Comprehensive study of mass modifications of light mesons in nuclear matter in the three-flavor extended Linear Sigma Model

Daiki Suenaga\textsuperscript{1,}\textsuperscript{*} and Phillip Lakaschus\textsuperscript{2,}\textsuperscript{1}

\textsuperscript{1}Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
\textsuperscript{2}Institute for theoretical physics, Max-von-Laue Str. 1, D-60438 Frankfurt am Main, Germany

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We study mass modifications of scalar, pseudo-scalar, vector, and axial-vector mesons in nuclear matter comprehensively using the three-flavor extended Linear Sigma Model (eLSM) and the two-flavor Parity Doublet Model (PDM). Nuclear matter is constructed by the one-loop approximation of the nucleon together with the meson mean fields. Correspondingly, we include one-loop corrections by the nucleons as well as the meson mean fields to define the meson masses in medium. As a result, we find all spin-0 meson masses except the pion, kaon, and the lightest scalar-isoscalar ones decrease at finite baryon density. The mass reduction of the $\eta'$ meson in nuclear matter supports the possibility of the formation of $\eta'$ mesic nuclei. For spin-1 mesons, masses of all axial-vector mesons decrease in medium, whereas density dependences of $\rho$ and $\omega$ meson masses sufficiently depend on the value of chiral invariant mass ($M_0$). Also, our results suggest $M_0 \approx 0.8\,\text{GeV}$ is preferable.

I. INTRODUCTION

One of the most important phenomena of Quantum Chromodynamics (QCD) is the spontaneous breakdown of chiral symmetry. This effect is essential to explain the masses of light mesons as well as the interactions among them in the vacuum [1, 2]. In nuclear matter, on the other hand, chiral symmetry is believed to tend to restore, which is expected to provide significant effects to the modifications of light meson properties in medium. Therefore, investigating meson properties such as masses gives us clues to a better understanding of the partial (incomplete) restoration of chiral symmetry (see Ref. [3, 4] for reviews and references therein).

While the small masses of pion and kaon can be understood by the spontaneous breakdown of chiral symmetry well, the large mass of $\eta'$ meson is mainly explained by the $U(1)\,A$ axial anomaly effect [5]. The $U(1)\,A$ anomaly is related to the existence of the instanton which can play an important role in the color confinement [6]. When we access to finite baryon density, some previous studies suggest the strength of $U(1)\,A$ anomaly can be changed, but still it is under discussion whether the magnitude of the anomaly is strengthened or weakened [7–9]. In association with the change of $U(1)\,A$ anomaly in nuclear matter, a mass reduction of the $\eta'$ meson leads to the possibility of the formation of $\eta'$ mesic nuclei as well [10–14]. Thus, this work is expected to give one suggestion to this issue.

Experimentally, the spectroscopy of pionic atom experiment at GSI was performed to observe the partial restoration of chiral symmetry in nuclear matter, whose result suggests a reduction of the chiral order parameter: $f_+^{\pi}(p_0)/f_0^{\pi} \approx 0.64$ at normal nuclear density $p_B = p_0$ [15]. Also, the fixed-target experiments for vector meson masses modifications in nuclei at J-PARC [16] and at Jefferson Laboratory [17] were operated. In these experiments, due to a complexity by a broadening of vector mesons, the results are still under discussion. Then, another experiment called E16 experiment is planned at J-PARC. Besides, the $\eta'$ mesic nuclei experiments at GSI [18–20] and at the University of Bonn [21–24], and $pp \rightarrow pp\eta'$ reaction experiment at the COSY accelerator complex [25] have been operated, regarding the investigation of the change of $U(1)\,A$ axial anomaly in nuclear matter.

Another $\eta'$ mesic nuclei experiments is ongoing at SPring-8 [26] (see Ref. [27] for a review and references therein).

In the present work, we study the mass modifications of light scalar, pseudo-scalar, vector, and axial-vector mesons in nuclear matter comprehensively to provide useful information of the partial restoration of chiral symmetry and the change of $U(1)\,A$ anomaly in medium to the existing and/or forthcoming experiments. For this purpose, we employ the three-flavor extended Linear Sigma Model (eLSM) established in Ref. [32], in which vector and axial-vector mesons are incorporated in addition to the ordinal scalar and pseudo-scalar mesons respecting global chiral symmetry and scale invariance [28–34]. The three-flavor version of the eLSM [32] could successfully reproduced the meson properties in the vacuum such as masses and decay widths \textsuperscript{1}, which is expected to be a powerful tool to examine the masses of light mesons in nuclear matter as well.

In this study, the nucleons are introduced by the two-flavor Parity Doublet Model (PDM) [38–42], and nuclear matter is constructed by the one-loop approximation of the nucleon together with the meson mean fields [43–46] obtained from the eLSM. In our approach, not only the

\textsuperscript{*}suenaga@mail.ccnu.edu.cn
\textsuperscript{1}lakaschus@th.physik.uni-frankfurt.de

\textsuperscript{1}Other extensions of the Linear Sigma Model can be possible by including tetraquark states [35–37], for example.
vacuum properties of nucleons but also the nuclear matter properties such as the saturation density, the binding energy per a nucleon, and the incompressibility are successfully reproduced. The PDM contains two types of the nucleons by employing the mirror assignment, which allows us to construct a nucleon mass term without violating the chiral symmetry. The PDM predicts the existence of a nucleon mass that does not originate from chiral symmetry breaking, which is the so-called chiral invariant mass \( M_0 \). The existence of \( M_0 \) is also suggested by lattice calculations in the context of a parity doubling, however, its precise value is still under discussion [47–49]. Therefore, our work is able to give information of the value of \( M_0 \) as well.

This paper is organized as follows. In Sec. II, the three-flavor eLSM is introduced and the determined model parameters are shown provided by Ref. [32]. In Sec. III, we show the two-flavor PDM and construct nuclear matter by combining the eLSM and the PDM. In Sec. IV, the remaining parameters are determined and numerical results of the density dependence of the meson masses are presented. In Sec. V, we conclude our work.

## II. EXTENDED LINEAR SIGMA MODEL (ELSM)

To study mesons in nuclear matter comprehensively, we employ the three-flavor eLSM in the present analysis for describing the light mesons. This model was established in Ref. [32], and successfully reproduced the meson properties in the vacuum such as masses and decay widths. In this section, we introduce the eLSM and show determined parameters by this reference.

The Lagrangian of the eLSM maintains a scale invariance except for the current quark mass effects, in which a dilaton is responsible for the violation of scale invariance of QCD, which is given by

\[
\mathcal{L}_{\text{ELSM}} = \mathcal{L}_{\text{diil}} + \mathcal{L}_{\text{int}} \left[ (D_{\mu}\Phi)^4 (D^\mu\Phi) \right] - m_0^2 \left( \frac{G}{G_0} \right)^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda_1 \left( \text{Tr}[\Phi^\dagger \Phi] \right)^2 - \lambda_2 \text{Tr}[\Phi^\dagger \Phi] + \text{Tr}[H(\Phi^\dagger + \Phi)] + \frac{1}{4} \text{Tr}\left[ L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} \right] + \text{Tr}\left[ \left( \frac{m_1^2}{2} \left( \frac{G}{G_0} \right)^2 + \Delta \right) (L^2_{\mu} + R^2_{\mu}) \right] + c_1 \left( \text{det} \Phi - \det \Phi^* \right)^2 + i \frac{g_2}{2} \left( \text{Tr}[L_{\mu\nu}[L^\mu, L^\nu]] + \text{Tr}[R_{\mu\nu}[R^\mu, R^\nu]] \right) + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[L^2_{\mu} + R^2_{\mu}] + h_2 \text{Tr}[L^2_{\mu} \Phi^\dagger \Phi + L^2_{\mu} \Phi^\dagger \Phi + 2h_3 \text{Tr}[L_{\mu} \Phi R^\mu \Phi^\dagger]] + g_3 \left( \text{Tr}[L_{\mu} L_{\nu} L^\mu L^\nu] + \text{Tr}[R_{\mu} R_{\nu} R^\mu R^\nu] \right) + g_4 \left( \text{Tr}[L_{\mu} L_{\nu} R^\mu L^\nu] + \text{Tr}[R_{\mu} R^\mu R_{\nu} R^\nu] \right) + g_5 \text{Tr}[L_{\mu} L^\nu] \text{Tr}[R_{\nu} R^\nu] + g_6 \left( \text{Tr}[L_{\mu} L^\nu] \text{Tr}[L_{\nu} L^\nu] + \text{Tr}[R_{\mu} R^\mu] \text{Tr}[R_{\nu} R^\nu] \right),
\]

where the meson nonets \( \Phi, L_{\mu}, \) and \( R_{\mu} \) are

\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\sigma_N + a_0^0 + i(a_N + \pi^0)}{\sqrt{2}} & a_1^+ & K_0^+ + iK^+
\frac{a_1^- + i\pi^-}{\sqrt{2}} & \frac{\sigma_N - a_0^0 + i(a_N - \pi^0)}{\sqrt{2}} & \frac{K_0^- + iK^-}{\sqrt{2}}
K_0^- + iK^- & \frac{K_0^+ + iK^+}{\sqrt{2}} & \frac{\sigma_S + i\eta_S}{\sqrt{2}}
\end{pmatrix},
\]

\[
L_{\mu} = \frac{\sqrt{2}}{2} \begin{pmatrix}
\frac{\omega_N + a_0^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K^+
\frac{\rho^- + a_1^-}{\sqrt{2}} & \frac{\omega_N - a_0^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0
K^{*-} + K_1^- & \frac{K^{*0} - K_1^0}{\sqrt{2}} & \frac{\omega_S + f_1S}{\sqrt{2}}
\end{pmatrix}_\mu,
\]

\[
R_{\mu} = \frac{\sqrt{2}}{2} \begin{pmatrix}
\frac{\omega_N + a_0^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+
\frac{\rho^- - a_1^-}{\sqrt{2}} & \frac{\omega_N - a_0^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0
K^{*-} - K_1^- & \frac{K^{*0} - K_1^0}{\sqrt{2}} & \frac{\omega_S - f_1S}{\sqrt{2}}
\end{pmatrix}_\mu.
\]

\( G \) is the dilaton field and its kinetic term and self-
interaction terms are included in $\mathcal{L}_{\text{dir}}$ \footnote{Explicitly, $\mathcal{L}_{\text{dir}}$ is of the form \cite{50-55}
\[\mathcal{L}_{\text{dir}} = \frac{1}{2} \partial_\mu G \partial^\mu G - \frac{1}{4} \frac{m_G^2}{G_0} \left( G^A \ln \frac{G}{G_0} - G^A \right). \]}

The chiral transformation laws for $\Phi$, $L_\mu$, $R_\mu$ are

\[ \Phi \rightarrow g_L \Phi g^L_R, \quad L_\mu \rightarrow g_L L_\mu g^L_R, \quad R_\mu \rightarrow g_R R_\mu g^R_R, \] \tag{4}

where $g_L$ ($g_R$) is an element of $U(3)_L$ ($U(3)_R$) chiral group. $L_{\mu\nu}$ and $R_{\mu\nu}$ in Eq. (1) are the field strengths

\[ L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu, \quad R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu, \] \tag{5}

employed as the kinetic term for removing unphysical degrees of freedom properly and making the term chirally invariant. The covariant derivative is $D_\mu \Phi = \partial_\mu \Phi - ig_1 (L_\mu \Phi - R_\mu)$ \footnote{In fact, $L_\mu$ and $R_\mu$ are not “gauge fields” as can be understood}. The matrices $H$ and $\Delta$ are responsible for the explicit breaking of chiral symmetry which take forms of $H = \text{diag}(h_N^0, h_N^0, h_S^0)$ and $\Delta = \text{diag}(\delta_N, \delta_N, \delta_S)$, respectively.

In the following analysis, we will regard the dilaton as the $f_0(1710)$ whose mass is larger than the other light meson ones so that the dilaton dynamics will be ignored below, i.e., the dilaton field in Eq. (1) is simply replaced by its mean field: $G \rightarrow G_0$. Also, we assume the large-$N_c$ suppression works well for interactions containing the spin-1 mesons, which allows us to drop the single-trace terms \cite{57, 58}. Hence, the reduced three-flavor eLSM reads

\[ \mathcal{L}_{\text{eLSM}}^{\text{red}} = \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}[\Phi^4] - \lambda_1 \left( \text{Tr}[\Phi^4] \right)^2 - \lambda_2 \text{Tr}[\Phi^2] + \text{Tr}[H(\Phi^2)] - \frac{1}{4} \text{Tr}[L_{\mu\nu} L^{\mu\nu}] + \text{Tr}[R_{\mu\nu} R^{\mu\nu}] + \text{Tr} \left[ \frac{m^2}{2} + \Delta \right] (L^2 + R^2) + c_1 (\text{det} \Phi - \text{det} \Phi^1)^2 + h_2 \text{Tr}[L^2 \Phi^4] + R^2 \Phi^4] + 2h_3 \text{Tr}[L_\mu \Phi R^4 \Phi^1] + g_{4p} \text{Tr}[L_{\mu\nu} L^\mu L^\nu] + \text{Tr}[R_\mu R_\nu R^\mu R^\nu] + g_{4p} \text{Tr}[L_{\mu\nu} L^\mu L^\nu] + \text{Tr}[R_\mu R_\nu R^\mu R^\nu]. \] \tag{6}

We should note that $g_2$ term is dropped as well, although this term can provide mass modifications to spin-1 kaon sector in nuclear matter due to the $\omega_N$ mean field. This is because we expect such effects are small since the $\omega_N$ mean field is suppressed in comparison to the $\sigma_N$ or $\sigma_S$ meson fields. We also note that due to the lack of information of four-point interactions of spin-1 mesons, we have taken $g_3 = g_4 \equiv g_{4p}$ for simplicity.

The values of the model parameters determined in Ref. \cite{32} are listed in Table I. These parameters are fixed in order to reproduce the masses and decay widths of the light mesons in the vacuum, with the mean fields of $\sigma_N$ and $\sigma_S$: $\phi_N \equiv \langle \sigma_N \rangle_{\text{vac}}$ and $\phi_S \equiv \langle \sigma_S \rangle_{\text{vac}}$, satisfying gap equations \footnote{Throughout this paper, we use a symbol “X” for referring to a vacuum value of the quantity $X$}. The detailed procedure to fix the parameters are given in Ref. \cite{32}. In this reference, the values of $\lambda_1$ and $h_1$ were not fixed due to a large uncertainty of the $f_0$ (scalar-isoscalar) meson sector. In the present study, we take $h_1 = 0$ by the large-$N_c$ suppression while $\lambda_1$ is still left as a free parameter. Besides, as we have mentioned in the previous paragraph, the value of $g_{4p}$ remains as free as well. These parameters $\lambda_1$ and $g_{4p}$ will be determined by fitting the nuclear matter properties in Sec. IV A. In fact, $g_{4p}$ can play a significant role to reproduce the incompressibility of nuclear matter \cite{43}.

| Parameters in eLSM | Values |
|-------------------|--------|
| $C_1$ [GeV$^2$]   | -0.9183 |
| $C_2$ [GeV$^2$]   | 0.4135  |
| $c_1$ [GeV$^{-2}$]| 450.5   |
| $\delta_N$ [GeV$^2$] | 0.1511 |
| $g_1$             | 5.8433  |
| $\phi_N$ [GeV]    | 0.1646  |
| $\phi_S$ [GeV]    | 0.1262  |
| $h_{0N}$ [GeV$^3$]| 0.001135 |
| $h_{0S}$ [GeV$^3$]| 0.02138 |
| $h_2$             | 9.880   |
| $h_3$             | 4.867   |
| $\lambda_1$       | 68.30   |

Table I. Parameters extracted from the eLSM in Ref. \cite{32}. Here, $C_1 = m_0^2 + \lambda_1 (\phi_N^2 + \phi_S^2)$ and $C_2 = m_1^2$ with $\phi_N$ and $\phi_S$ being the VEV of $\sigma_N$ and $\sigma_S$ in the vacuum

### III. CONSTRUCTION OF NUCLEAR MATTER

#### A. Parity Doublet Model (PDM)

In this study, the nuclear matter is constructed by a one-loop of the nucleon with the meson mean fields by combining the three-flavor eLSM and the two-flavor PDM. Although light mesons containing (anti-)strange quarks can couple with nucleon in principle when the strange and baryon numbers are conserved, we do not include such interactions to study meson masses in nuclear matter in a transparent way. Hence, the Lagrangian of the PDM is given by \cite{38, 42} by the transformation laws in Eq. (4), such that these vector mesons do not need to couple with $\Phi$ by a covariant derivative unlike in the context of Hidden Local Symmetry (HLS) \cite{56}. The remnant contributions are provided by $h_2$ and $h_3$ terms.
\[ \mathcal{L}_N = \bar{\psi}_1 (i\partial + \mu B \gamma_0 + g_V \tilde{R}) \psi_1 + \bar{\psi}_1 (i\partial + \mu B \gamma_0 + g_V \tilde{L}) \psi_1 + \bar{\psi}_2 (i\partial + \mu B \gamma_0 + h_V \tilde{L}) \psi_2 + \bar{\psi}_2 (i\partial + \mu B \gamma_0 + h_V \tilde{R}) \psi_2 \\
+ \tilde{g}_{11} V \left( Tr[\tilde{R}_\mu] \bar{\psi}_1 \gamma^\mu \psi_1 + Tr[\tilde{L}_\mu] \bar{\psi}_1 \gamma^\mu \psi_1 \right) + \tilde{g}_{22} V \left( Tr[\tilde{R}_\mu] \bar{\psi}_2 \gamma^\mu \psi_2 + Tr[\tilde{L}_\mu] \bar{\psi}_2 \gamma^\mu \psi_2 \right) \\
+ \tilde{h}_{11} V \left( Tr[\tilde{R}_\mu] \bar{\psi}_2 \gamma^\mu \psi_2 + Tr[\tilde{L}_\mu] \bar{\psi}_2 \gamma^\mu \psi_2 \right) + \tilde{h}_{22} V \left( Tr[\tilde{R}_\mu] \bar{\psi}_2 \gamma^\mu \psi_2 + Tr[\tilde{L}_\mu] \bar{\psi}_2 \gamma^\mu \psi_2 \right) \\
- M_0 [\bar{\psi}_1 \psi_2 - \bar{\psi}_2 \psi_1 - \bar{\psi}_2 \psi_1 + \bar{\psi}_1 \psi_2] - k_1 (\det \tilde{\Phi} + \det \tilde{\Phi}^\dagger) [\bar{\psi}_1 \psi_2 - \bar{\psi}_2 \psi_1 - \bar{\psi}_2 \psi_1 + \bar{\psi}_1 \psi_2] \\
- k_2 (\det \tilde{\Phi} - \det \tilde{\Phi}^\dagger) [\bar{\psi}_1 \psi_2 + \bar{\psi}_2 \psi_1 + \bar{\psi}_2 \psi_1 + \bar{\psi}_1 \psi_2] \\
- G_1 [\bar{\psi}_1 \tilde{\Phi}^\dagger \psi_1 + \bar{\psi}_1 \tilde{\Phi} \psi_1] - G_2 [\bar{\psi}_2 \tilde{\Phi}^\dagger \psi_2 + \bar{\psi}_2 \tilde{\Phi} \psi_2], \quad (7) \]

in which \( \psi_{1,1} \) is the naive-assigned nucleon and \( \psi_{2,1} \) is the mirror-assigned one, i.e., these nucleons transform under the \( U(2)_L \times U(2)_R \) chiral transformation as

\[
\psi_1 \rightarrow \tilde{g}_L \psi_1, \quad \psi_2 \rightarrow \tilde{g}_R \psi_2, \quad \psi_1 \rightarrow \tilde{g}_L \psi_2, \quad \psi_2 \rightarrow \tilde{g}_R \psi_1,
\]

with \( \tilde{g}_L \in U(2)_L \) and \( \tilde{g}_R \in U(2)_R \). \( \tilde{\Phi} \), \( \tilde{\Phi} \), and \( \tilde{L}_\mu \) are two-flavor projected light meson fields given by

\[
\tilde{\Phi} = \sigma_N + i \sigma^\alpha \tau^\alpha, \quad \tilde{V}_\mu = \frac{\tilde{L}_\mu + \tilde{R}_\mu}{2} = \frac{1}{2} (\omega_N + \rho^\alpha \tau^\alpha)\mu, \quad \tilde{A}_\mu = \frac{\tilde{L}_\mu - \tilde{R}_\mu}{2} = \frac{1}{2} (f_1 N + a^\alpha \tau^\alpha)\mu, \quad (9)
\]

with \( \tau^\alpha \) the Pauli matrices. In Eq. (7), \( \mu_B \) is a baryon number chemical potential put to access the finite baryon density. Unfamiliar terms are the \( k_1 \) and \( k_2 \) ones which include determinants in terms of the flavor. Although the mass dimensions of \( k_1 \) and \( k_2 \) are \([\text{MeV}^{-1}]\) in the two-flavor case, these terms are allowed by the \( U(1)_A \) axial anomaly in principle. Especially the \( k_2 \) term is essential to reproduce the decay width of \( N^+(1535) \rightarrow N \eta \) decay in the vacuum [59]. The \( k_1 \) term provides an additional contribution to the nucleon masses as will be observed soon. Under the chiral symmetry breaking at finite density, \( \sigma_N \) and the time-component of \( \omega_N^\mu \) possess the mean field values \( \phi_N \) and \( \omega_N^\mu \) (and \( \phi_N \) as well), respectively. These values are reduced to \( \phi_N \rightarrow \phi_N \) and \( \omega_N^\mu \rightarrow 0 \) (and \( \phi_N \rightarrow \phi_N \) in the vacuum). Note that the trace terms proportional to \( \tilde{g}_{11} V \), \( \tilde{h}_{11} V \), \( \tilde{g}_{22} V \), and \( \tilde{h}_{22} V \) allowed by the chiral symmetry are included to provide a difference between \( \rho NN \) coupling and \( \omega NN \) coupling, which make the density dependences of \( \rho \) and \( \omega_N \) meson masses differ.

In Lagrangian (7), although \( \psi_{1,1} \) and \( \psi_{2,1} \) are convenient to observe the chiral symmetric properties of the Lagrangian, these fields are not mass eigenstates. The mass eigenstates \( N_+ \) and \( N_- \) are obtained by introducing a mixing angle \( \theta \) as

\[
\begin{pmatrix}
N_+ \\
N_-
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \gamma_5 \sin \theta \\
-\gamma_5 \sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix}, \quad (10)
\]

with \( \theta \) satisfying

\[
\tan 2\theta = \frac{2 (M_0 + \frac{k_1}{2} \phi_N^2)}{(G_1 + G_2) \phi_N^2}, \quad (11)
\]

and the corresponding mass eigenvalues are

\[
m_\pm = \frac{1}{2} \sqrt{(G_1 + G_2) \phi_N^2 + 4 \left( M_0 + \frac{k_1}{2} \phi_N^2 \right)^2}, \quad (12)
\]

for \( N_\pm \) (double-sign correspondence). \( N_+ \) carries a positive-parity while \( N_- \) carries a negative-parity, then, we regard \( N_+ \) as the nucleon \( N^*(939) \) and \( N_- \) as the \( N^*(1535) \) as a proper assignment. Eqs. (11) and (12) show that the direct \( U(1)_A \) anomaly effect to the nucleons \( (k_1) \) can modify both the mixing angle \( \theta \) and the mass eigenvalues \( m_\pm \). At the chiral restoration point \( \phi_N = 0 \), Eq. (12) yields \( m_\pm \rightarrow M_0 \) which shows that the nucleons masses can be generated without the chiral symmetry breaking. This is why \( M_0 \) is often refereed to as a chiral invariant mass.

By assuming \( \tilde{g}_{11} V = \tilde{h}_{11} V = \tilde{g}_{22} V = \tilde{h}_{22} V = \tilde{g} \) (but still \( \tilde{g}_V \) and \( \tilde{h}_V \) are not identical) for simplicity, the Lagrangian (7) is rewritten into
\[ \mathcal{L}_N = \bar{N}_+ i\theta N_+ + \bar{N}_- i\theta N_- - m_+ \bar{N}_+ N_+ - m_- \bar{N}_- N_- \]
\[ + (g_V \cos^2 \theta + h_V \sin^2 \theta) \bar{N}_+ \bar{N}_+ N_+ + (g_V \cos^2 \theta - h_V \sin^2 \theta) \bar{N}_+ \bar{N}_- N_+ \]
\[ - (g_V - h_V) \sin \theta \cos \theta \bar{N}_+ \bar{N}_- \bar{N}_+ N_- - (g_V + h_V) \sin \theta \cos \theta \bar{N}_- \bar{N}_+ N_- \]
\[ - (g_V - h_V) \sin \theta \cos \theta \bar{N}_- \bar{N}_- \bar{N}_- N_+ - (g_V + h_V) \sin \theta \cos \theta \bar{N}_- \bar{N}_- N_- \]
\[ + (g_V \sin^2 \theta + h_V \cos^2 \theta) \bar{N}_- \bar{N}_- \bar{N}_- N_- + (g_V \sin^2 \theta - h_V \cos^2 \theta) \bar{N}_- \bar{N}_- \bar{N}_- N_- \]
\[ + 2 \bar{N}_+ \bar{N}_+ N_+ + 2 \bar{N}_- \bar{N}_- N_- \]
\[ - k_1 (\det \Phi + \det \Phi^\dagger) \left\{ \sin 2 \theta \bar{N}_+ N_+ + \cos 2 \theta \bar{N}_+ \bar{N}_+ N_+ + \sin 2 \theta \bar{N}_- N_- \right\} \]
\[ - k_2 (\det \Phi - \det \Phi^\dagger) \left\{ \sin 2 \theta \bar{N}_- N_+ + \cos 2 \theta \bar{N}_- \bar{N}_+ N_+ + \sin 2 \theta \bar{N}_- N_- \right\} \]
\[ - g_{NN} \bar{N}_+ (\sigma_N + a_0^* \tau^a) N_+ - g_{NN} \bar{N}_+ i \gamma_5 (\eta_N + \pi^a \tau^a) N_+ \]
\[ + g_{NN} \bar{N}_- (\sigma_N + a_0^* \tau^a) N_+ - g_{NN} \bar{N}_- i \gamma_5 (\eta_N + \pi^a \tau^a) N_+ \]
\[ - g_{NN'} \bar{N}_+ (\sigma_N + a_0^* \tau^a) N_+ - g_{NN'} \bar{N}_+ i \gamma_5 (\eta_N + \pi^a \tau^a) N_+ \]
\[ - g_{NN'} \bar{N}_- (\sigma_N + a_0^* \tau^a) N_+ - g_{NN'} \bar{N}_- i \gamma_5 (\eta_N + \pi^a \tau^a) N_+ \],

which are summarized in Table II (Recall the symbol "X" stands for a vacuum value of the quantity X). In our model, the axial charges and \( N^* (1535) \rightarrow N \eta \) decay width are calculated as \[ 28 \]
\[ \hat{g}_{N^+}^A = \frac{\hat{\phi}_N}{m_+} \times \]
\[ \left( \hat{g}_{NN} + (g_V \cos^2 \theta - h_V \sin^2 \theta) \frac{g_1}{m_{a_1}^2} \hat{\phi}_N \hat{m}_+ \right), \]
\[ \hat{g}_{N^-}^A = \frac{\hat{\phi}_N}{m_-} \times \]
\[ \left( \hat{g}_{NN'} + (g_V \sin^2 \theta - h_V \cos^2 \theta) \frac{g_1}{m_{a_1}^2} \hat{\phi}_N \hat{m}_- \right), \]

and

\[ \hat{\Gamma}_{N^* (1535) \rightarrow N \eta} = \frac{1}{8 \pi \hat{m}_-^2} \hat{G} \left( \hat{m}_+^2 + \hat{m}_-^2 - \hat{m}_N^2 \right), \]

with

\[ |\tilde{p}_N| \left[ \hat{m}_+^2 - (\hat{m}_+ + \hat{m}_-)^2 \right] [\hat{m}_-^2 - (\hat{m}_+ + \hat{m}_-)^2] \]
\[ 2 \hat{m}_- \]

\[ \hat{G} = - \hat{Z}_{NN} \left( k_2 \hat{\phi}_N \cos 2 \theta - \hat{g}_{NN'} \cos \theta \right), \]

respectively. Here, the renormalization factor \( \hat{Z}_{NN} \) and the mixing angle \( \theta \) are given in Eq. (A17) and Eq. (A19), respectively. As already mentioned, the parameter \( k_2 \) is crucial to fit the \( N^* (1535) \rightarrow N \eta \) decay in Eq. (17) \[ 59 \]. The list of the determined parameters after fitting nuclear matter properties will be summarized in Table IV in Sec. IV A.

### B. Nuclear matter

Here, we construct nuclear matter by combining the eLSM and the PDM provided in Sec. II and Sec. III A,
and fit the remaining parameters. The matter is described by the one-loop approximation of the nucleon together with the meson mean fields:

\[ \phi_N \equiv \langle \sigma_N \rangle, \quad \phi_S \equiv \langle \sigma_S \rangle, \quad \omega_N \equiv \langle \omega_N^{\mu=0} \rangle. \]  

Therefore, the grand potential (per volume) reads \(^5\)

\[ \frac{\Omega}{V} = -\frac{1}{4\pi^2} \left\{ \frac{2}{3} \left( k_F^2 + m_+^2 k_F^2 - \sqrt{k_F^2 + m_+^2 k_F m_+^2} + m_+^4 \ln \left( \frac{k_F + \sqrt{k_F^2 + m_+^2}}{m_+} \right) \right) \right\} \\
+ \left( \frac{m_0^2}{2} (\phi_N^2 + \phi_S^2) + \frac{\lambda_1}{4} (\phi_N^2 + \phi_S^2)^2 + \frac{\lambda_2}{8} (\phi_N^4 + 2\phi_S^4) - h_0 N \phi_N - h_{0S} \phi_S - \frac{m_+^2}{2} \omega_N \omega_N - \frac{g_{\omega N}}{2} \omega_N \right) \\
- \left( \frac{m_0^2}{2} (\phi_N^2 + \phi_S^2) + \frac{\lambda_1}{4} (\phi_N^2 + \phi_S^2)^2 + \frac{\lambda_2}{8} (\phi_N^4 + 2\phi_S^4) - h_0 N \phi_N - h_{0S} \phi_S \right), \]  

(20)

in which the effective chemical potential is

\[ \mu_B^* \equiv \mu_B - g_\omega \omega_N \]  

(21)

with

\[ g_\omega = -\frac{1}{2} (g_V \cos^2 \theta + g_V \sin^2 \theta) - 2 \bar{g} . \]  

(22)

The Fermi momentum \( k_F \) is defined via the relation \( \mu_B^* = \sqrt{k_F^2 + m_+^2} \), and the baryon number density \( \rho_B \) is given by \( \rho_B = 2 \pi^2 k_F^3 \). Note that the vacuum contributions to the thermodynamic potential has been subtracted to measure the thermodynamic quantities properly. The thermodynamic potential in Eq. (20) and gap equations with respect to \( \phi_N, \phi_S \) and \( \omega_N \),

\[ \frac{\partial \Omega}{\partial \phi_N} = 0, \quad \frac{\partial \Omega}{\partial \phi_S} = 0, \quad \frac{\partial \Omega}{\partial \omega_N} = 0, \]  

(23)

are essential to get nuclear matter quantities and determine density dependences of the meson masses consistently with the vacuum ones. We should note, in the current approach, the \( \omega NN \) coupling \( (g_\omega) \) in Eq. (22) depends on the density via the mixing angle \( \theta \) because we have assumed \( g_V \neq h_V \).

In this study, the saturation condition at the normal nuclear density \( \rho_0 = 0.16 \text{ fm}^{-3} \): \( \frac{\partial}{\partial \rho_B} \left( \frac{E}{N} \right) \bigg|_{\rho_0} = 0 \) (\( E \) is the total energy and \( N \) is the mass number), the binding energy per a nucleon \( \frac{E}{N} \bigg|_{\rho_0} - \frac{\mu_B}{\rho_B} \bigg|_{\rho_0} = 0 \), and the incompressibility \( K = 0.24 \text{ GeV} \) are chosen as input parameters by nuclear matter properties. First, the saturation condition reads

\[ \frac{\partial}{\partial \rho_B} \left( \frac{E}{N} \right) \bigg|_{\rho_0} = \frac{P}{\rho_B^2} \bigg|_{\rho_0} = 0, \]  

(24)

\[ \frac{\partial}{\partial \rho_B} \left( \frac{E}{N} \right) \bigg|_{\rho_0} - \frac{\mu_B}{\rho_B} \bigg|_{\rho_0} = 0. \]  

\[ E = -PV + \mu_B N, \]  

(25)

and

\[ dE = \mu_B dN, \]  

(26)

with \( PV = 0 \). Namely, we arrive at

\[ \left. P \right|_{\rho_0} = 0. \]  

(27)

The pressure of the medium is simply defined by \( P \equiv -\Omega/V \) with Eq. (20). Next, thanks to Eq. (27), the condition of the binding energy per a nucleon is reduced into

\[ \frac{E}{N} \bigg|_{\rho_0} = \mu_B \bigg|_{\rho_0} = 0.923 \text{ GeV}, \]  

(28)

together with Eq. (25) and \( \hat{m}_+ = 0.939 \text{ GeV} \). Finally, again by using Eqs. (25), (26) and (27), the incompressibility is

\[ K = 9 \rho_0^2 \frac{\partial^2}{\partial \rho_B^2} \left( \frac{E}{N} \right) \bigg|_{\rho_0} = 9 \rho_0 \left( \frac{\partial \mu_B}{\partial \rho_B} \right) \bigg|_{\rho_0} = 0.24 \text{ MeV}. \]  

(29)

The input parameters in terms of the nuclear matter properties are summarized in Table III.

| Inputs in nuclear matter | Values |
|------------------------|--------|
| \( P \bigg|_{\rho_0} \) | 0 |
| \( \mu_B \bigg|_{\rho_0} \) [GeV] | 0.923 |
| \( K \) [GeV] | 0.24 |

\(^5\) Here, contributions from \( N_\omega \) are absent. This is true as far as we stick to lower density \( 2\rho_0 \lesssim \rho_B \) (\( \rho_0 \) is the normal nuclear matter density).
A. Parameter determination

Before showing numerical results of meson mass modifications in nuclear matter, we determine the remaining model parameters by fitting the inputs in Table II and Table III. Our procedure leaves two free parameters. Hence, we select $M_0$ (chiral invariant mass) and $k_1$ (the strength of direct $U(1)_A$ anomaly effect to the nucleons) as free parameters.

The fixed parameters with a given set of $M_0$ and $k_1$ are summarized in Table IV. Due to strong constraints by nuclear matter properties, the allowed range of $M_0$ is $0.6 \text{ GeV} \lesssim M_0 \lesssim 0.8 \text{ GeV}$. In terms of $k_2$, we could find two solutions since the formula to calculate the $N^*(1535) \to N\eta$ decay width in Eq. (17) includes a quadratic term of $k_2$. Here, we pick up the solution of which the absolute value is smaller as done in Ref. [59]. In fixing the parameters, although the value of $\tilde{g}_A^{N^*}$ includes a large uncertainty as in Table II, we take $\tilde{g}_A^{N^*} = 0.2$ here. The case when we change the value will be discussed after plotting the results with $\tilde{g}_A^{N^*} = 0.2$. We should note that when the four-point interaction among spin-1 mesons ($g_{4p}$ term in Eq. (6)) is absent, we fail to reproduce the incompressibility. We assume the sign of $g_{4p}$ should be positive. Otherwise, we could find a nonzero $\tilde{\omega}_N$ even in the vacuum which is forbidden by the Lorentz invariance. Also, to determine the value of $\tilde{g}$, we have chosen a solution in such a way that the value of $\tilde{\omega}_N$ is always positive. We emphasize that we could confirm the first order liquid-gas phase transition takes place at $\mu_B = 0.923 \text{ GeV}$ [61].

A simply way to define the pion and kaon decay constants is [62]

$$ (f_\pi)^\text{med} = \frac{\phi_N}{Z_\pi}, \ (f_K)^\text{med} = \frac{\sqrt{2}\phi_N + \phi_N}{2Z_K}, \quad (30) $$

as a naive extension of the vacuum ones: $(f_\pi)^\text{vac} = \phi_N/Z_\pi$ and $(f_K)^\text{vac} = (\sqrt{2}\phi_N + \phi_N)/(2Z_K)$ ($Z_\pi$, $Z_K$, $\tilde{Z}_\pi$, and $\tilde{Z}_K$ are renormalization factors defined in Eqs. (B4) and (A15)). By employing Eq. (30), the density dependences of $(f_\pi)^\text{med}$ and $(f_K)^\text{med}$ for $M_0 = 0.8 \text{ GeV}$ and $k_1 = 0$ are depicted in Fig. 1. This figure clearly shows the partial restoration of chiral symmetry at finite baryon density [63, 64]. The reduction ratios of $f_\pi$ and $f_K$ to the vacuum ones are $(f_\pi)^\text{med}/(f_\pi)^\text{vac} \approx 85\%$ and $(f_K)^\text{med}/(f_K)^\text{vac} \approx 87\%$, respectively.

IV. RESULTS

B. Numerical results

Here, we calculate self-energies of scalar, pseudo-scalar, vector, and axial-vector mesons, and show the resultant mass modifications in nuclear matter. The grand potential (or equivalently the effective action) has been obtained by one-loop of the nucleon with the meson mean fields in Eq. (20), and the “ground state” of the system has been determined by solving the gap equations with respect to $\phi_N$, $\phi_S$, and $\tilde{\omega}_N$ in Eq. (23), respectively. Therefore, when we expand a perturbation series around this “ground state”, we need to include one-loop of the nucleon together with the meson mean fields from the viewpoint of a correct treatment of the original global chiral symmetry [46, 65].

A self-energy for the meson $X$ by one-loops in the momentum space generally depends on the external momentum: $\Pi_X(q_0, 0)$ (we have taken $q = 0$). In our approach, since the one-loops are regarded as corrections to the mean field approximations, we reduce the self-energy into a local form approximately as $\Pi_X(q_0, 0) \to \Pi_X(m_X, 0), \quad (31)$
with $m_X$ a mass of meson $X$ in the mean field level defined in Eqs. (A9) - (A12). Because self-energies of the mesons are intricate, detailed calculations of one-loop self-energies of the mesons are provided in Appendix C for spin-0 mesons and in Appendix D for spin-1 mesons. If the meson $X$ is not affected by any mixings, then the medium mass of meson $X$ is simply given by $(m^2_X)_{\text{med}} \equiv m^2_X + \Pi_X(m_X, \vec{0})$. In fact, some mesons have complex mixing structures to other mesons as in Ref. [32], which forces us to solve them. We summarize the procedure to solve the mixings and each meson mass formula in medium in Appendix B.

The resultant plots with the several choices of $M_0$ and $\tilde{k}_1$ ($\tilde{k}_1 = k_1 \hat{\phi}_N$) are depicted in Fig. 2 - Fig. 5. Although the reproduction of nuclear matter properties allows the value of $\tilde{k}_1$ to be up to $\tilde{k}_1 \approx 5$, when we take $\tilde{k}_1 \approx 5$, the mass of $\eta$ meson drops significantly and becomes negative at the lower density regime $1.5 \lesssim \rho/\rho_0$, caused by the contact interaction with the nucleon (described by the second term in Eq (C3)). This behavior leads to an $\eta$ meson condensation phase that breaks the parity, which should be discarded. The figures clearly show that the mass reduction of the $\eta$ meson is sensitive to the value of $\tilde{k}_1$. Experimentally, while the $\eta$-nucleus interaction is known to be attractive, its binding energy has not been determined well [27]. Our results suggest $\tilde{k}_1 \approx 0$ is preferable so as to get an appropriate mass reduction at finite density.

Also, we find the mass of $f^H_0$ is $(m^0_{f^H_0})_{\text{vac}} = 0.18 \text{ GeV} - 0.27 \text{ GeV}$ in the vacuum, and increases as we access to the finite density. The Particle Data Group (PDG) shows the mass of the lightest scalar-isoscalar meson ($f^0_{0}(500)$) is in a range of $m_{f^0_{0}(500)} = 400 - 500 \text{ MeV}$ [66], which contradicts our results. However, the decay width of $f^0_{0}(500)$ is large as well: $\Gamma_{f^0_{0}(500)} = 400 - 700 \text{ MeV}$, such that we need to include the dynamical processes to estimate the $f^0_{0}(500)$ mass properly by the $f^H_0$ one in our model. We discuss this issue in Sec. V in detail.

All figures indicate the masses of $f^H_0$, $K^*_0$, $a_0$, $\eta'$, $\eta$ mesons decrease at finite density. Especially, the mass reduction of $\eta'$ is about 200 MeV at the normal nuclear density $\rho_B = \rho_0$ for any cases, which is larger than the previous works [10–12] where the estimation is about 100

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*In these figures, $f^H_0$ and $f^0_0$ indicate the scalar-isoscalar mesons with smaller and larger masses in our model, respectively. $f^H_0$ and $f^0_0$ are the axial-vector-isoscalar mesons without and with the (anti-)strange quark, respectively. Also, $\omega_N$ and $\omega_S$ correspond to $\omega$ meson and $\phi$ meson in the Particle Data Group (PDG) notation.*
FIG. 4. (color online) The density dependence of spin-0 (left) and spin-1 (right) meson masses with $M_0 = 0.7$ GeV and $\tilde{k}_1 = 0$.

FIG. 5. (color online) The density dependence of spin-0 (left) and spin-1 (right) meson masses with $M_0 = 0.7$ GeV and $\tilde{k}_1 = -5$.

| $\hat{m}$ [GeV] | $M_0$ [GeV] | $k_1$ | $G_1$ | $G_2$ | $k_2$ | $g_V$ | $h_V$ | $\tilde{g}$ | $\lambda_1$ | $g_{4P}$ |
|------------------|-------------|-------|-------|-------|-------|-------|-------|---------|-----------|--------|
| 1.4              | 0.79        | 0     | 3.839 | 6.639 | 0     | 4.060 | -8.973 | -2.531  | -23.01    | 3.420  |
| 1.65             | 0.8         | 0     | 4.023 | 8.343 | 0     | 3.661 | -9.372 | -2.792  | -22.95    | 89.84  |

TABLE V. Parameters for $\hat{m}_- = 1.4$ GeV and $\hat{m}_- = 1.65$ GeV for the plots in Fig. 6 and Fig. 7.

FIG. 6. (color online) The density dependence of spin-0 (left) and spin-1 (right) meson masses with $\hat{m}_- = 1.4$ MeV, $M_0 = 0.79$ GeV, and $k_1 = k_2 = 0$. 
MeV. Such a large reduction of $\eta'$ meson mass support a possibility of formation of $\eta'$ mesic nuclei as well as the effective restoration of $U(1)_A$ axial anomaly.

Regarding the spin-1 mesons, $f_{1S}, K_1, f_{1N}, a_1, \omega_S, K^*$ masses decrease as the density increase, whereas $\rho$ and $\omega_N$ mesons masses depend on the parameter choices. For $M_0 = 0.8$ MeV, the $\rho$ meson mass decreases slightly while the $\omega_N$ meson mass scarcely changes at $\rho_B = \rho_0$. On the other hand, for $M_0 = 0.7$ MeV, the $\rho$ meson mass does not change while $\omega_N$ meson mass increases at $\rho_B = \rho_0$, and both masses increase at higher density. The experimental result shows the mass reduction of $\omega_N$ meson is $-29$ MeV at normal nuclear density whereas a large imaginary part of the optical potential of $70$ MeV is also expected [27], such that $M_0 = 0.8$ GeV is preferable in this sense. In fact, we have confirmed that the smaller value of $M_0$ we take, the more rapidly the $\omega_N$ meson mass increases as we access to the finite density. It is worth noticing that the difference of the density dependence of $\rho$ and $\omega_N$ mesons masses are induced by the difference between $\rho NN$ coupling and $\omega_N NN$ coupling which is allowed by the chiral symmetry as mentioned in Sec. IIIA.

The $\rho$ meson is regarded as a chiral partner to the $a_1$ meson within the two-flavor chiral symmetry such that the $\rho$ meson mass shift in nuclear matter is significant to study the partial restoration of chiral symmetry in medium [67, 68]. Although all figures show that the $\rho$ meson $a_1$ meson tends to degenerate at higher density, to study the mass shift more precisely, we need to include a decay width and a broadening effect [69–71].

The mass reduction of $f_{1N}$ meson at normal nuclear density is about $150 - 200$ MeV, which is larger than the result in Ref. [72] by the QCD sum rule approach in which the mass reduction is estimated from $55$ MeV to $130$ MeV. In this reference, the $\omega$ and $f_{1N}$ mesons are regarded as a chiral partner in the context of two-flavor chiral symmetry within the large-$N_c$ limit. In our calculation, although the chiral symmetry is not restored sufficiently, we can clearly observe a tendency of degeneracy of the $\omega_N$ and $f_{1N}$ mesons. In terms of $\omega_N$ meson, we find a mass reduction by a few $\%$, which is comparably consistent with the experiment [16]. This small mass reduction is realized within our model by assuming the large-$N_c$ suppression, i.e., by dropping $k_1$ term in Eq. (1).

In obtaining Table IV and plotting the results, we have assumed that the axial charge of $N^*(1535)$ is $g_A' = 0.2$ while the value includes a large uncertainty as indicated in Table II. Therefore, we have varied the value from $g_A' = -0.1$ to $g_A' = 0.5$. As a result, we confirm that all masses are insensitive to the value of $g_A'$.

### C. The other choices for $N_-$

While we have assigned the $N^*(1535)$ to $N_-$ in Sec. IVB, it is possible to regard another nucleon as the chiral partner to the nucleon $N(939)$. To examine such a possibility, we simply change the input parameter $\tilde{m}_- \rightarrow \tilde{m} = 1.2$ GeV, $\tilde{m}_- = 1.4$ GeV and $\tilde{m}_- = 1.65$ GeV whereas the other inputs except the $N_- \rightarrow N_+ \eta$ decay width are unchanged, since mass modifications of the mesons are less sensitive to $g_V$ and $h_V$. The values of $k_1$ and $k_2$ are fixed to be zero here for simplicity.

| $\tilde{m}_-$ [GeV] | Range of $M_0$ [GeV] |
|---------------------|-----------------------|
| 1200                | 0.57 - 0.75           |
| 1400                | 0.60 - 0.79           |
| 1535                | 0.61 - 0.81           |
| 1650                | 0.62 - 0.82           |

TABLE VI. The allowed value of $M_0$ for each $\tilde{m}_-$. A range of allowed value of $M_0$ for each choice of $\tilde{m}_-$ is listed in Table VI. This figure shows the smaller value of $m_A$ we take, the smaller value of range of $M_0$ we can obtain, as can be anticipated naively. We also plot the resultant density dependences of meson masses in Fig. 6 with $\tilde{m}_- = 1.4$ MeV, $M_0 = 0.79$ GeV, and in Fig. 7 with $\tilde{m}_- = 1.65$ MeV, $M_0 = 0.8$ GeV. Fig. 6 shows a rather reasonable value of $\omega_N$ meson mass at $\rho_0$. When
we plot the result with \( \hat{m} = 1.2 \) GeV, the \( \eta \) mass turns into imaginary below \( \rho_B \approx 1.3\rho_0 \) which is unphysical for any allowed values of \( M_0 \).

V. CONCLUSIONS

In this study we have investigated masses of scalar, pseudo-scalar, vector, and axial-vector mesons in nuclear matter comprehensively, within the three-flavor extended Linear Sigma Model (eLSM) in which vacuum properties of mesons such as masses and decay widths are successfully reproduced, and the two-flavor Parity Doublet Model (PDM). Nuclear matter is constructed by the one-loop of the nucleon with meson mean fields. To fix the model parameters, vacuum properties of the nucleons as well as normal nuclear matter properties are used as inputs as in Table II and Table III. Due to a strong restriction by the latter inputs, we find the value of \( M_0 \) should be in a range of \( 0.6 \) GeV \( \lesssim M_0 \lesssim 0.8 \) GeV, when we regard \( N^*(1535) \) as the chiral partner to the nucleon.

In calculating meson masses in nuclear matter, we have included the nucleon one-loop in addition to the mean fields. The results show that all spin-0 meson masses except the \( \pi, K \), and the lightest scalar-isoscalar (\( f_{L}^{0} \)) ones decrease at finite baryon density. Especially, a mass reduction of \( \eta \) meson is about 200 MeV, which is larger than the previous works \([10–12]\). Also, in terms of the direct \( U(1)_A \) axial anomaly effect to the nucleons, we find \( k_1 \approx 0 \) (\( k_1 \) is given in Eq. (7)) so as to obtain an appropriate mass reduction of \( \eta \) meson at the normal nuclear density.

For spin-1 mesons, all axial-vector meson masses decrease at finite density, while density dependences of \( \rho \) and \( \omega \) mesons depend on the value of \( M_0 \). Especially, \( \omega_N \) meson mass increases at finite density when we take a smaller value of \( M_0 \). The experimental result suggests a small reduction of \( \omega_N \) meson mass at the normal nuclear density \([27]\), hence, to reproduce such behavior, \( M_0 \approx 0.8 \) GeV is preferable within our framework. Unlike for spin-0 mesons, the large-\( N_c \) suppression has been assumed in our approach so that the chiral partner structure of the \( \omega_N \) and \( f_{1N} \) can be observed as proposed in \([72]\).

We expect our results provide the existing and/or forthcoming experiments with useful information on meson mass shift in nuclear matter from the viewpoint of the partial restoration of chiral symmetry and \( U(1)_A \) axial anomaly restoration.

Below, we discuss topics which is not covered in this paper. The small vacuum mass of the \( f_{L}^{0} \) and the large mixing with \( f_{0}^{H} \) might be an indication that another scalar-isoscalar resonance is needed to obtain correct vacuum values for the \( f_0 \) mesons. For instance, in Ref. \([44]\) the authors found that at nonzero density a light tetraquark has a strong influence on the medium properties of the system due to the interplay of two condensates, the tetraquark and the chiral condensate. A tetraquark degree of freedom \( \chi \) in the leading order of the large-\( N_c \) expansion in the two-flavor case can be incorporated into this model using the following interaction terms \([37]\):

\[
\mathcal{L}_{\chi \Phi \Phi} = \frac{c}{2} \chi \left( \sigma_N^2 + \bar{\pi}^2 - \sigma_0^2 - \eta_N^2 \right),
\]

\[
\mathcal{L}_{\chi \text{AV}} = \frac{d}{2} \chi \left( \rho_{S}^2 + \alpha_1^2 - \omega_0^2 - f_{1,N}^2 \right),
\]

where the new eLSM Lagrangian would be given as:

\[
\mathcal{L}_{\text{eLSM}} \rightarrow \mathcal{L}_{\text{eLSM}} + \frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi - \frac{m_{\chi}^2}{2} \chi^2 + \mathcal{L}_{\chi \Phi \Phi} + \mathcal{L}_{\chi \text{AV}}.
\]

In this framework the tetraquark is assumed to be mostly \( f_0(500) \), while the \( \sigma_N \) and \( \sigma_S \) correspond most likely to \( f_0(1370) \) and \( f_0(1700) \), respectively, as previously eLSM studies have shown \([29, 30, 37]\). Note, while in Ref. \([37]\) the authors found a negligible tetraquark condensate, the situation here might be different due to the inclusion of the \( \phi_S \) condensate which is not considered in Ref. \([37]\). Modification of our model by including the tetraquark as in Eq. (33) will be left as a future work.

In the current work, we have employed the two-flavor PDM for describing the couplings of the mesons and the nucleons, so that kaons have not directly coupled with nuclear matter, although such couplings are not forbidden when the strange number and the baryon number conserve \([73]\). One way to incorporate the missing couplings is to employ the three-flavor PDM \([74, 75]\), whereas the three-flavor PDM can include an uncertainty in terms of the assignment of chiral representations to baryons. We leave this improvement as a future work.

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Appendix A: Meson masses in the vacuum

Here, we show mass formulae of scalar, pseudo-scalar, vector, and axial-vector mesons in the vacuum obtained by the three-flavor eLSM in Eq. (6). By expanding the quadratic terms with respect to the meson fields in Eq. (6) with the mean fields

\[
\phi_N \equiv \langle \sigma_N \rangle, \quad \phi_S \equiv \langle \sigma_S \rangle, \quad \omega_N \equiv \langle \omega_N^{\mu=0} \rangle,
\]

first we find
\[ \mathcal{L}_{\text{red}}^{(2)} = \mathcal{L}_{\sigma_0}^{(2)} + \mathcal{L}_{\eta_0}^{(2)} + \mathcal{L}_{\eta_0}^{(2)} + \mathcal{L}_{\phi_0}^{(2)} + \mathcal{L}_{\kappa_0 K_1}^{(2)} + \cdots, \]  
(A2)

with

\[ \mathcal{L}_{\sigma_0}^{(2)} = \frac{1}{2} \partial_\mu \sigma_0 \partial^\mu \sigma_0 - \frac{m_{\sigma_0}^2}{2} \sigma_0^2 - \frac{1}{2} \partial_\mu \sigma_0 \partial^\mu \sigma_0 - \frac{m_{\sigma_0}^2}{2} \sigma_0^2 - m_{\sigma_0}^2 \sigma_0 \sigma_0 - \frac{m_{\sigma_0}^2}{2} \sigma_0^2 \]
\[ - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} - \frac{m_{\phi_0}^2}{2} \omega_{\mu\nu}^2 + 2 g_4 \omega_{\mu}^2 (\omega_{\mu}^{\mu=0})^2 - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{m_{\phi_0}^2}{2} \omega_{\mu\nu} \omega^{\mu\nu} , \]  
(A3)

\[ \mathcal{L}_{\eta_0}^{(2)} = \frac{1}{2} \partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{m_{\eta_0}^2}{2} \eta_0^2 - \frac{1}{2} \partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{m_{\eta_0}^2}{2} \eta_0^2 - m_{\eta_0}^2 \eta_0 \eta_0 - \frac{m_{\eta_0}^2}{2} \eta_0^2 - 2 g_4 \omega_{\mu}^2 (\eta_{\mu}^{\mu=0})^2 , \]  
(A4)

\[ \mathcal{L}_{\eta_0}^{(2)} = \partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{m_{\eta_0}^2}{2} \eta_0^2 - \frac{1}{2} \partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{m_{\eta_0}^2}{2} \eta_0^2 - m_{\eta_0}^2 \eta_0 \eta_0 - \frac{m_{\eta_0}^2}{2} \eta_0^2 - g_4 \omega_{\mu}^2 (\eta_{\mu}^{\mu=0})^2 , \]  
(A5)

\[ \mathcal{L}_{\phi_0}^{(2)} = \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{m_{\phi_0}^2}{2} \phi_0^2 - \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{m_{\phi_0}^2}{2} \phi_0^2 - m_{\phi_0}^2 \phi_0 \phi_0 - \frac{m_{\phi_0}^2}{2} \phi_0^2 - g_4 \omega_{\mu}^2 (\phi_{\mu}^{\mu=0})^2 , \]  
(A6)

\[ \mathcal{L}_{\kappa_0 K_1}^{(2)} = \partial_\mu \kappa_0 \partial^\mu K_1 + i g_1 \omega_{\mu} \partial_\mu K_1 - m_{\kappa_0}^2 \kappa_0 K_1 - m_{\kappa_0}^2 \kappa_0 K_1 - g_1 \phi_0 \phi_0 \kappa_0 K_1 + i g_1 \phi_0 \phi_0 \kappa_0 K_1 - \frac{1}{2} K_1 \mu K_1 \mu + \frac{1}{2} g_4 \omega_{\mu}^2 (\phi_{\mu}^{\mu=0})^2 , \]  
(A7)

\[ \mathcal{L}_{K_1 K_1}^{(2)} = \partial_\mu K_1 \partial^\mu K_1 + i g_1 \omega_{\mu} \partial_\mu K_1 - m_{K_1}^2 K_1 K_1 - g_1 \phi_0 \phi_0 K_1 K_1 - \frac{1}{2} g_4 \omega_{\mu}^2 (\phi_{\mu}^{\mu=0})^2 , \]  
(A8)

where we have defined

\[ m_{\sigma_0}^2 = m_0^2 + \left( 3 \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \lambda_1 \phi_S^2 + \frac{1}{2} (h_2 + h_3) \omega_N^2 , \]
\[ m_{\sigma_0}^2 = m_0^2 + \lambda_1 \phi_N^2 + 3 (\lambda_1 + \lambda_2) \phi_S^2 , \]
\[ m_{\sigma_0}^2 = 2 \lambda_1 \phi_S^2 , \]
\[ m_{\eta_0}^2 = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \lambda_1 \phi_S^2 - \frac{1}{2} (h_2 + h_3) \omega_N^2 , \]
\[ m_{\eta_0}^2 = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \lambda_1 \phi_S^2 - \frac{1}{2} (h_2 + h_3) \omega_N^2 , \]
\[ m_{\phi_0}^2 = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \left( \lambda_1 + \lambda_2 \right) \phi_S^2 + \frac{1}{2} \phi_N \phi_S - g_4 \omega_N^2 - \frac{1}{4} (h_2 + h_3) \omega_N^2 , \]  
(A9)

for scalar mesons,

\[ m_{\eta_0}^2 = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \lambda_1 \phi_S^2 + c_1 \phi_N \phi_S^2 - \frac{1}{2} (h_2 + h_3) \omega_N^2 , \]
\[ m_{\eta_0}^2 = m_0^2 + \lambda_1 \phi_N^2 + (\lambda_1 + \lambda_2) \phi_S^2 + \frac{c_1}{4} \phi_S^2 , \]
\[ m_{\eta_0}^2 = \frac{c_1}{2} \phi_S^2 , \]
\[ m_{\phi_0}^2 = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \lambda_1 \phi_S^2 - \frac{1}{2} (h_2 + h_3) \omega_N^2 , \]
\[ m_{\phi_0}^2 = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \left( \lambda_1 + \lambda_2 \right) \phi_S^2 - \frac{1}{2} \phi_N \phi_S - g_4 \omega_N^2 - \frac{1}{4} (h_2 + h_3) \omega_N^2 , \]  
(A10)
for pseudo-scalar mesons,
\[
m_{\omega_N}^2 = m_N^2 + \frac{1}{2}(h_2 + h_3)\phi_N^2 + 2g_4\omega_N^2 ,
\]
\[
m_{\omega_S}^2 = m_S^2 + (h_2 + h_3)\phi_S^2 + 2\delta_S ,
\]
\[
m_\rho^2 = m_{\omega_N} ,
\]
\[
m_K^2 = m_N^2 + \frac{1}{4}(g_1^2 + h_2)\phi_N^2 + \frac{1}{2}(g_1^2 + h_2)\phi_S^2 + \frac{1}{\sqrt{2}}(h_3 - g_1^2)\phi_N\phi_S + \delta_S + g_4\omega_N^2 ,
\]
for vector-mesons, and
\[
m_{f_{1N}}^2 = m_N^2 + \frac{1}{2}(2g_1^2 + h_2 - h_3)\phi_N^2 + 2g_4\omega_N^2 ,
\]
\[
m_{f_{1S}}^2 = m_N^2 + (2g_1^2 + h_2 - h_3)\phi_S^2 + 2\delta_S ,
\]
\[
m_{a_1}^2 = m_{f_{1N}}^2 ,
\]
\[
m_{K_1}^2 = m_N^2 + \frac{1}{4}(g_1^2 + h_2)\phi_N^2 + \frac{1}{2}(g_1^2 + h_2)\phi_S^2 - \frac{1}{\sqrt{2}}(h_3 - g_1^2)\phi_N\phi_S + \delta_S + g_4\omega_N^2 ,
\]
for axial-vector mesons, respectively. With respect to \(K\) (\(K\)) and \(K_0\) (\(K_0\)) mesons, we have defined the mass at finite chemical potential by the same manner for the “effective mass” in Ref. [76] 7.

In Eqs. (A4) - (A8), a term proportional to \(|\nu^\mu=0|^2\) (\(\nu = \rho, K, f_1, f_{1S}, a_1, K_1\)) is present. Field theoretically, one simple way to remove the unphysical mode of massive spin-1 meson is to start on a perturbation series by a Proca-type Lagrangian. Hence we discard such a problematic term.

The vacuum masses of \(a_0, \omega_N, \omega_S, \rho, K, f_1, f_{1S}, a_1, K_1\) are straightforwardly obtained as
\[
(m_{a_0}^2)^{\text{vac}} = \hat{m}_{a_0}^2 ,
\]
\[
(m_{\omega_N}^2)^{\text{vac}} = \hat{m}_{\omega_N}^2 ,
\]
\[
(m_{\omega_S}^2)^{\text{vac}} = \hat{m}_{\omega_S}^2 ,
\]
\[
(m_{\rho}^2)^{\text{vac}} = \hat{m}_{\rho}^2 ,
\]
\[
(m_{K}^2)^{\text{vac}} = \hat{m}_K^2 M ,
\]
\[
(m_{f_{1N}}^2)^{\text{vac}} = \hat{m}_{f_{1N}}^2 ,
\]
\[
(m_{f_{1S}}^2)^{\text{vac}} = \hat{m}_{f_{1S}}^2 ,
\]
\[
(m_{a_1}^2)^{\text{vac}} = \hat{m}_{a_1}^2 ,
\]
\[
(m_{K_1}^2)^{\text{vac}} = \hat{m}_{K_1}^2 ,
\]
\[
\text{(A13)}
\]

where \(\hat{m}_X^2 (X = \sigma_N, \sigma_S, a_0, \ldots)\) represents the corresponding quantities in Eqs. (A9)-(A12) replaced by \(\phi_N \rightarrow \phi_N, \phi_S \rightarrow \phi_S\) and \(\omega_N \rightarrow 0\). For the other mesons, we need to solve the mixings. As done in Ref. [32], by introducing mixing angles and redefining the spin-1 meson fields appropriately, we find
\[
(m_{a_0}^2)^{\text{vac}} = 2\hat{m}_a L ,
\]
\[
(m_{\omega_N}^2)^{\text{vac}} = 2\hat{m}_{\omega_N} L ,
\]
\[
(m_{\omega_S}^2)^{\text{vac}} = 2\hat{m}_{\omega_S} L ,
\]
\[
(m_{\rho}^2)^{\text{vac}} = 2\hat{m}_\rho L ,
\]
\[
(m_{K}^2)^{\text{vac}} = 2\hat{m}_K L ,
\]
\[
\text{(A14)}
\]

7 If we define the mass by \(m_K \equiv m_{\omega_K}/(|\bar{K}| = 0)\) (\(K = K, K_0, K_0\)) with \(\omega_K\) the dispersion, then the masses of \(K\) and \(\bar{K}\) (\(K_0\) and \(\bar{K}_0\)) split.
and
\[
\begin{pmatrix}
\eta
\end{pmatrix} = \begin{pmatrix}
\cos \theta_\eta & -\sin \theta_\eta \\
\sin \theta_\eta & \cos \theta_\eta
\end{pmatrix} \begin{pmatrix}
\eta_N \\
\eta_S
\end{pmatrix},
\tag{A19}
\]
by introducing mixing angles \(\theta_\sigma\) and \(\theta_\eta\) satisfying
\[
\tan 2\theta_\sigma = \frac{2\tilde{m}^2_{\sigma \sigma} - \tilde{m}^2_{\sigma N}}{\tilde{m}^2_{\sigma S} - \tilde{m}^2_{\sigma N}}, \quad \tan 2\theta_\eta = \frac{2\tilde{m}^2_{\eta \eta} - \tilde{m}^2_{\eta N}}{\tilde{m}^2_{\eta S} - \tilde{m}^2_{\eta N}}.
\tag{A20}
\]

**Appendix B: Meson masses in nuclear matter**

In this appendix, we discuss general properties of definition of meson masses in nuclear matter. In the present analysis, we define the meson masses in nuclear matter as a pole of each propagator with vanishing three-momentum in which one-loop corrections by the nucleons as well as the meson mean fields are included. A self-energy by one-loops in the momentum space generally depends on the external momentum: \(\Pi_X(q_0, 0)\). In our approach, since the one-loops are regarded as corrections to the mean field approximations, we reduce the self-energy into a local form approximately as \(\Pi_X(q_0, 0) \approx \Pi_X(m_X, 0)\) with \(m_X\) a mass of meson \(X\) in the mean field level defined in Eqs. (A9)-(A12). The concrete expressions of \(\Pi_X(q_0, 0)\) will be given in Appendix C and Appendix D.

For convenience, let us define the quantity
\[
\tilde{m}^2_X = m^2_X + \Pi_X(m_X, 0).
\tag{B1}
\]
Then, as a naive extension of Eq. (A13), \(a_0, \omega_N, \omega_S, \rho, K^*, f_1, f_1, a_1, \) and \(K_1\) masses in nuclear matter are easily provided by
\[
\begin{align*}
(m^2_{a_0})^\text{med} &= \tilde{m}^2_{a_0}, \quad (m^2_{\omega_N})^\text{med} = \tilde{m}^2_{\omega_N}, \quad (m^2_{\omega_S})^\text{med} = \tilde{m}^2_{\omega_S}, \\
(m^2_{\rho})^\text{med} &= \tilde{m}^2_{\rho}, \quad (m^2_{K^*})^\text{med} = \tilde{m}^2_{K^*}, \quad (m^2_{f_1})^\text{med} = \tilde{m}^2_{f_1}, \\
(m^2_{f_1, a_1})^\text{med} &= \tilde{m}^2_{f_1, a_1}, \quad (m^2_{K})^\text{med} = \tilde{m}^2_{K}.
\end{align*}
\tag{B2}
\]
For the other mesons, we must solve the mixings as done in Appendix A. Namely, as in Eqs. (A14) \(\pi, K_0^*,\) and \(K\) masses in nuclear matter are
\[
\begin{align*}
(m^2_{\pi})^\text{med} &= Z_{\pi}^2 \tilde{m}^2_{\pi}, \quad (m^2_{K_0^*})^\text{med} = Z_{K_0^*}^2 \tilde{m}^2_{K_0^*}, \\
(m^2_{K})^\text{med} &= Z_{K}^2 \tilde{m}^2_{K},
\end{align*}
\tag{B3}
\]
with
\[
\begin{align*}
Z_{\pi} &= \frac{\tilde{m}_{a_1}}{\sqrt{\tilde{m}^2_{a_1} - g_1^2 \phi^2_N}}, \\
Z_{K_0^*} &= \frac{2\tilde{m}_{K^*}}{\sqrt{4\tilde{m}^2_{K^*} - g_1^2 (\phi_N - \sqrt{2}\phi_S)^2}}, \\
Z_{K} &= \frac{2\tilde{m}_{K}}{\sqrt{4\tilde{m}^2_{K} - g_1^2 (\phi_N + \sqrt{2}\phi_S)^2}}.
\end{align*}
\tag{B4}
\]
For \(\sigma_N, \sigma_S, \eta_N,\) and \(\eta_S\), we find
\[
\begin{align*}
(m^2_{f_1/f_0})^\text{med} &= \frac{1}{2} \left( \tilde{m}^2_{\sigma S} + \tilde{m}^2_{\sigma}\right) \\
&\pm \sqrt{(\tilde{m}^2_{\sigma N} - \tilde{m}^2_{\sigma S})^2 + 4\tilde{m}^4_{\sigma N \sigma S}}, \\
(m^2_{\eta})^\text{med} &= \frac{1}{2} \left( \tilde{m}^2_{\eta N} + \tilde{m}^2_{\eta S}\right) \\
&\pm \sqrt{4\tilde{m}^4_{\eta N \eta S} - (\tilde{m}^2_{\eta N} - \tilde{m}^2_{\eta S})^2},
\end{align*}
\tag{B5}
\]
where \((m^2_{\eta N})^\text{med} = Z_{\eta N}^2 \tilde{m}^2_{\eta N}, \quad (m^2_{\eta S})^\text{med} = Z_{\eta S}^2 \tilde{m}^2_{\eta S}\) and \((m^4_{\eta N \eta S})^\text{med} = Z_{\eta N}^2 Z_{\eta S}^2 \tilde{m}^4_{\eta N \eta S}\) with
\[
\begin{align*}
Z_{\eta N} &= \frac{\tilde{m}_{f_1}}{\sqrt{\tilde{m}^2_{f_1} - 2g_1^2 \phi^2_S}}, \\
Z_{\eta S} &= \frac{\tilde{m}_{f_1}}{\sqrt{\tilde{m}^2_{f_1} - 2g_1^2 \phi^2_S}}.
\end{align*}
\tag{B6}
\]
as in Eq. (A16) by introducing an appropriate mixing angles.

**Appendix C: Self-energies for the spin-0 mesons**

Here, we list explicit forms of self-energies for the spin-0 mesons in nuclear matter with vanishing spatial momentum. The coupling manner with the nucleons for each meson can be read by the Lagrangian (13). By defining \(E_k = \sqrt{|\vec{k}|^2 + m^2_k}\), we can get the following results \(^8\):

---

\(^8\) As stated in the text, only the nucleon \(N(939)\) forms a Fermi surface since we stick to lower density.
\[
\Pi_{\sigma N}(q_0, \vec{0}) = \frac{8(k_1 \phi_N \sin 2\theta + g_{NN\sigma})^2}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^4}{E_k (4E_k^2 - q_0^2)} + \frac{2m_+ k_1 \sin 2\theta}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k} \\
- \frac{4(k_1 \phi_N \cos 2\theta - g_{NN\sigma})^2}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k} (2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2) + 2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2) \frac{2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2)}{E_k} + (m_+^2 - m_-^2)^2 . \] (C1)

\[
\Pi_{\pi N}(q_0, \vec{0}) = \frac{8k_2 \phi_N \sin 2\theta + g_{NN\pi}}{2\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k (4E_k^2 - q_0^2)} - \frac{2m_+ k_1 \sin 2\theta}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k} \\
- \frac{4(k_2 \phi_N \cos 2\theta - g_{NN\pi})^2}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k} (2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2) + 2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2) \frac{2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2)}{E_k} + (m_+^2 - m_-^2)^2 \] . (C2)

\[
\Pi_{\pi N}(q_0, \vec{0}) = \frac{8k_2 \phi_N \sin 2\theta + g_{NN\pi}}{2\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k (4E_k^2 - q_0^2)} - \frac{2m_+ k_1 \sin 2\theta}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k} \\
- \frac{4(k_2 \phi_N \cos 2\theta - g_{NN\pi})^2}{\pi^2} \int_0^{k_F} d|\vec{k}| \frac{|\vec{k}|^2}{E_k} (2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2) + 2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2) \frac{2|\vec{k}|^2 q_0^2 + m_+ m_-)(q_0^2 - (m_+ - m_-)^2)}{E_k} + (m_+^2 - m_-^2)^2 \] . (C3)

The remaining spin-0 mesons do not couple with the nucleons directly so that \( \Pi_{\sigma N}(q_0, \vec{0}) = \Pi_{\sigma N}'(q_0, \vec{0}) = 0 \).

Appendix D: Self-energies for the spin-1 mesons

In this appendix, we show the explicit forms of self-energies of spin-1 mesons in nuclear matter with vanishing spatial momentum. Before showing the results, to begin with, we discuss a general property of a spin-1 meson propagator in medium.

First, let us assume a propagator of a free spin-1 meson with mass \( m \) in the vacuum takes the form of “unitary gauge”:

\[
D_{\mu\nu}^0(q) = \frac{-i}{q^2 - m^2} \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2} \right) , \quad (D1)
\]

then, the inverse propagator is given by

\[
(D_{\mu\nu}^0)^{-1}(q) = i(q^2 - m^2)g^{\mu\nu} - iq^\mu q^\nu \quad (D2)
\]

\( (q^2 = q_0^2 - |\vec{q}|^2) \). Next, let us denote the self-energy in medium by

\[
\Pi^{\mu\nu}(q_0, \vec{q}) = \Pi^{T}(q_0, \vec{q}) P_{\mu\nu}^T + \Pi^{L}(q_0, \vec{q}) P_{\mu\nu}^L \equiv \Pi^T(q_0, \vec{q}) g^{\mu\nu} \]

\( + \Pi^L(q_0, \vec{q}) v^\mu v^\nu \),

\[
(\Pi^T(q_0, \vec{q}) v^\mu v^\nu \), \quad (D3)
\]

where the three dimensional transverse and longitudinal projection operators are defined by

\[
P_{\mu\nu}^T = g_{\mu\nu} \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right) g_{\nu j} , \quad P_{\mu\nu}^L = \frac{2v_\mu v_\nu}{q^2} - g_{\mu\nu} - P_{\mu\nu}^T , \quad (D4)
\]

and \( v_\mu \) fixes the reference frame chosen to be \( v_\mu = (1, \vec{0}) \). Hence, according to the Dyson equation \( (D_{\mu\nu}^{-1}(q_0, \vec{q}) = (D_{\mu\nu}^{-1}(q_0, \vec{q}) - i\Pi^{\mu\nu}(q_0, \vec{q}) \), we find the full propagator in medium expressed in terms of \( \Pi^{T}(q_0, \vec{q}) \), \( \Pi^{L}(q_0, \vec{q}) \), \( \Pi^T(q_0, \vec{q}) \), \( \Pi^L(q_0, \vec{q}) \) as

\[
D_{\mu\nu}^{\mu\nu}(q_0, \vec{q}) = \frac{X^T(q_0, \vec{q})}{X^T(q_0, \vec{q})} \frac{P_{\mu\nu}^T}{X^L(q_0, \vec{q})} + \frac{X^L(q_0, \vec{q})}{X^L(q_0, \vec{q})} \frac{P_{\mu\nu}^L}{X^L(q_0, \vec{q})} + \frac{1}{q^2} \left( \Pi^{T}(q_0, \vec{q}) - \Pi^{L}(q_0, \vec{q}) \right) v^\mu v^\nu \]

\( + \frac{i}{q^2} \left( \Pi^T(q_0, \vec{q}) - \Pi^L(q_0, \vec{q}) \right) v^\mu v^\nu \), \quad (D5)

with

\[
X^T(q_0, \vec{q}) \equiv q^2 - m^2 + \Pi^T(q_0, \vec{q}) - \Pi^L(q_0, \vec{q}) , \quad (D6)
\]

\( X^L(q_0, \vec{q}) \equiv q_0^2(m^2 + \Pi^T(q_0, \vec{q}))(q^2 - m^2) + \Pi^T(q_0, \vec{q}) - \Pi^L(q_0, \vec{q}) \)

\( - \Pi^L(q_0, \vec{q}) - \Pi^T(q_0, \vec{q}) (q^2 - m^2) + \Pi^L(q_0, \vec{q}) - \Pi^T(q_0, \vec{q}) \).

The mass of spin-1 meson is defined by the pole position of the full propagator in Eq. (D5) with vanishing spatial momentum: \( X^T(L)(q_0, \vec{0}) = 0 \). In calculating
Eq. (D6), practically, it is useful to employ the following relations [77]:

\[
\Pi^T(q_0, \vec{q}) = \frac{1}{2} D^{\mu \nu} \Pi_{\mu \nu} + \frac{q_0 q^i}{|q|^2} \Pi^{00} - \frac{q^i q^j}{|q|^2} \Pi^{ij},
\]

\[
\Pi^L(q_0, \vec{q}) = \frac{q^2 q^i}{|q|^2} \Pi^{00},
\]

\[
\Pi^s(q_0, \vec{q}) = \frac{q_0 q^i}{|q|^2} \Pi^{00} - \frac{q^i q^j}{|q|^2} = \frac{q^i}{|q|^2} q_\mu \Pi^{\mu 0},
\]

\[
\Pi^t(q_0, \vec{q}) = \Pi^{00} - \frac{q_0}{q_0} \Pi^{00} = \frac{1}{q_0} q_\mu \Pi^{\mu 0}.
\]

(D7)

At first glance, \(X^T(q_0, \vec{0})\) and \(X^L(q_0, \vec{0})\) do not coincide by Eq. (D6), which allows us to define the two kinds of masses of spin-1 meson in medium. However, according to the explicit calculations in our model, we find \(\Pi^F(q_0, \vec{0}) = \Pi^L(q_0, \vec{0}) = \Pi^V(q_0, \vec{0})\) and the full propagator (D5) turns into

\[
D^{\mu \nu}(q_0, \vec{q}) = \frac{i}{q_0^2 - m^2 + \Pi^V(q_0, \vec{0}) - \Pi^s(q_0, \vec{0})} P_T^{\mu \nu} + \frac{i}{q_0^2 - m^2 + \Pi^V(q_0, \vec{0}) - \Pi^s(q_0, \vec{0})} P_L^{\mu \nu} + \cdots,
\]

(D8)

which clearly shows that the masses of spin-1 meson in transverse and longitudinal components are identical as naively expected.

In the following we will show the results of self-energies of spin-1 mesons in nuclear matter. The interaction terms are extracted by Eq. (13) as in Appendix C. We should note the self-energy \(\Pi_X (q_0, \vec{0})\) here is defined by \(\Pi_X (q_0, \vec{0}) = -\Pi_X^V(q_0, \vec{0}) + \Pi_X^s(q_0, \vec{0})\). The results are

\[
\Pi_{\omega \nu}(q_0, \vec{0}) = \frac{8}{3 \pi^2} \left( \frac{1}{2} (q_0^2 \cos^2 \theta + h_\nu \sin^2 \theta) + 2 \tilde{g} \right)^2 \int_{0}^{k_F} d[\tilde{k}] \left[ \frac{\tilde{k}^2 + 2 |\tilde{k}|^2 + 3m_+^2}{E_k - q_0} \right].
\]

(D9)

\[
\Pi_{\rho}(q_0, \vec{0}) = \frac{8}{3 \pi^2} \left( \frac{1}{2} (q_0^2 \cos^2 \theta + h_\nu \sin^2 \theta) \right)^2 \int_{0}^{k_F} d[\tilde{k}] \left[ \frac{\tilde{k}^2 + 2 |\tilde{k}|^2 + 3m_+^2}{E_k - q_0^2} \right] \frac{3m_+ (m_+ - m_-)(m_+ + m_-)^2 - q_0^2 + 2|\tilde{k}|^2(m_+^2 - m_-^2 - 2q_0^2)}{q_0^4 - 2(2|\tilde{k}|^2 + m_+^2 + m_-^2)q_0^2 + (m_+^2 - m_-^2)^2}.
\]

(D10)

\[
\Pi_{f_{1,1}}(q_0, \vec{0}) = \frac{16}{3 \pi^2} \left( \frac{1}{2} (q_0^2 \cos^2 \theta - h_\nu \sin^2 \theta) \right)^2 \int_{0}^{k_F} d[\tilde{k}] \left[ \frac{\tilde{k}^2 + 2 |\tilde{k}|^2}{E_k - q_0^2} \right] \frac{3m_+ (m_+ - m_-)(m_+ + m_-)^2 - q_0^2 + 2|\tilde{k}|^2(m_+^2 - m_-^2 - 2q_0^2)}{q_0^4 - 2(2|\tilde{k}|^2 + m_+^2 + m_-^2)q_0^2 + (m_+^2 - m_-^2)^2}.
\]

(D11)

\[
\Pi_{\omega_1}(q_0, \vec{0}) = \Pi_{f_{1,1}}(q_0, \vec{0}).
\]

(D12)

The remaining spin-1 mesons do not couple with the nucleons directly so that \(\Pi_{\omega \omega}(q_0, \vec{0}) = \Pi_{K_1}(q_0, \vec{0}) = 0\).
