On- and Off-Policy Monotonic Policy Improvement

Ryo Iwaki  
Department of Adaptive Machine Systems  
Graduate School of Engineering  
Osaka University  
2-1, Yamadaoka, Suita city, Osaka, Japan  
ryo.iwaki@ams.eng.osaka-u.ac.jp

Minoru Asada  
Department of Adaptive Machine Systems  
Graduate School of Engineering  
Osaka University,  
2-1, Yamadaoka, Suita city, Osaka, Japan  
asada@ams.eng.osaka-u.ac.jp

Abstract

Monotonic policy improvement and off-policy learning are two main desirable properties for reinforcement learning algorithms. In this study, we show that the monotonic policy improvement is guaranteed from on- and off-policy mixture data. Based on the theoretical result, we provide an algorithm which uses the experience replay technique for trust region policy optimization. The proposed method can be regarded as a variant of off-policy natural policy gradient method.

1. Introduction

Reinforcement learning (RL) aims to optimize the behavior of an agent which interacts sequentially with an unknown environment in order to maximize the long term future reward. There are two main desirable properties for RL algorithms: monotonic policy improvement and off-policy learning.

If the model of environment is available and the state is fully observable, a sequence of greedy policies generated by policy iteration scheme are guaranteed to improve monotonically. However, in the approximate policy iteration including RL, the generated policy could perform worse and lead to policy oscillation or policy degradation (Bertsekas, 2011; Wagner, 2011, 2014). In order to avoid such phenomena, there are some efforts to guarantee the monotonic policy improvement (Kakade and Langford, 2002; Pirotta et al., 2013; Schulman et al., 2015; Thomas et al., 2015b; Abbasi-Yadkori et al., 2016).

On the other hand, the use of off-policy data is also very crucial for real world applications. In an off-policy setting, a policy which generate the data is different from a policy to be optimized. There are many theoretical efforts to efficiently use off-policy data (Precup et al., 2000; Maei, 2011; Degris et al., 2012; Zhao et al., 2013; Silver et al., 2014; Thomas et al., 2015a; Harutyunyan et al., 2016; Munos et al., 2016). Off-policy learning methods enable the agent to, for example, optimize huge function approximators effectively (Mnih et al., 2015; Wang et al., 2017) and learn a complex policy for humanoid robot control in real world by reusing very few data (Sugimoto et al., 2016).
In this paper, extending the approach by Pirotta et al. (2013), we show that monotonic policy improvement is guaranteed from on- and off-policy mixture data. Based on the theoretical result, we also provide an trust region policy optimization (TRPO) method (Schulman et al., 2015) with experience replay (Lin, 1992). The proposed method uses the Kullback-Leibler divergence as a metric for constraint, thus this method can be regarded as a variant of off-policy natural policy gradient method.

2. Preliminaries

We consider an infinite horizon discounted Markov decision process (MDP). An MDP is specified by a tuple \((\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \rho_0, \gamma)\). \(\mathcal{S}\) is a finite set of possible states of an environment and \(\mathcal{A}\) is a finite set of possible actions which an agent can choose. \(\mathcal{P} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}\) is a Markovian state transition probability distribution, \(\mathcal{R} : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}\) is a bounded reward function, \(\rho_0 : \mathcal{S} \rightarrow \mathbb{R}\) is an initial state distribution, and \(\gamma \in (0, 1)\) is a discount factor. We are interested in the model-free RL, thus we suppose that \(\mathcal{P}\) and \(\mathcal{R}\) are unknown.

Let \(\pi\) be a policy of the agent; if the policy is deterministic, \(\pi\) denotes the mapping between the state and action spaces, \(\pi : \mathcal{S} \rightarrow \mathcal{A}\), and if the policy is stochastic, \(\pi\) denotes the distribution over the state-action pair, \(\pi : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}\). For each policy \(\pi\), there exists an unnormalized \(\gamma\)-discounted future state distribution for the initial state distribution \(\rho_0\),

\[
\rho_\pi(s) = \sum_{t=0}^{\infty} \gamma^t \Pr(s_t = s)\rho_0(a_0 = a).
\]

We define the state value function \(V^\pi(s)\), the action value function \(Q^\pi(s, a)\), and the advantage function \(A^\pi(s, a)\) for the policy \(\pi\) as follows:

\[
V^\pi(s) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t \mathcal{R}(s_t, a_t) s_0 = s \right],
\]

\[
Q^\pi(s, a) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t \mathcal{R}(s_t, a_t) s_0 = s, a_0 = a \right],
\]

\[
A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s).
\]

Note that the following Bellman equations hold:

\[
V^\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left( \mathcal{R}(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s, a)V^\pi(s') \right),
\]

\[
Q^\pi(s, a) = \mathcal{R}(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s, a) \sum_{a' \in \mathcal{A}} \pi(a'|s')Q^\pi(s', a').
\]

Furthermore, we define the advantage of a policy \(\pi'\) over the policy \(\pi\) for each state \(s\):

\[
\tilde{A}^\pi_{\pi'}(s) = \sum_{a \in \mathcal{A}} \pi'(a|s)A^\pi(s, a) = \sum_{a \in \mathcal{A}} \left( \pi'(a|s) - \pi(a|s) \right) Q^\pi(s, a).
\]

The purpose of the agent is to find a policy \(\pi^*\) which maximizes the expected discounted reward \(\eta(\pi)\):

\[
\pi^* \in \arg \max_{\pi} \eta(\pi),
\]

2
where
\[ \eta(\pi) = \sum_{s \in S} \rho_0 V^\pi(s) = \sum_{s \in S} \rho_\pi(s) \sum_{a \in A} \pi(a|s) R(s, a). \]

In the following, we use the matrix notation for the previous equations as in Pirotta et al. (2013):
\[ v^\pi = \Pi^\pi (r + \gamma P v^\pi) = r^\pi + \gamma P v^\pi, \]
\[ q^\pi = r + \gamma P \Pi^\pi q^\pi, \]
\[ \bar{A}^\pi = \Pi^\pi' \Lambda^\pi = \left( \Pi^\pi' - \Pi^\pi \right) q^\pi, \]
\[ \eta(\pi) = \rho_0^\top v^\pi = \rho_0^\top (I - \gamma P^\pi)^{-1} r^\pi = \rho^\pi^\top r^\pi, \tag{1} \]

where \( \eta(\pi) \) is a scalar, \( r^\pi, v^\pi, \rho_0, \rho^\pi \) and \( \bar{A}^\pi \) are vectors of size \(|S|\), \( r, q^\pi \) and \( \Lambda^\pi \) are vectors of size \(|S||A|\), \( P \) is a stochastic matrix of size \(|S||A| \times |S|\) which contains the state transition probability distribution: \( P((s, a), s') = P(s'|s, a) \), \( \Pi^\pi \) is a stochastic matrix of size \(|S| \times |S||A|\) which contains the policy: \( \Pi^\pi(s, (s, a)) = \pi(a|s) \), and \( P^\pi = \Pi^\pi P \) is a stochastic matrix of size \(|S| \times |S|\) which represents the state transition matrix under the policy \( \pi \). Let \( M \) be a matrix whose entries are \( m_{ij} \), then \( \|M\|_1 = \max_i \sum_j |m_{ij}|, \|M\|_\infty = \max_j \sum_i |m_{ij}| \) and \( \|M\|_1 = \|M^\top\|_\infty \).

3. On- and Off-Policy Monotonic Policy Improvement Guarantee

In this section, we show that the monotonic policy improvement is guaranteed from on- and off-policy mixture data.

First, we introduce two lemmas to provide the main theorem. The first lemma states that the difference between the performances of any policies \( \pi \) and \( \pi' \) is given as a function of the advantage.

**Lemma 1.** (Kakade and Langford, 2002, lemma 6.1) Let \( \pi \) and \( \pi' \) be any stationary policies. Then:
\[ \eta(\pi') - \eta(\pi) = \rho^\pi^\top \bar{A}^\pi. \]

The second lemma gives a bound on the inner product of two vectors.

**Lemma 2.** (Haviv and Heyden, 1984, Corollary 2.4) Let \( e \) be a column vector of all entries are one. For a vector \( x \) such that \( x^\top e = 0 \) and any vector \( y \), it holds that
\[ |x^\top y| \leq \|x\|_1 \max_{i,j} |y_i - y_j| \frac{1}{2}. \]

Next we provide a bound to the difference between the \( \gamma \)-discounted state distributions for any stationary policies \( \pi, \pi' \) and \( \beta \).
Lemma 3. Let \( \pi, \pi' \) and \( \beta \) be any stationary policies for an infinite-horizon MDP with state transition probability \( P \). For \( \alpha \in [0, 1] \), the \( L_1 \)-norm of the difference between the \( \gamma \)-discounted state distributions is upper bounded as follows:

\[
\| \rho^\pi - (\alpha \rho^\pi + (1 - \alpha) \rho^\beta) \|_1 \\
\leq \frac{\gamma}{1 - \gamma} \left( \alpha \| P^{\pi'} - P^\pi \|_\infty + (1 - \alpha) \| P^{\pi'} - P^\beta \|_\infty \right) \left\| (I - \gamma P^{\pi'})^{-1} \right\|_\infty.
\]

Proof Eq. (1) indicates that for any policy \( \pi \) and any initial state distribution \( \rho_0 \), \( \gamma \)-discounted state distribution \( \rho^\pi \) satisfies

\[
\rho^\pi = \rho_0^\top + \gamma \rho^\pi^\top P^\pi.
\]

It follows that

\[
\begin{align*}
\left( \rho^\pi - \left( \alpha \rho^\pi + (1 - \alpha) \rho^\beta \right) \right)^\top \\
= \gamma \rho^\pi^\top P^{\pi'} - \left( \alpha \gamma \rho^\pi^\top P^\pi + (1 - \alpha) \gamma \rho^\beta^\top P^\beta \right) \\
= \gamma \left( \rho^{\pi'} - \left( \alpha \rho^\pi + (1 - \alpha) \rho^\beta \right) \right)^\top P^{\pi'} + \gamma \left( \alpha \rho^\pi^\top \left( P^{\pi'} - P^\pi \right) + (1 - \alpha) \rho^\beta^\top \left( P^{\pi'} - P^\beta \right) \right) \\
= \gamma \left( \alpha \rho^\pi^\top \left( P^{\pi'} - P^\pi \right) + (1 - \alpha) \rho^\beta^\top \left( P^{\pi'} - P^\beta \right) \right) \sum_{t=0}^\infty \left( \gamma P^{\pi'} \right)^t \\
= \gamma \left( \alpha \rho^\pi^\top \left( P^{\pi'} - P^\pi \right) + (1 - \alpha) \rho^\beta^\top \left( P^{\pi'} - P^\beta \right) \right) \left( I - \gamma P^{\pi'} \right)^{-1}.
\end{align*}
\]

The equality (2) follows from the successive substitution. Since \( P^{\pi'} \) is a stochastic matrix, the inverse of \( I - \gamma P^{\pi'} \) exists for any \( \gamma < 1 \), thus Neumann series converges and (3) follows. Therefore, it follows that

\[
\begin{align*}
\| \rho^{\pi'} - \left( \alpha \rho^\pi + (1 - \alpha) \rho^\beta \right) \|_1 \\
= \left\| \left( \rho^{\pi'} - \left( \alpha \rho^\pi + (1 - \alpha) \rho^\beta \right) \right)^\top \right\|_\infty \\
= \gamma \left( \alpha \| \rho^{\pi'}^\top \left( P^{\pi'} - P^\pi \right) + (1 - \alpha) \rho^\beta^\top \left( P^{\pi'} - P^\beta \right) \right) \left\| (I - \gamma P^{\pi'})^{-1} \right\|_\infty \\
\leq \gamma \left( \alpha \| P^{\pi'} - P^\pi \|_\infty + (1 - \alpha) \| P^{\pi'} - P^\beta \|_\infty \right) \left\| (I - \gamma P^{\pi'})^{-1} \right\|_\infty \\
\leq \frac{\gamma}{1 - \gamma} \left( \alpha \| P^{\pi'} - P^\pi \|_\infty + (1 - \alpha) \| P^{\pi'} - P^\beta \|_\infty \right) \left\| (I - \gamma P^{\pi'})^{-1} \right\|_\infty.
\end{align*}
\]

The following corollary gives a looser but model-free bound.

Corollary 4. Let \( \pi, \pi' \) and \( \beta \) be any stationary policies. For \( \alpha \in [0, 1] \), the \( L_1 \)-norm of the difference between the \( \gamma \)-discounted state distributions is upper bounded as follows:

\[
\| \rho^{\pi'} - \left( \alpha \rho^\pi + (1 - \alpha) \rho^\beta \right) \|_1 \leq \frac{\gamma}{(1 - \gamma)^2} \left( \alpha \| \Pi^{\pi'} - \Pi^\pi \|_\infty + (1 - \alpha) \| \Pi^{\pi'} - \Pi^\beta \|_\infty \right).
\]
Proof From Lemma 3, it follows that
\[
\| \rho^{\pi'} - (\alpha \rho^{\pi} + (1 - \alpha) \rho^{\beta} ) \|_1 \\
\leq \frac{\gamma}{1 - \gamma} \left( \alpha \| P^{\pi'} - P^{\pi} \|_{\infty} + (1 - \alpha) \| P^{\pi'} - P^{\beta} \|_{\infty} \right) \| (I - \gamma P^{\pi'})^{-1} \|_1 \\
\leq \frac{\gamma}{1 - \gamma} \left( \alpha \| \Pi^{\pi'} - \Pi^{\pi} \|_{\infty} + (1 - \alpha) \| \Pi^{\pi'} - \Pi^{\beta} \|_{\infty} \right) \| P \|_\infty \sum_{t=0}^\infty \gamma \| P^{\pi'} \|_t \\
\leq \frac{\gamma}{(1 - \gamma)^2} \left( \alpha \| \Pi^{\pi'} - \Pi^{\pi} \|_{\infty} + (1 - \alpha) \| \Pi^{\pi'} - \Pi^{\beta} \|_{\infty} \right).
\]

The main theorem is given by combining Lemma 1, Lemma 2, and Corollary 4.

Theorem 5. (On- and Off-Policy Monotonic Policy Improvement Guarantee) Let \( \pi \) and \( \pi' \) be any stationary target policies and \( \beta \) be any stationary behavior policy. For \( \alpha \in [0, 1] \), the difference between the performances of \( \pi' \) and \( \pi \) is lower bounded as follows:
\[
\eta(\pi') - \eta(\pi) \geq \alpha \rho^{\pi \top} \tilde{A}^{\pi}_{\pi'} + (1 - \alpha) \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'}, \\
- \frac{\gamma}{(1 - \gamma)^2} \left( \alpha \| \Pi^{\pi'} - \Pi^{\pi} \|_{\infty}^2 + (1 - \alpha) \| \Pi^{\pi'} - \Pi^{\beta} \|_{\infty} \| \Pi^{\pi'} - \Pi^{\beta} \|_{\infty} \right) \| q^{\pi} \|_{\infty}.
\]

Proof From Lemma 1, it follows that
\[
\eta(\pi') - \eta(\pi) = \rho^{\pi' \top} \tilde{A}^{\pi}_{\pi'} \\
= \rho^{\pi' \top} \tilde{A}^{\pi}_{\pi'} + \alpha \left( \rho^{\pi \top} \tilde{A}^{\pi}_{\pi'} - \rho^{\pi' \top} \tilde{A}^{\pi}_{\pi'} \right) + (1 - \alpha) \left( \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'} - \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'} \right) \\
= \alpha \rho^{\pi \top} \tilde{A}^{\pi}_{\pi'} + (1 - \alpha) \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'} - \left( \rho^{\pi'} - (\alpha \rho^{\pi} + (1 - \alpha) \rho^{\beta} ) \right)^\top \tilde{A}^{\pi}_{\pi'},
\]
where \( \alpha \in [0, 1] \). Note that for any policy \( \pi \), the \( \gamma \)-discounted state distribution \( \rho^{\pi \top} e = 1 \), thus \( (\rho^{\pi'} - (\alpha \rho^{\pi} + (1 - \alpha) \rho^{\beta} ))^\top e = 0 \). Therefore from Lemma 2 and Corollary 4, it follows that
\[
\eta(\pi') - \eta(\pi) \\
= \alpha \rho^{\pi \top} \tilde{A}^{\pi}_{\pi'} + (1 - \alpha) \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'} - \left( \rho^{\pi'} - (\alpha \rho^{\pi} + (1 - \alpha) \rho^{\beta} ) \right)^\top \tilde{A}^{\pi}_{\pi'} \\
\geq \alpha \rho^{\pi \top} \tilde{A}^{\pi}_{\pi'} + (1 - \alpha) \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'} - \left( \rho^{\pi'} - (\alpha \rho^{\pi} + (1 - \alpha) \rho^{\beta} ) \right)^1_2 \frac{\epsilon}{2} \\
\geq \alpha \rho^{\pi \top} \tilde{A}^{\pi}_{\pi'} + (1 - \alpha) \rho^{\beta \top} \tilde{A}^{\pi}_{\pi'} - \frac{\gamma}{(1 - \gamma)^2} \left( \alpha \| \Pi^{\pi'} - \Pi^{\pi} \|_{\infty} + (1 - \alpha) \| \Pi^{\pi'} - \Pi^{\beta} \|_{\infty} \right) \frac{\epsilon}{2},
\]
where \( \epsilon = \max_{s,s'} | \tilde{A}^{\pi}_{\pi'}(s) - \tilde{A}^{\pi}_{\pi'}(s') | \). The theorem follows by upper bounding \( \epsilon/2 \):
\[
\frac{\epsilon}{2} \leq \| \tilde{A}^{\pi}_{\pi'} \|_{\infty} = \left( \| \Pi^{\pi'} - \Pi^{\pi} \| q^{\pi} \|_{\infty} \right) \leq \| \Pi^{\pi'} - \Pi^{\pi} \|_{\infty} \| q^{\pi} \|_{\infty}.
\]

\[\blacksquare\]
Note that for any stochastic policies,
\[ \left\| \Pi' - \Pi \right\|_{\infty} = \max_{s \in S} \sum_{a \in A} |\pi'(a|s) - \pi(a|s)| = \max_{s \in S} \sum_{a \in A} |\pi(a|s) - \pi'(a|s)| \]

is identical to the maximum total variation distance between the policies with respect to the state \( \rho \), \( D_{TV}^{\max}(\pi || \pi') = \max_{s \in S} D_{TV}(\pi(\cdot|s)||\pi'(\cdot|s)) \). Thus Pinsker’s inequality,
\[ \frac{1}{2} D_{TV}(\pi || \pi')^2 \leq D_{KL}(\pi || \pi'), \]

where \( D_{KL}(\pi || \pi') \) is the Kullback-Leibler divergence between two policies, yields following corollary.

**Corollary 6.** Let \( \pi \) and \( \pi' \) be any stochastic stationary target policies and \( \beta \) be any stochastic stationary behavior policy. For \( \alpha \in [0, 1] \), the difference between the performances of \( \pi' \) and \( \pi \) is lower bounded as follows:
\[ \eta(\pi') - \eta(\pi) \geq \alpha \rho \pi', \bar{A}_\pi \gamma + (1 - \alpha) \rho \beta, \bar{A}_\pi \gamma - \frac{2\gamma}{(1 - \gamma)^2} \left( \alpha D_{KL}^{\max}(\pi || \pi') + (1 - \alpha) \left( D_{KL}^{\max}(\pi || \pi') D_{KL}^{\max}(\beta || \pi') \right)^{1/2} \right) \|q^\pi\|_\infty. \] (4)

**Remark 7.** Corollary 6 states that the penalty to the policy improvement is governed by \( D_{KL}^{\max}(\pi || \pi') \), \( D_{KL}^{\max}(\beta || \pi') \) and \( \alpha \). \( D_{KL}^{\max}(\beta || \pi') \) indicates the ‘off-policy-ness’, which possibly takes large value. However, in the penalty term, \( D_{KL}^{\max}(\beta || \pi') \) is multiplied by \( D_{KL}^{\max}(\pi || \pi') \) and \( 1 - \alpha \), thus, the monotonic policy improvement could be established with sufficiently small \( D_{KL}^{\max}(\pi || \pi') \) and appropriate value of \( \alpha \).

### 4. TRPO with Experience Replay

In this section, based on the theoretical result in the previous section, we argue that experience replay technique is directly applicable to the trust region policy optimization scheme. Note that the method presented here is just one possible implementation to perform monotonic policy improvement approximately from on- and off-policy mixture data.

#### 4.1 Relaxation of Constraint

First, we propose to use a heuristic approximation of the expected KL divergence instead of the maximum value as a constraint:
\[ \alpha \mathbb{E}_{s \sim \rho^\pi} \left[ D_{KL}(\pi || \pi') \right] + (1 - \alpha) \left( \mathbb{E}_{s \sim \rho^\pi} \left[ D_{KL}(\pi || \pi') \right] \mathbb{E}_{s \sim \rho^\beta} \left[ D_{KL}(\beta || \pi') \right] \right)^{1/2} \leq \delta. \] (5)

Furthermore, as discussed in Remark 7, the monotonic policy improvement could be established with sufficiently small \( D_{KL}^{\max}(\pi || \pi') \). Thus, in the following we consider the relaxed constraint:
\[ \mathbb{E}_{s \sim \rho^\pi} \left[ D_{KL}(\pi || \pi') \right] \leq \delta. \]

Note that this metric is identical to the one used in the literature of the natural policy gradient (Kakade, 2001; Bagnell and Schneider, 2003; Peters et al., 2003; Morimura et al., 2005), thus proposed method can be regarded as an off-policy natural policy gradient method.

---

1. \( D_{TV}(\pi || \pi') = \frac{1}{2} \sum_{a \in A} |\pi(a|s) - \pi'(a|s)| \) is another common definition of the total variation distance.
4.2 Sample Based Optimization of Parameterized Policy

Suppose that we would like to optimize the policy $\pi_\theta$ with parameter $\theta$. As done in TRPO (Schulman et al., 2015), the constrained optimization problem we should solve to update $\theta$ is:

$$
\max_{\theta'} L(\theta', \theta, \beta, \alpha) = \alpha E_{s \sim \rho^\theta, a \sim \pi_\theta} \left[ \frac{\pi_\theta(a|s)}{\pi_\theta(a|s)} A^{\pi_\theta}(a|s) \right] + (1 - \alpha) E_{s \sim \rho^\beta, a \sim \pi_\beta} \left[ \frac{\pi_\beta(a|s)}{\beta(a|s)} A^{\pi_\beta}(a|s) \right]
$$

subject to $E_{s \sim \rho^\theta} [D_{KL}(\pi_\theta(\cdot|s)\|\pi_\theta'(\cdot|s))] \leq \delta$. (6)

By setting $\alpha = 1$, proposed method reduces to TRPO. Note that $\alpha$ and $\beta$ can be varied at each update. Determining $\alpha$ depending on $E_{s \sim \rho^\beta} [D_{KL}(\beta\|\pi')]$ is an interesting choice. As a sampling from $\rho^\beta$ and $\beta$, one of the possible implementations is to use experience replay (Lin, 1992). Due to the constraint (6), the change of the policy at each update is not large; thus, if the replay buffer is appropriate size, either $E_{s \sim \rho^\beta} [D_{KL}(\beta\|\pi')]$ would not get large and then the policy could enjoy the monotonic improvement approximately.

5. Related Works

The theoretical result and proposed method are extension of the works by Pirotta et al. (2013); Schulman et al. (2015) to on- and off-policy mixture case. Thomas et al. (2015) proposed an algorithm with monotonic improvement which can use off-policy policy evaluation technique. However, its computational complexity is high. Gu et al. (2017a,b) proposed to interpolate TRPO and deep deterministic policy gradient (Lillicrap et al., 2016), and Gu et al. (2017b) showed the performance bound for on- and off-policy mixture update as well. In our notation, the penalty terms in their performance bound has $D_{KL}^{\max}(\pi\|\beta)$, which is a constant with respect to the policy after update, $\pi'$. Furthermore, their bound is directly penalized by the error of action value estimate. In contrast, in Corollary 6, our penalty terms has $D_{KL}^{\max}(\beta\|\pi')$ instead of $D_{KL}^{\max}(\pi\|\beta)$, and which is multiplied by $D_{KL}^{\max}(\pi\|\pi')$ and $1 - \alpha$. Thus all the penalty terms in Eq. (4) can be controlled in a well-designed optimization procedure to find $\pi'$.

6. Conclusion and Outlook

In this study, we showed that monotonic policy improvement is guaranteed from on- and off-policy mixture data. Based on the theoretical result, we also provided the TRPO method with experience replay. The proposed method is a variant of off-policy natural policy gradient method. An important future work is to deal with the full constraint (5).
References

Yasin Abbasi-Yadkori, Peter L. Bartlett, and Stephen J. Wright. A fast and reliable policy improvement algorithm. In *In Artificial Intelligence and Statistics*, pages 1338–1346, 2016.

J. Andrew Bagnell and Jeff Schneider. Covariant policy search. In *International Joint Conference on Artificial Intelligence*, pages 1019–1024, 2003.

Dimirti P. Bertsekas. Approximate policy iteration: A survey and some new methods. *Journal of Control Theory and Applications*, 9(3), 2011.

Thomas Degris, Martha White, and Richard S. Sutton. Off-policy actor-critic. In *International Conference on Machine Learning*, 2012.

Shixiang Gu, Timothy Lillicrap, Zoubin Ghahramani, Richard E Turner, and Sergey Levine. Q-prop: Sample-efficient policy gradient with an off-policy critic. In *International Conference on Learning Representations*, 2017a.

Shixiang Gu, Timothy Lillicrap, Zoubin Ghahramani, Richard E Turner, Bernhard Schölkopf, and Sergey Levine. Interpolated policy gradient: Merging on-policy and off-policy gradient estimation for deep reinforcement learning. In *Advances in Neural Information Processing Systems*, 2017b.

Anna Harutyunyan, Marc G. Bellemare, Tom Stepleton, and Rémi Munos. Q(λ) with off-policy corrections. In *International Conference on Algorithmic Learning Theory*, pages 305–320, 2016.

Moshe Haviv and Ludo Van Der Heyden. Perturbation bounds for the stationary probabilities of a finite markov chain. *Advances in Applied Probability*, 16(4), 1984. URL http://www.jstor.org/stable/1427341

Sham Kakade. A natural policy gradient. In *Advances in Neural Information Processing Systems*, volume 14, 2001.

Sham Kakade and John Langford. Approximately optimal approximate reinforcement learning. In *International Conference on Machine Learning*, volume 2, 2002.

Timothy P. Lillicrap, Jonathan J. Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. In *International Conference on Learning Representations*, 2016.

Long-Ji Lin. Self-improving reactive agents based on reinforcement learning, planning and teaching. *Machine Learning*, 8(3/4):69–97, 1992.

Hamid Reza Maei. *Gradient Temporal-Difference Learning Algorithms*. PhD thesis, University of Alberta, 2011.

Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, Georg Ostrovski, et al.
Human-level control through deep reinforcement learning. *Nature*, 518(7540):529–533, 2015.

Tetsuro Morimura, Eiji Uchibe, and Kenji Doya. Utilizing natural gradient in temporal difference reinforcement learning with eligibility traces. In *International Symposium on Information Geometry and Its Applications*, pages 256–263, 2005.

Rémi Munos, Tom Stepleton, Anna Harutyunyan, and Marc G. Bellemare. Safe and efficient off-policy reinforcement learning. In *Advances in Neural Information Processing Systems*, 2016.

Jan Peters, Sethu Vijayakumar, and Stefan Schaal. Reinforcement learning for humanoid robotics. In *Third IEEE-RAS International Conference on Humanoid Robots*, pages 1–20. American Association for Artificial Intelligence, 2003.

Matteo Pirotta, Marcello Restelli, Alessio Pecorino, and Daniele Calandriello. Safe policy iteration. In *International Conference on Machine Learning*, pages 307–315, 2013.

Doina Precup, Richard S. Sutton, and Satinder Singh. Eligibility traces for off-policy policy evaluation. In *International Conference on Machine Learning*, 2000.

John Schulman, Sergey Levine, Philipp Moritz, Michael Jordan, and Pieter Abbeel. Trust region policy optimization. In *International Conference on Machine Learning*, pages 1889–1897, 2015.

David Silver, Guy Lever, Nicolas Heess, Thomas Degris, Daan Wierstra, and Martin Riedmiller. Deterministic policy gradient algorithms. *International Conference on Machine Learning*, pages 387–395, 2014.

Norikazu Sugimoto, Voot Tangkaratt, Thijs Wensveen, Tingting Zhao, Masashi Sugiyama, and Jun Morimoto. Trial and error: Using previous experiences as simulation models in humanoid motor learning. *IEEE Robotics & Automation Magazine*, 23(1):96–105, 2016.

Philip S. Thomas, Georgios Theocharous, and Mohammad Ghavamzadeh. High confidence off-policy evaluation. In *AAAI*, 2015a.

Philip S. Thomas, Georgios Theocharous, and Mohammad Ghavamzadeh. High confidence policy improvement. In *International Conference on Machine Learning*, 2015b.

Paul Wagner. A reinterpretation of the policy oscillation phenomenon in approximate policy iteration. In *Advances in Neural Information Processing Systems*, 2011.

Paul Wagner. Policy oscillation is overshooting. *Neural Networks*, 52:43–61, 2014.

Ziyu Wang, Victor Bapst, Nicolas Heess, Volodymyr Mnih, Remi Munos, Koray Kavukcuoglu, and Nando de Freitas. Sample efficient actor-critic with experience replay. In *International Conference on Learning Representations*, 2017.

Tingting Zhao, Hirotaka Hachiya, Voot Tangkaratt, Jun Morimoto, and Masashi Sugiyama. Efficient sample reuse in policy gradients with parameter-based exploration. *Neural Computation*, 25(6):1512–1547, 2013.