Impurity-induced dephasing of Andreev states

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A study is presented concerning the influence of flicker noise in the junction transparency on coherent transport in Andreev states. The amount of dephasing is estimated for a microwave-activated quantum point contact and its electromagnetic environment are discussed.

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I. INTRODUCTION

How much can flicker noise in the junction transparency affect coherent transport in Andreev states present in a superconducting quantum point contact (SQPC)? The assumption of coherent transport in Andreev states is widely used in theoretical work, see e.g. Refs. 1 and 2. However, in realistic systems interaction with a dynamical environment will always introduce some amount of dephasing, see Refs. 3 and 4 for a review.

In the so-called microwave-activated quantum interferometer, full coherency of Andreev states is assumed and a method for Andreev-level spectroscopy is presented. The Andreev levels are probed with a microwave field, resulting in an interference pattern in the current. If dephasing is present this interference pattern will begin to deteriorate. This connection between dephasing and current makes the microwave-activated quantum interferometer a suitable system to study the effect of flicker noise in the junction transparency on transport through Andreev states. It also provides an excellent opportunity to probe the coupling between a SQPC and its electromagnetic environment.

In the following we study the role of dephasing induced by flicker noise in the normal state junction transparency, \( D \), of a SQPC in the transport through Andreev states. The concrete system which will be considered is the above presented interferometer. Flicker noise can be caused by the presence of an impurity atom close to the junction which has two states of almost equal energy to choose from. When the atom tunnels between its two states the junction transparency will fluctuate. Another source of flicker noise is the tunneling of an electron between impurity atoms in a doped region. If there are two such neighboring defects with available states, a hybrid two-level state is formed and the electron can hop between the two. This hopping will then add a fluctuation to the junction transparency. The amplitude of these fluctuations depend on the distance between the defects and the junction. From now on we will refer to these dynamic defects as two-level elementary fluctuators (EFs).

The microwave-activated quantum interferometer (MAQI) is based on a short, weakly biased SQPC which is subject to microwave irradiation. Confined to the contact area there are current carrying Andreev states. The corresponding energy levels – Andreev levels – are found in pairs within the superconductor energy gap \( \Delta \), one below and one above the Fermi level. When a SQPC is short (\( L \ll \xi_0 \) where \( L \) is the length of the junction while \( \xi_0 \) is the superconductor coherence length), there is only one pair of Andreev levels and their positions depend on the order parameter phase difference, \( \phi \), across the contact as

\[
E_{\pm} = \pm E(\phi) = \pm \Delta \sqrt{1 - D \sin^2(\phi/2)}.
\]

Within this pair, the two states carry current in opposite directions and in equilibrium only the lower state is populated. The applied bias, \( V \), through the Josephson relation \( \phi = 2eV/h \), forces the Andreev levels to move adiabatically within the energy gap with a period of \( T_p = h\pi/eV \), see Fig. 1.

![FIG. 1. Time evolution of the Andreev levels within the energy gap of a single-mode SQPC. The wavy lines connecting points \( A_1, A_2 \) and \( B_1, B_2 \) symbolize resonant transitions between the levels induced by an applied microwave field. The symbol \( \zeta \) defines the position of the resonances.](image-url)

The microwave field induces Landau-Zener (LZ) transitions between the Andreev levels (symbolized by wavy lines in Fig. 1). If the upper level is populated after the second transition a delocalized quasiparticle excitation

- \( A_1 \)
- \( A_2 \)
- \( B_1 \)
- \( B_2 \)

FIG. 1. Time evolution of the Andreev levels within the energy gap of a single-mode SQPC. The wavy lines connecting points \( A_1, A_2 \) and \( B_1, B_2 \) symbolize resonant transitions between the levels induced by an applied microwave field. The symbol \( \zeta \) defines the position of the resonances.
will be created when this Andreev level merges with the continuum. The result will be a dc contribution to the current. Further, this current exhibits an interference pattern since there are two paths available to the upper level. It is this “interference effect” which is utilized in the MAQI for Andreev-level spectroscopy.

In order to estimate the influence of flicker noise on the interference pattern we continue with a quantitative presentation of the interferometer and the model used for the fluctuations.

II. MODEL AND METHOD

Consider a short single-mode SQPC which is subject to a high frequency microwave field \((\hbar \omega = 2E(\phi) \leq 2\Delta)\). Let the contact, placed at \(x = 0\), be characterized by an energy-independent transparency \(D\). A weak bias, \(e V \ll \Delta\), is applied across the junction. We choose to describe the quasiparticles in the contact region with the following wave function,

\[
\Psi(x, t) = u_+(x, t)e^{ik_Fx} + u_-(x, t)e^{-ik_Fx},
\]

where the envelope functions \(u_\pm(x, t)\), left and right movers, are two-component vectors in electron-hole space. To simplify notation we introduce the four-component vector \(\mathbf{u} = [u_+, u_-]\). This vector satisfies the time-dependent Bogoliubov-de Gennes equation

\[
\begin{align*}
i\hbar \partial \mathbf{u}/\partial t & = [\mathcal{H}_0 + \sigma_z V_g(t)]\mathbf{u}, \quad (3) \\
\mathcal{H}_0 & = -i\hbar v_F\sigma_z \tau_z \partial /\partial x + \Delta [\sigma_x \cos(\phi(t)/2) \\ & + \text{sgn}(x) \sigma_y \sin(\phi(t)/2)], \quad (4)
\end{align*}
\]

where \(\sigma_i\) and \(\tau_i\) denote Pauli matrices in electron-hole space and in \(\pm\) space, respectively, while \(V_g(t) = V_e \cos(\omega t)\) is the time-dependent gate potential. We assume \(e V_\omega \ll \Delta\).

The boundary condition at \(x = 0\) is

\[
\mathbf{u}(+0) = D^{-1/2} \left[ 1 - \tau_y \right] (1 - D)^{1/2} \mathbf{u}(-0).
\]

As a result of the applied high frequency field there will be resonant transitions between the Andreev levels. These transitions introduce a mixed state which can be described within the resonance approximation as

\[
\mathbf{u}(x, t) = \sum \mathbf{u}_\pm(x) e^{\mp \omega t/2},
\]

where \(\mathbf{u}_+(x)\) and \(\mathbf{u}_-(x)\) are the envelope functions of the upper and lower Andreev states, while \(b^+\) and \(b^-\) are the corresponding probability amplitudes. The final result is a dc current through the SQPC,

\[
I_{dc} = \frac{e(2\Delta - \hbar \omega)}{\hbar \pi} \left| b^+ \left( \frac{\hbar \pi}{e V} \right) \right|^2 = 2I_0 \sin^2(\Theta + \Phi),
\]

\[
I_0 \equiv (2e/\hbar \pi) r^2(1 - r^2)(2\Delta - \hbar \omega),
\]

where \(r^2\) is the LZ transition amplitude, which depends on the bias voltage and the amplitude of the perturbation \((\Theta\) is the phase of the LZ transition, which can be considered constant). The phase \(\Phi\) which inhibits the interference is calculated through

\[
\Phi = \frac{1}{2eV} \int_{\phi_A}^{\phi_B} \left( E(\phi) - \frac{\hbar \omega}{2} \right) d\phi.
\]

Flicker noise of the junction transparency, \(D\), which is the subject to be discussed now, enters through \(E(\phi)\), Eq. (4), in this expression.

A. Flicker noise in the junction transparency

The main topic of this work is to study the effect of fluctuations in the junction transparency on the MAQI. For simplicity we choose to model the sources of these fluctuations, the EFs, with the so-called random telegraph process. This process is characterized by a random quantity \(\xi(t)\) which has the value +1 or −1 depending on whether the upper or lower EF state is occupied. We assume that the probability of each state is the same, namely 1/2. This is acceptable since EFs with inter-level distances, \(E_i\), smaller than \(k_B T\), will be “frozen” — they behave as static impurities which do not affect the dynamic fluctuations of \(D\). In this model the EF switches between its two states randomly in time. Physically, switching is a result of interactions between the EF and phonons or electrons in the contact area.

In the presence of EFs the junction transparency will be modulated. In other words, \(D \rightarrow D + D_f(t)\), where \(D_f(t)\) is assumed to be small. Generally

\[
D_f(t) = \sum_i A_i \xi_i(t),
\]

with \(A_i\) being the coupling strength of the \(i\)-th EF. We assume that the random processes in different EFs are not correlated. Consequently, after a change in variables \(t\) to \(\phi\) we can specify the random telegraph processes \(\xi_i(t)\) through the correlation function,

\[
\langle \xi_i(\phi_1)\xi_j(\phi_2) \rangle = \delta_{ij} e^{-2\gamma_i |\phi_2 - \phi_1|}, \quad (7)
\]

where \(\gamma_i\) is the switching rate of \(i\)-th EF in units of the Josephson frequency \(\omega_J = 2eV/h\). It is related to the dimensional switching rate \(\Gamma\) as \(\gamma = \hbar \Gamma/2eV\).

Depending on the construction of the SQPC there can be any number of EFs which are “in range” to influence the transparency. In junctions which are very small it is probable that only one single EF will be in the vicinity of the contact. In this case, the coupling constant \(A\) and the switching rate \(\Gamma\) can be directly evaluated from the measured telegraph noise intensity in the normal state,

\[
S(\tau) \equiv \langle I(t + \tau)I(t) \rangle_t - \langle I(t) \rangle_t^2.
\]
Indeed, the current through a single mode QPC at low
Temperatures can be expressed according to the Landauer
formula as $I(t) = 2e^2V D(t)/h$. Consequently, the ran-
dom telegraph noise intensity is equal to

$$S(\tau) = (2e^2V A/h)^2 \exp(-2\Gamma \tau), \quad (8)$$

and both $A$ and $\Gamma$ can be extracted from measured $S(\tau)$.
A possible approach for extracting these model parameters
from noise measurements in the case of many fluctu-
tators in the QPC area will be discussed later.

III. SINGLE EF

In the case of a very small contact it is possible to con-
sider only one EF and put $i = 1$. We start by decom-
posing Eq. (8) as

$$E(\phi) = E_+ (\phi) + E_- (\phi) \xi(\phi),$$

$$E_\pm = \frac{1}{2} [E(\phi|D_1) \pm E(\phi|D_- 1)], \quad (9)$$

where $D_{\pm 1}$ are the two different values the transparency
fluctuates between. Further, we assume that both $\gamma$ and
$g_{fs} \equiv E_0 / eV$ are much smaller than the reduced inter-
level distance $(E_+ \approx \Delta) / eV$. This means that all devi-
tations in time are much longer than the Andreev level
formation time, which is of the order $\hbar / 2\Delta$.

Fortunately, the expression above, Eq. (8), is linear in $\xi$
and we can write the effect of the EF as an additive
contribution to the accumulated phase, Eq. (8), without making any approximations. Namely, $\Phi \to \Phi + \Phi_f$, with

$$\Phi_f = \frac{1}{2} \int_{\phi_A}^{\phi_B} g_{fs}(\phi) \xi(\phi) d\phi. \quad (10)$$

with $g_{fs}(\phi) = E_-(\phi) / eV$. After averaging over the real-
izations of the random process $\xi(t)$, the expression (8)
for the MAQI current is replaced by

$$I_{dc} = I_0 \left[ 1 - W \cos(2\Theta + 2\Phi) \right], \quad (11)$$

where $W \equiv \langle e^{2i\Phi(t)} \rangle$ contains the dephasing.
Without dephasing $W = 1$ and the effect of the phase $\Phi(\phi_A, \phi_B)$
is a modulation of the dc current between 0 and $2I_0$, the
modulation depth

$$(I_{max} - I_{min})/(I_{max} + I_{min}) = |W| \quad . \quad (12)$$

being equal to one. This modulation of the current is the
interference effect utilized in the MAQI. When dephas-
ing enters, the quantity $W(\phi_A, \phi_B)$ will decrease and the
modulation $|W|$ of the current envelope, i.e. the in-
terference pattern, will in turn decrease.

To facilitate the calculation of the dephasing term, $W$,
we define the auxiliary function,

$$\Psi(\phi) = \left\langle \exp \left[ \int_{\phi_A}^{\phi} d\phi' g_{fs}(\phi') \xi(\phi') \right] \right\rangle. \quad (13)$$

The quantity of interest, $W$, is related to $\Psi(\phi)$ as $W \equiv \Psi(\phi_B)$. Further, it can be shown, cf. with Ref. 6, that Eq. (13) satisfies the differential equation

$$g_{fs}^2(\phi) \Psi(\phi) = \left[ 2\gamma - \frac{d \ln g_{fs}(\phi)}{d\phi} \right] \frac{d\Psi}{d\phi} + g_{fs}^2(\phi) \Psi = 0, \quad (14)$$

with the initial conditions

$$\Psi(\phi_A) = 1, \quad \frac{d\Psi}{d\phi}|_{\phi = \phi_A} = 0. \quad (15)$$

Let us consider the following two cases in more detail: (i) the “slow EF”, $\gamma \ll g_{fs}$, which corresponds to low
Temperatures, and (ii) the “fast EF”, $\gamma \gg g_{fs}$, which
 corresponds to relatively high temperatures.

A. Slow switching EF

In the low-temperature limit the EF will slowly switch
between its two states, and we can let $\gamma \to 0$ in Eq. (14).
Further, by introducing the function,

$$\Xi(\phi) = \int_{\phi_A}^{\phi_B} d\phi g_{fs}(\phi) \Psi(\phi), \quad (16)$$

and applying the initial conditions (15) we obtain the
integral equation,

$$\Xi^2 + \Psi^2 = 1. \quad (17)$$

The solution follows as $\Xi^{(0)}(\phi) = \cos \Phi(\phi)$ where $\Phi(\phi) \equiv (\int_{\phi_A}^{\phi_B} d\phi' g_{fs}(\phi'))$. Consequently, $W^{(0)} = \cos \Phi_{\xi}$ with

$$\Phi_{\xi} = \Phi(\phi_B) = \int_{\phi_A}^{\phi_B} d\phi g_{fs}(\phi). \quad (18)$$

A better approximation can be found by looking for the
solution in the form $\Psi^{(1)}(\phi) = u(\phi) \Xi^{(1)}(\phi)$ and assum-
ing $u(\phi)$ to be a slow function. Neglecting the second
derivative of $u(\phi)$ we obtain the differential equation

$$du/d\phi = -\gamma \eta(\phi) u \quad (19)$$

for $u(\phi)$ with

$$\eta(\phi) \equiv \left( 1 + \frac{1}{2g_{fs}^2 \tan \Phi(\phi)} \frac{d g_{fs}}{d\phi} \right)^{-1}. \quad (20)$$

Defining

$$\nu_{\xi} = \frac{1}{2\gamma} \int_{\phi_A}^{\phi_B} \eta(\phi) d\phi \quad (21)$$

we arrive at the following expression for the oscillating
part of the current,

$$W \cos \Phi = e^{-2\nu_{\xi}} \cos \Phi_{\xi} \cos \Phi$$

$$= e^{-2\nu_{\xi}} [\cos(\Phi + \Phi_{\xi}) + \cos(\Phi - \Phi_{\xi})] / 2. \quad (22)$$
For a constant \( g_{fs}, v_{\zeta} = 1 \) and \( \Phi_{\zeta} = 2g_{fs}\zeta \), where \( \zeta = (\phi_B - \phi_A)/2 \) is equal to half the distance between the resonance positions, see Fig. 1. In the general case these quantities are increasing functions of \( \zeta \).

At \( \gamma = 0 \), the current is split into two interference patterns of equal magnitude shifted by the phase \( \Phi_{\zeta} \). A plot of \( \Phi_{\zeta} \) as a function of \( \zeta \) is presented in Fig. 2. The transparency \( D \) and the strength of the fluctuator are shown in the inset. There is no dephasing, only a distortion of the interference pattern, the modulation being \( |W| = |\cos \Phi_{\zeta}| \).

This splitting into two patterns of equal magnitude follows from the assumption that the occupation probability is the same for the two EF states. The general case of arbitrary probabilities for the EF states can be solved numerically. At finite \( \gamma \) dephasing takes place and the amplitude of the interference oscillations decreases by \( e^{-2\gamma v_{\zeta} \zeta/2} \). The physical reason of dephasing is the finite life time of an EF in a given state. We have calculated \( W \) for \( \gamma = 0.1 \), comparing the analytical approximation above against a numerical solution of Eq. (14), cf. with Fig. 3. (The plot for the analytical case when \( D = 0.6, A = 0.05 \) is missing because of numerical difficulties).

**B. Fast switching EF**

In the case of fast switching the differential equation (14) can be approximated as,

\[
\frac{d\Psi}{d\phi} = -\frac{g_{fs}^2(\phi)}{2\gamma}\Psi(\phi). \tag{22}
\]

The solution in this case is easily obtained, as

\[
W = \Psi(\phi_B) = \exp\left[-K(\phi_B, \phi_A)\right], \tag{23}
\]

with

\[
K(\phi_B, \phi_A) = \frac{1}{2\gamma} \int_{\phi_A}^{\phi_B} d\phi g_{fs}^2(\phi). \tag{24}
\]
Here we find an exponential decay of the interference term. Expression (24) describes an effect which is similar to motional narrowing of spectral lines. When the EF fluctuates rapidly enough compared to the “energy resolution” $E_n/h$, influence from the difference between two EF states is smeared and dephasing will be of a diffusive character, with an effective, time-dependent diffusion constant $g_f^2/2\gamma$. A calculation of the factor $K$ for $\gamma = 3$ is shown in Fig. 3 for three different sets of values of the junction transparency and the EF’s strength. The effect of dephasing from flicker noise will decrease for higher bias voltages, since a higher bias, through the Josephson relation, leads to a shorter time between the MAQI interference pattern as a function of $E$.

Let us consider a large number of EFs with varying switching rates distributed in the contact area. For simplicity, we shall assume that only the fluctuators with inter-level spacings $U_i \lesssim kT$ are important, and that their distribution is uniform, $P_0(U) = P_0 \mathcal{V}$. Here $\mathcal{V}$ is the sample volume. Further, we assume that the switching rates $\gamma_i$ are the same for both transition directions (up or down) between the EF’s levels. This assumption is natural because the ratio between the corresponding transition rates is $\exp(-U_i/kT)$. Within the assumptions discussed above, the final results are substantially simplified while preserving the essential dependence on temperature and the resonance position. These approximations agree with a general theory developed in Ref. 8 for the case of dephasing by two-level systems (TLS) in glasses.

The first step now is to linearize the SQPC’s transparency with respect to $\xi_i$ as $D \rightarrow D + D_f(\xi)$. This allows us to once again find an additive contribution to the accumulated phase, Eq. (23), which in this case will be,

$$\Phi_f \approx \sum_i A_i \int_{\phi_A}^{\phi_B} \xi_i(\phi) g_f(\phi) d\phi.$$

Here we have defined $g_f(\phi) = (1/2eV)dE(\phi)/dD$. In the same manner as in Sect. 11, but this time averaging over $A$'s, $\gamma$'s and $\xi$'s, we can express the modified MAQI current through expression (14) with

$$W = \left\langle e^{i \sum_i A_i \int_{\phi_A}^{\phi_B} g_f(\phi) \xi_i(\phi) d\phi} \right\rangle_{A,\gamma,\xi}.$$

To approximate this average we use the Holtsmark method[10] which is valid in the limit of many fluctuators, $N = P_0 \mathcal{V} kT \gg 1$. This allows us to rewrite Eq. (27) as the average over the contributions from single EFs,

$$W_s(A, \gamma) = \left\langle \exp \left( i A \int_{\phi_A}^{\phi_B} g_f(\phi) \xi(\phi) d\phi \right) \right\rangle_{\xi}.$$

Since the number of EFs is assumed to be large, to keep dephasing at a reasonable level it is important to keep $\langle 1 - W_s \rangle$ small.

With the solutions for $W_s$ found in Sect. 11 the average $\langle 1 - W_s \rangle$ remains to be calculated. To calculate this average one has to specify the distributions of the parameters $A$ and $\gamma$. The simplest and most natural assumption is that these two quantities are not correlated. Consequently, the distribution $P(A, \gamma)$ can be decoupled as $P(A) P_{A}(\gamma)$. To specify the distribution $P_A$ let us assume that the EFs are uniformly distributed in space. An EF behaves like a dipole, either electric or elastic, this allows us to specify it’s interaction strength as $A(r) = A_0/r^3$, where $r$ is the distance between the contact and a given EF, while $A_0$ is a coupling constant dependent on a specific interaction mechanism. Note that the quantity $A_0$ has dimension of volume. Within this model we arrive at the normalized distribution function $P_A(A) = 4\pi A_0/3 \mathcal{V} A^2$ (see Appendix B for details). The distribution $P(\gamma)$ is specified in a manner which is commonly used in glasses. Namely, the logarithm of $\gamma$ is assumed to be uniformly distributed. Hence, $P(\gamma) \propto \gamma^{-1}$, see Appendix B. To normalize it let us take into account that for a given energy spacing $U$ there is a maximal switching rate. Since we are interested in the fluctuators with $U_i \lesssim kT$, we can specify the maximal switching rate as $\gamma_T$, which is a function of the temperature. The actual temperature dependence is determined by the specific interaction mechanism between the EF and its environment. If the transitions between the EF states are
caused by interaction with phonons, then $\gamma_T \propto T^3$, while if the transitions are caused by the electrons excitations, then $\gamma_T \propto T^{1/2}$. Therefore, the normalized distribution can be specified as $P_\gamma(\gamma) = (L\gamma)^{-1}$, where $L = \ln(\gamma_T/\gamma_{min}) \gg 1$. Here we have introduced the minimal switching rate, $\gamma_{min}$. To express the decay in a more clear form let us introduce the dimensionless frequency $\nu_d$ corresponding to the interaction strength for an EF separated from the contact by an average distance to the active fluctuators, $\bar{r} \equiv (4\pi P_0 kT/3)^{-1/3}$, divided by the Josephson energy $2eV$. We can specify $\nu_d$ as

$$\nu_d = 4\pi P_0 kT A_0/3 = A_0/\bar{r}^3. \quad (30)$$

The decay rate $K = -\ln W$ is then given by the expression

$$K = \nu_d \int_0^\infty \frac{dA}{A^2} \int_{\gamma_{min}}^{\gamma_T} d\gamma \left[1 - W_s(A, \gamma)\right]. \quad (31)$$

Note that $g_f$ has to be replaced with $Ag_{fm}$ in the expressions for $W_s$ in the many EF case, because of differences in notation.

To estimate the amount of dephasing let us take into account that the asymptotic expressions for $W_s$ in the many EF case, because of differences in notation.

$$\gamma \bar{\nu}_s \approx (A\bar{g}_c)^2 \eta_k/2\gamma, \quad (32)$$

where $\bar{\nu}_s$ differs from $\nu_s$ defined in Eq. (20) by the replacement $g_f \rightarrow Ag_{fm}$ in the differential equation (18), while

$$\bar{g}_c = \frac{1}{2\zeta} \int_{A_c}^{\phi_B} g_{fm}(\phi), \quad \eta_k = \frac{1}{2\zeta^2} \int_{A_c}^{\phi_B} g_{fm}^2(\phi) d\phi. \quad (33)$$

Note that $\bar{\nu}_s$ is a function of $A$, $\bar{\nu}_s(A)$. Consequently, Eq. (32) should be treated as an equation to determine the characteristic value of the coupling $A$. Defining the solution of Eq. (32) as $A_\zeta$ and splitting the integration over $A$ in Eq. (31) as

$$\int_{\gamma_{min}}^{\gamma_T} d\gamma \left(\int_0^{A_c} + \int_{A_c}^{\infty}\right) \frac{dA}{A^2} \left[1 - W_s(A, \gamma)\right]$$

we can use the expression (23) in the first interval and the expression (21) in the second one. Since both integrals are determined by $A_c$ we arrive at the estimate

$$-\ln W \approx 3\nu_d \bar{g}_c f_{\xi}, \quad (35)$$

where $f_{\xi} \equiv \bar{g}_c \sqrt{\eta_k \bar{\nu}_s(A_c)}$ is some function of $\zeta$, rather smooth if $D$ is not close to $1$. We see that the interference pattern decays exponentially with an increasing distance, $2\zeta$, between the resonances. Generally the $\zeta$-dependence of $f_{\xi}$ can be calculated numerically for a given transparency $D$ using the analytical expressions (20) and (32). For a constant $g_{fm}$, $f_{\xi} = g_{fm}$. We do not analyze here the function $f_{\xi}$ in detail.

A. Non-optimal EFs

In the previous sections it has been assumed that the system size is infinite. A consequence of this assumption is that, independent of temperature, the EFs which have the strongest effect on the junction transparency will always be included in the estimates. From the method used to obtain the general estimate for many EFs, Eq. (33), one can conclude that the EFs which have the most effective coupling fulfill $A = A_\zeta$, this corresponds to a spatial distance $r_s = (A_0/A_\zeta)^{1/3} \approx (A_0\bar{g}_c/\gamma)^{1/3}$. A further point is that the rate $\gamma$ is confined to the interval between $\gamma_{min}$ and $\gamma_T$. Thus we have actually assumed that the size of the region where EFs reside has a size greater than $r_{max} = (A_0\bar{g}_c/\gamma_{min})^{1/3}$, and that there is no “excluded region” without EFs near the contact with the size less that $r_{min} = (A_0\bar{g}_c/\gamma_T)^{1/3}$.

What happens if this “optimum” EF is out of the range? This can occur if the system is limited in size, or if there is a specifically pure region around the contact.

1. Role of finite size of the sample

Let us first discuss the role of a finite size, $R$, of the region containing EFs. If $R \lesssim r_{min} = (A_0/A_\zeta)^{1/3}$ then all EFs will act as “slow” ones. To estimate $\ln W$ in this case one can use the expression $W_s = \cos \Phi_s$ for the whole integration region over $A$ in Eq. (21), arriving at

$$-\ln W = 2\nu_d \bar{g}_c\zeta F(2A_0\bar{g}_c\zeta/R^3). \quad (36)$$

Here

$$F(x) = \int_x^\infty \frac{1 - \cos x}{x^2} dx, \quad (37)$$

which is a decreasing function of its argument. This expression (37) is valid if its right-hand side is less the right-hand side of Eq. (35).

At intermediate values of $R$,

$$(A_0\bar{g}_c/\gamma_{min})^{1/3} \gg R \gg (A_0\bar{g}_c/\gamma_T)^{1/3},$$

as in the previous case, only the second integral in the expression (34) does exist, lowest limit should also be replaced by $A_0\bar{g}_c/R^2$. However, the approximation $W_s = \cos \Phi_s$ is not valid any more. Using the approximation (21) one can obtain

$$W = \exp \left[-\nu_d f_{\xi} \frac{\ln(\gamma_T R^3/A_0\bar{g}_c)}{\ln(\gamma_T/\gamma_{min})}\right]. \quad (38)$$

This means that when the size of system is limited the amount of dephasing can be less than estimated for an infinite system.
2. Role of the spacer

Let us now discuss the role of a “pure” region (spacer) near the SQPC where there are no EFs. If a typical diameter $r_0$ of such region is large enough, such that $r_0 > r_{\text{max}} = (A_0 \xi_\gamma / \gamma_{\text{min}})^{1/3}$, then all EFs act as “fast” ones.

The single EF solution, $W_s$, in this limit is given by Eq. (23) in Sect. III. After calculating the average in Eq. (29) we arrive at

$$- \ln W \approx (\nu_d \bar{\zeta} / L) \sqrt{4\pi \zeta \xi / \gamma_{\text{min}}}.$$  

(39)

It is difficult to estimate the actual amount of dephasing. We have to restrict our conclusions to interpreting how the amount of dephasing will change depending on the parameters $\gamma_T$, $\gamma_{\text{min}}$, $\zeta$ and $T$. We already know that dephasing will increase with temperature. Generally one can also say that dephasing will increase with $\zeta$. However, at large enough temperatures when $\gamma_{\text{min}}$ appears large enough, the dephasing will decrease. The free parameter of the theory, $A_0$, can be estimated only roughly through comparison with the noise measurements in the normal state. To map the parameter $A_0$ to the noise let us employ the theory of flicker noise in a QPC to the case of a single mode contact. According to that theory, results for the noise intensity $S(\tau)$ are substantially dependent on the relationship between the maximal and minimal distances between the EFs and the QPC. The simplest case, which is quite realistic, is when these distances are of the same order of magnitude. When $\Gamma_T^{-1} \ll |\tau| \ll \Gamma_{\text{min}}^{-1}$, the noise intensity can be expressed as, cf. with Ref. [4],[3]

$$S(\tau) \approx \left( \frac{2e^2 V}{h} \right)^2 \left( \frac{4\pi P_0 k T A_0}{3} \right)^2 \ln(1/\gamma_{\text{min}}) / \ln(\Gamma_T / \Gamma_{\text{min}}).$$  

(40)

By obtaining estimates for $\Gamma_T / \gamma_{\text{min}}$ from noise spectra in the normal state one can, in principle, estimate the coupling parameter $A_0$. A key point is to make measurements of both the MAQI interference pattern and the normal-state noise spectra in a rather large frequency range. This combination does not look too simple.

V. CONCLUSIONS

We have presented a method for investigating the influence of flicker noise in the junction transparency of a SQPC on coherent Andreev states. This is done by estimating the effect of these fluctuations on the so-called microwave-activated quantum interferometer (MAQI).

For a small contact when only a single EF is in range to affect the junction transparency there can be either a distortion or a decay of the MAQI interference pattern. A distortion appears for very slow switching rates of the EF and a weak decay (dephasing) for fast rates. It is possible to confirm our model in the fast switching limit experimentally. The only unknown parameter is the switching rate $\gamma$ which can be found by measuring the telegraph noise of the contact in the normal state. This is best done by driving the system into the normal state with a magnetic field, since $\gamma$ is temperature dependent. When $\gamma$ is known it is then possible to calculate the amount of dephasing and compare with experimental results.

In the limit when the influence of many EFs has to be considered we have arrived at more general results. It is not possible to make any exact predictions since the distribution and coupling strength of the EFs are sample dependent. However, our calculations show that in the presence of many EFs there will always be an exponential decay of the MAQI interference pattern. The strength of this dephasing should be about the same for all switching rates. One exception is when there is an impurity-free region near the SQPC, in which case dephasing will decrease for higher rates.

Finally, we note that this paper together with work in Ref. [4] presents a framework which can be used to investigate the coupling of a SQPC to its electromagnetic environment.

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APPENDIX A: DERIVATION OF EQUATION FOR $W$

In this appendix the differential equation, Eq. (4) for $W$ is derived. The following is valid for the random telegraph process $\xi(t)$,

$$\langle \xi(\phi_1)\xi(\phi_2)\ldots\xi(\phi_n) \rangle = e^{-2\gamma|\phi_2-\phi_1|}\langle \xi(\phi_1)\xi(\phi_4)\ldots\xi(\phi_n) \rangle,$$

(A1)

when $\phi_1 \geq \phi_2 \geq \ldots \geq \phi_n$. Further, all averages with an odd number of $\xi$s are equal to zero. This follows from the assumption of equal probability for both EF states.

To facilitate the calculation of $W$, the auxiliary function

$$\Psi(\phi) = \exp \left[ i \int_{\phi_A}^{\phi} d\phi' g_{\text{eff}}(\phi') \xi(\phi') \right],$$

(A2)

is defined. By expanding the auxiliary function in a Taylor series and utilizing the relation in Eq. (A1) above, it is possible to rewrite the auxiliary function as
\[ \Psi(\phi) = 1 + i^2 \int_{\phi_1}^{\phi} d\phi_1 \int_{\phi_A}^{\phi_1} d\phi_2 g_{fs}(\phi_1) g_{fs}(\phi_2) e^{-2|\phi_2 - \phi_1|} \Psi(\phi_2). \] (A3)

A derivation with respect to \( \phi \) leads to the following integro-differential equation

\[ \frac{d\Psi}{d\phi} = -g_{fs}(\phi) \int_{\phi_1}^{\phi} d\phi_1 g_{fs}(\phi') e^{-2|\phi' - \phi|} \Psi(\phi'), \] (A4)

and a second derivation provides the following differential equation

\[ \frac{d^2\Psi}{d\phi^2} + \left[ 2\gamma - \frac{2 \ln g_{fs}(\phi)}{d\phi} \right] \frac{d\Psi}{d\phi} + g_{fs}(\phi) \Psi = 0, \] (A5)

with the initial conditions

\[ \Psi(\phi_A) = 1, \quad d\Psi/d\phi|_{\phi=\phi_A} = 0. \] (A6)

This equation makes it possible to find approximate analytical solutions for \( W \) in the single EF case.

**APPENDIX B: DISTRIBUTIONS, \( P_\gamma(\gamma) \) AND \( P_A(A) \)**

Here we outline the derivation for the distribution functions \( P(\gamma) \) and \( P(A) \) which are necessary when averaging in the many EF case. Let us begin with \( P(A) \), where \( A \) is the coupling strength between an EF and the contact. By assuming that an EF can be modeled with a dipole field, we can state that \( A \propto A_0/r^3 \) where \( r \) is the distance from the EF. The normalized distribution is then found through the following integral, \( P(A) = V^{-1} \int dV \delta(A - A_0/r^3) \) as

\[ P(A) = (4\pi/3V) (A_0/A^3). \] (B1)

To find the distribution of \( \gamma \) a closer look at the structure of the two level system (TLS) which is assumed for the EFs is necessary. The Hamiltonian for a TLS can be \( H = \Delta \sigma_2 - \Lambda \sigma_x \), where \( \Delta \) is the energy difference between the minima of the two states and \( \Lambda \) is the tunneling coupling between the two states. After diagonalization the excitation energy is found to be \( U = \sqrt{\Delta^2 + \Lambda^2} \). By assuming that the tunneling coupling decays exponentially we have that \( P(A) \propto 1/A \). Further, the hopping rate \( \gamma \) depends on \((\Lambda/U)^2\) and a term \( U^3/hE_c^2 \) where the latter term comes from assuming that hopping is phonon mediated (\( E_c \) is the parameter characterizing the coupling energy between the phonons and the EFs). Within these assumptions \( P(\gamma) \) is given by,

\[ P_\gamma(\gamma) = \int d\Delta d\Lambda P(\Lambda) \delta(U^2 - \Delta^2 - \Lambda^2) \times \delta \left( \gamma - \frac{\Lambda^2 U^3}{U^2 hE_c^2} \right) \propto \frac{1}{\gamma}. \] (B2)

After normalization we have,

\[ P(\gamma) = \frac{1}{L \gamma}, \] (B3)

where \( L = \ln(\gamma_T/\gamma_{\min}) \) with \( \gamma_{T,\min} \) being limiting values for the EF switching rate \( \gamma \).

In the general case \( P = P(U, \gamma, A) \) and the number of EFs will be given by \( N = \int dU d\gamma dA P(U, \gamma, A) \). By assuming a constant distribution for \( U \), we label \( P_0 \), we arrive at the number of EFs as \( N = R_0 V kT \).

The final distribution function for \( A \) and \( \gamma \) is then

\[ P(A, \gamma) = \frac{4\pi R_0 kT A_0}{3} \frac{1}{A^2 L \gamma}. \] (B4)

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