Spontaneous symmetry breaking and masses numerical results in Doplicher-Fredenhagen-Roberts noncommutative space-time

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Abstract – With the elements of the Doplicher-Fredenhagen-Roberts (DFR) noncommutative formalism, we have constructed a standard electroweak model. We have introduced the spontaneous symmetry breaking and the hypercharge in DFR framework. The electroweak symmetry breaking was analyzed and the masses of the new bosons were computed.

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Introduction. – The fact that we have infinities that destroy the concluding results of several calculations in QFT have motivated theoretical physicists to ask if a continuum space-time would be really necessary. One of the possible solutions would be to create a discrete space-time with a noncommutative (NC) algebra, where the position coordinates would be promoted to operators $\hat{X}^\mu (\mu = 0, 1, 2, 3)$ and they must satisfy commutation relations

$$[\hat{X}^\mu, \hat{X}^\nu] = i \ell \delta^{\mu\nu},$$

where $\ell$ is a length parameter, $\theta^{\mu\nu}$ is an antisymmetric constant matrix. In this way, we would have a kind of fuzzy space-time where, from this commutator, we have an uncertainty in the position coordinate. In order to put these ideas together, Snyder [1] has published the first work that considers the space-time as being NC. However, the frustrated result has doomed Snyder’s NC theory to years of ostracism [2]. After the relevant result that the algebra obtained from string theory embedded in a magnetic background is NC, a new flame concerning noncommutativity (NCY) was rekindled [3]. One of the paths (the most famous at least) for introducing NCY is through the Moyal-Weyl product where the NC parameter, i.e. $\theta^{\mu\nu}$, is an antisymmetric constant matrix, namely,

$$f(x) \ast g(x) = e^{i\theta^{\mu\nu} \partial_\mu \partial'_\nu} f(x) g(x') \bigg|_{x' = x},$$

However, at superior orders of calculations, the Moyal-Weyl product turns out to be highly nonlocal. This fact forced us to work with low orders in $\theta^{\mu\nu}$. Although it keeps the translational invariance, the Lorentz symmetry is not preserved [4]. For instance, concerning the case of the hydrogen atom, it breaks the rotational symmetry of the model, which removes the degeneracy of the energy levels [5]. Other subjects, where the objective is to introduce NC effects in gravity [6–15], in anyon models [16–18], and symmetries [19–21], NCY through path integrals and coherent states was devised in [22]. For more generalized NC issues and reviews, the interested reader can look at [19] and the references therein.

One way to work with the NCY was introduced by Doplicher, Fredenhagen and Roberts (DFR). They have considered the parameter $\theta^{\mu\nu}$ as an ordinary coordinate of the system [23,24]. This extended and new NC space-time has ten dimensions: four relative to the Minkowski space-time and six relative to $\theta$-space. Recently, in [25–28] it was demonstrated that the DFR formalism has in fact a canonical momentum associated with $\theta^{\mu\nu}$ [27,29–33] (for a review see [34]). The DFR framework is characterized by a field theory constructed in a space-time with extra-dimensions (4 + 6), and which does not necessarily require the presence of a length scale $\ell$ localized into the six dimensions of the $\theta$-space where, from (1), we can see that $\theta^{\mu\nu}$ has the dimension of length-square, when we consider $\ell = 1$. By taking the limit with no such scale, the usual algebra of the commutative space-time
is recovered. Besides the fact that the Lorentz invariance was recovered, we obviously hope that causality aspects in QFT in this \((x + \theta)\) space-time must be preserved, too [35–37].

**The NC Yang-Mills symmetry revisited.** – Considering a NC Yang-Mills model in the DFR framework, we have analyzed the gauge invariance of a fermion action under the star gauge symmetry transformations. The fermion Lagrangian coupled to non-NC gauge fields is given by

\[
\mathcal{L}_{\text{Spinor}} = \bar{\psi} \left( i\gamma^\mu D_\mu \psi + \frac{i}{2} \Gamma^{\mu\nu} D_{\mu\nu} \psi - m \right),
\]

where \(\Gamma^{\mu\nu} := i[\gamma^\mu, \gamma^\nu]/4\) is the rotation generator of the fermions, directly attached to the \(\theta\)-space.

Here we have defined the NC covariant derivative as being \(D_\mu \psi = \partial_\mu \psi + igA_\mu \psi\), and \(D_{\mu\nu} \psi\) is a new antisymmetric star-covariant derivative associated with the \(\theta\)-space,

\[
D_{\mu\nu} \psi := \lambda \partial_{\mu\nu} + ig' B_{\mu\nu} \psi,
\]

where the field \(B_{\mu\nu}\) is an antisymmetric tensor, \((B_{\mu\nu} = -B_{\nu\mu})\), for six independent components. The Lagrangian (3) is manifestly invariant under the star-gauge transformations,

\[
\psi \mapsto \psi' = U \psi, \quad A_\mu \mapsto A'_\mu = U^\dagger A_\mu U, \quad B_{\mu\nu} \mapsto B'_{\mu\nu} = U^\dagger B_{\mu\nu} U,
\]

since we have imposed that the element \(U\) is a unitary-star, that is, \(U^\dagger U = 1\). The Moyal product of two unitary matrix fields is always unitary, but in general \(\det(U \star U^\dagger) \neq \det(U) \star \det(U^\dagger)\), that is, \(\det U \neq 1\).

Therefore, the group that represents the previous star gauge is unitary but not special, say \(U^*(N)\). The structure of \(U^*(N)\) is the composition \(U^*(N) = U^1_1(N) \times SU^*(N)\) of a NC Abelian group with another NC special unitary gauge. In the gauge symmetry (5), we have obtained two NC gauge sectors: the first one with a vector gauge field, and the second one with a tensor gauge field. Hence, this gauge symmetry is composed of two unitary groups, say \(U^*(N)_{A^\mu} \times U^*(N)_{B^{\mu\nu}}\), and \(U\) is the element of both groups. The gauge fields \((A_\mu, B_{\mu\nu})\) are Hermitian and they can be expanded in terms of the Lie algebra generators in the adjoint representation as \(A_\mu = A^{a}_\mu \xi^a\) and \(B_{\mu\nu} = B^{a}_{\mu\nu} \xi^a\), where, by satisfying the Lie algebra commutation relation, we have that \([\xi^a, \xi^b] = i\epsilon^{abc} \xi^c\) \((a, b, c = 1, \ldots, N^2 - 1)\).

The dynamics of the star gauge sector is introduced by the star commutators

\[
F_{\mu\nu} = -\frac{i}{g} \left[ D_\mu B_\nu - D_\nu B_\mu - ig [A_\mu, A_\nu] \right],
\]

\[
G_{\mu\nu\rho\sigma} = -\frac{i}{g'} \left[ D_\mu B_{\nu\rho\sigma} - D_\nu B_{\mu\rho\sigma} - ig' [B_{\mu\nu}, B_{\rho\sigma}] \right].
\]

By construction, they have the gauge transformations

\[
F_{\mu\nu} \mapsto F'_{\mu\nu} = U \star F_{\mu\nu} \star U^\dagger,
\]

\[
G_{\mu\nu\rho\sigma} \mapsto G'_{\mu\nu\rho\sigma} = U \star G_{\mu\nu\rho\sigma} \star U^\dagger.
\]

We have established a consistent NC gauge symmetry in the DFR framework. The sector of fermions and non-Abelian gauge fields have an extended symmetry \(U(N)_{A} \times U(N)_{B}\). In the next section we apply this composite symmetry for the case of a standard electroweak model to calculate the masses of NC gauge bosons.

**The model** \(U^1_1(2) \times U^1_1(1) \times U^1_1(2) \times U^1_1(2)\) – Based on the symmetry \(U^*(N)_{A^\mu} \times U^*(N)_{B^{\mu\nu}}\), we will construct an electroweak model for the NC DFR framework. The candidate for this goal is the composite group \(U^1_1(2) \times U^1_1(1) \times U^1_1(2) \times U^1_1(2)\), in which we have left- and right-handed sectors concerning gauge vector fields, and the analogous one for the gauge tensor fields. This composite model is the analogous version of the Glashow-Salam-Weinberg model for the electroweak interaction in the context of DFR NCY.

In this way, firstly, we will define the left-handed fermions doublets, neutrinos and leptons, that transform into the fundamental representation of \(U^1_1(2)\), and in the antifundamental representation of \(U^1_1(1)\) as

\[
\Psi_L = \left( \begin{array}{c} \nu_L \\ \ell_L \end{array} \right) \mapsto \Psi'_L = U \star \Psi_L \star V_2^{-1},
\]

where \(U\) is the element of any group \(U^1_1(2)\), and \(V_2\) is the element of \(U^1_1(2)\). For the right sector \(U^1_1(1)\), the fermions transformation in the antifundamental representation is

\[
\ell_R \mapsto \ell'_R = \ell_R \star V_2^{-1}.
\]

The covariant derivatives acting on fermions in the left and right sectors of the model are defined by

\[
D_{L\mu} \Psi_L = \partial_\mu \Psi_L + i g_1 A_\mu \Psi_L - i J_{Lg'_1} \Psi_L \star B_\mu,
\]

\[
D_{L\mu} \Psi_L = \lambda \partial_\mu \Psi_L + i g_2 B_{\mu\nu} \star \Psi_L - i J_{Lg'_2} \Psi_L \star X_{\mu\nu},
\]

\[
D_{R\mu} \ell_R = \partial_\mu \ell_R - i J_{Rg'_2} \ell_R \star B_\mu,
\]

\[
D_{R\mu} \ell_R = \lambda \partial_\mu \ell_R - i J_{Rg'_2} \ell_R \star X_{\mu\nu}.
\]

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where $A_\mu = A^0_\mu \mathbb{1}_2 + A^a_\mu \sigma^a_2$ and $B_{\mu \nu} = B^0_{\mu \nu} \mathbb{1}_2 + B^a_{\mu \nu} \sigma^a_2$ are the non-Abelian gauge fields of $U^*_R(2)_A$ and $U^*_R(2)_B$, and $B_\mu$ and $X_{\mu \nu}$ in (11) are the Abelian gauge fields of $U^*_R(1)_B$ and $U^*_R(1)_X$. We have used the symbol $J$ as the generator of $U_R(1)$. Imposing the gauge transformations analogously to (5), we can construct the leptons Lagrangian invariant under the previous gauge transformations,

$$\mathcal{L}_{\text{Leptons}} = \bar{\Psi}_L \star \iota \gamma^\mu D_{\mu \nu} \star \Psi_L + \bar{\ell}_R \star \iota \gamma^\mu D_{\mu \nu} \star \ell_R. \quad (12)$$

The introduction of both left- and right-handed components wipes out the terms of propagation in the $\theta$-space together with all the interactions between the fermions and the sector of the gauge tensor fields. This puzzle can be bypassed when we introduce Yukawa interactions in the Higgs sector to break the gauge symmetry of $U^*_R(2)_B$. In the gauge fields sector, the field strength tensors of the bosons are defined by

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_1 [A_\mu, A_\nu],$$

$$H_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + iJ g^1 [B_\mu, B_\nu],$$

$$G_{\mu \nu \rho \lambda} = \lambda \partial_\mu B_{\nu \rho} - \lambda \partial_\nu B_{\rho \lambda} + ig_2 [B_{\mu \nu}, B_{\rho \lambda}],$$

$$X_{\mu \nu \rho \lambda} = \lambda \partial_\mu X_{\nu \rho} - \lambda \partial_\nu X_{\rho \lambda} + iJ g_2 [X_{\mu \nu}, X_{\rho \lambda}]. \quad (13)$$

The invariant Lagrangian of the invariant gauge fields is given by

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{2} \text{tr} (F_{\mu \nu} \star F^{\mu \nu}) - \frac{1}{4} H_{\mu \nu} \star H^{\mu \nu} - \frac{1}{2} \text{tr} (G_{\mu \nu \rho \lambda} \star G^{\mu \nu \rho \lambda}) - \frac{1}{4} X_{\mu \nu \rho \lambda} \star X^{\mu \nu \rho \lambda} - \text{tr} (F_{\mu \nu} \star G^{\mu \nu}) - \frac{1}{2} H_{\mu \nu} \star X^{\mu \nu}. \quad (14)$$

The interaction terms that appear in (12) reveal that the leptons and neutrinos can interact with the gauge fields components. Initially, we will write these interactions as

$$\mathcal{L}^\text{int}_{\text{Leptons-Gauge}} = -\frac{g_1}{2} \bar{\nu}_{\mu \nu} \star \gamma^\mu \left( A_\mu^1 - i A_\mu^2 \right) \star \ell_L$$

$$-\frac{g_1}{2} \bar{\ell}_L \star \gamma^\mu \left( A_\mu^1 + i A_\mu^2 \right) \star \nu_{\mu \nu}$$

$$-\frac{1}{2} \bar{\Psi}_L \star \gamma^\mu (g_1 A_\mu^1 I^3 + g_1 A_\mu^0 - J_L g^1 I B_\mu) \star \Psi_L$$

$$+ \bar{\ell}_R \star \gamma^\mu (J_R g^1 I B_\mu) \star \ell_R$$

$$+ \bar{\Psi}_L \star \gamma^\mu \left( \bar{\Psi}_L, J_L g^1 I B_\mu \right) + \bar{\ell}_R \star \gamma^\mu \left( \ell_R, J_R g^1 I B_\mu \right), \quad (15)$$

where we have defined $I^3 = \sigma^3/2$, for simplicity. Looking at these terms, we can ask which one is the hypercharge generator of the model. In the next section, we will use a Higgs mechanism to define the hypercharge in DFR space-time.

The first SSB and the hypercharge. – In order to identify the hypercharge, we will introduce the first Higgs sector coupled to the NC Abelian gauge vector field and consequently, we will eliminate the residual symmetry $U^*_R(1)$, i.e., the Abelian subgroup of $U^*_R(2)_A$. This Higgs sector is also coupled to the gauge tensor field of the non-Abelian sector $U^*_R(2)_B$ to provide new antisymmetric bosons. We will denote this Higgs field as the Higgs-one field. After this first spontaneous symmetry breaking (SSB), we will obtain

$$U^*_R(2)_X \times U^*_R(2)_B \times U^*_R(1)_X \times U^*_R(1)_B \times U^*_R(1)_Y.$$ \quad (16)

To do that, we will introduce the Higgs-one Lagrangian

$$\mathcal{L}_{\text{Higgs}}^{(1)} = (D_\mu \Phi_1) \star (D^\mu \Phi_1) + \frac{1}{2} (D_\mu \Phi_1) \star (D^\mu \Phi_1) - \frac{1}{2} \mu_1^2 \left( \Phi_1 \star \Phi_1 \right) - \frac{1}{2} \mu_2^2 \left( \Phi_1 \star \Phi_1 \right), \quad (17)$$

where $\mu_1$ and $\mu_2$ are real parameters. The covariant derivatives of (17) act on the Higgs-one field as

$$D_\mu \Phi_1 = \partial_\mu \Phi_1 + ig_1 A_\mu^1 \star \Phi_1 - iJ g^1 \Phi_1 \star B_\mu,$$

$$D_\mu \Phi_1 = \lambda \partial_\mu \Phi_1 + ig_2 B^0_\mu \star \Phi_1 + ig_2 B^a_\mu \sigma^a_2 \star \Phi_1. \quad (18)$$

The field $\Phi_1$ is a scalar doublet of both groups $U^*_R(2)_X$. In the antisymmetric sector, the Higgs-one transforms into the fundamental representation of $U^*_R(2)_B$ as

$$\Phi_1 \mapsto \Phi^{(1)}_1 = V_1 \star \Phi_1 \star V_2^{-1}, \quad (20)$$

where $V_1$ is the element of the subgroup $U^*_R(1)$

When the Higgs potential acquires a non-trivial vacuum expected value (VEV), say $v_1 \neq 0$, we can choose the usual parametrization in the unitary gauge, so that the massive terms in (17) are given by

$$\mathcal{L}_{\text{mass}}^{(1)} = \frac{1}{2} m_{B^+}^2 B^+_{\mu \nu} B^{\mu \nu} - \frac{v_1^2}{2} (g_1 A^0_\mu - J g^1_\mu B_\mu)^2 + \frac{v_2^2}{4} \left( \frac{1}{2} g_2 B^3_{\mu \nu} + g_2 B^0_{\mu \nu} \right)^2. \quad (21)$$

Note the emergence of a new charged field $\sqrt{2} B^+_{\mu \nu} = B^1_{\mu \nu} + iB^2_{\mu \nu}$, where the mass is given by

$$m_{B^+} = \frac{1}{2} g_2 v_1. \quad (22)$$

To define the hypercharge, the other mass terms suggest us to introduce the orthogonal transformations

$$A^0_\mu = \cos \alpha G_\mu + \sin \alpha Y_\mu,$$

$$B_\mu = -\sin \alpha G_\mu + \cos \alpha Y_\mu,$$

$$B^0_{\mu \nu} = \cos \beta G_{\mu \nu} + \sin \beta Y_{\mu \nu},$$

$$X_{\mu \nu} = -\sin \beta G_{\mu \nu} + \cos \beta Y_{\mu \nu}. \quad (23)$$
where $\alpha$ and $\beta$ are the mixing angles, and $\tan \alpha = J g'_1 / g_1$.
Here, the fields $Y$ set the gauge fields associated with the hypercharge generator, where we can define $g' Y_\mu = g_2 \sin \beta$, and the hypercharge of the Higgs is $Y_\mu = 1/2$.
Therefore, the Lagrangian (21) is rewritten as
\[
\mathcal{L}_{\text{Mass}}^{(1)} = \frac{1}{2} m_{B^\pm}^2 B_{\mu\nu}^{\pm} B^{\mu\nu-} + \frac{1}{2} m_{G_\mu}^2 G_\mu G^\mu + \frac{v^2}{4} \left[ g_2 \cos \beta G_{\mu\nu} + \frac{1}{2} \left( g' Y_{\mu\nu} - g_2 B_{\mu\nu}^1 \right) \right]^2.
\] (24)
where we can obtain the mass of $G_\mu$ given by the expression
\[
m_{G_\mu} = v_1 \sqrt{g_1^2 + (J g'_1)^2} = \frac{g_1 v_1}{\cos \alpha}.
\] (25)
The last term in (24) suggests us to make the second orthogonal transformation
\[
B_{\mu\nu}^1 = \cos \theta_2 Z_{\mu\nu} + \sin \theta_2 A_{\mu\nu},
Y_{\mu\nu} = -\sin \theta_2 Z_{\mu\nu} + \cos \theta_2 A_{\mu\nu},
\] (26)
where $\theta_2$ is another mixing angle, satisfying the condition $\tan \theta_2 = 2 \sin \beta$, so we can write that
\[
\mathcal{L}_{\text{Mass}}^{(1)} = \frac{1}{2} m_{B^\pm}^2 B_{\mu\nu}^{\pm} B^{\mu\nu-} + \frac{1}{2} m_{G_\mu}^2 G_\mu G^\mu + \frac{g_2^2 v^4}{4} \left( \cos \beta G_{\mu\nu} - \frac{1}{2} \sec \theta_2 Z_{\mu\nu} \right)^2.
\] (27)
We will diagonalize the last term in order to obtain the mass of $Z_{\mu\nu}$ which is
\[
m_{Z_{\mu\nu}} = \sqrt{\frac{5}{2} g_2 v_1},
\] (28)
while $G_{\mu\nu}$ remains massless in this SSB. Comparing the masses of $B^\pm$ and $Z_{\mu\nu}$, we will obtain the relation $m_{Z_{\mu\nu}} = \sqrt{\frac{5}{2} m_{B^\pm}}$.

The interactions between leptons-neutrinos and gauge vector bosons in (15) can be written in terms of the fields $G^\mu$ and $Y^\mu$ to identify the hypercharge generators of the left-right sectors as $J_R g'_1 \cos \alpha = -g Y_R, g_1 \sin \alpha - J_L g'_1 \cos \alpha = +g Y_L$. These definitions give us
\[
\mathcal{L}_{\text{Leptons-GY}}^\text{int} = -\Psi_L \star \gamma^\mu \left( g_1 A_1^\mu I^3 + g Y_L Y_\mu \right) + \Psi_L \star \ell_R \star \gamma^\mu \left( -g Y_R Y_\mu + \ell_R \right) + \Psi_L \star \gamma^\mu \left[ \Psi_L, (g_1 \sin \alpha - g Y_L) Y_\mu \right] + \ell_R \star \gamma^\mu \left[ \ell_R, (g Y_R Y_\mu) Y_\mu \right] - \Psi_L \star \gamma^\mu \left( g_1 \sec \alpha - g Y_L \tan \alpha \right) G_\mu \star \Psi_L + \ell_R \star \gamma^\mu \left[ \ell_R, g Y_R \tan \alpha \right] G_\mu + \Psi_L \star \gamma^\mu \left[ \Psi_L, (g_1 \sin \alpha + g Y_L \tan \alpha) G_\mu \right] + \ell_R \star \gamma^\mu \left[ \ell_R, g Y_R \tan \alpha \right] G_\mu.
\] (29)
Now we are ready to analyze how the mixing $A_1^\mu - Y_\mu$ defines the physical particles $Z^0$ and the massless photon, and posteriorly, the electric charge of the particles. To this end, we need to break the resting symmetry of this SSB. We will present this second mechanism in the next section.

**The electroweak symmetry breaking.** — Until now we have constructed a model for the NC electroweak interaction using a Higgs sector to eliminate the residual symmetry $U^*(1)$, and we have defined the hypercharge of the Abelian sector. Now we are going to introduce a second Higgs sector $\Phi_2$ in order to break the electroweak symmetry. We write the Lagrangian of the second Higgs-$\Phi_2$ as the scalar sector, that is,
\[
\mathcal{L}_{\text{Higgs}}^{(2)} = (D_\mu \Phi_2)^+ \star D^\mu \Phi_2 + \frac{1}{2} \left( D_\mu \Phi_2 \right)^+ \star D^\mu \Phi_2 - \mu_2^2 \left( \Phi_2^\dagger \Phi_2 \right) - g_{H2} \left( \Phi_2^\dagger \Phi_2 \right)^2,
\] (30)
where $\mu_2$ and $g_{H2}$ are real parameters. The field $\Phi_2$ is a complex scalar doublet that has the gauge transformation analogous to that from $\Phi_1$. The covariant derivatives act on $\Phi_2$ as
\[
D_\mu \Phi_2 = \partial_\mu \Phi_2 + i g_1 A_1^\mu \Phi_2 + i g_1 A_\mu^\mu \sigma^\mu \Phi_2,
D_\mu \Phi_2 = \lambda \partial_\mu \Phi_2 + i g_2 B_{\mu\nu}^1 \Phi_2 - i J g_2 \Phi_2 \times Y_{\mu\nu}.
\] (31)
Using the transformations (23), the term in $Y_\mu$ in the covariant derivative suggests that $g Y_{\mu \nu} := g_1 \sin \alpha$. We use a similar parametrization to first SSB to obtain the result
\[
\mathcal{L}_{\text{Mass}}^{(2)} = \frac{m_{W^\pm}}{2} W^\mu W^\nu - \frac{v^2}{4} \left( g_2 B_{\mu\nu}^0 - J g_2 Y_{\mu\nu} \right)^2 + \frac{v^2}{2} \left( g_1 \cos \alpha G_\mu + \frac{1}{2} \left( g Y_\mu - g_1 A_\mu^\mu \right)^2 \right),
\] (32)
where $v$ is the VEV that defines the scale for this SSB. As in the usual case, the mass of $W^\pm = g_1 v/2$. The mass terms of the neutral bosons in (32) motivate us to introduce the orthogonal transformations
\[
A_1^\mu = \cos \theta_W Z_\mu + \sin \theta_W A_\mu,
Y_\mu = -\sin \theta_W Z_\mu + \cos \theta_W A_\mu.
\] (33)
We can find all the mass terms in this Lagrangian
\[
\mathcal{L}_{\text{Mass}} = \frac{m_{W^\pm}}{2} W^\mu W^\nu + \frac{1}{2} m_{B^\pm}^2 B_{\mu\nu}^{\pm} B^{\mu\nu-} + \frac{g_1^2 v^4}{2} \left( \frac{1}{2} \sec \theta_W Z_\mu - \cos \alpha G_\mu \right)^2 + \frac{1}{2} \cos^2 \alpha G_\mu G_\mu + \frac{1}{4} \frac{g_2^2 v^2}{\beta} G_\mu G^\mu G^{\mu\nu} + \frac{1}{4} m_{Z^\nu}^2 Z_{\mu\nu} Z^{\mu\nu}.
\] (34)
Here we have taken into account the mass terms from the first SSB of the Higgs-$\Phi_1$. Note that the fields $A_1^\mu$ and $A_\mu^\mu$ are not present in the Lagrangian (32). They are the massless gauge fields remaining in the model after SSBs, namely, we have the final symmetry
\[
U_L^* (2) A_\mu^\mu \times U_R^* (1) B^\mu \times U_R^* (2) B^\nu \times U_L^* (1) X_{\mu\nu}
\] (41)
\[
SU_L^* (2) \times U_R^* (1) \times U^* \times U_R (1)
\] (42)
\[
U_{em} (1) \times U^* (1) A_{\mu\nu},
\] (35)
where $A_\mu$ is the photon field and $A_{\mu\nu}$ is its antisymmetric correspondent in the $\theta$-space. It is important to explain that the $Z$-field, which came from (33) is not the $Z^0$-particle of the standard electroweak model. The $Z^0$-particle will be defined by means of the mixing with the $G$-field in (34). Since we had established the scale $v_1 \gg v$, we have diagonalized the mixing term $Z - G$, so the masses of $Z$, $G$ and their antisymmetric pairs up to the second order in $v/v_1$ are given by

$$m_{Z^0} = \frac{g_1 v}{2 \cos \theta_W} \left( 1 - \frac{v^2}{2v_1^2} \cos^4 \alpha + \ldots \right),$$

$$m_G = \frac{g_1 v_1}{\cos \alpha} \left( 1 + \frac{v^2}{2v_1^2} \cos^4 \alpha + \ldots \right).$$

(36)

Substituting (33) into (29), we can identify the fundamental charge by the parametrization

$$e = g_1 \sin \theta_W = g \cos \theta_W,$$

where the electric charge is given by

$$Q_{em} = t^3 + Y.$$  

(38)

We will use the VEV of this SSB as the electroweak scale, that is, $v \simeq 246$ GeV, and considering the experimental value of $\sin^2 \theta_W \simeq 0.23$, the masses of $W^\pm$ and $Z^0$ are estimated at the tree level to give the values

$$m_{W^\pm} = \frac{37}{\sin \theta_W} \simeq 77 \text{ GeV},$$

$$m_{Z^0} = \frac{74}{\sin 2\theta_W} \left( 1 - \frac{v^2}{2v_1^2} \cos^4 \alpha + \ldots \right) \simeq 89 \text{ GeV} \left( 1 - \frac{v^2}{2v_1^2} \cos^4 \alpha + \ldots \right).$$

(39)

To estimate the values for the masses of $Z_{\mu\nu}$, $B^\pm$ and $G_{\mu\nu}$, we have to examine the 3-line and 4-line vertex of the bosons $B^\pm$ interacting with the $A^\mu$-photon.

Using the universality of the electromagnetic interaction, the coupling constant of this vertex is given by the fundamental charge, so we find the relation $g_2 = g_1 = e \csc \theta_W$, and the $\alpha$-angle is connected to $\theta_W$ by $\sin \alpha = \tan \theta_W$, so we obtain $\sin^2 \alpha \simeq 0.33$. This is the result of the NC standard model in the framework of $\theta^{\mu\nu}$-constant [38,39]. In the NC model, the scale of NCY has a lower bound of $\Lambda_{NC} \gtrsim 10^3$ GeV, so we use this scale to represent the first VEV, that is, $v_1 \simeq 1$ TeV. Consequently, the masses of the bosons $B^\pm$ and $Z_{\mu\nu}$ can be computed as

$$m_{G_{\mu}} \simeq 770 \text{ GeV}, \quad m_{B_{\mu}} \simeq 310 \text{ GeV},$$

$$m_{Z_{\mu\nu}} \simeq 699 \text{ GeV}, \quad m_{G_{\mu\nu}} \simeq \frac{154}{\cos \beta} \text{ GeV} > 154 \text{ GeV}.$$  

(40)

To localize our results inside an experimental scenario with the results obtained in LHC [40], we can compare them with the masses obtained for the NC gauge bosons, for instance. Firstly, we use the VEV $v_1$-scale as the NC scale $\Lambda_{NC}$ estimated in the literature $v_1 \sim \Lambda_{NC} \sim 1$ TeV. Within this scale, let us consider the values listed in eq. (40). The NC model that we are dealing with this letter has $(3 + 1 + 6)$-dimensions, in which the 6 extra dimensions are compactified. The bosons $B^\pm$ and $Z^\mu\nu$ are, specifically, due to these six extra dimensions, and we compare their masses to the $W^\pm$ and $Z^0$ in [40]. If we adopt the VEV-$v_1$ as the TeV-scale for [40], i.e., $v_1 \sim \sqrt{s} = 8$ TeV, we obtain the following masses for our results in (40), $m_{G_{\mu}} \simeq 6.1$ TeV, $m_{B_{\mu}} \simeq 2.5$ TeV and $m_{Z_{\mu\nu}} \simeq 5.5$ TeV. The mass of $W^\pm$, and a lower limit on the mass of $Z^0$, according to the ATLAS and CMS experiment, are given by

$$m_{W^\pm} = 2.5 \text{ TeV} \quad \text{and} \quad m_{Z^0} = 2.95 \text{ TeV},$$

(41)

which means that our results in (40) have same TeV order of energy. Notice that the NC contributions increased these numbers, which is a reasonable result.

In this way, we have analyzed some elements of the NC standard model such as the electroweak standard model. Since the position and $\theta$ coordinates are independent variables, the Moyal-Weyl product keeps its associative property and it is the basic product, as usual in canonical NC models.

Hence, we have introduced new ideas and concepts in DFR formalism and we began with the construction of the symmetry group $U^*_7(2) \times SU^*_5(1)$, which is the DFR version of the GSW model concerning the electroweak interaction, in order to introduce left- and right-handed fermionic sectors. Some elements such as covariant derivatives, gauge transformations and gauge invariant Lagrangians were constructed, and the interactions between leptons and gauge fields were discussed.

After that we have introduced the first Higgs sector to break one of the two Abelian NC symmetries in order to destroy the residual model’s $U^*_1(1)$ symmetry. The spontaneous symmetry breaking was discussed and, in this way, the Higgs Lagrangian was introduced. We have seen that in the context of the NC DFR framework, the Abelian gauge field associated with $U^*_1(1)$ has acquired a mass term. Besides, thanks to the NC scenario, some fields are massive and others massless. Also in the NC context we have obtained 3-line and 4-line vertex interactions and the renormalizability of the model was preserved. The residual symmetry $U^*_1(1)$ was eliminated via the use of the Higgs sector.

Moreover, we have introduced a second Higgs sector in order to break the electroweak symmetry and the masses of the old and new bosons were computed with the NC contributions. Since the Weinberg angle was identified as the basic angle to calculate the masses of the $W^\pm$ and $Z^0$, we have used the experimental value of the sine of the Weinberg angle in order to calculate the $W^\pm$ and $Z^0$ masses in a NC scenario. We have used the lower bound for the first SSB scale given by $v_1 \simeq 1$ TeV. Finally, we have obtained the masses for new antisymmetric bosons of the DFR framework.

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