The Matter Density Distribution for Mesons and Baryons

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The matter density distribution inside a hadron is evaluated using gauge-invariant correlation functions within the quenched and the unquenched theory. Comparison with the charge density distribution suggests that hadron deformation is a consequence of the relativistic motion of the quarks.

1. Introduction

The charge and matter density distributions encode detailed information on hadron structure. In the non-relativistic limit both charge and matter density distributions reduce to the square of the wave function. Therefore differences between them give a measure of relativistic effects. Hadron deformation can be searched via the matter density distribution as well as via the charge density distribution. There are strong experimental indications from measurements of the electromagnetic transition form factors in $\gamma^* N \to \Delta$ \cite{1} that the nucleon is deformed. It is thus interesting to address the issue of hadron deformation within lattice QCD and understand its physical origin. Comparison of matter and charge density distributions can help discriminate among possible mechanisms. A number of phenomenological studies use the bag-model to describe hadron deformation. For this reason we compare the charge and matter distributions to the bag-model results.

2. Observables

We evaluate equal time two-current correlators

$$C_\Gamma(\mathbf{r}, t) = \int d^3 \mathbf{r'} \langle h | j^n_\Gamma(\mathbf{r'} + \mathbf{r}, t) j^n_\Gamma(\mathbf{r'}, t) | h \rangle$$

with the current operator given by the normal order product $j^n_\Gamma(\mathbf{r}, t) = : \bar{u}(\mathbf{r}, t) \Gamma u(\mathbf{r}, t) :$ and $\Gamma = \gamma_0, 1$ for the charge and matter correlators respectively. We note that no gauge ambiguity arises in this determination of hadron wave functions unlike Bethe-Salpeter amplitudes. In the case of baryons two relative distances are involved and three current insertions are required. However we may consider integrating over one relative distance to obtain the one-particle density distribution that involves two current insertions and it is thus evaluated via Eq. 1.

3. Results

All the results presented here have been obtained on lattices of size $16^3 \times 32$. We analyze, for the quenched case, 220 NERSC configurations at $\beta = 6.0$ for pion to rho mass ratio $m_\pi/m_\rho = 0.88, 0.84, 0.78$ and 0.70 and, for the unquenched, 150 SESAM configurations \cite{4} for $\kappa = 0.156$ and 200 for $\kappa = 0.157$ with $m_\pi/m_\rho = 0.83$ and 0.76 respectively. The physical volume of the quenched and unquenched lattices is approximately the same.

In Eq. 1 the current couples to the quark at fixed time separation $t$ from the source, which must be large enough to sufficiently isolate the hadron ground state. Using local and Wuppertal smeared sources we first checked that we obtain the same value for the effective mass at $t = 8a$. This value agrees with the one extracted using data with Dirichlet boundary conditions utilizing the whole time extent of our lattice. In contrast, wall sources even at maximal separation $t = 8a$ still show a sizable excited states contamination. In addition, we checked, by comparing data at different $t$, that higher momenta are sufficiently suppressed in the current-current correlators since summing over the spatial volume of the source is not possible. Also, results obtained using Wup-
pertal smearing which suppresses high momenta and local sources which allow them, are in agreement at $t = 8a$. This means that contamination from high momentum states is negligible at $t = 8a$ allowing us to analyze standard full QCD configurations that employ anti-periodic b.c. Since, for our parameters, local sources produce the same results for $C_I(r)$ as smeared ones and carry no gauge noise they are preferable for this study.

The quenched matter density distribution shown in Fig. 1 broadens for the pion as the quarks become lighter. For the rho, the nucleon and the $\Delta^+$ essentially no variation is seen over the range of naive quark masses $\sim 300 - 100$ MeV.

In a previous study of the charge density distribution [2,3] we obtained a non-zero quadrupole moment for the rho or a $z - x$ asymmetry where the $z$-axis is taken along the spin of the rho as shown in Fig. 2. An angular decomposition of the wave function, as shown in Fig. 3, corroborates a non-zero charge deformation by producing a non-zero $L = 2$ component. No such deformation is seen for the matter density distribution in Figs. 2, 3. Since the matter and charge operators have the same non-relativistic limit, this strongly suggests that hadron charge deformation is a relativistic effect. This result has strong implications for the validity of various phenomenological models used in the study of nucleon deformation and $\gamma^*N \rightarrow \Delta$ form factors.

In Fig. 4 compares quenched and unquenched results for $C_I(r)$ at $m_\pi/m_\rho \approx 0.83$. Unquenching increases $C_I(r)$ at short distances in the case of the pion and the rho whereas for the baryons no significant changes are seen. Both quenched and unquenched matter distributions show a faster fall off than the charge distribution. Furthermore, whereas unquenching tends to lead to a small increase in the rho charge asymmetry, it has no effect on the matter density distribution.
This again suggests a relativistic origin for the rho charge deformation.

4. Comparison of lattice and bag model results

We consider only the lowest mode of the free Dirac field in a spherical bag of radius \( R \) that is chosen so as to minimize the mass of the hadron under consideration. For the comparison presented here, we use the lattice values for the masses of the rho, the nucleon and the \( \Delta \) to fix the bag parameters, \( B, Z_0 \) and \( \alpha_{bag} \), using as an input the naive quark mass. Expanding the quark fields in terms of bag eigenmodes we obtain for the charge and matter density distributions

\[
\langle \hat{j}^u_\mu (r) \hat{j}^d_\mu (r') \rangle_{MB} = \mp C \frac{(f(r)^2 + g(r)^2)}{(f(r')^2 + g(r')^2)} \quad (2)
\]

\[
\langle \hat{j}^I_\mu (r) \hat{j}^I_\mu (r') \rangle_{MB} = C' \frac{(f(r)^2 - g(r)^2)}{(f(r')^2 - g(r')^2)} \quad (3)
\]

with \( C, C' \) constants. \( M \) denotes mesons and \( B \) baryons. These expressions show explicitly that the difference between charge and matter density distributions is due to the opposite sign of the lower components of the Dirac spinor i.e. a relativistic effect.

From Fig. 5 we see that the bag model fails to reproduce the correct radial dependence of the lattice results for the charge and matter density distributions and that the Ansatz \( \exp(-mr) \) provides the best description of the data. A measure of the width of the distributions is provided by the root mean square (rms) radius. The ratio of the charge to the matter rms radius in the bag-model at \( \kappa = 0.153 \) and 0.154 is: 1.16 and 1.17 for the rho and 1.16 for the nucleon. The corresponding ratios for the lattice data are 1.15(3) and 1.16(8) for the rho and 1.18(4) and 1.20(8) for the nucleon. Thus, despite the failure of the bag model in describing the individual radial shape of the distributions it produces reasonable results for the relative widths of the charge to the matter distribution. Both lattice and bag model consistently predict a broader charge than matter distribution, with a very weak mass dependence.

5. Conclusions

For the lattice parameters used in this study, local sources are suitable interpolating fields for the evaluation of density distributions, since they have less gauge noise than smeared ones and the temporal extent of our lattice is large enough to obtain the ground state. Quenched and unquenched results for both the charge and matter density distributions show insignificant differences. The charge density distribution is, in all cases, broader than the matter density. For baryons, the lattice indicates a charge radius about 20\% larger than the matter radius. This effect is well reproduced by the bag model. The deformation seen in the rho charge distribution is absent in the matter distribution, both in the quenched and the unquenched theory. This observation suggests a relativistic origin for the deformation.

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