Numerical simulation of an anisotropic heat transfer in magnetized neutron stars with 3D basic operators method

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Abstract.

We have solved numerically a three dimensional boundary-value problem for a heat transfer equation in a magnetized neutron star crust with an updated tensorial heat conductivity coefficient. The temperature distribution in a neutron star crust in presence of a magnetic field was simulated. To calculate the surface temperature distribution, we have constructed a local one-dimensional plane-parallel model of a magnetized outer envelope of the neutron star and used it as an outer boundary condition for 3D problem to find a self-consistent solution.

This problem was solved with our extension of a basic (support) operators numerical method on a tetrahedral mesh. The idea of operator approach consists of inclusion the boundary conditions into difference form of the solving problem and its formulation as one operator equation. The finite difference operators are constructed in the way to fulfill corresponding relations between continuous operators (for example, div(curl)=0, div is conjugated to -grad so div(grad) is self-conjugated etc.). Such approach allows to obtain completely conservative implicit finite difference schemes. Efficient iterative methods can be used to find the solution because constructed matrixes have good properties, such as symmetry and positive definiteness.

1. Introduction

Neutron star (NS) surface magnetic fields can be $\sim 10^{12-13}\text{G}$. One of the possible ways to study the surface magnetic fields is observations of thermal radiation in a soft X-ray band [1, 2]. Periodic changes of spectra from some single NSs may indicate a nonuniform temperature distribution on their surfaces. Such heterogeneities are determined by anisotropic thermal conductivity of degenerate matter with presence of a strong magnetic field, because a heat flux is suppressed sufficiently across the magnetic field lines in the outer layers of magnetized neutron stars. Comparing results of simulations of the heat transfer processes with observational data can help to determine magnetic field structure on the NS surface.

Outer layers of the NS consist of a plasma of degenerate (and also ultrarelativistic at density $\rho \gg 10^6 \text{g/cm}^3$) electrons and almost nondegenerate nonrelativistic ions. We separate outer
layers to the crust \((10^{10} < \rho < 2 \cdot 10^{14} \text{ g/cm}^3)\) and an outer envelope \((\rho < 10^{10} \text{ g/cm}^3)\). Pressure is determined mostly by the electrons, and the matter can form a state of Coulomb crystal or liquid, and a heat also transfers mostly by the electrons in degenerate layers and by radiation in nondegenerate layer near the surface. A degree of the heat flux depression across the magnetic field is determined by a so called magnetization parameter \(\omega \tau\), where \(\tau\) is an average time between electron-nucleon collisions, and \(\omega = eB/m_e^*c\) is a cyclotron plasma frequency, \(m_e^* = m_e \sqrt{1 + p_f e^2/m_e c^2}\) is an effective electron mass, \(p_f = \hbar(3\pi^2 n_e)^{1/3}\) is an electron Fermi momentum, \(e\) is an electron charge, \(c\) is a speed of light, \(\hbar\) is a reduced Planck constant, \(B\) is a magnetic induction. A thermal conductivity tensor for the plasma with degenerate electrons is derived as a solution of a Boltzmann equation with a Chapman-Enskog method in [3].

In this work we have found a stationary solution of a three-dimensional heat transfer equation with anisotropic thermal conductivity. Because of a sufficient nonlinearity of a radiative boundary condition \((F \sim T^4\), where \(F\) is a heat flux, \(T\) is a temperature), physical parameters also drastically change through the thin crustal and envelope regions. In such a case a full 3D approach can not be applied to the problem. Fortunately, the heat flux in the envelope is mostly radial, and the temperature distribution in an envelope region can be calculated separately from the crust but selfconsistently with it [4]. We have built a local one-dimensional model of a thermal structure of the outer envelope.

In 2D approach this problem was solved in many works with different magnetic fields and microphysical input (e.g. [4, 5], and [6] for a review).

2. Physical input

2.1. Heat transfer in a magnetic field

Temperature distribution is defined by the heat transfer equation

\[
C \frac{\partial T}{\partial t} = \nabla \cdot \kappa \cdot \nabla T + f
\]

where \(C\) is a heat capacity, \(\kappa\) is a thermal conductivity tensor, \(f\) is defined by heat sources and sinks (neutrino emission, Joule heating, etc.). We look for a stationary solution \((\frac{\partial T}{\partial t} = 0)\) and also assume an absence of sources and sinks and consider \(f = 0\).

Thermal conductivity tensor \(\kappa\) for strongly degenerate electrons in a magnetic field was obtained in [3, 7] using a Chapman-Enskog method for a Boltzmann equation. This tensor takes into account heat fluxes along and across the magnetic field, and also a Hall heat flux. In the Cartesian coordinates it is written as follows

\[
\begin{align*}
\kappa_{ij} &= \frac{k_B T n_e}{m_e^*} \tau \left( \kappa^{(1)} \delta_{ij} + \kappa^{(2)} \epsilon_{ijk} \frac{B_k}{T} + \kappa^{(3)} B_i B_j \right) \\
&= \frac{5 \pi^2}{6} \omega_T \left( \frac{1}{1 + (\omega_T)^2} - \frac{6}{5} (\omega_T)^2 \right) \\
\kappa^{(1)} &= \frac{5 \pi^2}{6} (\omega_T)^2 \left( \frac{1}{1 + (\omega_T)^2} - \frac{3}{2} (\omega_T)^2 \right) \\
\kappa^{(2)} &= \frac{5 \pi^2}{6} (\omega_T)^2 \left( 1 + (\omega_T)^2 \right) \\
\kappa^{(3)} &= \frac{5 \pi^2}{6} (\omega_T)^2 \left( \frac{1}{1 + (\omega_T)^2} + \frac{6}{5} (\omega_T)^2 \right)
\end{align*}
\]

where \(n_e = \frac{\rho Z}{4 m_u}\) is an electron concentration, \(\tau = \frac{3}{32 \pi^2 m_e^* Z e^4 \Lambda} \omega_T\) is an average time between electron-nucleon collisions, \(k_B\) is a Boltzmann constant, \(\hbar\) is a Planck constant, \(\Lambda\) is a Coulomb logarithm. Parameter \(\omega_T\) changes drastically in the crust and the envelope of the NS, for the density \(\rho \sim 10^{10} \text{ g/cm}^3\), the value of the parameter \(\omega_T \sim 1\) when the value of a magnetic field induction \(B \sim 10^{13}\text{G}\). Approximately \(\omega_T \sim B/\rho^{2/3}\) in the crust, where \(\rho = 10^{10} - 2 \cdot 10^{14}\text{g/cm}^3\). As it follows from (2), heat conductivity coefficients across and along the magnetic field can be written in the following form:

\[
\begin{align*}
\kappa_{\perp} &= \frac{k_B T n_e}{m_e^*} \tau \kappa^{(1)} \\
\kappa_{\parallel} &= \frac{k_B T n_e}{m_e^*} \tau \kappa^{(1)} + \kappa^{(3)}
\end{align*}
\]
Through this work magnetic field is assumed to be dipolar
\[ B = \frac{B_p R_{NS}^3}{2} \frac{3(d \cdot r) r - d r^2}{r^5}, \]  
(4)

where \( R_{NS} \) is a NS radius, \( d \) is a unit vector aligned to magnetic dipole in the centre of a star, and \( B_p \) is a magnetic field induction on the pole.

The degree of the heat flux supression across the field (\( \kappa^{(1)} \) in (2) and (3)) is stronger, than in previous works \([8, 9]\), where a relation between heat fluxes along and across the magnetic field is \( F_\parallel / F_\perp = 1 + (\omega \tau)^2 \).

2.2. Equation of state
Density is explicitly appears in thermal conductivity (2). We have built a NS model solving the Tolman-Oppenheimer-Volkoff equations of hydrostatic equilibrium to get a density profile in the crust. For the NS interior we used moderately stiff equation of state SLy4 \([10]\), which is based on the microscopic calculations with an effective nuclear potential \([11]\). We have chosen central density \( \rho_c = 1 \times 10^{15} \) g/cm\(^3\). NS mass is \( M_{NS} = 1.42M_\odot \), where \( M_\odot \) is a solar mass, and inner and outer radii of NS crust are \( R_{in} = 10.594 \) km at \( \rho = \rho_{in} = 2 \times 10^{14} \) g/cm\(^3\) and \( R_{out} = R_{NS} = 11.618 \) km at \( \rho = 10^{10} \) g/cm\(^3\) respectively.

For the outer envelope (see next subsection) we assumed nonquantizing ideal plasma with nondegerate nonrelativistic ions (nuclei) and degenerate relativistic electrons:

\[ P = P_{id}^{(N)} + P_{id}^{(e)}, \]

where \( P_{id}^{(N)} = n_N k_B T \) is an ion pressure, and the pressure of the electrons of an arbitrary degree of degeneracy can be written in terms of Fermi-Dirac integrals \([12, 13]\):

\[ P_{id}^{(e)} = \frac{(2m_e)^{3/2}}{3\pi^2 \hbar^3 \beta^{5/2}} \left( I_{3/2}(\chi, \tau) + \frac{\tau}{2} I_{5/2}(\chi, \tau) \right) \]  
(5)

where \( \beta = (k_B T)^{-1}, \chi = \beta \mu_{id}^{(e)} \) is an electron chemical potential, normalized on \( k_B T \), \( \tau = (\beta m_e c^2)^{-1} \), and the Fermi-Dirac integral is defined as follows:

\[ I_{\nu}(\chi, \tau) = \int_0^\infty \frac{u^\nu \sqrt{1 + \tau u^2}}{\exp(u - \chi) + 1} du, \]  
(6)

here \( u = \beta m_e c^2 (\sqrt{1 + \frac{p^2 c^2}{m_e^2 c^2}} - 1) \), and \( p \) is an electron momentum. In most of the envelope and crust electron gas is strongly degenerate, and pressure is written as (e.g. \([14]\))

\[ P_{id}^{(e)} = \frac{m_e^4 c^5}{32\pi^2 \hbar^3} \left( \frac{1}{3} \sinh \xi - \frac{8}{3} \sinh \frac{\xi}{2} + \xi \right), \]  
(7)

where \( \xi = 4 \sinh^{-1} \left( \frac{3\pi^2 n_e}{m_e^2 c^2} \right)^{1/3} \), and in non-degenerate layer near the surface in ideal plasma approximation electron pressure (5) approaches the imit of ideal gas \( P_{id}^{(e)} = n_e k_B T \). We use analytical approximation for Fermi-Dirac integrals, which is considered in \([12, 13]\).
2.3. Thermal structure of an outer envelope

The outer envelope of the NS is a thin layer (∼ 10 – 100 metres) of plasma with degenerate electrons and nondegenerate ions. The temperature decreases by 2–3 orders of magnitude radially across this envelope, while surface temperature variations are in limits of one order of magnitude, so in the first approximation, heat flux is assumed to be only radial through the envelope, and it’s local value is constant. In such approach the thermal structure equation for the envelope reads [15, 16]:

$$\frac{dT}{dP} = \frac{3K T_s^4}{16g_s T^3}, \quad (8)$$

where $T_s$ is a local surface temperature, $K = K(B, \theta_B, T, \rho)$ is an effective opacity, $\theta_B$ is a magnetic field inclination angle to a normal to the surface, $g_s = GM_{NS}/(R_{NS}^2 \sqrt{1 - r_g/R_{NS}})$ is a surface gravity acceleration, where $G$ is a gravitational constant, and $r_g$ is a gravitational radius. For the effective opacity we have taken into account electron thermal conductivity (3), and radiative opacities for free-free and bound-free transitions and electron Thompson scattering. More detailed discussion can be found in [17]. Equation (8) can be solved as a Cauchy problem for the given values of surface temperature $T_s$ and surface pressure $P_s$, which is calculated from Eddington approximation $P_s \approx \frac{2g_s}{3K(B, \theta_B, T, \rho)}$ [18], using equation of state $P_s = P(\rho_s, T_s)$ to obtain $\rho_s$, where $\rho_s$ is a density at the NS surface. We solve this equation from the NS surface to the bottom of the envelope (crust-envelope boundary) at $\rho_b = 10^{10}$ g/cm$^3$ and obtain a temperature $T_b$ at this density. Solving equation (8) numerically for different magnetic fields, inclination angles and surface temperatures leads to so-called $T_s - T_b$-relationship, tabulated or analytically approximated function, which relates the surface temperature $T_s$ with the temperature on the crust-envelope boundary $T_b$. Results of several calculations are presented on Table 1. We used tabulated $T_s - T_b$-relationship as a boundary condition for the heat transfer equation in the crust (see next section).

Table 1. Surface temperature on a NS magnetic pole ($\theta_B = 0$) and on an equator ($\theta_B = \pi/2$) and their relations for $T_b = 1 \cdot 10^8 K$ and dipolar magnetic field (4).

| lg $B_p$ | 11  | 12  | 13  |
|----------|-----|-----|-----|
| $T_s(\theta = 0)/10^6 K$ | 1.02 | 1.03 | 1.16 |
| $T_s(\theta = \pi/2)/10^6 K$ | 0.71 | 0.35 | 0.18 |
| $T_{s\parallel}/T_{s\perp}$ | 1.43 | 2.94 | 6.44 |

3. Boundary value problem

We assume, that heat flux radiates from the surface of the NS, and a temperature on the inner radius of the crust $T_{core}$ is constant through the core because of sufficiently large value of a heat capacity. This temperature decreases with time as the core cools very slowly, after the fast neutrino cooling stage, so the thermal evolution of the NS can be considered as a sequence of cooling models with a stationary temperature distribution over the star.

In such consideration a value of the surface temperature $T_s$ is determined by a solution of a stationary heat transfer equation in a spherical layer with a given temperature $T_{core}$ on the inner boundary, and a radiative black-body boundary condition on the outer one, which is described by a Stephan-Boltzmann law $F_s = \sigma T_s^4$. In the outer envelope, because of smallness of its mass, the radial heat flux $F_s$ is assumed to be local constant. For this reason, this heat flux on the outer boundary of the crust (inner boundary of the envelope), is equal to the radial part of heat
flux calculated from the crust by the solution of the heat transfer equation. Boundary conditions read as

\[ T_{in} = T_{core}, \quad \kappa(B, \rho, T) \nabla_r T + F_s|_{out} = 0, \]  

(9)

where index ”in” corresponds to the temperature value on the inner boundary of the crust with \( r = R_{in} \), and index ”out” corresponds outer one with \( r = R_{out} \). Continuity of the temperature on the crust-envelope boundary is used with the \( T_s - T_b \)-relationship for different \((B, \theta_B)\), also \( T_b = T_{out} \). As a result, we obtain inambiguous dependence \( T_s(T_{out}, B, \theta_B) \), which determines surface temperature distribution in the magnetized NS. In spherical layer \( R_{in} \leq r \leq R_{out} \) we solve boundary-value problem for the heat transfer equation

\[ \nabla \cdot \kappa(B, \rho, T) \cdot \nabla T = 0 \]  

(10)

with boundary conditions (9). This problem is solved with our extension [19] of the Basic operators methods on unstructured tetrahedral mesh.

4. Numerical technique
An operator approach in numerical methods [20] was proposed by Soviet and Russian mathematician Alexander Andreyevich Samarskii. His idea consists of the inclusion of boundary conditions into a finite-difference form and formulation a finite-difference problem as an operator equation. It allows to study operators properties and use corresponding efficient methods for the solution of the numerical problem. The idea of the basic (support) operators method consists of the grid analogue construction of one of the continuous operator (e.g. \( \text{div} \)), which approximates corresponding continuous operator. The grid analogue of the relative conjugate operator (e.g. \( -\text{grad} \)) is constructed using a corresponding integral relation. In this way operator-difference equations will have the same properties as the differential problem. For example, self-conjugated continuous operator remains symmetrical in the grid form, and sparse and symmetrical matrix corresponds to it. Also basic operators method allows to obtain completely conservative finite-difference schemes. After obtaining the grid formulation of the solving problem, the sparse and symmetrical matrixes can be inversed efficiently by modern iterative or direct methods. Two dimensional operator-difference completely conservative Lagrangian method for the magnetohydrodynamical equations was built by Ardeljan et al. in the series of papers [21, 22, 23, 24] for triangular grid of a variable structure. 3D grid analogues for the differential operators in Cartesian coordinates were described in [19]. Following the operator formalism from [21, 22, 23, 24], the 3D grid analogues of basic differential operators were suggested and some test calculations were provided. In the next two subsections gradient and divergence operator are described.

4.1. Grid analogue of the gradient
We use the grid which consists of the tetrahedra. So-called cell-node approximation is used: node grid functions are defined in the grid nodes, and cell grid functions are defined to be constant in the grid cells and the boundary nodes. Following [21, 22, 23], let’s introduce linear spaces of cell grid functions, node functions and boundary node functions. This formal approach allows to study the approximation and stability of the numerical schemes using efficient modern methods.

First we introduce the grid gradient operator, which converts the scalar node function into the cell vector function. Conjugated differential operators can be considered in pairs (for example, \( \text{div} \) and \( \text{grad} \)), so, in this approach we have to obtain the approximation of one operator from
the pair naturally. In this case, we use the following invariant definition of the $\text{grad}$ operator:

$$
\nabla p = \lim_{V \to 0} \frac{1}{V} \int_S p \, d\vec{S}.
$$

(11)

Let $p$ is a scalar node grid function. Inside each cell we additionally define it as a linear interpolation of node values of the cell’s nodes. After integration of this linear interpolation, taking into account mean value theorem, we get the definition of operator $\text{grad}$ grid analogue of:

$$
(\nabla \Delta p)_i = \frac{1}{V_i} \sum_{k=1}^{4} (\bar{p}_k S_k \vec{n}_k)_i.
$$

(12)

Here $\bar{p}_k = \frac{(p_1 + p_2 + p_3)}{3}$ - is an average interpolated value of $p$ in the $k$-th tetrahedron face, indexes 1,2 and 3 correspond to $p$ node values on the $k$-th face; $\vec{n}_k$ is a unit external normal to the $k$-th face, $S_k$ is a square of the $k$-th face and $V_i$ is a volume of the cell with the index $i$. Expression (12) is the first order approximation for the differential operator $\text{grad}$. The grid analogues of other (nodes to cells) operators (such as $\text{div}(\text{vector})$, $\text{grad}(\text{vector})$, $\text{curl}(\text{vector})$ etc.) can be derived in a similar way.

4.2. Corresponding divergence operator

Let us introduce the scalar product in the linear grid spaces $(p, g)_\alpha = \sum_s U_s p_s g_s$, $\alpha$ corresponds to node and cell spaces. For the cells $U_s = V_i$, for nodes $U_s = W_j$, here $W_j = \frac{1}{4} \sum_{k=1}^{K_j} V_k$ is a "node volume" (see [21] and references in it).

The grid analogue for divergence operator is constructed to be conjugated to the $-\text{grad}$ grid operator from (12). The Green formula and its grid analogue was used (for the case when all functions vanish on the domain boundary or the domain is infinite):

$$
\int p \nabla \cdot \vec{v} dV + \int \vec{v} \cdot \nabla p dV = 0
$$

(13)

$$(\vec{v}, \nabla \Delta p) + (\nabla \cdot \vec{v}, p) = 0; \quad \sum_{l=1}^{N_i} \nabla_x \cdot \vec{v}_l p_l W_l = - \sum_{k=1}^{K_j} \vec{v}_k \nabla \Delta p_k V_k.
$$

(14)

Grid analogue of the Green formula (14) is written in terms of a scalar product in grid spaces. After substitution the $\nabla \Delta$ from (12) and rearrangement of terms for the $j$-th node the expression for the grid operator $\text{div}$ can be written in the form:

$$
(\nabla_x \cdot \vec{v})_j = -\frac{1}{3W_j} \sum_{k=1}^{K_j} \tilde{v}_k \cdot (\vec{n}_1 S_1 + \vec{n}_2 S_2 + \vec{n}_3 S_3)_k,
$$

(15)

here $K_j$ is a number of adjacent cells for $j$ node, $\tilde{v}_k$ - value of the cell function $\vec{v}$ in the $k$-th cell. Indexes 1, 2 and 3 correspond only to those cell faces that contain the $j$-th node. Summation is done over all cells adjacent to the $j$-th node.

Difference analogues of differential operators like (12) and (15) with corresponding boundary operators $\Phi$ (see e.g. [19, 21, 22]) allow to formulate grid analogues of boundary-value problems for various systems of partial differential equations.
4.3. Numerical implementation of the solving problem

To include boundary conditions in to the grid operator equation we follow the technique, suggested in [21, 22]. Operator-difference formulation of the problem (10) reads:

\[(I - \delta_1)\nabla \times \kappa \cdot \nabla \Delta (I - \delta_1)T + (I - \delta_1)\nabla \times \kappa \cdot \nabla \Delta T_{\text{core}} - \delta_2 \Phi \cdot \vec{n}T_s^4 = 0,\]  

(16)

where \(\Phi\) is a boundary operator, \(I\) is a unit operator, and operator \(\delta_k\) is defined as follows: it equals to 1 in the boundary nodes with the boundary sort \(k\) (\(k = 1\) for inner boundary nodes and \(k = 2\) for outer ones), and to 0 in other case.

This problem has to be solved self-consistently, because surface temperature \(T_s\) in the outer boundary condition is a function of a temperature in the crust itself. We implemented iterational procedure of relaxation: the problem is solved with boundary conditions of first type on the inner boundary and of the second type on the outer one \(n\) times until inequality \(\max |T_n^s - T_{n-1}^s| < \epsilon\) is not satisfied, where \(n\) is an iteration number, \(\epsilon\) is a some small number. After each iteration the value of \(T_s\) is specified with the \(T_s - T_0\)-relationship with the surface temperature distribution, obtained from the previous iteration. The value \(T_s^0\) for the boundary condition on the first iteration is obtained from the initial approach of the crust temperature. In some sense, this procedure is equivalent to a solving a time-dependent heat transfer equation with boundary conditions of the first and the third types, while the value of the heat flux density \(\sigma T_s^4(T)\) in the outer boundary condition from (10) is taken from the previous ”time”-step.

Operator-difference equation (16) is nonlinear at each ”time”-step (tensor \(\kappa\) is a function of temperature) and it should be solved with Newton method of solving systems of nonlinear equations, and appeared system of linear algebraic equations on the each Newton method iteration is solved by the iterative Seidel method.

5. Results

In this work we have self-consistently obtained the temperature distribution inside the volume of the crust and on the surface of the NS with a three-dimensional code. On Fig.1 temperature distribution in the crust is shown for the densities \(\rho = 10^{10} - 2 \cdot 10^{14} \text{ g/cm}^3\) with the core temperature \(T_{\text{core}} = 2 \cdot 10^8 K\) for the dipolar magnetic field (4) with induction \(B_p = 10^{13} \text{ G}\) on the pole.

The temperature distribution at the outer boundary of the crust is inverted in comparison with the temperature distribution at the neutron star surface, the cause is as follows. The most crucial suppression of the heat flux is in the envelope, where the parameter \(\omega \tau \gg 1\). In the envelope the suppressed heat flux on the magnetic equator causes decrease of a temperature gradient in the crust, so that a variation of the crust temperature on the magnetic pole is higher, than on the equator.

On Fig.2 surface temperature distribution is presented for the same parameters, as on Fig.1 with the semi-analytical temperature distribution from [25] with calculated pole and equator surface temperature values:

\[T_s(B, \theta) = [T_s^4(0) \cos^2 \theta_B + T_s^4(\pi/2) \sin^2 \theta_B]^{1/4}\]  

(17)

It is seen from Fig.2, that for a pure-dipolar magnetic field configurations an angular temperature dependence is in a good agreement with the formula (17) from [25]. Polar surface temperature exceeds equatorial one in a factor of \(\sim 5\).

In this work we solved axially symmetric 2D heat transfer problem with an updated electron-ion thermal conductivity and an original three-dimensional extension of Basic operators method as a first step of our studies. Consideration of 3D magnetic field configurations and modelling of observational manifestations of arising 3D effects will be published elsewhere.
**Figure 1.** Crust temperature distribution for the dipolar magnetic field (4) with $B_p = 10^{13}$ G, $T_{\text{core}} = 2 \cdot 10^8$ K (temperature $T$ is in units of $10^6$ K) for the density values $\rho = 10^{10} - 2 \cdot 10^{14}$ g/cm$^3$. left - cross-section in Y-Z plane, right - crust-envelope surface at $\rho_{\text{out}} = 10^{10}$ g/cm$^3$.

**Figure 2.** Surface temperature distribution (in units of $10^6$ K) for the same physical parameters as on Fig.1, $\Theta$ is a polar angle. Solid line - numerically calculated distribution, dashed line - calculated with the formula (17).
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