Allometric scaling and accidents at work

Czesław Cempel*, Maciej Tabaszewski¹ and Szymon Ordysiński²

¹Central Institute for Labour Protection – National Research Institute (CIOP-PIB), Poland; ²Poznań University of Technology, Poland

Allometry is the knowledge concerning relations between the features of some beings, like animals, or cities. For example, the daily energy rate is proportional to a mass of mammals rise of 3/4. This way of thinking has spread quickly from biology to many areas of research concerned with sociotechnical systems. It was revealed that the number of innovations, patents or heavy crimes rises as social interaction increases in a bigger city, while other urban indexes such as suicides decrease with social interaction. Enterprise is also a sociotechnical system, where social interaction and accidents at work take place. Therefore, do these interactions increase the number of accidents at work or, on the contrary, are they reduction-driving components? This article tries to catch such links and assess the allometric exponent between the number of accidents at work and the number of employees in an enterprise.

Keywords: accidents at work; allometric scaling; statistical data; size of an enterprise

1. Introduction

One of the universal (or ubiquitous) phenomena in nature is scaling, which concerns natural and living entities as well as systems created by humans, such as technical, complex socioeconomic and sociotechnical metasystems. In geometry, scaling simply means transformation that enlarges or diminishes objects, but what we have in mind here is not just geometrical scaling.[1,2] Scaling law is expressed simply as a power relation between two features Y and M, e.g., \( Y = Y_0 \cdot M^\beta \) (where \( Y_0 \) = constant keeping the dimension and correcting numeric); it describes the scale invariance found in many natural and human-made phenomena. Scaling can be isometric when both quantities are simply proportional (\( \beta = 1 \)), and allometric otherwise (when \( \beta \neq 1 \)). All researchers find the latter case most interesting.

The whole research problem was detected in biology in the first half of the 20th century,[3] and the most eminent researcher of this time seems to be Max Kleiber.[4] His famous scaling relation of the metabolic rate to the mass of the body of any animal with the exponent \( \beta = 3/4 \) is well known. This way of thinking spread from biology, releasing and uniting different branches of natural, technical and social sciences. It has shown that allometric scaling is a ubiquitous phenomenon, allowing one to describe and explain the behaviour of a running machine,[5] as well as indices of growing metropolises around the world.[6]

This article starts from this vast area of research of socioeconomic indexes of cities in particular,[7] but aims to describe in the same way accidents at work. The article also makes an attempt at grasping the influence of social interactions on the number of accidents at work.

2. Allometric scaling: principles

One of the most generalized approaches to scaling laws has been given by Barenblatt in his monograph ‘Scaling’ devoted mostly to physical and natural phenomena,[8] which are called multiscaling and can be expressed simply:

\[ Y = A \cdot X_1^{\alpha_1} \cdot X_2^{\alpha_2} \cdot X_n^{\alpha_n}. \]  

(1)

One can see from Equation (1) that there can be many independent variables and many scaling exponents. But the most used allometric scaling equation has already been given in Section 1 and it will be good to show it again here:

\[ Y = Y_0 \cdot M^\beta, \]  

(2)

where in early research efforts \( M \) stands only for the mass of the animal or other entity and \( Y \) for one of its features, measured along the population of entities. Taking into account the metabolic rate (kcal/day) of animals as a function of their mass, the first area of Kleiber’s research, one can find a simple allometric exponent \( \beta = 3/4 \).[9]

The simplicity and greatness of this finding was the reason why this type of research was extended to other areas of science. That is one of the reasons why in a recent paper on allometric scaling of countries,[10] the independent variable \( M \) has many more meanings, e.g., the area of the country, its population and gross domestic...
product (GDP). Following this trace, it is sometimes much better to divide Equation (2) by \( M \) to obtain a new meaning of allometric scaling, e.g., GDP per capita, and others as in Equation (3):

\[
\frac{Y}{M} = Y_o \cdot M^{\beta-1}.
\] (3)

On the other hand, there are some phenomena in nature, like the frequency \( F \) of a forest fire or a city fire, which indicate also the allometric scaling behaviour, but mostly having a negative exponent like in Equation (4):

\[
F = F_o \cdot S^{-\alpha},
\] (4)

where \( F_o \) = initial dimensional constant and \( S \) = symptom of the fire, e.g., burned area in a forest or the number of deaths in a city.

If we consider a forest fire in China, one can find, according to Lu et al.\[11\] two allometric exponents: \( \alpha_1 = 1.05 \) for the burned area of the forest, and \( \alpha_1 = 1.34 \) for the losses calculated in yuan (CNY). It is interesting to notice from those data that the different symptoms of the same fire phenomena, like burned area of forest or calculating loss in money, have quite a different allometric exponent. It is almost the same as the metabolic rate of animals from the beginning of this section with \( \beta = 3/4 \), and the heart rate of mammals, where it has quite a different and negative exponent (\( \beta = -1/4 \)).

The negative exponents of forest fires and heart rate remind the author that similar relations can be encountered in machine condition monitoring, where most observed vibration symptoms have a Pareto or Weibull type of distribution. The same can be observed when studying symptoms of an energy processor, like a model of a working machine, or other energy processing systems.\[12\]

3. Allometry in sociotechnical systems

The great success of allometric scaling in biology, also with human data, was why this approach has been transferred to other areas of human activity, e.g., enterprises, cities, metropolises and countries. The leading role in this way of thinking and research belongs to G.B. West and L.M.A. Bettencourt of Santa Fe Institute, NM, USA. Having data for cities and metropolises of many countries, they have calculated so-called urban indicators, which characterize the social and technical development of any city.\[13\] In this paper, the result of over 20 urban indicators are presented which can be divided into three groups: sublinear with \( \beta < 1 \) concerning technical infrastructure (length of the roads, cables, etc.), linear with \( \beta \approx 1 \) usually associated with individual human needs (house, job, water consumption) and superlinear with \( \beta > 1 \) associated with social human interaction in a city and reflected in the number of innovations, new patents, etc. However, one can find there negative social influences, e.g., the number of new AIDS cases and serious crimes. However, it is remarkable what the authors ascertain as a conclusion in this paper: ‘From this perspective, cities are concentrations not just a people, but rather of social interactions’.\[6,p.1439\]

The leading role of social interactions acting in cities and metropolitan areas, the good and the bad, was also analysed by these authors in another paper.\[14\] where, using the data for 360 US metropolitan areas, one common exponent \( \beta \approx 1.15 \) has been found for four urban indicators describing the evolution of metropolises. It is interesting to find that these indicators concern crime, personal income, the number of patents and the GDP of a metropolis.

Quite recently, a paper on the social influence on accidents in cities and metropolitan areas was published.\[15\] Melo et al. found, on the basis of Brazilian data from 2009, confirmation of an allometric superlinear exponent of crimes (\( \beta = 1.24 \)); homicides to be precise. However, road accidents have a linear allometric exponent (\( \beta = 0.99 \)), and with much surprise there is sublinear scaling of suicides (\( \beta = 0.84 \)). This may mean that the bigger the city, the lower the probability of suicide due to the mediating and calming interaction of surrounding people.

Having these data in mind, especially the positive influence of social interactions on suicides, one can now ask: what about accidents at work? Should one expect a superlinear scaling exponent like in the case of AIDS, or hopefully a sublinear one, like in the case of suicides? This question is our main research problem of this article.

4. Allometric scaling of accidents at work: preliminary considerations

Bearing in mind our research question, let us define exactly the research problem. One can see from Melo et al.’s paper\[15\] that if living in a bigger city, one is less likely to commit suicide. So, when asking about the number of accidents at work, we will look for accidents in different enterprises with a known number of employees, from one self-employed person up to several thousands of employees, e.g., in big state-owned enterprises.

In Poland, the only source of such data is the Central Statistical Office of Poland (GUS), where one can find data on accidents at work, initially grouped for some other purposes. For example, one can find cumulated data on accidents at work for 3 years there (Figure 1).\[16–18\]

Figure 1 shows that the classification of enterprises is very imprecise; one-employee enterprises at the far left, and enterprises with over 500 employees on the right (we will not consider the last right-hand number as it is very misleading). However, looking at the interval scale on the right-hand side of the figure, one can define on this basis six artificial enterprises as an average of all interval boundaries. This will reduce our accident data to a matrix with two columns and six rows (different enterprises). Expecting some allometric scaling, like in Equation (2), let us present our transformed data as a linear regression on a log-log...
Figure 1. Semi-qualitative data on accidents at work for 2008–2010 presented for different groups of enterprises.[16–18]
Note: SME = small and medium-sized enterprises.

Figure 2. Accidents at work versus the number of employees in an enterprise: data transformed for 2008 (see Figure 1) and calculated with a linear regression program.
Note: model \( y = bx^\beta \), where \( x \) = number of employees in enterprise, \( y \) = number of accidents, \( b \) = 802.4808, \( \beta \) = 0.6406; coefficient of determination \( R^2 \) = 0.8085, reduced major axis exponent \( \beta_{rma} \) = 0.8992.

5. Allometric scaling of accidents at work from original raw data

In Poland, accidents at work are registered by the GUS, which records all accidents at work of persons employed in almost all of Poland’s economy, regardless of whether the accident has resulted in work disability or not.[20] Statistical results presented in this article are based on data registered by GUS in 2005–2012 (over 750,000 records).

Expecting allometric scaling between the number of accidents at work and the number of employees, like in Equation (2), the data were first transformed using the logarithm function and analysed using a linear regression model on a log–log scale. All statistical analyses were calculated with a program in Matlab version R2009a as previously stated. First, let us look at the data for 2012 (Figure 3).

When analysing the linear regression model of accident data in Figure 3, one can notice two important facts. The allometric scaling exponent is not sublinear but, to our surprise, it is negative, which was impossible to predict, even from Melo et al.’s suicide data.[15] Secondly, the accident data have great dispersion, so they need some introductory transformation — smoothing — before calculating the allometric exponent \( \beta \) and the determination coefficient \( R^2 \). Because of this dispersion we have used additional smoothing with the kernel Nadaraya–Watson method, as in Melo et al.’s paper.[15] Figure 4 shows the result.

When comparing Figures 3 and 4 one can notice that smoothing the data lowers the allometric exponent \( \beta \) a little, and raises the determination coefficient \( R^2 \) much more. This is why we processed accident data for 2005–2012 in the same way. Figure 5 shows the results.

One can now infer that there is no error in our calculations; the allometric exponent is again negative and it has almost the same value, just about 1 (\( \beta = -1.063 \)), and the determination coefficient is again high (\( R^2 = 0.833 \)). But this can be much higher if we admit that our allometric
Figure 3. Accidents at work versus the number of employees in 2012, presented on a log–log scale, and data of a calculated model of linear regression. Please note the great dispersion of data. Note: model: \( y = bx^{\beta} \), where \( x \) = number of employees in enterprise, \( y \) = number of accidents, \( b = 17955.56 \), \( \beta = -1.0857 \); coefficient of determination \( R^2 = 0.8085 \).

Figure 4. A linear regression model of the number of accidents at work versus the number of employees in an enterprise in 2012, calculated from smoothed data with the Nadaraya–Watson method (cf. Figure 3). Note: model: \( y = bx^{\beta} \), where \( x \) = number of employees in enterprise, \( y \) = number of accidents, \( b = 10458.92 \), \( \beta = -0.9641 \); coefficient of determination \( R^2 = 0.8424 \).

Figure 5. A linear regression model of the number of work accidents versus the number of employees in an enterprise for 2005–2012 calculated from smoothed data (as before). Note: model: \( y = bx^{\beta} \), where \( x \) = number of employees in enterprise, \( y \) = number of accidents, \( b = 1174.28 \), \( \beta = -1.0637 \); coefficient of determination \( R^2 = 0.8330 \).

Figure 6. A reverse regression model of the number of accidents at work and the number employees in an enterprise for 2012. Note: model: \( y = bx^{\beta} \), where \( x \) = number of accidents, \( y \) = number of employees in enterprise, \( b = 273.80 \), \( \beta = -1.0662 \); coefficient of determination \( R^2 = 0.8268 \).

model in Equation (2) is mostly valid for over 10,000 and under 30,000 workers (see Figures 4 and 5).

Up to now we have calculated allometric exponents with ordinary linear regression, which means we expect errors only in the \( y \) coordinate (see Equation (2)), and with accidents at work data we also have some error associated with calculating the number of employees, so both coordinates are erroneous. In such a case, the real value of the allometric exponent may be different from that we have calculated before, and we should use another method to assess it. This method is called rma regression.[17] In this article one can also find a simple equation to calculate the real allometric exponent. This formula is simple: a geometric mean of allometric exponents taken from the direct \( D \) and reverse \( R \) regressions with proper signs, i.e.,

\[
\beta_{rma} = - (\beta_D \cdot \beta_R)^{1/2}.
\]  

In such a case we need reverse regression of accident data for 2005–2012 as presented in Figures 6 and 7.
Figure 7. A reverse regression model of the number of accidents at work and the number employees in an enterprise for 2005–2012 with smoothed data.

Note: model: $y = bx^\beta$, where $x =$ number of accidents, $y =$ number of employees in enterprise, $b = 1549.93$, $\beta = -1.002$; coefficient of determination $R^2 = 0.8427$.

Hence, we can calculate symmetric allometric exponents according to Equation (5), and by doing this one can obtain immediately:

$\beta_{2012} = -(0.96409 \cdot 1.0663)^{1/2} = -1.014$;
$\beta_{2005-2012} = -(1.0637 \cdot 1.002)^{1/2} = -1.032$.

As one can see, both exponents are negative and of order 1, so we can admit that the allometric exponent of accidents at work is $-1$ ($\beta \approx -1$) in our first approach.

This means that social interaction in the workplace is a dominating component in creating the number of accidents, so the bigger the enterprise, the less probability of an accident at work. Of course the data are greatly dispersed, but the tendency is unique; there are fewer accidents at a bigger workplace. This conclusion cannot be found in scientific literature on accidents at work, so more research should be done with more recent data on accidents.

6. Prospects of human interaction in the workplace and in a sociotechnical system

At the beginning of this article our attention was turned to social interaction\(^1\) in cities, where because of this influence we have additional innovations, greater GDP, etc., with superlinear allometric exponents. Now we have discovered statistically that the social interaction is also the main component in creating a lower probability of accidents in a bigger enterprise. The association of these two effects of social interaction is of profound importance. Hence it seems to be good to illustrate this allometric property of social interaction in a special figure, compared with isometric growth $\beta = 1$ (Figure 8).

This seems to be a very important finding of our article, but we are not claiming that the allometric exponent of accidents at work should always be $\beta = -1$ – less than 1 as in the case of suicides in Melo et al.’s paper may sometimes also be good.[15] However, we need more raw data on accidents at work to confirm this statistical relation.

7. Summary

The main aim of this article was to introduce allometric scaling into research on accidents at work, which take place in every enterprise; a small or large sociotechnical system. After an introductory presentation of contemporary allometric research results, special attention has been paid to the social interaction effects in cities as sociotechnical systems. On the one hand, we have here superlinear scaling exponents of the number of patents and inventions in metropolises versus population; on the other, a sublinear scaling exponent of suicides.

The introductory interval data on accidents at work in Poland (Figures 5 and 6) confirm similarly sublinear scaling exponents, so they confirm the influence of social interaction in the workplace. However, when processing two large raw databases on accidents at work, it was discovered that social influence in the workplace understood in this way is very decisive, giving negative allometric exponents, close to 1. This finding cannot be found in the scientific literature; it will be verified later with some other databases of accidents at work. However, it is promising to find such a possibility of reducing the number of accidents at work.

Note

1. See earlier quotation from [6,p.1439].
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