A new design concept of fully grouted rock bolts in underground construction

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Abstract. The main problem after excavating an underground excavation is to maintain the stability of the excavation for a certain period of time. Failure in meeting this demand is a threat to safety of men and equipment. Support and reinforcement are different instruments with different mechanisms. Among the common support systems in tunnelling and mining, rock bolts have been widely used to reinforce rock mass and also to reduce geological hazards. Furthermore rock bolts can be applied under varying different geological conditions with cost-effectiveness. Although different methods are developed for grouted rock bolts design until now, the interaction mechanism of the rock bolts and rock mass is still very complicated issue. The paper addresses an analytical model for the analysis and design of fully grouted rock bolts based on the reinforcement principle. According to this concept the jointed rock mass reinforced by grouted rock bolts is considered as composite material which includes rock mass, the grout material and the bolt shank. The mechanical properties of this composite material depend on the ratio of the components. The closed-form solution was developed based on the assumption that the rock mass around a circular tunnel remained elastic after installing fully grouted rock bolts. The main parameters of the rock-bolt system (the diameter and length of bolt shank, the space between the bolts) are then easily estimated from the obtained solution.

1. Introduction

Rock bolts are nowadays an effective and widely used support measure and an element of reinforcement in civil engineering and mining. Furthermore, rock bolts can be applied under very different geological and technical conditions to stabilize the rock mass. Rock bolts are cost effective because of their simple composition and their relatively low labour and energy consumption.

Although a great number of empirical, experimental, analytical and numerical methods have been developed to simulate bolting effects and at the end to design rock bolts as a support means in underground excavations, informed and discussed in many published articles, the interaction mechanisms of the rock bolts and the rock mass stay still a very complicated issue and are not well understood. There isn’t a concrete theoretical concept for determining all the parameters of a rock bolts system. Design of systematic rockbolt reinforcement in tunnelling is normally done by application of empirical and rational approaches.
Several analytical models have been developed to study the interaction behavior of bolts and rock mass, and to evaluate the effect of the bolts on stress and strain behavior of the rock around the underground opening [1-17]. Samit and Anand [18] proposed an analytical model using beam-column theory to analyze the reinforcement by the rockbolts. They found that the critical buckling load of the rock beam is influenced by the rock modulus. Another analytical model has been suggested by [5] in order to describe the reinforcement effect of rockbolts. Indraratna and Kaiser [3] have also been developed an analytical model using convergence - confinement method to analyze the effect of the bolt pattern on the extent of the yield zone and tunnel deformation. Carranza-Torres [15] has been proposed an analytical model of rockbolt reinforcement around tunnels and found that reinforcement can have a significant mechanical effect: increasing the confinement and decreasing the convergences. A simple analytical method was proposed by [12] for the analysis of the load on a single rockbolt. Those methods are based on rock-support interaction theory or convergence-confinement approach or based on composite material concept.

The analytical solutions for the composite element of the rock bolts and rock mass have been found in elsewhere [3], [9], [12], [19-20]. However, such analytical models did not consider the material components of rock bolt include the rock mass, steel bar and grout in detail.

The paper presents at first a simple analytical method for design of rock bolts based on the reinforcement principle and proposes a procedure for design of rock bolts system.

2. Analytical solution

2.1. Simplify model and concept

The problem to be solved is a circular tunnel of radius R excavated at a depth H in a homogeneous, isotropic and initially elastic rock mass. The tunnel is subjected to a hydrostatic stress field \( \sigma_0 = \gamma H \), where \( \gamma \) is the ground unit weight. The rock mass in primary state is elastic and the mechanical properties are described by the Young modulus \( E_{rm} \), Poisson’s ratio \( \nu_{rm} \) and axial compressive strength \( \sigma_{crm} \). After excavation a plastic zone would be built around the tunnel perimeter, at a certain distance behind the face if no support is installed. So a grouted rock bolts system is to be installed just in time in order to keep the rock mass elastic based on the assumption that the rock bolts system could improve the strength and decrease the deformability of the supported rock mass as a composite material. There are two zones that exist around the tunnel now, a zone of elastic rock composite and a zone of elastic rock mass (Figure 1). The rock composite has then the Young modulus \( E_{rm-wb} \) the Poisson’ ratio \( \nu_{rm-wb} \) and axial compressive strength \( \sigma_{c-wb} \). For simplification it’s assumed that \( \nu_{rm} = \nu_{rm-wb} = 0.5 \).

To evaluate the stability of the rock mass before excavation a stability factor \( (n_0) \) is used in simple way based on the elastic solution for stress and displacement distribution. The stability factor \( (n_0) \) is defined as the ratio between axial compressive strength of rock mass \( \sigma_{crm} \) and the maximal tangential stress on the perimeter \( \sigma_p = 2\sigma_n \) from the well-known solution as following:

\[
(n_0) = \frac{\sigma_{crm}}{2\sigma_n}
\]  

(1)

The rock mass is stable, if \( n_0 \geq 1 \) and is not stable if \( n_0 < 1 \) and needs to support, in these case for example with grouted rock bolts.

The grouted bolts with the length of L are placed along the radial direction around the tunnel. The mechanical properties of the rock mass with bolts are characterized by elastic modulus \( E_{rm-wb} \), Poisson’s ratio \( \nu \) and axial compressive strength \( \sigma_{c-wb} \) and are calculated from the mechanical parameters of the rock mass, the grouted material, the bolt rank and the geometrical parameters of the borehole, the steel bar and the opening based on the homogenization theory at any point as a function of r as follows [6]:

\[
\frac{E_{rm-wb}}{E_{rm}} = \frac{\sigma_{c-wb}}{\sigma_r} = \alpha = 1 + \frac{\pi d^2_h}{4} \frac{(E_{sb} - E_{sb}) + d^2_b(E_{sb} - E_{sb})}{a^2 E_{rm}} \frac{R}{r} = 1 + \frac{A_s R}{r}
\]  

(2)
$$A_i = \frac{\pi d_{bh}^2 (E_{grt} - E_{m}) + d_{sb}^2 (E_{sb} - E_{m})}{a^2 E_{m}}$$

with

Where: $d_{bh}$, $d_{sb}$, $E_{grt}$, $E_{sb}$, $a$ and $\alpha$ are the diameter of the borehole, the diameter of the steel bar, the elastic modulus of the grout, the elastic modulus of steel bar, the distance between the rock-bolts and the reinforcement factor respectively.

Figure 1. The axisymmetric tunnel problem

At this point it is to be mentioned that the concept of strengthening the rock masses by rock bolts as well as the use of homogenised model is not new. There are already some suggestions based on theoretical reasoning [3], [12], [18], [20] or experiment results [21-23], however there is no relationship between the reinforcement factor with the rock bolts parameter as in the formula (2).

The tunnel has then a support with uniform bolts on the perimeter of the tunnel, and it is subjected to far-field stresses $\sigma_0 = \gamma H$. The rock bolts system is to design so that the supported rock mass will be stable and remain elastic.

Solving this boundary problem, we obtain the stress field distribution in the rock mass zone with bolts surrounding the tunnel in polar coordinates as follows:

$$\sigma_{r-wb} = 2\sigma_0 \left[ \frac{r}{4} - \frac{A}{2r} + \frac{1}{3r^3} + \frac{A}{2r^2} + \frac{1}{3r} \right]$$

$$\sigma_{\theta-wb} = 2\sigma_0 \left[ \frac{r}{4} - \frac{1}{2r^2} + \frac{2A}{3r^3} + \frac{1}{2r^2} + \frac{A}{3r^2} \right]$$

$$\sigma_{r-wlb} = \sigma_0 - \sigma_0 - \sigma_0 \left[ 1 - \frac{1}{(R+L)^2} \frac{1}{B} \right] \left[ \frac{R}{r} \right]^2$$

$$\sigma_{\theta-wlb} = \sigma_0 + \sigma_0 - \sigma_0 \left[ 1 - \frac{1}{(R+L)^2} \frac{1}{B} \right] \left[ \frac{R}{r} \right]^2$$

Where: $\sigma_{r-wb}$ is the radial stresses in rock mass with bolt-grout support; $\sigma_{\theta-wb}$ is the tangential stresses in rock mass with bolt-grout support; $\sigma_{r-wlb}$ is the radial stresses in rock mass without bolt-grout support; $\sigma_{\theta-wlb}$ is the tangential stresses in rock mass without bolt-grout support.
\begin{equation}
B = \frac{1}{R^2} + \frac{2}{3} \frac{A(R + L)^3 - R^3}{R(R + L)^2}
\end{equation}

The tangential stress at the perimeter of the tunnel is determined by substitution \( r = R \) into equation (5):

\begin{equation}
\sigma_{\theta - \text{ub} - R} = \frac{1 + A_h}{1 + \frac{2}{3} A_h \frac{(R + L)^3 - R^3}{(R + L)^2}} = \frac{(1 + A_h)(R + L)^3}{1 + \frac{2}{3} A_h (R + L)^3 - \frac{2}{3} A_h R^3}
\end{equation}

The normal displacement at the perimeter of the tunnel can be estimated by the following equations:

\begin{equation}
u_{\theta - \text{ub} - R} = \frac{3\sigma_{\theta}}{2E_{\text{on}} 1 + \frac{2}{3} A_h \left(1 - \frac{R^3}{(R + L)^3}\right)} = \nu_0 \frac{1}{1 + \frac{2}{3} A_h \left(1 - \frac{R^3}{(R + L)^3}\right)}
\end{equation}

Where: \( \nu_0 \) is the elastic radial displacement of the tunnel perimeter after excavation.

The compressive strength of the rock mass after bolting is determined from equation (2) as follows:

\begin{equation}\sigma_{\text{cm - ub}} = (1 + A_h)\sigma_{\text{cm}}\end{equation}

The tangential stress and normal displacements at the perimeter of the tunnel before bolting can be expressed by using elastic solution as follows:

\begin{equation}\sigma_{\theta 0} = 2\sigma_0 \end{equation}

\begin{equation}\nu_0 = \frac{3\sigma_{\theta}}{2E_{\text{on}}} R \end{equation}

Comparison of the stress and displacement in the rock mass with bolts and without bolts, it is shown that:

\begin{equation}\sigma_{\theta - \text{ub} - R} > \sigma_{\text{cm - ub}} ; \nu_{\theta - \text{ub} - R} < \nu_0 \end{equation}

This demonstrates that the displacement of the rock mass surrounding tunnel after bolting is smaller than rock mass before bolting. But the stress is greater. Therefore, the stability of the rock mass after bolting needs to be estimated carefully by calculating and adjusting the bolt parameters.

Assuming that \( n_0 \) and \( n \) are the safety factor in terms of the rock mass before and after bolting on the perimeter of the tunnel respectively, defined as follows:

\begin{equation}n_0 = \frac{\sigma_{\text{cm}}}{\sigma_{\theta 0}} = \frac{\sigma_{\text{cm}}}{2\sigma_0} \end{equation}

\begin{equation}n = \frac{\sigma_{\text{cm - ub}}}{\sigma_{\theta - \text{ub} - R}} \end{equation}

If \( n_0 < 1 \), the rock mass surrounding tunnel is considered instable and needs to be bolted to maintain stable with \( n > 1 \). Hence, if the safety factor \( n \) for the rock mass is given, it is clearly that the bolt parameters (length, density, diameter of bolt, diameter of borehole, etc.) can be easily determined in relation of the mechanical properties of the rock mass and bolts.

After substituting the expressions (10) and (12) into equation (17), the safety factor of rock mass on the perimeter of the tunnel after bolting can be presented as follows:

\begin{equation}n = \frac{(1 + A_h)\sigma_{\text{cm}}}{(1 + A_h)\sigma_{\theta 0}} = \frac{(1 + A_h)\frac{\sigma_{\text{cm}}}{2\sigma_0}}{(1 + A_h)\frac{\sigma_{\text{cm}}}{2\sigma_0}} = \frac{(1 + \frac{2}{3} A_h (R + L)^3 - \frac{2}{3} A_h R^3}{(1 + A_h)(R + L)^3} = \left(\frac{1 + A_h}{(1 + A_h)(R + L)^3}\right) = \frac{1 + A_h}{(1 + A_h)(R + L)^3} \end{equation}
\[
\frac{(R + L)}{R^2} = \frac{2}{3} A_n \rho_n
\]

\(\Rightarrow\)

The length of bolt can be expressed from equation (19) as follows:

\[
L = R \left( \frac{2}{3} A_n \rho_n \right) \left( 1 - 1 + \frac{2}{3} A_n \rho_n - n \right)
\]

(20)

Because the length of the bolt must be greater than 0 \((L > 0)\), so that means:

\[
\left( 1 + \frac{2}{3} A_n \right) \rho_n - n > 0
\]

(21)

\(\Rightarrow\)

Combination of equation (3) and (21) leads to determine the distance between the bolts at the perimeter of tunnel based on the diameter of bolt, diameter of borehole, grout properties as follows:

\[
a < 4 \frac{\pi d_{sb} (E_{cs} - E_{cm}) + d_{bh} (E_{cs} - E_{cm})}{\left( \frac{3}{2} \frac{n}{\rho_n} - 1 \right) E_{cm}}
\]

(23)

However, the distance between the bolts can be estimated also based on experience. It can be seen from (21) that the safety factor \(n\) of the rock mass surrounding the tunnel after bolting can be reached maximum value as following expression:

\[
n_{max} = \left( 1 + \frac{2}{3} A_n \right) \rho_n
\]

(24)

That means the safety factor \(n\) of the rock mass after bolting is varied in the range of

\[1 < n < n_{max}\]

(25)

2.2. Calculation example

The case analyzed corresponds to a deep circular tunnel with \(R = 4.0\) m, excavated at a depth \(H=300\) m in a homogeneous rock mass. The rock mass has following elastic properties: axial compressive strength, \(\sigma_{cr} = 60\) MPa; unit weight, \(\gamma = 27\) kN/m\(^3\); rock mass rating index, \(RMR = 72\); elastic modulus, \(E_r = 0.5\) GPa. The support of the tunnel is provided by rock bolts uniformly distributed along the perimeter of the tunnel. The properties of the rock bolts are: elastic modulus of steel bar, \(E_{sb} = 210\) GPa; elastic modulus of grout, \(E_{grt} = 30\) GPa; diameter of steel bar, \(d_{sb} = 25\) mm; diameter of borehole, \(d_{bh} = 42\) mm. The strength of the rock mass can be estimated according to Hoek-Brown criterion [1] as follows:

\[
\sigma_{cm} = \sigma_c \sqrt{s}
\]

(26)

Where

\[
s = \exp \left( \frac{RMR - 100}{9} \right) = \exp \left( \frac{72 - 100}{9} \right) = 0.0466
\]

\(\Rightarrow\) \(\sigma_{cm} = 60 \sqrt{0.0466} = 12.96\) MPa

Assuming that the tunnel subjected to a hydrostatic stress field, the normal stress at the perimeter of excavated tunnel is calculated as follows:
\[
s = \exp\left( \frac{RMR - 100}{9} \right) = \exp\left( \frac{72 - 100}{9} \right) = 0.0466
\]

The safety factor of the rock mass before bolting is given as follows:

\[
n_b = \frac{\sigma_{cm}}{\sigma_{v0}} = \frac{12.96}{16.2} = 0.8
\]

Substitute the given parameters into equation (23), it is obtained the distance between bolts at the perimeter of the tunnel:

\[
a < 0.829 \text{ m}
\]

Using the equations (3) and (24) to determine the values of \(A_0\) and \(n_{\text{max}}\) with the distance between bolts are 0.8 m, 0.7 m and 0.6 m. The safety factor of the rock mass requirement after bolting is \(n = 1.0\). The length of rock bolts can be estimated then by equation (20). Table 1 lists the calculated results of rock bolts length with safety factor and distance between rock bolts are given.

**Table 1. The results for the length of rock bolts**

| a (m) | \(A_0\) | \(n_{\text{max}}\) | n | L (m) |
|-------|--------|----------------|---|-------|
| 0.6   | 0.717  | 1.18           | 1 | 0.5   |
| 0.7   | 0.527  | 1.08           | 1 | 1.1   |
| 0.8   | 0.403  | 1.01           | 1 | 2.9   |

The calculated rock bolts parameters indicated that it is quite consistent with the reality conditions and other methods.

3. Conclusion

The rock bolts are widely used as supporting system in mining and tunnelling. Rockbolt systems are designed until now by application of various empirical and rational approaches. An analytical model and its theoretical solutions are presented in this paper, and a procedure for design of fully grouted rockbolt system based on the reinforcement principle is developed.

A significant difference to other methods is that the calculation proceeds with a desired or predetermined safety factor and the strengthed rock mass remain elastic after installation of the rock bolts. And that is also the limit of the method in application.

The calculated rock bolts parameters based on this method indicated that it is quite consistent with the reality conditions and other methods. Therefore, it is completely possible to apply the proposed model for designing rock bolts system.

4. References

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