Optimization method for photovoltaic integration in residential houses

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Abstract. Good design and sizing of photovoltaic (PV) systems is very important in order to effectively harvest energy and minimize the investment cost. The optimum tilt and azimuth angles at which a PV system should be installed are often debated. This paper evaluates the trade-off between annual energy generation and payback period reduction through the analysis of a small house with pitched roof integrated PVs in both East and West sides. Validated irradiance and PV models were used for the analysis. The optimum tilt and azimuth angles are found to be 35° and 10° respectively. Finally, a contour map plot illustrating all possible tilt and orientation angles, corresponding to a payback period less than 20 years, is provided. The results are valid for different building typologies and locations with similar climate conditions as Prague.

1. Introduction

Photovoltaics (PV) is one of the most deployed technologies for generating clean energy and reducing CO₂ emissions. The total worldwide installed capacity has been exponentially increased during last decade, while one Tera Watt peak (TWₚ) of total installed PV capacity is forecasted to be reached in the next few years.

A good design and sizing of PV systems is very important in order to effectively exploit energy and reduce the payback period of the investment. Tilt and azimuth angles of an installation are key parameters that determine the amount of solar radiation received on the PV modules and thus the yield of the PV system. The assessment of the optimum tilt angle for the maximization of solar energy harvesting is still subject of interest, and researchers are still actively working on the topic [1-4].

This paper evaluates the trade-off between annual energy losses and possible electricity generation cost reductions through adapting PV installation angles on residential buildings in Prague, Czech Republic and other locations with similar conditions. A small house with pitched roof was used for the analysis of the PV generation in relation to the tilt angle and orientation of the roof based on validated irradiance and PV models. Finally, considering the actual costs of the selected thin-film PV system, conclusions are drawn indicating the range of tilt and azimuth angles limits in such houses for making a PV system investment beneficial and at least have a payback period equal to a PV module lifetime.

2. Experimental setup description

An experimental house located near Prague (Latitude: 50°09'24.2"N, Longitude: 14°10'10.5"E) with pitched roof, depicted in Fig. 1a, was used for the analysis of the behaviour of two thin-film PV systems (CdTe) integrated in different orientations (East-West). The PV systems outputs and weather data are
being continuously monitored in five-minute timesteps since September 2018. There are two Si-RS485TC-2T-MB irradiance sensors (for both East and West) with external temperature sensors glued to the back side of PV modules. Additionally, there is a Kipp & Zonen SMP11 pyranometer installed on the West side of the roof. All the sensors are measuring the irradiance at the roofs’ tilt angle of 30°.

The PV system size in each part of the roof is 1.92 kWp, and it is composed of 24 CdTe PV panels connected to two micro-inverters. Each micro-inverter has four channels, and every channel (connecting one string composed of three PV modules) is monitored separately. Additionally, as the house is at an azimuth angle of -22° (turned to East), measured data from various irradiance sensors (Fig. 1b) in different azimuth and tilt angles—Horizontal, South (90°, 60° and 30°), East (60° and 30°), West (60° and 30°)—installed 30 meters from the house were used for the calibration of the irradiance model.

2.1. Measured solar irradiance
One-year monitored data in one-minute timesteps obtained from the irradiance sensors shown in Fig. 1b, were used for the analysis of the solar potential in different orientations and tilt-angles. Fig. 2 shows the hourly average data of the measured irradiance profiles for summer and winter days. It can be seen that the highest irradiance values in winter are obtained for the tilt-angle 60° South. However, in summer, the tilt-angle with higher solar intensities is 30° South. Tilt-angles of 30° and 60° for East and West orientations provide better performance in summer time with maximum values that are 40% higher compared to the winter period. Regarding the horizontal (0°S) and vertical irradiance (90°S), profiles obtained for summer and winter days, show that, the vertical surfaces receive high irradiance in winter compared to summer, while the opposite is observed for the horizontal surfaces.

2.2. Measured PV power
The monitored PV outputs from the East and West roofs indicate that the West PV system is performing better than the East one. Table 3 shows that the East-facing PV system generates up to 45% less energy per month compared to the West-facing system. The difference between the East and West energy generation is mainly due to the orientation of the house (22° to the East).
The low East PV system energy generation affects the payback period of the system. Thus, the present work is conducted to assess and find the maximum azimuth and tilt angles limits in order to get a payback period lower than the lifetime of the project.

### Table 1. Monitored PV outputs and on-plane solar irradiance (G).

| Months | PV (kWh) | G (kWh/m²) | PV (kWh) | G (kWh/m²) |
|--------|----------|------------|----------|------------|
| Sep-18 | 99.2     | 62         | 155.2    | 86.7       |
| Oct-18 | 47.3     | 29.7       | 84.6     | 50         |
| Nov-18 | 27.3     | 18.2       | 52       | 30.9       |
| Dec-18 | 17.8     | 13.2       | 30.1     | 19         |
| Jan-19 | 24.1     | 17.5       | 42.7     | 26.1       |
| Feb-19 | 55.9     | 32.8       | 107.9    | 59.9       |
| Mar-19 | 112.6    | 66.6       | 153.8    | 86.7       |
| Apr-19 | 175.2    | 103.2      | 271.7    | 151.7      |

### 3. Methodology

The optimum angles at which a PV system should be mounted are often debated. In this context, an irradiance model was utilized for the analysis of solar radiation on tilted surfaces using a typical meteorological year (TMY) of a Global Horizontal Irradiance (GHI) for the city of Prague. One-year data of solar irradiance obtained from the setup (see Fig. 1b) were also used for the analysis and for the validation of the model. The effects of tilt and azimuth angles are presented on contour plots, which are convenient for cost analysis and the determination of the annual insolation on building surfaces.

#### 3.1. Irradiance model

The estimation of the total radiation incident on each surface requires knowledge of total and diffuse (or beam) radiation on a horizontal surface as well as the sun’s position. In general, the total tilted surface radiation is calculated by estimating and adding beam, diffuse and reflected radiation components on the tilted surface. The contribution of beam radiation on a tilted surface can be calculated by using a geometric factor dependent on the horizontal tilt, surface azimuth, declination angle and latitude (Eq.1). The contribution of reflected radiation on a tilted surface is calculated by assuming the ground acts as an isotropic reflector (Eq.2). Finally, the diffuse radiation on a tilted surface is determined by using Perez model [5]. This model accounts for circumsolar, horizon brightening, and isotropic diffuse radiation by empirically derived “reduced brightness coefficients (F₁ and F₂)” given in Eq.3.

\[
I_{b,T} = \frac{I_{h,b}}{\sin \alpha} (\sin \alpha \cos \beta + \cos \alpha \sin \beta \cos |\gamma - \gamma_n|) \tag{1}
\]

\[
I_{g,T} = I_G \left( \frac{1 - \cos \beta}{2} \right) \times \rho_g \tag{2}
\]

\[
I_{d,\text{tilt}} = I_a \left( \frac{1 + \cos \beta}{2} \right)(1 - F_1) + F_1 \cos \theta + F_2 \sin \beta \tag{3}
\]

where, \(I_{h,b}\) is the horizontal beam radiation (Wh/m²), \(\beta\) is the inclination angle, \(I_G\) is horizontal global radiation (Wh/m²), \(\alpha\) is the solar altitude (rad), \(\rho_g\) is the ground reflectance, \(\gamma\) is solar azimuth, \(\gamma_n\) is the azimuth angle of the normal of the surface (rad), \(\theta\) and \(\theta_0\) are the zenith angle and angle of incidence.

#### 3.2. PV model

The Sandia Array Performance Model (SAPM) was used for the estimation of the output power generated by the PV array at the maximum power points (MPP) [6]. This model is empirical and is described by the following equations:

\[
E_e = \frac{g}{g^*} \tag{4}
\]

\[
I_m = N_{pg} \left[ I_{m,0}(C_0 E_e + C_1 E_e^2) \left( 1 + \alpha_{imp}(T_c - T_c^*) \right) \right] \tag{5}
\]
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\[ V_{mp} = \frac{N_{sg} \left[ V_{mo} + C_2 N_s \delta(T_c) \log(E_c) + C_3 N_s (\delta(T_c) \log(E_c))^2 + \beta_{Vmp} E_c (T_c - T_c^r) \right]}{n k (T_c + 273.15)/q} \]

where, \( I_{mo} \) (A) and \( V_{mo} \) (V) are the PV module current and voltage of the MPP at standard test condition (STC). \( C_2 \) and \( C_3 \) are empirically determined coefficients (dimensionless) which relate \( I_{mp} \) to the effective irradiance. \( \alpha_{mp} \left( \circ^\circ \right) \) is the normalized temperature coefficient for \( I_{mp} \), \( C_1 \) (dimensionless) and \( C_3 (V^{-1}) \) are empirical coefficients which relate \( V_{mp} \) to the effective irradiance, \( \delta(T_c) \) is the thermal voltage per cell at temperature \( T_c \), \( q \) is the elementary charge, \( 1.60218 \times 10^{-19} \) (coulomb), \( k \) is the Boltzmann’s constant, \( 1.38066 \times 10^{-23} \) (J/K) and \( \beta_{Vmp} \left( V/\circ \right) \) is the module temperature coefficient \( V_{mp} \) at STC.

The SAPM reposes on some coefficients \( (C_0, C_1, C_2, n, \alpha_{mp}, \beta_{Vmp}) \) in order to reproduce the real behaviour of the PV module/array. The extraction of these coefficients is done from real measured profiles of the PV module/array following the same procedure published previously in [7].

3.3. Integration of PV and economic analysis

Apart from energy yield, it is very important to investigate the economic profitability and provide guidelines for the optimal integration of the PV system considering the tilt angle and orientations of the pitched roof. In this context, the Net Present Value (NPV) – defined as the sum of present incoming (benefits) and outgoing cash flows over the lifetime of the project – was calculated according to Eq.8.

\[ NPV = -C + \sum_{i=1}^{N} \frac{F(t)}{(1+i)^t} \]

where, \( C \) is the initial investment cost (€), \( F(t) \) is the annual generated income (€/year), \( N \) is the lifetime of the PVs and \( i \) is the real rate of interest for the Czech Republic. The initial investment includes the cost of the PV modules, electrical components and claddings. Considering the actual size of the installation in each part of the roof (1.92 kWp), a typical value of 1200 €/kWp was assumed representing the current PV costs in the Czech Republic [8]; 1%/initial cost was also considered for O&M.

The annual cash inflows are calculated considering the annual PV generation (100% of PV self-consumption) and PV degradation rate (%/year) [9], as well as the energy price and the discount rate valid for the particular moment in the Czech Republic [10]. Then, cash flow over the life-time of the PV system (25 years-expected) is calculated and results are given for the NPV and payback time.

4. Results and discussion

4.1. Models validation

The Matlab® environment was used for the implementation of the irradiance and PV models previously described. The validation of the models showing simulated data versus measured ones is given in Fig. 3. The irradiance model was validated by a measured data obtained for a tilt angle of 30° facing the South. Selected ten days’ data represent different weather conditions, and it can be observed that the model performs very well in reproducing the real data. Regarding the validation of the PV model, monitored data of one channel – one string composed of three PV modules– and same data length considering different weather conditions were used. A good match between simulated and measured data can be seen. Calculated Root Mean Square Error (RMSE) values, for both models are less than 5%.

The validated irradiance model was used for the estimation of the solar radiation on surfaces with different tilt angles varying from 0° to 90°, considering 360° azimuth angle variations. In that way, yearly irradiation values can be obtained for each set of tilt and azimuth angles. The obtained results for the location of the studied house are given in Fig. 4 (Left) as a contour plot. The South, East, West are represented by 0°, -90° and 90° respectively. The highest solar radiation of 1200 kWh/m² is obtained for the azimuth angles varying from -10° to 30° and for tilt angles between 25° and 45°.

The annual PV energy generation was also calculated based on the irradiance obtained for each set of tilt and azimuth angles. The PV energy generation is calculated for the CdTe PV modules technology in kWh per unit area. The obtained annual PV energy values shown in Fig. 4 (Right) represent the real PV outputs including all power losses. The results show that the PV energy values are affected similarly...
as the irradiance by tilt and azimuth angles. Moreover, the effect of temperature is also present, and leads to a broader range of tilt and azimuth angles associated with the maximum PV generation (in the range of 90 kWh/m² per year).

Figure 3. Validation of the irradiance model, and PV model.

Figure 4. Contour plots of annual solar yield (Left side) and annual PV energy generation (Right side)

4.2. Optimal integration of PV

Results from the assessment were also plotted as contour maps indicating the limits of azimuth and tilt angle for positive value of the NPV (Fig. 5). In accordance with the PV generation, maximum NPV was obtained for 35° tilt and 10° azimuth angle, leading to 13.5 year payback time. Fig. 5 (Left) shows an optimal position slightly shifted to the West. However it can be seen that NPV is relatively insensitive to minor deviations (between -30° to +30°) of azimuth from the sub-optimal orientation. Regarding the tilt angle, integration is suggested up to 10° regardless the orientation. When orientation is concerned, maximum benefits can be achieved for tilt angles between 20° and 45°, limiting the payback time under the 15 years.

Finally, the right side of Fig. 5 depicts all the possible values of tilt and azimuth angles which give a payback period lower than 20 years. These values were obtained from the analysis of one side of the roof (1.92 kWp). Thus, in order to get a payback period less than 20 years for a building with a symmetric pitched roof, its orientation should be 10°, and the roofs inclination angle equal or lower than 30°.

5. Conclusions

The present work showed the effect of azimuth angle on the performance of PV systems integrated to the pitched roof of an experimental house located near the capital of Czech Republic, Prague. The analysis carried out for the determination of the optimum tilt and azimuth angles is based on a validated irradiance and PV models. The accuracy of the calibrated model throughout real measured data showed a RMSE values lower than 5%. The obtained results from the analysis of the variation of tilt and azimuth angles revealed that, the optimum angles for maximizing the solar yields correspond to tilt = 35° and azimuth = 10°. For the maximization of the yearly energy PV generation, it was found that there is a specific range of tilt and azimuth angles which can give the same maximum values. Finally, based on
the actual costs of the studied thin-film PV system, the maximum NPV value that can be obtained after 25 years of operation under optimum tilt and azimuth angles is of 1156 Euros.

![Figure 5. NPV sensitivity to tilt and azimuth angles and relative limits for increased profitability.](image)

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