Fingerprinting the contribution of colored scalars to the $H^+W^-Z(\gamma)$ vertex

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ABSTRACT: Color-octet scalars arise in various Grand Unification scenarios and also other models of new physics. They are also postulated for minimal flavour violation. Purely phenomenological imprints of such scalars are therefore worth looking at. Motivated by this, we perform a complete one-loop calculation of the $H^+ \to W^+ Z(\gamma)$ decay in a two Higgs doublet model augmented by a color octet $SU(2)_L$ scalar doublet. The computation is conveniently segregated into colorless and colored components. The color-octet part being scaled by the color-factor provides an overall enhancement to the form factors. Crucial constraints from perturbative unitarity, positivity of the scalar potential, oblique parameters, Higgs signal strengths and direct search of a charged Higgs and color-octet scalars are folded-in into the analysis. Sensitivity of the loop-induced $H^+ \to W^+ Z(\gamma)$ vertex to other model parameters is elucidated. Finally, the prospect of observing a loop-induced $H^+ \to W^+ Z(\gamma)$ interaction at the future hadronic collisions is also discussed.
1 Introduction

The discovery of a Higgs boson at the Large Hadron Collider (LHC) [1, 2] completes the particle spectrum of the Standard Model (SM). Further, the interaction strengths of the discovered boson with the SM fermions and gauge bosons are found to be in agreement with the corresponding SM values. However, issues such as a non-zero neutrino mass, the existence of dark matter (DM), the observed imbalance between matter and antimatter in the universe, and, the instability (or metastability) of the electroweak (EW) vacuum in the
SM [3–5] hint towards additional dynamics beyond the SM. Interestingly, extending only the Higgs sector of the SM appropriately can suffice to address all the aforementioned issues. Moreover, a scalar of mass 125 GeV with interactions mimicking those of the SM Higgs can be extracted out of such extended Higgs sectors by virtue of additional symmetries or appropriate fine tuning. This therefore motivates putting forth extended Higgs sectors as prototypes of beyond-the-SM (BSM) physics.

A two-Higgs doublet model (2HDM) [6, 7] is one of such extensions of the SM Higgs sector. Motivated by the minimal supersymmetric standard model (MSSM) in part, it potentially shuts off the flavour changing neutral currents (FCNC), furnishes additional sources of CP-violation that can eventually explain the observed matter-antimatter imbalance, and, poses a solution to the strong CP problem. A 2HDM is in fact the smallest SU(2)_L multiplet to predict a singly charged Higgs H^+. The more well known collider search channels of the same are in fact its fermionic decays. The charged Higgs has been searched at the LHC through different production and decay modes. The H^+ → t̄b decay mode is considered in the search of a heavy H^+ whereas the preferred search channel for a light one is the H^+ → τν channel. However, probes in such channels are generally swamped by a heavy QCD background. An alternative therefore is to search for the bosonic decays H^+ → W^+h, W^+Z, W^+γ. That said, the last two of the aforementioned modes are prima facie more intriguing since they arise only at one-loop in multi-Higgs doublet models. The absence of the H^+W^−γ coupling at the tree level is an artefact of an unbroken. On the other hand, a tree level H^+W^−Z coupling is absent owing to the isospin symmetry of the kinetic terms of the Higgs sector. Since both these characteristics are, in general, broken at one loop level through effects from other sectors that do not respect the custodial invariance, these vertices are induced at loop level. Momentum dependent interactions appear therein consequently. It therefore gets clear that the strength of the H^+W^−Z interaction captures the custodial symmetry breaking effects in the model embedding it.

An also interesting extension of the SM Higgs sector first proposed in [8] consists of a scalar multiplet transforming as (8,2,1/2) under the SM gauge group. The motivation for the same is two-fold. The first is minimal flavour violation (MFV), which is a framework for having flavour-dependent masses without introducing unwanted flavour changing neutral currents (FCNCs). It assumes all breaking of the underlying approximate SU(3)_c × SU(2)_L × U(1)_Y flavour symmetry of the SM is proportional to the up- or down-quark Yukawa matrices. It has been shown in [8] that the only scalar representations under SU(3)_c × SU(2)_L × U(1)_Y complying with MFV are (1,2,1/2) and (8,2,1/2). Secondly, color octet scalars can emerge from a plethora of BSM scenarios. One such example grand unification models [9–12]. Other examples include topcolour scenarios [13], models with extra dimensions [14, 15] and chiral color models. Loop effects of the isodoublet color octet were looked at in [16–20] More on the TeV-scale phenomenology of color-octet scalars can be found in [21–30]. In fact, an (8,2,1/2) scalar multiplet also proved handy in explaining certain anomalous results seen at the Tevatron [31, 32] and the Runs I [19, 33, 34] and II [35] of the LHC that had stirred up excitement in those times.

The quest for non-minimal Higgs sectors led to the proposing of a hybrid scenario combining a 2HDM with a color-octet scalar multiplet [36]. Relevant theoretical and ex-
Experimental constraints were used in [36] to carve out an allowed parameter region. The more stringent requirements of high scale perturbative unitarity and vacuum stability under renormalisation group were imposed in a subsequent study [37]. We pick up this model to enumerate the strength of the $H^+W^−Z(γ)$ vertex at one-loop. While a similar calculation was done for the minimal supersymmetric standard model (MSSM) [38], a $Z_2$ symmetric 2HDM [39, 40], an aligned 2HDM [41] and a particular version of 3HDM containing two active and one inert doublet [42], we lay particular emphasis on the contribution coming from the color-octet scalars. The main features of the present study are outlined below.

- To our understanding, this is the first investigation of the impact of colored scalars on the $H^+W^−Z$ vertex. While computing the one-loop amplitude coming from the color-octet, an enhancement by a color-factor is expected. In tandem, also expected is an exclusion limit on the color-octet mass scale from direct searches at the LHC that will tend dilute this enhancement. We probe the interplay of the two aforementioned effects here.

- We adopt the non-linear gauge to get rid of unphysical vertices involving goldstones. This ends up simplifying the calculation. Further, we present certain simplified expressions for the one-loop form factors that make decoupling/non-coupling of the colored scalars from the $H^+W^−Z$ vertex apparent.

- As a phenomenological application, we also discuss how such new contributions change the decay branching fractions of the $H^+ → W^+Z(γ)$ mode, and, consequently, the production cross sections involving these decay processes at the LHC.

The paper is organised as follows. The model is introduced in Section 2. A detailed discussion on the non-linear gauge chosen and the analytic calculation of the various form factors is reported in Section 3. The constraints applicable to this scenario are elaborated in Section 4 and the numerical values of the form factors and branching ratios obtained are reported in Section 5. Section 6 outlines the observability of the $H^+W^−Z$ vertex at the LHC. The study is summarised in Section 7. Various important formulae are relegated to the Appendix.

2 Model description

The model considered replaces the scalar sector of the SM by three $SU(2)_L$ scalar doublets: two color singlets ($Φ_1, Φ_2$) and one color-octet $S$. As observed in [15], the only possible extensions of the scalar sector that do not transform under the flavour group and that satisfy minimal flavour violation are $SU(2)_L$ doublets that are singlets or octets under $SU(3)_c$.

The most general scalar potential involving $Φ_1, Φ_2$ and $S$ can be written as [36]:

$$V(Φ_1, Φ_2, S) = m_{11}^2 Φ_1^† Φ_1 + m_{22}^2 Φ_2^† Φ_2 - m_{12}^2 (Φ_1^† Φ_2 + Φ_2^† Φ_1) + \frac{λ_1}{2}(Φ_1^† Φ_1)^2 + \frac{λ_2}{2}(Φ_2^† Φ_2)^2 + λ_3(Φ_1^† Φ_1)(Φ_2^† Φ_2)$$
\[ + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left[ \left( \Phi_1^\dagger \Phi_2 \right)^2 + \left( \Phi_2^\dagger \Phi_1 \right)^2 \right] \]
\[ + 2m_S^2 \text{Tr} S^i S_i + \mu_1 \text{Tr} S^i S_j S^i S_j + \mu_2 \text{Tr} S^i S_j S^i S_j + \mu_3 \text{Tr} S^i S_j S^i S_j \]
\[ + \mu_4 \text{Tr} S^i S_j \text{Tr} S^i S_j + \mu_5 \text{Tr} S^j S_i \text{Tr} S^i S_j + \mu_6 \text{Tr} S_i S_j S^i S^j + \mu_7 \text{Tr} S_i S_j S^i S^j \]
\[ + v_1 \Phi_1^\dagger \Phi_1 \text{Tr} S^i S_j + v_2 \Phi_1^\dagger \Phi_1 \text{Tr} S^i S_j + \left( v_3 \Phi_1^\dagger \Phi_1 \text{Tr} S^i S_j + v_4 \Phi_1^\dagger \Phi_1 \text{Tr} S^i S_j + v_5 \Phi_1^\dagger \Phi_1 \text{Tr} S^i S_j + \text{h.c.} \right) \]
\[ + \omega_1 \Phi_2^\dagger \Phi_2 \text{Tr} S^i S_j + \omega_2 \Phi_2^\dagger \Phi_2 \text{Tr} S^i S_j + \left( \omega_3 \Phi_2^\dagger \Phi_2 \text{Tr} S^i S_j + \omega_4 \Phi_2^\dagger \Phi_2 \text{Tr} S^i S_j + \omega_5 \Phi_2^\dagger \Phi_2 \text{Tr} S^i S_j + \text{h.c.} \right) \]
\[ + \kappa_1 \Phi_1^\dagger \Phi_2 \text{Tr} S^i S_j + \kappa_2 \Phi_1^\dagger \Phi_2 \text{Tr} S^i S_j + \kappa_3 \Phi_1^\dagger \Phi_2 \text{Tr} S^i S_j + \text{h.c.,} \]  

(2.1)

where, \( \Phi_1, \Phi_2 \) and \( S \) can be written as,

\[ \Phi_i = \left( \frac{\phi_i^+}{\sqrt{2}(v_i + h_i + i z_i)} \right), \]

(2.2)

In the above, \( v_i \) is the vacuum expectation value (VEV) of \( \Phi_i \) with \( v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2 \). The ratio of two VEVs relates to the mixing angle \( \beta \) as \( \tan \beta = \frac{v_2}{v_1} \). Here the scalar potential parameters \( m_{11}^2, m_{22}^2, m_{12}^2, m_{5}^2, \mu_1 - 6, \lambda_{1 - 5}, \nu_1 - 5, \omega_{1 - 5}, \kappa_{1 - 3} \) are taken real to avoid \( CP \)-violation in the scalar sector. In Eq.(2.1), \( i, j \) and \( A, B \) respectively denote the fundamental \( SU(2) \) and adjoint \( SU(3) \) indices. One then defines \( S_i = S_i^A T_A \) (\( T_A \) being the \( SU(3) \) generators) and the trace in Eq.(2.1) is taken over the color indices. The colorless particle spectrum in this case is identical with the 2HDM that consists of the neutral CP-even Higgses \( h, H \), a CP-odd Higgs \( A \) and a charged Higgs \( H^+ \). The \( 2 \times 2 \) mass matrix corresponding to the CP-even scalars is brought into a diagonal form by the action of a mixing angle \( \alpha \).

Of these, the scalar \( h \) is taken to be the SM Higgs with mass \( 125 \text{ GeV} \). The masses of the neutral and charged mass eigenstate of the color octet can be written in terms of the quartic couplings \( \omega_i, \kappa_i, \nu_i \) and mixing angle \( \beta \) as [36]:

\[ M_{SR}^2 = m_S^2 + \frac{1}{4} v^2 \left( \cos^2 \beta (\nu_1 + \nu_2 + 2\nu_3) + \sin^2 \beta (\omega_1 + \omega_2 + 2\omega_3) \right), \]

(2.3a)

\[ M_{S\eta}^2 = m_S^2 + \frac{1}{4} v^2 \left( \cos^2 \beta (\nu_1 + \nu_2 - 2\nu_3) + \sin^2 \beta (\omega_1 + \omega_2 - 2\omega_3) \right), \]

(2.3b)

\[ M_{S+}^2 = m_S^2 + \frac{1}{4} v^2 \left( \nu_1 \cos^2 \beta + \kappa_1 \sin 2\beta + \omega_1 \sin^2 \beta \right). \]

(2.3c)

The Yukawa interactions in this model partition into the two following terms:

\[ \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_8 \]

(2.4)

where \( \mathcal{L}_{1(8)} \) involves the SM quarks and the color-singlet(octet) electroweak doublet. Here, \( \mathcal{L}_1 \) is chosen to coincide with the Yukawa Lagrangian in type I and type II 2HDM.
that essentially suppress the tree level flavour changing neutral currents (FCNC). On the other hand, the interaction with the color-octet has the form:

\[ \mathcal{L}_8 = -(y_u)_{pq} \bar{u}_R^p S_L^q Q_L^q - (y_d)_{pq} \bar{d}_R^p S_L^q Q_L^q + \text{h.c.} \]  

(2.5)

Here, \((p, q) = 1, 2, 3\) are the fermion generation indices. We however remark that \(\mathcal{L}_8\) shall not play a role in the present analysis and we retain it just for completeness.

3 One-loop form factors for \(H^+ \rightarrow W^+ Z(\gamma)\)

In this section we compute the \(H^+W^-Z(\gamma)\) vertex at one-loop for the present scenario. The various form factors are expressed in terms of the Passarino-Veltman functions and the publicly available library LoopTools [43] is used for numerical evaluation.

The amplitude for \(H^+ \rightarrow W^+ V(V = Z, \gamma)\) can be expressed as

\[ i\mathcal{M}(H^+ \rightarrow W^+ Z/\gamma) = ig m_W V_{\mu\nu}^\mu \epsilon_{\gamma\nu}^* (p_W) \epsilon^*_{\gamma V}(p_V). \]  

(3.1)

where,

\[ V_{\mu\nu}^\gamma = g^{\mu\nu} F_V + \frac{p_W^\mu p_V^\nu}{m_W^2} G_V + i\epsilon^{\mu\nu\rho\sigma} p_V^\rho p_W^\sigma H_V. \]  

(3.2)

Here \(p_W^\mu p_V^\nu\) are the incoming momenta of \(W^\pm\) and \(V\). Moreover, \(F, G\) and \(H\) are the form-factors corresponding to the respective Lorentz structures. For \(V = \gamma\), the Ward identity enforces the following condition:

\[ V_{\gamma\nu}^\mu p_{\gamma\nu} = 0 \]  

(3.3)

This ultimately leads to the following relation connecting \(F_\gamma\) and \(G_\gamma\):

\[ F_\gamma = \frac{G_\gamma}{2} \left( 1 - \frac{M_{H^+}^2}{M_W^2} \right) \]  

(3.4)

Scalars coming from both colorless and colored sectors, i.e, \(\phi_{1,2}\) as well as \(S\), contribute to \(H^+ \rightarrow W^+ V\). Accordingly, each form factor splits as

\[ X_V = X_{V_{2\text{HDM}}} + X_{V_S}, \]  

(3.5)

for \(X = F, G, H\). We now come to discussing the various one-loop diagrams comprising the amplitudes. For the colored part, the amplitude receives contributions from the following set of one-loop diagrams.

The amplitude corresponding to each set is UV-finite. We now invoke a gauge-fixing procedure that can simplify the calculation. To this end we introduce nonlinear gauge-fixing functions [44–50]:

\[ f^+ = \left( D^e_\mu + i g s_W^2 Z_\mu \right) W^{+\mu} - i \xi M_W G^+, \]  

(3.6a)
Figure 1: Set A of one-loop amplitudes for 2HDM and colored scalar.

Figure 2: Set B of one-loop amplitudes for 2HDM and colored scalar.

\[
f^Z = \partial_\mu Z^\mu - \xi M_Z G^0, \quad (3.6b)
\]

\[
f^A = \partial_\mu A^\mu. \quad (3.6c)
\]

with \(D^\mu_E\) the electromagnetic covariant derivative and \(\xi\) the gauge parameter. Note that \(f^+\) is nonlinear and transforms covariantly under the electromagnetic gauge group. The corresponding gauge fixing Lagrangian is given by

\[
\mathcal{L}_{GF} = -\frac{1}{\xi} f^+ f^- - \frac{1}{2\xi} (f^Z)^2 - \frac{1}{2\xi} (f^A)^2 \quad (3.7a)
\]

This gauge-fixing procedure is tailored to remove the unphysical \(G^+ W^- V\) vertices that arise in the Higgs kinetic-energy sector. Further into this, for \(\xi = 1\), the \(V_\mu (k) W_\nu^+ (p) W_\rho^- (q)\) (all momenta incoming) triple-gauge vertices have the following modified Feynman rules.

\[
\Gamma^{W^+ W^-}_{\mu\nu\rho} (k, p, q) = -ie \{ g_{\mu\nu} (k - p - q)_\rho + g_{\nu\rho} (p - q)_\mu + g_{\rho\mu} (q - k + p)_\nu \}, \quad (3.8a)
\]

\[
\Gamma^{ZW^+ W^-}_{\rho\nu\mu} (k, p, q) = -igc_W \{ g_{\mu\nu} (k - p + \frac{s_W^2}{c_W^2} q)_\rho + g_{\nu\rho} (p - q)_\mu \\
+ g_{\rho\mu} (q - k - \frac{s_W^2}{c_W^2} p)_\nu \}. \quad (3.8b)
\]
Therefore, by virtue of the aforementioned gauge fixing, the amplitudes in Set C vanish. Moreover, this implies that similar amplitudes coming from the colorless scalars would also vanish. We partition the remaining diagrams into UV-finite sets as displayed below.

We point out that Set $A'$ (Fig.4) is the color-singlet counterpart of Set A (Fig.1), both containing a scalar trilinear interaction on one vertex and gauge interactions on the other(s). Set $B'$ (Fig.5) can also be related to Set B (Fig.2) using similar arguments. The additional diagrams in Set $C'$ (Fig.6) feature $h(H)VV$ interactions and vanish in the $s_{\beta-\alpha} = 1$ limit. Finally, Fig.7 contains the fermionic one-loop diagrams.

The decay width of $H^+ \to W^+Z$ is given by

$$
\Gamma(H^+ \to W^+Z) = M_{H^+} \frac{\sqrt{\lambda(1,\omega,z)}}{16\pi} \sum_{i=L,T} |M_{ii}|^2,
$$

where $i = L(T)$ represents the longitudinal and transverse polarization and, $\lambda(a, b, c) = (a - b - c)^2 - 4abc$, $\omega = \left(\frac{M_W}{M_{H^+}}\right)^2$, $z = \left(\frac{M_Z}{M_{H^+}}\right)^2$.

The longitudinal and transverse contributions are given in terms of $F_V, G_V, H_V$ by,

$$
|M_{LL}|^2 = \frac{g^2}{4z} (1 - \omega - z) F_V + \frac{\lambda(1,\omega,z)}{2\omega} G_V^2,
$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Set $C$ of one-loop amplitudes for colored scalar.}
\end{figure}
\[ |M_{TT}|^2 = g^2(2\omega |F_V|^2 + \frac{\lambda(1, \omega, z)}{2\omega} |H_V|^2) . \] (3.10b)

For \( V = \gamma \), the relation \( F_{\gamma} = \frac{G_\gamma}{2} \left( 1 - \frac{M_{H^+}^2}{M_W^2} \right) \) is used in Eqs.(3.10a) and (3.10b) to obtain

\[ \Gamma_{H^+ \rightarrow W^+\gamma} = \frac{M_{H^+}^3}{8\pi v^2} \left( 1 - \frac{M_{W^+}^2}{M_{H^+}^2} \right)^3 \left( |G_{\gamma}|^2 + |H_{\gamma}|^2 \right) . \] (3.11)

4 Constraints applied

We discuss the relevant constraints on this model in this section.

4.1 Perturbativity

Demanding perturbativity leads to the following bounds on the couplings:

\[ \lambda_i \leq 4\pi \ (i = 1, 2, ...5), \ |\nu_j|, |\omega_j|, |k_j| \leq 4\pi \ (j = 1, 2, 3), \ y_i \leq \sqrt{4\pi} \ (i = t, b, \tau). \] (4.1a)
4.2 Stability conditions

To make the scalar potential bounded-from-below (BFB) in any direction in the field space, the following stability conditions are to be satisfied:

\[
\begin{align*}
\lambda_1 &\geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2}, \\
\lambda_3 + \lambda_4 - |\lambda_5| &\geq -\sqrt{\lambda_1 \lambda_2}, \\
\nu_1 &\geq -2\sqrt{\lambda_1 \mu_1}, \\
\omega_1 &\geq -2\sqrt{\lambda_2 \mu_1}, \\
\nu_1 + \nu_2 - 2|\nu_3| &\geq -2\sqrt{\lambda_1 \mu_1}, \\
\omega_1 + \omega_2 - 2|\omega_3| &\geq -2\sqrt{\lambda_2 \mu_1},
\end{align*}
\]
Among the above, Eqs. (4.2a) and (4.2b) correspond to the pure 2HDM. The rest involve the quartic couplings of the color octet and ensure positivity of the scalar potential in a hyperspace spanned by both colorless as well as colored fields.

### 4.3 High energy scattering unitarity

Additional constraints on the quartic couplings come from the unitarity. A tree-level $2 \rightarrow 2$ scattering matrix can be computed between various two particle states consisting of charged and neutral scalars [51]. Unitarity requires that the absolute value of each eigenvalue of the aforementioned scattering matrix must be $\leq 8\pi$. In the high energy limit, one such element of the scattering matrix is proportional to a quartic coupling. Therefore, demanding unitarity is tantamount to restricting the sizes of such couplings. Thus we end up with the following unitarity conditions for the present model.

\[
\left[ \frac{3}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2} \right] \leq 8\pi, \quad (4.3a)
\]
\[
\left[ \frac{1}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{1}{4}(\lambda_1 - \lambda_2)^2 + \lambda_4^2} \right] \leq 8\pi, \quad (4.3b)
\]
\[
\left[ \frac{1}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{1}{4}(\lambda_1 - \lambda_2)^2 + \lambda_5^2} \right] \leq 8\pi, \quad (4.3c)
\]
\[
(\lambda_3 + 2\lambda_4 - 3\lambda_5) \leq 8\pi, \quad (4.3d)
\]
\[
(\lambda_3 - \lambda_5) \leq 8\pi, \quad (4.3e)
\]
\[
(\lambda_3 + \lambda_4) \leq 8\pi, \quad (4.3f)
\]
\[
(\lambda_3 + 2\lambda_4 + 3\lambda_5) \leq 8\pi, \quad (4.3g)
\]
\[(\lambda_3 + \lambda_5) \leq 8\pi, \quad (4.3h)\]
\[|\nu_1| \leq 2\sqrt{2}\pi, \quad |\nu_2| \leq 4\sqrt{2}\pi, \quad |\nu_3| \leq 2\sqrt{2}\pi, \quad (4.3i)\]
\[|\omega_1| \leq 2\sqrt{2}\pi, \quad |\omega_2| \leq 4\sqrt{2}\pi, \quad |\omega_3| \leq 2\sqrt{2}\pi, \quad (4.3j)\]
\[|\kappa_1| \leq 2\pi, \quad |\kappa_2| \leq 4\pi, \quad |\kappa_3| \leq 4\pi. \quad (4.3k)\]

Eqs. (4.3a)-(4.3h) correspond to the unitarity limit for a pure two-Higgs doublet scenario [52–58]. More details on unitarising a spinless \((8,2,1/2)\) multiplet can be found in [28, 36].

### 4.4 Oblique parameters

The extended scalar sector modifies the oblique parameters [59] with respect to their SM contributions. The strongest constraint however comes from the \(T\)-parameter. Including the BSM contribution, the effective \(T\)-parameter can be written in terms of the SM and BSM contribution \(\Delta T\) as:

\[T = T_{\text{SM}} + \Delta T. \quad (4.4)\]

The most updated bound on the BSM contributions to \(T\)-parameter is [60]:

\[\Delta T = 0.07 \pm 0.12. \quad (4.5)\]

For the multi Higgs doublet models, the most constraining is the \(T\)-parameter [61] that restricts the mass splittings between the heavy neutral and charged scalars. In our case, the source of BSM contribution is two-fold, \(i.e.\) contribution arising from 2HDM and the scalar octet. Thus, \(\Delta T = T_{2\text{HDM}} + T_S\),

\[T_{2\text{HDM}} = \frac{1}{16\pi s_W^2 M_W^2} \left[ F(M_{H^+}^2, M_A^2) + s_W^2 \alpha \left( F(M_{H^+}^2, M_A^2) - F(M_{H^+}^2, M_A^2) \right) \right] + c_W^2 \alpha \left( F(M_{H^+}^2, M_A^2) - F(M_{A^0}^2, M^2) + F(M_{W^+}^2, M_H^2) - F(M_W^2, M^2) \right) + F(M_{Z^0}^2, M_{H^+}^2) - F(M_{Z^0}^2, M_{A^0}^2) + 4M_Z^2 B_0(M_Z^2, M_{H^+}^2, M_{A^0}^2) - 4M_W^2 B_0(M_W^2, M_{H^+}^2, M_{A^0}^2) \right], \quad (4.6a)\]

\[T_S = \frac{N_S}{16\pi s_W^2 M_W^2} \left[ F(M_{S^+}^2, M_{S^0}^2) + F(M_{S^+}^2, M_{S^0}^2) - F(M_{S^0}^2, M_{S^0}^2) \right], \quad (4.6b)\]

where,

\[F(x, y) = \frac{x + y}{2} - \frac{xy}{x - y} \ln \left( \frac{x}{y} \right) \quad \text{for} \quad x \neq y, \]

\[= 0 \quad \text{for} \quad x = y. \quad (4.7a)\]

\[B_0(m_1^2, m_2^2, m_3^2) = \frac{m_1\log(m_1^2) - m_2\log(m_2^2) - m_3\log(m_3^2)}{m_1^2 - m_3^2}. \quad (4.7b)\]

The principal way of observing colored states at the LHC is to look for their single production through gluon fusion and subsequent decay to dijets or a \(t\bar{t}\) pair. However, the corresponding cross section times branching ratio would depend on the choice of the Yukawa coupling of the colored scalar to a quark pair.
4.5 Flavour constraints

For a 2HDM with natural flavour conservation, the strongest constraint on the mass of the charged Higgs comes from the branching fraction for $B \to X_s \gamma$. In case of the type I 2HDM, this particular constraint supersedes the direct search constraints only for small $\tan\beta$. On the other hand, the type II 2HDM features a stringent $M_{H^+} > 580$ GeV bound [62] that is practically independent of $\tan\beta$.

4.6 Direct search

An $M_{H^+} > 100$ GeV bound for all types of 2HDM summarises the result for the direct search of $H^+$ in the $e^+e^- \to H^+H^-$ channel at LEP [63]. In the $M_{H^+} > M_t + M_b$ region, a measurement of $\sigma(pp \to tH^+ + X) \times BR(H^+ \to \tau^+ + \nu_{\tau})$ obviates $\tan\beta > 50$ for an $H^+$ of mass $\simeq 200$ GeV in case of type II 2HDM [64]. The corresponding constraint is further weak for type I 2HDM. In view of the aforementioned, we take $M_{H^+} \geq 150$ (600) GeV for a type I (type II) 2HDM throughout the present study.

We now come to discussing possible exclusion limits on the color-octet mass scale itself. Color-octet resonances have been searched for at the LHC in the $pp \to S \to jj$ [65–68] and $pp \to S \to tt$ [69–71] channels and a stringent bound of $\gtrsim 3$ TeV was subsequently pronounced. However, it was pointed out in [72] that the benchmark color-octet taken there led to a cross section several orders larger than what would be seen for the Manohar-Wise scenario. Ref.[72] showed that the bound weakens to $M_{S_R} > 700$ GeV upon tweaking the model parameters appropriately. On the other hand, pair-production of $S$, that occurs at the tree level itself, yields a cross section comparable to the loop-induced single $S$ production. The pair-production process was also studied at the LHC in the $4j, 4b, 4t, t\bar{t}b\bar{b}$ final states. Accordingly, ref.[72] showed that it is possible to salvage $M_{S_R} \simeq 800$ GeV in this model.

4.7 Higgs signal strengths

The parameters of the model are also constrained from the current LHC data, i.e. signal strengths in various Higgs decay modes. Denoting the signal strength for the channel $pp \to h, h \to i$ by $\mu_i$, it is defined as,

$$\mu_i = \frac{\sigma_{BSM}(pp \to h) \ BR_{BSM}(h \to i)}{\sigma_{SM}(pp \to h) \ BR_{SM}(h \to i)}. \quad (4.8)$$

Eq.(4.8) takes the form below upon expressing the branching fractions in terms of the decay widths.

$$\mu_i = \frac{\sigma_{BSM}(gg \to h)}{\sigma_{SM}(gg \to h)} \ \frac{\Gamma_{BSM}(h \to i)}{\Gamma_{BSM}^{tot}} \ \frac{\Gamma_{SM}^{tot}}{\Gamma_{SM}(h \to i)}. \quad (4.9)$$

Note that here the BSM contribution includes contributions from both 2HDM and the color octet. In particular, the modification induced by the color-octet is solely to the loop-induced $h \to gg, \gamma\gamma, Z\gamma$ decays.
Now the parton-level cross section of Higgs production through gluon fusion can be written as

\[ \sigma(gg \to h) = \frac{\pi^2}{8M_h} \Gamma(h \to gg) \delta(s - M_h^2), \]  

(4.10)

\(\sqrt{s}\) being partonic centre-of-mass energy. Using eqs. (4.8), (4.9) and (4.10), one can rewrite the \(\mu_i\) as:

\[ \mu_i = \frac{\Gamma_{BSM}^{h \to gg}}{\Gamma_{SM}^{h \to gg}} \frac{\Gamma_{BSM}^{h \to g}}{\Gamma_{SM}^{h \to g}} \frac{\Gamma_{SM}^{h \to h}}{\Gamma_{BSM}^{h \to h}}. \]  

(4.11)

In presence of color octet, both the production cross section of Higgs boson via gluon fusion and \(h \to gg\) decay width (Eq.(A.4b)) get modified. We have strictly imposed the alignment limit i.e. \((\beta - \alpha) = \frac{\pi}{4}\) throughout the analysis. We then compute the signal strengths in the tree level decay modes of Higgs, i.e. WW, ZZ, bb, \(\tau^+\tau^-\), following Eq.(4.11). The decay width and hence the signal strength in the loop induced decay channel of Higgs \(h \to \gamma\gamma\) are also altered owing to the presence of extra charged particles \((H^\pm, S^\pm)\) appearing in the loop. The expressions for the respective decay widths in the loop induced decay modes of \(h\), i.e. \(h \to \gamma\gamma\), \(h \to Z\gamma\), \(h \to gg\), are relegated to appendix A.1.

| \(\mu_i\) | ATLAS | CMS |
|----------------|--------|------|
| ZZ             | 1.20^{+0.16}_{-0.15}[73] | 0.94^{+0.08}_{-0.07}(stat.)^{+0.07}_{-0.07}(syst.)[74] |
| W^+W^-         | 2.3^{+1.2}_{-1.0}[75]   | 1.28^{+0.15}_{-0.17}[76]   |
| \(\gamma\gamma\) | 0.99 ± 0.14[77]      | 1.18^{+0.14}_{-0.14}[78]      |
| \(\tau\tau\)   | 1.09^{+0.18}_{-0.22}(stat.)^{+0.16}_{-0.11}(syst.)^{+0.16}_{-0.11}(theo syst)[79] | 1.09^{+0.27}_{-0.26}[80] |
| bb             | 2.5^{+1.3}_{-1.0}[81]   | 1.3^{+1.3}_{-1.0}[82]       |

Table 1: Latest limits on the \(h\)-signal strengths

The signal strength data from the ATLAS and CMS for a given channel can be combined to yield a resultant central value \(\mu\) and a resultant 1-sigma uncertainty \(\sigma\) as \(\frac{1}{\sigma^2} = \frac{1}{\sigma_{ATLAS}^2} + \frac{1}{\sigma_{CMS}^2}\) and \(\frac{\mu}{\sigma^2} = \frac{\mu_{ATLAS}}{\sigma_{ATLAS}^2} + \frac{\mu_{CMS}}{\sigma_{CMS}^2}\). In addition, we also require BR\(_{h \to Z\gamma}\) < 1% [83].

5 Numerical Results

We numerically evaluate the \(H^+W^-Z(\gamma)\) form factors and the corresponding branching ratios in this section taking type I and type II-like Yukawa interactions. Prior to that, we list out the independent parameters in this case. The 2HDM sector comprises of \(\tan\beta\), the quartic couplings \(\lambda_1\ldots\lambda_5\) and \(m_{12}\). Of these, \(\lambda_1\ldots\lambda_5\) can be traded off for the masses of the physical scalars and mixing angles \(\alpha, \beta\) using the following formulae:

\[ \lambda_1 = \frac{M_{H^+}^2c_\alpha^2 + M_H^2s_\alpha^2 - m_{12}^2\tan\beta}{v^2c_\beta^2} \]  

(5.1a)

\[ \lambda_2 = \frac{M_{H^+}^2s_\alpha^2 + M_H^2c_\alpha^2 - m_{12}^2/t_\beta}{v^2s_\beta^2} \]  

(5.1b)
The independent parameters in the color-octet sector are therefore (\(M_h, M_H, M_A, M_{H^+}, t_\beta\)). Similarly, the color octet sector comprises the mass parameter \(M_S\) and the quartic couplings \(\mu_{1-6}, \nu_{1-3}, \omega_{1-3}, \kappa_{1-3}\). We can exclude \(\mu_{1-6}\) here since they do not enter the current analysis. We again choose to trade off a few quartic couplings in terms of the masses of the colored scalars using:

\[
\lambda_3 = \frac{2M_{H^+}^2}{v^2} + \frac{s_\alpha}{s_\beta t_\beta} \left( \frac{M_H^2 - M_h^2}{v^2} \right) - \frac{m_{12}^2}{v^2 s_\beta c_\beta} \quad (5.1c)
\]

\[
\lambda_4 = \frac{M_A^2 - 2M_{H^+}^2}{v^2} + \frac{m_{12}^2}{v^2 s_\beta c_\beta} \quad (5.1d)
\]

\[
\lambda_5 = \frac{m_{12}^2}{v^2 s_\beta c_\beta} - \frac{M_A^2}{v^2} \quad (5.1e)
\]

The independent parameters coming from the 2HDM sector are therefore taken to be \((M_h, M_H, M_A, M_{H^+}, t_\beta)\). Similarly, the color octet sector comprises the mass parameter \(M_S\) and the quartic couplings \(\mu_{1-6}, \nu_{1-3}, \omega_{1-3}, \kappa_{1-3}\). We can exclude \(\mu_{1-6}\) here since they do not enter the current analysis. We again choose to trade off a few quartic couplings in terms of the masses of the colored scalars using:

\[
\omega_2 = \frac{2(M_{S_R}^2 + M_{S_l}^2 - 2M_{S^+}^2) - \nu_2 v^2 c_\beta^2 - \kappa_2 v^2 s_\beta^2}{v^2 s_\beta^2}, \quad (5.2a)
\]

\[
\omega_3 = \frac{M_{S_R}^2 - M_{S_l}^2 - \nu_3 v^2 c_\beta^2 - \kappa_2 v^2 s_\beta^2}{v^2 s_\beta^2}. \quad (5.2b)
\]

The independent parameters in the color-octet sector are therefore \((\nu_1, \nu_2, \nu_3, \kappa_1, \kappa_2, \kappa_3, \omega_1, M_{S_R}, M_{S_l}, M_{S^+})\). We further set \(M_{S^+} = M_{S_R}\) and \(M_{H^+} = M_H\) throughout the calculation since this leads to a vanishing contribution to the \(T\)-parameter from the colored scalars (see Eq.(4.6b)) and a manageable contribution from the 2HDM sector (see Eq.(4.6b)). We take \(M_{S_R} = 800\) GeV for this study.

More insight on the behaviour of the form factors coming from the color-octet can be gained by looking at their simplified expressions in the \(M_W, M_Z, M_{H^+} \ll M_{S_R}\) limit as shown below:

\[
F^A_{\gamma,S} = \frac{N_S}{16\pi^2 v_{CW}} [\lambda_{H^+ - S_R} f_1(r) + \lambda_{H^+ - S_l} f_2(r)] \quad (5.3a)
\]

\[
F^B_{\gamma,S} = -\frac{N_{SW}}{16\pi^2 v} \left( \frac{M_{H^+}^2 - M_W^2 + M_{S_R}^2}{M_{H^+}^2 - M_W^2} \right) [\lambda_{H^+ - S_R} f_3(r) + \lambda_{H^+ - S_l} f_4(r)] \quad (5.3b)
\]

\[
G^A_{\gamma,S} = \frac{N_S M_W^2}{16\pi^2 v_{CW} M_{S_R}^2} [\lambda_{H^+ - S_R} g_1(r) + \lambda_{H^+ - S_l} g_2(r)] \quad (5.3c)
\]

\[
F^A_{\gamma,S} = \frac{N_{SW}}{16\pi^2 v} [\lambda_{H^+ - S_R} f_5(r) + \lambda_{H^+ - S_l} f_6(r)] \quad (5.3d)
\]

\[
F^B_{\gamma,S} = \frac{N_{SW}}{16\pi^2 v} [\lambda_{H^+ - S_R} f_3(r) + \lambda_{H^+ - S_l} f_4(r)] \quad (5.3e)
\]

\[
G^A_{\gamma,S} = -\frac{4N_S M_W^2}{16\pi^2 v M_{S_R}^2} [\lambda_{H^+ - S_R} g_3(r) + \lambda_{H^+ - S_l} g_4(r)] \quad (5.3f)
\]

Here, \(r = \frac{M_{S_R}^2}{M_{S_R}^2}\). The functions \(f_i(r)\) and \(g_i(r)\) have the forms:

\[
f_1(r) = \frac{2r^2 \ln r + (4 - 3r)r - 1}{4(r - 1)^2}, \quad (5.4a)
\]
The loop functions and the trilinear interactions entering the color-octet form factors, i.e.,
\[ f_1(r) = \frac{1}{4} \left[ 4 \sin^2 \theta_W \left\{ r + r\left( -\ln r - 1 + \ln(M_{S_R}^2) \right) \right\} \right.
+ \frac{2 \cos 2 \theta_W \left\{ (r-1)(-2r+3) \ln(M_{S_R}^2 r) + 3r - 1 + (2 - 4r) \ln r \right\}}{(r-1)^2}
+ 2 \ln \left\{ -M_{S_R}^2(r-2) \right\} - \frac{r^2 + 2 \ln(2-r) - 1}{(r-1)^2} \right],
\[ f_3(r) = 0, \]
\[ f_4(r) = \frac{r^2 - 2r \ln r - 1}{2(r-1)^2}, \]
\[ g_1(r) = \frac{3(r-1)^4 \cos 2 \theta_W + 2(r-1) \{ r(11r-7) + 18(r-1)^2 \ln \left( \frac{1}{2} \right) + 2 \}}{36(r-1)^4}
+ \frac{12 \{ (r-3)(2r-3) - 3 \} \ln r}{36(r-1)^4}, \]
\[ g_2(r) = \frac{r^3 + \cos 2 \theta_W \left\{ 2r^3 + 3r^2 - 6r^2 \ln r - 6r + 1 \right\} - 9r + 6(r-2) \ln(2-r) + 8}{6(r-1)^4} \]
\[ g_3(r) = 0, \]
\[ g_5(r) = 0, \]
\[ f_6(r) = -f_4(r), \]
\[ g_3(r) = \frac{1}{24}, \]
\[ g_4(r) = \frac{(r-1)(2r+5) - 1 - 6r^2 \ln r}{12(r-1)^4} \]

It is seen that \( G_{Z(\gamma),S} \to 0 \) as \( M_{S_R} \to \infty \). This is only expected since \( G_{Z(\gamma),S} \) quantifies the strength of the dimension-6 operator connecting \( H^+, W^+ \) and \( Z(\gamma) \) that naturally decouples from the low-energy theory as the color-octet mass scale becomes too large. Moreover, one also notes \( F_{\gamma,S}^A + F_{\gamma,S}^B \sim M_{S_R}^2 \lambda_{H+S-S_1} \{ f_4(r) + f_6(r) \} = 0 \) in the \( M_{S_R} \to \infty \) limit. This clearly conforms to the Ward identity that requires \( F_{Z,S}^S \to 0 \) whenever \( G_{Z,S}^S \to 0 \). We conclude that the contribution coming from the color-octet sector in the \( H^+ \to W^+ \gamma \) decay width decouples for a very heavy color-octet mass scale. More so, this exercise also serves as a check of the full calculation. On the other hand, eq. (5.3a) and eq. (5.3b) ascertain that \( F_{Z,S}^S \) does not exhibit a decoupling behaviour, the most crucial difference between the \( H^+ W^-, Z \) and \( H^+ W^- \gamma \) form factors at one-loop. We plot \( f_i(r) \) and \( g_i(r) \) in Fig. 8 and find smooth variations. All \( |f_i(r)| \) register increments with increasing \( \frac{M_{S}^2}{M_{S_R}^2} \), and, therefore \( M_{S}^2 - M_{S_R}^2 \), has a role in determining the size of the \( H^+W^-Z(\gamma) \) interaction. The trilinear interactions entering the color-octet form factors, i.e., \( \lambda_{H+S-S_R} \) and \( \lambda_{H+S-S_1} \), have the following forms for \( M_{S^+} = M_{S_R} \).

\[ \lambda_{H+S-S_R} = -\frac{v}{4}(k_2 + k_3 + (\nu_2 + 2\nu_3) \cot \beta) \]
\[ \lambda_{H+S-S_1} = \frac{M_{S_R}^2 - M_{S_1}^2}{v} - \frac{v}{4}(k_2 - k_3 + (\nu_2 - 2\nu_3) \cot \beta) \]

An inspection of Fig. (8) and Eq. (5.5b) shows that both the loop functions and the trilinear interactions are sensitive to the mass-splitting between the CP-even and -odd members of the color octet. We scan \( \tan \beta \) and the color octet parameters in the following ranges:
Figure 8: Variation of $f_i(r)$ and $g_i(r)$ with $r$.

Figure 9: Left panel: Allowed parameter points in the $M_{S_I}$-$\tan\beta$ plane for $M_{S_R} = M_{S^+} = 800$ GeV. Here, "Theory" refers to the points allowed by perturbativity, unitarity and the BFB conditions. And "Theory + Expt" denotes the points allowed by all the theoretical and experimental constraints. Right panel: variation of $F_{Z,S}$ with $\tan\beta$ for $M_{S_I} - M_{S_R} = 50$ GeV (cyan), 100 GeV (blue), 150 GeV (red) in the $(M_{H^+}, M_{S_R}) = (200$ GeV,800 GeV) case. The color coding is explained in the legends.

\[ |\nu_1|, |\nu_2|, |\nu_3|, |\kappa_1|, |\kappa_2|, |\kappa_3|, |\omega_1| \leq 4\pi, \]
\[ 2 < \tan\beta < 10, \]
\[ M_{H^+} < M_A < M_{H^+} + 400 \text{ GeV}, \quad 0 < m_{12} < 1 \text{ TeV}, \]
\[ 0 < (M_{S_I} - M_{S_R}) < 150 \text{ GeV}. \]  

The constraints on the color octet sector restrict the mass splitting $(M_{S_I} - M_{S_R})$ to $\leq 150$ GeV for $M_{S_R} = 800$ GeV irrespective of $\tan\beta$ as depicted by Fig.(9). A scan of the color octet input parameters for $(M_{S_I} - M_{S_R})$ fixed to 50 GeV, 100 GeV and 150 GeV reveals that the higher the mass splitting, the larger $F_{Z,S}$ can become. This is precisely what Fig.(9)
shows.

With an insight now gained on the behaviour of the loop functions and the trilinear couplings, the $M_{H^+} > M_W + M_Z$ region is taken for the subsequent numerical analysis. The following values for the SM parameters are taken: $M_W = 80.3$ GeV, $M_Z = 91.2$ GeV, $M_t = 125.0$ GeV, $M_t = 173.2$ GeV, $M_b = 3.1$ GeV, $M_c = 1.6$ GeV, $M_\tau = 1.77$ GeV.

The form factors $F_Z, G_Z$ and $G_\gamma$ for an array of charged Higgs masses are computed next. We take $M_{H^+} = 175$ GeV, 200 GeV, 400 GeV, 600 GeV and 1 TeV for a type I 2HDM. In type II, however, masses below 600 GeV are omitted in view of the flavour constraints. A variation of the absolute values of these form factors with respect to $\tan \beta$ is depicted for type I and type II in Fig.10 and Fig.11. We have not shown the variation of $|F_\gamma|$ separately since it is not independent, and, is connected to $|G_\gamma|$ through Eq.(3.4). Contributions to $H_Z$ and $H_\gamma$ come from the 2HDM fermionic part only. Expressions for the form factors can be found in Appendix 3.

It can be inferred from Fig.10 and Fig.11 that the basic nature of the variations of $|F_Z|, |G_Z|$ and $|G_\gamma|$ with $\tan \beta$ are the same for different values of $M_{H^+}$. While the variation of the color octet contributions to these form factors with $\tan \beta$ does not show any particular trend, the corresponding 2HDM contributions, for both type I and type II 2HDM, decrease with increasing $\tan \beta$. This behaviour is due to the $\lambda_{HH^+H^-} \sim \cot \beta$ relation and the $tbH^+$ interaction being equal to $(M_t \cot \beta + M_b \cot \tan \beta)/\nu$ in type I(II) 2HDM. There lies a minute difference in these variations of different form factors in type I and type II 2HDM, originating from nonidentical fermionic contributions.

As mentioned earlier, in type I 2HDM, for lower $\tan \beta$, all the Yukawa couplings dominate because of $\cot \beta$ dependence. In the similar model, for higher $\tan \beta$ all the Yukawa couplings decrease simultaneously. But in type II 2HDM, for higher $\tan \beta$, Bottom quark and $\tau$-Yukawa couplings still continue to dominate owing to $\tan \beta$ dependence, while $y_t$ decreases.

Thus for type II, the tails of the form factors end at slight higher value at higher $\tan \beta$. Fig.10 represents the variation of $|F_Z|$, $|G_Z|$ (include contributions from 2HDM and color octet both), $|F_{Z,2HDM}|$, $|G_{Z,2HDM}|$ with $\tan \beta$ for different values of $M_{H^+}$. In the same figure, dark green, yellow, grey, red regions signify variations of $|F_{Z,2HDM}|$, $|F_Z|$, $|G_{Z,2HDM}|$, $|G_Z|$ respectively. The similar kind of variation has been depicted in Fig.11, for $|G_{\gamma,2HDM}|$ (light cyan region) and $|G_\gamma|$ (dark cyan region). As $M_{H^+}$ increases, all the form factors $|F_Z|$, $|G_Z|$, $|G_\gamma|$ decreases in both types of 2HDM. The maximum 2HDM contribution to the form factor $|F_Z|$ occur around low $\tan \beta \sim 2$. At these points, $|F_Z|$ is atleast 2-3 times $|F_{Z,2HDM}|$ at $M_{H^+} = 800$ GeV both for type I and type II 2HDM. As $M_{H^+}$ is lowered, the ratio of $|F_Z|$ to $|F_{Z,2HDM}|$ is lowered to $\sim 1.67$ for $M_{H^+} = 175$ GeV.

We next come to a discussion of the $H^+ \to W^+V$ branching fractions. This broad mass spectrum of $H^+$ is analysed by considering the piecewise intervals: (a) $(M_W + M_Z) \leq M_{H^+} < M_t + M_b$, (b) $M_t + M_b \leq M_{H^+} < 1$ TeV. Though interval (a) is narrow, it forbids the $H^+ \to tb$ mode. The branching ratio for $H^+ \to W^+Z$ (grey region in Fig.12(a)) becomes the dominant one among all, especially for large $\tan \beta$. At smaller $\tan \beta$, $BR(H^+ \to c\bar{s})$ (dark green region in Fig.12(a)) and $BR(H^+ \to \tau^+\nu_\tau)$ (violet region in Fig.12(a)) surpass $BR(H^+ \to W^+Z)$ owing to their $\cot \beta$ dependence. For illustration, we refer to Fig.12 (a),
in which \( \text{BR}(H^+ \to W^+ Z) \) and \( \text{BR}(H^+ \to W^+ \gamma) \) attains maximum value of \( \sim 80\% \) and \( \sim 5\% \) respectively for \( \tan \beta \sim 10 \) and \( M_{H^+} = 175 \text{ GeV} \). For interval (b), \( H^+ \to t \bar{b} \) opens up and corresponding branching ratio becomes the most dominant one \( (\sim 99\%) \) (brown region in Fig.12(a)). One must note that while \( H^+ \to W^+ H, W^+ A \) are kinematically closed upon setting \( M_{H^+} = M_H \) and \( M_A > M_H \) in the scan, \( H^+ \to W^+ h \) vanishes for the \( c_{\beta\alpha} = 0 \) limit chosen. Here it is worth mentioning that the interval (a) applies only to type I 2HDM. Whereas in interval (b), both type I and type II 2HDM can be accommodated together. The only difference in the form factors in two variants of 2HDM stems from the fermionic contribution only, since the scale factors \( (\zeta_t, \zeta_b) \) are different in two cases.

Lastly, in Fig.13, we show the impact of raising the color octet mass scale on the form factors. The form factors stripped of the \( \lambda_{H^+S-S_{R/I}} \) couplings will diminish in size and approach the limiting value given by Eq.(5.3a) and Eq.(5.3b). In addition, the higher is the color octet mass scale, the more constrained the trilinear couplings become from the theoretical constraints. A combination of these two effects leads \( |F_{Z,S}| \) to decrease by more than one order upon taking \( M_{S_R} = 1 \text{ TeV} \) to 5 TeV.

6 Sensitivity at the LHC

In this section, we comment on the observability of an \( H^+ \) at hadron colliders through the \( H^+ W^- Z \) interaction. We identify the following four processes that feature the \( H^+ W^- Z \) coupling at the production vertex, the decay vertex, or both: (a) \( pp \to H^+ jj \to W^+ ZZ \), (b) \( pp \to H^+ Z \to W^+ ZZ \), (c) \( pp \to tH^+ \to tW^+ Z \), (d) \( pp \to H^+ jj \to t\bar{b}jj \) (see Fig.14 for the Feynman diagrams.). The corresponding cross sections are computed for the following sample points (table 2).

| Sample point | \( M_{H^+}\) (GeV) | \( \tan \beta \) | \( |F_Z| \) | \( |G_Z| \) | \( \text{BR}(H^+ \to W^+ Z) \) | \( \text{BR}(H^+ \to t\bar{b}) \) |
|---------------|------------------|---------|---------|---------|----------------|----------------|
| SP1           | 175              | 8.12    | 1.79 × 10^{-2} | 3.93 × 10^{-4} | 8.54 × 10^{-4} | 0 |
| SP2           | 200              | 6.05    | 1.89 × 10^{-2} | 5.36 × 10^{-4} | 1.45 × 10^{-2} | 9.84 × 10^{-1} |
| SP3           | 600              | 5.65    | 2.19 × 10^{-2} | 3.58 × 10^{-4} | 8.26 × 10^{-4} | 9.92 × 10^{-1} |

Table 2: Sample parameter points proposed to compute cross sections at pp colliders.

We use MadGraph5 aMC@NLO [84] with the NN23LO1 parton distribution function to compute the LHC cross sections for these processes. The corresponding numbers for \( \sqrt{s} = 14 \text{ TeV} \) and 100 TeV [85–87] are respectively given in tables 3 and 4. The \( pp \to H^+jj \) process is generated using the 5-flavour scheme while imposing the cuts \( p_T^j > 20 \text{ GeV} \), \( |\eta_j| < 0.5 \) and \( \Delta R_{jj} > 0.4 \).

The most hefty event-yield is obviously obtained from \( pp \to tH^+ \) since this is a purely tree level production process in contrast to the others that involve the one-loop \( H^+W^-Z \) vertex. In SP3 for instance, \( H^+ \) having even a sizeable mass of 600 GeV, produced in association with an anti-\( t \), can decay to a gauge boson pair \( \sim 10^5 \) times in pp collisions at \( \sqrt{s} = 100 \text{ TeV} \) with an integrated luminosity \( L = 3000 \text{ fb}^{-1} \). The numbers are definitely

\(^1\) The very tiny branching ratio of \( H^+ \to W^+ b\bar{b} \) 3-body decay has been neglected in the calculation.
Figure 10: Variation of absolute value of the form factors $X = F_Z, G_Z, F_{Z,2HDM}, G_{Z,2HDM}$ with $\tan \beta$ for $M_{H^+} = 175, 200, 400, 600, 800, 1000$ GeV and $M_{S_R} = 800$ GeV for type I and type II 2HDM. Color coding is expressed in legends.

higher for a lighter $H^+$ in SP1 and SP2. A complete treatment however would entail considering the decays of the gauge bosons, and, a careful analysis of the signal and the SM backgrounds.

Despite offering lower event-yields than process (c), processes (a), (b) and (d) are interesting from a kinematical perspective. Since all three involve the $H^+ W^- Z$ interaction at the production vertex, the momentum dependent term in the amplitude (proportional
Figure 11: Variation of absolute value of the form factors $X = G_{\gamma}, G_{\gamma,2HDM}$ with $\tan \beta$ for $M_{H^+} = 175, 200, 400, 600, 800, 1000$ GeV and $M_{H^+} = 800$ GeV for type I and type II 2HDM. Color coding is expressed in legends.

to $G_Z$) becomes potentially important especially when a large momentum transfer is involved. This momentum dependence will distort various kinematical distributions to a degree controlled by the value of $G_Z$. Further amongst these, $H^+$ is produced via vector boson fusion (VBF) in (a) and (d) followed by decay to $t\bar{b}$ and $W^+Z$ respectively. It is then understood that (d) is useful to probe the $M_{H^+} > M_t + M_b$ region. As an example, SP3 would lead to the production of $5 \times 10^3$ $t\bar{b}$ pairs at the 100 TeV FCC-hh. On the other
Figure 12: Variation of branching ratio of $H^+$ into different decay modes with tan $\beta$ for $M_{H^+} = 175, 200, 400, 600, 800, 1000$ GeV and $M_{S_R} = 800$ GeV for type I and type II 2HDM.

Table 3: Cross sections of processes (a), (b), (c) and (d) at 14 TeV LHC.
Figure 13: $|F_{Z,S}|$ for $M_{H^+} = 200$ GeV and $M_{S_R} = 1$ TeV (yellow), 3 TeV (green) and 5 TeV (blue).

Figure 14: Feynman diagrams for the processes (a) (top left), (b) (top right), (c) (bottom left) and (d) (bottom right)

hand, the $M_W + M_Z < M_{H^+} < M_t + M_b$ window can be probed through (a). In fact, its cross section for SP1 is very similar to that of (d) for SP2. Further, when both $W^+$ and
$Z$ decay leptonically, process (d) leads to less hadronic activity compared to (a) enhancing the discernibility of the latter at the LHC. In this connection, process (b) can give rise to a hadron-free final state at the partonic level and this can potentially compensate for its lower cross section.

| Sample point | $\sigma_{pp\to H^+jj\to W^+Zjj}$ (fb) | $\sigma_{pp\to H^+Z\to W^+Z}$ (fb) | $\sigma_{pp\to H^+\to tWjj}$ (fb) | $\sigma_{pp\to H^+jj\to t\bar{b}jj}$ (fb) |
|--------------|---------------------------------|----------------------------------|---------------------------------|----------------------------------|
| SP1          | 4.001                           | 0.877                            | $1.412 \times 10^4$            | 0                                 |
| SP2          | 0.067                           | 0.013                            | 368.445                         | 4.57                              |
| SP3          | 0.012                           | $3.54 \times 10^{-4}$           | 31.15                           | 1.462                             |

Table 4: Cross sections of processes (a), (b), (c) and (d) at 100 TeV FCC-hh.

One must note that the aforementioned cross sections are much smaller than what would have been in case of the Georgi-Machacek (GM) model \[88–100\] that predicts $F_Z \sim O(0.1)$ at the tree level itself. However, Machine Learning techniques can be invoked to perform state-of-the-art event analyses and possibly distinguish the present scenario from the GM model by probing the momentum dependence of production amplitude discussed above.

7 Summary and conclusions

The weak isospin symmetry of the kinetic terms in models with $SU(2)_L$ scalar doublets forbids the $H^+W^-Z(\gamma)$ vertex at the tree level. However, radiative effects can give rise to this vertex at the one-loop level. It is therefore imperative to estimate the size of this vertex in such models and probe its observability at the energy frontier.

In this work, we have estimated the strength of the $H^+W^-Z(\gamma)$ interaction in context of an extended scalar sector comprising two color-singlet $SU(2)_L$ doublets and a color-octet $SU(2)_L$ doublet. While a Yukawa interaction between the SM fermions and the color-octet $SU(2)_L$ doublet is not relevant for the study, the corresponding ones involving the color singlet doublets are taken similar to the flavour conserving type I and type II 2HDM. The analysis takes into account all possible constraints. On the theoretical side, the constraints come from perturbativity of quartic couplings, unitarity of the Lee-Quigg-Thacker (LQT) eigenvalues and bounded-from-below criteria of the scalar potential. Experimental constraints include the bound on the charged Higgs mass $M_{H^+}$ from $B \to X_s\gamma$, Higgs signal strengths, oblique parameters and exclusion limits from LHC. We adopted the non-linear gauge to gauge-out the unphysical $G^+W^-Z(\gamma)$ vertex and the tadpoles. This simplified our calculation considerably. In order to throw more insight into the analysis, we presented simplified expressions of the form factors that are demonstrative of the decoupling/non-decoupling behaviour. We summarise below the key observations emerging from the analysis.

- The one-loop form factors borne out of the color-octet carry a color factor of 8.

\[2\] This is true also in some custodial variants of the GM model detailed in [96]
Amongst the various color octet form factors, \( F_{Z,S} \) clearly displays a non-decoupling behaviour. That is, it does not vanish in the \( M_{S_R} \rightarrow \infty \) limit. This is therefore identified as the source of a non-decoupled loop contribution to \( H^+ \rightarrow W^+Z \) partial width. On the contrary, \( F_{\gamma,S} \) and \( G_{\gamma,S} \) decouple thereby inducing a similar behaviour in \( H^+ \rightarrow W^+\gamma \) width.

The coupling \( \lambda_{H^+S-S_I} \) is sensitive to \( (M_{S_I}^2 - M_{S_R}^2) \) and so are the color octet form factors.

We have evaluated the various form factors and the \( H^+ \rightarrow W^+Z(\gamma) \) branching ratio for \( M_{H^+} \in [(M_W + M_Z), 1 \text{ TeV}] \). The color octet is found to enhance \( F_Z \) by up to a factor of \( \sim 3 \) with respect to the purely 2HDM value. While the 2HDM contribution registers a decrease with increasing \( \tan\beta \), the color octet contribution does not have such a behaviour. A larger relative enhancement is thus seen for \( \tan\beta \approx 10 \). The \( H^+ \rightarrow W^+Z \) branching fraction is found to touch the \( \mathcal{O}(1) \% \) ball-park for \( M_{H^+} > 200 \text{ GeV} \), the maximum being \( \approx 2\% \) for \( M_{H^+} = 1 \text{ TeV} \). On the other hand, the same branching fraction is seen to be as high as \( \sim 90 \% \) for \( M_{H^+} = 175 \text{ GeV} \) in which case the \( H^+ \rightarrow t\bar{b} \) mode is not open.

A closing remark is in order. Since the dominant contribution to the amplitude comes from the virtual effect of colored scalars, QCD corrections could be important. This point warrants further study and will be presented in a future work.

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### A Appendix

This section contains various analytical expressions related to form factors and partial widths.

#### A.1 Decay widths of \( h \)

##### A.1.1 \( h \rightarrow \gamma\gamma \)

\[
\begin{align*}
\mathcal{M}_{h \rightarrow \gamma\gamma}^{2\text{HDM}+S} &= \sum_f N_f Q_f^2 f_{hff} A_1/2 \left( \frac{M_h^2}{4M_W^2} \right) + f_{\phiVV} A_1 \left( \frac{M_h^2}{4M_W^2} \right) \\
&+ \frac{\lambda_{hH^+H^-}}{2M_{H^+}^2} A_0 \left( \frac{M_h^2}{4M_{H^+}^2} \right) + \frac{\lambda_{hS+S^+}}{2M_{S^+}^2} A_0 \left( \frac{M_h^2}{4M_{S^+}^2} \right), \\
\Gamma_{h \rightarrow \gamma\gamma}^{2\text{HDM}+S} &= \frac{G_F \alpha^2 M_h^3}{128\sqrt{2}\pi^3} |\mathcal{M}_{h \rightarrow \gamma\gamma}^{2\text{HDM}+S}|^2.
\end{align*}
\]
where \( G_F \) and \( \alpha \) denote respectively the Fermi constant and the QED fine-structure constant. The loop functions are listed below.

\[
A_{1/2}(x) = \frac{2}{x^2}((x + (x - 1)f(x)), \quad (A.2a)
\]
\[
A_1(x) = -\frac{1}{x^2}(2x^2 + 3x + 3(2x - 1)f(x)), \quad (A.2b)
\]
\[
A_0(x) = -\frac{1}{x^2}(x - f(x)), \quad (A.2c)
\]

with \( f(x) = \arcsin^2(\sqrt{x}); \quad x \leq 1 \)
\[
= -\frac{1}{4} \left[ \log \frac{1 + \sqrt{1 - x^{-1}}}{1 - \sqrt{1 - x^{-1}} - i\pi} \right]^2; \quad x > 1. \quad (A.2d)
\]

where \( A_{1/2}(x), A_1(x) \) and \( A_0(x) \) are the respective amplitudes for the spin-\( \frac{1}{2} \), spin-1 and spin-0 particles in the loop.

\[
f_{htt} = \frac{\cos \alpha}{\sin \beta}, \quad f_{hVV} = \sin(\beta - \alpha). \quad (A.3a)
\]

A.1.2 \( h \rightarrow gg \)

\[
\mathcal{M}_{h\rightarrow gg}^{2HDM+S} = \sum_f \frac{3}{4}f_{\phi ff} A_{1/2} \left( \frac{M_h^2}{4M_f^2} \right) + \frac{9\lambda hS_R S_R v}{8M_S^2} A_0 \left( \frac{M_h^2}{4M_S^2} \right)
\]
\[
+ \frac{9\lambda hS_L S_L v}{8M_L^2} A_0 \left( \frac{M_h^2}{4M_L^2} \right) + \frac{9\lambda hS_R S_R v}{4M_S^2} A_0 \left( \frac{M_h^2}{4M_S^2} \right), \quad (A.4a)
\]

\[
\Gamma_{h\rightarrow gg}^{2HDM+S} = \frac{G_F \alpha^2 M_h^3}{36 \sqrt{2} \pi^3} |\mathcal{M}_{h\rightarrow gg}^{2HDM+S}|^2. \quad (A.4b)
\]

Here, \( \alpha_s \) refers to the strong coupling constant.

A.1.3 \( h \rightarrow Z\gamma \)

\[
\mathcal{M}_{h\rightarrow Z\gamma}^{2HDM+S} = \sum_f N_f Q_f \left( \frac{2I_f - 4Q_f s^2_W}{c_W} \right) f_{\phi ff} B_{1/2} \left( \frac{M_h^2}{4M_f^2}, \frac{M_Z^2}{4M_f^2} \right) + f_{hVV} B_1 \left( \frac{M_h^2}{4M_Z^2}, \frac{M_Z^2}{4M_W^2} \right)
\]
\[
\quad + \frac{\lambda hH^- v}{2M_H^2} B_0 \left( \frac{M_h^2}{4M_H^2}, \frac{M_Z^2}{4M_H^2} \right) + N_S \frac{\lambda hS+S^- v}{2M_S^2} B_0 \left( \frac{M_h^2}{4M_S^2}, \frac{M_Z^2}{4M_S^2} \right), \quad (A.5a)
\]

\[
\Gamma_{h\rightarrow Z\gamma}^{2HDM+S} = \frac{G_F^2 \alpha M_h^3}{64 \pi^4} \left( 1 - \frac{M_W^2}{M_h^2} \right)^3 \left| \mathcal{M}_{h\rightarrow Z\gamma}^{2HDM+S} \right|^2, \quad (A.5b)
\]

\[
I_1(x, y) = -\frac{1}{2(x - y)} + \frac{f(x) - f(y)}{2(x - y)^2} + \frac{y(g(x) - g(y))}{2(x - y)^2}, \quad (A.6a)
\]
\[
I_2(x, y) = \frac{f(x) - f(y)}{2(x - y)}, \quad (A.6b)
\]
\[
B_0(x, y) = I_1(x, y), \quad (A.6c)
\]
\begin{equation}
B_{1/2}(x, y) = I_1(x, y) - I_2(x, y),
\end{equation}

\begin{equation}
B_1(x, y) = c_W \{4 \left(3 - \frac{s_W^2}{c_W^2}\right) I_2(x, y)
+ \left((1 + \frac{2}{x}) \frac{s_W^2}{c_W^2} - (5 + \frac{2}{x})\right) I_1(x, y)\},
\end{equation}

\begin{equation}
g(x) = \sqrt{x^{-1} - 1} \arcsin(x); \quad x \leq 1
= \frac{\sqrt{1 - x^{-1}}}{2} \left[ \log \frac{1 + \sqrt{1 - x^{-1}}}{1 - \sqrt{1 - x^{-1}}} - i\pi \right]; \quad x > 1
\end{equation}

A.2 $H^+$ decay widths

- $H^+ \to t\bar{b}$

\begin{equation}
\mathcal{M}_{H^+ \to t\bar{b}} = \frac{N_t}{v^2} \left[A_{tb}^2 \left(M_{H^+}^2 - (M_t + M_b)^2\right) + B_{tb}^2 \left(M_{H^+}^2 - (M_t - M_b)^2\right)\right],
\end{equation}

\begin{equation}
\Gamma_{H^+ \to t\bar{b}} = \frac{|\mathcal{M}|^2}{16\pi M_{H^+}} \sqrt{\lambda(1, M_{H^+}^2/M_{H^+}^2)}, \quad \text{for } M_{H^+} > M_t + M_b,
= 0, \quad \text{for } M_{H^+} < M_t + M_b.
\end{equation}

Here,

\begin{align}
\lambda(x, y, z) &= x^2 + y^2 + z^2 - 2xy - 2yz - 2zx, \\
A_{tb} &= M_b \zeta_b + M_t \zeta_t, \\
B_{tb} &= M_b \zeta_b - M_t \zeta_t.
\end{align}

Here, $\zeta_t$ and $\zeta_b$ are the scale factors with respect to SM. For type-I (type-II) 2HDM
$\zeta_t = \cot \beta$ ($\cot \beta$), $\zeta_b = -\cot \beta$ ($\tan \beta$).

- $H^+ \to c\bar{s}$

Decay width formula is same as $H^+ \to t\bar{b}$, with the replacement $M_t \to M_c$, $M_b \to M_s$.

- $H^+ \to \tau^+ \nu_\tau$

Decay width formula is same as $H^+ \to t\bar{b}$, with the replacement $M_t \to M_\tau$, $M_b \to 0$
, since $M_{\nu_\tau} = 0$.

- $H^+ \to W^+ \phi, \quad \phi = h, H, A$

\begin{equation}
\Gamma_{H^+ \to W^+ \phi} = \frac{a^2}{16\pi v^2 M_{H^+}^3} \lambda(1, M_{H^+}^2/M_{H^+}^2, M_{W^+}^2/M_{H^+}^2)^{3/2}
\end{equation}

where $a = c_{\beta - \alpha}$ for $\phi = h$ and $a = s_{\beta - \alpha}$ for $\phi = H$
A.3 Scalar trilinear vertices

\[ \lambda_{hS+S^-} = \frac{u}{2} (-\nu_1 c_\beta s_\alpha + \omega_1 s_\beta e_\alpha + \kappa_1 c_\beta + \alpha), \]  
(A.12a)

\[ \lambda_{hS_R S_R} = \frac{v}{2} \left\{ -(\nu_1 + \nu_2 + 2\nu_3) c_\beta s_\alpha + (\omega_1 + \omega_2 + 2\omega_3) s_\beta e_\alpha 
+ (\kappa_1 + \kappa_2 + \kappa_3) c_\beta + \alpha \right\}, \]  
(A.12b)

\[ \lambda_{hS_l S_l} = \frac{v}{2} \left\{ -(\nu_1 + \nu_2 - 2\nu_3) c_\beta s_\alpha + (\omega_1 + \omega_2 - 2\omega_3) s_\beta e_\alpha 
+ (\kappa_1 + \kappa_2 - \kappa_3) c_\beta + \alpha \right\}, \]  
(A.12c)

\[ \lambda_{H+S^- S_R} = \frac{1}{4} v \{ \sin \beta \cos \beta (\nu_2 - 2 \nu_3 + \omega_2 + 2 \omega_3) + (\kappa_2 + \kappa_3) \cos 2\beta \}, \]  
(A.12d)

\[ \lambda_{H+S^- S_l} = \frac{1}{4} v \{ \sin \beta \cos \beta (\nu_2 + 2 \nu_3 + \omega_2 - 2 \omega_3) + (\kappa_2 - \kappa_3) \cos 2\beta \}, \]  
(A.12e)

\[ \lambda_{hH+H^-} = v \{ (-\lambda_3 c_\beta^2 + (-\lambda_1 + \lambda_4 + \lambda_5)c_\beta s_\beta)e_\alpha 
+ (\lambda_3 s_\beta^2 + (\lambda_2 - \lambda_4 - \lambda_5)c_\beta^2 s_\beta)e_\alpha \}, \]  
(A.12f)

\[ \lambda_{hH+H^-} = \cos \alpha \{ \sin^2 \beta \cos \beta (\lambda_1 - \lambda_4 - \lambda_5) + \lambda_3 \nu \cos^3 \beta \} 
+ \sin \alpha \sin \beta \{ \nu \cos^2 \beta (\lambda_2 - \lambda_4 - \lambda_5) + \lambda_3 \nu \sin^2 \beta \}, \]  
(A.12g)

\[ \lambda_{hH+G^-} = \frac{v}{4} \{ (\lambda_2 - \lambda_3 + \lambda_4 + \lambda_5)c_\beta s_\alpha + (\lambda_2 - \lambda_3 + \lambda_4 + \lambda_5)c_\beta^2 s_\alpha 
+ (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5)s_\beta s_\alpha \}, \]  
(A.12h)

\[ \lambda_{hH+G^-} = \frac{v}{4} \{ (\lambda_2 - \lambda_3 + \lambda_4 + \lambda_5)c_\beta s_\alpha - (\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)c_\beta^2 s_\alpha 
- (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5)s_\beta s_\alpha \}, \]  
(A.12i)

A.4 Passarino-Veltman functions

Throughout the analysis the loop functions have been expressed in terms of Passarino-Veltman functions [101]. The integral formulæ of the functions relevant for our calculations, are illustrated below:

\[ B_0(p^2; M_1, M_2) = \int \frac{d^d k}{i \pi^2} \frac{1}{(k^2 - M_1^2)((k + p)^2 - M_2^2)}, \]  
(A.13a)

\[ p^\mu B_1(p^2; M_1, M_2) = \int \frac{d^d k}{i \pi^2} \frac{k^\mu}{(k^2 - M_1^2)((k + p)^2 - M_2^2)}. \]  
(A.13b)

We also have

\[ C_0(p_1^2, p_2^2, q^2; M_1, M_2, M_3) \]

\[ = \int \frac{d^d k}{i \pi^2} \frac{1}{(k^2 - M_1^2)((k + p_1)^2 - M_2^2)(k + q)^2 - M_3^2)}, \]  
(A.14a)

\[ (p_1^\mu C_{11} + p_2^\mu C_{12})(p_1^2, p_2^2, q^2; M_1, M_2, M_3) \]

\[ = \int \frac{d^d k}{i \pi^2} \frac{k^\mu}{(k^2 - M_1^2)((k + p_1)^2 - M_2^2)(k + q)^2 - M_3^2)}, \]  
(A.14b)

\[ (p_1^\mu p_2^\nu C_{21} + p_2^\mu p_3^\nu C_{22} + p_3^\mu p_3^\nu C_{23} + g^\mu \nu C_{24})(p_1^2, p_2^2, q^2; M_1, M_2, M_3) \]

\[ = \int \frac{d^d k}{i \pi^2} \frac{k^\mu k^\nu}{(k^2 - M_1^2)((k + p_1)^2 - M_2^2)(k + q)^2 - M_3^2)}, \]  
(A.14c)
The Passarino-Veltman functions have the closed forms below.

\[
B_0(p^2; M_1, M_2) = \text{div} - \int_0^1 dx \ln\Delta_B ,
\]
\[
B_1(p^2; M_1, M_2) = -\frac{\text{div}}{2} + \int_0^1 dx (1-x)\ln\Delta_B ,
\]
\[
C_0(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = -\int_0^1 dx \int_0^1 dy \frac{y}{\Delta_C} ,
\]
\[
C_{11}(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = -\int_0^1 dx \int_0^1 dy \frac{y(x-1)}{\Delta_C} ,
\]
\[
C_{12}(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = -\int_0^1 dx \int_0^1 dy \frac{y(y-1)}{\Delta_C} ,
\]
\[
C_{21}(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = -\int_0^1 dx \int_0^1 dy \frac{y(1-xy)^2}{\Delta_C} ,
\]
\[
C_{22}(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = -\int_0^1 dx \int_0^1 dy \frac{y(y-1)^2}{\Delta_C} ,
\]
\[
C_{23}(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = -\int_0^1 dx \int_0^1 dy \frac{y(1-xy)(1-y)}{\Delta_C} ,
\]
\[
C_{24}(p_1^2, p_2^2, q^2; M_1, M_2, M_3) = \frac{\text{div}}{4} - \frac{1}{2} \int_0^1 dx \int_0^1 dy y \ln\Delta_C .
\]

Where,
\[
\Delta_B = -x(1-x)p^2 + xM_1^2 + (1-x)M_2^2 ,
\]
\[
\Delta_C = y^2(p_1 x + p_2)^2 + y[x(p_2^2 - q^2 + M_1^2 - M_2^2) + M_2^2 - M_3^2 - p_2^2 + M_3^2] ,
\]
\[
\text{div} = \frac{2}{\epsilon} - \gamma_E + \ln4\pi + \ln\mu^2 .
\]

Here \(\epsilon = 4 - D\) is an infinitesimally small parameter in \(D\)-dimensional integral, \(\mu\) and \(\gamma_E\) are arbitrary dimensionful parameter and the Euler constant respectively. \(\text{div}\) becomes divergent in the limit \(D \to 4, \epsilon \to 0\). Here we adopt the following shorthand notations for brevity:

\[
B_i(p^2; M_A, M_B) = B_i(p^2; A, B) ,
\]
\[
C_{ij}(p_1^2, p_2^2, q^2; M_A, M_B, M_C) = C_{ij}(A, B, C) .
\]

### A.5 Form factors

\[
F_{Z,S}^A = \frac{N_S}{16\pi^2 c_W} \left[ \lambda_{H+S+} S_R \left[ (2 - 4s_W^2)C_{24}(S_R, S^+, S^+) - 2C_{24}(S^+, S_I, S_R) + s_W^2 B_0(q^2; S^+, S_R) \right] 
+ \lambda_{H+S+S} (2 - 4s_W^2)C_{24}(S_I, S^+, S^+) - 2C_{24}(S^+, S_R, S_I) + s_W^2 B_0(q^2; S^+, S_I) \right] ,
\]
\[
F_{Z,S}^B = \frac{N_S s_W^2}{16\pi^2 c_W} \left[ \lambda_{H+S-} S_R \left[ B_0(q^2; S^+, S_R) + 2B_1(q^2, S^+, S_R) \right] 
+ \lambda_{H+S-S} \left[ B_0(q^2; S^+, S_I) + 2B_1(q^2, S^+, S_I) \right] \right] ,
\]
\[ G^A_{Z,2HDM} = \frac{N_SM_W^2}{16\pi^2 v c_W} \left[ \lambda_{H+} S - S_r [ (2 - 4s_W^2)(C_{12} + C_{23})(S_R, S^+, S^+) + 2(C_{12} + C_{23})(S^+, S_R, S_I) ] + \lambda_{H+} S - S_I [ (2 - 4s_W^2)(C_{12} + C_{23})(S_I, S^+, S^+) - 2(C_{12} + C_{23})(S^+, S_R, S_I) ] \right], \quad (A.18c) \]

\[ F^A_{7,S} = \frac{N_S s_W}{16\pi^2 v} \left[ \lambda_{H+} S - S_R [ B_0(q^2, S^+, S_R) - 4C_{24}(S_R, S^+, S^+)] + \lambda_{H+} S - S_I [ B_0(q^2, S^+, S_I) - 4C_{24}(S_I, S^+, S^+) ] \right], \quad (A.19a) \]

\[ F^B_{7,S} = \frac{N_S s_W}{16\pi^2 v} \left[ \lambda_{H+} S - S_R [ B_0(q^2, S^+, S_R) + 2B_1(q^2, S^+, S_R)] + \lambda_{H+} S - S_I [ B_0(q^2, S^+, S_I) + 2B_1(q^2, S^+, S_I) ] \right], \quad (A.19b) \]

\[ G^A_{7,S} = -\frac{4N_S M_W^2 s_W}{16\pi^2 v} \left[ \lambda_{H+} S - S_R (C_{12} + C_{23})(S_R, S^+, S^+) + \lambda_{H+} S - S_I (C_{12} + C_{23})(S_I, S^+, S^+) \right], \quad (A.19c) \]

\[ F^A_{Z,2HDM} = \frac{1}{16\pi^2 v} \left[ \lambda_{H+} H - \cos(\beta - \alpha) \{ \frac{s^2_W}{c_W} B_0(q^2, h, H^+) - \frac{2}{c_W} C_{24}(H^+, A, h) + \frac{2 c_W}{c_W} C_{24}(h, H^+, H^+) \} \right. \]

\[ -\lambda_{H+} H - \sin(\beta - \alpha) \{ \frac{s^2_W}{c_W} B_0(q^2, H, H^+) - \frac{2}{c_W} C_{24}(H^+, A, H) + \frac{2 c_W}{c_W} C_{24}(h, H^+, H^+) \} \]

\[ +\lambda_{H+} G - \cos(\beta - \alpha) \{ \frac{s^2_W}{c_W} B_0(q^2, h, G^+) - \frac{2}{c_W} C_{24}(G^+, G_0, h) + \frac{2 c_W}{c_W} C_{24}(h, G^+, G^+) \} \]

\[ +\lambda_{AH+} G - \cos(\beta - \alpha) \sin(\beta - \alpha) \left\{ \frac{2}{c_W} \{ C_{24}(G^+, H, A) - C_{24}(G^+, h, A) \} \right\}, \]

\[ F^B_{Z,2HDM} = \frac{s^2_W}{16\pi^2 v c_W} \left[ -\lambda_{H+} H - \cos(\beta - \alpha) \left\{ B_0(q^2, H^+, h) + 2B_1(q^2, H^+, h) \right\} \right. \]

\[ +\lambda_{H+} H - \sin(\beta - \alpha) \left\{ B_0(q^2, H^+, H) + 2B_1(q^2, H^+, H) \right\} \]

\[ -\lambda_{H+} G - \sin(\beta - \alpha) \left\{ B_0(q^2, G^+, h) + 2B_1(q^2, G^+, h) \right\} \]

\[ -\lambda_{H+} G - \cos(\beta - \alpha) \left\{ B_0(q^2, G^+, H) + 2B_1(q^2, G^+, H) \right\} \], \quad (A.18c) \]

\[ G^A_{Z,2HDM} = \frac{1}{16\pi^2 v} \left[ \lambda_{H+} H - \cos(\beta - \alpha) \left\{ M_W^2 \{ (C_{12} + C_{23})(H^+, A, h) + \frac{2 c_W}{c_W} (C_{12} + C_{23})(h, H^+, H^+) \} \right\} \right. \]

\[ -\lambda_{H+} H - \sin(\beta - \alpha) \left\{ M_W^2 \{ (C_{12} + C_{23})(H^+, A, H) + \frac{2 c_W}{c_W} (C_{12} + C_{23})(h, H^+, H^+) \} \right\} \]

\[ +\lambda_{H+} G - \sin(\beta - \alpha) \left\{ M_W^2 \{ (C_{12} + C_{23})(G^+, G_0, h) + \frac{2 c_W}{c_W} (C_{12} + C_{23})(h, G^+, G^+) \} \right\} \]

\[ +\lambda_{AH+} G - \sin(\beta - \alpha) \left\{ M_W^2 \{ (C_{12} + C_{23})(G^+, H, A) + \frac{2 c_W}{c_W} (C_{12} + C_{23})(h, G^+, H) \} \right\} \].
\[ +\lambda_{\gamma H+H} \cos(\beta - \alpha) M_W^2 \left\{ -\frac{2}{c_W} (C_{12} + C_{23})(G^+, G^0, H) \right\} \]
\[ +\frac{2c_W}{c_W} (C_{12} + C_{23})(H, G^+, G^+) \}
\[ +\lambda_{\gamma H+G} \sin(\beta - \alpha) \cos(\beta - \alpha) M_W^2 \frac{2}{c_W} \left\{ (C_{12} + C_{23})(G^+, H, A) \right\} \]
\[ - (C_{12} + C_{23})(G^+, h, A) \right\} \right], \quad (A.20a) \]

\[ F_{Z,2HDM}^{A,F} = \frac{2N_t}{16\pi^2 v^2 c_W} \left[ M_t^2 \zeta_t (v_b + a_b) \left\{ 4C_{21}(t, b, b) - B_0(q^2, t, b) - B_0(p_W^2, b, t) - (2M_b^2 - M_Z^2) C_0(t, b, b) \right\} \right. \]
\[ - M_b^2 \zeta_b \left( v_b + a_b \right) \left\{ 4C_{21}(t, b, b) - B_0(p_W^2, b, b) - B_0(q^2, t, b) - (M_t^2 + M_b^2 - M_W^2) C_0(t, b, b) \right\} \]
\[ +M_b^2 \zeta_b \left( v_b + a_b \right) \left\{ B_0(p_W^2, b, b) + B_0(q^2, t, b) + (M_t^2 + M_b^2 - q^2) C_0(t, b, b) \right\} \]
\[ +2M_t^2 M_b^2 \zeta_t \left( v_b - a_b \right) C_0(t, b, b) \right] + (M_t, \zeta_t, v_b, a_b) \leftrightarrow (M_b, -\zeta_b, v_t, a_t), \quad (A.21a) \]

\[ F_{Z,2HDM}^{B,F} = \frac{4N_c M_W^2}{16\pi^2 v^2 c_W} \left[ M_t^2 \zeta_t (B_0 + B_1) - M_b^2 \zeta_b B_1 \right] (q^2, t, b), \quad (A.21b) \]

\[ G_{Z,2HDM}^{A,F} = \frac{4N_c M_W^2}{16\pi^2 v^2 c_W} \left[ M_b^2 \zeta_b (v_b + a_b) \left\{ 2C_{23} + 2C_{12} + C_{11} + C_0 \right\} \right. \]
\[ - M_b^2 \zeta_b (v_b + a_b) \left\{ 2C_{23} + C_{12} \right\} - M_b^2 \zeta_b \left( v_b - a_b \right) \left(C_{12} - C_{11}\right) \left(t, b, b\right) \]
\[ + M_t, \zeta_t, v_b, a_b \leftrightarrow (M_b, -\zeta_b, v_t, a_t), \quad (A.21c) \]

\[ H_{Z,2HDM}^{1PI} = \frac{4N_c M_W^2}{16\pi^2 v^2} \times \]
\[ \left[ M_t^2 \zeta_t \left( v_b + a_b \right) \left\{ C_0 + C_{11} \right\} - M_b^2 \zeta_b \left( v_b + a_b \right) C_{12} + M_b^2 \zeta_b \left( v_b - a_b \right) \left(C_{12} - C_{11}\right) \right] \left(t, b, b\right) \]
\[ + M_t, \zeta_t, v_b, a_b \leftrightarrow (M_b, +\zeta_b, v_t, a_t). \quad (A.22) \]

where, \[ v_f = I_f - s_W^2 Q_f, \quad a_f = I_f. \quad (A.23) \]

\[ F_{\gamma,2HDM}^A = \frac{1}{16\pi^2 v} \left\{ -\lambda_{\gamma H+H} \cos(\beta - \alpha) B_0(q^2, h, H^+) + \lambda_{\gamma H+H} \cos(\beta - \alpha) B_0(q^2, H, H^+) \right\} \]
\[ - \lambda_{\gamma H+G} \sin(\beta - \alpha) B_0(q^2, h, G^+) - \lambda_{\gamma H+G} \sin(\beta - \alpha) B_0(q^2, H, G^+) \right\} \]
\[ + 4 \lambda_{\gamma H+H} \cos(\beta - \alpha) C_{24}(h, H^+, H^+) - 4 \lambda_{\gamma H+H} \sin(\beta - \alpha) C_{24}(H, H^+, H^+) \right\} \]
\[ + 4 \lambda_{\gamma H+G} \cos(\beta - \alpha) C_{24}(h, G^+, G^+) \right\} \]
\[ + 4 \lambda_{\gamma H+G} \sin(\beta - \alpha) C_{24}(H, G^+, G^+) \right\}], \quad (A.24a) \]

\[ F_{\gamma,2HDM}^B = \frac{1}{16\pi^2 v} \left\{ -\lambda_{\gamma H+H} \cos(\beta - \alpha) s_W(2 B_1(q^2, H^+, h) + B_0(q^2, H^+, h)) \right\} \]
\[ + \lambda_{\gamma H+H} \sin(\beta - \alpha) s_W(2 B_1(q^2, H^+, H) + B_0(q^2, H^+, H))) \]
\[-\lambda_{hh+g-} \sin(\beta - \alpha) s_W (2 \, B_1(q^2, G^+, h) + B_0(q^2, G^+, h)) \]
\[-\lambda_{hh+g-} \cos(\beta - \alpha) s_W (2 \, B_1(q^2, G^+, H) + B_0(q^2, G^+, H)) \]
\[G_{\gamma,2HDM}^A = \frac{M_W^2}{16 \pi^2} \left[ 4 \, \lambda_{hh+H^-} s_W \cos(\beta - \alpha) (C_{12} + C_{23})(h, H^+, H^+) \right. \]
\[-4 \, \lambda_{HH+H^+} s_W \sin(\beta - \alpha) (C_{12} + C_{23})(H, H^+, H^+) \]
\[+4 \, \lambda_{hh+H^-} s_W \sin(\beta - \alpha) (C_{12} + C_{23})(h, H^+, G^+) \]
\[+4 \, \lambda_{HH+H^+} s_W \cos(\beta - \alpha) (C_{12} + C_{23})(H, H^+, G^+) \]
\[= \frac{4N_c Q_b M_W^2}{16 \pi^2 v^2_C W} [M^2_\xi (2C_{23} + 2C_{12} + C_{11} + C_0) - M^2_\zeta_b (2C_{23} + C_{12})] (t, b, b) + (M_t, \xi_t, Q_b) \leftrightarrow (M_b, -\xi_b, Q_t). \]
\[H_{\gamma,2HDM}^{1PI} = \frac{4N_c Q_b M_W^2}{16 \pi^2 v^2_C W} \times \]
\[M^2_\xi (C_0 + C_{11}) - M^2_\zeta_b C_{12} + M^2_\zeta_0 (C_{12} - C_{11})] (t, b, b) \]
\[+ (M_t, \zeta_t, Q_b) \leftrightarrow (M_b, +\zeta_b, Q_t) \]
\[F_Z = F_{Z,2HDM}^A + F_{Z,2HDM}^B + F_{Z,2HDM}^{A,F} + F_{Z,2HDM}^{A,F}, \]
\[G_Z = G_{Z,2HDM}^A + G_{Z,2HDM}^B + G_{Z,2HDM}^{A,F}, \]
\[H_Z = H_{Z,2HDM}^{1PI}, \]
\[F_\gamma = F_{\gamma,2HDM}^A + F_{\gamma,2HDM}^B + F_{\gamma,2HDM}^{A,F} + F_{\gamma,2HDM}^{A,F}, \]
\[G_\gamma = G_{\gamma,2HDM}^A + G_{\gamma,2HDM}^B + G_{\gamma,2HDM}^{A,F}, \]
\[H_\gamma = H_{\gamma,2HDM}^{1PI}. \]

References

[1] CMS collaboration, S. Chatrchyan et al., Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC, Phys. Lett. B716 (2012) 30 [1207.7235].

[2] ATLAS collaboration, G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B716 (2012) 1 [1207.7214].

[3] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto and A. Strumia, Higgs mass implications on the stability of the electroweak vacuum, Phys. Lett. B709 (2012) 222 [1112.3022].

[4] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori et al., Higgs mass and vacuum stability in the Standard Model at NNLO, JHEP 08 (2012) 098 [1205.6497].
[5] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio et al., *Investigating the near-criticality of the Higgs boson*, JHEP **12** (2013) 089 [1307.3536].

[6] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, *Theory and phenomenology of two-Higgs-doublet models*, Phys. Rept. **516** (2012) 1 [1106.0034].

[7] N. G. Deshpande and E. Ma, *Pattern of Symmetry Breaking with Two Higgs Doublets*, Phys. Rev. **D18** (1978) 2574.

[8] A. V. Manohar and M. B. Wise, *Flavor changing neutral currents, an extended scalar sector, and the Higgs production rate at the CERN LHC*, Phys. Rev. **D74** (2006) 035009 [hep-ph/0606172].

[9] P. Yu. Popov, A. V. Povarov and A. D. Smirnov, *Fermionic decays of scalar leptoquarks and scalar gluons in the minimal four color symmetry model*, Mod. Phys. Lett. **A20** (2005) 3003 [hep-ph/0511149].

[10] I. Dorsner and I. Mocioiu, *Predictions from type II see-saw mechanism in SU(5)*, Nucl. Phys. **B796** (2008) 123 [0708.3332].

[11] P. Fileviez Perez, R. Gavin, T. McElmurry and F. Petriello, *Grand Unification and Light Color-Octet Scalars at the LHC*, Phys. Rev. **D78** (2008) 115017 [0708.3332].

[12] P. Fileviez Perez, H. Iminniyaz and G. Rodrigo, *Proton Stability, Dark Matter and Light Color Octet Scalars in Adjoint SU(5) Unification*, Phys. Rev. **D78** (2008) 015013 [0803.4156].

[13] C. T. Hill, *Topcolor: Top quark condensation in a gauge extension of the standard model*, Phys. Lett. **B266** (1991) 419.

[14] B. A. Dobrescu, K. Kong and R. Mahbubani, *Leptons and Photons at the LHC: Cascades through Spinless Adjoints*, JHEP **07** (2007) 006 [hep-ph/0703231].

[15] B. A. Dobrescu, K. Kong and R. Mahbubani, *Massive color-octet bosons and pairs of resonances at hadron colliders*, Phys. Lett. **B670** (2008) 119 [0709.2378].

[16] Z. Heng and H. Zhou, *Higgs-strahlung production process $e^+e^- \rightarrow hZ$ at the future Higgs factory in the SM extension with color-octet scalars*, Chin. J. Phys. **54** (2016) 308.

[17] A. Kundu, T. De and B. Dutta-Roy, *The Role of color - octet isodoublet scalar bosons in the physics of K - anti-K, B(d) - B(d) mixing and the epsilon parameter*, Phys. Rev. **D49** (1994) 4793.

[18] A. Kundu, T. De and B. Dutta-Roy, *Role of isodoublet color - octet scalar bosons in radiative B decays*, Phys. Rev. **D49** (1994) 4801.

[19] G. D. Kribs and A. Martin, *Enhanced di-Higgs Production through Light Colored Scalars*, Phys. Rev. **D86** (2012) 095023 [1207.4496].

[20] G. Bhattacharyya, A. Kundu, T. De and B. Dutta-Roy, *Effects of isodoublet color - octet scalar bosons on oblique electroweak parameters*, J. Phys. **G21** (1995) 153.

[21] M. I. Gresham and M. B. Wise, *Color octet scalar production at the LHC*, Phys. Rev. **D76** (2007) 075003 [0706.0909].

[22] C. P. Burgess, M. Trott and S. Zuberi, *Light Octet Scalars, a Heavy Higgs and Minimal Flavour Violation*, JHEP **09** (2009) 082 [0907.2696].

[23] B. A. Dobrescu, G. D. Kribs and A. Martin, *Higgs Underproduction at the LHC*, Phys. Rev. **D85** (2012) 074031 [1112.2208].
Y. Bai, J. Fan and J. L. Hewett, Hiding a Heavy Higgs Boson at the 7 TeV LHC, JHEP 08 (2012) 014 [1112.1964].

X.-G. He, G. Valencia and H. Yokoya, Color-octet scalars and potentially large CP violation at the LHC, JHEP 12 (2011) 030 [1110.2588].

I. Dorsner, S. Fajfer, A. Greloj and J. F. Kamenik, Higgs Uncovering Light Scalar Remnants of High Scale Matter Unification, JHEP 11 (2012) 130 [1208.1266].

M. Reece, Vacuum Instabilities with a Wrong-Sign Higgs-Gluon-Gluon Amplitude, New J. Phys. 15 (2013) 043003 [1208.1765].

X.-G. He, H. Phoon, Y. Tang and G. Valencia, Unitarity and vacuum stability constraints on the couplings of color octet scalars, JHEP 05 (2013) 026 [1303.4848].

X.-G. He, Y. Tang and G. Valencia, Interplay between new physics in one-loop Higgs couplings and the top-quark Yukawa coupling, Phys. Rev. D88 (2013) 033005 [1305.5420].

X.-D. Cheng, X.-Q. Li, Y.-D. Yang and X. Zhang, $B_{s,d}^{−} → B_{s,d}^{+}$ mixings and $B_{s,d}^{−} → ℓ^{±}ℓ^{−}$ decays within the Manohar-Wise model, J. Phys. G42 (2015) 125005 [1504.00839].

L. M. Carpenter and S. Mantry, Color-Octet, Electroweak-Doublet Scalars and the CDF Dijet Anomaly, Phys. Lett. B703 (2011) 479 [1104.5528].

T. Enkhbat, X.-G. He, Y. Mimura and H. Yokoya, Colored Scalars And The CDF $W$+dijet Excess, JHEP 02 (2012) 058 [1105.2699].

J. M. Arnold and B. Fornal, Color octet scalars and high pT four-jet events at LHC, Phys. Rev. D85 (2012) 055020 [1112.0003].

J. Cao, P. Wan, J. M. Yang and J. Zhu, The SM extension with color-octet scalars: diphoton enhancement and global fit of LHC Higgs data, JHEP 08 (2013) 009 [1303.2426].

R. Ding, Z.-L. Han, Y. Liao and X.-D. Ma, Interpretation of 750 GeV Diphoton Excess at LHC in Singlet Extension of Color-octet Neutrino Mass Model, Eur. Phys. J. C76 (2016) 204 [1601.02714].

L. Cheng and G. Valencia, Two Higgs doublet models augmented by a scalar colour octet, JHEP 09 (2016) 079 [1606.01298].

L. Cheng and G. Valencia, Validity of two Higgs doublet models with a scalar color octet up to a high energy scale, Phys. Rev. D96 (2017) 035021 [1703.03445].

A. Arhrib, R. Benbrik, M. Chabab, W. T. Chang and T.-C. Yuan, CP violation in Charged Higgs Bosons decays $H^{\pm} → W^{\mp}(\gamma, Z)$ in the Minimal Supersymmetric Standard Model (MSSM), Int. J. Mod. Phys. A22 (2007) 6022 [0708.1301].

S. Kanemura, Enhancement of loop induced $H^{±}W^{±}Z^{0}$ vertex in two Higgs doublet model, Phys. Rev. D61 (2000) 095001 [hep-ph/9710237].

S. Kanemura, Possible enhancement of the $e^{+}e^{-} → H^{±} W^{±}$ cross-section in the two Higgs doublet model, Eur. Phys. J. C17 (2000) 473 [hep-ph/9911541].

G. Abbas, D. Das and M. Patra, Loop induced $H^{±} → W^{±}Z$ decays in the aligned two-Higgs-doublet model, Phys. Rev. D98 (2018) 115013 [1806.11035].

S. Moretti, D. Rojas and K. Yagyu, Enhancement of the $H^{±}W^{±}Z$ vertex in the three scalar doublet model, JHEP 08 (2015) 116 [1504.06432].
[43] T. Hahn and M. Perez-Victoria, *Automatized one loop calculations in four-dimensions and D-dimensions*, *Comput. Phys. Commun.* **118** (1999) 153 [hep-ph/9807565].

[44] K. Fujikawa, *Xi-limiting process in spontaneously broken gauge theories*, *Phys. Rev.* **D7** (1973) 393.

[45] M. Bace and N. D. Hari Dass, *Parity Violating Compton Amplitude in Unified Theories*, *Annals Phys.* **94** (1975) 349.

[46] M. B. Gavela, G. Girardi, C. Malleville and P. Sorba, *A Nonlinear R(xi) Gauge Condition for the Electroweak SU(2) X U(1) Model*, *Nucl. Phys.* **B193** (1981) 257.

[47] N. M. Monyonko, J. H. Reid and A. Sen, *Some Properties of Green’s Functions in the Nonlinear R(xi) Gauge*, *Phys. Lett.* **136B** (1984) 265.

[48] N. M. Monyonko and J. H. Reid, *ONE LOOP VACUUM POLARIZATION IN THE NONLINEAR R(XI) GAUGE*, *Phys. Rev.* **D32** (1985) 962.

[49] J. M. Hernandez, M. A. Perez, G. Tavares-Velasco and J. J. Toscano, *Decay Z → neutrino anti-neutrino gamma in the standard model*, *Phys. Rev.* **D60** (1999) 013004 [hep-ph/9903391].

[50] J. Hernandez-Sanchez, M. A. Perez, G. Tavares-Velasco and J. J. Toscano, *Decay H⁺ → W⁺ gamma in a nonlinear R(xi)-gauge*, *Phys. Rev.* **D69** (2004) 095008 [hep-ph/0402284].

[51] B. W. Lee, C. Quigg and H. B. Thacker, *Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass*, *Phys. Rev.* **D16** (1977) 1519.

[52] I. F. Ginzburg and I. P. Ivanov, *Tree-level unitarity constraints in the most general 2HDM*, *Phys. Rev.* **D72** (2005) 115010 [hep-ph/0508020].

[53] S. Kanemura, T. Kubota and E. Takasugi, *Lee-Quigg-Thacker bounds for Higgs boson masses in a two doublet model*, *Phys. Lett.* **B313** (1993) 155 [hep-ph/9303263].

[54] A. G. Akeroyd, A. Arhrib and E.-M. Naimi, *Note on tree level unitarity in the general two Higgs doublet model*, *Phys. Lett.* **B490** (2000) 119 [hep-ph/0006035].

[55] J. Horejsi and M. Kladiva, *Tree-unitarity bounds for THDM Higgs masses revisited*, *Eur. Phys. J.* **C46** (2006) 81 [hep-ph/0510154].

[56] B. Grimstein, C. W. Murphy and P. Uttayarat, *One-loop corrections to the perturbative unitarity bounds in the CP-conserving two-Higgs doublet model with a softly broken Z₂ symmetry*, *JHEP* **06** (2016) 070 [1512.04567].

[57] V. Cacchio, D. Chowdhury, O. Eberhardt and C. W. Murphy, *Next-to-leading order unitarity fits in Two-Higgs-Doublet models with soft Z₂ breaking*, *JHEP* **11** (2016) 026 [1609.01290].

[58] B. Gorczyca and M. Krawczyk, *Tree-Level Unitarity Constraints for the SM-like 2HDM*, 1112.5086.

[59] M. E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys. Rev.* **D46** (1992) 381.

[60] *Particle Data Group* collaboration, M. Tanabashi, K. Hagiwara, K. Hikasa, K. Nakamura, Y. Sumino, F. Takahashi et al., *Review of particle physics*, *Phys. Rev. D* **98** (2018) 030001.

[61] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, *The Oblique parameters in multi-Higgs-doublet models*, *Nucl. Phys.* **B801** (2008) 81 [0802.4353].
M. Misiak and M. Steinhauser, Weak radiative decays of the B meson and bounds on $M_{H^\pm}$ in the Two-Higgs-Doublet Model, *Eur. Phys. J.* **C77** (2017) 201 [1702.04571].

ALEPH, DELPHI, L3, OPAL, LEP collaboration, G. Abbiendi et al., Search for Charged Higgs bosons: Combined Results Using LEP Data, *Eur. Phys. J.* **C73** (2013) 2463 [1301.6065].

ATLAS collaboration, G. Aad et al., Search for charged Higgs bosons decaying via $H^\pm \to \tau^\pm \nu$ in fully hadronic final states using pp collision data at $\sqrt{s} =$ 8 TeV with the ATLAS detector, *JHEP* **03** (2015) 088 [1412.6663].

ATLAS collaboration, G. Aad et al., Search for new phenomena in the dijet mass distribution using $p - p$ collision data at $\sqrt{s} =$ 8 TeV with the ATLAS detector, *Phys. Rev.* **D91** (2015) 052007 [1407.1376].

ATLAS collaboration, T. A. collaboration, Search for New Phenomena in Dijet Events with the ATLAS Detector at $\sqrt{s} =$ 13 TeV with 2015 and 2016 data.

CMS collaboration, V. Khachatryan et al., Search for narrow resonances in dijet final states at $\sqrt{s} =$ 8 TeV with the novel CMS technique of data scouting, *Phys. Rev. Lett.* **117** (2016) 031802 [1604.08907].

CMS collaboration, V. Khachatryan et al., Search for narrow resonances decaying to dijets in proton-proton collisions at $\sqrt{s} =$ 13 TeV, *Phys. Rev. Lett.* **116** (2016) 071801 [1512.01224].

CMS collaboration, C. Collaboration, Search for $t\bar{t}$ resonances in boosted semileptonic final states in pp collisions at $\sqrt{s} =$ 13 TeV, *Phys. Rev. Lett.* **116** (2016) 071801 [1512.01224].

CMS collaboration, A. M. Sirunyan et al., Measurements of properties of the Higgs boson decaying to a W boson pair in pp collisions at $\sqrt{s} =$ 13 TeV, *Phys. Lett.* **B791** (2019) 96 [1806.05246].

ATLAS collaboration, M. Aaboud et al., Measurements of Higgs boson properties in the diphoton decay channel with 36 fb$^{-1}$ of pp collision data at $\sqrt{s} =$ 13 TeV with the ATLAS detector, *Phys. Rev.* **D98** (2018) 052005 [1802.04146].
[78] CMS collaboration, A. M. Sirunyan et al., Measurements of Higgs boson properties in the diphoton decay channel in proton-proton collisions at $\sqrt{s} = 13$ TeV, JHEP 11 (2018) 185 [1804.02716].

[79] ATLAS collaboration, T. A. collaboration, Cross-section measurements of the Higgs boson decaying to a pair of tau leptons in proton–proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, .

[80] CMS collaboration, A. M. Sirunyan et al., Observation of the Higgs boson decay to a pair of $\tau$ leptons with the CMS detector, Phys. Lett. B779 (2018) 283 [1804.02716].

[81] ATLAS collaboration, M. Aaboud et al., Search for Higgs bosons produced via vector-boson fusion and decaying into bottom quark pairs in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector, Phys. Rev. D98 (2018) 052003 [1807.08639].

[82] CMS collaboration, C. Collaboration, VBF H to bb using the 2015 data sample, .

[83] ATLAS collaboration, M. Aaboud et al., Searches for the $Z\gamma$ decay mode of the Higgs boson and for new high-mass resonances in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, JHEP 10 (2017) 112 [1708.00212].

[84] R. Frederix, S. Frixione, V. Hirschi, D. Pagani, H. S. Shao and M. Zaro, The automation of next-to-leading order electroweak calculations, JHEP 07 (2018) 185 [1804.10017].

[85] M. L. Mangano et al., Physics at a 100 TeV pp Collider: Standard Model Processes, CERN Yellow Rep. (2017) 1 [1607.01831].

[86] T. Golling et al., Physics at a 100 TeV pp collider: beyond the Standard Model phenomena, CERN Yellow Rep. (2017) 441 [1606.00947].

[87] N. Arkani-Hamed, T. Han, M. Mangano and L.-T. Wang, Physics opportunities of a 100 TeV proton–proton collider, Phys. Rept. 652 (2016) 1 [1511.06495].

[88] H. Georgi and M. Machacek, DOUBLY CHARGED HIGGS BOSONS, Nucl. Phys. B262 (1985) 463.

[89] M. S. Chanowitz and M. Golden, Higgs Boson Triplets With $M(W) = M(Z) \cos \theta_W$, Phys. Lett. 165B (1985) 105.

[90] J. F. Gunion, R. Vega and J. Wudka, Higgs triplets in the standard model, Phys. Rev. D42 (1990) 1673.

[91] M. Aoki and S. Kanemura, Unitarity bounds in the Higgs model including triplet fields with custodial symmetry, Phys. Rev. D77 (2008) 095009 [0712.4053].

[92] C.-W. Chiang and K. Yagyu, Testing the custodial symmetry in the Higgs sector of the Georgi-Machacek model, JHEP 01 (2013) 026 [1211.2658].

[93] K. Hartling, K. Kumar and H. E. Logan, The decoupling limit in the Georgi-Machacek model, Phys. Rev. D90 (2014) 015007 [1404.2640].

[94] K. Hartling, K. Kumar and H. E. Logan, Indirect constraints on the Georgi-Machacek model and implications for Higgs boson couplings, Phys. Rev. D91 (2015) 015013 [1410.5538].

[95] C.-W. Chiang, S. Kanemura and K. Yagyu, Novel constraint on the parameter space of the Georgi-Machacek model with current LHC data, Phys. Rev. D90 (2014) 115025 [1407.5053].

[96] H. E. Logan and V. Rentala, All the generalized Georgi-Machacek models, Phys. Rev. D92 (2015) 075011 [1502.01275].
[97] S. Blasi, S. De Curtis and K. Yagyu, Effects of custodial symmetry breaking in the Georgi-Machacek model at high energies, Phys. Rev. D96 (2017) 015001 [1704.08512].

[98] J.-Y. Cen, J.-H. Chen, X.-G. He, G. Li, J.-Y. Su and W. Wang, Searching for a charged Higgs boson with both $H^\pm W^\mp Z$ and $H^\pm tb$ couplings at the LHC, JHEP 01 (2019) 148 [1811.00910].

[99] N. Ghosh, S. Ghosh and I. Saha, Charged Higgs boson searches in the Georgi-Machacek model at the LHC, Phys. Rev. D101 (2020) 015029 [1908.00396].

[100] A. Banerjee, G. Bhattacharyya and N. Kumar, Impact of Yukawa-like dimension-five operators on the Georgi-Machacek model, Phys. Rev. D99 (2019) 035028 [1901.01725].

[101] G. Passarino and M. J. G. Veltman, One Loop Corrections for $e^+ e^-$ Annihilation Into $\mu^+ \mu^-$ in the Weinberg Model, Nucl. Phys. B160 (1979) 151.