Spin and Parity Assignments to Dipole Excitations of the Odd-Mass Nucleus $^{207}$Pb from Nuclear Resonance Fluorescence Experiments with Linearly-Polarized $\gamma$-Ray Beams

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Abstract. Pb($\gamma$, $\gamma'$) photon scattering reactions were studied [1] with the nearly monochromatic, linearly polarized photon beams at the High Intensity $\gamma$-ray Source (HI$\gamma$S) at the DFELL. Azimuthal scattering intensity asymmetries measured with respect to the polarization plane of the beam have been used for the first time to assign both the spin and parity quantum numbers of dipole excited states of $^{206,207,208}$Pb at excitation energies in the vicinity of 5.5 MeV. Evidence for dominant particle-core coupling is deduced from these results along with information on excitation energies and electromagnetic transition matrix elements.

Nuclear Resonance Fluorescence (NRF) is well established as a powerful method for investigation of nuclear dipole excitation (see [2–5] and References therein). However, assignments of spin and parity quantum numbers to excited states of odd-mass nuclei from NRF data has remained difficult up to now. Information on spin and parity quantum numbers must be obtained from anisotropies in angular intensity or polarization distributions.

For odd-A nuclei the anisotropies are typically small, in particular, when an unpolarized $\gamma$-ray beam is used for the experiments. The angular distribution function $W(\theta, \phi)$ can be expanded in terms of Legendre polynomials. In the absence of parity violation and third order multipole contributions for the $J^m_0(L_1, L'_1)J^s(L_2, L'_2)J^f_f$ cascade with unpolarized photons it can be written as:

$$W(\theta) = \sum_{\nu=0}^{\text{even}} B_{\nu}(\gamma_1)A_{\nu}(\gamma_2)P_{\nu}(\cos\theta)$$

where $P_{\nu}$ are the ordinary Legendre Polynomials of the $\nu$th order. The expansion coefficients $A$ and $B$ can be written, using the phase convention of Krane, Steffen, and Wheeler [6] for the
multipole mixing ratios $\delta_1$ and $\delta_2$ and in terms of tabulated F-coefficients [6] as:

$$A_\nu(\gamma_2) = \frac{1}{1 + \delta_2^2}[F_\nu(L_2, L_2, J_f, J) + 2\delta_2 F_\nu(L_2, L'_2, J_f, J) + \delta_2^2 F_\nu(L'_2, L'_2, J_f, J)]$$  \hspace{1cm} (2)

$$B_\nu(\gamma_1) = \frac{1}{1 + \delta_1^2}[F_\nu(L_1, L_1, J_0, J) - 2\delta_1 F_\nu(L_1, L'_1, J_0, J) + \delta_1^2 F_\nu(L'_1, L'_1, J_0, J)]$$  \hspace{1cm} (3)

Dipole transitions are the dominant mode of excitation in NRF processes. While in even-even nuclei (with $0^+$ ground state) a dipole excitation directly leads to a unique spin assignment $J = 1$ for the excited state, such an unambiguous assignment is not so straightforward in the case of an odd-mass nucleus with half-integer ground-state spin. Let us consider the case of a spin-1/2 ground state. Then, dipole excitations can excite states with spin quantum numbers $J = 1/2$ or $J = 3/2$. Figure 1 shows the angular distribution function (1) for pure dipole NRF cascades $1/2 \rightarrow 1/2 \rightarrow 1/2$ and $1/2 \rightarrow 3/2 \rightarrow 1/2$ using unpolarized $\gamma$-rays in the entrance channel. Unfortunately the differences of the maxima are not very significant in case of an odd-mass-nucleus, which is also mentioned in [7].

![Figure 1. Angular intensity distribution for unpolarized bremsstrahlung in case of the possible two dipole transitions in odd nuclei with $J = 1/2$ ground state. The differences in the intensity maxima are quite small, making spin assignments difficult.](image)

An alternative for assigning spin and parity quantum numbers in odd-nuclei is to investigate the target with linearly polarized photons. The intensity distribution function for polarized photons is given by

$$W(\theta, \phi) = \sum_{\nu=0}^{even} B_\nu(\gamma_1) A_\nu(\gamma_2) P_\nu(\cos\theta) + (\pm) L_1 \cos(2\phi) \sum_{\nu=0}^{even} B'_\nu(\gamma_1) A_\nu(\gamma_2) P_\nu^2(\cos\theta).$$  \hspace{1cm} (4)

Where $P_\nu$, A, B and $\delta$ are the same as in equation (1), $P_\nu^{(2)}$ is the unnormalized associated Legendre polynomial and $(\pm) L_1$ takes the value $+1(-1)$ in case of electric (magnetic) character of the leading multipole $L_1$ of the first transition. $B'$ can also be written in terms of the tabulated F-coefficients [6]:

$$B'_\nu(\gamma_1) = \frac{1}{1 + \delta_1^2}[\kappa_\nu(L_1, L_1) F_\nu(L_1, L_1, J_0, J) +$$

$$+ 2\delta_1^2 \kappa_\nu(L_1, L'_1)(L_1, L'_1, J_0, J) - \delta_1^2 \kappa_\nu(L'_1, L'_1) F_\nu(L'_1, L'_1, J_0, J)]$$  \hspace{1cm} (5)

The $\kappa$ coefficients can be found in Ref. [8]. For the most important cases of dipole and quadrupole radiation their values are $\kappa_2(1,1) = -1/2$, $\kappa_2(1,2) = -1/6$, $\kappa_2(2,2) = 1/2$ and
\( \kappa_4(2, 2) = -1/12 \). A \( 1/2^{\pi_0} \rightarrow 1/2^\pi \rightarrow 1/2^{\pi_0} \) NRF cascade has always an isotropic angular distribution, thus \( W(\theta, \phi)_{1/2\rightarrow1/2\rightarrow1/2} = 1 \). In the following we want to discuss the odd-mass nucleus \( ^{207}\text{Pb} \) with a ground-state spin \( J_0^{\pi_0} = 1/2^- \). An excited spin 1/2 state of the odd-mass nucleus \( ^{207}\text{Pb} \) therefore radiates isotropically and hence its angular distribution ratio at any two angles is 1. The intensity distribution function of a \( 1/2^- \rightarrow 3/2^+ \rightarrow 1/2^- \) photon scattering cascades using a polarized incident beam is given by

\[
W(\theta, \phi) = 1 + \frac{1}{4}[P_2(\cos\theta) - \frac{1}{2} \pi \cos(2\phi) P_2^{(2)}(\cos\theta)],
\]

with \( P_2^{(2)} \) being the unnormalized associated Legendre polynomial of second order and \( \pi \) being the parity quantum number of the excited state. For an excitation with positive parity, an E1 excitation, a maximum of NRF intensity shows up perpendicular to the polarization plane of the horizontally polarized photon beam. For an excitation with negative parity (M1), a maximum of intensity occurs in the polarization plane.

**Figure 2.** Angular distribution of the scattered photons in case of an E1 excitation in odd-mass nuclei for a horizontally polarized photon beam. The maxima of the intensity distribution occur at \( \phi = 0^\circ \) and \( \phi = 180^\circ \).

**Figure 3.** Angular distribution of the scattered photons at \( \theta = 90^\circ \). Depending on the character of the excitation, one obtains an azimuthal NRF asymmetry of 0.4.

Fig. 2 shows the angular distribution for \( 1/2^- \rightarrow 3/2^+ \rightarrow 1/2^- \) NRF cascade in an odd mass nucleus in case of a horizontally polarized photon beam. The analyzing power is maximum at a polar angle \( \theta = 90^\circ \). The maxima of NRF intensity occur for \( \phi = 0^\circ \) and \( \phi = 180^\circ \). In case of an E1 transition, the angular distribution is shown by the continuous curve, an M1 transition is shown by the dashed curve. Using a linearly polarized \( \gamma \)-ray beam for NRF studies of an odd-mass nucleus and placing detectors at the above-mentioned angles should make spin and parity assignments possible. The two states of \( ^{207}\text{Pb} \) observed at 5.490 and 5.597 MeV are of special interest. The assignment of their spin and parity quantum numbers represents a novel use of a linearly polarized photon-beam and, hence, it is discussed in more detail below.

Considering dipole excitations of the \( J_0^\pi = 1/2^- \) ground state of \( ^{207}\text{Pb} \), the spins and parities of the two excited states could be \( J^\pi = 1/2^+, 3/2^+ \) only. The corresponding dipole excitations of \( ^{207}\text{Pb} \) were previously investigated, using unpolarized bremsstrahlung at the High Intensity Photon Setup (HIPS) at S-DALINAC \(^7\). The measurements to assign the spin and parity quantum numbers in the 5.490 and 5.597 MeV states in \( ^{207}\text{Pb} \) have been performed at the High Intensity Gamma Ray Source (HI\(\gamma\)S) at Duke...
Free Electron Laser Laboratory (DFELL) using its nearly monoenergetic linearly polarized γ-ray beams. The H15S [9] is based on the Duke/OK-4 Storage Ring FEL. Head-on collisions of the FEL photons and the relativistic electrons in the storage ring generate γ-rays in the laboratory system by the Compton effect. The wavelength of the FEL photons and the energy of the electron beam in the ring are tunable over a large range of values, which allows the production of γ rays at a corresponding range of energies. An on-axis collimator selects a narrow cone of linearly polarized, nearly monoenergetic γ rays from 180° Compton back-scattering processes. That beam is tunable in energy in the MeV range and is available for nuclear resonance fluorescence reactions on target nuclei placed in the γ-ray beam. Fluorescence γ-quanta result from the decay of resonantly photo-excited nuclear levels and can be observed with high energy resolution using large volume HPGe semiconductor detectors. Details of the set-up and the method have been published previously in Refs. [9,10]. The OK-4 FEL was tuned to lase at wavelengths around 450 nm. Mean energies of the resulting γ-ray beams were $E_\gamma = 5.5$ and 5.6 MeV with a width of about 200 keV. The collimator size was 2.5 cm and the energy spread of the γ-beam was around 100 keV. The γ-beam intensity on target was of the order of $10^7$ γ per second. Data was taken for about 6 hours. The target consisted of a 5.65 g/cm$^2$-thick natural Pb sample with 25.4 mm diameter. (total mass 28 g). It was surrounded by four coaxial HPGe detectors with efficiencies of about 60% relative to a standard 3"x3" NaI detector at 1.3 MeV and they were mounted at a mean polar angle of $\theta = 90^\circ$ and at azimuthal angles of $\phi = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ with respect to the (horizontal) polarization plane of the incident γ beam.

![Figure 4](image_url)

**Figure 4.** Spectra of the Pb(γ, γ′) reaction at a mean polar angle $\bar{\theta} = 90^\circ$ relative to the incident photon beam and azimuthal angles $\phi_\parallel = (0^\circ)(\text{top})$ and $\phi_\perp = (90^\circ)$ (middle) relative to the polarization plane of the 5.6±1 MeV γ-ray beam at H15S. Big arrows (red) mark the two ground state transitions of $^{207}$Pb at 5.490 and 5.597 MeV. The difference of the intensity ratios of these two γ-rays is clearly visible. Other observed ground state decays are indicated by smaller arrows with the corresponding Pb isotopes labeled on top. The right most peak in the lower panel is a doublet caused by the 5.611 and 5.616 MeV lines from $^{207}$Pb and $^{207}$Pb, respectively. The spectrum at the bottom shows the mentioned states under $\theta = 135^\circ$ and $\phi = 0^\circ$.

Fig. 4 displays (gain-matched and added) photon scattering spectra at azimuthal angles $\phi_\parallel = (0^\circ, 180^\circ)$ in the polarization plane of the photon beam (top) and perpendicular to it.
Table 1. Excitation energy \( E_x \), spin and parity quantum numbers \( J^\pi \) from this work and from Ref. [7,11] of excited states of \(^{206,207,208}\text{Pb} \). Values for \( E_x \) are taken from Ref. [7] unless otherwise noted.

| \( E_x \) (MeV) | \( J^\pi \) | \( J^{\pi 0} \) |
|-----------------|-----------|-------------|
| \(^{206}\text{Pb} \) | 5.5251(3) | 1\(^-\) 1\(\pm\) |
| \(^{206}\text{Pb} \) | 5.5811(3) | 1\(^-\) 1\(\pm\) |
| \(^{206}\text{Pb} \) | 5.6161(3) | 1\(^-\) 1\(\pm\) |
| \(^{207}\text{Pb} \) | 5.4897(3) | 3\(^+\) 1\(\pm\) 3\(\pm\) |
| \(^{207}\text{Pb} \) | 5.5974(3) | 1\(^+\) 7\(\pm\) 3\(\pm\) |
| \(^{207}\text{Pb} \) | 5.611(2) | 3\(^+\) (1\(\pm\), 3\(\pm\) |
| \(^{208}\text{Pb} \) | 5.5121(3) | 1\(^-\) 1\(\pm\) |

\( \phi_\perp = (90^\circ, 270^\circ) \) (middle) using a linearly polarized incident photon beam with 5.6(1) MeV energy. The peaks at 5.490, 5.512, 5.597 and 5.611 MeV in the spectra at \( \phi_{\parallel} \) and \( \phi_{\perp} \) correspond to ground state decay transitions of excited states at these energies in \(^{207,208}\text{Pb} \). They are indicated by red arrows in the \( \phi = 0^\circ \) spectrum. In the spectrum at \( \phi_{\perp} \), the peaks at 5.525, 5.580 and 5.617 MeV represent ground state decay transitions of excited states at these energies in \(^{208}\text{Pb} \). These peaks are invisible in the \( \phi_{\parallel} \) spectrum, which proves their \( E1 \) character. To identify the spin and parity quantum numbers of the two states in \(^{207}\text{Pb} \), we form the double ratio (DR) of NRF intensities observed within the polarization plane \( (\phi_{\parallel}) \) and perpendicular to it \( (\phi_{\perp}) \). Neglecting small differences in the energy dependence of the detector efficiencies at these two nearby energies, the experimental DR is proportional to the DR of the angular distribution for one of the four possible scattering cascades \( \frac{1}{2}^- \rightarrow \frac{1}{2}^\pm \rightarrow \frac{1}{2}^- \) and \( \frac{1}{2}^- \rightarrow \frac{3}{2}^\pm \rightarrow \frac{1}{2}^- \). The theoretical DR is defined as

\[
\text{DR}_{J_1,J_2} = \frac{W_{J_1}(\phi_{\parallel})}{W_{J_1}(\phi_{\perp})} \cdot \frac{W_{J_2}(\phi_{\parallel})}{W_{J_2}(\phi_{\perp})},
\]

where \( W_J(\phi) \) for any \( \frac{1}{2}^- \rightarrow \frac{3}{2}^\pm \rightarrow \frac{1}{2}^- \) photon scattering cascades are given by \( W(90^\circ, \phi) \) in Eq. (6). Looking at the angular distributions at Figure 3, the maxima of the intensity for an \( E1 \) transition lies at 1.25 at \( \phi = 0^\circ \) and \( \phi = 180^\circ \). The relative intensity at \( \phi = 90^\circ \) and \( \phi = 270^\circ \), the minimum of intensity, amounts to 0.5. The DR for an \( E1 \) transition amounts to DR=0.4.

Table 2. Given is a total of 16 possible spin and parity combinations for the two states of \(^{207}\text{Pb} \), which correspond to different values of the double ratio (DR). Combinations restricted to \( E1 \) excitations of the two states are specified on the right.

| DR   | \( (J_1^{\pi}, J_2^{\pi}) \)                                      | for \( E1 \) only |
|------|------------------------------------------------------------------|------------------|
| 0.16 | \((\frac{3}{2}^+, \frac{3}{2}^-)\)                              | –                |
| 0.40 | \((\frac{3}{2}^+, \frac{1}{2}^\pm), (\frac{1}{2}^\pm, \frac{3}{2}^-)\) | \((\frac{3}{2}^+, \frac{1}{2}^+)\) |
| 1.00 | \((\frac{3}{2}^+, \frac{3}{2}^+), (\frac{1}{2}^-, \frac{3}{2}^-), (\frac{1}{2}^-, \frac{1}{2}^+), (\frac{3}{2}^+, \frac{3}{2}^+), (\frac{3}{2}^+, \frac{1}{2}^+), (\frac{1}{2}^+, \frac{3}{2}^+), (\frac{1}{2}^-, \frac{3}{2}^-)\) | – |
| 2.50 | \((\frac{3}{2}^-, \frac{1}{2}^\pm), (\frac{3}{2}^-, \frac{3}{2}^+), (\frac{1}{2}^+, \frac{3}{2}^-)\) | \((\frac{1}{2}^+, \frac{3}{2}^+), (\frac{1}{2}^-, \frac{3}{2}^-)\) |
| 6.25 | \((\frac{3}{2}^-, \frac{3}{2}^+)\)                                | –                |

Table 2 lists all possible DR values and the corresponding spin and parity combinations of
the two states. Experimentally one finds

\[ DR_{\text{expt}} = \frac{I_{5.490}(\phi_\parallel)}{I_{5.490}(\phi_\perp)} : \frac{I_{5.597}(\phi_\parallel)}{I_{5.597}(\phi_\perp)} = 0.39(4) . \]

Therefore, the quantum numbers \((J^\pi_{5.490}, J^\pi_{5.597})\) are restricted to the combinations \((3/2^+, 1/2^\pm)\) and \((1/2^\pm, 3/2^-)\), only.

From the transition rates of the two states at 5.490 and 5.597 MeV, the most likely multipolarity of their ground state decays is E1. Then the measured \(DR_{\text{expt}}\) and Table 2 leaves \(J^\pi = \frac{3}{2}^+\) for the state at 5.490 MeV and \(\frac{1}{2}^+\) for the state at 5.597 MeV.

In order to verify our conjecture of \(E1\) radiation character, we finally deduce the angular distribution perpendicular and parallel to the polarization plane for both transitions individually. With (6) one obtains

\[ W(90^\circ, \phi_\perp) = \begin{cases} 0.4 & J^\pi = \frac{3}{2}^- \\ 1 & J^\pi = \frac{1}{2}^\pm \\ 2.5 & J^\pi = \frac{3}{2}^+ \end{cases} \]  

\( (8) \)

Figure 5. Double ratio (DR) for the two states of \(^{207}\)Pb at 5.490 and 5.597 MeV on a logarithmic scale. Possible values of the DR are labeled on the horizontal lines in the figure. From the corresponding spin and parity combinations \((J^\pi_{5.490}, J^\pi_{5.597})\) for each DR, listed in Table 2, the measured DR=0.39(4) indicates that \((J^\pi_{5.490}, J^\pi_{5.597})\) must be \((3/2^+, 1/2^\pm)\) if \(E1\) radiation is considered, only.

Figure 6 displays the azimuthal intensity distribution ratios for the \(\gamma\)-ray transitions. The values are \(2.35(66)\) and \(0.92(25)\) for the states at 5.490 and 5.597 MeV, respectively. Comparing with (8) the data confirm the \((J^\pi_{5.490}, J^\pi_{5.597}) = (3/2^+, 1/2^+)\) assignment to better than two standard deviations. The isotropy of the angular distribution for spin-1/2 states prevents us from making parity assignments to these cases. For clarity we, therefore stress, that one out of these four quantum number assignments, namely the \(\pi = (+)\) assignment for the \(J=1/2^\pm\) state at 5.597 MeV, is not based on angular distribution of photon scattering intensity but on the large decay rate of the level which leaves \(E1\) multipolarity as the only likely possibility. These are the assignment that we adopt in Table 1. The level energies agree within 22 keV and 85 keV, respectively, with the excitation energy of the predominant bound \(E1\) excitation of the neighboring doubly-closed shell nucleus \(^{208}\)Pb at 5.512 MeV. I.e., the energy splitting of the doublet \(\Delta E_{\nu(p_{1/2}^-)}(5.512) = E_{3/2^-} - E_{1/2^-} = -107\) keV is less than 2% of the centroid value. This is comparable to the relative splitting of the well-established \(3^- \otimes \nu(p_{1/2}^-)\) weakly coupled doublet at 2.6 MeV \([12,13]\). However, literature values for transition rates indicate more pronounced deviation from weak coupling at 5.5 MeV. Like the \(1^-\) state of \(^{208}\)Pb at 5.512 MeV, that doublet of individual quantum states of \(^{207}\)Pb dominates its dipole excitation strength distribution, here, with \(E1\) decay rates to the ground state of 18(4) and 29(4) mW.u. \([7]\). While
these are the largest $E1$ strengths from the ground state to excited states up to 6.2 MeV, which is the energy up to where data on $^{207}$Pb are available, these values are a factor of three smaller than in the $^{208}$Pb core indicating a substantial deviation from a pure weak coupling.

In summary, we have demonstrated a new method for spin and parity quantum number assignments to highly excited particle-bound dipole excitations of odd-mass nuclei based on the measurement of azimuthal nuclear resonance fluorescence-intensity asymmetries about an incident linearly polarized $\gamma$-ray beam. This method has been used to identify doublet of energy levels of $^{207}$Pb that predominantly originates in the weak coupling of the $\nu(3p_{1/2})$ neutron hole to the strongest neutron-bound electric dipole excitation of the doubly closed shell $^{208}$Pb core at 5.51 MeV. The observed energy splitting of this $J = 1/2, 3/2$ doublet at an excitation energy of more than 30$A^{-1/3}$ MeV amounts to less than 2% of its excitation energy with the $J = 3/2$ state being the lower lying level. Deviations from the pure weak-coupling scenario show up in the absolute transition rates due to particle-core interaction.

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