Voids in the SDSS Galaxy Survey

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ABSTRACT

Using the method of searching for arbitrary shaped voids in the distribution of volume-limited samples of galaxies from the DR5 SDSS survey, we have identified voids and investigated their characteristics and the change in these characteristics with decreasing \(M_{lim}\) (from -19.7 to -21.2, \(H_0 = 100 \text{ km/s/Mpc}\) — the upper limit on the absolute magnitude of the galaxies involved in the construction of voids. The total volume of the 50 largest voids increases with decreasing \(M_{lim}\) with a break near \(M^* = -20.44\) — the characteristic value of the luminosity function for SDSS galaxies. The mean density contrast in voids increases with decreasing \(M_{lim}\) also with a weak break near \(M^*\) The exponent of the dependence of the volume of a void on its rank increases significantly with decreasing \(M_{lim}\) starting from \(M_{lim} \sim -20.4\) in the characteristic range of volumes, which reflects the tendency for greater clustering of brighter galaxies. The averaged profile of the galaxy density contrast in voids has a similar pattern almost at all \(M_{lim}\). The galaxies mostly tend to concentrate toward the void boundaries and to avoid the central void regions; the density contrast profile is flat in the intermediate range of distances from the void boundaries. The axial ratios of the ellipsoids equivalent to the voids are, on average, retained with changing \(M_{lim}\) and correspond to elongated and nonoblate void shapes, but some of the voids can change their shape significantly. The directions of the greatest void elongations change chaotically and are distributed randomly at a given \(M_{lim}\). The void centers show correlations reflecting the correlations of the galaxy distribution on scales \((35 - 70)h^{-1}\) Mpc. The galaxy distribution in the identified voids is nonrandom — groups and filaments can be identified. We have compared the properties of the galaxies in voids (in our case, the voids are determined by the galaxies with absolute magnitudes \(M_{abs} < M_{lim} = -20.44\), except for the isolated galaxies) and galaxies in structures identified using the minimum spanning tree. A bimodal color distribution of the galaxies in voids has been obtained. A noticeable difference is observed in the mean color indices and star formation rates per unit stellar mass of the galaxies in dense regions (structures) — as expected, the galaxies in voids are, on average, bluer and have higher \(\log(SFR/M_\text{star})\). These tendencies become stronger toward the central void regions.
1. Introduction

The distribution of galaxies is a complex cosmic network. The walls, filaments, and voids observed at the present epoch reflect both linear and nonlinear evolution of clustering. The pattern of the observed clustering out to $20 - 25h^{-1}$ Mpc can be described, for example, by a power law with a complex dependence of the exponent on the luminosity, color, and other properties of galaxies with the subsequent transition to homogeneous distribution, with the structures being traceable on scales exceeding the scale of homogeneity (Tikhonov 2006a, 2006b). The nature of such clustering depends on many small- and large-scale factors such as the cosmological parameters, the environments of galaxies and clusters, their formation history, the distribution of dark matter, and the scenario according to which the luminous and dark matter are related and evolve. The characteristics of voids have long been considered as tests of cosmological models. Regoes and Geller (1991) found that in their model for the formation of structures, certain initial conditions lead to the formation of a cellular structure with voids similar to those observed in galaxy surveys. Voids are the forming components of the large-scale structure. In recent years, various authors have considered in detail both observational and theoretical aspects of the existence and evolution of voids detected both in galaxy catalogs and in the dark matter halo distributions obtained in the ΛCDM model calculations of the N-body problem by Hoyle and Vogeley (2004), Gottlober et al. (2003), Shandarin et al. (2004), Croton et al. (2004), Benson et al. (2003), Colberg et al. (2005), and Patiri et al. (2006b). The void statistics are closely related to the methods of calculating the galaxy clustering; for example, VPF (Void Probability Function) provides information about the high-order correlation functions (Croton et al. 2004).

One important problem of the modern theory of the formation of structures is that according the ΛCDM model for the evolution of dark matter structures including the Λ-term attributable to the existence of dark energy, much matter must be present in voids, while the expected number of galaxies in voids is not observed (Peebles 2001).

Sheth and van de Weygaert (2004) developed a model for the distribution of void sizes and evolution in terms of the hierarchical clustering scenario. Furlanetto and Piran (2006) developed an analytical model that predicted the shape of the distribution of void sizes. In particular, they found that because of the so-called bias effect, the voids in the galaxy distribution are considerably larger than those in the dark matter distribution.

The shapes of voids are of interest along with the spectrum of their sizes. Based on
numerical calculations, Ike (1984) concluded that, in most cases, the voids between filaments must be nearly spherical in shape. Plionis and Basilakos (2002) analyzed the distribution of void sizes and shapes in the PSCz survey and compared them with the artificial distributions obtained in terms of various CDM models. Shandarin et al. (2004) found, in particular, that their large voids defined as regions with a density lower than a given value in the smoothed density field of the dark matter distribution are essentially nonspherical.

Patiri et al. (2006a) found that the distribution of galaxies in voids in the distribution of 2dFGRS galaxies differs significantly from a random one.

It has been firmly established that, compared to the general distribution, the galaxies in voids have bluer colors, lower luminosities, and higher star formation rates. In addition, a higher abundance of disk galaxies is observed among the galaxies in voids (Peebles 2001; see also Patiri et al. 2006b). Rojas et al. (2004, 2005) confirmed these tendencies by analyzing the photometric and spectroscopic properties of the galaxies in voids. Hogg et al. (2004) considered the dependence of the galaxy color and luminosity distributions on the density contrast and found that, on the one hand, the most luminous galaxies populate the densest regions, while the blue galaxies are present mostly in low-density regions, and, on the other hand, the mean parameters of the distributions in absolute magnitude and color change only slightly with over-density. Based on SDSS galaxies, Baldry et al. (2004) showed that the galaxy color distribution is bimodal and is described well by two Gaussians. They obtained a fit to the (u-r) — color absolute magnitude relation and compared the luminosity functions for red and blue galaxies. Hoyle et al. (2005) found significant differences between the luminosity functions of the galaxies in voids and dense regions.

Patiri et al. (2006b), who analyzed the voids in the DR4 SDSS survey, found no significant differences between the mean parameters of the field and void galaxies. In this paper, we use a different approach to selecting a check sample of galaxies for comparison with the properties of the galaxies in voids — we selected the galaxies of the check sample in high density structures located entirely outside the void boundaries.

In this paper, we also analyze the variations of void parameters with luminosity and perform a correlation analysis of the distribution of void centers.

2. THE DATA

The spectroscopic redshifts are expected to be obtained for about 106 galaxies and 105 quasars within the framework of the Sloan Digital Sky Survey (SDSS) based on photometric data for a sky region $10^4$ square degrees in area in the Northern Galactic Hemisphere in five
bands (u, g, r, i, z) with a limiting magnitude of $r = 22.5$ (York et al. 2000; Stoughton et al. 2002) once the program has been fully implemented. The photometric data were used for a homogeneous selection of various classes of objects to obtain their spectra. Two types of galaxies were chosen for determining the redshifts from the list of objects classified as extended ones: galaxies with a Petrosian magnitude $r < 17.77$ and a surface brightness exceeding $24 \text{ m/}\square\theta$. formed the Main Galaxy Sample (the number of objects in the final SDSS version is $\sim 900000$); the LRG (Luminous Red Galaxies) list includes galaxies with very red colors and $r < 19.5$ (the number of objects in the final SDSS version is $\sim 100000$). In this paper, we analyzed data from the fifth data release DR5 SDSS (www.sdss.org, Adelman-McCarthy et al. 2006).

When analyzing the DR5 data, we selected a rectangular part from the region of spectroscopic sky coverage for the convenience of allowance for the boundary conditions in determining the void boundaries and for ensuring sample completeness. In the $(\lambda, \eta)$ coordinate system of the survey, the selected region is $-48^\circ < \lambda < 48^\circ$, $6^\circ < \eta < 36.5^\circ$.

The Main Galaxy Sample is an apparent-magnitude-limited survey, which determines the method of constructing the volume-limited sample to eliminate incompleteness in radial coordinate – we set the limit on the $r$-band absolute magnitude for the sample galaxies equal to $M_{\text{lim}}^0 = r_{\text{lim}} - 25 - 5\log(R_{\text{max}}(1 + z_{\text{max}})) - K(z)$, where $r_{\text{lim}} = 17.77$ was taken as the limiting $r$-band magnitude, $K(z)$ is the $K$-correction, and $R_{\text{max}}$ is the chosen far boundary in radial coordinate corresponding to $z_{\text{max}}$. The $u$ and $r$ magnitudes used here were corrected for extinction.

To estimate the absolute magnitudes of the galaxies, we used a mean correction for SDSS galaxies in the form $K(z) = 2.3537z^2 + 0.5735z - 0.18437$ (Hickage et al. 2005; see also Blanton et al. 2003). The metric distances were recalculated from the redshifts with the Hubble parameter $H_0 = 100 \text{ km s}^{-1}\text{Mpc}^{-1}$, $h = H/H_0$, where $H$ is the true value of the Hubble constant, and the density parameters $\Omega_\Lambda = 0.7$, $\Omega_0 = 0.3$ (see, e.g., Hogg 1999).

3. THE METHOD

The void construction algorithm presented here has already been applied by Tikhonov (2006b) and Tikhonov and Karachentsev (2006) and is basically similar to the algorithm described by El-Ad and Piran (1997). The voids were constructed in the distribution of bright galaxies with absolute magnitudes $M_{\text{abs}}$ (in the $r$ band) lower than a certain value of $M_{\text{lim}}$. Here, we searched for voids containing a certain number of galaxies from the volume-limited sample with $M_{\text{abs}} < M_{\text{lim}}$. For the void-forming galaxies, we determined the mean
distance to the nearest neighbor $R_n$ and the standard deviation $\sigma_n$. If there was no neighbor with $M_{\text{abs}} < M_{\text{lim}}$ in the sphere of radius $R_n + \sigma_n$ around a particular galaxy of this sample, then this galaxy was excluded from the list of galaxies involved in the construction of voids. Thus, these excluded galaxies could fall into voids. The mean distance from the isolated galaxies to the nearest neighbor is considerably larger than that for isolated pairs (when two galaxies lie in the sphere of radius $R_n + \sigma_n$) — it is close to the mean distance between the galaxies for pairs. Thus, having eliminated the influence of isolated galaxies, we obtain more stable voids.

Next, we successively searched for galaxy-free seed spheres inside the sample volume (first, the largest sphere is searched for) and then expanded them by adding spheres whose centers are inside the already fixed part of the void and whose radii $R_{\text{sph}}$ are not smaller than the radius of the seed sphere multiplied by the coefficient $k = 0.9$ ($R_{\text{sph}} > 0.9 \cdot R_{\text{seed}}$, where $R_{\text{seed}}$ is the radius of the seed sphere). The voids are assumed to be located entirely within the geometrical boundaries of the sample.

The voids constructed in this way (at $k = 0.9$) have arbitrary shaped volumes. On the other hand, the voids are separated from each other and fairly thick throughout the volume, which allows them to be approximated by triaxial ellipsoids.

There exist other methods of searching for voids that are more commonly used in analyzing artificial dark matter distributions (see, e.g., Shandarin et al. 2006). In this approach, a smoothed (e.g., with a Gaussian filter) density field is constructed (the parameters of the resulting structures depend on the smoothing length) and a certain threshold local density that separates the low- and high-density regions is specified. In this case, the voids can be highly irregular in shape. A brief overview of the methods and references can be found, for example, in Tikhonov (2006b) and Patiri et al. (2006a).

4. THE DEPENDENCE OF VOID PROPERTIES ON LUMINOSITY

To analyze how the properties of the voids vary over a wide luminosity range of the galaxies forming them, we chose the redshifts limits $z_{\text{min}} = 0.02$ and $z_{\text{max}} = 0.1$ (sample A). The upper limit on the absolute magnitude of the galaxies in the volume-limited sample with these boundaries is $M_{\text{lim}}^0 = -19.67$. For this sample, the number of galaxies is $N = 47892$, the mean density is $\rho \approx 7 \cdot 10^{-3} h^3$, $R_n + \sigma_n \approx 3.0h^{-1}$ Mpc. The range of limits $M_{\text{lim}}$ on the absolute magnitudes of the galaxies involved in constructing the voids in which we analyzed the dependence is from $-19.7$ (40526 galaxies after the exclusion of isolated galaxies) to $-21.2$ (1241 galaxies). We chose this range in such a way that the sample was volume
limited and that it contained a sufficient number of galaxies. The resolution (the separation between the grid points) was about 1.7 Mpc in all cases.

In the same vein, we analyzed sample B with the redshift limits $z_{\text{min}} \approx 0.080$ and $z_{\text{max}} = 0.115$ chosen in such a way that the sample covered the same volume (about $6.82 \cdot 10^6 \cdot h^{-3}$ Mpc$^3$) as sample A. In this case, $M_{\text{lim}}^0 = -20.00$, $R_n + \sigma_n \approx 3.4 h^{-1}$ Mpc.

Figure 1 shows the total volume of the 50 largest voids in samples A and B and the number of void- forming galaxies as a function of $M_{\text{lim}}$. The increase in total volume occurs synchronously with the decrease in $M_{\text{lim}}$ and in the number of galaxies with $M_{\text{abs}} < M_{\text{lim}}$ (except for the isolated galaxies). A significant break in the dependence is observed near $M_{\text{lim}} = -20.5$, i.e., immediately after the characteristic value of $M^* = -20.44$ of the luminosity function for SDSS galaxies followed by a faster growth of the total volume. This is not just the result of the corresponding decrease in the number of void-forming galaxies (on the contrary, the decrease in the number of galaxies slows down). The 50 largest voids occupy from about 21\% ($M_{\text{lim}} = -19.7$) to 64\% ($M_{\text{lim}} = -21.1$) of the entire sample volume.

Figure 2 shows the dependence of the mean density contrast (contrast profile) for the galaxies of sample A $\delta\rho/\rho_{VL}$ (where $\rho_{VL}$ is the mean density of the galaxies with $M_{\text{abs}} < -19.67$ in sample A) that fell into a layer inside a particular void with the distance $r$ from the void boundaries normalized to the effective void radius $R_{\text{eff}} = (3 \cdot Vol/4 \cdot \pi)^{-1/3}$ ($Vol$ stands for the void volume) on the limiting absolute magnitude $M_{\text{lim}}$. We averaged the density contrast profile inside all voids with $R_{\text{seed}} > 15h^{-1}$ Mpc. The density contrast profile has common characteristic features for different $M_{\text{lim}}$: the galaxies concentrate to the void boundaries; there are virtually no galaxies in the central regions ($r/R_{\text{eff}} > 0.7 - 0.8$); the density contrast profiles of the galaxies in voids are flat up to $r/R_{\text{eff}} = 0.3 - 0.2$ followed by a significant increase in contrast. At $M_{\text{lim}} < -21.0$, the galaxy density near the void boundaries (in the first bin) is higher than the mean density ($\delta\rho/\rho_{VL} > 0$).

Figure 3 shows the dependence of the mean density contrast for the galaxies in voids $< \delta_v >$ with $R_{\text{seed}} > 15h^{-1}$ Mpc on $M_{\text{lim}}$ for four limits $M_1$ on the absolute magnitude of the galaxies retained in the volume-limited sample (from which the mean density in the formula for the density contrast $< \delta_v > = \delta\rho/\rho_{M_1}$) was obtained). In each of the four cases, the change in $M_{\text{lim}}$ began from $M_{\text{lim}} = M_1$. An indistinct break in the dependence can be distinguished at $M_{\text{lim}} \approx -20.6$. At fixed $M_{\text{lim}}$, the mean density contrast is lower (the voids become emptier) at lower $M_1$. Convergence is observed for all $M_1$ as $M_{\text{lim}}$ decreases. This indicates that the most luminous galaxies form a stable skeleton of the structure (the fraction of isolated galaxies decreases with decreasing $M_1$). In all cases, the mean density contrast $< \delta_v > < 0$, i.e., the identified voids are physically separated low-density regions.
Fig. 1.— Total volume of the 50 largest voids as a function of the limiting absolute magnitude $M_{\text{lim}}$ of the galaxies involved in constructing the voids for samples A and B. Also shown is the number of galaxies.

Fig. 2.— Mean overdensity of the galaxies in voids with $R_{\text{seed}} > 15$ Mpc with the distance $r$ from the void boundaries normalized to $R_{\text{eff}}$ versus $M_{\text{lim}}$. 
After compiling the list of voids (assigning the three-dimensional grid points to a particular void), we determined the void centers and calculated the moments of inertia of the bodies formed by the voids. We analyzed the void shapes based on the parameters of the equivalent ellipsoids. We constructed a $3 \times 3$ matrix of the moments of inertia $I_{ij}$ and used the condition $\text{det}(I_{ij} - \lambda \cdot E) = 0$, where $E$ is a $3 \times 3$ unit matrix, to find its eigenvalues $\lambda_i$, which are equal to the principal moments of inertia, from which the semiaxes of the equivalent ellipsoid were determined. The eigenvectors of matrix $I_{ij}$ give the directions of the semiaxes. The direction of the greatest void elongation coincides with that of the largest semiaxis of the equivalent ellipsoid.

The void shapes were analyzed for the first 20 identified voids. In general, the change in void configuration and shape with decreasing $M_{\text{lim}}$ is indicative of an irregular change in the configuration of the entire large-scale structure (in our approach, the influence of isolated galaxies was eliminated). The void centers are displaced significantly and the order of void identification changes.

The $b/a$ and $c/a$ axial ratios of the equivalent ellipsoid, where $a$, $b$ and $c$ are the largest, medium, and smallest axes, respectively, are correlated (see Fig.4 for $M_{\text{lim}} = -20.3$). Figure 5 shows the mean slopes $\phi$ with the errors of the linear fits to the $b/a$ and $c/a$ distributions for various $M_{\text{lim}}$. The dependence is stable starting from $M_{\text{lim}} = -20.3$ and is approximately equal to 1, i.e. the voids are predominantly in the shape of a slightly elongated cucumber.

The mean $c/a$ and $b/a$ show a small trend at $M_{\text{lim}} < -20.6$ (for the $c/a$ ratio, Fig.6 presents $\sigma$ of the distribution in the form of an error) — the ratios decrease with increasing luminosity and the voids, on average, become more elongated. At the same time, the relation between $c/a$ and $b/a$ is retained in the entire $M_{\text{lim}}$ range — the void oblateness is small and changes only slightly. For all $M_{\text{lim}}$, the cases where $c/a \leq 0.5$ for any of the first 20 voids are rare (the void has a significant elongation) most of the voids have $c/a \geq 0.6$ (the cases of circular voids with $c/a \leq 0.9$ are also rare).

Figure 7 presents the directions of the greatest elongations for the first 20 voids for the chosen $M_{\text{lim}}$ range. The void orientations change significantly even for neighboring values of $M_{\text{lim}}$ (although there are also cases where the void orientation is retained in a certain $M_{\text{lim}}$ range, e.g., near $\lambda = 25^\circ$, $\eta = -35^\circ$) and fill almost the entire area of possible directions rather uniformly.

We also considered the variation in the parameters of a single void that, for most values of $M_{\text{lim}}$, is the first void identified by the algorithm and its shape is determined only by the geometry of the distribution and does not depend on the boundaries of other voids. The volume and $R_{\text{seed}}$ of this void increase synchronously with decreasing $M_{\text{lim}}$ (Fig. 8); the
Fig. 3.— Mean overdensity $< \delta_v >$ of the galaxies in voids (relative to the mean density of the sample galaxies with $M_{\text{abs}} < M_1$) versus $M_{\text{lim}}$ for various $M_1$ (different symbols are used for different values).

Fig. 4.— Example of the correlation between the smallest- to-largest (c/a) and medium-to-largest (b/a) axial ratios of the equivalent ellipsoids of the first 20 identified voids. The slope of the linear fit is $\phi = 1.0$ and $M_{\text{lim}} = -20.3$. 
Fig. 5.— Slope \( \phi \) of the linear fit to the distribution of the \( b/a \) and \( c/a \) ratios (in the approximation of a triaxial ellipsoid) for the first 20 (at given \( M_{\text{lim}} \)) identified voids versus \( M_{\text{lim}} \).

Fig. 6.— Mean \( c/a \) and \( b/a \) ratios for the first 20 voids versus \( M_{\text{lim}} \). The distribution errors \( \sigma \) are given for the \( c/a \) ratios.
dependencies have a break at $M_{lim} = -20.6$ followed by a faster increase in $R_{seed}$ and the volume. The orientation of this void changes significantly and chaotically (Fig. 9). The c/a and b/a ratios increase irregularly with decreasing $M_{lim}$ (Fig. 10) — the void becomes more circular (except for $M_{lim} = -21.2$ at which the surrounding structure apparently changes greatly). The relation between c/a and b/a, along with the direction of the greatest elongation, changes significantly and irregularly.

The dependence of the void volume ($Vol$) on the void rank ($Rank$) at fixed $M_{lim}$ (the largest void has rank 1, the next void has rank 2, etc.) may have a simple interpretation (Gaite and Manrubia 2002). In particular, the break in this dependence reflects the scale of the transition from a power-law galaxy distribution to uniformity (Gaite 2005; Tikhonov 2006b). In this paper, we analyzed the change in the slope of the $\log(Vol) - \log(Rank)$ relation with decreasing $M_{lim}$ in the chosen range of volumes. We choose the range of volumes for each $M_{lim}$ between volume $V_1 \approx 10^4 h^{-3}$Mpc$^3$ (the relation exhibits a cutoff at the void ranks corresponding to volume $V_1$, which is probably determined by the constraint imposed on the minimum $R_{seed}$ of the identified voids) and volume $V_2$ corresponding to the break in the $\log(Vol) - \log(Rank)$ relation, which is interpreted as the beginning of the transition to uniformity in the galaxy distribution (Fig. 11). The power law segment of the relation between $V_1$ and $V_2$ with exponent $z$ reflects a power-law galaxy distribution on small scales. The exponent $z$ increases with decreasing $M_{lim}$ with a break near $M^*$ (Fig. 12). Staring from $M_{lim} = -21.6$, the values of $z$ become larger than unity, which allows the formal fractal dimension $D_z = 3/z$ of the galaxy distribution at given $M_{lim}$ to be estimated (this dimension is considered as a measure of the extent to which the sample volume is filled with galaxies). The significant increase in $z$ with decreasing $M_{lim}$ after the break reflects the well-known fact that more luminous galaxies are clustered more strongly (see, e.g., Tikhonov 2006a). The values of $z < 1$ and the nearly flat pattern of variation in $z$ to $M_{lim} = -21.4$ need a further study and an explanation. The effective radius of a void with volume $V_2$, $R_{eff} = (3 \cdot V_2 / 4 \cdot \pi)^{1/3}$, can be associated with the scale of the transition to uniformity at given $M_{lim}$. In the range of values under study, this scale ($R_{eff}$) lines in the range $19 - 27 \cdot h^{-1}$ Mpc and increases irregularly with luminosity.

5. THE DISTRIBUTION OF VOID CENTERS

In this section, we analyzed a sample with $z_{min} = 0.02$, $z_{max} = 0.12$, $M_{lim}^0 = -20.11$, in which 327 voids with $R_{seed} > 9h^{-1}$ Mpc were identified with a resolution of about 1.5 Mpc. The void centers were defined as the centers of mass of the set of grid points assigned to a given void. We analyzed the distribution of void centers using an cumulative correlation
Fig. 7.— Directions of the greatest elongations (the largest axes a of the equivalent ellipsoids) of the first 20 voids identified in sample for $M_{lim}$. The ranges of the directions corresponding to the geometrical boundaries of the sample are shown.

Fig. 8.— Volume and $R_{seed}$ of one first identified void versus $M_{lim}$.
Fig. 9.— Evolution of the direction of the elongation of one first void with $M_{lim}$.

Fig. 10.— $c/a$ and $b/a$ ratios of one first void versus $M_{lim}$. 
Fig. 11.— Void volume versus void rank at $M_{\text{lim}} = -20.3$. The power-law segment in the volume interval $V_1 - V_2$.

Fig. 12.— Evolution of the exponent of the void volume — rank” relation in the characteristic volume interval with $M_{\text{lim}}$. 
gamma function (conditional density) (Coleman and Pietronero 1992; Tikhonov 2006a). Once the signal has been stabilized (on small scales, the signal is absent, because the distances between the void centers are larger than 18 Mpc), the void centers exhibit a correlation on scales 35–70$h^{-1}$ Mpc with exponent $\gamma_v \sim 0.5$ (Fig. 13). The galaxies involved in determining the voids ($M_{abs} < 20.11$, except for isolated galaxies with $R_n + \sigma_n \approx 3.5$ Mpc) show a slightly weaker correlation, $\gamma_g \sim 0.4$, on these scales. These scales correspond to the range of scales in the galaxy distribution in which the transition to uniformity occurs (Tikhonov 2006a). The slightly larger exponent for the void centers is apparently obtained due to the empty regions near the boundaries, where there are no void centers (according to the definitions in the void search algorithm) and due to the differences in void volumes.

6. COMPARISON OF THE PROPERTIES OF GALAXIES IN VOIDS AND STRUCTURES

To compare the properties of the galaxies in voids and structures, we constructed a volume-limited sample with $z_{min} = 0.02$ and $z_{max} = 0.12$ with a limit on the absolute magnitude $M_{lim}^0 = -20.11$. The voids were determined in the distribution of galaxies with $M_{abs} < M_{lim} = -20.44$ in such a way that some the galaxies from the volume-limited sample could fall into voids. We then excluded isolated galaxies (without any neighbors at a distance smaller than $R_n + \sigma_n \approx 4.2$ Mpc) from the resulting list of void-forming galaxies. In this way, a certain number of isolated galaxies with $M_{abs} < -20.44$ fall into voids.

We identified 235 voids with $R_{seed} > 10h^{-1}$ Mpc. The voids were divided into large ones with $R_{eff} = (3 \cdot Vol / 4 \cdot \pi)^{1/3} > 20h^{-1}$ Mpc and small ones with $R_{eff} < 20h^{-1}$ Mpc. $R_{seed}$ and $R_{eff}$ are correlated: $R_{eff} = (1.26 \pm 0.02) \cdot R_{seed}$. However, the void with a smaller $R_{seed}$ (i.e., identified later) may have a larger volume. The galaxies that fell into voids were divided into bright galaxies of the volume-limited sample with $M_{abs} < -20.11$, i.e., those without any selection in the radial direction, and faint ones with $M_{abs} > -20.11$. A total of 2480 bright and 6104 faint galaxies fell into large voids. The mean overdensity of the galaxies from the volume-limited sample with $M_{abs} < -20.11$ that fell into voids is $\delta \rho / <\rho_{VL}> = -0.78$, where $<\rho_{VL}>$ is the mean density of the galaxies with $M_{abs} < M_{lim} = -20.11$.

The galaxy distribution in the identified voids shows the same features as the galaxy distribution in the entire volume (as was pointed out by Patiri et al., 2006b). Thus, for example, the galaxy distribution in void 2 in order of identification in a 20Mpc layer in $Z$ (in $\lambda$, $\eta$ coordinates) is essentially nonuniform (Fig. 14) — groups of galaxies and a filament crossing the void can be identified. The galaxies delineate the void boundaries and avoid the central region.
Fig. 13.— (a) Correlation functions of the galaxies (filled circles) and void centers (open circles). The right and left Y axes show the amplitudes of the correlation functions of the void centers and galaxies, respectively. (b) The distribution of void centers and galaxies in a ±15 Mpc layer along the Z-axis (in $\lambda$, $\eta$ coordinates).
To compare the galaxy properties, we selected a check sample containing galaxies in structures (dense regions). The galaxies in structures, i.e., those that do not fall into voids, were identified by constructing the minimum spanning tree. This tree consists of knots and edges and is constructed by appending new knots satisfying the condition for the distance to the already constructed part of the tree being at a minimum (Barrow et al. 1985). We constructed the minimum spanning tree in a sample with the angular boundaries $-30^\circ < \lambda < 30^\circ$, $10^\circ < \eta < 30^\circ$.

Once the minimum spanning tree has been constructed from bright galaxies with $M_{abs} < -20.11$, we identified the galaxies that were connected by the edges of lengths no larger than $L_{max}$ determined from the assumed limit on the overdensity. The relation between these parameters is defined by formula $\rho = 1/V = 3/4 \cdot \pi \cdot L_{max}^3$. As the lower limit, we chose the overdensity $\delta \rho/ < \rho_{VL} > = 2 (< \rho_{VL} > \approx 4.5 \cdot 10^3 \text{ Mpc}^{-3})$, which corresponds to an edge of length $L_{max} = 2.6h^{-1} \text{ Mpc}$. The mean distance to the nearest neighbor in this sample is $R_n \approx 2.0h^{-1} \text{ Mpc}$. When the edges smaller than $L_{max}$ are excluded from the tree, the tree breaks up into connected islands with $\delta \rho/ < \rho_{VL} >> 2$. For our analysis, we retained only those islands that contained more than 40 bright galaxies. In the sample under consideration, these structures contain a total of 3011 bright galaxies. The faint galaxies with $M_{abs} > -20.11$ offset by less than $L_{max}$ from the island galaxies were then appended to these structures. The number of faint galaxies attributed to structures turned out to be 3533.
The distribution of the faint galaxies that fell into voids and structures is subject to selection (incompleteness) in the radial direction, but we will consider their properties separately from those of the galaxies from the volume-limited sample — the influence of incompleteness is averaged to some degree, because the voids and structures are distributed quite uniformly over the sample volume.

Figure 15 presents the luminosity distribution for the galaxies in voids and structures. On average, the galaxies in structures are more luminous than those in voids — an excess of bright galaxies in structures and faint galaxies in voids is observed. The abrupt cutoff of the histogram for the galaxies in voids near $M_{abs} = -20.44$ results from the fact that only isolated galaxies with $M_{abs} < -20.44$ fall into voids according to the construction.

Figure 16 presents the galaxy number distribution in large voids as a function of the galaxy distance $r$ from the void boundary normalized to the effective void radius $R_{eff}$ for bright galaxies with $-20.44 < M_{abs} < -20.11$ and isolated galaxies with $M_{abs} < -20.44$ that fell into voids. Both subsamples show a similar increase in the number of galaxies toward the void boundaries. This, in particular, suggests that our approach to excluding isolated galaxies from the list of galaxies in the distribution of which we identified voids is legitimate.

The galaxies in the identified voids exhibit a binomial (red and blue) color distribution (Fig. 17). The mean color for bright galaxies in large voids $< u - r >_{VL} = 2.22$, $\sigma_{VL} = 0.50$. For ”faint” galaxies, $< u - r >_{dim} = 1.94$, $\sigma_{dim} = 0.60$. The systematic difference reflects the fact that the faint galaxies are, on average, bluer. Similar characteristics are observed for the galaxies in small voids. For bright and faint galaxies in structures, $< u - r >_{VL} = 2.47$, $\sigma_{VL} = 0.50$, $< u - r >_{dim} = 2.18$, $\sigma_{dim} = 0.57$. They are systematically redder than those in voids. The histograms for the galaxies in voids and structures differ significantly in shape — the fraction of blue galaxies is much larger in voids.

The data on the star formation rates per unit stellar mass for SDSS galaxies ($log(SFR/M_{star})$, below referred to as SFR) (Kauffmann 2003) were taken from the SDSS archive. Figure 18 compares $log(SFR/M_{star})$ for the galaxies in large voids and structures. The mean value and dispersion of $log(SFR/M_{star})$ for large voids for bright galaxies are $< SFR >_{VL} = -10.26$, $\sigma_{VL} = 0.49$, respectively. For faint galaxies, $< SFR >_{dim} = -10.09$, $\sigma_{dim} = 0.49$. Again the difference in the mean values stems from the fact that faint galaxies have, on average, higher star formation rates than bright galaxies. The bright and faint galaxies in structures have $< SFR >_{VL} = -10.43$, $\sigma_{VL} = 0.50$, $< SFR >_{dim} = -10.25$, $\sigma_{dim} = 0.49$, respectively. The star formation rates in structures are systematically lower.

The red $(u - r > 2)$ and blue $(u - r < 2)$ galaxies show the following characteristics
Fig. 14.— Distribution of galaxies in a ±10h⁻¹ Mpc layer along the Z-axis relative to the center of void No. 2 in order of identification. The asterisk indicates the void center.

Fig. 15.— Distribution of galaxies in voids and structures in absolute magnitudes (opposite hatching). \( M_{VL} = -20.11 \) is the boundary of the volume-limited sample and \( M_{lim} = -20.44 \) is the upper limit on the absolute magnitudes of the involved in constructing the voids.
Fig. 16.— Galaxy number distribution as a function of the distance from the void boundary normalized to the effective void radius, $R_{\text{eff}}$, for all voids with $R_{\text{eff}} > 20h^{-1}$ Mpc. Opposite hatching is used to denote the histograms for galaxies with $-20.44 < M_{\text{abs}} < -20.11$ and isolated galaxies in these voids with $M_{\text{abs}} < -20.44$.

Fig. 17.— $u - r$ colors of galaxies in voids and structures (the histograms with opposite hatching): (a) bright galaxies with $M_{\text{abs}} < -20.11$, (b) faint galaxies with $M_{\text{abs}} > -20.11$. 
for bright galaxies in large voids: \( < u - r >^v_{\text{blue}} = 1.68, \sigma^v_{\text{blue}} = 0.21; < u - r >^v_{\text{red}} = 2.52, \sigma^v_{\text{red}} = 0.42 \) and in structures: \( < u - r >^s_{\text{blue}} = 1.73, \sigma^s_{\text{blue}} = 0.20; < u - r >^s_{\text{red}} = 2.63, \sigma^s_{\text{red}} = 0.37 \). For faint galaxies in voids: \( < u - r >^v_{\text{blue}} = 1.56, \sigma^v_{\text{blue}} = 0.26; < u - r >^v_{\text{red}} = 2.46, \sigma^v_{\text{red}} = 0.39 \) and in structures: \( < u - r >^s_{\text{blue}} = 1.60, \sigma^s_{\text{blue}} = 0.26; < u - r >^s_{\text{red}} = 2.51, \sigma^s_{\text{red}} = 0.41 \).

The differences for voids and structures are found to be insignificant but larger than those obtained in a similar analysis by Patiri et al. (2006b).

For the galaxies in structures, the \((u - r)\) color — absolute magnitude (the trend with a slope of \(\text{slope}_1 = -0.24\) — brighter galaxies are, on average, redder), \(\log(SFR/M_{\text{star}})\) — absolute magnitude (\(\text{slope}_2 = 0.14\) — brighter galaxies have, on average, lower star formation rates), and color — \(\log(SFR/M_{\text{star}})\) (\(\text{slope}_3 = -0.59\) — bluer galaxies have, on average, higher star formation rates) relations are less pronounced than those for the galaxies in voids, for which \(\text{slope}_1 = -0.27, \text{slope}_2 = 0.16, \text{slope}_3 = -0.66\), respectively, although the differences are small.

The mean luminosity of the galaxies in all the identified voids is virtually independent of the galaxy distance from the void boundary normalized to the effective void radius \(R_{\text{eff}}\). The bright and faint galaxies in voids show opposite weak trends: on average, the galaxies with \(M_{\text{abs}} < -20.11\) become brighter and the galaxies with \(M_{\text{abs}} > -20.11\) become fainter as one goes from the boundaries to the central regions of the voids.

The (bright and faint) galaxies in voids, on average, have bluer colors (lower \(u - r\)) and higher star formation rates (the slopes of the linear fits to the \(((u - r) - r/R_{\text{eff}})\) \(\log(SFR/M_{\text{star}}) - r/R_{\text{eff}}\) relations are -0.12 and 0.06, respectively) as \(r/R_{\text{eff}}\) increases (where \(r\) is the distance from the boundary of a given void). Thus, the differences in the properties of the galaxies in voids and structures slightly increase even further when less dense regions inside the identified voids are considered.

7. CONCLUSIONS

In this paper, using an algorithm of identifying arbitrarily shaped voids, we analyzed the change in some of the void characteristics with luminosity of the galaxies (in the range of limits \(M_{\text{lim}}\) on the absolute magnitude -19.7 — -21.2) involved in the construction of voids and compared the properties of the galaxies in voids (mean density contrast) and the galaxies forming structures with an density contrast higher than 2.

The evolution of a number of void characteristics with luminosity shows a break near
$M^*$, in agreement with the change in exponent in a correlation analysis of the distribution of SDSS galaxies (Tikhonov 2006a). The structure of the galaxy distribution apparently changes qualitatively at $M_{abs} < M^*$.

As expected, the void volumes increase with decreasing $M_{lim}$ with a break near $M^*$ followed by a faster increase (in contrast, the decrease in the number of galaxies that form voids slows down), with the pattern being the same for two samples of equal volume covering different regions of space.

The mean density contrast in voids also increases with decreasing $M_{lim}$ with a weak break near $M^*$. The voids become emptier if we reduce the limit on the absolute magnitude of the galaxies under consideration at fixed $M_{lim}$.

The galaxies inside voids concentrate to the void boundaries and avoid the central regions. The density contrast profile is flat in intermediate regions. These results agree with those obtained previously from the galaxies (Patiri et al. 2006b) and the dark matter haloes in voids (Gottlober et al. 2003). Our study also confirms that the matter in voids is distributed irregularly and has the same features as the general distribution obtained by the above authors.

In general, the mean characteristics of the void shapes are retained with decreasing $M_{lim}$. We can only note a weak tendency: at $M_{lim} = -20.6$, the voids become, on average, slightly more elongated. There is also a weak tendency for the slope of the distributions of the medium-to-larger ($b/a$) smaller-to-larger ($c/a$) axial ratios of the ellipsoid equivalent to the void (these ratios are well correlated) to increase with decreasing $M_{lim}$. The slopes of the linear fits to these distributions are close to unity (0.8 – 1.1) in the entire $M_{lim}$ range, i.e., the voids are predominantly elongated and nonoblate at all $M_{lim}$. At the same time, the individually considered voids can change their shape with $M_{lim}$ significantly. The directions of the greatest void elongations are distributed quite uniformly and change chaotically with $M_{lim}$, which is indicative of an irregular change in the structure when more luminous galaxies are considered.

The exponent of the void volume – rank ($\log(\text{Vol}) - \log(\text{Rank})$) relation in the characteristic range of volumes increases significantly with decreasing $M_{lim}$ starting from $M_{lim} = -20.4$ (a break is again observed in the relation at a value close to $M^*$), which is a reflection of the tendency for more luminous galaxies to cluster more strongly. The scales of the beginning of the transition to uniformity (the break of the power-law segment in the $\text{Vol} – \text{Rank}$ relation at volume $V_2$ – see the text) determined from this relation increase with decreasing $M_{lim}$ and agree with the results of a correlation analysis of the galaxy distribution.

The distribution of void centers shows a certain correlation and reflects the correlations
of the galaxy distribution on the corresponding scales.

The derived differences in the properties of the galaxies in voids with a mean density contrast of -0.78 and the galaxies in structures with a density contrast higher than 2 (identified using the minimum spanning tree) agree qualitatively with the existing views: the galaxies in voids are, on average, bluer and have higher star formation rates per unit stellar mass. The last two tendencies become stronger when galaxies located closer to the central void regions are considered. However, quantitatively, these differences are not large enough to conclude that the formation histories of the galaxies in structures and voids differ fundamentally.

The need for a further study of the observed correlations and multiparameter tendencies and for determining the mean galaxy characteristics over wide ranges of density contrasts and luminosities and for various definitions of the characteristic galaxy environments is obvious. Our division into voids and structures is to some extent arbitrary and consists mainly in different density contrasts: there are also structures in the voids determined in this paper.

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Fig. 18.— Star formation rates per unit stellar mass (log(SFR/M$_{\text{star}}$)) for (a) bright and (b) faint galaxies in voids and structures.