Holographic compact stars meet gravitational wave constraints

Eemeli Annala,∗ Christian Ecker,† Carlos Hoyos,‡ Niko Jokela,§
David Rodríguez Fernández,¶ and Aleksi Vuorinen∗∗

1Department of Physics and Helsinki Institute of Physics
P.O. Box 64, FI-00014 University of Helsinki, Finland
2Institut für Theoretische Physik, Technische Universität Wien
Wiedner Hauptstr. 8-10, A-1040 Vienna, Austria
3Department of Physics, Universidad de Oviedo
Avda. Calvo Sotelo 18, ES-33007 Oviedo, Spain
4Institute for Theoretical Physics and Astrophysics, University of Würzburg
97074 Würzburg, Germany

We investigate a simple holographic model for cold and dense deconfined QCD matter consisting of three quark flavors. Varying the single free parameter of the model and utilizing a Chiral Effective Theory equation of state (EoS) for nuclear matter, we find four different compact star solutions: traditional neutron stars, strange quark stars, as well as two non-standard solutions we refer to as hybrid stars of the second and third kind (HS2 and HS3). The HS2s are composed of a nuclear matter core and a crust made of stable strange quark matter, while the HS3s have both a quark mantle and a nuclear crust on top of a nuclear matter core. For all types of stars constructed, we determine not only their mass-radius relations, but also tidal deformabilities, Love numbers, as well as moments of inertia and the mass distribution. We find that there exists a range of parameter values in our model, for which the novel hybrid stars have properties in very good agreement with all existing bounds on the bulk properties of compact stars. In particular, the tidal deformabilities of these solutions are smaller than those of ordinary neutron stars of the same mass, implying that they provide an excellent fit to the recent gravitational wave data GW170817 of LIGO and Virgo. The assumptions underlying the viability of the different star types, in particular those corresponding to absolutely stable quark matter, are finally discussed at some length.

PACS numbers: 21.65.Qr, 26.60.Kp, 11.25.Tq
Keywords: Neutron Star, Quark Matter, Gauge/Gravity Duality

I. INTRODUCTION

The nature and properties of compact stars is a topic of active research both on the observational and theoretical sides [1]. The standard picture is that all stars with densities comparable to the nuclear matter saturation density $n_s$ are neutron stars (NS), composed of hadronic matter of increasing density, or hybrid stars (HS) that in addition contain deconfined quark matter in their inner cores. This scenario is based on the assumption of nuclear matter being absolutely stable in vacuum, i.e., that it has a lower energy per baryon ratio at zero pressure than quark matter. Albeit a highly plausible assumption — after all, we know from observations that at least most of the compact stars detected so far appear to have masses and radii in the range predicted for NSs — the case for stable three-flavor quark matter and quark stars (QS) has not been settled yet [2–6]. In particular, a scenario with two separate families of compact stars with different mass-radius ($M–R$) branches remains viable [7–10]. Inherent in these proposals is the assumption that finite-size effects resolve problems related to, e.g., unobserved quark matter halos being formed around atomic nuclei.

On the theory side, the difficulty in excluding the existence of absolutely stable quark matter is related to the
fact that no robust first principles tools exist for studying this phase of Quantum ChromoDynamics (QCD) at moderate energy densities \[\text{[11]}\]. With the Sign Problem impeding lattice studies \[\text{[12]}\], and weak coupling methods being restricted to the high-density regime \[\text{[13, 14]}\], the options that remain include investigating simplified models of QCD (see, e.g., \[\text{[17]}\]) and deforming the theory to allow for a nonperturbative solution even at strong coupling. A prime example of the latter approach is naturally the gauge/gravity duality \[\text{[18–20]}\], which allows the description of strongly coupled theories with flavor \[\text{[21]}\].

Recently, the gauge/gravity duality was applied to the dense QCD matter inside compact stars in \[\text{[22]}\], where a holographic equation of state (EoS) for quark matter was combined with state-of-the-art nuclear theory results from Chiral Effective Theory (CET) to construct a set of NS matter EoSs. The holographic result was seen to contain exactly one free parameter, \(m_0\), corresponding to the three equal (constituent) quark masses, whose value was somewhat arbitrarily fixed to make the quark matter pressure vanish at the same baryon chemical potential as that of nuclear matter. This resulted in a strong first order deconfinement transition and the conclusion that the stars become unstable as soon as holographic quark matter begins to form inside their cores, so that no holographic HSs exist. It should, however, be noted that this approach neglected certain relevant physical effects, such as the differing bare masses of the quark flavors as well as quark pairing \[\text{[23, 24]}\], which has recently been approached using holography in \[\text{[26]}\].

In the paper at hand, we revisit the construction of compact stars by combining the “medium stiffness” CET EoS of \[\text{[27]}\] with the holographic quark matter EoS considered in \[\text{[22]}\], but this time relaxing the ad hoc assumption concerning the parameter \(m_0\). This is seen to lead to a rich phenomenology, with the variation of \(m_0\) generating four distinct types of compact stars, depicted in Fig. [1]. These include i) ordinary NSs, analogous to those constructed in \[\text{[22]}\]; ii) pure QSs, composed of absolutely stable quark matter; and iii) and iv) hybrid stars of the second and third kind (HS2, HS3), containing a nuclear matter core and either a quark matter crust (HS2), or a quark mantle and a nuclear matter crust (HS3). The viability of the solutions ii) and iii) is clearly subject to the assumptions about stable quark matter discussed above.

Inspecting the macroscopic properties of our compact stars, including their mass-radius relations, tidal deformabilities, as well as moments of inertia and mass distribution, we find that for a range of values of \(m_0\) our novel hybrid stars exhibit properties in excellent agreement with all known observational and theoretical bounds. Perhaps most interestingly, studying the tidal deformability of a 1.4 solar mass (\(M_\odot\)) star as a function of \(m_0\), we find the quantity to be minimized not by ordinary NSs, but by an HS2 solution with a quark crust. As we shall explain below, this implies that our hybrid stars are in excellent agreement with the recent gravitational wave observation GW170817 of LIGO and Virgo \[\text{[25]}\].

\section*{II. HOLOGRAPHIC MODEL AND SETUP}

The model we choose to describe quark matter with is based on \(\mathcal{N} = 4\) SU(\(N_c\)) supersymmetric Yang-Mills (SYM) theory with \(N_f = 3\) fundamental \(\mathcal{N} = 2\) matter hypermultiplets that we treat in the quenched approximation and identify as the flavor fields. To mimic finite quark density, we turn on a chemical potential for a \(U(1)\) component of the global \(U(\mathcal{N}_f) \sim U(1)_B \times SU(N_f)\) flavor symmetry of the theory. For simplicity, we set both the quark masses and chemical potentials to be equal, \(\mu_q \equiv \mu_B / N_c\), in which case the system is automatically both charge neutral and in beta equilibrium.

The thermodynamics of the above model has been extensively studied in a number of previous works \[\text{[29, 41]}\]. The free energy naturally splits into two contributions,

\[ F = F_{\mathcal{N}=4} + F_{\text{flavor}}, \]

where only the latter part depends on the quark density. Being primarily interested in quiescent compact stars, we set the temperature to zero, in which case we can safely ignore the \(\mathcal{N} = 4\) part above, while the flavor part takes the simple analytic form \[\text{[32, 41, 42]}\],

\[ F_{\text{flavor}} = -f_0(\mu_q^2 - m_0^2)^2. \]

Here, we have defined \(f_0 = \frac{N_c N_f}{4 \gamma \lambda_{YM}}\) with \(\gamma = \Gamma(7/6)\Gamma(1/3)/\sqrt{\pi}\), where \(m_0\) denotes the quark mass \[\text{[30]}\]. The pressure \(p\) and the energy density \(\varepsilon\) are further determined from Eq. \[\text{[2]}\] as \(p = -F_{\text{flavor}}, \varepsilon = \mu_q \frac{\partial p}{\partial \mu_q} - p\), which together lead to the EoS

\[ \varepsilon = 3p + 4m_0^2 \sqrt{f_0} \sqrt{\pi}. \]

It is worth noting that an EoS of this form can also be obtained as a special case of the MIT bag model EoS \[\text{[43]}\].

In our earlier work \[\text{[22]}\], we fixed the parameters of the above model to match the perturbative high-density limit of QCD, setting \(N_c = 3\) and \(\lambda_{YM} = \frac{3\pi}{16}\) \(\approx 10.74\), while corrections entering with inverse powers of \(N_c\) and \(\lambda_{YM}\) were altogether neglected. In the present paper, we follow the same conventions, but let the parameter \(m_0\) vary around the scale 310 MeV, where the nuclear matter pressure, chosen to follow the “medium stiffness” EoS of \[\text{[27]}\], vanishes. This EoS corresponds to charge neutral beta-equilibrated matter and follows the CET result of \[\text{[44]}\] up to 1.1\(M_\odot\), thereafter extrapolating it with an observationally constrained piecewise polytropic form.

\section*{III. COMPACT STAR SOLUTIONS}

As already noted, we construct NS matter EoSs by combining the medium stiffness nuclear matter EoS of \[\text{[27]}\] with a quark matter EoS obtained from the holographic model introduced in the previous section. At each value of the quark chemical potential, the phase
that is realized is taken to be the one with lower free energy, or larger pressure, so that potentially there can be even multiple phase transitions inside the star. These will generically be of first order, with a latent heat that can be determined from the difference of the energy densities of the two phases at the transition.

In Fig. 2, we show the pressure of the nuclear matter phase together with that of the holographic one, giving \( m_0 \) the values 260, 280, 300, 312, and 320 MeV, following the choices made in Fig. 1. These numbers have been chosen so that the cases displayed represent all of the four distinct scenarios we discover:

1. For \( m_0 \gtrsim 313.1 \) MeV, the nuclear matter pressure is dominant at low densities, but the quark matter phase takes over at a first order transition at some higher density, or chemical potential.

2. For \( 310.0 \) MeV \( \lesssim m_0 \lesssim 313.1 \) MeV, nuclear matter is still dominant at the lowest densities and quark matter at the highest, but between these regions there are not one but three first order transitions, so that counting from the lowest to the highest density, the phases of QCD matter are nuclear, quark, nuclear, and again quark matter.

3. For \( 261.4 \) MeV \( \lesssim m_0 \lesssim 310.0 \) MeV, quark matter turns out to be favored both at the lowest and highest densities (i.e., it is stable in vacuum), but at moderate densities there exists a density interval where the nuclear matter pressure is larger.

4. For \( m_0 \lesssim 261.4 \) MeV, the pressure of quark matter is larger at all densities.

The second and third of these scenarios are clearly non-standard. Upon closer inspection, their existence can be traced back to the functional form of our holographic EoS for quark matter, Eq. (3), which is seen to closely resemble that of the nuclear matter phase at low densities.

To study the properties of the compact stars built from the above EoSs — and to verify the claims made in the first section — we next proceed to solve the Tolman-Oppenheimer-Volkov equations that govern relativistic hydrostatic equilibrium inside the stars [45]. For each EoS, we not only solve the possible values of the stellar masses and radii, but in addition determine the stability of the configurations against infinitesimal adiabatic radial oscillations [46, 47], assuming the transitions to be fast [81]. The result of this exercise is shown in Fig. 3, where \( M-R \) curves corresponding to the five example EoSs of Fig. 2 are displayed.

Following the numbering of \( m_0 \) intervals introduced above, we now find:

1. For \( m_0 \gtrsim 313.1 \) MeV, the stars are always ordinary NSs, obeying an \( M-R \) relation fully determined by the results of [27]. Quark cores are excluded by stability arguments due to a strong first order deconfinement transition (cf. the discussion in [22]).

2. For \( 310.2 \) MeV \( \lesssim m_0 \lesssim 313.1 \) MeV, the stars are always of type HS3.

3. For \( 264.4 \) MeV \( \lesssim m_0 \lesssim 310.2 \) MeV, two stable solutions exist: QSs at large and HS2s at small radii.

4. For \( m_0 \lesssim 264.4 \) MeV, all the stars are QSs.

Of particular interest here are clearly those HS2s and HS3s, for which \( m_0 \) is only slightly below the critical value of 313.1 MeV. Zooming into these solutions, we observe the \( M-R \) relations to smoothly flow to that of the NSs, just as expected. It is interesting to note that qualitatively similar solutions have been found earlier based
on an MIT bag model EoS supplemented by a contribution from quark pairing [48, 49] (see also [50]).

Finally, note that shown in Fig. 3 are also a number of black dot-dashed curves, which correspond to analytic expansions of QS solutions, derived in the limit of small compactness $C = GM/(c^2 R)$ [82]. These solutions read

$$M \simeq \frac{M_0}{c_0} \left[ \frac{R_0 - R}{R_0} - \frac{c_1}{c_0} \left( \frac{R - R_0}{R_0} \right)^2 + \cdots \right], \quad (4)$$

where $c_0 \simeq 1.853$, $c_1 \simeq 2.948$, $R_0 = \pi/k$, $M_0 = c^2 R_0/G$, and $k^2 = 32\pi f_0 m_0^2 G/c^4$.

### IV. LIGO CONSTRAINTS AND UNIVERSAL RELATIONS

It has been suggested long ago that in a coalescing binary system of two NSs, or a black hole and a NS, the tidal forces between the two objects affect the gravitational wave signal in a way that can be measured using Earth-based gravitational wave detectors [51–59]. In the fall of 2017, such an effect was indeed observed by LIGO and Virgo in their analysis of gravitational wave data that very likely had their origins in the merger of two NSs [25] (see also the analyses of [43, 60, 70]). The measurement was reported in the form of a limit given for the tidal deformabilities of the two stars involved in the merger. This quantity is related to the Love numbers of the stars and measures their susceptibility to tidal forces that deform their shape. Importantly, both quantities are highly sensitive to the EoS of stellar matter, and it is of great interest to compute them for different candidate EoSs, including the ones introduced in our work.

Another reason to be interested in Love numbers is that they allow the verification of so-called universal relations, i.e., suggested correlations between different quantities characterizing compact stars that appear to be largely insensitive to the EoS of stellar matter. These relations, due to Yagi and Yunes [71], concern dimensionless ratios of the moment of inertia, the quadrupolar moment of the mass distribution, and the electric Love number of compact stars,

$$\bar{I} = \frac{c^4}{G^2 M^3} I, \quad \bar{Q} = -\frac{M}{T^2} \frac{Q}{\Omega^2/c^2}, \quad \bar{\lambda} = \frac{2}{3C^5} k_{2M}^4, \quad (5)$$

where $\Omega$ is the angular velocity and $C$ the compactness of the star. It is clearly worthwhile to check, whether these relations hold for our family of EoSs as well.

Given a specific EoS, the determination of Love numbers involves perturbing the metric of a spherically symmetric (non-rotating) star with a quadrupolar deformation [72, 73]. At the same time, to obtain the quantities $I$ and $Q$ requires considering stars rotating with a small angular velocity $\Omega$. The moment of inertia $I$ is then obtained from the ratio of the angular momentum and the angular velocity, while for $Q$ we must first determine the mass distribution inside the rotating star and then compute its second moment [74].

Beginning from the tidal deformability, we note that LIGO and Virgo provide the constraint $\lambda(1.4M_\odot) \leq 800$ for the likely case of slowly rotating stars (the low-spin prior) at a $90\%$ Bayesian probability level [28]. In addition to this, Fig. 5 of this reference gives both $90\%$ and $50\%$ probability contours for the independent tidal deformabilities of the two stars on a $\lambda_1$-$\lambda_2$ plane. To compare our results to these values, we first show in Fig. 4 how our example EoSs from Fig. 2 relate to these contours. Here, the curves have been generated by independently determining the tidal deformabilities for both stars involved in the merger, obtaining the possible mass pairs using the chirp mass of the event, $M = 1.188M_\odot$. Interestingly, the smallest deformabilities are obtained not for ordinary NSs but for the HS2s and HS3s.

To further inspect the rather surprising results observed, we next show in Fig. 3 the tidal deformability value of a star of mass $1.4M_\odot$ as a function of $m_0$. Indeed, we verify from here that the quantity is minimized around $m_0 = 304$ MeV, i.e., for HS2s with a ca. 2 km thick quark crust (cf. the inset of the figure). It is worth noting that the minimal value of $\lambda(1.4M_\odot) = 301$ is markedly smaller than that obtained for the NS solutions, $\lambda(1.4M_\odot) = 471$. This is very interesting to constrain with the recent claim of a lower bound existing for this quantity [68].

Moving next on to the universal relations, we have
quantitatively checked the relative accuracy, to which the I-Love-Q relations of Yagi and Yunes [73], concerning the correlations $\lambda - I$, $Q - I$, and $\lambda - Q$, are reproduced by our compact stars corresponding to different $m_0$ values. Inspecting the results, we find that the deviation from the universal limit is largest for the novel hybrid stars with relatively thick quark crusts, but quickly diminish as $m_0$ tends towards the critical value of 313.1 MeV. Although the deviations are never larger than 15%, this finding may suggest a way of distinguishing the HS2 and HS3 stars from NSs. Finally, we note that in the limit of small compactness, it is again possible to derive analytic results for the $\bar{I}$, $\bar{Q}$, and $\bar{\lambda}$ values of the QSs,

$$\bar{I} \simeq \frac{2}{3} \frac{\pi^2}{\pi^2 - 6} \frac{1}{C^2} \simeq 0.261 C^{-2}$$
$$\bar{Q} \simeq -\frac{32\pi^4}{9} \frac{(15 - \pi^2)}{24\pi (\pi^2 - 3)} C \simeq -30.35 C$$
$$\bar{\lambda} \simeq 0.173 C^{-5}.$$

V. CONCLUSIONS

As of today, holography remains the only first principles quantum field theory method capable of describing dense deconfined matter in a region where it is strongly coupled. While the list of theories with known gravity duals does not contain QCD yet, the strongly coupled regime of this theory covers practically all energies of phenomenological interest, including in particular the densities realized inside compact stars. Short of altogether circumventing the need to inspect the problematic density range inside NSs by interpolating between trusted low- and high-density EoSs [67] [75], it is thus advisable to seek insights from novel directions, including theories whose strong-coupling limits can be reliably investigated using the gauge/gravity duality.

In the paper at hand, we have approached the description of moderate-density quark matter by studying a supersymmetric cousin of QCD. Working in the large-$N_c$ and strong coupling limits of this theory, and boldly extrapolating our results to the physical three-color case, we derived a family of quark matter EoSs parameterized by exactly one free parameter, the mass of our three quark flavors, $m_0$. The result was then combined with the current best guess for the EoS of beta-equilibrated nuclear matter [27], and the outcome of the matching carefully applied to the construction of compact stars.

The results from the above exercise did not conform with the naive expectation of obtaining either pure quark or neutron stars. Instead, we found that for a range of values of $m_0$, the form of our quark matter EoS bears a striking similarity to that of the hadronic phase [27], giving rise to highly non-standard types of compact star solutions, reminiscent of what has been proposed earlier in [18] [49]. We dubbed these solutions hybrid stars of the second and third kind, HS2 and HS3, which were seen to contain layers of quark (HS2) or quark and nuclear (HS3) matter on top of a nuclear core. We stress that while the quantitative details of these solutions are naturally sensitive to the fine details of the nuclear matter EoS employed, the existence of the novel solutions is a robust consequence of the form of our holographic EoS, Eq. (6).

Taking the constructed set of EoSs at face value, it is interesting to study, whether some values of $m_0$ can be firmly ruled out by compact star measurements, assuming that all stellar observations correspond to the same branch of compact stars. To this end, in Secs. III and IV we embarked on a careful analysis of the properties of our compact stars, discovering a particularly interesting range of $m_0$, for which the hybrid stars display both $M - R$ relations and tidal deformabilities in good agreement with available observational data. In particular, we found the tidal deformability of a $1.4 M_\odot$ star to be minimized not by ordinary NSs but by an HS2 solution, exhibiting a ca. 2 km quark crust. The deformability values obtained for these stars were seen to be well within the 50% probability contours provided by LIGO and Virgo in their analysis of the gravitational wave data GW170817 [28].

A limitation of our approach is clearly that we are restricted to describing quark matter in the bulk, implying that the viability of the QS and HS2 scenarios, exhibiting absolutely stable strange quark matter, is subject to non-trivial assumptions concerning finite-size effects. In addition, in the present work we have only concentrated on the macroscopic properties of compact stars, leaving the study of, e.g., transport phenomena for the future. To gauge the robustness of our findings, it will finally be very interesting to repeat our analysis with a variety of nuclear matter EoSs as well as with holographic models exhibiting stiffer EoSs [70] [28].

Figure 5: The tidal deformability of a (lower-radius) $1.4 M_\odot$ star as a function of $m_0$. Shown here are also horizontal lines denoting the values $\lambda(1.4 M_\odot) = 800$ and 400, corresponding roughly to the 90% and 50% probability limits of LIGO and Virgo (cf. discussion in [67]). The cusp in the curve around $m_0 = 307$ MeV is due to the matched EoS becoming sensitive a small discontinuity in the hadronic EoS of [27]. Inset: internal structure of hybrid stars of mass $1.4 M_\odot$, with the orange (black) color again representing quark (nuclear) matter.
Acknowledgments

We are grateful to David Blaschke for helpful comments as well as for pointing out relevant existing literature, and to Kent Yagi for his advice concerning the analysis of the I-Love-Q relations. In addition, we thank Paul Chesler, Aleksi Kurkela, James Lattimer, and Joonas Nättilä for useful discussions. E.A. has been supported by the Finnish Cultural Foundation, and N.J. and A.V. by the Academy of Finland, grants no. 273545 and 1268023, as well as by the European Research Council, grant no. 725369. C.H. and D.R.F. have been supported by the Spanish grant MINECO-16-FPA2015-63667-P, and C.H. in addition by the Ramon y Cajal fellowship RYC-2012-10370 and D.R.F. by the GRUPIN 14-108 research grant from Principado de Asturias and by the C09 of SFB 1170 research grant by the DFG. C.E. has finally been supported by the Austrian Science Fund (FWF), projects no. P27182-N27 and DKW1252-N27.

[1] J. M. Lattimer and M. Prakash, Science 304, 536 (2004), arXiv:astro-ph/0405262 [astro-ph].
[2] N. Itoh, Prog. Theor. Phys. 44, 291 (1970).
[3] A. R. Bodmer, Phys. Rev. D4, 1601 (1971).
[4] H. Terazawa, in 2nd KEK Symposium on Radiation Dosimetry, Tsukuba, Japan, Mar 22-23, 1979 Tsukuba, Japan, March 22-23, 1979.
[5] E. Farhi and R. L. Jaffe, Phys. Rev. D30, 2379 (1984).
[6] E. Witten, Phys. Rev. D30, 372 (1984).
[7] E. S. Fraga, R. D. Pisarski, and J. Schaffner-Bielich, Phys. Rev. D63, 121702 (2001), arXiv:hep-ph/0101143 [hep-ph].
[8] F. Weber, Prog. Part. Nucl. Phys. 54, 193 (2005), arXiv:astro-ph/0407155 [astro-ph].
[9] S. Postnikov, M. Prakash, and J. M. Lattimer, Phys. Rev. D82, 024016 (2010), arXiv:1004.5098 [astro-ph.SR].
[10] A. Drago, A. Lavagno, and G. Pagliara, Phys. Rev. D98, 043014 (2014), arXiv:1309.7263 [nucl-th].
[11] N. Brambilla et al., Eur. Phys. J. C74, 2981 (2014), arXiv:1404.3723 [hep-ph].
[12] P. de Forcrand, PoS LAT2009, 010 (2009), arXiv:1005.0539 [hep-lat].
[13] B. A. Freedman and L. D. McLerran, Phys. Rev. D16, 1169 (1977).
[14] A. Vuorinen, Phys. Rev. D68, 054017 (2003), arXiv:hep-ph/0305183 [hep-ph].
[15] A. Kurkela, P. Romatschke, and A. Vuorinen, Phys. Rev. D81, 105021 (2010), arXiv:0912.1856 [hep-ph].
[16] A. Kurkela and A. Vuorinen, Phys. Rev. Lett. 117, 042501 (2016), arXiv:1603.00750 [hep-ph].
[17] M. Bülbül, Phys. Rept. 407, 205 (2005), arXiv:hep-ph/0402234 [hep-ph].
[18] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998), arXiv:9711200 [hep-th].
[19] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B428, 105 (1998), arXiv:hep-th/9802109 [hep-th].
[20] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998), arXiv:hep-th/9802150 [hep-th].
[21] A. Karch and E. Katz, JHEP 06, 043 (2002), arXiv:hep-th/0205236 [hep-th].
[22] C. Hoyos, D. Rodriguez Fernandez, N. Jokela, and A. Vuorinen, Phys. Rev. Lett. 117, 032501 (2016), arXiv:1603.02943 [hep-ph].
[23] M. G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B422, 247 (1998), arXiv:hep-ph/9711395 [hep-ph].
[24] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. B537, 443 (1999), arXiv:hep-ph/9804403 [hep-ph].
[25] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schifer, Rev. Mod. Phys. 80, 1455 (2008), arXiv:0709.4635 [hep-ph].
[26] A. F. Faedo, D. Mateos, C. Pantelidou, and J. Tarrio, JHEP 10, 139 (2017), arXiv:1707.06989 [hep-th].
[27] K. Hebeler, J. M. Lattimer, C. J. Pethick, and A. Schwenk, Astrophys. J. 773, 11 (2013), arXiv:1303.4662 [astro-ph.SR].
[28] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 119, 161101 (2017), arXiv:1710.05832 [gr-qc].
[29] D. Mateos, R. C. Myers, and R. M. Thomson, Phys. Rev. Lett. 97, 091601 (2006), arXiv:hep-th/0605046 [hep-th].
[30] S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers, and R. M. Thomson, JHEP 02, 016 (2007), arXiv:hep-th/0611099 [hep-th].
[31] D. Mateos, R. C. Myers, and R. M. Thomson, JHEP 05, 067 (2007), arXiv:hep-th/0701132 [hep-th].
[32] A. Karch and A. O’Bannon, JHEP 11, 074 (2007), arXiv:0709.0570 [hep-th].
[33] D. Mateos, S. Matsuura, R. C. Myers, and R. M. Thomson, JHEP 11, 085 (2007), arXiv:0709.1225 [hep-th].
[34] J. Erdmenger, M. Kaminski, P. Kerner, and F. Rust, JHEP 11, 031 (2008), arXiv:0807.2663 [hep-th].
[35] M. Ammon, J. Erdmenger, M. Kaminski, and P. Kerner, Phys. Lett. B680, 516 (2009), arXiv:0810.2316 [hep-th].
[36] F. Basu, J. He, A. Mukherjee, and H.-H. Shieh, JHEP 11, 070 (2009), arXiv:0810.3970 [hep-th].
[37] T. Faulkner and H. Liu, (2008), arXiv:0812.4278 [hep-th].
[38] M. Ammon, J. Erdmenger, M. Kaminski, and P. Kerner, JHEP 10, 067 (2009), arXiv:0903.1864 [hep-th].
[39] J. Erdmenger, V. Grass, P. Kerner, and T. H. Ngo, JHEP 08, 037 (2011), arXiv:1103.4145 [hep-th].
[40] N. Jokela and A. V. Ramallo, Phys. Rev. D92, 026004 (2015), arXiv:1503.04327 [hep-th].
[41] G. Itsios, N. Jokela, and A. V. Ramallo, Nucl. Phys. B909, 677 (2016), arXiv:1602.06106 [hep-th].
[42] A. Karch, M. Kulaxizi, and A. Parnachev, JHEP 11, 017 (2009), arXiv:0908.3493 [hep-th].
[43] E. Zhou, X. Zhou, and A. Li, (2017), arXiv:1711.04312 [astro-ph.HE].
[44] I. Tews, T. Kruger, K. Hebeler, and A. Schwenk, Phys. Rev. Lett. 110, 032504 (2013), arXiv:1206.0025 [nucl-th].
[45] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939).
[46] S. Chandrasekhar, Phys. Rev. Lett. 12, 114 (1964).
In the holographic model at $T=0$, there is no real distinction between the bare and constituent quark mass. We, however, suggest $m_0$ to be identified with the latter, as this conforms better with the intuition coming from QCD.

This amounts to the assumption that the phase changes instantaneously when the pressure fluctuates around the critical value. Should this not be the case, the stability of some of the configurations we find may change [79].

[79] E. Annala, C. Ecker, C. Hoyos, N. Jokela, D. Rodriguez Fernandez, and A. Vuorinen, In preparation.