ABSTRACT

We present a detailed, self-consistent model of radiatively driven stellar outflows which couples the radiative transfer and hydrodynamics equations. The circumstellar envelope, which consists of gas and dust, is described as a two-component fluid to account for relative drifts. Our results agree with both molecular line observations and infrared continuum spectra, and show that steady-state outflows driven by radiation pressure on dust grains adequately describe the surroundings of the majority of cool luminous evolved stars.

1. INTRODUCTION

Astronomical objects are often characterized by large linear scales, huge masses and powerful flows of energy. Such environments emphasize different physical effects than those encountered in a laboratory or in engineering applications. An example is the radiation pressure force. This force results from the transfer of momentum from the radiation field to the intervening medium. For isotropic interaction between the radiation and the matter, the acceleration due to radiation pressure is (1)

\[ a_{rp} = \frac{1}{c} \int_0^{\infty} \chi_\lambda F_\lambda d\lambda \]  

where \( c \) is the speed of light, \( F_\lambda \) is radiative flux vector, \( \chi_\lambda \) is the opacity per unit mass of the medium (including both absorption and scattering) and \( \lambda \) is wavelength. In the engineering applications this force is usually negligible, but in some astronomical environments it can dominate over all other forces.

Cool luminous evolved stars are one astronomical environment where radiation pressure plays a major role. These stars, which are in the last stages of their evolution, experience mass-loss. Although the exact nature of the mechanism that governs the initial ejection of mass from the stellar surface is still not clear \([2]\), subsequent outflow is driven by the radiation pressure force \([3,4]\). When the expanding gas reaches a certain distance from the star it becomes sufficiently cool that dust grains (solid particles with sizes \( \lesssim 1\mu m \)) begin to condense. Radiation pressure on the dust continues to push the material away from the star and a circumstellar shell, or envelope, is formed. Since the dust grains absorb stellar radiation and reradiate it in the infrared, the overall radiative spectral distribution is shifted toward longer wavelengths and many of these objects are invisible in the optical part of the spectrum.

In order to solve the radiation transfer in such envelopes, the mass density distribution needs to be known. However, a given density distribution implies a particular velocity field which, in turn, is determined by the radiation pressure force. Thus, the radiative transfer and the dynamics are inherently coupled and have to be solved simultaneously.

Our model assumes spherical symmetry and couples the momentum conservation and radiative transfer equations. The circumstellar envelope, which consists of gas and dust, is treated as a two-component fluid. We present an investigation of the dynamics and spectra of dusty envelopes around late-type stars, and discuss the implications for observations.

The rest of this paper is organized as follows: first we discuss the conditions for the radiation pressure force to be important \((\S 2)\), then we describe our model \((\S 3)\), and present the results \((\S 4)\). Some limitations of the model are discussed in \(\S 5\).

2. WHEN IS THE RADIATION PRESSURE FORCE IMPORTANT?

Eq. (1) shows that the acceleration due to the radiation pressure, \( a_{rp} \), is proportional to the radiation flux. This flux cannot be arbitrarily large because dust grains are radiatively heated, and in sufficiently intense radiation field can evaporate. Thus the upper bound on the radiation pressure force is set not by the intensity of the radiation field but by the evaporation temperature of the grains, \( T_{evap} \sim 1000 \text{ K} \).

The radiation pressure force is maximized when the grains are heated by a directional radiation field, \( |F_\lambda| = c u_\lambda \), where \( u_\lambda \) is the radiation energy density. In this case, the total (bolometric) flux absorbed by grains when they are heated to \( T_{evap} \) is:

\[ F \sim \left( \frac{T_{evap}}{1000 \text{ K}} \right)^4 \times 10^5 \text{ W m}^{-2} \]  

In deriving the numerical constant we have assumed small grains with opacity proportional to \( \lambda^{-1} \). With this flux, the acceleration due to the radiation pressure is

\[ a_{rp}^{max} \sim \left( \frac{T_{evap}}{1000 \text{ K}} \right)^4 \times 0.1 \text{ g} \]  

where \( g = 9.81 \text{ m s}^{-2} \). This is the maximal acceleration,
that a grain can experience due to the radiation pressure force, and is independent of the radiation field intensity. Dust grains are usually mixed with gas particles. In such a mixture the momentum which grains gain from the radiation field is transferred to gas particles via collisions [5]. In this case the effective acceleration of such a two-fluid system has to be multiplied by the dust-to-gas mass ratio (± 1), and thus \( a_{\text{rp}} \ll g \). As a result, in engineering applications the radiation pressure force is always negligible compared to the gravitational force.

When is the radiation pressure force important in astrophysical systems? The radiation pressure force will drive an outflow when it overcomes the gravitational force. Taking a typical circumstellar value for the dust-to-gas mass ratio of 0.002 [6], \( a_{\text{rp}} \approx 10^{-4} \, g \). Furthermore, this value can be even 10-100 times smaller when the dust drift effects become significant (this correction is proportional to \( \rho_{\text{gas}}^{1/2} \)) and puts a lower limit on the mass-loss rate [2], see §3.1.

The gravitational acceleration is

\[
a_{\text{g}} = G \frac{M}{r^2},
\]

where \( G \) is the gravitational constant, \( M \) is the stellar mass, and \( r \) is the distance from the star. Dust grains are heated to \( T_{\text{vap}} \) at distance

\[
r \sim 3 \times 10^{10} \text{ m} \left( \frac{L_*}{L_\odot} \right)^{1/2} \left( \frac{1000 \text{ K}}{T_{\text{vap}}} \right)^2 ,
\]

where \( L_* \) is the stellar luminosity, and \( L_\odot = 3.86 \times 10^{26} \text{ W m}^{-2} \) is the solar luminosity. The gravitational acceleration at the dust evaporation radius is thus

\[
a_{\text{g}} \sim 0.01 \frac{g}{L} \left( \frac{T_{\text{vap}}}{1000 \text{ K}} \right)^4,
\]

where \( M = M_*/M_\odot \) \( (M_\odot = 2 \times 10^{30} \text{ kg}) \) is the solar mass, and \( L = L_*/L_\odot \). Note that both \( a_{\text{rp}} \) and \( a_{\text{g}} \) scale with \( T_{\text{vap}}^4 \).

As the stellar luminosity increases the dust evaporation radius is pushed further out, and \( a_{\text{g}} \) decreases. For sufficiently small \( M/L \), \( a_{\text{g}} \) becomes smaller than \( a_{\text{rp}} \), and the radiation pressure force drives an outflow of dust and gas. Taking a typical value of \( M \approx 1 \), the minimum required luminosity is \( 10^3 \) to \( 10^4 \) \( L_\odot \). This is why a significant luminosity increase during the late stages of stellar evolution is accompanied by outflows; it is the small local \( a_{\text{g}} \), rather than the large \( a_{\text{rp}} \), which enables the radiatively driven stellar outflows.

### 3. A MODEL FOR RADIATION DRIVEN OUTFLOW

#### 3.1 The Mathematical Model

In this section we briefly present the basic equations describing the mathematical model. Detailed discussion of the model and the simplifications can be found in [2,7].

The equation of motion for the gas is

\[
v \frac{dv}{dr} = \frac{1}{r^2} \int_0^\infty \chi_\lambda F_\lambda d\lambda - \frac{GM_\star}{r^2}
\]

where \( v \) is the gas velocity. The first term on the right-hand side of eq. (3) describes the radiation pressure force, the second term is the gravitational force exerted by the star. The force due to the gas pressure gradient is omitted since it is negligible for the highly supersonic outflows described here. In cool stars, the radiation pressure force acts only on the dust grains (gas opacity is negligible). Since the dust flows out faster than the gas, the dust opacity decreases in proportion to the ratio of the gas and dust outflow velocities

\[
\chi_\lambda \propto \frac{v}{v + v_{\text{drift}}},
\]

where the dust drift velocity with respect to the gas flow is determined from

\[
v_{\text{drift}} = \sqrt{\frac{Q_F L_* v}{M_\star}}.
\]

Here \( Q_F \) is the flux averaged absorption efficiency, and \( M_\star \) is the mass-loss rate (for a detailed discussion of dust drift see [7]).

At first it appears from eq. (3) that outflows with arbitrary mass-loss rates can be driven by the radiation pressure force. However, the allowed mass-loss rate is limited from both below and above. The lower limit is set by the above mentioned drift effects: as the mass-loss rate, i.e. gas density, decreases, the dust-gas coupling decreases and with it the effective opacity. In the limit of large mass-loss rates, practically the entire stellar radiation is absorbed by dust, and shifted to longer, infrared wavelengths. As opacity is in general a decreasing function of wavelength, the effective opacity decreases with mass-loss rate in this regime, resulting in an upper limit on the mass-loss rate.

#### 3.2 The Solution Method

The coupled system of the dynamics and radiative transfer equations is solved by an iterative scheme. For a given velocity field, the radiative transfer is solved separately by an exact scheme described in [8]. With known radiation flux, an updated velocity field is calculated from eq. (3), and these steps are repeated until both the radiation and velocity fields converge. As an initial velocity field we take an analytic approximation obtained by neglecting drift and reddening effects, and gravitational force [9].

The radiative characteristics of the star are largely irrelevant for the solution. We assume that the star radiates as a black body with \( T_* = 2500 \text{ K} \), typical for late-type red giant stars. Similarly, we assume that the dust evaporation temperature \( T_{\text{vap}} = 1000 \text{ K} \). The dust optical properties are important because they determine the detailed signature of the emitted spectrum. In this work we consider the two major grain types that have been proposed for late-type stars. Dielectric coefficients for “astronomical silicate” are taken from Ossenkopf et al. [10], and for amorphous carbon from Hanner [11]. A more detailed study including 5 different grain types
and various mixtures is presented in [4]. Absorption and scattering efficiencies are evaluated using Mie theory, assuming the dust grains are homogeneous spheres following the size distribution proposed by Mathis et al. [12]. Carbon grains have smooth opacity, decreasing with wavelength, while astronomical silicate has characteristic peaks at 9.7 $\mu$m and 18 $\mu$m.

4. RESULTS

4.1 Scaling Properties of the Solution

For given dust optical properties, stellar temperature, dust evaporation temperature, gas-to-dust ratio, mass-loss rate, and luminosity, our model determines the outflow velocity field. Optical properties and the two temperatures are usually well constrained, so in the remainder of this paper we will consider gas-to-dust ratio $r_{gd}$, $v_e$, $\dot{M}$, and $L_*$ as the relevant quantities describing a model.

The velocity field can be described by terminal outflow velocity $v_e$, which sets the scale, and shape $u = v/v_e$. From the scaling properties of the system, $v_e$ can be expressed as the product of a velocity scale proportional to $L_*^{1/4} r_{gd}^{-1/2}$, and a dimensionless function of the dust optical depth [9]. Because the solution is characterized by a single parameter, it follows from dimensional analysis that the terminal outflow velocity is related to the input quantities through some (quite involved) function $\Phi$ such that

$$v_e = L_*^{1/4} r_{gd}^{-1/2} \Phi\left(\frac{\dot{M} L_*^{3/4}}{r_{gd}^{1/2}}\right)$$

The argument of function $\Phi$ follows from the dependence of optical depth on $\dot{M}$, $L_*$, and $r_{gd}$. That is, for given grains, there is a single family of self-similar solutions, $\Phi$, which relates $v_e L_*^{1/4} r_{gd}^{-1/2}$ to $M L_*^{3/4} r_{gd}^{-1/2}$. Strictly speaking, $\Phi$ also depends on $M_*$, and $\epsilon = v_1/v_e$, where $v_1$ is the velocity at the inner boundary, $r_1$, but detailed model calculations show that this dependence is negligible for all practical purposes. Solutions for two types of grains considered in this work are shown in figure 1 (obtained for $\epsilon = 0.1$ and $M_* = 1 M_\odot$). These solutions can be used to reconstruct the $v_e(\dot{M})$ relationship for arbitrary $L_*$ and $r_{gd}$.

We find from detailed models that the shape $u$ is well described by

$$u = \left(\epsilon^k + (1 - \epsilon^k)(1 - \frac{r_1}{r})\right)^{1/k}$$

The characteristic power index $k$ is a weak function of dust optical depth (fig. 2), ranging from 1.5 in optically thin envelopes to $\sim2.5$ in optically thick ones ($k=2$ corresponds to an idealized case with acceleration proportional to $r^{-2}$). Again, the dependence on $M_*$ and $\epsilon$ is negligible.
4.2 Dynamical Properties of the Solution

The predicted relationship between the terminal outflow velocity $v_e$, and mass-loss rate $\dot{M}$, can be used to compare the model to observations. Observations of molecular line emission (both thermal and maser) directly measure $v_e$, and with the aid of suitable emission models constrain $\dot{M}$. Straightforward comparison of model predictions with observed $v_e$ and $\dot{M}$ is hampered by uncertain values of $L^*$ and $r_{gd}$. Consequently, a direct utilization of the results presented in figure 1 is impractical because effects of changing $L^*$ and $r_{gd}$ are coupled together. Their impact can be disentangled by comparing the observed and predicted values of $\dot{M}v_e^{-3}r_{gd}^{-2}$ and $\dot{M}v_eL_4^{-1}$. Such a diagram is shown in figure 3. Lines are model predictions and symbols are the observed values for two samples of stars: oxygen rich (O) stars which are associated with silicate dust, and carbon rich (C) stars associated with carbon dust. We have assumed that all stars have $L_4 = L_*/10^4L_\odot = 1$, and constrained $r_{gd}$ by requiring best agreement (in a least-squares sense for the whole sample) between the modeled and observed values of $\dot{M}v_e^{-3}r_{gd}^{-2}$. Best values are 250 for O stars, and 400 for C stars. It is evident from figure 3 that these values are practically independent of assumed value for luminosity because different values of $L_*$ shift the data points only horizontally.

The obtained values for $r_{gd}$ are in agreement with other independent estimates (e.g. [6]). Close agreement between the model predictions and observations provides strong evidence that outflows around late-type stars are radiatively driven. Additional evidence is provided by the analysis of dust infrared emission.

4.3 Infrared Emission

An important property of the spherically symmetric radiative transfer problem is that, for given grains, the dimensionless spectral shape $f_\lambda = (\lambda F_\lambda)/F$, where $F = \int_0^\infty F_\lambda d\lambda$ is the bolometric flux, depends only on the radial density profile and overall optical depth [8]. The radial density profile for an outflow is fully determined by the velocity profile through the mass conservation relation. Since the velocity profile depends only on optical depth, it follows that for each grain type there is a family of spectra parametrized by optical depth. That is, irrespective of their individual values, any combination of relevant parameters produces the same spectral shape as long as the corresponding optical depth does not change.

A detailed comparison of observations with model spectra is given in [4], and here we only present results for silicate grains as an illustration. The six panels of
Dashed lines represent the direct stellar radiation emerging from the system, dotted lines are the dust emission and full lines are the total emission — the model predictions of observed spectra. In optically thin envelopes ($\tau_F < 1$) the strength of the 9.7 $\mu$m silicate emission feature increases with optical depth until $\tau_F$ reaches $\sim 1$. With further increase in $\tau_F$, this emission turns into an absorption feature whose depth is proportional to $\tau_F$. As an illustration, the inset in each panel displays the observed spectrum of an actual source whose IRAS number is indicated above the top left corner. Each panel also indicates the [25]-[12] color of the computed model while the number listed in large type in the inset is the IRAS color of the displayed source. The close agreement of spectral evolution between model predictions and observations is evident.

5. DISCUSSION

Since our models assume steady state, they can only describe the time-averaged behavior of the outflows. This is an adequate description as long as time variability occurs only on time scales shorter than the averaging time. Calculations show that most of the radiative acceleration takes place within $r \sim 10^{16}$ cm. With typical outflow velocities of $\sim 10$ km s$^{-1}$, this distance is covered in $\sim 300$ years, the relevant time scale for averaging density. Although all late-type stars display pulsational or irregular variability, those variations are characterized by time-scales of a few years at most, much shorter than the averaging time. Thus the assumption of steady state seems justified. Additional time dependence can be introduced by chemical evolution of the stellar atmosphere, possibly leading to gradients in the grain chemical composition. Though our models do not include such an effect, they adequately describe the envelopes as long as chemical gradients occur only at $r \gtrsim 10^{16}$ cm. The reason is that the grain composition in that region is largely irrelevant since all grains behave similarly at low temperatures and long wavelengths. And since spatial gradients with length scale $\lesssim 10^{16}$ cm imply temporal variations with time scales $\lesssim 100$ years, the durations of phases not covered by our models do not exceed $\sim 100$ years.$^\dagger$ If typical lifetimes in the mass-losing phase are $\sim 10^7$–$10^9$ years, only $\sim 0.1$–$1\%$ of all sources cannot be described by our models. Evolutionary effects characterized by time scales longer than $\sim 100$ years can be incorporated into our models as adiabatic changes of the relevant sources.

An important ingredient of our models is the assumption of spherical symmetry. Small departures from sphericity, such as a slight elongation, should not affect the results significantly. However, our models do not apply in the case of major deviations from spherical symmetry such as disk geometry or strong clumpiness with characteristic size scale less than $\sim 10^{16}$ cm. The success of our models in describing the infrared spectra and mean mass-loss rates indicates that such deviations may not be important for the majority of late-type stars.

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* Colors are convenient parameters used in astronomy to describe spectral shapes. Here we define [25]-[12] as $\log(F_{25}/F_{12})$, where $F_{12}$ and $F_{25}$ are fluxes at 12 $\mu$m and 25 $\mu$m, respectively.

† It has been recently suggested that mass-loss rate for some stars might change on similar time scales [14].