Information to energy conversion in quantum composite systems at finite temperature

N. V. Gnezdilov\textsuperscript{1}, A. I. Pavlov\textsuperscript{2}, V. Ohanesjan\textsuperscript{3}, Y. Cheipesh\textsuperscript{4}, and K. Schalm\textsuperscript{3}

\textsuperscript{1}Department of Physics, University of Florida, Gainesville, Florida 32601, USA
\textsuperscript{2}The Abdus Salam International Centre for Theoretical Physics (ICTP) Strada Costiera 11, I-34151 Trieste, Italy
\textsuperscript{3}Instituut-Lorentz, ∆-ITP, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
\textsuperscript{4}Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands

It is known that entanglement can be converted to work in quantum composite systems. In this paper we consider a quench protocol for two initially independent reservoirs $A$ and $B$ described by the quantum thermal states. For a free fermion model at low temperatures, the von Neumann entropy of each reservoir increases once the reservoirs are coupled. At the moment of decoupling there is an energy transfer to the system in the amount set by the von Neumann entropy accumulated during joint evolution of $A$ and $B$. This energy transfer appears as work produced by the quench to decouple the reservoirs. Once the reservoirs are disconnected, the information about their mutual correlations – von Neumann entropy – is stored in the energy increment of each reservoir. This result provides a possibility of a direct readout of quantum correlations at low temperature.

1 Introduction

Entanglement entropy is a measure of non-classical correlations in quantum composite systems. It is commonly defined as the von Neumann entropy of a subpart of a quantum composite system described by a pure quantum state [18]. Consideration of the later one requires zero temperature limit. Despite entanglement being an inherent characteristic of quantumness, the entanglement entropy itself is not easy to measure directly because of its nonlocal nature. Still, a few proposals are suggesting that the entanglement entropy can be related to the particle number fluctuations in free fermion systems [6, 7, 15, 16] and elicited from the low-energy states population dynamics in quantum many-body systems [1, 9]. A recent experiment reports that the entanglement entropy can be measured by an interference of two clones of a many-body state in ultra-cold atoms [14].

However, one may ask if there are generic consequences of quantum correlations between the subparts of a given system that are known to exist above zero temperature. This question may be answered in the framework of quantum thermodynamics – a rapidly developing field that relates general properties of quantum systems, such as work, heat, and entropy, irrespective of their microscopic nature [10]. Within this approach, the classical thermodynamic entities are extended to their quantum counterparts. In quantum thermodynamics, the energy of the evolving quantum state is usually expressed in terms of thermal

N. V. Gnezdilov: n.gnezdilov@ufl.edu
free energy and entropy production – a measure of the irreversibility of a quantum process defined as the relative entropy between an actual state of the system and a reference state \([10, 22]\). The latter is usually chosen as a thermal state of the system taken at the same time as the evolving one. This method has proven to be quite fruitful for thermodynamic analysis of quantum composite systems. In particular, it was shown that entanglement in quantum composite systems can be converted to work or heat \([8, 11, 12, 23]\). The conversion of quantum correlations to heat arises naturally for quantum systems in a form of decoherence due to interactions with environment \([20, 26]\) and governs a borderline between quantum and classical worlds \([27, 28]\). On the other hand, the conversion of quantum correlations to work is actively studied as it paves the way to quantum thermal engines potentially superior to their classical analogs \([2, 13, 21]\).

In this paper, we are interested in the way how quantum correlations may be converted to energy for quantum thermal states. We consider two arbitrary quantum systems, each of them initially in thermal equilibrium, coupled and then jointly evolving until decoupling. The coupling and decoupling process is controlled by a chosen quench protocol. We show that the energy acquired during the joint evolution by a subpart of a quantum composite system can be related to its entropies only, namely, the von Neumann entropy and the relative entropy between the evolved state of the subsystem and its initial thermal state. The von Neumann entropy accumulated during the joint evolution of the two systems sets the lower bound on the energy change. We apply this inherently quantum description to a quench-coupled system of non-interacting fermions. The composite free Fermi systems are particularly attractive since they allow to directly access the entanglement entropy between its subparts when coupled \([15, 16]\). For temperatures much lower than the Fermi energy in the system, the quantum correlations between the two subparts of the system, given by the change in the von Neumann entropy, are transferred to the subsystem’s energy. This energy increment originates from the external work produced by the quench to erase the correlations between the subsystems at the moment of decoupling. The von Neumann entropy accumulated during the joint evolution of the two subsystems can be directly read out from the energy increment.

2 Entropy to energy relation in quantum thermodynamics

We consider two isolated quantum systems \(A\) and \(B\), initially both in thermal equilibrium, that are instantaneously coupled by an arbitrary interaction \(V_{AB}\). The general time-dependent Hamiltonian is

\[
H(t) = H_A + H_B + g(t)V_{AB},
\]

(1)

where the function \(g(t)\) defines a quench protocol that turns on/off the interaction at a given time. The initial state of the full system is given by the product of two thermal density matrices

\[
\rho_0 = \rho_A \otimes \rho_B,
\]

(2)

\[
\rho_\alpha = \frac{1}{Z_\alpha} \sum_{n_\alpha} e^{-E_{n_\alpha}/T_\alpha} |n_\alpha\rangle \langle n_\alpha| = e^{-H_\alpha/T_\alpha}/Z_\alpha, \quad \alpha = A, B.
\]

(3)

Here \(|n_\alpha\rangle\) is an eigenstate of the Hamiltonian \(H_\alpha\) with an eigenenergy \(E_{n_\alpha}\), \(T_\alpha\) is the initial temperature, and \(Z_\alpha = \text{Tr}_\alpha e^{-H_\alpha/T_\alpha}\) is the partition function for \(\alpha = A, B\). Below we focus on the properties of the system \(A\) for brevity and use the units \(\hbar = k_B = 1\) everywhere.
Once the two systems are coupled they become correlated. A natural measure to study the correlations between $A$ and $B$ is the von Neumann entropy. The von Neumann entropy for the system $A$ is

$$S_{vN}(t) = -\text{Tr}_A \rho_A(t) \ln \rho_A(t),$$  \hspace{1cm} (4)

where $\rho_A(t) = \text{Tr}_B \rho(t)$ is the reduced density matrix. Here we are not limiting ourselves to the unitary evolution of the full density matrix $\rho(t)$. For example, it can be drawn from the Lindblad master equation in case the system (1) is dissipative.

Let us introduce the relative entropy, which is often used in both quantum information processing [18] and quantum thermodynamics [10] to distinguish two quantum states. For our purpose, we define the relative entropy between the evolved state $\rho_A(t)$ of the system $A$ from its initial thermal state $\rho_A$:

$$S(\rho_A(t)||\rho_A) = \text{Tr}_A \rho_A(t) (\ln \rho_A(t) - \ln \rho_A) \geq 0.$$  \hspace{1cm} (5)

As we shall use shortly, the relative entropy is defined as non-negative [18].

Now, we substitute the von Neumann entropy (4) into Eq. (5). Using that the initial state of $A$ is a thermal state at temperature $T_A$, we relate the expectation value of the Hamiltonian $H_A$ to the combination of the von Neumann and relative entropy

$$\text{Tr}_A \rho_A(t) H_A = \mathcal{F}_A + T_A (S_{vN}(t) + S(\rho_A(t)||\rho_A)).$$  \hspace{1cm} (6)

Here $\mathcal{F}_A = -T_A \ln Z_A$ is the initial thermal free energy of the system $A$. Subtracting the initial energy value $\text{Tr}_A \rho_A H_A = \mathcal{F}_A + T_A S_{vN}(0)$ from Eq. (6), we derive

$$\Delta E_A(t) = T_A (\Delta S_{vN}(t) + S(\rho_A(t)||\rho_A)),$$  \hspace{1cm} (7)

where $\Delta E_A(t) = \text{Tr}_A \rho_A(t) H_A - \text{Tr}_A \rho_A H_A$ and $\Delta S_{vN}(t) = S_{vN}(t) - S_{vN}(0)$. The relation (7) combines the first and the second law of thermodynamics for a subpart of an arbitrary quantum composite system. Being a thermodynamic statement, Eq. (7) should be understood as the equation of the energy-to-entropy balance after the process is over. In contrast with the previous studies, see e.g. Refs. [8, 11, 12, 20], we relate the energy of the system $A$ to its entropy properties solely. At low temperatures, the energy change is set by the amount of correlations with the other subpart of the full system, that emerge when the coupling is switched-on, and by its deviation from the initial state.

When the two systems are coupled, it is not possible to completely isolate them from each other to determine the actual energy shift in $A$ or $B$. However, one can count whether the energy of the system changes if $A$ and $B$ are decoupled later on. Since the relative entropy is non-negative, the von Neumann entropy provides the lower bound on the overall energy shift:

$$\Delta E_A \geq T_A \Delta S_{vN}.$$  \hspace{1cm} (8)

The inequality (8) is a generic thermodynamic property of quantum composite systems and yield quite intriguing consequences at low temperatures. Indeed, turning on the interaction between the subparts of a quantum composite system induces quantum correlations. Ergo, their von Neumann entropy increases in absence of thermal imbalance between the subsystems. According to Eq. (8), it necessarily leads to the energy increment when the systems are decoupled ($\Delta E_A > 0$). This prompts the question: can one directly detect quantum correlations between the subparts of a quantum composite system by measuring the resulting energy increment? Putting this in perspective, we explain the physics of this effect by taking the thermodynamic point of view on the quench-coupling/decoupling protocol in a free fermion system.
3 The case study: free fermions

The possibility of detection quantum correlations and entanglement entropy, in particular, is widely discussed for the free Fermi systems [6, 7, 15, 16]. In light of this, free fermions seem as a natural framework to proceed with our thermodynamic consideration.

Consider for systems $A$ and $B$ two isolated spinless fermionic reservoirs at temperatures $T_A$ and $T_B$. At time $t = 0$ we instantaneously couple the reservoirs together and then disconnect them at $t = t_0$. The coupling/decoupling process is controlled by the quench protocol $g(t) = \theta(t) - \theta(t - t_0)$ in the Hamiltonian (1). The Hamiltonian (1) reads

$$H_A = \sum_p \xi_p a_p^\dag a_p, \quad H_B = \sum_{p'} \xi_{p'} b_{p'}^\dag b_{p'}, \quad V_{AB} = \lambda \delta(x)a^\dag b(x) + h.c.,$$

where $A$ and $B$ are coupled locally. Here $a, a^\dag$ and $b, b^\dag$ are the fermionic operators, $\xi_p$ is the corresponding dispersion, and $\lambda$ is the coupling constant. Note that $[V_{AB}, H_A] \neq 0$ and $[V_{AB}, H_B] \neq 0$. Below we put both reservoirs at the same chemical potential to avoid electric currents.

The chosen quench protocol leads to the non-conservation of energy in our model. It immediately follows from the general form of the Hamiltonian (1) that the energy can be added to the system at the moment of turning on/off the interaction between $A$ and $B$:

$$\frac{d\langle H(t) \rangle}{dt} = g'(t)\langle V_{AB}(t) \rangle = \delta(t)\langle V_{AB}(0) \rangle - \delta(t - t_0)\langle V_{AB}(t_0) \rangle.$$  

(12)

The origin of this effect is quite simple. The quench can produce work to couple and decouple the reservoirs.

When $A$ and $B$ are coupled at $t = 0$ they become correlated and, hence, one can not measure their energy separately until the systems are disconnected at $t = t_0$. To evaluate the overall energy shift in the reservoir $A$ after decoupling from $B$, we compute the corresponding energy flux

$$\frac{d\langle H_A(t) \rangle}{dt} = i\langle [H, H_A] \rangle = -ig(t)\sum_{pp'} \xi_p(\lambda\langle a_{p'}^\dag b_{p'} \rangle - h.c.).$$

(13)

The correlation functions that define the energy flux (13) satisfy the equation

$$\lambda \frac{d\langle a_{p'}^\dag b_{p'} \rangle}{dt} = i\lambda(\xi_p - \xi_{p'})\langle a_{p'}^\dag b_{p'} \rangle - ig(t)|\lambda|^2 \sum_q \langle a_q^\dag a_q \rangle - \langle b_q^\dag b_q \rangle).$$

(14)

The exact solution of Eq. (14) requires notion of the correlation functions $\langle a_{p'}^\dag a_q \rangle$ and $\langle b_{p'}^\dag b_q \rangle$, the momenta-diagonal components of which are dynamic occupation numbers $n_\alpha(\xi_p, t)$ of fermions $\alpha = a, b$ with momentum $p$ at time $t$. We solve Eq. (14) perturbatively in the lowest order in $\lambda$ implying the equilibrium occupation numbers of the initial state of the system. Indeed, since there are no inter-momenta couplings before the quench, we use diagonal correlations $\langle a_{p'}^\dag a_q \rangle = \delta_{pq}\langle a_{p}^\dag a_p \rangle = \delta_{pq} n_A(\xi_p)$ and $\langle b_{p'}^\dag b_q \rangle = \delta_{qp'}\langle b_{p}^\dag b_p \rangle = \delta_{qp'} n_B(\xi_p)$ in Eq. (14), where $n_\alpha(\xi_p) = \left(e^{\xi_p \alpha / T_\alpha} + 1\right)^{-1}$ is the Fermi distribution function. The sought-for correlation function is

$$\lambda \langle a_{p'}^\dag b_{p'} \rangle = -i|\lambda|^2(n_A(\xi_p) - n_B(\xi_p)) \int_0^t dt' e^{i(\xi_p - \xi_{p'}) (t - t')} g(t').$$

(15)
It follows form the correlation function (15) that $\langle V_{AB}(0) \rangle = 0$ in Eq. (12), since there are no correlations between $A$ and $B$ at $t = 0$.

Substituting Eq. (15) into Eq. (13), we obtain the Fermi golden rule formula for the energy flux

$$\frac{d\langle H_A(t) \rangle}{dt} = -2g(t)|\lambda|^2 \int d\omega d\omega' \frac{\sin(\omega - \omega') t}{\omega - \omega'} (n_A(\omega) - n_B(\omega')).$$  (16)

Here we introduced the density of states (DoS) for both fermion species $\nu_A(\omega) = \sum_p \delta(\omega - \xi_p)$ and $\nu_B(\omega') = \sum_{p'} \delta(\omega' - \xi_{p'})$ and assume the two-dimensional reservoirs with constant DoS given by the inverse Fermi energy $\nu_\alpha \sim 1/\varepsilon_F$. Then the overall energy shift in $A$ is given by $\Delta E_A = \int_0^t dt \frac{d\langle H_A(t) \rangle}{dt}$. An identical computation of the energy flux can be done for the reservoir $B$.

Let’s consider $A$ and $B$ at zero temperature prior the quench. In equilibrium, there would not be an energy shift in either of the reservoirs. However, turning on the coupling entangles the states in the reservoirs and, therefore, results in presence of entanglement entropy between $A$ and $B$ that can be captured by the particle number fluctuations [15, 16].

So, does the energy of the $A$-reservoir remain unchanged once $A$ and $B$ are decoupled?

At zero temperature the distribution function $n_A(\omega)$ is a unit-step function $\theta(-\omega)$ for both reservoirs. Substituting the zero temperature distribution functions into the Fermi golden rule formula (16) and providing the UV cut-off $\sim \varepsilon_F$ for the frequency integrals, we derive the energy flux

$$\frac{d\langle H_A(t) \rangle}{dt} = 2g(t)|\lambda|^2 \sin(\varepsilon_F t) \frac{\sin^2(\varepsilon_F t/2)}{(\varepsilon_F t/2)^2}. $$  (17)

Accordingly, the overall energy shift in the reservoir $A$ is

$$\Delta E_A^Q = 2|\lambda|^2 \varepsilon_F \int_0^{\varepsilon_F t_0} d\zeta \sin \zeta \frac{\sin^2(\zeta/2)}{(\zeta/2)^2}. $$  (18)

As it is shown in Fig. 1 (blue solid curve), the energy of the system increases in absence of temperature or charge imbalance. The effect is suppressed in the continuous limit (small lattice spacing for the free fermions) where the Fermi energy is large. Here we use the superscript $Q$ in Eq. (18) to stress out that the effect is manifestly quantum since both systems are initially at zero temperature. The system $B$ has the same energy increase.

Now, we move on to finite temperature. To do so, we apply a temperature imbalance between the reservoirs and compute the energy flux. We restrict ourselves to the low temperature regime $\max(T_A, T_B) \ll \varepsilon_F$ and times much less than the inverse temperature. This limitation naturally arises from the perturbative origin of the Fermi golden rule formula (16). Indeed, at high temperatures the dynamics in the integral (16) is set by $t \sim 1/\max(T_A, T_B) \ll 1/\varepsilon_F$ but the energy flux formula itself is perturbative in $|\lambda|^2/\varepsilon_F^2$. The neglected dynamics of the occupation numbers appears in higher order in $|\lambda|^2/\varepsilon_F^2$. Therefore, once reaching the timescale of $t \sim 1/\max(T_A, T_B)$, the dynamics of the occupation numbers can not be considered slow and, hence, is no longer negligible. Still, consideration of small temperatures allows us to derive the energy dynamics perturbatively at the timescale $t \sim 1/\varepsilon_F \ll 1/\max(T_A, T_B)$. This logic applies to $t_0$, so that we disregard the thermalization of $A$ and $B$ while they are coupled.

We compute the energy flux (16) for $T_A = T \ll \varepsilon_F$ and $T_B \ll T$ using the equilibrium occupation numbers $n_A(\omega) = \left(e^{\omega/T} + 1\right)^{-1}$ and $n_B(\omega) = \theta(-\omega)$. This realises a quantum
Figure 1: Energy increment in the reservoir $A$ due to quench-coupling with the reservoir $B$ as a function of time at low temperature. The energy increment at zero temperature $\Delta E_A^Q \approx |\lambda|^2 / \varepsilon_F$ for $t_0 \ll 1/\varepsilon_F$ and reaches its maximum value of $3.2 |\lambda|^2 / \varepsilon_F$ at $t_m = \pi/\varepsilon_F$, which is defined by the first zero of the energy flux (17).

cooling protocol for the reservoir $A$. Since $A$ is at low temperature $T \ll \varepsilon_F$, we use the Sommerfeld expansion

$$\int_{-\varepsilon_F}^{\varepsilon_F} d\omega \frac{F(\omega, t)}{e^{\omega/T} + 1} \approx \int_{-\varepsilon_F}^{0} d\omega F(\omega, t) + \frac{\pi T^2}{6} F'(0, t),$$

where $F'(0, t)$ denotes the frequency derivative of $F$ at $\omega = 0$. Here the function $F(\omega, t) = \omega \int_{-\varepsilon_F}^{\varepsilon_F} d\omega' \frac{\sin(\omega - \omega')}{\omega - \omega'}$ has a well defined derivative $F'(\omega, t)$ in the considered energy interval, so that $F'(0, t) = 2 \int_0^{\varepsilon_F t} d\zeta \frac{\sin \zeta}{\zeta'^2}$. Finally, we reinstate the result in Eq. (18) with a small temperature correction:

$$\Delta E_A = \Delta E_A^Q - \frac{8 |\lambda|^2}{3 \varepsilon_F} \left(\frac{\pi T}{\varepsilon_F}\right)^2 \int_0^{\varepsilon_F t_0} d\zeta \int_0^{\varepsilon_F t_0} d\zeta' \frac{\sin \zeta'}{\zeta'^2}, \quad t_0 \ll 1/T. \quad (20)$$

In Fig. 1 (the dashed curve) we demonstrate that the overall energy change in the reservoir remains positive. Remarkably, cooling down the system at temperature $T \ll \varepsilon_F$ results in the energy increment of the refrigerated system if the system and the cold bath are coupled for $t_0 \sim 1/\varepsilon_F$.

Finally, we can consider $A$ and $B$ at the same temperature $T_A = T_B = T \ll \varepsilon_F$. We proceed the same way as above and use the Sommerfeld expansion (19) for both systems. This produces the energy increment

$$\Delta E_A = \Delta E_A^Q - \frac{8 |\lambda|^2}{3 \varepsilon_F} \left(\frac{\pi T}{\varepsilon_F}\right)^2 \sin^2 \left(\frac{\varepsilon_F t_0}{2}\right), \quad t_0 \ll 1/T. \quad (21)$$

The effect still persists when both systems evolve starting from thermal states, as seen in Fig. 1 (dotted curve).

In contrast to the subsystem’s energy, the time-dependent correlations between the subsystems initialized at $t = 0$ are well defined by the von Neumann entropy of the corresponding subsystem. The von Neumann entropy of the subpart of a free fermion system
is known to be expressed in terms of its occupation numbers $n_\alpha(\xi_p, t)$ \cite{15–17}. The von Neumann entropy of the reservoir $A$ is

$$S_{vN}(t) = -\sum_p (n_A(t, \xi_p) \ln n_A(t, \xi_p) + (1 - n_A(t, \xi_p)) \ln(1 - n_A(t, \xi_p))).$$  \hspace{1cm} (22)

Within the accuracy of the Fermi golden rule formula (16), the entropy flux can be computed perturbatively in the lowest order in $\lambda$:

$$\frac{dS_{vN}(t)}{dt} = -\sum_p \frac{dn_A(t, \xi_p)}{dt} \ln \frac{n_A(\xi_p)}{1 - n_A(\xi_p)},$$  \hspace{1cm} (23)

where the distribution function inside the logarithm is taken in equilibrium. Using that $\langle H_A(t) \rangle = \sum_p \xi_p n_A(t, \xi_p)$ and $n_A(\xi_p)/(1 - n_A(\xi_p)) = \exp(-\xi_p/T_A)$, we notice that the entropy and the energy fluxes are related as $T_A dS_{vN}(t)/dt = d\langle H_A(t) \rangle/dt$. Hence, when the systems are decoupled, we get

$$\Delta E_A = T_A \Delta S_{vN},$$  \hspace{1cm} (24)

that states that the overall energy change in $A$ (20) is set by the amount of correlations the system acquired during the joint evolution of $A$ and $B$. The equation (24) resembles the lower energy bound in the energy-entropy balance equation (7) with zero relative entropy

$$S(\rho_A(t_0)||\rho_A) = 0.$$  \hspace{1cm} (25)

The relative entropy (25) between the evolved and initial state of the reservoir $A$ saturates the lower bound in Klein’s inequality \cite{18}. Within the accuracy of our perturbation theory, this means that at $t = t_0$ the evolved state $\rho_A(t_0)$ is indistinguishable from its initial thermal state $\rho_A$, despite that the reservoirs $A$ and $B$ were evolving together from $t = 0$ to $t = t_0$. The same holds for the reservoir $B$. Therefore, the correlations between $A$ and $B$ are erased at the moment of decoupling. However, $t = t_0$ is a special moment when, according to Eq. (12), the energy is not conserved. Indeed, all the energy acquired by both systems is transferred at the precise moment of decoupling. Using the correlation function (15), we derive

$$\frac{d\langle H(t) \rangle}{dt} = -\delta(t - t_0)\langle V_{AB}(t_0) \rangle = \delta(t - t_0) (\Delta E_A + \Delta E_B).$$  \hspace{1cm} (26)

In our model the two systems become correlated at $t = 0$ and then are fully decoupled at $t = t_0$. At low temperatures, the amount of energy transferred to the system by the quench at $t = t_0$ is set by the von Neumann entropies of each system accumulated during joint evolution of $A$ and $B$, which captures the correlations between $A$ and $B$ up to the moment of decoupling. The work produced by the quench $-\Delta E_A - \Delta E_B$ is utilized to erase the correlations between the reservoirs. The information about the erased correlations is stored in the energy increment of each subsystem. As we illustrate in Fig. 1, the energy increment of the subsystem is well-described by its quantum part $\Delta E_A^Q$ for a short quench protocol $t_0 \lesssim 1/\varepsilon_F$ at low temperatures $\max(T_A, T_B) \ll \varepsilon_F$. As such, we refer to $\Delta E_A^Q + \Delta E_B^Q$ as the binding energy of quantum correlations between $A$ and $B$ — the amount of energy required to erase the quantum correlations between the subparts of a quantum composite system.
4 Conclusion

In this paper, we discuss non-equilibrium dynamics in quantum composite systems in a thermodynamic framework. We consider two arbitrary quantum systems $A$ and $B$ at finite temperature instantaneously coupled together. We show that the energy of a subpart of a quantum composite system can be related to its entropy properties solely. Specifically, the energy change of the system $A$ is set by the change of its von Neumann entropy and by the relative entropy between the evolved state of $A$ and its initial thermal state.

We consider a quench protocol for the free fermion system, where $A$ and $B$ are described by non-interacting fermions. Starting with zero temperature limit, we show that the subsystem’s energy increases in absence of charge or temperature imbalance. Proceeding to the finite temperature case, we find out that the energy increment is still present for the temperatures much less than the Fermi energy in the system. We demonstrate that the energy is transferred to the system in the moment of decoupling, while the amount of energy is given by change of the von Neumann entropy multiplied by the initial temperature. Here the decoupling quench acts as an external force which produces work to erase the correlations between $A$ and $B$. Meanwhile, the von Neumann entropy accumulated during the joint evolution of $A$ and $B$ is stored in the corresponding energy increment. Hereby, we refer to this effect as an information to energy conversion.

During the final stage of preparing this manuscript, we have noticed a preprint by Popovic et al. [20], which shows that there is a heat transfer from the system described by a quantum pure state to the thermal bath at the moment of decoupling. This heat dissipation is argued to originate from an energetic cost for a decoherence process. We note a similar energy transfer in the coupling/decoupling protocol for quantum thermal states, where the system and the bath are described by non-interacting fermions at low temperature. We argue that the amount of energy accumulated by a subpart of a quantum composite system is set by the amount of correlations with the other subpart developed by the moment of decoupling.

The effect of information to energy conversion is quite generic for quantum composite systems and appears as long as the Hamiltonian of the system contains non-commuting parts. The conversion happens when the subpart of the system is separated from the rest of the system as a consequence of the energetic cost of decoupling. The fingerprints of this effect are also present in strongly-correlated systems. In Ref. [21], a quantum system where non-local random interaction is turned on/off by a quench increases its energy drastically after the interaction is off. At the same time, the entropy flux presented in this paper qualitatively explains the anomalous growth of the energy flux in the evaporating Sachdev-Ye-Kitaev quantum dot [4, 25]. This subject will be addressed in details as well as its relation to the recent studies of the evaporating black holes [3, 5, 19] in the upcomming article [24].

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