Estimation of minimum miscibility pressure during CO₂ flooding in hydrocarbon reservoirs using an optimized neural network

Yapeng Tian¹,², Binshan Ju¹,³, Yong Yang⁴, Hongya Wang⁵, Yintao Dong¹, Nannan Liu¹, Shuai Ma¹ and Jinbiao Yu⁴

Abstract
CO₂ flooding recovery strongly depends on the minimum miscibility pressure (MMP). Conventional tests to determine gas–oil MMP such as rising bubble apparatus and slim tube displacement are either costly or time consuming. In order to propose a quick and accurate model to determine MMP, a back-propagation neural network is presented for MMP prediction during pure and impure CO₂ injections. Five new variables were screened as input parameters to the network. Next, the network was optimized using five evolutionary algorithms, and this work highlights that three of these evolutionary algorithms (e.g. Mind Evolutionary, Artificial Bee Colony, and Dragonfly) are firstly used to predict MMP. Then, data from the literature were input to the optimized network to train it. Statistical evaluation and graphical analyses were used to evaluate the performance of the proposed models and for comparison with published MMP correlates to obtain the optimal model for predicting MMP. The back-propagation model optimized using the dragonfly algorithm exhibited the highest accuracy among all those considered and MMP correlates; its coefficient of determination, average absolute percent relative error, root mean square error, and standard deviation were 0.965, 5.79%, 206.1, and 0.08, respectively.

¹China University of Geosciences (Beijing), School of Energy Resources, Beijing, China
²Key Laboratory of Marine Reservoir Evolution and Hydrocarbon Enrichment Mechanism, Ministry of Education, Beijing, China
³Key Laboratory of Geological Evaluation and Development Engineering of Unconventional Natural Gas Energy, Beijing, China
⁴Research Institute of Petroleum Exploration and Development of Shengli Oilfield, Sinopec Corp, Dongying, China
⁵Petrochina Coalbed Methane Company Ltd, Beijing, China

Corresponding author:
Binshan Ju, China University of Geosciences (Beijing), School of Energy Resources, Haidian District, Beijing 100083, China. Email: jubs2936@163.com

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In addition, reservoir temperature was determined as the strongest MMP predictor (Pearson correlation = 0.63) based on sensitivity analysis.

**Keywords**
Enhanced oil recovery, CO₂ injection, minimum miscibility pressure, back-propagation neural network, evolutionary algorithm

**Introduction**

The research of enhanced oil recovery (EOR) is one of the most important themes of development of reservoirs. Chemical flooding, gas flooding, thermal recovery, and microbial EOR have been generated as critical technologies for EOR (Hemmati-Sarapardeh et al., 2016). During recent years, the development of EOR by gas flooding is rapid, among which the development of CO₂ injection flooding is the fastest (Grigg and Schechter, 1997). During the process of CO₂ flooding, a large amount of CO₂ can be dissolved in crude oil, which results in a decrease in viscosity of crude oil and reduces the flow resistance of crude oil in porous media. Therefore, CO₂ flooding is widely applied in the tight reservoirs as an efficient method to improve oil recovery. Compared with conventional EOR technology, this technology can not only greatly improve oil recovery, but also store a large amount of CO₂ in the reservoir to reduce greenhouse effect, which is a win–win project (Ampomah et al., 2016). The CO₂ flooding can be divided into miscible flooding and immiscible flooding by flooding mechanism. The major difference between miscible and immiscible flooding is whether the formation pressure reaches the minimum miscible pressure (MMP). When the injection pressure reaches the MMP, the interface between crude oil and CO₂ disappears and a miscible zone is formed, which greatly reduces the capillary force and increases the recovery efficiency (Yuan and Johns, 2002).

MMP is an important parameter during the process of CO₂-EOR, because it determines the development mechanism of reservoirs. In one-dimensional displacement of two-phase flow systems such as gas and oil with a negligible dispersion, a piston-like displacement occurs when the pressure approaches MMP. In this case, the oil recovery will be very high (above 90%) after one pore volume gas injection, which is beneficial for improving oil production (Mohammad et al., 2017). For economic reasons, the choice of gas in the flooding operation for a given oil reservoir is based on the reservoir pressure and MMP. Consequently, an accurate prediction of MMP during CO₂ flooding process is essential for decreasing operational cost, increasing oil recovery, and screening reservoirs for CO₂ injection (Belhaj et al., 2013; Valluri et al., 2017).

The aim of this paper is to build a reliable and accurate model to quickly estimate the MMP during pure and impure CO₂ injections. To achieve this objective, back-propagation (BP) neural network is employed to develop a model of predicting MMP for pure and impure CO₂ streams. To reduce the number of oil components required and improve accuracy of predicting MMP, instead of directly using pseudo-components of oil as the input variables of BP network, a screening operation of input variables based on data sets from literature is implemented to select new input variables of BP network model to predict MMP. Moreover, BP network is optimized by five evolutionary algorithms (EAs) including...
genetic algorithm (GA), mind evolutionary algorithm (MEA), particle swarm optimization (PSO), artificial bee colony algorithm (ABC), and dragonfly algorithm (DA), and three of these EAs (e.g. MEA, ABC, and DA) firstly served as EAs of BP network model to obtain the optimal model for predicting MMP. Then 152 data sets from literature and the corresponding input variables are fed into the optimized BP model (i.e. EAs-BP) to obtain the optimal prediction model. In addition, statistical evaluation and graphical analyses are employed to assess the performance of each optimized BP model and to compare them with correlations in the literature. Finally, a sensitivity analysis is conducted to assess the influence of new input parameters proposed in this paper on CO₂–crude oil MMP.

**MMP determination**

*Laboratory methods*

For accurate determination of CO₂ MMP, several experiments have been implemented by previous authors. Rathmell et al. (1971) performed experiment of core flooding by CO₂ injection at different pressures and found that intermediates of crude oils can reduce the value of MMP. Zhou and Orr (1998) demonstrated, by performing rising-bubble experiments, that the bubble behavior is mainly influenced by changes in the interfacial tension (IFT) and used this result to determine the MMP values for gas drives. Rao and Lee (2002) focused on the investigation of IFT in estimation of MMP using a drop shape analysis technique. The results of investigation showed that disappearance of IFT between oil and gas can result in miscible flooding, which means that the pressure corresponding to zero IFT is MMP.

*Empirical methods*

As the aforementioned experiments were time-consuming and expensive, several researchers have attempted to build fast and robust correlations to determinate CO₂–oil MMP. Holm and Josendal (1974) established an MMP correlation based on the reservoir temperature as well as molecular weight of C₅⁺ in the reservoir oil. Yellig and Metcalfe (1980) presented an MMP correlation considering the effect of temperature and bubble-point pressure, and proposed that the sand-packed coil method can be used as an experimental method for determining the MMP. Later, Johnson and Pollin (1981) developed a correlation based on reservoir temperature and the composition of oil and injection gas to determine impure CO₂–oil MMP. Orr and Jensen (1984) showed a correlation to calculate MMP by means of analyzing pressure-composition phase diagrams, which can be used as a rough estimate of the MMP for low-temperature reservoirs. Glaso (1985) presented an MMP correlation considering the effect of intermediate fraction of crude oil, and variables affecting the CO₂ and N₂ MMP are reservoir temperature and C₇⁺ molecular weight of the oil. Alston et al. (1985) proposed an exponential equation including the volatile/intermediate ratio of oil to predict MMP, and correlated the equation with the weight average critical temperature of injection gas for impure CO₂–oil MMP. Unlike Alston’s correlation, Sebastian et al. (1985) presented a mole fraction average critical temperature as correction factor for predicting MMP of impure CO₂ stream. Wang and Orr (2000) proposed an analytical approach based on one-dimensional flow of oil and gas to estimate the MMP using the tie line intersection technique. Shokir (2007) used alternating conditional expectation to predict the
MMP based on some independent variables including reservoir temperature, the compositions of oil and injection gas. Ju et al. (2012) proposed an eight-parameter MMP model for pure and impure CO₂ injection through the statistics and regression method.

Artificial neural network methods

With the emergence of intelligent algorithms, many researchers have utilized them to solve the problem of prediction for nonlinear relationships between input variables and output variables. Huang et al. (2003) firstly used artificial neural network (ANN) approach to predict CO₂ MMP and impure CO₂ MMP factor throughout considering the concentration of contaminants in CO₂. Tatar et al. (2013) presented an intelligent model based on the Radial Basis Function Networks to estimate CO₂–oil MMP, and the outlier diagnosis is performed to determine the data sets to remove unrealistic data. Shokrollahi et al. (2013) proposed least-squares support vector machine to determine the value of MMP during the CO₂ injection. Chen et al. (2014) presented an approach to predict MMP for CO₂–oil system by BP ANN only optimized by GA and chose 10 parameters as input variables. Zhong and Carr (2016) developed the CO₂–MMP predicting model by support vector regression (SVR) with mixed kernel function, and the parameters of this model were optimized by PSO. Nevertheless, Bian et al. (2016) proposed a method for predicting CO₂–crude oil MMP by BP ANN model improved by GA, which showed that the proposed model for predicting the MMP is in excellent agreement with experimental data for pure and impure CO₂–oil MMP. Mohammad et al. (2017) developed the CO₂ MMP model using gene expression programming, which decreases the computational burden of mathematical methods for MMP determination.

The characteristics of the basis function of radial basis function (RBF) are mainly determined by the center of the basis function. However, basis function is constructed by arbitrarily selecting the center from the data points, which will result in uncertainty for predicting MMP. The classical support vector machine (SVM) gives only two kinds of classification algorithms, which cannot solve multidimensional problems very well for predicting MMP. Nevertheless, BP ANN model has powerful and effective ability to reflect the system’s complexity, and it has proven to be an attractive approach for system analysis and prediction. In addition, the BP network in this paper is optimized using five EAs to overcome the shortcomings of ANN that has local minimum and over-fitting problems.

Methodology

Back propagation neural network

ANN is a complex network system consisting of a large number of interconnected simple processing units (e.g. neurons) (Shafabakhsh et al., 2015). Each neuron generally utilizes a nonlinear function serving as an activation function. Based on error BP algorithm, BP neural network, a kind of feed forward network, was developed (Basheer and Hajmeer, 2000). The main characteristic of this network structure is the forward transmission of signals between neurons. Nevertheless, the error values of back propagation neural network (BP-NN) output compared with the expected output are back propagated to update the initial weights and thresholds. The training process of BP-NN continuously iterates until the sum of errors of BP-NN output obtains minimum. In this process, the weights and
thresholds of the neural network are adjusted by error BP; the adjustment is typically performed after each iteration.

A typical BP-NN consists of three layers: input layer, output layer, and hidden layer. The structure is shown in Figure 1. The standard BP-NN mainly includes two stages: forward propagation and backward propagation.

**Evolutionary algorithms**

*Genetic algorithm.* GA, based on theory of genetic and natural selection, is capable and efficient for tackling combinatorial optimization problems (Whitley, 1994). GA adopts probabilistic method to find the optimal solution, which can automatically obtain the optimized search space and adjust the search direction adaptively. Each individual is encoded with a real number string, which consists of four parts that are connection weight from input layer to hidden layer, hidden layer bias, connection weight from hidden layer to output layer and output layer bias. The fitness value, based on the prediction errors of GA-BPNN, of an individual in GA is calculated as shown in equation (1)

$$ F = \sum_{i=1}^{n} \left( e_{i}^{2} \right)^{-1/2} $$

(1)

where $n$ represents node number of output layer and $e_{i}$ denotes the error of the $i$th node.

*Mind evolutionary algorithm.* Aiming at the problems and shortcomings of evolutionary computation, such as premature convergence and slow convergence speed, MEA was originally put forward in the work of Sun and Sun (1998). The main concept of MEA strategy is that comparing the score of various individuals in superior group and temporary group to look for the optimal individual. Individuals of the initial population are randomly dispersed in the space, and the scores of each individual are calculated as follow
\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (e_i^2) \]  
\[ s = \frac{1}{\text{MSE}} \]

where \( \text{MSE} \) denotes the mean square error and \( s \) represents the score of individual.

**Particle swarm optimization.** PSO is a random searching algorithm based on swarm intelligence to obtain globally optimal solution, which simulates the migration and clustering behavior of birds in the process of foraging (Eberhart and Kennedy, 1995). In the process of iteration for PSO, velocity \( (v_i^t) \) and position \( (x_i^t) \) of the \( i \)th particle are calculated as follow

\[ v_{id}^{t+1} = v_{id}^t + c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (np_{id}^t - x_{id}^t), d = 1, 2, \cdots, D \]
\[ x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1}, d = 1, 2, \cdots, D \]

where \( t \) represents the iterations, \( r_1 \) and \( r_2 \) are random numbers evenly distributed in \([0,1]\), \( c_1 \) and \( c_2 \) denote the acceleration constant.

**Artificial bee colony algorithm.** Karaboga and Basturk (2008) firstly proposed ABC approach. The basic idea of ABC is originated from the bee colony to cooperate with each other in honey harvesting through individual division of labor and information exchange. ABC, which is a high-performance algorithm based on swarm intelligence, uses the real number coding to optimize the solution of real numbers in continuous space at first. In the searching process of ABC algorithm, the functions of three kinds of bee are different. Employed bees are used to find good solutions. Onlooker bees are employed to improve convergence speed, and scout bees are used to enhance the ability for preventing ABC algorithms from local optimum (Akay and Karaboga, 2012; Karaboga and Basturk, 2007).

The initial position of food source \( i \) is randomly generated in search space. The formula is as follows

\[ x_{id} = x_{d}^{\text{min}} + r(x_{d}^{\text{max}} - x_{d}^{\text{min}}) \]

where \( r \) is a random number in \([0, 1]\), \( x_{d}^{\text{max}} \) and \( x_{d}^{\text{min}} \) are the upper boundary and lower boundary of the \( d \)th dimension, respectively.

At the beginning of the search, the employed bees use the previous information of food sources (solution) to find new food sources around the food source \( i \); the formula is as follows

\[ v_{id} = x_{id} + \varphi(x_{id} - x_{jd}) \]

where \( d \) is a random number in \([1, 2, \ldots, D]\), which indicates that the employed bee randomly choose one dimension to search, \( \varphi \) is a random number in \([-1, 1]\), which
determinates magnitude of the perturbation. \( j \) is a random number in \( \{1, 2, \ldots, BN\} \) and different from \( i \).

**Dragonfly algorithm.** DA, which is a new optimization algorithm based on swarm intelligence proposed by Mirjalili (2016), is mainly inspired by the static and dynamic swarming behaviors of dragonflies in nature. There are five behaviors determining the position of individuals, which make individuals go forward to food sources and depart from enemies. These behaviors of dragonfly can be mathematically modeled as follows.

The separation is calculated as follows

\[
S_i = -\sum_{j=1}^{N} X - X_j
\]  

where \( X \) is the position of the current individual, \( X_j \) is the position of neighboring individual \( j \), and \( N \) is the number of neighboring individuals around current individual.

Alignment is determined by the following computation

\[
A_i = \frac{\sum_{j=1}^{N} V_j}{N}
\]  

where \( V_j \) denotes the velocity of neighboring individual \( j \).

The cohesion is calculated as shown in equation (10):

\[
C_i = \frac{\sum_{j=1}^{N} X_j}{N} - X
\]  

Attraction towards a food source is determined using position of the food source \((X^+)\) as described in equation (11)

\[
F_i = X^+ - X
\]  

Distraction outwards an enemy is defined using position of the enemy \((X^-)\) as follows

\[
E_i = X^- - X
\]  

Step \((\Delta X)\) is the direction vector of the movement of dragonfly in the search space, and position \((X)\) is the position vector of the dragonfly in the search space. The step and position of dragonflies are updated as follows

\[
\Delta X_{t+1} = (sS_t + aA_t + cC_t + fF_t + eE_t) + w\Delta X_t
\]  

\[
X_{t+1} = X_t + \Delta X_{t+1}
\]
where $s$ denotes the separation weight, $a$ indicates the alignment weight, $c$ is the cohesion weight, $f$ denotes the food factor, $e$ is the enemy factor, $w$ indicates the inertia weight, $S_i$ is the separation of the dragonfly $i$, $A_i$ shows the alignment of dragonfly, $C_i$ is the cohesion of the dragonfly $i$, $F_i$ is the food source of the dragonfly $i$, $E_i$ represents the position of enemy of the dragonfly $i$, and $t$ shows the iteration counter.

**Data analysis**

**Analysis and screening of influencing factors of MMP**

The CO$_2$ MMP cannot be predicted by a single parameter due to the diversity of crude oil components. The miscible mechanism and interaction between CO$_2$ and various components are different. Therefore, crude oil components are divided into three parts according to carbon number, and each part is considered as pseudo-component to research its effect on MMP in some previous publications (Alston et al., 1985; Shokir, 2007; Shokrollahi et al., 2013). The three pseudo-components generally include volatile component (CH$_4$ and N$_2$), intermediate component (C$_2$–C$_6$), and heavy component (C$_7$+). In the process of pure CO$_2$ injection, the mole percentages of pseudo-components serve as the influence factors to predict MMP. Besides, molecular weight of C$_5$+ fraction is also employed to predict the MMP, because the MMP increases with molecular weight of C$_5$+ fraction (Shokir, 2007). The mole percentages of multiple components need be determined in the above-mentioned process. In order to reduce the required number of oil components and improve prediction accuracy, screening operation is implemented to determine the high relevance parameters for prediction of MMP. Pearson correlation coefficient has been validated to be an effective coefficient for determining relevance (Chen et al., 2014). Therefore, in this paper, it is employed to evaluate the correlation between input parameters and output parameters. The formula of Pearson correlation coefficient ($r$) is expressed as follows

$$r(Inp_j, MMP) = \frac{\sum_{i=1}^{n} (Inp_{ji} - \bar{Inp}_j)(MMP_i - \bar{MMP})}{\sqrt{\sum_{i=1}^{n} (Inp_{ji} - \bar{Inp}_j)^2 \sum_{i=1}^{n} (MMP_i - \bar{MMP})^2}}$$

(15)

where $n$ is the number of data set, $Inp_{ji}$ indicates the $i$th input value of the $j$th input parameter, $MMP_i$ represents the $i$th MMP, $\bar{Inp}_j$ and $\bar{MMP}$ indicate the mean value for the $j$th input and the mean value of MMP, respectively.

To screen the high relevance parameters for prediction of MMP, 12 input parameters, which include the mole percentages of C$_1$–C$_6$, C$_7$+, CO$_2$, H$_2$S, and N$_2$, molecular weight of C$_5$+ fraction and temperature, are selected to implement screening operation. The correlation between input parameters and output parameter (MMP) of 52 data points obtained by pure CO$_2$ injection can be expressed by Pearson correlation coefficient in Table 1.

It is found that temperature has the highest positive correlation (0.822) with MMP. However, the mole percentage of C$_6$ has the highest negative correlation (–0.522) with MMP. When the absolute value of Pearson coefficient between both parameters is less than 0.4, it indicates that both parameters are generally weakly related. Therefore, the absolute value of Pearson coefficient between input parameters selected and MMP should
**Table 1.** Pearson correlation coefficient between each component and output parameter (MMP).

|       | T(F) | N2   | CO₂ | H₂S | C₁   | C₂   | C₃   | C₄   | C₅   | C₆   | C₇⁺  | C₅⁺MW | MMP(ψ) |
|-------|------|------|-----|-----|------|------|------|------|------|------|------|-------|--------|
| T(F)  | 1.000 |      |     |     |      |      |      |      |      |      |      |       |        |
| N₂    | 0.179 | 1.000|     |     |      |      |      |      |      |      |      |       |        |
| CO₂   | 0.267 | −0.209| 1.000|     |      |      |      |      |      |      |      |       |        |
| H₂S   | 0.255 | −0.084| 0.112| 1.000|      |      |      |      |      |      |      |       |        |
| C₁    | 0.416 | 0.045| 0.098| 0.215| 1.000|      |      |      |      |      |      |       |        |
| C₂    | 0.062 | −0.156| 0.472| 0.150| 0.310| 1.000|      |      |      |      |      |       |        |
| C₃    | −0.183| −0.122| 0.101| 0.043| 0.048| 0.724| 1.000|      |      |      |      |       |        |
| C₄    | −0.102| −0.013| 0.007| 0.084| −0.025| 0.405| 0.749| 1.000|      |      |      |      |       |        |
| C₅    | 0.063 | 0.114| −0.026| −0.023| −0.236| 0.048| 0.354| 0.601| 1.000|      |      |       |        |
| C₆    | −0.377| −0.118| −0.144| −0.016| −0.478| −0.160| 0.152| 0.258| 0.437| 1.000|      |       |        |
| C₇⁺   | −0.333| −0.002| −0.298| −0.288| −0.832| −0.691| −0.514| −0.394| −0.088| 0.246| 1.000|       |        |
| C₅⁺MW | 0.237 | 0.066| 0.082| −0.305| −0.030| −0.256| −0.352| −0.425| −0.304| −0.318| 0.290| 1.000|        |
| MMP(ψ)| 0.822 | 0.174| 0.244| 0.114| 0.523| −0.018| −0.313| −0.399| −0.202| −0.522| −0.284| 0.450| 1.000  |

MMP: minimum miscibility pressure.
be more than 0.4. There are four (temperature, the molecular weight of C5+ fraction and the mole percentages of C1 and C6) among all input parameters that attain this condition.

In order to compare the accuracy of two predictive models between the new four input parameters and the pseudo-components (Table 1), the standard BP-NN is used to build predictive model of MMP. The predictive results of two models are compared by some parameters, such as determination coefficient ($R^2$), average absolute percent relative error (AAPRE %), root mean square error (RMSE) and standard deviation (SD). They are calculated as follows

$$AAPRE = 100 \frac{1}{N} \sum_{i=1}^{N} \left| \frac{MMP^\text{exp}_i - MMP^\text{pred}_i}{MMP^\text{exp}_i} \right|$$

$$RSME = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{MMP^\text{exp}_i - MMP^\text{pred}_i}{MMP^\text{exp}_i} \right)^2}$$

$$SD = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left( \frac{MMP^\text{exp}_i - MMP^\text{pred}_i}{MMP^\text{exp}_i} \right)^2}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (MMP^\text{exp}_i - MMP^\text{pred}_i)^2}{\sum_{i=1}^{N} (MMP^\text{exp}_\text{ave} - MMP^\text{pred}_i)^2}$$

where $N$ is the number of data points, $MMP^\text{exp}_{\text{ave}}$ is the average value of all experimental data points, $MMP^\text{exp}_i$ and $MMP^\text{pred}_i$ are the $i$th experimental MMP value and predictive MMP value, respectively.

The results of statistical error analysis are reported in Table 2. This table reports the results of AAPRE, RMSE and $R^2$ show that predictive results of new input parameters model are better than that of pseudo-components model. For the impure CO2 injection, pseudo critical temperature of impure CO2 is typically served as a parameter in the process of estimation for MMP (Kamari et al., 2015; Shokrollahi et al., 2013). Therefore, in this study, the predictive model for MMP of pure and impure CO2–oil system is developed by six parameters: reservoir temperature, the molecular weight of C5+ fraction, the mole percentages of C1 and C6, pseudo critical temperature of injection gas and MMP.

**Data sources and scope**

In the ANN model, all data are divided into two parts for training and testing, respectively. The precision of model is strongly dependent on adequate data. In this study, 152 experimental values, including 52 pure CO2–oil MMPs and 100 impure CO2–oil MMPs, were collected from the open literatures (Adekunle and Hoffman, 2016; Bon et al., 2006; Cardenas et al., 1984; Dicharry et al., 1973; Dong et al., 2001; Eakin and Mitch, 1988; Graue and Zana, 1981; Holm and Josendal, 1974; Kanatbayev et al., 2015; Lai et al., 2017; Lashkarbolooki et al., 2017; Li and Luo, 2017; Li et al., 2012; Li et al., 2018; Metcalfe, 1982; Moudi et al., 2009; Harmon and Grigg, 1988; Sebastian et al., 1985; Spence et al., 1980;
Table 2. Statistical error analysis of different input parameters models for test, train, and total data.

|                        | Train set | Test set | Total  |
|------------------------|-----------|----------|--------|
|                        | $R^2$     | AAPRE    | RMSE   | SD     | $R^2$ | AAPRE | RMSE   | SD     | $R^2$ | AAPRE | RMSE   | SD     |
| New input parameters   | 0.958     | 7.459    | 259.580| 0.089  | 0.934 | 6.703 | 190.594| 0.117  | 0.938 | 7.347 | 247.809| 0.111  |
| model (new model)      |           |          |        |        |       |       |        |        |       |       |        |        |
| Pseudo-components      | 0.934     | 7.782    | 259.354| 0.102  | 0.933 | 6.873 | 245.250| 0.087  | 0.934 | 7.575 | 256.702| 0.099  |
| model (old model)      |           |          |        |        |       |       |        |        |       |       |        |        |

AAPRE: average absolute percent relative error; RMSE: root mean square error; SD: standard deviation.
Thakur et al., 1984; Zuo et al., 1993) to build a robust model for estimating MMP. Eighty percent (122) of the all experimental values are used for training and the remaining 20% (30) are employed for test. A detailed analysis of the experimental values includes that the reservoir temperature ranges from 303 to 414 K, the molecular weight of C$_5^+$ fraction ranges from 126 to 301 g/mol, the C$_1$ and C$_6$ mole percentage of the crude oil range from 0.07% to 62.88% and 0.21 to 7.35%, respectively, the pseudo critical temperature (T$_{cm}$) of injection gas ranges from 215.3 to 328.5 K, and the experimental MMP values range from 1086 to 6757 psi.

### Results and discussions

**Prediction results of EAs-BP model**

In this study, BP-NN is employed to build a model for predicting the MMP of CO$_2$–crude oil system, and the model is optimized by five EAs: GA, MEA, PSO, ABC, and DA. The flowchart of BP-NN model optimized by EAs is shown in Figure 2.

In order to evaluate the performance of the proposed models, quantitative analysis is performed by means of the aforementioned parameters. At first, the performances of optimized models are evaluated by a parametric statistical analysis. The results of statistical error analysis for each optimized model are reported in Table 3. The evaluating parameters include $R^2$, AAPRE, RMSE, and SD for different data sets: train set, test set, and the all data. These results indicate some useful points. Firstly, there is no overfitting for each optimized model, because the difference of results between train set and test set is inconspicuous. Secondly, the results of test set for each optimized model prove the feasibility of EAs-BP on the predicting CO$_2$–crude oil MMP. Thirdly, comparing the results of all proposed models, the DA-BP model has the highest accuracy for predicting MMP than the other optimized models. The DA-BP model predicts the MMP as the output with total values of 0.965, 5.79%, 206.1, and 0.080 for $R^2$, AAPRE, RMSE, and SD, respectively. In addition, it can be found from Table 3 that the precision of all optimized BP models in this work can be arranged in the following order: DA-BP > PSO-BP > MEA-BP > GA-BP > ABC-BP.

In order to show the stability of the best model proposed in this paper (i.e. DA-BP), frequencies of relative error of predicted MMP for training set and testing set are calculated as shown in Figure 3(a) and (b), respectively. It is found from the figure that errors of predicted MMP for almost all data sets are close to zero.

The validity of the DA-BP model is investigated by the trend analysis of the effects of input variables on MMP, such as reservoir temperature and critical temperature of injection gas. Figure 4 indicates the change of MMP against reservoir temperature ($T_r$) and critical temperature of injection gas ($T_{cm}$) in subplots (a) and (b), respectively. The figure shows that the MMP increases with the reservoir temperature, while the MMP decreases with the increase of critical temperature of injection gas. The trends of results predicted by the DA-BP model are similar to the trends of experimental values shown in the same figure.

**Comparison of EAs-BP model with published correlations**

In order to further demonstrate the accuracy and performance of the EAs-BP model proposed in this paper, four previously published correlations (including Alston et al., 1985; Sebastian et al., 1985; Orr and Jensen, 1984; Yellig and Metcalfe, 1980) and two
previous MMP models (GA-SVR and RBF Network) are employed to calculate MMP for comparing with the result of the EAs-BP model. Since some correlations only can predict the MMP of pure CO₂-oil, therefore, the modified correlation (Sebastian et al., 1985) is used for pure CO₂ MMP model to correct prediction results and estimate the MMP value of impure CO₂ injection. The statistical error analysis for the different MMP models and correlations is shown in Table 4. Three parameters, including AAPRE, RMSE, and R², are employed to evaluate the performance of different MMP models and correlations. As it

Figure 2. Flowchart of optimized BP model for predicting MMP.
BP: back propagation; EAs: evolutionary algorithms.
can be seen, the EAs-BP models have the better performance than published correlations for MMP estimation during pure and impure CO\textsubscript{2} injection. Among EAs-BP models, the DA-BP model has the least values of AAPRE and RMSE (5.79% and 206.1) as well as the highest R\textsuperscript{2} (0.965). These values indicate that the DA-BP model is more accurate than the other models for MMP estimation.

For better understanding the difference between EAs-BP models in the paper and published correlations, the AAPRE, the most widely accepted evaluating parameter, is displayed in Figure 5 for intuitively comparing performance of different models and correlations. It is noted from this figure that the AAPREs of EAs-BP models is far less than that of previously published correlations and MMP predicting models, which indicates

Table 3. Statistical error analysis of MMP models based on different algorithms for test, train, and total data.

|                | DA-BP | PSO-BP | MEA-BP | GA-BP | ABC-BP |
|----------------|-------|--------|--------|-------|--------|
| **Train set**  |       |        |        |       |        |
| R\textsuperscript{2} | 0.972 | 0.967  | 0.966  | 0.957 | 0.956  |
| AAPRE (%)      | 5.70  | 6.11   | 6.76   | 7.11  | 7.32   |
| RMSE           | 193.1 | 209.8  | 213.2  | 238.5 | 243.1  |
| SD             | 0.079 | 0.087  | 0.093  | 0.099 | 0.099  |
| **Test set**   |       |        |        |       |        |
| R\textsuperscript{2} | 0.915 | 0.869  | 0.886  | 0.909 | 0.888  |
| AAPRE (%)      | 6.17  | 7.66   | 7.39   | 6.49  | 6.69   |
| RMSE           | 252.1 | 316.4  | 336.6  | 267.0 | 297.8  |
| SD             | 0.086 | 0.094  | 0.103  | 0.087 | 0.091  |
| **Total**      |       |        |        |       |        |
| R\textsuperscript{2} | 0.965 | 0.957  | 0.954  | 0.951 | 0.946  |
| AAPRE (%)      | 5.79  | 6.42   | 6.88   | 6.99  | 7.21   |
| RMSE           | 206.1 | 234.7  | 242.6  | 244.4 | 254.8  |
| SD             | 0.080 | 0.087  | 0.094  | 0.096 | 0.097  |

AAPRE: average absolute percent relative error; ABC-BP: artificial bee colony algorithm-back propagation; DA-BP: dragonfly algorithm-back propagation; GA-BP: genetic algorithm-back propagation; MEA-BP: mind evolutionary algorithm-back propagation; PSO-BP: particle swarm optimization-back propagation; RMSE: root mean square error; SD: standard deviation.

Figure 3. Error distribution for (a) training set and (b) testing set.
that EAs-BP models are more accurate for estimating MMP. Among published correlations, Lee (Sebastian et al., 1985) correlation achieves the best accuracy, while accuracy of Alston correlation is the lowest.

Figure 6 shows the predicted MMP of each of the EAs-BP models and the best correlation (i.e., Lee (Sebastian et al., 1985)) in the published literature to more directly compare the results of each model. The 45° line (i.e., $X = Y$) is utilized to evaluate the relevancy between experimental and predicted MMP as well as the accuracy of each model. The calculated values of Lee correlation in the scatter plot are loosely distributed around 45° line, which means that there is a poor correlation between the predicted and the experimental values. However, the predicted values of EAs-BP models especially DA-BP are extremely

![Figure 4. Performance of the DA-BP model: Effects of $T_r$ and $T_{cm}$ on MMP.](image)

**Table 4.** The comparison of four correlations and the models developed in this study.

| Models               | Modified correlation | AAPRE (%) | RMSE  | $R^2$ |
|----------------------|----------------------|-----------|-------|-------|
| Lee                  | Sebastian et al.     | 26.71     | 965.3 | 0.421 |
| Orr and Jensen       | Sebastian et al.     | 27.37     | 984.4 | 0.403 |
| Yellig and Metcalfe  | Sebastian et al.     | 31.51     | 1609.8| 0.395 |
| Alston et al.        |                      | 34.92     | 1161.7| 0.414 |
| GA-SVR               |                      | 7.93      | 267.1 | 0.939 |
| RBF network          |                      | 7.42      | 263.5 | 0.941 |
| DA-BP                |                      | 5.79      | 206.1 | 0.965 |
| PSO-BP               |                      | 6.42      | 234.7 | 0.957 |
| MEA-BP               |                      | 6.88      | 242.6 | 0.954 |
| GA-BP                |                      | 6.99      | 244.3 | 0.951 |
| ABC-BP               |                      | 7.21      | 254.7 | 0.946 |

AAPRE: average absolute percent relative error; ABC-BP: artificial bee colony algorithm-back propagation; DA-BP: dragonfly algorithm-back propagation; GA-BP: genetic algorithm-back propagation; GA-SVR: genetic algorithm-support vector regression; MEA-BP: mind evolutionary algorithm-back propagation; PSO-BP: particle swarm optimization-back propagation; RBF: radial basis function; RMSE: root mean square error.
compacted in the nearest vicinity of 45° line. This means that DA-BP is capable of predicting a wide scope of MMP with high accuracy.

In order to visualize the difference between published correlations and the EAs-BP model, cumulative frequency is employed for analysis. Figure 7 shows the difference of the cumulative frequency of the absolute estimation error for the DA-BP model and two correlations. It is found from Figure 7 that the cumulative frequency of the DA-BP model is highest when the absolute estimation error is less than 25%. Furthermore, this model predicts approximately 97% MMP points with absolute relative estimation errors less than 20%. However, the MMP points in this error range are approximately 56% and 46% for Lee and Orr and Jensen (Sebastian et al., 1985), respectively. This demonstrates that the DA-BP predicts pure and impure CO₂–oil MMP with the most efficiency.

**Sensitivity analysis**

A sensitivity analysis is conducted in order to improve the insight of the influence of new input parameters proposed in this paper on CO₂–crude oil MMP. The aforementioned Pearson correlation coefficient (\( r \)) is employed in this investigation to determine the influence of each new input parameter on MMP. The influence of input variable on MMP increases, as the absolute value of \( r \) between input and output parameters increases. When the value of \( r \) is less than zero, the effect of input parameter on MMP is negative. When the value of \( r \) is more than zero, this effect is positive. Figure 8 shows the sensitivity of CO₂–crude oil MMP to five input parameters (\( T_r, T_{cm}, C_1, C_6, \) and \( MWC_{5+} \)).

Among all input parameters, there are three parameters (\( T_r, C_1, \) and \( MWC_{5+} \)) with positive effect on MMP. In other words, MMP increases with the values of these parameters.
Moreover, the reservoir temperature ($T_r$) has the most influence (0.63) on MMP. However, the $C_6$ percentage of the crude oil ($C_6 \text{ (mol)}$) and the pseudo critical temperature of injection gas ($T_{cm}$) have negative effects on MMP, which is beneficial for decreasing the MMP value. The effect of all input parameters on MMP can be arranged in the following order.

**Figure 6.** Comparison of the results between the proposed models and the selected literature models. ABC-BP: artificial bee colony algorithm-back propagation; DA-BP: dragonfly algorithm-back propagation; GA-BP: genetic algorithm-back propagation; MEA-BP: mind evolutionary algorithm-back propagation; MMP: minimum miscibility pressure; PSO-BP: particle swarm optimization-back propagation. 

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Conclusions
In this paper, a BP neural network model is presented for MMP prediction during pure and impure CO₂ flooding. In order to reduce the required components of oil and improve prediction accuracy, screening operation of input variables based on 152 MMP values is implemented, and five variables (i.e. reservoir temperature, the molecular weight of C₅⁺ fraction, the mole percentages of C₁ and C₆ and pseudo critical temperature of injection gas) are

\[ T_r > T_{cm} > C_6(\text{mol}) > C_1(\text{mol}) > \text{MWC}_5^+ \]
selected as new input parameters for BP network model to predict MMP. The main objective of this study was to develop the robust and accurate model for predicting MMP. Therefore, weights and thresholds of BP network are optimized by some different EAs including GA, MEA, PSO, ABC, and DA. Finally, the selected input parameters are fed into the optimized BP model (i.e. EAs-BP) to obtain the optimal model for predicting MMP. Moreover, statistical evaluation and graphical analyses are utilized to compare the performance of the optimized BP model and correlations in the literature. The results demonstrate that the proposed EAs-BP models are more accurate and efficient than published correlations for MMP estimation. The DA-BP model shows the best prediction results with the determination coefficient ($R^2$) of 0.965, AAPRE of 5.79%, and RMSE of 206.1. In addition, a sensitivity analysis for new input parameters shows that the reservoir temperature (the highest correlation coefficient of 0.63) has the most influence on MMP. To this end, the proposed DA-BP model is the most accurate and reliable model for predicting the pure/impure CO$_2$–crude oil MMP.

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ORCID iDs
Yapeng Tian https://orcid.org/0000-0003-4414-4543
Nannan Liu https://orcid.org/0000-0003-0403-2475

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