CONTROL OF PREY DISEASE IN STAGE STRUCTURE MODEL

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ABSTRACT

In this paper, a mathematical model consisting of the prey-predator model, prey is at risk of disease then become as susceptible and infected, while predator with different stage structure: immature and mature predator, the infected prey is at risk recover and harvest. The function of disease is proportionality function. At the beginning, the reasons of studying stage structure and the dynamic of nontrivial subsystems that may be lead to coexistence of these types of spieces explain and by using Maple software, Jacobean matrix, Routh-Hurwitz criterion, Bendixson-Dulac criterion and Lyapunov function to prove the existence, periodic solution, local and global stability. We concluded that the survival for two preys are possible through the non-periodic solution due to the Bendixson-Dulac criterion, also the immature predator neither attack preys nor yield offspring's and die when the mature predator extinction, the global stability conditions for the original system be stretch of global stability conditions for subsystems. Finally, Mathematica software employs to describe some results in numerical simulation.

1. Introduction

In classical models it is assumed that all populations have the same ability and this unrealistic in some times. Therefore, life history introduced by using stage structure models that classified into two categories: mature and immature individuals' because there are many different between them in age, power, body size, activity of some diseases in designated stage, the perfect ways of hunting, etc. As examples big fish differ in death rates from small fish, Alzheimer's disease, poliomyelitis, etc. In this chapter mature predator take his responsibility to research on food and reproductive ability while the immature predator does not yield offspring's and depend on mature predator in his food, we assume that the society of prey populations suffer from disease that transmitted by contact between them which is susceptible, infected individual, mature attack the weakly prey which is easy to catch according to Lotka-Volterra function and converts from susceptible preys only to immature according to Michaelis-Menten type. To make the model more realistic, use a suitable option for prevention, coexistence of the population which is known as Harvesting and SIS model, which means healing from disease with incomplete immunity as in cholera, influenza, etc. Another important and realistic factor is a disease which occurs throughout infection by some bacterial, various, or micro parasites. A disease spreading in interacting populations was first studied by [1]. [2] were studied a model for single stage structure. After that, two species models with stage structure was developed by [3]. [4] studied stage structure with infected Prey. While, [5] studied the effect of harvesting on predator with stage structure and establish the conditions for existence of a global stability for the positive equilibrium point, stage structure for preys in Prey-predator model was investigated by [6], also there are many studies on stage structure with cannibalism phenomenon prevalent in the tribes of South America, India, which occurs when prey and predator of the same species as [7].

The rest of paper is categorized in the following manner. In the next section, we introduced the model. The sufficient feasibly conditions for the existence with strategy of proofs utilizes for Boundedness and positivity of the solutions, local and global stability such as maple program-ing, periodic orbit, Routh Hurwitz criteria and Lyapunov function for some points of these systems was in sections three, four and five. In section six, meaning of threshold of stability explained. Section seven perform numerical
simulations. After that, the last section have some theoretical results on this new biological model.

2. Mathematical Model
An SIS model with stage structure for predators and linear harvesting for diseases that spreads horizontally in prey population takes the following form:

\[
\begin{align*}
\frac{dx}{dt} &= r x (1-x) - c \frac{xi}{x+i} + \delta i - \kappa_1 xz + \mu x, \\
\frac{di}{dt} &= c \frac{xi}{x+i} - \delta i - \alpha_2 iz - d_i i - qi, \\
\frac{dy}{dt} &= \kappa_2 \frac{xz}{1+\mu x} - (D + d_z) y, \\
\frac{dz}{dt} &= Dy + \alpha_3 iz - d_z z.
\end{align*}
\]  

(2.1)

This system consists four types of populations: susceptible prey \(x(t)\), diseased (infected) prey \(i(t)\), immature predator \(y(t)\) and mature predator \(z(t)\) at any time \(t\) respectively, where \(x', y', z'\geq 0\), all parameters are positive constants, \(r\) is the intrinsic growth rate of the susceptible prey, \(\mu\) represents the search rate multiplied by the handling time of the mature predators, \(\kappa_2\) represents the conversion coefficient of the mature predators, \(d_i, d_z\) and \(d_r\) are the death rate of the infected prey, immature and mature predators respectively, \(D\) represents the rate at which immature predator become mature, \(q\) is the rate of harvesting of infected prey, the function of disease is proportionality function \(\frac{ci}{x+i}\) where \(c\) is the transmissi-on rate coefficient, only mature predators can attack preys and eats the infected prey with Holling type 1 functional response \(iz\), \(\alpha_2\) is the rate of predation and \(\alpha_3\) is the conversion rate of predation of infected prey, while eating the susceptible prey with Michaela's Menten-Holling type functional response \(\frac{xz}{1+\mu x}\) and give to immature predators from its since they are neither predate nor reproductive ability, \(\kappa_2\) is the research rate of mature predator. Finally, \(\kappa_2\) is the conversion coefficient of mature predator.

3. Natural Solution

**Theorem 1:** All solutions of system (2.1) in \(R^4_+\) are uniformly and bounded.

**Proof:** Let \(N(t) = x(x) + i(t) + y(t) + z(t), \mu > 0\)

\[
\frac{dN}{dt} + \mu x = \frac{dx}{dt} + \frac{di}{dt} + \frac{dy}{dt} + \frac{dz}{dt} + \mu x,
\]

then

\[
\frac{dN}{dt} + \mu x = rx (1-x) - c \frac{xi}{x+i} + \delta i - \kappa_1 xz + \mu x
\]

\[-(\alpha_2 iz - qi - d_i i - d_z z + \mu x),
\]

Since \(\kappa_1 > \kappa_2\) and \(\alpha_2 > \alpha_1\), by [7].

Hence, \(\frac{dN}{dt} + \mu x \leq -rx^2 + (r + \mu)x, \frac{dN}{dt} \leq \mu (r + \mu)^2\), say \(w\). Then,

\[
0 < N(x(t),i(t),y(t),z(t)) \leq \frac{w}{\mu} (1-e^{-\mu t}) + (x,i,y,z)e^{-\mu t}, \forall t > 0.
\]

**Theorem 2:** In system 2.1, \(c > \delta + d_i + q\).

**Proof:** Suppose that \(c \leq \delta + d_i + q\), then \(\frac{dx}{dt} \leq 0\) that lead to negative equation, which is contradiction, then \(c > \delta + d_i + q\).

4. The Dynamic of Subsystems
There are many nontrivial subsystems of system (2.1). These subsystems occurs in the absence of one or two populations. The extinction of species in a particular environment is a natural and realistic command due to the inability of organism on adapt to the new circumstances such as adapting to the relations of competition and predation, overexploitation of it or its resources, the environmental pollution such as toxic chemicals, fertilizers and pesticides, water pollution, high or exhaustion water, spread of diseases, etc. Which lead to the death of many of them or migration to another environment in search of more favorable circumstances. Therefore, we investigate these subsystems as follows:

4.1. The Absence of Predators
When the predators-extinction, then model (2.1) consists of prey population only that grows to the maximum carrying capacity but spread of diseases with proportionality function in prey population isn’t good command, therefore we used harvesting to control on it as follows:

\[
\begin{align*}
\frac{dx}{dt} &= r x (1-x) - c \frac{xi}{x+i} + \delta i, \\
\frac{di}{dt} &= c \frac{xi}{x+i} - \delta i - d_i i - qi.
\end{align*}
\]  

(4.1)

4.1.1 Natural Solution

**Theorem 3:** All solutions of (4.1) are positive and bounded.

**Proof:** As in theorem 1, see figure 1:

Fig. 4.1: Subsystem (4.1) has no periodic orbit in \(R^2_+\)

**Theorem 4:** Subsystem (4.1) has no periodic orbit in \(R^2_+\).
Proof: To show there is no periodic orbit to this subsystem, we use Dulac’s criterion and first consider the \( x = 1 \) plane. Let

\[
M(x, i) = \frac{1}{x}, \quad g_i(x, i) = \kappa (1 - x) - c(x^i \frac{x}{x + i} + \delta i)
\]

g_i(x, i) = c\frac{x^i}{x + i} - \delta_i - d_i - q_i.

Clearly, \( M(x, i) \) be a positive in the interior of the positive quadrant of \( x = 1 \) plane. So we have

\[
\begin{align*}
M(x, i) &= \frac{r(1-x)}{i} - c\frac{x}{x + i} + \delta \quad \text{and} \\
g_i(x, i) &= c\frac{x^i}{x + i} - \delta_i - d_i - q_i.
\end{align*}
\]

Then

\[
\Delta(x, i) = \frac{\partial g_i(M)}{\partial x} - \frac{\partial (g_i(M))}{\partial t} = -r - \delta.
\]

It is brought to light that \( \Delta(x, i) \) remains with the same sign, therefore this point cannot have any periodic solution in the \( x = 1 \) plane.

4.1.2 Equilibrium Points and Stability:

To discuss the equilibrium points of subsystem (4.1), taking \( \frac{dx}{dt} = 0 \) and \( \frac{di}{dt} = 0 \) are equal to zero, then we get three equilibrium points as:

1. Vanishing equilibrium point \( P_{i,3}(0, 0) \) always exists.
2. Infection-free equilibrium point \( P_{i,3}(1, 0) \) always exists.
3. Interior equilibrium point \( P_{i,3}(\xi, \iota) \) exists, means the two preys survive in the absence of predators, where \( \xi = \frac{i}{c} (\delta + d + q) \) and \( \iota = \frac{r}{c} (1 - \xi) \) exists in \( R^2 \) of \( XI - \) plane. To study the stability of these points, use the following Jacobian matrix

\[
J_i = \begin{bmatrix}
-2\kappa - c & i^2 \\
\delta - c\frac{x^2}{x + i} & \frac{c}{x + i} \\
\end{bmatrix}
\]

(42)

The stability of the positive point discuss only, then the characteristic equation near \( P_{i,3}(\xi, \iota) \) is

\[
\lambda^2 + A_1\lambda + A_2 = 0,
\]

where \( A_1 = -\text{Tr}(J_{i,3}) > 0 \), if \( \xi > \frac{\sqrt{c}}{2} \).

\[
\lambda = \text{secondary diagonal of } J_{i,3} > 0, \quad \text{if } \delta < \frac{(\delta + d + q)^2}{c}.
\]

Therefore, from Routh Hurwitz criteria it's stable with above condition. Then, from these two conditions, this point is stability. In fact, here the factor of recovery of disease \( \delta \) plays an important role in the stability of this subsystem.

Theorem 5: The predator free equilibrium point \( P_{i,3}(\xi, \iota) \) is globally asymptotically stable in \( R^2 \) of \( x = 1 \) plane. Proof: The unique positive equilibrium point \( P_{i,3}(\xi, \iota) \) is locally asymptotically stable, all solutions of subsystem (4.1) are non-periodic in \( R^2 \) and uniformly bounded, hence by using Poincare-Bendixson theorem, \( P_{i,3}(\xi, \iota) \) is globally asymptotically stable.

4.2 The Absence of Infected-Prey

When the disease-extinction, then model (2.1) can be classified as an active, healthy society and reduced as follows:

\[
\begin{align*}
\frac{dx}{dt} &= \kappa_1 \frac{x}{1 + \mu_0} (1 - \frac{x}{D + d_1}) \quad \text{and} \\
\frac{dy}{dt} &= \kappa_2 \frac{x}{1 + \mu_2} (1 - \frac{x}{D + d_2}) \\
\frac{dz}{dt} &= Dz - dz.
\end{align*}
\]

4.2.1 Natural Solution

Theorem 6: All solutions of (4.4) is a positive and bounded.

Proof: As in theorem 1, and by Maple software take the following fixed point:

\[
\begin{align*}
r &= 0.915, k_1 = 0.41, \mu_0 = 0.463, k_2 = 0.269, D = 0.1235, d_1 = 0.034, d_2 = 0.161,
\end{align*}
\]

see figure 2:

![Positivity and bounded solutions](image)

Fig. 2: Positivity and bounded solutions

4.2.2 Equilibrium points and Stability

To discuss the equilibrium points, letting \( \frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0 \), then subsystem (4.4) has three equilibrium points as follows:

1. Trivial equilibrium point \( P_{i,0}(0, 0, 0) \) always exists.
2. The predator-extinction equilibrium point \( P_{i,2}(1, 0, 0) \) always exists.
3. Coexistence equilibrium point \( P_{i,2}(\xi', \iota', \zeta') \) exists, means the susceptible prey and both predators survive, where

\[
\zeta = \frac{x}{D(1 + \mu_0)(D + d_1)\mu_0} - \frac{1}{D} \quad \text{and} \\
\zeta' = \frac{x}{D(1 + \mu_2)(D + d_2)\mu_2}.
\]

With condition \( D\kappa_1 > d_1(1 + \mu_0)\mu_0 \).

Then, to study the stability of these points, use the following Jacobian matrix

\[
J_i = \begin{bmatrix}
-2\kappa - \kappa_1 \frac{z}{(1 + \mu_0)} & 0 & -\kappa_1 \frac{x}{1 + \mu_0} \\
\kappa_2 \frac{z}{(1 + \mu_2)} & -(D + d_1) & \kappa_2 \frac{x}{1 + \mu_2} \\
0 & D & -d_i
\end{bmatrix}
\]

(4.5)

Also, we will discuss the stability of the coexistence point only \( P_{i,2}(\xi', \iota', \zeta') \), then the characteristic
equation is
\[ \lambda^2 + A_x \lambda^2 + A_x \lambda + A_z = 0 \quad (4.6) \]
where \( A_x = -d_i (D + d) \left[ 1 - 2 \eta \left( \frac{1}{1 + \mu X} \right) \right] \); if \( \lambda \leq \omega \),
\[ A_x = d_i (D + d) - (D + d - d_i) \left( 1 - 2 \eta \left( \frac{1}{1 + \mu X} \right) \right) \].

Choose \( \lambda = \omega \), then
\[ A_x = \left( \frac{\omega}{1 + \mu X} \right) \left[ 1 - 2 \eta \left( \frac{1}{1 + \mu X} \right) \right] \).
Therefore, from Routh Hurwitz criteria its stable if \( A_x - A_i > 0 \).

**Theorem 7**: The interior equilibrium point \( P_{x,i}(\omega, \omega, \omega) \) is globally stable with conditions \( \rho_i \neq \rho_i \) and \( \rho_i > \rho_i \).

**Proof**: Since \( P_{x,i}(\omega, \omega, \omega) \) is locally asymptotically stable, then we define a Lyapunov function:
\[ V(x, y, z) = C_1 \left( x - \omega, \omega, \omega \right) \]
\[ V(y, z) = C_2 \left( y - \omega, \omega, \omega \right) \]
\[ V(x, z) = C_3 \left( x - \omega, \omega, \omega \right) \]

Choosing \( C_1 = C_2 = C_3 = \frac{\omega}{1 + \mu X} \), then we get
\[ \frac{dV}{dt} = -r C_1 (x - \omega, \omega, \omega) \left( \eta \left( \frac{1}{1 + \mu X} \right) \right) \]

**4.3 The Absence of Immature-Predator**
If all immature predators turns to mature, then system (2.1) become aging society for the predator population and feeding on both preys as follows:
\[ \frac{dx}{dt} = \rho \left( 1 - x \right) - c \frac{x \eta}{x + \omega} - \delta \omega \]
\[ \frac{dy}{dt} = c \frac{x \eta}{x + \omega} - \delta i \eta - \delta \omega \]
\[ \frac{dz}{dt} = \delta i \omega - d \omega \]

**4.3.1 Natural Solution**

**Theorem 8**: All solutions of (4.7) are a positive and bounded.

**Proof**: As in Theorem 1.

**Theorem 9**: If \( \rho_i \leq \rho_i \), then \( \lim_{t \to \infty} z(t) = 0 \) as \( t \to \infty \)

**Proof**: Assume that \( \rho_i < \rho_i \), then \( \rho_i < \rho_i \) and hence \( \frac{dz}{dt} < 0 \), therefore \( \lim_{t \to \infty} z(t) \) exists and nonnegative. We claim that \( \lim_{t \to \infty} z(t) = 0 \) as \( t \to \infty \). Otherwise, there is a positive constants \( B \) and \( \varepsilon \); such that \( \lim_{t \to \infty} z(t) = B > \varepsilon > 0 \), then there exist \( t_0 > 0 \); \( B - \varepsilon < z(t) < B + \varepsilon \) for some \( t > t_0 \). In addition there is a positive constant \( i_{\max} \) such that \( i_{\max} > i(t) \) for some \( t > t_0 \), then from the fourth equation of system (2.1) it follows that \( z(t) = z(t_0) e^{(\rho_i - d_i)(0)} \)
\[ z(t) = z(t_0) e^{-d_i(t_0 - t)} \] as \( t \to \infty \). Which is contradiction, then \( \lim_{t \to \infty} z(t) = 0 \).

**4.3.2 Stability of Equilibrium Points**
To study the equilibrium points, taking \( \frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0 \) and observed that subsystem (4.7) has three equilibrium points as follows:
1. Vanishing equilibrium point \( P_{x,i}(0,0,0) \) always exists.
2. Axial equilibrium point \( P_{x,i}(1,0,0) \) exists when the disease and mature predator disappear.
3. Coexistence equilibrium point \( P_{x,i}(\omega, \omega, \omega) \) exists, means the two preys and mature predator survive, where \( \omega = \frac{i (\delta + \alpha \omega + d_i + q)}{c - (\delta + \alpha \omega + d_i + q)} \), and
\[ \omega = \frac{1}{\kappa} \left( \frac{1 - \omega}{1 - \omega} \right) \]

with conditions \( r(1 - x) > (d_i + q) \) and \( c > (\delta + \alpha \omega + d_i + q) \). To discuss the local stability for these points, we used Jacobian matrix where the elements are
\[ \frac{\partial f_1}{\partial x} = r - \rho \left( x \right) - c \frac{x \eta}{x + \omega} - \delta \omega \]
\[ \frac{\partial f_2}{\partial y} = \delta \omega - \kappa \frac{x \eta}{x + \omega} \]
\[ \frac{\partial f_3}{\partial z} = \delta \omega - \delta \omega \]

However, the stability of coexistence point \( P_{x,i}(\omega, \omega, \omega) \) discuss only, then the characteristic equation is \( \lambda^2 + U_{ij} \lambda^2 + U_{ij} \lambda + U_{ij} = 0 \). Where,
\[ U_{ij} = -\left( j_{ij} + f_{ij} + f_{ij} \right) > 0 \] if \( \omega = \frac{1}{2} \)}
And

\( \frac{dx}{dt} = c \gamma - \delta - \mu x - d_y x - e y - r \delta \) \( \frac{dz}{dt} = \frac{e r - (\delta + \alpha z + d_y z)}{r (x + i)} \)

\( U_2 = j_{15} j_{25} - j_{25} j_{12} - j_{15} j_{25} j_{12} \),

\( U_1 = j_{15} j_{25} j_{12} - j_{15} j_{25} j_{12} > 0 \).

From Routh Hurwitz criteria this point is stable with above condition and if \( U_2, U_1 > 0 \).

**Theorem 10:** Assume that the coexistence equilibrium point is local stability, then it is globally stable with conditions \( x_i > x_t \) and \( x_i > x_1 \).

**Proof:** Let the function \( L(1,i,z) \) and let \( C_1, C_2 \) and \( C_3 \) are positive such that

\( L(1,i,z) + C_1 \left( x_i - \delta + \mu x_i \right) + C_2 \left( i - \delta + \mu i \right) + C_3 \left( z_i - x_i - \delta - d_q - d_q \right) \).

\( \lambda = \left( 1,0,0,0 \right) \),

\( \lambda = - \left( D + d_y \right) \) and \( \lambda = -d_y \).

Therefore, this point is a saddle since it's unstable along \( x \) direction and stable along \( i, y \) and \( z \) directions. In fact, near \( P_{125} \) the prey population grows to carrying capacity while other populations decline.

2. The fourth eigenvalues of \( J_{5,2} \) are \( -r.c - \delta - d_q - q \), \( -D + d_y \) and \( -d_y \), However, it is a saddle point with condition \( c > d_q + q \) and stable otherwise.

3. The characteristic equation of \( J_{5,2} \) is \( \lambda^2 + A \lambda + B \lambda^2 + C \lambda + D = 0 \).

Where, \( A = - (a_1 + a_2 + a_3 + a_4) \),

\( B = (a_1 a_2 + a_3 a_4 + a_2 a_4) + (a_1 + a_2) (a_3 + a_4) \),

\( C = (a_2 + a_4) (a_1 + a_2) + (a_3 + a_4) (a_1 + a_3) \),

\( D = a_1 a_2 a_3 a_4 - a_1 a_2 a_4 - a_1 a_3 a_4 - a_2 a_3 a_4 \). By Routh-Hurwitz theorem \( p_{25} \) is local stability if \( A, C, D > 0 \) and \( A > 0 \),

where \( A - 3D - A^2 \). However, A suitable Lyapunov function use to investigate global stability analysis for the non-trivial equilibrium points as follows:

**Theorem 11:** Axial equilibrium point \( P_{125}(0,0,0,0) \) is global stability in the first positive cone.

**Proof:** Since \( P_{125}(1,0,0,0) \) is locally asymptotically stable then we define a Lyapunov function as follows: \( F(x,y,z) = r(1-x) - \ln x + y + z + \), by [9].

It is clear that \( F(x,y,z) > 0 \) and \( F(x,y,z) > 0 \) \( \forall F(x,y,z) < P_{125} \).

Now by differentiating \( F \) with respect to time \( t \) and going some algebraic handling, given that:

\( \frac{dF}{dt} = \left[ \begin{array}{c} -c \frac{dy}{dt} \frac{dx}{dt} + x + \frac{dF}{dt} \end{array} \right] + c \frac{z}{x + i} + \frac{x}{x + i} \). \( \frac{dF}{dt} = \left[ \begin{array}{c} \frac{c z}{x + i} + \frac{x}{x + i} \end{array} \right] \). \( \frac{dF}{dt} = \left[ \begin{array}{c} \frac{c z}{x + i} + \frac{x}{x + i} \end{array} \right] \).

Now, due to the facts \( a_1 > a_2 \) and \( k_1 > k_2 \), we obtain that \( F_1 \) is Lyapunov.

So \( P_{125} \) is a globally asymptotically stable.

**Theorem 12:** The necessary conditions for global stability of interior point \( P_{125} \) are

\( x_i > x_i, x_i > x_i, y_i > x, z_i > x, x_i \) and \( y_i > z, y_i \).
**Proof**: Since $P_{4,2}$ is locally asymptotically stable, then a Lyapunov function chosen as:

$L(x,y,z,w) = C_1 \left( x - x, -x, \ln \frac{x}{x}, + C_2 \left( -i, -i, \ln \frac{i}{i} \right) \right)$

$C_1 \left( y - y, -y, \ln \frac{y}{y}, + C_2 \left( z - z, -z, \ln \frac{z}{z} \right) \right)$

$\frac{dL}{dt} = -rC_1 \left( x - x, -x, \ln \frac{x}{x}, + C_2 \left( -i, -i, \ln \frac{i}{i} \right) \right) +$

$c \left( C_1 \omega \left( x, -x \right), \ln \frac{x}{x}, + C_2 \left( -i, -i, \ln \frac{i}{i} \right) \right) +$

$\delta \left( z, -z, -z, \ln \frac{z}{z} \right)$.  

Suppose that $C_2 = C_2 \alpha$, and $C_1 = C_1 \alpha$. by some algebraic steps we get $\frac{dL}{dt} < 0$.

5. The Stability Threshold

From the observation of the first, second, third and fourth matrices and the stability of each subsystems and the original system, we can show that the conditions for stability of these points which represent the stability of the original model, in other word, stability of original system is necessary for stability of each subsystem and versa visa. Since all population dependent on prey and the condition $x \geq \frac{1}{2}$ is the condition to stability of all systems, then we can say that this condition is the stability threshold of each systems.

6. Numerical Simulation

In this section, the effect of harvesting discuss in two cases. First case when the model as SI and second the model as SIS. After several attempts and taking into account the conditions mentioned as well as taking advantage of some papers. Fixed the parameters as:

$r = 0.894, d_1 = 0.0, c = 0.133, D = 0.067, d_1 = 0.0, c = 0.036, \sigma = 0.019, k_1 = 0.34, \sigma = 0.091, k_2 = 0.0112, d_2 = 0.261, d_3 = 0.05, \sigma = 0.032, d_4 = 0.119$. First case, when susceptible prey become infected prey and $q = 0$. In this case all populations survive. The model is stable and solution go to the point $(0.76, 0.44, 0.82, 0.52)$, see figure 6.1:

Second case, discuss the SIS model. In this model susceptible prey become infected and become susceptible again. Without harvesting all populations survive and solution reached to equilibrium point $(0.83, 0.69, 0.4, 0.78, 0.47)$ see figure 6.3:

Finally, in any cases of these four cases, the size of prey is greater than or equal 0.5.
Conclusions
In this paper, many factors which effect on dynamical properties of biological and mathematical models are used such as: logistic growth function, functional response, disease, stage structure and harvesting. At the beginning, we studied the nontrivial subsystems that may be lead to coexistence of these types of spices by using Maple software, Jacobean matrix, Routh-Hurwitz criterion, Bendiixon-Dulac criterion and Lyapunov function to prove the existence, periodic solution, local and global stability. But, spreading of diseases in prey population had to use References

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In this paper, many factors which effect on dynamical properties of biological and mathematical models are used such as: logistic growth function, functional response, disease, stage structure and harvesting. At the beginning, we studied the nontrivial subsystems that may be lead to coexistence of these types of spices by using Maple software, Jacobean matrix, Routh-Hurwitz criterion, Bendiixon-Dulac criterion and Lyapunov function to prove the existence, periodic solution, local and global stability. But, spreading of diseases in prey population had to use

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