Convergence issues in the EPANET solver

J. Muranho\textsuperscript{a,b,c,*}, A. Ferreira\textsuperscript{d}, J. Sousa\textsuperscript{c,e}, A. Gomes\textsuperscript{a,b}, A. Sá Marques\textsuperscript{c,f}

\textsuperscript{a}Department of Informatics, University of Beira Interior, Rua Marquês d'Ávila e Bolama, 6201-001 Covilhã, Portugal
\textsuperscript{b}Instituto de Telecomunicações, University of Beira Interior, Covilhã, Portugal
\textsuperscript{c}MARE - Marine and Environmental Sciences Centre, University of Coimbra, Portugal
\textsuperscript{d}U.T.C. of Civil Eng., Polytechnic Institute of Castelo Branco, Av. do Empresário,6000-767 Castelo Branco, Portugal
\textsuperscript{e}Dep.of Civil Engineering, Polytechnic Institute of Coimbra, Rua Pedro Nunes, 3030-199 Coimbra, Portugal
\textsuperscript{f}Dep. of Civil Engineering, University of Coimbra, Rua Luís Reis Santos - Pólo II, 3030-788 Coimbra, Portugal

Abstract

EPANET assumes that the nodal outflows are constant regardless of the network pressures - known as the Demand-Driven Approach (DDA). This approach is quite accurate when the system operates under adequate positive pressures, but not when pressures are not sufficient enough. In scenarios of insufficient pressure is required an alternative approach to the DDA in order to compute the available demand as a function of nodal pressure, known as the Pressure-Driven Approach (PDA); for example, through the use of a pressure-demand relationship. However, the embedding of the pressure-demand relationship into the hydraulic solver can lead to convergence problems.

This paper details the measures taken in WaterNetGen to avoid the convergence issues, namely the use of relaxation coefficients and the use of a more stable linear system solver. The paper also analyses the DDA and the PDA performances and draws relevant conclusions.

© 2015 The Authors. Published by Elsevier Ltd.

Peer-review under responsibility of the Scientific Committee of CCWI 2015.

Keywords: EPANET, demand-driven, hydraulic solver, pressure-driven, WaterNetGen.

1. Introduction

EPANET [1] is one of the most used software for the simulation of water distribution systems. EPANET computes the nodal heads and link flows considering that the water demand assumes fixed and known values and...
are assigned to the network nodes. This is accomplished by solving simultaneously the mass conservation equation for each node and the energy equation for each link in the network. To compute the nodal heads, EPANET iteratively solves a linearized system of equations until some convergence criterion is satisfied — see Fig. 1.

![EPANET hydraulic simulation flowchart](image)

**Fig. 1** EPANET hydraulic simulation flowchart. (a) Extended Period Simulation; (b) Iterative computation of heads and flows.

To compute a new solution, that is, to compute new heads and flows (Fig. 1b) EPANET employs the Global Gradient Algorithm (GGA) [2] that linearizes the energy equations. As shown in Fig. 1a, the demands are updated before the new solution is computed. Therefore, the new solution is computed to always satisfy the required demand, even if this leads to negative pressures - EPANET is demand-driven. Obviously, the DDA only produces meaningful results if the pressure is enough to supply the required demand. One way to address this issue is to compute the available demand as a function of the current pressure. This requires a structural change in the flowchart of Fig. 1, moving the “Update demands” block to inside the loop of Fig. 1b, and eventually using a pressure-demand relationship. Computing the available demand as a function of the current pressure, inside the loop, brings instability to the iterative process: 1) new derivative terms must be included in the coefficients matrix; and 2) as the nodal pressure approaches zero, some sections of the network may have flow rates close to zero making the coefficients matrix semi-definite or even indefinite. This paper presents the approach followed in the WaterNetGen software [3] to deal with these new challenges, namely through the modification of the solver to deal with positive indefinite matrices and the use of relaxation coefficients for convergence purposes.

Besides the introduction, this paper includes a section of background work that briefly details the EPANET and WaterNetGen solvers. Then two example networks are used to study the effect of using the \( LDL^T \) factorization in the numerical solver and the incorporation of relaxation coefficients to improve convergence. The paper also includes a final section that summarizes and draws some conclusions.

### 2. Background work

#### 2.1. EPANET solver

The EPANET solver, as noted above, is based on the GGA [2, 4, 5]. The steady state formulation of the GGA for a hydraulic network composed of \( np \) pipes with unknown flow rates, \( nn \) nodes with unknown heads and \( n0 \) nodes with known heads is states as follows:
\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & 0
\end{bmatrix} \begin{bmatrix}
Q \\
H
\end{bmatrix} = \begin{bmatrix}
-A_{10}H_0 \\
-q
\end{bmatrix}
\] (1)

where \(Q=[Q_1, Q_2, ..., Q_{np}]^T\) is the column vector of unknown pipe flow rates; \(H=[H_1, H_2, ..., H_{mn}]^T\) is the column vector of unknown nodal heads; \(H_0=[H_{01}, H_{02}, ..., H_{0n0}]^T\) is the column vector of known nodal heads; \(q=[q_1, q_2, ..., q_{m}]^T\) is the column vector of known demands.

In equation (1), \(A_{11}\) is a \(np \times np\) diagonal matrix whose elements correspond to the pipe head losses; \(A_{12}=A_{21}^T\) and \(A_{10}=A_{01}^T\) are the topological incidence matrices of size \(np \times mn\) and \(np \times n0\), respectively.

Assuming a head loss function as follows:
\[h_i = R_i Q_i^{n_i-1}\] (2)

the \(A_{11}\) matrix takes the form:
\[
A_{11} = \begin{bmatrix}
R_1|Q_1|^{n_1-1} & & & \\
& R_2|Q_2|^{n_2-1} & & \\
& & & \cdots \\
& & & R_{np}|Q_{np}|^{n_{np}-1}
\end{bmatrix}
\] (3)

Considering \(D_{11}\) as the derivative of \(A_{11}\) with respect to pipe flows, the iterative formulation to find a solution of (1) can be stated as follows:
\[
\begin{align*}
A^k &= A_{21}(D_{11}^k)^{-1}A_{12} \\
F^k &= (A_{21}Q^k - q) - A_{21}(D_{11}^k)^{-1}(A_{10}H_0 + A_{11}Q^k) \\
H^{k+1} &= (A^k)^{-1}F^k \\
Q^{k+1} &= Q^k - (D_{11}^k)^{-1}(A_{11}Q^k + A_{12}H^{k+1} + A_{10}H_0)
\end{align*}
\] (4)

The computation of the new heads, \(H^{k+1}\), corresponds to solving a system of linear equations \(Ax = b\), with \(A = A^k\), \(x = H^{k+1}\), and \(b = F^k\), where the coefficients matrix \(A\) is sparse, symmetric and positive definite. Todini & Pilati [2] proposes solving this linear system using the Incomplete Cholesky Factorization/Modified Conjugate Gradient Algorithm. Instead of the Pre-conditioned Conjugate Gradient Algorithm, EPANET solves this linear system using a direct method (\(LL^T\) – Cholesky factorization) customized to sparse matrices [6].

2.2. WaterNetGen solver

WaterNetGen implements the pressure-driven version of the GGA [7, 8, 9], both for demands [10] and for leaks and bursts at pipe level [11], using expressions (5) and (6), respectively.

\[
q_i^{\text{min}}(P_i) = q_i^{\text{max}} \times \begin{cases} 
1 & P_i \geq P_i^{\text{ref}} \\
\left(\frac{P_i - P_i^{\text{min}}}{P_i^{\text{ref}} - P_i^{\text{min}}}\right)\alpha & P_i^{\text{min}} < P_i < P_i^{\text{ref}} \\
0 & P_i < P_i^{\text{min}}
\end{cases}
\] (5)
where \( q_{i}^{\text{avl}} \) is the available demand; \( P_{i}^{\text{ref}} \) is the reference (or service) pressure necessary to fully satisfy the required demand \( q_{i}^{\text{req}} \); \( P_{i}^{\text{min}} \) is the pressure below which no water can be supplied; \( \alpha \) (typically \( \alpha = 0.5 \)) is the exponent of the pressure-demand relationship; \( P_{i} \) is the current pressure at node \( i \).

\[
q_{k}^{\text{leak}}(P_{i}) = \begin{cases} 
\beta_{k} l_{k} (P_{i})^{\alpha_{k}} + C_{k} (P_{i})^{\delta_{k}} & P_{i} > 0 \\
0 & P_{i} \leq 0
\end{cases}
\]

(6)

where \( q_{k}^{\text{leak}} \) is the total leakage flow along pipe \( k \); \( l_{k} \) is the length of pipe \( k \); \( \alpha_{k} \) and \( \beta_{k} \) are parameters of the background leakage model; \( C_{k} \) and \( \delta_{k} \) are parameters of the bursts model (classical orifice flow formulas); and \( P_{i} \) is the average pressure in pipe \( k \) computed as the arithmetic mean of the pressure values of its end nodes.

Given equations (5) and (6), the iterative formulation for the pressure-driven approach is as follows [8]:

\[
D_{2}^{k} = D_{2}^{k} + DL_{2}^{k}
\]

\[
F^{k} = [A_{21}O^{k} - (q_{\text{avl}}^{k} - q_{\text{leak}}^{k})] - A_{21}(D_{2}^{k})^{-1} (A_{10}H_{0} + A_{11}Q^{k}) - D_{2}^{k}H^{k}
\]

\[
H^{k+1} = (A^{k})^{-1} F^{k}
\]

\[
Q^{k+1} = Q^{k} - (D_{1}^{k})^{-1} (A_{11}Q^{k} + A_{12}H^{k+1} + A_{10}H_{0})
\]

(7)

where \( DL_{\text{avl}} \) and \( D_{\text{avl}} \) are the derivatives of \( q_{\text{leak}} \) and \( q_{\text{avl}} \) elements with respect to pipe pressures and nodal pressures, respectively.

Several authors have employed relaxation coefficients to avoid convergence problems with PDA [8, 12]. In the same way, WaterNetGen applies the following update procedure for heads and flows:

\[
H^{k+1} = H^{k} + \phi^{k} (H^{k+1} - H^{k})
\]

\[
Q^{k+1} = Q^{k} + \lambda^{k} (Q^{k+1} - Q^{k})
\]

(8)

where \( \lambda^{k}, \phi^{k} \in [0, 1] \). The relaxation coefficients are updated as a function of the iteration errors (head and flow).

The pseudo-code of Fig. 2 illustrates the iterative process used to solve the system of equations, where the functions PDdemands() and PDleakages() are used to compute pressure-dependent demands and pipe leakage flow, respectively.

```plaintext
REPEAT
    PDdemands(); // compute pressure-dependent demand + derivative terms
    PDleakages(); // compute pressure-dependent leakages + derivative terms
    newcoeffs(); // compute coefficients of linearized network equations
    netsolve(...); // compute linear system solution (new heads)
    newflows(); // computes new flows
    update heads; // update heads using relaxation
    update flows; // update flows using relaxation
    update relaxation; // update relaxation coefficient
UNTIL convergence;
```

Fig. 2 – Pressure-driven implementation of the WaterNetGen solver.

EPANET uses Cholesky factorization to solve the linear system of equations. But, in the pressure-driven approach, some sections of the network may have flow rates close to zero making the coefficients matrix semi-definite or even indefinite (the matrix \( A_{ij} \), equation (3), may have zeros on the main diagonal). Therefore, in the
WaterNetGen solver, the $LL^T$ factorization is replaced by the $LDL^T$ factorization, where $L$ is a lower triangular matrix with unit diagonal and $D$ is a diagonal matrix. The $LDL^T$ factorization can be applied to positive indefinite matrices.

The replacement of the $LL^T$ factorization can be done, inside the solver C-language code, without major concerns because both factorizations use the same amount of computer memory, that is, the symbolic factorization based on the minimum degree ordering algorithm (used by EPANET) is the same for both $LL^T$ and $LDL^T$ factorizations.

3. Test cases

To study the suitability of the relaxation coefficients and the influence of the $LDL^T$ factorization, two test networks will be considered. The first network corresponds to a small gravity-fed system; the second one corresponds to a more complex network including pumps.

For this study it is considered that the solver reaches convergence (Fig. 1b) if the ratio of the sum of absolute values of pipe flow changes to the total flow is less than 0.001 (Accuracy).

3.1. Gravity-fed system

The gravity-fed network is composed by 1 reservoir, 200 junctions and 217 pipes – see Fig. 3. The network is distributed by five pressure zones (that is, sectors with buildings of different heights - number of storeys above ground), and must supply 63.66 L/s, which corresponds to 57.87 L/s of consumption and 10% of water losses (5.79 L/s).

For simulation purposes the water losses can be considered as one more demand category assigned to the nodes (as the consumption) or can be computed at pipe level, using equation (6), and assigned to the pipe end-nodes, half to each node. Since WaterNetGen allows the two possibilities, both are considered for the PDA.

For water losses (assumed as 10% of the consumption), and considering only the background leakage model of equation (6), the parameters for all pipes are $\beta_k = 2.76 \times 10^{-6}$ and $\alpha_k = 0.9$, which corresponds to 5.79 L/s under normal behaviour.
It will be considered only steady-state behaviour with four scenarios: normal behaviour, and one pipe closed, two pipes closed and three pipes closed (three faulty scenarios). The simulation types are label as DDA, for the DDA, PDA, for the PDA with water losses assigned directly to the nodes, and PDALoss, for PDA with losses at pipe level.

**Scenario 1 – Normal behaviour**

As illustrated in Fig. 3, under normal behaviour the steady-state nodal pressures are slightly above its reference pressure (minimum required pressure for the zone). In this scenario, both simulation approaches (PDA and DDA) are suitable.

Simulation results (iterations):

- **DDA + LDL^T**: 4
- **DDA + LL^T**: 4
- **PDA + LDL^T**: 9
- **PDA + LL^T**: 9
- **PDALoss + LDL^T**: 9
- **PDALoss + LL^T**: 9

**Scenario two – one pipe break/closed**

In case of insufficient pressures the DDA is not suitable. So only the PDA produces meaningful results. In this scenario one pipe is closed (to simulate a pipe break). Fig. 4 illustrates the pressures and flows obtained after one pipe is closed (pressure computed using PDA).

![Example network: pressures map resulting from one pipe break/closed.](image)

The network no longer can supply the required demand - Fig. 5 illustrates the network nodes that do not have enough pressure.

Simulation results (iterations):

- **PDA + LDL^T**: 17
- **PDA + LL^T**: 17
- **PDALoss + LDL^T**: 24
- **PDALoss + LL^T**: 27

With PDA the network supplies a total of 50.66 L/s and with PDALoss the network supplies 47.08 L/s for consumption and have a total of water losses of 3.86 L/s (the total flow equals 50.94 L/s).
Scenario three – two pipe breaks/closed

Fig. 6 illustrates the scenario considering two pipes simultaneously closed.

Simulation results (iterations):

\[
\begin{align*}
\text{PDA + LDL}^T & : \quad 88 & \quad \text{PDA + LL}^T & : \quad 77 \\
\text{PDALoss + LDL}^T & : \quad 27 & \quad \text{PDALoss + LL}^T & : \quad 27
\end{align*}
\]

With PDA the network supplies a total of 41.18 L/s and with PDALoss the network supplies 38.29 L/s for consumption and have a total of water losses of 3.34 L/s (the total flow equals 41.63 L/s).

Scenario four – three pipe breaks/closed

The Fig. 7 illustrates the scenario considering three pipes simultaneously closed.
Simulation results (iterations):

\[
\begin{align*}
&\text{PDA} + LDL^T: & 41 & \quad \text{PDA} + LL^T: & 58 \\
&\text{PDALoss} + LDL^T: & 105 & \quad \text{PDALoss} + LL^T: & 96
\end{align*}
\]

With PDA the network supplies a total of 35.37 L/s and with PDALoss the network supplies 32.97 L/s for consumption and have a total of water losses of 2.94 L/s (the total flow equals 35.91 L/s).

Resume - Analysis

Table 1 summarizes the results achieved for the above test scenarios.

| Scenario | Behaviour | Factor | Iterations | PDA Demand (Ratio %) | PDA Demand (L/s) | PDALoss Demand (L/s) | Losses (L/s) |
|----------|-----------|--------|------------|----------------------|------------------|----------------------|--------------|
| One      | Normal    | $LL^T$ | 9          | 7                    | 63.66            | 57.87                | 5.79         |
|          |           | $LDL^T$| 9          | 7                    | (100)            |                      |              |
| Two      | 1 Pipe closed | $LL^T$ | 17         | 27                   | 50.66            | 47.68                | 3.86         |
|          |           | $LDL^T$| 17         | 24                   | (79.6)           |                      |              |
| Three    | 2 Pipes closed | $LL^T$ | 88         | 27                   | 41.18            | 38.29                | 3.34         |
|          |           | $LDL^T$| 88         | 27                   | (64.7)           |                      |              |
| Four     | 3 Pipes closed | $LL^T$ | 58         | 96                   | 35.37            | 32.96                | 2.94         |
|          |           | $LDL^T$| 41         | 105                  | (55.6)           |                      |              |

As expected, from the analysis of Table 1 it can be concluded that as the pressure decreases less demand can be satisfied. For example, when three pipes are closed simultaneously only 55.6% of the required demand can be satisfied.

The DDA always reaches the convergence in 4 iterations. For the PDA and PDALoss simulations, with this example nothing can be concluded about the influence of the factorization type on the number of iterations (the behaviour is not well-defined).
One note about the use of relaxation coefficients: all simulations performed without employing relaxation coefficients did not converge. Therefore, the use of relaxation coefficients is mandatory for PDA simulations.

3.2. The C-Town network

The previous section shows that the use of relaxation coefficients is mandatory for PDA simulations. However, nothing can be concluded about the numerical factorization algorithm. This section uses a more complex network, the C-Town network – see Fig. 8, used in the Battle of Background Leakage Assessment for Water Networks - BBLAWN [13].

![Fig. 8 The C-Town network composed of a reservoir, seven tanks, eleven pumps, a control valve, a check valve, 432 pipes and 388 junctions.](image)

For the BBLAWN, WaterNetGen was used to define a pump scheduling for the C-Town network in order to minimize the operational costs (energy) and background leakage. The background leakage model follows equation (6). The BBLAWN simulation spans over one week (168 hours).

PDA with leakage at pipe level, PDALoss, was used to solve this problem, with $LL^T$ and $LDL^T$ factorizations. Although both factorizations performed a similar number of iterations to conclude the simulation period (around 19 000 iterations for 168 hours), $LDL^T$ always reached convergence (with a 0.001 accuracy) but in some time steps $LL^T$ simply didn’t converge.

The next step is to close some pipes (simulating breaks) and study the behaviour of both factorizations. The $LDL^T$ factorization always reach convergence when the $LL^T$ converges but the opposing not always occurs, that is there are cases where the $LDL^T$ converges but $LL^T$ fails.

In some cases, the PDA fails to converge (with both $LL^T$ and $LDL^T$). A closer look to the reason of the failure points to a less effective job of the relaxation coefficients, emphasizing the need of a more robust algorithm to guide the search.

4. Conclusions

The EPANET software uses a demand-driven approach to simulate water distribution systems. The EPANET solver uses the Cholesky decomposition ($LL^T$) to compute the solution of a system of linear equations. However,
under deficient pressure conditions the EPANET results are inaccurate. Therefore, in these scenarios a pressure-driven model is required.

Embedding a pressure-driven model in the EPANET solver allows the computation of the available demand as a function of the current pressure and also allows account for leakage at pipe level. But these increasing modelling capabilities have a side effect: new terms for the system coefficients matrix and the possibility of the matrix being positive indefinite. This requires a new approach to reach convergence: a numerical factorization applicable to indefinite matrices (LDLT) and relaxation coefficients to improve convergence.

The study conducted in this paper shows that the LDLT factorization can in fact improve the convergence cases and the incorporation of relaxation coefficients is mandatory for PDA simulations.

However, in some cases even the use of relaxation coefficients is not enough to guarantee convergence. So, as future development, other approaches need to be studied, for example, the use of a backtracking algorithm to assure global convergence for the Newton-Raphson method.

Availability

WaterNetGen can be downloaded from http://www.dec.uc.pt/~WaterNetGen.

References

[1] L.A. Rossman, EPANET2 users’ manual, Drinking Water Research Division, Risk Reduction Engineering Laboratory, Office of Research and Development, U.S. Environmental Protection Agency, Cincinnati, 2000.

[2] E. Todini, S. Pilati, A gradient algorithm for the analysis of pipe networks, in: B. Coulbeck, C.H. Orr (Eds), Computer Applications in Water Supply, Volume I - Systems analysis and simulation, John Wiley & Sons, London, 1988, pp. 1-20.

[3] J. Muranho, A. Ferreira, J. Sousa, A. Gomes, A. Sá Marques, WaterNetGen – an EPANET extension for automatic water distribution network models generation and pipe sizing, Water Science & Technology: Water Supply, IWA Publishing, 12 (2012), 117-123.

[4] E. Todini, Un metodo del gradiente per la verifica delle reti idrauliche, Bollettino degli Ingegneri della Toscana, 11 (1979), 11-14.

[5] R. Salgado, E. Todini, P.E. O’Connel, Comparison of the gradient method with some traditional methods for the analysis of water supply and distribution, in: B. Coulbeck, C.H. Orr (Eds), Computer Applications in Water Supply, Volume 1 - System analysis and simulation, John Wiley & Sons, London, 1988, pp. 38-62.

[6] A. George, J. W.-H. Liu, Computer Solution of Large Positive Definite Systems. Prentice-Hall, Englewood Cliffs, 1981, NJ.

[7] E. Todini, A more realistic approach to the “extended period simulation” of water distribution networks, in: C. Maksimovic, D. Butler, F. A. Memon (Eds.), Advances in water supply management, Balkema, Lisse, The Netherlands, 2003, pp. 173-184.

[8] O. Giustolisi, D.A. Savic, Z. Kapelan, Pressure-driven demand and leakage simulation for water distribution networks, Journal of Hydraulic Engineering, 134 (2008), 626-635.

[9] J.M. Wagner, U. Shamir, D.H. Marks, Water distribution reliability - Simulation methods, J. Water Resources Planning and Management, 114 (1988), 276-294.

[10] G. Germanopoulos, A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models, Civil Engineering Systems, 2 (1985), 171-179.

[11] J. Muranho, A. Ferreira, J. Sousa, A. Gomes, A. Sá Marques, Pressure-dependent Demand and Leakage Modelling with an EPANET Extension – WaterNetGen, Procedia Engineering, 89 (2014), 632 - 639.

[12] M.H. Hayuti, R. Burrows, D. Naga, Modelling water distribution systems with deficient pressure, ICE - Water Management, 160(2007), 215-224.

[13] J. Sousa, J. Muranho, A. Sá Marques, R. Gomes, WaterNetGen Helps C-Town, Procedia Engineering, 89 (2014), 103 - 110.