Radial Excitations of Heavy-Light Mesons

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1. Introduction

BaBar has recently announced the discovery of a new D_s state, which seems to be c\bar{s} state, \( D_{s0}(2860) \). Subsequent to this experiment, Belle has observed a new state of \( D_s^*(2715) \) whose spin and parity is determined to be 1\(^{-}\)\[2\]

We were the first in predicting 0\(^+\) and 1\(^+\) states of \( D_s \) and \( D \) particles, \( D_{s0}(2317) \), \( D_{s1}(2460) \), \( D_0^*(2308) \), and \( D_1^*(2427) \), \[3\], and we have also succeeded in reproducing higher resonances of \( B/B_s \) particles, \( B_1(5270) \), \( B_2^*(5745) \), and \( B_{s2}^*(5839) \), \[4\] using our semirelativistic quark potential model. Our model succeeds in reproducing these states with one to two percent of accuracy compared with the experiments, \( D_{s0}(2860) \) and \( D_s^*(2715) \), which are identified as 0\(^+\) and 1\(^-\) radial excitations \((n = 2)\). We also present calculations of radial excitations for \( B/B_s \) heavy mesons. Relation between our formulation and the modified Goldberger-Treiman relation is also described.

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To interpret the state \( D_{s0}(2860) \), there are arguments that this \( c\bar{s} \) state is explained to be a scalar by a coupled channel model, \[8\], or that it is a \( J^{PC} = 3^- \) state, \[9\], or that it can be explained by using a phenomenological interaction terms like our quark potential model, \[10\] or that it can be analyzed by using the \( ^3P_0 \) model. \[11\]

Starting from the astonishing discovery of \( D_{sJ} \) particles with narrow decay width by BaBar and CLEO, and confirmed by Belle, a series of successive experiments on the spectrum of a heavy-light system, i.e., heavy mesons, heavy quarkonium, and heavy baryons, stimulates theorists to explain all these spectra as well as their decay modes. See the recent reviews of Refs. \[12\], \[13\]. It seems that a new era of spectroscopy is opening, which is challenging to theorists to solve these spectra at the same time.

2 Numerical Calculation

Our model starts from a Hamiltonian with a scalar confining potential together with a Coulombic vector potential, \( \gamma_1 \). We expand the whole system, i.e., Hamiltonian, wave function, and eigenvalue is expanded in one to two percent of accuracy compared with the experiments, \( D_{s0}(2860) \) and \( D_s^*(2715) \), which are identified as 0\(^+\) and 1\(^-\) radial excitations \((n = 2)\). We also present calculations of radial excitations for \( B/B_s \) heavy mesons. Relation between our formulation and the modified Goldberger-Treiman relation is also described.

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Abstract. Recent discovery of \( D_s \) states suggests existence of radial excitations. Our semirelativistic quark potential model succeeds in reproducing these states within one to two percent of accuracy compared with the experiments, \( D_{s0}(2860) \) and \( D_s^*(2715) \), which are identified as 0\(^+\) and 1\(^-\) radial excitations \((n = 2)\). We also present calculations of radial excitations for \( B/B_s \) heavy mesons. Relation between our formulation and the modified Goldberger-Treiman relation is also described.
and $w_k(r)$ is a finite series of a polynomial in $r$, $w_k(r) = \sum_{i=0}^{N-1} a_i^k (r/a)^i$, which takes different coefficients for $w_k(r)$ and $v_k(r)$. In actual calculation, we have used $N = 7$. Hence we can in principle obtain seven different radial excitations.

In this paper, we calculate the most optimal values of parameters so that recently discovered and known $n = 2$ (the first radial excitation) $D_s$ particles are all fitted well around one percent of accuracy compared with the experiments. Here only a strong coupling $\alpha_s$ is modified and other parameters are kept the same as in [4], in which we have obtained $\alpha_s = 0.261$ both for $D$ and $D_s$. These are presented in Table 1 at the first order of calculation in $p/m_Q$ with $p$ being internal quark momentum and $m_Q$ heavy quark mass.

With these values of parameters, we obtain $n = 2$ masses of $D_s$ and $D$ at the same time. The results are shown in Table 3 for $D_s$. We also predict $n = 2$ states for $D, B$, and $B_s$ states, which are shown in Tables 4 and 5 assuming the same strong coupling $\alpha_s$ for $D_s$, which may actually be different for $B/B_s$ particles. In these Tables, $p_i$ and $n_i$ are $i$-th order positive and negative component contributions of a heavy quark, respectively, and $c_i = p_i + n_i$. When one carefully looks at these Tables, one notices that values of higher states $^3D_1$ and $^3D_2^-$ are not reliable even though we have listed in Tables for consistency with the former calculations.

### Table 1. Most optimal values of parameters.

| Parameters | $\alpha_s^{2s}$ | $a$ (GeV$^{-1}$) | $b$ (GeV) |
|------------|-----------------|-----------------|-----------|
| $m_s, d$ (GeV) | 0.112 | 0.0929 | 1.032 |
| $m_c$ (GeV) | 4.639 |

### Table 2. Theoretical mass gap. Values in brackets are experiments. Units are in MeV.

| Mass gap ($n = 1$) | $D$ | $D_s$ | $B$ | $B_s$ |
|-------------------|-----|-------|-----|-------|
| $0^+ - 0^-$       | 414 (441) | 358 (348) | 322 | 239 |
| $1^+ - 1^-$       | 410 (419) | 357 (348) | 320 | 242 |
| $n = 2$           | $D$ | $D_s$ | $B$ | $B_s$ |
| $0^+ - 0^-$       | 308 | 274 | 206 | 160 |
| $1^+ - 1^-$       | 350 | 327 | 216 | 171 |

are degenerate. When the light quark mass and a scalar potential are turned on, then the degeneracy due to chiral symmetry is resolved and the mass gap corresponding to the modified Goldberger-Treiman relation is given by

$$\Delta M = M_0(k = +1) - M_0(k = -1),$$

where $M_0(k)$ is a degenerate mass for a quantum number $k$, which appears in Tables 3-5 to distinguish states. This is described as $\Delta M = g_2 f_2 / G_A$ in [4]. They have assumed dominant interaction terms for hyperfine splitting due to $1/m_Q$ corrections so that mass gaps between $0^+ - 0^-$ and $1^+ - 1^-$ are almost equal to each other, which seems to hold in our formulation. Our hyperfine splittings due to $1/m_Q$ to this mass gap $\Delta M$ are calculated and we have added those to degenerate mass gaps between $0^+ - 0^-$ and $1^+ - 1^-$ for $D, D_s, B$, and $B_s$ with $n = 1$, which are given in Table 2. Our dynamical calculation supports the assumption that the mass gaps between $0^+ - 0^-$ and $1^+ - 1^-$ are almost equal to each other in the case of $n = 1$. In the second row of the same Table values for $n = 2$ are given, which is not as good as the case for $n = 1$.

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Fig. 1. Procedure how the degeneracy is resolved in our model. A quantum number \( k \) is defined in the main text.

### Table 3. \( D_s(n = 2) \) meson mass spectra (first order). Units are in MeV.

| \( 2^{4+1}L_J(J^P) \) | \( M_0 \) | \( c_1/M_0 \) | \( p_1/M_0 \) | \( n_1/M_0 \) | \( M_{\text{calc}} \) | \( M_{\text{obs}} \) |
|------------------------|-------------|-------------|-------------|-------------|--------------|--------------|
| \( ^1S_0(0^-) \)       | 2328        | 1.006 × 10^{-1} | 0.919 × 10^{-1} | 8.695 × 10^{-3} | 2563         | –            |
| \( ^3S_1(1^-) \)       | 2456        | 1.553 × 10^{-1} | 1.470 × 10^{-1} | 8.245 × 10^{-3} | 2837         | 2856         |
| \( ^3P_0(0^+) \)       | 2585        | 1.969 × 10^{-1} | 1.966 × 10^{-1} | 2.531 × 10^{-4} | 3094         | –            |
| \( ^3P_2(2^-) \)       | 2391        | 8.600 × 10^{-1} | 5.016 × 10^{-4} | 1.444 × 10^{-4} | 4449         | –            |
| \( ^3D_1(1^-) \)       | -4.287 × 10^{-1} | -4.287 × 10^{-1} | 5.482 × 10^{-7} | 1366         | –            |

### Table 4. \( D(n = 2) \) meson mass spectra (first order). Units are in MeV.

| \( 2^{4+1}L_J(J^P) \) | \( M_0 \) | \( c_1/M_0 \) | \( p_1/M_0 \) | \( n_1/M_0 \) | \( M_{\text{calc}} \) | \( M_{\text{obs}} \) |
|------------------------|-------------|-------------|-------------|-------------|--------------|--------------|
| \( ^1S_0(0^-) \)       | 2241        | 1.078 × 10^{-1} | 0.975 × 10^{-1} | 1.038 × 10^{-2} | 2483         | –            |
| \( ^3S_1(1^-) \)       | 2418        | 1.540 × 10^{-1} | 1.444 × 10^{-1} | 9.621 × 10^{-3} | 2791         | –            |
| \( ^3P_0(0^+) \)       | 2491        | 2.076 × 10^{-1} | 2.070 × 10^{-1} | 5.956 × 10^{-4} | 3008         | –            |
| \( ^3P_2(2^-) \)       | 2319        | 2.318 × 10^{-1} | 2.318 × 10^{-1} | 1.101 × 10^{-4} | 3069         | –            |
| \( ^3D_1(1^-) \)       | 2280        | 1.863 × 10^{-1} | 1.863 × 10^{-1} | 5.418 × 10^{-4} | 2706         | –            |
| \( ^3D_2(2^-) \)       | 2.109 × 10^{-1} | 2.099 × 10^{-1} | 1.203 × 10^{-4} | 2759         | –            |

### Table 5. \( B(n = 2) \) meson mass spectra (first order). Units are in MeV.

| \( 2^{4+1}L_J(J^P) \) | \( M_0 \) | \( c_1/M_0 \) | \( p_1/M_0 \) | \( n_1/M_0 \) | \( M_{\text{calc}} \) | \( M_{\text{obs}} \) |
|------------------------|-------------|-------------|-------------|-------------|--------------|--------------|
| \( ^1S_0(0^-) \)       | 5849        | 0.919 × 10^{-2} | 0.831 × 10^{-2} | 8.49 × 10^{-4} | 5902         | –            |
| \( ^3S_1(1^-) \)       | 6025        | 1.634 × 10^{-2} | 1.620 × 10^{-2} | 5.867 × 10^{-5} | 5944         | –            |
| \( ^3P_0(0^+) \)       | 6098        | 2.226 × 10^{-2} | 2.221 × 10^{-2} | 4.779 × 10^{-5} | 6160         | –            |
| \( ^3P_2(2^-) \)       | 5888        | 1.610 × 10^{-2} | 1.605 × 10^{-2} | 4.668 × 10^{-5} | 5982         | –            |
| \( ^3D_1(1^-) \)       | 1.809 × 10^{-2} | 1.808 × 10^{-2} | 1.037 × 10^{-5} | 5994         | –            |

### Table 6. \( B_s(n = 2) \) meson mass spectra (first order). Units are in MeV.

| \( 2^{4+1}L_J(J^P) \) | \( M_0 \) | \( c_1/M_0 \) | \( p_1/M_0 \) | \( n_1/M_0 \) | \( M_{\text{calc}} \) | \( M_{\text{obs}} \) |
|------------------------|-------------|-------------|-------------|-------------|--------------|--------------|
| \( ^1S_0(0^-) \)       | 5936        | 0.878 × 10^{-2} | 0.882 × 10^{-2} | 7.588 × 10^{-4} | 5988         | –            |
| \( ^3S_1(1^-) \)       | 6063        | 1.399 × 10^{-2} | 1.325 × 10^{-2} | 7.429 × 10^{-4} | 6148         | –            |
| \( ^3P_0(0^+) \)       | 6193        | 2.299 × 10^{-2} | 2.292 × 10^{-2} | 6.908 × 10^{-5} | 6202         | –            |
| \( ^3P_2(2^-) \)       | 5999        | 2.052 × 10^{-2} | 2.052 × 10^{-2} | 4.709 × 10^{-8} | 6320         | –            |
| \( ^3D_1(1^-) \)       | 7.631 × 10^{-2} | 7.627 × 10^{-2} | 4.449 × 10^{-5} | 6456         | –            |
| \( ^3D_2(2^-) \)       | -3.802 × 10^{-2} | -3.802 × 10^{-2} | 4.861 × 10^{-8} | 5770         | –            |