Cubic Hall viscosity in three-dimensional topological semimetals

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Introduction.

The discovery of hydrodynamic flow in two-dimensional (2D) metallic systems [1,2] has renewed interest in the study of nondissipative “Hall” viscosity. In rotationally invariant 2D fluids, there is a single Hall viscosity coefficient related to the topological properties of the occupied electronic states [3–11]. In clean systems, the Hall viscosity manifests in width-dependent corrections to the Hall conductance of mesoscopic channels, backflow corrections to the local current density near point contacts, and in moments of the semiclassical distribution function [12–15]. Local voltage measurements on graphene samples in magnetic fields have shown signatures of the Hall viscosity [16]. Hall viscosity also appears in classical fluids with broken time-reversal symmetry (TRS) like chiral active fluids [17,18]. This “momentum” Hall viscosity (MHV) describes a stress response that can be related to a change in momentum density. The MHV contains meaningful information even beyond the hydrodynamic regime [3,19–21].

In parallel, a related geometric response coefficient—the phonon Hall viscosity (PHV)—has gained attention. A response to dynamic strains via electron-phonon coupling, and also a rank four tensor, the PHV is expected to appear in the dispersion for acoustic phonons [20,22] and in spin-phonon-coupled systems through a contribution to thermal Hall conductance [23–26].

Beyond 2D, the role of nondissipative viscosity in transport remains largely unexplored. Reports of hydrodynamic behavior in topological semimetals (TSMs) [27], and the growing interest in magnetic TSMs [28], raise the question of how to generalize the Hall viscosity to 3D. Preliminary efforts have focused on quasi-2D transport [29–35], or made use of preferred “polar” directions such as the Weyl node separation direction in TSMs. Furthermore, octahedral symmetry forbids the presence of a nonzero Hall viscosity [19,36]. However, magnetic crystals may have nonpolar point group symmetries that are not octahedral; the nondissipative geometric response of such systems remains an open question.

Looking beyond 2D, in this work we find that tetrahedral symmetry allows for the appearance of a new, fundamentally 3D “cubic” Hall viscosity. To our knowledge this has not been encountered before in the literature and could be realized in a wide array of classical and quantum fluids with broken TRS.

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For uniaxial flows, this new viscosity gives a force perpendicular to the flow direction which vanishes when the velocity is constant along the direction of flow. As a proof-of-principle, we focus on a toy model in the experimentally interesting case of the cubic MSG P213 (No. 198.9), with TRS breaking chiral magnetism. Chiral multifold fermions such as these act as point sources of Berry curvature in the Brillouin zone [37–45], making them ideal models to explore topological response functions [40,46–48]. We compute the MHV and PHV for this model. For the PHV, we consider an electron-phonon coupling ansatz to derive the “phonon” strain coupling [20,22,34,49,50] which will yield us a resulting “phonon” stress tensor. For the ansatz to derive the “phonon” strain coupling [20,22,34,49,50], we consider an electron-phonon coupling with tetrahedral symmetry:

\[ f^\eta_j = -\frac{\partial}{\partial t} (\eta t^j) + \frac{\delta t^j}{\partial \theta} \delta^j_i \delta^i_j, \]

(7)

where \( f^\eta_j \) is the force density and \( \delta t^j_i \) is the viscous stress tensor. We find that \( \eta_1 \) and \( \eta_2 \) contribute additively to \( f^\eta_j \):

\[ f^\eta_j = \eta_1 + \eta_2 = \frac{\Lambda}{\sqrt{3}} (\epsilon_{m+i} J_{m+i}). \]

(8)

We see that the fully symmetric tensor \( \Lambda \), which is only invariant in systems with tetrahedral symmetry, plays a key role in generating the nondissipative forces. Contrast this with quasi-2D Hall viscous forces, which take the form,

\[ f_j^{\eta,2D} = \eta_{2D} \epsilon_{m+i} J_{m+i}. \]

(9)

and require a symmetry-breaking pseudovector \( \mathbf{B} \).

Finally, since only the sum \( \eta_1 + \eta_2 \) appears in the viscous forces, there must exist a divergenceless contact term which shifts between \( \eta_1 \) and \( \eta_2 \) in the bulk. This term is

\[ \delta t^j_i = C_0 \epsilon_{m+i} \Lambda_{m+j} J_{m+i}, \]

(10)

which shifts

\[ \eta_1 \rightarrow \eta_1 + \frac{\sqrt{3} C_0}{2}, \]

(11)

\[ \eta_2 \rightarrow \eta_2 - \frac{\sqrt{3} C_0}{2}, \]

(12)

analogous to the bulk redundancy between Hall viscosity and odd pressure in 2D systems [11,61]. We show the effects of \( \eta_{1,2} \) in Fig. 1.

**Tight-binding model.** Let us now consider a model for a cubic chiral magnetic system and compute \( \eta_{1,2} \) for both the MHV and the PHV as a proof-of-principle [62]. Our tight-binding Hamiltonian is

\[ H = \sum_{nm \mathbf{r} \mathbf{r}'} c_{nm}^\dagger \epsilon_{m+i} J_{m+i}, \]

(13)

consisting of \( s \)-type orbitals at 4a Wyckoff position of SG P213 (No. 198). The indices \( n \) and \( m \) label the four orbitals,
and $r, r'$ index the unit cells of the crystal with lattice spacing $a$. The nearest-neighbor hopping $t_{nm}^r$ has uniform magnitude $t$, and we break TRS with a cubic-symmetric magnetic flux $\phi$ via a Peierls substitution [63]. Shifting to momentum space and suppressing the orbital $n$ and $m$ indices, we write

$$H = \sum_{nmk} \hat{c}_k^\dagger f(k)c_k. \quad (14)$$

We give $f(k)$ explicitly in the SM [53], and we show the spectrum in Fig. 2(a). As a simple example of where to find nonvanishing cubic Hall viscosity, we focus on the spectrum in Fig. 2(a). As a simple example of where to find nonvanishing cubic Hall viscosity, we focus on the spectrum in Fig. 2(a). As a simple example of where to find nonvanishing cubic Hall viscosity, we focus on the spectrum in Fig. 2(a). As a simple example of where to find nonvanishing cubic Hall viscosity, we focus on the spectrum in Fig. 2(a).

To do so we first must define a stress tensor, $T_{\mu\nu} = \sum_{nmk} \hat{c}_k^\dagger T_{\mu\nu}^r(k)c_k$, corresponding to Eq. (13). For the PHV, we define the stress tensor by considering an electron-phonon coupling ansatz $[20, 22, 64]$ and perturbing the background lattice, yielding the phonon stress tensor. The phonon stress results from microscopically perturbing the lattice via phonons, relating to atomic displacements. For the MHV, we perturb the electronic degrees of freedom directly via coupling to background geometry $[10, 11]$, yielding the continuity stress tensor. This is a coarse-grained stress tensor that directly corresponds to momentum transport in the long-wavelength limit and can be identified with the stress tensor of fluid dynamics.

In the phonon method, strain is introduced into the model through small displacements of the orbital positions, modifying the hopping parameters $t_{nm}^r$ as

$$t_{nm}^r \rightarrow e^{-i(\delta r)_{nm}^r} + O(\delta r^2). \quad (19)$$

Above, $\delta r$ is the change in distance between orbitals given by the applied (unsymmetrized) strain as $u_{\mu\nu} = \partial_{\mu} \delta r_{\nu}$. Applying this prescription to Eq. (13) we define the phonon stress tensor as

$$T_{\mu\nu}^{(p)} = \frac{\delta H}{\delta u_{\mu\nu}}. \quad (20)$$

Given the structure of the viscosity tensor Eq. (5) and the fact that antisymmetric strains enter only at higher orders in $\delta r$ in Eq. (19), it suffices to consider “diagonal” strains (i.e., $u_{xx}$, $u_{yy}$, $u_{zz}$) for a system of finite size.
and $u_{cz}$) [66]. We find that to first order (see SM [53]),
\begin{equation}
\begin{aligned}
& t_{01}, t_{23} \rightarrow t + (u_x + u_y) t \\
& t_{02}, t_{13} \rightarrow t + (u_y + u_z) t \\
& t_{03}, t_{12} \rightarrow t + (u_z + u_x) t .
\end{aligned}
\end{equation}

The diagonal phonon stress tensor restricted to the spin-1 fermions is then
\begin{equation}
T_{xx}^{(p)}(k) = v_F \cos(\phi)(k_x L_x + k_y L_y) + v_F \sin(\phi)(k_y L_x + k_z L_z)
\end{equation}
\begin{equation}
= v_F \cos(\phi)(k_x L_x + k_y L_y) + m v_F \sin(\phi)(k_x L_x + k_y L_y),
\end{equation}
\begin{equation}
T_{xy}^{(p)}(k) = v_F \sin(\phi)(k_x L_x + k_y L_y) + m v_F \sin(\phi)(k_x L_x + k_y L_y).
\end{equation}
\begin{equation}
T_{yy}^{(p)}(k) = v_F \cos(\phi)(k_x L_x + k_y L_y) + m v_F \sin(\phi)(k_x L_x + k_y L_y).
\end{equation}

$T_{xx}^{(p)}$ transforms as a tensor in the point group 23, which is the point group describing both the underlying lattice and the $\Gamma$ point. Note that even when $\phi = 0$, although the Hamiltonian $h$ is invariant under $SO(3), T_{xx}^{(p)}$ is covariant only under the discrete group 23.

The continuity stress $T_{\mu}^{(c)}$ is defined via a lattice analog of the momentum continuity equation (see SM) [11], resulting in
\begin{equation}
T_{\mu}^{(c)}(k) = \left( \right.
\end{equation}
where $a, b = 1, 2$ as in Eq. (3).

We also emphasize that the MHV and PHV are responses to first order in $\phi$ and $\mu$. Using the energies and eigenvectors given in the SM [53], we find that the integrand for $\eta_1$ in Eq. (26) has an energy denominator that is odd in $k_\mu$, which suppresses the zeroth order contribution from the numerator. When the states are taken to zeroth order in $\phi$, the numerator is odd in $k_\mu$ and $k_\nu$, and when the states are taken to first order in $\phi$, the only nonvanishing matrix elements in the numerator are odd in $k_\mu$, all of which leads to $\eta_1 = 0$ [69].

The total MHV is
\begin{equation}
\eta_{\text{tot}}^{(c)} = \eta_2^{(c)} = \frac{v_F^2}{8\pi^3} \left( \right.
\end{equation}
where $\beta_2 = \frac{4\pi}{3\sqrt{3}} \approx 0.0310$. Around $\mu = 0$, the viscosity is discontinuous. This arises from the fact that, since the antisymmetric part of the continuity stress is linear in $\phi$, we must consider the unperturbed band structure in the energy denominators in Eq. (26). When $\phi = 0$, the band structure has a flat band bisecting two linearly dispersing bands. The filling of the flat band when $\mu$ passes through zero then causes the discontinuity in $\eta_2$, which we can attribute to the contribution of this band to the Hall viscosity. We plot $\eta_{\text{tot}}^{(c), p}$ in Fig. 2(c).

Similar to the Hall viscosity for Dirac fermions in 2D [11,20,70], we see that both $\eta_{\text{tot}}^{(p)}$ and $\eta_{\text{tot}}^{(c)}$ consist of two terms, one of which depends explicitly on the cutoff $\Lambda$. We can interpret the cutoff-independent contribution (or, more properly, its derivative with respect to chemical potential) as the Fermi surface contribution to the Hall viscosity, while the cutoff-dependent term parametrizes unknown contributions to the viscosity from occupied states at large momenta. Using the continuity stress tensor, we can go beyond this approximation to compute the MHV for the full tight-binding model numerically (see SM).

Conclusion. We have highlighted a manifestly 3D cubic Hall viscosity (MHV and PHV), which appears with tetrahedral symmetry. As a proof-of-concept, we have shown that these viscosities are nonzero for a threefold fermion at the $\Gamma$ point in MSG $P2_13$ (No. 198). Using our phonon and continuity methods to examine the stress response in this model, we found that the MHV and PHV were nonzero in this system. We also emphasize that the MHV and PHV are responses
defined for distinct stress tensors. The MHV corresponds to the "continuity" stress that exactly matches the symmetries at the Γ point, while the PHV corresponds to the "phonon" stress which is intimately connected to the elastic response of the underlying lattice model.

Beyond our proof-of-principle calculation, the manifestly 3D nature of the cubic Hall viscosity suggests that viscous transport in 3D magnetic materials can be phenomenologically different than in two dimensions. In particular, measuring the local flow profiles [2,31] or thermoelectric transport coefficients [27] in magnets in the tetrahedral SGs (Nos. 195–206) would reveal the signatures of our 3D viscosity. For example, outside of MSG 198.9, we could consider a 3D cubic magnet with approximate Galilean symmetry at low energies. For such a system the force tensor Eq. (7), and therefore the MHV [71], is proportional to the wave-vector-dependent Hall conductivity [10,57,72],

\[ \alpha^2 \partial_{ij} H \propto \eta_{tot} \delta^c_{m} q_{j} q_{i} \epsilon_{mij} \equiv \eta_{tot} \mathbf{V}_{i}^{c} q_{j} \epsilon_{mij}, \]  

where the vector \( \mathbf{V}(q) \) highlights the structural parallel with the natural optical activity of a crystal [40]. We can decompose \( \mathbf{V} \) into longitudinal and transverse components as \( \mathbf{V}_{ij} = \mathbf{q} \mathbf{\cdot} \mathbf{V} = \eta_{tot} \mathbf{q} \mathbf{\cdot} \mathbf{q} \mathbf{\cdot} \mathbf{q} \) and \( \mathbf{V}_{ij} = \mathbf{V} - \mathbf{V}_{ij} \). Then we see that \( \mathbf{V}_{ij} \) gives a \( q \)-dependent correction to natural optical activity, while \( \mathbf{V}_{ij} \) leads to a Hall current proportional to the longitudinal component of the electric field. Note, crucially, that \( \mathbf{V}_{ij} \) vanishes for plane waves at normal incidence. Furthermore, in tetrahedral systems without Galilean invariance, this wave-vector-dependent contribution to the conductivity need not be zero; the fate of the viscosity–conductivity relation in these systems (generalizing work such as Ref. [73]) is an interesting avenue for further study. Analogous considerations for flow in narrow channels suggest that \( \eta_{tot} \) may play a role in interaction-dominated transport in narrow channels [12,13].

The physical signatures of PHV proposed in spin-phonon and electron-phonon-coupled systems [22,24–26], such as contributions to thermal Hall conductance and modifications to phononic dispersion, could be probed as well in these systems to measure the cubic PHV. The frequency and disorder [74] dependence of the MHV and PHV could also yield interesting insights. As none of our results are specific to the hydrodynamic regime, we expect disorder that preserves the symmetry on average will not modify our qualitative conclusions.

Chiral magnets such as the family of MnIrSi [51,52] are promising platforms to study these effects. As shown in Ref. [28], this compound has a noncollinear magnetic configuration preserving the size of the unit cell; group theory analysis showed further that the ground-state magnetic order preserved all of the unitary symmetry operations consistent with MSG \( P2_{1}3 \). Another interesting candidate is MnTe\(_{2}\) in MSG \( Pa\bar{3} \) (No. 205.33) [75–77]. It has a reported noncollinear magnetic structure, with the magnetic moments of the four inequivalent manganese ions pointing along the cubic body diagonals. Although naturally a semiconductor, Ag-doping could increase the carrier concentration [78].

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