A Speculation About a Puzzled Result in 
Energy Spectrum of Cosmic-Ray Electrons 
Around TeV Energies

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Abstract

Nuclear Coulomb potential at completely ionized and extremely thin atmosphere can leaks to a macroscopic spatial scale. This effect is used to explain the difference between the energy spectra of cosmic-ray electrons around 1 TeV measured by different experimental groups. The result inspires us to review the traditional electromagnetic shower theory at the extreme conditions. It is also reminds us that the energy spectrum of cosmic-ray electrons, which are measured by Fermi-LAT and DAMPE at a higher altitude is more closer to a true signature.

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Energy spectra of all-electrons (electron+positron) at GeV-TeV energy band in cosmic-ray have been measured by Alpha Magnetic Spectrometer (AMS-02) [1], Fermi Large Area Telescope (Fermi-LAT)[2], DArk Matter Particle Explorer (DAMPE) [3] and Calorimetric Electron Telescope (CALET) [4]. The discovery of the excess (or break) of the spectra around $\sim 900 \text{ GeV}$ causes big interest because it may be related to the dark matter signal. However, a puzzled question is why the data of AMS and CALET are noticeably lower than that of DAMPE and Fermi-LAT in the $80 \text{GeV} \sim 1 \text{ TeV}$ energy band? The above observations have been accumulated and improved over many years. Besides, Fermi-LAT, DAMPE and CALET use the similar calorimeter, while AMS employed a completely different kind of magnetic spectrometer. Therefore, the above difference is seemingly not to be caused by the systematic or measurement errors.

We noticed that both AMS and CALET set on the international space station at $\sim 400 \text{ km}$, while Fermi-LAT and DAMPE are orbiting the earth at $560 \sim 500 \text{ km}$ altitude. A naive suggestion is that the signal of electron/positron is weakened by the electromagnetic shower caused by the extremely thin atmosphere in its transmission from $500 \text{ km}$ to $400 \text{ km}$. For this sake, we use the electromagnetic cascade equation to estimate the value of the radiation length $\lambda$, which may lead to the difference between the spectra as shown in Fig. 1. We find $\lambda \approx 5 \times 10^{-7} \text{g/cm}^2$. This value is eight-orders of magnitude smaller than the usual standard value $\lambda = 37 \text{g/cm}^2$.

Is it possible there exists such a big difference in the value of the radiation length? For this sake, we re-review the bremsstrahlung theory, which is a basis of electromagnetic shower. We find there is a big atomic size effect, which was neglected for a long time. The bremsstrahlung of an electron must contain the scattering of the incident electron on the nuclear electric field for the conservation of energy-momentum. It is well known that the total cross section of the Rutherford cross section either in the classical or quantum
Figure 1: Cosmic electron spectrum multiplied by $E^3$ as a function of energy. Data are taken from [1-4]. Black curves are input spectrum Eq. (7) and blue curves are cascade results through $0.2\lambda$. It implies the radiation length $\lambda \simeq 5 \times 10^{-7} \text{g/cm}^2$.

theory is infinite, which origins from following fact: the long-range $1/r$ potential has a significant contribution over all space. If the distance from the nucleus is large compared with the atomic radius, the Coulomb field of the positive nuclear charge is completely screened by the electrons of a neutral atom. Therefore, for the most events, the Coulomb cross section is limited in an atomic scale due to the screening effect. However, in the complete ionosphere (for example, at the magnetosphere) about $400-500 \text{km}$ height, the oxygen atoms are not only completely ionized, but the density is extremely thin. The ionized atoms in the magnetosphere are completely different from that at middle and lower atmospheres. At $400 \sim 500 \text{ km}$ high altitude, on average, there is only one atom per $1/1000000000$ cubic centimeter. This is a space with macroscopic scale $\sim 10^{-3} \text{cm}$, where the nuclear Coulomb potential may penetrate into a broader space. In such a low density atmosphere, the maximum screening length of oxygen ion-free electron system expands from a atomic scale $\sim 10^{-8} \text{ cm}$ to $\sim 10^{-3} \text{ cm}$ if the free electrons are uniformly
distributed. We will see that this may lead to a big correction to the total bremsstrahlung cross section.

We first consider cosmic-ray cascades specifically in the atmosphere of the earth. The basic high-energy processes that make up an electromagnetic cascade are pair production and bremsstrahlung \[5\]. We denote \(X\) and \(\lambda\) as the depth and the radiation length in unity \(g/cm^2\). The probability for an electron/positron of energy \(E_i\) to radiate a photon of energy \(\omega = zE_i\) in traversing \(dt = dX/\lambda\) is \(P_{e\rightarrow\gamma}(z)dtdz\),

\[
P_{e\rightarrow\gamma}(z) = z + \frac{1 - z}{z} \left( \frac{4}{3} + 2b \right).
\]  
(1)

The parameter \(b = 0.0122\). The corresponding probability per radiation length for \(e \rightarrow e\) is

\[
P_{e\rightarrow e}(z) = 1 - z + \frac{z}{1 - z} \left( \frac{4}{3} + 2b \right).
\]  
(2)

On the other hand, the probability for a photon to produce a pair of \(e^+e^-\), in which the electron has energy \(E_i = z\omega\) is \(P_{\gamma\rightarrow e}dtdz\),

\[
P_{\gamma\rightarrow e}(z) = \frac{2}{3} - \frac{1}{2}b + \left( \frac{4}{3} + 2b \right)(z - \frac{1}{2})^2.
\]  
(3)

The distributions of numbers of photon \(N_\gamma\) and electron/positron \(N_e\) satisfy the coupled equations for electromagnetic cascades

\[
\frac{dN_\gamma(\omega, t)}{dt} = \int \frac{d\omega'}{\omega'} P_{e\rightarrow\gamma} \left( \frac{\omega}{\omega'} \right) N_e(\omega', t) - N_\gamma(\omega, t) \int_0^1 dz P_{\gamma\rightarrow e}(z),
\]  
(4)

and

and
\[
\frac{dN_e(E_f, t)}{dt} = \int \frac{dE_i}{E_i} P_{e-e} \left( \frac{E_f}{E_i} \right) N_e(E_i, t) - N_e(E_f, t) \int_0^1 dz P_{e-e}(z) + 2 \int \frac{d\omega}{\omega} P_{\gamma \rightarrow e} \left( \frac{E_f}{\omega} \right) N_\omega(\omega, t). 
\]

(5)

The DAMPE data around \( E_e \sim 900 \) GeV have been parameterized by using a gluon condensation model in our previous work [6], which reads

\[
\Phi_j(E_j) = \Phi_0^j(E_j) + \Phi_j^{GC}(E_j),
\]

for \( j = e^- \) or \( e^+ \) and

\[
\Phi_j^{GC}(E_j) = \begin{cases} 
\frac{50C_{\pi}}{2\beta_p-1} E_{\pi}^{GC} \left( \frac{E_f}{E_{\pi}^{GC}} \right)^{-\beta_j} \left[ \frac{1}{\beta_j} \left( \frac{E_i}{E_{\pi}^{GC}} \right)^{-\beta_j} + \left( \frac{1}{\beta_j} + \frac{1}{\beta_e} \right) - \frac{1}{\beta_j} \right] & \text{if } E_j \leq E_{\pi}^{GC} \\
\frac{50C_{\pi}}{(2\beta_p-1)(\beta_j+2\beta_p-1)} \left( \frac{E_{\pi}^{GC}}{E_{\pi}^{GC}} \right)^{-\beta_j-\beta_\gamma-2\beta_p+1} & \text{if } E_j > E_{\pi}^{GC} 
\end{cases}
\]

(7)

The parameters \( E_{\pi}^{GC} = 880 \) GeV, \( \beta_p = 1.7, \beta_\gamma = 1.3, \beta_e = 0.6, C_{880 \text{ GeV}} = 1.15 \times 10^{-6} \).

The results present a smoothly broken power at 0.9 TeV as shown by black curves in Fig. 1.

We take Eq.(7) as the input distribution at height 500 km and evolve it using the cascade equation (4)-(5). We find that the distribution at \( t = 0.2 \) can fit the AMS-CALET data (see blue curves in Fig. 1). The corresponding transmission distance is 100 km. We get \( \lambda = 5 \times 10^{-7} g/cm^2 \) using \( \rho = 10^{-14} g/cm^3 \) at this altitude. Obviously, this value is much smaller than a normal value \( \lambda = 37 g/cm^2 \). It is a reason that the electromagnetic cascades above height 400 km are always be neglected. However, we doubt whether the result \( \lambda = 37 g/cm^2 \) is right if considering the special conditions of the magnetosphere.

The radiation length of the bremsstrahlung has been studied in detail by Bethe-Heitler in their classical works [7,8], the results are broadly refereed in the literatures. The cross section of the bremsstrahlung in a pure Coulomb field of a nucleus has two divergent sources: the logarithmic divergence at \( \omega \rightarrow 0 \) and the infinite total Coulomb scattering
cross section. The logarithmic divergence at the cascade equation (4)-(5)) is canceled by using the virtual corrections. On the other hand, the Coulomb divergence is restricted by the screening model.

The differential cross section of the Bethe-Heitler formula for the bremsstrahlung [7] is

\[
d\sigma_{B-H} = \frac{Z^2\alpha^3}{(2\pi)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|} \frac{d\omega}{\omega} d\Omega_{\omega} d\Omega_{k} \left[ \frac{\vec{p}_f^2 \sin^2 \varphi_f}{(E_f - |\vec{p}_f| \cos \varphi_f)^2} \left( 4E_i^2 - \vec{q}^2 \right) + \frac{\vec{p}_i^2 \sin^2 \varphi_i}{(E_i - |\vec{p}_i| \cos \varphi_i)^2} \left( 4E_f^2 - \vec{q}^2 \right) \right. \\
+ 2\omega^2 \frac{\vec{p}_i^2 \sin \varphi_i + \vec{p}_f^2 \sin \varphi_f}{(E_i - |\vec{p}_i| \cos \varphi_i)(E_f - |\vec{p}_f| \cos \varphi_f)} - 2 \frac{|\vec{p}_i||\vec{p}_f| \sin \varphi_i \sin \varphi_f \cos \phi}{(E_i - |\vec{p}_i| \cos \varphi_i)(E_f - |\vec{p}_f| \cos \varphi_f)} \left( 2E_i^2 + 2E_f^2 - \vec{q}^2 \right) \right],
\]

where the Coulomb potential

\[
A(x) = \left( Z \frac{1}{r}; \vec{0} \right),
\]

or

\[
A(q) = \int d^3x e^{i\vec{q} \cdot \vec{x}} A(x) = 4\pi Z e \frac{1}{|\vec{q}|^2}.
\]

are used. In Eq. (8) \(\varphi_i, \varphi_f\) are the angles between \(\vec{k}\) and \(\vec{p}_i, \vec{p}_f\) respectively; \(\phi\) is the angle between \((\vec{p}_i\vec{k})\) plane and \((\vec{p}_f\vec{k})\) plane. After integral over the angles, the differential cross section Eq. (8) at high energy \(E_i, E_f \gg m_e\) can be simplified as

\[
d\sigma_{B-H} \approx \frac{Z^2\alpha^3 d\omega}{m_e^2 \omega E_i} \left( E_i^2 + E_f^2 - \frac{2}{3} E_i E_f \right) \left( \log \frac{2E_i E_f}{m_e \omega} - \frac{1}{2} \right).
\]

Equations (8) and (11) use a pure Coulomb field. Its contributions to the total bremsstrahlung cross section is divergent. A question is how to consider the screening effect due to the outer electrons of a neutral atom? The works [7,8] take a following model. The largest
contribution in a bremsstrahlung process to the radiation cross section originates from the region where the momentum transfer \( \vec{q} = \vec{p}_f + \vec{k} - \vec{p}_i \) is least. This happens at

\[
|\vec{q}_{\text{min}}| = |\vec{p}_i| - |\vec{p}_f| - |\vec{k}|
\]

\[
= \sqrt{E_i^2 - m_e^2} - \sqrt{E_f^2 - m_e^2} - \omega \approx \frac{m_e^2 \omega}{2E_i E_f} = \frac{m_e^2}{2E_i} \frac{1 - \beta}{\beta}. \tag{12}
\]

where \( E_f \equiv \beta E_i \). In the coordinate space this corresponds to a impact parameter.

\[
r(\omega) = \frac{1}{|\vec{q}_{\text{min}}(\omega)|} = \frac{2E_i E_f}{m_e^2 \omega} \tag{13}
\]

Assuming the factor \( 2E_i E_f / m_e \omega \) in Eq.(11) just identifies with \( m_e r \). Then taking \( r \) as the atomic radius \( R \) for a neutral atom, i.e.,

\[
r \equiv R = a_0 Z^{-1/3}, \tag{14}
\]

where \( a_0 \) is Bohr’s radius. Thus, we have the total bremsstrahlung cross section for a complete screening atom

\[
\sigma_{B-H} \approx \frac{Z^2 \alpha^3}{m_e^2} \ln \omega_{\text{max}} \frac{4}{E_i^2} (E_f^2 + E_i^2 - \frac{2}{3} E_f E_i) \left( \log(m_e R) - \frac{1}{2} \right). \tag{15}
\]

It implies that the Bethe-Heitler cross section is proportional to \( \ln R \). Obviously, Eq. (15) can not provide an eight-orders of magnitude increase in the bremsstrahlung cross section even we take \( R \sim 10^{-3} \text{ cm} \).

However, Eq. (14) is a special model. The definition (13) has some arbitrariness. In fact, \( 2E_i E_f / m_e \omega \) is a variable rather than a fixed scale. As Bethe and Heitler have emphasized [7] that \( r \) in Eq. (13) can be regarded as the impact parameter \( r \) at given \( E_i, \omega \). If according to this understanding, the impact parameter \( r \) should be defined in a
different cross section $d\sigma_{B-H}/dE_f$, then to integral it on two-dimensional impact space, which will produce a different $R$-dependent total cross section.

A natural way is that the screening effect should be written as the screening potential in the S-matrix element at the beginning step. We use a screening potential to describe the leaked potential of a charged atom, i.e.,

$$A(x) = \left( Ze \frac{e^{-\mu r}}{r}; 0 \right),$$

or

$$A(q) = \int d^3x e^{iq\cdot x} A(x) = 4\pi Ze \frac{1}{|q|^2 + \mu^2}.$$

$\mu$ is a screening parameter with the dimension of mass and $1/\mu$ describes the range of atomic electric field.

Unfortunately, it’s very complicated to get the exact analytical solution of the cross section like Eq. (8). However, at the soft photon limit $\omega \to 0$ the Coulomb cross section can be factorized to a soft version of the Bethe-Heitler formula [9,10]

$$\frac{d\sigma_{\text{Soft Brems}}}{d\Omega} = \frac{d\sigma_{\text{Screening Coul}}}{d\Omega} \frac{2\alpha}{\pi} \ln \frac{\omega_{\text{max}}}{\omega_{\text{min}}} \left\{ \begin{array}{ll} \frac{4}{3} v^2 \sin^2 \frac{1}{2} \theta, & NR \\ \ln(-q^2/m_e^2) - 1, & ER \end{array} \right\}$$

The result of (18) is a special example of a more general soft-photon theorem [11], which states that in the limit $\omega \to 0$ the amplitude for any processes leading to photon emission can be factorized into two parts, one of them is the amplitude for the same process but without photon emission.

We calculate the bremsstrahlung cross section at a give initial energy $E_i$. For the nonrelativistic (NR) limit we obtain
\[ \sigma_{\text{Soft,NR}}^{\text{Brem}} \simeq \frac{8Z^2\alpha^3}{E_i^2} \ln \frac{E_i}{\omega_{\text{min}}} \left[ \ln \frac{\mu^2 + 4E_i^2}{\mu^2} - 1 \right], \quad (19) \]

which gives a similar \( \ln R \)-dependent cross section but with a \( 1/E_i^2 \)-suppression factor.

We consider the integral of Eq. (18) at the ER limit. Because of \( m_e^2 \) in \( \ln(-q^2/m_e^2) \) is not introduced as a cut-off parameter [7], the theory itself does not have any restrictions on the value of \( q^2 \). Therefore, \( -q^2 < m_e^2 \) is allowed. It implies a negative cross section.

If we regard this \( m_e \) as a cut-off parameter, the result shows that the scattering will be restricted inside a small range \( r \sim 1/|\vec{q}| < 1/m_e \sim 10^3 \text{ fm} \), which is smaller much than atomic radius \( \sim 10^5 \text{ fm} \), and the cross section becomes irrelevant to the screening parameter \( \mu \) (referring Eq. (21)) and \( m_e \gg \mu \). According to Ref. [12], \( m_e^2 \) in \( \ln(-q^2/m_e^2) \) should be replaced by an extra cut-off parameter \( \Lambda^2 \) using the following substitution

\[ \ln(-q^2/m_e^2) \to \ln((-q^2/m_e^2)/(-q_{\text{min}}^2/m_e^2)) = \ln \frac{-q^2}{\Lambda^2}. \quad (20) \]

\( \Lambda \) relates to the measurement resolution. Thus, the cross section at the ER limit reads

\[ \sigma_{\text{Soft,ER}}^{\text{Brem}} = \frac{Z^2\alpha^3\pi}{2E_i^2} \int_{\theta_{\text{min}}}^{\pi} d\theta \frac{\sin \theta}{\left(\sin^2 \frac{\theta}{2} + \frac{4E_i^2}{\Lambda^2}\right)^2} \frac{2\alpha}{\pi} \left[ \ln \left(\frac{-q^2}{\Lambda^2}\right) - 1 \right] \ln \frac{E_i}{\omega_{\text{min}}} \]

\[ \simeq 4Z^2\alpha^3 \ln \frac{E_i}{\omega_{\text{min}}} \left[ \frac{2}{\mu^2 + \Lambda^2} \ln \frac{E_i}{\Lambda} + \frac{1}{E_i^2} \left( \frac{E_i^2}{\mu^2 + \Lambda^2} \ln \left(\frac{\Lambda^2}{4E_i^2}\right) - \frac{E_i^2}{\mu^2} \ln \left(\frac{\Lambda^2}{\mu^2} + \Lambda^2\right) \right) \right] \]

\[ \simeq \left\{ \begin{array}{ll}
\frac{4Z^2\alpha^3}{\Lambda^2} \ln \frac{4E_i^2}{\Lambda^2} \ln \frac{E_i}{\omega_{\text{min}}}, & \Lambda \gg \mu \\
\frac{2Z^2\alpha^3}{\mu^2} \ln \frac{16E_i^2}{\mu^2} \ln \frac{E_i}{\omega_{\text{min}}}, & \Lambda = \mu \\
\frac{4Z^2\alpha^3}{\mu^2} \ln \frac{4E_i^2}{\mu^2} \ln \frac{E_i}{\omega_{\text{min}}}, & \Lambda \ll \mu 
\end{array} \right. \quad (21) \]

Interestingly, the parameters \( \mu \) and \( \Lambda \) may appear at a same factor \( 1/(\mu^2 + \Lambda^2) \) in the cross section. Since \( 1/\mu \) is a space scale of the screening Coulomb potential, \( 1/\Lambda \) should also be related to a space character about the scattering. In fact, \( \theta_{\text{min}} \) in Eq. (18) relates
to a maximum impact parameter $b_{\text{max}}$ for the scattering of electron on a central Coulomb potential. Therefore, $1/\Lambda \sim b_{\text{max}}$. Thus we have a several scales as shown in Eq. (21).

The result of Eq. (21) shows that as long as the resolution of the process is high enough (i.e., $1/\Lambda > $ atomic scale) at high energy, the bremsstrahlung cross section is almost proportional to the geometric area of the atomic Coulomb field $\sim R^2$, rather than a weaker $\ln R$-dependence that the Bethe-Heitler formula (15) predicted. We emphasize that the value of $\Lambda$ is process-dependent. For a detailed discussion see [12].

In the above discussion, we use the non-relativistic Rutherford cross section. At high energy limit, the Mott differential cross section should replace the Rutherford formula. The unpolarized Mott cross section is

$$\frac{\sigma_{\text{Mott}}}{d\Omega} = \frac{\sigma_{\text{Roth}}}{d\Omega} \left( 1 - v^2 \sin^2 \frac{\theta}{2} \frac{1}{2} \right). \quad (22)$$

It still gives a strong $R$-dependent total cross section, since the contributions of factor $\sim \sin^2(\theta/2)$ to the total cross section is $\sim \ln 1/\mu$-increase, which can be neglected if comparing with $1/\mu^2$ increase of Rutherford cross section at small $\mu$.

Finally, we roughly estimate the conditions for observing the strong $R$-dependent cross section. According to a toy model [13] for air shower we take $\beta = 1/2$ in Eq. (12). Atomic screening at 400 km $\sim$ 500 km altitude will significantly reduce the radiation intensity at

$$R \sim 1/|q_{\text{min}}| > 10^{-4} - 10^{-3} \text{ cm}. \quad (23)$$

Using Eqs.(12) and (13) one can find that it corresponds to $E_i > 100$ GeV and the Lorentz factor $\gamma \sim 10^6$. Although the Bethe-Heitler formula predicts the $\ln R$-dependent bremsstrahlung cross section, Eq.(21) shows that a much stronger $R$-dependence of cross section under some extreme conditions is possible. We think that for very high energy electrons ($E_i \sim 1$ TeV), this new effect is important. Of course, the value of $\mu$ can not
be infinitely small, incident electrons cannot 'feel' the electronic field at \( 1/\mu = \infty \). In most interstellar medium, the atomic density of gas decreases from \( 10^8/cm^3 \) to several per cubic centimeter. In these cases, we need to consider the value of \( \mu \) limit. We think that the energy band measured by [1-4] is an ideal range for observing a big increasing bremsstrahlung cross section. If our conclusion is correct, we predict that the energy spectrum of cosmic-ray electrons above \( \sim 600 \) km altitude will be obviously higher than the Fermi-LAT and DAMPE data.

Such huge effects in Eq. (21) caused by soft photon radiation not only suppress the lower energy part of the electron spectra, but also reduce its whole energy spectra and radiation length as we have shown in Fig. 1. Why so strong enhancement effect in the bremsstrahlung cross section is neglected during a long time? We try to understand the reasons.

(i) A main difference between Eqs. (15) and (21) is the \( R \)-dependence of the cross section. As we have known that lots of experiments have been used to test the screening effect based on the Bethe-Heitler formula (15), where the targets include solid, liquid and normal gas, none of them has found a big enhancement of the cross section. The possible reasons are as follows. The \( R^2 \)-dependent effect appears in the integrated cross sections \( d\sigma(E_i, \omega)/d\omega \) or \( \sigma(E_i) \), where the angle \( \theta \) is integrated. It implies that \( \Lambda \) may take a minimum value \( \Lambda < \mu \). Besides, in the neutral atomic matter \( 1/\mu \simeq a_0 Z^{-1/3} \sim 10^{-8} \) cm. While for the most ionized medium, \( R \sim \) Debye screening length. Both of them are the atomic-molecule scale and they have not a big difference either according to \( \sigma \sim \ln R \) or to \( \sigma \sim R^2 \ln R \). Therefore, we may not notice the difference. On the other hand, the number density of ionized interstellar gas is about \( < 1/cm^3 \) [14], which is far less than our considering density in this work. So the bremsstrahlung component of diffuse gamma ray is not obviously change.

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(ii) The evolution of the energy spectra of electrons/positrons in an air shower at high energy begins at the top of the atmosphere. The standard approach based on the ground uses the Monte Carlo computer code, which has millions of events that need precise tracking. Obviously, such Monte Carlo calculations becomes impractical. In addition, many simplified parameterizations are used. They may indistinct the phenomenon of the increasing cross section.

(iii) In contrast to the space-based detectors like the above mentioned four measurements, the ground-based HESS [15] and VERITAS [16] do not measure the primary particles (electrons, photons) directly. They detect the Cherenkov light generated by charged particles in air showers, which begin at the upper atmosphere. Because the original position of the shower is not fixed, we don’t use their data in this work.

In summary, the nuclear Coulomb potential of a completely ionized atom at 400 km ∼ 500 km altitude may expand to a macroscopic spatial scale ∼ 10⁻³ cm from an atomic radius ∼ 10⁻⁸ cm. It may result in a maximum 10⁸ times increase of the bremsstrahlung cross section of electron/positron at the 100 GeV ∼ 1 TeV energy band. Using this effect we explain the difference of the energy spectra measured by different experimental groups. The result inspires us to review the traditional electromagnetic shower theory at the extreme conditions. It is also reminds us that the energy spectrum of cosmic-ray electrons, which are measured by Fermi-LAT and DAMPE at a higher altitude is more closer to a true signature.

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