LOW-ENERGY EFFECTIVE ACTION OF SUPERSTRING THEORY

T.R. TAYLOR
Department of Physics, Northeastern University
Boston, MA 02115, U.S.A.

ABSTRACT

A fundamental task for the heterotic superstring theory is the determination of the effective action describing the physics of massless string excitations at low energies. This is necessary for the phenomenological applications of string theory, in particular for the unification of gauge interactions and for the gaugino condensation mechanism of supersymmetry breaking. In this talk, I report on the recent progress in computing the effective supergravity action from superstring scattering amplitudes, at the tree level and beyond. I discuss moduli-dependent string loop corrections to gauge and Yukawa couplings.

Talk presented at the 7th Meeting of the American Physical Society
Division of Particles and Fields, Fermilab, November 10-14, 1992

1. Introduction

The basic property of string theory, which makes it so attractive from the point of view of particle physics, is that the physical couplings and masses are in principle calculable. They are determined by the vacuum expectation values (VEVs) of massless scalar fields, like the dilaton and moduli. Any serious attempt to compute the low-energy parameters from string theory must address two basic questions: 1) How do masses and couplings depend on the VEVs of dilaton, moduli, Higgs scalars etc.? and 2) What fixes these VEVs? In the past several years, there has been some slow but steady progress towards answering the second question; however, the general perception is that the problem of scalar VEVs still escapes a satisfactory solution. Here, I have nothing new to say about this problem. What is however very surprising, taking into account this stalemate in superstring phenomenology, is the amount of progress regarding the first question, of the determination of the dependence of physical quantities on scalar VEVs, which I will generically call the problem of moduli-dependence of physical parameters.

A very efficient method of studying the moduli-dependence of low-energy parameters, developed in the last couple of years, relies on the computation of the effective supergravity action describing the physics of massless string excitations. The moduli dependence of the effective action can be determined by evaluating the appropriate superstring scattering amplitudes.\(^1\,^2\,^3\,^4\) In this process, supergravitational interactions are determined directly from superstring theory. The moduli-dependent loop corrections obtained in this way give rise to the so-called threshold corrections to superstring unification parameters,\(^5\) and may be phenomenologically relevant in the future,
once one understands the mechanism that fixes the moduli VEVs. They may also be relevant for
the gaugino condensation mechanism of supersymmetry breaking. In this talk, I will discuss the
structure of low-energy supergravity that emerges from direct string computations of the tree-level
and one-loop scattering amplitudes. I will also report the results of recent computations of threshold
corrections to Yukawa couplings.4

2. Superstring Supergravity

The massless spectrum of heterotic superstring theory contains the supergravity multiplet,
gauge multiplets, and a large number of chiral multiplets. In addition, there is a dilaton which
belongs to a very distinct supersymmetry multiplet, together with the two-index antisymmetric
tensor – the Kalb-Ramond field. The dilaton VEV plays the role of the string loop expansion
parameter. Since the Kalb-Ramond field is equivalent to a pseudoscalar axion, one usually represents
the dilaton and its supersymmetric partners by one chiral multiplet $S$.

2.1. Tree Level

The most general $N = 1$ supergravity action, describing local interactions involving up to
two derivatives, is characterized by three functions of chiral superfields: the Kähler potential $K$
which determines the kinetic terms, the analytic superpotential $W$ related to the Yukawa couplings,
and the analytic function $f$ associated with the gauge couplings. At the tree level, the general
structure of the $f$-function and the Kähler potential is common to all compactifications:6

$$f^{(0)} = kS, \quad K^{(0)} = -\ln(S + \bar{S}) + G^{(0)}(Z, \bar{Z}),$$

(1)

where $k$ is the level of the Kač-Moody algebra that generates the gauge group, and $Z$ denote chiral
superfields other than $S$. The function $G^{(0)}(Z, \bar{Z})$ as well as the superpotential $W(Z)$ depend on
the details of compactification and can be determined by using a number of different methods.
The method that I will discuss here is based on direct computation of superstring scattering
amplitudes. A typical amplitude that allows determination of the tree-level Kähler potential involves
four scalar fields. Such amplitudes have been explicitly computed at the tree level for orbifold
compactifications.1

Example. As an example, consider a class of orbifolds with the gauge group $E_8 \otimes E_6 \otimes U(1)^2$,
k = 1, which contain three untwisted families of 27’s, $A_j$, $j = 1, 2, 3$, in one-to-one correspondence
with the three untwisted moduli $T_j$. In this case,

$$W = A_1 A_2 A_3 + \ldots, \quad G^{(0)} = -\sum_{j=1}^3 \ln(T_j + \bar{T}_j - A_j \bar{A}_j) + \ldots$$

(2)

2.2. One Loop and Beyond

Since the dilaton VEV plays the role of the role of the string loop expansion parameter,
string loop corrections give rise to kinetic terms that mix the dilaton with the moduli. This com-
plicates the analysis of the moduli-dependence of the effective action. The problem can be avoided
by representing the dilaton and its supersymmetric partners by a linear supermultiplet.7,8 The linear
formulation makes direct use of the Kalb-Ramond field, which is very natural in view of the

corresponding string vertex operator.

The linear multiplet $L$ corresponds to a real vector multiplet of the form:

$$L = \{l, \chi, 0, 0, h^\mu, -\theta \chi, -\Box l\}, \quad (3)$$

where $l$ is the dilaton, $\chi$ the dilatino, and $h^\mu$ is the dual field strength of the antisymmetric tensor
field $b_{\lambda \rho}$:

$$h^\mu = \frac{1}{2} \epsilon^{\mu \nu \lambda \rho} \partial_\nu b_{\lambda \rho}. \quad (4)$$

The tree level Lagrangian defined by the gauge function and the Kähler potential of Eq.(4)
is equivalent to a Lagrangian constructed from the d-density of $(L - 2k\Omega)^{-1/2} e^{-G^{(0)}/2}$, where $\Omega$

\[\text{Here, I ignore the subtleties related to the so-called chiral compensator which I set } \Sigma_0 = 1.\]
is the Chern-Simons vector superfield (the vector component of $\Omega$ is the gauge topological current $\omega^\mu$, $\partial_\mu \omega^\mu = F \tilde{F}$). This can be shown by performing a supersymmetric generalization of the duality transformation between the Kalb-Ramond field and the pseudoscalar axion. Note that although the Chern-Simons field is not gauge invariant, the invariance of the Lagrangian is ensured by the appropriate transformation property of $L$, so that the combination $\hat{L} = L - 2k\Omega$ remains invariant. A simple power counting argument shows that the string loop expansion must generate a $d$-density of the form:

$$d = \hat{L}^{-1/2} e^{-G^{(0)}/2} + \hat{L} G^{(1)} + O(\hat{L}^{5/2}).$$

The one-loop term proportional to $\hat{L}$ is usually called the Green-Schwarz term because it can be interpreted as the compactification of the ten-dimensional term involved in the Green-Schwarz anomaly cancellation mechanism.

In the linear formulation, the gauge function depends on the chiral superfields $Z$ only, and by a similar argument it consists entirely of the one-loop contribution $f = f^{(1)}(Z)$. On the other hand, the superpotential does not receive any loop corrections: $W = W^{(0)}(Z)$, in agreement with the well known supersymmetric non-renormalization theorems. In the following, I will discuss the relation of the one-loop functions $G^{(1)}$ and $f^{(1)}$ to the physical parameters of superstring theory.

### 3. Effective Action and Physical Parameters in Superstring Theory

The tree-level effective action describes interactions of massless string excitations at energies below the string scale. These include contact interactions due to the propagation of heavy particles in one-(massive)-particle reducible diagrams. The masses of heavy particles depend on the moduli, e.g. the radii of compactified dimensions, therefore the induced massless particle interactions are also moduli-dependent. In order to compute the one-loop effective action, one should integrate all diagrams involving heavy particles propagating inside loops. In string theory, it is very difficult to separate heavy from massless particles in higher genus diagrams, therefore the computation of the effective action becomes a bit more subtle than in field theory. In order to explain the procedure employed in string theory, I will discuss the computation of the one-loop terms that determine the moduli-dependence of gauge couplings. These terms are contained in the interaction of the form $-\frac{1}{4} \Delta F_{\mu \nu} F^{\mu \nu}$, where $\Delta(l, z, \bar{z})$ is a real function which plays the role of a field-dependent $1/g^2$, $g$ being the gauge coupling constant.

In order to compute the function $\Delta$ one considers the two-point correlation function of gauge bosons. After inserting two gauge boson vertices on a world-sheet torus one obtains the on-shell $(Q^2 = 0)$ correlation function as an integral over the Teichmüller parameter $\tau$ of the torus. The integrand depends on the moduli, reflecting moduli-dependent masses of heavy particles propagating in the loop. Since there are also massless particles propagating in the loop, and the external particles are on-shell, the integral over the Teichmüller parameter diverges in the infrared. In the analogous background field-theoretical computations, such a logarithmic divergence is usually regulated by going off-shell, to momentum $Q^2 \neq 0$. These logarithmic $(\ln Q^2)$ terms play very important role in the effective field theory – they cause the “running” of gauge coupling constants. It is very important to realize that in string theory, as well as in quantum field theory, the momentum-dependence of gauge coupling constants is a purely infrared effect, therefore the corresponding beta function coefficient of the $Q^2 \to 0$ divergence depends on the massless particle content only. The momentum-dependence is governed by the usual renormalization group equations. Hence, as far as running is concerned, there is no difference between string and standard unifications.

The basic difference between string and standard unifications concerns the boundary conditions at the unification scale. Whereas in the standard case the boundary conditions are completely arbitrary, in string theory they are determined by dynamical VEVs. In the leading approximation, the tree-level part of the $d$-density gives $\Delta = k/l$ hence, at the string unification scale $M$,

\[b\text{The tree level gauge kinetic terms are contained in the d-density of Eq.(5); this will become clear in Eq.(6).} \]
to gauge couplings satisfies a non-renormalization theorem. 3

4. Threshold Corrections to Yukawa Couplings and Kähler metric

The one-loop supergravity action of Sec.2.2, dictates the following from of Yukawa interactions between chiral fermions $\psi$ and scalars $z$:

$$\mathcal{L}_Y = -\frac{1}{2} \sqrt{|\mathcal{G}|} \frac{e^{G(0)}}{2} W^{(0)}_{ijk} \psi^i \psi^j z^k + c.c.,$$

where the subscripts of denote differentiation with respect to the corresponding fields. Note that the above expressions depends on the tree-level quantities only. The physical Yukawa couplings defined by the fermion-scalar scattering amplitudes may receive however loop corrections. They arise from the wave the function renormalization factors, i.e. from the corrections to the Kähler metric.
In order to discuss loop corrections to the Kähler metric, it is sufficient to consider the bosonic part of the kinetic energy terms:\(^7\)\(^8\)

\[
\mathcal{L}_B = -\frac{1}{4t^2} \partial_{\mu} t^{\mu} l + \frac{1}{4t^2} b_{\mu} h^\mu - G_{ij} \partial_\mu z^i \partial^\mu \bar{z}^j - \frac{i}{2} (G^{(1)}_{ij} \partial_\mu z^i - G^{(1)}_{ij} \partial_\mu \bar{z}^j) h^\mu.
\] (9)

At the one-loop level, the Kähler metric is

\[
G_{ij} = G^{(0)}_{ij} + i G^{(1)}_{ij}.
\] (10)

The main advantage of using linear formulation of supergravity is that it provides a simple way of computing loop corrections to the Kähler metric, by considering a three-point amplitude involving two complex scalars and one Kalb-Ramond field.\(^4\) Inspection of the last term in Eq.(1) shows that

\[
\epsilon^{\mu \nu \lambda \rho} p_1 \lambda p_2 \rho G^{(1)}_{ij} = \langle z_i (p_1) \bar{z}_j (p_2) b_{\mu \nu \rho} (p_3) \rangle.
\] (11)

Although this amplitude vanishes for on-shell Minkowski momenta, it can be computed for complex Euclidean momenta, as it was done in similar computations for moduli and gauge bosons.\(^2\) As a result, one obtains:\(^4\)

\[
G^{(1)}_{ij} = \int \frac{d^2 \bar{z}}{2\pi} \int d^2 \zeta \bar{\eta}(\bar{\tau})^{-2} \langle \Psi_i (\zeta) \overline{\Psi}_j (0) \rangle,
\] (12)

where \(\Psi_i\) and \(\overline{\Psi}_j\) are the corresponding primary fields.

The integral over the Teichmüller parameter in Eq.(12) is infrared divergent. As in the case of gauge couplings, these divergences are due to massless particles propagating in the loop. The coefficients of divergent terms correspond to the one-loop anomalous dimensions. They can be extracted from Eq.(12) which is valid for a general compactification. The comparison with field-theoretical anomalous dimensions shows that the string computation implicitly uses a gauge in which the superpotential remains unrenormalized. Again as in the case of gauge couplings, the momentum-dependence of physical Yukawa couplings in string theory turns out as determined by the corresponding field-theoretical beta functions.\(^7\)\(^8\) The remaining finite part of \(G^{(1)}_{ij}\) gives the string threshold corrections to wave function factors. These corrections determine the boundary conditions for the physical Yukawa couplings \(\lambda_{ijk}\) at the unification scale:

\[
\lambda_{ijk}(M) = \lambda_{ijk}^{\text{tree}} [1 + \ell (Y_i + Y_j + Y_k)]^{-1/2},
\] (13)

where \(Y_i\) is defined as the finite part of \(G^{(0)ij}G^{(1)}_{ij}\) (no summation over \(i\)).

The one-loop Kähler metric and threshold corrections to Yukawa couplings can be explicitly computed from Eq.(12) in the case of orbifold compactifications. For the untwisted moduli, \(G^{(1)}_{ij}\) obtained in this way agrees with the corresponding contribution of the Green-Schwarz term to the threshold corrections to gauge couplings discussed in Sec.3, cf. Eq.(3).

For the untwisted 27’s and \(\overline{27}\)’s of \(E_6\) one obtains non-vanishing wave function corrections only if the orbifold group contains a subgroup that leaves invariant one of the three complex orbifold planes and preserves \(N = 2\) supersymmetry. The threshold correction \(Y_z\) for an untwisted \(z\)-field (27 or \(\overline{27}\)) associated with such a plane depends on its moduli \(T\), and does not depend on the moduli of other planes:

\[
Y_z = \frac{2\hat{\gamma}_z}{\text{ind}} \ln[|\eta(T)|^4 (T + \overline{T})] + y_z,
\] (14)

Here, the coefficients \(\hat{\gamma}_z\) are anomalous dimensions of the \(z\)-fields in the corresponding \(N = 2\) supersymmetric theory with the orbifold defined by the little group of the unrotated plane associated with the modulus \(T\), and \(\text{ind}\) is the index of this little group in the full orbifold group; \(y_z\) are moduli-independent constants. One can show that \(\hat{\gamma}_z = -\hat{\beta}_z/2\), where \(\hat{\beta}_z/2\) is the corresponding beta function coefficient of any gauge subgroup that transforms \(z\) non-trivially in the embedding \(N = 2\) theory. The threshold corrections to Yukawa couplings can be computed by substituting Eq.(14) into Eq.(13).
Example. As an example, consider the Yukawa coupling between three untwisted 27’s of $E_6$. At the tree-level, this coupling is:

$$\lambda_{ijk}^{\text{tree}} = \frac{g_{\text{tree}}}{\sqrt{2}} W_{ijk},$$

where $W_{ijk}$ are constants which are non-zero only if the three 27’s are associated with three different planes. In this case, Eq.(14) combined with Eq.(13) give the boundary condition:\(^4\)

$$\lambda_{ijk}(M) = \frac{g_{E_6}(M)}{\sqrt{2}} W_{ijk} [1 + g_{E_6}(M) y_{ijk}]^{-1/2},$$

where $y_{ijk}$ are moduli-independent constants, and $g_{E_6}(M)$ is the one-loop $E_6$ gauge coupling at the unification scale, cf. Eq.(6). As a result, the boundary relation between the untwisted Yukawa couplings and the $E_6$ gauge coupling at the unification scale does not receive any moduli-dependent corrections at the one-loop level.

5. Conclusions

It is a common opinion among high energy physicists that superstring theory is still far from becoming the theory of everything. There remain many fundamental problems to be solved before a reliable low-energy phenomenology can be developed. What remains however unquestionable is that superstring theory provides a unique example of a perfectly consistent unification of gravitational and gauge interactions, within the framework of local supersymmetry. The efforts described here have been motivated by the desire to understand the low-energy limit of such a consistent theory. As a result, we have now a very good understanding of the low-energy physics of supergravity theory that describes not only the classical limit of string theory, but also some interesting string loop effects, in particular the threshold corrections. This “bottom–up” approach to superstring theory may well provide some new insights into the Planck-scale physics.

Acknowledgements

It is my great pleasure to thank I. Antoniadis, E. Gava and K.S. Narain for enjoyable collaborations. This work was supported in part by the Northeastern University Research and Scholarship Fund and in part by the National Science Foundation under grant PHY-91-07809.

References

[1] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B 329 (1990) 27.
[2] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B 355 (1991) 649.
[3] I. Antoniadis, K.S. Narain and T.R. Taylor, Phys. Lett. B 267 (1991) 37; I. Antoniadis, E. Gava and K.S. Narain, Phys. Lett. B 283 (1992) 209; Nucl. Phys. B 393 (1992) 93.
[4] I. Antoniadis, E. Gava and K.S. Narain and T.R. Taylor, preprint NUB-3057 (1992).
[5] V.S. Kaplunovsky, Nucl. Phys. B 307 (1988) 145.
[6] E. Witten, Phys. Lett. B 155 (1985) 151.
[7] P. Adamietz, P. Binétruy, G. Girardi and R. Grimm, preprint ENSLAPP-A-388/92 (1992); M.K. Gaillard and T.R. Taylor, Nucl. Phys. B 381 (1992) 577; P. Binétruy, G. Girardi and R. Grimm, Phys. Lett. B 265 (1991) 111.
[8] S. Ferrara and M. Villasante, Phys. Lett. B 186 (1987) 85; S. Cecotti, S. Ferrara and M. Villasante, Int. J. Mod. Phys. A2 (1987) 1839.
[9] H.P. Nilles, Phys. Lett. B 180 (1986) 240.
[10] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B 372 (1992) 145; G.L. Cardoso and B.A. Ovrut, Nucl. Phys. B 369 (1992) 351.
[11] M.B. Green and J.H. Schwarz, Phys. Lett. B 149 (1984) 117.
[12] T.R. Taylor and G. Veneziano, Phys. Lett. B 212 (1988) 147.
[13] S.J. Gates, M. Grisaru, M. Roček and W. Siegel, *Superspace* (Benjamin/Cummings, 1983).
[14] J. Louis, in *Proceedings of the Second International Symposium on Particles, Strings and Cosmology*, P. Nath and S. Reucroft eds. (World Scientific, 1992), p. 751; S. Ferrara, C. Kounnas, D. Lüst and F. Zwirner, Nucl. Phys. B 365 (1991) 431; L. Ibáñez and D. Lüst, Nucl. Phys. B 382 (1992) 305.
[15] R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo and C.A. Savoy, Phys. Lett. B 115 (1982) 212.