Performance Improvement of Active MacPherson Suspension Using a Pneumatic Muscle and an Intelligent Vibration Compensator

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ABSTRACT This paper presents an intelligent control strategy for an active MacPherson suspension driven by a pneumatic muscle (PM) actuated system and evaluates its improved control performance on a self-developed test-rig. To fulfill the fully-functional test and analysis of active suspension systems, this test-rig comprises three units: 1. an active MacPherson suspension unit, which consists of a PM mechanism and two MacPherson struts, is built to isolate vibration from road disturbances; 2. a road profile generator (RPG) unit, which can produce a vertical force to lift the car-body according to various road profiles; 3. a PC-based control (PBC) unit, which computes, sends and receives signals for both of the active MacPherson suspension and the PC-based control unit. The objective of this study is to improve the MacPherson suspension in terms of the capability of road vibration isolation by using the PM actuation that can actively provide extra compensatory force for MacPherson struts. Then, for motion control of the PM, this study employs an interval type-2 adaptive fuzzy controller to approximate the optimal control law and adopts a self-tuning and fuzzy sliding mode compensator to compensate unmodeled dynamics for the active MacPherson suspension system. Three experiments are conducted to compare the active MacPherson suspension system with the original MacPherson struts through various road profiles on the test-rig. The results show the significant improvement for the proposed active MacPherson suspension system in suppressing the displacement and acceleration of the car-body.

INDEX TERMS Active suspension systems, interval type-2 fuzzy control, sliding mode compensator, pneumatic muscles, road vibration suppression.

I. INTRODUCTION

Vehicle suspension systems (VSS) are designed to isolate vibrations from the internal engine and external road disturbances to maintain driving comfort. VSS’s can be categorized into three general systems: passive, semi-active and active suspension. A passive suspension system has the simplest structure with common and basic suspension components, including springs and dampers. This has been proven to work smoothly in all types of road situations [1]. However, a passive suspension system is extremely sensitive to road surface bumps and irregularities. A semi-active suspension system is an electronic shock absorber that is capable of adjusting damping coefficients in real-time for different road conditions [2], [3]. Up to now, the Magneto Rheological (MR) damper is one of the most devices successfully applied to semi-active suspension systems [4]. An active suspension system with a force actuator absorbs shocks and maintains the chassis at an even level [5]–[7]. This is realized by monitoring the feedback forces from the road and generating movements that are perpendicular to the vehicle’s body. Most early designs for suspension systems used purely mechanical components or a simple combination of mechanical and hydraulic components. Current suspension systems usually involve a combination of mechanical, hydraulic, pneumatic and even electronic components. However, this combination
complicates the physical construction. In general, an active suspension system (ASS) contains an ordinary passive suspension component and active actuators. A hydraulic actuator [8], [9] is the most commonly used because it is strong and has a high power-to-weight ratio. However, fluid leaks may cause environmental pollutants. A hydraulic actuator is also large so it is not easily accommodated within the ASS. In recent years, pneumatic-based actuators have been the subject of many studies [10] and may replace hydraulic suspension due to the advantages such as low cost, compact structures, lightweight, and high reliability in operation. Zapateiro et al. [11] presented semi-active suspension control for vehicles using an MR damper. The nonlinearity in the rapid response of the MR damper was ignored while using back-stepping techniques. Banerjee et al. [12] designed and implemented a single axis DC attraction type suspension system in which four cascade lead compensators individually control four electromagnetic actuators. This suspension prototype achieved stable levitation with the desired operating gap by ignoring mutual cross-coupling. In [9], a hydraulic-driven active suspension system with fuzzy sliding controllers is presented to compensate for nonlinearity and time-varying behavior. This study simulated a suspension system on a test rig using a 20mm high bump. An ASS with a linear switched reluctance actuator is presented in [7]. The system incorporates a tracking differentiator (TD) into a PD controller to stabilize the system, but the displacement and the derivative of the sprung mass are acquired for the TD which significantly amplifies noise. To provide superior performance to a passive suspension and a PID-based ASS, a pneumatic-driven ASS test rig with four control feedback loops was proposed to improve force tracking and to compensate for disturbance [6], [7]. However, the maximum displacement of the sprung mass resulted only within 0.5cm high that limits the application in vehicle suspension systems. In [10], a state feedback control law based on the linear model of the prototype ASS was investigated and verified using a real-road-conditions generator. In 13,14,15, an interval type-2 fuzzy controller (IT2FC) was proposed for a nonlinear active suspension system that is subject to system modeling uncertainties, measurement noise and external disturbances. Such a controller minimizes the displacement and acceleration of the sprung mass that leads to more system robustness and reliability. However, it remains difficult to yield appropriate membership functions and fuzzy rules for the interval fuzzy controller to deal with system uncertainty and random vibrations. In 2016, we presented a prototype [16] of the PM-drive active suspension system and used an interval type-2 fuzzy controller with an adaptive fuzzy sliding compensator for it to compensate road disturbances and improve the ride comfort.

The Pneumatic Muscle (PM) was developed in the 1950s to actuate the artificial limbs [10], [17]–[19]. The retraction strength of a PM is dependent on the strength of the individual fibers in the woven shell. A PM has several advantages over a pneumatic actuator (PA): 1. if the cylinders have the same diameter, the weight of the PM is much lighter of that for a PA but it supplies much more initial force during contraction; 2. the physical model of the PM can be regarded as a spring that is under constant pressure or which has a constant volume (similar to a damping) and exhibits similar actions as the springs work; 3. a PM is light but powerful because it has a better ratio for power-to-weight and 4. a PM does not experience fluid leaks and has better performance in rough environments. The experimental results in [20] show that the FESTO PM performs well with the AVSS at frequencies of up to 8Hz and that the FESTO PM is applicable to suspension systems with a provided control law regardless of the complicated and highly nonlinear items in the system dynamics. However, a PM is very difficult to be compensated using conventional control approaches since it has a highly nonlinear mathematical and physical properties in nature [21], [22].

MacPherson Struts [23] have a similar physical layout to a real vehicle suspension. They are packaged with a spring and a damper and can suppress vibrations. However, it cannot actively provide actuator force against vibration while encountering road variations; besides, it contains nonlinearities and uncertainties in the mathematical model while analyzing its dynamic behavior. Therefore, it is critical to design an active actuator force for MacPherson Suspension to improve its performance on vibration suppression. This paper proposes an active MacPherson suspension system (AMSS) that utilizes a FESTO PM, named as AMSS-PM, as the force actuator for active suspension control. To generate real road profiles, the developed RPG unit consists of a wheel, rollers, an induction motor and a pneumatic cylinder. Because of the PM’s complicated dynamics, the designed AMSS-PM is nonlinear and uncertain. Thus, an interval type-2 adaptive fuzzy controller combined with a self-tuning fuzzy sliding mode compensator (IT2AFC-STFSMC) is presented to compensate for the system uncertainties and attenuate disturbances and noise. This paper expands from [16] to enhance its experimental results and stability analysis for the overall system. There are three different road conditions conducted to verify the feasibility of AMSS-PM in this paper, and stability proof in detail for an ASS with unknown uncertainties and disturbances is also addressed here. The major contributions of this paper are depicted as follows:

- The utilization of PM actuator achieves the compact design, low energy consumption, and cost-effective implementation of the AMSS that utilizes a FESTO PM for a quarter-car suspension platform.
- A fully-functional test-rig for a quarter car is developed to validate the feasibility and performance of the designed AMSS-PM under various road conditions as provided by the road profile generator.
- The proposed IT2AFC-STFSMC can dominate the unmodeled dynamics and parametric uncertainties caused by external disturbances for a general nth nonlinear uncertain suspension system while guaranteeing stability and robustness for the tracking control problem.
The adaptive law of the IT2AFC-STFSMC provides an instantly tuning scheme for the controller gains of the IT2AFC-STFSMC, so that the optimal parameters can be found according to changed road conditions and the system stability remains within the Lyapunov framework.

II. DESCRIPTION OF THE MATHEMATICAL MODEL

The PM is a long tube that is made of natural rubber with fibers wrapped inside and metal fittings attached at each end. Due to the reversible physical deformation, the PM can produce linear motion during contraction and expansion of the muscle. The next section presents the mathematical model for the PM.

A. MATHEMATICAL MODEL OF THE PM

The PM changes shape when the pressure of the air that flows into the interior of the rubber tube is changed, as shown in Fig. 1. Injecting compressed air into the rubber tube expands the PM and produces a force that drags the sprung mass downward. Otherwise, the sprung mass is restored to its original shape when air is released from the PM. The reversible physical deformation during contraction and expansion produces a linear motion: Firstly, if the PM is modeled using a common static physical model [24]–[26], the length and the diameter of the PM are formulated as:

\[ L = l \cdot \cos \theta \]  
\[ D = \frac{l}{n \cdot \pi} \cdot \sin \theta \]  

where \( L \) represents the actual length of the PM and is a variable of the pressure inside the PM, \( L_0 \) represents the original length of the PM, \( l \) represents the length of the thread, \( n \) represents the number of turns in the thread, \( D \) represents the diameter, and \( \theta \) represents the angle that is resulted by threads relative to the longitudinal axis. Consider the PM as the cylindrical shape, and its volume can be expressed as:

\[ V = \frac{\pi D^2 L}{4} \]  

Substituting Eq. (1) and Eq. (2) into Eq. (3) yields:

\[ V = \frac{l^3 (1 - \cos^2 \theta) \cos \theta}{4\pi n^2} \]  

For the PM, the effective area of the cylinder changes over time so it is considered to be a pneumatic cylinder with a variable volume. According to Principle of the Conservation of Energy, the simple geometric force \( F_a \) that is exerted by the PM is calculated as the product of the pressure and the change in the volume with respect to length, which is [24], [25]:

\[ F_a = -(P - P_e) \frac{dV}{dL} \]  

where \( P \) represents the pressure inside the PM and \( P_e \) represents atmospheric pressure. The term \( dV/dL \) can be expressed as an equivalent effective area:

\[ \bar{A} = \frac{3L^2 - l^2}{4\pi n^2} \]  

Substituting (6) into (5) yields:

\[ F_a = (P - P_e) \frac{3L^2 - l^2}{4\pi n^2} \]  

Due to the complicated dynamic behavior of the proportional pressure regulator (PPR), which regulates airflow into the PM, a high-order nonlinear dynamic equation is required to establish an accurate mathematical model. Fortunately, since the bandwidth of the PPR is much wider than that of the PM, the PPR model can be formulated as a first-order linear differential equation for the AMSS-PM. After simple manipulation, the transfer function between the input voltage and the output pressure is:

\[ P = \frac{K_v}{T_s s + 1} \bar{u} \]  

where \( K_v \) is a constant gain of the PPR, \( T_s \) is a time constant for the PPR and \( \bar{u} \) is the input voltage. Substituting Eq. (8) into Eq. (7) yields:

\[ F_a = \left(\frac{K_v}{T_s s + 1} \bar{u} - P_e\right) \frac{3L^2 - l^2}{4\pi n^2} \]  

B. MATHEMATICAL MODEL FOR A QUARTER CAR

The AMSS-PM uses two MacPherson struts and a PM to support the sprung mass. The PM also provides vertical movement to reduce vibration. Figure 2 illustrates a model of the AMSS-PM, where \( Z_s \) is the displacement of the sprung mass.
The principal parameters of quarter car model.

TABLE 1.

| Parameter | Description                  | Value   |
|-----------|------------------------------|---------|
| $K_s$     | MacPherson strut stiffness coefficient | 19.6 kN/m |
| $B_s$     | MacPherson strut damping coefficient | 1400 Ns/m |
| $M_s$     | Mass of the sprung mass      | 252 kg  |
| $K_t$     | Tire stiffness                | 160 kN/m |
| $M_e$     | Mass of the unsprung mass    | 55 kg   |
| $L_o$     | Original length of the PM     | 600 mm  |
usually not given, even we have the type of PM for the AMSS-PM. Besides, the PM shows the complexity and uncertainties in its dynamic behavior because of the air-compressibility and the hysteresis phenomenon. Also, for the AMSS-PM, the nonlinear effects in the passive spring force, damping force, and vertical tire force would be amplified in some situations [2].

Therefore, the AMSS-PM exhibits complex nonlinear and time-varying behavior due to the fact that it contains these uncertainties and unmodeled dynamics from the MacPherson struts, the PM and the tire bouncing. The traditional model-based controller is challenging to implement for the AMSS-PM. To overcome the difficulty referred to, this paper proposed IT2AFC+STFSMC to control the AMSS-PM and evaluated its control performance.

A. FEEDBACK LINEARIZATION OF THE INPUT-OUTPUT MAP

In practice, many nonlinear systems may not be represented in an equivalent linear system. The feedback linearization is a common approach used for transforming a nonlinear system into an equivalent linear system. A general nonlinear system is given by:

\[ \dot{z} = F(z, v), \quad o = h(z) \]  

where \( z = [z_1, z_2, \cdots, z_n]^T \in \mathbb{R}^n \) is the state vector, \( v \) and \( o \) are the input and output of the system, respectively, and \( F \) and \( h \) are nonlinear functions.

**Assumption 1:** If the general nonlinear system has a relative degree \( r \) and the control signal \( v \) linear with respect to the \( o^{(r)} \), then an equivalent linear system can be found, as given by:

\[ o^{(r)} = \bar{f}(z) + \bar{g}(z)v, \]  

where \( \bar{f} \) and \( \bar{g} \) are unknown functions, and \( \bar{g} \neq 0 \) for \( z \) in controllable region \( U_c \).

In this paper, some assumptions are made before linearization: (1) the dynamic equation (12) satisfies the assumption 1; (2) the external loading and the friction that is generated during the motion of the piston rod in the shock absorber are refined as uncertain, which are neglected for the purposes of linearization. Clearly, Eq. (12) can be expressed a 5th order general nonlinear system, as given by:

\[ \ddot{x} = f(x, \ddot{u}) \]
\[ \ddot{y} = h(x), \]  

where the state vector is defined as \( \chi = [x_1, x_2, x_3, x_4, x_5]^T \) and the corresponding vector fields \( f(\chi, \ddot{u}) \) and \( h(\chi) \) are described as shown at the bottom of the next page:

and

\[ h(\chi) = \ddot{y} = x_1 \]

in which \( f(\chi, \ddot{u}) \) and \( h(\chi) \) are unknown and smooth vector functions on the set \( \Omega \in \mathbb{R}^6 \). In order to derive the relative degree \( r \) with \( 0 \leq r < 5 \) of the nonlinear system (15), the output \( \ddot{y} \) is differentiated along the \( f \) until the output appears explicitly [27], such as \( \partial f/\partial u(\dot{L}_1 h(\chi)) = 0, \ i \in \{0, 1, 2\} \), \( \partial f/\partial u(\dot{L}_2 h(\chi)) \neq 0 \). This relieves that the relative degree is \( r = 3 < n = 5 \), which means that the Eq. (15) can be transformed into a 3rd-order affine system with the system state \( \ddot{x} = [\ddot{y}, \ddot{y}']^T \), as given by:

\[ \ddot{y}^{(3)} = F(\ddot{x}) + G(\ddot{x})u, \]  

where

\[ F(\ddot{x}) = \frac{-1}{M_s} [K_s(x_2 - x_4) - B_s(\frac{1}{M_s} + \frac{1}{M_s})] (x_1 - x_3) \]
\[ + \frac{3(L_0 + (x_1 - x_3))^2 - l^2}{4\pi n^2} x_5 \]
\[ + \frac{(3L_0 + (x_1 - x_3))^2 - l^2}{4\pi n^2} P_c \]
\[ + \frac{B_s K_s}{M_s} x_3 - \frac{B_s}{M_s} Z_r x_5 + \frac{6(L_0 + (x_1 - x_3)(x_2 - x_4))}{4\pi n^2 T_s} x_5 \]
\[ + \frac{6(L_0 + (x_1 - x_3)^2(x_2 - x_4))}{4\pi n^2} P_c \]

and \( G(\ddot{x}) = \frac{K_v}{M_s} \frac{3(L_0 + (x_1 - x_3))^2 - l^2}{4\pi n^2} \). The details of Eq. (16) is shown in Appendix B.

In Eq. (16), \( y \) is the displacement of the sprung mass, \( \ddot{x} \) is the state vector and \( \ddot{u} \) is the control input (voltage) from the PPR. It is worthy of note that both \( F(\ddot{x}) \) and \( G(\ddot{x}) \) are highly nonlinear and uncertain, so it is more difficult for the AMSS-PM to define the boundaries for the states and to determine accurate dynamic models. The interval type-2 FLC, which is a type of intelligent controller, gives satisfactory performance for systems that are uncertain and imprecise. In this paper, an additional compensator is necessary to attenuate the lump of disturbances. A self-tuning fuzzy sliding mode compensator is proposed in [8] that allows disturbance to be rejected. Hence, this study combines an intelligent interval type-2 adaptive fuzzy control (IT2AFC) and a self-tuning fuzzy sliding model compensation (STFSMC) deal with the nonlinearity and uncertainty that is inherent in the AMSS-PM.

B. DESCRIPTION OF THE IT2AFC-STFSMC

To extend the control problem to a general case, Eq. (16) can be expressed in the form of Eq. (14), as given by:

\[ y^{(n)} = F(x) + G(x)u + d(x), \]  

where \( d(x) \) is an external disturbance and the unmodeled friction force of the piston rod within the shock absorber, \( x = [y \ y \ \cdots \ y^{(n-1)}]^T \in \mathbb{R}^n \), \( u \in R \), \( y \in R \) be the state vector, the control input and the system output, respectively. It is assumed that \( |d(x)| \leq D \) for all states \( x \), \( F(x) \) and \( G(x) \) are partially unknown functions with uncertain time-varying parameters. The boundary conditions of \( G(x) \) are expressed as \( 0 < M_2(x) \leq G(x) \leq M_1(x) \) and \( |\dot{G}(x)| \leq M_2(x) \), where \( M_1(x), M_2(x) \) and \( M_3(x) \) are positive unknown...
functions. Without loss of generality, \( G(x) \) can be assumed to be strictly positive. The reference signals are defined as 
\[
x_d = [y_d \ y_d \cdots y_d^{(n-1)}]^T,
\]
so the tracking error vector is expressed as:
\[
e(t) = x_d - x = [e(t) \ e(t) \cdots e^{(n-1)}(t)]^T.
\]
(18)
The sliding surface is:
\[
S(t) = c_1 e(t) + c_2 \dot{e}(t) + \cdots + c_{n-1} \dot{e}^{(n-1)}(t),
\]
(19)
where \( c_i \) is specified such that \( \sum_{i=1}^{n} c_i \lambda^{i-1} \) is a Hurwitz polynomial and \( \lambda \) is a Laplace operator. Taking [28] as a reference, we have the ideal control law as:
\[
u^* = \frac{1}{G(x)}[\eta S_B(t) + A^T e(t) - F(x) - d(x) + \gamma_d(t)].
\]
(20)
where \( \eta > 0 \) is a constant, \( A_s = [c_1, c_2, \cdots, c_{n-2}]^T \) is the constant vector, and \( S_B(t) = S - \Phi \sigma(t)/\Phi \), for which \( \Phi \geq 0 \) is the width of the boundary layer of the sliding surface \( S \). The properties of the function \( S_B \) are described below [29]:

\textbf{Property 1:} As \( |S(t)| > \Phi \Rightarrow |S_B(t)| = |S(t)| - \Phi \) and \( S_B(t) \).

\textbf{Property 2:} As \( |S(t)| \leq \Phi \Rightarrow S_B(t) = S_B(t) = 0 \).

The conditions on Property 1 and Property 2 allow the adaptation terminates as researching the boundary layer. Differentiating Eq. (19) gives:
\[
\dot{S}(t) = A^T e(t) + \gamma_d(t) - F(x) - G(x)u - d(x).
\]
(21)
Substituting Eq. (20) into Eq. (21) gives:
\[
\dot{S}(t) + \eta S_B(t) = 0
\]
(22)
The convergence for \( e(t) = [e(t) \ e(t) \cdots e^{(n-1)}(t)]^T \) in Eq. (22) can be achieved while \( \eta > 0 \). However, some variables in Eq. (16) maybe unknown or perturbed in AMSS-PM, and \( d(x) \) may not be measurable, so the implementation of the ideal control law \( \nu^* \) is impossible for the AMSS-PM. To this regard, the proposed IT2AFC-based STFSMC yields the control force \( \hat{u}_{sc} \) to approximate the ideal control law and a compensator \( u_{comp}(S) \) to compensate for the disturbance and the modeling error, which is:
\[
u = \hat{u}_{sc}(S, \hat{a}) + u_{comp}(S).
\]
(23)

\section{C. DESIGN OF THE INTERVAL TYPE-2 ADAPTIVE FUZZY CONTROLLER (IT2AFC)}

In this section, a single-input IT2AFC is designed to formulate the control law \( \hat{u}_{sc}(S, \hat{a}) \), as mentioned in Eq. (23). For the IT2AFC with \( M \) rules, the \( i \)th fuzzy rule is designed as:
\[
R^i: \text{if } S \text{ is } \tilde{F}^i_{TS} \text{ then } \hat{u}_{sc} = \hat{a}^i_{TS}, \quad (i = 1, \ldots, M),
\]
(24)
where \( S \) is the input variable, \( \tilde{F}^i_{TS} \) is the interval type-2 fuzzy set, and \( \hat{a}^i_{TS} \) is the interval type-2 singleton fuzzy set. The output of the IT2AFC calculated by the singleton fuzzification, the product inference and the center-average defuzzification gives:
\[
\hat{u}_{sc}(S, \hat{a}) = \frac{y_1 + y_r}{2} = \frac{1}{2} \left[ \hat{a}^T_{TS} \hat{a}^T_{TS} \right] \left[ \hat{x}_1 \right] = \hat{a}^T_{TS} \hat{x},
\]
(25)
where \( y_1 \) and \( y_r \) respectively represent the farthest left and the farthest right points of the interval type-2 set. In (25), the weight vector \( \hat{a}^T = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_M] \) is used to estimate the optimal weight vector \( \hat{a}^* = [\hat{a}_1^*, \hat{a}_2^*, \ldots, \hat{a}_M^*] \) and the parameter \( \hat{a}^{*T} \) is reasonably assumed to be bounded. The farthest left point of the interval type-2 set is defined as:
\[
y_i = \frac{\sum_{i=1}^{L} \hat{u}_{F_{TS}^i} (S) \hat{a}_i + \sum_{i=L+1}^{M} \hat{u}_{F_{TS}^i} (S) \hat{a}_i}{\sum_{i=1}^{L} \hat{u}_{F_{TS}^i} (S) + \sum_{i=L+1}^{M} \hat{u}_{F_{TS}^i} (S)}
\]
\[
= \sum_{i=1}^{L} \hat{p}_i \hat{a}_i + \sum_{i=L+1}^{M} \hat{p}_i \hat{a}_i = \left[ \hat{a}^T \hat{a}^T \right] \left[ \hat{p}_i \right] = \hat{a}^T \hat{p}_i,
\]
(26)
where \( \hat{u}_{F_{TS}^i} \) and \( \hat{u}_{F_{TS}^i} \) represent the upper and lower degrees of the membership function, respectively. \( F_{TS}^i \) is the farthest left point of \( \hat{a}^T_{TS} \), \( \hat{p}_i = \hat{u}_{F_{TS}^i} (S) / W_i \), and \( p^i = \hat{u}_{F_{TS}^i} (S) / W_i \), in which
\[
W_i = \sum_{i=1}^{L} \hat{u}_{F_{TS}^i} (s) + \sum_{i=L+1}^{M} \hat{u}_{F_{TS}^i} (s).
\]
The farthest right point of the interval type-2 set is defined as:

\[ y_r = \frac{R}{\sum_{i=1}^{M} \bar{\mu}_{F_{r_{s}}}(s) \hat{\alpha}_r^i + \sum_{i=R+1}^{M} \mu_{F_{r_{s}}}(s) \hat{\alpha}_r^i} \]

\[ = \sum_{i=1}^{R} p_i \hat{\alpha}_r^i + \sum_{i=R+1}^{M} p_i \hat{\alpha}_r^i = \left[ \hat{\alpha}_r^T \quad \hat{\alpha}_r^T \right] \begin{bmatrix} \bar{p}_r \\ p_r \end{bmatrix} = \hat{\alpha}_r^T \xi_r, \quad (27) \]

where \( \hat{\alpha}_r \) is the farthest right point of \( \hat{\alpha}_r \), \( p_i = \bar{\mu}_{F_{r_{s}}}(s) / W_r \), and \( p_i = \mu_{F_{r_{s}}}(s) / W_r \), in which \( W_r = \sum_{i=1}^{R} \bar{\mu}_{F_{r_{s}}}(s) + \sum_{i=R+1}^{M} \mu_{F_{r_{s}}}(s) \). The parameters \( L \) and \( R \), respectively in (26) and (27), can be calculated by using type reduction [30]. The adaptive law for the IT2AFC is given as:

\[ \dot{\hat{\alpha}} = \eta_1 \cdot S_\Delta \cdot \dot{\xi}, \quad (28) \]

where \( \eta_1 > 0 \) is the adaptive learning rate.

**D. DESIGN OF THE SELF-TUNING FUZZY SLIDING MODE COMPENSATION (STFSMC)**

The STFSMC uses the sliding surface \( S \) as the input and the sliding control law \( u_{fs} \) as the output. Figure 3 shows the membership functions for \( S \) and \( u_{fs} \). The associated linguistic variables are:

\[ T(S) = \{ NB, NM, NS, ZO, PS, PM, PB \} \]

\[ = \{ f_1^1, f_2^1, f_3^1, f_4^1, f_5^1, f_6^1, f_7^1 \} \]

\[ T(u_{fs}) = \{ NB, NM, NS, ZO, PS, PM, PB \} \]

\[ = \{ f_1^{\alpha}, f_2^{\alpha}, f_3^{\alpha}, f_4^{\alpha}, f_5^{\alpha}, f_6^{\alpha}, f_7^{\alpha} \} \quad (29) \]

The fuzzy rules are simply expressed as:

\[ R^l : \text{if } S = f^l_k \text{ then } u_{fs} = f^{s-l}_{\alpha}, \quad l = 1, \ldots, 7. \quad (30) \]

The output of STFSMC is calculated by the singleton fuzzification, the max-min inference and the center-average defuzzification gives:

\[ u_{fs} = \int_{-1}^{1} F(u_{fs}) du_{fs}, \quad (31) \]

where \( F(u_{fs}) = \bigwedge_{i=1}^{7} \{ f_i^l(S) \land f^{s-l}_{\alpha}(u_{fs}) \} \). To avoid heavy computational cost in the general fuzzy control algorithm, the calculation of \( u_{fs} \) can be simplified as two following cases:

**C1** \( k = S/\Phi < 0 \):

\[ u_{fs} = \begin{cases} 1, & \text{if } k < -1 \\ \frac{7.5k^2 + 13.5k + 5}{9k^2 + 15k + 5}, & \text{if } -1 \leq k < -\frac{2}{3} \\ \frac{9k^2 + 11k + 2}{18k^2 + 18k + 2}, & \text{if } -\frac{2}{3} \leq k < -\frac{1}{3} \\ \frac{1.5k^2 + 1.5k}{9k^2 + 3k - 1}, & \text{if } -\frac{1}{3} \leq k < 0 \end{cases} \quad (32) \]

**C2** \( k = S/\Phi \geq 0 \):

\[ u_{fs} = \begin{cases} -1.5k^2 + 1.5k, & \text{if } 0 \leq k < \frac{1}{3} \\ \frac{9k^2 - 3k - 1}{18k^2 - 8k + 2}, & \text{if } \frac{1}{3} \leq k < \frac{2}{3} \\ -7.5k^2 + 13.5k - 5, & \text{if } \frac{2}{3} \leq k < 1 \\ -1, & \text{if } k \geq 1 \end{cases} \quad (33) \]

where \( \Phi > 0 \) denotes the width of the boundary layer. Notice that \( u_{fs} = -\text{sgn}(S) \) if \( |S| \geq \Phi \), and thereafter the STFSMC is designed as:

\[ u_{comp}(S) = -(k_c + \hat{\rho})u_{fs}, \quad (34) \]

where \( k_c \) is a compensation gain that is expressed as:

\[ k_c = \frac{M_0(x)}{2M_0^2(x)}, \quad (35) \]

where \( M_0(x) \) and \( M_2(x) \) are specified variables; \( \hat{\rho} \) is a compensation gain [31] that is represented as:

\[ \hat{\rho} = \eta_2 \cdot |S_\Delta|, \quad (36) \]

in which \( \eta_2 > 0 \) is a learning rate that is greater than zero. Figure 4 shows the overall system for the proposed IT2AFC-STFSMC. For the IT2AFC, the parameter \( g_a \) is used to ensure that the sliding surface \( S \) is within the range of the fuzzy input and the gain factor \( g_a \) is used to regulate the fuzzy output \( \hat{u}_{fs} \).

**Assumption 2**: The internal dynamics of the nonlinear system are stable under the influence of the IT2AFC+STFSMC.

**Theorem 1**: Consider the AMSS-PM in the form of an affine system (17) that satisfies Assumptions 1 to 2, the control law is designed as:

\[ u = \hat{u}_{fs}(\hat{\alpha}) + u_{comp}(S), \quad (37) \]

where \( \hat{u}_{fs}(\hat{\alpha}) \) represents the IT2AFC that is shown in (25) and \( u_{comp}(S) \) represents the STFSMC that is shown in (34),
with the adaptive laws (28) and (36), respectively. Then, (i) the system state $x$ and the control law $u$ are bounded and (ii) the tracking errors converge to 0 as $t \to \infty$. The proof of Theorem 1 is given in Appendix A.

**IV. TEST RIG DESIGN AND DEVELOPMENT**

The designed test-rig for a quarter of vehicle model is shown in Fig. 5. This test-rig consists of three main units: the AMSS-PM unit, the RPG unit and the PBC unit. The following details the construction of these three units.

### A. AMSS-PM UNIT

The main components of the AMSS-PM unit are constructed using two MacPherson struts and a PM. This unit also contains a sprung mass which includes a counterweight and an upper support frame to represent a car-body, as shown in Fig. 6. To measure the displacement and the acceleration of the sprung mass, a linear encoder and an accelerometer are installed on the upper support frame and a linear scale is installed in the plane of the under support frame to measure vertical displacement. The PM, in parallel with two
MacPherson struts, are installed between the vehicle body and the wheel-axle to generate control force to the suspension. A proportional pressure regulator (PPR) regulates air into the PM and the PM produces vertical movement to attenuate external vibration from irregular roads and to maintain the upper support frame in a stable position. With the voltage range from 0V to 10V of the PPR, the system initially sets 5V to maintain the PM in a half-stretched condition so that the PM can react to road variations immediately. Therefore, the zero variation for the displacement of the sprung mass is set to 5V the half-stretched condition.

**B. PBC UNIT**

The PPR is installed inside a control box in the PBC unit, as shown in Fig. 7. The PBC is implemented on a National Instruments (NI) cRIO-9074 integrated system with an industrial 400 MHz real-time processor, a 64GB memory, a NI-9263 D/A card, a NI-9215 A/D card and a NI-9411 encoder card. The NI-9215 A/D card receives data, including the displacement of the unsprung mass and under support frame, and the acceleration of the sprung mass, from the AMSS-PM unit and the RPG unit. The NI-9411 encoder card receives data for the displacement of the sprung mass and the NI-9263 D/A card sends control signals to the proportional directional control valve (PDCV), the inverter and the PPR. The PM, the pneumatic cylinder and the induction motor are driven according to these signals.

**C. RPG UNIT**

The RPG unit, as shown in Fig. 8, consists of a wheel, rollers, an induction motor and a pneumatic cylinder. The induction motor, which is controlled by a frequency converter, rotates the rollers to induce rotation of the wheel. The maximum speed that the wheel can reach is 35 km/hr. A PDCV regulates the flow of air into the cylinder to create vertical displacements to take account of road conditions. The maximum pressure that is provided by the air compressor is 6 bar. To connect the base and the frame for the rollers, the bottom of the pneumatic cylinder is fixed to the base and the rollers bear firmly on the tire. A linear scale which is installed on the support frame is applied to measure the vertical variation in the road profile that is created by the pneumatic cylinder.

**V. EXPERIMENTAL RESULTS**

With the experimental setup as detailed in Section IV, the objective of the proposed AMSS-PM is to isolate the vibration generated by road irregularities and offer a ride comfort performance. Primarily, this paper aims to use the PM to provide extra compensatory force for the MacPherson strut and enhance its performances on vibration reduction. In this section, two phases are presented here to show that the proposed AMSS-PM has much improvement than the MacPherson strut and enhance its performances on vibration reduction.

First, we examine the performance of the vibration reduction regarding their displacement variation and acceleration variation in the time-domain on three different road profiles, which are a sine wave road profile, a rough concave-convex road profile, and a two-bump excitation road profile. Secondly, we evaluate the performance of the ride comfort due to their car-body acceleration in the frequency-domain. Please note that the sine-wave road profile (Experiment 1) has a constant 0.5Hz frequency so that its frequency analysis can be neglected.

For these three experiments, the initial control parameters are chosen as $c_1 = 5$, $c_2 = 1$, $\Phi = 6.5$, $g_s = 0.4$, $g_d = 0.3$, $\eta_1 = 0.05$, $\eta_2 = 0.01$, and $k_c = 0.65$. The membership functions for the controllers $u_{f_1}$ and $u_{f_2}$ are given in TABLE 2, where $M(S)$ and $M(u_{f_1})$ for $u_{f_1}$ is the type-1 fuzzy sets, $M(S)$ for $u_{f_2}$ is the gaussian type-2 fuzzy set, and $M(\hat{u}_{f_2})$ for $\hat{u}_{f_2}$ is the singleton type-2 fuzzy set. For $M(S)$, $m_1$ and $m_2$ respectively stand for the mean of the upper and lower membership functions, and $\delta$ is the variation of the membership functions.

TABLE 3 shows the fuzzy parameters used for the classic
TABLE 2. The membership functions for $u_{fs}$ and $\dot{u}_{fs}$.

| Linguistic variables | Parameters of MFs |
|----------------------|------------------|
| Input Var. $M(S) = [\cdots, -2/3\Phi, -1/3\Phi, 0, 1/3\Phi, 2/3\Phi, \cdots]$ | Output Var. $M(u_{fs}) = [-1.0, -0.66, -0.33, 0, 0.33, 0.66, 1.0]$ |

TABLE 3. The membership functions for $u_{FLC}$.

| Linguistic variables | Parameters of MFs |
|----------------------|------------------|
| Input Var. $M(S) = [\cdots, -0.25, -0.75, 0.5, -0.91, -0.01, 0.5, \cdots]$ | Output Var. $M(u_{FLC})$ is evenly distributed in the interval of $[-11, 11]$. The number of $M(u_{FLC})$ is set to seven. |

TABLE 4. The principal parameters of pneumatic muscle.

| Description | Value |
|-------------|-------|
| Mass of the PM | 1.2kg |
| Nominal inside diameter of the PM | 40mm |
| Lifting force of the PM | 6000N |
| Maximum permissible contraction of the PM | 150mm |

FIGURE 9. Displacement of the sprung mass over a sine wave road profile.

FIGURE 10. Acceleration comparison of the sprung mass over a sine wave road profile.

FIGURE 11. Actuating pressure comparison of the FC and IT2AFC-STFSMC when the quarter car moves along a sine wave road profile.

fuzzy controller (FC) in experiments. The mean for $M_{FLC}(S)$ is chosen as the average of $m_1$ and $m_2$, and the variation is the same as the one for $M(\dot{u}_{fs})$. TABLE 4 shows the principal parameters of the PM. It is noted that the output (voltage) of the IT2AFC-STFSMC is within the interval of $[-5, 5]$ V. So, the output has to be shifted to the interval of $[0, 10]$ because the zero variation for the displacement of the sprung mass is set to 5V. The following three experiments are used to verify the effectiveness of the AMSS-PM on the test-rig.

Experiment 1: Vehicle Riding on a Sine Wave Road Profile

In the first experiment, a sinusoid with an amplitude of 15mm and a frequency of 0.5Hz is used to verify the performance on vibration reduction. The road profile in Experiment 1 is given by

$$Z_{r3}(t) = 15\sin(\pi t) \quad \text{for } t \in [0, 10].$$  (38)
FIGURE 12. Displacement of the sprung mass over a rough concave-convex road profile.

FIGURE 13. Acceleration comparison of the sprung mass over a rough concave-convex road profile.

FIGURE 14. Actuating pressure comparison of the FC and IT2AFC-STFSMC when the quarter car moves along a rough concave-convex road profile.

TABLE 5. The comparisons between the IT2AFC-STFSMC with Macpherson and FC.

| Suspections                  | MacPherson | AMSS-PM with FC | AMSS-PM with IT2AFC-STFSMC |
|-----------------------------|------------|-----------------|-----------------------------|
| Performance                 |            |                 |                             |
| Maximum Displacement        | <18mm      | <7mm            | <6mm                        |
| RMS                         | 11.45mm    | 4.07mm          | 2.57mm                      |
| Acceleration of the sprung mass | 0.64g      | 0.21g           | 0.26g                       |

As can be seen in Fig. 9, while traveling on the sinusoid road, the maximum displacement of the sprung mass with the AMSS-PM by using the IT2AFC-STFSMC strategy is less than 6mm (red dashed line), and it for the AMSS-PM with FC is less than 7mm, and it for the MacPherson struts is up to 18mm. The resulted RMS values for the displacement of the sprung mass are 2.57mm, 4.07mm and 11.45mm, respectively. Figure 10 shows the acceleration comparison of the sprung mass, and Figure 11 shows the actuating pressure comparison of the FC and IT2AFC-STFSMC when the quarter car moves along a sine wave road profile. The maximum acceleration for the sprung mass is bounded within 0.26g (g = 9.8 m/s²) for the AMSS-PM using the IT2AFC-STFSMC while the maximum acceleration is 0.64g for the MacPherson struts and it for the AMSS-PM using the FC is 0.21g. In this case, the IT2AFC-STFSMC provides the larger initial pressure than the FC for the AMSS-PM so that it has bigger maximum acceleration. The experimental results also show that the AMSS-PM with the IT2AFC-STFSMC strategy can effectively suppress the vibration of the sprung mass. The performance comparisons are shown in Table 5.

Experiment 2: Vehicle Riding on a Rough Concave-Convex Road Profile

This experiment verifies the stability and robustness of the AMSS-PM running on a rough concave-convex road profile. The road profile is composed of a bump and a hollow with a sinusoidal disturbance superimposed to simulate a rough road surface that can be represented by

\[
Z_1(t) = \begin{cases} 
-15.54(t - 3.5)^3 + 3.5(t - 3.5)^2 + d_1(t) & \text{for } t \in [3.5, 5) \\
15.54(t - 6.5)^3 + 3.5(t - 6.5)^2 + d_1(t) & \text{for } t \in [5, 6.5) \\
15.54(t - 8.5)^3 - 3.5(t - 8.5)^2 + d_1(t) & \text{for } t \in [8.5, 10) \\
-15.54(t - 11.5)^3 - 3.5(t - 11.5)^2 + d_1(t) & \text{for } t \in [10, 11.5) \\
d_1 & \text{else,}
\end{cases}
\] (39)
where \( d_{r1}(t) \) is the sinusoidal disturbance. To make the simulation more challenging, a disturbance signal \( d_{r1}(t) \) is assigned as \( d_{r1}(t) = 1.75 \sin(2\pi t) + 0.7 \sin(7.5\pi t) \) mm.

Figure 12 shows the variation in the displacement (red dashed line) of the sprung mass when the quarter car moves along a rough concave-convex road. As can be seen, the maximum displacement of the sprung mass for the AMSS-PM with the IT2AFC-STFSMC and the FC is less than 4.5mm and 8mm. Using the same test-rig and conditions, the displacement of the sprung mass using the MacPherson struts follows the variation in the road profile. The maximum displacement is 24mm because the MacPherson struts cannot supply sufficient opposing force to suppress vibration. The root-mean-square (RMS) values for the displacement of the sprung mass are 1.92mm, 2.85mm and 9.71mm for the AMSS-PM with and the FC, respectively. However, the RMS for the displacement of the sprung mass is 9.71mm for the MacPherson struts. Figure 13 shows the acceleration comparison of the sprung mass and Figure 14 shows the actuating pressure comparison of the FC and IT2AFC-STFSMC when the quarter car moves along a rough concave-convex road.
The maximum value for the acceleration of the sprung mass is around 0.11g for the AMSS-PM with the IT2AFC-STFSMC, 0.24g for the AMSS-PM with the FC, and 0.43g for the MacPherson suspension. The performance comparison is shown in Table 6.

For the ride comfort evaluation, the frequency analysis regarding body acceleration is supplemented in accordance with this road profile. Firstly, the frequency response of sprung mass acceleration subject to a rough concave-convex road condition for three control approaches is illustrated in Fig. 15. Figures 15(a) and 15(c) respectively show the magnitude and the power spectral density (PSD) for the road vertical variation excited by the RPG. As shown in Figs. 15(b) and 15(d), vibration and ride comfort for both sprung mass natural frequency range (1-2Hz) and human sensitive frequency range (4-8Hz) can be improved by the proposed controller since the magnitudes are reduced in the frequency domain. Besides, the proposed controller also can exhibit better performance on the ride comfort than the original McPherson and the FC.

**Experiment 3: Vehicle Riding on a Two-Bump Excitation Road Profile**

The road profile in Experiment 3 consists of two bumps: a 30mm high bump and a 2cm high bump. The road profile is given by

\[
Z_{r2}(t) = \begin{cases} 
-30 \times \sin(t/2 \times \pi) & \text{for } t \in [4, 6) \\
-20 \times \sin(t/2 \times \pi) & \text{for } t \in [12, 14) \\
0 & \text{else.}
\end{cases}
\]
Figure 16 shows the displacement of the sprung mass when the quarter car travels on a twin-bump excitation road using the AMSS-PM and MacPherson struts. The maximum displacement of the sprung mass for the AMSS-PM using the IT2AFC-STFSMC strategy is less than 8mm, while the maximum displacement of the sprung mass for the MacPherson struts is 27mm and it for the AMSS-PM using FC is around 12mm. Their RMS values for the displacement of the sprung mass are 1.77mm, 3.37mm and 7.98mm, respectively. Figure 17 shows the acceleration comparison of the sprung mass and Figure 18 shows the actuating pressure comparison of the AMSS-PM using the IT2AFC-STFSMC while the quarter car moves along a twin-bump excitation road profile. The maximum acceleration for the sprung mass is bounded within 0.22g for the AMSS-PM using the IT2AFC-STFSMC while the maximum acceleration is 0.42g for the MacPherson struts and it for the AMSS-PM using FC is around 0.27g. The performance comparisons between the IT2AFC-STFSMC with MacPherson and FC are shown in Table 7.

For the ride comfort evaluation, the frequency response of sprung mass acceleration subject to a twin-bump road condition for different control approaches is illustrated in Fig. 19. Figure 19(a) and Figure 19(c) respectively show the magnitude and the PSD for the road vertical variation excited by the RPG. From the observation in Figs. 19(b) and 19(d), the frequency response and the PSD of the proposed controller can outperform the original MacPherson and the FC for both sprung mass natural frequency range (1-2Hz) and human sensitive frequency range (4-8Hz). With Fig. 16, these results clearly indicate that the improvement in ride comfort but with less vibration can be achieved using the proposed controller.

VI. CONCLUSION

This study proposes a conceptual model of an ASS that is driven by a light and low-cost PM actuator with a high power-to-weight ratio, named as AMSS-PM. The AMSS-PM is implemented on a quarter car test rig and its feasibility in terms of the vibration isolation and improved ride comfort is demonstrated from the experiments. Since the PM is modeled as a pneumatic cylinder with a variable sectional area, this study firstly presents a mathematical model for a quarter car with the proposed AMSS-PM. To address the problem of highly nonlinearity and uncertainty of the suspension system dynamics, the IT2AFC-STFSMC is designed and its stability is proved with a Lyapunov stability analysis. The experimental results show that the proposed control strategy for AMSS-PM can significantly reduce the displacement and the acceleration of the sprung mass while facing different road surface variations. Concludingly, the developed AMSS-PM with the IT2AFC-STFSMC can substantially achieve improved performance of the MacPherson suspension with regard to advantageous vibration isolation and better ride comfort. The future work in this study will intend to explore further applications of this novel AMSS-PM mechanism on a half car or a full car suspension system.

APPENDIXES

APPENDIX A

This Theorem can be proven by two steps. In Step 1, the system state $x$ and the adaptive control $u$ are bounded when the IT2AFC-STFSMC is used for the AMSS-PM. Next, Step 2 shows that the tracking error will converge to zero asymptotically.

Step 1: Substituting Eqs. (20) and (23) into (21) yields

$$\dot{S}(t) + \eta S(\Delta(t)) = G[u^* - \hat{u}_c(S, \hat{a}) - u_{comp}(S)].$$  \hspace{1cm} (A1)

If Assumption 1 holds for Eq. (A1), a candidate Lyapunov function is specified as:

$$V(S(\Delta), \hat{a}, \hat{\rho}) = G(S)^2 + \frac{1}{2\eta_1} \dot{\hat{a}}^T \hat{a} + \frac{1}{2\eta_2} \hat{\rho}^2,$$  \hspace{1cm} (A2)

where $\hat{a}^T = a^T - \hat{a}^T$ and $\hat{\rho} = \rho^* - \hat{\rho}$ are respectively the approximation errors of the parameter vectors $a^T$ and $\rho^*$ and $\eta_1$ and $\eta_2$ are positive constants. Differentiating Eq. (A2) with respect to time yields:

$$\dot{V}(S(\Delta), \hat{a}, \hat{\rho}) = S_\Delta \dot{S}_\Delta - \frac{\dot{G}(S)^2}{2G^2(S)} + S_\Delta[u^* - \hat{u}_c(S, \hat{a})]$$

$$- \frac{1}{2} S_\Delta u_{comp}(S) + \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a} + \frac{1}{\eta_2} \hat{\rho} \dot{\hat{\rho}}$$

$$= -\eta S_\Delta^2 - \frac{\dot{G}(S)^2}{2G^2(S)} + S_\Delta[u^* - \hat{u}_c(S, \hat{a})]$$

$$- \frac{1}{2} S_\Delta u_{comp}(S) + \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a} + \frac{1}{\eta_2} \hat{\rho} \dot{\hat{\rho}} \leq -\eta S_\Delta^2 - \frac{M_2(S)^2}{2M_1(S)} + S_\Delta \hat{\rho} u_{\hat{\rho}} - \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a} - \frac{1}{\eta_2} \hat{\rho} \dot{\hat{\rho}}.$$  \hspace{1cm} (A4)

Because $S_\Delta u_{\hat{\rho}} = -|S_\Delta|$, Eq. (A4) is derived as:

$$-\eta S_\Delta^2 - |S_\Delta|\left[\frac{M_2(S)|S_\Delta|}{2M_1^2(S)} - k_c + |S_\Delta| |u^* - \hat{u}_c(S, \hat{a}^*)|\right] + S_\Delta \hat{\rho} u_{\hat{\rho}} - \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a} - \frac{1}{\eta_2} \hat{\rho} \dot{\hat{\rho}}$$

$$= -\eta S_\Delta^2 - |S_\Delta|\left[\frac{M_2(S)|S_\Delta|}{2M_1^2(S)} - k_c + |S_\Delta| (\rho^* - \hat{\rho})\right]$$

$$+ S_\Delta(\hat{a}^T - \hat{a}^T) \xi - \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a} - \frac{1}{\eta_2} \hat{\rho} \dot{\hat{\rho}}.$$  \hspace{1cm} (A5)

Then, because $\hat{a}^T = -\hat{a}^T$ and $\hat{\rho} = -\hat{\rho}$, Eq. (A5) becomes:

$$-\eta S_\Delta^2 - |S_\Delta|\left[\frac{M_2(S)|S_\Delta|}{2M_1^2(S)} - k_c + |S_\Delta| (\rho^* - \hat{\rho}) + S_\Delta \hat{a}^T \xi - \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a}$$

$$= -\eta S_\Delta^2 - |S_\Delta|\left[\frac{M_2(S)|S_\Delta|}{2M_1^2(S)} - k_c + |S_\Delta| (\rho^* - \hat{\rho}) + S_\Delta \hat{a}^T \xi - \frac{1}{\eta_1} \dot{\hat{a}}^T \hat{a} \right].$$
\[
- \frac{1}{\eta_2} \ddot{\hat{\rho}} = -\eta \frac{S^2}{M_1(x)} + |S| \left[ \frac{M_2(x) |S|}{2M_0^2(x)} - k_c \right] \\
+ \ddot{\bar{\alpha}}(S\Delta \bar{\xi}) - \frac{1}{\eta_1} \ddot{\bar{\alpha}} + \ddot{\hat{\rho}}(S\Delta |\bar{\rho}| - \frac{1}{\eta_2} \dot{\hat{\rho}}). \tag{A6}
\]

Substituting Eq. (28), (34), and (35) into Eq. (A6) yields:

\[
\dot{V}(S_\Delta, \bar{\alpha}, \hat{\rho}) \leq -\eta M_1(x) S_\Delta. \tag{A7}
\]

From Eq. (A7), \(S_\Delta(t), \bar{\alpha}(t), \text{ and } \hat{\rho}(t)\) are bounded if the initial values of \(S_\Delta(0), \bar{\alpha}(0)\) and \(\hat{\rho}(0)\) are bounded. If \(S_\Delta(t), \bar{\alpha}(t), \text{ and } \hat{\rho}(t)\) are bounded, it implies that \(e(t)\) is bounded. Consequently, all of the system states \(x(t)\) are bounded, and the adaptive control \(u(t)\) is also bounded.

**Step 2:** From Eqs. (A7) again, the designed control law including Eqs. (25), (28), (34)-(36) can guarantee that \(S_\Delta \in L_\infty\) and the error converges. Integrating both sides of Eq. (A7) yields:

\[
\int_0^\infty S_\Delta dt \leq \frac{V(S_\Delta(\infty), \bar{\alpha}, \hat{\rho}) - V(S_\Delta(0), \bar{\alpha}, \hat{\rho})}{\eta M_1(x)}. \tag{A8}
\]

The right side of Eq. (A8) is bounded, i.e., \(S_\Delta \in L_\infty\). Using Barbalat’s lemma [29], [32], it is proven that \(S_\Delta = 0\) as \(t \to \infty\), which means that the inequality \(|S| \leq \Phi\) holds and the tracking error \(e(t)\) will asymptotically converge to zero as \(t \to \infty\).

**APPENDIX B**

According to [27], we can linearize a SISO system by differentiating its output. For convenience in analysis, we can initially neglect the frictional force \(F_\mu\) and bring back later as uncertainties. By applying the feedback linearization theory and using \(\dot{\tilde{y}} = \dot{x}_1 = \dot{x}_2\), we can find the second derivative and the third derivative of \(x_1\), which are

\[
\ddot{\tilde{y}} = -\frac{1}{M_s}[K_s(x_1 - x_3) + B_s(x_2 - x_4)] \\
- \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} x_5 \\
+ \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} P_e. \tag{A9}
\]

and

\[
\dddot{\tilde{y}} = -\frac{1}{M_s}[K_s(x_1 - x_3) + B_s(x_2 - x_4)] \\
- \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} \dot{x}_5 \\
- \frac{[6L_0 + (x_1 - x_3)(\dot{x}_1 - \dot{x}_3)]}{4\pi n^2} x_5 \\
+ \frac{[6L_0 + (x_1 - x_3)^2\dot{x}_1 - \dot{x}_3]}{4\pi n^2} P_e. \tag{A10}
\]

Simply manipulating Eq. (A10), we can have

\[
\dddot{\tilde{y}} = -\frac{1}{M_s}[K_s(x_2 - x_4) - B_s \left( \frac{1}{M_s} + \frac{1}{M_0} \right) (K_s(x_1 - x_3) \\
+ B_s(x_2 - x_4) - \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} x_5 \\
+ \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} P_e)] \\
+ \frac{B_s K_s}{M_s} x_3 - \frac{B_s K_s}{M_0} x_5 + \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2 T_s} x_5 \\
+ \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} P_e \tag{A11}
\]

So, we can find

\[
\dddot{\tilde{y}}(x) = F(\tilde{x}) + G(\tilde{x}) \dot{u}, \tag{A12}
\]

where

\[
F(\tilde{x}) = -\frac{1}{M_s}[(K_s(x_2 - x_4) - B_s \left( \frac{1}{M_s} + \frac{1}{M_0} \right) (K_s(x_1 - x_3) \\
+ B_s(x_2 - x_4) - \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} x_5 \\
+ \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} P_e)] \\
+ \frac{B_s K_s}{M_s} x_3 - \frac{B_s K_s}{M_0} x_5 + \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2 T_s} x_5 \\
+ \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} P_e \\
\]

and

\[
G(\tilde{x}) = \frac{K_v}{M_s T_s} \left[ \frac{[3L_0 + (x_1 - x_3)^2 - l^2]}{4\pi n^2} \right].
\]

**REFERENCES**

[1] A. F. Naudé and J. A. Snyman, “Optimization of road vehicle passive suspension systems: Part I. Optimalization algorithm and vehicle model,” Appl. Math. Model., vol. 27, no. 4, pp. 249–261, Apr. 2003.

[2] H.-H. Chang, Y.-L. Chen, and K.-C. Hsu, “Optimized sensorless antivibration control for semiactive suspensions with cosimulation analysis,” IEEE/ASME Trans. Mechatronics, vol. 20, no. 4, pp. 1898–1911, Aug. 2015.

[3] J. C. Tudon-Martínez, C. A. Vivas-Lopez, D. Hernandez-Alcantara, R. Morales-Menendez, O. Sename, and R. A. Ramirez-Mendoza, “Full vehicle combinatory efficient damping controller: Experimental implementation,” IEEE/ASME Trans. Mechatronics, vol. 23, no. 1, pp. 377–388, Feb. 2018.

[4] X. Tang, H. Du, S. Sun, D. Ning, Z. Xing, and W. Li, “Takagi–Sugeno fuzzy control for semi-active vehicle suspension with a magnetorheological damper and experimental validation,” IEEE/ASME Trans. Mechatronics, vol. 22, no. 1, pp. 291–300, Feb. 2017.

[5] I. Eski and Ş. Yıldırım, “Vibration control of vehicle active suspension system using a new robust neural network control system,” Simul. Model. Pract. Theory, vol. 17, no. 5, pp. 778–793, May 2009.

[6] G. Priyandoko, M. Mailah, and H. Jamaluddin, “Vehicle active suspension system using skyhook adaptive neuro active force control,” Mech. Syst. Signal Process., vol. 23, no. 3, pp. 855–868, Apr. 2009.

[7] J. Lin, K. W. E. Cheng, Z. Zhang, N. C. Cheung, X. Xue, and T. W. Ng, “Active suspension system based on linear switched reluctance actuator and control schemes,” IEEE Trans. Veh. Technol., vol. 62, no. 2, pp. 562–572, Feb. 2013.

[8] S.-J. Huang and H.-Y. Chen, “Adaptive sliding controller with self-tuning fuzzy compensation for vehicle suspension control,” Mechatronics, vol. 16, no. 10, pp. 607–622, Dec. 2006.
[9] M. Hutter, P. Leemann, G. Hottiger, R. Figi, S. Tagmann, G. Rey, and G. Small, “Force control for active chassis balancing,” IEEE/ASME Trans. Mechatronics, vol. 22, no. 2, pp. 613–622, Apr. 2017.

[10] R. Kang, Y. Guo, L. Chen, D. T. Branson, and J. S. Dai, “Design of a pneumatic muscle based continuum robot with embedded tendons,” IEEE/ASME Trans. Mechatronics, vol. 22, no. 2, pp. 751–761, Apr. 2017.

[11] M. Zapateiro, N. Luo, H. R. Karimi, and J. Vehl, “Vibration control of a class of semiactive suspension system using neural network and backstepping techniques,” Mech. Syst. Signal Process., vol. 23, no. 6, pp. 1946–1953, Aug. 2009.

[12] S. Banerjee, D. Prasad, and J. Pal, “Design, implementation, and testing of a single axis levitation system for the suspension of a platform,” ISA Trans., vol. 46, no. 2, pp. 239–246, Apr. 2007.

[13] J. Cao, H. Liu, P. Li, and D. Brown, “Adaptive fuzzy logic controller for vehicle active suspensions with interval-type 2 fuzzy membership functions,” in Proc. IEEE Int. Conf. Fuzzy Syst. (IEEE World Congr. Comput. Intell.), Jun. 2008, pp. 83–89.

[14] R. S. Bijian, S. Mohammad, and R. Mehdi, “Control of active suspension system: An interval type-2 fuzzy approach,” World Appl. Sci. J., vol. 12, no. 12, pp. 2218–2228, 2011.

[15] M. M. Zirkohi and T.-C. Lin, “Interval type-2 fuzzy-neural network indirect adaptive sliding mode control for an active suspension system,” Nonlinear Dyn., vol. 79, no. 1, pp. 513–526, Jan. 2015.

[16] Y. H. Lo, R. P. Chen, L. W. Lee, I. H. Li, and Y. D. Pan, “Design and implementation of an Interval type-2 adaptive fuzzy controller for a novel pneumatic active suspension system,” in Proc. Int. Conf. Soft Comput. Intell. Syst., 2016, pp. 801–805.

[17] G. Waycaster, S. K. Wu, and X. Shen, “Design and control of a pneumatic artificial muscle actuated above-knee prosthesis,” J. Med. Devices, vol. 5, no. 3, 2011, Art. no. 031003.

[18] G. Belforte, G. Eula, A. Ivanov, and S. Sirolli, “Soft pneumatic actuators for rehabilitation,” Actuators, vol. 3, no. 2, pp. 84–106, May 2014.

[19] F. Daerden and D. Lefeber, “Pneumatic artificial muscles: Actuators for robotics and automation,” Eur. J. Mech. Environ. Eng., vol. 47, no. 1, pp. 11–21, 2000.

[20] V. Ostasevicius, J. Sapragonas, A. Rutka, and D. Staliulionis, “Investigation of active car suspension with pneumatic muscle,” in Proc. SAE Tech. Paper Ser., Aug. 2010, pp. 2002–2206.

[21] Y. Yuan, Y. Yu, and L. Guo, “Nonlinear active disturbance rejection control for the pneumatic muscle actuators with discrete-time measurements,” IEEE Trans. Ind. Electron., vol. 66, no. 3, pp. 2044–2053, Mar. 2019.

[22] H. Yang, Y. Yu, and J. Zhang, “Angle tracking of a pneumatic muscle actuator mechanism under varying load conditions,” Control Eng. Pract., vol. 61, pp. 1–10, Apr. 2017.

[23] S. D. Shinde, S. Maheshwari, and S. Kumar, “Literature review on analysis of various Components of McPherson suspension,” Mater. Today, Proc., vol. 5, no. 9, pp. 19102–19108, 2018.

[24] C.-P. Chou and B. Hannaford, “Measurement and modeling of McKibben pneumatic artificial muscles,” IEEE Trans. Robot. Automat., vol. 12, no. 1, pp. 90–102, Feb. 1996.

[25] F. Daerden, D. Lefeber, and P. Kool, “Using free radial expansion pneumatic muscles for control of a 1DOF robotic arm,” in Proc. 1st Int. Symp. Climbing Walking Robots, 1998, pp. 209–214.

[26] B. Tondu and P. Lopez, “Modeling and control of McKibben artificial muscle robot actuators,” IEEE Control Syst. Mag., vol. 20, no. 2, pp. 15–38, Apr. 2000.

[27] J. J. E. Slotine, and W. Li, Applied Nonlinear Control. Upper Saddle River, NJ, USA: Prentice-Hall, 1991.

[28] L. W. Wang, Adaptive Fuzzy Systems and Control: Design and Stability Analysis. Upper Saddle River, NJ, USA: Prentice-Hall, 1994.

[29] A. Trebi-Ollennu, B. A. Stacey, and B. A. White, “A multivariable decoupling design of an ROV depth control system (a direct adaptive fuzzy SMC approach),” J. Dyn. Syst., Meas., Control, vol. 119, no. 1, pp. 89–94, Mar. 1997.

[30] I.-H. Li and L.-W. Lee, “Interval type 2 hierarchical FNN with the H-infinity condition for MIMO non-affine systems,” Appl. Soft Comput., vol. 12, no. 8, pp. 1996–2011, Aug. 2012.

[31] M.-H. Chiang, L.-W. Lee, and H.-H. Liu, “Adaptive fuzzy controller with self-tuning fuzzy sliding-mode compensation for position control of an electro-hydraulic displacement-controlled system,” J. Intell. Fuzzy Syst., vol. 26, no. 2, pp. 815–830, 2014.

[32] L.-W. Lee and I.-H. Li, “Design and implementation of a robust FNN-based adaptive sliding-mode controller for pneumatic actuator systems,” J. Mech. Sci. Technol., vol. 30, no. 1, pp. 381–396, Jan. 2016.