Improving CP measurement with THEIA and muon decay at rest

Shao-Feng Ge\textsuperscript{1,2,a}, Chui-Fan Kong\textsuperscript{1,2,b}, Pedro Pasquini\textsuperscript{1,2,c}

\textsuperscript{1} Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{2} Key Laboratory for Particle Astrophysics and Cosmology (MOE) and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China

Received: 24 February 2022 / Accepted: 31 May 2022 / Published online: 30 June 2022
© The Author(s) 2022

Abstract We explore the possibility of using the recently proposed THEIA detector to measure the \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) oscillation with neutrinos from a muon decay at rest (\( \mu \)DAR) source to improve the leptonic CP phase measurement. Due to its intrinsic low-energy beam, this \( \mu \)THEIA configuration (\( \mu \)DAR neutrinos at THEIA) is only sensitive to the genuine leptonic CP phase \( \delta_D \) and not contaminated by the matter effect. With detailed study of neutrino energy reconstruction and backgrounds at the THEIA detector, we find that the combination with the high-energy DUNE can significantly reduce the CP uncertainty, especially around the maximal combination with the high-energy DUNE can significantly and backgrounds at the THEIA detector, we find that the combination with the high-energy DUNE can significantly reduce the CP uncertainty, especially around the maximal.

1 Introduction

The charge-parity (CP) symmetry violation is a key to understand the existence of baryon asymmetry in the Universe, namely, why there are more matter than anti-matter \cite{1–5}. There are at least two possible sources of CP violation in the Standard Model (SM) of particle physics: the CP phase in the quark mixing matrix \cite{6,7} and the leptonic CP phases in the neutrino mixing matrix \cite{8}. Especially, the leptonic CP phases at low energy play an important role \cite{9,10} in the leptogenesis mechanism \cite{11–13}. Both the Dirac and Majorana CP phases can contribute to the leptogenesis mechanism. However, only the Dirac CP phase manifests itself in neutrino oscillation and can be measured by oscillation experiments \cite{14}.

The nonzero reactor mixing angle (\( \theta_r \equiv \theta_{13} \)) measured by Daya Bay \cite{15} and RENO \cite{16} heralds the precision era of neutrino oscillation experiments. A nonzero \( \theta_r \) allows the Dirac CP phase \( \delta_D \) to have physical effect since these two variables always appear together as \( \sin \theta_r e^{\pm i \delta_D} \) in the standard parametrization \cite{8} of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix \cite{17,18}. Typically, the neutrino oscillations from the muon flavor to the electron flavor (\( \nu_\mu \rightarrow \nu_e \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)) are used by the long-baseline accelerator experiments to measure \( \delta_D \) \cite{19}.

The current long-baseline experiments T2K \cite{20} and NO\( \nuA \) \cite{21} are approaching the discovery threshold. The 2019 T2K result \cite{22} with \( \delta_D = 252^\circ_{-33}^{+28} \) and \( 281^\circ_{-31}^{+28} \) for the normal and inverted orderings (NO and IO), respectively, has excluded almost half of the parameter space except \([165^\circ, 358^\circ] \) and \([-214^\circ, 342^\circ] \) at 3\( \sigma \) confidence level (C.L.). It is interesting to see that the maximal CP phase \( \delta_D = 90^\circ \) is around the best-fit point. However, the 2019 NO\( \nuA \) result is \( \delta_D = 0^\circ_{-23}^{+74} \) \cite{23} for NO with best-fit value at vanishing CP phase, \( \delta_D = 0^\circ \). In 2021, T2K and NO\( \nuA \) updated their results with the best-fit value from T2K remaining the same, \( \delta_D = 252^\circ_{-33}^{+28} \) (NO) and \( 281^\circ_{-31}^{+28} \) (IO) \cite{24} while the NO\( \nuA \) best fit changes to \( \delta_D = 148^\circ_{-157}^{+49} \), for NO \cite{25}.

Although not significant, there is a tension between the T2K and NO\( \nuA \) data that they exclude each other at 1 or C.L. \cite{26,27}. New physics can explain the tension. Both belonging to accelerator neutrino experiments, T2K and NO\( \nuA \) have very different configurations. While the T2K baseline is 295 km and the peak energy is at 0.6 GeV, the NO\( \nuA \) baseline is 810 km and peak energy at 2 GeV. These differences in baseline and beam energy leave room for new physics. For example, the non-standard interaction (NSI) contributes extra matter potential \cite{28} and hence its effect on oscillation probabilities is energy dependent \cite{29,30} to provide a possible solution \cite{31,32}. The tension is reduced when the data are analyzed in the context of Lorentz invariance violation (LIV). However, it is accompanied by a new mild tension between the best-fit values of \( \sin^2 \theta_{23} \) \cite{33}. Besides, the
non-unitarity mixing due to heavy neutrinos [34–36] allows extra CP phases to fake the genuine CP effect which can also explain the tension [37]. However, a more recent work [38] points out the non-unitarity cannot explain the tension with the bounds on non-unitarity parameters from the combination of short- and long-baseline data. Similar thing happens for the light sterile neutrino scheme [39]. Whether this tension is truly new physics or not needs further investigation at current and future experiments.

Even if the current tension between T2K and NOνA measurements vanishes with more data, correct interpretation of CP measurements still faces intrinsic issues including event rate inefficiency, $\delta_D \leftrightarrow \pi - \delta_D$ degeneracy, and large CP uncertainty around the maximal values [40–42]. These issues still remain for the next-generation experiments like T2HK [43] and DUNE [44]. Although its wide spectrum can help to reduce the $\delta_D$ degeneracy, DUNE has much larger matter effect than T2K and NOνA due to higher energy peaking around 2.5 GeV [45]. At long-baseline experiments, the genuine CP effect can be faked by the ubiquitous matter effect, reducing the experimental sensitivity to $\delta_D$ [46–48]. In addition, the uncertainties in the matter effect can also reduce the CP sensitivity at DUNE [45].

In this paper, we propose $\mu$THEIA as combination of the THEIA detector [49–52] and a $\mu$DAR neutrino flux to improve the Dirac leptonic CP phase measurement together with DUNE. Section 2 summarizes the contamination of matter effect in the CP measurement and explains why the $\mu$DAR neutrino flux with lower energy can help. Then in Sect. 3, we describe the low-energy mode at $\mu$THEIA and the high-energy mode at DUNE, including selection criteria, energy reconstruction, smearing, and backgrounds. A combination of $\mu$THEIA and DUNE can significantly improve the CP sensitivity as illustrated in Sect. 4. Therein, we also give the details of simulation and $\chi^2$ analysis. Our $\mu$THEIA proposal is compared with the existing configurations/proposals in Sect. 4.4 and summarized in Sect. 5.

## 2 Improving CP measurement with multiple baselines and beam energies

The current T2K and NOνA experiment aims for the discovery of leptonic CP violation, namely, excluding $\delta_D = 0$ and $\pi$. Once a nontrivial $\delta_D$ is measured, the next step is precision measurement of its value. Several experimental configurations have been proposed to improve the CP measurement after the reactor mixing angle $\theta_r$ was measured by Daya Bay and RENO. The upgrade from existing experiments includes: (1) Intensity Upgrade: the beam intensity is significantly enhanced, such as T2K-II [53]; (2) Detector Upgrade: T2HK [43] has a much larger detector Hyper-K than Super-K; (3) Spectrum Upgrade: DUNE [44] adopts wider on-axis spectrum than the off-axis one of NOνA; and (4) Baseline Upgrade with longer baseline such as T2HK [54–56] and DUNE. In addition, there are also several new proposals: (5) the accelerator experiments such as P2O [57], ESSνSB [58], and MOMENT [59]; (6) CP measurement with sub-GeV atmospheric neutrino oscillation such as Super-PINGU [60,61], Super-ORCA [62], and even at JUNO [63] or DUNE [64]. Comparison among various experimental configurations can be found in [65–68].

### 2.1 Matter contamination on CP measurement

However, the neutrino energy for all these designs is not low enough to avoid contamination from the ubiquitous matter effect [45,69–74]. We will try to describe how the CP measurement is contaminated by the matter effect. Based on this, one can see the possible solutions.

The neutrino propagation through matter is described by the following Hamiltonian [28,75],

$$
\mathcal{H}_M = \frac{1}{2E_\nu} U \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{array} \right) U^\dagger + \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right). \tag{1}
$$

The first term is the vacuum Hamiltonian that is a product of the PMNS matrix $U$ [17,18] and the diagonal mass matrix with solar $\Delta m^2_{21} \equiv \Delta m^2_{\odot}$ and atmospheric $\Delta m^2_{31} \equiv \Delta m^2_{\text{atm}}$ mass squared differences. The neutrino energy $E_\nu$ in the denominator contributes as an overall factor. So it is not a suppression in the vacuum term but actually an enhancement of the matter potential in the second term.

Induced by the SM weak interaction, the matter potential $V = \sqrt{2} G_F n_e$ is proportional to the Fermi constant $G_F$ and the electron number density $n_e$,

$$
V(x) \approx \frac{\rho(x)}{g/c^3} Y_e(x) \times 7.56 \times 10^{-14} \text{eV}, \tag{2}
$$

and only appears in the first element of the potential matrix in (1) for the electron flavor [28]. The matter potential $V(x)$ is not just proportional to the matter density $\rho(x)$ but also the number of electrons per nucleon $Y_e(x)$. Both the matter density and chemical composition vary with position. Consequently, the matter potential is in general also a function of $x$. In the Earth crust, the electron fraction is approximately $Y_e \approx 0.5$ and the average matter density is $\bar{\rho} \equiv \langle \rho(x) \rangle \approx 2.845 g/cm^3$ [45]. For simplicity, we ignore the density variation and adopt the averaged matter density $\bar{\rho}$ as constant along the baseline of DUNE.

To make the CP and matter effect explicit, we perform a series expansion of the $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) oscillation probability in terms of the ratio between the two mass squared differences $\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31} \approx 3\%$ and the reactor mixing angle $\theta_r \equiv \sin \theta_r \approx 0.15$ [76–78],

\[ \text{Springer} \]
oscillation experiments is not negligible in the first place. Less, the matter effect at accelerator and atmospheric neutrino Earth intrinsically has multiple baselines [60–64]. Neverthe-

\[
P_{\nu \mu \rightarrow \nu e} \approx 2 \alpha^2 \sin^2 \theta_{\mu} \sin^2 \left(\frac{1 + A}{2} \Delta_{\mu} \right) + 4 \alpha^2 \sin^2 \left(\frac{1 + A}{2} \Delta_{\mu} \right)
\]

The sign \( \pm (\mp) \) is for neutrino (the upper one) and anti-

\[
P_{\nu_{\mu\rightarrow\nu e}} \rightarrow P_{\nu_{\mu\rightarrow\nu e}} \sin \Delta_{\mu} \sin \delta_D, \]

\[
\Delta_{\mu} \approx \frac{\Delta m^2_{\mu\mu}}{L} / 4 \varepsilon \nu_e / \varepsilon \mu_e, \]

The neutrino oscillation is modulated by the neutrino energy. The higher neutrino energy, the larger contamination on the CP measurement. Comparing with the 30% effect at T2K for \( E_\nu \approx 550 \) MeV [73], DUNE with peak energy around 2.5 GeV suffers from larger matter effect. So it is more urgent for the DUNE experiment to have a complementary baseline with low-energy beam to further improve the CP measure-

\[
\frac{P_{\nu e} \rightarrow \nu \mu}{P_{\mu e} \rightarrow \nu e} \sin \Delta_{\mu} \sin \delta_D, \]

\[
\Delta_{\mu} \approx \frac{\Delta m^2_{\mu\mu}}{L} / 4 \varepsilon \nu_e / \varepsilon \mu_e, \]

The neutrino measurement, the essential observable is the difference between the neutrino and anti-neutrino oscillation probabilities, \( P_{\nu_{\mu\rightarrow\nu e}} \rightarrow P_{\nu_{\mu\rightarrow\nu e}} \sin \Delta_{\mu} \sin \delta_D, \) that is proportional to \( \sin \delta_D \) in the absence of matter potential. However, a realistic measurement has sign difference in not just the Dirac CP phase \( \delta_D \) but also the matter term \( A \). With a typical size of \( A \approx (0.05, 0.1, 0.2) \) estimated with the peak neutrino energies (0.55, 2, 2.5) GeV at T2K/T2HK, NOvA, and DUNE, respectively, the matter effect on neutrino CP measurement cannot be ignored. The matter potential can fake the genuine CP violation and blur the CP measurement.

At a single long-baseline neutrino oscillation experiment, there is only one independent CP observable but two parame-

\[
\Delta_{\mu} \approx \frac{\Delta m^2_{\mu\mu}}{L} / 4 \varepsilon \nu_e / \varepsilon \mu_e, \]

The difference between the oscillation probabilities with and without matter, \( \delta P_{\mu e} \equiv P_{\mu e} (A) - P_{\mu e} (A = 0) \), in Fig. 1 shows explicitly the matter effect at the DUNE and \( \mu \)THEIA configurations. For DUNE (\( L = 1300 \) km with blue and yellow lines), the difference can be as large as \( \delta P_{\mu e} \approx 0.03 \) for \( E_\nu / L \approx 1.5 \) MeV/km which is roughly 52% (62%) of the CP-violating oscillation probability \( P_{\mu e} \approx 0.058 \) \((|P_{\mu e} - P_{\mu e}| = 0.048)\). Even for the neutrino energy peak at \( E_\nu / L \approx 1.9 \) MeV/km, the size of the matter effect is still as large as 0.025 and 33% (52%) of \( P_{\mu e} = 0.075 \) \((|P_{\mu e} - P_{\mu e}| = 0.048)\). It is interesting to see that the probability difference is almost independent of the Dirac CP phase \( \delta_\mu \). This is because the major matter effect, \( \delta P_{\mu e} \approx 8 \alpha^2 \Delta_{\mu}^2 / A \approx 0.02 \), comes from the second term of (3) at the oscillation peak without involving \( \delta_\mu \). Although the matter effect also appears through the third term of (3), the effect is further suppressed by a prefactor of \( \Delta_{\mu} \sin \theta_\mu / (\alpha / s_\alpha) \approx 0.3 \) and hence is a minor effect. Altogether, the matter effect at DUNE is at the same order as the genuine CP effect.

However, it is impossible to simply add a \( \mu \)DAR source and share the same liquid Argon detectors of DUNE in a similar way as TNT2K. This is because there are no free protons to provide inverse beta decay (IBD) for unique probe of the electron anti-neutrino and hence the \( \nu_\mu \rightarrow \nu_\mu \) oscillation. Besides, the \( \nu_\mu \rightarrow \nu_e \) cross section is too small to detect the \( \mu \)DAR flux at DUNE [44].

A new THEIA detector at the same site of SURF was recently proposed [49–52,93]. With a new technique of water-based liquid scintillator (WbLS), it is possible to use both scintillation and Cherenkov lights [94–98]. This opens the possibility of detecting the low-energy \( \mu \)DAR neutrino oscillation to supplement the high-energy mode at DUNE. For convenience, we call the combination of \( \mu \)DAR and THEIA as \( \mu \)THEIA.

The difference between the oscillation probabilities with and without matter, \( \delta P_{\mu e} \equiv P_{\mu e} (A) - P_{\mu e} (A = 0) \), in Fig. 1 shows explicitly the matter effect at the DUNE and \( \mu \)THEIA configurations. For DUNE (\( L = 1300 \) km with blue and yellow lines), the difference can be as large as \( \delta P_{\mu e} \approx 0.03 \) for \( E_\nu / L \approx 1.5 \) MeV/km which is roughly 52% (62%) of the CP-violating oscillation probability \( P_{\mu e} \approx 0.058 \) \((|P_{\mu e} - P_{\mu e}| = 0.048)\). Even for the neutrino energy peak at \( E_\nu / L \approx 1.9 \) MeV/km, the size of the matter effect is still as large as 0.025 and 33% (52%) of \( P_{\mu e} = 0.075 \) \((|P_{\mu e} - P_{\mu e}| = 0.048)\). It is interesting to see that the probability difference is almost independent of the Dirac CP phase \( \delta_\mu \). This is because the major matter effect, \( \delta P_{\mu e} \approx 8 \alpha^2 \Delta_{\mu}^2 / A \approx 0.02 \), comes from the second term of (3) at the oscillation peak without involving \( \delta_\mu \). Although the matter effect also appears through the third term of (3), the effect is further suppressed by a prefactor of \( \Delta_{\mu} \sin \theta_\mu / (\alpha / s_\alpha) \approx 0.3 \) and hence is a minor effect. Altogether, the matter effect at DUNE is at the same order as the genuine CP effect.

In contrast, the matter effect at \( \mu \)THEIA (\( L = 38 \) km with red and green lines) is negligibly small. Being essentially insensitive to the matter potential, \( \mu \)THEIA can focus on the
The oscillation probability $\frac{1}{2} P(A)$ and oscillation probability difference $\delta P_{\mu e} \equiv P_{\mu e}(A) - P_{\mu e}(A = 0)$ between the matter-induced and vacuum cases as a function of $E_\nu / L$ for both neutrino (solid) and anti-neutrino (dashed) modes. Both $\mu$THEIA ($L = 38\text{ km}$ with green line for $P_{\mu e}(A)/3$ and red one for $\delta P_{\mu e}$) and DUNE ($L = 1300\text{ km}$ with yellow line for $P_{\mu e}(A)/3$ and blue one for $\delta P_{\mu e}$) are illustrated. The pink band indicates the $\mu$THEIA energy range $[30, 50]\text{MeV}$ while the DUNE energy window spans the whole range.

As mentioned in the previous section, the low-energy $\bar{\nu}_\mu$ cannot be detected by the DUNE detector. But the THEIA detector is an ideal equipment. For the major oscillation channel $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ for CP measurement with $\mu$DAR neutrinos, the IBD process ($\bar{\nu}_\mu + p \rightarrow e^+ + n$) is an ideal detection method. With a large volume of WbLS, THEIA has a significant fraction of free protons (hydrogen) to allow IBD. Both positron and neutron in the final state are detectable by THEIA.

The WbLS allows THEIA to detect both scintillation and Cherenkov lights. There are several differences between the Cherenkov and scintillation lights that can be used for separation [94,97]: (1) the arrival time, i.e., Cherenkov light arrives nanoseconds earlier than the delayed scintillation light; (2) the angular topology, i.e., the Cherenkov light emission will cause a local enhancement on top of the isotropic scintillation signal; (3) the wave-length, i.e., the scintillation light has shorter wavelength while the Cherenkov light has relatively longer one.

For particle identification, the Cherenkov ring from $e^\pm$ and $\gamma$ has a smeared pattern. In contrast, the heavier muon or charged pion has much sharper Cherenkov ring. In addition, the production rates of Cherenkov and scintillation lights are different for different particles [94]. We will give more details once needed in later discussions.

Both Cherenkov and scintillation lights can be used for energy reconstruction. For low energy reactor and supernova $\bar{\nu}_e$ neutrinos, the energy is typically reconstructed with linear form, $E_\nu = E_e + m_n - m_p$ [100] where $m_n$ ($m_p$, $m_e$) is the neutron (proton, electron) mass and $E_e$ the positron energy. Nevertheless, this linear formula cannot reconstruct the neutrino energy very precisely. To see the deviation clearly, we simulate the IBD events with GENIE [101,102] and reconstruct the neutrino energy according to the linear formula. As shown in Fig. 2, the reconstructed energy spectrum for neutrinos with $E_\nu^\text{true} = 40\text{ MeV}$ spreads into a trapezoid (green solid line) and the central value shifts leftward by almost 2MeV. In other words, for the $\mu$DAR neutrinos with $O(10\text{MeV})$ energy, the neutrino energy reconstruction is no longer a linear dependence on the positron energy with a constant shift.

3.1.1 Signal at THEIA detector

As proposed above, the essential feature of the DUNE and $\mu$THEIA complex is a combination of different baselines and neutrino beams. In addition, the DUNE and THEIA detectors are also quite different with liquid Argon and WbLS targets, respectively. Each combination of neutrino beam and detector has its own characteristics in neutrino detection. For clarity, we elaborate separately the details of the “low-energy mode” (LEM) that the $\mu$DAR beam is detected by the THEIA detector in Sect. 3.1 as well as the the “high-energy mode” (HEM) that the LBNF beam is detected by both the DUNE and THEIA detectors in Sect. 3.2. We study in detail the event reconstruction for both signal and background to obtain their normalized transfer tables. The event rate including the information of flux, running time, cross section, and detector size can be found in the following Sect. 4 when estimating the CP sensitivity.

3.1 The low-energy mode

The $\mu$DAR neutrinos are produced by a cyclotron complex. For example, a typical 800 MeV proton beam hits a thick target to first generate pions. Although both $\pi^+$ can be produced, $\pi^-$ is mostly absorbed by the positively charged nuclei while $\pi^+$ decays at rest via $\pi^+ \rightarrow \mu^+\nu_\mu$. The decay product $\mu^+$ also loses its energy and decays at rest via $\mu^+ \rightarrow e^++\nu_e+\bar{\nu}_\mu$. During this process, three neutrinos ($\nu_\mu$, $\nu_e$, and $\bar{\nu}_\mu$) are produced. Of them, $\bar{\nu}_\mu$ experiences the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation that is of interest to CP measurement. Since $\mu^+$ decays at rest, $\bar{\nu}_\mu$ has a well-understood spectrum with maximum energy of 53 MeV [99].

![Fig. 1 The oscillation probability $\frac{1}{2} P(A)$ and oscillation probability difference $\delta P_{\mu e} \equiv P_{\mu e}(A) - P_{\mu e}(A = 0)$ between the matter-induced and vacuum cases as a function of $E_\nu / L$ for both neutrino (solid) and anti-neutrino (dashed) modes. Both $\mu$THEIA ($L = 38\text{ km}$ with green line for $P_{\mu e}(A)/3$ and red one for $\delta P_{\mu e}$) and DUNE ($L = 1300\text{ km}$ with yellow line for $P_{\mu e}(A)/3$ and blue one for $\delta P_{\mu e}$) are illustrated. The pink band indicates the $\mu$THEIA energy range $[30, 50]\text{MeV}$ while the DUNE energy window spans the whole range.](image-url)
Another possibility is adding a correction term with non-linear dependence [103],

\[ E_{\nu}^{\text{me}} = E_e + \Delta \frac{2E_e(E_e + \Delta) + \Delta^2 - m_e^2}{2m_p}, \tag{4} \]

where \( \Delta \equiv m_n - m_p \) is the difference between the neutron \((m_n)\) and proton \((m_p)\) masses. The blue solid line clearly shows that the reconstructed trapezoid spectrum shifts back to center around the true neutrino energy at \( E_{\nu}^{\text{true}} = 40 \text{ MeV} \). Nevertheless, the reconstructed spectrum is still a trapezoid even without smearing due to the detector resolution. With a half width being almost 1.3 MeV, the corresponding energy uncertainty can be as large as 3.25% which is even larger than the statistical uncertainty as elaborated below.

To get a precise reconstruction of the neutrino energy, we adapt the reconstruction formula from long-baseline experiments [104] to incorporate the scattering angle,

\[ E_{\nu}^{\text{me}} = \frac{m_n^2 - m_p^2 - m_e^2 + 2m_p E_e}{2(m_p - E_e + |p_e| \cos \theta_e)}, \tag{5} \]

where \( p_e \) is the electron momentum while \( \theta_e \) is the angle between the neutrino beam and positron direction. The directional information is a benefit due to the capability of detecting Cherenkov light for a WbLS detector. In the absence of detector resolution, (5) can obtain exactly the true neutrino energy and the reconstructed spectrum becomes a \( \delta \)-function (red solid line) in Fig. 2. Note that the positron energy \( E_e \) cannot be directly measured. The positron not just loses its kinetic energy but also annihilates with an environmental electron to produce two 511 keV \( \gamma \)'s. The total deposited energy that is visible in the detector receives an extra electron mass in addition to the positron energy, \( E_{\text{vis}} \equiv E_e + m_e \). In other words, the positron energy in the linear as well as non-linear energy reconstruction formula (4) and (5) is reconstructed as \( E_e = E_{\text{vis}} - m_e \).

With systematical uncertainty from the reconstruction formula eliminated, the remaining uncertainty mainly comes from the detector resolutions of the visible energy \( E_{\text{vis}} \) and scattering angle \( \theta_e \). Since the light yield is proportional to the deposited energy, the visible energy \( E_{\text{vis}} \) can be reconstructed from the number of photoelectrons (p.e.) which is roughly 80 (130) p.e./MeV for the Cherenkov (scintillation) light [97]. The relative statistical uncertainties are \( \langle \sigma_{E_{\text{vis}}}/E_{\text{vis}} \rangle_{\text{Ch}} = 1/\sqrt{80 \times E_{\text{vis}}/\text{MeV}} \) for the Cherenkov light and \( \langle \sigma_{E_{\text{vis}}}/E_{\text{vis}} \rangle_{\text{sc}} = 1/\sqrt{130 \times E_{\text{vis}}/\text{MeV}} \) for the scintillation light, respectively. The combined uncertainty is

\[ \sigma_{E_{\text{vis}}} = \frac{1}{\sqrt{210 \times E_{\text{vis}}/\text{MeV}}} \approx \frac{7 \%}{E_{\text{vis}}/\text{MeV}}. \tag{6} \]

Since the \( \mu \)DAR neutrino energy is typically \( \mathcal{O}(10 \text{ MeV}) \), the relative error \( \sigma_{E_{\text{vis}}}/E_{\text{vis}} \lesssim 2 \% \) is actually quite good. Note that the scintillation light yield is more efficient than the Cherenkov one and both can play important roles in energy reconstruction. At \( E_{\nu}^{\text{true}} = 40 \text{ MeV} \), the positron energy resolution leads to about 1.17% uncertainty on the reconstructed neutrino energy.

In addition, the scattering angle \( \theta_e \) in (5) also receives an imperfect detector resolution and blur the reconstructed neutrino energy. Although the actual resolution for the \( \mu \)DAR flux that has higher energy should be better, we take a conservative value \( \sigma_\theta = 10^\circ \) [51] estimated for supernova neutrino. The angular uncertainty leads to a 0.44% uncertainty on the neutrino energy reconstruction which is slightly less than the one from positron energy resolution. In Fig. 2, the dashed lines show the combined neutrino energy uncertainty, which is 1.28% for \( E_{\nu}^{\text{true}} = 40 \text{ MeV} \) assuming Gaussian smearing.

Once produced, the neutron from IBD would experience thermalization and lose energy before being captured. The scintillation light emitted during this process can mix with the light from positron. Consequently, the visible energy receives an extra contribution from the neutron kinetic energy \( T_n \), \( E_{\text{vis}} = E_e + m_e + Q_F \times T_n \) where \( Q_F \) is the neutron quenching factor. For the \( \mu \)DAR neutrino at 40 MeV, the neutron kinetic energy \( T_n \) is typically 3 MeV according to GENIE simulation. With WbLS, it is possible to reconstruct the neutron kinetic energy \( T_n \) by measuring the positron scattering angle [105]. The neutron quenching factor \( Q_F \) keeps decreasing with \( T_n \) and is 15% at \( T_n = 0.25 \text{ MeV} \) [105]. Then from the measured visible energy \( E_{\text{vis}} \), one can first solve the positron energy \( E_e = E_{\text{vis}} - m_e - Q_F \times T_n \) and put it into (5) to reconstruct \( E_{\nu} \). In principle, the neutron kinetic energy is not a problem for the IBD energy reconstruction. But the neutron quenching factor \( Q_F \) still remains to be measured in a WbLS as far as we know. So for simplicity, we omit the neutron kinetic energy in our current phenomenological study.
3.1.2 Background suppression with WbLS

As illustrated in Sect. 3.1.1, the $\nu_e$ signal contains not just a positron but also a neutron in the final state. Although a single positron can already allow precise energy reconstruction, it receives various backgrounds. The final-state neutrino is extremely important for selecting out the IBD signal with double coincidence.

The muon neutrino and anti-neutrino cannot experience charged-current scattering with not enough energy to produce a $\mu^\pm$ in the final state. However, all neutrinos and anti-neutrinos can elastically scatter with electron to produce an energetic electron in the final state. But these backgrounds can also be removed by requiring neutron capture [94,97].

**Atmospheric neutrinos:** The atmospheric neutrino flux [118,119] contains all neutrino flavors ($\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$, and $\bar{\nu}_\tau$) to contribute as background [87]. With much higher energy, the atmospheric neutrino backgrounds can experience all types of neutrino-nucleus scattering and hence need much more dedicated treatment. Fortunately, the excellent performance of the WbLS with both Cherenkov and scintillation detection can effective suppress these backgrounds. Except $\nu_e$ and $\bar{\nu}_\mu$, the other components can be removed at the THEIA detector as we elaborate below.

**Reactor, Solar, Geo-, Supernova, and DUNE Beam Neutrinos:** At low energy, there are four neutrino sources from reactor [109,110], solar [111], geo-radioactivity [112,113], and supernova [114,115]. While the solar neutrinos are not anti-neutrinos and hence cannot fake the $\nu_e$ IBD signal, the other three may be incorrectly identified as the signal. Fortunately, all these neutrinos typically have much lower energy than majority of the $\mu$DAR neutrinos. With an energy cut, $E_\nu > 30$ MeV, they can be safely removed [87]. On the other side, the LBNF beam can also contribute $\nu_\mu$ flux. With pulsed beam at much higher energy, a combined cut on the arrival time and energy, $E_\nu < 55$ MeV, will eliminate the contamination from the complementary DUNE experiment.

**Intrinsic $\mu$DAR Beam:** During the production of $\mu$DAR with cyclotron, various neutrinos can be produced via $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ and $\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \nu_\mu(\bar{\nu}_\mu)$. One needs to examine all possible intrinsic backgrounds. The $\nu_e$ flux can produce exactly the same IBD signal and becomes an intrinsic background from the same $\mu$DAR source. There is no way to remove or suppress this background on the detector side. Fortunately, the negatively charged $\pi^-$ is efficiently absorbed by the positively charged nuclei inside the target before decay. So the $\mu^-$ production is much smaller than $\mu^+$ with a suppression as large as $10^{-4}$ [87]. Since the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probability is typically $O(1\%)$ and hence two orders larger, the $\mu^-$ contamination can be safely neglected.

The second beam background is the electron neutrino $\nu_e$ from the same $\mu$DAR. After oscillation, most $\nu_e$ neutrinos can survive and scatter with oxygen ($\nu_e + ^{16}O \rightarrow e^- + ^{16}F$) or carbon ($\nu_e + ^{12}C \rightarrow e^- + ^{12}N$) \(^1\) to produce an electron in the final state. Fortunately, the scattering cross section with oxygen is much smaller than the IBD one in the $\mu$DAR energy range [116,117] and hence can be neglected. Even if some events can still be produced, the double coincidence of neutron capture can remove this background.

The muon neutrino and anti-neutrino cannot experience charged-current scattering with not enough energy to produce a $\mu^\pm$ in the final state. However, all neutrinos and anti-neutrinos can elastically scatter with electron to produce an energetic electron in the final state. But these backgrounds can also be removed by requiring neutron capture [94,97].

**$\nu_e$:** The charged-current quasi-elastic scattering (CC-QES) process $\nu_e + ^{16}O \rightarrow e^- + ^{16}F$ cannot produce a neutron. Then the neutron tagging with WbLS can efficiently remove this background. For the atmospheric $\nu_e$ with higher energies, 30% of the CC-QES scattering process can kick out a neutron and a proton from the nuclei via $\nu_e + ^{16}O \rightarrow e^- + n + ^{15}O^* + p$. Since energy is needed to kick out $n$ and $p$, the primary electron energy is much smaller than the neutrino energy. Mainly $E_\nu \in [80,130]$ MeV will produce electrons with energy reconstructed in the $\mu$DAR range [30,55] MeV. In addition, roughly half of the single neutron events are also accompanied by a monoenergetic photon ($\sim$6.2 MeV) due to the de-excitation of $^{15}O^*$, which can also serve as background veto. Both electron ($T_e$) and proton ($T_p$) kinetic energies deposit as visible energy $E_{vis}$. Requiring $E_{vis} \in [30,55]$ MeV, only 0.07 events per year at THEIA-25 can survive, which is a negligible amount.

Moreover, those neutrinos with even higher energy can also have resonant (RES) and deep inelastic (DIS) scatter-

\(^1\) Since the scintillator composition and fraction for THEIA are not decided yet and only a possible fraction 1% ~ 10% is mentioned [51], we leave the carbon contribution open and focus on the major water target.

© Springer
neutron. The events containing energy window, roughly 30% of CC-RES can have a single used to separate RES and DIS events. In the energy range with at least a pion (π⁰ or π⁺) starting around Eν ~ 200 MeV, and possibly a neutron in the final state. In addition, the interaction can also produce protons in the final state or kick off protons from the nucleus [120]. The neutron capture process emits a single 2.2 MeV photon [97] that can be used to separate RES and DIS events. In the energy range Eν ∈ [200, 600] MeV can contribute to the μDAR IBD energy window, roughly 30% of CC-RES can have a single neutron. The events containing π⁰ can be vetoed by energy cut Evis < 55 MeV, since the π⁰ decays immediately into a pair of photons each with energy Eγ ≥ mπ/2. This reduces the remaining atmospheric νe background by another 50% to only 15% of the total CC-RES. The charged π⁺ first deposits its kinetic energy as scintillation light and then decays at rest to produce a μ⁺, since its decay length is typically at least one order longer than the radiation length in water [121]. Then the energy deposition is composed of five parts: 1) the kinetic energy Tπ of the primary electron; 2) the proton kinetic energy Tp ≡ Eπ − mπ; 3) the π⁺ kinetic energy Tπ ≡ Eπ − mπ; 4) the μ⁺ kinetic energy Tμ = (mπ − mμ)^2/2mπ ≈ 4 MeV that is uniquely determined by the pion decay at rest; and 5) the e⁺ energy Tp⁺ ranging from 0 to mμ/2. Since the positron always annihilation with an environmental electron, we also include the released 2mμ photon energy in Tp⁺ for convenience. For a π⁺ with energy Eπ, the minimal total energy deposit is Eπ > mπ + Tμ + 2mμ ≈ Eπ − 134 MeV if both the primary and Michel electrons have vanishing momentum. Those events with Eπ ≥ 189 MeV can be vetoed by Evis ≥ 55 MeV. Requiring the primary and Michel electrons to be produced at rest is actually very stringent. Even with less energetic π⁺, the atmospheric νe background can be easily vetoed. As shown in the left figure of Fig. 3, once the π⁺, μ⁺, and e⁺ energies are considered, the νe CC-RES event rate is suppressed to negligible level with just around 8 × 10⁻⁴ events/year for Evis ∈ [30, 55] MeV at the THEIA-25 detector. Fig. 3 also shows the CC-DIS case which has only 20% for single neutron events and its contribution is only around 10⁻⁵ event per year. The typical value of the proton kinetic energy is around O(10 MeV) for sub-GeV neutrinos. It can further suppress the background by one order for RES and almost down to zero for DIS, shown as the purple curves of Fig. 3.

Fig. 3 The atmospheric νe (left) and νe (right) background event rate spectrum from CC-RES (solid) and CC-DIS (dashed) interactions as a function of the visible energy Evis at THEIA-25. With energy deposits from the primary electron (Tπ, red), charged π⁺ (+Tπ, blue), μ⁺ (+Tμ, green), the Michel electron (+Tμ, yellow), and proton (+Tp, purple) gradually added up, the νe event spectrum moves out of the IBD energy window [30, 55] MeV to higher energy. Similarly, the νe spectrum has the same feature with the primary electron (Tπ, red), charged π⁻ (+Tπ, blue) or with Cherenkov cut (w/ Cheren. cut, green), and proton (+Tp, purple). Multiplication factors have been used to make the highly suppressed spectrum visible.

νe: The electron anti-neutrino νe that scatters with Hydrogen via CC-QES (or precisely IBD) process is an irreducible background to the IBD signal and there is no experimental solution. It contributes the major background for μTHEIA. Since the direction of the incoming νe is unknown, the scattering angle θe in (5) cannot be correctly measured but only inferred from the μDAR source direction. This wrong scattering angle effect introduces significant smearing for the reconstructed neutrino energy Eν. With more details elaborated in App. A, Fig. 4 shows that the smearing can be as large as 3.4% ~ 5.5% at half height for Evis ∈ [30, 55] MeV. The wrong scattering angle effect is much larger than the detector resolution in Fig. 2 with known direction. With higher neutrino energy, the wrong scattering angle effect becomes more severe. The atmospheric νe CC-QES background is estimated as 1.1 event per year at THEIA-25.

The νe CC-RES and CC-DIS events mainly consist of a π⁰ or π⁻ in addition to the primary positron. Slightly higher than the νe mode, the single neutron events contribute around 50% (60%) of the total CC-RES (CC-DIS) events. The π⁰ can be vetoed by the energetic photon with Eγ ≥ mπ/2. Since the pion decay length is much longer than its radiation length in...
WbLS can identify the invisible muon with triple coincidence of 2.2 $\mu$spheric and [3900, 7150] scintillation ones, respectively. The atmospheric neutrino energy $E_{vis}^{\nu_e}$ corresponds to [2400, 4400] Cherenkov photons and the combined scintillation photon to satisfy $N_{ch}/N_{sc} > 0.4$, the background rate is suppressed to only 2% [94].

$\nu_e$/Cherenkov and scintillation lights, and the 2.2 MeV delayed $\gamma$ from neutron capture). The triple coincidence can essentially remove all the invisible muon background. Even for a conservative study by requiring the Cherenkov photon and the combined scintillation photon to satisfy $N_{ch}/N_{sc} > 0.4$, the background rate is suppressed to only 2% [94].

$\nu_\tau$: Although tau neutrinos also exist in the atmospheric flux, the primary $\tau^\pm$ from the CC scattering can decay into either electron or muon with roughly 17% branching ratio each. However, the $\tau^\pm$ lepton is very heavy with mass at 1.78 GeV. The electron and muon from tau decay is then much more energetic than the IBD signal. A simple cut on the visible energy can effectively veto the atmospheric tau neutrino backgrounds.

Neutral current: In addition to the CC events, the neutral-current (NC) scattering is also a potential background. Since the neutral current process is flavor blind, all flavors can contribute. First, the NC-QES scattering can not contribute as background since there is no positron in the final state. The NC-RES and NC-DIS processes both allow a single $\pi^\pm$ and $\pi^0$ production in the final state. The interaction can also produce neutron and proton. Since $\pi^0$ decays into a pair of photons each with energy larger than $m_\pi/2 \approx 70$ MeV, the deposit energy is already beyond the IBD window of $\mu$ DAR $\nu_e$. Among the remaining $\pi^\pm$, $\pi^-$ is absorbed by nuclei and only $\pi^+$ below Cherenkov threshold can decay through $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ to fake the IBD positron. For NC-RES (NC-DIS) events, around 43% (45%) have a single neutron while 16% (22%) have a single charged $\pi^+$ in the final state. The fraction reduces to 11% (18%) if requir-
Fig. 6 The major background spectrum to the $\mu$DAR IBD signal as a function of the reconstructed neutrino energy $E_{\text{rec}}^\nu$ at THEIA-25 (17 kt) with 10 years running and a baseline of $L = 38$ km. The remaining backgrounds are atmospheric $\bar{\nu}_e$ (blue), invisible muon (red), and the $\mu$DAR $\bar{\nu}_e$ (green).

Fig. 7 The event spectra of $\mu$DAR source at THEIA detector at baseline $L = 38$ km. The four solid lines are $\delta_D = 0^\circ$ (red), $90^\circ$ (blue), $180^\circ$ (green), and $270^\circ$ (purple). The filled regions stand for the backgrounds the atmospheric $\nu_e$ (blue), the atmospheric $\nu_\mu/$$\bar{\nu}_\mu$ (red) and the intrinsic $\nu_e$ from the $\mu$DAR beam (green).

ing both neutron and $\pi^+$. It has a probability for $E_{\text{vis}}$ being within the energy window of interest as shown in Fig. 5. Requiring $E_{\text{vis}} \in [30, 55]$ MeV, the integrated event number gives around 0.03 (0.06) events per year for NC-RES (NC-DIS) at THEIA-25 which is also a negligible amount. Since there is also proton, the background can be further reduced by at least one order if the proton kinetic energy is also taken into account.

Figure 6 shows the three survival background spectra as discussed above, including the beam $\bar{\nu}_e$ with $1.83 \times 10^{25}$ protons on target (POT) as well as the atmospheric $\bar{\nu}_e$ and $\nu_\mu/$$\bar{\nu}_\mu$. While the atmospheric invisible muon background dominates at a Cherenkov detector such as Super-K and Hyper-K [87], it is a small minor contribution at the WbLS detector THEIA even with a conservative selection procedure. The intrinsic $\mu$DAR beam background is even smaller. THEIA is an ideal detector for the CP measurement with $\mu$DAR source.

3.1.3 Event selection

As elaborated above, the IBD signal is quite distinctive at the THEIA detector. All backgrounds can be suppressed to negligibly small amount. Here we summarize the selection criteria for the IBD events,

1. Only one $e$-like Cherenkov ring;
2. For the total visible energy $E_{\text{vis}}$, the number of scintillation photons $N_{\text{sc}}$ is within the range of $[3900, 7150]$ and Cherenkov photons $N_{\text{ch}} \in [2400, 4400]$ that correspond to a $(30 \sim 55)$ MeV positron;
3. The ratio of Cherenkov and scintillation photons $N_{\text{ch}}/N_{\text{sc}}$ is larger than 0.4;
4. Existence of delayed $\gamma$s from neutron capture;

For the above requirements, the energy window used in the criterion (2) can remove reactor, solar, geo-, supernova and DUNE beam neutrino backgrounds. The criteria (2) and (4) as double coincidence is efficient in removing the $\mu$DAR flux background except the intrinsic $\bar{\nu}_e$. The combination of all criteria forms the triple coincidence which is even more powerful to reduce the invisible muon background. Finally, the criteria (1) and (2) can remove the atmospheric CC-RES, CC-DIS, and NC backgrounds.

Note that these criteria are already quite conservative. For the WbLS technique to be used by THEIA, more information can help to distinguish signal from background. Especially, the time information and pulse shape can be used to distinguish the Cherenkov and scintillation lights [94]. This could be extremely useful to further suppress the atmospheric invisible muon background with triple coincidence fully implemented.

In Fig. 7, we show the signal and background event rates for the low-energy mode at $\mu$THEIA. The WbLS can highly suppress the background especially for the “invisible muon” that is reduced to a negligible amount. With CP dependence dominating the event rate, we can expect improvement of the CP sensitivity, which is elaborated later in Sect. 4.

3.2 The high-energy mode

The high-energy LBNF neutrinos from Fermilab to SURF is a wide beam spanning from 0.5 to 5 GeV with peak at 2.5 GeV [44] for both neutrino and anti-neutrino modes. Each mode contains four different flavor components: $\nu_e$, $\bar{\nu}_e$, $\nu_\mu$ and $\bar{\nu}_\mu$. 

$\mu$THEIA-25

$L = 38$ km

$\bar{\nu}_e$

Atmospheric $\nu_e$

$\mu$DAR $\bar{\nu}_e$

$E_{\text{rec}}^\nu$ [MeV]

$0.0$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$

$30$ $35$ $40$ $45$ $50$ $55$

Fig. 6

$E_{\text{vis}}^\nu$ [MeV]

$0.0$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$ $0.7$ $0.8$ $0.9$ $1.0$

$30$ $35$ $40$ $45$ $50$ $55$

Fig. 7

$\delta_D = 0^\circ$

$\delta_D = 90^\circ$

$\delta_D = 180^\circ$

$\delta_D = 270^\circ$

atm $\bar{\nu}_e$

atm $\nu_\mu/$$\bar{\nu}_\mu$

$\mu$DAR $\bar{\nu}_e$

Event Rate [MeV$^{-1}$]

$0$ $1$ $2$ $3$ $4$ $5$ $6$ $7$ $8$ $9$

$E_{\text{rec}}^\nu$ [MeV]

$\bar{\nu}_e$

$\nu_e$/\$\bar{\nu}_e$

$\nu_\mu$/\$\bar{\nu}_\mu$

$\mu$DAR $\bar{\nu}_e$

$\nu_e$/\$\bar{\nu}_e$

$\nu_\mu$/\$\bar{\nu}_\mu$
Since the THEIA detector is at the same SURF experimental site as the DUNE far detectors, it can also probe the LBNF neutrinos. While the event reconstruction of high energy neutrinos at the DUNE liquid Argon detectors has already been studied carefully [122], we focus on the detection at THEIA.

The key element for the neutrino CP measurement is neutrino flavor reconstruction for $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$). It is interesting to see that the low-energy $\mu$DAR neutrinos outside the energy window [0.5 GeV, 5 GeV] cannot contribute as background. Although atmospheric neutrinos can overlap in energy, the pulse shape of the LBNF beam provides an efficient way to suppress the atmospheric backgrounds. Both signal and the remaining background actually come from the same LBNF beam.

With broad energy range, several types of CC scatterings with a target nuclei $N$ can happen. In addition to a charged lepton $\ell_\alpha$, the final state is either a single nuclei $N'$ for the quasi-elastic ($\nu_\alpha + N \rightarrow \ell_\alpha + N'$), nuclei plus mesons for the resonant ($\nu_\alpha + N \rightarrow \ell_\alpha + N' + meson$), nuclei plus hadrons for the deep-inelastic ($\nu_\alpha + N \rightarrow \ell_\alpha + N' + hadrons$) CC scatterings [117]. Typically CC-QES dominates below 1 GeV and CC-RES between 1.2 GeV and $5 \sim 7$ GeV, while CC-DIS takes over above 7 GeV. With the LBNF neutrino beam being below 5 GeV, most of the interactions are CC-QES and CC-RES. In order to make better neutrino reconstruction, it is desirable to distinguish these different CC scattering events.

The following discussions focus on the signal of appearance channels $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. Similar procedures shall also apply for the disappearance channels $\nu_\mu \rightarrow \bar{\nu}_e$ and $\bar{\nu}_\mu \rightarrow \nu_e$. Although the disappearance channels do not contribute significantly to the leptonic CP measurement, they can serve as supplementary probe of the other oscillation parameters and the neutrino flux. Both appearance and disappearance channels are taken into account in our GLoBES simulation in Sect. 4.

### 3.2.1 The CC-QES Category

**Signals:** The CC-QES process has a two-body final state with a primary lepton and a nuclei. A combination of Cherenkov and scintillation lights can achieve outstanding lepton identification as discussed in Sect. 3.1. Similar to the IBD case (5), the neutrino energy can be reconstructed from the charged lepton energy $E_\ell$ (or momentum $|p_\ell|$) and scattering angle $\theta_\ell$ [104],

$$E_\nu^{\text{rec}} = m_f^2 - m_\ell^2 + 2m_\ell^2 E_\ell \cos \theta_\ell,$$

(7)

where $m_f$ is the final-state nucleon mass. On the other hand, the initial nucleon mass $m_i$ always appears together with the binding energy $E_b$ for a nucleon inside $^{16}\text{O}$ nuclei as $m_i' \equiv m_i - E_b$. The binding energy $E_b = 42$ MeV (19 MeV) for the neutrino (anti-neutrino) mode is adopted to make the reconstructed energy peak at the true value for $E_\nu = 2.5$ GeV as shown in Fig. 8. This energy reconstruction formula is similar to the IBD one (5) with the only difference that the initial free proton (Hydrogen) mass $m_p$ is replaced by $m_i'$ to account for the binding energy. Experimentally, there is no efficient way to distinguish whether it is the Oxygen or Hydrogen nuclei that is scattered. Consequently, the neutrino energy reconstruction (7) for Oxygen target is used universally since the mass fraction of Oxygen is eight times larger than Hydrogen in the water target. Similar to the low energy mode, the same Gaussian smearing $7\%/\sqrt{E_{\text{vis}}/\text{MeV}}$ is used for simulating the detector resolution of the deposit visible energy. In addition, we take the angular resolutions $1.48^\circ$ for $e^\pm$ and $1.00^\circ$ for $\mu^\pm$ from SK-IV [123].

Figure 8 shows the reconstructed spectrum for a 2.5 GeV $\nu_e$ ($\nu_\mu$) at the THEIA detector. For the CC-QES events, the reconstructed neutrino energy spectrum is the narrowest whose width at half height is roughly 80 MeV. The high-energy CC-RES or CC-DIS event always has at least one $\pi^\pm/\pi^0$ in the final state which can be used to distinguish from the CC-QES signal. That means the CC-QES events can be separately from the CC-RES and CC-DIS counterparts. For illustration, Fig. 8 also shows the individual features of CC-RES and CC-DIS events separately. The fact that CC-RES and CC-DIS partially overlap with each other will be elaborated in later discussions.

**Backgrounds:** The LBNF beam can contribute intrinsic $\nu_e$ ($\bar{\nu}_e$) background $\nu_e \rightarrow \nu_e$ ($\bar{\nu}_e \rightarrow \bar{\nu}_e$) which is irreducible. For the neutrino (anti-neutrino) mode, the $\nu_e$ ($\bar{\nu}_e$) flux is two orders smaller than the dominant $\nu_\mu$ ($\bar{\nu}_\mu$) [122]. Considering the $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) oscillation probability that is

---

**Fig. 8** The reconstructed energies of a 2.5 GeV $\nu_\mu/\bar{\nu}_\mu$ neutrino at the THEIA detector for different CC scattering processes: QES (red), RES (blue), and DIS (green). The combined total spectrum is plotted in black color. For comparison, both $\nu_\mu$ (solid) and $\bar{\nu}_\mu$ (dashed) modes are plotted. Note that the total spectrum for neutrino mode is normalized and the others weighted by their corresponding cross sections.
typically \( \lesssim 5\% \), the intrinsic background can be comparable or roughly one order smaller than the signal. In other words, the intrinsic beam background is sizable and hence cannot be neglected. Moreover, the beam also has intrinsic \( \nu_e \) (\( \bar{\nu}_e \)) fluxes which are smaller by another order. Although the positron annihilation at the THEIA detector can in principle be used to distinguish the electric charge of the final-state leptons, we assume the \( \bar{\nu}_e \) (\( \nu_e \)) also contributes as background to be conservative. The intrinsic beam backgrounds are shown as the pink filled region in Fig. 9.

For the disappearance channel of \( \nu_\mu \) (\( \bar{\nu}_\mu \)), the misidentification of muon as electron is also a potential background. Nevertheless, the misidentification rate is 0.05% for a muon misidentified as an electron and 0.02% for the converse one [123]. This contribution is much smaller than the intrinsic \( \nu_e/\bar{\nu}_e \) background and hence can be neglected for simplicity.

As for the neutral-current events, the NC-QES does not have charged lepton in the final state which can serve as an effective veto. However, the NC-RES and NC-DIS scatterings have both single and multi-pion final states. The multi-pion background can be removed by simply imposing a single-ring cut. Single pion production has \( \pi^\pm \) or \( \pi^0 \) in the final state. The \( \pi^\pm \) above the Cherenkov threshold can produce a \( \mu^- \)-like ring, since both are heavy particles with similar masses. For \( \pi^+ \) below the Cherenkov threshold, it can experience a chain decay \( \pi^+ \rightarrow \mu^+ \rightarrow e^+ \) with both \( \pi^+ \) and \( \mu^+ \) decaying at rest. Although the final \( e^+ \) can produce an \( e^- \)-like ring, its maximal energy is only \( m_\mu/2 = 53 \) MeV and hence the reconstructed energy is far below the signal energy window \([0.5, 5]\) GeV. Moreover, the invisible \( \pi^+ \) and \( \mu^+ \) can actually be seen by the THEIA detector with scintillation lights before the \( e^+ \) Cherenkov ring, which provides an extra way to veto the invisible \( \pi^+ \) background. Furthermore, the invisible \( \pi^- \) is absorbed in the detector and hence cannot produce an electron to fake signal. Neither \( \pi^+ \) nor \( \pi^- \) can become background. However, the photon pair from \( \pi^0 \) decay can fake an electron. This happens if (1) the two photons have a small opening angle \( \theta_{\gamma\gamma} \leq 17^\circ \) and overlap with each other or (2) one photon is soft enough to be invisible [124]. Roughly 3.3% (16.2%) of NC-RES (NC-DIS) single \( \pi^0 \) events at the peak energy \( E_v = 2.5 \) GeV have overlapping photons. For the soft photon in case (2), we take a conservative \( E_\gamma < 30 \) MeV Cherenkov detection threshold [124]. The soft photon events with \( \theta_{\gamma\gamma} > 17^\circ \) contribute 7.6% (6.8%) of the NC-RES (NC-DIS) scatterings. In total, 10.9% (23.0%) of the NC-RES (NC-DIS) events have a \( \pi^0 \) to fake the electron. The \( \pi^0 \) direction and energy are then used as the electron information to reconstruct the neutrino energy via (7) for small opening angle case. Since the soft photon direction is difficult to reconstruct, the direction of the hard photon is used instead.

**Event selection:** As elaborated above, the CC-QES category has only one primary lepton in the final state. Being different from the IBD signal of the low-energy mode, there is no neutral for double coincidence to significantly suppress the background. The selection criteria for the CC-QES signal events are summarized below,

1. Only one primary \( e^- \)-like Cherenkov ring with \( |p_e| > 100 \) MeV. For the disappearance channel the primary muon is selected with \( \mu^- \)-like Cherenkov ring with \( |p_\mu| > 200 \) MeV.
2. Reconstructed neutrino energy inside the range \([0.25, 5]\) GeV.
3. No extra Cherenkov light from mesons.

The event spectra of the CC-QES category after selection and the corresponding backgrounds are shown as functions of the reconstructed neutrino energy \( E_v^{\text{rec}} \) in Fig. 9. For both neutrino and anti-neutrino modes, the signal lines clearly show the CP dependence. Taking \( \delta_D = 90^\circ \) for illustration, the corresponding blue line is at the bottom for the neutrino mode while it is at the top for the anti-neutrino one. The opposite feature happens for the other maximal CP phase \( \delta_D = 270^\circ \). This is a reflection of the fact that the CP-violating term \( \sin \delta_D \) in (3) differs by a sign. The NC background is contributed by all flavors and the CC one by the intrinsic \( \nu_e/\bar{\nu}_e \) beam background in addition to the misidentified \( \nu_\mu/\bar{\nu}_\mu \) comes from the disappearance channels. While the signal and CC backgrounds can extend to 5 GeV, the NC background mainly contributes below 3.5 GeV since typically more particles are produced to split the energy. The two NC background peaks here are produced due to the soft photon contribution (lower energy peak) and the small opening angle contribution (higher energy peak), respectively. For CC-QES, the CC background spectrum is quite flat and the NC backgrounds dominate below 2.5 GeV. Around the peak energy, \( E_v^{\text{rec}} = 2.5 \) GeV, the signals dominate to provide a good CP sensitivity. Between neutrino and anti-neutrino modes, the typical event rate differs by a factor around 2.5.

### 3.2.2 The CC-RES category

**Signals:** The CC-RES process for the appearance signal \( \nu_\mu \rightarrow \nu_e \) (\( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)) has 1 electron (positron) plus 1 pion (\( \pi^\pm \) or \( \pi^0 \)). Therefore, the CC-RES event can be distinguished from the CC-QES one by ring counting if the final-state pion is a charged one \( \pi^\pm \). For \( \pi^0 \), the major contribution to its decay final state is two resolved photons. Nevertheless, both CC-RES and CC-DIS can produce pion in the final state to have some overlap with each other. But it is still possible to partially distinguish CC-RES CC-DIS events by the number of pions. Our simulation shows that for the neutrino (anti-neutrino) mode at the peak energy of 2.5 GeV, only 15% (11%) of the CC-RES events have multiple pions, while CC-
DIS reaches 64% (48%). Hence the CC single-pion event can be categorized as CC-RES while the multi-pion one as CC-DIS.

The energy reconstruction formula (7) for CC-QES can no longer apply due to different particles in the final state. Instead of having nucleons in the initial and final states, CC-RES has heavy Delta baryons as intermediate resonance. Most of the pions are produced from Delta decays $\Delta^{++} \rightarrow p + \pi^+$, $\Delta^{+} \rightarrow p + \pi^0$, and $\Delta^{-} \rightarrow n + \pi^-$ [120]. Usually the proton is not energetic enough to produce a Cherenkov ring and hence cannot be uniquely identified but only leaves some scintillation lights. Observationally, the $\nu_e/\bar{\nu}_e$ CC-RES has a primary lepton and a pion ($\pi^0$ or $\pi^\pm$). A reasonable neutrino energy reconstruction for CC-RES is [104],

$$E_{\nu}^{\text{rec}} = \frac{m^2_\Delta - m^2_p - m^2_{\pi^\pm} + 2m_p E_\ell}{2(m_p - E_\ell + |p_\ell| \cos \theta_\ell)}. $$

Comparing with (7), the final-state nucleon mass is replaced by the Delta baryon mass, $m_\Delta = 1.232$ GeV. As demonstrated with blue lines in Fig. 8, the CC-RES energy reconstruction formula (8) gives a correct peak position. However, for the high-energy scattering process, not only $\Delta$ is produced but also other resonant particles like $N(1440)$ which can also produce pion particles. So (8) cannot describe all the resonant processes exactly and gives a wider distribution than CC-QES.

**Backgrounds:** The beam background for the CC-RES category has three major contributions. First, the beam electron-flavor neutrinos (anti-neutrinos) of the disappearance channel $\nu_e \rightarrow \nu_e$ ($\bar{\nu}_e \rightarrow \bar{\nu}_e$) via either CC-RES or CC-DIS scatterings contributes as irreducible background with the same $1\ell\pi\tau$ final state. Being depicted as the pink regions in Fig. 10, this type of background for neutrino (left) and anti-neutrino (right) modes contribute roughly 13.9% and 14.9% of the total detected events for $\delta_D = -90^\circ$.

The second beam background comes from the muon neutrinos $\nu_\mu$ interacting via CC-RES or CC-DIS to produce $\mu^-$ and $\pi^0$ in the final state. The $\pi^0$ can fake electron if one of the decay photons is soft ($E_{\gamma} < 30$ MeV) or the two photons are almost collinear ($\theta_{\gamma\gamma} < 17^\circ$). For the peak energy $E_\nu = 2.5$ GeV, only 7.6% (13%) of such $\pi^0$ can fake an electron. In addition, the muon needs to be misidentified as a charged pion. With both misidentification, the beam $\nu_\mu$ can fake the CC-RES signal. Although the new reconstruction algorithm of fitQun [123] shows some capability of separating pion from muon at T2K using the hadronic kinks in the pion propagation [125], we assume $\mu^-$ and $\pi^+$ cannot be separated to be conservative. Not to say, $\mu^-$ has an electron but $\pi^+$ has a positron in their decay products, respectively, and the $e^+e^-$ annihilation photons from $\pi^\pm$ can also provide a distinguishable feature. Note that there is no beam $\bar{\nu}_\mu$ CC background for the anti-neutrino mode. This is because the $\pi^-$ in the $\bar{\nu}_e$ CC-RES signal is absorbed in the detector and does not produce a delayed Michel electron which is different from the beam $\bar{\nu}_\mu$ CC background for the anti-neutrino mode. This is the case because the $\pi^-$ in the $\bar{\nu}_e$ CC-RES signal is absorbed in the detector and does not produce a delayed Michel electron which is different from the beam $\bar{\nu}_\mu$ CC background for the anti-neutrino mode. This is the reason why the $\bar{\nu}_e$ CC-RES signal is absorbed in the detector and does not produce a delayed Michel electron which is different from the beam $\bar{\nu}_\mu$ CC background for the anti-neutrino mode.

The third part of the background comes from NC-RES and NC-DIS with two pions ($2\pi^0$ or $\pi^0 + \pi^\pm$) in the final state. In such case, one $\pi^0$ decays into unresolved photons to fake...
the primary electron signal while the other pion coincides with the one in the signal. The estimated NC background is shown as the orange region of Fig. 10. We can see that the reconstructed NC background mostly occur at $E_{\text{rec}}^\nu < 3$ GeV.

**Event selection:** In summary, the selection criteria for CC-RES signal is,

1. Only one primary $e$-like Cherenkov ring with $|p_e| > 100$ MeV.
2. Reconstructed neutrino energy inside the range [0.25, 5] GeV.
3. Single pion particle in the final state. To be conservative, the events with a charged $\pi^\pm$ or a neutral $\pi^0$ are not mixed into a single CC-RES category. More detailed study is necessary for maximizing the CP sensitivity.

Figure 10 depicts the $\nu_e$ (left) and $\bar{\nu}_e$ (right) CC-RES signal and the corresponding backgrounds. Comparing the two panels, we can see that the neutrino event rates are typically 4 ~ 5 times larger than its anti-neutrino counterparts. This is a direct consequence of the relatively larger cross section for neutrino than anti-neutrino. For backgrounds, the neutrino mode has four components while the anti-neutrino mode has only three. This is because the disappearance channel $\nu_\mu \rightarrow \bar{\nu}_\mu$ has $\mu^+$ and Michel $e^+$ to be distinguished from $\pi^-$ of the $\bar{\nu}_e$ CC-RES signal. For neutrino mode, all backgrounds contribute roughly the same size while for anti-neutrino the NC and intrinsic $\nu_e$ background dominate. Around the major peak, $E_{\text{rec}}^\nu \approx 2$ GeV, the signal is slightly larger than the background for neutrino and anti-neutrino modes.

3.2.3 The CC-DIS category

**Signals:** In order to enhance the statistics, we also include CC-DIS events as a separate category. As mentioned in the previous CC-RES section, the events identified as CC-DIS are those with an $e$-like ring accompanied with more than one pion in the final state. The majority of events contains two pions. The 2-pion events include any of the following combination $(\pi^\pm, \pi^\pm)$, $(\pi^\pm, \pi^0)$ and $(\pi^0, \pi^0)$. Similar to the CC-RES category, only a pair of resolved photons are identified as $\pi^0$. For events with one or more $\pi^0$ and consequently 3 or more $e$-like rings in the final state, we take the most energetic $e$-like ring as the primary electron/positron. Then the neutrino energy can be reconstructed using the same (8). Nevertheless, the scattering process of CC-DIS differs a lot from CC-RES and the reconstructed neutrino energy is almost flat as shown with green curves in Fig. 8.

**Backgrounds:** The background of the CC-DIS process also has three major contributions. The first is the irreducible one from the electron-flavor neutrinos via the CC-RES or CC-DIS with the $1eN\pi$ final state with $N > 1$. Being depicted as the pink regions in Fig. 11, this background is the largest contribution for $E_{\text{rec}}^\nu > 1.5$ GeV and can be as large as 45% of the total events at 2.5 GeV for $\delta_D = -90^\circ$.

The second component comes from muon neutrinos $\nu_\mu$ interacting via CC-RES or CC-DIS to produce a $\mu^-$, a $\pi^0$ and one or two other pions of any type. While $\mu^-$ is misidentified as a pion, the $\pi^0$ can be misidentified as an electron if it has a soft photon $E_{\gamma} < 30$ MeV or the two photons are almost collinear, $\theta_{\gamma\gamma} < 17^\circ$. The multi-pion production of two or three pions occurs for 4% of the total $\nu_\mu$ CC events. In addition, the misidentification of the $\pi^0$ as an electron occurs in
Fig. 11 The event spectra of CC-DIS signal and its background at the THEIA-25 detector with LBNF neutrino (left) and anti-neutrino (right) beams. Four solid lines are used for $\delta_D = 0^\circ$ (red), $90^\circ$ (blue), $180^\circ$ (green), and $270^\circ$ (purple). For comparison, the filled regions stand for the backgrounds of the intrinsic $\nu_e/\bar{\nu}_e$ beam CC (pink), the $\nu_\mu$ CC (green), the $\nu_e/\bar{\nu}_e$ CC-RES (blue), and the NC multi-pion events (yellow). Both neutrino and anti-neutrino modes have 6.5 years running

17% (9%) of the events for 2-pion (3-pion) final state. This $\nu_\mu$ background is shown as green curve in the left panel of Fig. 11 only for the neutrino mode. Again, there is no $\nu_\mu$ CC background due to the delayed Michel electron veto.

The third component is the NC multi-pion background of $\pi^0\pi^0\pi^\pm$ with $x, y = 0, \pm$. The triple-pion final state is reconstructed as background if one $\pi^0$ is misidentified as electron. This occurs for only 0.1% of the total NC events at the 2.5 GeV peak energy, which is represented as the orange region in Fig. 11. It is most relevant at low energies, $E_{\text{rec}} \nu < 1.5$ GeV where the CP value has small impact.

**Event selection:** We can see that the CC-DIS category has much similarity as the CC-RES one with single-pion final state replaced by the multiple one. Although the CC-DIS signal would not contribute much as shown in Fig. 8, this separation reveals the feature of various channels. Below we summarize the selection criteria for the CC-DIS signal,

1. Only one primary $e^-$ like Cherenkov ring with $|p_e| > 100$ MeV. In the presence of multiple $e^-$ like rings, the most energetic one is identified as the primary lepton.
2. Reconstructed neutrino energy inside the range $[0.25, 5]$ GeV.
3. At least two pions in the final state with energy $\pi (|p_\pi|) > 200$ MeV. Similar to the CC-RES category, all CC-DIS events are put into a single category without division according to the final-state pions. This conservative treatment can be further improved with more careful studies.

The CC-DIS signals and their backgrounds are shown in Fig. 11. Different from the CC-QES in Fig. 9 and CC-RES in Fig. 10, the CC-DIS signal can be smaller than the background, especially for the neutrino mode. Around the peak, $E_{\text{rec}} \nu \approx 2.5$ GeV, the dominant background comes from the CC-RES multi pion background which can reach more than 50% of the total events at $E_{\text{rec}} \nu \approx 2$ GeV. The anti-neutrino case is slightly better with signal still dominating around the peak energy. In addition, the three background components for the $\bar{\nu}_e$ mode all have sizable contributions. The NC background background dominates for $E_{\text{rec}} \nu < 2$ GeV, while the CC-RES dominates at intermediate energies $2\, \text{GeV} < E_{\text{rec}} \nu < 3.5$ GeV and the intrinsic $\nu_e$ dominates for $E_{\text{rec}} \nu > 3.5$ GeV. Nevertheless, $\nu_e$ events are 7 times larger than $\bar{\nu}_e$.

### 4 CP sensitivity with $\mu$THEIA and DUNE

As discussed in Sect. 2, the combination of $\mu$THEIA and DUNE is expected to improve the CP measurement. Below we give a quantitative estimation. We first summarize the experimental setups in Sect. 4.1 for completeness and then establish the $\chi^2$ formalism in Sect. 4.2. In order to maximize the CP sensitivity, Sect. 4.3 explores the optimal baseline between the $\mu$DAR source and the THEIA detector for $\mu$THEIA. Based on these, the CP sensitivity and the influence of matter effect are studied in Sect. 4.4.

#### 4.1 Experimental setups

The $\mu$DAR neutrinos are typically produced by cyclotrons [126]. We adopt the configuration that the $\mu$DAR flux is generated by a 9 mA proton beam with 800 MeV protons hitting the high-Z target to deliver $1.83 \times 10^{25}$ POT in 10 years [87].
Since the μDAR cyclotron is not a pulsed beam, it is possible to achieve almost full duty factor. Of the charged pions produced by proton hitting the target, π− is mainly absorbed by the positively charged nuclei while π+ first loses energy in the thick material and then decays at rest to produce μ+. Similar process of decay at rest also happens for μ+. Consequently, the μDAR neutrino spectra are well defined and predicted by the SM interactions.

For DUNE, the LBNF neutrinos are produced by 120 GeV protons with 1.2 MW beam power and the flux can reach 1.1 × 10^{21} POT/year. We take 6.5 years running for each neutrino and anti-neutrino mode [44]. With completely different energy windows [30, 55] MeV for μDAR and [0.5, 5] GeV for LBNF, there is no energy overlap between the low-energy μDAR flux and the high-energy LBNF one. So the two fluxes can run simultaneously.

For THEIA detector we consider two possibilities, THEIA-25 with a fiducial mass of 17 kt and THEIA-100 with a fiducial mass of 70 kt [51]. The DUNE detector has four modules, each with 10 kt fiducial mass [122]. Both THEIA and DUNE detectors are at the same SURF site and 1289 km away from the LBNF source. To install the THEIA detector, one of the DUNE modules needs to be replaced. As mentioned in Sect. 2, the average matter density 2.85 g/cm^3 is used for both DUNE and μTHEIA.

In the following, we consider three combinations: (1) the full DUNE configuration with 4 far detector modules, (2) the reduced DUNE with 3 far detector modules and μTHEIA-25, and (3) the reduced DUNE and μTHEIA-100. The high-energy mode (HEM) can be detected by both the DUNE and THEIA detectors while the low-energy mode (LEM) only applies for the THEIA detector as discussed in Sect. 3. The energy window, fiducial mass, baseline, and running time of each experimental configurations are summarized in Table 1.

4.2 Simulation and \( \chi^2 \) analysis

We then use GLoBES [127, 128] to simulate the event rates and evaluate the CP sensitivities. A quantitative evaluation is realized by minimizing the \( \chi^2 \) function that contains three contributions,

\[
\chi^2 \equiv \chi^2_{\text{stat}} + \chi^2_{\text{sys}} + \chi^2_{\text{para}},
\]

for statistical (\( \chi^2_{\text{stat}} \)) and systematical (\( \chi^2_{\text{sys}} \)) uncertainties in addition to the prior constraint on the oscillation parameters (\( \chi^2_{\text{para}} \)).

For event rate > 10 in a single bin, a Gaussian \( \chi^2 \) is much more convenient with analytical fit [100, 129, 130]. However, the event rates considered in this paper are not large enough.

To make the sensitivity evaluation exact, the statistical part in the first term takes the Poisson form,

\[
\chi^2_{\text{stat}} \equiv 2 \sum_{i} \left[ (1 + a_{\text{sig}})N_{i}^{\text{sig}} + (1 + a_{\text{bkg}})N_{i}^{\text{bkg}} - N_{i}^{\text{data}} \right] ^2 - N_{i}^{\text{data}} \ln \left( \frac{(1 + a_{\text{sig}})N_{i}^{\text{sig}} + (1 + a_{\text{bkg}})N_{i}^{\text{bkg}}}{N_{i}^{\text{data}}} \right),
\]

where \( N_{i}^{\text{data}} \), \( N_{i}^{\text{sig}} \) and \( N_{i}^{\text{bkg}} \) are the pseudo data, signal and background event numbers in the \( i \)th bin, respectively. The coefficients \( a_{\text{sig}} \) and \( a_{\text{bkg}} \) are nuisance parameters for the signal and background normalizations, respectively.

The \( \chi^2_{\text{sys}} \) term contains the uncorrelated Gaussian priors of signal and background normalizations. We take \( \sigma_{\text{sig}} = 5\% \) and \( \sigma_{\text{bkg}} = 10\% \) for each channel of both the low- and high-energy modes that are observed at THEIA detector. For the low-energy mode, the normalization uncertainties are the same as the configuration given in [87] while the uncertainties are more conservative than the values (\( 2\% \) for \( \nu_e \) and \( 5\% \) for \( \bar{\nu}_e \)) used in [51]. The systematics of the LBNF beam detection by the DUNE detector are described by the official configuration files [122].

Finally, \( \chi^2_{\text{para}} \) contains the prior information on the oscillation parameters. Their best fit values are obtained from the global fit result [131].

\[
\begin{align}
\sin^2 \theta_4 &= 0.318, \quad \sin^2 \theta_0 = 0.574, \quad \sin^2 \theta_r = 0.022, \\
\Delta m^2_1 &= 7.50 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_2 = 2.55 \times 10^{-3} \text{ eV}^2,
\end{align}
\]

where we take NO in our study.

Among these oscillation parameters, the solar mass squared difference \( \Delta m^2_1 \) and the solar mixing angle \( \theta_4 \) are kept fixed throughout our analysis. On one hand, the contribution of the solar mass squared difference \( \Delta m^2_2 \) enters via a coefficient parameter \( \alpha \equiv \Delta m^2_2 / \Delta m^2_3 \) as shown in (3). The parameter \( \alpha \) has a small value (\( \approx 0.03 \)) and the error in \( \Delta m^2_2 \) has an even smaller contribution. Hence it can be neglected comparing with the \( O(1) \) uncertainty on the CP phase. On the other hand, the solar mixing angle appears in the first and the third term on the right-hand side of (3). Since the current prior on the solar mixing angle is roughly 3\%, it can also be neglected for the study of the large CP phase uncertainty.

Moreover, the next generation reactor neutrino experiment like JUNO [63] will provide a sub-percent uncertainty on \( \theta_4 \) that is negligibly small.

For the other mixing parameters, the reactor mixing angle \( \theta_r \), the atmospheric mixing angle \( \theta_0 \), and the atmospheric mass-square difference \( \Delta m^2_{\text{atm}} \) are treated as free parameters. We use the marginalized one-dimensional \( \chi^2 \) curves [131] as our priors, \( \chi^2_{\text{para}} \equiv \chi^2_{\theta_r} + \chi^2_{\theta_0} + \chi^2_{\Delta m^2_{\text{atm}}} \). Due to the existing tension between the T2K and NOvA results, which is discussed in Sect. 1, we do not include any prior on \( \delta_D \).
Table 1 The energy window, fiducial mass, running time, baseline, and matter density for the three experimental setups considered in this paper

|                     | DUNE          | DUNE + μTHEIA-25/100 |
|---------------------|---------------|----------------------|
|                     | (HEM)         | (HEM)                |
|                     | (HEM)         | (LEM)                |
| Energy Window (GeV) | [0.5, 5]      | [0.5, 5]             |
|                     | [0.03, 0.055] |                      |
| Fiducial Mass (kt)  | 40            | 30+17/70             |
|                     | 17/70         | 17/70                |
| Running Time (νy, νy) | (6.5, 6.5)  | (6.5, 6.5)           |
|                     | (0, 10)       |                      |
| Baseline (km)       | 1289          | 1289                 |
|                     | ??            |                      |
| Density (g/cm³)     | 2.85          | 2.85                 |

Fig. 12 The CP phase uncertainty, ΔδD, for the combination of DUNE and THEIA as a function of the μTHEIA baseline L for true values δDtrue = 0° (red), 90° (blue), 180° (green), 270° (purple), and 150° (orange). While the LBNF flux runs for 6.5 years each in the neutrino and anti-neutrino modes, μDAR provides 10 years for the anti-neutrino mode. The larger DUNE + μTHEIA-100 (thick lines) has much smaller CP uncertainty than DUNE + μTHEIA-25 (thin lines). To illustrate the matter effect on the CP uncertainty, simulations with fixed q = 1 and a 10% prior on q are shown in the left and right panels, respectively.

Since the matter effect is a natural source of fake CP, we take its uncertainty by including a parameter q to scale its average value defined in Sect. 2, A → qA [132]. The average matter density corresponds to q = 1 while the vacuum case takes q = 0. Note that q is undetermined and hence treated as the fifth free parameter. In the following discussions, we consider two different scenarios for its uncertainty: fixed q = 1 (no uncertainty) and a conservative 10% Gaussian uncertainty. For comparison, previous studies have used 1% ∼ 2% uncertainty [45,133–135].

The CP uncertainty ΔδD is defined as the half-width of the Δχ² = χ²(δD) − χ²min = 1 band where χ²min corresponds to the best-fit value δDBF of the CP phase. Since our simulation uses pseudo-data, the best-fit value is the same as the true value, δDtrue. Note that it is not necessary for the χ²(δD) function to be symmetric around the minimum. For this case, the previous definition, Δχ² = 1, gives two boundaries below (δD) and above (δD) the best-fit value δDBF. Then we take the average deviation as the CP uncertainty, ΔδD ≡ (δD − δD)/2.

4.3 Baseline options of μTHEIA

The baseline between the μDAR source and the THEIA detector can significantly affect the CP uncertainty ΔδD. Figure 12 shows ΔδD as a function of the μTHEIA baseline L in the range from 10 km to 80 km. Since the true value of the CP phase δDtrue is unknown, it needs to be varied. Four typical CP values δDtrue = 0°, 90°, 180° and 270° are chosen for illustration. In addition, δDtrue = 150° is not only preferred by the current result of NOνA but also significantly affected by the matter effect uncertainty as elaborated in Sect. 4.4. So we also show the δDtrue = 150° curve for comparison. Both THEIA-25 (thin lines) and THEIA-100 (thick lines) are considered. With larger detector size, the CP uncertainty decreases but the baseline dependence follows the same trend.

To see the impact of matter effect on the optimal baseline, the left panel of Fig. 12 is obtained by fixing q = 1 while the right one takes q = 1 ± 0.1. For both cases, the result shows two local minima in the CP uncertainty for maximal CP violation. One is around L = 38 km and the other around L = 55 km. The longer one, L = 55 km, is a local optimal...
option for $\delta_D = 270^\circ$ that is preferred by the T2K measurement. Although the true local minimum for vanishing CP violation cases $\delta_D = 0^\circ$ and $180^\circ$ actually happens with $L \gtrsim 65$ km, the difference in $\chi^2$ is not significant while the maximal CP violation cases $\delta_D = 90^\circ$ and $270^\circ$ (or equivalently $-90^\circ$) become much worse. Since the data-driven $\delta_D = -90^\circ$ is of larger interest, $L = 55$ km is preferred than the longer 65 km. For the shorter one, the choice is more difficult. The global minimum around $L \approx 38$ km for $\delta_D = \pm 90^\circ$ is very close to the global maximum for the vanishing CP violation cases. So choosing $L = 38$ km needs to pay too much price and we take $L = 30$ km to balance among various CP values. Our simulations take these three baselines $L = 30$ km, 38 km, and 55 km as possible options. The final choice is up to the on-going T2K and NOvA experiments. The comparison between the left (fixed $q = 1$) and right (10% uncertainty around $q = 1$) panels of Fig. 12 shows that although the uncertain matter effect contaminates the CP sensitivity, increasing $\Delta \delta_D$ by 3 or 4 degrees to be exact, it does not affect the optimal baselines.

The optimal baseline options $L = (30, 38, 55)$ km for DUNE $+$ 3THEIA are all different from the TNT2HK one $L = 23$ km [87]. Not just the baseline length is different, but also TNT2K/TNT2HK obtains only a single local minimum. The key feature is the atmospheric invisible muon background. Since both SK and HK are water Cherenkov detectors, the atmospheric invisible muon dominates the background. As $L$ increases, the beam flux decreases with $1/L^2$, but the atmospheric background remains the same and eventually dominates the statistics. So the relatively large amount of the atmospheric background limits the optimal baseline length. For comparison, 3THEIA with WbLS reduces the invisible muons to negligible amount as shown in Fig. 7. Therefore, 3THEIA can have longer baseline than TNT2K/TNT2HK to optimize the CP sensitivity.

While the options $L = 30$ km and 38 km are not so far from the 23 km of TNT2K/TNT2HK and hence easier to understand, the longer baseline $L = 55$ km also achieving comparable CP sensitivities seems counter-intuitive. From 30 to 55 km, the flux decreases quadratically with distance and is suppressed by a factor $(30/55)^2 \approx 0.3$. However, this flux reduction is compensated by the increasing oscillation amplitude. To make this feature explicit, we show in the left panel of Fig. 13 the oscillation probability difference, $\Delta P_{\nu_\mu \rightarrow \bar{\nu}_e}(\equiv P_{\nu_\mu \rightarrow \bar{\nu}_e}^{\text{fit}} - P_{\nu_\mu \rightarrow \bar{\nu}_e}^{\text{true}})$, between the true ($\delta_D^{\text{true}}$) and fit ($\delta_D^{\text{fit}}$) CP values,

$$\Delta P_{\nu_\mu \rightarrow \bar{\nu}_e} = 16\Delta \delta_D \sin\Delta \delta_D \sin \left( \frac{2\Delta_a - 2\delta_D^{\text{true}} - \Delta \delta_D}{2} \right).$$ (12)

Note that this formula is obtained from (3) in the vacuum limit ($A \rightarrow 0$). The oscillation and CP phases are defined as $\Delta_{a,s} \equiv \Delta m^2_{a,s} L/4E_\nu$ and $\Delta \delta_D \equiv \delta_D^{\text{fit}} - \delta_D^{\text{true}}$. To further illustrate the CP sensitivity, the parameter $(\Delta P_{\nu_\mu \rightarrow \bar{\nu}_e}/P_{\nu_\mu \rightarrow \bar{\nu}_e})^2$ has a similar form as the $\chi^2$ calculation is also calculated in the right panel of Fig. 13. For convenience, we name $(\Delta P)^2/P$ as pseudo-$\chi^2$ at the oscillation probability level. Both variables are shown as a function of the atmospheric oscillation phase $\Delta_a (\equiv \Delta m^2_{a} L/4E_\nu)$ for five different true CP values, $\delta_D^{\text{true}} = 0^\circ, 90^\circ, 180^\circ, 270^\circ$, and $150^\circ$. The differ-

Fig. 13 Left: The oscillation probability difference $\Delta P_{\nu_\mu \rightarrow \bar{\nu}_e}$ between the true ($\delta_D^{\text{true}}$) and fitting ($\delta_D^{\text{fit}}$) CP values as a function of the atmospheric oscillation phase $\Delta_a (\equiv \Delta m^2_{a} L/4E_\nu)$. Five different true CP values $\delta_D^{\text{true}} = 0^\circ$ (red), $90^\circ$ (blue), $180^\circ$ (green), $270^\circ$ (purple), and $150^\circ$ (orange) are shown for comparison. The fitting values $\delta_D^{\text{fit}}$ are assigned 10$^\circ$ larger than the corresponding $\delta_D^{\text{true}}$ values. The neutrino energy varies within [30, 55]MeV while the baseline is fixed at three different baselines, $L = 30$ km (red region), 38 km (purple region), and 55 km (light green region). Right: The relative difference $(\Delta P_{\nu_\mu \rightarrow \bar{\nu}_e}^2/P_{\nu_\mu \rightarrow \bar{\nu}_e}^2)$ with the same setups.
ence between the fitting and true CP values are assigned to have $\Delta'\delta_D = 10^\circ$ for illustration.

The left panel shows that the second peak of the oscillation probability difference $\Delta P$ is much larger than the first one and the relative size between the pseudo-$\chi^2$ peaks in the right panel is also significantly enhanced. Take the $\delta_D^{\text{true}} = 0^\circ$ curve in the right panel as an example, the second peak is 3 times of the first one which can roughly compensate the flux suppression ($\sim 0.3$). Similar feature applies also for the other true CP values. This explains why the local minimum around $L = 55$ km has roughly the same value at $L = 30$ km.

The three filled regions correspond to different baselines $L = 30$ km, 38 km, and 55 km, respectively, while the $\mu$DAR neutrino energy $E_\nu$ spans a wide range of $[30, 55]$ MeV. With $\Delta_a$ inversely proportional to $E_\nu$, the left boundary of each region corresponds to the upper energy limit 55 MeV and the right one to the lower limit 30 MeV. Since the $\bar{\nu}_\mu$ spectrum from $\mu$DAR source peaks at the upper limit [87], the left sides of the filled regions give the largest contribution. This important feature can explain the location of those local minimums in Fig. 12. For example, the left side of the pink region for $L = 30$ km covering the first oscillation peak/valley of $\delta_D^{\text{true}} = 0^\circ$ in Fig. 13 explains why this baseline corresponds to the best sensitivity of this true CP value in Fig. 12. The same thing happens for $L = 38$ km with the left side of the purple region covering the $\delta_D^{\text{true}} = \pm 90^\circ$ peak/valley in Fig. 13 to justify the local minimum in Fig. 12. Not to say the left side of the light green region of $L = 55$ km covers the second peak/valley of $\delta_D^{\text{true}} = \pm 90^\circ$ in Fig. 13.

4.4 CP sensitivity and matter effect

As emphasized in earlier discussions, the CP sensitivity suffers from matter effect contamination. The DUNE + $\mu$THEIA configuration we propose in this paper can overcome this issue to provide a clean measurement of the Dirac CP phase $\delta_D$. Figure 14 shows the CP uncertainty $\Delta \delta_D$ as a function of the true value $\delta_D^{\text{true}}$ for three $\mu$THEIA benchmark baselines, $L = 30$ km (top), 38 km (middle), and 55 km (bottom) found in the previous Sect. 4.3. For each baseline, we consider several scenarios: (1) DUNE alone with 4 modules, DUNE with 3 modules and (2) $\mu$THEIA-25 or (3) $\mu$THEIA-100. More details are summarized in Table 1.

In all panels, the DUNE configuration contains the full 40 kt detector and the blue curves do not change with varying $L$ since it does not depend on $\mu$THEIA baseline. For fixed matter effect ($q = 1$), the CP uncertainty peaks around $\delta_D^{\text{true}} \approx 70^\circ$ and $250^\circ$ which are consistent with $[65, 133, 135, 136]$ where either fixed matter effect or just 2% uncertainty is adopted. Slightly shifted peaks at $\delta_D^{\text{true}} \approx 90^\circ$ and $270^\circ$ are obtained and attributed to different treatment of systematics in [137].

![Fig. 14](image-url)
From DUNE alone to DUNE + \( \mu \)THEIA-25, the CP uncertainty \( \Delta D \) significantly reduces by roughly 1/4. This is especially true around the maximal CP values, \( D_{\text{true}} = \pm 90^{\circ} \) since the \( \mu \)DAR spectrum is especially wide to provide both \( \cos D \) and \( \sin D \) terms. With only \( \sin D \) term, the CP uncertainty around the maximal CP phase is intrinsically large, \( \Delta D \propto 1/\cos D \). But a wide spectrum can also introduce a large enough \( \cos D \) term to make the CP uncertainty decrease. Previous study shows that this feature is expected to appear when \( \mu \)DAR flux is added to supplement the narrow beam accelerator experiments, such as TNT2K/TNT2HK \([40–42,87]\). Nevertheless, the result turns out that this is also true for the addition of \( \mu \)THEIA to DUNE, although the LBNF flux spectrum is already quite wide. Adding a larger \( \mu \)THEIA-100 can even further reduce the CP uncertainty to almost only 5° for the best case.

As expected, the matter effect can fake the CP violation and hence its uncertainty can significantly modify the CP uncertainty. In addition to the fixed \( q = 1 \) scheme (solid lines), Fig. 14 also shows the results obtained with 10% uncertainty in the matter effect or equivalently \( q = 1 \pm 0.1 \) (dashed lines). With \( q \) relaxed, the CP uncertainty becomes much worse especially around the vanishing CP violation, \( D_{\text{true}} \approx 0^{\circ} \) or 180°. The most significantly affected point is around \( D_{\text{true}} \approx 150^{\circ} \) or 330°. With the uncertainty of matter effect taken into account, the improvement brought by \( \mu \)THEIA is even more significant. This is exactly because of the fact that matter effect plays more important role at DUNE with much higher energy than the low-energy \( \mu \)THEIA. With \( \mu \)THEIA added, even switching on matter effect uncertainty would not make the situation much worse. It also is interesting to see that the CP uncertainty around the maximal CP violation is almost not affected by switching on/off the matter effect uncertainty. The \( \mu \)THEIA improvement is quite stable against matter effect.

To understand the interplay between the Dirac CP phase and the matter effect qualitatively, Fig. 15 shows the partial derivative \( \partial P/\partial \delta_D \) of oscillation probability (3) with respect to the Dirac CP phase \( \delta_D \) and \( \partial P/\partial q \) to the matter effect parameter \( q \) at \( q = 1 \) as a function of neutrino energy \( E_\nu \). The matter effect mimics the CP effect quite well at \( D_{\text{true}} = 150^{\circ} \) with the two derivative curves having similar shapes and peak positions. For comparison, the two derivatives have very different features at 70°.

Note that the green curves with DUNE and \( \mu \)THEIA-100 are much more flat than the original DUNE alone. For most of the parameter space, the CP uncertainty is better than 8°. Among the three panels of Fig. 14, the \( L = 38 \) km one has the most flat CP uncertainty curves for the DUNE + \( \mu \)THEIA-100 configuration. Especially, the CP certainty is always better than 8° no matter what is the value of \( D_{\text{true}} \). In this sense, \( L = 38 \) km is probably the optimal baseline for \( \mu \)THEIA.

To further illustrate the advantages of the DUNE + \( \mu \)THEIA combination, we compare with other existing experiments or designs in Fig. 16. The first two rows show the latest measurements from the NOvA (\( \delta_D = 148^{\circ}+49^{\circ}-157^{\circ} \)) \([25]\) and T2K (\( \delta_D = -108^{\circ}+40^{\circ}-33^{\circ} \)) \([24]\). The T2K experiment has two major upgrades: T2HK \([43]\) with a much larger Hyper-K detector and T2HKK \([56]\) with another detector at the second oscillation peak. For both of them, a matter effect uncertainty of 6% is considered \([56]\). Since T2HK is in construction and T2HKK still being planned, there is no real data yet. We take two typical values \( \delta_D = -90^{\circ} \) and 150° for illustration. At these two true values, the CP uncertainty at T2HK (T2HKK) can reach 22°(13°) and 10°(7°), respectively \([43,56]\).

The experiments listed in Fig. 16 are sorted according to their CP uncertainties. After T2K and their upgrades, the next one is the DAE\( \mu \)ALUS experiment that uses \( \mu \)DAR neutrinos. With three cyclotrons, its CP uncertainty touches down to 18° (28°) at the chosen typical CP phase \( \delta_D = -90^{\circ} \) (150°) \([84]\). Another \( \mu \)DAR experiment design \([85]\) uses the JUNO detector \([63]\). The combination of DAE\( \mu \)ALUS + JUNO can achieve even better sensitivity than DAE\( \mu \)ALUS alone with an uncertainty of 18° (21°) at \( \delta_D = -90^{\circ} \) (150°) \([86]\).

The next group is the accelerator-based DUNE \([44]\) and MOMENT \([59]\). The CP uncertainty at DUNE is derived from our own simulation with a conservative 10% uncertainty in the matter potential, as shown in Fig. 14. Different from the \( \mu \)DAR flux, the neutrino flux from muon decay in flight is adopted by the MOMENT experiment to 15° \([138]\).

Finally, TNT2HK and the two DUNE+\( \mu \)THEIA-25/100 configurations use \( \mu \)DAR neutrinos to supplement the accel-
The current $\delta_D$ measurements at T2K and NOvA. For comparison, the projected CP sensitivities at various future and planned experiments are sorted by the size of CP uncertainty. For T2K and NOvA that are already running, their latest results from real experimental data are shown while for those still in plan or design, we simply list the theoretical/phenomenological. We did not generate new data. We only included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not covered by the terms of the Creative Commons licence, permissions are required. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

5 Conclusion and outlook

The leptonic CP phase measurement at accelerator-based neutrino oscillation experiments suffers from the contamination of matter effect. The higher neutrino energy, the more severe contamination. In this paper, we put forward a possible combination of intrinsically low-energy $\mu$DAR neutrinos and the recently proposed THEIA detector to overcome this problem.

Our simulation shows that the THEIA detector using WbLS has very good capability of particle identification. This is especially useful for suppressing the atmospheric invisible muon background, which was the major background at the T2NTK/TNT2HK configuration, to negligible amount. Then the $\mu$DAR $\nu_\mu \rightarrow \bar{\nu}_e$ oscillation leaves a very clear IB signal in the detector. In addition, the high-energy LBNF flux can also be measured at the THEIA detector in addition to DUNE.

With essentially a background free measurement, the enhancement on CP sensitivity from $\mu$THEIA is significant. The CP uncertainty around the maximal CP violation $\delta_D = \pm 90^\circ$ reduces up to 20% (40%) when compared to the standard DUNE configuration. Especially, the CP uncertainty is controlled to be below 8° and the best case can be as good as 6° for the baseline $L = 38$ km. In addition, the dependence of CP uncertainty on the true CP phase value is largely mitigated. If realized, either the DUNE + $\mu$THEIA-25 or DUNE + $\mu$THEIA-100 configuration can bring the CP measurement into a precision era.

Acknowledgements The authors are grateful to Constantinou Andreopoulos, Junting Huang, Robert Svoboda, Julia Tena Vidal, Zhe Wang, and Guang Yang for valuable discussions and helps. The authors are supported by the Double First Class start-up fund (WF220442604) provided by Tsung-Dao Lee Institute & Shanghai Jiao Tong University, the Shanghai Pujiang Program (20PJ1407800), and National Natural Science Foundation of China (No. 12090064). This work is also supported in part by Chinese Academy of Sciences Center for Excellence in Particle Physics (CCEPP).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Our work is theoretical/phenomenological. We did not generate new data. We only simulated future experiments.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Supported by SCOAP3.
Appendix A: The wrong scattering angle effect

As demonstrated in Sect.3.1.1, a more precise energy reconstruction for the IBD signal requires both momentum and the scattering angle $\theta_e$ in (5). Reconstructing the scattering angle is possible for the $\mu$DAR neutrinos with fixed source location. Unfortunately, the direction of the incoming atmospheric neutrino is unknown. So the atmospheric neutrino background suffers from the wrong scattering angle effect which can further blur the neutrino energy reconstruction.

Figure 17 shows the geometry of the atmospheric IBD background. With O(10) km baseline, the $\mu$DAR neutrinos essentially travels horizontally, providing a natural definition of the $x$-axis while the other horizontal direction is the $y$-axis of the lab frame (cyan). Then, the zenith angle $\theta_z$ is measured from the vertical $z$-axis. Given $\theta_z$, the incoming atmospheric neutrino direction is parametrized by the azimuth angle $\phi_z$, measured from the $x$-axis. The final-state lepton typically has a nonzero scattering angle $\theta_e$ from the neutrino direction. For convenience, a neutrino frame (blue) is established around the neutrino momentum $p_e$ with $x'$, $y'$, and $z'$-axes.

The wrong scattering angle is defined as the one between the direction of $e^-/e^+$ (blue arrow) and the direction of the $\mu$DAR flux (cyan $x$-axis). For comparison, the true scattering angle $\theta_e$ and the corresponding azimuth angle $\phi_e$ are also plotted in the figure. To calculate the wrong angle as a function of $\theta_z$, $\theta_e$, $\phi_e$, and $\phi_z$, one need to first establish the connection between the horizontal and neutrino frames. The $z'$ direction can be easily read out from the figure, $z' = -\sin \theta_e \cos \phi_z x - \sin \theta_e \sin \phi_z y - \cos \theta_e z$. But the $x'$- and $y'$-axes can be randomly set, as long as they satisfy $\vec{n}_{x'} \cdot \vec{n}_{x'} = \vec{n}_{y'} \cdot \vec{n}_{y'} = \vec{n}_{x'} \cdot \vec{n}_{y'} = 0, |\vec{n}_{x'}| = |\vec{n}_{y'}| = 1$ and $\vec{n}_{x'} \times \vec{n}_{y'} = \vec{n}_{z'}$. Accordingly, the following transformation matrix between two coordinate systems is chosen,

$$
\begin{pmatrix}
    x' \\
y' \\
z'
\end{pmatrix} =
\begin{pmatrix}
-\cos \theta_e \cos \phi_z & -\cos \theta_e \sin \phi_z & \sin \theta_e \\
-\sin \phi_z & \cos \phi_z & 0 \\
-\sin \theta_e \cos \phi_z & -\sin \theta_e \sin \phi_z & \cos \theta_e
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}.
$$

The $e^-/e^+$ direction in the neutrino frame is $(\sin \theta_e \cos \phi_e, \sin \theta_e \sin \phi_e, \cos \theta_e)$. Using the frame transformation (A1), the $e^-/e^+$ direction in the lab frame is

$$
\vec{n}_e = \begin{pmatrix}
-\cos \theta_e \cos \phi_z & \cos \theta_e \sin \phi_z & -\sin \theta_e \\
\sin \phi_z & \cos \phi_z & 0 \\
\sin \theta_e \cos \phi_z & \sin \theta_e \sin \phi_z & \cos \theta_e
\end{pmatrix}
\begin{pmatrix}
\cos \phi_e & 0 & \sin \phi_e \\
0 & 1 & 0 \\
-\sin \phi_e & 0 & \cos \phi_e
\end{pmatrix}
\begin{pmatrix}
\sin \theta_e \cos \phi_e \\
\sin \theta_e \sin \phi_e \\
\cos \theta_e
\end{pmatrix}.
$$

where we have used shorthand notations, $(\phi_{e,z}, \phi_{e,s}) \equiv (\cos \phi_e, \sin \phi_e)$ and $(\theta_{e,z}, \theta_{e,s}) \equiv (\cos \theta_e, \sin \theta_e)$.

With $\mu$DAR flux in the direction of $\vec{n}_{\mu\text{DAR}} = (1, 0, 0)$, the cosme of the wrong scattering angle $\theta_w$ is given by

$$
\cos \theta_w \equiv \vec{n}_e \cdot \vec{n}_{\mu\text{DAR}} / |\vec{n}_e| |\vec{n}_{\mu\text{DAR}}| = -\cos \phi_e \sin \theta_e \cos \phi_z - \sin \phi_e \sin \phi_z.
$$

To see the effect of a wrong scattering angle, we use Monte Carlo method to randomly generate the scattering process. As seen from Fig. 17, there are four different angles in the whole scattering process and each of them has specific probability distribution. First, the zenith angle of the atmospheric neutrino flux $\theta_z$ is defined as the angle between the local zenith and the direction of the atmospheric neutrino flux [139]. Its probability distribution is sampled according to the low energy Gran Sasso flux downloaded from the Honda website [119]. Given a zenith angle $\theta_z$, the azimuth angle $\phi_z$ is isotropically sampled since the location of the $\mu$DAR source is not known yet.

From the atmospheric neutrino scattering with target, the scattering angle $\theta_e$ of the final-state charged lepton distributes according to the GENIE simulation. One important feature is that the $\cos \theta_e$ distribution depends on the neutrino energy $E_{\nu}$. Moreover, the lepton azimuth angle $\phi_e$ is isotropic.

In addition, the atmospheric neutrino flux is affected by the neutrino oscillation through the Earth which is a function of the propagation length $2R \cos \theta_e$, where $R = 6371$ km is the Earth radius. We use the PREM Earth model [140] that is implemented in GLoBES to calculate the oscillation probability to calculate the modified atmospheric neutrino flux.

As expected, the wrong scattering angle significantly affects the energy reconstruction since the scattering angle in the energy reconstruction formula (5) plays an important role as we discussed in Sect.3.1.1. The reconstructed energy spectra at several different typical energies are shown in Fig. 4.
50. V. Fischer [Theia], Theia: a multi-purpose water-based liquid scintillator neutrino experiment. J. Phys. Conf. Ser. 1468(1), 012125 (2020). https://doi.org/10.1088/1742-6596/1468/1/012125

51. C. Boehm et al. [Hyper-Kamiokande], Hyper-Kamiokande design report. arXiv:1805.04163 [physics.ins-det]

52. R. Acciarri et al. [DUNE], Super-ORCA: measuring the leptonic CP-phase with atmospheric neutrinos. Phys. Rev. D 98(1), 015025 (2018). https://doi.org/10.1103/PhysRevD.98.015025. arXiv:1802.06784 [hep-ph]

53. K. Abe et al. [Hyper-Kamiokande], Hyper-Kamiokande: faint objects in motion or the new CP-violation. Phys. Rev. D 104(7), 075030 (2021). https://doi.org/10.1103/PhysRevD.104.075030. arXiv:2103.01998 [hep-ph]

54. S. S. Choubey, M. Ghosh, D. Pramanik, Sensitivity study of proton to ORCA (P2O) experiment: effect of antineutrino run, background and systematics. Eur. Phys. J. C 79(7), 603 (2019). https://doi.org/10.1140/epjc/s10052-019-7064-i. arXiv:1812.02608 [hep-ph]

55. A. Alekou et al., [ESSnuSB], Updated physics performance of the ESSnuSB experiment: ESSnuSB collaboration. Eur. Phys. J. C 81(12), 1130, 151803 (2021). https://doi.org/10.1140/epjc/s10052-019-0985-8. arXiv:2107.07585 [hep-ex]

56. K. Cao, M. He, Z.L. Hou, H.T. Jing, Y.F. Li, Z.H. Li, Y.P. Song, J.Y. Tang, Y.F. Wang, Q.F. Wu, et al. Muon-decay medium-baseline neutrino beam facility. Phys. Rev. ST Accel. Beams 17, 090101 (2014). https://doi.org/10.1103/PhysRevSTAB.17.090101. arXiv:1401.8125 [physics.acc-ph]

57. S. Razzaque, A.Y. Smirnov, Super-PINGU for measurement of the leptonic CP-phase with atmospheric neutrinos. JHEP 05, 139 (2015). https://doi.org/10.1007/JHEP05(2015)139. arXiv:1406.1407 [hep-ph]

58. S. Razzaque, A.Y. Smirnov, Super-PINGU for measuring CP-violation. Nucl. Part. Phys. Proc. 265–266, 183–185 (2015). https://doi.org/10.1016/j.nuclphysbps.2015.06.046. arXiv:1501.03145 [hep-ph]

59. J. Hofestädt, M. Bruchner, T. Eberl, Super-ORCA: measuring the leptonic CP-phase with atmospheric neutrinos and beam neutrinos. PoS ICRC2019, 911 (2020). https://doi.org/10.22323/1.358.0911. arXiv:1907.12983 [hep-ex]

60. F. An et al. [JUNO], Neutrino physics with JUNO. J. Phys. G 43(3), 030401 (2016). https://doi.org/10.1088/1361-648X/43/3/030401. arXiv:1507.05613 [physics.ins-det]

61. K.J. Kelly, P.A. Machado, I. Martinez Soler, S.J. Parke, Y.F. Perez Gonzalez, Sub-GeV atmospheric neutrinos and CP-violation in DUNE. Phys. Rev. Lett. 123(8), 081801 (2019). https://doi.org/10.1103/PhysRevLett.123.081801. arXiv:1904.02751 [hep-ph]

62. P. Ballett, S.F. King, S. Pascoli, N.W. Prouse, T. Wang, Sensitivities and synergies of DUNE and T2HK. Phys. Rev. D 96(3), 033003 (2017). https://doi.org/10.1103/PhysRevD.96.033003. arXiv:1612.07275 [hep-ph]

63. S.K. Raut, Matter effects at the T2HK and T2HKK experiments. Phys. Rev. D 96(7), 075029 (2017). https://doi.org/10.1103/PhysRevD.96.075029. arXiv:1703.07136 [hep-ph]

64. K. Chakraborty, K.N. Deepthi, S. Goswami, Spotlighting the sensitivities of Hyper-Kamiokande, DUNE and ESSnuSB experiment: ESSnuSB collaboration. Eur. Phys. J. C 81(7), 491 (2021). https://doi.org/10.1007/JHEP05(2015)139. arXiv:1406.1407 [hep-ph]

65. M. Ghosh, T. Ohlsson, A comparative study between ESSnuSB experiment: ESSnuSB collaboration. Eur. Phys. J. C 81(7), 491 (2021). https://doi.org/10.1140/epjc/s10052-021-09227-0. arXiv:2107.07585 [hep-ex]

66. K. Arafune, M. Koike, J. Sato, CP violation and matter effect in long baseline neutrino oscillation experiments. Mod. Phys. Lett. A 35, 1081 (2020). https://doi.org/10.1142/S0217732320500583 arXiv:1906.05779 [hep-ph]

67. J. Hofestädt, M. Bruchner, T. Eberl, Super-ORCA: measuring the leptonic CP-phase with atmospheric neutrinos and beam neutrinos. PoS ICRC2019, 911 (2020). https://doi.org/10.22323/1.358.0911. arXiv:1907.12983 [hep-ex]

68. J. Arafune, M. Koike, J. Sato, CP violation and matter effect in long baseline neutrino oscillation experiments. Phys. Rev. D 98(1), 015025 (2018). https://doi.org/10.1103/PhysRevD.98.015025. arXiv:1802.06784 [hep-ph]

69. K. Minakata, H. Nunokawa, Effects of matter density fluctuation in long baseline neutrino oscillation experiments. Phys. Rev. D 57, 9443–9417 (1998). https://doi.org/10.1103/PhysRevD.57.9443. arXiv:hep-ph/9705208

70. H. Minakata, H. Nunokawa, CP violation versus matter effect in long baseline neutrino oscillation experiments. Phys. Rev. D 57, 4403–4417 (1998). https://doi.org/10.1103/PhysRevD.57.4403. arXiv:hep-ph/9705208

71. C. Boehm et al. [Theia], Theia: faint objects in motion or the new astrometry frontier. arXiv:1707.01348 [astro-ph.IM]

72. V. Fischer [Theia], Theia: a multi-purpose water-based liquid scintillator detector. arXiv:1809.05987 [physics.ins-det]

73. M. Askins et al. [Theia], THEIA: an advanced optical neutrino detector. Eur. Phys. J. C 80, 416 (2020). https://doi.org/10.1140/epjc/s10052-020-7977-8. arXiv:1911.03501 [physics.ins-det]

74. D. Guffanti [THEIA Proto], Prospects for THEIA: an advanced liquid scintillator neutrino experiment. J. Phys. Conf. Ser. 1468(1), 012124 (2020). https://doi.org/10.1088/1742-6596/1468/1/012124

75. K. Abe, H. Aihara, A. Ajmi, J. Amey, C. Andreopoulos, M. Antonova, S. Aoki, A. Atherton, S. Ban, F.C.T. Barbato, et al. Proposal for an extended run of T2K to 20 $\times$ 10^{21} POT. arXiv:1609.04111 [hep-ex]

76. K. Hagiwara, N. Okamura, K.I. SendA, Solving the neutrino parameter degeneracy by measuring the T2K off-axis beam in Korea. Phys. Lett. B 637, 266–273 (2006) (Erratum: Phys. Lett. B 641, 491 (2006)). https://doi.org/10.1016/j.physletb.2006.09.003. arXiv:hep-ph/0504061

77. K. Hagiwara, N. Okamura, K.I. SendA, Physics potential of T2KK: an extension of the T2K neutrino oscillation experiment with a far detector in Korea. Phys. Rev. D 76, 093002 (2007). https://doi.org/10.1103/PhysRevD.76.093002. arXiv:hep-ph/0607255

78. K. Abe et al. [Hyper-Kamiokande], Physics potentials with the second Hyper-Kamiokande detector in Korea. PTEP 2018(6), 063C01 (2018). https://doi.org/10.1093/ptep/pty044. arXiv:1611.06118 [hep-ex]
iments. Nucl. Phys. B 671, 483–497 (2003). https://doi.org/10.1016/j.nuclphysb.2003.08.009, arXiv:hep-ph/0303078
73. S.F. Ge, A.Y. Smirnov, Non-standard interactions and the CP phase measurements in neutrino oscillations at low energies. JHEP 10, 138 (2016). https://doi.org/10.1007/JHEP10(2016)138 arXiv:1607.08513 [hep-ph]
74. J. Datta, M. Nizam, A. Ajmi, S.U. Sankar, Matter vs vacuum oscillations in atmospheric neutrinos. Nucl. Phys. B 961, 115251 (2020). https://doi.org/10.1016/j.nuclphysb.2020.115251 arXiv:1907.08966 [hep-ph]
75. S.P. Mikheev, A.Y. Smirnov, Resonance amplification of oscillations in matter and spectroscopy of solar neutrinos. Sov. J. Nucl. Phys. 42, 913–917 (1985)
76. M. Freund, Analytic approximations for three neutrino oscillation parameters and probabilities in matter. Phys. Rev. D 64, 053003 (2001). https://doi.org/10.1103/PhysRevD.64.053003 arXiv:hep-ph/0103300
77. E.K. Akhmedov, R.Johansson, M. Lindner, T. Ohlsson, T. Schwetz, Series expansions for three flavor neutrino oscillation probabilities in matter. JHEP 04, 078 (2004). https://doi.org/10.1088/1126-6708/2004/04/078, arXiv:hep-ph/0402175
78. P. Coloma, A. Donini, E. Fernandez-Martinez, P. Hernandez, Precision on leptonic mixing parameters at future neutrino oscillation experiments. JHEP 06, 073 (2012). https://doi.org/10.1007/JHEP06(2012)073 arXiv:1203.5651 [hep-ph]
79. H.J. He, X.J. Xu, Connecting the leptonic unitarity triangle to neutrino oscillation with CP violation in the vacuum and in matter. Phys. Rev. D 95(3), 033002 (2017). https://doi.org/10.1103/PhysRevD.95.033002 arXiv:1606.04054 [hep-ph]
80. M. Cho, Y. Choi, H.S. Lee, Y.M. Lee, S.K. Raut, Neutrino oscillations at dual baselines. arXiv:1907.01185 [hep-ph]
81. S.F. Ge, A.Y. Smirnov, Non-standard interactions and the CP phase measurements in neutrino oscillations at low energies. JHEP 10, 138 (2016). https://doi.org/10.1007/JHEP10(2016)138 arXiv:1607.08513 [hep-ph]
82. H. Wei, Z. Wang, S. Chen, Discovery potential for supernova relic neutrinos with slow liquid scintillator detectors. Phys. Lett. B 769, 255–261 (2017). https://doi.org/10.1016/j.physletb.2017.03.071, arXiv:1607.01671 [physics.ins-det]
83. Z. Guo, M. Yeh, R. Zhang, D.W. Cao, M. Qi, Z. Wang, S. Chen, Slow liquid scintillator candidates for MeV-scale neutrino experiments. Astropart. Phys. 109, 33–40 (2019). https://doi.org/10.1016/j.astrophys.2019.02.001, arXiv:1708.07781 [physics.ins-det]
84. J. Caravaca, B.J. Land, M. Yeh, G.D. Orebi Gann, Characterization of water-based liquid scintillator for Cherenkov and scintillation separation. Eur. Phys. J. C 80(9), 867 (2020). https://doi.org/10.1140/epjc/s10052-020-8418-4, arXiv:2006.00173 [physics.ins-det]
85. J. Sawatzki, M. Wurm, D. Kresse, Detecting the diffuse supernova neutrino background in the future water-based liquid scintillator detector theia. Phys. Rev. D 103(2), 023021 (2021). https://doi.org/10.1103/PhysRevD.103.023021, arXiv:2007.14705 [physics.ins-det]
86. B.J. Land, Z. Bagdasarian, J. Caravaca, M. Smiley, M. Yeh, G.D. Orebi Gann, MeV-scale performance of water-based and pure liquid scintillator detectors. Phys. Rev. D 103(5), 052004 (2021). https://doi.org/10.1103/PhysRevD.103.052004, arXiv:2007.14999 [physics.ins-det]
87. A. Aguilar-Arevalo et al. [LSND], Evidence for neutrino oscillations from the observation of νe appearance in a νμ beam. Phys. Rev. D 64, 112007 (2001). https://doi.org/10.1103/PhysRevD.64.112007, arXiv:hep-ex/0104049
88. S.F. Ge, K. Hagiwara, N. Okamura, Y. Takaesu, Determination of mass hierarchy with medium baseline reactor neutrino experiments. JHEP 05, 131 (2013). https://doi.org/10.1007/JHEP05(2013)131, arXiv:1210.8141 [hep-ph]
89. C. Andreopoulos, A. Bell, D. Bhattacharya, F. Cavanna, J. Dobson, S. Dytman, H. Gallagher, P. Guzowski, R. Hatcher, P. Kehayias et al., The GENIE Neutrino Monte Carlo generator. Nucl. Instrum. Methods A 614, 87–104 (2010). https://doi.org/10.1016/j.nima.2009.12.009, arXiv:0905.2517 [hep-ph]
90. C. Andreopoulos, C. Barry, S. Dytman, H. Gallagher, T. Golan, R. Hatcher, G. Perdue, J. Yarba, The GENIE Neutrino Monte Carlo generator: physics and user manual. arXiv:1510.05494 [hep-ph]
91. L. Thierry, https://indico.cern.ch/event/147174/contributions/181786/attachments/142219/201806/INSS2011_Lasserre_Lecture1.pdf Reactor Antineutrino, July 18–30, 2011. International Neutrino Summer School, Cartigny
92. K. Abe et al. [T2K], Measurement of neutrino and antineutrino oscillations by the T2K experiment including a new additional sample of νe interactions at the far detector. Phys. Rev. D 96(9), 092006 (2017) (Erratum: Phys. Rev. D 98(1), 019902 (2018)). https://doi.org/10.1103/PhysRevD.96.092006, arXiv:1707.01048 [hep-ex]
interactions. JHEP **08**, 163 (2021). https://doi.org/10.1007/JHEP08(2021)163 arXiv:2106.04597 [hep-ph]

138. J. Tang, S. Vihonen, T.C. Wang, Precision measurements on $\delta_{\text{CP}}$ in MOMENT. JHEP **12**, 130 (2019). https://doi.org/10.1007/JHEP12(2019)130 arXiv:1909.01548 [hep-ph]

139. M. Honda, M. Sajjad Athar, T. Kajita, K. Kasahara, S. Midorikawa, Atmospheric neutrino flux calculation using the NRLMSISE-00 atmospheric model. Phys. Rev. D **92**(2), 023004 (2015). https://doi.org/10.1103/PhysRevD.92.023004. arXiv:1502.03916 [astro-ph.HE]

140. A.M. Dziewonski, D.L. Anderson, Preliminary reference earth model. Phys. Earth Planet. Inter. **25**, 297–356 (1981). https://doi.org/10.1016/0031-9201(81)90046-7