Model independent determination of the spins of the $P_c(4440)$ and $P_c(4457)$ from the spectroscopy of the triply charmed dibaryons

Ya-Wen Pan,¹ Ming-Zhu Liu,¹ Fang-Zheng Peng,¹ Mario Sánchez Sánchez,² Li-Sheng Geng,¹,3 and Manuel Pavon Valderrama¹

¹School of Physics and Nuclear Energy Engineering & International Research Center for Nuclei and Particles in the Cosmos & Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China
²Centre d’Études Nucléaires, CNRS/IN2P3, Université de Bordeaux, 33175 Gradignan, France
³School of Physics and Engineering, Zhengzhou University, Zhengzhou, Henan 450001, China

The LHCb collaboration has recently observed three narrow pentaquark states — the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ — that are located close to the $D\Sigma_c$ and $D^{*}\Sigma_c$ thresholds. Among the so-far proposed theoretical interpretations for these pentaquarks, the molecular hypothesis seems to be the preferred one. Nevertheless, in the theoretical works prefer the opposite identification [20–22]. In this letter we point out that heavy antiquark-diquark symmetry induces a model-independent relation between the spin-splitting in the masses of the $P_c(4440)$ and $P_c(4457)$ $D^{*}\Sigma_c$ pentaquarks and the corresponding splitting for the $0^+$ and $1^+ \Xi_c$, $\Sigma_c$ triply charmed dibaryons. This is particularly relevant owing to a recent lattice QCD prediction of the $1^+$ triply charmed dibaryon, which suggests that a calculation of the mass of its $0^+$ partner might be within reach. This in turn would reveal the spins of the $P_c(4440)$ and $P_c(4457)$ pentaquarks, providing a highly nontrivial test of heavy-quark symmetry and the molecular nature of the pentaquarks. Furthermore, the molecular interpretation of the hidden-charm pentaquarks implies the existence of a total of ten triply charmed dibaryons as $\Xi_c^+(\Sigma_c^0)$ molecules, which, if confirmed in the lattice, will largely expand our understanding of the non-perturbative strong interaction in the heavy-quark sector.

PACS numbers: 13.60.Le, 12.39.Mk,13.25.Jx

Recently the LHCb collaboration has reported the observation of three pentaquark states [1] — the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ — very close to two meson-baryon thresholds, the $D\Sigma_c$ in the case of the $P_c(4312)$ and the $D^{*}\Sigma_c$ in the case of the $P_c(4440)$ and $P_c(4457)$. This interesting coincidence has been promptly interpreted by a series of theoretical works [2–8] as evidence for their molecular nature, a hypothesis which is further fueled by the predictions a few years ago of the existence of these pentaquarks [9–14]. In addition to the molecular picture, there are other explanations, e.g., hadro-charmonium [15] or compact pentaquark states [16–18]. It is also worth noticing that the $P_c(4440)$ and $P_c(4457)$ were previously identified as a single state, the $P_c(4450)$ [19].

However the molecular interpretation cannot unambiguously determine the spins of the $P_c(4440)$ and $P_c(4457)$, which can be either 1/2 or 3/2. Initial studies showed a preference for the $P_c(4440)$ and $P_c(4457)$ being identified with $J = 1/2$ and $3/2$ molecules respectively [2,4,6], while recent theoretical works prefer the opposite identification [20,22].

Given this situation, one might turn to lattice QCD for a model independent answer. But meson-baryon systems are difficult to study directly in the lattice, more so if there are coupled channels. After the 2015 pentaquark peaks [19], there have been only two lattice QCD studies on $J/\phi N$ and $\eta N$ interactions [23,24], none of which found pentaquarks. These studies, which were performed within a single-channel approximation, concluded that further calculations including coupled-channels were needed.

To circumvent the aforementioned difficulties, we propose to learn more about the $D^{(*)}\Sigma_c^{(*)}$ pentaquarks from the $\Xi_c^{(*)}\Sigma_c^{(*)}$ dibaryons, which are connected by heavy antiquark-diquark symmetry (HADS).

Hadronic molecules containing heavy quarks ($c, b$) are constrained by heavy-quark symmetry (HQS) [25,26], which can be exploited to predict unobserved states [27–30] or explain relations among experimentally known states [31–33]. Two manifestations of HQS are relevant for the present work: heavy-quark spin symmetry (HQQS) (the interaction between heavy hadrons is independent of the orientation of the heavy quark spin) and HADS [34] (a pair of heavy-quarks behaves like a heavy antiquark). Regarding the pentaquark states, HQSS predicts the existence of seven molecular states [4,6,11,20,21,35]. Namely, from HQSS and the LHCb pentaquarks [1] one expects four more unobserved hidden-charm pentaquarks.

But the latest discovery of the LHCb pentaquarks leads to more consequences. From HADS, we expect the light-quark structures of the $D^{(*)}$ and the $\Xi_c^{(*)}$ to be almost identical: if the $D^{(*)}\Sigma_c^{(*)}$ system binds, the $\Xi_c^{(*)}\Sigma_c^{(*)}$ system should also bind. Specifically we can update the triply charmed dibaryon spectrum, from the four dibaryons originally predicted in Ref. [36] to ten dibaryons in the present manuscript. As we will show, this could help determine the spins of the $P_c(4440)$ and $P_c(4457)$. Besides, it seems to be much easier for lattice QCD to study the $\Xi_c^{(*)}\Sigma_c^{(*)}$ systems than the $D^{(*)}\Sigma_c^{(*)}$ systems, which up to now lattice QCD studies have not been able to simulate. On the other hand, a recent lattice QCD study [37] has reported the likely existence of a triply charmed $\Xi_c \Sigma_c$ dibaryon with spin-parity $J^P = 1^*$, isospin $I = \frac{1}{2}$ and a binding energy of $B_2 = (8 \pm 17)$ MeV, where Ref. [37] indi-
cates that owing to the large uncertainties its existence is not guaranteed. Here we predict this dibaryon to have a binding energy of \( B_2 \sim (15 - 30) \text{ MeV} \), which is compatible within its large error with the results of Ref. \[37\], but clearly leans towards the conclusion that the \( 1^+ \Xi_c \Sigma_c \) dibaryon binds. More importantly, as we will show below, if lattice QCD is able to calculate the mass of the \( J^P = 0^+ \) configuration or the mass splitting between the \( 0^- \) and \( 1^+ \Xi_c \Sigma_c \) dibaryons, it will provide a model-independent determination of the spins of the \( \frac{P_c}{4440} \) and \( \frac{P_c}{4457} \).

Finally, the (singly) heavy baryons can be represented with a contact-range Lagrangian involving a light-quark and light-diquark subfield as

\[
\mathcal{L}_{\text{contact}} = C_a(q^c_L q^c_L)(d^c_L d^c_L)
+ \ C_b(q^c_L \bar{\sigma}_L q^c_L) \cdot (d^c_L \bar{S}_L d^c_L),
\]

where \( \bar{\sigma}_L \) are the Pauli matrices for the \( q_L \) subfield and \( \bar{S}_L \) the spin-1 matrices for the \( d_L \) subfield. From this Lagrangian, we end up with the non-relativistic potential

\[
V_C = C_a + C_b \bar{\sigma}_L \cdot \bar{S}_L.
\]

This is supplemented by a series of rules to translate the light-quark spin operators into the heavy hadron ones. For the heavy antimeson fields we have

\[
(\bar{D} | \bar{\sigma}_L | \bar{D}) = 0 \quad \text{,} \quad (\bar{D}^* | \bar{\sigma}_L | \bar{D}^*) = \bar{S}_1,
\]

where \( \bar{S}_1 \) are the spin-1 matrices as applied to the \( \bar{D}^* \) meson. For the doubly heavy baryons

\[
\langle \Xi_c^c | \bar{\sigma}_L | \Xi_c^c \rangle = - \frac{1}{3} \bar{\sigma}_1 \quad \text{,} \quad \langle \Xi_c^c | \bar{\sigma}_L | \Xi_c^c \rangle = + \frac{2}{3} \bar{\sigma}_2,
\]

where \( \bar{\sigma}_1 \) and \( \bar{\sigma}_2 \) are the Pauli and spin-\( \frac{1}{2} \) matrices as applied to the \( \Xi_c^c \) and \( \Xi_c^c \) baryons. Finally for the singly heavy hadrons

\[
\langle \Sigma_c | \bar{S}_L | \Sigma_c \rangle = + \frac{2}{3} \bar{\sigma}_2 \quad \text{,} \quad \langle \Sigma_c^c | \bar{S}_L | \Sigma_c^c \rangle = + \frac{2}{3} \bar{\sigma}_2,
\]

with \( \bar{\sigma}_2 \) and \( \bar{\sigma}_2 \) as in Eq. \[4\], but applied to the \( \Sigma_c \) and \( \Sigma_c^c \) baryons. With these rules and the contact-range potentials of Eq. \[2\] we arrive at the potentials of Table I. We only consider contact-range interactions, as previous studies of heavy hadron-hadron systems \[40,\ 41\], indicate that pion exchanges are perturbative in the charm sector.

The potentials of Table I are subject to a certain degree of uncertainty. They are derived from HQS, which is only exact in the limit of infinite heavy-quark mass, \( m_Q \rightarrow \infty \). For finite \( m_Q \) we expect small violations, which for HQSS are of the order of \( \Lambda_{QCD}/m_Q \) with \( \Lambda_{QCD} \sim (200 - 300) \text{ MeV} \) the QCD scale, i.e., we expect a 15% error in the charm sector for the pentaquark potentials. For HADS, the uncertainty is of the order of \( \Lambda_{QCD}/(m_Q v) \) \[34\], with \( v \) the expected velocity for the heavy-quark pair. From the estimation of Ref. \[42\], \( m_Q v \sim 0.8 \text{ GeV} \) for a charm quark pair, we arrive at a 25–40% uncertainty for HADS. This figure has been conjectured to be larger, up to the point that HADS might not be applicable in the charm sector \[43\]. But recent lattice QCD calculations \[44–47\] of the mass splitting between the \( J = \frac{1}{2} \) and \( J = \frac{3}{2} \) doubly charmed baryons are usually 20–25% smaller than the HADS prediction, suggesting our lower estimation for the HADS uncertainty. From this we will settle on a 25% error for the dibaryon potentials.

TABLE I: The lowest-order contact range potentials for the heavy antimeson - heavy baryon and doubly heavy baryon - heavy baryon systems, which depend on two unknown coupling constants \( C_a \) and \( C_b \).

| State | \( J^P \) | \( V \) | State | \( J^P \) | \( V \) |
|-------|--------|------|-------|--------|------|
| \( D \Sigma_c \) | 1/2\(^-\) | \( C_a \) | \( \Xi_c \Sigma_c \) | 0\(^-\) | \( C_a + \frac{2}{3} \bar{\sigma}_2 \) |
| \( \bar{D} \Sigma_c \) | 3/2\(^-\) | \( C_a \) | \( \Xi_c \Sigma_c \) | 1\(^-\) | \( C_a + \frac{2}{3} \bar{\sigma}_2 \) |
| \( D^* \Sigma \Sigma_c \) | 1/2\(^-\) | \( C_a - \frac{2}{3} \bar{\sigma}_2 \) | \( \Xi_c \Sigma_c \) | 1\(^-\) | \( C_a - \frac{2}{3} \bar{\sigma}_2 \) |
| | 3/2\(^-\) | \( C_a + \frac{2}{3} \bar{\sigma}_2 \) | \( \Xi_c \Sigma_c \) | 2\(^-\) | \( C_a + \frac{2}{3} \bar{\sigma}_2 \) |
| \( \bar{D}^* \Sigma \Sigma_c \) | 1/2\(^-\) | \( C_a - \frac{2}{3} \bar{\sigma}_2 \) | \( \Xi_c \Sigma_c \) | 0\(^-\) | \( C_a - \frac{2}{3} \bar{\sigma}_2 \) |
| | 3/2\(^-\) | \( C_a - \frac{2}{3} \bar{\sigma}_2 \) | \( \Xi_c \Sigma_c \) | 3\(^-\) | \( C_a - \frac{2}{3} \bar{\sigma}_2 \) |

To obtain concrete predictions we have to solve a non-relativistic bound state equation with the contact-range potentials of Table I. If we work in momentum space we simply solve the Lippmann-Schwinger equation for the bound state pole, which reads

\[
\phi(k) + \frac{\int d^3 p \langle k | V(p) | p \rangle (2\pi)^3 p \mu}{B + \mu} = 0,
\]

where \( \phi(k) \) is the vertex function, \( B \) the binding energy and \( \mu \) the reduced mass. To solve this equation we first regularize the potential

\[
\langle p | V_A | p' \rangle = C(\Lambda) f\left( \frac{p}{\Lambda} \right) f\left( \frac{p'}{\Lambda} \right),
\]

with \( \Lambda \) the cutoff, \( f(x) \) a regulator, and \( C \) the linear combination of the \( C_a \) and \( C_b \) couplings that corresponds to the
particular molecular state we are interested in. Notice that $C = C(A)$, i.e., the couplings depend on the cutoff, which is necessary if we want to properly renormalize the calculations, i.e., we want the predictions to depend on the cutoff only moderately, where cutoff variations represent the uncertainty coming from subleading order terms that we have not explicitly taken into account. We will choose a Gaussian regulator of the type $f(x) = e^{-x^2}$ and for the cutoff we will use the range $\Lambda = 0.5 - 1$ GeV, where the average within this range corresponds to the $\rho$ meson mass.

For the masses of heavy hadrons, we use $m_D = 1867$ MeV, $m_{\Sigma_c} = 2009$ MeV, $m_{\Xi_c} = 2454$ MeV, $m_{\Omega_c} = 2518$ MeV, $m_{\Xi_{cc}} = 3621$ MeV, and $m_{\Xi_{c}} = 3727$ MeV, where the mass of $\Xi_{c}$ has been deduced from the HADS relation $m_{\Xi_{cc}} - m_{\Xi_c} = 3(m_D - m_{\Xi})/4$ [42].

Now we still have to determine the couplings $C_a$ and $C_b$. For this, we notice that if the $P_c(4440)$ and $P_c(4457)$ are indeed $\bar{D}\Sigma_c$ bound states, their contact-range potentials read

$$V(\bar{D}\Sigma_c, J = \frac{1}{2}) = C_a - \frac{4}{3} C_b, \quad (8)$$

$$V(\bar{D}\Sigma_c, J = \frac{3}{2}) = C_a + \frac{2}{3} C_b. \quad (9)$$

Since the spins of the hidden-charm pentaquarks are not experimentally known yet, there are two possible identifications: the $P_c(4440)$ is the spin $\frac{1}{2}$ state and the $P_c(4457)$ the spin $\frac{3}{2}$ one (scenario A), or vice versa (scenario B) [4]. This in principle gives us a consistency test in terms of the postdiction of the $P_c(4312)$ as a $D\Sigma_c$ bound state, see Table I for details. The outcome is inconclusive though: scenario A is marginally preferred over scenario B. In both scenarios, we arrive at the conclusion that the ten possible triply charmed dibaryons, with binding energies of 10 – 50 MeV, see Fig. 1 for a graphical representation of the full dibaryon spectrum and more details can be found in Table II.

One immediately notices that there is a strong correlation between the ordering of the triply charmed baryons and that of the pentaquark states, particularly that of $P_c(4440)$ and $P_c(4457)$. For instance, if the $0^+ \Sigma_c \Xi_c$ state had a larger mass than its $1^+$ counterpart, then scenario A would be preferred. The difference in their binding energies is about 10 MeV, which might be achievable in the lattice. In addition, one can study (one of) the three extra multiplets to confirm the conclusion. For instance, a decrease of the mass as a function of the spin in the $\Xi_c \Sigma_c (0^+, 1^+)$ multiplet and an increase in the $\Xi_c \Sigma_c (1^+, 2^+)$ multiplet will both be unambiguous signals that scenario A is preferred.

More concretely, from Table I and assuming that the $C_b$ coupling is perturbative, we obtain at the relation

$$M(1^+) - M(0^+) = -\frac{4}{9} (M(1^+) - M(\frac{1}{2}^+)) = \pm (7.6 \pm 1.9) \text{ MeV}, \quad (10)$$

with the $-/+$ sign respectively for scenarios A and B and where we have used the experimental pentaquark masses to obtain the number on the second line (with the error corresponding to the HADS uncertainty). It is worth noticing that in our numerical study with $\Lambda = 0.5$ GeV, we obtain $M(1^+) - M(0^+) = -8.4$ MeV and +9.7 MeV for scenarios A and B, respectively.

In summary, the experimental discovery of the three pentaquark states — the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ — which are conjectured to be hadronic molecules, not only makes it possible to predict the full spectrum of S-wave triply charmed dibaryons ($\Xi_c^*(\Sigma_c^*)$ molecules) but also opens the possibility of a model-independent determination of the spins of the pentaquarks from lattice QCD simulations of the dibaryons, which are definitely feasible in the near future. The basis for the prediction of the triply charmed dibaryons are HQSS and HADS. We exploited these two symmetries within the framework of a contact-range EFT, which offers us a series of theoretical advantages including systematicity and controlled error estimations. The binding energies of the triply charmed dibaryons are predicted to lie in the $(10 - 50)$ MeV range, where the details depend on the cutoff and the spins of the pentaquarks. Among them, we predict a $1^+ \Xi_c \Sigma_c$ dibaryon with a binding energy of $15 - 30$ MeV, to be compared with $(8 \pm 17)$ MeV in the lattice [34]. The spin-splitting of the dibaryon masses together with HADS, i.e., Eq. (10), indicates that a lattice QCD calculation of the dibaryon spectrum will provide a model-independent determination of the quantum numbers of the LHCb pentaquarks. The recent calculation of Ref. [37] implies that such studies are within reach of current state of the art lattice QCD simulations.

I. ACKNOWLEDGMENTS

This work is partly supported by the National Natural Science Foundation of China under Grant No.11735003, the fundamental Research Funds for the Central Universities, and the Thousand Talents Plan for Young Professionals.

[1] R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 222001 (2019), 1904.03947.
[2] H.-X. Chen, W. Chen, and S.-L. Zhu (2019), 1903.11001.
[3] R. Chen, Z.-F. Sun, X. Liu, and S.-L. Zhu (2019), 1903.11013.
[4] M.-Z. Liu, Y.-W. Pan, F.-Z. Peng, M. Sánchez Sánchez, L.-S. Geng, A. Hosaka, and M. Pavon Valderrama, Phys. Rev. Lett. 122, 242001 (2019), 1903.11560.
[5] F.-K. Guo, H.-J. Jing, U.-G. Meißner, and S. Sakai, Phys. Rev. D99, 091501 (2019), 1903.11503.
[6] C. W. Xiao, J. Nieves, and E. Oset (2019), 1904.01296.
[7] Y. Shimizu, Y. Yamaguchi, and M. Harada (2019), 1904.00587.
[8] Z.-H. Guo and J. A. Oller, Phys. Lett. B793, 144 (2019), 1904.00851.
[9] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010), 1007.0573.
[10] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. C84,
FIG. 1: Predicted masses of pentaquark states and hexaquark (dibaryon) states in scenario A (upper), in which the $P_c (4440)/P_c (4457)$ is identified with the $J = \frac{3}{2}^+$ $D^*\Sigma_c$ molecule, and scenario B (lower), where the opposite assignment is made. The dotted horizontal lines indicate the central experimental masses of the $P_c (4312)$, $P_c (4380)$, $P_c (4440)$, and $P_c (4457)$. The black solid lines denote the masses obtained with $\Lambda = 1$ GeV while the red bands denote the uncertainties originated from HQS (15%) and HADS (25%) breaking. The lattice QCD result for the $1^+ \Xi_c \Sigma_c$ state is calculated using the binding energy of Ref. [13] and the experimental masses for $\Xi_c$ and $\Sigma_c$. 
TABLE II: Binding energies for the pentaquarks composed of a charmed baryon and a charmed antimeson in the contact-range EFT defined in Eqs. [1] and [2]. The two coupling constants, $C_\nu$ and $C_{\overline{\nu}}$, are determined by reproducing the $P_c(4440)$ and $P_c(4457)$ as $\frac{1}{2}$- and $\frac{1}{2}^- D^\ast \Sigma_c$ molecules (scenario A) or $\frac{1}{2}^-$ and $\frac{3}{2}^- D^\ast \Sigma_c$ molecules (scenario B), respectively. The results are obtained with a cutoff of $\Lambda = 0.5(1.0)$ GeV.

| State       | $J^P$ | $\Lambda$(GeV) | Scenario A | Scenario B |
|-------------|-------|-----------------|------------|------------|
| $P_c(4312)(D^\ast \Sigma_c)$ | $\frac{1}{2}^-$ | 4320.8 | 0.5(1) | 8.8$^{+5.2}_{-4.5}$ (7.6$^{+2.8}_{-2.0}$) | 14.4$^{+5.8}_{-5.0}$ (12.8$^{+11.8}_{-10.5}$) |
| $P_c(4440)(D^\ast \Sigma_c)$ | $\frac{3}{2}^-$ | 3385.3 | 0.5(1) | 9.1$^{+5.5}_{-5.0}$ (8.1$^{+3.5}_{-3.0}$) | 14.7$^{+6.0}_{-5.5}$ (13.5$^{+12.0}_{-11.5}$) |
| $P_c(4440)(\bar{D}^\ast \Sigma_c)$ | $\frac{1}{2}^-$ | 4462.2 | 0.5(1) | Input(4$^-$) | Input(4$^+$) |
| $P_c(4457)(D^\ast \Sigma_c)$ | $\frac{3}{2}^-$ | 4526.7 | 0.5(1) | 25.6$^{+9.0}_{-8.4}$ (26.3$^{+16.6}_{-15.6}$) | 3.0$^{+2.8}_{-2.5}$ (3.4$^{+6.2}_{-5.5}$) |
| $P_c(4457)(\bar{D}^\ast \Sigma_c)$ | $\frac{1}{2}^-$ | 4526.7 | 0.5(1) | 15.8$^{+6.1}_{-5.5}$ (15.9$^{+12.6}_{-11.8}$) | 10.0$^{+5.2}_{-4.9}$ (10.0$^{+10.2}_{-9.5}$) |
| $P_c(4457)(\bar{D}^\ast \Sigma_c)$ | $\frac{3}{2}^-$ | 4526.7 | 0.5(1) | 3.0$^{+5.5}_{-5.0}$ (3.4$^{+6.1}_{-5.5}$) | 25.6$^{+9.0}_{-8.4}$ (26.3$^{+16.6}_{-15.6}$) |

TABLE III: Same as Table II but for the dibaryons composed of a doubly charmed baryon ($\Xi_{cc}$, $\Xi_{cc}$) and a singly charmed baryon ($\Sigma_c$, $\Sigma_c$).

| State       | $J^P$ | $\Lambda$(GeV) | Scenario(A) | Scenario(B) |
|-------------|-------|-----------------|------------|------------|
| $\Xi_{cc}$, $\Xi_{cc}$ | 0$^+$ | 6074.9 | 0.5(1) | 10.0$^{+17.9}_{-6.4}$ (17.9$^{+26.9}_{-14.4}$) | 30.4$^{+15.2}_{-14.2}$ (43.2$^{+33.1}_{-27.2}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 1$^+$ | 6074.9 | 0.5(1) | 18.4$^{+11.3}_{-6.1}$ (28.3$^{+26.6}_{-14.4}$) | 20.7$^{+12.1}_{-10.4}$ (31.2$^{+27.7}_{-21.6}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 1$^+$ | 6139.5 | 0.5(1) | 11.3$^{+15.4}_{-7.0}$ (20.0$^{+25.4}_{-17.2}$) | 29.6$^{+15.5}_{-13.8}$ (42.8$^{+32.7}_{-26.9}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 2$^+$ | 6139.5 | 0.5(1) | 20.0$^{+11.8}_{-10.4}$ (30.7$^{+27.2}_{-21.6}$) | 20.0$^{+11.8}_{-10.4}$ (30.8$^{+27.2}_{-21.6}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | $\frac{1}{2}^+$ | 6180.9 | 0.5(1) | 28.2$^{+14.7}_{-13.4}$ (41.0$^{+32.0}_{-26.0}$) | 12.2$^{+8.8}_{-7.6}$ (21.0$^{+22.2}_{-16.2}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | $\frac{3}{2}^+$ | 6180.9 | 0.5(1) | 10.2$^{+8.0}_{-6.4}$ (18.5$^{+21.0}_{-14.4}$) | 30.7$^{+15.5}_{-14.2}$ (44.1$^{+33.3}_{-27.2}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 0$^+$ | 6245.5 | 0.5(1) | 35.0$^{+18.8}_{-15.6}$ (50.2$^{+35.6}_{-28.9}$) | 7.6$^{+6.3}_{-5.3}$ (15.8$^{+19.2}_{-13.0}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 1$^+$ | 6245.5 | 0.5(1) | 29.9$^{+15.2}_{-11.9}$ (43.7$^{+22.9}_{-17.2}$) | 11.5$^{+8.5}_{-7.0}$ (20.7$^{+22.0}_{-15.9}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 2$^+$ | 6245.5 | 0.5(1) | 20.3$^{+10.5}_{-9.5}$ (31.6$^{+27.3}_{-21.6}$) | 20.3$^{+11.8}_{-10.5}$ (31.6$^{+27.3}_{-21.6}$) |
| $\Xi_{cc}$, $\Xi_{cc}$ | 3$^+$ | 6245.5 | 0.5(1) | 7.6$^{+6.3}_{-5.3}$ (15.8$^{+19.2}_{-13.0}$) | 35.0$^{+18.8}_{-15.6}$ (50.2$^{+35.6}_{-28.9}$) |
015202 (2011), 1011.2399.
[11] C. W. Xiao, J. Nieves, and E. Oset, Phys. Rev. D88, 056012 (2013), 1304.5368.
[12] M. Karliner and J. L. Rosner, Phys. Rev. Lett. 115, 122001 (2015), 1506.06386.
[13] W. L. Wang, F. Huang, Z. Y. Zhang, and B. S. Zou, Phys. Rev. C84, 015203 (2011), 1101.0453.
[14] Z.-C. Yang, Z.-F. Sun, J. He, X. Liu, and S.-L. Zhu, Chin. Phys. C36, 6 (2012), 1105.2901.
[15] M. I. Eides, V. Y. Petrov, and M. V. Polyakov (2019), 1904.11616.
[16] A. Ali and A. Ya. Parkhomenko, Phys. Lett. B793, 365 (2019), 1904.00446.
[17] Z.-G. Wang (2019), 1905.02892.
[18] J.-B. Cheng and Y.-R. Liu (2019), 1904.00446.
[19] M. Pavon Valderrama (2019), 1905.02892.
[20] M. J. Savage and M. B. Wise, Phys. Lett. B232, 113 (1989).
[21] M. J. Savage and M. B. Wise, Phys. Lett. B237, 527 (1990).
[22] N. Isgur and M. B. Wise, Phys. Rev. D91, 094502 (2015), 1502.01845.
[23] N. Isgur and M. B. Wise, Phys. Lett. B322, 113 (1989).
[24] N. Isgur and M. B. Wise, Phys. Lett. B327, 527 (1990).
[25] M. Pavon Valderrama, Phys. Rev. D86, 056004 (2012), 1204.2790.
[26] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, and M. P. Valderrama, Phys. Rev. D88, 054014 (2013), 1305.4052.
[27] C. W. Xiao, J. Nieves, and E. Oset (2019), 1906.09010.