Abstract—Future cellular networks are intended to have the ability to sense the environment by utilizing reflections of transmitted signals. Multi-dimensional sensing brings along the crucial advantage of being able to resort to multiple domains to resolve targets, enhancing detection capabilities compared to one-dimensional (1D) estimation. However, estimating parameters jointly in 5G New Radio systems poses the challenge of limiting the computational complexity while preserving a high resolution. To that end, we make use of channel state information (CSI) decimation for Multiple Signal Classification (MUSIC)-based joint range-angle of arrival estimation. We further introduce multi-peak search routines to achieve additional detection capability improvements. Simulation results with orthogonal frequency-division multiplexing (OFDM) signals show that we attain higher detection probabilities for closely spaced targets than with 1D range-only estimation. Moreover, we demonstrate that for our considered 5G setup, we are able to significantly reduce the required number of computations due to CSI decimation.

Index Terms—OFDM radar, MUSIC, Sensing, Localization

I. INTRODUCTION

Joint Communication and Sensing (JCAS) is expected to be one of the main features of Beyond 5G (B5G) and 6G cellular networks and is currently attracting a lot of attention in the research community [1]. Future networks should be able to extract information about the physical world from the channel, performing de-facto sensing operations. Road user protection [2] and positioning in indoor scenarios [3] are two of the various sensing applications. In communication systems, the channel state information (CSI) is estimated as part of the necessary steps for communication purposes. Orthogonal frequency-division multiplexing (OFDM) radar [4] can be used to exploit this information by means of the reflections from objects illuminated by the transmitted signal to estimate their ranges and angles of arrivals (AoAs). Among possible estimation techniques, we consider the Multiple Signal Classification (MUSIC) algorithm [5], as it offers super-resolution capabilities over the Periodogram algorithm and allows coping with generic antenna array shapes.

Estimating range and AoA jointly offers the major advantage that targets can be resolved in two dimensions and consequently improves the ability to detect closely spaced targets compared to one-dimensional (1D) estimation. In general, joint estimation of multiple parameters is a well-researched field and has been discussed e.g., in [6] and [7]. For the particular case of jointly estimating range and AoA with MUSIC and OFDM signals, SpotFi [8] has made use of the spatial smoothing technique [9]. Differently from [8], which deals with WiFi signals, we consider 5G New Radio (NR) numerologies [10], requiring a large number of subcarriers to achieve the necessary bandwidth for a high range resolution. The resulting increased CSI matrix dimension would lead to a complexity that is too high to be handled in real-time applications.

To that end, we enhance the spatial smoothing technique by properly decimating the estimated CSI, adopting the approach used in [11] for joint range-Doppler estimation in the context of automotive radars. Our proposal enables high resolution in 5G sensing applications while drastically reducing the computational complexity. We also formulate a necessary condition for detecting multiple targets with multi-dimensional spatial smoothing. Similar to the motivation of earlier works ([12], [13]), we want to avoid an inefficient grid search in the resulting two-dimensional (2D) MUSIC spectrum. We therefore present a peak search routine using Powell’s algorithm [14] that can detect multiple targets at once. Moreover, we can reduce the missed detection probability by coherently removing the contribution of previously detected targets utilizing subspace tracking methods [15]. All our algorithms can straightforwardly be extended to higher dimensions for estimating additional parameters and can directly be implemented on top of 5G NR architectures.

The paper is structured as follows: after defining the signal model in Section II, we present our algorithms in Section III. Section IV introduces simulation scenario and assumptions and discusses our method’s benefits based on the results. The paper is wrapped up with a conclusion (Section V).

II. SIGNAL MODEL

We consider a single TX antenna illuminating the environment by transmitting phase-modulated constant envelope complex symbols $s = [s_1, s_2, \ldots, s_N]$, modulated onto $N$ subcarriers of an OFDM signal with subcarrier spacing $\Delta f$. The carrier wavelength is denoted by $\lambda$. While we only investigate static targets in this paper, the extension to Doppler estimation could be accomplished by considering multiple OFDM symbols.

The RX uniform linear array (ULA) is co-located with the transmitter and comprises $K$ antennas with element spacing $d$. Let the received symbols at array element $k$ be written as a row
vector $y_k = [y_{k,1}, y_{k,2}, \ldots, y_{k,N}]$. Assuming knowledge of the TX symbols, the frequency domain CSI at each RX antenna is obtained by carrying out an element-wise division

$$c_{k,n} = \frac{y_{k,n}}{s_n}.$$ \hspace{1cm} (1)

Stacking the CSI of all antenna elements vertically yields the $K \times N$ CSI matrix $C = [c_1, c_2, \ldots, c_K]^T$. Assuming $Q$ impulsive scatterers as targets, each of them generates a reflection according to azimuth AoA $\theta_q$ and range $r_q = \frac{\tau_q}{c}$, where $\tau_q$ is the delay of the reflection caused by the $q$-th target and $c$ the speed of light. The CSI matrix can be expressed as the superposition of the channel contributions of all $Q$ targets

$$C = \sum_{q=1}^{Q} h_q b(\theta_q) a(r_q)^T + Z,$$ \hspace{1cm} (2)

where $h_q$ is the complex coefficient of the $q$-th target and $Z$ the $K \times N$ random complex additive white Gaussian noise (AWGN) matrix. Note that we neglect the influence of self-interference, as it is not the scope of this work and has previously been addressed in literature [16]. Assuming that $N\Delta f \ll f_c$ (carrier frequency) and $r_q \gg Kd$, the vectors $b(\theta_q)$ and $a(r_q)$ are given as

$$b(\theta_q) = \left[1, e^{j2\pi \frac{\theta_q}{\lambda}}, \ldots, e^{j2\pi(K-1) \frac{\theta_q}{\lambda}}\right]^T \hspace{1cm} (3)$$

$$a(r_q) = \left[1, e^{-j2\pi \Delta f \cdot 2k}, \ldots, e^{-j2\pi(N-1) \Delta f \cdot 2k}\right]^T \hspace{1cm} (4)$$

and represent the linear phase shifts in the antenna and subcarrier dimension induced by $\theta_q$ and $r_q$ of the $q$-th target.

III. PROPOSED ALGORITHM

A. Spatial Smoothing with CSI Decimation

Applying MUSIC in two dimensions couples range and AoA estimates to the same target and therefore improves the ability of discriminating different targets. However, previous work [9] states that for detecting multiple targets, a necessary condition is that the number of independent measurements for computing the sample covariance matrix $\tilde{R}_y$ must be greater or equal than $Q$. To achieve this with a single snapshot, [8] makes use of the spatial (in the algebraic sense) smoothing technique by generating $L$ sub-arrays $[C_{s,1}, C_{s,2}, \ldots, C_{s,L}]$ from $C$. The $M \times L$ smoothed CSI matrix $\hat{C}$ is then constructed as

$$\hat{C} = \left[\text{vec}(C_{s,1}), \text{vec}(C_{s,2}), \ldots, \text{vec}(C_{s,L})\right]$$

$$= \left[c_{s,1}, c_{s,2}, \ldots, c_{s,L}\right],$$ \hspace{1cm} (5)

with $M$ being the number of samples per sub-array and vec() the vectorization operator applied in row-major order. As an example, all elements in the orange and blue boxes of Fig. 1 form two independent sub-arrays. The sample covariance matrix can be computed as

$$\tilde{R}_y = \frac{1}{M} \hat{C} \hat{C}^H.$$ \hspace{1cm} (6)

After performing the Eigenvector decomposition (EVD) of $\tilde{R}_y$, the eigenvectors are partitioned into the $M \times Q$ signal subspace $U_S$ corresponding to the $Q$ strongest eigenvalues and the complementary $M \times (M - Q)$ noise subspace $U_N$. For estimating $Q$, the minimum description length method [17] is used. The 2D MUSIC spectrum is obtained by computing

$$P_{MU}(r, \theta) = \frac{1}{(b(\theta) \otimes a(r))\text{H} U_N U_N^T (b(\theta) \otimes a(r))},$$ \hspace{1cm} (7)

where $b(\theta)$ and $a(r)$ are the steering vectors for the trial range-azimuth pair $(r, \theta)$ and $\otimes$ is the Kronecker product.

Preserving the domain apertures of $C$ leads to the dimensions of the sub-arrays growing rapidly, rendering the approach computationally expensive. Obviously, one could simply consider small sub-arrays and limit the complexity in this way. However, in modern 5G systems, even with $\Delta f = 60$ kHz, the number of subcarriers $N$ could already exceed one thousand. To preserve the range resolution, it is prerequisite to have sub-arrays with a large frequency domain aperture $A_f$, as the range resolution $\Delta r$ of an OFDM radar is

$$\Delta r = \frac{c}{2A_f \Delta f}.$$ \hspace{1cm} (8)

Therefore, the range resolution is inversely proportional to the frequency aperture. With fixed $\Delta f$, one can only increase $A_f$, thus $M$, to improve ranging performance, quickly leading to computationally infeasible situations.

As introduced in [11] for range-Doppler estimation, we decimate the sub-arrays, allowing us to keep a large aperture in both frequency (i.e., resulting effective bandwidth) and spatial (i.e., aperture of the antenna array) dimension. Besides apertures $A_f$ and $A_s$, we parametrize the generation of sub-arrays by defining decimation and stride between them, which shall be denoted as $D$ and $S$, respectively, with subscripts $f$ and $s$ representing frequency and spatial dimension. The antenna and subcarrier indices of the $\ell$-th sub-array of the smoothed CSI matrix $C$ are denoted as vectors of length $M$

$$a_\ell = 1_{S_f} \otimes [\hat{a}_f, \hat{a}_f + D_f, \ldots, \hat{a}_f + (N_f - 1)D_f]^T \hspace{1cm} (9)$$

$$f_\ell = [\hat{f}_s, \hat{f}_s + D_s, \ldots, \hat{f}_s + (N_s - 1)D_s]^T \otimes 1_{S_s} \hspace{1cm} (10)$$

where $\hat{f}_s$ and $\hat{a}_f$ are the respective initial indices, $N_f = [A_f/D_f]$ and $N_s = [A_s/D_s]$ the numbers of subcarriers and antennas per sub-array, and $1_{S_f}$ and $1_{S_s}$ all-ones vectors of length $N_f$ and $N_s$. In total, $L = S_f S_s$ sub-arrays can be obtained, where $S_f = [(N - A_f)/D_f + 1]$ and $S_s = [(K - A_s)/S_s + 1]$ denote the number of different subcarrier and antenna sets as a result of striding. Referring to Fig. 1, the initial full sub-arrays can be sampled by selecting only the colored elements. After generating $\hat{C}$ in this way, the 2D MUSIC spectrum is estimated using (5) - (7), with the steering vectors being dependent on the sub-array definition parameters

$$b_{\phi_a}(\theta) = \left[1, e^{j\phi_a \sin(\theta)}, \ldots, e^{j\phi_a \sin(\theta)}\right]^T \hspace{1cm} (12)$$

$$a_{\phi_f}(r) = \left[1, e^{j\phi_f \cdot 2\pi}, \ldots, e^{j\phi_f \cdot 2\pi}\right]^T, \hspace{1cm} (13)$$
have the same coordinates over \( V \). Therefore, the contribution of these sub-arrays to estimate \( R \) equal, differing only by constant phase shifts, according to \( v \) in the subcarrier domain reduces the unambiguous range reducing the computational complexity. However, decimating \( Q \) sub-arrays with ca. \( D \) sub-arrays, and thus resolution, can be achieved with \( A \) reduced from \( V - Q \). In the worst-case, the targets have equal coordinates in \( V \) dimensions, all sub-arrays generated by striding over these dimensions will be equal, differing only by constant phase shifts, according to (2). Therefore, the contribution of these sub-arrays to estimate \( R_y \) in (6) will be the same, leading to \( \text{rank}(R_y) = Q' < Q \).

An extensive proof with an example is given in [18].

\[ \text{Subcarriers} \]
\[
\begin{bmatrix}
C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} & C_{1,5} & C_{1,6} & C_{1,7} & C_{1,8} \\
C_{2,1} & C_{2,2} & C_{2,3} & C_{2,4} & C_{2,5} & C_{2,6} & C_{2,7} & C_{2,8} \\
C_{3,1} & C_{3,2} & C_{3,3} & C_{3,4} & C_{3,5} & C_{3,6} & C_{3,7} & C_{3,8} \\
C_{4,1} & C_{4,2} & C_{4,3} & C_{4,4} & C_{4,5} & C_{4,6} & C_{4,7} & C_{4,8} \\
C_{5,1} & C_{5,2} & C_{5,3} & C_{5,4} & C_{5,5} & C_{5,6} & C_{5,7} & C_{5,8} \\
C_{6,1} & C_{6,2} & C_{6,3} & C_{6,4} & C_{6,5} & C_{6,6} & C_{6,7} & C_{6,8}
\end{bmatrix}
\]

Fig. 1: Illustration of two different sub-arrays obtained by proposed sub-array generation with apertures \( A_f = 7 \) and \( A_a = 5 \), decimations \( D_f = 3 \) and \( D_a = 2 \), and strides \( S_f = S_a = 1 \).

where \( \phi_a = 2\pi D_a q^2 \) and \( \phi_f = -2\pi D_f \Delta f \) to ease notation. Due to decimation, the number of elements per sub-array \( M \) is reduced from \( A_f A_a \) to \( N_f N_a \). Therefore, an equal frequency domain aperture, and thus resolution, can be achieved with sub-arrays with ca. \( D_f D_a \) times less elements, drastically reducing the computational complexity. However, decimating in the subcarrier domain reduces the unambiguous range \( r_{max} = \frac{\pi D_f \Delta f}{2} \) by \( D_f \). We account for this by choosing the ranges in (13) within \( r_{max} \) to avoid aliasing.

Given that we want to discriminate \( Q \) targets in a multi-dimensional space, at least \( Q \) independent sub-arrays are necessary to compute \( \hat{R}_y \). If the targets are not resolvable in one domain, e.g., they have the same range, we notice that striding only in the corresponding dimension, in that case subcarriers, does not generate independent measurements. We can then formalize a necessary condition for the separation of \( Q \) targets in a multi-dimensional space.

**Theorem 1.** Consider signals generated by periodically sampling across \( V \) different dimensions of interest, e.g., subcarriers and antennas in Eq. (2). To separate \( Q \) targets, that can be solved in at least one dimension of interest, with multi-dimensional spatial smoothing, at least \( Q \) independent sub-arrays must be used w.r.t. each dimension of interest.

**Proof.** We need to separate \( Q \) targets in a \( V \)-dimensional MUSIC spectrum. This means that \( \hat{R}_y \) must be of rank \( Q \). In the worst-case, the \( Q \) targets have equal coordinates in \( V - 1 \) dimensions, but can be discriminated in dimension \( v \). Assume we have \( Q' < Q \) independent sub-arrays, by striding over dimension \( v \), plus an arbitrary number of sub-arrays by striding over the other \( V - 1 \) dimensions. Since the targets have the same coordinates over \( V - 1 \) dimensions, all sub-arrays generated by striding over these dimensions will be equal, differing only by constant phase shifts, according to (2). Therefore, the contribution of these sub-arrays to estimate \( \hat{R}_y \) in (6) will be the same, leading to \( \text{rank}(\hat{R}_y) = Q' < Q \). \( \Box \)

\[ c_q = \sum_{q=1}^{Q} c_q \]

\[ \hat{c}_q = c_q - U_N U_N^H c_q. \]

An extensive proof with an example is given in [18].

\[ \text{Peak Search Routine} \]

In order to find targets in the 2D space, the maxima of the MUSIC spectrum defined in (7) need to be found. Avoiding an exhaustive grid search and allowing to detect multiple targets at once are the motives behind a proper peak search routine. To find peaks in the spectrum, Powell’s algorithm [14] implemented in Python’s SciPy library is utilized. For the algorithm to converge to the 2D MUSIC spectrum maxima, suitable starting points for the search are critical. For this, we first evaluate (7) in a coarse grid, where the sampling period in each dimension is equal to one half of the corresponding domain resolution. The locations of the \( N_{\text{start}} \) highest MUSIC spectrum values are selected as starting points. Each of them is used by Powell’s algorithm to find a peak that is compared against a constant false alarm rate (CFAR) noise threshold \( \gamma \) defined by a predefined probability of false alarm \( p_{FA} \) [19]. If the peak value is below \( \gamma \), it is discarded.

\[ c_q = \sum_{q=1}^{Q} c_q \]

\[ \hat{c}_q = c_q - U_N U_N^H c_q. \]

After normalizing the L2-norm \( \hat{c}_q, \text{norm} = \frac{\hat{c}_q}{||\hat{c}_q||} \), \( \hat{c}_q, \text{norm} \) is added to \( U_N \) to obtain the updated noise subspace \( \hat{U}_N = [u_1, u_2, \ldots, \hat{c}_q, \text{norm}] \) to be used in (7) to re-estimate the spectrum. Note that this subspace tracking approach does not require re-calculate the EVD of \( \hat{R}_y \) [15]. Fig. 2a shows an example, where the detected target of the initial spectrum on the left (red cross) is canceled out coherently and thus its contribution does not show up in the updated spectrum on the right. However, Fig. 2b shows that if two targets are not resolvable in any dimension, removing the contribution of one target leads to a slight displacement of the remaining target w.r.t. to its true location. While this peak can still be detected, the displacement degrades the estimation accuracy. Using multiple starting points helps alleviating this issue, as it enables detecting close peaks. Nonetheless, cancelation enhances the detection capabilities, as it facilitates detecting weak peaks after removing the contribution of stronger ones.

\[ \text{D. Investigated Peak Selection Routines} \]

Combining the optionalities to either detect one or multiple targets at once and to iteratively remove their contributions, we define three peak selection routines to be investigated.

- **Single.** Only the strongest point (\( N_{\text{start}} = 1 \)) of the coarse grid is used as a starting point for the peak search. If this leads to a detection, the contribution of the target is canceled out before re-computing the spectrum for the next iteration.

\[ \text{Proof.} \]

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Fig. 2: Comparison of 2D MUSIC spectra before (left) and after (right) cancelation of a target. In (a), targets are resolvable, whereas in (b) the residual spectrum is displaced from the true target due to the targets being non-resolvable.

- **Multiple.** To detect multiple targets at each iteration, the $N_{\text{start}}$ strongest points of the coarse grid are used as starting points. All targets are removed before re-estimating the spectrum.
- **Off.** This mode also considers $N_{\text{start}} > 1$, but no coherent target cancelation is performed. Therefore, computation time is saved at the expense of detection capability. Routines single and multiple iterate until no targets are left, while off only performs a single peak detection iteration.

### IV. RESULTS AND DISCUSSION

#### A. 1D vs. 2D Estimation

We consider two targets in our simulations as this suffices to show the improved resolution capability using joint estimation. However, the algorithm can be applied to an arbitrary number of targets in practice red and future work should include more complex channel models or channels generated by ray tracing tools, e.g., [20]. The targets are initially placed at $J = 10000$ random positions resulting in the same range ($< r_{\text{max}} = 25$ m, in line with a typical factory environment) between targets and RX ULA. From there, one target is incrementally moved away from the RX such that the target range difference is increased by 0.1 m. For each range difference, the same random AoAs uniformly drawn between -60° and 60° are kept. The signal-to-noise ratio (SNR) at RX antenna $k$ is defined as $\text{SNR} = \frac{\mathbf{E}(y_k^2)}{\sigma_N^2}$, where $\sigma_N^2$ is the noise variance. We consider free-space path loss to model the attenuation of the impulsive scatterers. Note that the fading is modeled through the interference among them. Table I lists the 5G compliant OFDM signal specification, the chosen parameters for our proposed algorithm, and the resulting radar characteristics. We consider a 4-antenna ULA with $\lambda/2$ element spacing and only decimate in the frequency domain (i.e., $D_a = 1$) to avoid spatial aliasing. According to Theorem 1, we choose $A_a = 3$ to allow striding in the spatial domain such that two targets with the same range can still be separated if they are resolvable via their angles.

| Number of subcarriers $N$ | 1500 |
|---------------------------|------|
| Carrier frequency $f_c$   | 3.5 GHz |
| Subcarrier spacing $\Delta f$ | 60 kHz |
| Number of RX antennas $K$ | 4 |
| Antenna spacing $d$       | $\lambda/2 \approx 0.05$ m |
| Frequency aperture $A_f$  | 1401 |
| Frequency decimation $D_f$ | 100 |
| Antenna aperture $A_a$    | 3 |
| Antenna decimation $D_a$  | 1 |
| Frequency stride $S_f$    | 1 |
| Antenna stride $S_a$      | 1 |
| Starting points for peak search $N_{\text{start}}$ | 10 |
| Range resolution $\Delta r$ | 1.78 m |
| Unambiguous range $r_{\text{max}}$ | 25 m |

We define the root-mean-square error (RMSE) for parameter $\Theta$ (either range $r$ or azimuth AoA $\theta$) as

$$\text{RMSE} = \sqrt{\frac{1}{2J} \sum_{j=1}^{J} \sum_{q=1}^{2} |\hat{\Theta}_q - \Theta_q|^2},$$

where $\hat{\Theta}_q$ and $\Theta_q$ are estimate and true value of the respective parameter for target $q$. For computing the RMSE, we put the main focus on the target closer to the ULA (first target), i.e., in case of a single detection we compute the error to the first target. If both targets are detected, the estimate $\hat{r}_q$ that is closer to the ULA is assigned to the first target and the remaining estimate to the second one. Note that we assign detected targets to true targets only based on the range estimate. To compute errors in case of missed detections, we either use the maximum of the pre-computed coarse grid as the estimate for the first target (in case of zero detections) or remove the contribution of the first detected target and use the maximum of the resulting coarse grid as the estimate for the second one. We discard the 1% highest and lowest errors for the RMSE calculation, as outliers can be removed with tracking techniques [2].

First, the general advantages of joint estimation are shown by outlining the performance differences of 1D range-only estimation with MUSIC to our algorithm. For 1D estimation we use the same parameters (Table I), except choosing $A_a = 1$. Fig. 3a shows the probability of missed detection for 1D range-only and 2D estimation at SNRs of 5 dB and 15 dB. To limit the number of curves we only plot peak selection routine multiple for 1D estimation. It can be seen that for both SNRs, the 2D estimation methods already achieve low probabilities of missed detection for targets with the same range, whereas the 1D range-only estimation can only detect a single target.
in such cases. Nonetheless, 1D estimation is more robust for higher target range differences, especially at an SNR of 5 dB. Observing the performance at 15 dB clarifies the benefit of recomputing the spectrum after removing the contributions of detected targets, as 2D off (no target cancelation) shows the worst performance. Once the targets are fully resolvable in range, the 2D estimation routines attain missed detection probabilities of roughly 0.006, while 1D multiple performs slightly better and achieves circa 0.004. Overall, 2D multiple exhibits the best performance of the investigated 2D routines.

The range RMSE curves (Fig. 3b) show similar progressions. All 2D peak selection routines display a comparable RMSE performance. At 15 dB, both 1D and 2D techniques converge to error floors of roughly 0.02 m. The problem of target detection is complex to analyze analytically, thus we plot the achievable range resolution Δr from Eq. (8) with our setup as a reference. In general, our algorithms allow to discriminate the targets earlier than the theoretical resolution.

B. Benefits of CSI Decimation

To demonstrate the benefits of decimating in the subcarrier domain, we compare the parametrization in Table I against setups with i) \( A_f = 15, D_f = 1 \), ii) \( A_f = 141, D_f = 10 \), and iii) \( A_f = 701, D_f = 50 \). The number of samples per sub-array is identical (\( M = 45 \)), but the achievable range resolution is reduced by a factor equal to the reduction in \( A_f \). To keep the computational effort equal, \( L = 200 \) sub-arrays were used for all setups. For this experiment we place the two targets at random positions within \( r_{\text{max}} \) without a fixed range difference. Figs. 4 and 5 show that a higher range resolution improves both the RMSE and the missed detection probability performance significantly. Nonetheless, one can observe that a higher SNR is necessary to achieve similar missed detection probability and RMSE capabilities as in Fig. 3. This can be explained by trials where the targets are either placed so close that they can not be resolved in either domain, or so far apart that the second target can not be detected due to the path-loss attenuation making them fall below the detection threshold. We therefore as a reference included the curves for the first target only (1st), which for \( D_f = 100 \) (corresponding to 2D multiple in Fig. 3) converge to about 0.01 m and 3°, respectively.

C. Computational Complexity Comparison

Finally, we demonstrate the savings in computational complexity by comparing the parametrization in Table I (\( D_f = 100 \)) with the smoothing technique without decimation (\( D_f = 1 \) ) [8]. The setups result in \( M = 45 \) and \( M = 4203 \) elements per sub-array, respectively. The number of floating-point operations (FLOPs), i.e., complex multiplications and summations, for a single evaluation of (7) is approximated as \( 2M^2(M - Q) \), thus \( O(M^3) \), given \( Q \) is small. This leads to \( \approx 148 \text{ GFLOPs for } D_f = 1 \) and \( \approx 174 \text{ kFLOPs for } D_f = 100 \). Note that the EVD’s complexity can also be assumed to be \( O(M^3) \), but is only computed once, while (7) is evaluated for all points in the coarse grid, making it the bottleneck. The numbers show that we can reduce the FLOPs by a factor of \( \approx 8.5 \cdot 10^5 \) in a 5G system operating at 3.5 GHz with 90 MHz bandwidth. The gain comes at the price of a reduced unambiguous range, but in typical indoor scenarios the targets’ maximum ranges have a ceiling, making this disadvantage tolerable. Moreover, a slight processing gain enhancement due to the higher \( M \) can be achieved by not decimating.

V. CONCLUSION

We presented an algorithm for joint estimation of range and AoA using MUSIC and OFDM signals. CSI decimation significantly reduces the required number of computations for the considered 5G scenario and makes joint multi-dimensional estimation feasible in practical systems. We demonstrated that our joint approach improves the probability of detection for
Fig. 4: Range and azimuth RMSE curves for decimations $D_f = 1$, $D_f = 10$, $D_f = 50$, and $D_f = 100$ and routine multiple. All setups consider sub-arrays with the same number of samples. Hence, higher $D_f$ leads to a higher frequency aperture $A_f$.

(closely spaced targets compared to 1D range-only estimation, while achieving similar range errors. Moreover, we have proposed efficient single and multi-target detection algorithms.

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REFERENCES

[1] S. Mandelli, M. Arnold, M. Henninger, F. Schaich, and T. Wild, “From sensor networks to network as a sensor,” ITG News, no. 4, 2021.
[2] S. Saur, M. Mizmizi, J. Otterbach, T. Schlitter, R. Fuchs, and S. Mandelli, “5GCAK demonstration: Vulnerable road user protection through positioning with synchronized antenna signal processing,” in WSA 24th International ITG Workshop on Smart Antennas, 2020.
[3] C. Geng, T. E. Abrudan, V.-M. Kolmonen, and H. Huang, “Experimental study on probabilistic ToA and AoA joint localization in real indoor environments,” in ICC 2021-IEEE International Conference on Communications, 2021.
[4] C. Sturm, T. Zwick, and W. Wiesbeck, “An OFDM system concept for joint radar and communications operations,” in IEEE VTC Spring 69th Vehicular Technology Conference, 2009.
[5] R. Schmidt, “Multiple emitter location and signal parameter estimation,” IEEE transactions on antennas and propagation, vol. 34, no. 3, 1986.
[6] M. C. Vanderveen, C. B. Papadias, and A. Paulraj, “Joint angle and delay estimation (JADE) for multipath signals arriving at an antenna array,” IEEE Communications Letters, vol. 1, no. 1, 1997.
[7] L. Taponencco, A. A. D’Amico, and U. Mengali, “Joint TOA and AOA estimation for UWB localization applications,” IEEE Transactions on Wireless Communications, vol. 10, no. 7, 2011.
[8] M. Potaru, K. Joshi, D. Bharadia, and S. Katti, “SpotFi: Decimeter level localization using WiFi,” in ACM SIGCOMM Conference on Special Interest Group on Data Communication, 2015.
[9] T.-J. Shan and T. Kailath, “Adaptive beamforming for coherent signals and interference,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 33, no. 3, 1985.
[10] 3GPP, “NR: Physical channels and modulation,” Technical Specification (TS) 38.211, 2020, version 16.2.0.
[11] J. B. Sanson, P. M. Tomé, D. Castanheira, A. Gameiro, and P. P. Monteiro, “High-resolution delay-Doppler estimation using received communication signals for OFDM radar-communication system,” IEEE Transactions on Vehicular Technology, vol. 69, no. 11, 2020.
[12] K. V. Rangarao and S. Venkatnarasimhan, “Gold-MUSIC: A variation on MUSIC to accurately determine peaks of the spectrum,” IEEE Transactions on Antennas and Propagation, vol. 61, no. 4, 2012.
[13] J.-f. Chen and H. Ma, “An accurate real-time algorithm for spectrum peaks search in 2D MUSIC,” in IEEE 2011 International Conference on Multimedia Technology, 2011.
[14] M. J. Powell, “An efficient method for finding the minimum of a function of several variables without calculating derivatives,” The computer journal, vol. 7, no. 2, 1964.
[15] B. Yang, “Projection approximation subspace tracking,” IEEE Transactions on Signal processing, vol. 43, no. 1, 1995.
[16] C. D. Nwankwo, L. Zhang, A. Quddus, M. A. Imran, and R. Tafazoli, “A survey of self-interference management techniques for single frequency full duplex systems,” IEEE Access, vol. 6, 2017.
[17] J. Rissanen, “Modeling by shortest data description,” Automatica, vol. 14, no. 5, 1978.
[18] M. Henninger, S. Mandelli, M. Arnold, and S. ten Brink, “A computationally efficient 2D MUSIC approach for 5G and 6G sensing networks,” arXiv preprint arXiv:2104.15132, 2021.
[19] K. M. Braun, “OFDM radar algorithms in mobile communication networks,” Ph.D. dissertation, KIT-Bibliothek, 2014.
[20] M. Arnold, M. Bauhofer, S. Mandelli, M. Henninger, F. Schaich, T. Wild, and S. ten Brink, “MaxRay: A raytracing-based integrated sensing and communication framework,” arXiv preprint arXiv:2112.01751, 2021.