Nuclear pairing interaction plays a crucial role in both macroscopic-microscopic and fully macroscopic descriptions of nuclei. In the present study we discuss different pairing interactions (monopole and $\delta$ pairing forces) and the methods allowing for the particle number symmetry restoration in addition to the customary BCS treatment of pairing correlations in the context of $\alpha$-decay half-lives for superheavy nuclei. The calculations are done in the macroscopic-microscopic framework for even-even nuclei with $Z > 110$.

1. Introduction

In the macroscopic-microscopic treatment of nuclear energy one needs to know the dependence of the total energy of the nucleus on deformation parameters (e.g., an elongation). In this method the energy is a sum of the macroscopic part ($E_{\text{macro}}$) and microscopic energy consisting of shell ($\delta E_{\text{shell}}$) and pairing ($\delta E_{\text{pair}}$) energies

$$V(\text{def}) = E_{\text{macro}} + \delta E_{\text{shell}} + \delta E_{\text{pair}}.$$  \hspace{1cm} (1)

The influence of the type of macroscopic energy models on both fission and on $\alpha$-decay was discussed recently.\textsuperscript{1,2} For the macroscopic part we choose the Yukawa+Exponential model (YpE).\textsuperscript{3} The shell energy depends only on the single particle model – here Woods-Saxon potential.\textsuperscript{4} The third component of the total energy is the pairing energy customary obtained with the monopole pairing force, i.e., the pairing force with constant matrix elements ($g = \text{const}$) in the BCS model.
As it is known, the usual BCS treatment of correlations leads to the particle number non-conservation. To correct the solutions one may apply e.g., exact projection techniques, i.e., variation after projection method or use some approximative approaches like solving Lipkin-Nogami equations. Both these methods are used here in the case of seniority pairing.

In the following a short description of Lipkin-Nogami (LN) model and a variation after projection (VAP) method are presented together with the consequences of both these techniques on $\alpha$-decay energies ($Q_\alpha$-values) and $\alpha$-half-lives ($T_\alpha$) in the region of superheavy nuclei ($Z > 110$).

2. Pairing models

In this chapter we give a short review of the pairing models which will be used in our calculations. At each stage of the presentation we will also show some intermediate results concerning pairing properties of considered nuclei.

State dependent $\delta$-pairing

The $\delta$ type pairing interaction induces the state dependence of the pairing gaps. In this case one has to solve a large number of BCS equations in order to obtain all the gaps in the pairing window (in this paper we solve $N + 1$ equations for neutrons and $Z + 1$ for protons). The strength of the $\delta$-force which reproduces pairing gaps and masses in superheavy region of nuclei is 225 MeV fm$^3$ for both neutrons and protons. In the case of monopole pairing we have used $g$ parameters reported in Ref. 5.

Figures 1 and 2 show the Fermi level pairing gaps and pairing energies in both seniority BCS (BCSg) and state dependent BCS (BCS$\delta$) models. The differences in pairing gaps are of the order of 0.1 MeV. The deviations in the pairing energy are in some cases larger than 0.5 MeV.

Lipkin-Nogami method

In the case of Lipkin-Nogami (LN) model one reduces the fluctuations of the particle number by adding to the Hamiltonian $\hat{H}$ the terms $-\lambda \hat{N}$ and the quadratic term $-\lambda^2 (\hat{N} - \langle \hat{N} \rangle)^2$ and one minimizes the average energy with respect to $\lambda$. The procedure described in e.g., Ref. 6,7 gives the following expression for the new corrected energy

$$ E_{LN} = E - \lambda^2 \langle \Psi | \Delta \hat{N} | \Psi \rangle, \quad (2) $$

where

$$ \lambda^2 = \frac{\langle \Psi | \hat{H} (\Delta \hat{N})^2 | \Psi \rangle}{\langle \Psi | (\Delta \hat{N})^2 | \Psi \rangle}, \quad \Delta \hat{N} = \hat{N} - \langle \hat{N} \rangle, \quad (3) $$

$\hat{N}$ is the number operator and $\Psi$ is the BCS ground state.
Fig. 1. Fermi pairing gaps in the case of BCSg and BCSδ for isotopes of Z=112.

Fig. 2. Pairing corrections ($\delta E_{\text{shell}}$) in the case of BCSg and BCSδ for isotopes of Z=114.
Variation After Projection approach

Variation after projection (VAP) or Full BCS (FBCS) is an exact particle number projection method. Particle number projected wave function is

$$\Psi = C \int d\zeta \zeta^{-n_0-1} \prod_{\nu} (u_\nu + v_\nu \zeta a^+_\nu a^0_\nu) \Phi_0,$$

where $\Phi_0$ is the vacuum state for particles, $n_0$ the number of nucleonic pairs, and $C$ the normalization constant

$$|C|^2 = 1/(4\pi^2) R_0^0,$$

where $R_0^0$ is defined in Eq. (8).

The energy of the system of $N$ interacting fermions defined as

$$E = \langle \Psi | \hat{H} | \Psi \rangle / \langle \Psi | \Psi \rangle,$$

is given by

$$E = \frac{1}{R_0^0} \left( \sum_{k>0}(2\epsilon_k - g_{kk})v_k^2 R_1^1(k) - \sum_{p,q} g_{pq}u_p v_p u_q v_q R_2^2(p,q) \right).$$

The $R_n^N(k_1, \ldots, k_n)$ are the residues of contour integrals on the complex plane

$$R_n^N(k_1, \ldots, k_N) = \frac{1}{2\pi i} \int dz \frac{z^n}{z^{n_0+1}} \prod_{k \neq k_1, \ldots, k_N} (u_k^2 + z v_k^2).$$

Here $v_k$ and $u_k$ are the parameters such that $u_k^2 + v_k^2 = 1$ and the number of pairs of nucleons reads

$$n_0 = \sum_k v_k^2 R_1^1(k)/R_0^0.$$
3. $\alpha$ half-lives

Both the approximate LN and exact VAP projection methods have been applied to calculate macroscopic-microscopic masses, $\alpha$-decay energies ($Q$-values) and half-lives ($T_\alpha$) for even-even isotopes of superheavy nuclei: $112 \leq Z \leq 120$. The $\alpha$ half-lives are determined from modified Viola-Seaborg formula.\textsuperscript{11} In the following section we discuss the results of these calculations.

The $Q_\alpha$-values

$$Q_\alpha(Z, N) = [M(Z, N) - M(Z - 2, N - 2) - M(2, 2)]c^2,$$

where $M$ is the nuclear mass, are depicted in Figure 3 for all considered models. The pattern of $Q_\alpha$ vs. neutron number $N$ is similar in all of the cases. However, in the vicinity of $N = 162$ and $N = 184$ one observes some discrepancies which will influence the calculations of $\alpha$-half-lives. This effect can be noticed on the Figure 4 where the logarithms of the half-lives (in years) are displayed. The largest differences occur in the vicinity of magic numbers where the standard BCSg and BCS$\delta$ methods collapse yielding no superfluid solutions.

Figure 5 shows the discrepancy between the half-lives (logarithms) vs. the neutron number $N$ in the case of all of the models relative to the VAP results. The largest differences in $\log T_\alpha$ are observed in the case of very heavy isotopes ($N > 180$).
Fig. 4. Alpha half-lives for all models for isotopes of Z = 116 isotopes.

Fig. 5. The differences $\log(T_\alpha/y) - \log(T_\alpha/y)_{\text{VAP}}$ versus neutron number $N$ for all models.
Similarly, Figure 6 displays the correlations between the half-lives data for the \( \log(T_{\alpha}/y) \) versus the data of the VAP model. The largest model discrepancies which reaches 4 orders of magnitude concern the long-living (\( \log(T_{\alpha}/y) > -8 \)) isotopes-this corresponds to the region of superheavy nuclei which is of the main concern of both the experiment and the theory.

4. Summary

We have calculated \( \alpha \)-decay half-lives using four pairing models: seniority (BCSg), \( \delta \)-pairing (BCS\( \delta \)), Lipkin-Nogami model and variation after projection method. As expected, both the kind of pairing interaction involved and the method of solution influence the \( \alpha \)-decay half-lives.

The largest discrepancies measured for 70 nuclei relative to the results of the VAP method are observed in the case of Lipkin-Nogami approach (an average deviation \( \sigma_{LN} = 0.717 \text{ MeV} \)) and BCS with constant pairing interaction, where \( \sigma_{BCSg} = 0.654 \text{ MeV} \). Surprisingly, the \( \delta \)-pairing BCS (BCS\( \delta \)) model results are closer to the variation after projection(VAP) data in comparison to those obtained from the Lipkin-Nogami approach. In the latter case the mean deviation from VAP data \( \sigma_{\delta} = 0.438 \text{ MeV} \).

The half-lives of long lived superheavy \( (N > 180) \) nuclei are determined with
the largest uncertainty.

The problem of the description of the alpha decay of superheavy nuclei is still present in the realm of superheavy game and is a challenge for the theory to invent new, more powerful alpha decay models describing the phenomenon.

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