A Feasibility Study for a Self-Oscillating Loop for a Three Degree-of-Freedom Coupled MEMS Resonator Force Sensor

Chun Zhao\textsuperscript{a}, Graham S. Wood\textsuperscript{a}, Suan H. Pu\textsuperscript{a, b}, Michael Kraft\textsuperscript{c}, * 

\textsuperscript{a}Nano Research Group, University of Southampton, Southampton, SO17 1BJ, United Kingdom 
\textsuperscript{b}University of Southampton Malaysia Campus, Nusajaya, Johor, 79200, Malaysia 
\textsuperscript{c}Montefiore Institute, University of Liege, Liege, B-4000, Belgium

Abstract

For the first time, we investigate a self-oscillating control loop for a three degree-of-freedom (DoF) weakly coupled MEMS resonator sensor for force sensing applications. This is an important step towards real-time measurements using such a sensor. The simulated results successfully demonstrated that, without any external drive signals, the proposed self-oscillating loop is able to automatically lock to the desired mode frequency for the sensing applications. The amplitude ratios from the simulation showed good agreement with the theoretical values.

1. Introduction

MEMS coupled resonator sensors have been studied by an increasing number of researchers in recent years due to their distinct advantages, namely enhanced sensitivity [1] and common mode rejection abilities [2], compared to conventional single resonator sensors [3]. Coupled resonator sensors utilize a mode localization effect [4] that requires the measurement of the vibrational amplitudes change at a particular mode frequency. A promising resonator sensor

* Corresponding author. Tel.: +32-(0)-4-366-2616 ; fax: +32-(0)-4-366-2620. 
E-mail address: m.kraft@ulg.ac.be
consisting of three weakly coupled MEMS resonators [5] demonstrated a four orders of magnitude improvement in stiffness change sensitivity, by measuring the amplitude ratios of the out-of-phase mode.

However, to find the desired mode frequency, e.g. the out-of-phase mode for the 3DoF resonator sensor, current approaches include manually adjusting the frequency of the drive signal [5], which is time consuming and unsuitable for practical applications. Implementing a self-oscillation loop structure [6] is an important step to enable real time measurement for emerging coupled resonator sensors.

In this paper, we shall illustrate the feasibility, through simulation results, of realizing such a self-oscillating loop utilizing a positive electrical feedback, based on an oscillator loop structure for a single MEMS resonator [7].

2. Transfer function of MEMS oscillators

For a self-sustained oscillator as shown in Figure 1a, the oscillation start-up condition is derived from the Barkhausen criterion [9]:

\[ |H(j\omega_0)G(j\omega_0)| > 1 , \angle|H(j\omega_0)G(j\omega_0)| = 0 \]  

(1)

where \( \omega_0 \) is the oscillator’s desired angular frequency. For a 3DoF weakly coupled resonator system modelled by Figure 1b, we assume weak coupling with \( |M_c|<K/10<K_2-K \), and symmetrical physical parameters except: \( K_1=K+\Delta K_1, K_3=K+\Delta K, \Delta K_1>0, \Delta K<0 \) and \( K_2\geq 2K \). These assumptions are made based on our previous work on a force sensor (measuring \( \Delta K_1 \)) with a negative bias stiffness perturbation \( \Delta K \) [8]. The detailed transfer functions can be found in [8]. For our particular interest, the out-of-phase mode frequency, where ideally resonator 1 and resonator 3 vibrate out-of-phase to each other if damping is neglected, can be derived:

\[ \omega_{op} \approx \frac{1}{\sqrt{\frac{1}{M}[K+K_c+\frac{1}{2}(\Delta K_1-K) - \frac{2K_c^2}{K_2-K+K_c} + \sqrt{\frac{(\Delta K_1-K)^2 + (\frac{2K_c}{K_2-K+K_c})^2}}]} } \]  

(2)

If we assume \( |\gamma \Delta K/K|>10 \) and \( \Delta K>0 \), and suppose that the velocity of resonator 1, \( v_1=d(x_1)/dt \), is the input of the electrical feedback and \( F_1 \) as the only actuation force, we can obtain the following at the out-of-phase mode frequency \( \omega_{op} \):

\[ |H(j\omega_{op})| = \frac{|v_1(j\omega_{op})|}{|F_1(j\omega_{op})|} \approx \frac{1}{c} \]  

(3)

\[ \angle H(j\omega_{op}) \approx -\arctan\left(\frac{\gamma/Q}{\sqrt{\gamma^2 + 4}}\right) - \arctan\left(\frac{\sqrt{\gamma^2 + 4}}{\gamma/Q}\right) + \frac{\pi}{2} \approx 0 \]  

(4)
Where \( \gamma = K(K_2 - K + K_c)/(Kc)^2 \) and \( c \) is the damping of the resonators. Due to the similar conclusions of a single resonator, we can transplant the electrical feedback circuit structure in [7] to realize the self-oscillation loop for a 3DoF resonator sensor.

### 3. Simulation

Fig. 2 Figure showing the schematics including: equivalent electrical circuit model of the 3DoF MEMS resonator with stiffness perturbations \( \Delta K \) and \( \Delta K_1 \). \( \Delta K/K = -0.001, \Delta K_1/K = 0.001 \) and changes to \( \Delta K_1/K = 0.0002 \) from \( t = 1s \) and onwards; variable gain stage and amplitude detection.
For the self-oscillation loop simulation, we have modelled the 3DoF weakly coupled resonator sensor into equivalent RLC circuit [5], shown in Figure 2. The equivalent values used for the simulations are also shown in Figure 2. The values are chosen close to the real physical device [8]. The standard transimpedance amplifier (TIA) shown in Figure 2 was used to convert the motional currents into voltages. The values of the feedback resistors and capacitors were chosen to be the same as the characterization circuit in our previous work [8]. A pulsed current source lasting for 100ns was used to inject the initial energy to start the oscillation.

The electrical feedback also includes a variable gain structure in conjunction with an amplitude detector, with both shown in Figure 2. Due to the 180° phase shift of the TIA, a further 180° phase shifter is required to ensure that the phase delay of the electrical feedback is 0°. The voltage controlled gain stage was realized using a p-channel enhancement mode transistor BSS84 due to the low on-state resistance and fast switching. A bias stage was used to make sure that the PMOS is always in the triode region. In Figure 2, before the amplitude detection, an amplification stage was used to make sure that the increase in amplitude can be detected. The amplitude detector that followed generates the control signal for the variable gain stage. This control signal ensures that the loop gain magnitude $|H(j\omega_0)G(j\omega_0)|$ decreases from approximately 2 to 1 as the amplitude of the resonator 1 increases, achieving the automatic gain control (AGC) function for the self-oscillating loop [7].

4. Simulation results

The simulation results in Figure 3 show that the self-oscillation loop is capable of achieving self-oscillation without actuation voltages, and auto-adjusting when stiffness perturbation $\Delta K_1$ is introduced. It can be seen that the resonators start to oscillate from approximately $t=0.2s$, and the amplitudes of the resonators can settle within approximately 0.5s.

To ensure the resonators were oscillating at the desired out-of-phase mode, the mode frequencies were calculated from the results and compared to the theory (Equation 2). The simulated period of one cycle for each stiffness perturbation $\Delta K_1$ was calculated by evaluating the time period for approximately 140 cycles (which is within a 0.01 second window) in the simulation, and dividing that time by the number of cycles. The oscillation frequencies were then calculated from the period of a cycle. The calculated values of the out-of-phase mode frequencies from the simulation results in Figure 4a show that the proposed loop structure is able to track the out-of-phase mode frequency with different perturbations, with an error smaller than 25ppm.

To verify the functionality of the force sensor [8], amplitude ratios as an output signal are also verified. The amplitude ratios from simulations can be calculated by taking the amplitudes of resonator 1 and 3 after 1s of the introduction of the stiffness perturbation, so that the amplitudes are within 1% of difference across a 0.01 second window. The obtained simulated amplitude ratios and theoretically estimated values [8] are plotted in Figure 4b. The simulated amplitude ratios agreed well with the theoretical estimations with relative errors within 2%.

Fig. 3 Simulated transient response of resonators 1 and 3 within the self-oscillation loop, with the stiffness perturbation $\Delta K_1/K$ changes from 0.001 to 0.0002 from $t=1s$: (a) transient response of resonator 1; (b) transient response of resonator 3.
5. Conclusions and outlook

To conclude, in this paper, we have demonstrated a feasible self-oscillation loop structure for the emerging type of resonator sensor with 3DoF weakly coupled resonators. The simulation results showed that the structure was able to track the out-of-phase mode frequency of the 3DoF resonator sensor, while maintaining the functionality of the sensor.

The future work should be focusing on the implementation of the physical self-oscillation loop structure discussed in this paper. The main design challenges will be the time delay for the resonators to settle, as in current simulation results, the time delay for resonator 3 to settle is approximately 0.5s, which still needs further improvements.

References

[1] M. Spletzer, A. Raman, A. Q. Wu, X. Xu, R. Reifenberger (2006): Ultrasensitive mass sensing using mode localization in coupled microcantilevers. Applied Physics Letters, 88, 254102.
[2] P. Thiruvenkatanathan, J. Yan, A. A. Seshia (2010): Differential amplification of structural perturbations in weakly coupled mems resonators, IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency control, Vol. 57, No. 3, 690-697
[3] M. A. Schmidt, R. T. Howe (1987): Silicon resonant microsensors, 14th Automotive Materials Conference: Ceramic Engineering and Science Proceedings, Volume 8, Issue 9/10, 1019–1034
[4] P. W. Anderson (1958): Absence of diffusion in certain random lattices. Physical Review, 109(5):1492–1505
[5] C. Zhao, G. Wood, J. Xie, H. Chang, S. Pu, M. Kraft (2015): A sensor for stiffness change sensing based on three weakly coupled resonators with enhanced sensitivity, Micro Electro Mechanical Systems (MEMS), IEEE 28th International Conference on, 881-884.
[6] A. A. Seshia, M. Palaniapan, T. A. Roessig, R. T. Howe, R. W. Gooch, R. W. Schimert, S. Montague (2002): A vacuum packaged surface micromachined resonant accelerometer, Journal of Microelectromechanical Systems (MEMS), Vol. 11, No. 6, 784-793
[7] C. T. C. Nguyen, R. T. Howe (1999): An integrated cmos micromechanical resonator high-Q oscillator, IEEE Journal of Solid-State Circuits, Vol. 34, No. 4, 440-455
[8] C. Zhao, G. S. Wood, J. B. Xie, H. L. Chang, S. H. Pu and M. Kraft (2015): A force sensor based on three weakly coupled resonators with ultrahigh sensitivity. Sensors and Actuators A: Physical, 232, 151-162
[9] F. He, R. Ribas, C. Lahuec, M. Jezequel (2009): Discussion on the general oscillation startup condition and the Barkhausen criterion. Analog Integrated Circuits and Signal Processing, 59(2):215-221