Influence of Intercluster Connection Mode on Cluster Network Synchronization

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Abstract. Aiming at the cluster network composed of BA scale-free network, this paper proposes two one-way driving inter-cluster interconnection methods. One inter-cluster interconnection method is connected to the generous node of the driven sub-network, referred to as BDN mode. The other is a small node connected to the driven subnet, referred to as SDN mode. Using the second eigenvalue and the synchronization dynamics to compare the synchronization capabilities of cluster networks in different inter-cluster interconnection modes, it is found that the cluster networks interconnected by BDN mode have stronger synchronization capabilities.

Keywords: Power network; cluster network; complex network; synchronization dynamics.

1. Introduction

In the real world, many actual networks have the structure of interconnecting subnets. The subnets are independent of each other and related to each other. This complex network has involved various fields, such as biological sciences. Scholars in various fields have developed this research. At present, many researchers have put their eyes on the research of network synchronization and control, and have achieved a series of valuable research results in recent years [1-8]. Literature [2] studies the problem of synchronous sampling control in time-delay complex dynamic networks. The experimental simulation results show that the given sufficient conditions for time-delay complex dynamic network synchronization are less conservative and the data sampling controller is feasible. Literature [4] studied the problem of complex network fault-tolerant synchronization control with actuator faults, and designed a dynamic output feedback controller using linear matrix inequality technology, and verified the feasibility and effectiveness of the proposed method through numerical simulation. Literature [5] uses the adaptive control method to study the problem of finite time synchronization control of complex dynamic networks, and uses numerical examples to verify the validity of the conclusions of this paper. Literature [6] based on complex network synchronization, this paper studies the problem of coordinated control of urban expressway multiple on-ramps. The cell transmission model is used to establish a complex network dynamic model of urban expressway node coupling, and the effectiveness of the coordinated control method is verified by specific example simulations. It can achieve the purpose of restraining traffic congestion and improving road traffic efficiency at the cost of a smaller control range, and the control effect is better than traditional coordinated control methods. Literature [7] mainly
investigates the globally exponential synchronization of complex networks (CNs) via delayed impulsive control, where the stabilizing effects of time-delay in impulses are fully considered. Two examples are given to illustrate the effectiveness of the theoretical results. Since changes in the complex network structure will affect the synchronization dynamics behavior of the complex system, this article starts with the network structure, studies the synchronization behavior of the complex network, reveals the evolution mechanism of the synchronization process of the complex system, and helps us better understand the different complex systems. The difference in behavior will help us to take measures to optimize the operation of complex network systems.

2.  Cluster network modeling with scale-free subnet structure

2.1. BA scale-free model

The cluster network model used in this paper is based on the BA scale-free network as a subnet. According to Barábasi and Albert, a scale-free network model is proposed, which is called BA scale-free model [9]. They believe that the previous network model did not consider the two important properties of the real network: growth and preferred connectivity. The former means that new nodes are continuously added to the network, and the latter means that after a new node comes in, the node with a larger degree in the network is preferred for connection. Based on the growth and preferential connection characteristics of the network, the construction algorithm of the BA scale-free network model is as follows:

1. Initial: Firstly, \(m_0\) nodes are given, and it is fully connected with the number of connected edges is \(m_0(m_0 - 1)/2\);
2. Growth: At each time step, add a new node and \(m\) new edges connected to the node, here \(m \leq m_v\);
3. Optimal selection: The probability \(p_i\) that the newly added node is connected to the existing node \(i\) in the network satisfies:

\[
p_i = \frac{k_i}{\sum_j k_j}
\]

After \(t\) steps, you can get a network with \(N=t+m_0\) nodes and \(mt+m_0(m_0-1)/2\) edges.

This paper uses the above-mentioned BA scale-free network model construction algorithm to build two BA scale-free networks with the same structure but different node sizes. Each subnet is connected by unweighted and undirected edges, and clusters are interconnected by unweighted and directed edges. The cluster network modeling diagram with three scale-free subnet structures is shown in Figure 1.

Fig.1 Cluster network modeling diagram of three scale-free subnet structures
2.2. Type I network synchronization capability

The synchronization area corresponding to Type I network is unbounded [10]. When the network is a simple connected network without weights and directions, the coupling matrix A is a symmetric matrix. If it is specified that when node i is connected to node j, $a_{ij}=1$, and when there is no connection $a_{ij}=0$, the diagonal elements are:

$$a_{ii} = \sum_{j=1}^{N} a_{ij}$$  \hspace{1cm} (2)

The eigenvalues of A are all non-positive real numbers, which can be recorded as $0 = \lambda_1 > \lambda_2 > \ldots \geq \lambda_N$. At this time, the synchronization ability of the network is determined by the largest non-zero eigenvalue $\lambda_1$ of matrix A. The smaller the value of $\lambda_2$, the stronger the synchronization ability of the network. When node i and node j are connected $a_{ij}=-1$, when there is no connection $a_{ij}=0$, the diagonal elements are:

$$a_{ii} = \sum_{j=1}^{N} a_{ij}$$  \hspace{1cm} (3)

The eigenvalues of A are all non-negative real numbers, denoted as $0 = \lambda_1 < \lambda_2 < \ldots \leq \lambda_N$. At this time, the synchronization capability of the network is determined by the smallest non-zero eigenvalue $\lambda_2$ of matrix A. When the connection of nodes in the network has directional or weighted value, the coupling matrix A is generally an asymmetric matrix, and its eigenvalue is expressed as $\lambda_i = \lambda_i^r + j\lambda_i^i, (i = 1,2,\ldots,N)$. At this time, The synchronization capability of the network can be determined by the maximum (or minimum) non-zero eigenvalue $\lambda_2$.

2.3. Kuramoto model

The Kuramoto model was proposed by Kuramoto in [11], and it is one of the most classic models for studying the synchronization of periodic oscillator networks. In this model, each oscillator moves at a random natural frequency, and different oscillators are coupled with each other through the sine of the phase difference. The expression of this model is very simple, but it can show a variety of different synchronization patterns.

The mathematical expression of the Kuramoto model is as follows:

$$\dot{\theta}_i = \omega_i + K \sum_{j=1}^{N} a_{ij} \sin(\theta_j - \theta_i), \quad (i = 1,2,\ldots,N)$$ \hspace{1cm} (4)

$\theta_i$ represents the phase of the node i and $\dot{\theta}$ is a variable with a period of $2\pi$. $\omega_i$ represents the natural frequency of each oscillator. $\omega_i$ is usually a single peak. It is usually a unimodal, symmetrical distribution function $g(\omega)$. N represents the total number of nodes, K the coupling strength $a_{ij}$, the coupling relationship between the nodes in the system.

To describe the synchronization behavior of node oscillators in the network, we define:

$$r(t)e^{i\phi(t)} = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j(t)}$$ \hspace{1cm} (5)

$r(t)$ is called the order parameter, $\phi(t)$ is the average phase of the node oscillator. The range of values $r(t)$ is $0 \leq r(t) \leq 1$, and its size represents the internal consistency of the dynamic behavior of a large number of phase oscillators. The larger the r, the more orderly the system [12]. At $r=1$, the phase of all oscillators in the system is the same, that is, complete synchronization is achieved. hence, the size of the $r(t)$ describes the degree of synchronization of the oscillator network.
3. Cluster network synchronization research under different connection modes between clusters

A subnet composed of a BA scale-free network, the node degree distribution of each subnet follows the power law distribution form, that is, a small number of nodes have a larger degree value, and most nodes have a smaller degree value. For such a network with uneven degree distribution, we study the synchronization of the cluster network composed of two unidirectionally driven inter-cluster interconnections from the perspective of node degree values. One is the large node connected to the driven promoter network, called BDN; the other is the small node connected to the driven promoter network, called SDN.

We construct the number of small network nodes NS=50, the number of large subnet nodes NB=100. When the kuramoto synchronization model is used for experimental simulation, the value of coupling strength in the network model is obtained as K=6. The natural frequency $\omega_i$ of each oscillator is 0, the initial phase of small network is uniform in $[-\pi, \pi]$, and the initial phase of large network is uniform in $[-\pi, \pi]$.

3.1. Effect of network structure on synchronization ability of cluster networks

By changing the inter-cluster connection value $L$, the simulation analysis of the synchronization capability of the two types of cluster networks changes with the inter-cluster connection number $L$, and the second characteristic value $\lambda_2$ is used to reflect the network synchronization capability. The experimental results are shown in Figure 2.

![Fig.2 Changes of the second eigenvalue $\lambda_2$ of the cluster network with the number of inter-cluster connections $L$ in two types of inter-cluster connections](image-url)
The second eigenvalue $\lambda_2$ of the BDN cluster connection mode is smaller than that of the SDN cluster connection mode. The smaller the $\lambda_2$ value, the stronger the synchronization ability of the network. With the increase of the number of connections $L$ between clusters, the synchronization ability of cluster network composed of BDN mode and SDN mode is gradually enhanced. Therefore, we want the cluster network to have the strongest synchronization ability. From the perspective of network structure, it is obtained that the driving network is preferentially connected to the node with a large degree of the driven network.

3.2. Dynamics research on the synchronization ability of cluster networks

The kuramoto oscillator model is used to study the synchronization dynamics of the BA power network. To begin with, we construct the number of small subnet nodes is $N_S=50$, the number of large subnet nodes is $N_B=500$, and the number of inter-cluster connections is $L=5$. The connection between clusters is unidirectional.

![The variation of the sequence parameter $r(t)$ of the cluster network with time $t$ under the two types of inter-cluster connection](image)

Figure 3 uses the value of the sequence parameter $r(t)$ to describe the synchronization capability of the node oscillators in the network over time. In the initial period of time, each vibrator in the network is mainly affected by the intra-cluster coupling, and the intra-cluster coupling of the two networks is the same, but the effect of the inter-cluster coupling is very small, so the change of the sequence parameter value $r(t)$ of the cluster network in the initial time period is very similar in the two inter-cluster connections. With the delay of time, the effect of inter-cluster coupling on the overall synchronization capability of each network is also different, resulting in different time for the two networks to achieve complete synchronization. The value of the sequence parameter $r(t)$ corresponding to the cluster network in the BDN mode changes to 1 more quickly, indicating that the cluster network in this mode has a strong synchronization capability and each node has reached complete synchronization.
4. Conclusion

This paper focuses on the synchronization performance research of cluster networks under different inter-cluster connection modes. First, the second characteristic value $\lambda_2$ of the synchronization capability evaluation index based on the network topology is used to compare the cluster network synchronization conditions under different inter-cluster connection modes, and it is found that when the number of connections between clusters is determined, the second characteristic value $\lambda_2$ of the cluster network in the BDN mode is small, indicating that its synchronization capability is better than the SDN mode. Then, the cluster network synchronization capability is studied from the kuramoto model evaluation index sequence parameter $r(t)$ based on node synchronization dynamics. The experimental simulation results show that the overall synchronization speed of the cluster network is faster in the BDN mode. Therefore, in order to better enhance the overall synchronization performance of the cluster network, we choose to drive the subnet to connect to the large degree node of the driven subnet. This research result has certain guiding significance for the stable operation of the complex network formed by the interconnection of subnets.

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