Grid-characteristic numerical method for medical ultrasound

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Abstract. Grid-characteristic method (GCM) is a fast and reliable numerical method that allows to model wave effects in viscoelastic media with high accuracy, including surface and contact waves. This research is dedicated to the application of GCM to the problem of medical ultrasound. Calculations for High-Intensity Focused Ultrasound (HIFU) were performed on 3D model statements for homogeneous and inhomogeneous media, and a qualitative correspondence with experimental data was achieved. Numerical results include estimation of consumed energy (based on Maxwell viscosity model), velocity vector and stress tensor components. Various material parameters were considered, including relaxation time and inclusions of different types.

1. Introduction
One of the promising applications of ultrasound in medicine is associated with its ability for a quick and controlled thermal destruction of tissue proteins, caused by absorbing focused acoustic waves [1]. When using linear phased arrays, the intensity of the acoustic wave in the focal plane increases significantly, which makes it possible to use powerful ultrasound, for example, to destroy tumors [2], [3] or stop internal bleeding non-invasively, i.e. without conventional surgery. In clinical practice, this method appeared in the ’40s of the last century [4], however, the first mention of the use of High-Intensity Focused Ultrasound (HIFU) for operational purposes was described in [5].

The main mechanism of the currently used HIFU is thermal [2]. The absorption of mechanical energy of ultrasound in soft tissues causes an increase in temperature above the limiting temperature of protein denaturation, which leads to cell necrosis as a result of coagulation. Although most of the initial cell death in tissues exposed to HIFU fields is caused by cell necrosis as a result of thermal damage, HIFU can also induce apoptosis. In apoptotic cells, the cell nucleus self-destructs with rapid DNA degradation by endonucleases [6]. Apoptosis may be an important delayed bioeffect in tissues exposed to low energy surgical ultrasound, especially in poorly regenerating cells such as neurons. Therefore, there may be a wider area of cell death than just the target area of HIFU, and this mechanism could be a potential limitation of the method.

Although HIFU offers great potential for non-invasive treatment of malignant neoplasms, especially widespread or inoperable tumors, its effectiveness has limitations and there are a number of risks that can lead to adverse outcomes. First of all, the interaction of ultrasound, its thermal effects and soft tissues needs a further research. Secondly, all the solid inclusions like bones and implants have a high acoustic contrast with surrounding tissues. It causes multiple
reflections and artifacts that cannot be explained by an acoustic material model [7]. Thirdly, all the soft tissues also have different material parameters and under certain circumstances can affect the ultrasound waves pattern, shifting or blurring the focal area.

Numerical modeling of HIFU encounters a number of problems, including a lack of reliable data on material parameters and clinical data in open access. Nevertheless, a considerable amount of research is dedicated to the numerical modeling of ultrasound, and a wide variety of methods is used. A finite-difference method is used in [8]. A method based on the Rayleigh Sommerfeld model and the multi-Gaussian beam (MGB) model is introduced in [9]. A finite integration technique is introduced in [10]. For numerical modeling of strong acoustic beams, the Khokhlov-Zabolotskaya-Kuznetsova equation is also used [11], [12]. A wavefront construction method [13] for biological applications was presented in [14, 15]. This article uses a grid-characteristic numerical method and presents numerical results for problems of HIFU interaction with a heterogeneous medium.

2. Material model and numerical method

This work uses a viscoelasticity model to describe the media:

\[ \rho \dot{\vec{v}} = \nabla \cdot \sigma \]
\[ \dot{\sigma} = \lambda (\nabla \cdot \vec{v}) I + \mu (\nabla \otimes \vec{v} + (\nabla \otimes \vec{v})^T) - \frac{\sigma}{\tau_0}. \]

In these equations \( \vec{v} \) is a velocity vector, \( \sigma \) is a stress tensor, \( I \) is an identity matrix. The rheology of the media is described by Lame coefficients \( \lambda \) and \( \mu \), medium density \( \rho \), viscous relaxation time parameter \( \tau_0 \).

A numerical solution is obtained using a grid-characteristic method. This method is designed specifically for problems of wave propagation in a heterogeneous media. It was used successfully for various dynamic problems, including impacts on composite structures [16, 17], seismic migration [18], wave propagation in a fractured [19] and multicomponent [20] media. Cross-validation with other numerical methods was also performed [21].

An implementation of the method for biomedical problems was presented and discussed in [7]. A modification that allows to model a viscoelastic material behavior in homogeneous soft tissues was described in [22].

3. Numerical results

3.1. Problem statement

We consider a model problem for a propagation of an ultrasound pulse from a linear phased array through a homogeneous medium with a cylindrical inclusion. The calculation area is a parallelepiped 20x10x10 mm, the general view is presented on figure 1. The linear phased array of 30 elements is modeled as a set of border conditions on a YZ surface. Linear elements are parallel to the Y axis. The focusing is performed in the XZ plane. The rest of a surface with the phased array is a free border. Other surfaces have a consuming border condition, effectively modeling an infinite bulk of material. For the background material, parameters are as follows: \( \lambda = 20000 \) Pa, \( \mu = 10000 \) Pa, \( \rho = 1.25 \) g/mm\(^3\). For \( \tau_0 \) we consider values 0.1 s\(^{-1}\) and 10.0 s\(^{-1}\).

The inclusion is a cylinder with its axis directed along the Y axis. Cylinder radius is 3.5 mm. Two types of inclusions are considered. The first is a ”bonelike” material – faster and less viscous: \( \lambda = 40000 \) Pa, \( \mu = 20000 \) Pa, \( \rho = 1.35 \) g/mm\(^3\), \( \tau_0 = 100.0 \) s\(^{-1}\). Its material parameters are less different from the background material than actual bones, but it still allows us to look at effects from this inclusion on a qualitative level. The second is a ”tumorlike” material – slower and more viscous: \( \lambda = 15000 \) Pa, \( \mu = 10000 \) Pa, \( \rho = 1.15 \) g/mm\(^3\), \( \tau_0 = 0.01 \) s\(^{-1}\).
Ultrasound pulse parameters are as follows. Frequency is 300 Hz, characteristic pulse length is 1 ms, pulse amplitude is 100 Pa.

Figure 1. General view of the calculation area.

3.2. Pulse propagation and absorbed energy without inclusions

Figure 2 shows the wave pattern for $\tau_0 = 0.1 \; s^{-1}$. The wave pattern for $\tau_0 = 10.0 \; s^{-1}$ looks similar and is not included here. The decrease of amplitude is slightly smaller, but forms and positions of wavefronts are the same. It coincides with a general description of the viscosity model and with clinical and technological practice concerning the ultrasound. Complex non-linear behavior of soft tissues manifests itself during conventional procedures of determining material parameters due to relatively slow loading. The behavior of soft tissues during a fast loading, where loading time is comparable to acoustic time scale, is close to linear except for the attenuation. In diagnostic ultrasound the attenuation is accounted for by the time-gain compensation and does not change the most important parameters of the resulting image – shapes and positions of analyzed objects.

Figure 2. Velocity modulus during the calculation for $\tau = 0.1 \; s^{-1}$. From left to right – focal distance 1 mm, 5 mm, 10 mm, 20 mm, 50 mm and infinite. From top to bottom – consecutive time steps.

On the other hand, in surgical ultrasound the attenuation affects the resulting absorbed energy distribution. Despite the similarity of wave patterns, the absorbed energy distribution
for different $\tau_0$ differs significantly, as you can see on figure 3. This difference means that the analysis of a wave pattern by itself without considering the viscous behavior of soft tissues can not be an effective method for any development or improvement of the ultrasound equipment. Also, this figure shows that the HIFU is not suitable for a very viscous media – the ultrasound signal is absorbed in the upper part of the material, close to the transducer, and the increase of focal distance does not improve the situation.

The focal area has a typical elongated shape that can be seen in [23] and [24]. The next step in the direction of comparing numerical results with given clinical data is to consider changes in a material caused by the absorbed energy during the same calculation. It can include a change of material parameters due to a higher temperature or a change of the material model itself due to chemical reactions or a mechanical damage.

![Figure 3](image_url)

**Figure 3.** Energy absorption pattern for $\tau = 0.1$ s$^{-1}$ (on top) and $\tau = 10.0$ s$^{-1}$ (on bottom). From left to right – focal distance 1 mm, 5 mm, 10 mm, 20 mm, 50 mm and infinite.

3.3. Cylindrical inclusion in the middle of the specimen

Figure 4 shows the wave pattern for a specimen with a cylindrical inclusion in the center of the specimen for two types of inclusion material. As you can see, the inclusion in the middle of the specimen drastically changes the wave pattern.

![Figure 4](image_url)

**Figure 4.** Velocity modulus during the calculation for different types of inclusions: "bonelike" (on top) and "tumorlike" (on bottom). From left to right – consecutive time steps. Focal distance is 10 mm.

Figures 5 and 6 show the absorbed energy pattern for this statement. In both cases the inclusion blocks the ultrasound in terms of heating – although the wave pattern shows that some waves actually travel through and around, they are not concentrated enough to cause any noticeable heating.

Figure 5 also shows a non-uniform absorption pattern which is caused by the interference of direct and reflected waves. The reflected waves are hard to notice by the naked eye on figure 4, but even inclusions that are not highly contrasting and does not change the general shape of wavefronts reflect waves that can be noticed, for example, on the ultrasound transducer.

Figure 6 shows that an inclusion of a material with high absorption leads to a heating of this inclusion even the signal is focused in front of it and the background medium also has
a relatively high absorption. If the background medium has a low viscosity, all the energy absorption happens in the inclusion regardless of our attempts in focusing.

![Figure 5](image1.png)

**Figure 5.** Energy absorption pattern for $\tau = 0.1 \, s^{-1}$ (on top) and $\tau = 10.0 \, s^{-1}$ (on bottom). From left to right – focal distance 1 mm, 5 mm, 10 mm, 20 mm, 50 mm and infinite. Material of the inclusion is "bonelike".

![Figure 6](image2.png)

**Figure 6.** Energy absorption pattern for $\tau = 0.1 \, s^{-1}$ (on top) and $\tau = 10.0 \, s^{-1}$ (on bottom). From left to right – focal distance 1 mm, 5 mm, 10 mm, 20 mm, 50 mm and infinite. Material of the inclusion is "tumorlike".

### 3.4. Cylindrical inclusion on the side of the phased array

Figure 7 shows the wave pattern for a specimen with a cylindrical inclusion on a side from the phased array for two types of inclusion material. Despite the inclusion being on the side and not crossing the focal areas that appear without inclusion (figure 2), there is a noticeable difference in forms and positions of wavefronts.

![Figure 7](image3.png)

**Figure 7.** Velocity modulus during the calculation for different types of inclusions: "bonelike" (on top) and "tumorlike" (on bottom). From left to right – consecutive time steps. Focal distance is 10 mm.

Figures 8 and 9 show the absorbed energy pattern for this statement. The difference in elastic wave patterns leads to a considerable difference in absorbed energy patterns, the focal area is deformed and displaced.
4. Conclusions
In this research, we use the grid-characteristic numerical method on irregular grids. This implementation was designed for dynamic problems in solid mechanics and showed good results in impact damage modeling, composites ultrasound and medical diagnostic ultrasound.

This method was applied to the modeling of high-intensity focused ultrasound, and solutions of model statements show typical patterns that can be observed in experimental data from open access.

Numerical results also show that the presence of acoustically contrasting objects can affect the results of HIFU procedure. The slow "tumorlike" inclusion is highly heated in any statement, despite its position and focusing. Amplitude, frequency and length of the ultrasound pulse must be adjusted in a way that allows pulses to travel a long distance in soft tissues without being absorbed in upper layers. At the same time, it leads to a wider spreading of stray waves that can affect highly absorbing inclusions even if they are located on a distance from the ultrasound transducer. The "bonelike" inclusion is not heated due to the low absorption, but it changes the shape and position of focal area even if it does not cross the focal area of the phased array without inclusions. For example, this effect can complicate the HIFU procedure on objects that are located under ribs or close to them.

Generally, the modeling in biomechanics encounters a number of problems concerning material parameters. Any complication of a material model that is necessary for considering complex non-linear behavior leads to a necessity of measuring a set of parameters that may or may not have a direct physical meaning and be measurable in a simple and transparent way. This research is limited to a simple viscosity model, but it shows a qualitative correspondence with experimental data, and the method of measuring the absorbed energy allows to implement any complex viscosity model.

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References

[1] Tyshlek D, Aubry J F, Ter Haar G, Hananel A, Foley J, Eames M, Kassell N and Simonin H.H 2014 Focused ultrasound development and clinical adoption: 2013 update on the growth of the field *Journal of Therapeutic Ultrasound* 2

[2] Tempany C M C, McDannold N J, Hynynen K and Jolesz F A 2011 Focused Ultrasound Surgery in Oncology: Overview and Principles *Radiology* 259 pp 39–56

[3] Yang R, Reilly C R, Rescorla F J, et al. 1991 High-intensity focused ultrasound in the treatment of experimental liver cancer *Arch. Surg.* 126 pp 1002-9

[4] Lynn J G, Zwenner R L, Chick A J and Miller A E 1942 A new method for the generation and use of focused ultrasound in experimental biology *J. Gen. Physiol.* 26 pp 179–93

[5] Wood R W and Loomis A L 1927 The physical and biological effects of high frequency sound waves of great intensity *Philos. Mag. Ser. 7* 4 pp 417–436

[6] Lagneaux L, de Meulenaer E C, Delforge A, et al. 2002 Ultrasonic low-energy treatment: a novel approach to induce apoptosis in human leukemic cells *Exp. Hematol.* 30 pp 1293-301

[7] Beklemysheva K A, Danilov A A, Petrov I B, Salamatova V Yu, Vassilevski Yu V and Vasyukov A V 2015 Virtual blunt injury of human thorax: age-dependent response of vascular system *Russ. J. Numer. Anal. Math. Model.* 30 pp 259-68

[8] Mast T D, Hinkelman L M, Metlay A L, Orr M J and Waag R C 1999 Simulation of ultrasonic pulse propagation, distortion, and attenuation in the human chest wall *J. Acoust. Soc. Am.* 106 pp 3665-77

[9] Kazuyuki N and Naoyuki K 2012 3-D Modelings of an Ultrasonic Phased Array Transducer and Its Radiation Properties in Solid *Ultrasonic Waves* pp 59-80

[10] Fellinger P, Marklein R, Langenberg K J and Klaholz S 1995 Numerical modeling of elastic wave propagation and scattering with EFIT - elastodynamic finite integration technique *Wave motion* 21 pp 47–66

[11] Meaney P, Cahill M D and ter Haar G R 2000 The intensity dependence of lesion position shift during focused ultrasound surgery *Ultrason Med. Biol.* 26 pp 441–50

[12] Curra F P, Mourad P D, Khokhlova V A, Cleveland R O and Crum L A 2000 Numerical simulations of heating patterns and tissue temperature response due to high-intensity focused ultrasound *IEEE UFFC* 47 pp 1077–89.

[13] Lee K J 2005 Efficient ray tracing algorithms based on wavefront construction and model based interpolation method (Doctoral Dissertation. M. S., Texas A&M University)

[14] Vassilevski Yu V, Beklemysheva K A, Grigoriev G K, Kulberg N S, Petrov I B and Vasyukov A V 2017 Numerical modelling of medical ultrasound: phantom-based verification *Russian Journal of Numerical Analysis and Mathematical Modelling* 32 pp. 339-46

[15] Beklemysheva K A, Grigoriev G K, Kulberg N S, Petrov I B, Vasyukov A V and Vassilevski Yu V 2018 Numerical simulation of aberrated medical ultrasound signals *Russian Journal of Numerical Analysis and Mathematical Modelling* 33 pp 277-88

[16] Beklemysheva K A, Vasyukov A V, Kazakov A O and Petrov I B 2018 Grid-Characteristic Numerical Method for Low-Velocity Impact Testing of Fiber-Metal Laminates *Lobachevskii J. Math.* 39 pp 874–83

[17] Beklemysheva K A, Golubev V I, Petrov I B and Vasyukov A V 2021 Determining effects of impact loading on residual strength of fiber-metal laminates with grid-characteristic numerical method *Chinese Journal of Aeronautics* 34 pp 1-12

[18] Golubev V I 2019 The Usage of Grid-Characteristic Method in Seismic Migration Problems *Smart Innovation, Systems and Technologies* 133 pp 143–55

[19] Golubev V, Nikitin I and Ekimenko A 2020 Simulation of seismic responses from fractured MARMOUSI2 model *AIP Conference Proceedings* 2312 paper No 050006

[20] Golubev V, Shevchenko A and Petrov I 2020 Simulation of Seismic Wave Propagation in a Multicomponent Oil Deposit Model *International Journal of Applied Mechanics* 12 paper No 2050084

[21] Biryukov V A, Miyahara V A, Petrov I B and Khokhlov N I 2016 Simulation of Elastic Wave Propagation in Geological Media: Intercomparison of Three Numerical Methods *Computational Mathematics and Mathematical Physics* 56 pp 1086–95

[22] Beklemysheva K A and Petrov I B 2020 Numerical Modeling of High-Intensity Focused Ultrasound with Grid-Characteristic Method *Lobachevskii J. Math.* 41 pp 2638–47

[23] Chan A H, Fujimoto V Y, Moore D E, Martin R W and Vaezy S An image-guided high intensity focused ultrasound device for uterine fibroids treatment *Medical Physics* 29 2611

[24] Maxwell A, Sapozhnikov O, Bailey M et al. 2012 Disintegration of tissue using high intensity focused ultrasound: Two approaches that utilize shock waves *Acoustics today* 8 pp 24–36