Role of Four Gravitational Constants in Nuclear Structure

U V S Seshavatharam* and S Lakshminarayana†

Abstract

This paper attempts to understand the role of the four gravitational constants in the nuclear structure which helps in understanding the nuclear elementary charge, the strong coupling constant, nuclear charge radii, nucleon magnetic moments, nuclear stability, nuclear binding energy and Neutron life time. The three assumed atomic gravitational constants help in understanding neutron-proton stability. Electromagnetic and nuclear gravitational constants play a role in understanding proton-electron mass ratio, Bohr radius and characteristic atomic radius. With reference to the weak gravitational constant, it is possible to predict the existence of a weakly interacting fermion of rest energy 585 GeV, called Higg’s fermion. Cosmological ‘dark matter’ research and observations can be carried out in this direction also.

Keywords: Four Gravitational Constants, Nuclear Structure, Higgs’s Fermion

1. Introduction

The most desirable cases of any unified description are:
   a) to implement gravity in microscopic physics and to estimate the magnitude of the Newtonian gravitational constant ($G_N$)
b) to develop a model of microscopic quantum gravity  
c) to simplify the complicated issues of known physics  
d) to predict new effects arising from a combination of the fields inherent in the unified description

In this context, with respect to the available literature pertaining to large gravitational coupling constants \([1-6]\), we propose the existence of four different gravitational constants assumed to be associated with the observed four fundamental interactions and study their possible role in understanding nuclear stability and binding energy \([7-12]\) for light, medium, heavy and super heavy atomic nuclides. Even though our approach to nuclear physics is speculative, proposed assumptions and relations show a wide range of applications embedded with in-depth low energy nuclear physics, high energy nuclear physics, and final unification.

### 2. Four Assumptions

With reference to recent paper publications and conference proceedings \([13-30]\), we propose the following four assumptions:

1) There exist four different gravitational constants associated with gravitational, weak, electromagnetic and strong interactions.

2) The nuclear gravitational constant \(G_s\) is very large in such a way that,

\[
R_0 \approx \frac{2G_s m_p}{c^2}
\]  

(1)

3) Strong coupling constant \([31,32]\) can be expressed with,

\[
\alpha_s \approx \left( \frac{\hbar c}{G_s m_p^2} \right)^2
\]  

(2)

4) There exists a strong elementary charge in such a way that,

\[
e_s \approx \left( \frac{G_s m_p^2}{\hbar c} \right) e \cong \frac{e}{\sqrt{\alpha_s}}
\]  

(3)
3. To Fix the Magnitudes of \((G_s, \alpha_s\text{ and } e_s)\)

Considering neutron, proton and electron rest masses, and based on the relation (11), the proposed nuclear gravitational constant can be estimated. Further, on the basis of that, other values can be estimated.

\[
\begin{align*}
G_s &\approx 3.3293665 \times 10^{28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \\
R_0 &\approx \frac{2G_s m_p}{c^2} \approx 1.2392185 \text{ fm} \\
\alpha_s &\approx 0.1152072 \\
e_s &\approx 4.7203105 \times 10^{-19} \text{ C}
\end{align*}
\]

(4)

4. Interplay Among the Four Gravitational Constants

According to Roberto Onofrio [5], electroweak scale gravitational constant is roughly \(10^{33}\) times the Newtonian gravitational constant.

Let, Weak gravitational constant= \(G_w\)

Electromagnetic gravitational constant = \(G_e\)

Newtonian gravitational constant = \(G_N\)

We noticed that,

\[
\frac{m_p}{m_e} \approx \left( \frac{G_s m_p^2}{hc} \right) \left( \frac{G_e m_e^2}{hc} \right) \approx \left( \frac{e_s G_s}{eG_N} \right)^{\frac{1}{12}}
\]

(5)

\[
\frac{m_p}{m_e} \approx \left( \frac{G_s}{G_N^{2/3} G_e^{1/3}} \right)^{\frac{1}{7}}
\]

(6)

\[
\frac{G_w}{G_s} \approx \frac{G_m e^2}{hc}
\]

(7)

\[
\frac{G_w}{G_N} \approx \left( \frac{m_p}{m_e} \right)^{10}
\]

(8)
\[
\frac{G_s^2}{G_e G_w} \approx \frac{G_s m_p m_e}{\hbar c} \quad (9)
\]

By knowing the magnitudes of \( G_s \) and \( \left( \frac{m_p}{m_e} \right) \), \( (G_e,G_w,G_N) \)
can be estimated. Based on the proposed relations (5 to 9),
\[
G_s \approx 6.679077 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \\
G_e \approx 2.374474 \times 10^{-7} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \\
G_w \approx 2.909406 \times 10^{-12} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} 
\quad (10)
\]

5. New Concepts and Semi-Empirical Relations

It can be suggested that:

1) Fine structure ratio can be addressed with,
\[
\alpha \approx \left( \frac{e_s^2}{4\pi\varepsilon_0 G_s m_p^2} \right) \frac{\hbar c}{G_s m_p^2} \approx 7.297352533 \times 10^{-3}
\]

2) Proton magnetic moment can be addressed with
\[
\mu_p \approx \frac{e \hbar}{2m_p} \approx \frac{e G_s m_p}{2c} \approx 1.488055 \times 10^{-26} \text{ J.T}^{-1}
\]

3) Neutron magnetic moment can be addressed with
\[
\mu_n \approx \frac{(e_s - e) \hbar}{2m_n} \approx 9.816235 \times 10^{-27} \text{ J.T}^{-1}.
\]

4) Nuclear unit radius can be expressed
as, \( R_o \approx \frac{2G m_p}{c^2} \approx \left( \frac{e_s}{e} \right) \left( \frac{\hbar}{m_p c} + \frac{\hbar}{m_n c} \right) \)

5) Root mean square nuclear charge radii [33] can be addressed with,
\[
R_{(Z,A)} \approx \left\{ 1 - 0.349 \left( \frac{N - Z}{N} \right) \right\} N^{1/3} \times 1.262 \text{ fm} \\
\approx \left\{ Z^{1/3} + \left( \sqrt{Z(A - Z)} \right)^{1/3} \right\} \left( \frac{G_s m_p}{c^2} \right)
\]
6) Nuclear potential energy can be understood with,
\[
\frac{e^2}{4\pi \varepsilon_0 \left( G_m c^2 \right)} \approx 20.17225 \text{ MeV}
\]

7) Close to stable mass numbers, nuclear binding energy can be understood with a single energy co-efficient \[29\],
\[
\frac{e^2 G m_p^3}{8 \pi \varepsilon_0 \hbar} \approx \frac{e e}{8 \pi \varepsilon_0 \left( \hbar / m_p c \right)} \approx \frac{e^2 \varepsilon_0}{8 \pi \varepsilon_0 \left( G m_p c^2 \right)} \approx 10.086124 \text{ MeV}
\]

8) With reference to \( (h/2) \), useful quantum energy, a constant can be expressed with,
\[
E_{(h/2)} \approx \left( \frac{e^2 G m_p^3}{4 \pi \varepsilon_0 \left( h/2 \right)^2} \right) \approx 80.6889925 \text{ MeV}
\]

9) Close to magic and semi-magic proton numbers \[29\], nuclear binding energy seems to approach
\[
\left[ 2.531 \left( n + \frac{1}{2} \right) \right]^2 10.09 \text{ MeV where } n = 0, 1, 2, 3, \ldots \text{ and }
\left( m_p - m_p / m_p \right) = 2.531.
\]

10) The characteristic melting temperature associated with a proton can be expressed with,
\[
T_{proton} \approx \frac{\hbar c^3}{8 \pi k_B G_s m_p} \approx 0.15 \times 10^{12} \text{ K}
\]

11) Characteristic nuclear-neutral mass unit \[30\] can be addressed with,
\[
\sqrt{\frac{\hbar c}{G_s}} \approx 546.6365 \text{ MeV/c}^2.
\]

12) Fermi’s weak coupling constant \[5,32\] can be addressed with,
\[
G_w \approx \frac{\hbar c}{4G \left( \frac{h}{c^3} \right)} \approx \frac{4G \hbar}{c^3} \text{ where } \sqrt{\frac{4G \hbar}{c^3}} \text{ can be called as the Schwarzschild radius of weak scale Planck mass,}
\]
\[
\sqrt{\frac{\hbar c}{G_w}} \approx 584.983 \text{ GeV/c}^2
\]
13) Bohr radius of a Hydrogen atom can be addressed with,

\[ a_0 \approx \left( \frac{4\pi\varepsilon_0 G_n m_e}{e^2} \right) \left( \frac{G_n m_e}{c^2} \right) \approx 5.297 \times 10^{-11} \text{ m} \]

14) Characteristic atomic radius can be addressed with,

\[ R_{\text{atom}} \approx \sqrt[2]{\frac{G_n G_m}{c^2}} \approx 33.094 \text{ pico meters.} \]

6. To Fit Neutron-Proton Mass Difference

Neutron-proton mass difference can be understood with:

\[ \left( \frac{m_n c^2 - m_p c^2}{m_e c^2} \right) \ln \frac{E_{(h/2)}}{m_e c^2} \approx \ln \frac{4e^2 G_n m_p^3}{\varepsilon_0 h^2 m_e c^2} \]

\[ (11) \]

7. To Fit Neutron Life Time

The Neutron lifetime \( t_n \) can be understood with the following relation:

\[ t_n \approx \exp \left( \frac{E_{(h/2)}}{(m_n - m_p)c^2} \right) \times \left( \frac{\hbar}{m_e c^2} \right) \approx 871.62 \text{ sec} \]

\[ (12) \]

This can be compared with the recommended value [32] of the Neutron lifetime, (880.2±1.0) sec

8. Understanding Proton-Neutron Stability with Three Atomic Gravitational Constants

Let,

\[ s \approx \left[ \left( \frac{e}{m_p} \right) \pm \left( \frac{e}{m_e} \right) \right] \approx \left( \frac{G_s m_p m_e}{\hbar c} \right) \]

\[ \approx \frac{G_s^2}{G_s G_w} \approx 0.00160454 \]

\[ (13) \]

Nuclear beta stability line can be addressed with a relation of the form [relation 8 of reference 9].
\[ A_s \approx 2Z + s(2Z)^2 \approx 2Z + (4s)Z^2 \]
\[ \approx 2Z + 0.00642Z^2 \approx Z(2 + kZ) \]  \hspace{1cm} (14)

where \( k \approx 4s \approx 0.00642 \)

By considering a factor like \( 2\sqrt{k} \), likely possible range of \( A_s \) can be addressed with,

\[
\begin{align*}
A_s & \approx Z[(2 \pm 0.08) + kZ] \\
\left\{ \begin{array}{l}
(A_s)_{\text{lower}} \approx Z(1.92 + kZ) \\
(A_s)_{\text{mean}} \approx Z(2.0 + kZ) \\
(A_s)_{\text{upper}} \approx Z(2.08 + kZ)
\end{array} \right.
\end{align*}
\]  \hspace{1cm} (15)

See Table-1. An interesting point to be noted is that, for \( Z=112,113 \) and 114, estimated lower stable mass numbers are 296, 299, and 302 respectively. Corresponding neutron numbers are 184, 186, and 188. These neutron numbers are very close to the currently believed shell closure at \( N=184 \). It needs further study [33].

**Table 1:** Likely Possible Range of \( A_s \) for \( Z=5 \) to 118

| Proton number | \( (A_s)_{\text{lower}} \) | \( (A_s)_{\text{mean}} \) | \( (A_s)_{\text{upper}} \) |
|---------------|-----------------|-----------------|-----------------|
| 5             | 10              | 10              | 11              |
| 6             | 12              | 12              | 13              |
| 7             | 14              | 14              | 15              |
| 8             | 16              | 16              | 17              |
| 9             | 18              | 19              | 19              |
| 10            | 20              | 21              | 21              |
| 11            | 22              | 23              | 24              |
| 12            | 24              | 25              | 26              |
| 13            | 26              | 27              | 28              |
| 14            | 28              | 29              | 30              |
| 15            | 30              | 31              | 33              |
| 16            | 32              | 34              | 35              |
| 17            | 34              | 36              | 37              |
| 18            | 37              | 38              | 40              |
| 19            | 39              | 40              | 42              |
| 20            | 41              | 43              | 44              |
| 21            | 43              | 45              | 47              |
| 22            | 45              | 47              | 49              |
| 23            | 48              | 49              | 51              |
|   |   |   |   |
|---|---|---|---|
|24 |50 |52 |54 |
|25 |52 |54 |56 |
|26 |54 |56 |58 |
|27 |57 |59 |61 |
|28 |59 |61 |63 |
|29 |61 |63 |66 |
|30 |63 |66 |68 |
|31 |66 |68 |71 |
|32 |68 |71 |73 |
|33 |70 |73 |76 |
|34 |73 |75 |78 |
|35 |75 |78 |81 |
|36 |77 |80 |83 |
|37 |80 |83 |86 |
|38 |82 |85 |88 |
|39 |85 |88 |91 |
|40 |87 |90 |93 |
|41 |90 |93 |96 |
|42 |92 |95 |99 |
|43 |94 |98 |101 |
|44 |97 |100 |104 |
|45 |99 |103 |107 |
|46 |102 |106 |109 |
|47 |104 |108 |112 |
|48 |107 |111 |115 |
|49 |109 |113 |117 |
|50 |112 |116 |120 |
|51 |115 |119 |123 |
|52 |117 |121 |126 |
|53 |120 |124 |128 |
|54 |122 |127 |131 |
|55 |125 |129 |134 |
|56 |128 |132 |137 |
|57 |130 |135 |139 |
|58 |133 |138 |142 |
|59 |136 |140 |145 |
|60 |138 |143 |148 |
|61 |141 |146 |151 |
|62 |144 |149 |154 |
|63 |146 |151 |157 |
|64 |149 |154 |159 |
| 65  | 152 | 157 | 162 |
|-----|-----|-----|-----|
| 66  | 155 | 160 | 165 |
| 67  | 157 | 163 | 168 |
| 68  | 160 | 166 | 171 |
| 69  | 163 | 169 | 174 |
| 70  | 166 | 171 | 177 |
| 71  | 169 | 174 | 180 |
| 72  | 172 | 177 | 183 |
| 73  | 174 | 180 | 186 |
| 74  | 177 | 183 | 189 |
| 75  | 180 | 186 | 192 |
| 76  | 183 | 189 | 195 |
| 77  | 186 | 192 | 198 |
| 78  | 189 | 195 | 201 |
| 79  | 192 | 198 | 204 |
| 80  | 195 | 201 | 207 |
| 81  | 198 | 204 | 211 |
| 82  | 201 | 207 | 214 |
| 83  | 204 | 210 | 217 |
| 84  | 207 | 213 | 220 |
| 85  | 210 | 216 | 223 |
| 86  | 213 | 219 | 226 |
| 87  | 216 | 223 | 230 |
| 88  | 219 | 226 | 233 |
| 89  | 222 | 229 | 236 |
| 90  | 225 | 232 | 239 |
| 91  | 228 | 235 | 242 |
| 92  | 231 | 238 | 246 |
| 93  | 234 | 242 | 249 |
| 94  | 237 | 245 | 252 |
| 95  | 240 | 248 | 256 |
| 96  | 243 | 251 | 259 |
| 97  | 247 | 254 | 262 |
| 98  | 250 | 258 | 265 |
| 99  | 253 | 261 | 269 |
| 100 | 256 | 264 | 272 |
| 101 | 259 | 267 | 276 |
| 102 | 263 | 271 | 279 |
| 103 | 266 | 274 | 282 |
| 104 | 269 | 277 | 286 |
| 105 | 272 | 281 | 289 |
9. **Nuclear Binding Energy at Stable Mass Numbers**

Important points to be noted are:

1. With reference to electromagnetic interaction, and based on proton number, \((1/\alpha_s) \approx 8.68\) can be considered as the maximum strength of nuclear binding energy.

2. \(Z \approx 30\) seems to represent a characteristic reference number in understanding the nuclear binding of light and heavy atomic nuclides.

Based on these points, at stable mass numbers of \(Z\), nuclear binding energy can be expressed by the following simple empirical relation.

\[
(B)_{A_s} \approx \gamma \times Z \times (m_n - m_p) e^2
\]  

(16)

If \((Z < 30)\), coefficient, \(\gamma \approx \left[ \left( \frac{1}{\alpha_s} + 1 \right) + \sqrt{Z} \right] \)

If \((Z \geq 30)\), \(\gamma \approx \left( \frac{1}{\alpha_s} + 1 \right) + \sqrt{30} \equiv 15.157\)

and \(15.157 \times 1.29333 \text{ MeV} \equiv 19.6033 \text{ MeV}\)

Thus, for, \((Z < 30)\)

\[
(B)_{A_s} \approx [9.68 + \sqrt{Z}] \times Z \times 1.2933 \text{ MeV}
\]

(17)
for, \( (Z \geq 30) \)

\[
(B)_{A_i} \cong Z \times 19.6033 \text{ MeV}
\]  

(18)

Close to the stable mass numbers, the binding energy is estimated with relations (14) and (16) and compared with Semi-empirical mass formula (SEMF) (See Table 2). It needs further study with respect to its surprising results against a single energy coefficient! It may also be noted that understanding nuclear binding energy with a single energy coefficient is a challenging task and needs in-depth study. To improve accuracy, we tried to understand nuclear binding energy with two simple terms with the same energy coefficient (See sec-11).

**Table 2:** Nuclear Binding Energy Close to Stable Mass Numbers of Z=2 to 100

| Proton number | Est. Mass number close to stability | Est. BE (MeV) | SEMF BE (MeV) | Error (MeV) |
|---------------|-----------------------------------|---------------|---------------|-------------|
| 2             | 4                                 | 28.7          | 22.0          | -6.7        |
| 3             | 6                                 | 44.3          | 26.9          | -17.4       |
| 4             | 8                                 | 60.4          | 52.9          | -7.6        |
| 5             | 10                                | 77.1          | 62.3          | -14.8       |
| 6             | 12                                | 94.1          | 87.4          | -6.7        |
| 7             | 14                                | 111.6         | 98.8          | -12.8       |
| 8             | 16                                | 129.4         | 123.2         | -6.2        |
| 9             | 19                                | 147.6         | 148.9         | 1.3         |
| 10            | 21                                | 166.1         | 167.5         | 1.4         |
| 11            | 23                                | 184.9         | 186.1         | 1.2         |
| 12            | 25                                | 204.0         | 204.7         | 0.7         |
| 13            | 27                                | 223.4         | 223.2         | -0.2        |
| 14            | 29                                | 243.0         | 241.6         | -1.4        |
| 15            | 31                                | 262.9         | 260.0         | -2.9        |
| 16            | 34                                | 283.1         | 290.8         | 7.7         |
| 17            | 36                                | 303.5         | 305.1         | 1.6         |
| 18            | 38                                | 324.1         | 327.2         | 3.1         |
| 19            | 40                                | 345.0         | 341.5         | -3.5        |
| 20            | 43                                | 366.1         | 371.6         | 5.5         |
| 21            | 45                                | 387.4         | 389.6         | 2.2         |
| 22            | 47                                | 408.9         | 407.5         | -1.4        |
|   |   |   |   |
|---|---|---|---|
| 23 | 49 | 430.6 | 425.2 | -5.4 |
| 24 | 52 | 452.5 | 454.6 | 2.0  |
| 25 | 54 | 474.7 | 468.9 | -5.8 |
| 26 | 56 | 497.0 | 489.6 | -7.4 |
| 27 | 59 | 519.5 | 515.2 | -4.3 |
| 28 | 61 | 542.2 | 532.5 | -9.7 |
| 29 | 63 | 565.0 | 549.7 | -15.4|
| 30 | 66 | 588.1 | 577.9 | -10.2|
| 31 | 68 | 607.7 | 592.0 | -15.7|
| 32 | 71 | 627.3 | 619.8 | -7.5 |
| 33 | 73 | 646.9 | 636.6 | -10.3|
| 34 | 75 | 666.5 | 653.3 | -13.2|
| 35 | 78 | 686.1 | 677.9 | -8.2 |
| 36 | 80 | 705.7 | 697.0 | -8.7 |
| 37 | 83 | 725.3 | 721.3 | -4.0 |
| 38 | 85 | 744.9 | 737.6 | -7.3 |
| 39 | 88 | 764.5 | 761.6 | -2.9 |
| 40 | 90 | 784.1 | 780.2 | -3.9 |
| 41 | 93 | 803.7 | 803.9 | 0.2  |
| 42 | 95 | 823.3 | 819.7 | -3.6 |
| 43 | 98 | 842.9 | 843.2 | 0.2  |
| 44 | 100| 862.5 | 861.2 | -1.3 |
| 45 | 103| 882.1 | 884.4 | 2.2  |
| 46 | 106| 901.7 | 909.6 | 7.9  |
| 47 | 108| 921.3 | 922.7 | 1.4  |
| 48 | 111| 940.9 | 947.6 | 6.7  |
| 49 | 113| 960.5 | 962.8 | 2.3  |
| 50 | 116| 980.2 | 987.5 | 7.3  |
| 51 | 119| 999.8 | 1009.7| 9.9  |
| 52 | 121| 1019.4| 1024.6| 5.2  |
| 53 | 124| 1039.0| 1046.5| 7.6  |
| 54 | 127| 1058.6| 1070.4| 11.9 |
| 55 | 129| 1078.2| 1085.1| 6.9  |
| 56 | 132| 1097.8| 1108.7| 11.0 |
| 57 | 135| 1117.4| 1130.1| 12.7 |
| 58 | 138| 1137.0| 1153.3| 16.3 |
| 59 | 140| 1156.6| 1165.6| 9.0  |
| 60 | 143| 1176.2| 1188.5| 12.3 |
| 61 | 146| 1195.8| 1209.3| 13.5 |
| 62 | 149| 1215.4| 1231.9| 16.5 |
| 63 | 151| 1235.0| 1245.9| 10.9 |
|   |   |   |   |   |
|---|---|---|---|---|
| 64 | 154 | 1254.6 | 1268.2 | 13.6 |
| 65 | 157 | 1274.2 | 1288.4 | 14.2 |
| 66 | 160 | 1293.8 | 1310.4 | 16.6 |
| 67 | 163 | 1313.4 | 1330.4 | 17.0 |
| 68 | 166 | 1333.0 | 1352.0 | 19.0 |
| 69 | 169 | 1352.6 | 1371.7 | 19.1 |
| 70 | 171 | 1372.2 | 1385.1 | 12.9 |
| 71 | 174 | 1391.8 | 1404.5 | 12.7 |
| 72 | 177 | 1411.4 | 1425.7 | 14.2 |
| 73 | 180 | 1431.0 | 1444.8 | 13.8 |
| 74 | 183 | 1450.6 | 1465.7 | 15.0 |
| 75 | 186 | 1470.2 | 1484.6 | 14.3 |
| 76 | 189 | 1489.8 | 1505.1 | 15.3 |
| 77 | 192 | 1509.4 | 1523.7 | 14.3 |
| 78 | 195 | 1529.0 | 1544.0 | 14.9 |
| 79 | 198 | 1548.6 | 1562.4 | 13.7 |
| 80 | 201 | 1568.2 | 1582.3 | 14.1 |
| 81 | 204 | 1587.8 | 1600.5 | 12.6 |
| 82 | 207 | 1607.4 | 1620.2 | 12.7 |
| 83 | 210 | 1627.0 | 1638.1 | 11.0 |
| 84 | 213 | 1646.7 | 1657.5 | 10.8 |
| 85 | 216 | 1666.3 | 1675.2 | 8.9 |
| 86 | 219 | 1685.9 | 1694.3 | 8.5 |
| 87 | 223 | 1705.5 | 1718.6 | 13.1 |
| 88 | 226 | 1725.1 | 1737.5 | 12.4 |
| 89 | 229 | 1744.7 | 1754.6 | 10.0 |
| 90 | 232 | 1764.3 | 1773.2 | 9.0 |
| 91 | 235 | 1783.9 | 1790.2 | 6.3 |
| 92 | 238 | 1803.5 | 1808.5 | 5.1 |
| 93 | 241 | 1823.1 | 1830.2 | 7.1 |
| 94 | 245 | 1842.7 | 1848.3 | 5.6 |
| 95 | 248 | 1862.3 | 1864.8 | 2.5 |
| 96 | 251 | 1881.9 | 1882.6 | 0.7 |
| 97 | 254 | 1901.5 | 1898.9 | -2.6 |
| 98 | 258 | 1921.1 | 1922.7 | 1.6 |
| 99 | 261 | 1940.7 | 1938.7 | -2.0 |
| 100 | 264 | 1960.3 | 1956.1 | -4.2 |
10. Understanding Nuclear Binding Energy of Deuteron

If it is assumed that, there exists no strong interaction in between proton and neutron, nuclear binding energy of deuteron can be expressed as,

\[ BE \text{ of } ^2_1H \cong 2 \times (m_n - m_p) c^2 \cong 2.59 \text{ MeV} \]  

(19)

where,

\[ \left( \frac{1}{\alpha_s} + 1 \right) \cong 1 \]

\[ \xrightarrow{\alpha_s \to 0} \left( \frac{e_s}{e} \right)^2 \to 0 \Rightarrow e_s \to 0 \]

This can be compared with the experimental value of 2.225 MeV.

11. Understanding Nuclear Binding Energy with Two Terms (Close to Stable Mass Numbers)

Based on the new integrated model proposed by N. Ghahramany et al [11,12],

\[ B(Z,N) = \left\{ A - \left( \frac{N^2 - Z^2}{3Z} + \delta(N - Z) + 3 \right) \right\} \frac{m_e c^2}{\gamma} \]  

(20)

where \( \gamma = \text{Adjusting coefficient} \approx (90 \text{ to } 100) \).

if \( N \neq Z, \delta (N - Z) = 0 \) and if \( N = Z, \delta (N - Z) = 1 \).

Readers are encouraged to see references in [11,12] for the derivation part. Point to be noted is that, close to the beta stability line, \( \left[ N^2 - Z^2 \right] \) takes care of the combined effects of coulombic and asymmetric effects. In this context, we would like to suggest that,

\[ \frac{m_e c^2}{\gamma} \cong \frac{m_e c^2}{90 \text{ to } 100} \cong \text{Constant} \]

\[ \cong \frac{e^2}{8\pi \epsilon_0 \left( G_s m_p / e^3 \right)} \cong 10.09 \text{MeV} \]  

(21)
Proceeding further, with reference to relation (14), it is also possible to show that, for \( Z \approx (40\) to \(83)\), close to the beta stability line,

\[
\left[ \frac{N_i^i - Z^i}{Z} \right] \approx kA_i Z \tag{22}
\]

\[
\left[ \frac{N_i^i - Z^i}{3Z} \right] \approx kA_i Z \tag{23}
\]

Based on the above relations and close to the stable mass numbers of \((Z \approx 5\) to \(118)\), with a common energy coefficient of 10.06 MeV, we would like to suggest two terms for fitting and understanding nuclear binding energy.

The first term helps in increasing the binding energy and can be considered as,

\[
\text{Term}_1 = A_y \times 10.06 \text{ MeV} \tag{24}
\]

The second term helps in decreasing the binding energy and can be considered as,

\[
\text{Term}_2 = \left( \frac{kA_i Z}{2.531} + 3.531 \right) \times 10.06 \text{ MeV} \tag{25}
\]

where

\[
\left( \frac{m_n - m_p}{m_c^2} \right) \approx \ln \left( \frac{1}{\sqrt{k}} \right) \approx 2.531.
\]

\[
3.531 \approx 1 + 2.531 \approx 1 + \ln \left( \frac{1}{\sqrt{k}} \right)
\]

Thus, binding energy can be fitted with,

\[
B_{A_i} \approx \left[ A_y - \left( \frac{kA_i Z}{2.531} + 3.531 \right) \right] \times 10.06 \text{ MeV} \tag{26}
\]

See the following figure 1. The dotted red curve plotted with relations (14) and (26) can be compared with the green curve.
plotted with the standard semi-empirical mass formula (SEMF). For medium and heavy atomic nuclides, the fit is excellent. It seems that some correction is required for light atoms.

![Figure 1: Binding energy per nucleon close to stable mass numbers of Z = 5 to 118](image)

We are working on understanding and estimating the binding energy of mass numbers above and below the stable mass numbers. With trial and error, we have developed a third term of the form 

$$\left[ \frac{(A_s - A)^2}{A_s} \right] \times 10.06 \text{ MeV}.$$ 

Using this term, approximately, it is possible to fit the binding energy of isotopes in the following way.

$$B_A \approx \left[ A - \left( \frac{kAZ}{2.531} + 3.531 \right) \right] - \left[ \frac{(A_s - A)^2}{A_s} \right] \times 10.06 \text{ MeV} \quad (27)$$

Figure 2 shows the estimated isotopic binding energy of Z=50. The dotted red curve plotted with relations (14) and (27) can be compared with the green curve plotted with SEMF.

For Z=50 and A=100 to 130, with reference to SEMF, there is not much difference in the estimation of binding energy. With reference to SEMF, when \(A > 130\), estimated binding energy seems to be increasing and when \(A \geq 212\), estimated binding energy seems to be decreasing rapidly. It needs further study and refinement.
12. To Fix the Magnitude of Fermi’s Weak Coupling Constant

With trial-error, we noticed that,

$$ R_{\text{m}} \approx \frac{2G_{s}m_{p}}{c^{2}} \approx \left( \frac{m_{p}}{m_{e}} \right) \sqrt{\frac{G_{s}}{\hbar c}} $$

$$ \Rightarrow \left( \frac{2G_{s}m_{p}}{c^{2}} \right) \approx \sqrt{\frac{G_{s}}{\hbar c}} \quad (28) $$

where $G_{s}$ is the Fermi’s weak coupling constant [5,31,32] and $\sqrt{\frac{G_{s}}{\hbar c}}$ is the characteristic length associated with a weak interaction.

Based on the relation (28),

$$ \alpha_{s}G_{s} \approx \frac{4h^{3}m_{p}^{2}}{m_{s}^{2}c} \quad (29) $$

$$ G_{s} \approx \left( \frac{1}{\alpha_{s}} \right) \frac{4h^{3}m_{p}^{2}}{m_{s}^{2}c} \equiv \frac{4G_{s}^{2}m_{s}^{2}h}{c^{3}} \quad (30) $$

$$ \equiv \hbar c \left( \frac{2G_{s}m_{p}}{c^{2}} \right)^{2} \approx 1.4400414 \times 10^{-12} \text{ J.m}^{2} $$
Recommended value of $G_r \approx 1.43586 \times 10^{-36}$ J.m$^3$. It may be noted that relations (29) and (30) seem to play a key role in understanding the basics of final unification and needs further study.

13. To Fix the Magnitude of Newtonian Gravitational Constant

With reference to Planck scale and considering the following semi-empirical relation, magnitude of the Newtonian gravitational constant ($G_N$) can be fitted.

$$\left( \frac{m_p}{m_r} \right) \approx \left( \frac{G m^3}{\hbar c} \times \frac{G_r}{G_N} \right)^{\frac{1}{\pi}} \approx \left( \frac{e G_r}{e G_N} \right)^{\frac{1}{\pi}}$$

(31)

Based on relations (28) to (31),

$$\left( \frac{G_r}{G_N} \right) \approx \sqrt{\alpha_r} \left( \frac{m_p}{m_r} \right)^{12} \approx \sqrt{\frac{4h m^2 c}{m^4 c F_w}} \left( \frac{m_p}{m_r} \right)^{12}$$

(32)

$$\left( \frac{G_N}{G_r} \right) \approx \frac{1}{2} \left( \frac{m}{m_p} \right)^{10} \left[ \frac{G_r}{\hbar c} \left( \frac{h}{m c} \right) \right]$$

$$\rightarrow \left\{ \begin{array}{l}
G_N \approx \left( \frac{m}{m_p} \right)^{10} \left[ \frac{G m^2}{\hbar c} \right] G_r \\
G_r \approx \left( \frac{m}{m_p} \right)^3 \left( \frac{G c h}{m^2} \right)
\end{array} \right.$$  

(33)

where $\frac{h}{m c} \approx$ Compton wavelength of electron.

Based on the recommended and estimated values of $G_r$,

If, $G_r \approx 1.43586 \times 10^{-36}$ J.m$^3$,
\[ G_N \approx 6.66937197 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \]

If, $G_r \approx 1.440414 \times 10^{-36}$ J.m$^3$,
\[ G_N \approx 6.679076 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \]

Average value can be expressed as,
\[ G_N \approx 6.674224 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}. \]
In terms of the nuclear charge radius,

$$G_s \approx \frac{1}{4} \left( \frac{m_e}{m_p} \right) \sqrt{c^3 G_s R_e^3 / \hbar^3}$$  \hspace{1cm} (34)$$

Accuracy of \((G_s)\) seems to depend on \((G_s, R_e, \alpha, G_r)\).

14. To Understand the Nuclear Charge Radius and Fermi’s Weak Coupling Constant

Based on the relation (8),

$$R_e \approx \left( \frac{m_e}{m_p} \right) \sqrt{\frac{4G_s \hbar}{c^3}} \approx \left( \frac{m_e}{m_p} \right) \sqrt{\frac{4G_s \hbar}{c^3}}$$  \hspace{1cm} (35)$$

where \(\sqrt{\frac{4G_s \hbar}{c^3}}\) can be called as the Schwarzschild radius of weak scale Planck mass

$$G_s \approx \hbar c \left( \frac{4G_s \hbar}{c^3} \right)^{1/2} \approx \frac{4G_s \hbar^3}{c^2}$$  \hspace{1cm} (36)$$

Characteristic electroweak mass and its Schwarzschild radius can be expressed as,

$$M_w \approx \frac{\sqrt{G_s \hbar}}{G_w} \approx 584.983 \text{ GeV/c}^2$$  \hspace{1cm} (37)$$

$$\frac{2G_u M_w}{c^3} \approx \sqrt{\frac{4G_u \hbar}{c^3}} \approx 6.74642 \times 10^{-19} \text{ m}$$  \hspace{1cm} (38)$$

$$\frac{m_e}{M_w} \approx \frac{G m_e m_p}{\sqrt{G_s \hbar^3}}$$  \hspace{1cm} (39)$$

Based on the relation (14), relation (39) can be given some consideration in understanding neutron-proton stability.

$$\frac{M_w}{m_e} \approx \frac{G_s}{G_w}$$  \hspace{1cm} (40)$$

15. To Understand the Important Strong Interaction Parameters

Based on the above relations, strong interaction range and strong coupling constant can be understood with the following relation.
\( R_0 \approx 2 \left( \frac{m_p}{m_r} \right) \left( \frac{G_v}{G_i} \right) \left( \frac{h}{m_c} \right) \)  \hspace{1cm} (41)

\( e_e \approx \frac{\sqrt{\alpha}}{e} \approx \frac{m_p^2}{M_m m_r} \)

\( \rightarrow m_r \approx \left( \frac{e}{e} \right) \sqrt{M_m m_r} \)  \hspace{1cm} (42)

One strange approximation is,

\( \left( \frac{m}{m_r} \right)^{10} \approx \exp \left( \frac{1}{\alpha_i} \right) \)  \hspace{1cm} (43)

4.356 \times 10^{32} \approx 5.259 \times 10^{32}

If so,

\( \frac{G_v}{G_i} \approx \exp \left( \frac{1}{\alpha_i} \right) \)  \hspace{1cm} (44)

Based on the above relations, strong interaction range can be understood with the following relation.

\( R_0 \approx 2 \exp \left( \frac{1}{\alpha_i} \right) \left( \frac{m_p}{m_r} \right) \left( \frac{G_v}{G_i} \right) \left( \frac{h}{m_c} \right) \)  \hspace{1cm} (Or)

\( R_0 \approx \exp \left( \frac{1}{\alpha_i} \right) \left( \frac{m_p + m_n}{m_r} \right) \left( \frac{G_v}{G_i} \right) \left( \frac{h}{m_c} \right) \)  \hspace{1cm} (45)

It seems interesting to infer that,

a) \( \left( \frac{1}{\alpha_i} \right) \) and \( \exp \left( \frac{1}{\alpha_i} \right) \) play a crucial role in deciding the strong interaction range.

b) An increase in the value of \( \alpha_i \) help in decreasing the interaction range. This may be an indication of a more strongly bound nuclear system.

c) A decrease in the value of \( \alpha_i \) help in increasing the interaction range. This may be an indication of the more weakly bound nuclear system.

d) Proportionality constant being \( \exp \left( \frac{1}{\alpha_i} \right) \).
According to current literature [34], nuclear charge radii can be expressed with the following formulae,

\[
R_0 \propto \left( \frac{m_p + m_n}{m_e} \right)
\]
\[
R_0 \propto \left( \frac{G_s}{G} \right)
\]
\[
R_0 \propto \left( \frac{\hbar}{m_c} \right)
\]

Based on these relations, by adjusting the coefficients 0.349 and 0.015 and bringing the value of \( R_0 \) close to 1.24 fm, magnitudes of \( (G_s, G) \) can also be fitted.

## 16. Discussion

According to Rosi et al. [35], there is no definitive relationship between \( G_N \) and the other fundamental constants and no theoretical prediction for its value to test the experimental results. Improving the knowledge of \( G_N \) not only has a pure metrological interest but also plays a key role in theories of gravitation, cosmology, particle physics, astrophysics, and geophysical models.

By following the works of Sivram, De Sabbata, and Gasperini [36-39] and with respect to the partial numerical success of the proposed relations, we are trying to understand the very nature of the four interactions in terms of tensors, vectors and axial vectors.

Interaction constants are connected both with global phenomena of physics and with phenomena at small distances, such as quantum gravity. Therefore, the search for relations among the constants of the four types of interactions is important, relevant and necessary. At present, there exist no basic formulae or mechanisms using by which one can develop at least models with ad hoc relations for
estimating the Newtonian gravitational constant. It would be important to consider in detail such theories as microscopic quantum gravity and a combination of the fields inherent in the unified description of the four interactions.

Clearly speaking, even though materialistic atoms have an independent existence, they are not allowing scientists and engineers to explore the secrets of gravity at the atomic scale. This may be due to incomplete unification paradigm, the inadequacy of known physics and technological difficulties etc. When heavenly bodies are made up of tiny atoms, it is imperative to find correlations that might exist among ‘atoms’ and ‘heavenly body’ as a whole. In this challenging scenario, one fundamental question to be answered is: Is Newtonian gravitational constant having any physical existence? We would like to suggest that, it is a man created empirical constant and is having no physical existence. Clearly speaking, it is not real but virtual. For understanding the secrets of large scale gravitational effects, scientists consider it as a physical constant. In the same way, each atomic interaction can be allowed to have its own gravitational constant. With further study, their magnitudes can be refined for a better understanding of their nature.

17. Conclusion

With reference to the famous semi-empirical mass formula having 5 different energy terms and 5 different energy coefficients, qualitatively and quantitatively, our proposed relations (14), (16), (26) and (27) are very simple to follow and a special study seems to be required for understanding the binding energy of isotopes above and below the stability line. We are working in this direction.

Considering relations (36 to 42), it is possible to predict that there exists a weakly interacting fermion of rest energy 585 GeV. It can be called as Higg’s fermion. Cosmological ‘dark matter’ research and observations [40] can be carried out in this direction also.

With further research and considering relations (1 to 10) and (28 to 46), current nuclear models and strong interaction concepts can be studied in a unified manner with respect to strong nuclear gravity. In this context, relation (35) can be given some consideration.
Finally, the value of the Newtonian gravitational constant can successfully be estimated with nuclear elementary physical constants.

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