An Inversion Method for Geoacoustic Parameters of Multi-layer Seabed in Shallow Water

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Abstract. To solve the problem of obtaining geoacoustic parameters of multi-layer seabed in shallow water, a geoacoustic inversion method based on Bayesian theory is studied in this paper. In this study, the seabed is assumed to be a horizontal layered elastic medium, and the error function is established according to the sound pressure field in the water layer and bayesian theory. The global optimization algorithm is used to obtain the three target parameters of wave velocity, density and wave velocity attenuation in each layer of the seabed. The designed optimization method can correct the parameters in the process of finding the solution according to the physical characteristics of the upper and lower layers. The simulation results show that this method greatly improves the stability and accuracy of the inversion results for the multi-layer seabed model.

1. Introduction

In shallow water waveguides, the acquisition of geoacoustic parameters has always been a hot issue in the field of underwater acoustic engineering. Compared with the traditional direct measurement methods, the parameters of seabed can be obtained quickly and cheaply by acoustic methods. However, most of the existing inversion models consider the liquid seabed and do not consider the seabed layered. In addition, the inversion of geoacoustic parameters by bayesian method has become a research hotspot, which can qualitatively and quantitatively analyse the uncertainty of parameter inversion results from a statistical point of view, but it is still not separated from the setting that the seabed is regarded as a semi-infinite liquid seabed. Therefore, taking the complex sound pressure $p$ as the matching object, improves the optimization algorithm according to the physical law of seabed acoustic impedance, and presents an inversion method for geoacoustic parameters of multi-layer seabed in shallow water. The feasibility and effectiveness of the inversion method are verified and analysed by numerical simulation.
2. Overview of Inversion Methods

2.1. Shallow Water Multi-Layer Seabed Acoustic Field Model

In order to solve the theoretical value of complex sound pressure \( p \) of the object studied in this paper. According to wave theory and Fast Field Method (FFM) establish the prediction model of underwater sound field based on Pekeris waveguide [1]. The model shown in the figure 1, the seabed is regarded as \( n \)-layer elastic medium, the \( z \)-axis represents the ocean depth, and the \( r \)-axis represents the propagation direction of the sound signal, \( z_s \) represents the sound source position, \( f_0 \) is the sound source frequency, \( c_1 \) and \( \rho_1 \) represent the sound velocity and density in the water. \( \phi_1, \phi_{p,i}, \psi_{s,i}(i=2, 3, \ldots, n) \) respectively water layer compression wave displacement potential function, seabed layer compression wave and transverse wave displacement potential function. The inversion parameters in each medium include the compression wave velocity \( c_{pn} \), transverse wave velocity \( c_{sn} \), seabed density \( \rho_n \), compression wave attenuation \( \alpha_{pn} \), transverse wave attenuation \( \alpha_{sn} \) and each layer depth \( h_n \) of each layer as the inversion target parameters.

\[
\begin{align*}
O & \quad f_0 \quad r \quad z = 0 \\
& \quad \rho_1 \quad c_1 \quad \phi_1 \\
& \quad z = H_1 \\
& \quad \rho_{b1} \quad c_{p2} \quad c_{s2} \quad \phi_{p2} \quad \psi_{s2} \\
& \quad z = H_2 \\
& \quad \rho_{b2} \quad c_{p3} \quad c_{s3} \quad \phi_{p3} \quad \psi_{s3} \\
& \quad z = H_3 \\
& \quad \vdots \quad \vdots \\
& \quad \rho_{bn} \quad c_{pn} \quad \phi_{pn} \quad \psi_{sn} \\
& \quad z = H_n \\
Z & \quad \downarrow
\end{align*}
\]

Figure 1. \( n \)-layer waveguide model.

According to the potential functions and boundary conditions of each layer, a set of equations for solving the potential function under the waveguide model shown in figure 1 can be established. Through the solution of the equations, the potential function values of each layer in figure 1 can be obtained. The specific formula derivation is detailed in the references [1].

In this paper, the complex sound pressure \( p \) and the fluid layer displacement potential equation (1) satisfy the following relationship, and the following equation is solved by the FFM.

\[
p(r, z, \omega) = \rho_1 (2\pi f_0)^2 \phi_1
\]

2.2. Error Function

Bayesian method is used to inversion geoacoustic parameters [2], and the inversion solution is expressed by the Posterior Probability Density (PPD) of the parameters. Under bayesian theory, the observation data vector \( d \) and the model parameter vector \( m \) are regarded as random, according to the bayesian criterion, as shown in equation (2).

\[
P(m | d) = P(d | m)P(m) / P(d)
\]

Where \( P(m | d) \) represents PPD, \( P(d | m) \) as a function of \( m \) under a certain measurement data \( d \) is defined as a likelihood function \( L(m) \) that is determined by the form of the data and the statistical
distribution of data errors (including measurement errors and model errors). Considering that it is difficult to obtain independent estimation of error statistics in practice, the assumption of unbiased Gaussian error is adopted in the processing process [3], the form of likelihood function as shown equation (3):

$$L(m) = P(d|m) \propto \exp(-E(m))P(m)$$

(3)

Therefore, the equation (4) express the final error function $E(m)$ is as follows:

$$E(m) = K \sum_{f} \ln[B^{f}(m)] \left| p^{f}_{\text{mea}} \right|^{2}$$

(4)

$$B^{f}(m) = 1 - \left| \frac{p^{f}_{\text{FFM}(m)}}{p^{f}_{\text{mea}} \left| p^{f}_{\text{FFM}(m)} \right|^{2}} \right|^{2}$$

(5)

where $p^{f}_{\text{FFM}}$ is the predicted sound pressure field calculated by equation (1), $p^{f}_{\text{mea}}$ represents the actual measured sound pressure field, $f$ represents the sound source frequency, and $K$ represents the number of hydrophones. The matching degree between the sound pressure value given by the prediction model and the measured sound pressure is determined by the minimum value of $E(m)$.

2.3. Improved Optimization Algorithm

The application of bayesian theory to geoacoustic parameter inversion is explain the inversion parameters from the point of view of probability, the error function established according to bayesian theory as a criterion to retain better parameters in the process of optimization can well reflect the inversion parameters from the point of view of probability [4].

![Figure 2. Improved algorithm flow chart.](image-url)
In Bayesian theory, the Maximum A Posterior (MAP) model are estimated by Simulated Annealing Algorithm (SA). Then the PPD values of the parameters are obtained by Metropolis-Hasting sampling as the inversion solution.

The improved inversion flow chart is shown in figure 2, where \( m_1 \) and \( m_2 \) represents the wave velocity, density, wave velocity attenuation and sedimentary depth of the first and second layers of the seabed. In this paper, the part in the range of dotted line is improved [5-6]. According to the physical law of geoaoustic impedance, the geoaoustic impedance of the second layer is larger than that of the first layer, so micro-forward disturbance is added in the optimization to make the inversion process more in line with the physical law [7-8].

### 3. Simulation Analysis

According to the above-mentioned seabed model, two-layer seabed model with a layer of elastic sediment on the semi-infinite seabed is selected for analysis and discussion. In the simulation, the depth of the sound source and the depth of the receiving point are set to 20 m. The environmental parameters and inversion interval are shown in table 1.

| Inversion parameters | True value | Discussion value | Before improvement | After improvement |
|----------------------|------------|------------------|--------------------|-------------------|
| \( h_{/m} \)         | 100        | /                | /                  | /                 |
| \( c_{/m/s^{-1}} \)   | 1500       | /                | /                  | /                 |
| \( \rho_{/g/cm^3} \)  | 1.00       | /                | /                  | /                 |
| \( c_{/m/s^{-1}} \)   | 2000       | 1800-2200        | 1998.711±74.281    | 2001.781±21.340   |
| \( c_{/m/s^{-1}} \)   | 1000       | 900-1100         | 995.771±40.048     | 1003.152±17.623   |
| \( \rho_{/g/cm^3} \)  | 1.50       | 1.00-2.00        | 1.549±0.181        | 1.529±0.069       |
| \( a_{/dB/km} \)     | 0.10       | 0.10-0.11        | 0.099±0.004        | 0.100±0.001       |
| \( a_{/dB/km} \)     | 0.10       | 0.00-0.11        | 0.100±0.004        | 0.100±0.001       |
| \( h_{/m} \)         | 20         | 15-25            | 20.111±1.487       | 19.972±0.742      |
| \( c_{/m/s^{-1}} \)   | 2500       | 1800-2800        | 2296.279±198.416   | 2429.019±59.653   |
| \( c_{/m/s^{-1}} \)   | 1200       | 900-1400         | 1130.888±101.266   | 1205.547±28.319   |
| \( \rho_{/g/cm^3} \)  | 1.70       | 1.00-2.00        | 1.521±0.199        | 1.670±0.048       |
| \( a_{/dB/km} \)     | 0.10       | 0.09-0.11        | 0.100±0.004        | 0.100±0.001       |
| \( a_{/dB/km} \)     | 0.10       | 0.09-0.11        | 0.100±0.004        | 0.100±0.001       |

Through table 1, we can see that the mean and variance of the parameters have been significantly improved after the improved algorithm. The changes in wave velocity and density on the second floor are particularly obvious. From the point of view of mean deviation, \( \rho_3 \) has decreased from 8.1% to 5.8%, \( c_3 \) has decreased from 5.8% to 4.6%, and \( \rho_3 \) has decreased from 10.5% to 1.7%. From the point of view of variance change, the variance of \( c_3 \) decreased from 198.416 to 59.653, the variance of \( c_3 \) decreased from 101.266 to 28.319, and the variance of \( \rho_3 \) decreased from 0.199 to 0.048.

Figure 3 and 4 present the one dimension probability density function distribution of the corresponding parameters of the two-layer seabed model, which the red line represents the true value of each parameter. It can be seen from the diagram that after the improvement, the distribution of the parameters in the discussion area is narrower, the uncertainty of the parameters is less, and the effect of the parameters of the second layer is more obvious. Figure 5 shows that the sound transmission loss curve calculated by the average value of the inversion results of each parameter before and after the improved algorithm is compared with the transmission loss curve calculated by the true value. As seen in this comparison the transmission loss curve obtained by the improved optimization algorithm is in better agreement with the transmission loss curve under the true value, which further verifies the feasibility of the improved algorithm.
Figure 3. Posterior probability distribution of the first layer of parameters.

Figure 4. Posterior probability distribution of the second layer of parameters.
4. Conclusion
This paper studies a method of inversion of geoacoustic parameters of shallow water layered seabed model, selects a two-layered seabed model covered by a layer of elastic sediment on the semi-infinite seabed for simulation analysis. The optimization algorithm of the two-layer seabed model is improved by comparing the mean, variance and transmission loss of the inversion results before and after the improvement, it shows that the improved algorithm greatly improves the accuracy and stability of the inversion results, making the inversion result more in line with the setting of normal distribution of inversion results under ideal simulation conditions. Suitable for inversion of geoacoustic parameters of two-layer seabed model.

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