Parametrically driven Kerr cavity solitons

Nicolas Englebert1✉, Francesco De Lucia1,2, Pedro Parra-Rivas1, Carlos Mas Arabí1,2, Pier-John Sazio2, Simon-Pierre Gorza1 and François Leo1

Cavity solitons are optical pulses that propagate indefinitely in nonlinear resonators. They are attracting attention, both for their many potential applications and their connection to other fields of science. Cavity solitons differ from laser dissipative solitons in that they are coherently driven. So far the focus has been on driving Kerr solitons externally, at their carrier frequency, with the aim of creating a stable localized solution for fixed parameters. Here we experimentally demonstrate Kerr cavity solitons driving at twice their carrier frequency, using an all-fibre optical parametric oscillator. In that configuration, called parametric driving, two backgroundless solitons of opposite phase may coexist. We harness this multiplicity to generate a string of random bits, thereby extending the pool of applications of Kerr cavity solitons to random number generators and Ising machines. Our results are in excellent agreement with a seminal amplitude equation, highlighting connections to hydrodynamic and mechanical systems, among others.

The spontaneous formation of patterns is encountered across many fields of science. Spatially extended nonlinear systems may be brought away from equilibrium, where spatiotemporal patterns emerge. Examples include convection rolls in heated fluids, vegetation patches in arid regions and localized structures in vibrated layers of sand. Such complex patterns can often be described by relatively simple reaction/diffusion equations that capture most of the nonlinear dynamics. These so-called amplitude equations have been shown to be universal. Very different systems in terms of microscopic physical laws can, under some conditions, be governed by the same macroscopic equation, providing important connections between distinct fields of science.

One such class of equations are the driven, damped, one-dimensional nonlinear Schrödinger equations (NLSEs), which describe pattern formation in charge-density condensates, driven plasmas, surface waves and optical resonators among others (see refs. 1–6 and references therein). The conservative NLSE admits exact soliton solutions and similar solitary waves can be found when dissipation and forcing are added. Two well-known configurations are external driving, also known as a.c.-driving in the context of plasma physics, and parametric driving. In the former, the system is described by equation (1), known as the Lugiato–Lefever equation in optics. In the latter, the system is described by equation (2), commonly called the parametrically driven, damped NLSE (or PDNLSE).

The solitary waves of the PDNLSE have been observed in water tanks and in pendulum lattices, where the driving consists of periodically varying a parameter of the system at twice its oscillation frequency. In optics, they have been predicted to exist in amplifier chains and resonators, where the parametric driving term finds its origin in the nonlinear polarization. Such parametrically driven Kerr cavity solitons (PDCSs) constitute a subclass of optical dissipative solitons along with temporal cavity solitons (CSs), which are solutions of equation (1), have attracted a lot of attention over the past decade and have been shown to underpin the formation of ultra-coherent optical frequency combs in micro-resonators. In that context, they are now commonly referred to as dissipative Kerr solitons (or DKWs).

Both CSs and PDCSs are sech-shaped pulses phase locked to a driving laser, but there are important differences between the two. CSs are sustained by additive driving, which forms a homogeneous background next to the soliton. The soliton is phase locked to the background and a single attractor exists for fixed detuning and driving power. PDCSs, on the other hand, are driven through phase-sensitive amplification, implemented, for instance, via four-wave mixing or three-wave mixing. The soliton lacks a homogeneous background and two stable, out-of-phase, solutions may coexist. The multiplicity opens up several new avenues for soliton coalescence as already demonstrated in hydrodynamics. Moreover, stable optical pulses of opposite phases can be used to implement random number generators and Ising machines.

Despite recent interest in temporal patterns of optical parametric oscillators and patterns excited through optical Faraday instabilities, PDCSs have never been experimentally characterized. Soliton formation may have played a role in demonstrations of all-optical storage using phase-sensitive amplification but no direct observation of solitons was reported.

Here, using parametric down conversion as the driving process, we experimentally investigate PDCSs. We implement an all-fibre, singly resonant, degenerate OPO with both quadratic and cubic nonlinearities. We measure backgroundless sech-shaped optical waves at the signal frequency and show that solutions with different phases may coexist in the resonator. As a proof-of-principle experiment, we generate a short series of random numbers using PDCSs.

Bifurcation analysis

We start by theoretically examining the dynamics of soliton formation in degenerate OPOs incorporating a Kerr section. We use the dimensionless PDNLSE (equation (2)). The derivation of the equation and its normalization are detailed in Supplementary Section 1. In this equation, there are only two independent parameters: the detuning Δ and the driving strength μ. They determine the two-dimensional parameter space, plotted in Fig. 2a, where we show the different nonlinear attractors of the system. The degenerate OPO threshold is located at $\mu = \sqrt{1 + 4\Delta^2}$ and corresponds to a pitchfork bifurcation of the trivial state. For negative detunings, that bifurcation is supercritical and the trivial state is modulationally unstable above $\mu = 1$ (ref. 7). The patterns emerging beyond this instability correspond to...
non-degenerate oscillation, which hence has a lower threshold than degenerate emission in that region. For positive detunings, the trivial solution is stable up to $\mu = \sqrt{1 + \Delta^2}$ and the pitchfork bifurcation is subcritical. An unstable homogeneous state emerges from the trivial solution and folds at the saddle-node bifurcation $SN_{h,s}$ located at $\mu = 1$ (Fig. 2a). Beyond the fold, the upper branch is modulationally unstable, creating a region where a trivial solution and a modulated pattern coexist. In that region ($\mu > 1$), the PDNLSE admits exact solitary waves of the form $u = \sqrt{2\beta} \text{sech}(\beta \tau) \exp(i\phi)$ where $\cos(2\phi) = \mu^{-1}$ and $\beta^2 = \Delta + \mu \sin(2\phi)$ (refs. 7,15,15,19). There are two solitons of different amplitude and each can have one of two opposite phases. Both branches, defined as the soliton peak power, are shown in Fig. 2b as a function of the driving amplitude. They connect at the saddle-node bifurcation $SN_{h}$ ($\mu = 1$). The solutions corresponding to $\sin(2\phi) > 0$ are always unstable. These soliton branches are reminiscent of the ones describing CSs\textsuperscript{4}. Conversely, when plotted as a function of the detuning, see Fig. 2c, both the homogeneous and

Fig. 1 | Illustration of the differences between externally driven and parametrically driven cavity solitons. a, CSs are solutions of the externally driven NLSE (equation (1)), where $u$ is the intracavity field envelope, $T$ is a slow time, $r$ is a time reference travelling at the soliton group velocity and $S$ is the intracavity driving field envelope. The forcing is provided through constructive interference with a driving laser at the soliton carrier frequency $\omega_s$. The coherent interference between the soliton and the driving laser results in the formation of dark pulses in the through port. In the frequency domain, the additive driving corresponds to providing energy to a single longitudinal mode (red dotted arrow). The other modes are sustained by parametric wave mixing (black dotted arrows). b, PDCs are backgroundless sech-shaped solutions of the parametrically driven NLSE (equation (2)), where $\mu$ is the driving field and $u^*$ denotes the complex conjugate of $u$. They are sustained by the same parametric processes as CSs but in this case the driving itself is also parametric (red dotted arrows). The energy is provided directly from a driving laser at $\omega_s$ to all the longitudinal modes composing the soliton through phase-sensitive amplification (PSA), which can be implemented in a $\chi^{(3)}$ or $\chi^{(2)}$ medium. Here we use degenerate parametric down conversion as the nonlinear driving process ($\omega_p = 2\omega_s$).

Fig. 2 | Bifurcation structure of the PDNLSE. a, Phase diagram in the $(\Delta, \mu)$-parameter space showing the main dynamical regions of the system. The bifurcation lines are the pitchfork bifurcation (PB, green line), corresponding to the degenerate OPO threshold, and the Hopf bifurcation (HB, grey line). The black line at $\mu = 1$ corresponds to the saddle-node bifurcation of both the non-trivial homogeneous state $SN_h$ and the soliton state $SN_s$ for $\Delta > 0$, and to modulation instability (MI) for $\Delta < 0$. b, Bifurcation diagram showing the soliton branches (red line) as well as the homogeneous states (black line) as a function of $\mu$ for $\Delta = 1.2$. The solid lines correspond to stable states, the dashed lines correspond to homogeneously unstable states and the dotted line to modulationally unstable states. c, Bifurcation diagram as a function of $\Delta$ for $\mu = 1.37$. The soliton develops breathing behaviour in-between the HBs (dash-dotted grey line). The colours and line descriptions are as in b.
soliton branches differ from those of CSs. Unlike tilted resonances, the stable and saddle solutions do not connect, making the branches infinitely long. Along the main soliton branch, there are a couple of Hopf bifurcations. Between these bifurcations, the PDCSs are unstable and localized oscillatory behaviour as well as complex spatiotemporal dynamics can be found. In what follows, we focus on the low detuning region where the formation of stable solitons is predicted.

Experimental set-up

For our experimental investigation of the PDCS, we introduce an all-fibre degenerate OPO (see Fig. 3), with a signal oscillating around 1,550 nm. The cavity is composed of three main sections made of different fibres, each of a different length ($L_i$): a periodically poled fibre (PPF; $L_1 = 21$ m), a standard single-mode fibre (SMF; $L_2 = 21$ m) and an erbium-doped fibre (EDF; $L_3 = 0.52$ m). The first two fibres provide, separately, the quadratic and cubic nonlinearities while the EDF is used to compensate the intracavity loss. The 775 nm driving signal is generated by frequency doubling a highly coherent 1,550 nm laser, which can be phase- and amplitude-modulated, in a periodically poled lithium niobate (PPLN) crystal. It is sent into the cavity through a wavelength division multiplexer (WDM) and removed after the PPF. The total intracavity loss ($\Lambda \approx 40\%$) is measured by removing the doped fibre $\Lambda \approx 40\%$. The intracavity amplifier is pumped with 2 W at 1,480 nm, leading to a single-pass amplification $gL_2 \approx 35\%$ where $g$ is the fibre gain. The effective intracavity loss around 1,550 nm ($\Lambda_2 = \Lambda - gL_2 \approx 5\%$) corresponds to a finesse of 122 ($Q$-factor $= 2.6 \times 10^4$). Note that this effective finesse is limited by gain dispersion in our experiment.$^{[41]}$ Increasing the amplifier factor leads to lasing at shorter wavelengths ($\sim 1,548$ nm), out of range of the highly coherent driving laser.

The oscillation threshold of the OPO, $\mu = 2k\sqrt{P_pL_1/\Lambda_e} = 1$, where $k$ is the effective second-order nonlinearity of the PPF, corresponds to a 775 nm driving power $P_p = 5.4$ W. We drive the cavity with short flat-top pulses, synchronized to the cavity free spectral range around 1,550 nm (9.2 MHz), to minimize the average driving power and to ensure that the average intracavity power remains well below the saturation power of the intracavity amplifier (600 mW).

The phase detuning of the intracavity degenerate signal ($\delta_0$) is controlled by tuning the frequency of the 1,550 nm driving laser. Light propagation in this synchronously driven, singly resonant OPO is, under some conditions (see Supplementary Section 1), described by equation (2).

Characterization of the parametrically driven Kerr cavity soliton

In a first experiment, we use 650-ps-long, 10-W-peak driving pulses (corresponding to $\mu = 1.37$) and scan the laser frequency ($\sim 230$ kHz ms$^{-1}$). Our results are shown in Fig. 4. The signal resonance, measured around 1,550 nm, is reminiscent of that observed in externally pumped Kerr resonators.$^{[20]}$ The signal average power gradually increases until it reaches the bistable region where it suddenly drops, indicating the formation of localized structures. The small plateau emerging at that point corresponds to the soliton branch shown in Fig. 2c. In the context of externally driven Kerr resonators, it is often called the soliton step as pulses tend to merge one by one, leading to a stair-shaped transmission curve.$^{[20]}$ Additional higher-resolution measurements of the nonlinear transmission of the cavity, including multi-soliton steps, are shown in Supplementary Fig. 2. We readily note an important difference between our experimental scans and the analytical branch shown in Fig. 2c. The soliton step in our experiments has a finite extension while the theoretical branch grows indefinitely with increasing $\Delta$. First, we stress that frequency scans are inherently dynamic in the context of externally driven Kerr resonators.$^{[20]}$ The signal average power gradually increases until it reaches the bistable region where it suddenly drops, indicating the formation of localized structures. The small plateau emerging at that point corresponds to the soliton branch shown in Fig. 2c. In the context of externally driven Kerr resonators, it is often called the soliton step as pulses tend to merge one by one, leading to a stair-shaped transmission curve.$^{[20]}$ Additional higher-resolution measurements of the nonlinear transmission of the cavity, including multi-soliton steps, are shown in Supplementary Fig. 2. We readily note an important difference between our experimental scans and the analytical branch shown in Fig. 2c. The soliton step in our experiments has a finite extension while the theoretical branch grows indefinitely with increasing $\Delta$. First, we stress that frequency scans are inherently dynamic such that the measured output power is not necessarily representative of steady-state solutions at the corresponding detuning. Second, higher order effects, not included in equation (2), such as parametric gain saturation, limit the soliton existence range. In our experiment, the soliton collapse is due to the 5 nm, flat-top intracavity filter we use to prevent lasing at shorter wavelengths.$^{[20]}$ As the detuning is ramped up, so is the soliton’s spectral width, such that the filter eventually prevents stable soliton propagation.

Next, we use a control signal to stabilize the system in the soliton region (see Methods). The average output power when the detuning is set to $\Delta = 1.2$ (corresponding to a phase detuning $\delta_0 = 0.03$) is shown in Fig. 4a. A high-resolution (80 ps) recording of the
ARTICLES

a random sequence of bits. For this demonstration, we phase modulate the pump beam so as to excite a series of equally spaced single solitons. The physics behind soliton attraction to phase maxima is similar to that of CSs[14] and is detailed in Supplementary Section 3. A low modulation frequency (4.6 GHz) is chosen to be able to resolve individual solitons on the oscilloscope. We extract a portion of the 1.550 nm driving laser before its frequency doubling and use it as a local oscillator for coherent detection (see Fig. 5a). We excite two solitons in the cavity and send both the reference and the combined beams to a fast photodetector. The results are shown in Fig. 5b–c. As expected, the reference, corresponding to the intensity, displays identical traces separated by 220 ps. After interfering with the local oscillator, however, two different amplitudes are measured. These measurements confirm that solitons of different phases are excited in the cavity. In a second series of experiments, we expand the pulse width to host four solitons and perform three distinct resonance scans. Our results are shown in Fig. 5d–f. By assigning a binary value to each soliton, our results correspond to a series of four-bit random numbers, highlighting the potential of PDCSs for applications. Moreover, our measurements confirm that the solitons are phase locked, as only two distinct amplitudes are measured across the different scans.

**Discussion**

In summary, we investigated Kerr soliton formation in singly resonant OPOs. We built a system that is well described by the semi-parametric NLSE when driven with a frequency close to twice that of a longitudinal mode. We theoretically showed that a couple of stable solitons exist in a broad region of experimental parameters. Our measurements confirm the existence of a backgroundless, sech-shaped and phase-locked optical pulse in that region. Its temporal and spectral profiles are in excellent agreement with the soliton solution of the PDNLSE. The same profile corresponds to the corresponding cavity output is shown in Fig. 4b. A resolution-limited pulse can be seen exiting the cavity every roundtrip time. Further temporal (Fig. 4c) and spectral (Fig. 4d) characterizations confirm that a short (3.6 ps) pulse is circulating in the cavity. The agreement with the analytic soliton solution of the PDNLSE is excellent. The experimental spectral background corresponds to the intracavity-amplified spontaneous emission[42]. These measurements confirm that our novel system is governed by the PDNLSE in that region and constitute an experimental observation of its well-known soliton in optics.

**Random bit generation**

PDCSs are phase locked to a driving laser, as are externally driven CSs that attract a lot of attention because of their inherent stability. The additional advantage of the PDCS is its multiplicity. Owing to the $\mathbb{Z}_2$-symmetry of the PDNLSE, two attractors, which have the same amplitude but are out of phase, may coexist in the cavity, adding a degree of freedom to Kerr resonators. In particular, it opens the possibility of using Kerr solitons in applications that require two different attractors, such as random bit generators[29] and Ising machines[49]. To confirm this potential, we design a proof-of-principle experiment of random number generation. The concept is simple. When a soliton is spontaneously excited, it has a 50% chance of locking to the pump with one of the two possible phase relations. By exciting multiple solitons, and extracting the phase, we can generate a random sequence of bits. For this demonstration, we phase modulate the pump beam so as to excite a series of equally spaced single solitons. The physics behind soliton attraction to phase maxima is

![Figure 4 | Characterization of the PDCS.](image)

**Fig. 4 | Characterization of the PDCS.** a. Forward scan (black line) through a resonance for $P_p=10\,\text{W}$. The dot highlights the stabilization setpoint ($\Delta=1.2$). The blue line corresponds to the output power when the cavity is actively stabilized around that level. b. Oscilloscope recording—taken several seconds after the excitation process—showing a stable, resolution-limited pulse exiting the cavity. c. Experimental (blue line) and theoretical (red line) autocorrelation traces. The inset shows the theoretical profile of the corresponding background-free soliton. d. Experimental (blue line) and theoretical (red line) spectra at the cavity output. The narrow peak corresponds to back-reflections of the control signal.

![Figure 5 | Random bit generation.](image)

**Fig. 5 | Random bit generation.** a. Experimental set-up for coherent detection. CW, continuous wave. b. Direct detection of two PDCSs. c. Coherent detection of two PDCSs, highlighting the two different phases. d–f. Sequences of four random bits generated through PDCS formation.
Moreover, we showed that applications of PDCSs go beyond free phase modulation, can be generated in a long fibre cavity. Because of individual spins is limited by the repetition rate of the pump laser. Our results show that a grid of individual spins, as dense as the input phase modulation, can be generated in a long fibre cavity. Because the number of potential connections scales as $N^2$, a 40 GHz phase modulation would lead to a three orders of magnitude increase in the number of spin–spin couplings as compared to the state of the art.

Online content
Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41566-021-00858-z.

Received: 25 January 2021; Accepted: 12 July 2021; Published online: 20 September 2021

References
1. Gross, M. C. & Hohenberg, P. C. Pattern formation outside of equilibrium. Rev. Mod. Phys. 65, 851–1112 (1993).
2. Ahlers, G., Grossmann, S. & Loche, D. Heat transfer and large scale dynamics in turbulent Rayleigh–Bénard convection. Rev. Mod. Phys. 81, 503–537 (2009).
3. Lejeune, O., Tlidi, M. & Couderon, P. Localized vegetation patches: a self-organized response to resource scarcity. Phys. Rev. E 66, 010901 (2002).
4. Umhanbowar, P. B., Melo, F. & Swinney, H. L. Localized excitations in a vertically vibrated granular layer. Nature 382, 793–796 (1996).
5. Barashenkov, I. V. & Smirnov, Y. S. Existence and stability chart for the ac-driven, damped nonlinear Schrödinger solitons. Phys. Rev. E 54, 5707–5725 (1996).
6. Barashenkov, I. V., Bogdan, M. M. & Korobov, V. I. Stability diagram of the phase-locked solitons in the parametrically driven, damped nonlinear Schrödinger equation. Europhys. Lett. 15, 113–118 (1991).
7. Bondila, M., Barashenkov, I. V. & Bogdan, M. M. Topography of attractors of the parametrically driven nonlinear Schrödinger equation. Physica D 87, 314–320 (1995).
8. Zakharov, V. E. & Shabat, A. B. Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. Sov. J. Exp. Theor. Phys. 34, 62–69 (1972).
9. Lugiato, L. A. & Lefever, R. Spatial dissipative structures in passive optical systems. Phys. Rev. Lett. 58, 2209–2211 (1987).
10. Haelterman, M., Trillo, S. & Wabnitz, S. Dissipative modulation instability. Physica D 87, 314–320 (1995).
11. Coen, S., Riva, P. & De Cao, G. Temporal cavity solitons in one-dimensional Kerr media as bits in an all-optical buffer. Nat. Photonics 4, 471–476 (2010).
12. Herr, T. et al. Temporal solitons in optical microresonators. Nat. Photonics 8, 145–152 (2014).
13. Coen, S., Randle, H. G., Sylvestre, T. & Erkintalo, M. Modeling of octave-spanning Kerr frequency combs using a generalized mean-field Lugiato–Lefever model. Opt. Lett. 38, 37–39 (2013).
14. Parra-Rivas, P., Gomila, D., Matias, M. A., Coen, S. & Gelens, L. Dynamics of localized and patterned structures in the Lugiato–Lefever equation determine the stability and shape of optical frequency combs. Phys. Rev. A 89, 043813 (2014).
15. Kimpenberg, T. J., Gaeta, A. L., Lipson, M. & Gorodetsky, M. L. Dissipative Kerr solitons in optical microresonators. Science 361, eaan8083 (2018).
16. Nozaki, K. & Bekki, N. Chaotic solitons in a plasma driven by an RF field. J. Physical Soc. Japan 54, 2363–2366 (1985).
17. Miles, J. W. Parametrically excited solitary waves. J. Fluid Mech. 148, 451–460 (1984).
18. Trillo, S. & Haelterman, M. Excitation and bistability of self-trapped signal beams in optical parametric oscillators. Opt. Lett. 23, 1514–1516 (1998).
19. Wang, X. et al. Universal mechanism for the binding of temporal cavity solitons. Optica 4, 855–863 (2017).
20. Cole, D. C., Lamb, E. S., De’Haye, P., Diddams, S. A. & Papp, S. B. Soliton crystals in Kerr resonators. Nat. Photonics 11, 671–676 (2017).
21. Marandi, A., Leindecker, N. C., Vodopyanov, K. L. & Byer, R. L. All-optical quantum random bit generation from intrinsically binary phase of parametric oscillators. Opt. Express 20, 19322–19330 (2012).
22. Inagaki, T. et al. A coherent Ising machine for 2000-node optimization problems. Science 354, 603–606 (2016).
23. Pierangeli, D., Marcucci, G. & Conti, C. Large-scale photonic Ising machine by spatial light modulation. Phys. Rev. Lett. 122, 213902 (2019).
24. Mosca, S. et al. Modulation instability induced frequency comb generation in a continuously pumped optical parametric oscillator. Phys. Rev. Lett. 121, 093903 (2018).
25. Bruch, A. W. et al. Pockels soliton microcomb. Nat. Photonics 15, 21–27 (2021).
26. Tararov, N., Perez, A. M., Chursin, D. V., Stiawanus, K. & Turitsyn, S. K. Mode-locking via dissipative Faraday instability. Nat. Commun. 7, 12441 (2016).
27. Bessis, F. et al. Gain-through-filtering enables tuneable frequency comb generation in passive optical resonators. Nat. Commun. 10, 4489 (2019).
28. Copie, F., Conforti, M., Kudlinski, A., Musot, A. & Trillo, S. Competing Turing and Faraday instabilities in longitudinally modulated passive resonators. Phys. Rev. Lett. 116, 143901 (2016).
29. Bartolini, G. D., Sekeris, L. & Kumpf, P. All-optical storage of a picosecond-pulse packet using parametric amplification. In Optical Amplifiers and Their Applications (eds Zervas, M. et al.) FAW’17 (Optical Society of America, 1997).
30. Pérez-Arjona, I., Roldán, E. & de Valcarcel, G. J. Theory of quantum fluctuations of optical dissipative structures and its application to the squeezing properties of bright cavity solitons. Phys. Rev. A 75, 063802 (2007).
31. Scroggie, A. J. et al. Pattern formation in a passive Kerr cavity. Chaos Solitons Fractals 4, 1323–1354 (1994).
32. Coen, S. & Erkintalo, M. Universal scaling laws of Kerr frequency combs. Opt. Lett. 38, 1790–1792 (2013).
33. De Lucia, F., Keefer, D. W., Corbari, C. & Sazio, P. J. A. Thermal poling of silica optical fibers using liquid electrodes. Opt. Lett. 42, 69–72 (2017).
34. Englund, N., Arieti, C. M., Parra-Rivas, P., Gorza, S.-P. & Leo, F. Temporal solitons in a coherently driven active resonator. Nat. Photonics 15, 536–541 (2021).
35. Anderson, M., Leo, F., Coen, S., Erkintalo, M. & Murdoch, S. G. Observations of spatiotemporal instabilities of temporal cavity solitons. Optica 3, 1071–1074 (2016).
36. Jang, J. K., Erkintalo, M., Coen, S. & Murdoch, S. G. Temporal tweezing of light through the trapping and manipulation of temporal cavity solitons. Nat. Commun. 6, 7370 (2015).
37. Riemensberger, J. et al. Massively parallel coherent laser ranging using a soliton microcomb. Nature 581, 164–170 (2020).
38. Udum, T., Holzworth, R. & Hänsch, T. W. Optical frequency metrology. Nature 416, 233–237 (2002).

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

© The Author(s), under exclusive licence to Springer Nature Limited 2021
Methods

Linear stability analysis. The temporal linear stability of the steady-state solutions, shown in Fig. 2, has been computed by solving the eigenvalue problem \( Ly = \lambda y \), obtained from the linearization of equation (2) around a given steady state, where \( L \) is the linear operator evaluated at such a state. The eigenvalues and eigenvectors of \( L \) can be easily solved analytically for the homogeneous state as shown in Supplementary Section 2. For the soliton state stability, we have adopted a numerical approach. We compute the eigenvalues of the Jacobian matrix obtained from \( L \) after spatial discretization in an \( N = 1,024 \) points grid.

Experimental set-up. The all-fibre OPO is made of a section ( \( L_o = 27 \) cm) of PPF, a section ( \( L = 21 \) cm) of standard telecommunication single-mode silica fibre (SMF-28) and a section ( \( L = 52 \) cm) of EDF. The PPF has a second-order nonlinear parameter of \( \kappa = 0.04 \text{ W}^{-1/2} \text{m}^{-1} \) and a phase-matching wavelength of 1,548.8 nm at room temperature. This wavelength is increased up to 1,549.72 nm by the tuning range of the driving laser. The output power is attenuated by 1/25 (SMF-28) and a section (FTGS-100) of standard telecommunication single-mode silica fibre outside diameter 27 μm and a numerical aperture NA = 0.17. Two 27-μm diameter channels run adjacent to the fibre core at a distance of 13.6 μm and 7.2 μm, respectively, from the core’s edges. The fibre is first thermally poled in single-anode configuration at 265°C with an electric potential of 8 kV applied to the embedded electrode, for two hours. The second-order nonlinearity created via thermal poling is then erased periodically by means of a continuous-wave argon-ion laser frequency doubled to 244 nm, equipped with an acousto-optic modulator used to modulate the laser output.

The laser is focused to a circular spot, 20 μm in diameter, while the poled fibre is clamped onto a linear stage using two fibre rotator clamps. The laser is modulated using the acousto-optic modulator while translating the fibre core through the spot to achieve a grating of the desired duty cycle and period. The latter was chosen to be 55 μm to have quasi-phase matching at a wavelength of around 1,530 nm.

Coherent detection measurement. To demonstrate the existence of PDCSs with opposite phases, the cavity is synchronously pumped with 1 ns or 1.9 ps flat-top pulses. On these driving pulses, we also imprint a 4.6 GHz phase modulation using a phase modulator. As with CSs, PDCSs are attracted by phase-modulation maxima (see Supplementary Section 3). When scanning the resonance, we generate up to four PDCSs, separated by 220 ps. Using a 90/10 coupler, most of the cavity output power \( P_{\text{out}} \) is sent to a 10 GHz photodiode (that is, the reference beam in Fig. 5a). The remaining power is combined with part of the driving laser power, obtained by bypassing the frequency shifter, through another 90/10 coupler. The result of the interference is sent to a 45 GHz photodiode for coherent detection.

Data availability

The data that support the findings of this study are available from the corresponding author on reasonable request.

References

47. Li, Z. et al. Experimental observations of bright dissipative cavity solitons and their collapsed snaking in a Kerr resonator with normal dispersion driving. Optica 7, 1195–1203 (2020).

48. De Lucia, F. et al. Single is better than double: theoretical and experimental comparison between two thermal poling configurations of optical fibers. Opt. Express 27, 27761–27776 (2019).

49. Jang, J. K. et al. Controlled merging and annihilation of localised dissipative structures in an AC-driven damped nonlinear Schrödinger system. New J. Phys. 18, 033034 (2016).

Acknowledgements

We are grateful to M. Fita Codina for the manufacturing of experimental components and to P. Kockaert and G. Corbari for fruitful discussions. This work was supported by funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement no. 757890) and from the Fonds Emile Defay. N.E. acknowledges the support of the Fonds pour la formation à la Recherche dans l’Industrie et dans l’Agriculture (FRIA). F.D.L. acknowledges the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement no. 842675. F.L. and P.P.R. acknowledge the support of the Fonds de la Recherche Scientifique (FNRS).

Author contributions

N.E. designed and performed the experiments, supervised by S.-P.G. Both F.D.L. and F.-J.S. manufactured the periodically poled fibre. N.E. derived and simulated the mean-field model. P.P.-R. and C.M.A. performed the bifurcation and linear stability analysis of the mean-field model. F.L. supervised the overall project and wrote the manuscript. All authors discussed the results and contributed to the final manuscript.

Competing interests

N.E., S.-P.G. and F.L. have filed patent applications on the active resonator design and its use for frequency conversion (European patent office, application number EP20188731.2). The remaining authors declare no competing interests.

Additional information

Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41566-021-00858-z.

Correspondence and requests for materials should be addressed to Nicolas Englebert.

Peer review information Nature Photonics thanks the anonymous reviewers for their contribution to the peer review of this work.

Reprints and permissions information is available at www.nature.com/reprints.
Supplementary information

Parametrically driven Kerr cavity solitons

In the format provided by the authors and unedited
Supplementary Information – Parametrically driven Kerr cavity solitons

Nicolas Englebert,1 Francesco De Lucia,1,2 Pedro Parra-Rivas,1 Carlos Mas Arábi,1 Pier-John Sazio,2 Simon-Pierre Gorza,1 and François Leo1
1Service OPERA-Photonique, Université libre de Bruxelles (ULB), 50 Avenue F. D. Roosevelt, CP 194/5, B-1050 Brussels, Belgium
2Optoelectronics Research Centre, University of Southampton, SO17 1BJ, United Kingdom

This document contains the Supplementary Information for the manuscript entitled "Parametrically driven Kerr cavity solitons". We derive the parametrically driven nonlinear Schrödinger equation and perform a stability analysis of its stationary states. We also theoretically describe how parametrically driven cavity solitons (PDCSs) behave when the driving field is phase-modulated. Finally, additional experimental results are given.

I. MEAN-FIELD MODEL

Pattern formation in our singly resonant OPO can be described by a single mean-field equation at the signal frequency \( \omega_s \). We here detail its derivation starting from the full lumped model. The cavity boundary conditions for both the signal field \( A \) at \( \omega_s \) and the pump field \( B \) at \( 2\omega_s \) read \[ \]

\[
A_{m+1}(0, \tau) = \sqrt{T} A_m(L, \tau) e^{i\varphi_0}, \quad (S1)
\]

\[
B_{m+1}(0, \tau) = B_m, \quad (S2)
\]

where \( m \) is the roundtrip number, \( T = \prod_k T_k \) denotes the total insertion loss of all cavity components (polarization controllers, coupler, WDMs, ...), \( B_m \) is the driving field and \( \varphi_0 \) is the linear phase accumulated by the signal over one roundtrip.

In what follows, we assume that the undepleted pump approximation is verified, i.e. \( \partial B_m/\partial z = 0 \). Moreover, we consider that degenerate parametric down conversion (second harmonic generation) is quasi phase matched in the periodically poled fiber (PPF). The propagation in the PPF is then governed by the following equation \[ \]

\[
\frac{\partial A_m}{\partial z} = -\left( \frac{\alpha(1)}{2} + i\frac{\beta(1)}{2} \frac{\partial^2}{\partial \tau^2} \right) A_m + i\kappa B_m A_m^* \quad (S3)
\]

We introduce the superscript (1) for the parameters of the PPF at the signal wavelength. We will later use (2) for the single mode fibre (SMF) and (3) for the erbium doped fibre (EDF). For clarity, we drop the superscripts when defining the parameters in what follows. \( z \) is the position along the fibre, \( \tau = t - \beta_1 z \) with \( \beta_1 = [d\beta(\omega)/d\omega]|_{\omega_s} \) where \( \beta(\omega) \) is the propagation constant, \( \Delta \beta = 2\beta(\omega_s) - \beta(2\omega_s) \) is the phase mismatch and \( \beta_2 = [d^2 \beta(\omega)/d\omega^2]|_{\omega_s} \) is the group velocity dispersion coefficient. \( \alpha_s \) is the loss coefficient and \( \kappa \) is the second-order nonlinear parameter of the fibre. The third order nonlinearity of the PPF is neglected.

The evolution of the signal in the SMF is described by the nonlinear Schrödinger equation (NLSE) \[ \]

\[
\frac{\partial A_m}{\partial \tau} = \left( \frac{\alpha(2)}{2} + i\frac{\beta(2)}{2} \frac{\partial^2}{\partial \tau^2} - i\gamma |A_m|^2 \right) A_m, \quad (S4)
\]

where \( \gamma \) is the third-order nonlinear coefficient of the fibre. Finally, the propagation in the EDF is described by

\[
\frac{\partial A_m}{\partial z} = \left( \frac{g_0}{2} - i\frac{\beta(3)}{2} \frac{\partial^2}{\partial \tau^2} \right) A_m, \quad (S5)
\]

where \( g_0 \) is the unsaturated gain. We neglect the saturation of the gain, which is justified by the low average intracavity power in our synchronously pumped cavity \[ \]

The third order nonlinearity of the EDF is also neglected. The set of equations (S1)-(S5) constitute the full lumped model of the system. It is often referred to as a generalised Ikeda map \[ 5 \]. Under the approximation of high finesse, this map can be reduced to a single mean-field equation following the approach described in \[ 2,6 \]. We first integrate, through the Euler method, each propagation equation over the respective length of the fibre. We find

\[
A_m(L) - A_m(0) \approx \left( \frac{\alpha(1)}{2} + i\frac{\beta(1)}{2} \frac{\partial^2}{\partial \tau^2} \right) L_1 A_m(0) + i\kappa B_m L_1 A_m^*(0)
\]

\[
- \left( \frac{\alpha(2)}{2} + i\frac{\beta(2)}{2} \frac{\partial^2}{\partial \tau^2} - i\gamma |A_m(0)|^2 \right) L_2 A_m(0)
\]

\[
+ \left( \frac{g_0}{2} - i\frac{\beta(3)}{2} \frac{\partial^2}{\partial \tau^2} \right) L_3 A_m(0), \quad (S6)
\]

where \( L_1 \) is the length of the PPF, \( L_2 \) is the length of the SMF and \( L_3 \) is the length of the EDF. \( L = L_1 + L_2 + L_3 \) is the total cavity length. Second, we write the boundary conditions (S1) as

\[
A_{m+1}(0) \approx \left( 1 - \frac{R}{2} - i\delta_0 \right) A_m(L), \quad (S7)
\]

*Electronic address: nicolas.englebert@ulb.be
where $\mathcal{R} = 1 - T$ and we introduced the detuning $\delta_0 = 2k\pi - \varphi_0$ where $k$ is an integer. By substituting Eq. (S6) in (S7), we find the mean-field equation known as the parametrically driven, damped nonlinear Schrödinger equation (PDNLSE) [21,22]

$$tk \frac{\partial A}{\partial T} = \left(-\frac{\Lambda_c}{2} - i \frac{\beta_2 L}{2} \frac{\partial^2}{\partial t^2} - i \delta_0\right) A + \kappa B_{in} L_1 A^* + i \gamma L_2 |A|^2 A \tag{S8}$$

where we introduced the slow-time $t = nt_R$ ($t_R$ is the roundtrip time), the effective loss $\Lambda_c = \alpha_{(1)}^2 L_1 + \alpha_{(2)}^2 L_2 + \mathcal{R} - g_0 L_3$ and the average accumulated dispersion $\beta_2 L = \beta_2^{(1)} L_1 + \beta_2^{(2)} L_2 + \beta_2^{(3)} L_3$. Introducing the dimensionless parameters, $T \rightarrow (\Lambda_c T)/(2t_R)$, $\tau \rightarrow \sqrt{\frac{1}{2^2 \pi^2 T}}$, $\Delta = 2\delta_0/\Lambda_c$, $u = \sqrt{\frac{2\xi L_3}{\Lambda_c}} \mu$ and $\mu = 2\kappa B_{in} L_1/\Lambda_c$. We find the normalized PDNLSE

$$\frac{\partial u}{\partial T} = \left(-1 + i(|u|^2 - \Delta) + i \frac{\partial^2}{\partial \tau^2}\right) u + \mu u^*, \tag{S9}$$

which is Eq. (2) of the manuscript.

II. LINEAR STABILITY ANALYSIS

To compute the linear stability of the homogeneous state solutions ($\partial u_h/\partial T = 0$), against generic perturbations $\xi(\tau, T)$, we first linearize the system around $u_h$ by introducing the ansatz $u(\tau, T) = u_h + \xi(\tau, T) + c.c.$ ($|\xi| < 1$) in Eq. (S9). Keeping all the terms at first order in $\epsilon$, we obtain the linear equation

$$\frac{\partial T}{\partial T} \begin{bmatrix} \xi \\ \xi^* \end{bmatrix} = \begin{bmatrix} A & B \\ B^* & A^* \end{bmatrix} \begin{bmatrix} \xi \\ \xi^* \end{bmatrix}, \tag{S10}$$

where

$$A \equiv -(1 + i\Delta) + i\delta_0^2 + 2i|u_h|^2, \quad B \equiv iu_h^2 + \mu. \tag{S11}$$

To solve this equation, we consider modulated perturbation modes of the form $\xi(\tau, T) = a_\Omega e^{\sigma \tau + i\Omega \tau + c.c.}$, with $\Omega$ and $\sigma$ being the frequency and growth rate of the perturbation, respectively. This ansatz then leads to the linear system

$$\begin{bmatrix} A_\Omega - \sigma & B \\ B^* & A_\Omega^* - \sigma \end{bmatrix} \begin{bmatrix} a_\Omega \\ \bar{a}_\Omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{S12}$$

with $A_\Omega \equiv -(1 + i\Delta) - i\Omega^2 + 2(|u_h|^2)^2$, which has a non-trivial solution if

$$\sigma^2 + 2\sigma + f(\Omega) = 0, \quad f(\Omega) = |A_\Omega|^2 - |B|^2, \tag{S13}$$

is satisfied. This condition yields the dispersion relation

$$\sigma(\Omega) = -1 + \sqrt{1 - f(\Omega)}, \tag{S14}$$

which relates the growth of the perturbation $\xi$ with its frequency. If $\text{Re}[\sigma]$ is negative, the modulated perturbation $\xi$ decays, and $u_h$ is stable. However, when the contrary holds, the perturbation grows exponentially and $u_h$ is unstable. The transition occurs at the critical frequency $\Omega_c$ which satisfies simultaneously the conditions (i) $\sigma(\Omega)|_{\Omega_c} = 0$ and (ii) $\sigma'(\Omega)|_{\Omega_c} = 0$, with $'$ denoting derivation with respect to $\Omega$.

A. Stability of the trivial state

For the trivial state $u_h = 0$, $f(\Omega) = 1 - \mu^2 + (\Delta + \Omega^2)^2$, and the conditions (i)-(ii) lead respectively to

$$\mu^2 = 1 + (\Delta + \Omega^2)^2, \quad (\Delta + \Omega^2)\Omega_c = 0, \tag{S15}$$

which define the onset of the instability and the critical frequency $\Omega_c$ of the growing perturbation at the instability. Depending on $\Omega_c$, two different instabilities take place. For $\Omega_c = 0$, a pitchfork bifurcation, occurs at $\mu = \mu_p \equiv 1 + \Delta^2$, such that for $\mu < \mu_p$, $u_h = 0$ is stable against homogeneous perturbations and unstable otherwise. This instability corresponds to the degenerate OPO threshold.

In contrast, when $\Omega_c = \Omega_T \equiv \sqrt{-\Delta}$ and $\Delta < 0$, a Turing or modulation instability crops up at $\mu = \mu_T \equiv 1$, where $u_h = 0$ becomes unstable against modulated perturbations of frequency $\Omega_T$. This instability corresponds to the non-degenerate OPO threshold. Figure S11 shows the stability of the trivial state using solid (dashed) lines for the stable (unstable) states, for $\Delta < 0$ (see Fig. S11a) and $\Delta > 0$ (see Fig. S11b).

B. Stability of the non-trivial state

The non-trivial homogeneous state can be written as $u_h = \sqrt{X} e^{i\phi}$, where $X$ satisfies $\mu^2 = 1 + (X - \Delta)^2$, and $\cos(2\phi) = \mu^{-1}$. For $\Delta < 0$ only one nontrivial state exists corresponding to $X^+ \equiv \Delta + \sqrt{\mu^2 - 1}$, that we label $u_h^+$ [see Fig. S11a]. However, for $\Delta > 0$ two nontrivial states $u_h^\pm$ appear corresponding to $X^\pm \equiv \Delta \pm \sqrt{\mu^2 - 1}$. These solution branches are plotted in Fig. S11.

In this case, $f = (\Delta - 2X + \Omega^2)^2 - \Delta^2$, and the dispersion relation reads

$$\sigma = -1 + \sqrt{\Delta^2 + 1 - (\Delta - 2X + \Omega^2)^2}. \tag{S16}$$

At this stage it is convenient to analyse the stability by means of the marginal instability curve (MIC) which defines the region of instability of $u_h$ against perturbation modes of frequency $\Omega$. The MIC is obtained from the condition $\sigma(\Omega, X, \Delta) = 0$, which leads to the branches

$$X_T^\pm = \frac{1}{2}(\Omega^2 + \Delta \pm \Delta). \tag{S17}$$
The instability regions associated to the non-uniform states shown in Fig. S1b and Fig. S1d are plotted in Fig. S1c and Fig. S1e respectively. For a fixed $\Delta$, the non-trivial states $u_h$ within the region $(X^+_f, X^-_f)$ are unstable, and stable otherwise.

Figure S1 shows the growth rate $\sigma$ of the modulated perturbation for different values of $X$ in the supercritical regime ($\Delta < 0$). They correspond to the same color points shown in Fig. S1. Increasing $X$, the band of unstable modes shifts to higher values of $\Omega$, as does the maximum of $\sigma$ remains constant and equal to $\sigma_{\text{max}} = -1 + \sqrt{\Delta^2 + 1}$. The boundaries of the band of unstable frequencies are determined by $X^+_f$ [see Fig. S1c]. Thus, the growth rate curves shown in Fig. S1 correspond to the slices of constant $X$ in Fig. S1a. The most unstable frequency is marked with a dash-dotted line in Fig. S1b.

Similarly, Fig. S1f shows the instability region in the subcritical regime corresponding to Fig. S1a, whereas Fig. S1b depicts the growth rates corresponding to three values of constant $X$ marked in Figs. S1d and Figs. S1f. For low values of $X$, $u_h$ is unstable with respect to homogeneous perturbation ($\Omega = 0$), see red curve in Fig. S1f. However, increasing $X$, $u_h$ eventually becomes unstable to modulated perturbations of maximal frequency $\Omega_u$ (green curve in Fig. S1f). Above SN$_h$, occurring at $(\mu, X) = (1, \Delta)$, $u_h$ is always modulational unstable.

### III. PHASE MODULATION

Cavity solitons (CS) are known to be attracted to phase maxima of the driving beam [11]. In our experiment, we use a similar technique to lock the parametrically driven Kerr cavity solitons (PDCSs) on a 4.6 GHz grid. We imprint a periodic phase modulation $\psi(\tau)$ onto the driving laser prior to its frequency doubling. The resulting beam is phase modulated at twice the initial modulation frequency (see Fig. 3). Writing $\mu(\tau) = \mu_0 e^{i2\psi(\tau)}$, the mean-field equation (S9) becomes

$$\frac{\partial u}{\partial T} = \left(-1 + i(|u|^2 - \Delta) + i \frac{\partial^2}{\partial \tau^2}\right) u + \mu_0 e^{i2\psi(\tau)} u^*.$$

By substituting $u = \tilde{u} e^{i\psi(\tau)}$ into equation (S18), we find

$$\frac{\partial \tilde{u}}{\partial T} = \left(-(1 + \psi'' + i(|\tilde{u}|^2 - (\Delta + \psi'^2)) + i \frac{\partial^2}{\partial \tau^2} - 2\psi' \frac{\partial}{\partial \tau}\right) \tilde{u} + \mu_0 \tilde{u}^*,$$

where $\psi' = \frac{d\psi}{d\tau}$ and $\psi'' = \frac{d^2\psi}{d\tau^2}$. The impact of $\psi''$ (resp. $\psi'$) over the total losses (resp. detuning) is small [11]. The term $2\psi' \frac{\partial}{\partial \tau}$ provides a local $\tau$-dependent variation of the group velocity. When $\psi' > 0$ (resp. $\psi' < 0$), the soliton suffers a delay (resp. advancement) with respect
to the carrier frequency reference frame. This implies that individual PDCSs will lock to a phase modulation maximum.

[1] Mosca, S. et al. Modulation Instability Induced Frequency Comb Generation in a Continuously Pumped Optical Parametric Oscillator. Physical Review Letters 121, 093903 (2018). URL https://link.aps.org/doi/10.1103/PhysRevLett.121.093903 Publisher: American Physical Society.

[2] Leo, F. et al. Walk-Off-Induced Modulation Instability, Temporal Pattern Formation, and Frequency Comb Generation in Cavity-Enhanced Second-Harmonic Generation. Physical Review Letters 116, 033901 (2016). URL https://link.aps.org/doi/10.1103/PhysRevLett.116.033901 Publisher: American Physical Society.

[3] Agrawal, G. Nonlinear Fiber Optics (Academic Press, 2013).

[4] Englebert, N., Mas Arabí, C., Parra-Rivas, P., Gorza, S.-P. & Leo, F. Temporal solitons in a coherently driven active resonator. Nature Photonics 1–6 (2021). URL https://www.nature.com/articles/s41566-021-00807-w Publisher: Nature Publishing Group.

[5] Ikeda, K. Multiple-valued stationary state and its instability of the transmitted light by a ring cavity system. Optics Communications 30, 257–261 (1979). URL http://www.sciencedirect.com/science/article/pii/0030401879900907

[6] Haelterman, M., Trillo, S. & Wabnitz, S. Dissipative modulation instability in a nonlinear dispersive ring cavity. Optics Communications 91, 401–407 (1992). URL http://www.sciencedirect.com/science/article/pii/0030401892903672

[7] Miles, J. W. Parametrically excited solitary waves. Journal of Fluid Mechanics 148, 451–460 (1984). URL https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/parametrically-excited-solitary-waves/F6E726F7DD524A2F8FB89F3EC0CBEAF4 Publisher: Cambridge University Press.

[8] Denardo, B. et al. Observations of localized structures in nonlinear lattices: Domain walls and kinks. Physical Review Letters 68, 1730–1733 (1992). URL https://link.aps.org/doi/10.1103/PhysRevLett.68.1730 Publisher: American Physical Society.

[9] Longhi, S. Ultrashort-pulse generation in degenerate optical parametric oscillators. Optics Letters 20, 695–697 (1995). URL https://www.osapublishing.org/ol/abstract.cfm?uri=ol-20-7-695 Publisher: Optical Society of America.

[10] Longhi, S. & Geraci, A. Modulational instability oscillation and solitary waves in a nonlinear dispersive cavity with parametric gain. Applied Physics Letters 67, 3060–3062 (1995). URL https://aip.scitation.org/doi/10.1063/1.114864 Publisher: American Institute of Physics.

[11] Jang, J. K., Erkintalo, M., Coen, S. & Murdoch, S. G. Temporal tweezing of light through the trapping and manipulation of temporal cavity solitons. Nature Communications 6, 7370 (2015). URL https://www.nature.com/articles/ncomms8370 Number: 1 Publisher: Nature Publishing Group.
Fig. S2. Spontaneous parametrically driven cavity solitons generation. a, Cavity resonance measured with a 200 kHz photodiode. The scan leads to the generation of a multiple soliton-step, i.e. the spontaneous generation of multiple PDCSs. b, Similar scan performed with a 12 GHz detection system. The oscillation is initiated on the edges of the driving pulse, after which the signal broadens to reach the same duration as the pump pulse and eventually collapses on the soliton state. c, Corresponding pump pulse profile. It remains unchanged throughout the scan which validates the constant pump approximation used in our model. d, Pump spectrum below and above (e) the oscillation threshold. The absence of spectral broadening further confirms that cascaded nonlinearities do not play a significant role in our system.