Phenomenological aspects of the exotic $T$ quark in 331 models

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In the context of 331 models we analyze the phenomenology of exotic $T$ quarks with electric charge $2/3$. We establish bounds for the corresponding masses and mixing angles and study the decay modes $T \rightarrow bW, tZ$ and $qH$. It is found that the decays into scalars are strongly dependent on the model parameters, and can be the dominant ones in a scenario with approximate flavor symmetry.

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I. INTRODUCTION

In addition to the main goal of identifying a light Higgs boson [1], an important challenge of the LHC is the observation of clear signals of physics beyond the Standard Model (SM). In fact, the general consensus is that the SM is not the ultimate theory for strong and electroweak interactions, and many models have been proposed throughout the last three decades attempting to solve existing theoretical puzzles such as hierarchy problems, replication of fermion families, coupling unification, etc [2]. Since most of these models include new physics already at the TeV scale, it is likely that the corresponding effects could be observed at the LHC at 100 fb$^{-1}$ luminosity. In general, the various theories of new physics predict the presence of new ("exotic") fermions, gauge bosons and scalar bosons.

In particular, many models include exotic $T$ quarks with electromagnetic charge $2/3$ [3]. This is e.g. the case of little Higgs theories [4], in which an extra $T$ is introduced in order to cancel the quadratic divergence in the Higgs selfenergy coming from the ordinary top quark, and the case of theories including extra dimensions, in which one has towers of quark singlets $T_{L,R}^{(n)}$ that can be associated to the left and right handed components of the SM top quark [5]. Extra $T$ quarks are also predicted in the context of "331" models, in which the standard SU(2)$_L$ $\otimes$ U(1)$_Y$ electroweak gauge symmetry is enlarged to SU(3)$_L$ $\otimes$ U(1)$_X$ [6]. In these models, extra fermions have to be added to the ordinary SM quarks in order to complete the corresponding SU(3)$_L$ triplets. In the LHC, a pair of these exotic fermions can be produced through gluon fusion or quark-antiquark annihilation [7] in the reaction $pp \rightarrow TT$, or a single $T$ quark can come out through the reaction $bq \rightarrow Tq'$ [8]. Estimations in Refs. [3, 9] show that these reactions can be distinguished in the LHC at 100 fb$^{-1}$ luminosity, even for exotic fermion masses of the order of 1 TeV. The corresponding background is basically given by SM top quark production, offering the possibility of finding a clear signature of new physics [10]. Thus, the analysis of $T$ production and decay in definite models is a subject that deserves detailed study.

In this work we concentrate on the study of $T$ phenomenology in 331-symmetric models. These schemes offer an explanation to the puzzle of family replication, since the requirements of anomaly cancellation imply a relation between the number of fermion families and the number of colors [6, 11]. In addition, in this context it is possible to fit the observed neutrino masses and mixing angles [12]. Owing to the enlarged gauge symmetry, the models include new gauge bosons and exotic quarks that behave as singlets under standard SU(2)$_L$ transformations. Different classes of 331 models include either new quarks with ordinary 2/3 and -1/3 electric charges or even more exotic fermions with charges 5/3 and -4/3 that come together with doubly charged gauge bosons and scalar fields (in fact, this is the case of the original versions of the model, see Ref. [6]). In order to break the gauge symmetry it is necessary to introduce a minimal scalar sector of two triplets (this is called the "economical" model [13, 14]), while other 331 models consider three scalar triplets and even an additional scalar SU(3)$_L$ sextet [15, 16, 17].

Since we are interested in studying the presence of an exotic top-like quark, we consider here a 331 model in which fermions have ordinary electric charges [17, 18]. We take into account the general situation of a scalar sector that includes three triplets, analyzing the couplings of the $T$ with the gauge bosons and the experimental constraints on the mixing angles between the $T$ and the ordinary quarks. These bounds allow us to constrain the expected widths for $T$ decays into $tZ$ and $bW$ states for definite values of the $T$ mass. Finally, we consider the Yukawa sector of the model, studying the decays of the $T$ into an ordinary quark and a neutral or charged scalar boson. The predictions for the relative magnitude of these decays depends on several unknown model parameters. However, it is possible to get estimations of the expected rates in the context of specific schemes for the fermion mass matrices.

The article is organized as follows. In Sect. II we briefly describe the model structure, while in Sect. III we study in more detail the couplings of the exotic $T$ quark with the ordinary gauge bosons. Sect. IV and V are devoted to
The analysis of $T$ decays into gauge boson and scalar channels respectively. Finally, in Sect. VI we sketch the main outlines of this work.

### II. MODEL

As stated, in 331 models the SM gauge group is enlarged to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. The fermions are organized into $SU(3)_L$ multiplets, which include the standard quarks and leptons, as well as exotic particles. Though the criterion of anomaly cancellation leads to some constraints in the fermion quantum numbers, still an infinite number of 331 models is allowed [11]. In general, the electric charge can be written as a linear combination of the diagonal generators of the group,

$$ Q = T_3 + \beta T_8 + X , $$

where $\beta$ is a parameter that characterizes the specific 331 model particle structure and quantum numbers.

If one requires that the new quarks have ordinary electric charges $2/3$ and $-1/3$, the values of $\beta$ are restricted only to $\pm 1/\sqrt{3}$ [11, 18, 19]. Here we focus in the model with $\beta = -1/\sqrt{3}$, which includes only one extra $T$, thus the quark mixing in the $Q = 2/3$ sector is enlarged minimally. The fermion sector is completed with two extra quarks $B_{1,2}$ with charge $-1/3$ and a heavy neutrino associated to each lepton family. The situation is sketched in Table I where $Q$ and $X$ quantum numbers are explicitly indicated. Indices $i$ and $j$ run from 1 to 3, while $m$ runs from 1 to 2.

| $q_{mL}$ | $Q$ | $X$ |
|----------|-----|-----|
| $\begin{pmatrix} D_m \\ -U_m \\ B_m \end{pmatrix}_L$ | $3^*$ | $\begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$ | 0 |
| $q_{mL}$ | $3$ | $\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$ | 1 |
| $u_{iR}, d_{iR}, b_{mR}, t_R$ | $1 \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}$ |
| $\ell_{jL}$ | $3$ | $\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ | $-\frac{1}{4}$ |
| $e_{jR}, N_{jR}$ | $1$ | $-1, 0$ | $-1, 0$ |

Table I: Fermion content and $Q$ and $X$ quantum numbers for the 331 model with $\beta = -1/\sqrt{3}$. Index $m$ runs from 1 to 2, while $i, j$ run from 1 to 3.
charge states can be defined according to
\[
W^\alpha_G = \frac{1}{2} \begin{pmatrix}
W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} W^+_\mu & \sqrt{2} Y^0_\mu \\
\sqrt{2} W^-_\mu & -W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} Y^0_\mu \\
\sqrt{2} Y^-_\mu & \sqrt{2} Y^+_\mu & -\frac{2}{\sqrt{3}} W^8_\mu
\end{pmatrix},
\] (2)
while a neutral vector boson \(B_\mu\) is associated with the \(U(1)_X\) group. The fields \(W_3, W_8\) and \(B\) can be conveniently rotated into states \(A, Z\) and \(Z'\) according to
\[
A_\mu = S_W W^3_\mu + C_W \left( -\frac{1}{\sqrt{3}} T_W W^8_\mu + \sqrt{1 - \frac{1}{3} T_W^2} B_\mu \right),
\]
\[
Z_\mu = C_W W^3_\mu - S_W \left( -\frac{1}{\sqrt{3}} T_W W^8_\mu + \sqrt{1 - \frac{1}{3} T_W^2} B_\mu \right),
\]
\[
Z'_\mu = -\sqrt{1 - \frac{1}{3} T_W^2} W^8_\mu - \frac{1}{\sqrt{3}} T_W B_\mu,
\] (3)
where \(A\) and \(Z\) are identified with the usual photon and \(Z\) boson of the SM. In Eq. (3) we have introduced the Weinberg angle \(\theta_W\), defined by
\[
S_W = \frac{\sqrt{3} g'}{\sqrt{3 g^2 + 4 g'^2}},
\] (4)
where \(g\) and \(g'\) correspond to the coupling constants of the \(SU(3)_L\) and \(U(1)_X\) groups, respectively. \(S_W\) stands for \(\sin \theta_W\), etc.

This scheme clearly requires an enlarged scalar sector. We consider the model that includes three scalar triplets
\[
\chi = \begin{pmatrix}
\chi^0_1 \\
\chi^0_2 \\
\xi_X + i \xi_X
\end{pmatrix}_{(X=-1/3)} \quad \rho = \begin{pmatrix}
\rho^+_1 \\
\rho^0_2 + i \xi_\rho \\
\rho^+_3
\end{pmatrix}_{(X=2/3)} \quad \eta = \begin{pmatrix}
\eta^0_1 \\
\eta^0_2 \\
\eta^0_3
\end{pmatrix}_{(X=-1/3)},
\] (5)
where the \(X\) quantum numbers are indicated explicitly (notice that \(\chi^0_1\) and \(\eta^0_1\) are complex fields, while the remaining neutral fields are real). The corresponding scalar potential is given in Ref. [10]. The breakdown of the electroweak symmetry proceeds in two steps: firstly, the \(SU(3)_L \otimes U(1)_X\) group is broken to \(SU(2)_L \otimes U(1)_Y\) through a nonzero vacuum expectation value (VEV) \(\nu_\chi\) of the field \(\xi_X\). This induces the (heavy) masses of the exotic fermions and gauge bosons. Secondly, the VEVs \(\nu_\rho, \nu_\eta\) of the neutral fields \(\xi_\rho\) and \(\xi_\eta\) in the \(\rho\) and \(\eta\) triplets break the symmetry to \(U(1)_{em}\), providing masses to the standard quarks, leptons and gauge bosons. We assume that there is a hierarchy between the first and second breakdown scales, i.e.
\[
\nu_\chi \gg \nu_\rho, \nu_\eta.
\] (6)
As stated, \(\nu_\rho\) and \(\nu_\eta\) are responsible for the \(W^\pm\) and \(Z\) boson masses, thus \(\nu_\rho^2 + \nu_\eta^2 \approx \sqrt{2}/4 G_F \approx (175 \text{ MeV})^2\).

The approximate scalar mass eigenstates and their corresponding masses are sketched in Table I. It can be seen that the states \(\chi^0_1, \eta^0_1\) and \(\rho^+_3\) are heavy scalars, with masses of the order of the large scale \(\nu_\chi\), while \(H^0\) is a light scalar that can be associated with the SM Higgs boson. The remaining physical scalars \(H^0\), \(A^0\) and \(H^\pm\) have masses of the order of \(\sqrt{f} \nu_\chi\), where \(f\) is a dimensionful parameter that drives a trilinear coupling \(\epsilon_{ijk} \chi_i \rho_j \eta_k\) in the scalar potential (we have assumed that \(|f| \leq \nu_\chi\), so as to avoid the introduction of a new dimension scale). The mixing angle \(\beta\) in Table I is given by \(\tan \theta_3 = \nu_\rho/\nu_\eta\).

The scalars couple to fermions through Yukawa like interaction terms. In general, these can be written as
\[
\mathcal{L}_Y = \sum_{q', \Phi} \left( \sum_{m=1}^2 h_{q' m}^q \bar{q}'_m \Phi_1 q_1 \Phi_2 + h_{q' 3}^q \bar{q}'_3 L \Phi_2 + \text{h.c.} \right),
\] (7)
where the sum extends over all quarks \(q'\) and scalar triplets \(\Phi = \eta, \rho, \chi\). In view of the quantum numbers in Table I the \(U(1)_X\) invariance constrain these couplings to those that satisfy \(X_{\Phi} + X_{q'} = 0\) and \(X_{\Phi} + X_{q} = 1/3\) for the first and second term in the parentheses, respectively. Thus the allowed combinations are \(q_{1R}^q \Phi = D_{1R} \chi^*, B_{mR} \chi^*, D_{1R} \eta^*, B_{mR} \eta^*, U_{1R} \rho^*, T_{1R} \rho^*\) for the first term, and \(q_{3L} \Phi = D_{1R} \rho, B_{mR} \rho, U_{1R} \eta, T_{1R} \eta, U_{1R} \chi, T_{1R} \chi\) for the second one. In the standard quark sector, it can be seen that the scenario is similar to that obtained in the Two-Higgs Doublet Model (THDM) type III [21, 22]. As expected, the nonzero VEVs of the scalar fields lead to a \(4 \times 4\) and a \(5 \times 5\) mass matrices in the up and down quark sectors, respectively.
| Mass eigenstate | Mass squared | Feature |
|-----------------|--------------|---------|
| $G_{Z'}^0 \simeq -\xi_8$ | 0 | $Z_{L'}$ Goldstone |
| $G_{Z'}^0 \simeq S_\beta \xi_\rho - C_\beta \xi_8$ | 0 | $Z_\rho$ Goldstone |
| $G_{WZ}^0 = S_\beta \rho_1^0 - C_\beta \eta_2^0$ | 0 | $W_{\rho}^0$ Goldstone |
| $G_{K^0}^0 \simeq -\chi_1^0$ | 0 | $K_{\mu}^0$ Goldstone |
| $G_{K^0}^0 \simeq -\chi_2^0$ | 0 | $K_{\mu}^0$ Goldstone |
| $h_\pm \simeq S_\beta \xi_\rho + C_\beta \xi_8$ | $\sim \nu_\eta^2, \nu_\mu^2$ | SM-like scalar |
| $A_1^0 \simeq C_\beta \xi_\rho + S_\beta \xi_8$ | $\sim |f| \nu_\chi$ | physical |
| $H^0 \simeq -C_\beta \xi_\rho + S_\beta \xi_8$ | $\sim |f| \nu_\chi$ | physical |
| $H^\pm = C_\beta \rho_1^\pm + S_\beta \eta_2^\pm$ | $\sim |f| \nu_\chi$ | physical |
| $\xi_1^0$ | $\sim \nu_\mu^2$ | physical |
| $\eta_1^0, \eta_3^0$ | $\sim \nu_\chi^2$ | physical |
| $\rho_3^0$ | $\sim \nu_\chi^2$ | physical |

Table II: Approximate mass eigenstates in the scalar sector.

III. T QUARK COUPLINGS TO W AND Z BOSONS

As stated, in the 331 model with $\beta = -1/\sqrt{3}$, one has two exotic quarks $B_1$ and $B_2$ with electric charge $Q = -1/3$ and one exotic quark $T$ with $Q = 2/3$. These nonstandard fermions can be organized together with the ordinary up- and down-like quarks $U_i = u, c, t$ and $D_i = b, s, d$ into fermion vectors

$$
\mathcal{U}^0 = \begin{pmatrix}
U^0 \\
T^0
\end{pmatrix} = \begin{pmatrix}
U^0 \\
T^0
\end{pmatrix}, \quad \mathcal{D}^0 = \begin{pmatrix}
D^0 \\
B^0
\end{pmatrix} = \begin{pmatrix}
D^0 \\
B^0
\end{pmatrix},
$$

(8)

where the superindex 0 indicates that we are working with weak current eigenstates. Using this notation, the usual SM charged weak interactions can be written as

$$
\mathcal{L}^{(cc)} = -\frac{g}{\sqrt{2}} \mathcal{U}_L^0 \gamma^\mu \mathcal{P} \mathcal{D}_L^0 W_\mu^+ + \text{h.c.},
$$

(9)

where, in order to project over the ordinary quark sector, we have introduced a $4 \times 5$ matrix $\mathcal{P}$ defined by

$$
\mathcal{P} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}.
$$

(10)

Notice that exotic quarks transform as singlets under $SU(2)_L$ transformations, thus they do not couple with the $W^\pm$ gauge bosons.

We change now to the mass eigenstate basis $\mathcal{U}, \mathcal{D}$ by introducing unitary $4 \times 4$ and $5 \times 5$ rotation matrices for the up- and down-like quark sectors, respectively:

$$
\mathcal{U}_L^0 = V_u^U \mathcal{U}_L, \quad \mathcal{D}_L^0 = V_d^D \mathcal{D}_L.
$$

(11)

It is useful to group the elements of $V_u^U$ into conveniently defined non-quadratic submatrices,

$$
V_u^U = \begin{pmatrix}
V_u^{(3 \times 3)} & V_u^{(3 \times 1)} \\
V_u^{(1 \times 3)} & V_T
\end{pmatrix} \quad \text{and} \quad V_d^D = \begin{pmatrix}
V_d^{(3 \times 3)} & V_d^{(3 \times 2)} \\
V_d^{(2 \times 3)} & V_B^{(2 \times 2)}
\end{pmatrix}.
$$

(12)
Thus, in the basis of quark mass eigenstates the couplings in Eq. (9) read
\[
\mathcal{L}^{(cc)} = - \frac{g}{\sqrt{2}} U_L \gamma^\mu V^u_{L}^\dagger \mathcal{P} V^d_L D \mathcal{L} W^\mu_\mu + \text{h.c.}
\]
\[
= - \frac{g}{\sqrt{2}} \left[ \tilde{U}_L \gamma^\mu V_{CKM} D L + \tilde{U}_L \gamma^\mu V^u_0^{\dagger} V^d_B L + \tilde{T}_L \gamma^\mu V^{u_t}_0^{\dagger} D L + \tilde{T}_L \gamma^\mu V^{u_t}_0^{\dagger} V^d_B L \right] W^\mu_+ + \text{h.c.} \quad (13)
\]
Notice that the mixing matrix \( V_{CKM} = V_{0}^{u_t} V^{u_d}_0 \) that acts over the SM quark sector is not in general unitary.

Owing to the enlargement of the gauge symmetry group, 331 models include also exotic gauge bosons. In the models with \( \beta = \pm 1/\sqrt{3} \) these have electric charge 0 or 1. In our case one has a heavy charged gauge boson \( Y^+ \), that couples with the fermions according to
\[
\mathcal{L}_{Y^+} = - \frac{g}{\sqrt{2}} \tilde{T}_L \gamma^\mu \left[ \cos \theta \left( V_X^{u_t} V^d_L + V_X^{u_t} V^d_B L \right) + \sin \theta \sum_{i=1}^5 K_i D_{Li} \right] W^\mu_+ + \text{h.c.} \quad (14)
\]
Notice that quarks \( u^0, c^0 \) couple to \( Y^+ \), whereas \( t^0 \) does not. This is a consequence of the structure of the model, where one of the quark families belongs to a different \( SU(3)_L \) representation than the other two. It is also important to point out that in general \( W \) and \( Y \) are not mass eigenstates. They become mixed by a small mixing angle \( \theta \), which turns out to be suppressed by the weak symmetry breaking scale ratio \( [14] \), \( \theta \sim \mathcal{O}(\nu/\nu_\chi) \).

In particular we are interested here in the couplings that involve the exotic quark \( T \). After a redefinition of the mass states \( W^+ \) and \( Y^+ \) we obtain
\[
\mathcal{L}^{(cc:T)} = - \frac{g}{\sqrt{2}} \tilde{T}_L \gamma^\mu \left[ \cos \theta \left( V_X^{u_t} V^d_L + V_X^{u_t} V^d_B L \right) + \sin \theta \sum_{i=1}^5 K_i D_{Li} \right] W^\mu_+ + \text{h.c.} \quad (15)
\]
where \( K_i = V_T^{V^d_{3i}} + V_X^{u_t} V^d_{3i} + V_X^{u_t} V^d_{3i} \). Phenomenologically, it is well known that \( V_{CKM} \) is compatible with a unitary matrix. Therefore, it is natural to expect the matrices \( V^u_0 \) and \( V^d_0 \) to be approximately unitary, which implies \( |V^u_0| \ll |V_T| \approx 1 \). If we also approximate \( \cos \theta \) to 1, the couplings between the \( T \) quark and the ordinary down quarks can be written as
\[
\mathcal{L}^{(cc:T)} = - \frac{g}{\sqrt{2}} \tilde{T}_L \gamma^\mu V^{(T)}_i D_{Li} W^\mu_+ + \text{h.c.} \quad (16)
\]
with the definition
\[
V^{(T)}_i = \sum_{j=1}^3 V_X^{u_t j} V^d_{3j} + V_T \sin \theta V^d_{3i} \quad (17)
\]

Let us now perform a similar analysis for the neutral currents. The couplings between the quarks and the \( Z \) boson in the 331 model with \( \beta = -1/\sqrt{3} \) read \([14][19]\)
\[
\mathcal{L}^{(nc)} = \frac{-g}{2C_W} \left[ (1 - \frac{4}{3} S_W^2) \tilde{U}_L \gamma^\mu U^0_L - \frac{4}{3} S_W^2 \tilde{U}_L \gamma^\mu U^0_L + \frac{1}{2} S_W^2 \tilde{D}_L \gamma^\mu D^0_L \right]
\]
\[
+ \frac{2}{3} S_W^2 \tilde{B}_L \gamma^\mu B^0_L - \frac{4}{3} S_W^2 \tilde{U}_R \gamma^\mu U^0_R + \frac{2}{3} S_W^2 \tilde{D}_R \gamma^\mu D^0_R \right] Z_\mu
\]
\[
= \frac{-g}{2C_W} \left[ \tilde{U}_L \gamma^\mu U^0_L - \tilde{D}_L \gamma^\mu D^0_L - \frac{4}{3} S_W^2 \tilde{U}_R \gamma^\mu U^0_R + \frac{2}{3} S_W^2 \tilde{D}_R \gamma^\mu D^0_R \right] \quad (18)
\]
In the last expression, only the first two terms in the r.h.s. lead to a mixing between current eigenstates when one rotates to the mass basis. The interactions involving the exotic quark \( T \) read then
\[
\mathcal{L}^{(nc:T)} = \frac{-g}{2C_W} \tilde{T}_L \gamma^\mu V^{(T)}_i U^0_L Z_\mu + \text{h.c.} \quad (19)
\]
As in the case of the charged states \( W^+ \) and \( Y^+ \), the \( Z \) state becomes mixed with other neutral gauge bosons. However, the interactions between the \( T \) quark and the standard up-like quarks mediated by the exotic neutral gauge bosons are expected to suffer a twofold suppression: on one hand, the mixing angles between the gauge bosons suffer a strong suppression \( \mathcal{O}(\nu^2/\nu_\chi^2) \)[14][19], and on the other hand, for the neutral currents one expects a mechanism of suppression of flavor changing currents in order to be compatible with experimental constraints (which mainly come from the down-like quark sector). In this way, the contributions to \( T \) decay arising from this mixing will be neglected in our analysis.
IV. BOUNDS FOR $T \to U, Z$ AND $T \to D, W$ DECAY WIDTHS

The mass of the $T$ quark is expected to be of the order of the $\nu_\chi$ scale, i.e., at the TeV range, therefore we are not able to establish bounds for the mixing angles from direct $T$ production before having at our disposal the results of forthcoming experimental devices as the LHC or ILC \cite{24}. However, it is possible to set bounds for the $T$ decay widths by taking into account contributions from virtual $T$ quarks to lower energy processes. Here we concentrate in the observables $\Delta m_K$, $\Delta m_{B_q}$ and $\Delta m_{B_s}$, which typically lead to the most stringent constraints for the presence of new physics. Charged currents involving $T$ quarks will contribute to these magnitudes through one-loop box diagrams including one or two virtual $T$'s. Moreover, these contributions are expected to be enhanced by the large $T$ mass, just as happens in the case of the top quark.

The theoretical expressions for the contributions to the mentioned observables driven by box diagrams can be written as

\[
\Delta m_K = m_{K_L} - m_{K_S} = \frac{G_F^2}{12\pi^2} m_W^2 m_K f_K^2 B_K C_K
\]

\[
\Delta m_{B_q} = m_{B^{0}_{M_{U}} } - m_{B^{0}_{L} } = \frac{G_F^2}{12\pi^2} m_{W}^2 m_{B_q} f_{B_q}^2 B_{B_q} \left| C_{B_q}^{(SM)} + C_{B_q}^{(T)} \right|, \quad q = d, s
\]

(20)

where $f_P$ are the $P$ meson weak decay constants, and the parameters $B_P$ account for the theoretical uncertainties related with the evaluation of matrix elements that involve hadronic states \cite{24,25,26}. The coefficients $C_K$ and $C_{B_q}$ are given by the sum of the contributions of boxes that include ordinary and exotic quarks. Explicitly one has

\[
C_K = \sum_{f,f'=u,c,t} \lambda_{f,sd} \lambda_{f',sd} E_\Box(x_f, x_{f'}) + 2 \sum_{f=u,c,t} \lambda_{f,sd} \lambda'_{s,d} E_\Box(x_f, x_T) + \lambda'_{sd}^2 E_\Box(x_T, x_T)
\]

\[
C_{B_q} = \sum_{f,f'=u,c,t} \lambda_{f,bq} \lambda_{f',bq} E_\Box(x_f, x_{f'}) + 2 \sum_{f=u,c,t} \lambda_{f,bq} \lambda'_{b,q} E_\Box(x_f, x_T) + \lambda'_{b,q}^2 E_\Box(x_T, x_T)
\]

(21)

where we have introduced the definitions

\[
\lambda_{U_i,D_j,D_k} = (V_{CKM}^*)_{ij} (V_{CKM})_{ik}, \quad \lambda'_{D_i,D_j} = V_i^{(T)*} V_j^{(T)}, \quad x_f = \frac{m^2_{f}}{m_{W}^2},
\]

(22)

together with the previous associations $(D_1 D_2 D_3) = (d s b), (U_1 U_2 U_3) = (u c t)$. The Inami-Lim function $E_\Box(x, y)$ is given by \cite{27}

\[
E_\Box(x, y) = \frac{4 - 7xy}{4(x-1)(y-1)} + \left[ \frac{x^2(4-8x+xy)\log x}{4(x-1)^2(x-y)} + (x \to y) \right].
\]

(23)

Since in our case the $V_{CKM}$ matrix is not unitary, we cannot introduce the usual unitarity relations to take into account the GIM cancellations in Eqs. (21). However, owing to the unitarity of the $V_{L}^{c}$ and $V_{R}^{d}$ rotation matrices, the following relation is found to hold:

\[
\sum_{i=1,3} \lambda_{U_i,D_j,D_k} + \lambda_{D_i,D_j} = \sum_{i=1,3} V_{0,i}^{d*} V_{0,i}^{d} = -V_{X_j}^{d*} V_{X_k}^{d}.
\]

(24)

Using this relation the coefficients $C_K, C_{B_q}$ can be written as

\[
C_K \simeq \lambda_{c,sd}^2 S(x_c, x_c) + \lambda_{l,sd}^2 S(x_l, x_l) + 2 \lambda_{c,sd} \lambda_{l,sd} S(x_c, x_l) + 2 \lambda_{c,sd} \lambda'_{s,d} S(x_c, x_T) + 2 \lambda_{s,d} \lambda'_{s,d} S(x_l, x_T) + \lambda_{s,d}^2 S(x_T, x_T) - 2 V_{X_j}^{d*} V_{X_k}^{d} \left[ \lambda_{c,sd}(E_\Box(0, x_c) - 1) + \lambda_{l,sd}(E_\Box(0, x_l) - 1) + \lambda_{s,d}^2(E_\Box(0, x_T) - 1) \right]
\]

(25)

\[
C_{B_q} \simeq \lambda_{l,bq}^2 S(x_l, x_l) + 2 \lambda_{l,bq} \lambda'_{b,q} S(x_l, x_T) + \lambda'_{b,q}^2 S(x_T, x_T) - 2 V_{X_j}^{d*} V_{X_k}^{d} \left[ \lambda_{c,sd}(E_\Box(0, x_c) - 1) + \lambda'_{s,d}(E_\Box(0, x_T) - 1) \right]
\]

(26)

where $S(x, y)$ is the Inami-Lim function usually considered in Standard Model calculations,

\[
S(x, y) = -\frac{3xy}{4(x-1)(y-1)} + \left[ \frac{xy(4-8x+x^2)\log x}{4(x-1)^2(x-y)} + (x \to y) \right].
\]

(27)
In the limit $x \to y$ one has \cite{28}

$$S(x, y) \to S_0(x) = \frac{4x - 11x^2 + x^3}{4(x - 1)^2} + \frac{3x^3 \log x}{2(x - 1)^3} \ . \quad (28)$$

In Eqs. (25) and (26) we have taken the limit $x_u \to 0$, and we have neglected the contributions to $C_{B_i}$ driven by $\lambda_c$ and a term proportional to $(V'_{X k}^{d} V'_{X k})^2$. Notice that in both equations the first line corresponds to the usual SM contribution, the second line includes the contribution from exotic quarks and the third line is a residual contribution that arises from the nonunitarity of the $V'_{ij}$ matrix. Concerning this last term, it is worth to point out that the experimental values of $\Delta m_K$ and $\Delta m_{B_s}$ also provide constraints on the nondiagonal elements of $V'_{ij}$. Indeed, as shown in Ref. \cite{29}, the latter lead to tree level FCNC mediated by the $Z'$ gauge boson. In addition, in Eqs. (25) and (26) we have neglected perturbative QCD corrections. These are typically below a 30% level \cite{28}, therefore they are not relevant in order to estimate the order of magnitude of the bounds for the couplings involving the $T$ quark.

Now, taking into account the experimental measurements for the $\Delta m_P$ observables, and assuming that there is no fortunate cancellations with other contributions from nonstandard physics (e.g. the mentioned $Z'$-mediated FCNC), we can establish bounds for the products $|\lambda'_{D_i D_j}| = |V_i^{(T)} V_j^{(T)}|$, $ij = 12$, 13 and 23, for a given value of the $T$ quark mass. To do this, we cannot take into account the usual estimations of SM contributions, which assume the unitarity of $V_{C K M}$. Indeed, the top quark mixing angles in the SM are basically determined from the same observables we want to use to constrain the new parameters. Thus, in order to estimate upper bounds for the $T$ mixing angles, we will just take into account the $c$ quark contribution to $\Delta m_K$ ($\lambda_{c s, s d}$ can be measured from direct observation), while the remaining SM contributions will be set to zero.

For the numerical analysis we will use the experimental results \cite{30,31}

$$\begin{align*}
\Delta m_K &= m_{K_L} - m_{K_S} = (5.292 \pm 0.009) \times 10^{-3} \ \text{ps}^{-1} \\
\Delta m_{B_d} &= m_{B^0_d} - m_{B^0_s} = 0.507 \pm 0.005 \ \text{ps}^{-1} \\
\Delta m_{B_s} &= m_{B^0_{s d}} - m_{B^0_{s s}} = 17.77 \pm 0.12 \ \text{ps}^{-1} \ . \quad (29)
\end{align*}$$

Taking the central values of these measurements (errors are negligible at the level of accuracy of our estimations) we obtain the results shown in the left panel of Fig. 1 where we plot the bounds for $|\lambda'_{D_i D_j}|$ as functions of the $T$ quark mass. It can be seen that the values obtained are of the order of $10^{-3}$, and they decrease for increasing $m_T$. This can be understood by noting that $S_0(x_T) \sim x_T/4$ for large values of $x_T$. In addition, one can relate the bounds for $|\lambda'_{D_i D_j}|$ with the corresponding bounds for the decay widths of the exotic $T$ quark into a $W^+$ boson and a down-like quark $d$, $s$ or $b$. These widths are given by

$$\Gamma(T \to D_i W^+) = \frac{G_F m_T^3}{\sqrt{2} \ 8 \pi} \ |V_i^{(T)}|^2 \ (1 - 3 y_W^4 + 2 y_W^6) \ , \quad (30)$$

Figure 1: Upper bounds for $|\lambda'_{D_i D_j}|$ (left panel) and $\Gamma(T \to D_i W^+)$ as functions of the $T$ quark mass.
where $y_W \equiv m_W/m_T$. Since the $\Delta m_P$ observables involve products $|V_i^{(T)} V_j^{(T)}|^*$, one can establish upper bounds for the products $\Gamma(T \to D_i W^+) \Gamma(T \to D_j W^+)$, for $i \neq j$. The corresponding numerical results are shown in the right panel of Fig. 1 where we plot these bounds as functions of the exotic $T$ quark mass.

The experimental values of $\Delta m_P$ do not allow us to establish separate bounds for the $|V_i^{(T)}|$ parameters. However, it is interesting to consider the case in which all three experimental constraints are saturated simultaneously. In this situation one finds the values for $|V_i^{(T)}|$ shown in the left panel of Fig. 2 (as before, we show the plots as functions of $m_T$). As expected, the couplings between the exotic $T$ quark and the ordinary $d$, $s$, and $b$ quarks are suppressed according to the usual family hierarchy. Notice that in principle one could have $Tb$ mixing angles as large as $\sim 0.1$ for $T$ quark masses of a few TeV. Finally, from the values of $|V_i^{(T)}|$ one can immediately obtain the corresponding $T \to D_i W^+$ decay widths. These are quoted in the right panel of Fig. 2 as functions of the $T$ quark mass. We notice that the dependence on $m_T$ vanishes for the ratios between the decays into down-like quarks of different families, the corresponding branching ratios obeying approximate relations

$$\frac{\text{BR}(T \to d W^+)}{\text{BR}(T \to s W^+)} \approx \frac{1}{30}, \quad \frac{\text{BR}(T \to d W^+)}{\text{BR}(T \to b W^+)} \approx \frac{1}{150},$$

which arise from the ratios between $V^{(T)}$ matrix elements.

![Figure 2: Left: values of $|V_i^{(T)}|$ that simultaneously saturate the experimental bounds of the observables $\Delta m_K$, $\Delta m_{B_d}$ and $\Delta m_{B_s}$. Right: $T \to D_i W^+$ decay widths that correspond to the values in the left panel.](image)

Let us now analyze the $T$ decays into a $Z$ boson and an ordinary up-like quark. From the currents in Eq. (19) we have

$$\Gamma(T \to t Z) = \frac{G_F m_T^3}{\sqrt{2}} |\lambda_{T1}^2| \left( 1 + y_T^2 - 2 y_T^2 - 2 y_T^2 y_T^2 + y_T^4 \right) \sqrt{1 - (y_T + y_T)^2} \sqrt{1 - (y_T - y_T)^2}$$

$$\Gamma(T \to U_i Z) = \frac{G_F m_T^3}{\sqrt{2}} |\lambda_{T U_i}^2| \left( 1 - 3 y_T^4 + 2 y_T^6 \right), \quad i = 1, 2,$$

where $y_t = m_t/m_T$, $y_Z = m_Z/m_T$, and we have defined

$$\lambda_{T U_i}^2 = \sum_{i=1}^3 V_{X_i}^{u} V_{0 j i}^{u}.$$

In principle, both the order of magnitude of the matrix elements $V_{X_i}^{u}$ and $V_{0 j i}^{u}$ cannot be constrained independently from present experimental measurements. However, in order to have an estimation of the possible size of the decay widths, we can take into account the values of $|V_i^{(T)}|$ obtained above, together with some assumptions on the mixing.
matrices $V_{\alpha}^{u,d}$. In this sense, most popular models of quark mass matrices assume that off-diagonal elements of mixing matrices satisfy family hierarchies given by

$$V_{0ij}^q \sim \left( \frac{m_q}{m_{q_j}} \right)^{1/2},$$

for $q = U, D$. From Eq. (17), and taking into account the results shown in the left panel of Fig. 2, one has then

$$|\lambda_{T_{1u}}^q| \sim |V_{X1}^q| \sim 0.01$$
$$|\lambda_{T_{1c}}^q| \sim |V_{X2}^q| \sim 0.05$$
$$|\lambda_{T_{1t}}^q| \sim |V_{X3}^q| \sim 0.1.$$  (35)

For the decay widths of the exotic $T$ quark into $U_i Z$ states, $U_i = u, c, t$, one obtains approximately the same dependence on $m_T$ as in the case of $\Gamma(T \rightarrow D_i W^+)$ processes, together with a global kinematical factor $\simeq 1/2$. Thus we have for each family

$$\Gamma(T \rightarrow D_i W^+) \simeq 2 \Gamma(T \rightarrow U_i Z).$$

The above relations provide a couple of hints on what we can expect from exotic $T$ decays if they are observed in future colliders: on one hand, for any value of $m_T$ the branching ratios for $T \rightarrow U_i Z$ and $T \rightarrow D_i W^+$ should be of the same order of magnitude, being $U_i$ and $D_i$ up and down quarks of the same family. On the other hand, the decay widths should obey family hierarchies, as stated in Eq. (31). As stated, these relations correspond to the situation in which the 331 contributions saturate the bounds on the $\Delta m^2$ mass differences.

Let us recall that we have considered here the 331 model with $\beta = -1/\sqrt{3}$, in which one has only one exotic quark $T$ with electric charge $Q = 2/3$. One can also study the model with $\beta = +1/\sqrt{3}$, in which one has two exotic quarks of this kind, $T_{1,2}$. Though the theoretical treatment remains qualitatively similar, in this case one has to deal with more unknown parameters (masses and mixing angles), and the phenomenological analysis is obscured. Therefore we have concentrated here on the first possibility. Another important difference between both models is that in the case $\beta = -1/\sqrt{3}$ the theory includes extra heavy neutrinos, while for $\beta = +1/\sqrt{3}$ one has exotic charged leptons.

V. $T \rightarrow qH$ DECAYS

Let us analyze the decays of a $T$ quark into an ordinary quark and a scalar field. As stated in Sect. II, the scalar eigenstates can be separated into those with masses at the $\nu_\chi$ scale and a set of fields that can be associated with the scalars of a THDM. Here we concentrate on the decays of the $T$ into these lighter states, assuming that the other channels are largely suppressed or even forbidden owing to the large scalar masses.

The quark-scalar vertices arise from the Yukawa couplings in Eq. (7), which include a large number of unknown parameters. Consequently, in order to get an estimation of the expected order of magnitude of the relevant couplings it is necessary to rely on a definite Ansatz for the quark mass matrices. A natural option in this sense is to consider a scenario with approximate flavor symmetry such as that proposed by Cheng and Sher [22, 32], now extended to include the heavy $T$ quark. This scenario is consistent with the assumption introduced in the previous section, see Eq. (34). Thus, for the up-like quark sector we will write the coupling constants in Eq. (7) as

$$h_{Uj}^\Phi = \lambda_{Uj}^\Phi \sqrt{m_{Uj} m_{\Phi}}/\nu_{\Phi},$$

with $\lambda_{Uj}^\Phi \simeq O(1)$. Within this Ansatz, the dominant $T$ decays in the scalar sector arise from the terms driven by the couplings $h_T^{\mu\rho}$ and $h_T^{3\eta}$. Now, if flavor symmetry is approximately conserved, current quark eigenstates and mass eigenstates are approximately the same. Let us identify the top and bottom quarks with the quarks in the 3 representation, i.e. $U_3$ and $D_3$. Then the relevant couplings for the $T$ quark decays are

$$h_T^{3\eta}[\bar{t}_L T_R (\cos \theta_\beta h^0 + \sin \theta_\beta H^0 + i \sin \theta_\beta A^0) + \bar{b}_L T_R \sin \theta_\beta H^-].$$

Here we have neglected higher orders in $t - T$ mixing, which in the framework of approximate flavor symmetry means to work at the leading order in $(m_t/m_T)^2$. In this limit the corresponding decay widths are given by

$$\Gamma(T \rightarrow t h^0) = \frac{|h_T^{3\eta}|^2}{32\pi} m_T \cos^2 \theta_\beta (1 - y_{h^0})^2$$
$$\Gamma(T \rightarrow q \phi) = \frac{|h_T^{3\eta}|^2}{32\pi} m_T \sin^2 \theta_\beta (1 - y_{\phi})^2,$$  (39)
Table III: Approximate branching ratios for $T$ decays into scalars and gauge bosons in a Cheng-Sher-like scenario.

|                  | $m_\phi \ll m_T$ | $m_\phi = \frac{m_T}{2}$ | $m_\phi > m_T$ |
|------------------|------------------|------------------|------------------|
| $\tan \theta_\beta = 0.1$ | 0.14             | 0.15             | 0.18             |
| $\tan \theta_\beta = 1$  | 0.36             | 0.32             | 0.30             |
| $\tan \theta_\beta = 10$ | 0.00             | 0.00             | 0.00             |

where $y_X = (m_X/m_T)^2$, and in the second equation $q_\phi = t H^0, t A^0, b H^+$. If in addition we assume $\nu_\alpha \approx \nu_\rho$, the global coupling constant $g_T^{3\phi}$ can be approximated by $|h_T^{3\phi}|^2 \approx |\lambda_T^{3\phi}|^2 8 m_t m_T G_F/\sqrt{2}$.

As stated, we have identified the top and bottom quarks with those in the $3\phi$ representation. This election is in principle arbitrary. If, instead, we had chosen the $t$ and $b$ quarks to belong to one of the families in the $3^*$, the results in Eqs. (48) and (49) would be qualitatively similar, with the interchange $\cos \theta_\beta \leftrightarrow \sin \theta_\beta$, $\eta \leftrightarrow \rho$. Since $\theta_\beta$ is an unknown parameter, it is seen that the family choice is not relevant in order to obtain a rough numerical estimation of the size of the decays.

To present some numerical results for the expected relative decay widths of the $T$ quark, we will neglect the mass of the light Higgs boson $h^0$ compared with $m_T$, and we will take $m_H^0 \simeq m_A^0 \simeq m_H^\pm$ (in fact these masses are expected to be of the same order of magnitude, see Table II). Finally, the sizes of $T \to tZ$ and $T \to bW^+$ decays will be approximated taken into account the assumption in Eq. (44), which leads to $|\lambda_T^{3\phi}|^2 \approx |V_T^{3\phi}|^2 \approx m_t/m_T$. One gets in this way

$$\Gamma(T \to tZ) : \Gamma(T \to bW^+) : \Gamma(T \to h^0) : \Gamma(T \to q_\phi) \approx \frac{1}{2} : 1 : 2 \cos^2 \theta_\beta : 2 \sin^2 \theta_\beta \left(1 - m_\phi^2/m_T^2\right)^2,$$  

(40)

where, as before, $q_\phi = t H^0, t A^0, b H^+$. Notice that in this Cheng-Sher-like scenario the relative sizes of the decay widths do not depend (at the leading order) on the $T$ quark mass. Only the phase space factor $(1 - m_\phi^2/m_T^2)$ appears in the case of $T \to q_\phi$ decays.

Results from the relations in Eq. (40) are shown in Table III, where we quote approximate values for the branching ratios taking different values for $\tan \theta_\beta$ and the intermediate scalar masses $m_\phi$. We recall that one expects $m_\phi^2 \sim |f| \nu_\lambda$, where $f$ is a parameter that drives a trilinear coupling in the scalar potential. The values in the table have been obtained after many assumptions and approximations, therefore they should be taken only as illustrative. However, since this corresponds to a plausible scenario, it is interesting to notice that the decays into scalars might be the most important ones in the search for an exotic $T$ quark.

### VI. SUMMARY

In summary, we have studied here the phenomenology of exotic $T$ quarks in the framework of 331-symmetric models. We have concentrated on the models with $\beta = -1/\sqrt{3}$, in which one has a single $T$ with charge $2/3$ that in general mixes with the ordinary $u$, $c$ and $t$ quarks. We have studied in detail the couplings of this $T$ quark with the ordinary gauge bosons, establishing bounds for $T \to qW$ decays from the measured values of neutral $K$, $B_d$ and $B_s$ mass differences. Then we have analyzed the decays $T \to qZ$, considering the situation in which the previous bounds are saturated, together with some assumptions on the hierarchies in the quark mixing angles. As expected, the bounds are in agreement with family hierarchies. The dependence with the $T$ quark mass is shown in Figs. 1 and 2. Finally we have analyzed the decays of the $T$ quark into a scalar and an ordinary quark. Though the results are strongly dependent on model parameters that are in principle unknown, it is possible to present some estimations for the widths by considering a definite scenario in which one has approximate flavor symmetries. By performing plausible assumptions on the order of magnitude of coupling constants and mass scales, it can be seen that the decays into fermion-scalar states may be indeed the dominant ones.

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[1] ATLAS Collaboration, Technical Design Report, CERN-LHCC-99-15; CMS Collaboration, Technical Proposal, CERN-LHCC-94-38; G. Weiglein et. al. [LHC/LC study Group], hep-ph/0410364.

[2] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by D. Freedman et. al. (North-Holland, Amsterdam, 1980); T. Yanagida, in Proceedings of the Workshop on Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980); J.R. Ellis et. al. Phys. Lett. B150, 142 (1985); L. J. Hall and M. Suzuki, Nucl. Phys. B231, 419 (1984).

[3] P. H. Frampton, P. Q. Hung and M. Sher, Phys. Rep. 330, 263 (2000).

[4] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001); V.D. Barger, M.S. Berger and R. J. N. Phillips, Phys. Rev. D 52, 1663 (1995).

[5] F. del Aguila and J. Santiago, JHEP 0203, 010 (2002).

[6] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992); P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47, 4158 (1993).

[7] F. del Aguila, L. Ametler, G.L. Kane and J. Vidal, Nucl. Phys. B 334, 1 (1990).

[8] G. Azuelos et. al., Eur. Phys. J. C 395213 (2005).

[9] J. A. Aguilar-Saavedra, Phys. Lett. B 625, 234 (2005) [Erratum-ibid. B 633, 792 (2006)].

[10] F. del Aguila, L. Ametler, G.L. Kane and J. Vidal, Nucl. Phys. B 334, 1 (1990).

[11] F. del Aguila, L. Ametler, G.L. Kane and J. Vidal, Nucl. Phys. B 334, 1 (1990).

[12] F. Pisano and V. Pleitez, Phys. Lett. B 625, 234 (2005) [Erratum-ibid. B 633, 792 (2006)].