Trans-sonic propeller stage

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Abstract

We follow the approach used by Davies and Pringle (1981) and discuss the trans-sonic substage of the propeller regime. This substage is intermediate between the supersonic and subsonic propeller substages. In the trans-sonic regime an envelope around a magnetosphere of a neutron star passes through a kind of a reorganization process. The envelope in this regime consists of two parts. In the bottom one turbulent motions are subsonic. Then at some distance \( r_s \) the turbulent velocity becomes equal to the sound velocity. During this substage the boundary \( r_s \) propagates outwards till it reaches the outer boundary, and so the subsonic regime starts.

We found that the trans-sonic substage is unstable, so the transition between supersonic and subsonic substages proceeds on the dynamical time scale. For realistic parameters this time is in the range from weeks to years.

Keywords: compact objects, isolated neutron stars, evolution

1 Introduction

Observational appearances of a neutron star (NS) are mainly determined by its interaction with the surrounding plasma. The following main stages (regimes) can be distinguished (see a very detailed description in Lipunov 1992 or in Lipunov et al. 1996)

- **Ejector.** At this stage plasma is swept away by low-frequency electromagnetic radiation or/and by a flow of relativistic particles. Matter is stopped further than the so-called light cylinder radius \( r_L \).

- **Propeller.** If a NS is in the propeller regime than matter can penetrate inside \( r_L \), but it is stopped by a rapidly rotating magnetosphere of the NS.

- **Accretion.** Finally the NS is slowed down and the centrifugal barrier disappears, so if matter cools fast enough then it can fall down onto the surface of the compact object.

Normally a NS is born at the stage of ejection (a radio pulsar is a classical example of a NS at the ejector stage). Then as the spin period increases the NS passes
propeller and accretor stages. For NSs with large (>400 km/s) spatial velocities another stage — Georotator — can appear.

In a simplified model it is possible to define transitions between different stages by comparing external and internal pressure (Fig. 1). The external one can be roughly approximated as the ram pressure of a flow of the interstellar medium (far from the NS) or as the pressure of matter falling down onto the NS in its gravitational field (for distances smaller than the so-called gravitational capture radius $r_G$). The internal one inside the light cylinder $r_\ell$ can be estimated as a pressure connected with the magnetic field of a NS.

In this paper we discuss the propeller stage. An existence of it was recognized long ago (initially by Shvartsman (1970) and later on by Illarionov and Sunyaev (1975). However still this stage is not well understood. Here we introduce and discuss a new substage of this regime of magneto-rotational evolution of NSs.

2 General features of the propeller stage

Davies et al. (1979) and Davies, Pringle (1981) note that at the stage of propeller there can be a significant energy release at the magnetospheric boundary. Energy can be large enough to form a kind of a turbulent quasistatic atmosphere. These authors distinguished three substages of the propeller regime.

1. Very rapid rotator:

\[ c_s(r_{in}) \simeq r_{in}\Omega \gg v_{ff} \, . \]  

here $c_s$ — sound velocity, $\Omega = 2\pi/P$ — spin frequency, $v_{ff}$ — free-fall velocity.

2. Supersonic propeller:

\[ r_{in}\Omega \gg c_s(r_{in}) \, . \]
3. Subsonic propeller:

\[ r_{\text{in}} \Omega \ll c_s(r_{\text{in}}) \]  

and

\[ v_t(r) < c_s(r) \quad \text{in} \quad r_{\text{in}} < r < r_{\text{out}} = r_G , \]  

here \( r_{\text{in}} \) and \( r_{\text{out}} \) — are internal and external radii of an envelope, \( v_t \) — turbulent velocity.

We will not discuss the substage of the very rapid propeller here. Mainly we focus on super- and subsonic substages and on the transition between them. The supersonic regime can be considered a classical propeller where accretion is impossible due to a centrifugal barrier. At the subsonic stage the magnetospheric (Alfven) radius is smaller than the corotation radius \( r_c \). Accretion does not start because temperature is too high (see original discussion in Davies, Pringle, 1981 and later proposals in Ikhsanov 2003).

Braking laws (for the spin-down) are different at different substages. In the supersonic regime energy loss rate does not depend on the spin period (Davies, Pringle 1981), however different formulas for the spin-down at this stage were suggested, see a list for example in (Lipunov, Popov 1995). In the subsonic one the spin-down is always slower: \( P \propto \rho t \).

3 Why does the intermediate regime exist?

In general supersonic and subsonic regimes cover all possible values of the rotation velocity \( \Omega = 2\pi/P \). The supersonic propeller formally operates till \( r_{\text{in}}^{\text{super}} \Omega \geq c_s(r_{\text{in}}^{\text{super}}) \). Here \( r_{\text{in}}^{\text{super}} = r_G 2^{9/7} r_M^{7/9} \) (Davies, Pringle 1981), \( r_M = (\mu^2/\dot{M}\sqrt{2GM})^{2/7} \) — magnetospheric (Alfven) radius.

The subsonic regime is on when

\[ r_{\text{in}}^{\text{sub}} \Omega \leq c_s(r_{\text{in}}^{\text{sub}}) \]  

and \( v_t(r) \leq c_s(r) \) for \( r_{\text{in}}^{\text{sub}} \leq r \leq r_{\text{out}} = r_G \),

here \( r_{\text{in}}^{\text{sub}} = r_M \) (Davies, Pringle 1981). As for the subsonic stage \( v_t(r_{\text{in}}^{\text{sub}}) \approx r_{\text{in}}^{\text{sub}} \Omega \) and \( v_t/c_s \propto r^{1/3} \), then this regime is valid for \( r_{\text{in}}^{\text{sub}} \Omega \leq c_s(r_{\text{in}}^{\text{sub}}) (r_{\text{in}}^{\text{sub}}/r_G)^{1/3} \).

Note the following properties of the stages.

1. At both stages internal radii of an envelope \( r_{\text{in}} \) do not depend on \( \Omega \), and always \( r_{\text{in}}^{\text{super}} > r_{\text{in}}^{\text{sub}} \).

2. The structure of the envelope on the two substages is different.

3. It is easy to check that the end of the supersonic substage and the beginning of the subsonic one both correspond to:

\[ \Omega = \sqrt{2GMr_M^{-7/6}r_G^{-1/3}} , \]  

\[ P \sim 1.15 \times 10^{14} \mu_{30}^{2/3} \rho_{-6}^{1/3} \rho_{-24}^{1/3} \]
Even in the frame of Davies and Pringle model an intermediate regime during which the structure of the envelope is reorganized is inevitable. We call this intermediate stage trans-sonic propeller. As we show below this is a short nonequilibrium episode in a life of a NS, and the spin frequency does not change significantly during this reorganization process.

4 Trans-sonic propeller

Let us consider the structure of a quasistationary envelope at the intermediate trans-sonic propeller substage. In the atmosphere around such a NS $c_s(r) \approx v_f$ (Davies, Pringle 1981). (This condition is valid also for super– and subsonic propeller.) Processes in the lower part of the atmosphere are similar to the ones on the subsonic stage:

$$v_t(r_{in}) \simeq r_{in} \Omega < c_s.$$  \hfill (7)

If the envelope (atmosphere) is adiabatic then for this bottom part of it the polytropic index is equal to $n = 3/2$ and

$$\rho(r) \propto r^{-3/2}, \quad p(r) \propto r^{-5/2}.$$ \hfill (8)

We assume following Davies and Pringle (1981) that the rotational energy of the NS is dissipated at the magnetospheric boundary and that it is transported outwards by turbulence. For such assumptions we have:

$$v_t(r) \propto r^{-1/6}$$ \hfill (9)

and the turbulent Mach number is:

$$M_t(r) \equiv \frac{v_t(r)}{c_s(r)} \propto r^{1/3}.$$ \hfill (10)

Till $M_t < 1$, i.e. while $r < r_s$ where $r_s$ is the boundary between two parts of the envelope ($r_{in} < r_s < r_G$) the structure of the envelope is not changed. For large radii turbulence becomes supersonic. Small-scale shock waves are formed and they quickly dissipate part of the energy, so that turbulent velocity decreases down to the sound velocity. In the range $r_s < r < r_G$ the envelope structure is different from the bottom part:

$$M_t(r) \approx 1,$$ \hfill (11)

$$\rho(r) \propto r^{-1/2}, \quad p(r) \propto r^{-3/2}.$$ \hfill (12)

In the outer part of the envelope physical conditions are similar to the ones on the supersonic substage.

To determine parameters of the whole atmosphere it is necessary to calculate the position of the boundary between two parts of the envelope, $r_s$, and position of the inner boundary of the bottom part, $r_{in}$ (during the transition it decreases from $r_G^{2/9} r_M^{7/9}$ to $r_M$). To do it it is necessary to solve the following system of equations:
Trans-sonic propeller stage

\[
\begin{align*}
\frac{\mu^2}{8\pi r_{in}^6} & = \frac{1}{2} \frac{\dot{M} v_\infty}{4\pi r_G^7} \left( \frac{r_G}{r_s} \right)^{3/2} \left( \frac{r_s}{r_{in}} \right)^{5/2} \\
\Omega r_{in}^{7/6} r_s^{-1/6} & = \sqrt{\frac{2GM}{r_s}} \tag{13}
\end{align*}
\]

However, the system is degenerate and each equation can be reduced to: \( r_s \propto r_{in}^{-7/2} \).

If the following equation is fulfilled:

\[
\frac{\mu^2}{8\pi} = \left( \frac{1}{2} \frac{\dot{M} v_\infty}{4\pi r_G^{1/2}} \right) \frac{(2GM)^{3/2}}{\Omega^3} \tag{14}
\]

then the system is compatible, i.e. for all \( r_{in} \) in the range \( r_M < r_{in} < r_M^{7/9} r_G^{2/9} \) there is some \( r_s \) that is a solution of the eq. \[13\].

The compatibility condition is fulfilled at the end of the supersonic substage. Later (during the transition) at any given moment (for any \( \Omega \)) left-hand side of eq \[14\] is smaller than the right-hand one. It means that the magnetospheric pressure and the envelope pressure have the same dependences on \( r_{in} \), but the latter one is always larger (the first equation of the system \[13\]).

During the trans-sonic stage the period is not changing significantly (a typical value is determined by eq. \[6\]), so in terms of the rotational evolution the subsonic substage nearly immediatelly follows the supersonic one. The spin-down law for the trans-sonic propeller is the same as for the subsonic regime.

An energy release during the transition stage is negligible. Estimates for realistic isolated NS parameters give a value \( \Delta E \lesssim 10^{30} \text{ erg} \).

5 Discussion

We want to note that calculation similar to the ones presented above are just rough estimates. There are several reasons for that.

The first is connected with uncertainties in many parameters, even in their determinations. For example the accretion rate is usually taken as \( \dot{M} = \pi r_G^2 \rho_\infty v_\infty \). This is just an estimate, and for different velocities it can be different from the actual value by a factor a few. Small changes is some parameters can lead to significant changes in others. For example Ikhsanov (2001) discusses the value of the critical period which determines the end of the subsonic propeller stage (and so the accretion stage begins). The obtained value is different from the one found in the original paper by Davies and Pringle (1981) by a factor of 7.5. Correspondently all time scales are also significantly changed. But note, that this fact is due to a change in the magnetospheric radius \( r_M \) only by a factor of \( \sim 2! \).

The second one is connected with idealizations. Even if all parameters can be well defined, then it is necessary to take into account such details as non-spherical form of the magnitosphere, inclination of the magnetic axis relative to the spin axis, angular moment in the infalling matter (even for cases when the condition for disc formation is not fulfilled), etc. For example even if \( r_M > r_c \) part of the magnetosphere is inside \( r_c \) as the corotation radius is the radius of a cylinder, not a
sphere. In that sense the process of alignment (see for example Regimbau, de Freitas Pacheco 2001 and Beskin et al. 1993) during the magneto-rotational evolution of a NS can be important in a destiny of a NS.

A low rate accretion can proceed even at the propeller stage due to several reasons. One of them is diffusion of plasma. Such form of accretion was discussed in details by Ikhsanov (2003). For long spin periods luminosity due to such an accretion can be larger than the dissipation of the rotational energy on the boundary of the magnetosphere.

An important question is connected with the whole time of evolution prior the accretor stage $t_A$. Obviously

$$ t_A = t_E + t_P, \quad (15) $$

where $t_E$ is the time which a NS spends at the stage of ejector, and $t_P$ is the duration of the propeller stage. Even $t_E$ is not well determined. Usually authors assume that spin-down at this stage is determined by the magneto-dipole formula with the braking index equal to 3. However, direct measurement for many radio pulsars show that the braking index is smaller than 3. Also an evolution of the angle between spin and magnetic axes is usually not taken into account.

As the propeller stage consists of the three substages then it is necessary to write:

$$ t_P = t_{super} + t_{trans} + t_{sub}. \quad (16) $$

If $t_E$ and $t_P$ are determined the fate of NSs for different parameters can be easily shown on $t_E - t_P$-diagram suggested in (Popov 2004). In the Fig.2 we show an example of such a plot. For this illustration we assume that the accretion regime starts at the period:

$$ P_{br} = 4.5 \cdot 10^{7} \mu_{30}^{-1/2} \cdot M_8^{-5/7} m^{-4/21}, \quad (17) $$

$\dot{M}_8 = \dot{M}/10^{8} \text{gs}^{-1}$, $m = M_{NS}/M_\odot$. Here we renormalize the value by Ikhsanov (2001).

Time scales are determined by (see Popov 2004):

$$ t_E = 0.8 \cdot 10^{9} \mu_{30}^{-1/2} n^{-1/2} v_6 \text{yr}, \quad (18) $$

here $n = \rho m_p^{-1}$ – interstellar medium number density, $m_p$ – proton mass.

$$ t_{super} = 1.3 \cdot 10^{6} \mu_{30}^{-8/7} n^{-3/7} v_6^{9/7} \text{yr}, \quad (19) $$

this is a very efficient spin-down suggested by Shakura (1975). so our estimate of $t_{super}$ is in some sense a low bound.

$$ t_{trans} + t_{sub} = 10^{3} \mu_{30}^{-2} m P_{br} \text{yr}. \quad (20) $$

The mass of a NS is assumed to be $M_{NS} = 1.4 M_\odot$.

In the plot we show lines for $t_E + t_P$ equal to 1, 5 and 10 Gyrs. Eight symbols corresponds to eight combintations of $n, v, \mu$ (see the table).
Figure 2: $t_E - t_P$-diagram for isolated NSs. Quasi-linear nature of the distribution of model points on the log $t_E - \log t_P$ plane was discussed in detail in (Popov 2004).

6 Conclusions

In this paper we showed that an intermediate substage of the propeller regime – the trans-sonic propeller – should exist. However, the stage is non-stationary and very short. As it was shown above the existence of this intermediate stage does not change the timescale of the evolution prior to the accretor stage (see Ikhsanov 2003 for a discussion of the timescales). Other conclusions can be summarized as follows:

- The intermediate trans-sonic propeller substage in unstable.
- The duration of the transition can be roughly estimated as $\sim r_G/v_f$ (from weeks to years for realistic isolated NSs).
- The spin frequency is nearly unchanged during this transition.
- The energy release during the transition is small.
S.B. Popov and M.E. Prokhorov

Table 1: \( t_E \) and \( t_P \) for typical values of \( n \), \( v \) and \( \mu \)

| Number | \( n \)  | \( v \), km s\(^{-1}\) | \( \mu \) | \( \log t_E \), yrs | \( \log t_P \), yrs |
|--------|---------|----------------|-------|----------------|----------------|
| 1      | 0.1     | 20             | 1     | 9.70           | 9.39           |
| 2      | 0.1     | 20             | 10    | 8.70           | 8.15           |
| 3      | 0.1     | 40             | 1     | 10.0           | 10.0           |
| 4      | 0.1     | 40             | 10    | 9.01           | 8.79           |
| 5      | 1.0     | 20             | 1     | 9.20           | 8.68           |
| 6      | 1.0     | 20             | 10    | 8.20           | 7.44           |
| 7      | 1.0     | 40             | 1     | 9.51           | 9.32           |
| 8      | 1.0     | 40             | 10    | 8.51           | 8.08           |

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