I. INTRODUCTION

The theoretical tools for the description of exclusive processes are the hard-scattering amplitude which describes the process-dependent quark-gluon interaction within perturbative QCD and the probability amplitude for finding the lowest twist quark state in each hadron. The total amplitude is represented by the convolution of these two parts, assuming factorization of highly off-shell or large transverse momentum regions of phase space from regions of low momenta necessary to form bound states. Higher-twist components, corresponding to a higher number of partons (quark-pairs and gluons) are supposed to be suppressed by powers of the momentum transfer $Q^2$. Recent progress in Sudakov-suppression techniques provides support for the conjectured infrared protection of the perturbative picture. The focus in this talk will be on recent theoretical developments on exclusive reactions involving nucleons.

II. PERTURBATIVE ASPECTS

The $Q^2$-dependence of hadronic wave functions is determined by the renormalization group equation. To leading order, the dynamical evolution of the lowest twist hadronic distribution amplitude, which is the hadronic wave function integrated over transverse momenta up to a resolution scale $Q^2$, is described by the one-gluon exchange kernel subsumming leading logarithms of ladder graphs. Specifically, the nucleon distribution amplitude $\Phi_N$ is obtained as the solution to the integrodifferential equation

$$\left\{ Q^2 \frac{\partial}{\partial Q^2} + \frac{3C_F}{2\beta} \right\} \Phi(x_i, Q^2) = \frac{C_B}{\beta} \int_0^1 [dy] \, V(x_i, y_i) \, \Phi(y_i, Q^2) $$

\[ f_0^1[dx] \equiv \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \delta(1 - x_1 - x_2 - x_3), \] where $C_F$ and $C_B$ are the Casimir operators of the fundamental and adjoint representations of $SU(3)$, respectively, and $\beta$ is

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the Gell-Mann and Low function. The leading-order expression for the integral kernel $V$ has been calculated in [1].

To solve the evolution equation (1), $\Phi_{N}(x_{i}, Q^{2})$ has to be expanded in terms of the eigenfunctions of the integral kernel $V$. These eigenfunctions correspond to three-quark operators which are multiplicatively renormalizable, i.e., to operators with definite anomalous dimensions [4]. The latter can be determined by diagonalizing the evolution equation within some appropriate basis. To this end, one expresses the solution of (1) in the form

$$\Phi_{N}(x_{i}, Q^{2}) = \Phi_{\text{as}}(x_{i}) \sum_{n=0}^{\infty} B_{n} \tilde{\Phi}_{n}(x_{i}) \left( \frac{\alpha_{s}(Q^{2})}{\alpha_{s}(\mu^{2})} \right)^{\gamma_{n}},$$

(2)

where $\{\Phi_{n}\}_{n=0}^{\infty}$ are orthonormalized eigenfunctions of the interaction kernel with in a truncated basis of Appell polynomials of maximum degree $M$. $\Phi_{\text{as}}(x_{i}) = 120 x_{1} x_{2} x_{3}$ is the asymptotic form of the nucleon distribution amplitude [1]. Because the $\gamma_{n}$ are positive fractional numbers increasing with $n$, higher terms in (2) are gradually suppressed. The Appell polynomials are polynomials of two independent variables, say $x_{1}$ and $x_{3}$. Thus one can expand $\{\tilde{\Phi}_{n}\}$ in terms of the polynomial basis $\{x_{1}^{i} x_{3}^{j}\}_{i,j=0}^{\infty}$:

$$\tilde{\Phi}_{n} = \sum_{i,j=0}^{\infty} a_{n}^{ij} x_{1}^{i} x_{3}^{j}.$$

(3)

Then defining moments

$$\Phi_{N}^{[\text{as}]}(\mu^{2}) = \int_{0}^{1} [dx_{1} x_{2}^{0} x_{3}^{0}] \Phi_{N}(x_{i}, \mu^{2}),$$

(4)

the expansion coefficients $B_{n}$ can be formally determined by inverting (2) to obtain

$$\frac{B_{n}(Q^{2})}{\sqrt{N_{n}}} = \sqrt{N_{n}} \left( \frac{\alpha_{s}(Q^{2})}{\alpha_{s}(\mu^{2})} \right)^{\gamma_{n}} \sum_{i,j=0}^{\infty} a_{n}^{ij} \Phi_{N}^{[\text{as}]}(\mu^{2}).$$

(5)

An explicit calculation of the expansion coefficients $B_{n}(\mu^{2})$ involves the orthonormalization of polynomials with two variables—a problem with no unique solution because the procedure depends on the order in which it is performed. To determine the anomalous dimensions $\gamma_{n} = \left( \frac{3}{2} \frac{C_{F}}{N_{c}} + 2 \eta_{n} \frac{C_{F}}{N_{c}} \right)$, one has to compute first the zeros $\eta_{n}$ in the characteristic polynomial that diagonalizes the evolution kernel. Such a program has been carried out in [5,6] and a complete eigenfunction basis has been constructed up to polynomial order 9. All previous calculations had been restricted to maximum order 3 [1,7]. It is noteworthy that up to order 7, we have performed the diagonalization of the evolution kernel analytically. Below order 3, our anomalous dimensions coincide with those computed by Peskin [4]; those of order 3 confirm the recently published (numerical) estimates of [7]. Our results are listed in Table I; for more details we refer to [3]. The large number of computed eigenvalues—a total of 54 corresponding to 29 symmetric and 25 antisymmetric eigenfunctions—enables the evaluation of a well-defined pattern (Fig. 1). The trend line of this pattern follows the empirical power law (solid line) $\gamma_{n} = 0.37 O(n)^{0.565}$. There is, certainly, no evidence that the ensuing global
behavior of baryon anomalous dimensions coincides with that of mesons (dotted lines), as prematurely suggested in [6]. The proposed method can be used to consistently generalize the ansatz for $\Phi_N$ to higher orders by retaining the correct evolution behavior [6].

The calculation of moments of the nucleon distribution amplitude resides on nonperturbative techniques and will be considered in the next chapter.
TABLE I. Orthogonal eigenfunctions $\tilde{\Phi}_n(x_1, x_2, x_3) = \sum_{l_k} a^n_{kl} x_1^l x_2^l x_3^l$ of the nucleon evolution equation (represented by the coefficient matrix $a^n_{kl}$ with $a^n_{kl} = S_n a^n_{lk}$; $a^n_{22} = 0$ for all $n$). The normalization is given by $\int_0^1 [dx] x_1 x_2 x_3 \tilde{\Phi}_k(x_1) \tilde{\Phi}_n(x_1) = (N_n)^{-1} \delta_{kn}$. 

| $n$ | $M$ | $S_n$ | $a^n_{00}$ | $a^n_{01}$ | $a^n_{02}$ | $a^n_{10}$ | $a^n_{11}$ | $a^n_{20}$ | $a^n_{21}$ | $N_n$ |
|-----|-----|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-------|
| 0   | 0   | 1     | $\frac{1}{2}$ | -1        | $\frac{1}{4}$ | 0         | 0         | 0         | 0         | 120   |
| 1   | 1   | -1    | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | 1         | 0         | 0         | 0         | 1260  |
| 2   | 1   | 1     | $\frac{3}{4}$ | -1        | $\frac{3}{4}$ | -1        | 1         | 0         | 0         | 420   |
| 3   | 2   | -1    | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | 1         | -1        | 1         | 0         | 756   |
| 4   | 2   | 1     | $\frac{3}{4}$ | -1        | $\frac{3}{4}$ | -1        | 1         | -1        | 1         | 34020 |
| 5   | 2   | -1    | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | 1         | -1        | 1         | -1        | 1944  |
| 6   | 3   | 1     | $\frac{115 + \sqrt{7}}{162}$ | -1        | $\frac{115 + \sqrt{7}}{162}$ | 1         | $\frac{79 + \sqrt{7}}{4801}$ | -1        | $\frac{79 + \sqrt{7}}{4801}$ | 4620 ($485 \pm 11 \sqrt{77}$) |
| 7   | 3   | -1    | $\frac{159 + \sqrt{7}}{162}$ | $\frac{79 + \sqrt{7}}{4801}$ | -1        | $\frac{159 + \sqrt{7}}{162}$ | -1        | $\frac{159 + \sqrt{7}}{162}$ | 27720 ($3360 \pm 247 \sqrt{77}$) |
| 8   | 3   | -1    | $\frac{159 + \sqrt{7}}{162}$ | $\frac{79 + \sqrt{7}}{4801}$ | 1         | -1        | $\frac{159 + \sqrt{7}}{162}$ | -1        | $\frac{159 + \sqrt{7}}{162}$ | 27720 ($3360 \pm 247 \sqrt{77}$) |
| 9   | 3   | -1    | $\frac{159 + \sqrt{7}}{162}$ | $\frac{79 + \sqrt{7}}{4801}$ | 1         | -1        | $\frac{159 + \sqrt{7}}{162}$ | -1        | $\frac{159 + \sqrt{7}}{162}$ | 4620  |
| 10  | 4   | -1    | $\frac{346 - \sqrt{97}}{3801}$ | -1        | $\frac{346 - \sqrt{97}}{3801}$ | 0         | 0         | 0         | 0         | 0     |
| 11  | 4   | -1    | $\frac{346 - \sqrt{97}}{3801}$ | -1        | $\frac{346 - \sqrt{97}}{3801}$ | 0         | 0         | 0         | 0         | 0     |

| $n$ | $a^n_{00}$ | $a^n_{01}$ | $a^n_{02}$ | $a^n_{10}$ | $a^n_{11}$ | $a^n_{20}$ | $a^n_{21}$ | $a^n_{30}$ | $a^n_{31}$ | $a^n_{40}$ | $a^n_{41}$ | $N_n$ |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-------|
| 0   | 1         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 1   | 0         | 1         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 2   | -2        | 3         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 3   | 2         | -7        | 8         | 4         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 4   | 0         | 1         | -4        | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 5   | 2         | -7        | 14        | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 6   | 1         | -6        | $\frac{41 + \sqrt{77}}{4}$ | $\frac{31 - \sqrt{77}}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | 0     |
| 7   | 1         | -6        | $\frac{41 + \sqrt{77}}{4}$ | $\frac{31 + \sqrt{77}}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | $\frac{16}{4}$ | 0     |
| 8   | 0         | 1         | -3        | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 9   | 0         | 1         | -3        | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 10  | 0         | 1         | -5        | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
| 11  | 0         | 1         | -5        | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0         | 0     |
FIG. 1. Eigenvalues of the evolution equation for symmetric (left-hand side) and antisymmetric eigenfunctions (right-hand side).

III. NONPERTURBATIVE ASPECTS

The derivation of the nucleon distribution amplitude from QCD is intimately connected with confinement and employs nonperturbative methods, e.g., QCD sum rules [8], lattice gauge theory [9] or the direct diagonalization of the light-cone Hamiltonian within a discretized light-cone setup [10]. To determine the moments $\Phi_N^{(n_1n_2n_3)}$, a short-distance operator product expansion is performed at some spacelike momentum $\mu^2$ where quark-hadron duality is valid [8]. One considers ($z$ is a lightlike auxiliary vector with $z^2 = 0$)

$$
\left. (iz \cdot \frac{\partial}{\partial z_i})^{n_i} \Phi_N(z_1 \cdot p) \right|_{z_i = 0} = \prod_{i=1}^{3} \left. (iz \cdot \frac{\partial}{\partial z_i})^{n_i} \int_0^1 [dx] \ e^{-i \sum_{i=1}^{3} (z_i \cdot p) z_i \Phi_N(x_i)} \right|_{z_i = 0} = (z \cdot p)^{n_1+n_2+n_3} \Phi_N^{(n_1n_2n_3)}
$$

and evaluates correlators of the form [11,12]

$$
I^{(n_1n_2n_3,m)}(q,z) = i \int d^4x \ e^{iq \cdot x} < \Omega|T(O_\gamma^{(n_1n_2n_3)}(0) \hat{O}_\gamma^{(m)}(x))|\Omega > (z \cdot \gamma)_{\gamma'\gamma}
$$

where the factor $(z \cdot \gamma)_{\gamma'\gamma}$ serves to project out the leading twist structure in the correlator, and

$$
O^{(n_1n_2n_3)} = (z \cdot p)^{-\sum_{i=1}^{3} n_i} \prod_{i=1}^{3} \left. (iz \cdot \frac{\partial}{\partial z_i})^{n_i} O(z_1 \cdot p) \right|_{z_i = 0}
$$

are appropriate three-quark operators containing derivatives. Their matrix elements

$$
< \Omega|O_\gamma^{(n_1n_2n_3)}(0)|P(p) > = f_N(z \cdot p)^{n_1+n_2+n_3+1} N_\gamma O^{(n_1n_2n_3)}
$$

are related to moments of the covariant distribution amplitudes [13] $V$, $A$, and $T$: $\Phi_N(x_i) = V(x_i) - A(x_i)$, $\Phi_N(1,3,2) + \Phi_N(2,3,1) = 2T(1,2,3)$ with $V(1,2,3) = V(2,1,3)$, $A(1,2,3) = A(2,1,3)$, and $T(1,2,3) = T(2,1,3)$. Here $f_N$ denotes the “proton decay constant”.

On the basis of such QCD sum-rule calculations, useful theoretical constraints on the moments of nucleon distribution amplitudes have been obtained and various models [8,11,12,14,15] have been proposed. Examples of physical observables calculated from these models are given in [8,11,14] and more recently in [15,17]. Our project differs from previous ones in that we use a “hierarchical” $\chi^2$-criterion to parametrize the deviations from the sum-rule constraints [21]. This affords to determine optimized versions of previous model distribution amplitudes as well as to find a new solution (we labeled heterotic) [15] which hybridizes morphological and dynamical features of COZ-type [11] and GS-type [14] amplitudes providing results corroborated by the available data. The “hierarchical” treatment of
the sum rules takes into account the higher stability of the lower-level moments [16] and does not overestimate the significance of the still unverified constraints [1] for the third-order moments. The simple but important assumption is that the model space can be safely truncated at states with bilinear correlations of fractional momenta because adding higher-order contributions should only refine the initial approximation. The advantage of our method becomes apparent by taking a more global approach, i.e., looking for solutions on the scale of the whole validity range of the sum rules. This treatment leads to a characteristic series of local minima shown in Fig. 2.

FIG. 2. Large scale pattern of nucleon distribution amplitudes complying with existing QCD sum rules.

They constitute a pattern characterized by a smooth and finite orbit which is completely specified by joint values of $B_4$ and $R \equiv |G^a_M|/G^p_M$. This striking scaling behavior seems to pertain to solutions [22] which incorporate higher-order eigenfunctions (Appell polynomials)—not used in our fit procedure (see inset in Fig. 2). The robustness of the fiducial orbit suggests that many of the features of nucleon distribution amplitudes that seemed unrelated actually fit together into a coherent overall structure, hence leveraging our knowledge of specific characteristics into a more general context. Isolated samples in the $(B_4, R)$ plane are relegated to spurious solutions, either because they exhibit unrealistic large oscillations in the longitudinal momentum fractions [22] (stars) or because they yield a wrong evolution behavior for the nucleon form factors [7] (light upside-down triangles). The presented curves are fits to the local minima of the COZ sum rules (+ labels) and to a combined set of KS and COZ sum rules (○ labels) restricted within the intervals $0.104 \div 0.4881$ and $0.0675 \div 0.482$, respectively: $R = 0.437338 - 0.006016B_4 - 0.000176B_4^2$ (solid line) and $R = 0.431303 - 0.00752B_4 - 0.000241B_4^2 + 3.851221 \times 10^{-6}B_4^3$ (dotted line). Cross-type solutions show up across the fiducial orbit and they seem to undergo a complete metamorphosis as they pass from COZ-type solutions ($R = \leq 0.5$) to the heterotic one (smallest possible value of $R$ still compatible with the sum-rule constraints). The passage between these two types of solutions is smooth and the changed shapes follow an orderly sequence of grada-
tions characterized by comparable values of $\chi^2$. All solutions are organized by the same kind of dependence between $R$ and $B_4$. The solution denoted $Het'$, past the COZ-cluster, has “mirror”-image characteristics relative to the heterotic solution [21][18] but is unstable.

IV. SUMMARY AND CONCLUSIONS

We have attempted to provide a unified view of nucleon distribution amplitudes derived on the basis of QCD sum rules. The pattern that emerges is not one of widely dispersed solutions across the parameter space, but one of a smooth and finite curve (an orbit) in the $(B_4, R)$ plane, as underlined in Figure 2. On the perturbative side, a systematic theoretical procedure to calculate orthonormalized eigenfunctions of the leading-order nucleon evolution kernel has been developed and a complete set of the first 54 terms has been explicitly evaluated.

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