\[ \text{NN} \rightarrow \text{NN}\pi \] from the effective field theory point of view: short comings and gains

C. Hanhart

Department of Physics, University of Washington,
Box 351560, Seattle, WA 98195-1560

Based on a counting scheme for the application of chiral perturbation theory to pion production in NN collisions p-wave production is studied. It is demonstrated that, contrary to the s-wave production where loops enter at too low an order to allow the scheme predictive power, there are certain p-wave observables where parameter free predictions are possible. These predictions are consistent with experimental data. We show that the investigation of charged pion production yields a constraint on a contribution to the three-nucleon force with a spin-isospin structure that could resolve the \( A_y \) puzzle in \( nd \) scattering.

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In the last few years, the various \( \text{NN} \rightarrow \text{NN}\pi \) reactions have been studied both experimentally and theoretically with a focus on near-threshold energies. However, although the first low energy data where published a decade ago, the s-wave pion production is still not fully understood [2].

The pion dynamics are largely controlled by chiral symmetry constraints, and the hope that the use of Chiral Perturbation Theory (\( \chiPT \)) would yield insights led to the use of tree-level \( \chiPT \) to calculate the cross sections close to threshold [4, 5, 6, 7, 8]. The scale of typical internal momenta is set by the initial nucleon momentum \( p_i \geq \sqrt{M_N m_\pi} \). Ref. [4] emphasized that the diverse contributions to the \( Ss \) (capital letters denotes the \( NN \) angular momentum, whereas small letters denote the relative angular momentum of the pion with respect to the \( NN \) center of mass) final states thus have to be ordered in powers of \( \sqrt{m_\pi/M_N} \). Because of this large scale the static approximation cannot be employed for nucleons. More generally, as was pointed out in Ref. [4, 9], there is a mix among interactions of different chiral indexes.

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Let us begin with a short discussion of s-wave production. Due to a lack of space only $\pi^0$ production will be looked at here. At leading order ($\mathcal{O}(\sqrt{m_\pi M_N})$) direct emission off the nucleon and the delta contribute (c.f. Fig. 1i & ii). Unfortunately the leading order turns out to be small for two reasons that are not under control of the power counting: i) a cancelation of the nucleonic and delta diagram and ii) the presence of a zero in the half-off-shell NN T-matrix in the $^1S_0$ partial wave at off-shell momenta $p_{off} \approx p_i$ for small on shell momenta $1$. At next-to leading order ($\mathcal{O}(\sqrt{m_\pi M_N}^2)$) already loops enter. Some of those are depicted in figure 1. This explains the discrepancy of the results of [5, 4] and the data. A check of the convergence of the series for the s-waves is thus technically difficult.

The situation is much more pleasant for the p-wave production as was pointed out in ref. [10]. Let us do the expansion at values of typical external momenta $q_{ex} = m_\pi$. First of all the expansion starts at $\mathcal{O}(1)$, since the leading operators lead to p-wave pions. Secondly, the next non vanishing order ($\mathcal{O}(\sqrt{m_\pi M_N}^2)$) is free of loops, since for p-wave production not all the pion momenta in the loop can be internal ones. Thus the order of the loops gets enhanced by a factor of $q_{ex}/p_{typ} = m_\pi/\sqrt{m_\pi M_N}$ compared to the s-wave case. In figure 2 we show the complete set of diagrams contributing to the p-wave $\pi^0$ production up to next-to-leading order. Thus we can employ chiral perturbation theory efficiently in the study of pion production.

The Lagrangian relevant for the present investigation is given in ref. [10]. As usual in effective field theories a number of parameters appear whose strength is not constrained by symmetry. In our case these are, besides the $\pi$-nucleon coupling constant and $f_\pi$, the parameters in front of terms with two pion fields and two nucleon fields ($c_3$ and $c_4$, which where fixed in $\pi N$ scattering [11]) and two further parameters in front of terms containing four nucleon fields and one pion field – $d_1$ and $d_2$. These are as

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1 That is true for all so called realistic potentials we are aware of.
yet unknown. It was pointed out in ref. [12] that the latter structures can lead to a significant contribution to the three nuclear force. As will be shown below \( \pi \)-production in \( NN \) collisions can constrain the linear combination \( d = d_1 + 4d_2 \) and thus will help us to get a consistent understanding of low energy nuclear physics.

Before we move on, we first have to show that the series converges, as claimed above. The crucial observation to make this test possible is, that the short range \( d_i \) terms only support \( S \to S'p \) transitions. We can therefore make parameter free predictions up to \( \mathcal{O}(m_\pi/m_N) \) for all those observables where the lowest \( \pi \) partial wave allowed is \( p \) and not both \( NN \) states are in a relative S-wave. Such an observable exists, namely the \( ^3\sigma_1 \) (we use the notation of ref. [13]: \( ^{2S+1}\sigma_m \)) cross section in neutral-pion production with \( Pp \) as the lowest partial waves contributing. This observable was recently measured at IUCF [13]. It was found in ref. [10] that indeed there is reasonable convergence in the p-wave production.

Thus we can move on to calculate an observable that is sensitive to the short range operators proportional to \( d \). Note that the effective field theory not only constrains the possible coupling structures but also gives an order of magnitude estimate for the low energy constants. We can thus rewrite \( d = \delta/(f_\pi^2M_N) \), where we expect \( \delta \) to be a number of order 1. The desired observable is the amplitude of the \(^1S_0 \to ^3S_1p \) transition \( (a_0) \), recently extracted from data of the reaction \( pp \to pn\pi^+ \) [14]. In Fig. 3 we show the sensitivity of \( a_0 \) to the strength of the short range interactions. Obviously, with better data pion production can put a strong bound at a possible three body force originated in the \( d_i \) terms.

Based on the convergence of the chiral expansion in p-wave pion production we demonstrated that data on this reaction can be used to extract information about the three-nucleon force. It is clear that more accurate data would be very useful. We find it very gratifying that chiral symmetry provides a direct connection between pion production at energies \( \sim 350 \) MeV (IUCF) and \( Nd \) scattering at energies \( \sim 10 \) MeV (Madison, TUNL).
Fig. 3. $a_0$ of $pp \to np\pi^+$ in chiral perturbation theory. The different lines correspond to values of the parameter related to the three-nucleon force: $\delta = 1$ (long dashed line), $\delta = 0$ (dot-dashed line), $\delta = -0.2$ (solid line), and $\delta = -1$ (short dashed line). Data are from Ref. [14].

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