Electric Characteristics of Rotational States positive parity in isotopes $^{170,172,174}\text{Yb}$

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Abstract

Accounting for Coriolis mixing of experimentally known rotational bands with $K^\pi < 3^+$, non-adiabatic effects in energy and electric characteristics of excited states are investigated, within phenomenological model. The energy and wave function structure of excited states are calculated. The finding reveals that the bands mixing has been found to have considerable impact on the wave function of low-lying states $0^+$ and $2^+$ bands. In addition, the probabilities of $E2$– transitions have been calculated. The values from calculations of $B(E2)$ transitions from $0^+_2$, $0^+_3$, $2^+_1$, and $2^+_2$ bands are compared with the experimental data.

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I. INTRODUCTION

Despite the fact that the structure of deformed nuclei and nature of low excited levels have been substantially studies over more than four decades, this still occupies a central part of today’s research [1]-[3].

An extensive research interest in the properties of deformed nuclei has risen in recent years with the exploration of a new collective isovector magnetic dipole mode $[4, 5]$. The measured values of excited energy of magnetic mode are found to be not so high in an excited spectrum, and consideration of mixing with low-lying exciting states appear to lead to an interesting physical phenomena $[6, 7]$.

The nuclei $^{170,172,174}\text{Yb}$ have been well studied. It is important to note that these are investigated in a number of ways such as radioactive decay of $^{170,172,174}\text{Lu}$, and different nuclear reactions. In these isotopes, many $1^+$ states and $K^\pi = 0^+$, $2^+$ bands have been observed. For instance, the excited energy $K^\pi = 0^+$ and $2^+$ it rises with the increase in number of neutrons (see Figure 1).

The values of probability of $B(E2)$ with low-lying levels of $K^\pi = 0^+$, $2^+$ bands, and also Rasmusson’s parameter value $X_1(E0/E2)$, dimensionless units matrix element $\rho(E0)$ for the $E0$– transition and a multipole mixture coefficients $\delta(E2/M1)$ are defined experimentally $[8-10]$.

Numerous conducted experiments on defining spectroscopic characteristics of low-lying exciting states, particular $K^\pi = 1^+$ in deformed nuclei $[5]$, have motivated the further theoretical investigations. In this case, investigations influence of $K^\pi = 1^+$ states to the properties of low-lying levels is actual.

FIG. 1: (Color online) The energy of $I = 2$ states of $K^\pi = 0^+_2$, $0^+_3$, $0^+_4$, $2^+_1$, $2^+_2$ and $2^+_3$ bands in isotopes $^{170,172,174}\text{Yb}$.
II. THE MODEL

To analyze the properties of low-lying positive parity states in $^{170,172,174}$Yb, the phenomenological model [3] is exploited. This model takes into account the mixing of states of the $K^\pi = 0^+, 2^+$ and $1^+$ bands. The Hamiltonian model is

$$H = H_{rot}(I^2) + H_{K,K'}$$

(1)

$$H^K_{K',K}(I) = \omega_K \delta_{K,K'} - \omega_{rot}(I)(j_z)_{K,K'}(I,K)\delta_{K,K'\pm 1}$$

(2)

where $\omega_K$ - bandhead energy of rotational band, $\omega_{rot}(I)$ - an angular frequency of rotational nucleus, $(j_z)_{K,K'}$ - matrix elements which describe Coriolis mixture between rotational bands and

$$\zeta(I,0) = 1 \quad \zeta(I,2) = \left[1 - \frac{2}{I(I+1)}\right]^\frac{1}{2}$$

The eigenfunction of Hamiltonian model (1) is

$$\Psi_{MK} = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \sqrt{2}\psi_{gr,K}^{I}D_{MK}^{I}(\theta) + \sum_{K'} \frac{\psi_{K',K}^{I}}{\sqrt{1 + \delta_{K',0}}} \left[ D^{I}_{K,K'}(\theta)b_{K'}^{+} + (-1)^{I+K'}D^{I}_{K'-K}(\theta)b_{-K'}^{+} \right] \right\} |0>$$

(3)

here $\psi_{K',K}^{I}$ is the amplitude of mixture of basis states.

The rotational part of Hamiltonian $H_{rot}(I)$ is diagonal by wave functions (4). Note that $H_{rot}(I)$ is determined by exploiting Harris parameterization for energy and angular momentum [11]

$$E_{rot}(I) = \frac{1}{2} \Im_{0}\omega_{rot}^2(I) + \frac{3}{4} \Im_{1}\omega_{rot}^4(I)$$

(4)

$$[I(I+1)]^{1/2} = \Im_{0}\omega_{rot}(I) + \Im_{1}\omega_{rot}^3(I)$$

(5)

where $\Im_{0}$ and $\Im_{1}$ - are the inertia parameters of the rotational core.

The rotational frequency of the core $\omega_{rot}(I)$ is found by solving cubic equation (6). This equation has two imaginary roots and one real root. The real root is as follows

$$\omega_{rot}(I) = \left\{ \frac{1}{233} + \left( \frac{1}{233} + \left( \frac{3}{663} \right)^3 \right)^{1/4} \right\}$$

(6)

$$H^{\sigma}_{K,K} \Psi_{K,K'}^{I} = E_{K}^{I}(I)\Psi_{K,K'}^{I}$$

(7)

where $\sigma = \pm 1$.

A. Energy spectra and structures of the states

The calculations have been carried out for the isotopes $^{170,172,174}$Yb. All experimentally known rotational bands of positive parity with $K^\pi < 3^+$ have been included in basis Hamiltonian states.

The experiment suggests that $m = 5$ band with $K^\pi = 0^+$, one band $\ell = 1$ with $K^\pi = 2^+$, and $\nu = 19$ with $K^\pi = 1^+$ states in $^{170}$Yb [3]. These all $n = m+\ell+m = 25$ rotational bands have been included in the basis states of Hamiltonian (1). For the isotopes $^{172,174}$Yb, basis states of Hamiltonian include $n = 15$ ($m = 5$, $\ell = 2$ and $\nu = 8$) and $n = 22$ ($m = 5$, $\ell = 2$ and $\nu = 15$), correspondingly [3, 12].

The parameters of inertia $\Im_{0}$ and $\Im_{1}$ are estimated by exploiting Harris parameterization [11], and using the experimental data for energy up to spin $I \leq 8\hbar$ for ground band [12].

The Hamiltonian (2) has transformational properties, that the state $\sigma = \pm 1$ signature, which imposes restrictions on angular momentum values.

$$(-1)^{I}\sigma = 1.$$  

For the states with negative signature $\sigma = -1$, Hamiltonian (2) has dimension $n = \ell + \nu$, as in bands with $K^\pi = 0^+_m$ the are no condition states with odd spins $I$. For the states with positive signature $\sigma = +1$, Hamiltonian (2) has dimension $n = m + \ell + \nu$.

The model parameters are described as follows:

a) the bandhead energy ground $\omega_{01}$ and $K^\pi = 0^+_m$ bands has taken from experiment, as they are not revoluted by Coriolis force. Bandhead energy of $1^+_m$ bands
are also defined from an experiment [8]

\[ \omega_{1\nu} = E_{1\nu}^{exp}(I = 1) - E_{rot}(I = 1); \]

b) matrix elements \((j_x)_{2\nu} = (j_x)_{2\nu 1}\) and bandhead energy of \(2^+_1\) bands \(\omega_{2\nu}\) are determined from the most favored experimental and theoretical spectrum of energy states with a negative signature \(\sigma = -1\), e.a. for energy state for even spins \(I;\)

c) the matrix elements \((j_x)_{0m} = (j_x)_{0m 1}\) defined by the least square method from the best fitted of theoretical energy spectra state with positive signature \(\sigma = +1\) with experimental data.

The obtained values of model parameters are presented in Table 1.

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| \(A\) | \((j_x)_{01,1}\) | \((j_x)_{02,1}\) | \((j_x)_{03,1}\) | \((j_x)_{04,1}\) | \((j_x)_{05,1}\) | \((j_x)_{21,1}\) | \((j_x)_{22,1}\) |
|-------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 170   | 0.1864           | 0.3936           | 0.6586           | 0.9081           | 0.0009           | 0.7278           |
| 172   | 0.2754           | 0.9777           | 0.7176           | 0.11             | 0.30             | 0.325            | 0.21             |
| 174   | 0.185            | 0.4              | 0.25             | 0.15             | 0.20             | 0.085            | 0.1              |

Note: \((j_x)_{K',K}^\pm\) are matrix elements of the Coriolis interactions.

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Calculation comparison of energy with experimental values for \(^{170,172,174}\)Yb is illustrated in Figures 2, 3 and 4, correspondingly.

Apparently, one may see from the Figures that the model qualitatively reproduces experimental energy of rotational states up to energy \(3\text{MeV}\). However, in high spin values \(I \geq 12\hbar\) noticeable deviation has been observed in calculated values of energy and that obtained from experiment. Note that this deviation increases with the growth of angular momentum \(I\). This is probably due to the fact that the influence of rotation on internal nuclei structure has not been considered in this model. In future, we will study electromagnetic properties of low-lying states \(I < 10\hbar\).

Amplitude of the states \(\Psi_{K',K}^\pm\) for \(K^\pi = 2^+_1\) and \(0^+_2\) bands for \(^{170,172,174}\)Yb, are provided in Figures 5, 6, 7, 8, 9 and 10, respectively. The components which have small values are not illustrated in Figure. Also the components \(\Psi_{1p,K}\) band are not given except for the first \(1^+_1\). The values for others \(\Psi_{1^+_1,K}\) states are define as follows

\[
\Psi_{1\nu,K} = \frac{\omega_{1\nu} - \omega_K}{\omega_{1\nu} - \omega_K}. \tag{9}
\]

From Figures 5 and 7, we can see that \(K^\pi = 0^+_2\) and \(K^\pi = 2^+_1\) bands states in \(^{170}\)Yb, \(K^\pi = 0^+_3\) and \(K^\pi = 2^+_1\) bands in \(^{172}\)Yb are mixed strongly even in low spin values \(I\). It is associated with the close location to each other (see Figure 1). In isotopes \(^{170,172}\)Yb, considerable deviation in signature of the states \(K^\pi = 2^+_1\) band can be observed. This reflects in the values of probability electromagnetic transitions. In this case, description of quantum number \(K\) is difficult for these states. Thus, in \(^{170}\)Yb, a number of research works [13] in this context note that the states with \(I = 2^+\) (1.1386 MeV) and \(I = 2^+\) (1.1454 MeV) \(K = 2\) and \(K = 0\), respectively. On the other hand, some works [8] document that \(K = 0\) and \(K = 2\), correspondingly. In case \(^{174}\)Yb, the mixture effect is not so strong.

III. ELECTRIC QUADRUPOLE TRANSITIONS

With the wave functions calculated by solving the Shrödinger equation [7], reduced probabilities of \(E2\)–transitions between states \(I_iK_i\) and states of ground band \(I_f01\) are calculated [8].

\[
B(E2; I_iK_i \rightarrow I_f0_1) = \left\{ \frac{5}{16\pi} e Q_0 \left[ \Psi_{I_f}^{I_f} \Psi_{0_1}^{I_f} C_{I_f0_1}^{I_f} + \sum_n \Psi_{K_n0_1}^{I_f} \Psi_{K_nK}^{I_f} C_{I_fK_n}^{I_f} \right] \right\}^2 \tag{10}
\]
Table II: The values of the parameters $m_K$ and the intrinsic quadrupole moment $Q_0$, which are used in calculations (in $\text{fm}^2$)

| $A$ | $m_{0;1}$ | $m_{0;3}$ | $m_{0;5}$ | $m_{1;\nu}$ | $m_{2;1}$ | $m_{2;3}$ | $Q_0$ |
|-----|-----------|-----------|-----------|-------------|-----------|-----------|-------|
| 170 | 2         | 24        | 3         | 8           | -5        | 19        | 8     |
|     | 172       | 10        | 1         | 6.9         | 8         | -5        | 8     |
|     | 174       | 8         | 1         | -6.9        | 8         | -10       | 8     |

Table 3 compares reduced probability $E2$– transitions with existing experimental data [8-11, 14]. Moreover, reduced matrix elements of $E2$– transitions for $^{172}\text{Yb}$ are provided in Table 4. In a similar vein, these values are also compared with experimental values as well as values found by using other models [15-17].

Table III: Reduced probability of $E2$– transitions in the isotopes $\text{Yb} (\text{e}^2 \text{fm}^4)$

| $A$ | $I_fK_f \rightarrow I_iK_i$ | Exp. Theory | $I_fK_f \rightarrow I_iK_i$ | Exp. Theory |
|-----|-----------------------------|-------------|-----------------------------|-------------|
| $^{170}\text{Yb}$ | 22, $\rightarrow$ 00, 00 | 151(35)[8] | 90 | 00, $\rightarrow$ 20, 00 | 60(15)[8] |
|     | $\rightarrow$ 20 | 269(60)[8] | 60 | 00, $\rightarrow$ 20, 20 | 567(118)[8] |
|     | $\rightarrow$ 40 | 27(6)[8] | 10 | 00, $\rightarrow$ 20, 40 | 18 |
| $^{172}\text{Yb}$ | 22, $\rightarrow$ 00, 75.6(63)[9] | 74.6(57)[14] | 82 | 00, $\rightarrow$ 20, 00 | 205(60)[9] |
|     | $\rightarrow$ 20 | 121(12)[14] | 130 | 20, $\rightarrow$ 00, 14(1)[9] | 14(1)[14] |
|     | $\rightarrow$ 40 | 6.8(7)[14] | 8.6 | 20, $\rightarrow$ 20, 45(7)[9] | 52(8)[14] |
|     | $\rightarrow$ 42 | 398(284)[9] | 15 | 40, $\rightarrow$ 40, 142(20)[9] | 140(20)[14] |
|     | $\rightarrow$ 40 | 739(512)[9] | 81 | 00, $\rightarrow$ 20, 40 | 0.14(3)[9] |
|     | 32, $\rightarrow$ 20 | 152(11)[14] | 154 | 20, $\rightarrow$ 00, 0.4(1)[9] | 3.4(2)[14] |
|     | $\rightarrow$ 40 | 79.6(14) | 73 | 20, $\rightarrow$ 20, 0.6(4)[9] | 11.9(8)[14] |
|     | $\rightarrow$ 22 | 20(4)[9] | 32(4)[14] | 23 | 20, $\rightarrow$ 40 | 1.0(1)[14] |
|     | $\rightarrow$ 20, 31(2)[9] | 51(7)[14] | 38 | 00, $\rightarrow$ 20 | 0.25(9) |
|     | $\rightarrow$ 40 | 3.3(4)[14] | 2.2 | 20, $\rightarrow$ 00, 10(6)[9] |
|     | $\rightarrow$ 32 | 54(7)[14] | 42 | 00, $\rightarrow$ 20 | 0.27(9) |
|     | $\rightarrow$ 40 | 22(3)[14] | 21 | 20, $\rightarrow$ 00, 19(9)[9] |
| $^{174}\text{Yb}$ | 00, $\rightarrow$ 20 | 81(50)[10] | 64 | 22, $\rightarrow$ 20, 144(30)[10] | 133 |

It is important to note that our results are obtained consecutively. In the initial step, energy and wave function the states are computed. Further, by utilizing these...
wave functions, reduced probability of $E2$– transitions are calculated. From the Table 4, one may gather that

performed calculations within our model provide a better correspondence with experiment data.

TABLE IV: Reduced Matrix Elements of $E2$– transitions in $^{172}\text{Yb}$, calculated within our model which comparison with experimental data [15] and are calculated using the rotational-vibrational model (RVM2) [16] and the IBA-1 model [17] (eb)

| $I_1 K_1 \rightarrow I_f K_f$ | Exp. | RVM2 | IBA-1 | Theory | $I_1 K_1 \rightarrow I_f K_f$ | Exp. | RVM2 | IBA-1 | Theory |
|-----------------------------|------|------|-------|--------|-----------------------------|------|------|-------|--------|
| $20_1 \rightarrow 20_1$     | -2.92| -2.92| -2.93 |        | $20_1 \rightarrow 00_1$     | 2.45 | 2.45 | 2.45 | 2.45 |
| $40_1 \rightarrow 40_1$     | -3.69| -3.74| -3.74 |        | $40_1 \rightarrow 20_1$     | 3.76 | 3.93 | 3.91 | 3.93 |
| $60_1 \rightarrow 60_1$     | -4.46| -4.46| -4.46 |        | $60_1 \rightarrow 40_1$     | 5.34 | 4.97 | 4.90 | 4.96 |
| $80_1 \rightarrow 80_1$     | -5.08| -5.08| -5.08 |        | $80_1 \rightarrow 60_1$     | 5.90 | 5.80 | 5.60 | 5.80 |
| $100_1 \rightarrow 100_1$   | -5.63| -5.63| -5.63 |        | $100_1 \rightarrow 80_1$    | 6.71 | 6.54 | 6.29 | 6.54 |
| $120_1 \rightarrow 120_1$   | -6.15| -6.15| -6.15 |        | $120_1 \rightarrow 100_1$   | 7.01 | 7.19 | 6.79 | 7.20 |
| $140_1 \rightarrow 140_1$   | -6.62| -6.62| -6.62 |        | $140_1 \rightarrow 120_1$   | 8.12 | 7.80 | 7.18 | 7.81 |
| $22_1 \rightarrow 00_1$     | 0.20+0.24 | 0.21 | 0.20+0.24 | 0.203 | $22_1 \rightarrow 20_1$     | 0.16 | 0.27 | 0.01 |        |
| $22_1 \rightarrow 20_1$     | 0.25+0.016 | 0.25 | 0.31+0.25 | 0.255 | $22_1 \rightarrow 40_1$     | 0.16 | 0.26 | 0.082 |       |
| $22_1 \rightarrow 40_1$     | 0.063+0.009 | 0.062 | 0.10+0.066 | 0.106 | $22_1 \rightarrow 60_1$     | -1.62+0.071 | 0.19 | -0.31 | 0.108 |   |
| $22_1 \rightarrow 60_1$     | 0.22+0.07 | 0.20 | 0.13+0.11 | 0.11 | $22_1 \rightarrow 80_1$     | -0.08 | 0.27+0.02 | 0.26 | 0.19 | 0.13 |
| $22_1 \rightarrow 80_1$     | 0.46+0.08 | 0.38 | 0.45+0.27 | 0.27 | $22_1 \rightarrow 100_1$    | -0.13 | -0.08 | -0.08 | -0.08 | |
| $22_1 \rightarrow 100_1$    | 0.32(11) | -0.328 |        |        | $22_1 \rightarrow 120_1$    | -0.26 | -0.08 | -0.08 | -0.08 | |
| $22_1 \rightarrow 120_1$    | 0.235(6) | -0.226 |        |        | $22_1 \rightarrow 140_1$    | -0.27 | -0.08 | -0.08 | -0.08 | |

$^a$) This matrix element was used to normalize the results of the model calculations.

To evaluate the degree of nonadiabaticity, manifested in the reduced probabilities of $E2$– transitions, in Table 5 theoretical ratios $R_{IK}$ has been compared with their adiabatic values $R_{IK}^{ad}$ as well as experimental data [8, 10, 13, 14, 18] which is determined as follows

$$R_{IK} = \frac{I_{\gamma}(IK \rightarrow I_1 0_1)}{I_{\gamma}(IK \rightarrow I_3 0_1)} \left( \frac{E_{\gamma}(IK \rightarrow I_2 0_1)}{E_{\gamma}(IK \rightarrow I_1 0_1)} \right)^\ell (12)$$

In $^{170}\text{Yb}$, experimental ratio $R_{102}$ for $E2$– transitions from states $K^\pi = 0^+_2$ band differ from adiabatic theory considerably (10–40 times). This is associated with mixing $0^+_2$ and $2^+_1$ bands. An important question can raised in this regard. Why do the ratios $R_{102}$ for transitions from $2^+_1$ bands differ not so strongly from adiabatic theory with respect to $R_{102}$?

This results can be explained by the fact that the matrix element $m_{02}$ is greater about 10 times than that of $m_{10}$ (see Table 2). One may see from this comparison that the mixing effect of states low-lying bands plays a crucial role which considerably demonstrates that $E2$– transitions even in low values of angular momentum $I$.

IV. CONCLUSION

In the present work, non-adiabatic effects in energies and electric characteristics of excited states are studied within the phenomenological model which taking into account Coriolis mixing of all experimentally known rotational bands with $K^\pi < 3^+$.

The energy and structure of wave functions of excited states are calculated. And also the reduced probabilities of $E2$– transitions is calculated. The ratio of $E2$– transitions probability from $K^\pi = 0^+_m$ and $2^+_1$ bands are calculated and compared with experimental data which gives the satisfactory result.

If matrix elements of $E2$– transitions $m_K$ one of two strongly mixing bands $K$ is less than matrix element of
TABLE V: The ratios of reduced probabilities of $E2$– transitions $R_{iK} = B(E2; I_iK_i → I_f0_1)/B(E2; I_iK_i → I_f'0_1)$ in isotopes $^{22}Yb$

| $A$ | $I_iK_i$ | $I_fK_f$ | $x$ | Experiments | Theory | Alaga |
|-----|----------|----------|-----|-------------|-------|-------|
| $^{174}Yb$ | $22^+_1$ | $20^+_1$ | $00^+_1$ | $1.77(8)$ | $1.86$ | $1.43$ |
|      | $40^+_1$ | $20^+_1$ |       | $0.998(11)$ | $0.043$ | $0.050$ |
|      | $32^+_1$ | $40^+_1$ | $20^+_1$ | $0.784(4)$ | $0.75$ | $0.40$ |
|      | $52^+_1$ | $60^+_1$ | $40^+_1$ | $1.39(46)$ | $1.50$ | $0.57$ |
|      | $72^+_1$ | $80^+_1$ | $60^+_1$ | $1.27(24)$ | $2.42$ | $0.67$ |

| $A$ | $I_iK_i$ | $I_fK_f$ | $x$ | Experiments | Theory | Alaga |
|-----|----------|----------|-----|-------------|-------|-------|
| $^{176}Yb$ | $22^+_1$ | $20^+_1$ | $00^+_1$ | $1.94(52)$ | $1.1$ | $1.43$ |
|      | $40^+_1$ | $20^+_1$ |       | $3.73(90)$ | $1.82$ | $0.91$ |
|      | $60^+_1$ | $60^+_1$ | $40^+_1$ | $10.7(19)$ | $1.67$ | $0.81$ |
|      | $80^+_1$ | $80^+_1$ | $60^+_1$ | $25.3(85)$ | $2.45$ | $0.77$ |
|      | $100^+_1$ | $100^+_1$ | $80^+_1$ | $29.9(71)$ | $29.2$ | $0.74$ |
|      | $120^+_1$ | $120^+_1$ | $100^+_1$ | $2.5$ | $5.75$ | $2.57$ |

$^{m_{K'}}$, of $K'$ ($m_K < m_{K'}$), then the difference in the ratio $R_{iK}$ for the first band $K$ from Alaga rule is bigger than the difference in $R_{iK'}$ from Alaga rule. In other words, if $m_K < m_{K'}$, nonadiabaticity in the ratio $R_{iK}$ is stronger than that of $R_{iK'}$.

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FIG. 2: (Color online) Comparison of the calculated and experimental energy spectra of positive-parity states for $^{170}$Yb.
FIG. 3: (Color online) Comparison of the calculated and experimental energy spectra of positive-parity states for $^{172}$Yb.

FIG. 4: (Color online) Comparison of the calculated and experimental energy spectra of positive-parity states for $^{174}$Yb.
FIG. 5: (Color online) Structure of the wave-functions of $2^+_1$–band states for $^{170}$Yb.

FIG. 6: (Color online) Structure of the wave-functions of $0^+_2$–band states for $^{170}$Yb.
FIG. 7: (Color online) Structure of the wave-functions of $2^+_1$–band states for $^{172}$Yb.

FIG. 8: (Color online) Structure of the wave-functions of $0^+_2$–band states for $^{172}$Yb.
FIG. 9: (Color online) Structure of the wave-functions of $2^+_1$–band states for $^{174}$Yb.

FIG. 10: (Color online) Structure of the wave-functions of $0^+_2$–band states for $^{174}$Yb.