THE ENSEMBLE PHOTOMETRIC VARIABILITY OF ~25,000 QUASARS IN THE SLOAN DIGITAL SKY SURVEY

Daniel E. Vanden Berk,1,2 Brian C. Wilhite,2,3 Richard G. Kron,2,3 Scott F. Anderson,4 Robert J. Brunner,5 Patrick B. Hall,6,7 Željko Ivezic,8 Gordon T. Richards,6 Donald P. Schneider,9 Donald G. York,1,9 Jonathan V. Brinkmann,10 Don Q. Lamb,11 Robert C. Nichol,11 and David J. Schlegel8

Received 2003 August 13; accepted 2003 October 1

ABSTRACT

Using a sample of over 25,000 spectroscopically confirmed quasars from the Sloan Digital Sky Survey, we show how quasar variability in the rest-frame optical/UV regime depends on rest-frame time lag, luminosity, rest wavelength, redshift, the presence of radio and X-ray emission, and the presence of broad absorption line systems. Imaging photometry is compared with three-band spectrophotometry obtained at later epochs spanning time lags up to about 2 yr. The large sample size and wide range of parameter values allow the dependence of variability to be isolated as a function of many independent parameters. The time dependence of variability (the structure function) is well fitted by a single power law with an index $\gamma = 0.246 \pm 0.008$, on timescales from days to years. There is an anticorrelation of variability amplitude with rest wavelength—e.g., quasars are about twice as variable at 1000 Å as at 6000 Å—and quasars are systematically bluer when brighter at all redshifts. There is a strong anticorrelation of variability with quasar luminosity—variability amplitude decreases by a factor of about 4 when luminosity increases by a factor of 100. There is also a significant positive correlation of variability amplitude with redshift, indicating evolution of the quasar population or the variability mechanism. We parameterize all of these relationships. Quasars with ROSAT All-Sky Survey X-ray detections are significantly more variable (at optical/UV wavelengths) than those without, and radio-loud quasars are marginally more variable than their radio-quiet counterparts. We find no significant difference in the variability of quasars with and without broad absorption line troughs. Currently, no models of quasar variability address more than a few of these relationships. Models involving multiple discrete events or gravitational microlensing are unlikely by themselves to account for the data. So-called accretion disk instability models are promising, but more quantitative predictions are needed.

Subject headings: galaxies: active — quasars: general — techniques: photometric

1. INTRODUCTION

The luminosities of quasars and other active galactic nuclei (AGNs) have been observed to vary from X-ray to radio wavelengths and on timescales from several hours to many years. The majority of quasars exhibit continuum variability on the order of 10% on timescales of months to years. A minority of AGNs, broadly classified as blazars, vary much more dramatically on much shorter timescales. The mechanisms behind quasar variability are not known, although in principle variability is a powerful means of constraining models for the energy source of AGNs. The most promising models (for nonblazar variability) include accretion disk instabilities (e.g., Rees 1984; Kawaguchi et al. 1998), so-called Poissonian processes, such as multiple supernovae (e.g., Terlevich et al. 1992) or star collisions (Courvoisier, Paltani, & Walter 1996; Torricelli-Ciamponi et al. 2000), and gravitational microlensing (e.g., Hawkins 1993). Only recently have the various models become quantitative enough for meaningful comparison with observations. A consensus on the observational trends with variability is emerging, but disagreements remain, and even the most fundamental relationships need better characterization.

Several dozen studies of quasar optical broadband variability have appeared in the literature. A number of the more important studies are summarized in tabular form by Helfand et al. (2001) and Giveon et al. (1999). Most ensemble studies have focused on establishing correlations between variability (defined in various ways as a measure of the source brightness change) and a number of parameters, most importantly time lag, quasar luminosity, rest-frame wavelength, and redshift. Characteristic timescales of variability range from months to years (e.g., Collier & Peterson 2001; Cristiani et al. 1996; di Clemente et al. 1996; Smith & Nair 1995; Hook et al. 1994; Trèvese et al. 1994). The amplitude of variability rises quickly on those timescales, but it may slow or even level off on longer timescales.

An anticorrelation between quasar variability and luminosity was reported by Angione & Smith (1972) and confirmed in numerous subsequent studies (Uomoto, Wills, & Wills 1976; Pica & Smith 1983; Lloyd 1984; O’Brien, Gondhalekar, & Wilson 1988; Hook et al. 1994; Trèvese et al. 1994;
PHOTOMETRIC VARIABILITY OF QUASARS IN SDSS

Cid Fernandes, Aretxaga, & Terlevich 1996; Cristiani et al. 1996, 1997; Paltani & Courvoisier 1997; Giveon et al. 1999; Garcia et al. 1999; Hawkins 2000; Webb & Malkan 2000). Such an anticorrelation is expected in Poissonian models, although an anticorrelation is expected in Poissonian models, although any differences are insignificant to the results of this study. Throughout this paper we use a flat, cosmological constant–dominated cosmology with parameter values $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$, and $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

1. THE QUASAR DATA SET

2.1. The Sloan Digital Sky Survey

The SDSS is a project to image $10^4 \text{ deg}^2$ of sky, mainly in the northern Galactic cap, in five broad photometric bands ($u, g, r, i,$ and $z$) to a depth of $r \sim 23$ and to obtain spectra of $10^6$ galaxies and $10^5$ quasars observed in the imaging survey (York et al. 2000). All observations are made with a dedicated 2.5 m telescope at Apache Point Observatory in New Mexico. Images are taken with a large mosaic CCD camera (Gunn et al. 1998) in a drift-scanning mode. Absolute astrometry for point sources is accurate to better than 100 mas (Pier et al. 2003). Site photometricity and extinction monitoring are carried out simultaneously with a dedicated 20 inch (0.51 m) telescope at the observing site (Hogg et al. 2001). The imaging data are reduced and calibrated using the PHOTO software pipeline (Lupton et al. 2001). In this study we use the point-spread function (PSF) magnitudes, which are determined by convolving the reduced imaging data with a model of the spatial PSF. The PSF magnitudes are more stable than aperture magnitudes for point sources, because they are less dependent on seeing variations and because the PSF background noise is lower within the survey seeing limit (which is $1.7\text{''}$). The SDSS photometric system is normalized so that the $u, g, r, i,$ and $z$ magnitudes are on the AB system (Smith et al. 2002; Fukugita et al. 1996; Oke & Gunn 1983). The photometric zero-point calibration is accurate to better than 1% (rms) in the $g, r,$ and $i,$ bands and to better than 2% in the $u$ and $z$ bands, measured by comparing the photometry of objects in scan overlap regions. The SDSS image reduction and calibration routines have evolved throughout the course of the survey, and the imaging runs have been reprocessed accordingly. Thus, the object imaging magnitudes deemed “best,’’ and that we use in this study, may be slightly different than those used for the spectroscopic target selection, although any differences are insignificant to the results of this study. Throughout this paper we use magnitudes corrected for Galactic extinction according to Schlegel, Finkbeiner, & Davis (1998).
Objects are selected for spectroscopic follow-up as candidate galaxies (Strauss et al. 2002; Eisenstein et al. 2001), quasars (Richards et al. 2002), and stars (Stoughton et al. 2002). The spectroscopic targets are grouped by 3" diameter areas or "tiles" (Blanton et al. 2003). For each tile, an aluminum plate is drilled with holes corresponding to the sky locations of the targets, along with holes for blank sky, calibration stars, and guide stars. The plates are placed at the focal plane of the telescope, and optical fibers run from the hole positions to two spectrographs, each of which accepts 320 fibers allowing for the simultaneous observation of 640 objects. For each plate, approximately 500 galaxies, 50 quasars, and 50 stars are observed. Spectroscopic observations generally occur up to a few months, but occasionally years, after the corresponding imaging observations, depending on scheduling constraints. The spectroscopic data for this study come from 479 spectroscopic plates observed and processed through 2002 September; 284 of the plates are part of the SDSS First Data Release (DR1; Abazajian et al. 2003), publicly available since 2003 April. Seven of the 291 DR1 plates are not included in this study, since the DR1 plate list was not finalized until after the sample for this study had been gathered.

2.2. Quasar Target Selection and Sample Definition

Quasar candidates are selected from the imaging sample by their nonstellar colors from the five-band photometry, as well as by matching SDSS point sources with FIRST radio sources (Stoughton et al. 2002). The selection is similar to that described by Richards et al. (2002), but the formal implementation of their algorithm was imposed after the cutoff date for the DR1 quasar sample and much of the post-DR1 data used in this study. About two-thirds of the candidates are confirmed to be quasars from the spectroscopic survey. Ultraviolet-excess quasars are targeted to a limit of $i = 19.1$, and higher redshift quasars are targeted to $i = 20.2$. These criteria give a sample that is estimated to be over 90% complete (Richards et al. 2002). Additional quasars are targeted as part of the SERENDIPITY and ROSAT classes (Stoughton et al. 2002) or (incorrectly) as stars.

Quasars are identified from their spectra using a combination of both automated classification (about 94%) and manual inspection (about 6%) of those objects flagged by the spectroscopic pipeline as being less reliably identified. For the purposes of this study, we define "quasar" to mean any extragalactic object with broad emission lines (pipeline-measured purposes of this study, we define "quasar" to mean any ex-
PSF magnitude can be easily understood and are almost entirely correctable.

There are three primary sources for the magnitude difference discrepancy: the inclusion of objects with bad PSF magnitudes, an aperture effect relating the finite fiber diameters to the PSF magnitudes, and what may be a very small but significant sky undersubtraction in the spectroscopic data. Occasionally, point sources in the images can have poor photometry if they are closely blended with other objects, occur where the seeing has changed very rapidly, or lie where there may be other problems in the imaging data. The long tails in the histograms of Figure 1 are populated mainly by the measurements of these objects. Because the objects will have unusual measured colors, they are sometimes selected as high-redshift quasar candidates for spectroscopic follow-up but turn out to be normal stars upon examination of the spectra. Therefore, spectroscopically confirmed stars that were selected as high-redshift quasar candidates are removed from the stellar data set for spectrophotometric refinement. Late-type stars identified by the spectroscopic pipeline are also rejected, because they are often variable.

The median $\Delta m$ offset from zero is simply an aperture effect, whereby the $3''$ spectroscopic fibers subtend a smaller fraction of the total object image than the PSF used to measure the PSF magnitudes in the imaging data. The spectroscopic fiber flux density to PSF flux density ratio is nearly constant (but somewhat dependent on seeing at the spectroscopic epoch; see below), so the magnitude difference will also be nearly constant and nonzero.

The downward trend of $\Delta m$ with PSF magnitude seen for each band is most easily accounted for by a small overestimation of the flux density in the spectroscopic data, possibly caused by a slight undersubtraction of the sky level. Tests of the imaging photometric calibration show that the effect is not likely to be caused by sky oversubtraction in the imaging data. Further tests will have to be done to determine the cause with certainty. The correction for a flux density overestimation, combined with a fiber aperture correction, gives

$$\Delta m = m_s - m_p = -2.5 \log \left( \frac{f_s}{f_p} \right) = -2.5 \log \left[ a \left( f_p + b \right) / f_p \right] = -2.5 \log \left( a + ab10^{(m_p - C)/2.5} \right),$$

where $a$ is the aperture correction, $b$ is the correction for the flux density offset, and $C$ is the zero-point constant used in

![Fig. 1.—Uncorrected spectroscopic minus photometric magnitudes vs. imaging PSF magnitudes for stars observed on the same spectral plates as the quasars (left). Results for all three passbands are shown. The curves show the binned median trends and the upper and lower 68.3% confidence envelopes. The panels on the right show the uncorrected magnitude difference histograms. The 68.3% confidence half-widths, $\sigma$, are given for a spectroscopic S/N of 10.](image-url)
converting flux density to magnitude. Assuming that $a$ and $b$ are constants, the function has two adjustable parameters, and fits to the data provide reasonably good descriptions of the $\Delta m$ versus $m_p$ trend. However, in order to account for any other effects, expected or unexpected, we use the following, more flexible three-parameter function:

$$\Delta m = \Delta m_0 - \exp\left[\left(m_p - m_0\right)/m^*\right],$$  \hspace{1cm} (2)$$

where $\Delta m_0$, $m_0$, and $m^*$ are constants to be determined from the fits to the data. For example, the Malmquist bias (Malmquist 1924) will add to the magnitude difference approximately as the square of the PSF magnitude uncertainty, $\sigma_p^2$. Since $\sigma_p$ ranges from about 0.01 to 0.05, the Malmquist bias is expected to affect the magnitude difference by at most a few percent. In the absence of this or other higher order effects, the three-parameter function would almost exactly reproduce the two-parameter logarithmic fit. The three-parameter functions are fitted to the data in each band separately, then subtracted from the magnitude differences.

After this correction, offsets from zero remain for the mean $\Delta m$ values for stars on the same plate, in excess of those expected from statistical uncertainty. These plate-to-plate offsets are due to differences in the SEDs of the stars used as initial spectrophotometric calibrators relative to BD +17°4708 and due to differences in the seeing at the epochs of the plate observations. The former effect applies to “half-plates,” corresponding to the two sets of 320 fibers running separately to each spectrograph. To correct for these effects, we reject stars outside of the 99% confidence envelope resulting from the $\Delta m$ corrections described above and work only with half-plates that have at least five remaining stars (the average number is about 20). The median $\Delta m$ offsets are calculated for each half-plate and subtracted from the values for each of the stars observed with that half-plate.

The final corrected $\Delta m$ distributions are shown in Figure 2 as a function of spectral S/N. The width of the stellar $\Delta m$ distribution is correlated with magnitude, but the better correlation is with spectral S/N. The reason for this is that while magnitude and S/N are correlated, it is the S/N that is directly related to the quality of a spectrum. As a whole, the 68.3% confidence half-width (nominally 1 $\sigma$) is $\approx$0.08 in each band at a spectroscopic S/N of 10, which is a substantial improvement over the initial widths of $\approx$0.13. Fits to the 68.3%
confidence half-width as a function of spectral S/N are shown in Figure 2, for which we used a function of the form

$$\sigma_{S/N} = a_0 + a_1 \exp(a_2(S/N)), \quad (3)$$

where $a_0$, $a_1$, and $a_2$ are constants. These fits are used as statistical measurement uncertainties for the quasars (see § 3).

The same spectrophotometric corrections applied to the stars are also applied to the quasars. The resulting distribution of quasar magnitude differences as a function of spectral S/N and the histograms of magnitude differences are shown in Figure 3. The mean corrected magnitude differences for the quasars are 0.002, −0.004, and −0.011 for the $g$, $r$, and $i$ bands, respectively. These values are small compared with the measurement uncertainties derived from the stars. It is possible that small differences in the SEDs of stars and quasars affect the recalibration of the quasar photometry. However, any effect is expected to be quite small, since the same filter transmission curves are used for both the imaging and the spectroscopic photometry, and the majority of the stars used for the recalibration were selected as quasar candidates in the color-selected survey, which guarantees that the SEDs are very similar. The 68.3% confidence limit half-widths of the $g$, $r$, and $i$ magnitude difference distributions are 0.134, 0.119, and 0.114, respectively, at an S/N of 10, substantially larger than those of the stars. The stars and quasars were selected to be point sources, observed simultaneously with the same instrument, and often were selected with the same algorithm. The larger magnitude differences among the quasars, therefore, demonstrate the variable nature of the quasars in the sample. The following sections quantify the variability and its dependence on many quasar parameters.

3. ENSEMBLE VARIABILITY MEASUREMENT—THE STRUCTURE FUNCTION

The magnitude difference histograms from Figures 1–3 show that the quasars are significantly more variable as a class than the stars. Assuming no stellar variability, we can use the distribution of the stellar magnitude differences to quantify the statistical measurement uncertainties. Removing the width of the stellar magnitude difference distribution in quadrature, the average quasar magnitude differences (at a spectral S/N of 10) due to variability in the sample are 0.103, 0.086, and 0.080 in the $g$, $r$, and $i$ bands, respectively. Measurement uncertainties must be taken into account, because they are comparable to the values of the variability itself. The large sizes of the samples (both the quasars and the comparison stars) and the good agreement of the photometric and spectroscopic measurements lead us to believe that the photometric variability is due to real variations in the quasar brightness, which are observable in a large and well-selected sample.
stars) allow the measurement uncertainty to be effectively removed.

We first show the absolute values of the measured quasar magnitude variations, uncorrected for measurement uncertainty, in Figures 4–7, as a function of quasar rest-frame time lag (Fig. 4), absolute magnitude in the rest-frame $i$ band (Fig. 5), rest-frame wavelength (Fig. 6), and redshift (Fig. 7). Data in each of the three photometric bands are shown separately. Average values in a set of bins are also shown. Because flux densities in the Ly$\alpha$ forest region are not representative of the true quasar flux, we have omitted data in each band at redshifts beyond which the Ly$\alpha$ forest covers the band: $z = 2.5, 4.75, \text{and} 6.0$ for the $g, r,$ and $i$ bands, respectively. The number of measurements rejected for each band because of the Ly$\alpha$ forest are 742, 45, and 0 for the $g, r,$ and $i$ bands, respectively. The figures show that there are several apparent correlations even in the uncorrected data. In particular, the average magnitude difference increases with time lag in all three bands, and the magnitude difference decreases with more negative absolute magnitude (decreases with luminosity). No trends are apparent at this stage between magnitude difference and rest wavelength or redshift. Again, it is important to account for measurement uncertainty before making any claims about the dependence of variability on any parameter.

The definition of variability used here is a statistical measure of the magnitude difference, taking into account measurement uncertainty. The first application is to the dependence of variability on rest-frame time lag—the so-called structure function. Historically, the structure function has been the primary measure of variability for studies of both individual quasars and quasar ensembles. For individual quasars with multiple sampling epochs, the structure function is comprised of the values of the magnitude differences for each pair of time lags in the data set, and it is closely related to the autocorrelation function (e.g., Simonetti, Cordes, & Heeschen 1985). In the ensemble case, here with only two sampling epochs, the structure function is simply the average value of the magnitude difference for all objects with the same (or nearly the same) time lags. The error analysis is simpler in the ensemble case, since all of the data points are independent.

We define the ensemble variability, $V$, of a set of quasars as

$$V = \sqrt{\frac{\pi}{2} E[(\Delta m)^2 - \langle \sigma_{S/N}^2 \rangle]},$$

where $\Delta m$ is the measured magnitude difference, $\sigma_{S/N}$ is the statistical measurement uncertainty of $\Delta m$ (as a function of spectral S/N) derived from the fits to the star measurements in $\S$ 2.4, and the brackets denote average quantities. The average absolute value of the magnitude difference, along with the scaling factor of $\pi/2$, is more robust against the presence of outliers in the data than the average of the square of the differences. The values $V$ as a function of rest-frame time lag $\Delta \tau$ define the structure function, $V(\Delta \tau)$. The same relation has been used for the structure function in previous variability studies (e.g., di Clemente et al. 1996).

The binned structure function for all of the quasars in the sample for each of the three photometric bands is shown in

![Figure 4](https://via.placeholder.com/150)

**Figure 4.**—Magnitude difference (uncorrected for measurement uncertainties) vs. rest-frame time delay in each of the three photometric passbands. The binned points show the mean values, while the error bars show the rms deviations divided by the square root of the number of objects in a bin.
Figure 8 with logarithmic axes. The bins were chosen to have equal intervals in logarithmic rest-frame time lag and to have reasonably large numbers of objects. The number of objects per bin ranges from 241 for the shortest time lag bin, covering 7–11 days in the $g$ band, to 7919 for the $i$-band bin covering time lags from 111 to 176 days. Quasars with magnitude differences larger than 0.75—just over 5 times the 1 $\sigma$ width of the distribution for quasars—were rejected from the analysis in order to remove outliers. This step removes about 1% of the quasars, which is more than would be expected for a truly normal distribution. In a related paper (Z. Ivecić et al. 2004, in preparation), the distribution of ultraviolet-excess quasar magnitude differences, including very large differences, is discussed in more detail. The apparently highly variable quasars may be optically violent variables and are valuable for follow-up studies, but the focus here is on “typical” quasar variability. The error bars were determined by propagating the rms errors, $\sigma$, in the average magnitude difference and measurement uncertainty in quadrature,

$$\sigma(V) = \frac{1}{2} V^{-1} \sqrt{\pi^2 (\langle |\Delta m| \rangle)^2 \sigma^2 (\langle |\Delta m| \rangle) + \sigma^2 (\langle \sigma^2_{S/N} \rangle)}.$$  (5)

Two trends are obvious: First, the structure function increases as a function of time lag—the magnitude differences are greater the longer the time between measurements. Second, the amplitude of variability is greater in the $g$ (bluest) band than in the others, and the $r$-band amplitude is generally greater than that in the $i$ (reddest) band. This is the variability anticorrelation with wavelength found in a several previous studies. We quantify the wavelength dependence explicitly in § 4.4, accounting for the dependence on other parameters. For the purposes of the remainder of this section, the clear wavelength dependence means that the analysis of the structure function addresses the three bands individually.

The correlation of variability with time lag has been found in numerous previous studies; however, the form of the structure function has remained a topic of debate. We fitted the binned structure functions with the two most common parameterizations. The first is a power law,

$$V(\Delta \tau) = \left( \frac{\Delta \tau}{\Delta \tau_0} \right)^{\gamma}$$  (6)
(e.g., Hook et al. 1994), where $\Delta \tau_0$ and $\gamma$ are constant parameters to be determined. This will appear as a straight line in a log-log plot, such as Figure 8. The second is an asymptotic function—a constant minus an exponential—which is the most common parameterization of the structure function (e.g., Bönoli et al. 1979; Trèves et al. 1994; Hook et al. 1994; Enya et al. 2002),

$$V(\Delta \tau) = V_0 - \Delta \tau_0 e^{-\Delta \tau/\Delta \tau_0}, \quad (7)$$

where again $V_0$ and $\Delta \tau_0$ are to be determined. The “time-scales,” $\Delta \tau_0$, whatever their values, are simply parameters of the functions to be fitted to the data and cannot necessarily be directly compared with physical characteristic timescales, such as those associated with accretion disks, starbursts, or gravitational lens dynamics.

Parameter values, uncertainties, and $\chi^2$ values for each of the functions in each of the photometric bands are given in Table 1. Based on the $\chi^2$ values, the functional form that best fits the structure function in each band is a power law. The power-law slopes in each of the three bands—0.293 ± 0.030, 0.336 ± 0.033, and 0.303 ± 0.035 for $g$, $r$, and $i$, respectively—are consistent with each other within one standard deviation of the difference. The power-law scale factors (where the structure functions would have a value of 1) are not well constrained, mainly because the observed time lag only extends to about 700 days. The shape of the structure function at much longer time lags is sometimes observed to “flatten” somewhat (Cristiani et al. 1997; Hook et al. 1994; Trèves et al. 1994; see Hawkins 2002 for a counterexample), but the data at long time lags do not yet favor one parameterization over another. In any case, what we can say from this study is that a two-parameter power law is a good description of the data—and a better description than a two-parameter exponential—up to time lags of about 2 yr. As the SDSS proceeds, the range of time lags will eventually reach up to about 5 yr. The rest-frame time sampling will continue to improve on all scales, and the power-law form can be even more stringently tested on longer timescales.

The wavelength dependence of the structure function becomes clearer at longer time lags (and should become even clearer by survey end), as would be expected from either a power-law or an exponential fit. The distributions of quasar properties—e.g., luminosity and rest-frame wavelength—vary with rest-frame time lag because of survey selection effects and the dependence of these properties on redshift. In the next section, we disentangle the dependence of variability on four primary quasar parameters.

4. VARIABILITY DEPENDENCE ON TIME LAG, LUMINOSITY, WAVELENGTH, AND REDSHIFT

4.1. Selection Function

The structure function calculated in the previous section describes the variability of the full data set with respect to time lag. However, variability is almost certainly also a function of quasar luminosity, rest-frame wavelength, and possibly redshift. In order to separate the dependences of variability on multiple parameters, the selection biases must be taken into account. Even the structure function may not give the true
dependence of variability on rest-frame time lag. The set of quasars within a narrow range of time lags will be populated with objects with wide ranges of the other parameters. In addition, since rest-frame time lag is dependent on redshift, as are the other parameters, the distributions of quasar parameter values will be correlated with the time lag. For example, high-redshift quasars will generally have shorter time lags in the rest frame than lower redshift quasars.

The selection function—that is, the region of parameter space occupied by the data set—is shown in projected planes in Figure 9. A number of artificial correlations are evident and are due to both the survey selection criteria and the dependence of luminosity, rest-frame time lag, and rest wavelength on redshift. Variability information can obviously only be obtained in the regions of the four-dimensional parameter space containing a statistically sufficient number of objects.

In order to determine how variability is related to a single parameter, the space was divided into small regions in three dimensions and the variability calculated as a function of the remaining parameter in each of the slices. The condition that there be enough quasars to reliably measure variability was the primary limiting factor for the bin sizes. For each parameter there is then a set of variability relations, each set representing the results of restricting the ranges of the other three parameters. As shown in the remainder of this section, in most cases variability trends are clear even in independent, restricted data sets.

If we make the assumption that the equations describing the multiparameter dependence of variability are separable, the results from each of the slices may be scaled in the single-parameter ranges where they overlap, in order to find
the variability dependence on a single parameter. That is, the form of the variability dependence is assumed to be

\[ V(\Delta \tau, M_i, \lambda, z) = v(\Delta \tau)v(M_i)v(\lambda)r(z), \quad (8) \]

where \( V \) is calculated as in \( \S \) 3. While this form is not necessarily correct, it greatly simplifies the analysis, and the relatively simple relations found for each parameter suggest that it is not far from reality. In the following subsections, we show the unscaled variability relations for single parameters in the independent, restricted data sets, then show the results after scaling the independent sets together, assuming equation (8).

The scaled relations are fitted with relatively simple descriptive functions for each parameter.

### 4.2. Time Lag

We focus first on the dependence of variability on rest-frame time lag, independent of the other three parameters. The full quasar sample was first separated into six redshift bins, each with an equal spacing in logarithmic \( 1+z \). The redshift bin sides are 0.185, 0.499, 0.895, 1.395, 2.028, 2.829, and 3.840. The quasar sample in each redshift bin was then divided into two halves separated at the median absolute magnitude of the quasars in the redshift bin: \( M_i, \text{median} = -22.96, -24.07, \)

| Parameter Values for Fits to the Binned Structure Functions |
|------------------------------------------------------------|
| Band | \( V_0 \) (mag) | \( \Delta \tau_0 \) (days) | \( \gamma \) | \( \chi^2 \) |
|-----|----------------|-----------------|-----|-----|
| **Power Law** | | | | |
| \( g \) | \( 0.168 \pm 0.005 \) | 51.9 \pm 6.0 | \( \ldots \) | 20.5 |
| \( r \) | \( 0.155 \pm 0.006 \) | 74.7 \pm 8.9 | \( \ldots \) | 24.8 |
| \( i \) | \( 0.139 \pm 0.005 \) | 62.6 \pm 8.3 | \( \ldots \) | 39.3 |
| **Exponential** | | | | |
| \( g \) | \( 0.168 \pm 0.005 \) | 51.9 \pm 6.0 | \( \ldots \) | 20.5 |
| \( r \) | \( 0.155 \pm 0.006 \) | 74.7 \pm 8.9 | \( \ldots \) | 24.8 |
| \( i \) | \( 0.139 \pm 0.005 \) | 62.6 \pm 8.3 | \( \ldots \) | 39.3 |

Fig. 9.—Projected parameter values for all of the quasars. This effectively shows the selection function in the parameter space given by rest-frame time lag, \( \Delta \tau \), redshift, \( z \), absolute \( i \)-band magnitude, \( M_i \), and rest wavelength, \( \lambda_{\text{rest}} \). The passbands are indicated when necessary. For clarity, only the \( r \)-band data are shown for the middle plots.
Taking each photometric band separately for the quasars in each of the 12 redshift/absolute magnitude bins restricted the quasar rest wavelengths to small ranges. The procedure produced 36 independent data sets confined to small ranges of redshift, absolute magnitude, and rest wavelength but unrestricted with respect to rest-frame time lag. The variability amplitude and uncertainty as a function of time lag were determined as in \( \S 3 \) for the quasars in each of the 36 data sets independently. The time lag bins were set to have a constant logarithmic time width, as in \( \S 3 \), but with twice the width, to accommodate the smaller number of objects per bin. The rest-frame time lag bin sides, in days, are 7.0, 17.6, 44.3, 111.1, 278.7, and 699.2. Unphysical (imaginary) values of the variability amplitude occurred in a small number (three) of cases in which the number of quasars was relatively small. In most cases when this occurs the number of quasars is five or fewer. For all further analysis, binned data sets containing fewer than 10 quasars or which produce imaginary values of the variability amplitude are rejected. For each of the 36 data sets, the variability with respect to rest-frame time lag is an independent structure function, over which the absolute magnitude, rest wavelength, and redshift do not vary greatly. The results are shown in Figure 10. Results for the three photometric bands, corresponding to restricted ranges of rest wavelength, are shown in separate panels. The average redshift of the quasars contributing to each structure function is indicated by color, with redder colors representing progressively higher redshifts. The structure functions containing the more luminous halves of the quasar data sets are shown with filled points, and the less luminous halves with open points.

It is clear from Figure 10 that variability is an increasing function of rest-frame time lag at all redshifts, absolute magnitudes, and rest wavelengths. Two other trends can also be seen: the less luminous quasars vary more than their more luminous counterparts (nearly all of the open points lie above the filled points within the same redshift bin), and quasars vary more at shorter wavelengths, confirming what was shown by the unrestricted structure functions in \( \S 3 \).

Under the assumption that the variability as a function of time lag is separable from the other dependencies, the individual structure functions can be scaled together in the time lag regions where they overlap. Using as a reference a structure function near the middle of the redshift, luminosity, and wavelength distributions, all 36 structure functions (excluding bins with too few objects) were scaled so that the sum of the products of the amplitudes and the time lag bin widths (the areas under the curves) were equal. The scaled points are shown in the bottom right-hand panel of Figure 10, along with the best-fit power law. The parameter fits to a power law and an exponential are given in Table 2. A two-parameter power law provides a very good fit and is better than the asymptotic (exponential) form. The power-law fit has a slope of \( \gamma = 0.246 \pm 0.008 \), which is comparable to but shallower

![Figure 10](image-url)
than the values found for the unrestricted structure functions in § 3. Scaling the data points will change the characteristic timescale of the function (the time lag at which the power-law amplitude would be unity), but not the power-law slope. That the slope is relatively close to those found for the unrestricted structure functions is due to the offsetting variability dependencies on luminosity and rest wavelength. From Figure 9, longer time lags are statistically populated by more luminous objects, which vary less but at shorter wavelengths, where the variability is greater. The significance of the power-law slope in relation to variability models is discussed in § 7.

4.3. Absolute Magnitude

The luminosity (absolute magnitude) dependence of variability is separated from the other parameters in a way similar to that used for the time lag in the previous subsection. The full quasar sample was separated into the same six redshift bins, and each separate quasar sample was further divided into two halves at the median rest-frame time lag of the quasars in the redshift bin. The median rest-frame time lags, in days, for each redshift bin are 249.9, 192.0, 157.1, 122.9, 100.5, and 73.6. Again, considering the three photometric bands separately restricted the rest wavelengths of each data set to small ranges. This produced 36 independent quasar subsamples unrestricted with respect to absolute magnitude. The variability amplitude as a function of absolute magnitude, in bins 1 mag in width, is shown for each quasar subsample in Figure 11. The variability amplitude is an increasing function of absolute magnitude (brighter objects vary less) for nearly every subsample. Also seen in Figure 11 are the time lag dependence—variability amplitudes are greater at longer time lags (filled points) than short time lags (open points)—and a wavelength dependence, seen most easily in the amplitude differences between the g and i bands. This is a clear demonstration that the well-known luminosity-variability anticorrelation is not simply due to time lag or rest-wavelength selection effects.

The data sets were again scaled so that the areas under the curves were equal. In each redshift bin, the six individual sets (three wavelength and two time lag bins) were scaled to have identical sums of the product of the absolute magnitude bin width and variability amplitude. Then, proceeding from the lowest to highest redshifts, the scaled redshift sets were rescaled to the adjacent redshift set so that the areas under the

| Parameter | Power Law | Exponential |
|-----------|-----------|-------------|
| $V_0$     | $\ldots$ | 0.144 ± 0.001 |
| $\Delta \tau_0$ (days) | $(5.36 \pm 1.46) \times 10^5$ | 40.4 ± 1.4 |
| $\gamma$ | 0.246 ± 0.008 | $\ldots$ |
| $\chi^2$ | 299.0 | 443.4 |

Fig. 11.—Similar to Fig. 10, but for variability as a function of absolute magnitude. Filled and open points show results from slices with longer and shorter time lags, respectively, within the same redshift range. The bottom right-hand panel shows all of the scaled data points, along with a best-fit generalized Poissonian function (solid line) and Poissonian function with the power-law index fixed at $\frac{1}{2}$ (dashed line).
curves were equal in regions where the absolute magnitude coverage overlapped. The scaled data points are shown in the bottom right-hand panel of Figure 11. A straight line can be fitted to the data points, but such a description is unphysical, since at large luminosities (large negative absolute magnitudes) the function becomes negative. In so-called Poissonian, or discrete-events, models the relative luminosity variability, \( \delta L/L \), is expected to vary with luminosity as \( \delta L/L \propto L^{-\beta} \), where \( \beta = \frac{1}{2} \) in general (e.g., Cid Fernandes et al. 2000). This relationship translates into the absolute magnitude form

\[
\nu(M_i) \propto 10^{0.4M_i/2.5}.
\] (9)

This function and one in which \( \beta \) is held fixed at 0.5 were fitted to the data. Both fits are shown in Figure 11. The logarithmic equation, with a best fit of \( \beta = 0.246 \pm 0.005 \), fits the data as well as a straight line and avoids the problem of negative values. The Poissonian prediction of \( \beta = \frac{1}{2} \) gives a poor fit and is clearly inconsistent with the data. Scaling the individual data sets, which accounts for arbitrary contributions from variability dependencies on time lag, rest wavelength, and redshift, does not change the value of \( \beta \). We discuss the implications for Poissonian models further in § 7.

4.4. Rest Wavelength

The rest-wavelength dependence of variability was isolated for subsamples selected to cover small ranges in redshift, time lag, and absolute magnitude. The redshift and absolute magnitude bins are the same ones used to isolate the time lag dependence (12 separate bins; see § 4.2). The data in each of these bins were divided into three separate samples in the plane of absolute magnitude and time lag, according to the following lines: \( M_i = 0.085\Delta \tau - 31.25 \) and \( M_i = 0.024\Delta \tau - 31.42 \).

This gives 36 independent data sets covering small ranges of redshift, absolute magnitude, and time lag. All but one of the data sets contain enough objects to compute reliable variability measurements. Each data set samples three separate rest-wavelength points given by the effective rest wavelengths of the three photometric bands. The variability amplitude as a function of the rest-frame wavelength for each set is shown in Figure 12. The average rest-frame time lag in each set is color-coded (longer time lags are redder), the more luminous half of a redshift/time lag bin is shown with solid points, and each redshift subset is shown in a separate panel.

In most cases, the variability amplitude decreases with wavelength, as expected from previous analysis. The cases in which the opposite happens occur at short time lags and very low or very high redshifts, but there are too few cases to make any claims about deviations from the general trend. The time lag and luminosity dependencies are also evident from Figure 12.

The data points were scaled in a manner similar to that in § 4.3. For each redshift bin, the six sets of points (three time lag bins and two absolute magnitude bins) were scaled to the same area under the curves and then, moving from low to high

---

**Fig. 12.—**Variability as a function of rest-frame wavelength for independent slices of data. Colors indicate average rest-frame time lag, with redder colors showing longer time lags, according to the color key. Filled and open points show results from slices with more luminous and less luminous quasars, respectively, within the same redshift and time lag ranges. Results from the six redshift slices are shown in separate panels.
redshift, the points were rescaled to match the appropriate area in the adjacent redshift bin. In this case, since the rest-wavelength bin limits are not equal for the separate redshift bins, the three wavelength points in each set were connected by two straight lines, and the area under the lines was calculated in the regions where they overlapped the wavelengths of the adjacent redshift bin points. The scaled points as a function of rest wavelength are shown in Figure 13. If the variability is due to a simple change in the index of a single power law, we would expect the wavelength dependence to be

$$v(\lambda) = -2.5\Delta \alpha \lambda \log \lambda + C,$$

(10)

where $\Delta \alpha$ is the difference in the wavelength power-law index and $C$ is a constant related to the pivot wavelength (where the two power laws intersect), presumed to be much longer than the observed wavelengths. Figure 13 is plotted with a logarithmic wavelength axis, and so the relation would be seen as a straight line. A single straight line is an adequate fit from the shortest wavelengths up to about 4000 Å, but it does not account well for the longer wavelength end. Contamination from host galaxy light at longer wavelengths would have the opposite effect—to cause the variability to decrease even faster with wavelength. A three-parameter exponential function, although physically unmotivated, fits the data well:

$$v(\lambda) = a_0 \exp(-\lambda/\lambda_0) + a_1,$$

(11)

with parameter values $a_0 = 0.616 \pm 0.056$, $\lambda_0 = 988 \pm 60$ Å, and $a_1 = 0.164 \pm 0.003$.

4.5. Redshift

The redshift dependence of variability is more difficult to isolate from the time lag, absolute magnitude, and wavelength dependencies. The reason for this can be seen from inspection of Figure 9. For example, samples restricted to a narrow range of rest wavelengths will have three independent redshift intervals (taking the three bands separately), but the corresponding absolute magnitude ranges for the redshift intervals may not overlap appreciably. Therefore, to isolate the redshift dependence, we first found a region of wavelength—absolute magnitude space that is covered by quasar data in all three bands. Figure 14 shows the wavelength—absolute magnitude plane and the selected region, which is bounded by a triangle with corners $(\lambda, M_i)$ at $(1250, -27.0)$, $(1250, -29.4)$, and $(3491, -23.8)$. Outside of this region, quasar data are generally available for only one or two of the bands. The data contained in this region were then separated into 16 wavelength bins, shown in Figure 14, at intervals of 100, 200, 400, or 800 Å, depending on the number of objects contained in the bin. For each bin of wavelength-separated data, objects were selected from a single range of time lags, chosen so that the average absolute magnitudes, time lags, and wavelengths were about equal for data in each of the three photometric bands. Taking each of the photometric bands separately for a restricted data set gives a wide range of redshifts, while keeping the ranges of time lag, absolute magnitude, and rest wavelength nearly constant.

The variability amplitudes for all 16 data sets (each with three redshift points) are shown in Figure 15. The number of objects contributing to each point ranges from 65 to 1150, with a mean number of 239. Lines connect the points belonging to data sets with nearly the same parameter values but at different redshifts. The color of the points and lines corresponds to the average absolute magnitude of the data set, with bluer colors representing brighter absolute magnitudes. The point sizes correspond to the average time lag of the objects contributing to the points. The absolute magnitude correlation, discussed above, is evident in Figure 15, but it is partly counteracted by the nearly monotonically increasing rest wavelength with average absolute magnitude (seen from Fig. 14) and the generally longer average time lags associated with fainter average absolute magnitudes.
What is of interest here is the dependence of the variability on redshift. The results are fairly noisy, and it is difficult to detect any clear trend with redshift. The sets of points were scaled by matching the areas under the curves of adjacent data sets, as in the previous subsections, starting with the sample with the shortest rest wavelengths. The scaled data points are shown in Figure 16. There is a correlation between the scaled variability and redshift—quasars appear to be more variable at higher redshifts. The Spearman rank correlation probability that the points are uncorrelated is less than $10^{-4}$, even after accounting for the reduction of the number degrees of freedom by the number of restricted data sets (16). A straight-line fit to the data points (linear in redshift and variability amplitude) gives

$$v(z) = (0.019 \pm 0.002)z + (0.037 \pm 0.005).$$  \hspace{1cm} (12)

The correlation, although significant, is weak enough that it could easily have gone unnoticed in previous variability studies, especially since most of them suffer from a lack of sufficient sample overlap to test the redshift relationship independently of other parameters. The redshift evolution of variability would have serious consequences for a number of currently proposed models. If the effect is intrinsic, the quasar population or the variability mechanism is changing over time. External causes are also possible, such as gravitational microlensing, which could increase with redshift, since more potential lenses would be available. The variability correlation with redshift is discussed further in § 7.

5. COLOR DEPENDENCE

Evidence from previous ensemble studies (Trèvese et al. 2001; Giveon et al. 1999; Edelson et al. 1990) suggests that the SEDs of quasars become harder (bluer) in bright phases. Indirect evidence also comes from the fact that there is a strong wavelength dependence on variability (§ 4.4). This could happen, for example, if the index of a power-law component of the continuum changes with luminosity (Trèvese & Vagnetti 2002).

Quasar colors are a strong function of redshift (Richards et al. 2001), since various spectral features move into and out of the photometric passbands with redshift changes. A pure power-law spectrum would have a single set of colors independent of redshift. The observed quasar color structure is mainly due to the presence of strong emission features, especially broad Fe ii complexes, as well as the Ly$\alpha$ forest. Figure 17 shows the average imaging photometric colors of quasars as a function of redshift in two samples selected to be either brighter or fainter by at least 3 $\sigma$ in at least one of the $g$, $r$, or $i$ bands relative to the spectrophotometry. We use the imaging photometric colors rather than the spectroscopic, since they are more precise and two more colors are available. Although the color differences are small (~0.03), at most redshifts up to at least 2.5 and for each color the bright-phase sample is bluer than the faint-phase sample. Also shown in Figure 17 are the color differences of the bright-phase minus the faint-phase samples as a function of redshift. Both the binned and average color differences are shown. The color differences are increasingly larger for shorter wavelength bands; i.e., quasars in bright phases are bluer than those in faint phases, and they are even bluer at shorter wavelengths.

That the color change persists at high redshift also indicates that it cannot be accounted for solely by a nonvariable red spectral component, such as the quasar host galaxy. Such a component would contribute a higher fraction of the total quasar light in the faint phases, making quasars appear redder than in the bright phases. Any reasonable host galaxy SED and luminosity would contribute very little light to the bluest bands and would quickly be redshifted out of the other bands. By a redshift of 0.5, there should be almost no significant contamination from the host galaxies in any of the passbands. A host galaxy component cannot account for the wide range of redshifts over which the color difference is significant.

5. COLOR DEPENDENCE

Evidence from previous ensemble studies (Trèvese et al. 2001; Giveon et al. 1999; Edelson et al. 1990) suggests that the SEDs of quasars become harder (bluer) in bright phases. Indirect evidence also comes from the fact that there is a strong wavelength dependence on variability (§ 4.4). This could happen, for example, if the index of a power-law component of the continuum changes with luminosity (Trèvese & Vagnetti 2002).

Quasar colors are a strong function of redshift (Richards et al. 2001), since various spectral features move into and out of the photometric passbands with redshift changes. A pure power-law spectrum would have a single set of colors independent of redshift. The observed quasar color structure is mainly due to the presence of strong emission features, especially broad Fe ii complexes, as well as the Ly$\alpha$ forest. Figure 17 shows the average imaging photometric colors of quasars as a function of redshift in two samples selected to be either brighter or fainter by at least 3 $\sigma$ in at least one of the $g$, $r$, or $i$ bands relative to the spectrophotometry. We use the imaging photometric colors rather than the spectroscopic, since they are more precise and two more colors are available. Although the color differences are small (~0.03), at most redshifts up to at least 2.5 and for each color the bright-phase sample is bluer than the faint-phase sample. Also shown in Figure 17 are the color differences of the bright-phase minus the faint-phase samples as a function of redshift. Both the binned and average color differences are shown. The color differences are increasingly larger for shorter wavelength bands; i.e., quasars in bright phases are bluer than those in faint phases, and they are even bluer at shorter wavelengths.

That the color change persists at high redshift also indicates that it cannot be accounted for solely by a nonvariable red spectral component, such as the quasar host galaxy. Such a component would contribute a higher fraction of the total quasar light in the faint phases, making quasars appear redder than in the bright phases. Any reasonable host galaxy SED and luminosity would contribute very little light to the bluest bands and would quickly be redshifted out of the other bands. By a redshift of 0.5, there should be almost no significant contamination from the host galaxies in any of the passbands. A host galaxy component cannot account for the wide range of redshifts over which the color difference is significant.

5. COLOR DEPENDENCE

Evidence from previous ensemble studies (Trèvese et al. 2001; Giveon et al. 1999; Edelson et al. 1990) suggests that the SEDs of quasars become harder (bluer) in bright phases. Indirect evidence also comes from the fact that there is a strong wavelength dependence on variability (§ 4.4). This could happen, for example, if the index of a power-law component of the continuum changes with luminosity (Trèvese & Vagnetti 2002).

Quasar colors are a strong function of redshift (Richards et al. 2001), since various spectral features move into and out of the photometric passbands with redshift changes. A pure power-law spectrum would have a single set of colors independent of redshift. The observed quasar color structure is mainly due to the presence of strong emission features, especially broad Fe ii complexes, as well as the Ly$\alpha$ forest. Figure 17 shows the average imaging photometric colors of quasars as a function of redshift in two samples selected to be either brighter or fainter by at least 3 $\sigma$ in at least one of the $g$, $r$, or $i$ bands relative to the spectrophotometry. We use the imaging photometric colors rather than the spectroscopic, since they are more precise and two more colors are available. Although the color differences are small (~0.03), at most redshifts up to at least 2.5 and for each color the bright-phase sample is bluer than the faint-phase sample. Also shown in Figure 17 are the color differences of the bright-phase minus the faint-phase samples as a function of redshift. Both the binned and average color differences are shown. The color differences are increasingly larger for shorter wavelength bands; i.e., quasars in bright phases are bluer than those in faint phases, and they are even bluer at shorter wavelengths.

That the color change persists at high redshift also indicates that it cannot be accounted for solely by a nonvariable red spectral component, such as the quasar host galaxy. Such a component would contribute a higher fraction of the total quasar light in the faint phases, making quasars appear redder than in the bright phases. Any reasonable host galaxy SED and luminosity would contribute very little light to the bluest bands and would quickly be redshifted out of the other bands. By a redshift of 0.5, there should be almost no significant contamination from the host galaxies in any of the passbands. A host galaxy component cannot account for the wide range of redshifts over which the color difference is significant.
6. VARIABILITY OF RADIO, X-RAY, AND BAL QUASARS

There is evidence from previous studies that the variability amplitude of quasars varies among different subclasses, such as those with radio emission (Helfand et al. 2001; Eggers et al. 2000; Garcia et al. 1999; Pica & Smith 1983) or BAL systems (Sirola et al. 1998). The entire class of highly variable blazars, for example, is defined in part by X-ray and radio emission (e.g., Ulrich et al. 1997). Here we examine the variability of broad subclasses of quasars in comparison to carefully selected control samples.

6.1. Radio-detected Quasars

Some SDSS quasar candidates are selected as optical matches to radio sources in the FIRST survey (White et al. 1997; Ivezić et al. 2002). About 10% of the verified quasars in the sample have counterparts in the FIRST survey. In the areas of the SDSS sample covered by the FIRST survey at the time the quasar candidates were selected, there are 1553 verified quasars with FIRST catalog matches. To test whether quasars with detected radio emission are more or less variable than those without, we have extracted a control sample of quasars without matches in the FIRST catalog. The non–radio-detected control sample was constructed to have the same redshift, luminosity, and time lag distribution as the radio-detected sample, by matching each radio quasar with a nonradio quasar with nearly the same redshift, magnitude, and time lag. The standard deviations of the redshift, magnitude, and time lag differences are \( \sigma(\Delta z) = 0.03 \), \( \sigma(\Delta m) = 0.04 \) mag, and \( \sigma(\Delta \tau) = 3.6 \) days, and in no case were the differences allowed to be greater than 0.5, 0.5 mag, and 40 days, respectively. Of the FIRST-matched quasars, 1376, 1389, and 1388 could be matched with counterparts in the non-FIRST sample in the \( g, r, \) and \( i \) bands, respectively. Kolmogorov-Smirnov tests comparing the redshift, magnitude, and time lag distributions of

![Graph showing variability of faint-phase vs. bright-phase colors of quasars as a function of redshift (left).](image)
the radio and control samples show that they are statistically indistinguishable. This also guarantees that the wavelength coverages of the samples are nearly identical.

A radio-loud (not simply radio-detected) subsample and its corresponding radio-quiet control sample were also generated. Radio loudness is defined here as the ratio of the rest-frame 5 GHz to 4500 Å flux densities (e.g., Sramek & Weedman 1980), and a quasar is deemed radio-loud if the ratio is at least 100. The sample sizes of the radio-loud quasars and the matched radio-quiet control quasars are 492, 530, and 546 objects for the g, r, and i bands, respectively.

The matched time lag structure functions for the full radio-detected and non–radio-detected samples are shown in Figure 18. There is no significant difference in the binned structure functions of the radio-detected and radio-undetected quasars. On the other hand, the matched structure functions for the radio-loud subsample, shown in Figure 19, are about 1.3 times higher than that for the radio-quiet subsample. However, only the difference in the i band is significant (matched-pair t-test probability of 1% if the samples were not truly different). Thus, there is marginal evidence that radio-loud quasars are more optically variable than radio-quiet quasars, but a larger sample will be needed to confirm this.

The qualitative result that the radio-loud quasars are more variable agrees with most of the past suggestions (Helfand et al. 2001; Eggers et al. 2000; Garcia et al. 1999; Pica & Smith 1983). Since most blazars are radio-loud (e.g., Ulrich et al. 1997), the higher variability amplitude of radio-loud quasars may reflect a higher fraction of blazars. There is not enough information from this survey to reliably classify individual objects as blazars (at the very least, more detailed light curves are needed). Further subdivision of the sample by finer radio loudness currently yields too few quasars for meaningful comparisons. In any case, the evidence for a correlation between radio loudness and UV/optical variability amplitude is suggestive, but not conclusive.

6.2. X-Ray–detected Quasars

As with radio quasars, some of the SDSS quasars are selected for spectroscopic follow-up as matches to sources in the ROSAT All-Sky Survey (RASS; Voges et al. 1999, 2000). A detailed analysis of RASS source matches to the SDSS data is given by Anderson et al. (2003). About 5% of the verified quasars can be matched with RASS sources, giving about 1300 X-ray quasars in our sample. We constructed the X-ray and control samples in the same way as for the radio sample and its control. The numbers of matched objects in each of the g, r, and i bands are 1010, 1008, and 1009, respectively. Again, Kolmogorov-Smirnov tests show that with respect to redshift, luminosity, and time lag, the two sets of samples are indistinguishable.

The time lag structure functions for the X-ray–detected and X-ray–nondetected samples are shown in Figure 20. The X-ray sample is more variable than the non–X-ray control sample at time lags up to about 250 days, after which the differences of variability amplitudes are much smaller. Overall the X-ray sample amplitudes are larger by a factor of ≈10%. The matched-pair t-test probabilities of the differences occurring by chance are 9.1%, 0.8%, and 0.4% for the g, r, and i bands, respectively. The data therefore show that X-ray–detected quasars are significantly more variable than their X-ray–nondetected counterparts. However, the difference may become less significant at longer time lags and at longer wavelengths.

The higher X-ray variability amplitude is probably not surprising, given the high fraction of X-ray–detected objects...
among blazars. As for the radio sample, the X-ray sample was selected purely by optical matches to catalog sources; no information about the variability of the objects in the sample was used beforehand. This is the first time that a large X-ray–detected sample of quasars has been examined for UV/optical variability. The SDSS sample will soon be large enough to subdivide it further by X-ray brightness. In the meantime, a correlation between X-ray emission and UV/optical variability amplitude can be claimed. We discuss this further in § 7.

6.3. BAL Quasars

BAL quasars (BALQSOs) are defined by the presence of very strong, blueshifted absorption troughs in their spectra. About 5% of the quasars in the SDSS sample can be classified as BALQSOs, a fraction that is heavily redshift- and color-dependent. The largest systematically selected samples of BALQSOs are those of Reichard et al. (2003) and Tolea, Krolik, & Tsvetanov (2002), each of which contains close to the same set of objects drawn (in somewhat different ways) from the SDSS Early Data Release quasar sample. Both sets contain about 200 objects, and for the present purposes the samples are indistinguishable; we have used the Reichard et al. (2003) sample, since the selection process is more automated and is likely to be used for future BALQSO catalogs. A control sample of non-BAL quasars was designed to have consistent redshift, luminosity, and time lag distributions, in the manner described above. The matched sample of BALQSOs contains 178, 189, and 190 objects in the g, r, and i bands, respectively.

The matched structure functions are shown in Figure 21. The time lag binning is necessarily coarse because of the relatively small sample sizes. At the level of sensitivity of this sample, there is no difference in the variability amplitudes of BALQSOs and non-BAL quasars.

Currently favored models of the BAL phenomena attribute the absorption to high-opacity gas, either as clouds or flows, usually viewed near the plane of an accretion disk. If the presence of BALs is purely a viewing angle effect, then continuum variability, if due to the central quasar engine, is unlikely to be correlated. However, if variability is due to the presence of obscuring dust of varying attenuation crossing the sight line to a quasar, BAL quasars may be expected to be more highly variable at optical and UV wavelengths. The issue will need to be settled with a larger sample, but the current results do not support any correlation between the presence of BAL features and UV/optical variability.

7. DISCUSSION

To summarize our results, we have separated the dependence of variability on a number of parameters and found a power-law dependence on time lag, anticorrelations with wavelength and luminosity, and a correlation with redshift. All of these relationships have been parameterized. Radio-loud and X-ray quasars also appear to be more variable than their quiet counterparts. There is currently no model of quasar continuum variability at optical and UV wavelengths that addresses all of the relationships described here, and until recently, there were virtually no quantitative predictions. Current models can be classified broadly into three groups: accretion disk instabilities, discrete-event, or Poissonian, processes, and gravitational microlensing. We ignore other evidence for or against the
theories and describe how variability as an independent phenomenon may constrain the models.

The Poissonian models postulate that quasar luminosity, or at least a significant fraction of it, is generated by some type of multiple discrete and independent energetic events, such as supernovae or star collisions (e.g., Terlevich et al. 1992; Torricelli-Ciamponi et al. 2000). The statistical superposition of the light curves of the randomly occurring events determines the luminosity at any given time. As discussed in §4.3, the simplest Poissonian models predict a luminosity-dependent power-law slope of $\beta = \frac{1}{2}$, which is inconsistent with our results. More detailed models in which the event rate, energy, timescale, and background contribution are adjustable parameters can produce a wide range of slopes (Cid Fernandes et al. 2000), but a value of $\beta = \frac{1}{2}$ is still difficult to avoid in models invoking supernovae and their remnants as the events (Paltani & Courvoisier 1997; Aretxaga, Cid Fernandes, & Terlevich 1997). Another apparently unavoidable consequence of the Poissonian models is that the variable luminosity component is not wavelength-dependent (Cid Fernandes et al. 2000), and any color changes must be the result of a nonvariable component (such as a host galaxy), which must be red to qualitatively account for the wavelength correlation found here and in other studies. We have shown that the variable source itself is wavelength-dependent, and a host galaxy component alone cannot account for the color changes. Quantitative predictions for the power-law slope of the structure function in the starburst (supernova) model (Kawaguchi et al. 1998) range from $\gamma \approx 0.7$ to 0.9, which is also quite inconsistent with the value we find ($\gamma = 0.246$). Thus, based on predictions for the time lag, wavelength, and luminosity dependence of variability, current Poissonian models are inconsistent with the observational results of this study. It remains to be seen if non-Poissonian processes, for example, those in which the events are not independent or random, can account for the observations.

The idea that the motions of intervening matter along the geodesics to quasars may cause flux variations (microlensing) was discussed as early as the late 1970s (Chang & Refsdal 1979), but few quantitative predictions have been worked out with regard to typical quasar variability. Using the simulated microlensing light curves of Lewis et al. (1993) and Schneider & Weiss (1987), Hawkins (2002) generated structure functions that have a power-law form with slopes ranging from $\gamma \approx 0.23$ to $\gamma = 0.31$, which is consistent with what we find. However, the slopes depend on the unknown lens mass distribution function, velocity distribution, and source size (Wyithe & Turner 2001), so a wide range of values is possible. There is little doubt that microlensing of quasar images does happen, and it has likely been detected in at least two cases, Q2237+0305 (Schmidt et al. 2002 and references therein) and Q0957+561 (Refsdal et al. 2000). However, in each case, the quasar is microlensed by a foreground galaxy, which means that the geodesics selectively pass through regions of relatively high density. Wyithe & Turner (2002) showed that the probability of microlensing by stars among single-image (not macrolensed) sources is very small; dark matter composed of compact objects can improve the probability, but at most only about 10% of sources are expected to be microlensed at any given time. In addition, since unresolved macrolensed quasars—which appear more luminous than they really are—are more likely to be microlensed, the anticorrelation of variability amplitude with luminosity is opposite to the trend that would
be expected from microlensing. While the deflection of light by gravity is achromatic, the wavelength dependence of variability is not necessarily evidence against the microlensing hypothesis, as long as quasar-emitting regions are smaller and brighter with decreasing wavelength. It is difficult, however, to see why there would be any dependence on the radio or X-ray properties of the quasar sample. The amplitude of quasar variability changes with redshift, but it is nearly as strong at low redshifts as it is at high redshifts, which also shows that microlensing cannot be the primary cause of variability, since microlensing events should be extremely rare at low redshift. Finally, reverberation mapping studies (Peterson 2001) show that the quasar broad-line region varies in response to changes in the continuum luminosity, showing that a large fraction of variability must be intrinsic to quasars. If some quasar variability is due to microlensing, it will be important to isolate it from the other sources, since it has the potential to constrain the components of dark matter and the structure of quasars.

It is widely accepted that quasar luminosity is generated through some set of processes related to the accretion of matter from a disk onto a supermassive black hole (e.g., Rees 1984). It is therefore natural to consider mechanisms associated with changes in these processes as a source of quasar variability. Qualitatively, most schemes would tend to follow the trends we find here, in particular the anticorrelations of variability with wavelength and luminosity. For example, the disk emission spectrum of the standard optically thick, geometrically thin accretion disk model (e.g., Shakura & Sunyaev 1973) is more luminous and bluer when the accretion rate is higher, and the relative luminosity change would be lower in more luminous objects for a given change in accretion rate. However, it is not known how the accretion rate would change or how the resultant luminosity changes would propagate through the disk. While there has been much theoretical discussion of the various possibilities for the emission mechanisms and their stabilities (e.g., Wallinder, Kato, & Abramowicz 1992; Schramkowski & Torkelsson 1996), there are currently few quantitative predictions that can be compared to observational results. Kawaguchi et al. (1998) generated structure functions for the cellular automaton model for disk instability and found power-law forms with slopes ranging from $\gamma = 0.41$ to 0.49. While this range is inconsistent with our results, the model is necessarily simplified, and a number of assumptions need to be made. The complexity of possible accretion disk (or jet) instability models is likely what has prevented more quantitative predictions. Disk instability models are clearly promising, but as yet it is difficult to compare them to the observations.

In summary, the weight of the observational evidence seems to disfavor gravitational microlensing and generic Poissonian processes as the primary source of quasar variability. Accretion disk instability models have yet to be adequately developed quantitatively for direct comparison with our results. It is also plausible that a combination of sources produces variations in quasar light curves, and no single model can be completely eliminated at this time.

8. CONCLUSIONS

We have examined the ensemble broadband photometric variability of a very large and homogeneous sample of quasars from the SDSS—the largest sample ever used to study variability. The three-band spectrophotometry of each object was compared directly to the imaging photometry obtained at an
earlier epoch. Because of the large number of objects and wide coverage of parameter space, the dependences of variability amplitude on time lag, luminosity, wavelength, and redshift could be disentangled for the first time. The variability amplitude increases with time lag (up to about 2 yr) as a power law with a slope of $\gamma = 0.25$. In terms of the variability amplitude, more luminous quasars are less variable, shorter wavelengths are more variable, and more distant quasars are somewhat more variable; all of these relationships are parameterized. Radio-loud quasars appear to be more variable than their radio-quiet counterparts, and quasars with detectable X-ray emission (in the \textit{ROSAT} survey) are more variable than those without. It is difficult to explain the results in the context of models involving discrete events (Poissonian models) and gravitational microlensing. Accretion disk instability models are promising, but more quantitative predictions are needed to test them against the observational results.

Funding for the creation and distribution of the SDSS Archive has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Aeronautics and Space Administration, the National Science Foundation, the US Department of Energy, the Japanese Monbukagakusho, and the Max Planck Society. The SDSS Web site is http://www.sdss.org.

The SDSS is managed by the Astrophysical Research Consortium (ARC) for the Participating Institutions. The Participating Institutions are the University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, The Johns Hopkins University, Los Alamos National Laboratory, the Max Planck Institute for Astronomy (MPIA), the New Mexico State University, the University of Pittsburgh, Princeton University, the United States Naval Observatory, and the University of Washington.
Voges, W., et al. 1999, A&A, 349, 389
———. 2000, IAU Circ., 7432, 2
Wallinder, F. H., Kato, S., & Abramowicz, M. A. 1992, A&A Rev., 4, 79
Webb, W., & Malkan, M. 2000, ApJ, 540, 652
White, R. L., Becker, R. H., Helfand, D. J., & Gregg, M. D. 1997, ApJ, 475, 479

Wyithe, J. S. B., & Turner, E. L. 2001, MNRAS, 320, 21
———. 2002, ApJ, 575, 650
York, D. G., et al. 2000, AJ, 120, 1579