Comparison of finite element modelling of a magnetic field by the example of solving the magnetostatics problem

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Abstract. Two approaches for calculating the magnetic field in the structural elements of an electromagnetic engine using the finite element method are considered. Comparison of programs in test mode is proposed on the example of calculating the power characteristics of a linear electromagnetic engine with an axisymmetric construction and a complex profile of the magnetic circuit. Programs focused on solving field problems in two-dimensional areas. For assess the accuracy of the results of finite element modelling, a comparison with the force characteristics obtained on the physical model was made. Based on the results of testing the programs, the errors in the calculations and the further use of programs for solving magnetostatic problems are estimated.

1. Introduction

One of the main tasks in the construction of linear electromagnetic engines is to ensure the calculation of the necessary strength of the magnetic interaction of the armature and stator [1–4]. The calculated power characteristic is determining when choosing an electromagnetic engine to drive any electromechanical device.

Researches of various constructions of electromagnetic engines show that for the same volume useful work and the magnitude of the magnetic interaction can vary significantly [5–8]. An accurate calculation of the power characteristics can be obtained if there is a rigorous mathematical model of the device that adequately responds to changes in the input parameters of a device that has a complex geometry profile of the ferromagnetic sections making up the magnetic circuit.

In the presence of complex spatial geometry of the magnetic circuit profile, analytical estimates of the distribution of the magnetic field in the vicinity of the ferromagnetic sections and the determination of the strength characteristics of their magnetic interaction can have a significant error.

Force interactions in electromagnetic engines are accomplished through a magnetic field. Therefore, to study force interactions to solve construction problems, the most promising method is field methods for calculating the magnetic field [9, 10].

The existing software packages for the numerical calculation of magnetic fields should be considered as a universal tool for solving various problems, including magnetostatics [11–13]. The differences between existing programs lead to various adaptations of numerical calculations of magnetic fields to solve specific problems. The advantage of programs that use field approaches is the ability to calculate the magnetic interaction of ferromagnetic figures directly.
Thus, the numerical calculation of magnetic fields is an effective tool for calculating electromagnetic devices, including linear electromagnetic engines, which have a complex profile of the geometry of the magnetic circuit [14, 15].

As a rule, the calculation of force interactions between the armature and the stator of the motor is determined based on a preliminary calculation of the magnetic field with taking into account the geometry of the magnetic circuit and the saturation of the ferromagnetic sections.

The most common methods used for calculating magnetic fields are finite element methods [16]. The main differences between the existing programs are the limitations associated with the calculation of the magnetic field for a certain type of problem.

One of such programs for solving magnetostatic problems is a program for calculating a two-dimensional magnetic field in an axisymmetric construction of an electromagnetic engine with two working air gaps and a combined composite armature. The basis of the program is the software package created at the Department of Applied Mathematics of Novosibirsk State Technical University (NSTU) [17, 18].

Another program for calculating a two-dimensional field is a widely available package FEMM (Finite Element Method Magnetics) [19, 20]. The advantage of the package is visual design tools that facilitate the construction of new models.

The aim of the work is to assess the applicability of these programs by comparing the obtained results with each other and with experimental data on the example of solving the magnetostatic problem for the axisymmetric construction of an electromagnetic engine that has a complex profile of the magnetic circuit geometry.

2. Materials and methods

In the program created at the Department of Applied Mathematics (NSTU) [17] to describe the magnetostatic problem in the axisymmetric construction of an electromagnetic motor, the equation:

$$\text{rot} \frac{\vec{B}}{\mu(B)} = \vec{J},$$

(1)

where $\vec{J}$ – current density vector in the winding, $\vec{B}$ – magnetic field induction vector, $\mu$ – magnetic permeability, generally depending on the module of the magnetic induction vector $B = \sqrt{B_r^2 + B_\theta^2 + B_z^2}$.

Vector potential introduced $\vec{A}$ such that $\vec{B} = \text{rot} \vec{A}$. Then the equation (1) the authors [17] transform to:

$$\text{rot} \frac{1}{\mu(B)} \text{rot} \vec{A} = \vec{J}.$$  

(2)

For axisymmetric construction, the equation (2) solved in cylindrical coordinates $(r, z)$. In the set coordinates, the calculation area $\Omega$ includes half of the vertical section of the structure. The left border $\Omega$ is the axis of symmetry of the structure, and the remaining three of its boundaries are considered remote so that the effect of the magnetic field on the boundaries is negligible.

With the axisymmetric construction, it is believed that the current density and vector potential $\vec{A}$ have only one nonzero $\varphi$-component, which depends on coordinate $r$ and $z$. Then instead of the vector equation (2) scalar equation applies:

$$-\text{div} \left( \frac{1}{\mu(B)} \text{grad} A_\varphi \right) + \frac{A_\varphi}{\mu(B)r^2} - \frac{A_\varphi}{r} \frac{\partial}{\partial r} \left( \frac{1}{\mu(B)} \right) = J_\varphi$$

(3)

On the outer border $G$, including the axis of symmetry and three remote boundaries of the computational domain $\Omega$, it is believed that the potential $A_\varphi$:

$$A_\varphi |_G = 0$$

(4)
According to the solution found from the equation (3) with boundary conditions (4) values \( A_\varphi \) the components of the magnetic induction vector are calculated in the form

\[
B_r = -\frac{\partial A_\varphi}{\partial z}, \quad B_z = \left(\frac{\partial A_\varphi}{\partial r} + \frac{A_\varphi}{r}\right).
\] (5)

Based on the found values (5) of the components of the magnetic field induction vector, the force acting on the armature is calculated.

To solve the problem (3), (4) finite element method is used. It takes into account that the calculation area \( \Omega \) in the cylindrical coordinate system contains only rectangular subdomains, then rectangular finite elements are used for discrediting it. The choice of rectangular elements is due to a simpler construction of a computational scheme for subdomains with \( \mu = \mu(B) \) and good accuracy of the resulting solution. For solve the nonlinear problem, the Newton method was used, which is based on the linearization of a nonlinear finite element system by expanding in a row of Taylor.

In the program the force \( F_z^M \) acting on the anchor, is determined by the volumetric \( F_z^{MV} \) and superficial \( F_z^{MS} \) components. Force component \( F_z^{MV} \) is determined by the formula [17]

\[
F_z^{MV} = \int_{\Omega} \frac{1}{2} \left( \frac{\mu(B)}{\mu_0} - \frac{\mu_0}{\mu(B)} \right) dB_z dz d\Omega,
\]

where \( \Omega_{\text{arm}} \) – subregion of the computational domain \( \Omega \), corresponding to the anchor. The expression for force \( F_z^{MS} \) obtained from the formula (6) taking into account the gap of the tangent component of the induction vector \( \vec{B} \) on the border between steel and air:

\[
F_z^{MS} = \alpha \int \left( \frac{\mu(B)}{\mu_0} - 1 \right) \left( \frac{B_z}{\mu(B)} \right)^2 \mu_0 dS,
\]

where \( S \) – unification of horizontal boundaries between the anchor and the air gap, \( \alpha = 1 \) on the lower borders and \( \alpha = -1 \) on the upper borders of the anchor; \( B_z \) – induction vector component \( \vec{B} \), taken on \( S \) from the side of the iron.

The resulting force acting on the anchor

\[
F_z = F_z^{MS} + F_z^{MV} \tag{6}
\]

The FEMM program implements on the basis of the finite element method an equation describing a magnetic field in a two-dimensional formulation [19]

\[
\nabla \times \left[ \frac{1}{\mu(B)} \nabla \times \vec{A}_\varphi \right] = \vec{J}_\varphi,
\] (7)

where \( \vec{A}_\varphi \), \( \vec{J}_\varphi \) – components of the vector magnetic potential and the current density vector.

Magnetic permeability is a function of induction \( B \) and determined through the magnetization curve of the material

\[
\mu = \frac{B}{H(B)}.
\]

To solve (7) the FEMM program generates a finite element structure consisting of elements forming triangles.

When solving (7) in the meridian plane \((r, z)\), one of the calculated boundaries is the central axis \( z \). The remaining boundaries when calculating the magnetic field are considered remote outside the core, where the magnetic field is very small. The computational domain of the axisymmetric model includes only half of the vertical section of the structure.
The resulting interaction force between the armature and the stator is determined through the Maxwell tensor

\[ F = \frac{2\pi}{\mu_0} \int r B_r B_z dl, \]

where \( B_r, B_z \) – radial and axial components of magnetic induction, \( l \) – integration circuit.

### 3. Analysis of the results of the calculation of force interactions

As examples in Figures 1 and 2 according to the results of magnetic field calculations, field patterns are shown in the form of lines of equal level for magnetic flux. Pictures of the field are presented on the symmetric halves of the cross-section of the electromagnetic engine. The calculation of the field is performed with the same position of the armature and the magnitude of the magnetizing force of the field winding.

![Figure 1](image1.png) **Figure 1.** Picture of the field of magnetic flux lines in the program of NSTU

![Figure 2](image2.png) **Figure 2.** Picture of the field of magnetic flux lines in the FEMM program

For a program created in NSTU [17] equal level lines according to the equation (3) presented in Figure 1. For a FEMM program [19, 20] equal level lines are obtained by the equation (7) and presented in Figure 2.

As an object of research, the axisymmetric design of an electromagnetic engine with two working air gaps and a combined composite armature was considered. The constructions of the electromagnetic
engine contain the following parameters (Figure 2): \( r_1 = 33.5 \cdot 10^{-3} \text{m} \), \( r_2 = 48 \cdot 10^{-3} \text{m} \), \( r_3 = 5.9 \cdot 10^{-2} \text{m} \), \( r_4 = 12 \cdot 10^{-3} \text{m} \), \( r_5 = 15 \cdot 10^{-3} \text{m} \), \( h_1 = 0.1 \text{m} \), \( h_2 = h_3 = 12 \cdot 10^{-3} \text{m} \), \( h_4 = h_5 = 16 \cdot 10^{-3} \text{m} \). The number of coils of the field winding: \( v = 780 \). Field current: 15 A. The magnetic circuit of the engine is made of structural steel grade Steel-3 [2, 18].

Pictures of magnetic flux lines in Figure 1 have differences from magnetic flux lines in Figure 2. The existing differences can be explained by choice of rectangular elements of the finite element grid in Figure 1 and triangular elements of the finite element grid in Figure 2.

Dependencies of the resulting forces (Figure 3) calculated by the formulas (6) and (8) from the coordinates of the position of the anchor relative to the stator are presented in Figure 3. The obtained dependencies demonstrate a high level of agreement between the curves of electromagnetic forces obtained by the formulas (6) (curve 1) and (8) (curve 2) in the entire range of the working stroke of the anchor. With the existing difference in the calculations, the force curves quite accurately coincide with the experimental curve 3 (Figure 3) of electromagnetic force obtained on a physical model. Small differences with the experimental curve are observed in the zone of small forces. The calculated values of the efforts here are higher than the experimental ones. This behaviour of the curves can be explained by the dominant forces of mechanical friction of the anchor of the physical model. In the calculations, according to the formulas (6) and (8) mechanical friction forces in models are not taken into account.

![Figure 3. Comparison of calculated and experimental characteristics of electromagnetic forces](image)

Differences in the calculation of electromagnetic forces when using different finite element modelling and experiment programs do not exceed 3%. The error corresponds to the level of measurement errors.

4. Conclusion
Based on the testing of finite element modelling of the magnetic field, a comparative estimation of the calculation error is given by the example of solving magnetostatics problems. The reliability of the numerical results of solving the problems of magnetostatics is confirmed by the example of calculating the power characteristics of an electromagnetic motor of an axisymmetric construction, which has a complex profile of the magnetic circuit. The established calculation error of both programs corresponds to the level of measurement errors and does not exceed 3%. Existing universal package FEMM has a higher adaptation level when performing numerical calculations. Contains universal visual design tools that facilitate the construction of new models compared to the software package created in NSTU.
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