Look-up table approaches for TCSC impedance control considering thyristor conduction characteristic

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Abstract: Thyristor-controlled series capacitor (TCSC) characteristic curves of fundamental frequency impedance change with line current considering thyristor conduction characteristic, which results in experimental deviation when using the open-loop impedance control. To end this, this study derives the TCSC fundamental frequency impedance mathematic model to reveal the theoretical mechanism that fundamental frequency impedance changes with line currents. This study proposes two new look-up table approaches to mitigate deviation of the least square look-up table approach using the three-point quadratic interpolation and linear interpolation algorithms. The proposed methods have been validated via TCSC physical simulation experiments.

1 Introduction

Thyristor-controlled series capacitor (TCSC), as the first-generation flexible alternative current transmission systems (FACTS) equipment, is one of the most popular FACTS applications in real projects [1–4]. The physical simulation experiments of TCSC can reflect the real TCSC operational features and identify practical issues of control strategies, which are significant for real TCSC projects [5–7]. Normally, the controller of TCSC consists of three levels, i.e. the upper control level, middle control level, and lower control level. The upper control layer includes system control strategies, e.g. transient stability control and subsynchronous resonance control. The middle control level is the impedance control, which transforms the thyristor conduction direction from the upper control into thyristor trigger angle for the lower control. The lower control level is in charge of triggering the thyristors. The middle impedance control of TCSC plays a significant role in the hierarchical control architecture of TCSC, which directly affects the effectiveness of upper control strategies [8–13].

According to TCSC digital simulation under the line current synchronisation mode, this paper has found that TCSC characteristic curves of fundamental frequency impedance change with line current values, which causes experimental deviation when using the open-loop impedance control. In this paper, a fundamental frequency impedance model of TCSC is derived considering thyristor conduction characteristic under the line current synchronisation mode. This model shows the theoretical mechanism that TCSC characteristic of fundamental frequency impedance changes with line currents. Two new look-up table approaches are proposed to address issues of existing least square look-up table method, which are validated using physical simulation experiments.

2 Establishment of fundamental frequency impedance model considering thyristor conduction characteristic

2.1 Analysis of thyristor conduction characteristic of TCSC

Typical TCSC physical simulation device applies a pair of anti-parallel and low-power thyristor module. The conduction characteristic of the whole thyristor can be approximately equivalent to the constant resistance and the constant voltage (the same as thyristor conduction direction). Since the voltage level of physical simulation experiment is relatively low comparing to actual projects, the rated line current is only a dozen amperes and the thyristor conduction current is also relatively low. Therefore, the mathematical model ignores constant resistance characteristic of thyristor conduction and only considers constant voltage characteristic (i.e. around 1 V).

However, in the real project, both the current flowing through thyristors and the voltage are high. It is necessary to consider both constant voltage and resistance characteristic of thyristor conduction. If the thyristor conduction current is several hundred amperes, then the voltage of the entire reactor branch is probably between several hundred volts to several thousand volts at this time, and constant voltage of thyristor series is about tens of volts. Therefore, the thyristor voltage cannot be completely ignored. In this scenario, the model of constant voltage and resistance should be applied. If the thyristor current is so large that 1 V voltage can be completely ignored, the thyristor conduction characteristic can be replaced by a constant resistance model.

2.2 Fundamental frequency impedance model without considering thyristor conduction characteristic

The current relationship between fundamental frequency impedance of TCSC \((X_{\text{tcsc}})\) and trigger lead angle \((\beta)\) is as follows, without considering thyristor conduction characteristic [14]

\[
X_{\text{tcsc}} = \frac{1}{\omega L} \left[ 1 - K^2(2\beta + \sin 2\beta) \right] + \frac{4K^2 \cos^2 \beta}{\pi (K^2 - 1)^2} \left[ K \tan K \beta - \tan \beta \right]
\]

(1)

Suppose \(\alpha_s\) is trigger delay angle (trigger angle), trigger lead angle \(\beta\) is half of conduction angle, \(\beta = \pi - \alpha_s\), natural resonance angular frequency \(\omega_0 = 1/\sqrt{LC}\), \(K = \omega_0 / \omega = 100\pi \text{ rad/s}\).
It can be obtained from (1) that TCSC equivalent fundamental frequency impedance only relates to trigger angle when $L$ and $C$ are given. Thyristor delay angle corresponding with resonant point called resonant trigger angle $\alpha_{res}$ which is determined by the inductance and capacitance parameters. Equation (1) is the relationship between fundamental frequency impedance and trigger angle without considering thyristor conduction characteristic.

2.3 Proposed fundamental frequency impedance model considering thyristor conduction characteristic

Owing to discontinuous conduction control of TCSC thyristor, the TCSC circuit has non-linear characteristics. However, during the period of thyristor conduction or blocking, the changing state is linearity and periodicity. Thus, we present the topology analysis approach to derive fundamental frequency impedance model of TCSC considering thyristor conduction characteristic, which analyses characteristics of the circuit for solving linear differential equations.

The steady-state waveform of TCSC under the line current synchronisation mode is shown in Fig. 1. $i_{TH1}$, $i_{TH2}$, and $u_c$ represent the thyristor current, line current, and capacitor voltage, respectively. $i_{TH1}$ means thyristor peak current, $\alpha$ is trigger angle in line current synchronisation mode, $\sigma$ is the conduction angle, and $\sigma = \pi_1 + \pi_2$ (assuming $\beta_1 = \beta_2$). Taking the positive peak time of the line current as the zero point of time axis, the steady-state capacitor voltage is odd function, as well as the line current and thyristor current are even functions, all which have half-cycle symmetry. If the half-cycle steady-state calculation has been derived, then the calculation waveform on the whole time axis can be obtained based on this characteristic. The line current is represented by a current source with constant magnitude. That is

$$i_{line}(t) = I_0 \cos \omega t$$

(2)

2.3.1 Mathematical model and solution of thyristor conduction: When there is one thyristor in the conduction state ($\omega t \in [-\beta, \beta]$), TCSC equipment has the structure of series capacitor in parallel with the reactor. Considering constant voltage characteristic of thyristor ($U_{TH0}$) and assuming that the reactor is pure inductive element, the differential equations to describe TCSC circuit dynamic characteristics are shown in (3). Parameters $C$ and $L$ are the capacitance of series capacitor and inductance of the reactor, respectively

$$\begin{cases}
C \frac{d}{dt}u_c(t) = i_{TH1}(t) - i_{TH2}(t) \\
L \frac{d}{dt}i_{TH1}(t) = u_c(t) + U_{TH0}
\end{cases}$$

(3)

Solving (2) and (3), (4) is obtained

$$\begin{align*}
\frac{d^2}{dt^2} u_c(t) - \frac{1}{LC} \frac{d}{dt} u_c(t) &= -\frac{\omega L_0}{C} \sin \omega t - \frac{U_{TH0}}{LC} \\
\frac{d^2}{dt^2} i_{TH1} + \frac{i_{TH1}}{LC} &= \frac{I_0}{C} \cos \omega t
\end{align*}$$

(4)

The boundary conditions are as follows:

$$\begin{align*}
u_{C(0)} &= 0 \\
i_{TH1(0)} &= i_{TH2(0)} = 0
\end{align*}$$

(5)

The capacitor voltage expression considering thyristor conduction ($\omega t \in [-\beta, \beta]$) can be obtained via solving second-order constant coefficient linear differential (4), denoted as $u_C(t)$

$$u_C(t) = u_C(\theta) = \frac{I_0}{\omega C} \frac{1}{K^2 - 1} \left(\frac{K \cos \beta}{\cos K\beta} \sin K\omega t - \sin \omega t\right) + U_{TH0} \cos K\omega t$$

(6)

where $K = \omega_0/\omega$, $\omega_0 = 1/\sqrt{LC}$, and $\omega = 100\pi \text{ rad/s}$.

2.3.2 Mathematical model and solution of thyristor blocking: When two anti-parallel thyristors are in the block state ($\omega t \in [\beta, \pi - \beta]$), the TCSC circuit is equivalent to a series capacitor branch. Its state is described by the following first-order differential equation

$$C \frac{d}{dt} i_{line}(t) = i_{line}(t)$$

(7)

The boundary conditions are shown as follows:

$$u_C(\pi) = u_C(\pi/\omega) = u_C(\beta/\omega)$$

(8)

The capacitor voltage expression considering thyristor blocking ($\omega t \in [\beta, \pi - \beta]$) can be obtained via solving first-order differential equation (7), denoted as $u_C(t)$

$$u_C(t) = u_C(\theta) = \frac{I_0}{\omega C} \frac{K \cos \beta}{K^2 - 1} \left(\tan K\beta - K \tan \beta\right) + \sin \omega t + U_{TH0} \cos K\beta - 1$$

(9)

The TCSC capacitor voltage is a period variable with half-cycle and odd symmetry characteristics when in the steady-state operation. The fundamental wave amplitude $A_1$ of capacitor voltage can be calculated using Fourier transform, as shown in the following equation

$$A_1 = \frac{2}{\pi} \int_{0}^{\pi} u_C(t) \sin \omega t dt$$

$$= \frac{2}{\pi} \int_{-\pi}^{\pi} u_C(t) \sin \omega t dt - \int_{-\pi}^{\pi} u_C(t) \sin \omega t dt$$

(10)

Equation (11) can be obtained with (6) and (9)

$$A_1 = \frac{I_0}{\omega C} \left[1 - \frac{2}{\pi} \frac{K^2}{K^2 - 1} \left(2\beta + \sin 2\beta\right) + \frac{4K^2 \cos^2 \beta}{\pi(K^2 - 1)} \left(K \tan K\beta - \tan \beta\right)\right]$$

(11)

Equation (12) is the relationship between fundamental frequency

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**Fig. 1 Steady-state waveforms of TCSC electric parameters**

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impedance and trigger angle, which can be obtained from (11)

$$\begin{align*}
X_{\text{TCSC}} &= \frac{A_1}{I_m} = \frac{1}{6\pi} \left( 1 - \frac{K^2}{\pi(K^2 - 1)} \right) (2\beta + \sin 2\beta) \\
& \quad + \frac{4K^2 \cos^2 \beta}{\pi(K^2 - 1)} (K \tan K \beta - \tan \beta) \\
& \quad - \frac{4U_{\text{THD}}}{\pi} \cos \beta (1 - \cos K \beta)
\end{align*}$$

where $A_1$ represents fundamental amplitude of capacitor voltage, $I_m$ means the peak value of line current, and $U_{\text{THD}}$ represents constant voltage characteristic of thyristor conduction.

Equations (12) and (1) are the relationship between fundamental frequency impedance and trigger angle with and without thyristor conduction characteristic, respectively. It is observed that the section including $U_{\text{THD}}$ in (12) is the theoretical reason that TCSC characteristic of fundamental frequency impedance changes with line currents. The less the amplitude of line current is, the larger the deviation of relationship curve of fundamental frequency impedance and firing angle is. If the line current amplitude is large enough, the thyristor voltage can be ignored.

### 3 Proposed look-up table methods

In real TCSC projects, the middle impedance control of TCSC is difficult to calculate the trigger angle ($\alpha = \pi - \beta$) based on (1) with the restriction of real-time requirement in the impedance control. According to (1), a relationship table of fundamental frequency impedance and trigger angle is created to look up trigger angle for the open-loop impedance control, as shown in Fig. 2.

Therefore, the relationship curve between impedance and trigger angle is the key for the implementation of TCSC control.

#### 3.1 Issue of LSM-based look-up table approach

The open-loop control model in TCSC middle impedance control is shown in Fig. 1. The look-up table is used to transform the order impedance from the upper control into thyristor trigger angle, and then the thyristors are triggered in the lower control. However, during both PSCAD/EMTDC digital simulation and physical simulation experiments of TCSC, we have found that there is non-ignorable deviation in the open-loop impedance control according to the relationship curve of impedance and trigger angle using the least square method (LSM).

In the previous TCSC physical simulation device, LSM is adopted as the look-up table approach of the open-loop impedance control without considering thyristor conduction characteristic. Firstly, the relationship curve between the fundamental frequency impedance and trigger angle is generated using LSM, i.e. fitting polynomial coefficients according to an offline file with fundamental frequency impedance and trigger angle data. Secondary, when the middle control receives order impedance values from the upper control, the program in the middle control will call a function to calculate a trigger angle according to the polynomial coefficients.

According to aforementioned analysis, in terms of the small capacity TCSC physical simulation device without considering the thyristor conduction voltage drop, there are certain errors when only fundamental frequency impedance and trigger angle are utilised to build the table. In addition, the issue on large look-up table deviation has been found when the LSM is adopted in the open-loop control. Fig. 3 shows relationship curves of fundamental frequency impedance and firing angle in capacitive region under capacitor voltage synchronisation mode.

#### 3.2 Three-point quadratic interpolation algorithm

A new look-up table approach using three-point quadratic interpolation algorithm is proposed to address issues of existing look-up table method.

Given the function value of node $(x_i, y_i)(i = 0, 1, ..., n - 1; j = 0, 1, ..., m - 1)$ in rectangular domain $n \times m$ as $z_{ij} = z(x_i, y_j)$, the function value of the specific interpolating point $(u, v)$ is $w = z(u, v)$.

Assuming that the coordinates of $n \times m$ nodes in the rectangular domain are as follows:

$$\begin{align*}
x_0 &< x_1 < \cdots < x_{n-1} \\
y_0 &< y_1 < \cdots < y_{m-1}
\end{align*}$$

The corresponding function value is

$$z_{ij} = z(x_i, y_j) \quad (i = 0, 1, ..., n - 1; j = 0, 1, ..., m - 1)$$

For several nearest nodes of interpolating point $(u, v)$, the coordinates in two directions are as follows:

$$\begin{align*}
x_i &< x_{i+1} < x_{i+2} \\
y_j &< y_{j+1} < y_{j+2}
\end{align*}$$

The three-point quadratic interpolation algorithm is as follows:

$$z(x, y) = \sum_{p=0}^{p=2} \sum_{q=0}^{q=2} a_{pq} (x - x_0) (y - y_0)$$

In the practical application, $(x_i, y_j)$ and $z_{ij}$ are replaced by $(X_i, I_j)$ and $A_{pq}$, respectively. Here, $X_i$ is the fundamental frequency impedance, $I_j$ means line current, and $A_{pq}$ represents firing angle of thyristor. The original interpolation data are from MultipleRun simulation of PSCAD/EMTDC. The interpolation algorithm is implemented using C.
program. Giving interpolation point, i.e. some order impedance value, the firing angle can be found by calling the interpolation program.

3.3 Three-point quadratic interpolation algorithm

Another look-up approach is also proposed using linear interpolation algorithm. Fig. 4 shows relationship curves of fundamental frequency impedance and firing angle in capacitive region under different line currents. Fig. 5 is a three-dimensional diagram of line current, fundamental frequency impedance, and firing angle in capacitive region. In terms of the same fundamental frequency impedance, the larger line current is, the greater trigger angle is. Therefore, we can use the linear interpolation algorithm to look up table. Firstly, multiple relationship curves between fundamental frequency impedance and trigger angle under different line current values are generated. Secondly, the adjacent two line current curves are found according to the specific line current value. Then, according to the order impedance, the adjacent impedance values are identified using dichotomy from the fundamental frequency impedance curves corresponding to each adjacent line current. It is supposed that the curve between the two points is a straight line. The trigger angle associated with order impedance can be found by calling the interpolation program. This is an open access article published by the IET under the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0/)

4 Experimental results

The aforementioned two new look-up table approaches are proposed to address issues of exiting least square look-up table method, which are validated using physical simulation experiments.

### Table 1 Results by three-point quadratic interpolation and linear interpolation algorithm

| Line current, A | Order impedance, Ω | Three-point quadratic interpolation (trigger angle °) | Linear interpolation (trigger angle °) | Angle difference (°) |
|-----------------|--------------------|--------------------------------------------------|--------------------------------------|----------------------|
| 1.25            | 5.0                | 63.107854                                         | 62.501874                            | 0.6060               |
| 1.5             | 6.0                | 58.025278                                         | 58.650446                            | 0.6252               |
| 2.3             | 6.7                | 58.850125                                         | 58.971209                            | 0.1211               |
| 2.7             | 9.1                | 54.204653                                         | 54.060341                            | 0.1461               |
| 3.5             | 8.2                | 57.327046                                         | 57.119764                            | 0.2073               |
| 4.2             | 6.4                | 63.500124                                         | 63.583950                            | 0.0838               |
| 5.0             | 10.0               | 56.330243                                         | 56.028110                            | 0.3021               |
| 5.7             | 12.5               | 53.007981                                         | 52.857934                            | 0.1500               |
| 6.0             | 7.5                | 61.001955                                         | 61.590011                            | 0.5881               |
| 8.0             | 5.1                | 74.078142                                         | 74.191041                            | 0.1129               |

![Fig. 4 Relationship curves of fundamental frequency impedance and firing angle in capacitive region under different line currents](image)

The experimental results are demonstrated in Table 1 and Fig. 6. In the physical simulation experiment, ten relationship curves of impedance and firing angle are generated with different line currents (1, 1.5, 2, 2.5, 3, 4, 5, 6, 7, 8 A), as shown in Fig. 6. According to these line currents and order impedance values, the corresponding trigger angles are attained using both three-point quadratic interpolation and linear interpolation algorithms, as shown in Table 1. In Fig. 6, ‘+’ and ‘-’ are the interpolation results (i.e. trigger angles) using the proposed three-point quadratic interpolation and linear interpolation algorithms, respectively. The proposed look-up approaches can realise efficient and effective look-up for TCSC impedance control considering thyristor conduction characteristic, which have been applied in the open-loop impedance control of a real TCSC physical simulation device.

5 Conclusions

The digital simulation and physical simulation investigations of TCSC experiment equipment have found that TCSC characteristic curves of fundamental frequency impedance change with current values considering the impact of the TCSC thyristor conduction characteristic, which brings experimental deviation when using the open-loop impedance control. This paper presents the fundamental frequency impedance mathematic model of TCSC considering thyristor conduction characteristic under the line current synchronisation mode. It reveals the theoretical mechanism why TCSC characteristic of fundamental frequency impedance changes with line currents. In this paper, the three-point quadratic interpolation and linear interpolation algorithms-based new
look-up table approaches are proposed to mitigate deviation of existing least square look-up table method. The proposed look-up table approaches have been validated and verified using physical simulation experiments of the TCSC device.

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7 References
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