Determination of sign during phase correction of sign-variable modulation spectra of intersubband light absorption in GaAs/AlGaAs quantum wells

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Abstract. Modulation of intersubband light absorption in GaAs/AlGaAs quantum wells under external impact may contain different sign peaks. In this work, we show that fast Fourier transform may give mistakes in negative modulation intensity during the phase correction of obtained interferogram of absorption modulation. We have demonstrated that information about the spectrum sign could be extracted from the phase spectrum. Sign mistake could be eliminated using the phase correction with modified phase spectra during the fast Fourier transform.

1. Introduction

Infrared spectroscopy is a powerful analytic method which is used in fundamental and applied research of the optical properties of materials (monitoring of production processes, medicine, environmental monitoring and characterization of different materials) [1 – 4]. Nowadays the widely used method of spectral analysis in the infrared and terahertz spectral ranges is Fourier spectroscopy. It has several advantages over traditional dispersive infrared spectroscopy: the ability to simultaneously record the entire spectrum (radiation of all wavelengths from a source reaches the detector at the same time) and no need to use the narrow slits to increase the spectral resolution. One of the important advantages of the infrared Fourier spectroscopy is the possibility of multiplexing and receiving more powerful light emission, which allows to obtain a higher resolution as compared with traditional dispersive spectral instruments [5].

Fourier spectroscopy could be used in experimental study of intersubband and intracenter light absorption in quantum wells. Carrier transitions between the ground and excited impurity states in GaAs/AlGaAs quantum wells belong to the terahertz spectral range. The transitions between the resonant and localized states belong to the mid-infrared range. Also in this range lies intersubband transitions. Analysis of the spectra of intersubband and intracenter absorption of polarized light in electric field allows to estimate the contribution of partial impurity ionization by an electric field to the possibility of the emission of radiation related to intracenter hole transitions. In such experiment the sign of the modulation intensity could be both positive and negative. Standard automatic phase correction methods assign the positive sign to all extremes in spectrum and it is necessary to use the phase correction with stored phase which we demonstrate in this work. This work precedes the study of the intracenter light absorption in beryllium doped GaAs/AlGaAs quantum wells.
2. Basic theoretical approach

Fourier spectrometers usually operate with positive signals only, namely with photoresponse of detector excited with incident light. The result of the fast Fourier transform (FFT) of the measured interferogram is the complex spectral function of amplitude $C(\omega)$, but not the real spectral function $S(\omega)$ obtained using traditional dispersive spectrometers ($S$ and $s$ in this case are different values):

$$C(\omega) = s(\omega) \exp(i \varphi(\omega)),$$

(1)

where $\varphi(\omega)$ is the phase of the sign-variable signal, which depends on the light frequency.

Today, there are a number of phase correction methods (methods of obtaining a real spectral function $S$): multiplication of the spectrum by the complex conjugate function, the Foreman method, the Mertz method etc. Less sensitive to interferogram noise is the Mertz method [6], which consists in taking the real part of the multiplications of $C(\omega)$ by an inverted phase factor:

$$S(\omega) = \text{Re}[C(\omega) \exp(-i \varphi(\omega))] = \text{Re}[s(\omega)] = s(\omega).$$

(2)

It is assumed in the standard methods of Mertz and Forman that $S(\omega) > 0$ at all frequencies [7]. Sometimes, this condition is not valid. Standard phase correction of signal which has extremes of different sign leads to an incorrect result. Negative spectral function of amplitude can be represented in two ways:

$$-s(\omega) \exp(i \varphi(\omega)) = s(\omega) \exp(i \varphi(\omega + \pi)).$$

(3)

Let's consider how the formula (2) will change if the phase $\varphi$ is shifted to $\pi$ (the peak is negative):

$$C_-(\omega) = -s(\omega) \exp(i \varphi(\omega)) = s(\omega) \exp(i \varphi(\omega + \pi)),$$

(4)

$$S_-(\omega) = \text{Re}[C_-(\omega) \exp(-i \varphi(\omega + \pi))] =$$

$$= \text{Re}[s(\omega) \exp(i \varphi(\omega + \pi)) \exp(-i \varphi(\omega + \pi))] = \text{Re}[s(\omega)].$$

(5)

Comparing the equations (2) and (5) we see that the sign is lost:

$$S_-(\omega) = S(\omega).$$

(6)

So, the standard phase correction method gives a positive sign to all extremes and the resulting spectrum have incorrect sign of negative peaks.

3. Results and discussions

In this work, at first we used PC and simulated the direct and inverse fast Fourier transforms of two model spectra containing peaks of the same sign and peaks of the different sign. Carrying out the standard Mertz phase correction of the spectra with the same sign leads to the correct results of direct and inverse FFT. Another result was demonstrated for the fast Fourier transforms of the spectra with negative peak in the central part of the spectra. After direct and inverse FFT the information about sign of central peak was missed and the entire amplitude spectrum of $S(\omega)$ turned out to be of the same sign (graph 1 in figure 1). So, any standard phase correction method may contain a phase error by $\pi$. In order to obtain information about proper sign of the signal during the FFT, we also calculated the phase spectrum $\varphi(\omega)$ (graph 2 in figure 1) from the interferogram. One can see that there is a phase...
shift by $\pi$ (180 degree) in the region of the central peak, indicating a different sign of these peaks in amplitude spectrum. Figure 2 shows a part of the interferogram for a spectrum with peaks of the same sign (graph 2) and with peaks of the different sign (graph 1). One interferogram is out-phasing to another. This is associated with a phase shift. So, in order to obtain the proper sign of the peaks in the amplitude spectra, during carrying out the phase correction by the formula (2) it is necessary to replace the inverted phase factor with sharp phase shift by $\pi$ with a smooth (obviously positive) phase spectrum without shift. The phase spectrum is determined by the interferometer and the experimental setup parameters. So, the smooth phase spectrum of positive sign must be saved during receiving the sign-variable spectrum under the condition of high accuracy of reproducibility of the location of moving mirror. Sometimes it is possible, but this is very hard and information of the sign change can be obtained only from the phase spectrum. Our theoretical modeling shows that full information about the signal sign can be easy obtained from the phase spectrum. The proper spectrum can be achieved by FFT with phase correction by manually removed the 180 degree phase shift (graph 2 in figure 1).

![Figure 1](image1.png)  
**Figure 1.** Amplitude spectrum $S_-(\omega)$ containing an error in phase on $\pi$ (graph 1, left axis), phase spectrum $\phi(\omega)$ (graph 2, right axis).

![Figure 2](image2.png)  
**Figure 2.** Interferogram for spectra with peaks of the same sign (graph 2) and with peaks of the different signs (graph 1).

We used abovementioned Mertz phase correction method with smoothed stored phase in investigation of the modulation spectrum of intersubband light absorption in tunnel-coupled GaAs/AlGaAs quantum wells in longitudinal and transverse electric fields. Effect of modulation is associated with heating of charge carriers and their redistribution between quantum wells [8, 9, 10]. The experimentally obtained spectrum of absorption modulation is absolutely positive (graph 1 in figure 3). It can be seen from the phase spectrum of the absorption change, that there are $\pi$ shifts of phase (graph 2 in figure 3). Therefore, in these points the sign of the signal should change. To obtain the proper modulation spectrum of absorption, the interferogram of the constant amplitude spectrum (the phase spectrum in this case is smooth) was measured simultaneously with the measurement of the interferogram of the sign-variable spectrum. The phase spectrum of the first interferogram, obviously, does not have sharp phase shift. We can invert this phase spectrum and use in equation (2) during the phase correction in FFT of the second interferogram. Taking into account equation (1) the inverted phase factor of the constant spectrum:

$$
\varphi'(\omega) = \arctg \left( \frac{\text{Im}[C'(\omega)]}{\text{Re}[C'(\omega)]} \right),
$$

(7)

where $C'(\omega)$ is complex spectral function of amplitude of the constant spectrum.

The phase spectrum $\varphi'(\omega)$ of the interferogram of the constant spectrum was used during the phase correction in FFT of the modulated spectrum according to formulas (2) and (5). The inverted phase
factor containing the $\pi$ phase shift was replaced with the phase factor of the smooth phase spectrum $\phi'(\omega)$. The resulting proper modulation spectrum shows peaks of different signs. This was confirmed by analyzing the time-resolved modulation spectra of the intersubband absorption in GaAs/AlGaAs quantum wells in electric fields found by subtracting the radiation intensity in the absence of field from the transmitted radiation intensity in an electric field. In such experiment both spectra were definitely positive.

Our present modeling shows that the simultaneous measuring of two signals like in [8, 9, 10] can be avoided. It is sufficient to measure one interferogram and obtain the phase and amplitude spectra from it.

Obtained results can be used in analyzing the modulation of intersubband light absorption in electric field in beryllium doped quantum wells GaAs/AlGaAs. We also will apply this model to interpret the emission spectra associated with plasmons in GaN/AlGaN heterojunction [11] and GaAs epilayers [12, 13] in electric field.

![Figure 3](image)

**Figure 3.** Amplitude spectrum (graph 1, right axis) and the corresponding phase spectrum (graph 2, left axis) of intersubband absorption modulation of tunnel-coupled GaAs/AlGaAs quantum wells under transverse electric field.

4. Conclusion

We showed that in analyzing the sign-variable signal from Fourier spectrometer it is necessary to record phase spectrum and to use it manually at phase correction during fast Fourier transform. These results can be useful in studies of intersubband and intracenter absorption in multipass geometry in electric field at low temperatures in beryllium doped GaAs/AlGaAs quantum wells.

Acknowledgments

This work was supported by the Russian Science Foundation under grant No. 18-72-00034.

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