Comparison of the new data for the negative muon capture in $^{28}\text{Si}$ with the recent shell model calculations using a full 1s-0d model space and the USD empirical effective interaction and the one-nucleon weak current provides the induced pseudoscalar $g_P$ substantially smaller than the value predicted by PCAC and pion-pole dominance. We find that adding the effect from the one-pion exchange axial charge density and using the same scheme of calculations does not change situation to the value of $g_P$.

1 Introduction

The four-current describing the axial part of the weak interaction of the muon and the proton consists of two parts:

$$J_{\lambda,\mu}^{a}(p',p,q) = i\bar{u}(p')[-g_{A}(q)\gamma_{\mu}\gamma_{5} + i\frac{g_{P}(q)}{m_{\mu}}q_{\nu}\gamma_{5}]\frac{\tau^{a}}{2}u(p).$$

(1)

The constant of the induced pseudoscalar, $g_P$, is given by PCAC, pion-pole dominance and the Goldberger-Trieman relation as

$$g_{P}(q) = \frac{2Mm_{\mu}}{m_{\pi}^{2} + q^{2}}g_{A}(q),$$

(2)

where $M, m_{\pi}$ and $m_{\mu}$ is the nucleon, pion and muon mass respectively and $g_{A}(0) = -1.2601 \pm 0.0025$. 

1
It is known that the experimental value of $g_P$ is the least known constant of the four constants ($g_V, g_A, g_W, g_P$) defining the weak nucleon current. The best measurement of the ordinary muon capture by the proton yields $g_P$ with an uncertainty of 42% and the world average reduces this to 22%. The recent precise measurement of the transition rate for the muon capture by $^3\text{He}$ leads to the extraction of $g_P$ with an accuracy of 19%.

2 Studies of $g_P$ in $A = 28$ Nuclei

2.1 $\gamma - \nu$ Correlations

An interesting attempt is being made for many years, to study $g_P$ in the reaction

$$\mu^- + ^{28}\text{Si}(0^+) \rightarrow \nu_\mu + ^{28}\text{Al}^*(1^+; 2201\text{ keV}) \rightarrow \gamma + ^{28}\text{Al}^*(0^+; 972.4\text{ keV}),$$

by measuring the $\gamma - \nu$ correlation. The formula for the $\gamma - \nu$ correlations is

$$W = 1 + a^2_0 P_2(\hat{k} \cdot \hat{\nu}) + (\hat{P}_\mu \cdot \hat{k})(\hat{k} \cdot \hat{\nu}) \left[ a^0 + \frac{2}{3} c^0_1 + b_2^0 \right] P_2(\hat{k} \cdot \hat{\nu}),$$

and it is the coefficient $a^2_0$ which is measured. The correlation coefficient $a^2_0$ is written in terms of reduced matrix elements (r.m.e.) of multipole amplitudes between initial and final states as

$$a^2_0 = F \frac{1 - x^2}{2 + x^2}, \quad x = \frac{|A_{1, f i}|}{|M_{1, f i}|},$$

where the coefficient, $F = 1$, is a function of the nuclear spin sequence in the $\gamma$-transition, $1^+ \rightarrow 0^+$. The multipole operators $\hat{A}_1$ and $\hat{M}_1$ can be written in terms of the standard multipoles

$$i\hat{A}_1 = \frac{C}{f} (\hat{\mathcal{L}}^A_1 - \hat{\mathcal{M}}^A_1),$$

$$i\hat{M}_1 = \frac{C}{\sqrt{2f}} (\hat{T}^A_{1, el} - \hat{T}^V_{1, mag}).$$

Since $a^2_0$ depends on the ratio of the r.m.e. it is expected that it depends on the nuclear models only weakly. However, the calculations show that this statement is only partially true.

The value of $a^2_0$ has recently been obtained

$$a^2_0 = 0.360 \pm 0.059.$$
2.2 Impulse Approximation

In order to compare the calculations with the result (8), one should write down the currents. The leading terms of the one-nucleon weak vector and axial currents in the q-space are taken from

\[ \hat{\mathbf{\jmath}}_A(q) = \tau^{-1} \left[ g_A \vec{\sigma} - \frac{g_P}{2M m_\mu} \vec{q} \cdot \vec{q} \right], \]  
\[ \hat{\rho}_A(q) = \tau^{-1} \left[ \frac{g_A}{2M} \vec{\sigma} \cdot (\vec{p}' + \vec{p}) - \frac{g_P}{2M m_\mu} q_0 (\vec{\sigma} \cdot \vec{q}) \right], \]  
\[ \hat{\mathbf{\jmath}}_V(q) = \tau^{-1} \left[ \frac{g_V}{2M} (\vec{p}' + \vec{p}) + \frac{g_V + g_M}{2M} i (\vec{\sigma} \times \vec{q}) \right]. \]

(9)

(10)

(11)

Here \( \vec{q} = \vec{p}' - \vec{p} \) and \( q_0 = m_\mu - \nu \).

The total weak current is equal to the sum of the vector and axial currents. In the configuration space,

\[ \hat{\mathbf{\jmath}}_A(\vec{x}) = \tau^{-1} \left[ g_A \vec{\sigma} \delta(\vec{r} - \vec{x}) + \frac{g_P}{2M m_\mu} \nabla_x (\vec{\sigma} \cdot \vec{r} \delta(\vec{r} - \vec{x})) \right], \]  
\[ \hat{\rho}_A(\vec{x}) = \tau^{-1} \left[ \frac{g_A}{2M} \{ (\vec{\sigma} \cdot \vec{p}) \delta(\vec{r} - \vec{x}) \} - i \frac{g_P}{2M m_\mu} q_0 (\vec{\sigma} \cdot \vec{r} \delta(\vec{r} - \vec{x})) \right], \]  
\[ \hat{\mathbf{\jmath}}_V(\vec{x}) = \tau^{-1} \left[ \frac{g_V}{2M} \{ \vec{p}, \delta(\vec{r} - \vec{x}) \} + \frac{g_V + g_M}{2M} (\vec{\sigma} \times \vec{r} \delta(\vec{r} - \vec{x})) \right]. \]

(12)

(13)

(14)

The Coulomb multipole, \( \hat{M}^A_{LM}(1) \), for the one-nucleon weak axial charge density and in the second quantized formalism is

\[ \hat{M}^A_{LM}(1) = \int d^3 x \, j_L(q x) Y_{LM} \hat{\rho}_A(\vec{x}) = \sum_{\alpha' \alpha} \hat{M}^A_{LM}(\alpha', \alpha) c^{\dagger}_{\alpha'} c_\alpha. \]

(15)

The r.m.e. of the Coulomb multipole expressed in terms of the single particle r.m.e.'s and one-body density-matrix elements as defined by Donnelly and Sick in Eq. (4.86) of Ref. is

\[ M^A_{LM, f_1}(1) \equiv \langle f || \hat{M}^A_{LM}(1) || i \rangle = \sum_{j', j} \langle j' ; 1 \frac{1}{2} || \hat{M}^A_{LM}(1) || j ; 1 \frac{1}{2} \rangle \langle f || \frac{1}{L \sqrt{3}} \left[ c^{\dagger}_{j'} \otimes \hat{c}_j \right]_{L; 1} || i \rangle, \]

(16)

where

\[ \hat{c}_{j m \tau} = (-1)^{j + m} (-1)^{j + \tau} c_{j - m - \tau}. \]

(17)

Similar equations can be obtained for other multipoles entering Eq. and Eq. (6).
3 Exchange Charge Density

Let us remind ourselves that the spontaneous breaking of the chiral symmetry is accompanied by the appearance of pions as Goldstone bosons. The production and absorption of these bosons in the electroweak interactions on the nucleon is described by the low energy theorems\textsuperscript{16,17}. As a consequence, the time component of the one-nucleon and of the weak axial one-pion exchange currents are of the same order in $1/M$. This fact makes the weak axial one-pion exchange charge density a favourable object for studying the pionic degrees of freedom in nuclei\textsuperscript{18}.

The leading term of the weak axial one-pion exchange charge density is\textsuperscript{19}

$$\hat{\rho}_A^{\text{exch}} = -i\sqrt{2} (\sigma_1 \times \sigma_2)_1 (\frac{g}{2M})^2 \frac{m^2}{g_A} \Delta_F(q_i^2) \Delta_F(q_i^2) \times F_{\pi NN}(q_i^2) F_{\rho NN}(q_i^2) (\sigma_1 \cdot \sigma_2) + (1 \leftrightarrow 2),$$  \hfill (18)

where

$$\Delta_F(q_i^2) = \frac{1}{m_B^2 + q_i^2}, \quad F_{\pi NN}(q_i^2) = \left(\frac{\Lambda_B^2 + m^2_B}{\Lambda_B^2 + q_i^2}\right)^{n_B}. \hfill (19)$$

The parameters $\Lambda_B$ and $n_B$ from the potential OBEPQB\textsuperscript{20} extended by the $a_1$ exchange\textsuperscript{21} are used in the calculations.

The corresponding Coulomb multipole operator is

$$\hat{M}^A_{LM}(\text{exch}) = \int d^3x j_L(qx) Y_{LM} \hat{\rho}_A(x; \text{exch}) = \sum_{\alpha_1',\alpha_2',\alpha_1,\alpha_2} \hat{M}^A_{LM}(\alpha_1',\alpha_2',\alpha_1,\alpha_2; \text{exch}) c^\dagger_{\alpha_1'} c^\dagger_{\alpha_2'} c_{\alpha_2} c_{\alpha_1}. \hfill (20)$$

The r.m.e. of the operator (20) is defined again in accordance with Donnelly and Sick (see Eq. (4.89) of Ref.\textsuperscript{14}),

$$M^A_{L,j_1}(\text{exch}) \equiv \langle i || \hat{M}^A_{LM}(\text{exch}) || f \rangle = \sum_{j_1',j_2',j',T,j_1,j_2,J_T} \langle [j_1' \otimes j_2']_{j',T} || \hat{M}^A_{LM}(\text{exch}) || [j_1 \otimes j_2]_{j,T} \rangle \times$$

$$\langle i || \frac{-1}{L\sqrt{3}} \left\{ \left[ c^\dagger_{j_1'} \otimes c^\dagger_{j_2'} \right]_{j',T} \otimes \left[ \tilde{c}_{j_1} \otimes \tilde{c}_{j_2} \right]_{j,T} \right\} \rangle || f \rangle. \hfill (21)$$

As it is seen, the two-body matrix elements calculated in the single-particle basis are separated from the two-body density matrix elements.
4 Results and Discussion

The results of the experiments\cite{7,8} have been compared to numerical results\cite{22,23} for specific nuclear models with contradictory conclusions. The comparison of the recent experimental data\cite{9,10} for the correlation coefficient (5) with the results of the calculations\cite{22,24} for the nuclear shell model\cite{25} has shown that these results yield the value of $g_P$ substantially lower in comparison with the PCAC prediction. Moreover, the most recent analysis\cite{26} based on the wave functions of a full 1s-0d model space obtained from the USD residual interaction with the OXBASH code\cite{27} gives $g_P/g_A = 0.0 \pm 3.2$.

Our calculations using the currents (12)-(14) and the one-body density matrix elements obtained from OXBASH code with the W residual interaction are presented in Fig. 1. Comparing the full curve with the full curve of Fig. 3 of Ref.\cite{10} shows a weak model dependence within one type of calculations. However, the results for various classes of models can differ considerably, yielding quite a large model dependence of the extracted $g_P$. The dashed curve corresponds to $g_A(0) = -1$. This value of $g_A$ is advocated in Refs.\cite{28,29,30,31}. However, another approach\cite{32,33} predicts only 5%-10% damping of $g_P$ in $A = 28$ nuclei. The dotted curve is obtained with $g_A(0) = -1$ and without the velocity dependent term in the one-nucleon axial charge density (13). It is clear that the damping of the effect of this term shifts the value of $g_P$ to the PCAC prediction.

In searching for a possible compensation of this velocity dependent term we turned our attention to the fact that in the same type of the transition, $\Delta T = 1, 0^+ \rightarrow 1^+$, in $A = 12$ nuclei\cite{34}, the almost exact chiral symmetry of the strong interactions manifests itself via a large effect of the weak axial one-pion exchange charge density. Indeed, as it is seen from Table 6 of Ref\cite{34}, the matrix element of the weak axial charge density is enhanced after including the contribution of the soft pion exchange charge density by almost 40%, which is just an amount demanded by the data. There is no reason to believe that the effect of the exchange charge density does not take place also in the case of reaction (3), unless some other large nuclear physics effect (core polarization etc.) does not come into play and acts in a opposite direction. The two-body density matrix elements were calculated again using the OXBASH code and the W residual interaction. However, in contrast to the case of $A = 12$ nuclei\cite{34}, in our scheme of calculations the effect of the weak axial one-pion exchange charge density turns out to be negligibly small. This result may be attributed to the fact that the wavefunctions and the operators of nuclear currents are not constructed consistently. This point seems to be proved at least for the one-nucleon currents in the recent report\cite{35}, where Siiskonen et
Figure 1: Dependence of the correlation coefficient $a_0^2$ on $g_P/g_A$; full curve corresponds to
$g_A = -1.26$, dashed curve is for $g_A = -1$ and the dotted curve is obtained with $g_A = -1$
and with the velocity dependent term in the one-nucleon axial charge density omitted.

anal constructed consistently renormalized one-particle transition operators and
an effective interaction starting from a realistic NN interaction and the G-
matrix appropriate for the 1s-0d shell. This leads to a result in the range
$4.4 \leq g_P/g_A \leq 5.9$ thus leaving space for an effect of $\approx 30\%$ of the weak
axial one-pion exchange charge density. Let us note that the authors of Ref.\textsuperscript{35}
use the value $g_A(0) = -1$.

In conclusion we note that the present calculation fails to give a value of
$g_P$ consistent with that obtained from PCAC and the pion-pole dominance. A
possible reason for this result may be traced to an inconsistent treatment of
the wavefunctions and the transition operators and to the fact that many of
the usual renormalization effects have not been taken into account.
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