Prediction of Directional Young's Modulus of Particulate Reinforced MMC using Finite Element Methods

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Abstract. The aim of this present study is to predict the Young’s Moduli of metal matrix composite (MMC) by three dimensional (3-D) Finite Element Analysis (FEA) and compare the results with Halpin-Tsai (HT) model. In the present work, cubic shaped Representative Volume Element (RVE) has been considered to model the MMC. Particle is taken as cylindrical in shape. The parameters considered in present study are aspect ratio (ratio of height and diameter) and volume fraction. Further, a wide range of the modulus ratios (ratios of Young’s moduli of particle and matrix) have been considered to investigate their effects on effective Young’s modulus of the composite. It is observed from the study that the modulus ratio, aspect ratio and volume fraction of particle are the most dominating factors in directional effective Young’s modulus of metal matrix composite.

1. Introduction

Metals are normally alloyed with other elements to improve their physical and mechanical properties and wide ranges of alloy composition are available. Final properties are strongly influenced by thermal and mechanical treatment which determine the microstructure [1-2]. Metal matrix composite (MMC) has been used in industries like aerospace, automotive, electronics and infrastructure, because of their attractive properties such as light weight, stronger, stiffer material properties, high wear resistance and its high specific strength and stiffness at room or at elevated temperature than those of base metal counterparts [1-2]. Generally two approaches are adopted to determine the properties of composites; macro-mechanical analysis where composite material is considered as a homogeneous orthotropic continuum and micro-mechanical analysis where composite material is at the fibre/particle and matrix level. The particles which are used as reinforcement in matrix, play a vital role in controlling such mechanical properties of MMCS. Particle volume fraction, particle size and shape, particle aspect ratio (ratio of height and diameter), orientation of particle in matrix field; interfacial bonding are the key factors of controlling the composite properties [3]. During the last few decades, several attempts have been made to explore the relationship between the structure and the deformation of particulate metal matrix composite (PMMC). Continuum models exhibit some detailed quantitative results about the composite strength revealing the effect of aspect ratio, volume fraction and strain hardening component of matrix [4-5]. Typically, the unit cell technique combined with the known material properties of fibre and matrix is used to determine the overall behaviour of the composite [6]. In this context, Ganesh and Chawla [7] correlated the material microstructures with their macroscopic properties in order to understand the material behaviour of existing materials as well as to develop new materials. Chawla and Chawla [8] reveals about the micro-structural study of reinforcement in metal matrix to understand the material morphology. Numerical simulation methods like Finite Element Analysis (FEA) offer an alternative way to understand the behaviour of composites.
2. Methodology

In the present study, parameters considered for the FE analysis are; modulus ratio, \(\lambda_1\) (ratio of Young’s moduli of particle and matrix); particle aspect ratio, \(\lambda_2\) (ratio of particle height and diameter); particle height fraction, \(\lambda_3\) (ratio of particle height and RVE length); particle area fraction, \(\lambda_4\) (ratio of particle cross-section and RVE cross-section), particle volume fraction (ratio of particle volume and RVE volume, \(V_f=\lambda_1\lambda_4\)). Generally the effective elastic response of a composite with perfect particle–matrix bonding under load is evaluated by the conventional Rule of Mixtures (ROM) [1], where particles are continuous (\(\lambda_3=1\)) in matrix field and can be formulated as:

\[
E_c = E_pV_p + E_mV_m = E_m(\lambda_4 + 1 - \lambda_4)
\]  

(1)

where \(E\) and \(V\) are the Young’s modulus and volume fraction for the matrix (\(m\)) and the particles (\(p\)), respectively. ROM does not account for the effect of aspect ratio. But Young’s modulus varies with particle shape and size even for a constant volume fraction in discontinuous (\(\lambda_3<1\)) particle reinforced composites. Hence, ROM is not effective for estimating effective Young’s modulus of composite for discontinuous particle reinforced MMCs. In order to estimate the longitudinal Young’s modulus (\(E_c\)), the following theoretical semi empirical model has been proposed by Halpin-Tsai (HT) model [9] for a perfectly oriented discontinuous reinforcement in the composite, where the load is applied parallel to the reinforcement orientation.

\[
E_c = \frac{E_m(1+2sqV_p)}{1-qV_p} = \frac{E_m(1+2q\lambda_2\lambda_3\lambda_4)}{1-q\lambda_3\lambda_4}
\]  

(2)

where \(E_c\), \(E_p\) and \(E_m\) are the Young’s modulus of composite, particle and matrix, respectively; \(s\) is the aspect ratio (\(\lambda_2\)); \(V_p\) is volume fraction of the particle (\(\lambda_1\lambda_4\)); \(q\) is a geometrical parameter that is expressed as,

\[
q = \left(\frac{E_p}{E_m}\right)^{\frac{1}{2s}} = \frac{\lambda_4-1}{\lambda_4+2\lambda_2}
\]  

(3)

3. Finite Element Analysis

In this present study, three dimensional (3-D) Finite Element Analysis (FEA) are being carried out with Representative Volume Element (RVE) as cubic shape to represent major features of the microstructure. RVE is considered as high symmetry in nature where single particle in cylindrical shape is located at centre of the RVE. Total volume of the RVE is taken as 5080 \(\mu\text{m}^3\) [10] for all FE analyses. Due to symmetry of RVE, only one eight volume of the RVE is being modelled in this analysis. Three symmetry boundary conditions are considered for three respective faces of RVE where out of plane translational degrees of freedom (DOF) are being restricted. RVE is under tensile load in terms of 1% strain in longitudinal direction in which particle longitudinal axis is being oriented. Wide range of modulus ratios (\(\lambda_1\)) is being considered (0, 0.1, 5.54, 10, 20, 100) to investigate their effects on effective Young’s modulus of composite. Alfonso et al. [10], investigated the effect of directional strength of composite considering particle as SiC (Young’s modulus and Poisson’s ratio of 401.4 GPa and 0.18) and matrix as Aluminium Al356 (Young’s modulus and Poisson’s ratio of 72.4 GPa and 0.33) with modulus ratio \(\lambda_1=5.54\). However, in the present study in addition to SiC/Al356 composite, different modulus ratios (\(\lambda_1\)) are considered with particle and matrix Poisson’s ratio kept as 0.3. Both particle and matrix are considered as isotropic in nature. To investigate the effect of aspect ratio of particle (\(\lambda_2\)), particle volume is taken as 610 \(\mu\text{m}^3\) (\(V_f=12\%\)) [10] and particle dimensions are shown in Table 1. Figure 1 shows schematic representation of full RVE and 1/8th RVE. Second order brick elements of ANSYS (solid186) of approximate size of 0.2 \(\mu\text{m}\) with reduced integration are used. Particle aspect ratio (\(\lambda_2\)) varies from 0.2 to 1.8. To investigate effect of volume fraction, different particle geometries are considered with aspect ratios (\(\lambda_3\)) of 0.8 and 1.4 as shown in Table 2. Table 3 shows particle geometries for different particle diameters with particle height of 6.53 \(\mu\text{m}\) (\(\lambda_3=0.380\)) and 11.50 \(\mu\text{m}\) (\(\lambda_3=0.669\)). Further, different particle geometries (shown in Table 4) are also considered with constant diameter of 8.21 \(\mu\text{m}\) (\(\lambda_3=0.179\)) and 10.89 \(\mu\text{m}\) (\(\lambda_3=0.315\)) to understand the effect of
particle height in estimating Young’s modulus of the composite. Directional Young’s modulus of composite under axial load can be calculated using equation, $E_c = \frac{R}{A\varepsilon}$, where $R$ is the axial reaction force at support extracted from FEA, $A$ is cross-sectional area of RVE and $\varepsilon$ is the average strain of 1% along loading direction.

![Schematic representation of (a) Full RVE and (b) 1/8th RVE.](image)

**Figure 1.** Schematic representation of (a) Full RVE and (b) 1/8th RVE.

**Table 1.** Dimensions of Particle for different Aspect Ratios ($\lambda_2$) [10]

| Aspect Ratio ($\lambda_2$) | Diameter, $d_p$ (µm) | Height, $h_p$ (µm) |
|---------------------------|----------------------|-------------------|
| 0.2                       | 15.71                | 3.14              |
| 0.6                       | 10.89                | 6.53              |
| 1.0                       | 9.19                 | 9.19              |
| 1.4                       | 8.21                 | 11.50             |
| 1.8                       | 7.55                 | 13.59             |

**Table 2.** Particle height for different Aspect Ratios

| Aspect Ratio | Volume Fraction, % $V_f$ ($\lambda_3 \lambda_4$) |
|--------------|-----------------------------------------------|
| $\lambda_2=0.8$ | 4.695 5.915 6.771 7.453 8.028 8.531 8.981 9.389 |
| $\lambda_2=1.4$ | 6.818 8.590 9.833 10.823 11.658 12.389 13.042 13.636 |

**Table 3.** Particle Aspect Ratios for different diameter

| Height Fraction | Volume Fraction, % $V_f$ ($\lambda_3 \lambda_4$) |
|----------------|-----------------------------------------------|
| $\lambda_3=0.380$ | 1.31 0.93 0.76 0.66 0.59 0.54 0.50 0.46 |
| $\lambda_3=0.669$ | 3.07 2.17 1.77 1.53 1.37 1.25 1.16 1.08 |

**Table 4.** Particle Aspect Ratios for different height

| Area Fraction | Volume Fraction, % $V_f$ ($\lambda_3 \lambda_4$) |
|---------------|-----------------------------------------------|
| $\lambda_3=0.179$ | 0.29 0.58 0.88 1.17 1.46 1.75 |
| $\lambda_3=0.315$ | 0.13 0.25 0.38 0.50 0.63 0.75 |
4. Results and Discussions

4.1. Effect of particle Aspect Ratio for constant particle Volume Fraction

Variations of normalised Young’s modulus ($\lambda$, defined as the ratio of Young’s moduli of composite and matrix) with aspect ratio ($\lambda_3$) are shown in Figure 2. Volume fraction of particle $V_f$ ($\lambda_2$,$\lambda_3$) is taken as 12%. Results show that the composite exhibits higher value of $\lambda$ with increasing $\lambda_3$. This is applicable for all values of $\lambda_1$ (0-100). When $V_f$ remains constant and $\lambda_3$ is higher there is an increase in height (increase of $\lambda_1$) and reduction of area (decrease of $\lambda_4$) of the particle. Higher value of $\lambda_3$ indicates long particle or fiber. If particle and matrix are considered as springs connected together, in case of $\lambda_3$=1 (i.e. continuous fiber) and $\lambda_4$$<1$, system is equivalent to two springs are connected in parallel and ROM is applicable and effective stiffness of composite is defined as $K_m + K_p$, where K, m and p, represent stiffness, matrix and particle, respectively. But in case of $\lambda_4$=1 and $\lambda_3$$<1$, springs are connected in series, effective stiffness of composite is $\frac{K_m K_p}{K_m + K_p}$. Hence, parallel connection is better in enhancement of Young’s modulus of composite. From Figure 2, it can be noticed that rate of increase of $\lambda$ is more in case of composite having higher value of $\lambda_1$. This behaviour is like composites with higher value of $\lambda_1$ and $\lambda_2$ (long stiffer particle as fiber like carbon nanotube) are more effective in enhancement of stiffness of composite [1]. Normalised Young’s modulus; $\lambda$, extracted from FEA and HT model shows good agreement for $\lambda_3$ between 0.6-1.4 along with $\lambda_1$ range of 0-20. However, FEA results are higher than HT model with $\lambda_1$=100. In the case when particle is void and $\lambda_4$=0.2, it acts like a sharp crack. As void does not carry any load, there will be redistribution of load path along the loading direction due to presence of sharp crack and effective stiffness of the composite is less. When $\lambda_4$=0 and the MMC has high value of $\lambda_2$ and less value of $\lambda_3$, this results in less severity of crack due to less deviation of load path and loads are being carried out by surrounding matrix material and FEA results matches well with that of HT model.

\[ V_f = 12\% \]

**Figure 2.** Young’s modulus versus aspect ratio with volume fraction ($V_f$) of 12%.

4.2. Effect of particle Volume Fraction for constant particle Aspect Ratio

The effect of volume fraction in overall modulus of composite with aspect ratios ($\lambda_2$) of 0.8 and 1.4 are shown in Figures 3(a) and 3(b), respectively. Figures show that with the increase in volume fraction, strength of composite in longitudinal direction increases for $\lambda_1$$>1$. Further, rate of change of $\lambda$ is more in case of $\lambda_2$=1.4 than 0.8 for $\lambda_1$$>1$ (maximum value of $\lambda$ are 1.65 and 2.45, respectively). HT model is well suited for all MMC composite of $\lambda_2$=0.8 and $\lambda_1$$>1$. When aspect ratio of particle $\lambda_2$=1.4 and $1<\lambda_1<20$, HT and FEA results match up to $V_f$ of 12.5%. But in case of $\lambda_2$=0.8 and $\lambda_1$$<1$, HT model predicts higher value than that of FEA. This may be due to non-accounting of crack severity in HT model. For material having very low strength ratio, most of the load is being carried out by the matrix and as volume fraction of particle increases overall stiffness of composite decreases and curve shows a downward slope with volume fraction. It can be concluded that both material strength ratio and aspect ratio have a big role as volume fraction increases. Halpin and Tsai also predicted a geometric parameter ($q$) depends on $\lambda_1$ and $\lambda_2$ for short particle/fiber [9]. Results also show that the limitation of HT model with higher values of $\lambda_1$, $\lambda_2$ and $V_f$ ($\lambda_3$,$\lambda_4$).
Figure 3. Young’s modulus versus volume fraction with particle aspect ratio ($\lambda_2$) of (a) 0.8 and (b) 1.4.

4.3. Effect of particle Diameter for constant particle Height

Effect of particle diameter are being investigated considering particle height as 6.53 $\mu$m ($\lambda_3$=0.380) and 11.5 $\mu$m ($\lambda_3$=0.669). The only way to increase volume fraction is to increase diameter. If diameter increases with constant height ($\lambda_3$ as constant), $\lambda_4$ increases and volume fraction ($V_f = \lambda_3 \lambda_4$) increases. For $\lambda_1$>1, higher value of $V_f$ results more amount of particle with stiffer material in composite and hence higher value of $\lambda$. But decrease of aspect ratio ($\lambda_2$) exhibits adverse effect in rate of increase of $\lambda$. For same volume fraction, $\lambda$ is less in case of Figure 4(a) compared to Figure 4(b) due to lesser value of $\lambda_2$ in former case for $\lambda_1$>1 (maximum value of $\lambda$ are 1.45 and 1.91, respectively for $V_f$ of 20%). Aspect ratio decreases from 1.31 to 0.46 in case of particle height of 6.53 $\mu$m whereas it varies from 3.07 to 1.08 for 11.5 $\mu$m (see Table 3). When $\lambda_4$>1, HT model under predicts the FEA results as shown in Figure 4(b) due to higher aspect ratio ($\lambda_2$). When $\lambda_4$=0, i.e. particle as void, value of $\lambda$ is less for particle height of 6.53 $\mu$m (Figure 4a) as compared to the same with particle height of 11.5 $\mu$m (Figure 4b). This is due to the effect of porosity in the MMC. The similar trend is also observed for $\lambda_1$=0.1.

Figure 4. Young’s modulus versus volume fraction with particle height ($h_p$) of (a) 6.53 $\mu$m and (b) 11.5 $\mu$m.

4.4. Effect of particle Height for constant particle Diameter

Variations of overall response in terms of Young’s modulus of composite are shown in Figures 5(a) and 5(b) with particle diameter of 8.21 $\mu$m ($\lambda_4$=0.179) and 10.89 $\mu$m ($\lambda_4$=0.315), respectively. Due to the increase in height of the particle, volume fraction as well as strength of composite increases for $\lambda_1$>1. Rate of change of $\lambda$ is more in case of Figure 5(a) than Figure 5(b) due to higher aspect ratio ($\lambda_2$) of former case (variation of $\lambda_2$ is 0.29 to 1.75 and 0.13 to 0.75, respectively, Table 4). Maximum values of $\lambda$ obtained from FEA are 2.39 and 1.43, respectively for $V_f$ of 15%. It is observed that the deviation in the prediction of HT model as compared with the FE model is more for higher aspect ratio.
of the particle when $\lambda_1 > 1$. If height of the particle is increased with constant diameter, effect of volume fraction and aspect ratio together enhance the overall stiffness of the composite due to parallel distribution of stiffness between particle and matrix. FE results are matching well with HT model for $\lambda_2<1$ and $\lambda_1 > 1$ but more deviation has been found for $\lambda_2>1$ and $\lambda_1 > 1$. However, FE analysis results with soft particle ($\lambda_1=0, 0.1$) are in close agreement with HT model prediction for $\lambda_1=0.179$ (Figure 5a) and deviates for $\lambda_1=0.315$ (Figure 5b).

![Figure 5. Young’s modulus versus volume fraction with particle diameter (dp) of (a) 8.21 µm and (b) 10.89 µm.](image)

5. Conclusions

Present study shows that the modulus ratio (ratio of Young’s moduli of particle and matrix) and aspect ratio of particle (ratio of height and diameter) are the most dominating factors in directional effective Young’s modulus of metal matrix composite (MMC) which is further influenced by volume fraction of the particle. It is also demonstrated by the current study that HT model prediction for MMC matches well with FE analysis results for limited range of modulus ratios, aspect ratio and volume fraction of the particle.

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