A-term inflation and the MSSM

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Abstract. The parameter space for A-term inflation is explored with $W = \lambda_p \phi^p / p M_p^{p-3}$. With $p = 6$ and $\lambda_p \sim 1$, the observed spectrum and spectral tilt can be obtained with soft mass of order $10^2$ GeV but not with a much higher mass. The case $p = 3$ requires $\lambda_p \sim 10^{-9}$ to $10^{-12}$. The ratio $m/A$ requires fine-tuning, which may be justified on environmental grounds. An extension of the MSSM to include non-renormalizable terms and/or Dirac neutrino masses might support either A-term inflation or modular inflation.
Recently, it has been proposed that the field content of the minimal supersymmetric
standard model (MSSM) describes both the low energy physics that is observed at
colliders, and inflation after the observable Universe leaves the horizon [1]. The inflaton
is supposed to correspond to a flat direction. To achieve this, a new kind of inflation
model is formulated, in which the potential is of the form

\[ V = \frac{1}{2} m^2 \phi^2 - A \frac{\lambda_p \phi^p}{p M^{p-3}_P} + \lambda^2_p \phi^{2(p-1)} \]

The last term corresponds to a minimal globally supersymmetric theory with
superpotential

\[ W = \frac{\lambda}{p} \phi^p / M^{p-3}_P. \]

The first two terms are soft supersymmetry breaking parameters. Supersymmetry-
breaking is taken to be gravity-mediated so that \( A \sim m \). For parameter values that
allow inflation, \( m \gg H \) where \( H \) is the Hubble parameter, and as a result the phase of
\( \phi \) adjusts to minimize the potential, giving Eq. (1).

This kind of inflation model is further explored in [2], where it is called \( A \)-term
inflation. The model is interesting in its own right, and might apply with \( \phi \) a gauge
singlet, or a flat direction of any gauge group. In this note, the parameter space for
\( A \)-term inflation is explored, and a fine-tuning issue is addressed.

The model works because the parameters can be chosen so that \( V' \) and \( V'' \) are
very small at some point, allowing inflation to take place near that point. Indeed,
\( V' = V'' = 0 \) exactly at the point

\[ \phi_0 = \left( \frac{m M^{p-3}_P}{\lambda \sqrt{2p-2}} \right)^{1/(p-2)}, \]

provided that

\[ m^2 = \frac{A^2}{8(p-1)}. \]

More generally, inflation can occur near \( \phi_0 \) if

\[ m^2 = \frac{A^2}{8(p-1)} \left( 1 + \frac{\delta^2}{m^2} \right) \]

with the parameter \( \delta^2/m^2 \) sufficiently small. To first order in this parameter, \( \delta^2 \) is the
shift in \( m^2 \) away from the value (4), at fixed \( A \). If the parameter is positive then \( V' \)
is positive for all \( \phi \). If it is negative, \( V \) has a maximum to the left of \( \phi_0 \), and viable
inflation can take place only to the left of the maximum.

This model is stable against loop corrections, supergravity corrections and
corrections from higher-order terms in the tree-level potential. Indeed [2], such
corrections just multiply the right hand side of Eq. (4) (hence of Eq. (5)) by some
factor \((1 + f)\) with \(|f| \ll 1\), and the right hand side of Eq. (3) by some factor \((1 + g)\)
with \(|g| \ll 1\). These factors have a negligible effect on the predictions and would be
invisible on our plots, and so we ignore them in the following.
For a given $p$, the parameters of the model are $\lambda_p$, $m^2$ and $\delta^2$. Assuming that the inflaton perturbation generates the curvature perturbation, constraints on the model are provided by the observed values of the spectrum of the curvature perturbation and of spectral index:

$$P_{\zeta}^{1/2} = 4.8 \times 10^{-5}$$

$$n = 0.948 \pm 0.015$$

The uncertainty in the spectrum is negligible for the present purpose, and the uncertainty on $n$ is the current 1-σ value for models (like the present one) which give a negligible tensor perturbation.

These constraints are evaluated analytically in the Appendix. They depend on the number $N$ of $e$-folds of inflation after the observable Universe leaves the horizon, and we have set $N = 50$ which corresponds to continuous radiation domination between the end of inflation and the present matter-dominated era. That situation is expected because the flat direction is expected to quickly decay and thermalize at the end of inflation. Delayed reheating would not alter $N$ much, but one or two bouts of thermal inflation could reduce it by 10 or 20, which as we see would significantly alter the constraints.

The constraints are plotted in Figure 1 in which lines of constant $n$, $\phi_0$ and $\delta^2/m^2$ are shown. For each point in the $m$-$\lambda_p$ plane, we choose $\delta^2$ to give the observed spectrum. The panels, from left to right, correspond to $p = 6$, $p = 4$ and $p = 3$, respectively.

![Figure 1](image_url)

**Figure 1.** The graphic shows lines of constant $n$ (solid line), $\phi_0$ (dashed line) and $\delta^2/m^2$ (thick solid line) for the cases $p = 6$ (left hand panel), $p = 4$ (central panel) and $p = 3$ (right hand panel), as obtained from Eqs. (3), (A.8) and (A.9), in the $m$-$\lambda_p$ plane ($\phi_0$ and $m$ are expressed in GeV). Also shown the region where the spectral index is within the observational limits, along with lines $n = 1.5$ and $n = 0.92$ (corresponding to $\delta^2/m^2 = 0$) added for reference. The amount of fine-tuning $\delta^2/m^2$ ranges between $\sim 10^{-22}$ and $\sim 10^{-12}$.

We discuss first the $p = 6$ results. If the ultra-violet cutoff of the theory is $M_p$ one generally expects $\lambda_p$ roughly of order 1, or perhaps of order $1/p! \sim 10^{-3}$. If instead there is a GUT corresponding to a cutoff $10^{-2}M_p$ these expected values are increased by a factor $10^{2(p-3)} = 10^6$. The MSSM places $m$ in the range $10^2$ to $10^3$ GeV.
Our plot confirms the remarkable finding of [1], that the spectrum and the spectral index can be consistent with observation for values of $\lambda_p$ and $m$ within the expected range. In particular, $\delta^2 = 0$ gives $n = 1 - 4/N$. With $N = 50$ this is a bit low compared with observation, though reducing $N$ by 20 or so would bring it up to the central value.

Our plot shows two things which were not anticipated. First, the ratio $m^2/A^2$ must be fine-tuned to the value (4) with extraordinary accuracy of order $10^{-20}$, in order to reproduce the observed spectrum. Second, a further modest fine-tuning is needed to get $n$ within the range 0.90 to 1.00, which observation surely requires. These fine-tunings make $A$-term inflation very different from inflation with potentials of the usual form [6, 7, 8]. For those potentials the predictions are not very sensitive to the parameters (though one should remember that the potentials are usually obtained by setting equal to zero parameters that might reasonably have been expected to be significant). Also, $n$ is automatically a bit below 1 if $\ln V$ is strongly concave-downwards as is the case for several well-motivated shapes of the potential.

Our plot shows that the constraints cannot be satisfied (with $\lambda_p$ in the expected range) if $m$ is increased by several orders of magnitude. However, this is consistent with the observed normalization of the spectrum; it is the high value of $n$ that is inconsistent.

What about the fine-tuning? In recent years, there has been increasing interest in the idea that there exists a landscape of values, for parameters which would formerly have been regarded as fixed. The landscape is supposed to correspond to possible solutions of the equations of a fundamental theory, which might perhaps be realised somewhere in the universe. The fundamental theory is usually supposed to be string theory [9], or in some cases just field theory.

The landscape might allow fine-tuning to be justified on environmental (anthropic) grounds. In the context of cosmology, the landscape was proposed to allow fine-tuning of the axion misalignment angle [10], of the cosmological constant [11] or of the value of a curvaton field [12]. In the context of particle physics, it might explain the one-percent fine-tuning of the MSSM [13], or the drastic fine-tuning of Split Supersymmetry [14] or no supersymmetry at all [15]. Let us see how the landscape might justify the fine-tuning of $A$-term inflation.

The possibility arises because environmental considerations require that $P^{1/2}_\zeta$ be within a factor 10 or so of its observed value [16]. The predictions for $P^{1/2}_\zeta$ at these limits are shown in Figure 2. To get a feel for what is going on, consider the values $m = 10^3$ GeV and $\delta^2 = 0$ with $\lambda_p$ chosen to give the observed value of $P^{1/2}_\zeta$. Now allow $\delta^2$ to vary keeping $m$ and $\lambda_p$ fixed; we see that requiring $P^{1/2}_\zeta$ to be within its anthropic range requires $|\delta^2/m^2| \lesssim 10^{-20}$. The situation is similar for other choices of $m$ and $\lambda_p$.

Of course, to allow a landscape of values for $m$ raises the question of why it has the low value corresponding to the MSSM. (As we already remarked, the low value is essential so that the spectral index has a chance of being compatible with observation.)

It was pointed out in [2] that inflation per se requires the value to be tuned with accuracy of order $10^{-4}$, but observational constraints were not considered there.
Interestingly, the question may possibly be answered by considering the initial condition for inflation. For negative \( \delta^2 \), corresponding to \( n < 0.92 \) and low \( m^2 \), inflation with this potential can start with eternal inflation near the maximum [17]. The indefinite duration of eternal inflation arguably justifies such an initial condition. For positive \( \delta^2 \), we see in the Appendix that there is no regime of eternal inflation except for a negligible interval (which does not allow the spectral index to become significantly different above \( n = 0.92 \)). The initial condition may still be justified in that case if, as is quite reasonable, there is an early phase of primordial inflation at the usual high scale through one of the usual mechanisms, but with the parameters such that the curvature perturbation generated then is much less than the observed one.

Indeed, after such inflation \( \phi \) may take on a range of values within the inflated patch, but the environmental constrain on \( P_\zeta \) means that we can live only in selected regions. (Such inflation may also begin with eternal inflation, which arguably makes it irrelevant whether A-term inflation supports eternal inflation.) Now comes the crucial point; as \( m^2 \) increases at fixed \( \lambda_p \), the viable range of initial values of \( \phi \) becomes smaller, and so does the possible amount of inflation. Arguably, this means that the probability that we live in a region with such a value decreases. Hence low values of \( m^2 \) may be favoured. Finally, values below \( 10^2 \) GeV or so may be disfavoured since the MSSM does not then reproduced the SM as a good approximation. Provided that \( \lambda_p \) is fairly close to 1, the combined effect of these arguments is to favour \( m \sim 10^2 \) GeV.
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Instead of fixing $\lambda_p$, we might consider a landscape of values for both it and $m$. Then the viable range of initial values of the inflaton field, and the total amount of inflation, increases as we move down and to the left in the plot of Figure 2. We lose the environmental argument for low $m$ in that case.

Now we move on to the case $p = 3$. There is no factor $M_P$ in Eq. (11) so that $\lambda_p$ is a renormalizable coupling. Figure 1 confirms the finding of [18], that the observational constraints require $\lambda_p \sim 10^{-12}$ if $m$ is of order $10^2$ to $10^3$ GeV. As was pointed out there, the small $\lambda_p$ is the one required to generate a Dirac neutrino mass of the observed value, if $\phi$ is a suitable flat direction of the MSSM extended to make the neutrino field a Dirac field. The fine-tuning is about the same as in the $p = 6$ case. Now the viable range of initial values of the inflaton field, and the total amount of inflation, increases as we move up and to the left in the corresponding plot of Figure 1. If we could find a reason for favouring $m \sim 10^2$ GeV, the value $\lambda_p$ would be anthropically favoured, and so would the observed value of the neutrino mass in the model of [18].

This discussion of small $\lambda_p$ in the case $p = 3$ reminds us, somewhat uncomfortably, that the received wisdom of expecting $\lambda_p \sim 1$ for $p > 3$ lacks a firm foundation. In the case of $p = 3$ couplings, one is forced to accept a wide range of values to explain the wide range of particle masses generated by the Higgs mechanism (down to $10^{-5}$ or so for the coupling that determines the electron mass). There is no generally accepted scheme for explaining this range, and none for justifying the expectation $\lambda_p \sim 1$ for $p > 3$ which is really made just on grounds of simplicity.

In fact, a completely different view about these coefficients is also quite natural; that the potential in a flat direction is of the form $V = V_0 f(\phi/M_P)$ with $f$ and its low derivatives having magnitude of order 1 at a typical point in the interval $0 < \phi \lesssim M_P$. This possibility is mentioned in [23] in the context of Affleck-Dine baryogenesis. In the context of string theory it would correspond to $\phi$ being a modulus with the origin a point of enhanced symmetry [19]. A typical term in the power series expansion of $V$ is now of order $\pm V_0 (\phi/M_P)^n$, and all terms are important at $\phi \sim M_P$. The mass is of order $V_0^{1/2}/M_P$.

Inflation with this type of potential [20] is usually called modular inflation. The inflaton is usually supposed to be a gauge singlet, but as was noticed in [2] it could as well be a flat direction of a gauge multiplet. The fundamental assumption for modular inflation is that the potential has a maximum at $\phi \sim M_P$. (Given the form of $f$ there could hardly be a maximum at $\phi \ll M_P$, but of course that does not mean that there must be a maximum at $\phi \sim M_P$.)

At the maximum, the slow roll parameter $\eta$ is equal to $f''$. This would typically be of order $-1$ which would correspond to fast-roll inflation [21], which is viable only if the curvature perturbation is generated from the vacuum fluctuation of a field different from the inflaton [22]. The idea of modular inflation is that one gets lucky, so that at the maximum $|f''| \lesssim 10^{-2}$. Then one can expect [6] [7] [8] the power-series expansion of $V$ around the maximum to be dominated by higher powers, giving an effective potential $V \simeq V_0 [1 - (\phi/\mu)^p]$ with $p \gtrsim 3$ not necessarily an integer. This allows the curvature
perturbation to be generated from the vacuum fluctuation of the inflaton if \( V_0 \) is of order \((10^{15} \text{ GeV})^4\), corresponding to \( m \sim 10^{12} \text{ GeV} \), and \( n \) is then automatically within the observed range. We conclude that inflation with an MSSM flat direction corresponding to a modulus can generate the observed curvature perturbation if the mass is large corresponding to Split Supersymmetry. (In contrast A-term inflation is not viable in the context of Split Supersymmetry since an A term is forbidden in that case by observational constraints.)

Returning to A-term inflation, we have dealt so far with the cases \( p = 6 \) and \( p = 3 \) that alone are possible within the MSSM. As seen in Figure 2 the case \( p = 4 \) satisfies the observational constraints only for a unique coupling \( \lambda_p \simeq 10^{-8} \). The intermediate case \( p = 5 \) (not shown) is similar to \( p = 6 \) but the slope of the allowed regime is steeper and corresponds to smaller \( \lambda_p \). We have not considered cases \( p > 6 \), because as \( p \) increases the situation that the term with that power actually dominates begins to look increasingly unlikely, owing to the extraordinarily strong suppression of lower powers that this requires [24].

In conclusion, we have explored the parameter space for A-term inflation, and have identified a severe fine-tuning. We have presented some environmental considerations that may help to justify this tuning. While recognising that such arguments are extremely controversial, we feel that they are worthy of consideration now that the idea of a landscape is under active discussion.

By way of closing, we would like to remind the reader of a fundamental point. For A-term inflation with \( p > 3 \), as well as for modular inflation, the potential involves non-renormalizable terms. Such terms are negligible in the low-energy theory describing astrophysical and terrestrial processes. This means that A-term or modular inflation using a flat direction of the MSSM cannot be tested in the laboratory; such models are invoking an extension of the MSSM, not to include more fields but to include more interactions. Only the proposal of [18] can be so tested, through its assertion that the neutrino is a Dirac particle.

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Appendix

Appendix A.1. Constraints from CMB normalization

Near \( \phi = \phi_0 \) we may write

\[
V(\phi) = V(\phi_0) + \frac{1}{6} V'''(\phi_0)(\phi - \phi_0)^3,
\]

(A.1)
where \( V''''(\phi_0) = 2(p - 2)^2 \frac{m^2}{\phi_0} \) and \( V(\phi_0) = \frac{(p-2)^2}{2p(p-1)} m^2 \phi_0^2 \). If \( m^2 \) is reduced by a small amount \( \delta^2 \equiv \delta(m^2) \) the potential becomes

\[
V \simeq V(\phi_0) + \frac{1}{6} V''''(\phi_0) \chi^3 + \delta^2 \phi_0 \chi,
\]

where we have introduced the field \( \chi = \phi - \phi_0 \). In order for the flat direction to be responsible for the observed curvature perturbation we impose the CMB normalization

\[
P^{1/2}_\zeta = \frac{1}{\sqrt{12\pi^2} M_p^3 V'(\chi^*_s)},
\]

which at \( p \) and \( m \) fixed determines the field value \( \chi^*_s \) when the observable Universe leaves the horizon in terms of \( \delta^2 \) and \( \phi_0 \). We obtain

\[
\frac{|\chi^*_s|}{\phi_0} = \left[ \frac{1}{P^{1/2}_\zeta \sqrt{12\pi^2}} \frac{p - 2}{(2p(p - 1))^{3/2}} M_p \left( \frac{\phi_0}{M_p} \right)^2 - \frac{1}{(p - 2)^2 m^2} \delta^2 \right]^{1/2}.
\]

In the slow-roll approximation, the number of \( e \)-foldings \( N \) after the observable Universe leaves the horizon is given by

\[
\frac{1}{p - 2} \sqrt{\frac{\delta^2}{m^2}} \cot \left[ \frac{2p(p - 1)}{p - 2} N \left( \frac{M_p}{\phi_0} \right)^2 \sqrt{\frac{\delta^2}{m^2}} \right] = -\frac{\chi^*_s}{\phi_0}.
\]

When \( \delta^2 \) becomes negative the corresponding equation can be readily found by analytic continuation.

Once a particular value \( \lambda_p \) is selected, and having fixed \( m \) and \( p \), there is only one possible value \( \delta^2 \) for which the flat direction generates the observed spectrum of perturbations.

In order to solve Eq. (A.5) for \( \delta^2 \) and \( \phi_0 \) we introduce the variables

\[
\Delta^2 \equiv \frac{\delta^2}{m^2} \left( \frac{M_p}{\phi_0} \right)^4 \quad \text{and} \quad \varphi \equiv \left( \frac{\phi_0}{M_p} \right)^2,
\]

where it should be noted that \( \Delta^2 \) carries the same sign as \( \delta^2 \). In these variables Eq. (A.5) leads to a parametrisation of \( \varphi \) in terms of \( \Delta^2 \). Using then Eq. (A.5), written as \( \varphi = \varphi(\Delta^2) \), the spectral index \( n \approx 1 + 2\eta \) becomes

\[
n = 1 - \frac{8p(p - 1)}{p - 2} \sqrt{\Delta^2} \cot \left[ \frac{2p(p - 1)}{p - 2} N \sqrt{\Delta^2} \right],
\]

which allows us to parametrise implicitly \( \Delta^2 = \Delta^2(n, N, p) \). With this we can use \( \varphi = \varphi(\Delta^2) \) to obtain \( \delta^2 \) and \( \phi_0 \)

\[
\frac{\delta^2}{m^2} = \Delta^2 \varphi^2 = \left( \frac{(p - 2)^3}{(2p(p - 1))^{3/2}} \frac{1}{P^{1/2}_\zeta \sqrt{12\pi^2} M_p} \right)^2 \frac{1}{\Delta^2} \sin^4 \left[ \frac{2p(p - 1)}{p - 2} N \sqrt{\Delta^2} \right],
\]

\[
\left( \frac{\phi_0}{M_p} \right)^2 = \varphi = \left( \frac{(p - 2)^3}{(2p(p - 1))^{3/2}} \frac{1}{P^{1/2}_\zeta \sqrt{12\pi^2} M_p} \right)^2 \Delta^2 \sin^2 \left[ \frac{2p(p - 1)}{p - 2} N \sqrt{\Delta^2} \right].
\]

When \( \Delta^2 < 0 \) it follows that \( \delta^2 \) has the same sign as \( \Delta^2 \) and that \( \phi_0 \) is always positive, as they must.

\[\text{Here we neglect the contribution from } \chi \text{ at the end of inflation, } \chi_e. \text{ This approximation holds until } |\eta| \sim 1 \text{ and amounts to taking the limit } |\chi_e| \to \infty.\]
Appendix A.2. Eternal inflation?

Now we determine the condition for the Universe to undergo a phase of eternal inflation when $\delta^2$ is positive, thus not resulting in the formation of any extrema in the potential. A region of the Universe undergoes eternal inflation if

$$|V'| < \frac{3}{2\pi} H^3$$  \hspace{1cm} (A.10)

in it. When the $A$-term does not generate a local minimum $V'$ finds its minimum value at $\phi_0$: $V'(\phi_0) = \phi_0 \delta^2$. In this case Eq. (A.10) turns into

$$\frac{\delta^2}{m^2} < \frac{3}{2\pi} \left( \frac{(p-2)^2}{6p(p-1)} \right)^{3/2} \frac{m}{M_P} \left( \frac{\phi_0}{M_P} \right)^2.$$  \hspace{1cm} (A.11)

Using now Eqs. (A.7), (A.8) and (A.9) the equation above becomes

$$\sin^2 \left[ \frac{2p(p-1)}{p-2} N \sqrt{\Delta^2} \right] < \mathcal{P}_\zeta^{1/2}.$$  \hspace{1cm} (A.12)

Keeping the first term in a series expansion, this implies $\Delta^2 < \mathcal{O}(10^{-10})$ for the cases $p = 3, 4$ and $p = 6$, which in view of Fig. A1 can only be satisfied for $n \approx 0.92$.

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