Calculation of the off-axis in-phantom scatter functions

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Abstract. Semi-empirical algorithms used for dose calculation in high energy photon beams are usually based in central axis data. The applicability of them to points off axis has been assessed only for old calculation techniques, such those using tissue air ratio functions. Recently, a new formulation based on phantom scatter quantities i.e. the \( \frac{D}{Y} \) formalism, has been introduced for use in central axis and homogeneous media. The extension of this calculation methodology for points off axis has to deal mainly with spectral changes that must to be accounted for. In this work, a new multiplicative function \( P(d, r) \) is introduced to account for variations in the phantom scatter quantities, which depends on the phantom depth (d) and on the off-axis position (r). This function was calculated from measurements of dose profiles (off-axis profiles in water) and in-air profiles corresponding to photon beams of 6 and 15 MV. Additionally, the function \( \frac{D}{Y} \) was determined experimentally and compared with the theoretical prediction, with an agreement better than 2% in any case. This difference could be attributed to experimental uncertainties and is within the clinical tolerances for dose calculation.

1. Introduction

Radiotherapy is one of the most effective treatments for cancer. It uses the biological effects of the ionizing radiation to kill or control malignant cells. However, radiotherapy is a complex multi-step process and a number of vital stages are required, for instance, tumours staging, definition of treatment strategies, imaging acquisition, volumes delineation, simulation and patient appointments, planning and dose calculation, set-up and delivery treatment, and finally, following up the patient.

Absorbed dose calculation in clinical situation is not trivial because the process of the energy deposition is itself very complex. In addition, the irradiation techniques and the new technologies of equipments add complexities to the dose distribution calculation. There are different methods to predict the absorbed dose in irradiated tissues. They can be grouped in three general categories: semi-empirical methods, semi-analytic techniques (convolution/superposition) and explicit treatment of the radiation (Monte Carlo).

In order to determinate the dose at a given point in the medium the energy spectrum of the incident beam must be known \(^{[1-5]}\). However, its experimental determination in clinical beams is highly impractical. Monte Carlo determination is also an awkward procedure: it requires some in-phantom measurements to validate or adjust the results. In order to avoid this problem, most dosimetry
protocols, based on both air kerma standards and absorbed dose to water standards, have defined different quality specifiers for high energy x-rays which can easily be calculated in clinical situations and can be used to characterize different beams.

Different Protocols have recommended the Tissue-Phantom Ratio, TPR\(_{20,10}\) as a quality index of a high-energy photon beam\(^{[1-5]}\). TPR\(_{20,10}\) is defined as the ratio of water absorbed doses on the beam axis at the depths of 20 cm and 10 cm in a water phantom, obtained with a constant source-detector distance SDD (100 cm) and a 10 cm \(\times\) 10 cm field size at the position of the detector.

However, in high energy photon beam the TPR\(_{20,10}\) changes as a function of the position in the beam, due to the change in the energy spectrum within the beam. The raw beam exiting from a target window of an accelerator is strongly forward peaked due to the shape of the angular cross section of bremsstrahlung production\(^{[6,7]}\). The primary aim of the flattening filter is to modulate the energy fluence exiting from the electron target such that a uniform dose distribution is delivered at a depth in the patient. Due to this fluence modulation the energy spectrum is hardened showing an average energy lower along rays situated away from the central axis. Hence, the TPR\(_{20,10}\) decreases with increasing distance from the central axis (off-axis points)\(^{[7]}\). Consequently, the applications of algorithms originally conceived for dose calculation at central axis have to be modified for off-axis points, since the beam in both situations is different\(^{[7,8]}\). Therefore, the objective in this study is to extend a semi-empirical dose calculation algorithm valid at the central axis to points off-axis.

2. Formalism for dose calculation at off-axis points and in homogeneous medium

The approach developed in this work is an extension of the formalism of Venselaar \textit{et al}\(^{[9]}\) whose phase space description has been recently explained by Sanz \textit{et al}\(^{[10]}\). Both are defined at central axis as:

\[
D_{\text{irreg}}(d, DFP) = D_{\text{ref}} \cdot \Psi_{\text{rel,irreg}}(DFP) \cdot \left[\frac{D}{\Psi_{\text{rel,irreg}}(d)}\right]
\]  

(1)

where \(D_{\text{irreg}}(d, DFP)\) represent the absorbed dose at a depth \(d\) and at a distance SPD away from the x-rays source. \(D_{\text{ref}}\) is the referent dose measured in referent conditions (collimator setting 10 cm \(\times\) 10 cm, SSD = 100 cm and \(d = 10\) cm). \(\Psi_{\text{rel,irreg}}(DFP)\) is the in-air relative energy fluence measured with a mini-phantom at DFP and the \(\left[\frac{D}{\Psi_{\text{rel,irreg}}(d)}\right]\) is the in-phantom relative scatter function measured at a depth \(d\). The "irreg" subscript denotes an arbitrary irregular field configuration.

Adopting an arrangement similar to the equation (1) the absorbed dose at off-axis points can be defined as:

\[
D_{\text{irreg}}(d, DFP, r) = D_{\text{ref}} \cdot \Psi_{\text{rel,irreg}}(DFP, r) \cdot \left[\frac{D}{\Psi_{\text{rel,irreg}}(d, r)}\right]
\]  

(2)

where \(r\) is the distance from the central axis to the off-axis point. In this context \(r\) is evaluated at the isocentric plane, despite the calculation point can be in any other plane.

In principle the absorbed dose at an off-axis point can be separated in primary and secondary components. The primary component is composed by the "focal" radiation from the x-ray target, without any interaction in intervening structures, and the "extrafocal" radiation that have been scattered at least once in the head. The secondary component is due to the phantom scatter radiation.

\[
D_{\text{rel}}(d, DFP, r) = D_{\text{rel,prim}}(d, DFP, r) + D_{\text{rel,sec}}(d, DFP, r)
\]  

(3)

where the "prim" subscript denotes the primary component and the "sec" subscript denotes the secondary component.
Let define a hypothetical quantity \( c(r) \) which is measured in an irregular field configuration at an off-axis position \( r \) from the central axis. Another quantity, defined as \( c_0(r) \), is determined in the same off-axis point than \( c(r) \) but now, the point’s position is considered as part of the central axis beam. With this approach all relative distances to the edge field are preserved. So, the \( c_0(r) \) and \( c(r) \) are measured in the same point with respect to the irregular field but in different points with respect to the central axis. The geometrical relationships are shown in figure (1):

**Figure 1.** Schematic representations of the quantities \( c(r) \) and \( c_0(r) \). The left side represents the actual positions of the central axis and the off-axis point where the \( c(r) \) quantity is measured. In the right side the same off-axis point is shown as a part of the central axis beam.

A new quantity \( C(r) \) can be derived from the relation between \( C(r) = c(r)/c_0(r) \). This idea allows supposing that \( C(r) \) allows to determinate the quantity \( c(r) \) with data obtained in the central axis beam. In other words, it allows knowing the quantity at off-axis points from the quantity measured in the central axis beam.

From the equation (3) and applying the above idea we define:

\[
D_{rel}(r) = p(r) \cdot D_{rel,prim,0}(r) + s(r) \cdot D_{rel,sec,0}(r)
\]  

where only the variable \( r \) is considered and denotes the distance away from the central axis. The other variables such as \( d \), DFF, size and shape of the field and energy are equal in all cases and they are omitted for simplicity. Considering the equation (4) we define: \( p(r) \) = ratio between relative primary dose at an off-axis point to relative primary dose but considering this point as the central axis beam. \( p(r) = D_{rel,prim}(r) / D_{rel,prim,0}(r) \). The “0” subscript denotes the off-axis point as part of the central axis. \( s(r) \) = ratio between relative secondary dose at an off-axis point to relative secondary dose but considering this point as the central axis beam. \( s(r) = D_{rel,sec}(r) / D_{rel,sec,0}(r) \).

From equation (4) and applying some mathematical calculations we have:

\[
D_{rel}(r) = p(r) \cdot \left[ D_{rel,prim,0}(r) + D_{rel,sec,0}(r) \right] + D_{rel,sec,0}(r) \cdot \left[ s(r) - p(r) \right]
\]  

where the second term on the right-hand side can be neglected, because \( s(r) \) and \( p(r) \) are close to unity at the central axis beam. In fact, this second term is considered as the error term of the method.

Analyzing the in-air relative energy fluence we define:

\[
\Psi_{rel}(r) = p \cdot \Psi(r) \cdot \Psi_{rel,0}(r)
\]

where \( \Psi_{rel}(r) \) is the in-air relative energy fluence at an off-axis point. \( p \cdot \Psi(r) \) is the ratio between in-air relative energy fluence at an off-axis to the in-air relative energy fluence at an off-axis point but
considering this point as the central axis beam, \( p \Psi(r) = \Psi_{\text{rel}}(r)/\Psi_{\text{rel,0}}(r) \). Using the quantities defined in equation (5) and (6) we can write:

\[
\frac{D_{\text{rel}}(r)}{\Psi_{\text{rel}}(r)} = p(r) \cdot \left[ \frac{D_{\text{rel,0}}(r)}{\Psi_{\text{rel,0}}(r)} \right]
\]  

(7)

and, after rearrangements:

\[
\frac{D_{\text{rel}}(r)}{\Psi_{\text{rel}}(r)} = \left[ \frac{D}{\Psi} \right]_{\text{rel}}(r) = p(r) \cdot \left[ \frac{D}{\Psi} \right]_{\text{rel,0}}(r)
\]  

(8)

where \([D/\Psi]_{\text{rel}}(r)\) is the in-phantom scatter function at an off-axis point defined at distance \( r \). In turn, \([D/\Psi]_{\text{rel,0}}(r)\) is the in-phantom scatter function at an off-axis point but considering the point as part of the central axis beam. \( P(r) \) is the \( p(r) \) to \( p \Psi(r) \) ratio. This function converts the \([D/\Psi]_{\text{rel}}(r)\) into the actual \([D/\Psi]_{\text{rel,0}}(r)\) at off-axis points.

Introducing the \( P(r) \) quantity into the equation (2) we obtain:

\[
D_{\text{irreg}}(d,DFP,r) = D_{\text{ref}} \cdot \Psi_{\text{rel,irreg}}(DFP,r) \cdot P(r) \cdot \left[ \frac{D}{\Psi} \right]_{\text{rel,irreg,0}}(d,r)
\]  

(9)

With the new expression the absorbed dose at off-axis points can be determined using functions measured on the central axis. The energy fluence \( \Psi_{\text{rel,irreg}}(DFP,r) \) can be determined at off-axis points using any of the methods described in the literature (see for example Johnsson et al\(^{[11]} \)).

In order to calculated the \( P(d,r) \) the equation (9) can be used:

\[
P(d,r) = \frac{OAR_{\text{water}}(d,DFP,r,L_d)}{OAR_{\text{prim}}(DFP,r,L_c) \cdot OAR\left[ \frac{D}{\Psi} \right]_{\text{rel,0}}(d,r,L_d)}
\]  

(10)

where \( OAR \) means off-axis ratio and:

- \( OAR_{\text{water}}(d,DFP,r,L_d) \): Absorbed dose at an off-axis point (located at a distance \( r \) away from the central axis), relative to the absorbed dose in central axis, both at the same plane with distance \( DFP \) from the source and depth in phantom \( d \), when the beam generate a field size \( L_d \) in the plane of interest.

- \( OAR_{\text{prim}}(DFP,r,L_c) \): In-air energy fluence at an off-axis point (located at a distance \( r \) away from the central axis), relative to the in-air energy fluence in central axis, both at the same plane with distance \( DFP \) from the source, when the beam generate a field size \( L_c \) in the isocentric plane.

- \( OAR\left[ \frac{D}{\Psi} \right]_{\text{rel}}(d,r,L_d) \): In-phantom scatter function \([D/\Psi]_{\text{rel}}\) at an off-axis point (located at a distance \( r \) away from the central axis), relative to the in-phantom scatter function \([D/\Psi]_{\text{rel,0}}\) in central axis (\( r = 0 \)), both at the same plane with distance \( DFP \) from the source and depth in phantom \( d \), when the beam generate a field size \( L_d \) in the plane of interest. Both function are obtained using in-phantom scatter function data valid in central axis beam.

3. Experimental method

All the measurements were performed for 6 and 15 MV x-ray beams from a Saturne 41 linear accelerator (General Electric, USA), using a 0.125 cm\(^3\) MULTIDATA type 9732-2 ionization chamber connected to KEITHLEY 35617 EBS electrometer. In addition, for relative measurements a dual channel MULTIDATA electrometer connected to a computer was used. The in-phantom profile were
measured using an automatic MULTIDATA scanner and the in-air measurements were performed with a narrow cylindrical beam-coaxial aluminium mini-phantom of 2 cm diameter, with the chamber in upright position.

4. Results

The \( P(r) \) function was obtained using the equation (8). A comparison of calculated \( P(r) \) for different depths are shown in Figure (2) and (3) at 6 MV and 15 MV, respectively.

![Figure 2](image1)

Figure 2. The \( P(r) \) quantity at 6 MV was plotted as a function of different eccentricities from the central axis. The \( P(r) \) was evaluated for four depth, 1.6 cm (□), 10 cm (●), 20 cm (Δ) and 30 cm (★).

![Figure 3](image2)

Figure 3. The \( P(r) \) quantity at 15 MV was plotted as a function of different eccentricities from the central axis. The \( P(r) \) was evaluated for four depth, 3.1 cm (□), 10 cm (●), 20 cm (Δ) and 30 cm (★).

The curves of figures (2) and (3) have different trends according to the related depth. For depths larger than 10 cm the function \( P(r) \) decreases with the distance from the central axis and reciprocally for shallower depths. This behaviour is due to a combination of the “horn” of the primary fluence and
beam softening. At shallow depths dominates the horn of the primary fluence, whereas deeper in the phantom the effect of beam softening becomes dominant.

Direct measurements of $[D/\Phi]_{rel}$ where compared with the results of the theoretical model for different field sizes, depths and distances from the central axis. In particular, measurements where performed for the 5x5, 10x10 and 20x20 field sizes, at depths of dose maximum, 10, 20 and 30 cm, and distances from the central axis 0, 5, 10 and 15 cm. Comparison with our model was in agreement to within 2.1 % showing that the method could be in the clinical routine.

Another aspect to take into account is the variation of the $P(r)$ in function of the depths and the eccentricities. From shallower depths than 10 cm the maximum difference are less than 4.3% at 15 MV and 2.7% at 6 MV ($r=15$ cm). From shallow depths than 10 cm the maximum difference are less than 5.5% and 6.8% at 15 MV and 6 MV ($r=15$ cm and $d=30$ cm), respectively.

5. Conclusion
In this work, we have developed an algorithm to directly determine the absorbed dose at off-axis points extending a semi-empirical algorithm valid at central axis. The $P(r)$ multiplicative function was defined to predict the in-phantom scatter function at off-axis points using as input data the in-phantom scatter function measured on the central axis. Comparison of results of this model with direct measurement of the expected quantities shown good agreement and, consequently, the formalism presented could be used for clinical purposes.

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