Gravitational and relativistic deflection of X-ray superradiance

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Einstein predicted that clocks at different altitudes tick at various rates under the influence of gravity. This effect has been observed using $^{57}$Fe Mössbauer spectroscopy over an elevation of 22.5 m (ref. 1) or by comparing accurate optical clocks at different heights on a submetre scale. However, challenges remain in finding novel methods for the detection of gravitational and relativistic effects on more compact scales. Here, we investigate a scheme that potentially allows for millimetre- to submillimetre-scale studies of the gravitational redshift by probing a nuclear crystal with X-rays. Also, a rotating crystal can force interacting X-rays to experience inhomogeneous clock tick rates within it. We find that an association of gravitational redshift and special-relativistic time dilation with quantum interference is manifested by a time-dependent deflection of X-rays. The scheme suggests a table-top solution for probing gravitational and special-relativistic effects, which should be within the reach of current experimental technology.

Modern X-ray science has opened up an entirely new era of quantum optics. The vast and unexplored field of X-ray quantum optics provides many possibilities for both fundamental research and applications. In particular, studies of the interaction between X-rays and nuclear condensed-matter systems have led to novel control of X-ray photons. Furthermore, the well-known Pound–Rebka experiment shows that the Mössbauer effect is remarkably useful for testing gravitational redshift by detecting the influence of Earth’s gravity on the nuclear absorption of X-rays. Another result obtained with a rotating nuclear crystal demonstrates that the interaction between X-rays and nuclei is also useful for testing special relativity. Therefore, X-ray quantum optics with nuclei may offer a new approach to the exploration of gravity and special relativity in quantum systems. The gravitational redshift indicates that clocks at high altitudes run faster than those at sea level under the influence of Earth’s gravity. Also, according to special relativity, a moving clock ticks at a slower rate than a fixed one in an observer’s rest frame. Thus, within a crystal, identical particles at different lattice sites form an array of clocks that tick at inhomogeneous rates when they are influenced by gravity or by time dilation while subject to inhomogeneous motion. This opens up the possibility of treating a tiny crystal as a probe to reveal the properties of space at the scale of femtometres. Probing gravitational or relativistic effects on small length scales may not only demonstrate their influence on quantum systems (such as the performance of optical clocks), but may also test the ansatz of quantized space and time in theories of quantum gravity that are inconsistent with the continuous version in general relativity.

Here, we investigate a scenario that manifests distinct deflections of a single photon under the action of gravity or special relativity and quantum interference. Considering long-lived nuclear states, one could shine an X-ray on a thin nuclear crystal to create a collective excitation that is simultaneously perturbed by relativistic time dilatation. Because the excitation is delocalized over the whole ensemble of nuclei, it evolves with an inhomogeneous rate caused by Earth’s gravity. We find that the inhomogeneous evolution of a delocalized excitation causes a deflection of the reemitted single photon. This time-dependent deflection suggests that the photon trajectory can be influenced by Earth’s gravity, even though it is stored as a stationary quantum excitation in a crystal. An analogue in the optical domain with slow light propagation in media (≈100 m s$^{-1}$) proposes a light deflection of $\sim 10^{-9}$ o (ref. 24), but might raise challenges for experimental detection.

A quantum collective excitation occurs when a single photon is absorbed by a collection of $N$ particles. This single photon is then shared by $N$ particles, which leads to a delocalized collective excitation state as depicted in Fig. 1a. The collective excitation state can be written as

$$|E\rangle = \frac{1}{\sqrt{N}} \sum_{\ell} e^{i\tilde{k}_0 \cdot \vec{r}_\ell} e^{i\nu_G - \omega_G t}|g\rangle|\ell\rangle \tag{1}$$

Here, $|g\rangle|\ell\rangle$ denotes that particle $\ell$ at position $\vec{r}_\ell$ is in its excited state $|e\rangle$, while the other $N-1$ particles remain in the ground state $|g\rangle$ as illustrated in Fig. 1b. The excitation energy of particle $\ell$ is $\hbar \omega_G$, where $\hbar$ is the reduced Planck constant, and $\nu_G$ and $\tilde{k}_0$ are the angular frequency and wavevector of the incident single photon, respectively. As a result of quantum interference between the emission from each crystal site, a directional reemission of a single photon, namely, superradiance, follows the decay of state $|E\rangle$ along the direction $\tilde{k}_0$ of an incident photon. Directional superradiance is routinely observed in nuclear forward scattering of X-rays using a nuclear solid-state crystal (typically a few micrometres thick and a few millimetres in diameter). These tiny dimensions therefore give the upper bound of the spatial scale where relativistic effects could be probed by our scheme.

A gradient perturbation, as depicted in Fig. 1c, can be utilized to control the direction of superradiance. As some external gradient perturbation is applied to the whole ensemble, the originally constant $\omega_G$ becomes inhomogeneous $\omega_G(\vec{R} + \vec{r}_\ell)$, where $\vec{R}$ is the position of the ensemble relative to the origin of the perturbation and $\vec{r}_\ell$ is the particle position in the ensemble coordinates. Assuming that the ensemble size is much smaller than $|\vec{R}|$, one can use the Taylor expansion $\omega_G(\vec{R} + \vec{r}_\ell) \approx \omega_G(\vec{R}) + \nabla \omega_G(\vec{R}) \cdot \vec{r}_\ell$. By substituting the expanded $\omega_G(\vec{R} + \vec{r}_\ell)$ into equation (1), the collective excitation state (see Methods and Supplementary Section 3) becomes

$$|E\rangle = \frac{1}{\sqrt{N}} \sum_{\ell} e^{i\tilde{k}_0(t) \cdot \vec{r}_\ell} e^{i(\nu_G - \omega_G(\vec{R})t)}|g\rangle|\ell\rangle \tag{2}$$

where the new time-dependent wavevector $\tilde{k}_0(t) = \tilde{k}_0 - t \nabla \omega_G(\vec{R})$ indicates the deflection of superradiance. If $\tilde{k}_0$ and $\nabla \omega_G(\vec{R})$ are...
**Figure 1 | Superradiant single photon.** a. An incident single photon with wavevector $\vec{k}_0$ is absorbed and shared by an ensemble of two-level quantum particles. Without knowing which one is excited, a collective excitation, which is a superposition of all possibilities, will form. The decay of such a delocalized collective excitation will be followed by directional emission of a single photon along the incident $\vec{k}_0$. This coherently reemitted photon is called a superradiant single photon. Yellow and red dots illustrate particles in the ground and excited states, respectively, and red arrows depict both the incident and the reemitted single photon. b. A collective excitation is created in a collection of two-level particles while a particle is excited from its ground level $|g\rangle$ to a higher energy level $|e\rangle$ by absorbing a single photon with wavevector $\vec{k}_0$. c. A superradiant single photon is deflected by a gradient perturbation depicted by the brown upward arrow. $\phi$ is the deflection angle between the incident wavevector $\vec{k}_0$ and deflected wavevector $\vec{k}_S$. 

**Figure 2 | Gravitational deflection of X-ray superradiance.** a. Earth’s gravity deflects the superradiant single X-ray photon from its incident direction with time-dependent angle $\phi_g$. The green cuboid depicts a fixed nuclear crystal in the gravitational field of the Earth, the red and blue arrows represents the incident and reemitted single photon, respectively, and yellow horizontal bars illustrate the count numbers at different positions of a position-sensitive detector. b. Influence of gravity on proper time. Clocks closer to the Earth run slower. The same happens to fixed nuclei (yellow dots) in a crystal (depicted by the green rectangle). Earth, crystal size and clock ticks are not in scale.
Table 1 | Maximum deflection angles of superradiant photons induced by Earth’s gravity $\phi_g(t_{\text{coh}})$ and by the crystal’s inhomogeneous speed in a rotor $\phi_C(t_{\text{coh}})$.  

| Crystal | $E_s$ (keV) | Coherence time $t_{\text{coh}}$ | $\phi_g(t_{\text{coh}})$ (°) | $\phi_C(t_{\text{coh}})$ (°) |
|---------|-------------|-------------------------------|-----------------------------|-----------------------------|
| $^{55}$Sc | 12.4 | 459 ms | $8.6 \times 10^{-7}$ | 90 |
| $^{57}$Fe | 14.41 | 141 ns | $2.6 \times 10^{-5}$ | 90 |
| $^{67}$Zn | 93.31 | 13.09 µs | $2.5 \times 10^{-11}$ | 2.4 $\times 10^{-3}$ |
| $^{73}$Ge | 13.28 | 4.21 µs | $7.9 \times 10^{-12}$ | 7.8 $\times 10^{-4}$ |
| $^{109}$Ag | 88.03 | 57.13 s | $1.1 \times 10^{-4}$ | 90 |
| $^{181}$Ta | 6.24 | 8.73 µs | $1.6 \times 10^{-4}$ | 1.6 $\times 10^{-3}$ |
| $^{182}$Ta | 16.27 | 408 ms | $7.7 \times 10^{-5}$ | 90 |
| $^{229}$Th:CaF$_2$ | 0.0078 | 1 ms | $1.9 \times 10^{-9}$ | 0.18 |

$\tau_{\text{coh}}$ is the coherence time of the corresponding nuclear transition. The parameters of the rotor are $R = 5$ mm and $A = 2\pi \times 70$ kHz. $E_s$ is the nuclear excited-state energy, which also corresponds to the used photon energy$^{25}$.

perpendicular to one another, the deflection angle $\phi(t)$ between $\vec{k}_0$ and $\vec{k}(t)$ can be written as

$$\phi(t) = \tan^{-1}\left[\frac{|\nabla \omega_0(R)| t}{|\vec{k}_0|}\right]$$

(3)

In this Letter, we demonstrate that the superradiant deflection described by equations (2) and (3) may be introduced by Earth’s gravity and by the time dilation of special relativity in a rotating system.

The studied system (on the surface of the Earth) is depicted in Fig. 2. Trains of monochromatic X-ray synchrotron radiation pulses or single X-ray photons resonant to a nuclear transition impinge on the crystal along the red arrow$^2$. The X-ray pulses are spaced at intervals longer than the coherence time $t_{\text{coh}}$, defined by, for example, the lifetime of a nuclear excited state. Each weak synchrotron radiation pulse or single X-ray photon mostly excites a single nucleus in a crystal and so creates the above collective excitation state $|E\rangle$ defined by equation (2)$^{12,13,14,16}$. Because of gravity, particles at various crystal sites evolve differently in time, depending on their position. The gravitational redshift deduced from the Schwarzschild metric gives $|\nabla \omega_0(R)| = \omega \times 1.09 \times 10^{-19}/\text{m}$ on the Earth’s surface, which is consistent to a good degree with the experimental redshift found by Pound and Rebka$^1$. Because of this inhomogeneous quantum phase evolution, the superradiant photons are reemitted along the blue arrow in Fig. 2a, deflected by the time-dependent angle (Supplementary Sections 1 and 3)

$$\phi_g(t) \approx \tan^{-1}\left[\frac{GM_E t}{c^2 \sqrt{1 - \frac{2GM_E}{c^2 R_E^3}}\tau_{\text{coh}}^2}\right]$$

(4)

where $G$ is the gravitational constant, $M_E$ is the mass of the Earth, $c$ is the speed of light in vacuum, and $\tau_{\text{coh}}$ is its radius. Because gravity is a weak force, the resulting angular deflection velocity is $\delta\phi_g \approx 1.9 \times 10^{-6}$ s$^{-1}$, so $\tau_{\text{coh}} > 0.5$ s is required to observe a resolvable deflection angle of $10^{-6}$° with modern X-ray optics. As listed in Table 1, some candidate nuclei transitions (such as those of $^{45}$Sc, $^{109}$Ag and $^{182}$Ta) already give large enough $\phi_g$ to be observed.

Figure 3 | X-ray superradiance under the influence of special relativity. a, X-ray photon deflection by time dilation of special relativity, where the crystal (green cuboid) is attached to a rotor with radius $R$ and is subject to rotation with angular frequency $\Lambda$. Under the influence of inhomogeneous relativistic time dilation, the incident single X-ray photon (red arrow) is bent along the blue arrow with a time-dependent angle $\phi_C$ and time-dependent reemission probability (yellow bars on the position-sensitive detector). The red circle on the detector illustrates the trajectory of the registered signals without considering the inhomogeneous relativistic time dilation. b, According to the time dilation effect, the tick rate of the proper time of each nucleus at different radii, $r$, depends on the local velocity, $|\nabla \ell_0| = \Lambda r$, within a crystal. Also, green arrows illustrate local velocities of nuclei (yellow dots) at different sites of a rotating crystal (green rectangle).
In particular, the $\phi_{\text{coh}}$ of $1.1 \times 10^{-4}$ s given by $^{109}$Ag allows for a larger divergence angle of superradiance. This leads to a tiny length scale of $\approx 37 \mu m$ on which the gravitational effect can be observed, which may require considerable experimental effort (Supplementary Sections 6 and 9). This gravitational deflection depends on the strength of gravity. For instance, if the Earth was compressed to a compact size of 1.3 km in diameter, $\phi_{\text{coh}}$ would speed up to about 180° s$^{-1}$. In what follows, we show that such a strong deflection can be achieved by an inhomogeneous time dilatation of special relativity.

We turn to the realization of a larger deflection of superradiance by a fast rotating crystal (Fig. 3). An inhomogeneous clock tick rate in a crystal (caused by time dilatation of special relativity) mimics the time gradient of a gravitational field$^{17}$. A nuclear crystal is fixed on a rotor with a rotating angular frequency $\Lambda$ and radius $R$. As the entire system rotates, a particle at site $r_C$ of the crystal moves at velocity $\mathbf{v}(t) = \Lambda (R + r_C) \hat{e}_z$, as shown in Fig. 3b. According to special relativity$^{17}$, the transition angular frequency of a particle at site $r_C$ of a crystal becomes $\omega_{\text{coh}} = \omega_{\text{coh}} \sqrt{1 - (\Lambda (R + r_C) / c)^2}$, and therefore a spatial gradient of $\omega_{\text{coh}}$ points in the radial direction of the rotor in this configuration. Here, $\omega_{\text{coh}}$ is the transition angular frequency of a particle in its rest frame. To create a collective excitation in a rotating crystal as depicted in Fig. 3, the crystal is also illuminated by a train of X-rays spaced by $\tau_{\text{coh}}$ along the red arrow. Subsequently, single X-ray photons scattered off the crystal are registered by a position-sensitive detector along the blue arrow in Fig. 3a with a time-dependent deflection angle (Supplementary Sections 2 and 3):

$$\phi_{\text{coh}}(t) = \tan^{-1} \left[ \frac{RA^2 t}{\sqrt{c^2 - \Lambda^2 R^2}} \right]$$

A rotor with a radius of 5 mm and rotation frequency of 70 kHz, as previously used in the so-called nuclear lighthouse effect$^{1}$, allows an angular velocity of $\omega_{\text{coh}} \approx 185° \text{s}^{-1}$, yielding distinct deflection angles with a requirement of only $\tau_{\text{coh}} > 10 \mu s$. As demonstrated in Table 1, many nuclear transitions (for example, $^{57}$Fe and $^{62}$Zn) are suitable for that purpose. Remarkably, some transitions with $\tau_{\text{coh}} > 0.3 \mu s$ may even allow $\phi_{\text{coh}} > 90°$ and deliver an unmistakable signature of relativistic deflections. We emphasize that the key parameter for the present effect is the coherence time of an interacting transition with photons. Coherence times of a few milliseconds, $1 \mu s$ and $40 \mu s$ have been proposed or experimentally achieved in systems with $^{228}$Th-CaF$_2$ (ref. 26), diamond nitrogen-vacancy centres$^{27}$ and Pr$^{3+}$: Y$_2$SiO$_5$ (ref. 28), respectively. These quantum memories, together with the recently demonstrated diamond silicon-vacancy centre$^{29}$, deserve attention and may be suitable for our scheme at different energy scales. Moreover, a semiclassical analysis shows that the deflection angle can be enhanced by using an optically dense crystal (Supplementary Section 5.2).

In conclusion, we have investigated a scheme for the special-relativistic and gravitational deflection of X-ray superradiance on a millimetre to submillimetre scale. Given that such a small length scale is difficult to access with the Pound–Rebka experiment, the present set-up would provide an improvement for the demonstration of gravitational effects. Gravitational superradiant deflection may also provide a novel method for measuring locally the gravitational acceleration demonstrated by equation (4). Moreover, the special-relativistic photon deflection set-up could become a new type of X-ray optics for delivering high-energy photons into tunable angles with a wide range. This could be done by adjusting the radius $R$ or the rotation frequency $\Lambda$ of the used rotor, as described by equation (5). This scheme therefore suggests an innovative way not only to explore quantum light–matter interactions with gravity and special relativity on a tiny scale, but also to turn fundamental physics into applications.

**Methods**

We have used the well-known Weiskopf–Wigner theory$^{23}$ to calculate the relativistic deflection of superradiant emission following the decay of a collective excitation state $|E\rangle$ according to equation (2). We treat the gravitational redshift as a space-dependent detuning in flat background space–time (Supplementary Section 1 and Supplementary equations (1) and (6)). Equations (2) and (3) give an intuitive picture of our scheme, but further details need to be taken into account to describe a more general system, as illustrated in Fig. 1c. We align our coordinate system, such that $\hat{e}_z$ is the unit vector parallel to the incident wavevector $\mathbf{k}_e$, whereas $\hat{e}_x$ is the unit vector perpendicular to $\mathbf{k}_e$. The latter unit vector is also parallel to the direction of Earth’s gravity or the radial direction of a rotor. Moreover, a time-independent perturbation (for example, Earth’s gravity) perpendicular to $\mathbf{k}_e$ and a time-dependent perturbation induced by some external fields parallel to $\mathbf{k}_e$ are considered. One could then define $\mathbf{k}_{\text{coh}} = \mathbf{k}_e - \epsilon \mathbf{Q}$, $\mathbf{Q} = \mathbf{Q} \parallel \omega(t) (\mathbf{R}, \mathbf{t}) d\mathbf{r}$, where a constant $\mathbf{Q}$ and a time-dependent $P(t)$ are used. Accordingly, the time-dependent superradiant wavevector reads

$$\mathbf{k}_{\text{coh}}(t) = \mathbf{k}_e - \epsilon \mathbf{Q}(t)$$

Note that, without $P(t)$, $|k_{\text{coh}}|$ increases in time and may break energy and momentum conservation, preventing coherent emission after a certain time. This problem can be solved by introducing $P(t) = \mathbf{k}_e - \epsilon \mathbf{Q}(t)$ in order to maintain $|k_{\text{coh}}| = |k_e|$. With this refinement, the deflection angle is given by

$$\phi_{\text{coh}}(t) = \tan^{-1} \left[ \frac{Q t}{\sqrt{\mathbf{k}_e^2 - Q^2 t^2}} \right]$$

which is used to calculate $\phi_{\text{coh}}(t_{\text{coh}})$ and $\phi_{\text{coh}}(r_{\text{coh}})$ in Table 1 (see Supplementary Sections 1 to 4 and Supplementary Table 2 for the explicit formulae). The time-dependent $P(t)$ is the reason for introducing the time-dependent $\omega(t) (\mathbf{R}, \mathbf{t})$ above and for modifying the frequency gradient $P(t)$ and $Q t$ into time integral expressions. In practice, $P(t)$ can be introduced, for example, by an inhomogeneous Zeeman shift$^{24}$. A time-dependent magnetic gradient field $\mathbf{B}(t, \mathbf{r}) = \epsilon \mathbf{Q}(t) \mathbf{Q}(t) - \epsilon \mathbf{Q}(t) \mathbf{Q}(t)$ can therefore be utilized to reveal the deflected superradiant emission, where $\mathbf{Q}(t)$ is the magnetic moment of the corresponding particle. Finally, the oscillating behaviour of photon counts on a space-sensitive detector, illustrated by yellow bars in Figs 2 and 3, reveals the multiple scattering of X-ray photons in a crystal. This is also calculated by using either Weiskopf–Wigner theory or the Maxwell–Bloch equation (Supplementary Section 5).
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**Author contributions**
W.-T.L. and S.A. contributed equally to this work.

**Additional information**
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to W.-T.L.’s current address is Max-Planck-Institut für Physik komplexer Systeme and Center for Free-Electron Laser Science, Germany.

**Competing financial interests**
The authors declare no competing financial interests.