Stable Deflection in Ferroelectric Negative-Capacitance Hybrid MEMS Actuator With Cubic Nonlinear Spring

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Abstract— Electrostatic microelectromechanical systems (MEMS) actuators suffer from an instability called pull-in, wherein the movable electrode snaps onto the fixed electrode beyond a certain applied voltage. Thus, the entire allowed range is not utilized for stable operation. We propose pull-in free, low-voltage operation by using a cubic nonlinear spring with a ferroelectric negative-capacitance hybrid MEMS actuator. We use a physics-based framework based on the energy landscape to illustrate stability improvement. This framework uses energy-displacement and voltage-displacement plots for analysis. We predict that the actuator can operate in three distinct modes: 1) monostable; 2) bistable; and 3) always-stable, based on the value of the cubic spring constant. We also estimate the threshold values of the cubic spring constant that demarcate the three modes of operation. By proper choice of the cubic spring constant, we predict pull-in free, low-voltage operation of the hybrid actuator, when compared to the standalone MEMS actuator. The results obtained are in agreement with the numerical simulations. This work will aid in the design of electrostatic MEMS actuators for low-voltage applications without pull-in instability.

Index Terms— Electrostatic microelectromechanical systems (MEMS) actuator, ferroelectric negative-capacitance, pull-in instability, travel range.

I. INTRODUCTION

ELECTROSTATIC microelectromechanical systems (MEMS) actuators are the key elements in a wide range of applications like RF MEMS switches, display devices, and resonators [1]. Electrostatic MEMS actuators employ a Coulombic force of attraction between a movable electrode and a fixed electrode to induce a displacement in the movable electrode. These actuators, however, suffer from instability, known as pull-in. Beyond a certain applied voltage, called the pull-in voltage, the movable electrode snaps down onto the fixed electrode [2]. As a result, the maximum distance traveled by the movable electrode before it snaps down, termed as travel range, is limited to a fraction of the air gap between the two electrodes. Therefore, the entire allowed range in the air gap is not available for the stable operation of the electrostatic MEMS actuator.

Several methods have been proposed in the literature to improve the pull-in instability and thus extend the travel range of the electrostatic MEMS actuator. For example, pull-in can be avoided by connecting a feedback capacitor or a metal–oxide–semiconductor (MOS) capacitor operating in depletion mode, in series with the MEMS actuator [3], [4]. Pull-in can also be eliminated by using a specially designed nonlinear spring that exactly counteracts the nonlinear electrostatic force. For instance, an electrostatic MEMS actuator with a properly designed cam suspension can avoid pull-in [5], [6]. Here, the nonlinear elastic spring is obtained from two parallel cantilevers whose length is effectively shortened by bending them over identical curved cams of a specific profile. Another technique to extend the travel range is to use the effect of spring-stiffening [7], [8]. Replacing the planar electrodes with electrically reconfigurable nanostructured electrodes [9] can also extend the travel range. This technique uses a nonplanar and nonfixed geometrical structure, having an array of electrically connected cylinders and spheres. However, in the above-mentioned techniques, the improved stability is achieved at the cost of increased supply voltage, when compared to the actuation voltage of the standalone MEMS actuator [3], [4], [6], [8], [9].

Our goal is to illustrate improved stability by eliminating pull-in, while operating at a lower voltage when compared to the standalone MEMS actuator. We begin with the idea of a ferroelectric negative-capacitance hybrid MEMS actuator for low-voltage operation, based on [10]. This hybrid actuator is a series combination of a ferroelectric capacitor exhibiting negative capacitance and the MEMS actuator. This is similar to negative capacitance-field effect transistors (NC-FETs) wherein the ferroelectric is in series with the FET. Kobayashi [11], for example, reviews the theory and experimental results related to NC-FETs. The negative capacitance can provide internal voltage amplification if its magnitude is matched to the capacitance of the MEMS actuator or FET, thereby enabling low-voltage operation [10], [12]. Although the hybrid actuator in [10] is predicted to have a low-voltage...
operation, pull-in instability is not eliminated. Moreover, it is predicted to have a reduced travel range when compared to the standalone MEMS actuator [10], [13].

In this work, we propose the elimination of pull-in instability in the hybrid actuator by adding a cubic nonlinear spring to it. Cubic nonlinearity in the spring can be introduced by spring-stiffening effect [8] and/or by using various nonlinear micro-flexures [6], [14], [15]. We use a physics-based framework based on the energy landscape [16] to illustrate the stability improvement. The framework uses graphical energy–displacement and voltage–displacement plots to analyze the hybrid MEMS actuator. Depending on the value of the cubic nonlinear spring constant and the applied input voltage, we predict that the hybrid actuator can work in three distinct modes: 1) monostable; 2) bistable; and 3) always stable. The monostable mode suffers from pull-in instability. The bistable mode mimics the operation of a standalone electrostatic bistable MEMS actuator [17], wherein, a snap-through behavior and hysteresis are observed in its static displacement–voltage characteristics. The always-stable mode depicts the elimination of the pull-in instability in the hybrid actuator. We show that the results obtained are in good agreement with numerical simulations. We also estimate the threshold values of the cubic nonlinear spring constant that demarcate the three modes of operation (the procedure to obtain the threshold values is explained in Section IV-B). By proper choice of the cubic spring constant, such that it is close to but greater than the threshold value, we predict that the hybrid actuator can operate at a lower voltage, and without pull-in (always-stable mode), when compared to the standalone actuator.

This article is organized as follows. Section II reviews the pull-in instability in various electrostatic MEMS configurations. Section III explains the Hamiltonian of the hybrid actuator. Section IV presents the analysis of the hybrid actuator with linear and cubic nonlinear springs, based on the energy-landscape method. Finally, Section V presents our conclusions.

II. REVIEW OF PULL-IN IN ELECTROSTATIC MEMS

The standalone MEMS actuator considered is a clamped–clamped beam with fixed–fixed flexure based on [18] and [19], as shown in Fig. 1(a). It is modeled using a single degree of freedom (1-DOF) model as shown in Fig. 1(b). This is a lumped parameter model that approximates the MEMS actuator as a variable parallel plate capacitor, consisting of a fixed bottom electrode and a movable top electrode separated by an air gap $g_o$ [2]. The dynamical variable $x$ denotes the midpoint deflection of the movable top electrode in the clamped–clamped structure. The inertia and energy dissipation of the device are modeled using an effective mass $m$ and a damper with a damping coefficient $c$, respectively. The stiffness is modeled using a spring having a linear spring constant $k_1$ and a cubic nonlinear spring constant $k_3$ (characteristic of a Duffing spring), such that the spring force is $F_{\text{spring}} = k_1x + k_3x^3$. Mid-plane stretching in a clamped–clamped beam can result in such a cubic nonlinearity [18]. The MEMS actuator has a pair of stoppers, made of an insulating material, so that the top and bottom electrodes are not shorted electrically [20], [21]. The top electrode is therefore limited to move from 0 to $g_o - h_s$, where $h_s$ denotes the height of the stopper [see Fig. 1(b)]. Since the pull-in phenomenon is not affected by surface forces [16], [22], we neglect them in our analysis. The damping coefficient $c$ does not affect the analysis as we investigate the static response. Table I lists the values of the MEMS parameters used in this work. The dimensions listed are fairly typical for MEMS clamped–clamped beams [19], [23], [24].

The static response of the MEMS actuator is obtained by applying a slowly varying input [2], [25]. When the input voltage exceeds the pull-in voltage, the top electrode snaps down, resulting in pull-in. The maximum distance in the air gap up to which the actuator can attain a stable equilibrium is called the travel range. For the standalone MEMS actuator with a linear spring ($k_3 = 0$), we have the pull-in voltage $V_{\text{PI1}}$, and travel range $X_{\text{PI1}}$, as [2]

$$V_{\text{PI1}} = \sqrt{\frac{8k_1g_o^2}{27\epsilon_oA_M}}; \quad X_{\text{PI1}} = \frac{g_o}{3}$$ (1)

where $\epsilon_o$ is the permittivity of free space and $A_M$ is the area. The travel range can be extended beyond $g_o/3$ by including a nonlinear spring [8], however, with an increased pull-in voltage. For example, with a cubic nonlinear spring added ($k_3 \neq 0$), the extended travel range, $X_{\text{PI2}} = \zeta g_o$, is obtained by solving [8]

$$\frac{1}{\zeta^3} - \frac{1}{\zeta} = \frac{3}{5} \zeta + \frac{5}{3} \eta - \frac{1}{3} \eta = 0$$ (2)

where $\eta = k_1/(k_3g_o^2)$. The maximum possible travel range in this case is $0.6 g_o$, which is obtained for a perfectly cubic spring with $\eta = 0$ [8]. Fig. 2(a) depicts the static characteristics of the standalone MEMS actuator with linear ($k_3 = 0$) spring and cubic nonlinear ($k_3 \neq 0$) spring, plotted using the parameters from Table I. Based on the design rule given in [8], for the MEMS actuator with cubic nonlinear spring, we have chosen $\eta = 0.1$. This gives a travel range, $X_{\text{PI2}} \approx 95\%$ of $0.6 g_o$ (note that $0.6 g_o$ is the maximum possible travel range, achieved when $\eta = 0$). The stable (unstable) region of operation is characterized by a positive (negative) slope in its static displacement–voltage characteristics [2].
TABLE I
PARAMETERS OF THE HYBRID MEMS ACTUATOR USED IN THIS WORK

| Parameter                | Value                  |
|--------------------------|------------------------|
| Beam material            | Gold (Au)              |
| Length of the beam, L    | 140 μm                 |
| Width of the beam, W     | 120 μm                 |
| Thickness of the beam, T | 0.5 μm                 |
| Actuation area, A_M      | 1.44 × 10⁻⁸ m²        |
| Young’s modulus, E       | 78 GPa                 |
| Density, D               | 19280 kg/m³            |
| Mass, m = 0.35 × D × volume | 5.6 × 10⁻¹¹ kg [18]  |
| Width of the support, w_s | 20 μm                  |
| Length of the support, l_s | 80 μm                 |
| Linear spring constant, ٢_١ | 1.52 N/m [18]       |
| Cubic nonlinear spring constant, ٢_٣ | Variable               |
| Initial air gap, g_o     | 2 μm                   |
| Stopper height, h_s      | 0.15 μm                |
| Permittivity of free space, ٢_٠ | 8.854 × 10⁻¹² F/m    |

Ferroelectric material HfO₂ [16]

α_F = −2.88 × 10⁻⁹ m/F
β_F = 3.56 × 10⁻¹¹ m⁶/V²/C²
γ_F = 0 m⁸/V⁴/C⁴
Ferroelectric thickness, t_F = 45.24 nm
Ferroelectric area, A_F = 9.87 μm²

1. Standalone MEMS (Linear spring)
2. Standalone MEMS (Cubic non-linear spring)
3. MEMS with feedback capacitor
4. MEMS with cam suspension
5. Hybrid MEMS (Linear spring)

Fig. 2. (a) Static characteristics of the standalone MEMS actuator, feedback capacitor-MEMS actuator, and MEMS actuator with cam suspension. Pull-in instability is present in standalone MEMS with linear and nonlinear springs, while it is eliminated in MEMS with a feedback capacitor and in MEMS with cam suspension. (b) Static characteristics of hybrid MEMS actuator with linear spring. Although the pull-in voltage is reduced, pull-in is not eliminated in this configuration.

instability is not avoided in the standalone MEMS with linear and cubic nonlinear springs as there exists an unstable region, as depicted in Fig. 2(a).

A feedback capacitor connected in series with the MEMS actuator with linear spring (٢_٣ = 0) can avoid pull-in [3]. In this configuration, the effective electrical air gap is increased to g_o + C_o/C_f_b, where C_o is the zero-bias MEMS capacitance and C_f_b is the series feedback capacitance. By proper design of the feedback capacitor C_f_b, the entire allowed range (=g_o − h_s) can be traversed by the movable electrode without pull-in [3]. In this case, the voltage required to close the gap is denoted as V_GC. Fig. 2(a) also shows the static characteristics of the feedback capacitor-MEMS configuration. Based on the design criteria to avoid pull-in given in [3] and [26] and for the parameters listed in Table I, we obtain C_f_b = 35.91 fF. Note that the gap-closing voltage V_GC is significantly greater than the pull-in voltage of the standalone MEMS actuator V_PI1, as shown in Fig. 2(a).

Pull-in can also be eliminated by designing a nonlinear spring that counteracts the nonlinear electrostatic force [6]. This nonlinear spring is realized by using a cam suspension. The design of the cam suspension is detailed in [5] and [6]. The static response for this case is plotted in Fig. 2(a). The static response follows the response of the standalone MEMS actuator with linear spring up to a displacement Δ = g_o/4. Note that Δ should be chosen to be less than g_o/3. Beyond Δ, the displacement–voltage response is linear, and thus, pull-in is avoided [6]. As in the case of the MEMS with a feedback capacitor, the gap-closing voltage here, V_GCΔ, is also greater than the pull-in voltage of the standalone MEMS actuator V_PI1, as shown in Fig. 2(a).

A detailed analysis of this configuration is presented in Section IV. The pull-in voltage V_PI5 is less than the pull-in voltage V_PI1 of the standalone MEMS actuator in Fig. 2(a). However, the travel range Δ_PI5 is reduced when compared to the standalone MEMS actuator and pull-in instability is not eliminated [10], [13], as depicted in Fig. 2(b). Table II summarizes pull-in in various MEMS configurations.

In this work, we propose the elimination of pull-in in the hybrid actuator using a cubic nonlinear spring, as explained in the following sections.

III. HAMILTONIAN OF THE HYBRID ACTUATOR

The Hamiltonian (total energy) Hₜ of the 1-DOF electrostatic MEMS actuator, driven by a voltage source V_M, neglecting
damping, is given by [8], [26]

\[ H_M(x, \dot{x}) = \frac{1}{2} m \dot{x}^2 + U_M(x) \]  

(3)

where \( \dot{x} \) is the velocity. The first term on the right-hand side denotes the kinetic energy. The total potential energy \( U_M \) stored in the linear and cubic nonlinear springs and in the capacitor formed by the top and bottom electrodes is expressed as

\[ U_M(x) = \frac{1}{2} k_1 x^2 + \frac{1}{4} k_3 x^4 - \frac{1}{2} \epsilon_o A_M \frac{V_M^2}{(g_o - x)}. \]  

(4)

In order to extract analytical results, we assume the ferroelectric capacitor to behave as a single homogeneous domain. This assumption has been used to describe experimental results on negative capacitance with different ferroelectrics for thicknesses up to \( \approx 100 \) nm [29]–[35]. In the case where the ferroelectric material is inhomogeneous, the single domain assumption could describe an averaged response, using an effective value of the ferroelectric coefficients [34]. With the single domain assumption, the energy associated with the ferroelectric capacitor is given by [10]

\[ U_F(q) = -\frac{1}{2} \alpha q^2 + \frac{1}{4} \beta q^4 + \frac{1}{6} \gamma q^6 - V_F q \]  

(5)

where \( q \) is the charge. The voltage across the ferroelectric capacitor \( V_F \) is given by [10]

\[ V_F = -\alpha q + \beta q^3 + \gamma q^5 \]  

(6)

\[ \alpha = -\frac{\alpha_F t_F}{A_F}, \quad \beta = \frac{\beta_F t_F}{A_F}, \quad \gamma = \frac{\gamma_F t_F}{A_F^2} \]  

(7)

where \( \alpha_F, \beta_F, \) and \( \gamma_F \) are ferroelectric anisotropy coefficients, and \( t_F \) and \( A_F \) are the thickness and area of the ferroelectric, respectively.

Based on our earlier work in [26], we use a mapping function between the displacement \( x \) and the charge \( q \) as

\[ q = \frac{\epsilon_o A_M V_M}{(g_o - x)}. \]  

(8)

This mapping function is based on the charge–voltage relationship of a parallel plate capacitor. From Fig. 1(c), we have, \( V_M = V_{in} - V_F \). Therefore, (8) can be modified as

\[ q = \frac{\epsilon_o A_M \left[ V_{in} - (-\alpha q + \beta q^3 + \gamma q^5) \right]}{(g_o - x)}. \]  

(9)

The above equation is solved to obtain the charge \( q \) as a function of applied voltage \( V_{in} \) and displacement \( x \). This charge is then substituted in (5) and (6) to obtain the energy associated with the ferroelectric \( U_F \), in terms of displacement \( x \). Therefore, we obtain the Hamiltonian \( H_H \) of the hybrid actuator as \( H_H(x, \dot{x}) = U_F(x) + H_M(x, \dot{x}) \). The potential energy of the hybrid actuator is thus obtained as

\[ U_H(x) = U_F(x) + U_M(x). \]  

(10)

\[ q = \frac{\epsilon_o A_M \left[ V_{in} - (-\alpha q + \beta q^3 + \gamma q^5) \right]}{(g_o - x)}. \]  

\[ \frac{dU_M(x)}{dx} = 0. \]

At static equilibrium, \( \frac{dU_M(x)}{dx} = 0 \). Therefore, we have

\[ k_1 x + k_3 x^3 = \frac{q^2}{2 \epsilon_o A_M}. \]  

(12)

The equilibrium charge \( Q_{eq} \) is obtained as

\[ Q_{eq}(x) = \sqrt{2 \epsilon_o A_M (k_1 x + k_3 x^3)}. \]  

(13)

The equilibrium voltage across the MEMS capacitor \( V_{Meq} \) is expressed as \( V_{Meq}(x) = Q_{eq}(x)/C_M = Q_{eq}(x)/(\epsilon_o A_M) \). Using (6), the equilibrium voltage across the

\[ U_F(q) = -\frac{1}{2} \alpha q^2 + \frac{1}{4} \beta q^4 + \frac{1}{6} \gamma q^6 - V_F q \]  

\[ V_F = -\alpha q + \beta q^3 + \gamma q^5 \]  

\[ \alpha = -\frac{\alpha_F t_F}{A_F}, \quad \beta = \frac{\beta_F t_F}{A_F}, \quad \gamma = \frac{\gamma_F t_F}{A_F^2} \]  

\[ q = \frac{\epsilon_o A_M \left[ V_{in} - (-\alpha q + \beta q^3 + \gamma q^5) \right]}{(g_o - x)}. \]  

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Fig. 4. (a) Static characteristics of the hybrid actuator for different values of the cubic nonlinear spring constant \( k_3 \). For an applied voltage, the number of stable and unstable equilibrium displacements depend on the value of \( k_3 \), giving rise to three possible modes of operation. (b) Monostable mode for \( k_3 < k_{th1} \), bistable mode for \( k_{th1} < k_3 < k_{th2} \), and always-stable mode for \( k_3 > k_{th2} \).

ferroelectric capacitor \( V_{Fe} \) is given by \( V_{Fe}(x) = -\alpha \frac{Q_{eq}(x)}{\epsilon_0} + \beta Q^3_{eq}(x) + \gamma Q^5_{eq}(x) \). Therefore, the equilibrium voltage across the hybrid actuator \( V_{H_{eq}} \) is obtained as

\[
V_{H_{eq}}(x) = V_{M_{eq}}(x) + V_{Fe}(x). \tag{14}
\]

Using (14), we plot the equilibrium voltage \( V_{H_{eq}} \) as a function of the displacement \( x \) for different values of \( k_3 \) to obtain the static characteristics of the hybrid actuator, as shown in Fig. 4(a). For an applied voltage, depending on the value of \( k_3 \), we can predict three possible modes of operation: 1) monostable; 2) bistable; and 3) always-stable.

We calculate the threshold values of the cubic nonlinear spring constant \( k_3 \) that demarcate the different modes of operation. Equation (14) is expanded as

\[
V_{H_{eq}}(x) = Q_{eq} \left( \frac{g_0 - x}{\epsilon_0 A_M} - \alpha Q_{eq} + \beta Q^3_{eq} + \gamma Q^5_{eq} \right). \tag{15}
\]

We solve for \( k_3 \), satisfying the condition \( dV_{H_{eq}}(x)/dx = 0 \) \( \forall \ x \in [0, g_0 - h_1] \). The resultant solution (see Appendix for details), as a function of displacement \( x \), is plotted in Fig. 4(b). The value of \( k_3 \) at the stopper end (at \( x = g_0 - h_1 \)) is denoted as \( k_{th1} \). This threshold value represents the boundary between monostable and bistable modes. The maximum value of \( k_3 \), denoted as threshold value \( k_{th2} \), represents the boundary between the bistable and always-stable modes, as shown in Fig. 4(b). Thus, monostable mode occurs for \( k_3 < k_{th1} \). For \( k_{th1} < k_3 < k_{th2} \), the hybrid actuator operates in bistable mode. Finally, for \( k_3 > k_{th2} \), the hybrid actuator is in always-stable mode. For the chosen parameters from Table I, we get \( k_{th1} = 2.55 \times 10^{10} \text{ N/m}^3 \) and \( k_{th2} = 4.87 \times 10^{10} \text{ N/m}^3 \).

1) Monostable Mode \( (k_3 < k_{th1}) \): This mode has one stable and one unstable equilibrium displacement for an applied voltage. In this mode, the hybrid actuator suffers from the pull-in instability, as in the case of the hybrid actuator with a linear spring. In the static characteristics plotted in Fig. 4(a), the region with a positive slope represents stable operation, while the region with a negative slope represents an unstable region of operation. For example, for \( k_3 = 1 \times 10^{10} \text{ N/m}^3 \), the hybrid actuator is in the monostable mode, having both stable \( dV_{H_{eq}}(x)/dx > 0 \) and unstable \( dV_{H_{eq}}(x)/dx < 0 \) regions of operation. The energy–displacement landscape in this mode resembles Fig. 3. Thus, pull-in is not eliminated in this mode of operation.

2) Bistable Mode \( (k_{th1} < k_3 < k_{th2}) \): This mode has two stable and one unstable equilibrium displacements and resembles the operation of a standalone electrostatic bistable MEMS actuator [17]. For instance, the static characteristics for \( k_3 = 4 \times 10^{10} \text{ N/m}^3 \) is shown in Fig. 5(a) (i). The hybrid actuator, in this case, shows bistable operation. Note that such bistable operation is obtained without the need for any buckling or initial curvature of the clamped-clamped beam [37]. Fig. 5(a) (ii) shows the corresponding potential energy–displacement plot, obtained using (10). In the bistable regime, with an increase in voltage, a snap-through from one stable equilibrium displacement to another occurs, similar to the standalone electrostatic bistable MEMS actuator [37]. However, pull-in instability is avoided, because, after the snap-through, the actuator continues to operate in the stable region. This is also verified by comparing the results obtained from the energy-landscape method with the numerical simulation based on [13]. We have added the cubic nonlinear spring module in the simulation framework of [13], to enable the comparison. Fig. 5(b) shows the static characteristics obtained using both the methods. The snap-through and hysteresis, which are the characteristic features of the bistable operation, are observed. Note that the energy-landscape approach captures both the stable (positive slope) and unstable (negative slope) regions of operation in the static characteristics, whereas the numerical simulation captures only the stable operation of the actuator. After the snap-through, the actuator continues to settle at the stable equilibrium displacement corresponding to the applied voltage, thereby, avoiding pull-in. The bistable mode of operation is useful for applications like memories [38], switches [39] and filters [40].

3) Always-Stable Mode \( (k_3 > k_{th2}) \): Finally, for higher values of the cubic nonlinear spring constant \( k_3 \) greater than \( k_{th2} \), the hybrid actuator has only one stable equilibrium displacement for an applied voltage, thereby, avoiding the pull-in instability (always stable). For example, the static characteristics for \( k_3 = 5 \times 10^{10} \text{ N/m}^3 (> k_{th2}) \) is shown in Fig. 6(a) (i). The corresponding potential energy–displacement plot is shown in Fig. 6(a) (ii). For any applied voltage, there exists only one stable equilibrium displacement and hence, the hybrid actuator...
is always stable (without pull-in). The results obtained using the energy-landscape approach are also in agreement with the numerical simulation based on [13], as depicted in Fig. 6(b). The cubic nonlinearity in the spring force is inherently present in clamped–clamped beams [41]. This implies the possibility of avoiding pull-in in the hybrid actuator with a properly designed clamped–clamped beam with appropriate value of $k_3$.

With increase in $k_3$, over and above $k_{th_2}$, the voltage required to close the gap, increases. For example, in Fig. 4(a), comparing the plots with $k_3 = 5 \times 10^{10}$ and $6 \times 10^{10}$ N/m$^3$, a lower voltage is required to close the gap ($= g_o - h_s = 1.85 \mu m$) in the case with $k_3 = 5 \times 10^{10}$ N/m$^3$. Thus, low-voltage, pull-in free operation of the hybrid actuator, when compared to the standalone MEMS actuator, can be ensured by choosing $k_3$ close to but greater than $k_{th_2}$. For instance, in Fig. 6(a), the gap-closing voltage in the hybrid actuator, $V_{GC6} = 1.21$ V, is less than the static pull-in voltage of the standalone MEMS actuator with linear spring [see Fig. 2(a)], $V_{PI1} = 5.32$ V. The low-voltage, pull-in free, always-stable mode can find use in various analog positioning applications such as MEMS varactors [18], micromirrors [14], optical switches [42], tunable filters [43], and so on. For instance, variable analog capacitors (or varactors) realized using MEMS actuators [18] precisely control the separation between the top and bottom electrodes, based on an applied voltage. These devices are restricted to operating in the stable range of displacement of the movable electrode. In the case of the standalone MEMS actuator with a linear spring, the MEMS capacitance becomes $1.5 \times C_o$ when the applied voltage equals the pull-in voltage $V_{PI1}$, as shown in Fig. 7. Here, $C_o$ is the zero-bias MEMS capacitance. When the applied voltage exceeds $V_{PI1}$, pull-in occurs and therefore, the capacitance cannot be controlled. Thus, pull-in instability limits the capacitance ratio to a theoretical value of 1.5 in the standalone MEMS actuator with a linear spring. The elimination of pull-in in the proposed hybrid MEMS actuator (operating in always-stable mode) could increase the capacitance ratio beyond 1.5, due to the increase in stable displacement of the movable plate in the varactor. Furthermore, this extended capacitance tuning range could be achieved at low voltages when compared to the standalone MEMS actuator, as depicted in Fig. 7.

Various techniques to obtain nonlinear spring force are reported in the literature. For instance, contact points are used to implement a nonlinear spring profile [44]. Partitioned beams and springs with decreasing coil lengths are used to implement this technique [44]. Springs with several folds are also used to obtain the nonlinearity in the spring force [45]. The nonlinearity is determined by the choice of the folding angle. In [46], the gap between the anchor and the proof-mass is varied to introduce the nonlinearity. Boisseau et al. [47]
reports the use of H-shaped springs to obtain cubic nonlinearity. In [48], tuning of the nonlinear cubic spring is achieved using the spring topology variation. Hajati [49] illustrates this extended capacitance tuning range is achieved at low voltages when compared to the standalone MEMS varactor.

Fig. 7. Capacitance ratio in MEMS varactor using standalone MEMS with linear spring and hybrid MEMS with cubic nonlinear spring (operating in always-stable mode). Note that the capacitance ratio is increased beyond 1.5 in the hybrid MEMS varactor since pull-in is eliminated. Furthermore, this extended capacitance tuning range is achieved at low voltages when compared to the standalone MEMS varactor.

V. CONCLUSION

We have proposed a low-voltage, pull-in-free, stable operation of a ferroelectric negative-capacitance hybrid MEMS actuator with a cubic nonlinear spring. We have used a physics-based framework based on the energy-landscape to analyze the response of the hybrid actuator. The framework analyzes the hybrid actuator using graphical energy–displacement and voltage–displacement plots. For an applied voltage, based on the value of the cubic nonlinear spring constant \( k_3 \), the hybrid actuator has three distinct modes of operation. We have also estimated the threshold values of the cubic nonlinear spring constant \( k_{th1} \) and \( k_{th2} \), that demarcate the three modes of operation. Pull-in is not eliminated in the monostable mode. The bistable mode mimics the operation of a standalone electrostatic bistable MEMS actuator, depicting snap-through and hysteresis in the static characteristics. Pull-in is eliminated in the always-stable mode, wherein there is only one stable equilibrium displacement, for any value of applied voltage. The results obtained using the energy-based framework are in agreement with the numerical simulations and analytical predictions. By proper choice of the cubic nonlinear spring constant, close to but greater than \( k_{th} \), low-voltage operation without pull-in can be ensured in the hybrid actuator. This work, therefore, can aid in the design of electrostatic MEMS actuators for low-voltage applications that require an extended travel range without the pull-in instability.

APPENDIX

MODES OF OPERATION IN THE HYBRID ACTUATOR

The boundary between the different modes of operation in Fig. 4(b) is obtained by imposing \( \frac{dV_{H_0}(x)}{dx} = 0 \) \( \forall x \in [0, g_o - h_1] \), to obtain \( k_3 \) as a function of \( x \). This condition is equivalent to \( \frac{dV_{H_0}(x)}{dQ_{eq}(x)} = 0 \) \( \forall x \in [0, g_o - h_1] \), based on the one-to-one relationship between charge and displacement, at equilibrium, as discussed in [26]. From (13) and (15), we get

\[
\frac{dV_{H_0}(x)}{dQ_{eq}(x)} = -\alpha + 3\beta Q_{eq}^2(x) + 5\gamma Q_{eq}^4(x)
\]

with

\[
\frac{dQ_{eq}(x)}{dx} = \frac{\epsilon_0 A_M (k_1 + 3k_3^2)}{2 \epsilon_0 \epsilon_r (k_1 x + k_3^3)}
\]

which yields

\[
-\alpha + 6\beta \epsilon_r A_M (k_1 x + k_3^3) + 5\gamma (\epsilon_r A_M (k_1 x + k_3^3))^2 + \frac{g_o - x}{\epsilon_0 \epsilon_r A_M} - \frac{2}{\epsilon_0 \epsilon_r A_M} k_1 x + k_3^3 = 0.
\]

The above equation is solved (ignoring \( \gamma \)) to obtain an analytical expression for \( k_3 \) as a function of \( x \) as \( k_3 = (1/36\beta \epsilon_r^2 A_M^2 x^3)(p_1 + (p_2)^{1/2}) \) where

\[
p_1 = 3x^2(\alpha \epsilon_r A_M - g_o) + x^3(5 - 24k_1 \beta \epsilon_r^2 A_M^2) \tag{19a}
\]

\[
p_2 = p_1^2 + p_3 \tag{19b}
\]

\[
p_3 = -72k_1 \beta \epsilon_r^2 A_M^2 x^5(g_o - \alpha \epsilon_r A_M) + \cdots + 216k_1 \beta \epsilon_r^2 A_M^2 x^6(1 - 2k_1 \beta \epsilon_r^2 A_M^2). \tag{19c}
\]

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