CP Violating Observables in $e^-e^+ \rightarrow W^-W^+$

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ABSTRACT

We consider various integrated lepton charge-energy asymmetries and azimuthal asymmetries as tests of CP violation in the process $e^-e^+ \rightarrow W^-W^+$. These asymmetries are sensitive to different linear combinations of the CP violating form factors in the three gauge boson $W^-W^+$ production vertex, and can distinguish dispersive and absorptive parts of the form factors. It makes use of purely hadronic and purely leptonic modes of $W$’s decays as well as the mixed modes. The techniques of using the kinematics of jets or missing momentum to construct CP–odd observables are also employed. These CP violating observables are illustrated in the generalized Left-Right Model and the Charged Higgs Model.

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I. INTRODUCTION

Since the discovery \([1]\) of CP violation (CPV) in 1964, a satisfactory understanding of its origin has eluded us. One reason for this is that there is very little experimental information about CPV. The measurement of the \(\epsilon\) parameter in the \(K^0 - \bar{K}^0\) system remains the only evidence of CPV. The searches for CP violating effects in other physical systems (such as the \(\epsilon'\) parameter or the electric dipole moments of neutron or electron) have produced only negative constraints. The detection of new CP violating effects will certainly greatly enhance our understanding of this phenomenon. One potential stage for such new effects are high energy collisions in existing or future colliders. This is an exciting possibility because it is well-known that within the Standard Kobayashi–Maskawa (KM) Models \([2]\) the possibility of detecting CPV in high energy collisions is very small. Therefore, any evidence we can record in this arena will be a window into the physics beyond the Standard Model.

In this paper we shall explore the possibility of detecting CPV in the production of \(W^+W^-\) pairs in the leptonic colliders. Among the various interactions that can contribute to this process, the vertices associated with the couplings of \(W\) gauge bosons to photons or \(Z\) bosons have not been probed strongly by the present experimental data. Therefore we shall consider these vertices as the potential sources of the new CP violating phenomena. One can start by writing down the most general three vector boson coupling \([3,4]\) consistent with gauge and Lorentz invariance. It was found that \([3,4]\) the effective vertices for \(W^+W^-V\), where \(V\) is either a photon or \(Z\) boson, may be parametrized in terms of seven form factors for each \(V\), three linear combinations for each case are CP violating.

These CP violating form factors are zero even at one loop level in the KM Model. However, when we introduce new physics such as Left-Right Symmetry, Supersymmetry, or additional Higgs multiplets, CP violation can appear at one loop and lead to significant CP non-conservation expressed in the three gauge boson vertex. However, one should always keep in mind that CPV models using renormalizable gauge theories are just simple examples of what can happen beyond the Standard Model. Since CP is such a fundamental symmetry, the fact that one can probe these form factors is already very interesting even if one can not produce a renormalizable gauge theory that gives large form factors. The LEP–II collider at CERN and its potential upgrades are ideal for studying the properties of the three gauge boson vertices including CPV.

The pair production process \(e^-e^+ \rightarrow x\bar{x}\), where \(x\) is some particle, is characterized only by the angle between the \(x\) momentum and the \(e^-\) momentum in the CM frame, and the helicities of the particles. If the particle helicities are averaged over, one can easily show that the process is C, P and CP self-symmetric. For this reason the helicities need to be determined or statistically analyzed to observe violations of these discrete symmetries in the pair production process. Here we shall assume that the electron and positron beam are unpolarized. We shall employ the dependence of final state momenta on the helicity of the decaying \(W\)’s to probe CP property.

There are basically two different types of helicity analyzers. The leptonic decay of the \(W\) is an example of the first type. A \(W^-\) boson at rest with its spin along the \(z\)-axis preferentially emits a lepton in the \(-z\) direction. When we boost the \(W^-\) in the \(+z\)-direction, the leptons \(\ell^-\) from the \(W^-\) with positive helicity are on average softer than those with zero or negative helicities. Hence, if there is a CP violating asymmetry in the helicity
states of the $W^-W^+$ pair, it may show up as a difference in the number of positive and negative leptons in each lepton energy bin. This is investigated in Section III. A second way to get information about the helicity is to zoom into the azimuthal angular dependence of the outgoing fermions. This can be done easily for the outgoing charged leptons. When the outgoing fermions are quarks, one has to look at the relative azimuthal angular dependence of the decay planes of the two $W$ bosons without sensitivity to the charge of the associated $W$ bosons; this also allows detection of CP violation at the production vertex as we show in Section IV. However, for the purpose of detecting CP violating signals whatever the source is, it may be more natural to start from kinematic variables that are easier to detect. Some examples of this are emphasized in Section V and VI.

CP violating effects in $e^-e^+ \rightarrow W^-W^+$ have been analyzed before [3,5,6]. In this article we make a comprehensive analysis of some integrated observables which are also sensitive to all of the parameters of the gauge boson vertices. Some of these are just the integrated or partially-integrated versions of the asymmetries considered in Ref. [5]. Some of them were never considered before. These new observables are somewhat more intuitive than the weighting approach of Ref. [5]. They also allow one to make use of all the decay modes of $W$ pairs. The process by which these observables are constructed is quite general and may be applied to other systems [7–11]. In section II, we describe the effective $ZWW$ and $\gamma WW$ vertices. In section III, we show the energy asymmetry between the lepton and the anti–lepton from the decay of $W^\pm$. In section IV, we setup a framework to study the CP–odd, CP$\hat{T}$-even angular asymmetry. We further pursue other CP violating observables in section V using the purely hadronic mode, and section VI (purely leptonic mode). Models which give significant CP violation in the vertices are discussed in section VII.

II. THE THREE GAUGE BOSON VERTEX AND HELICITY AMPLITUDES

Hagiwara et al. have made a detailed analysis [3] of the $WW$ production processes. In this section we shall give a summary of the general operator structure involved. The three gauge boson effective vertex for a vector boson $V$ coupling to two $W$ bosons is

$$\Gamma_{V^\ast(P)\rightarrow W^-(q)W^+(\bar{q})}^{\alpha\beta\mu} = f_1^V(q-\bar{q})^\mu g^{\alpha\beta} - f_2^V(q-\bar{q})^\mu P^\alpha P^\beta + f_3^V(P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) + i f_4^V(P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^V \varepsilon^{\mu\alpha\beta\rho}(q-\bar{q})_\rho - f_6^V \varepsilon^{\mu\alpha\beta\rho} P_\rho - (f_7^V/m_W^2)(q-\bar{q})^\mu \varepsilon^{\alpha\beta\rho\sigma} P_\rho (q-\bar{q})_\sigma.$$

These are the most general operator structure for on-shell $W$ bosons. In the above, $P$ is the momentum of the vector particle $V$ into the vertex, and $q$ and $\bar{q}$ are the outgoing momenta of the $W^-$ and $W^+$ respectively. $\alpha$ and $\beta$ are the polarization indices of the $W^-$ and $W^+$ respectively. All seven form factors, $f_1-f_7$, can have absorptive and dispersive parts depending on the model and the kinematics. The Standard Model predicts, at tree level,

$$f_1 = 1, \quad f_2 = 2, \quad f_{2,4,5,6,7} = 0.$$

Form factors $f_1,f_2,f_3$ and $f_5$ are CP even couplings while $f_4$, $f_6$, and $f_7$ are CP odd. Our first task is to decode both the real and the imaginary parts of these form factors $f_{4,6,7}$ from the CP violating observables constructed out of the asymmetries in the scattering kinematics.
The second task is to identify potential CP violation renormalizable gauge models in which these form factors are induced significantly. In Standard KM Model, contributions to these form factors are very small because they are not generated until two or three loop level and they come with the typical light quark mass and mixing angle suppression factors. In non-standard models, $f_4$ and $f_6$ are easily generated at one loop. Gauge models are discussed in Section VII.

When investigating the asymmetries induced by these form factors, one should be aware of the constraint imposed by the CPT theorem and unitarity condition. It is well-known that it is very difficult to check directly the effect the time reversal symmetry, $T$, or the CPT symmetry in process such as ours. This is because both symmetries require one to interchange the initial and the final states. However, in the Born approximation, the unitarity of the $S$-matrix implies that the transition matrix, $M$, is hermitian, and this allows one to flip the initial state into the final state as long as the amplitude is also complex conjugated. We shall define a pseudo time reversal symmetry, which we shall call $\hat{T}$, that transforms only the kinematic observables of both the initial and final states according to time reversal but does not transform the initial and final state into each other as required by time reversal. Therefore as long as the transition matrix is hermitian, which is always the case in Born approximation, the CPT constraint can be reduced to

$$\langle f|M|i\rangle = \langle \text{CP}\hat{T}(f)|M|\text{CP}\hat{T}(i)\rangle^*, \quad (3)$$

where $|\text{CP}\hat{T}(j)\rangle$ represents the state $|j\rangle$ with its kinematic variables and quantum number numbers transformed by CP$\hat{T}$. We shall call the transformation from $\langle f|M|i\rangle$ into $\langle \text{CP}\hat{T}(f)|M|\text{CP}\hat{T}(i)\rangle^*$ the CPT transformation. Therefore the CPT theorem allows us to detect whether the transition matrix is hermitian or not by looking at the same process with its kinematic variables transformed by CPT. The nonhermiticity of the transition matrix occurs when the contributions beyond Born approximation are included in which some intermediate states can be on-shell. Such contributions are traditionally called final state interactions (FSI), (even though initial state rescattering can also give rise to a similar effect). In our case, since the initial state is CP self conjugate, we can label our observables according to their CP and CP$\hat{T}$ properties. Clearly if an observable is CP$\hat{T}$-odd, FSI will be required for its observation. Similarly, if it is CP$\hat{T}$-even, there is no need of FSI for its observation. Note however that in renormalizable gauge theory the fact that one needs FSI does not always mean that the effect will contain an additional suppression factor. Since CP violating effects are usually loop effects, it is possible to incorporate the CP violation and the rescattering (FSI) effects into the same one loop diagram. The loop diagram then interferes with the tree level process to produce an observable result. In the form factor approach which we are adopting, nonhermiticity of the transition matrix, which is the hallmark of the final state rescattering effect, can be represented as the imaginary parts of the form factors. Therefore, to measure both the real and the imaginary parts of the form factors directly one needs both CP–odd, CP–even and CP–odd, CP–odd observables.

Helicity amplitudes for the $W$ pair production are given in section 3.1 of Ref. 3. They can be written as

$$\mathcal{M}_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}(\Theta) = \sqrt{2} e^2 \mathcal{M}^\text{max}[|\Delta\sigma|,|\Delta\lambda|](\Theta), \quad (4)$$

where $\sigma, \bar{\sigma}(= \pm 1)$ are the electron and positron helicities, $\lambda, \bar{\lambda}$ are the $W^-$ and $W^+$ helicities respectively. The angle $\Theta$ is the angle between the electron and the $W^-$. $|\Delta\sigma| = |\frac{1}{2}(\sigma - \bar{\sigma})|$, and
Δλ = λ − ̄λ. The coordinate system is shown in Fig. 1. Note that our definition differs from that of Ref [3] by a factor of ε = Δσ(−1)λ; this convention leads to simpler CP properties [4]. For the density matrices of the W’s in section [V]. To be complete, we list the relevant d functions,

\[ d_{1,±2}^2(\Theta) = -d_{1,±\overline{2}}^2(\Theta) = ±\frac{1}{2}(1 ± \cos Θ) \sin Θ, \]
\[ d_{1,±1}^1(\Theta) = d_{1,±\overline{1}}^1(\Theta) = \frac{1}{2}(1 ± \cos Θ), \]
\[ d_{1,0}^1(\Theta) = -d_{1,0}^1(\Theta) = -d_{0,1}^1(\Theta) = -d_{0,-1}^1(Θ) = -\frac{1}{\sqrt{2}} \sin Θ. \]

Due to the angular momentum conservation, the amplitude is non–vanishing only when θ = −σ in the high energy limit \( \sqrt{s} \gg m_e \). Therefore only Δσ = ±1 case concerns us.

\[ \hat{M}_{\sigma,−σ;λ,λ} = \frac{β}{\sin^2 θ_W}(-\frac{1}{2}δ_{σ,−1} + \sin^2 θ_W)A^Z_{λ,λ} \left( \frac{s}{s - m^2_Z} - βA^γ_{λ,λ} \right) \]
\[ + \frac{δ_{σ,−1}}{2β \sin^2 θ_W}[B_{λ,λ} - \frac{C_{λ,λ}}{1 + β^2 - 2β \cos Θ}], \]

with \( β^2 = 1 − γ^2, \gamma = \frac{1}{2}\sqrt{s/m_W} \) and \( \sin^2 θ_W = 0.23 \). Coefficients \( A^γ \) and \( A^Z \) are related to amplitudes due to \( γ^*, Z^* \) in the s–channel. Coefficients \( B \) and \( C \) are related to the t–channel neutrino exchange diagram. Note that, for Δλ = ±2, \( A_{λ,λ} = 0 \) and \( B_{λ,λ} = 0 \), only coefficients \( C \) from the ν contribution survive. We shall tabulate \( A_{λ,λ}, B_{λ,λ}, \) and \( C_{λ,λ} \) in the matrix forms. First, we denote \( \hat{A} \) as the contribution to \( A \) at the tree level in the Standard Model, and \( δA \) as the deviation due to the CP violating form factors.

\[ A^V_{λ,λ} = \hat{A} + δA^V_{λ,λ} + \cdots \quad (V = A, Z). \]

The corrections due to other CP conserving form factors are hidden in the dots. In this paper, we are not interested in them. In the basis (−, 0, +), the matrices are,

\[ \hat{A} = \begin{pmatrix} 1 & 2γ & 0 \\ 2γ & 1 + 2γ^2 & 2γ \\ 0 & 2γ & 1 \end{pmatrix}, \]
\[ δA = \begin{pmatrix} -i(β^{-1}f_6^V + 4γ^2βf_1^V) & -iγ(f_4^V + β^{-1}f_6^V) & 0 \\ -iγ(-f_1^V + β^{-1}f_6^V) & 0 & iγ(f_4^V + β^{-1}f_6^V) \\ 0 & iγ(-f_1^V + β^{-1}f_6^V) & i(β^{-1}f_6^V + 4γ^2βf_7^V) \end{pmatrix}. \]

Under CP transformation [13],

\[ M_{σ,−σ;λ,−λ}(θ) → M_{−σ,−σ;−λ,−λ}(θ), \]

and therefore \( A_{λ,−λ} \rightarrow A_{−λ,−λ}, \) or \( A_{i,j} \rightarrow A_{j,i} \) in present notation. Therefore form factors \( f_{4,6,7}, \) as appeared in Eq. (8), already parametrize the most general CP–odd part of A. Note also that, under CPT, \( A_{λ,λ} \rightarrow A^*_{−λ,−λ}, \) therefore the real parts of \( f_{4,6,7} \) are CPT–even, but their imaginary parts are CPT–odd.
The matrix $B$ is just given by $\hat{A}$ with the factor 1 at the 00 entry removed, i.e. $B_{\lambda,\bar{\lambda}} = \hat{A}_{\lambda,\bar{\lambda}} - \delta_{\lambda,0} \delta_{\bar{\lambda},0}$. The matrix $C$ is given below,

$$C = \begin{pmatrix} \gamma^{-2} & (2 - 2\beta)/\gamma & 2\sqrt{2}\beta \\ (2 + 2\beta)/\gamma & 2/\gamma^2 & (2 - 2\beta)/\gamma \\ 2\sqrt{2}\beta & (2 + 2\beta)/\gamma & \gamma^{-2} \end{pmatrix}.$$  \hspace{1cm} (10)

The CP violation in the three gauge boson vertex affects terms with $\lambda + \bar{\lambda} \neq 0$ only. There are 6 complex phenomenological CP violating parameters, namely the three form factors, $f_1$, $f_6$, and $f_7$, each for photon and for $Z$ couplings.

As stated earlier, the CP information is carried by the helicities of the $W$'s. They can be decoded from the decay products of the $W$ bosons. Assuming the standard $V-A$ interaction in the $W$ decay, the decay amplitudes are

$$\mathcal{M}^- (W^- (\lambda) \rightarrow f_1 \bar{f}_2) \sim l_\lambda^- = d_{\lambda,1}(\psi)e^{i\lambda\phi},$$

$$\mathcal{M}^+ (W^+ (\bar{\lambda}) \rightarrow f_3 \bar{f}_4) \sim l_{\bar{\lambda}}^+ = d_{-\bar{\lambda},1}(\bar{\psi})e^{-i\bar{\lambda}\bar{\phi}}.$$ \hspace{1cm} (11)

The normalization is not important in our following discussion. The polar angle $\psi$ and the azimuthal angle $\phi$ of $f_1$ are defined in the $W^-$ rest frame. Similarly, the polar angle $\bar{\psi}$ and the azimuthal angle $\bar{\phi}$ of $f_4$ are defined in the $W^+$ rest frame. These two rest frames of $W^\pm$ are constructed by merely boosting (without rotation) the $e^-e^+$ CM frame along the common $z$ axis, which is defined here as pointing in the direction of motion of $W^-$. In our convention, there is a sign difference in $\mathcal{M}^+$ when $\bar{\lambda} = 0$ from that in Eq.(4.8b) of Ref. [3]. (This is consistent with the removal of the sign factor $\varepsilon$ from Eq.(11) mentioned earlier).

These decay amplitudes have to be folded with the production amplitude in Eq.(4) to obtain the amplitude for the overall process, $e^-e^+ \rightarrow W^-W^+$ followed by $W^- \rightarrow f_1 \bar{f}_2$ and $W^+ \rightarrow f_3 \bar{f}_4$. Following Ref. [3], the differential cross section averaged on the initial fermion polarizations and summed over the final state fermion polarizations can be written as

$$\sigma_0 P_{\lambda',\bar{\lambda}'}^{\lambda,\bar{\lambda}}(l^-_\lambda l'^-_{\lambda'})(l^+_\lambda l'^+_{\lambda'}),$$ \hspace{1cm} (12)

where $\sigma_0$ is a factor independent of the $W$ polarizations which does not concern us here. The matrix $P$ is the general density matrix of $W^-W^+$ boson pair defined as

$$P_{\lambda',\bar{\lambda}'}^{\lambda,\bar{\lambda}} = \mathcal{N}^{-1} \sum_{\sigma,\bar{\sigma}} \mathcal{M}_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}(\Theta) \mathcal{M}^*_{\sigma,\bar{\sigma};\lambda',\bar{\lambda}'}(\Theta).$$ \hspace{1cm} (13)

Here $\mathcal{N}$ is the normalization such that $\text{Tr}P=1$. The azimuthal-angle dependence of Eq.(12) has been worked out in Ref. [3]. Under CP transformation

$$P_{\lambda',\bar{\lambda}'}^{\lambda,\bar{\lambda}} \rightarrow P_{-\lambda',-\lambda'}^{-\lambda,-\bar{\lambda}},$$ \hspace{1cm} (14)

while under CPT

$$P_{\lambda',\bar{\lambda}'}^{\lambda,\bar{\lambda}} \rightarrow P_{-\lambda,-\bar{\lambda}}^{-\lambda',-\lambda'} = (P_{-\lambda,\lambda}^{-\lambda',-\lambda'})^*.$$ \hspace{1cm} (15)

It seems straightforward to measure the asymmetry in event rates between the two CP conjugated configurations,
However, these angle variables may not be completely reconstructed because (i) the identity of the quark cannot be fully retained in the final jet configuration, or (ii) the missing neutrinos introduce ambiguity in event reconstruction.

In addition, in its totally differential form in Eq. (16), it is not easy to make any simple physical interpretation. The fact that events scatter over a multivariable domain also lowers statistics which makes it harder to establish the CP asymmetry. In the following, we shall try to construct a few CP–odd observables which are more accessible to measurement. For the decay modes in which event reconstruction is possible (in particular, the mixed lepton–hadron modes), we focus on integrated CP asymmetries that have simple and intuitive interpretations kinematically. For the modes in which only a partial reconstruction is possible, such as the purely hadronic modes or the purely leptonic modes, we shall construct CP–odd observables based on partial information.

Eq. (16) provides another way of looking at the final state (CP ˆT-odd) effect. It is well-known that the observation of CP violation requires a source of complex phase in addition to the one that is due to CP violation. Observation of Eq. (16) implies that it is possible to dig out a CP violating signal from either the ψ dependence or from the φ dependence. The complex φ dependence in Eq. (11) provides automatically the additional source of complex phase one needs. The resulting CP–odd observables are therefore can be made CP ˆT even. They will be investigated in Section IV and later. To decode the signal from the ψ dependence alone, one needs the form factors to be complex to provide the additional source of complex phase needed since the ψ-dependent d—functions in Eq. (11) are real. Therefore, corresponding CP–odd observables are CP ˆT-odd.

III. LEPTONIC ENERGY ASYMMETRY (CP–ODD AND CP ˆT–ODD)

In this section we shall start with an CP–odd observable that is CP ˆT–even and therefore requires FSI. In our case, it means only the imaginary parts of the form factors contribute. For a process in which the initial state is CP self conjugate and the final states are heavy particles with spin, one can easily form a CP–odd quantity using different helicity states of the final particles. This idea was applied recently to many examples such $gg \rightarrow t\bar{t}$ [14], $e^-e^+ \rightarrow W^+W^-$ [9], or Higgs decay to $t\bar{t}$ or gauge boson pairs [7,15]. To observe this helicity asymmetry one has to rely on the kinematics of the heavy particle decay to decode the helicities of the decaying particles. Luckily for the top quark and the $W^\pm$ boson this can be done by the asymmetry in the energy spectrum of the charged leptons in their semileptonic and leptonic decay respectively. We shall apply this idea here to $e^-e^+ \rightarrow W^-W^+$. Note that this helicity asymmetry in heavy particle production is C–odd (for charged heavy particle pairs), CP–odd, and CP ˆT-odd. Therefore FSI are required to get a nonvanishing result and one can only test the imaginary parts of the form factors this way. We shall keep the real parts of the form factors at their tree level Standard Model value in this section.

Let us begin with a simple illustration of CP violation due to the absorptive parts of $f_4$, $f_6$ and $f_7$. For the $e^-e^+$ initial configuration, only coefficients $A$ contribute to the production amplitudes in Eq. (11). It is straightforward to see that the amplitudes $\mathcal{M}_{+,--;\lambda,\bar{\lambda}}^V$ due to $V$ contribution in the $s$–channel are proportional to coefficients as follows,
\[
\begin{align*}
\mathcal{M}^{V}_{+,+-,0} & \sim 2 + \text{Im} \ f_4^V - \text{Im} \ f_6^V / \beta, \\
\mathcal{M}^{V}_{+,--,-} & \sim 2 - \text{Im} \ f_4^V + \text{Im} \ f_6^V / \beta, \\
\mathcal{M}^{V}_{+,--,-} & \sim 2 - \text{Im} \ f_4^V - \text{Im} \ f_6^V / \beta, \\
\mathcal{M}^{V}_{+,+-,0} & \sim 2 + \text{Im} \ f_4^V + \text{Im} \ f_6^V / \beta, \\
\mathcal{M}^{V}_{+,+-,0} & \sim 1/\gamma - \text{Im} \ f_6^V / (\beta \gamma) - 4\gamma \beta \text{Im} \ f_7, \\
\mathcal{M}^{V}_{+,+-,0} & \sim 1/\gamma + \text{Im} \ f_6^V / (\beta \gamma) + 4\gamma \beta \text{Im} \ f_7. 
\end{align*}
\] (17)

The presence of the absorptive parts of \( f_{4,6,7}^V \) produces asymmetry in the production rates between the CP conjugate states \((\lambda, \bar{\lambda})\) and \((-\lambda, -\bar{\lambda})\), i.e. \((+, 0)\) and \((0, -)\); \((0, +)\) and \((-0, -)\); or \((+, +)\) and \((-0,-)\). Such CP asymmetry also happens in the \(e^-e^+\) channel although the formulas become much more lengthy because of the involvement of the \(\nu\)-exchange diagram. However, it is just as straightforward to study the amplitudes in Eqs. (14) numerically. This is what we shall do here, instead of producing the complete analytic formula.

Since we are going to integrate over the azimuthal angles \(\phi\) and \(\bar{\phi}\), and assuming that the \(W\) bosons are produced on shell, the resulting \(W^-W^+\) production cross section is proportional to

\[
\sigma_{\lambda,\bar{\lambda}} = \mathcal{N} \sum_{\lambda',\bar{\lambda}'} P_{\lambda',\bar{\lambda}'}^{\lambda,\bar{\lambda}} \delta_{\lambda,\lambda'} \delta_{\bar{\lambda},\bar{\lambda'}} \sum_{\sigma,\bar{\sigma}} \mathcal{M}_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}(\Theta) \mathcal{M}^*_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}(\Theta). 
\] (18)

The interference effect between different \((\lambda, \bar{\lambda})\) configurations drops away because the integration over the azimuthal angles kills the “off–diagonal” contributions. There are three CP violating rate differences, namely, \(\sigma_{+,0} - \sigma_{0,-}\), \(\sigma_{0,+} - \sigma_{-0}\), and \(\sigma_{+,+} - \sigma_{-,-}\). The detailed relationship between \(\sigma_{\lambda,\bar{\lambda}}\) and the form factors is very tedious and has been worked out by Gounaris et. al. in Ref. [16]. Here we shall concentrate on the contributions \(\sigma_{\lambda,\bar{\lambda}}\) to the lepton energy asymmetry. However one should note that in general \(\sigma_{+,0} - \sigma_{0,-}\) depends only on the combination \(\text{Im} \ f_4^V - \text{Im} \ f_6^V / \beta\) and \(\sigma_{0,+} - \sigma_{-,0}\) only on \(\text{Im} \ f_4^V + \text{Im} \ f_6^V / \beta\) and \(\sigma_{+,+} - \sigma_{-,-}\) only on \(\text{Im} \ f_6^V / (\beta \gamma) + 4\gamma \beta \text{Im} \ f_7\).

Consider events with one of the \(W\) bosons decaying leptonically. The other \(W\) can decay either hadronically or leptonically. The energy spectra of the lepton \(l\) from \(W^-\) and the anti–lepton \(\bar{l}\) from \(W^+\) will be different due to the above asymmetry. In the CM frame of the collider, the energies of the lepton \(l\) and the anti–lepton \(\bar{l}\) are determined by the variables \(\psi\) and \(\bar{\psi}\) respectively.

\[
E(l^-) = \frac{1}{4} \sqrt{s} (1 + \beta \cos \psi), \quad E(l^+) = \frac{1}{4} \sqrt{s} (1 - \beta \cos \bar{\psi}). 
\] (19)

Define the energy fraction \(x = 4E/\sqrt{s} \in [1 - \beta, 1 + \beta]\). The lepton (anti–lepton) from the decay of a right handed \(W^-_{\lambda=1}\) (left handed \(W^+_{\lambda=-1}\)) will have a softer energy spectrum,

\[
f(x) = \frac{3}{8} \beta^{-3} (x - 1 - \beta)^2. 
\] (20)

This normalized function is derived based on Eq. (11). Similarly, the lepton (anti–lepton) from the decay of a left handed \(W^-_{\lambda=-1}\) (right handed \(W^+_{\lambda=1}\)) will have a harder energy spectrum,
\[ g(x) = \frac{3}{8}\beta^{-3}(x-1+\beta)^2. \]  

And, the lepton (anti-lepton) from the decay of a longitudinal \( W_{\lambda=0}^- (W_{\lambda=0}^+) \) will have an energy spectrum symmetric with respect to the average value \( x = 1 \),

\[ n(x) = \frac{3}{4}\beta^{-3}(\beta^2 - 1 + 2x - x^2) = \frac{3}{2}\beta^{-1} - f(x) - g(x). \]  

We obtain the energy asymmetry

\[ a_E(x) \equiv \frac{1}{N^-} \frac{dN^-}{dx(\ell^-)} - \frac{1}{N^+} \frac{dN^+}{dx(\ell^+)} = \left( \sum_{\lambda,\bar{\lambda}} \sigma_{\lambda,\bar{\lambda}} \right)^{-1} \left\{ (\sigma_{+,+} - \sigma_{-,+})[f(x) - g(x)] 

+ (\sigma_{+,0} - \sigma_{-,0})[f(x) - n(x)] - (\sigma_{0,+} - \sigma_{-,0})[g(x) - n(x)] \right\} 

= \left( \sum_{\lambda,\bar{\lambda}} \sigma_{\lambda,\bar{\lambda}} \right)^{-1} \left\{ (\sigma_{+,0} - \sigma_{-,0} + \sigma_{0,+} - \sigma_{-,0} + 2\sigma_{+,+} - 2\sigma_{-,+}) \frac{3}{2}[f(x) - g(x)] 

+ (\sigma_{+,0} - \sigma_{-,0} - \sigma_{0,+} + \sigma_{-,0}) \frac{3}{2}[f(x) + g(x) - \beta^{-1}] \right\}. \]  

Here distributions are compared at the same energy for the lepton and the anti-lepton, \( x = x(\ell^+) = x(\ell^-) \). The count \( N^- (N^+) \) includes events with a prompt lepton (anti-lepton) from \( W^- (W^+) \) decay with \( W^+ (W^-) \) decays arbitrarily. Here the cross sections \( \sigma_{\lambda,\bar{\lambda}} \) are still in general a function of the variable \( \Theta \). This equation also implies that there are only two CP-odd combinations of \( \sigma_{\lambda,\bar{\lambda}} \) one can probe. One of them is P-even. The other one is P-odd. To improve the observability, one may as well integrate \( a_E(x) \) over ranges of \( x \). It is also obvious that the asymmetry will cancel if we integrate \( a_E(x) \) over the complete range of \( x \), namely from \( 1 - \beta \) to \( 1 + \beta \). One easy way to define the integrated energy asymmetry, \( A_E \) to is integrate \( a_E(x) \) over the upper half range, i.e. from 1 to \( 1 + \beta \),

\[ A_E \equiv \left( \int_{1}^{1+\beta} - \int_{1-\beta}^{1} \right) a_E(x)dx 

= -\frac{3}{4}(\sigma_{+,0} - \sigma_{-,0} + \sigma_{0,+} - \sigma_{-,0} + 2\sigma_{+,+} - 2\sigma_{-,+})/\left( \sum_{\lambda,\bar{\lambda}} \sigma_{\lambda,\bar{\lambda}} \right). \]  

Assuming that the CP odd form factors are small perturbation from the CP conserving ones, we can compute the expected energy asymmetry per unit of \( \text{Im} \ f_i \) as a function of \( \cos \Theta \). This is plotted in Fig. 2a, 2b for different center-of-mass energy. The asymmetry \( A_E \) is smaller in the forward region, \( \cos \Theta \sim 1 \), because in that region the CP conserving neutrino mediated diagrams dominate. In the backward region, \( \cos \Theta \sim -1 \), both \( \text{Im} \ f_i^{\gamma,Z} \) give positive contributions while \( \text{Im} \ f_i^6 \) and \( \text{Im} \ f_i^7 \) give negative contributions.

Also note that \( A_E \) is a parity-odd (C-even) operator. Therefore, if one considers the S-channel photon-mediated diagrams alone, the form factor \( \text{Im} \ f_i^\gamma \) will not contribute. However, since the lowest order diagrams also include the Z-mediated graphs which have a different parity property, \( \text{Im} \ f_i^\gamma \) still contributes even if the lowest order neutrino mediated diagram is negligible.

The integration limits are arbitrary. If one integrates over \( x \) symmetrically about the midpoint 1, then the contribution from \( \sigma_{+,+} - \sigma_{-,+} \) will be eliminated because \( f(x) - g(x) \) is antisymmetric about \( x = 1 \). Therefore there is no contribution from \( \text{Im} \ f_i^\gamma \) in this case.
These types of combinations are parity-even and C-odd. However, just as for $\text{Im } f_i^\gamma$ in the previous case, $\text{Im } f_6^\gamma$ also contributes even if the neutrino dominated diagrams are negligible. For example, one can define

$$A'_E(\alpha) \equiv \left( \int_{1-\alpha}^{1+\alpha} - \int_{1-\beta}^{1-\alpha} - \int_{1+\alpha}^{1+\beta} \right) a_E(x) dx$$

$$= -n_\alpha (\sigma_{+,0} - \sigma_{0,-} - \sigma_{0,+} + \sigma_{-,0}) / \left( \sum_{\lambda,\bar{\lambda}} \sigma_{\lambda,\bar{\lambda}} \right),$$

(25)

where $0 < \alpha < \beta$ and $n_\alpha$ is a numerical constant. For $\alpha = \beta/3$, $n_\alpha = \frac{12}{37}$. The contribution of various $\text{Im } f_i$ to $A'_E$ is plotted in Fig.2c as a function of $\cos \Theta$. Just as in the case of asymmetry $A_E$, $A'_E$ is smaller in the forward region because of the neutrino mediated diagrams. In the backward region, the contributions of both $\text{Im } f_i^\gamma, Z^4$ are positive while those of $\text{Im } f_i^\gamma, Z^6$ are negative.

From Eq.(25), one can not probe all three independent CP–odd combinations of $\sigma_{\lambda,\bar{\lambda}}$. Therefore the lepton energy asymmetry alone can not probe all three independent imaginary parts of the form factors, $\text{Im } f_{4,6,7}$. We shall discuss another probe of the these imaginary parts later. In addition, two choices of the integrated asymmetry presented above are just examples which may not be optimal. A careful weighting of events to avoid cancellations may still significantly improve the sensitivity of the asymmetry.

In Eq.(23) and in the figures, we have summed over the total decay modes of the other $W$ boson. However, there may be other contributions to the leptonic energy asymmetry not going through the two $W$ intermediate states. If one does not use at least one of the $W$ bosons as a tag to eliminate other contributions, one will still have to investigate the other potential contributions before the measurement can be interpreted. However, one can also tag the other $W$ boson by selecting only its purely hadronic decay modes and use the jet energy and momentum measurement to eliminate the other potential contributions which are not due to $W$ pair production. Of course this will require a sacrifice in event rate. The detection uncertainty in jet energy and momentum also has to be taken into account. However, even with this uncertainty, the efficiency in eliminating the non-$WW$ background can be still quite high.

IV. UNCORRELATED UP–DOWN ASYMMETRY.

It is known that explicit CP violation requires the CP nonconserving vertex as well as additional complex amplitudes. In the previous case, this complex structure comes from the absorptive part due to the final state interactions. However, the complex structure can also come from other sources. One of these is the azimuthal phase $\exp(iLz\phi)$ in the decay process. This will produce a CP–odd up–down asymmetry even with only the dispersive part of CP violating vertex, $\text{Re } f_{4,6,7}$.

We concentrate our attention to those events that one of the $W$ decays leptonically and the other one decays into quark jets, i.e. either $f_1 = \ell^-$ or $\bar{f}_4 = \ell^+$. CP violation could appear as the difference of two separate azimuthal distributions for $\ell^-$ and $\ell^+$. This kind of angular asymmetry is very simple. We only look at the azimuthal angle of the lepton or the anti-lepton from one $W$. The recoiling jets from the other $W$ are only used to define the
reaction plane. In this way, we are looking at the separate (uncorrelated) density matrices for $W^-$ and $W^+$.

The angular distribution of $\ell^-$ from the $W^-$ decay is specified by the the spin density matrix $\rho_{\lambda,\lambda'}$ of the $W$ boson.

$$\rho(\Theta)_{\lambda,\lambda'} = N(\Theta)^{-1} \sum_{\sigma,\bar{\sigma},\bar{\lambda}} M(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}) M^*(\sigma, \bar{\sigma}, \lambda', \bar{\lambda}) = \sum_{\lambda} P_{\lambda,\bar{\lambda}}^{\lambda,\bar{\lambda}}. \quad (26)$$

Here $N$ is the normalization such that $\text{Tr} \rho = 1$. $\rho$ is hermitian by definition. The normalized distribution for $\ell^-$ is given by

$$\frac{dN(\ell^-)}{d\phi d\cos \psi} = \frac{1}{4 \pi} \frac{3}{4} \left[ (1 - \cos \psi)^2 \rho_{++} + (1 + \cos \psi)^2 \rho_{--} + 2 \rho_{00} \sin^2 \psi 
- 2\sqrt{2} \text{Re} \rho_{+0} (1 - \cos \psi) \sin \psi \cos \phi + 2\sqrt{2} \text{Im} \rho_{+0} (1 - \cos \psi) \sin \phi 
- 2\sqrt{2} \text{Re} \rho_{-0} (1 + \cos \psi) \sin \psi \cos \phi - 2\sqrt{2} \text{Im} \rho_{-0} (1 + \cos \psi) \sin \phi 
+ 2 \text{Re} \rho_{+-} (1 - \cos^2 \psi) \cos 2\phi - 2 \text{Im} \rho_{+0} (1 - \cos^2 \psi) \sin 2\phi \right]. \quad (27)$$

Similarly, the angular distribution of $\ell^+$ from the $W^+$ decay is specified by another spin density matrix, $\bar{\rho}_{\lambda,\lambda'}$, of the $W^+$ boson.

$$\bar{\rho}(\Theta)_{\lambda,\lambda'} = N'(\Theta)^{-1} \sum_{\sigma,\bar{\sigma},\bar{\lambda}} M(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}) M^*(\sigma, \bar{\sigma}, \lambda', \bar{\lambda'}) = \sum_{\lambda} P_{\lambda,\bar{\lambda}}^{\lambda,\bar{\lambda}}. \quad (28)$$

The normalized distribution for $\ell^+$ is given by

$$\frac{dN(\ell^+)}{d\phi d\cos \psi} = \frac{1}{4 \pi} \frac{3}{4} \left[ (1 - \cos \bar{\psi})^2 \bar{\rho}_{++} + (1 + \cos \bar{\psi})^2 \bar{\rho}_{--} + 2 \bar{\rho}_{00} \sin^2 \bar{\psi} 
+ 2\sqrt{2} \text{Re} \bar{\rho}_{+0} (1 - \cos \bar{\psi}) \sin \bar{\psi} \cos \bar{\phi} + 2\sqrt{2} \text{Im} \bar{\rho}_{+0} (1 - \cos \bar{\psi}) \sin \bar{\phi} 
+ 2\sqrt{2} \text{Re} \bar{\rho}_{-0} (1 + \cos \bar{\psi}) \sin \bar{\psi} \cos \bar{\phi} - 2\sqrt{2} \text{Im} \bar{\rho}_{-0} (1 + \cos \bar{\psi}) \sin \bar{\phi} 
+ 2 \text{Re} \bar{\rho}_{-+} (1 - \cos^2 \bar{\psi}) \cos 2\bar{\phi} + 2 \text{Im} \bar{\rho}_{+0} (1 - \cos^2 \bar{\psi}) \sin 2\bar{\phi} \right]. \quad (29)$$

In our present phase convention, if CP were conserved, we would have the following identities, first noticed by Gounaris et al. [5,13],

$$\rho(\Theta)_{\lambda,\lambda'} = \bar{\rho}(\Theta)_{-\lambda,-\lambda'}. \quad (30)$$

Under CP conjugation, we exchange $\ell^-$ and $\ell^+$ with angle substitutions $\bar{\psi} \leftrightarrow \pi - \psi$, $\bar{\phi} \leftrightarrow \pi + \phi$. The distribution in Eq.(29) is transformed into Eq.(27) provided Eq.(30) is satisfied. On the other hand, the CPT invariance implies [13],

$$\rho(\Theta)_{\lambda,\lambda'} = \bar{\rho}(\Theta)_{-\lambda,-\lambda'}^*. \quad (31)$$

When the effect of the CP violating form factors are included in the analysis, one can form the following CP or CPT eigen–combinations:

$$R(\pm)(\Theta)_{\lambda,\lambda'} = \text{Re} \rho(\Theta)_{\lambda,\lambda'} \pm \text{Re} \bar{\rho}(\Theta)_{-\lambda,-\lambda'}.$$
and

\[ I(\pm)(\Theta)_{\lambda,\lambda'} = \text{Im} \rho(\Theta)_{\lambda,\lambda'} \pm \text{Im} \bar{\rho}(\Theta)_{-\lambda,-\lambda'} \].

Among them, \( R(+) \) and \( I(+) \) are CP\( \hat{T} \) odd; \( R(-) \) and \( I(-) \) are CP odd and the others are either CP\( \hat{T} \)- or CP–even respectively. The observation of CP–odd \( R(-) \) requires final state interactions due to CP\( \hat{T} \). Therefore it does not need to involve the complex phase of the azimuthal dependence in Eqs.\((27, 29)\). It can be decoded by analyzing the polar angular dependence of \( \psi \) or \( \bar{\psi} \) in these equations. In the collider M frame these dependences can be translated into the energy dependence of the corresponding lepton in the final state, which has already been studied in the previous section.

Here we shall focus on \( I(-) \) which does not require final state interactions. Since \( \rho \) is hermitian, the only nonzero components of \( I(-) \) is \( I(-)_{+,0}, I(-)_{+,0}, I(-)_{0,-} \). They are related to Re \( f_{4,6,7} \).

One can in principle make detailed angular analysis of the difference between Eq.\((27)\) and its CP conjugate in Eq.\((29)\), similar to what was done for the case of \( e^-e^+ \to W^-W^+ \) by Gounaris et al. \( [5] \). However, since all experimental measurements of angles have finite resolution, such analysis is not very useful in practice. We wish to find simpler observables which may be more intuitive and may be easier to measure, we shall consider the following up–down asymmetry

\[ A_{u.d.}(\Theta) = \frac{dN(\ell^+, \text{up}) + dN(\ell^+, \text{up}) - dN(\ell^-, \text{down}) + dN(\ell^+, \text{down})}{[dN(\ell^+, \text{up}) + dN(\ell^+, \text{up})] + [dN(\ell^-, \text{down}) + dN(\ell^+, \text{down})]} . \tag{32} \]

It is evaluated for each scattering angle \( \Theta \). The branching fraction of the \( W \) semileptonic decay cancels in the ratio. Integrating on \( \psi \) and \( \bar{\psi} \) over the full range and on \( \phi, \bar{\phi} \) over up or down hemispheres, we obtain \( A_{u.d.}(\Theta) \) in a very simple form from Eqs.\((27, 29)\),

\[ A_{u.d.}(\Theta) = \frac{3}{8} \sqrt{2} \left[ \text{Im} \rho(\Theta)_{+,0} - \text{Im} \bar{\rho}(\Theta)_{-,0} - \text{Im} \rho(\Theta)_{-,0} + \text{Im} \bar{\rho}(\Theta)_{+,0} \right] \]

\[ = \frac{3}{8} \sqrt{2} \left[ I(\cdot)_{+,0} - I(\cdot)_{-,0} \right] . \tag{33} \]

As we sum up contributions from \( \ell^\pm \) in each square bracket of Eq.\((32)\), the asymmetry is insensitive to the sign of charge, it is obvious that a non-vanishing value of \( A_{u.d.}(\Theta) \) is a genuine signal of CP violation. Though, the angular distributions of the leptons derived in Eqs.\((27, 29)\) will have corrections from other CP conserving final state interactions, however, these corrections cannot fake the CP asymmetry as the effects due to the CP conserving sources cancel away in the differences.

To make distinction between \( I(-)_{+,0} \) and \( I(-)_{-,0} \) one can not integrate over the full range of \( \psi \) or \( \bar{\psi} \). If we restrict \( dN(\ell^\pm) \) in the numerator of Eq.\((32)\) to count only events with \( E(\ell^\pm) > E_0 \) for some energy \( E_0 \) in the physical range, \( \frac{1}{4} \sqrt{s}(1 + \beta) \geq E_0 \geq \frac{1}{4} \sqrt{s}(1 - \beta) \), and keep the denominator unchanged, then one obtain a different integrated asymmetry \( A_{u.d.'} \). One can show that

\[ A_{u.d.'}(\Theta) = \frac{3 \sqrt{2}}{4\pi} \left\{ (a(E_0) - b(E_0))(\text{Im} \rho(\Theta)_{+,0} - \text{Im} \bar{\rho}(\Theta)_{-,0}) \right\} \]

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which gives a different combination of $I(-)_{+,0}$ and $I(-)_{-,0}$. Here, $a(E_0) = -\frac{1}{2} \sin \eta \cos \eta + \frac{i}{2} \eta$, and $b(E_0) = \frac{1}{3} \sin^3 \eta$ with $E_0 = \frac{1}{4} \sqrt{s} (\beta \cos \eta + 1)$. For example, if we use $E_0 = \frac{1}{4} \sqrt{s}$, the average energy, then $\eta = \pi/2$ and $a(E_0) = \pi/4$, $b(E_0) = 1/3$.

To probe the effect of the CP violating quantity $I(-)_{+, -}$ one has to look into the azimuthal dependence. For example, in another integrated asymmetry such as

$$A^{n.d.}(\Theta) = \frac{dN(\ell^-, I+III) + dN(\ell^+, II+IV)}{dN(\ell^-, I+II+III+IV) + dN(\ell^+, I+II+III+IV)}.$$

Here the range of the azimuthal angle has been divided into four usual quadrants I,II,III and IV. It can be shown that

$$A^{n.d.}(\Theta) = -\frac{1}{\pi} \left( \text{Im} \rho(\Theta)_{+, -} - \text{Im} \bar{\rho}(\Theta)_{-, +} \right).$$

The above three independent measurements are enough to extract the three CP–odd (CPT–even) parameters $I(-)_{+, -}$, $I(-)_{+, 0}$, $I(-)_{-, 0}$ which are related to the elements of the density matrices.

The relationship between $I(-)_{+, -}$, $I(-)_{+, 0}$, $I(-)_{-, 0}$ and the CP violating form factors are too lengthy to be useful to present explicitly here. In Fig. 3(a,b,c), we plot the contributions to $I(-)_{+, 0}$, $I(-)_{-, 0}$, $I(-)_{+, -}$ due to various form factors. Re $f_7^{\gamma,Z}$ give rise to the largest contributions to $I(-)_{+, 0}$, especially in the region $\cos \theta < 0$; while Re $f_6^{\gamma,Z}$ give the largest contributions to $I(-)_{-, 0}$ especially in the region $\cos \theta < 0$.

V. CP VIOLATION IN PURELY HADRONIC W DECAYS

When the $W$ decays into quarks which turn into jets, the information about the charges and flavors of the quarks is lost. The useful definitions of the angles depend on the amount of information which can be measured. In the ideal case, when the quark charge and flavor can be detected, then unambiguous definitions of the angles used in previous sections can be carried over to the hadronic case by substituting a quark doublet for a lepton doublet. However, in reality, the flavor and charge cannot be identified event by event, so none of the observables discussed in previous sections can be uniquely defined. A better starting point is to use the angles that can be defined purely kinematically and which do not refer to the flavors or the charges of the underlying quarks or $W$ bosons. For example, one can define the variables by singling out one of the jets kinematically. Such an approach has been proposed before for other processes [11]. In this section we wish to present two different but similar observables for this purpose.

To start, one can define $W^{(>)}$ to be the $W$ which has the the hardest jet. Since the quark with the largest energy belongs to the $W$ boson with the largest $|\cos \psi|$, where $\psi$ of the decay products is the polar angle at the $W$ rest frame defined after Eq.(11). $W^{(>)}$ is either $W^{-}$ if $|\cos \psi| > |\cos \bar{\psi}|$, or $W^{+}$ if $|\cos \psi| < |\cos \bar{\psi}|$. In practice, one has to impose some cuts to exclude events for which $|\cos \psi| \approx |\cos \bar{\psi}|$. Once the $W^{(>)}$ is selected one can
use it to define an unique orientation for the production plane. One can define uniquely a new scattering angle $\Theta^{(>)}$, using the $W^{(>)}$ instead of the undetermined $W^-$. The C, P, CP, and CPT transformations of a typical $e^-e^+ \rightarrow W^-W^+ \rightarrow q\bar{q}g3\bar{q}4 \rightarrow 4$-jet event is demonstrated in Fig. 4. One very simple CP violating observable that one can easily identify is the forward-backward asymmetry of the most energetic jet. The observable is also CPT-odd, therefore can be used to study the imaginary parts of the CP violating form factors. It is also possible to study the real parts of the form factors, however one has to look into the azimuthal dependence of jets. First, we look at the asymmetry in the fully differential cross section $\sigma^{(>)}(\Theta^{(>)}; \psi_>, \phi_>; \psi_<, \phi_<)$ where $\psi_>, \phi_>$ are the CM angles of $W^{(>)}$ and $\psi_<, \phi_<$ are the corresponding angles of the other $W$. One can define the general CP-odd asymmetry $A_{CP}$ as

$$A_{CP} = \sigma^{(>)}(\Theta^{(>)}; \psi_>, \phi_>; \psi_<, \phi_<) - \sigma^{(>)}(\pi - \Theta^{(>)}; \psi_>, \pi - \phi_>; \psi_<, \pi - \phi_<).$$

(37)

Meanwhile, under CPT

$$\sigma^{(>)}(\Theta^{(>)}; \psi_>, \phi_>; \psi_<, \phi_<) \rightarrow \sigma^{(>)}(\pi - \Theta^{(>)}; \psi_>, \pi + \phi_>; \psi_<, \pi + \phi_<),$$

(38)

which clearly indicates that if one integrate over full range of $\psi_>, \phi_>; \psi_<$ and $\phi_<$ the resulting asymmetry is CPT-odd. One can also form a more general CP-odd, CPT-odd differential asymmetry to measure the absorptive form factors such as

$$A_i = \sigma^{(>)}(\Theta^{(>)}; \psi_>, \phi_>; \psi_<, \phi_<) - \sigma^{(>)}(\pi - \Theta^{(>)}; \psi_>, \pi - \phi_>; \psi_<, \pi - \phi_<)$$

$$- \sigma^{(>)}(\pi - \Theta^{(>)}; \psi_>, \pi + \phi_>; \psi_<, \pi + \phi_<) - \sigma^{(>)}(\Theta^{(>)}; \psi_>, -\phi_>; \psi_<, -\phi_<),$$

(39)

Since CP and CPT transformations differ in the azimuthal distributions one has to dig into this dependence to probe the CP-odd, CPT-even part. For example one can form the corresponding CPT-odd differential asymmetry which measured the real part of the CP violating form factors:

$$A_r = \sigma^{(>)}(\Theta^{(>)}; \psi_>, \phi_>; \psi_<, \phi_<) - \sigma^{(>)}(\pi - \Theta^{(>)}; \psi_>, \pi - \phi_>; \psi_<, \pi - \phi_<)$$

$$+ \sigma^{(>)}(\pi - \Theta^{(>)}; \psi_>, \pi + \phi_>; \psi_<, \pi + \phi_<) - \sigma^{(>)}(\Theta^{(>)}; \psi_>, -\phi_>; \psi_<, -\phi_<).$$

(40)

After integrating decay angles, the differential cross section $\sigma^{(>)}$ for producing $W^{(>)}$ with the hardest jet at an angle $\Theta^{(>)}$ can be related to an average of the underlying $W^+W^-$ production cross section. It can be written as

$$\sigma^{(>)}(\Theta^{(>)}) = \sum_{\lambda, \bar{\lambda}} \sigma_{\lambda, \bar{\lambda}}(\Theta^{(>)}) \int_0^1 d\zeta \int_0^1 d\bar{\zeta} D_{\lambda}(\zeta)D_{\bar{\lambda}}(\bar{\zeta}) \vartheta(\zeta - \bar{\zeta})$$

$$+ \sum_{\lambda, \bar{\lambda}} \sigma_{\lambda, \bar{\lambda}}(\pi - \Theta^{(>)}) \int_0^1 d\zeta \int_0^1 d\bar{\zeta} D_{\lambda}(\zeta)D_{\bar{\lambda}}(\bar{\zeta}) \vartheta(\bar{\zeta} - \zeta),$$

(41)

where the first sum is the contribution when the hardest jet is originated from $W^-$ while the second sum is the contribution when the hardest jet is originated from $W^+$. The decay probability function $D_{\lambda}(\zeta)$ is related to those decay amplitudes in Eq. (14),

$$D_{\lambda}(\zeta) = \frac{3}{2} B_h |d_{\lambda,-1}(\psi)|^2 + |d_{\lambda,-1}(\pi - \psi)|^2 = \begin{cases} \frac{3}{2} B_h (1 + \zeta^2) & \text{if } \lambda = \pm 1, \\
\frac{3}{2} B_h (1 - \zeta^2) & \text{if } \lambda = 0. \end{cases}$$

(42)
where we relate $\zeta = \cos \psi$, etc. The coefficients, $\frac{3}{2}$ or $\frac{3}{4}$, normalize the functions to one after integration over $\zeta$. $B_h$ is the branching fraction for the $W$ boson decaying into the purely hadronic mode. We have normalized $D_\lambda(\zeta)$ to $B_h$ when it is integrated over $0 \leq \zeta \leq 1$. The step function $\vartheta(\zeta)$ is unity if $\zeta > \epsilon_c$, zero otherwise. The threshold $\epsilon_c$ should correspond to the minimum jet energy difference detectable corresponding to the experimental resolution, but for simplicity, we illustrate the ideal case that $\epsilon_c = 0$.

Using the following integrals

$$\int_0^1 d\zeta \int_0^1 d\zeta' (1 + s\zeta^2)(1 + s'\zeta'^2) \vartheta(\zeta - \zeta') = \frac{1}{4}[(2 + s) + s'(\frac{1}{3} + s\frac{2}{3})], \tag{43}$$

where $s, s'$ are two sign factors taking values $\pm 1$. The forward–backward asymmetry is proportional to the difference of the following two integrals,

$$\int_0^1 d\zeta \int_0^1 d\zeta' (1 + s\zeta^2)(1 + s'\zeta'^2) \left[ \vartheta(\zeta - \zeta') - \vartheta(\zeta' - \zeta) \right] = \frac{1}{6}(s - s'). \tag{44}$$

In this equation, the sign $s = \pm 1$ corresponds to $|\lambda| = 1$ and $|\lambda| = 0$ respectively, and similarly, the sign $s' = \pm 1$ corresponds to either $|\lambda| = 1$ and $|\lambda| = 0$ respectively. Note that the contribution vanishes unless $\lambda \bar{\lambda} = 0$. Therefore, this asymmetry is not going to give us information about the CP–odd $\sigma_{+,+} - \sigma_{-,+}$. This is because after one averages over the angles $\psi$ and $\pi - \psi$, there is no difference between the decay of a $W^+$ with helicity $+$ and a $W^-$ with helicity $-$. For this reason, the asymmetry is insensitive to $\text{Im} \, f_7$. With this, we can derive the forward–backward asymmetry of the hardest jet analytically as

$$\sigma^{(>)}(\Theta^{(>)} - \sigma^{(>)}(\pi - \Theta^{(>)} = \frac{3}{8}B_h^2 \left[ \sigma_{-,0}(\Theta^{(>)})) - \sigma_{0,+}(\Theta^{(>)})) \\
\quad + \sigma_{+,0}(\Theta^{(>)})) - \sigma_{0,-}(\Theta^{(>)})) \\
\quad - (\Theta^{(>)} \rightarrow \pi - \Theta^{(>)})) \right]. \tag{45}$$

The asymmetry is usually defined as the ratio of the above quantity to the symmetric sum as below,

$$\sigma^{(>)}(\Theta^{(>)} + \sigma^{(>)}(\pi - \Theta^{(>)} = B_h^2 \sum_{\lambda, \bar{\lambda}}[\sigma_{\lambda,\bar{\lambda}}(\Theta^{(>)})) + \sigma_{\lambda,\bar{\lambda}}(\pi - \Theta^{(>)})). \tag{46}$$

The branching fraction will drop away in the ratio. This asymmetry is plotted in Fig. 5 for various form factors. The $Z$ boson mediated diagrams give larger contributions than the photon mediated diagrams. The form factors $\text{Im} f_6^Z$ gives the largest contribution. Therefore this measurement may be most effective in constraining $\text{Im} f_6^Z$ and less effective in constraining $\text{Im} f_4^Z$ and $\text{Im} f_6^Z$.

One of the shortcomings of the asymmetry described above is that the angle $\Theta^{(>)}$ is defined with respect to one of the $W$ bosons. Therefore one has to rely on good resolution on the $W$ boson momentum to define the angle. That is not a necessity as far as detecting CP violation is concerned. As analyzed earlier, the forward–backward asymmetry of the hardest jet itself, instead of the associated $W$ boson, is already a genuine CP–odd, CPT–even observable. Indeed this is a better observable to use in practice because it allows one
to make less severe cuts in order to make sure the contributions due to the non-$W^+W^-$ source are gotten rid of. In fact, if the goal is simply to detect CP violation, it is not necessary to check whether the event is due to the $W$ boson or not. The forward–backward asymmetry of the hardest jet itself of any three-jets or four-jets events is already a genuine CP–odd, CPT–even observable. The $W^+W^-$ intermediate state is only one of the potential mechanisms giving rise to such an asymmetry. It is only in the context of studying three gauge boson form factors that one wishes to eliminate the non–$W^+W^-$ background. To study the asymmetry in the context of $W^+W^-$ intermediate state, one has to go back to the general equation for cross section, $\sigma(\Theta, \psi, \phi, \bar{\psi}, \bar{\phi})$, in Eq. (12). For a given $\Theta$ the resulting jet can be either forward or backward. Given the angles $\psi$ and $\phi$ of a parton the criterion for a jet to be forward is

$$f(\Theta, \psi) = \gamma \frac{\beta + \cos \psi}{\sin \psi} \cot \Theta > \cos \phi.$$  \hspace{1cm} (47)

One can use this criterion to integrate over the region of phase space for fixed $\Theta$ for which the hardest jet is forward and subtract those region in which the hardest jet is backward. That is,

$$A_{FB}(\Theta) = \int_{-1}^{1} d\zeta \int_{-1}^{1} d\bar{\zeta} \left( \int_{F}^{\phi} - \int_{B}^{\phi} \right) d\phi \left( \frac{1}{2\pi} \right) \sigma \phi (|\zeta| - |\bar{\zeta}|)$$

$$+ \int_{-1}^{1} d\zeta \int_{-1}^{1} d\bar{\zeta} \int_{0}^{2\pi} d\phi \left( \int_{F}^{\phi} - \int_{B}^{\phi} \right) d\bar{\phi} \sigma \phi (|\bar{\zeta}| - |\zeta|),$$  \hspace{1cm} (48)

where $F$ indicates integration region in which $\cos \phi < f(\Theta, \psi)$ when $\zeta = \cos \psi > 0$, or, $\cos(\pi + \phi) < f(\Theta, \pi - \psi)$ when $\zeta < 0$ while $B$ represent the rest of the $\phi$ integration region. For $\bar{F}$ and $\bar{B}$, the function $f$ should be changed to $f(\Theta, \bar{\psi}) = \gamma(-\beta + \cos \bar{\psi}/\sin \bar{\psi}) \cot \Theta > \cos \bar{\phi}$.

The first term in the above equation is the contribution when the $W^-$ provides the hardest jet, while the second term is when the hardest jet is originated from $W^+$ decay. The total forward backward asymmetry will be an integration over $\Theta$. In practice, it is much more practical to study the distribution by Monte–Carlo simulation. In Fig. 6a,b, we plot the difference of event distributions versus the cosine of the polar angle of the hardest jet. The size of error bars comes from the numerical fluctuation in the simulation. Comparing this with the corresponding curve in Fig. 5, one finds that the asymmetries, even though different in the two cases, are quite similar with the one defined with respect to the hardest jet directly larger by about a factor of two. This indicates that there is indeed no unwanted cancellation when one uses the second, more directly measurable definition of asymmetry.

Unfortunately, defining the angles based on the hardest jet will not expose the dispersive parts of the form factors. This is reflected in Eq. (45) where only the absorptive parts are relevant. One can try to use the CPT–even asymmetry, $A_r$, defined earlier, however, it is an awkward observable to use.

As an alternative, one can avoid the reference to the polar angles in the $W$ decays altogether. Then the system of $e^-e^+ \rightarrow W^-W^+ \rightarrow (\text{four jets})$ looks like an unoriented production plane and two decay half-planes. Each half-plane extends in the direction of the $W$ boson with which it is associated. One can define an angle $\varphi$ between a half-plane and the production plane by rotating a half-plane along its axis (defined by the direction of the momentum of its $W$) in a right-handed sense until it coincides with the production plane.
The angle of this rotation defines $\varphi$ taking value in $(0, \pi)$ in a unique way. There are two such azimuthal angles corresponding to the two $W$ bosons, and they must be distinguished. By convention, one can define the azimuthal angles so that $\varphi$ corresponds to the $W$ emitted in the forward direction with respect to the incoming electron momentum. The $\varphi'$ corresponds to the azimuthal angle in the jets of the recoiling $W$.

To conform to previous definition of the azimuthal angle, we shall also define $\varphi'$ by the right-handed rule along the direction of the forward $W$ just as in the definition of $\varphi$. The $W^+W^-$ production angle $\Theta_{JP}$ in this case, by definition, only extends from $0$ to $\pi/2$, and both $\varphi, \varphi' \in (0, \pi)$. This coordinate system will be referred to as the Jet-Plane (JP) system. The decay polar angles will be integrated out. As before, a detailed simulation should set the threshold variable $\epsilon$ to reflect the experimental jet energy and direction resolution. Here, it is assumed that only two tiny holes in the phase space are cut out corresponding to polar angles of 0 and $\pi$.

Now one can ask what the effect of a CP transformation is on a system with this configuration. One can convince oneself that under CP

$$\text{CP} : (\Theta_{JP}, \varphi, \varphi') \rightarrow (\Theta_{JP}, \varphi', \varphi).$$

(49)

Under $\text{CP}\hat{T}$,

$$\text{CP}\hat{T} : (\Theta_{JP}, \varphi, \varphi') \rightarrow (\Theta_{JP}, \pi - \varphi', \pi - \varphi).$$

(50)

Define $n(\Theta_{JP}, \varphi, \varphi')$ to be the differential rate in JP variables. At the quark level, the rate is $\sigma_q(\Theta, \varphi, \bar{\varphi})$ for a $W^-$ at a scattering angle $\Theta$ with the $W^-$ decay azimuthal angle $\varphi$ and the $W^+$ decay azimuthal angle $\bar{\varphi}$. A summation must be made over quark configurations which lead to the same jet angles, $\Theta_{JP}$, $\varphi$, and $\varphi'$.

$$n(\Theta_{JP}, \varphi, \varphi') = \left[ \sigma_q(\Theta_{JP}, \varphi, \varphi') + \sigma_q(\Theta_{JP}, \varphi + \pi, \varphi') + \sigma_q(\Theta_{JP}, \varphi, \varphi' + \pi) + \sigma_q(\Theta_{JP}, \varphi, \varphi' + \pi) \right]$$

$$+ \left[ \Theta_{JP} \rightarrow \pi - \Theta_{JP}, \varphi \rightarrow -\varphi', \varphi' \rightarrow -\varphi \right].$$

(51)

The last summing bracket interchanges the $W^+$ and $W^-$. One way of constructing the integrated CP–odd asymmetry is to split the range of $\varphi$ into two quadrants and define the following asymmetry:

$$A_{JP}(\Theta_{JP}) = \frac{n(\Theta_{JP}, \text{I,II}) - n(\Theta_{JP}, \text{II,II})}{n(\Theta_{JP}, \text{all,all})},$$

(52)

where I and II refer to the two quadrants of the azimuthal spaces for both $\varphi$ and $\varphi'$. Note that this asymmetry is $\text{CP}\hat{T}$-even and therefore probes only the dispersive parts of the form factors. Fig. 7 shows the numerical result for various form factors. The form factors $\text{Re} \ f^7_6$, $\text{Re} \ f^7_7$ and $\text{Re} \ f^7_Z$ give rise to larger (positive) asymmetry than the other three form factors especially for $\cos \Theta_{JP} < 0.5$. The numerator above is half of the following expression

$$n(\Theta_{JP}, \text{I,all}) + n(\Theta_{JP}, \text{all,II}) - n(\Theta_{JP}, \text{II,all}) - n(\Theta_{JP}, \text{all,I}),$$

which has the same form as the numerator in Eq.(35). Therefore, we have the relation,
of partial reconstruction will reduce statistics. some of the asymmetry will be averaged away by this ambiguity and relying on this mode plane agree that a charged lepton was “above”, or “below”, the production plane. However, termination — there is a region of phase space in which both solutions for the production $W$ pair production are possible.

We can indeed apply this analysis to any $W$ boson purely due to the boost of the $W$ bosons as long as the quarks in the final state are light. Therefore, the $W$–partner of the hardest jet must be the softest one among the four. One can indeed apply this analysis to any $e^−e^+ \to$ four–jets events by pairing simply the hardest jet with the softest one in setting up one of the “decay planes” for the definition of the $Θ_{JP}$.

Of course in that case, one cannot conclude that the CP–odd signal is purely due to the CP violating signal, this may be an advantage instead of a set–back.

VI. CP ASYMMETRY IN THE PURELY LEPTONIC MODE

In the process $e^-e^+ \to W^-W^+ \to (\ell^-\bar{\nu})(\ell'^+\nu)$, the two missing neutrinos are not detectable. It is known that kinematics can only be re–constructed with a two–fold ambiguity. To be self–contained, we shall explain this re–construction geometrically. The vectors $\vec{p}_{W^-}$ and $\vec{p}_{W^+}$ lie on the surface of a sphere $S$ of radius $\frac{1}{2}(s – 4M_W^2)^{\frac{1}{2}}$. The direction of $\vec{p}_{\ell^-}$ fixes a point $P$ on $S$. Since the magnitude the missing $\vec{p}_\nu$ is just $E_\nu = \frac{1}{2}\sqrt{s} – E_{\ell^-}$, the angle $\theta$ between $\vec{p}_{\ell^-}$ and $\vec{p}_{W^-}$ is fixed to be $\cos \theta = ([\vec{p}_{\ell^-}][\vec{p}_{W^-}] – [\vec{p}_W^-]E_\nu/\vec{p}_{\ell^-}||\vec{p}_{W^-}||$. The closed triangle described by the vectors $\vec{p}_{\ell^-}$, $\vec{p}_{W^-}$, and $\vec{p}_\nu$ has a fixed shape, but it is free to swing about the axis $\vec{p}_{\ell^-}$. Therefore, the locus of $\vec{p}_{W^-}$ is a circle $C$ on the sphere $S$ about point $P$. similarly, the locus of $-\vec{p}_{W^-}$ another circle $C'$ on the sphere $S$ about point $P'$, which corresponds to the direction of $-\vec{p}_{\ell'^+}$. Since $W^+$ and $W^−$ are recoiling one another, the solution of the physical directions of $W^−$ and $W^+$ are locations of the intersections of these two circles $C, C'$. Generally, these two circles have two common intersecting points, which give rise to the two folded uncertainty that we cannot resolve without knowing the direction of each neutrino momentum.

If we wish to measure the up–down asymmetries in Eqs.(32–36), we need to know whether $\ell^−$ or $\ell'^+$ in each event is above or below the production plane. The two–folded ambiguity in fixing the $W^+W^−$ production plane will not always prevent us from making such determination — there is a region of phase space in which both solutions for the production plane agree that a charged lepton was “above”, or “below”, the production plane. However, some of the asymmetry will be averaged away by this ambiguity and relying on this mode of partial reconstruction will reduce statistics.
An alternative is to use CP violating observables which depend only on the observed momenta $\vec{p}_{e^-}$, $\vec{p}_{e^+}$, $\vec{p}_{\ell^-}$, and $\vec{p}_{\ell^+}$. For example, we can measure the overall asymmetry as follows,

$$\frac{N(\vec{p}_{e^-} \cdot \vec{p}_{e^+} \times \vec{p}_{\ell^-} > 0) - N(\vec{p}_{e^-} \cdot \vec{p}_{e^+} \times \vec{p}_{\ell^-} < 0)}{N(\vec{p}_{e^-} \cdot \vec{p}_{e^+} \times \vec{p}_{\ell^-} > 0) + N(\vec{p}_{e^-} \cdot \vec{p}_{e^+} \times \vec{p}_{\ell^-} < 0)}.$$  \hfill (54)

This quantity is C-even, P-odd and $\hat{T}$-odd therefore only the real parts of the CP–odd form factors contribute. In Fig. 8a,b, we use Monte–Carlo simulation to illustrate this asymmetry in the event distribution versus the variable $\chi = (8/sM_W)\vec{p}_{e^-} \cdot \vec{p}_{\ell^+} \times \vec{p}_{\ell^-}$. Only $\text{Re} f_{4,6}^{Z}$ are used as illustrations. They show that $\text{Re} f_{6}^{Z}$ generally give rise to larger contributions than $\text{Re} f_{4}^{Z}$.

One can also understand this asymmetry geometrically by constructing two oriented planes. The first plane is specified by $\vec{p}_{\ell^\pm}$. The other plane is constructed by the beam direction and the total missing momentum. The relevant observable is the angle $\Phi$ between these two planes. CPV will appear in a way that the event distribution is not symmetric under the transformation $\Phi \leftrightarrow -\Phi$. Under CPT, $\Phi$ is invariant. In Fig. 9a,b, we use Monte–Carlo simulation to illustrate this asymmetry in the event distribution versus the variable $S = \sin \Phi$ using $\text{Re} f_{4}$ and $\text{Re} f_{6}$ as examples. Just as in Fig. 8a,b, $\text{Re} f_{6}^{Z}$ generally give rise to larger contributions than $\text{Re} f_{4}^{Z}$.

Note that this CP violating observable can be applied to any process with $e^-e^+ \rightarrow \ell^-\ell^+X$ where $X$ is some neutral object. The $W^+W^-$ intermediate state is only one of the mechanisms that can give rise to this asymmetry in general. For example, another process that can contribute to this asymmetry is $e^-e^+ \rightarrow Z + Z^* \rightarrow \ell^-\ell^+\nu\bar{\nu}$. More exotic, the signal may originate from wino pair production in supersymmetric models. If one wishes to investigate only the $W^+W^-\gamma(Z)$ form factors one can do further kinematic cuts based on the determination of $W$ momenta up to a two-fold ambiguity. Alternatively, one can insist that $\ell'$ and $\ell$ are of different flavor. In that case the non–$W$–pair background can be greatly reduced.

**VII. MODELS OF CP VIOLATION**

In specific gauge models of CP violation, the natural values for the CP–odd form factors considered here are expected to be $10^{-2}$ or smaller due to the one loop suppression factor. That makes observation of the CP–odd effects discussed here a very difficult task. In this section we briefly discuss several possibilities of having models which can lead to asymmetries which may be observable. As noted earlier, the standard, KM model is not expected to give significant CP violation signals at high energy colliders. One reason is that the CP violation will always be proportional to the sines of the mixing angles, an a priori suppression of about $10^{-3}$ over CP conserving processes, even at high energy. In other models such as multi-Higgs doublet models, where CP violation arises in the Higgs sector, CP violation is suppressed at low energy because the mass of the Higgs bosons is so high. At higher energies, such suppression effect disappears. Similarly, one can also imagine SUSY models in which CP symmetry is broken in the couplings of heavy superpartners to gauge bosons.

Since $f_{6}$ and $f_{7}$ contain the parity violating $\varepsilon$ symbol, a general argument [12] shows that it takes at least one fermion loop to generate them even in non-standard models. Among the
CP odd form factors, $f_7$ is the one that is least likely to be generated at the one loop level in non-standard models. To generate $f_7$ form factor, in addition to the fermion loop, there have to be enough factors of momentum from the fermion propagators to provide three powers of external momenta. To achieve this, one has to avoid the mass insertion in the three fermion propagators. It is easy to show that if the neutral gauge boson vertex is flavor neutral, for the one loop graph to be CP violating, it requires coexistence of the left the right handed couplings in the $W$ coupling vertex. To achieve CP violation, it is not possible avoid mass a insertion in fermion propagators. Therefore, there are insufficient momenta in one loop to generate the three powers required. In contrast, $f_4$ and $f_6$ typically can be generated at one loop in most non-standard models. The parity–even $f_4$ can even be induced in purely bosonic loop graphs.

He, Ma, and McKellar [17] have calculated the one-loop contributions to the CP–odd form factors in the two Higgs doublet and left-right symmetric models. In the two Higgs doublet model, there are three neutral and one charged Higgs boson which can all contribute from within the loop shown in Fig. 10. He et al. find that only $f_4^Z$ is generated and that it can be as large as $10^{-3}$. Note that the presence of an absorptive part in the form factor depends on the mass of the Higgs bosons; Higgs bosons lighter than $\frac{1}{2}\sqrt{s}$ can go on shell in the loop. However, one also have to take into account the constraints on the masses in the Higgs sector from flavor changing neutral current (FCNC) bounds. To the degree that the natural flavor conserving discrete symmetry is broken, CP violation may appear in the the Higgs sector. Therefore the constraint on FCNC also tends to limit CP violation as well.

The Left-Right symmetric model is an example of a class of models in which the CP violating form factors are generated at one loop by fermion loops such as those in Fig. 11. These models have have left- and right-handed couplings to $W$ bosons each with a different phase. To generate $f_4$ the axial coupling is required, hence there is no $f_4^\gamma$ induced. To generate $f_6^Z$ the vector coupling is required. In a Left-Right symmetric model in which CP violation arises in the mixing of the $SU(2)_L$ and $SU(2)_R$ gauge bosons, Ref [17] find that at $\sqrt{s} = 200$ GeV, $f_4^Z \sim 10^{-4}$, $f_6^Z \sim 10^{-4}$, and $f_6^\gamma \sim 10^{-3}$. The mixing angle is constrained by low energy physics [18] to be less than 0.0028 and this puts $f_4^Z$ and $f_6^Z$ beyond LEP–II’s limit of detectability, and $f_6^\gamma$ is on the very edge.

There are other models in this class which may be more promising because they are less constrained experimentally. For example, [19] one can imagine that a subset of the supersymmetric spectrum consisting of winos and photinos with similar couplings to those of the quarks in the Left-Right model generates a larger $f_4^Z$, $f_6^Z$ and $f_6^\gamma$ (Fig. 11 replacing quarks with $W$ and $\gamma$ superpartners). This is because there is no constraint on the CP violating source in such models. The masses of the winos or photinos can be accommodated by current data in the range less than $\frac{1}{2}\sqrt{s}$ and larger than the current bounds, thus providing an absorptive part to the above form factors.

**VIII. CONCLUSION**

At LEP II, the $W^+W^-$ production cross section reach the maximum of $\sigma \approx 20\, pb$ for $\sqrt{s} = 200$ GeV which can provide about $10^4$ $W^+W^-$ pairs per year for the design luminosity of $5 \cdot 10^{31}cm^{-2}sec^{-1}$. The branching ratio of the $W$ decay into each lepton channel is about...
while each of the light quark channel is about \( \frac{1}{3} \). This gives us a lot of events for both the leptonic and the hadronic channels. As we have shown in the paper, it is possible to test CP symmetry in purely leptonic, purely hadronic or and mixed channels of the two W boson decays. While the event statistics probably will not be large enough to test some of the popular alternative gauge models of CP violation, it is nevertheless sufficient to provide nontrivial constraints on the CP-odd form factors in the three gauge boson couplings.

Even though in this paper we have concentrated on decoding the CP-odd form factors of the three gauge boson couplings in the process \( e^-e^+ \rightarrow W^-W^+ \), many of the techniques we used in the analysis can also be applied to other high energy collision processes which may be relevant to some special models of CP violation. This is especially true for the observables we analyzed in Section V and VI. It is interesting to measure the CP-odd signal in events with four jets or with two charged leptons and missing momentum in \( e^-e^+ \) colliders, independent of whatever the intermediate states that may be used to interpret such signals. In case of null signals, they can be translated into constraints about various form factors in different models.
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Figure Caption

Fig. 1 The coordinate system for the process $e^-e^+ \rightarrow W^-W^+ \rightarrow (f_1\bar{f}_2)(f_3\bar{f}_4)$ Fig.1a is the $W$ pair production plane. Fig.1b is the $W^-$ decay kinematics. Note that the angles $\psi$ and $\phi$ are defined in the rest frame of the decaying $W$, while the angle $\Theta$ is defined in the center of mass system. The $x$, $y$, and $z$ axes which define the angles $\bar{\psi}$ and $\bar{\phi}$ for the $W^+$ decay also share the same directions as those depicted in the figure; again, the angles are defined in the rest frame of the $W^+$.

Fig. 2 (a) $A_E$, per unit of $\text{Im} f$, at $\sqrt{s} = 190$ GeV, (b) $A'_E$, per unit of $\text{Im} f$, at $\sqrt{s} = 250$ GeV, (c) $A'_E$ per unit of $\text{Im} f$, at $\sqrt{s} = 190$ GeV.

Fig. 3 (a) $\text{Im } \rho_{+,0} - \text{Im } \bar{\rho}_{-,0}$, (b) $\text{Im } \rho_{-,0} - \text{Im } \bar{\rho}_{+,0}$, and (c) $\text{Im } \rho_{+,+-} - \text{Im } \bar{\rho}_{-,+,}$, per unit of $\text{Re } f$, versus $\Theta$ at $\sqrt{s} = 190$ GeV.

Fig. 4 Discrete transformations of a typical $e^-e^+ \rightarrow W^-W^+ \rightarrow 4$-jet event.

Fig. 5 The forward–backward asymmetry of $W$ boson that is associated with the most energetic jet in $e^-e^+ \rightarrow W^-W^+ \rightarrow 4$-jets. Each curve corresponds to an individual CPV form factor having an absorptive part while the others are set to zero.

Fig. 6 (a) Result from Monte–Carlo simulation for the forward–backward asymmetry of the most energetic jet itself in $e^-e^+ \rightarrow W^-W^+ \rightarrow 4$-jets versus the cosine of the polar angle $\Theta_{HJ}$ of the jet. Asymmetry here is measured per unit of $\text{Im } f_4^Z$ or $\text{Im } f_6^Z$ with other CPV form factors tuned off. (b) Same as (a) for the form factors $\text{Im } f_4^Z$ and $\text{Im } f_6^Z$. For readability, smooth Monte–Carlo results are joined with a smooth curve.

Fig. 7 The jet plane asymmetry for $e^-e^+ \rightarrow W^-W^+ \rightarrow 4$-jets as a function of $\cos \Theta_{JP}$. Each curve corresponds to an individual form factor having an dispersive part while the others are held at zero.

Fig. 8 (a) Result from Monte–Carlo simulation for the purely leptonic mode. The distribution difference $n_\chi(\chi) - n_\chi(-\chi)$ is plotted at $\sqrt{s} = 190$ GeV per unit of $\text{Re } f_4^Z$ or $\text{Re } f_6^Z$. Here $\chi = (8/sM_W)p_{\ell^-} \cdot \vec{p}_{\ell^-} \times \vec{p}_{\ell^-}$, and $n_\chi(\chi) = N^{-1}dN/d\chi$. (b) Same as (a) for the form factors $\text{Re } f_4^Z$ and $\text{Re } f_6^Z$. For readability, smooth Monte–Carlo results are joined with a smooth curve.

Fig. 9 (a) Result from Monte–Carlo simulation for the distribution difference $n_S(S) - n_S(-S)$ is plotted at $\sqrt{s} = 190$ GeV as a function of $S$ per unit of $\text{Re } f_4^Z$ or $\text{Re } f_6^Z$. Here $S = \sin \Phi$ and $n_S(S) = N^{-1}dN/dS$. (b) Same as (a) for the form factors $\text{Re } f_4^Z$ and $\text{Re } f_6^Z$.

Fig. 10 Diagrams responsible for $f_4^Z$ in the two Higgs doublet model.

Fig. 11 CP violating Fermion loop diagrams in the Left-Right symmetric model.
\( \sqrt{s} = 190 \text{ GeV} \)

- Dash-dot-dot-dot Re \( f \)
- Dash-dot-dot Re \( f \)
- Dot Re \( f \)
- Dash Re \( f \)
- Solid Re \( f \)

[Im \( \rho_+ \) - Im \( \rho_- \)]/Re \( f \)
Figure 1a.
Figure 1b
Figure 4.

Hard Jet Into (UP)  
[Original]

Hard Jet Out (DOWN)  
[P conjugate]

Hard Jet Into (DOWN)  
[C, CPT conjugate]

Hard Jet Out (UP)  
[CP conjugate]
$n_z(z) - n_z(-z)$ (per unit of Im f)

Forward-Backward asymmetry of the Hardest Jet

$\sqrt{s}=190$ GeV

(b)

Fig. 6
Jet Plane asymmetry per unit of Re f

Fig. 7

\( \sqrt{s} = 190 \text{ GeV} \)
\( n_\chi(\chi) - n_\chi(-\chi) \) per unit of Re \( f \)
$n_{\chi}(\chi) - n_{\chi}(-\chi)$ per unit of $\text{Re } f$

$\chi$

$\sqrt{s} = 190$ GeV

$N^{-1} dN/d\chi$

Fig. 8
\( n_s(S) - n_s(-S) \) (per unit Re \( f \))

\[ S = \sin(\phi) \]

[Diagram showing graph with axes and annotations]
\( (\Phi)_{\text{vis}} = S \)

\( \Re \ell^4 \neq 0 \)

\( \Re \ell^6 \neq 0 \)

\( \Lambda \equiv 190 \text{ GeV} \)

\( (\Phi)_{\text{vis}} = S \)

\( (SP/\mathcal{NP}_{\Lambda - N}) = (\cdot)^S u \)

(a)
Figure 10.

Figure 11.
