A quasi-one-dimensional model is proposed to predict a thermal choked flow-field with a pseudo-shock wave system (ramjet-mode operation) in diverging dual-mode combustors. To predict pressure-rise upstream of the injector through the pseudo-shock wave system, the model used an empirical correlation between the penetration length of the pseudo-shock wave system and the rise in pressure. In addition, to predict the thermal choking location in diverging combustors, the model used the combustion efficiency distribution against a streamwise location, which was deduced from the combustion test results reported in the present study. Results indicate that the model simulates ramjet-mode operation with a reasonable degree of accuracy by correlating of the pseudo-shock wave system and combustion efficiency distribution.

Key Words: Ramjet-mode Operation, Pseudo-shock Wave System, Thermal Choking

Nomenclature

\( A(A_{\text{inj}}) \): cross-sectional area (at injector location)
\( A_{\text{wet}} \): wet area
\( B \): expansion ratio
\( C_f \): skin friction coefficient
\( C_W \): Stanton number
\( C_p \): specific heat at constant pressure
\( D \): hydraulic diameter
\( F_{\text{fr}i}, F_{\text{th}} \): skin friction force, thrust
\( H \): enthalpy
\( K_1, K_2 \): constant number in Eq. (15)
\( L_{\text{inj}} \): length from injection location
\( m \): mass flow rate
\( M \): Mach number
\( M_w \): Molecular weight
\( n \): molecular concentration
\( N \): constant number in Eq. (13) and Eq. (14)
\( p(p_{\text{w}}) \): static (wall) pressure
\( p_{\text{peak}} \): peak pressure
\( P_t \): total pressure
\( Pr \): Prandtl number
\( Q_w \): wall heat transfer
\( R \): gas constant
\( Re_{\text{st}} \): Reynolds number of momentum thickness
\( T, T_i \): static temperature, total temperature
\( T_{\text{adw}} \): adiabatic wall temperature
\( T_w \): wall temperature
\( V \): velocity
\( x \): streamwise location from divergence origin

\( X_{\text{PSW}} \): penetration length of pseudo-shock wave system
\( \alpha \): constant exponent in Eq. (16)
\( \varepsilon \): constant number in Eq. (12)
\( \phi \): equivalence ratio
\( \eta_c \): combustion efficiency
\( \theta \): momentum thickness
\( \rho \): density

Subscripts

2: reacted gas
a: airflow
f: fuel
i: certain gas
O2: oxygen
st: origin of pseudo-shock wave system

1. Introduction

Ramjet-mode operation of a scramjet flow-pass is a vital part of rocket-based, combined-cycle engines (i.e., rocket engines embedded within the scramjet flow-pass) proposed by the Japan Aerospace Exploration Agency (JAXA).\(^1\) Ramjet-mode operation provides high thrust in a wide flight-speed range, through ascent trajectory to low-earth orbit, thereby providing a highly effective speciﬁc impulse.\(^2\) To attain ramjet-mode combustion and the resulting high-pressure level within its diverging combustor, it is necessary to choke the airflow with heat input through the addition of combustion and mass. This so-called “pseudo-shock wave system (PSW),”\(^3\) would decelerate the incoming supersonic airflow to subsonic prior to the combustion zone to reduce the friction loss within the PSW,\(^4\) and this reduction would allow further heat addition under the choking condition. The streamwise choking location within the diverging combustor was found to have a sizable effect on the speciﬁc impulse,\(^5\) and therefore, the thermal choking location should be controlled according to the flight Mach number. However, intro-
ducings a specific setup to attain choking at a specific location (e.g., insertion of mechanical throttling) is not suitable for thermal choking location control.

As our target is to apply a system capable of operation ranging from acceleration to low-earth orbit, a handy engine performance prediction method for system optimization is needed. It should be noted that three-dimensional computational fluid dynamics (CFD) requires too much time and resources. In addition, the simulation of such complicated flow-field, especially flow separation to form the shock-train and combustion to cause thermal choking, is still beyond the current CFD capacity. However, one-dimensional calculation is still useful for performance prediction, especially at the system analysis level.

In the past, many studies have been conducted on the one-dimensional performance prediction of the scramjet flow-pass, including that during ramjet-mode operation. Waltrup and Billig proposed an empirical equation for pressure distribution of the PSW within constant-area cylindrical ducts. After that, Sullins et al. proposed a modified version of the empirical equation for rectangular ducts that worked reasonably well for the PSW up to a maximum pressure rise of 80%. The former was for constant-area ducts; whether or not it could be applied for the diverging duct was unknown.

Recently, Torrez et al. conducted a series of works on quasi-one-dimensional performance predictions for thermal choking conditions. In their method, they modeled the PSW with the core flow compressed using a flow tube to squeeze by the flow separation zone. However, this model did not match with previous findings that a rather uniform subsonic flow was observed prior to the combustion zone during ramjet-mode operation. In addition, their numerical technique to manage the thermal choking needed to identify the choking location (e.g., the exit of constant-area combustor in their case), which was not possible in our case. Tian et al. have proposed a through technique to predict pressure distribution within the diverging duct. They applied the empirical equation to predict the PSW penetration length within the diverging duct by assuming that 10% of the rise in pressure was due to divergence; however, this number was not rationalized in their study. Additionally, they applied a “uniform heat release” model to simulate heat release, thereby enabling the position of thermal choking to be located easier, while logarithmic variation would rather be expected in real combustors.

In the present study, a simple quasi-one-dimensional performance prediction model is proposed for the diverging dual-mode combustor during ramjet-mode operation. In this model, combustion pressure prediction based on a singular solution as proposed by Torrez et al. is addressed, while the thermal choking location is set arbitrarily. For this purpose, two relations are given: 1) a relation between the PSW penetration length (i.e., length between the pressure-rise origin due to combustion and the injection location in the streamwise direction) and pressure-rise, and 2) the combustion efficiency distribution in the streamwise direction (i.e., the heat release distribution). Both relations applicable to diverging ducts did not exist in the reference, so these two relations were experimentally examined using hydrogen injection results of a diverging combustor.

2. Modeling and Experimental Methods

2.1. Prediction model

Figure 1 shows the flowchart of the present model. At first, flow states without fuel injection (i.e., “no-fuel” case) are calculated under the following assumptions; 1) quasi-one-dimensional flow and cross-sectional area of the duct are a function of the axial distance along the combustor, and 2) a steady flow behave as an ideal gas. In the calculation for the no-fuel case, mass, momentum and energy conservation equations and the equation of state are solved from the combustor entrance using the initial airflow conditions as the input. The skin friction and heat transfer are calculated using the van Driest method and Chilton-Colburn analogy. Next, assuming the peak pressure (p_{peak}), the streamwise pressure distribution in the PSW of the diverging combustor is deduced using the PSW correlation, which is described later. The skin friction within the PSW was zero due to separation flows in the PSW. When the pressure and initial air properties are given, one of three equations (i.e., mass, momentum and energy) must be neglected. In the present study, mass and momentum equations are conserved, while the energy equation is neglected. Thus, thrust and heat release within the PSW region are calculated. Next, after adding fuel, flow states downstream of the injector are calculated to-
ward the combustor exit by giving the combustion efficiency distribution in the streamwise direction and solving the following equations,

\[ m_a + m_f = m_2 (= \rho_2 V_2 A_2) \]  
\[ IF_a + IF_f - \sum dF_{fi} + \sum dF_{iu} = m_2 V_2 + p_2 A_2 \]  
\[ H_a(T_{iu}) + H_f(T_{iu}) - \sum dQ_w = H_2(T_2) + \frac{1}{2} m_2 V_2^2 \]  
\[ p_2 = \rho_2 R_2 T_2 \]

where, subscript \( a, f \) and \( i \) denote airflow (at the combustor entrance), fuel and reaction gas, respectively. Note that the momentum equation in Eq. (2) is taken in account only for the streamwise direction. Enthalpy \( (H) \) and gas constant \( (R) \) of a certain gas \((i)\) are calculated using

\[ H_i(T) = \sum n_{i,j} \int_{T_{ref}}^{T} C_{pi} dT + \sum n_{i,j} \Delta H(T_{ref}) \]

\[ R_i = \frac{\sum n_{i,j} M_{wi,j}}{\sum n_{i,j}} \]

where, \( \Delta H(T_{ref}) \) denotes the standard heat of formation, and subscript \( j \) denotes the gas species (i.e., CO, CO₂, H, H₂, H₂O, N, NO, NO₂, N₂, O, OH, O₂ and Ar in this study). In the present study, combustion efficiency \( (\eta_c) \) is defined as the ratio of reaction \( O_2 \) flow rate to total \( O_2 \) flow rate capable of reacting, as described in Eq. (5). Local skin friction (\( dF_{fi} \)), local thrust (\( dF_{th} \)) and local heat transfer (\( dQ_w \)) downstream of the injector are deduced from the last reaction gas states as the following equations,

\[ dF_{fi} = \frac{1}{2} \rho V^2 C_f A_{wet} \]

\[ dF_{th} = \int pdA \]

\[ dQ_w = \dot{C}V \rho A_{wet} (T_{aw} - T_w) \]

where, \( A_{wet} \) denotes the wet area in \( dx \). The adiabatic wall temperature \( (T_{aw}) \) is deduced from

\[ T_{aw} = T_s + Pr \frac{V^2}{2C_f} \]

Prandtl number \( (Pr) \) is a constant of 0.75 in this study, which was deduced by REFPROP.\(^{17}\) when heated air species were input at the combustor entrance. The value of \( Pr \) calculated is quite reasonable compared to the turbulent Prandtl number of 0.70 ± 0.07.\(^{18}\) The origin of the boundary layer behind the PSW, which is required for deducing the skin friction coefficient \( (C_f) \), is reset at the injector location. For the present calculation, the streamwise mesh size \( (dx) \) is 1 mm.

Finally, whether the calculation successfully provides a singular solution within the diverging duct is judged. A singular solution is the state where subsonic flows behind the PSW accelerate to sonic speed (i.e., thermal choking) as the result of heat release from fuel reacting and then to supersonic speed as the result of area expansion. With the occurrence of thermal choking in diverging combustors, the conditions for attaining a singular solution are more complicated than that in constant-area combustors; in subsonic flows, heat release effects need to exceed the flow divergence effect for acceleration, while in supersonic flows after thermal choking, flow divergence effects need to exceed the heat release effects to attain more acceleration. Therefore, a singular solution can be found when combustion efficiency distribution is given.

In the present model, a singular solution was obtained by iterating the \( p_{peak} \) assumed; that is, changing total pressure of the airflow. Generally, the PSW penetration length increases as back-pressure (i.e., peak pressure) increases. Assuming skin friction in the PSW region is negligible, total skin friction (from combustor entrance to injection location) will decrease as peak pressure increases. Thus, changing peak pressure leads to changing total pressure.

Figure 2 shows typical pressure distributions downstream of the injector if the peak pressure assumed is 1) adequate (i.e., singular solution), 2) low or 3) high. Assuming a low \( p_{peak} \), the impulse function would be not sufficient to cause thermal choking without a singular solution. Assuming an excessive \( p_{peak} \), deceleration due to area expansion overcomes acceleration, so that the flow remains subsonic through the diverging combustor, thereby causing a mismatch against pressure at the combustor exit to occur.

Two relations are required for the model proposed: the relation between the PSW penetration length and pressure-rise, and the streamwise combustion efficiency distribution with the diverging combustor. Researchers have presented an empirical formula and calculation results regarding the relations in the constant-area ducts.\(^{8,9,19}\) However, there have been few effective relations reported regarding the diverging combustor case. Their relations therefore were experimentally investigated from combustion tests in diverging ducts.

### 2.2. Experimental apparatus and measurements

Figure 3 shows a schematic diagram of the test apparatus.
The combustor is directly connected to a blow-down-type wind tunnel facility with a vitiation air heater to obtain Mach 2.5 flows. The airflow total temperature ($T_{t,a}$) and total pressure ($P_{t,a}$) are 2200 ± 20 K and 0.55 ± 0.04 MPa, respectively. The cross-sectional area at the combustor entrance is 94.3 mm in height and 51.0 mm in width. The test section consists of a constant-area section (540 mm) and a diverging-area section (600 mm) with a diverging angle of 3.0 deg on one side only. In this study, the streamwise location from the origin of diverging section is given by $x$. There are wall injectors (3.0 mm in diameter, four on each sidewall) installed in the diverging section at streamwise locations of $x = 75$ mm (P1), 225 mm (P2), and 375 mm (P3) (i.e., arrows showing their locations in Fig. 3). The fuel, room-temperature hydrogen, is injected through a perpendicular injection scheme during sonic speed, and the total orifice discharge coefficient is about 0.89.

The wall pressure was measured with electrical-scanning transducers (System 8400, Pressure System Inc., range: 0–689 kPa, error: ±0.2% full scale). To mitigate run-to-run deviation in test conditions, pressure measurements are normalized using the (total) pressure measured in the chamber of the vitiation air heater. The repeatability of the non-dimensionalized pressure measurements was within ±5%. To evaluate combustion efficiency distribution, quasi-one-dimensional analysis is used in this study.\(^{20}\) By giving the wall pressure measured and initial conditions of both the airflow and the fuel, combustion efficiency can be evaluated at each wall pressure tap by solving mass, momentum, and enthalpy equations. The discrepancy between the efficiency evaluation near the combustor exit and the efficiency evaluated from the cross-sectional gas sampling data was 7% at most.\(^{20}\) which is the result of uncertainty in estimating efficiency.

Additionally, the previous experimental findings during ramjet-mode operation are used,\(^{21}\) where hydrogen was injected into diverging combustors on both sides. The test section of the previous combustor consisted of a constant-area section (295 mm) with 2 mm steps, and a diverging combustor section (600 mm). The diverging half-angle was basically 3.1 deg, the upstream portion of diverging combustor being replaceable with 1.55 or 6.2 deg diverging ducts. For more details, refer to Tomioka et al.\(^{20}\)

### 3. Results and Discussion

#### 3.1. Experimental results

Figure 4(a) shows normalized wall pressure distributions using various fuel equivalence ratios ($\phi$). Pressure-rise due to flow separation at the combustor exit was observed for the no-fuel case (closed circle) and the fuel injection case because the pressure was too low compared to the atmosphere. For the one-dimensional calculation, over-estimation of combustion efficiency occurred due to taking the pressure recovery as the results of fake heat release. Therefore, the pressure distribution between the origin of the diverging section (or the peak pressure location for the combustion case) and the origin of separation was curve-fitted using Crocco’s equation,\(^{22}\)

$$pA^{\epsilon/(\epsilon-1)} = \text{const.}$$

(12)

to obtain the reference pressure distribution. Constant $\epsilon$ was 3.7 for the no-fuel case, while $\epsilon$ was 1.6–1.8 for the P1-injection case. Figure 4(b) shows corrected pressure distributions. At a $\phi$ of 0.57, peak pressure-rise was observed downstream of the injector together with slight pressure propagation across the injector location. With increasing $\phi (>0.85)$, the peak location of pressure-rise moved to the injector location, after which pressure-rise was observed upstream of the injector (i.e., shock-train generation). On the other hand, at $x > 300$ mm, pressure distribution increased slightly as $\phi$ increased, but differences among pressure distributions in the range of $0.85 < \phi < 1.12$ were narrowing.

During ramjet-mode operation, a so-called precombustion shock-train\(^{23}\) is generated upstream of the injector location due to interaction between shock waves (to decelerate the in-
coming supersonic flow) and the wall boundary layer. As a result, the pressure-rise upstream of the injector shown in Fig. 4(b) was observed. Moreover, the precombustion shock-train moves upstream due to a rise in back-pressure caused by the increasing amount of heat release (i.e., increasing equivalence ratio), as observed in Fig. 4(b).

Figure 5 shows deduced Mach number distribution deduced using various $\phi$. Note that the Mach number upstream of the injector was deduced using only mass and momentum equations and the pressure measured. The Mach numbers at the injector location were below unity, in the range of $0.85 < \phi < 1.12$. Judging from the pressure-rise features and the Mach number distribution, ramjet-mode operation was confirmed from $\phi$ more than 0.85 for the P1-injection case.

Figures 6(a) and 6(b) show normalized wall pressure and Mach number distributions deduced with P1, P2 and P3 injection cases at $\phi = 1.1$. As same as Fig. 4, pressure distributions were curve-fitted with Eq. (12). The pressure distribution for the P1-injection case in Fig. 6 is the same as that in Fig. 4, being reproduced for comparison. The location of peak pressure-rise was observed around the injector regardless of the injection configuration, while the peak pressure value decrease as the injector location was moved downstream. The origin of the PSW was almost the same ($x = 0\text{ mm}$) for P2 and P3 injections, the PSW penetration length for P2-injection being shorter than that for P3-injection. In the previous study, it was observed that the backward-facing step anchored the PSW. For the P2-injection case, it was assumed that an expansion wave, generated from the origin of the diverging duct, prevented PSW penetration. The Mach number at the injection location was below unity for all of the cases, while it increased downstream of the injector as shown in Fig. 6(b). For the P3-
thickness was deduced applying Tucker pressure measurements. The increasing rate of momentum evaluated 12 mm at most at the facility nozzle exit using Pitot approximately 1.0 mm because the boundary layer is estimated at the combustor entrance is estimated to be (Eq. (14)).

**3.2. Modeling of pseudo-shock wave system**

Here, the relation between the PSW penetration length and pressure-rise in diverging combustors to apply to the prediction model is discussed. At first, the empirical equation for constant-area ducts proposed by Waltrup and Billig \(^7,^8\) is referenced, and it is written as

\[
\frac{X_{PSW}(M_{ps}^2 - 1)Re_{\theta st}^N}{\sqrt{D_{st} \theta st}} = 50 \left( \frac{P}{P_{st}} - 1 \right) + 170 \left( \frac{P}{P_{st}} - 1 \right)^2
\]  

(13)

where, subscript \(st\) denotes the state of the PSW origin. \(Re_{\theta}\) and \(\theta\) denote the Reynolds number based on momentum thickness and momentum thickness, respectively. Momentum thickness at the combustor entrance is estimated to be approximately 1.0 mm because the boundary layer is estimated 12 mm at most at the facility nozzle exit using Pitot pressure measurements. The increasing rate of momentum thickness was deduced applying Tucker’s method.\(^{23}\)

Figure 7 shows plots of pressure distributions for distance from the PSW origin \((X_{PSW})\) for the cases with constant-area and diverging sections. Pressure is normalized with pressure at the origin of the PSW (i.e., peak pressure normalized with the pressure further upstream), resulting in a lower normalized pressure-rise further downstream. Therefore, Eq. (13) could not be applied for the case using diverging ducts.

Next, a focus was placed expansion ratio \((B)\), which was defined as the ratio of cross-sectional area at the peak pressure location and that at the origin of the PSW. Penzin experimentally investigated the effects of expansion ratio on peak pressure-rise under similar diverging angles,\(^{24}\) and found that the rise in peak pressure decreases as the expansion ratio increases. In this test configuration, the diverging angle of the combustor was constant for all of the injection configurations, with a \(B\) being 1.03, 1.15 and 1.33 for P1, P2 and P3 injection configurations at a \(\phi\) of 1.1, respectively. Figure 7 shows a trend that the peak pressure is lower as the injection configuration moves further downstream (i.e., increasing \(B\)). Therefore, the equation proposed by Waltrup and Billig (Eq. (13)) was modified as the following correlation by adding a \(B\) factor,

\[
\frac{X_{PSW}(M_{ps}^2 - 1)Re_{\theta st}^{0.25}}{B \sqrt{D_{st} \theta st}} = 50 \left( \frac{P}{P_{st}} - 1 \right) + 170 \left( \frac{P}{P_{st}} - 1 \right)^2
\]  

(14)

Figure 8 shows plots of pressure-rise against the left-hand side of Eq. (14). All of the data fell close to the correlation shown in Eq. (14). However, correlation of Eq. (14) results in a large error due to the uncertainty of each parameter, mainly momentum thickness. In the present case, the momentum thickness differs between the diverging and non-diverging sides. In addition, the boundary layer is not influence
enced by the corner effects of the rectangular ducts. The error of the boundary layer affects estimating the Reynolds number and Mach number at the origin of the PSW. Figure 9 shows the peak pressure-rise with various equivalence ratios at the diverging angle of 3.0° (present data) and at the half-diverging angle of 1.55°, 3.1° and 6.2°, respectively.21)

The deviation from Eq. (14) is approximately /C6% at the most, as shown by the dotted line in Fig. 9.

3.3. Streamwise combustion efficiency distribution

Figure 10 shows the one-dimensionally deduced combustion efficiency distributions in the streamwise direction for the P1-injection configuration. Combustion efficiency increased as the distance from the injection location increased, and then reached a peak. Although only a little difference in combustion efficiency gradient was observed at 50–200 mm downstream of the injector, combustion efficiency distributions were similar during ramjet-mode operation for the P1-injection case. The value of combustion efficiency decreased as φ came close to unity when more than 200 mm downstream of the injector, showing a mixing-controlled feature.25)

Figure 11 shows the combustion efficiency distributions for P1, P2 and P3 injection configurations at a similar φ of 1.1. Compared to the P1-injection case, combustion efficiency increased more gradually against the streamwise distance for the P2-injection case, and the efficiency for P2-injection finally became consistent with that for P1-injection when 350 mm downstream. On the other hand, the combustion efficiency for P3-injection increased along the same gradient as that for P1-injection, but the combustion efficiency for P3-injection was higher than that for P1-injection due to the high initial combustion efficiency for P3-injection.

To express the difference in combustion efficiency distribution against the injection configuration in diverging ducts, a focus was placed on cross-sectional area ratio (A/Ainj) because dominant parameters for mixing (i.e., combustion) such as velocity ratio and density ratio between airflow and fuel interacted based on the change in A/Ainj. For thermal choking in diverging ducts, the first portion of combustion efficiency distribution (i.e., /C17c distribution from the injector to the thermal choking location) is important because thermal choking does not occur when combustion efficiency is constant. Here, a simple linear correlation for the combustion efficiency distribution using diverging ducts is provided,

\[ \eta_c = K_1 \phi^\alpha L_{inj} + K_2 \]  

where, \( K_1 \) and \( K_2 \) are constant, and \( L_{inj} \) denotes the distance from the injection location. Constant coefficient (α) is varied using fuel-rich or lean conditions, written as

\[ \alpha = \begin{cases} 
0.3 \ln(2 - \phi) & (\phi \leq 1) \\
0.3 & (\phi > 1) 
\end{cases} \]

which determines that the value of \( \phi^\alpha \) is minimum and symmetrical with \( \phi \) of unity.

Figure 12 shows the comparison between correlation and experimental results using various equivalence ratios for P1, P2, and P3 injection cases. When constant \( K_1 \) and \( K_2 \) were 0.003992 and 0.4018, respectively. The discrepancy from the correlation was ±8% at most, showing that streamwise...
Combustion efficiency distribution can be predicted within an error of $\pm 8\%$ for P1, P2 and P3 injection cases. Note that pressure near the injector ($L_{inj}$ less than 50 mm) could not exist due to test configuration limits. Thus, it is unknown whether or not combustion efficiency can be guaranteed near the injector. Unfortunately, a physical explanation could not be found for the correlation. Therefore, identifying the dominant factor for combustion efficiency in diverging ducts is needed. In the present model, the correlation of Eq. (15) was used to estimate the streamwise combustion efficiency distribution.

### 3.4. Comparison with experimental results

Finally, the flow states during ramjet-mode operation are predicted using our present model (shown in Fig. 1) using the two correlations proposed in Eqs. (14) and (15). Calculation in the region 0–50 mm downstream of the injector was skipped because it was unknown whether or not combustion efficiency deduced by Eq. (15) can be guaranteed near the injector.

Figure 13 shows the pressure distributions for the experimental (i.e., solid line with symbols) and predicted (i.e., dash line) results for the P1-injection configuration at a $\phi$ of 1.12, 0.98 and 0.85, respectively. The result for the no-fuel case is also shown. A singular solution was found for all fuel-injection cases, and thermal choking occurred at an $x$ of 179 mm ($\phi = 1.12$), 175 mm ($\phi = 0.98$) and 172 mm ($\phi = 0.85$). The peak pressure calculated and the origin of the PSW were in good agreement with the experimental results; however, the model could not resolve the local pressure fluctuations associated with the two-dimensional weak shock waves inside the PSW.

Figure 14 shows the pressure distributions of experimental (i.e., solid line with symbols) and predicted (i.e., dash line) results for P2 and P3 injection configurations at a $\phi$ of 1.1. For a singular solution, thermal choking occurred at $x = 370$ mm (P2) and 550 mm (P3), showing that the model satisfies the thermal choking location is set arbitrarily. The peak pressure could be predicted within 4.8%. However, the PSW penetration length was under-estimated, especially for the case of predicting the P3-injection configuration. Compared to the results, under-estimation of the location for the pressure-rise origin was approximately 12%. For the P3-injection case, the origin of the PSW was near the connection between the constant-area section and the diverging section. Accordingly, the expansion wave from the connection point influenced the local Mach number and boundary layer thickness, causing this rather large discrepancy. The uncertainty in predicting both predicted peak pressure and the PSW penetration length is caused by the uncertainty of

### Table 1. Sensitivity analysis of Eq. (14) and Eq. (15) to the model.

|                        | Peak pressure, $p_{peak}/p_{atm}$ | PSW penetration length, $X_{PSW}$ (mm) |
|------------------------|-----------------------------------|---------------------------------------|
|                        | Plus error Base Minus error       | Plus error Base Minus error           |
| Correlation between PSW penetration length and pressure-rise, Eq. (14) | 0.246 (6.9%) 0.230 (−8.8%) | 325 (7.6%) 301 (−9.6%) |
| Streamwise combustion efficiency distribution, Eq. (15) | 0.243 (5.4%) 0.214 (−6.8%) | 325 (7.6%) 301 (−9.6%) |

Fig. 12. Normalization of combustion efficiency distributions.

Fig. 13. Pressure comparison for the P1-injection configuration ($\phi = 0.85$–1.12).

Fig. 14. Pressure comparison for P2 and P3 injection configurations ($\phi = 1.1$).
input two correlations: Eq. (14) and Eq. (15). As mentioned above, correlation discrepancy between the PSW penetration length and pressure-rise (Eq. (14)) was ±35% at most. On the other hand, the discrepancy for streamwise combustion efficiency distribution (Eq. (15)) was ±8% at most. Here, a sensitivity analysis was conducted to evaluate the effects of Eq. (14) and Eq. (15) uncertainty on the predicted peak pressure and the PSW penetration length. As the sample condition, the P3-injection configuration at α of 1.1 was used. The peak pressure and the PSW penetration length were predicted for cases where 1) error on the left-hand side of Eq. (14) is ±35%, and 2) error of combustion efficiency in Eq. (15) is ±8%, respectively. Table 1 summarizes the results of sensitivity analysis. The results of the analysis revealed that the error in Eq. (14) has an impact on predicting the peak pressure and the PSW penetration length within approximately ±10%. Not only that, error in Eq. (15) also impacts predicting the peak pressure and the PSW penetration length within approximately ±10%.

Judging from comparisons with experiments and the sensitivity analysis, the present model could predict the flow states during ramjet-mode operation in a diverging combustor. The levels of error were within ±9% for peak pressure and within ±12% for the PSW penetration length. For a more accurate prediction of flow states during ramjet-mode operation in diverging ducts, more accurate correlations regarding the PSW and combustion efficiency are required.

4. Conclusion

A quasi-one-dimensional model was proposed to predict the flow states of diverging combustors, and the predictions were compared to hydrogen combustion experiments during ramjet-mode operation. From the experiments, a correlation between pressure-rise and the penetration length of the pseudo-shock wave system (PSW) was found. In addition, combustion efficiency distribution in the streamwise direction was normalized using equivalence ratios and cross-sectional area ratio between the streamwise location and the injector, resulting in a simple linear correlation. Using pseudo-shock wave correlation and combustion efficiency distribution, a singular solution (i.e., subsonic to supersonic in a diverging combustor) was found by solving conservations of mass, momentum and energy through iterating peak pressure. Compared to experimental results, pressure predicted is in good agreement, especially downstream of the thermal choking location. Peak-pressure is within ±9% and the PSW penetration length is within ±12%. For more accurate prediction of flow states during ramjet-mode operation in diverging ducts, more accurate relations regarding the PSW and combustion efficiency distribution are required.

References

1) Kanda, T. and Kudo, K.: Conceptual Study of a Combined-Cycle Engine for an Aerospace Plane, J. Propul. Power, 19 (2003), pp. 859–867.
2) Fry, R. S.: A Century of Ramjet Propulsion Technology Evolution, J. Propul. Power, 20 (2004), pp. 27–58.
3) Matsuo, K., Miyazato, Y., and Kim, H. D.: Shock Train a Pseudo-Shock Phenomena in Internal Gas Flow, Prog. Aerospace Sci., 35 (1999), pp. 33–100.
4) Shchetinikov, E. S.: Piecewise-one-dimensional Models of Supersonic Combustion and Pseudo Shock in a Duct, Combustion, Explosion and Shock Waves, 9 (1973), pp. 409–417.
5) Tomioka, S., Tani, K., Masumoto, R., and Ueda, S.: Dual-Mode Operation of a Rocket-Ramjet Combined Cycle Engine, Trans. JSASS Aerospace Technology Japan, 8, iss 27 (2010), pp. 13–18.
6) Tian, Y., Xiao, B., Zhang, S., and Xing, J.: Experimental and Computational Study on Combustion Performance of a Kerosene Fueled Dual-mode Scramjet Engine, Aerospace Sci. Technol., 46 (2015), pp. 451–458.
7) Walterp, P. J. and Billig, F. S.: Structure of Shock Waves in Cylindrical Ducts, AIAA J., 11 (1973), pp. 1404–1408.
8) Billig, F. S.: Research on Supersonic Combustion, J. Propul. Power, 9 (1993), pp. 499–514.
9) Sullins, G. and McLafferty, G.: Experimental Results of Shock Trains in Rectangular Ducts, AIAA Paper 92-5103, 1992.
10) Torrez, S. M., Driscoll, J. F., Ihme, M., and Fotta, M. L.: Reduced-Order Modeling of Turbulent Reacting Flows with Application to Ramjets and Scramjets, J. Propul. Power, 27 (2011), pp. 371–382.
11) Torrez, S. M., Dulle, D. J., and Driscoll, J. F.: New Method for Computing Performance of Choked Reacting Flows and Ram-to-Scram Transition, J. Propul. Power, 29 (2013), pp. 433–445.
12) Kobayashi, K., Tomioka, S., Kato, K., Kudo, K., Murakami, A., and Mitani, T.: Performance of a Dual-Mode Combustor with Multi-Staged Fuel Injection, J. Propul. Power, 22 (2006), pp. 518–526.
13) Tian, L., Chen, L., Chen, Q., Li, F., and Chang, X.: Quasi-One-Dimensional Multimodes Analysis for Dual-Mode Scramjet, J. Propul. Power, 30 (2014), pp. 1559–1567.
14) Diskin, G. S. and Northam, G. B.: Effect of Scale on Supersonic Combustor Performance, AIAA Paper 87-2164, 1987.
15) van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids, J. Aeronaut. Sci., 18 (1951), pp. 145–160.
16) Colburn, A. P.: A Method of Correlating Forced Convection Heat Transfer Data and a Comparison with Fluid Friction, Int. J. Heat Mass Transfer, 7 (1964), pp. 1359–1384.
17) Reference Fluid Thermodynamic and Transport Properties Database (REFPROP), National Institute of Standards and Technology, 2018.
18) Shapiro, A. H.: The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. II, Ronald Press Co., New York, 1954, pp. 1087–1088.
19) Bement, D. A., Stevens, J.R., and Thompson, M. W.: Measured Operating Characteristics of a Rectangular Combustor/Inlet Isolator, AIAA Paper 90-2221, 1990.
20) Tomioka, S., Murakami, A., Kudo, K., and Mitani, T.: Combustion Tests of a Staged Supersonic Combustor with a Strut, J. Propul. Power, 17 (2001), pp. 293–300.
21) Tomioka, S., Kobayashi, K., Kudo, K., Murakami, A., and Kanda, T.: Performance Supersonic Combustors with Fuel Injection in Diverging Section, J. Propul. Power, 22 (2006), pp. 111–119.
22) Crocco, L.: One Dimensional Treatment of Steady Gas Dynamics, Fundamentals of Gas Dynamics, Vol. III, Princeton Legacy Library, Princeton, 1958, pp. 105–130.
23) Tucker, M.: Approximate Calculation of Turbulent Boundary-Layer Development in Compressible Flow, NACA Tech. Note, No. 2337, 1951.
24) Penzin, V. I.: Deceleration of Supersonic Flows in Smoothly Diverging-Area Rectangular Ducts, Scramjet Propulsion, Progress in Astronautics and Aeronautics, Vol. 189, 2000, pp. 321–337.
25) Mitani, T., Chineze, N., and Kanda, T.: Reaction and Mixing Controlled Combustion in Scramjet Engines, J. Propul. Power, 17 (2001), pp. 308–314.

In-Seuck Jeung
Associate Editor