Adaptive Friction Compensation Control for a Roll-Isolated Strapdown Inertial Navigation System

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Abstract. In order to meet the requirements of miniaturization and low cost of rolling missiles, the roll-isolated strapdown inertial navigation systems began to be applied to such weapon systems. This paper proposes an adaptive control approach based on LuGre friction torque model to eliminate the influence of the nonlinear characteristics of the friction torque of the roll-isolated platform at low speed. Two nonlinear state observers are used to estimate the unknown state variables in LuGre model, and the parameters of the model are adaptively estimated. The global asymptotic stability of the system is proved by Lyapunov method. The simulation results show that compared with the traditional control approaches, this approach improves the control precision and low-speed performance of the roll-isolated platform, and can meet the performance requirements of the roll-isolated strapdown inertial navigation system.

1. Introduction

A rolling missile is a self-rotating flight vehicle whose rotational speed can range from several to more than a dozen revolutions per second [1]. Due to its characteristic of self-rotating, the use of strapdown inertial navigation systems on such weapon is severely limited. Angular velocity sensors generally do not have high dynamic measurement ranges. Some laser or fiber optic gyroscopes have sufficiently large ranges, while such characteristics make their resolution lower [2]. Considering the cost constraint, it is uneconomical to directly apply the strapdown inertial navigation system to the rolling missile, so the strapdown inertial navigation system can be installed on a stable platform that is roll-isolated [3]. Based on the three-axis stable inertial platform, the system eliminates the stability axes in the direction of yaw and pitch, and only retains in the roll direction. Its characteristics are closer to the strapdown inertial navigation system, so it is called roll-isolated strapdown inertial navigation system [4].

Since the rolling missile has a wide range of rotational speed during the whole flight, and has a fast acceleration process in the active segment, the platform stable loop is required to have high dynamic accuracy, sufficient capability for dynamic response and excellent anti-interference ability. Due to these characteristics of the platform stability loop, it is difficult to obtain better control effects by the method based on the classical control theory. In recent years, some scholars have proposed platform stability loop control methods based on sliding mode variable structure control [5] and optimal auto disturbance rejection control [6]. In order to reduce the cost and volume of the system, an DC servo motor is used to directly drive the roll-isolated platform. The rotation speed of the rolling missile is much lower than that of a servo motor, so the driving motor of the platform will always work in its
low speed section. When the motor is running at low speed, the nonlinear characteristic of the friction torque is obvious, which has a great influence on the static and dynamic performance of the system. It mainly manifested as the limit cycle oscillation and low-speed crawling at steady state \[7\]. In this paper, based on the LuGre friction torque model \[8\], an adaptive control approach is proposed to compensate the nonlinear friction torque during the working process of the platform. It is verified that the control method can guarantee the stability and asymptotic convergence of the system, and the control effect is simulated and analyzed.

2. Model Description
A simplified model of the platform control system is shown in figure 1. \(L\) is the inductance for the motor armature winding, \(R_o\) is the resistance of the armature winding, \(K_m\) is the moment coefficient, \(K_e\) is the back EMF coefficient, \(J\) is the total moment of inertia on the motor shaft, \(r\) is the commanding rotation angle, \(\theta\) is the actual angle of rotation of the platform, \(U\) is the armature voltage of the motor, \(\omega\) is the torque provided by the motor, \(M_f\) is the friction torque of the system, \(F(s)\) is the transfer function of the controller.

The mathematical model of the system can be described as follow:

\[U = L \dot{i} + R_o \dot{i} + K_e \dot{\theta}\]  
\[J \ddot{\theta} = K_m \dot{\theta} - M_f\]  

\[0\]

\[F(s)\]

\[U\]

\[\frac{1}{Ls + R_o}\]

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\[ a \ddot{\theta} = U + b \dot{\theta} - e_0 \dot{\theta} - e_1 z + e_2 |\dot{\theta}| g(\dot{\theta})^{-1} z \]  
(5)

where \( a = JR_0 K_{m1}^{-1} \), \( b = - K_e \), \( e_0 = (\sigma_1 + \mu b) R_0 K_{m1}^{-1} \), \( e_1 = \sigma_0 R_0 K_{m1}^{-1} \), \( e_2 = \sigma_1 R_0 K_{m1}^{-1} \). The parameters \( a \) and \( b \) are only related to the characteristics of the platform and motor. Selecting \( e_0, e_1 \) and \( e_2 \), which are related to the friction torque, as the unknown adaptive parameters. Obviously, \( a, e_0, e_1 \) and \( e_2 \) are positive, while \( b \) is negative.

The roll-isolated platform is a position control system with maximum speed limit, and the position tracking error \( e \) is described as follow:

\[ e = \theta - r \]  
(6)

Taking Derivatives twice on the both sides of (6):

\[ \ddot{e} = \ddot{\theta} - \dddot{r} \]  
(7)

Then equation (5) can be expressed as:

\[ a \ddot{e} = U + b \dot{\theta} - e_0 \dot{\theta} - e_1 z + e_2 |\dot{\theta}| g(\dot{\theta})^{-1} z - a \dddot{r} \]  
(8)

Since the state variables introduced by LuGre friction model are unknown and unmeasurable, two state observers are designed to estimate \( z \). The equations of the state observer are shown as follow:

\[ \frac{d\hat{z}_0}{dt} = \dot{\hat{\theta}} |\dot{\theta}| g(\dot{\theta})^{-1} \hat{z}_0 + f_0 \]  
(9)

\[ \frac{d\hat{z}_1}{dt} = \dot{\hat{\theta}} |\dot{\theta}| g(\dot{\theta})^{-1} \hat{z}_1 + f_1 \]  
(10)

where \( \hat{z}_0 \) and \( \hat{z}_1 \) are the estimates of \( z \) , \( f_0 \) and \( f_1 \) are the dynamic terms of the observer. Making \( \hat{z}_0 = z_0 - \hat{z}_0 \), \( \hat{z}_1 = z_1 - \hat{z}_1 \), then the estimation errors of the observer can be written as:

\[ \frac{d\hat{z}_0}{dt} = -|\dot{\theta}| g(\dot{\theta})^{-1} z_0 - f_0 \]  
(11)

\[ \frac{d\hat{z}_1}{dt} = -|\dot{\theta}| g(\dot{\theta})^{-1} z_1 - f_1 \]  
(12)

Using \( \hat{e}_0, \hat{e}_1, \hat{e}_2 \) as the estimated value of the unknown parameter \( e_0, e_1, e_2 \) respectively, the control rate is established as follow:

\[ U = -c \dot{e} - b \dot{\theta} + \hat{e}_0 \dot{\theta} + \hat{e}_1 \hat{z}_0 - \hat{e}_1 z_0 + \hat{e}_2 |\dot{\theta}| g(\dot{\theta})^{-1} \hat{z}_1 + a \dddot{r} \]  
(13)

where \( c \) is a positive design constant. Equation (8) can be written as follow, by adding equation (13) into it:

\[ a \ddot{e} = -c \dot{e} - e_0 \dot{\theta} - e_1 \hat{z}_0 - e_1 z_0 + e_2 |\dot{\theta}| g(\dot{\theta})^{-1} \hat{z}_1 + e_2 \hat{z}_1 \]  
(14)

where \( \ddot{e}_0, \ddot{e}_1, \ddot{e}_2 \) are the estimated errors of the unknown parameters \( e_0, e_1, e_2 \) respectively, which are defined as \( \ddot{e}_0 = e_0 - \hat{e}_0 \), \( \ddot{e}_1 = e_1 - \hat{e}_1 \), \( \ddot{e}_2 = e_2 - \hat{e}_2 \).

The chosen Lyapunov function is as follow:

\[ V(t) = \frac{1}{2} a \dot{e}^2 + \frac{1}{2} e_0^2 \hat{z}_0^2 + \frac{1}{2} e_1^2 \hat{z}_1^2 + \frac{1}{2} e_2^2 \hat{z}_1^2 + \frac{1}{2} \alpha_0 \dot{\theta}^2 + \frac{1}{2} \alpha_1 \ddot{\theta}^2 + \frac{1}{2} \alpha_2 \dddot{\theta}^2 \]  
(15)

where \( \alpha_0, \alpha_1, \alpha_2 \) are positive design constants. The derivative of the Lyapunov function is:
\[ V(t) = -c e^2 - \dot{e}_0 \left( e_0 e_0^{-1} + \dot{\theta} e \right) - \dot{z}_0 \left( e_0 e_0^{-1} + \dot{z}_0 e \right) - \dot{e}_1 \left( e_0 e_0^{-1} \right) - e_2 \left( \dot{\theta} \right) g(\dot{\theta})^{-1} \dot{z}_0 e - e_1 \dot{z}_0 \left( f_0 + e \right) \]

\[ -e_2 \dot{z}_0 \left( f_1 \dot{\theta} g(\dot{\theta})^{-1} e \right) - e_1 \dot{\theta} g(\dot{\theta})^{-1} z_0 e - e_2 \dot{\theta} g(\dot{\theta})^{-1} z_0 e \]

Equation (16) can be expressed as follow:

\[ V(t) = -c e^2 - e_1 \dot{\theta} g(\dot{\theta})^{-1} z_0 e - e_2 \dot{\theta} g(\dot{\theta})^{-1} z_0 e \]

As \( c, e_1, e_2 \) and \( g(\dot{\theta}) \) are positive, \( V(t) \leq 0 \), according to the Lyapunov stability theory, the designed controller is globally asymptotically stable.

4. Simulation Analysis

In order to verify the effect of the adaptive compensation control based on LuGre friction torque model for the roll-isolated strapdown inertial navigation system, the static parameters of the LuGre model of actual platform are identified. Table 1 lists the identification results.

| Parameter | Value | Unit |
|-----------|-------|------|
| \( \sigma_0 \) | 30 | mN·m |
| \( \sigma_1 \) | 0.3 | mN·m |
| \( \mu_b \) | 0.03 | mN·m·s/rad |
| \( M_c \) | 2.8 | mN·m |
| \( M_b \) | 3.4 | mN·m |
| \( \dot{\theta} \) | 0.1 | mrad/s |

The parameters of the driving motor of the platform are presented in table 2.

| Parameter | Value | Unit |
|-----------|-------|------|
| \( L \) | 3.09 | mH |
| \( R_a \) | 28.6 | \( \Omega \) |
| \( K_m \) | 23.8 | mN·m/A |
| \( K_e \) | 23.8 | mV·s/rad |
| \( J \) | 5.1 | g·cm² |

Figure 2 shows the position tracking errors of the stable response curve of the roll-isolated platform to a sinusoidal command with the amplitude of 0.314 rad and frequency of 15 Hz, using a PD controller and an adaptive friction compensation controller respectively. The parameters of the PD controller are \( k_p = 400 \), \( k_d = 10 \), while the parameters of the adaptive friction compensation controller are \( c = 10 \), \( \alpha_0 = \alpha_1 = \alpha_2 = 1 \). It is obvious that compared with the PD controller, the motor of the roll-isolated platform can obtain higher precision and better smoothness while working at a low speed, controlled by the adaptive friction compensation controller.
In order to examine the ability of the roll-isolated platform with the adaptive friction compensation controller to respond to a rapid increase of velocity, which may happen when a rolling missile launches, different values of $c$ are used and the tracking errors of velocity are shown in figure 3. It can be seen that compared with the controller with $c = 5$, larger value of $c$ can reduce the amplitude of velocity tracking error, but it may lead to a convergent oscillation with high frequency, which is unexpected.

Figure 3. Velocity tracking errors of slope response.

Figure 4 shows when the parameters of the LuGre model increase by 20%, the position tracking error of the same control command shown in figure 2, using the same adaptive friction compensation controller in it simultaneously. The result shows that the change of model parameter has little impact on position tracking error, because of the adaptive compensation effect of the controller.

Figure 4. Position tracking errors of different parameters of LuGre model.

5. Conclusion
In this paper, the dynamic model of the platform on the roll-isolated strapdown inertial navigation system is established based on LuGre friction torque model. Two state observers are used to estimate the unknown state variables in LuGre model, and an adaptive friction compensation control approach is proposed. Then based on the Lyapunov stability theory, the global asymptotic stability of the closed-loop system is proved. After parameter identification of the LuGre model on the actual roll-isolated platform system, the simulation research is carried out. The simulation results show that the
adaptive friction compensation controller effectively improves the platform's low-speed running stability and tracking accuracy, and can meet the performance requirements of the roll-isolated strapdown inertial navigation systems on the rolling missiles.

6. References

[1] Shi Y, Wang B, Dong M and Gao Z 2012 *Advanced Materials Research* **383** 4115-4120

[2] Zhou Q, Qin Y and Zhao C 2009 *Journal of Chinese Inertial Technology* **17** 383-387

[3] Qian X, Lin R and Zhao Y 2006 *Flight Mechanics of Missile* (Beijing: Beijing Institute of Technology Press) p 107

[4] Shi R, Guo Z, Yang Y, Chai J and Gao W 2017 *Aerospace Shanghai* **34** 161-168

[5] Yuri B S 1995 *Journal of Guidance, Control, and Dynamics* **18** 773-781

[6] Zhou Q, Qin Y and Yang P 2009 *Measurement & Control Technology* **28** 95-98+100

[7] Armstrong H B, Dupont P and Wit C C 1994 *Automatica* **30** 1083-1138

[8] Liu Q, Hu H and Er L 2002 *Electric Drive* **32** 10-13

[9] Wit C C, Olsson H, Astrom K J and Lischinsky P 1995 *IEEE Transactions on Automatic Control* **40** 419-425