Formation of Kuiper Belt Binaries

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It appears that at least several percent of large Kuiper belt objects are members of wide binaries. Physical collisions are too infrequent to account for their formation. Collisionless gravitational interactions are more promising. These provide two channels for binary formation. In each, the initial step is the formation of a transient binary when two large bodies penetrate each other’s Hill spheres. Stabilization of a transient binary requires that it lose energy. Either dynamical friction due to small bodies or the scattering of a third large body can be responsible. Our estimates favor the former, albeit by a small margin. We predict that most objects of size comparable to those currently observed in the Kuiper belt are members of multiple systems. More specifically, we derive the probability that a large body is a member of a binary with semi-major axis of order $a$. The probability depends upon $\sigma$, the total surface density, $\Sigma$, the surface density of large bodies having radius $R$, and $\theta_\odot \approx 10^{-4}$, the angle subtended by the solar radius as seen from the Kuiper belt. For $(\sigma/\Sigma)R < a < R/\theta_\odot$, the probability is just $(\Sigma/\rho R)\theta_\odot^{-2}$, the optical depth of the large bodies divided by the solid angle subtended by the Sun. For $R < a < r_u \equiv (\sigma/\Sigma)R$, it varies inversely with semi-major axis and reaches $\sim (\sigma/\rho R)\theta_\odot^{-2}$ at $a \approx R$. Based on current surveys of the Kuiper belt, we estimate $\Sigma/\rho \sim 3 \times 10^{-4}$ cm and $R \sim 100$ km. We obtain $\sigma/\rho \sim 0.3$ cm by extrapolating the surface density deduced for the minimum mass solar nebula. Rough predictions are: outside of the critical separation $r_u/a_\odot \sim 3''$, the binary probability is $\sim 0.3\%$; at separations of $0.2''$, comparable to current resolving capabilities, it reaches $\sim 5\%$, in agreement with results from the HST binary survey by Brown.
1 Introduction

The Kuiper belt\(^1\) is the best solar system laboratory for studies of the early stages of runaway accretion. Runaway accretion in the Kuiper belt terminated when the velocity dispersion of its members was increased by an as yet undetermined process.\(^2\) Unlike the asteroid belt, its largest members have suffered little collisional evolution since.

The discovery that a substantial fraction of its largest members are in binaries with wide separations and order unity mass ratios\(^3-9\) is the latest of many surprises provided by the Kuiper belt. Collisions coupled with tidal evolution, mechanisms that may explain other solar system binaries, fail to account for the formation of the Kuiper belt binaries.

The low frequency of collisions among the large Kuiper belt objects\(^10\) implies that binaries formed by collisionless interactions mediated by gravity. These were most effective earlier when dynamical friction due to small bodies limited the velocity dispersion of the large ones. This situation pertains during runaway accretion.

In the following section §2, we outline a simple model for runaway accretion to set the stage for binary formation. We are guided by more detailed formalisms implemented in numerical simulations. Values of relevant parameters are estimated based on the numbers and sizes of objects deduced from Kuiper belt surveys, and by the extrapolation of the surface density in the minimum mass solar nebula. We estimate the binary formation rate and derive the semi-major axis distribution in section §3. In the final section §4, we discuss the observational implications of our results and mention several open issues.

2 Preliminaries

In this section, we introduce a simple set of equations to describe the major processes that occur during the growth of planetesimals. These equations are abstracted from more complete treatments in the literature.\(^11,12\) The main simplification in our approach is the identification of two groups of bodies, small ones, containing most of the total mass, and large ones, contributing a small fraction of it. The latter group, which dominates the stirring of all bodies, is identified with the currently observed population of Kuiper belt objects.

Gravitational interactions determine the accretion rates and velocity dispersions of both large
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and small bodies. The governing equations for these processes read:

\[
\frac{1}{\Omega_\odot} \frac{1}{R} \frac{dR}{dt} \sim \frac{\Sigma}{\rho R^2} F_{M-M} + \frac{\sigma}{\rho R} F_{M-m},
\]

(1)

\[
\frac{1}{\Omega_\odot} \frac{1}{s} \frac{ds}{dt} \sim \frac{\sigma}{\rho s},
\]

(2)

\[
\frac{1}{\Omega_\odot} \frac{1}{v} \frac{dv}{dt} \sim \frac{\Sigma}{\rho R^2} F_{M-M} - \frac{\sigma}{\rho R} F_{M-m}^2,
\]

(3)

\[
\frac{1}{\Omega_\odot} \frac{1}{u} \frac{du}{dt} \sim \frac{\Sigma}{\rho R^2} F_{M-m}^2 - \frac{\sigma}{\rho s},
\]

(4)

where \( R \) and \( s \) are the radii of the large and small bodies, \( \Sigma \) and \( \sigma \) are their surface mass densities, \( v \) and \( u \) are their velocity dispersions,\(^*\) \( \rho \) is the material density\(^†\) and \( \Omega_\odot \) is the orbital frequency around the Sun.\(^‡\)

The expressions \( \Sigma \Omega_\odot / \rho R \) and \( \sigma \Omega_\odot / \rho s \) give the kinematic (ignoring gravitational focusing) collision rates of large bodies onto large bodies and small bodies onto small bodies; \( \sigma \Omega_\odot / \rho R \) is the kinematic collision rate of small bodies onto a large one multiplied by the mass ratio \( (s/R)^3 \). The factors \( F_{M-M} \) and \( F_{M-m} \) in equation (1) account for the enhancements of the physical collision rates due to gravitational focusing by large bodies of the trajectories of incoming large and small bodies respectively. Cross sections for large deflections exceed those for physical collisions when gravitational focusing is significant. The factors \( F_{M-M}^2 \) and \( F_{M-m}^2 \) that appear in equations (3) and (4) are the ratios of those cross sections to the geometrical ones. Appropriate expressions for \( F_{M-m} \) read:

\[
F_{M-m} \sim \begin{cases} 
1 & v_{esc} < u \\
(v_{esc}/u)^2 & v_H < u < v_{esc} \\
(v_{esc}^2/v_H u) \theta_\odot^{1/2} & v_{H} < u < v_{H}
\end{cases}
\]

(5)

where \( \theta_\odot \approx 10^{-4} \) is the angle subtended by the solar radius as seen from the Kuiper belt, \( v_{esc} \) is the escape velocity from the large bodies and \( v_H \) is their Hill velocity defined in terms of the Hill radius,

\[
R_H \sim \left( \frac{M}{M_\odot} \right)^{1/3} \frac{a_\odot}{\theta_\odot},
\]

(6)

\(^*\)We are making an additional simplification by using a single velocity dispersion in place of three that are needed to characterize a triaxial velocity ellipsoid.

\(^†\)We use the same density for the Kuiper belt objects as for the Sun.

\(^‡\)In writing equations (1)-(4) we implicitly assume that \( u > v \).
The escape velocity is related to the Hill velocity by

$$v_{\text{esc}} \sim (G\rho)^{1/2} R \sim v_H/\theta_\odot^{1/2}.$$  \hspace{1cm} (8)

$F_{M-M}$ is obtained by replacing $u$ with $v$ in the above expressions and in their ranges of validity.

Each of equations (1)-(4) has a simple interpretation. In equation (1), the two terms on the rhs account for the radius growth of the large bodies by the accretion of large and small bodies. The single term on the rhs of equation (2) describes how the radii of small bodies increase as the result of coalescence under conditions of negligible gravitational focusing. In equation (3), the first term describes the viscous stirring of large bodies due to mutual gravitational deflections, whereas the second term accounts for the damping of the large bodies’ velocity dispersion by dynamical friction resulting from their gravitational interactions with the small bodies. Equation (4) shows how the velocity dispersion of the small bodies evolves under viscous stirring by large bodies and damping due to collisions between small bodies.

Values of the dimensionless parameters $\Sigma/\sigma \ll 1$ and $\theta_\odot$ determine the appropriate regime of runaway accretion. At the Kuiper belt $a_\odot \approx 40$ AU so $\theta_\odot \approx 10^{-4}$. Based on Kuiper belt surveys\textsuperscript{13-15} which detect only the large bodies, we estimate $\Sigma/\rho \sim 3 \times 10^{-4}$cm and $R \approx 100$ km. Extrapolating the minimum mass solar nebula surface density to 40 AU, yields $\sigma/\rho \sim 0.3$ cm. The resulting ratio of $\Sigma/\sigma \sim 10^{-3}$ is compatible with Kuiper belt simulations.\textsuperscript{16-18} In the rest of this paper we assume that

$$\theta_\odot < \frac{\Sigma}{\sigma} < 1.$$  \hspace{1cm} (9)

In this regime of runaway accretion, the rates of gravitational stirring and dynamical friction acting on the large bodies are much greater than the large bodies’ growth rates. Moreover, provided the radii of the small bodies satisfy

$$s > \frac{\Sigma}{\sigma} R \sim 0.1 \text{ km},$$  \hspace{1cm} (10)

their growth rates are negligible compared to those of the large bodies and collisional damping of their velocities is unimportant on the timescale of viscous stirring.\textsuperscript{8} Under these approximations,"
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dynamical friction on the large bodies balances their viscous stirring, and the stirring of the small bodies proceeds with negligible collisional losses. Equations (1)-(4) yield

\[
\frac{v}{v_H} \sim \frac{1}{\theta_\odot} \left( \frac{\Sigma}{\sigma} \right)^{3/2} \sim 0.3
\]  

(11)

and

\[
\frac{u}{v_H} \sim \left( \frac{1}{\theta_\odot} \frac{\Sigma}{\sigma} \right)^{1/2} \sim 3.
\]  

(12)

Thus, the velocity ratio

\[
\frac{u}{v} \sim \theta_\odot^{-1/2} \frac{\sigma}{\Sigma} \sim 10.
\]  

(13)

These results are in accord with the most recent simulations. 18

Given this solution, the balanced rates of viscous stirring and dynamical friction for large bodies satisfy

\[
-\frac{1}{v} \frac{dv}{dt} \bigg|_{df} = \frac{1}{v} \frac{dv}{dt} \bigg|_{vs} \sim \frac{\sigma}{\rho R} \left( \frac{\sigma}{\Sigma} \right)^2 \Omega_\odot.
\]  

(14)

Large bodies grow predominantly by accreting small bodies; The small bodies’ velocity dispersion evolves mainly by viscous stirring provided by the large bodies. These two processes proceed at equal rates:

\[
\frac{1}{R} \frac{dR}{dt} \sim \frac{1}{u} \frac{du}{dt} \sim \frac{\sigma}{\rho R} \left( \frac{\sigma}{\Sigma} \right) \Omega_\odot.
\]  

(15)

3 Binaries

Because \( v < \Omega_\odot R_H \), the gravity of a large body significantly deflects other large bodies at separations smaller than \( R_H \), whereas small bodies are only affected at separations \(^*\) smaller than

\[
r_u \sim \frac{GM}{u^2} \sim \frac{\sigma}{\Sigma} R.
\]  

(16)

Since \( v_H < u < v_{esc} \), we have \( R < r_u < R_H \).

For separations \( r < r_u \), the velocity dispersion of the small bodies is

\[
\left( \frac{GM}{r} \right)^{1/2} \sim \left( \frac{r_u}{r} \right)^{1/2} u.
\]  

(17)

In addition, since the impact parameter for a small body to arrive at radius \( r \) is \( b \sim (r_u r)^{1/2} \), the continuity equation implies that the small bodies’ number density is enhanced by a factor

\[
\frac{b^2 u}{s^2 u (r_u/s)^{1/2}} \sim \left( \frac{r_u}{s} \right)^{1/2}.
\]  

(18)

\(^*\)We assume that the small bodies are on hyperbolic orbits with respect to the large body.
3.1 binary formation

We identify two distinct channels for binary formation. Both begin when two large bodies penetrate each other's Hill spheres. This occurs at a rate

$$\frac{\Sigma}{M} R_H^2 \Omega_\odot \sim \frac{\Sigma}{\rho R} \left( \frac{1}{\theta_\odot} \right)^2 \Omega_\odot \sim 10^{-4} \, \text{y}^{-1},$$

per large body. Stabilization of such a transient binary requires energy loss on timescale $\Omega_\odot^{-1}$. This can be achieved either by dynamical friction from small bodies or by interaction with a third large body.

Dynamical friction from small bodies during time $\Omega_\odot^{-1}$ results in a fractional energy loss

$$\frac{\Delta E}{M v_H^2} \sim \frac{\sigma}{\rho R} \left( \frac{\sigma}{\Sigma} \right)^2 \sim 0.03.$$ (20)

This is also the fraction of transient binaries that become bound. Numerical simulations that verify this assertion are described in the figure. Thus, this mechanism leads to a binary formation rate per large body

$$R_1 \sim \left( \frac{\sigma}{\rho R} \right)^2 \left( \frac{\sigma}{\Sigma} \right)^2 \Omega_\odot \sim 3 \times 10^{-6} \, \text{y}^{-1}.$$ (21)

The probability that a third large body joins the Hill sphere of a transient binary during its lifetime is

$$\frac{\Sigma}{\rho R} \left( \frac{1}{\theta_\odot} \right)^2 \sim 3 \times 10^{-3}.$$ (22)

A significant fraction of these triplets will result in a bound binary. Therefore, the binary formation rate per large body via this channel is

$$R_2 \sim \left( \frac{\Sigma}{\rho R} \right)^2 \left( \frac{1}{\theta_\odot} \right)^4 \Omega_\odot \sim 3 \times 10^{-7} \, \text{y}^{-1}.$$ (23)

With our parameters the ratio

$$\frac{R_2}{R_1} \sim \left( \frac{\Sigma}{\sigma} \right)^3 \left( \frac{1}{\theta_\odot} \right)^2 \sim 0.1$$ (24)

is smaller than unity. Thus from here on we take $R = R_1$.

The ratio of $R$ to $R^{-1}dR/dt$ is

$$\frac{\sigma}{\rho R} \left( \frac{1}{\theta_\odot} \right)^2 \sim 3,$$ (25)

so binaries form at a rate comparable to the large bodies’ growth rate.
3.2 binary fraction and semi-major axis distribution

Let \( p(a) \) be the differential probability distribution of finding a large body in a binary with semi-major axis \( a \). In steady state\(\|^\)\n\[
   a \ p(a) \sim \frac{\mathcal{R}}{a^{-1}|da/dt|}
\]
(26)
For \( r_u < a < R_H \), \( a^{-1}da/dt \) equals the energy decay rate due to dynamical friction given by equation (14). Thus
\[
   a \ p(a) \sim \frac{\Sigma}{\rho R} \left( \frac{1}{\theta_{\odot}} \right)^2 \sim 3 \times 10^{-3},
\]
(27)
which is independent of \( a \).

For \( R < a < r_u \), two competing effects come into play. The number density of small bodies is enhanced by a factor \((r_u/a)^{1/2}\) and their velocity dispersion is increased by the same factor. Therefore \( a^{-1}|da/dt| \) is reduced by a factor \( a/r_u \) in comparison to its constant value for \( r_u < a < R_H \). Thus in this interval of semi-major axis
\[
   a \ p(a) \sim \frac{\sigma}{\rho R} \left( \frac{1}{\theta_{\odot}} \right)^2 \frac{R}{a} \sim 3 \left( \frac{R}{a} \right),
\]
(28)
which increases inwards as \( a^{-1} \).

The timescale for a binary to spiral in until contact is achieved is equal to \( R(dR/dt)^{-1} \). Contact occurs on the same time scale as that during which large bodies grow by accretion of small ones. Thus, if we define a critical separation
\[
   a_{\text{crit}} \sim \frac{\sigma}{\rho \theta_{\odot}^2} \sim 300 \text{ km},
\]
(29)
inside of which \( a \ p(a) > 1 \), then accretion by binary inspiral might make a substantial contribution before \( R \) reaches \( a_{\text{crit}} \). As long as \( R < a_{\text{crit}} \), we may also expect to find more than one companion to a given large body. However, both these assertions are uncertain since they require the stability of clusters of high multiplicity. For our parameters, \( R \sim a_{\text{crit}} \).

Exchange reactions, in which the lighter member of a binary is replaced by a heavier body which passes through the system, are rare occurrences. The rate at which large bodies pass through an existing binary of semimajor axis \( r_H \) is given by equation (19). This is smaller than the rate of orbital decay. For smaller semimajor axes, even fewer large bodies pass through during an orbital decay time.

\(\|^\) By steady state we mean during a time over which \( R \) is sensibly constant.
4 Discussion

We propose that the wide binaries observed in the Kuiper belt formed during runaway accretion. A fraction of the large bodies that entered each other’s Hill spheres became bound as the result of energy lost to small bodies by dynamical friction.** The time scale for a large body to become bound to a similar companion was

\[
\left(\frac{\rho R}{\sigma}\right)^2 \left(\frac{\Sigma}{\sigma}\right) \frac{\theta^2}{\Omega_\odot} \sim 3 \times 10^5 \text{y},
\]  

(30)

Further dynamical friction hardened the binaries. The timescale to achieve contact was that during which isolated large bodies grew:

\[
\frac{\rho R}{\sigma} \left(\frac{\Sigma}{\sigma}\right) \frac{1}{\Omega_\odot} \sim 10^6 \text{y}.
\]  

(31)

Initially the inspiral was at a constant rate, but it slowed down inside \(r_u\) where the small bodies’ number density and velocity dispersion were enhanced above their background values. We deduce that the probability that a large body is part of a binary with angular separation greater than \(r_u/a_\odot \sim 3''\) is

\[
\frac{\Sigma}{\rho R} \left(\frac{1}{\theta_\odot}\right)^2 \sim 3 \times 10^{-3}.
\]  

(32)

Inward of \(r_u\), the binary probability per logarithmic interval of semimajor axis increases inversely with semimajor axis. Close to contact, the probability exceeds unity for \(R < a_{crit} \sim 100 \text{km}\), which implies that systems with higher multiplicities would exist if such are stable. For the resolution of the HST survey \(\sim 0.2''\), we predict a binary fraction of about 5%, roughly compatible with observations (Brown, private communication). Our prediction that close binaries are common could be tested by monitoring the brightness of Kuiper belt objects for evidence of eclipses and/or fast rotation. The latter is a consequence, but not a unique signature, of binary mergers.

In our analysis we estimate the surface density ratio of large to small bodies from the observational census of Kuiper belt objects together with an extrapolation of the surface density of the minimum mass solar nebula. It would be an improvement to have a theoretical understanding of how this ratio evolves during runaway accretion. Also, we elaborated the binary evolution scenario with a given set of parameters, specifically \(\theta_\odot\) and \(\Sigma/\sigma\). Different scalings that apply for other parameter regimes remain to be worked out.

**Interaction with a third large body was a less important channel for binary formation.
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Additional predictions suitable for testing by future observations of Kuiper belt binaries could be made by extending our model in a variety of ways. Relaxing the restriction to two mass groups would allow us to investigate the mass ratio distribution of binary components and its dependence on semi-major axis. We predict the formation of multiple systems for bodies with \( R < a_{\text{crit}} \). Numerical experiments can establish what types of systems survive disruption due to mutual interactions of their members. We could also predict eccentricity and inclination distributions. This would require a more detailed look at the anisotropy of the velocity distribution of small bodies, especially for \( a < r_u \). Anisotropy leads to tensorial dynamical friction in which the force is not antiparallel to the velocity. Binney\(^{19}\) has investigated a related problem, the evolution of the orbits of galaxies in anisotropic clusters. We can already say that the eccentricities are likely to be large. They begin large following binary capture. Moreover, since the magnitude of the dynamical friction force increases outward for \( a < r_u \), they should remain large in this region.

Simple numerical simulations of the Hill problem that incorporate dynamical friction would help pin down the dependence of the capture probability on the velocity dispersion and on the strength of dynamical friction. Computations involving three massive bodies are needed for evaluation of capture probabilities via that channel. Three body capture cross sections have been computed for stars in globular clusters,\(^{20}\) but in our regime the Hill sphere must be taken into account.

The two largest known Kuiper belt objects, Pluto and Charon, form a binary. Collision coupled with tidal evolution is the generally accepted mode of formation for this system. Our scenario for binary formation provides another candidate. It may also apply to the formation of some binaries in the asteroid belt.

In closing, we caution that our numerical results are crude. Dynamical processes have been estimated to order of magnitude. We have ruthlessly discarded numerical coefficients and logarithmic factors. More credence should be accorded to the functional dependences of our results than to the precise numerical predictions.

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Figure 1. Numerical simulations of binary formation by dynamical friction. Two equal mass bodies approach each other on initially circular orbits. We integrate the equations of motion including dynamical friction under the Hill approximation in a frame rotating at the average mean motion. The center of mass is fixed at (0,0). We plot only the trajectories of one of the bodies; the trajectories of the other body are related by reflection through the origin. Dashed red trajectories did not form binaries, and solid blue ones did. The axes are in units of $a_\odot (M/M_\odot)^{1/3}$. The Lagrange points of the corresponding restricted 3-body problem are marked by green dots and the Hill sphere by a dashed green circle. Dynamical friction is modelled as a force antiparallel and proportional to the velocity measured relative to the Keplerian shear. Every capture orbit is plotted but only a sample of the non-capture orbits are shown. It is evident that capture is only possible within distinct ranges of impact parameters, whose widths are roughly proportional to the drag. The symbol $C$ denotes the ratio of the linear measure of the impact parameters that result in capture to $a_\odot (M/M_\odot)^{1/3}$. The symbol $D$ denotes the fractional decrease in velocity due to drag over a time $\Omega_\odot^{-1}$, essentially half the rhs of equation (20). Clearly $C \approx D$ to better than a factor of two. This verifies our assertion that the fractional energy loss over time $\Omega_\odot^{-1}$ is also the fraction of transient binaries that become bound. A sample of similar calculations starting from orbits with finite velocity $v < v_H$ gives similar results.