Certain conclusions of Gordon decomposition

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ABSTRACT

We use a new view to the our reality which is presented by Guts-Deutsch multiverse. In this article, we consider some conclusions of Gordon decomposition of Dirac current.
Introduction

In paper [1] we suppose that ghost spinors are same to Deutsch shadow particles [2]. From this suggestion some questions are appeared. In this article such questions are investigated. We assume that our reality is the Guts-Deutsch multiverse [2, 3]. In paper [4] some aspects of quantum particles interference were studied. In result the study of shadow particle current is interested. The last exists because in [2] we have a new explanation of interference in known quantum mechanics experiments. Deutsch assumes that shadow electrons act upon a real electron. Herewith shadow electrons are electrons in parallel universes and they are not observed by detectors. The mathematical model of Guts-Deutsch multiverse was presented in [3]. So a problem of studied of shadow particles, or in other words ghost spinors, is appeared.

1 Gordon decomposition

Let us consider Dirac equation

\[ i\hbar \gamma^k \left( \frac{\partial \psi}{\partial x^k} - \Gamma_k \psi \right) - mc\psi = 0, \]

the spin connection \( \Gamma_k \) is founded by formula:

\[ \Gamma_k = \frac{1}{4} g_{ml} \left( \frac{\partial \lambda_{(s)}^l}{\partial x^k} \lambda_{(s)}^r - \Gamma_{rk} \right) s^{mr}, \]

where

\[ s^{mr} = \frac{1}{2} \left( \gamma^m \gamma^r - \gamma^r \gamma^m \right). \]

Moreover we take into account that

\[ \gamma^k \equiv \lambda^{(i)}_{(i)} \gamma^{(i)}, \tag{1} \]

here \( \lambda_{(i)}^{(i)} \) is a \( i \)-th vector of tetrad and \( \gamma^{(i)} \) are Dirac matrixes for which we have the next presentation with matrixes of Pauly:

\[ \gamma^{(0)} = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \quad \gamma^{(\alpha)} = \begin{bmatrix} 0 & \sigma_\alpha \\ -\sigma_\alpha & 0 \end{bmatrix}, \]

\[ \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \]
Herewith the stress-energy tensor is defined by expression:

\[
T_{ik} = \frac{i\hbar c}{4} \left\{ \psi^* \gamma^{(0)} \gamma_i \left( \frac{\partial \psi}{\partial x^k} - \Gamma_k \psi \right) - \left( \frac{\partial \psi^*}{\partial x^k} \gamma^{(0)} + \psi^* \gamma^{(0)} \Gamma_k \right) \gamma_i \psi + 
\right.
\]

\[
+ \psi^* \gamma^{(0)} \gamma_k \left( \frac{\partial \psi}{\partial x^i} - \Gamma_i \psi \right) - \left( \frac{\partial \psi^*}{\partial x^i} \gamma^{(0)} + \psi^* \gamma^{(0)} \Gamma_i \right) \gamma_k \psi \right\}. \tag{2}
\]

For two solutions of Dirac equation \( \psi_1(x) \) and \( \psi_2(x) \) we have Gordon decomposition [5, p. 45]:

\[
c\psi_2^* \gamma^{(0)} \gamma^{(k)} \psi_1 = \frac{i\hbar}{2m} \left[ \psi_2^* \gamma^{(0)} \frac{\partial \psi_1}{\partial x^k} - \frac{\partial \psi_2^*}{\partial x^k} \gamma^{(0)} \psi_1 \right] + \frac{\hbar}{2m} g_{lm} \frac{\partial}{\partial x^m} \left[ \psi_2^* \gamma^{(0)} \sigma^{kl} \psi_1 \right],
\]

where \( \sigma_{ik} = \frac{i}{2} [\gamma_i, \gamma_k] \). As known the Dirac current is defined by expression:

\[
j^{(k)} = c \lambda^{(k)}_i \psi^* \gamma^{(0)} \gamma^i \psi.
\]

Let us use the formula (1), then first expression for Dirac current will take a form:

\[
j^{(k)} = c \lambda^{(k)}_i \psi^* \gamma^{(0)} \lambda^{(m)}_i \gamma^{(m)} \psi.
\]

If we take into consideration the property of tetrads vectors \( \lambda^{(k)}_i \lambda^{(m)}_i = \delta^k_m \) and summarize last expression then we have

\[
j^{(k)} = c \psi^* \gamma^{(0)} \gamma^{(k)} \psi.
\]

Now let us take \( \psi_1 = \psi_2 \) then we will get the Gordon decomposition of Dirac current [5]

\[
j^{(k)} = \frac{i\hbar}{2m} \left[ \psi^* \gamma^{(0)} \frac{\partial \psi}{\partial x^k} - \frac{\partial \psi^*}{\partial x^k} \gamma^{(0)} \psi \right] + \frac{\hbar}{2m} g_{lm} \frac{\partial}{\partial x^m} \left[ \psi^* \gamma^{(0)} \sigma^{kl} \psi \right]. \tag{3}
\]

The first summand is a relativistic analog of displacement current

\[
j = \frac{i\hbar}{2m} \left[ \varphi \nabla \varphi^* - \varphi^* \nabla \varphi \right],
\]

here \( \varphi \) is a solution of Shrödinger equation. The second summand in (3) corresponds to spin current [5]. So we have following. A Dirac current and a momentum are not proportionals to one another. We will show that in some cases the displacement current of ghost spinors equals to zero though the Dirac current of ghost spinors not vanishes.
2 Shadow particles currents

Shadow particles are solutions of Dirac equation. Ones are defined by vanished stress-energy tensor and a non-zero Dirac current. Now if we take a track of the stress-energy tensor \([2]\) and also if we take into account the Dirac equation, then the next result will be take placed \([6]\):

\[
T_i^j = mc^2\psi^*\gamma^{(0)}\psi.
\]  \(\text{(4)}\)

Herewith we used the conjugated Dirac equation:

\[
i\hbar \left( \frac{\partial \psi^+}{\partial x^k} + \psi^+\Gamma_k \right) \gamma^k = -mc\psi^+,
\]

where we taken the standard denotation of Dirac-conjugated spinor \(\psi^+ = \psi^*\gamma^{(0)}\). In case of solutions of Dirac equation for shadow particles the stress-energy tensor equals to zero. Then the track of the last is equal to zero too. So from formula \([4]\) the next result was got.

**Proposition 1. (Ghost spinor necessities condition)**

*If a solution of Dirac equation be a ghost spinor then*

\[
\psi^+\psi = \psi^*\gamma^{(0)}\psi = 0.
\]

Hereinafter we will take into consideration Minkowsky spacetime. So we have that \(\gamma^k = \gamma^{(k)}\) and the spin connection \(\Gamma_k\) is equal to zero.

The next theorem was proved in \([4]\).

**Theorem 1.** Let \(\psi = u \cdot G(x)\) be a solution of Dirac equation. Herewith a spacetime geometry is defined by Minkowsky metric. Let \(\psi^*\psi \neq 0\) and

\[
G(x) = f(x) + i \cdot g(x),
\]

where \(f(x)\) and \(g(x)\) are smooth real functions. Moreover

\[
u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix},
\]

where \(\forall \ i \ u_i \in \mathbb{R}\). In considered conditions \(\psi\) will be a ghost spinor if and only if \(g(x) = a \cdot f(x)\), where \(a = \text{const} \in \mathbb{R}\).
So if a solution of Dirac equation is presented by expression $\psi = u \cdot G(x)$ then it will be a ghost spinor if and only if $G(x)$ is a real function. For these wave functions the next statement about current, which is defined by expression

$$j^k_p = \frac{ih}{2m} \left[ \psi^* \gamma^{(0)} \frac{\partial \psi}{\partial x^k} - \frac{\partial \psi^*}{\partial x^k} \gamma^{(0)} \psi \right], \quad (5)$$

can be formulated.

**Theorem 2.** Let a solution of Dirac equation $\psi$ be a ghost spinor. Also let us assume that $\psi$ satisfies one of the next conditions:

1) $\psi = u \cdot f(x)$, where $x = (x^0, x^1, x^2, x^3)$, herewith $f(x)$ is a real function and bispinor $u$ has complex components,

2) $\forall k \frac{\partial \psi}{\partial x^k} = g_k(x) \psi$, where $g_k(x)$ are complex functions,

then the displacement current in Gordon decomposition of Dirac current is equal to zero everywhere.

**Proof.** Let us take into account the formula (5). It is not difficult to see that we must prove only the equality

$$\psi^* \gamma^{(0)} \frac{\partial \psi}{\partial x^k} - \frac{\partial \psi^*}{\partial x^k} \gamma^{(0)} \psi = 0. \quad (6)$$

Let a first case will be take placed. Then a shadow particle will be defined by expression $\psi = u \cdot f(x)$. Thus if we insert corresponded solution of Dirac equation to (6) then we get the expression

$$u^* \gamma^{(0)} u \left( f \frac{\partial f}{\partial x^k} - \frac{\partial f}{\partial x^k} f \right) = 0.$$

So in this case the displacement current is equal to zero.

Let now a second case will be take placed:

$$\forall k \frac{\partial \psi}{\partial x^k} = g_k(x) \psi.$$

Then the left part of the equality (6) has the form $^1$

$$g_k(x) \psi^+ \psi - \overline{g_k(x)} \psi^+ \psi.$$

In general case, for example if we take real particle, this expression not equals to zero. For shadow particles, as we have proposition 1, both summands equal to zero. The proof is finished.

$^1$We have indication $\overline{\gamma}(x)$ for complex conjugation function
From these results some corollaries are appeared. Let us consider two solutions of Dirac equation \( \psi_1 \) and \( \psi_2 \). If these particles interact then for current of the result wave \( \psi = \psi_1 + \psi_2 \). Both Dirac current and displacement current are disintegrated to the sum which contains summands of free particles and summand of interaction. If for example \( \psi_2 \) is a ghost spinor then the next result exists.

Let a displacement current \( j^k_{ip} \) corresponds to the wave function \( \psi_i \). So the displacement current of the result wave satisfies to

\[
j^k_p = j^k_{1p} + j^k_{2p} + j^k_{12p},
\]

where in our case \( j^k_{2p} = 0 \) and third summand is defined by interaction of fields. Herewith

\[
j^k_{12p} = \frac{i\hbar}{2m} \left[ \psi_1^* \gamma^{(0)} \frac{\partial \psi_2}{\partial x^k} - \frac{\partial \psi_1^*}{\partial x^k} \gamma^{(0)} \psi_2 + \psi_2^* \gamma^{(0)} \frac{\partial \psi_1}{\partial x^k} - \frac{\partial \psi_2^*}{\partial x^k} \gamma^{(0)} \psi_1 \right]
\]

and in general case the corresponding expression not equals to zero. Thus we have \( j^k_p \neq j^k_{1p} \). So we can see that in considering case interaction occurs at the expense of spin current.

### 3 Wave-corpuscle duality

As known, quantum mechanics is based on corpuscle and wave properties of quantum particles. Let us assume that our reality is a multiverse. Then corresponded duality may be explained by following.

In first let us notice that a square of module of probability amplitude of shadow particle, which is a own particle of the some real particle, is equal to a square of module of probability amplitude of the corresponded real particle.

Let ”wave” consists of one real particle and many own shadow particles. A shadow particle not can be detected. So in any moment of time only one particle can be detected, it be a real particle. This will correspond to corpuscle properties of our ”wave.” Herewith such wave property as interference will be explained by shadow patities exxistance [1, 2, 3, 4].

### Conclusion

If a shadow particle interacts with a real particle then the displacement current of a real particle is changed, but the displacement current of a shadow particle,
in considering cases, is equal to zero. The displacement current is in proportion to the momentum. So we can conclude that the interaction, which is between a real particle and a shadow particle, acts upon a location of a real particle in space. In spite of the fact that the displacement current of a shadow particle and the momentum of a shadow particle are not equal to zero. But as for real as for shadow particle the Dirac current, which is conservable value, be non-zero. This influence will be exist as the spin current of a shadow particle is not equal to zero. These results are agreed with Deutsch assumptions [2].

In paper [11] in Minkowsky spacetime, ghost spinors were found. In [7 8 9 10] in curved spacetimes, ghost neutrinos were found. So we can see that not only in flat spacetime the shadow particles can exist.

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