Degeneracy on a two-Higgs $CP$ non-invariant Higgs sector

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Abstract. Many models with extended Higgs sectors have states where masses are quasi-degenerate at tree-level. Higher order corrections may separate this masses, but they may also bring them closer, opening the possibility of total or near degeneracy of the physical masses. If, in addition, the Higgs sector does not conserve $CP$ symmetry, then the neutral Higgs physical states are no longer defined with a $CP$ charge, they would be a mixture of $CP$-defined states given at tree level. Near degeneracy in the heavy neutral Higgs system with $CP$ violation could imply mixing of coherent overlapping states, which at exact degeneracy will show up as a double pole in the propagator matrix of the neutral Higgs sector. This possibility has consequences on the phenomenology and in the type of expected experimental signatures for the nearly and exact Higgs degeneracy, for example, the lineshape will be different than expected for the fermion scattering amplitude via $CP$-Higgs, $\sigma(\mu\bar{\mu} \to H^{\pm\pm} \to f^* f^{'*})$.

1. Introduction

It is well known that CP (charge-parity) is not an exact symmetry of the hadronic sector and, at higher energies, the non-conservation of CP could be present in other sectors as well, for instance the scalar Higgs sector. Besides this natural reason, new sources of CP violation are a required condition for baryogenesis [1]. Models that consider direct $CP$ non-invariance in the Higgs sector imply mixing of $CP$-even and $CP$-odd neutral states.

In the case of the Minimal Supersymmetric Standard Model (MSSM), CP conservation at first order could be broken by higher order terms when the soft-SUSY couplings are complex numbers. For the Two Higgs Doublet Model (THDM) the couplings are not as restricted as in the MSSM and some can be considered complex even at tree level.

In the MSSM the tree level Higgs states $H^0$ and $A^0$ are nearly degenerate in mass and have been widely studied [2]. In the context of the radiative corrected CP non-invariant self-energies, the exact degeneracy of the non-physical states $H^0$ and $A^0$ has been considered [3]. The Argand diagram of a complex degeneracy is calculated in a THDM with a top dominating diagram and also for a supersymmetric model considering stops contributions. This $CP$ system could affect $\gamma\gamma$ Higgs production. In [4] stop and sbottom as dominant radiative corrections using Renormalization Group Equations (RGE) to $CP$ neutral Higgs bosons masses have been performed,
they considering values for the parameters which move the physical states away from degeneracy.

A renormalized procedure using the Feynman Diagrammatic Approach and dominant $m_t^4$ diagrams $\mathcal{CP}$ in the Higgs mass matrix was performed in [5]. The most complete work done so far considering all the one-loop diagrams and dominant two-loop, to obtain renormalized Higgs masses in this $\mathcal{CP}$ MSSM (or CMSSM) context is presented in [6], where the authors found that the real part of the propagator has an important contribution from the off-diagonal neutral Higgs masses.

The $\mathcal{CP}$ Higgs neutral system might show coherence of Higgs states as a manifestation of a geometrical degeneration. A geometrical degeneracy is characterized by coalescence of eigenvalues and of the corresponding eigenstates [7, 8, 9]. Associated to this degeneracy a branch point or exceptional point appears in the non-Hermitian Hamiltonian spectra as function of the parameters of the model [10]. In this specific point of non-Hermitian degeneracy the basis of the space states is not complete, the system is no longer diagonalizable by a similarity transformation with a unitary matrix, and implies that the states are self-orthogonal.

One of the first studies which shed light on the presence of this branch point singularities was done by C. Bender and T. Wu [12] in a $\lambda\phi^4$ field theory. In this model the perturbation series for the ground state energy diverges. The authors carried out a study for the structure of energy levels extending the analyticity to the complex $\lambda-$plane based on a WKB method and the property of self-orthogonality of the functions, and found that the energy levels $E(\lambda)$ have an infinite number of branch points. For each branch point a crossing energy level occurs (two-fold degeneracy) associated to a self-orthogonal state.

This property of branch points has been studied in the THDM model in ref. [11]. The effects of interference due to widths of two or more near neutral Higgs resonances in a general multi-Higgs doublet models with CP non invariance has been studied in [13].

The presence of branch points have been studied in nuclear physics [21] and meson physics, [22], as well as in string theory [23]. Their physical properties have been observed in non-relativistic physical systems, as for example in microwave cavities [14, 15, 16, 17, 18], in electronic systems [19], and in molecular photo-dissociation [20].

The goal of this work is to study the manifestation of accidental degeneracy in the Higgs heavy neutral system. In section 3 we calculate the transition matrix for the s-channel via Higgs $\mathcal{CP}$ considering a top dominant loop showing the analytical form at exact degeneracy. The coherence of the Higgs boson states might show up as non-hermitian resonances in the Higgs lineshape via fermionic scattering. Considering a complete calculation with $s-$dependence will be relevant when analyzing possible $\mathcal{CP}$ experimental signals at colliders.

### 2. Masses as poles of the propagator

The first order squared mass matrix $M^{(0)2}$ of neutral Higgs fields, which is Hermitian and symmetric by $CPT$ invariance, is readily derived from the second derivative of the Higgs potential. In order to renormalize the masses it has to be supplemented by the anti-hermitian decay matrix $-i M \Gamma(s)$ [26, 27], which includes the widths of the Higgs states in the diagonal elements, as well as the transition matrix elements for any combination of pairs of states in the off diagonal elements, these last elements are non zero in the $\mathcal{CP}$ case.

$$M^2(s) = M^2(s) - i M \Gamma(s) = M^{(0)2} + \hat{\Pi}(s),$$

where $M^{(0)2}$ is the bare mass and $\hat{\Pi}(s)$ contains the quantum corrections. The mass matrix elements $\hat{\Pi}(s)$ are constructed explicitly from the contribution of the self-energy diagrams, i.e.
radiative mass corrections. It is important to notice that the physical masses, defined as the poles of the propagator of the transition matrix, where the complete renormalized mass matrix should be considered, will depend on the model as new loop corrections coming from extra particles may contribute to Higgs radiative corrections; it will also depend on the way non-Hermitian couplings appear in this mass matrix.

To obtain renormalized masses there are different approaches [28]: a) The effective potential method, b) The renormalization group method, c) Complete one-loop calculation.

Specifically for the case of CP violation of neutral Higgs sector this issue has been treated with a variety of approaches in different models:

(i) Only Standard Model loop contributions resumed in the Pinch Technique framework [29, 30].
(ii) In the MSSM considering that the CP contributions are the diagrams dominated by complex trilineal A-terms couplings to the stop and sbottom [31].
(iii) Considering domain $m_4^t$ contributions [5]
(iv) All complex-MSSM contributions using Feynman Diagrammatic approach to the self-energies considering complex couplings were calculated numerically by Frank et al. [6].

Whichever way it is calculated, the mass matrix should be gauge independent, display only physical thresholds, have the correct unitarity, analyticity and renormalization group properties and satisfy the equivalence principle.

For the purpose of this paper we will consider the Higgs masses on a simplified toy model given in [3]. A complete analysis of degeneracy for the renormalized Higgs masses in the CMSSM will be presented in a forthcoming paper.

2.1. Neutral Higgs bosons as s-channel resonances

The detailed study of the $H_2 − H_3$ system is particularly interesting because in most beyond the Standard Model scenarios, such as SUSY, the masses of these two particles are almost degenerate. Therefore, their resonant production is expected to be detected as two relatively closely spaced or even overlapping resonances in processes such as the production of fermion and anti-fermion pairs in collisions of muons and anti-muons. In the case of non-coherence, the lineshape of these resonances (fig. (1)) in the muon cross section via Higgs boson exchange, would indicate the presence or absence of CP mixing in the $H_2 − H_3$ system [32]. The lineshape of non-complex couplings on the MSSM is given in ref. [34].

\[ \mathcal{T} = \mathcal{T}^s + \mathcal{T}^t = V_i^P \Delta_{ij} V_j^D + \mathcal{T}^t, \]  

Figure 1. s-channel one-loop muon cross section through neutral CP Higgs

The transition amplitude for the production of a fermion-antifermion pair $f \text{ and } \bar{f}$ in a $\mu \mu$ collision is a two channel contribution and may be written as
where \( i, j = A, H \) for the known \( CP \) invariant basis and \( i, j = H_2, H_3 \) for the physical basis. \( V^P_i \) and \( V^D_j \) are the production and decay amplitudes of the processes. At one-loop level, the production and decay vertices, \( V^P_i(s) \) and \( V^D_j(s) \) coincide with the corresponding tree level vertices.

In eq. (2), \( \Delta_{ij} \) is the propagator matrix in the presence of mixing in the \( s \)-channel, and \( T^t \) represents all the \( t \)-channel and box diagrams. In the resonant region, on which we concentrate in this section, the \( t \)-channel and box diagrams are sub-dominant and can be safely ignored [30].

Then, in the electroweak basis, the transition amplitude matrix via resonant excitation of \( CP \)-non-conserving Higgs exchange, with \( CP \) violation at one loop is given by

\[
T^s(s) = V^P \frac{1}{s - M^2_H} V^D, \tag{3}
\]

where we have consider the decoupling limit approximation for the neutral Higgs system, in which we obtain the light Higgs boson \( h^0 \) as a light SM-like Higgs, leaving two neutral heavy Higgs states which at tree level are almost degenerate. Then the mixing of \( CP \) is within the two heavier states and the mass matrix in eq. (3) is given by

\[
M^2_H = M^2_{H_2 H_3}(s) = \begin{pmatrix} m_{H^0}^2 + \hat\Pi_{H H}(s) & \hat\Pi_{H A}(s) \\ \hat\Pi_{H A}(s) & m_{A}^2 + \hat\Pi_{A A}(s) \end{pmatrix}. \tag{4}
\]

This decoupling limit given in terms of the parameters of the Higgs potential, is defined by the inequality [33]

\[
m_{A}^2 >> |\lambda_i|v^2, \tag{5}\]

where \( \lambda_i \) are the couplings of the Lagrangian [3]. This means that the mixing between the light state, \( H_1 \approx h^0 \), and the heavy states, \( H_2 \) and \( H_3 \), is small, compared with the mixing of the nearly degenerate heavy Higgs states \( H_2 \) and \( H_3 \). Now, the physical masses of the neutral heavy Higgs bosons are identified with the poles of the propagator matrix \( \Delta_{H_2 H_3}(s) \). Hence, the masses of the neutral, heavy Higgs bosons are defined as the solutions of the implicit equation

\[
\det \left[ \Delta_{H_2 H_3}^{1}(s) \right] = \det \left[ (s)1_{2 \times 2} - M^2_{H_2 H_3}(s) \right] = 0, \tag{6}\]

is the mass matrix of the neutral and heavy Higgs bosons in the standard \( CP \) defined states basis, where the notation of lower case \( m_{H_i} \) implies tree level mass. In the physical basis, \( M^2_{H_2 H_3}(s) \) is diagonal with mass eigenvalues given by

\[
m_{H_j}^2(s^*) + \hat\Pi_{H_j}(s^*) := \mu^2_{H_j}(s^*), \quad j = 2, 3. \tag{7}\]

We identify \( s_i^* \) with the solution of eq. (6), i.e. the value of the pole mass of the neutral heavy Higgs bosons, where \( s_i^*(x_1, x_2) \) is a function of two or more free parameters of the Lagrangian of the particular model. Thus, the two heavy neutral Higgs bosons are mass degenerate if there exists an \( s^* \) such that

\[
\mu^2_{H_2}(s^*) = \mu^2_{H_3}(s^*). \tag{8}\]

As the determinant is an invariant quantity, we may write eq.(6) in terms of the eigenvalues of this squared mass matrix which, at the pole position \( s^* \), gives

\[
(s_2^* - \mu^2_{H_2}(s^*)) (s_3^* - \mu^2_{H_3}(s^*)) = 0, \quad s_2^* \neq s_3^*. \tag{9}\]
where $s^i(x_1, x_2)$ is a function of two or more free parameters in $\mathcal{L}$. In our case we use the simplified model where the parameters would be $\lambda$ as the magnitude of all complex couplings and $\phi$ as the complex phase.

Finally, in the resonant region, the dispersive (real) part of the self-energy (occurring in the Breit-Wigner denominator of the propagator) can be re-absorbed in the real part of the mass matrix, as is expressed in eq. (1), and the imaginary part will correspond to

$$i \Im m \Pi_{ii}(s) = i M_i \Gamma_i(s), \, \, i = H, A.$$  \hfill (10)

Then the masses of the neutral heavy Higgs bosons can be written as

$$\mu_{H_i}^2(s) = M_{H_i}^2 - i M_i \Gamma_i = \frac{1}{2} T \pm \sqrt{\tilde{R} - i \tilde{\Gamma}}^2$$  \hfill (11)

with $j = 2, 3$ and

$$\tilde{R} = \frac{1}{2} \left[ M_{H_i}^2 - M_A^2 \right], 0, \Re e \left[ \Pi_{HA}(s) \right], \quad \tilde{\Gamma} = \left( \frac{1}{2} \left[ M_H \Gamma_H(s) - M_A \Gamma_A(s) \right], 0, \Im m \left[ \Pi_{HA}(s) \right] \right)$$

which, in the case of the $CP$ Higgs heavy sector may change the structure of the physical states near degeneracy and should be taken into account carefully. In next section we will obtain the mass matrix elements explicitly using a simplified THDM to account for the 1 loop corrections.

3. Branch point in simplified THDM

The branch point or exceptional point is associated to the degeneracy of the masses in the physical basis. At exact degeneracy, the eigenstates and eigenvalues of the mass matrix are equal, and we can find the values of the parameter model at which this occurs.

We will be using the mass matrix parametrization following ref. [3] where one-loop top quark is taken to be dominant. for quantum radiative corrections The two parameters $\lambda$ and $\phi$ are the THDM model $CP$ violation parameters coming from the Lagrangian, $\lambda = |\lambda_i|$ as a generalized magnitude of complex couplings and $\phi$ the generalized phase in the couplings.

The degeneracy can be analyzed through the difference of the physical masses, which may vanish for specific values of the parameters. Other analysis of this kind of singularity can be found in [8, 9, 10]. The complex masses are unfolded generating hypersurfaces which show the branch point behavior of the singularity. The real and imaginary parts of the masses have branch cuts which begin in the same exceptional point but they extend in opposite directions, as is shown in Figure 1 of [11].

The difference of the mass eigenstates (11) can be given in terms of the non-physical basis as

$$\mu_2^2 - \mu_3^2 = (M_{H_2}^2 - M_{H_3}^2) - i (M_{H_2} \Gamma_{H_2} - M_{H_3} \Gamma_{H_3})$$

$$= \left\{ \left[ (M_{H_i}^2 - M_A^2)^2 - (M_H \Gamma_H(s^*) - M_A \Gamma_A(s^*))^2 \right] + 4 \left[ (\Re e \Pi_{HA})^2 - (\Im m \Pi_{HA})^2 \right] - 2i \left[ (M_{H_i}^2 - M_A^2)(M_H \Gamma_H(s^*) - M_A \Gamma_A(s^*)) \right] - M_A \Gamma_A(s^*) \right\}^{1/2}. \hfill (12)$$

Considering no $s$-dependence, the non-physical mass matrix elements differences are given in terms of the Lagrangian parameters by

$$M_{H_i}^2 - M_A^2 = \lambda \nu^2 \cos \phi,$$
\[ |M_H \Gamma_H - M_A \Gamma_A| = \frac{1}{32\pi} (\Delta_t + 9\lambda^2 v^2 \cos 2\phi). \]  

(13)

Here the top quark contribution is contained in \( \Delta_t = -12M^2_{H/A} (m_t/v)^2 (1 - \beta_t^2) \beta_t \), with \( \beta_t \) the rapidity of the top quark. Using the same values of the masses and decay width involved, proposed in the same reference: \( M_H = 520\text{GeV}, \ M_A = 500\text{GeV}, \ \Gamma_H = 2.58\text{GeV} \) and \( \Gamma_H = 2.58\text{GeV} \).

The off-diagonal terms are parametrized as

\[ \text{Re} \Pi_{HA} = -\frac{1}{2} \lambda v^2 \sin \phi, \quad \text{Im} \Pi_{HA} = -\frac{9}{64\pi} \lambda^2 v^2 \sin 2\phi, \]  

(14)

In terms of this model parametrization, the difference of the squared masses of the heavy neutral system \( H_2 - H_3 \) becomes

\[ \mu_2^2 - \mu_3^2 = \frac{1}{2} \left\{ \left[ \lambda^2 v^4 - \left( \frac{9}{32\pi} \right)^2 \lambda^4 v^2 \right] - \left( \frac{1}{32\pi} \right)^2 \left( \Delta_t^2 + 18\Delta_t \lambda^2 v^2 \cos 2\phi \right) \right. \\
- \left. i \frac{1}{16\pi} \left( \Delta_t \lambda v^2 \cos \phi + 9\lambda^3 v^4 \cos \phi (1 - 4 \sin^2 \phi) \right) \right\}^{1/2} \]  

(15)

Solving for \( \mu_2^2 - \mu_3^2 = 0 \) we find the numerical values of \( \lambda \) and \( \phi \) which accomplish degeneracy conditions

\[ \lambda^* = 0.1075 \quad \text{and} \quad \phi = \frac{\pi}{2} \]  

(16)

which are physically acceptable.

3.1. Analytic structure of the singularity

To determine the nature of the mass singularity in the resonance crossing, it is convenient to unfold the exceptional point in surfaces around it. The term under the square root in eq. (15) is a regular function of its arguments and may be expanded in a Taylor series around the exceptional point as

\[ \frac{1}{2} \sqrt{c^{(1)}_1 (\lambda - \lambda^*) + c^{(1)}_2 (\phi - \phi^*)}, \]  

(17)

where the \( c^{(1)}_k \)'s are the derivatives of the term in the square root in eq. (15). In order to examine the precise nature of the singularity of functions \( \mu^2_{2,3}(\lambda, \phi) \) we introduce the following notation in the parameter space

\[ \vec{R} = \begin{pmatrix} \text{Re} c_1^{(1)} \\ \text{Re} c_2^{(1)} \end{pmatrix}, \quad \vec{I} = \begin{pmatrix} \text{Im} c_1^{(1)} \\ \text{Im} c_2^{(1)} \end{pmatrix}, \quad \vec{\zeta} = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} \lambda - \lambda^* \\ \phi - \phi^* \end{pmatrix}, \]  

(18)

and

\[ \text{sgn}(\text{Re} \mu^2_{2,3}) \text{sgn}(\text{Im} \mu^2_{2,3}) = \text{sgn}(\vec{I} \cdot \vec{\zeta}), \]  

(19)

then we obtain the real and imaginary parts for \( \mu^2_{2,3}(\lambda, \phi) \) as

\[ \text{Re} \mu^2_{2,3}(\lambda, \phi) = \pm \frac{1}{2\sqrt{2}} \left[ \sqrt{(\vec{R} \cdot \vec{\zeta})^2 + (\vec{I} \cdot \vec{\zeta})^2 + (\vec{R} \cdot \vec{\zeta})^2} \right]^{1/2}, \]  

(20)
\[ \Im m \mu_2^2(\lambda, \phi) = \pm \frac{1}{2\sqrt{2}} \left[ \sqrt{(\vec{R} \cdot \vec{\zeta})^2 + (\vec{I} \cdot \vec{\zeta})^2 - (\vec{R} \cdot \vec{\zeta})^2} \right]^{1/2}, \tag{21} \]

From equation (20) it can be seen that near the exceptional point \( \Re e \mu_2^2(\lambda, \phi) \), is a two branch function of \( (\lambda, \phi) \), which can be represented by a two fold surface \( \Sigma_R \) in a tridimensional euclidean space with coordinates \( (\Re e \mu_2^2, \lambda, \phi) \). The two branches of \( \Re e \mu_2^2(\lambda, \phi) \) are represented by two sheets, which are copies of the \( (\lambda, \phi) \) plane branches represent twofold sheets as copies of the \( (\lambda, \phi) \) having a cut along the line where the two branches join smoothly. The cut is defined as the locus of the point where the argument of the square root function vanishes.

Close to the origin of coordinates, this locus is defined by a unit vector \( \vec{\zeta}_c \) in the \( (\lambda, \phi) \) plane, such

\[ \vec{I} \cdot \vec{\zeta}_c = 0 \quad \text{and} \quad \vec{R} \cdot \vec{\zeta}_c = -|\vec{R} \cdot \vec{\zeta}_c|. \tag{22} \]

Therefore, the real part of the function \( \mu_2^2(\lambda, \phi) \), as a function of the real parameters \( (\lambda, \phi) \), has an algebraic branch point of square root type at exceptional point, and a branch cut along a line that start at the exceptional point and extend in a positive direction defined by a unit vector \( \vec{\zeta}_c \) satisfying eq.(22).

A similar analysis shows that \( \Im m \mu_2^2(\lambda, \phi) \), also has an algebraic point of square root type at the exceptional point with coordinates \( (\lambda_d, \phi_d) \), and also has a branch cut along a line that start at exceptional point and extends in the negative direction defined by the unit vector \( \vec{\zeta}_c \), see Fig. 1 of ref. [11]

Discussion about this analysis can be seen in [11] where we show the complex surfaces for the real and imaginary part of the neutral heavy Higgs masses near degeneracy and where the branch point is illustrated.

4. Lineshape of \( CP \) neutral Higgs bosons

The s-channel resonance cross section, \( \sigma(\mu \bar{\mu} \rightarrow H^{CP} \rightarrow l\bar{l}) \) is obtained from the squared of the transition matrix eq. (3). The differential cross section will be, up to the phase space constants, proportional to the square of the transition amplitude

\[ d\sigma \propto \left| T^d(s) \right|^2; \tag{23} \]

or by the Optical Theorem

\[ d\sigma(s) \propto -i \left| T^d(s) - T^d(s)^\dagger \right|. \tag{24} \]

We will now discuss the form of the transition matrix both off degeneracy and at degeneracy to show its continuity and the consequences on Higgs lineshape of going through the singular point of exact degeneracy.

4.1. Transition matrix with non-coherence of states

Once we are able to write the neutral heavy \( CP \) Higgs bosons mass matrix (4) in the physical states basis, the transition matrix in eq. (3) as the general transition matrix off-degeneracy for
Figure 2. The behavior of non-coherent states, the interference of two resonances associated to Breit-Wigner propagators add constructively.

The transition matrix off-degeneracy has the expected behavior in the resonance region, two simple poles in the $s$–plane located at the physical masses of the heavy neutral Higgs bosons $H_2$ and $H_3$. It shows no coherence of the states and the interference is constructive, as can be seen as in figure 2.

Within the THDM with only top loop [3] there is no $s$-dependence on the mass matrix, we thus obtain a simplified expression for the transition matrix (25), as

$$\left(\mu^2_{H_i}(s_i^*)\right)' = \left(\frac{d\mu^2_{H_i}(s)}{ds}\right)_{s_i} = 0$$

$$\mathcal{T}_{res} = \frac{1}{\mu^2_{H_3} - \mu^2_{H_2}} \left[ \frac{1}{s - \mu^2_{H_2}} - \frac{1}{s - \mu^2_{H_3}} \right] \times \left\{ V_H^P (s - M_A^2 + i M_A \Gamma_A) V_H^D + V_A^P (s - M_H^2 + i M_H \Gamma_H) V_A^D + V_H^P \Delta^2_{HA} V_H^D - V_A^P \Delta^2_{HA} V_A^D \right\}.$$
Figure 3. Off degeneracy and non-coherence of states, the heavy and neutral $\mathcal{CP}$ Higgs system will behave as constructive interference of resonances. We are taking $\alpha = \pi/6$ and $\tan \beta = 15$.

$$\left| T_{\text{res}} \right|^2 = \frac{1}{(s - \mu_{H_3}^2)[s - \mu_{H_2}^2]} \times \left\{ V^P_H (s - M_A^2 + i M_A \Gamma_A) V^P_H + V^P_H (s - M_H^2 + i M_H \Gamma_H) V^D_H + V^P_H \Delta^2_{HA} V^D_H - V^P_H \Delta^2_{HA} V^D_H \right\}. \quad (26)$$

For the numerator, the vertices correspond to the Yukawa couplings.

$$V^P_H = Y^H_{\mu} = \frac{g m_{\mu} \cos \alpha}{2 M_W \cos \beta} = V^P_H^\dagger,$$
$$V^D_H = Y^H_{l} = \frac{g m_{l} \cos \alpha}{2 M_W \cos \beta} = V^D_H^\dagger;$$
$$V^P_A = Y^A_{\mu} = i \gamma^5 \frac{g m_{\mu} \tan \beta}{2 M_W} = -V^P_H^\dagger,$$
$$V^D_A = Y^A_{l} = i \gamma^5 \frac{g m_{l} \tan \beta}{2 M_W} = -V^D_H^\dagger. \quad (27)$$

Then we find the squared transition matrix will have the form

$$\left| T_{\text{res}} \right|^2 = \frac{1}{|s - \mu_{H_3}^2|^2 |s - \mu_{H_2}^2|^2} \frac{g^2 m_{\mu} m_{l}}{4 M_W^2 \cos^2 \beta} \left\{ [(s - M_A^2)^2 + (M_A \Gamma_A)^2] \cos^4 \alpha \right. + \left. [(s - M_H^2)^2 + (M_H \Gamma_H)^2] \sin^4 \beta - 2 \cos^2 \alpha \sin^2 \beta \Re \left[ \tilde{M}_A^2 \tilde{M}_H^2 \right] \right\}$$

$$= \frac{1}{|s - \mu_{H_3}^2|^2 |s - \mu_{H_2}^2|^2} \frac{g^2 m_{\mu} m_{l}}{4 M_W^2 \cos^2 \beta} \left[ \tilde{M}_A^2 \cos^2 \alpha - \tilde{M}_H^2 \sin^2 \beta \right], \quad (28)$$

where we have defined $\tilde{M}_A^2 = (s - M_A^2 + i M_A \Gamma_A)$ and $\tilde{M}_H^2 = (s - M_H^2 + i M_H \Gamma_H)$, and $\Re \left[ \tilde{M}_A^2 \tilde{M}_H^2 \right] = [(s - M_A^2)(s - M_H^2) + M_A \Gamma_A M_H \Gamma_H]$. We can see that off-degeneracy the interference of two particles behave the usual way.

We now run the energy $\sqrt{s}$ for $\left| T_{\text{res}} \right|^2$ to find the heavy neutral $\mathcal{CP}$ Higgs state near the accidental degeneracy.

In the case of $\mathcal{CP}$ conserving neutral Higgs bosons there are not a coalescence of the states and the states simply overlap as for example in figure 22 of ref. [34].
The next step is to find the continuity of the transition matrix at exact degeneracy and take this into account for the calculation of the neutral heavy Higgs lineshape.

4.2. Transition matrix at degeneracy

Because of the coalescence of the corresponding eigenstates the transition matrix is no longer diagonalizable by a similarity transformation, so we write it as a Jordan block, and we find a generalized eigenstate to complete the basis. As we mentioned the masses can be written as functions of the complex parameters of the Lagrangian and at exact degeneracy will show up as a rank one algebraic branch point. In this section we write down the structure of this singularity which comes from the mixing in the propagator of the neutral and heavy Higgs states $H$ and $A$. The propagator has a double pole in the non-physical sheet of the complex plane of the squared energy $s$. We show the form of the propagator at exact degeneracy.

Now, at exact degeneracy $M_{H_2} = M_{H_3} = M_{H_d}$, the propagator matrix in the $H - A$ representation takes the form

$$
\Delta^{(d)}_{H_2 H_3}(s) = \frac{1}{s - M_d^2 + iM_d \Gamma_d} 1_{2 \times 2} + \frac{1}{(s - M_d^2 + iM_d \Gamma_d)^2} M_d \Gamma_d(s^*) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}. \tag{29}
$$

The matrix on the right hand side of this equation may be brought to a Jordan block form by means of a similarity transformation,

$$
\Delta^{(d)}_{H_2 H_3}(s) = 
\begin{pmatrix}
\frac{1}{s - M_d^2 + iM_d \Gamma_d} & iM_d \Gamma_d(s^*) \\
0 & \frac{1}{s - M_d^2 + iM_d \Gamma_d}
\end{pmatrix}, \tag{30}
$$

then, the resonant transition matrix at degeneracy, in the mass representation takes the form

$$
T^d(s) = \begin{pmatrix} \tilde{V}_H^P & \tilde{V}_A^P \end{pmatrix} \Delta^{(d)}_{H_2 H_3}(s) \begin{pmatrix} \tilde{V}_H^D \\ \tilde{V}_A^D \end{pmatrix} \tag{31}
$$

Calculating the squared of this matrix considering parameter values $\lambda$ and $\phi$ off-degeneracy, but close to it, and taking also into account fermions in and out of the process we have a quite different behavior of the heavy neutral CP Higgs lineshape, figure 4.2. Even at not exact degeneracy but close enough, the interference on the coalescent states will cause a lineshape reduction down to zero.

This behavior was analyzed previously by [13] in simplified multi-Higgs models, but it was not considered in previous CP non invariant analysis of neutral heavy Higgs, as for example in figure 3 of ref. [32].

5. Summary and conclusions

In a CP conserving multi-Higgs models, the neutral scalar states will have CP-even charge and pseudoscalar will have CP-odd charge. In general, for a CP non-conserving multi-Higgs models the CP identification of neutral Higgs states is no longer possible. Specifically for models with Higgs complex doublets they mix so having even and odd components in their wave function. In the 2HDM, the neutral Higgs bosons spectrum includes a light state $h_1$, the SM-like Higgs boson, and two heavy neutral states, $H, A$. In a CP context the two heavy neutral Higgs states are no longer CP defined and can strongly mix $H_2 - H_3$. They may be degenerate not only in its mass
Figure 4. Near degeneracy the coalescence of the states may interfere destructively in the resonance. Considering the values of the parameters not at exact degeneracy but close, the lineshape reduces drastically to zero at the pole for the CP Higgs heavy neutral bosons. We are taking $\alpha = \pi/6$ and $\tan \beta = 15$. but in states. These properties manifest themselves as mixing of two coherent resonances in the transition amplitude for the production of a fermion-antifermion pair in a $\mu^+\mu^-$ collision, for example. From the analysis of the effective mass matrix in the propagator of the heavy neutral system $H_2 - H_3$, we found that, at the degeneracy point, the hypersurfaces that represent the physical masses (pole masses) as functions of the parameters of the system have a rank one algebraic branch point, and the real and imaginary parts of the mass have branch cuts both starting at the same exceptional point but extending in opposite directions. Associated with this singularity, the propagator of the mixed heavy neutral Higgs system, $H_2 - H_3$, has a double pole in the complex $s$-plane. We also examined the continuity of the transition amplitude at the point of exact degeneracy of the two masses. We analyzed the physical masses and possible degeneracy of the heavy neutral Higgs boson system in a THDM with complex couplings. At exact coalescence of two neutral Higgs states, the propagator matrix has two single poles and one double pole. More details on this calculation can be found in ref. [11]. A more detailed CMSSM analysis is in progress.

We analyzed and displayed the behavior of a degeneracy of a non-hermitian matrix which is radically different from a hermitian system. We showed that the transition matrix is continuous at the exact coalescence, allowing thus to calculate the CPV in a specific cross section at the exact degeneracy.

The heavy neutral Higgs and CP lineshape would have a radically different behavior than the ones showed in the literature. As an example the process $\mu\mu \rightarrow H^{CP} \rightarrow ff$ in the s-channel resonance goes down to zero at exact degeneracy. This analysis suggests that a detailed experimental study of the lineshape of the resonances associated with the excitation of this Higgs system may provide valuable information on the CP nature of the underlying theory.

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