DEVELOPMENTS IN THE THEORY OF THE QUANTUM HALL EFFECT

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ABSTRACT

The past few years have produced major advances in our understanding of the quantum Hall effects—quantized and unquantized. Theories based on a mathematical transformation, where the electrons are replaced by a set of fermions interacting with a Chern-Simons gauge field, have been useful for explaining and predicting observations at even-denominator filling fractions where quantized Hall plateaus are not observed, as well as for giving new insight into the most prominent fractional quantized Hall states at odd-denominator fractions. Other theoretical approaches have led to important advances in our understanding of edge-excitations for systems in a fractional quantized Hall state, of phases and phase transitions in bilayer systems, of tunneling phenomena in the quantum Hall regimes, and of disorder-induced transitions between “neighboring” quantum Hall plateaus. Some highlights of these developments will be reviewed.

1. Introduction

During the past few years, there has been very substantial progress, both theoretical and experimental, in our understanding of the behavior of electrons in a partially-filled Landau level. Some of this work has focused on the nature of the quantized Hall states, in which the Hall resistance $\rho_{xy}$ is locked to a rational multiple of the unit $h/e^2$, while the longitudinal resistivity $\rho_{xx}$ is found to vanish in the limit of low temperatures. Other work, at least equally interesting, has focused on Landau-level filling-fractions $f$ where the quantized Hall effect is not observed. A number of remarkable phenomena have been observed under these conditions, and we find that the effects of electron-electron interactions in a two-dimensional electron system in a strong magnetic field, can be quite subtle, even when the Hall conductance is not quantized. It seems appropriate to denote these phenomena, collectively, as the unquantized quantum Hall effect (UQHE).

In the following sections, I will try to outline developments in several selected
areas that I have found to be particularly exciting or challenging. The references
cited are representative examples intended to give the reader an entrance into the
literature of these subjects. It would not be possible in the space allotted to give
anything approaching a complete listing of the significant contributions, nor has it
been possible for me to devote the time that would be necessary to assemble such
a listing, or to conduct the thorough critical evaluation that might allow for an
optimal use of the reference space that is available. I apologize in advance for the
references omitted.

2. Fermion Chern-Simons Theory

The unquantized Hall effect has been especially interesting for very high mobil-
ity samples at or near an even-denominator fraction such as $f = 1/2$. The theoretical
approach which has proved most useful in this case is based on a mathematical
transformation to a system of fermions (“composite fermions”) interacting with a
fictitious gauge field of the Chern-Simons type.\(^1\)\(^-\)\(^3\) (Composite fermions were orig-
inally introduced by Jain to explain the most prominent fractional quantized Hall
states, at fractions of the form $f = p/(2p + 1)$, where $p$ is an integer.\(^4\)\(^-\)\(^6\) Theories
based on a transformation to a system of bosons interacting with a Chern Simons
field had also been used previously to discuss the quantized Hall effect.\(^5\),\(^7\) A key re-
sult has been the realization that at $f = 1/2$, and at various other even denominator
filling fractions, there are fermionic quasiparticles which move in straight lines, and
behave in many ways like quasiparticles of a Fermi liquid in zero magnetic field.\(^1\) For
magnetic fields which deviate by a small amount $\Delta B$ from the magnetic field $B_{1/2}$
which corresponds to $f = 1/2$, the quasiparticles move in circles whose radius $R_c^*$
is equal to the cyclotron radius for a particle in the effective field $|\Delta B|$. This has
important experimental consequences, which have been beautifully demonstrated
by several experiments during the past year.\(^8\)\(^-\)\(^10\)

Later in this session, Bob Willett will describe surface acoustic wave experi-
ments which show a resonance feature when the effective cyclotron diameter $2R_c^*$
coincides with a theoretically-predicted multiple of the acoustic wavelength.\(^8\) Other
experiments have seen effects in transport properties when $2R_c^*$ is related to geo-
metric features of an imposed lithographic structure.\(^9\),\(^10\) Taken together, these
experiments confirm that fermionic quasiparticles exist and follow the prescribed
trajectories at least over distances of several microns, a hundred times larger than
the electron-electron separation or the true cyclotron diameter of an electron in the
lowest Landau level.

An important open question is whether some type of modified Fermi liquid
theory can hold, in principle, all the way down to zero energy and infinite length
scales, at zero temperature, precisely at $f = 1/2$, in an ideal sample with no impurity
scattering. If so, is there a finite effective mass for the fermions, or is the effective
mass $m^*$ singular in the limit $|E - E_F| \to 0$? The original analysis of Ref. 1
suggested that $m^*$ should diverge as $\ln |E - E_F|$, if the electron-electron repulsion
has the Coulomb form, $\propto 1/r$ for large separations $r$. More recent investigations do
not necessarily agree with this conclusion, however.\(^11\) It is also possible that different
definitions could lead to different results for the effective mass. One choice, which is at least well defined in principle, is to use the energy gap $E_g$ of the fractional quantized Hall state at $f = p/(2p + 1)$, where $p$ is an integer, to define the mass at energy scale $E_g$, through the asymptotic relation (presumed valid at large $p$)\(^1\)

$$E_g \sim \frac{B e h}{(2p + 1) c m^*(E_g)}.$$  \(1\)

It is difficult to apply this expression to actual experimental data for the energy gap, however, because of the large effects of impurity scattering,\(^12\) for which there is no proper theory. Values of the effective mass have also been obtained recently from the amplitude of Shubnikov-de Haas oscillations at higher temperatures; however, these results are obtained using a theory of Shubnikov-de Haas oscillations based on ordinary Fermi liquid theory which may or may not be correct near $f = 1/2$ for the temperatures in question.\(^13\)

Within the composite fermion picture, at the mean field level, the ground state of the fractional quantized Hall state at $f = p/(2p + 1)$ is just an integer quantized Hall state, with $|p|$ filled Landau levels, as originally noted by Jain. The effective field $\Delta B$ is equal to $B/(2p + 1)$ in this case, and the gap (1) is identified with the cyclotron energy of a particle of mass $m^*$ in the field $\Delta B$. In order to properly obtain the dispersion relation for neutral excitations (quasi-exciton modes) or to study the linear response functions at finite $q$ and $\omega$, it is necessary to go beyond the mean field approximation, at least to the level of random phase approximation, or, better, to modifications of the RPA based on Landau Fermi-liquid theory.\(^1,6,14\)

### 3. Effects of Disorder

A different aspect of the unquantized Hall effect is the transition from one quantized Hall plateau to another under conditions where impurity scattering plays a dominant role. When impurities are important we must distinguish more carefully between the filling fraction $f$, which is defined in terms of the electron-density $n_e$ by

$$f = \frac{n_e h c}{B e}, \quad (2)$$

and the dimensionless Hall conductance $\nu$, which is defined by

$$\sigma_{xy} = \frac{\nu e^2}{h}. \quad (3)$$

If the impurity scattering is sufficiently strong, relative to the electron-electron interaction, there should be a direct transition from one integer Hall plateau to another, in an interval of magnetic fields (or of $f$) that becomes infinitely narrow in limit of temperature $T \to 0$. If the impurity scattering is reduced, then the first fractional Hall plateau appears, say at $\nu = 1/3$. In this case, according to recent theoretical analyses,\(^15\) we should find a reentrant phase diagram, where in the limit
of $T \to 0$, as the magnetic field decreases and $f$ increases, we find a sequence of sharp transitions: first from an insulating state with $\nu = 0$ to a fractional quantized Hall state with $\nu = 1/3$, then back down to $\nu = 0$, followed by a sharp transition to $\nu = 1$. (A direct transition from $\nu = 1/3$ to $\nu = 1$ is not allowed.) If the impurity scattering is decreased further, we should expect to see new plateaus appear at $\nu = 2/3$, $\nu = 2/5$, etc. If the impurity scattering is reduced sufficiently, we reach the situation where at least for the lowest attainable temperatures, there is a smooth variation of the Hall conductance near $f = 1/2$, and the phenomena characteristic of an impurity-free system begin to appear.

Recent theoretical work has dealt with such questions as: Which transitions are allowed to be direct at $T = 0$? What are the transition widths at $T \neq 0$? What is the value of $\rho_{xx}$ at the peak of the transition?\textsuperscript{15–17} Other work has focused on the conductivity at $T \neq 0$ in the field region of a quantized Hall plateau, and on the possibility that mesoscopic density inhomogeneities may be responsible for the observed values of $\rho_{xx}$ at higher temperatures, where Hall plateaus have disappeared.\textsuperscript{18,19}

4. Edge States

If a sample is in a quantized Hall state, so that there is an energy gap for delocalized excitations in the “bulk” of the sample, there must nevertheless be zero-energy excitations (edge states) along the sample boundaries. For noninteracting electrons, in an integer quantized Hall state, the edge states may be understood as arising because the occupied Landau levels are pushed up through the Fermi level by the confining potential at the boundary. (See Fig. 1, left panel). Electrons in edge states at a given boundary have a group velocity, parallel to the boundary, in a single direction, arising from $\vec{E} \times \vec{B}$ drift of the orbits in the electric field of the confining potential.\textsuperscript{20} An alternate view is to think about each edge state as the dividing line between two incompressible quantum Hall fluids with different values of $\nu$. An extra particle at the edge causes a bulge in the boundary, which propagates with a well-defined velocity and direction as an “edge magnetoplasmon.”\textsuperscript{21–23} (See right panel of Fig. 1).

Recent theories have extended these considerations to fractional quantized Hall states, and have characterized the possible combinations of edge states that may occur.\textsuperscript{24–27} In the fractional case, quantum fluctuations have a major effect, particularly on the tunneling of charged quasiparticles into or out of an edge state. The effects of these fluctuations can be understood using the various techniques previously applied to conventional one-dimensional metals, and the resulting description of the edge is characterized as a “chiral Luttinger liquid.”\textsuperscript{22,27} Recent tunneling experiments confirm key predictions of the theory.\textsuperscript{28}

In most actual samples, the electron profile drops gradually to zero at the edge, on a scale large compared to the electron-electron separation. In a number of recent papers, the authors have attempted to calculate self-consistently the electron profile near a sample edge, and have discussed some of the differences that may occur between properties of gradual and sharp edges.\textsuperscript{23,29}
5. Bilayer Systems

A variety of interesting theoretical and experimental questions arise when there are two parallel electron layers. (A wide single quantum well may also act like a two layer system because the self-consistent Coulomb potential develops a peak at the center of the well.) Depending on the separation between the layers, the Coulomb interaction between electrons in different layers may be relatively weak or may become comparable in strength to the interaction between electrons in the same layer. Depending on the height as well as the thickness of the barrier between the layers, tunneling of electrons between the two layers may be more or less important. Depending on these parameters, and on the filling factors of the layers, a variety of quantized Hall states, as well as unquantized states, have been found to occur.\(^{30-33}\)

One interesting experimental result has been the observation of a quantized Hall plateau at total filling \(f = 1/2\) for certain ranges of the system parameters.\(^{30-32}\) Another interesting phase can occur at total fillings \(f = 1\), (and at various other filling factors), where the total filling is locked at a quantized value, but the relative fraction of electrons in each layer is free to vary. Interest has focussed on characterizing the possible phases that can occur, and understanding their properties, as well as on predicting the occurrence of transitions between different phases as the system parameters are varied at a fixed filling factor \(f\).\(^{30-36}\)

Application of a magnetic field parallel to the surface introduces, effectively, a spatial variation in the phase of the tunneling matrix element between the two layers. An unexpected phase transition, observed in a bilayer system with \(f = 1\), in a relatively weak parallel field, has been explained in terms of this effect.\(^{33,37}\)

6. Other Topics

Theoretical progress has also been made on a variety of other aspects of two-dimensional electron systems in strong magnetic fields. I cite but a few examples.

In any given sample, for sufficiently strong magnetic fields, at low enough temperatures, one expects to find an insulating phase where \(\rho_{xx}\) becomes very large, and \(\rho_{xy}\) is small compared to \(\rho_{xx}\). This could occur because of electron-electron interaction (formulation of a Wigner crystal), because of electron impurity interactions (carrier freeze-out) or because of some complicated combination of these effects. There has been considerable theoretical and experimental effort concerning the transition to the insulating state under various circumstances.\(^{38}\)

Theoretical arguments have been advanced to explain the observations of a pseudogap for tunneling into a layer (or between two layers) in the unquantized Hall regime.\(^{39}\)

Electron-electron interactions have been invoked to explain the splittings and shifts of the cyclotron resonance line at very low filling factors.\(^{40}\)

Invited papers in other sessions of this conference will describe experiments in these and other areas of the quantum Hall effect, and no doubt they will also give further references to theoretical work in the field.
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8. References

1. B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B47, 7312 (1993).
2. V. Kalmeyer and S.-C. Zhang, Phys. Rev. B46, 9889 (1992); E. Rozayi and N. Read, Phys. Rev. Lett. 72, 900 (1994).
3. See also G. Moore and N. Read, Nucl. Phys. B360, 362 (1991); M. Greiter and F. Wilczek, Mod. Phys. Lett. B4, 1063 (1990).
4. J.K. Jain, Phys. Rev. Lett. 63, 199 (1989); Phys. Rev. 40, 8079 (1989); Phys. Rev. B41, 7653 (1990); J.K. Jain, S.A. Kivelson, and N. Trivedi, Phys. Rev. Lett. 64, 1297 (1990); X.G. Wu, G. Dev and J.K. Jain, Phys. Rev. Lett. 71, 153 (1994).
5. A. Lopez and E. Fradkin, Phys. Rev. 44, 5246 (1991).
6. E. Fradkin, Field Theories of Condensed Matter Systems (Addison-Wesley, Redwood City, CA, 1991), Chap. 10.
7. S.M. Girvin and A.H. MacDonald, Phys. Rev. Lett. 58, 1252 (1987); S.-C. Zhang, H. Hansson, and S. Kivelson, Phys. Rev. Lett. 62, 82 (1989); Phys. Rev. Lett. 62, 980(E), (1989); S.-C. Zhang, Int. J. Mod. Phys. B6, 25 (1992); D.-H. Lee and M.P.A. Fisher, Phys. Rev. Lett. 63, 903 (1989).
8. R.L. Willett, R.R. Ruel, K.W. West, and L.N. Pfeiffer, Phys. Rev. Lett. 71, 3846 (1993).
9. W. Kang, H.L. Stormer, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 71, 3850 (1993).
10. V.J. Goldman, B. Su, and J.K. Jain, Phys. Rev. Lett. 72, 2065 (1994); V.J. Goldman and B. Su, preprint.
11. Y.B. Kim, A. Furusaki, X.-G. Wen, and P.A. Lee, preprint; B.L. Altshuler, A. Houghton, and J.B. Marston, preprint; J. Gan and E. Wong, Phys. Rev. Lett. 71, 4226 (1993); D.V. Kveshchenko and P.C.E. Stamp, Phys. Rev. Lett. 71, 2118 (1993); C. Nayak and F. Wilczek, Nucl. Phys. B417, 359 (1994); J. Polchinski, preprint.
12. R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 70, 2944 (1993).
13. D.R. Leadley, R.J. Nicholas, C.T. Foxon, and J.J. Harris, Phys. Rev. Lett. 72, 1906 (1994); R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Solid State Commun. 90, 71 (1994).
14. A. Lopez and E. Fradkin, Phys. Rev. B47, 7080 (1993); S.H. Simon and B.I. Halperin, Phys. Rev. B48, 17368 (1993); S. He, S.H. Simon, and B.I. Halperin, Phys. Rev. B50, 1823 (1993).
15. S. Kivelson, D.-H. Lee, and S.-C. Zhang, Phys. Rev. B46, 2223 (1992).
16. D.B. Chklovskii and P.A. Lee, *Phys. Rev.* B48, 18060 (1993); D.-H. Lee, Z. Wang, and S. Kivelson, *Phys. Rev. Lett.* 70, 4130 (1993); A.M. Dykhne and I.M. Ruzin, *Phys. Rev.* B50, 2369 (1994); I.M. Ruzin and S. Feng, preprint.

17. H. Levine, S. Libby, and A. Pruisken, *Phys. Rev. Lett.* 51, 1915 (1983); J.T. Chalker and P.D. Coddington, J. Phys. C21, 2665 (1988); Y. Huo and R.N. Bhatt, *Phys. Rev. Lett.* 68, 1375 (1992); A.W.W. Ludwig, M.P.A. Fisher, R. Shantar, and G. Grinstein, preprint.

18. D.G. Polyakov and B.I. Shklovskii, *Phys. Rev.* B48, 1167 (1993), and preprints

19. I.M. Ruzin, *Phys. Rev.* B47, 15727 (1993); S.H. Simon and B.I. Halperin, preprint; J. Hajdu, M. Metzler, and H. Moraal, preprint; M.B. Isichenko, *Rev. Mod. Phys.* 64, 961 (1992).

20. B.I. Halperin, *Phys. Rev.* B25, 2185 (1982).

21. D.B. Mast, A.J. Dahm, and A.L. Fetter, *Phys. Rev. Lett.* 54, 1706 (1985); D.C. Glattli, E.Y. Andrei, G. Deville, J. Portrenaud, and F.B.I. Williams, *Phys. Rev. Lett.* 54, 1710 (1985).

22. X.-G. Wen, *Phys. Rev.* B44, 5708 (1991).

23. I.L. Aleiner and L.I. Glazman, preprint.

24. A.H. MacDonald, *Phys. Rev. Lett.* 64, 222 (1990); M.D. Johnson and A.H. MacDonald, *Phys. Rev. Lett.* 67, 2060 (1991).

25. A. Cappelli, C.A. Trugenberger, and G.R. Zembla, *Nucl. Phys.* B396, 465 (1993), and preprint.

26. L. Brey, preprint; D.B. Chklovskii, preprint.

27. C. de C. Chamon and X.-G. Wen, *Phys. Rev. Lett.* 70, 2605 (1993); K. Moon, H. Yi, C.L. Kane, S.M. Girvin, and M.P.A. Fisher, *ibid.* 71, 4381 (1993); C.L. Kane, M.P.A. Fisher, and J. Polchinski, *ibid.* 72, 4129 (1994).

28. F.P. Milliken, C.P. Umbach, and R.A. Webb, preprint.

29. D.B. Chklovskii, B.I. Shklovskii, and L.I. Glazman, *Phys. Rev.* B46, 4026 (1992); C.W.J. Beenaker, *Phys. Rev. Lett.* 64, 216 (1990); A.M. Chang, *Solid State Commun.* 74, 871 (1990); J. Dempsey, B.Y. Gelfand, and B.I. Halperin, *Phys. Rev. Lett.* 70, 3639 (1993); C. de C. Chamon and X.-G. Wen, preprint.

30. Y.W. Suen, L.W. Engel, M.B. Santos, M. Shayegan, and D.C. Tsui, *Phys. Rev. Lett.* 68, 1379 (1992).

31. J.P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, K.W. West, and S. He, *Phys. Rev. Lett.* 68, 1383 (1992).

32. Y.W. Suen, H.C. Manoharan, X. Ying, M.B. Santos, and M. Shayegan, *Surface Science* (305), 13 (1994).

33. S.Q. Murphy, J.P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, and K.W. West, *Phys. Rev. Lett.* 72, 728 (1994).

34. T. Chakraborty and P. Pietiläinen, *Phys. Rev. Lett.* 59, 2784 (1987); S. He, S. Das Sarma, and X.C. Xie, *Phys. Rev.* B47, 4394 (1993); M. Greiter, X.-G. Wen, and F. Wilczek, *Phys. Rev.* B46, 9586 (1992); B.I. Halperin, *Surface Science* 305, 1 (1994).

35. X.-G. Wen and A. Zee, *Phys. Rev.* B47, 2265 (1993); J. Fröhlich and A. Zee, *Nucl. Phys.* B364B, 517 (1991).
36. D. Schmeltzer and J.L. Birman, *Phys. Rev.* **B47**, 10939 (1993); Z.F. Ezawa and A. Iwazaki, *Int. J. Mod. Phys.* **B19**, 3205 (1992); *Phys. Rev. B47*, 7295 (1993).
37. K. Yang, K. Moon, A.H. MacDonald, S.M. Girvin, D. Yoshioka, and S.-C. Zhang, *Phys. Rev. Lett.* **72**, 732 (1994).
38. See, e.g., R. Price, P.M. Platzman, S. He, and X. Zhu, *Surface Science** **305**, 126 (1994); X. Zhu and S.G. Louie, *Phys. Rev. Lett.* **70**, 335 (1993); K.S. Esfarjani, S.T. Chui, and X. Qiu, *Phys. Rev. B46*, 4638 (1992); L. Ziang and H. Fertig, *Phys. Rev. B50* (in press).
39. S.R.E. Yang and A.H. MacDonald, *Phys. Rev. Lett.* **70**, 4110 (1993); Y. Hatsugai, P.A. Bares, and X.-G. Wen, *Phys. Rev. Lett.* **71**, 424 (1993); S. He, P.M. Platzman, and B.I. Halperin, *Phys. Rev. Lett.* **71**, 777 (1993); P. Johansson and J.M. Kinaret, *Phys. Rev. Lett.* **71**, 1435 (1993); A.L. Efros and F.G. Pikus, *Phys. Rev. B48*, 14694 (1993); Y.B. Kim and X.-G. Wen, preprint; I.L. Aleiner, H.U. Baranger, and L.I. Glazman, preprint.
40. N.R. Cooper and J.T. Chalker, *Phys. Rev. Lett.* **72**, 2057 (1994).
Figure 1. Alternate descriptions of edge states, for noninteracting spinless electrons, with two filled Landau levels. The left panel shows the single-electron energy spectrum, $\epsilon_{kn}$, in the Landau gauge, near a sample boundary. The wave function with wavevector $k$ in the $x$ direction is localized about $y = y_k \equiv k\hbar c/Be$. Heavy lines indicate occupied states; arrows point to edge states at the Fermi level. The right panel shows regions of space where there are respectively 2, 1 and 0 occupied Landau levels. An electron added to the inner edge state is here indicated as a bulge, which propagates to the right, as indicated. Electron-electron interactions modify the velocities, but not the directions of propagation.
