EFFECTS OF FORMATION EPOCH DISTRIBUTION ON X-RAY LUMINOSITY AND TEMPERATURE FUNCTIONS OF GALAXY CLUSTERS

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ABSTRACT

We investigate statistical properties of galaxy clusters in the context of a hierarchical clustering scenario, taking into account their formation epoch distribution; this study is motivated by the recent finding by Fujita and Takahara that X-ray clusters form a fundamental plane in which the mass and the formation epoch are regarded as two independent parameters. Using the formalism that discriminates between major mergers and accretion, the epoch of a cluster formation is identified with that of the last major merger. Since tiny mass accretion following formation does not much affect the core structure of clusters, the properties of X-ray emission from clusters are determined by the total mass and density at their formation time. Under these assumptions, we calculate X-ray luminosity and temperature functions of galaxy clusters. We find that the behavior of the luminosity function differs from the model that does not take into account formation epoch distribution; the behavior of the temperature function, however, is not much different. In our model, the luminosity function is shifted to a higher luminosity and shows no significant evolution up to $z \sim 1$, independent of cosmological models. The clusters are populated on the temperature-luminosity plane, with a finite dispersion. Since the simple scaling model in which the gas temperature is equal to the virial temperature fails to reproduce the observed luminosity-temperature relation, we also consider a model that takes into account the effects of preheating. The preheating model reproduces the observations much more accurately.

Subject headings: cosmology: theory — galaxies: clusters: general — X-rays: galaxies

1. INTRODUCTION

Galaxy clusters are the largest virialized objects in the universe and should be useful cosmological probes, since the properties of clusters are considered to depend on cosmological parameters and reflect the structure formation history. Galaxy clusters contain a diffuse hot gas that emits X-rays via thermal bremsstrahlung. The X-ray temperature and luminosity functions of clusters and their evolution have been used to determine the density of the universe and the amplitude of the rms density fluctuation. If there exist unique mass-temperature ($T$) and mass-luminosity ($L$) relations, one can predict X-ray temperature and luminosity functions of clusters at redshift $z$ from a theoretical mass function. Therefore, it is important to investigate correlations among physical quantities of clusters and examine if such unique relations exist. Recently, Fujita & Takahara (1999a) found that clusters at low redshifts ($z < 0.1$) form a plane (the fundamental plane) in three-dimensional space ($\log \rho_\text{gas,0}, \log r_c, \log T$), where $\rho_\text{gas,0}$ is the central gas density and $r_c$ is the core radius of clusters. The data on the plane are still correlated and form a band on the plane. The observed relation $L \propto T^3$ turns out to be the cross section of the band normal to the major axis. The existence of the fundamental plane implies that the clusters form a two-parameter family, which suggests that the physical quantities of clusters are determined by the halo mass and density at the time of their formation and that no unique mass-temperature or mass-luminosity relation exists (Fujita & Takahara 1999b). In fact, using $N$-body simulations, Navarro, Frenk, & White (1997) found that the structure of clusters is related to their formation epoch, although they claim that the clusters are a one-parameter family, assuming a unique relation between the mass and formation redshift. In this paper, motivated by the finding of the fundamental plane, we present a formulation of statistics of clusters in the context of hierarchical structure formation theory, taking into account the formation epoch distribution.

So far, Press & Schechter's (1974, hereafter PS) mass function has been widely used to calculate temperature and luminosity functions (e.g., Eke, Cole, & Frenk 1996). Although the PS mass function provides a number of dark halos at a given time, it does not provide information about the formation time of halos. Thus a conventional way to compute the temperature and luminosity functions is to simply assume that the formation time is the same as the observed time. Since, in this case, clusters form a one-parameter family about the mass, an extension of PS theory is necessary to investigate the two-parameter nature of clusters, taking into account the effect of the formation epoch distribution. Using the merger probabilities in an extended Press & Schechter clustering model (Bond et al. 1991; Bower 1991), Lacey & Cole (1993, hereafter LC) derived a formation epoch distribution function in an enlightening way, although LC did not calculate predictions of X-ray luminosity and temperature functions. The formation time in their model is defined by that time when the halo mass becomes half of that at the observed epoch; this definition does not discriminate between tiny and notable relative mass capture or between accretion and mergers. In the hierarchical clustering scenario, low-mass objects successively merge with one another to build up ever more massive objects. However, major deviations from equilibrium and subsequent relaxation take place only when halos of comparable masses merge, while tiny mass captures have little effect on the capturing halos, so it is desirable to devise such
formulations. Kitayama & Suto (1996a, 1996b) have attempted to describe the formation and destruction of halos within the extended PS prescription by discriminating between accretion and mergers. They define the halo formation and destruction rates and the formation epoch distribution by utilizing the survival probability. Although they predict various statistical properties of X-ray clusters, they assume an empirical relationship between temperature and luminosity in such predictions, which is not satisfactory from our point of view.

In this paper, we adopt another model of the formation epoch distribution, proposed by Salvador-Solé, Solanes, & Manrique (1999, hereafter SSM). They developed a modified version of the extended PS clustering model (Bond & Frenk 1998), we also investigate a simple preheating formulation between gas and dark matter results in an temperature functions. Since it is well known that a simple scaling model of X-ray clusters, and we calculate the $L$-scaling relation that differs from observations (Eke, Navarro, & Frenk 1998), we also investigate a simple preheating model. The paper is organized as follows. The formulation and destruction rates and the formation epoch distribution but also the halo mass at formation time. In order to obtain statistics of galaxy clusters, we assume that the halo mass and density at the last major merger determine the structure of the clusters. The temperature and luminosity of galaxy clusters are calculated in terms of the halo mass and density at formation time without resorted to empirical relations. On the basis of these prescriptions, we construct a simple scaling model of X-ray clusters, and we calculate the $L-T$ distribution and the luminosity and temperature functions. Since it is well known that a simple scaling model between gas and dark matter results in an $L-T$ relation that differs from observations (Eke, Navarro, & Frenk 1998), we also investigate a simple preheating model.

### 2. FORMULATION

#### 2.1. Formation Time Distribution

In this subsection, we summarize the SSM formalism. To follow the formation and evolution of halos, SSM used a modified version of the extended PS clustering model (Bond et al. 1991; Bower 1991; LC) and made a schematic distinction between minor and major mergers by defining the formation of a halo as the last major merger it experienced. This definition does not affect the abundance of halos at a given time, although it affects the description of their growth. Thus, the mass function is equal to the PS mass function, $M' > M$, or specific merger rate, is

$$ r^n_{L}(M \rightarrow M', t)dM' = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M} \frac{\delta(t)}{\sigma^2(M)} \left| \frac{d\sigma(M)}{dM} \right| \frac{d\delta(t)}{dt} \left[ \frac{1}{\sigma^2(M')} \right] dM' $$

$$ \times \left[ 1 - \frac{\sigma^2(M')}{\sigma^2(M)} \right]^{-3/2} \exp \left\{ - \frac{1}{2} \frac{\delta^2(t)}{\sigma^2(M)} \right\} dM' . \quad (2) $$

In the SSM formalism, it is assumed that a halo with mass $M$ experiences a major merger and is destroyed when the relative mass increment $\Delta M/M = (M' - M)/M$ exceeds a certain threshold $\Delta_m$. The major merger is regarded as the formation of a new halo. On the other hand, when $\Delta M/M < \Delta_m$, the event is regarded as continuous accretion; the halo keeps its identity and its core structure. Thus, from the specific merger rate (eq. [2]), the mass accretion rate $\dot{M}_{\text{acc}}(M, t) = \frac{dM}{dt}$ of halos with mass $M$ at time $t$ is defined as

$$ R_{\text{mass}}(M, t) = \int_{M}^{M+\Delta_M} \Delta M r^n_{L}(M \rightarrow M', t)dM'. $$

The destruction rate is defined as

$$ r^d(M, t) = \int_{M+\Delta_m}^{\infty} \Delta M r^n_{L}(M \rightarrow M', t)dM'. \quad (4) $$

The formation rate should be written as

$$ r'[M(t), t] = \frac{d\ln n[M(t), t]}{dt} + r^d[M(t), t] + \dot{\Delta}_M R_{\text{mass}}[M(t), t], \quad (5) $$

from the conservation equation for the number density of halos per unit mass, along the mean of mass accretion tracks $M(t)$, which is the solution of the differential equation

$$ \frac{dM}{dt} = R_{\text{mass}}[M(t), t]. \quad (6) $$

From this formation rate, one can obtain the distribution of formation time $t_f$ for halos with mass $M$ at time $t$:

$$ \Phi_f(t_f; M, t) dt_f = r'[M(t_f), t_f] \times \exp \left\{ - \int_{t_f}^{t} r'[M(t'), t'] dt' \right\} dt_f . \quad (7) $$

From equation (6), the halo mass at formation time, $M_f = M(t_f)$, becomes

$$ M_f = M - \int_{t_f}^{t} R_{\text{mass}}[M(t'), t'] dt'. \quad (8) $$

The value of $\Delta_m$ is fixed by the fits to the empirical mass-density (or mass-radius) correlation, obtained by $N$-body simulations (Navarro, Frenk, & White 1996, 1997). To be precise, $M_f$ for fixed $M$ and $z_f$ should have scatter around the mean accretion track (eq. [6]). Therefore, $M_f$ in equation (8) is an approximation of the mean forma-
tion mass of true distribution. Although we ignore this scatter for the sake of simplicity, it could be estimated by using the algorithm given in Nusser & Sheth (1999).

In this paper, we investigate two cosmological models, the standard cold dark matter (SCDM) model and the lambda cold dark matter (LCDM) model. In Table 1, we tabulate the cosmological density parameter ($\Omega_0$), the cosmological constant ($\lambda_0$), the Hubble constant ($h$) in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the present rms density fluctuation ($\sigma_8$) in spheres of radius $8 h^{-1} \text{ Mpc}$. Figure 1 shows the formation time distributions (eq. [7]) for the SCDM and LCDM models for several values of the present halo mass. Since in the LCDM model the universe is in the accelerated expanding stage at $z \sim 0$, the growth of density fluctuation has stopped. Thus, the slope of the formation time distribution at $z \sim 0$ in the LCDM model is less steep than it is in the SCDM model (Fig. 1). Figure 2 shows halo mass at its formation time (eq. [8]) for the halo with the present mass of $10^{15} M_\odot$.

Note that, in the PS formalism, there is no distinction between mergers and accretion. Any mass capture is regarded as a merger, i.e., $t_f = t$. Consequently, the formation time distribution is

$$\Phi_f(t_f; M, t) = \delta^D(t_f - t),$$

where $\delta^D(x)$ is Dirac’s delta function.

### 2.2. X-Ray Cluster Model

In the SSM formalism, the mass function of dark halos at a fixed epoch is given by the PS mass function. On the other hand, for a given mass, the dark halos range in formation time as depicted by equation (7). Thus, dark halos form a two-parameter family. SSM found that the characteristic density and the scale radius of a halo at the present epoch are proportional to the critical density of the universe and the virial radius of the halo at the formation time $t = t_f$, respectively. This means that, in between major mergers, halos grow gradually with the accretion of surrounding matter, while their centers remains unchanged. Considering the two-parameter family nature of clusters, we assume that the physical quantities of galaxy clusters are represented by the mass and the virial density at their formation time or the time of their last major merger. On this assumption, the temperature and luminosity of a cluster are determined by the formation redshift, $z_f$, and the halo mass, $M_f$, at $z_f$; $T = T[z_f, M_f(M, z_f)]$, $L = L[z_f, M_f(M, z_f)]$. Here the relation between $t_f$ and $z_f$ is straightforwardly determined once the cosmological model is specified. In order to obtain $L$ and $T$, we use the spherical collapse model (Tomita 1969; Gunn & Gott 1972), and we assume that the cluster is spherically symmetric and intracluster gas is in an isothermal hydrostatic equilibrium in the gravitational potential of dark matter halo.

![Figure 1](image1.png)

**Fig. 1.**—Formation redshift distribution function, $\Phi(z_f); M, z = 0$, for several values of the present halo mass in the (a) SCDM and (b) LCDM models.

In the spherical collapse model, the virial density, $\rho_f$, of a halo at formation redshift, $z_f$, is given by

$$\rho_f = \rho_0(z_f)\Delta_c(z_f) = \rho_c(1 + z_f)^3 \frac{\Omega_{\text{cr}}(1 + z_f)^3}{\Omega(z_f)},$$

where $\rho_0(z_f)$ is the critical density of the universe, $\Delta_c(z_f)$ is the ratio of the virial density to the critical density, and $\Omega(z_f)$ is the cosmological density parameter. The index 0 refers to the values at $z = 0$. We use the fitting formula of

| TABLE 1 |
| --- |
| **Cosmological Parameters of the Models** |
| Model | $\Omega_0$ | $\lambda_0$ | $h$ | $\sigma_8$ |
| SCDM | 1.0 | 0.0 | 0.5 | 1.0 |
| LCDM | 0.3 | 0.7 | 0.7 | 1.0 |
where \( \rho_{\text{gas},0} \) is the central gas density and \( r_c \) is the core radius. The central gas density is calculated through the relation

\[
\int_0^{r_c} \rho_{\text{gas}}(r) 4\pi r^2 \, dr = M_{\text{gas}} ,
\]

where \( M_{\text{gas}} \) is the total gas mass. From now on, we assume that \( r_c = r_f/8 \) (Fujita & Takahara 1999b).

Thus, from equations (14) and (15), the luminosity is given by

\[
L = 1.79 \times 10^{44} \left( \frac{k_B T}{1 \text{ keV}} \right)^{1/2} \left( \frac{M_f}{10^{15} M_\odot} \right) \times \left[ \frac{h^2 \Delta_c \Omega_0}{\Omega_0(z_f)} (1 + z_f)^{2/3} \right] f_m \text{ ergs s}^{-1} ,
\]

where \( f_m \equiv M_{\text{gas}}/M_f \) is the gas mass fraction, and we assume here \( \beta = 2/3 \) for simplicity. Equations (13) and (17) show that the temperature and luminosity are indeed functions of the halo mass and formation redshift, when one specifies the cosmological model and the relation between \( T \) and \( T_{\text{vir}} \).

2.3. Temperature and Luminosity Function

Next, we formulate statistics of galaxy clusters. Now that the temperature and luminosity are expressed as functions of \( t_f \) and \( M_f \), such that \( T = T[t_f, M_f(M, t_f)] \) and \( L = L[t_f, M_f(M, t_f)] \), we can derive temperature and luminosity functions using transformation of the two variables from \( (t_f, M) \) to \( (T, L) \) as follows.

For a mass range, \( M - M + dM \) at \( t \), the comoving number density of clusters formed at \( t_f \sim t_f + dt_f \) is given by

\[
n(t_f, M; t) dt_f dM = n(M, t) \Phi_f(t_f; M, t) dt_f dM .
\]

Thus the comoving number density of clusters at \( t \), with \( T \sim T + dT \) and \( L \sim L + dL \) is given by

\[
n(T, L; t) dT dL = n(t_f, M; t) \Phi_f(t_f; M, t) dt_f dM .
\]

Accordingly,

\[
n(T, L; t) = n(t_f, M; t) \frac{\partial (t_f, M)}{\partial (T, L)} .
\]

Since the comoving number density, \( n(T, L; t) \), gives the distribution function on the \( L-T \) plane, it reflects the \( L-T \) relation of clusters.

The temperature function and luminosity function are given, respectively, by

\[
n(T; t) = \int n(T, L; t) dL ,
\]

\[
n(L; t) = \int n(T, L; t) dT .
\]

It is to be noted that these functions take into account the distribution of cluster formation time \( t_f \).

Thus, given the relations \( T = T[t_f, M_f(M, t_f)] \) and \( L = L[t_f, M_f(M, t_f)] \), one can calculate the \( L-T \) distribution
function, the temperature function, and the luminosity function.

3. RESULTS AND DISCUSSION

In this section, we derive the temperature function, the luminosity function, and the $L$-$T$ distribution function. We assume that the gas mass fraction of clusters $f_m$, is the same as the cosmic baryon ratio $\Omega_b/\Omega_c$, where $\Omega_b$ is the density parameter of baryon, and we adopt $\Omega_b = 0.0125 h^{-2}$ to be consistent with the primordial nucleosynthesis.

3.1. Scaling Model

First, we examine the case where the gas temperature $T$ is equal to the virial temperature $T_{\text{vir}}$. In this case, the structure is self-similar, and we call this model the scaling model. We calculate the temperature and luminosity functions based on the SSM formalism (eq. [7]) and compare them with those based on the PS formalism (eq. [9]). The temperature function and redshift evolution are shown in Figure 3, where thin lines and symbols denote SSM and PS predictions, respectively. As is evident in the figure, there is little difference between the predictions of the two formalisms. This can be explained as follows. When we consider the distribution of the formation time, physical quantities of clusters are affected by two factors. First, if the object formed earlier, its virial density becomes larger (eq. [10]). Second, if the object formed earlier, its mass at formation time becomes smaller, for a given mass at $z = 0$ (Fig. 2). Gas temperature depends on the virial density and mass at the formation time, as $T \propto M_f/r_f \propto \rho_f^{1/3} M_f^{1/3}$, and these two factors tend to cancel out. This behavior of the temperature function is the same as that found in previous studies (e.g., Kitayama & Suto 1996b). This feature is common to both of the cosmological models that we investigate. The $z_f$ dependence of temperature, $T = T(z_f; M = 10^{15} M_\odot)$, is shown in Figure 4, which explicitly shows that $T$ is roughly constant for $z_f \lesssim 2$. Note that Mathiesen (1999) also finds no evidence for a correlation between X-ray temperature and formation time in his simulation of X-ray clusters. The
difference between the SCDM and LCDM models is mainly caused by the dependence on the time evolution of virial density (eq. [10]).

The luminosity function and redshift evolution are shown in Figure 5, where thin lines and symbols denote SSM and PS predictions, respectively. As is seen, the SSM formalism predicts larger number densities than the PS formalism; for the SSM formalism, there is little evolution, even in the SCDM model, in contrast with the PS formalism. This result is different from that of the temperature function because of the difference between the dependence of

![Fig. 3.—Temperature function at $z = 0$ and $z = 1$ in the (a) SCDM and (b) LCDM models. Symbols, predictions based on the PS formalism; thin lines, predictions based on the SSM formalism; thick line, power-law fit to the observed low-redshift temperature function, obtained by Henry (2000).]

![Fig. 4.—Temperature-formation redshift relation: $T = T(z_f; M = 10^{15} M_\odot)$. Thin lines, relation in the scaling model; thick lines, relations in the preheating model.]
temperature and luminosity on virial density and mass at formation time. Since \( L \propto \rho_f^2 r_f^3 T^{1/2} \propto \rho_f M_f T^{1/2} \propto \rho_f^{7/6} M_f^{2/3} \), the luminosity depends more strongly on the virial density than the temperature. The \( z_f \) dependence of luminosity, \( L(z_f, M = 10^{15} M_\odot) \), is presented in Figure 6, which shows that \( L \) increases with \( z_f \). Therefore, earlier formed dense clusters contribute to the increase of the luminosity function shown in Figure 5. Moreover, the increase of \( L \) with \( z_f \) explains that there is little evolution of the luminosity function from \( z = 1.0 \) to \( z = 0 \), in both the SCDM and LCDM universes. Since the growth of density fluctuations in the SCDM is more rapid than that in the LCDM model, one might think that the luminosity function in the SCDM model should evolve rapidly. However, the present result implies that this is not the case if we consider the effect of the distribution in the formation redshift. Thus, recent observational evidence suggesting little evolution of the luminosity function (Rosati et al. 1998; De Grandi et al. 1999; Nichol et al. 1999) does not necessarily mean that the SCDM is disfavored against LCDM model. It is also noted that if we consider the effect of formation epoch distribution, smaller values of \( \sigma_8 \) are needed to reproduce the observations, as compared to the PS formalism.

When we compare the predictions with the observations, one should note that the amplitude of temperature and luminosity functions can be adjusted by changing \( \sigma_8 \), so that we are concerned with their shape. In Figure 3, the thick line is the best power-law fit to the observed low-redshift temperature function obtained by Henry (2000). The predicted shape of the temperature function is consistent with observations for both the SCDM and LCDM models, although agreement is better for the LCDM model. In Figure 5, the thick line is the best-fitting Schechter function to the observed bolometric luminosity function, within \( z = 0.3 \), obtained by Ebeling et al. (1997). The predicted shape is much steeper than the observations for both SCDM and LCDM models. This is another representation of the well-known discrepancy of the \( L-T \) relation.

Next, we investigate the distribution on the \( L-T \) relation. For several values of \( n[\log (T), \log (L); z = 0] \), we plot iso-density contours on the \( L-T \) plane in Figure 7. From the relations \( T \propto \rho_f^{1/3} M_f^{2/3} \) and \( L \propto \rho_f^{7/6} M_f^{2/3} \), the luminosity

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**Fig. 5.—** Luminosity function at \( z = 0 \) and \( z = 1 \) in the (a) SCDM and (b) LCDM models. Symbols, predictions based on the PS formalism; thin lines, predictions based on SSM formalism; thick line, best-fitting Schechter function to the observed bolometric luminosity function for galaxy clusters within \( z = 0.3 \) obtained by Ebeling et al. (1997).

**Fig. 6.—** Luminosity-formation redshift relation: \( L = L(z_f, M = 10^{15} M_\odot) \). Thin lines, relation in the scaling model; thick lines, relations in the preheating model.
behaves as \( L \propto \rho_f^{1/2} T^2 \). As discussed in § 1, in the PS formalism, it is assumed that the observed redshift of a cluster is equal to the formation redshift \( z = z_f \). Thus, most of the observed clusters \( z \sim 0 \) have nearly the same virial density \( \rho_f(z) \), and their physical quantities depend only on mass. Therefore, in the PS formalism, clusters form a one-parameter family, shown by the thick line in Figure 7. On the other hand, in the SSM formalism, the virial densities have a wide range of values because \( \rho_f \) depends on the formation redshift. Thus, clusters form a two-parameter family. The scatter of the \( L-T \) relation, shown in Figure 7, reflects the dispersion of the halo formation time. Scharf & Mushotzky (1997) also pointed this out. Since \( \rho_f \) tends to be distributed more widely for smaller clusters, the scatter of \( L-T \) relation is larger at lower \( T \) and \( L \). Thus, the slope of \( L-T \) relation in the SSM formalism becomes shallower than \( L \propto T^2 \). This is, however, in conflict with the observed correlation \( L \propto T^3 \) (e.g., David et al. 1993). As long as we assume that the ratio of the core gas density to the virial density is constant, this tendency persists. To resolve this discrepancy, this ratio should vary such that less massive clusters have smaller baryon fraction at the cluster core, from which much of the X-ray emission originates (Metzler & Evrard 1994; Kay & Bower 1999; Valageas & Silk 1999; Wu, Fabian, & Nulsen 2000). Many authors have attributed such behavior to the preheating of intracluster gas (Kaiser 1991; Evrard & Henry 1991; Cavaliere, Menci, & Tozzi 1998; Balogh, Babul, & Patton 1999); i.e., they suggest that the intracluster gas was heated before the cluster formed. On the other hand, some authors claim that intracluster gas is heated after the formation of the cluster (Loewenstein 2000; Brighenti & Mathews 2001). Moreover, Bryan (2000) argued that the cooling of gas and the resultant galaxy formation are sufficient to explain the lower gas fraction in small clusters and groups without additional heating. Since these models give qualitatively similar gas distributions, we will adopt a preheating model in § 3.2. It is to be noted that future observations may discriminate between the heating and galaxy formation models (e.g., Fujita 2001).

### 3.2. Preheating Model

We examine the effects of preheating, using a simple model of Fujita & Takahara (2000) that is based on the models of Cavaliere et al. (1998). They combine the effects of shock heating and preheating, so that gas temperature is higher than the virial temperature, such that

\[
T = T_{\text{vir}} + \frac{3}{2} T_i ,
\]

where \( T_i \) is a given preshock temperature. In this model, \( \beta \) is given by

\[
\beta = \frac{T_{\text{vir}}}{T_{\text{vir}} + (3/2) T_i} .
\]

If \( T_i \) is not negligible compared to \( T_{\text{vir}} \), \( \beta \) becomes smaller, which means that hot gas expands and the gas density in the core decreases under the condition that the total gas mass of the cluster is constant. As a result, X-ray luminosity becomes smaller, which results in a steeper \( L-T \) relation. Comparison with observations of clusters suggests that the preheated temperature \( T_i \) is about 0.5–2 keV (Fujita & Takahara 2000). Here we adopt \( T_i = 1 \) keV.

Using equation (15), we define the normalized central gas density as

\[
f_d(\beta) = \frac{\rho_{\text{gas},0}}{\rho_f} = \frac{4 \pi n_m}{3 I_3(\beta) (r_c)} ,
\]

where

\[
I_3(\beta) = 4 \pi \int_0^{r_c} x^2 \left( 1 + x^2 \right)^{3/2} \frac{dx}{\left( 1 + x^2 \right)^{3/2}} dx .
\]
From equations (14), (15), and (25), the luminosity is written, using \( f_d(\beta) \), as

\[
L = 3.2 \times 10^{40} \left( \frac{k_b T}{1 \text{ keV}} \right)^{1/2} \left( \frac{M_f}{10^{15} M_\odot} \right) \\
\times \left[ \frac{h^2 \Delta_c}{\Omega_0 / \Omega(z_f)} \right] (1 + z_f)^3 f_d(\beta)^2 I_2(\beta) \text{ ergs s}^{-1},
\]

(27)

where we assume \( r_f/r_c = 8 \) and

\[
I_2(\beta) \equiv 4\pi \int_0^8 \frac{x^2}{(1 + x^2)^{3/2}} dx.
\]

(28)

Because \( \beta \) is a function of \( T_{\text{vir}} \) once \( T_e \) is fixed, \( T \) and \( L \) are functions of \( z_f \) and \( M_f \) in this preheating model, as is the case in the scaling model. Thus, we can calculate the \( z_f \) dependences of the temperature and luminosity functions using the formulation constructed in § 2.3. The \( z_f \) dependence of temperature, \( T = T(z_f, M = 10^{15} M_\odot) \), and that of the luminosity, \( L = L(z_f, M = 10^{15} M_\odot) \), are shown by thick lines in Figures 4 and 6, respectively. As a result of preheating, gas temperature becomes higher and luminosity becomes lower, as compared with the scaling model predictions for this case. The temperature and luminosity functions are shown in Figures 8 and 9, respectively. For several values of \( n[\log (T), \log (L); z = 0] \), isodensity contours on the \( L-T \) plane are plotted in Figure 10. As a result of the preheating, the gas temperature is raised and the gas expands, as compared to the case without preheating. Thus,
When the effect of preheating is large, the central gas density, $f_{\text{g}}$, is decreased, and luminosity is lowered. When $T_{\text{vir}} \sim T_{\text{i}}$, the effect of preheating is large, and luminosity is greatly decreased. On the other hand, when $T_{\text{vir}} \gg T_{\text{i}}$, a cluster is not much affected by the preheating, and $\beta \sim 1$. Since we assumed $\beta = \frac{2}{3}$ in the scaling model, the luminosity in the preheating model is larger than that in the scaling model in spite of preheating. Therefore, the slope of the $L$-$T$ relation in the preheating model is steeper than $L \propto T^2$ (Fig. 10), the number of clusters with $L > 10^{44}$ ergs s$^{-1}$ increases (Fig. 9), and the number of clusters with $L < 10^{44}$ ergs s$^{-1}$ decreases (Fig. 9). In the preheating model, there is also little evolution of luminosity function. The reason is the same as that for the scaling model.

In Figures 8 and 9, the thick lines are the observed temperature and luminosity functions, the same as in Figures 3 and 5. In both the SCDM and LCDM models, the slope of the luminosity function, based on the SSM formalism, is near to that observed, although it is still slightly steeper in the LCDM model. On the other hand, the slope of the temperature function is steeper that observed. Better data and preheating models are needed to resolve this mismatch. In Figure 10, the thick solid lines show the PS prediction, which becomes as steep as the observed slope because of the preheating effect. The thick dotted lines represent the observed $L$-$T$ relation obtained by David et al. (1993). Note that the observed $L$-$T$ relation has a large dispersion that is comparable to the predicted width. In both the SCDM and LCDM models, the predicted $L$-$T$ relation can match the observed relation rather well.

4. CONCLUSIONS

We have investigated the effects of formation epoch distribution on the statistical properties of galaxy clusters in the context of an hierarchical structure formation scenario. The mass and formation redshift of galaxy clusters constitute two independent parameters. In this way, using the formalism of Salvador-Solé, Solanes, & Manrique (1999), with a few plausible assumptions, we have derived temperature and luminosity functions and the distribution on the $L$-$T$ plane. First, we investigated a simple scaling model, in which gas temperature is equal to virial temperature. The temperature function is almost the same as that in the PS formalism, while the luminosity function is shifted to a higher luminosity and shows no significant evolution independent of the cosmological model because earlier formed clusters have denser intracluster gas in the cluster core. The luminosity-temperature relation becomes a band with a broad width instead of a line, but its slope becomes a little flatter than that in the PS formalism, which is inconsistent with the observations. Second, we have examined a simple preheating model in this framework. Preheating makes the gas distributions of poor clusters flatter than those of rich clusters, which reduces the X-ray luminosity. The resultant $L$-$T$ relation is steeper than that in the scaling model and becomes consistent with observations. Although the temperature and luminosity functions are broadly consistent with observations, better observations and preheating models are needed for quantitative comparisons.

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Fig. 10.—Isodensity contours on the $L$-$T$ plane in the (a) SCDM and (b) LCDM models. The values of $\log \{n/\log (T), \log (L); z = 0\}$ are separated at equal logarithmic intervals by 1.25 and range from $-4$ to $-12.75$. Thick line, predicted $L$-$T$ relation based on the PS formalism, corresponding to the case of $z_f = 0$; dotted line, power-law fit to the observed $L$-$T$ relation obtained by David et al. (1993).
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