Study on thermal characteristics of deep groove ball bearing based on Green’s Function and COMSOL

Kai Zhang¹*, Hanping Hu¹ and Xiaojie Liu²

¹Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, Anhui 230027, China
²Key Laboratory of Ocean Energy Utilization and Energy Conservation of the Ministry of Education, Dalian University of Technology, Dalian, 116024, China

*Corresponding author: zk1314@mail.ustc.edu.cn

Abstract. Thermal characteristics seriously affect the performance of bearing, which analyzing the temperature field of bearing is inevitable. This paper presents a better solution based on Green’s Function. It is more efficient and accurate than the integral transform method provided in current literature. The finite element method (FEM) with COMSOL is applied to simulate in the three-dimensional model. By comparing analytical solution and FEM, the tendency of temperature is reasonable. And the maximum temperature locates at the roller is 41.2476°C. Beside, little difference in temperature between inner and outer raceway and the former is little high. Moreover, heat flux only transfers from rollers to raceway, which causes thermal load on the contact region. According to the analytical method, deep groove needs equivalent treatment to reduce error. Then 40.7995°C obtained by Green’s Function and 40.8592°C obtained by COMSOL in the same position on inner raceway. This paper mainly provides an advantage analytical method to verify the temperature field in deep groove ball bearing, and it is valuable for studying thermal reliability of bearing.

1. Introduction

Several papers have provided related solutions to obtain the rolling bearing temperature. Some scholar used finite element analysis, which Mujumdar[1] just introduced the application of numerical method in this field, Xiu[2] and Tarawnh[3] applied ANSYS to analyze the thermal characteristics in bearing but the result relies on construction, contact parameters and the assumption condition. Others took thermal resistance method as solution, which Harris[4] established an equivalent thermal resistance model based on the thermal resistance method to predict the steady-state temperature field. Winer[5] provided thermal resistance method to verify the temperature of tapered roller bearing. These solutions cost lots of time in building the network or the complex equation. In contrast, the analytical solution is the best, which make sure accuracy and less calculation time. Baïri[6] provide a three-dimensional heat transfer model, but the boundary conditions are not flexible enough. Hannon[7-9] provide another analytical solution, which established a steady-state heat conduction equation on the inner and outer raceways respectively based on integral transform, but the temperature field depends on integrated area and the ability of integration. Cole[10] used the one-dimensional Green's function solution to calculate the steady-state temperature, which can't solve more complicated model.

This paper proposes a two-dimensional Green's function to solve the bearing’s temperature, which can get the temperature value at any position in the model with MATLAB program. And the result
compares with COMSOL to verify each other, which has a better consistent.

2. Steady-analytical model

This paper establishes a hollow cylindrical model based on the radial and circumferential dimension to simulate the ring of bearing, and applies the Green’s Function relatively. The process begins by making the following assumption:

1. The solution is assumed to be steady-state
2. Thermal contact resistance is negligible
3. The rings have enough stiffness
4. The bearing raceway geometries can be equivalent

There is no internal heat source and no influence of initial condition. And the Green's function will be obtained from the control equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{k} g(r, \theta, t) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$  \hspace{1cm} (1)

Boundary conditions:

$$T = f_1(r, \theta), \quad r=a, \quad \theta \in [0,2\pi]$$ \hspace{1cm} (2)
$$\frac{\partial T}{\partial r} = f_2(r, \theta), \quad r=b, \quad \theta \in [0,2\pi]$$ \hspace{1cm} (3)

Initial conditions:

$$T=0 \quad t < t'$$ \hspace{1cm} (4)

The differential equation of Green's function based on the formula (1) is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial G}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} + \frac{1}{k} \delta(r-r') \delta(\theta-\theta') \delta(t-t') = \frac{1}{\alpha} \frac{\partial G}{\partial t}$$ \hspace{1cm} (5)

Boundary conditions of auxiliary problems:

$$G = 0, \quad r=a, \quad \theta \in [0,2\pi]$$ \hspace{1cm} (6)

Initial conditions:

$$G=0 \quad t < t'$$ \hspace{1cm} (7)

According to the conservation of energy, the heat absorption from t to t + t’ in any spatial position is equal to the amount of internal heat source:

$$\rho C_p d\nu \left[ G|_{t=0} - G|_{t=t'} \right] \delta(r-r') \delta(\theta-\theta') dt$$ \hspace{1cm} (8)

With the initial conditions of the auxiliary problem,

$$G|_{t=t'} = \frac{1}{\rho C_p} \delta(r-r') \delta(\theta-\theta')$$ \hspace{1cm} (9)

The instantaneous point heat source \( \delta(r-r') \delta(\theta-\theta') \delta(t-t') \) is equivalent to the initial temperature distribution at the moment t', which is \( \frac{\alpha}{k} \delta(r-r') \delta(\theta-\theta') \). Further transform needs in terms of condition, and the result is as (10).

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial G}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} = \frac{1}{\alpha} \frac{\partial G}{\partial t}$$ \hspace{1cm} (10)

By separating the space-time variables:

$$G(r, \theta, t) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} e^{-\alpha \beta_m^2 t} (A_{mn} \sin n\theta + B_{mn} \cos n\theta) R_v(\beta_m, r)$$ \hspace{1cm} (11)

According to the orthogonality of \( R_v(\beta_m, r) \) and the operator \( \int_a^b R_v(\beta_m, r) dr \) and the property of the \( \delta \) function, obtains the formula (12).

$$A_{mn} \sin n\theta + B_{mn} \cos n\theta = \frac{1}{\pi N(\beta_m)} \int_a^b \int_0^{2\pi} r' R_v(\beta_m, r') \frac{\alpha}{k} \delta(r-r') \delta(\theta-\theta') \delta(t-t') * \cos(v(\theta-\theta')) d\theta' dr'$$ \hspace{1cm} (12)

The steady state solution can be obtained from the transient equation.

$$G(r, \theta) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{1}{\rho m + \pi N(\beta_m)} r' R_v(\beta_m, r') \cos v(\theta-\theta') + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{1}{\rho m + \pi N(\beta_m)} r' R_v(\beta_m, r') \cos v(\theta-\theta')$$ \hspace{1cm} (13)

The derivative of Bessel function:

$$I_v(\beta_m b) = 0.5 \left( I_{v-1}(\beta_m b) - I_{v+1}(\beta_m b) \right)$$ \hspace{1cm} (14)
\[ Y'_v(\beta_m b) = 0.5 \left( Y_{v-1}(\beta_m b) - Y_{v+1}(\beta_m b) \right) \] (15)

Substitute the above conditions, and obtain complete two-dimensional Green's function solution:

\[
G(r, \theta | r', \theta') = \frac{\pi}{4k} \sum_{v=0}^{\infty} \sum_{m=1}^{\infty} \frac{J_v^2(\beta_m a)}{1 - \left( \frac{r}{\beta_m b} \right)^2} \frac{J_v'(\beta_m a) - J_v'(\beta_m b)}{J_v(\beta_m a) - J_v(\beta_m b)} \* r' \* \left[ J_v(\beta_m r) \* Y'_v(\beta_m b) - J'_v(\beta_m b) \right] * Y_v(\beta_m r') \]

\[
\sum_{m=1}^{\infty} \sum_{v=0}^{\infty} \frac{J_v^2(\beta_m a)}{1 - \left( \frac{r}{\beta_m b} \right)^2} \frac{J_v'(\beta_m a) - J_v'(\beta_m b)}{J_v(\beta_m a) - J_v(\beta_m b)} \* r' \* \left[ J_v(\beta_m r) \* Y'_v(\beta_m b) - J'_v(\beta_m b) \right] * Y_v(\beta_m r') \* \cos(v(\theta - \theta'))
\] (16)

The temperature field is obtained from the general solution of Green's function

\[
T(r, \theta) = \sum_{i=1}^{\infty} \int_{s_i}^{r} \theta | r' = r_i \* f_i(r, \theta) \* ds_i = k * \left( \frac{1}{h_a} \int_0^{2\pi} \frac{\partial g(r, \theta | r', \theta')}{\partial r} \* \right) \left| r'=a \* f_1(r', \theta') \* d\theta' + \right.

\[
\frac{1}{k_b} \int_0^{2\pi} g(r, \theta | r', \theta') \* f_2(r', \theta') \* d\theta'
\] (17)

This paper programs the Green's Function with MATLAB. In order to compare with these two methods, this paper uses COMSOL to simulate. The red line stands for the result of analytical equation, and the green one is the simulation’s result. The result is shown in Figure 1. It can be seen that the results are basically the same. At the location close to the heat source, the little difference is caused by the order of the eigenfunction and the number of eigenvalues in the analytical equation or radius of heat source. When taking 100 eigenvalues and 100 levels, the result is 1.4635°C, which takes 11s. Analyzing the results of COMSOL, when the surface heat source radius is 0.008m, the result is 1.4650°C, which takes 44s. Obviously, the analytical solution calculates faster than simulation with better accuracy.

Figure 1 Comparison of MTLAB and COMSOL

3. Finite element simulation

3.1 Physical model

This paper took a research on the deep groove bearing. We used the CREO to establish the structure displayed in Figure 2, and simplified from omitting the cages and chamfers. The dimension is 20 mm x 40 mm x 12 mm. each row has 9 rolling elements, and the detail parameters is in the Table 1, and the structure displays in Figure 3:

| Table 1. The basis parameters of bearing |
|-----------------------------------------|
| Inside diameter                        | 20 mm |
| Outside diameter                       | 40 mm |
| Width                                  | 12 mm |
| Number of rolling element each row     | 9 mm  |
| Pitch diameter                         | 31 mm |
| Roller element diameter                | 6.35 mm |
3.2. Calculation of heat generation

In this paper, we take the simple and exact model to calculate heat generation of bearing, which is Palmgren model[11]. According to the friction torque $M_f$, lubrication torque $M_l$ and speed of bearing $N_0$, the total power loss $H_t$ can obtain.

$$H_t = 1.047 \times 10^{-4}(M_f + M_l) \times N_0 \quad (18)$$

$$M_f = f_0 P_0 D_0 \quad (19)$$

$$M_l = f (\mu N_0)^2 D_0^3 \times 10^{-7} (\mu N_0 \gg 2000) \quad (20)$$

$$M_l = 160 \times 10^{-7} f D_0^3 (\mu N_0 < 2000) \quad (21)$$

Some parameters about above formulas, which $f_0$ is coefficient of bearing’s structure, $P_0$ is equivalent dynamic load of bearing, $D_0$ is pitch diameter, $f$ is coefficient about types and lubrication of bearing, $\mu$ is Dynamic viscosity of lubricating oil. Then obtain the total power loss of the bearing is 1.17W, which works at the speed 1500r/min and the radial load in inner ring 200N. According to Burton and Steph[12], the ratio of power loss between roller elements and raceway is 1:1. And omit skidding in the contact region.

![Figure 3 Structure of deep groove bearing](image)

3.3. Calculation of convection coefficient

Many types of calculation on the convection coefficient, and the most is empirical formula. Harris provided an approximate formula[13].

$$A_c = 0.332 k_0 L^{-1} Pr^{-3} Re^{-2} (Re < 5 \times 10^5) \quad (22)$$

There are parameters, $L$ is characteristic length, $k_0$ is coefficient of lubrication, $Pr$ is Prandtl number of lubricating oil, $Re$ is Reynolds number of lubricating oil. Apply convective heat transfer coefficient to the inner and outer ring and rolling element surface, set the initial ambient temperature of the bearing to 23°C, and temperature of convection environment is 40°C. The coefficient displayed in Table 2.

| Table 2. Heat coefficient of convection $W/(m^2 \cdot ^\circ C)$ |
|-----------------|-----------------|
| Roller element  | 463             |
| Inner ring      | 335             |
| Outer ring      | 272             |

And COMSOL could not establish contact region automatically. Thus the contact between roller elements and raceway should add to make sure good heat transfer, which set the roller as the target body and raceways as contact body, and display clearly in Figure 4 and Figure 5. And the model is divided into 263025 elements with 52845 mesh vertex.
3.4. analysis of simulation

According to Figure 6, the maximum temperature is 41.2476 °C located at the roller, and temperature of the two rings is similar. But temperature in the inner ring is higher than the outer. From Figure 7, the temperature is uniform at the contact region, which satisfies the steady-temperature field. For the rolling element, heat transfers into both raceway and the transfer rate of radial dimension is faster than the axial.

The Figure 8, Figure 9 and Figure 10 are the components of bearing, which display the detail distribution of temperature. The maximum temperature locates at the groove is 40.8593°C, and the maximum temperature of outer ring is 40.6894°C, and it is 41.2476°C at contact surface.
According to the Figure 11, there is almost no heat flux transfer from raceway into the rollers. Thus, take the better conductivity material of the contact region is necessary, which can accelerate heat loss, for convection has little effect in the contact region.

3.5. comparison of analytical result and the simulation result

In order to obtain the exact result of Green’s Function, some equivalent treatment will take for the inner ring, especially, the deep groove. According to Figure 2, temperature is uniform in circumference. Thus, the equivalent heat flux should be calculated, which needs the contact area of inner ring. The length of groove in the raceway is 4.1991 mm, and the equivalent heat flux is 899.5948 W/m².

According to the MATLAB program, the final temperature converges to 0.0376 under per unit heat flux with the equivalent radius 12.665 mm, when the number of eigenvalues is 66. And the result is 40.7601°C on the basis of formula (12). Besides, the maximum temperature of inner raceway is 40.8545°C from Figure 9, and the error is 6.9483% of the two results. The reason of error may be the difference of equivalent heat flux and radius.

4. Conclusion

This paper provides a new analytical solution of obtaining the temperature field of deep groove ball bearing, which has consistent with simulation by COMSOL. The Green’s Function can calculate the temperature of bearing with any shape region based on certain heat source, and heat source in this paper is even. Besides, the program computes fast than the simulation, which costs 0.025s in this model. In terms of post-treatment, COMSOL is better than ANSYS, and according to simulation, the maximum temperature is on the roller and little temperature difference between inner ring and the outer. From analyzing direction of heat flux, which the heat flux just flows from roller to raceway. Thus, raceway has the highest thermal load, for almost no convection occurs at this region, which needs better thermal conductivity materials to dissipate heat. The study of this paper provides a theoretical method to obtain temperature field correspondingly. And simulation of COMSOL is instructive for the engineering Applications.

References
[1] Mujumdar, A.s. and M. Hasan 2010 Drying Technol 34.
[2] Xiu, S.C., S.Q. Gao, and Z.L. Sun 2010 Adv Mater Res 118-120
[3] Tarawneh, C.M., A.A. Fuentes, J.A. Kypuros, L.A. Navarro, A.G. Vaipan, and B.M. Wilson 2012 J Therm Sci Eng Appl 4 3.
[4] A, H.T. and K.M. N 2007 Advanced Concepts of Bearing Technology, Trans Eng Sci Vol. 2.
[5] Winer, W.O., S. Bair, and B. Gecim 1986 A S L E Transactions 29 4.
[6] Baïri, A., N. AliLAT, J.G. Bauszin, and N. Laraqi 2004 Int J Therm Sci 43 6.
[7] Hannon, W.M. 2015 J Tribol 137 3.
[8] Hannon, W.M. 2015 J Tribol 137 3.
[9] Hannon, W.M., T.A. Barr, and S.T. Froelich 2015 J Tribol 137 3.
[10] Cole, K.D., C.M. Tarawneh, A.A. Fuentes, B.M. Wilson, and L. Navarro 2010 *Int J Heat Mass Transfer* **53** 9-10.

[11] A, P. 1959 *Ball and Roller Bearing Engineering*. BurbankPress.

[12] Burton, R.A. and H.E. Staph 1967 *Asle Transactions* **10** 4.

[13] Harris, T.A. and M.N. Kotzalas 2010 *Analysis of rolling bearing*. 