Multipion decays of $\omega(782)$ and $\phi(1020)$.

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Using the chiral model of pseudoscalar, vector, and axial vector mesons based on the hidden local symmetry added with the terms induced by the Wess-Zumino anomaly, the results of calculations of the branching fractions of the decays $\omega(782)$ and $\phi(1020)$ mesons to the $2\pi^+2\pi^-\pi^0$, $\pi^+\pi^-3\pi^0$ multipion states are presented.

1. INTRODUCTION

The theory aimed at describing low energy hadron processes should be formulated in terms of effective colorless degrees of freedom. They can be introduced on the basis of spontaneously broken approximate chiral symmetry $SU(3)_L \times SU(3)_R$ which is the symmetry of QCD Lagrangian

$$L_{QCD} = -\frac{1}{4} \left( \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf_{abc} G^b_\mu G^c_\nu \right)^2 + \sum_{q=u,d,s,c,b,t} \bar{q} \times \left[ \gamma_\mu \left( i\partial_\mu - \frac{\lambda^a}{2} G^a_\mu \right) - m_q \right] q, \quad (1)$$

([\lambda^a, \lambda^b] = 2if_{abc}\lambda^c)$ relative independent rotations of right and left fields of approximately massless $u, d, s$ quarks:

$$q_L = \frac{1 + \gamma_5}{2} q \rightarrow V_L q_L,$$
$$q_R = \frac{1 - \gamma_5}{2} q \rightarrow V_R q_R, \quad (2)$$

where $V_{L,R} \in SU(3)_{L,R}$. The pattern of the spontaneous breaking is $SU(3)_L \times SU(3)_R \Rightarrow SU(3)_{L+R}$. According to the Goldstone theorem, spontaneous breaking of global symmetry results in appearance of massless fields. In our case they are light $J^P = 0^-$ mesons $\pi^+, \pi^-, \pi^0, K^+, K^0, K^-, K^0$, $\eta$. The transformation law $U \rightarrow V_L U V_R^\dagger$ where $U = \exp (i\Phi \sqrt{2}/f_\pi)$, and

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad (3)$$

fixes the Lagrangian of interacting Goldstone mesons:

$$L_{GB} = \frac{f^2}{4} \text{Sp} \left( \partial_\mu U \partial_\mu U^\dagger \right) + \cdots.$$ 

Dots mean the terms with higher derivatives. Upon adding the term $\propto m^2 \text{Sp}(U + U^\dagger)$ which explicitly breaks chiral symmetry, Goldstone bosons become massive. The Wess-Zumino (WZ) term

$$\Gamma_{WZ} = -\frac{im_c}{240\pi^2} \int_{M_5} \text{Sp} \left( dUU^\dagger \right)^5$$

removes spurious selection rule forbidding processes with odd number of Goldstone mesons.

Pseudoscalar mesons are produced in $e^+e^-$ annihilation via vector resonances, hence one should include vector mesons in a chiral invariant way. The problem of testing chiral models of the vector meson interactions with Goldstone bosons is acute because in well studied decays $\rho \rightarrow 2\pi$, $\omega \rightarrow 3\pi$ the final pions are not soft enough to rely on the lowest derivative tree effective Lagrangian. The multiple pion decays are most promising because pions are truly soft.

2. THE MODEL

There are a number of various chiral models incorporating non-anomalous interactions of
pseudoscalar, vector, and axial vector mesons which are equivalent at the level of lowest number of derivatives [12]. However, the anomalous processes are most conveniently treated in the framework of the generalized hidden local symmetry (GHLS) approach [3]. Choosing the specific gauge and restricting to the sector of nonstrange mesons it looks as

\[ \mathcal{L} = a_0 f_\pi^2 \text{Tr} \left( \frac{\partial \mu \xi^\dagger + \partial \nu \xi^\dagger}{2i} - g V_\mu \right)^2 + b_0 f_\pi^2 \text{Tr} \left( \frac{\partial \mu \xi^\dagger - \partial \nu \xi^\dagger}{2i} + g A_\mu \right)^2 + c_0 f_\pi^2 g A_\mu^2 + d_0 f_\pi^2 \text{Tr} \left( \frac{\partial \mu \xi^\dagger - \partial \nu \xi^\dagger}{2i} \right)^2 + \frac{f_\pi^2}{2} \text{Tr} \left( F_{\mu \nu}^{(V)} + F_{\mu \nu}^{(A)} \right)^2 - \frac{1}{2} \text{Tr} \left( F_{\mu \nu}^{(V)} \right)^2 - i \alpha_4 g \text{Tr} [A_\mu, A_\nu] F_{\mu \nu}^{(V)} + 2 i \alpha_5 g \times \text{Tr} \left[ \frac{\partial \mu \xi^\dagger - \partial \nu \xi^\dagger}{2i}, A_\nu \right] + [A_\mu, A_\nu] \times (\omega_\mu \partial_\nu \pi) - g (c_1 + c_2 - c_3) \omega_\mu \partial_\nu \pi + 4 c_3 g f_\pi^2 \omega_\mu \omega_\nu \left[ [\rho_\lambda \partial_\sigma \pi], [\rho_\lambda \partial_\sigma \pi] \right] + \frac{1}{6} \frac{f_\pi^2}{f_\pi^2} \omega_\mu \omega_\nu \left[ (\rho_\lambda \partial_\sigma \pi) - \frac{(\omega_\mu \partial_\sigma \pi)}{f_\pi^2} \right] \left( 1 + \frac{\pi^2}{f_\pi} \right). \]

\[ (4) \]

where \( \xi = \exp i \frac{\pi \rho_\lambda}{f_\pi} \).

\[ F_{\mu \nu}^{(V)} = \partial_\mu V_\nu - \partial_\nu V_\mu - i [V_\mu, V_\nu] - i [A_\mu, A_\nu], \]
\[ F_{\mu \nu}^{(A)} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [V_\mu, A_\nu] - i [A_\mu, V_\nu], \]
\[ V_\mu = \left( \frac{\tau}{2} \cdot \rho_\mu \right) + \frac{\omega_\mu}{2}, \]
\[ A_\mu = \left( \frac{\tau}{2} \cdot A_\mu \right). \]

(5)

The following steps are necessary in order to obtain the minimal effective Lagrangian describing non-anomalous processes:

- To exclude axial vector-pseudoscalar mixing by means of introduction of physical \( a_1(1260) \) meson field \( a_1 \):

\[ A_\mu = a_\mu - \frac{b_0}{g(b_0 + c_0)} \frac{\partial \mu \xi^\dagger - \partial \nu \xi^\dagger}{2i}. \]

(6)

Here the total nonlinear combination of pion fields is rotated away. This provides the fulfillment the Adler condition for decay amplitudes with the intermediate \( a_1 \) meson independently of its mass.

- To renormalize according to \( f_\pi \rightarrow Z^{-1/2} f_\pi, \pi \rightarrow Z^{-1/2} \pi \), \( a_0, b_0, c_0, d_0 = Z(a, b, c, d) \), where

\[ \left( d_0 + \frac{b_0 c_0}{b_0 + c_0} \right) Z^{-1} = 1. \]

- To make the choice \( \alpha_4 = -\alpha_5 = -1, a = b = c = 2, d = 0 \) which removes the higher derivative \( \rho \pi \pi \) coupling, results in univerality \( g_{\rho \pi \pi} = g \), vector dominance, KSRF \( 2 g_{\rho \pi \pi}^2 f_\pi^2 / m_\pi^2 = 1 \).

Anomalous vertices \( \omega \rightarrow 3 \pi, 5 \pi, \rho \pi, \rho \rho \pi \) etc. are treated using the lagrangian induced by the Wess-Zumino anomaly equation. We give the necessary lowest derivative terms of effective lagrangian:

\[ \mathcal{L}^\text{an} = \frac{n_c g}{32 \pi^2 f_\pi^2} \xi \mu \lambda \sigma \left\{ c_1 - c_2 - c_3 + \frac{\pi^2}{4 f_\pi^2} \times \left[ \frac{3}{2} (c_2 + c_3) - c_1 \right] \omega_\mu \partial_\nu \pi - g (c_1 + c_2 - c_3) \omega_\mu \times \left[ \omega_\mu \partial_\nu \pi / f_\pi^2 \right] \left( 1 + \frac{\pi^2}{f_\pi} \right) \right\}. \]

(7)

The diagrams describing the \( \omega \rightarrow 5 \pi \) decay amplitude are shown in Fig. 1. The obtained decay amplitudes satisfy the Adler condition. See Refs. [15,16] for more detail.

The parameters \( c_1, c_2, c_3 \) are arbitrary. They are fixed in accord with the condition of absence of the pointlike \( \omega \rightarrow \pi^+ \pi^- \pi^0 \) vertex, hence \( c_1 - c_2 - c_3 = 0 \). Then \( c_3 \) is fixed by the coupling constant \( g_{\omega \pi \pi} = -\frac{n_c g^2 / f_\pi}{8 \pi^2 f_\pi^2} \). Next we take \( c_1 + c_2 - c_3 = 0 \), which seems arbitrary, but within the accuracy to percent the results are robust to wide variations breaking this condition [1]. So, our choice is \( c_1 = c_3, c_2 = 0 \).

3. THE RESULTS

The \( \omega, \phi \rightarrow 5 \pi \) decay width is represented as 8-dimensional integral over Mandelstam-like Kumar variables:

\[ \Gamma_{5 \pi} = \frac{\pi \rho_{\text{sym}}}{24 N_{\text{sym}}(2 \pi)^{11}} \int_{s_{1-}}^{s_{1+}} d s_{1-} \times \]

(8)
The notations are given in Ref. [7]. The results of the evaluation of the branching fractions of the ω → 5π decay modes under assumption [7] are collected in Table 1.

The case of the φ → 5π decay is more subtle. If the φ transitions to nonstrange mesons are due to φω mixing then the results of the ω → 5π calculations are translated to the case of φ:

\[ \Gamma_{\phi \rightarrow 5\pi}(m_\phi^2) = |\varepsilon_{\phi\omega}|^2 \Gamma_{\omega \rightarrow 5\pi}(m_\phi^2), \]

\[ |\varepsilon_{\phi\omega}|^2 = \frac{\Gamma_{\phi \rightarrow 3\pi}(m_\phi^2) W_{\omega \rightarrow 3\pi}(m_\phi^2)}{\Gamma_{\omega \rightarrow 3\pi}(m_\phi^2) W_{\phi \rightarrow 3\pi}(m_\phi^2)} = 3 \times 10^{-3}, \]

where \( W_{\phi \rightarrow 3\pi} \) is the dynamical phase space volume. In the opposite case when φ goes to nonstrange mesons directly [8], one could construct the effective Lagrangian guided by the property of the decay amplitude being chiral invariant, by analogy with the ω → 5π case and with the same assumptions about free parameters [4]. The results of calculations are collected in Table 2.

Two remarks are in order. First, the KLOE data on φ → 3π [9] allow possible nonzero pointlike vertex. Taking this vertex into account results in ±8% deviation from the figures obtained under declared assumption about free parameters of effective Lagrangian. Second, the sum of the non-resonant diagrams Fig. 1(b), (c), (d), (e) contributes about 20% to the φ → 5π partial width. This sum is incoherent with the contribution of the dominant diagram Fig. 1(a). Hence, within the accuracy ±20% the dynamics of the decay φ → 5π is dominated by the process φ → ρπ followed by the decay ρ → 4π [8] with the resonant ρ and thus is insensitive neither to arbitrary parameters nor to the φω mixing model.

The mass spectra of the four pion subsystem in the final five-pion state are shown in Fig. 2 and 3. Both spectra has the peak due to the ρ pole. The spectrum of π⁺π⁻π⁻π⁰ has the second peak due to the combination of the strong

| Table 1 |
|---|---|---|
| \( m_{a_1} \) (GeV) | \( B_{\omega \rightarrow 5\pi^-3\pi^0} \) | \( B_{\omega \rightarrow 2\pi^02\pi^-\pi^0} \) |
| 1.09 | \( 4.2 \times 10^{-9} \) | \( 3.8 \times 10^{-9} \) |
| 1.23 | \( 4.1 \times 10^{-9} \) | \( 3.7 \times 10^{-9} \) |
| no \( a_1 \) | \( 3.6 \times 10^{-9} \) | \( 3.3 \times 10^{-9} \) |

\[ (1 - \xi_2^2)^{-1/2} \int_{t_{5-}}^{t_{3+}} dt_3 |M_{\omega,\phi \rightarrow 5\pi}|^2 \times [\lambda(s,t_2,t'_2)(1-\xi_3^2)(1-\eta_5^2) \times (1-\xi_2^2)]^{-1/2} \times \]

\[ \int_{t_{2-}}^{t_{12+}} dt_2 |\lambda(s,t_1,t'_1)(1-\xi_2^2)(1-\eta_5^2)|^{-1/2} \times \]

\[ \int_{u_{2-}}^{u_{3+}} \frac{du_3}{u_3} \times \int_{u_{1-}}^{u_{4+}} \frac{du_1}{u_1} \times \int_{s_{2-}}^{s_{3+}} ds_2 \times \int_{s_{3-}}^{s_{4+}} ds_3 \times \]

\[ \int_{s_{2-}}^{s_{4+}} |\lambda(s_2, s_3, s_4)|^{-1/2} |\lambda(s, m_3^2, u_2)|^{-1/2} \times \]

\[ \left( 1 - \xi_2^2 \right)^{-1/2} \left( 1 - \xi_3^2 \right)^{-1/2} \left( 1 - \eta_5^2 \right)^{-1/2} \left( 1 - \xi_4^2 \right)^{-1/2} \left( 1 - \eta_4^2 \right)^{-1/2} \]
Table 2
The same as in Table 1, but for $\phi \to 5\pi$.

| $m_{a_1}$ [GeV] | $B_{\phi \to \pi^+\pi^-3\pi^0}$ | $B_{\phi \to \pi^+2\pi^-\pi^0}$ |
|-----------------|----------------------------------|----------------------------------|
| 1.09            | $4.4 \times 10^{-7}$             | $8.8 \times 10^{-7}$             |
| 1.23            | $3.9 \times 10^{-7}$             | $7.7 \times 10^{-7}$             |
| no $a_1$        | $2.5 \times 10^{-7}$             | $5.0 \times 10^{-7}$             |

energy dependent anomaly induced contribution $\rho^- \to \omega\pi^- \to \pi^+\pi^-\pi^-\pi^0$ in the decay $\phi \to \rho^-\pi^+ \to \pi^+\pi^-\pi^-\pi^0$ and the phase space kinematical restriction. Such contribution is absent in the decay $\phi \to \rho^-\pi^+ \to \pi^+\pi^-\pi^0\pi^0\pi^0$.

Figure 2. The mass spectrum of $\pi^+\pi^-\pi^-\pi^0\pi^0$ in the decay $\phi \to \pi^+\pi^-\pi^-\pi^0\pi^0$ normalized to the respective $5\pi$ width; $\sqrt{s} = m_{\phi}$, $m^2_{2345} = (q_2 + q_3 + q_4 + q_5)^2 \equiv s_1$.

With the total KLOE statistics $\int L dt \approx 500 \text{ pb}^{-1}$ (circa 2003) one can have the number of events of the $\phi \to 5\pi$ decay approximately 1340, 2070, 2360, respectively, in the model without $a_1$, in the model incorporating $a_1$ with the mass $m_{a_1} = 1.23$ GeV, 1.09 GeV.

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