On Capacity Computation for the Two-User Binary Multiple-Access Channel: Solutions by Cooperation

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Abstract—This paper deals with the computation of the boundary of the capacity region for the two-user memoryless multiple-access channel, which is equivalent to solving a difficult non-convex optimization problem. We study properties of the optimality conditions for a relaxation (cooperation) approach suggested in the literature. We give conditions under which a solution to the relaxed problem has the same value as the actual optimal solution and show that these conditions can in some cases be applied to construct solutions for a restricted class of discrete multiple-access channel using convex optimization.

I. INTRODUCTION

For some multiuser channels, the capacity region can be characterized in terms of mutual information expressions. However, even for channels where such a single-letter representation is available, evaluation of the capacity region is often a hard problem since computation of the capacity region boundary is generally a difficult and nonconvex optimization problem. For the single-user discrete memoryless channel, computation of the capacity is a convex problem, and several numerical methods that allow to calculate the capacity within arbitrary precision have been developed, e.g. the Arimoto-Blahut algorithm [1][2]. For the discrete memoryless multiple-access channel (MAC), no algorithms for the computation of the capacity region boundary are known. A fundamental step in this direction has been taken in [3], where a numerical method for calculating the sum-rate capacity (also called total capacity) of the two-user MAC with binary output has been developed. This was achieved by showing that the calculation of the sum capacity can be reduced to the calculation of the sum capacity for the two-user MAC with binary input and binary output and by giving necessary and sufficient conditions for sum-rate optimality by a partial modification of the Kuhn-Tucker conditions. Unfortunately, further generalizations [4][5] of this approach to the most general \((N_1, \ldots, N_m; M)\)-MAC (with \(m\) users, each with an alphabet of size \(N_k\) and output alphabet of size \(M\)) is partially incorrect, as demonstrated in [6]. The works in [6] and [7] consider the computation of not only the sum capacity, but of the whole capacity region of the two-user discrete MAC.

In [6], it was shown that for a restricted class of \((2, 2; 2)\)-MACs (the 3-parameter \((2, 2; 2)\)-MAC) and weight vectors, depending on an ordering property of the channel matrix, the optimal solution is located on the boundary, or the objective function has at most one stationary point in the interior of the domain. In [7], it is shown that the only non-convexity in the problem stems from the requirement of the input probability distributions to be independent, i.e. from the constraint for the probability matrix specifying the joint probability input distribution to be of rank one. An approximate solution to the problem is proposed by removing this independence constraint (i.e. relaxation of the rank-one constraint), obtaining an outer bound region to the actual capacity region. By projecting the obtained probability distribution to independent distributions by calculating the marginals, one obtains an inner bound region. Even though the authors present some examples where this approach gives the actual capacity region (i.e. the outer bound region, the inner bound region and the capacity region coincide), the result is often suboptimal, and it is not clarified when the actual capacity region is obtained. Consequently, the solution of the capacity computation problem for the discrete memoryless MAC remains an interesting unsolved problem, even for the case of two users and binary alphabets.

Contributions. In this paper, we study properties of the optimality conditions for the relaxation (cooperation) approach suggested in [7]. We derive conditions under which a solution to the relaxed problem has the same value as the actual optimal solution and show how these conditions can in some cases be applied to construct solutions for a restricted class of \((2, 2; 2)\)-MACs using convex optimization. Opposed to this, we demonstrate by examples that even for these channels, the marginal approach suggested in [7] generally offers only suboptimal solutions. We remark that in addition, these results offer a convenient characterization of the capacity region for this channel subclass which can be used for further studies such as the investigation of duality relations between the discrete multiple access and broadcast channel [8].

Organization. The paper is organized as follows: Section II introduces the problem formulation. In section III, we give a sufficient condition under which the relaxation approach from [7] results in an optimal solution. Using this condition,
II. PROBLEM FORMULATION

The communication model under study is the discrete and memoryless two-user multiple-access channel, specified by input alphabets $X_1 = \{1, \ldots, N_1\}$, $X_2 = \{1, \ldots, N_2\}$, the output alphabet $Y = \{1, \ldots, M\}$ and conditional channel transition probabilities $Q(y|ij) = \Pr[Y = y|X_1 = i, X_2 = j]$ (collectively denoted by the channel transition matrix $Q$) for $y \in Y, i \in X_1, j \in X_2$. This channel is referred to as the $(N_1, N_2; M)$-MAC in the following. Let $P_1 := \{p \in \mathbb{R}^{N_1}_+ : \sum_{k=1}^{N_1} p_k = 1\}$ for $i = 1, 2$. It is well-known that the capacity region $C_{(N_1, N_2; M)}$ of the $(N_1, N_2; M)$-MAC is given by [9]-[11]

$$C_{(N_1, N_2; M)} = \operatorname{Co} \left( \bigcup_{(p_1, p_2) \in P_1 \times P_2} \mathcal{A}(p_1, p_2) \right), \tag{1}$$

where

$$\mathcal{A}(p_1, p_2) = \{ R \in \mathbb{R}^{2}_{++} : \begin{align*}
R_1 &\leq I(X_1;Y|X_2), \\
R_2 &\leq I(X_2;Y|X_1), \\
R_1 + R_2 &\leq I(Y;X_1, X_2) \}.$$ \tag{2}

Here, $p_1, p_2$ specify the input distribution by $\Pr[X_u = s] = p_{us}$, where $p_{us}$ denotes the $s$-th component of $p_u$. $\operatorname{Co}$ denotes the convex closure operation and $I$ is mutual information.

The problem we consider in this work is computing the boundary of the capacity region, or equivalently, since the capacity region is convex, the maximization of the weighted sum-rate in the capacity region for a given weight vector $w = (w_1, w_2)^T \geq 0$ (c.f. Figure 1):

$$\max_{r \in C_{(N_1, N_2; M)}} w^T r. \tag{3}$$

We remark that one of the difficulties of the problem stems from the fact that, unlike for the Gaussian case [11], there generally are no input distributions $p_1, p_2$ that jointly optimize the mutual information bounds in (2).

Each polyhedron region $\mathcal{A}(p_1, p_2)$ is specified by the corner points $C_1(p_1, p_2) := (I(Y;X_1), I(Y;X_2|X_1))^T$ and $C_2(p_1, p_2) := (I(Y;X_1|X_2), I(Y;X_2))^T$. In order to explicitly formulate the problem in the probability domain, it is easily verified that the weighted sum-rate optimization problem formulated above can be stated in terms of optimization over the region defined by the $C_1, C_2$ points as follows: For $w_1 \leq w_2$, it holds that

$$\max_{r \in C_{(N_1, N_2; M)}} w^T r = \max_{(p_1, p_2) \in P_1 \times P_2} \Psi(p_1, p_2), \tag{4}$$

where $\Psi(p_1, p_2) := w^T C_1(p_1, p_2)$. Clearly, for $w_1 > w_2$, the optimization can similarly be performed by optimizing over the $C_2$ points, and for $w_1 = w_2$, the problem reduces to the sum capacity problem studied in [3]-[5].

III. SOLUTIONS BY USER COOPERATION

We now consider the relaxation approach proposed in [7]. In this work, it has been shown that the only non-convexity of the problem is due to the requirement of the input probability distributions to be independent, i.e. the constraint for the probability matrix specifying the joint probability input distribution to be of rank one. For this, the following functions for matrices $P \in \mathbb{R}^{N_1 \times N_2}$ and vectors $p_1 \in \mathbb{R}^{N_1}, p_2 \in \mathbb{R}^{N_2}$ are defined:

$$f_1(P, p_2) = \sum_{y,i,j} P_{ij} Q(y|ij) \ln \frac{Q(y|ij)p_{2j}}{\sum_k P_{kj} Q(y|kj)}, \tag{5}$$

$$f_2(P, p_1) = \sum_{y,i,j} P_{ij} Q(y|ij) \ln \frac{Q(y|ij)p_{1i}}{\sum_k P_{ik} Q(y|ik)}, \tag{6}$$

$$f_{12}(P) = \sum_{y,i,j} P_{ij} Q(y|ij) \ln \frac{Q(y|ij)}{\sum_{k,l} P_{kl} Q(y|kl)}. \tag{7}$$

Here, $P$ models the joint input probability distribution (not necessarily independent for the two users), $p_1$ and $p_2$ specify the marginal distributions for each user and $f_1, f_2, f_{12}$ are obtained by a slight modification of the mutual information expressions (allowing dependent distributions). The capacity computation problem can then, for a weight vector $w = (\theta, 1 - \theta)^T \geq 0$, be equivalently formulated as

$$\min_{R_1, R_2, P, p_1, p_2} \left( -\theta R_1 + (1 - \theta) R_2 \right) \tag{8}$$

subject to

$$R_1 \geq 0, R_2 \geq 0$$

$$R_1 \leq f_1(P, p_2) \tag{9}$$

$$R_2 \leq f_2(P, p_1) \tag{10}$$

$$R_1 + R_2 \leq f_{12}(P) \tag{11}$$

$$P_1 = p_1, P^T 1 = p_2, P \geq 0, 1^T P 1 = 1 \tag{12}$$

rank($P$) = 1.

It has been shown in [7] that $f_1, f_2, f_{12}$ are concave functions, so that nonconvexity in the above problem is only caused by the rank one constraint (12) which forces the joint distribution specified by $P$ to be independent. By removing this constraint, and hence allowing cooperation of the users, one obtains a convex problem, which is efficiently solvable using standard

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1In the following, $I$ denotes the column vector with all entries equal to 1, $0$ is the zero matrix (or vector), and for vectors or matrices $x, y$, we denote component-wise inequalities as $x \leq q$. 

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![Fig. 1. Illustration of the capacity computation problem: Weighted sum-rate optimization for the weight vector $w$.](image-url)
convex optimization techniques. Subsequently, we refer to this problem as the relaxed problem. The objective function value \( f_{\text{relaxed}} \) at a solution \((R_1, R_2, P, p_1, p_2)\) for the relaxed problem gives a lower bound to the objective value \( f_{\text{optimal}} \) of the actual (non-relaxed) problem. The matrix \( P \) obtained by solving the relaxed problem is generally not of rank one. In [7], it has been proposed to use the product distribution \( P' = p_1'p_2 \) (which is of rank one and obtained by forming the independent distribution specified by the marginals of \( P \)) as a heuristic solution, called the marginal approach in this paper. One fixes \( P', p_1, p_2 \) in the problem above and solves the convex problem in \( R_1, R_2 \), resulting in an objective function value \( f_{\text{marginal}} \), which is an upper bound to the actual optimum value. We remark that it is easy to see that this solution is, for \( \theta \leq \frac{1}{2}, \) given by \( R_1 = f_{12}(P') - f_2(P', p_1), \) \( R_2 = f_2(P', p_1). \) From this, it also follows in this case that \( f_{\text{marginal}} = -\Psi(p_1, p_2). \) For \( \theta > \frac{1}{2}, \) one similarly obtains \( R_1 = f_1(P', p_2), \) \( R_2 = f_{12}(P') - f_1(P', p_2). \)

In the following, we first give, for the general case, a sufficient condition for optimality. Introducing Lagrange multipliers \( \lambda \in \mathbb{R}^5, \alpha \in \mathbb{R}^N, \beta \in \mathbb{R}^N, \mu \in \mathbb{R}^{N_1 \times N_2}, \tau \in \mathbb{R}, \) the Lagrangian function [12] for the relaxed problem is given by

\[
\mathcal{L}(R_1, R_2, P, p_1, p_2, \lambda, \alpha, \beta, \mu, \tau) = - (\theta R_1 + (1 - \theta)R_2) - \lambda_1 R_1 - \lambda_2 R_2 + \lambda_3 (R_1 - f_1(P, p_2)) + \lambda_4 (R_2 - f_2(P, p_1)) + \lambda_5 (R_1 + R_2 - f_{12}(P)) + \alpha^T(P1 - p_1) + \beta^T(P^T 1 - p_2) - \sum_{i,j} \mu_{i,j} p_{i,j} + \tau(1^T P1 - 1).
\]

In the following, we define

\[
A_{ij} = \frac{\partial}{\partial P_{ij}} f_1(P, p_2) = \sum_{y} Q(y||i) \ln \frac{Q(y||i)p_{2j}}{\sum_k P_{kj} Q(y||kj)} - 1
\]

\[
B_{ij} = \frac{\partial}{\partial P_{ij}} f_2(P, p_1) = \sum_{y} Q(y||i) \ln \frac{Q(y||i)p_{1j}}{\sum_k P_{ki} Q(y||ik)} - 1
\]

\[
f_{12}(P) = \sum_{y} Q(y||i) \ln \frac{Q(y||i)}{\sum_{k,l} P_{ki} Q(y||kl)} - 1
\]

and

\[
D_i = \frac{\partial}{\partial p_{i}} f_1(P, p_2) = \sum_{p_{i}} P_{ij}, E_i = \frac{\partial}{\partial p_{i}} f_2(P, p_1) = \sum_{p_{i}} P_{ji}.
\]

The stationarity conditions of the Lagrangian at the KKT point are given by the set of equations

\[
-\lambda_1 + \lambda_3 + \lambda_5 = \theta \tag{14}
\]

\[
-\lambda_2 + \lambda_4 + \lambda_5 = 1 - \theta \tag{15}
\]

\[
\forall i: -\lambda_i D_i - \alpha_i = 0 \tag{16}
\]

\[
\forall i : -\lambda_3 E_i - \beta_i = 0 \tag{17}
\]

\[
\forall i, j : -\lambda_3 A_{ij} - \lambda_4 B_{ij} - \lambda_5 C_{ij} + \alpha_i + \beta_j - \mu_{ij} + \tau = 0 \tag{18}
\]

Subsequently, we restrict to the case \( \theta \leq \frac{1}{2}, \) the other case can, by symmetry, be treated similarly. Consider an optimal solution \((R_1, R_2, P, p_1, p_2)\) with \( R_1 > 0, R_2 > 0. \) From complementary slackness, we have \( \lambda_1 = \lambda_2 = 0. \) Furthermore, it is clear that at least one of the constraints (9)-(11) must be active for this solution. First consider the case that (9) is not active. Then \( \lambda_3 = 0, \) implying \( \lambda_4 = 1 - 2\theta > 0, \lambda_5 = \theta > 0, \) so that (10) and (11) are active. If (10) is not active (\( \lambda_4 = 0), \) then \( \lambda_3 = 1 - \theta \) and \( \lambda_5 = 2\theta - 1 < 0, \) so that (10) must always be active. For inactive (11), it follows that (9) and (10) must be active. Summarizing, there are three cases possible:

1) (9) active, (10) active, (11) active
2) (9) inactive, (10) active, (11) active
3) (9) active, (10) inactive, (11) active

Fig. 2. Relative performance loss using the marginal approach in comparison to the optimal solution for example I.

Fig. 3. Relative performance loss using the marginal approach in comparison to the optimal solution for example II.
3) (9) active, (10) active, (11) inactive

Now consider the case 2) where (9) is inactive and \( P > 0 \). Clearly, at the optimum, \( D_1 = E_1 = 1 \) and by complementary slackness, \( \mu = 0 \), simplifying the system (14)-(18) to

\[
\forall i, j : (1 - 2\theta)B_{ij} + \theta C_{ij} = 2\theta - 1 + \tau
\]

(19)

Now, for each \( P \in \mathbb{R}_{+}^{N_1 \times N_2} \) and channel transition matrix \( Q \), we define the sets of matrices

\[
S(Q, P) := \left\{ P \in \mathbb{R}_{+}^{N_1 \times N_2} : \bar{P} \geq 0, \bar{P}1 = P1, \bar{P}^T1 = 1, \forall y : \sum_{k,l} (P_{kl} - \bar{P}_{kl})Q(y|kl) = 0, \forall i, y : \sum_{k} (P_{ik} - \bar{P}_{ik})Q(y|ik) = 0 \right\}
\]

(20)

and \( S^1(Q, P) := \left\{ \bar{P} \in S(Q, P) : \text{rank}(\bar{P}) = 1 \right\} \).

The set \( S(Q, P) \) consists of the positive solutions of a linear system defined by the channel matrix \( Q \) and the solution matrix \( P \). One obtains the following sufficient conditions for \( f_{\text{relaxed}} = f_{\text{optimal}} \):

**Proposition 1:** Let \( \theta \leq \frac{1}{2} \) and \((R_1, R_2, P, p_1, p_2)\) be a solution to the relaxed problem such that \( f_1(P, p_2) < \hat{R}_1 \) (i.e. (9) is inactive), \( P > 0 \) and \( R_1, R_2 > 0 \). If \( \hat{P} \in S^1(Q, P) \), then \((\hat{R}_1, \hat{R}_2, \hat{P}, \hat{P}^T1)\) is a solution to the non-relaxed problem, where \( \hat{R}_1 = f_{12}(\hat{P}) - f_2(\hat{P}, \hat{P}^T1) + f_2(P, P^T1) \).

**Proof:** The expressions \( B_{ij} \) and \( C_{ij} \) are kept invariant by the matrices in \( S(Q, P) \). Hence, choosing the same Lagrange multipliers as for \((R_1, R_2, P, p_1, p_2)\), by inspection of the stationarity condition (19) and observing that the complementary slackness conditions hold, it follows that \((\hat{R}_1, \hat{R}_2, \hat{P}, \hat{P}^T1)\) also satisfies the KKT conditions. Finally, note that \( \hat{R}_1 \geq 0 \) from the properties of mutual information.

**IV. THE 3-PARAMETER (2, 2, 2)-MAC**

For some channels, it is possible to explicitly construct matrices in \( S^1(Q, P) \), which will be demonstrated in the following. The channels that we consider now are \((2, 2, 2)\)-MACs with the restriction \( Q(1|1, 1) = Q(1|1, 2) \) on the channel transition probability matrix, which were also studied in [6]. We call such a channel a 3-parameter \((2, 2, 2)\)-MAC. The information-theoretic interpretation of these channels is as follows: Conditioned on the event that user 1 transmits the symbol 1, the channel that user 2 observes is the single-user antisymmetric binary channel, which has zero capacity. In other words, whenever user 1 transmits 1, the symbol of user 2 cannot be distinguished at the receiver. In fact, it is easily verified that \( I(Y; X_2|X_1 = 1) = 0 \).

For these channels, we have the following result:

**Lemma 1:** For the 3-parameter \((2, 2, 2)\)-MAC with channel transition matrix \( Q \), it holds that for each \( P \in \mathbb{R}_{+}^{2 \times 2} \), \( P > 0 \), the set \( S^1(Q, P) \) contains the matrix

\[
\bar{P} := P + \frac{\text{det}(P)}{P_{11} + P_{22}} \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}
\]

(21)

**Proof:** It is easily verified that \( \bar{P} \in S(Q, P) \). Moreover, one quickly checks that \( \text{det}(P) = 0 \), implying \( \text{rank}(P) = 1 \) since \( P \neq 0 \).

From this, we get:

**Theorem 1:** For the 3-parameter \((2, 2, 2)\)-MAC and \( \theta \leq \frac{1}{2} \), it holds that if the solution \((R_1, R_2, P, p_1, p_2)\) to the relaxed problem satisfies \( f_1(P, p_2) < \hat{R}_1 \), \( P > 0 \) and \( R_1, R_2 > 0 \), then \((\hat{R}_1, \hat{R}_2, \hat{P}, \hat{P}^T1)\) is a solution to the (non-relaxed) problem (8), where \( \hat{R}_1 = f_{12}(\hat{P}) - f_2(P, P^T1) \) and yields the same objective value, i.e. \( f_{\text{optimal}} = f_{\text{relaxed}} \).

To summarize, the following solution approach for the 3-parameter \((2, 2, 2)\)-MAC for \( \theta \leq \frac{1}{2} \) can be applied:

1) Solve the relaxed problem, obtaining a solution \((R_1, R_2, P, p_1, p_2)\)
2) Check if \( \text{rank}(P) = 1 \) (if yes, then a solution to the non-relaxed problem has been found)
3) If \( f_1(P, p_2) < \hat{R}_1 \), \( P > 0 \) and \( R_1, R_2 > 0 \), then a solution to the non-relaxed problem is \((\hat{R}_1, \hat{R}_2, \hat{P}, \hat{P}^T1)\)
with \( \tilde{R}_1 = f_{12}(P) - f_2(P, P_1), \tilde{R}_2 = f_2(P, P_1), \) i.e. optimal input probability distributions for problem (4) are given by \( P_1 = P_1, P_2 = P_2. \)

Opposed to this, the approach of obtaining a rank one solution via the marginals (i.e. using \( P' = P_1^1 P_2 \) as described above) is generally suboptimal even for the 3-parameter (2; 2; 2)-MAC. We demonstrate this by two examples in the following. The first example (example I) is given by the 3-parameter (2; 2; 2)-MAC with channel parameters \( Q(1|11) = Q(1|12) = 0.7, Q(1|21) = 0.6, Q(1|22) = 0.1. \) For the second example (example II), we use the parameters \( Q(1|11) = Q(1|12) = 0.4, Q(1|21) = 0.1, Q(1|22) = 0.36. \) Here, we choose a discretization of weight values \( \theta \) in the interval \((0, \frac{1}{4})\) of step size \( 10^{-4} \). For each such value \( \theta \), we first solve the relaxed (convex) problem using the MATLAB function fmincon, resulting in a solution \((R_1, R_2, P, p_1, p_2)\) with objective function value \( f_{\text{relaxed}} \). The marginal approach results in the objective function value \( f_{\text{marginal}} = \sum \theta(p_1, p_2) \) and rate pairs \((R_1^\text{marginal}, R_2^\text{marginal})\). If the relaxed solution satisfies the conditions in Theorem 1, then \( f_{\text{optimal}} = f_{\text{relaxed}} \) and the optimal input distributions are computed as \( P_1^* = P_1, P_2^* = P_2. \) The corresponding optimal rates are given by \( R_1^\text{optimal} = R_1, R_2^\text{optimal} = R_2. \) For the two examples and the different \( \theta \) values, we compare the objective function values \( f_{\text{optimal}} \) and \( f_{\text{marginal}} \), the rate regions given by the points \((R_1^\text{optimal}, R_2^\text{optimal})\) and \((R_1^\text{marginal}, R_2^\text{marginal})\), respectively, and the input probability distributions obtained.

Figures 2 and 3 show the relative performance loss (in percentage with respect to the optimal solution) of the marginal approach in comparison to the optimal solution for example I and example II, respectively. In each of the figures, it is displayed for a range of \( \theta \) values (which suffices to trace out the boundary of the capacity regions). We remark that for all values in this range, the conditions in Theorem 1 hold. One sees that there is a small, but notable performance loss for the marginal approach as compared to the optimal solution. Figures 4 and 5 display a section of the rate regions obtained, illustrating that the boundary obtained via the marginal approach is strictly contained inside the actual capacity boundary. Finally, Figure 6 displays the input probability distributions \( p_1, p_2 \) and \( p_1^*, p_2^* \) for both examples. For each example, the curves show the pairs of the first components of the vectors \( p_1, p_2 \) and \( p_1^*, p_2^* \) respectively. The figure demonstrates that the input probability distributions obtained by the marginal approach generally differ considerably from the optimal ones.

V. CONCLUSIONS

In this paper, we have studied the problem of computing the boundary of the capacity region for the two-user memoryless multiple-access channel, which consists in solving a difficult nonconvex optimization problem. The relaxation (user cooperation) suggested in [7] identifies a rank one constraint as the only non-convexity of the problem, allowing to find suboptimal solutions via convex optimization by removing this constraint. We derived conditions under which a solution to the (convex) relaxed problem has the same value as the actual optimal solution. For the class of 3-parameter (2; 2; 2)-MAC, we applied these conditions to show that, in some cases, it is possible to obtain the optimal solution by first solving the convex problem and then computing an optimal solution for the actual (non-relaxed) problem. By means of two examples, we demonstrated that even for these channels, the marginal approach suggested in [7] offers only suboptimal solutions. We finally note that these results also provide a convenient characterization of the capacity region for the class of 3-parameter (2; 2; 2)-MACs, which can be used for further studies such as on duality relations for discrete memoryless multiple access and broadcast channels [8].

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