Electron Rescattering in a Bicircular Laser Field

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Abstract. We investigate high-order above-threshold ionization (HATI) of krypton atoms by a bicircular laser field, which consists of two coplanar co- or counter-rotating circularly polarized fields of frequencies \( r\omega \) and \( s\omega \). We show that the photoelectron spectra in the HATI process, presented in the momentum plane, exhibit the same discrete rotational symmetry as the driving field. We also analyze HATI spectra for various combinations of the intensities of two field components for co- and counter-rotating fields. We find that the appearance of high-energy plateau for the counter-rotating case is vary sensitive to the laser intensity ratio, while the plateau is always absent for the co-rotating bicircular field.

1. Introduction
High-order harmonics generation (HHG) and high-order above threshold ionization (HATI) by the so-called bicircular field are very interesting and useful tools in the laser-atom/molecule interaction. The bicircular field consists of two coplanar counter- or co-rotating circularly polarized fields having different angular frequencies. For linear polarization, HHG and HATI processes have been observed with high precision and theoretically analyzed with various methods [1, 2, 3, 4, 5]. Both processes can be explained within the three-step model [6]: in the first step the electron is ionized, then it is driven back to the origin by the laser field, and, finally, in the third step, the electron recombines to the ground state (HHG) or scatters elastically of the parent ion (HATI). For circular polarization, there is no rescattering because when a classical electron is released in the laser-field environment with zero initial velocity, it will never return to the site of its release. For a bicircular field, however, the harmonic-generation efficiency is surprisingly high, which indicates that the absence of rescattering for one circularly polarized field does not extend to the superposition of two such fields.

The first experimental and theoretical studies of HHG by bicircular fields are dating back to the mid and late nineties. HHG spectra of argon atoms in bichromatic linearly and circularly polarized fields, were investigated for the first time experimentally in [7]. In fact, it was observed that strong high-harmonic emission can be achieved using a counter-rotating bicircular field having the frequencies \( \omega \) and \( 2\omega \). Calculations based on a zero-range potential model have confirmed this [8]. The influence of a strong magnetic field on HHG spectra induced by two color laser fields was studied theoretically by solving time-dependent Schrödinger equation (TDSE) for \( \text{H}_2^+ \) molecular ion [9]. Schemes for generation of circularly polarized high-order harmonics by an \( \omega - 2\omega \) circularly polarised laser field were proposed in [10]. In the following...
time, a series of theoretical studies of HHG in two color fields were reported. Namely, the HHG process in a bicircular field was explained using a semiclassical three-step model based on the quantum-orbit formalism [11]. It was found that the harmonic-emission efficiency is very high and the harmonics are circularly polarized with alternating helicities [12]. Besides that, the two-dimensional electron trajectories responsible for HHG process were clearly identified. Experimental confirmation of the circular nature of harmonics, generated in a bicircular field, was reported in 2014 [13]. Modern theoretical and experimental studies of HHG in bicircular laser fields can be found in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

The (H)ATI process by a bicircular field was considered in Ref. [24], where a superposition of two counter-rotating circularly polarized pulses, one long and one few-cycle, having the same frequency, was considered. It was shown that high-energy electrons, generated in ionization by such a combination of pulses, are emitted in a direction correlated with the carrier-envelope phase. Angle-resolved electron energy spectra in strong-field ionization by a bicircular field of arbitrary frequencies \(r\omega\) and \(s\omega\) were analyzed in [25, 26]. For the case where \(r = 1\) and \(s = 2\), the predicted three-lobed shape of these spectra was recently confirmed experimentally [27]. Results on high-energy spectra obtained theoretically and experimentally have been reported recently [28, 29]. A detailed analysis of the HATI process, based on quantum-orbit theory, is given in [30].

Other atomic processes such as laser-assisted recombination [31], non-sequential double ionization [32, 33, 34], the possibility of introducing spin into attoscience [35], etc. were also investigated in strong bicircular fields.

2. Strong-Field Approximation Theory
The Strong-Field Approximation (SFA) theory of (H)ATI process in a bicircular field is described in detail in [30]. The observable quantity is the averaged differential ionization rate given by

\[
\dot{w}_{\text{pi}}(n) = \frac{N_e}{2l+1} \sum_{m=-l}^{l} \dot{w}_{\text{pilm}}(n),
\]

(1)

where \(N_e\) is the number of equivalent electrons in the ionizing shell and \(w_{\text{pilm}}(n) = 2\pi p |T_{\text{pi}}(n)|^2\), while \(n\) is the number of absorbed photons such that \(n\omega = E_p + U_p + I_p\), with \(I_p\) the ionization potential and \(U_p\) the ponderomotive energy. The \(T\)-matrix element \(T_{\text{pi}}(n)\) can be written as

\[
T_{\text{pi}}(n) = \int_0^T \frac{dt}{T} e^{i[p\cdot \alpha(t) + U_1(t)]} \left[ T_{\text{p}}^{(0)}(t) + T_{\text{p}}^{(1)}(t) \right] e^{in\omega t},
\]

(2)

where

\[
T_{\text{p}}^{(0)}(t) = \langle p + A(t) | r \cdot E(t) | \psi_0 \rangle
\]

(3)

is the part of the transition amplitude describing the ATI process, while

\[
T_{\text{p}}^{(1)}(t) = -i \int_0^\infty \left( \frac{2\pi}{it} \right)^{3/2} \langle p | V | k_s \rangle \langle k_s + A(t - \tau) | r \cdot E(t - \tau) | \psi_i \rangle e^{i[S_{k_s(t-\tau)} - S_{k_s(t)}]}.
\]

(4)

allows for one additional interaction of the liberated electron with its parent ion (HATI process). The phase factor in the previous equation is given by

\[
S_{k_s(t)} = \int_0^t dt' \{ [k_s + A(t')]^2/2 + I_p \} = (k_s^2/2 + U_p + I_p)t + k_s \cdot \alpha(t) + U_1(t), \text{ with } U_1(t) = \int_0^t A^2(t') dt'/2 - U_pt \text{ and } \alpha(t) = \int_0^t A(t') dt' .
\]
3. Numerical results

We are going to present numerical results of (H)ATI spectra and momentum distributions for krypton atoms exposed to a strong bicircular laser field. A bicircular laser field is a superposition of two coplanar circularly polarized counter-rotating or co-rotating fields having the intensities $I_1 = E_1^2$ and $I_2 = E_2^2$ and the angular frequencies $r \omega$ and $s \omega$, which are integer multiples of the same fundamental frequency $\omega$. It is defined by

$$E_x(t) = [E_1 \sin(r \omega t + \phi_1) + E_2 \sin(s \omega t + \phi_2)] / \sqrt{2},$$

$$E_y(t) = [-E_1 \cos(r \omega t + \phi_1) \pm E_2 \cos(s \omega t + \phi_2)] / \sqrt{2},$$

where the counter-rotating (co-rotating) fields correspond to the sign $+1$ (-1). A change of the phase $\phi_2$, for a fixed value of $\phi_1$, corresponds to a rotation of the field around the $z$ axis by the angle $\alpha = s \phi_1 / (r + s)$. In the present work we choose $\phi_1 = \phi_2 = 0$. Denoting $A_1 = E_1 / (r \omega)$ and $A_2 = E_2 / (s \omega)$, we obtain for the ponderomotive energy $E_p = (A_1^2 + A_2^2) / 4$. Normalized electric-field vector $E(t)$ and the corresponding normalized vector potential $A(t)$ for various combinations of $(r, s) \in (1, 2), (1, 3), (1, 4)$, equal intensities of the field components and counter-rotating case are presented in the upper panels of Fig. 1.

In the bottom panels of Fig. 1 we present momentum distributions of HATI electrons obtained by ionising Kr atoms by a bicircular field having equal components intensities $I_1 = I_2 = 2 \times 10^{14}$ W/cm$^2$ for the combinations $(r, s) = (1, 2), (1, 3), (1, 4)$. The fundamental wavelength is 800 nm. The counter-rotating bicircular fields obeys dynamical symmetry, with the result that the rotation by the angle $\alpha_j = -r \omega \tau_j$ about the $z$ axis is equivalent to a translation in time by $\tau_j = j T / (r + s)$. It has been shown [30] that both, the direct and the rescattering $T$-matrix elements, are invariant with respect to a rotation by the angle $\alpha_j = 2 \pi j r / (r + s)$ up to the same phase factor. Momentum distributions, presented in the bottom panels of Fig. 1, clearly obeys $(r + s)$-fold rotational symmetry of the differential ionization rate which confirms previous conclusions. If we compare polar diagrams of the vector potentials, shown by the blue dashed curves in top panels of Fig. 1, with the corresponding differential ionization rate, shown in the bottom panels of Fig. 1, we see that the rate is maximal for $p = -A(\tau)$, i.e. the electrons are predominantly emitted opposite to the direction of the vector potential at the ionization time $\tau$.

One way to enhance the process of electron-ion recombination and the harmonic emission rate is to vary the relative intensity between the two components of counter-rotating bicircular field [29]. Here we present the results for ATI spectra obtained by ionizing Kr atoms using a $\omega - 3 \omega$ bicircular field. In Fig. 2 we plot the electron yield as function of the electron kinetic energy for various combinations of the relative intensity $I_{3\omega}/I_\omega$. The spectra of direct electrons, calculated using $T$-matrix element given by (3), are presented by black solid curves, while the overall spectra, which include direct and rescattered electrons are shown by red dashed lines. It is clear that the photoelectron distributions in this case are extremely sensitive to the intensity ratio of the driving laser fields. For low values of the ratio $I_{3\omega}/I_\omega$ the total spectra are comparable to the spectra of the direct electrons. The absence of a plateau in the spectra indicates that the rescattering does not occur. As the $I_{3\omega}/I_\omega$ ratio is increased to the values of 1, 3 or 9, the shape of the spectra is changing and a plateau begins to form, which is an indication of the rescattering of the released electrons. The cutoff energy in the rescattering plateau takes its maximal value for the ratio $I_{3\omega}/I_\omega = 9$. In this case, the ponderomotive energies of the two components, $U_{p1} = A_1^2 / 4$ and $U_{p2} = A_2^2 / 4$, have the same value.

In order to consider the ionization by a co-rotating bicircular field, we present photoelectron spectra for various values of the intensity ratio $I_{3\omega}/I_\omega$. Similar to the counter-rotating case, we plot on the same panels of Fig. 2 the electron spectra including only the direct electrons (green
Figure 1. Upper panels: normalized electric-field vector $E(t)$ (black solid lines) and vector potential $A(t)$ (blue dashed lines) of the $r\omega - s\omega$ counter-rotating bicircular laser field for equal laser field component intensities and $(r,s) \in (1,2), (1,3), (1,4)$, plotted for $0 \leq t \leq T$, $T = 2\pi/\omega$. Lower panels: the logarithm of the differential ionization rate of Kr atoms presented in false colors in the electron momentum plane for ionization by a counter-rotating bicircular laser field with $(r,s) \in (1,2), (1,3), (1,4)$ and equal intensity of both components $I_1 = I_2 = 2 \times 10^{14}$ W/cm$^2$. The fundamental wavelength is 800 nm.

dotted lines) and the overall spectra consisting of the direct as well as rescattered electrons (blue long-dashed lines). It is clear that the electron spectra do not change significantly with the change of the intensity ratio $I_3\omega/I_\omega$. The spectra are dominated by the direct electrons for all intensity ratios, which indicates the absence of the rescattering event in the co-rotating bicircular field and confirms previous results [29, 30].

4. Conclusions
We presented momentum distributions of the electrons, freed in ionization by a strong bicircular laser field of frequencies $r\omega$ and $s\omega$. The presented spectra, obtained by applying SFA theory, exhibit characteristic symmetry properties, which are a consequence of the dynamical symmetry of the counter-rotating bicircular field. Furthermore, we calculated photoelectron momentum distributions for a broad range of intensity ratios of the two field components for the $(r,s) = (1,3)$ case. We found that the photoelectron spectra for the counter-rotating bicircular field change significantly with the change of the intensity ratio $I_3\omega/I_\omega$ and optimize for the value of $I_3\omega/I_\omega = 9$. For lower values of the ratio $I_3\omega/I_\omega$ the spectra are dominated by direct electrons, while for values higher than 1 a plateau begins to form. The photoelectron spectra obtained in a co-rotating bicircular field are dominated by the direct electrons for all intensity ratios. This behaviour indicates the absence of the rescattering event in the co-rotating bicircular field.
Figure 2. Photoelectron distributions of Kr atoms ionized by co- and counter-rotating $\omega - 3\omega$ bicircular fields, obtained for various intensity ratios of the field components. We present spectra of direct electrons for co-rotating (counter-rotating) field by green dotted (black solid) lines as well as the overall spectra consisting of direct and rescattered electrons by blue, long-dashed (red dashed) lines. The laser field parameters are the same as in Fig. 1.

References

[1] Becker W, Grasbon F, Kopold F, Milošević D B, Paulus G G and Walther H 2002 Adv. At., Mol., Opt. Phys. 48 35
[2] Agostini P and DiMauro L F 2004 Rep. Prog. Phys. 67 813
[3] Scrinzi A, Ivanov M Y, Kienberger R and Villeneuve D M 2006 J. Phys. B 39 R1
[4] Milošević D B, Paulus G G, Bauer D and Becker W 2006 J. Phys. B 39 R203
[5] Krausz F and Ivanov M 2009 Rev. Mod. Phys. 81 163
[6] Corkum P B 1993 Phys. Rev. Lett. 71 1994
[7] Eichmann H, Egbert A, Nolte S, Momma C, Wellegehausen B, Becker W, Long S and McIver J K 1995 Phys. Rev. A 51 R3414
[8] Long S, Becker W and McIver J K 1995 Phys. Rev. A 52 2262
[9] Zuo T and Bandrauk A D 1995 J. Nonlinear Opt. Phys. Mater. 04 533
[10] Becker W, Chichkov B N and Wellegehausen B 1999 Phys. Rev. A 60 1721
[11] Milošević D B, Becker W and Kopold R 2000 Phys. Rev. A 61 063403
[12] Milošević D B and Sandner W 2000 Opt. Lett. 25 1532
[13] Fleischer A, Kfir O, Diskin T, Sidorenko P and Cohen O 2014 Nat. Photon. 8 543
[14] Pisanty E, Sukiasyan S and Ivanov M 2014 Phys. Rev. A 90 043829
[15] Milošević D B 2015 J. Phys. B 48 171001
[16] Milošević D B 2015 Opt. Lett. 40 2381
[17] Medišauskas L, Wragg J, van der Hart H and Ivanov M. Y. 2015 Phys. Rev. Lett. 115 153001
[18] Milošević D B 2015 Phys. Rev. A 92 043827
[19] Kfir O et al. 2015 Nat. Photonics 9 99
[20] Fan T et al. 2015 Proc. Natl. Acad. Sci. USA 112 14206
[21] Chen C et al. 2016 Sci. Adv. 2 e1501333
[22] Reich D M and Madsen L B 2016 Phys. Rev. A 93 043411
[23] Odžak S, Hasović E and Milošević D B 2016 Phys. Rev. A 93 043413
[24] Hasović E, Milošević D B and Becker W 2006 Laser Phys. Lett. 3 200
[25] Kramo A, Hasović E, Milošević D B and Becker W 2006 Laser Phys. Lett. 4 279
[26] Hasović E, Kramo A and Milošević D B 2008 Eur. Phys. J. Spec. Top. 160 205
[27] Mancuso C A et al. 2015 Phys. Rev. A 91 031402(R)
[28] Hasović E, Becker W and Milošević D B 2016 Opt. Express 24 6413
[29] Mancuso C A et al. 2016 Phys. Rev. A 93 053406
[30] Milošević D B and Becker W 2016 Phys. Rev. A 93 063418
[31] Odžak S and Milošević D B 2015 Phys. Rev. A 92 053416
[32] Chaloupka J L and Hickstein D D 2016 Phys. Rev. Lett. 116 143005
[33] Mancuso C A et al. 2016 Phys. Rev. Lett. 117 133201
[34] Eckart S et al. 2016 Phys. Rev. Lett. 117 133202
[35] Milošević D B 2016 Phys. Rev. A 93 051402(R)