Heralded generation of symmetric and asymmetric entangled qudits with weak cross-Kerr nonlinearity

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Abstract

High dimensional entangled states have attracted much more attentions, due to their strong nonlocality and much powerful capability for quantum information processing. By the methods presented in this paper, arbitrary forms entangled qudits including symmetric and asymmetric forms could be generated with the weak cross-Kerr nonlinearity. These schemes are heralded by the detections of single-photon detectors. If all the detectors do not register any single photons, the generation is success with the probability $1/n^M$ determined by dimension $n$ and partite $M$. Furthermore, these schemes work well even with the common photon number nonresolving detectors, therefore they are feasible with the current experimental technology.

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I. INTRODUCTION

Quantum entanglement acts as crucial resources in the area of quantum information processing. It makes various quantum information tasks to be possible, while these tasks, such as quantum teleportation \[1\], quantum dense coding \[2\], etc., are impossible for classical information. The generation of quantum entanglement will continue throughout the development of quantum information processing. How to increase the length of quantum entanglement and reinforce the ability to generate various forms of quantum entanglement have been a long hotter topics in the research of quantum entanglement generation. Recently, many exciting progress are reported to set the new records of the length of quantum entanglement. In 2009, T. Monz et al reported the creation of 14-qubit entanglement with trapped ions \[3\], and later in 2011, X. C. Yao et al \[4\] and Y. F. Huang et al \[5\] had reported the generation of 8-photon entanglement, respectively. On the other hand, in the past years, various forms of quantum entanglement have also been generated in optical system, e.g., GHZ state \[6, 7\], W state \[8, 9\], cluster state \[10\], etc. Besides of these traditional entangled qubits, there is a special form of quantum entanglement, the so-called high dimensional entangled states, or called entangled qudits. With the qudits and entangled qudits, we could increase the security of quantum cryptography \[11–17\] and the efficiency of quantum logic gates \[18, 19\]. Moreover, high dimensional universal resources (e.g. AKLT states) for measurement based quantum computation can be related to the ground states of certain many-body strongly correlated Hamiltonian \[20–24\].

In this paper, we will focus on the optical system and consider how to generate entangled qudit efficiently. There are many qudit definitions in optical systems, what we concern is the polarization degree of freedom of multi-photon qudit \[25–35\]. Using the photon number of vertical polarization as encoding, the definition of polarization qudit could be expressed as \(|j\rangle_n \equiv |(n-j-1)H,jV\rangle\), for \(j = 0, \cdots, n-1\), where \(n\) is the dimension, and H and V represent the horizontal and vertical polarizations. This definition requires that the \(n-1\) photons are in the same spatial and temporal mode, therefore it is hard to operate each photon respectively, which increases the difficulty to generate various forms of entangled qudits. If the independent qudits are available, the success probability of entangled qudit generation is only \(1/n\) \[32, 36\]; while only the qubits and entangled qubits are available, the success probability of entangled qutrit generation is \(3/16\) in Ref. \[34\] or \(1/4\) in Ref. \[37\].
Moreover, almost all the former entangled qudit generation schemes only involved the case of symmetric entangled qudits, which could be expressed as follows,

$$\sum_{j=0}^{n-1} c_j |j\rangle_n \otimes |j\rangle_n,$$

where $\sum_{j=0}^{n-1} |c_j|^2 = 1$. However, the following asymmetric entangled qudits are also valuable,

$$\sum_{j=0}^{n-1} c_j |j\rangle_n \otimes |(j + k) \mod n\rangle_n,$$

where $k = 1, \cdots, n - 1$, and $\sum_{j=0}^{n-1} |c_j|^2 = 1$. Especially, when $c_j = \frac{1}{\sqrt{n}} \tau^j$ (where $\tau = e^{i2\pi/n}$), the asymmetric entangled qudits and the symmetric entangled qudits constitute the maximally entangled basis of two qudits.

Here we will use the cross-phase modulation (XPM) approach [38–40] to develop the generation of asymmetric entangled qudits. Briefly, the XPM approach bases on the interaction between a Fock state $|n\rangle$ and a coherent state $|\alpha\rangle$, resulting in the transformation $|n\rangle |\alpha\rangle \rightarrow |n\rangle |\alpha e^{i\theta}\rangle$, where the phase shift of coherent state is determined by the photon number of the Fock state. Assisted with the XPM approach, the generation scheme is heralded, and then the generated entangled qudit could be used flexibly in the further quantum information processing. We should note that our approach is available for the generation of symmetric entangled qudits as well, and could be generalized to the case of multi-partite, but not limit to only two-partite.

The rest of the paper is organized as follows. In Section II, we first introduce the ancilla single-photon qudit and then use the XPM approach to generate asymmetric entangled qutrits. Then in the next section, we will develop the approach to generate asymmetric entangled qudits. Sec. IV is for discussion and conclusion remark.

II. GENERATION OF ASYMMETRIC ENTANGLED QUTRITS

Before we outline the generation scheme, we will first introduce a single-photon qudit, which is encoded by the spatial modes of the single photon. After that, we will use the single-photon qudit as ancilla to generate entangled qudits.

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FIG. 1: (Color online) Generation of single-photon qudit. A single photon in the state $|H\rangle$ is injected into a series of proper unitary operations and polarized beam spitters, the balanced single-photon qudit encoded by the spatial modes could be generated.

A. Single-photon qudit

The balanced single-photon qudit could be expressed as follows,

$$|\phi\rangle_s^n = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} |j\rangle_s,$$

where $|j\rangle_s$ denotes the spatial mode of the single-photon. This state could be generated easily by the setups shown in the Fig. 1. The input state is initially prepared as the state $|H\rangle$, and then will be transformed into the state $\sqrt{\frac{n-1}{n}} |H\rangle + \frac{1}{\sqrt{n}} |V\rangle$ by the first single-photon unitary operation $U_0 = \left(\begin{array}{cc} \sqrt{\frac{n-1}{n}} & -\frac{1}{\sqrt{n}} \\ \frac{1}{\sqrt{n}} & \sqrt{\frac{n-1}{n}} \end{array}\right)$. After that, the state is injected into a polarized beam splitter (PBS), which let the horizontal polarization $|H\rangle$ to be passed and the vertical polarization $|V\rangle$ to be reflected (denote this spatial mode to be $|0\rangle_s$). Repeating the process, associated with the single-photon unitary operations $U_j = \left(\begin{array}{cc} \sqrt{\frac{n-j-1}{n-j}} & -\frac{1}{\sqrt{n-j}} \\ \frac{1}{\sqrt{n-j}} & \sqrt{\frac{n-j-1}{n-j}} \end{array}\right)$ ($j = 1, \cdots, n-1$) and $\sigma_x$ (used in the final spatial mode and then the polarization of all the spatial modes are the same to be $|V\rangle$), the balanced single-photon qudit could be generated.

B. Asymmetric entangled qutrit

To show our generation scheme clearly, we first use the generation of asymmetric entangled qutrit as example and in follows will generalize it to generate asymmetric entangled qudit. The balanced single-photon qutrit, as well as two qubus beams $|\alpha\rangle |\alpha\rangle$ are required as ancilla. The initial state is supposed to be an independent bi-photon qutrit as
FIG. 2: (Color online) Generation of asymmetric entangled qutrit. Introduce a balanced single-photon qudit as ancilla, which is coupled to two qubus beams through cross-phase modulation (XPM) processes, associated with two independent polarized qutrits. With the proper design of XPM phase shifts, the asymmetric entangled qutrits could be heralded generated with the success probability $1/9$. In addition, if more independent qutrits are introduced and we repeat the same processes, multi-partite entangled qutrits could be generated as well.

$$|\psi_1\rangle = a_0|0\rangle_3 + a_1|1\rangle_3 + a_2|2\rangle_3, \quad \text{where} \quad \sum_{j=0}^{2} |a_j|^2 = 1.$$  These independent qutrits could be created by higher-order parametric down-conversion process [33], by the Hong-Ou-Mandal (HOM) interference of two single-photon qubits [41], or by the transformation from single-photon qutrit [42]. This qutrit is injected into a PBS and the upper mode, associated with the modes $|1\rangle_s, |2\rangle_s$ of the single-photon qutrit, will be coupled to one of the qubus beam as depicted in Fig.2. The XPM phase shifts induced by the couplings of the upper mode and the mode $|1\rangle_s$ are supposed to be $\theta$, while that of the mode $|2\rangle_s$ is supposed to be $2\theta$. After that, we will get the following state,

$$\begin{align*}
&\frac{1}{\sqrt{3}} (a_0|0\rangle_3|0\rangle_s + a_1|1\rangle_3|1\rangle_s + a_2|2\rangle_3|2\rangle_s) |\alpha\rangle |\alpha e^{i2\theta}\rangle \\
&+ \frac{1}{\sqrt{3}} (a_0|0\rangle_3|1\rangle_s + a_1|1\rangle_3|2\rangle_s) |\alpha\rangle |\alpha e^{i3\theta}\rangle + \frac{1}{\sqrt{3}} a_0|0\rangle_3|2\rangle_s |\alpha\rangle |\alpha e^{i4\theta}\rangle \\
&+ \frac{1}{\sqrt{3}} (a_1|1\rangle_3|0\rangle_s + a_2|2\rangle_3|1\rangle_s) |\alpha\rangle |\alpha e^{i\theta}\rangle + \frac{1}{\sqrt{3}} a_2|2\rangle_3|0\rangle_s |\alpha\rangle |\alpha\rangle.
\end{align*}$$  (4)

Implement a displacement $-2\theta$ to the second qubus beam and let the two qubus beams interfered on the 50:50 beam splitter (BS), which will implement the transformation
\[ |\alpha_1\rangle |\alpha_1\rangle \rightarrow \left| \frac{\alpha_1 - \alpha_2}{\sqrt{2}} \right\rangle \left| \frac{\alpha_1 + \alpha_2}{\sqrt{2}} \right\rangle \] will yield the following state,

\[
\frac{1}{\sqrt{3}} \left( a_0 |0\rangle_3 |0\rangle_s + a_1 |1\rangle_3 |1\rangle_s + a_2 |2\rangle_3 |2\rangle_s \right) |0\rangle \left| \sqrt{2} \alpha \right\rangle
\]

\[
+ \frac{1}{\sqrt{3}} \left( a_0 |0\rangle_3 |1\rangle_s + a_1 |1\rangle_3 |2\rangle_s \right) |\alpha^+_s\rangle \left| \alpha^+_s\right\rangle + \frac{1}{\sqrt{3}} a_0 |0\rangle_3 |2\rangle_s \left| \alpha^2_s\right\rangle
\]

\[
+ \frac{1}{\sqrt{3}} \left( a_1 |1\rangle_3 |0\rangle_s + a_2 |2\rangle_3 |1\rangle_s \right) |\alpha^{-1}_s\rangle \left| \alpha^{-1}_s\right\rangle + \frac{1}{\sqrt{3}} a_2 |2\rangle_3 |0\rangle_s \left| \alpha^{-2}_s\right\rangle \left| \alpha^{-2}_s\right\rangle, \quad (5)
\]

where \( |\alpha^k_s\rangle = \left| \frac{\alpha^{\pm} e^{\pm ik\theta}}{\sqrt{2}} \right\rangle (k = \pm 1, \pm 2) \). If the vacuum component \(|0\rangle\) can be well distinguished from the other components \(|\alpha^{\pm1,\pm2}_s\rangle\), we could achieve the following state,

\[ a_0 |0\rangle_3 |0\rangle_s + a_1 |1\rangle_3 |1\rangle_s + a_2 |2\rangle_3 |2\rangle_s, \quad (6) \]

with the success probability 1/3. In this case, the qubus beams could be recycled, since they are the same as the initial ones. The desired discrimination could be realized by an idea photonic number non-resolving detector (PNND) (on/off detector with quantum efficiency \(\eta = 1\)) and the corresponding error probability due to the overlap of the vacuum component and the other components is

\[ P_E = \frac{4}{9} e^{-2|\alpha|^2 \sin^2 \frac{\theta}{2}} + \frac{2}{9} e^{-2|\alpha|^2 \sin^2 \theta}, \quad (7) \]

which could tend to 0 under the conditions \(|\alpha|^2 \sin^2 \frac{\theta}{2} \gg 1 \) and \(|\alpha|^2 \sin^2 \theta \gg 1\). This requirement could be satisfied by increasing the amplitude of coherent state \(\alpha\), when it works in the weak nonlinearity regime \((\theta \ll 1)\).

Evidently, if we introduce another independent bi-photon qutrit and repeat the above processes, we could get the symmetric entangled qutrit. On the other hand, if we want to obtain the asymmetric entangled qutrit, the experimental setups must be adjusted slightly, which is shown in the right side of Fig.2. Firstly, we introduce another bi-photon independent qutrit which is prepared as \(|\psi_2\rangle = b_0 |0\rangle_3 + b_1 |1\rangle_3 + b_2 |2\rangle_3\), where \(\sum_{j=0}^2 |b_j|^2 = 1\). Similarly, this qutrit is injected into a PBS and the upper mode, associated with the modes \(|0\rangle_s\), \(|1\rangle_s\) of the ancilla single-photon qutrit, is coupled to the second qubus beam as depicted in Fig.2. After that, the following state could be achieved,

\[ (a_0 b_1 |0\rangle_3 |1\rangle_3 |0\rangle_s + a_1 b_2 |1\rangle_3 |2\rangle_3 |1\rangle_s + a_2 b_0 |2\rangle_3 |0\rangle_3 |2\rangle_s) |\alpha\rangle |\alpha e^{i2\theta}\rangle + rest., \quad (8) \]

where \(rest.\) denote the other components that the qubus beam catches different phase shifts other than \(2\theta\). One more displacement \(-2\theta\) is implemented and the interference of two
qubus beams will yield the state,

\[ (a_0 b_1 |0\rangle_3 |1\rangle_3 |0\rangle_s + a_1 b_2 |1\rangle_3 |2\rangle_3 |1\rangle_s + a_2 b_0 |2\rangle_3 |0\rangle_3 |2\rangle_s) |0\rangle \sqrt{2} \alpha + \text{rest}. \]  

(9)

Obviously, using an ideal PNND to detect the first coherent state component, the above state could be projected into the following state,

\[ a_0 b_1 |0\rangle_3 |1\rangle_3 |0\rangle_s + a_1 b_2 |1\rangle_3 |2\rangle_3 |1\rangle_s + a_2 b_0 |2\rangle_3 |0\rangle_3 |2\rangle_s, \]  

(10)

with the success probability \(|a_0 b_1|^2 + |a_1 b_2|^2 + |a_2 b_0|^2\). If the coefficients \(a_i = b_i = 1/\sqrt{3}\), for \(i = 0, 1, 2\), the success probability is also 1/3. Moreover, it is easy to find that the corresponding error probability is the same as Eq. (7).

Finally, to achieve the desired asymmetric entangled qutrit, we should erase the ancilla single-photon qutrit without changing anything else. We firstly perform the following Fourier transformation by a so-called linear optical multi-port interferometer (LOMI) \[43\] on the single-photon qutrit,

\[ |j\rangle_s = \frac{1}{\sqrt{3}} \sum_{k=0}^{2} e^{2\pi i j k/3} |k\rangle_s, \]  

(11)

where \(j, k\) denote the spatial modes. After performing some phase shifts controlled by the detection on this single photon through the classical feedforward, we could get the desired asymmetric entangled qutrit,

\[ a_0 b_1 |0\rangle_3 |1\rangle_3 + a_1 b_2 |1\rangle_3 |2\rangle_3 + a_2 b_0 |2\rangle_3 |0\rangle_3. \]  

(12)

By properly setting the coefficients \(a_i, b_i\), the following maximally entangled qutrits could be obtained,

\[ \frac{1}{\sqrt{3}} \left( |0\rangle_3 |1\rangle_3 + \tau^m |1\rangle_3 |2\rangle_3 + \tau^{2m} |2\rangle_3 |0\rangle_3 \right), \]  

(13)

where \(\tau = e^{2\pi i/3}\) and \(m = 0, 1, 2\). Totally, the success probability is 1/9. Moreover, exchanging the order of two qutrits and properly setting the coefficients, we could achieve the following maximally entangled qutrits,

\[ \frac{1}{\sqrt{3}} \left( |0\rangle_3 |2\rangle_3 + \tau^m |1\rangle_3 |0\rangle_3 + \tau^{2m} |2\rangle_3 |1\rangle_3 \right). \]  

(14)

These six asymmetric entangled qutrits associated with the other three symmetric entangled qutrits constitute the maximally entangled basis of two qutrits. We should note here that the generation scheme does not require any postselection processes, that is it is heralded
FIG. 3: (Color online) Generation of asymmetric entangled qudit. Similar to the generation of asymmetric entangled qutrits, a balanced single-photon qudit is introduced as ancilla. Two independent polarized qudits are coupled to the two qubits beams, associated with the ancilla single-photon qudit as depicted in Fig. 3. By the proper design of XPM phase shifts, the asymmetric entangled qudits could be heralded generated with the success probability $1/n^2$ determined by the dimension $n$. Here we suppose $k \geq 1$, which corresponds to the case of asymmetric entangled qudits generation. If the XPM phase shifts induced to the two independent qudits are the same, the symmetric entangled qudits could be generated. Moreover, if more independent qudits are introduced and the similar processes are repeated, multi-partite entangled qudits could be generated with the success probability $1/n^M$ determined by the dimension $n$ and partite $M$.

by the detection of the coherent state component. If the detector registers any signals, the generation scheme is failure, while no detection means the success of the generation. Therefore, the generated entangled qutrit could be used flexibly in the further quantum information processing.

III. GENERATION OF ENTANGLED QUDITS

The above scheme could be generalized to realize the generation of arbitrary forms of entangled qudits (expressed in Eq. (2)). At first, we introduce a single-photon qudit $|\phi\rangle_n$ as ancilla. Suppose an independent multi-photon qudit is prepared as $\sum_{j=0}^{n-1} a_j |j\rangle_n$, where $\sum_{j=0}^{n-1} |a_j|^2 = 1$. After the multi-photon qudit is injected into a PBS, its upper mode associated
with the spatial modes of the single-photon qudit are coupled to the second qubus beam as depicted in Fig.4. The XPM phase shift induced by the upper mode is supposed to be θ and those by the spatial modes |j⟩ₙ (j = 0, · · · , n − 1) are supposed to be jθ. After the interaction, we will get the following state,

\[ \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} a_j |j⟩ₙ |j⟩ₙ |α⟩ |αe^{i(n-1)θ}⟩ + rest., \]  

(15)

where rest. denote the components that the qubus beam catches the phase shifts other than (n − 1)θ. After that, implement a displacement −(n − 1)θ to the second qubus beam and let the two qubus beams interfered on a 50:50 BS. The above state will be transformed into the follows,

\[ \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} a_j |j⟩ₙ |j⟩ₙ |0⟩ |\sqrt{2α}⟩ + rest. \]  

(16)

Using an ideal PNND to detect the first coherent state component will project the above state to

\[ \sum_{j=0}^{n-1} a_j |j⟩ₙ |j⟩ₙ, \]  

(17)

when the detection is \( p_n = 0 \). The corresponding success probability is 1/n.

Similarly, we should introduce another independent multi-photon qudit, which is supposed to be \( \sum_{j=0}^{n-1} b_j |j⟩ₙ \). If we want to achieve the symmetric entangled qudit, what we should do is to repeat the above processes with the same experimental setups. While we want to achieve the asymmetric entangled qudit, we should adjust the experimental setups. As shown in the right side of Fig.3, the corresponding XPM phase shifts induced by the upper mode of second multi-photon qudit is also supposed to be θ, while the other XPM phase shifts induced by the spatial modes of the single-photon qudit are supposed to be \( [(j + k) \mod n]θ \). After the interaction, we could get the following state,

\[ \sum_{j=0}^{n-1} a_j b_{(j+k) \mod n} |j⟩ₙ |(j + k) \mod n⟩ₙ |j⟩ₙ |α⟩ |αe^{i(n-1)θ}⟩ + rest. \]  

(18)

Next we implement a displacement −(n − 1)θ to the second qubus beam and let two qubus beams interfered on a 50:50 BS. Then the following state could be achieved after detecting the first coherent state component with the result \( p_n = 0 \),

\[ \sum_{j=0}^{n-1} a_j b_{(j+k) \mod n} |j⟩ₙ |(j + k) \mod n⟩ₙ |j⟩ₙ. \]  

(19)
Finally, the ancilla single-photon qudit must be erased before we get the desired quantum state. To complete the erasure, we first implement the following Fourier transformation,

$$|j\rangle_s = \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} e^{2\pi ijk/n} |k\rangle_s,$$

(20)

by a $n$-port LOMI \[43\]. After that, detecting the single-photon qudit and using the detection to control the conditional phase shifts through the classical feedforward will yield the desired asymmetric entangled qudit,

$$\sum_{j=0}^{n-1} a_j b_{(j+k) \mod n} |j\rangle_n |(j + k) \mod n\rangle_n.$$  

(21)

By properly setting the coefficients, we could get the following states,

$$\frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} \tau^{jm} |j\rangle_n \otimes |(j + k) \mod n\rangle_n,$$

(22)

where $m, k = 0, \cdots, n - 1$ and $\tau = e^{i2\pi/n}$. These states constitute to be the maximally entangled basis of two qudits and the corresponding success probability is $1/n^2$.

Furthermore, if we introduce one more multi-photon qudit and repeat the above processes, we could achieve three-partite entangled qudits. In other words, the above scheme could be easily generalized to create any forms of multi-partite entangled qudits, including symmetric and asymmetric multi-partite entangled qudits. The corresponding success probability is $1/n^M$, which determined by the dimension $n$ and partite $M$.

IV. DISCUSSION AND CONCLUSION

In this paper, we propose a scheme to generate entangled qudits, including the symmetric and asymmetric forms. Using a balanced single-photon qudit encoded by its spatial modes as ancilla, the forms of generated entangled qudits could be flexible by setting the proper XPM phase shifts. The success probability for maximally entangled qudits is determined by the dimension and the partite as $1/n^M$. It looks that to be not high. However, this scheme is heralded, but not based on the post-selection. If any of the ideal PNNDs register any signals, the scheme is failure; otherwise, it is successful. While many other optical schemes are based on the post-selection, and then when we use the generated quantum entanglement, we will encounter many limitations, e.g., no further interference is allowed.
In other words, only single-photon operations are allowed to performed on the generated quantum entanglement. Obviously, the generated entangled qudits in our scheme could be used without any limitations in the further quantum information tasks, since the generation is heralded.

Actually, the probabilistic quantum gate is still valuable for quantum information processing. For example, the universal resource, e.g., cluster state could be generated efficiently even with the probabilistic quantum gate [44]. Developing the idea in Ref. [44], we could generate the high dimension universal resource, e.g., AKLT states efficiently by the probabilistic gate presented in this paper as well.

Now, we discuss the feasibility of the present scheme briefly. The ideal PNND used in this scheme seems unrealistic, however, we could use the common PNND, e.g. silicon avalanche photodiode (APD), to replace the ideal PNND. Though the quantum efficiency of common PNND is lower than 1 (η_{APD} \sim 0.7), it could works well in our scheme. What we want is to distinguish the vacuum state from the other coherent components. If we choose the proper parameters, e.g., the XPM phase shift \theta = 0.01 and the amplitude of qubus beam |\alpha| = 500, the average photon number of the coherent components |\alpha_{\pm 1}\rangle and |\alpha_{\pm 2}\rangle are about 13 and 50, respectively. In this case, the vacuum state could be well separated from the coherent components even with the common PNND, though the corresponding error probability \( P_E \ll 1 \).

Finally, the core element of our scheme is the proper design of XPM phase shifts based on the weak cross-Kerr nonlinearity. Since we had theoretically demonstrated that a small XPM phase shift with high fidelity is feasible in the weak nonlinearity regime [45], even the so-called multi-mode effect is considered, then the weak nonlinearity in the idealized single-mode picture is valid. Associated the mature linear optical techniques, e.g. the interference of two qubus beams, the detection of single photons, etc., our scheme is feasible with the current experimental technology.

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