Abstract:

Recent HERA low $Q^2$ data show that the logarithmic slope of the proton structure function ($\frac{\partial F_2}{\partial \ln Q^2}$) is significantly different from perturbative QCD expectations for small values of $Q^2$ at exceedingly small values of $x$. We show that shadowing (screening) corrections provide a natural explanation for this experimental observation.

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Recent HERA data on $Q^2$ and $x$ dependence of the logarithmic $Q^2$ derivative of the proton structure function $F_2(x,Q^2)$, $\frac{\partial F_2}{\partial \ln Q^2}$, have become available \[1\] \[2\]. This is shown in Fig.1 where each point corresponds to a different values of $Q^2$ and $x$. As seen, this observable rises steeply with decreasing $x$ to values of $x$ of approximately $10^{-4}$ with values of $Q^2$ larger than a few $GeV^2$. However, in the exceedingly small $x < 10^{-4}$ and low $Q^2 < 5 GeV^2$ values, the behaviour of the $F_2$ slope is completely different - decreasing rapidly with $x$ and/or $Q^2$ getting smaller (see Fig.1). This dramatic change of $\frac{\partial F_2}{\partial \ln Q^2}$ is not compatible with the prediction of perturbative QCD (pQCD). This is shown in Fig.1 where the HERA data \[1\] is compared with the theoretical DGLAP expectations \[3\] based on the GRV’94 parton distribution input \[4\]. Specifically, in the small $x$ limit of the DGLAP equations we have

$$\frac{\partial F_2^{DGLAP}(x,Q^2)}{\partial \ln(Q^2/\Lambda^2)} = \frac{2\alpha_s}{9\pi} xG^{DGLAP}(x,Q^2),$$  \hspace{1cm} (1)$$

where the expected rise of the $F_2$ slope is associated with the logarithmic $Q^2$ growth of $xG(x,Q^2)$ implied by the DGLAP equations in the small $x$ limit.

The new data pose a severe theoretical challenge as it requires a successful description of DIS in the transition region characterized by intermediate distances. These distances are smaller than the confinement scale $\frac{1}{\Lambda}$, where $\alpha_s(\Lambda^2) \gg 1$, but are still not small enough to justify a reliable pQCD calculation. For sufficiently small $Q^2$ we expect non-perturbative contributions to be important in DIS. However, if the transition region corresponds to larger values of $Q^2$ as it appears from Fig.1, we presume that these non-perturbative contributions to DIS will depend on several average characteristics such as vacuum expectation of the square of gluon tensor and the correlation length of two gluons within the hadron. These properties are reflected in the QCD Sum Rules approach \[5\].

In order to assess the significance of the new data we examine both $\frac{\partial F_2}{\partial \ln Q^2}$ and $F_2$. A change in the functional $Q^2$ dependence of $\frac{\partial F_2}{\partial \ln Q^2}$ at small $Q^2$ values can be deduced from general arguments. Since $\sigma_t = \frac{4\pi^2\alpha_{em}}{Q^2} F_2$ is finite (non zero) for real photoproduction ($Q^2 = 0$ and $x = 0$), we conclude that at small $Q^2$ values $F_2 \propto Q^2$ (see Ref. \[6\] for a more detail discussion). Accordingly, an eventual change in the functional $x$ dependence of $\frac{\partial F_2}{\partial \ln Q^2}$ in the limit of exceedingly small $x$ is expected. However, the particular $x$ and $Q^2$ location of this change, its dynamical mechanism and the relevance to pQCD are not \textit{apriori} clear. We note that the $F_2$ data in the HERA kinematic region at low $x$ and $Q^2 \geq 1 GeV^2$ are well reproduced by the DGLAP evolution equations with input parton distributions \[4\] \[7\] \[8\] starting from very low $Q^2$ values. The importance of the new $\frac{\partial F_2}{\partial \ln Q^2}$ data is that it opens a new window through which a transition region is observed in the $Q^2$ range of $1 - 4 GeV^2$ and $x < 10^{-4}$. This transition is apparently not resolved through
the study of $F_2$ in the same $x$ and $Q^2$ domain. This transition is not predicted by the DGLAP evolution (see Eq. (1)) as can readily be seen in Fig.1.

The objective of this letter is to show that shadowing (screening) corrections (SC) to $F_2$ account for the deviation from the DGLAP predictions and provide a natural explanation for the observed experimental phenomena.

![Figure 1](image-url)

Figure 1: The HERA data and GRV'94 prediction (triangles) on the $F_2(x, Q^2)$ slope. The data are taken from Ref. [2].

There are two different types of SC that contribute to change of the $F_2$ slope: (i) SC due to passage of the quark-antiquark pair through the nucleon, which lead to
a more general equation for the $F_2$ slope than Eq. (1) and (ii) SC to the gluon structure function in Eq. (1). We first discuss the quark-antiquark sector.

1. Closed formulae for the penetration of a $q\bar{q}$ - pair through the target in the Eikonal (Glauber) approach were written many years ago [9] [10] and have been discussed in detail over the past few years [11]. For the $F_2$ slope these formulae lead to

$$\frac{\partial F_2(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = \frac{Q^2}{3\pi^2} \int db_i^2 \left\{ 1 - e^{-\kappa(x, Q^2, b_i^2)} \right\},$$

(2)

where

$$\kappa(x, Q^2, b_i^2) = \frac{2\pi\alpha_S}{3Q^2} \Gamma(b_i) xG^{DGLAP}(x, Q^2).$$

(3)

Here, we have used the main property of the DGLAP evolution equations, which allows us to factor out the impact parameter ($b_i$) dependence from $x$ and $Q^2$ (see Ref. [12]). $Q_0^2$ is the photon virtuality scale from which we apply pQCD for our calculation. $\Gamma(b_i)$ denotes the Fourier transform of the two gluon nonperturbative form factor of the target, which is independent of the incident particle. The relation between the profile $\Gamma(b_i)$ and the two gluon form factor reads

$$\Gamma(b_i) = \frac{1}{\pi} \int e^{-i(\vec{b}_i \cdot \vec{q}_t)} F(t) \, d^2 q_t,$$

(4)

with $t = -q_t^2$. Note, that factorization in Eq. (3) is valid only for $|t| \leq Q_0^2$. To simplify the calculations we approximate

$$\Gamma(b_i) = \frac{1}{R^2} e^{-\frac{R^2}{R^2}}.$$

(5)

An impressive property of Eq. (2) is the fact that the SC depend on the gluon structure function at short transverse distances $r_\perp^2 = \frac{4}{Q^2}$. This means that for the $F_2$ slope we can calculate the SC with guaranteed theoretical accuracy in pQCD, while the SC for the deep inelastic structure function ($F_2$) which originate from large distances $r_\perp^2 \geq \frac{4}{Q^2}$ and are, thus, not well defined and could lead to errors in the calculations.

The nonperturbative QCD (npQCD) information we need is given in Eq. (3) by the nonperturbative profile $\Gamma(b_i)$. In Fig.2 we show the lowest order SC to $F_2$ in Eq. (2) which are proportional to $\kappa^2$. In the additive quark model (AQM) we have two diagrams shown in Fig.2 which are of order $\kappa^2$. One can see that in AQM we have two scales for the integration over $q_\perp$: the distance between two constituent quarks in a nucleon (the first diagram in Fig.2) and the size of the constituent quark (the second diagram in Fig.2) (see Refs. [13] [14] for details). Eq. (2) is the simplest formula in which we can assume that two gluons inside a nucleon have no other correlation than their being confined in a nucleon with size $R$. A more general formula for the two radii model of the nucleon was obtained in Ref. [14].
For the $F_2$ slope

$$\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = \frac{Q^2}{3 \pi^2} \int dB t \left\{ 1 - e^{-\kappa_1(x, Q^2; b_t^2)} + \frac{\kappa_2^2(x, Q^2; b_t^2)}{\kappa_1(x, Q^2; b_t^2) - \kappa_2(x, Q^2; b_t^2)} e^{-\kappa_1(x, Q^2; b_t^2)} \right. $$

$$\left. + \frac{\kappa_2^2(x, Q^2; b_t^2)}{(\kappa_1(x, Q^2; b_t^2) - \kappa_2(x, Q^2; b_t^2))^2} \left( e^{-\kappa_1(x, Q^2; b_t^2)} - e^{-\kappa_2(x, Q^2; b_t^2)} \right) \right\}, \tag{6}$$

where (see Ref. [14] for details)

$$\kappa_1(x, Q^2; b_t) = \frac{2\pi\alpha_S}{3Q^2R_1^2} xG_{DGLAP}(x, Q^2) e^{-\frac{b_t^2}{R_1^2}} = \kappa_1(x, Q^2) e^{-\frac{b_t^2}{R_1^2}};$$

$$\kappa_2(x, Q^2; b_t) = \kappa_1(x, Q^2) \frac{R_1}{R_2} e^{-\frac{b_t^2}{R_2^2}}. \tag{7}$$

To evaluate the influence of the SC we introduce a damping factor ($D(\kappa)$)

$$D_Q(\kappa) = \frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} \frac{Q^2}{3 \pi^2} \int dB t \kappa(x, Q^2; b_t^2), \tag{8}$$

where $\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ is calculated from Eq. (4) for a one radius model of the nucleon and from Eq. (3) for a two radii model. The denominator is the first term of the $F_2$ slope which corresponds to the DGLAP equations. Note, that in such a calculation for a two radii model $R = R_1$. 

Figure 2: The first order SC $\propto \kappa^2$ for $F_2(x, Q^2)$. 

For the $F_2$ slope

$$\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = \frac{Q^2}{3 \pi^2} \int dB t \left\{ 1 - e^{-\kappa_1(x, Q^2; b_t^2)} + \frac{\kappa_2^2(x, Q^2; b_t^2)}{\kappa_1(x, Q^2; b_t^2) - \kappa_2(x, Q^2; b_t^2)} e^{-\kappa_1(x, Q^2; b_t^2)} \right. $$

$$\left. + \frac{\kappa_2^2(x, Q^2; b_t^2)}{(\kappa_1(x, Q^2; b_t^2) - \kappa_2(x, Q^2; b_t^2))^2} \left( e^{-\kappa_1(x, Q^2; b_t^2)} - e^{-\kappa_2(x, Q^2; b_t^2)} \right) \right\}, \tag{6}$$

where (see Ref. [14] for details)

$$\kappa_1(x, Q^2; b_t) = \frac{2\pi\alpha_S}{3Q^2R_1^2} xG_{DGLAP}(x, Q^2) e^{-\frac{b_t^2}{R_1^2}} = \kappa_1(x, Q^2) e^{-\frac{b_t^2}{R_1^2}};$$

$$\kappa_2(x, Q^2; b_t) = \kappa_1(x, Q^2) \frac{R_1}{R_2} e^{-\frac{b_t^2}{R_2^2}}. \tag{7}$$

To evaluate the influence of the SC we introduce a damping factor ($D(\kappa)$)

$$D_Q(\kappa) = \frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} \frac{Q^2}{3 \pi^2} \int dB t \kappa(x, Q^2; b_t^2), \tag{8}$$

where $\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}$ is calculated from Eq. (4) for a one radius model of the nucleon and from Eq. (3) for a two radii model. The denominator is the first term of the $F_2$ slope which corresponds to the DGLAP equations. Note, that in such a calculation for a two radii model $R = R_1$. 

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Using this damping factor \( D(\kappa) \) we can write the \( F_2 \) slope in the form

\[
\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = D_Q(\kappa) \frac{\partial F_2^{DGLAP}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)}.
\]  

(9)

The calculated damping factor of Eq. (8) is plotted in Fig.3 for a one radius model with \( R^2 = 10 GeV^{-2} \) versus \( \kappa \) ( upper curve ) and for two a radii model with two sets of radii: (i) \( R_1^2 = 10 GeV^{-2} \) and \( R_2^2 = 3 GeV^{-2} \) and (ii) \( R_1^2 = 6 GeV^{-2} \) and \( R_2^2 = 2 GeV^{-2} \) versus \( \kappa_1 \) ( see Eq. (7) ). We note that two radii sets of curves are almost undistinguishable from one another as a function of \( \kappa_1 \).

![Figure 3: The damping factor \( D_Q(\kappa) \) versus \( \kappa(\kappa_1) = 0.03 + 0.05 \times n \) for a one radius model with \( R^2 = 10 GeV^{-2} \) ( upper curve) and for a two radii model with two sets of radii: (i) \( R_1^2 = 10 GeV^{-2} \) and \( R_2^2 = 3 GeV^{-2} \) and (ii) \( R_1^2 = 6 GeV^{-2} \) and \( R_2^2 = 2 GeV^{-2} \).](image)

One can see that the two radii model of the nucleon leads to sufficiently large \( SC \) which depend on the set of radii chosen. Note that the value of \( \kappa_1 \) is inversly proportional to \( R_1^2 \).

2. The Glauber ( Eikonal ) formula for the SC in the gluon structure function was obtained by Mueller [14] and discussed in details in Ref.[15].

\[
xG^{SC}(x, Q^2) = \frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_{Q_0^2}^{Q^2} dQ'^2 \int db_1^2 \left\{ 1 - e^{-\kappa_G(x', Q'^2, b_1^2)} \right\},
\]

(10)
with \( \kappa_G(x', Q^2; b_t^2) = \frac{3}{4} \kappa(x', Q^2; b_t^2) \) where \( \kappa \) is taken from Eq. (3). In the case of a two radii model the formula can be derived as a direct generalization of the procedure suggested in Ref. [14].

\[
xG(x, Q^2) = \frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_{Q_0^2}^{Q^2} dQ'^2 \int db_t^2 \{ 1 - e^{-\kappa_{G1}(x', Q'^2; b_t^2)} + \kappa_{G2}(x', Q'^2; b_t^2) - \kappa_{G2}(x', Q'^2; b_t^2) \} e^{-\kappa_{G1}(x', Q'^2; b_t^2)} + \frac{\kappa_{G1}(x', Q'^2; b_t^2) - \kappa_{G2}(x', Q'^2; b_t^2)}{\left( \kappa_{G1}(x', Q'^2; b_t^2) - \kappa_{G2}(x', Q'^2; b_t^2) \right)^2} (e^{-\kappa_{G1}(x', Q'^2; b_t^2)} - e^{-\kappa_{G2}(x', Q'^2; b_t^2)}) \},
\]

where \( \kappa_{G1} = \frac{9}{4} \kappa_1 \) and \( \kappa_{G2} = \frac{9}{4} \kappa_2 \) where \( \kappa_1 \) and \( \kappa_2 \) are defined in Eq. (4).

The gluon damping factor is defined as

\[
DG(x, Q^2) = \frac{xG^{SC}(x, Q^2)}{\frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_{Q_0^2}^{Q^2} dQ'^2 \int db_t^2 \kappa_G(x', Q'^2; b_t^2)},
\]

where \( xG^{SC} \) is calculated from Eq. (10) for a one radius model and from Eq. (11) for a two radii model. For a two radii model \( \kappa_G = \kappa_{G1} \) in the dominator of Eq. (12). It is important to note that the \( Q^2 \) integration in Eq. (11) and Eq. (12) spans all distances, including long distances dominated by nonperturbative dynamics. This is very different from Eq. (4) and Eq. (5) where the SC depend on the gluon density at short transverse distances. Since a theoretical approach to npQCD is still lacking, we eliminate the long distance contributions to Eq. (11) and Eq. (12) by imposing a low cutoff on the \( Q^2 \) integration. With this cutoff we neglect the contributions due to transverse distances \( r_\perp > \frac{1}{Q_0} \). This procedure makes our calculation of \( DG(x, Q^2) \) less reliable than our calculation of \( D_Q(x, Q^2) \). To evaluate the errors due to this source we have calculated \( xG(x, Q^2) \) with two different cutoff values: \( Q_0^2 = 0.4 \text{GeV}^2 \) and \( Q_0^2 = 1 \text{GeV}^2 \), using the GRV'94 parameterization [4] for the solution of the DGLAP evolution equations. The resulting gluon damping factors, \( DG(x, Q^2) \) differ by about 10% which lends a reasonable credibility to our calculation. As we have noted in the introduction, we are of the opinion that the GRV'94 parameterization does not contain SC. This is due to the fact that most of the experimental sample used to fix the GRV'94 parameters has values of \( x \geq 10^{-3} \), where we estimate the SC are very small.

The formula which we use to compare with HERA experimental data is

\[
\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)} = \frac{D_Q(x, Q^2)}{DG(x, Q^2)} \frac{\partial F_2^{GRV}(x, Q^2)}{\partial \ln(Q^2/Q_0^2)},
\]
Figure 4-a: The $F_2$ slope for SC in quark sector only. Triangles are the GRV’94 prediction. Stars are the result of SC calculations in the one radius model for a nucleon with $R^2 = 10 GeV^2$, squares are for the SC in the two radii model with $R_1^2 = 10 GeV^2$ and $R_2^2 = 3 GeV^2$ and circles are for the SC in the two radii model with $R_1^2 = 6 GeV^2$ and $R_2^2 = 2 GeV^2$. 
Figure 4-b: The $F_2$ slope for SC both in quark and gluon sectors. Notations are the same as in Fig.4-a.
where $F_2^{GRV}$ is the deep inelastic structure function calculated in the DGLAP evolution approach with the GRV’94 parameterization. Our results compared with the experimental data are shown in Fig. 4. Since we use the GRV’94 input our calculations can be carried out only for $Q^2 > 0.4\, GeV^2$.

To summarize, the main points of this letter are:

1) The deviation of $\frac{\partial F^{SC}(x,Q^2)}{\partial \ln(Q^2/Q_0^2)}$ from the behaviour predicted by the DGLAP evolution in the small $Q^2$ and exceedingly small $x$ region is associated with SC applied both to the passage of quark-antiquark pair through the nucleon and the gluon density within the nucleon target. When SC are applied good agreement with the new HERA data, with $Q^2 > 0.8\, GeV^2$, is obtained.

2) SC for the quark-antiquark interaction with the target nucleon is concentrated at short distances $r_\perp \approx \frac{2}{Q}$ and, therefore, can be calculated to a good degree of accuracy in pQCD. The effects of these SC are sufficiently large to account for most of the difference between the DGLAP prediction and experimental data, unlike the case of the deep inelastic structure function $F_2$.

3) The calculated SC for the gluon structure function are large and contain uncertainties due to long distance contributions which have not been included in the calculation. At present it is not possible to evaluate the errors which arise from nonperturbative contribution. We have checked the relative contribution coming from $3\, GeV^2 \geq r_\perp^2 \geq 1\, GeV^2$ by changing $Q_0^2$ - the lower limit of $Q^2$ integration in the calculation of $xG^{SC}(x, Q^2)$. The resulting change in $D_G(x, Q^2)$ is not more than 10%.

4) Our determination of the two radii of the nucleon rests on $J/\Psi$ photo and DIS production data. The present analysis suggests that better data on the $F_2$ logarithmic $Q^2$ slope may provide an independent determination of these radii as well as additional knowledge on the role of long distance nonperturbative contributions to the SC.

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