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MSC:
33D45 Basic orthogonal polynomials and functions (Askey-Wilson polynomials, etc.)
05A30 q-calculus and related topics
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
60E05 Probability distributions: general theory

Keywords:
q-Hermite; Al-Salam-Chihara; Rogers-Szegő; Chebyshev polynomials; q-ultraspherical; continuous q-Hahn; Askey-Wilson polynomials; connection coefficients; summing kernels; generalization of Poisson-Mehler kernel; generalization of Kibble-Slepian formula; q-Wiener processes; conditional moments; joint; marginal; conditional distributions

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