Study on the prediction method of supporting force distribution and experiment

Hongfeng Ma¹, Gaojie Liu²

¹School of Mechanical and Aerospace Engineering, Jilin University, Changchun, 130022, China
²Taizhou First Technician College, Taizhou, 318020, China
*Corresponding author’s e-mail: 290297738@qq.com

Abstract. The supporting force distribution is very important for the vehicle to maintain safety and stability, especially for the heavy-duty refitting vehicle with eccentric load. A simple and generic model to compute the supporting force distribution is essential in the design stage of refitting vehicle, whereas there is little research involving in solving this problem by now. This article presents an efficient and simple method to predict the supporting force of tire (supporting point). Firstly, the model for the vehicle with three or four axles is simplified. The mathematical matrix is established by force equilibrium equation, moment equilibrium equation and deformation compatibility equation. With the known parameters of load, position and dimension, the force of every supporting point can be calculated, and so the supporting force distribution is acquired. Then, a corresponding experiment is designed and implemented. According to the results of the comparison, the results indicate that the general prediction method to calculate supporting force distribution presented in this paper is accurate and validated in the pre-design stage of the refitting vehicle.

1. Introduction

The refitting vehicles are very popular in engineering applications, majority of them have four or three axles, multi tires (supporting points) are supporting on the ground. It is the statically indeterminate system, the force distribution of supporting points is very important to ensure the safety and stability of the whole vehicle. Though the FEA for the whole vehicle can solve this problem, whereas, in the initial stage of design, the frequent improvement of design needs the FE model changes frequently, resulting in wasting a lot of time. So a simple and generic mathematical model to compute the force distribution is essential in the design stage of refitting vehicle. This paper presents an efficient and simple method to predict the supporting force of tire (supporting point).

There is little research involving in proposing simple and generic model to compute the force distribution for the vehicle by now. Some researchers studied the axle load and solved engineering problems in the recent years [1-4]. Timm et al. proposed a mixed distribution model of two or more theoretical distributions to accurately characterize axle load spectra [5]. Battini et al. used a plane finite element analysis to study the axle load distribution, and based on this, a triangular load distribution was proposed [6]. Law et al. presented a new moving load identification method to identify a system of general moving loads or interaction forces between the vehicle and the bridge deck [7]. Gerbert et al. presented a theory for determining the distribution of the belt tension and the tooth load in timing belts [8]. Zhang et al. showed that the distribution of the belt tension and the tooth load in metal timing belts
was theoretically analyzed [9]. Nirut and Kitjapat presented an analytical method for determining the load distribution of single-column multibolt connection [10].

This article presents an efficient and simple method to predict the supporting force of tire (supporting point). The subsequent sections are organized as follows. In section 2, the model for the vehicle with three or four axles is simplified. The mathematical matrix is established by force equilibrium equation, moment equilibrium equation and deformation compatibility equation. In section 3, an experiment is designed and carried out, and the FEA for the experimental model is also implemented. Finally, the work is summarized in section 4.

2. Mathematical model

For the vehicle with four axles, a balanced suspension is typically designed between the rear two axles to homogenize the supporting force, as shown in Figure 1. According to the force analysis, the vehicle with four axles can be simplified to the force state of the vehicle with three axles. Each axle has two supporting points of tire, so there are 8 supporting points for the vehicle with four axles, there are 6 supporting points for the vehicle with three axles. In the following study, the calculation formula of the force distribution will be deduced based on the force model of the vehicle with three axles.

\[ G + F = \sum_{i=1}^{6} N_i \quad (i=1, 2, \ldots, 6) \]  

where \( G \) is the gravity of the whole vehicle, \( F \) is the load, and \( N_i \) represents the force of the \( i \)-th supporting point. \( N_1 \) and \( N_2 \) are the supporting forces of the front axle, respectively. \( N_3 \) and \( N_4 \) are the supporting forces of the mid axle, respectively. \( N_5 \) and \( N_6 \) are the supporting force of the rear axle, respectively.

\[ N_f = N_1 + N_2 \]  
\[ N_m = N_3 + N_4 \]  
\[ N_r = N_5 + N_6 \]  

where \( N_f \) is the supporting force of the front axle, \( N_m \) is the supporting force of mid axle, and \( N_r \) is the supporting force of the rear axle. The calculation model is simplified, the deformation form of the vehicle is assumed to be the configuration shown in Figure 2, and the stiffness of the leaf spring and the tire is assumed linear elasticity. The point \( o \) represents the center of gravity, \( M \) represents the moment caused by \( F \), \( \delta_f \) is the deformation of front axle, \( \delta_m \) is the deformation of mid axle, and \( \delta_r \) is the deformation of rear axle. \( L \) means the distance from the center of gravity to the front axle, \( L_m \) means the distance from the mid axle to the front axle, and \( L_r \) means the distance from the rear axle to the front axle.
The static force equilibrium equation can be defined as

$$N_f + N_m + N_r = G + F$$

The moment equilibrium equation can be defined as

$$N_r L_r + N_m L_m = (G + F) L - M$$

According to the geometric relation, the deformation compatibility equation can be defined as

$$\frac{\delta_m}{\delta_r} - \frac{\delta_n}{\delta_r} = \frac{L_r - L_m}{L_r}$$

The stiffness of the leaf spring and the tire is assumed linear elasticity, the elastic equation can be defined as

$$N_f = k_f \delta_f$$

$$N_m = k_m \delta_m$$

$$N_r = k_r \delta_r$$

where \(k_f\), \(k_m\) and \(k_r\) represent the stiffness of the front axle, the stiffness of the mid axle and the stiffness of the rear axle, respectively.

The mathematical matrix can be acquired based on the Eq. (5-10), as shown in Eq. (11). For the structure designed in the initial stage, the parameters of \(G\), \(F\), \(L\), \(M\), \(k_f\), \(k_m\) and \(k_r\) can be acquired from geometric model and the stiffness of the leaf spring and the tire. So \(\delta_f\), \(\delta_m\) and \(\delta_r\) are obtained by calculating the mathematical matrix, then the supporting force of each axle is acquired by the Eq. (8-10).

$$\begin{bmatrix} k_f & k_m & k_r \\ k_f L_r & k_m L_m & 0 \\ L_m & -L_r & L_r - L_m \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_m \\ \delta_r \end{bmatrix} = \begin{bmatrix} G + F \\ (G + F) L - M \\ 0 \end{bmatrix}$$

The force state in left view of the vehicle is presented in Figure 3. The point \(o\) is the gravity center of the chassis of the vehicle. The point \(o'\) is the equivalent loading position. When the load is on the symmetric plane, the forces of the two supporting points in every axle are the same. While the eccentric load occurs, the additional moment is formed, the forces of the two supporting points in every axle are different, resulting in every force of supporting points of the vehicle is different.
As shown in Figure 3, $G$ is the gravity of the chassis, $F$ is the load, and $a$ is the distance from the symmetric plane to the supporting point. Based on the force translation theorem, the additional moment can be equivalent from point $o$ to point $o'$, so

$$l' = \frac{M'}{(G + F)}$$  \hspace{1cm} (12)

where $l'$ is the equivalent distance from the point $o$ to the point $o'$, $M'$ is the additional moment caused by the eccentric load. $(G+F)$ is the total load applied to the vehicle. As the structure of the chassis is assumed to be non-deformed, so the distance $l'$ is also equivalent and applicable to the force state of each axle, as shown in Figure 4.

Figure 3: Force analysis diagram in the left view

Figure 4: The force analysis for each axle

where $F'$ is the load applied on the axle, $N'$ and $N''$ are the forces of the two supporting points of each axle, so the static force equilibrium equation can be defined as

$$N = N' + N''$$  \hspace{1cm} (13)

$$N = F'$$  \hspace{1cm} (14)

The moment equilibrium equation can be defined as

$$N'' \times 2a = F'(a - l')$$  \hspace{1cm} (15)

$N'$ and $N''$ can be calculated by Eq. (12), Eq. (13) and Eq. (15)

$$N' = \frac{N + \frac{M'}{2(G + F)a}}{N}$$  \hspace{1cm} (16)

$$N'' = \frac{N + \frac{M'}{2(G + F)a}}{N}$$  \hspace{1cm} (17)

where $N$ is the supporting force of each axle, so the force distribution of the vehicle is as follows
\[
\begin{bmatrix}
N_1 = N_f - \frac{M'}{2} \frac{1}{G + F} N_f \\
N_2 = N_f - \frac{M'}{2} \frac{1}{G + F} N_f \\
N_3 = N_f + \frac{M'}{2} \frac{1}{G + F} N_f \\
N_4 = N_f - \frac{M'}{2} \frac{1}{G + F} N_f \\
N_5 = N_f + \frac{M'}{2} \frac{1}{G + F} N_f \\
N_6 = N_f - \frac{M'}{2} \frac{1}{G + F} N_f \\
\end{bmatrix}
\]

(18)

For the vehicle with four axles, the force distribution of the vehicle is as follows

\[
\begin{bmatrix}
N_1 = N_f - \frac{M'}{2} \frac{1}{G + F} N_f \\
N_2 = N_f - \frac{M'}{2} \frac{1}{G + F} N_f \\
N_3 = N_f + \frac{M'}{4} \frac{1}{G + F} N_f \\
N_4 = N_f - \frac{M'}{4} \frac{1}{G + F} N_f \\
N_5 = N_f + \frac{M'}{4} \frac{1}{G + F} N_f \\
N_6 = N_f - \frac{M'}{4} \frac{1}{G + F} N_f \\
\end{bmatrix}
\]

(19)

3. Experiment and comparison

3.1. Experiment in the structural laboratory

In order to verify the validity and accuracy of the above mathematical model, an experiment is designed and implemented in the structural laboratory. As the main factors influenced on the supporting force of the vehicle are the leaf spring and tire, and their stiffness are assumed to be linear, so in the laboratory, the springs are utilized to simulate the leaf spring and tire of the vehicle. As shown in Figure 5, the experimental model has 3 axles, and 6 supporting points are laid on 6 weighing sensors. The rectangular beam are simulated as axle, the welded structure is simulated as the chassis. Two actuators are set in the two side of experimental model, which are used to apply symmetrical load and eccentric load. The spring near the supporting point is simulated as the tire, whose stiffness is 700 N/mm. The spring between the axle and the chassis is simulated as the leaf spring, whose stiffness is 500 N/mm. The weighing sensors connecting to data acquisition equipment are set under every supporting point. The precision value of the weighing sensor is 3‰. The test process is as following, six weighing sensors are reset and placed on the ground in the laboratory. Then, the whole experimental model is laid on the weighing sensors by the staff utilizing lifting equipment. In the end, the symmetrical load or eccentric load is applied on the experimental model by the actuators, and there are 3 load cases in all, listed in Table 1.
Figure 5: The experimental model

Table 1: Load cases

| Load case    | Description                           | Remarks                  |
|--------------|---------------------------------------|--------------------------|
| Load case 1  | Gravity 9.22 kN, left load 10kN, right load 10kN | With symmetrical load    |
| Load case 2  | Gravity 9.22 kN, left load 20kN, right load 15kN | With eccentric load      |
| Load case 3  | Gravity 9.22 kN, left load 25kN, right load 15kN | With eccentric load      |

3.2. Calculated result and comparison
The geometric parameters can be acquired from the geometric model, as shown in Figure 6. $L_f=2074$ mm, $L_m=2074$ mm, $L=2074$ mm, $M=-1862240$ N*mm, $G=9220$ N, $a=435$ mm. For the load case 1, $F=20000$ N, for the load case 2, $F=35000$ N, for the load case 3, $F=40000$ N. As the stiffness of the spring and the tire are 500 N/mm and 700 N/mm respectively, so

$$k_f = k_m = k_h = 2 \times \frac{770 \times 500}{(770 + 500)} = 583 \text{ N/mm}$$ (20)

Based on the Eq. 10, the supporting force of each axle ($\delta_f, \delta_m$ and $\delta_r$) can be acquired. Then, the force distribution of the vehicle can be calculated by the Eq. 18. For the load case 1, $M'=0$, for the load case 2, $M'=6465000$ N*mm, and for the load case 3, $M'=12930000$ N*mm.

Figure 6: The geometric model of experiment

The calculated and experimental results are presented in Table 2, the calculated result means the supporting force calculated by the mathematical model, and the error means the calculated accuracy of supporting force relative to the experimental result. According to the data and the comparison, the maximum error is 5.9%, while the minimum error is 1.5%. And the average error is only 3.9%, which is not high for the engineering design. In brief, the experimental results are close to the calculated results in all the load cases, and this verifies the validity of the mathematical model proposed in this paper.
Table 2: The comparison between calculation and experiment

| Supporting points | 1         | 2         | 3         | 4         | 5         | 6         |
|-------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| **Load case 1**   | **Calculation / kN** | 4.840     | 4.840     | 4.908     | 4.908     | 4.862     | 4.862     |
|                   | **Experiment / kN**  | 4.735     | 4.665     | 4.755     | 4.835     | 4.995     | 5.125     |
|                   | **Error**            | 2.2%      | 3.8%      | 3.2%      | 1.5%      | 2.7%      | 5.1%      |
| **Load case 2**   | **Calculation / kN** | 10.053    | 4.995     | 9.585     | 4.763     | 9.901     | 4.920     |
|                   | **Experiment / kN**  | 9.565     | 4.720     | 9.815     | 5.010     | 9.755     | 5.215     |
|                   | **Error**            | 5.1%      | 5.8%      | 2.3%      | 4.9%      | 1.5%      | 5.6%      |
| **Load case 3**   | **Calculation / kN** | 13.501    | 3.334     | 12.720    | 3.141     | 13.247    | 3.272     |
|                   | **Experiment / kN**  | 12.745    | 3.165     | 13.25     | 3.285     | 13.465    | 3.455     |
|                   | **Error**            | 5.9%      | 5.3%      | 4.0%      | 4.4%      | 1.6%      | 5.3%      |

4. Conclusion
The main factor affecting the supporting force is the vertical stiffness of the supporting point, so in this study, the vertical stiffness of leaf spring and tire is considered and assumed linear. Simultaneously, the structure of the chassis is assumed to be non-deformed. When the vehicle is in the static and stable state the mathematical matrix is established by force equilibrium equation, moment equilibrium equation and deformation compatibility equation. The force distributions of the vehicle with 3 axles and 4 axles are presented in Eq. 18 and Eq. 19. To verify the validity of the mathematical model, the experiment is designed and implemented in the laboratory.

The mathematical model proposed in this paper can solve the problem of force distribution for the refitting vehicle, however, there are some assumptions in this prediction method, and they are also the limiting factors for the application of this model. Firstly, the chassis or the frame should have enough stiffness, so that the structural deformation is very small. Secondly, the nonlinear characteristic of stiffness for the leaf spring and tire is not big. To further improve the calculation model proposed in this paper, nonlinear factors need to be further considered and studied.

References
[1] Wu S Q, Law S S. Vehicle axle load identification on bridge deck with irregular road surface profile. Engineering Structures, 2011, 33(2):591-601.
[2] Mutton P J, Epp C J, Dudek J. Rolling contact fatigue in railway wheels under high axle loads. Wear, 1991, 144(1-2):139-152.
[3] Pinkaew T. Identification of vehicle axle loads from bridge responses using updated static component technique. Engineering Structures, 2006, 28(11):1599-1608.
[4] Mutton P J, Alvarez E F. Failure modes in aluminothermy rail welds under high axle load conditions. Engineering Failure Analysis, 2004, 11(2):151-166.
[5] Timm D H, Tisdale S M, Turochy R E. Axle Load Spectra Characterization by Mixed Distribution Modelling. Journal of Transportation Engineering, 2005, 131(2):83-88.
[6] Battini J M, Mahir Åker-Kaustell, Syk A, et al. Effect of Axle Load Spreading and Support Stiffness on the Dynamic Response of Short Span Railway Bridges. Structural Engineering International, 2014, 24(4):1-10.
[7] Law S S, Bu J Q, Zhu X Q, et al. Vehicle axle loads identification using finite element method. Engineering Structures, 2004, 26(8):1143-1153.
[8] Gerbert G, H. Jönsson H, Persson U, et al. Load Distribution in Timing Belts. Journal of Mechanical Design, 1978, 100(2):208.
[9] Zhang W, Koyama T. Load Distribution of Metal Timing Belts. ASME. J. Mech. Des. 1999, 123(1):98-103.
[10] Nirut K, Kitjapat P. An Analytical Method for Determining the Load Distribution of Single-Column Multibolt Connection. Advances in Civil Engineering, 2017, 2017:1-19.