The combined effectiveness of magnetic force and heat/mass transfer on peristaltic transportation "Hyperbolic Tangent" Nanofluid in a Slopping Non-Regular Non-symmetric Channel.

T Sh Ahmed
Collage of education for pure science (ibn al Haitham) university of Baghdad, Iraq
Email: Tamaraalshareef@yahoo.com

Abstract. In the present article, we present the peristaltic motion of "Hyperbolic Tangent nanofluid" by a porous area in a two dimensional non-regular a symmetric channel with an inclination under the impact of inclination angle under the impact of inclined magnetic force, the convection conditions of "heat and mass transfer " will be showed. The matter of the paper will be further simplified with the assumptions of long wave length and less "Reynolds number ", we are solved the coupled non-linear equations by using technical analysis of "Regular perturbation method "of series solutions. We are worked out the basic equations of continuity, motion, temperature, and volume fraction particles for the recently fluid. The impact of incoming parameters on the inflow features have been studied and painted.

1. Introduction

Peristaltic motion has been intended with large attention by many researchers for Newtonian and non-Newtonian fluids because of its implementations in biology and modern technology. Peristaltic movement of nano fluids is considerable of transport in cancer therapy and applications in drug delivery. Obtainable publications and reviews signalize that literature centering the peristaltic transport of nano fluids are very restricted (e.g., [1, 2, 3, 4, and 5]). Non-Newtonian fluid have great applications in industrial and biological operations. Moreover, the significance of thermal impacts of blood in the processes like oxygenation and hemodialysis makes the study about the achievement of heat transfer in peristalsis very serious. Much interest has focused on non-Newtonian fluid models as a base fluid of nano fluids.

Recently, hamed M. et al [6] studied the influence of slip and convective boundary conditions on peristaltic transport of ‘‘nano hyperbolic tangent fluid’’ in an inclined a symmetric channel. Nadeem S. et al [7] made an inspiration of induced magnetic field on ‘‘nano hyperbolic tangent fluid’’ in curved channel.

Now, in this paper we analyzed the mixed convective peristaltic flow of ‘‘nano hyperbolic tangent fluid’’ in an inclined non-uniform asymmetric channel under the effect of an inclined magnetic force through porous area. The governing equations contain the emulation effects of Brownian motion and
thermo phonetic diffusion of nano particles. The inflow type or style are studied under the hypothesis of long wave length and then we solved them by using perturbation analytic method of series solutions. The analysis of the results are submitted for the distribution of velocity, temperature, nanoparticles concentration and the phenomenon of trapping. Numerical results will be presented for the ‘‘average rise in pressure ’’ of the above fluid.

2. Mathematical Model

Let us show the peristaltic inflow of an ‘‘incompressible electrically conducting’’ ‘‘nano hyperbolic tangent fluid’’ into a two-dimensional inclined a symmetric non-uniform duct filled with porous material. Let \( F = H_i \) and \( F = H_f \) be severally the lower and upper wall boundaries of the channel. We have a tendency to assume the infinite wave train traveling with speed \( C \) through the non-regular walls. We select a ‘‘rectangular coordinates’’ system for the channel with \( \overline{X} \) through the direction of wave diffusion and parallel to the centerline and \( \overline{Y} \) transverse to it. An sloping or an inclined magnetic field \( (B_x \sin \beta, B_y \cos \beta) \) will be applied in \( \overline{XY} \)-direction . ‘‘Heat and mass transfer of the fluid ’’ will be analyzed. The geometry of walls of the channel are given by the equations:

\[
\begin{align*}
H_i(x,t) &= -d - m\overline{x} - a_i \sin \left[ \frac{2\pi}{\lambda} (x - ct) + \phi \right], \text{lower wall} \\
H_f(x,t) &= d + m\overline{x} + a_i \sin \left[ \frac{2\pi}{\lambda} (x - ct) \right], \text{upper wall}
\end{align*}
\]

Where \( 2d \) is the width of the duct’’, \( \phi \) denotes the phase difference which varies in the range \( 0 \leq \phi \leq \pi \). \( \phi = 0 \) Represents to symmetric channel with waves out of phase, (i.e.) both walls move outward or inward simultaneously. Also \( a_i, a_f, d \) and \( \phi \) satisfies the condition:

\[
a_i^2 + a_f^2 + 2a_i a_f \cos \phi \leq (2d)^2. \tag{3}
\]

The temperature and construction at the lower and upper walls are assumed to be \( T_i, c_i \) and \( T_f, c_f \) respectively. The system of ‘‘continuity equations’’, ‘‘motion equations’’, ‘‘nano-particle temperature’’ and ‘‘volume- fraction’’ for an incompressible (MHD) "hyperbolic tangent nano fluid" are:

![Figure (1)](image-url)
\[
\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{V}}{\partial X} = 0
\]

(4)

\[
\rho \frac{\partial \vec{U}}{\partial t} + \frac{\partial \rho \vec{V}}{\partial X} = -\frac{\partial \bar{P}}{\partial X} + \frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial \bar{\rho}}{\partial \bar{X}} + \frac{\partial \bar{\rho}}{\partial \bar{X}}
\]

\[-\sigma B_0^2 \cos \beta (\vec{U} \cos \beta - \vec{V} \sin \beta) \frac{\mu \bar{T}}{k_0} + \rho \bar{g} \sin \alpha.
\]

(5)

\[
\rho \frac{\partial \vec{V}}{\partial t} + \frac{\partial \rho \vec{U}}{\partial X} = -\frac{\partial \bar{P}}{\partial X} + \frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial \bar{\rho}}{\partial \bar{X}} + \frac{\partial \bar{\rho}}{\partial \bar{X}}
\]

\[+ \sigma B_0^2 \sin \beta (\vec{U} \cos \beta - \vec{V} \sin \beta) \frac{\mu \bar{T}}{k_0} - \rho \bar{g} \cos \alpha.
\]

(6)

\[
(pC' \gamma) \frac{\partial \bar{T}}{\partial t} + \vec{U} \frac{\partial \bar{T}}{\partial X} + \vec{V} \frac{\partial \bar{T}}{\partial Y} = k \left[ \frac{\partial^2 \bar{T}}{\partial X^2} + \frac{\partial^2 \bar{T}}{\partial Y^2} \right] - \frac{\partial \bar{q}_\tau}{\partial t} + (pC' \gamma_{\tau}) \rho \frac{\partial \bar{C}}{\partial X} + \frac{\partial \bar{C}}{\partial \bar{X}} + \frac{\partial \bar{C}}{\partial \bar{Y}} + \frac{\partial \bar{C}}{\partial \bar{Y}}
\]

(7)

\[
\frac{D_T}{T_m} \left[ \frac{\partial \bar{T}}{\partial \bar{X}} \right]^2 + \left[ \frac{\partial \bar{T}}{\partial \bar{Y}} \right]^2 - \sigma B_0^2 (\vec{U} \cos \beta - \vec{V} \sin \beta)^2
\]

(8)

Where the “radiative heat flux” \( q_r \) is taken into account negligible within the X-direction in comparison with Y-direction. Thus by victimization Roseland approximation for radiation, the “radiative heat flux” is given by:

\[
q_r = \frac{16 \sigma \bar{T}_0^3}{3K^*} \frac{\partial \bar{T}}{\partial \bar{Y}}
\]

(9)

The extra stress tensor in "hyperbolic tangent fluid" is given by: [25]

\[
\tau = -\mu_\tau + (\mu_\tau + \mu_\alpha)(1 - \Gamma \gamma)^{-1}\tau
\]

(10)

and \( \Gamma \gamma = \sqrt{\frac{1}{2} \sum_{ij} y_{ij} \bar{y}_{ij}} = \sqrt{\frac{1}{2}} \bar{\Gamma} \bar{\gamma} \)

For \( \mu_\tau = 0 \) and \( \Gamma \gamma < 1 \), equation (10) can written as:

\[
\tau = -\mu_\beta (1 - \Gamma \gamma)^{-1} \tau = -\mu_\beta (1 - \Gamma \gamma) \tau
\]

(11)

The “stress components” \( \tau_{XX}, \tau_{XY}, \tau_{YY} \) and \( \tau_{TT} \) can got through the following relations:

\[
\tau_{XX} = \mu_\beta [(1 + n (\Gamma \gamma - 1)] \frac{\partial \vec{U}}{\partial X} \frac{\partial \vec{U}}{\partial X}
\]

(12)

\[
\tau_{XY} = \mu_\beta [(1 + n (\Gamma \gamma - 1)] \frac{\partial \vec{U}}{\partial Y} \frac{\partial \vec{V}}{\partial X}
\]

(13)

\[
\tau_{YY} = \mu_\beta [(1 + n (\Gamma \gamma - 1)] \frac{\partial \vec{V}}{\partial Y} \frac{\partial \vec{V}}{\partial X}
\]

(14)

The appropriate boundary conditions comparing wall no-slip and convective boundary conditions are given as follows:
\( \bar{U} = 0, -K_a \frac{\partial T}{\partial Y} = h_s (T_o - T) \) and \( -K_a \frac{\partial C}{\partial Y} = h_m (C_o - C) \) at \( \bar{Y} = \bar{H}_1 \) \( (15) \)

\( \bar{U} = 0, -K_a \frac{\partial T}{\partial Y} = h_s (T - T_i) \) and \( -K_a \frac{\partial C}{\partial Y} = h_m (C - C_i) \) at \( \bar{Y} = \bar{H}_2 \) \( (16) \)

We consider the following non-dimensional quantities:

\[
\begin{align*}
&x = \frac{X}{h},
&y = \frac{Y}{h},
&t = \frac{t_i}{c},
&u = \frac{U}{c},
&v = \frac{V}{c},
&\delta = \frac{d}{\lambda},
&h_1 = \frac{H_1}{d},
&h_2 = \frac{H_2}{d},
&\theta = \frac{T - T_o}{T_1 - T_0},
&p = \frac{d \rho}{c h \mu_0},
&y = \frac{d}{c}.
\end{align*}
\]

We define:

\[
\begin{align*}
&x_{xx} = \frac{\partial^2 x}{\partial x^2},
&t_{xx} = \frac{\partial^2 t}{\partial x^2},
&t_{yy} = \frac{\partial^2 t}{\partial y^2},
&Re = \frac{\rho cd \mu_0}{\mu_c},
&\phi = \frac{C - C_o}{C_1 - C_o},
&B_h = \frac{h_1 d}{K_h}.
\end{align*}
\]

\[
\begin{align*}
&\frac{P}{P_t} = \frac{\mu_c \rho}{\mu_c \rho},
&a_1 = \frac{a_2}{m},
&m = \frac{m \lambda}{\mu_c},
&N_b = \frac{\mu_0}{\rho_f},
&\phi_c = \frac{(\rho C)_{C_1}}{(\rho C)_{C_0}},
&N_t = \frac{\sigma \rho_f (1 - \rho C_f) M}{\mu_0},
&R_n = \frac{16 \sigma f t_0^3}{3 K' \mu C_f},
&Sc = \frac{v}{D_B},
&E_c = \frac{c^2}{D_B},
&Pr = \frac{c^2}{D_B},
&k^2 = \frac{1}{D_a},
&u = \frac{\partial y}{\partial x},
&v = \frac{\partial y}{\partial x}.
\end{align*}
\]

Using the above dimensionless variables, continuity equation has automatically achieved and equations from (5) - (8) become:

\[
\begin{align*}
&\text{Re} \delta \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x \partial y} \right) = -\frac{\partial p}{\partial x} + \delta \frac{T}{x} \frac{\partial x}{\partial x} + \delta \frac{T}{x} \frac{\partial y}{\partial x} - N_1 \frac{\partial \psi}{\partial x} - M^2 \cos \beta
\end{align*}
\]

\[
\begin{align*}
&\sin \beta \delta \frac{\partial y}{\partial x} \frac{Re}{Fr} \sin \alpha
\end{align*}
\]

\[
\begin{align*}
&\text{Re} \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} \right) = -\frac{\partial p}{\partial x} + \delta \frac{T}{x} \frac{\partial x}{\partial x} + \delta \frac{T}{x} \frac{\partial y}{\partial x} + \frac{M^2}{cos \beta}
\end{align*}
\]

\[
\begin{align*}
&\sin \beta \delta \frac{\partial y}{\partial x} + N_1 \delta \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{Fr} \cos \alpha
\end{align*}
\]

\[
\begin{align*}
&\text{Re} \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial x} \right) + R_n \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} + N_1 \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \right) \right)
\end{align*}
\]

\[
\begin{align*}
&N_1 \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} + \left( \frac{\partial \psi}{\partial y} \right)^2 \right) + \left( \frac{\partial \psi}{\partial y} \right)^2 + 2 \delta \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} + \delta \frac{\partial \psi}{\partial y} \right)
\end{align*}
\]

With non-dimensional stress components are:

\[
\begin{align*}
&m_1 = \mu \cos \beta;
&m_2 = \mu \sin \beta;\quad m_3 = \mu \cos \beta \sin \beta;
&m_4 = \mu \sin \beta;\quad \delta = \delta \frac{\partial \psi}{\partial y}.
\end{align*}
\]
\[ t_{xx} = 2\{(1 + n(We - 1))\] \( \frac{\partial u}{\partial x} \) \]

(23)

\[ t_{xy} = (1 + n(We - 1))\] \( \frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x} \)

(24)

\[ t_{yy} = 2\delta\{(1 + n(We - 1))\] \( \frac{\partial v}{\partial y} \)

(25)

\[ \dot{\psi} \{y\} = \sqrt{2\delta^2 \left[ \frac{\partial u}{\partial x}^2 + \frac{\partial v}{\partial y}^2 \right] + \left( \frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x} \right)^2} \]

(26)

Similarly Eqs. (15) and (16) become:

\[ \frac{\partial \psi}{\partial y} = 0, \] \( \frac{\partial \theta}{\partial y} = B_n \psi \) \text{ and } \( \frac{\partial \phi}{\partial y} = B_n \phi \) \text{ at } \( y = h_1 \)

(27)

\[ \frac{\partial \psi}{\partial y} = 0, \] \( \frac{\partial \theta}{\partial y} = B_n (1 - \theta) \) \text{ and } \( \frac{\partial \phi}{\partial y} = B_n (1 - \phi) \) \text{ at } \( y = h_2 \)

(28)

Where \( x \) is the non-dimensional axial coordinate, \( y \) is the non-dimensional transverse coordinate; \( t \) is the dimensionless time.

Eqs. (18)- (26) Under long wavelength and less “Reynolds number” approximation yield following set of equations:

\[ \frac{\partial p}{\partial x} = \frac{\partial \psi}{\partial y} t_{xy} - N_1 \frac{\partial \psi}{\partial y} + \frac{\text{Re} \sin \alpha}{\text{Fr}} \]

(29)

\[ \frac{\partial p}{\partial y} = 0 \]

(30)

\[ (1 + R_s \text{ Pr}) \frac{\partial^2 \theta}{\partial y^2} + \text{Pr} [N_s \frac{\partial \theta}{\partial y}] + N_i \left( \frac{\partial \theta}{\partial y} \right)^2 + B m_i \left( \frac{\partial \psi}{\partial y} \right)^2 = 0 \]

(31)

\[ \frac{\partial^2 \theta}{\partial y^2} + N_s \frac{\partial \theta}{\partial y} = 0 \]

(32)

With

\[ t_{xx} = 2\{(1 + n(We - 1))\] \( \frac{\partial u}{\partial x} \) \]

(33)

\[ t_{xy} = (1 + n(We - 1))\] \( \frac{\partial u}{\partial y} \)

(34)

\[ t_{yy} = 0 \]

(35)

\[ \dot{\psi} \{y\} = \frac{\partial u}{\partial y} \]

(36)

Eliminating pressure from Eqs. (29) and (30) we get the following system:
\[
\frac{\partial^2 \psi}{\partial y^2} + N_1 \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{37}
\]

\[
t_{xy} = 2(1-n) \frac{\partial^2 \psi}{\partial x \partial y} + 2nWe \frac{\partial^2 \psi}{\partial y^2} \frac{\partial^2 \psi}{\partial x^2} \tag{38}
\]

\[
t_{xy} = (1-n) \frac{\partial^2 \psi}{\partial y^2} + nWe \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \tag{39}
\]

\[
\psi = \frac{-F}{2} \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = B_n \phi \quad \text{at} \quad y = h_1 = -1 - mx - a \sin \left[ \frac{2\pi}{\lambda} (x - t) + \varphi \right] \tag{40}
\]

\[
\psi = \frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = B_n (1 - \theta) \quad \text{and} \quad \frac{\partial \phi}{\partial y} = B_n (1 - \phi) \quad \text{at} \quad y = h_2 = 1 + mx + b \sin [2\pi (x - t)] \tag{41}
\]

\[
(1 + Pr \frac{\partial}{\partial x}) \frac{\partial^2 \theta}{\partial y^2} + PrN_1 \frac{\partial \theta}{\partial y} + Brm\left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 = 0 \tag{42}
\]

Where \( A = \frac{1}{2} (B_n + \frac{N_1}{N_b}) \)

The "non-dimensional average rise in pressure \( \Delta p \)" per wavelength is given by:

\[
\Delta p = \int_0^1 \int_{\alpha_0}^{\alpha_1} \frac{\partial p}{\partial x} \, dx \, dt \tag{43}
\]

### 3. Perturbed system and perturbation solutions

The equations (29), (32) and (42) are not linear and their exact solution are not possible. Hence, we utilize the perturbation technique with small weissenberg number in the form of

\[
\psi = \psi_0 + We \psi_1 + o(We^2)
\]

\[
\theta = \theta_0 + We \theta_1 + o(We^2)
\]

\[
F = F_0 + We F_1 + o(We^2)
\]

\[
p = p_0 + We p_1 + o(We^2)
\]

\[
\phi = \phi_0 + We \phi_1 + o(We^2)
\]

\[
u = u_0 + We u_1 + o(We^2)
\]

Such that \( u_0 = \frac{\partial \psi_0}{\partial y}, \quad u_1 = \frac{\partial \psi_1}{\partial y} \)

#### 3.1 Perturbed system

##### 3.1.1 Zeroth-Order System

\[
(1-n) \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) - N_1 \frac{\partial^2 \psi_0}{\partial y^2} = 0 \tag{45}
\]
\[ \frac{\partial \rho_0}{\partial x} = \frac{\partial}{\partial y} \left[ (1 - n) \frac{\partial^2 \psi_0}{\partial y^2} \right] - N_1 \frac{\partial^2 \psi_0}{\partial y^2} + \frac{Re}{Fr} \sin \alpha \] (46)

\[ (1 + Pr_n) \frac{\partial^2 \theta_0}{\partial y^2} + Pr N_e A \frac{\partial \theta_0}{\partial y} + B m_1 \frac{\partial \psi_0}{\partial y} = 0 \] (47)

\[ \frac{\partial^2 \psi_0}{\partial y^2} + N_e \frac{\partial^2 \theta_0}{\partial y^2} = 0 \] (48)

With corresponding boundary conditions:

\[ \psi_0 = -\frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0, \quad \frac{\partial \theta_0}{\partial y} = B_1 \theta_0, \quad \frac{\partial \phi_0}{\partial y} = B_\alpha \phi_0 \quad \text{at} \quad y = h_1 \]

\[ \psi_0 = \frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0, \quad \frac{\partial \theta_0}{\partial y} = B_1 (1 - \theta_0), \quad \frac{\partial \phi_0}{\partial y} = B_\alpha (1 - \phi_0) \quad \text{at} \quad y = h_2 \] (49)

3.1.2 First-order system

\[ (1 - n) \frac{\partial^2 \psi_1}{\partial y^2} - n \frac{\partial^2 \psi_1}{\partial y^2} - N_1 \frac{\partial \psi_1}{\partial y} = 0 \] (50)

\[ \frac{\partial p_1}{\partial x} = \frac{\partial}{\partial y} \left[ (1 - n) \frac{\partial^2 \psi_1}{\partial y^2} + n \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) \right] - N_1 \frac{\partial \psi_1}{\partial y} \] (51)

\[ (1 + Pr_n) \frac{\partial^2 \theta_1}{\partial y^2} + Pr N_e A \frac{\partial \theta_1}{\partial y} + 2 B m_1 \frac{\partial \psi_0}{\partial y} \frac{\partial \psi_1}{\partial y} = 0 \] (52)

\[ \frac{\partial^2 \theta_1}{\partial y^2} + N_e \frac{\partial^2 \theta_1}{\partial y^2} = 0 \] (53)

With corresponding boundary conditions:

\[ \psi_1 = -\frac{F_1}{2} \frac{\partial \psi_1}{\partial y} = 0, \quad \frac{\partial \theta_1}{\partial y} = B_1 \theta_1, \quad \frac{\partial \phi_1}{\partial y} = B_\alpha \phi_1 \quad \text{at} \quad y = h_1 \]

\[ \psi_1 = \frac{F_1}{2} \frac{\partial \psi_1}{\partial y} = 0, \quad \frac{\partial \theta_1}{\partial y} = -B_1 \theta_1, \quad \frac{\partial \phi_1}{\partial y} = -B_\alpha \phi_1 \quad \text{at} \quad y = h_2 \] (54)

3.2 Perturbation Solutions

3.2.1 Zeroth-Order Solution

The solutions of equations from (45-48) are given by:

\[ \psi_0 = a_1 + e^{-ny} (a_2 + a_n e^{2ny}) n_2 + a_4 y; \] (55)

\[ u_0 = a_2 - a_n e^{-ny} n_2 + a_4 e^{ny} n_2; \] (56)

\[ \frac{\partial \rho_1}{\partial x} = \frac{a_4 \text{Fr} (-1 + n) n_2^2 + \text{ReSin} [\alpha]}{\text{Fr}}; \] (57)
\[ \theta_0 = c_4 + B \left( e^{-i \lambda y} + B_d c_i e^{i \lambda y} + \frac{1}{2} B e^{2i \lambda y} (B_4 + B_2 e^{3i \lambda y} + B_1 e^{4i \lambda y}) m_1^2 n_2 - B y B m_1^2 y \right); \quad (58) \]

\[ \sigma_0 = c_5 + e^{x_3} N (B n m_1^2 n_2 - 4a_1 a_d D_1 e^{6i \lambda y} + a_2^2 D_2 e^{5i \lambda y} n_2 + a_1 D_4 e^{3i \lambda y}) \]

\[ \frac{(4a_1 D_4 + a_2 D_0 e^{2i \lambda y} n_2) N b + 2c_1 D_0 e^{2i \lambda y}}{2 AD_b} \]

\[ + c \frac{1}{2 AD_b} + \frac{1}{2 AD_b} \left( 1 + \frac{1}{Pr} R_n \right) \left( \frac{c_6 y}{2 AD_b} \right); \quad (59) \]

Where \( a, c, \) (i, j = 1, 2, 3, 4) are constant can be determined. Furthermore, the “non-dimensionless average rise” in pressure \( \Delta p \) per “wavelength” at this order is

\[ \Delta p_0 = \int_a^b \frac{dp}{dx} dx \]

### 3.2.2 First-order solution

Substituting the zeroth-order solution (55) into Eqs. (50-53) and then solving the resulting system with the corresponding boundary conditions, we get:

\[ \psi_1 = e^{-i \lambda y} \left( \frac{a_2^2 m_1^2 n_2^2}{3(-1 + n)} + e^{i \lambda y} \left( \frac{a_1^2 e^{i \lambda y} m_1^2 n_2^2}{3(-1 + n)} + \frac{3(-1 + n)}{e^{2i \lambda y} b_1 + b_2} \right) \right) + b_3 \]

\[ u_1 = e^{-i \lambda y} \left( \frac{-3b_1 e^{i \lambda y} (-1 + n) + 3b_0 e^{i \lambda y} (-1 + n) + n_1 (3b_1 e^{2i \lambda y} (-1 + n) - 2a_1^2 e^{4i \lambda y} m_1^2 n_2^2)}{3(-1 + n)} \right); \quad (61) \]

\[ \frac{\partial p_1}{\partial x} = b_2 (-1 + n) n_2^2; \quad (63) \]

\[ \theta_1 = c_4 + A_1 e^{-i \lambda y} + A_2 e^{i \lambda y} + A_3 e^{i \lambda y} + A_4 e^{i \lambda y} + A_5 e^{i \lambda y} + A_6 e^{i \lambda y} + A_7 e^{i \lambda y}; \quad (64) \]

\[ \text{Pr}(6a_1 B r E_m^2 m_2^2 + 3b_2 e^{i \lambda y} E_2 (-1 + n) + c_1 e^{i \lambda y} e^{i \lambda y} + \frac{3(-1 + n)}{b_1 + b_2}); \quad (65) \]

Where \( b, c, \) (i, j = 1, 2, 3, 4) are constant can be determined the “non-dimensionless average rise” in pressure \( \Delta p \) per “wavelength” at this order is given as follows:

\[ \Delta p_1 = \int_a^b \frac{\partial p_1}{\partial x} dx \]

### 4. Graphical Results and discussion

The section shows the graphical analysis of different “parameters” on the flow of hyperbolic tangent nano fluid through porous area in the slopping non-symmetric non-uniform duct. the analysis of “average pressure rise”, “axial velocity”, temperature distribution, “nano particle distribution” and stream trapping are studied in terms of graphs and then discussed physically. To employ the manner of
solution, “numerical calculation” for distinct values of “amplitudes of upper and lower walls" (b and a), non-uniform parameter of the duct (m), the inclination angle of magnetic field and channel \(\beta, \alpha\) respectively, “Reynolds number” \((Re)\), “Fraud number” \((Fr)\), “Hartmann number” \((M)\), “porosity parameter” \((k)\), the weissenberg number \((we)\) and power law index of “hyperbolic tangent fluid” \((n)\). To analyze and discuss the results, let the instantaneous volume rate of the flow \(F(x,t)\) periodic in \((x-t)\) as:

\[
F(x,t) = \theta' + a \sin(2\pi(x-t)+\phi) + b \sin(2\pi(x-t))
\]

(67)

### 4.1 Velocity profile

The velocity expression is given in the Eqs. (56) and (62). To see the influence of non-uniform parameter \(m\), phase angle \(\phi\), inclination angle of magnetic field \(\beta\), hyperbolic tangent \((we)\), Hartmann number \(M\), porosity parameter \((k)\), amplitudes of walls \((a\ and\ b)\) and volume flow rate on the “axial velocity”, the graphs are plotted in figs.(2-11) at the constant \(x = 0.3\) and \(t = 0.5\). We noted from the graph that the velocity profile for the non-uniform parameter \(m\) is sketched in Fig.(2), we noted that the velocity of the fluid will be increased in the central part for the a convergent channel \((m < 0)\) and it is increased in the edges of the divergent channel \((m > 0)\) which is high compared to uniform channel \((m = 0)\). Fig. (3) Showed the influence of \(\phi\) on velocity. It is found that the value of \(u\) will be increased in the side of lower wall of the channel. The effect of \(a\) is plotted in fig. (4), it is noticed that an increase in a lead to drop the velocity at all parts of the channel. A similar behavior of \(b\) on the velocity, which has shown in fig. (5). the axial velocity for the Hartmann number \(M\) is displayed in fig. (6), it is examined that the velocity of fluid will be run-down in the center of channel but the flow is inverted at the points of the region at \(y \in [-0.4075, -0.4075]\) and thus the velocity will be rise in the boundaries of the channel. Like action of porosity parameter and the flow will be overturn at the point \((0.4432, -0.4642)\) and its action will be noted in fig. (7). On the other hand, the impact of slopping angle of magnetic force \((\beta)\) is opposite of effect of \(M\) on the flow, that is the velocity will be grewed in the focus of the channel and inflected at the region of \(y \in [0.4625, -0.4652]\) and its effect will appeared in fig. (8). the effectiveness of flow rate \((\theta)\) is plotted in fig. (9), it is observed that the magnitude of velocity will be rising at all parts of channel Fig. (10) Will be offer the impress of hyperbolic tangent fluid parameter \((we)\), it is examined that the axial velocity will be increased in upper wall and reversal situation will be noted in the lower part of channel and we observed that the graph is asymmetric by clear way in the parties of the channel. The influence of power law index of above fluid \((n)\) is designed in, fig. (11), we found that the velocity will be less in the center of channel and will be bigger in the upper wall where the flow will be inflected at the point \((0.4139, 0.4831)\)
The Eqs. (58) and (64) show the temperature expression. Temperature distributions are function of y are plotted in figures (12-26) to study the effects of $m, \varphi, a, b, M, k, n, Br, Rn, N_a, N_f, B_m, B_n, \beta$ and $\theta$ with $x = 0.3$ and $t = 0.5$. Figs. ((12)-(18)) are drawn to study the effect of $m, \varphi, Br, Pr, N_a, N_f$ and $\theta$ on the temperature distribution we see that the temperature increases with an increasing of last parameters. The opposite effect for the variables $a, b$ and $Rn$ on the temperature where their graphs are displayed in Figs. ((19), (20) and (21)) respectively. From Fig. (22). we observed that the temperature grows with an increase of (M). Reversal impact for the parameter $\beta$ on temperature distribution and its graph is shown in Fig. (23). Similar conduct for the effect of porosity parameter ($k$) on the temperature and it is shown in Fig. (24), Fig. (25) is designed for the parameter ($B_m$), it is observed that an increase in ($B_m$) causes less in

**4.2 Temperature Profile**

Figure (1-11): Effect of parameters on velocity profiles

$$m = 0.20, t = 0.50, \varphi = \pi/6, a = 0.20, b = 0.10,$$

$$M = 5, we = 0.1, n = 0.55, \beta = \pi/4, k = 1, \theta = 1$$
temperature at the region of flow of \((-2.5, -1.5)\) and it has bigger in the region of \((-1, -2)\). Fig. (26) Shows that the temperature will be a rise in the region of \((-2, 2.5)\) with an increase of \(B_m\).

\[
\begin{align*}
\text{Fig. (12)} & \quad m = 0.10 \\
\text{Fig. (13)} & \quad \varphi = \pi/6 \\
\text{Fig. (14)} & \quad \varphi = \pi/4 \\
\text{Fig. (15)} & \quad \varphi = \pi/3 \\
\text{Fig. (16)} & \quad m = 0.20 \\
\text{Fig. (17)} & \quad m = 0.30 \\
\text{Fig. (18)} & \quad B_r = 1 \\
\text{Fig. (19)} & \quad B_r = 1.5 \\
\text{Fig. (20)} & \quad B_r = 2 \\
\text{Fig. (21)} & \quad P_r = 1 \\
\text{Fig. (22)} & \quad P_r = 12 \\
\text{Fig. (23)} & \quad P_r = 1.5 \\
\text{Fig. (24)} & \quad N_b = 10 \\
\text{Fig. (25)} & \quad N_b = 12 \\
\text{Fig. (26)} & \quad N_b = 14 \\
\text{Fig. (27)} & \quad N_b = 15 \\
\text{Fig. (28)} & \quad N_b = 2 \\
\text{Fig. (29)} & \quad \theta = 1 \\
\text{Fig. (30)} & \quad \theta = 1.3 \\
\text{Fig. (31)} & \quad \theta = 1.6 \\
\text{Fig. (32)} & \quad \theta = 1.8 \\
\text{Fig. (33)} & \quad \theta = 2 \\
\text{Fig. (34)} & \quad \theta = 2.2 \\
\text{Fig. (35)} & \quad \theta = 2.4 \\
\text{Fig. (36)} & \quad \theta = 2.6 \\
\text{Fig. (37)} & \quad \theta = 2.8 \\
\text{Fig. (38)} & \quad \theta = 3 \\
\text{Fig. (39)} & \quad \theta = 3.2 \\
\text{Fig. (40)} & \quad \theta = 3.4 \\
\text{Fig. (41)} & \quad \theta = 3.6 \\
\text{Fig. (42)} & \quad \theta = 3.8 \\
\text{Fig. (43)} & \quad \theta = 4 \\
\text{Fig. (44)} & \quad \theta = 4.2 \\
\text{Fig. (45)} & \quad \theta = 4.4 \\
\text{Fig. (46)} & \quad \theta = 4.6 \\
\text{Fig. (47)} & \quad \theta = 4.8 \\
\text{Fig. (48)} & \quad \theta = 5 \\
\text{Fig. (49)} & \quad \theta = 5.2 \\
\text{Fig. (50)} & \quad \theta = 5.4 \\
\text{Fig. (51)} & \quad \theta = 5.6 \\
\text{Fig. (52)} & \quad \theta = 5.8 \\
\text{Fig. (53)} & \quad \theta = 6 \\
\text{Fig. (54)} & \quad \theta = 6.2 \\
\text{Fig. (55)} & \quad \theta = 6.4 \\
\text{Fig. (56)} & \quad \theta = 6.6 \\
\text{Fig. (57)} & \quad \theta = 6.8 \\
\text{Fig. (58)} & \quad \theta = 7 \\
\text{Fig. (59)} & \quad \theta = 7.2 \\
\text{Fig. (60)} & \quad \theta = 7.4 \\
\text{Fig. (61)} & \quad \theta = 7.6 \\
\text{Fig. (62)} & \quad \theta = 7.8 \\
\text{Fig. (63)} & \quad \theta = 8 \\
\text{Fig. (64)} & \quad \theta = 8.2 \\
\text{Fig. (65)} & \quad \theta = 8.4 \\
\text{Fig. (66)} & \quad \theta = 8.6 \\
\text{Fig. (67)} & \quad \theta = 8.8 \\
\text{Fig. (68)} & \quad \theta = 9 \\
\text{Fig. (69)} & \quad \theta = 9.2 \\
\text{Fig. (70)} & \quad \theta = 9.4 \\
\text{Fig. (71)} & \quad \theta = 9.6 \\
\text{Fig. (72)} & \quad \theta = 9.8 \\
\text{Fig. (73)} & \quad \theta = 10 \\
\text{Fig. (74)} & \quad \theta = 10.2 \\
\text{Fig. (75)} & \quad \theta = 10.4 \\
\text{Fig. (76)} & \quad \theta = 10.6 \\
\text{Fig. (77)} & \quad \theta = 10.8 \\
\text{Fig. (78)} & \quad \theta = 11 \\
\text{Fig. (79)} & \quad \theta = 11.2 \\
\text{Fig. (80)} & \quad \theta = 11.4 \\
\text{Fig. (81)} & \quad \theta = 11.6 \\
\text{Fig. (82)} & \quad \theta = 11.8 \\
\text{Fig. (83)} & \quad \theta = 12 \\
\text{Fig. (84)} & \quad \theta = 12.2 \\
\text{Fig. (85)} & \quad \theta = 12.4 \\
\text{Fig. (86)} & \quad \theta = 12.6 \\
\text{Fig. (87)} & \quad \theta = 12.8 \\
\text{Fig. (88)} & \quad \theta = 13 \\
\text{Fig. (89)} & \quad \theta = 13.2 \\
\text{Fig. (90)} & \quad \theta = 13.4 \\
\text{Fig. (91)} & \quad \theta = 13.6 \\
\text{Fig. (92)} & \quad \theta = 13.8 \\
\text{Fig. (93)} & \quad \theta = 14 \\
\text{Fig. (94)} & \quad \theta = 14.2 \\
\text{Fig. (95)} & \quad \theta = 14.4 \\
\text{Fig. (96)} & \quad \theta = 14.6 \\
\text{Fig. (97)} & \quad \theta = 14.8 \\
\text{Fig. (98)} & \quad \theta = 15 \\
\text{Fig. (99)} & \quad \theta = 15.2 \\
\text{Fig. (100)} & \quad \theta = 15.4 \\
\text{Fig. (101)} & \quad \theta = 15.6 \\
\text{Fig. (102)} & \quad \theta = 15.8 \\
\text{Fig. (103)} & \quad \theta = 16 \\
\text{Fig. (104)} & \quad \theta = 16.2 \\
\text{Fig. (105)} & \quad \theta = 16.4 \\
\text{Fig. (106)} & \quad \theta = 16.6 \\
\text{Fig. (107)} & \quad \theta = 16.8 \\
\text{Fig. (108)} & \quad \theta = 17 \\
\text{Fig. (109)} & \quad \theta = 17.2 \\
\text{Fig. (110)} & \quad \theta = 17.4 \\
\text{Fig. (111)} & \quad \theta = 17.6 \\
\text{Fig. (112)} & \quad \theta = 17.8 \\
\text{Fig. (113)} & \quad \theta = 18 \\
\text{Fig. (114)} & \quad \theta = 18.2 \\
\text{Fig. (115)} & \quad \theta = 18.4 \\
\text{Fig. (116)} & \quad \theta = 18.6 \\
\text{Fig. (117)} & \quad \theta = 18.8 \\
\text{Fig. (118)} & \quad \theta = 19 \\
\text{Fig. (119)} & \quad \theta = 19.2 \\
\text{Fig. (120)} & \quad \theta = 19.4 \\
\text{Fig. (121)} & \quad \theta = 19.6 \\
\text{Fig. (122)} & \quad \theta = 19.8 \\
\end{align*}
\]
4.3 Concentration Profiles

The Eqs. (59) and (65) explain the concentration expression. Concentration distributions are function study the effects of \( m, \phi, a, b, M, k, n, Br, Rn, Pr, N_\beta, N_\gamma, B_m, B_n, \beta \) and \( \theta \) with \( x = 0.3 \) and \( t = 0.5 \). Opposite behavior for concentration profile is noted when compared with temperature. The influence of \( m, \phi, M, Br, Pr, N_\beta, N_\gamma, B_m, B_n, \beta \) and \( \theta \) on the concentration are noted in Figs. (27-34) which is observed that the magnitude of concentration will be less with the comparison of increasing of above last parameters. Inverse effects of the variables \( a, b, k, Rn, Pr, N_\beta, N_\gamma, B_m \) and \( \beta \) on concentration which; is noticed that the concentration is an increasing function of these variables and their graphs are displayed in figs. (35-41).

Figure (12-26): influence of parameters on profile of Temperature

\[
\begin{align*}
\text{Figure (24)}: & \quad k = 1, k = 1.5, k = 2 \\
\text{Figure (25)}: & \quad B_\lambda = 0.1, B_\lambda = 0.11, B_\lambda = 0.12 \\
\text{Figure (26)}: & \quad B_m = 0.10, B_m = 0.20, B_m = 0.30 \\
\text{Figure (27)}: & \quad m = 0.10, m = 0.2, m = 0.30 \\
\text{Figure (28)}: & \quad \phi = \pi/6, \phi = \pi/4, \phi = \pi/3 \\
\text{Figure (29)}: & \quad M = 1, M = 1.5, M = 2 \\
\text{Figure (30)}: & \quad Br = 1, Br = 1.5, Br = 2 \\
\text{Figure (31)}: & \quad Pr = 1, Pr = 12, Pr = 1.5 \\
\text{Figure (32)}: & \quad N_\gamma = 10, N_\gamma = 12, N_\gamma = 14
\end{align*}
\]
4.4 Pressure rise of Pumping Characteristic

Figs. (42-47) shows the changes of “$\Delta p$”against “time mean flow rate”$\theta’$”. The whole region is consered into five parts:

1. Peristaltic Pumping region where $(\Delta p > 0, \theta’ > 0)$
2. Augmented pumping (co-pumping) region where $(\Delta p < 0, \theta’ > 0)$
3. When $(\Delta p > 0, \theta’ < 0)$, then it is a retrograde pumping region.
4. There is a co-pumping region where $(\Delta p < 0, \theta’ < 0)$
5. $(\Delta p = 0)$ Corresponds to the free pumping region.

Fig. (42) Give the effect of M on “average pressure rise $\Delta p$”, it are often seen from the graph that the “pumping” rate will be decrease in the retrograde, co-Pumping and augmented regions and the
relation between the regions of pumping is linear. Fig. (43) Examined the influence of \( \text{we} \) on "pressure rise", it is noted that the rate of pumping will be less in the retrograde and augmented regions and the relation between them is not linear. The effect of \( \beta \) on pressure rise is opposite of Hartmann's number behavior and its graph will be shown in fig. (44), Fig.(45) and fig. (46) Shows the effects of Reynolds number and inclination angle of channel \( \alpha \), we observed that pressure rise will be rise in all region of pumping. Reversal observation is examined for the influence of Froude number \( \text{Fr} \) and its diagram is noticed in fig. (47).

4.5 Trapping Phenomenon

From Eqs. (55) and (61) we have calculated the stream function in terms of \( y \). The phenomenon of trapping is another interesting area in peristaltic transport. In a reference wave frame, the formation of an internally circulating bolus of fluid by closed stream lines is called trapping and this trapped bolus is pushed a head a long with that bolus also appear in the fixed frame which may probably be due to the influence of non-zero time-average of the flow average period of the wave. The trapping for different values of \( m \), \( M \), \( \text{we} \) and \( \beta \) are shown in Figs. (48)- (51), at fixed values of \( t=0.5 \).Figs. (48)- (49), shows the stream line behavior of \( m \) and \( \beta \), we found that the volume and number of trapping bolus increase with an increase of last variables. From figs. (50)-(51), it is found that the size and number of bolus decreases with an increase of \( M \) and \( \text{we} \) .
Figure (48): Effect of m on trapping

\[ M = 5, \omega_e = 0.0001, \beta = \frac{\pi}{\sqrt{3}} \]

(a) \( m = 0 \), (b) \( m = 0.2 \), (c) \( m = 0.4 \)

Figure (49): Effect of (\( \beta \)) on trapping

\[ m = 0.2, M = 5, \omega_e = 0.0001 \]

(a) \( \beta = \frac{\pi}{6} \), (b) \( \beta = \frac{\pi}{4} \), (c) \( \beta = \frac{\pi}{3} \)

Figure (50): Effect of (\( M \)) on trapping

\[ \beta = \frac{\pi}{\sqrt{3}}, m = 0.2, \omega_e = 0.0001 \]

(a) \( M = 1 \), (b) \( M = 3 \), (c) \( M = 5 \)
5. Deduction of the problem

A mathematical mode has been taken to review the peristaltic motion of nano hyperbolic tangent fluid with in the inclined non-uniform asymmetric duct under the impact of inclined magnetic force and porous medium with convective conditions. The duct asymmetry tapered is created by selecting the peristaltic wave train on the non-regular walls to have distance amplitudes and phase. A long-wave length and low-“Reynolds number” approximations are dependent. A regular perturbation method is utilized to get the expression for “stream function”, “axial velocity”, temperature distribution and concentration distribution. Numerical study has been conducted for the common rise within the physical science parameters of the fluid with peristaltic flow is mentioned, the most results will be summarized as follows:

- The velocity of the fluid increase at the core part of the channel in the case of convergent channel \( m < 0 \) and decrease at the walls of channel in the case of divergent channel \( m > 0 \).
- The velocity of the fluid will be increased at \( \theta \) and the opposite case is satisfied with an increase of \( a \).
- The velocity is increased in the lower part of the channel with an increase of \( \phi \), reversal situation with an increase of \( b \).
- It found that the axial velocity decreases at the center of the channel and increase at the two parts of the channel with an increase of \( M \) and \( k \), inverse case is satisfied for an increase of \( \beta \).
- It is observed that the velocity of fluid will be increase in the upper wall of channel with an increase of \( n \) and \( we \) and will be decrease at the lower part of the channel.
- It is found that the axial take parabolic paths.
- It is observed that the size and number of bolus will be larger with an increase of \( m, a, b, \beta, \phi \) and \( \theta \), opposite case is noted for an increase of \( M, we, n \) and \( k \).
- An increase in Hartmann number causes reduction in value of velocity of fluid, in fact it is due that magnetic field applied in transverse direction yield resistance to fluid particles which decrease the velocity.
- An increase in porosity causes decrease in magnitude of velocity of fluid, this emphasis that the velocity reduced in the center of duct with porous area in it, so the more porous in a porous area will be less resistance to the inflow of fluid, this make a velocity is the more value, this is what happened with the velocity at the walls of channel.
• Temperature is the median kinetic energy of the particles and kinetic energy depends on velocity, so the temperature will be increase with an increase of $m, \varphi$, M, Br, Pr, $N_B$, $N_f$, and $\theta$ opposite behavior is observed for an increase of $a$, $b$, k, Rn, $\varphi$ and $\beta$.

• It is observed that an increase in value of Br causes an increase in the magnitude of temperature of fluid, in fact it is due that an increase in Br increase the resistance showed by shear in flow which in turn increases the heat generation due to the viscous dissipation influence and hence the temperature of the fluid increases, similar reason for rising the temperature of fluid with an increase of Pr.

• The reason behind an increase the temperature of fluid in the central line due to the presence of viscous dissipation term, the viscosity of the fluid in the viscous fluid flow will get kinetic energy from the motion of the fluid and converted it into internal energy of the fluid that raises the fluids temperature, this process is called viscous dissipation.

• An increase in the value of Brownian number $N_B$ lead to enhancement in temperature is due to increase the kinetic energy of the nanoparticles and transforms into internal energy that raises nano fluids temperature.

• Concentration distribution is reversal case of the temperature analysis for more parameters which impact the problem.

• The concentration of nanoparticles of fluid will be enhanced with an increase of $a$, $b$, k, Rn, $N_B$, $B_m$ and $\beta$, the reversal case is satisfied for an increase of $m, \varphi$, M, Br, Pr, $N_f$, $B_m$ and $\theta$.

• $B_h$ and $B_m$ Have wobbling increase on the temperature of fluid.

• The rate of pressure rise will be less with an increase of $M_{we}$ and Fr and the reversal case is verified with an increase of $\beta$, Re and $\alpha$.

• The relationship between the regions of pumping is linear in the case of increasing of $M, \beta, Fr, Re, B_h$ and $\alpha$ and it is non-linear with an increase of $we$.

• The symmetry of graphs of velocity is rather small with the effect of $m, M$ and $\varphi$.

• The symmetry of velocity's graphs is a little bit with the effect of $a$ and $b$.

• The symmetry of velocity's graphs is pretty good with the influence of $k$.

• The symmetry of velocity's graphs is satisfied by clear way with the influence of $\theta$, reversal situation is verified with an influence of $\varphi, we$ and $n$.

6. References

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Appendix

\[
\begin{align*}
E_{11} &= (A\dot{N}b \dot{Pr}^4 + A \dot{N}b^4 \dot{Pr}^4 (1 + Pr Rn) - 13A \dot{N}b^3 \dot{Pr}^3 (n_1 + n_1 Pr Rn)^3) + 32 A NbPr^2 Pr (1 + Pr Rn) + 72 A NbPr^2 Pr^2 (1 + Pr Rn); \\
B_1 &= \frac{2a_2 a_4 B m^2 n 2}{n_1 - ANb Pr + n_1 Pr Rn}; \\
B_2 &= \frac{2n_1 1 + ANb Pr + n_1 Pr Rn}{a_1 n_1 2}; \\
B_3 &= \frac{2 a_4 a_2 n_1 2 n_2}{A Nb Pr^2 n_1 1 + Pr Rn}; \\
B_4 &= \frac{(a_4^2 - 2 a_2 a_1 2 n^2 2)}{A Nb Pr^2 (1 + Pr Rn)}; \\
B_5 &= \frac{-A Nb Pr}{(1 + Pr Rn)}; \\
B_6 &= \frac{-A Nb Pr}{(1 + Pr Rn)}; \\
E_{12} &= (-A \dot{N}b^3 \dot{Pr}^4 - 2A^2 n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn) + 9A Nb Pr (n_1 + n_1 Pr Rn) + 18 (n_1 + n_1 Pr Rn)^3); \\
E_{13} &= (-A \dot{N}b^3 \dot{Pr}^4 + (1 + n_1 Pr Rn)^2); \\
E_{14} &= (-A \dot{N}b^3 \dot{Pr}^4 + 6A^2 n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn) - 11A Nb Pr (n_1 + n_1 Pr Rn)^3) + 6(n_1 + n_1 Pr Rn); \\
n_{55} &= \frac{(A Nb Pr + 6n_1 (1 + Pr Rn))}{1 + Pr Rn}; \\
E_{15} &= (A \dot{N}b^3 \dot{Pr}^4 + 3A n_1 Nb Pr (1 + Pr Rn) + 2(n_1 + n_1 Pr Rn)^3); \\
E_{16} &= (A Nb Pr + 3n_1 (1 + Pr Rn)); \\
E_{17} &= (A \dot{N}b^3 \dot{Pr}^4 + 6A^2 n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn) + 11A Nb Pr (n_1 + n_1 Pr Rn)^3) + 6(n_1 + n_1 Pr Rn); \\
E_{18} &= (A \dot{N}b^3 \dot{Pr}^4 + 4(n_1 + n_1 Pr Rn)^2); \\
n_{44} &= \frac{(n_1 + A Nb Pr + n_1 Pr Rn)}{1 + Pr Rn}; \\
E_7 &= (n_1 + A Nb Pr + n_1 Pr Rn); \\
E_8 &= (-A Nb Pr^2 + 2n_1 (1 + Pr Rn)); \\
E_9 &= (A Nb Pr + 2n_1 (1 + Pr Rn)); \\
n_4 &= \frac{A Nb Pr}{1 + Pr Rn}; \\
E_{19} &= (A \dot{N}b^3 \dot{Pr}^4 + 3A^2 n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn) - 5A \dot{N}b^3 \dot{Pr}^3 (n_1 + n_1 Pr Rn)^2 - 15A \dot{N}b^3 \dot{Pr}^3 (n_1 + n_1 Pr Rn)^2 + 4A Nb Pr (n_1 + n_1 Pr Rn)^3) + 12(n_1 + n_1 Pr Rn)^3); \\
n_5 &= \frac{(A Nb Pr + 2n_1 (1 + Pr Rn))}{1 + Pr Rn}; \\
D_2 &= (A \dot{N}b^3 \dot{Pr}^4 - 5A^2 n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn)^2 + 4(n_1 + n_1 Pr Rn)^3); \\
n_6 &= \frac{A Nb Pr}{1 + Pr Rn}; \\
D_2 &= (-A \dot{N}b^3 \dot{Pr}^4 - 2A n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn) + A Nb Pr (n_1 + n_1 Pr Rn)^2 + 2(n_1 + n_1 Pr Rn)^3); \\
n_5 &= \frac{(n_1 + A Nb Pr + n_1 Pr Rn)}{1 + Pr Rn}; \\
D_3 &= (-A \dot{N}b^3 \dot{Pr}^4 - A^2 n_1 Nb^3 \dot{Pr}^3 (1 + Pr Rn) + 4A Nb Pr (n_1 + n_1 Pr Rn)^3) + 4(n_1 + n_1 Pr Rn); \\
\end{align*}
\]

| Symbol | Nomenclature | Symbol | Nomenclature |
|--------|--------------|--------|--------------|
| \((x, y)\) | “Cartesian coordinates in a wave frame” | \(\Phi\) | “Non-dimensional concentration of the nano particles of the fluid” |
| \((x, y)\) | “Cartesian coordinates in a fixed frame” | \(k_1\) | “Thermal conductivity of the fluid” |
| Symbol | Definition |
|--------|------------|
| $p$    | “The pressure in the wave of reference” |
| $\overline{p}$ | “The pressure in the fixed frame of reference” |
| $d$    | “Mean-half width of the channel” |
| $\overline{t}$ | “Time in the fixed frame of reference” |
| $(u,v)$ | “Velocity components of wave frame of reference” |
| $(\overline{u},\overline{v})$ | “Velocity components of fixed frame of reference” |
| $a_1, a_2$ | “Amplitude of the channel at the lower and upper walls respectively” |
| $Da$   | “Darcy number” |
| $Re$   | “The Reynolds number” |
| $M$    | “Hartmann Number” |
| $We$   | “The Weissenberg number” |
| $\lambda$ | “Wave length of the channel” |
| $c$    | “Wave speed” |
| $\theta$ | “Volume flow rate in a wave frame” |
| $B_0$  | “Constant transverse magnetic field” |
| $Q(x,t)$ | “Instantaneous flux in a fixed frame” |
| $k_0$  | “permeability” |
| $F$    | “The non-dimensional mean flows of wave frame” |
| $\mu_\infty$ | “The infinity shear rate viscosity” |
| $\mu_0$ | “The zero shear rate viscosity” |
| $\tau$ | “Stress tensor extra” |
| $\delta$ | “The wave length” |
| $\Gamma$ | “The time constant” |
| $\alpha$ | “Angle of inclination of channel” |
| $\beta$ | “Angle of inclination of magnetic field” |
| $\rho_f$ | “The density of the fluid” |
| $\rho_p$ | “The density of particle” |
| $\sigma$ | “The electrical conductivity of the fluid” |
| $g$    | “Acceleration to gravity” |
| $k$    | “The porosity parameter” |
| $\sigma^*$ | “Mean absorption coefficient” |
| $K^*$ | “The coefficient of thermal conductivity” |
| $\lambda$ | “The thermal conductivity” |
| $\mu$  | “Viscosity” |
| $\nu$  | “Transverse velocity” |
| $\Psi$ | “Stream function” |
| $u$    | “Axial velocity” |
| $v$    | “Transverse velocity” |
| $N_b$  | “The Brownian motion parameter” |
| $N_s$  | “The thermophoresis parameter” |
| $R_n$  | “The thermal radiation parameter” |
| $B_{h_b}$ | “The heat and mass transfer Biot number respectively” |
| $h_{\alpha}, h_{\beta}$ | “The coefficient of heat and mass transfer” |
| $\gamma$ | “The ratio of the effective heat capacity of nano particle material and heat capacity of the fluid” |
| $n$    | “Power-law index” |
| $a, b$ | “Non-dimensional amplitudes of the lower and upper walls respectively” |
| $c_{\alpha}$ | “Concentration of the nano particle of the fluid at upper wall” |
| $c_{\beta}$ | “Concentration of the nano particle of the fluid at lower wall” |
| $T$    | “Temperature of the fluid at upper wall” |
| $T_0$  | “Temperature of the fluid at lower wall” |
| $c$    | “Concentration of the particle” |
| $\rho$ | “The density of the particle” |
| $\sigma$ | “The electrical conductivity of the fluid” |
| $g$    | “Acceleration to gravity” |
| $k$    | “The porosity parameter” |
| $\sigma^*$ | “Mean absorption coefficient” |
| $K^*$ | “The coefficient of thermal conductivity” |
| $\lambda$ | “The thermal conductivity” |
| $\mu$  | “Viscosity” |
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