Computation of Horizontal Correlation of Sound in Presence of Internal Waves in Deep Water and Long Distances

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ABSTRACT

Numerical solutions are given for a parabolic approximation of the acoustic wave equation at 200 and 250 Hz in two and three spatial dimensions to determine if azimuthal coupling in the cross-range coordinate significantly affects horizontal correlation in the presence of internal gravity waves in the sea. No evidence for coupling is found for distances of 4000 km and less. This implies that accurate solutions are possible using computations from uncoupled vertical slices. Shapes of horizontal correlation are closer to shapes given by two theories than at lower frequencies.

I. INTRODUCTION

The horizontal correlation of low frequency sound in the deep ocean is important for practical problems including how long an array can be built to yield gain in signal-to-noise ratio via beamforming. Applications include Navy detection and location systems and international programs for the detection and location of nuclear blasts in the sea. It has long been believed that internal gravity waves may set the limit for horizontal correlation in these circumstances, and comparison between models and data seem to back this up [1, 2, 3, 4, 5]. The models in these studies did not solve a three spatially dependent wave equation (3D) because of the very large computer demands in doing so. Instead, the models computed two-dimensional solutions of a wave equation along vertical slices through a three dimensional sound speed field. This is known as a N x 2D approach and is much more efficient than computing a 3D solution. In any case, the fact that the N x 2D solutions resemble the measurements indicates a 3D solution may not be needed in such cases. The purpose of this study is to see if there is any significant difference between a N x 2D solution and a 3D solution at frequencies of 200 and 250 Hz and distances up to 4000 km. Previous numerical studies found no significant difference up to 150 Hz [6, 7], which is consistent with analysis of signals near 75 Hz and a few thousand kilometers distance [1, 2, 3, 4]. Comparisons between
2D and 3D solutions have not been made at higher frequencies such as 250 Hz where experimental measurements of horizontal correlation have been made [5]. Due to the unusual availability of a large amount of computer time at a supercomputer center, the comparison at these higher frequencies was made. There are dozens of experiments up to 250 Hz and at basin-scales where horizontal arrays have collected data, so the current study is relevant for considering whether or not a 3D solution is needed to predict horizontal correlation.

Solution of the linear acoustic wave equation is impractical on a supercomputer at 250 Hz and 4000 km distance. Instead, a parabolic equation (PE) is solved that is barely practical to implement. There are many PEs that could be used for this calculation, and the one used here is very accurate in the vertical dimension, yielding accurate travel times of all acoustic paths at thousands of kilometers distance, regardless of the launch angle of the paths at the source [8]. At long distances, an accurate PE is needed in this application because these launch angles go from roughly -15 to +15 degrees. On the other hand, the horizontal correlation of sound is of order 1 km at a few hundred Hertz and order 1000 km distance, so the effect is associated with a horizontal angle of about 1/1000 = 0.001 radians. All of the PE approximations are reported to be valid at small angles, and the one we choose is a standard small angle approximation for the horizontal component [9]. No PE approximations would get the phase difference right at hydrophones separated by fractions of kilometers or more at distances of several thousands of kilometers, not even the sound-speed insensitive approximation if it could be made to work in both the horizontal and vertical planes with coupling. But it does not seem important to get the absolute phase differences right to get an accurate estimate of the horizontal correlation due to fluctuations of the sound speed field. Its the relative changes in the phases with time at two points that matter for computing horizontal correlation. It is not known through any mathematical theorem if any existing PE approximation is accurate enough to yield reliable estimates of the sought-after effects. Another possible approach is to utilize the Thomson-Chapman wide angle PE approximation in the vertical and horizontal planes [10]. It is not known if it is any more accurate than the approximation used here. With the wide-angle approximation, for example, travel times of some multipath at 1000 km can be in error by order one second because the single reference speed needed for the approximation makes the travel time right for only one equivalent vertical launch angle. In summary, the approximations used here are reported to be valid for the magnitude of the effects being investigated. Results from this paper could be checked in the future if someone invents a computationally practical method yielding more accurate solutions for the wave equation.

II. LIMITED HORIZONTAL DOMAIN AND CONVERGENCE

The calculations follow the same methods used before [6] [7], including the method by Smith [11] that was followed meticulously in all respects to make the problem computationally practical by confining the numerical solution to a limited horizontal domain
We elaborate on two additional details not previously reported. Firstly, Smith explains the computational domain near the source must first be computed over the entire 360 degrees of azimuth for a few acoustic wavelengths. Afterward, the computational domain may be restricted horizontally (e.g. Fig. 1 in Ref. [7]). We verified that the size of the 360 degree domain was large enough so that further enlarging had insignificant affect on the computed solution at the receivers. Secondly, we validated numerically that the rather narrow-looking damping regions on the periphery were wide enough to attenuate significant reflections from the horizontal boundaries of the domain (e.g. Fig. 1 in Ref. [7]). The overall horizontal extent of the domain was set to four Fresnel radii as that was wide enough to not contaminate with boundary reflections the central computational region for assessing horizontal correlation (Eq. 10 in Ref. [7]). Numerical convergence of the PE at 200 and 250 Hz was obtained with horizontal and vertical grids of 50 m and 1 m respectively (Table I). The PEs that were solved are exactly the same as shown in [6, 7] except a typographical error is corrected here by placing a plus sign after the $n^2$ term in Eq. 3.

![Figure 1](image.png)

Figure 1: Horizontal domains for computing parabolic approximation of the acoustic wave equation at 200 and 250 Hz. Horizontal and vertical axes indicate distances along and perpendicular to the central geodesic. The central geodesic is that between the source and receiver on the midpoint of the horizontal array at 4000.74 km. Values of the acoustic field are undamped within dashed lines. Damping occurs between dashed and solid lines.

III. RESULTS

At 200 and 250 Hz, and at all distances between 1000 and 4000 km, there are no significant differences between 2D and 3D solutions of the PEs (e.g. Fig. 2). Com-
Table 1: Range step and number of depths needed in computational grid for 3D parabolic approximation to obtain convergence of solution at indicated frequencies. The number of depths are those in the computational domain, which extends from 0 to 8000 m depth. Approximate times for a single uncoupled and coupled azimuth run are given for an IBM Power5+ processor at 1.9 GHz. Computations at 200 and 250 Hz require about 2.5 GB of memory per cpu. Codes are written in Fortran and make use of IBM’s Engineering Scientific Subroutine Library (ESSL) and Mathematical Acceleration Subsystem (MASS) libraries. Jobs were run using the Message Passing Interface (MPI) with 16 cpus.

| f(Hz) | ∆r (km) | # Depths | Computer CPU Hours |
|-------|---------|----------|--------------------|
|       |         |          | Uncoupled | Coupled |
| 200   | 0.05    | $2^{13}$ | 1200      | 2000    |
| 250   | 0.05    | $2^{13}$ | 1600      | 2400    |

Computations at 200 and 250 Hz required about 200,000 cpu hours on an IBM Power 5+ processor at 1.9 GHz. Statistical convergence was obtained with 17 snapshots of the internal wave field at 200 Hz, and 37 snapshots at 250 Hz. For each snapshot, the acoustic field was computed on an array of length 14 km. The interval between snapshots was one-half day where the 3D internal wave field was temporarily evolved using the linear dispersion relation. The results in Fig. 2 have about 1727 degrees of freedom (37 snapshots × 14 km array length/0.3 km correlation length).

Voronovich and Ostashev[12, 13] developed theories to estimate when effects of azimuthal coupling were significant. Their theories have not been used to draw boundaries in frequency/transmission-distance space that designate when the 3D solution is required. The second of their theories [13] appears to be the most advanced, and they report a distance of validity up to about 1000 km. The numerical results here might be used to compare with that or other theories.

The numerical calculations for the shape of horizontal correlation are sometimes parametrized by the form $\exp(-y/y_*)^p$ where $y$ is horizontal separation and $y_*$ and $p$ are constants derived from theory or fit to the numerical calculations. The first theory [14] yields $p = 1.5$ and the second [13, 15] yields $p = 2$. The numerical calculations at 200 (not shown) and 250 Hz (Fig. 3) yield values of $p$ from about 1.4 at 500 km to about 1.9 at 4000 km, so they do not help confirm one theory over another. Numerical calculations at 150 Hz and lower [6, 7] yield values of $p$ between about 1 and 1.4, which are inconsistent with both theories. It is not known why the numerical values come closer to theoretical results at higher frequencies.

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Figure 2: 95% confidence limits via the bootstrap [16] for horizontal correlation at 250 Hz at 500 m depth and 4000 km distance from source for uncoupled azimuth (dark lines) and coupled azimuth (light lines) solutions of the 3D wave equation. The dotted line is at $e^{-1}$.

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Figure 3: Least-squares fit of numerically-computed horizontal correlation at 250 Hz with \(\exp[-(y/y_*)^p]\) where minimum is found over all values of \(p\) and \(y_*\). Results are given for indicated distance from source at each of four depths (100, 500, 1000, and 3000 m). The fit is made only to model values with correlation exceeding 0.3.

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