Structures and turbulent statistics in a rotating plane Couette flow

Takahiro Tsukahara
Department of Mechanical Engineering, Tokyo University of Science, 2641 Yamazaki, Noda-shi, Chiba, 278-8510 Japan
E-mail: tsuka@rs.tus.ac.jp

Abstract. This paper presents direct numerical simulations of various turbulent states and structures in a rotating plane Couette flow. The formations of patterns or roll cells were investigated in a wide range of the rotation number $\Omega$, using a large domain size of $204.8 \times 2\delta \times 102.4\delta$. The controlling parameters are $Re = U_w\delta/\nu = 750$ and $\Omega = 2\Omega_z\delta^2/\nu$, where $U_w$ is half the wall velocity difference and $\Omega_z$ is the spanwise-rotation angular velocity. The simulations reproduced many key features in the experiment of Tsukahara et al. (JFM, 648: 5–33). For stabilizing rotation with $\Omega \leq -20$, a striped pattern of laminar-turbulent regions was observed. For destabilizing rotation with $\Omega = 30$, turbulent eddies seem to be contained in the three-dimensional roll cells. The wavy motion of the roll cells in the turbulent background reveals a $30\delta$ periodicity in the streamwise direction, and the spacing of the turbulent stripe is of the order of $200\delta$. A computational domain needs to be large enough compared with these structures induced by the system rotation to ensure reliable quantitative results.

1. Introduction

It is known that system rotation gives rise to a Coriolis force, which may be stabilizing or destabilizing, depending on the direction of rotation. If the mean flow vorticity is of the opposite sign with respect to the system rotation vector then the flow becomes unstable, whereas the flow becomes stabilized if they have the same sign. Studies of turbulence and transition in rotating reference frames have been undertaken by a number of researchers, in regard to, for example, rotating fluid-machinery applications or geophysical flows. Lezious & Johnston (1976) performed linear stability calculations for a spanwise rotating laminar channel flow—both for pressure-driven Poiseuille flow and plane Couette flow (PCF)—and investigated experimentally those flows further in turbulent regimes. Alfredsson & Persson (1989) reported the existence of longitudinal roll cells and their sequence in the spanwise direction and also showed secondary instabilities and transition to turbulence. Such a rotation-induced instability may occur at Reynolds numbers almost two orders of magnitude smaller than the value at the Tollmien-Schlichting waves would be unstable in the non-rotating case. Hiwatashi et al. (2007) studied the rotating PCF at low Reynolds numbers to confirm stable three-dimensional (3D) roll cells, which was predicted theoretically/numerically by Nagata (1998). As in the turbulent case, there are a few direct numerical simulation studies of this flow. Bech & Andersson (1996) carried out direct numerical simulations (DNSs) of weakly rotating turbulent PCF. They revealed that rotations in either direction damped the turbulence and that...
the destabilized flow was more energetic than the non-rotating flow due to the accompanying two-dimensional (2D) roll cells. The paper by Komminaho et al. (1996) was mainly devoted to the non-rotating case; however, they also showed that a turbulent flow can be stabilized and even relaminarized by rather weak negative rotation.

Recently, Tsukahara et al. (2010) reported the results of a systematic experimental investigation into the rotating PCF, and uncovered a rich variety of different flow regimes, as shown in figure 1. The flow-state diagram shows that, with increasing Reynolds number and/or angular speed, the flow undergoes a series of transitions from the laminar Couette flow, to spanwise periodic 2D roll cells, as previously predicted by Lezius & Johnston (1976), to a state with waves on the vortices in the form of 3D roll cells as reported by Hiwatashi et al. (2007), and further to chaotic and turbulent flow. Interestingly, even at much higher Reynolds numbers than the critical Reynolds number, some multiplicity appeared depending on the system rotation: i.e. when the destabilizing rotation is imposed, a tertiary flow in the form of a 3D roll-cell state occurs via a secondary flow with a 2D roll cell; under the stabilizing rotation, the elongated roll cell is found to be damped. However, those flow states were distinguished on the basis of the visualization by adding reflective flakes (tiny platelet particles) into the working fluid. The approach of DNS is a promising alternative for clarifying the detailed velocity fields and structures in the turbulent flows.

The present study is aimed at investigating the flow patterns (organized structures) in the turbulent PCF subjected to system rotation, as well as giving accurate turbulent statistics.

2. Numerical procedure

The direct numerical simulations (DNSs) of PCF with system rotation, as in figure 2, performed here were continuations of the non-rotating PCF simulation reported by Tsukahara et al. (2006). We used a finite-difference code to advance the incompressible Navier-Stokes equations in time. The boundary conditions are periodic in streamwise ($x$) and spanwise ($z$) and no-slip at the walls ($y = \pm \delta$). All present DNS were run at a Reynolds number of $Re = U_w \delta / \nu = 750$, based
on half the velocity difference between the walls $U_w$, the channel half width $\delta$, and the kinematic viscosity $\nu$. The additional control parameter is a rotation number $\Omega = 2\Omega_z\delta^2/\nu$, where $\Omega_z$ is the rotation speed along the spanwise ($z$) axis. In some earlier studies, a rotation number defined as $Ro = 2\Omega_z\delta/U_w = \Omega/Re$ was alternatively used; however, we chose $\Omega$ to characterize the effect of rotation for the sake of easy comparison with the previous experimental work (Tsukahara et al., 2010). Because of the negative mean-flow spanwise vorticity throughout the channel ($\omega_z = -\partial u/\partial y < 0$), negative $\Omega$ means cyclonic background vorticity, while positive rotation numbers denote anticyclonic background vorticity. The former case gives rise to stabilization of the flow.

The details regarding computational conditions are summarized in table 1. Seven different values of $\Omega$ were tested: the spatial resolutions for the simulations at these rotation numbers were constant with respect to the channel width, but they slightly altered in terms of the viscous wall unit depending on $\Omega$, as shown in the table. For instance, the horizontal grid spacing is $0.1\delta$ for every case, which corresponds to $3.6(\nu/u_\tau)$ for $\Omega = -45$ and $6.5(\nu/u_\tau)$ for $\Omega = 45$. Here, $u_\tau$ is the friction velocity. These resolutions can be considered to be small enough that the dependence on the spatial and temporal resolutions can be neglected. A non-uniform grid was applied in the $y$-direction. The computational box was relatively large as compared to DNSs in the literature (Bech & Andersson, 1996; Komminaho et al., 1996; Tsukahara et al., 2006).

The initial flow field was constructed from a featureless turbulent non-rotating PCF at $Re = 750$. Once the rotation number was changed to the intended value with the fixed Reynolds number (see figure 1 for the tested conditions), statistical data and visualized fields were obtained after a flow field had reached statistical-steady state. In each case, we monitored the signal of the wall shear stress on both walls during the simulation, and ensured that the flow had reached a statistically stationary state.

| Table 1. Computational conditions. |
|-----------------------------------|
| Reynolds number, $Re$             | 750 |
| Rotation number, $\Omega$         | 0, ±20, ±30, ±45 |
| Domain size, $(L_x \times L_y \times L_z)$ | $204.8\delta \times 2\delta \times 102.4\delta$ |
| Grid points, $(N_x \times N_y \times N_z)$ | $2048 \times 64 \times 1024$ |
| Grid spacing in $x$ and $z$ directions | 3.6–6.5 |
| Minimum grid spacing in $y$ direction | 0.14–0.26 |
| Maximum grid spacing in $y$ direction | 2.00–3.64 |
| Time increment                      | 0.0086–0.029 |
3. Result and discussion

3.1. Structures of 3D roll cells in turbulent background and turbulent stripe

In this subsection, we investigate the flow structures in rotating turbulent PCF in detail. Emphasis is placed on the discussions of large-scale organized structures rather than of small-scale turbulent structures near walls and of the related effects. Figures 3–6 depict typical snapshots of instantaneous flow fields, which were obtained for different values of the rotation number, but at the same Reynolds number. Note that the flow fields visualized here are pieces of the computational domain for the sake of making it easy to observe the relationship between large-scale structures and small-scale eddies (or detailed velocity field). Here, the velocity fluctuations are normalized by $U_w$, while the value of $II'$, the second invariant of the velocity gradient tensor, is by $u_\tau$ and $\nu$. The superscript of ‘+’ denotes a dimensionless quantity in wall units.

To explore the structural characteristics of the roll cells and small-scale turbulent eddies in three dimensional space, we plot in figure 3 the iso-surfaces of $II'^+$ and the contour of the streamwise velocity fluctuation in an $x$-$z$ plane, and in figure 5 the contour of $II'^+$ and the velocity vectors in a cross-section, for positive (destabilizing) rotations at $\Omega = 20$ and 30. For the non-rotating turbulent case, it is well known that the central part contains large-scale structures (high/low speed regions) extending over a streamwise distance longer than twenty or thirty times the channel width (Komminaho et al., 1996; Tsukahara et al., 2006): see figure 4. These high/low momentum regions of $u'$ are mostly induced by elongated streamwise vortical
motions and allow us to estimate the size and pattern of the vortical motions. The structures are of finite size and not stationary in either time or in space, and thereby the clear roll cells mentioned later were not observed in the flow without system rotation. Moreover, small-scale turbulent eddies, indicated by green iso-surfaces, seem randomly distributed in the horizontal directions. When destabilizing rotation was imposed, the flow was found to have meandering (or infinite) streaks due to the coexistence of 2D or 3D roll cells, as shown in figure 3. In particular, vortices in the flow at $\Omega = 30$ are unevenly distributed and clustered between streaks and the flow may be considered to contain turbulence in the roll cells'. Figure 5 shows the cross-sectional view of the flow fields. Vortical motions of the size of the channel width can be clearly observed even in the instantaneous flow field at $\Omega = 30$, as marked by dotted circles in figure 5(b). In this flow, small-scale eddies exhibit an uneven distribution in the spanwise direction. They are observed to concentrate in several large-scale vortical motions, corresponding to the inside of the roll cells. As for the weaker rotation at $\Omega = 20$, the distribution of vortices is revealed to be intermittent compared to the non-rotating case according to figure 3(a), but is different from those for $\Omega = 30$, that is, the large-scale vortical motions are not clearly observed in figure 5(a). Instantaneously, there appears to be a larger population of turbulent eddies and they are more randomly distributed in the spanwise direction. In terms of the flow state, these findings are in accordance with experimental observations (Tsukahara et al., 2010).

When subjected to stabilizing system rotation, the flows for $\Omega \leq -20$ exhibit the stripe pattern of oblique bands, which stretch across the channel and consist of a periodic alternation of turbulent and laminar bands, as shown in figure 6. The stripe inclination angle seems to be unchanged with enhanced rotation effect, although its orientation changes from case to case (depending on the initial condition). Small-scale streamwise streaks as well as eddies (not visualized in the figure) appear within the large-scale turbulent bands. Those streaks appear to be elongated with an increase in the negative rotation number from $\Omega = -20$ to $-30$. It can be conjectured from figure 6(b) that, with a stronger stabilizing effect, the distance between turbulent bands should become larger so that the area of laminar region increases and finally the flow will be relaminarized.

3.2. Two-point correlation and pre-multiplied energy spectra
The lengths of those large-scale structures caused by the system rotations were also examined quantitatively using the means of the two-point velocity correlation and energy spectra analysis.
Figure 6. Same as figure 3, but for negative rotations: contour shows $u'/U_w \geq 0.3$ (red) and $u'/U_w \leq -0.3$ (blue). The field shown here is a quarter portion of the computational domain, namely, of $L_x/2 \times L_z/2$.

Figure 7. Two-point correlation coefficient of the streamwise velocity fluctuation as a function of the streamwise direction at the channel center.

Figure 7 shows the streamwise two-point correlation for the streamwise velocity component (that is, $u'$) with respect to streamwise distance $\Delta x$ at the channel center, which is defined as
the following:

\[ R_{uu}(\Delta x) = \frac{u'(x, \delta, z)u'(x + \Delta x, \delta, z)}{u'_{\text{rms}}^2}, \]  

(1)

where \( u'_{\text{rms}} \) denotes the root-mean-square value of \( u' \), i.e., the streamwise turbulence intensity. If the computational box length is long enough compared to intrinsic structures in the turbulent flow, the correlation generally falls to zero at long distance, as in the present non-rotating case (\( \Omega = 0 \)): see figure 7(a). There is a periodic component present in the destabilized flows (under positive system rotation) with a period of approximately 30\( \delta \). This streamwise wavelength is very consistent with those obtained by the experimental visualization using flake particles (Tsukahara et al., 2010). Visualizations by the present DNS show that the periodicity is probably due to a meandering pattern of positive and negative velocity fluctuations (figure 3). As for the stabilized flow, \( R_{uu} \) falls to negative values at the maximum separation (half the domain size) for both cases of \( \Omega = -20 \) and \( -30 \). This shows that one long-wavelength structure is captured with the computational box, although several turbulent bands seem to appear instantaneously in the flow at \( \Omega = -20 \), as shown in figure 6(a). A further elongation of the domain should be required to achieve the decorrelation at a long streamwise distance.

The pre-multiplied energy spectra \( k_x E_{uu}(k_x) \), based on which the contained energy of each wavelength is readily evaluated, are given in figure 8 as a function of wavelength, \( \lambda_x = 2\pi/k_x \). Note that a peak position of the pre-multiplied energy spectra corresponds to a mean spacing of
energetic structure. For all of the present simulations, a peak is located at around 10δ (marked as (i) in the figure), which is approximately the same size as the well-known near-wall streaks. With respect to the large-scale motions such as the roll cells and the stripes, they appear as an additional spike at a long wavelength in the spectra. Figure 8(a) shows the spike of (ii) at λx = 29δ, which should be related to the length of the 3D roll cells, while the peak of (iii) at λx = 205δ in figure 8(b) reveals the spacing of the turbulent stripe. It is interesting to note that the (ii)-wavelength has more energy for Ω = 20 than for Ω = 30. As for the negative rotation, the (iii)-wavelength was found to be dominant when the rotation rate was increased. Although not shown in this paper, the same tendency can be assumed for the spanwise direction.

3.3. Various turbulent statistics

Figure 9 shows the change in the mean velocity profile due to system rotation. Also shown is the profile obtained by Komminaho et al. (1996), who employed the spectral method, and this coincides well with our result. The apparent effect is the change of the mean shear in a wide region away from the walls. In positive rotations for Ω ≥ 20, the shear rate ∂u/∂y at the channel center is very small, with nearly-zero absolute vorticity. On the other hand, the profile for negative rotation comes close to the linear velocity profile. Figure 10 shows the variation of the friction Reynolds number Reτ = μτδ/ν as Ω is
successively changed. Although Komminaho et al. (1996) reported a monotonic decrease in $Re_\tau$ as a function of $\Omega$, a fitted curve based on the present results deviates from that and exhibits more complex dependence. It is difficult to use the fitting curve to estimate the lowest value of $\Omega$, at which turbulence exists in the rotating PCF, but at least it can be conjectured that turbulence would persist locally even at $\Omega = -50$ for $Re = 750$. The obtained values of $Re_\tau$ for $\Omega = -30$ and $-45$ are apparently far from the laminar-flow value. The disagreement between the present and earlier DNSs may be due to the difference of the domain size. Komminaho et al. (1996) used the computational domain of $28\pi\delta \times 2\delta \times 8\pi\delta$, which has been found to be insufficiently large to capture the turbulent stripe.

The effect of positive system rotation is to enhance the turbulence intensities (both of $u'^{+}_{rms}$ and $w'^{+}_{rms}$) near the walls as can be readily seen in figure 11. An increase of turbulence intensity throughout the channel is also found in the case of $\Omega = -30$. This is probably due to the influence of the turbulence stripe. Once the stripe pattern of turbulence appears, the flow field is accompanied by the secondary flow, including non-zero mean spanwise velocity (figure not shown). To ensure reliable quantitative results, the intrinsic phenomena, such as roll cells and turbulent stripes, must be captured.

4. Concluding remarks

In the present paper, we have focused on the large-scale structures and spatial intermittency in the rotating plane Couette flow at $Re = 750$ and on their influences on turbulence statistics. We demonstrated various flow states by changing the rotation number and confirmed consistency with an experimental observation by Tsukahara et al. (2010).

In destabilized flows with positive rotations, the in-plane distribution of the streamwise velocity exhibits the appearance of large-scale (infinite-length) streaks meandering with a wavelength of about $30\delta$. This wavelength is a few times longer than that of 3D roll cells in laminar flows predicted by Nagata (1998). In view of the energy contained in different wavelengths of the velocity fluctuation, the energy of the roll cells is most intense at the moderate rotation $\Omega = 20$, while it is reduced, as is shorter-wavelength energy (turbulent eddies), for high rotation numbers. As observed in the experimental observation, for $Re = 750$ and $\Omega = 30$, most turbulent eddies are found to exist inside the roll cells by observing a cross-sectional view of the flow field.

As for stabilized flows, we found that the turbulence stripe occurred spontaneously and maintained itself. The streamwise wavelength of the stripe pattern, normalized by the channel half width, was slightly larger than those in a non-rotating flow at low Reynolds numbers (Tsukahara et al., 2009; Duguet et al., 2010), while the inclination angle of the pattern seemed to be unchanged. In terms of turbulence statistics, the use of a large computational domain is crucial, because it allows us to reproduce the spatio-temporal intermittency structures of the turbulent stripe. The patterned flow can sustain itself as locally turbulent for higher negative rotation numbers than those previously reported by Komminaho et al. (1996), resulting in a larger frictional coefficient compared to the laminar value.

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