Defective fission correlation data from the 2E-2v method

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(Dated: June 27, 2018)

The double-energy double-velocity (2E-2v) method allows assessing fission-fragment mass yields prior to and after prompt neutron emission with high resolution. It is, therefore, considered as a complementary technique to assess average prompt neutron multiplicity as a function of fragment properties. We have studied the intrinsic features of the 2E-2v method by means of event-wise generated fission-fragment data and found severe short-comings in the method itself as well as in some common practice of application. We find that the 2E-2v method leads to large deviations in the correlation between the prompt neutron multiplicity and pre-neutron mass, which deforms and exaggerates the so called ‘sawtooth’ shape of \( \bar{\nu}(A) \). We have identified the treatment of prompt neutron emission from the fragments as the origin of the problem. The intrinsic nature of this deficiency, risk to render 2E-2v experiments much less interesting. We suggest a method to correct the 2E-2v data, and recommend applying this method to previous data acquired in 2E-2v experiments, as well.

PACS numbers: 25.85.-w,29.85.Fj
Keywords: 2E, 2E-2v, fission, neutron multiplicity

Fission-fragment (FF) spectroscopy has been carried out since the very beginning of the discovery of nuclear fission. It represents an important gateway for understanding the dynamics of fission. In particular the number of prompt neutrons emitted from each highly excited FF and its correlation with FF properties (mass and total kinetic energy) is a key to better understand how the excitation energy is shared between the fragments. Together with other observables, e.g., isomeric yield ratios and prompt \( \gamma \)-rays, they may provide a deeper insight into the dynamics of the fission process. High-resolution fragment yields may reveal the underlying nuclear structure, observed, e.g., through the so-called odd-even staggering in the observed mass yields.

One principal technique for investigating FF properties, e.g., pre-neutron mass yields and total kinetic energy (TKE) distributions, is based on the measurement of both fragment kinetic energies; the so-called double-energy (2E) method [1]. The conducted 2E studies are numerous, but rarely with a pre-neutron mass resolution no better than 4 u.

Two techniques bringing major improvements to the mass resolution are the energy-velocity method, exploited by the LOHENGRIN recoil mass separator [2], and the double-energy double-velocity method (2E-2v), which was introduced via the concept of COSI-FAN-TUTTE [3]. The 2E–2v method is currently under development world-wide, e.g., VERDI [4], SPIDER [5] (and future MegaSPIDER), FALSTAFF [6] and STEFF [7]. All the present realizations of the 2E–2v method promise a resolution of the pre-neutron mass better than 2u.

While the 2E method relies on prompt neutron emission data as an input to the data analysis, the 2E–2v technique promises provision of those data from the independent analysis of the experimental data. A focus is put on prompt-neutron FF correlations, which merit more and more attention by theoreticians and modelists. As many experiments have used a 2E analysis for calibration purposes, we studied aspects of this technique too. We will demonstrate inherent short-comings of both the techniques and how they affect the 2E–2v data. We also suggest a way to remedy both the 2E–2v method and previously obtained 2E–2v data.

To facilitate a clean test of the analysis methods, they are not tested with experimental data. Instead we generated realistic synthetic data, in order to know the true values of all variables exactly. The generated data do not need to replicate reality perfectly since we merely want to know how well the analysis method will reproduce the synthetic data. However, it is beneficent to work with reasonably realistic data.

The GEF code [8] was used to generate \( 1 \times 10^{6} \) fission events of the studied fissioning system. The fragment recoil upon neutron emission is not correctly handled by GEF, so the post neutron state was recalculated based on the pre-neutron state as well as a list of Center-of-Momentum (CoM) energies of the emitted neutrons for both fragments. In both cases the information was supplied by GEF. For each neutron energy in the list, a random direction was selected isotropically in the fragment’s
CoM frame, and the emission kinematics was calculated in full.

Since the GEF output only contained the atomic and mass numbers, the fragment masses were looked up in The Ame2012 atomic mass evaluation [9]. The processed events were written to a ROOT file [10], later to be analyzed by the 2E–2v method, as well as the 2E method, both described below.

In our ‘ideal’ study of the 2E–2v method no resolution or other detector effects are included. In the analysis relativistic kinematics are used, but for simplicity, we show the classical equivalent expressions for how the relevant quantities are derived:

\[
M_{1,2}^{\text{post}} = M_{\text{sum}}M_{1,2}^{\text{pre}} \frac{v_{1,2}^{\text{pre}}}{v_{1,2}^{\text{pre}} + v_2^{\text{pre}}} \quad \text{(1)}
\]

\[
M_{1,2}^{\text{pre}} = \frac{2E_{1,2}^{\text{post}}}{(v_{1,2}^{\text{post}})^2} \quad \text{(2)}
\]

\[
E_{1,2}^{\text{pre}} = M_{1,2}^{\text{pre}}v_{1,2}^{\text{pre}}^2 \frac{1}{2} \quad \text{(3)}
\]

\[
\nu_{1,2} = \frac{M_{1,2}^{\text{pre}} - M_{1,2}^{\text{post}}}{m_n} \quad \text{(4)}
\]

where \(M_{\text{sum}} = M_{\text{pre}}^1 + M_{\text{pre}}^2 = M_{\text{Cf252}} - \text{TKE}^{\text{pre}}/c^2\), \(M\) denotes mass, \(\nu\) velocity, \(E\) energy and \(\nu\) neutron multiplicity. The momentum of the incoming particle has been neglected since we have only studied spontaneous fission or fission induced by thermal neutrons.

The 2E–2v method makes the assumption of isotropic neutron emission in the CoM frame, which almost leads to a conservation of the average values of the velocities as pointed out by Stein [11]. It can be shown that the difference is less than 0.01%. The velocities are not conserved event-wise, but since \(\nu^{\text{pre}}\) is not measured, the 2E–2v method approximates \(\nu^{\text{pre}}\) by \(\nu^{\text{post}}\) in Eqs. (1) and (3). The error of this estimation of \(\nu^{\text{pre}}\) is, in the 252Cf(sf) case, approximately equivalent to a 1% normally distributed random error in \(\nu^{\text{pre}}\).

Contrary to the 2E–2v method, the 2E method requires additional information since only the two energies are measured. The 2E analysis needs the neutron multiplicity \(\nu\) to be parametrized based on information acquired from previous measurements or model calculations. In our case, the information comes from GEF rather than experimental studies.

For each event, the 2E method iterates Eqs. (5) to (7) until the masses (and energies) converge.

\[
M_{1,2}^{\text{post}}(i+1) = M_{1,2}^{\text{pre}}(i) - \nu_{1,2}m_n \quad \text{(5)}
\]

\[
E_{1,2}^{\text{pre}}(i+1) = E_{1,2}^{\text{post}}(i) - \frac{M_{1,2}^{\text{pre}}(i)}{M_{1,2}^{\text{post}}(i+1)} \quad \text{(6)}
\]

\[
M_{1,2}^{\text{pre}}(i+1) = M_{\text{sum}}E_{1,2}^{\text{pre}}(i+1) + \frac{E_{1,2}^{\text{pre}}(i+1)}{E_{1,2}^{\text{pre}}(i+1) + E_2^{\text{pre}}(i+1)} \quad \text{(7)}
\]

When all masses changed by less than \(1 \times 10^{-10}\) u between two subsequent iterations, convergence was considered reached. Typically, only a few iterations were needed.

In Eq. (6), two terms for each emitted neutron have been left out:

\[
\frac{m_n}{M_{\text{post}}}E_n^{\text{CoM}} \quad \text{and} \quad m_n\nu^{\text{pre}}\nu_n^{\text{CoM}}.
\]

The first term is in the order of tens of keV, while the second term vanishes on average, assuming a isotropic neutron emission in the CoM frame of the fragment. Just as in the 2E–2v case, this approximation is only strictly valid for average quantities.

Both the 2E–2v and the 2E method have been found to, on average, reconstruct the proper mass within about 0.1 u. A slight overestimation of \(M^{\text{pre}}\) is expected due to the excitation energy of the FFs, which is unaccounted for in Eqs. (1) and (7). Although both methods reproduce the one-dimensional mass and energy spectra well, discrepancies start to show as one looks at other derived quantities. In FIG. 1 the resulting \(\nu(M^{\text{pre}})\) from the 2E–2v method is plotted together with the synthetic ‘truth’. It is seen to deviate from the correct value especially around symmetry and in the wings of the mass distribution. If \(\nu^{\text{pre}}\) is never approximated by \(\nu^{\text{post}}\) in Eqs. (1) and (3), the 2E–2v analysis of the synthetic data reproduces the correlations to high precision. This proves that the problem lies in this approximation, but \(\nu^{\text{pre}}\) will of course not be available in an actual experimental situation.

The assumption of an unchanged fragment velocity, before and after neutron emission, causes each of the velocities to be over- or underestimated with equal chances. This directly leads to an over- and underestimation of the pre-neutron masses in Eq. (1). Since there is no approximation in Eq. (2), \(M_1^{\text{post}}\) and \(M_2^{\text{post}}\) will always be calculated correctly. It is then clear from Eq. (4) that an overestimation of \(M^{\text{pre}}\) directly leads to an overestima-
If calibration of-

For example, the ratio in the range of 30 ns to 80 ns. Since the ToF could not be reproduced, the ratio must be incorrect.

FIG. 2. (Color online) The mass-wise positive correlation between the calculated \( M^{\text{pre}} \) and \( \bar{\nu} \) for events with mass number 110, 120 and 130, respectively.

The discrepancies are the largest around symmetry, so let us observe \( \bar{\nu} \) for a mass slightly lower than the mass of symmetric fission (where \( \bar{\nu}(M^{\text{pre}}) \) is overestimated in FIG. 1). Due to the large yield differences between neighboring masses, the calculated \( \bar{\nu} \), will be dominated by events that have overestimated \( M^{\text{pre}} \), i.e., events with a lower true \( M^{\text{pre}} \). Since we concluded that these events also overestimates \( \nu \), the \( \bar{\nu} \) evaluated for this mass will be the average of predominately overestimated neutron multiplicities. Thus, \( \bar{\nu} \) itself will be overestimated. The same effect is mirrored for masses at the other side of symmetry, and the argument applies equally well on the discrepancies in the wings of the mass distribution, where the magnitude of the mass-yield derivative with respect to \( M^{\text{pre}} \) is also large. The end result is the exaggerated saw tooth shape one sees in FIG. 1.

We suspect that the effect of this correlation can be found in previous measurements. Although the assumption that the velocities are conserved is central to the 2\( E \)–2\( \nu \) method, its implications seem to have gone unnoticed in several measurements. In the \( ^{239}\text{Pu} (n_{\text{th}}, f) \) measurements of Nishio et al. [12] the velocity assumption is not given much attention and is not even included in the error estimation. Brinkmann et al. comments on the errors introduced by neglecting neutron evaporation as “possibly serious” but never encounter the correlation problem since they do not present \( \bar{\nu}(M^{\text{pre}}) \) in their paper [14]. Müller et al. report on a 2\( E \)–2\( \nu \) measurement of \( ^{235}\text{U} (n_{\text{th}}, f) \) where the over- and undershoot for \( \bar{\nu}(M^{\text{pre}}) \) in the symmetry region was interpreted as a “better mass resolution” (FIG. 11 and 12 in ref. [15]). They attributed a pre-neutron mass uncertainty of 0.325 u to the neutron emission, but they never made a reference to any correction for the correlation problem presented here, so we must assume that their results were affected by it.

Due to the lower neutron multiplicity of \( ^{235}\text{U} (n_{\text{th}}, f) \) compared to \( ^{252}\text{Cf} (sf) \), the exaggeration of \( \bar{\nu}(M^{\text{pre}}) \) is less severe in the Müller et al. data, but we observed the same trend of over- and undershoots by running our simulation also for the \( ^{235}\text{U} (n_{\text{th}}, f) \) case.

By having established the exact cause and effect of the problem, we can now focus on solving it. We will show that it is possible to correct \( \bar{\nu}(M^{\text{pre}}) \), by outlining a simple proof-of-concept correction procedure that does not rely on any new data, only on the measured (uncor-
rected) pre-neutron mass distribution and neutron multiplicity. The correction is applied to $\bar{\nu}(M^{\text{pre}})$ as a whole, not event-wise, and can therefore also be used to correct previous measurements.

The algorithm works by deconvoluting binned data. In our tests, a bin width of 0.1 u has been used, which is much smaller than any feasible experimental mass resolution. The origin of the problem is that an actual pre-neutron mass, $M^c$, can be incorrectly assigned a different mass $M^m$ due to the neutron emission. Therefore, we construct a response matrix $R$ such that

$$M^m_i = \sum_j R_{ij} M^c_j. \quad (8)$$

The response can be closely modeled by a normal distribution, where the mass dependent width can be estimated by kinematic calculations using the measured $\bar{\nu}(M^{\text{pre}})$. In the case of $^{235}\text{Cf}(sf)$ the standard deviation due to neutron emission was on average 0.8 u. In addition, each matrix element $R_{ij}$ is weighted by the mass yield $Y(M^c_j)$.

To express the inverse relationship,

$$M^c_j = \sum_i R_{ij} M^m_i. \quad (9)$$

it was assumed that the transpose of $R$ could approximate $R^{-1}$, even though $R$ is not necessarily strictly orthogonal.

We calculate a corrected $\bar{\nu}$,

$$\bar{\nu}^c_j = \frac{1}{m_n} (M^c_j - M^{\text{post}}_j) = \frac{1}{m_n} \sum_i R_{ij} (M^m_i - (M^{\text{post}}_i + \Delta M_{ij})), \quad (10)$$

where $\Delta M_{ij}$ is just the distance between the bins $i$ and $j$ in mass units, and Eq. (9) has been used to rewrite $M^c_j$.

We recognize that $\bar{\nu}^m_i = (M^m_i - M^{\text{post}}_i)/m_n$ and arrive at our final expression:

$$\bar{\nu}^c_j = \sum_i R_{ij} \left( \bar{\nu}^m_i - \frac{\Delta M_{ij}}{m_n} \right). \quad (11)$$

Figure 4 shows the resulting corrected $\bar{\nu}(M^{\text{pre}})$. The synthetic ‘true’ $\bar{\nu}(M^{\text{pre}})$ is reproduced remarkably well, considering the somewhat crude correction method. The method also conserves the total average neutron multiplicity. Unfortunately, a smearing effect is present, leaving us unable to resolve all structures in the synthetic $\bar{\nu}(M^{\text{pre}})$.

In FIG. 5 we show the result of testing the correction on $^{235}\text{U}(n_{th},f)$ instead of $^{252}\text{Cf}(sf)$. The synthetic $\bar{\nu}$ generated by the GEF code shows much more structure than one observes in a real experiment on this reaction. However, this makes it an even better test to see how much of the structure in the synthetic data our suggested correction method reproduces.

It has been known from the start that the $2E$ method has limited resolution due to the neutron emission. But it is not only a resolution problem. A pure $2E$ measurement deduces at least eight quantities, even though only two energies are measured. That is the great success of the method, but it also leaves many degrees of freedom to be incorporated in calibrations or substituted by previously measured data. Reproducing known quantities and correlations is no guarantee that every possible correlation deduced by the $2E$ method is correct. Caution is advised whenever a new correlation is derived and interpreted. The same analysis used for the experimental data should be tested also with synthetic data, and the
dependence of, e.g., the input data and neutron emission should be thoroughly investigated. Only then one is able to decide, whether the quality of the data meets the requirements for the purpose one has in mind.

One such purpose was mentioned previously. Using a $2E$ analysis to determine the PDT calibration for a $2E-2\nu$ setup, might risk to fail. The deduced ToF deviates systematically from the true value, and neither the $M_{\text{pre}}$ nor the $E_{\text{post}}$ functional dependence are correctly reproduced. The errors would be less significant if a longer flight path was used but that would diminish the solid angle coverage, which is usually already small. Luckily, there are other ways for $2E-2\nu$ setups to determine the PDT. Correcting for the PDT is not easy, but examples in literature exist where the PDT corrections did not rely on the $2E$ method [14, 15].

Measuring $\bar{\nu}$ in a straightforward and independent way is one of the highlights of the $2E-2\nu$ method. Inability to reproduce the correct shape of $\bar{\nu}(M_{\text{pre}})$ would have made the whole method much less interesting. We found that the intrinsic problem of the $2E-2\nu$ method, due to the approximation $v_{\text{pre}} \approx v_{\text{post}}$, is correctable.

Considering how much effort is currently put into developing the $2E-2\nu$ method on various locations around the world, it is of uttermost importance that the consequences of neutron emission gets the attention they deserve, and that the nuclear physics community keep improving on the $2E-2\nu$ method itself.

ACKNOWLEDGMENTS

This work was supported by the European Commission within the Seventh Framework Programme through Fission-2013-CHANDA (project no.605203).

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