Softening of the nuclear equation-of-state by kinetic non-equilibrium in heavy ion collision

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Highly compressed nuclear matter created in relativistic heavy collisions is to large extent governed by local non-equilibrium. As an idealised scenario colliding nuclear matter configurations are studied with an effective in-medium interaction based on the microscopic DBHF model. It is found that on top of the repulsive momentum dependence of the nuclear forces kinetic non-equilibrium leads to an effective softening of the equation of state as compared to ground state nuclear matter. The separation of phase space which is the basic feature of such anisotropic configurations has thereby a similar influence as the introduction of a virtual new degree of freedom.

One major goal of relativistic heavy ion physics is to explore the behaviour of the nuclear equation-of-state (EOS) far away from saturation, i.e. at high densities and non-zero temperature. Over the last three decades a large variety of observables has been investigated both, from the experimental and theoretical side motivated by the search for the nuclear EOS. The collective particle flow is thereby intimately connected to the dynamics during the compressed high density phase of such reactions\textsuperscript{1}.\textsuperscript{2}. E.g., the elliptic flow which develops in the early compression phase is thought to be a suitable observable to extract information on the EOS\textsuperscript{3}. But also the production of strange particles is a good probe to study dense matter\textsuperscript{4}. Recent precision measurements of the $K^+$ production at SIS energies strongly support the scenario of a relatively soft EOS\textsuperscript{5,6}.

The temporal evolution of the collision from a highly anisotropic initial configuration in phase space to an – at least partially – equilibrated final configuration is successfully described by microscopic transport models like BUU\textsuperscript{7} or QMD\textsuperscript{8}. In this type of models the nuclear mean field is usually based on phenomenological parameterisations\textsuperscript{7,10}. Such parameterisations allow different extrapolations to high densities, subsummed by referring to a ‘hard’ or a ‘soft’ equation-of-state\textsuperscript{1}. which can be tested in heavy ion collisions. A fundamental question is, however, if such a procedure is sufficient to to extract well defined information on the EOS of equilibrated nuclear matter (NM) from heavy ion collisions. The mean field used in transport simulations refers to \textit{locally equilibrated nuclear matter}. Anisotropy effects of the local phase space are usually neglected for the density dependent (or local) part of the mean field although it is clear that they should be included from a theoretical point of view\textsuperscript{11,14}. Such a treatment appears to be justified if non-equilibrium effects are small, have a short lifetime, or if the mean field is not much affected. As we will discuss in the present work none of this three conditions is fulfilled in relativistic heavy ion reactions in the SIS energy range: 1. The initial phase space distribution in the participant zone is that of two currents of nuclear matter colliding with beam velocity. The local momentum distributions of colliding nuclear matter configurations, called CNM in the following, are given by two Fermi ellipsoids, i.e. two boosted Fermi spheres separated by a relative velocity\textsuperscript{3}. 2. The relaxation time needed by the system to equilibrate coincides more or less with the high density phase of the reaction. Hence, anisotropy effects are present all over the compression phase where one essentially intends to study the EOS at high densities. Experimental evidence for incomplete equilibration even in central collisions at SIS energies has recently been reported in\textsuperscript{13}. 3. The impact of phase space anisotropies on the nuclear EOS is large and leads to a considerable softening of the effective EOS seen in heavy ion collisions.

To obtain a more quantitative measure for the size and relevant time scales for phase space anisotropies in Fig. 1 the time evolution of the quadrupole moment of the energy-momentum-tensor

$$Q_{zz} = \frac{2T_{33} - T_{11} - T_{22}}{T_{33} + T_{11} + T_{22}}$$

(1)

at the collision centre of central (b=0 fm) \textit{Au + Au} reactions at two typical beam energies, i.e. 0.6 A.GeV and 6.0 A.GeV, respectively, is compared to the time evolution of the corresponding baryon density. The quantities were obtained in relativistic BUU calculations\textsuperscript{14}. $Q_{zz}$ is a measure for the anisotropy in beam (z) direction. At 6.0 A.GeV $Q_{zz}$ reaches already the asymptotic value of 2 where the Fermi momentum can be neglected compared to the beam velocity. Fig. 1 demonstrates that the relaxation time to reach equilibrium configurations ($Q_{zz}$ \approx 0) coincides more or less with the high density phase, independent of the beam energy. Therefore non-equilibrium effects should be taken into account on the level of the effective in-medium interaction which means to determine the mean field used in transport calculations consistently for colliding nuclear matter.
The effective mass \( M_{\rho} \) ple by non-linear equations. \( \Theta \) is the step function, potential) of [18] with nuclear matter saturation properties (Bonn A in the present work is based on recent DBHF calculations. However, already this approximation invokes a contrast to the local density approximation where the energy per particle is defined in the usual way as 

\[ E = \frac{1}{2} m v^2 + \frac{1}{2} k v^2 \]

\[ u \text{ GeV/nucleon.} \]

In the present work we want to study the impact of such non-equilibrium effects on the nuclear EOS probed by heavy ion collisions. The configurations are idealised by colliding nuclear matter which describes well the local phase space during the reaction. For comparison also the central densities (in units of \( \rho_{\text{sat}} \)) are shown.

The general repulsive character of the momentum dependence of the DBHF self-energies are extrapolated to the CNM configurations and the mean field is constructed by a superposition of the contributions from the two currents. However, already this approximation invokes a self-consistency problem for the CNM configuration

\[ \Theta_{12} = \Theta_{1} + \Theta_{2} - \Theta_{1} \cdot \Theta_{2} \]  

composed by the two currents \( \Theta_{i} = \Theta(\mu^i(k_{Fi}^F) - k_{Fi}^F u^i) \). The effective mass \( M^* = M + \Sigma_{S_{12}} \), the scalar density \( \rho_{s_{12}} = < M^*/E^* >_{12} >_{\rho_{sat}} \) and the configuration (2) are coupled by non-linear equations. \( \Theta \) is the step function, \( k_{Fi} \) are the Fermi momenta and \( u^i = (\gamma_i, \gamma_i u_i) \) are the streaming velocities of the two subsystem currents. The last term in eq. (2) ensures that the Pauli principle is fulfilled for small velocities where the two ellipsoids might overlap. For details see [13].

\[ < X >_{12} = \frac{1}{(2\pi)^3} \int d^3 k X(k) \Theta_{12}(k) \]  

(3)

denotes the summation over all occupied states. In spin-isospin saturated nuclear matter the phase space occupancy factor is \( \kappa = 4 \). The energy momentum tensor in CNM is given by \( [3] \).

\[ T_{12}^{\mu\nu} = \kappa^2 k^{*\mu}/E^* >_{12} - V_{12}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \{ \Sigma_{S_{12}} \rho_{s_{12}} - V_{12}^{\lambda} \} \]  

(4)

with the scalar contribution

\[ \Sigma_{S_{12}} = \Sigma_{S_{12}}^{\mu\nu}/E^* >_{12} >_{\rho_{sat}} \]  

(5)

and the terms arising from the vector field

\[ V_{12}^{\mu\nu} = \Sigma_{12}^{\mu\nu} k^{*\nu}/E^* >_{12} \]  

(6)

The scalar self-energy \( \Sigma_{S_{12}} \) enters in a Hartree form averaged over the explicit momentum dependence into \( [3] \). In an analogous way a Hartree vector self-energy \( \Sigma_{V_{12}} = V_{12}^{\mu\lambda} j_{12}^{\lambda} / j_{12}^{2} \) can be defined. The full self-energy has the form \( \Sigma_{12}(k_{F_1}, k_{F_2}, u_1, u_2) = \Sigma_{S_{12}} + \gamma^i \Sigma_{U_{12}}^i \).

In contrast to the local density approximation where the self-energy is a function of the total Fermi momentum \( k_{F_{12}} \) the self-energy in CNM depends on the densities of the two subsystems and their streaming velocities\[ [3] [4].

In the following we will only consider the symmetric case \( (k_{F_1} = k_{F_2}) \). In Fig. 2 the equation-of-state’s in symmetric colliding nuclear matter are shown for streaming velocities of the subsystem currents \( u = |u_i| = 0.2/0.4/0.6/0.8 \) (in units of \( c \)) which correspond to incident laboratory energies \( E_{lab} = 0.08/0.36/1.05/3.34 \) GeV/nucleon. \( u = 0 \) is the isotropic case (ground state NM). The energy per particle is defined in the usual way as

\[ E_{12}(\varrho_{12}, u) = T_{12}^{00} / \varrho_{12} - M \]  

(7)

with \( \varrho_{12} = \sqrt{j_{12}^{2}} >_{12} \) |c.m. the invariant baryon density in the c.m. frame of the two currents, i.e. the frame where \( j_{12} = 0 \). At high streaming velocities the “EOS” is significantly stiffer then in ground state nuclear matter because the energy per particle \( E_{12} \) includes the contribution form the relative motion of the two currents. The general repulsive character of the momentum dependent part of the nuclear interaction leads to a strongly enhanced repulsive component which increases with an increasing amount of relative motion in the system.
FIG. 2. EOS in nuclear matter (solid) and colliding nuclear matter determined in the DBHF model. The upper part shows the total energy per particle $E_{12}$ as a function of the c.m. total density. The streaming velocities are $u = 0.2$ (dotted), $0.4$ (dashed), $0.6$ (long-dashed), $0.8$ (dot-dashed). The lower part shows the effective EOS, i.e. the binding energy per particle $E_{12}^{\text{bind}}$ where the kinetic energy of the relative motion in CNM has been subtracted.

However, a meaningful discussion of non-equilibrium effects with respect to the ground state EOS should be based on the binding energy and thus the contribution from the relative motion of the two currents has to be subtracted. This leads to an effective EOS in colliding nuclear matter which is directly linked to the hydrodynamical picture. To do so, we subtract the kinetic energy of a nucleon inside the medium $E_{\text{kin}}^{\ast} = E^{\ast} - M^{\ast} = \sqrt{k^{2} + M^{2}} - M$ calculated in CNM or NM, respectively, is thereby averaged of the corresponding configurations. Thus one obtains the binding energy per particle

$$E_{12}^{\text{bind}}(\varrho_{12}, u) = T_{12}^{00} - \mathcal{E}_{\text{rel}} - M$$

as a function of the total c.m. density $\varrho_{12}$ and the c.m. streaming velocity $u$. The binding energy, i.e. the effective EOS in colliding nuclear matter, is shown in the lower part of Fig. 2. The effective EOS appears softer and even more attractive compared to ground state nuclear matter. This is a general feature of colliding nuclear matter and can be understood by a very transparent and model independent argument:

Let us consider two currents with sufficiently high $u$, i.e. well separated Fermi ellipsoids in momentum space $(\Theta_{1} \cdot \Theta_{2} = 0)$. For the following discussion we assume that the self-energies have no explicit momentum dependence, like in relativistic mean field (RMF) or density dependent RMF theory. Then the energy density (in the c.m. frame) is given by

$$T_{12}^{00}(\varrho_{12}, u) = <E^{\ast}>_{12} - \frac{1}{2}(\Sigma_{S_{12}} \varrho_{s_{12}} + \Sigma_{0_{12}} \varrho_{12})$$

and the non-locality of the system, i.e. the high relative momenta of the separated ellipsoids enter only via the momentum dependence of $E^{\ast}$. The separation of projectile and target nucleons in momentum space increases the phase space volume since in both currents states are occupied up to $k_{F} = 0.5k_{F_{\text{sat}}}$. For a purely local interaction which is insensitive to the relative momenta of the currents this increase of phase space would have the same effect as the introduction of an additional virtual degree of freedom as illustrated in Fig. 3. Thus, in a mean field approach the effective binding energy can be approximated by a modification of the corresponding expression for single nuclear matter

$$E_{12}^{\text{bind}}(\varrho_{12}, u) \approx \left[<E^{\ast}>_{12} - \frac{1}{2}(\Sigma_{S_{12}} \varrho_{s_{12}} + \Sigma_{0_{12}} \varrho_{12}) \right] / \varrho_{1} - M$$

FIG. 3. Schematic representation of the phase space in nuclear matter (a), colliding nuclear matter (b) and in colliding nuclear matter as experienced by a local potential (c).
$\kappa = 8$ in Eq. (8). For the vector density this leads to a linear dependence $q_{1}(k_{F}, \kappa = 8) = 2q_{1}(k_{F}, \kappa = 4)$ which restores the total density. The dependence of the scalar density $q_{s}$, is non-linear. However, the total c.m. vector density in colliding nuclear matter is still enhanced by a $\gamma$-factor, i.e. $q_{2}(k_{F}, k_{F}, u) = \gamma(u)q_{1}(k_{F}, \kappa = 8)$ which does not completely cancel in the effective binding energy and leads to a stronger repulsion originating from the vector field as compared to the approximation \[1\]. In Fig.4 the corresponding EOS \[1\] is shown as obtained in the density dependent RMF approach \[21\] to the DBHF model. Here we neglect the explicit, but weak momentum dependence of the DBHF self-energies. Varying in these calculations $\kappa$ from 4 to 8 illustrates the phase space effects. There is, of course, no exact agreement of the simplified ansatz of Eq. \[1\] with full self-consistent CNM calculations (Fig.2), in particular at high densities, which is expected. With increasing density self-consistency effects between the effective mass $M^{*}$ and the configuration \[2\] itself become more important. A reduced in-medium mass $M^{*}$ leads to reduced momenta $k_{M}^{\mu} = u_{\mu}M^{*}$ at fixed velocities and to a shift of the Fermi ellipsoids. Thus at high densities Pauli blocking effects persist for interacting two-Fermi-ellipsoid configurations even at high relative velocities \[3\]. Furthermore, the relativistic mean field approximation is non-local through the momentum dependence via $E^{*}$ in \[1\]. However, it becomes clear from this comparison that the leading order effect for the EOS in colliding nuclear matter is the separation of phase space.

$$U(\varrho, \mathbf{k}) = U_{\text{loc}}(\varrho) + U_{\text{nonloc}}(\varrho, \mathbf{k})$$ (12)

are usually composed by a local, density dependent potential $U_{\text{loc}}(\varrho)$ and a non-local momentum dependent part $U_{\text{nonloc}}(\varrho, \mathbf{k}) = \int d^{3}k' f(k')V(k-k')$ with $V$ an effective two-body-interaction. In Hartree-Fock approximation these two terms correspond to the direct (Hartree) and the exchange (Fock) part of the potential. In ground state nuclear matter the Fock terms give usually small corrections to the EOS (mean field dominance) \[22\]. In colliding nuclear matter the non-local part of the interaction is responsible for the strong repulsion seen in Fig.2. Applying potentials of the form \[12\] in transport calculations for heavy ion collisions the Fock part $U_{\text{nonloc}}(\varrho, \mathbf{k})$ accounts by definition properly for the actual momentum space configurations $f(k')$, given e.g. by testparticle distributions. The mean field or local part is, however, decoupled from the anisotropy of the phase space since it is parameterized as a function of the total density. Consequently, $U_{\text{loc}}(\varrho)$ reflects a density dependence which is only correct in equilibrated nuclear matter but does not apply to anisotropic momentum space configurations.

To summarise, the equation of state probed by the compression phase in energetic heavy ion reactions is to large extent governed by local non-equilibrium. The corresponding separation of phase space can be regarded as the introduction of an effectively new degree of freedom which lowers the binding energy per particle and makes the effective EOS seen in heavy ion reactions significantly softer. We conclude that this “trivial” but leading order effect should be taken into account when conclusions on the EOS are drawn from heavy ion collisions.

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