Static black holes and strictly static spacetimes in Einstein-Gauss-Bonnet gravity with gauge field

Marek Rogatko
Institute of Physics
Maria Curie-Sklodowska University
20-031 Lublin, pl. Marii Curie-Sklodowskiej 1, Poland
marek.rogat@poczta.umcs.lublin.pl
rogat@kft.umcs.lublin.pl
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We examine strictly static asymptotically flat spacetimes in Einstein-Gauss-Bonnet gravity with $U(1)$-gauge field, revealing that, up to the small curvature corrections, static conformally flat slices of the spacetime in question are of Minkowski origin. We consider uncharged and charged black hole solutions in the theory showing that up to the small curvature limit, they are diffeomorphic to Schwarzschild-Tangherlini or Reissner-Nordström solutions, respectively.

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I. INTRODUCTION

Emergence of black holes and gravitational collapse in generalized Einstein theories of gravity attract much attention last years. These results are closely related to the problem of classification of non-singular black hole solutions. The pioneering investigations of the mathematical topics bounded with the black hole equilibrium states were attributed to Israel [1]. Then they were developed in Refs. [2]-[3]. The alternative proof of the black hole uniqueness theorem was established by Bunting [4] and its generalizations [5]-[7]. Removing the condition of non-degeneracy, the complete classification of vacuum black hole solutions as well as Einstein-Maxwell black holes under the condition that all degenerate components of the event horizon should carry charge of the same signs, was established [8]. This assumption was get rid of in Ref.[9], were the near-horizon geometry of the black hole was investigated.

On the other hand, it turned out that a construction of the uniqueness black hole theorem for stationary axisymmetric spacetime was far more complicated task [10]. Nevertheless, the complete proof was devised by Mazur [11] and Bunting [12] (for various aspects of the black hole uniqueness theorem story see [13] and references therein).

In the recent years there was also a resurgence of mathematical works concerning black hole equilibrium states in higher dimensional spacetime. Higher dimensional black objects like black holes, black rings and black Saturns were widely examined. The complete classification of $n$-dimensional charged black holes both with non-degenerate and degenerate component of the event horizon was obtained in Refs.[14]-[16]. On the other hand, some partial results for the nontrivial case of $n$-dimensional rotating black hole uniqueness theorem were provided in [17]. Matter field behaviors in the spacetime of higher dimensional black hole were examined in Ref.[18].

Attempts of unifying all forces of Nature and constructing quantum theory of gravity also triggered the researches in the realm of the low-energy string theory. They encompass studies of the black hole uniqueness theorem in dilaton gravity, Einstein-Maxwell-axion-dilaton (EMAD)-gravity as well as supergravities theories [19]. On the other hand, the strictly stationary static vacuum spacetimes in Einstein-Gauss-Bonnet theory were discussed in [20]. The Chern-Simons (CS) modified gravity emerging in string theory as an anomaly-canceling term in Green-Schwarz mechanism [21], was treated from the point of view of the uniqueness of black holes. Namely, it was revealed that a static asymptotically flat black hole solution is unique to be Schwarzschild spacetime in CS-gravity [22], while static black holes with $U(1)$-gauge are isomorphic to Reissner-Nordström one [23].

Black holes are also key ingredients of the AdS/CFT attitude [24], in the context of a possible matter configuration in AdS spacetime. In Ref.[25] it was found that strictly stationary AdS spacetime did not allow for the existence of nontrivial configurations of complex scalar or form fields, while the generalization of the aforementioned problem, in EMAD-gravity with negative cosmological constant was accomplished in Ref.[26].

On the other hand, there has been also a renewed interest in theories with higher curvature terms, which arise naturally in various contexts, e.g., string theory, braneworld physics (see e.g., [27]), holographic superconductors. Among the theories which involve higher derivative curvature terms, the so-called Einstein-Gauss-Bonnet and its Lovelock generalization are of particular interests. Their equations of motion contain derivatives of the metric of order no higher than the second and therefore it has been proven to be ghosts free when expanding about flat spacetime, evading any problem with unitarity [28].

Motivated by the aforementioned researches we shall consider the problem of strictly static slices (without Killing horizons) in Einstein-Gauss-Bonnet gravity as well as the uniqueness of uncharged and charged black holes in this
theory. In our considerations we confine our attention to small curvature corrections, i.e., to the $O(\alpha)$-order, where $\alpha$ is a Gauss-Bonnet coupling constant.

The paper is organized as follows. In Sec.II we review the system under considerations. Sec.III will be devoted to the conditions which should be satisfied for strictly static slices in Einstein-Gauss-Bonnet gravity with $U(1)$-gauge field. In Sec.IV one considers the uniqueness theorem for both uncharged and charged black holes in the theory in question, restricting our attention to small curvature corrections. In Sec.V we conclude our investigations.

II. EINSTEIN-GAUSS-BONNET GRAVITY WITH $U(1)$ GAUGE FIELD

In this section we shall consider Einstein gravity with Gauss-Bonnet term and $U(1)$-gauge field. It is provided by the action

$$I = \kappa \int d^{n+1}x \sqrt{-g} \left[ R + \alpha \left( R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4 R_{\alpha\beta}R^{\alpha\beta} + R^2 \right) - F_{\mu\nu}F^{\mu\nu} \right],$$  \hspace{1cm} (1)

where $\alpha$ is the coupling constant of the Gauss-Bonnet term, regarded as the inverse of the string tension. It naturally arises as the next leading order of the $\alpha$-expansion of heterotic string theory and is positively defined \cite{28}. The dimension of this coefficient is (length)$^2$. In what follows we confine our attention to the case when $\alpha > 0$. On this account, the field equations obtained by variation of the action (1) imply

$$R_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu} \frac{T}{n-1} - \alpha \tilde{H}_{\mu\nu} - g_{\mu\nu} \frac{\alpha}{1-n} \left( R_{\sigma\rho\gamma\delta}R^{\sigma\rho\gamma\delta} - 4 R_{\gamma\delta}R^{\gamma\delta} + R^2 \right),$$  \hspace{1cm} (2)

$$\nabla_\mu F^{\mu\nu} = 0,$$  \hspace{1cm} (3)

where we have denoted by $\tilde{H}_{\mu\nu}$ the following expression:

$$\tilde{H}_{\mu\nu} = 2 R_{\rho}^{\alpha\beta\gamma} R_{\mu\alpha\beta\gamma} - 4 R_{\mu\alpha\beta} R_{\mu\rho\alpha\beta} - 4 R_{\rho\alpha\beta} R_{\mu\rho\alpha\beta} + 2 R R_{\mu\rho\alpha\beta}.$$  \hspace{1cm} (4)

On the other hand, the energy momentum tensor $T_{\mu\nu} = -\frac{\delta S}{\delta g^{\mu\nu}}$ of matter fields in question yields

$$T_{\alpha\beta}(F) = 2 F_{\alpha\gamma}F_{\beta}^{\gamma} - \frac{1}{2} g_{\alpha\beta} F_{\mu\nu}F^{\mu\nu}.$$  \hspace{1cm} (5)

We consider static spacetime in which the asymptotically timelike Killing vector field $k_\alpha = (\frac{2}{V})_\alpha$ orthogonal to the $n$-dimensional hypersurface $\Sigma$ of constant time, is defined. The line element of static spacetime subject to the asymptotically timelike Killing vector field $k_\alpha$ and $V^2 = -k_\mu k^\mu$ is provided by the following relation:

$$ds^2 = -V^2 dt^2 + g_{ij} dx^i dx^j,$$  \hspace{1cm} (6)

where $V$ and $g_{ij}$ are independent of the $t$-coordinate as the quantities defined on the hypersurface $\Sigma$ of constant $t$. Additionally, we suppose that on $\Sigma$ the electromagnetic potential will be of the form $A_0 = \psi \, dt$.

Taking the form of static metric into account, the corresponding equations of motion yield

$$V(n) \nabla_i (n) \nabla^i V = T_{00} + \frac{V T}{n-1} + 4 \alpha \frac{n-3}{n-1} (n) \nabla_i (n) \nabla_j V (n) G^{ij} - \frac{V}{n-1} \mathcal{L}_{GB},$$  \hspace{1cm} (7)

$$\nabla_i (n) \nabla^i \psi = \frac{1}{V(n)} \nabla_i \psi (n) \nabla^i V,$$  \hspace{1cm} (8)

$$(n+1) R_{ij} = T_{ij} - g_{ij} \frac{T}{n-1} - \alpha \left( 4 V^2 (n) \nabla_i (n) \nabla_m V (n) \nabla_j (n) \nabla^m V 
- \frac{4}{V^2} (n) \nabla_i (n) \nabla_j V (n) \nabla_m (n) \nabla^m V \right) - \tilde{H}_{ij} + g_{ij} \frac{\alpha}{n-1} \mathcal{L}_{GB},$$  \hspace{1cm} (9)

where in the above relations covariant derivative with respect to the metric tensor $g_{ij}$ is denoted by $(n)\nabla$, while $(n)G_{ij}(g)$ is the Einstein tensor defined on $n$-dimensional hypersurface $\Sigma$. 

Let us assume further that we take into account the asymptotically flat spacetime, i.e., the spacetime contains a data set \((\Sigma_{\text{end}}, g_{ij}, K_{ij})\) with gauge fields such that \(\Sigma_{\text{end}}\) is diffeomorphic to \(\mathbb{R}^n\) minus a ball and the following asymptotic conditions are fulfilled:

\[
|g_{ij} - \delta_{ij}| + r|\partial_a g_{ij}| + \ldots + r^k|\partial_{a_1 \ldots a_k} g_{ij}| + r |K_{ij}| + \ldots + r^k |\partial_{a_1 \ldots a_k} K_{ij}| \leq O\left(\frac{1}{r^{n-4}}\right),
\]

\[
|F_{\alpha\beta}| + r |\partial_a F_{\alpha\beta}| + \ldots + r^k |\partial_{a_1 \ldots a_k} F_{\alpha\beta}| \leq O\left(\frac{1}{r^{n-2}}\right).
\]

Consequently, under the above assumptions, there is a standard coordinates system in which we have the usual asymptotic decay properties. Namely, let us assume that each of the quantities \(V, \psi, \) and \(g_{ij}\), has the asymptotic expansion given by in the form as follows:

\[
V \simeq 1 - \frac{\mu}{r^{n-2}} + O\left(\frac{1}{r^{n-3}}\right),
\]

\[
\psi \simeq \frac{Q}{r^{n-2}} + O\left(\frac{1}{r^{n-4}}\right),
\]

\[
g_{ij} \simeq 1 + \frac{2}{(n-2)r^{n-2}} + O\left(\frac{1}{r^{n-1}}\right),
\]

where \(Q\) and \(\mu\) are constant and represent electric charge and the ADM mass, respectively.

### III. STRICTLY STATIC SPACETIME WITH MATTER

To begin with we examine the strictly stationary case, when in static spacetime under considerations one has no Killing horizons. Having in mind equation (7) we obtain the following:

\[
(n)\nabla^i \left((n)\nabla_i V - 4 \frac{n-3}{n-1} (n)\nabla_j V (n)G_{ij}\right) = -\frac{\alpha}{n-1} V (n)L_{GB} + \frac{2(n-2)}{V (n-1)} (n)\nabla_a \psi (n)\nabla^a \psi,
\]

where by \((n)L_{GB}\) we have denoted

\[
(n)L_{GB} = (n)R_{ijkl} (n)R^{ijkl} - 4 (n)R_{ij} (n)R^{ij} + (n)R^2
\]

\[
= (n)C_{ijkl} (n)C^{ijkl} - 4 \frac{n-3}{n-2} \left[(n)R_{ab} (n)R^{ab} - \frac{n}{4(n-1)} (n)R^2\right].
\]

In equation (16), by \((n)C_{ijkl}\) one denotes the \(n\)-dimensional Weyl tensor.

To proceed further, let us examine the limit of the above relation in asymptotically flat spacetime near the spatial infinity, i.e., when \(r \to \infty\). One commences with the limit behaviour of \(n\)-dimensional Ricci curvature tensor \((n)R_{ij}\). Namely, taking into account relation (7) and the asymptotic limits given by (12)-(14), one arrives at the following expression:

\[
(n)\nabla^i V (n)R_{ij} \leq O\left(\frac{x_j}{r^{2(n-1)}}\right) + O(\alpha).
\]

Next, the right-hand side of equation (15) will be provided by the limits of the forms

\[
(n)R_{ij} (n)R^{ij} \leq O\left(\frac{1}{r^{2(n-1)}}\right) + O(\alpha),
\]

\[
(n)R^2 \leq O\left(\frac{1}{r^{2(n-2)}}\right).
\]

Restricting our considerations to the case when slices are conformally flat, i.e., \((n)C_{ijkl} = 0\), one reaches to the expressions

\[
(n)L_{GB} \simeq O\left(\frac{1}{r^{2(n-1)}}\right),
\]

\[
(n)\nabla_a \psi (n)\nabla^a \psi \simeq O\left(\frac{1}{r^{2(n-1)}}\right).
\]
we arrive at the conclusion that the volume integral can be rewritten in terms of the surface integral at the spatial infinity. It can be readily verify that it implies

\[
\int_{\text{vol}} dV \, (n)\nabla^i \left( (n)\nabla_i V + \alpha \, \mathcal{O}\left( \frac{1}{r^{2n-1}} \right) \right) = \int_{S_\infty} dS^i \left( (n)\nabla_i V + \alpha \, \mathcal{O}\left( \frac{1}{r^{2n-1}} \right) \right). \tag{22}
\]

One can easily see that the integrands do not influence on the value of the integral in question and \((n)\nabla^i (n)\nabla_i V = 0\) which leads to the conclusion that \(V\) is constant and in the spacetime under consideration the ADM mass is equal to zero.

On this account it is customary to ask about the behaviour of the Ricci scalar curvature on the considered conformally flat hypersurfaces. The sign of the Ricci scalar tensor plays the crucial role in the rigid positive energy theorem \cite{29}. Returning to the exact form of it which can be provided by

\[
(n) R = \frac{2}{V}\nabla^i (n)\nabla_i V - \frac{2}{n-1} T - \alpha \left( \frac{n-3}{n-1} \right) \mathcal{L}_{\text{GB}}, \tag{23}
\]

one can reveal that

\[
(n) R = 2 \frac{(n)\nabla^i (n)\nabla_i V}{V^2} \left[ 1 - \alpha \, \frac{4n (n-3)}{(n-1)(n-2)} \frac{(n)\nabla^i \psi (n)\nabla_i \psi}{V^2} \right] + 4 \frac{\alpha}{n-2} \frac{n-3}{n-2} R_{ij} (n) R^{ij} + \mathcal{O}(\alpha^2). \tag{24}
\]

The second term is manifestly greater than zero, and because of the fact that we shall restrict our consideration to \(\mathcal{O}(\alpha)\)-order, one reveals that \((n) R \geq 0\). Taking the above discussion into account, we can assert that the following theorem holds:

**Theorem:**

Assume that one considers the asymptotically flat strictly static spacetime being the solution of Einstein-Gauss-Bonnet equations of motion with \(U(1)\)-gauge matter sector. Let us suppose further that the fall-off of the considered matter field in of the form given by the relations (12)-(14). Restricting our considerations to the case of static conformally flat slices of the spacetime in question, one states that up to the order of \(\mathcal{O}(\alpha)\) the spacetime is the Minkowski one, where \(\alpha\) is a coupling constant of the Gauss-Bonnet curvature correction.

**IV. BLACK HOLES IN GAUSS-BONNET GRAVITY**

In this section we shall consider static black hole spacetime in the theory under consideration. The properties of black hole solutions of Einstein-Gauss-Bonnet gravity were intensively examined from various points of view. The thermodynamics of such black holes was investigated in Refs.\cite{30}, the spherical symmetric null dust collapse or scalar field in Gauss-Bonnet gravity was elucidated in Refs.\cite{31}, while Vaidya type solutions were studied in \cite{32}. On the other hand, black hole solutions for Einstein-Gauss-Bonnet gravity were first achieved in \cite{28,33}, while for a charged case in Ref.\cite{34}. The generalization of the above researches to the Gauss-Bonnet theory with a cosmological constant were presented in \cite{35}, while the spacetime structure of \(n\)-dimensional static solution was elaborated in Refs.\cite{36}. The above investigations were strengthened to the case of dilatonic Einstein-Gauss-Bonnet theory \cite{37}, where asymptotically flat and asymptotically AdS black holes in various dimensions were studied. For a complete history of black holes in higher order gravity theories see, e.g., \cite{38} and references therein.

To proceed further, let us assume that in the spacetime in question one can find appropriate coordinate system in which the asymptotic limits determined by the relations (12)-(14) will be provided. Moreover, we suppose that the black hole event horizon is a Killing horizon located at the level surface \(V = 0\). We shall consider the non-degenerate black hole Killing horizon case. In the next step, one examines the conformal transformation which implies

\[
\tilde{g}_{ij}^{\pm} = \Omega_\pm^2 \, g_{ij}. \tag{25}
\]

The conformal factor is provided by

\[
\Omega_\pm = \left[ \left( \frac{1 \pm V}{2} \right)^2 - \frac{C^2}{4} \psi^2 \right]^{1/2}, \tag{26}
\]

where \(C = (2(n-2)/(n-1))^{1/2}\). In the next step the standard procedure used in the proofs of the black hole uniqueness theorem can be implemented. Namely, we obtain two manifolds \((\Sigma_-, \tilde{g}_{ij}^-)\) and \((\Sigma_+, \tilde{g}_{ij}^+)\). Next, we paste \((\Sigma_\pm, \tilde{g}_{ij}^\pm)\) along the surface \(V = 0\). This in turn enables one to construct a complete regular hypersurface \(\Sigma = \Sigma_- \cup \Sigma_+ \cup \{p\}\). On
the other hand, if \((\Sigma, \hat{g}_{ij})\) is an asymptotically flat solution of the underlying equations of motion with non-degenerate black hole event horizon, then our next step will be to study the total gravitational mass on hyperspace \(\Sigma\). In order to do it we shall inspect the Ricci scalar curvature given on the hypersurface in question. It leads us to two manifolds \((\Sigma^\pm, \hat{g}^\pm_{ij})\).

The Ricci scalar curvature tensor \((n)\hat{R}\) on the adequate hypersurfaces \(\Sigma^\pm\) is of the form

\[
\frac{\Omega_+^4 (n)\hat{R}}{[\left(\frac{1+V}{2}\right)^2 - \frac{C^2}{\psi^2}]}^{2\mp 2} = \frac{2}{V^4} \left[ \frac{1}{4} \left( 1 - V^2 - C^2 \psi^2 \right) (n)\nabla^i\psi + \frac{\psi V}{2} (n)\nabla^i V \right]^2 + \alpha \left[ \left( \frac{1+V}{2} \right)^2 - \frac{C^2}{4} \psi^2 \right]^2 \Theta, \tag{27}
\]

where we have denoted by \(\Theta\) the following:

\[
\Theta = \frac{n - 3}{(n-1)(n-2)} \left( 4 (n-1) (n)R_{ij} - n (n)R^2 \right) + (n)C_{ijkl}^2. \tag{28}
\]

In what follows we shall discuss the two cases of the black hole solutions in Einsein-Gauss-Bonnet gravity, namely uncharged and charged ones.

### A. Uncharged black hole solution

In the case in question one has \(\psi = 0\) in the above relations. From equation \((30)\) we get that the crucial point is the sign of it. Because of the fact that \((n)R_{ij}^2\) and \((n)C_{ijkl}^2\) are manifestly greater than zero, we focus our attention on the term \(-n (n)R^2\). From Ref. [22] one gets that it is equal to

\[
(n)\hat{R} = 4 \alpha \frac{n - 3}{n - 2} (n)R_{ij}^2 + \mathcal{O}(\alpha^2). \tag{29}
\]

Having in mind the exact form of relation \((27)\), we conclude that \(-n (n)R^2\) is of order \(\mathcal{O}(\alpha^2)\) and up to the \(\alpha\)-order can be neglected. Hence, it reveals that \((n)\hat{R}\) is greater or equal to zero and the positive energy theorem \([29]\) can be implemented in our proof. Following the method presented in the next subsection one can prove the uniqueness of uncharged asymptotically flat black holes in Gauss-Bonnet gravity up to \(\mathcal{O}(\alpha)\) order. The above arguments enable one to formulate the conclusion:

**Theorem:**

Let us consider a static solution to Gauss-Bonnet gravity equation of motion with an asymptotically timelike Killing vector field \(k_\mu\). Suppose that the manifold in question consists of a connected and simply connected spacelike hypersurface \(\Sigma\) to which \(k_\mu\) is orthogonal. Then, up to \(\mathcal{O}(\alpha)\), where \(\alpha\) is a Gauss-Bonnet coupling constant, there exist a neighborhood of \(\Sigma\) which is diffeomorphic to an open set of \(n + 1\)-dimensional Schwarzschild-Tangherlini black hole solution.

### B. Charged black hole solution

In the charged black hole case the situation is far much more involved. Having in mind the positive mass theorem, one has that the hypersurface \(\Sigma\) must be flat and additionally the relation provided by

\[
(V^2 + C^2 \psi^2 - 1) (n)\nabla^i \psi = 2 \psi V (n)\nabla^i V, \tag{30}
\]

should be satisfied. The above equation yields that the level surface \(V\) and the potential of \(U(1)\)-gauge field \(\psi\) coincide, i.e., the physical Cauchy hypersurface \(\Sigma\) is conformally flat.

As far as the term bounded with \(\Theta\) is concerned, after tedious computations one obtains

\[
\Theta = (n)C_{ijkl}^2 + 4 \frac{n - 1}{V^2} \left( (n)\nabla_i (n)\nabla_j V - 2 \frac{(n)\nabla_i \psi (n)\nabla_j \psi}{V} \right)^2 + \frac{n - 3}{(n-1)(n-2)} W(n) \left( \frac{(n)\nabla_i \psi (n)\nabla_j \psi}{V} \right)^2 + \mathcal{O}(\alpha), \tag{31}
\]

where we have written

\[
W(n) = \frac{16(n-2)^2 + n(n-1)^2 - 2(n-1)(n-2)(n-3) - 3n(n-3)^2}{(n-1)^2}. \tag{32}
\]
A close inspection of the above polynomial reveals that for the spacetime dimensions equal to 5, 6, 7 its value is greater than zero but when $n \geq 8$ is less than zero.

As in the previous case we confine our attention to the $O(\alpha)$-order and use the positive energy theorem. It is clear from equation (27) that if one takes into account the small curvature corrections, the Ricci scalar curvature tensor $\ddot{R}(\tilde{g})$ on the hypersurface $\Sigma$ is equal to non-negative value. Moreover, using the asymptotic behaviour of $\tilde{g}_{ij+}$ on $\Sigma_+$, i.e., $\tilde{g}_{ij+} = \delta_{ij} + O\left(\frac{1}{r^{n-1}}\right)$, one concludes that the total mass on the hypersurface $\Sigma$ vanishes. Consequently, having in mind the positive mass theorem [29], the manifold $\Sigma$ is isometric to a flat manifold. This fact enables us to rewrite the metric tensor $g_{ij}$ in a conformally flat form provided by the following expression [14]

$$g_{ij} = U^{-\frac{1}{n}} \delta_{ij}.$$  

(33)

In the above relation we have defined a smooth function $U = \frac{2}{\sqrt{1 + n r^{-2}}} \, , \, \Omega$ on the $n$-dimensional Euclidean manifold $\nabla_i \nabla^i U = 0$, where $\nabla$ is the connection on a flat manifold. Consequently one can endorse for the metric $\delta_{ij}$ in the flat base space the line element provided by

$$\delta_{ij} dx^i dx^j = \hat{\rho}^2 d\Omega^2 + \hat{h}_{AB} dx^a dx^B.$$  

(34)

To begin with we shall examine the case of the single black hole event horizon. The event horizon is located at $U = 2$ and it can be shown that the embedding of $\mathcal{H}$ into the Euclidean $n$-dimensional space is totally umbilical [30]. Moreover, the embedding must be hyperspherical, which means that each of the connected components of the horizon $\mathcal{H}$ is a geometric sphere of a certain radius determined by the value of $\rho \mid \mathcal{H}$, where $\rho$ is the coordinate introduced on the hypersurface $\Sigma$ by the line element of the form

$$g_{ij} dx^i dx^j = \rho^2 d\Omega^2 + h_{AB} dx^A dx^B.$$  

Without loss of generality, one connected component of the horizon can be always located at $r = r_0$ surface. Thus, we have to do with a boundary value problem for the Laplace equation on the base space $\Omega = E^n / B^n$ with the Dirichlet boundary condition $U \mid \mathcal{H} = 2$ and the asymptotic decay condition $U = 1 + O\left(\frac{1}{r^{n-2}}\right)$. Let us suppose further, that $U_1$ and $U_2$ be two solutions subject to the boundary value problem. Using the Green identity and integration over the volume element one arrives at the following expression:

$$\left( \int_{r=\infty} - \int_{\mathcal{H}} \right) (U_1 - U_2) \partial_r (U_1 - U_2) dS = \int_{\Omega} \nabla (U_1 - U_2) |^2 d\Omega.$$  

(35)

The boundary conditions make the left-hand side of the above relation vanish and we conclude that two solutions must be identical.

In order to get rid of the assumption about a single event horizon, one should elaborate the evolution level surface in Euclidean space [14,13]. The Gauss equation in Euclidean space allows us to receive the evolution relations concerning the behaviour of shear $\sigma_{AB}$. Then, the harmonicity of $U$ function leads us to the conclusion that

$$\sigma_{AB} = 0, \quad \dot{U} A \rho = 0, \quad \dot{U} A k = 0,$$  

(36)

where $\dot{D}_A$ denotes the covariant derivative on each level set of $V$, $k_{AB}$ is the second fundamental form of the level set. Consequently, this effects that each level surface of the function $U$ is totally umbilic and therefore spherically symmetric.

It worth mentioning that the above results are local [14,15], because of the fact that we have examined only the region without saddle points of the harmonic function in question. As was claimed in the above references the global result can be achieved by taking into account the assumption about analyticity. Summing it all up, we arrive at the main statement of this subsection:

**Theorem:**

Consider a static solution to $(n+1)$-dimensional Gauss-Bonnet gravity with an Abelian $U(1)$-gauge field equations of motion. Suppose that there is defined an asymptotically timelike Killing vector field $k_i$, orthogonal to the simply connected spacelike hypersurface $\Sigma$. Then, up to the $O(\alpha)$-order, where $\alpha$ is a Gauss-Bonnet coupling constant, it turns out that there is a neighborhood of the hypersurface in question which is diffeomorphic to an open set of $(n + 1)$-dimensional Reissner-Nordström non-extreme black hole solution.
V. CONCLUSIONS

In our paper we have elaborated the strictly static asymptotically flat spacetimes in Einstein-Gauss-Bonnet gravity theory with $U(1)$-gauge matter field. It was found that up to the small curvature corrections and the adequate decay property of the matter field, static conformally flat slices of the manifold in question were of Minkowski origin. In the same limit we analyzed static black hole solutions, both uncharged and charged ones. It was revealed that in the considered spacetime there were neighborhoods which were diffeomorphic to $(n+1)$-dimensional Schwarzschild-Tangherlini and non-extreme Reissner-Nordström black hole spacetimes.

Of course there are many remaining issues like, e.g, getting rid of the $O(\alpha)$-order limit or to consider another type of black hole, stationary axisymmetric solution, in the theory in question. We hope to consider these problems elsewhere.

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