The check-bits-weight-adjustable scheme of rate compatible modulation based on the systematic Polar code

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Abstract — Rate compatible modulation is a novel adaptive transmission technology that does not require channel state information feedback, can achieve rate adaptive adjustment, and smoothly approach channel capacity. However, due to the nature of incremental decoding, the decoding complexity is too high and the transmission performance is reduced under bad channel conditions. Based on the feature of the systematic Polar code that separates the information bits and the check bits, this paper optimizes the design of the hybrid cascade of the systematic Polar code and rate-compatible modulation, and proposes a feasible scheme for adjusting the corresponding weight of check bits. The simulation results show that the proposed scheme can achieve better transmission performance in a wider range of signal-to-noise ratio with relatively lower decoding overhead.

1. Introduction

How to improve systematic reliability and frequency band utilization under the condition of limited channel resources to meet the increasingly diverse communication business needs is a key technical problem that needs to be solved in wireless communication. Adaptive modulation is one of the technologies to effectively solve this problem. It can adjust the transmission rate in time according to the changes of channel conditions. However, traditional adaptive modulation can only achieve stairs-shaped rate adjustment, and requires timely feedback of channel state information. In order to solve these problems, Cui et al. proposed rate compatible modulation (RCM) in literature [1] in 2011. The sending end of the RCM first groups the source bit sequence, and then generates RCM symbols by random weighting through the mapping matrix. The random mapping method allows different symbols to carry part of the same information. The receiving end continuously accumulates the symbols passing through the channel. When a sufficient number of symbols are accumulated, the information can be decoded with great probability. If the decoding is successful, an ACK signal representing the successful decoding will be sent to the sending end, which will stop sending this group of information and start sending the next group of information after receiving it. If the decoding is unsuccessful, the receiving end will continue to accumulate more symbols to participate in the decoding until the decoding is successful. When the channel quality is good, the receiving end only needs to accumulate a few symbols to successfully decode with a high probability. When the channel quality is poor, the receiving end needs to accumulate more symbols to recover the source information. The more the number of symbols required for decoding at the receiving end, the lower the transmission rate and the greater the decoding complexity.

The systematic code is a kind of code containing the input information bit sequence in the output codeword. Arikan first proposed the systematic Polar code and its decoding algorithm in the literature [2]. Since the information bits of the systematic code can be directly observed through the channel during
decoding, a reasonable and accurate estimate can be obtained without a lengthy decoding process, and the correctness of the received source symbol can be quickly determined \[^{[3]}\]. Compared with the traditional non-systematic polarization codes, systematic polarization codes have better bit error rate performance. Therefore, the systematic Polar code gradually becomes the first choice for engineering applications.

In order to overcome the problem of low transmission rate and high decoding complexity of RCM when channel quality is poor, this paper proposes a scheme of cascading systematic Polar codes with RCM, based on the systematic Polar code's characteristic of separating information bits and parity bits. In this scheme, we redesign the mapping matrix and the transmission mode of the coded symbols, so that the error correction capability of the check bits to the information bits is enhanced correspondingly with the channel quality decreases. In this way, the number of symbols required for RCM demodulation is reduced, the transmission rate is increased, and the decoding complexity is reduced.

### 2. Related work

#### 2.1. Implementation method of weight adjustable scheme

From the analysis of the actual transmission process of RCM, it can be seen that in order to improve the transmission performance, it is necessary to reduce the number of symbols required for decoding as much as possible. We cascade the systematic Polar code with the RCM, and use the systematic Polar code to encode the source bit sequence first to separate the information bits from the check bits. Since the number of check bits is much smaller than the number of information bits, it can be considered to reduce the number of transmissions of symbols corresponding to information bits and increase the number of transmissions of symbols corresponding to check bits. In the transmission of each group of code words, the fixed weight set is first used to carry out RCM mapping for the information bit, and with the increase of the transmission symbol, the variable weight set from small to large is gradually used to carry out RCM mapping for the check bits. Since the sender sends out a certain number of symbols generated only by information bit mapping, if it has not received an ACK signal from the receiver, it indicates that the quality of the channel is relatively poor, and it is necessary to use the check bits to assist decoding. A larger weight set is used to map the check bits to generate subsequent symbols to enhance the error correction ability to the demodulated information bits. The specific principle block diagram of cascade RCM check weight adjustable scheme based on systematic Polar code is shown in Figure 1.

![Figure 1](image.png)

**Figure 1.** The principle block diagram of RCM check weight adjustable scheme based on systematic Polar code.

It can be seen from Figure 1 that the source bit sequence at the sending end is encoded by the systematic Polar code to generate a codeword sequence, and the information bits and check bits are separated. The information bits are RCM-modulated by a mapping matrix with a fixed weight set, and the check bits are RCM-modulated by a hybrid mapping matrix with adjustable combinations of multiple weight sets. At the receiving end, the symbol sequences corresponding to the information bits and the
check bits are respectively demodulated by RCM, and the demodulated soft information is combined together and decoded by the systematic Polar decoder.

It can be seen from the above that the check-bits-weight-adjustable scheme is not simply adding a systematic Polar code encoding process directly before the RCM process, but is equivalent to reconstructing the RCM mapping matrix of the cascaded systematic Polar code. Suppose the code rate of the system Polar code is \( R = \frac{K}{N} \), that is, in a code word sequence of the system Polar code with the code length \( N \), the number of information bits is \( K \), and the number of check bits is \( N-K \). The equivalent cascaded mapping matrix \( G_c \) of the RCM check weight adjustable scheme based on the systematic polar code is shown in Figure 2.

\[
G_c = \begin{bmatrix}
G_e & 0 \\
0 & G_v
\end{bmatrix}
\]

Figure 2. Construction of equivalent cascade mapping matrix.

The RCM transmission process is a process of continuously accumulating transmission symbols to achieve decoding conditions. The number of transmission symbols is related to the channel quality. It can be seen from Figure 2 that the equivalent cascaded mapping matrix of the scheme is composed of two block matrices, which are \( G_e \) and \( G_v \) respectively. When the number of generated symbols is less than \( M_1 \), the block matrix corresponding to the check bits is an all-zero matrix, and the check bits is not mapped to these symbols, that is, the first \( M_1 \) symbols only contain information bits. In other words, when the channel quality is good, without channel coding can also get very good transmission performance. As the channel quality deteriorates, the number of symbols required for demodulation increases. When the number of symbols is greater than \( M_1 \), the check bits come into play. \( G_v \) uses a mapping matrix with weight set \( \omega_1 \) to map the check bits into these symbols. When the channel quality continues to deteriorate, the number of symbols required for demodulation continues to increase. When the number of symbols is greater than \( M_2 \), \( G_v \) uses a mapping matrix with weight set \( \omega_2 \), and the weight value of \( \omega_2 \) is greater than that of \( \omega_1 \). And so on, as the number of transmitted symbols increases, the mapping matrix corresponding to the check bits uses a larger weight set.

2.2. Selecting the weight sets and setting the adjustment nodes

Literature [4] pointed out that as the elements of the weight set increase, the number of symbol types generated by mapping also increases, which makes the demodulation complexity increase accordingly. According to the weight set design criteria of the RCM's mapping matrix proposed in the literature [1], we take the original weight set of RCM \{±1,±2,±4\} to perform simple element deletion to obtain several simpler weight sets: \( \omega_1 \), \( \omega_2 \), \( \omega_3 \). Where, \( \omega_1 \) directly uses the original weight set of RCM. On the basis of \( \omega_1 \), replace a pair of ±4 with ±1, ±2, and get \( \omega_2 = \{±1,±2,±1,±2\} \). Then delete one ±1 and one ±2 in \( \omega_2 \), and get \( \omega_3 = \{±1,±2\} \). The advantage of such a design of the weight set is that the change is simple,
and the next weight set can be obtained by simply transforming the previous weight set, which is convenient for unified design and implementation. Therefore, the weight set $\omega_1$ has the highest demodulation complexity, $\omega_2$ is in the middle, and $\omega_3$ has the lowest complexity.

Next, we simulate and compare the number of symbols required for the demodulation of the systematic Polar code cascaded RCM scheme. The code rate of the systematic Polar code is 3/4, the code length is 512, and the information bit length is 384. The size of mapping matrix $G_r$ corresponding to the information bits is 384x384, using the original weight set. The mapping matrix corresponding to the check bits uses weight sets $\omega_1$, $\omega_2$ and $\omega_3$ in turn, and the size of the mapping matrix corresponding to each weight set is 128x128. The simulation channel is set to additive white Gaussian noise channel (AWGN). The receiving end uses the Log-likelihood ratio Belief Propagation (LLR-BP) \cite{5} demodulation algorithm of RCM and the Successful Cancellation List (SCL) decoding algorithm of the systematic Polar code. The simulation result is shown in Figure 3.

![Figure 3](image_url)

**Figure 3.** The number of symbols required for demodulation using different weight sets.

It can be seen from the above figure that as $E_s/N_0$ decreases, the number of symbols required for successful demodulation increases, and the large weight set mapping matrix used by the check bit can effectively reduce the number of symbols required for demodulation. When the number of symbols is less than 160 and $E_s/N_0$ is greater than 14dB, the number of symbols required for demodulation of the three weight sets corresponding to the check bits is almost the same; When the number of symbols is greater than 160 and less than 290, and $E_s/N_0$ is greater than 7dB and less than 14dB, the number of symbols required for demodulation using the weight set $\omega_1$ and $\omega_2$ is almost the same, both less than $\omega_3$; When the number of symbols is greater than 290 and $E_s/N_0$ is less than 7dB, the number of symbols required for demodulation using the weight set $\omega_1$ is less than $\omega_2$ and $\omega_3$. Combined with the above analysis of the demodulation complexity, the demodulation complexity corresponding to the weight set is $\omega_1 > \omega_2 > \omega_3$. Therefore, when the number of symbols is less than 160, the mapping matrix corresponding to the check bits selects weight set $\omega_3$; when the number of symbols is greater than 160 and less than 290, the weight set is selected $\omega_2$; when the number of symbols is greater than 290, the weight set is selected $\omega_1$.

### 3. Simulation results and performance analysis

In order to verify the RCM check-bits-weight-adjustable scheme based on the systematic Polar code proposed in this paper, we simulated the throughput performance and Bit Error Rate (BER) performance of the scheme.

The simulation uses the systematic Polar code with code length of 512 and code rate of 3/4 to be cascaded with RCM. The mapping matrix $G_r$ corresponding to the information bits uses the original
weight set and the size is 384×384. The mapping matrix $G_v$ corresponding to the check bits is composed of a block matrix with three weight sets $\omega_1$, $\omega_2$ and $\omega_3$. The weight set $\omega_3$ is used for rows 1 to 160 of $G_v$, the weight set $\omega_2$ is used for rows 161 to 290, and the weight set $\omega_1$ is used for rows larger than 290. The simulation channel is set to AWGN channel. The receiving end use LLR-BP demodulation algorithm of RCM and SCL decoding algorithm of the systematic Polar code.

3.1.BER performance comparison

To simulate BER performance, we first need to fix the transmission rate. We set the length of the source bit sequence to 384, and select a symbol number M in each of the three symbol intervals determined in the previous section, which is 384, 256 and 128 respectively. At each Es/N0, 107 bits are simulated for each weight set. In this paper, the BER performance of the check-bits-weight-adjustable scheme (This scheme is referred to as SPolar-RCM-CWA in the simulation diagram) under the given three weight sets are compared with that of the traditional cascade scheme (This scheme is referred to as Polar-RCM in the simulation diagram) in literature [6].

Figures 4 to 6 show BER curves under three transmission rates.

![Figure 4](image1.png)

Figure 4. Simulation comparison of BER performance when M=384.

![Figure 5](image2.png)

Figure 5. Simulation comparison of BER performance when M=256.
It can be seen from this figure that in the check-bits-weight-adjustable scheme designed in this paper, the mapping matrix corresponding to the check bits uses the above three weight sets to achieve better BER performance than the traditional cascade scheme. The simulation result further proves that the increase of the matrix weight set corresponding to the check bits will improve the BER performance.

3.2. Throughput performance comparison

This paper compares the throughput performance of the proposed scheme with the traditional cascade scheme. The simulation results are shown in Figure 8. It can be seen from the curve in Figure 8 that the throughput performance of the check-bits-weight-adjustable scheme is better than the traditional cascade scheme in the entire channel range.

4. Conclusions

In this paper, the mapping matrix and transmission mode of systematic Polar code and RCM cascade scheme are optimized by using the property of systematic codes, which separates the information bits and the check bits. We propose an optimization scheme with adjustable weights of the mapping matrix corresponding to the check bits. The scheme adopts the method of first transmitting information bits and then transmitting the check bits with adjustable weight, so that the error correction ability of the check bits to the information bits changes dynamically with the channel quality. Through simulation comparison, it can be seen that the RCM check-bits-weight-adjustable scheme based on the systematic Polar code can achieve better transmission performance in a wider channel range with relatively lower decoding overhead.
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