Reconfigurable Intelligent Surface Aided NOMA Networks

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Abstract

Reconfigurable intelligent surfaces (RISs) constitute a promising performance enhancement for next-generation (NG) wireless networks in terms of enhancing both their spectrum efficiency (SE) and energy efficiency (EE). We conceive a system for serving paired power-domain non-orthogonal multiple access (NOMA) users by designing the passive beamforming weights at the RISs. In an effort to evaluate the network performance, we first derive the best-case and worst-case of new channel statistics for characterizing the effective channel gains. Then, we derive the best-case and worst-case of our closed-form expressions derived both for the outage probability and for the ergodic rate of the prioritized user. For gleaning further insights, we investigate both the diversity orders of the outage probability and the high-signal-to-noise (SNR) slopes of the ergodic rate. We also derive both the SE and EE of the proposed network. Our analytical results demonstrate that the base station (BS)-user links have almost no impact on the diversity orders attained when the number of RISs is high enough. Numerical results are provided for confirming that: i) the high-SNR slope of the RIS-aided network is one; ii) the proposed RIS-aided NOMA network has superior network performance compared to its orthogonal counterpart.

Index Terms

NOMA, passive beamforming, reconfigurable intelligent surface.

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I. INTRODUCTION

The demand for next-generation (NG) networks having high energy efficiency (EE) has been rapidly increasing [1]. A variety of sophisticated wireless technologies have been proposed for NG networks, including massive multiple-input multiple-output (MIMO) and millimeter wave (mmWave) communications [2]. Recently, cost-efficient reconfigurable intelligent surfaces (RISs) have been proposed for cooperative NG networks [3]–[5].

To enhance both the spectrum efficiency (SE) and EE of NG networks, non-orthogonal multiple access (NOMA) has been proposed as a promising technique of opportunistically capitalizing on the users’ specific channel state information (CSI) differences [6]–[8]. NOMA networks are capable of serving multiple users at different quality-of-service (QoS) requirements in the same time/frequency/code resource block [9]–[11]. Hence, inspired by the aforementioned benefits of NOMA and RIS techniques, we explore the network’s performance enhanced by the intrinsic integration of the power-domain NOMA and RIS techniques in the downlink (DL).

A. Prior Work

In recent years, RIS based techniques have received considerable attention owing to their beneficial applications [12]–[14]. The RIS aided system comprises an array of intelligent surface units, each of which can independently absorb energy and shift the phase of the incident signal. By appropriately adjusting the reflection angles and amplitude coefficients of RIS elements, the electromagnetic signal can be reconfigured for wireless transmission. The performance of RIS-aided and relay-assisted networks was compared in [15], indicating that RIS-aided networks may have better network performances, provided that the number of RISs is high enough. The associated energy consumption model was proposed in [16], [17], where the EE of the proposed network was optimized. Numerous application scenarios, such as RISs aided physical layer security relying on cooperative jamming techniques have also been considered [18], [19]. The RIS components are capable of blocking the signal of eavesdroppers, hence enhancing the secrecy performance. RIS assisted simultaneous wireless information and power transfer (SWIPT) was proposed in [20] for the users located in coverage-holes. In the 5G new radio (NR) standard, the coverage area is significantly reduced for carriers beyond 6GHz [21]. Hence, a sophisticated

1In this article, we use NOMA to refer to power-domain NOMA for simplicity.
signal alignment strategy was employed at the RISs for coverage area enhancement in mmWave scenarios \[22\]. However, in most previous research, continuous amplitude coefficients and phase shifts were assumed at the RISs \[23\], whilst in practice the phase shifts of RISs may not be continuous. Thus discrete phase shifts were considered in \[24\] for a multiple-input single-output (MISO) assisted RIS network. The channel capacity of a RIS-aided MIMO network was maximized, where both analog and digital beamforming, as well as hybrid beamforming were considered \[25\]. Furthermore, focusing on the user’s fairness, a fairness-oriented design (FOD) was proposed in a RIS-aided MIMO network \[26\].

To further enhance both the SE and EE of the DL, NOMA and RIS techniques were integrated in \[27\]. The RISs can be deployed for enhancing the power level of the cell-edge users, where the cell-center users treat the reflected signal as interference \[27\]. Both continuous and discrete phase shifters were used in a RIS-aided MISO NOMA network \[28\]. Naturally, the BS-user link plays a key role \[29\]. A RIS-aided NOMA network was also investigated in \[30\], whilst the BS-user link and the BS-RIS link, as well as the RIS-user link were assumed to experience Rayleigh fading. The associated bit error ratio (BER) was evaluated in the case of Rayleigh fading in \[31\]. However, both the BS and RISs are part of the infrastructure, and the RISs are typically positioned for exploiting the line-of-sight (LoS) path with respect to the fixed BS in NG networks for increasing the received signal power. Hence, the impact of fading environments on RIS networks has also attracted attention \[32\]. A fairness-oriented algorithm was proposed in a RIS-aided NOMA network \[32\], where Rician fading channels were used for modelling the channel gains. Note that when the Nakagami and Rice fading parameter obey the following constraint \[m = \frac{(K+1)^2}{2K+1}\], these fading channels are identical \[33\] eq. (3.38)].

**B. Motivations and Contributions**

The above-mentioned papers mainly studied the network’s fairness, whilst there is a paucity of investigations on the SE improvement of NOMA networks. To comprehensively analyze the network’s performance enhanced by RISs, a RIS-aided SISO-NOMA network is proposed. Motivated by the potential joint benefits of RISs and NOMA networks, whilst relying on analog beamforming \[34\], in this article we will analyse the performance of a RIS-aided NOMA DL scenario, where a priority-oriented design (POD) is proposed, which is also capable of enhancing the SE. In the proposed POD, we improve the performance of the user having the
best channel gain, where all other users rely on RIS-aided beamforming. In contrast to previous contributions [26], we will show that the proposed POD outperforms the FOD in terms of its SE.

Against to above background, our contributions can be summarized as follows:

- We propose a novel RIS-aided NOMA network, where a POD is employed for enhancing the SE. The impact of the LoS transmission on the reflected BS-RIS-user links are exploited. Furthermore, the impact of the proposed design on the attainable performance is characterized in terms of its outage probability (OP), ergodic rate, SE and EE.
- Explicitly, we derive closed-form expressions of both the OP and of the ergodic rate for the proposed RIS-aided NOMA network. Both the best-case and worst-case of the OP and of the ergodic rate are derived. Both accurate and approximate closed-form expressions are derived. Furthermore, both the diversity orders and high-SNR slopes are obtained based on the OP and ergodic rate. The results confirm that the diversity order can be enhanced by increasing the number of RISs.
- The simulation results confirm our analysis, illustrating that: 1) the BS-user link can be ignored when the number of RISs is high enough; 2) the RIS-aided NOMA network relying on the optimal power allocation factors is capable of outperforming its OMA counterpart; 3) the SE of the proposed POD can be significantly enhanced compared to the FOD, when the number of RISs is high enough.

C. Organization and Notations

The rest of the paper is organized as follows. In Section II, the model of RIS-aided NOMA networks is discussed. Our analytical results are presented in Section III, while our numerical results are provided in Section IV for verifying our analysis, followed by our conclusions in Section V. The distribution of a circularly symmetric complex Gaussian (CSCG) random variable with mean $x$ and covariance matrix $k$ is denoted by $\mathcal{CN}(x, k)$; and $\sim$ stands for “distributed as”. $\mathbb{P}(\cdot)$ and $\mathbb{E}(\cdot)$ represent the probability and expectation, respectively.

II. System Model

Let us consider the RIS-aided NOMA DL, where a BS equipped with a single transmit antenna (TA) is communicating with $W$ users, each equipped with a single receive (RA) antenna. We
have $N > 1$ intelligent surfaces at the appropriate location. By appropriately adjusting the reflection angles and amplitude coefficients of the RIS elements, the electromagnetic signal can be beneficially manipulated. Fig. 1 illustrates the wireless communication model for a single BS.

A. RIS-Aided SISO-NOMA Network

We first provide a fundamental model to illustrate the network performance affected by RISs. In order to illustrate the LoS links between the BS and RISs, the small-scale fading vector is defined as

$$
\mathbf{h} = \begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix},
$$

where $\mathbf{h}$ is a $(N \times 1)$-element vector whose elements represent the Nakagami fading channel gains. The probability density function (PDF) of the elements can be expressed as

$$
f_1(x) = \frac{m_1^{m_1} x^{m_1-1}}{\Gamma(m_1)} e^{-m_1 x},
$$

where $m_1$ denotes the fading parameter, and $\Gamma(\cdot)$ represents the Gamma function. Note that $\Gamma(m_1) = (m_1 - 1)!$ when $m_1$ is an integer.

It is assumed that there are a total of $W$ users in the cluster, where the pair of users, user
$W$ and user $v$ with $1 \leq v < W$, are superimposed for DL transmission in NOMA. Hence, the small-scale fading vector between the RISs and user $W$ is defined as

$$\mathbf{g}_W = \left[ g_{W,1} \cdots g_{W,N} \right].$$  \hspace{1cm} (3)

Similarly, the small-scale fading vector between the RISs and user $v$ is given by

$$\mathbf{g}_v = \left[ g_{v,1} \cdots g_{v,N} \right], \hspace{1cm} (4)$$

where $\mathbf{g}_W$ and $\mathbf{g}_v$ are $(1 \times N)$-element vectors whose elements represent the Nakagami fading channel gains having fading parameters of $m_W$ and $m_v$, respectively.

Due to the strong scattering environment, the BS-user link between the BS and user $W$ as well as that between the BS and user $v$ are modelled by Rayleigh fading, which can be expressed as $r_W$ and $r_v$, respectively.

In DL transmission, the BS sends the following signal to the paired NOMA users:

$$\mathbf{s} = \alpha_v^2 s_v + \alpha_W^2 s_W, \hspace{1cm} (5)$$

where $s_v$ and $s_W$ denote the signal intended for user $v$ and user $W$, respectively, with $\alpha_v^2$ and $\alpha_W^2$ representing the power allocation factors of user $v$ and user $W$, respectively. Based on the NOMA protocol, $\alpha_v^2 + \alpha_W^2 = 1$.

Without loss of generality, we focus our attention on user $W$, and the signal received by user $W$ from the BS through RISs is given by

$$y_w = \left( \mathbf{g}_W \Phi d_1^{-\alpha_1} d_{2,w}^{-\alpha_2} + r_w d_{3,w}^{-\alpha_3} \right) p \mathbf{s} + N_0, \hspace{1cm} (6)$$

where $p$ denotes the transmit power of the BS, $\Phi \triangleq \text{diag} [\beta_1 \phi_1, \beta_2 \phi_2, \cdots, \beta_N \phi_N]$ is a diagonal matrix, which accounts for the effective phase shift applied by all intelligent surfaces, $\beta_n \in (0, 1]$ represents the amplitude reflection coefficient of RISs, while $\phi_n = \exp(j \theta_n), j = \sqrt{-1}, \forall n = 1, 2, \cdots, N$, and $\theta_n \in [0, 2\pi)$ denotes the phase shift introduced by the $n$-th intelligent surface. It is assumed that the CSIs are perfectly known at the RIS controller [24]. $d_1$ and $d_{2,w}$ denote the distance between the BS and RISs as well as that between the RISs and user $w$, while $d_{3,w}$ denotes the distance between the BS and user $w$. Furthermore, $\alpha_1$ and $\alpha_n$ denote the path loss exponent of the BS-RIS-user links and BS-user links. Finally, $N_0$ denotes the additive white
Gaussian noise (AWGN), which is modeled as a realization of a zero-mean complex circularly symmetric Gaussian variable with variance \( \sigma^2 \).

III. RIS DESIGN FOR THE PRIORITIZED USER IN NOMA NETWORKS

In this section, we first design the phase shifts and reflection amplitude coefficients for the RISs. Our new channel statistics, OPs, ergodic rates, SE and EE are illustrated in the following subsections.

A. RIS Design

When the direct BS-user signal and reflected BS-RIS-user signals are co-phased, the channel gain of user \( W \) is given by

\[
|h_W|^2 = |g_W \Phi h d_1^{-\alpha_1} d_2^{-\alpha_2} + r_W d_3^{-\alpha_3}|^2. \tag{7}
\]

It is assumed that there are \( W \) users in the cluster, and then the achievable channel gains of users \( 1 \cdots W \) are ordered as follows [35]:

\[
|h_1|^2 < |h_2|^2 < \cdots < |h_W|^2. \tag{8}
\]

We then turn our attention to the RIS design. It is assumed that the RISs mainly focus on providing maximum channel gain to the prioritized user for enhancing the SE. Without loss of generality, we assume that the prioritized user is the one having the best ordered channel gain.

In this article, in order to simultaneously control multiple RISs, the global CSI is assumed to be perfectly available at the RIS controller. Since user \( W \) is the prioritized user, we aim for maximizing users’ received power by designing the phase shifts and reflection amplitude coefficients of RISs as follows:

\[
Max(g_W \Phi h + r_W)
\]

subject to \( \beta_1 \cdots \beta_N = 1 \)

\[
\theta_1 \cdots \theta_N \in [0, 2\pi).
\]

Thus by utilizing our signal alignment technique, our objective can be achieved by phase-shifting the signals received at the RISs, which is capable of significantly improving the received power.
Thus, we first define a channel vector as follows:

\[ \tilde{h} = \begin{bmatrix} g_{W,1}h_1\phi_1 & \cdots & g_{W,N}h_N\phi_N \end{bmatrix}. \] (10)

Hence, the design of the \( n \)-th RIS can be expressed as

\[ \Theta(\tilde{h}_n) = \Theta(r_W), \] (11)

where \( \Theta(\cdot) = \arg(\cdot) \) denotes the angle of the element, and \( \tilde{h}_n \) denotes the \( n \)-th element of \( \tilde{h} \).

We then generate the effective vector of user \( W \) as follows:

\[ \bar{h} = \begin{bmatrix} g_{W,1}h_1 & \cdots & g_{W,N}h_N \end{bmatrix}. \] (12)

Thus, the phase shifts of the RISs can be further transformed into

\[ \Phi = \Theta(\bar{h})^{-1}\Theta(r_W). \] (13)

Since the phase shifts are designed for the prioritized user \( W \), the effective channel gain for user \( v \) can be written as

\[ |\tilde{h}_v|^2 = \left( \sum_{n=1}^{N} g_{v,n}\theta_n h_n d_1^{-\alpha_1} d_2^{-\alpha_2} + r_v d_3^{-\alpha_3} \right)^2, \]

which cannot be evaluated. However, for the effective channel gain we have:

\[ 0 \leq |\tilde{h}_v|^2 \leq |\tilde{h}_W|^2. \] (14)

We then consider the situation that two users, i.e. user \( W \) and user \( v \) having indexes of \( v < W \), are paired to perform NOMA.

**B. New Channel Statistics**

In this subsection, we derive new channel statistics for the proposed RIS-aided NOMA network, which will be used for evaluating the OPs and ergodic rates in the following subsections.

**Lemma 1.** Let us assume that the fading parameters of the elements in \( h \) and \( g_W \) are \( m_1 \) and \( m_W \), respectively. The elements of the channel vectors are independently and identically distributed (i.i.d.). On the one hand, the worst-case distribution of the effective channel gain of
user $W$ can be formulated as

$$\left| \tilde{h}_{W,l} \right|^2 \sim \Gamma \left( \frac{\left( N(d_1d_2W)^{-\alpha_l} + d_{3,W}^{-\alpha_n} \right)^2}{Nm_l(d_1d_2W)^{-2\alpha_l} + d_{3,W}^{-2\alpha_n}}, \frac{Nm_l(d_1d_2W)^{-2\alpha_l} + d_{3,W}^{-2\alpha_n}}{N(d_1d_2W)^{-\alpha_l} + d_{3,W}^{-\alpha_n}} \right),$$  \hspace{1cm} (15)

where $\Gamma (\cdot, \cdot)$ represents the Gamma distribution, and $m_l = \frac{(1+m_l+m_W)}{m_1m_W}$. On the other hand, the best-case distribution of the effective channel gain of user $W$ can be expressed by

$$\left| \tilde{h}_{W,u} \right|^2 \sim \Gamma \left( \frac{\left( N^2(d_1d_2W)^{-\alpha_l} + d_{3,W}^{-2\alpha_n} \right)^2}{N^2m_u(d_1d_2W)^{-2\alpha_l} + d_{3,W}^{-2\alpha_n}}, \frac{N^2m_u(d_1d_2W)^{-2\alpha_l} + d_{3,W}^{-2\alpha_n}}{N^2(d_1d_2W)^{-\alpha_l} + d_{3,W}^{-\alpha_n}} \right),$$  \hspace{1cm} (16)

where $m_u = \frac{1+Nm_1+Nm_W}{m_1m_W}$.

**Proof:** Please refer to Appendix A.

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C. Outage Probability

In this article, the OP of user $W$ is defined by

$$P_w = \mathbb{P} \left( \log_2(1 + \text{SINR}_W) < R_w \right) + \mathbb{P} \left( \log_2(1 + \text{SINR}_W) > R_w, \log_2(1 + \text{SINR}_W) < R_W \right),$$  \hspace{1cm} (17)

where $R_W$ and $R_v$ represent the target rates of user $W$ and user $v$, respectively.

We then focus our attention on the SINR analysis of user $W$ having the best channel gain. The cell-centre user $W$ first decodes the signal of the cell-edge user $v$ with the following SINR:

$$\text{SINR}_{W \rightarrow v} = \frac{\left| \tilde{h}_W \right|^2 p\alpha_v^2}{\left| \tilde{h}_W \right|^2 p\alpha_W^2 + \sigma^2}.$$  \hspace{1cm} (18)

Once the signal of user $v$ is decoded successfully, user $W$ decodes its own signal at an SINR of:

$$\text{SINR}_W = \frac{\left| \tilde{h}_W \right|^2 p\alpha_W^2}{\sigma^2}.$$  \hspace{1cm} (19)

Let us now turn our attention to calculating the OP of user $W$ based on Theorems 1 and 2.

**Theorem 1.** Assuming that $\alpha_v^2 - \alpha_W^2 > 0$, the worst-case of the closed-form OP expression
of user $W$ can be expressed as

$$P_{W,u} = \frac{\gamma \left( k_1, \frac{I_{W}}{\lambda_1} \right)}{\Gamma(k_1)}^W,$$

(20)

where $I_{W*} = \max \{ I_{w}, I_v \}$, $I_{v} = \frac{\varepsilon_v \sigma_v^2}{p(\alpha_v^2 - \alpha_W^2 \varepsilon_v)}$, $I_{W} = \frac{\varepsilon_W \sigma_W^2}{p\alpha_W}$, $\varepsilon_v = 2^{R_v} - 1$, $\varepsilon_W = 2^{R_W} - 1$, $k_1 = \frac{N(\alpha^2 + d_{W})}{N \alpha_W(d_{W})^{2\alpha_W}}$, and $\gamma(\cdot)$ represents the lower incomplete Gamma function.

**Proof:** Please refer to Appendix B.

**Theorem 2.** Assuming that $\alpha_v^2 - \alpha_W^2 \varepsilon_v > 0$, the best-case of the closed-form OP expression of user $W$ can be expressed as

$$P_{W,l} = \frac{\gamma \left( k_2, \frac{I_{W}}{\lambda_2} \right)}{\Gamma(k_2)}^W,$$

(21)

where $k_2 = \frac{N^2(\alpha^2 + d_{W})}{N \alpha_W(d_{W})^{2\alpha_W}}$, and $\lambda_2 = \frac{N^{2\mu}(d_{W})^{2\alpha_W}}{N^2(\alpha^2 + d_{W})^{2\alpha_W}}$.

**Proof:** Similar to Appendix B. Theorem 2 can be readily proved.

It is however quite challenging to directly obtain engineering insights from (20) and (21) due to the $W$-th power of the lower incomplete Gamma function. Thus, in order to gain further insights in the high-SNR regime, the approximate behaviors are analyzed, when the SNR is sufficiently high, i.e. when the transmit SNR obeys $\frac{P}{\sigma^2} \to \infty$.

**Corollary 1.** Assuming that $\alpha_v^2 - \alpha_W^2 \varepsilon_v > 0$, the worst-case and best-case of the OP can be approximated in closed form by

$$\bar{P}_{W,u} = \left( \sum_{s=0}^{\infty} \frac{1}{\Gamma(k_1 + s + 1)} \left( \frac{I_{W*}}{\lambda_1} \right)^s \right)^W \sum_{j=0}^{W} \left( \begin{array}{c} W \\ j \end{array} \right) (-1)^j \left( \frac{I_{W*}}{\lambda_1} \right)^{k_{1W} + j},$$

(22)

and

$$\bar{P}_{W,l} = \left( \sum_{s=0}^{\infty} \frac{1}{\Gamma(k_2 + s + 1)} \left( \frac{I_{W*}}{\lambda_2} \right)^s \right)^W \sum_{j=0}^{W} \left( \begin{array}{c} W \\ j \end{array} \right) (-1)^j \left( \frac{I_{W*}}{\lambda_2} \right)^{k_{2W} + j}.$$
OP in the following Propositions.

**Proposition 1.** Based on Corollary 1, the diversity orders can be determined by using the approximate results, explicitly the worst-case and best-case on the diversity order of user $W$ supported by the proposed RIS-aided NOMA network are given by

$$d_{W,u} = - \lim_{\frac{P}{\sigma^2} \to \infty} \frac{\log \bar{P}_{W,u}}{\log \frac{P}{\sigma^2}} \approx k_1 W, \quad (24)$$

and

$$d_{W,l} = - \lim_{\frac{P}{\sigma^2} \to \infty} \frac{\log \bar{P}_{W,l}}{\log \frac{P}{\sigma^2}} \approx k_2 W. \quad (25)$$

**Remark 1.** The results of (24) demonstrate that the diversity orders can be approximated by $\frac{NW}{m_l}$ for the prioritized user $W$, when the number of RISs is high enough. It is also demonstrated that increasing the number of RISs and carefully pairing the NOMA users is capable of significantly improving the outage performance.

**Remark 2.** Assuming that $m_1 \to \infty$, which indicates a strong LoS link between the BS as well as the RISs, and provided that the number of RISs is high enough, the diversity orders on both the best-case and worst-case of the prioritized user $W$ can be approximated by $NWm_W$.

**Remark 3.** Since the users are ordered by their effective channel gain, and based on the results of (24), in order to minimize the OP of the paired NOMA users, it is preferable to pair the users having the best and the second best effective channel gains.

**Remark 4.** Assuming that the number of RISs is high enough, and based on the results of (25), the diversity order of the worst-case on the OP can be further approximated by $\frac{NWm_1m_W}{m_1+m_W}$. Again, assuming that $m_1 \to \infty$, both the worst-case and best-case on the diversity order of user $W$ can be approximated by $NWm_W$, which indicates that the diversity order of both the best-case and worst-case of the OP are identical.

**Remark 5.** Assuming that the BS-user link of user $W$ is the dominant component, where the path loss exponent $\alpha_N = \alpha_L$ as well as $d_1d_{2,W} >> d_{3,W}$, the diversity order of both the best-case and worst-case are $W$.

Due to the impact of hostile fading environments in NG networks, it is worth mentioning
that no BS-user link may be available between the BS and the paired NOMA users, and the approximate result mainly depends on the 0-th ordered element in (22) and (23). Thus, we continue by providing basic numerical insights using the following Corollary.

**Corollary 2.** Due to the hostile fading environment between the BS and the users in NG networks, and assuming that \( \alpha_v^2 - \alpha_{Wv}^2 > 0 \), the 0-th ordered elements in terms of the worst-case and best-case on the approximate OP of user \( W \) are given by

\[
\hat{P}_{W,u} = \frac{1}{\Gamma (\varphi_1 + 1)^W} \sum_{j=0}^{W} \binom{W}{j} (-1)^j \left( \frac{I_{W*}}{m_l(d_1d_2,W)^{-\alpha_l}} \right)^{-(\varphi_1W+j)},
\]

and

\[
\hat{P}_{W,l} = \frac{1}{\Gamma (\varphi_2 + 1)^W} \sum_{j=0}^{W} \binom{W}{j} (-1)^j \left( \frac{I_{W*}}{m_u(d_1d_2,W)^{-\alpha_l}} \right)^{-(\varphi_2W+j)},
\]

where \( \varphi_1 = \frac{N}{m_l} \), and \( \varphi_2 = \frac{N^2}{m_u} \).

**Remark 6.** Assuming that no BS-user links are expected between the BS and the prioritized NOMA user, based on results of (26), the best-case and worst-case on the diversity orders of user \( W \) are seen to be \( \frac{N^2W}{m_u} \) and \( \frac{NW}{m_l} \), respectively.

**D. Ergodic Rate**

The ergodic rate is a salient performance metric related to the SE and EE. Therefore, we focus our attention on analyzing the ergodic rate of user \( W \). The approximate ergodic rate expressions of user \( W \) are given in the following Theorems.

**Theorem 3.** Assuming that \( N \) RISs simultaneously serve user \( W \), and \( \alpha_v^2 - \alpha_{Wv}^2 > 0 \), the worst-case on the ergodic rate of user \( W \) can be expressed in closed form as follows:

\[
R_{W,l} = \sum_{s=0}^{W} \binom{W}{s} (-1)^s \frac{1}{\bar{k}} \prod_{a_1+\ldots+a_{\bar{k}}=s} \left( \frac{s}{a_1, \ldots, a_{\bar{k}}} \right) \prod_{t=1}^{\bar{k}} \left( \frac{(C)^{t-1}}{(t-1)!} \right)^{a_t} \\
\times \exp(Cs)Ei(-Cs) + \sum_{i=1}^{(t-1)a_{\bar{t}}} (-1)^{i-1}(i-1)! (Cs)^i,
\]

where \( C = \frac{\alpha_v^2}{\lambda_1p_0\alpha_{Wv}} \), and \( \bar{k} = [k_1] \) is obtained by rounding \( k_1 \) to the nearest integer.
Proof: Please refer to Appendix D.

Similarly, the best-case on the ergodic rate of user $W$ is formulated in the following Theorem.

**Theorem 4.** Assuming that $N$ RISs simultaneously serve user $W$, and $\alpha_v^2 - \alpha_W^2 \epsilon_v > 0$, the best-case on the ergodic rate of user $W$ can be expressed in closed form as follows:

$$R_{W,u} = \sum_{s=0}^{W} \binom{W}{s} (-1)^s \frac{1}{k} \sum_{a_1+...+a_k_s=s} \binom{s}{a_1, \ldots, a_k_s} \prod_{t=1}^{k} \left( \frac{(C_u)^{t-1}}{(t-1)!} \right)^{a_t}$$

$$\times \left( \exp(C_u s) Ei(-C_u s) + \sum_{i=1}^{(t-1)a_t} (-1)^{i-1}(i-1)!(C_u s)^i \right),$$

where $C_u = \frac{\sigma_v^2}{\lambda_{2pW}}$, and $\bar{k}_u = [k_2]$.

*Proof: Similar to Appendix D, the results in (29) can be readily obtained.*

To gain deep insights into the system’s performance, the high-SNR slope, as the key parameter determining the ergodic rate in the high-SNR regime, is worth estimating. Therefore, we first express the high-SNR slope as

$$S^W_{\infty} = - \lim_{\frac{R_W}{\sigma^2} \to \infty} \frac{R_W}{\log_2 (1 + \frac{R_W}{\sigma^2})}. \tag{30}$$

**Proposition 2.** By substituting (28) and (29) into (30), the high-SNR slope of user $W$ is given by

$$S^W_{\infty} = 1. \tag{31}$$

**Remark 7.** The results of (31) illustrate that the slope of the ergodic rate in the proposed RIS-aided NOMA network is one, which is not affected by the number of RISs.

Based on the passive beamforming weight design at the RISs, the distribution of NOMA user $v$, having the lower received power, cannot be evaluated. Hence, we only provide the associated SINR analysis for simplicity. By relying on the NOMA protocols, user $v$ treats the signal from user $W$ as interference, and the SINR is given by

$$SINR_v = \frac{(|g_v \Phi h + r_v|^2)\alpha_v^2 p}{(|g_v \Phi h + r_v|^2)\alpha_W^2 p + \sigma^2}. \tag{32}$$
Since the elements in $\Phi$ are gleaned from random variables, and based on the insights from [34], the SINR of user $v$ can be further approximated as:

$$\text{SINR}_v = \frac{(|g_v h|^2 G_N(\bar{\theta}) + |r_v|^2) \alpha_v^2 \epsilon_v}{(|g_v h|^2 G_N(\bar{\theta}) + |r_v|^2) \alpha_W^2 \epsilon_v + \sigma^2},$$

(33)

where $G_N(\bar{\theta})$ denotes the normalized Fejér Kernel function with parameter $N$. Note that $G_N(\bar{\theta})$ has a period of two, hence $\bar{\theta}$ is uniformly distributed over $[-1, 1]$. Thus, the ergodic rate of user $v$ can be expressed as follows.

**Theorem 5.** Assuming that $N$ RISs simultaneously serve user $W$, and $\alpha_v^2 - \alpha_W^2 \epsilon_v > 0$, the worst-case and best-case on the ergodic rate of user $v$ can be expressed as follows:

$$R_{v,l} = \frac{1}{\ln(2)} \int_0^{\alpha_W^2} \frac{1 - F_{v,l}(x)}{1 + x} dx,$$

(34)

and

$$R_{v,u} = \frac{1}{\ln(2)} \int_0^{\alpha_W^2} \frac{1 - F_{v,u}(x)}{1 + x} dx,$$

(35)

where $F_{v,l}(x) = \left(\frac{\gamma(k_1 C_{v,l})}{\Gamma(k_1)}\right)^v$, $F_{v,u}(x) = \left(\frac{\gamma(k_2 C_{v,u})}{\Gamma(k_2)}\right)^v$, $C_{v,l} = \frac{\sigma^2 x}{\lambda_1 G_N(\bar{\theta}) p(\alpha_v^2 - \alpha_W^2 \epsilon_v)}$, and $C_{v,u} = \frac{\sigma^2 x}{\lambda_2 G_N(\bar{\theta}) p(\alpha_v^2 - \alpha_W^2 \epsilon_v)}$.

**Proof:** Similar to Appendix D, the results can be readily derived.

**Remark 8.** Let us assume that $\bar{\theta} \to 0$, indicating that the paired NOMA users share an identical channel vector, the Fejér Kernel function can be considered as one. Hence, based on the insights in [36], the best-case and worst-case on the ergodic rate of user $v$ may approach $R_v = \log_2 \left(1 + \frac{\alpha_v^2}{\alpha_W^2}\right)$ in the high-SNR regime.

In order to provide further insights for RIS-aided NOMA networks, the ergodic rate of the paired users is also analysed in the OMA scenario using TDMA. The OMA benchmark adopted in this article relies on supporting user $W$ and user $v$ in a pair of identical time slots. In each time slot, the RISs provide access only for one of the users. Thus, the channel capacity of user $W$ in the OMA scenario can be expressed as

$$R_{W,O} = \mathbb{E}\left\{\frac{1}{2} \log_2 (1 + \text{SNR}_{W,O})\right\},$$

(36)
where $SINR_{W,O} = \frac{\left(\sum_{n=1}^{N} |g_{W,n}h_n|\right)^2 (d_1d_2,W)^{-\alpha_l} + |r_W|^2 d_3^{-\alpha_n}/\sigma^2}{p}$. Similarly, the channel capacity of user $v$ can be expressed as

$$R_{v,O} = \mathbb{E} \left\{ \frac{1}{2} \log_2 (1 + SINR_{v,O}) \right\},$$

(37)

where $SINR_{v,O} = \frac{\left(\sum_{n=1}^{N} |g_{v,n}h_n|\right)^2 (d_1d_2,v)^{-\alpha_l} + |r_v|^2 d_3^{-\alpha_n}/\sigma^2}$. 

### E. Spectrum Efficiency and Energy Efficiency

Based on the analysis of the previous two subsections, a tractable SE expression can be formulated in the following Proposition.

**Proposition 3.** The SE of the proposed RIS-aided NOMA network is given by

$$S = R_v + R_W.$$  

(38)

In NG networks, EE is an important performance metric. Thus, based on insights gleaned from [37], we first model the total power dissipation of the proposed RIS-aided NOMA network as

$$P_e = P_{B,s} + 2P_U + p\varepsilon_b + NP_L,$$

(39)

where $P_{B,s}$ is the static hardware power consumption of the BS, $\varepsilon_b$ denotes the efficiency of the power amplifier at the BS, $P_U$ is the power consumption of each user, and $P_L$ represents the power consumption of each RIS controller. Thus, the EE of the proposed network is given by the following Proposition.

**Proposition 4.** The EE of the proposed RIS-aided NOMA network is

$$\Theta_{EE} = \frac{S}{P_e},$$

(40)

where $S$ and $P_e$ are obtained from (38) and (39), respectively.
IV. Numerical Studies

In this section, numerical results are provided for the performance evaluation of the proposed network. Monte Carlo simulations are conducted for verifying the accuracy of our analytical results. The bandwidth of the DL is set to $BW = 1$ MHz, and the power of the AWGN is set to $\sigma^2 = -174 + 10\log_{10}(BW)$ dBm. The power attenuation at the reference distance is set to -30 dB, and the reference distance is set to 1 meter. Note that the LoS and NLoS links are indicated by the Nakagami fading parameter, where $m = 1$ and $m > 1$ are for NLoS and for LoS links, respectively. The target rates are $R_W = 1.5$ and $R_v = 1$ bits per channel use (BPCU). The power allocation factors of the paired NOMA users are set to $\alpha_v^W = 0.6$ and $\alpha_v^W = 0.4$. The number of users is set to $W = 2$, and $v = 1$. The fading environments are set to $m_1 = m_W = m_v = 1$. The length of the BS-RIS link is set to $d_1 = 60$ m. The length of the RIS-user links are set to $d_{2,W} = 80$ m and $d_{2,v} = 100$ m, and these of the BS-user links are set to $d_{3,W} = d_{3,v} = 100$ m. The path loss exponents of the reflected BS-RIS-user and the direct BS-user links are set to $\alpha_l = 2.2$ as well as $\alpha_n = 3.5$, respectively, unless otherwise stated.

1) Impact of the Number of RISs: In Fig. 2, we focus our attention on the OP of the RIS-aided NOMA network. The solid curves and dashed curves represent the worst-case and best-case of the analytical results, respectively. We can see that as the number of RISs serving user $W$ increases, the OP decreases. This is due to the fact that, as more RISs are employed, the received signal power can be significantly increased as a benefit of the increased diversity order. Observe that
Fig. 3: OP of the RIS-aided NOMA network versus the SNR parameterized by fading factors. The number of RISs is set to $N = 3$.

Fig. 4: OP of the RIS-aided NOMA network versus the SNR parameterized by the number of users. The number of RISs is set to $N = 3$. The path loss exponents are set to $\alpha_n = \alpha_l = 3$.

the slope of the curves increases with the number of RISs, which validates our Remark 1. Let us assume that $d_{2,W} = d_{3,W}$ and $\alpha_N = \alpha_L$, then the minimum diversity order that can be obtained is 1 for the case of $m_1 = m_W = 1$ and $N = 1$, which is identical to that of the non-RIS-aided networks. Observe that as expected the simulation results are located between the best and worst cases, which verifies Remark 4.

2) Impact of Fading Environments: In Fig. 3, we evaluate the OP of the prioritized user $W$ in different fading environments. As expected, with the transmit power increases, the OP decreases. Observe that both the BS-RIS as well as RIS-user links have an impact on the OP, which is in contrast to the FOD of [26], where the fading environment of the RIS-user link has almost no effect on the OP.
3) Impact of the Number of Users: Let us now study the impact of the number of users in Fig. 4. Observe that it is preferable to pair the users having the best effective channel gains for minimizing the OP. Based on the results in the high-SNR regime, the diversity order is seen to be significantly enhanced by increasing the number of users, because they experience independent fading channels. It is also worth noting that the diversity order is $W$, which verified by the insights gleaned from Remark 2. This is because when the path loss exponent is $\alpha_l = \alpha_n = 3$, the power received from links reflected by the RISs can be nearly ignored.

4) Ergodic Rate: Fig. 5 compares the ergodic rates of paired NOMA users versus the SNR parameterized by the fading parameters and by the number of RISs. Several observations can be drawn as follows: 1) Based on the curves in Fig. 5(a) we can observe that the LoS links of both the BS-RIS as well as of the RIS-user links increase the ergodic rate of user $W$, where the ergodic rate approaches the best-case for the case of $m_1 \to \infty$. 2) The triangles are between the best-case and worst-case, which verify the accuracy of our results. 3) As seen from the figure, the high-SNR slope of user $W$ is one, which also verifies Remark 8. 4) The ergodic rate can be significantly increased by employing more RISs, which is because the spatial diversity gain can be significantly increased upon increasing the number of RISs. 5) The ergodic rate of conventional NOMA dispensing with RISs is provided as the benchmark schemes, which can be calculated by setting the number of RISs to $N = 0$. 6) Fig. 5(b) evaluates the ergodic rate of the non-prioritized user $v$. Observe that in the high-SNR regime, the slope of user $v$ approaches zero.
TABLE I:
DIVERSITY ORDER AND HIGH-SNR SLOPE

| Access Mode       | Rx | D   | S   |
|-------------------|----|-----|-----|
| RIS-aided NOMA    | $W$| $\frac{N \cdot m_1 \cdot W}{m_1 + m_W}$ | 1   |
|                   | $v$| $\frac{N \cdot m_1 \cdot v}{m_1 + m_v}$ | 0   |
| Conventional NOMA | $W$| $W$ | 1   |
|                   | $v$| $v$  | 0   |
| OMA               | $W$| $W$ | 0.5 |
|                   | $v$| $v$  | 0.5 |

Fig. 6: SE of both the RIS-aided NOMA and OMA networks versus the SNR and the number of RISs. The fading parameters are set to $m_1 = m_W = m_v = 3$.

in Fig. 5(b) which indicates that the number of RISs has no significant impact on the ergodic rate of user $v$. In TABLE I we use D and S to represent the diversity order and high-SNR slope for the case that $N$ is large enough, respectively. It is worth noting that the diversity order of the non-prioritized user $v$ is the optimized result, which can only be obtained by setting $\tilde{\theta} \to 0$.

5) Comparing the RIS-aided NOMA to an OMA Network: In Fig. 6 we then evaluate the SE of our RIS-aided NOMA network, as well as that of its OMA counterpart. The results of the RIS-aided NOMA and OMA networks are derived by $R_W + R_v$ and $R_{W,O} + R_{v,O}$, respectively. We can see that the RIS-aided NOMA network is capable of outperforming its OMA counterpart in terms of its SE by appropriately setting the power allocation factors. Observe that the SE gap between the RIS-aided NOMA network and its OMA counterpart becomes higher, when the number of RISs is increased, which indicates that it is preferable to employ more RISs for enhancing the SE.
Fig. 7: SE of the proposed RIS-aided NOMA network versus the SNR and the number of RISs. The results of POD and FOD are calculated from [38] and [26]. The fading parameters are set to $m_1 = m_W = m_v = 3$.

6) Comparing the POD to the FOD: In Fig. 7 we evaluate the SE of the proposed POD. The SE of the FOD in [26] is provided as the benchmark schemes. Observe from the figure that the SE of the POD is higher than the FOD of [26], which indicates that the proposed POD becomes more competitive compared to the FOD. This is due to the fact that the proposed POD is conceived for attaining the maximum network throughput for the prioritized user, which is capable of providing higher SE. By contrast, the FOD is mainly focused on the fairness, which can provide higher throughput for the cell-edge users.

7) Comparing Half-duplex and Full-duplex Relay Networks:
In order to provide further engineering insights, combined with the insights inferred from [38]–[40], the network throughputs of alternative full-duplex (FD) and half-duplex (HD) cooperative networks are evaluated. We consider the classic relaying protocols, where the transmissions of HD-relaying are divided into two identical phases. By contrast, the FD-relay is suffering from self-interference. It is assumed that both the BS and relay, as well as the users are equipped by a single antenna. Similar to (7), we first rank the entire set of \( W \) users according to their effective channel gains. We then evaluate the SE of the FD network, where the FD-relay has to decode the signal of the paired NOMA users. Based on the insights from Remark 6 for simplicity, it is assumed that no BS-user link exists. Hence, the FD-relay first decodes the signal of user \( v \), achieving the following expectation:

\[
R_{F,v} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{p|h_{R,1}|^2 d_1^{-\alpha} \alpha_v^2}{p|h_{R,1}|^2 d_1^{-\alpha} \alpha_W^2 + p_d \epsilon_H + \sigma^2} \right) \right\}, \tag{41}
\]

where \( |h_{R,1}|^2 \) denotes the channel gain between the BS and the FD-relay, \( \epsilon_H \) denotes the self-interference coefficient of the FD-relay itself, and \( p_d \) denotes the transmit power of the relay. Then the FD-relay can decode the signal of user \( W \) as follows:

\[
R_{F,W} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{p|h_{R,1}|^2 d_1^{-\alpha} \alpha_W^2}{p_d \epsilon_H + \sigma^2} \right) \right\}, \tag{42}
\]

Similarly, assuming that SIC can be also invoked successfully by the paired NOMA users, and thus the non-prioritized user \( v \) treats the signal of user \( W \) as interference, and the expected data rate can be given by

\[
R_v = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{p_d|h_{R,v}|^2 d_2^{-\alpha} \alpha_v^2}{p_d|h_{R,v}|^2 d_2^{-\alpha} \alpha_W^2 + \sigma^2} \right) \right\}, \tag{43}
\]

where \( |h_{R,v}|^2 \) denotes the channel gain between the FD-relay and user \( v \). On the other hand, by utilizing SIC technique, the transmission rate of user \( W \) is given by

\[
R_W = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{p_d|h_{R,W}|^2 d_2^{-\alpha} \alpha_W^2}{\sigma^2} \right) \right\}. \tag{44}
\]

More specifically, the size of data rate for user \( v \) and user \( W \) depend on four kinds of data rates, such as 1) the data rate for the relay to detect user \( v \); 2) the data rate for the relay to detect user \( W \); 2) The data rate for user \( v \); and 3) the data rate for user \( W \). Among the FD-relay in
the network, based on (41) to (44), the expected rate of the paired NOMA users in the FD-relay network can be given by

$$\bar{R}_{F,v} = \min \{ R_{F,v}, R_v \}, \quad (45)$$

and

$$\bar{R}_{F,W} = \min \{ R_{F,W}, R_W \}. \quad (46)$$

We then consider the HD-relay network, where the expected data rate of the paired NOMA users at the HD-relay can be given by

$$R_{H,v} = \mathbb{E} \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{p|h_{R,1}|^2 d_1^{-\alpha_l} \alpha_v^2}{p|h_{R,1}|^2 d_1^{-\alpha_l} \alpha_W^2 + \sigma^2} \right) \right\}, \quad (47)$$

and

$$R_{H,W} = \mathbb{E} \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{p|h_{R,1}|^2 d_1^{-\alpha_l} \alpha_W^2}{\sigma^2} \right) \right\}. \quad (48)$$

By applying the classic SIC technique, the expected data rate of paired NOMA users can be written as

$$\bar{R}_v = \mathbb{E} \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{p_d|h_{R,v}|^2 d_2^{-\alpha_l} \alpha_v^2}{p_d|h_{R,v}|^2 d_2^{-\alpha_l} \alpha_W^2 + \sigma^2} \right) \right\}, \quad (49)$$

and

$$\bar{R}_W = \mathbb{E} \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{p_d|h_{R,W}|^2 d_2^{-\alpha_l} \alpha_W^2}{\sigma^2} \right) \right\}. \quad (50)$$

Thus, the expected rate of the paired NOMA users in the HD-relay network is given by

$$\bar{R}_{H,v} = \min \{ R_{H,v}, \bar{R}_v \}, \quad (51)$$

and

$$\bar{R}_{H,W} = \min \{ R_{H,W}, \bar{R}_W \}. \quad (52)$$

In Fig. 8 we evaluate the network throughput of our RIS-aided NOMA network, as well as of the HD-relay and FD-relay aided networks. The results of the FD-relay and HD-relay are given by $\bar{R}_{F,v} + \bar{R}_{F,W}$ and $\bar{R}_{H,v} + \bar{R}_{H,W}$, respectively. The transmit power of the HD-relay and FD-relay are set to $p_d = (p - 10)$ dBm. We can see that the network throughput gap between the RIS-aided NOMA network and the other pair of relay aided networks becomes smaller, when
the number of RISs is increased. Observe that for the case of $N = 18$, the proposed RIS-aided NOMA network is capable of outperforming two relay aided networks, which indicates that the RIS-aided NOMA network becomes more competitive, when the number of RISs is high enough.

8) Energy Efficiency: Fig. 9 evaluates the EE of the proposed RIS-aided NOMA network versus the number of RISs. On the one hand, it is can be observed that the EE improves as the number of RISs increases. However, observe that the slope of the EE curve is decreasing, which indicates that there exists an optimal value of the number of RISs that maximizes the EE. Furthermore, in contrast to the conventional relay networks [41], it is worth noting that the EE can be increased upon increasing the transmit power at the BS.

V. CONCLUSIONS

In this article, we first reviewed the recent advances in RIS-aided networks. In order to illustrate the impact of RISs, we adopted a SISO network, where the passive beamforming weights of the RISs were designed. Both the best-case and worst-case of our new channel statistics, OPs, ergodic rates, EEs and SEs were derived in closed-form for characterizing the system performance. In NG networks, an important future direction is to extend the proposed model to RIS-aided MIMO-NOMA network, where the active beamforming and passive beamforming, as well as detection vectors can be jointly designed for improving the network’s throughput.
Appendix A: Proof of Lemma

In wireless communications, a pair of wave sources may be deemed perfectly coherent if they only have a constant phase difference but the same frequency, as well as the same waveform. In this context, a RIS array exhibit spatial coherence because its elements at the opposite ends of the array have a fixed phase relationship. Therefore, there are two potential scenarios for modeling the channel gain of RIS-aided networks: 1) When the length of the BS-user link and BS-RIS-user link is nearly identical, the received signals can be coherent by considered, and thus the channel gain is given by

\[ |\tilde{h}_W|^2 = |g_W \Phi h d_1^{-\alpha_l} d_2^{-\alpha_l} + r_W d_3^{-\alpha_l}|^2. \]  

(A.1)

2) On the other hand, since in practice, the length of the BS-user and BS-RIS-user links usually varies substantially, the received signals cannot be considered as coherent waves \cite{42}. Thus, when the reflected BS-RIS-user and the direct BS-user signals are co-phased, the BS-user and the BS-RIS-user signals can be boosted at the prioritized user \( W \) by utilizing the classic maximum ratio combining (MRC) technique having the following channel gain:

\[ |\tilde{h}_W|^2 = |g_W \Phi h|^2 (d_1 d_2, W)^{-\alpha_l} + |r_W|^2 d_3^{-\alpha_l}. \]  

(A.2)

Note that for the first scenario, where the direct BS-user and the reflected BS-RIS-user links are coherent waves, the results of (A.1) can be transformed into (A.2), provided that the number of RISs is high enough. Thus, in the rest of this article, we only analyze the network’s performance for the second scenario.

Based on the passive beamforming design in (9), the effective channel gain of the prioritized user \( W \) can be written as

\[ |\tilde{h}_W|^2 = \left( \sum_{n=1}^{N} |g_{W,n} h_n| \right)^2 (d_1 d_2, W)^{-\alpha_l} + |r_W|^2 d_3^{-\alpha_l}. \]  

(A.3)

By exploiting the fact that the elements of \( |g_W| \) and \( |h| \), as well as of \( |r_W| \) are i.i.d., the worst-case of effective channel gain matrix can be transformed into
\[ |\tilde{h}_W|^2 = \sum_{n=1}^{N} |g_{W,n}h_n|^2 (d_1 d_{2,W})^{-\alpha_l} + |r_W|^2 d_{3,W}^{-\alpha_l} \]  
\[ = \sum_{n=1}^{N} |g_{W,n}|^2 |h_n|^2 (d_1 d_{2,W})^{-\alpha_l} + |r_W|^2 d_{3,W}^{-\alpha_l}. \]  

(A.4)

Note that the elements of the channel matrix \(|g_W|\) and \(|h|\) obey the Nakagami distribution having the fading parameters \(m_W\) and \(m_1\), respectively. By exploiting the property of random variables, we obtain the mean and variance as follows

\[ E_1 = \mathbb{E}(|g_{W,n}|^2) \mathbb{E}(|h_n|^2) = 1, \]

(A.5)

and

\[ V_1 = \left( \mathbb{E}(|g_{W,n}|^2)^2 + V(|g_{W,n}|^2) \right) \left( \mathbb{E}(|h_n|^2)^2 + V(|h_n|^2) \right) \]

\[ - \mathbb{E}(|g_{W,n}|^2)^2 \mathbb{E}(|h_n|^2)^2 = \frac{(1 + m_1 + m_W)}{m_1 m_W}. \]

(A.6)

Thus, the distribution can be written as

\[ |g_{W,n}h_n|^2 \sim \mathcal{CN}(1, m_l), \]

(A.7)

where \(m_l = \frac{1 + m_1 + m_W}{m_1 m_W}\), and thereby the mean and variance of the effective channel gain is given by

\[ \sum_{n=1}^{N} |g_{W,n}h_n|^2 (d_1 d_{2,W})^{-\alpha_l} \sim \mathcal{CN}(N(d_1 d_{2,W})^{-\alpha_l}, Nm_l(d_1 d_{2,W})^{-2\alpha_l}). \]

(A.8)

Due to the fact that user \(W\) also detects the signal transmitted from the direct BS-user link, the worst-case of the effective channel gain of user \(W\) can be rewritten as

\[ |\tilde{h}_{W,l}|^2 \sim \mathcal{CN}(N(d_1 d_{2,W})^{-\alpha_l} + d_{3,W}^{-\alpha_l}, Nm_l(d_1 d_{2,W})^{-2\alpha_l} + d_{3,W}^{-2\alpha_l}). \]

(A.9)

After some algebraic manipulations, we obtain the effective channel gain in a more elegant form in (15).

We then turn our attention to the best-case of the effective channel gain of user \(W\). By
exploiting Cauchy-Schwarz inequality \[43\], the effective channel gain can be written as
\[
\left( \sum_{n=1}^{N} |g_{W,n}h_n| \right)^2 \leq \left( \sum_{n=1}^{N} |g_{W,n}|^2 \right) \left( \sum_{n=1}^{N} |h_n|^2 \right). \tag{A.10}
\]

Similar to the procedures in (A.5) to (A.9), the best-case of the distribution can be obtained as follows:
\[
\left| \tilde{h}_{W,l} \right|^2 \sim \mathcal{CN} \left( N^2(d_1d_{2,W})^{-\alpha_l} + d_3^{2\alpha_l}, N^2m_u(d_1d_{2,W})^{-2\alpha_l} + d_3^{-2\alpha_l} \right). \tag{A.11}
\]

Hence, the best-case distribution can be derived in terms of \[16\], and the proof is complete.

**APPENDIX B: PROOF OF THEOREM 1**

Let us first consider the worst-case of the prioritized user $W$. Based on the OP defined in \[17\], the worst-case of the OP can be rewritten as
\[
P_W = \mathbb{P} \left( \left| \tilde{h}_{W,l} \right|^2 < I_v \right) + \mathbb{P} \left( I_v < \left| \tilde{h}_{W,l} \right|^2 < I_W \right). \tag{B.1}
\]

Since the users are ordered based on their effective channel gain, the marginal PDF of user $W$ is given by \[44\]
\[
f_W(x) = W \tilde{f}_W(x) \left( \tilde{F}_W(x) \right)^{W-1}, \tag{B.2}
\]
where $\tilde{f}_W(x)$ and $\tilde{F}_W(x)$ represent the PDF and cumulative density function (CDF) of the unordered effective channel gain associated with $\tilde{f}_W(x) = \frac{x^{k_1-1}}{\Gamma(k_1)\lambda_1^{k_1}} \exp(-\frac{x}{\lambda_1})$ and $\tilde{F}_W(x) = 1 - \frac{\gamma(k_1, \frac{x}{\lambda_1})}{\Gamma(k_1)}$.

Based on the marginal PDF given in (B.2), the OP is given by
\[
P_W = W \int_{0}^{I_{W^*}} \tilde{f}_W(x) \left( \tilde{F}_W(x) \right)^{W-1} dx. \tag{B.3}
\]

After some algebraic manipulations, the OP of the prioritized user $W$ in \[20\] can be obtained. The proof is complete.
**APPENDIX C: PROOF OF COROLLARY**

In order to glean further engineering insights, we first expand the lower incomplete Gamma function as follows [45]:

\[
\gamma(k_1, \frac{I_{W*}}{\lambda_1}) = \sum_{s=0}^{\infty} \frac{\Gamma(k_1)}{\Gamma(k_1 + s + 1)} \left( \frac{I_{W*}}{\lambda_1} \right)^{k_1+s} \exp \left( -\frac{I_{W*}}{\lambda_1} \right). \tag{C.1}
\]

In the high-SNR regime, recall that \( \lim_{x \to 0} (1 - e^{-x}) \approx x \). Hence the OP of user \( W \) can be approximated in the high-SNR regime as follows:

\[
P_W = \left( \sum_{s=0}^{\infty} \frac{1}{\Gamma(k_1 + s + 1)} \left( \frac{I_{W*}}{\lambda_1} \right)^{W \cdot s} \left( 1 - \frac{I_{W*}}{\lambda_1} \right) \right)^W. \tag{C.2}
\]

Upon involving the binomial expansion and after some algebraic manipulations, the approximate result can be further transformed into

\[
\tilde{P}_W = \left( \sum_{s=0}^{\infty} \frac{1}{\Gamma(k_1 + s + 1)} \left( \frac{I_{W*}}{\lambda_1} \right)^{W \cdot s} \sum_{j=0}^{W} \binom{W}{j} \left( -1 \right)^j \left( \frac{I_{W*}}{\lambda_1} \right)^{k_1W+j} \right). \tag{C.3}
\]

Thus, after some algebraic manipulations, the results in (22) can be obtained, and the proof is complete.

**APPENDIX D: PROOF OF THEOREM**

Let us commence by expressing the worst-case on the ergodic rate of the prioritized user \( W \) as follows:

\[
R_{W,l} = \mathbb{E} \{ \log_2 [1 + SINR_W (x)] \} = - \int_0^{\infty} \log_2(1 + x) d[1 - F(x)]
\]

\[
= \frac{1}{\ln(2)} \int_0^{\infty} \frac{1 - F(x)}{1 + x} dx. \tag{D.1}
\]

The CDF of user \( W \) can be calculated as

\[
F(x) = \left( \frac{\gamma(k_1, Cx)}{\Gamma(k_1)} \right)^W. \tag{D.2}
\]

In order to derive the closed-form expression of the worst-case, we first round the shape parameter to the closest integer, i.e., \( \bar{k} = [k_1] \). Hence, the lower incomplete Gamma function
can be further expanded to
\[
\frac{\gamma (\bar{k}, Cx)}{\Gamma(\bar{k})} = \left(1 - \sum_{i=0}^{\bar{k}-1} \frac{(Cx)^i}{i!} e^{-Cx}\right).
\] (D.3)

By utilizing the binomial expansion, the result in (D.3) can be further transformed into
\[
\left(1 - \sum_{i=0}^{\bar{k}-1} \frac{(Cx)^i}{i!} e^{-Cx}\right)^W = \sum_{s=0}^{W} \binom{W}{s} (-1)^s \left(\sum_{i=0}^{\bar{k}-1} \frac{(Cx)^i}{i!} e^{-Cx}\right)^s.
\] (D.4)

Thus, the ergodic rate can be written as
\[
R_{W,l} = \frac{1}{\ln(2)} \sum_{s=1}^{W} \frac{W}{s} (-1)^s \int_{0}^{\infty} e^{-Cx} \left(\sum_{i=0}^{\bar{k}-1} \frac{(Cx)^i}{i!} e^{-Cx}\right)^s \frac{1}{1 + x} \, dx.
\] (D.5)

We then expand (D.5) by using the multi-nomial theorem as follows [46]:
\[
\left(\sum_{i=0}^{\bar{k}-1} \frac{(Cx)^i}{i!} e^{-Cx}\right)^s = \sum_{a_1 + \ldots + a_{\bar{k}} = s} \binom{s}{a_1, \ldots, a_{\bar{k}}} \prod_{t=1}^{k} \left(\frac{(Cx)^{t-1}}{(t-1)!}\right)^{a_t},
\] (D.6)

where \(\binom{s}{a_1, \ldots, a_{\bar{k}}} = \frac{s!}{a_1! \ldots a_{\bar{k}}!}\). Thus, the result can be rewritten as
\[
R_{W,l} = \sum_{s=0}^{W} \frac{W}{s} (-1)^s \frac{1}{k} \sum_{a_1 + \ldots + a_{\bar{k}} = s} \binom{s}{a_1, \ldots, a_{\bar{k}}} \prod_{t=1}^{k} \left(\frac{(Cx)^{t-1}}{(t-1)!}\right)^{a_t}
\times \int_{0}^{\infty} x^{(t-1)a_t} \exp \left(-Csx\right) \frac{1}{1 + x} \, dx.
\] (D.7)

Hence, the tractable approximate results can be derived as
\[
R_{W,l} = \sum_{s=0}^{W} \frac{W}{s} (-1)^s \frac{1}{k} \sum_{a_1 + \ldots + a_{\bar{k}} = s} \binom{s}{a_1, \ldots, a_{\bar{k}}} \prod_{t=1}^{k} \left(\frac{(Cx)^{t-1}}{(t-1)!}\right)^{a_t}
\times \left(\exp(Cs) Ei(-Cs) + \sum_{i=1}^{(t-1)a_t} (-1)^{i-1} (i-1)! (Cs)^i \right).
\] (D.8)
Thus, the worst-case on the ergodic rate of user $W$ is obtained in (28), and the proof is complete.

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