Supersymmetrization of generalized Freedman-Townsend models

Friedemann Brandt and Ulrich Theis

Institut für Theoretische Physik, Universität Hannover, Appelstr. 2, D–30167 Hannover, Germany

Abstract: We review briefly generalized Freedman-Townsend models found recently by Henneaux and Knaepen, and provide supersymmetric versions of such models in four dimensions which couple 2-form gauge potentials and ordinary gauge fields in a gauge invariant and supersymmetric manner. The latter models have the unusual feature that, in a WZ gauge, the supersymmetry transformations do not commute with all the gauge transformations.

1 Motivation

We report on work [1] which was motivated by two recent developments. One of these is the construction of new four dimensional $N=2$ supersymmetric gauge theories [2] by gauging the central charge of $N=2$ vector-tensor multiplets [3]. The other one is a classification of possible gauge invariant interactions of $p$-form gauge potentials [4] in any spacetime dimension. It turned out that these two developments overlap: non-supersymmetric limits of the models [2], obtained by neglecting the fermions and freezing scalars to constants, are contained in a class of gauge theories found in [4]. The latter gauge theories generalize $n$-dimensional Freedman-Townsend models [5]: in addition to the $(n-2)$-form gauge potentials present in Freedman-Townsend models, they contain $p$-form gauge potentials with $p < n-2$ interacting with each other and with the $(n-2)$-form gauge potentials. We call such gauge theories Henneaux-Knaepen models. Specified to four spacetime dimensions, they give interactions between 2-form gauge potentials and ordinary gauge fields. The $N=2$ supersymmetric models [2] may thus be regarded as supersymmetric versions of special Henneaux-Knaepen models. A natural question in this context is whether more general Henneaux-Knaepen models can be supersymmetrized too. Our work provides $N=1$ supersymmetric versions of such models in four spacetime dimensions.

2 Interactions between $p$-form gauge potentials

We first briefly review some of the results obtained by Henneaux and Knaepen [4]. Their starting point is the free action for a set of $p$-form gauge potentials $A^a$ with various...
form-degrees \( p_a \) in \( n \) spacetime dimensions,
\[
S^{(0)} = \frac{1}{2} \int \delta_{ab} F^a \wedge * F^b, \quad F^a = dA^a, \quad A^a = \frac{1}{p_a!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{p_a}} A^{a}_{\mu_1 \ldots \mu_{p_a}}. \tag{1}
\]

The free action is of course invariant under the standard abelian gauge transformations
\[
\delta_\epsilon^{(0)} A^a = de^a \tag{2}
\]
where \( e^a \) is an arbitrary \( (p_a - 1) \)-form.

The classification of gauge invariant interactions between the \( A^a \) performed in [2] proceeds along the lines of [4]. One seeks the possible consistent deformations \( S \) and \( \delta_\epsilon \) of the free action and its gauge symmetries. These are deformations such that the deformed action is invariant under the deformed gauge transformations,
\[
S = S^{(0)} + g S^{(1)} + g^2 S^{(2)} + \ldots \quad \delta_\epsilon = \delta_\epsilon^{(0)} + g \delta_\epsilon^{(1)} + g^2 \delta_\epsilon^{(2)} + \ldots \quad \delta_\epsilon S = 0, \tag{3}
\]
where \( g \) is a continuous deformation parameter. A deformation is called trivial if it can be removed by local field redefinitions of the \( A^a_{\mu_1 \ldots \mu_{p_a}} \).

To first order in \( g \), (3) imposes that \( S^{(1)} \) be \( \delta_\epsilon^{(0)} \)-invariant on-shell. Hence, the general form of \( S^{(1)} \) is
\[
S^{(1)} = S^{(1)}_{\text{inv}} + \hat{S}^{(1)}, \quad \delta_\epsilon^{(0)} S^{(1)}_{\text{inv}} = 0, \quad \delta_\epsilon^{(0)} \hat{S}^{(1)} \approx 0 \tag{4}
\]
where \( \approx \) denotes weak (on-shell) equality.

\( S^{(1)}_{\text{inv}} \) contains polynomials in the field strengths \( F^a_{\mu_1 \ldots \mu_{p_a+1}} \) and derivatives thereof. Moreover it contains the possible abelian Chern-Simons terms that can be constructed from the \( A^a \) present in the free model under study. A remarkable result is the following.

**Theorem** [4]. If \( p_a \in \{2, \ldots, n - 2\} \) \( \forall a \), then \( \hat{S}^{(1)} \) can be assumed to be a linear combination of terms of the form
\[
\int (\ast F^{a_1}) \wedge \ldots \wedge (\ast F^{a_r}) \wedge F^{a_{r+1}} \wedge \ldots \wedge F^{a_{r+s}} \wedge A^{a_{r+s+1}}, \quad r \neq 0, \quad \sum_{i=1}^r (n - p_{a_i} - 1) + \sum_{i=r+1}^{r+s} (p_{a_i} + 1) + p_{a_{r+s+1}} = n. \tag{5}
\]

In (5), \( r \neq 0 \) guarantees that (3) does not reduce to a Chern-Simons term (the latter is already contained in \( S^{(1)}_{\text{inv}} \)). The second condition in (5) expresses the requirement that the integrand of (5) be a volume-form.

The above theorem does not hold in presence of 1-form gauge potentials. Namely, then \( \hat{S}^{(1)} \) can contain additional terms which cannot be cast in the form (5). An example is the cubic Yang-Mills vertex \( f_{abc} \int A^a \wedge A^b \wedge * F^c (f_{abc} = f_{[abc]} \neq 0) \) which contains the product of two undifferentiated gauge potentials, in contrast to (5).

Note that (3) imposes conditions on the interaction terms (5). These conditions can be rather severe. Consider for instance the case that all the \( p \)-form gauge potentials have the same degree, i.e., \( p_a = p \) for all \( a \). Then (4) yields \( (r-1)(n-2-p) + s(p+1) = (2-r) \) whose left hand side is nonnegative for \( p \leq n - 2 \) whereas the right hand side is negative for \( r > 2 \). Examining the two remaining cases \( r = 1, 2 \), one finds that \( p = n - 2, r = 2, s = 0 \) is the only solution with \( p \notin \{0, n\} \). This solution yields precisely the Freedman-Townsend vertices. When, in addition to \( (n - 2) \)-forms, there are also \( p \)-forms with other degrees, then (4) has further solutions of a similar type, namely \( (r, s) = (2, 0), p_1 = n - 2, p_2 = p_3 \) arbitrary. The corresponding vertices (5) are given by
\[
\int (\ast F^a) \wedge (\ast F^b) \wedge A^c, \quad p_a = n - 2, \quad p_b = p_c. \tag{7}
\]
As also shown in [3], linear combinations \( k_{abc} f(*F^a) \wedge (*F^b) \wedge A^c \) of these vertices can be completed to a consistent deformation of the free theory to all orders in \( g \) if the coefficients \( k_{abc} \) with \( p_a = p_b = p_c \) are the structure constants of an arbitrary Lie algebra \( G \) (not necessarily compact) and the other coefficients \( k_{abc} \) define representation matrices of \( G \). Furthermore, the complete deformation can be elegantly constructed in first order form by means of auxiliary 1-forms.

In four spacetime dimensions, the resulting models couple 2-form gauge potentials and ordinary gauge fields. Henceforth we denote the former by \( B_A \), the latter by \( A^a \), and the auxiliary 1-forms by \( V^A \),

\[
B_A = \frac{1}{2} dx^\mu \wedge dx^\nu B_{\mu\nu A} , \quad A^a = dx^\mu A^a_\mu , \quad V^A = dx^\mu V^A_\mu .
\]  

In the first order formulation, the deformed action and gauge transformations read

\[
S = f[-F^A(V) \wedge B_A + \frac{1}{2} \hat{F}^a \wedge *\hat{F}_a + \frac{1}{2} V^A \wedge *V_A] \]

\[
\delta_\epsilon B_A = d\epsilon_A - g f_{BC}^A V^B \wedge \epsilon_C + 2g T^a_{AB,*} \hat{F}_a^b \]

\[
\delta_\epsilon A^a = d\epsilon^a + g T^a_{AB} V^A \epsilon^b , \quad \delta_\epsilon V^A = 0.
\]

Here, \( \epsilon_A \) and \( \epsilon^a \) are arbitrary 1-forms and 0-forms respectively, and

\[
F^A(V) = dV^A + \frac{1}{2} g f_{BC}^A V^B \wedge V^C , \quad \hat{F}^a = dA^a + g T^a_{AB} V^A \wedge A^b
\]

\[
*\hat{F}_a = \frac{i}{4} \delta_{ab} dx^\mu \wedge dx^\nu \varepsilon_{\mu\nu\rho\sigma} \hat{F}^{\rho\sigma b} , \quad *V_A = \frac{1}{6} \delta_{AB} dx^\mu \wedge dx^\nu \wedge dx^\rho \varepsilon_{\mu\nu\rho\sigma} V^B
\]

where \( f_{AB}^C \) and \( T_A \) are structure constants and representation matrices of some Lie algebra \( G \),

\[
f_{[AB}^D f_{C]D} = 0 , \quad [T_A , T_B] = f_{AB}^C T_C .
\]

3 \quad **D=4, N=1 supersymmetric Henneaux-Knaepen models**

We shall now briefly outline our method [1] to supersymmetrize all the models defined through [2] [3]. We first construct a superspace version of these models, generalizing earlier work [4] on supersymmetric Freamon-Townsend models. To that end we associate an appropriate superfield with each of the forms in (8),

\[
B_A \rightarrow \Psi^\alpha_A , \quad A^a \rightarrow A^a , \quad V^A \rightarrow V^A , \quad (8)
\]

where \( \Psi^\alpha_A \) is a chiral spinor-superfield as in [7], while \( A^a \) and \( V^A \) are real vector-superfields. From \( A^a \) and \( V^A \), we construct two chiral superfields, \( Y^a_\alpha \) and \( W^A_\alpha \),

\[
Y^a_\alpha = -\frac{i}{4} \hat{D}^2 (e^{-2i\nu D_\alpha} e^{i\nu A})^a , \quad \nu = g V^A T_A
\]

\[
g W^A_\alpha T_A = -\frac{i}{4} \hat{D}^2 (e^{-2i\nu D_\alpha} e^{i\nu V}) .
\]

Thanks to the chirality of \( \Psi^\alpha_A \), \( Y^a_\alpha \) and \( W^A_\alpha \), the following Lagrangian is manifestly supersymmetric,

\[
L = \int d^2 \theta [ W^A \Psi_A + \delta_{ab} Y^a Y^b + d^2 \bar{\theta} \delta_{AB} V^A V^B] + c.c.
\]

This Lagrangian gives indeed a supersymmetric version of (8). This is seen by working out \( L \) in component form and by verifying that it is gauge invariant, up to a total derivative,
under the following transformations of $\Psi^A$, $A^a$ and $V^A$ which are the counterparts of the gauge transformations (10) and (11):

$$
\delta \Psi^A = i \bar{D}^2 (e^{-2i\hat{V}} e^{i\hat{C}}) A - 2ig \delta_{ab} Y^a T^b_{Ac} \Lambda^c, \quad \hat{V}^B_A = -g V^C f_{CA}^B \tag{18}
$$

$$
\delta A^a = i (e^{iV} \Lambda^a - e^{-i\bar{V}} \bar{\Lambda})^a, \quad \delta V^A = 0. \tag{19}
$$

Here $C_A$ and $\Lambda^a$ are arbitrary real vector superfields and chiral superfields respectively ($D_{\alpha} \Lambda^a = 0$).

Now, the gauge transformations (18) and (19) act as shift symmetries on some of the component fields of the superfields $\Psi^A$ and $A^a$. As usual, this means that the action can actually be written in terms of fewer fields, with a correspondingly reduced gauge invariance and modified supersymmetry transformations. Such a “WZ gauged” version of the above models is also constructed in [1]. The surviving component fields of $\Psi^A$ are those of a real linear multiplet, i.e., a real scalar field $\varphi_A$, a real 2-form gauge potential $B_{\mu\nu}^A$, and a Weyl spinor $\chi_A$. The surviving component fields of $A^a$ are a gauge field $A^{a}_{\mu}$, a Weyl spinor $\lambda^a$ and a real auxiliary field $D^a$. The component fields of $V^A$ are auxiliary fields which may be eliminated using the equations of motion. The supersymmetry algebra closes only modulo gauge transformations. All these features are expected from the experience with other supersymmetric gauge theories in the WZ gauge, such as standard super Yang-Mills theories. However, there is also a remarkable difference, as compared to other supersymmetric gauge theories: in the WZ gauge constructed in [1], the supersymmetry transformations do not commute with all the gauge transformations, not even on-shell!

References

[1] F. Brandt and U. Theis, D=4, N=1 Supersymmetric Henneaux-Knaepen Models, hep-th/9811180.

[2] P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, Phys. Lett. B 373 (1996) 81 (hep-th/9512143);
P. Claus, B. de Wit, M. Faux and P. Termonia, Nucl. Phys. B 491 (1997) 201 (hep-th/9612203);
N. Dragon and U. Theis, Phys. Lett. B 446 (1999) 314 (hep-th/9805199);
N. Dragon, E. Ivanov, S. Kuzenko, E. Sokatchev and U. Theis, Nucl. Phys. B 538 (1999) 411 (hep-th/9805152).

[3] M.F. Sohnius, K.S. Stelle and P.C. West, Phys. Lett. B 92 (1980) 123;
B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, Nucl. Phys. B 451 (1995) 53 (hep-th/9504006).

[4] M. Henneaux and B. Knaepen, Phys. Rev. D 56 (1997) 6076 (hep-th/9706119).

[5] D. Freedman and P.K. Townsend, Nucl. Phys. B 177 (1981) 282.

[6] G. Barnich and M. Henneaux, Phys. Lett. B 311 (1993) 123 (hep-th/9304057).

[7] W. Siegel, Phys. Lett. B 85 (1979) 333.

[8] T.E. Clark, C.-H. Lee and S.T. Love, Mod. Phys. Lett. A 14 (1989) 1343.