDA data \[5\], together with our fit, are presented in the upper left panel of Figure 1. We use \(f^{-1}\) as a unit of time, and scaling all lengths with the deformation radius \(L_d = \sqrt{g H_0 / f}\), where \(H_0\) is the equivalent depth of the atmospheric layer.

3. Linear stability analysis of the Martian polar vortex

Linearising (1) about the above-described vortex, and looking for solutions of the linearised equation in the form of azimuthal harmonics \(\propto e^{i(l\theta - \omega t)}\), leads to a linear eigenproblem for the frequencies \(\omega\). Eigenvalues with non-zero imaginary part correspond to unstable modes. The resulting eigenproblem is discretised in radial direction on a stretched Chebyshev grid, and solved by the pseudo-spectral collocation method. The linear stability analysis shows that the Martian polar vortex is unstable. The results are presented in Figure 1, where the typical structure of the unstable mode is displayed, together with dependence of the growth rates on \(l\). The most
unstable mode, thus, has azimuthal wavenumber \( l = 3 \). This mode will be used for initialisation of nonlinear simulations below.

**4. Improving the model with diabatic effects**

As already said we are interested in nonlinear saturation of just displayed instability. The previous studies [1], [2], [3], however, show that diabatic effects, which are absent in (1) can be important. We, thus, improve the mode by including them. The first, and generic for the atmospheres, diabatic effect is thermal relaxation to an equilibrium state through radiative cooling/heating. In terms of pressure it may be represented as a relaxation of the pressure to the equilibrium value \( H(r) \). Hence

\[
\frac{Dh}{Dt} + h \nabla H \cdot \mathbf{v} = -\frac{h - H(r)}{\tau_r}. \tag{2}
\]

The relaxation time \( \tau_r \) is of the order of one Martian (solar) day.

The second diabatic effect is specific to Mars, with its almost pure CO\(_2\) atmosphere. At winter temperatures at the pole, CO\(_2\) undergoes a gas - solid deposition phase transition, if pressure (i.e. \( h \) in the present context) is high enough. The deposition process leads to diminishing the mass of gaseous CO\(_2\), and can be thought of as a relaxational sink in the equation for \( h \) of the form

\[
\frac{h - h_{th}}{\tau_c} H(h - h_{th}),
\]

where \( h_{th} \) is a threshold value over the equilibrium pressure profile, and \( \tau_c \ll \tau_r \). To be efficient the deposition needs nuclei, which are provided by dust particles. We can consider that deposition starts when the bulk number of nuclei in the atmospheric column \( N \) reaches some threshold \( N_{th} \), and increases with the number of nuclei. Hence, for the sink in the \( h \) equation we get

\[
C = \frac{h - h_{th}}{\tau_c} H(h - h_{th})(N - N_{th})H(N - N_{th}). \tag{3}
\]

The dust particles are simply advected by the flow. Upon vertical averaging, this gives the conservation equation (we neglect the changes in \( N \) due to deposition):

\[
\frac{DN}{Dt} + N \nabla H \cdot \mathbf{v} = 0.
\]

At the same time, the deposition process leads to latent heat release, and related vertical convective flux. This flux also acts as a sink of \( h \), cf [4], and is proportional to \( C \). Hence, the resulting equation for \( h \) combining thermal relaxation and deposition effects, is:

\[
\frac{Dh}{Dt} + h \nabla H \cdot \mathbf{v} = -\frac{h - H}{\tau_r} - \beta C, \tag{4}
\]

where the coefficient \( \beta \) can be incorporated in \( \tau_c \).

**5. Nonlinear saturation of the instability. Adiabatic vs diabatic scenarios**

Nonlinear saturation of the instability is studied by direct numerical simulations initialised with the main vortex configuration with superimposed most unstable mode of weak amplitude (several per cent). Simulations are made with finite-volume numerical scheme used in the above-cited references. A useful diagnostics of the outputs of such simulations is provided by potential vorticity (PV) \( q = \zeta + \frac{f}{r} \), which is a Lagrangian invariant of the adiabatic system. Here \( \zeta = \partial_x v - \partial_y u \) is relative vorticity. We present in Figure 2 a comparison of adiabatic and diabatic saturations of the instability in terms of PV. A pronounced difference between adiabatic and diabatic saturations is that, while in the former the vortex loses its annular structure, it keeps it in the latter. The \( l = 2 \) component is manifest in the diabatic evolution with thermal relaxation (2nd row in the Figure). The vortex structure at the late stages in this simulation is in striking resemblance with the data (cf. [5]). As follows from Figure, the diabatic evolution with
Figure 2. Evolution of PV of the vortex perturbed by the most unstable mode in adiabatic - 1st row, diabatic with thermal relaxation only - 2nd row, diabatic with deposition without thermal relaxation - 3rd row, diabatic with thermal relaxation and deposition - 4th row, simulations. Dashed line indicates initial radius of maximum wind.

deposition and without thermal relaxation also maintains the annular structure of the vortex, but rather favours appearance of the $l = 3$ mode, and not of the $l = 2$ one. Yet, the zones of enhanced PV in the annulus, which are typical for the Martian polar vortex, cf. [5], but absent without deposition, appear.

6. Summary and discussion
Thus, the simplest possible vertically integrated model of Martian atmosphere allows, after inclusion of roughly parametrised effects of radiative cooling and deposition phase transition, to understand some salient features of Martian winter polar vortex, such as maintenance of annular structure, loss of axial symmetry, and appearance of zones of enhanced potential vorticity. Such crude representation of dynamics and thermodynamics of Martian polar atmosphere can, obviously, give only a first idea of the dynamical role of diabatic effects, which is nevertheless to be born in mind in more detailed modelling.

References
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