Theoretical cross sections in the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction

M. Katsuma
Advanced Mathematical Institute, Osaka City University, 558-8585 Osaka, Japan
E-mail: mkatsuma@sci.osaka-cu.ac.jp

Abstract. The $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction seems to be explained by the simple reaction mechanism of the direct-capture potential model. The theoretical cross section at $E_{\text{c.m.}} = 0.3$ MeV is found to be enhanced with the $E2$ transition. This is caused by the tail of the subthreshold $2^+_1$ state ($E_x = 6.92$ MeV). The total $S$-factor is consistent with the previous studies. The photodisintegration of $^{16}\text{O}$ is also found to be dominated by the $E2$ excitation in the vicinity of the $\alpha$-particle threshold.

1. Introduction
The $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction is considered as a key reaction for the carbon-oxygen ratio in the universe, and it has a great influence on the stellar evolution of massive stars and nucleosynthesis of elements [1]. However, because of the Coulomb barrier, the cross section is too small to measure at the centre-of-mass energy $E_{\text{c.m.}} = 0.3$ MeV, corresponding to the helium burning temperature. To understand the reaction rates, the experimental efforts have been made with the various methods, e.g. [2, 3, 4, 5], including indirect measurements. The cross section of the inverse reaction is expected to be larger than that of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$. The experimental projects with the high-intensity laser are in progress to reveal the tiny cross section, e.g. [6].

Under the circumstances, I have recently calculated the low-energy cross section of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ by using the direct-capture potential model (PM) [7], and have converted them into the reaction rates [8]. In this presentation, I report the theoretical low-energy extrapolation of the cross section and examine the additional resonant contribution to PM [9]. In addition, I discuss the photodisintegration of $^{16}\text{O}$ for the future experiments by the same model [10].

In the next section, the PM is described, briefly. The low-energy cross section and the reaction rates are discussed in § 3. The photoelectric cross section of $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$ is also illustrated. The summary is in § 4.

2. Potential model (PM)
The PM describes the fundamental process of radiative capture reactions, and it works in the direct reaction mechanism, where only a few degrees of freedom of motion are activated.

The $\gamma$-ray angular distribution for $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ at low energies is given by $d\sigma/d\Omega_\gamma = \sum_{j=0}^{4} c_j P_j(cos \theta_\gamma)$, where $P_j$ is the Legendre polynomials, $c_j$ is the coefficient of the expansion, which is calculated from the matrix element of the electromagnetic operator between the initial wave and the final bound state. To generate the initial wave, i.e. the $\alpha$+$^{12}\text{C}$ continuum states, I use the internuclear potential to reproduce the phase shifts of elastic scattering [11] and the $\alpha$-particle binding energy of $2^+_1$ ($E_x = 6.92$ MeV). $E_x$ denotes the excitation energy of $^{16}\text{O}$. The
From Fig. 2(a), the $p$-wave (dashed curve) is predominant. At low energies, the $d$-wave (dotted curve) seems to become more important because the angular distribution has two peaks. The experimental data are taken from [2, 3].

The radiative capture cross section is given by

$$\sigma_{\gamma\alpha} = \frac{4\pi e^2}{\bar{\omega}^2} \sigma_{\alpha\gamma},$$

where $k_\gamma$ is the wavenumber of photons $k_\gamma = E_\gamma/(\hbar c); E_\gamma = E_{c.m.} + 7.162$ MeV [10].

3. Low-energy cross sections of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

In this section, I show the calculated results of the cross section from PM. After discussing the additional contribution, I also illustrate the predicted cross section of the inverse reaction.

The calculated $\gamma$-ray angular distribution is compared with the experimental data [2, 3] in Fig. 1. The trend of the peak and valley appears to be reproduced by the solid curve obtained from PM. At $E_{c.m.} = 2.267$ MeV, the $\gamma$-ray angular distribution has a single peak, that means the $p$-wave (dashed curve) is predominant. At low energies, the $d$-wave (dotted curve) seems to become more important because the angular distribution has two peaks. The $E2$ and $E1$ components of the $S$-factor are shown in Figs. 2(a) and (b) as a function of the incident energy. From Fig. 2(a), the $E2$ transition is found to be enhanced at low energies. This is caused by the subthreshold $2^+_1$ state that has the $\alpha+^{12}\text{C}$ cluster structure [9, 15]. For the $E1$ transition, the $1^+_2$ molecular resonance is found at $E_{c.m.} \approx 2.4$ MeV ($E_x \approx 9.6$ MeV) in Fig. 2(b).
Table 1. Astrophysical $S$-factors at $E_{c.m.} = 0.3$ MeV, listed in keV b unit.

|       | PM [7, 8] | [3] | [4] | [14] |
|-------|-----------|-----|-----|------|
| $E1$  | 3         | 76 ± 20 | 79 ± 21 | 79 ± 21 |
| $E2$  | $150^{+41}_{-17}$ | 85 ± 30 | 70 ± 70 | 120 ± 60 |
| Cascade | $18^{+45}_{-14}$ | 4 ± 4 | 16 ± 16 |
| Total | $171^{+46}_{-22}$ | 165 ± 54 | 165 ± 107 | 199 ± 81 |

![Figure 3. E1+E2 S-factor of $^{12}$C($\alpha,\gamma$)$^{16}$O. The additional resonant contributions are included in the solid curve (PM+BW). The dotted curve is from PM. (Exp.:[2, 3, 5])](image1.png)

![Figure 4. Comparison of the reaction rates between PM and PM+BW. The shade area is the uncertainty of the rates estimated from PM [8]. The result is shown in ratio.](image2.png)

The $\alpha+^{12}$C cluster states play a crucial role in the discussion about the reaction mechanism of $^{12}$C($\alpha,\gamma$)$^{16}$O. The potential used here reproduces the elastic scattering data for $E_{c.m.} < 5$ MeV [7], and it describes the $\alpha+^{12}$C rotational bands very well [15]. The $\gamma$-ray angular distribution shown in Fig. 1 seems to be made by the interference between $2^+$ and $1^−$. The extrapolated $S$-factor at $E_{c.m.} = 0.3$ MeV is listed in Table 1. As comparison, the results from [3, 4, 14] are also listed. The total $S$-factor from PM is consistent with that from [3, 4, 14]. In contrast, the $E1$ and $E2$ components are different. It should be, however, noted that the present result is based on the analysis of the $\gamma$-ray angular distribution, that looks good in Fig. 1. The resulting reaction rate resembles the previous results because the total $S$-factor is comparable. The deduced reaction rate is available in [8].

To examine the contribution from other resonances, the Breit-Wigner (BW) type of the experimental resonances [16] is temporarily appended to PM. Figure 3 shows the sum of the $E1$ and $E2$ $S$-factors for $^{12}$C($\alpha,\gamma$)$^{16}$O. The dotted curve is the result of PM. The solid curve is the result with the experimental resonances (PM+BW). The contribution from the $1^−$ state ($E_x = 7.12$ MeV) is hindered by the isospin selection rule. Under the weak coupling [7], $1^−$ does not have the large $\alpha$-particle width. The large difference between two curves can be seen above $E_{c.m.} = 3$ MeV. However, the difference between the derived reaction rates is very small, as shown in Fig. 4. To compare the models, the resulting reaction rate is displayed in ratio. From Figs. 3 and 4, I confirm that the additional resonances do not have the important contribution. I, therefore, consider that the $^{12}$C($\alpha,\gamma$)$^{16}$O reaction is described by PM successfully enough.

Finally, I illustrate the photoelectric cross section of $^{16}$O($\gamma,\alpha$)$^{12}$C in Fig. 5. The solid curve is the result from PM. The dashed and dotted curves are the $E1$ and $E2$ components, respectively. From Fig. 5, the $E2$ excitation is found to dominate the reaction in the vicinity of the $\alpha$-particle
threshold. I also predict that the coupling between the $1_1^-$ and $1_2^-$ states is weak, and that the angular distribution of the emitted $\alpha$-particle at $E_\gamma \approx 8.5$ MeV is described by the interference between $2_1^+$ and $1_2^-$ as well as the counterpart of $^{12}$C($\alpha,\gamma$)$^{16}$O shown in Fig. 1.

4. Summary
I have discussed the theoretical cross section in the $^{12}$C($\alpha,\gamma$)$^{16}$O reaction obtained from PM. The potential used here is concordant with the optical potential reproducing the elastic scattering data over the wide energy region. In addition, I have shown the calculated result of $^{16}$O($\gamma,\alpha$)$^{12}$C.

The $^{12}$C($\alpha,\gamma$)$^{16}$O reaction seems to be explained by the direct reaction mechanism of PM. The theoretical cross section at $E_{c.m.} = 0.3$ MeV is found to be enhanced with the $E2$ transition. This is caused by the tail of the subthreshold $2_1^+$ state having the $\alpha+^{12}$C cluster structure. The PM makes the relatively large $E1$ transition from the $1_2^-$ molecular resonance at $E_{c.m.} \approx 2.4$ MeV. The $\gamma$-ray angular distribution at $E_{c.m.} \approx 1.3$ MeV appears to be made by the interference between $2_1^+$ and $1_2^-$. The total $S$-factor at $E_{c.m.} = 0.3$ MeV is consistent with the previous studies. The contribution from other resonances is negligible. The photodisintegration of $^{16}$O is also found to be dominated by the $E2$ excitation at the interest energies.

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