Unraveling the role of magnetic anisotropy on the thermoelectric response: a theoretical and experimental approach

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Abstract
Magnetic anisotropies have a key role in tailoring magnetic behavior in ferromagnetic (FM) systems for specific sensor applications. Furthermore, they are also essential elements for manipulating the thermoelectric response in anomalous Nernst effect (ANE) and longitudinal spin Seebeck effect (LSSE) systems at unsaturated field regimes. Here, we propose a theoretical approach and explore the role of magnetic anisotropies on the magnetization and thermoelectric response of noninteracting multidomain FM systems. The magnetic behavior and the thermoelectric curves are calculated from a modified Stoner–Wohlfarth model for an isotropic system, a uniaxial magnetic system, and for a system with a mixture of uniaxial and cubic magnetocrystalline magnetic anisotropies. The changes in the thermoelectric response caused by the magnetic anisotropy are remarkable. Furthermore, the fingerprints of the energy contributions to the thermoelectric response are disclosed. To test the robustness of our theoretical approach, we engineer films with specific magnetic properties and directly compare experimental data with the theoretical results. Thus, experimental evidence is provided to confirm the validity of our theoretical approach. The results go beyond the traditional reports focusing on magnetically saturated films and show how the thermoelectric effect behaves during the whole magnetization curve. Our findings reveal a promising way to explore the ANE and LSSE as powerful tools for studying magnetic anisotropies, as well as employing systems with magnetic anisotropy as sensing or elements in technological applications.

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(Some figures may appear in color only in the online journal)
1. Introduction

Thermoelectric effects driven by spin currents that are thermally activated have attracted increasing interest not just due to their relevance in the context of fundamental physics of phenomena associated with spin caloritronics [1–5], but also due to their potential relevance in a wide variety of technological applications [6–8]. Remarkably, the anomalous Nernst effect (ANE) [9–12] and the longitudinal spin Seebeck effect (LSSE) [7, 13–17] arise as the most studied effects in the field. The latter specifically allows us to study pure spin currents that do not present charge carriers. This is an exciting alternative for producing low-power devices. The ANE and LSSE are observed when a magnetic material is simultaneously submitted to a temperature gradient $\nabla T$ and a magnetic field $\vec{H}$ perpendicular to each other. However, while the ANE appears exclusively in ferromagnetic (FM) metals, the LSSE is often disclosed for bilayers consisting of an insulating ferromagnetic (FMI) or FM layer capped by non-magnetic (NM) metal with high spin–orbit coupling [7, 13–17].

It is worth highlighting that, in FM metallic bilayer heterostructures, the thermoelectric response of the whole system is a result of the combination of both the ANE and the LSSE. Hence, to circumvent this fact and suppress any contribution associated with the ANE or thermomagnetic effects, thus obtaining a pure LSSE signal, systems are commonly engineered, taking into consideration a FMI insulator. For instance, the well-known $\text{Y}_3\text{Fe}_5\text{O}_{12}$ (YIG) material is used to compose the bilayers heterostructure [18–21]. Indeed, the magnetic and electrical nature of YIG suggests the YIG/NM bilayers as natural candidates for investigations addressing the LSSE.

Despite the suitability of the YIG/NM bilayers for such a goal, the experimental procedures needed to induce the FMI phase in thin YIG films often involve several steps, including reactive magnetron sputtering and/or annealing, the latter typically performed in a controlled atmosphere [22]. In addition, the magnetic behavior of the YIG/NM systems is commonly found to be isotropic in the film-plane. These features make the achievement of YIG/NM bilayers combining specific structural and magnetic properties a complex task.

Given these issues raised by YIG/NM bilayers, a different path would be highly convenient. Thus, we bring to light that FM/NM or FIM/NM heterostructures [7, 18–24] arise as the most studied effects in the field.

Moreover, the influence of the cubic magnetocrystalline anisotropy of $\text{Co}_{86.7}\text{Fe}_{27.8}\text{Al}_{13.44}$ Heusler alloys, grown onto substrates with distinct orientations, on thermoelectric effects has been studied by the LSSE [25]. For both situations, the results corroborate that the thermoelectric curves are strongly dependent on the magnitude and orientation of the magnetic field with respect to the anisotropies [23–25]. In addition, the thermoelectric measurements may be yet used to probe the magnetic anisotropy, as well as to identify the field ranges in which the anisotropic and Zeeman energy terms represent the major contribution to the magnetic free energy density governing the magnetization process [25].

In this work, we propose a theoretical approach from a modified Stoner–Wohlfarth model [26] and explore the role of magnetic anisotropies on the magnetic and thermoelectric responses of noninteracting multidomain FM systems. The magnetic behavior has been calculated together with the thermoelectric response for an isotropic system, a uniaxial magnetic system, as well as for a system with a mixture of uniaxial and cubic magnetocrystalline magnetic anisotropies. Modifications of the magnetic behavior with the anisotropy have been verified and it is shown that the thermoelectric response is strongly affected by these changes. To test the robustness of the developed theoretical approach, films have been engineered with the specific magnetic properties and a direct comparison between the experimental data and the theoretical results has been carried out. Our findings allow us to infer how the thermoelectric measurements of a given magnetic system responds to a hysteretic magnetic field, an essential feature for sensor applications.

2. Theoretical approach

2.1. Thermoelectric effects

The ANE corresponds to the generation of a voltage by applying a temperature gradient to a FM metal or a semiconductor [9, 10]. The electrical field associated with the ANE can be expressed as

$$\vec{E}_{\text{ANE}} = -\lambda_{\text{N}} \mu_0 (\vec{m} \times \nabla T),$$

in which $\lambda_{\text{N}}$ is the anomalous Nernst coefficient, $\mu_0$ is the vacuum magnetic permeability, $\vec{m}$ is the magnetization vector, and $\nabla T$ is the temperature gradient.

The spin Seebeck effect (SSE), in turn, consists of the generation of a voltage by applying a temperature gradient to FM/NM or FIM/NM heterostructures [7, 27]. The electrical field due to the SSE generated in the NM metal is described by

$$\vec{E}_{\text{SSE}} = -\lambda_{\text{SSE}} (\vec{\sigma} \times \nabla T).$$

Here, $\lambda_{\text{SSE}}$ is the spin Seebeck coefficient, $\vec{\sigma}$ is the spin polarization, the orientation of which is given by the magnetization direction, and $\nabla T$ represents the temperature gradient.

From equations (1) and (2), the remarkable similarity of the effects also coming from the experimental configuration can
be observed, where $\nabla T$ and $\vec{H}$ are perpendicular to each other, as represented in figure 1. In the present case, it is assumed that $\nabla T$ is perpendicular to the plane of the film, while $\vec{H}$ remains in the plane of the film, as depicted in figure 1. For the calculations, the thermoelectric voltage $V$ detected at the electrical contacts at the ends of the main axis of the film is given by:

$$V = -\frac{L}{0} \vec{E} \cdot d\vec{r},$$

(3)

the limits of integration being related to the distance between the electrical contacts used in the experiment. Here we assume $y$ as the direction for the detection of $V$.

The SSE voltage is measured by means of the inverse spin Hall effect [28]. The relation between the spin current $J_s$, generated from the temperature gradient, and the charge current $J_c$, produced in the NM metal layer, is given by:

$$\vec{J}_c = \theta_{SH} \left( \frac{2e}{h} \right) \left( \vec{J}_s \times \hat{\sigma} \right),$$

(4)

in which $\hat{\sigma}$ is the spin-polarization direction and $\theta_{SH}$ is the spin Hall angle that measures the efficiency of the spin-charge conversion.

### 2.2. Magnetic free energy density

From the magnetic free energy density $\xi$ describing the system with a given anisotropy configuration [29], a routine for the energy minimization provides the values of the equilibrium angles of the magnetization at each magnetic field $\vec{H}$. Then, once the magnetization response is obtained for a specific field orientation, the thermoelectric voltage $V$ can be obtained. This numerical procedure enables us to achieve the dependence of the thermoelectric voltage for systems with distinct anisotropy configurations.

To proceed with the magnetic and thermoelectrical calculations, the film defining the magnetic system is first described. Here, a theoretical system is considered that is divided into 50 noninteracting magnetic domains, which allows representing a sample with magnetic anisotropy dispersion. The magnetization of the whole system is then obtained averaging the

magnetization of the domains. The magnetic free energy density for each domain is based on a modified Stoner–Wohlfarth model [30, 31], in which the magnetic anisotropy is considered. Here, we focus on systems with uniaxial and cubic magnetocrystalline anisotropies, as well as on an isotropic one. For the sake of simplicity, the magnetization of each domain is considered in the plane of the film and, consequently, the free energy density can be written by

$$\xi_i = -m_i H \cos(\varphi_{H} - \varphi_{M}) - k_i \sin^2(\varphi_{M}) - \frac{1}{4} \xi_{c11},$$

(5)

The first term is the Zeeman interaction, the second describes the uniaxial magnetic anisotropy, and the last one is related to the cubic magnetocrystalline anisotropy. For the numerical calculations, $\theta_{M} = 90^\circ$ and $\varphi_{M}$ are defined as the equilibrium angles of the magnetization for each domain at a given magnetic field $\vec{H}$. At the same time, $\theta_{M}$ is the angle of the magnetization vector with respect to the $z$ axis, while $\varphi_{M}$ is the angle between the projection of the $\vec{m}_i$ in the $xy$ plane with respect to the $x$ axis, as indicated in figure 2. Similarly, $\theta_{H}$ and $\varphi_{H}$ are defined as the angles describing the orientation of the magnetic field $\vec{H}$. In addition, $m_i$ is the unit vector of the magnetization, $m_i$ is the saturation magnetization, and $k_i = m_i^2 / 2$, where $h_{ij}$ is the anisotropy field related to the uniaxial magnetic anisotropy in the $h_{ij}$ direction for each domain, defined by $\theta_{H}$ and $\varphi_{H}$. For all quantities, the subscript $i$ indicates that they are associated to the $i$th magnetic domain composing the system. Finally, $\xi_{c11}$ describes the in-plane component of cubic magnetocrystalline anisotropy, which can be written as

$$\xi_{c11} = k_{c11} \left( \alpha_{11}^2 \alpha_{21}^2 + \alpha_{12}^2 \alpha_{32}^2 + \alpha_{22}^2 \alpha_{33}^2 \right),$$

(6)

with

$$\alpha_{11} = \cos \varphi_{M} \sin \theta_{M},$$

$$\alpha_{21} = \sin \varphi_{M} \sin \theta_{M},$$

(7)
Looking at equation (5), a system with uniaxial magnetic anisotropy is represented taking $k_{ui} \neq 0$ and $k_{ci1} = 0$. For our concern here, we may mention that taking $k_{ui} \neq 0$ and $k_{ci1} \neq 0$, a system is described having both uniaxial and cubic magneto-crystalline anisotropies, while by considering $k_{ui} \neq 0$, $\varphi_m = \varphi_H$ and $k_{ci1} = 0$, a system is described in which the magnetization is aligned to the magnetic field, i.e. having isotropy in the film-plane.

2.2.1. Magnetic system with isotropy in the film-plane. Figure 3 presents the magnetic and thermoelectric responses for a magnetic system with magnetic isotropy in the plane of the film. As previously mentioned, it is assumed $k_{ui} \neq 0$, $\varphi_H = \varphi_H$, and $k_{ci1} = 0$ to describe the magnetic free energy density of this case study. For our numerical calculations, a system is considered consisting of a single magnetic domain with the following general parameters: $m_u = 143$ emu cm$^{-3}$ and $h_u = 30$ Oe, with $\theta_u = 90^\circ$, $\varphi_u = \varphi_H$, and $\theta_i = 90^\circ$. Notice that the use of the multidomain structure is not required for this case. For the thermoelectric calculation, it is assumed $\Delta T = 30$ K and an effective thermoelectric coefficient of $\lambda_{eff} = 1.3 \times 10^{-5}$ V K$^{-1}$[23]. Here, $\lambda_{eff}$ brings the contributions of $\lambda_N$ and $\lambda_{SSE}$ to the system. For a system in which the LSSE is also present, such as YIG/NM, it is assumed $\lambda_{eff} = \lambda_{SSE}$.

For the sake of simplicity, the results are addressed in terms of the normalized magnetization and thermoelectric voltage values. To this end, the quantities $m/m_i$ and $V/V_{max}$ are defined, where $m_i$ and $V_{max}$ are the maximum values obtained for the magnetization and the thermoelectric voltage, respectively, when $\varphi_H = 0^\circ$.

Figure 3(a) shows the magnetization behavior as a function of the field for distinct $\varphi_H$ values. As expected, all the curves have squared shapes, with a coercive field of 30 Oe, being similar irrespective of the field orientation given by $\varphi_H$ due to the constant alignment between the $m_u$ and $H$ for isotropic systems.

Figure 3(b) in turn shows the thermoelectric response for such an isotropic magnetic system. The shape of the curves mirrors the magnetization loops, even with varying $\varphi_H$. However, it is observed that there is a significant reduction in the amplitude of the curve as $\varphi_H$ increases. This feature is directly related to the decrease of the component of the induced electric field along the V detection direction defined by the electrical contacts.

Figure 3(c) presents the angular dependence of the $V/V_{max}$ for the constant magnetic field value. Here we show the curves for two magnetic field values, 300 Oe and 2000 Oe. For both situations, the sample is magnetically saturated, and the signal response has a cosine-shaped form, being proportional to $\cos \varphi_H$.

Finally, it is worth emphasizing that the results for the isotropic magnetic system are in agreement with those found in previous reports for FMI materials, such as the YIG/Pt bilayers, samples that are widely explored in investigations addressing the LSSE [7, 32].

2.2.2. Magnetic system with uniaxial magnetic anisotropy. For a magnetic system with uniaxial magnetic anisotropy, it is assumed $k_{ui} \neq 0$ and $k_{ci1} = 0$. Specifically, a system is considered with the following parameters: $m_u = 1330$ emu cm$^{-3}$, $h_u = 100$ Oe, $\theta_u = 90^\circ$, $\theta_i = 90^\circ$, and $\lambda_{eff} = 1.3 \times 10^{-5}$ V K$^{-1}$. In addition, the multidomain structure is now taken into account in order to mimic the magnetic anisotropy dispersion. The uniaxial anisotropy direction is first set at $\varphi_u = 90^\circ$, with a linear dispersion of $5^\circ$. This procedure enables us to represent an FM system in which the uniaxial magnetic anisotropy is affected/dispersed by, for instance, the stress stored in a film during deposition.

Figure 4 shows the evolution of the magnetic and thermoelectric responses with $\varphi_H$ for a magnetic system with uniaxial magnetic anisotropy, in which the easy magnetization axis lies in the plane of the film. For the thermoelectric calculations, it is also assumed $\Delta T = 30$ K.

Figure 4(a) shows the magnetization curves for distinct values of $\varphi_H$. From a general point of view, well-known behavior is verified, with a near-squared shape for the curves calculated at $\varphi_H = 80^\circ$, revealing an easy magnetization axis located nearby this orientation. For $\varphi_H = 0^\circ$, the form of
the magnetization curve directly allows us to infer a hard magnetization axis, corroborating the behavior for a uniaxial magnetic system when the field is perpendicular to the easy axis. Remarkably, a small coercive field is observed for $\varphi_H = 0^\circ$, a feature arising from the magnetic domains with anisotropy dispersion. In this sense, the presented theoretical approach considering multidomains appears as an appropriate tool to describe the details of the magnetic response of the systems.

Considering the thermoelectric behavior, presented in figure 4(b), the shape of the curves is not strongly modified as the $\varphi_H$ increases. For $\varphi_H = 0^\circ$, the thermoelectric curve mirrors the one observed for magnetization. This feature is expected, since the induced electrical field is along the $V$ detection direction given by the electrical contacts for this configuration. However, although the shape of the curves remains unchanged even with increasing $\varphi_H$ (from $0^\circ$ to $90^\circ$), a decrease in the $V/V_{\text{max}}$ value at the maximum field is observed.

Figure 4(c) depicts the dependence of $V/V_{\text{max}}$ with $\varphi_H$ for a constant magnetic field. Here, an interesting feature is observed with respect to the shape of the curves.

For an external magnetic field of 2000 Oe, it is observed that the response of $V/V_{\text{max}}$ as a function of $\varphi_H$ is perfectly described by a cosine, as expected according to theory for a magnetically saturated sample. For an external field of 300 Oe in turn, a change in the profile of the curve is verified, and the sample is magnetically saturated at this smaller field value, an intriguing fact at a first glance.

At 2,000 Oe, the field value is much above the saturation and anisotropy fields. As a consequence, the Zeeman energy is, at least, one order of magnitude higher than the uniaxial anisotropy energy, representing a major contribution to the magnetization dynamics. Hence, the Zeeman energy defines the magnetization direction, which follows the orientation of the field, leading to a dependence of $V/V_{\text{max}}$ with $\cos \varphi_H$.

The most striking finding here resides in the fact that the deviation from the cosine shape is found when the field is 300 Oe. This distortion can be interpreted in terms of the competition of two energy contributions. Specifically, at a field value right above the saturation, the Zeeman energy and the uniaxial magnetic anisotropy energy are of the same order of magnitude, in a sense that the response of the whole system is a result of the balance of both of terms. Given this, small deviations of the magnetization are identified through the thermoelectric experiment.

Remarkably, although the magnetometric techniques are not efficient enough to probe small deviation of the magnetization at saturated states, in which the field is right above the saturation field, the ANE or LSSE experiments provide an increased sensitivity for such a task.

To explore the modification on the thermoelectric curves when the uniaxial anisotropy direction is modified, now $\varphi_{\text{LH}} = 0^\circ$ is considered, with a linear dispersion of 5°, as used in the previous case. Figure 5(a) shows the magnetization curves, allowing us to observe the quasi-static magnetic behavior for a uniaxial system with in-plane magnetic anisotropy. Instead of the previous situation ($\varphi_{\text{LH}}$), the magnetization curves start from a squared shape for $\varphi_{\text{LH}} = 0^\circ$. Besides, the modification of $\varphi_H$ shows similar curves to those observed in the previous case, in which $\varphi_{\text{LH}} = 90^\circ$. On the other hand, remarkable modifications are observed in the thermoelectric curves, as shown in figure 5(b). While the shape of the curves is unchanged in the previous situation with varying $\varphi_{\text{LH}}$, here the thermoelectric curves present a strong dependence on $\varphi_H$ values. For $\varphi_H = 0^\circ$, the thermoelectric curves mirror the magnetization curves, with similar coercive field values, as expected, since the electric field is aligned with the electrical contacts. However, as $\varphi_H$ increases, a decrease in the thermoelectric intensity at high fields is observed, keeping the maximum values reached by the thermoelectric voltage unchanged when $\bar{H}$ is close to $h_{\text{LH}}$. The shape of the curves is a result of the competition between the Zeeman and the uniaxial anisotropy, which is expected given that the sample is not saturated. For the curves at higher $\varphi_H$ values and high $\bar{H}$ intensities, the magnetization is aligned with the magnetic field direction. However, as the magnetic field intensity decreases, the uniaxial magnetic anisotropy leads to a rotation of the magnetization and, consequently, of the electric field in the LSSE or ANE experiments. This feature leads to an increase in the thermoelectric voltage for fields near to $h_{\text{LH}}$ values.
Once again, for the $V/V_{\text{max}}$ as a function of the $\varphi_H$ curves (figure 5(c)), a slight distortion is observed on the cosine curve for an external field of 300 Oe, a feature associated with the competition of the Zeeman and uniaxial anisotropy. However, the cosine shape is observed for the curves in which the field is 2000 Oe, in agreement with the previous discussions.

Considering the remarkable modification observed in the thermoelectric curves when the $\varphi_H$ values are changed (figures 4 and 5), the magnetic and thermoelectric curves have been calculated for a fixed $\varphi_H$, and $\varphi_H$ varying from $0^\circ$ up to $180^\circ$; these curves are depicted and discussed in figure S1 (available online at stacks.iop.org/JPD/55/025001/mmedia) of the supplementary material.

### 2.2.3. Magnetic system with uniaxial and cubic magnetocrystalline anisotropies

A more complex magnetic anisotropy configuration was further explored. In the following, a magnetic system with both uniaxial and cubic magnetocrystalline anisotropies is addressed. Such a system discloses very interesting magnetization curves and thermoelectric responses, with potential for technological applications, such as in magnetic logic keys.

Specifically, a system is considered with the parameters previously employed for the uniaxial one, $m_u = 1330$ emu cm$^{-3}$, $h_u = 100$ Oe, $H_u = 90^\circ$, $\theta_H = 90^\circ$, and $\lambda_{\text{eff}} = 1.3 \times 10^{-5}$ V K$^{-1}$, with the additional quantity of $k_{\text{cu}} = 2.8k_u$, associated with the cubic magnetocrystalline anisotropy. The multidomain structure is also kept to represent the magnetic anisotropy dispersion, by setting the uniaxial anisotropy direction at $\varphi_u = 90^\circ$, with a linear dispersion of $5^\circ$.

Figure 6 presents the numerical calculations for the magnetization curves and thermoelectric responses for $\varphi_u = 0^\circ$, $\varphi_u = 45^\circ$, and $\varphi_u = 90^\circ$.

Considering $\varphi_u = 0^\circ$, figures 6(a.I) and (a.II) depict the magnetization and thermoelectric behavior as a function of the external magnetic field for distinct $\varphi_H$ values. The addition of the cubic magnetocrystalline anisotropy yields, besides the easy and hard magnetization axes, the intermediate one at $\varphi = 45^\circ$. The emergence of such a configuration of magnetic anisotropy and the competition of anisotropy axes lead to magnetization curves with interesting behavior, in particular for high $\varphi_H$ values. Given that the uniaxial anisotropy axis is along the $V$ detection direction, the richness of details found in the magnetization curves is not observed in the thermoelectric ones.

However, when $\varphi_H \neq 0^\circ$, the refined structure of the magnetization curves, full of details associated with the magnetization process, is also identified in the thermoelectric curves. For $\varphi_H = 45^\circ$, changes in both magnetization and thermoelectric curves are observed as the $\varphi_H$ is modified (see figures 6(b.I) and (b.II)). For $\varphi_H$ values smaller than 45°, the thermoelectric curves present a similar shape to those observed on the magnetization curves. On the other hand, for $\varphi_H$ values higher than $\varphi_H = 45^\circ$, the shapes of the thermoelectric and magnetic curves disclose fundamental differences, with a decrease in $V/V_{\text{max}}$. Taking into account $\varphi_u = 90^\circ$ (figures 6(c.I) and (c.II)), the mirror of the magnetization curves and thermoelectric one is verified for the $\varphi_H = 0^\circ$ and $\varphi_H = 20^\circ$ field directions. On the other hand, a remarkable modification on the thermoelectric curves is verified for $\varphi_H$ above 45. This behavior reflects the modifications observed in the previous situations. From a general point of view, important modifications are observed in the shape of the thermoelectric curves when $\varphi_H$ is higher than 45°, irrespective of the $\varphi_u$ direction. This behavior corroborates previous results observed in magnetostrictive systems, in which the effective magnetic anisotropy has been modified by stress applications [23].

It is worth remarking that the addition of the cubic magnetocrystalline to a uniaxial anisotropy axis (keeping constant the anisotropy parameters considered in the previous sections and assuming $k_{\text{cu}} = 2.8k_u$) describes a system with harder magnetic properties, if compared with those found in figures 3(a), 4(a), and 5(a). This fact becomes evident when the coercive fields are observed in figures 6(a.I), (b.I), and (c.I), as well as through the fact that most of the curves in figure 6 do not achieve magnetic saturation at 300 Oe.

Such behavior is reflected in the dependence of $V/V_{\text{max}}$ with $\varphi_H$, as presented in figure 6(d.I) for a system with $\varphi_u =$
To highlight this feature, figure S2 in the supplementary material presents calculations of the magnetization and thermoelectric response for a field of ±2000 Oe. To highlight the influence of the magnetic state on the thermoelectric response, figure 6(d.II) brings the evolution of the \(V/V_{\text{max}}\) at 300 Oe, with \(\phi_H\) for distinct \(\phi_L\) values. From this, the competition of the anisotropies and Zeeman energies in this complex magnetic system can be disclosed.

3. Comparison with experimental results

Three magnetic systems with distinct anisotropy configurations were produced to test the robustness of the theoretical approach.

The first sample is a YIG (300 nm)/Pt (6 nm) bilayer grown onto a [111]-oriented GdGaO\(_3\) substrate, a sample with isotropic magnetic properties. The sample geometry and the employed materials make this sample suitable for the study of pure LSSE. The second sample is a Co\(_{0.7}\)Fe\(_{0.3}\)B\(_{20}\) (300 nm) film grown onto a glass substrate, corresponding to a uniaxial magnetic system. In this case, given the sample structure, the thermoelectric voltage is associated with the pure ANE response. The third sample consists of a Co\(_{58.3}\)Fe\(_{27.8}\)Al\(_{12.9}\) (53 nm)/W (2 nm) bilayer deposited onto a [100]-oriented GaAs substrate. Such a system has cubic magnetocrystalline and uniaxial magnetic anisotropies. Here, the thermoelectric response is a result of the mixing of the LSSE and ANE. Although the LSSE and ANE have different physical origins, their correspondence with respect to hysteretic behavior is similar. The experimental procedures used for sample preparation and characterization are presented in the supplementary material.

The investigated samples allow verification not just of the dependence of the magnetic and thermoelectric features with the magnetic anisotropy, but they also provide tools to show that the ANE and LSSE experiments present very similar behavior and can be described by the developed theoretical approach.

Figure 7 shows the experimental and theoretical results for the YIG (300 nm)/Pt (6 nm) bilayer. The measurements are obtained with \(\Delta T = 27\) K. For the calculations, it is considered \(m_s = 143\) emu cm\(^{-3}\), \(\lambda_{\text{eff}} = 5.2 \times 10^{-6}\) V K\(^{-1}\), and \(h_{\text{ff}} = 4.8\) Oe. Further, it is assumed \(\phi_L = 30^\circ + \phi_H\). This value allows us to reproduce the experimental results, for which a deviation is observed between \(\phi_L\) and \(\phi_H\). Anyway, the isotropic behavior of the YIG (300 nm)/Pt (6 nm) bilayer as a function of the \(\phi_H\) is evident and corroborates the decrease of \(V/V_{\text{max}}\) with increasing \(\phi_H\) and the absence of modifications in the shape of the curve with varying field orientation. Considering the curves obtained at \(\phi_H = 90^\circ\) (not shown), no thermoelectrical signal is observed. This feature is associated with the perpendicular alignment of the electric field and the electric contact direction (see figure 1). It is important to point out that the surface anisotropy contribution can be present in this case. This contribution is due to the insertion of \(\phi_L = 30^\circ + \phi_H\). It is well known that the YIG has small magnetocrystalline anisotropy. This surface magnetic anisotropy can affect the curve...
shape of the SSE as a function of the magnetic field due to the perpendicular magnetic anisotropy that arises close to the YIG surface. The effect of the surface magnetic anisotropy on the SSE can remain even when the roughness is very slight, meaning that the anisotropy is an intrinsic feature of the YIG surface [33, 34].

Considering the uniaxial magnetic system, the experimental results for a Co$_{30}$Fe$_{70}$B$_{20}$ film deposited onto a glass substrate are presented. During the deposition, a magnetic field of 1.0 kOe has been applied to induce the uniaxial magnetic behavior. Figure 8 presents thermoelectric curves for the selected $\phi_H$ values of 0°, 30°, and 60°. For the numerical calculations, $m_s = 1026$ em cm$^{-3}$, $k_{ui} = 7.072 \times 10^4$ ergs cm$^{-3}$, $k_{ci} = 0$, $\Delta T = 27$ K, $\lambda_{eff} = 1.3 \times 10^{-5}$ V K$^{-1}$, and $\phi_{th} = 85^\circ$ have been used.

From figure 8, a very interesting evolution in the shape of the thermoelectric response with $\phi_H$ is observed. In particular, this feature is a consequence of the change in the orientation of the field, which is rotating from the easy magnetization axis direction to the hard magnetization one. For $\phi_H = 0^\circ$, i.e. along the easy axis, a mirror is verified between the thermoelectric and magnetization curves (the latter not shown). This behavior is due to the fact that the detection direction of the thermoelectric voltage, defined by the electrical contacts, is perpendicular to the easy magnetization axis. By rotating the magnetic field to $\phi_H = 30^\circ$ and $\phi_H = 60^\circ$, at a high magnetic field regime, the component of the induced electric field along the direction of the electrical contacts decreases, leading to a reduction of the thermoelectrical voltage. On the other hand, at a low magnetic field regime, the uniaxial magnetic anisotropy leads the magnetization to the easy axis, rotating the electric field and, consequently, increasing the thermoelectrical voltage, as confirmed from the results. This mechanism is observed in both the experimental results and the numerical calculations. Here, it is worth pointing out the alignment between the electric field and contact direction is essential to describe the experimental curves. In addition, although a multidomain system is simulated, the complex domain structure of a real system leads to an increase in the coercive field for a real sample. This feature is responsible for the small discrepancy between the coercivity values of the experimental and theoretical results observed as $\phi_H$ increases.
Further, the fingerprints of the energy contributions to the thermoelectric response is strongly affected by these changes. Remarkable modifications of the magnetic behavior with the uniaxial and cubic magnetocrystalline magnetic anisotropies. The approach has been tested for an isotropic system, a uniaxial approach based on a modified Stoner–Wohlfarth model. Main FM systems has been addressed, based on a theoretical and thermoelectric responses of noninteracting multilayer systems. In summary, the role of magnetic anisotropies on the magnetic and thermoelectric response, an issue associated with the anisotropy configuration, reviewed, and edited the manuscript. All authors have read and agreed to the published version of the manuscript. M A C is responsible for the conceptualization, methodology, writing, and editing. All authors have written, reviewed, and edited the manuscript.

Figure 9 shows the experimental and theoretical results obtained for the thermoelectric curves of Co$_{85.73}$Fe$_{27.83}$Al$_{13.44}$/W bilayers grown onto [100]-oriented GaAs substrates and submitted to a 1.0 kOe during the deposition (details on the experimental procedure can be found in the supplementary material). The employed substrates and the annealing process allows us to obtain a magnetic system with uniaxial and cubic magnetocrystalline anisotropies. The numerical calculations, $m_s = 1330$ emu cm$^{-3}$, $k_{uw} = 1.66 \times 10^5$ ergs cm$^{-3}$, $k_{ci} = 2.8k_{uw}$, $\Delta T = 27$ K, and $\varphi_{lt} = 90^\circ$, $\lambda_{eff} = 3.79 \times 10^{-8}$ V K$^{-1}$ has been applied. From a general point of view, the evolution of the curve's shape as $\varphi_{lt}$ increases has been observed. The curves present a remarkable modification on the shape when $\varphi_{lt}$ is higher than 45°. This behavior is similar to that observed in the theoretical results presented in Figure 6. As discussed before, at $\varphi_{lt} = 45^\circ$, the studied system shows an intermediate magnetization axis. For this reason, the $\varphi_{lt}$ value is an important angle. For $\varphi_{lt}$ values smaller than this critical value, the thermoelectric curves seem to present similar behavior to that presented by the magnetization. On the other hand, for $\varphi_{lt}$ values larger than 45°, the shape changes drastically. Hence, the cross of $\varphi_{lt}$ through this anisotropy axis leads to modifications in the shape of the thermoelectric response, an issue associated with the anisotropy configuration, discussed in detail in [23].

4. Conclusion

In summary, the role of magnetic anisotropies on the magnetic and thermoelectric responses of noninteracting multidomain FM systems has been addressed, based on a theoretical approach based on a modified Stoner–Wohlfarth model. The approach has been tested for an isotropic system, a uniaxial magnetic system, and for a system with a mixture of uniaxial and cubic magnetocrystalline magnetic anisotropies. Remarkable modifications of the magnetic behavior with the anisotropy have been verified and it has been shown that the thermoelectric response is strongly affected by these changes. Further, the fingerprints of the energy contributions to the thermoelectric response have been disclosed, revealing that the anisotropy energy terms represent a fundamental contribution to the energy balance even in saturated states when the field is right above the saturation field. To test the robustness of the developed theoretical approach, three films have been engineered with specific magnetic properties and have been compared to the corresponding magnetic and thermoelectric experimental data. From the comparison and the remarkable agreement between experiment and theory, experimental evidence has been provided to confirm the validity of the theoretical approach. The results go beyond the traditional reports focusing on magnetically saturated films and show how the thermoelectric effect behaves during the whole magnetization curve. The results emphasize the importance of selecting specific field alignments and field amplitudes to provide reliable interpretation of the ANE and LSSE experiments. Our findings reveal a suitable and robust way to explore the ANE and the LSSE as powerful tools for studying magnetic anisotropies, as well as for employing systems with magnetic anisotropy as sensing elements for technological applications.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Author contributions

All authors have read and agreed to the published version of the manuscript. M A C is responsible for the conceptualization, methodology, writing, and editing. All authors have written, reviewed, and edited the manuscript.

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Conflict of interest

The authors declare no conflict of interest.

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