An integrated maintenance planning, warranty policy, technology level and pricing model considering time value of money in a three-level servicing contract

Ali Salmasnia\textsuperscript{1*}, Ahmad Hatami\textsuperscript{2}

\textsuperscript{1}Department of Industrial Engineering, Faculty of Engineering, University of Qom, Qom, Iran, Tel: +98-912-2865720
\textsuperscript{2}Department of Industrial Engineering, Faculty of Engineering, University of Qom, Qom, Iran, Tel: +98-913-6478943

Abstract

A common way to address customer concerns in the post-warranty period is to provide an extended warranty. Although sometimes the manufacturer is reluctant to offer an extended warranty, an agent takes on this task to maintain market share. For this purpose, the warranty policy is presented as a three-level servicing contract with the objectives of maximizing the manufacturer’s profit, agent’s profit, and customer satisfaction. The model considers two approaches to control the number of product failures: (1) technology level used in manufacturing as an effective factor in product reliability; and (2) non-periodic maintenance activities to maintain the product reliability at an acceptable level. To calculate the costs imposed on each side of the contract more accurately, the time value of money is considered in the calculation of financial flows. To illustrate the effectiveness of the approach, three comparative studies are provided. The first one shows the impact of the presence of the agent and the provision of an extended warranty period, while the second one proves the importance of preventive maintenance to reduce costs and increase the interests of each side. The results of the last one shows the effect of considering time value of money in calculating cash flows.

Keywords: three-level servicing contract, non-periodic preventive maintenance, two-dimensional warranty, technology level, time value of money.

* Corresponding author, Associate Professor, E-mail addresses: a.salmasnia@qom.ac.ir, TEL:989122865720
1. Introduction

Nowadays, business models have undergone many changes to meet the needs of customers, and organizations have to revise business processes and management strategies to maintain their markets. One of the tools that manufacturers use to increase market share and customer satisfaction is providing after-sale services in the warranty period which leads to an assurance in customers’ minds. In most models, warranty period is presented with time limit and customer consumption rate is not considered. These models not only do not differentiate customers from each other, but also bear high costs due to high consumption of some customers. In order to eliminate this weakness, the warranty period is considered to be two-dimensional in a way that two-dimensional warranties consider both age and consumption as well as the potential combination between them. Singpurwalla and Wilson. [1] modeled the satisfaction of the customer and the manufacturer in terms of the number of product failures in the two-dimensional warranty period using game theory. Huang et al. [2] developed a two-dimensional warranty policy by considering a bivariate Weibull process to analyze the breakdown process of a repairable product simultaneously in terms of time and usage. Tong et al. [3] aims to determine two-dimensional warranty limits under a one-dimensional preventive maintenance. Ye et al. [4] suggested a strategy that offers buyers a two-dimensional warranty menu to provide buyer-friendly warranties. For more details regarding two-dimensional warranty, interested readers are referred to Gertsbakh and Kordonsky. [5], Chun and Tang. [6] and Baik et al. [7].

Today, extended warranties are used as a solution to enhance the customer's assurance to cover possible product failures after the end of the basic warranty period. Wu and Longhurst. [8] investigated the imposed costs to manufacturer during both basic and extended warranty periods.

In some circumstances, the manufacturer is not able to provide extended warranty for some reasons, including restrictions on the circulating money. In order to maintain the customer’s assurance, the manufacturer allows the agent to take the
privilege of this period. In this regard, Esmaeili et al. [9] introduced the agent to support the manufacturer's product during the post-warranty period under a three-level servicing contract. Asgharizadeh et al. [10] developed a stochastic model to study the impact of equipment reliability and the number of buyers being serviced on the agent's optimal strategies using a game theoretic formulation. Readers interested in extended warranty can study Bouguerra et al. [11], Chang and Lin [12], Lam et al. [13] and Hartman et al. [14].

Performing maintenance activities and choosing the appropriate level of technology are known as two effective factors to reduce the cost of product failure. Maintenance policies can be divided into periodic and non-periodic categories. In the periodic \textit{pm} policy, time interval between each two consecutive \textit{pm} activities are the same, while non-periodic maintenance policies aim to maintain the reliability of the product at an appropriate level. interested readers can refer to Chien (2008), Salmassnia and Yazdekhasti [15] and Kim et al. [16] for more study regarding periodic \textit{pm} policies, while Park et al. [17], Su and Wang [18] and Huang et al. [19] are the latest studies of non-periodic maintenance policies.

In spite of what is given in previous paragraph, in many articles, maintenance is the only factor to reduce failures, while the fact that the product technology level also has a significant effect on the number of failures and attraction of customer attention, is ignored. In other words, when introducing a new product to the market, technical key variables such as design and reliability should be taken into account. In this regard, Darghouth et al. [20] developed a model to obtain optimal design, warranty and price for products sold with a maintenance service contract. DeCroix. [21] proposed a model based on game theory for companies that are forced to set warranty policy, reliability parameters, and prices for their products.

Considering the time value of money is very important to calculate financial flows more accurately, especially in processes that the costs are imposed in different times. Articles by Luciano and Peccati. [22], Van der Laan. [23], Disney et al. [24] and Lin et al. [25] highlight the importance of the time value of money. Therefore,
taking the net present value into account is necessary in the analysis of manufacturer, agent and customer costs. In the warranty literature, however, according to our knowledge, we can only refer to Teng [26], which calculated the manufacturer cost under an extended warranty considering the time value of money. In Table 1. Literature review), in order to illustrate the existing gaps in research in a better way, they have been briefly introduced and summarized.

In order to fill the research gaps and overcome the above problems, a three-level model including the manufacturer profit, agent profit and customer satisfaction under two-dimensional warranty policy with capability of being extended is developed. In addition, the appropriate technology level used in manufacturing the product and implementation of non-periodic maintenance activities are taken as two approaches to reduce product failure. On the other hand, in an effort to increase the accuracy in calculating the costs imposed on the sides of the contract over time, the time value of money is taken into account in the calculation of financial flows.

The rest of this study is as follows: in the next section, problem definition and its assumptions are fully explained. In Section 3, a mathematical model is introduced considering technology level used in manufacturing, two-dimensional extended warranty and applying non-periodic maintenance policy in a three-level service contract. In Section 4, solution approach is illustrated. Section 5 is called experimental results which includes three parts: (1) numerical example; (2) sensitivity analysis and (3) comparative study. Eventually, the conclusions are expressed in Section 6.

Please insert Table 1 here
Problem definition
In a competitive world, the manufacturers moved from providing warranty just in time dimension to two-dimensional warranties in terms of both time and usage in a way that each dimension which reaches its ending point first, the warranty ends, and this is a trick to reduce the cost of the warranty service provider. On the other hand, it is important to address the concerns of customers about the cost of the post-warranty period. To tackle this problem, the extended warranty has attracted the attention of many manufacturers as an attractive policy for customers. However, sometimes the manufacturer is not willing to provide an extended warranty, and an external unit will be responsible for this task. According to what is mentioned, a two-dimensional warranty contract is provided based on age and usage including three sides of manufacturer, customer and agent.

As we know, the level of technology used in manufacturing the product has a significant effect on the cost of manufacturing, the number of failures and reliability. Consequently, in this study, the technology level is considered as a decision variable. Furthermore, it is assumed that the time-to-failure follows a Weibull distribution, which describes the proper behavior of the failure of mechanical products. According to what previously mentioned, the number of product failures during its lifetime is affected by three factors of time, usage rate, and technology level. As a result, the failure rate can be expressed as Figure 1: factors affecting the failure rate.

Providing customers with warranty imposes a considerable cost on the warranty service provider. An effective way to reduce the cost of providing services is to carry out maintenance activities. These activities are effective when they respond to the failure rate trend, and are run to maintain product reliability at an acceptable level during flexible intervals. As can be seen in Figure 2: illustration of the relationship
between the manufacturer and the agent in the provision of warranty service and non-periodic maintenance policy, a non-periodic maintenance policy is presented in which the cost of \textit{pm} activity is incurred by the manufacturer in the basic warranty period, while it is incurred by the agent during the extended warranty period. Also, with regard to the lengthy periods of basic and extended warranty, taking the time value of money into account is important for more accurate calculation of the costs imposed on manufacturer and the agent. For this reason, Net Present Value (NPV) is used to increase the accuracy in calculating maintenance and minimal repair costs.

\textit{Please insert Figure 2 here}

Since the manufacturer’s profit and the agent’s profit are respectively related to product sales and extended warranty sales, on the other hand, customer satisfaction is reduced due to the increase in product purchase price and extended warranty, in the model, the product price and the extended warranty price are considered as two decision variables in order to create a balance to maximize the three objective functions.

2.1. Notations

Prior to developing the presented mathematical model, the notations used in the problem formulation are provided in Table 2. Notations. As shown in Table 2. Notations, notations are divided into three categories of indices, decision variables and parameters.

\textit{Please insert Table 2 here}
1.2. **Problem assumptions**

The assumptions in this problem are outlined as follow:

- The product can be repaired, and depreciation is caused by age and usage, and in case of lack of maintenance operations, product failure rate increases.

- Product depreciation behavior can be explained by the Weibull process.

- All product failures during the warranty period are fixed with minimal repairs. This means that the reliability of the product after repairs is the same as that of before the failure.

- Maintenance strategy is implemented by the manufacturer and the agent during basic warranty and extended warranty, respectively.

- Preventive maintenance activities are imperfect, which means the repaired product is better than that of before the failure and worse than the new product.

- All failures during the warranty period are rectified by a minimal repair and the customer does not pay any fees.

- The repair time for a defective product, like the preventive maintenance time, is negligible and can be ignored.

- Market conditions are such that the price and warranty period do not change throughout the product life cycle.

- The length of the extended warranty period is from the end of the basic warranty period to the end of the useful lifetime of the product.

2. **Model description**

In this section, firstly, failure rate function is introduced, and then, the cost of each preventive maintenance is calculated. Afterwards, the costs imposed on the manufacturer and agent during two-dimensional basic and extended warranty periods are obtained. Then, the values of decision variables including product price $P_p$, the price of extended
warranty $PW_e$, the design variable $\theta$, time of the first non-periodic maintenance activity $\tau_1$, and maintenance activity level $m$, profit functions of manufacturer, agent and customer satisfaction are obtained.

3.1. **Product failure modeling based on technology level**

Since the usage rate $r$ is a non-negative random variable and varies from one customer to the next, a uniform distribution function $G(r), g(r)$ is considered for this variable. Also, since modeling is done for mechanical products, it is assumed that the time to failure follows a Weibull distribution function. In other words, the number of product failures follows a non-homogeneous Poisson process (NHPP) which means the product failure rate is an ascending function of time and customer usage rate. In addition, it is a known fact that failure rate is a function of the technology level used to produce the product. Therefore, the scale parameter of Weibull distribution is considered as a function of the technology level $\theta$ and usage rate of $r$. This concept is inspired by Darghouth et al. [20] and is explained as follows:

$$
\beta(r, \theta) = \frac{\gamma_0}{\gamma_1 r + \gamma_2 \theta}
$$

(1)

where $\gamma_0, \gamma_1, \gamma_2$ parameters have positive values which can be estimated by means of historical data of recorded warranty information in the factory. Finally, failure rate can be expressed as follows:

$$
\lambda(t) = \frac{f(t)}{1 - F(t)}
$$

(2)

$$
\lambda(t|r, \theta) = \frac{f(t|r, \theta)}{1 - F(t|r, \theta)} = \alpha \left( \frac{t}{\beta(r, \theta)} \right)^{n-1}
$$

3.2. **Modeling the imperfect maintenance strategy**
In this study, the concept of virtual age is used to illustrate the effect of *pm* activities, which was introduced by Kijima et al. [27] for the first time and then developed by Kijima [28]. On the other hand, in order to maintain the reliability of the product at a predetermined level, a non-periodic maintenance policy is used in which the expended number of failures of the product in the *pm* intervals remains constant. Thus, the time for performing the *j*-th *pm* is as follows:

\[
\int_{t_{j-1}}^{t_j} \lambda(v_{j-1} + t - \tau_{j-1}) dt = \int_0^n \lambda(t) dt \\
\int_{t_{j-1}}^{t_j} \frac{\alpha}{\beta} \left( \frac{v_{j-1} + t - \tau_{j-1}}{\beta} \right)^{(a-1)} dt = \int_0^n \frac{\alpha}{\beta} \left( \frac{t}{\beta} \right)^{(a-1)} dt \\
\frac{\alpha}{\beta} \left( \frac{1}{\beta} \right)^{(a-1)} \int_{t_{j-1}}^{t_j} (v_{j-1} + t - \tau_{j-1})^{(a-1)} dt = \frac{\alpha}{\beta} \left( \frac{1}{\beta} \right)^{(a-1)} \int_0^n (t)^{(a-1)} dt \tag{3}
\]

\[
\left( v_{j-1} + t - \tau_{j-1} \right)^{(a)} \bigg|_{t_{j-1}}^{t_j} = \left[ t^{(a)} \right]_0^n \Rightarrow (v_{j-1} + \tau_j - \tau_{j-1})^a - v_{j-1}^a = \tau_1^a \\
\Rightarrow v_{j-1} + \tau_j - \tau_{j-1} = (\tau_1^a + v_{j-1}^a)^{1/a} \\
\]

where *v* is the virtual age and *t* is the actual age of the product in (*j-1*)-th maintenance activity.

And finally:

\[
\tau_j = \left( \tau_1^a + v_{j-1}^a \right)^{1/a} - v_{j-1} + \tau_{j-1} \tag{4}
\]

Hence, for a certain *pm* level, product virtual age after *j*-th *PM* is obtained by:

\[
v_1 = \delta(m).\tau_1 \\
v_j = v_{j-1} + \delta(m)(\tau_j - \tau_{j-1}) \quad j \geq 2, 0 \leq m \leq M \tag{5}
\]

where \(\delta(m)\) denotes age-reduction coefficient and is a descending function of *m* as \(\delta(m) = (1 + m)e^{-m}\). So, \(\delta(m)\) can vary in the interval [0,1] and higher value of *m* signifies lower \(\delta(m)\). In particular, for an infinite maintenance effort *M* (impossible to achieve in practice), the product is restored to as good as it was following the last *pm* \(\delta(\infty) = 0\). Hence, this model does not allow *pm* to be perfect. If *m* = 0, then \(\delta(0) = 1\).
\( v_j = \tau_j, j \geq 1 \), and practically \( PM \) activity has not been effective. While in a general view, if \( 0 < m < M \), item is rectified to some extent and the failure rate is reduced.

In this research, it is assumed that \( PM \) level is fixed during warranty period. In this situation, Equation (5) can be rewritten recursively as follows:

\[
\begin{align*}
v_j &= v_{j-1} + \delta(m)(\tau_j - \tau_{j-1}) \\
&= v_{j-2} + \delta(m)(\tau_j - \tau_{j-2}) \\
&= ... \\
&= v_0 + \delta(m)(\tau_j - \tau_0)
\end{align*}
\]  

(6)

3.3. Warranty period ranges

In this paper, a rectangular two-dimensional warranty with \([0,W] \times [0,U]\) limits, as can be seen in Figure 3: Two-dimensional warranty policy, is used. In other words, warranty ends when the product real age reaches \( W \), or the real usage reaches \( U \) (of course each one which happens first). Consequently, two modes \( r \leq \eta \) and \( r > \eta \) according to the customer’s usage rate are resulted.

Please insert Figure 3 here

Based on Figure 3: Two-dimensional warranty policy, if the usage rate is \( r > \eta \), usage limit comes forth sooner than time limit, while if the usage rate is \( r \leq \eta \), time limit comes forth sooner than usage limit, and the warranty period finishes.

3.4. Preventive maintenance cost

With regard to Darghouth et al. [20], for a given maintenance activity at level \( m \), \( PM \) cost is calculated by means of Equation (7):

\[
C_{p_j}(m) = a + b(m) + c_j, j = 1, 2, ... .
\]

(7)

where \( a \) is the fixed cost and \( b(m) \) denotes the contribution of the improvement level \( m \) in the \( pm \) cost, which is obtained via:
where $\beta$ and $\phi$ are two positive constants. According to Equation (8), if $m = 0$, no improvement has been made, and it results in $b(m) = 0$. And if $m \rightarrow M$ then $b(m) \rightarrow \infty$, which means the product returns to as good as new state. $c_j$ denotes the cost of gaining knowledge and practical skills necessary for doing maintenance activities. By introducing this parameter, the effect of learning in calculating the cost of each $pm$ activity is reflected actually. In fact, with each $PM$ activity, costs of $PM$ provider will reduce due to the increase in the experience. Based upon Wright [29], these costs can be expressed in form of learning curve:

$$c_j = c_1 j^\xi \rightarrow \xi = \frac{\log 0.8}{\log 2}, \ j = 1, 2,...$$

where $c_1$ is a fixed value denoting learning cost for the first $pm$ activity. It is mentionable that parameters $c_j$ and $a$ are independent from $PM$ level.

Eventually, non-periodic $PM$ cost during basic warranty by considering the time value of money is calculated via Equation (10):

$$n_{pm_1} = \max \left\{ j \mid \tau_j < W, j \geq 0 \right\} \quad r \leq \eta$$

$$n_{pm_2} = \max \left\{ j \mid \tau_j < \frac{U}{r}, j \geq 0 \right\} \quad r > \eta$$

$$C_{pm_1}(m) = \sum_{j=1}^{n_{pm_1}} Cp_j(m) \times e^{-ir\tau_j}, \ x=1,2$$

3.5. Non-periodic preventive maintenance strategy in a two-dimensional warranty space

A non-periodic preventive maintenance policy is implemented in order to reduce the number of failures during warranty period.
In accordance with the equation $u = rt$ and warranty limits, the reference usage rate can be defined as $\eta = \frac{U}{W}$, which results in the following two modes:

1. $r \leq \eta$
2. $r > \eta$

$r \leq \eta$: For this option, as can be seen in Figure 4, option $r \leq \eta$, warranty period ends when actual age and usage reach $W$ and $rW$, respectively.

Please insert Figure 4 here

In this option, the number of $PM$ activities in the warranty period is:

$$n_{pm} = \max\left\{ j \mid r_j < W, j \geq 0 \right\}$$  \hspace{1cm} (11)

In other words, maintenance activities are carried out $n_{pm}$ times in warranty period, and after doing $n_{pm}^{th}$ non-periodic $PM$ activity and the end of warranty period, warranty provider does no extra $PM$.

As mentioned before, it is assumed that time-to-failure follows a Weibull distribution. Using Equations (1) and (5), the expected number of failures can be calculated for a constant usage rate on condition that $r \leq \eta$ in the warranty period according to Equation (12):

$$E\left[ N(\Omega) \mid r \leq \eta \right] = \sum_{j=1}^{n_{pm}} \int_{v_j}^{v_j+1} \lambda(t \mid r, \theta) dt + \int_{v_{pm}}^{v_{pm} + W-v_{pm}} \lambda(t \mid r, \theta) dt$$  \hspace{1cm} (12),

Considering the time value of money under this option, the expected cost of the warranty service for a fixed usage rate on condition that $r \leq \eta$ is obtained as follows:
\[ E[C_{WB} | r \leq \eta] = C_r \sum_{j=1}^{n_{pm2}} \int_{v_{j-1}}^{v_j} \lambda(t|r, \theta)e^{-ir} dt + C_r \int_{v_{n_{pm2}}}^{v_{pm2} + \frac{U}{r} - \tau_{pm2}} \lambda(t|r, \theta)e^{-ir} dt + C_{pm2}(m) \] (13)

\[ r > \eta: \text{ For this option, in accordance with Figure (5), time and usage of warranty termination are } \frac{U}{r} \text{ and } U, \text{ respectively.} \]

**Please insert Figure 5 here**

In addition, the number of non-periodic preventive maintenance activities conducted in the warranty period is calculated using Equation (14).

\[ n_{pm2} = \max \left\{ j \mid \tau_j < \frac{U}{r}, j \geq 0 \right\} \] (14)

Therefore, the expected number of failures for a fixed usage rate during warranty period on condition that \( r > \eta \) is obtained through:

\[ E[N_{WB} | r > \eta] = \sum_{j=1}^{n_{pm2}} \int_{v_{j-1}}^{v_j} \lambda(t|r, \theta)dt + \int_{v_{n_{pm2}}}^{v_{pm2} + \frac{U}{r} - \tau_{pm2}} \lambda(t|r, \theta)dt \] (15)

The expected cost of the warranty service for a fixed usage rate on condition that \( r > \eta \) and based on the time value of money for this option is equal to:

\[ E[C_{WB} | r > \eta] = C_r \sum_{j=1}^{n_{pm2}} \int_{v_{j-1}}^{v_j} \lambda(t|r, \theta)e^{-ir} dt + C_r \int_{v_{n_{pm2}}}^{v_{pm2} + \frac{U}{r} - \tau_{pm2}} \lambda(t|r, \theta)e^{-ir} dt + C_{pm2}(m) \] (16)

\[ E[C_{WB}] = \int_0^\eta E[C_{WB} | r \leq \eta] g(r)dr + \int_\eta^\infty E[C_{WB} | r > \eta] g(r)dr \] (17)

Ultimately, total expected cost of services is calculated using Equation (17).
3.6. Mathematical modeling of the agent

As mentioned earlier, for the cost of extended warranty services in the proposed model, the agent is in charge. To calculate this cost, first, the warranty cost for the entire lifetime $[0, W + W_e] \times [0, U + U_e]$ of the product is obtained using Equations (11) to (17), then the cost of the basic warranty period is subtracted from it.

$$E\left[C_{WL}\right] = \begin{cases} E[C_{WL}] & r \leq \eta \\ E[C_{WL}] & r > \eta \end{cases}, \eta = \frac{L}{K}$$

(18)

$$E[C_{WE}] = E[C_{WL}] - E[C_{WB}]$$

Agent’s revenue is the result of selling extended warranty which is considered as a decision variable, and is expressed as follows:

$$RSC_A = PW_E$$

(19)

According to the explanations in the previous sub-sections, time of warranty termination is equal to $W$ and $\frac{U}{r}$ in modes $r \leq \eta$ and $r > \eta$, respectively. In other words, in accordance with Figure 6: Two-dimensional warranty and lifetime policy, warranty period ends when the production age reaches $\min\left\{W, \frac{U}{r}\right\}$. The lifetime of the product is similarly formulated as $\min\left\{W_{life}, \frac{U_{life}}{r}\right\}$ when $W_{life}$ and $U_{life}$ as can be seen in the figure denote product lifecycle limits in two dimensions of age and usage, respectively.

**Please insert Figure 6 here**

In the post-base warranty period, the costs imposed on the customer in absence of the agent, when the extended warranty is not provided, are calculated as Equation (20).
According to these equations, agent income must comply with the constraint (21).

\[ RSC_A < \text{COSTNOPM} \]  

The expected agent benefit is equal to the subtraction of the extended warranty costs from the revenue resulted from selling the extended warranty. It is modeled as follows.

\[ \Pi_A = RSC_A - E[C_{WE}] \]

\[ \Pi_A = (1 - \rho) \times \Pi_A \quad 0 \leq \rho < 1 \]  

where \( \rho \) is the percentage of the agent’s profit that manufacturer takes from agent as the concession of offering warranty.

### 3.7. Mathematical modeling for the manufacturer

As mentioned before, Manufacturer’s income per product depends on two factors of the product price and the agent’s profit as follows:

\[ RSC_M = P_p + \rho \times \Pi_A \quad 0 \leq \rho < 1 \]  

The cost of manufacturing the product is proportional to the technology level, so the higher the technology level used in manufacturing, the higher the cost of manufacturing.

\[ C_p = \psi_0 + \psi_1 \theta e^{(\theta_{\text{max}} - \theta_{\text{min}})} \]  

where \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) are design variable limits and Parameters \( \psi_0 \) and \( \psi_1 \) are the fixed cost of manufacturing the product and the sensitivity coefficient of the design variable.

The cost of providing warranty services by the manufacturer is obtained based on Equations (11) to (17):
\[ E[C_{WB}] = \begin{cases} E[C_{WB}] & r \leq \eta \\ E[C_{WB}] & r > \eta \end{cases}, \quad \eta = \frac{U_B}{W_B} \]  

(25)

The manufacturer's expected profit can be obtained by subtraction of the cost of manufacturing and the provision of the basic warranty, from the incomes of selling the product and giving the concession to the agent. It is modeled as follows:

\[ \Pi_M = RSC_M - E[C_{WB}] - C_p \]  

(26)

3.8. **Mathematical modeling for customer**

Customer satisfaction can be considered as a function of design variable which means that the higher technology level leads to the more customer revenue based on the equation \( R = \varphi \frac{1}{\theta} \), where \( \varphi \) is the technology level sensitivity coefficient of the product, which it’s multiplication by the level of product manufacturing technology \( \frac{1}{\theta} \) results in the customer revenues per time unit. By multiplying this number by the average product availability time for the customer, customer’s revenue over the product life time is calculated, and because the repair times are assumed to be zero or very close to zero, the availability time can be considered to be equal to the product life time. It is mathematically expressed as follows:

\[ \omega(r) = \min \left\{ \frac{U_B + U_E}{r}, W_B + W_E \right\} \]  

\[ L = \int_{r_{\text{min}}}^{r_{\text{max}}} \omega(r) \cdot g(r) \, dr \]  

(27)

\[ RSC_c = R \cdot L \]

The expected customer satisfaction is equal to the subtraction of the cost of purchasing the product and the extended warranty from customer’s revenue. This is modeled as bellow:

\[ \Pi_c = RSC_c - Pp - PW_E \]  

(28)
3.9. The aggregation approach based on desirability concept

Knowing that in this problem, we aim to maximize profit of manufacturer, profit of agent and customer satisfaction simultaneously. For this purpose, desirability function approach according to Salmasnia et al. [30] is applied. In this approach, each objective function is converted to a scale less value in the interval \([0,1]\) in the form of \(d(\Pi_i)\), \(i=M,A,C\) which is named desirability. It increases as the corresponding objective value increases.

\[
d(\Pi_i) = \begin{cases} 
1 & Z_i \geq Z_i^{max} \\
\frac{Z_i - Z_i^{min}}{Z_i^{max} - Z_i^{min}} & Z_i^{min} < Z_i < Z_i^{max} \\
0 & Z_i \leq Z_i^{min}
\end{cases}
\]

In Equation (29), \(Z_i^{max}\) and \(Z_i^{min}\) denote the acceptable maximum and minimum values, respectively for the \(i^{th}\) objective function. In this approach, we calculate \(Z_i^{max}\) by maximizing the objective function solely, and \(Z_i^{min} = \psi_i \times Z_i^{max}\), \(i=M,A,C\) is considered, where \(\psi_i\) is a coefficient in the interval \([0,1]\) and is selected based on the opinion of the decision maker.

In order to integrate three desirability functions in form of a unified phrase, the geometric mean as Equation (30) is used. Clearly \(D\) can vary in the interval \([0,1]\) and \(D\) increases as the balance of the objective functions becomes more favorable. The other important feature of this Equation (30) is that if any \(d(\Pi_i) = 0\), (that is, if one of the objective functions is unacceptable) then \(D = 0\).

\[
D = \text{Max}\left[ d(\Pi_M) \times d(\Pi_A) \times d(\Pi_C) \right]
\]

4. Solution approach

The model presented in Section 3, due to the fact that decision variables in the objective function are in integral bounds, has complexity which doesn’t allow it to be solved by
means of exact methods. Meta-heuristic algorithms are one of the best methods to solve problems with high complexity because these algorithms can achieve suitable solutions in reasonable times.

In this paper, in order to achieve optimal values of decision variables, Particle Swarm Optimization (PSO) is implemented because of its good performance in optimizing non-linear models, unique searching mechanism, simplicity in concept, computational efficiency, and ease of implementation. It is one of the most popular meta-heuristic algorithms that recently has been widely applied by some researchers such as Salmasnia et al. [31] and Salmasnia et al. [32].

The solution representation is a key factor in developing the PSO algorithm which could be a string of both integers and real numbers. The solution representation for the proposed model includes five-dimensional vectors that each dimension refers to a certain decision variable. In the presented model, the preventive maintenance level \( m \) is an integer, while the values of decision variables including \( P_p \), product price, \( PW_E \) the price of extended warranty, the design variable \( \theta \), time of the first non-periodic maintenance activity \( \tau_1 \) are real numbers. For generating the initial value of each continuous decision variable, a uniformly distributed random value is generated between lower and upper limits of the considered decision variable. Furthermore, in order to generate an initial value for discrete decision variables, a random value from a uniform distribution in the interval \([0,1]\) is generated. The values of discrete variables, i.e. the preventive maintenance level \( m \), is obtained according to Equation (31).

\[
m = \min(n_{\text{min}} + \text{floor}((m_{\text{max}} - n_{\text{min}} + 1) \times R), m_{\text{max}})
\]

where, \( n_{\text{min}} \) and \( n_{\text{max}} \) are the lower and upper limits of \( n \), respectively. Furthermore, \( R \) is a random number from a uniform distribution in the interval \([0,1]\).

5. **Experimental results**
The aim of this study is to maximize manufacturer’s profit, agent’s profit and customer satisfaction under a three-level servicing contract. In this section, a numerical example is presented to demonstrate the applicability of the proposed model. The rest of this section is divided into three parts as follows:

Section (5.1) presents the mathematical programming and the assigned values to model parameters. Then, the PSO algorithm is used to optimize the model. In Section (5.2), the effect of parameters on the objective functions are analyzed. In Section (5.3), three comparisons are presented to illustrate the effectiveness of the proposed model.

5.1. Numerical example

Assume a product for which the manufacturer offers two-dimensional basic-warranty and the agent provides two-dimensional extended-warranty. To control the number of failures, on one side a non-periodic maintenance policy is used and on the other side the technology level used in manufacturing the product is considered. The value of parameters which are extracted from Esmaeili et al. [9] and Darghouth et al. [20] are shown in Table 3. The values of the parameters in the numerical example.

Please insert Table 3 here

With regard to complexity of the model, this problem is solved in MATLAB2017 by using PSO algorithm, and the optimized results are provided in Table 4. values of decision variables and objective functions.

Please insert Table 4 here

Based on the obtained results, the design variable used in the manufacturing is suggested to be $\theta = 0.2442$ and the first maintenance activity is implemented at time $\tau_1 = 0.2124$ with level $m = 4$. In addition, the price by which the product is sold must be $P_p = 9.7731$ and, the price of selling the extended warranty must be $PW_E = 2.5976$ so that the objective functions are maximized.
5.2. Sensitivity analysis

Sensitivity analysis is very common for the domain of optimization. In general, it is the task of understanding the behavior of the final solution of an optimization problem because of changes in the input parameters Foumani et al. [33]. In this section, the effect of the parameters $C_r, \beta, U_u, W_u, U_{life}, W_{life}$ on three objective functions are investigated. In this regard, Taguchi experimental design $L_{27}$ as a tool for generating problems by making targeted changes in the model parameters is used. For example, $C_r$ takes three values of 0.25, 0.3, 0.35. As can be seen in Table 5, Problems generated by Taguchi design $L_{27}$, the parameters take each one of their levels in 9 different problems generated by Taguchi $L_{27}$. For example, the $U_u$ parameter takes its low, medium and high levels in problem sets (1, 2, 3, 16, 17, 17, 18, 22, 23, 23, 24), (4, 5, 6, 10, 10, 11, 11, 12, 25, 26, 26, 27) and (7, 8, 9, 13, 14, 14, 15, 19, 19, 20, 21), respectively. It is noteworthy that the values of the other parameters in each of the twenty-seven problems remain constant according to Table 3. The values of the parameters in the numerical example.

Please insert Table 5 here

the average of the obtained values for the agent’s Profit, manufacturer’s profit and customer satisfaction in the twenty-seven instances for each level of each parameter are recorded in Table 6. Sensitivity analysis of the parameters on agent’s profit, respectively. For example, the number 3.866511 in Table 6. Sensitivity analysis of the parameters on agent’s profit is the average values of the manufacturer profit in instances 1, 2, 3, 10, 11, 12, 19, 20, 21 in which parameter $\beta'$ is at its first level. Then the difference between the obtained maximum and minimum values for three levels of each parameter is considered as $\Delta$. Finally, $\Delta$ values are ranked, and the effect of the parameters on the objective function is reported in order in the Rank line.

Please insert Table 6 here
According to Table 6. Sensitivity analysis of the parameters on agent’s profit when product lifetime $W_{life}$ increases, three objective functions increase as well. Because the increase in parameter $W_{life}$ directly affects customer satisfaction in accordance with Equation (28), and an increase in customer satisfaction to achieve a balance in three objective functions leads to an increase in product sales price and the extended warranty price, and eventually the agent and manufacturer’s profit are also affected and increased in accordance with Equations (22) and (26), respectively. The rank lines in Table 6. Sensitivity analysis of the parameters on agent’s profit confirm that lifetime $W_{life}$ has the greatest impact and $U_B$ has the least effect on the three objectives of the manufacturer profit, the agent profit and customer satisfaction compared to the other parameters.

5.3. Comparative study

In this section, three comparative studies to illustrate the effectiveness of the suggested model are presented. In the first comparative study, the effect of the presence of an agent and the provision of an extended warranty on the profit of the manufacturer and the customer is investigated. In the second comparative study, the effect of carrying out preventive maintenance activities is analyzed. Ultimately, in the final study, the effect of taking the time value of money into account on the calculation of the costs imposed on each side of the contract is examined.

Comparative study 1: Agent presence in extended warranty

One of customers’ concerns is the cost of the post-warranty period. When the product age increases, the number of failures increases as well, as a result, more repair costs are imposed on the customer. In this case, the extended warranty is a customer-friendly offer to reduce customer concerns. However, in some circumstances, the manufacturer is not able to provide this period because of the limited circulation of funds. As a result, the
presence of an agent is planned to reduce concerns. This comparative study examines the importance of the presence of an agent in warranty contracts. In this regard, the proposed model is compared with the model without the presence of an agent.

**Please insert Table 9 here**

**Please insert Figure 7 here**

As it was expected, based on Figure 7: Graphical representation of the improvement percentage in the comparative study (1), the proposed mathematical model has a better performance in all three objective functions compared to the model without the agent. The results from Table 9. Comparison of the proposed model with the model in which the agent is absent indicate the situation in which the agent is not present for the extended warranty and in the post-warranty period the maintenance activities are not performed. This leads to an increase in the number of product failures. To make a balance between two objective functions of manufacturer’s profit and customer satisfaction, since lack of implementing maintenance activities in the post-warranty period entails a lot of costs to the customer, the manufacturer is forced to lower the selling price of the product, which results in a reduction in the manufacturer’s profit. For example, according to the results in Table 9. Comparison of the proposed model with the model in which the agent is absent, in instance 27 in which repair cost and lifetime are at their highest levels, the most improvement percentages in both the manufacturer’s profit and customer satisfaction among 27 instances are obtain.

**Study of comparison 2: Implementing maintenance activities**

In this comparative study, in order to show the effectiveness of the implementing maintenance activities on the objective functions, the proposed model is compared in 27 instances presented in Table 5. Problems generated by Taguchi design \( L_{27} \) with the same model with the difference that maintenance activities are not implemented.

**Please insert Table 10 here**
Based on Figure 8: Graphical representation the improvement percentage in the comparative study (2), for 27 instances, the proposed model for all three objective functions of the manufacturer’s profit, the agent’s profit and customer satisfaction, have had better performance in comparison to the model without pm activities. In this regard, according to Table 10. Comparison of the proposed model with the model without preventive maintenance (pm), the most improvement is related to agent’s profit, with the minimum, maximum and average of 18.2553, 599.1795 and 79.5093 percent in 27 instances. The greatest improvement is related to instance 3, because the extended warranty period has the longest duration compared to the other instances, which increases the costs of the agent and the effect of performing pm activities in reducing costs is well visible. In other words, the agent’s profit in the proposed model is 3.9912, while, not implementing maintenance activities results in an increase in the number of failures, leading to an increase in failure costs and consequently agent’s profit decrease to 0.5704. The average of improvement in customer satisfaction and manufacturer’s profit are 46.0421% and 24.5417%, respectively. Improving customer satisfaction with an average of 46.0421%, means that when maintenance activities are not implemented, the number of failures is increased resulting in an increase in the basic and extended warranty costs. As a result, the manufacturer and the agent, to compensate for the issue, increase the sales price of the product and the sales price of the extended warranty, which reduce customer satisfaction.

Study of comparison 3: Considering Time Value of Money
In this comparative study, in order to show the impact of considering the time value of money on objective functions, the presented model is compared with the same model without considering the time value of money in 27 instances given in Table 5. Problems generated by Taguchi design $L_{27}$. 

Please insert Figure 8 here

Please insert Table 11 here
It is remembered that in the proposed model, the time value of money is considered by the NPV method. In other words, the cost of providing the base and the extended warranty (minimal repairs and maintenance activities) imposed on the manufacturer and the agent are returned to the present value. In contrast, the manufacturer profit and the agent profit depend on the product price and the extended warranty price respectively that are received at the beginning of period. Consequently, when the NPV method is used, the cost of providing the warranty service is less sensible. In this regard, to create a balance among the three objective functions, the product price and the extended warranty price are reduced. Finally, according to Figure 9: Graphical representation the improvement percentage in the comparative study (3), the NPV method leads to an increase in each three of the manufacturer’s profit, agent’s profit, and customer satisfaction. According to Table 11. Comparison of the proposed model with the model lacking consideration of the time value of money., in the 27 solved instances, the average of improvement for the manufacturer's profit, the agent profit and customer satisfaction are 11.5, 11.27, 20.72 percent, respectively.

6. Conclusion

To fill the gaps in the warranty literature, a two-dimensional warranty model with the possibility of extension under a three-level service contract was developed. It aimed to maximize the three objective functions of the manufacturer profit, the agent profit and customer satisfaction, simultaneously. In addition, the technology level used in manufacturing and pm level were considered as two effective factors on the product failure rate. Subsequently, sensitivity analysis was performed on six parameters \( C, \beta, U_B, W_B, U_{life}, W_{life} \). The results showed that the parameter \( W_{life} \) has the highest, and the parameter \( U_B \) has the least effect on the three objective functions. Moreover, to prove the effectiveness of the proposed model, three comparative studies were conducted with the following objectives: in the first comparative study, provision of the extended warranty
by the agent was investigated. The results, as expected, showed that reduction in the number of failures due to implementing the maintenance activities in the post-warranty period, results in the reduction of costs and the increase in customer satisfaction, which means that this factor leads to an increase in manufacturer’s profit due to an increase in the price of the product. In the second comparison, the importance and necessity of implementation of maintenance activities was investigated in saving costs and increasing profits. The results of the comparison showed that the implementation of maintenance activities had the strongest effect on the agent profit with an improvement of 79.5093 percent on average. In the last comparison, the effect of taking the time value of money into account was examined, this comparison showed that not considering the time value of money creates deceptive satisfaction and benefits for the customer, the agent and the manufacturer, which is far from reality.

As future research we suggest to extend this paper in two directions: first, implementing a pro-rata warranty policy and sharing maintenance and repair costs during the warranty period between manufacture and customer, and second, providing services to customer as options, using appropriate tools such as game theory.

References

[1] Singpurwalla, N. D., and Wilson, S., “The warranty problem: its statistical and game-theoretic aspects”, SIAM review, 35, 17-42 (1993).
[2] Huang, Y. S., Gau, W. Y., and Ho, J. W., “Cost analysis of two-dimensional warranty for products with periodic preventive maintenance”, Reliability Engineering & System Safety, 134, 51-58 (2015).
[3] Tong, P., Z. Liu and F. Menet al., “Designing and Pricing of Two-dimensional Extended Warranty Contracts Based on Usage Rate”, International Journal of Production Research 52, 6362–6380 (2014).
[4] Ye, Z. S., and Murthy, D. P., “Warranty menu design for a two-dimensional warranty”, Reliability Engineering & System Safety, 155, 21-2 (2016).
[5] Gertsbakh, I. B., and Kordonsky, K. B., “Parallel time scales and two-dimensional manufacturer and individual customer warranties.” *IIE transactions*, 30, 1181 (1998).

[6] Chun, Y. H., and Tang, K., “Cost analysis of warranty policies based on the product usage rate”, *IEEE Transactions on Engineering Management*, 46, 201-209 (1999).

[7] Baik, J., Murthy, D. N. P., and Jack, N., “Two-dimensional failure modeling with minimal repair”, *Naval Research Logistics (NRL)*, 51, 345-362 (2004).

[8] Wu, S., and Longhurst, P., “Optimizing age-replacement and extended non-renewing warranty policies in lifecycle costing”, *International Journal of Production Economics*, 130, 262-267 (2011).

[9] Esmaeili, M., Gamchi, N. S., and Asgharizadeh, E., “Three-level warranty service contract among manufacturer, agent and customer: A game-theoretical approach”, *European Journal of Operational Research*, 239, 177-186 (2014).

[10] Asgharizadeh, E., and Murthy, D. D., “Service contracts: A stochastic model”, *Mathematical and Computer Modelling*, 31, 11-20 (2000).

[11] Bouguerra, S., Chelbi, A., and Rezg, N., “A decision model for adopting an extended warranty under different maintenance policies”, *International Journal of Production Economics*, 135, 840-849 (2012).

[12] Chang, W. L., and Lin, J. H., “Optimal maintenance policy and length of extended warranty within the life cycle of products”, *Computers & Mathematics with Applications*, 63, 144-150 (2012).

[13] Lam, Y., and Lam, P. K. W. “An extended warranty policy with options open to consumers”, *European Journal of Operational Research*, 131, 514-529 (2001).

[14] Hartman, J. C., and Laksana, K., “Designing and pricing menus of extended warranty contracts”, *Naval Research Logistics (NRL)*, 56, 199-214 (2009).

[15] Salmasnia, A., and Yazdekhashi, A., “A bi-objective model to optimize periodic preventive maintenance strategy during warranty period by considering customer satisfaction”, *International Journal of System Assurance Engineering and Management*, 8, 770-781 (2017).

[16] Kim, C. S., Djamaludin, I., and Murthy, D. N. P., “Warranty and discrete preventive maintenance”, *Reliability Engineering & System Safety*, 84, 301-309 (2004).
[17] Park, M., Jung, K. M., and Park, D. H., “A Generalized Age Replacement Policy for Systems Under Renewing Repair-Replacement Warranty”, *IEEE Trans. Reliability*, 65, 604-612 (2016).

[18] Su, C., and X. Wang., “Optimizing Upgrade Level and Preventive Maintenance Policy for Second-hand Products Sold with Warranty. Proceedings of the Institution of Mechanical Engineers”, *Journal of Risk and Reliability* 228, 518–528 (2014).

[19] Huang, Y. S., Huang, C. D., and Ho, J. W., “A customized two-dimensional extended warranty with preventive maintenance”, *European Journal of Operational Research*, 257(3), 971-978 (2017).

[20] Darghouth, M. N., Aït-Kadi, D., and Chelbi, A., “Joint optimization of design, warranty and price for products sold with maintenance service contracts”, *Reliability Engineering & System Safety*, 165, 197-208 (2017).

[21] DeCroix, G. A., “Optimal warranties, reliabilities and prices for durable goods in an oligopoly”, *European Journal of Operational Research*, 112, 554-569 (1999).

[22] Luciano, E., and Peccati, L., “Capital structure and inventory management: The temporary sale price problem”, *International Journal of Production Economics*, 59, 169-178 (1999).

[23] Van der Laan, E., “An NPV and AC analysis of a stochastic inventory system with joint manufacturing and remanufacturing”, *International Journal of Production Economics*, 81, 317-331 (2003).

[24] Disney, S. M., Warburton, R. D., and Zhong, Q. C., “Net present value analysis of the economic production quantity”, *IMA Journal of Management Mathematics*, 24, 423-435 (2013).

[25] Lin, J., Pulido, J., and Asplund, M., “Reliability analysis for preventive maintenance based on classical and Bayesian semi-parametric degradation approaches using locomotive wheel-sets as a case study”, *Reliability Engineering & System Safety*, 134, 143-156 (2015).

[26] Teng, H. M., “Extended warranty pricing considering the time of money”, *Journal of Information and Optimization Sciences*, 27, 401-409 (2006).
[27] Kijima, M., H. Morimura, and Y. Suzuki., “Periodical Replacement Problem without Assuming Minimal Repair”, European Journal of Operational Research 37: 194–203 (1988).

[28] Kijima, M., “Some results for repairable systems with general repair”, Journal of Applied Probability, 26, 89–102 (1989).

[29] Wright, T. P., “Factors Affecting the Cost of Airplanes”, Journal of Aeronautical Sciences, 3, 122-128 (1936).

[30] Salmasnia, A., Bashiri, M., and Salehi, M., “A robust interactive approach to optimize correlated multiple responses”, The International Journal of Advanced Manufacturing Technology, 67(5-8), 1923-1935 (2013).

[31] Salmasnia, A., Kaveie, M., and Namdar, M., “An integrated production and maintenance planning model under VP-T2 Hotelling chart”, Computers & Industrial Engineering, 118, 89-103 (2018).

[32] Salmasnia, A., Namdar, M., and Noroozi, M., “Robust design of a VP-NCS chart for joint monitoring mean and variability in series systems under maintenance policy”, Computers & Industrial Engineering, 124, 220-236 (2018).

[33] Foumani, M., Razeghi, A., and Smith-Miles, K., “Stochastic optimization of two-machine flow shop robotic cells with controllable inspection times: From theory toward practice”, Robotics and Computer-Integrated Manufacturing, 61, 101822 (2020).

[34] Chen, J. A., and Chien, Y. H., “Renewing warranty and preventive maintenance for products with failure penalty post- warranty”, Quality and Reliability Engineering International, 23, 107-121 (2007).

[35] Chien, Y. H., “Optimal age-replacement policy under an imperfect renewing free-replacement warranty”, IEEE Transactions on Reliability, 57, 125-133 (2008).

[36] Jack, N., Iskandar, B. P., and Murthy, D. P., “A repair–replace strategy based on usage rate for items sold with a two-dimensional warranty”, Reliability Engineering & System Safety, 94, 611-617 (2009).

[37] Shafiee, M., Chukova, S., Maintenance models in warranty: A literature review”, European Journal of Operational Research, 229, 561-572 (2013).
[38] Wang, J., Zhou, Z., and Peng, H., “Flexible decision models for a two-dimensional warranty policy with periodic preventive maintenance”, *Reliability Engineering & System Safety*, 162, 14-27 (2017).

**Biography:** Ali Salmasnia is currently an Associate Professor in University of Qom, Qom, Iran. His research interests include quality engineering, reliability, applied multivariate statistics and multi-criterion decision making. He is the author or co-author of various papers published in Computers & Industrial Engineering, Journal of Manufacturing Systems, Applied Soft Computing, Neurocomputing, Applied Mathematical Modelling, Expert Systems with Applications, Applied Stochastic Models in Business and Industry, IEEE Transactions on Engineering Management, International Journal of Information Technology and Decision Making, TOP, Quality and Reliability Engineering International, Communications in Statistics-Simulation and Computation, Operational Research, International Journal of Advanced Manufacturing Technology, and Scientia Iranica.

**Biography:** Ahmad Hatami is a MS student in Department of Industrial Engineering, Faculty of Engineering, University of Qom. His current research interests include maintenance, warranty and multi-criterion decision making.
Figures and Tables

**Figure 1**: factors affecting the failure rate

**Figure 2**: illustration of the relationship between the manufacturer and the agent in the provision of warranty service and non-periodic maintenance policy
Figure 3: Two-dimensional warranty policy

Figure 4: option $r \leq \eta$

Figure 5: option $r > \eta$
Figure 6: Two-dimensional warranty and lifetime policy

Figure 7: Graphical representation of the improvement percentage in the comparative study (1)
Figure 8: Graphical representation the improvement percentage in the comparative study (2)
Figure 9: Graphical representation the improvement percentage in the comparative study (3)
| papers                        | point of view | Optimization | pricing |
|------------------------------|---------------|--------------|---------|
| Decroix. [21]                | ✓             | ✓            |         |
| Chun and Tang [6]            | ✓             | ✓            | ✓       |
| Lam et al. [13]              | ✓             | ✓            | ✓       |
| Baik et al. [7]              | ✓             | ✓            |         |
| Kim et al. [16]              | ✓             | ✓            |         |
| Teng. [26]                   | ✓             | ✓            | ✓       |
| Chen and Chien. [34]         | ✓             | ✓            | ✓       |
| Chien. [35]                  | ✓             | ✓            |         |
| Jack et al. [36]             | ✓             | ✓            |         |
| Hartman et al. [14]          | ✓             | ✓            |         |
| Chang and Lin. [12]          | ✓             | ✓            |         |
| Bouguerra et al. [11]        | ✓             | ✓            | ✓       |
| Shafiee et al. [37]          | ✓             | ✓            |         |
| Esmaeili et al. [9]          | ✓             | ✓            | ✓       |
| Su and Wang. [18]            | ✓             | ✓            | ✓       |
| Huang et al. [2]             | ✓             | ✓            | ✓       |
| Huang et al. [19]            | ✓             | ✓            | ✓       |
| Lin et al. [25]              | ✓             | ✓            |         |
| Park et al. [17]             | ✓             | ✓            |         |
| Salmasnia and Yazdekhashi [15]| ✓             | ✓            |         |
| Darghouth et al. [20]        | ✓             | ✓            | ✓       |
| Wang et al. [38]             | ✓             | ✓            | ✓       |
| This research                | ✓             | ✓            | ✓       |
Table 2. Notations

| Notations | Descriptions |
|-----------|--------------|
| **Index** |              |
| $j$       | Index of preventative activity |
| **Decision variables** | |
| $m$       | preventive maintenance level, $0 \leq m < M$ and integer; |
| $\tau_i$  | Time of the first non-periodic maintenance activity |
| $\theta$  | Technology level |
| $PW_E$    | extended warranty price |
| $P_P$     | Sale price of the product |
| **Parameter** | |
| $L,K$     | Lifetime and Life use of product |
| $U,W$     | Age and usage limits of two-dimensional warranty |
| $v(t)$    | Virtual age after implementing a $pm$ activity |
| $\delta(m)$ | Age reduction factor of $pm$ with level $m$ |
| $ir$      | Interested rate |
| $f(t|r,\theta)$ | time to failure density function |
| $F(t|r,\theta)$ | time to failure cumulative distribution function |
| $\beta(r,\theta)$ | Scale parameter for Weibull distribution as a function of $r,\theta$ |
| $\lambda(t|\sigma,\theta)$ | Conditional failure rate |
| $E[N_Y|\sigma]$ | the expected number of failures per unit item conditional on $R = \sigma$ |
| $E[CW_r|\sigma]$ | the expected warranty servicing cost per unit item conditional on $R$ |
| $RSC_X$   | the average revenue of the service contract under option $X$ |
| $C_{p_j}(m)$ | cost for the $j^{th}$ $pm$ action with level $m$ |
| $C_p$     | Production cost |
| $r$       | usage rate ($\frac{u}{l}$) |
| $\Pi_l$   | Profit of manufacturer and agent ($l = M, A$) |
| $\Pi_c$   | Customer satisfaction |
### Table 3. The values of the parameters in the numerical example

| parameter | $\alpha$ | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $a$ | $\phi$ | $c_1$ |
|-----------|-----------|------------|------------|------------|-----|-------|-------|
| value     | 2         | 0.23       | 0.8        | 0.5        | 0.002 | 0.1   | 0.005 |

| parameter | $\xi$ | $\alpha_e$ | $ir$ | $\psi_0$ | $\psi_1$ | $\theta_{\text{min}}$ | $\theta_{\text{max}}$ |
|-----------|-------|------------|------|----------|----------|-----------------------|-----------------------|
| value     | -0.35 | 0.28       | 0.15 | 1.5      | 1.5      | 0.1                   | 0.4                   |

| parameter | $\varphi$ | $Z_{\text{max}}^m$ | $Z_{\text{max}}^A$ | $Z_{\text{max}}^C$ | $Z_{\text{min}}^m$ | $Z_{\text{min}}^A$ | $Z_{\text{min}}^C$ |
|-----------|-----------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| value     | 0.85      | 10.6641              | 4                    | 39.022               | 1.5                  | 0.5                  | 1                    |

| parameter | $C_r$ | $\beta^i$ | $U_B$ | $W_B$ | $U_{\text{life}}$ | $W_{\text{life}}$ |
|-----------|-------|-----------|-------|-------|-------------------|-------------------|
| value     | 0.3   | 0.005     | 3     | 3     | 6                 | 6                 |

### Table 4. values of decision variables and objective functions

| decision variable | $P_p$ | $PW_E$ | $\theta$ | $\tau_i$ | $m$ |
|-------------------|-------|--------|----------|----------|-----|
| value             | 9.7731| 2.5976 | 0.2442   | 0.2124   | 4   |

| objective function | $\Pi_M$ | $\Pi_A$ | $\Pi_C$ |
|--------------------|----------|----------|----------|
| value              | 3.6031   | 2.6022   | 3.1012   |

### Table 5. Problems generated by Taguchi design $L_{27}$

| Instance | $C_r$ | $\beta^i$ | $U_B$ | $W_B$ | $U_{\text{life}}$ | $W_{\text{life}}$ | value of objective functions |
|----------|-------|-----------|-------|-------|-------------------|-------------------|-----------------------------|
| value   | agent profit | manufacturer profit | Customer satisfaction |
|---------|----------------|----------------------|-----------------------|
| 1       | 0.25 | 0.002     | 2     | 2     | 4.5               | 4.5               | 1.1768 | 2.5747 | 1.1789 |
| 2       | 0.25 | 0.002     | 2     | 2     | 6.0               | 6.0               | 2.6867 | 4.0881 | 2.6854 |
| 3       | 0.25 | 0.002     | 2     | 2     | 7.5               | 7.5               | 3.9912 | 5.3885 | 3.994 |
| 4       | 0.25 | 0.005     | 3     | 3     | 4.5               | 4.5               | 1.3744 | 2.7723 | 1.3763 |
| 5       | 0.25 | 0.005     | 3     | 3     | 6.0               | 6.0               | 2.6429 | 4.0443 | 2.6415 |
| 6       | 0.25 | 0.005     | 3     | 3     | 7.5               | 7.5               | 3.9538 | 5.351 | 3.9565 |
| 7       | 0.25 | 0.008     | 4     | 4     | 4.5               | 4.5               | 1.0092 | 2.9939 | 1.5207 |
| 8       | 0.25 | 0.008     | 4     | 4     | 6.0               | 6.0               | 2.5283 | 3.9283 | 2.5285 |
| 9       | 0.25 | 0.008     | 4     | 4     | 7.5               | 7.5               | 3.9166 | 5.3138 | 3.9192 |
| 10      | 0.3  | 0.002     | 3     | 4     | 4.5               | 6.0               | 1.7962 | 3.1977 | 1.7947 |
| 11      | 0.3  | 0.002     | 3     | 4     | 6.0               | 7.5               | 3.2097 | 4.6065 | 3.2127 |
| 12      | 0.3  | 0.002     | 3     | 4     | 7.5               | 4.5               | 2.112 | 3.5097 | 2.1143 |
| 13      | 0.3  | 0.005     | 4     | 2     | 4.5               | 6.0               | 1.5712 | 2.9691 | 1.5732 |
| 14      | 0.3  | 0.005     | 4     | 2     | 6.0               | 7.5               | 3.168 | 4.566 | 3.1716 |
| 15      | 0.3  | 0.005     | 4     | 2     | 7.5               | 4.5               | 1.8241 | 3.2249 | 1.8232 |
| 16      | 0.3  | 0.008     | 2     | 3     | 4.5               | 6.0               | 1.5188 | 2.919 | 1.5186 |
| 17      | 0.3  | 0.008     | 2     | 3     | 6.0               | 7.5               | 3.1292 | 4.5237 | 3.1303 |
| 18      | 0.3  | 0.008     | 2     | 3     | 7.5               | 4.5               | 2.2497 | 3.6469 | 2.2525 |
| 19      | 0.35 | 0.002     | 4     | 3     | 4.5               | 7.5               | 2.2651 | 3.664 | 2.2662 |
Table 6. Sensitivity analysis of the parameters on agent’s profit

| Level | $C_r$    | $\beta'$ | $U_B$    | $W_B$    | $U_{life}$ | $W_{life}$ |
|-------|----------|----------|----------|----------|------------|------------|
| 1     | 2.586656 | 2.4776   | 2.456511 | 2.361244 | 1.658789   | 1.658789   |
| 2     | 2.286544 | 2.432289 | 2.4358   | 2.466067 | 2.488378   | 2.488378   |
| 3     | 2.390578 | 2.353889 | 2.371467 | 2.436467 | 3.077633   | 3.116611   |
| $\Delta$ | 0.300111 | 0.123711 | 0.085044 | 0.104822 | 1.418844   | 1.457822   |
| Rank  | 3        | 4        | 6        | 5        | 2          | 1          |

Table 7. Sensitivity analysis of the parameters on manufacturer’s profit

| Level | $C_r$    | $\beta'$ | $U_B$    | $W_B$    | $U_{life}$ | $W_{life}$ |
|-------|----------|----------|----------|----------|------------|------------|
| 1     | 4.050544 | 3.866511 | 3.855156 | 3.760322 | 3.122689   | 3.122689   |
| 2     | 3.684833 | 3.831456 | 3.834789 | 3.854511 | 3.878667   | 3.878667   |
| 3     | 3.780056 | 3.817467 | 3.825489 | 3.9006   | 4.466256   | 4.514078   |
| $\Delta$ | 0.365711 | 0.049044 | 0.029667 | 0.104822 | 1.343567   | 1.391389   |
| Rank  | 3        | 5        | 6        | 4        | 2          | 1          |

Table 8. Sensitivity analysis of the parameters on customer satisfaction

| Level | $C_r$    | $\beta'$ | $U_B$    | $W_B$    | $U_{life}$ | $W_{life}$ |
|-------|----------|----------|----------|----------|------------|------------|
| 1     | 2.644556 | 2.4787   | 2.457378 | 2.362389 | 1.716656   | 1.716656   |
| 2     | 2.2879   | 2.555167 | 2.4368   | 2.467111 | 2.488133   | 2.488133   |
| 3     | 2.391133 | 2.411667 | 2.429411 | 2.494089 | 3.079      | 3.1188     |
| $\Delta$ | 0.356656 | 0.1435   | 0.027967 | 0.1317   | 1.362344   | 1.402144   |
| Rank  | 3        | 4        | 6        | 5        | 2          | 1          |
Table 9. Comparison of the proposed model with the model in which the agent is absent

| Model                                      | Instance | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------------------------------------------|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| The proposed model                         |          |     |     |     |     |     |     |     |     |     |
| manufacturer’s profit                      |          | 2.57| 4.08| 5.38| 2.77| 4.04| 5.35| 2.99| 3.92| 5.31|
| customer satisfaction                      |          | 1.17| 2.68| 3.99| 1.37| 2.64| 3.95| 1.52| 2.52| 3.91|
| The model without the presence of an agent |          |     |     |     |     |     |     |     |     |     |
| manufacturer’s profit                      |          | 2.35| 3.39| 4.24| 2.25| 3.41| 4.057| 2.73| 3.67| 4.53|
| customer satisfaction                      |          | 1.04| 1.99| 2.84| 1.05| 2.00| 2.25| 1.41| 2.16| 2.77|
| Improvement percentage of manufacturer’s profit |          | 9.56| 20.27| 26.97| 23.13| 18.57| 31.87| 9.57| 6.82| 17.11|
| Improvement percentage of customer satisfaction |          | 12.38| 34.478| 40.292| 30.85| 31.52| 75.190| 7.530| 16.649| 41.003|

Table 10. Comparison of the proposed model with the model without preventive maintenance (pm)

| Model                                      | Instance | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------------------------------------------|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| The proposed model                         |          |     |     |     |     |     |     |     |     |     |
| agent’s profit                             |          | 1.17| 2.68| 3.99| 1.37| 2.64| 3.95| 1.01| 2.53| 3.92|
| manufacturer’s profit                      |          | 2.57| 4.09| 5.39| 2.77| 4.04| 5.35| 2.99| 3.92| 5.31|
| customer satisfaction                      |          | 1.19| 2.68| 3.99| 1.38| 2.64| 3.96| 1.52| 2.53| 3.92|
| The model lacking pm                       |          |     |     |     |     |     |     |     |     |     |
| agent’s profit                             |          | 0.98| 1.59| 0.57| 1.14| 1.91| 1.75| 0.64| 2.14| 2.83|
The proposed model with the model lacking consideration of the time value of money.
| Model                                | Instance | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|--------------------------------------|----------|----|----|----|----|----|----|----|----|----|
| The proposed model                   | agent’s profit | 1.79 | 3.20 | 2.11 | 1.57 | 3.1 | 1.82 | 1.51 | 3.12 | 2.24 |
|                                      | manufacturer’s profit | 3.19 | 4.60 | 3.50 | 2.96 | 4.5 | 3.22 | 2.91 | 4.52 | 3.64 |
|                                      | customer satisfaction | 1.79 | 3.21 | 2.11 | 1.57 | 3.1 | 1.82 | 1.51 | 3.13 | 2.25 |
| The model lacking consideration of the time value of money | agent’s profit | 1.76 | 2.93 | 1.61 | 1.20 | 3.0 | 1.70 | 1.13 | 2.94 | 2.00 |
|                                      | manufacturer’s profit | 2.76 | 4.18 | 2.95 | 2.55 | 4.4 | 2.56 | 2.48 | 4.11 | 3.22 |
|                                      | customer satisfaction | 1.39 | 2.69 | 1.61 | 1.20 | 3.10 | 1.19 | 1.13 | 2.59 | 1.24 |
| Improvement percentage of agent’s profit | 1.73 | 9.45 | 30.5 | 30.34 | 2.24 | 6.84 | 33.76 | 6.10 | 12.1 |
| Improvement percentage of manufacturer’s profit | 15.63 | 9.95 | 18.65 | 16.04 | 1.58 | 25.7 | 17.49 | 9.91 | 13.18 |
| Improvement percentage of customer satisfaction | 28.57 | 19.11 | 30.65 | 30.32 | 2.15 | 53.21 | 33.91 | 20.62 | 80.98 |
| Model                                | Instance | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| The proposed model                   | agent’s profit | 2.26 | 1.80 | 3.25 | 2.21 | 1.92 | 3.21 | 2.19 | 1.46 | 3.17 |
|                                      | manufacturer’s profit | 3.66 | 3.20 | 4.56 | 3.61 | 3.32 | 4.61 | 3.59 | 2.86 | 4.57 |
|                                      | customer satisfaction | 2.26 | 1.80 | 3.25 | 2.22 | 1.91 | 3.21 | 2.19 | 1.46 | 3.17 |
| The model lacking consideration of the time value of money | agent’s profit | 1.82 | 1.59 | 3.20 | 2.17 | 1.78 | 3.20 | 2.12 | 0.94 | 3.16 |
|                                      | manufacturer’s profit | 3.31 | 2.53 | 4.43 | 3.38 | 3.18 | 4.24 | 3.51 | 2.23 | 4.19 |
|                                      | customer satisfaction | 2.07 | 1.80 | 3.19 | 1.99 | 1.78 | 2.72 | 2.12 | 0.94 | 2.64 |
| Improvement percentage of agent’s profit | 24.37 | 13.09 | 1.73 | 2.03 | 7.56 | 0.45 | 3.582 | 54.78 | 0.46 |
| Improvement percentage of manufacturer’s profit | 10.54 | 26.27 | 3.11 | 6.74 | 4.40 | 8.78 | 2.253 | 27.68 | 8.99 |
| Improvement percentage of customer satisfaction | 9.310 | 0.121 | 1.847 | 11.09 | 7.52 | 17.85 | 3.461 | 53.73 | 19.88 |

Average, maximum and minimum of improvement percentage at the agent’s profit in all of the instances: 11.5039, 54.7839, 0.2605
Average, maximum and minimum of improvement percentage at the manufacturer’s profit in all of the instances: 11.0229, 27.6818, 1.5813
Average, maximum and minimum of improvement percentage at the customer satisfaction in all of the instances: 20.7283, 80.9856, 0.1219