Distinguishing technicolor models via $t\bar{t}$ productions at polarized photon colliders

Bin Zhang, Yuanning Gao

Center for High Energy Physics, Tsinghua University, Beijing 100084, China

Yu-Ping Kuang

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China and
Center for High Energy Physics, Tsinghua University, Beijing 100084, China*

We study top quark pair productions at a polarized photon collider from an $e^+e^-$ linear collider (LC) in various improved technicolor model, namely, the one-family walking technicolor model, the top-color-assisted technicolor model, and the top-color-assisted multiscale technicolor model. Recent constraint on the top-pion mass from the precision data of $R_b$ is considered. It is shown that, considering only statistical errors, a polarized photon collider from a 500 GeV LC with an integrated luminosity of 500 fb$^{-1}$ is sufficient for distinguishing the three improved technicolor models experimentally.

PACS number(s): 12.60.Nz, 13.40.-f, 14.65.Ha

Although the standard model (SM) has successfully passed the precision electroweak tests, its electroweak symmetry breaking (EWSB) mechanism is still unclear. The Higgs boson has not been found, and the LEP2 bound on the Higgs boson mass is 114.4 GeV [1]. Furthermore, the SM Higgs sector suffers from the well-known problems of triviality and unnatureness arising from the elementary Higgs field. There have been many new physics models of the EWSB mechanism proposed for avoiding the above problems. An attractive idea of completely avoiding triviality and unnatureness is to abandon the elementary Higgs field(s), such as various kinds of improved technicolor models [2, 3, 4], top quark seesaw models [5], and certain little Higgs models [6]. In a previous paper [7], we studied the tests of various seesaw models [2, 3, 4], top quark seesaw models [5], and certain little Higgs models [6].

In this paper, we study the same processes at polarized photon colliders, and take into account the recent bound on the top-pion mass from the precision data of $R_b$ [8]. We shall see that, considering only statistical uncertainties, a polarized photon collider built from a $\sqrt{s} = 500$ GeV LC with an integrated luminosity of 500 fb$^{-1}$ is sufficient for distinguishing the improved technicolor models.

It has been shown that the polarization of the initial laser beam and the electron beam will significantly affect the photon spectrum at the photon collider [9]. Let $P_e$ be the polarization of the initial laser, $\lambda_e$ be the polarization of the electron in the first beam, and $P_{e'}$ and $\lambda_{e'}$ be the corresponding parameters in the second beam, respectively. It is shown in Ref. [9] that the colliding photons will peak in a narrow region near the high energy end (80% of the electron energy) if $2\lambda_e P_e = -1$. This improves the monochromatization and enhances the effective energy of the colliding photons at the photon collider. We shall see that this effect leads to the possibility of distinguishing different improved technicolor models at a polarized photon collider built from a $\sqrt{s} = 500$ GeV LC.

Let $E_c$, $\omega_0$, $\omega$, and $\sqrt{s}/\sqrt{s}$ be the incident electron energy, the laser-photon energy, the backscattered photon energy, and the center-of-mass energies of $e^+e^-(\gamma\gamma)$, respectively. The photon luminosity $dL_{\gamma\gamma}$ is [10]

$$dL_{\gamma\gamma} = 2zdz \int_{z^{1/2}/x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} F_{\gamma/e}(x) F_{\gamma/e}(s^2/x). \quad (1)$$

with $z = \sqrt{s}/s$, and $x_{\text{max}} = \omega_{\text{max}}/E_c$,

$$F_{\gamma/e}(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2} \right]$$

* Mailing address
helicity is $+1$ and $k$, where

$$D(\xi) = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2}\right) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{1(1 + \xi)^2},$$

where $\xi = 4E_\gamma\omega_0/m_e^2$. In order to avoid the creation of $e^+e^-$ pairs by the interaction of the incident and backscattered photons, $\xi$ should not be larger than 4.8. As in Ref. [7], we take $\xi = 4.8$, then $x_{\text{max}} \approx 0.83$ and $D(\xi) = 1.8$. The formula for the counting rate of $\gamma\gamma \rightarrow X$ at a polarized photon collider has been given in Ref. [9]. It is

$$d\tilde{N}_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma}(d\sigma + \Delta d\tau + \zeta_2 \zeta_2 d\sigma_{20} + \zeta_2 d\sigma_{02}),$$

where $\zeta_i \ (\tilde{\zeta}_i), \ i = 1, 2, 3$ are Stokes parameters [3],

$$\zeta_2 = \frac{C_{20}}{C_{00}},$$

$$C_{00} = \frac{1}{1 - x} + 1 - x - 4r(1 - r) - 2\lambda_r r\xi(2r - 1)(2 - x),$$

$$C_{20} = 2\lambda_r r\xi(1 + (1 - x)(2r - 1)^2) - P_c(2r - 1)(\frac{1}{x} + 1 - x),$$

$$x = \frac{\omega}{E_e}, \quad r = \frac{\xi}{\xi(1 - x)},$$

$$\sigma_{ij}, \ i, j = 0, 1, 2, 3 \text{ are cross sections defined in Ref. [11],}$$

$$\Lambda = \langle \zeta_3 \tilde{\zeta}_3 - \zeta_1 \tilde{\zeta}_1 \rangle$$

and $d\sigma$, $d\sigma^a$ are [2]

$$d\sigma = \frac{1}{4} \left(\left|M_{++}\right|^2 + \left|M_{\ldots\ldots}\right|^2 + \left|M_{\ldots+}\right|^2 + \left|\bar{M}_{++}\right|^2\right) d\Gamma,$$

$$d\sigma^a = \frac{1}{4} \left(\left|\bar{M}_{++}\right|^2 + \left|\bar{M}_{\ldots\ldots}\right|^2 - \left|\bar{M}_{\ldots+}\right|^2\right) d\Gamma,$$

in which the subscripts $+$ and $-$ indicate that the photon helicity is $+1$ and $-1$, respectively, and

$$d\Gamma = \frac{(2\pi)^4}{4k} \delta(k + \tilde{k} - \sum_j p_f) \prod_j d^3p_f / (2\pi)^3 2\epsilon_f,$$

where $k$, $\tilde{k}$ and $p_f$ are four momenta of the colliding photons and the $f$-th final state particle, respectively. After averaging over the azimuthal angles, $d\sigma_{20}$ and $d\sigma_{02}$ vanish, and $\Delta d\sigma^a$ is negligibly small. So Eq. (2) becomes

$$d\tilde{N}_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma}(d\sigma + \zeta_2 \zeta_2 d\sigma^a).$$

The corresponding cross section $\sigma(s)$ at an LC with center-of-mass energy $\sqrt{s}$ can be obtained by further integrating Eq. (4) over the parameter $z$ [11]

$$\sigma(s) = \int_{2m_t/\sqrt{s}}^{s_{\text{max}}} \frac{d\tilde{N}_{\gamma\gamma \rightarrow X}}{dz} dz.$$
of them not being tagged is 25%. Therefore our actual b-tagging efficiency in this case is 75%.

(b) One tagged b (or $\bar{b}$) jet is in the b-tagging effective region $30^\circ < \theta < 150^\circ$, and the untagged b (or $\bar{b}$) jet is in the detectable but not b-tagging effective region $20^\circ < \theta < 30^\circ$. The other four jets are in the detectable region $20^\circ < \theta < 160^\circ$. Our calculation shows that the probability of satisfying this requirement is 10%. In this case the b-tagging efficiency is 50%.

Taking into account both of these two possible cases and the W decay branching ratio $B(W \rightarrow q\bar{q}) = 68\%$, our final detection efficiency is

$$(68\%)^2 \times 47\% \times (58.3\% \times 75\% + 10\% \times 50\%) \approx 10\%.$$  

In recent years, two of the $e^+e^-$ linear collider projects have been actively pushed. They are the DESY TeV Energy Superconducting Linear Accelerator (TESLA) with the designed luminosity of $3.4 \times 10^{34}$ cm$^{-2}$sec$^{-1}$ corresponding to $\int \frac{Ldt}{yr} = 340$ fb$^{-1}$, and the KEK Joint Linear Collider (JLC) with the designed luminosity of $(8 - 9) \times 10^{33}$ cm$^{-2}$sec$^{-1}$ corresponding to $\int \frac{Ldt}{yr} = (80 - 90)$ fb$^{-1}$. As usual, we shall take an integrated luminosity of 500 fb$^{-1}$ and the 10% detection efficiency to estimate the numbers of events ($N_{events}$) of $\gamma\gamma \rightarrow t\bar{t}$. In this paper, only statistical errors are taken into account.

In the following, we calculate the helicity amplitudes in Eq. (3) for $\gamma\gamma \rightarrow t\bar{t}$ in various improved technicolor models. As what we did in Ref. 5, for avoiding singularities arising from the very forward or very backward scatterings, we take the rapidity and transverse momentum cuts

$$|y| < 2.5, \quad p_T > 20 \text{ GeV}$$  

which will also increase the relative corrections.

As in Ref. 2, we study the PGB contributions to $\gamma\gamma \rightarrow t\bar{t}$ in three technicolor models, namely Model A: the one-family walking technicolor model by Appelquist and Terning 2, Model B: the top-color-assisted technicolor model by Hill 3, and Model C: the top-color-assisted multiscale technicolor model by Lane 4. The PGBs in these three models are quite different. This is the main reason why the three models can be distinguished. The formulas for the PGB-$t\bar{t}$ couplings in the three models and the production amplitudes are given in Ref. 2.

Model A:
In Model A, color-singlet PGBs are composed of technileptons, which do not couple to $t\bar{t}$. Thus there is no s-channel resonance contribution in this model. The relevant PGBs are the color-octet technipions containing the $SU(2)_W$-singlet $\Pi^0_a$ and the $SU(2)_W$-triplet $\Pi^+_a$. Their masses are in the few hundred GeV range, and their decay constant is $f_a \approx 140$ GeV 2. The coupling of these PGBs to $t(b)$-quark is

$$\frac{\sqrt{2} m_t}{f_a} \left( \frac{\lambda^a}{2} t\Pi^0_a + it\gamma_5 \frac{\lambda^a}{2} t\Pi^+_a + \frac{1}{\sqrt{2}} \bar{b}(1 - \gamma_5) \frac{\lambda^a}{2} b\Pi^+ _a \right).$$  

TABLE I: Technipion corrections to the $\gamma\gamma \rightarrow t\bar{t}$ cross section $\Delta\sigma$, the relative correction $\Delta\sigma/\sigma_0$, the total cross section $\sigma = \sigma_0 + \Delta\sigma$, and $N_{events}$ for an integrated luminosity of 500 fb$^{-1}$ taking account of the 10% detection efficiency at a 500 GeV $e^+e^-$ linear collider for with 2$\lambda_c P_c = -1$ for various values of $m_{t_{\Pi_a}}$ in Model A (The SM cross section is $\sigma_0 = 196$ fb).

| $m_{t_{\Pi_a}}$(GeV) | $\Delta\sigma$ (fb) | $\Delta\sigma/\sigma_0$ (%) | $\sigma$ (fb) | $N_{events}$ |
|--------------------|------------------|------------------|---------------|--------------|
| 250                | -31              | -15.8            | 165           | 8250         |
| 300                | -26              | -13.3            | 170           | 8500         |
| 350                | -22              | -11.2            | 174           | 8700         |
| 400                | -19              | -9.7             | 177           | 8850         |
| 500                | -14              | -7.1             | 182           | 9100         |

These color-octet PGBs contribute to $\gamma\gamma \rightarrow t\bar{t}$ only through radiative corrections which are small. We list, in TABLE I, the obtained $\Pi_a$ corrections to the $\gamma\gamma \rightarrow t\bar{t}$ cross section $\Delta\sigma$, the relative correction $\Delta\sigma/\sigma_0$ ($\sigma_0$ stands for the SM cross section), the total cross section $\sigma = \sigma_0 + \Delta\sigma$, and $N_{events}$ for an integrated luminosity of 500 fb$^{-1}$ at a polarized photon collider from a 500 GeV LC with the polarization $2\lambda_c P_c = -1$ for $m_{t_{\Pi_a}}$ in the range $250$ GeV $< m_{t_{\Pi_a}} < 500$ GeV given in Ref. 2.

We see that the corrections $\Delta\sigma$ are negative which are mainly from the interference between the PGB-amplitude and the SM-amplitude. The absolute square of the PGB-amplitude is not large in this model. We see from TABLE I that the total cross sections $\sigma$ are much larger than those given in Ref. 2 due to the enhancement of the effective photon energy in the high energy region at the polarized photon collider. We see that $N_{events}$ for an integrated luminosity of 500 fb$^{-1}$ taking account of the 10% detection efficiency are $(8 - 9) \times 10^3$. The corresponding 95% statistical uncertainties are about 2%. Comparing with the relative corrections $\Delta\sigma/\sigma_0 \sim -(7$-16)$\%$ listed in TABLE I, we see that the $\Pi_a$ correction effect can be clearly detected.

Model B:
Model B contains a technicolor sector and a top-color sector. The technicolor sector only contributes a small portion of the top quark mass, so $m_t' = \epsilon m_t$ ($\epsilon \ll 1$), while most of the top quark mass is contributed from the top-color sector, say $m_t - m_t' = (1 - \epsilon) m_t$. The $b \rightarrow s\gamma$ experiment requires $\epsilon < 0.1$ 5. In this paper, we take
a typical value $\epsilon = 0.08$, i.e., $m_t' = 0.08m_t \approx 14$ GeV as an example to do the study.

In the technicolor sector, the coupling of the color-octet technipion to $t(b)$-quark is similar to Eq. (10) but with $m_f/f_\Pi$ replaced by $m_t'/f_\Pi$, i.e.,

$$\frac{\sqrt{2}m_t'}{f_\Pi} \left( i\bar{t}\gamma_5\frac{\lambda^a}{2}\Pi^0_a + i\bar{t}\gamma_5\frac{\lambda^b}{2}\Pi^3_a + \frac{1}{\sqrt{2}}(1 - \gamma_5)\frac{\lambda^a}{2}b\Pi^+_a + \frac{1}{\sqrt{2}}\bar{b}(1 + \gamma_5)\frac{\lambda^b}{2}t\Pi^-_a \right).$$

(11)

In addition to the color-octet technipions, there are also color-singlet technipions $\Pi^0$ and $\Pi^3$, composed of techniquarks, with the decay constant $F_{\Pi} \approx 120$ GeV, and masses in the few hundred GeV range. The coupling of these color-singlet PGBs to $t(b)$-quarks is

$$c_t\frac{m_t'}{\sqrt{2f_\Pi}} \left( i\bar{t}\gamma_5\Pi^0 + i\bar{t}\gamma_5\Pi^3 + \frac{1}{\sqrt{2}}(1 - \gamma_5)b\Pi^+ + \frac{1}{\sqrt{2}}\bar{b}(1 + \gamma_5)t\Pi^- \right), \quad c_t = 1/\sqrt{6}. \quad (12)$$

In the top-color sector, there are color-singlet top-pions $\Pi^0_t$ and $\Pi^3_t$ with the decay constant $F_{\Pi_t} \approx 50$ GeV. The coupling of top-pions to $t(b)$-quark is

$$m_{t} - m_t'/\sqrt{2f_{\Pi_t}} \left( i\bar{t}\gamma_5\Pi^0_t + \frac{1}{\sqrt{2}}(1 - \gamma_5)b\Pi^+_t + \frac{1}{\sqrt{2}}\bar{b}(1 + \gamma_5)t\Pi^-_t \right).$$

(13)

Recently, it has been pointed out that the LEP precision data of $R_b$ gives important constraint on the top-pion mass [8]. With $\epsilon = 0.08$, the $2\sigma$ bound from $R_b$ on the top-pion mass in this model is roughly 300 GeV $\leq m_{\Pi_t} \leq 900$ GeV [8]. The naturalness of the model favors lower values of $m_{\Pi_t}$. So we take 300 GeV $\leq m_{\Pi_t}$ $\leq 500$ GeV in this study. In Ref. [2], $m_{\Pi_t}$ was taken to be in the range of 180 – 300 GeV according to the original paper, Ref. [8]. Such a range is below the recent lower bound. We shall see that the updated heavier $\Pi_t$ will make the situation quite different at the polarized photon collider from the 500 GeV LC.

The color-singlet PGBs $\Pi^0$, $\Pi^3$, and $\Pi^0_t$ couple to the initial state photons through triangle fermion loops (techniquark loops and top quark loops), so they can contribute $s$-channel resonances in $\gamma\gamma \rightarrow tt$. The triangle fermion loops are enhanced by the anomaly, so that the $s$-channel resonance contributions can be of the order of tree level contributions. These $s$-channel resonance contributions are dominant in Model B and Model C. If the PGB mass is greater than $2m_t$, it can decay into $t\bar{t}$. The decay rate is determined by the PGB-$t - \bar{t}$ coupling strengths given in Eqs. (11) and (12). Since $m_t - m_t'/f_\Pi$, in Eq. (13) is large, the width of $\Pi^0_t$ will be large if its mass is greater than $2m_t$. In this case, the $s$-channel resonance effect from $\Pi^0_t$ is not so significant. On the other hand, $m_t'/f_\pi$ in Eq. (11) is small, so that the resonance effects from $\Pi^0$ and $\Pi^3$ are significant even their masses are greater than $2m_t$. The width of $\Pi^0_t$ is very small which is hard to detect experimentally [8]. So we concentrate on examining the $s$-channel resonance effect of $\Pi^0$, and simply take a typical mass of $m_{\Pi^0} = 300$ GeV for $\Pi^3$.

The obtained $\Delta \sigma$, $\Delta \sigma/\sigma_0$, $\sigma$, and $N_{\text{events}}$ for an integrated luminosity of 500 fb$^{-1}$ taking account of the 10% detection efficiency at a 500 GeV $e^+e^-$ linear collider for $2\lambda_eP_e = 1$ for $m_t' = 14$ GeV, $m_{\Pi^0} = 300$ GeV, and various values of $m_{\Pi^0}$ and $m_{\Pi^3}$ in Model B (The SM cross section is $\sigma_0 = 196$ fb).

| $m_{\Pi^0}$ (fb) $\Delta \sigma/\sigma_0$ (%) $\sigma$ (fb) $N_{\text{events}}$ |
|-----------------|----------|--------|-----------------|
| 300             | 86.2     | 44.0   | 110             |
| 400             | 47.3     | 24.1   | 149             |
| 500             | 87.4     | 44.6   | 109             |
| $m_{\Pi^3} = 400$ GeV |
|-----------------|----------|--------|-----------------|
| 300             | 112      | 57.1   | 308             |
| 400             | 158.3    | 80.8   | 354             |
| 500             | 115      | 58.7   | 311             |

The obtained $\Delta \sigma$, $\Delta \sigma/\sigma_0$, $\sigma$, and $N_{\text{events}}$ for an integrated luminosity of 500 fb$^{-1}$ for $m_{\Pi^0} = 300$ and 400 GeV, and $m_{\Pi^0} = 300$, 400, and 500 GeV in this model are listed in TABLE II. We see that, in the case of $m_{\Pi^0} = 300$ GeV, $\Delta \sigma$ is more negative than in Model A because there is $s$-channel $\Pi^0_t$ and $\Pi^3$ resonance contributions in addition to the radiative corrections, and the $s$-channel resonance contributions are mainly from the interferences between the PGB-amplitudes and the SM-amplitude. In the case of $m_{\Pi^0} = 400$ GeV, $\Delta \sigma$ becomes positive. This is because that the absolute squares of the $\Pi_t$ and $\Pi^0$ resonance amplitudes are large in this case due to the enhancement of the photon spectral luminosity in the region around 400 GeV (80% of the $e^+e^-$ energy) in the case of $2\lambda_eP_e = -1$ at the polarized photon collider [4].

To have an insight of the detailed situation, we plot the $t\bar{t}$ invariant mass distributions in Fig. 1 and Fig. 2 for the six sets of parameters in TABLE II. Fig. 1 shows the invariant mass distributions for $m_{\Pi^0} = 300$ GeV, $m_{\Pi^0} = 300$, 400, and 500 GeV. We see that all curves are enhanced in the region around 400 GeV which is just the effect of the photon spectral luminosity. In the $m_{\Pi^0} = 400$ GeV distribution, we see a clear peak at 400 GeV. In the $m_{\Pi^0} = 300$ and 500 GeV distributions no clear peaks can be seen. This is because that the probability
of the center-of-mass energy of the two colliding photons being 300 or 500 GeV is very small as can be seen from the photon spectral luminosity. We cannot see the top-pion resonance since the width of top-pion is much larger. Fig. 2 shows the invariant mass distributions for $m_{\Pi_t}=400$ GeV. The behaviors are similar but the enhancement around 400 GeV is stronger due to the s-channel contribution of the $m_{\Pi_t} = 400$ GeV top-pion.

In the case of $m_{\Pi_t} = 300$ GeV, $N_{\text{events}}$ is $(5.4 - 7.5) \times 10^3$ (cf. TABLE II). The 95% C.L. statistical uncertainties are thus (2 - 3)%. Comparing with the relative corrections $\Delta \sigma/\sigma_0 = (24 - 45)\%$ in TABLE II, we see that these correction effects can be very clearly detected. For distinguishing Model B from Model A, we see that the relative difference between the total cross sections in Model A and Model B is $[\sigma(A) - \sigma(B)]/\sigma(A)$ = (15 - 40)\%, so that these two models can be very clearly distinguished.

In the case of $m_{\Pi_t} = 400$ GeV, $N_{\text{events}}$ is $(1.5 - 1.7) \times 10^4$. The corresponding 95% C.L. statistical uncertainties are (1.5 - 1.6)\%. Now the relative corrections are $\Delta \sigma/\sigma_0 = (57 - 81)\%$ in TABLE II, so that these correction effects can be very clearly detected. The relative difference between the total cross sections in Model A and Model B is now $[\sigma(A) - \sigma(B)]/\sigma(A) = -(71 - 100)\%$, thus these two models can be very clearly distinguished.

Model C:

Model C is similar to Model B, but the decay constant of the color-singlet technipions is $f_1 \approx 40$ GeV rather than $f_1 \approx 120$ GeV. Moreover, the coupling constant $c_1$ in Eq. (11) and the $\Pi^0\gamma\gamma$ coupling are also different from those in Model B [4, 7]. The smallness of $f_1$ makes the coupling constants $m_0/f_1$ in Eqs. (10) and (11) larger than those in Model B by a factor of 3. These changes enhance the the s-channel $\Pi^0$ resonance effect in Model C significantly. Furthermore, the $2\sigma R$ bound on the top-pion mass in this model is roughly 250 GeV $\leq m_{\Pi_t} \leq 560$ GeV. To compare with Model B, we also take the range 300 GeV $\leq m_{\Pi_t} \leq 500$ GeV in this study.

TABLE III: Technipion and top-pion corrections to the $\gamma\gamma \rightarrow t\bar{t}$ cross section $\Delta \sigma$, the relative correction $\Delta \sigma/\sigma_0$, the total cross section $\sigma = \sigma_0 + \Delta \sigma$, and $N_{\text{events}}$ for an integrated luminosity of 500 fb$^{-1}$ taking account of the 10% detection efficiency at a 500 GeV $e^+e^-$ linear collider for $2\lambda_\gamma^2P_c = -1$ for $m_\lambda=14$ GeV and various values of $m_{\Pi_t}$ and $m_{\Pi_0}$ in Model C with $m_{\Pi_0} = 300$ GeV (The SM cross section is $\sigma_0 = 196$ fb).

| $m_{\Pi_0}$ | $\Delta \sigma$ (fb) | $\Delta \sigma/\sigma_0$ (%) | $\sigma$ (fb) | $N_{\text{events}}$ |
|-------------|----------------------|-----------------------------|----------------|-------------------|
| 300         | -22.1                | -11.3                       | 174            | 8700              |
| 400         | 469.2                | 230.4                       | 665            | 33250             |
| 500         | -92.3                | -47.1                       | 104            | 5200              |

| $m_{\Pi_0}$ | $\Delta \sigma$ (fb) | $\Delta \sigma/\sigma_0$ (fb) | $\sigma$ (fb) | $N_{\text{events}}$ |
|-------------|----------------------|-------------------------------|----------------|-------------------|
| 300         | 79.1                 | 40.4                          | 275            | 13750             |
| 400         | 1647.3               | 840.5                         | 1843           | 92150             |
| 500         | 125.7                | 64.1                          | 322            | 16100             |

As in the case of Model B, we examine the cases of $m_{\Pi_t} = 300$ and 400 GeV, and $m_{\Pi_0} = 300, 400, 500$ GeV, with a typical value of the $\Pi^3$ mass, $m_{\Pi_0} = 300$ GeV. The obtained values of $\Delta \sigma$, $\Delta \sigma/\sigma_0$, and $\sigma$ are listed in TABLE III. We see that $\Delta \sigma$ is negative only in the case of $m_{\Pi_t} = m_{\Pi_0} = 300$ GeV. In all other cases, $\Delta \sigma$ is positive because the absolute square of the $\Pi^0$ resonance amplitude is large due to the largeness of the coupling constant $m_0/f_1$. This effect is very significant in the case of $m_{\Pi_t} = m_{\Pi_0} = 400$ GeV.

In Figs. 3 and 4, we plot the $t\bar{t}$ invariant mass distributions in Model C for $m_{\Pi_t} = 300$ and 400 GeV and $m_{\Pi_0} = 300, 400, 500$ GeV. Again, we see the clear
The only case which needs to be studied values of the PGB masses, Model C can be distinguished production cross sections. and Model B can be clearly distinguished from the production cross sections. Since the numbers of events are around 8700–8850, it is possible to measure the $t\bar{t}$ invariant mass $(M_{tt})$ distribution. In Fig. 5 we plot the $M_{tt}$ distributions for the two cases.

We see from TABLE III and TABLE I that, for most values of the PGB masses, Model C can be distinguished from Model A. The only case which needs to be studied more carefully is distinguishing the cases of Model C with $m_{\Pi_a} = m_{\Pi^a} = 300$ GeV from Model A with $m_{\Pi_a} = 350–400$ GeV. From TABLE III and TABLE I, we see that the total cross sections for these two cases are almost the same, so that they cannot be distinguished by merely measuring the total cross sections. According to Ref. 10, the energy resolution is $\Delta E/E = 0.33/\sqrt{E}$. We then smear the calculated $M_{tt}$ distributions for the above two cases are shown in Fig. 6. We see that they are different, especially in the vicinities of 365 GeV and 400 GeV where the differences are of the level of $(10–40)\%$. According to the energy resolution, the resolution

In the case of $m_{\Pi_a} = 300$ GeV, $N_{events}$ is $(5.2–33) \times 10^3$. The corresponding 95% C.L. statistical uncertainties are then $(1–3)\%$. Compared with the large relative corrections $\Delta \sigma/\sigma_0$ listed in TABLE III, we see that the PGB effects in Model C can be clearly detected. Comparing the relative corrections $\Delta \sigma/\sigma_0$ listed in TABLE III and TABLE II, we see that the difference between Model C and Model B is significant for $m_{\Pi^0} = 300$ and 400 GeV. For $m_{\Pi^0} = 500$ GeV, the relative difference is $\Delta \sigma(B) = \sigma(C))/\sigma(B) = [109–104]/109 = 5\%$ which is also beyond the statistical uncertainties. Thus model C and Model B can be clearly distinguished from the production cross sections.

![FIG. 3: $t\bar{t}$ invariant mass distribution in Model C for $m_{\Pi_a} = 300$ GeV and $m_{\Pi} = 300, 400, and 500$ GeV.](image1)

![FIG. 4: $t\bar{t}$ invariant mass distribution in Model C for $m_{\Pi_a} = 400$ GeV and $m_{\Pi} = 300, 400, and 500$ GeV.](image2)

![FIG. 5: Comparison of the $t\bar{t}$ invariant mass distributions in Model C for $m_{\Pi_a} = m_{\Pi} = m_{\Pi_a} = 300$ GeV and Model A for $m_{\Pi_a} = 350$ GeV.](image3)

![FIG. 6: Comparison of the smeared $t\bar{t}$ invariant mass distributions in Model C for $m_{\Pi_a} = m_{\Pi} = m_{\Pi_a} = 300$ GeV and Model A for $m_{\Pi_a} = 350$ GeV according to the resolution $\Delta E/E = 0.33/\sqrt{E}$.](image4)
for measuring the $M_{t\bar{t}}$ distribution around 400 GeV is $\Delta M_{t\bar{t}} = 0.33 \times \sqrt{400 \text{ GeV}} = 6.6 \text{ GeV}$. So that the above two cases in model A and Model C can be distinguished by separately measuring the numbers of events in the regions $355 \text{ GeV} \leq M_{t\bar{t}} \leq 375 \text{ GeV}$ and $390 \text{ GeV} \leq M_{t\bar{t}} \leq 410 \text{ GeV}$.

In the case of $m_{\Pi_{t}}=400 \text{ GeV}$, we can see that Model C is very different from the SM, and all the three models can be distinguished.

In summary, we have studied the possibility of testing and distinguishing three improved technicolor models (Models A, B, and C) via $t\bar{t}$ productions at a polarized photon collider with $2\lambda_e P_c = -1$ from a 500 GeV $e^+e^-$ linear collider with an integrated luminosity of $500 \text{ fb}^{-1}$.

The signal contains six jets from $t \to W^+b$ and $W \to q\bar{q}'$. Backgronds can be suppressed by taking the cut [9] and tagging a $b$ quark jet. Considering the possible detection ability of the detector and the usual $b$-tagging efficiency, the detection efficiency is 10%. We see that, considering only the statistical error, the three improved technicolor models can all be well tested and can be distinguished from each other by measuring the $\gamma\gamma \to t\bar{t}$ production cross section and the $t\bar{t}$ invariant mass distribution.

**Acknowledgement:** We would like to thank C.-P. Yuan for discussions. This work is supported by the National Natural Science Foundation of China under the grant number 90103008.

[1] S. Eidelman, al., (Particle Data Group), Phys. Lett. B 592, 1 (2004).
[2] T. Appelquist and J. Terning, Phys. Lett. B 315, 139 (1993).
[3] C.T. Hill, Phys. Lett. B 345, 483 (1995); K. Lane and E. Eichten, *ibid*. 352, 382 (1995); G. Buchalla, G. Burdan, C.T. Hill, and D. Kominis, Phys. Rev. D 53, 5185 (1996).
[4] K. Lane, Phys. Lett. B 357, 624 (1995).
[5] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. 81, 2634 (1998); R. S. Chivukula, B. A. Dobrescu, H. Georgi, and C. T. Hill, Phys. Rev. D 59, 075003 (1999); H.-J. He, C. T. Hill, and T. Tait, Phys. Rev. D 65, 055006 (2002);
[6] N. Arkani-Hammed, A.G. Cohen, and H. Georgi, Phys. Lett. B 513 (2001) 232; N. Arkani-Hammed, A.G. Chen, E. Katz, A.E. Nelson, T Gregoire, and J.G. Wacker, JHEP 0208 (2002) 021; J.G. Wacker, [hep-ph/0208235]; N. Arkani-Hammed, A.G. Cohen, E. Katz, and A.E. Nelson, JHEP 0207 (2002) 034; T. Gregoire and J.G. Wacker, [hep-ph/0207164]; I. Low, W. Skiba, and D. Smith, [hep-ph/0207243].
[7] H.-Y. Zhou, Y.-P. Kuang, C.-X. Yue, H. Wang, and G.-R. Lu, Phys. Rev. D 57, 4205 (1998).
[8] C.-X. Yue, Y.-P. Kuang, X.-L. Wang, and W.B. Li, phys. Rev. D 62, 055005 (2000).
[9] I.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo, and V.I. Telnov, Nucl. Instr. and Methods in Phys. Res. 219, 5 (1984); B. Badelek et al., Part VI of TESLA Technical Design Report, DESY 2001-011.
[10] O.J.P. Eboli et al., Phys. Rev. D 47, 1889 (1993); K. Cheung, *ibid*. 47, 3750 (1993).
[11] V.M. Budnev, I.F. Ginzburg, G.V. Meledin, and V.G. Serbo, Phys. Rep. 15 C, 181 (1975).
[12] Particle Physics Experiments at JLC (JLC Technical Design Report), KEK Report 2001-11.
[13] M. Baillargeon et al., in $e^+e^-$ Collisions at TeV Energies: The Physics Potential, Part D, edited by P. Zerwas, DESY 96-123D (1995); see also Part VI of TESLA Technical Design Report, DESY 2001-011.
[14] Part I of TESLA Technical Design Report, DESY 2001-011.
[15] B. Balaji, Phys. Rev. D 53, 1699 (1996).
[16] Part IV of TESLA Technical Design Report, DESY 2001-011.