Instability of local supersonic regions over a flat-sided airfoil with a blunt trailing edge

A. Kuzmin

St. Petersburg State University, 28 University Ave., St. Petersburg, 198504 Russia
E-mail: a.kuzmin@spbu.ru

Abstract. The two-dimensional turbulent transonic flow over a symmetric flat-sided airfoil with a blunt trailing edge is studied numerically. Solutions of the Reynolds-averaged Navier-Stokes equations are obtained with a finite-volume solver on a fine computational mesh. The non-uniqueness of flow field in certain bands of the given free-stream Mach number and angle of attack is demonstrated. Intricate dependence of the lift coefficient on the free-stream parameters is discussed. Adverse free-stream parameters, which admit abrupt changes of the flow structure and lift, are identified.

1. Introduction

In late 1980s, a discussion on the non-uniqueness of inviscid transonic flows ended up with a conclusion that the non-uniqueness had been caused by drawbacks of the potential flow model or numerical schemes [1,2]. Meanwhile, in 1991 A. Jameson obtained convincing non-unique solutions of the Euler equations for four asymmetric airfoils designed with an optimization tool [3]. The non-uniqueness was associated with a flow hysteresis under variation of the angle of attack. Later, multiple solutions of the Euler equations were obtained for several symmetric airfoils at zero angle of attack in narrow bands of the free-stream Mach number $M_{\infty}$ [4–6].

It was pointed out in [6] that the non-uniqueness occurs when an airfoil comprises a flat or nearly flat part. The small curvature of this part causes a formation of closely spaced supersonic regions whose coalescence/rupture proceeds abruptly and results in instability of the flow structure. This phenomenon was documented for inviscid and turbulent flows over a number of symmetric and asymmetric airfoils [7–9]. Also, instability of the flow structure was demonstrated for airfoils with aileron or spoiler deflections that produce a small local curvature of the profile. Transonic flows over 3D wings whose surfaces comprise nearly flat parts were examined in [10–12].

A detailed interpretation of the phenomena associated with the coalescence/rupture of supersonic regions was recently given in [13, 14] for inviscid flow over a flat-sided double wedge. In addition, it was mentioned in [13] that truncations of the wedge produce expansions of the bifurcation band, though trigger flow oscillations in the rear region and wake.

In the present paper we study in detail the turbulent flow over the 20 percent truncation of the double wedge considered in [13]. In Section 2 we formulate the problem and outline a numerical method. Then in Sections 3 and 4 we discuss the turbulent flow behavior at various $M_{\infty}$ and angles of attack.
It is worth mentioning that airfoils with blunt trailing edges have recently become of growing practical interest in view of a coolant injection technology through the edges for a thermal protection of turbine blades [15].

2. Formulation of the problem and numerical method

The airfoil under consideration is given by the expressions as follows:

(a) nose: \( y(x) = \pm 0.4x/3 \) at \( 0 \leq x < 0.3 \),
(b) middle part: \( y(x) = \pm 0.04 \) at \( 0.3 \leq x \leq 0.7 \),
(c) rear part: \( y(x) = \pm 0.4(1-x)/3 \) at \( 0.7 \leq x < 0.8 \),
(d) base: \(-0.08/3 \leq y \leq 0.08/3\) at \( x=0.8 \),

where \((x,y)\) are Cartesian coordinates non-dimensionalized by the length \( l=0.5 \text{ m} \) of full wedge [13].

The fully turbulent flow over airfoil (1) is governed by the unsteady Reynolds-averaged Navier-Stokes equations (URANS) with respect to the static temperature \( T(x,y,t) \), density \( \rho(x,y,t) \), and velocity components \( U(x,y,t) \), \( V(x,y,t) \), where \( t \) is time. The static pressure \( p(x,y,t) \) is related to \( \rho(x,y,t) \) and \( T(x,y,t) \) by the equation of state \( p=\rho R T \), where \( R=c_p-c_v/\gamma \) is the specific gas constant. The air is treated as a perfect gas whose specific heat at constant pressure \( c_p \) is 1004.4 J/(kg K), the ratio of specific heats \( \gamma \) is 1.4.

A lens-type computational domain, whose outer boundary is composed by arcs \( \Gamma_1 \) and \( \Gamma_2 \), extends from \(-100 \) to \(100\) in the \( y\)-direction and from \(-40 \) to \(120\) in the \( x\)-direction, see Fig. 1. On the inflow part \( \Gamma_1 \) we prescribe the temperature \( T_\infty=250 \text{ K} \), angle of attack \( \alpha \), free-stream Mach number \( M_\infty<1 \), and turbulence level of 1\% . The velocity components are determined by the relations

\[
U_\infty=M_\infty a_\infty \cos \alpha \quad \text{and} \quad V_\infty=M_\infty a_\infty \sin \alpha,
\]

where \( a_\infty=\sqrt{\gamma R T_\infty} \) is the sound speed.

On the outflow part \( \Gamma_2 \) of boundary, we impose the static pressure \( p_\infty=3\times10^5 \text{ N/m}^2 \). The vanishing heat flux and free-slip condition are given on airfoil (1). Initial data are either uniform free stream or a nonuniform flow calculated for other values of \( M_\infty \) and \( \alpha \).

The URANS equations were solved with ANSYS-18.2 CFX finite-volume solver [16] using the SST \( k-\omega \) turbulence model [17]. A computational mesh was constituted by quadrilaterals in 40 layers on profile (1) and by triangles in the rest of computational domain. The dimensionless thickness \( y^+ \) of the first mesh layer on the profile was less than 1. The cells were clustered near the profile for an accurate resolution of the boundary layer and shocks. Mesh cells located at \( y^+>0 \) and \( y^+<0 \) were perfectly symmetric about the \( x\)-axis. To create 3D meshes used by the CFX solver, we extruded the 2D meshes in the \( z\)-direction from 0 to \( l_z=0.002 \text{ m} \). Test computations on uniformly refined meshes of about
2.3×10^5, 4.7×10^5, and 9.9×10^5 cells only showed a small discrepancy between flow fields obtained on the second and third meshes. That is why, to study transonic flow at various \( M_{\infty} \) and \( \alpha \), we used the second mesh. An implicit backward Euler scheme is used for the global time-stepping. The time step of \( 10^{-5} \) s ensured the root-mean-square Courant-Friedrichs-Lewy number smaller than 2. The solver was validated by computation of several benchmark transonic flow problems [12].

3. Symmetric and asymmetric flows at zero angle of attack

First, we solved the problem at 0.839≤\( M_{\infty} \)≤0.8448, \( \alpha=0 \) using the free-stream parameters for initialization of solutions. Computations showed formation of a flow pattern with an oscillating boundary layer at the rear of airfoil and a vortex wake looking like a classic von Karman street. The temporally averaged flow is symmetric about the x-axis and exhibits two local supersonic regions on each side of airfoil, see Fig. 2. The lift coefficient \( C_L = 2L/(\rho_{\infty}U_{\infty}^2l,0.8l) \) oscillates in time between \( C_{L,\text{min}} \) and \( C_{L,\text{max}} \), where lift \( L \) is calculated by integration of the static pressure over the airfoil. This flow regime corresponds to curves 1 in Fig. 3 which show margins of the lift coefficient oscillations as functions of \( M_{\infty} \).

![Figure 2. Temporal snapshot of Mach number contours \( M(x,y) = (U^2+V^2)^{1/2}/a =\text{const} \) in the symmetric flow at \( M_{\infty}=0.842, \alpha=0 \).](image)

![Figure 3. Margins of the lift coefficient oscillations versus \( M_{\infty} \) at \( \alpha=0 \) in turbulent flow over airfoil (1). Computations on a mesh of 988,904 cells. Sketches indicate the number and locations of supersonic regions in the flow regimes 1 - 4.](image)
Then we solved the problem at $M_\infty=0.849$, $\alpha=0$ using again the free-stream parameters for initialization of solution. The calculated asymmetric flow exposed a single supersonic region on each side of airfoil. This solution was used for flow computations at decreasing $M_\infty$ step-by-step from 0.849 to 0.8442, see curves 2 in Fig. 3; at each step, initial data were flow parameters obtained at the previous step.

There is a hysteresis of the symmetric solutions in the range $0.8442 < M_\infty < 0.8448$. Meanwhile, in this range, small perturbations trigger a transition from a symmetric flow to asymmetric one with positive or negative lift, see curves 3 and 4 in Fig. 3. To ensure a transition, e.g., to the asymmetric flow with $C_L > 0$, one can apply a positive impulse of $\alpha$ on the boundary $\Gamma_1$. Using the calculated asymmetric flow at $\alpha=0$ and changing step-by-step the Mach number $M_\infty$, we obtained asymmetric flows in the band

$$0.8400 \leq M_\infty \leq 0.8465.$$  \hfill (2)

Figure 4 illustrates, for example, Mach number contours in the asymmetric flow with positive lift at $M_\infty=0.842$ (cf. Fig. 2). The bifurcation band (2) is noticeably longer than the one obtained for flow over the double wedge with the sharp trailing edge [14]. This is explained by a closer correlation between the upper and lower flow domains via the near wake in the case at hand.

Frequencies of lift coefficient oscillations lie in the range between 2000 and 2300 Hz for the free-stream Mach numbers under consideration.

![Figure 4. Temporal snapshot of Mach number contours in the asymmetric flow with $C_L > 0$ at $M_\infty=0.842$, $\alpha=0$.](image)

4. Behavior of local supersonic regions under variation of the angle of attack at $M_\infty=0.8455$

Figure 5 shows the lift coefficient as a function of the angle of attack at $M_\infty=0.8455$. Curves 2 illustrate an evolution of $C_L$ when initial data are parameters of the symmetric flow obtained at $\alpha=0$ (indicated by the triangles in Fig. 3). In this regime, the qualitative flow pattern with a single supersonic region on each side of airfoil persists under variation of $\alpha$ in the band $-0.225^\circ \leq \alpha \leq 0.225^\circ$. Meanwhile if $\alpha$ exceeds $0.225^\circ$, then the supersonic region on the lower side of airfoil ruptures, and $C_L$ jumps from curves 2 to curves 3. The flow pattern corresponding to curves 3 holds on at $0.2^\circ \leq \alpha \leq 0.6^\circ$. Similarly, when $\alpha$ drops below $-0.225^\circ$, the supersonic region on the upper side of airfoil ruptures, and $C_L$ jumps from curves 2 to curves 1.

Curves 4 illustrate the evolution of $C_L$ when initial data are parameters of the asymmetric flow with $C_L > 0$ obtained at $\alpha=0$ (indicated by the quadrangles in Fig. 3). This regime with three local supersonic regions takes place at $-0.05^\circ \leq \alpha \leq 0.01^\circ$. Meanwhile if $\alpha$ exceeds $0.01^\circ$, or drops below $-0.05^\circ$, then the bow supersonic region on the lower side of airfoil shifts downstream and gets into the
coalescence with the rear supersonic region. That is why the flow topology abruptly changes, and $C_L$ jumps from curves 4 to curves 2.

![Figure 5. Margins of the lift coefficient oscillations versus the angle of attack at $M_\infty=0.8455$ in turbulent flow over airfoil (1).](image)

The obtained results make it possible to identify adverse free-stream conditions, in which the flow topology is unstable, and $C_L$ exhibits considerable jumps. For example, this occurs at $M_\infty=0.8455$ and variation of the angle of attack near $\alpha=\pm 0.01^\circ$, see Fig. 5. Another example follows from Fig. 3 which shows that, in the asymmetric flow at $M_\infty=0.840$ and $\alpha=0$, the lift coefficient oscillates about 0.075 or $-0.075$; however, a decrease of $M_\infty$ to 0.8395 triggers a jump of the mean value of $C_L$ to zero.

5. Conclusion
The numerical simulation of transonic flow over airfoil (1) with the blunt trailing edge demonstrated flow behavior that is different from the one over airfoils with sharp trailing edges. At zero angle of attack $\alpha=0$, the band of free-stream Mach numbers admitting asymmetric flow regimes is noticeably longer than that in the case of sharp trailing edge. At $M_\infty=0.8455$ the dependence of $C_L$ on $\alpha$ exhibits several discontinuities with increasing angle $\alpha$ from -0.6° to 0.6°. The discontinuities are caused by abrupt changes of the flow topology due to rupture/coalescence of local supersonic regions on the airfoil.

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