RECENT ADVANCES IN ODDERON PHYSICS

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Abstract
We present in this talk the phenomenological and theoretical advances in
Odderon physics which occurred since the last EDS Blois Workshop (Seoul,
1997).
1 Introduction

The elastic and diffractive scattering is controlled by two singularities located near $J = 1$ in the complex $J$-plane: the Pomeron $F_+$, in the even-under-crossing amplitude $F_+$, and the Odderon $F_-$, in the odd-under-crossing amplitude $F_-$. The Odderon idea was long time forced to stay in the Purgatory, because it contradicted the belief, founded on the dominant approach of the 70’s - the Regge-pole model, that $F_-$ is dominated by singularities located near $J = 1/2$ ($\rho$ and $\omega$ Regge poles and their cuts). In spite of its rediscovery in QCD in the 80’s, and of its solid theoretical foundation in the framework of asymptotic theorems and derivative relations, the Odderon continued to be considered as an heretical concept. As late as in October 1990, here, in Dubna, André Martin did not hesitate to associate the words "revolution" and "Odderon". The complete theoretical legitimacy of the Odderon came only in the last few years from the calculation of the Odderon intercept in QCD. In the first part of my talk I will discuss the last results in this field.

From the phenomenological point of view, the interest clearly shifted from the somewhat biased study of Odderon effects in $\bar{p}p$ and $pp$ scattering towards HERA Odderon physics. The surprisingly rich activity in Odderon phenomenology in the last two years will be described, before drawing conclusions, in the second part of the talk.

2 Calculation of the Odderon intercept in QCD

The most important recent result is, beyond any doubt, the discovery by Janik and Wosiek, and, independently, by Lipatov, of an exact solution for the Odderon intercept in LLA.

In QCD the Odderon is a C-odd state of 3 reggeized gluons which interact pairwise with a well-defined potential (Fig. 1). The problem is to find an operator $\hat{q}_3$,

$$\hat{q}_3 = -r_{12}r_{23}r_{31}p_1p_2p_3,$$

which commutes with the Odderon Hamiltonian $H$,

$$[\hat{q}_3, H] = 0$$

and has a much simpler form than $H$.

The exact solution of the problem is formulated in terms of the eigenvalue equation

$$\hat{q}_3 f = q_3 f$$

where

$$f = \left(\frac{\rho_{12}\rho_{13}\rho_{23}}{\rho_{10}^2\rho_{20}^2\rho_{30}^2}\right)^\mu \Phi(z),$$
with $\mu = h/3$, $h$ being the conformal weight. The function $\Phi$ satisfies a third order linear equation

$$a(z) \frac{d^3}{dz^3} \Phi(z) + b(z) \frac{d^2}{dz^2} \Phi(z) + c(z) \frac{d}{dz} \Phi(z) + d(z) \Phi(z) = 0,$$

(5)

where

$$a(z) = z^3(1 - z)^3,$$

(6)

$$b(z) = 2z^2(1 - z)^2(1 - 2z),$$

(7)

$$c(z) = z(z - 1)(z(z - 1)(3\mu + 2)(\mu - 1) + 3\mu^2 - \mu),$$

(8)

$$d(z) = \mu^2(1 - \mu)(z + 1)(z - 2)(2z - 1) - iq_3(z(1 - z),$$

(9)

$$z = \frac{\rho_{12}\rho_{30}}{\rho_{10}\rho_{32}}, \rho_k = x_k + iy_k (k = 1, 2, 3), \rho_{ij} = \rho_i - \rho_j.$$  

(10)

In Ref. 7, the numerical solution

$$q_3 = -0.20526i$$

(11)

was found, corresponding to the Odderon energy

$$\epsilon = 0.16478.$$  

(12)

The relation between the Odderon energy $\epsilon$ and the Odderon intercept $\alpha_O(0)$ is given by the equation

$$\alpha_O(0) = 1 - (9\alpha_s/2\pi)\epsilon.$$  

(13)

For realistic values of $\alpha_s$ ($\alpha_s \simeq 0.19$) one gets

$$\alpha_O(0) = 0.94.$$  

(14)

In collaboration with M.A. Braun and P. Gauron we recently performed a direct calculation of the lower bound for the Odderon intercept $\alpha_O(0)$ in the framework of the variational approach we formulated earlier in collaboration with L. Lipatov [10]. In this variational approach the Odderon energy is defined as

$$\epsilon = E/D.$$  

(15)

In Eq. (15) $D$ is a normalization constant and the energy functional is given by

$$E = \sum_{n=\pm\infty} \int_{-\infty}^{\infty} d\nu \epsilon_n(\nu)|\alpha_n(\nu)|^2,$$

(16)
where
\[ \epsilon_n(\nu) = 2 \text{Re}[\psi\left(\frac{1+|n|}{2} + i\nu\right) - \psi(1)], \quad (17) \]

\[ \alpha_n(\nu) = \int_0^\infty dr r^{-2 - 2\nu} \int_0^{2\pi} d\phi e^{-i\phi} \left( i\nu + \frac{n+1}{2} + r e^{i\phi}(h - i\nu - \frac{n-1}{2}) \right) \left( i\nu - \frac{n-1}{2} \right) \]
\[ \left( -\tilde{h} + i\nu - \frac{n-1}{2} \right)Z(r,\phi), \quad (18) \]

\[ h = 1/2 + n/2 - i\nu, \quad \tilde{h} = 1/2 - n/2 + i\nu, \]
\[ -\infty < \nu < \infty, n = \ldots, -1, 0, 1, \ldots, \quad (19) \]

\[ Z(z, z^*) = |z(1-z)|^{2h/3} \Psi(z, z^*), \quad (20) \]

The function \( \Psi \) in Eq. (21) is invariant under the transformations \( z \to 1 - z \) and \( z \to 1/z \) (Bose symmetry in the 3 gluons). The following trial functions \( \Psi \) were used in Ref. 9:
\[ \Psi = \sum_{k=1}^{N-N_1} c_k a^{k/2-1/6} + \sum_{k=1}^{N_1} d_k a^{k-1/6} \ln a, \quad (21) \]

where
\[ a = \frac{r^2 r_1^2}{(1 + r^2)(1 + r_1^2)(r^2 + r_1^2)}. \quad (22) \]

The result is
\[ \epsilon = 0.22269, \quad (23) \]
a value which has to be compared with Eq. (12). The corresponding Odderon intercept is
\[ \alpha_O(0) = 0.96. \quad (24) \]

By comparing the values (24) and (14), one sees that there is only a 2% difference between the “exact” result and the variational one.

We draw from this section the conclusion that the Odderon intercept is very close to 1, i.e. is much higher than the 1/2 (\( \rho, \omega \)) intercept. We therefore expect important Odderon effects at high energy.

The LLA result shows that the gap \( \alpha_O(0) - 1 \) is surprisingly small and therefore very sensitive to higher order corrections. The crucial problem of knowing if \( \alpha_O(0) \) is bigger than 1, smaller than 1 or just equal to one, is therefore
still an open problem. The last case in the one I favor. The best way of solving
this problem is, in my opinion, the study of the non-perturbative Odderon.
Promising results on this line were already obtained[1].

Let me add, before closing this section, that very recently, G.P. Korchemsky
and J. Wosiek obtained a new representation for the Odderon wave function[12],
which is in agreement with the results of Refs. 7 and 8 and which allows
identification of a new quantum number - triality - associated with the Odderon.

An intriguing problem is if there is or not a Pomeron-Odderon exchange-
degeneracy, like in the non-leading reggeon $\rho - \omega - f - A_2$ sector. The study of
the C-even state of 3 reggeized gluons is crucial in this context[13].

3 The Odderon phenomenology

3.1 What is the problem with the Odderon?

A quarter of century after its birth the Odderon has still an uncertain existential
status. Of course, there are some interesting experimental indications of its
existence:

- the difference between $pp$ and $\bar{p}p$ differential cross-sections in the dip-
  shoulder region at ISR energies[14];

- the unusual shape of the polarisation in $\pi^- p \rightarrow \pi^- n$ at low energies[15] (in-
  dicating a $\rho$-type Odderon, distinct from the $\omega$-type Odderon as required
  in LLA);

- the extraction of the semi-theoretical $\rho$-parameter from the $dN/dt$ UA4/2
  $\bar{p}p$ data in the presence of oscillations at very small $t$ and high energies or
  of a more complicated phase of the forward scattering amplitude[16].

However, these experimental indications are either isolated or controversial.
The real problem with the Odderon is the paradoxical scarcity of the high-
energy data in hadron-hadron scattering, leading to an excessive focus on the
$pp$ and $\bar{p}p$ scattering. In other words we fit high-energy parameters by using
mainly low-energy data and, even worse, we draw conclusions about the Odderon
based only on $pp$ and $\bar{p}p$ scattering. The folklore about the "suppression" of the
Odderon has its source in these facts. The recent shift in attention from $pp$ and
$\bar{p}p$ scattering towards $ep$ scattering is, in this context, very positive.

3.2 HERA-Odderon phenomenology

The most active team in the Odderon phenomenology in the last two years is, of
course, the Heidelberg group (H.G. Dosch, O. Nachtmann, E.R. Berger) and its
associates (A. Donnachie, P.V. Landshoff, W. Kilian, M. Rueter). In a series of
papers[17,18], they produced impressive results with a solid theoretical ground.

For example, the pseudoscalar meson production

$$e^\pm p \rightarrow e^\pm pPS, \text{ where } PS = \pi^0, \eta, \eta', \eta_c$$  (25)
at HERA ($\sqrt{s} = 300.6$ GeV) constitutes a direct probe for the Odderon. The Odderon is here in competition with the photon only (see Fig. 2) : the Odderon contribution is not obscured by the huge Pomeron contribution as in the hadron-hadron reactions.

By taking, as a toy model, the Odderon as a Regge pole located near $J = 1$,

$$\eta_O \beta(t) \left( \frac{s}{s_0} \right)^{\alpha_O(t) - 1} \xi(t)$$

(26)

where $\alpha_O(0) \simeq 1$, $\xi(t)$ is the signature factor, $\beta|t|$-the Odderon residue and $\eta_O = \pm 1$ (due to the absence of a positivity property for the Odderon contribution), one gets important Odderon effects in a variety of observables. I give just one example in Fig. 3 : the $p_{\perp}$ distribution for pion production in the photoproduction region. One can see from Fig. 3 (where $c_O$ is proportional to $\eta_O$) that dramatic effects are induced by the presence of the Odderon. Moreover, one can see that the effects for the case $\eta_O = +1$ are drastically different from the ones for $\eta_O = -1$ : the HERA data can give important indications on the sign of $\Delta \sigma$ - the difference between hadron-hadron and hadron-antihadron total cross-sections.

An important theoretical ingredient in several works of the Heidelberg group\cite{18} is the Stochastic Vacuum Model (SVM)\cite{19} which established a very interesting connection between the topological Y-shape of the baryons and the coupling of the Odderon. If the angle between two sheets of the baryon is very small (i.e. the baryon has a diquark-quark structure) the Regge-pole Odderon is suppressed. An interesting process is the photoproduction of pions with single dissociation (breakup of the target proton), because this process is independent of the particular structure of the baryon. Its cross-section is $\simeq 300$ nb, i.e. 50 times larger for the corresponding one in the elastic photoproduction.

Of course, taking the Odderon as a Regge-pole could be too simplistic : for example, perturbative QCD indicates a more complicate singularity structure in the complex $J$-plane. Moreover, we showed longtime ago\cite{20} that the Regge-pole Odderon induces an overall shift of the low-energy data for the real parts which are already very well described by the Pomeron and the secondary Regge poles. Therefore, the Regge-pole Odderon is the worst case to be considered as a possible Odderon singularity. However, as a toy model, it can be still used as an illustration of typical Odderon effects, present even if the Odderon coupling is very much suppressed as compared with the Pomeron one.

A very interesting Odderon effect was recently studied by S.J. Brodsky et al.\cite{9} : the asymmetry in the fractional energy of charm versus anticharm jets in the diffractive photoproduction $\gamma p \rightarrow c\bar{c}Y$ at HERA. This asymmetry is very sensitive to the Pomeron-Odderon interference (Fig. 4) : it measures the Odderon amplitude linearly. Namely

$$A(t, M_z^2, z_c) \simeq \epsilon \frac{\sin[(\pi/2)(\alpha_O - \alpha_P)]}{\cos((\pi/2)\alpha_O)} \left( \frac{s_{\gamma p}}{M_z^2} \right)^{\alpha_O - \alpha_P}$$

(27)

where $z_c = E_c(E_{\gamma})$ and $\epsilon = \pm 1$. It can be seen from eq. (27) that the sign of the asymmetry is controlled by the gap $(\alpha_O - 1)$ and the energy dependence
by the gap \((\alpha_O - \alpha_P)\). By taking, as a numerical illustration, \(\alpha_P = 1.13\) and \(\alpha_O = 0.95\), Brodsky et al. get a lower bound for the asymmetry equal to 15%.

Before closing this section, let me mention other interesting phenomenological studies of the Odderon: diffractive \(C = +\) neutral meson production from virtual photons\(^{22}\), exclusive \(\eta_c\) photo- and electroproduction\(^{23}\), exclusive \(f_2\) leptoproduction\(^{24}\) and single-spin asymmetries for small-angle pion production in hadron collisions\(^{25}\).

### 4 Conclusions: the Odderon in the future

As we understood from the talk of S. Weisz at this Conference\(^{26}\), the TOTEM experiment at LHC will not be very helpful for the Odderon physics: at least in the initial stage of the experiment, priority will be given to just measuring the \(pp\) total cross-section.

Great hopes arise from the talk of S.B. Nurushev\(^{27}\) concerning the R7 experiment at RHIC\(^{28}\) with a higher luminosity that the UA4/2 experiment at CERN. Several measurements concern directly the Odderon physics: extraction of the \(\rho\) parameter with a precision \(\delta \rho = 0.01\); research for oscillation at very small \(t\) \((|t| \simeq (1-4) \cdot 10^{-4} \text{ GeV}^2\); the evolution of the dip in \(d\sigma/dt\); the polarisation parameters. In connection with this last measurement, E. Leader and T.L. Trueman showed in a very recent paper\(^{29}\) the high sensitivity of the spin dependence of \(pp\) scattering, particularly the parameter \(A_{NN}\), to the Odderon.

We had a nice surprise learning that the \(\bar{p}p\) option at RHIC is realistic. That would allow the detection of the Odderon through the measurement of the difference \(\Delta \sigma\) of the \(\bar{p}p\) and \(pp\) total cross-sections: the Regge-pole model predicts \(\Delta \sigma = 40 \mu \text{b}\) at RHIC while the maximal-Odderon approach\(^{2}\) predicts \(\Delta \sigma = -2.4 \text{ mb}\). The expected precision at RHIC being \(\Delta \sigma = 0.5 \text{ mb}\), one can clearly detect the presence of the Odderon through \(\Delta \sigma\).

Let me also stress the importance of the systematic study of the energy-dependence of \(\sigma_T(s)\) in the RHIC range \(50 \text{ GeV} \leq \sqrt{s} \leq 500 \text{ GeV}\). A huge gap in the high-energy hadron data will be filled.

However, we have not to wait till the next millenium in order to get a long awaited evidence of the Odderon. The results of the H1 experiment on the pseudoscalar meson production at HERA\(^{30}\) will be soon available.

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Fig. 1. The Odderon in QCD as a C-odd state of 3 reggeized gluons.

Fig. 2. The Odderon exchange in $e^\pm p \rightarrow e^\pm pPS$.

Fig. 3. The $p_\perp$ distribution for pion production in the photoproduction region (Ref. 17).

Fig. 4. The Pomeron-Odderon interference in $\gamma p \rightarrow c\bar{c}p'$ (Ref. 21).