General Formulation for Proton Decay Rate in Minimal Supersymmetric SO(10) GUT

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Abstract

We make an explicit formulation for the proton decay rate in the minimal renormalizable supersymmetric (SUSY) SO(10) model. In this model, the Higgs fields consist of $\mathbf{10}$ and $\mathbf{126}$ SO(10) representations in the Yukawa interactions with matter and of $\mathbf{10}$, $\mathbf{126}$, $\mathbf{26}$, and $\mathbf{210}$ representations in the Higgs potential. We present all the mass matrices for the Higgs fields contained in this minimal SUSY SO(10) model. Finally, we discuss the threshold effects of these Higgs fields on the gauge couplings unification.

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I. INTRODUCTION

Proton decay would be a smoking gun signature for Grand Unified Theories (GUTs). Unfortunately, no such signal has been seen. In fact, very strong experimental limits have been set for this process, placing the minimal GUTs in a very precarious position. SuperKamiokande (SuperK) has set a lower limit on the proton lifetime in the channel $p \rightarrow K^{+}\bar{\nu}$ as

$$\tau(p \rightarrow K^{+}\bar{\nu}) \geq 2.2 \times 10^{33} \text{[years]},$$

at the 90% confidence level [1]. This has already placed stringent constraints on SU(5). In fact, minimal renormalizable SUSY SU(5) model is almost absolutely excluded [2]. Thus the realistic unified model builders must seriously consider the proton lifetime constraints.

Now, SO(10) GUTs have been mainly discussed in connection with the neutrino oscillations since this part reveals the physics beyond the Standard Model. In this connection, SO(10) GUTs have some advantages over SU(5) GUTs. One of them is that they incorporate the right-handed neutrinos as the member of the $16$ dimensional spinor representation together with the other standard model fermions, and provide the natural explanation of the smallness of the neutrino masses through the seesaw mechanism [4]. In this paper, we consider the minimal renormalizable SUSY SO(10) model. This model contains two Higgs fields $10$ and $126$ in the Yukawa interactions with matter [5] [6]. This is a minimal model in the sense that it contains only the renormalizable operators at the GUT scale and it has the minimal contents of the Higgs fields compatible with the low-energy experimental data. If we relax the renormalizability at the GUT scale, the different minimal SO(10) models are also possible to consider [7] [8]. In this paper, we restrict our arguments within the renormalizable theory at the GUT scale, and use the name of minimal in this restricted sense.

1If we take some textures of the mass matrices for fermions and sfermions, we may get a safe region for the proton life time in a minimal renormalizable SUSY SU(5) model [3].
As was shown in [5] [6], this theory is highly predictive. However, the recent data [9] [10] showed that one of our prediction about the neutrino mass square ratio $\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} \sim 0.19$ [6] is out of $3\sigma$ allowed region. So we need to make re data-fitting compatible with the up to date experimental data well. However, the development of GUTs and rich experimental data drive us to a new stage of precision calculation. That is, we must do the precise estimations to include the ambiguities, coming from the input data and the threshold corrections. The former comes from the fact that there are many input data and they have rather large error bars (see, the strange quark mass, for instance). We must scan these many possibilities systematically. This is tedious but rather technical and is on going in a separate form. The latter is more fundamental. It depends on the details of the superpotential of the Higgs sector, whose effects are not confined in the low-energy data predictions. In order to investigate the proton decay rate and the gauge couplings unification in precise, we have to determine all the mass spectra of the Higgs fields in terms of the parameters presented in this model. This is a very complicated task itself and is the main motivation of this work. Even in the minimal model, there are so many free parameters. So, in practical analysis of the proton decay rate and also the gauge couplings unification, one has to reduce the number of free parameters. That means we should consider the smallest number of the Higgs contents. Thus, we introduce our Higgs system as the most simplest one, $\{10 \oplus 126 \oplus 126 \oplus 210\}$. The meaning of the introduction of these representations will be revealed in the next section. Since our results are the general one for the "minimal" renormalizable SO(10) models, it can be applicable to any parameter regions. For instance, even if we fix the type of the Yukawa couplings in the matter sector and also the Higgs potential, the result is not unique. If we restrict the values of the parameters in the superpotential to some restricted region, we may get the two different types of the seesaw mechanism, type-I [5] [6] or type-II [11]. In this paper, we do not explain the way to save the model from the proton decay rate of SuperK, explicitly. Our main purpose of this paper is to produce all the mass spectra of the Higgs fields including all the Clebsch-Gordan (CG) coefficients and to propose a general formulation which is applicable to any parameter choices. In these applications, our theory
might be found to be insufficient. Even if it is the case, our theory is very useful for more elaborate theory.

This paper is organized as follows. In Sec.II, we give the explicit form of the superpotential in our model. In Sec.III, a very brief description of the symmetry breaking procedure and the decomposition of the original Higgs fields into the minimal supersymmetric standard model (MSSM) are given. In Sec.IV, using these techniques, we can get the mass matrices for a variety of fields, especially for the would-be Nambu-Goldstone (NG) modes. Then we can check that the appropriate NG modes do appear in the mass spectra. In Sec.V, we check the mass matrices for the electroweak Higgs doublets and consider the conditions for two Higgs doublets to remain light. In Sec.VI, we derive the formulae for the evaluation of the proton decay rate. In Sec.VII, we finally check the remaining mass matrices and the effects of the threshold corrections on the gauge couplings unification. In Appendices, we list up all the coefficients of dimension-five and -six operators, which are relevant to proton decay. The applications to a more elaborate model will be given in a separate publication.

II. MINIMAL SO(10) GUT

In this section, we explain the minimal renormalizable SUSY SO(10) model. As mentioned in the introduction, it contains two Higgs fields in the Yukawa interactions with matter [5] [6]. In the SO(10) models, the left- and right-handed fermions in a given i-th generation are assigned to a single irreducible representation $16_i \equiv \Psi_i$. Since $16 \otimes 16 = 10 \oplus 120 \oplus 126$, the fermion masses are generated when the Higgs fields of $10$, $120$, and $126$ dimensional representations develop nonvanishing vacuum expectation values (VEVs). The use of only one Higgs field, $10$ in the Yukawa interactions with matter is obviously ruled out for the description of the realistic quark and lepton mass matrices. Furthermore, the use of $126$ dimensional Higgs field has desirable properties for providing masses of the right-handed Majorana neutrinos. Also it was found that $10 (\equiv H)$ and $126 (\equiv \Delta)$ are suitable for the mass matrices since they satisfy the Georgi-Jarlskog relation. In order to preserve super-
symmetry, we must also include the Higgs field \( \Delta \) of 126 dimensional representation. The Higgs field \( \Phi \) of 210 dimensional representation is introduced to break the SO(10) gauge symmetry \([12]\) and to make mix the Higgs doublets included in \( H \) and \( \overline{\Delta} \) \([5]\). Then the minimal Yukawa coupling becomes

\[
W_Y = Y_{10}^{ij} \Psi_i H_j + Y_{126}^{ij} \Psi_i \overline{\Delta} \Psi_j,
\]

(2.1)

and the minimal Higgs superpotential is \([12] [13] [14]\)

\[
W = m_1 \Phi^2 + m_2 \Delta \overline{\Delta} + m_3 H^2 + \lambda_1 \Phi^3 + \lambda_2 \Phi \Delta \overline{\Delta} + \lambda_3 \Phi \Delta H + \lambda_4 \Phi \overline{\Delta} H.
\]

(2.2)

The interactions of 210, 126, 126 and 10 lead to some complexities in decomposing the GUT representations to the MSSM and in getting the low energy mass spectra. Particularly, the CG coefficients corresponding to the decompositions of \( SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \) have to be found. This problem was first attacked by Xiao-Gang He and one of the present authors (SM) \([15]\) and further by Lee \([13]\). But they did not present the explicit form of mass matrices for a variety of Higgs fields and also did not perform a formulation of the proton life time analysis. In this paper we will complete that program in the frame of our minimal SO(10) model.

### III. SYMMETRY BREAKING

In order to discuss the symmetry breaking pattern, here we briefly summarize our conventions for the SO(10) indices. SO(10) indices \( \alpha = 1, 2, \cdots, 9, 0 \) are divided into two parts \( \alpha = 1, 2, 3, 4 \) for \( SO(4) \cong SU(2) \times SU(2) \) and \( \alpha = 5, 6, 7, 8, 9, 0 \) for \( SO(6) \cong SU(4) \). For the \( SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \) decompositions it is very useful to define a "Y diagonal basis"; \( 1^\prime = 1 + 2i, \ 2^\prime = 1 - 2i, \ 3^\prime = 3 + 4i, \ 4^\prime = 3 - 4i, \ 5^\prime = 5 + 6i, \ 6^\prime = 5 - 6i, \ 7^\prime = 7 + 8i, \ 8^\prime = 7 - 8i, \ 9^\prime = 9 + 0i, \ 0^\prime = 9 - 0i \) (up to normalization factor, \( 1/\sqrt{2} \)). Hereafter we use this Y diagonal basis and omit the dashes: The 10 dimensional irreducible representation, \( H \) is spanned by the states \( \alpha = 1, 2, \cdots, 9, 0 \). The 210 dimensional irreducible representation, \( \Phi \) and the \( T_{26} \oplus 126 \) dimensional reducible representation \( \overline{\Delta} + \Delta \), are spanned by
the anti-symmetric tensors of the fourth rank \((\alpha\beta\gamma\delta)\) and the anti-symmetric tensors of the fifth rank \((\alpha\beta\gamma\delta\epsilon)\), respectively. Here and below the bracket \((\cdots)\) represents the total anti-symmetrization of the indices within the bracket.

The Higgs fields of the minimal \(SO(10)\) model contain five directions which are singlets under \(SU(3)_C \times SU(2)_L \times U(1)_Y\). The corresponding VEVs are defined by

\[
\langle \Phi \rangle = \sum_{i=1}^{3} \hat{\phi}_i \hat{\phi}_i, \quad \langle \Delta \rangle = v_R \hat{v}_R, \quad \langle \Delta \rangle = \overline{v}_R \overline{\hat{v}}_R,
\]

where \(\hat{\phi}_i (i = 1, 2, 3)\) are included in \(210\),

\[
\hat{\phi}_1 = -\frac{1}{\sqrt{24}} (1234), \quad \hat{\phi}_2 = -\frac{1}{\sqrt{72}} (5678 + 5690 + 7890), \quad \hat{\phi}_3 = -\frac{1}{\sqrt{144}} (1256 + 1278 + 1290 + 3456 + 3478 + 3490),
\]

\(\hat{v}_R\) is in \(126\)

\[
\hat{v}_R = \frac{1}{\sqrt{120}} (13579),
\]

and \(\hat{v}_R\) is in \(126\)

\[
\hat{v}_R = \frac{1}{\sqrt{120}} (24680).
\]

Notice that

\[
\hat{\phi}_i \cdot \hat{\phi}_j = \delta_{ij} \quad (i, j = 1, 2, 3),
\]

\[
\hat{v}_R \cdot \hat{v}_R = \overline{v}_R \cdot \overline{v}_R = 0,
\]

\[
\hat{v}_R \cdot \hat{v}_R = 1.
\]

Due to the D-flatness condition the absolute values of the VEVs, \(\overline{v}_R\) and \(v_R\) are equal,

\[
|\overline{v}_R| = |v_R|.
\]
Now we write down the VEV conditions which preserve supersymmetry, with respect to the directions $\hat{\phi}_1$, $\hat{\phi}_2$, $\hat{\phi}_3$, and $\hat{v}_R$, respectively.

\begin{align}
2m_1\phi_1 + 3\lambda_1 \frac{\phi_1^2}{\sqrt{6}} + \lambda_2 \frac{v_R \cdot \bar{v}_R}{10\sqrt{6}} &= 0, \\
2m_1\phi_2 + 3\lambda_1 \left( \frac{\phi_1^2 + \phi_2^2}{9\sqrt{2}} \right) + \lambda_2 \frac{v_R \cdot \bar{v}_R}{10\sqrt{2}} &= 0, \\
2m_1\phi_3 + 3\lambda_1 \left( \frac{\phi_1\phi_3}{3\sqrt{6}} + \frac{\sqrt{2}\phi_2\phi_3}{9} \right) + \lambda_2 \frac{v_R \cdot \bar{v}_R}{10} &= 0, \\
\left\{ m_2 + \lambda_2 \left( \frac{\phi_1}{10\sqrt{6}} + \frac{\phi_2}{10\sqrt{2}} + \frac{\phi_3}{10} \right) \right\} \cdot v_R &= 0.
\end{align}

(3.10)–(3.13)

Here we consider only the solutions with $|v_R| \neq 0$. Eliminating $v_R \cdot \bar{v}_R$, $\phi_1$ and $\phi_2$ from Eqs. (3.10)–(3.13), one obtains a fourth-order equation in $\phi_3$,

\begin{align}
\left( \phi_3 + \frac{M_2}{10} \right) \left\{ 8\phi_3^3 - 15 \mathcal{M}_1\phi_3^2 + 14 \mathcal{M}_1^2 \phi_3 - 3 \mathcal{M}_1^3 + (\phi_3 - \mathcal{M}_1)^2 \mathcal{M}_2 \right\} &= 0,
\end{align}

(3.14)

where

\begin{align}
\mathcal{M}_1 &\equiv 12 \left( \frac{m_1}{\lambda_1} \right), \quad \mathcal{M}_2 \equiv 60 \left( \frac{m_2}{\lambda_2} \right).
\end{align}

(3.15)

Any solution of the cubic equation in $\phi_3$ is accompanied by the solutions

\begin{align}
\phi_1 &= -\frac{\phi_3 (\mathcal{M}_1^2 - 5\phi_3^2)}{\sqrt{6} (\mathcal{M}_1 - \phi_3)^2}, \\
\phi_2 &= -\frac{1}{\sqrt{2}} \frac{(\mathcal{M}_1^2 - 2\mathcal{M}_1\phi_3 - \phi_3^2)}{(\mathcal{M}_1 - \phi_3)}, \\
v_R \cdot \bar{v}_R &= \frac{5}{3} \left( \frac{\lambda_1}{\lambda_2} \right) \frac{\phi_3 (\mathcal{M}_1 - 3\phi_3) (\mathcal{M}_1^2 + \phi_3^2)}{(\mathcal{M}_1 - \phi_3)^2}.
\end{align}

(3.16)

The linear term gives the solution of the fourth-order equation (3.14) which is very simple, $\phi_3 = -6 \left( \frac{m_2}{\lambda_2} \right)$. It leads to $\phi_1 = -\sqrt{6} \left( \frac{m_2}{\lambda_2} \right)$, $\phi_2 = -3\sqrt{2} \left( \frac{m_2}{\lambda_2} \right)$ and $\sqrt{v_R \cdot \bar{v}_R} = \sqrt{60} \left( \frac{m_2}{\lambda_2} \right) \sqrt{2 \left( \frac{m_2}{m_1} \right)} - 3 \left( \frac{\lambda_1}{\lambda_2} \right)$. This solution preserves the $SU(5)$ symmetry. Therefore, it is physically not interesting. The cubic term solutions lead to the true $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry.
IV. WOULD-BE NG BOSONS

In order to check the number of NG modes we write down the mass matrices for the Higgs(ino) fields which transmute the non-MSSM $SO(10)$ gauge fields into very massive gauge fields. At first, we list the quantum numbers of the would-be NG modes under $SU(3)_C \times SU(2)_L \times U(1)_Y$.

- $[(\mathbf{3}, 2, \frac{5}{6}) \oplus (\mathbf{3}, 2, -\frac{5}{6})]$,
- $[(\mathbf{3}, 2, -\frac{1}{6}) \oplus (\mathbf{3}, 2, \frac{1}{6})]$,
- $[(\mathbf{3}, 1, -\frac{2}{3}) \oplus (\mathbf{3}, 1, \frac{2}{3})]$,
- $[(1, 1, 1) \oplus (1, 1, -1)]$,
- $[(1, 1, 0)]$.

Total number of the NG degrees of freedom is: $12 + 12 + 6 + 2 + 1 = 33$. In the following subsections we give explicit expressions for the mass matrices and check that their determinants are zero. The mass matrices receive contributions from the $F$ terms in the Higgs potential. The matrix elements of the mass matrices comprise the CG coefficients which appear as coefficients of the triple products of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ components of the Higgs superfields. For the calculation of the CG coefficients, one must first find the explicit expressions for the $SU(3)_C \times SU(2)_L \times U(1)_Y$ components of the Higgs superfields. We will publish the complete tables of the CG coefficients of a more general Higgs sector in a separate publication and we will list only the mass matrices in this paper.

Note that the mass matrix for every irreducible representation under $SU(3)_C \times SU(2)_L \times U(1)_Y$ with $Y \neq 0$ and the mass matrix for the corresponding complex conjugate representation are equal up to transposition. Therefore, only one of the two accompanied mass matrices is listed. Of course, when enumerating the total degrees of freedom, one has to be careful to include all the mass eigenvalues (472 in total). The mass matrices define the mass part of the superpotential as a bilinear form of the fields and corresponding complex
conjugate fields. The basis for the mass matrix is defined as a row of the fields multiplying the mass matrix form the left.

\[ A. \begin{bmatrix} (3, 2, \frac{2}{6}) & \oplus & (3, 2, -\frac{2}{6}) \end{bmatrix} \]

In the basis \( \left\{ \Phi^{(3,2,-\frac{2}{6})}_{(6,2,2)}, \Phi^{(3,2,-\frac{2}{6})}_{(10,2,2)} \right\} \) (here and hereafter the lower indicies indicate \( SU(4)_C \times SU(2)_L \times SU(2)_R \) and the upper \( SU(3)_C \times SU(2)_L \times U(1)_Y \) in the case of double indices), the mass matrix is written as

\[
\begin{pmatrix}
2m_1 - \frac{\lambda_1 \phi_3}{6} & \frac{\lambda_1 \phi_2}{3\sqrt{2}} \\
\frac{\lambda_1 \phi_3}{3\sqrt{2}} & 2m_1 + \frac{\lambda_1 \phi_2}{3\sqrt{2}} - \frac{\lambda_1 \phi_3}{6}
\end{pmatrix}.
\]

This determinant is indeed zero assuming the VEV conditions, Eqs. (3.10)-(3.13).

\[ B. \begin{bmatrix} (3, 2, -\frac{1}{6}) & \oplus & (3, 2, \frac{1}{6}) \end{bmatrix} \]

In the basis \( \left\{ \Phi^{(3,2,-\frac{1}{6})}_{(6,2,2)}, \Phi^{(3,2,\frac{1}{6})}_{(10,2,2)}, \Lambda^{(3,2,\frac{1}{6})}_{(15,2,2)}, \overline{\Lambda^{(3,2,\frac{1}{6})}_{(15,2,2)}} \right\} \), the mass matrix is written as

\[
\begin{pmatrix}
2m_1 + \frac{\lambda_1 \phi_3}{6} & \frac{\lambda_1 \phi_2}{3\sqrt{2}} & -\frac{\lambda_1 \nu_R}{10\sqrt{3}} & 0 \\
\frac{\lambda_1 \phi_3}{3\sqrt{2}} & 2m_1 + \frac{\lambda_1 \phi_2}{3\sqrt{2}} + \frac{\lambda_1 \phi_3}{6} & -\frac{\lambda_1 \nu_R}{5\sqrt{6}} & 0 \\
-\frac{\lambda_1 \nu_R}{10\sqrt{3}} & -\frac{\lambda_1 \nu_R}{5\sqrt{6}} & m_2 + \frac{\lambda_2 \phi_2}{30\sqrt{2}} + \frac{\lambda_2 \phi_3}{20} & 0 \\
0 & 0 & 0 & m_2 + \frac{\lambda_2 \phi_2}{30\sqrt{2}} + \frac{\lambda_2 \phi_3}{60}
\end{pmatrix}.
\]

This determinant is also equal zero assuming the VEV conditions.

\[ C. \begin{bmatrix} (3, 1, -\frac{2}{3}) & \oplus & (3, 1, \frac{2}{3}) \end{bmatrix} \]

In the basis \( \left\{ \Phi^{(3,1,-\frac{2}{3})}_{(15,1,1)}, \Phi^{(3,1,\frac{2}{3})}_{(15,1,3)}, \underline{\Lambda^{(3,1,\frac{2}{3})}_{(10,1,3)}} \right\} \), the mass matrix is written as

\[
\begin{pmatrix}
2m_1 + \frac{\lambda_1 \phi_3}{3\sqrt{2}} & \frac{\lambda_1 \phi_2}{3\sqrt{2}} & -\frac{\lambda_1 \nu_R}{10\sqrt{3}} \\
\frac{\lambda_1 \phi_3}{3\sqrt{2}} & 2m_1 + \frac{\lambda_1 \phi_2}{3\sqrt{2}} + \frac{\lambda_1 \phi_3}{6} & -\frac{\lambda_1 \nu_R}{5\sqrt{6}} \\
-\frac{\lambda_2 \nu_R}{10\sqrt{3}} & -\frac{\lambda_2 \nu_R}{5\sqrt{6}} & m_2 + \frac{\lambda_2 \phi_2}{10\sqrt{6}} + \frac{\lambda_2 \phi_3}{30} + \frac{\lambda_2 \phi_3}{30}
\end{pmatrix}.
\]

This determinant is also equal zero assuming the VEV conditions.
D. \([1,1,1] \oplus (1,1,-1)\]

In the basis \(\{\Phi^{(1,1,1)}_{(15,1.3)}, \Delta^{(1,1,1)}_{(10,1,3)}\}\), the mass matrix is written as

\[
\begin{pmatrix}
2m_1 + \frac{\lambda_1 \phi_1}{\sqrt{6}} + \frac{\sqrt{2} \lambda_2 \phi_2}{3} & -\frac{\lambda_2 v_R}{10} \\
-\frac{\lambda_2 v_R}{10} & m_2 + \frac{\lambda_2 \phi_1}{10 \sqrt{6}} + \frac{\lambda_2 \phi_2}{10 \sqrt{2}}
\end{pmatrix}
\] (4.4)

This determinant is also equal zero assuming the VEV conditions.

E. \([1,1,0]\]

In the basis \(\{\Phi^{(1,1,0)}_{(1,1,1)}, \Phi^{(1,1,0)}_{(15,1.3)}, \Phi^{(1,1,0)}_{(10,1,3)}, \Delta^{(1,1,0)}_{(10,1,3)}, \Delta^{(1,1,0)}_{(10,1,3)}\}\), the mass matrix is written as

\[
\begin{pmatrix}
2m_1 & 0 & \frac{\lambda_1 \phi_1}{\sqrt{6}} & -\frac{\lambda_2 v_R}{10 \sqrt{6}} & -\frac{\lambda_2 v_R}{10 \sqrt{2}} \\
0 & 2m_1 + \frac{\sqrt{2} \lambda_1 \phi_2}{3} & \frac{\sqrt{2} \lambda_1 \phi_2}{3} & -\frac{\lambda_2 v_R}{10 \sqrt{2}} & -\frac{\lambda_2 v_R}{10} \\
\frac{\lambda_1 \phi_1}{\sqrt{6}} & \frac{\sqrt{2} \lambda_1 \phi_2}{3} & 2m_1 + \frac{\lambda_1 \phi_1}{\sqrt{6}} + \frac{\sqrt{2} \lambda_1 \phi_2}{3} & -\frac{\lambda_2 v_R}{10} & -\frac{\lambda_2 v_R}{10} \\
-\frac{\lambda_2 v_R}{10 \sqrt{6}} & -\frac{\lambda_2 v_R}{10 \sqrt{2}} & -\frac{\lambda_2 v_R}{10} & e4 & 0 \\
-\frac{\lambda_2 v_R}{10 \sqrt{6}} & -\frac{\lambda_2 v_R}{10 \sqrt{2}} & -\frac{\lambda_2 v_R}{10} & 0 & e4
\end{pmatrix}
\] (4.5)

Here \(e4 \equiv m_2 + \lambda_2 \left(\frac{\phi_1}{10 \sqrt{6}} + \frac{\phi_2}{10 \sqrt{2}} + \frac{\phi_3}{10}\right)\) is nothing but Eq. (3.13) and Eq. (4.5) has one zero eigenvalue.

V. ELECTROWEAK HIGGS DOUBLET

In the standard picture of the electroweak symmetry breaking, we have the Higgs doublets which give masses to the matter. These masses should be less than or equal to the electroweak scale. Since we approximate the electroweak scale as zero, we must impose a constraint that the mass matrix should have one zero eigenvalue.

We define

\[
H^u_{10} \equiv H^{(1,2, \frac{1}{2})}_{(1,2,2)}, \quad \Sigma_u \equiv \Sigma^{(1,2, \frac{1}{2})}_{(15,2,2)}, \quad \Delta_u \equiv \Delta^{(1,2, \frac{1}{2})}_{(15,2,2)}, \quad \Phi_u \equiv \Phi^{(1,2, \frac{1}{2})}_{(10,2,2)},
\] (5.1)

and
\[ H_d^{10} \equiv H_{(1,2,2)}^{(1,2,-\frac{1}{2})}, \quad \Delta_d \equiv \Delta_{(15,2,2)}^{(1,2,-\frac{1}{2})}, \quad \Phi_d \equiv \Phi_{(10,2,2)}^{(1,2,-\frac{1}{2})}. \]  

(5.2)

In the basis \( \{ H_u^{10}, \bar{\Delta}_u, \Delta_u, \Phi_u \} \), the mass matrix is written as

\[ M_{\text{doublet}} \equiv \begin{pmatrix}
2m_3 & \lambda_3 \phi_2 - \lambda_4 \phi_3 \\
\lambda_4 \phi_2 - \lambda_5 \phi_3 & m_2 + \lambda_2 \phi_2 - \frac{\lambda_3 \phi_3}{30} & 0 & 0 \\
\frac{\lambda_3 \phi_2}{10} - \frac{\lambda_4 \phi_3}{2\sqrt{5}} & 0 & m_2 + \frac{\lambda_2 \phi_2}{15\sqrt{2}} + \frac{\lambda_3 \phi_3}{30} & -\frac{\lambda_5 \phi_3}{10} \\
\frac{\lambda_4 \phi_2}{2\sqrt{5}} & 0 & -\frac{\lambda_2 \phi_2}{10} & 2m_1 + \frac{\lambda_1 \phi_2}{\sqrt{2}} + \frac{\lambda_5 \phi_3}{2}
\end{pmatrix}. \]

(5.3)

The corresponding mass terms of the superpotential read

\[ W_m = \left( H_u^{10}, \bar{\Delta}_u, \Delta_u, \Phi_u \right) M_{\text{doublet}} \left( H_d^{10}, \Delta_d, \bar{\Delta}_d, \Phi_d \right)^T. \]

(5.4)

The requirement of the existence of a zero mode leads to the following condition.

\[ \det M_{\text{doublet}} = 0. \]  

(5.5)

For instance, in case of \( \lambda_3 = 0 \), \( m_2 + \frac{\lambda_2 \phi_2}{15\sqrt{2}} - \frac{\lambda_3 \phi_3}{30} = 0 \), we obtain a special solution to Eq. (5.5), while it keeps a desirable vacuum and it does not produce any additional massless fields. However, we proceed our arguments hereafter without using this special solution.

We can diagonalize the mass matrix, \( M_{\text{doublet}} \) by a bi-unitary transformation.

\[ U^* M_{\text{doublet}} V^\dagger = \text{diag}(0, M_1, M_2, M_3). \]

(5.6)

Then the mass eigenstates are written as

\[ \left( H_u, h_u^1, h_u^2, h_u^3 \right) = \left( H_u^{10}, \bar{\Delta}_u, \Delta_u, \Phi_u \right) U^T, \]

\[ \left( H_d, h_d^1, h_d^2, h_d^3 \right) = \left( H_d^{10}, \Delta_d, \bar{\Delta}_d, \Phi_d \right) V^T. \]

(5.7)

The representations 45 and/or 54, and higher dimensional operators, are not included in our minimal model. Therefore, we must set the "Doublet-Triplet splittings" by hand as Eq. (5.5). In the case of \( \lambda_3 = \lambda_4 \), Eq. (5.3) becomes symmetric, and \( H_u \) and \( H_d \) have the same
coefficients in Eq. (5.7). This cannot be accepted since it leads to the formal singularity in
the low-energy Yukawa couplings (A matrix in Eq. (6.14)). Namely, it leads to the equality
\( Y_u = Y_d \), and therefore only the ratio of \( Y_{10} \) and \( Y_{126} \) can be determined from Eq. (6.14). So
we set \( \lambda_3 \neq \lambda_4 \) hereafter.

By making the inverse transformation of Eq. (5.7), the following expressions are obtained,
\[
H_{10}^u = \alpha_u H_u + \cdots, \quad H_{10}^d = \alpha_d H_d + \cdots, \quad \Delta_u = \beta_u H_u + \cdots, \quad \Delta_d = \beta_d H_d + \cdots, \tag{5.8}
\]
where "+\cdots" represent the heavy Higgs fields, \( h^{i}_{u,d} (i = 1, 2, 3) \) which are integrated out
when considering the low-energy effective superpotential.

Precisely, we can read off from Eq. (5.7) as
\[
\alpha_u = (U^*)_{11}, \quad \beta_u = (U^*)_{12}, \quad \alpha_d = (V^*)_{11}, \quad \beta_d = (V^*)_{13}. \tag{5.9}
\]
Using the two pairs of the Higgs doublets, \( H_{u,d}^{10} \) and \( \Delta_{u,d} \), the Yukawa couplings of Eq. (2.1)
are rewritten as
\[
W_Y = u^c_i \left( Y_{10}^{ij} H_u^{10} + Y_{126}^{ij} \Delta_u \right) q_j + d^c_i \left( Y_{10}^{ij} H_d^{10} + Y_{126}^{ij} \Delta_d \right) q_j \\
+ \nu^c_i \left( Y_{10}^{ij} H_u^{10} - 3Y_{126}^{ij} \Delta_u \right) \ell_j + e^c_i \left( Y_{10}^{ij} H_d^{10} - 3Y_{126}^{ij} \Delta_d \right) \ell_j \\
+ \nu^c_i \left( Y_{126} v_R \right) \nu^c_j. \tag{5.10}
\]
By using Eq. (5.8), we obtain the low-energy effective superpotential which is described by
only the light Higgs doublets \( H_u \) and \( H_d \),
\[
W_\text{eff} = u^c_i \left( \alpha_u Y_{10}^{ij} + \beta_u Y_{126}^{ij} \right) H_u q_j + d^c_i \left( \alpha_d Y_{10}^{ij} + \beta_d Y_{126}^{ij} \right) H_d q_j \\
+ \nu^c_i \left( \alpha_u Y_{10}^{ij} - 3\beta_u Y_{126}^{ij} \right) H_u \ell_j + e^c_i \left( \alpha_d Y_{10}^{ij} - 3\beta_d Y_{126}^{ij} \right) H_d \ell_j \\
+ \nu^c_i \left( Y_{126} v_R \right) \nu^c_j + \mu_\text{eff} H_u H_d. \tag{5.11}
\]
Here we have assumed that some mechanism, like the Giudice-Masiero mechanism [16] in
supergravity, may produce the effective \( \mu \) term, \( \mu_\text{eff} \) for the light Higgs doublets.
VI. PROTON DECAY

After the symmetry breaking from SO(10) to \(SU(3)_C \times SU(2)_L \times U(1)_Y\), the generic Yukawa interactions between the matter fields and the color triplet Higgs fields are given by

\[
W_Y = Y_{10}^{ij} H_T (q_i \ell_j + u_i^c d_j^c) + Y_{126}^{ij} \Delta_T (q_i \ell_j + u_i^c d_j^c) + Y_{126}^{ij} \Delta_T (q_i \ell_j + u_i^c d_j^c)
\]

Here we have defined

\[
H_T \equiv H^{(3,1,\frac{1}{3})}_{(6,1,1)}, \quad H_T \equiv H^{(3,1,-\frac{1}{3})}_{(6,1,1)}, \quad \Delta_T \equiv \Delta^{(3,1,\frac{1}{3})}_{(6,1,1)}, \quad \Delta_T \equiv \Delta^{(3,1,-\frac{1}{3})}_{(6,1,1)}, \quad \Delta_T' \equiv \Delta^{(3,1,\frac{1}{3})}_{(10,1,3)}, \quad \Delta_T' \equiv \Delta^{(3,1,-\frac{1}{3})}_{(10,1,3)}.
\]

For later use we define

\[
\Delta_T \equiv \Delta^{(3,1,\frac{1}{3})}_{(6,1,1)}, \quad \Delta_T \equiv \Delta^{(3,1,-\frac{1}{3})}_{(6,1,1)}, \quad \Delta_T' \equiv \Delta^{(3,1,\frac{1}{3})}_{(10,1,3)}, \quad \Phi_T \equiv \Phi^{(3,1,\frac{1}{3})}_{(15,1,3)}, \quad \Phi_T' \equiv \Phi^{(3,1,-\frac{1}{3})}_{(15,1,3)}.
\]

In the basis \(\{H_T, \Delta_T, \Delta_T', \Phi_T, \Delta_T'\}\), the mass matrix reads

\[
M_{\text{triplet}} = \begin{pmatrix}
2m_3 & \frac{-\lambda_1 \phi_1}{\sqrt{10}} & \frac{-\lambda_2 \phi_2}{\sqrt{30}} & \frac{\lambda_3 \phi_1}{\sqrt{10}} & \frac{-\lambda_2 \phi_2}{\sqrt{10}} \\
\frac{-\lambda_1 \phi_1}{\sqrt{10}} & m_2 & 0 & \frac{-\lambda_1 \phi_1}{10 \sqrt{3}} & \frac{\lambda_2 \phi_2}{10 \sqrt{2}} \\
\frac{-\lambda_1 \phi_1}{\sqrt{10}} & \frac{\lambda_2 \phi_2}{\sqrt{30}} & 0 & m_2 & 0 \\
\frac{\lambda_3 \phi_1}{\sqrt{5}} & \frac{-\lambda_2 \phi_2}{10 \sqrt{3}} & 0 & m_{44} & \frac{-\lambda_2 \phi_2}{5 \sqrt{6}} \\
\frac{-\sqrt{3} \lambda_1 \phi_1}{\sqrt{15}} & \frac{\lambda_2 \phi_2}{15 \sqrt{2}} & 0 & \frac{-\lambda_1 \phi_1}{5 \sqrt{6}} & m_{55}
\end{pmatrix},
\]

where \(m_{44} \equiv 2m_1 + \frac{\lambda_1 \phi_1}{\sqrt{6}} + \frac{\lambda_1 \phi_2}{3 \sqrt{2}} + \frac{2 \lambda_1 \phi_1}{3} + \frac{2 \lambda_1 \phi_2}{30 \sqrt{2}}\) and \(m_{55} \equiv m_2 + \frac{\lambda_2 \phi_1}{10 \sqrt{6}} + \frac{\lambda_2 \phi_2}{30 \sqrt{2}}\).

The corresponding mass terms of the superpotential read

\[
W_m = (H_T, \Delta_T, \Delta_T', \Phi_T, \Delta_T') M_{\text{triplet}} (H_T, \Delta_T, \Delta_T', \Phi_T, \Delta_T')^T.
\]

Here we integrate out the color triplet Higgs fields, \(\Delta_T\) and \(\Phi_T\), which do not appear in the Yukawa interaction with matter, Eq. (2.1),
Moreover, integrating out the color triplet Higgs field $\Delta_T$, we obtain the effective Yukawa interactions between the matter fields and the color triplet Higgs fields as

$$W_Y = Y_{ij}^{ij_6} \ H_T \ (q_i \ell_j + u_i d_j^c) + Y_{ij}^{ij} \ \Delta_T \ (q_i \ell_j + u_i d_j^c)$$

$$+ Y_{ij}^{ij} \ H_T \ \frac{1}{2} q_i q_j$$
\[\begin{align*}
&+ \left( Y_{10}^{ij} - \frac{m_{31}}{m_{33}} Y_{126}^{ij} \right) H_T \left( u_i^c e_j^c + d_i^c \nu_j^c \right) \\
&+ Y_{126}^{ij} \Delta_T \frac{1}{2} q_i q_j \\
&+ \left( 1 - \frac{m_{32}}{m_{33}} \right) Y_{126}^{ij} \Delta_T \left( u_i^c e_j^c + d_i^c \nu_j^c \right). \tag{6.9}
\end{align*}\]

Then the effective mass terms for the remaining color triplet Higgs fields are written as

\[ W_{\text{eff}}^m = H_T \left( a H_T + b \Delta_T \right) \]
\[ + \Delta_T \left( c H_T + d \Delta_T \right) \]
\[ \equiv \left( H_T, \Delta_T \right) M_T \begin{pmatrix} H_T \\ \Delta_T \end{pmatrix}, \tag{6.10} \]

where \( a, b, c, d \) are defined by

\[ a \equiv m_{11} - \frac{m_{13}}{m_{33}} \cdot m_{31}, \quad b \equiv m_{12} - \frac{m_{13}}{m_{33}} \cdot m_{32}, \]
\[ c \equiv m_{21} - \frac{m_{23}}{m_{33}} \cdot m_{31}, \quad d \equiv m_{22} - \frac{m_{23}}{m_{33}} \cdot m_{32}. \tag{6.11} \]

Combining the Eqs. (6.9) and (6.10) leads to the effective dimension-five interactions after integrating out the remaining color triplet Higgs fields \([17]\),

\[-W_5 = C_{ijkl}^{ijkl} \frac{1}{2} \frac{q_i q_j q_k q_l}{q_i q_j q_k q_l} + C_{ijkl}^{ijkl} u_i^c e_j^c u_k^c d_l^c, \tag{6.12} \]

inducing the dangerous proton decay. Here, \( C_L \) and \( C_R \) are given by the Yukawa coupling matrices at the GUT scale, \( M_G \)

\[ C_L^{ijkl}(M_G) = \left( Y_{10}^{ij}, Y_{126}^{ij} \right) M_T^{-1} \begin{pmatrix} Y_{10}^{kl} \\ Y_{126}^{kl} \end{pmatrix}, \]
\[ C_R^{ijkl}(M_G) = \left( Y_{10}^{ij} - \frac{m_{13}}{m_{33}} Y_{126}^{ij}, \ 1 - \frac{m_{32}}{m_{33}} \right) Y_{126}^{ij} M_T^{-1} \begin{pmatrix} Y_{10}^{kl} \\ Y_{126}^{kl} \end{pmatrix}. \tag{6.13} \]

Note that

\[ \begin{pmatrix} Y_{10} \\ Y_{126} \end{pmatrix} = \begin{pmatrix} \alpha_u & \beta_u \\ \alpha_d & \beta_d \end{pmatrix}^{-1} \begin{pmatrix} Y_u \\ Y_d \end{pmatrix} \]
\[ \equiv A^{-1} \begin{pmatrix} Y_u \\ Y_d \end{pmatrix}. \tag{6.14} \]
Thus we have

\[
C_{ij}^{ijkl} = \left( Y_{ij}, Y_{ij} \right) \left( A M_T A^T \right)^{-1} \begin{pmatrix} Y_{u}^{kl} \\ Y_{d}^{kl} \end{pmatrix}.
\] (6.15)

We make use of this expressions in order to evaluate the renormalization group effects on the Wilson coefficients \(C_{ij}^{ijkl}\) and \(C_{ij}^{ijkl}\). Without loss of generality, we can use the basis where \(Y_u\) is real and diagonal,

\[
Y_u = \frac{1}{v \sin \beta} \text{diag}(m_u, m_c, m_t),
\] (6.16)

with \(v \simeq 174.1\) [GeV]. Since \(Y_d\) is a symmetric matrix, it can be described as

\[
Y_d = \frac{1}{v \cos \beta} \overline{V}_{\text{CKM}}^\dagger \text{diag}(m_d, m_s, m_b) V_{\text{CKM}},
\] (6.17)

by using a unitary matrix

\[
\overline{V}_{\text{CKM}} \equiv e^{i\alpha_1} e^{i\alpha_2 \lambda_3} e^{i\alpha_3 \lambda_8} V_{\text{CKM}}^{\dagger} e^{i\beta_2 \lambda_3} e^{i\beta_3 \lambda_8},
\] (6.18)

where \(\lambda_3, \lambda_8\) are the Gell-Mann matrices and \(V_{\text{CKM}}\) is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [18].

The complete anti-symmetry in the color indices requires that the dimension-five operator Eq. (6.12) possesses the flavor non-diagonal indices [19]. As a consequence, the dominant decay mode is \(p \to K^{+}\bar{\nu}\). This fact implies that the chargino dressing diagrams dominate over the gluino and the neutralino dressing diagrams [20].

In the components form, the dimension-five operators at the SUSY breaking scale, \(M_{\text{SUSY}}\) are written as

\[
\mathcal{L}_5 = C_{L}^{(\bar{u}d) XY ij} \bar{u}_X \tilde{d}_Y u_{Li} \nu_{Lj} + C_{L}^{(\bar{u}d) XY ij} \frac{1}{2} \bar{u}_X \bar{u}_Y d_{Li} \nu_{Lj} \\
+ C_{R}^{(\bar{u}d) XY ij} \bar{u}_X \tilde{d}_Y u_{Ri} \nu_{Rj} + C_{R}^{(\bar{u}d) XY ij} \frac{1}{2} \bar{u}_X \bar{u}_Y d_{Ri} \nu_{Rj} \\
+ C_{L}^{(\bar{d}d) XY ij} \bar{d}_X \tilde{d}_Y d_{Li} \nu_{Lj} + C_{L}^{(\bar{d}d) XY ij} \frac{1}{2} \bar{d}_X \bar{d}_Y u_{Li} \nu_{Lj}
\]

\[\text{In Ref. [6], we set these phases } \alpha_i \ (i = 1, 2, 3), \ \beta_i \ (i = 2, 3) \text{ to zero or } \pi.\]
\begin{align}
+ C_{L}^{(\tilde{u}\tilde{e}ud)XYij}\tilde{u}_{X}\tilde{e}_{Y}u_{Li}d_{Lj} + C_{L}^{(\tilde{d}\tilde{e}uu)XYij}\frac{1}{2}\tilde{d}_{X}\tilde{e}_{Y}u_{Li}u_{Lj} \\
+ C_{R}^{(\tilde{u}\tilde{e}ud)XYij}\tilde{u}_{X}\tilde{e}_{Y}u_{Ri}d_{Rj} + C_{R}^{(\tilde{d}\tilde{e}uu)XYij}\frac{1}{2}\tilde{d}_{X}\tilde{e}_{Y}u_{Ri}u_{Rj} \\
+ C_{L}^{(\tilde{d}\tilde{e}ud)XYij}\tilde{d}_{X}\tilde{e}_{Y}u_{Li}d_{Lj} + C_{L}^{(\tilde{d}\tilde{e}uu)XYij}\frac{1}{2}\tilde{u}_{X}\tilde{e}_{Y}d_{Li}d_{Lj}.
\end{align}

(6.19)

The coefficients are obtained from the coefficients of the original dimension-five operators including their renormalization from $M_G$ to $M_{\text{SUSY}}$. Their explicit forms are found in Appendix A. After the sparticles dressing, we obtain the following type of dimension-six operators causing nucleon decays,

\begin{align}
L_6 &= \frac{1}{16\pi^2} \left[ C_{LL}^{(\tilde{u}\tilde{d}ue)ij}(u_{Li}d_{Lj})(u_{Li}e_{Lj}) + C_{RL}^{(\tilde{u}\tilde{d}ue)ij}(u_{Ri}d_{Rj})(u_{Le}e_{Lj}) \\
&+ C_{LR}^{(\tilde{d}\tilde{e}ud)ij}(u_{Li}d_{Lj})(u_{Ri}e_{Rj}) + C_{RR}^{(\tilde{d}\tilde{e}ud)ij}(u_{Ri}d_{Rj})(u_{Re}e_{Rj}) \\
&+ C_{LL}^{(u\tilde{d}d\nu)ijk}(u_{Li}d_{Lj})(d_{Lj}\nu_{Lk}) + C_{RL}^{(u\tilde{d}d\nu)ijk}(u_{Ri}d_{Rj})(d_{Lj}\nu_{Lk}) \\
&+ C_{RL}^{(d\tilde{d}u\nu)ijk}\frac{1}{2}(d_{Ri}d_{Rj})(u_{Li}\nu_{Lk}) \right].
\end{align}

(6.20)

These operators should be renormalized from $M_{\text{SUSY}}$ to $M_Z$ and further to the hadronization scale ($\mu_{\text{had}} \approx 1$ [GeV]). Then the effective four-Fermi Lagrangian is converted to a hadronic Lagrangian by using the chiral Lagrangian method [21] [22]. Details are given in Appendices B and C.

For the decay mode $p \to K^+\bar{\nu}_i$, the partial decay rate is given by the formula

\begin{equation}
\Gamma(p \to K^+\bar{\nu}_i) = \frac{m_p}{32\pi} \left( 1 - \frac{m_{K^+}}{m_p^2} \right)^2 \frac{1}{f_\pi} |A(p \to K^+\bar{\nu}_i)|^2.
\end{equation}

(6.21)

Here $m_p = 0.938$ [GeV] is the proton mass, $m_{K^+} = 0.493$ [GeV] is the kaon mass and $f_\pi = 0.131$ [GeV] is the pion decay constant.

The amplitude $A(p \to K^+\bar{\nu}_i)$ for $p \to K^+\bar{\nu}_i$ reads [23]

\begin{align}
A(p \to K^+\bar{\nu}_i) &= \left[ \beta C_{LL}^{(u\tilde{d}d\nu)^{21i}} + \alpha C_{RL}^{(u\tilde{d}d\nu)^{21i}} \right] \frac{2m_p}{3m_B} D \\
&+ \left[ \beta C_{LL}^{(u\tilde{d}d\nu)^{12i}} + \alpha C_{RL}^{(u\tilde{d}d\nu)^{12i}} \right] \left[ 1 + \frac{m_p}{3m_B} (3F + D) \right] \\
&+ \alpha C_{RL}^{(d\tilde{d}u\nu)^{12i}} \left[ 1 - \frac{m_p}{3m_B} (3F - D) \right].
\end{align}

(6.22)
Here $m_B = 1.150 [\text{GeV}]$ is an averaged baryon mass, $F = 0.44$, $D = 0.81$ are the parameters in terms of which the octet-baryon axial-vector form factors are expressed and $\alpha, \beta$ are the hadron matrix elements which are defined by [24]

$$
\alpha u_L(k) = \langle 0 | d_R u_R u_L | p(k) \rangle,
$$
$$
\beta u_L(k) = \langle 0 | d_L u_L u_R | p(k) \rangle.
$$

(6.23)

$u_L(k)$ denote the left-handed components of the proton wave function. It is known that $|\alpha| = |\beta|$, and $\beta$ is in the range [24]

$$
0.003 [\text{GeV}^3] \leq \beta \leq 0.03 [\text{GeV}^3].
$$

(6.24)

From the recent lattice calculations, one group reported that [25]

$$
\alpha = -(0.015 \pm 0.001) [\text{GeV}^3],
$$
$$
\beta = 0.014 \pm 0.001 [\text{GeV}^3].
$$

(6.25)

But the other group reported the smaller values [26]

$$
\alpha = -(0.006 \pm 0.001) [\text{GeV}^3],
$$
$$
\beta = 0.007 \pm 0.001 [\text{GeV}^3].
$$

(6.26)

VII. GAUGE COUPLINGS UNIFICATION

In general, the gauge couplings unification imposes constraints on the mass spectrum of many varieties of Higgs fields [27]. Our strategy is a generic one that all of the dimensionless coefficients should remain of order one to preserve the perturbative limit and put all the VEVs at the GUT scale in order to realize the simple gauge couplings unification picture. For the numerical evaluation, we use the one-loop renormalization group equations (RGEs)
in the $\overline{\text{DR}}$ scheme \cite{28}, \footnote{\overline{\text{DR}} uses dimensional regularization through dimensional reduction with modified minimal subtraction.} \footnote{Here we assume, for simplicity, all the mass eigenvalues of the Higgs fields are smaller than $M_G$ and all the masses of the gauge fields lie around $M_G$. In the other cases, the formula becomes quite complicated.}

\[
\frac{1}{\alpha_i (M_G)} = \frac{1}{\alpha_i (M_Z) |_{\overline{\text{MS}}}} - \frac{C_2 (G_i)}{12 \pi} + \frac{1}{2 \pi} \left[ b_i \log \left( \frac{M_Z}{M_G} \right) + \sum_\zeta b_\zeta^i \log \left( \frac{\text{det}' M_\zeta}{M_G^{\text{rank}(M_\zeta)}} \right) \right], \quad (7.1)
\]

where $C_2$ is the quadratic Casimir operator; $C_2 (SU(3)) = 3$, $C_2 (SU(2)) = 2$, $C_2 (U(1)) = 0$, and $\zeta$ denotes the Higgs fields which have the corresponding gauge quantum numbers. $M_\zeta$ is it’s mass matrix and ”det’” means that the determinant should be taken excluding the zero modes. $b_i$ and $b_\zeta^i$ are the $\beta$ function coefficients ; $b_3 = -3$, $b_2 = 1$, $b_1 = \frac{33}{3}$, and $b_\zeta^i$ are given in Tables I and II. For $\alpha_i (M_Z) |_{\overline{\text{MS}}}$, we use the following values.

\[
\alpha_3 (M_Z) |_{\overline{\text{MS}}} = \alpha_s (M_Z), \quad (7.2)
\]
\[
\alpha_2 (M_Z) |_{\overline{\text{MS}}} = \alpha (M_Z) / \sin^2 \theta_W (M_Z), \quad (7.3)
\]
\[
\alpha_1 (M_Z) |_{\overline{\text{MS}}} = \frac{5}{3} \alpha (M_Z) / \left( 1 - \sin^2 \theta_W (M_Z) \right), \quad (7.4)
\]

with \cite{29}

\[
\alpha_s (M_Z) = 0.1172, \quad \alpha (M_Z) = 1/128.92, \quad \sin^2 \theta_W (M_Z) = 0.23113. \quad (7.5)
\]

Excluding the fields which mix with the would-be NG fields and the fields with $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers, $[(1, 2, \frac{1}{2}) + \text{h.c.}]$ (”Higgs doublet”) and $[(3, 1, \frac{1}{3}) + \text{h.c.}]$ (color triplet Higgs fields), the massive fields are given as follows.

For $\overline{126}$ and $126$ representation fields, their quantum numbers, the masses and their $\beta$ function coefficients are given in Table I.

For $210$ representation field, their quantum numbers, the masses and their $\beta$ function coefficients are given in Table II.
TABLE I. The mass matrices and the β function coefficients for 126 and 126.

| quantum numbers | mass matrices, or mass eigenvalues                                                                 | $b_3^\zeta$ | $b_2^\zeta$ | $b_1^\zeta$ |
|-----------------|----------------------------------------------------------------------------------------------------------------|------------|-------------|-------------|
| $(8, 2, \frac{1}{2}) + \text{h.c.}$ | $\begin{pmatrix} m_2 - \frac{\lambda_2 \phi_2}{30\sqrt{2}} - \frac{\lambda_2 \phi_3}{60} & 0 \\ 0 & m_2 - \frac{\lambda_2 \phi_2}{30\sqrt{2}} + \frac{\lambda_2 \phi_3}{60} \end{pmatrix}$ | 12         | 8           | $\frac{24}{5}$ |
| $(6, 3, \frac{1}{3}) + \text{h.c.}$ | $m_2 - \frac{\lambda_2 \phi_1}{10\sqrt{6}} - \frac{\lambda_2 \phi_2}{30\sqrt{2}}$ | 15         | 24          | $\frac{12}{5}$ |
| $(6, 1, \frac{4}{3}) + \text{h.c.}$ | $m_2 + \frac{\lambda_2 \phi_1}{10\sqrt{6}} - \frac{\lambda_2 \phi_2}{30\sqrt{2}} + \frac{\lambda_2 \phi_3}{30}$ | 5          | 0           | $\frac{64}{5}$ |
| $(5, 1, \frac{2}{3}) + \text{h.c.}$ | $m_2 + \frac{\lambda_2 \phi_1}{10\sqrt{6}} - \frac{\lambda_2 \phi_2}{30\sqrt{2}}$ | 5          | 0           | $\frac{16}{5}$ |
| $(6, 1, \frac{1}{3}) + \text{h.c.}$ | $m_2 + \frac{\lambda_2 \phi_1}{10\sqrt{6}} - \frac{\lambda_2 \phi_2}{30\sqrt{2}}$ | 5          | 0           | $\frac{4}{5}$  |
| $(3, 1, \frac{1}{3}) + \text{h.c.}$ | $m_2 - \frac{\lambda_2 \phi_1}{10\sqrt{6}} + \frac{\lambda_2 \phi_2}{30\sqrt{2}}$ | 3          | 12          | $\frac{6}{5}$  |
| $(3, 2, \frac{7}{6}) + \text{h.c.}$ | $\begin{pmatrix} m_2 + \frac{\lambda_2 \phi_2}{30\sqrt{2}} - \frac{\lambda_2 \phi_3}{60} & 0 \\ 0 & m_2 + \frac{\lambda_2 \phi_2}{30\sqrt{2}} - \frac{\lambda_2 \phi_3}{20} \end{pmatrix}$ | 2          | 3           | $\frac{48}{5}$ |
| $(3, 1, \frac{4}{3}) + \text{h.c.}$ | $m_2 + \frac{\lambda_2 \phi_1}{10\sqrt{6}} + \frac{\lambda_2 \phi_2}{30\sqrt{2}}$ | 1          | 0           | $\frac{32}{5}$ |
| $(1, 3, 1) + \text{h.c.}$ | $m_2 - \frac{\lambda_2 \phi_1}{10\sqrt{6}} + \frac{\lambda_2 \phi_2}{10\sqrt{2}}$ | 0          | 4           | $\frac{18}{5}$ |
| $(1, 1, 2) + \text{h.c.}$ | $m_2 + \frac{\lambda_2 \phi_1}{10\sqrt{6}} + \frac{\lambda_2 \phi_2}{10\sqrt{2}} - \frac{\lambda_2 \phi_3}{10}$ | 0          | 0           | $\frac{24}{5}$ |
TABLE II. The mass matrices and the $\beta$ function coefficients for $210$.

| quantum numbers | mass matrices, or mass eigenvalues | $b_3^\zeta$ | $b_2^\zeta$ | $b_1^\zeta$ |
|-----------------|-----------------------------------|------------|------------|------------|
| (8, 3, 0)       | $2m_1 - \frac{\lambda_1 \phi_1}{\sqrt{6}} - \frac{\lambda_1 \phi_2}{3\sqrt{2}}$ | 9          | 16         | 0          |
| (8, 1, 1) + h.c. | $2m_1 + \frac{\lambda_1 \phi_1}{\sqrt{6}} - \frac{\lambda_1 \phi_2}{3\sqrt{2}}$ | 6          | 0          | $\frac{48}{5}$ |
| (8, 1, 0)       | $\begin{pmatrix} 2m_1 - \frac{\lambda_1 \phi_2}{3\sqrt{2}} & \frac{\lambda_1 \phi_3}{3\sqrt{2}} \\ \frac{\lambda_1 \phi_3}{3\sqrt{2}} & 2m_1 + \frac{\lambda_1 \phi_1}{\sqrt{6}} - \frac{\lambda_1 \phi_2}{3\sqrt{2}} \end{pmatrix}$ | 3          | 0          | 0          |
| (6, 2, $\frac{5}{2}$) + h.c. | $2m_1 - \frac{\lambda_1 \phi_2}{3\sqrt{2}} - \frac{\lambda_1 \phi_3}{6}$ | 10         | 6          | 10         |
| (6, 2, $\frac{1}{2}$) + h.c. | $2m_1 - \frac{\lambda_1 \phi_2}{3\sqrt{2}} + \frac{\lambda_1 \phi_3}{6}$ | 10         | 6          | $\frac{2}{5}$ |
| (3, 3, $\frac{2}{3}$) + h.c. | $2m_1 - \frac{\lambda_1 \phi_1}{\sqrt{6}} + \frac{\lambda_1 \phi_2}{3\sqrt{2}}$ | 3          | 12         | $\frac{24}{5}$ |
| (3, 1, $\frac{5}{3}$) + h.c. | $2m_1 + \frac{\lambda_1 \phi_1}{\sqrt{6}} + \frac{\lambda_1 \phi_2}{3\sqrt{2}} - \frac{2\lambda_1 \phi_3}{3}$ | 1          | 0          | 10         |
| (1, 3, 0)       | $2m_1 - \frac{\lambda_1 \phi_2}{\sqrt{6}} + \sqrt{7}\frac{\lambda_1 \phi_3}{3}$ | 0          | 2          | 0          |
| (1, 2, $\frac{3}{2}$) + h.c. | $2m_1 + \frac{\lambda_1 \phi_2}{\sqrt{2}} - \frac{\lambda_1 \phi_3}{2}$ | 0          | 1          | $\frac{27}{5}$ |
Putting these values into Eq. (7.1), the unification condition produces two individual equations,

\[ \alpha_3 (M_G) = \alpha_2 (M_G), \quad (7.6) \]

and

\[ \alpha_3 (M_G) = \alpha_1 (M_G). \quad (7.7) \]

Setting all VEVs at the GUT scale, \( \phi_1 \sim \phi_2 \sim \phi_3 \sim |v_R| \sim M_G \), and the remaining dimensionless coefficients of order one, we can search whether Eqs. (7.6) and (7.7) have a solution for \( M_G \) below the Planck scale, \( M_G \leq M_{\text{Planck}} \). If such a solution exists, it would limit the parameters in the superpotential Eq. (2.2) to some restricted region.

**VIII. CONCLUSION**

We find the general formulation for the proton decay rate in the minimal renormalizable SUSY SO(10) models. Using this generic formulation one can find whether the minimal SUSY SO(10) grand unified theory has been excluded.

Recently, using their Yukawa couplings (Eqs. (8) and (9) in Ref. [30]), Goh-Mohapatra-Nasri-Ng obtained the allowed region of \((x, y, z)\) which correspond to \( \left( \frac{a}{d}, -\frac{b}{d}, -\frac{c}{d} \right) \) in our notation. However, they did not discuss the concrete form of the superpotential and, therefore, compatibilities of their superpotential with the other constraints are not clear in their paper. Also, as we have mentioned above, there appears a non zero \( x \) value even without the \( 54 \) dimensional Higgs field. Further, besides the color triplet Higgs fields, there is a much richer Higgs particle contents. These additional Higgs fields may cause a pathology of the gauge couplings unification. This paper presents a relationship among these comprehensive but tightly connected problems.
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NOTE ADDED

In the recent paper [arXiv:hep-ph/0402122] and the revised version of the paper [arXiv:hep-ph/0204097], the authors claimed that they disagree with our results for mass matrices. We point out that our results satisfy all possible consistency checks. Namely, for arbitrary couplings $m_{1,2,3}$ and $\lambda_{1,2,3,4}$ in Eq. (2.2) there are solutions characterized by $|v_R| = 0$, particularly $SU(5) \times U(1)$, $SU(5) \times U(1)$ flipped, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ symmetry preserving vacua. (see, Eqs. (3.10)–(3.12) with $|v_R| = 0$). In all the above four symmetry breakings and in all the mass matrices, the field $\Phi$ decouples from the other set of fields, $H$, $\Delta$ and $\Delta$. Moreover, all our mass eigenvalues for the multiplets coming from the field $\Phi$, coincide with the corresponding results in [15]. Furthermore, for the $SU(5)$ symmetry breaking case, $|v_R| \neq 0$ (see, the end of Sec.III), all our mass eigenvalues (472 in total) and the corresponding multiplets under $SU(3)_C \times SU(2)_L \times U(1)_Y$ are grouped according to the $SU(5)$ irreducible representations with the correct would-be NG fields. The details will be published in a separate publication.

APPENDIX

A. Dimension-five operators

In this appendix, we list the explicit form of the various interaction coefficients.

We use the following notations for the mixing matrices which diagonalize the squark, slepton mass-squared matrices and chargino, neutralino mass matrices. Squark, slepton
mass-squared matrix $M_f^2$, chargino and neutralino mass matrices $M_C$ and $M_N$ are diagonalized by the unitary matrices $U_f$, $O_L$, $O_R$ and $O_N$, respectively.

$$U_f M_f^2 U_f^\dagger = \text{diag}(m_{f_1}^2, ..., m_{f_6}^2),$$

$$O_R M_C O_L^\dagger = \text{diag}(m_{\chi_1}^-, m_{\chi_2}^-),$$

$$O_N^* M_N O_N^\dagger = \text{diag}(m_{\chi_1}^0, m_{\chi_2}^0, m_{\chi_3}^0, m_{\chi_4}^0).$$

(A.1)

For the dimension-five operators, we have the following expressions \(^5\)

$$C_L^{(\bar{u}\bar{d}\bar{u}e)}_{XYij} \equiv C_L^{ijkl}(U_u^* X_k(U_d^*)_Y l, \text{(A.2)}$$

$$C_L^{(\bar{u}\bar{d}\bar{d}e)}_{XYij} \equiv C_L^{[ijkl]m}(U_u^* X_k(U_u^*)_Y l(V_{CKM})_im, \text{(A.3)}$$

$$C_R^{(\bar{u}\bar{d}u)}_{XYij} \equiv (C_R^{eijkl} - C_R^{eijkl})(U_u^* X_k(U_d^*)_Y l, \text{(A.4)}$$

$$C_R^{(\bar{d}\bar{d}u)}_{XYij} \equiv (C_R^{eijkl} - C_R^{eijkl})(U_u^* X_k(U_d^*)_Y l, \text{(A.5)}$$

$$C_L^{(\bar{u}\bar{u}u)}_{XYij} \equiv (C_L^{mnkl} - C_L^{kmnl})(U_u^* X_k(U_u^*)_Y l, \text{(A.6)}$$

$$C_L^{(\bar{d}\bar{d}u)}_{XYij} \equiv (C_L^{mnkl} - C_L^{kmnl})(U_u^* X_k(U_u^*)_Y l, \text{(A.7)}$$

$$C_R^{(\bar{u}\bar{d}u)}_{XYij} \equiv (C_R^{eijkl} - C_R^{eijkl})(U_u^* X_k(U_d^*)_Y l, \text{(A.8)}$$

$$C_R^{(\bar{d}\bar{d}u)}_{XYij} \equiv (C_R^{eijkl} - C_R^{eijkl})(U_u^* X_k(U_d^*)_Y l, \text{(A.9)}$$

$$C_R^{(\bar{u}\bar{d}u)}_{XYij} \equiv (C_R^{eijkl} - C_R^{eijkl})(U_u^* X_k(U_d^*)_Y l, \text{(A.10)}$$

$$C_R^{(\bar{d}\bar{d}u)}_{XYij} \equiv (C_R^{eijkl} - C_R^{eijkl})(U_u^* X_k(U_d^*)_Y l, \text{(A.11)}$$

$$C_L^{(\bar{u}\bar{u}u)}_{XYij} \equiv (C_L^{mnkl} - C_L^{kmnl})(U_u^* X_k(U_u^*)_Y l(V_{CKM})_jm, \text{(A.12)}$$

$$C_L^{(\bar{d}\bar{d}u)}_{XYij} \equiv (C_L^{mnkl} - C_L^{kmnl})(U_u^* X_k(U_u^*)_Y l(V_{CKM})_jm, \text{(A.13)}$$

In Eqs. (A.6) and (A.7), it should be noticed that the neutrinos in the final states should be rotated from the flavor eigenstates to the mass eigenstates by using the Maki-Nakagawa-Sakata (MNS) mixing matrix [31], $U_{\text{MNS}}$.

\(^5\)We use a notation for an anti-symmetric tensor, $A^{[ijkl]} \equiv A^{ijkl} - A^{kijl}$. 

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B. Sparticles interactions

We use the following notations for the quark-gluino-squark, quark(lepton)-chargino-squark(slepton) and quark(lepton)-neutralino-squark(slepton) interactions,

- quark-gluino-squark interactions

\[
\mathcal{L}_{int} = -i \sqrt{2} u_i^c \left[ G_{iX}^{L(u)} P_L + G_{iX}^{R(u)} P_R \right] \bar{g} \tilde{u}_X - i \sqrt{2} d_i^c \left[ G_{iX}^{L(d)} P_L + G_{iX}^{R(d)} P_R \right] \bar{g} \tilde{d}_X + \text{h.c.} \quad (B.1)
\]

- quark(lepton)-chargino-squark(slepton) interactions

\[
\mathcal{L}_{int} = u_i^c \left[ C_{iAX}^{L(u)} P_L + C_{iAX}^{R(u)} P_R \right] \chi_A^+ \tilde{d}_X + d_i^c \left[ C_{iAX}^{L(d)} P_L + C_{iAX}^{R(d)} P_R \right] \chi_A^+ \tilde{u}_X + \nu_i^c C_{iAX}^{R(\nu)} P_R \chi_A^+ \tilde{\nu}_X + e_i^c \left[ C_{iAX}^{L(e)} P_L + C_{iAX}^{R(e)} P_R \right] \chi_A^+ \tilde{e}_X + \text{h.c.} \quad (B.2)
\]

- quark(lepton)-neutralino-squark(slepton) interactions

\[
\mathcal{L}_{int} = u_i^c \left[ N_{iAX}^{L(u)} P_L + N_{iAX}^{R(u)} P_R \right] \chi_A^0 \tilde{u}_X + d_i^c \left[ N_{iAX}^{L(d)} P_L + N_{iAX}^{R(d)} P_R \right] \chi_A^0 \tilde{d}_X + \nu_i^c N_{iAX}^{R(\nu)} P_R \chi_A^+ \tilde{\nu}_X + e_i^c \left[ N_{iAX}^{L(e)} P_L + N_{iAX}^{R(e)} P_R \right] \chi_A^0 \tilde{e}_X + \text{h.c.} \quad (B.3)
\]

Explicitly, we have the following expressions

\[
G_{iX}^{L(u)} \equiv g_3 (U_{u}^*)_{X,i+3}, \quad (B.4)
\]
\[
G_{iX}^{R(u)} \equiv g_3 (U_{d}^*)_{X,i}, \quad (B.5)
\]
\[
G_{iX}^{L(d)} \equiv g_3 (U_{u}^*)_{X,i+3}, \quad (B.6)
\]
\[
G_{iX}^{R(d)} \equiv g_3 (U_{d}^*)_{X,k} (V_{CKM})_{ik}, \quad (B.7)
\]
\[
C_{iAX}^{L(u)} \equiv g \frac{m_{u_i}}{\sqrt{2} M_W \sin \beta} (O_{R}^*)_{A2} (U_{d}^*)_{X,i}, \quad (B.8)
\]
These expressions are found in [32], but only for the quark sector. So here we write them explicitly.
C. Dimension-six operators

For the dimension-six operator, we divide the coefficients into three parts according to the dressed sparticles,

\[ C^{(udue)}_{LL} = C^{(udue)}_{LL}(g) + C^{(udue)}_{LL}(\bar{\chi}^0) + C^{(udue)}_{LL}(\bar{\chi}^\pm), \]  

(C.1)

etc. Then we have the following expressions. These expressions have the same forms as [23]. However, ours are different from them in the neutrino sector as was mentioned in the end of Appendix. A.

\[ C^{(udue)}_{LL}(g) = \frac{4}{3} \frac{1}{m_g} C_L^{(udue)XY} G_{1X}^{R(u)} G_{1Y}^{R(d)} F \left( \frac{m^2_{\chi}}{m^2_{uX}}, \frac{m^2_{\chi}}{m^2_{dY}} \right), \]  

(C.2)

\[ C^{(udue)}_{LL}(\bar{\chi}^\pm) = \frac{1}{m_{\chi^\pm}} \left[ -C_L^{(udue)XY} G_{1X}^{R(u)} G_{1Y}^{R(d)} F \left( \frac{m^2_{\chi^\pm}}{m^2_{uX}}, \frac{m^2_{\chi^\pm}}{m^2_{dY}} \right) \right. \]

\[ + C_L^{(udue)XY} G_{1X}^{R(u)} G_{1Y}^{R(d)} F \left( \frac{m^2_{\chi^\pm}}{m^2_{uX}}, \frac{m^2_{\chi^\pm}}{m^2_{dY}} \right) \]  

(C.3)

\[ C^{(udue)}_{LL}(\bar{\chi}^0) = \frac{1}{m_{\chi^0}} \left[ C_L^{(udue)XY} G_{1X}^{R(u)} G_{1Y}^{R(d)} F \left( \frac{m^2_{\chi^0}}{m^2_{uX}}, \frac{m^2_{\chi^0}}{m^2_{dY}} \right) \right. \]

\[ + C_L^{(udue)XY} G_{1X}^{R(u)} G_{1Y}^{R(d)} F \left( \frac{m^2_{\chi^0}}{m^2_{uX}}, \frac{m^2_{\chi^0}}{m^2_{dY}} \right) \]  

(C.4)

\[ C^{(udue)}_{RL}(g) = \frac{4}{3} \frac{1}{m_g} C_L^{(udue)XY} G_{1X}^{L(u)} G_{1Y}^{L(d)} F \left( \frac{m^2_{\chi}}{m^2_{uX}}, \frac{m^2_{\chi}}{m^2_{dY}} \right), \]  

(C.5)

\[ C^{(udue)}_{RL}(\bar{\chi}^\pm) = -\frac{1}{m_{\chi^\pm}} C_L^{(udue)XY} G_{1X}^{L(u)} G_{1Y}^{L(d)} F \left( \frac{m^2_{\chi^\pm}}{m^2_{uX}}, \frac{m^2_{\chi^\pm}}{m^2_{dY}} \right), \]  

(C.6)

\[ C^{(udue)}_{RL}(\bar{\chi}^0) = \frac{1}{m_{\chi^0}} \left[ C_L^{(udue)XY} G_{1X}^{L(u)} G_{1Y}^{L(d)} F \left( \frac{m^2_{\chi^0}}{m^2_{uX}}, \frac{m^2_{\chi^0}}{m^2_{dY}} \right) \right. \]

\[ + C_L^{(udue)XY} G_{1X}^{L(u)} G_{1Y}^{L(d)} F \left( \frac{m^2_{\chi^0}}{m^2_{uX}}, \frac{m^2_{\chi^0}}{m^2_{dY}} \right) \]  

(C.7)

\[ C^{(udue)}_{LR}(g) = \frac{4}{3} \frac{1}{m_g} C_L^{(udue)XY} G_{1X}^{R(u)} G_{1Y}^{R(d)} F \left( \frac{m^2_{\chi}}{m^2_{uX}}, \frac{m^2_{\chi}}{m^2_{dY}} \right), \]  

(C.8)

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\[
C_{LR}^{(udue)}(x^\pm) = \frac{1}{m_{\chi_A}^2} \left[ -C_{R}^{(udue)XYij} C_{1AY}^{(R(u)} C_{iAX}^{(R(d)} \left( \frac{m_{z_1}^2}{\chi_A}, \frac{m_{z_2}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYij} C_{1AX}^{(R(e)} C_{iAY}^{(R(d)} \left( \frac{m_{z_1}^2}{\chi_A}, \frac{m_{z_2}^2}{\chi_A} \right) \right] ,
\]
\[
C_{LR}^{(udue)}(x^\pm) = \frac{1}{m_{\chi_A}^2} \left[ C_{R}^{(udue)XYij} N_{1AX}^{(R(u)} N_{iAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYij} N_{1AX}^{(R(e)} N_{iAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
C_{RR}^{(udue)}(\tilde{g}) = \frac{4}{3} \frac{1}{m_{\chi_A}^2} C_{R}^{(udue)XYij} G_{L}^{(u)} C_{iAY}^{(R(d)} \left( \frac{m_{z_1}^2}{\chi_A}, \frac{m_{z_2}^2}{\chi_A} \right) ,
\]
\[
C_{RR}^{(udue)}(\chi^\pm) = \frac{1}{m_{\chi_A}^2} \left[ C_{R}^{(udue)XYij} N_{1AX}^{(R(u)} N_{iAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYij} N_{1AX}^{(R(e)} N_{iAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
C_{LL}^{(udue)}(\tilde{g}) = \frac{4}{3} \frac{1}{m_{\chi_A}^2} \left[ C_{L}^{(udue)XYjk} G_{LX}^{(R(u)} G_{iY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYjk} G_{iX}^{(R(d)} G_{jY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
C_{LL}^{(udue)}(\chi^\pm) = \frac{1}{m_{\chi_A}^2} \left[ -C_{L}^{(udue)XYjk} G_{1AY}^{(R(u)} G_{iAX}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYjk} G_{jAY}^{(R(d)} G_{kAX}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
C_{LL}^{(udue)}(\chi^\pm) = \frac{1}{m_{\chi_A}^2} \left[ C_{L}^{(udue)XYjk} N_{1AX}^{(R(u)} N_{iAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYjk} N_{1AX}^{(R(e)} N_{iAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYjk} N_{jAY}^{(R(d)} N_{kAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[
+ C_{L}^{(dudue)XYjk} N_{jAY}^{(R(d)} N_{kAY}^{(R(d)} \left( \frac{m_{z_0}^2}{\chi_A}, \frac{m_{z_0}^2}{\chi_A} \right) \right] ,
\]
\[ + C^{(u dd vv) jk}_{RL} N^{R(u)}_{i AX} N^{R(v)}_{k AY} F_{L} \left( \frac{m^2_{\chi_0}}{m^2_{u_X}}, \frac{m^2_{\chi_0}}{m^2_{d_Y}} \right), \]  
\[ (C.16) \]

\[ C^{(u dd vv) jk}_{RL}(g) = \frac{4}{3} \frac{1}{m_{\tilde{g}}} C^{(u dd vv) jk}_{L} G_{1X}^{L(u)} G_{i Y}^{L(d)} F_{L} \left( \frac{m^2_{\chi_0}}{m^2_{u_X}}, \frac{m^2_{\chi_0}}{m^2_{d_Y}} \right), \]  
\[ (C.17) \]

\[ \begin{align*} 
C^{(u dd vv) jk}_{RL}(\chi^\pm) &= \frac{1}{m_\chi^\pm} \left[ -C^{(u dd vv) jk}_{L} C^{L(u)}_{1 A Y} C^{L(d)}_{i A X} F_{L} \left( \frac{m^2_{\chi_0}}{m^2_{u_X}}, \frac{m^2_{\chi_0}}{m^2_{d_Y}} \right) \right] + C^{(u ed u) jk(R)}_{R} C^{R(u)}_{j A X} C^{R(d)}_{k A Y} F_{L} \left( \frac{m^2_{\chi}}{m^2_{d_X}}, \frac{m^2_{\chi}}{m^2_{d_Y}} \right), 
\end{align*} \]  
\[ (C.18) \]

\[ \begin{align*} 
C^{(u dd vv) jk}_{RL}(\chi^0) &= \frac{1}{m_\chi^0} C^{(u dd vv) jk}_{L} N^{L(u)}_{1 A X} N^{L(d)}_{i A Y} F_{L} \left( \frac{m^2_{\chi_0}}{m^2_{u_X}}, \frac{m^2_{\chi_0}}{m^2_{d_Y}} \right), 
\end{align*} \]  
\[ (C.19) \]

\[ \begin{align*} 
C^{(dd uv) jk}_{RL}(g) &= \frac{4}{3} \frac{1}{m_{\tilde{g}}} C^{(dd uv) jk}_{L} G_{1X}^{L(u)} G_{j Y}^{L(d)} F_{L} \left( \frac{m^2_{\tilde{g}}}{m^2_{d_X}}, \frac{m^2_{\tilde{g}}}{m^2_{d_Y}} \right), 
\end{align*} \]  
\[ (C.20) \]

\[ C^{(dd uv) jk}_{RL}(\chi^\pm) = 0, \]  
\[ (C.21) \]

\[ \begin{align*} 
C^{(dd uv) jk}_{RL}(\chi^0) &= \frac{1}{m_\chi^0} C^{(dd uv) jk}_{L} N^{L(d)}_{j A Y} N^{L(u)}_{i A X} F_{L} \left( \frac{m^2_{\chi_0}}{m^2_{d_X}}, \frac{m^2_{\chi_0}}{m^2_{d_Y}} \right). 
\end{align*} \]  
\[ (C.22) \]

Here we have defined a loop function,

\[ F(x, y) \equiv \frac{x y}{x - y} \left( \frac{1}{1 - x} \log x - \frac{1}{1 - y} \log y \right). \]  
\[ (C.23) \]
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