Large eddy simulation of supersonic turbulent mixing layer

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Large eddy simulation of supersonic turbulent mixing layer

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Abstract. A Large Eddy Simulation (LES) method is developed to simulate the supersonic turbulence mixing layer. The 7th-order upwind compact difference scheme and the modified Smagorinsky eddy viscosity model are adopted in the LES method developed here. Four sets of grid have been used to investigate for the investigating of the grid sensitivity for the LES. As a finer grid is used, a greater fraction of the turbulence spectrum is obtained directly. In comparison with the RANS method, the pulsating characteristic of the supersonic mixing layer is well simulated by the LES method.

1. Introduction

Free shear layers could be found in many situations in nature including aviation industrial applications, such as the film cooling system of the aero-optical window (figure 1). Detailed understanding of the physics of free shear layer and get the unsteady density variations is very important, as the optical rays are refracted by the turbulent flow and the fine-scale structure of the turbulence [1-4]. On these purpose, a high-order LES method is developed to perform the supersonic turbulent mixing layer. The method developed here is intended for configurations in which a dominant structural feature provides an unsteady mechanism to drive the turbulent development in the mixing layer.

Even though the RANS (Reynolds Average Navier-Stokes) method has developed for many years, it has nature defects to describing the unsteady flow field and it also needs a proper mixing model for simulating the mixing layer. The 7th-order upwind compact difference and the modified Smagorinsky eddy viscosity model are adopted in the LES method developed here. Four sets of grid would be calculated for the investigating of the grid sensitivity in LES. As a finer grid is used, a greater fraction of the turbulence spectrum is directly calculated.

The RANS method is also used to simulate the mixing layer compared with the LES method. It shows that though the structure feature of the flow field is similar, the mixing layer of RANS' result is thinner than the LES' result, and the RANS method can't provide the unsteady density pulse which is very important to the aero-optic effect simulation. The LES method shows the great advantage in describing this problem.
2. Numerical method

2.1. Governing equations

In LES, the large scale field can be obtained from the solution of the filtered Navier-Stokes equations, and the sub-grid scales are formulated according to the modified Smagorinsky eddy-viscosity model [5]. To simulate the hypersonic mixing layer, a density-weighted Favre-filter operator is introduced, and the Favre-filtered Navier-Stokes equations are described as follows:

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_i}{\partial x_i} = 0
\]

\[
\frac{\partial \bar{\rho} \bar{u}_i}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_i \bar{u}_j}{\partial x_j} = \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \Gamma_{ij}}{\partial x_j} + \frac{\partial \bar{\rho} \bar{e}}{\partial x_i} \frac{\partial \bar{u}_j}{\partial x_j} - \frac{\partial \bar{\rho} \bar{u}_i \bar{u}_j}{\partial x_j} \frac{\partial \bar{u}_j}{\partial x_j}
\]

\[
\frac{\partial \bar{\rho} \bar{e}}{\partial t} + \frac{\partial \bar{\rho} \bar{e}}{\partial x_j} \frac{\partial \bar{u}_j}{\partial x_j} = \frac{\partial \bar{p}}{\partial x_i} (\theta_i + \Gamma_{ij} \bar{u}_i)
\]

And \( \bar{p} = \bar{\rho} \bar{R} \bar{T}, \Gamma_{ij} = \bar{\sigma}_{ij} + \tau_{ij}, \theta_j = \bar{a}_j + \bar{q}_j \)

\[
\bar{\rho} \bar{e} = \bar{\rho} \bar{R} \bar{T}, \Gamma_{ij} = \bar{\sigma}_{ij} + \tau_{ij}, \theta_j = \bar{a}_j + \bar{q}_j
\]

The following is modified Smagorinsky eddy-viscosity model

\[
\bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]

\[
\mu_t = C_R \bar{\rho} \Delta^2 \sqrt{\bar{S}_{ij} \bar{S}_{ij}}
\]

\[
\tau_{ij} = 2 \mu_t \left( \bar{S}_{ij} - \frac{1}{3} \bar{S}_{kk} \delta_{ij} \right)
\]

\[
\bar{q}_j = \mu_t \frac{\partial \bar{T}}{\partial x_j}
\]

where \( C_R = 0.00423 \) is the model coefficient, and \( \Delta \) is the length scale defined by

\[
\Delta = \left[ \frac{1}{2} (V_L + V_R) \right]^{\frac{1}{2}} D
\]

where \( V_L \) and \( V_R \) are the tetrahedral volumes to the left and right of the cell face and \( D = 1 - \exp(-n^+/A^+) \) is the van Driest damping factor, where \( n^+ = n u_e/v_w \) is the normal distance to the
nearest solid boundary normalized by the viscous length scale $u_t/v_w$, $v_w$ is the kinematic viscosity evaluated at the wall, and $u_t$ is the local friction velocity. $A^+ = 26$ is the van Driest constant. The 7th-order upwind compact difference \cite{6} has been used to solve the three-dimensional compressible non-dimensional Favre-filtered Navier-Stokes equations numerically. The compact storage 3rd-order explicit Runge-Kutta method is applied for time-integration.

2.1.1. Computation model. The computation model is described as figure 2. The zone A is the main flow field, with $H=20\text{km}$, $Ma=6$, $Re=1e7/m$. The jet gas is air, and with an exit flow of $Ma=3$, $T_{\text{total}}=300\text{K}$, $P_{\text{static}}=4800\text{Pa}$, $Re=3.8e7/m$. The zone B is the mixing layer region.

![Figure 2. Computational field.](image)

3. Results and discussion
For the results would be affected by the grid scale, four sets of grid as the table 1 are used. The results are shown in figure 3 and figure 4. With the refinement of the grid, the turbulent boundary layer of the upstream flat region develops more fully, and the structure is more abundant. The downstream compression shock wave interference, shock wave and turbulence structure area of the mixing layer are more clear, and the non physical high frequency oscillation is not found. These results show that the LES method developed here is applicable to simulate the supersonic turbulent mixing layer.

| Name    | Mesh (million) | $\Delta x_{\text{min}}$/m | $\Delta y_{\text{min}}$/m | $\Delta z_{\text{min}}$/m |
|---------|----------------|---------------------------|---------------------------|---------------------------|
| Grid a  | 3              | $2\times10^{-5}$          | $3\times10^{-5}$          | $8\times10^{-5}$          |
| Grid b  | 5              | $1.5\times10^{-3}$        | $3\times10^{-5}$          | $8\times10^{-3}$          |
| Grid c  | 10             | $1\times10^{-5}$          | $3\times10^{-5}$          | $8\times10^{-3}$          |
| Grid d  | 20             | $1\times10^{-5}$          | $3\times10^{-5}$          | $4\times10^{-5}$          |

Figure 5 show the vorticity iso-surface of the grid c result. And we can see the development history of the upstream flat’s turbulent boundary layer, which changes from “Λ” vortex to “hairpin” vortex, and then broken up into the mixing layer’s vortex, and finally to be fully turbulence.

Figure 7 and figure 9 are the results of the time averaged flow field of the LES. Figure 6 and figure 8 are the results of flow field calculated under the same conditions using the RANS method (S-A turbulence model). Grid from (a) to (d) corresponds to four sets of grids respectively. By the comparison of flow structures, the results of the two methods are similar. The mixing shear layer of LES method is thicker than that of RANS method, because the crushing and mixing of large eddy are considered in LES method, which makes gas mixing more fully. Figures 10 is the comparison of the velocity profile of the two methods.
Figure 3. Temporal density contours.

(Pulsatile value is instantaneous value minus time averaged value)

Figure 4. Pulsatile density contours.
Figure 5. Instantaneous turbulent structures visualised using the Q criterion.

(a) Grid a
(b) Grid b
(c) Grid c
(d) Grid d

Figure 6. RANS' density contours.

Figure 7. LES' time averaged density contours.
Figure 8. RANS’ temperature contours.

Figure 9. LES’ time averaged temperature contours.
4. Conclusions
The supersonic turbulent mixing layer is simulated by LES method, and the development process of
turbulent shear layer is obtained, the “Λ” vortex, “hairpin” vortex and the broken up of the mixing
shear layer vortex are reproduced. The comparison of the calculation results with the RANS method
and the LES is made, which shows the LES method can simulate the fluctuation characteristics of
mixing shear layer, and the thickness of the shear layer is thicker than the RANS results in the fully
developed turbulent region. For the further work, experiment would be conducted on the mixing shear
layer study.

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