A comparison of designs of off-axis Gregorian telescopes for mm-wave large focal plane arrays

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We compare the diffraction-limited field of view (FOV) provided by four types of off-axis Gregorian telescopes: the classical Gregorian, the aplanatic Gregorian, and designs that cancel astigmatism and both astigmatism and coma. The analysis is carried out using telescope parameters that are appropriate for satellite and balloon-borne millimeter and sub-millimeter wave astrophysics.

We find that the design that cancels both coma and astigmatism provides the largest flat FOV, about 21 square degrees. We also find that the FOV can be increased by about 15% by optimizing the shape and location of the focal surface.

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1. **Introduction**

Future advances in mm and sub-mm wave astronomy critically depend on the design of optical systems. Detector technology has matured to the point where detector sensitivity is limited by photon noise from the source or from unavoidable photons in the light-path, such as the atmosphere and the mirrors. Only an increase in the number of photometers can significantly increase the overall sensitivity of an instrument.

Arrays of detectors with tens and hundreds of elements have recently come on line\textsuperscript{1,2} and it is widely expected that the construction of such instruments will accelerate as the fabrication of both semiconductor-based and superconducting-based bolometers becomes more
uniform and more automated through the use of standard micro-lithography techniques.\textsuperscript{3-7}

These large focal plane arrays need to be coupled to telescopes that provide a correspondingly large, diffraction-limited field of view (FOV). The need for optical systems with large useable FOV was not acute in the past when typically only a few photometers were coupled to the telescope. Because of the long wavelength, it is relatively easy to design an optical system for use at the millimeter wave band that is diffraction limited near the center of the FOV. The current challenge is to provide for the largest useable FOV in order to accommodate large arrays.

An increase in the available FOV of off-axis Gregorian telescopes is of particular interest because such telescopes have higher aperture efficiency and lower side-lobe response than Cassegrain or on-axis Gregorian reflecting telescopes.\textsuperscript{8} It is also interesting to analyze in detail Gregorian telescopes that have a low f-number and a small number of mirrors because such systems find wide use in satellites and balloon borne payloads which require compact and simple optical systems. For example, Gregorian telescopes have been used extensively in recent years in ground based and balloon borne experiments to characterize the anisotropy of the cosmic microwave background radiation (CMB).\textsuperscript{9-12} Both NASA’s Microwave Anisotropy Probe (MAP) satellite and the European Space Agency’s Planck satellite, two missions designed to map CMB temperature fluctuations, employ off-axis Gregorian telescopes with low f-number.\textsuperscript{13,14}

The primary source of aberrations in Gregorian and Cassegrain telescopes are coma and astigmatism. Several designs have been proposed to improve on the classical Gregorian (CG) design, which has a parabolic primary and an elliptical secondary. In an aplanatic Gregorian (AG) telescope coma is cancelled without creating spherical aberration. The primary mirror
is slightly ellipsoidal and the secondary is a slightly more eccentric ellipsoid than in the similar CG. The conic constant of one mirror is chosen to eliminate coma and the conic constant of the other mirror is adjusted to compensate the spherical aberration introduced by the change in the first mirror. Aplanatic designs are fairly common, for example, aplanatic versions of Cassegrain telescopes were used in the Hubble Space Telescope\textsuperscript{15} and the Keck 10-meter telescopes.\textsuperscript{16}

Dragone\textsuperscript{17, 18} has described designs for off-axis Gregorian systems which eliminate astigmatism, and both astigmatism and coma; hereafter we refer to these designs as D1 and D2, respectively. These designs also greatly reduce instrumental polarization for field points near the center of the FOV.\textsuperscript{19} The reduction in instrumental polarization is of benefit for antennas for communication systems, which use polarization as a method to increase bandwidth,\textsuperscript{20, 21} and for experiments designed for detecting polarized signals. For example, intense efforts are now being made by a number of experiments to discover the CMB polarization anisotropy.\textsuperscript{22–26}

The AG, D1 and D2 designs present progressively improved image quality near the center of the field of view, however it is not clear which of the systems provides a larger useable FOV, which is the quantity of interest for millimeter wavelength focal plane arrays. In this paper we quantitatively compare the size of the diffraction-limited FOV provided by these three optical designs. Because we are interested in potential applications for millimeter-wave astrophysics, and CMB research in particular, we perform our analysis with telescopes that provide $\sim 8$ arcminute full-width at half-maximum beam size at 150 GHz ($\lambda = 2$ mm). We comment on the applicability of our analysis to other wavelengths in Section 3.

We use the telescope of the Archeops balloon borne experiment\textsuperscript{9} as a baseline for compar-
ison. Archeops is designed to observe the CMB with an array of 24 bolometric photometers distributed in four frequency bands between 143 and 545 GHz with beam sizes between 8 and 5 arcminutes, respectively. The focal plane array is a prototype of the High Frequency Instrument, one of two focal plane instruments on board ESA’s Planck satellite. The satellite is scheduled to be launched around 2007. Archeops has an off-axis tilted Gregorian telescope following the D1 design, and is similar in its physical parameters to the Planck telescope.\textsuperscript{14}

2. Method of Comparison

To compare the three optical designs in terms of their useable field of view we held the focal ratio $f$, aperture $a$, and off-axis distance of the chief ray $y$ constant with values as those of the Archeops telescope. (The off-axis distance of the chief ray is measured from the primary mirror paraboloid axis, see Figure 1). These values are given in Table 1. The magnitude of coma and astigmatism aberrations scale as $A_c(\theta y^2/f^2)$ and $A_a(\theta^2 y/f)$ respectively, where $\theta$ is the field angle and $A_c$ and $A_a$ are coefficients that depend on other parameters of the telescope, such as magnification, and mirror conic constants. By keeping $f$, $a$, and $y$ constant we ensure that all three telescopes have the same angular resolution and very similar physical size, allowing us to examine how the different designs affect the coefficients $A_c$ and $A_a$.

We use the diffraction-limited FOV (DLFOV) as the figure of merit by which we compare the different optical systems. As is customary,\textsuperscript{27} we call the image of a field point “diffraction- limited” if the root mean square wavefront error (WFE) is less than 1/14 of a wave. We measure WFEs on a grid of elevation and cross-elevation using the optical design software CodeV. To convert a set of WFE measurements to a FOV we determine the angular interval in elevation and cross-elevation at which the image is diffraction limited. We
approximate the diffraction-limited region of the focal plane as an ellipse with axes defined by these intervals and calculate the area of this ellipse as: \( \text{DLFOV area} = \left( \frac{\pi}{4} \right) [\text{elevation interval} \times \text{cross-elevation interval}] \). Detailed analysis showed that in all cases the approximation of the DLFOV as an ellipse was within 5% of the DLFOV calculated numerically using a finely spaced grid of WFE measurements. We have chosen the DLFOV as a figure of merit because it provides a direct measure of the number of photometers that can be coupled to the telescope.

We compare the three telescopes using two classes of focal surfaces, a nominal focal surface and an optimized focal surface. The nominal focal surfaces for the AG, D1, and D2 systems are flat and positioned at the final focus of each system. The optimized focal surfaces are curved and displaced from the position of the nominal surface in order to provide larger DLFOVs. To determine the optimized focal surface we evaluate the WFE numerically at 25 locations on the surface as a function of its radius of curvature, defocus, and tilt, and the optimal surface is the one that provides the largest DLFOV. We choose these three parameters for optimizing the focal surface because they provide a broad range of uncomplicated focal surface configurations. It is possible to further improve telescope performance by increasing the complexity of the focal surface, for example, by specifying a position-dependent defocus. However, such solutions are too specific to a given photometer-array configuration to be useful for a comparison of the general properties of telescope design.

Although the focal surface optimization tends to degrade the image at the center of the focal plane and to improve the performance at the edge, the WFE at the center of the focal plane in all three optimized systems remains far below the diffraction limit. Figure 2 shows the 25 field points used for each optimization; all points are weighted equally. We have
tested many different weighting schemes and concluded that the DLFOV obtained with each point weighted equally is within one or two percent of the best DLFOV achieved with more complicated weighting schemes.

3. Results and Conclusions

The results, summarized in Table 2, show that the D2 design provides the largest available FOV and is a good choice as a telescope that needs to accommodate a large array of photometers. The AG design provides the smallest FOV, although it is still considerably better than the classical Gregorian telescope upon which all of these systems are based. With our telescope parameters, the optimized D1 and D2 designs provide $\sim 20\%$ and $\sim 50\%$ larger DLFOV than the optimized AG design, respectively. As expected, a larger DLFOV is obtained in all systems by optimizing the parameters of the focal surface, but this improvement decreases from $\sim 60\%$ in the case of the AG design to only about $20\%$ for the D2 design. Because the D2 system with a flat focal plane provides a usable FOV that is almost as large as the one with an optimized focal surface, it is very suitable for arrays of detectors that are fabricated on flat silicon wafers.$^{3-7}$ For this system the physical lengths of the axes of the diffraction limited region of the focal plane are 17.7 and 16.2 cm in the elevation and cross-elevation directions, respectively.

The DLFOV that we found for each of the three telescope designs is the area in which the effects of aberrations are small compared to diffraction for a frequency of 150 GHz and an aperture that gave a single mode beam size of 8 arcminutes. It is straightforward to show that for a fixed beam size and in the single mode optics limit the DLFOV will be larger at higher frequencies. For single mode optics the aperture area $A$, beam solid angle $\Omega$, and
frequency $\nu$ are related through $A\Omega = C/\nu^2$, so for a fixed beam size $A \propto \nu^{-2}$; at higher frequencies the aperture area is smaller. Since the diffraction spot (or Airy disk) size scales as $1/(A\nu^2)$, it is a constant as a function of frequency under these assumptions. Ray aberrations, however, decrease as $A$ decreases so the ratio of aberration size to diffraction spot size also decreases with increasing $\nu$. Because a single mode system of constant beamsize becomes increasingly diffraction limited at higher frequencies, the DLFOV is likely to increase.

Since the D1 and D2 designs can significantly improve the available area in the FOV it is instructive to assess the shape of the mirrors that these designs require. The D1 mirrors are conic sections, an ellipsoid and a paraboloid, identical to those that define the CG, but the ellipsoid axis is tilted relative to the paraboloid axis. This tilt is chosen according to conditions outlined by Dragone; in the case of the Archeops telescope the angle is $15^\circ$ (see Table 1 and Figure 1). In the D2 design, localized corrections are applied to the shape of the mirrors of the D1 design. These corrections are designed to cancel coma near the center of the FOV. The magnitude of the local surface corrections are given by $K r^4$, where $K$ is a constant that is different for each of the two mirrors and $r$ is the perpendicular distance from the the segment of the optical axis between the two mirrors, see Figure 1.

The constant $K$ depends on the distance between the mirrors along the optical axis, and for Gregorian telescopes the corrections are such that they curve the primary toward the secondary and the secondary away from the primary (see Figure 1). For the Archeops system that we have discussed in this paper $K = 3.54 \times 10^{-9}$ and $7.76 \times 10^{-8}$ cm$^{-3}$ for the primary and secondary, respectively. Given the sizes of the two Archeops mirrors the largest correction of the primary is 3.4 mm and is 2.4 mm for the secondary, values which are neither very large compared to the size of the mirrors, nor so small such as to make
accurate machining difficult.
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28. The tilt is about an axis perpendicular to the plane of the optical axis.

29. Our optimization was performed using the CodeV optimization routine, using our own optimization criteria intended to maximize DLFOV.
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Fig. 1. The D1 and D2 systems compared in this paper. The correction applied to the D2 system has been increased by a factor of 25 to make it visible in this plot. Some parameters of the D1 system given in Table 1 are labeled.

Fig. 2. The field points used in the focal plane optimizations. The crosses (+) mark the AG field points and the diamonds (⋄) mark the field points used for both the D1 and D2 designs.
Figure 1, S. Hanany and D. P. Marrone.
Figure 2, S. Hanany and D. P. Marrone.
Table 1. Various parameters of the Archeops telescope.

| Parameter                                      | Value     |
|------------------------------------------------|-----------|
| Aperture ($a$)                                 | 1500 mm   |
| Focal Ratio ($f$)                              | 1.33      |
| Off-Axis Distance ($y$)                        | 997.79 mm |
| Relative Tilt of Mirror Axes                   | 15°       |
| Primary Mirror:                                |           |
| Physical Dimensions                            | 1500 mm × 1768 mm |
| Shape                                         | Paraboloid|
| Focal Length                                   | 800 mm    |
| Secondary Mirror:                              |           |
| Physical Dimensions                            | 790 mm × 841 mm |
| Shape                                         | Ellipsoid |
| Semi-major Axis                                | 650 mm    |
| Conic Constant                                 | -0.1837   |
Table 2. Elevation and cross-elevation intervals, and diffraction limited field of view (DLFOV) for various designs of off-axis Gregorian telescopes

| System          | Elevation (deg.) | Cross-Elevation (deg.) | DLFOV<sup>a</sup> (sq. deg.) (10<sup>-3</sup> sr.) |
|-----------------|------------------|------------------------|-----------------------------------------------|
| Nominal CG<sup>b</sup> | 1.30             | 2.20                   | 2.20 0.67                                      |
| Nominal AG<sup>c</sup> | 3.85             | 3.30                   | 10.0 3.05                                     |
| Optimized AG     | 4.85             | 4.40                   | 16.8 5.12                                     |
| Nominal D1<sup>d</sup> | 4.95             | 4.60                   | 17.9 5.45                                     |
| Optimized D1     | 5.35             | 4.70                   | 19.7 6.00                                     |
| Nominal D2<sup>d</sup> | 5.45             | 5.00                   | 21.4 6.52                                     |
| Optimized D2     | 5.95             | 5.20                   | 24.3 7.40                                     |

<sup>a</sup> The DLFOV is given in both square degrees and steradians

<sup>b</sup> Classical Gregorian

<sup>c</sup> Aplanatic Gregorian

<sup>d</sup> D1 and D2 are defined in the text