Determination of stress concentration factors on flat plates of structural steel

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Abstract. The aim of this study was to determine stress concentration factors using finite element software; within the most outstanding results is ANSYS software validation for the determination of concentration factors in flat plates with a central hole subjected to axial load; since it could be statistically demonstrated that there aren’t significant differences between the theoretical data and the data obtained through simulation. Were also obtained plots of stress concentration factors for flat plates with three holes, under the same load; it is noted that if the holes are transversely located, the concentration factor decreases to the extent that the relationship P3 / d increases, but to ensure this correlation it should be prevented that the hole diameter and the hole spacing exceeds a limit value, since if this occurs the point of maximum stress changes the inner to the outer area of the hole because the separation between the hole and the edge of the plate decreases significantly. If the arrangement of the holes is longitudinal it can be seen that the concentration factor decreases to the extent that the ratio d / b increases and additionally the magnitude of the factor is unaffected by the distance between centers of the holes and are quite similar to present a single flat plate with central hole.

1. Introduction
Abrupt changes originated from irregularities in the distribution of stresses are known as stress concentrators; these are presented for all types of stress, axial, bending or shear in the presence of fillets, holes, grooves, keyways, splines, tool marks or accidental scrapes. The inclusions or defects within the material over the surface also serve as "stress risers". The first mathematical study on stress concentration was published shortly after 1900, with the aim of working with other very simple different cases, experimental methods were developed to measure local efforts. In recent years they have started using computer simulations based on finite elements.

The information available in the literature is quite limited and is restricted to simple conditions of stress concentrators, such as a hole, a steak or a groove, to validate the use of finite element software for obtaining this parameter in complex conditions represents a significant contribution to the design of mechanical elements, as it would allow to predict of a better way the stresses in the component parts of a machine due to the action of different load.
2. Conceptual framework

2.1. Stress concentrators
The stress concentrators are geometrical irregularities that cause an increase in the average effort that should be present in regions near these discontinuities, the relationship between the maximum stress that occurs and the average effort that should occur is defined as stress concentration factor; which is determined by experimental or analytical methods and presented in graphical form for ease interpretation.

![Figure 1](image.png)

**Figure 1.** Stress distribution for a plate subjected to tensile load (a) away from the hole; (b) in the section of central hole [1].

A typical example of a stress concentrator is a rectangular bar with a central hole, subjected to tensile load, as shown in figure 1 (a). If the bar is cut in the cross section of the hole, the tensile stress will be as shown in figure 1 (b), the stress distribution along the cut surface is substantially uniform until reaching the vicinity of the hole, where efforts suddenly increase. The maximum value of stress at points is found by multiplying the average effort by a $K$ factor of stress concentration [1, 2].

2.2. Stress concentration factors
The stress concentration factor for static load can be determined as the relation of the actual maximum real stress in the discontinuity and the average stress, and it is obtained through the equation 1 [3].

$$K = \frac{\text{Actual maximum stress}}{\text{Average stress}} \quad (1)$$

The average stress is determined through the basic equations and is defined according to the type of load that is acting on the element. In the case of an axial load that causes tension or compression, this value is calculated by equation 2 [4].

$$\sigma = \frac{\text{Axial Force}}{\text{Cross section area}} \quad (2)$$

The stress concentration factors are the type of discontinuity on the geometry of the discontinuity and the type of load is experienced.

2.3. Finite elements
The general idea of the finite element method is the division of a continuous, in a smaller set of elements interconnected by a series of points called nodes. The equations governing the behavior of continuous also govern the element. Of this way is achieved to pass from a continuous system (infinite degrees of freedom), which is governed by a differential equation or a system of differential equations to a system with a number of finite degrees of freedom whose behavior is modeled by a system of linear or non-linear equations [5, 6].

On nodes are materialized the fundamental unknowns of the problem; in the case of structural elements the fundamental unknowns are nodal displacements, and from these can be calculated the remaining unknowns of interest: stresses, deformations, etc. The finite element method and mathematical formulation is relatively new, although its basic structure has been known from long time, in recent years it has undergone a great development because of advances in computer technology. These advances have been precisely those who have made available to the user many programs.
implementing finite element calculations. The correct handling of this type of program requires a deep knowledge not only of the material with which we work, but also the beginnings of the MEF; only in this case it will be able to ensure that the results of analyzes are in accordance with reality.

3. Methodology
The methodology for the development of the Project consisted of three phases:

3.1. Software simulations for determination of equivalent Von Misses stresses in sheets with central holes
The steps carried out for conducting the simulations were the following:
- Definition of the model that was used for simulation; in this case was defined a static model.
- Deposit the mechanical properties of the material to the database of the software and select it for the simulation; the data provided was: elastic modulus (E = 205 GPa), the yield strength in tension or compression (S_y = 250 MPa) and ultimate strength (S_u = 460 MPa).
- Realization of the graphic representation of the element to simulate; It was drawn a rectangular sheet of 0.00635 m thick; 0.0254 or 0.0508 m width and 0.4 m long. Additionally was performed 1 or 3 holes of different diameters and arrangements in the central zone.
- Realization of meshing process, the zones near discontinuities were refined.
- Definition of loads (magnitude and direction) and the supports that guarantee de static equilibrium.
- Perform the simulation and obtain the distribution of stresses present on the sheet.

3.2. Validation of the use of ANSYS software, in the obtainment of stress concentration factors
The software validation was done by comparing the data obtained from simulations performed on a sheet with a central hole and subjected to axial load; and the theoretical data found in the literature for the same conditions of stress concentration.

3.3. Determination of the graphics of stress concentrator factors in flat plates with three central holes, under axial load
The simulations to determine the stress concentration factor in flat plates with three central holes were performed for Wheel base holes of 0.015875 and 0.0254 m, and over a plate of 0.0508 m wide; 0.00635 m thick and 0.4 m length. The arrangement of the holes in the plate was made of two different forms, longitudinally and transversely; the diameters used were: 0.001016, 0.002032, 0.003048, 0.004064, 0.00508, 0.006096, 0.007112, 0.008128, 0.009144, 0.01016 y 0.011176 m; for the following loads: 5000, 10000, 15000, 20000N; these forces were applied on a timely basis in the centroid of the cross section of the plate.

4. Results obtained

4.1. Software validation
The results of the maximum stresses obtained from the simulations were used to determine the stress concentration factor in flat plates with a central hole, and subjected to axial load; these data are reflected in figure 2.
To validate the use of Ansys software in obtainment of concentration factors, we proceeded to determine the % error present between the theoretical data and those obtained through simulations, which are listed in figure 2; In addition was carried out an statistical analysis that showed a decision variable t of experimental student of 0.237, value that is below the 2.1448 obtained from the chart of this variable for a reliability of 95%, which allows us to affirm that there are no significant differences between the theoretical and simulated data.
4.2. Stress concentration factors in flat plates with three holes under axial load
The behavior of stress concentration factors of three holes located transversely on a flat plate, as expected, decreases as the relation between three times the diameter of the holes divided by the width of the plate increases; however this behavior doesn’t remain constant through the range but reaches a value which presents an inflection point and starts to increase, as shown in figure 3; this tendency may be due the weakening that suffers the plate in the zones near the upper and lower edges, and according with diameter of the hole increases, this region decreases and consequently the flow lines of stress in this area will be closer together.

Figure 2. Comparison between the concentration factor data obtained through the simulation and the theoretical found in the bibliography.

Figure 3. Stress concentration factors for a rectangular plate with three transverse holes.
The behavior of the stress concentration factor in three longitudinal holes located on a flat plate is shown in figure 4, as expected this parameter decreases as the relation between the diameter of the hole divided by the width of the plate increases; Also one can observe that the values of this parameter are independent of the distance between centers of the holes, because the data obtained are nearly equal for the two simulated separation conditions.

![Figure 4](image)

**Figure 4.** Stress concentration factors for a rectangular plate with three longitudinal holes.

### 5. Conclusions

The finite element software can be used as a tool to determine graphical stress concentration factors for flat plates with a central hole, and subjected to axial load, as could be statistically demonstrated that aren’t significant differences between the theoretical data and data obtained through simulation. The maximum calculated error between these data was 5.42% when geometric relationship between the hole diameter and the width of the plate is 0.004.

The stress concentration factors in flat plates with three holes located transversely and under axial load decreases when the relation 3r/d increases, which indicates that the larger the diameter of the hole the concentration factor is lower; but the maximum stress on the cross section increases. However, to ensure this correlation should be prevented that the hole diameter and the hole spacing exceeds a limit value, if this occurs the point of maximum stress changes the inner to the outer zone of the hole because the separation between the hole and the edge of the plate decreases significantly.

The stress concentration factors in flat plates with three holes located longitudinally and subjected to axial load decreases to the extent that the relation d/b increases and additionally the magnitude of the factor is unaffected by the distance between centers of the holes.

### References

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