Vortex unpinning due to crustquake initiated neutron excitation and pulsar glitches

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ABSTRACT

Pulsars undergoing crustquake release strain energy, which can be absorbed in a small region inside the inner crust of the star and excite the free superfluid neutrons therein. The scattering of these neutrons with the surrounding pinned vortices may unpin a large number of vortices and effectively reduce the pinning force on vortex lines. Such unpinning by neutron scattering can produce glitches for Crab like pulsars and Vela pulsar of size in the range $\sim 10^{-8}$ – $10^{-7}$, and $\sim 10^{-9}$ – $10^{-8}$, respectively. Although we discuss here the crustquake initiated excitation, the proposal is very generic and equally applicable for any other sources, which can excite the free superfluid neutrons, or can be responsible for superfluid - normal phase transition of neutron superfluid in the inner crust of a pulsar.

Key words: stars: neutron, pulsars: general, scattering, gravitational waves.

1 INTRODUCTION

Pulsars are known to be excellent time-keepers. However, a significant number of pulsars show sporadic spin-up events, namely glitches. A total of 555 glitches in 190 pulsars have been catalogued$^1$ and reported to date (Espinoza et al. 2011). The size of glitches lie in the range $\sim 10^{-12}$ – $10^{-5}$, with a typical interglitch time of a few years. Although, the models associated with pinning-unpinning of superfluid vortices (Anderson & Itoh 1975) are considered to be the most popular models for explaining glitches, the crustquake (Ruderman 1969) model finds its place in the literature quite regularly in the study of glitches or otherwise (see Haskell & Melatos 2015 for a detailed review). There have been discussions in the literature suggesting the involvement of crustquake in neutron star physics, such as an explanation for the giant magnetic flare activities observed in magnetars (Thompson & Duncan 1995; Lander et al. 2015), as a possible source of gravitational waves (GW) from isolated pulsars (Keer & Jones 2015; Layek & Yadav 2020). On the other hand, though the basic picture of the superfluid model is well accepted, it has a few issues which are not understood with certainty. For instance, the value of a very important quantity used in this model, namely, the pinning strength is not calculated from first principles. Similarly, the precise mechanism which triggers vortex unpinning is not known with certainty. In fact, there are suggestions (Melatos et al. 2008; Eichler & Shaisultanov 2010) that crustquake itself might act as a trigger mechanism for sudden angular momentum transfer by vortices to the crust. Similarly, Akbar & Alpar (2015), have proposed that the movement of the broken crustal plate caused due to crustquake, and hence, the motion of vortices attached with the plate is responsible for glitches. Here we should also mention an interesting study by Link & Epstein (1996), where the authors have proposed thermally driven pulsar glitches, caused by sudden deposition of energy in the inner crust. The deposited energy propagates as thermal waves through some parts of the inner crust and raises the local temperature. As per their study, this thermal fluctuations caused by energy depositions affect the coupling between neutron superfluid and the rigid outer crust causing the star to spin-up. Crustquake has been assumed to be one of the sources for such energy depositions. Although our proposed mechanism of glitches is unpinning of vortices, as we discuss below, in contrast to the picture as proposed by Link & Epstein (1996), the work is worth mentioning from the perspective of energy deposition into the crust caused by crustquake.

In this work, we propose a novel mechanism of unpinning of superfluid vortices. In our scenario, crustquake and superfluid vortices are both considered to be responsible for pulsar glitches. We propose that strain energy released ($\Delta E = B\Delta \epsilon$) due to crustquake (Baym & Pines 1971) excites the unbound superfluid neutrons that exist in the inner crust. These excited neutrons should share their energy with the pinned vortices through scattering. As a result, a large number of vortices ($\sim 10^{13}$) existing in the
neighbourhood of the quake site can be released, causing the star to spin-up. Here, the size of the glitch depends on the number of vortices released due to the excitations. For Crab like pulsars and Vela, the size of the glitches will be shown to lie in the range $\sim 10^{-8} - 10^{-7}$ and $\sim 10^{-9} - 10^{-8}$, respectively.

The paper is organized in the following manner. In section 2, we briefly review the relevant features of the crustquake model. We present our work in the subsequent sections. In section 3, we determine the number of vortices that can be unpinned through neutron-vortex scattering. Here, we will provide the expression for glitch size. The mechanism for unpinning of vortices and the time of occurrence of glitches will be discussed in sections 4 and 5, respectively. We will present our results in section 6. Here, we will also make a brief comment on the future scope of this study. Finally, we conclude our work in section 7.

2 CRUSTQUAKE: THE BASIC FEATURES

The crustquake in a neutron star is caused due to the existence of a solid elastic deformed crust of thickness about 1 km (Ruderman 1969; Smoluchowski & Welch 1970). The deformation of the crust is characterised by its ellipticity/oblateness $\epsilon = \frac{I_{xx} - I_{zz}}{I_{zz}}$, where $I_{xx}$, $I_{zz}$ and $I_0$ are the moment of inertia about z-axis (rotation axis), x-axis and of the spherical star, respectively (Baym & Pines 1971). At an early stage of formation, the crust solidified with initial oblateness $\epsilon_0$ (unstrained value) at a much higher rotational frequency of the star. As the star slows down, the oblateness $\epsilon(t)$ decreases, leading to the development of strain in the crust. Once the critical stress is reached, crust cracks followed by a sudden change of oblateness $\Delta \epsilon$. As a result, the moment of inertia (MI) of the star decreases, which leads to an increase in its rotational frequency. In the crustquake model, the glitch size is related to $\Delta \epsilon$ through $\Delta \epsilon / \epsilon_0 = \Delta \Omega / \Omega_0 = d \epsilon / \epsilon_0$, and it is completely determined by the extent to which $\Delta \epsilon$ is changed due to crustquake. The interglitch time being proportional to $\Delta \epsilon$, is also determined by the change of oblateness. Note that for crustquake to be a successful model for glitches, it should be a regular event for a pulsar, with a frequency of once in a few years. This requires the crustal strain to be released partially in a crustquake event. Equivalently, the fractional strain release, $\eta = \frac{\Delta \epsilon}{\epsilon_0}$ should always be less than unity. In fact, smaller the value of this fraction, larger the number of crustquake events that are likely to occur during the rotational history of a pulsar. In our model, we will take a fixed value of $\Delta \epsilon = 10^{-8}$ motivated by the crustquake model of glitches for Crab like pulsars. By 'Crab like', we mean pulsars with the characteristic age $\epsilon_{21}$ and glitch size similar to that of Crab pulsar. We will show below that the condition $\eta < 1$ will be satisfied with the realistic values of star’s ellipticity $\epsilon$, which is determined by the critical (breaking) strain $u_{cr}$ that a star can sustain without breaking.

We will now present the relationship between the critical strain $u_{cr}$ and the ellipticity of the star $\epsilon$ by mentioning a few relevant parameters (see Baym & Pines 1971; Jones 2002 for details). The total energy of the deformed pulsar is given by $\langle E \rangle = E_0 + \frac{L^2}{2I} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2$. Where $E_0$ is the contribution of gravitational potential energy of the spherical pulsar. $L$ and $I$ are the angular momentum and the moment of inertia of the deformed pulsar, respectively. The coefficient $A$ ($\simeq 10^{53}$ erg) arises as a correction of gravitational energy due to deviation from sphericity. The coefficient $B$ ($\simeq 10^{48}$ erg) is related to the modulus of rigidity of the star’s crust (Baym & Pines 1971; Jones & Andersson 2001). Within an approximation $B << A$, the upper bound on star’s ellipticity can be written in terms of the critical strain $u_{cr}$ as $\epsilon < B/A u_{cr} \approx 10^{-7} u_{cr}$.

Using the above equation, the possible upper bound on $\epsilon$ can be obtained from the values of $u_{cr}$, as estimated theoretically in several works (Horowitz & Kadu 2009; Chugunov & Horowitz 2010; Baiko & Chugunov 2018; Horowitz & Kadu 2009) have done detailed molecular dynamics simulations to estimate the magnitude of crustal breaking strain of neutron star. Simulations were performed through modelling the crust as monocrystalline and polycrystalline materials and they obtained the critical strain $u_{cr} = 0.1$. Substituting this in Eq. (2), we get $\epsilon < 10^{-6}$. Recently, Baiko & Chugunov (2018) have followed a semi-analytical approach to study the crustal properties of a neutron star, including the analysis on crustal breaking strain. For polycrystalline materials, they have obtained the value $u_{cr} = 0.04$. For this value, the upper bound of ellipticity is given by $\epsilon = 0.4 \times 10^{-6}$. From the observational perspective, there were several attempts (Abadie et al. 2011; Aasi et al. 2013, 2014; Abbott et al. 2020) to constrain star’s ellipticity by observations. Any asymmetric mass distribution of pulsar relative to its rotation axis, such as triaxiality (Jones 2002), mountains (Haskell et al. 2006; Bhattacharyya 2020) that can be characterised by ellipticity parameter are the source of continuous gravitational waves. As the strain amplitude of such gravitational waves is proportional to $\epsilon$, their non-observation naturally puts an upper limit on the ellipticity of the star. Here, we mention recent results by Abbott et al. (2020), that are based on the analysis of LIGO and VIRGO data obtained from the searches of continuous gravitational waves from a few isolated pulsars. The results were presented for three recycled pulsars, along with two relatively young pulsars Crab and Vela. We will quote the results for Crab and Vela that are relevant in the context of our present model. As per the analysis in Abbott et al. (2020), the upper limits of $\epsilon$ were constrained at $10^{-7}$ and $10^{-8}$ for Crab and Vela, respectively.

The typical fractional strain released $\eta = \frac{\Delta \epsilon}{\epsilon_0}$ can now be obtained for a fixed value of $\Delta \epsilon = 10^{-8}$ and putting the values of $\epsilon$ as quoted above. Firstly, within the theoretical uncertainties in the estimate of $u_{cr}$, the values of $\epsilon$ in the range $(1.0 - 0.4) \times 10^{-6}$ provides $\eta$ in the range $\sim 0.01 - 0.02$. For the values of $\epsilon$ as constrained by the observations, $\eta$ will be even smaller. For Crab and Vela, the values are given by $\eta = 10^{-3}$ and $\eta = 10^{-4}$, respectively.
As we see from above, the set of values of \( \eta \) satisfy the condition \( \eta < 1 \) quite comfortably. Hence, we will take \( \Delta \epsilon = 10^{-8} \) throughout this work to be consistent with the conditions that are required in the crustquake model, i.e., the glitch size, interglitch time and the fractional strain released. The corresponding value of strain energy is then given by \( \Delta E = B \Delta \epsilon \approx 10^{40} \text{ erg} \). We assume that the released energy is absorbed in the inner crust and excites the neutrons in the bulk neutron superfluid. We will show that the excited neutrons, in turn, can unpin a large number of vortices through neutron-vortex scattering from a local region in the equatorial plane. We calculate the number of unpinned vortices and estimate the glitch size. Note, with the fixed value of \( \Delta \epsilon = 10^{-8} \), the interglitch time is always fixed to be about one year, irrespective of the glitch size produced by the local unpinning in our model. We will show that for Crab like pulsars and Vela, the glitch size still can vary in the range \( \sim 10^{-9} - 10^{-7} \), without affecting the interglitch time.

3 EXCITATION OF SUPERFLUID NEUTRONS AND GLITCHES

We assume that superfluid vortices are pinned at \( t = 0 \), and a fraction of these vortices get unpinned by neutron excitations caused by crustquake at \( t = t_p \). The interglitch time \( t_p \) is expected to be of the same order as the time duration of successive crustquake events. We take the picture that crustquake occurs in a local region around the equatorial plane in the outer crust of the star (see Fig. 1). We choose the quake site to be in the equatorial plane motivated by the picture proposed by Baym & Pines (1971) in their work on the crustquake model for glitches. There was also a detailed study (Franco et al. 2000) on the development of the crustal strain, which arises due to the slowing down of the star. By including the effects of the magnetic field, the authors have calculated the strain angle and found out that the strain angle is maximum in the equatorial plane, making it most likely place for the quake site.

The absorption of strain energy \( \Delta E = B \Delta \epsilon \) should excite the free superfluid neutrons that exist outside the nuclei surrounding the pinned vortices. The sharing of energy through scattering by these excited neutrons with the vortex core neutrons should result in the unpinning of vortices causing the glitch event. Assume \( \Omega_c \) is the angular velocity of the pinned vortices that remains fixed during \( t = 0 \) to \( t = t_p \). \( \Omega_c(t) \) is the angular velocity of the corotating crust-core coupled system with \( \Omega_c(0) = \Omega_c \). The development of differential angular velocity \( \delta \Omega = \Omega_p - \Omega_c(t) \) between vortices and the rest follows the time evolution of the star and can be written as (at \( t = t_p \))

\[
\frac{\Omega_p(t_p) - \Omega_c(t_p)}{\Omega_c(t_p)} = \left( \frac{\delta \Omega}{\Omega} \right)_{t_p} \simeq \frac{2}{\tau} t_p,
\]

where \( \tau = -\frac{\Omega}{\delta \Omega} \) is the characteristic age of pulsar and we assume \( t_p \ll \tau \). For ease of notation, from now onward we assume, \( (\frac{\delta \Omega}{\Omega})_{t_p} \equiv \frac{\Omega_p}{\Omega} \). The glitch size can be estimated applying the model of superfluid vortices,

\[
\frac{\Delta \Omega}{\Omega} = \left( \frac{\delta \Omega}{\Omega} \right) \left( \frac{N_v}{N_{cr}} \right),
\]

where \( \frac{\delta \Omega}{\Omega} \) is the MI ratio of bulk neutron superfluid in the inner crust to the rest of the star (Ruderman 1976). \( N_{cr} \) is the total number of pinned vortices in the equatorial plane in the inner crust. The ratio \( \frac{N_v}{N_{cr}} \) is incorporated since only a fraction of vortices is expected to be affected by the excited neutrons. In standard superfluid model (Anderson & Holz 1975), the above ratio is almost unity and \( \delta \Omega \) should be replaced by its critical value \( \delta \Omega_{cr} \). Where \( \delta \Omega_{cr} \) is the maximum value of \( \delta \Omega \) at which the magnus force balances the pinning force. The magnus force per unit length on a vortex line is given by \( f_m = \rho \kappa R \Omega \). Equating this with the pinning force per unit length \( f_p = \frac{E_p}{\kappa R} \), we get (Alpar et al. 1984),

\[
\delta \Omega_{cr} = \frac{E_p}{\rho \kappa R \Omega_c},
\]

where, \( E_p \) and \( \rho \) are the pinning energy per pinning site (to be estimated later) and the local mass density, respectively. \( \kappa = \frac{\hbar}{2m} \) is the quantum vorticity with \( m \) being the neutron mass. \( \xi \simeq 10 \text{ fm} \) is the coherence length of the bulk superfluid, and the nucleus-nucleus distance is denoted by \( b \) (\( \approx 100 \text{ fm} \)). \( R \) (\( \approx 10 \text{ km} \)) is the distance of the inner crust from the centre of the star. The numerical value of \( \delta \Omega_{cr} \) will be estimated in the next section. We now estimate the number of vortices \( N_v \) that are expected to be affected by the neutron excitation. First, we take a region of volume \( V_p \) within which the free neutrons should be excited. The relevant volume can be estimated by energy balance as

\[
B \Delta \epsilon = N_v \Delta f = \frac{\Delta^2}{E_f} n_f V_p.
\]

or equivalently,

\[
V_p = \frac{B \Delta \epsilon}{n_f \Delta f},
\]

where the free neutron number density in the superfluid state is denoted by \( n_f \), and \( \Delta f \) denotes the superfluid free energy gap of the neutrons. \( N_v \) is the number of excited neutrons. The mass density \( \rho \) of the inner crust lies in the range \( \sim (10^{11} - 10^{14}) \text{ gm-cm}^{-3} \). The Fermi momentum of the free neutrons at \( \rho \approx 5 \times 10^{11} \text{ gm-cm}^{-3} \) has been calculated by several authors (Pastore et al. 2011) and found to be of order \( k_f \approx 0.2 \text{ fm}^{-1} \). The corresponding value of neutron density is given by

\[
n_f = \frac{k_f^3}{2 \pi^2} = 2.7 \times 10^{-4} \text{ fm}^{-3},
\]

which increases as one goes deeper in the crust. For our case, the relevant region of interest is the outer part of the inner crust, and we will take the above value for the estimate of \( V_p \). The superfluid gap parameter \( \Delta f = 0.06 \text{ MeV} \) for \( k_f \approx 0.2 \text{ fm}^{-1} \) (Pastore et al. 2011) and (Chamel & Haensel 2008) and (Sinha & Sedrakian 2015). Putting the values of various quantities in Eq. (7), we get \( V_p = 5.1 \times 10^5 \text{ m}^3 \). Note that the calculation of \( V_p \) assumes the isotropic distribution of energy from the quake site, and the geometry is taken to be a cubical shape as shown in Fig. 1.

The isotropic distribution of energy from the quake site is an assumption in estimating the volume of the
affected region. In principle, the volume of the affected region should depend on the Fermi energy ($E_f$) and pairing energy ($\Delta_f$) of the neutron superfluid in that region. Although, the distances to which the energy transports in the azimuthal direction (with respect to the rotation axis) and along the altitude of the star are expected to be the same, the distance across the inner crust should be different from the other two directions. In this work, we will not take into account such anisotropy and assume cubical geometry only. Here, we mention the works of Lander et al. (2015) and Akbal & Alpar (2018), where the authors have considered a cubical geometry in their respective studies. Lander et al. (2015) have studied the crustquake event due to the development of magnetic strain as a result of internal magnetic field evolution of the star. In their study, the authors have considered a cubical crustquake geometry to calculate the required magnetic field strength for crust breaking. Similarly, Akbal & Alpar (2018) have studied vortex unpinning (Although, the unpinning mechanism is completely different from ours.) due to the crustal plate movement triggered by the quake. In their work, the size of the broken plate and the number of unpinned vortices are calculated by modelling cubical shape as one of the quake site geometries. The number of unpinned vortices estimated by the authors turned out to be of a similar order, irrespective of the assumed geometries in their work. In this spirit and for simplifying calculations, we consider a cubical geometry to test our model by estimating the volume $V_p$ of the affected region, number of unpinned vortices $N_v$, and the glitch size $\Delta\Omega / \Omega$.

Now denoting $\Delta l$ as the length of each side of the cube, we now express the volume $V_p$ in terms of number of vortices $N_v$ as

$$V_p = (\Delta l)^3 = N_v \frac{\Delta l}{n_v},$$

where $n_v = \frac{2m_n \Omega}{\hbar^2} = 10^3 \text{ cm}^{-2} \left( \frac{\Omega}{s^{-1}} \right)$ is the areal density of vortices. Number of vortices in the equatorial plane, which are expected to be unpinned due to neutron excitation, can be estimated using Eq. (7) and Eq. (8) as

$$N_v = \frac{B \Delta \epsilon n_v E_f}{\Delta l n_f \Delta_f^2} = 3.1 \times 10^{11} \left( \frac{\Omega}{s^{-1}} \right).$$

In the above equation, the numerical factor has been calculated by taking $\Delta l = (V_p)^{1/3} = 172$ m and the values of other parameters are taken as mentioned earlier. Note that for Crab/Vela $\Omega \approx 10^{-2}$, there are about $N_v \approx 10^{13}$ vortices which can be released from the volume $V_p$. Finally, using Eq. (9) and Eq. (10), the glitch size is obtained as

$$\frac{\Delta \Omega}{\Omega} = \left( \frac{6.2 \times 10^{11}}{N_{\text{cr}}} \right) \left( \frac{I_p}{I_c} \right) \left( \frac{t_p}{7} \right),$$

where the total number of vortices in the crust is given by

$$N_{\text{cr}} \approx (2\pi R \Delta R) n_v = \left( \frac{6.3 \times 10^{11}}{\Omega} \right).$$

Here, $\Delta R \approx 1$ km is the thickness of the crust. Substituting the value of $N_{\text{cr}}$ in Eq. (10), we get

$$\frac{\Delta \Omega}{\Omega} \approx 10^{-3} \left( \frac{I_p}{I_c} \right) \left( \frac{t_p}{7} \right).$$

By taking $t_p$ of the same order as typically observed inter-glitch time of pulsars, we estimate the glitch size using Eq. (12) and results are discussed in section 6. Of course, the glitch will arise provided $N_v \approx 10^{13}$ vortices are unpinned by neutron excitation from the region of our interest. We now discuss the mechanism which ensures the unpinning of the vortices.

4 MECHANISM OF LOCAL UNPINNING

We assumed that the crustquake occurs in the outer crust in the vicinity of the equatorial plane and the energy released by this event is distributed isotropically from the quake site. The energy $\Delta E \approx 10^{40}$ erg absorbed in the volume $V_p = 5.1 \times 10^6$ m$^3$ should excite a $N_v = \frac{2m_n \Omega}{\hbar^2} V_p \approx 10^{47}$ number of neutrons from the bulk neutron superfluid. Ignoring small finite temperature ($kT \approx 0.01$ MeV) correction, each of these excited neutrons has approximately $E_f = 0.83$ MeV amount of energy. If the total energy of the excited neutrons is more than the pinning energy of all vortices enclosed in volume $V_p$ (see Fig. 1), then the inelastic scattering of these neutrons with the vortex core neutrons should unpin the vortices. Note that the pinning energy $E_p$ acts as the binding energy of the vortex-nucleus system, and it arises due to the interaction of the vortex with the nucleus. Thus the sharing of energy by the excited neutrons with the vortex core neutrons increases the kinetic energy of the later. In fact, the energy of the excited neutrons equivalently can be treated as the activation energy, which helps to overcome the pinning barrier. We will show below that the excited neutrons have the required energy to overcome the barrier. The inelastic collision can be represented as,

$$\text{excited neutron} \overset{\sim}{\rightarrow} E_f \rightarrow \text{de-excited neutron} \overset{E_f - E_P}{\rightarrow} \text{free vortex}.$$

The quantities in brackets denote the energy of various objects. The negative sign in front of $E_p (>0)$ signifies the binding energy of the pinned vortex. It should be noted that
following unpinning; the vortex is free to move (i.e., free vortex) outward with radial velocity \(v_r\). Here we make a few comments on the possibility of repinning of the outward moving vortices. Since the study of repinning initiated by Sedrakian (1995), the mechanism and consequences of repinning have been discussed often in the literature, whether in the context of creep theory (Alpar et al. 1984), or in the standard theory of superfluid vortices (Anderson & Itoh 1975). One of the important consequences of repinning, namely, the acoustic radiation is quite relevant to our present model. As per the studies in Warszawski et al. (2012), the acoustic radiation caused by repinning (so-called ‘acoustic knock-on’ as per the terminology used in Warszawski et al. (2012)) is believed to play an important role in the process of vortex avalanche. The avalanche can be a viable process for our model to produce large size glitches through acoustic knock-on (and proximity knock-on) caused by repinning of outgoing vortices. In this present work, we skip the studies of vortex avalanche (which will be explored in future) except making a few comments in the results & discussion section.

We now compare the pinning energy with the energy of excited neutrons. The pinning mechanism as per discussion in Ref. [Alpar et al. 1984] depends on the density \(\rho\). It was suggested that for \(\rho > 10^{13}\) gm cm\(^{-3}\), the vortex lines are pinned to lattice nuclei with pinning energy per site (Alpar et al. 1984, 1989). For the density \(\rho > 10^{13}\) gm cm\(^{-3}\), the lines are preferably pinned in between nuclei (interstitial pinning) and the pinning energy \(E_p\) has been calculated to be of the order 1 KeV per site (Link & Epstein 1991).

\[
E_p = \frac{3}{8} \frac{\Delta^2}{\gamma} E_f n_f V \simeq 5.8 \times 10^{-4} \text{ MeV.} \tag{13}
\]

Where \(V = \frac{4}{3} \pi \xi^3\) is the overlap volume between the vortex and the nucleus. The size of vortex core \(\xi\) (\(\approx 10\) fm) is the same order as the nuclear radius. The numerical value of \(\gamma\) is of order unity [Alpar et al. 1984, 1989]. For the density \(\rho < 10^{13}\) gm cm\(^{-3}\), the lines are unpinned, the pinning force on the whole vortex lines is of order unity (Alpar et al. 1984, 1989). For the density \(\rho = 10^{13}\) gm cm\(^{-3}\), the lines are unpinned, the pinning force is decreased enough for the vortex lines to pass through the volume \(V\).

The decrease of pinning force per unit length due to the neutron-vortex scattering. Since all the vortices that lie inside the volume \(V\), are unpinned, the pinning force on the whole vortex lines passing through \(V\) is reduced. The fractional decrease in pinning force per unit length is given by

\[
\frac{\Delta f_i}{f_i} = \frac{\Delta l}{l} \simeq 0.1, \tag{16}
\]

where \(f_i\) is the pinning force per unit length [Anderson & Itoh 1975, Pizzochero 2011] of vortex lines and \(\Delta f_i\) is the decrease of pinning force per unit length due to the effects as mentioned above. The above numerical value in Eq. (16) has been calculated by taking the length of a vortex line (assuming straight), threaded in the inner crust (see Fig. 1) as, \(l \simeq \sqrt{2R\Delta l} \simeq 1855\) m (\(R = 10\) km).

The decrease in pinning force per unit length for a vortex line must reduce the critical differential angular frequency, \(\delta \Omega_{cr}\). The numerical value of \(\delta \Omega_{cr}\) can be estimated from Eq. (5). Taking \(E_p = 5.8 \times 10^{-4}\) MeV from Eq. (13) and \(\rho = 5 \times 10^{11}\) gm cm\(^{-3}\), we get \(\delta \Omega_{cr} = 0.09\) rad s\(^{-1}\). The fractional decrease of the above quantity should be of the same order as \(\Delta f_i/l\), i.e.,

\[
\frac{\Delta (\delta \Omega_{cr})}{\delta \Omega_{cr}} \simeq 0.1. \tag{17}
\]

If the build-up differential angular frequency \(\delta \Omega\) at \(t_p\) does not differ much from its critical value \(\delta \Omega_{cr}\), then the magnus force effectively should be able to move the vortex lines from the pinning site. The relative difference can be estimated as

\[
\frac{\delta \Omega - \delta \Omega_{cr}}{\delta \Omega_{cr}} = 1 - \left(\frac{2\Omega}{\delta \Omega_{cr}}\right) = 1 - x. \tag{18}
\]

The numerical value of \((1 - x)\) can be determined by taking the typical values for Crab/Vela, \(t_p = 1\) year, \(\tau = (10^3 - 10^4)\) years and \(\Omega \simeq 10^2\) s\(^{-1}\). For these set of values, the numerical value of \((1 - x)\) turns out to be of a similar order, such that Eq. (17) is satisfied. Note, the purpose of the above exercise is to check whether the pinning force is decreased enough for the vortex lines passing through the volume \(V\) to move under magnus force, even when \(\delta \Omega\) at \(t = t_p\) is less than the critical value \(\delta \Omega_{cr}\). It is indeed true, as suggested by the above set of arguments. We conclude this part by saying that crustquake initiated neutron excitations can unpin a large number of vortex lines which pass through the volume \(V\) and hence produces glitch through the local unpinning.

5 THE TIME OF OCCURRENCE OF GLITCHES FOLLOWING CRUSTQUAKE

Now we estimate the time of occurrence of glitches \(t_g\) following the crustquake event. The time \(t_g\) is the sum of the time taken for unpinning followed by the time for the vortex to move outward and share their excess angular momentum to the crust. First, we estimate the time for unpinning, which is determined by the relaxation time scale \((\tau_{\text{unpin}})\) of neutron-vortex scattering. The calculation
of $\tau_{nn}$ deserves a separate work, and we will provide here an order of magnitude estimate following the approach of Feibelman (1971). To understand postglitch behaviour of pulsars, the author (Feibelman 1971) has worked out a detailed calculation of relaxation time scale $\tau_{nn}$ of electrons by considering the scattering of thermal electrons with the vortex core neutrons. The contribution of neutron-vortex scattering was not considered in their calculation. It is mainly due to the presence of very few thermally excited neutrons in the bulk superfluid at a lower temperature ($kT \simeq 0.01$ MeV) of the star. Note that the probability of excited neutrons relative to superfluid neutrons is given by $e^{-\Delta f/\mu}$, and the scattering of these neutrons with the vortex core neutrons should not be suppressed, and it should naturally provide a finite relaxation time scale $\tau_{nn}$.

We will now estimate $\tau_{nn}$ following the expression of Feibelman (1971),

$$
\tau_{nn} = \left( \frac{\Omega_{en}}{\Omega} \right) \left( \frac{4}{3\pi^2 g_n} \right) \left( \frac{E_f}{E_{en}} \right)^2 \left( \frac{E_{en}}{\Delta_e} \right) \left( \frac{E_{en}}{2m_e c^2} \right)^{1/2} \times \left( \frac{\hbar}{\Delta_e} \right) \left( \frac{\exp(\frac{\Delta_e}{\sqrt{\beta f}})}{K_0(\frac{1}{\sqrt{\beta f}})} \right) \exp(\frac{\pi \Delta_e^2}{4 \Delta f E_f}) \text{s}. \tag{19}
$$

Here, $\frac{\Omega_{en}}{\Omega}$ is the ratio of upper critical angular speed of neutron fluid ($\Omega_\text{en} = 10^{20}$) to the angular speed of the star. For Crab/Vela the ratio is of order $10^{-8}$.

The coupling strength associated with neutron-neutron interactions can be described by the neutron $g$ factor, $g_n = -1.91$. $K_0(\frac{1}{\sqrt{\beta f}})$ is the zero order Bessel function. $E_{f}$ and $\Delta_e$ denote the Fermi energy and superfluid gap parameter associated with the neutron vortex core, respectively. The factor $\beta = \frac{\sqrt{\pi}}{2}$ in the calculation of $\tau_{nn}$ Feibelman (1971) is due to the finite temperature probability distribution of electrons. In our case, $\beta$ should be replaced by bulk superfluid energy gap $\Delta f$.

For the case of nuclear-vortex pinning, we take the approximation $E_{fv} \simeq E_f = 0.83$ MeV and $\Delta_{fv} \simeq \Delta_f \simeq 0.06$ MeV. For interstitial pinning, the approximation will be replaced by equality. Substituting these quantities in Eq. (19), we get

$$
\tau_{nn} = 3.0 \times 10^{-5} \text{s}. \tag{20}
$$

Thus, as we see from Eq. (20), the unpinning time scale for electron $\tau_{en}$ has been estimated Feibelman (1971) to be on the order of days to years. The shorter time scale of $\tau_{nn}$ as compared to $\tau_{en}$ is expected due to the difference of coupling strength in electron-neutron and neutron-neutron interactions. The former is dipole-magnetic moment interaction and hence the strength of the interaction is proportional to $\alpha g_{en}$, where $\alpha = \frac{e^2}{\hbar} = \frac{1}{\sqrt{\pi}}$. The later interaction is solely due to magnetic moment of the neutrons with the interaction strength proportional to $g_n^2$. We should mention that a detailed calculation is required for the precise estimate of $\tau_{nn}$. From the perspective of the occurrence of glitches following crustquake, as we see below that even a few orders of magnitude change in the value of $\tau_{nn}$ will have negligible contribution to $t_g$. Next, we determine the time ($t_g$) taken by the unpinned vortices to move toward the outer crust and share their excess angular momentum to the crust. The radial velocity $v_r$ of the unpinned vortices is $v_r = \frac{\Delta \Omega}{\Omega} = \frac{2\pi}{\sqrt{\beta}}$ $\Delta R \simeq \left(10^{-8} - 10^{-9}\right)$ cm$^{-1}$ s$^{-1}$, where we have used Eq. (21), and $\tau \simeq \left(10^3 - 10^4\right)$ years are the age of Crab and Vela, respectively. Thus the time $t_g \simeq \frac{2\pi}{\sqrt{\beta}}$ lies in the range $\sim (0.17 - 1.7)$ s. We see that the glitch due to vortex unpinning occurs at $t_g = \tau_{nn} + t_c \simeq (0.17 - 1.7)$ s after the crustquake. The implication of the time of occurrence $t_g$ of pulsar glitches in our model will be discussed in the next section.

Note that the change of oblateness of the star due to crustquake is also expected to produce a glitch (as per the crustquake model) of order $10^{-5}$. The glitch rise time $\Delta t$ is approximately determined Ruderman (1971), Haskell et al. (2015) by the speed of shear wave $v = \sqrt{\rho c} = 3 \times 10^8$ cm s$^{-1}$. Where $\mu = 10^{30}$ dynes-cm$^2$ is the shear modulus of the crust and $\rho \simeq 10^{13}$ gm-cm$^{-3}$ is the average crust density. Thus, the time for the shear wave to propagate along the stellar surface of the radius ($R$) is given by $\Delta t \simeq \pi R/v = 0.01$ s Baym & Pines (1971). From the perspective of distinguishability of the glitches produced by two different sources, the glitch rise time $\Delta t$ in the crustquake model needs to be compared with the time of occurrence of glitch $t_g$ produced by local unpinning. We will discuss this issue in the next section.

6 RESULTS AND DISCUSSION

We have discussed (in section 4) our novel mechanism of local unpinning of vortices in a given region caused by sharing of energy by the excited neutrons with the vortex core neutrons. We will now estimate the glitch size $\Delta \Omega$ caused by the local unpinning for Crab like pulsars and Vela pulsars using the Eq. (12). The glitch size depends on the number of vortices $N_v$ released due to local unpinning. For the fixed input energy $B \Delta \epsilon$, this number depends on the properties of bulk neutron superfluid via $E_f$ and $\Delta_f$. The values of $E_f$ and $\Delta_f$ are taken from the literatures as noted in section 3. The energy input is provided by the strain energy released $B \Delta \epsilon$ due to the crustquake. The value of this energy ($\sim 10^{40}$ erg) is set based on the arguments provided in section 2. The interglitch time $t_g$ is set by the frequency of occurrence of crustquake events and is proportional to the change of oblateness $\Delta \epsilon$ due to crustquake. We choose $\Delta \epsilon = 10^{-5}$ in accordance with the interglitch time of approximately one year. The ratio of moment of inertia of superfluid component in the inner crust to the rest of the star is of order $t_g^2 \simeq 10^{-2}$ as per the evidences through several studies (see, for example, Ref. Ruderman (1976)). Putting these values in Eq. (12) and by taking the typical characteristic age of Crab and Vela in the range $\tau \simeq \left(10^{-8} - 10^{-3}\right)$ years, we get the glitch size ($t_g \simeq 1$ year) as

$$
\frac{\Delta \Omega}{\Omega} \simeq 10^{-8} - 10^{-9}, \tag{21}
$$
where the relatively larger (smaller) value of glitch size is for Crab (Vela) pulsar.

The above estimate of glitch size corresponds to unpinning of $N_c \approx 10^{13}$ vortices (Eq. (9)), out of total $N_c \approx 10^{17}$ vortices (Eq. (11)) present in the inner crust. Note that the MI ratio $\frac{I_p}{I_c}$ is taken as $10^{-2}$ for the estimate of glitch size. It was suggested (Ruderman 1976) that this ratio takes different values depending on the presence or absence of normal neutron fluid (called as ‘transition region’) between the inner crust and the interior neutron superfluid. In the presence of a normal layer, the unpinned vortices are required to share their excess angular momentum to a relatively larger part of the corotating system and hence, $\frac{I_p}{I_c} \approx 10^{-5}$ is relatively smaller. In the absence of such layer, the above ratio is increased and approximately is given by $\frac{I_p}{I_c} = 0.1$. In fact, Piekarewicz et al. (2014) suggested that within theoretical uncertainties in the equation of state, the neutron star can have $\frac{I_p}{I_c} = 0.1$. For this value, there will be about one order of magnitude enhancement in the glitch size. For Crab like pulsars and Vela, the glitch size can be of order $10^{-7}$ and $10^{-5}$, respectively. Interestingly, such glitches have been observed for Crab (Basu et al. 2020) & Melatos 2012b) fit quite well in our model. In proximity knock-on, presence of the azimuthal component of the vortex velocity can make an individual vortex knock-on the other vortices present in the equatorial plane and nearby the cubical volume $V_c$. Note that for completely outward motion, the vortices should not encounter vortices along its trajectory. In our future work, we would like to determine the trajectory of unpinned vortices, estimate the number of vortices, and $\Delta t \sim 6$ s.

For this value, there will be about one order of magnitude enhancement in the glitch size. For Crab like pulsars and Vela, the glitch size can be of order $10^{-7}$ and $10^{-5}$, respectively. Interestingly, such glitches have been observed for Crab (Basu et al. 2020) & Melatos 2012b). We proposed a novel mechanism for the unpinning of vortices (Melatos et al. 2008) which will be explored in our future work. First, we anticipate that almost instantaneous release of about $10^{13}$ vortices may act as a trigger mechanism to unpin the nearby vortices, which lie in the equatorial plane. Among various suggestions on vortex avalanches (Melatos et al. 2008; Warszawski & Melatos 2012b; Akbal & Alpar 2018), we find that the knock-on pictures (Melatos et al. 2008; Warszawski & Melatos 2012b) fit quite well in our model. In proximity knock-on, presence of the azimuthal component of the vortex velocity can make an individual vortex knock-on the other vortices present in the equatorial plane and nearby the cubical volume $V_c$. Note that for completely outward motion, the vortices should not encounter vortices along its trajectory. In our future work, we would like to determine the trajectory of unpinned vortices, estimate the number of vortices, and $\Delta t \sim 6$ s. The acoustic knock-on caused by repinning of vortices can also be a viable process in our model. In future, we would like to implement these mechanisms to study the avalanche process.

Other than glitches, another interesting phenomenon of inhomogeneous vortex line movement could be the generations of gravitational waves from an isolated pulsar (Jones 2002; Bagchi et al. 2018; Layek & Yadav 2020). We would like to explore this possibility in future following the approach of Warszawski & Melatos (2012a), and estimate the strain amplitude associated with the gravitational waves.

7 CONCLUSION

We proposed a novel mechanism for the unpinning of superfluid vortices in the inner crust of a pulsar. It occurs through the scattering of excited neutrons with the vortex core neutrons. The excitation of neutrons are caused by the
absorption of strain energy released due to the crustquake event. We take a cubical shape region near the most probable quake site around the star’s equatorial plane and determine the volume (∼ 10^6 m^3) where a fraction (∼Δv/v) of bulk superfluid neutrons are excited. The scattering of these excited neutrons with the vortex core neutrons results in the unpinning of vortices from the above volume. The Crab and Vela pulsar with Ω ≃ 10^9 s^{-1} can release about 10^{13} vortices as a result of local unpinning. The size of the glitches have been estimated to lie in the range 10^{-8} − 10^{-7}, and ∼ 10^{-5} − 10^{-8} for Crab and Vela pulsar, respectively. The glitches, though vary in size, have the same frequency of occurrence of about once in a year.

We estimated the relaxation time scale of excited neutrons through neutron-vortex scattering and the value of τ_{nn} ≃ 10^{-3} s justifies the absence of multiple glitches within the time interval of a few days or months. The glitch rise time t_{c} ∼ (0.2 − 2) s in our model also turns out to be consistent with the typical feature of the glitch profile (sudden spin-up event). At the same time, this common feature (i.e., small glitch rise time) of all crustquake initiated glitch models makes it difficult to choose one among various models [Ruderman 1991 Link & Epstein 1996 Akbal & Alpar 2018].

The model for unpinning proposed here has the potential to explore further by implementing the knock-on picture to study vortex avalanches. Also, sudden release of a large number of vortices can have consequences on the emission of gravitational radiation. We would like to explore these in our future work. Finally, though we have discussed the excitation of neutron superfluid initiated by crustquake, this proposal is very generic. The approach used here should be applicable for any other sources, which have the potential to excite the superfluid neutrons, or can make superfluid - normal phase transition in the inner crust of a pulsar. It will be interesting to look for such sources.

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9 DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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