Gravitational-wave Sources from Mergers of Binary Black Holes Catalyzed by Flyby Interactions in the Field

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Abstract

Several scenarios were suggested for the origins of gravitational-wave (GW) sources from mergers of stellar binary black holes (BBHs). Here we propose a novel origin through catalyzed formation of GW sources from ultra-wide binaries in the field. Such binaries experience perturbations from random stellar flybys that excite their eccentricities. Once a wide binary is driven to a sufficiently small pericenter approach, GW emission becomes significant, and the binary inspirals and merges. We derive an analytic model and verify it with numerical calculation to compute the merger rate to be \( \sim 1 \times f_{\text{wide}} \text{Gpc}^{-3} \text{yr}^{-1} \) \( (f_{\text{wide}} \text{ is the fraction of wide BH-binaries}) \), which is a relevant contribution to the observationally inferred rate. The observational signatures from this channel include spin-orbit misalignment; preference for high mass ratio BBH; preference for high velocity dispersion host galaxies; and a uniform delay-time distribution.

Unified Astronomy Thesaurus concepts: Gravitational wave sources (677); Binary stars (154); Wide binary stars (1801); Interstellar dynamics (839)

1. Introduction

Extensive theoretical studies over the past few decades have proposed the existence of gravitational-wave (GW) sources arising from the mergers of two compact objects, and provided a wide range of predicted production rates of such sources (e.g., Belczynski et al. 2002, 2004, 2007, 2008, 2016; Antonini & Perets 2012; Dominik et al. 2012; Antognini et al. 2014; Antonini et al. 2014; de Mink & Belczynski 2015; Petrovich & Antonini 2017, and more). Observationally, 11 confirmed GW mergers have been detected by aLIGO and VIRGO since their initial operation. These include 10 mergers of binary black holes (BBHs) and a single merger from a binary neutron-star (NS) The LIGO Scientific Collaboration et al. (2018). The currently inferred BBH-merger rate from these observations (in the local universe) is \( R_{\text{BBH}} = 9.7-101 \text{Gpc}^{-3} \text{yr}^{-1} \), while the merger rates of binary NS is \( R_{\text{BNS}} = 110-3840 \text{Gpc}^{-3} \text{yr}^{-1} \).

Three main evolutionary channels were proposed in the context of GW mergers. The first deals with mergers in dense environments such as galactic centers or globular clusters (e.g., Rodriguez et al. 2016, 2018; Banerjee 2018; Fragione & Kocsis 2018; Hamers et al. 2018; Leigh et al. 2018; Rasskazov & Kocsis 2019), where binary mergers are catalyzed by strong interactions with stars in these dense environments. In such environments, strong three-body interactions lead to harden compact binaries and excite their eccentricities. Such models predict GW-production rates in the range of \( \sim 2-20 \text{Gpc}^{-3} \text{yr}^{-1} \).

The second deals with the isolated evolution of initially massive close binary stars (e.g., Belczynski et al. 2008, 2016; Dominik et al. 2012, 2015). Some of the massive close binaries strongly interact through one or two common envelope phases (e.g., Dominik et al. 2012), which may lead to the production of a short period binary. A fraction of the post-CE binaries are sufficiently close to merge via GW emission within Hubble time. A different merger path is through the “chemically homogeneous channel” (Mandel & de Mink 2016). The large uncertainties in the evolutionary parameters and processes of the massive stars give rise to a wide range of expected GW-source production rates in the range of \( \sim 10^{-2}-10^{3} \text{Gpc}^{-3} \text{yr}^{-1} \).

The third evolutionary channel deals with mergers induced by secular evolution of triple systems either in the field (Antonini et al. 2017) or in dense environments (e.g., Antonini & Perets 2012; Petrovich & Antonini 2017; Antonini et al. 2018; Hoang et al. 2018; Samsing & D’Orazio 2018; Samsing 2018; Fragione et al. 2019; Hamilton & Rafikov 2019). In this channel the secular perturbations by a third companion (Lidov–Kozai evolution Kozai 1962; Lidov 1962) can drive BBHs into high eccentricities such that they merge within a Hubble-time; the rates expected in this channel are \( \sim 0.5-15 \text{Gpc}^{-3} \text{yr}^{-1} \).

Here we present a fourth evolutionary channel, in which we focus on wide (SMA > 1000 au) BBHs in the field perturbed by random flyby interactions of field stars in their host galaxy. Kiib & Raymond (2014) and Michaely & Perets (2016) showed that wide binary systems can be significantly affected by flyby interactions of field stars, and effectively experience a collisional evolution.

We show that collisional evolution in the field could also be important for the formation of GW sources. We analyze the evolution of wide-orbit BBHs and show that a fraction of these can be driven into close pericenter distances due to interaction with stellar perturbers. In cases where the pericenter distance of a given binary is driven into a sufficiently small distance GW emission becomes significant and the binary rapidly loses angular momentum and energy due to GW emission, and eventually inspirals and merges as a GW-source detectable by aLIGO/VIRGO.

This paper is organized as follows. In Section 2 we present the analytic model, the basic assumptions, and the calculations. In Section 3 we present the numerical verification. We discuss the results and summarize in Section 4. The numerical procedure, the equations we integrate, and the data analysis are described at length in Appendix A.
2. Analytic Model

2.1. Formation Scenario

We consider a wide BBH with semimajor axis (SMA) \(a > 10^3\) au. The binary resides in the field of the host galaxy and is therefore affected by short duration dynamical interactions with field stars. The dynamical encounters can be modeled through the impulse approximation, namely \(t_{\text{int}} \equiv b/v_{\text{enc}} \ll P\) (where \(b\) is the closest approach to the binary and \(v_{\text{enc}}\) is the velocity of the perturbing mass) and \(P\) is the orbital period. These perturbations can change the binary SMA, \(a\) and the binary eccentricity, \(e\). It has been shown (Lightman & Shapiro 1977; Merritt 2013) that angular momentum change is more significant than energy change for this process, hence we focus on the eccentricity change. If these interactions produce a sufficiently small pericenter passage, then the system can merge via GW emission within Hubble time.

There are four relevant timescales for this impulsive treatment: the interaction timescale \(t_{\text{int}} \equiv b/v_{\text{enc}}\), the binary orbital period \(P\); the merger time from a specific binary configuration via GW emission \(t_{\text{merger}}\); and the time between two consecutive encounters of the system and a flyby perturber, \(t_{\text{enc}} = 1/f = (n_s \sigma v_{\text{enc}})^{-1}\) where \(n_s\) is the stellar number density, and \(\sigma\) is the geometric cross-section of the binary and the flyby.

We restrict our model to the impulsive regime, \(t_{\text{int}} \equiv b/v_{\text{enc}} \ll P\). This gives an upper bound to the closest approach distance, \(b \ll P \times v_{\text{enc}}\) and hence limits the average time between encounters. Specifically, for a BBH with a total mass of \(20M_\odot\), SMA of \(a \sim 10^3\) au (hence \(P \approx 3 \times 10^5\) yr) and a typical velocity encounter of \(v_{\text{enc}} = 50\) km s\(^{-1}\) (velocity dispersion in the field) we restrict \(b\) such that \(t_{\text{int}} \ll P\). We chose the interaction time to be one order of magnitude less than the period \(t_{\text{int}} \ll P/10\) where numerical simulations show that the impulse approximation still holds. Hence we get \(b = t_{\text{int}} \times v_{\text{enc}} = 3 \times 10^2\) au, this corresponds to \(t_{\text{enc}} \approx 1\) Myr. Therefore we consider only systems that merge within this timescale such that \(t_{\text{merger}} < t_{\text{enc}}\) which insures that the merger occurs before the next impulsive interaction. Farther out flybys can also perturb the system and further excite the system, but at very large separations the interaction becomes adiabatic and the effects become small. We neglect the intermediate regime in which the perturbation time and the orbital times are comparable, which are likely to somewhat enhance the perturbation rates explored here.

2.2. Analytic Description

We consider the evolution of an ensemble of wide BBH binaries with initial separations \(a > 10^3\) au and comparable component masses \(m_1 \sim m_2 = m_{\text{BH}}\). For simplicity we assume all binaries to have the same SMA, and a thermal distribution of orbital eccentricities, \(f(e)de = 2ede\). In the following we derive the fraction of merging systems within this ensemble and find its dependence on the SMA of the binaries, \(a\) and the environmental conditions, namely the stellar density \(n_s\) and velocity dispersion \(\sigma_v = v_{\text{enc}}\).

The timescale for a GW merger of an isolated binary is given by Peters (1964)

\[
t_{\text{merger}} \approx a^4/\beta' \times (1 - e^2)^{3/2}
\]

with \(\beta' = (85/3)G^2m_1m_2(m_1 + m_2)e^{-5}\), where \(G\) is Newton’s constant and \(c\) is the speed of light and \(m_1 = m_2 = m_{\text{BH}}\). Given a binary with SMA \(a\) we can solve Equation (1) for the critical eccentricity \(e_\text{c}\) required for the binary to merge within any merger time, we set the merger time to be the time between encounters, \(t_{\text{merger}} = t_{\text{enc}} = T\).

All systems with eccentricities equal to or greater than \(e_\text{c}\) would therefore merge within this time frame \(T\). Hence, given a thermal distribution of eccentricities we find the fraction of a system that merged within a time \(T\) and was lost from the ensemble to be:

\[
F_q = \int_{e_c}^{1} 2ede = 1 - e_c^2 = \left(\frac{\beta'}{a^4}\right)^{2/7}.
\]

Following previous studies we term this “loss” region the “loss-cone” (see, e.g., Hills 1981; Nicholson & Nicholson 1983; Merritt 2013); after time \(T\) all binaries in the loss-cone merge via GW emission and this phase-space region becomes empty. However, binaries outside the loss-cone that do not merge within this timescale can be perturbed by a flyby encounter, changing their angular momentum, and thereby enter and replenish the loss cone. The average size of the phase-space region into which stars are perturbed during a single orbital period is termed the smear cone, defined by \(\theta = (\Delta v)/v_k\), where \(v_k\) is the Keplerian velocity of the binary. The value of \(v_k\) can be calculated given that the average separation of a Keplerian orbit is \(r = a(1 + 1/2e^2)\) and we approximate \(e \rightarrow 1\), namely \(v_k = (Gm_b/3a)^{1/2}\).

\(\Delta v\) is the average change in the velocity over an orbital period due to perturbations (Hills 1981). Let us consider flyby interactions using the impulse approximation. Hills (1981) showed that on average the velocity change (for a binary with SMA, \(a\), to the binary components is of the order of \(\Delta v \approx 3Gm_p/v_{\text{enc}}b^2\) where \(v_{\text{enc}}\) is the velocity of the flyby star with respect to the binary center of mass, \(m_p\) is the perturber mass, and \(b\) is the closest approach distance of a flyby. Therefore, the square of the angular size of the smear cone cause by the impulse of the flyby on the binary is

\[
\theta^2 = \frac{9G^2a^2m_b^2}{(v_{\text{enc}}b^2)^2} \frac{3a}{27G^2a^2m_b^2} = \frac{27Ga^2m_b^2}{m_b(v_{\text{enc}}b^2)^2}.
\]

and for \(\theta \ll 1\) we get the fractional size of the smear-cone velocity space over the 4\(\pi\) sphere to be after a single passage of the perturber

\[
F_s = \frac{\pi\theta^2}{4\pi} = \frac{27}{4} \left(\frac{m_p}{m_b}\right)^2 \left(\frac{Gm_b}{av_{\text{enc}}^2}\right) \left(\frac{a}{b}\right)^4.
\]

It is evident from (4) that for a given binary the size of the smear cone depends on the perturber quantities, i.e., mass, velocity, and the closest approach. The ratio of \(F_s\) to \(F_q\) indicates the fraction of the loss-cone filled after a single flyby.

\[
\frac{F_s}{F_q} = \frac{27}{4} \left(\frac{m_p}{m_b}\right)^2 \left(\frac{Gm_b}{av_{\text{enc}}^2}\right) \left(\frac{a}{b}\right)^4 \left(\frac{a^4}{\beta'^4}\right)^{2/7}.
\]

In the case where the loss-cone is continuously full \((F_s = F_q)\) the depletion rate, \(\dot{L}(a, n_s)\) only depends on the loss-cone size, \(F_q(a)\), and the merger time, \(T\). Hence the depletion rate for the full loss-cone (FLC) is \(\dot{L}_q(a, n_s) = F_q/T\).
Note that for the FLC the depletion rate is independent of the stellar density. Furthermore, the FLC depletion rates scale like \( L \propto F_{q} \propto a^{-8/7} \), i.e., the FLC depletion rate decreases with increasing SMA.

On the other hand, tighter binaries are less susceptible for change due to a flyby, this is evident from Equation (4). Therefore tighter than a critical SMA we expect that the loss-cone will not be full all the time, in this "empty loss cone (ELC)" case the depletion rate depends on the rate of orbits being kicked into the loss-cone, namely \( f = n_{w} \sigma_{\text{enc}} \), where \( n_{w} \) is the stellar density, \( \sigma = \pi b^{2} \) is the geometric cross-section.

The condition for the loss-cone to be continuously full is that the loss-cone orbits are replenished at least as fast as they are depleted due to the GW emission. This occurs when the rate of flyby’s that enter orbits to the loss-cone, \( f \), is equal to the rate of which orbits are depleted from the loss-cone, \( 1/T \), namely \( n_{w} \pi b^{2} \sigma_{\text{enc}} = T^{-1} \).

A stellar flyby is sufficiently strong to replenish the loss-cone if
\[
(v_{\text{enc}} b^{2})^{2} \leq \frac{27}{4} \frac{G m_{p}^{2} a^{29/7}}{m_{b} (3\beta^{2} T)^{2/7}}.
\]

Plugging this into the continuously full condition we get an equation for the critical SMA that separates the ELC and the FLC regimes:
\[
a_{\text{crit}} = \left[ \frac{4 m_{p} (3\beta^{2} T)^{-12/7}}{27 G m_{b}^{2} \pi^{2} a^{3}} \right]^{7/29}.
\]

Using the critical SMA we can calculate the merger probability in each of these regimes, \( a < a_{\text{crit}} \) (empty) and \( a > a_{\text{crit}} \) (full). We denote \( F_{q} \) as the fraction of wide binaries destroyed after time \( T \), and therefore \( 1 - F_{q} \) represents the fraction of binaries that survive as wide binaries at the relevant timescale. For the ELC regime \( a < a_{\text{crit}} \) the relevant timescale is \( 1/f \); for the FLC regime \( a > a_{\text{crit}} \) the relevant timescale is \( T \). Therefore, \( (1 - F_{q}) \) is a monotonically decreasing function of time, and the probability for a merger of a wide binary is
\[
L(a, n_{w})_{a < a_{\text{crit}}} = 1 - (1 - F_{q}(a))^{f(a, n_{w})},
\]
where \( t \) is the time since birth of the binary. As one can expect the probability only depends on the size of the loss cone and the rate of interactions.

In the FLC regime the limiting factor is not the value of \( f \), but the merger timescale \( T \). Therefore, the full expression for the loss probability for \( a > a_{\text{crit}} \) is
\[
L(a)_{a > a_{\text{crit}}} = 1 - (1 - F_{q}(a))^{1/T}.
\]

The above treatment neglects the fact that perturbations may also "ionize" a binary and destroy it, namely, the binary is disrupted by the random flybys. Such an ionization process decreases the available number of wide binaries. To account for the ionization process we consider the finite lifetime of wide binaries due to flybys using the approximate relation given by Bahcall et al. (1985) for \( f_{1/2} \), the half-lifetime of a wide binary evolving through encounters
\[
 t_{1/2} = 0.00233 \frac{v_{\text{enc}}}{G m_{p} n_{w} a^{-1}}.
\]

We account for this and taking the leading term of Equations (8) and (9) to get
\[
 L_{a < a_{\text{crit}}} = \tau n_{w} m_{p} a^{13/14} \times \left( \frac{27 G (3\beta T)^{2/7}}{4 m_{b}} \right)^{1/2} (1 - e^{-1/T})
\]
and
\[
 L_{a > a_{\text{crit}}} = \tau a^{-8/7} \times T^{-5/7} (1 - e^{-1/T}),
\]
where \( \tau = t_{1/2} / \ln 2 \) is the mean-lifetime of the binary.

In order to estimate the number of systems observable within a year in aLIGO we first calculate the number of systems merging in a Milky Way (MW)–like galaxy per unit time. In order to do that we need to integrate over all SMA in a given stellar density and over all stellar densities in the galaxy we model. We follow a similar calculation from Michaely & Perets (2016). We model the Galaxy in the following way, let \( dN(r) = n_{w}(r) \cdot 2\pi \cdot r \cdot h \cdot dr \) be the number of stars in a region \( dr \) (and scale height \( h \)), located at distance \( r \) from the center of the Galaxy. Following Kaib & Raymond (2014) and references within we model the Galactic stellar density in the Galactic disk as follows \( n_{w}(r) = n_{0} e^{-r-r_{0}}/h_{0} \), where \( n_{0} = 0.1 \text{ pc}^{-3} \) is the stellar density near our Sun, \( R_{l} = 2.6 \text{ kpc} \) (Juric et al. 2008) is the galactic length scale and \( r_{0} = 8 \text{ kpc} \) is the distance of the Sun from the galactic center.

Integrating over the stellar densities throughout the disk of the Galaxy \( (r = 0.5–15 \text{ kpc}) \) we can obtain the total number of mergers through this process. Next we account for the fraction of wide BBH systems from the entire population of stars in the Galaxy, \( f_{\text{BBH}} \). We use the following standard values. Given a Kroupa initial mass function (Kroupa 2001), the fraction of the stars that evolve to become BHs is \( f_{\text{primary}} \approx 10^{-3} \). If we assume most BHs form without any natal-kick (similar assumptions were taken in other works e.g., Belczynski et al. 2016, Mandel 2016), we can expect all BHs to be in binary (or higher multiplicity) systems and the fraction \( f_{\text{bin}} = 1 \), consistent with the binary fraction inferred for the O-stars progenitors of BHs (Duchêne & Kraus 2013; Sana et al. 2014; Moe & Di Stefano 2016). Next we assume two sets of mass ratio distributions: first, uniform, \( Q \in (0.1, 1) \) (Duchêne & Kraus 2013); second, random pairing with \( Q \propto m^{-2} \) (Moe & Di Stefano 2016). The former yields a fraction of secondaries that evolve into BHs of \( f_{\text{secondary}} \approx 0.4 \) while the latter gives \( f_{\text{secondary}} \approx 0.04 \). We also consider two SMA distribution functions for completeness. A log-uniform distribution (Opik law; Duchêne & Kraus 2013; Moe & Di Stefano 2016), \( f_{a} \propto a^{-1} \), and a log-normal (Duchêne & Kraus 2013) (with mean SMA of 50 au and dispersion of \( \sigma_{\text{log} a} \approx 2.3 \)). The fraction of systems with SMA larger than \( 10^{3} \text{ au} \) is \( f_{\text{wide}} \approx f_{\text{bin}} \times 0.2 \) for log-uniform while \( f_{\text{wide}} \approx f_{\text{bin}} \times 0.15 \) for the log-normal. These values are actually a lower limit for massive binaries, recently Igoshev & Perets (2019) found that a wide binary fraction for massive B-stars to be \( f_{\text{wide}} \approx 0.5 \), and theoretical models suggest that the fraction of wide binary O-stars and BHs could be close to unity (Perets & Kouwenhoven 2012). \( f_{\text{wide}} = 1 \), and we therefore expect a wide-binary fraction in the range 0.2–1, in the following we use \( f_{\text{wide}} = 0.5 \)
\[
f_{\text{BBH}} \approx 2 \times 10^{-4} \left( \frac{f_{\text{primary}}}{10^{-3}} \right) \left( \frac{f_{\text{secondary}}}{0.4} \right) \left( \frac{f_{\text{wide}}}{0.5} \right). \]
The number of merging BBH per megayear from this channel with a SMA log-uniform distribution for a MW-like Galaxy is

\[ \Gamma = \int_{0.5}^{15 \text{kpc}} \int_{10^5 \text{au}}^{10^9 \text{au}} L(a, n_s) \times 10^{10} \text{yr}^{-1} f_a f_{\text{BBH}} \text{d}a \text{d}N(r) \approx 0.41 \text{Myr}^{-1}. \]  

(14)

While the value for log-normal distribution is \( \Gamma \approx 0.25 \text{Myr}^{-1} \).

Following Belczynski et al. (2016) we calculate the merger rate, \( R \) per \( \text{Gpc}^3 \) by using the following estimate

\[ R = 4.8(0.48) \left( \frac{f_{\text{primary}}}{10^{-3}} \right) \left( \frac{f_{\text{secendary}}}{0.4(0.04)} \right) \left( \frac{f_{\text{wide}}}{0.5} \right) \text{Gpc}^3 \text{ yr}^{-1} \]  

(15)

while the merger rate for log-normal distribution is

\[ R = 2.9(0.29) \left( \frac{f_{\text{primary}}}{10^{-3}} \right) \left( \frac{f_{\text{secendary}}}{0.4(0.04)} \right) \left( \frac{f_{\text{wide}}}{0.5} \right) \text{Gpc}^3 \text{ yr}^{-1}, \]  

(16)

where \( \rho_{\text{gal}} \) is local density of the MW-like galaxies with the value of \( \rho_{\text{gal}} = 0.0116 \text{ Mpc}^{-3} \) (e.g., Kopparapu et al. 2008) and \( \Gamma \) is given in units of \( \text{Myr}^{-1} \).

### 3. Numerical Calculation

In this section we briefly describe the numerical calculation we perform. We simulate the evolution of a binary BH with masses of \( m_1 = m_2 = 10M_\odot \). For 10 Gyr, we treat the evolution by considering both the evolution of the binary between encounters, and in particular the effects of GW emission, as well as the change of the binary orbital elements due to the impulsive flyby encounters with field stars. For each ensemble we calculate the fraction that merged out of the initial binaries.

The numerical results are presented in Figure 1; the numerical result are highly consistent with the result of the analytic model. A more detailed technical description of the numerical procedure, equations, and analysis is given in the Appendices A, B, and C.

### 4. Discussion and Summary

The merger rate we calculate strongly depends on the natal-kicks given to BHs, which are poorly constrained (e.g., Repetto et al. 2012, 2017) although others assume differentially (e.g., Gualandris et al. 2005; Willems et al. 2005; Fragos et al. 2009; Janka 2013). Particularly, it is still unknown whether a BH receives a momentum kick at birth like an NS, or forms without any natal-kick following a failed supernova or a large amount of fallback (e.g., Belczynski et al. 2004, 2008; Ertl et al. 2015). Previous models that were able to produce rates comparable to the rate inferred from observations had typically taken similar assumptions of zero kick velocities (for all BHs, or at least for all BHs more massive than 10\( M_\odot \)), while models assuming higher natal kicks produced significantly lower rates (Belczynski et al. 2008; Dominik et al. 2015). In our case low natal kicks can unbind the wide binaries, lowering their fraction. A typical orbital velocity or escape velocity from a cluster is on the order of few \( \text{km s}^{-1} \), hence any natal-kick larger than 10 \( \text{km s}^{-1} \) would unbind binaries or eject from the cluster, thus excluding the capture scenario. In principle, in models where wide binaries form following the dispersal of their birth cluster on longer timescales (Kouwenhoven et al. 2010; Perets & Kouwenhoven 2012), BH may acquire wide companions well after their formation. Nevertheless, even in these cases the BHs need to be retained in the cluster until its dispersal, and therefore the natal kick needs to be sufficiently low for a BH not to escape the cluster. We conclude that adapting similar no-kick assumptions for BHs (as done by other potentially successful scenarios) suggests the wide-binary channel explored here can give rise to a high production rate of GW sources from BBH mergers of perturbed ultra-wide binaries. In the future we intend to explore the rates of forming wide binaries from stellar evolution of close binaries that experience mass loss, Blaauw-kick and none-zero natal kicks (Michaey et al. 2016).

Our proposed evolutionary channel gives rise to specific characteristics of the BBH mergers, which together can provide a distinct signature for this channel, as we discuss in the following.

Equations (11) and (12) describe the probability dependency for a given environment, namely the stellar density \( n_s \) and the encounter velocity \( v_{\text{enc}} \). We note that in both equations there is a \( \tau(1 - e^{-t/\tau}) \) dependency. Hence following Equation (10) the merger probability increases with the encounter velocity. For example, taking the same environment as that assumed in Section 2.2 but with \( v_{\text{enc}} = 200 \text{ kms}^{-1} \) gives a factor of \( \sim 1.92 \) higher rate of BBH GW sources. We therefore expect a preference for host galaxies with higher velocity dispersion.

This model is sensitive to extreme mass ratios, \( Q = m_2/m_1 \) where \( m_2 \leq m_1 \). The equations that govern the rates depend on \( \beta \). When the binary mass is kept constant but the mass ratio \( Q \) is varied we get: \( \beta' \propto Q/(1 + Q)^2 \), and since this is a monotonically increasing function, equal mass components have the highest probability to merge. Moreover, the merger rate also has a monotonic dependence on the total binary mass, due to the complex mass dependence in the loss-cone analysis and the effects of ionization (see Equations 11 and 12). Hence overall we expect a preference toward GW sources from more massive binaries and higher mass ratios. Furthermore, in this channel the spins of the BHs are likely uncorrelated given the origin of the BH components from very wide separations (or a random capture) and we therefore expect the spins of the merging BBH components to be randomly (mis)aligned, in contrast with, e.g., the isolated binary evolution channel (e.g., Mandel & de Mink 2016). Given the long timescale for inspiral from large separations we also expect BBHs to fully circularize by the time they reach the aLIGO band and to not produce any eccentric binaries at these frequencies, in contrast with some of the dynamical channels. Finally, in our scenario the distribution of merger time since star formation, the delay-time distribution differs from the isolated binary channel. The latter predicts a \( \propto \tau^{-1} \) dependence (Dominik et al. 2015) while our model, which have no time dependency on the merger probability, generally predicts a uniform delay-time distribution.

In summary, the wide-binary origin for BBH GW mergers can give rise to a potential rate of \( \sim 1 \times f_{\text{wide}} \text{ yr}^{-1} \text{ Gpc}^{-3} \) (where \( f_{\text{wide}} \) can plausibly reside in the range of 0.2–1), which contributes to (the lower range of) the observationally inferred rate of \( \sim 10–110 \text{ yr}^{-1} \text{ Gpc}^{-3} \) from aLIGO/VIRGO detections, and is strongly dependent on the natal kicks imparted to BHs at birth. It can be characterized by the following signatures: (1) A slight preference for high mass ratio BBH GW sources. (2) A preference for more massive BBH. (3) Typically randomly
misaligned spin-orbit BHs. (4) Circular orbits in the aLIGO band. (5) Preference for high velocity dispersion host galaxies/environments. (6) A uniform delay-time distribution.

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Appendix A
Appendix: Numerical Scheme

In order to calculate the average time between encounters we use the rate \( f = n_s \sigma \langle v_{\text{enc}} \rangle \), where \( n_s \) is the stellar number density, taken to be the solar neighborhood value of \( n_s = 0.1 \text{ pc}^{-3} \); \( \langle v_{\text{enc}} \rangle \) is the velocity of the perturber as measured from the binary center of mass, where we set \( \langle v_{\text{enc}} \rangle = 50 \text{ km s}^{-1} \) similar to the velocity dispersion in the solar neighborhood; and \( \sigma \) is the interaction cross-section. We focus on the impulsive regime, namely \( t_{\text{int}} \ll P \) (see Section 2.1). With these values the largest closest approach distance \( b \) for which an encounter can be considered as impulsive is \( b_{\text{max}} = 5 \times 10^4 \text{ au} \). The average time between such impulsive encounters is given by \( t_{\text{enc}} = 1/f \approx 1 \text{ Myr} \). Therefore we randomly sample the time between encounters from an exponential distribution with a mean \( f \) (due to the Poisson distribution of encounter times).

We initialize the wide binary with an SMA \( a \) and eccentricity \( e \). At each step we first find the next encounter time \( t_{\text{enc}} \), and evolve the binary for \( t_{\text{enc}} \) through the well known equations of motion given by Peters (1964) for \( \dot{a} \) and \( \dot{e} \). If the binary did not merge through GW emission by the time of the next encounter we simulate the impulsive interaction with a perturber with velocity \( v_{\text{enc}} \) drawn from a Maxwellian distribution with velocity dispersion \( \langle v_{\text{enc}} \rangle \) and a mass of \( m_p = 0.6 M_\odot \), typical for stars in the field. After changing the binary orbital parameters due to the encounter we continue to evolve the binary until the next encounter and so on, until the binary merges or disrupts, or the maximal simulation time of 10 Gyrs is reached.

Appendix B
Post-interaction Orbital Elements

In order to calculate the post-interaction orbital elements of a perturbed binary, we apply the following procedure. We first randomize the perturber trajectory; we randomly sample the closest approach point, \( b = bb \) of the perturber from an isotropic distribution by randomizing the two spherical coordinate angles, uniformly distributed \( \cos \alpha \in (-1, 1) \) and uniformly distributed \( \beta \in (0, 2\pi) \), which determine a plane perpendicular to the closest approach vector \( b \). The perturber trajectory can have any direction within this plane, hence we

![Figure 1. Merger probability of BBH with \( m_b = 20M_\odot \) with flyby mass \( m_p = 0.6M_\odot \) and \( v_{\text{enc}} = 50 \text{ km s}^{-1} \). The stellar density number is \( n_s = 0.1 \text{ pc}^{-3} \). The theoretical probability it calculated after \( t = 10 \text{ Gyr} \) since the BBH was formed. The peak probability is achieved at \( a = a_{\text{crit}} \). The merger time \( T = 1 \text{ Myr} \). Red dashed line without accounting for ionization; blue solid line accounting for ionization. The black circles are the estimated probabilities from the numerical simulation (see Section (3) for details). The error bar represent one standard deviation from the estimated value.](image-url)
randomize an additional angle, uniformly distributed in the range \( \xi \in (0, 2\pi) \) as to choose an arbitrary axis in the plane. Next we randomize \( b \) by setting the distribution function to be \( f(b) \propto b_{\text{max}} \).

We randomize the state of the binary by randomizing its mean anomaly, \( M \) from a uniform distribution between \((0, 2\pi)\) and calculate the post-interaction binary orbital elements \( a_{\text{post}} \), \( e_{\text{post}} \) and \( \theta_{\text{post}} \) using the impulse approximation. In the impulse approximation we neglect any motion of the binary during the passage of perturber. In this approximation we can calculate the velocity change of each of the components of the binary (Rickman 1976; Collins & Sari 2008). The velocity vector change for \( m_1 \) is given by

\[
\Delta v_1 = \int_{-\infty}^{\infty} \frac{G m_p r_p}{|r_p|^3} \, dt = \frac{2 G m_p v_{\text{enc}} b}{v_{\text{enc}} b},
\]

where \( r_p \) is the position of the flyby perturber set to be at the closest approach at \( t = 0 \). In the impulse regime we can approximate the trajectory of the flyby during the interaction as a straight line thus

\[
r_p = b + v_{\text{enc}} b t.
\]

The velocity change for \( m_2 \) is then

\[
\Delta v = \int_{-\infty}^{\infty} \frac{G m_p (b_2 + v_{\text{enc}} t)}{|b_2 + v_{\text{enc}} t|^3} \, dt = \frac{2 G m_p v_{\text{enc}} b_2}{v_{\text{enc}} b_2},
\]

where \( b_2 \) is the closest approach of the flyby perturber. We can find the relation between \( b \) and \( b_2 \) in the straight line trajectory approximation by (Collins & Sari 2008):

\[
b_2 = b - r + v_p (r \cdot v_p).
\]

The change in the relative velocity is simply

\[
\Delta v_r = \Delta v_2 - \Delta v_1.
\]

Given the relative velocity, \( v \) (as a function of the SMA, eccentricity and the mean anomaly), and the relative velocity change vector change, we can calculate the post-interaction eccentricity vector

\[
e_{\text{post}} = \frac{(v + \Delta v_r) \times (r \times (v + \Delta v_r))}{G m_p} - \frac{r}{r},
\]

to find the post-interaction eccentricity vector that is the norm of the eccentricity vector \( e_{\text{post}} = |e| \).

For the post-interaction SMA we calculate the change in orbital energy. The energy change is due to the velocity kick imparted by the perturber. In the impulse approximation we model the interaction only via a velocity change and not by a change in the separation itself during the encounter. The specific orbital energy is

\[
\varepsilon = \frac{v^2}{2} - \frac{G m_b}{r} = -\frac{G m_b}{2a},
\]

where \( r \) is the separation. The velocity change \( \Delta v_r \) can be written as a sum of the parallel (to the instantaneous orbital velocity) and the perpendicular vectors \( \Delta v_r = \Delta v_{r,\parallel} + \Delta v_{r,\perp} \). Hence the specific energy change is given by

\[
\Delta \varepsilon = \frac{(v + \Delta v_r)^2}{2} - \frac{v^2}{2} = \frac{\Delta v_{r,\parallel}^2 + 2v \cdot \Delta v_{r,\perp}}{2},
\]

which translates to change in the SMA of

\[
\Delta a = -a \cdot \frac{\Delta \varepsilon}{\varepsilon}
\]

to give us the final post-interaction SMA \( a_{\text{post}} = a + \Delta a \).

### Appendix C

**Numerical Results**

In order to validate our analytic calculation we numerically verify Equations (11) and (12) as plotted in Figure 1. We calculate the merger probability for the same binary setup and stellar environment, specifically for a BBH with \( m_b = 20M_{\odot} \), perturbed by stars of typical mass \( m_p = 0.6M_{\odot} \) and velocity dispersion of \( \langle v_{\text{enc}} \rangle = 50 \text{ kms}^{-1} \). The specific energy change is taken to be \( \varepsilon = 0.3 \text{ pc}^{-3} \). The final merger probability is calculated after \( t = 10 \text{ Gyr} \) since the BBH was initialized. We calculate the GW merger probability for several SMA values, \( a \approx a_{\text{crit}} = 2 \times 10^4 \text{ au} \) and \( a = 3 \times 10^4 \text{ au}, 1 \times 10^5 \text{ au} \). For each value of \( a \) we consider a range of initial eccentricities \( e \), and follow the evolution of each of the systems for 10 Gyr. In Table 1 we present the number of systems for each combination of \( a \) and \( e \). For each of the modeled initial conditions we calculate the fraction of merged systems in the following way. We record the number of merged systems \( n(a, e) \) to find the fraction of merged systems out of the total

| \( a (10^4 \text{ au}) \) | \( e = 0.8 \) | \( e = 0.9 \) | \( e = 0.95 \) | \( e = 0.99 \) | \( e = 0.995 \) | \( e = 0.999 \) | \( e = 0.9995 \) | \( e = 0.9999 \) |
|----------------|---------|---------|---------|---------|---------|---------|---------|---------|
| \( a = 0.5 \)  | \( n \) | ...     | ...     | 0       | 34      | 470     | 722     | 1515    |
| \( N \)        | ...     | ...     | ...     | 24927   | 11348   | 24587   | 24682   | 2470    |
| \( a = 1 \)    | \( n \) | \( N \) | ...     | 5       | 135     | 472     | 1801    | 158     | 3759    |
| \( a = 2 \)    | \( n \) | \( N \) | ...     | 100000  | 250430  | 99434   | 100367  | 94240   | 7491    | 125087  |
| \( a = 3 \)    | \( n \) | \( N \) | ...     | 99950   | 99588   | 96550   | 94873   | 94195   | 24643   | 2494    |
| \( a = 4 \)    | \( n \) | \( N \) | ...     | 2       | 62      | 85      | 207     | 64      | 61      |

Note. In all runs we chose \( b_{\text{max}} = 5 \times 10^4 \text{ au} \) to ensure we simulate the impulsive regime. The stellar number density is \( n_s = 0.1 \text{ pc}^{-3} \) and the velocity dispersion is \( \langle v_{\text{enc}} \rangle = 50 \text{ kms}^{-1} \). The perturber mass is \( m_p = 0.6M_{\odot} \). \( n \)-represents the number of mergers due to GW out of \( N \) simulations with the same initial setup, namely \( a \) and \( e \). The ratio of \( n/N \) is the estimate of the probability of merger for a specific \( a \) and \( e \).
In order to calculate the overall probability, we weigh \( f_{\text{merged}} \) with a thermal distribution of the initial eccentricities. Numerically we approximate the integral by the following estimate:

\[
P(f) = \int f_{\text{merged}}(a, e) \times f(e) \, de,
\]

where \( f(e) = 2e \) is the thermal distribution of the initial eccentricities. Numerically we approximate the integral by the following sum,

\[
P_1(a) = \sum_i \frac{1}{2} \left( f(a, e_{i+1}) + f(a, e_i) \right) \times f_{\text{merged}}(e_{i+1}) \times \Delta e_i,
\]

where \( \Delta e_i = e_{i+1} - e_i \) and \( \max \{ e_i \} < 1 \). Another approximation is

\[
P_2(a) = \sum_i f(a, e_i) \times f(e_i) \times \Delta e_i,
\]

where \( \Delta e_i = e_{i+1} - e_i \) and \( \max \{ e_i \} = 1 \). It is clear that \( P_1 \) overestimates the integral while \( P_2 \) underestimates \( P \). Hence we take the arithmetical mean of \( P_1 \) and \( P_2 \) and assign it as the calculated probability, \( P \).

In order to calculate the standard deviation we use the following estimate:

\[
\sigma_{\text{integrated}} = \left( \sum_i \sigma(a, e_i)^2 \times f(e_{i+1}) \times \Delta e_i \right)^{1/2}.
\]
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