Median Filtering Forensics Scheme for Color Images Based on Quaternion Magnitude-Phase CNN

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Abstract: In the paper, a convolutional neural network based on quaternion transformation is proposed to detect median filtering for color images. Compared with conventional convolutional neural network, color images can be processed in a holistic manner in the proposed scheme, which makes full use of the correlation between RGB channels. And due to the use of convolutional neural network, it can effectively avoid the one-sidedness of artificial features. Experimental results have shown the scheme’s improvement over the state-of-the-art scheme on the accuracy of color image median filtering detection.

Keywords: Quaternion transform, convolutional neural network, median filtering forensics, color image.

1 Introduction

As one of the most widely used image carriers, color image is widely used in information dissemination, social networking and other fields. With the rapid development of network technology and the widespread use of social software, the modification of images has become increasingly simple, and the authenticity detection of images becomes more and more difficult. Digital image forensics technology is an important means to verify the originality and authenticity of the image, which can test the originality and authenticity of an image by distinguishing the fingerprints left by tampering operations. Most of the forensics researches on image tampering focus on copy-move [Fridrich, Soukal and Lukas (2003)], image splicing [Ng and Chang (2004)], double-compression [Pevny and Fridrich (2008)], and blurring [Avcibas, Sankur and Memon (2006)], etc. Median filtering, as a non-linear smoothing filter, is often used for denoising. Forgers usually use median filtering to destroy the evidence of tampering. Therefore, implementing median filtering detection can recover the history of image tampering, which yields useful information for forensic analysis. The special property of median filtering is increasingly attracting researchers’ interest and attention.

The existing median filtering forensics methods are mainly divided into two categories. In the first type, features are obtained from the spatial domain of image. In Kirchner et al. [Kirchner and Fridrich (2010)], Fridrich et al. introduced a feature called subtractive pixel

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adjacency matrix (SPAM), which is obtained from the joint distribution of first-order difference images. Yuan [Yuan (2011)] used the statistical properties measured in a median filtering window of size 3×3, and proposed a feature called the Median Filtering Feature (MFF), which achieves better detection results for non-compressed images and can effectively distinguish the median filtering with other kinds of filtering, such as mean filtering. Kang et al. [Kang, Stamm, Peng et al. (2013)] used a new feature called median filtering residual (MFR) for the AR model to improve the accuracy of the algorithm. Chen et al. [Chen, Kang, Liu et al. (2015)] used a convolutional neural network to train and learn the MFR. In [Bayar and Stamm (2016)], Bayar et al. proposed a new kind of convolutional layer, which was a great improvement. Chuang et al. [Chuang, Swaminathan and Wu (2009)] introduced a new detection method based on empirical frequency response (EFR), which can detect multiple methods of tampering.

It can be found that the median filtering forensic algorithms mentioned above are mostly used for gray image detection. They ignore the relevant information of color image’s RGB channels. Regarding the forensics research of color images, Schauerte et al. [Schauerte and Stiefelhagen (2012)] used the quaternion discrete cosine transform to detect faces. Carre et al. [Soulard and Carre (2010); Soulard and Carré (2011); Carré and Denis (2006)] first used color quaternion wavelet transform (CQWT) to encode color images. Compared with DWT, CQWT has the characteristics of high compression ratio and low distortion. Subsequently, they used CQWT and its coefficients of magnitude and three phases to classify color image textures. Due to CQWT’s continuous multi-scale analysis characteristics, they achieved higher classification accuracy than DWT and CWT with the same features. Rizo et al. [Rizo-Rodriguez and Ziou (2015)] made research on face recognition using illumination invariance and quaternion-based Local Binary Patterns. The method simply defines LBP in the form of quaternion, which solves the problem of redundant information between the center point and adjacent point of traditional LBP. Rzadkowski et al. [Rzadkowski and Snopek (2015)] proposed a watermarking algorithm based on quaternion Fourier transform. Li et al. [Li, Li and Fu (2013)] proposed the feature of magnitude and phases based on the quaternion wavelet transform and multi-level Copula features to study image texture, the accuracy was improved by 4.9% compared with traditional wavelet transform. Ouyang et al. [Ouyang, Shu, Wen et al. (2014)] proposed a robust watermarking in quaternion transform domain by applying the unified logarithmic coordinate mapping in the quaternion Fourier transform domain. Wang et al. [Wang, Li, Shi et al. (2017)] applied the quaternion wavelet to distinguish computer generated images from natural images.

In the paper, a quaternion transformation based convolutional neural network is proposed to directly detect the median filtering forgery of color images. The contribution of the scheme is summarized as follows:

(1) Quaternion is used to represent color images and the correlation between the three channels is fully exploited.

(2) Polar representation of quaternion is applied, which solves the problem of mismatch in data format and becomes the foundation of Magnitude CNN and Phase CNN.

(3) The merging of Magnitude CNN and Phase CNN outperforms the state-of-the-art method in the detection of color image median filtering.
The remaining part of the paper is organized as follows. In Section 2, we will briefly introduce some related work. Then the proposed scheme is introduced in Section 3. The performance of our experiment and the results are provided in Section 4. Finally, Section 5 provides the conclusion.

2 Related work

2.1 Chen’s scheme

In Chen et al. [Chen, Kang, Liu et al. (2015)], Chen et al. proposed adding a filter layer as a pre-processing layer into the conventional convolutional neural network. Through the filter layer, the median filtering residual (MFR) of the image is obtained, and then input to the convolutional neural network. By using the filter layer to suppress the interference caused by image content, the trace left by tampering can be investigated.

The definition of MFR is to apply a median filtering window of size \( w \times w \) on the test image and obtain the output image, as seen in the following equation:

\[
d(i, j) = \text{med}_w(x(i, j)) - x(i, j) = y(i, j) - x(i, j)
\]

(1)

where \( x(i, j) \) is the original value at point \( (i, j) \), \( y(i, j) \) is median filtered value of \( x(i, j) \) and \( d(i, j) \) means the MFR, which is the difference between \( y(i, j) \) and \( x(i, j) \).

Then the MFR is used as the input into the convolutional neural network. By adjusting hyperparameters when training, the model starts to learn the differences between the original images and the tampered ones, afterwards, a classifier for median filtering forensics is generated.

2.2 Bayar’s scheme

In Bayar et al. [Bayar and Stamm (2016)], Bayar et al. found that the existing convolutional neural network cannot be used directly, so they proposed a new form of convolutional layer to force convolutional neural network to learn the features needed for detection of median filtering. The inspiration of the strategy comes from a variety of previous schemes in the field of image forensics and steganalysis, i.e. many forensics and steganalysis algorithms have operations similar to the following: Predict the pixel value of a point based on the pixels surrounding it, and calculate the difference between the predicted value and the actual one, then create features based on the difference matrix. It can be found in most of schemes used in image forensics, steganalysis, and resizing, as well as the scheme proposed by Chen above, is based on the strategy.

Therefore, a kind of prediction error filter is used as the kernel of the first convolutional layer. The implementation strategy of prediction error filter is the same as described above, as shown below:

\[
\begin{aligned}
\sigma(0,0) &= -1 \\
\sum_{x,y \neq 0} \sigma(x,y) &= 1
\end{aligned}
\]

(2)

It is known that the weights of the filters are constantly updated according to the back-propagation algorithm in convolutional neural network when training. However, in the
scheme, the weights of the first layer are always constrained to meet the requirements above, so that the entire convolutional neural network can continuously learn median filtering forensics features.

The scheme had overcome the deficiency of Chen’s scheme and made some improvements, but it is still based on gray images. To make full use of the correlation between RGB channels, a method based on quaternion transformation is proposed.

3 The proposed scheme

In this section, the framework of the proposed scheme is presented, followed by detailed explanation of each part.

3.1 Quaternion discrete cosine transformation

3.1.1 Quaternion

Quaternion was introduced by Hamilton [Hamilton (1969)] in 1843. A quaternion consists of four components, including one real part and three imaginary parts, as shown in the following figure:

\[ q = a + bi + cj + dk \{ a, b, c, d \in R \} \]  

And i, j and k meet the following multiplication rules:

\[ i^2 = j^2 = k^2 = -1, \]
\[ i * j = -j * i = k, j * k = -k * j = i, \]
\[ k * i = -i * k = j \]  

The polar representation of quaternion is defined as follows:

\[ q = |q| e^{i\varphi} e^{j\theta} e^{k\psi} \]  

Among them,

\[ |q| = \sqrt{a^2 + b^2 + c^2 + d^2} \]

where \(|q|\) is the magnitude, \(\varphi\), \(\theta\), \(\psi\) are the three phases of the quaternion, and \(\varphi \in [-\pi, \pi]\), \(\theta \in [-\pi / 2, \pi / 2]\), \(\psi \in [-\pi / 4, \pi / 4]\). The formulas of the three phases are as follows:

\[ \varphi = a \tan(2(cb + ad), a^2 - b^2 + c^2 - d^2) / 2 + k\pi \]

\[ \theta = a \tan(2(bd + ac), a^2 + b^2 - c^2 - d^2) / 2 \]

\[ \psi = \arcsin(2(ab - bc)) / 2 \]
3.1.2 Quaternion discrete cosine transformation

The original feature in QDCT (Quaternion Discrete Cosine Transformation) domain proposed in Feng et al. [Feng and Hu (2008)] is very remarkable. They can be summarized following the four steps below.

Firstly, transform the given quaternion matrix $f_q(m, n)$ into the Cayley-Dickson form, i.e.,

$$f_q(m, n) = A(m, n) + B(m, n)j$$

where $A(m, n) = a + bi$ and $B(m, n) = c + di$.

Secondly, calculate the DCT (Discrete Cosine Transformation) of $A(m, n)$ and $B(m, n)$ respectively, the result can be written as $DCT_A(m, n)$ and $DCT_B(m, n)$.

Thirdly, using $DCT_A(m, n)$ and $DCT_B(m, n)$ to form a quaternion:

$$F_{jq}(m, n) = DCT_A(m, n) + DCT_B(m, n)j$$

Finally, multiply $F_{jq}$ with the quaternionization factor $u_q$ to obtain the final result:

$$QDCT[f_q(m, n)] = u_q \ast F_{jq}(m, n)$$

where $u_q = 1/\sqrt{3}(i + j + k)$, which is called a unit pure quaternion.

It is adopted to use a quaternion matrix $f_q(m, n)$ to represent a color image, i.e.

$$f_q(m, n) = f_R(m, n)\ast i + f_G(m, n)\ast j + f_B(m, n)\ast k$$

where $f_R(m, n)$, $f_G(m, n)$, $f_B(m, n)$ represent the R, G, B components of the color image.

3.2 The proposed framework

The framework of the proposed QDCT features based scheme is shown in Fig. 1. Given a digital image, we do not simply use it as an input into convolutional neural network just like Bayar et al. [Bayar and Stamm (2016)], but transfer it into QDCT domain to extract richer features for color image forensics. Furthermore, when compared with the scheme proposed in Feng et al. [Feng and Hu (2008)], the features extracted in QDCT domain in our scheme are expanded ones, in order to meet with the data format when being input into the convolutional neural network followed. Afterwards, due to the use of convolutional neural network—a useful tool that can automatically learn different kinds of kernels, more specific features are novelty obtained, with the aim to have a better identification in more aspects rather than only one.
In the proposed scheme, considering the loss of the correlation between RGB channels when converting color images to gray images, a new kind of convolutional neural network based on quaternion transformation is proposed, so we can have a better classification when dealing with color images.

3.3 Magnitude CNN

After performing QDCT on the image, the QDCT coefficient will be obtained. The coefficient, specifically a matrix, consists of M×N (M and N are the width and height of the processed image respectively) quaternions. As suggested previously, the obtained coefficient cannot be directly input into convolutional neural network due to their mismatch in the data format. Therefore, the method to transfer the coefficient matrix into the polar representation is used.

First the magnitude matrix of the coefficient is calculated using formula (6), while many elements of the magnitude matrix are found to be 0. In this case, the value of the corresponding places in other three phase matrices are all set to 0. And the remaining ones are all calculated using formula (7), as shown in Tab. 1.

**Table 1:** Magnitude and phase calculation

| Algorithm |
|-----------|
| 1. Input: $f_q(m, n)$ |
| 2. Magnitude = $\sqrt{a^2 + b^2 + c^2 + d^2}$ |
| 3. For each Magnitude in Magnitude Do |
| 4. If Magnitude = 0 then |
| 5. $\phi_i = \Theta_i = \Psi_i = 0$ |
| 6. Else: |
| 7. $\phi_i = \text{atan}(2(cd+ad), a^2-b^2+c^2-d^2)/2$ |
| 8. $\Theta_i = \text{atan}(2(bd+ac), a^2+b^2-c^2-d^2)/2$ |
| 9. $\Psi_i = \arcsin(2(ad-bc))/2$ |
| 10. End If |
| 11. End For |
At last, one magnitude matrix and three phase matrices are obtained, all of which are consisted of \( M \times N \) real numbers. They are saved separately, which aims at reducing computational complexity in later processing. Fig. 2 shows the result of our method, using the picture of Lena as an example.

![Fig. 2: Lena and its four corresponding matrices of QDCT coefficient](image)

As mentioned above, a color image produces four matrices, which can be used directly as input into convolutional neural network. Therefore, Magnitude CNN is proposed using the magnitude matrix as input. Here is how Magnitude CNN is designed. It consists of 1 input layer, 3 convolutional layers, 2 max-pooling layers and 3 fully-connected layers followed by a softmax classifier.

The first convolutional layer has 12 kernels with size of 5\( \times \)5, while the second have 64 kernels of size 7\( \times \)7 and the third have 48 kernels of size 3\( \times \)3. Rectified Linear Units (ReLUs) [Nair and Hinton (2010)] is selected as activation function. Both the second and third convolutional layers are followed by an overlapping max-pooling [Boureau, Bach, Lecun et al. (2010)] layer with a kernel size of 3\( \times \)3 and a stride of 2, which can reduce complexity and enhance robustness of our model.

In addition, Local Response Normalization (LRN) [Jarrett, Kavukcuoglu, Ranzato et al. (2010)] is used after the max-pooling layer where the central value in each neighborhood is normalized by the surrounding pixel values, which helps to improve the generalization ability of the model. The dropout technique [Hinton, Srivastava, Krizhevsky et al. (2012)] is adopted in layer FC1 and FC2 which sets to zero the neurons with probability of 0.5. It forces a neuron to work with other randomly selected neurons to achieve good results so
that overfitting can be prevented. Finally, a softmax function is applied at the end of our model to do the classification.

And as mentioned, the result of unconstrained convolutional neural network applied directly on the median filtering detection is not good. A variety of strategies have been simulated and we eventually adopt the same constraints as [Bayar and Stamm (2016)], i.e., the kernels of the first convolution layer are constrained to satisfy the requirements in (2).

To demonstrate the performance of Magnitude CNN, one of the feature maps produced by the first convolutional layer is shown in Fig. 3, compared with another one produced by using gray image as input. It can be seen that the first one contains mainly texture information of the original image while the second image involves mostly morphological information. Therefore, Magnitude CNN concentrates on learning various features of image’s texture information, which can get rid of the influence of image content and leads to better classification. The experimental results are shown in Tab. 2.

![Figure 3: The feature maps for comparison](image)

### 3.4 Phase CNN

Meanwhile, several groups of convolutional neural network using different phases as input are proposed. The detailed design is the same as Magnitude CNN. Three feature maps of different phases produced by the first convolutional layer are shown in Fig. 4. The experimental results are shown in Tab. 3.

![Figure 4: The feature maps produced by the first convolutional layer](image)
Taking the image of phi in Fig. 4 as an example, Phase CNN also involves mainly texture information. Due to the limit of value ranging, Phase CNN is able to capture features of texture information from other aspects.

3.5 Magnitude-Phase CNN

Because of the complementarity between magnitude and phases, Magnitude-Phase CNN is proposed, which uses all the four magnitude and phases as input into convolutional neural network to train, and employs all of obtained feature maps before fully-connected layer as a union to do the learning and classification. All the design is the combination of Magnitude CNN and Phase CNN, except one additional merging layer is set before fully-connected layer to make a union of upper layer’s output.

It is known to us all that each output of different magnitude and phases before the merging layer has four dimensions, corresponding to the batch, image row, image column and channel. It is necessary to determine which dimension to merge from that can get better results when merging. Therefore, experiments have been implemented to determine the best choice in this situation. Detailed results are conducted in Tab. 4.

4 Experimental results

In the Section, experiments are performed to verify the effectiveness of our scheme for median filtering forensics detection. Comparative experiments are also made between our scheme and other state-of-the-art methods on our dataset.

4.1 Image database

An experimental database of original and tampered images is built to evaluate the performance of Magnitude-Phase CNN. Our experimental database is collected from CASIA TIDE v2.0 [Dong, Wang and Tan (2013)] and the UCID database [Schaefer (2003)], making up the original part. The CASIA TIDE v2.0 database contains 12,614 color images with size ranging from 240×160 to 900×600 pixels in JPEG, BMP and TIFF formats, and the UCID database consists of 886 images of size 384×512 and 512×384. Next, a set of tampered images are generated by applying median filtering on the original images with a 5×5 window. Finally, we got a database of 13500 original images and 13500 tampered images, which are all color images. After separating them with a 2:1 ratio, 18000 images are used for training while the remaining 9000 images are used for testing. Our images are all cropped to the size of 256×256 during training and testing.

4.2 Implementation details

In the experiment, these four reference indexes will be obtained: Training accuracy, training loss, test accuracy and test loss. Among them, the test accuracy is mainly concern, and the remaining ones are used to determine whether the model is overfitting.

In addition, all our experiments are implemented using one NVIDIA GeForce GTX 1080Ti GPU. Stochastic gradient descent is employed to optimize our network with hyperparameters set as follows: momentum=0.9, decay=5×10^{-4}, and learning rate=10^{-6}.
4.3 Performance of the proposed scheme

In order to obtain a model for better median filtering forensics detection, self-comparative experiments are designed based on the scheme. As mentioned above, after applying the quaternion discrete cosine transform to the image, one magnitude and three phases can be obtained. However, it is not clear which ones can improve the detection accuracy and which ones will have a negative effect. Therefore, experiments are conducted separately, in order to acquire the most perfect combination. The experimental results are shown in Tab. 5.

**Table 2: Accuracy on Magnitude Compared with Gray Image**

|                  | Accuracy  |
|------------------|-----------|
| Magnitude        | 98.01%    |
| Gray image       | 97.72%    |

**Table 3: Accuracy on Different Phases**

|         | Accuracy  |
|---------|-----------|
| Phi     | 60.56%    |
| Theta   | 74.56%    |
| Psi     | 74.11%    |

**Table 4: Accuracy on Different Axes when Merging**

|         | Accuracy  |
|---------|-----------|
| Batch   | 99.40%    |
| Image Row | 99.35%  |
| Image Column | 99.32% |
| Channel | 99.38%    |

The experimental results have shown that using batch as the axis of merging is slightly better than other results. It is analysed that this method is similar to using a larger batch of images to train the network instead of higher, wider and more channels.

**Table 5: Accuracy on Different Combinations**

|                     | Accuracy  |
|---------------------|-----------|
| Magnitude+Phi       | 98.25%    |
| Magnitude+Theta     | 98.39%    |
| Magnitude+Psi       | 98.31%    |
| Magnitude+Phi+Theta | 99.15%    |
| Magnitude+Phi+Psi   | 99.20%    |
| Magnitude+Theta+Psi | 99.25%    |
| All                 | 99.40%    |
It can be seen that the experimental accuracy is not good when there is no magnitude but only phase. Therefore, we no longer perform the combination between the phases, but use the magnitude directly with different phases to combine. Moreover, it is found that among the three phases, phi has the lowest improvement on the detection accuracy, but it is not useless, and only if all the four magnitude and phases are adopted will the best result be achieved. The result of the experiment helps to guide the design of Magnitude-Phase CNN.

4.4 Comparison with other methods

The convergence performance of the proposed scheme is firstly displayed, and Fig. 5 shows the performance on our database.

![Graph](a) Bayar’s scheme

![Graph](b) The proposed scheme

**Figure 5:** The Convergence Performance Comparison

Accuracy comparison between our scheme and other methods is also made. Tab. 6 shows the detection performance comparison of the proposed scheme with other two methods,
i.e. Chen et al. [Chen, Kang, Liu et al. (2015); Bayar and Stamm (2016)] on our datasets. It can be observed that the proposed scheme is superior to other methods. To the best of our knowledge, the method in Chen et al. [Bayar and Stamm (2016)] is the state-of-the-art method. Our scheme outperforms the one in Chen et al. [Chen, Kang, Liu et al. (2015)] by 2.64%, and the one in Bayar et al. [Bayar and Stamm (2016)] by 1.03%, respectively.

Table 6: Accuracy on Different Schemes

| Scheme               | Accuracy  |
|----------------------|-----------|
| Chen’s scheme        | 96.76%    |
| Bayar’s scheme       | 98.37%    |
| The proposed scheme  | 99.40%    |

5 Conclusion

In this article, a quaternion transformation based convolutional neural network is proposed to detect the median filtering forgery of color images. First, a QDCT coefficient is obtained, which cannot be used directly into convolutional neural network. Second, the polar form of quaternion is applied on the QDCT coefficient, which yields a magnitude and three phases. Then Magnitude CNN and Phase CNN are proposed to do detection from different aspects. Finally, the Magnitude-Phase CNN is proposed. It can be seen from the proposed method that quaternion does make full use of the correlation between RGB channels. However, it should be noted that the reason why we have to use the polar form of quaternion is optional, i.e., QDCT coefficient can be transferred to polar form to satisfy the need of convolutional neural network, while convolutional neural network can also be modified to satisfy the need of QDCT coefficient. This remains a problem to be solved.

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