We propose a phenomenological unified model for dark matter and dark energy based on an equation of state parameter $w$ that scales with the arctan of the redshift. The free parameters of the model are three constants: $\Omega_0, \alpha$ and $\beta$. Parameter $\alpha$ dictates the transition rate between the matter dominated era and the accelerated expansion period. The ratio $\beta/\alpha$ gives the redshift of the equivalence between both regimes. Cosmological parameters are fixed by observational data from Primordial Nucleosynthesis (PN), Supernovae of the type Ia (SNIa), Gamma-Ray Bursts (GRB) and Baryon Acoustic Oscillations (BAO). The calibration of the 138 GRBs events is performed using the 580 SNIa of the Union2.1 data set and a new set of 79 high-redshift GRBs is obtained. The various sets of data are used in different combinations to constraint the parameters through statistical analysis. The unified model is compared to the $\Lambda$CDM model and their differences are emphasized.

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## I. INTRODUCTION

The recent technological improvement in the space observations deeply altered Cosmology. In the end of the 90’s, the measurement of the luminosity distance of type Ia supernovae (SNIa) unveiled an accelerating cosmic expansion at recent times $[1,2]$. This result was later confirmed by other works using different sets of data $[3]$ such as the cosmic microwave background radiation (CMB) data $[4]$, baryon acoustic oscillations (BAO) $[5,6]$ and even the relatively recent Gamma-Ray burst (GRB) data $[8-10]$. As long as one assumes a homogeneous and isotropic cosmological background, the cosmic acceleration at low redshifts seems an indisputable observational truth $[11]$. The simplest theoretical way of describing cosmic acceleration is through the cosmological constant $\Lambda$, a negative energy density uniformly distributed throughout the cosmos. The resulting $\Lambda$CDM cosmological model $[18]$ is a plethora of cosmological models based on the same idea $[19]$; they are built either based on theoretical motivations $[20-30]$ or on phenomenological ones $[31-39]$. Our model is built on phenomenological grounds.

The UM is a dynamical model developed from a specific functional form chosen for the parameter $w$ of the equation of state $p = w\rho$, where $p$ is the pressure related to the cosmic component of density $\rho$: $w$ is given in terms of the arctan function. This way, the universe filled with the unified fluid passes smoothly from a matter-like behavior ($w \approx 0$) to a dark-energy-like dynamics ($w \approx -1$). This property is justified theoretically once the history of the universe demands a matter-dominated era with decelerated expansion ($2 \lesssim z \lesssim 10000$) followed by an accelerated period dominated by dark energy ($z \lesssim 2$) $[11]$. Our goal is to treat dark matter and dark energy on the same footing.

The unified scenario for the dark components is meaningful only if one can constraint the free parameters of the Unified Model by using a large number of observational data. For this end, we will use the already mentioned SNIa, BAO and GRB data plus information on the baryon density parameter $\Omega_0$ coming from primordial nucleosynthesis (PN) data $[4]$.

We used Union2.1 compilation $[40]$ for obtaining the distance modulus $\mu$ of the supernovae as a function of

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4. Inhomogeneous cosmological model, such as those in Refs. $[11-17]$, present alternative explanation to the apparent present-day cosmic acceleration.
their redshift $z$. For the GRB, we employed data in Ref. [11, which include 29 GRBs in addition to the set of 109 GRBs of Ref. [12]. Also, we payed special attention to the construction of the calibration curve of the GRB. The procedure involved an interpolation to the points in the plot of $\mu$ as function of $z$. We noticed that the common interpolation methods, such as linear and cubic interpolation techniques, are not the best-quality ones. In fact, Akima’s method [13] is the one which provides a curve that naturally connects the observational points without bumps or discontinuities. We devoted special care to the GRB data as they rise as new good candidates for standard candles at very high redshifts, with great potential of revealing additional cosmological information.

The paper is organized as follows. Sect. II presents our Unified Model (UM) for the dark sector of the universe; also, the basic equation of the $\Lambda$CDM model are reviewed. This prepares the ground for data fitting aiming to constraint the free parameters of both UM and $\Lambda$CDM. Sect. III discusses the cosmological data sets used for this end, and the statistical treatment is finally performed in Sect. IV. The physical consequences of the data fit for the various combinations of data (PN, SNIa, GRB and BAO) are addressed in Sect. IV E and further discussed in Sect. V where we also point out our final comments.

II. COSMOLOGICAL SET UP

This section presents the two cosmological models that are constrained by observational data in this paper. The first one is a phenomenological model that we call Unified Model (UM). The second one is the fiducial $\Lambda$CDM model, considered here for the sake of comparison.

A. Unified model

Our framework will be a flat universe filled with baryonic matter and a unified component of dark matter and dark energy. The Hubble function for this model is

$$H = H_0 \sqrt{\Omega_U (z, \alpha, \beta) + \Omega_{b0} (1 + z)^3} ,$$  

(1)

where $\Omega_U (z, \alpha, \beta)$ is the density parameter of the unified fluid,

$$\Omega_U (z) = \Omega_{U0} \exp \left\{ \int_0^z \frac{[1 + w_U (z')]}{1 + z'} dz' \right\} ,$$

which is subjected to the constraint

$$\Omega_{U0} + \Omega_{b0} = 1$$  

(2)

and depends on the redshift $z$ and three free parameter $\Omega_{b0}$, $\alpha$ and $\beta$ to be determined from adjustment to the available observational data.

The parameter $w_U$ of the equation of state is a function of $z$ and describes the transition from the matter dominated dynamics to the acceleration domination epoch. It is convenient to define

$$w_U = \frac{1}{\pi} \arctan (\alpha z - \beta) - \frac{1}{2}. $$  

(3)

The idea to propose a phenomenological parametrization which unifies the dark components is not new. For instance, in the works [36, 37] the authors use an expression exhibiting plots resembling those built with Eq. (3); however, there is an important conceptual difference between their reasoning and ours. Whereas in this paper we adopt a dynamical approach, the authors of [36, 37] use a kinematic one. The advantage of a kinematic model in which one chooses to parametrize the deceleration parameter $q$ in terms of the redshift $z$ – as that of Ref. [37] – is that very few assumptions on the nature of the dark components are taken a priori. On the other hand, dynamical models parameterizing $w (z)$ are more physical in the sense that they enable a meaningful perturbation theory (once they presuppose Einstein’s equation of gravity and standard cosmological assumptions).

Parameter $\alpha$ gives the transition rate between the decelerated expansion and the recent accelerated phase of the universe’s evolution. Parameter $\beta$ provides the value for $w_U$ today (null redshift). Moreover,

$$z_{eq} = \frac{\beta}{\alpha} $$  

(4)

is the redshift corresponding to the equivalence between the dark energy and the dark matter energy densities. This expression is obtained by taking $w_U = -1/2$, the average of the values $w = 0$ and $w = -1$.

Fig. 1a shows that the larger is $\alpha$ the greater is the transition rate (if $\beta$ is kept constant). Fig. 1b illustrates the fact that the value of the redshift of equivalence grows with $\beta$ (for a given $\alpha$).

A distinguishing feature of the unified model is the asymptotic behavior of $w_U$ in the future infinity, when the cosmic time $t$ diverges ($t \rightarrow \infty$), or equivalently, $z = -1$: for the concordance $\Lambda$CDM model, one gets $w_{eff} = -1$ whereas $w_U (z = -1) > -1$ [4].

B. $\Lambda$CDM

We shall fit the concordance $\Lambda$CDM model to the observations using the same data sets and techniques applied to our unified model for comparison.

The $\Lambda$CDM Hubble function for the flat universe is:

$$H (z) = H_0 \sqrt{\Omega_{b0} + \Omega_{\Lambda0} (1 + z)^3 + (1 - \Omega_{b0} - \Omega_{\Lambda0})} ,$$  

(5)

$\omega_{eff}$ is the ratiotion between the pressure and the energy density.
where $\Omega_{b0}$ is the density parameter for the baryonic matter and $\Omega_{d0}$ is the density parameter for the dark matter component. In the $\Lambda$CDM cosmology, the constant $\Omega_{\Lambda} = (1 - \Omega_{b0} - \Omega_{d0})$ is the energy density of the dark energy, interpreted as a cosmological constant. The term $\Omega_{\Lambda}$ overcomes the contribution coming from the dark matter content when it is greater than $\Omega_{b} = \Omega_{b0} (1 + z)^3$.

This occurs at

$$z_{eq} = \left( \frac{1 - \Omega_{b0} - \Omega_{d0}}{\Omega_{d0}} \right)^{1/3} - 1.$$

The above formulas will be useful in Sec. IV when we obtain parameters $\Omega_{b0}$ and $\Omega_{d0}$ using the observational data.

### III. COsmological DATA SETS

The free parameters in Eqs. (1) and (3) will be estimated using four different data sets: Primordial Nucleosynthesis, type Ia Supernovae, Baryon Acoustic Oscillations and Gamma-Ray Bursts.

#### A. Primordial Nucleosynthesis data

According to the Big Bang model, the nuclei of the light elements — hydrogen (H), deuterium (D), $^3$He, $^4$He and $^7$Li — were created in the first minutes of the Universe during a phase known as the primordial nucleosynthesis [44]. The abundances of these light elements depend on the present-day value of the baryon density parameter $\Omega_{b0}$ and on the Hubble constant $H_0$ [45]. In fact, it is possible to obtain $\Omega_{b0}h^2$ through a precise measurement of the primordial abundance ratio for any two light nuclei species.

Among those nuclei formed during the primordial nucleosynthesis, the simplest to be measure is the deuterium to hydrogen abundance ratio $(D/H)$ [46]. Ref. [47] suggests to determine this ratio using information from a special type of high-redshift quasar (QSO), more specifically through damped Lyman alpha systems (DLA) spectra [48, 49, 50]. The deuterium to hydrogen abundance ratio was given as $(D/H) = (2.335 \pm 0.05)$ by Ref. [45]. This result follows from the DLA QSO SDSS J1419+0829 spectrum. The above value for $(D/H)$ leads to

$$\Omega_{b0}h^2 = 0.0223 \pm 0.0009.$$  \hspace{1cm} (7)

Refs. [52] e [53] discuss measurements of the Hubble constant $H_0$ with a negligible dependence on the cosmological model. These two sources enable one to obtain the normalized Hubble constant $h$ in Eq. (7) as:

$$\begin{cases}
h_R = 0.738 \pm 0.028 & \text{(Riess)}; \\
h_F = 0.743 \pm 0.015(\text{sta}) \pm 0.021(\text{sys}) & \text{(Freedman)}. 
\end{cases}$$

(8)

On the order hand, the $H_0$ measured for Planck satellite [41] indicates $h = 0.673 \pm 0.012$. So, there is a noticeable $2.5\sigma$ discrepancy between the value for $h$ given by Riess and Freedmann and the one measure by Planck satellite. In this work we shall adopt a conservative stance and use $h = 0.74$. Nevertheless we use $\sigma_h = 0.07$ as uncertainty in order to accommodate Planck’s value with a confidence interval of 1$\sigma$. Using the data in (8) and Eq. (7), one can estimate the baryon density parameter and define the quantity

$$\chi^2_{PN} = \frac{(\Omega_{b0} - \Omega_{b0}^{PN})^2}{(\sigma_{\Omega_{b0}^{PN}})^2},$$

(9)

where $\Omega_{b0}^{PN} = (0.0407 \pm 0.0079)$.

\[\text{We emphasize that only the QSO with the characteristics discussed in [48, 49, 50] can be used to determine the abundance ratio (D/H).}\]
B. SNIa data

The supernovae are super-massive star explosions with intense luminosity. Among them, type Ia supernovae (SNIa) are the most important for cosmology since they can be taken as standard candles due to their characteristic luminosity curves.

In order to estimate the cosmological parameters of the unified model, we will employ the 580 SNIa compilation available in Ref. [40] by the Supernova Cosmology Project (SCP). Union2.1 data set presents the redshift $z$ of each supernova and the related distance modulus $\mu$ accompanied by its uncertainty $\sigma_\mu$.

The distance modulus is a logarithmic function of the luminosity distance $d_L$:

$$\mu = 5 \log_{10} \frac{d_L}{\text{Mpc}} + 25.$$  

It can be expressed in terms of

$$d_h \equiv \frac{H_0}{c} d_L = (1 + z) \int_0^z \frac{dz'}{E(z'; \vec{\theta})}$$ (10)

as

$$\mu \left( z; \vec{\theta}, \mathcal{M} \right) = 5 \log d_h \left( z; \vec{\theta} \right) + \mathcal{M},$$ (11)

where

$$H \left( z; H_0, \vec{\theta} \right) = H_0 E \left( z; \vec{\theta} \right)$$

is the Hubble function, $\mathcal{M}$ is a constant depending on the Hubble constant $H_0$, the speed of light $c$ and the absolute magnitude of the standard supernova in the regarded band [53]. $\vec{\theta}$ is the vector of parameters for the particular cosmological model under consideration. We shall not discuss the quantities encapsulated in $\mathcal{M}$ since they are not of our concern here; in fact, $\mathcal{M}$ will be marginalized in the statistical treatment of the data.

C. GRB data

The SNIa data provide us with reliable cosmological information till redshifts of the order of 1.7 (c.f. [55]). On the other hand, cosmic microwave background anisotropy measurements permit us to access information about the large scale universe at $z \sim 1000$ [56]. In between, there is a large redshift interval observationally inaccessible; scientific community is making great effort to collect astronomical data to fill in this gap. Perhaps the most promising candidates for this scope are the Gamma-Ray Bursts. It is expected that a fraction $\gtrsim 50\%$ of the observed GRB have $z > 5$ and the redshift values of these objects may be as large as 10 or even greater [57].

Even though we do not fully understand GRB emission mechanism, they are considered excellent candidates to standard candles because of their intense brightness [8, 58]. That is the reason why many authors have been proposing empirical luminosity correlation functions that standardize GRB as distance indicators [10, 59–61].

An additional problem to the use of GRB is the so called circularity problem. Unlike what happens in the supernovae case, there is no data set that is completely model independent and which could be used to calibrate GRBs distance curves [8, 9]. A number of different statistical methods were suggested to overcome this model dependence; e.g. see Refs. [8, 42, 62–68].

This work make use of the 138 Gamma-Ray Bursts compiled in Ref. [11]. They were calibrated according to the method described in Ref. [42], which tries to eliminate model dependence. Two different groups of GRB were considered: the low-redshift set has $z < 1.4$; the high-redshift one presents events with $z > 1.4$.

The distance modulus of the low-redshift GRB were determined using the SNIa data in the following way. We built the plot of the distance modulus versus the redshift for the 580 supernovae of the Union2.1 data set. The supernovae with the same redshift had their distance modulus values averaged. The points in the plot $z \times \mu$ were interpolated to provide a function $\mu = \mu(z)$ with domain $0 \leq z \leq 1.4$. There are many interpolation techniques such as linear, cubic and Akima’s interpolations [43]. We used these three methods and chose the last one for building the function $\mu(z)$ because Akima’s technique is the one giving a curve that intercepts the points in a more smooth and natural way – see Appendix A. With these SNIa low-redshift $\mu(z)$ it is possible to estimate the distance modulus of each one of the 59 low-redshift GRBs. These values of $\mu$ are then substituted in

$$\mu = 5 \log_{10} \frac{d_L}{\text{Mpc}} + 25$$ (12)

to give the associated luminosity distances $d_L$. They, in turn, appear in the expression for the isotropically radiated equivalent energy:

$$E_{\text{iso}} = \frac{4 \pi S_{\text{bolo}} d_L^2}{1 + z},$$ (13)

where $S_{\text{bolo}}$ is the GRB observed bolometric fluency. In the work [10], Amati noticed the correlation between the energy peak of the GRB spectrum ($E_p$) and the isotropically radiated energy ($E_{\text{iso}}$), formulating the equation:

$$\log_{10} \frac{E_{\text{iso}}}{\text{erg}} = \lambda + b \log_{10} \frac{E_{p,i}}{300 \text{ keV}},$$ (14)

which is known as the Amati’s relation. We determine parameters $\lambda$ and $b$ by using the low-redshift GRB data set, with the $E_{p,i}$ data available in Ref. [44] and the $E_{\text{iso}}$...
obtained from the SNIa calibration curve. Parameters $\lambda$ and $b$ are obtained from a linear fit to the Amati’s relation. The usual linear fit procedures in astronomy are the ordinary least-squares regression of the dependent variable $Y$ against the independent variable $X = \text{OLS}(Y|X)$ – and the ordinary least-squares of $X$ on $Y = \text{OLS}(X|Y)$. However, if there is a domain within which occurs an intrinsic scattering of the data with respect to the individual uncertainties, it is preferable to use the OLS bisector method, as described in Ref. [9]. Following the procedure in this reference, we performed linear regressions using the three methods above; the values obtained for the parameters $b$ and $\lambda$ of the Amati’s relation are displayed in Table I, the straight lines built from those parameters are show in Fig. 2.

| Method          | $b$    | $\sigma_b$ | $\lambda$ | $\sigma_\lambda$ |
|-----------------|--------|------------|-----------|------------------|
| OLS(X|Y)          | 1.564  | 0.084     | 52.74     | 0.06             |
| OLS(Y|X)          | 1.861  | 0.099     | 52.79     | 0.06             |
| OLS bissector   | 1.703  | 0.053     | 52.77     | 0.06             |

We decided to adopt the values of $\lambda$ and $b$ given by the OLS bisector method once the intrinsic dispersion of the data is dominant over the observational errors. Then, we calculated the quantity $\log_{10} E_{\text{iso}}$ for the high-redshift GRBs and their distance modulus

$$
\mu = \frac{5}{2} \log_{10} \frac{E_{\text{iso}}}{\text{erg}} + \frac{5}{2} \log_{10} \left( \frac{1+z}{4\pi S_{\text{bolo}}} \right) + 25
$$

with an associated uncertainty

$$
\sigma_{\mu} = \sqrt{\frac{5}{2} \sigma_{\log_{10} E_{\text{iso}}}^2 + \left( \frac{5}{2} \sigma_{S_{\text{bolo}}} \right)^2}.
$$

The uncertainty related to $\log_{10} E_{\text{iso}}$ is given by:

$$
\sigma^2_{\log_{10} E_{\text{iso}}} = \sigma^2_{\mu} + \left( \frac{b}{\ln 10} \frac{\sigma_{E_{p,i}}}{E_{p,i}} \right)^2 + \left( \frac{b}{\ln 10} \frac{\sigma_{E_{p,i}}}{E_{p,i}} \right)^2 + \sigma^2_{E_{\text{sys}}}.
$$

This equation is obtained from Amati’s relation through error propagation. We also added the contribution of the systematic error $\sigma_{E_{\text{sys}}}$ coming from extra dispersion in the luminosity relations. This systematic error is a free parameter and can be estimated by imposing $\chi^2_{\text{red}} = 1$ on the curve fitting to the luminosity plots. This was done in Ref. [9], and the value obtained is: $\sigma^2_{E_{\text{sys}}} = 0.39$.

After performing the GRB calibration using Union2.1 SNIa data, one obtains a set of values for the distance modulus $\mu (z)$ (and its uncertainty $\sigma_{\mu}$) for 79 high-redshift GRB. This set of values for $\mu (z) \pm \sigma_{\mu}$ is shown in Appendix B.

**D. BAO data**

Before the last scattering, the baryon-photon plasma weakly coupled oscillated due to a competition between the gravitational collapse and the radiation pressure [11]. According to [70], the velocity of the resulting sound waves in the plasma is $c_s = 1/\sqrt{3(1 + 3\rho_b/4\rho_T)}$. The stagnation of these waves after the decoupling lead to an increase of the baryon density at the scales corresponding to the decoupling covered by the acoustic wave until the decoupling time. This effect produces a peak of baryon acoustic oscillation (BAO) in the galaxy correlation function. BAO picks data present very small systematic uncertainties when compared to the other cosmological data sets [5] [71]. This is clearly an advantage to be used.

The baryon release marks the end of the Compton drag epoch and occurs at the redshift $z_{\text{drag}} \simeq 1059$ [4]. The sound horizon $r_s$ determines the location of the length scale of the BAO pick. It is given by:

$$
r_s (z_{\text{drag}}) = \frac{1}{\sqrt{3}} \int_{z_{\text{drag}}}^{\infty} \frac{c \, dz}{\sqrt{1 + \frac{3}{1+z} \frac{\Omega_{\text{b}}}{\Omega_0}}}.
$$

The original Hubble function of the unified model, Eq. (1), must be modified to

$$
\tilde{H} (z) = H_0 \left[ \Omega_0 (z, \alpha, \beta) + \Omega_{\text{b}} (1+z)^3 + \Omega_{\gamma,0} (1+z)^4 \right]^{1/2}.
$$

in order to include the radiation-like term $\Omega_{\gamma,0} (1+z)^4$. This is necessary here because we are dealing with the $z > 1000$, corresponding to the baryon-photon decoupling epoch, when the radiation was by no means negligible. The fact that $\lim_{z \to \infty} \tilde{H} (z) = \tilde{H} (z)$ guarantees that $\tilde{H} (z)$ describes the same unified model we have been discussing from the beginning of the paper.

For the sake of comparison, we shall study the sound horizon $r_s$ for the $\Lambda$CDM model. The Hubble function for this case is:

$$
\tilde{H} (z) = H_0 \left[ (1 - \Omega_{\text{b}} - \Omega_{\text{d}} - \Omega_{\gamma,0}) + (\Omega_{\text{b}} + \Omega_{\gamma,0}) (1+z)^3 + \Omega_{\gamma,0} (1+z)^4 \right]^{1/2}.
$$

The density parameter $\Omega_{\gamma,0}$ describes the contributions from the photons as well as that from the ultra-relativistic neutrinos. In accordance with [72] [73],

$$
\Omega_{\gamma,0} = \Omega_{\gamma,0} (1 + 0.227 \times N_{\text{eff}}),
$$

where $N_{\text{eff}} = 3.046$ is the effective number of neutrinos. The present-day value of the photon density parameter is $\Omega_{\gamma,0} = 5.46 \times 10^{-6}$, cf. Ref. [73].

When we substitute [19] into [18], the sound horizon turns out to be a function of the free parameters $\alpha$ and $\beta$.
where $d_s$ is the sound horizon, $D_A(z)$ is the angular diameter distance in the following way. BAO data allow us to obtain the angular diameter distance $D_A(z)$, achieved from the observation of the clustering perpendicular to the line of sight, and the Hubble function $H(z)$, measured through the clustering along the line of sight. However, $D_A(z)$ and $H(z)$ are not obtained independently, but through the distance scale ratio

$$d_z = \frac{r_s(z_{\text{drag}}, \bar{\theta})}{D_v(z, \bar{\theta})},$$

where

$$D_v(z, \bar{\theta}) = \left[ \left(1 + z \right)^2 D_A^2\left(z, \bar{\theta} \right) \frac{cz}{H\left(z', \bar{\theta} \right)} \right]^{1/3}$$

is the efective distance ratio, and

$$D_A\left(z, \bar{\theta} \right) = \frac{1}{(1 + z)} \int \frac{cdz'}{H\left(z', \bar{\theta} \right)}.$$

We perform a data fit to the seven values measured for distance scale ratio $d_z$ — see Table 1.

### IV. ANALYSIS AND RESULTS

In this section we will use the maximum likelihood method for estimating the three cosmological parameter of the unified model, namely $\alpha$, $\beta$ and $\Omega_{\Lambda0}$. Four different likelihood functions will be built considering the following data sets: (SNIa + PN); (SNIa + PN+BAO); (SNIa + PN + GRB); (SNIa + PN + GRB + BAO). Using the same data sets we also determine parameters $\Omega_{\Lambda0}$ and $\Omega_{b0}$ which characterize the ΛCDM model.

#### A. SNIa and PN data sets

Using (1) we define

$$\chi^2_{\text{SN+N}}\left(\bar{\theta}\right) = \chi^2_{\text{SN, m}}\left(\bar{\theta}\right) + \chi^2_{\text{PN}}$$

where the function $\chi^2_{\text{SN, m}}$ comes from the $\chi^2_{\text{SN}}$ of Union2.1 supernovae data,

$$\chi^2_{\text{SN}}\left(\bar{\theta}, \mathcal{M}\right) = \sum_{i=1}^{580} \frac{\left[\mu_i - 5 \log d_h\left(z_i; \bar{\theta}\right) - \mathcal{M}\right]^2}{\sigma_i^2 + \sigma_{\text{lens}}^2 + \sigma_{\mu_v}^2},$$

after analytic marginalization of the parameter $\mathcal{M}$ [77]. Notice that there are other sources of errors in addition to the uncertainties $\sigma_\mu$ in the distance modulus of each supernova [40]. In fact, we are taking into account the uncertainties $\sigma_{\text{lens}}$ due to gravitational lensing [78] and the uncertainties $\sigma_{\mu_v}$ related to peculiar velocities [79]. They are evaluated by the relations:

$$\sigma_{\text{lens}} = 0.093z \quad \text{and} \quad \sigma_{\mu_v} = \frac{2.172 \sigma_v}{z c},$$

where it is adopted $\sigma_v = 300 \text{ km s}^{-1}$. Uncertainty $\sigma_{\text{lens}}$ accounts for possible changes in the observed flux of a
supernova due to gravitational lensing [80]. The uncertainty \( \sigma_{\mu} \) is included in order to consider the peculiar velocities related to the host galaxy. This source of error is modeled according to Ref. [79].

Minimizing \( \chi^2_{S+N} (\hat{\theta}) \) for Unified Model (UM) and \( \Lambda \)CDM model we estimate the parameters \( \hat{\theta}_{\text{UM}} = (\Omega_{b0}, \alpha, \beta) \) and \( \hat{\theta}_{\Lambda \text{CDM}} = (\Omega_{b0}, \Omega_{\Lambda}) \). The values of the parameters for both models are found in Table III. The probability distribution functions and the confidence regions graphs (with 1, 2, 3 \( \sigma \) confidence levels) are displayed in Fig. 3. Using the Monte Carlo approach to perform the propagation of uncertainties [81], one obtains \( z_{\text{eq}, \text{UM}} = 0.368_{-0.065}^{+0.110} \) for the equivalence redshift of the Unified Model; on the other hand, \( z_{\text{eq}, \Lambda \text{CDM}} = 0.442_{-0.057}^{+0.063} \) is the equivalence redshift for the \( \Lambda \)CDM model.

The quantity \( \chi^2 \) for SNIa, GRB and PN data is:

\[
\chi^2_{S+N+G} (\hat{\theta}) = \chi^2_{S+N} + \chi^2_{\text{GRB,m}} ,
\]

where \( \chi^2_{\text{GRB,m}} \) is Eq. (28) with \( \mathcal{M} \) marginalized and \( \chi^2_{S+N} \) is given by Eq. (25).

Once again, the parameters of the unified model and \( \Lambda \)CDM model are obtained through minimizing Eq. (29). The best-fit values and single-parameter estimates are displayed in Table IV. Fig. 4 displays the confidence regions related to the two-parameter estimates with 1, 2, 3 \( \sigma \) confidence regions. These sets of data led to \( z_{\text{eq}, \text{UM}} = 0.372_{-0.066}^{+0.117} \) and

\[
z_{\text{eq}, \Lambda \text{CDM}} = 0.444_{-0.055}^{+0.062} \text{ for the dark components equivalent redshift associated to the UM and \( \Lambda \)CDM model respectively.}

### TABLE III. Parameters of the Unified Model (UM) and of the \( \Lambda \)CDM model obtained through the fits to SNIa and Primordial Nucleosynthesis data. The confidence contours are shown in Fig. 3.

| Model | Parameter Estimates | Single-Parameter Best-fit |
|-------|---------------------|---------------------------|
| UM    | \( \Omega_{b0} \)   | 0.0412_{-0.0079}^{+0.0079} | 0.0407 |
|       | \( \alpha \)        | 2.2_{-1.2}^{+1.8}         | 2.0    |
|       | \( \beta \)         | 0.96_{-0.37}^{+0.51}      | 0.91   |
|       | \( \chi^2_{\text{red}} \) | -                      | 0.886  |
| ACMD  | \( \Omega_{b0} \)   | 0.237_{-0.022}^{+0.023}   | 0.237  |
|       | \( \chi^2_{\text{red}} \) | -                      | 0.885  |

#### B. GRB, SNIa and PN data sets

In this section, we shall use as input to our statistical treatment three sets of data, namely Union2.1 data; the value of \( \Omega_{\text{PN}}^{\text{PN}} \) coming from primordial nucleosynthesis; and, the GRB data duly calibrated.

We will use the 79 high-redshift GRBs (Appendix B) and implement the same script of Sect. IV A in order to estimate the cosmological parameters. For doing so we define \( \chi^2_{\text{GRB}} \) as

\[
\chi^2_{\text{GRB}} (\hat{\theta}, \mathcal{M}) = \sum_{i=1}^{79} \frac{\left[ \mu_i - 5 \log d_h (z_i; \hat{\theta}) - \mathcal{M} \right]^2}{\sigma_i^2 + \sigma_{\mu_i}^2} . \tag{28}
\]

There are studies [9, 68, 82] indicating that the dispersion due to gravitational lensing are negligible for GRB. Therefore, the uncertainties \( \sigma_{\text{lens}} \) appearing in Eq. (27) are ignored here.

The quantity \( \chi^2 \) for SNIa, GRB and PN data is:

\[
\chi^2_{S+N+G} (\hat{\theta}) = \chi^2_{S+N} + \chi^2_{\text{GRB,m}} , \tag{29}
\]

where \( \chi^2_{\text{GRB,m}} \) is Eq. (28) with \( \mathcal{M} \) marginalized and \( \chi^2_{S+N} \) is given by Eq. (25).

Once again, the parameters of the unified model and \( \Lambda \)CDM model are obtained through minimizing Eq. (29). The best-fit values and single-parameter estimates are displayed in Table IV. Fig. 4 displays the confidence regions related to the two-parameter estimates with 1, 2, 3 \( \sigma \) confidence regions. These sets of data led to \( z_{\text{eq}, \text{UM}} = 0.372_{-0.066}^{+0.117} \) and

\[
z_{\text{eq}, \Lambda \text{CDM}} = 0.444_{-0.055}^{+0.062} \text{ for the dark components equivalent redshift associated to the UM and \( \Lambda \)CDM model respectively.}

### TABLE IV. Parameters of the UM and of the \( \Lambda \)CDM model obtained through the fits to SNIa, GRB and Primordial Nucleosynthesis data. The confidence contours are shown in Fig. 4.

| Model | Parameter Estimates | Single-Parameter Best-fit |
|-------|---------------------|---------------------------|
| UM    | \( \Omega_{b0} \)   | 0.0412_{-0.0079}^{+0.0079} | 0.0407 |
|       | \( \beta \)         | 0.93_{-0.35}^{+0.49}      | 0.88   |
|       | \( \chi^2_{\text{red}} \) | -                      | 0.871  |
| ACMD  | \( \Omega_{b0} \)   | 0.237_{-0.022}^{+0.023}   | 0.237  |
|       | \( \chi^2_{\text{red}} \) | -                      | 0.870  |

#### C. BAO, SNIa and PN data sets

The function \( \chi^2_{\text{BAO}} \):

\[
\chi^2_{\text{BAO}} (\hat{\theta}) = \sum_{i=1}^{79} \frac{1}{\sigma_i^2} \left[ \frac{r_s (z_{\text{drag}, \hat{\theta}})}{D_V (z_i, \hat{\theta})} \right]^2 , \tag{30}
\]

is calculated using the data in Table I. We add to this function the expression for \( \chi^2_{S+N} \), Eq. (29), so that we take into account BAO data together with SNIa and PN data sets. As a result we get \( \chi^2_{S+N+G} \):

\[
\chi^2_{S+N+G} (\hat{\theta}) = \chi^2_{\text{BAO}} + \chi^2_{S+N} . \tag{31}
\]

By minimizing (31), one finds the best-fit values and the single-parameter estimates shown in Table IV. Fig. 4 displays the confidence regions related to the two-parameter estimates. Through propagation of uncertainties by the Monte Carlo approach, one obtains \( z_{\text{eq}, \text{UM}} = 0.431_{-0.057}^{+0.064} \) for the redshift of equivalence according to the unified model. On the other hand, \( z_{\text{eq}, \Lambda \text{CDM}} = 0.454_{-0.057}^{+0.058} \) is the value for this equivalence redshift if one considers the \( \Lambda \)CDM best-fit values.
FIG. 3. Confidence regions considering SNIa and PN data sets for the UM and ΛCDM model.

FIG. 4. Confidence contours considering SNIa, PN and GRB data sets.

TABLE V. Parameters of the UM and of the ΛCDM model obtained through the fits to BAO, SNIa and Primordial Nucleosynthesis data. The confidence regions are shown in Fig. 5.

| Model | Parameter | Estimates | Single-Parameter Best-fit |
|-------|-----------|-----------|--------------------------|
| UM    | Ω₀₀      | 0.0400^{+0.0077}_{-0.0076} | 0.0412 |
|       | α        | 2.17^{+0.43}_{-0.35} | 2.15 |
|       | β        | 0.97^{+0.16}_{-0.15} | 0.95 |
|       | χ²_red   | - | 0.879 |
| ΛCDM  | Ω₀₀      | 0.0404^{+0.0053}_{-0.0051} | 0.0405 |
|       | Ω₀₀      | 0.234^{+0.022}_{-0.021} | 0.234 |
|       | χ²_red   | - | 0.877 |

D. BAO, SNIa, GRB and PN data sets

Our final statistical analyzes takes into account all the data sets: type Ia supernovae, primordial nucleosynthesis constraint, Gamma-Ray Bursts and baryonic acoustic oscillations. In this case, the complete χ² function is

$$\chi^2_{S+N+G+B} = \chi^2_{S+N+G} + \chi^2_{BAO},$$

where χ²_{S+N+G} and χ²_{BAO} are given by (29) and (30) respectively. By minimizing χ²_{S+N+G+B}, the best-fit parameter are obtained; these are shown in Table V. Fig. 6 show the PDF and confidence regions for the set of parameters. The probability distribution functions of the cosmological parameters provide $z_{\text{eq, UM}} = 0.432^{+0.064}_{-0.054}$ and $z_{\text{eq, ΛCDM}} = 0.459^{+0.062}_{-0.053}$ for the redshifts of equivalence in the context of UM and ΛCDM model.
FIG. 5. Confidence regions for SNIa, PN and BAO data sets.

FIG. 6. Confidence contours for all data sets (SNIa + PN + GRB + BAO).

TABLE VI. Parameters of the UM and of the ΛCDM model obtained through the fits to all sets of data (SNIa + PN + GRB + BAO). The confidence contours are shown in Fig.6

| Model | Parameter | Single-Parameter Estimates | Best-fit |
|-------|-----------|-----------------------------|----------|
| UM    | Ω_{b0}   | 0.0402 ± 0.0077             | 0.0414   |
|       | α        | 2.15 ± 0.44                | 2.14     |
|       | β        | 0.97 ± 0.16                | 0.95     |
|       | Χ^2_{red} | -                         | 0.864    |
| ΛCDM  | Ω_{b0}   | 0.0405 ± 0.0053            | 0.0405   |
|       | Ω_{d0}   | 0.234 ± 0.022              | 0.234    |
|       | Χ^2_{red} | -                         | 0.863    |

E. Discussion

All data set include SNIa and PN data. We shall take them as a basis for comparison with the results coming from the addition of GRB and BAO data.

Tables III and IV show the estimates for the parameters of our unified model and of ΛCDM model obtained with the sets (PN + SNIa) and (PN + SNIa + GRB). By comparing those values, we conclude that the inclusion of GRB data to the original set (PN + SNIa) causes little impact on the parameters estimates. For example, the discrepancy between \( z_{eq} \) with and without GRB is less than 2% while the 1σ relative corresponding uncertainties are at least of 17%. The double-parameter estimate for \( \alpha \) and \( \beta \) is also insensitive to the inclusion of GRB data to the set. Indeed, the 3σ-confidence region of the set (PN + SNIa + GRB) shown in Fig.7a is only slightly smaller than the one for the set (PN + SNIa).

The set including SNIa, PN and BAO is rather restric-
tive in comparison with the results obtained with SNIa and PN only. According to the values shown in Tables III and IV, by considering the BAO picks in the statistical treatment we reduced considerably the 1σ-confidence interval of the single-parameter estimates and the 3σ-confidence region of the two-parameter estimates (cf. Fig. 7b). Besides, the redshift of equivalence \( z_{eq} \) changed in a sensible way for Unified Model. Indeed, for the UM, we had \( z_{eq} = 0.368 \) before including BAO, and we obtained \( z_{eq} = 0.431 \) with the set (PN + SNIa + BAO).

Subsect. IV D deals with the statistical analysis performed using all sets of data. Once again, it is clear that the GRB data were not restrictive enough to modify significantly the confidence regions: compare the central regions in the plots of Figs. 7b and 7c. For convenience, these regions are displayed together in Fig. 7d.

Assuming that the set (PN + SNIa + GRB + BAO) gives the most realistic values for the cosmological parameters, we use the best-fit results for \( \alpha \) and \( \beta \) in Eq. (3) in order to obtain \( w_{\text{eff}} \) of the dark sector of the universe according to the Unified Model. We also obtain \( w_{\text{eff}} \) of the dark components in the \( \Lambda \)CDM model, using \( \Omega_{d0} = 0.23 \) e \( \Omega_{b0} = 0.04 \), for comparison. Both models are characterized by \( w_{\text{eff}} = w_{\text{dark}}(z) \) whose behavior are shown in Fig. 8a. In the distant future, one anticipates \( a \to \infty \), which implies \( z \to -1 \). The \( \Lambda \)CDM model gives \( w_{\text{dark}}(z = -1) = -1 \) while the UM leads to \( w_{\text{dark}}(z = -1) = -0.901 \). Notice that there are no big differences between the models in the region of small redshifts (0 \( \lesssim \) \( z \lesssim \) 0.5). In addition, the present-day values (\( z = 0 \)) are \( w_{\text{dark},0} = -0.760 \) for the \( \Lambda \)CDM model and \( w_{\text{dark},0} = -0.746 \) for the UM. The functions \( w_{\text{dark}}(z) \) of both models are equal in the region of \( z \approx 2.5 \) and slightly different elsewhere. Fig. 8b shows that the transition rates \( dw_{\text{dark}}/dz \) for the two models are well distinguished. The peak of the transition rate for the \( \Lambda \)CDM model is \( (dw_{\text{dark}}/dz)_{\text{max}} = 0.57 \) and occurs at \( z_{\text{max}} = 0.17 \). For the UM we get \( (dw_{\text{dark}}/dz)_{\text{max}} = 0.68 \) at the larger redshift of \( z_{\text{max}} = 0.45 \). Both model interchange the quality of being the one with the larger transition rate depending

![Figure 7](image)
V. FINAL COMMENTS

This work presented a cosmological model unifying dark matter and dark energy through a parametrization in terms of function $\arctan$. The three parameters of the model, $\alpha$, $\beta$ and $\Omega_{\text{tot}}$, were estimated admitting flat spacial curvature and using four observational data set, namely: Primordial Nucleosynthesis, SNIa, GRB and BAO. The same combination of data was employed to constraint the $\Lambda$CDM model. This was used as standard with respect to which our model was compared.

The results were analyzed in two distinct ways: (i) the influence of the inclusion of the GRB and BAO data in the estimates of the parameters of the UM and the $\Lambda$CDM model was discussed, and (ii) the direct comparison of UM and $\Lambda$CDM model was performed. In regard to point (i), it can be said that the inclusion of GRB data to the basic set (PN plus SNIa) barely modifies the confidence contours. In fact, there is no noticeable difference between the curves in Fig. 7a) even after increasing the number of GRB (by including 29 GRB to the set presented in [42]) and improving the interpolation technique of the calibration procedure. This indicates that, in spite of been promising as standard candles, GRB events are still not competitive in comparison to other sets of data such as the one for supernovae. Unlike the GRB data, the inclusion of BAO significantly restricts the parameter space; this is particularly true for the Unified Model (see Fig. 8b). With respect to point (ii), we can say that the UM and $\Lambda$CDM model exhibit statistically equivalent results for the baryon density $\Omega_{b}$ and $\Omega_{\text{tot}}$ occurs in their evolution toward the future, for $-1 < z < 0$. In fact, our parametrization leads to $\lim_{z \to -1} w_{\text{dark}} = -0.901$ and not to $\lim_{z \to -1} w_{\text{dark}} = -1$ as in the fiducial model. We can not affirm at the current stage of our investigation, if this difference between models is a physical effect due to the unification of the dark components in the UM or only an artifact of the parametrization for $w$ that we have chosen.

Future perspectives include two important subjects. The first concerns the dependence of the results on the specific parametrization for $w(z)$ chosen in our Unified Model. In particular, the arctan parametrization does not contain the $\Lambda$CDM model, i.e. there is no combination of the values of $\alpha$ and $\beta$ leading to the $w_{\text{dark}}$ of the $\Lambda$CDM model. This issue might be overcome by employing other parametrization such as one based on function $\tanh$. A second matter of investigation would be a possible UM-$\Lambda$CDM equivalence in a perturbative level. Indeed, could the statistical equivalence encountered in our data analysis (performed on the background) show up in a perturbative approach as well? These two question shall be addressed in further works.

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Appendix A: Akima’s interpolation method

Akima proposed in [43] a new interpolation technique aiming to overcome a difficulty shared by other interpolation methods, namely: the curve intercepting the data set does not present a natural evolution, as if it were drawn by hand. Typically, these other methods violate the continuity of the function or of its first-order derivative in some region of the domain; even if this flaw does not occur, the resulting curve presents undesirable oscillations or instabilities.

Ref. [43] establishes an interpolation method based in a piecewise function built with third-degree polynomials. The continuity of the composite function and its derivative are guaranteed by geometrical arguments. The slope $t$ of a given intermediate point among five neighboring points is calculated by

$$t = \frac{m_{2} |m_{4} - m_{3}| + m_{3} |m_{2} - m_{1}|}{|m_{4} - m_{3}| + |m_{2} - m_{1}|}, \quad (A1)$$

where $m_{i}$ is the slope of the straight line connecting the $i$-th point (among the five points of the set) to the $(i+1)$-th point. For instance, $m_{2}$ is the angular coefficient of the straight line connecting the second and third points. The slopes uncertainty can be easily obtained through the method of propagation of uncertainties.

By using Eq. (A1), one estimates the slopes for a set with $N$ points $(x_{i}, y_{i})$ except for the four points at the ends. Then, a third-degree polynomial is interpolated to the neighboring points respecting their coordinates and the determined slopes. Notice that by knowing the two coordinates and the two derivatives associated to a pair of points we are able to interpolate a third-degree polynomial, which has four degrees of freedom. However, we can not estimate the rate of change of the two last points at the ends using (A1). These extremal points are interpolated to their internal neighbors, whose coordinates and slopes are known.

Fig. 9 shows part of the interpolation curves built according to linear, cubic and Akima’s interpolation methods. The zoom includes Union2.1 data from redshift 1 to 1.4. The linear interpolation produces a curve connecting the points in a direct form; but the first-order derivative of the function describing the curve is not continuous at the points.
FIG. 8. Comparison between fiducial $\Lambda$CDM model ($\Omega_{b0} = 0.04; \Omega_{d0} = 0.23$) and the unified model with the best-fit values of parameters $\alpha, \beta$ and $\Omega_{b0}$ obtained with SNIa, GRB, BAO and PN data sets ($\alpha = 2.150; \beta = 0.9743; \Omega_{b0} = 0.04020$). Plots of (a) $\omega_{\text{dark}}$ and (b) of $d\omega_{\text{dark}}/dz$ with respect to redshift for both models.

On the other hand, cubic interpolation generates a smooth function with a continuous first-order derivative. However, huge instabilities and oscillations show up (such as those between point number 10 and point number 11 in the sample). This makes this method unsuited for the process of calibrating GRB curves.

For this end, Akima’s interpolation is the most adequate because it gives a smooth and continuous function; this function has continuous derivative; and, the interpolated function follows the natural tendency of the points and in between them (i.e. there are no spurious oscillations).

Appendix B: High-redshift GRB distance modulus

In subsection III C, we have calibrated the 79 high-redshift GRB compiled in Ref. [41]. Here, the results are presented in Table VII.

| $z$  | $\mu$ (MPc) | $\sigma_{\mu}$ (MPc) |
|------|-------------|----------------------|
| 1.44 | 43.68       | 1.02                 |
| 1.44 | 44.18       | 1.08                 |
| 1.46 | 44.41       | 1.00                 |
| 1.48 | 43.97       | 1.00                 |
| 1.49 | 45.43       | 1.12                 |
| 1.52 | 43.26       | 1.04                 |
| 1.55 | 44.48       | 1.04                 |
| 1.55 | 46.33       | 1.05                 |
| 1.56 | 43.15       | 1.77                 |
| 1.60 | 44.60       | 1.13                 |
| 1.60 | 47.03       | 1.04                 |
| 1.61 | 47.38       | 1.13                 |
| 1.62 | 44.77       | 1.02                 |
| 1.64 | 45.31       | 1.01                 |
| 1.71 | 47.45       | 1.66                 |
| 1.73 | 43.64       | 1.05                 |

Continued on next page.
| $z$ | $\mu$ (MPc) | $\sigma_{\mu}$ (MPc) |
|-----|--------------|----------------------|
| 1.80 | 45.86        | 1.04                 |
| 1.82 | 45.25        | 1.00                 |
| 1.90 | 46.25        | 1.19                 |
| 1.95 | 46.95        | 1.16                 |
| 1.97 | 45.07        | 1.06                 |
| 1.98 | 44.94        | 1.08                 |
| 2.07 | 44.35        | 1.03                 |
| 2.10 | 47.16        | 1.37                 |
| 2.11 | 47.42        | 1.01                 |
| 2.14 | 45.19        | 1.03                 |
| 2.15 | 47.83        | 1.15                 |
| 2.20 | 46.81        | 1.17                 |
| 2.22 | 45.32        | 1.18                 |
| 2.30 | 45.91        | 1.22                 |
| 2.30 | 46.59        | 1.31                 |
| 2.35 | 47.27        | 1.22                 |
| 2.35 | 46.74        | 1.36                 |
| 2.43 | 46.82        | 1.06                 |
| 2.43 | 47.35        | 1.18                 |
| 2.45 | 47.86        | 1.21                 |
| 2.51 | 46.92        | 1.05                 |
| 2.58 | 45.55        | 1.03                 |
| 2.59 | 46.62        | 1.04                 |
| 2.61 | 46.32        | 1.07                 |
| 2.65 | 46.02        | 1.07                 |
| 2.69 | 46.44        | 1.12                 |
| 2.71 | 45.27        | 1.33                 |
| 2.75 | 45.85        | 1.13                 |
| 2.77 | 45.99        | 1.00                 |
| 2.82 | 47.05        | 1.01                 |
| 2.90 | 45.73        | 1.11                 |
| 3.00 | 46.63        | 1.18                 |
| 3.04 | 46.55        | 1.03                 |
| 3.04 | 45.38        | 1.25                 |
| 3.08 | 47.55        | 1.20                 |
| 3.20 | 46.23        | 1.18                 |
| 3.21 | 45.96        | 1.19                 |
| 3.34 | 47.49        | 1.06                 |
| 3.35 | 48.09        | 1.03                 |
| 3.36 | 45.82        | 1.04                 |
| 3.37 | 47.81        | 1.32                 |
| 3.42 | 47.45        | 1.07                 |
| 3.43 | 47.18        | 1.02                 |
| 3.53 | 47.15        | 1.03                 |
| 3.57 | 46.35        | 1.06                 |
| 3.69 | 45.74        | 1.07                 |
| 3.78 | 49.24        | 1.41                 |
| 3.91 | 46.71        | 1.18                 |
| 4.05 | 48.52        | 1.04                 |
| 4.11 | 47.39        | 1.27                 |
| 4.27 | 48.13        | 1.23                 |
| 4.35 | 47.57        | 1.10                 |
| 4.41 | 48.47        | 1.07                 |
| 4.50 | 46.55        | 1.29                 |
| 4.90 | 47.43        | 1.25                 |
| 5.11 | 48.67        | 1.07                 |
| 5.30 | 47.89        | 1.05                 |
| 5.60 | 48.45        | 1.02                 |

Continued on next page.
TABLE VII – continued from previous page

| z    | μ (MPC) | σ_μ (MPC) |
|------|---------|-----------|
| 6.29 | 50.02   | 1.20      |
| 6.70 | 50.27   | 1.39      |
| 8.10 | 49.75   | 1.29      |

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