Energy Method on Dynamic Buckling of Cylindrical Shell with Simply-Fixed Supported subjected to Axial Impact of Rigid Mass

Zhaowei XIN1,a, Zhijun HAN2,b

College of Mechanical and Vehicle Engineering Taiyuan University of Technology, Taiyuan, Shanxi, China
College of Biomedical Engineering, Taiyuan University of Technology, Taiyuan, Shanxi, China

b hanzhijun@tyut.edu.cn
a 714932824@qq.com, b 13073578750@126.com

Abstract—Based on the energy method, considering the stress wave effect, the buckling problem of cylindrical shell subjected to impact is studied. Substituting the Lagrange function and the trial function satisfied the boundary condition into the second Lagrange equation. After the second-order linear partial differential equation is obtained, the analytical expression of the critical buckling speed of a rigid mass impacting cylindrical shell is obtained by analyzing the properties of the equations. The influence of critical length, impact mass, axial mode number, circumferential mode number and diameter-thickness ratio on buckling is discussed by analyzing the examples. The results show that the stress wave effect, initial kinetic energy and diameter-thickness ratio have a significant effect on the dynamic buckling of cylindrical shells; the higher-order modes of buckling and the buckling of cylindrical shell are easily motivated by high-speed impact.

1. INTRODUCTION

Cylindrical shell’s buckling failure under strong dynamic load impact has a wide range of applications in various industrial fields and defense. At present, the research on the static buckling problem is quite sufficient, but the research on the dynamic buckling problem is still full of challenges. Therefore, there is a strong engineering application value, theoretical value and significance to do the research on the dynamic buckling of cylindrical shells.

LEPIK L [1], Karagiozova D. [2] and Gui Y F. [3] analyzed the dynamic buckling of cylindrical shells under axial step loads, whose results show that stress wave has an effect on the dynamic buckling of cylindrical shells. Bisagni C [4] studied the dynamic buckling of cylindrical shells by numerical analysis and experiment. Ignoring the stress wave effect, the results show that the initial flaws and load duration have an effect on dynamic buckling. Gao K. [5] studied the dynamic buckling of cylindrical shells with different diameter-thickness ratios under full load. Ignoring the stress wave effect, considering initial defect, a motion equation was derived. The above research shows that step load loading is generally used when considering the stress wave effect, and initial defects are generally used when ignoring the stress wave effect and loading is performed at full load. The methods for researching the problem
include theoretical analysis, numerical simulation or computer simulation, and experimental research. The conclusions obtained are different due to different angles of consideration and methods used; and there are few and insufficient researches on the dynamic buckling of masses with engineering application backgrounds impacting cylindrical shells. Karagiozova D.\cite{6} and Chen Y T \cite{7} considered the stress wave effect and carried out finite element numerical simulation of a cylindrical shell under axial impact load, thereby obtaining different buckling modes. Ma J.\cite{8} carried out an experimental study on the impact of a mass on a cylindrical shell and simulated it using LS-DYNA. The buckling study of the above-mentioned mass impacting the cylindrical shell shows that the propagation of stress waves is complicated, which brings some difficulties to the theoretical research. The current research mainly focuses on experiments and computer simulation analysis.

Based on these, considering the stress wave effect, the buckling problem of cylindrical shell subjected to impact is studied. Substituting the Lagrange function and the trial function satisfied the boundary condition into the second Lagrange equation. After the second-order linear partial differential equation is obtained, the analytical expression of the critical buckling speed of a rigid mass impacting cylindrical shell is obtained by analyzing the properties of the equations. The influence of critical length, impact mass, axial mode number, circumferential mode number and diameter-thickness ratio on buckling is discussed by analyzing the examples.

2. DEDUCTION OF ANALYTICAL EXPRESSION OF CRITICAL BUCKLING SPEED

As shown in figure. 1, the left end of a homogeneous steel cylindrical shell is hinged and the right edge is fixed. The length is L, Radius is R, Thickness is h, Modulus of elasticity is E, Density is ρ, Poisson ratio μ, the cross-sectional area is A, select cylindrical coordinate system x,y,z, its corresponding displacement is u,v,w, the critical length is lcr, the critical time is tcr, the propagation speed of the stress wave is c, where $l_{cr} = t_{cr} \times c$. The left end was struck by a rigid mass of mass M.

![Figure 1: Cylindrical shell impacting by a rigid mass block](image)

Stress wave propagates in a cylindrical shell \cite{9}:

$$\sigma(x,t) = \begin{cases} \rho v_c \left( \frac{\partial u}{\partial t} + \frac{\partial v}{\partial x} \right), & 0 \leq x \leq c t \\ 0, & c t < x \leq L \end{cases}$$

(1)

When the stress wave reaches lcr, ignoring the axial inertia, based on the geometric relationship of the cylindrical shell under the small deflection theory, sorting out the expressions of system kinetic energy T, system strain energy U, and system work W gives:

$$T = \frac{1}{2} \rho h \int_0^{2\pi} \int_0^L \left[ \frac{2E}{R^2} \left( \frac{\partial u}{\partial x} \right)^2 + \frac{2E}{R^2} \frac{\partial v}{\partial x} \frac{\partial v}{\partial x} + \frac{1}{E} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{2(1-\mu)}{E} \frac{\partial w}{\partial x} \frac{\partial w}{\partial x} \right] R \, dx \, d\theta$$

$$U = \frac{1}{2} \omega h \int_0^{2\pi} \int_0^L \left[ \frac{2E}{R^2} \left( \frac{\partial u}{\partial x} \right)^2 + \frac{2E}{R^2} \frac{\partial v}{\partial x} \frac{\partial v}{\partial x} + \frac{1}{E} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{2(1-\mu)}{E} \frac{\partial w}{\partial x} \frac{\partial w}{\partial x} \right] R \, dx \, d\theta$$

$$W = -\frac{1}{2} \int_0^{2\pi} \int_0^L \left[ \sigma \left( \frac{\partial u}{\partial x} \right) \right] R \, dx \, d\theta$$

(2)

Let the test function be:

$$w = \left[ a \sin \frac{n \pi x}{2L} + b \sin \frac{m \pi x}{2L} \right] T(t)$$

(3)

Where n is the number of axial modes, m is the number of circumferential modes, and a and β are the relevant undetermined coefficients. Equation (3) should satisfy the following boundary conditions.
Substituting equations (3) into equation (4) gives

\[
\begin{align*}
\begin{bmatrix}
\frac{n_1 \pi}{2} + \beta \frac{n_2 \pi}{2} = 0 \\
\frac{n_1 \pi}{2} - \beta \frac{n_2 \pi}{2} = 0 \\
\frac{n_1 \pi}{2} + \beta \frac{n_2 \pi}{2} = 0
\end{bmatrix}
\end{align*}
\]

(5)

If equation (5) has a solution, then \( \sin n_1 \pi \) and \( \sin n_2 \pi \) must both be odd numbers, that \( n_1 = 2n - 1, n_2 = 2n + 1 \). Because \( a = \beta \), if \( a = 1, \beta = 1 \). The effective trial function for obtaining a cylindrical shell is:

\[
w = \frac{\sin(n_{2n-1}) \pi - x + \sin(n_{2n+1}) \pi}{2} \sin m \xi \cdot T(r) \quad (n = 1, 2, 3, \ldots)
\]

(6)

Let the Lagrange function of the system be \( L \), and

\[
L = T - U + W
\]

(7)

Lagrange's equation of the second kind:

\[
\frac{\partial L}{\partial T} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{T}} \right) = 0
\]

(8)

Substituting equation (2), (6), (7) into equation (8) gives:

\[
- \rho h \int_0^x \int_0^{2\pi} \left( \frac{d^2 \ddot{r}(t, \theta)}{dt^2} \right)^2 \frac{d^2 \ddot{r}(t, \theta)}{d\theta^2} \frac{d^2 \ddot{r}(t, \theta)}{dt^2} d\theta d\phi - \beta \int_0^x \int_0^{2\pi} \left( \frac{d^2 \ddot{r}(t, \theta)}{dt^2} \right)^2 \frac{d^2 \ddot{r}(t, \theta)}{d\theta^2} d\theta d\phi + \frac{1}{R^2} \left( \int_0^x \int_0^{2\pi} \frac{d^2 \ddot{r}(t, \theta)}{dt^2} d\theta d\phi \right)^2 - \frac{k}{R^2} \int_0^x \int_0^{2\pi} \left( \frac{d^2 \ddot{r}(t, \theta)}{dt^2} \right)^2 d\theta d\phi T = 0
\]

(9)

Organize the equation:

\[
C_1 \ddot{T} - (C_2 - C_3) T = 0
\]

(10)

Substituting equation (6) into equations (10):

\[
C_1 = - \rho h R \pi l_m c_v, \quad C_2 = C_3 = \frac{\pi k_l c_v R}{\rho h R c_v} + D \gamma_2 + \gamma_3 + \gamma_4, \quad C_3 = \frac{\rho h R \pi l_m c_v}{\rho h R c_v} + D \gamma_2 + \gamma_3 + \gamma_4.
\]

(11)

where:

\[
\begin{align*}
\gamma_2 &= \frac{\rho h R \pi l_m c_v}{\rho h R c_v}, \\
\gamma_3 &= \frac{\pi k_l c_v R}{\rho h R c_v}, \\
\gamma_4 &= \frac{\rho h R \pi l_m c_v}{\rho h R c_v} + D \gamma_2 + \gamma_3 + \gamma_4, \\
\gamma_1 &= \frac{\pi c_v R}{\rho h R c_v} \frac{(2n_{2n-1}) + (2n_{2n+1})}{(2n_{2n-1}) + (2n_{2n+1})}, \\
\gamma_2 &= \frac{\pi c_v R}{\rho h R c_v} \frac{(2n_{2n-1}) + (2n_{2n+1})}{(2n_{2n-1}) + (2n_{2n+1})}, \\
\gamma_3 &= \frac{\pi c_v R}{\rho h R c_v} \frac{\pi c_v R}{\rho h R c_v}, \\
\gamma_4 &= \frac{\pi c_v R}{\rho h R c_v} \frac{(2n_{2n-1}) + (2n_{2n+1})}{(2n_{2n-1}) + (2n_{2n+1})},
\end{align*}
\]

(12)

Analyzing the nature of the solution of formula (10):

When \( C_2 - C_3 < 0 \), the solution obtained diffuses exponentially over time, indicating that the cylindrical shell has buckled;
When $C_2-C_3 > 0$, the solution obtained is a trigonometric function over time, indicating that the cylindrical shell does not buckle;

When $C_2-C_3 = 0$, it is in the critical state of buckling. At this time, the critical condition of buckling can be obtained, which is:

$$v_{cr} = \frac{Rn\omega^2 + D(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4)}{4\rho c^2 \pi^2 (1 - \nu^2)\delta_1 - \rho c^2 \pi^2 \gamma_2 \delta_2}$$

Equation (13) is the analytical expression of critical speed of cylindrical shell buckling.

3. ANALYSIS OF EXAMPLE

In the analysis of the example, take the cylindrical shell length $L=1m$, radius $R=2m$, thickness $h=0.02m$, diameter to thickness ratio $k=2R/h$, $E=2\times10^{11}Pa$, $\mu=0.3$, $\rho=7800kg/m^3$. Figure. 2 shows the relationship between $v_{cr}$ and $l_{cr}$ in the case of $m = 1$, and $n = 1, 2, 3$. Figure.3 shows the relationship between $v_{cr}$ and $l_{cr}$ in the case of $n = 1$, and $m = 1, 2, 3, (k = 200, l_0=10)$

These graphs show that the $v_{cr}$ decreases with the increase of $l_{cr}$, which also indicates that the stress wave effect has a significant effect on dynamic buckling. When $l_{cr}$ is the same, the $v_{cr}$ increases with the increase of $n$. It shows that the higher-order modes of cylindrical shell buckling are easily excited by high-speed impact. When $m$ are different, the curves almost coincide in figure. 3, indicating that the buckling mode transition is not obvious at high speed. Comparing figure.2 and figure.3 shows that the effect of $n$ on buckling is more significant than that of $m$. 
Figure 4 shows the relationship between \( v_{cr} \) and \( l_0 \) when \( m = 1, n = 1, 2, 3 \). Figure 5 shows the relationship between \( v_{cr} \) and \( l_0 \) when \( n = 1, m = 1, 2, 3 \) (\( k = 200, \ lcr = 1m \)).

When the \( l_0 \) is the same, \( v_{cr} \) and the initial impact kinetic energy both decrease as \( n \) increases, showing that the initial kinetic energy has a significant effect on buckling. When \( m \) are different, the curves almost coincide in Figure 5. It shows that the buckling modal transition of a cylindrical shell under high-speed impact is not obvious, which is different from the influence of \( n \) on buckling.

Figure 6 shows the relationship between \( v_{cr} \) and \( l_{cr} \) when \( l_0 = 10 \). Figure 7 shows the relationship between \( v_{cr} \) and \( l_0 \) when \( l_{cr} = 1m \) (\( n = 1, m = 1, k = 100, 150, 200 \)).
The graph shows that the vcr decreases with the increase of lcr, which also indicates that the stress wave effect has a significant effect on dynamic buckling. When lcr or the l0 is the same, vcr increases with the decrease of k, which indicates that the smaller k is, the more easily the buckling of the cylindrical shell is excited by high-speed impact.

4. CONCLUSIONS
The paper draws the following conclusions through theoretical research and example analysis:

(1) Based on the energy method and considering the stress wave effect, the analytical expression of the critical buckling speed of a rigid mass impacting cylindrical shell is obtained by analyzing the properties of the equations.

(2) In the analysis of examples, the effects of lcr, l0, n, m and k on the dynamic buckling are discussed. The results show that:

It can be clearly seen that vcr decreases with the increase of lcr, which indicates that the stress wave effect has a significant effect on the dynamic buckling of cylindrical shells. When the l0 is the same, the initial impact kinetic energy decreases as the number of modes increases, which indicates that the initial kinetic energy has a significant effect on buckling; however, when m are different, the curves almost coincide. It shows that the transition of buckling mode under high-speed impact is not obvious, which is different from the influence of n on buckling.

When k is different, the vcr increases as k decreases. It shows that the smaller k is, the easier the buckling of the cylindrical shell can be excited by high-speed impact.

ACKNOWLEDGMENT
Sponsors: National Natural Science Foundation (11372209)

REFERENCES
[1] Ü LO LEPIK, “Bifurcation analysis of elastic–plastic cylindrical shells,” Nonlin. Mech, vol. 34(2), pp. 299–311, 1999.
[2] Karagiozova D, Alves Marcilio, “Transition from progressive buckling to global bending of circular shells under axial impact-Part I: Experimental and numerical observations,” J .Solid & Structure, vol. 41, pp. 1565-1580, 2004.
[3] GUI Y F,MA J M, “Buckling of step cylindrical shells under axial impact load,” Vib. Shock , vol. 38(01), pp. 208-213, 2019.
[4] Bisagni C, “Dynamic buckling of fiber composite shells under impulsive axial compression,” Thin-Walled Structures, vol. 43(3) pp. 499-514, 2005.
[5] Gao K, Gao W, Wu D, “Nonlinear dynamic buckling of the imperfect orthotropic E-FGM circular cylindrical shells subjected to the longitudinal constant velocity,” Mech. Sciences, vol. 138-139(4)), pp. 199-209, 2018.
[6] Karagiozova D, Jones N, “On dynamic buckling phenomena in axially loaded elastic–plastic cylindrical shells,” Nonlin. Mech. vol. 37(7) pp.1223-1238, 2002.

[7] Chen Y T, Zheng G T, “New Method for Determining the Second Critical speed of Thin-wall Cylindrical Shells Under Axial Impact,” Vib. Shock., vol.01, pp. 52-54., 2007.

[8] MA J, YAN Y, YANG L, “Experiments and Finite Element Analysis of Laminated Composite Cylindrical Shells with Circular Hole Subjected to Dynamic Loads,” Acta Aeronautica ET Astronautica Sinica. vol. 33(5) pp. 871-878, 2012.

[9] Zhang G Y. “Numerical Method of Dynamic Buckling of Rod Impacted by A Rigid Body,” Taiyuan: Taiyuan University of Technology, 2019