Analysis of mathematical problem-solving ability based on metacognition on problem-based learning

Mulyono1,* and R Hadiyanti1

1Mathematics Department, Universitas Negeri Semarang, Semarang, Indonesia
*Corresponding author: mulyono.mat@mail.unnes.ac.id

Abstract. Problem-solving is the primary purpose of the mathematics curriculum. Problem-solving abilities influenced beliefs and metacognition. Metacognition as superordinate capabilities can direct, regulate cognition and motivation and then problem-solving processes. This study aims to (1) test and analyzes the quality of problem-based learning and (2) investigate the problem-solving capabilities based on metacognition. This research uses mixed method study with The subject research are class XI students of Mathematics and Science at High School Kesatrian 2 Semarang which divided into tacit use, aware use, strategic use and reflective use level. The collecting data using scale, interviews, and tests. The data processed with the proportion of test, t-test, and paired samples t-test. The result shows that the students with levels tacit use were able to complete the whole matter given, but do not understand what and why a strategy is used. Students with aware use level were able to solve the problem, be able to build new knowledge through problem-solving to the indicators, understand the problem, determine the strategies used, although not right. Students on the Strategic ladder Use can be applied and adopt a wide variety of appropriate strategies to solve the issues and achieved re-examine indicators of process and outcome. The student with reflective use level is not found in this study. Based on the results suggested that study about the identification of metacognition in problem-solving so that the characteristics of each level of metacognition more clearly in a more significant sampling. Teachers need to know in depth about the student metacognitive activity and its relationship with mathematical problem solving and another problem resolution.

1. Introduction
The purpose of education is to prepare students for solving problems faced every day. The subject that can lead students to solve daily problems in Mathematics. Mathematics is a curriculum content that has a function as a tool of problem deepening and problem-solving to help people solve problems in life [1]. This statement is by the opinion of Orton [2] which states that problem-solving is part of a math curriculum that is very important to apply in solving a problem that is not routine which is needed in everyday life. The importance of students’ mathematical problem-solving skills is the focus of world education. The National Council of Teachers of Mathematics (NCTM) pays excellent attention to problem-solving abilities. Problem-solving skills are a focus in the mathematics curriculum [3]. Problem-solving is the center of both inquiry and application, so it must be established throughout the content of the mathematics curriculum. Such ability is necessary for learning and applying mathematical ideas.

The mathematics curriculum in Indonesia begins to focus on problem-solving abilities. This ability shows that 75% of the composition of national exam math problems in 2016 is a matter of problem-
solving. The impact of the change is the result of the national exam of SMA/MA in 2016 has decreased from the average of 61.29 to 54.78.

The absorption capacity of calculus on the 2016 national examination occupies the lowest position when compared with the absorption of other mathematical content. It shows the lack of students' ability to master the material of calculus. Another fact that can be considered is to look at the results of student work for the middle of the even semester test. The results for the mid-term of the even semester mathematics test in class XI SMA Kesatrian 2 in 2016/2017 academic year, show average for each class is still far from the minimum mastery criterion that is 41. Based on the information from one of the mathematics teachers in SMA Kesatrian 2 Semarang, most students have difficulty when faced with math problem-solving. The difficulties can be seen from the mistakes made by students in the process of solving the problem. Data on student work in question for problem-solving pointed out that students make many mistakes. Various difficulties faced by students when working on problem-solving are difficulty understanding the problem, writing the known variables, changing the variables into the mathematical language, and application of the formula used. This is an indication that the reality on the field shows that the problem-solving activities of mathematics have not been used as the main activity in learning mathematics. The ability of Indonesian students to solve problems that require the ability to examine, reason, and communicate effectively, and solve and interpret problems in various situations is still lacking [4].

The most complex mathematical competence among education levels from kindergarten, elementary, junior to senior high school is the competence of mathematics for senior high school students. This is because senior high school students are at the stage of formal operational thinking [2]. The formal thinking stage allows students to think at a high level, where problem-solving abilities are included. Mathematics in the class XI of senior high school, possesses core competencies to understand, apply, and analyze factual, conceptual, procedural, and metacognitive knowledge based on his or her knowledge of science, technology, art, culture and humanities with the insights of humanity, nationality, state and civilization on the causes of phenomena and events, and apply procedural knowledge to specific areas of study in accordance with their talents and interests to solve problems. In the process of solving problems involving metacognitive knowledge. Metacognitive knowledge is within the scope of metacognition. Metacognition consists of "meta" as a prefix and "cognition". Meta is the prefix for the word "cognition" which means after cognition [5]. The prefix in the word "cognition" is used to reflect the idea that the metacognition is "about" or "after" the cognition. This means that metacognition is cognition about cognition. The essential metacognitive skill in problem-solving is predicting, planning, monitoring, and evaluating [6]. There are two important aspects of metacognitive skill, namely self-monitoring and planning. The two experts agree that planning and monitoring are two essential aspects of problem-solving, but there is a slightly different opinion between them.

Metacognition, in general, can mean thinking about thinking, becoming more concerned with thoughts, feelings, and actions and taking into account the effects on others. Dawson [7] states that metacognition is an important component of interpersonal intelligence. Costa and Kallick [8] stated that metacognition is one of the indicators of habits of mind as the highest intelligent behavior characteristic that arises when the man is confronted with a problem which the solution is not immediately known. Similarly, Pintrich suggests that metacognition is characterized by a superordinate ability to direct and regulate cognitive, motivational, and problem-solving processes to achieve certain goals [9]. Metacognition is a person's ability to know what is known and unknown. This is the first step before planning a strategy to generate what information is needed, to be aware of the steps and strategies during the problem-solving process, and to reflect on and evaluate the productivity of our thinking. Therefore, to improve the problem-solving ability of a person, it needs efforts to increase metacognition.

Levels of metacognition or awareness of a person in the process of thinking according to Fisher [5] include: (a) tacit use, i.e. the type of thinking in making decisions without thinking about the decision and students only try or answer inconsequentially in solving the problem; (b) aware use, a type of thinking that indicates a person is aware of "what" and "when" he is doing something, and the student is aware of everything done in solving the problem; (c) strategic use, a type of thinking that shows a
person organizing his thinking by realizing specific strategies that improve the accuracy of thinking, and students can use and be aware of the right strategy for solving problems; and (d) reflective use, a type of thinking that shows a person reflecting on his thinking by considering the acquisition and how to improve it, and the student is able to realize or correct the mistakes made.

Problem-based learning facilitates students learning to overcome problems, trying to plan, evaluate and manage their use of strategies in solving problems, in which those are activities that can increase metacognition. Adesoji [10] and Dogru [11] stated that problem-based learning could develop positive attitudes toward science, improve scientific work skills, provide an appropriate learning environment to enhance students' metacognition.

Based on the above description, the purposes of this research are (1) to test the quality of problem-based learning on learning of mathematics integral material of class XI and (2) analyze problem-solving ability regarding student metacognition.

2. Methods
The research used is mix methods with the concurrent embedded model. The research method of mix methods is a research approach that combines or correlates qualitative research methods with quantitative [12]. This research was conducted in SMA Kesatrian 2 Semarang. The sample for the quantitative data in this study is the students of class XI MIPA 2 as the experimental group with problem-based learning and XI MIPA 3 as the control group. The subject of research consists of 6 students taken from each level of metacognition in an experimental class by using purposive sampling technique.

The instruments used in this research consist of metacognition scale, problem-solving test, observation sheet, student response, teacher response and interview result sheet. Analysis of research data includes mastery of problem-solving ability test, difference average of solving problem ability test, and the test for difference proportion mastery of problem-solving ability. Data analysis steps according to [13] are data reduction, data presentation (data display), and conclusion drawing.

3. Result and Discussion
The first step is taken before the learning is to group students based on their metacognitive level. The result of metacognition scale and rubric analysis obtained four students at strategic use level, 20 students on the level of aware use, four students at tacit use level and eight students unidentified metacognition level. No students found on the level of reflective use. Subsequently, select two research subjects from each level of metacognition. The learning quality test is carried out covering the planning, process and outcome phases. At the preparation stage, all learning tools have been validated by experts, considered in a very good category, meaning learning tools can be used for research. In the implementation stage of learning is said to be of quality if the observations of the implementation of learning and student activities go into the category of at least good. The overall averages of teacher performance observations, student response results, and teacher responses are (1) the number of students who responded positively was 80% over 75%; (2) teachers respond with score 4.32 including very good category; and (3) the ability of teachers to manage learning with a score of 3.72 included in the high category.

Test of the learning quality in the outcome stage is to assess the effectiveness of problem-based learning to problem-solving skills. Analysis of final data of this research consists of the prerequisite test that is normality test and homogeneity test, mastery of problem-solving ability test, a difference of two average test, and test of difference of proportion. The following is the empirical data result of descriptive analysis of problem-solving abilities data on experimental class and control class (Table 1).
Table 1. Descriptive Analysis of Problem Solving Abilities Data

| No | Descriptive Statistics | Experiment | Control |
|----|------------------------|------------|---------|
| 1  | Sum of Students        | 36         | 35      |
| 2  | Highest Score          | 89         | 88      |
| 3  | Lowest Score           | 56         | 45      |
| 4  | Averages               | 71.47      | 66.11   |
| 5  | Range                  | 33         | 43      |
| 6  | Variance               | 99         | 105.10  |
| 7  | Standard Deviation     | 9.95       | 10.25   |
| 8  | Learning Mastery       | 78%        | 51%     |

From the empirical results then performed the mastery test. The mastery test of problem-solving ability in this research uses one party t-test (right side test) and z-test with the calculation using Ms. Excel. Based on right side t-test, \( t = 3.903 > 2.030 \) is obtained. This means the average score of problem-solving abilities in the experimental class is more than 65. The classical mastery test uses the z-test (right side test). Value of \( z \)-table is obtained \( 1.665 > 1.64 \). It can be concluded that the proportion of students who reached the threshold mastery is more than 65%. The difference test of two average in this study using independent sample t-test. In the calculation of the value of \( t \) for independent samples, obtained \( t \) value = 2.235 > 1.995 then the average problem-solving ability of students in the experimental class is better than the average problem-solving ability of control-class students. Test of the difference proportion in this research using test of \( z \). Based on the calculation results, obtained the value of \( z \) = 2.324 > 1.64 then the number of students who achieve mastery in the experimental class more than the number of students who reach the minimum limit on the control class. Based on the results of an analysis for learning quality from the preparation stage, the implementation and the results obtained the conclusion that problem-based learning has good quality.

The result of problem-solving ability analysis based on metacognition on problem-based learning on four students on tacit use level, 20 students on the level of aware use and four students on a scale of strategic use can be seen in Table 2.

Table 2. Student distribution on achievement of NCTM problem-solving indicator based on metacognition level

| Metacognition Level | Problem Solving Indicator |
|--------------------|---------------------------|
| Tacit Use          | 30% 35% 0% 25%            |
| Aware Use          | 70% 40% 80% 60%           |
| Strategic Use      | 90% 45% 85% 70%           |

From Table 2 it can be said that after the application of problem-based learning model to students on Strategic Use level achieve a higher score on each problem-solving indicator than the student on Tacit Use and Aware Use level. And the obtainment of the highest score on the results of the problem-solving test also obtained by students on the Strategic Use level of metacognition. While students with the lowest problem-solving score are on the metacognition level of Tacit Use.

The NCTM problem-solving indicator that can be achieved by Tacit Use students is only the first indicator to build new mathematics through problem-solving, while the other three indicators cannot be achieved. Aware Use students can build new math through problem-solving, use and adapt a variety of appropriate strategies to solve problems and observe and describe processes in solving problem, but have not mastered solving mathematical problems involving other contexts. Finally, students with Strategic Use levels reach all four of the problem-solving indicators well.
Data analysis of problem-solving abilities based on Polya solving steps from each level of metacognition obtained from the results of student tests and interviewing problem-solving abilities. The summary of the analysis of problem-solving interviews based on students' metacognition after problem-based learning is shown in Table 3.

### Table 3. Description of problem-solving steps at the student's metacognition level

| Problem Solving Steps | Metacognition Level |
|-----------------------|---------------------|
|                       | Tacit use           | Aware use           | Strategic use              |
| Understanding the problem | Write down information that is known and being asked even if it is not complete. | Understanding the deficiency and advantages, able to write the information correctly and clearly. | Can understand the problem, but less thoroughly |
| Planning strategies | The chosen strategy is not precise and unclear. | Have not been able to plan strategies for solving math problems that involve other contexts. | Plan the problem-solving strategy correctly and completely. |
| Do the calculations | Do not calculate according to plan. | Less careful in doing the calculations. | Make calculations according to plan. |
| Reflection | Often do not check. | Checking results, but not in the process. | Less careful in-process checking and outcome of problem-solving. |

### 4. Conclusion

Based on the result of the research, it can be concluded that: (1) Problem-based learning has a good quality to problem-solving ability, (2) high level of student metacognition is directly proportional to students' mathematics problem-solving ability. The higher the level of students' metacognition then the problem-solving ability of students will also be better. The result shows that the students with levels tacit use was able to complete the whole matter was given but do not understand what and why a strategy used. Students with aware use level were able to solve the problem, be able to build new knowledge through problem-solving to the indicators understand the problem, determine the strategies used, although not good. Students on strategic use level able to use and adopt a wide variety of appropriate strategies to solve problems and achieve re-examine indicators of process and outcome. The student with reflective use level is not found in this study.

### References

[1] Riedesel C A 1996 *Teaching Elementary School Mathematics* (MA: A Simon and Schuster Company)

[2] Orton A 2004 *Learning Mathematics: Issues, Theory, and Classroom Practice. Third Edition* (New York: Continuum)

[3] NCTM 2000 *Principles and Standards for School Mathematics*. Reston (VA: NCTM)

[4] Stacey K 2010 *J Math Educ (IndoMS-JME)* 2 1

[5] Fisher R 1998 *Early Child Dev Care* 141 1

[6] Lucangeli D and Cornoldi C 1997 *Math Cogn* 3 121

[7] Dawson T L 2008 *Metacognition and learning in adulthood. Prepared in response to tasking from ODNI/CHCO/IC Leadership Development Office*, Developmental Testing Service, LLC

[8] Costa A L and Kallick B 2000 *Describing 16 Habits of Mind Habits of Mind: A Developmental Series* Alexandria (VA: Association for Supervision and Curriculum Development)

[9] García T, Marisol Cueli M, Rodríguez C, Krawec J and González-Castro P 2015 *Revista Psicodidactica J* 20 209

[10] Adesoji F A 2008 *Anthropologist* 10 21
[11] Dogru M 2008 *J Environ Sci Educ* **3** 9
[12] Creswell J W 2013 *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (California: SAGE Publications, Inc)
[13] Milles M B, Huberman A M and Saldaña S 2013 *Qualitative Data Analysis* (New Delhi: SAGE Publications, Inc)