The intercept of the BFKL pomeron from Forward Jets at HERA

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Abstract

Recently the H1 and ZEUS collaborations have presented cross sections for DIS events with a forward jet. The BFKL formalism is able to produce an excellent fit to these data. The extracted intercept of the hard pomeron suggests that when all higher order corrections are taken into account the cross section will still rise very rapidly as expected for low $x$ dynamics.

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1 Introduction

Using a physical gauge, deep inelastic scattering processes can be represented within perturbative QCD with ladder diagrams like the one shown in figure 1. In this picture the interaction between an electron and a proton is mediated through the exchange of a virtual photon of four momentum squared $q_\mu q^\mu = -Q^2$, which couples to a quark–antiquark box at the top of the parton ladder inside the proton.

The DGLAP \cite{1} and BFKL \cite{2} schemes select different leading logarithmic regions of the phase space to describe the partonic evolution along this ladder.

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The DGLAP approach takes into account the leading terms in $\ln Q^2$, but neglects those in $\ln(1/x)$. In this case the square of the transverse momentum of the partons along the ladder are strongly ordered, $k_i^2 \gg k_{i-1}^2$ (cf. fig. 1), whereas the longitudinal momenta obey $x_i < x_{i-1}$. On the other hand, in the BFKL formalism the $\ln(1/x)$ are resummed in the region of $k_i^2 \approx k_{i-1}^2$ and $x_i \ll x_{i-1}$. Due to the inherent approximations involved the domain of applicability of the DGLAP equations is medium to high $x$–Bjorken, but the BFKL approach should only be used at small $x$–Bjorken.

The electron–proton collider HERA opened up the study of DIS to new domains of small values of $x$–Bjorken and still sizeable virtuality of the photon. It was thus expected that experiments at HERA would be able to measure the transition from the region of applicability of DGLAP to that of BFKL. Among the different proposed observables to test the BFKL approach to DIS, the measurement of forward jets is considered to be one of the most promising.

Higher order corrections to the BFKL formalism have been recently calculated [3]. They turned out to be sizable and pointed out the need of still higher order corrections. This reduced the quantitative predictive power of the leading logarithmic approximation to BFKL calculations, but the qualitative behavior at small $x$ may still remain the same. Taking this into account a fit to HERA data, which of course resumes all orders, may shed light into the problem of the numerical stability of the BFKL kernel under higher order corrections. In this spirit the intercept of the BFKL pomeron is extracted from a fit to the forward jet data of the H1 and ZEUS collaborations.

2 Forward jets at HERA

Back in 1990 A. H. Mueller [6] proposed to look for DIS events with one jet—other than the jet originating from the struck quark—fulfilling the following characteristics (cf. fig. 1):

- **$x$ small.** The selection of the smallest $x$–Bjorken experimentally possible implies going away from the domain of validity of DGLAP and into a phase space area governed by BFKL.
- **$x_J$ large.** This, along with the previous item, provides the phase space for parton evolution and has the extra advantage to enable the use of parton
density functions (PDF) of the proton in a region where they have already been measured, so no extrapolation is needed.

• $k_J^2 \approx Q^2$. This requirement suppresses DGLAP evolution without affecting the BFKL dynamics. It also provides a big scale at both ends of the ladder, avoiding thus the dangerous infrared regions of phase space and increasing the solidity of the analytic predictions.

The process so selected provides a well defined topology for the experimentalist. One has to look for a DIS event with a jet at high rapidities and enforce that the virtuality of the proton is of the same scale as the transverse momentum squared of the jet. As the direction of the proton is termed by the HERA experiments the forward direction, this kind of events are known as forward jets events. Recently both the H1 [4] and the ZEUS [5] collaborations have presented cross sections for this observable. The H1 collaboration selects DIS events requiring the energy ($E_e$) and polar angle ($\theta_e$) of the scattered electron to be $E_e > 11$ GeV and $160^\circ < \theta_e < 173^\circ$. ZEUS only requires $E_e > 10$ GeV and the polar angle $\theta_e$ is restricted by the rest of the kinematic constrains. Both of them select events with $y_{Bj} > 0.1$ to avoid the jet from the struck quark going forward. In the jet side, selected with a cone algorithm of radius 1, the cuts used are for H1: $k_J > 3.5$ GeV, $x_J \approx E_J/E_{beam} = E_J/820\text{GeV} > 0.035$ and $7^\circ < \theta_J < 20^\circ$. ZEUS ask for jets with $k_J > 5.0$ GeV, $x_J > 0.036$ and $8.5^\circ < \theta_J$. Both collaborations require that $0.5 < k_J^2/Q^2 < 2.0$. The luminosity used is 2.8 and 6.4 pb$^{-1}$ for the H1 and ZEUS collaborations respectively. The H1 collaboration also reports a similar search, where all cuts were kept the same, except that $k_J > 5.0$ GeV was required. The measured cross sections for different bins in $x$–Bjorken are presented in the first column of tables 1 and 2. Note that both collaborations presented slightly asymmetrical systematic errors. The errors presented in tables 1 and 2 are an average of the systematic errors added in quadrature to the statistical error of the measurement. Note also that the measurements at the lowest $x$–Bjorken have not been included. This is because, due to the $0.5 < k_J^2/Q^2 < 2.0$ cut, the experiments ran out of phase space. Thus these points mix dynamic effects with phase space restrictions and are not useful for the analysis presented here.
3 BFKL fits to the forward jet data

The cross section for DIS events containing a forward jet has been calculated at leading logarithmic approximation within the BFKL formalism in references [7–9]. There the following form has been found:

\[
\frac{d^4\sigma}{dx dQ^2 dx_J dk^2_J} = C\alpha_s(Q^2)F(x_J, Q^2)\sqrt{\frac{Q^2}{k^2_J}} \exp[(\alpha_p - 1) \ln x_J/x] \left(\ln x_J/x\right)^{1/2}
\]  

(1)

where \(\alpha_s(Q^2)\) is the QCD coupling constant at the scale \(Q^2\) and \(F\) is a generic parton distribution in the proton given by

\[
F(x_J, Q^2) = x_J G(x_J, Q^2) + \frac{4}{9}[x_J q(x_J, Q^2) + x_J \bar{q}(x_J, Q^2)].
\]  

(2)

In [7–9] an explicit form for the parameters \(C\) and \(\alpha_p\) can be found. Recently the NLO corrections to the BFKL kernel have been presented [3]. These corrections turn out to be very large implying that NNLO calculations are needed. In particular the corrections affected the intercept of the BFKL pomeron \(\alpha_p\), reducing the predictive power of the explicit forms given in [7–9] at LO and in [3] at NLO. This means that the exact power of the leading behavior of equation (1),

\[
\left(\frac{x_J}{x}\right)^{\alpha_p - 1}
\]  

(3)

is not numerically known from BFKL calculations. Nonetheless given that the first experimental data for this process are available, it is tempting to test if the form of equation (1) does indeed describe the measurement, and if so, which intercept of the BFKL pomeron is favored by the data.

To perform the fit there are some other ingredients needed. Standard parton density functions are required to evaluate \(F\). Also as equation (1) is a four differential expression, values for \(x\), \(Q^2\), \(x_J\) and \(k^2_J\) are needed in each \(x\)–Bjorken bin. Both collaborations, H1 and ZEUS, report that the Monte Carlo generator Ariadne [10] describes very well, not only the \(x\)–Bjorken dependence, but all distributions involved in the analysis. So a similar analysis to that reported by both collaborations has been performed at the hadron level of the Ariadne Monte Carlo. It has been checked that the cross section obtained...
with this procedure agrees with both, the reported data and also with the expectations from the Ariadne Monte Carlo as given by the H1 and ZEUS collaborations. Using this Monte Carlo data set for forward jet events the mean values of the variables $x$, $Q^2$, $x_J$ and $k_J$ have been estimated for each measured point of both collaborations. The results are presented in tables 1 and 2 along with the measured data points and their errors.

Using the input of tables 1 and 2, the PDFs given by GRV-LO [11], GRV-HO [12], CTEQ-4M [13] and MRS-R1 [14] –last three calculated in the $\overline{\text{MS}}$ scheme–, and an $\alpha_s$ value consistent with each of the different parton density functions, a 2 parameter fit was performed separately to the H1 and the ZEUS points using formula (1). The $\chi^2$ and the value of $\alpha_p$ obtained are shown in table 3.

4 Discussion

The results of the fits can be summarized as follows:

1. All data points could be successfully fit to the form of equation (1). This is quite encouraging because that was the main motivation for this kind of measurements.

2. The fits are insensitive to the PDF used. This result is also as expected. As a matter of fact this was one of the main advantages of the forward jet proposal.

3. Different values of the exponent $\alpha_p$ are found when using LO or NLO PDF. On the one hand, being the formula (1) a LO approximation to BFKL, one is tempted to consider only the use of LO PDFs. On the other hand, the whole idea of performing the fits is to have a data driven estimation of the effects of higher order corrections to the BFKL kernel. The actual variation of the value of $\alpha_p$ is expected in the basis of it being proportional to $\alpha_s$ in LO. As the value of $\alpha_s$ decreases in going from LO to NLO, the value of $\alpha_p$ must compensate this trend and increase from one case to the other.

4. The values of $\alpha_p$ found using H1 data are different to those found using ZEUS measurements. On the one hand ZEUS data is more precise having a
factor of 3 more luminosity. This allowed the ZEUS collaboration to measure the cross section in a region definitely dominated by DGLAP, having thus a clean transition from the box diagram at the top of figure 1 to the case when the box is complemented with a ladder. This greatly constrains the fit to equation (1). On the other hand H1 points reach smaller $x$, which is the interesting region for BFKL studies, although still with huge errors. One possible explanation for the difference in the values obtained using the data of both collaborations, could be again the $\alpha_s$ dependence of $\alpha_p$. Note that the average $Q^2$ is a lot bigger in the ZEUS measurement than in the H1 case. In the LO approximation this would naively produce a 15 to 20% increase of $\alpha_p$ from H1 data with respect to that from ZEUS cross sections.

5. The formula (1) is at parton level, i.e. it does not includes hadronization effects. As reported by both collaborations these are uncertain. Different models yield not only different normalization, but may also yield a $x$ dependence (see for example figure 8 on reference [5]) of the correction from hadron to parton level.

6. Using the dipole approach to BFKL, it has been shown that the measurements of the structure function $F_2$ can be described using an intercept of $\alpha_p = 1.28$ [15]. Some care is necessary when comparing this value with the one obtained here. The question of infrared divergencies due to the random walk generated by the BFKL kernel [16] is quite sensitive for the $F_2$ case, but it does not appear in the case of forward jets. Nevertheless is quite comforting that two so different observables yield results compatible with the BFKL formalism.

7. The possibility to experimentally reach lower $x$ at still sizable $Q^2$ is very important. The fits presented here, although they could not used the lowest $x$ points due to phase space constrains, have shown that the BFKL dynamics enforces a steep rise of the cross section for a process governed by a hard pomeron inspite the NLO corrections to the BFKL kernel. Reaching smaller $x$ will allow to access the saturation region of hot spots in the proton, which was one of the primary motivations of the forward jet proposal. This goal may still be reachable at HERA.
5 Conclusion

A fit was performed of a BFKL prediction to forward jet production as measured by the H1 and ZEUS collaborations. All data were consistent with the assumption of using the BFKL formula for this process. Difference in the intercept of the pomeron obtained with different sets of data, may be assigned to its $\alpha_s$ dependence. The fits support the idea of a hard pomeron and point in the direction that when all higher order corrections are taken into account the forward jet cross section will still be rising quite rapidly.

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Table 1
Cross sections for the forward jet production measured by the H1 [4] collaboration along with average values for the kinematic variables obtained from the Ariadne MC [10]. Note that the quoted errors are an average of the slightly asymmetric cuts reported by the collaboration.

| $\sigma$ [nb] | $<x>$ | $<Q^2>$ [GeV$^2$] | $<E_J>$ [GeV] | $<k_J>$ [GeV] |
|---------------|-------|-------------------|--------------|--------------|
| H1 $k_J > 3.5$ GeV |
| 342 ± 55      | 0.00073 | 21.5              | 34.4         | 5.0          |
| 224 ± 32      | 0.0012  | 26.9              | 35.4         | 5.5          |
| 138 ± 25      | 0.0017  | 31.4              | 36.9         | 5.8          |
| 67 ± 11       | 0.0024  | 38.1              | 38.1         | 6.3          |
| 32 ± 5        | 0.0035  | 47.0              | 38.8         | 6.9          |
| H1 $k_J > 5.0$ GeV |
| 132 ± 20      | 0.0012  | 32.2              | 37.9         | 6.3          |
| 96 ± 20       | 0.0017  | 334.8             | 39.3         | 6.5          |
| 55 ± 10       | 0.0024  | 40.1              | 39.4         | 6.7          |
| 28 ± 6        | 0.0035  | 48.2              | 39.6         | 7.2          |

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List of Figures

1. Representation of a deep inelastic event using a physical gauge within perturbative QCD
Table 2
Cross sections for the forward jet production measured by the ZEUS [5] collaboration along with average values for the kinematic variables obtained from the Ariadne MC [10]. Note that the quoted errors are an average of the slightly asymmetric cuts reported by the collaboration.

| $\sigma$ [nb] | $<x>$ | $<Q^2>$ [GeV$^2$] | $<E_J>$ [GeV] | $<k_J>$ [GeV] |
|---------------|-------|-------------------|--------------|--------------|
| 77.8 ± 7.6    | 0.0019| 50.7              | 39.9         | 7.6          |
| 34.4 ± 3.6    | 0.0033| 75.6              | 43.8         | 8.7          |
| 14.1 ± 2.1    | 0.006 | 113.6             | 49.6         | 10.4         |
| 6.5 ± 0.7     | 0.010 | 176.4             | 58.5         | 12.9         |
| 2.7 ± 0.4     | 0.018 | 244.7             | 67.3         | 15.1         |
| 0.6 ± 0.3     | 0.031 | 366.8             | 78.8         | 18.8         |

Table 3
Values obtained for $\alpha_p$ from a fit of the BFKL formalism to the data on forward jet production using different PDFs.

| PDF           | H1 $k_J > 3.5$ GeV | H1 $k_J > 5.0$ GeV | ZEUS         |
|---------------|--------------------|--------------------|--------------|
|               | $\chi^2$ $\alpha_p$ | $\chi^2$ $\alpha_p$ | $\chi^2$ $\alpha_p$ |
| GRV–LO        | 1.05 1.6 ± 0.4 0.24 | 1.7 ± 0.5 0.81     | 1.15 ± 0.13  |
| GRV–HO        | 1.0 1.8 ± 0.4 0.24  | 1.8 ± 0.5 0.76     | 1.24 ± 0.14  |
| CTEQ–4M       | 0.99 1.8 ± 0.4 0.22 | 1.8 ± 0.5 0.79     | 1.26 ± 0.14  |
| MRS–R1        | 1.0 1.8 ± 0.4 0.24  | 1.8 ± 0.5 0.91     | 1.26 ± 0.14  |
Fig. 1. Representation of a deep inelastic event using a physical gauge within perturbative QCD.