Numerical simulation of effects of sand grain diameters and volume fractions on mass transferring from the water-liquid to the water-vapor

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Abstract. The paper analyzed the effects of sand grain diameters and volume fractions on the mass transferring from the water-liquid to the water-vapor in a two-dimensional nozzle. Based on the mixture model, $k-\varepsilon$ turbulence model and Schnerr-Sauer cavitation model, the solid-liquid-vapor three phases’ cavitation flows were simulated. When the grain mean diameters were defined as constants, volume fractions were changed to investigate the effects of them. The grain mean diameters were 0.013 mm, 0.025 mm and 0.05 mm. Volume fractions were 0.02, 0.04, 0.05, 0.07 and 0.10. Results indicated that cavitation occurred at the beginning spots of the narrow part of the nozzle, low pressure regions. With the different grain mean diameters and volume fractions, effects of the sand on the mass transferring from the water-liquid to the water-vapor were diverse, proved by the curves of the cavitation numbers with the volume fractions of the sand and the curves of the volume fractions of the water-vapor with the volume fractions of the sand, reflecting the distinctions of interactions between the bubbles and the sand grains.

Introduction

Cavitation is that the water-vapor bubbles are created in the low pressure regions in the fluid flow field, causing the destruction of materials, initiating the vibration and the noise and making the performances of the machines decrease. Cavitation also is a complex multiphase flow. Different cavitation models were built by Singhal, Kunz and Schnerr&Sauer, to predict and analyze the features...
of it. Nowadays, two different methods have been used to study cavitation, including the experiment and numerical simulation. However, exact solutions of some certain parameters can’t be gained by experiments, so numerical simulation is applied widely. In this article, this will be followed by a description of the effects of the sand on the mass transferring from the water-liquid to the water-vapor in a 2D flow field.

Nozzle is one kind of significant experiment instrument to simulate the cavitation flow. The study of the cavitation occurring in the nozzle has been done by some prestigious scholars. The k-ω SST two-equation turbulence model and k-kl-ω turbulence model were employed by Meixin Xue to simulate the non-cavitation and cavitation flow in a diesel nozzle [3]. Some parameters of the cavitation were changed by Hengzhou Wo to research incipient cavitation in orifices [7]. A cavitation water jet flow field of the convergent-divergent nozzle was discussed by Yiyu Lu [4]. The fluid-solid two phases jet flow field in a nozzle was acquired by Mingbo Wang [5]. The effects of different shapes of the nozzle on the cavitation were considered by Liming Yao [6].

To obtain the accurate results of three phases’ cavitation flows, the preparation work was model foundation and grid partition. In these simulations, the grain mean diameters of the sand were 0.013mm, 0.025mm and 0.05mm and each volume fraction of the sand was 0.02, 0.04, 0.05, 0.07 and 0.10. With the grain mean diameters and volume fractions of the sand having changed respectively, based on the mixture model, two phases’ flows of fluid-solid were numerically simulated in this nozzle. When these flows converged, based on the Schnerr-Sauer cavitation model, water-vapor was added to these simulations to investigate the mass transferring from the water-liquid to the water-vapor in the solid-liquid-vapor three phases’ cavitation flows.

1. Physical and mathematical equations in multiphase Flow

1.1. Basic equations in multiphase flow [14]

Continuity equation:

$$\frac{\partial \rho_j}{\partial t} + \frac{\partial (\bar{\rho}_j \bar{u}_j)}{\partial x_j} = 0$$  \hspace{1cm} (1)

Momentum equation:

$$\frac{\partial}{\partial t} (\bar{\rho}_j \bar{u}_j) + \frac{\partial}{\partial x_j} (\bar{\rho}_j \bar{u}_j \bar{u}_j) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} (\bar{t} \bar{u}_j) + \bar{p}_j \bar{u}_j$$  \hspace{1cm} (2)

Energy equation:

$$\frac{\partial}{\partial t} (\bar{\rho}_j \bar{T}_j) + \frac{\partial}{\partial x_j} (\bar{\rho}_j \bar{u}_j \bar{T}_j) = \frac{\partial}{\partial x_j} (\lambda \frac{\partial \bar{T}_j}{\partial x_j}) + \dot{q}_j - \bar{q}_r$$  \hspace{1cm} (3)

Composition equation:

$$\frac{\partial}{\partial t} (\bar{\rho}_j \bar{Y}_j) + \frac{\partial}{\partial x_j} (\bar{\rho}_j \bar{u}_j \bar{Y}_j) = \frac{\partial}{\partial x_j} \left( D_p \frac{\partial \bar{Y}_j}{\partial x_j} \right) - \bar{\omega}_j$$  \hspace{1cm} (4)
1.2. Standard $k - \varepsilon$ two-equation model \[15\]

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho ku)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_i + G_h - \rho \varepsilon - Y_k + S_k
\]

\[5\]

\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \frac{\varepsilon}{k} (G_i + C_f G_h) - C_o \rho \frac{e^2}{k} + S_k
\]

\[6\]

1.3. Rayleigh-Porisky equation \[14\]

\[
r_b \frac{d^2 r_b}{dt^2} + \frac{3}{2} \left( \frac{d r_b}{dt} \right)^2 = \left( \frac{p_b - p_v}{\rho_i} \right) - \frac{4\nu}{r_b} \frac{d r_b}{dt} \frac{2\sigma}{\rho \rho_b}
\]

\[7\]

1.4. Schnerr-Sauer cavitation model \[10\]

Bubble radius:

\[
r_b = \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{3}} \frac{3}{4\pi n_b}
\]

\[8\]

Source terms:

\[
R_c = \frac{\rho_i \rho_b}{\rho} \alpha \left( 1 - \alpha \right) \frac{2 \left( p - p_v \right)}{3 \rho_i} \left( \frac{p}{\rho} \right) \left( \frac{p}{\rho_v} \right)^{\frac{1}{3}} \quad (p \leq p_v)
\]

\[
R_c = \frac{\rho_i \rho_b}{\rho} \alpha \left( 1 - \alpha \right) \frac{2 \left( p - p_v \right)}{3 \rho_i} \left( \frac{p}{\rho} \right) \left( \frac{p}{\rho_v} \right)^{\frac{1}{3}} \quad (p \geq p_v)
\]

\[9\]

2. Numerical simulation

2.1. Basic assumption

The water was incompressible and the sand was the continuous medium. Physical properties of every phase were invariant. Shape of the sand grain was regarded as sphere. It was inserted and was not insulated in water. Phase transition of the sand can be neglected. The flowing state was steady.

2.2. Geometric model and grid partition

The geometric model had the same sizes with the nozzle coming from the ANSYS Fluid Dynamic Verification Manual. The geometric parameters were that the inlet diameter was 23mm and the outlet diameter was 8mm; the wide length was 16mm and the narrow length was 32mm. \[16\]. Quadrilateral structured grids were adopted to discrete the computational domain.

Figure 1. Geometric model of the nozzle. Figure 2. Computational domain and grids.
2.3. Method of numerical simulation and boundary conditions

Pressure was defined at the inlet and the outlet and values were 250,000,000Pa and 95,000Pa. The value of vaporization pressure was 3540Pa. And the temperature was 300K. [16]

Physical parameters used in numerical simulations were that density of the water-liquid was $\rho_l=1000 \text{ kg/m}^3$ and the kinematic viscosity of the water-liquid was $\mu_l=0.001 \text{ kg/(m·s)}$; Values of the density and the kinematic viscosity of the water-vapor were $\rho_v=0.02558 \text{ kg/m}^3$ and $\mu_v=1.26\times10^{-6} \text{ kg/(m·s)}$. Density of the sand was $\rho_s=2650 \text{ kg/m}^3$. The bubble diameter was $r_b=1.0\times10^{-5} \text{ m}$ and the bubble density was $n_b=1.0\times10^{13}$. [16]

Commercial Fluent software was used to simulate and analyze the cavitation flow field. Standard wall functions were applied to treat the flow near the wall. “Implicit Body Force” was employed to speed up the convergence. Drag coefficient was set from the sand to the water-liquid and from the water-vapor to the water-liquid. The level of the turbulent intensity was 10%. For the sand and the water-vapor, the backflow volume fraction was zero. All the residuals were less than $1.0\times10^{-4}$.

Numerical simulations were performed by varying the volume fractions of the sand when the grain mean diameter of the sand was a constant.

3. Results and Analysis

3.1. Contours of volume fraction of water-liquid or water-vapor

![Contours of volume fraction of water-liquid or water-vapor](image)

Figure 3. Contour of volume fraction of the water-liquid from the ANSYS Fluid Dynamic Verification Manual and simulated.

Figure 3 exemplified the contour of volume fraction of the water-liquid from the ANSYS Fluid Dynamic Verification Manual. Figure 4 indicated that the contour of volume fraction of the water-liquid simulated under the same boundary conditions. Compared these two contours, regions of the low pressure were the same, verifying that the mass transferring from the water-liquid to the water-vapor occurring at the same place.

![Contours of volume fraction of water-vapor](image)

Figure 4. Contours of volume fraction of the water-vapor with mean diameter being 0.013mm
Axial symmetrical distribution of the contours of the water-vapor was one obvious feature of these contours. The water-vapor was mainly concentrated on the beginning spots of the narrow part of the nozzle, for example, spot A and B, demonstrating that cavitation mainly occurred at these spots. With the distance increasing gradually, contents of the water-vapor became very little. At the outlet, contents reduced nearly to zero. When the grain mean diameter and volume fraction of the sand was set to 0.05mm and 0.1, changing of the water-vapor was more markedly. A possible explanation was that the decrease of contents of the water-vapor was made by the inter-collisions of sand grains with bubbles. When the grain mean diameters were gotten larger and larger and the volume fractions were grown more massively, inter-collisions developed with far more aggravation and intensification.

3.2. Curves of cavitation numbers with volume fractions of the sand

The cavitation number is

$$\sigma = \frac{p_v - p_e}{\frac{1}{2} \rho v^2}$$

(10)

So, curves of the cavitation numbers with the volume fractions of the sand could be got.

Figure 7. Curve of cavitation numbers with the volume fractions of the sand.

Figure 7(a) shows that when the sand grain mean diameter was 0.013mm and the volume fractions
were grown gradually, cavitation numbers were a steady decline, owning to a linear-change-rate. So equation of a degree could be used to fit this curve approximately.

Figure 7(b) reveals that when the grain mean diameter was set to 0.025mm and the volume fractions were increased gradually, cavitation numbers made a substantial decrease and when the volume fractions were ranged from 0.02 to 0.07, linear-variety-rate could be used to describe this trend approximately. There was a sharp drop of the cavitation numbers when the volume fractions were varied from 0.07 to 0.10.

Figure 7(c) indicates that the condition was that the grain mean diameter was 0.05mm. When the volume fractions of the sand were grown constantly, the changing trend of cavitation numbers looked like a parabola. When the volume fraction was 0.04, the cavitation number reached its highest point, reflecting that it had a profound impact on the cavitation flow. So the parabola equation could be utilized to fit this curve approximately.

3.3. Curves of volume fractions of the water-vapor with volume fractions of the sand

Figure 8(a) conveys that when the volume fractions of the sand were increased gradually, the whole changing trend of the water-vapor looked like a “W”. When the volume fraction of the sand was 0.05, volume fractions of the water-vapor reached its maximum. However, when the volume fraction of the sand was set to 0.07, volume fractions of the water-vapor became its minimum. The fluctuation varied between 0.089186 and 0.088035. Figure 8(b) illustrates that when the grain mean diameter was set to 0.025mm and the volume fractions of the sand were increased constantly, the substantial decrease of the volume fractions of the water-vapor was one feature of this curve.

Figure 8(c) explains that compared with Figure 11 and Figure 12, this figure showed a completely different trend. The prerequisite was that the grain mean diameter was 0.05mm. When the volume fractions of the sand were changed from 0.02 to 0.07, a slight decline was one character of the volume fractions of the water-vapor. However, when the volume fractions of the sand were ranged from 0.07 to 0.10, there was a slump in the volume fractions of the water-vapor. So, it was palpable that when the volume fraction of the sand reached 0.10, the volume fraction of the water-vapor reached its lowest point.

4. Summary and Conclusions

Based on the Schnerr-Sauer cavitation model, the effects of sand grain mean diameters and volume fractions on the mass transferring from the water-liquid to the water-vapor have been explored. It was
manifest to comprehend the effects on the cavitation flow and these simulations could be applied to investigate solid-liquid-vapor three phases’ cavitation flow in the centrifugal pump.

From the presented results, contours and curves, conclusions are that:

1. Axial symmetrical distribution was one feature of all contours of the water-vapor.
2. When the grain mean diameter was 0.05mm and the volume fraction was 0.1, sand had a profound impact on the cavitation flow.
3. When the grain mean diameters and volume fractions were distinct and diverse, distributions of the water-vapor and values of the cavitation numbers were different, reflecting the distinctions of interactions between the bubbles and the sand grains.

Nomenclature

\[ u = \text{velocity}; \ p = \text{pressure}; \ \rho = \text{density}; \ \rho_v = \text{density of water-vapor}; \ \rho_m = \text{density of mixture}; \]
\[ p_s = \text{static pressure}; \ \tau_{k,ji} = \text{component of stress tensor}; \ T = \text{temperature}; \]
\[ g = \text{body force per unit volume}; \ C = \text{specific heat}; \]
\[ \dot{\lambda} = \text{thermal conductivity}; \ q_c = \text{radiant heat}; \ q_r = \text{reaction heat}; \]
\[ \dot{\omega} = \text{instantaneous value}; \ k = \text{turbulent kinetic energy}; \]
\[ \mu = \text{dynamic viscosity}; \ \mu_t = \text{turbulent viscosity}; \]
\[ C_{2k} = \text{empirical constant}; \]
\[ \sigma_k = \text{empirical constant}; \]
\[ \sigma_\varepsilon = 1.0; \ b = \text{bubble}; \]
\[ \rho_b = \text{bubble wall pressure}; \]
\[ \sigma_b = \text{bubble wall tension}; \]
\[ \alpha = \text{fraction of water-vapor}; \]
\[ \lambda = \text{thermal conductivity}; \]
\[ q_c = \text{radiant heat}; \]
\[ q_r = \text{reaction heat}; \]
\[ \mu_t = \text{turbulent viscosity}; \]
\[ C_{2k} = \text{empirical constant}; \]
\[ \sigma_k = 1.0; \]
\[ \rho_w = \text{free stream pressure}; \]
\[ \rho_i = \text{density of water-liquid}; \]
\[ r_b = \text{bubble radius}; \]
\[ n_b = \text{bubble density}; \]
\[ \nu = \text{kinematic viscosity}; \]
\[ \rho_v = \text{density of water-vapor}; \]
\[ p_vap = \text{vaporization pressure}. \]

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