Heavy-Meson Observables via Dyson-Schwinger Equations

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ABSTRACT: We summarise a Dyson-Schwinger-equation-based calculation of an extensive range of light- and heavy-meson observables, characterised by heavy-meson leptonic decays, semileptonic heavy-to-heavy and heavy-to-light transitions - \( B \to D^* \), \( D \to K^*, K, \pi \), radiative and strong decays - \( B^* \) \( (s) \to B \) \( (s) \) \( \gamma \); \( D^* \) \( (s) \to D \) \( (s) \) \( \gamma \), \( D \pi \), and the rare \( B \to K^* \gamma \) flavour-changing neutral-current process. In the calculation the heavy-quark mass functions are approximated by constants, interpreted as their constituent-mass: \( \hat{M}_c = 1.32 \text{ GeV} \) and \( \hat{M}_b = 4.65 \text{ GeV} \).

KEYWORDS: Dyson-Schwinger Equations, Hadron Physics, Heavy-Quark Observables.

1. Introduction

The Dyson-Schwinger equations (DSEs) [1] provide a nonperturbative, Poincaré-covariant, field theoretical approach to the calculation of hadronic matrix elements, and they have been widely applied to the phenomena of continuum strong QCD [2]. Many applications have focused on nonhadronic electroweak interactions because the electroweak probe is well understood and the interactions therefore explore the structure of the hadronic target. These are just the phenomena of interest to this community.

In Refs. [3–6] both light- and heavy-mesons are represented as bound states of a dressed-quark and -antiquark, with the quarks’ dressing described by a DSE: the QCD gap equation. The general form of the solution of this equation is

\[
S_f(p) = Z_f(p^2)/(i\gamma \cdot p + M_f(p^2)),
\]

(1.1)

\( f = u, d, s, c, b \) is the flavour label, and extensive studies have revealed [5,6] that while the mass function of a light-quark: \( M_{u,d,s} \), is a rapidly-varying function, that of an heavy-quark is almost momentum-independent: see Fig. 1.

The behaviour of \( M_{c,b} \) (and also that of \( Z_{c,b} \), which is not illustrated here) suggests that the
heavy-quark propagator can be well-approximated by:
\[
S_Q(p) = \frac{1}{[i\gamma \cdot p + M_Q]}, \quad Q = c, b,
\]
where \( M_Q \) is a constituent-heavy-quark mass parameter. A good description of observable phenomena requires \( M_Q \approx M_Q^f \), see below.

The dressed-quark propagator can also be written in the form
\[
S_f(p) = -i\gamma \cdot p \sigma^f_S(p^2) + \sigma^f_S(p^2),
\]
and the contrasting, significant momentum-dependence of the light-quark propagator is efficaciously represented via the algebraic parametrisation introduced in Ref. \[8\]
\[
\bar{\sigma}^f_S(x) = 2\bar{m}_f \mathcal{F}(2(x + \bar{m}_f^2))
\]
\[
+ \mathcal{F}(b_1 x) \mathcal{F}(b_2 x) \left[ b_0^2 + b_1^2 \mathcal{F}(\epsilon x) \right],
\]
\[
\bar{\sigma}^f_S(x) = \frac{1}{x + m^2} \left[ 1 - \mathcal{F}(2(x + \bar{m}_f^2)) \right],
\]
(1.4) (1.5)
where \( f = u, s \) (isospin symmetry is assumed), \( \mathcal{F}(y) = \frac{1 - e^{-y}}{y}, x = p^2/\lambda^2; \bar{m}_f = m_f/\lambda \); and
\[
\bar{\sigma}^f_S(x) = \lambda \sigma^f_S(p^2), \quad \bar{\sigma}^f_S(x) = \lambda^2 \sigma^f_S(p^2),
\]
with \( \lambda = 0.566 \text{GeV} \) a mass scale. This algebraic form combines the effects of confinement and DCSB with free-particle behaviour at large spacelike \( p^2; \sigma^f_S(p^2) \sim 1/p^2 \) and \( \sigma^f_S(p^2) \sim m^2/p^2 \). (With \( S(p) \) an entire function a sufficient condition for confinement is satisfied; i.e., the absence of a Lehmann representation for coloured Schwinger functions \[2\]. \( \epsilon = 10^{-4} \) is introduced in Eq. \[4\] only to decouple the large- and intermediate-\( p^2 \) domains: it is not a fitting parameter.

Bound states in quantum field theory are described by Bethe-Salpeter amplitudes whose momentum-dependence implements the necessary restriction of the relative momenta of the constituents in a bound state. This means that a further, commonly-used approximation to Eq. \[2.2\]
\[
S_Q(k + P) = \left( \frac{1}{2} \left| k \cdot v - E_H \right| + \mathcal{O} \left( \frac{|k|}{M_Q}, \frac{E_H}{M_Q}, \frac{\bar{M}_Q}{M_Q} \right) \right) \mathcal{F}(\epsilon k v - E_H)
\]
where \( P_\mu =: m_H v_\mu =: (\bar{M}_Q + E_H) v_\mu, m_H \) is the hadron’s mass and \( k \) is the momentum of the lighter constituent, can only be reliable if both the momentum-space width of the Bethe-Salpeter amplitude, \( \omega_H \), and the binding energy, \( E_H \), are significantly less than \( \bar{M}_Q \).

2. Heavy-Quark Limit

The DSE framework reproduces all the acknowledged consequences of heavy-quark symmetry. In addition, one obtains explicit expressions for the physical observables in the heavy quark limit; e.g., the leptonic decay constants is given by
\[
f_P = f_V = \frac{\kappa_f}{\sqrt{m_H}} \frac{N_c}{2\sqrt{2}\pi^2} \int_0^\infty du \left( \sqrt{u} - E_H \right)
\]
\[
\times \varphi_H(z) \left\{ \sigma^f_S(z) + \frac{1}{2} \sqrt{u} \sigma^f_V(z) \right\},
\]
(2.1)
\[
\frac{1}{\kappa_f} = \frac{N_c}{4\pi^2} \int_0^\infty dw \varphi_H^2(z) \left\{ \sigma^f_S(z) + \sqrt{u} \sigma^f_V(z) \right\},
\]
where \( z = u - 2E_H \sqrt{u} \), \( f \) labels the meson’s lighter quark and \( \varphi_H(z) \) is the scalar function characterising the dominant Dirac-covariant in the heavy-meson’s Bethe-Salpeter amplitude; e.g., the \( \gamma_5 \)-term for the pseudoscalar and \( \gamma_\mu \)-term for the vector meson.

As another example, the semileptonic heavy-to-heavy pseudoscalar transition form factors \( (P_1 \rightarrow P_2 \ell \nu) \) acquire a particularly simple form in the heavy-quark symmetry limit:
\[
f_{\pm}(t) := \frac{m_{P_2} \pm m_{P_1}}{2\sqrt{m_{P_2} m_{P_1}}} \xi_f(w),
\]
(2.2)
\[
\xi_f(w) = \frac{\kappa_f^2 N_c}{4\pi^2} \int_0^\infty \frac{\tau}{W} \int_0^\infty du \varphi_H^2(z_W) \left[ \sigma^f_S(z_W) + \sqrt{u} \sigma^f_V(z_W) \right],
\]
(2.3)
with \( W = 1 + 2\tau(1 - \tau)(w - 1), z_W = u - 2E_H \sqrt{u/W} \) and
\[
w = m_{P_1}^2 + m_{P_2}^2 - t = -v_{P_1} \cdot v_{P_2}.
\]
(2.4)

The canonical normalisation of the Bethe-Salpeter amplitude automatically ensures that
\[
\xi_f(w = 1) = 1
\]
(2.5)
and it follows [3] from Eq. (2.3) that

$$\rho^2 := - \left. \frac{d \xi_f}{d w} \right|_{w=1} \geq \frac{1}{3}. \quad (2.6)$$

Similar analysis for the heavy-to-heavy transitions with vector mesons in the final state and for heavy-to-light transitions yields relations between the form factors that coincide with those observed in Ref. [9]; i.e., in the heavy-quark limit these form factors too are expressible solely in terms of $\xi_f(w)$.

3. Results

This phenomenological application of DSE methods to the calculation of heavy-quark observables is founded on a large body of work that has focused on light-meson physics. References [2,10] provide an overview. Herein we summarise results for an extensive but not exhaustive body of observables: heavy-meson leptonic decays, semileptonic heavy-to-heavy and heavy-to-light transitions - $B \to D^*$, $D, \rho, \pi$; $D \to K^*$, $K, \pi$; radiative and strong decays - $B_s^* \to B_s \gamma$; $D_s^* \to D_s \gamma$, $D \pi$, and the rare $B \to K^* \gamma$ flavour-changing neutral-current process.

In Ref. [6] an algebraic characterisation of the dressed-quark propagators and bound state Bethe-Salpeter amplitudes employing ten parameters, plus the four quark masses, was used in a $\chi^2$-fit to $N_{\text{obs}} = 42$ heavy- and light-meson observables. That yielded: $\chi^2/\text{d.o.f} = 1.75$ and $\chi^2/N_{\text{obs}} = 1.17$, and the quality of the fit is illustrated in Tables 1 and 2. Using an approximating algebraic representation of the functions involved materially simplifies the calculation of observables, bypassing the repeated solving of nonlinear, coupled integral equations. It is a useful but not necessary artefact, as Refs. [7,11–13] make plain.

The fitting yielded dressed-quark-propagator parameter values

$$\begin{array}{l|lll}
\bar{m}_f & b^f_0 & b^f_1 & b^f_2 \\
\hline
u & 0.00948 & 2.94 & 0.733 \\
s & 0.210 & 3.18 & 0.858
\end{array} \quad (3.1)$$

with $b^{u,s}_0 = 0.131$, $b^{u,s}_1 = 0.105$, $b^{u,s}_2 = 0.185$, which were not varied i.e., $b^{u,s}_0$ retain the values fixed in previous studies of light-meson observables [14].

The dimensionless $u, s$ current-quark masses in Eq. (3.1) correspond to $m_u = 5.4 \text{ MeV}$ and $m_s = 119 \text{ MeV}$, and these algebraic propagators yield Euclidean constituent-light-quark masses: $M_{u}^E = 0.36 \text{ GeV}$ and $M_{s}^E = 0.49 \text{ GeV}$.

The fitted constituent-heavy-quark mass parameters are

$$\hat{M}_c = 1.32 \text{ GeV} \quad \text{and} \quad \hat{M}_b = 4.65 \text{ GeV}, \quad (3.2)$$

consistent with the estimates reported in Ref. [15] and hence the heavy-meson binding energy is large:

$$E_D := m_D - \hat{M}_c = 0.67 \text{ GeV}, \quad E_B := m_B - \hat{M}_b = 0.70 \text{ GeV}. \quad (3.3)$$

These values yield $E_D/\hat{M}_c = 0.51$ and $E_B/\hat{M}_b = 0.15$, and provide an indication that while an heavy-quark expansion, Eq. (1.7), will be accurate for the $b$-quark it will provide a poor approximation for the $c$-quark. The constituent-heavy-quark-masses in Eq. (3.2), obtained in a Poincaré covariant approach [6], are, respectively, $\sim 25\%$ and $\sim 10\%$ smaller than the values used in non-relativistic models.

Reference [6] represented the dominant scalar function in the light-pseudoscalar-meson Bethe-Salpeter amplitude as

$$\mathcal{E}_P(k^2) = \frac{1}{f_P} B_P(k^2), \quad P = \pi, K; \quad (3.4)$$

constructed via $B_P := B_{P|k^2}|_{k^0 = b^P_0}$ using Eq. (1.3) with, e.g., $f_\pi = f_\pi/\sqrt{2}$. This Ansatz follows from the constraints imposed by the axial-vector Ward-Takahashi identity and the fit yielded

$$b_0^\pi = 0.204, \quad b_0^K = 0.319. \quad (3.5)$$

The exploration of light-vector-meson properties is less extensive than that of light-pseudoscalar-mesons, and this is not peculiar to DSE analyses. Hence, following, e.g., Refs. [16], the algebraic characterisation of Ref. [6] used

$$\phi(k^2) = f/(1 + k^4/\omega_f^4); \quad (3.6)$$

i.e., a one-parameter form to describe the dominant scalar function in the vector Bethe-Salpeter amplitude. This is merely a simple, efficacious
Table 1: The 16 dimension-GeV quantities used in $\chi^2$-fitting the model parameters. The values in the “Obs.” column are taken from Refs. [7,15,17]. (Table adapted from Ref. [6].)

|        | Obs. | Calc. |        | Obs. | Calc. |
|--------|------|-------|--------|------|-------|
| $f_\pi$ | 0.131| 0.146 | $m_\pi$ | 0.138| 0.130 |
| $f_K$  | 0.160| 0.178 | $m_K$  | 0.496| 0.449 |
| $\langle \bar{u}u \rangle^{1/3}$ | 0.241| 0.220 | $\langle ss \rangle^{1/3}$ | 0.227| 0.199 |
| $\langle \bar{q}q \rangle^{1/3}$ | 0.245| 0.255 | $\langle \bar{q}q \rangle^{1/3}_K$ | 0.287| 0.296 |
| $f_\rho$ | 0.216| 0.163 | $f_{K^*}$ | 0.244| 0.253 |
| $\Gamma_{\rho\pi}$ | 0.151| 0.118 | $\Gamma_{K^*}(K\pi)$ | 0.051| 0.052 |
| $f_D$  | 0.200| ± 0.030| 0.213 | $f_{D_s}$ | 0.251| ± 0.030| 0.234 |
| $g_{BK\gamma\gamma\hat{M}_B}$ | 0.170| ± 0.035| 0.182 | 2.03 ± 0.62| 2.86 |

Table 2: The 26 dimensionless quantities used in $\chi^2$-fitting the model parameters. The values in the “Obs.” column are taken from Refs. [7,15,18–22]. The light-meson electromagnetic form factors are calculated in impulse approximation [8,14,23] and $\xi(w)$ is obtained from $f_{B\rightarrow D}(t)$ via Eq. (2.2). (Table adapted from Ref. [6].)

|        | Obs. | Calc. |        | Obs. | Calc. |
|--------|------|-------|--------|------|-------|
| $f^B_{\rightarrow D}^{K}(0)$ | 0.73 | 0.58 | $f_\pi r_\pi$ | 0.44 ± 0.004| 0.44 |
| $F_\pi^{(3.3 \text{GeV}^2)}$ | 0.097| ± 0.019| 0.077 | $B(B \rightarrow D^*)$ | 0.0453 ± 0.0032| 0.052 |
| $\rho^2$ | 1.53| ± 0.36| 1.84 | $\alpha^{B\rightarrow D^*}$ | 1.25 ± 0.26| 0.94 |
| $\xi(1.1)$ | 0.86| ± 0.03| 0.84 | $A_{FB}^{D\rightarrow D^*}$ | 0.19 ± 0.031| 0.24 |
| $\xi(1.2)$ | 0.75| ± 0.05| 0.72 | $B(B \rightarrow \pi)$ | (1.8 ± 0.6)_{x10^{-4}}| 2.2 |
| $\xi(1.3)$ | 0.66| ± 0.06| 0.63 | $f^B_{\pi}(14.9 \text{GeV}^2)$ | 0.82 ± 0.17| 0.82 |
| $\xi(1.4)$ | 0.59| ± 0.07| 0.56 | $f^B_{\pi}(17.9 \text{GeV}^2)$ | 1.19 ± 0.28| 1.00 |
| $\xi(1.5)$ | 0.53| ± 0.08| 0.50 | $f^B_{\pi}(20.9 \text{GeV}^2)$ | 1.89 ± 0.53| 1.28 |
| $B(B \rightarrow D)$ | 0.020| ± 0.007| 0.013 | $B(B \rightarrow \rho)$ | (2.5 ± 0.9)_{x10^{-4}}| 4.8 |
| $B(D \rightarrow K^*)$ | 0.047| ± 0.004| 0.049 | $f_{D\rightarrow K}(0)$ | 0.73 | 0.61 |
| $\frac{V(0)}{A_1(0)}$ | 1.89| ± 0.25| 1.74 | $f_{D\rightarrow \pi}(0)$ | 0.73 | 0.67 |
| $\frac{V(0)}{A_1(0)}$ | 1.93| ± 0.13| 1.17 | $g_{B^*B\pi}$ | 23.0 ± 5.0| 23.2 |
| $\frac{A_2(0)}{A_1(0)}$ | 0.73| ± 0.15| 0.87 | $g_{D^*D\pi}$ | 10.0 ± 1.3| 11.0 |

The ordering in magnitude is qualitatively understandable: the heavier the meson the smaller the spacetime volume occupied. In addition, the result: $\omega_D = \omega_B$, which means that the Compton wavelength of the $c$-quark is greater than the length-scale characterising the bound state’s extent, emphasises that Eq. (3.7) must provide a poor approximation for the $c$-quark.

With the model’s parameters fixed, it is possible to calculate a wide range of other light- and heavy-meson observables. Some of the results are summarised in Tables[3,4] while Figs. [2,3] depict the calculated $t$-dependence of the semileptonic transition form factors that are the hadronic manifestation of the $b \rightarrow c$, $b \rightarrow u$, $c \rightarrow s$ and $c \rightarrow d$ transitions. The form factors can be approximated by the monopole
Figure 2: Left panel: calculated semileptonic $B \to D$ and $B \to D^*$ form factors. Right panel: the semileptonic $B \to \pi$ and $B \to \rho$ form factors with, for comparison, data from a lattice simulation [21] and a vector dominance, monopole model: $f_{B \to \pi}^+(t) = 0.46/(1 - t/m_{B^*}^2)$, $m_{B^*} = 5.325$ GeV, the light, short-dashed line. Monopole fits to the model’s results are given in Eqs. (3.9) and (3.10). (Figure adapted from Ref. [6].)

Figure 3: Calculated semileptonic $D \to K$ and $D \to K^*$ (left panel), $D \to \pi$ and $D \to \rho$ (right panel) form factors. Monopole fits to the calculated results are given in Eqs. (3.9) and (3.10). (Figure adapted from Ref. [6].)

with $h(0)$ given in Tables 4 and 5, and $h_1$, in GeV$^2$, listed in Eq. (3.10).

| $B \to D, D^*$ | $h_f^{D^*}$ | $h_A^{D^*}$ | $h_f^D$ | $h_A^D$ |
|---------------|-------------|-------------|--------|--------|
| $B \to \pi, \rho$ | $(4.63)^2$ | $(5.73)^2$ | $(4.64)^2$ | $(4.61)^2$ |
| $D \to K, K^*$ | $(5.58)^2$ | $-(21.53)^2$ | $(6.94)^2$ | $(7.06)^2$ |
| $D \to \pi, \rho$ | $(2.31)^2$ | $(6.70)^2$ | $(3.09)^2$ | $(2.78)^2$ |

With these calculated results it is possible to check the fidelity of heavy-quark symmetry limits. The universal function characterising semileptonic transitions in the heavy-quark symmetry limit, $\xi(w)$, can be obtained most reliably from $B \to D, D^*$ transitions, if it can be obtained at all. Using Eq. (2.2) to extract it from $f_{B \to D}^+(t)$ one obtains

$$\xi_f^+(1) = 1.08,$$

which is a measurable deviation from Eq. (2.3). The calculated form of $\xi_f^+(w)/\xi_f^+(0)$ is depicted in Fig. 6 and compared with two experimental fits [20]:

$$\xi(w) = 1 - \rho^2(w - 1),$$

$$\rho^2 = 0.91 \pm 0.15 \pm 0.16,$$

$$\xi(w) = \frac{2}{w + 1} \exp \left[ (1 - 2\rho^2) \frac{w - 1}{w + 1} \right].$$
The calculated result is well approximated by
\[
\rho^2 = 1.53 \pm 0.36 \pm 0.14.
\]

The calculated result is well approximated by
\[
\xi^{f_+}(w) = \frac{1}{1 + \tilde{\rho}^2_{f_+}(w-1)}, \quad \tilde{\rho}^2_{f_+} = 1.98.
\] (3.14)

The semileptonic \( B \to D^* \) transition can also be used to extract \( \xi(w) \). That yields [6]
\[
\xi^{A_1}(1) = 0.987, \quad \xi^{A_2}(1) = 1.03, \quad \xi^{V}(1) = 1.30,
\] (3.15)
and an \( w \)-dependence well-described by Eq. (3.14), but with
\[
\tilde{\rho}^2_{A_1} = 1.79, \quad \tilde{\rho}^2_{A_2} = 1.99, \quad \tilde{\rho}^2_{V} = 2.02.
\] (3.16)

These and other results in Ref. [6] furnish a measure of the degree to which heavy-quark symmetry is respected in \( b \to c \) processes. Combining them it is clear that even in this case, which is the nearest contemporary realisation of the heavy-quark symmetry limit, corrections of \( \lesssim 30\% \) must be expected.

4. Epilogue

Herein we have summarised a direct extension of DSE-based phenomenology to experimentally accessible heavy-meson observables [6]. That extension explored the fidelity of a simple approximation, Eq. (1.2), to the dressed-heavy-quark propagator, and yields a unified and uniformly accurate description of an extensive body of light- and heavy-meson observables. Algebraic analysis proves [6] that in the heavy-quark limit pseudoscalar meson masses grow linearly with the mass of their heaviest constituent; i.e., \( m_P \propto \hat{m}_Q \), while the numerical results indicate that corrections to the heavy-quark symmetry limit of \( \lesssim 30\% \) are encountered in \( b \to c \) transitions and that these corrections can be as large as a factor of two in \( c \to d \) transitions.

The calculation of the semileptonic transition form factors for \( B^- \) and \( D^- \)-mesons on their entire kinematic domain and with the light-quark sector well constrained is potentially useful in the experimental extraction of the CKM matrix elements \( V_{cb}, V_{ub} \). That is also true of the calculation of the leptonic decay constants; e.g., accurate knowledge of \( f_B \) can assist in the determination of \( V_{td} \). The calculations show that the leptonic decay constants for \( D_f \)-mesons do not lie on the heavy-quark \( 1/\sqrt{\hat{m}_Q} \)-trajectory, and provide an estimate of the total width of the \( D^{+}\)- and \( D^{*0} \)-mesons, for which currently there are only experimental upper-bounds.

The model we have summarised employs simple parametrisations for the dressed-quark propagators and meson Bethe-Salpeter amplitudes. Its efficacy supports the interpretation that heavy- and light-mesons are both simply finite-size bound states of dressed-quarks and -antiquarks; i.e., they are not qualitatively different. Furthermore, this efficacy demonstrates that the qualitative elements of the Poincaré-covariant DSE-framework are rich enough to account for the gamut of strong interaction phenomena; i.e., as observed too in Ref. [26], no essential element is inherently lacking.

Naturally the model can be improved; e.g., via a wider study of light-vector-meson observables, so as to more tightly constrain their properties, perhaps using direct Bethe-Salpeter equation studies like Refs. [11,12] as a foundation for improved models of the vector meson Bethe-Salpeter amplitudes. Along this path, a more significant extension is the development of a Ward-
Table 3: Calculated values of a range of observables not included in fitting the model’s parameters. The “Obs.” values are extracted from Refs. [15,17,18,24]. (Table adapted from Ref. [6].)

| Obs. | Calc. | Obs. | Calc. |
|------|-------|------|-------|
| $f_K r_K$ | 0.472 ± 0.038 | $\frac{1}{2} f_K r_K$ | (0.19 ± 0.05)$^2$ |
| $g_{r\pi}$ | 6.05 ± 0.02 | $\Gamma_{D^{s0}}$ (MeV) | < 2.1 |
| $g_{K^+ K^0}$ | 6.41 ± 0.06 | $\Gamma_{D^{s+}}$ (keV) | < 131 |
| $g_\rho$ | 5.03 ± 0.012 | $\Gamma_{D^{s0}}$ (MeV) | < 1.9 |
| $f_{D^*}$ (GeV) | 0.290 | $\Gamma_{B^{+}B^+\gamma}$ (keV) | 0.030 |
| $f_{D^{*}_c}$ (GeV) | 0.298 | $\Gamma_{B^{+}0^{+}}$ (keV) | 0.015 |
| $f_{B_s}$ (GeV) | 0.195 ± 0.035 | $\Gamma_{B^{+}B_s\gamma}$ (keV) | 0.011 |
| $f_{B^+}$ (GeV) | 0.200 | $B(D^{*+} \rightarrow D^+\pi^0)$ | 0.306 ± 0.025 |
| $f_{D^*}$ (GeV) | 0.209 | $B(D^{*+} \rightarrow D^0\pi^+)$ | 0.683 ± 0.014 |
| $f_{D^*}/f_D$ | 1.10 ± 0.06 | $B(D^{*+} \rightarrow D^+\gamma)$ | 0.011 ± 0.007 |
| $f_{B^*}/f_B$ | 1.14 ± 0.08 | $B(D^{*0} \rightarrow D^0\pi^0)$ | 0.619 ± 0.029 |
| $f_{D^*}/f_D$ | 1.36 | $B(D^{*0} \rightarrow D^0\gamma)$ | 0.381 ± 0.029 |
| $f_{B^*}/f_B$ | 1.10 | $B(B \rightarrow K^*\gamma)$ | (5.7 ± 3.3)$^{10^{-5}}$ |

Table 4: Calculated values of some $b \rightarrow c$ and $b \rightarrow u$ transition form factor observables not included in fitting the model’s parameters. The “Obs.” values are extracted from Refs. [15,18]. (Table adapted from Ref. [6].)

Takahashi-identity-preserving Bethe-Salpeter kernel applicable to the study of heavy-meson masses. That would provide further insight into the structure of heavy-meson bound state amplitudes, an integral part of these calculations for which only rudimentary models are currently available. It would also assist in constraining DSE phenomenology via a comparison with calculations and models of the heavy-quark potential.

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Table 5: Calculated values of some $c\to s$ and $c\to d$ transition form factor observables not included in fitting the model’s parameters. The “Obs.” values are extracted from Refs. [15,18]. (Table adapted from Ref. [6].)

| Obs. | Calc. | Obs. | Calc. |
|------|-------|------|-------|
| $B(D^+ \to \rho^0)$ | 0.032 | $\alpha_{D^+\to\rho}$ | 1.03 |
| $B(D^0 \to K^0)$ | 0.037 ± 0.002 | 0.036 | $B(D^0\to\rho^0)$ | 0.044 ± 0.034 | 0.065 |
| $A_1^{D\to K^+}(0)$ | 0.56 ± 0.04 | 0.46 | $A_1^{D\to K^+}(t_{\text{max}})$ | 0.66 ± 0.05 | 0.47 |
| $A_2^{D\to K^+}(0)$ | 0.39 ± 0.08 | 0.40 | $A_2^{D\to K^+}(t_{\text{max}})$ | 0.46 ± 0.09 | 0.44 |
| $V^{D\to K^+}(0)$ | 1.1 ± 0.2 | 0.80 | $V^{D\to K^+}(t_{\text{max}})$ | 1.4 ± 0.3 | 0.92 |
| $R_1^{D\to K^+}(1)$ | 1.72 | 1.74 | $R_2^{D\to K^+}(1)$ | 0.83 |
| $R_1^{D\to \rho}(1)$ | 0.103 ± 0.039 | 0.098 | $R_2^{D\to \rho}(w_{\text{max}})$ | 0.87 |
| $B(D^0\to\pi)$ | 1.2 ± 0.3 | 1.10 | $f_{D^+\to\pi}(t_{\text{max}})$ | 1.31 ± 0.04 | 1.11 |
| $B(D^0\to K^0)$ | 1.2 ± 0.3 | 1.10 | $f_{D^+\to\rho}(t_{\text{max}})$ | 2.18 |
| $A_1^{D^0\to \rho}(0)$ | 0.69 | 0.60 | $A_1^{D^0\to \rho}(t_{\text{max}})$ | 0.58 |
| $A_2^{D^0\to \rho}(0)$ | 0.54 | 0.64 | $A_2^{D^0\to \rho}(t_{\text{max}})$ | 0.64 |
| $V^{D^0\to \rho}(0)$ | 1.22 | 1.51 | $V^{D^0\to \rho}(t_{\text{max}})$ | 1.51 |
| $R_1^{D^0\to \rho}(1)$ | 2.08 | 0.88 | $R_2^{D^0\to \rho}(1)$ | 0.88 |
| $R_1^{D^0\to \rho}(w_{\text{max}})$ | 2.03 | 0.91 | $R_2^{D^0\to \rho}(w_{\text{max}})$ | 0.91 |

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