The influence of chiral surface states on the London penetration depth in \( \text{Sr}_2\text{RuO}_4 \)

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The London penetration depth for the unconventional superconductor \( \text{Sr}_2\text{RuO}_4 \) is analyzed assuming an order parameter which breaks time reversal symmetry and parity simultaneously. Such a superconducting state possesses chiral quasiparticle states with subgap energies at the surface. We show that these subgap states can give a significant contribution to the low-temperature behavior of the London penetration depth yielding a \( T^2 \) power-law even though bulk quasiparticle spectrum is gapped. The presence of several electron bands gives rise to interband transition among the subgap surface states and influences the properties of the surface impedance. Furthermore, the surface states lead also to a non-linear Meissner effect.

KEYWORDS: \( \text{Sr}_2\text{RuO}_4 \), \( p \)-wave superconductor, time-reversal breaking state, chiral surface state

Several years of intense experimental research have established the unconventional nature of superconductivity in \( \text{Sr}_2\text{RuO}_4 \).\(^1,2\) This compound has a layered perovskite structure representing a basically two-dimensional metal with three almost cylindrical Fermi surfaces. The symmetry of the superconducting state is very likely odd in parity, which implies the spin-triplet configuration analogous to superfluid \( ^3\text{He} \).\(^3,4\) Muon spin rotation experiments provide evidence for broken time reversal symmetry,\(^5\) a fact that strongly suggests that the gap function has the basic form,

\[
d(k) = \Delta_0 \frac{k_x \mp i k_y}{k_F} \tag{1}
\]

which is a chiral \( p \)-wave state, here written in the vector representation, assuming cylindrical symmetry. The Cooper pairs possess an internal orbital angular momentum which is oriented along the \( z \)-axis. A consequence of this topological property of the superconducting phase is the presence of chiral surface states at the surface.\(^6-8\) While the chiral \( p \)-wave state has a basically gapful quasiparticle spectrum, the surface states correspond to subgap quasiparticle excitations with a continuous spectrum down to zero energy.\(^7,8\) These quasiparticle states are Andreev bound states and extend only over a coherence length towards the bulk. In this letter we consider the contribution of these states to the temperature dependence of the London penetration depth. The London penetration depth \( \lambda || \) for currents within the plane and the in-plane coherence length \( \xi || \) are very similar giving a Ginzburg-Landau parameter \( \kappa = \lambda ||/\xi || \approx 2.6 \). Therefore, the presence of the surface states can lead to a visible reduction of the screening effect and could even dominate the low-temperature behavior \( \lambda || \), in particular, if the bulk quasiparticle spectrum is gapped. We show here that power-law temperature dependence can result from the surface states, which is usually taken as an evidence for nodes in the bulk quasiparticle gap.

The discussion of the London penetration depth requires a careful analysis of the current-current response to an external field which can be written in general as

\[
j_{\mu}(r, t) = -\frac{c}{4\pi} \sum_{\nu} \int dt' \int d^3r' K_{\mu\nu}(r, t; r', t') A_{\nu}(r', t') \tag{2}
\]

where only the transverse component of \( A_{\nu}(r', t') \) enters. The kernel \( K_{\mu\nu}(r, t; r', t') \) is obtained from the current-current correlation function. In our case this response consists of two contributions: the bulk part due to the continuum of quasiparticle states above the gap and the part due to the surface states. We consider from now on the specific case of a surface with normal vector along the \( z \)-axis and an external field parallel to the \( z \)-axis. Consequently we have to deal with the transverse vector potential and screening current along the \( y \)-axis. The relevant terms are then,

\[
j_y(r, t) = -\frac{1}{c} \int d^3r' \left[ \Pi_{yy}(r; r') - \frac{e^2 \delta^{(4)}(r - r')}{4\pi \lambda_0^2} \right] A_y(r') \tag{3}
\]

where the integral runs of the four coordinates \( r' = (r', t') \) and \( \lambda_0 \) corresponds to the bare “London penetration depth” of the bulk regime which can be considered as basically temperature-independent for very low temperatures. For this bulk part we take the local approximation, while for the first part connected with the surface states nonlocality is important, as we will see below.

The surface states can be easily described within the Bogolyubov-de Gennes formalism if we neglect self-consistency of the gap \( \Delta_0 \) which we choose to be constant everywhere inside the superconductor. For the sake of simplicity we assume also that the surface provides specular reflection of quasiparticles and the gap has no anisotropy on the Fermi surface. The electron band in our model has cylindrical symmetry and is represented by the parabolic form, \( \varepsilon_k = \{(k_x^2 + k_y^2) - k_F^2\}/2m \) neglecting any dispersion along the \( z \)-axis. Using these simplifi-
cations the wave function of the subgap states localized at the surface is given by
\[
\begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} \simeq \sqrt{\frac{2}{\xi_0 L_y d}} e^{ik_y y - \frac{i}{\xi_0} \sin(k_y x)} \begin{pmatrix} 1 \\ -i \end{pmatrix}.
\]
(4)

Since only states very close to Fermi surface are important for the low-temperature properties the wave vector can be represented essentially as \((k_x, k_y) = k_F(\cos \theta, \sin \theta)\) for \(k_F \xi_0 \gg 1\). Further, \(L_y\) is the extension of the system along the \(y\)-direction with periodic boundary conditions and \(d\) is the interlayer spacing (the wave function is renormalized per layer). The energy of the surface states is given by \(E_{k_y} = \eta \Delta_0 k_y / k_F\) with \(\eta := \pm 1\) denoting the sign of the chirality of \(\mathbf{d}(k) = z(k_x \pm i k_y)/k_F\).\(^9\) Defining the current operators as \(j_{\alpha}(r) = (he/2m i)(\bar{\psi} \sigma_{\alpha} \psi - h.c.)\), where
\[
\bar{\psi}(r) = \begin{pmatrix} \psi^\dagger(r) \\ \psi^\dagger(r) \end{pmatrix} = \sum_k \begin{pmatrix} u_k(r) & -i v_k^*(r) \\ v_k^*(r) & i u_k^*(r) \end{pmatrix} \begin{pmatrix} \gamma^\dagger_k \\ \gamma^\dagger_k \end{pmatrix},
\]
is the Nambu field operator with \(\gamma^\dagger_{\kappa}\) the Bogolyubov quasi-particle operator, we can express the current-current correlation function \(\Pi_{yy}(r' r')\) as
\[
\Pi_{yy}(r' r') = \frac{\hbar^2 e^2}{4m^2 L_y d} \lim_{r_1 \to r_2} \begin{pmatrix} \frac{d}{\omega_n - sE_k} \end{pmatrix} \mathbf{G}(r_1; r_2) \mathbf{G}(r_2; r_1) \mathbf{G}(r_1; r_2) \mathbf{G}(r_2; r_1),
\]
(5)
where \(\mathbf{G}(r; r')\) is the Nambu-Gor'kov Green’s function in real space and \(\partial_{\eta}\) denotes the derivative with respect to the spatial \(y\)-coordinate. The Green’s function can be expressed as
\[
\mathbf{G}(r, r'; i \omega_n) \simeq \phi(x)i\phi(x') L_y d \sum_{0 < k < k_F} \sum_{s = k} \frac{\partial_0 - s\partial_2}{i \omega_n - sE_k} e^{iks(y-y')} \sin(k_F x)
\]
(7)
with \(\phi(x) = \sqrt{2/\xi_0} \exp^{-x/\xi_0} \sin^2(k_F x\xi_0)\), \(\partial_0\) the unit matrix and \(\partial_2\) the second Pauli matrix, and \(\omega_n\) being the fermionic Matsubara frequency. Using Eqs. (6) and (7), we calculate the current-current correlation function. The translational invariance along the \(y\)-direction allows us to transform the \(y\)-coordinate into momentum space,
\[
\Pi_{yy}(x, x'; q, i \Omega_n) \simeq -\frac{32\hbar^2 e^2}{m^2 L_y d} g(x)g(x') \sum_{0 < k < k_F} k^2 \frac{f(E_{k+q}) - f(E_k)}{i \Omega_n - E_{k+q} + E_k}
\]
\[
\simeq -8\hbar^2 k_b^2 \frac{(k_B T/\Delta_0)^2}{3dm^2 \Delta_0} \frac{2}{1 - i \Omega_n k_F q/\Delta_0} \frac{g(x)g(x')}{1 - i \Omega_n k_F q/\Delta_0},
\]
(8)
in the limit \(T \ll T_c\), where \(g(x) \simeq \exp(-2k_F^2 x^2/\xi_0)\) is the square of the amplitude of the surface state wave function and \(\Omega_n\) is the bosonic Matsubara frequency. In deriving Eq. (8), we restrict ourselves to the leading contribution for \(q \ll k_F\) and \(k_B T \ll \Delta_0\). The nonlocal nature of response enters via the product form \(g(x)g(x')\) which accounts for the fact that each of the quasiparticle state is localized at the surface. The field at the particular point \((x', y', z')\) couples to the surface state in the same layer with a weight \(g(x')\) and yields consequently a response at any other point \((x, y, z')\) with weight \(g(x)\). This is a feature of the effectively one-dimensional character of the surface states within each layer. A local approach in this place would underestimate the role of the surface states in the low-temperature response.

Combining Eqs. (3) and (8), where we further use the analytic continuation \(\Omega_n \to \hbar \omega + i\delta\) with the Maxwell equation \(\nabla^2 A_y(r, \omega) = -\frac{4\pi e}{c} j_y(r, \omega)\) we obtain an integro-differential equation for \(\bar{A}_y(r, t)\). The boundary condition is given by \(\partial_x A_y(r, \omega)|_{x = 0} = B_z(q, \omega)\), where \(B_z(q, \omega)\) is the external magnetic field at the surface parallel to the \(z\)-axis. We solve this equation using an approximation \(g(x) \simeq \exp(-2x/\xi_0)/2\xi_0\) in the integrand (we ignore the fast oscillations), which is certainly valid for \(k_F \xi_0 \gg 1\). This allows us to calculate the surface impedance, \(Z(q, \omega) = 4\pi e E_y(x = 0, q, \omega)/B(q, \omega)\). We then obtain the penetration depth using the relation \(4\pi \omega \lambda(q, \omega) = \text{Im}Z(q, \omega)\). Taking a static limit \(\hbar \omega k_F \xi_0 \to 0\) and \(q \to 0\), we find in the regime of \(k_B T \ll \Delta_0, \Delta \lambda(T) = \lambda(T) - \lambda(0)\) has \(T^2\)-behavior:
\[
\Delta \lambda(T)/\lambda_0 \simeq \frac{4\pi^2}{3} \frac{e}{(2k + 1)^2} (k_B T/\Delta_0)^2.
\]
(9)
Setting \(\kappa \simeq 2.6\), which is a typical value of \(\text{Sr}_2\text{RuO}_4\), we obtain \(\Delta \lambda(T)/\lambda_0 \simeq 0.14 \times (T/T_c)^2\), if we assume the weak-coupling relation \(\Delta_0 = 1.76 k_B T_c\). We ignored the temperature dependence of \(\lambda_0\) as it is exponential in the low-temperature regime in our model.

It is important to notice that this contribution is independent of the sign of the chirality and the charge of

Fig. 1. Schematic spectrum of the chiral surface states in the multi-band case of \(\text{Sr}_2\text{RuO}_4\). The Brillouin zone contains three Fermi surfaces. The electron-like \(\beta\) and \(\gamma\)-Fermi surfaces yield a chiral surface state spectrum centered around \(k_y = 0\), while the spectrum of the surface state due to the hole-like \(\alpha\)-Fermi surface located at the Brillouin zone boundary with opposite sign of chirality. The gap magnitudes are in general different. We assigned, however, the same magnitude for \(\alpha\) and \(\beta\)-Fermi surface for simplicity.
the prefactor of the case of several superconducting bands the contributions mains of the two superconducting states would not lead to a significant change of the result. Furthermore, for the case of several superconducting bands the contributions of the surface states of each band add up to enlarage the prefactor of the $T^2$-law. The coherence length as the extension of the surface states towards the interior is different for each band, since the Fermi velocities and the gap magnitudes are different. The coherence length, experimentally determined via the measurement of $H_{c2}$, giving $\kappa \approx 2.6$ is the shortest among all. Therefore, the enhancement of the surface state contribution can be sizable in the multiband case. We consider here the case of three bands, as schematically shown in Fig.1, where each band yields its own surface state described by a Green’s function $G^{(j)}(r; r')$, (the superscript $j$ labels the $j^{th}$band). We assume that the reflection of quasiparticles on the surface does not lead to transitions among the different bands. Each band is characterized by a Fermi vector $k_{F}^{(j)}$, the effective band mass $m_j$ and the superconducting gap $\Delta_j$. We now use Eq.(6) to calculate the contribution of each band to the current-current correlation function and then analyze the resulting equation for the transverse vector potential as in the single band case. This leads to the low-temperature behavior of the London penetration depth,

$$\Delta \lambda(T) \approx \frac{4\pi^2}{3} \sum_{j=1}^{3} \frac{\kappa^{(j)}}{(2\kappa^{(j)} + 1)^2} \left( \frac{\Delta_0}{\lambda_j \Delta_j} \right)^2 \left( \frac{k_{F}^{(j)}}{\Delta_0} \right)^2 \tag{10}$$

where

$$\frac{1}{\lambda_j^2} = \sum_{j} \frac{1}{\lambda_j^2} = \sum_{j} \frac{2e^2k_{F}^{(j)}^2}{m_j c^2 d} \tag{11}$$

and $\kappa^{(j)} = \lambda_0/\xi^{(j)}$ with $\xi^{(j)} = h^2k_{F}^{(j)}/m_j \Delta_j$. We choose $\Delta_0 = 1.76k_B T_c$. The contribution of all three bands can easily give a prefactor to the $(T/T_c)^2$-law of order one, consistent with recent measurements for fields along the $z$-axis.

In the discussion of the multi-band situation we neglected the interband effects. Oscillatory fields, for example appearing in microwave experiments, can yield interband transitions. The matrix elements for the transition depends on various details of the orbital and band structure. We do not go into these complex details here, but assume that the interband transition can be described by an ordinary current operator, $j^{(i,j)}_{\mu}(x) = (\hbar/2m_i')i(\Psi^{(i)}\partial_\mu \Psi^{(j)} - h.c.)$ where $m'$ is a phenomenological parameter accounting for the matrix element, $\Psi^{(i)}$ is the Nambu field operator of the $i$-th band as in Eq. (5). In the small-momentum ($q$) and small-frequency ($\omega$) limit only surface states are important which share the same zero-energy momentum. As shown in Fig.1 this is the case for the $\beta$- and $\gamma$-band. The $\alpha$-band is unimportant because for interband transitions a large momentum transfer ($q \sim \pi$) is necessary. Analogous to Eq.(8) we can derive the correlation function,

$$\Pi^{(\beta,\gamma)}_{xy}(x_1, x_2, q; i\Omega_n) \approx \frac{16\hbar^2e^2}{m^2_dL_y} \tilde{g}(x_1)\tilde{g}(x_2) \sum_k k^2 \times \left[ f(E_k^{(i)}) - f(E_k^{(f)}) \right] \left( \frac{\Omega_n - E_k^{(i)} + i\delta}{\Omega_n - E_k^{(i)} + E_k^{(f)}} \right) \left( \frac{\Omega_n - E_k^{(f)}}{\Omega_n - E_k^{(f)} + E_k^{(f)}} \right)$$

where $\tilde{g}(x) \approx \exp(-2x/\tilde{\zeta})$ and $\tilde{\zeta} = \tilde{\xi}(\beta) + \xi(\gamma)$. The surface state spectra in the two bands is approximated by $E_k^{(i)} = v_\beta k$, with $v_\beta = \Delta_i/k_F^{(i)}$. We now take the analytic continuation $\Omega_n \rightarrow \hbar \omega + i\delta$ and set $q = 0$ in Eq.(12). For the limit of very small $\hbar \omega (\ll k_BT)$ the surface impedance gets the following contribution from the interband transitions,

$$\text{Im}Z_{ib}(\omega) = \frac{4\pi^3\xi^{(\gamma)}_0 \xi^{(\gamma)}}{3\lambda^{(\gamma)^2}} \left( \frac{\Delta_0 m_j}{\Delta_\gamma m'} \right)^2 \omega \frac{v_\gamma}{16\hbar^2}$$

$$\times \left[ \left( \frac{k_BT}{\Delta_\gamma} \right)^2 \left( \frac{v_\gamma v_\beta - 1}{\hbar \omega (\Delta_\gamma)} \right)^2 \frac{\hbar \omega}{\Delta_\gamma} \right]$$

for the imaginary part which yields in the zero-frequency-limit also a $T^2$-contribution to the London penetration depth ($\kappa = \lambda/\xi$ and $v = v_\gamma - v_\beta > 0$). For the real part we obtain,

$$\text{Re}Z_{ib}(\omega) = \frac{2\pi^3\xi^{(\gamma)}_0 \xi^{(\gamma)}}{3\lambda^{(\gamma)^2}} \left( \frac{\Delta_0 m_j}{\Delta_\gamma m'} \right)^2$$

$$\times \left( \frac{v_\gamma}{16\hbar^2} \right)^3 \left( \frac{\hbar \omega}{\Delta_\gamma} \right)^2 \omega \left( \frac{\hbar \omega}{k_BT} \right)^2$$

for given $T$ and small $\hbar \omega (\ll k_BT \ll \Delta_\gamma)$. The imaginary part, the inductive resistance, shows a $\omega$-linear plus $\omega^3$-behavior, while the real part, the surface resistance follows an $\omega^4$-law. In the opposite limit where $k_BT \ll \hbar \omega \ll \Delta_\gamma$ the surface impedance due to interband transitions has to vanish. The reason is that for $q = 0$ the initial and final states are either both empty or occupied in the zero-temperature limit. A simple analysis shows the following behavior,

$$\text{Re}Z_{ib}(\omega) \propto \omega^4 \left( \frac{\omega}{T} \right)^2 e^{-\frac{\omega + v_\beta + \phi_{ib}}{2\Delta_\gamma}}, \text{Im}Z_{ib}(\omega) \propto T^4 \frac{\omega}{\omega^2}$$

The surface resistance and the inductive resistance vanish exponentially and with a power-law, respectively, in the zero-temperature limit.

We now consider the possibility of a so-called nonlinear Meissner effect. The application of a magnetic field introduces a Doppler shift which changes the quasiparticle energy,

$$E_k' = E_k^{(sg)} + \frac{e v_{fy}}{c} A_y$$

if we again consider the case of $n = (1, 0, 0)$. We can expand the current-current correlation function for small $A_y$ and analyze the contribution to the London penetration depth in the same way as done above. Restricting
to the single band model we obtain,
\[ \frac{\Delta \lambda(T)}{\lambda_0} \simeq \frac{4\pi^2}{3} \left( \frac{k_B T}{\Delta_0} \right)^2 \left( 1 - \eta \frac{3\kappa}{2} \frac{H}{H_c^2} \right) \]
which leads to a non-linear correction in the external field. This effect is a consequence of the angular momentum of the Cooper pairs coupling to the field along the $z$-axis. The sign of this correction depends on the chirality (direction of angular momentum) and the character of the Fermi surface, i.e., a different sign appears for the $\alpha$-band than for the $\beta$- and $\gamma$-band in Fig.1. Therefore, the presence of electron-like and hole-like Fermi surfaces as well as the formation of domains of the two chiral states lead to compensations which diminish the effect.

In recent experiments by Bonalde et al., the temperature dependence of the London penetration depth was determined using a self-inductive technique.\(^\text{10}\) It was found that the low-temperature behavior is indeed governed by a $T^2$-behavior. This is not compatible with a simple interpretation in terms of line nodes in the gap as originally proposed based on the $T$-power laws in specific heat and NQR,\(^\text{11}\) since this would lead to a linear $T$-dependence.\(^\text{12}\) The more sophisticated approach based on a nonlocal response theory by Kostin and Leggett (KL) (for the reason that $\kappa$ is small), however, would yield a $T^2$-behavior.\(^\text{13}\) On the other hand, in this letter we propose an alternative mechanism for a $T^2$-behavior based on the contributions of the surface states. In both theories it is expected that this power-law behavior is absent for $\lambda_\perp$, for screening currents flowing along the $z$-axis. The $z$-axis current is not proportional to the surface state energy as required to obtain the $T^2$-behavior in our theory. Furthermore, $\kappa_\perp$ is about 20 times larger than the in-plane $\kappa$ so that contribution of the surface states as well as the nonlocal effect by KL are rather small. However, the measurements for fields in the plane, probing the $z$-axis current show a similar $T^2$-behavior. This is in apparent conflict with both interpretations. Since in this case, however, not only $\lambda_\perp$ but also the contribution from in-plane currents from the surfaces normal to the $z$-axis are involved, the final answer will be given only when these geometrical aspects have been thoroughly investigated.\(^\text{14}\)

In our model the gap size is isotropic on the Fermi surface. Anisotropy in turn modifies the surface state spectrum to $E_{k_y} = v_{ky} + v_3 k_y^2 + \ldots$ without destroying the particle-hole symmetry ($vv' < 0$ in general). This yields an additional $T^3$-contribution which may not be so small. Together with other correction in this order this leads to
\[ \frac{\Delta \lambda(T)}{\lambda_0} = a(T/T_c)^2 + b(T/T_c)^4 + \ldots \]
with $a \sim b > 0$. In an intermediate range the second term generates a $T$-dependence which over some temperature range appears to be close to a $T^3$-behavior and only at rather low temperatures the $T^2$-law would dominate. For one sample Bonalde et al. could indeed fit their data reasonably well with a $T^3$-curve.\(^\text{14}\) While this sample happened to be dirty it is not clear from the experiment what is the intrinsic origin for the apparently different power-law. Therefore, the additional $T^4$-contribution, which due to the surface orientation and disorder is larger than usual, may be one possible explanation.

A further aspect noteworthy here is that our mechanism is active at the surface only, while the KL scheme also applies also in the bulk of the superconductor. Therefore, the London penetration depth governing the magnetic interaction between vortices should have different temperature dependence in the two scenarios. Measurements of in the mixed phase by \(\mu\)SR suggest that London penetration depth saturates faster than $T^2$ at low temperature.\(^\text{15}\) In contrast to the surface-sensitive experiment mentioned above,\(^\text{10}\) \(\mu\)SR is indeed a bulk probe. Unfortunately, it has less accuracy in determining the temperature dependence of the London penetration depth so that we cannot draw a strong conclusion to date. Nevertheless, the present experimental result is consistent with the interpretation based on surface states.

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