Fractal Analysis of Rock Joint Profiles

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Abstract. Surface reliefs of rock joints are analyzed in geotechnics when shear strength of rocky slopes is estimated. The rock joint profiles actually are self-affine fractal curves and computations of their fractal dimensions require special methods. Many papers devoted to the fractal properties of these profiles were published in the past but only a few of those papers employed a convenient computational method that would have guaranteed a sound value of that dimension. As a consequence, anomalously low dimensions were presented. This contribution deals with two computational modifications that lead to sound fractal dimensions of the self-affine rock joint profiles. These are the modified box-counting method and the modified yardstick method sometimes called the compass method. Both these methods are frequently applied to self-similar fractal curves but the self-affine profile curves due to their self-affine nature require modified computational procedures implemented in computer programs.

1. Introduction
Many authors in the past tried to determine joint rock coefficients $JRC$ introduced by Barton and Choubey [1, 2] by means of the fractal dimensions $D$ of joint rock profiles. Usually, they introduced intuitive functional relationships $JRC = f(D)$ optimized by a regression method, e.g. by the least square method. A comprehensive overview of existing empirical fitting patterns $JRC = f(D)$ can be found in the recently published paper by Li and Huang [3]. This paper refers to 19 empirical fitting patterns that can be divided into 5 groups:

$$JRC = \sum_{n=0}^{N} c_n D^n, \quad N = 1 \text{ or } N = 3$$ (1)

$$JRC = \sum_{n=0}^{N} c_n (D-1)^n, \quad N = 2$$ (2)

$$JRC = c(D-1)$$ (3)

$$JRC = c(D-1)^\beta$$ (4)

$$JRC = F(D-1)$$ (5)
where the coefficients $c$, $c_n$, $\beta$ and others that are hidden in the fractional functions $F$ associated with group (5) are fitting constants optimized by various regression methods. The papers referred to in the mentioned overview [3] presented fractal dimensions of rock joint profiles derived from either the standard Barton's profiles [1, 2] or the profiles of natural specimens. The vast majority of those dimensions are anomalously low $D \in (1, 1.09)$. The values about $D \approx 1.09$ represent very smooth surfaces. In practice, when analysing natural rock joints some researchers used to find essentially larger $D$ values. For example, Car [4] when analysing rock surfaces at Yucca Mountain in Nevada found power spectrum fractal dimensions reaching up to 1.467, whereas the yard-stick fractal dimensions of the same surfaces did not exceed the value 1.032. The reason why such a low value of $D$ was estimated consisted in the fact that the computations did not employ a convenient method that would have been adapted to self-affine profiles. Brown [5] was probably the first researcher who expressed the opinion that such anomalously low dimensions may have caused by inconvenient computational procedures. The valuable discussions of the problems concerning the computations of fractal dimensions of self-affine curves can be found in the papers by Brown [5] and Malinverno [6].

In the present paper we briefly describe how to modify the conventional box-counting and yard-stick methods to obtain sound values of fractal dimensions for self-affine profiles and, in addition, we present several results performed by these modified methods.

2. Modified computational methods

The problem of the self-affine curves consists in the fact that in contrast to self-similar curves the self-affine ones are not identically scaled in $x$- and $y$-directions and the common computational methods have to be adapted to this self-affine property. The different scaling in $x$- and $y$-directions means that when the $x$-coordinates of a self-affine curve are multiplied by a positive constant $b$, then the corresponding $y$-variables have to be multiplied by a different positive constant $Hb$ in order to receive statistically similar curves [8], where $H$ is the so-called Hurst exponent [8] and $b$ is the so-called scaling constant.

When exploring scanned curves of fractured surfaces like Barton profiles (figure 1), it is not a priory known their scaling pattern and thus it is necessary to test it. The most often type of tests consists in subsequent multiplying the $y$-coordinates of the self-affine curve by large numerical values and in computing corresponding fractal dimensions up to their limiting stable magnitudes [5], [6]. It means that prior to computations of fractal dimensions by the box-counting method or the yard-stick method it is necessary to multiply the $y$-coordinates by a larger numerical value and then start with computations of fractal dimension. By repeating this procedure with larger numerical values, a set of growing fractal dimensions are obtained that asymptotically approach a limit, i.e. a stable fractal dimension.

As has been mentioned in the foregoing paragraph, the modified computational procedures require the scanned self-affine profiles to be pre-processed in the vertical direction (modified $y$-coordinates). Afterwards, he pre-processed self-affine curves may be subjected to standard fractal computations to find a stable limiting dimension.

In the next section, a numerical example of the modified computational procedure is presented to illustrate the set of subsequently growing fractal dimensions. Besides these modified computations, a normal unmodified computations will be augmented to offer a comparison between modified and unmodified computational procedures.
3. Results and discussions

We implemented the compass method within the Matlab environment and computed the fractal dimension of the Barton self-affine profile JRC (16-18). Subsequently multiplying y-coordinates by a series of large numbers (1x, 50x, 100x, 1000x), we received a set of growing dimensions (1.014, 1.294, 1.299, 1.301). These results show that the unmodified computation (1x) indeed results in anomalously low dimension (1.014) whereas the limiting stable computation (1000x) leads to rather higher dimension (1.301). We have also repeated these computations by the box-counting method and have obtained the stable dimension quite close to that of the modified compass method, i.e. ~1.3. These results may be easily correlated with the irregularity of the investigated Barton curve JRC (16-18) shown in figure 1. This curve is not smooth enough to be ascribable to anomalously low dimension 1.014 that represents an almost perfectly smooth straight line. The value 1.3 is more realistic due to higher irregularity of this self-affine curve. This example straightforwardly illustrates that an improper computational procedure applied to self-affine curves inevitably leads to anomalously low dimensions that are far from reality. In the past, the compass method and the box-counting method were frequently applied to non-modified profile curves and, as a consequence, many authors reported very low values of fractal dimensions.

Figure 1. Standard JRC profiles after Barton and Choubey [2]
Many papers with anomalously low dimensions have been summarized in the review by Li and Huang [3]. According to Mandelbrot [8], each self-affine fractal possesses both local and global dimensions. The regions of local and global dimensions are separated by the so-called crossover boundary (i.e. by the crossover point when a self-affine curve is investigated). As soon as the computations overcome the crossover point and run in the global region, the dimension successively approaches 1.

4. Conclusions
The present contribution has discussed the problem concerning correct computations of fractal dimensions of rock joint profiles. These profiles are used in geotechnical practice for assessing joint rock coefficients. Since rock joint profiles are self-affine curves, they require a special treatment to determine correct values of fractal dimensions. When using standard compass or box-counting methods, the pre-processing of the curves is necessary. The y-coordinates of these curves are repeatedly multiplied by large numbers and corresponding dimensions are computed. The computations are finished as soon as the limiting value of the dimension is reached. Such a computational procedure takes into account different scaling properties in vertical and horizontal directions of the two dimensional self-affine profiles. Neglecting these self-affine properties leads to anomalously low values of fractal dimensions.

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