Precision calculations for the Higgs decays \( H \to ZZ/WW \to 4 \) leptons

A. Bredenstein\(^a\), A. Denner\(^b\), S. Dittmaier\(^c\) and M.M. Weber\(^d\)

\(^a\)High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
\(^b\)Paul Scherrer Institut, Würenlingen und Villigen, CH-5232 Villigen PSI, Switzerland
\(^c\)Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), D-80805 München, Germany
\(^d\)Bergische Universität Wuppertal, D-42097 Wuppertal, Germany

Extending earlier work, we provide predictions obtained with the Monte Carlo generator \( \text{PROPHECY4f} \) for the decays \( H \to ZZ/WW \to 4l \) including the complete electroweak \( \mathcal{O}(\alpha) \) corrections and some higher-order improvements. The gauge-boson resonances are described in the complex-mass scheme. Here, particular attention is paid to a comparison of different final states with identical charges, such as \( e^+e^−\mu^+\mu^- \) and \( \mu^+\mu^−\mu^+\mu^- \).

1. INTRODUCTION

The primary task of the LHC will be the detection and the study of the Higgs boson. If it is heavier than 140 GeV, it decays dominantly into gauge-boson pairs, i.e. into 4 fermions. These decays offer the largest discovery potential for a Higgs boson with a mass \( M_H \gtrsim 130 \) GeV \(^1\), and the decay \( H \to ZZ \to 4l \) will allow for the most accurate measurement of \( M_H \) above 130 GeV \(^2\). At an \( e^+e^- \) linear collider, these decays will enable measurements of the corresponding branching ratios and couplings at the few-per-cent level.

A kinematical reconstruction of the Higgs boson and of the virtual W and Z bosons requires the study of distributions. Thereby, it is important to include radiative corrections, in particular real photon radiation. In addition, the verification of the spin and the CP properties of the Higgs boson relies on the study of angular and invariant-mass distributions \(^3\). As a consequence a Monte Carlo generator for \( H \to ZZ/WW \to 4f \) including electroweak corrections is needed.

In the past, only the electroweak \( O(\alpha) \) corrections to decays into on-shell gauge bosons \( H \to ZZ/WW \) \(^4\) and some leading higher-order corrections were known. However, in the threshold region the on-shell approximation becomes unreliable. Below the gauge-boson-pair thresholds only the leading order was known until recently.

\( \text{PROPHECY4f} \) \(^6\) is a recently constructed Monte Carlo event generator for \( H \to ZZ/WW \to 4f \) that includes electroweak corrections as well as some higher-order improvements. Since the process with off-shell gauge bosons is consistently considered without any on-shell approximations, the obtained results are valid above, near, and below the gauge-boson pair thresholds. Parallel to our work, another generator for \( H \to ZZ \to 4f \), including electromagnetic corrections only, has been introduced in Ref. \(^7\).

In this note we briefly describe the structure of \( \text{PROPHECY4f} \) and of the underlying calculations. Moreover, we extend the numerical results of Ref. \(^6\) obtained with this code by paying particular attention to final states that differ only in the generation quantum numbers of the leptons.

2. CALCULATIONAL DETAILS

We have calculated the complete electroweak \( O(\alpha) \) corrections to the processes \( H \to 4f \). This includes both the corrections to the decays \( H \to ZZ \to 4f \) and \( H \to WW \to 4f \) and their interference. The calculation of the one-loop diagrams has been performed in the conventional ’t Hooft–Feynman gauge and in the background-field formalism using the conventions of Refs. \(^8\) and \(^9\), respectively. The masses of the external fermions
have been neglected whenever possible, i.e. everywhere but in the mass-singular logarithms.

For the implementation of the finite widths of the gauge bosons we use the “complex-mass scheme”, which was introduced in Ref. [10] for lowest-order calculations and generalized to the one-loop level in Ref. [11]. In this approach the W- and Z-boson masses are consistently considered as complex quantities, defined as the locations of the propagator poles in the complex plane. The scheme fully respects all relations that follow from gauge invariance. A brief description of this scheme can also be found in Ref. [12].

The amplitudes have been generated with FeynArts, using the two independent versions 1 and 3, as described in Refs. [13] and [14], respectively. The algebraic evaluation has been performed in two completely independent ways. One calculation is based on an in-house program implemented in Mathematica, the other has been completed with the help of FormCalc [15]. The amplitudes are expressed in terms of standard matrix elements and coefficients of tensor integrals as described in the appendix of Ref. [13].

The tensor coefficients are evaluated as in the calculation of the corrections to $e^+e^- \to 4$ fermions [11][17]. They are recursively reduced to master integrals at the numerical level. The scalar master integrals are evaluated for complex masses using the methods and results of Refs. [18][19][20]. UV divergences are regulated dimensionally and IR divergences with an infinitesimal photon mass. Tensor and scalar 5-point functions are directly expressed in terms of 4-point integrals [21]. Tensor 4-point and 3-point integrals are reduced to scalar integrals with the Passarino–Veltman algorithm [22] as long as no small Gram determinant appears in the reduction. If small Gram determinants occur, two alternative schemes are applied [23]. One method makes use of expansions of the tensor coefficients about the limit of vanishing Gram determinants and possibly other kinematical determinants. In the second, alternative method we evaluate a specific tensor coefficient, the integrand of which is logarithmic in Feynman parametrization, by numerical integration. Then the remaining coefficients as well as the standard scalar integral are algebraically derived from this coefficient. The results of the two different codes, based on the different methods described above are in good numerical agreement.

Since corrections due to the self-interaction of the Higgs boson become important for large Higgs-boson masses, we have included the dominant two-loop corrections to the decay $H \to VV$ proportional to $G_F^2 M_H^4$ in the large-Higgs-mass limit which were calculated in Ref. [24].

The matrix elements for the real photonic corrections are evaluated using the Weyl–van der Waerden spinor technique as formulated in Ref. [25]. The soft and collinear singularities are treated both in the dipole subtraction method following Ref. [26] and in the phase-space slicing method following Ref. [27]. For the calculation of non-collinear-safe observables we use the extension of the subtraction method introduced in Ref. [28]. Final-state radiation beyond $O(\alpha)$ is included at the leading-logarithmic level using the structure functions given in Ref. [30] (see also references therein).

The phase-space integration is performed with Monte Carlo techniques. One code employs a multi-channel Monte Carlo generator similar to the one implemented in RacoonWW [10] and Coffer$\gamma$ [20][31], the second one uses the adaptive integration program VEGAS [32].

3. NUMERICAL RESULTS

We use the $G_{\mu}$ scheme, i.e. we define the electromagnetic coupling by $\alpha_{G_{\mu}} = \sqrt{2} G_{\mu} M_W^2 s_w^2 / \pi$. Our lowest-order results include the $O(\alpha)$-corrected width of the gauge bosons. In the distributions the photon has been recombined with the nearest charged fermion if the invariant mass of the photon–fermion pair is below 5 GeV. More details about the setup, all input parameters, and more detailed results are provided in Ref. [33].

In Table 1 the partial decay width including $O(\alpha)$ corrections is shown for different decay channels and different values of the Higgs-boson mass. In parentheses the statistical error of the phase-space integration is indicated, and $\delta = \Gamma / \Gamma_0 - 1$ labels the relative corrections.
Table 1

Partial decay widths for $H \rightarrow 4$ leptons including $\mathcal{O}(\alpha)$ and $\mathcal{O}(G^2 M_H^2)$ corrections and corresponding relative corrections for various decay channels and different Higgs-boson masses (taken from Ref. [6]).

| $H \rightarrow$ | $M_H$ [GeV] | $\Gamma$ [MeV] | $\delta$ [%] | $\Gamma$ [MeV] | $\delta$ [%] | $\Gamma$ [MeV] | $\delta$ [%] |
|----------------|--------------|----------------|-------------|----------------|-------------|----------------|-------------|
| $e^+e^-\mu^-\mu^+$ | 140          | 0.0012628(5)   | 2.3         | 0.0920162(7)   | 2.7         | 0.8202(2)     | 4.4         |
| $e^-e^+\mu^-\mu^+$ | lowest order | 0.0012349(4)   |             | 0.019624(5)    |             | 0.78547(8)    |             |
| $l^-l^+l^-l^+$   | corrected    | 0.0006692(2)   | 2.1         | 0.010346(3)    | 2.7         | 0.41019(8)    | 4.4         |
| $l^-l^+l^-l^+$   | lowest order | 0.0006555(2)   |             | 0.010074(2)    |             | 0.39286(4)    |             |
| $\nu_e e^-\mu^-\bar{\nu}_\mu$ | corrected | 0.04807(2)     | 3.7         | 4.3109(9)      | 6.2         | 12.499(3)     | 5.0         |
| $\nu_e e^-\mu^-\bar{\nu}_\mu$ | lowest order | 0.04638(1)     |             | 4.0610(7)      |             | 11.907(2)     |             |
| $\nu_l l^+l^-l^-\bar{\nu}_l$ | corrected | 0.04914(2)     | 3.7         | 4.344(1)       | 6.1         | 14.133(3)     | 5.0         |
| $\nu_l l^+l^-l^-\bar{\nu}_l$ | lowest order | 0.04738(2)     |             | 4.0926(8)      |             | 13.458(2)     |             |

The first two channels, $e^+e^-\mu^-\mu^+$ and $l^-l^+l^-l^+$, $l = e, \mu$, result from the decay $H \rightarrow ZZ \rightarrow 4f$. The partial widths for only electrons or muons in the final state are equal in the limit of vanishing external fermion masses, since for collinear-safe observables, such as the partial widths, the fermion-mass logarithms cancel. The width for $H \rightarrow l^-l^+l^-l^+$ is smaller by about a factor 2, because it gets a factor $1/4$ for identical particles in the final state and it proceeds in lowest order via two Feynman diagrams that are related by the exchange of two outgoing electrons and that have only a small interference. The channel $\nu_e e^-\mu^-\bar{\nu}_\mu$ results from the decay $H \rightarrow WW \rightarrow 4f$, while the last channel $\nu_l l^+l^-l^-\bar{\nu}_l$ receives contributions from both the decay into W and into Z bosons.

As can be seen from the table, the relative corrections to $e^+e^-\mu^-\mu^+$ and $l^-l^+l^-l^+$ practically coincide, demonstrating that the corrections are not significantly influenced by the interference between the lowest-order diagrams that exist for $l^-l^+l^-l^+$. Similarly, there is hardly any difference between the corrections to $\nu_e e^-\mu^-\bar{\nu}_\mu$ and $\nu_l l^+l^-l^-\bar{\nu}_l$. Plots showing the dependence of the partial decay widths on $M_H$ in the range $M_H = 120$–700 GeV, together with the corresponding corrections, can be found in Ref. [6].

As examples for differential cross-sections we consider distributions in angles between outgoing charged leptons. We note that the applied photon recombination does not significantly change the angular distributions, because recombining soft or collinear photons with fermions does not change their directions significantly.

In the decay $H \rightarrow \nu_l l^+l^-l^-\bar{\nu}_l$ neither the Higgs-boson nor the W-boson momenta can be reconstructed from the decay products. Thus, angular distributions can only be studied upon including the Higgs-production process. If the Higgs boson was, however, produced without transverse momentum, or if the transverse momentum was known, the angle between $l^+$ and $l^-$ in the plane perpendicular to the beam axis could be studied without knowledge of the production process. We define the transverse angle between $l^+$ and $l^-$ in the frame where $k_{H,T} = 0$ as

$$
\cos(\phi_{l^+l^-}) = \frac{k_{l,T} \cdot k_{l^'-T}}{|k_{l,T}||k_{l^'-T}|},
$$

$$
\text{sgn}(\sin(\phi_{l^+l^-})) = \text{sgn}\{e_z \cdot (k_{l,T} \times k_{l^'-T})\},
$$

where $k_{l^0,T}$ are the fermion momenta transverse w.r.t. the unit vector $e_z$, which could be identified with the beam direction of a Higgs-production process. The corresponding distribution, together with the influence of the corrections, is shown in Figure 1. For $M_H > 2M_Z$ the interference between the WW- and ZZ-initiated diagrams turns out to be negligible, and the corrections look similar for the different final states, in particular for those that result from virtual W bosons.

Finally, we investigate the distribution of the angle between like-sign leptons in the decays $H \rightarrow e^-e^+\mu^-\mu^+$ and $H \rightarrow \mu^-\mu^+\mu^-\mu^+$ in the rest frame of the Higgs boson. Figure 2 shows that the corrections tend to reduce the well-
A. Bredenstein, A. Denner, S. Dittmaier and M.M. Weber

\[ \frac{d\Gamma}{d\phi_{l^+l^-}} \text{ [MeV]} \quad M_H = 200 \text{ GeV} \]

\[ \nu_\mu \mu^+ \mu^- \bar{\nu}_\mu \quad \nu_e e^+ \mu^- \bar{\nu}_\mu \quad \nu_\tau \nu_e \mu^- \mu^+ \]

\[ \text{interference} \times 10 \]

\[ \cos \theta_{l^+l^-} \quad M_H = 200 \text{ GeV} \]

\[ \delta \% \quad M_H = 200 \text{ GeV} \]

\[ \nu_\mu \mu^+ \mu^- \bar{\nu}_\mu \quad \nu_e e^+ \mu^- \bar{\nu}_\mu \quad \nu_\tau \nu_e \mu^- \mu^+ \]

\[ \text{interference} \]

Figure 1. Corrected distribution in the transverse angle between the charged leptons (l.h.s.) and corrections (r.h.s.) in the decays of the type $H \rightarrow \nu_l l^+ l^- \bar{l}$ for $M_H = 200$ GeV. We define \[ |\text{interference}| = \left| (\nu_\mu \mu^+ \mu^- \bar{\nu}_\mu) - (\nu_e e^+ \mu^- \bar{\nu}_\mu)(\nu_\tau \nu_e \mu^- \mu^+) \right| \]

\[ \frac{d\Gamma}{d\cos \theta_{l^+l^-}} \text{ [MeV]} \quad M_H = 200 \text{ GeV} \]

\[ e^- e^+ \mu^- \mu^+ \quad \mu^- \mu^+ \mu^- \mu^+ \]

\[ \text{interference} \times 10 \]

\[ \cos \theta_{l^+l^-} \quad M_H = 200 \text{ GeV} \]

\[ \delta \% \quad M_H = 200 \text{ GeV} \]

\[ e^- e^+ \mu^- \mu^+ \quad \mu^- \mu^+ \mu^- \mu^+ \]

Known enhancement in forward direction. Interference terms in the two Born diagrams for $H \rightarrow \mu^- \mu^+ \mu^- \mu^+$ are very small for $M_H > 2M_Z$, and the relative corrections for the final states
Electroweak corrections to the Higgs decays $H \to ZZ/WW \to 4\ell$

$
\mu^+\mu^-\mu^+\mu^- \text{ and } e^+e^-\mu^+\mu^- \text{ practically coincide.}
$

4. CONCLUSIONS

We have presented further results of the generator PROPHECY4f which calculates the complete electroweak $O(\alpha)$ radiative corrections to the Higgs-boson decays $H \to ZZ/WW \to 4f$. Particular attention has been paid to different final states with identical charges, such as $e^+e^-\mu^+\mu^-$ and $\mu^+\mu^-\mu^+\mu^-$.  

REFERENCES

1. S. Asai et al., Eur. Phys. J. C 32S2 (2004) 19 [hep-ph/0402254]; S. Abdullin et al., Eur. Phys. J. C 39S2 (2005) 41.

2. L. Zivkovic, Czech. J. Phys. 54 (2004) A73.

3. V.D. Barger, K.M. Cheung, A. Djouadi, B.A. Kniehl and P.M. Zerwas, Phys. Rev. D 49 (1994) 79 [hep-ph/9306270].

4. S.Y. Choi, D.J. Miller, M.M. Muelleitner and P.M. Zerwas, Phys. Lett. B 553 (2003) 61 [hep-ph/0210077].

5. J. Fleischer and F. Jegerlehner, Phys. Rev. D 23 (1981) 2001; B.A. Kniehl, Nucl. Phys. B 352 (1991) 1 and Nucl. Phys. B 357 (1991) 439; D.Y. Bardin, P.K. Khristova and B.M. Vilensky, Sov. J. Nucl. Phys. 54 (1991) 833 [Yad. Fiz. 54 (1991) 1366].

6. A. Bredenstein, A. Denner, S. Dittmaier and M. M. Weber, hep-ph/0604011 to appear in Phys. Rev. D.

7. C. M. Carloni Calame et al., Nucl. Phys. Proc. Suppl. 157 (2006) 73 [hep-ph/0604033].

8. A. Denner, Fortsch. Phys. 41 (1993) 307.

9. A. Denner, G. Weiglein and S. Dittmaier, Nucl. Phys. B 440 (1995) 95 [hep-ph/9410338].

10. A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B 560 (1999) 33 [hep-ph/9904472].

11. A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Nucl. Phys. B 724 (2005) 247 [hep-ph/0505042].

12. A. Denner and S. Dittmaier, hep-ph/0605312 these proceedings.

13. J. Küblbeck, M. Böhm and A. Denner, Comput. Phys. Commun. 60 (1990) 165; H. Eck and J. Küblbeck, Guide to FeynArts 1.0, University of Würzburg, 1992.

14. T. Hahn, Comput. Phys. Commun. 140 (2001) 418 [hep-ph/0012260].

15. T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. 118 (1999) 153 [hep-ph/9807565]; T. Hahn, Nucl. Phys. Proc. Suppl. 89 (2000) 231 [hep-ph/0005029].

16. A. Denner, S. Dittmaier, M. Roth and M.M. Weber, Nucl. Phys. B 660 (2003) 289 [hep-ph/0302198].

17. A. Denner, S. Dittmaier, M. Roth and L.H. Wieders, Phys. Lett. B 612 (2005) 223 [hep-ph/0502063].

18. G. 't Hooft and M. Veltman, Nucl. Phys. B 153 (1979) 365.

19. W. Beenakker and A. Denner, Nucl. Phys. B 338 (1990) 349.

20. A. Denner, U. Nierste and R. Scharf, Nucl. Phys. B 367 (1991) 637.

21. A. Denner and S. Dittmaier, Nucl. Phys. B 658 (2003) 175 [hep-ph/0212259].

22. G. Passarino and M. Veltman, Nucl. Phys. B 160 (1979) 151.

23. A. Denner and S. Dittmaier, Nucl. Phys. B 734 (2006) 62 [hep-ph/0509141].

24. A. Ghinculov, Nucl. Phys. B 455 (1995) 21 [hep-ph/9507240]; A. Frink, B.A. Kniehl, D. Kreimer and K. Riesselmann, Phys. Rev. D 54 (1996) 4548 [hep-ph/9606310].

25. S. Dittmaier, Phys. Rev. D 59 (1999) 016007 [hep-ph/9805445].

26. T. Stelzer and W.F. Long, Comput. Phys. Commun. 81 (1994) 357 [hep-ph/9401258].

27. S. Dittmaier, Nucl. Phys. B 565 (2000) 69 [hep-ph/9904440].

28. M. Böhm and S. Dittmaier, Nucl. Phys. B 409 (1993) 3 and Nucl. Phys. B 412 (1994) 39; U. Baur, S. Keller and D. Wackeroth, Phys. Rev. D 59 (1999) 013002 [hep-ph/9807417].

29. A. Bredenstein, S. Dittmaier and M. Roth, Eur. Phys. J. C 44 (2005) 27 [hep-ph/0506005].

30. W. Beenakker et al., hep-ph/9602351.

31. A. Bredenstein, S. Dittmaier and M. Roth, Eur. Phys. J. C 36 (2004)
341. [hep-ph/0405169](http://arxiv.org/abs/hep-ph/0405169).

32. G.P. Lepage, J. Comput. Phys. **27** (1978) 192.