**Kepler Microlens Planets and Parallaxes**

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**ABSTRACT**

*Kepler*’s quest for other Earths need not end just yet: it remains capable of characterizing cool Earth-mass planets by microlensing, even given its degraded pointing control. If *Kepler* were pointed at the Galactic bulge, it could conduct a search for microlensing planets that would be virtually non-overlapping with ground-based surveys. More important, by combining *Kepler* observations with current ground-based surveys, one could measure the “microlens parallax” \(\pi_E\) for a large fraction of the known microlensing events. Such parallax measurements would yield mass and distance determinations for the great majority of microlensing planets, enabling much more precise study of the planet distributions as functions of planet and host mass, planet-host separation, and Galactic position (particularly bulge vs. disk). In addition, rare systems (such as planets orbiting brown dwarfs or black holes) that are presently lost in the noise would be clearly identified. In contrast to *Kepler*’s current primary hunting ground of close-in planets, its microlensing planets would be in the cool outer parts of solar systems, generally beyond the snow line. The same survey would yield a spectacular catalog of brown-dwarf binaries, probe the stellar mass function in a unique way, and still have plenty of time available for asteroseismology targets.

*Subject headings:* gravitational lensing: micro — planetary systems

1. **Introduction**

The *Kepler* satellite has found more than 3000 planetary candidates, the overwhelming majority of which are real planets ([Batalha et al., 2013](https://www.jpl.nasa.gov/cgi-bin/nph-read.arcabs?dset=mnras2013&B=1&J=471&F=3&pg=976&pr=0)). To give one example of the new parameter space probed, *Kepler* has discovered 231 “Earth-radius” planets (within 25% of

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Earth’s radius). To date *Kepler* has detected planets only by the transit method, and as a result it is highly biased toward close-in planets. For example, the median period of the “Earth-radius” sample is 5.2 days, and the maximum period is 69 days.

Here we propose to apply *Kepler* to characterizing much colder planets in the outer parts of their solar systems, using the microlensing technique (Gaudi 2012). We show that although *Kepler* is not optimally designed for this task, it can be competitive with existing and under-construction ground-based surveys in terms of finding planets.

However, what *Kepler* would add that is fundamentally new would be microlens parallaxes for a large fraction of microlensing events, including almost all of those with planetary signals (whether detected by *Kepler* or from the ground). In the great majority of cases, such parallax measurements would enable determination of the host mass and distance, and thus also the planet mass, which would greatly enhance the value of both groups of planets.

The requirements of a *Kepler* microlensing survey are well-matched to the limitations on its performance due to loss of pointing stability. In order to be an effective transit-search tool, *Kepler* had to monitor $\sim 10^5$ stars. Given data-transmission constraints, this implied relatively long (30 min) integrations on each star, which in turn required high pointing stability. However, to be an effective microlensing-planet tool, it need only observe $\sim 10^3$ stars. Hence the same data-transmission constraints are compatible with much shorter exposures.

The photometric requirements of microlensing planet searches are very different from *Kepler’s* transit survey. Planetary deviations are typically tens of percent, compared to $\lesssim 1\%$ for transits. However, the source stars are much fainter, typically $18 \lesssim I \lesssim 16$, compared to $V \lesssim 16$ for the transit survey. Microlensing events typically last a few weeks to months. They are usually quickly identified from the ground, but *Kepler* would have to be notified of these identification to conduct its search. Planetary deviations due to Jupiter-mass planets typically last one day, while those due to Earth-mass planets typically last about one hour. Hence somewhat shorter cadences are needed than *Kepler’s* traditional 30 min in order to get full sensitivity to the lowest-mass planets.

The photometric requirements for microlensing parallax measurements are substantially less restrictive than for finding planets because the parallax signal extends over the entire event, not just a few hours or days. This is important: it means that even if the photometric challenges prove too difficult to find a large number of planets on its own, *Kepler’s* main contribution of precise characterization of ground-based planets can remain intact.
2. Observation Strategy

At present, roughly 2000 microlensing events are discovered per year by the Optical Gravitational Lens Experiment (OGLE) and Microlensing Observations for Astrophysics (MOA) collaborations. The overwhelming majority of these are found in a region that could fit in a single pointing of the 105 deg² *Kepler* camera. Thus, the first element of the strategy would be simply to point *Kepler* at this field, when permitted by its 55° Sun exclusion angle. Whenever a new microlensing event was found (from Earth), it would be added to the list of *Kepler* targets. Most events are detected at least several days before they do anything interesting, so such “uploads” of new targets could be grouped in batches, if necessary. Microlensing events could also be removed from the list when they returned to baseline.

We note that *Kepler* is in a $P = 372.5\text{ d}$ orbit and so drifting behind Earth at $7.2^\circ\text{ yr}^{-1}$ and hence is now roughly 1 month (0.5 AU) behind Earth. This is an excellent position to create a large baseline for “parallactic viewing” while still having a strongly overlapping “bulge season” with Earth.

3. Unique Impact: Microlensing Parallaxes

The observational strategy outlined above would accomplish two aims: measure the “microlens parallax” of a large fraction of events and detect planets in a subset. We will argue below that the planet-finding capability is comparable but not qualitatively superior to ground-based capabilities. Hence, we focus first on what is unique about a *Kepler* microlensing survey: parallaxes.

3.1. What is microlensing parallax, $\pi_E$?

The magnitude of the microlens parallax, $\pi_E$ is simply the lens-source relative parallax, $\pi_{\text{rel}}$, scaled to the angular *Einstein* (1936) radius $\theta_E$:

$$\pi_E = \frac{\pi_{\text{rel}}}{\theta_E}; \quad \theta_E^2 = \kappa M \pi_{\text{rel}}; \quad \kappa \equiv \frac{4G}{c^2 \text{AU}} = 8.1 \frac{\text{mas}}{M_\odot}$$  \hspace{1cm} (1)
3.2. Parallax: Rosetta Stone for microlensing planets

The significance of a parallax measurement is that if \( \theta_E \) is also measured, then one can immediately derive \( M \) and \( \pi_{\text{rel}} \),

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M = \frac{\theta_E}{\kappa \pi_E}, \quad \pi_{\text{rel}} = \frac{\text{AU}}{D_L} - \frac{\text{AU}}{D_S} = \theta_E \pi_E, \quad (2)
\]

Since the source distance \( D_S \) is usually known quite well, measuring \( \pi_{\text{rel}} \) immediately gives the lens distance \( D_L \).

While in general it is quite difficult to measure \( \theta_E \), such measurements are almost always possible in planetary lensing events. This is because the source must pass over or near a “caustic” caused by the planet if the planet is to be detected. The lightcurve deviation is therefore a function of \( \rho \equiv \theta_*/\theta_E \), where \( \theta_* \) is the angular source size, which means that \( \rho \) can almost always be measured from the lightcurve of planetary events. Since \( \theta_* \) can be routinely measured from the source color and magnitude (Yoo et al. 2004), \( \theta_E = \theta_*/\rho \) can also be measured.

Hence, microlens parallax is a Rosetta Stone for planetary microlensing events, turning what was initially thought to be a purely statistical technique (Gould & Loeb 1992) into individual planet-mass and distance measurements.

3.3. How is microlens parallax measured?

To date, the overwhelming majority of microlens parallax measurements have relied on observing lightcurve deviations induced by the accelerated motion of Earth during the event (Gould 1992; Alcock et al. 1995; Poindexter et al. 2003). However, because most microlensing events are short compared to the time required for Earth to move a radian (yr/2\( \pi \sim 58 \text{ d} \)), such “orbital” parallax measurements are quite rare. Another approach is to simultaneously observe the event from two locations on Earth (Hardy & Walker 1995; Holz & Wald 1996), but since the projected Einstein radius \( \bar{r}_E \equiv \text{AU}/\pi_E \) is typically several AU, this is only practical for extreme magnification events \( A \gtrsim 1000 \) (Gould 1997), and in fact there are only two such cases (Gould et al. 2000; Yee et al. 2009).

Therefore, the only method that can routinely return microlens parallaxes is to combine...
observations from a satellite at $\mathcal{O}(\text{AU})$ from Earth and so enable simultaneous observations from two locations separated by a distance that is comparable to $\tilde{r}_E$ (Refsdal 1966; Dong et al. 2007).

Kepler therefore possesses two tremendous advantages for a microlens parallax survey: it is already in solar orbit and it can observe essentially all ongoing microlensing events simultaneously.

3.4. Parallax degeneracies

However, it also faces challenges. Some of these are related to its relatively large point spread function (PSF), which we will discuss below. But one challenge is rooted in the nature of space-based parallax measurements: degeneracy. As already noted by Refsdal (1966) and discussed more thoroughly by Gould (1994), space-based parallax measurements are subject to a four-fold discrete degeneracy. Basically, Earth and satellite see the same event, but displaced in the Einstein ring, and so having different peak times $t_0$ and different impact parameters $u_0$. The microlens parallax, is then given essentially by

$$\pi_E = \frac{\text{AU}}{D_{\perp,\text{sat}}(\Delta \tau, \Delta \beta)}; \quad \Delta \tau \equiv \frac{t_{0,\text{sat}} - t_{0,\oplus}}{t_E}; \quad \Delta \beta \equiv u_{0,\text{sat}} - u_{0,\oplus},$$

where $D_{\perp,\text{sat}}$ is the Earth-satellite separation projection onto the plane of the sky and $t_E$ is the Einstein timescale. The problem is that while $t_0$ is uniquely determined from the lightcurve, $u_0$ is a signed quantity whose magnitude is measured but not its sign. Thus, $\pi_E$ can take on four values depending on the signs of $u_0$ as seen from Earth and the satellite. However, since the mass depends only on the magnitude of $\pi_E$, only a two-fold degeneracy is really of major interest. That is, do $u_{0,\text{sat}}$ and $u_{0,\oplus}$ have the same or opposite signs? Or, equivalently: is the source seen projected on the same or opposite side of the lens as seen from the two observatories? See Figure 1 and also Figures 1 and 2 from Gould (1994).

Gould (1995) showed that this degeneracy could be broken because the timescales of the events as seen from Earth and the satellite are slightly different, and this difference is a function of $\Delta \beta$. Gaudi & Gould (1997) then investigated how well this degeneracy could be broken for events seen toward the Galactic Bulge. Their assumptions were far more conservative than those likely to apply to Kepler observations. First, they considered a narrow-angle pointed mission (rather than a wide-angle survey), in which the observations of each target would be limited to a relatively few epochs, whereas Kepler observations would be continuous. Second, at the time it was believed that the source flux in the space-filter could not be accurately determined from the ground-based lightcurve, whereas subsequently
Yee et al. (2012) have shown that this indeed is possible to at least 1% precision. See also Gould (2013) and Yee (2013).

Undoubtedly, there will be microlensing events discovered that are so faint that the parallax degeneracy will not be broken. However, few planets are likely to be found in such faint events. Moreover, depending on the geometry of the event, it is sometimes not necessary to actually break the degeneracy to derive good mass estimates (e.g., if $|\Delta \tau| \gg |\Delta \beta|$).

4. **Planetary Science with *Kepler* Microlens Parallaxes**

At present, most microlensing planet detections return $\theta_E$ and hence the product $M\pi_{\text{rel}} = \theta_E^2/\kappa$, but not the mass and distance separately. Hence, these quantities are estimated only statistically for most events. The estimates make use of Galactic models together with various pieces of information, such as the geocentric lens-source relative proper motion $\mu = \theta_E/t_E$ and upper limits on the lens flux from blended light. But generally these estimates are accurate to only a factor of two, and of course can be radically incorrect in cases of unusual or unexpected systems. In particular, there is only one planet out of about 30 detected to date that is known to be in the Galactic bulge with good confidence, even though the majority of lenses are in the bulge. Hence, it is very difficult to disentangle the distributions of planets as functions of controlling properties, such as planet mass, host mass, distance from host, and Galactic position.

In one fell swoop, *Kepler* could resolve all of these uncertainties, and it could do so for the several dozen planets per year that will be discovered with current and in-construction experiments. For example, standard core-accretion theory predicts a dip in the planet mass function between Neptunes and Jupiters. By sharpening the mass resolution of microlens planets, *Kepler* could directly test this prediction for ice and gas giants found beyond the snow line, which is presumably their birth place.

Not only would this increased precision be of direct use in better understanding the planets that microlensing is discovering, it would also put them “on the same playing field” as the planets, mostly much closer to their hosts, discovered by other techniques.

5. **Kepler Cold Planets**

In addition to measuring microlens parallaxes, *Kepler* observations will probe a virtually independent set of microlensing planets from those detected from the ground. This is because it is displaced from Earth by $D_{\perp,\text{sat}}/\tilde{r}_E$ in the Einstein ring, which will typically be of order
10%. Since planetary perturbations are usually much smaller than this, planets detected from Earth will generally not be detected by Kepler and vice-versa.

Here, we estimate the general competitiveness of Kepler relative to the Korea Microlensing Telescope Network (KMTNet, Poteet et al. 2012) which is the largest ground-based microlensing experiment currently under construction. KMTNet will have three 1.6m telescopes, each with a 4 deg² field of view, located in Chile, South Africa, and Australia. Like Kepler, therefore, it is in principle capable of near-continuous coverage for the fraction of the year when the Sun is well away from the bulge. KMTNet will cycle through four fields, observing each for 2 min out of 10. Considering weather at these sites, it will have a combined duty cycle of perhaps 2/3. In addition, Kepler has a “white-light” response compared to I-band filters that are needed from the ground. Taking account of the increase in both signal and noise implies a factor 1.6 advantage. Altogether, these factors give Kepler an advantage of a factor 12.

However, Kepler has disadvantages as well, and these are overall stronger. First, its aperture is smaller by a factor 1.6². Most important, its PSF is much larger. At best, the FWHM ~ 3.1′′, whereas average KMTNet seeing is likely to be ~ 1.2′′. Since almost all photometry is likely to be below sky in either case, these two disadvantages combine to a factor (1.6×3.1/1.2)² ∼ 17. Finally, the problems posed by field drift are difficult to estimate without detailed simulations. The exposures can be made short enough that this drift does not affect individual images, but the undersampled PSF, in conditions of crowded bulge fields is likely to increase the photometric noise beyond the above naive calculation. Thus, Kepler will find fewer planets within the 16 deg² probed by KMTNet, which contain the richest planet hunting ground. By the same token, of course, a Kepler microlens planet search would fall far short of one by WFIRST (Green et al. 2012; Spergel et al. 2013). However, Kepler will also find planets in outlying regions, which are being surveyed by OGLE and MOA at lower cadence. And, as emphasized above, virtually all the planets that it does find will be undetected from the ground.

Nevertheless, this calculation shows that the planets found by Kepler are not reason enough, by themselves, for it to do a microlensing survey. Moreover, since it will be looking only at events found by others, it will be useless for finding free-floating planets (FFP), which are a unique capability of microlensing (Sumi et al. 2011). Rather, its principal value is to

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³It will, however, be useful for vetting the main contaminant of the FFP signal, stellar microlensing events whose timescales are exceptionally short due to small πrel despite high mass M. If FFP events can be alerted to Kepler within ~ 1d, they will appear similarly for Kepler because their parallaxes will be small πE = (πrel/κM)¹/². However, if they are due to planets, then Kepler will see no event at all because the large parallax puts the source well outside the Einstein ring from Kepler’s perspective.
obtain microlens parallaxes, which would enormously enhance the value of planets detected in ground-based surveys.

6. Other Microlensing Applications

Microlensing surveys are also a powerful probe of binaries, in particular low-mass binaries that are difficult or impossible to detect by other methods. For example, Choi et al. (2013) discovered two brown dwarf binaries, which obeyed the binding-energy floor found previously using standard brown-dwarf search techniques, but at much lower mass and tighter separation. That these binaries yielded mass measurements (and so could even be recognized as brown dwarfs, not stars) was only due to the fact that they were unusually nearby (few kpc) and so had large, easily measurable parallaxes. Like planetary events, binary events routinely yield $\theta_E$, so that *Kepler* microlens parallaxes would give masses and distances for all binaries, and so sift out these brown-dwarf binaries, which are otherwise generally unrecognizable. Moreover, binaries, in contrast to planets, would often be detected by both *Kepler* and ground observatories, which would provide detailed information on their orbits. Finally, while the point-lens events would not generally yield $\theta_E$ (and so masses), their mass function could be studied statistically from a *Kepler* microlens parallax survey (Han & Gould 1995).

7. Non-microlensing Applications

The number of microlensing targets to be observed is not large, at most 2000 in a season, and not all must be observed all season. The exposure times must be fairly short because of *Kepler*’s degraded pointing stability, but it is unlikely that they need to be 100 times shorter than *Kepler*’s traditional 30 min exposures. From a microlensing standpoint, there are no drivers for exposures shorter than about 5 minutes. Thus, it is likely that microlensing targets will absorb only a small fraction of the available data-transmission capability. Other objects, such as bright asteroseismology targets could therefore be observed. In particular, since short exposures are needed due to stability problems, one could target bright dwarfs, which have higher-frequency oscillations than the giant-star targets on which *Kepler* has concentrated to date.

We thank Scott Gaudi and Jennifer Yee for stimulating discussions. Work by AG was supported by NSF grant AST 1103471 and NASA grant NNX12AB99G. KH is supported by a Royal Society Leverhulme Trust Research Fellowship.
REFERENCES

Alcock, C., Allsman, R.A., Alves, D., et al. 1995, ApJ, 454, L125
Batalha, N.M. Rowe, J.F., Bryson, S.T., et al., ApJS, 204, 24
Choi, J.-Y., Han, C., Udalski, A., et al. 2013, ApJ, 768, 129
Dong, S., Udalski, A., Gould, A., et al. 2007, ApJ, 664, 862
Einstein, A. 1936, Science, 84, 506
Gaudi, B.S. 2012, ARA&A, 50, 411
Gaudi, B.S. & Gould, A. 1997, ApJ, 477, 152
Gould, A. 1992, ApJ, 392, 442
Gould, A. 1994, ApJ, 421, L75
Gould, A. 1995, ApJ, 441, L21
Gould, A. 1997, ApJ, 480, 188
Gould, A. 2004, ApJ, 606, 319
Gould, A. 2013, ApJ, 763, L35
Gould, A. & Loeb, A. 1992, ApJ, 396, 104
Gould, A., Udalski, A., Monard, B. et al. 2009, ApJ, 698, L147
Green, J., Schechter, P., Baltay, C. et al. 2012, Wide-Field InfraRed Survey Telescope (WFIRST) Final Report, arXiv:1208.4012
Han, C. & Gould, A. 1995, ApJ, 447, 53
Hardy, S.J. & Walker, M.A. MNRAS, 276, L79
Holz, D.E. & Wald, R.M. 1996, ApJ, 471, 64
Poteet, W.M., Cauthen, H.K., Kappler, N., Kappler, L.G., Park, B.-G., Lee, C.-U., Kim, S.-L., Cha, S.-M. 2013, SPIE, 8444, 5
Poindexter, S., Afonso, C., Bennett, D.P., Glicenstein, J.-F., Gould, A, Szymański, M.K., & Udalski, A. 2005, ApJ, 633, 914
Refsdal, S. 1966, MNRAS, 134, 315

Sumi, T., Kamiya, K., Bennett, D.P., et al. 2011, Natur, 473, 349

Spergel, D., Gehrels, N., Breckinridge, J., et al. 2013, Wide-Field InfraRed Survey Telescope-Astrophysics Focused Telescope Assets WFIRST-AFTA Final Report, arXiv:1305.5422

Yee, J.C. 2013, ApJ, in press, arXiv1303.6957

Yee, J.C., Udalski, A., Sumi, T., et al. 2009, ApJ, 703, 2082

Yee, J.C., Shvartzwald, Y., Gal-Yam, A., et al. 2012, ApJ, 755, 102

Yoo, J., et al. 2004, ApJ, 603, 139
Fig. 1.— Illustration of four-fold degeneracy derived from comparison of Kepler and ground based lightcurves. Upper panel shows two possible trajectories of the source relative to the lens for each of Kepler (red) and Earth (blue) observatories. Each set would give rise to the same point-lens lightcurve in the lower panel (same colors), leading to an ambiguity in the Earth-Kepler separation (distance between red circle and blue square) relative to the Einstein ring. In this particular case, the planet causes deviations to both lightcurves (green), thus proving that the trajectories are on the same side of the Einstein ring. More generally, the planet would appear in only one curve, leaving the ambiguity open. In this case, it would be resolved by more subtle differences in the Einstein timescale. See Gould (1994, 1995).