Vector field and rotational curves in dark galactic halos

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Abstract. We study equations of a non-gauge vector field in a spherically symmetric static metric. The constant vector field with a scale arrangement of components: the temporal component about the Planck mass $m_{Pl}$ and the radial component about $M$ suppressed with respect to the Planck mass, serves as a source of metric reproducing flat rotation curves in dark halos of spiral galaxies, so that the velocity of rotation $v_0$ is determined by the hierarchy of scales: $\sqrt{2}v_0^2 = M/m_{Pl}$, and $M \sim 10^{12}$ GeV. A natural estimate of Milgrom’s acceleration about the Hubble rate is obtained.

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1. Introduction

In contrast to a scalar field appropriately elaborated in the cosmology lately, a vector field has remained beyond intensive studies of astro-particle problems.

Indeed, a slowly rolling scalar field drives an inflation [1], which produces a correct spectrum of inhomogeneous perturbations in a matter density as was observed in an anisotropy of cosmic microwave background [2]. Next, a scalar field naturally gives a state with a negative pressure, a quintessence [3] [4] [5], so that it can cause an accelerated expansion of Universe [6] with a dynamical dark energy. Moreover, some authors try to ascribe a dark matter in halos of galaxies [7] [8] [9] to a scalar field, too, though properties of such the fields at the galaxy scales should be rather different from those of quintessence at the cosmic scale [10] [11] [12] [14]. So, the scalar fields provide a preferable, more or less successful theoretical treatment of important astro-particle phenomena‡. Though some tuning procedures might be required due to an arbitrariness of scalar potentials. So, the scalar particle of quintessence should have a mass of Hubble constant scale, for instance. In addition, the dark matter distribution in the spiral galaxies falls as $1/r^2$, which could be a problem for a theoretical explanation in terms of both a scalar field or exotic weak interacting particles.

However, as was recently observed [16], a vector field dynamics possesses features, which make it attractive for the cosmological studies. So, in the evolution of flat isotropic homogeneous universe a vector field gains a dynamical mass proportional to

‡ An example of alternative description due to a modification of gravity is given in [15].
the Hubble constant at any, even trivial, potential\textsuperscript{§}. Therefore, we do not need any synthetic assumptions to get such the characteristic property in the sector of scalar fields\textsuperscript{||}, but instead it is enough to introduce an isotropic vector field in the cosmology, and covariant derivatives automatically generate such the dynamical mass term.

We study a simple Lorentz-invariant form of lagrangian for a vector field interacting gravitationally only\textsuperscript{¶},

\[ \mathcal{L}_V = \frac{1}{2} g^{\mu\nu} \left( \nabla_\mu \phi^m \right) \left( \nabla_\nu \phi^n \right) g_{mn} - V(\phi^2), \]  

(1)

where \( \xi \) is a vector field signature\textsuperscript{+}, that can be normal (\( \xi = -1 \)) or phantomic (\( \xi = +1 \)), respectively\textsuperscript{*}.

Since a temporal component of the four-vector has a negative sign of its kinetic term at the normal signature \( \xi = -1 \), we will not treat (1) as a fundamental lagrangian in a sense of axiomatic field theory, but we address it as an effective phenomenological lagrangian with a phantom: the field possessing the negative kinetic term. So, we accept the following \textbf{postulate of validity} for the phantom phenomenology:

- the phantom component is not observable in the flat space-time, i.e. it does not interact with matter field (one can use a standard technique in order to introduce a gauge invariant lagrangian for a vector field);
- in gravity, the phantom is valid in regions \textit{beyond physical singularities}, if exist.

In this respect, a motivation for the lagrangian of (1) could be manyfold.

First, a phenomenology of a scalar quintessence shows that a state parameter of the quintessence, i.e. the ratio of a pressure to a density of energy \( w = p/\rho \), can take values less than \(-1\) \textsuperscript{3}:

\[ w < -1, \]

that, probably, puts the quintessence into the phantom stage: the field with a negative sign of kinetic energy \textsuperscript{5}\textsuperscript{21}\textsuperscript{22}. The lagrangian of vector field in (1) \textit{naturally} contains a phantom component, the temporal component at normal signature \( \xi = -1 \). An important physical difference from the case of scalar phantom is a non-trivial covariant derivative of the vector field.

However, the negative kinetic energy of a fundamental field is not restricted from below, so that the phantom cannot be considered as a fundamental field. Such the treatment of scalar phantom involves a cut-off for the kinetic term or equivalently higher derivative terms providing a low boundary of negative energy. For example, we can add a term of the form

\[ \delta \mathcal{L} = \frac{1}{\Lambda^2} \dot{\phi}^4, \quad \dot{\phi} = \frac{d\phi}{dt}, \]

which guarantees a ‘stabilization’ of negative kinetic term \( \mathcal{L}_0 = -1/2 \dot{\phi}^2 \) for the scalar phantom. Nevertheless, greater characteristic time-scale of field changes with respect to the inverse stabilization scale \( 1/\Lambda \) is more accurate leading approximation

\textsuperscript{§} Note, that a reasonable form of potential for a vector field is more strictly constrained than that of a scalar field.

\textsuperscript{||} The interacting scalar fields could give a dynamical mass of Hubble scale as was shown in [17].

\textsuperscript{¶} A gauge vector field with a global symmetry was investigated in [18], while a non-linear electrodynamics, which leads to an acceleration of universe, was studied in [19].

\textsuperscript{+} The signature of metric is assigned to (+, −, −, −).

\textsuperscript{*} Note that the four-vector is generically composed by spin-1 and spin-0 components (see ref. [20] for discussion on a covariant object decomposition into components with definite spins).
by the small negative kinetic term. So, we can expect that at cosmological and
galactic scales the effective phantom theory with negative kinetic term could be
rather sound, if the stabilization scale is microscopic enough. Decay instabilities of
phantom in the presence of several phantoms were considered in [3]. Thus, standard
arguments justifying the study of scalar phantoms are applicable for the introduction
of lagrangian (1) for the vector field.

Second, theories with extra dimensions generate cosmological equations including
specific terms quadratic in energy-momentum tensors of matter propagating in 4 di-
dimensions in addition to ordinary energy-momentum term [23]. This fact can be con-
sidered as an effective redefinition of energy-momentum tensor:

\[ \rho \rightarrow \rho + \frac{1}{2\sigma} \rho^2, \]

where \( \sigma \) is a bare cosmological constant (a brane tension). So, the negative kinetic
term of phantom should generate the term quadratic in the kinetic energy, which
effectively reproduces the cut-off mentioned above.

Third, let us show that in contrast to a naive expectation a free phantom
interacting with a gravity only, cannot propagate in a homogeneous isotropic space-
time at all. Indeed, in the Friedmann–Robertson–Walker metric

\[ ds^2 = dt^2 - a^2(t) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2], \]

(2)

we get the following independent field equations for the time-dependent homogeneous
scalar phantom \( \phi \):

\[ H^2 = \frac{8\pi G}{3} \left( -\frac{1}{2} \dot{\phi}^2 + V_0 \right), \quad \ddot{\phi} + 3H \dot{\phi} = 0, \quad H \equiv \frac{\dot{a}}{a}, \]

(3)

where we have added a cosmological constant \( V_0 \) for reference. At flat limit of \( V_0 = 0 \)
we get evident condition

\[ H^2 \geq 0 \quad \Rightarrow \quad \dot{\phi} \equiv 0, \]

and the phantom is a constant field, which does not evolve (or propagate). Moreover,
in the case of positive cosmological constant \( V_0 > 0 \), the ‘pathological’ kinetic energy
is automatically restricted

\[ \dot{\phi}^2 \leq 2V_0. \]

Such the kind of the restriction remains valid also in the presence of ordinary matter
with a density \( \rho \) and pressure \( p \) satisfying the conservation law for the tensor of energy-
momentum

\[ \dot{\rho} + 3H (\rho + p) = 0, \]

in the isotropic homogeneous expanding universe, so that a finite density of energy is
falling down, and the permitted region of phantom energy becomes more narrow. The
same arguments are valid for the temporal component of the vector field, too.

In addition, we emphasize that the metric of (2) substituted into the Einstein–
Hilbert action of gravity

\[ S_{EH} = -\frac{1}{16\pi G} \int R \sqrt{-g} \ d^4x \]

after the integration by parts, takes the form

\[ S_{FRW} = -\frac{3}{8\pi G} \int \dot{a}^2 a \ d^4x, \]

(4)
which is the action for a scalar field $a(t)$ up to a normalization factor with a ‘phantom’ sign of kinetic term. That is why a cosmological singularity is possible. A reason for a consistency of such the theory of gravity is due to the FRW metric does not give ‘free gravitational waves’, but it refers to the coupled case, i.e. we deal with a virtual gravitational field, while the harmonic oscillations are well behaved. Therefore, the same features can be ascribed to fields of ‘gravitational nature’, as we will mean on the vector field.

Next, we consider a vector field, which Lagrangian is not invariant under gauge transformations. For comparison, any gauge field $A_\mu$, for example, an abelian field, contains unphysical degree, i.e. purely gauge component:

$$A_\mu = \partial_\mu f.$$  

If the energy-momentum tensor of gauge field is gauge invariant, then the purely gauge component does not propagate. If we add a gauge-fixing term, the gauge invariance guarantees that the longitudinal component preserves its bare propagator, i.e. it is not renormalized. Hence, the longitudinal field undetectable by gauge interactions remains arbitrary and gauge-dependent, anyway. So, it would be rather reasonable to study gauge-noninvariant vector field interacting with gravity, only, in order to touch main effects due to such the field.

Finally, a current-current interaction

$$\mathcal{L}_{\text{int}} = j^\mu A_\mu$$

preserves its gauge independence, if the matter current $j^\mu$ is transversal:

$$\nabla_\mu j^\mu = 0.$$  

This note could be important, since it gives a chance for transversal components of the vector field interact with the matter, if they do not kinematically mixed with longitudinal ones.

Further, the observed quasi-isotropic cosmic microwave background (CMB) fixes a universe rest frame, i.e it forms an ether. Indeed, experimentalists measure a velocity of Earth motion in CMB in order to extract a small true anisotropy (so-called dipole subtraction). Thus, the expanding Universe determines a specific rest frame, which allows one to introduce fields, corresponding to the symmetry of the frame. Particularly, the vector field preserving the homogeneous and isotropic metric should have a temporal component, only: $\phi^m = (\phi_0, 0)$. It was a priori clear, that the curved metric of expanding Universe breaks the Lorentz-invariance, of course. That is why the vector field components have a fixed form required by the symmetry of the physical system. In addition, we expect that an effective potential of the vector field should be of a gravitational origin, so that its dimensional parameters are posed in the Planck mass range. If the potential has a stable point, we expect that a corresponding value of temporal component $\phi_0$ is given by $\phi_0 \sim m_{\text{Pl}}$. We will see that the expansion causes a small time dependence of $\phi_0(t)$ in vicinity of $\phi_0$. So, the variation of $\phi_0$ at galactic scales of distance and time is negligible. This fact means that we get a factorization: the field equations give a true function of $\phi_0(t)$, which can be considered as a constant external field $\phi_0$ at galactic level.

Furthermore, given the external vector field source $\phi_0 = \phi_0$, we can allow small stochastic fluctuations of spatial components $\phi$, only. These fluctuations could be related with matter fields. For instance, a complex scalar matter field $\chi$ can develop a vacuum fluctuations (condensates) at a characteristic scale $M \ll m_{\text{Pl}}$, so that the spatial derivatives $\nabla \chi$ coupled to the spatial components of vector field,
can stochastically get nonzero vacuum expectations $\langle \partial_i \chi^1 \partial_j \chi \rangle$ at the same scale $M$, and the effective potential of spatial vector field could acquire a form yielding a stochastically stable point of $\langle \phi^2 \rangle = \phi_0^2 \sim M^2$, too. Thus, we arrive at the position with two scales of expectation values for the vector field: the temporal component $\phi_0$ of the order of $m_{Pl}$ and the spatial component $\phi_0$ of the order of $M \ll m_{Pl}$. This small parameter is characteristic for the problem, and the arrangement of scales is valid, say, at $M$ determined by a spontaneous symmetry breaking in a great unification theory, $M \sim M_{GUT}$. The constant vector field with the scale arrangement serves as the external source in the gravity equations at the galactic scale.

The consideration is essentially transformed in the spherically symmetric case, since the symmetry of the system allows the spatial component directed along the radius-vector: $\phi = \phi_0 \mathbf{n}$ with the unit vector $\mathbf{n} = \mathbf{r}/r$. Therefore, the external vector field takes the form $\phi^m = (\phi_0, \phi_0 \mathbf{n})$, which is purely gauge field with a gauge function $f(t, r) = \phi_0 t - \phi_0 r$ and $\phi_m = \partial_m f(t, r)$ in a flat space-time limit, so that this field can be detected gravitationally, only. In addition, this point gives an extra argument for the lagrangian of $\Upsilon$, since one could completely substitute a scalar field derivative for the vector field: $\phi^m = \partial_m f(t, r)$, so that the potential near the extremal point, say, $V \approx V_0 - \mu^2 (\phi_0^2 - \phi^2)$, would be transformed to an ordinary kinetic term of scalar field $f$ up to a normalization factor, while the other terms represent higher derivative contributions in an affective lagrangian.

In this paper we study a vector field dynamics in a static spherically symmetric metric, which should be in halos of spiral galaxies. Rotational curves in such the galaxies become flat in the regions of dark halos, that corresponds to $1/r^2$-dependence of dark matter density on the distance from the galaxy center. We show that the covariant derivatives of vector fields naturally and uniquely generate such the dependence. A physical reason for the conclusion is rather simple. First, the vector field gains a dynamical mass term determined by a spatial curvature. Second, the problem introduces a small parameter, a constant velocity of rotation in the dark halos $v_0$. Third, following the factorization of cosmological and galactic scales, at large distances we have to reach a cosmological limit: the vector field negligibly slowly evolving with time and distance, i.e. the constant field at the galaxy scale with the hierarchy of expectations $\kappa = \phi_0/\phi_0 \ll 1$. We find the relation between two small parameters, the velocity and scale ratio: $v_0^2 = \kappa/\sqrt{2}$. Then, the curvature should fall as $1/r^2$, only, that reproduces the flat rotational curves.

The paper is organized as the following: In section 2 we study Einstein equations with the vector field in a static spherically symmetric metric

$$ds^2 = f(r) dt^2 - \frac{1}{h(r)} dr^2 - r^2 [d\theta^2 + \sin^2 \theta d\varphi^2], \quad (5)$$

by deriving the energy-momentum tensor. Then we remind a condition following from the observation of flat rotation curves: the function $f(r)$ should take a specific form up to small corrections neglected, while $h(r)$ should be close to 1. Then we expand in small $v_0^2$ and $\kappa$ and find the solution$^6$:

$$v_0^2 = \kappa/\sqrt{2}, \quad h(r) = 1 - 2v_0^2, \quad f(r) = \frac{2v_0^2}{r},$$

giving a $1/r^2$-profile of the curvature and a flat asymptotic behavior for a velocity.

$^6$ We use an ordinary notation for the derivative with respect to the distance by the prime symbol $\partial_r f(r) = f'(r)$. 

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5. Vector field and rotational curves
of rotation††, so that the velocity squared is determined by the small ratio of spatial component to the temporal one, i.e. the ratio of characteristic matter scale to that of gravity, the Planck mass, that gives \( M \sim 10^{-7} m_{\text{Pl}} \sim 10^{12} \text{ GeV} \), a scale in the range of GUT breaking†. In section 3 we analyze the factorization. Section 4 is devoted to the description of applicability region for the flat rotation curves in the framework of constant vector fields, that gives a natural estimate of Milgrom’s acceleration. Other approaches are shortly discussed for comparison. The results are summarized in Conclusion.

2. Generic equations

General expressions for the Christoffel symbols, Ricci tensor and scalar curvature for the spherically symmetric static metric of (5) are listed in Appendix A.

Then, a tensor entering the Einstein equations

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu},
\]

i.e.,

\[
G^\mu_{\nu} = R^\mu_{\nu} - \frac{1}{2} \delta^\mu_{\nu} R,
\]

takes the form

\[
G^t_t = \frac{1 - h}{r^2} - \frac{h'}{r}, \quad G^r_r = \frac{1 - h}{r^2} - \frac{f'}{r} \left( \frac{f'}{f} - \frac{b'}{b} \right),
\]

\[
G^\theta_\theta = G^{\phi}_\phi = -\frac{1}{2} \frac{h}{f} \frac{f'^2}{f} - \frac{1}{4} \frac{h}{f} \left( \frac{f'}{f} + \frac{b'}{b} \right) + \frac{1}{4} \frac{h}{f} \frac{f'}{f} \left( \frac{f'}{f} - \frac{b'}{b} \right) + \frac{r}{2} \phi^2_f - \frac{1}{2} \frac{h'}{h} \phi^2_r,
\]

In polar coordinates \( x^m = (t, r, \theta, \phi) \), covariant derivatives of vector field \( \phi^m = (\phi_0, \phi_r, 0, 0) \) in the metric of (5) are equal to

\[
\phi^t_t = \frac{1}{2} f' \phi_r, \quad \phi^t_r = \phi'_0 + \frac{1}{2} \frac{f'}{f} \phi_0,
\]

\[
\phi^r_t = \frac{1}{2} b f' \phi_0, \quad \phi^r_r = \phi'_r - \frac{1}{2} \frac{b'}{b} \phi_r,
\]

\[
\phi^\theta_\theta = \phi^{\phi}_\phi = \frac{1}{r} \phi_r, \quad \sqrt{-g} = \sqrt{f/b} r^2 \sin^2 \theta,
\]

where we have also shown the determinant of the metric, too.

Squaring \((D\phi)^2 \equiv \phi_{\mu}^m \phi_{\nu}^m g_{mn} g^{\mu\nu}\) gives

\[
(D\phi)^2 = \frac{1}{2} \frac{f'}{f} \phi^2_r + \left( \phi'_r - \frac{1}{2} \frac{b'}{b} \phi_r \right) \phi^2_r + \frac{2}{r^2} \phi^2_f - \frac{1}{4} \left( \frac{f'}{f} \right)^2 \phi^2_0 - \frac{1}{4} \frac{h}{f} \left( \frac{f'}{f} + \frac{b'}{b} \right) \phi^2_0.
\]

††The same metric was exactly first found by Nucamendi, Salgado and Sudarsky (the last reference of [10]). They also studied the light bending and gravitational lensing, which do not conflict with observations, so that we will not concern for this question in the present paper.

† One can also put \( M \sim \mu^2/m_{\text{Pl}} \) yielding \( \mu \sim 10^{16} \text{ GeV} \), which is closer to the GUT scale. However, this treatment suffers from ‘tending to desirable result’, I think.
The action of the vector field in the problem is written down as
\[ S_V = -\frac{1}{2} \int dt \; dr \; d\theta \; d\phi \; (D\phi)^2 \sqrt{-g}, \]
where we have not displayed a potential, which properties have been described in the Introduction, so that it makes dominant contribution to the field equations for the vector field, that result in the mentioned quasi-constant fields as will be discussed in section 3.

To the moment we remind the restriction following from the flatness of rotation curves. In this way, a particle motion in the metric of (5) is determined by the Hamilton–Jacobi equations
\[ g^\mu\nu \partial_\mu S \partial_\nu S - m^2 = 0, \tag{11} \]
where \( m \) denotes the particle mass. Following the general framework, we write down the solution in the form, which incorporates two integrals of the motion in the spherically symmetric static gravitational field,
\[ S = -\mathcal{E} t + \mathfrak{M} \theta + S(r), \tag{12} \]
where \( \mathcal{E} \) and \( \mathfrak{M} \) are the conserved energy and rotational momentum, respectively. Then, from (11) we deduce
\[ \left( \frac{\partial S}{\partial r} \right)^2 = \frac{1}{\hbar} \mathcal{E}^2 - \frac{1}{\hbar} \left( \frac{\mathfrak{M}^2}{r^2} + m^2 \right), \tag{13} \]
which results in
\[ S(r) = \int_{r_0}^{r(t)} dr \; \frac{1}{\sqrt{\hbar}} \sqrt{\mathcal{E}^2 - V^2(r)}, \tag{14} \]
where \( V^2 \) is an analogue of potential,
\[ V^2(r) = \int \left( \frac{\mathfrak{M}^2}{r^2} + m^2 \right). \]
The trajectory is implicitly determined by the equations
\[ \frac{\partial S}{\partial \mathcal{E}} \bigg|_{\theta} = \text{const} = -t + \int_{r_0}^{r(t)} dr \; \frac{1}{\sqrt{\hbar}} \frac{\mathcal{E}}{\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{15} \]
\[ \frac{\partial S}{\partial \mathfrak{M}} \bigg|_{\theta} = \text{const} = \theta - \int_{r_0}^{r(t)} dr \; \frac{\mathfrak{M}}{r^2} \frac{1}{\sqrt{\hbar}} \sqrt{\mathcal{E}^2 - V^2(r)}. \tag{16} \]
Taking the derivative of (15) and (16) with respect to the time \( t \), we get
\[ 1 = \dot{r} \frac{\mathcal{E}}{\sqrt{\hbar} \sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{17} \]
\[ \dot{\theta} = \frac{\dot{r}}{r^2} \frac{\mathfrak{M}}{\sqrt{\hbar} \sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{18} \]
and, hence,
\[ \mathcal{E} = \frac{\dot{r}}{r^2} \mathfrak{M}, \tag{19} \]
\(^\dagger\) As usual \( \partial_t f(t) = \dot{f} \).
relating the energy and the rotational momentum, where we have introduced the velocity
\[ v \overset{\text{def}}{=} r \dot{\theta}. \]

The points of return are determined by
\[ \dot{r} = 0, \quad \Rightarrow \quad \mathcal{E}^2 - V^2 = 0, \quad \Rightarrow \quad 2\mathcal{N}^2 = m^2 r^2 \frac{v^2}{f - v^2}. \]

The circular rotation takes place, if two return points coincide with each other, i.e. we have the stability of zero \( \dot{r} \) condition. Introducing a ‘proper distance’ \( \lambda \) by
\[ \frac{\partial}{\partial \lambda} = \frac{\partial r}{\partial \lambda} \frac{\partial}{\partial r} = \sqrt{\frac{\mathcal{N}}{f}} \frac{\partial}{\partial r} \]
we deduce the wave-equation with spectral parameter \( \mathcal{E}^2 \) and ‘potential’ \( V^2 \)
\[ \left( \frac{\partial S}{\partial \lambda} \right)^2 = \mathcal{E}^2 - V^2, \quad (20) \]
so that the stability of circular motion implies the stability of potential,
\[ \frac{\partial V^2}{\partial r} = 0. \quad (21) \]

Then, we get
\[ v^2 = \frac{1}{2} \frac{d f(r)}{d \ln r}. \quad (22) \]

Introducing a re-scaled velocity with respect to the proper time,
\[ \varepsilon^2 = \frac{1}{f} v^2, \]
we get the result of \[ 24 \]
\[ \varepsilon^2 = \frac{1}{f} \frac{d \ln f}{d \ln r}. \]

An accuracy of observations do not allow us to distinguish \( v \) from \( \varepsilon \), since \( f(r) \rightarrow 1 \) at large distances in dark halos. So, in the halo we put
\[ v = v_0 = \text{const.} \]

Therefore,
\[ f' = 2v_0^2 \frac{v_0}{r} \quad (23) \]
gives the profile of flat rotation curves\dagger. Nonrelativistically, we get
\[ \mathcal{H} = 1 - q, \quad q \ll 1, \quad (24) \]
while the form of dependence on the distance is not fixed by the flatness.

Let us show that for the vector field serving as an external source, there is a solution of Einstein equations with
\[ q' \approx 0, \quad (25) \]
where the approximation means the leading order in small parameter of \( v_0^2 \), which is, in practice, \( v_0^2 \sim 10^{-7} \), since the characteristic velocity in the halos is about \( 100 - 150 \) km/s, i.e \( v_0 \sim (1/3 - 1/2) \times 10^{-3}. \)

\dagger More specified consideration can be found in \[ 25 \], for instance.
Remember, that we consider the following limit for the external vector field in a spiral galaxy:

\[
\phi_0 = \phi_*, \quad \phi_r = \phi_*, \quad \kappa = \frac{\phi_*}{\phi_*} \ll 1,
\]

so that

\[
\phi_0' = 0, \quad \phi_r' = 0,
\]

and we refer the system of (24)–(27) as a C-surface condition. Then

\[
(D\phi)^2|_C = -\frac{1}{2} \frac{b}{f} (f')^2 \phi_*^2 + \frac{1}{4} \left(\frac{f'}{f}\right)^2 \phi_*^2 + \frac{2}{r^2} \phi_*^2,
\]

so that we get terms quadratic in the field, that implies the appearance of ‘induced’ mass. This fact is characteristic for the vector field dynamics in the curved space-time. Combining the C-surface with (23) gives a C-surface.

It is a straightforward task to calculate the energy-momentum tensor for the vector field

\[
T_\nu^\mu = 2 g_\nu^\alpha \sqrt{-g} \frac{\delta S_V}{\delta g_\mu^\alpha},
\]

so that

\[
T^t_t = -2 f \frac{\sqrt{b/f}}{r^2 \sin \theta} \frac{\delta S_V}{\delta f}, \quad T^r_r = 2 h f \frac{\sqrt{b/f}}{r^2 \sin \theta} \frac{\delta S_V}{\delta h},
\]

while the angle components are given by

\[
T_\theta^\theta = T_\phi^\phi = -\frac{\sqrt{b/f}}{r^2 \sin \theta} \frac{\delta S_V}{\delta \lambda},
\]

where \(\delta \lambda\) is the dilatation of \(\varphi\):

\[
\delta \lambda \varphi = \delta \lambda \cdot \varphi.
\]

Direct calculations result in

\[
T^t_t|_C = -\frac{1}{2} \frac{\phi_*^2}{r^2} \left[ -2 h f'' + \frac{1}{2} (f')^2 \frac{b}{f} - \frac{4}{r} h f' \right] - \frac{1}{8} \frac{\phi_*^2}{r^2} \left[ -\frac{8}{r^2} - 3 \left(\frac{f'}{f}\right)^2 + \frac{8}{r} \frac{f'}{f} + 4 \frac{f''}{f} \right].
\]

Applying C-surface in the leading order over \(v_0 \ll 1\) and \(\kappa \ll 1\) we get

\[
T^t_t|_C \approx \frac{\phi_*^2}{r^2} (2v_0^2 + \kappa^2),
\]

and

\[
G^t_t|_C \approx \frac{q}{r^2}.
\]

The Einstein equation for the temporal components reads off

\[
G^t_t|_C = 8\pi G T^t_t|_C \Rightarrow q = 8\pi G \phi_*^2 (2v_0^2 + \kappa^2).
\]
transformed to
\[
T_{rr}^C \approx \frac{\phi^2}{r^2} (v_0^4 + \kappa^2),
\]
while
\[
G_{rr}^C \approx \frac{q - 2v_0^2}{r^2},
\]
and
\[
G_{rr}^C = 8\pi G T_{rr}^C \Rightarrow q - 2v_0^2 = 8\pi G \phi^2 (v_0^4 + \kappa^2).
\]
Finally, the angle component is equal to
\[
T_{\phi\phi}^C = T_{\theta\theta}^C \approx \phi^2 \frac{r^2}{v_0^4} (\kappa^2 - v_0^4),
\]
yielding
\[
T_{\phi\phi}^C \approx \frac{\phi^2}{r^2} (\kappa^2 - v_0^4),
\]
so that
\[
G_{\phi\phi}^C = \frac{\phi^2}{r^2},
\]
and
\[
G_{\phi\phi}^C = 8\pi G T_{\phi\phi}^C \Rightarrow v_0^4 = 8\pi G \phi^2 (\kappa^4 - v_0^4).
\]
Equations (32), (35) and (38) are satisfied at
\[
\kappa = \sqrt{2v_0^2 + \mathcal{O}(v_0^4)),
\]
\[
q = 2v_0^2 + \mathcal{O}(v_0^4),
\]
\[
8\pi G \phi^2 = 1 + \mathcal{O}(v_0^2).\]
Therefore, this solution results in the temporal component of the energy-momentum tensor dominates and has the required profile with the distance:
\[
T_{rr} \sim T_{\phi\phi} \sim T_{\theta\theta} \sim \mathcal{O}(v_0^2) \cdot T_t \sim \mathcal{O} \left( \frac{1}{r^2} \right).
\]
Numerically, we get
\[
\kappa \sim 10^{-7} \Rightarrow M \sim 10^{12} \text{ GeV},
\]
hence, the characteristic scale of matter influence on the vector field is in the range of GUT and effective scale responsible for the small neutrino masses.

Thus, the small ratio of two natural energetic scales determines the rotation velocity in dark galactic halos. In addition, we have found condition (41), which is analogous to a definition. Let us test this problem in the next section.
3. Factorization

The previous section treats the vector field as an external source. That would be valid, if the vector-field equations are satisfied. In that case we should take into account the field potential as we suggest that the characteristic scales in this potential much greater than the scales induced by the size of galactic halos as well as the Hubble rate or large-scale inhomogeneous structures.

So, first, suppose that we deal with the constant isotropic homogeneous vector field. It means that the radial component is equal to zero, while the temporal component is posed at the extremal point of its potential:

\[ \frac{\partial V}{\partial \phi_0} \bigg|_C = 0, \quad \frac{\partial^2 V}{\partial \phi_0^2} \bigg|_C = m_0^2, \quad |m_0| \sim \phi_0 = \phi_* \sim m_{Pl}. \]

If we consider an expanding isotropic homogeneous Universe, then the potential gets an additional term, determined by the Hubble rate [16]:

\[ \delta V = \frac{3}{2} H^2 \phi_0^2, \]

so that the temporal component acquires a slow variation with the time due to the displacement of stable point, since the field equation takes the form

\[ \ddot{\phi}_0 + 3H \dot{\phi}_0 - 3H^2 \phi_0 - \frac{\partial V}{\partial \phi_0} = 0, \]

and neglecting the time derivative, we get

\[ 3H^2 \phi_0 + \frac{\partial V}{\partial \phi_0} = 0. \]

Expanding in \( \phi_0 \) at \( \phi_0 = \phi_* + \delta \phi_0 \) gives

\[ 3H^2 \phi_* + (3H^2 + m_0^2) \delta \phi_0 = 0, \]

so that at \( H \ll m_{Pl} \)

\[ \frac{\delta \phi_0}{\phi_*} \approx -\frac{3H^2}{m_0^2}, \]  

which is really small correction, we justify. The induced time-dependence is due to the Hubble rate

\[ \delta \dot{\phi}_0 \approx -H \phi_* \frac{H^2}{m_0^2} \left( \frac{1}{H^2} \frac{\dot{a}}{a} - 1 \right) \Rightarrow |\delta \dot{\phi}_0 / \phi_0 H| \ll 1. \]

Analogous arguments are valid for the spatial component of the vector field: the corresponding effective mass of radial component is \( m_r \sim M \), introduced above, while the correction due to the covariant derivative

\[ \delta_r V = \frac{2}{r^2} \phi_r^2 \]

generates a small correction at macroscopic scales \( 1/r \ll M \), so, an induced dependence of \( \phi_r \) on the distance is actually suppressed, and \( \phi_r \approx \phi_* \).

If both the expansion and the radial component are nonzero, then the covariant derivatives

\[ \phi_r^r = H \phi_0, \quad \phi_r^\theta = \phi_r^\varphi = \frac{1}{r} \phi_r + H \phi_0, \]  

(43)
induce the correction to the potential

$$\delta_H V = \frac{1}{2} H^2 \phi_0^2 + \left( \frac{1}{r} \phi_r + H \phi_0 \right)^2,$$

which gives similar restrictions on the suppressed variations of vector field, of course.

Therefore, both the evolution equations and Einstein equations at galactic scales can be considered with the constant vector field serving as the external source of gravity.

Note, that as was shown in [16], the dependence of $\phi_0$ on time generates a variation of effective gravitational constant with the time. However, such the dependence should be small as forced by the experimental observations. We have seen that in the scheme described above the time-dependence of vector field is negligible.

Finally, let us discuss the relation

$$8\pi G \phi_0^2 = 1,$$

which looks like a definition, but the constraint. Introduce a bare gravitational constant $G_0$ and an extra interaction of vector field with the gravity, so that we consider the FRW metric, while generically instead of $\phi_0^2$ one should introduce the invariant square of vector field in the interaction, of course.

$$3 \left( \frac{1}{8\pi G_0} + \frac{c}{2} \phi_0^2 \right) H^2 = \rho - \frac{3}{2} H^2 \phi_0^2,$$

which reproduces the ordinary form, if we put

$$\frac{1}{G} = \frac{1}{G_0} + (1 + c) 4\pi \phi_0^2.$$

Taking into account [15], we get

$$G_0 = \frac{2}{1 - c} G.$$

For instance, at $c = 1$ we have $1/G_0 = 0$, and we find that $\phi_0^2$ defines the gravitational constant, or the Planck mass.

Thus, we have shown that preferably gives a defining relation for the gravitational constant, which has been used as the starting point for the scale arrangement. This statement seems to be justified. On the other hand, it points to a deep connection of the vector field dynamics with the gravity.

### 4. Indicating the Milgrom’s acceleration

As we have just shown in the previous section, the expansion of Universe causes the time-dependence of metric, which produces a negligible time-dependence of temporal component of the vector field. Therefore, the ‘cosmological limit’ of vector field is consistently reached. However, the radial component of vector field can induce the...
angle components of covariant derivatives resulting in the anisotropic potential of (44). The anisotropy can be neglected at large distances, when
\[ \left| \frac{1}{r} \phi_r \right| \ll H \phi_0. \]

Then, the ‘cosmological limit’ of constant vector field can be disturbed by the potential of (44), at distances less than \( r_0 \) defined by
\[ \frac{1}{r_0} \phi_r = \varepsilon H \phi_0 \Rightarrow \frac{1}{r_0} \phi_* = \varepsilon H_0, \quad (46) \]
where \( H_0 = H(t_0) \) is the value of Hubble constant at the current moment of time \( t_0 \), and \( \varepsilon \) is a parameter of order of \( 1 - 0.1 \). Thus, at distances less than \( r_0 \) the flatness can be disturbed, since the vector field can acquire a distance dependence\( \dagger \). Substituting the ratio \( \phi_*/\phi_0 = \sqrt{2} v_0^2 \) into (46), we get
\[ \frac{v_0^2}{r_0} = \frac{\varepsilon}{\sqrt{2}} H_0, \quad (47) \]
while the quantity
\[ a_0 = \frac{v_0^2}{r_0} \quad (48) \]
is the centripetal acceleration at a ‘critical radius’ of \( r_0 \). Then, the critical acceleration is determined by the Hubble rate\( + \),
\[ a_0 = \frac{\varepsilon}{\sqrt{2}} H_0, \quad (49) \]
and it determines the acceleration below which the limit of flat rotation curves becomes justified\( * \). That is exactly a direct analogue of the critical acceleration introduced by M. Milgrom in the framework of modified Newtonian dynamics (MOND) \[26\]. In MOND, the Milgrom’s acceleration \( a_0 \) separates two regimes, the Newtonian and modified ones, so that at gravitational accelerations \( a < a_0 \), the dynamics reaches the limit reproducing the non-Newtonian flat rotation curves. By empirical data, Milgrom surprisingly found that \( a_0 \sim H_0 \), which is quite an amazing relation.

Then, equations (46)–(49) shows that the scale of Milgrom’s acceleration naturally appears in the framework of vector field embedded into the theory of gravitation.

Further, we could suppose that in the case, when the gravitational acceleration produced by the visible matter in the galactic centers exceeds the critical value, we cannot reach the limit of flat rotation curves. Indeed, in that case the distance dependence cannot be excluded for the vector field. The Newtonian acceleration at the ‘critical scale’ \( r_0 \) is equal to
\[ a_0^* = \frac{G M}{r_0^2}, \]
where \( M \) is a visible galactic mass. According to (49), the critical acceleration is a universal quantity slowly depending on the time, while (48) implies that the ‘critical distance’ can be adjusted by variation of parameter \( v_0 \). Therefore, we should put
\[ a_0^* = a_0, \quad (50) \]
\( \dagger \) The actual parameter is the ratio of radial component to the distance, of course.
\( \dagger \) The dependence of radial component should dominate.
\( + \) In units with the speed of light equal to 1.
\( * \) In practice, \( \varepsilon \approx 1/4. \)
which yields
\begin{equation}
v_0^4 = GMa_0.
\end{equation}

The galaxy mass is proportional to an H-band luminosity of the galaxy $L_H$, so that \( 51 \) reproduces the Tully–Fisher law
$$L_H \propto v_0^4.$$  

Then, other successes of MOND can be easily incorporated in the framework under consideration, too.

Nevertheless, one could look at \( 50 \) as a coincidence, which can be treated twofold. Indeed, it implies that
$$\phi^2 \sim \frac{GM}{H} \quad \Rightarrow \quad \phi^2 \sim MH.$$  

So, first, the extremal point of potential for the radial component is fixed by the galaxy mass and the Hubble rate, or, second, if we arrange the scale of $\phi_\ast \sim M \sim 10^{12}$ GeV, then the characteristic masses of galaxies should be given by $M \sim M^2/H$. The preference for one of two viewpoints is subjective. Despite of that, a possible challenge is a coincidence problem for the radial field adjusted to get a critical acceleration.

For completeness, let us discuss two modern approaches incorporating the Milgrom’s acceleration, too. First, the empirical suggestion of MOND was reformulated as a nonrelativistic mechanics in the Lagrange form \[27\], where the relativistic theory as a scalar-tensor gravity was also given, though superluminal velocities of the scalar field were found as well as extragalactic gravitational lensing is too weak. A ‘stratified’ theory by Sanders \[28\] involves a priori constant time-like vector field as a disformal extension of curved metric to a physical one. Recently, a tensor-vector-scalar theory for the MOND paradigm succeeded by J.D.Bekenstein \[29\] was presented as a consistent theory with dynamical fields in agreement with all observational data. It involves an unknown function, which guarantees the MOND effect of critical acceleration in a relativistic theory. In this theory the Milgrom’s acceleration is ad hoc quantity, while in the framework we have presented its scale is naturally obtained. Note, that our consideration of Tully–Fisher law has repeated the arguments of J.D.Bekenstein taken from \[29\].

The same note concerns for the second approach: a nonsymmetric gravitation theory (NGT) by J.W.Moffat \[30\], where the scale of Milgrom’s acceleration has been fixed empirically in order to extract the model parameters, say, a characteristic scale. The advantage of NGT is a direct possibility to explicitly fit the astronomical data on the rotation curves by theoretical formulas, though the nonsymmetric rank-2 tensor itself (instead of metric) is rather an exotic object. In addition, the NGT points to an exponential decrease of rotation velocity at infinitely large distances.

Thus, in this section we have got a natural estimate of critical acceleration, the Milgrom’s acceleration determining the region of consistency for the flat rotation curves.

5. Conclusion

In this paper we have found that the spherically symmetric static Einstein equations with the source given by the energy-momentum tensor of constant vector field have the solution characterized by the metric corresponding to flat rotation curves in spiral galaxies at large distances. The feature of such the vector field is the scale hierarchy:
the temporal component is of the order of Planck mass $m_{Pl}$, while the radial component about $M$ should be suppressed, so that the small parameter $\kappa = M/m_{Pl} \ll 1$ determines the square of small rotation velocity $v_0^2 = \kappa/\sqrt{2}$, implying $M \sim 10^{12}$ GeV. The arrangement of scales should be caused by a specific potential of vector field, so that its dynamics at cosmological scales is factorized from the dynamics at galactic scales. Thus, the vector field can give the explanation for the dark matter in galactic halos. This statement is enforced by the natural estimate of Milgrom’s acceleration, below which the flatness can be consistently justified.

We have to point also to a possibility that the vector field, which spatial components in the asymptotic region is directed along the radius-vector (in agreement with the symmetry of the problem), can be a manifestation of a monopole (see papers by Nucamendi, Salgado and Sudarsky in [10]). In that case the magnitude of radial component can fall to zero in vicinity of galactic centers, that can be exploited for an explanation of decrease for the dark matter contribution into the rotation velocity near the galactic centers.

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A. Appendix: Metric values

The spherically symmetric metric of (5) produces the following Christoffel symbols:

\[
\begin{align*}
\Gamma^t_{tr} &= \frac{1}{2} \frac{f'}{f}, & \Gamma^r_{rr} &= -\frac{1}{2} \frac{b'}{b}, & \Gamma^r_{tt} &= \frac{1}{2} \frac{f'}{f} b, \\
\Gamma^\theta_{tr} &= \Gamma^\phi_{tr} = \frac{1}{r}, & \Gamma^r_{\theta\theta} &= -r b, & \Gamma^r_{\phi\phi} &= -r \sin^2 \theta b, \\
\Gamma^\theta_{\phi\phi} &= -\sin \theta \cos \theta, & \Gamma^\phi_{\theta\phi} &= \frac{\cos \theta}{\sin \theta},
\end{align*}
\]

(52)

while the symbols are symmetric over the contra-variant indices, and other symbols not listed above are equal to zero. In eqs.(52) we do not explicitly show the dependence of metric components on the distance.

Then, the non-zero elements of Ricci tensor are given by

\[
\begin{align*}
R_{tt} &= \frac{1}{2} \frac{b f'''}{f} + \frac{1}{r} \frac{b f'}{f} - \frac{1}{4} \left( \frac{f'}{f} - \frac{b'}{b} \right) b f', \\
R_{rr} &= -\frac{1}{2} \frac{f'''}{f} - \frac{1}{r} \frac{b'}{b} + \frac{1}{4} \left( \frac{f'}{f} - \frac{b'}{b} \right) f', \\
R_{\theta\theta} &= 1 - b - r b' - \frac{1}{2} r b \left( \frac{f'}{f} - \frac{b'}{b} \right), & R_{\phi\phi} &= R_{\theta\theta} \sin^2 \theta,
\end{align*}
\]

(53)

while the scalar curvature is equal to

\[
R = \frac{b}{f} f''' - \frac{1}{2} \frac{b}{f} \left( \frac{f'}{f} - \frac{b'}{b} \right) f' + \frac{b}{r} \left( \frac{f'}{f} + \frac{b'}{b} \right) - \frac{2}{r^2} (1 - b).
\]

(54)
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