The application of genetic algorithm optimization on quadratic investment portfolio without a risk-free asset under Value-at-Risk

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Abstract. In this paper, we performed the Genetic Algorithm within problems of quadratic investment portfolio without a risk-free asset under Value-at-Risk. The limitation of this study is that the risk of an investment portfolio measured by Value-at-Risk, and each investor has the nature of risk aversion. To solve these problems: First, we established the mean vector and covariance matrix. The second step was to define the vector mean and covariance matrices for the formulation of Value-at-Risk of the investment portfolio. Third, using the mean vector and Value-at-Risk established the model. To complete the optimization problem, we performed the Genetic Algorithm. The results show that the trade-off between risk and expected return does not only depend on the type of investor but also on the size of the investment. The Genetic Algorithm certifies us the robust solution in the optimization problem because of its natural ability to locate the global minimal. Moreover, genetic algorithm can be used as an effective way in numerical completion of the optimization of quadratic investment portfolio. In a realistic investment situation, it has likely more constraints. For example, the restriction on short-selling, is need to be considered.

Keywords: Genetic algorithm, mathematical economics, investment portfolio, Value-at-Risk, short-selling

1. Introduction

The capital market takes an important role in economic activity of development countries. It is indicated by the activities of investing in stocks in capital market [5]. An investor who will invest in stock market should be able to understand the risks, which contains uncertainty in the future. This means that it contains an element of risk for investors [3], [11]. By this background, the investors should have an accurate information in making a decision of selecting portfolio stocks in capital market. In this paper, both deterministic and heuristic approach has been tested to the portfolio selection problem based on Markowitz 1952 to address an optimal choice of the portfolio, which has a maximum return and the minimum risk. Markowitz model has an effectiveness to give the steps to
investors in preparing its portfolio and also gives the weight of the allocation of funds in the market shares. Markowitz model is often known as the model of Mean-Variance [7], [12]. Mean-Variance is a quadratic portfolio model, because variance is in the form of a quadratic function [4]. In this research, The Value at Risk (VaR) (heuristic) and the Genetic Algorithm (GA) are addressed to set the optimum portfolio [6], [2].

VaR is a method of calculating market risk to determine the risk of loss that can occur in a portfolio, either single instrument or multi instrument [16]. The advantage of VaR are that this method focuses on downside risk, independent of the distribution assumption of return, and its flexibility be applied to all traded financial products. The number obtained from the measurement by this method is the result of aggregate or comprehensive calculation of product risks as a whole [13]. VaR provides an estimation of the likelihood or probability of incurring losses that is greater than the number of losses that have been determined. VaR also takes into account to changes in prices of existing assets and their effects on other assets. This allows the measurement of the reduced risk resulted by the diversification of some investment or portfolio instruments [1], [17].

The Genetic Algorithm (GA) has been widely known as the robust method in solving many scientific problem which has a non-linierity issue and complex solution [10]. The GA is one of a heuristic method that is a branch of the evolutionary algorithm, which is a technique to solve complex optimization problems by mimicking the process of evolution of living things [9]. The Algorithm can be applied to many studies of sciences. For example in chemical problem to estimate the fittest parameters in kinematic model [15], and the optimization process in chemical reactions and for solving the complex model in accelerator physics by implicating the GA to its multi-objectives optimization [14]. Nowadays, The GA can be also applied in economics studies such as a model of cobweb-type in which has the series of cyclical nature of prices and quantities and the Optimization in market of stocks and the model of portfolio [15].

GA does not involve deterministic calculations, Goldberg argues that GA has characteristics that need to be known, so that it can be distinguished from other searching or optimization procedures: (1) GA’s works with encoding of problem solution sets based on predefined parameters and not the parameters themselves. (2) GA’s conducting a search for a population of a number of individuals who are the solution to problems not just of an individual. (3) GA’s is the information of objective function (fitness), as a way to evaluate individual individuals who have the best solution, not the derivatives of a function. (4) GA’s use the rules of opportunity transition, not deterministic rules. The following advantages make GA differ compared to other deterministic methods in the problem of optimization. In this study, GA has been applied to the optimization problems to provide optimization results with various possible combinations of w values as the weight factors for each stock which are being close to the most optimum solution.

The essence of forming a portfolio is to allocate capital to various investment alternatives, so that overall investment risk can be minimized. The optimum portfolios are categorized as efficient if they have the same level of risk, yet able to provide a higher profit rate, or are able to generate the same level of profit, but with a lower risk. The comparative study is presented by simultaneously defining the solution under Value at Risk (VaR) and the GA methods to determine the optimum weight combination of a portfolio with no risk-free assets. As a numerical illustration, some stocks are traded on the Indonesian capital market.

2. Mathematical models

In this part, the basic concepts of the mathematical model are explained which is to be used in the next part. Furthermore, the description moves to discuss the model of VaR investment portfolio and The GA optimization.

2.1. Basic concept
Suppose the length of time of the investment is expressed by \( t_0 = 0 \), and \( t_1 = 1 \), (the initial investment and the end investment). Investors set a portfolio with expectation of values \( E[V_1] \) at time \( t_1 = 1 \), is high. Since \( V_1 \) fluctuates, it is expected that the risk \( Var[V_1] \) is minimum. Suppose \( V_0 \) initial investment, and a simple asset market with \( n \geq 2 \) risky assets with spot prices \( S^k_t \), where \( t = 0,1 \) and \( k = 1, ..., n \). Note that \( S^k_0 \) is known, whereas \( S^k_1 \) is not. We also allow positions in a risk-free zero-coupon bond that cost \( B_0 \) at time 0, and pays one unit of the chosen currency at time 1 [8], [11].

A position in the risky assets is represented by a vector \( h = (h_1, ..., h_n)^T \in \mathbb{R}^n \), where \( h_k \) is the number of units of asset number \( k \) which is held over the time period by the investor. We let \( h_0 \) denote the position in the risk-free bond. The prices or market values at time \( t = 0 \) and \( t = 1 \) of an affordable portfolio are [8]:

\[
h_0B_0 + \sum_{k=1}^{n} h_kS^k_0 \leq V_0 \quad \text{and} \quad V_1 = h_0 + \sum_{k=1}^{n} h_kS^k_1.
\]

If no-risk-free bond is available, then we simply set \( h_0 = 0 \). The next we take the initial monetary value of the position in the \( k^{th} \) asset, \( w_k = h_kS^k_0 \) and \( w_0 = h_0B_0 \). With monetary portfolio weights the current and future portfolio values can be expressed as [8]:

\[
w_0 + \sum_{k=1}^{n} w_k \leq V_0 \quad \text{and} \quad w_0 \frac{1}{B_0} + \sum_{k=1}^{n} \frac{w_k}{S^k_0}, \quad \text{(1)}
\]

It is seen that determining the optimal allocation of initial capital \( V_0 \) requires the knowledge of the expected value \( \mu \) and covariance matrix \( \Sigma \) of the vector \( \mathbf{R} \), where [8]:

\[
\mathbf{R}^T = \left[ \begin{array}{c} s_1^1 \\ s_2^1 \\ \vdots \\ s_n^1 \\ s_0^0 \end{array} \right].
\]

With \( R_0 = 1/B_0 \) and \( \mathbf{w}^T = (w_1, ..., w_n)^T \) we may write \( V_1 = w_0R_0 + \mathbf{w}^T \mathbf{R} \), and therefore [11]:

\[
E[V_1] = w_0R_0 + \mathbf{w}^T \mathbf{\mu}, \quad \text{(2)}
\]

and

\[
Var[V_1] = \mathbf{w}^T \Sigma \mathbf{w}. \quad \text{(3)}
\]

We assume that the covariance matrix \( \Sigma = \text{Cov}(\mathbf{R}) = E[(\mathbf{R} - \mu)(\mathbf{R} - \mu)^T] \) is positive definite: \( \mathbf{w}^T \Sigma \mathbf{w} > 0 \) for all \( \mathbf{w} \neq 0 \). By definition, any covariance matrix is symmetric and also positive-semidefinite: for any \( \mathbf{w} \neq 0 \), \( \mathbf{w}^T \Sigma \mathbf{w} = \text{Var}(\mathbf{w}^T \mathbf{R}) \geq 0 \). Therefore, assuming that \( \Sigma \) is positive definite is equivalent to assuming that \( \Sigma \) is invertible, or equivalently, that all the eigenvalues of \( \Sigma > 0 \) [8], [11].

2.2. The Model of portfolio investment based on Value-at-Risk

In this part, we apply the VaR method to the optimization of portfolio without a Risk-Free asset. It is assumed that the return has a certain distribution, and the risk of portfolio is measured by the VaR method according to Down in 2002, model of Risk measurement [16]. VaR assessment model for the portfolio \( p \) is defined as \( \text{VaR}_p = -V_0(\mu_p + z_\alpha \sigma_p) \). Since \( w_0R_0 \) is risk-free asset and constant, therefore, referring to equation (2) and (3), The VaR of portfolio \( p \) can be written as [6]:

\[
\text{VaR}_p = -V_0(\mu_p + z_\alpha \sigma_p).
\]
\[ VaR_p = -V_0 \{ w^T \mu + z_\alpha (w^T \Sigma w)^{1/2} \} . \]  \hspace{1cm} (4)

Where negative sign represents a loss, \( V_0 \) the initial capital invested, and \( z_\alpha \) the percentile of the standard normal distribution when the level of significance is given as \( (1-\alpha)\% \).

Thus, the objective function of the investment portfolio model is to maximize \( \{ \mu_p - \rho VaR_p \} \) or maximize \( \left\{ w^T \Sigma + \frac{c}{2V_0} V_0 [w^T \Sigma + z_\alpha (w^T \Sigma w)^{1/2}] \right\} \), hence the investment portfolio optimization model that needs to be completed is [16]:

\[
\text{Maximum} \left\{ (1 + \frac{c}{2})w^T \mu + \frac{c}{2} z_\alpha (w^T \Sigma w)^{1/2} \right\}. \hspace{1cm} (5)
\]

Subject to \( w^T I \leq V_0 \)

Assume that there are two investments with return \( R \) and \( \tilde{R} \), investors who invested \( V_0 \) have expectations the same result, then the value of the constant \( c \), can be determined through the equality that [8]:

\[ E[V_0 R] - \frac{c}{2V_0} VaR[V_0 R] = E[V_0 \tilde{R}] - \frac{c}{2V_0} VaR[V_0 \tilde{R}], \]

So we get

\[
c = \frac{2\{E[V_0 R] - E[V_0 \tilde{R}]\}}{\{VaR[V_0 R] - VaR[V_0 \tilde{R}]\}}. \hspace{1cm} (6)
\]

2.3. The Model of portfolio investment based on genetic algorithm

In order to solve the equation (5), we applied the GA proposed by Holland in 1975 [14]. The GA initially generates a random value of ‘gen’ \( w^T \) within a certain interval, which is associated in a ‘chromosome’. This algorithm allows the competition between each chromosome [15], which brings each potential solution to the optimization problem. A set of chromosome called ‘population’ hence performed an iteration (generation) to determine the fittest parameters of the ‘objective function’. In this case, the equation (5) is the objective function. The generation process consist of evaluating the ‘fitness function’ adapted to create a new population until the optimum chromosomes has been addressed. This operation is made by setting the genetic operator: number of population and Chromosome are 100 and 5, number of generation is 1000, the cross over and mutation rate are set to be 0.25.

For an ideal state, we assume that \( w^T I \) is equal to \( V_0 \), hence ‘controlled’ fitness function is

\[
w^T I = V_0 \hspace{1cm} (6)
\]

\[
w^T I - V_0 = 0 \]

where \( I^T = (1,1,1,1) \). Which refer to the equation (5) as the objective functions.

Since we wanted to maximize the objective function, we adopted the ‘Roulette-Wheel’ selection, so that the value of \( e^U \) are greater than 0. Where \( U \) is the objective functions. The selection of the new individual follows this formula [15]

\[
w_i = \frac{f_i}{\sum_{i=1}^{n} f_i} \hspace{1cm} (7)
\]
where \( w_i \) is the new individual of each \( w \) and \( f \) is the fitness value for each individual. \( n \) is the size of population. In this case, we applied a random cross over and mutation algorithm which has a probability of 0.25 for each gen in the chromosomes to be crossed over or mutated [15].

3. Result and discussion

In this part, the result of the numerical data are presented included the GA result and the discussion section.

3.1. Numerical data

In this part, we intended to show how the application of the model has been formulated, to analyze stock data traded on the capital market. Stock data analyzed can be accessed through the website http://www.finance.go.id/. The data consists of 5 (five) selected shares, for the period of January 2, 2016 up to April 28, 2017. The data includes shares: INDF, DEWA, AALI, LSIP, and ASII.

The average return value of the five stocks is given in a vector respectively \( \mu^T = (0.0154, 0.0390, 0.0033, 0.0088, 0.0003) \). While the value of variance of return with the value of covariance between returns of the five shares is given in the form of covariance matrix \( \Sigma \) and its inverse \( \Sigma^{-1} \),

\[
\Sigma = \begin{bmatrix}
0.0026 & 0.0001 & -0.0022 & 0.0002 & 0.0001 \\
0.0001 & 0.0028 & 0.0003 & 0.0000 & 0.0001 \\
-0.0002 & 0.0003 & 0.0013 & 0.0006 & 0.0004 \\
0.0002 & 0.0000 & 0.0006 & 0.0019 & 0.0003 \\
0.0001 & 0.0001 & 0.0004 & 0.0003 & 0.0002 \\
\end{bmatrix}
\]

\[
\Sigma^{-1} = \begin{bmatrix}
0.0450 & -0.0022 & 0.0365 & -0.0018 & -0.0916 \\
-0.0022 & 0.0370 & -0.0092 & 0.0039 & -0.0048 \\
0.0365 & -0.0092 & 0.2310 & -0.0022 & -0.4723 \\
-0.0018 & 0.0039 & -0.0022 & 0.0694 & -0.1008 \\
-0.0916 & -0.0048 & -0.4723 & -0.1008 & 1.6441 \\
\end{bmatrix}
\]

The \( \mathbf{I}^T = (1, 1, 1, 1, 1) \) is the unit vector, where the initial capital is assumed as Misalkan \( V_0 = 1 \) currency unit. Next, vectors \( \mu^T \), vectors \( \mathbf{I}^T \), and inverse covariance matrices \( \Sigma^{-1} \), are simultaneously used for the calculation of investment portfolio optimization. Here, it is assumed that the sale and purchase of short selling stock is not permitted. This optimization process is done by using Mean-VaR model. In this optimization process, the values of risk aversion constants \( c > 0 \) are determined sequentially from the smallest value to the largest value as can be seen in figure 1. The optimization process of the portfolio investment has been done referring to \( \alpha = 5\% \) which give \( \bar{z}_{5\%} = -1.645 \). The risk aversion constants \( c > 0 \) are which satisfies the assumption that short sales are not allowed is \( 8 \leq c \leq 20 \).

3.2. Genetic Algorithm Result

Based on the numerical data and the information discussed in section 3.1, then the data and all related parameters are used to optimize the investment portfolio. The process of investment portfolio optimization is done by using genetic algorithm to determine the weight composition, for any given risk aversion, which has an interval value of \( 8 \leq c \leq 20 \). After generating 100 random samples of \( w \), we have 13 chromosomes, which have the best fitness functions as shown in Table 1.

| \( c \)  | Weight for stocks of | Sum of weight | Mean | Variance | VaR | Ratio |
|------|----------------------|--------------|------|----------|-----|-------|
| 8    | INDF 0.0115 DEWA 0.0936 AALI 0.4092 LSIP 0.0644 ASII 0.4213 | 1.0000 | 0.0059 | 0.0003 | 0.0222 | 0.2642 |
| 9    | 0.1487 0.1825 0.2646 0.0570 0.3472 | 1.0000 | 0.0109 | 0.0012 | 0.0470 | 0.2316 |
| 10   | 0.2083 0.2191 0.2038 0.0538 0.3150 | 1.0000 | 0.0130 | 0.0022 | 0.0646 | 0.2013 |
| 11   | 0.2443 0.2446 0.1638 0.0518 0.2955 | 1.0000 | 0.0144 | 0.0030 | 0.0755 | 0.1905 |
| 12   | 0.2690 0.2606 0.1377 0.0505 0.2822 | 1.0000 | 0.0153 | 0.0036 | 0.0832 | 0.1837 |
| 13   | 0.2872 0.2723 0.1186 0.0495 0.2724 | 1.0000 | 0.0160 | 0.0041 | 0.0889 | 0.1794 |
The listed number of the \( w \) chromosomes in Table 1 are hence being evaluated for each given risk aversion \( c \) to determine the mean, variance, and the VaR of investment portfolio using the equation (2), (3), and (4). The result are presented in Table 1 as well. The Ratio represents the proportion of the Mean toward VaR relatively.

The efficient-frontier curve are generated by plotting the Mean versus VaR as shown in Figure 1. This efficient frontier curve is a distribution of coordinate points (Mean, VaR), for each risk aversion value within interval \( 8 \leq c \leq 20 \).

A detail explanation of the Figure 1 are presented in the next section.

3.3. Discussion

As it can be seen in Table 1, the different values for each risk aversion are addressed to the shift of composition of Investment portfolio weight. As a result, it appears that the values of the mean and Value-at-Risk portfolio are also changing. The greater values of the investment portfolio mean are followed by the greater Values-at-Risk, which is in this case, it is a measurement of investment portfolio risk. This indicates that the greater the value of investment portfolio expectations, it is always followed by the greater investment risk.

The distribution of 'red-dots’ in the efficient-frontier curve in Figure 1. Represents the appropriate points for investors to make investments, according to the level of risk preferences described as risk aversion. Throughout the efficient-frontier curve, the minimum point occurs when the value of risk aversion \( c = 8 \), and the composition of the investment portfolio weight is as vector \( w^T = (0.0115, 0.0936, 0.4092, 0.0644, 0.4213) \). This minimum point, also refer to the minimum portfolio. This minimum portfolio gives a mean value of 0.0059, and Value-at-Risk of 0.0222. While the maximum point occurs when the value of risk aversion \( c = 20 \), and the composition of the investment portfolio weight is as vector \( w^T = (0.3462, 0.3106, 0.0564, 0.0463, 0.2405) \). This maximum point, also refer to the maximum portfolio. This maximum portfolio gives a mean value of 0.0181, and Value-at-Risk of 0.1076. Throughout the efficient-frontier curve, in addition to the minimum portfolio and maximum portfolio, there is also an optimum portfolio. This optimum portfolio is indicated by the ratio between mean and Value-at-Risk where it has a greatest value, which is 0.2642. The optimum portfolio occurs.
when the value of risk aversion $c = 8$, and the composition of the investment portfolio weight is as vector $\mathbf{w}^T = (0.0115, 0.0936, 0.4092, 0.0644, 0.4213)$. In this case, the optimum portfolio is also a minimum portfolio.

This analysis is expected to provide an overview for investors, who will invest funds especially on investment portfolios in the five stocks discussed in this paper. However, this study can be a good references to make a self-analysis for other stocks that are available in the market.

4. Conclusion
We have done the portfolio optimization modeling without risk-based on Value-at-Risk and the Genetic Algorithm. According to the research, it can be concluded as follows. By applying the algorithm upon the optimization of investment portfolio, the optimum portfolio is located when the value of risk aversion $c = 8$ and the composition of the investment portfolio weight is as vector $\mathbf{w}^T = (0.0115, 0.0936, 0.4092, 0.0644, 0.4213)$. This optimum portfolio gives the expected return at 0.0059, and the Value-at-Risk at 0.0222. In this study, the optimum portfolio play a role as maksimum global portfolio, which has the greatest ratio of mean to Value-at-Risk at 0.2642. This optimum portfolio is located in the minimum portfolio point which means that the minimum portfolio is the optimum portfolio as well. The maximum portfolio has the value of mean at 0.0181, and the Value-at-Risk at 0.2642 where the value of risk aversion $c = 20$. However, since its ratio is lower than 0.02642 (only 0.1683). this maximum portfolio is not the maximum global. The application of VaR gives a robust result where the solution is unique. In contrary, The GA gives not only one robust solution, but also offers many alternative solutions to the problem. This might be close to the real problem that can be complex and needs many alternative choices since the global stock market has lots of uncertainty. The GA’s can generate lots of potential solution according to its ability based on random search of solution. However, in the optimization problem, we expect a unique solution which gives us the optimum robust solution. Therefore, we can obtain the solution by evaluating all of the potential solution by calculating each fitness. The higher the fitness value of the chromosome $\mathbf{w}^T$ then the chromosome election probability will be higher. In this study the result of $\mathbf{w}^T$ has the highest fitness value. This global optimum portfolio has the largest ratio of the mean to the risk value.

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