Enhanced transmission of terahertz radiation through a periodically modulated slab of layered superconductor

D V Kadygrob\textsuperscript{1}, N M Makarov\textsuperscript{2}, F Pérez-Rodríguez\textsuperscript{2}, T M Slipchenko\textsuperscript{1} and V A Yampol'skii\textsuperscript{1,2,3,4}

\textsuperscript{1} A Ya Usikov Institute for Radiophysics and Electronics, National Academy of Sciences of Ukraine, 61085 Kharkov, Ukraine
\textsuperscript{2} Benemérita Universidad Autónoma de Puebla, Puebla, Puebla 72000, Mexico
\textsuperscript{3} V N Karazin Kharkov National University, 61077 Kharkov, Ukraine
E-mail: yam@ire.kharkov.ua

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Abstract. We predict the enhanced transparency of a modulated slab of layered superconductor for terahertz radiation due to the diffraction of an incident wave and the resonance excitation of eigenmodes. The electromagnetic field is transferred from the irradiated side of the slab to the other by excited waveguide modes (WGMs) which do not decay in layered superconductors, in contrast to metals, where the enhanced light transmission is caused by the excitation of evanescent surface waves. We show that a series of resonance peaks can be observed in the dependence of transmittance on the incidence angle when the dispersion curve of the diffracted wave crosses successive dispersion curves for the WGMs.

\textsuperscript{4} Author to whom any correspondence should be addressed.

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1. Introduction

Since the first observation by Ebbesen et al [1], enhanced light transmission (ELT) through metal films perforated by subwavelength holes has been the focus of attention of many research groups (see, e.g., reviews [2, 3] and references therein). This phenomenon is observed in films with thicknesses significantly exceeding the skin depth, and the transmission coefficient is found to be much larger than that predicted by Bethe’s theory of electromagnetic diffraction at small apertures [4]. The ELT is related to the coupling of surface plasmons resonantly excited at both sides of a perforated film. A discussion of this and some alternative mechanisms of the extraordinary transmission can be found in the review by Zayats et al [5]. Recent interest in the aforementioned effect is due to its possible applications for light control, photovoltaics and detection and filtering of radiation in the visible and far-infrared frequency ranges.

To observe the ELT in sufficiently thick metal films, a mechanism should exist for the transfer of electromagnetic energy from the irradiated side of the film to the other. In this connection, we would like to mention two forgotten methods to make metal films transparent for electromagnetic waves. Both of them are characteristic of pure metals at low temperature. The first is the so-called anomalous penetration of an electromagnetic field by the chains of the Larmor electron orbits (see, e.g., [6]). In the external dc magnetic field parallel to the sample surface, the electrons carry the electromagnetic field out from the skin layer and form an additional current layer at a distance of the Larmor diameter from the sample surface. Another group of Larmor electrons carry the electromagnetic field deeper into the sample and form the next current layer, and so on. Thus, the Larmor electrons transmit the electromagnetic energy through metal films.

The second mechanism is related to the generation of weakly decaying electromagnetic waves: helicons, dopplerons, cyclotron waves, etc (see, e.g., [7–9]). Under certain conditions, in the presence of an external dc magnetic field, they can propagate in a metal and transfer electromagnetic energy from the irradiated side of a film to the other.

In this paper, we suggest a novel mechanism for the ELT, which is a combination of the two mechanisms discussed above. Specifically, we make an analytical study of the enhanced transmission of terahertz radiation through a slab of layered superconductor with a periodically modulated value of maximum density \( J_c \) of the \( e \)-axis Josephson current. Such a modulation can be implemented either by irradiating a standard \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta \) sample covered by a modulated mask [10] or by pancake vortices controlled by an out-of-plane magnetic field [11]. As in the case of the ELT in metals, the modulation results in the generation of diffracted waves, which under the resonance conditions excite electromagnetic eigenmodes.
The Josephson current along the crystallographic c-axis interacts with the electromagnetic field inside the insulating dielectric layers, giving rise to the Josephson plasma waves (JPWs) (see, e.g., the review [12] and references therein). Of great importance is that the characteristic frequencies of the JPWs belong to the terahertz range, which is relevant for different applications. What is more essential to us is the ‘hyperbolic’ form of the dispersion law for JPWs [12],

$$\frac{k_{sx}^2 \lambda_c^2}{\omega^2/\omega_J^2} - 1 - k_{sz}^2 \lambda_{ab}^2 = 1.$$  \hspace{1cm} (1)

Here \(\omega\) is the wave frequency; \(k_{sx}\) and \(k_{sz}\) are the components of the wave vector along and across the superconducting layers; \(\lambda_c = c/\omega_J \varepsilon^{1/2}\) and \(\lambda_{ab}\) are the magnetic-field penetration depths along and across the layers, respectively; \(\omega_J \sim 1\) THz is the Josephson plasma frequency; \(\varepsilon\) is the interlayer dielectric constant; and \(c\) is the speed of light.

Equation (1) shows that JPWs with \(\omega > \omega_J\) can propagate across the layers as the longitudinal component \(k_{sx}\) of the wave vector is sufficiently large

$$k_{sx} > k_c = \frac{1}{\lambda_c} \sqrt{\left(\omega/\omega_J\right)^2 - 1}.$$  \hspace{1cm} (2)

This specific feature of the JPWs is responsible for the principal difference of the ELT in metals and in layered superconductors. In metals, not only the basic wave with \(k_{sx} = (\omega/c)\sin \theta\), but all diffracted waves with \(k_{sx} = (\omega/c)\sin \theta + ng\) exponentially decay in a sample (here \(\theta\) is the incidence angle, \(g\) is the period of the reciprocal lattice of modulation and \(n\) is an integer). In contrast, for layered superconductors, while the basic wave with \((\omega/c)\sin \theta < k_c\) exponentially decays into the sample, the diffracted waves, having \(|(\omega/c)\sin \theta + ng| > k_c\), propagate across the layers. Therefore, the diffracted waves serve as carriers of electromagnetic energy, similarly to the helicons, dopplerons and cyclotron waves described in [7–9]. Thus, the diffraction in metals can result in the resonant excitation of the symmetric or antisymmetric evanescent surface waves, whereas the diffraction in the layered superconductors provides more effective resonant excitation of waveguide modes (WGMs) (see [13, 14]), which do not decay inside the slab but oscillate across the layers.

In this paper, we analytically study the ELT in layered superconductors with weak harmonic modulation for the case where the inequalities \((\omega/c)\sin \theta < k_c < |(\omega/c)\sin \theta \pm g|\) hold true. The transmittance versus incidence angle \(\theta\) displays a series of resonance peaks emerging from the successive intersection of different dispersion curves for the WGMs with the dispersion curve of the diffracted wave.

2. Electromagnetic field in vacuum and layered superconductor

Consider a slab of layered superconductor of thickness \(d\) surrounded by vacuum (see figure 1). The crystallographic ab-plane coincides with the xy-plane, and the c-axis is directed along the z-axis. The plane \(z = 0\) is the top edge of the slab. Suppose that the maximum c-axis Josephson current density \(J_c\) and, therefore, the Josephson plasma frequency \(\omega_j\) are periodically modulated in the x-direction with a spatial period \(L\),

$$\omega_j(x) = \omega_j [1 + f \cos (gx)] , \quad g = 2\pi/L, \quad f \ll 1.$$  \hspace{1cm} (3)

A plane electromagnetic wave of transverse magnetic polarization, \(\mathbf{E}^{\text{inc}} = \{E_x^{\text{inc}}, 0, E_z^{\text{inc}}\}\) and \(\mathbf{H}^{\text{inc}} = \{0, H_z^{\text{inc}}, 0\}\), is incident from the vacuum onto the top side, \(z = 0\), of the
superconductor at an angle $\theta$. The in-plane and out-of-plane components of its wave vector $k$ are

$$k_x \equiv q = k \sin \theta, \quad k_z = k \cos \theta, \quad k = \omega/c. \quad (4)$$

The in-plane periodic modulation results in generation of the diffracted waves with the in-plane wave numbers $q_n = q + ng$. For simplicity, we derive the electromagnetic field distribution taking into account only the first-order diffracted wave with $q_1 = q + g$. This wave provides the ELT as $q_1$ is close to the wave vector of one of the eigenmodes. The results are then generalized to the minus-first diffraction order, i.e. for $q_{-1} = |q - g|$.

We assume the diffracted wave in the vacuum to be evanescent ($q_1 > \omega/c$), and its in-plane wave number $q_1$ to be close to the corresponding wave number of one of the eigenmodes. Thus, the electromagnetic field in the vacuum above the superconductor ($z < 0$) is presented as a sum of the incident wave (with unit amplitude), the specularly diffracted ($n = 0$) wave and the first-order diffracted ($n = 1$) evanescent wave. The magnetic $H^V_{\text{top}}(x, z)$ and the tangential electric $E^V_{x\top}(x, z)$ fields read

$$H^V_{\text{top}}(x, z) = \exp(i q x + i k z \cos \theta) + R_0 \exp(i q x - i k z \cos \theta) + R_1 \exp(i q_1 x + \kappa^V_1 z), \quad (5)$$

$$E^V_{x\top}(x, z) = \cos \theta \left[ \exp(i q x + i k z \cos \theta) - R_0 \exp(i q x - i k z \cos \theta) \right] - \frac{i \kappa^V_1}{k} R_1 \exp(i q_1 x + \kappa^V_1 z), \quad (6)$$

where the attenuation coefficient $\kappa^V_1 = \sqrt{q_1^2 - k^2} > 0$.

The electromagnetic field under the superconductor ($z > d$) is a sum of the transmitted wave and the first-order diffracted evanescent wave,

$$H^V_{\text{bot}}(x, z) = T_0 \exp[i q x + i k(z - d) \cos \theta] + T_1 \exp[i q_1 x - \kappa^V_1 (z - d)], \quad (7)$$

$$E^V_{x\bot}(x, z) = T_0 \cos \theta \exp[i q x + i k(z - d) \cos \theta] + \frac{i \kappa^V_1}{k} T_1 \exp[i q_1 x - \kappa^V_1 (z - d)]. \quad (8)$$

Figure 1. Schematic geometry of the problem; $k^i$, $k^r$, and $k^t$ are, respectively, the wave vectors of the incident, reflected and transmitted waves; $q_1$ is the longitudinal wave vector of the excited WGM.
The electromagnetic field inside the layered superconductor is determined by the distribution of the gauge-invariant phase difference \( \varphi(x, z, t) \) of the order parameter between the layers (see, e.g., [12]),

\[
\frac{\partial H^s}{\partial x} = - \frac{\mathcal{H}_0}{\lambda_c \omega^2} [\omega_j^2(x)(1 - i \Gamma_c) - \omega^2] \varphi, \quad \mathcal{H}_0 = \frac{\Phi_0}{2\pi D \lambda_c}.
\]

\[
E^s_z = i k (1 + i \Gamma_{ab}) \lambda_{ab}^2 \frac{\partial H^s}{\partial z}, \quad E^s_z = -i \mathcal{H}_0 k \lambda_c \varphi.
\]  

(9)

Here, \( \omega_j(x) \) is given by equation (3), \( \omega_j = (8\pi e DJ_c/\hbar)^{1/2} \) is the Josephson plasma frequency unperturbed by the modulation, \( J_c \) is the maximum value of the Josephson current density \( j_c = j_c \sin \varphi \), \( D \) is the spatial period of the layered structure and \( \Phi_0 = \pi c \hbar / e \) is the magnetic flux quantum. The dimensionless relaxation frequencies \( \Gamma_{ab} = 4\pi \sigma_{ab} \omega \lambda_{ab}^2 / \epsilon \omega_0 \lambda_c^2 \) and \( \Gamma_c = 4\pi \sigma_c \omega / \epsilon \omega_0 \lambda_c^2 \) are proportional to the average quasiparticle conductivities \( \sigma_{ab} \) (along the layers) and \( \sigma_c \) (across the layers). Note that \( \Gamma_c \) can actually be neglected due to the smallness of out-of-plane conductivity \( \sigma_c \).

The phase difference \( \varphi \) is governed by a set of coupled sine-Gordon equations (see, e.g., the review [12] and references therein). For linear JPWs in the continuum limit, the coupled sine-Gordon equation is written as

\[
\left[ 1 - (1 + i \Gamma_{ab}) \lambda_{ab}^2 \frac{\partial^2}{\partial z^2} \right] [\omega_j^2(x) - \omega^2] \varphi - \lambda_j^2 \omega_j^2 \frac{\partial^2 \varphi}{\partial x^2} = 0.
\]  

(10)

Note that, in general, the electric component \( E^s_z \) induces a charge in the superconducting layers. This charge originates an additional interlayer coupling (the so-called capacitive coupling) resulting in the appearance of an additional term in equation (10) and modification of the relation between \( E^s_z \) and \( \varphi \) in equations (9). The corresponding dispersion equation for the linear JPWs in the presence of capacitive coupling was obtained in [15]. According to this dispersion equation, the capacitive coupling affects the properties of the longitudinal JPWs with the wave vector perpendicular to the layers. However, it can be safely neglected in our case, when the wave-vector has a large enough component \( q \gg \omega / \epsilon \) along the layers. The latter is provided by the smallness of the parameter \( \alpha = \epsilon R_0^2 / s D \) with \( R_0 \) being the Debye length for the induced charge in a layered superconductor and \( s \) being the thickness of the superconducting layers.

We solve equation (10) perturbatively in a small modulation amplitude \( f \ll 1 \), see equation (3). Then, equation (9) yields the magnetic field inside the layered superconductor \( 0 < z < d )

\[
H^s(x, z) = \Psi_0(x) \left[ C_0^+ \exp[p_0(z - d)] + C_0^- \exp(-p_0 z) \right] + \Psi_1(x) \left[ C_1^+ \exp(ik_1^z z) + C_1^- \exp(-ik_1^z z) \right]
\]  

(11)

with

\[
\Psi_0(x) = \exp(i qx) - F_{01} \exp(i q_1 x), \quad \Psi_1(x) = \exp(i q_1 x) + F_{01} \exp(i qx),
\]  

(12)
\[ p_0 = \frac{1}{\lambda_{ab}} \left( 1 - \frac{i \Gamma_{ab}}{2} \right) \sqrt{1 - \frac{\lambda_{ab}^2 q_0^2}{\Omega^2 - 1} \left( 1 + \frac{F F_{01}}{2} \frac{q_0^2}{\Omega^2 - 1 - q_0^2} \right)}, \]
\[ \kappa_1^s = \frac{1}{\lambda_{ab}} \left( 1 - \frac{i \Gamma_{ab}}{2} \right) \sqrt{\frac{\lambda_{ab}^2 q_1^2}{\Omega^2 - 1} - 1 \left( 1 + \frac{F F_{01}}{2} \frac{q_1^2}{\Omega^2 - 1} \right)}, \]
\[ F = \frac{f}{\Omega^2 - 1}, \quad F_{01} = \frac{q_1 q_0}{q_1^2 - q_0^2}, \quad \Omega = \frac{\omega}{\omega_0}. \quad (13) \]

The tangential component of the electric field in the layered superconductor reads
\[ E_\parallel(x, z) = \frac{1}{i} \left\{ a_0 \Psi_0(x) \left[ C_0^+ \exp[p_0(z - d)] - C_0^- \exp[-p_0 z] \right] + a_1 \Psi_1(x) \left[ C_1^+ \exp[i \kappa_1^s z] - C_1^- \exp[-i \kappa_1^s z] \right] \right\}, \quad (14) \]
where
\[ a_0 = k \lambda_{ab} \sqrt{1 - \frac{\lambda_{ab}^2 q_0^2}{\Omega^2 - 1}} \quad a_1 = k \lambda_{ab} \sqrt{\frac{\lambda_{ab}^2 q_1^2}{\Omega^2 - 1} - 1} \quad (15) \]
are the small surface impedances for the basic and the first-order diffracted waves, respectively.

3. Transmittance and reflectance

Matching the tangential components of electric and magnetic fields at the boundaries, \( z = 0 \) and \( d \), we obtain eight linear algebraic equations for eight unknown amplitudes, \( R_0, R_1, T_0, T_1, C_0^+, C_0^-, C_1^+, C_1^- \). Solving these equations yields the transmittance \(|T_0|^2\) and reflectance \(|R_0|^2\) of the superconducting slab,
\[ |T_0|^2 = \frac{4 F_{01}^4 a_2^2 \cos^2 \theta}{D^2 + B^2}, \quad (16) \]
\[ |R_0|^2 = \frac{D^2 + (\kappa_1^s d \Gamma_{ab}/2)^2}{D^2 + B^2}, \quad (17) \]
where
\[ D = \tan \left( \kappa_1^s d \right) - 2 \frac{k}{\kappa_1^s} a_1, \quad (18) \]
\[ B = \frac{\kappa_1^s d}{2} \Gamma_{ab} + 2 F_{01}^2 \frac{a_1}{\cos \theta}. \quad (19) \]

The equation \( D = 0 \) with \( f = 0 \), i.e. with \( F = 0 \) in equations (13), defines the spectrum of symmetric and antisymmetric waveguide eigenmodes [14]. The weak modulation of the plasma frequency (3) gives rise to a shift of the dispersion curves and to additional damping due to the leakage of the eigenmode energy via a diffracted wave. This leakage is specified by the second term in equation (19).

Equations (16) and (17) describe the resonant enhancement in transmittance and, accordingly, the suppression in reflectance due to the excitation of the WGM by the first-order diffracted wave. Similar expressions can be derived for the case of the resonance WGM.
Figure 2. The transmittance $|T_0|^2$ and reflectance $|R_0|^2$ versus the incidence angle $\theta$ for $d = \lambda_c = 10 \lambda_{ab}$, $\epsilon = 16$, $\Gamma_{ab} = 10^{-4}$, $f = 0.2$, $\Omega = 1.1$, $gc/\omega = 3$. The first number in the brackets shows the diffraction order responsible for the resonance and the second is the number of the resonantly excited WGM.

excitation in the minus-first diffraction order. To this end, one should substitute the wave number $q_{-1} = |k \sin \theta - g|$ instead of $q_1 = g + k \sin \theta$ in all formulae. Figure 2 shows six resonance peaks in the dependence of transmittance $|T_0|^2$ and reflectance $|R_0|^2$ on the incidence angle $\theta$. Three of them are due to the first-order resonance diffraction when the argument $\kappa_1^S d$ of the tangent in equation (18) is close to $m\pi$ with $m = 8, 9, 10$; and the others are related to the resonances in the minus-first diffraction order at $\kappa_{-1}^S d \approx m\pi$ with $m = 4, 5, 6$. Namely, these branches of the dispersion curves for the WGMs are crossed by the dispersion curves of the diffracted waves in the first and minus-first orders when changing the incidence angle $\theta$ from 0 to $\pi/2$. 

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Figure 3. The magnetic-field distribution: (a) the nonresonance case at $\theta = 45^\circ$, (b) the resonance diffraction for the order $n = +1$, $\theta = 66.1^\circ$. The other parameters are the same as in figure 2. The superconducting slab is magnified tenfold in the $z$-direction.

We also illustrate the ELT effect via the distribution of the magnetic field in figure 3. Under the nonresonant conditions, the interference pattern is seen in the vacuum region above the superconductor, and the transmitted wave is absent. In the resonance, the reflected wave is totally suppressed, the interference pattern in the far-field disappears, and the transmitted wave is clearly seen under the layered superconductor.

4. Conclusions

The enhanced transparency of a modulated slab of layered superconductor has been predicted for terahertz radiation. It stems from the diffraction of an incident wave and the resonant excitation of WGMs. The diffracted wave does not decay in the superconducting slab, in contrast to metals, where the ELT is caused by the excitation of evanescent surface waves. A series of resonance peaks, associated with different WGMs, arises in the dependence of transmittance $|T_0|^2$ on incidence angle $\theta$.

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