Generating Searchable Public-Key Ciphertexts with Hidden Structures for Fast Keyword Search

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Abstract—Existing semantically secure public-key searchable encryption schemes take search time linear with the total number of the ciphertexts. This makes retrieval from large-scale databases prohibitive. To alleviate this problem, this paper proposes Searchable Public-Key Ciphertexts with Hidden Structures (SPCHS) for keyword search as fast as possible without sacrificing semantic security of the encrypted keywords. In SPCHS, all keyword-searchable ciphertexts are structured by hidden relations, and with the search trapdoor corresponding to a keyword, the minimum information of the relations is disclosed to a search algorithm as the guidance to find all matching ciphertexts efficiently. We construct a simple SPCHS scheme from scratch in which the ciphertexts have a hidden star-like structure. We prove our scheme to be semantically secure based on the decisional bilinear Diffie-Hellman assumption in the Random Oracle (RO) model. The search complexity of our scheme is dependent on the actual number of the ciphertexts containing the queried keyword, rather than the number of all ciphertexts. Finally, we present a generic SPCHS construction from anonymous identity-based encryption and collision-free full-identity malleable Identity-Based Key Encapsulation Mechanism (IBKEM) with anonymity. We illustrate two collision-free full-identity malleable IBKEM instances, which are semantically secure and anonymous, respectively, in the RO and standard models. The latter instance enables us to construct an SPCHS scheme with semantic security in the standard model.

Index Terms—Public-key searchable encryption, semantic security, identity-based key encapsulation mechanism, identity based encryption

I. INTRODUCTION

PUBLIC-KEY encryption with keyword search (PEKS), introduced by Boneh et al. in [1], has the advantage that anyone who knows the receiver’s public key can upload keyword-searchable ciphertexts to a server. The receiver can delegate the keyword search to the server. More specifically, each sender separately encrypts a file and its extracted keywords and sends the resulting ciphertexts to a server; when the receiver wants to retrieve the files containing a specific keyword, he delegates a keyword search trapdoor to the server; the server finds the encrypted files containing the queried keyword without knowing the original files or the keyword itself, and returns the corresponding encrypted files to the receiver; finally, the receiver decrypts these encrypted files. The authors of PEKS [1] also presented semantic security against chosen keyword attacks (SSCKA) in the sense that the server cannot distinguish the ciphertexts of the keywords of its choice before observing the corresponding keyword search trapdoors. It seems an appropriate security notion, especially if the keyword space has no high min-entropy. Existing semantically secure PEKS schemes take search time linear with the total number of all ciphertexts. This makes retrieval from large-scale databases prohibitive. Therefore, more efficient searchable public-key encryption is crucial for practically deploying PEKS schemes.

One of the prominent works to accelerate the search over encrypted keywords in the public-key setting is deterministic encryption introduced by Bellare et al. in [2]. An encryption scheme is deterministic if the encryption algorithm is deterministic. Bellare et al. [2] focus on enabling search over encrypted keywords to be as efficient as the search for unencrypted keywords, such that a ciphertext containing a given keyword can be retrieved in time complexity logarithmic in the total number of all ciphertexts. This is reasonable because the encrypted keywords can form a tree-like structure when stored according to their binary values. However, deterministic encryption has two inherent limitations. First, keyword privacy can be guaranteed only for keywords that are a priori hard-to-guess by the adversary (i.e., keywords with high min-entropy to the adversary); second, certain information of a message leaks inevitably via the ciphertext of the keywords since the encryption is deterministic. Hence, deterministic encryption is only applicable in special scenarios.

A. Our Motivation and Basic Ideas

We are interested in providing highly efficient search performance without sacrificing semantic security in PEKS.

1Since the encryption of the original files can be separately processed with an independent public-key encryption scheme as in [1], we only describe the encryption of the keywords (unless otherwise clearly stated in the paper).
A closer look shows that there is still space to improve search performance in PEKS without sacrificing semantic security if one can organize the ciphertexts with elegantly designed but hidden relations. Intuitively, if the keyword-searchable ciphertexts have a hidden star-like structure, as shown in Figure 1, then search over ciphertexts containing a specific keywords may be accelerated. Specifically, suppose all ciphertexts of the same keyword form a chain by the correlated hidden relations, and also a hidden relation exists from a public Head to the first ciphertext of each chain. With a keyword search trapdoor and the Head, the server seeks out the first matching ciphertext via the corresponding relation from the Head. Then another relation can be disclosed via the found ciphertext and guides the searcher to seek out the next matching ciphertext. By carrying on in this way, all matching ciphertexts can be found. Clearly, the search time depends on the actual number of the ciphertexts containing the queried keyword, rather than on the total number of all ciphertexts.

To guarantee appropriate security, the hidden star-like structure should preserve the semantic security of keywords, which indicates that partial relations are disclosed only when the corresponding keyword search trapdoor is known. Each sender should be able to generate the keyword-searchable ciphertexts with the hidden star-like structure by the receiver’s public-key; the server having a keyword search trapdoor should be able to disclose partial relations, which is related to all matching ciphertexts.

Observe that a keyword space is usually of no high min-entropy in many scenarios. Semantic security is crucial to guarantee keyword privacy in such applications. Thus the linear search complexity of existing schemes is the major obstacle to their adoption. Unfortunately, the linear complexity seems to be inevitable because the server has to scan and test each ciphertext, due to the fact that these ciphertexts (corresponding to the same keyword or not) are indistinguishable to the server.

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Semantic security is preserved 1) if no keyword search trapdoor is known, all ciphertexts are indistinguishable, and no information is leaked about the structure, and 2) given a keyword search trapdoor, only the corresponding relations can be disclosed, and the matching ciphertexts leak no information about the rest of ciphertexts, except the fact that the rest do not contain the queried keyword.

B. Our Work

We start by formally defining the concept of Searchable Public-key Ciphertexts with Hidden Structures (SPCHS) and its semantic security. In this new concept, keyword-searchable ciphertexts with their hidden structures can be generated in the public key setting; with a keyword search trapdoor, partial relations can be disclosed to guide the discovery of all matching ciphertexts. Semantic security is defined for both the keywords and the hidden structures. It is worth noting that this new concept and its semantic security are suitable for keyword-searchable ciphertexts with any kind of hidden structures. In contrast, the concept of traditional PEKS does not contain any hidden structure among the PEKS ciphertexts; correspondingly, its semantic security is only defined for the keywords.

Following the SPCHS definition, we construct a simple SPCHS from scratch in the random oracle (RO) model. The scheme generates keyword-searchable ciphertexts with a hidden star-like structure. The search performance mainly depends on the actual number of the ciphertexts containing the queried keyword. For security, the scheme is proven semantically secure based on the Decisional Bilinear Diffie-Hellman (DBDH) assumption [3] in the RO model.

We are also interested in providing a generic SPCHS construction to generate keyword-searchable ciphertexts with a hidden star-like structure. Our generic SPCHS is inspired by several interesting observations on Identity-Based Key Encapsulation Mechanism (IBKEM). In IBKEM, a sender encapsulates a key $K$ to an intended receiver $ID$. Of course, receiver $ID$ can decapsulate and obtain $K$, and the sender knows that receiver $ID$ will obtain $K$. However, a non-intended receiver $ID'$ may also try to decapsulate and obtain $K'$. We observe that, (1) it is usually the case that $K$ and $K'$ are independent of each other from the view of the receivers, and (2) in some IBKEM the sender may also know $K'$ obtained by receiver $ID'$. We refer to the former property as collision freeness and to the latter as full-identity malleability. An IBKEM scheme is said to be collision-free full-identity malleable if it possesses both properties.

We build a generic SPCHS construction with Identity-Based Encryption (IBE) and collision-free full-identity malleable IBKEM. The resulting SPCHS can generate keyword-searchable ciphertexts with a hidden star-like structure. Moreover, if both the underlying IBKEM and IBE have semantic security and anonymity (i.e. the privacy of receivers’ identities), the resulting SPCHS is semantically secure. As there are known IBE schemes [4], [5], [6], [7] in both the RO model and the standard model, an SPCHS construction is reduced to collision-free full-identity malleable IBKEM with anonymity. In 2013, Abdalla et al.
proposed several IBKEM schemes to construct Verifiable Random Function (VRF) [8]. We show that one of these IBKEM schemes is anonymous and collision-free full-identity malleable in the RO model. In [9], Freire et al. utilized the "approximation" of multilinear maps [10] to construct a standard-model version of Boneh-and-Franklin (BF) IBE scheme [11]. We transform this IBE scheme into a collision-free full-identity malleable IBKEM scheme with semantic security and anonymity in the standard model. Hence, this new IBKEM scheme allows us to build SPCHS schemes secure in the standard model with the same search performance as the previous SPCHS construction from scratch in the RO model.

C. Other Applications of Collision-Free Full-Identity Malleable IBKEM

We note that collision-free full-identity malleable IBKEM is of independent interest. In addition to being a building block for the generic SPCHS construction, it may also find other applications, as outlined in the sequel.

Batch identity-based key distribution. A direct application of collision-free full-identity malleable IBKEM is to achieve batch identity-based key distribution. In such an application, a sender would like to distribute different secret session keys to multiple receivers so that each receiver can only know the session key to himself/herself. With collision-free full-identity malleable IBKEM, a sender just needs to broadcast an IBKEM encapsulation in the identity-based cryptography setting, e.g., encapsulating a session key $K$ to a single user $ID$. According to the collision-freeness of IBKEM, each receiver $ID'$ can decapsulate and obtain a different key $K'$ with his/her secret key in the identity based crypto-system. Due to the full-identity malleability, the sender knows the decapsulated keys of all the receivers. In this way, the sender efficiently shares different session keys with different receivers, at the cost of only a single encapsulation and one pass of communication.

Anonymous identity-based broadcast encryption. A slightly more complicated application is anonymous identity-based broadcast encryption with efficient decryption. An analogous application was proposed respectively by Barth et al. [12] and Libert et al. [13] in the traditional public-key setting. With collision-free full-identity malleable IBKEM, a sender generates an identity-based broadcast ciphertext $(C_1, C_2, (K_1 || SE(K_2, F_1)), ..., (K_2 || SE(K_N, F_N)))$, where $C_1$ and $C_2$ are two IBKEM encapsulations, $K_1$ is the encapsulated key in $C_1$ for receiver $ID_1$, $K_2$ is the encapsulated key in $C_2$ for receiver $ID_1$, and $SE(K_2, F_1)$ is the symmetric-key encryption of file $F_1$ using the encapsulated key $K_2$. In this ciphertext, the encapsulated key $K_1$ is not used to encrypt anything. Indeed, it is an index to secretly inform receiver $ID_1$ on which part of this ciphertext belongs to him. To decrypt the encrypted file $F_1$, receiver $ID_1$ decapsulates and obtains $K_1$ from $C_1$, finds out $K_1 || SE(K_2, F_1)$ by matching $K_1$, and finally extracts $F_1$ with the decapsulated key $K_2$ from $C_2$.

It can be seen that the application will work if the IBKEM is collision-free full-identity malleable. It preserves the anonymity of receivers if the IBKEM is anonymous. Note that trivial anonymous broadcast encryption suffers decryption cost linear with the number of the receivers. In contrast, our anonymous identity-based broadcast encryption enjoys constant decryption cost, plus logarithmic complexity to search the matching index in a set $(K_1^1, ..., K_N^1)$ organized by a certain partial order, e.g., a dictionary order according to their binary representations.

D. Related Work

Search on encrypted data has been extensively investigated in recent years. From a cryptographic perspective, the existing works fall into two categories, i.e., symmetric searchable encryption [14] and public-key searchable encryption.

Symmetric searchable encryption is occasionally referred to as symmetric-key encryption with keyword search (SEKS). This primitive was introduced by Song et al. in [15]. Their instantiated scheme takes search time linear with the size of the database. A number of efforts [16], [17], [18], [19], [20] follow this research line and refine Song et al.'s original work. The SEKS scheme due to Curtmola et al. [14] has been proven to be semantically secure against an adaptive adversary. It allows the search to be processed in logarithmic time, although the keyword search trapdoor has length linear with the size of the database. In addition to the above efforts devoted to either provable security or better search performance, attention has recently been paid to achieving versatile SEKS schemes as follows. The works in [14], [21] extend SEKS to a multi-sender scenario. The work in [22] realizes fuzzy keyword search in the SEKS setting. The work in [23] shows practical applications of SEKS and employs it to realize secure and searchable audit logs. Chase et al. [24] proposed to encrypt structured data and a secure method to search these data. To support the dynamic update of the encrypted data, Kamara et al. proposed the dynamic searchable symmetric encryption in [25] and further enhanced its security in [26] at the cost of large index. The very recent work [27] due to Cash et al. simultaneously achieves strong security and high efficiency.

Following the seminal work on PEKS, Abdalla et al. [28] fills some gaps w.r.t. consistency for PEKS and deals with the transformations among primitives related to PEKS. Some efforts have also been devoted to make PEKS versatile. The work of this kind includes conjunctive search [29], [30], [31], [32], [33], [34], range search [35], [36], [37], subset search [37], time-scope search [28], similarity search [39], authorized search [49], [50], equality test between heterogeneous ciphertexts [51], and fuzzy keyword search [52]. In addition, Arriaga et al. [53] proposed a PEKS scheme to keep the privacy of keyword search trapdoors.

In the above PEKS schemes, the search complexity takes time linear with the number of all ciphertexts. In [24],
an oblivious generation of keyword search trapdoor is to maintain the privacy of the keyword against a curious trapdoor generation. A chain-like structure is described to speed up the search on encrypted keywords. One may note that the chain in (40) cannot be fully hidden to the server and leaks the frequency of the keywords (see Supplemental Materials A for details). To realize an efficient keyword search, Bellare et al. [2] introduced deterministic public-key encryption (PKE) and formalized a security notion “as strong as possible” (stronger than onewayness but weaker than semantic security). A deterministic searchable encryption scheme allows efficient keyword search as if the keywords were not encrypted. Bellare et al. [2] also presented a deterministic PKE scheme (i.e., RSA-DOAEP) and a generic transformation from a randomized PKE to a deterministic PKE in the random oracle model. Subsequently, deterministic PKE schemes secure in the standard model were independently proposed by Bellare et al. [41] and Boldyreva et al. [42]. The former uses general complexity assumptions and the construction is generic, while the latter exploits concrete complexity assumptions and has better efficiency. Brakerski et al. [43] proposed the deterministic PKE schemes with better security, although these schemes are still not semantically secure. So far, deterministic PEKS schemes can guarantee semantic security only if the keyword space has a high min-entropy. Otherwise, an adversary can extract the encrypted keyword by a simple encrypt-and-test attack. Hence, deterministic PEKS schemes are applicable to applications where the keyword space is of a high min-entropy.

E. Organization of this Article

The remaining sections are as follows. Section II defines SPCHS and its semantic security. A simple SPCHS scheme is constructed in Section III. A general construction of SPCHS is given in Section IV. Two collision-free full-identity malleable IBKEM schemes, respectively in the RO and standard models, are introduced in Section V. Section VI concludes this paper.

II. MODELING SPCHS

We first explain intuitions behind SPCHS. We describe a hidden structure formed by ciphertexts as \((\mathcal{C}, \text{Pri}, \text{Pub})\), where \(\mathcal{C}\) denotes the set of all ciphertexts, \(\text{Pri}\) denotes the hidden relations among \(\mathcal{C}\), and \(\text{Pub}\) denotes the public parts. In case there is more than one hidden structure formed by ciphertexts, the description of multiple hidden structures formed by ciphertexts can be \((\mathcal{C}, (\text{Pri}_1, \text{Pub}_1), ..., (\text{Pri}_N, \text{Pub}_N))\), where \(N \in \mathbb{N}\). Moreover, given \((\mathcal{C}, \text{Pub}_1, ..., \text{Pub}_N)\) and \((\text{Pri}_1, ..., \text{Pri}_N)\) except \((\text{Pri}_i, \text{Pri}_j)\) (where \(i \neq j\)), one can neither learn anything about \((\text{Pri}_i, \text{Pri}_j)\) nor decide whether a ciphertext is associated with \(\text{Pub}_i\) or \(\text{Pub}_j\).

In SPCHS, the encryption algorithm has two functionalities. One is to encrypt a keyword, and the other is to generate a hidden relation, which can associate the generated ciphertext to the hidden structure. Let \((\text{Pri}, \text{Pub})\) be the hidden structure. The encryption algorithm must take \(\text{Pri}\) as input, otherwise the hidden relation cannot be generated since \(\text{Pub}\) does not contain anything about the hidden relations. At the end of the encryption procedure, the \(\text{Pri}\) should be updated since a hidden relation is newly generated (but the specific method to update \(\text{Pri}\) relies on the specific instance of SPCHS). In addition, SPCHS needs an algorithm to initialize \((\text{Pri}, \text{Pub})\) by taking the master public key as input, and this algorithm will be run before the first time to generate a ciphertext. With a keyword search trapdoor, the search algorithm of SPCHS can disclose partial relations to guide the discovery of the ciphertexts containing the queried keyword with the hidden structure.

Definition 1 (SPCHS). SPCHS consists of five algorithms:

- **SystemSetup**\((1^k, \mathcal{W})\): Take as input a security parameter \(1^k\) and a keyword space \(\mathcal{W}\), and probabilistically output a pair of master public-and-secret keys \((\text{PK}, \text{SK})\), where \(\text{PK}\) includes the keyword space \(\mathcal{W}\) and the ciphertext space \(\mathcal{C}\).

- **StructureInitialization**\((\text{PK})\): Take as input \(\text{PK}\), and probabilistically initialize a hidden structure by outputting its private and public parts \((\text{Pri}, \text{Pub})\).

- **StructuredEncryption**\((\text{PK}, \mathcal{W}, \text{Pri})\): Take as inputs \(\text{PK}\), a keyword \(W \in \mathcal{W}\) and a hidden structure’s private part \(\text{Pri}\), and probabilistically output a keyword-searchable ciphertext \(C\) of keyword \(W\) with the hidden structure, and update \(\text{Pri}\).

- **Trapdoor**\((\text{SK}, W)\): Take as inputs \(\text{SK}\) and a keyword \(W \in \mathcal{W}\), and output a keyword search trapdoor \(T_W\) of \(W\).

- **StructuredSearch**\((\text{PK}, \text{Pub}, C, T_W)\): Take as inputs \(\text{PK}\), a hidden structure’s public part \(\text{Pub}\), all keyword-searchable ciphertexts \(C\) and a keyword search trapdoor \(T_W\) of keyword \(W\), disclose partial relations to guide finding out the ciphertexts containing keyword \(W\) with the hidden structure. An SPCHS scheme must be consistent in the sense that given any keyword search trapdoor \(T_W\) and any hidden structure’s public part \(\text{Pub}\), algorithm StructuredSearch\((\text{PK}, \text{Pub}, C, T_W)\) finds out all ciphertexts of keyword \(W\) with the hidden structure \(\text{Pub}\).

In the application of SPCHS, a receiver runs algorithm **SystemSetup** to set up SPCHS. Each sender uploads the public part of his hidden structure and keyword-searchable ciphertexts to a server, respectively by algorithms **StructureInitialization** and **StructuredEncryption**. Algorithm **Trapdoor** allows the receiver to delegate a keyword search trapdoor to the server. Then the server runs algorithm **StructuredSearch** for all senders’ structures to find out the ciphertexts of the queried keyword.

The above SPCHS definition requires each sender to maintain the private part of his hidden structure for algorithm **StructuredEncryption**. A similar requirement appears in symmetric-key encryption with keyword search (SEKS) in which each sender is required to maintain a
secret key shared with the receiver. This implies interactions via authenticated confidential channels before a sender encrypts the keywords to the receiver in SEKS. In contrast, each sender in SPCHS just generates and maintains his/her private values locally, i.e., without requirement of extra secure interactions before encrypting keywords.

In the general case of SPCHS, each sender keeps his/her private values $\text{Pri}$. We could let each sender be stateless by storing his/her $\text{Pri}$ in encrypted form at a server and having each sender download and re-encrypt his/her $\text{Pri}$ by storing his/her secret key shared with the receiver. This implies interactions before encrypting keywords.

The semantic security of SPCHS is to resist adaptively chosen keyword and structure attacks (SS-CKSA). In this security notion, a probabilistic polynomial-time (PPT) adversary is allowed to know all structures’ public parts, query the trapdoors for adaptively chosen keywords, query the private parts of adaptively chosen structures, and query the ciphertexts of adaptively chosen keywords and structures (including the keywords and structures which the adversary would like to be challenged). The adversary will choose two challenge keyword-structure pairs. The SS-CKSA security means that for a ciphertext of one of two challenge keyword-structure pairs, the adversary cannot determine which challenge keyword or which challenge structure the challenge ciphertext corresponds to, provided that the adversary does not know the two challenge keywords’ search trapdoors and the two challenge structures’ private parts.

**Definition 2 (SS-CKSA Security).** Suppose there are at most $N \in \mathbb{N}$ hidden structures. An SPCHS scheme is SS-CKSA secure, if any PPT adversary $A$ has only a negligible advantage $\text{Adv}_{\text{SPCHS},A}$ to win in the following SS-CKSA game:

- **Setup Phase**: A challenger sets up the SPCHS scheme by running algorithm SystemSetup to generate a pair of master public-and-secret keys $(\text{PK}, \text{SK})$, and initializes $N$ hidden structures by running algorithm StructureInitialization $N$ times (let $\text{PSet}$ be the set of all public parts of these $N$ hidden structures); finally the challenger sends $\text{PK}$ and $\text{PSet}$ to $A$.
- **Query Phase 1**: A adapitively issues the following queries multiple times.
  - Trapdoor Query $Q_{\text{Trap}}(W)$: Taking as input a keyword $W \in \mathcal{W}$, the challenger outputs the keyword search trapdoor of keyword $W$;
  - Privacy Query $Q_{\text{Pri}}(\text{Pub})$: Taking as input a hidden structure’s public part $\text{Pub} \in \text{PSet}$, the challenger outputs the corresponding private part of this structure;
  - Encryption Query $Q_{\text{Enc}}(W, \text{Pub})$: Taking as inputs a keyword $W \in \mathcal{W}$ and a hidden structure’s public part $\text{Pub}$, the challenger outputs an SPCHS ciphertext of keyword $W$ with the hidden structure $\text{Pub}$.
- **Challenge Phase**: A sends two challenge keyword-and-structure pairs $(W^*_0, \text{Pub}^*_0) \in \mathcal{W} \times \text{PSet}$ and $(W^*_1, \text{Pub}^*_1) \in \mathcal{W} \times \text{PSet}$ to the challenger; The challenger randomly chooses $d \in \{0, 1\}$, and sends a challenge ciphertext $C^*_d$ of keyword $W^*_d$ with the hidden structure $\text{Pub}^*_d$ to $A$.

- **Query Phase 2**: This phase is the same as **Query Phase 1**. Note that in **Query Phase 1** and **Query Phase 2**, $A$ cannot query the corresponding private parts of $\text{Pub}^*_0$ and $\text{Pub}^*_1$ and the keyword search trapdoors both of $W^*_0$ and $W^*_1$.
- **Guess Phase**: $A$ sends a guess $d'$ to the challenger. We say that $A$ wins if $d = d'$. And let $\text{Adv}_{\text{SS-CKSA},A}^{\text{SS-CKSA}} = \text{Pr}[d = d'] - \frac{1}{2}$ be the advantage of $A$ to win in the above game.

A weaker security definition of SPCHS is the selective-keyword security. We refer to this weaker security notion as SS-sK-CKSA security, and the corresponding attack game as SS-sK-CKSA game. In this attack game, the adversary $A$ chooses two challenge keywords before the SPCHS scheme is set up, but the adversary still adaptively chooses two challenge hidden structures at **Challenge Phase**. Let $\text{Adv}_{\text{SS-sK-CKSA},A}^{\text{SS-sK-CKSA}}$ denote the advantage of adversary $A$ to win in this game.

### III. A SIMPLE SPCHS SCHEME FROM SCRATCH

Let $\gamma \xleftarrow{} \mathbb{R}$ denote an element $\gamma$ randomly sampled from $\mathbb{R}$. Let $G$ and $G_1$ denote two multiplicative groups of prime order $q$. Let $g$ be a generator of $G$. A bilinear map $\hat{e} : G \times G \rightarrow G_1$ is an efficiently computable and non-degenerate function, with the bilinearity property $\hat{e}(g^a, g^b) = \hat{e}(g, g)^{ab}$, where $(a, b) \xleftarrow{} \mathbb{Z}_q^*$ and $\hat{e}(g, g)$ is a generator of $G_1$. Let $\text{BGen}(1^k)$ be an efficient bilinear map generator that takes as input a security parameter $1^k$ and probabilistically outputs $(q, G, G_1, g, \hat{e})$. Let keyword space $\mathcal{W} = \{0, 1\}^*$. A simple SPCHS scheme secure in the random oracle model is constructed as follows.

- **SystemSetup**($1^k, \mathcal{W}$): Take as input $1^k$ and the keyword space $\mathcal{W}$, compute $(q, G, G_1, g, \hat{e}) = \text{BGen}(1^k)$, pick $s \xleftarrow{} \mathbb{Z}_q^*$, set $P = g^s$, choose a cryptographic hash function $H : \mathcal{W} \rightarrow G_1$, set the ciphertext space $\mathcal{C} \subseteq G_1 \times G \times G_1$, and finally output the master public key $\text{PK} = (q, G, G_1, g, \hat{e}, P, H, \mathcal{W}, \mathcal{C})$, and the master secret key $\text{SK} = s$.
- **StructureInitialization**($\text{PK}$): Take as input $\text{PK}$, pick $u \xleftarrow{} \mathbb{Z}_q^*$, and initialize a hidden structure by outputting a pair of private-and-public parts $(\text{Pri} = (u), \text{Pub} = g^u)$. Note that Pri here is a variable list formed as $(u, \{(W, Pt[u, W])|W \in \mathcal{W}\})$, which is initialized as $(u)$.
- **StructuredEncryption**($\text{PK}, W, \text{Pri}$): Take as inputs $\text{PK}$, a keyword $W \in \mathcal{W}$, a hidden structure’s private part $\text{Pri}$, pick $r \xleftarrow{} \mathbb{Z}_q^*$ and do the following steps:
  1. Search $(W, Pt[u, W])$ for $W$ in $\text{Pri}$;
  2. If it is not found, insert $(W, Pt[u, W]) \xleftarrow{} \mathbb{G}_1$ to $\text{Pri}$, and output the keyword-searchable ci-
phertext \( C = (\hat{e}(P, H(W))^u, g^r, \hat{e}(P, H(W))^r \cdot P^t[u, W]); \)

3) Otherwise, pick \( R \in \mathbb{G}_1 \), set \( C = (P^t[u, W], g^r, \hat{e}(P, H(W))^r \cdot R) \), update \( P^t[u, W] = R \), and output the keyword-searchable ciphertext \( C \);

- \( \text{Trapdoor}(SK, W) \): Take as inputs \( SK \) and a keyword \( W \in W \), and output a keyword search trapdoor \( T_W = H(W)^a \) of keyword \( W \).

- \( \text{StructuredSearch}(PK, Pub, C, T_W) \): Take as inputs \( PK \), a hidden structure’s public part \( Pub \), all keyword-searchable ciphertexts \( C \) (let \( C[i] \) denote one ciphertext of \( C \), and this ciphertext can be parsed as \( \langle C[i, 1], C[i, 2], C[i, 3] \rangle \in \mathbb{G}_1 \times \mathbb{G}_1 \) and a keyword trapdoor \( T_W \) of keyword \( W \), set \( C' = \phi \), and do the following steps:

1) Compute \( Pt' = \hat{e}(Pub, T_W); \)

2) Seek a ciphertext \( C[i] \) having \( C[i, 1] = Pt' \); if it exists, add \( C[i] \) into \( C' \);

3) If no matching ciphertext is found, output \( C' \);

4) Compute \( Pt' = \hat{e}(C[i, 2], T_W)^{-1} \cdot C[i, 3] \), and go to Step 2.

Figure 2 shows a hidden star-like structure, which is generated by the SPCHS instance. When running algorithm \( \text{StructuredSearch}(PK, Pub, C, T_W) \), it discloses the value \( \hat{e}(P, H(W_i))^u \) by computing \( \hat{e}(Pub, T_W_i) \), and matches \( \hat{e}(P, H(W_i))^u \) with all ciphertexts to find out the ciphertext \( \hat{e}(P, H(W_i))^u, g^r, \hat{e}(P, H(W_i))^r \cdot Pt[u, W_i] \).

Then the algorithm discloses \( Pt[u, W_i] \) by computing \( \hat{e}(g^r, T_W_i)^{-1} \cdot \hat{e}(P, H(W_i))^r \cdot Pt[u, W_i] \), and matches \( Pt[u, W_i] \) with all ciphertexts to find out the ciphertext \( Pt[u, W_i], g^r, \hat{e}(P, H(W_i))^r \cdot R \). By carrying on in this way, the algorithm will find out all ciphertexts of keyword \( W_i \) with the hidden star-like structure, and stop the search if no matching ciphertext is found.

**Consistency.** Roughly speaking, algorithm \( \text{StructuredSearch} \) repetitively discloses the value of \( Pt' \) and matches the value with all ciphertexts’ first parts to find out the matching ciphertexts. Since all disclosed values of \( Pt' \) are either collision-free (due to the hash function \( H \)) and random (according to algorithm \( \text{StructuredEncryption} \)), no more than one ciphertext matches in each matching process. The found ciphertexts should contain the queried keyword, since given a keyword search trapdoor, algorithm \( \text{StructuredSearch} \) only can disclose the values of \( Pt' \), which are corresponding to the queried keyword. Formally, we have Theorem 1 on consistency whose proof can be found in Supplemental Materials B.

**Theorem 1.** Suppose the hash function \( H \) is collision-free, except with a negligible probability in the security parameter \( k \). The above SPCHS instance is consistent, also except with a negligible probability in the security parameter \( k \).

**Semantic Security.** The SS-CKSA security of the above SPCHS scheme relies on the DBDH assumption in \( \text{BGen}(1^k) \). The definition of DBDH assumption \( [3] \) is as follows.

**Definition 3 (The DBDH Assumption).** The DBDH problem in \( \text{BGen}(1^k) = (g, \mathbb{G}, \mathbb{G}_1, g, \hat{e}) \) is defined as the advantage of any PPT algorithm \( B \) to distinguish the tuples \( (g^a, g^b, g^c, \hat{e}(g, g)^{abc}) \) and \( (g^a, g^b, g^c, \hat{e}(g, g)^{abc}) \), where \( (a, b, c, y) \in \mathbb{Z}_q^4 \). Let \( Adv_{\text{DBDH}}^B(1^k) = Pr[B(g^a, g^b, g^c, \hat{e}(g, g)^{abc}) = 1] - Pr[B(g^a, g^b, g^c, \hat{e}(g, g)^{abc}) = 1] \) be the advantage of algorithm \( B \) to solve the DBDH problem. We say that the DBDH assumption holds in \( \text{BGen}(1^k) \), if the advantage \( Adv_{\text{DBDH}}^B(1^k) \) is negligible in the parameter \( k \).

In the security proof, we prove that if there is an adversary who can break the SS-CKSA security of the above SPCHS instance in the RO model, then there is an algorithm which can solve the DBDH problem in \( \text{BGen}(1^k) \). Formally we have Theorem 2 whose proof can be found in Supplemental Materials C.

**Theorem 2.** Let the hash function \( H \) be modeled as the random oracle \( \mathcal{Q}_H(\cdot) \). Suppose there are at most \( N \in \mathbb{N} \) hidden structures, and a PPT adversary \( A \) wins in the SS-CKSA game of the above SPCHS instance with advantage \( Adv_{\text{SS-CKSA}}^A \) in which \( A \) makes at most \( q_i \) queries to oracle \( \mathcal{Q}_{\text{Trap}}(\cdot) \) and at most \( q_o \) queries to oracle \( \mathcal{Q}_{\text{Pri}}(\cdot) \). Then there is a PPT algorithm \( B \) that solves the DBDH problem in \( \text{BGen}(1^k) \) with advantage

\[
Adv_{\text{B}}^B(1^k) \approx \frac{27}{(q_i \cdot q_o)^3} \cdot Adv_{\text{SS-CKSA}}^A,
\]

where \( e \) is the base of natural logarithms.

**Forward and Backward Security.** Even in the case that a sender gets his local privacy \( \text{Pri} \) compromised, SPCHS still offers forward security. This means that the existing hidden structure of ciphertexts stays confidential, since the local privacy only contains the relationship of the new generated ciphertexts. To offer backward security with SPCHS, the sender can initialize a new structure by algorithm \( \text{StructureInitialization} \) for the new generated ciphertexts. Because the new structure is independent of the old one, the compromised local privacy will not leak the new generated structure.

**Search Complexity.** All keyword-searchable ciphertexts can be indexed by their first parts’ binary bits. Assume that there are in total \( n \) ciphertexts from \( n_i \) hidden structures, and the \( i \)-th hidden structure contains \( n_{w,i} \) ciphertexts of keyword \( W_i \in W \). With the \( i \)-th hidden structure, the search complexity is \( O(n_{w,i} \log n) \). For all hidden structures, the sum search complexity is \( O((n_i + n_w) \log n) \), where \( n_w = \sum_{i=1}^{n_i} n_{w,i} \). Since \( n = \sum_{W \in W} n_w \) and \( n_w = \sum_{i=1}^{n_i} n_{w,i} \), we have that \( n_r \ll n_w \ll n \). Thus the above SPCHS instance allows a much more efficient search than existing PEKS schemes, which have \( O(n) \) search complexity.

One may note that SPCHS loses its significant advantage in search performance compared with PEKS if \( n_i = n \) holds. However, this special case seldom happens. In practice, a sender will extract several keywords from each of
When \( Pr[u,W_i] \notin Pri (i \in [1,L]) \) the SPCHS ciphertexts are

\[
\begin{align*}
&\text{Implies the following value} \quad \text{Implicit the following value} \\
&\hat{e}(P,H(W_i)^y), g^r, \hat{e}(P,H(W_i)^y) \cdot Pr[u,W_i] \\
&\text{Have the same value} \\
&Pr[u,W_i] \cdot g^r, \hat{e}(P,H(W_i)^y) \cdot R \\
&\text{Have the same value} \\
&Pr[u,W_i] \cdot g^r, \hat{e}(P,H(W_i)^y) \cdot R
\end{align*}
\]

Note that, in each ciphertext, the value \( R \) and the value \( r \) are randomly chosen. For \( i \in [1,L] \), \( Pt[u,W_i] \) is initialized with a random value when generating the first ciphertext of keyword \( W_i \), and it will be updated into \( R \) after generating each subsequent ciphertext of keyword \( W_i \).

Figure 2: Hidden star-like structure generated by the above SPCHS instance

| Hardware         | Intel CPU E5300 @ 2.60GHz |
|------------------|-----------------------------|
| OS and compiler  | Win XP and Microsoft VC++ 6.0 |
| Program library  | MIRCACL version 5.4.1       |
| Parameters of bilinear map | \( y^x = x^3 + A \cdot x + B \cdot x \) |
| Elliptic curve   | \( t^m + t^a + t^b + t^c + 1 \) |
| Pentanomial basis| \( 2^m \) \( m = 379 \) |
| Group order: \( q \) | \( 2^m + 2^{m+1}/2 + 1 \) |
| a                | 315 |
| b                | 301 |
| c                | 287 |

The default unit is decimal.

Figure 3: Time cost of SPCHS

his files. So we usually have \( n_s \ll n \) even if each sender only has one file. In addition, most related works on SEKS and PEKS assume that each file has several keywords.

**Experiment.** We coded our SPCHS scheme, and tested the time cost of algorithm **StructuredSearch** to execute its cryptographic operations for different numbers of matching ciphertexts. We also coded the PEKS scheme \([1]\). Table 1 shows the system parameters including hardware, software, and the chosen elliptic curve. Assume there are in total \( 10^4 \) searchable ciphertexts. PEKS takes about 53.8 seconds search time per keyword, since it must test all ciphertexts for each search. Figure 3 shows the experimental results of SPCHS. It is clear that the time cost of SPCHS is linear with the number of matching ciphertexts, whereas for PEKS it is linear with the number of total ciphertexts. Hence, SPCHS is much more efficient than PEKS.

**IV. A GENERIC CONSTRUCTION OF SPCHS FROM IBKEM AND IBE**

In this section, we formalize collision-free full-identity malleable IBKEM and a generic SPCHS construction from IBKEM and IBE.

**A. Reviewing IBE**

Before the generic SPCHS construction, let us review the concept of IBE and its Anonymity and Semantic Security both under adaptive-ID and Chosen Plaintext Attacks (Anon-SS-ID-CPA).

**Definition 4 (IBE \([11]\)).** IBE consists of four algorithms:

- **Setup**\(_{\text{IBE}}(1^k, \mathcal{ID}_{\text{IBE}})\): Take as inputs a security parameter \( 1^k \) and an identity space \( \mathcal{ID}_{\text{IBE}} \), and probabilistically output the master public-and-secret-key pair \((\mathbf{PK}_{\text{IBE}}, \mathbf{SK}_{\text{IBE}})\), where \( \mathbf{PK}_{\text{IBE}} \) includes the message space \( \mathcal{M}_{\text{IBE}} \), the ciphertext space \( \mathcal{C}_{\text{IBE}} \) and the identity space \( \mathcal{ID}_{\text{IBE}} \).
- **Extract**\(_{\text{IBE}}(\mathbf{SK}_{\text{IBE}}, \mathcal{ID})\): Take as inputs \( \mathbf{SK}_{\text{IBE}} \) and an identity \( \mathcal{ID} \in \mathcal{ID}_{\text{IBE}} \), and output a decryption key \( \hat{S}_{1\mathcal{ID}} \) of \( \mathcal{ID} \).
- **Enc**\(_{\text{IBE}}(\mathbf{PK}_{\text{IBE}}, \mathcal{ID}, M)\): Take as inputs \( \mathbf{PK}_{\text{IBE}} \), an identity \( \mathcal{ID} \in \mathcal{ID}_{\text{IBE}} \) and a message \( M \), and probabilistically output a ciphertext \( \hat{C} \).
- **Dec**\(_{\text{IBE}}(\hat{S}_{1\mathcal{ID}'}, \hat{C})\): Take as inputs the decryption key \( \hat{S}_{1\mathcal{ID}'} \) of identity \( \mathcal{ID}' \) and a ciphertext \( \hat{C} \), and output a message or \( \perp \), if the ciphertext is invalid.

An IBE scheme must be consistent in the sense that for any \( \hat{C} = \text{Enc}_{\text{IBE}}(\mathbf{PK}_{\text{IBE}}, \mathcal{ID}, M) \) and \( \hat{S}_{1\mathcal{ID}'} = \text{Dec}_{\text{IBE}}(\hat{S}_{1\mathcal{ID}'}, \hat{C}) \) (Anon-SS-ID-CPA)
Extract_{IBKEM}(SK_{IBKEM}, ID'), \textbf{Dec}_{IBKEM}(S_{1D'}, \hat{C}) = M \text{ holds if } ID' = ID, \text{ except with a negligible probability in the security parameter } k.

In the Anon-SS-ID-CPA security notion of IBE, a PPT adversary is allowed to query the decryption keys for adaptively chosen identities, and adaptively choose two challenge identity-and-message pairs. The Anon-SS-ID-CPA security of IBE means that for a challenge ciphertext, the adversary cannot determine which challenge identity and which challenge message it corresponds to, provided that the adversary does not know the two challenge identities’ decryption keys. The Anon-SS-ID-CPA security of an IBE scheme is as follows.

**Definition 5** (Anon-SS-ID-CPA security of IBE [46]). An IBE scheme is Anon-SS-ID-CPA secure if any PPT adversary \( B \) has only a negligible advantage \( Adv_{\text{IBKEM}}^{\text{Anon-SS-ID-CPA}} \) to win in the following Anon-SS-ID-CPA game:

- **Setup Phase**: A challenger sets up the IBE scheme by running algorithm Setup_{IBE} to generate the master public-and-secret-keys pair \((PK_{ibe}, SK_{ibe})\), and sends \( PK_{ibe} \) to \( B \).
- **Query Phase 1**: Adversary \( B \) adaptively issues the following query multiple times.
  - **Challenge Key Query** \( \mathcal{Q}_{\text{IBE}}^{\text{DBKEM}}(ID) \): Taking as input an identity \( ID \in \mathcal{I}_{DBKEM} \), the challenger outputs the decryption key of identity \( ID \).
  - **Challenge Phase**: Adversary \( B \) sends two challenge identity-and-message pairs \((ID_0, M_0) \) and \((ID_1, M_1) \) to the challenger; the challenger picks \( d \leftarrow \{0, 1\} \), and sends the challenge IBE ciphertext \( \hat{C}_d = \text{Enc}_{\text{IBE}}(PK_{\text{IBE}}, ID_d, M_d) \) to \( B \).
- **Query Phase 2**: This phase is the same as Query Phase 1. Note that in Query Phase 1 and Query Phase 2, \( B \) cannot query the decryption key corresponding to the challenge identity \( ID_0 \) or \( ID_1 \).
- **Guess Phase**: Adversary \( B \) sends a guess \( \hat{d} \) to the challenger. We say that \( B \) wins if \( \hat{d} = d \). Let \( Adv_{\text{IBKEM}}^{\text{Anon-SS-ID-CPA}} = Pr[\hat{d} = d] - \frac{1}{2} \) be the advantage of \( B \) to win in the above game.

**B. The Collision-Free Full-Identity Malleable IBKEM**

Our generic construction also relies on a notion of collision-free full-identity malleable IBKEM. The following IBKEM definition is derived from [47]. A difference only appears in algorithm Encaps_{IBKEM}. In order to highlight that the generator of an IBKEM encapsulation knows the chosen random value used in algorithm Encaps_{IBKEM}, we take the random value as an input of the algorithm.

**Definition 6** (IBKEM). IBKEM consists of four algorithms:

- **Setup_{IBKEM}(1^k, \mathcal{I}_{DBKEM})**: Take as inputs a security parameter \(1^k\) and an identity space \(\mathcal{I}_{DBKEM}\), and probabilistically output the master public-and-secret-keys pair \((PK_{IBKEM}, SK_{IBKEM})\), where \(PK_{IBKEM}\) includes the identity space \(\mathcal{I}_{DBKEM}\), the encapsulated key space \(\mathcal{K}_{IBKEM}\) and the encapsulation space \(\mathcal{C}_{IBKEM}\).
- **Extract_{IBKEM}(SK_{IBKEM}, ID')**: Take as inputs \(SK_{IBKEM}\) and an identity \(ID \in \mathcal{I}_{DBKEM}\), and output a decryption key \(S_{ID}\) of \(ID\).
- **Encaps_{IBKEM}(PK_{IBKEM}, ID, r)**: Take as inputs \(PK_{IBKEM}\) and an identity \(ID \in \mathcal{I}_{DBKEM}\) and a random value \(r\), and deterministically output a key-and-encapsulation pair \((K, C)\) of \(ID\).
- **Decaps_{IBKEM}(S_{ID'}, C)**: Given as inputs the decryption key \(S_{ID'}\) of identity \(ID'\) and an encapsulation \(C\), and output an encapsulated key or \(\perp\), if the encapsulation is invalid.

An IBKEM scheme must be consistent in the sense that for any \((K, C) = \text{Encaps}_{IBKEM}(PK_{IBKEM}, ID, r)\), \(\text{Decaps}_{IBKEM}(S_{ID'}, C) = K\) holds if \(ID' = ID\), except with a negligible probability in the security parameter \(k\).

The collision-free full-identity malleable IBKEM implies the following characteristics: all identities’ decryption keys can decapsulate the same encapsulation; all decapsulated keys are collision-free; the generator of the encapsulation can also compute these decapsulated keys; the decapsulated keys of different encapsulations are also collision-free.

**Definition 7** (Collision-Free Full-Identity Malleable IBKEM). IBKEM is collision-free full-identity malleable, if there is an efficient function \(FIM\) that for any \((K, C) = \text{Encaps}_{IBKEM}(PK_{IBKEM}, ID, r)\), the function \(FIM\) satisfies the following features:

- **(Full-Identity Malleability)** For any identity \(ID' \in \mathcal{I}_{DBKEM}\), the equation \(FIM(ID', r) = \text{Decaps}_{IBKEM}(S_{ID'}, C)\) always holds, where \(S_{ID'} = \text{Extract}_{IBKEM}(SK_{IBKEM}, ID')\).
- **(Collision-Free)** For any identity \(ID' \in \mathcal{I}_{DBKEM}\) and any random value \(r'\), if \(ID \neq ID' \lor r \neq r'\), then \(FIM(ID', r') \neq FIM(ID', r')\) holds, except with a negligible probability in the security parameter \(k\).

A collision-free full-identity malleable IBKEM scheme may preserve semantic security and anonymity. We incorporate the semantic security and anonymity into Anon-SS-ID-CPA secure IBKEM. But this security is different from the traditional version [47] of the Anon-SS-ID-CPA security due to the full-identity malleability of IBKEM. The difference will be introduced after defining that security. In that security, a PPT adversary is allowed to query the decryption keys for adaptively chosen identities, and adaptively choose two challenge identities. The Anon-SS-ID-CPA security of IBKEM means that for a challenge key-and-encapsulation pair, the adversary cannot determine the correctness of this pair and the challenge identity of this pair, given that the adversary does not know the two challenging identities’ decryption keys. The Anon-SS-ID-CPA security of a collision-free full-identity malleable IBKEM scheme is as follows.

**Definition 8** (Anon-SS-ID-CPA security of IBKEM). An IBKEM scheme is Anon-SS-ID-CPA secure if any PPT adversary \( B \) has only a negligible advantage \(Adv_{\text{IBKEM}}^{\text{Anon-SS-ID-CPA}}\) to win in the following Anon-SS-ID-CPA game:
• **Setup Phase:** A challenger sets up the IBKEM scheme by running algorithm $\text{Setup}_{\text{ibkem}}$ to generate the master public-and-secret-keys pair $(PK_{\text{ibkem}}, SK_{\text{ibkem}})$, and sends $PK_{\text{ibkem}}$ to $B$.

• **Query Phase 1:** $B$ adaptively issues the following query multiple times.
  - **Decryption Key Query $Q_{\text{DK}}^{\text{IBKEM}}(ID):** Taking as input an identity $ID \in TD_{\text{ibkem}}$, the challenger outputs the decryption key of identity ID $K^*_0$.

• **Challenge Phase:** $B$ sends two challenge identities $ID^*_0$ and $ID^*_1$ to the challenger; the challenger picks $d \in \mathbb{G}_p \setminus \{0,1\}$, computes $(K^*_0, C^*_0) = \text{Encaps}_{\text{ibkem}}(PK_{\text{ibkem}}, ID^*_0, r_0)$ and $(K^*_1, C^*_1) = \text{Encaps}_{\text{ibkem}}(PK_{\text{ibkem}}, ID^*_1, r_1)$, and sends the challenge key-and-encapsulation pair $(K^*_d, C^*_0)$ to $B$, where $r_0$ and $r_1$ are randomly chosen.

• **Query Phase 2:** This phase is the same as Query Phase 1. Note that in Query Phase 1 and Query Phase 2, $B$ cannot query the decryption keys both of the challenge identities $ID^*_0$ and $ID^*_1$.

• **Guess Phase:** $B$ sends a guess $d'$ to the challenger. We say that $B$ wins if $d' = d$. Let $Adv_{\text{Anon}-\text{SS-ID-CPA}}^{\text{IBKEM}, B}$ be $Pr\{d' = d\} - \frac{1}{2}$ be the advantage of $B$ to win in the above game.

In the above definition, the anonymity of the encapsulated keys is defined by the indistinguishability of $K^*_0$ and $K^*_1$. But we do not define the anonymity of the IBKEM encapsulations (i.e., the challenge key-and-encapsulation pair consists of $C^*_0$ instead of $C^*_1$), since the full-identity malleability of IBKEM implies that any IBKEM encapsulation is valid for all identities. A weaker security definition of IBKEM is the selective-identity security, referred to as the Anon-SS-sid-CPA security. The corresponding attack game is called the Anon-SS-sid-CPA game in which the adversary must commit to the two challenge identities before the system is set up.

**C. The Proposed Generic SPCHS Construction**

Let keyword space $W \subset TD_{\text{ibkem}} = TD_{\text{ibc}}$. Our generic SPCHS construction from the collision-free full-identity malleable IBKEM and IBE is as follows.

• **SystemSetup($1^k, W$):** Take as inputs a security parameter $1^k$ and the keyword space $W$, run $(PK_{\text{ibkem}}, SK_{\text{ibkem}}) = \text{Setup}_{\text{ibkem}}(1^k, TD_{\text{ibkem}})$ and $(PK_{\text{ibc}}, SK_{\text{ibc}}) = \text{Setup}_{\text{ibc}}(1^k, TD_{\text{ibc}})$, and output a pair of master public-and-secret keys $(PK = (PK_{\text{ibkem}}, PK_{\text{ibc}}), SK = (SK_{\text{ibkem}}, SK_{\text{ibc}}))$. Let the SPCHS ciphertext space $C = \mathcal{K}_{\text{ibkem}} \times \mathcal{C}_{\text{ibc}}$, and $\mathcal{K}_{\text{ibkem}} = \mathcal{M}_{\text{ibkem}}$.

• **StructureInitialization(PK):** Take as input PK, arbitrarily pick a keyword $W \in W$ and a random value $u$, generate an IBKEM encapsulated key and its encapsulation $(K, C) = \text{Encaps}_{\text{ibkem}}(PK_{\text{ibkem}}, W, u)$, and initialize a hidden structure by outputting a pair of private-and-public parts $(\text{Pri} = (u), \text{Pub} = (C))$. Note that Pri here is a variable list formed as $(u, \{(W, PT[u, W])|W \in W\})$, which is initialized as $(u)$.

(In the above, an IBKEM encapsulation and its related random value are respectively taken as the public-and-private parts of a hidden structure. To generate these two parts, an arbitrary keyword have to be chosen to run algorithm $\text{Encaps}_{\text{ibkem}}$.)

• **StructuredEncryption(PK, W, Pri):** Take as inputs PK, a keyword $W \in W$, a hidden structure’s private part Pri, and do the following steps:

  1) Search $(W, PT[u, W])$ for $W$ in Pri;

  2) If it is not found, insert $(W, PT[u, W] \leftarrow M_{\text{db}})$ to Pri, and output the keyword-searchable ciphertext $C = (\text{FIM}(W, u), \text{Enc}_{\text{ibc}}(PK_{\text{ibc}}, W, PT[u, W]));$

  3) Otherwise, pick $R \leftarrow M_{\text{db}},$ set $C = (PT[u, W], \text{Enc}_{\text{ibc}}(PK_{\text{ibc}}, W, R))$, update $PT[u, W] = R$, and output the keyword-searchable ciphertext $C$;

• **Trapdoor(SK, W):** Take as inputs SK and a keyword $W \in W$, run $S_W = \text{Extract}_{\text{ibkem}}(SK_{\text{ibkem}}, W)$ and $\hat{S}_W = \text{Extract}_{\text{ibc}}(SK_{\text{ibc}}, W)$, and output a keyword search trapdoor $T_W = (\hat{S}_W, S_W)$ of keyword $W$.

• **StructuredSearch(PK, Pub, C, T_W):** Take as inputs PK, a hidden structure’s public part Pub, all keyword-searchable ciphertexts $C$ (let $\mathcal{C}[i]$ denote one ciphertext of $C$, and this ciphertext can be parsed as $(\mathcal{C}[i, 1], \mathcal{C}[i, 2]) \in C = \mathcal{K}_{\text{ibkem}} \times \mathcal{C}_{\text{ibc}}$) and a keyword trapdoor $T_W = (\hat{S}_W, S_W)$ of keyword $W$, set $C' = \phi$, and do the following steps:

  1) Compute $PT' = \text{Decaps}_{\text{ibkem}}(\hat{S}_W, \text{Pub})$;

  2) Seek a ciphertext $C'[i]$ having $\mathcal{C}[i, 1] = PT'$; if it exists, add $\mathcal{C}[i]$ into $C'$;

  3) If no matching ciphertext is found, output $C'$;

  4) Compute $PT' = \text{Dec}_{\text{ibc}}(\hat{S}_D', C[i, 2])$, go to step 2;

Figure 4 shows a hidden star-like structure generated by the generic SPCHS construction. When running algorithm $\text{StructuredSearch}(PK, Pub, C, T_W)$, the full-identity malleability of IBKEM allows the algorithm to disclose the value $\text{FIM}(W_i, u)$ by computing $\text{FIM}(W_i, u) = \text{Decaps}_{\text{ibkem}}(\hat{S}_W, \text{Pub})$ and find out the ciphertext $(\text{FIM}(W_i, u), \text{Enc}_{\text{ibc}}(PK_{\text{ibc}}, W_i, PT[u, W_i]))$. Then the consistency of IBE allows the algorithm to disclose $PT[u, W_i]$ by decrypting $\text{Enc}_{\text{ibc}}(PK_{\text{ibc}}, W_i, PT[u, W_i])$ and find out the ciphertext $(PT[u, W_i], \text{Enc}_{\text{ibc}}(PK_{\text{ibc}}, W_i, R))$. By carrying on in this way, the consistency of IBE allows the algorithm to find out the rest of ciphertexts of keyword $W_i$ with the hidden star-like structure, and stop the search if no more ciphertexts are found.

**Consistency.** When running the above algorithm $\text{StructuredSearch}(PK, Pub, C, T_W)$, the consistency and full-identity malleability of IBKEM assures that $\text{FIM}(W, u) = \text{Decaps}_{\text{ibkem}}(\hat{S}_W, \text{Pub})$ holds. The collision-freeness of IBKEM assures that only one ciphertext containing keyword $W$ has the value $\text{FIM}(W, u)$ as
its first part. Therefore the algorithm can find out the first ciphertext of keyword $W$ with the hidden structure $\text{Pub}$. Then the consistency of IBE allows the algorithm StructuredSearch to find out the rest of ciphertexts containing keyword $W$ with the hidden structure $\text{Pub}$. Formally we have Theorem 3. The proof can be found in Supplemental Materials D.

**Theorem 3.** The above generic SPCHS scheme is consistent if its underlying collision-free full-identity malleable IBKEM and IBE schemes are both consistent.

**Semantic Security.** The SS-sK-CKSA security of the above generic SPCHS construction relies on the Anon-SS-sID-CPA security of the underlying IBKEM and the Anon-SS-ID-CPA security of the underlying IBE. In the security proof, we prove that if there is an adversary who can break the SS-sK-CKSA security of the above generic SPCHS construction, then there is another adversary who can break the Anon-SS-ID-CPA security of the underlying IBKEM or the Anon-SS-ID-CPA security of the underlying IBE. Theorem 4 formally states the semantic security of our generic SPCHS construction. The proof can be found in Supplemental Materials E.

**Theorem 4.** Suppose there are at most $N \in \mathbb{N}$ hidden structures, and a PPT adversary $A$ wins in the SS-sK-CKSA game with advantage $\text{Adv}^\text{SS-sK-CKSA}_{\text{IBKEM},A}$. Then there is a PPT adversary $B$, who utilizes the capability of $A$ to win in the Anon-SS-ID-CPA game of the underlying IBKEM or the Anon-SS-ID-CPA game of the underlying IBE with advantage $\frac{1}{2} \cdot \text{Adv}^\text{SS-sK-CKSA}_{\text{IBKEM},A}$.

V. TWO COLLISION-FREE FULL-IDENTITY MALLEABLE IBKEM INSTANCES

**The Instance in the RO Model.** Abdalla et al. proposed several VRF-suitable IBKEM instances in [8]. An IBKEM instance is VRF-suitable if it provides unique decapsulation. This means that given any encapsulation, all the decryption keys corresponding to the same identity decapsulate out the same encapsulated key, and the key is pseudo-random. Here, the decryption key extraction is probabilistic and for the same identity, different decryption key may be extracted in different runs of the key extraction algorithm. It is clear that our proposed collision-free full-identity malleability not only implies unique decapsulation, but also implies that the generator of an encapsulation knows what keys will be decapsulated by the decryption keys of all identities. In Supplemental Materials F, we prove that the VRF-suitable IBKEM instance proposed in Appendix A.2 of [8] is collision-free full-identity malleable. Even though this IBKEM scheme has the traditional Anon-SS-ID-CPA security, we further prove that this IBKEM scheme is Anon-SS-ID-CPA secure based on the DBDH assumption in the RO model according to Definition 8.

**The Instance in the Standard Model.** In [9], Freire et al. utilized the “approximation” of multilinear maps to construct a programmable hash function in the multilinear setting (MPHF). Then Freire et al. utilized this hash function to replace the traditional hash functions of the BF IBE scheme in [11] and reconstructed this IBE scheme in the multilinear setting. They finally constructed a new IBE scheme with semantic security in the standard model. We find that this new IBE scheme can be easily transformed into a collision-free full-identity malleable IBKEM scheme with Anon-SS-ID-CPA security in the standard model. To simplify the description of this IBKEM scheme, we do not consider the “approximation” of multilinear maps. This means that we will leave out the functions that are the encoding of a group element, the re-randomization of an encoding and the extraction of an encoding. Some related definitions are reviewed as follows.

**Definition 9 (Multilinear Maps [9]).** An $\ell$-group system in the multilinear setting consists of $\ell$ cyclic groups $\mathbb{G}_1, \ldots, \mathbb{G}_\ell$ of prime order $p$, along with bilinear maps $\hat{e}_{i,j} : \mathbb{G}_i \times \mathbb{G}_j \rightarrow \mathbb{G}_{i+j}$ for all $i, j \geq 1$ with $i+j \leq \ell$. Let $g_i$ be a generator of $\mathbb{G}_i$. The map $\hat{e}_{i,j}$ satisfies $\hat{e}_{i,j}(g_i^a, g_j^b) = g_{i+j}^{ab}$ (for all $a, b \in \mathbb{Z}_p$). When $i, j$ are clear, we will simply use $\hat{e}_{i,j}$ to denote $\hat{e}_{i,j}(g_i^a, g_j^b).$
write \( \hat{\epsilon} \) instead of \( \hat{\epsilon}_{i,j} \). It will also be convenient to abbreviate \( \hat{\epsilon}(h_1, \ldots, h_j) = \hat{\epsilon}(h_1, \hat{\epsilon}(h_2, \ldots, \hat{\epsilon}(h_{j-1}, h_j) \cdots)) \) for \( h_j \in G_1 \) and \( \hat{\epsilon} = (t_1 + t_2 + \cdots + t_j) \leq \ell \). By induction, it is easy to see that this map is \( j \)-linear. Additionally, we define \( \hat{\epsilon}(g) = g \). Finally, it can also be useful to define the group \( G_0 = Z_{G_1}^{\ell} \) of exponents to which this pairing family naturally extends. In the following, we will assume an \( \ell \)-group system \( MPG_\ell = \{ \{G_1\}_{\ell \in [1,\ell+1]} p, \{e_{i,j}\}_{i,j \geq 1, i+j \leq \ell+1} \} \) generated by a multilinear maps parameter generator \( MG_\ell \) on input a security parameter \( \ell \).

**Definition 10** (The \( \ell \)-MDDH Assumption [9]). Given \( (g, g^{Z_2}, \ldots, g^{Z_{\ell+1}}) \) (for \( g \not\in G_1 \) and uniform exponents \( x_1, \ldots, x_{\ell+1} \)), the \( \ell \)-MDDH assumption is that the element \( \hat{\epsilon}(g^{x_1}, \ldots, g^{x_{\ell+1}}) \in G_\ell \) is computationally indistinguishable from a uniform \( G_1 \)-element.

**Definition 11** (Group hash function [9]). A group hash function \( H \) into \( G \) consists of two polynomial-time algorithms: the probabilistic algorithm \( HGen(1^k) \) outputs a key \(hk\), and \( HEval(hk, X)\) (for a key \(hk\) and \( X \in \{0,1\}^\ell\)) deterministically outputs an image \( H_{hk}(X) \in G \).

**Definition 12** (MPHF [9]). Assume an \( \ell \)′-group system \( MPG_{\ell'} \) as generated by \( MG_{\ell'}(1^k) \). Let \( H \) be a group hash function into \( G_{\ell'}(\ell' \leq \ell) \), and let \( m, n \in \mathbb{N} \). We say that \( H \) is an \( (m,n) \)-programmable hash function in the multilinear setting \((m,n)\)-MPHF if there are PPT algorithms \( TGen \) and \( TEval \) as follows.

- \( TGen(1^k, c_1, \ldots, c_1, h) \) (for \( c_1, h \in G_1 \) and \( h \neq 1 \)) outputs a key \( kh \) and a trapdoor \( td \). We require that for all \( c_1, h \), that distribution of \( kh \) is statistically close to the output of \( HGen \).
- \( TEval(td, X) \) (for a trapdoor \( td \) and \( X \in \{0,1\}^k \)) deterministically outputs \( a_X \in Z_{p^n} \) and \( B_X \in G_{\ell-1} \) with \( H_{hk}(X) = \hat{\epsilon}(c_1, \ldots, c_1) \cdot \hat{\epsilon}(B_X, h) \). We require that there is a polynomial \( p(k) \) such that for all \( h, X_1, \ldots, X_m, Z_1, \ldots, Z_n \in \{0,1\}^k \) with \( \{X_1\}_1 \cap \{Z_1\}_1 = \emptyset \), \( P_{hk, \{X_i\}, \{Z_i\}} = \Pr[\{a_X = \cdots = a_{X_m} = 0\} \land (\hat{\epsilon}(a_{Z_1}, \ldots, a_{Z_n}) \neq 0)] \geq 1/p(k) \), where the probability is over possible trapdoors \( td \) output by \( TGen \) along with the given \( hk \). Furthermore, we require that \( P_{hk, \{X_i\}, \{Z_i\}} \) is close to statistically independent of \( hk \). (Formally, \( P_{hk, \{X_i\}, \{Z_i\}} - P_{hk', \{X_i\}, \{Z_i\}} \leq v(k) \) for all \( hk \) and \( hk' \) in the range of \( TGen \), all \( \{X_i\}, \{Z_i\} \), and negligible \( v(k) \).)

We say that \( H \) is a \((poly,n)\)-MPHF if it is a \((q(k),n)\)-MPHF for every polynomial \( q(k) \). Note that \( TEval \) algorithm of an \( MPHF \) into \( G_1 \) yields \( B_X \in G_0 \), i.e., exponents \( B_X \).

Let identity space \( ID_{\text{IBKEM}} = \{0,1\}^k \). The IBKEM instance in the standard model is as follows.

- **Setup_{\text{IBKEM}}(1^k, ID_{\text{IBKEM}}):** Take as input a security parameter \( 1^k \) and the identity space \( ID_{\text{IBKEM}} \), generate an \((\ell+1)\)-group system \( MPG_{\ell+1} = \{\{G_1\}_{\ell \in [1,\ell+1]} p, \{e_{i,j}\}_{i,j \geq 1, i+j \leq \ell+1} \} \rightarrow MG_{\ell+1}(1^k) \), generate a \((poly,2)\)-MPHF \( H \) into \( G_\ell \) and \( hk \leftarrow HGen(1^k) \), choose \( h \not\in G_1 \) and \( x \not\in Z_p \), set the encapsulated key space \( K_{\text{IBKEM}} = G_{\ell+1} \), set the encapsulation space \( C_{\text{IBKEM}} = G_1 \), and output the master public key \( PK_{\text{IBKEM}} = (MGP_{\ell+1}, hk, H, h, x, TD_{\text{IBKEM}}, K_{\text{IBKEM}}, C_{\text{IBKEM}}) \) and the master secret key \( SK_{\text{IBKEM}} = (hk, x) \).
- **Extract_{\text{IBKEM}}(SK_{\text{IBKEM}}, ID):** Take as inputs \( SK_{\text{IBKEM}} \) and an identity \( ID \in TD_{\text{IBKEM}} \), and output a decryption key \( \hat{S}_{ID} = H_{hk}(ID)^x \) of \( ID \).
- **Encaps_{\text{IBKEM}}(PK_{\text{IBKEM}}, ID, r):** Take as inputs \( PK_{\text{IBKEM}} \), an identity \( ID \in TD_{\text{IBKEM}} \) and a random value \( r \in Z_p \), and output a key-encapsulation pair \((\hat{K}, \hat{C})\), where \( \hat{K} = \hat{\epsilon}(H_{hk}(ID), h)^r \) and \( \hat{c} = h^r \).
- **Decaps_{\text{IBKEM}}(\hat{S}_{ID}, \hat{C}):** Take as inputs the decryption key \( \hat{S}_{ID}' \) of identity \( ID' \) and an encapsulation \( C \), and output the encapsulated key \( \hat{K} = \hat{\epsilon}(\hat{C}, \hat{S}_{ID}') \) in \( G_{\ell+1} \) if \( C \in G_1 \) or output \( \perp \) otherwise.

**Consistency.** According to Definitions 9 and 11 it is very easy to verify the consistency of the above IBKEM scheme.

**Collision-Free Full-Identity Malleability.** Let the function \( FIM(ID, r) = \hat{\epsilon}(h^x, H_{hk}(ID))r \) for any identity \( ID \in TD_{\text{IBKEM}} \) and any random value \( r \in Z_p \). Given any \((\hat{K}, \hat{C}) \leftarrow Encaps_{\text{IBKEM}}(PK_{\text{IBKEM}}, ID, r)\), we clearly have that: (1) for any identity \( ID' \), equation \( FIM(ID', r) = Decaps_{\text{IBKEM}}(\hat{S}_{ID'}, \hat{C}) \) holds; (2) for any identity \( ID' \) and any random value \( r' \), if \( ID' \neq ID \) and \( r' \neq r \) holds, equation \( FIM(ID, r) \neq FIM(ID', r') \) holds except with a negligible probability. So the above IBKEM scheme is collision-free full-identity malleable.

**Anon-SS-ID-CPA Security.** In [9], Freire et al. utilized a \((poly,1)\)-MPHF to construct a standard-model version of the BFIBE scheme with the SS-ID-CPA security. On the contrary, we use a \((poly,2)\)-MPHF in constructing the above IBKEM scheme, since this kind of MPHP is more useful in proving the Anon-SS-ID-CPA security. Theorem 5 formally states the Anon-SS-ID-CPA security of the above IBKEM scheme. The proof can be found in Supplemental Materials G.

**Theorem 5.** Assume the above IBKEM scheme is implemented in an \((\ell+1)\)-group system, and with a \((poly,2)\)-MPHF \( H \) into \( G_\ell \). Then, under the \((\ell+1)\)-MDDH assumption, this IBKEM scheme is Anon-SS-ID-CPA secure.

According to Theorem 4 and 5, the generic SPCHS construction implies a SPCHS instance with SS-sk-CKSA security in the standard model. Indeed, this SPCHS instance can be provably SS-CKSA secure.

**VI. CONCLUSION AND FUTURE WORK**

This paper investigated as-fast-as-possible search in PEKS with semantic security. We proposed the concept of SPCHS as a variant of PEKS. The new concept allows keyword-searchable ciphertexts to be generated with a hidden structure. Given a keyword search trapdoor, the search algorithm of SPCHS can disclose part of this hidden structure for guidance on finding out the ciphertexts of the
queried keyword. Semantic security of SPCHS captures the privacy of the keys and the invisibility of the hidden structures. We proposed an SPCHS scheme from scratch with semantic security in the RO model. The scheme generates keyword-searchable ciphertexts with a hidden star-like structure. It has search complexity mainly linear with the exact number of the ciphertexts containing the queried keyword. It outperforms existing PEKS schemes with semantic security, whose search complexity is linear with the number of all ciphertexts. We identified several interesting properties, i.e., collision-freeness and full-identity malleability in some IBKEM instances, and formalized these properties to build a generic SPCHS construction. We illustrated two collision-free full-identity malleable IBKEM instances, which are respectively secure in the RO and standard models.

SPCHS seems a promising tool to solve some challenging problems in public-key searchable encryption. One application may be to achieve retrieval completeness verification which, to the best of our knowledge, has not been achieved in existing PEKS schemes. Specifically, by forming a hidden ring-like structure, i.e., letting the last hidden pointer always point to the head, one can obtain PEKS allowing to check the completeness of the retrieved ciphertexts by checking whether the pointers of the returned ciphertexts form a ring.

Another application may be to realize public key encryption with content search, a similar functionality realized by symmetric searchable encryption. Such kind of content-searchable encryption is useful in practice, e.g., to filter the encrypted spams. Specially, by forming a hidden tree-like structure between the sequentially encrypted words in the encrypted spams. Specially, by forming a hidden tree-like structure, i.e., letting the last hidden pointer always point to the head, one can obtain PEKS allowing to check the completeness of the retrieved ciphertexts by checking whether the pointers of the returned ciphertexts form a ring.

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A sender generates the searchable ciphertexts of any keyword \( W_i \in \mathcal{W} \) by the following steps:

1) The first time to encrypt keyword \( W_i \), he uploads
\[
PEKS(Pub, W_i, K_{i1} || P_{i1}^1), P_{i1}^1 || E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3), P_{i1}^2.
\]
to the server, and asks the server to store \( E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \) in position \( P_{i1}^1 \) and store a flag in position \( P_{i1}^2 \).

Note: algorithm \( PEKS(Pub, W_i, K_{i1} || P_{i1}^1) = IBE(Pub, W_i, K_{i1} || P_{i1}^1 || C_2) || C_2 \) takes public parameter \( Pub \), identity \( W_i \) and plaintext \( K_{i1} || P_{i1}^1 || C_2 \) as inputs and generates an IBE ciphertext, and finally outputs the IBE ciphertext and \( C_2 \), where the symmetric key \( K_{i1} \) and \( C_2 \) are randomly chosen.

Algorithm \( E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \) denotes using the symmetric key \( K_{i1}^1 \) to encrypt \( P_{i1}^2 || K_{i2}^2 || P_{i1}^3 \), where the symmetric key \( K_{i1}^1 \) is randomly chosen, and \( P_{i1}^1 \) denotes the parameters for private information retrieval (they will be used to retrieve the corresponding data when the keyword \( W_i \) is queried).

2) The second time to encrypt keyword \( W_i \), he uploads \( P_{i2}^2 || E(K_{i2}^2, P_{i2}^3 || K_{i3}^3 || P_{i2}^4) \) to the server, and asks the server to store \( E(K_{i2}^2, P_{i2}^3 || K_{i3}^3 || P_{i2}^4) \) in position \( P_{i2}^4 \) and store the flag in position \( P_{i2}^3 \).

3) The subsequent encryptions of keyword \( W_i \) are similar to Step 2.

Figure 5: Procedure to generate keyword searchable ciphertexts in [40].

In Fig. 5 we first review how to generate keyword-searchable ciphertexts according to [40] such that the ciphertexts of the same keyword form a chain. Then we analyze why the chain of any keyword is visible in the view of the server, and give a straightforward method to make the chain invisible. But this method seems to be impractical.

According to the first step in Fig. 5, the server trivially knows the relation between ciphertexts \( PEKS(Pub, W_i, K_{i1} || P_{i1}^1) \) and \( E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \), and knows that if a subsequent ciphertext is stored in the position \( P_2 \), this subsequent ciphertext is related to \( E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \). So in the second step, the server knows the relation between ciphertexts \( E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \) and \( E(K_{i2}^2, P_{i2}^3 || K_{i3}^3 || P_{i2}^4) \), and knows that if another subsequent ciphertext is stored in the position \( P_3 \), this subsequent ciphertext is related to \( E(K_{i2}^2, P_{i2}^3 || K_{i3}^3 || P_{i2}^4) \). By the same method, the server will know the chain of keyword \( W_i \) even without the keyword search trapdoor of keyword \( W_i \). Furthermore, the length of the chain leaks the frequency of keyword \( W_i \).

A sender generates the searchable ciphertexts of any keyword \( W_i \in \mathcal{W} \) by the following steps:

1) At the setup phase, he uploads \{ \( PEKS(Pub, W_i, K_{i1} || P_{i1}^1) || i \in [1, |\mathcal{W}|] \} \) to the server, where \(|\mathcal{W}|\) denotes the size of keyword space \( \mathcal{W} \).

2) The first time to encrypt keyword \( W_i \), he uploads \( P_{i1}^1 || E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \) to the server, and asks the server to store \( E(K_{i1}^1, P_{i1}^2 || K_{i2}^2 || P_{i1}^3) \) in position \( P_{i1}^1 \).

3) The second time to encrypt keyword \( W_i \), he uploads \( P_{i2}^2 || E(K_{i2}^2, P_{i2}^3 || K_{i3}^3 || P_{i2}^4) \) to the server, and asks the server to store \( E(K_{i2}^2, P_{i2}^3 || K_{i3}^3 || P_{i2}^4) \) in position \( P_{i2}^4 \).

4) The subsequent encryptions of keyword \( W_i \) are similar to Step 3.

Figure 6: New procedure to generate keyword-searchable ciphertexts for [40].

In order to keep the privacy of the chain, a straightforward method is to generate the PEKS ciphertexts for all keywords at the setup phase and delete the flag. The specific procedure is given in Fig. 6. This method hides the relation between the PEKS ciphertext and the symmetric-key ciphertext of any keyword, and the relation between two symmetric-key ciphertexts of any keyword also is hidden. But it seems that this method is impractical from a performance viewpoint, since each sender must generate the PEKS ciphertexts for all keywords at the setup phase and remember lots of private information which are encrypted by these PEKS ciphertexts.

### B. Proof of Theorem 7

**Proof:** Without loss of generality, it is sufficient to prove that given the keyword-searchable trapdoor \( T_{W_i} = H(W_i)^s \) of keyword \( W_i \) and the hidden structure’s public part \( Pub = g^t \), algorithm \( StructuredSearch(PK, Pub, C, T_{W_i}) \) only finds out all ciphertexts of keyword \( W_i \) with the hidden structure \( Pub \). Note that \( P = g^s \).

Algorithm \( StructuredSearch(PK, Pub, C, T_{W_i}) \) computes \( Pt' = \hat{e}(Pub, T_{W_i}) \) in its first step. Since \( \hat{e}(Pub, T_{W_i}) = \hat{e}(P, H(W_i))^s \), algorithm \( StructuredSearch(PK, Pub, C, T_{W_i}) \) finds out the ciphertext \( (\hat{e}(P, H(W_i))^s, g^r, \hat{e}(P, H(W_i))^r \cdot Pt[u, W_i]) \) by matching \( Pt' \) with all ciphertexts’ first part in its second step. Moreover,
due to the collision-freeness of hash function $H$, only keyword $W_i$ has $Pt' = \hat{e}(P, H(W_i))^u$, except with a negligible probability in the security parameter $k$. So only the ciphertext $(\hat{e}(P, H(W_i))^u, g^r, \hat{e}(P, H(W_i))^r \cdot Pt[u, W_i])$ is found with overwhelming probability in this step.

Then algorithm StructuredSearch($PK, Pub, \mathbb{C}, T_W$) discloses $Pt[u, W_i]$ from the ciphertext $(\hat{e}(P, H(W_i))^u, g^r, \hat{e}(P, H(W_i))^r \cdot Pt[u, W_i])$ by computing $Pt' = Pt[u, W_i] = \hat{e}(g^r, T_W)^{-1} \cdot \hat{e}(P, H(W_i))^r \cdot Pt[u, W_i]$.

Recall that in algorithm StructuredEncryption, $Pt[u, W_i]$ was randomly chosen in $G_1$ and taken as the first part of only one ciphertext of keyword $W_i$ with the hidden structure $Pub$. So when algorithm StructuredSearch($PK, Pub, \mathbb{C}, T_W$) goes back to its second step, only the ciphertext $(Pt[u, W_i], g^r, \hat{e}(P, H(W_i))^r \cdot R)$ is found with overwhelming probability.

By carrying on in this way, algorithm StructuredSearch($PK, Pub, \mathbb{C}, T_W$) only finds out all ciphertexts of keyword $W_i$ with the hidden structure $Pub$, except with a negligible probability in the security parameter $k$. And the algorithm will stop, since the random value $R$ contained in the last found ciphertext does not match any other ciphertext’s first part.

C. Proof of Theorem

Proof: To prove this theorem, we will construct a PPT algorithm $B$ that plays the SS-CKSA game with adversary $A$ and utilizes the capability of $A$ to solve the DBDH problem in $BG_{\text{Gen}}(k^*)$ with advantage approximately $\left(\frac{27}{q_r q_s q_t} - p\right)$.

Adversary $A$’s $Adv_{\text{SS-CKSA}(G_{\text{SS-CKSA}})}$: Let $Coin \overset{\$}{\leftarrow} \{0, 1\}$ denote the operation that picks $Coin \in \{0, 1\}$ according to the probability $Pr[Coin = 1] = \sigma$ (the specified value of $\sigma$ will be decided latter). The constructed algorithm $B$ in the SS-CKSA game is as follows.

- Setup Phase: Algorithm $B$ takes as inputs $(q, G, G_1, g, e, g^a, g^b, g^c, Z)$ (where $Z$ equals either $\hat{e}(g, g)^{abc}$ or $\hat{e}(g, g)^{ab}$) and the keyword space $\mathcal{W}$, and performs the following steps:

  1) Initialize the three lists $Pt = \emptyset \subseteq \mathcal{W} \times \mathfrak{G} \times G_1$, $\mathbb{S}\mathbb{L}\mathbb{I}\mathbb{S} = \emptyset \subseteq \mathcal{W} \times G \times Z_q^* \times \{0, 1\}$ and $\mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S} = \emptyset \subseteq \mathcal{W} \times \mathfrak{G} \times Z_q^* \times \{0, 1\}$;

  2) Set the ciphertext space $\mathcal{C} = G_1 \times \mathfrak{G} \times G_1$ and $PK = (q, G, G_1, g, \hat{e}, P = g^a, W, \mathcal{C})$;

  3) Initialize $N$ hidden structures by repeating the following steps for $i \in [1, N]$:

    a) Pick $u_i \overset{\$}{\leftarrow} Z_q^*$ and $Coin_i \overset{\$}{\leftarrow} \{0, 1\}$;

    b) If $Coin_i = 1$, compute $Pub_i = g^{u_i}$;

    c) Otherwise, compute $Pub_i = g^{u_i}$;

  4) Set $P\mathbb{S}\mathbb{E}\mathbb{S} = \{Pub_i|i \in [1, N]\}$ and $\mathbb{S}\mathbb{L}\mathbb{I}\mathbb{S} = \{(Pub_i, u_i, Coin_i)|i \in [1, N]\}$;

  5) Send $PK$ and $P\mathbb{S}\mathbb{E}\mathbb{S}$ to adversary $A$.

- Query Phase 1: Adversary $A$ adaptively issues the following queries multiple times.

  - Hash Query $Q_H(W)$: Taking as input a keyword $W \in \mathcal{W}$, algorithm $B$ does the following steps:

    1) Pick $x \overset{\$}{\leftarrow} Z_q^*$ and $Coin \overset{\$}{\leftarrow} \{0, 1\}$;

    2) If $Coin = 0$, add $(W, g^x, x, Coin)$ into $\mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S}$ and output $g^x$;

    3) Otherwise, add $(W, g^{c^x}, x, Coin)$ into $\mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S}$ and output $g^{c^x}$;

  - Trapdoor Query $Q_{T_{\text{trap}}}(W)$: Taking as input a keyword $W \in \mathcal{W}$, algorithm $B$ does the following steps:

    1) If $(W, \ast, \ast, \ast) \notin \mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S}$, query $Q_H(W)$;

    2) According to $W$, retrieve $(W, X, x, Coin)$ from $\mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S}$;

    3) If $Coin = 0$, output $g^{a-x}$; otherwise abort and output $\perp$;

  - Privacy Query $Q_{P_{\text{Pr}}}(Pub)$: Taking as input a structure’s public part $Pub \in P\mathbb{S}\mathbb{E}\mathbb{S}$, algorithm $B$ does the following steps:

    1) According to $Pub$, retrieve $(Pub, u, Coin)$ from $\mathbb{S}\mathbb{L}\mathbb{I}\mathbb{S}$;

    2) If $Coin = 0$, output $u$; otherwise abort and output $\perp$;

  - Encryption Query $Q_{E_{\text{Enc}}}(W, Pub)$: Taking as inputs a keyword $W \in \mathcal{W}$ and a structure’s public part $Pub$, algorithm $B$ does the following steps:

    1) If $(W, \ast, \ast, \ast) \notin \mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S}$, query $Q_H(W)$;

    2) According to $W$ and $Pub$, retrieve $(W, X, x, Coin)$ and $(Pub, u, Coin')$ respectively from $\mathbb{H}\mathbb{L}\mathbb{I}\mathbb{S}$ and $\mathbb{S}\mathbb{L}\mathbb{I}\mathbb{S}$;

    3) Pick $r \overset{\$}{\leftarrow} Z_q^*$, and search $(W, Pub, Pt[u, W]) \overset{\$}{\leftarrow} G_1$ to $Pt$ and $Pub$ in $Pt$;

    4) If $W$ is not found, insert $(W, Pub, Pt[u, W]) \overset{\$}{\leftarrow} G_1$ to $Pt$ and do the following steps:

      a) If $Coin = 1 \land Coin' = 1$, output $C = (Z^{x-u}, g^r, \hat{e}(g^a, X)^r \cdot Pt[u, W])$;

      b) If $Coin = 0 \land Coin' = 1$, output $C = (\hat{e}(g^{a}, g^{b^u})^r, g^r, \hat{e}(g^a, X)^r \cdot Pt[u, W])$;

      c) If $Coin' = 0$, output $C = (\hat{e}(g^a, X)^u, g^r, \hat{e}(g^a, X)^r \cdot Pt[u, W])$;

    5) Otherwise, pick $R \overset{\$}{\leftarrow} G_1$, set $C = (Pt[u, W], g^r, \hat{e}(g^a, X)^r \cdot R)$, update $Pt[u, W] = R$ and output $C$;
• **Challenge Phase**: Adversary $A$ sends two challenge keyword-structure pairs $(W_0^*, Pub_0^*) \in W \times PSet$ and $(W_1^*, Pub_1^*) \in W \times PSet$ to algorithm $B$; $B$ picks $d \leftarrow \{0, 1\}$, and does the following steps:

1. According to $Pub_0^*$ and $Pub_1^*$, retrieve $(Pub_b^0, u_0^*, Coin_0^*)$ and $(Pub_b^1, u_1^*, Coin_1^*)$ from SList; and if $Coin_0^* = 0 \lor Coin_1^* = 0$, then abort and output $\perp$;
2. If $(W_d^*, u_0, u_1) \notin HList$, query $Q_H(W_d^*)$;
3. According to $W_d^*$, retrieve $(W_d^*, X_d^i, x_d^i, Coin)$ from HList; and if $Coin = 0$, then abort and output $\perp$;
4. Search $(W_d^*, Pub_b^*, Pt[u_0^*, W_d^*])$ for $W_0^*$ and $Pub_b^*$ in $Pt$;
5. If it is not found, insert $(W_d^*, Pub_b^*, Pt[u_0^*, W_d^*]) \leftarrow G_1$ to $Pt$, and send $C_d^* = (Z_{d^*}^u, g^k, Z_{d^*}^a \cdot Pt[u_0^*, W_d^*])$ to adversary $A$;
6. Otherwise, pick $R \leftarrow \mathbb{G}_1$, set $C_d^* = (Pt[u_0^*, W_d^*], g^k, Z_{d^*}^a \cdot R)$, update $Pt[u_0^*, W_d^*] = R$, and send $C_d^*$ to adversary $A$.

• **Query Phase 2**: This phase is the same as **Query Phase 1**. Note that in **Query Phase 1** and **Query Phase 2**, adversary $A$ cannot query the corresponding private parts both of $Pub_0^*$ and $Pub_1^*$ and the keyword search trapdoors both of $W_0^*$ and $W_1^*$.

• **Guess Phase**: Adversary $A$ sends a guess $d'$ to algorithm $B$. If $d = d'$, $B$ output 1; otherwise, output 0.

Let $\overline{Abort}$ denote the event that algorithm $B$ does not abort in the above game. Next, we will compute the probabilities $Pr[\overline{Abort}], Pr[B = 1|Z = \hat{g}(g, g)^{abc}]$ and $Pr[B = 1|Z = \hat{g}(g, g)^y]$, and the advantage $Adv_B^{BBDDH}(1^k)$.

According to the above game, the probability of the event $\overline{Abort}$ only relies on the probability $\sigma$ and the number of times that adversary $A$ queries oracles $Q\mathcal{TR}_{trap}(\cdot)$ and $Q\mathcal{PK}_{\mathcal{C}}(\cdot)$. We have that $Pr[\overline{Abort}] = (1 - \sigma)^{q_t \cdot q_p} \cdot \sigma^3$. Let $\sigma = \frac{1}{3 \cdot q_t \cdot q_p}$. We have that $Pr[\overline{Abort}] \approx \left(\frac{1}{e \cdot q_t \cdot q_p}\right)^3$, where $e$ is the base of natural logarithms.

When $Z = \hat{\epsilon}(g, g)^{abc}$ and the event $\overline{Abort}$ holds, it is easy to find that algorithm $B$ simulates a real SS-CKSA game in adversary $A$’s mind. So we have

$$Pr[d = d'|\overline{Abort} \land Z = \hat{\epsilon}(g, g)^{abc}] = (Adv_{SS-CKSA}^{SPCHS, A} + \frac{1}{2})$$

When $Z = \hat{\epsilon}(g, g)^y$ and the event $\overline{Abort}$ holds, algorithm $B$ generates a challenge ciphertext, which is independent of the challenge keywords $W_0^*$ and $W_1^*$. So we have

$$Pr[d = d'|\overline{Abort} \land Z = \hat{\epsilon}(g, g)^y] = \frac{1}{2}.$$

Now, we can compute the advantage $Adv_B^{BBDDH}(1^k)$ as follows:

$$Adv_B^{BBDDH}(1^k) = Pr[B = 1|Z = \hat{\epsilon}(g, g)^{abc}] - Pr[B = 1|Z = \hat{\epsilon}(g, g)^y]$$

$$= Pr[d = d'|\overline{Abort} \land Z = \hat{\epsilon}(g, g)^{abc}] - Pr[d = d'|\overline{Abort} \land Z = \hat{\epsilon}(g, g)^y]$$

$$= Pr[d = d'|\overline{Abort} \land Z = \hat{\epsilon}(g, g)^{abc}] \cdot Pr[\overline{Abort}|Z = \hat{\epsilon}(g, g)^{abc}]$$

$$- Pr[d = d'|\overline{Abort} \land Z = \hat{\epsilon}(g, g)^y] \cdot Pr[\overline{Abort}|Z = \hat{\epsilon}(g, g)^y]$$

$$\approx (Adv_{SS-CKSA}^{SPCHS, A} + \frac{1}{2}) \cdot \left(\frac{27}{(e \cdot q_t \cdot q_p)^3} - \frac{1}{2}\right)$$

$$\approx \left(\frac{27}{(e \cdot q_t \cdot q_p)^3}\right) \cdot Adv_{SS-CKSA}^{SPCHS, A}$$

In addition, it is clear that algorithm $B$ is a PPT algorithm, if adversary $A$ is a PPT adversary. In conclusion, if a PPT adversary $A$ wins in the SS-CKSA game of the above SPCHS instance with advantage $Adv_{SS-CKSA}^{SPCHS, A}$, in which $A$ makes at most $q_t$ queries to oracle $Q\mathcal{TR}_{trap}(\cdot)$ and at most $q_p$ queries to oracle $Q\mathcal{PK}_{\mathcal{C}}(\cdot)$, then there is a PPT algorithm $B$ that solves the DBDH problem in $BGen(1^k)$ with advantage approximately

$$Adv_B^{BBDDH}(1^k) \approx \left(\frac{27}{(e \cdot q_t \cdot q_p)^3}\right) \cdot Adv_{SS-CKSA}^{SPCHS, A}$$

where $e$ is the base of natural logarithms.

\[\square\]

**D. Proof of Theorem 2**

Proof: Without loss of generality, it is sufficient to prove that given the keyword-searchable trapdoor $T_{W_i} = (\hat{S}_{W_i}, \hat{S}_{W_i})$ of keyword $W_i$ and the hidden structure’s public part $Pub = C$, algorithm StructuredSearch($PK, Pub, C, T_{W_i}$) only finds out all ciphertexts of keyword $W_i$ with the hidden structure $Pub$, where $\hat{S}_{W_i} = Extract_{\mathcal{BKEM}}(SK_{\mathcal{BKEM}}, W_i), \hat{S}_{W_i} = Extract_{\mathcal{SKin}}(SK_{\mathcal{SKin}}, W_i), C$ is from $(K, C) = \mathcal{Encaps}_{\mathcal{BKEM}}(PK_{\mathcal{BKEM}}, W, u)$, keyword $W$ is arbitrarily chosen in $\mathcal{W}$, and $u$ is a random value.
AlgorithmStructuredSearch(PK, Pub, C, T_W) computes P ′ = DecapsIBKEM( ˜S_W, Pub) in its first step. According to the full-identity malleability of IBKEM in Definition 7, we have FIM(W_i, u) = DecapsIBKEM( ˜S_W, Pub). So algorithm StructuredSearch(PK, Pub, C, T_W) finds out the ciphertext (FIM(W_i, u), Enc_{ibkem}(PK_{ibkem}, W_i, Pt[u,W_i])) by matching P ′ with all ciphertexts’ first part in its second step. Moreover, due to the collision-freeness of IBKEM in Definition 7, there is no keyword W_j ( ≠ W_i) to meet FIM(W_i, u) = FIM(W_j, u), and no hidden structure Pub′ ( ≠ Pub) to meet FIM(W_i, u) = FIM(W_i, u′), where Pub′ is generated by algorithm StructureInitialization(PK) with the random value u′. So only the ciphertext (FIM(W_i, u), Enc_{ibkem}(PK_{ibkem}, W_i, Pt[u,W_i])) is found in this step, except with a negligible probability in the security parameter k. Then, according to the consistency of IBE, algorithm StructuredSearch(PK, Pub, C, T_W) can decrypt Pt[u,W_i] by algorithm Dec_{ibkem}( ˜S_W, Enc_{ibkem}(PK_{ibkem}, W_i, Pt[u,W_i])).

Recall that in algorithm StructuredEncryption, Pt[u,W_i] was randomly chosen in G_1 and taken as the first part of only one ciphertext of keyword W_i. So when StructuredSearch(PK, Pub, C, T_W) goes back to its second step, only the ciphertext (Pt[u,W_i], Enc_{ibkem}(PK_{ibkem}, W_i, R_i)) is found, except with a negligible probability in the security parameter k.

By carrying on in the same way, algorithm StructuredSearch(PK, Pub, C, T_W) only finds out all ciphertexts of keyword W_i, with the hidden structure Pub, except with a negligible probability in the security parameter k. And the algorithm will stop, since the random value R contained in the last found ciphertext of keyword W_i fails to match any other ciphertext’s first part.

E. Proof of Theorem 4
Proof: Let G_1 and G_2 be the challengers respectively in the Anon-SS-sID-CPA game of the underlying IBKEM scheme and the Anon-SS-ID-CPA game of the underlying IBE scheme. A constructed adversary B in the SS-sK-CKSA game of the generic SPCHS construction is as follows.

• Setup Phase: In this phase,
  1. A sends two challenge keywords (W_0′, W_1′) to B.
  2. B arbitrarily picks I_i ← (TD_{ibkem}−W), and sends two challenge identities (W_0′, I_i) to G_1. (The I_i is existing, since we have W ⊂ TD_{ibkem}.)
  3. G_1 generates (PK_{ibkem}, SK_{ibkem}) by algorithm Setup_{ibkem} and sends PK_{ibkem} to B.
  4. B queries G_1 for the challenge key-and-encapsulation pair.
  5. G_1 picks d ∼ {0, 1}, generates (K_{0,d}, C_0) = Encaps_{ibkem}(PK_{ibkem}, W_0′, r_0) and (K_{1,d}, C_1) = Encaps_{ibkem}(PK_{ibkem}, I_i, r_1), and sends (K_{d} ,C_0) to B, where r_0 and r_1 are randomly chosen.
  6. B adds C_0 into the set PSet ⊆ C_{ibkem}.
  7. G_2 generates (PK_{ibkem}, SK_{ibkem}) by algorithm Setup_{ibkem}, and sends PK_{ibkem} to B.
  8. B initializes the two lists SList = ∅ ⊆ C_{ibkem} × {0, 1} and Pt = ∅ ⊆ W × C_{ibkem} × M_{ibkem}, and initializes N − 1 hidden structures by repeating the same steps for i ∈ [1, N-1]:
    a) Pick a random value u_i and an arbitrary keyword W_i ∈ W;
    b) Generate (K_i, C_i) = Encaps_{ibkem}(PK_{ibkem}, W_i, u_i), add Pub_i = C_i into the set PSet, and add (Pub_i, u_i) into SList;
  9. B finally sends PK and PSet to A.

• Query Phase 1: In this phase, adversary A adaptively issues the following queries multiple times.
  - Trapdoor Query Q_{Trap}(W): Taking as input a keyword W ∈ W, B forwards the query W both to the decryption key oracles S_W = Q_{DBKEM}^I(W) and ˜S_W = Q_{DBK}^E(W), and sends T_W = ( ˜S_W, S_W) to A.
    (In this query, A cannot query the keyword search trapdoor corresponding to the challenge keyword W_0′ or W_1′. In addition, one may find that B cannot respond the query Q_{Trap}(I_i). However, this is not a problem, since we let I_i ⊆ (TD_{ibkem}−W). So A never issues that query.)
  - Privacy Query Q_{priv}(Pub): Taking as input a structure’s public part Pub ∈ PSet, B aborts and outputs ⊥ if Pub = C_0; otherwise, B retrieves (Pub, u) from SList according to Pub and outputs u.
  - Encryption Query Q_{Enc}(W, Pub): Taking as inputs a keyword W ∈ W and a structure’s public part Pub, B does the following steps:
    a) Search (W, Pub, Pt[u,W]) for W and Pub in Pt;
      (Note that u is not a really known value. It is just a symbol to denote the random value used to generate Pub = C_0.)
    b) If it is not found, query ˜S_W = Q_{DBK}^I(W), insert (W, Pub, Pt[u,W] ∼ M_{ibkem}) to Pt and output C = (Decaps_{ibkem}( ˜S_W, Pub), Enc_{ibkem}(PK_{ibkem}, W, Pt[u,W]));

Abort

Win

B

game of the underlying IBE scheme under the condition that $Q$:

\textbf{Guess Phase}.

\textbf{Let Challenge Phase}.

guess adversary $A$; this phase is the same as $\text{IBE, IBKEM, or IBKE}$ to challengers $\prime$ denote the event that adversary $A$ does not abort in the above game. Suppose adversary $A$ totally queries $Q_{\text{enc}}$ for $q_p$ times. Then we have $Pr[\text{Abort}] = \frac{N}{\sum_{i=0}^{\infty} \frac{n}{N}} = \frac{N}{N}$. Note that $q_p \leq (N-2)$ always holds, since adversary $A$ cannot query $Q_{\text{enc}}$ for the challenge structures $(\text{Pub}, \text{Pub}^*)$.

\textbf{Let $Win_{\text{SS-ID-CPA}}$ denote the event that $B$ wins in the Anon-SS-ID-CPA game of the underlying IBKEM scheme under the condition that $B$ does not abort. Let $Win_{\text{IBKEM}}$ denote the event that $B$ wins in the Anon-SS-ID-CPA game of the underlying IBE scheme under the condition that $B$ does not abort. Let $Adv_B$ be the advantage of $B$ to have $Win_{\text{IBKEM}}$, $Win_{\text{SS-ID-CPA}}$, $Win_{\text{IBKEM,B}}$, $Win_{\text{SS-ID-CPA,B}}$, $Win_{\text{IBKEM,B}}$, $Win_{\text{SS-ID-CPA,B}}$ holds. Since $B$ has the probability no less than $\frac{3}{4}$ to have $Win_{\text{IBKEM}}$ or $Win_{\text{SS-ID-CPA}}$ holds under the condition that $B$ does not abort, we clearly have...
Let \( \text{Belong} \) denote the event that \( (W^*_u, \text{Pub}^*_u, P[u^*, W^*_u]) \notin \text{Pt} \) holds in the above Challenge Phase. On the contrary, let \( \text{Win} \) denote the event that \( (W^*_0, \text{Pub}^*_0, P[u^*, W^*_0]) \in \text{Pt} \) holds in the above Challenge Phase.

We compute the probability \( Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] + Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] \) as follows.

\[
Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] + Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] = (Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] + Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}]) - Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] - Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] = Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] + Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}]
\]

We compute the probability \( Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] + Pr[\text{Win}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM,B}} | \text{Abort}] \) as follows.
Theorem 6. Let the hash function \( H \) be modeled as the random oracle \( Q_H(\cdot) \). Suppose a PPT adversary \( A \) wins in the Anon-SS-ID-CPA game of the above IBKEM instance with advantage \( \text{Adv}^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A} \), in which \( A \) makes at most
Then there is a PPT algorithm $B$ that solves the DBDH problem in $\text{BGen}(1^k)$ with advantage approximately

$$
Adv_B^{DBDH}(1^k) \approx \frac{4}{(e \cdot q_p)^2} \cdot Adv_{IBKEM,A}^{Anon-SS-ID-CPA}
$$

where $e$ is the base of natural logarithms.

Proof: To prove this theorem, we will construct a PPT algorithm $B$ that plays the Anon-SS-ID-CPA game with adversary $A$ and utilizes the capability of $A$ to solve the DBDH problem in $\text{BGen}(1^k)$ with advantage approximately $\frac{4}{(e \cdot q_p)^2} \cdot Adv_{IBKEM,A}^{Anon-SS-ID-CPA}$. Let $\text{Coin} \leftarrow \{0, 1\}$ denote the operation that picks $\text{Coin} \in \{0, 1\}$ according to the probability $Pr[\text{Coin} = 1] = \sigma$ (the specified value of $\sigma$ will be decided latter). The constructed algorithm $B$ in the Anon-SS-ID-CPA game is as follows.

- **Setup Phase**: Algorithm $B$ takes as inputs $(q, G, G_1, g, \hat{e}, g^a, g^b, Z)$ (where $Z$ equals either $\hat{e}(g, g)^{ab}$ or $\hat{e}(g, g)^y$) and the identity space $\mathcal{D}_{\text{ibkem}}$, and does the following steps:
  1. Initialize a list $\mathcal{HList} = \emptyset \subseteq \mathcal{D}_{\text{ibkem}} \times G \times Z \times \{0, 1\}$;
  2. Set the encapsulated key space $\mathcal{K}_{\text{ibkem}} = G_1$, the encapsulation space $\mathcal{C}_{\text{ibkem}} = G$ and $\mathcal{PK}_{\text{ibkem}} = (g, G, G_1, g, \hat{e}, P = g^a, \mathcal{D}_{\text{ibkem}}, \mathcal{K}_{\text{ibkem}}, \mathcal{C}_{\text{ibkem}})$;
  3. Send $\mathcal{PK}_{\text{ibkem}}$ to adversary $A$;

- **Query Phase 1**: Adversary $A$ adaptively issues the following queries multiple times.
  - Hash Query $\mathcal{Q}_H(ID)$: Taking as input an identity $ID \in \mathcal{D}_{\text{ibkem}}$, algorithm $B$ does the following steps:
    1. Pick $x \leftarrow Z^*_q$ and $\text{Coin} \leftarrow \{0, 1\}$;
    2. If $\text{Coin} = 0$, add $(ID, g^x, x, \text{Coin})$ into $\mathcal{HList}$ and output $g^x$;
    3. Otherwise, add $(ID, g^{-x}, x, \text{Coin})$ into $\mathcal{HList}$ and output $g^{-x}$;
  - Decryption Key Query $\mathcal{Q}_{\text{IBKEM}}^{(ID)}$: Taking as input an identity $ID \in \mathcal{D}_{\text{ibkem}}$, algorithm $B$ does the following steps:
    1. If $(ID, **, **, *) \notin \mathcal{HList}$, query $\mathcal{Q}_H(ID)$;
    2. According to $ID$, retrieve $(X, x, \text{Coin})$ from $\mathcal{HList}$;
    3. If $\text{Coin} = 0$, output $g^x$; otherwise, abort and output $\perp$;

- **Challenge Phase**: Adversary $A$ sends two challenge identities $ID^0 \in \mathcal{D}_{\text{ibkem}}$ and $ID^1 \in \mathcal{D}_{\text{ibkem}}$ to algorithm $B$; $B$ picks $d \leftarrow \{0, 1\}$, and does the following steps:
  1. If $(ID^0, **, **, *) \notin \mathcal{HList}$, query $\mathcal{Q}_H(ID^0)$;
  2. If $(ID^1, **, **, *) \notin \mathcal{HList}$, query $\mathcal{Q}_H(ID^1)$;
  3. According to $ID^0$ and $ID^1$, retrieve $(ID^0, X^0, x^0, \text{Coin}^0)$ and $(ID^1, X^1, x^1, \text{Coin}^1)$ from $\mathcal{HList}$;
  4. If $\text{Coin}^0 = 0 \lor \text{Coin}^1 = 0$, then abort and output $\perp$;
  5. Finally send the challenge key-and-encapsulation pair $(Z^d, g^d)$ to adversary $A$;

- **Query Phase 2**: This phase is the same as **Query Phase 1**. Note that in **Query Phase 1** and **Query Phase 2**, adversary $A$ does not query the decryption key corresponding to the challenge identity $ID^0$ or $ID^1$.

- **Guess Phase**: Adversary $A$ sends a guess $d'$ to algorithm $B$. If $d' = d$, $B$ output 1; otherwise, output 0.

Let $\text{Abort}$ denote the event that algorithm $B$ does not abort in the above game. Next, we will compute the probabilities $Pr[\text{Abort}], Pr[\text{B} = 1|Z = \hat{e}(g, g)^{ab}]$ and $Pr[\text{B} = 1|Z = \hat{e}(g, g)^y]$, and the advantage $Adv_B^{DBDH}(1^k)$.

According to the above game, the probability of the event $\text{Abort}$ only relies on the probability $\sigma$ and the number of times of adversary $A$ to query oracle $\mathcal{Q}_{\text{IBKEM}}^{DK}(ID)$. We have that $Pr[\text{Abort}] = (1 - \sigma)^{2 \cdot q_p} \cdot \sigma$. We have that $Pr[\text{Abort}] \approx \frac{4}{(e \cdot q_p)^2}$, where $e$ is the base of natural logarithms.

When $Z = \hat{e}(g, g)^{ab}$ and the event $\text{Abort}$ holds, it is easy to find that algorithm $B$ simulates a real Anon-SS-ID-CPA game in adversary $A$’s mind. So we have $Pr[d = d'|\text{Abort} \land Z = \hat{e}(g, g)^{ab}] = (Adv_{IBKEM,A}^{Anon-SS-ID-CPA} + \frac{1}{2})$.

When $Z = \hat{e}(g, g)^y$ and the event $\text{Abort}$ holds, algorithm $B$ generates an incorrect challenge ciphertext, and it is independent of the challenge identities $ID^0$ and $ID^1$. So we have $Pr[d = d'|\text{Abort} \land Z = \hat{e}(g, g)^y] = \frac{1}{2}$.

Now, we can compute the advantage $Adv_B^{DBDH}(1^k)$ as follows:

$$
Adv_B^{DBDH}(1^k) = Pr[\text{B} = 1|Z = \hat{e}(g, g)^{ab}] - Pr[\text{B} = 1|Z = \hat{e}(g, g)^y]
$$

$$
= Pr[d = d'|\text{Abort} \land Z = \hat{e}(g, g)^{ab}] - Pr[d = d'|\text{Abort} \land Z = \hat{e}(g, g)^y] - Pr[d = d'|\text{Abort} \land Z = \hat{e}(g, g)^{ab}] \cdot Pr[\text{Abort}]\cdot Z = \hat{e}(g, g)^{ab}] - Pr[d = d'|\text{Abort} \land Z = \hat{e}(g, g)^y] \cdot Pr[\text{Abort}]\cdot Z = \hat{e}(g, g)^y]
$$

$$
\approx (Adv_{IBKEM,A}^{Anon-SS-ID-CPA} + \frac{1}{2}) \cdot \frac{4}{(e \cdot q_p)^2} - \frac{1}{2} \cdot \frac{4}{(e \cdot q_p)^2} = \frac{4}{(e \cdot q_p)^2} \cdot Adv_{IBKEM,A}^{Anon-SS-ID-CPA}
$$
In addition, it is clear that algorithm $B$ is a PPT algorithm, if adversary $A$ is a PPT adversary. In conclusion, if a PPT adversary $A$ wins in the Anon-SS-ID-CPA game of the above IBKEM instance with advantage $Adv^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A}$, in which $A$ makes at most $q_p$ queries to oracle $Q^{\text{IBKEM}}_{\text{DK}}(\cdot)$, then there is a PPT algorithm $B$ that solves the DBDH problem in $\text{BGen}(1^k)$ with advantage approximately

$$Adv^{\text{DBDH}}_B(1^k) \approx \frac{4}{(e \cdot q_p)^2} \cdot Adv^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A}$$

where $e$ is the base of natural logarithms.

\section*{G. Proof of Theorem 5}

\textbf{Proof:} Suppose a PPT adversary $A$ wins in the Anon-SS-ID-CPA game of the above IBKEM instance with advantage $Adv^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A}$, in which $A$ makes at most $q_p$ queries to oracle $Q^{\text{IBKEM}}_{\text{DK}}(\cdot)$. To prove this theorem, we will construct a PPT algorithm $B$ that plays the Anon-SS-ID-CPA game with adversary $A$ and utilizes the capability of $A$ to break the $(\ell + 1)$-MDDH assumption in $\text{MG}_{\ell + 1}(1^k)$. The constructed algorithm $B$ in the Anon-SS-ID-CPA game is as follows.

\begin{itemize}
  \item \textbf{Setup Phase:} Algorithm $B$ gets as input an $(\ell + 1)$-group system $\text{MPG}_{\ell + 1}$ and group elements $g, g^{x_1}, \ldots, g^{x_{\ell + 2}} \in \mathbb{G}_1$ and $S \in \mathbb{G}_{\ell + 1}$, where either $S = \hat{e}(g^{x_1}, \ldots, g^{x_{\ell + 2}})$ (i.e., $S$ is real) or $S \in \mathbb{G}_{\ell + 1}$ uniformly (i.e., $S$ is random).
  \item $B$ generates a $(q_p, 2)$-MPHF $\mathbf{H}$ into $\mathbb{G}_1$, sets up the master public key as $\mathbf{PK} = (\text{MPG}_{\ell + 1}, h_k, \mathbf{H}, h, h', \mathbb{T}_{\mathbf{D}}, \mathbf{K}, \mathbf{C})$ for $(h, h') = (g, g^{x_1})$ and $(h, tk) \leftarrow \text{TGen}(1^k, g^{x_1}, \ldots, g^{x_2}, g)$, finally sends $\mathbf{PK}_{\text{IBKEM}}$ to adversary $A$. Here, we use the $\text{TGen}$ and $\text{TEval}$ algorithms of the $(q_p, 2)$-MPHF property of $\mathbf{H}$.
  \item \textbf{Query Phase 1:} Adversary $A$ adaptively issues the following query multiple times.
    \begin{enumerate}
      \item Decryption Key Query $Q^{\text{IBKEM}}_{\text{DK}}(\text{ID})$: Taking as input an identity $\text{ID} \in \mathbb{T}_{\text{IBKEM}}$, algorithm $B$ does the following steps:
        \begin{enumerate}
          \item Compute $\text{TEval}(td, \text{ID}) = (a_{ID}, B_{ID})$;
          \item If $a_{ID} = 0$, return $\hat{S}_{ID} = \hat{e}(B_{ID}, h')$; otherwise, abort and output $\bot$;
        \end{enumerate}
      \item Note that we have $\hat{S}_{ID} = \hat{e}(B_{ID}, h') = \hat{e}(B_{ID}, h^{x_1 + 1} = \mathbf{H}_{kk}(\text{ID})^{x_1 + 1}$. So $B$ can answer a $Q^{\text{IBKEM}}_{\text{DK}}(\text{ID})$ query of $A$ for identity $\text{ID}$ precisely when $a_{ID} = 0$.
    \end{enumerate}
  \item \textbf{Challenge Phase:} Adversary $A$ sends two challenge identities $\text{ID}_0^A \in \mathbb{T}_{\text{IBKEM}}$ and $\text{ID}_1^A \in \mathbb{T}_{\text{IBKEM}}$ to algorithm $B$; $B$ picks $\hat{d} \overset{\$}{\leftarrow} \{0, 1\}$, and does the following steps:
    \begin{enumerate}
      \item Compute $\text{TEval}(td, \text{ID}_0^A) = (a_{ID_0^A}, B_{ID_0^A})$ and $\text{TEval}(td, \text{ID}_1^A) = (a_{ID_1^A}, B_{ID_1^A})$;
      \item If $a_{ID_0^A} = 0 \vee a_{ID_1^A} = 0$, then abort and output $\bot$;
      \item Send the challenge key-and-encapsulation pair $(\hat{K}_d^A = S^{a_{ID_0^A}} \cdot \hat{e}(B_{ID_0^A}, g^{x_1 + 1}, g^{x_{\ell + 2}}), \hat{C}_0^A = g^{x_{\ell + 2}})$ to adversary $A$;
    \end{enumerate}
  Suppose algorithm $B$ does not abort (i.e., both $a_{ID_0^A} \neq 0$ and $a_{ID_1^A} \neq 0$ hold), we have $\mathbf{H}_{kk}(\text{ID}_0^A) = \hat{e}(g^{x_1}, \ldots, g^{x_{\ell + 1}}) \cdot \hat{e}(B_{ID_0^A}, h)$ and $\mathbf{H}_{kk}(\text{ID}_1^A) = \hat{e}(g^{x_1}, \ldots, g^{x_{\ell + 1}}) \cdot \hat{e}(B_{ID_1^A}, h)$. Furthermore, if $S = \hat{e}(g^{x_1}, \ldots, g^{x_{\ell + 1}})$, we have $K_{\hat{d}} = S^{a_{ID_0^A}} \cdot \hat{e}(B_{ID_0^A}, h^{x_1 + 1}, g^{x_{\ell + 2}}) = \hat{e}(\mathbf{H}_{kk}(\text{ID}_0^A), g^{x_{\ell + 1}})^{x_1 + 2}$. This implies that the challenge key-and-encapsulation pair $(\hat{K}_d^A, \hat{C}_0^A)$ is a valid one in this case. Otherwise, $\hat{K}_d^A$ contains no information about $\hat{d}$.
  \item \textbf{Query Phase 2:} This phase is the same as \textbf{Query Phase 1}. Note that in \textbf{Query Phase 1} and \textbf{Query Phase 2}, adversary $A$ cannot query the decryption key corresponding to the challenge identity $ID_0^A$ or $ID_1^A$.
  \item \textbf{Guess Phase:} Adversary $A$ sends a guess $\hat{d}$ to algorithm $B$. Let $\overline{\text{Abort}^L}$ denote the event that $B$ does not abort in the previous phases. Let $\mathcal{I} = \{I_{D_1}, \ldots, I_{D_{q_B}}, I_{D_0^A}, I_{D_1^A}\}$ be the set of the queried IDs by $A$ and the challenge identities $ID_0^A$ and $ID_1^A$. Let $P_2 = Pr[\overline{\text{Abort}^L} | \mathcal{I}]$, which will be decided later. As in [9, 43], $B$ “artificially” aborts with probability $1 - (1 - (P_2 \cdot p(k)))$ for the polynomial $p(k)$ from Definition 3, and outputs $\bot$. If it does not abort, $B$ uses the guess of $A$. This means that if $\hat{d} = \hat{d}'$, $B$ outputs 1, otherwise it outputs 0.
  \item In \textbf{Guess Phase}, $B$ did not directly use the guess of $A$, since event $\overline{\text{Abort}^L}$ might not be independent of the identities in $\mathcal{I}$. So $B$ “artificially” aborts to achieve the independence. Let $\overline{\text{Abort}^L}$ be the event that $B$ does not abort in the above game. We have that $Pr[\overline{\text{Abort}^L}] = 1 - Pr[\overline{\text{Abort}^L} | \mathcal{I}] - Pr[\overline{\text{Abort}^L} | (\mathcal{I} \cdot (1 - (1 - (P_2 \cdot p(k))))) = 1/p(k)$. Hence, we have $Pr[B = 1 | S \text{ is real}] = Pr[\overline{\text{Abort}^L} \cdot (1/2 + Adv^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A})]$, and $Pr[B = 1 | S \text{ is random}] = Pr[\text{Abort}^L] \cdot \frac{1}{2}$, where $\frac{1}{2} + Adv^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A}$ is the probability that $A$ succeeds in the Anon-SS-ID-CPA game of IBKEM. Further, we have $Pr[B = 1 | S \text{ is real}] - Pr[B = 1 | S \text{ is random}] = \frac{1}{p(k)} \cdot Adv^{\text{Anon-SS-ID-CPA}}_{\text{IBKEM},A}$.

Hence, $B$ breaks the $(\ell + 1)$-MDDH assumption if and only if $A$ breaks the Anon-SS-ID-CPA security of the above IBKEM scheme.

Finally, to evaluate $P_2$, we can only approximate it (up to an inversely polynomial error, by running $\text{TEval}$ with freshly generated keys sufficiently often), which introduces an additional error term in the analysis. We refer to [43] for details on this evaluation.