Vorticity and Magnetic Field Generation from Initial Anisotropy in Ultrarelativistic Gamma-Ray Burst Blastwaves

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Because conical segments of quasispherical ultrarelativistic blastwaves are causally disconnected on angular scales larger than the blastwave inverse Lorentz factor, astrophysical blastwaves can sustain initial anisotropy, imprinted by the process that drives the explosion, while they remain relativistic. We show that initial angular energy fluctuations in ultrarelativistic blastwaves imply a production of vorticity in the blastwave, and calculate the vortical energy production rate. In gamma-ray burst (GRB) afterglows, the number of vortical eddy turnovers as the shocked fluid crosses the blastwave shell is about unity for marginally nonlinear anisotropy. Thus the anisotropy must be nonlinear to explain the magnetic energy density inferred in measured GRB spectra.

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Special relativistic shocks occur where the gravity of a compact object, such as a black hole, accelerates plasma to ultrarelativistic velocities. Emission from such shocks is observed in astrophysical sources on many length, time, and energy scales, and is often attributed to the synchrotron process. The composition of the plasma, the magnitude of the preshock magnetic field, and the Lorentz factor of the shock are generally unknown. In the best-studied system—the radiative afterglow that follows the bright flash of γ-rays in gamma-ray burst (GRB) phenomenon—the structure of the emitting region can be modeled from the observed emission. In the standard GRB afterglow model, the radiation is produced in a relativistic blastwave shell propagating into a weakly magnetized plasma. The afterglow emission is then the synchrotron radiation from nonthermal electrons gyrating in a strong magnetic field of the shocked plasma. Detailed studies of GRB spectra and light curves have shown that the magnetic energy density in the emitting region is a fraction \( \epsilon_B \sim 10^{-2} \) to \( 10^{-3} \) of the internal energy density \( \epsilon_\gamma \) and that the magnetic field must be present over at least a few percent of the blastwave thickness \( \lambda_s \). The origin of this downstream magnetic field is a longstanding important open question.

Compressional amplification of the weak pre-existing microgauss magnetic field of the circumburst medium merely yields \( \epsilon_B \sim 10^{-9} \). The strong small-scale field generated in collisionless plasma instabilities in the shock transition was considered as a candidate for the post-shock field \( \lambda_s \). However, simulations of shock transition layers indicate that the small-scale field decays rapidly over few plasma skin depths \( \lambda_s \), and does not persist on distances from the shock transition \( \sim 10^3 \lambda_s \) where the emission originates. Suggestions have been made that a persistent magnetic field develops in the shock precursor due to the streaming of shock-accelerated protons \( \epsilon_B \) or cascade-generated \( \epsilon^+ \) pairs \( \epsilon_B \). The feasibility of these scenarios depends on the insufficiently understood details of particle acceleration and streaming instabilities.

Recently, Sironi & Goodman showed that if the unshocked medium is strongly inhomogeneous, significant vorticity is produced in the shock transition. The resulting downstream turbulence amplifies the weak seed magnetic field of the preshock medium to the observed level. The density of the interstellar medium may vary at the required level in GRB associated with the iron core collapse in massive stars (i.e., “long”-type GRBs), but is not expected in the GRB associated with old progenitors (i.e., “short”-type GRBs). While the strength of the magnetic field in the afterglows of short-type GRBs is not well-constrained, the similarity of the afterglow light curves and spectra in the long- and short-type GRBs suggests that the magnetic field is amplified by the same process in both systems.

Here we propose a source of vortical energy in the shock downstream that could generically be present in any ultrarelativistic blastwave: the initial angular anisotropy of the energy carried by the blastwave. A unique feature of ultrarelativistic blastwaves is that they are composed of many small, causally-disconnected patches. The patches lose causal contact when the driving outflow (e.g., an electromagnetic jet launched near a black hole) accelerates the swept-up ambient medium, and evolve independently until the blastwave decelerates sufficiently (see below). In GRBs, an additional source of angular inhomogeneities is the phase in which the initial (“prompt”) γ-rays are emitted, when a sizable fraction of the outflow energy is dissipated and emitted as γ-rays. This takes place after the relativistic outflow achieves its maximal Lorentz factor, and thus any variation in the dissipation in causally disconnected regions translates directly into angular energy fluctuations in the shock wave that ultimately blasts into the ambient medium.

There is also direct observational motivation for initial angular anisotropy in GRB outflows. Significant angular fluctuations have been invoked to explain the large vari-
atation of the $\gamma$-ray luminosity between bursts [10] and the intraburst variability observed in many afterglows [11].

The afterglow polarization [12] indicates a breaking of axial symmetry; the correlatedness of this polarization with large amplitude variability suggests blastwave anisotropy as the source of variability [12, 14]. E.g., the afterglow in GRB 021004 exhibits a strong light curve variability correlated with a variable linear polarization; anisotropy on scales $\sim 2\pi/l$ with $l \sim 200$ and a nonlinear energy contrast $\sim 3$ can explain the data [14].

We proceed to estimate the conditions for successful turbulent magnetic field amplification in quasispherical ultrarelativistic blastwaves with initial angular energy fluctuations. Consider an ultrarelativistic point explosion with total isotropic-equivalent energy $E$ propagating into a medium of uniform density $\rho_0$. We work in the rest frame of the unshocked fluid and the explosion center. In spherical symmetry, the Lorentz factor of the strong shock wave that forms at the leading edge of the blastwave decays in time $t$ as $\Gamma = \left(\frac{17 \pi^5}{8} E / T^2 \rho_0 \right)^{1/2}$ and its position is located at $R = t (1 - \frac{1}{3} \Gamma^{-2})$ (the speed of light is unity). The Lorentz factor in the shock downstream $\gamma = \Gamma / (2 \chi)$ is expressed in terms of the Blandford-McKee [15] variable $\chi(r,t) = (r - r_0) / (r - R) + O(\Gamma^{-2})$. Here, $r_0$ is the radius from the center of the explosion and $\Gamma > 1$ is assumed.

Consider initial $(t = 0)$ linear fluctuations in the energy of the explosion, $E(\theta, \phi) = [1 + \delta_E(\theta)]E$, where at no loss of generality we assume axial symmetry, $\delta_\phi \to 0$ and $\delta_E(\theta) = \delta_E Y_0(\theta)$. Here $Y_{lm}(\theta, \phi)$ is the spherical harmonics; the case with an azimuthal dependence, $m \neq 0$, can be treated equivalently. Conical angular segments of the blastwave separated by $\Delta \theta > 2 \Gamma^{-1}$, which corresponds to $l \lesssim 3 \pi \Gamma$, have not been in causal contact since the beginning of the explosion, and are evolving independently as fragments of spherical explosions with energy fluences displaced from the spherical average. Following [10, 16], we expand the radial Lorentz factor $\gamma(r, \theta, t) = \gamma(r, t)[1 + \delta_p(r,t)Y_0(\theta)]$, the pressure $p(r, \theta, t) = \rho(r, t)[1 + \delta_p(r,t)Y_0(\theta)]$, the shock radius $R(\theta, t) = R(t)[1 + \delta_p(t)Y_0(\theta)]$, and the velocity of the fluid in the $\theta$ direction $u(r, \theta, t) = u(r,t)\partial_\theta Y_0(\theta)$, which is here assumed to be Newtonian, as linear perturbations around their spherical averages. Then, at times $t > t_{\text{crit}} \equiv \left(\frac{15 \pi^5}{8} E / T^2 \rho_0 \right)^{1/3}$, i.e., when $\Delta \theta > \frac{2}{3} \pi \Gamma^{-1}$ and for $\chi(r,t) \sim 1$, the fluctuations simply inherit their initial values and are independent of time and radius, $\delta_p = -\frac{1}{2} \delta_E$, $\delta_\theta = \frac{1}{4} \Gamma^{-2} \delta_E$, $\delta_\gamma = 0$, and $u = 0$.

Gruzinov [16] linearizes the equations of relativistic hydrodynamics, $\partial_\theta T^{\alpha \beta} = 0$, where $T^{\alpha \beta}$ is the energy-momentum tensor, and changes variables $(r, t) \to (\xi, \tau)$, where $\xi \equiv \frac{1}{2} \ln \chi(r,t)$ and $\tau \equiv -\frac{1}{2} \ln \Gamma(t)$, to derive equations governing the evolution of the fluctuations

$$\dot{\delta}_p + 3\delta_\gamma - \frac{3}{2} \delta_\gamma - \delta_p + \frac{1}{2} l(l+1) u = 0,$$

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$$\dot{\delta}_p + 3\delta_\gamma - \frac{3}{2} \delta_\gamma - \delta_p + \frac{1}{2} l(l+1) u = 0,$$

where the overdot and the prime denote derivatives with respect to $\tau$ and $\xi$, respectively, and the equations are valid for $\xi \ll -\frac{2}{3} \pi/2$ and $\tau < 0$, as the blastwave is Newtonian at larger $\xi$ or $\tau$. At the shock transition ($\xi = 0$), the continuity of the energy-momentum flow across the transition implies the shock jump conditions [10]

$$\dot{\delta}_p = 2\delta_\gamma - \frac{10}{3} \delta_\gamma, \quad u = \frac{3}{10} e^{\alpha_+ \tau} \delta_p - 2\delta_\gamma. \quad (2)$$

The first two of Eq. (1) can be expressed in terms of the Riemann variables $f_{\pm} \equiv 2\delta_\gamma \pm \frac{\sqrt{3}}{2} \delta_p$ propagating along the $C^\pm$ characteristics with velocities $c^\pm = 3 \mp 2\sqrt{3}$ [10]

$$f^\pm + c^\pm (f^\pm)' - \alpha_+ (f^+ + f^-) + \zeta^\pm u = 0, \quad (3)$$

where $\alpha_+ \equiv \frac{2}{3} \mp \frac{5}{3}$ and $\zeta^\pm \equiv (1 \mp \sqrt{3})(l+1)$. Following [16], we adopt the notation $f \equiv f_+$ and $g \equiv f_-$. Dynamics of the shocked fluid governed by Eq. (1) is easily understood: an initial energy-pressure fluctuation $\delta_p$ sources transverse ($\delta_p, u$) and longitudinal ($\delta_\gamma, \delta_\theta$) acoustic fluctuations, which oscillate after the mode comes within the causal horizon at $t = t_{\text{crit}}$ ($\delta_\gamma$ equals the radial velocity perturbation in the local fluid rest frame). This is analogous to the horizon re-entry of superhorizon cosmological fluctuations in the early universe.

The initial value problem (Eq. 1) subject to the boundary conditions (Eq. 2) is well-posed if a boundary condition at the origin of the $C^+$ characteristics, at some distance $\xi_{\text{max}}$ from the shock is specified. Gruzinov [16] reports that the solution near the shock is independent of the boundary condition if $\xi_{\text{max}}$ is large enough. The independence can be understood as follows. Since $\alpha_+ / c^+ > 0$, the fluctuations $f^+$ are damped from large to small $\xi$; this is because the explosion energy is concentrated near the shock [15]. The reverse $f^-$ fluctuations grow much slower than the $f^+$ decay, so the former do not seed the latter at large $\xi$. Having specified a boundary condition, e.g., $f(\xi_{\text{max}}) = 0$, the evolution of perturbations in the shock downstream is uniquely determined and can be solved for, e.g., after Taylor expansion $\psi(\xi, \tau) = \sum_{n=0}^\infty (n!)^{-1} \xi^n \psi_n(\tau)$, where $\psi = f, g, u$.

An approximate solution is obtained after assuming that the perturbations decay after they start to oscillate, and that the decay is complete everywhere except within few wavelengths $N\lambda^+$ from the shock, where $\lambda^+ = 2\pi e^{c_+}/k^+$, and $k^+$ is the frequency of oscillations of fluctuations propagating along the $C^+$ characteristic. The frequency is estimated by substituting the last of Eq. (1) in the first of Eq. (2) which yields $N \lambda^+ \approx \frac{1}{2} (1 - \frac{1}{3\sqrt{3}})(l+1)^{1/2} e^{3\eta^2/2}$. Thus we set $\xi_{\text{max}} \sim N \lambda^+ \propto e^{-3\eta^2/2}$; the decay proceeds from large to small distances from the
shock. Expanding to the linear order yields
\[
\begin{align*}
\dot{f}_0 + e^+ f_1 - \alpha^+ (f_0 + g_0) + \zeta^+ u_0 &= 0, \\
\dot{g}_0 + e^- g_1 - \alpha^- (f_0 + g_0) + \zeta^- u_0 &= 0, \\
\dot{u}_0 + u_1 - \frac{14}{3} u_0 + \frac{1}{2} \varepsilon_{3r} \left( f_0 - g_0 \right) &= 0, \\
\dot{f}_0 - g_0 - \dot{f}_0 + g_0 + \frac{5}{3} f_0 + g_0 &= 0, \\
\dot{f}_0 + \varepsilon_{\text{max}}(t) f_1 &= 0, \\
u_0 - \frac{3}{10} \varepsilon_{3r} \left( f_0 - g_0 \right) &= 0,
\end{align*}
\]
where the first three equations are the original equations of motions for \(f, g,\) and \(u,\) and the last three are the two constraints imposed at \(\xi = 0\) (the shock jump conditions), and a constraint imposed at \(\xi = \varepsilon_{\text{max}}.\) Integrating the ODEs in Eq. 4 with initial conditions \(f_0(-\infty) = -g_0(-\infty)\) and \(u_0(-\infty) = 0\) and \(N = 1,\) we find that the resulting approximate solution \(\delta_p(\xi = 0, \tau) = \frac{4}{3}\sqrt{3} \iota_0(\tau - g_0(\tau))\) closely matches the solution in [10], thus validating our approximations.

We employ the definition of relativistic vorticity \(\tilde{\omega} \equiv \vec{\nabla} \times \dot{H},\) which is conserved, \(\partial_t \tilde{\omega} - \vec{\nabla} \times (\vec{v} \times \tilde{\omega}) = 0,\) in the smooth (i.e., shock-free) part of the flow in an ideal, barotropic fluid with ultrarelativistic equation of state. Here, \(\dot{H} \equiv p^{1/3} \gamma \vec{v},\) where as before, \(p\) is the fluid pressure, \(\gamma = (1 - v^2 - u^2)^{-1/2}\) is its Lorentz factor, and we have decomposed fluid velocity into \(\vec{v} = \vec{v}^\tau + u \hat{\theta}.\) Since \(u \ll v,\) \(\gamma \approx (1 - v^2)^{-1/2}\) is the radial Lorentz factor. Vorticity vanishes in an unperturbed, spherically-symmetric blastwave, but in a blastwave with initial anisotropy, it is produced at the shock and advected into the downstream. In the \((\xi, \tau)\) frame, the vorticity conservation equation becomes \(\dot{\omega} + \omega = 9\omega + O(\Gamma^{-2}) = 0,\) where \(\omega\) is the amplitude of the sole nonvanishing component of the vorticity, \(\tilde{\omega} = \omega \partial_t Y_0 \hat{\phi}.\) The vorticity decreases into the downstream, \(\omega(\xi, \tau) = e^{-3\gamma \omega(0, \tau - \xi)} \) in terms of the fluid perturbation the vorticity equals
\[
\omega \approx \frac{p^{1/3} \gamma}{t} \left[ \frac{41}{6} u - 2u' \right] e^{-3\tau - 4\xi} - \delta_\gamma - \frac{1}{4} \delta_p \right],
\]
where we have kept only the leading-order terms. The vorticity in Eq. 5 is conserved if the fluid perturbations evolve according to Eq. 1. Because \(\delta_p(\tau \rightarrow -\infty) \neq 0\) while \(\delta_\gamma \sim 0\) and \(u \sim 0,\) vorticity does not vanish identically prior to the horizon entry of the mode.

We follow the prescription in [17] to calculate the vortical energy, which is defined in the local, noninertial rest frame of the shocked fluid; we denote the quantities in this frame with tilde, \(\tilde{\gamma} = \gamma(r - vt)\) and \(\tilde{t} = \gamma(t - vr).\) Vorticity transforms as a 2-form \(\Omega = \omega_{\theta \phi} \partial r \wedge d(\rho \gamma)\). Since \(d r \wedge d(\rho \gamma) \rightarrow -\gamma^{-1} d\vec{r} \wedge d(\rho \gamma)\) to leading order in \(\gamma^{-1}\) (terms involving \(d\vec{r}\) do not contribute to vorticity), we have \(\dot{\omega} \approx \omega/\gamma,\) and the \(\varepsilon_{\omega vort} \) dependence of the vorticity is given by \(\dot{\omega} = e^{-3\gamma/2} \omega(0, \tau - \xi).\)

The fluid is subject to differential acceleration, and thus there is not a single inertial frame in which we can calculate the vortical energy. As an approximation, we work in the instantaneous inertial rest frame of the fluid element at \(\xi = 0.\) Simultaneity in this frame is equivalent to \(d\tilde{r}(\xi, \tau) = 0,\) which implies that \(d\tilde{r} \approx \frac{3}{3} \Gamma^{-1} d\tilde{\xi}.\) This is not exact; the numerical coefficient depends on the approximation, and thus the forthcoming estimates are crude. We also replace covariant derivatives with normal derivatives in what follows. We carry out approximate projection of the vorticity onto the shock plane
\[
\dot{\varepsilon}_{\text{vort}} = \frac{t}{3\gamma} \int_0^\infty \dot{\omega} d\xi \approx \frac{2}{33\gamma} \dot{\omega}(\xi = 0).
\]

The vortical energy density in the local fluid rest frame equals \([17] \dot{\varepsilon}_{\text{vort}} = \frac{4}{3} \rho |\vec{H}_{\text{sol}}|^2 p^{-1/2},\) where \(\vec{H}_{\text{sol}}\) is the solenoidal component of \(\vec{H},\) and \(\rho\) is the proper energy density. The solenoidal component can be written as a curl of a vector potential \((\vec{H}_{\text{sol}} = \nabla \times \vec{A}).\) Setting \(\vec{\nabla} \cdot \vec{A} = 0,\) the vorticity is related to the potential via \(\nabla^2 \vec{A} = -\tilde{\omega}.\) Recall that \(\tilde{\omega} = \partial_p Y_{0},\) which is not an eigenfunction of the angular component of the Laplacian.

However, for \(l \gg 1,\) at no loss of generality we can restrict analysis to a small patch of the spherical shell, e.g., near the equator, \(\theta - \frac{\pi}{2} \lesssim l^{-1},\) where \(Y_{0}(\theta) \approx \frac{1}{2} \cos(\theta)\) and
the shell is locally quasi-planar. Then \( \tilde{A} = A \sin(\theta) \tilde{\phi} \) and \( A = 1/(2\pi e^{-1r/Y}) \) is the solution of (we freely set the origin of the coordinate \( \tilde{r} = 0 \))

\[
\frac{\partial^2 A}{\partial \tilde{r}^2} - \frac{l^2}{\tilde{r}^2} A = \frac{1}{\pi} \tilde{\omega}(\tilde{r}, \tilde{\theta}) \approx \frac{1}{\pi} \tilde{\sigma}(\tilde{r}),
\]

where \( \delta(\tilde{r}) \) is the \( \delta \)-function. Close to the shock we take curl to obtain \( \langle \tilde{H}_{\text{sol}} \rangle^2 \approx \langle \frac{1}{3\pi} \tilde{\sigma} (\tilde{\omega}/\gamma) \rangle^2 \), where the angular brackets denote an average over \( \cos \theta \). Substituting in \( \varepsilon_{\text{vort}} \) and diving by the internal energy \( \rho \) yields the fractional vortical energy \( \varepsilon_{\text{vort}} \equiv \varepsilon_{\text{vort}}/\rho \)

\[
\varepsilon_{\text{vort}} \approx K \frac{l_2^2}{l_3^2} \left( \frac{41}{6} u - 2u' \right) e^{-3\varepsilon - 4\xi - \delta_\gamma - 1/4} \varepsilon_\rho \]

where \( K = \frac{8}{3} \left( \frac{1}{35} \right) \approx 2.5 \times 10^{-4} \) is a numerical constant sensitive to the specifics of our approximations.

Fig. [1] shows the evolution of the fluid perturbation variables, the fluid-frame vorticity and the fractional energy in vortical motions. The latter oscillates and reaches peak average amplitude \( \varepsilon_{\text{vort}} \sim 3 \times 10^{-2} \delta_{p,0}^2 \) after an \( l \)-mode becomes causal, for \( \Gamma \approx (0.01 - 0.1) \times l \). Here, \( \delta_{p,0} \equiv \delta_p (\Gamma = \infty) \) is the initial fractional pressure fluctuation. The plots in Fig. [3] are scaled to be independent of \( l \), but absicissa range satisfying the premise that the blastwave is ultrarelativistic does depend on \( l \).

A fluid with nonvanishing vorticity develops eddies which give rise to magnetic field amplification via the turbulent dynamo mechanism [18]. Strong amplification is expected when the number of eddy turnovers is larger than unity. The number is estimated as the ratio \( N_{\text{eddy}} \sim t_{\text{cross}}/t_{\text{eddy}} \), of the time scale \( t_{\text{cross}} \sim 1/t/\gamma \) on which the fluid crosses the shell of the blastwave to the eddy turnover time \( t_{\text{eddy}} \sim \Gamma l^3/4l (l + 1) \). Using \( \langle (\partial_\theta Y_0)^2 \rangle^2 = 4\pi l (l + 1) \), we have

\[
N_{\text{eddy}} \sim \frac{1}{4\sqrt{2\pi}} \frac{l}{\Gamma} \frac{t_{\text{cross}}}{l_{\text{eddy}}} \approx 6.3 \sqrt{\varepsilon_{\text{vort}}} \sqrt{2 \times 10^{-4}/K}.
\]

For \( \varepsilon_{\text{vort}} \sim 0.03 \), we expect \( N_{\text{eddy}} \sim 1 \). Therefore, within the limits of our approximations, a nonlinear initial perturbation, \( \delta_{p,0} \gtrsim 1 \), is necessary to amplify the magnetic field. If amplification happens, the final energy density in the field will be in approximate equipartition with the vortical motions, \( \varepsilon_B \sim \varepsilon_{\text{vort}} \). With \( \delta_{p,0} \sim 1 \) we find that \( \varepsilon_B \sim 3 \times 10^{-2} \), consistent with the typical values inferred from GRB spectra. The anisotropy of GRB blastwaves could be fully nonlinear. Since our treatment is inadequate in the nonlinear regime (see [19] for an analytic approach to aspherical blastwaves), numerical simulations of relativistic blastwave hydrodynamics are needed to study the nonlinear turbulence produced in the shock.

Fig. [1] shows that the vorticity associated with an \( l \)-mode persists through an increase by \( \times 10^3 \) in \( \Gamma^2 \), which corresponds to an increase by \( \times 10^4 \) in the observer time (\( t_{\text{obs}} \propto \Gamma^{-8/3} \)). Thus a single mode \( l \sim 500 \) produces vorticity during the observed GRB afterglow (\( 10^{2} - 10^{9} \) s after the burst). Since \( \Gamma_{\text{max}} \sim 100 \), we indeed expect such an \( l \) mode from causality considerations. Our estimates indicate that \( \delta_E \sim 1 \) is required for strong magnetic field amplification, although in this regime, our linear treatment is not strictly applicable. A measurable signature of such angular anisotropy is afterglow flux variability at the level \( \sim \delta_E \) on time scales \( t_{\text{obs}} \) for which \( l \sim 2\pi \Gamma \).

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