Effects of the generalized uncertainty principle on the thermal properties of Kemmer oscillator

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Abstract: A series of aspects of the quantum gravity predict a modification in the Heisenberg uncertainty principle to the generalized uncertainty principle (GUP). In the present work, using the momentum space representation, we study the behavior of the Kemmer oscillator in the context of the GUP. The wave function, the probability densities, and the energy spectrum are obtained analytically. Furthermore, the thermodynamic properties of the system are investigated by using the numerical method and the influence of GUP on thermodynamic functions are also discussed.

Keywords: Kemmer oscillator; GUP; Epstein zeta function

PACS Nos.: 03.65.Pm; 04.20.Jb

1. Introduction

Almost all approaches to quantum gravity predict the existence of a minimum measurable length scale of the order of the Planck scale in the spacetime, these approaches usually include the string theory [1–3], the loop quantum gravity [4], the black hole physics [5], the doubly special relativity [6, 7] and the noncommutative field theories [8], and it is widely believed that the GUP theory is a common characteristic of these approaches. Indeed, the application of GUP in quantum mechanics overcomes the shortcomings of the Heisenberg Uncertainty Principle (HUP) at high energies. Nowadays, various aspects have been studied in connection with the GUP. As typical examples: the Lifshitz field theories [9], the path integral quantization [10], the time crystals [11], the supersymmetry breaking [12], the Black Hole Thermodynamics [13], the cosmological constant from a deformation of the Wheeler–DeWitt equation [14], the deformation of the Dirac equation [15] and the GUP-Corrected Thermodynamics for all Black Objects and the Existence of Remnants [16] are studied. In addition, more works in this context can be found in [17–28].

On the other hand, Kemmer equation plays a very important role in relativistic quantum mechanics. Recently, many research works are devoted to discuss about it [29–31], especially for the interest in the quark–anti-quark bound state problem. In the present work, we are interested to study the effects of the GUP on the thermal properties of the Kemmer oscillator. This work is organized as follows: in Sect. 2 we analyze the solutions of the Kemmer oscillator in the context of the GUP and obtain the energy spectrum as well as the corresponding probability density. Subsequently, the thermodynamic properties of the Kemmer oscillator are investigated by employing the zeta Epstein function in Sect. 3. Moreover, in order to have an intuitive understanding for the statistical properties of the physical system, we depict the numerical results of the thermodynamic functions with several figures and discuss the effects of the GUP parameter on thermal properties in Sect. 4. Finally, Sect. 5 is our conclusion.

2. The Kemmer oscillator in the context of the GUP

The relativistic Kemmer equation for spin one particles can be written as [32–34]
\[
\left( \beta^u \hat{p}_u - Mc \right) \psi_k = 0,
\]
where \( M \) is the total mass of the two same spin one-half particles. The Kemmer matrices \( \beta^u \) (\( u = 0, 1, 2, 3 \)) satisfy the following algebra relation:
\[
\beta^u \beta^v + \beta^v \beta^u = g^{uv} \beta^1 + g^{1u} \beta^v,
\]
with \( g^{uv} = \gamma^u \otimes \hat{I} + \hat{I} \otimes \gamma^v \),
where \( \gamma^u \) are the Dirac matrices, \( \hat{I} \) is a \( 4 \times 4 \) identity matrix, and \( \otimes \) expresses a direct product.

Considering the configuration of the Dirac oscillator potential in one dimension case, the operator \( \hat{p}_x \) should be substituted by \( \hat{p}_x - \imath MBw x \) in the free Kemmer equation, with \( w \) being the oscillator frequency. Thus, the Kemmer equation in the presence of a Dirac oscillator interaction is given by
\[
\left[ (\gamma^0 \otimes \hat{I} + \hat{I} \otimes \gamma^0) E - c (\gamma^0 \otimes \hat{\sigma}_x + \hat{\sigma}_x \otimes \gamma^0) \right] (\hat{p}_x - \imath MBw x) - Mc^2 \gamma^0 \otimes \gamma^0) \psi_k = 0.
\]

The stationary state \( \psi_k \) of the Eq. (4) can be described by a four-component spinor as
\[
\psi_k = \psi_D \otimes \psi_D = (\psi_1 \psi_2 \psi_3 \psi_4)^T,
\]
substituting Eq. (5) into Eq. (4), we can easily obtain four linear algebraic equations
\[
(2E - Mc^2) \psi_1 - c (\hat{p}_x + \imath MBw x) \psi_2 - c (\hat{p}_x + \imath MBw x) \psi_3 = 0,
- c (\hat{p}_x - \imath MBw x) \psi_1 + Mc^2 \psi_2 + c \psi_3 = 0,
- c (\hat{p}_x - \imath MBw x) \psi_1 + Mc^2 \psi_3 + c \psi_4 = 0,
- c (\hat{p}_x + \imath MBw x) \psi_2 + c \psi_3 - (2E + Mc^2) \psi_4 = 0.
\]

From the above equations, it is not difficult to obtain the following relations
\[
\psi_2 = \psi_3, \quad \psi_1 = \frac{2c}{2E - Mc^2} (\hat{p}_x + \imath MBw x) \psi_2, \quad \psi_4 = \frac{2c}{2E + Mc^2} (\hat{p}_x + \imath MBw x) \psi_2,
\]
combination Eqs. (5) and (6) we have
\[
\left\{ \hat{p}_x^2 + M^2 w^2 x^2 + \frac{Mc^2}{k_1 + k_2} - \imath MBw [x, p_x] \right\} \psi_2 = 0,
\]
where \( k_1 = 2c^2/(Mc^2 - 2E), k_2 = 2c^2/(Mc^2 + 2E) \). Here, it should be noted that in the momentum space representation we have \( \hat{p} = p \) and \( \hat{x} = i\hbar (1 + \beta p^2) \).

In addition, according to the deformed Heisenberg algebra \( [1] \), the commutation relation between position vector and momentum vector satisfy
\[
[x, p] = i\hbar (1 + \beta p^2).
\]

Substituting Eq. (9) into Eq. (8) we obtain
\[
\left[ (1 + \beta p^2) \frac{\partial^2}{\partial p^2} + 2\beta p (1 + \beta p^2) \frac{\partial}{\partial p} - \frac{(1 + Mw\beta)}{M^2 w^2 h^2} p^2 \right] \psi_2(p) = 0,
\]
with \( \epsilon = \frac{Mc^2}{k_1 + k_2} \).

With the aid of the variable \( q \) defined by \( p \in (-\infty, +\infty) \rightarrow q \in \left( -\frac{\pi}{2Mw\sqrt{\beta}}, +\frac{\pi}{2Mw\sqrt{\beta}} \right) \), with \( q = \frac{\tan^{-1} p\sqrt{\beta}}{Mw\sqrt{\beta}} \) we can rewrite the Eq. (10) as
\[
\psi_2(q) = \left[ \frac{\partial^2}{\partial q^2} - \frac{1 + Mw\beta}{\beta} \sin^2 (Mw\beta\sqrt{q}) \right] \psi_2(q) = 0.
\]

Here we introduce an auxiliary function \( \psi_2(q) = [1 - \sin^2 (Mw\beta\sqrt{q})]^n \left[ \sin (Mw\beta\sqrt{q}) \right] \) for the sake of simplification, let us assume that
\[
\frac{\psi_2(q) - (1 + Mw\beta) \psi_2(q)}{M^2 w^2 h^2 \beta^2} = 0.
\]

Actually, this equation will lead to the following expression of \( \psi \), i.e.,
\[
\psi_1 = \frac{1 + \sqrt{1 + \frac{4(1 + Mw\beta)}{M^2 w^2 h^2 \beta^2}}}{2} \quad \text{and} \quad \psi_2 = \frac{1 - \sqrt{1 + \frac{4(1 + Mw\beta)}{M^2 w^2 h^2 \beta^2}}}{2}.
\]

Moreover, the polynomial solution to Eq. (13) is obtained by demanding the following condition:
\[
-\psi - \frac{\epsilon + Mw\beta}{M^2 w^2 h^2 \beta} = n(1 + 2\epsilon)
\]
with \( n \) being a non-negative integer.

The Eq. (13) can be rewritten as
\[
\left[ 1 - \sin^2 (Mw\beta\sqrt{q}) \right] \left[ \sin (Mw\beta\sqrt{q}) \right] - (2\epsilon + 1) \sin (Mw\beta\sqrt{q}) f \left[ \sin (Mw\beta\sqrt{q}) \right] = 0.
\]

Obviously, its solution can be expressed in terms of
Gegenbauer’s polynomials as \( f[\sin(M\omega\sqrt{\beta}q)] = NC_n^\pm[\sin(M\omega\sqrt{\beta}q)] \), with \( N \) being a normalization constant. Then the momentum eigenfunction of Kenmer oscillator in the context of the GUP is given by

\[
\psi_2(p) = N \left[ 1 - \sin^2 \left( M\omega\sqrt{\beta}q \right) \right] C_n^\pm[\sin(M\omega\sqrt{\beta}q)] \nonumber,
\]

\[
= N \frac{1}{1 + \beta p^2} C_n^\pm \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right), \quad (16)
\]

Besides, by using the following property of Gegenbauer’s polynomials \([35]\)

\[
\frac{dC_n^\pm[\sin(M\omega\sqrt{\beta}q)]}{dsin(M\omega\sqrt{\beta}q)} = 2\pi C_{n-1}^\pm[\sin(M\omega\sqrt{\beta}q)], \quad (17)
\]

we finally obtain

\[
\psi_1(p) = \frac{2\pi}{2E - Mc^2} \left[ p_x - \hbar M\omega(1 + \beta p^2) \frac{\partial}{\partial p_x} \right] \psi_2(p) \nonumber,
\]

\[
= \frac{2\pi}{2E - Mc^2} N \frac{1}{1 + \beta p^2} \left\{ \left( p + \hbar M\omega\beta p \right) C_n^\pm \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right) - 2\pi \hbar M\omega(1 + \beta p^2) C_{n-1}^\pm \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right) \right\} \nonumber,
\]

\[
\psi_2(p) = \psi_1(p), \quad \psi_3(p) = \frac{2\pi}{2E + Mc^2} \left[ p_x + \hbar M\omega(1 + \beta p^2) \frac{\partial}{\partial p_x} \right] \psi_2(p) \nonumber,
\]

\[
= \frac{2\pi}{2E + Mc^2} N \frac{1}{1 + \beta p^2} \left\{ \left( p + \hbar M\omega\beta p \right) C_n^\pm \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right) - 2\pi \hbar M\omega(1 + \beta p^2) C_{n-1}^\pm \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right) \right\} \nonumber,
\]

\[
(18)
\]

Therefore, the wave function of the system can be written as

\[
\psi_k = N \left( \frac{2\pi}{2E - Mc^2} \left( \frac{p_x + i\hbar M\omega x}{1 + \beta p^2} \right) \right) \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right) C_n^\pm \left( \frac{p\sqrt{\beta}}{\sqrt{1 + \beta p^2}} \right). \quad (19)
\]

At this stage, we determine the normalization constant \( N \) by using the following condition:

\[
(\psi_k, \psi_k) = \int_{-\infty}^{+\infty} \frac{1}{1 + \beta p^2} \psi_k^* (\gamma^0 \otimes \gamma^0) \psi_k dp = 1, \quad (20)
\]

and according to the relation \[\int_{-1}^{1} dy(1 - y^2)^{\lambda - \frac{3}{2}} [C_n(y)]^2 = \frac{2^{2\lambda - 1} \Gamma(2\lambda + n)}{n! \Gamma(\lambda) \Gamma(2\lambda)}, \]
we have

\[
N = \beta^2 \pi^{\frac{1}{2}} 22^{-1} \int \left\{ -\Gamma(2\zeta + n) \frac{\Gamma(2\zeta + n)}{n!(n + \zeta)(\zeta)^2} + \frac{2\zeta(2E^2 + 2\zeta M\omega\hbar \beta)}{(2E - Mc^2)^2 (2E + Mc^2)^2} \right\} \left\{ \frac{1}{\beta - 4\zeta M\omega \hbar} (1 + M\omega\zeta \beta) \frac{\Gamma(2\zeta + n)}{n!(n + \zeta)(\zeta)^2} \right. \]

\[
+ \frac{\zeta M\omega \hbar^2 \beta}{(n + 1)!(n + \zeta + 1)(\zeta + 1)^2} \left. \right\}^{-\frac{1}{2}}, \quad (21)
\]

then the corresponding probability density of every component is given by

\[
P_i'(t) = \left\{ \int_{-\infty}^{+\infty} \psi_i^* (\gamma^0 \otimes \gamma^0) \psi_i dp \right\}, \quad i = 1, 2, 3, 4. \quad (22)
\]

Furthermore, we derive the energy spectrum from the Eq. (14) which leads to

\[
E_n^2 = c^2 M^2 w^2 h^2 \beta c^2 + \left( \frac{2Mw c^2 \sqrt{1 + M\omega\beta}}{c^2 M^2 w^2 h^2 \beta} + \frac{c^2 M^2 w^2 h^2 \beta}{2} + \frac{c^4 M^2}{4} + M\omega h c^2 + M\omega h c^2 \sqrt{1 + M\omega\beta} \right) \nonumber,
\]

\[
n = 0, 1, 2, \ldots \quad (23)
\]

In view of the obscuring and complexity of Eq. (23), we decide to depict the numerical results aiming to show the effect of the GUP in the energy spectra. In Fig. 1, the energy spectra \( E \) versus the principal quantum number for different values of the GUP parameter are plotted. Positive and negative energy levels correspond to the case of a particle and antiparticle, respectively. It shows that for the same principal quantum number, the energy \( E \) increases monotonically with the increase of the GUP parameter. The effect of the GUP parameter on the energy levels is observable, where \( \beta = 0 \) corresponding to the case of the normal quantum mechanics, and this result is rigorously consistent with the Ref. [36].
3. The thermal functions of Kemmer oscillator under the influence of GUP

As we know, all thermodynamic quantities can be obtained from the partition function \( Z \), therefore, in the following work, the partition function of the system is calculated firstly. We start with the following eigenvalues of Kemmer oscillator in the context of GUP as

\[
E_n = \sqrt{\kappa n^2 + \eta n + 1} \frac{\sqrt{M^2 c^2 w^2 \hbar^2 \beta}}{2} + \frac{c^2 M^2}{4} + c^2 M w \hbar + c^2 M w \hbar \sqrt{1 + M w \hbar \beta},
\]

(24)

where \( \kappa = \frac{M w \hbar \beta}{2 \sqrt{1 + M w \hbar \beta}} \) and \( \eta = \frac{M w \hbar \beta}{2 \sqrt{1 + M w \hbar \beta}} \).

Given the energy spectrum, we can define the partition function via

\[
Z = \sum_n e^{-\beta E_n},
\]

(25)

where \( \beta = \frac{1}{k_B T} \) with \( k_B \) is the Boltzmann constant. In this case, the Eq. (25) can be written as

\[
Z = \sum_n e^{-\frac{1}{2} \sqrt{\kappa n^2 + \eta n + 1}},
\]

(26)

with \( T = \frac{M \omega_{0} \hbar}{4} \). Motivated by the previous work [37], now we set \( \tau = \frac{\sqrt{1 + M w \hbar \beta}}{M \omega_{0}} \), \( t = \frac{1}{\tau} \sqrt{\kappa n^2 + \eta n + 1} \), and by using the following relation [38]

\[
e^{-t} = \frac{1}{2\pi i} \int_C ds^{-\gamma} \Gamma(s),
\]

(27)

then the sum can be expressed as

\[
\sum_{n} e^{-\frac{1}{2} \sqrt{\kappa n^2 + \eta n + 1}} = \frac{1}{2\pi i} \int_C ds^{-\gamma} \sum_{n} (\kappa n^2 + \eta n + 1)^{-\gamma} \Gamma(s) = \frac{1}{2\pi i} \int_C ds^{-\gamma} J(s) \Gamma(s),
\]

(28)

where \( J(s) = \sum_{n} \frac{1}{Q(1,n)^{\gamma}} \) and \( \Gamma(s) \) are the Euler and one-dimensional Epstein zeta function respectively, with \( Q(1,n) = \kappa n^2 + \eta n + 1 \). For the sake of convenience, we set \( x = \frac{1}{2} \) and \( y = \sqrt{\frac{4\kappa n^2}{x^2}} \), then the expression of \( J(s) \) can be rewritten as

\[
J(s) = 2\kappa^{-\frac{\gamma}{2}} \zeta(s) + \frac{2\kappa^{-\frac{\gamma}{2}} y \Gamma(s-1) \Gamma\left(\frac{s}{2} - \frac{1}{2}\right)}{\Gamma\left(\frac{s}{2}\right)} H\left(\frac{s}{2}\right).
\]

(29)

Now, the final partition function turns into

\[
Z = \frac{2^{\gamma}}{2\pi i} \int_C ds^{-\gamma} J(s) \Gamma(s) + \frac{1}{2\pi i} \int_C ds^{-\gamma} 2\kappa^{-\gamma/2} y \frac{2^{1/2}}{\pi} \zeta(s-1) \Gamma(s-1) \Gamma\left(\frac{s}{2} - \frac{1}{2}\right) \Gamma\left(\frac{s}{2}\right) + \frac{1}{2\pi i} \int_C ds^{-\gamma} 2\kappa^{-\gamma/2} y \frac{2^{1/2}}{\pi} \zeta(s-1) \Gamma(s-1) \Gamma\left(\frac{s}{2} - \frac{1}{2}\right) H\left(\frac{s}{2}\right) \Gamma(s),
\]

(30)

Next, by applying the residues theorem, we have

\[
Z = 2 \zeta(0) + \frac{2}{\sqrt{k}} (\zeta(1) + \zeta(0)) T + \frac{2\pi T^2}{k y}. \]

(31)

In addition, it should be noted that the last integral in Eq. (30) goes to the zero because of the following relation

\[
\frac{1}{\Gamma(s)} = se^{s} \prod_{n=1}^{\infty} \left\{ 1 - \frac{x}{n} \right\} e^{-\frac{x}{n}},
\]

(32)

where \( r \) is Euler’s constant expressed as \( r = \lim_{n \to \infty} \left( \frac{\sum_{k=1}^{n} \frac{1}{k} - \log(n)}{n} \right) \), thus the final partition function of Kemmer oscillator in the context of GUP becomes

\[
Z(T, \kappa) = \frac{2\pi}{\kappa \sqrt{k} - 1} T^2 + \frac{1}{\sqrt{k}} T - 1.
\]

(33)

Then the thermodynamic properties of the physical system, such as free energy, mean energy, specific heat, and entropy, can be calculated from the following expressions
F = -T \ln(Z),
U = T^2 \frac{\partial \ln(Z)}{\partial T},
C = 2T \frac{\partial \ln(Z)}{\partial T} + T^2 \frac{\partial^2 \ln(Z)}{\partial T^2},
S = \ln(Z) + T \frac{\partial \ln(Z)}{\partial T}.

In order to perform our analysis on the thermodynamics of the Kemmer oscillator, we will restrict ourselves to stationary states of positive energy. And from the Eq. (34), we predict that the thermodynamic functions will be very complicated. In this case, in order to have an intuitive understanding for thermodynamic properties of the Kemmer oscillator in the context of the GUP, in the following, we briefly depict our numerical results on the evaluation of the thermodynamic functions, i.e. free energy, mean energy, specific heat, and entropy, via the numerical partition function Z.

4. Results and discussions

Before starting this part, it should be noted that all profiles of the thermodynamic quantities as a function of dimensionless temperature variable \( \tau \) for different values of \( \kappa \), i.e. \( \kappa = 1.5, 2.3, 3.4 \) are plotted in Figs. 2, 3, 4, 5, and 6, and in these figures, the natural unit \( \hbar = c = M = 1 \) is employed.

From the result shown in Fig. 2, it shows that the partition function \( Z \) increases monotonically with dimensionless variable \( s \), and for a fixed value of \( s \), the partition function decreases with the increase of the deformed parameter \( \kappa \). In Fig. 3, the free energy \( F \) is shown, we see that the free energy decreases with \( \kappa \) growing, and for a fixed value of \( \tau \), the profile of the curves decreases monotonically with the temperature for both cases. We plot the mean energy \( U \) versus \( \tau \) for different values of the deformed parameter \( \kappa \) in Fig. 4, it also shows that for a fixed \( \tau \) the mean energy decreases when \( \kappa \) grows. The profile of heat capacity \( C \) as a function of \( \tau \) for different values of \( \kappa \) is depicted in Fig. 5. We show that the heat capacity increases for increasing \( \tau \) at first and then remain...
invariant for a same value with $\tau$ growing, it means that the
effect of the GUP on the heat capacity can be negligible at
high temperature. The curves of the numerical entropy
versus $\tau$ for different values of $\kappa$ is plotted in Fig. 6.
It shows that the tendency of entropy rapidly increases at first
for increasing $\tau$ and then slowly grows for large $\tau$ values,
and for a fixed value of $\tau$, the entropy of the system
decreases when the deformed parameter $\kappa$ grows.

5. Conclusions

This paper was devoted to studying the thermodynamic
properties of the Kemmer oscillator in the context of the
GUP. We first analyzed the Kemmer oscillator in the
context of the GUP and obtained the wave function,
the corresponding probability density of every component as
well as the energy spectrum by employing the Gegenbauer
polynomial. Subsequently, we investigated the thermody-
namic properties of the system by employing the Epstein
zeta function and depicted our numerical results for the
corresponding thermodynamic functions through the asso-
ciated partition function $Z$, and the effects of the GUP
parameter on thermodynamic properties were also repor-
ted. They show that the wave function, the probability
densities, the energy spectrum and the corresponding
thermodynamic functions of Kemmer oscillator depend
explicitly on the deformed parameter $\beta$ which characterizes
the effects of the GUP on this considered physics system.

Acknowledgements This work is supported by the National Natural
Science Foundation of China (Grant Nos. 11465006, 11565009).

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