Effect of interlayer exchange coupling in spin-torque nano oscillator

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(Dated: 16 August 2022)

The dynamics of the magnetization of the free layer in a spin-torque nano oscillator (STNO) influenced by a noncollinear alignment between the magnetizations of the free and pinned layers due to an interlayer exchange coupling has been investigated theoretically. The orientations of the magnetization of the free layer with that of the pinned layer have been computed through the macrospin model and they are found to match well with experimental results. The bilinear and biquadratic coupling strengths make the current to switch the magnetization between two states or oscillate steadily. The expressions for the critical currents between which oscillations are possible and the critical bilinear coupling strength below which oscillations are not possible are derived. The frequency of the oscillations is shown to be tuned and increased to or above 300 GHz by the current which is the largest to date among nanopillar-shaped STNOs.

I. INTRODUCTION

Since the discovery of the spin transfer torque (STT) effect by Slonczewski and Berger, tremendous and continuous interest has been shown in the study of magnetization in STT-based memories because of its potential applications in magnetic memory technologies. Compared to the conventional STT, creating and detecting spin polarized currents in the bilayer structures, consisting of heavy metal and ferromagnetic layers, and trilayer structures where the ferromagnetic layers get separated by a non-magnetic layer, are important for the study of frequency enhancement, energy efficiency, fast magnetization switching, and spintronic-based neuromorphic computing systems. In recent times, modern spintronic devices utilize multilayer structures where the coupling between the magnetic layers has to be controlled to get enhanced magnetic properties. In an STNO the tunability in the thickness of the 3d, 4d and 5d nonmagnetic metallic spacer layer leads to ferromagnetic or antiferromagnetic coupling between the free and pinned ferromagnetic layers. This interlayer exchange coupling has been extensively investigated since 1980s. Recent results in this context show that the presence of exchange coupling in a magnetic layer structure is mostly applicable to modern STT magnetic random-access memory (MRAM) and sensor devices. In particular, the study of the interlayer exchange coupling effect in synthetic magnetic multilayer systems might provide a general strategy for beyond-CMOS electronic devices.

Very recently, Nunn et al. have reported that a noncollinear alignment between the magnetizations of the free and pinned layers can be achieved and precisely controlled when they are coupled across a spacer layer consisting of a non-magnetic material (Ru) alloyed with a ferromagnetic material (Fe). This noncollinear alignment happens due to the biquadratic coupling which can be tuned by changing the composition and thickness of the spacer layer. This Co/RuFe/Co structure is useful for the noncollinear spintronic design with magnetic moments tilted to the film plane. Nevertheless, there is a lack of understanding about the impact of noncollinear alignment on the dynamics of the magnetization of the free layer in an STNO which is yet to be understood.

Therefore, in this paper, we theoretically investigate the influence of the noncollinear alignment due to the bilinear and biquadratic couplings in a Co/RuFe/Co STNO by numerically solving the underlying Landau-Lifshitz-Gilbert-Slonczewski (LLGS) equation. Our investigations confirm the existence of switching and high-frequency oscillations of the magnetization of the free layer due to current, in the presence of the bilinear and biquadratic exchange couplings. These studies open up new possibilities for the applications which utilize the current induced switching and high-frequency oscillations due to interlayer exchange coupling for the development of spintronic devices and neuromorphic computing devices. Sec. II addresses the governing equation, namely the Landau-Lifshitz-Gilbert-Slonczewski (LLGS) equation, for the magnetization dynamics of the free layer in Co/RuFe/Co STNO along with the bilinear and biquadratic exchange couplings. The current induced switching, oscillations of the magnetization of the free layer and its tunability by current and interlayer exchange coupling along with the expressions for the critical values of current and bilinear exchange coupling for the possibility of oscillations are presented in Sec. III. Finally, concluding remarks are made in Section IV. In Appendix A we have presented a methodology to identify the equilibrium orientation of the magnetization in the absence of the current using a microwave field. In Appendix B, the derivation for the critical values of the current and finally in Appendix C, we have discussed the output power and its enhancement.
II. MODEL

We consider an STNO with a spacer layer Ru$_{100-x}$Fe$_x$ consisting of a nonmagnetic material (Ru) alloyed with a ferromagnetic material (Fe). This layer is sandwiched between two ferromagnetic layers, which includes a free layer where the direction of the magnetization can change and a pinned layer where the magnetization is fixed along the positive $x$-direction. The plane of the layers is considered to be aligned with the $xy$-plane. The dynamics of the unit magnetization vector of the free layer $\mathbf{m}$ is given by the LLGS equation

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{eff} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt} + \gamma H_S \mathbf{m} \times (\mathbf{m} \times \mathbf{p}),$$

(1)

In Eq. (1), $\mathbf{m} = m_x \mathbf{e}_x + m_y \mathbf{e}_y + m_z \mathbf{e}_z$ or $\mathbf{m} = (m_x, m_y, m_z)$, $|\mathbf{m}| = 1$. Here, $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit vectors along $x$, $y$ and $z$ directions, respectively. In spherical polar coordinates $m_x = \sin \theta \cos \phi$, $m_y = \sin \theta \sin \phi$ and $m_z = \cos \theta$, where $\theta$ and $\phi$ are polar and azimuthal angles, respectively. $\gamma$ is the gyromagnetic ratio, $\alpha$ is the damping parameter and $H_S = H_{S0}/(1 + \lambda \mathbf{m} \cdot \mathbf{p})$.

where $H_{S0} = \hbar I/2eM_s V$ and $\mathbf{p} = e \mathbf{e}_z$. Here $I$ is the current passing through the free layer, $\hbar(=h/2\pi)$ is the reduced Plank’s constant, $e$ is the electron charge, $M_s$ is the saturation magnetization and $V$ is the volume of the free layer. The dimensionless parameters $\eta$ and $\lambda$ determine the magnitude and angular dependence of the spin-transfer torque. The energy density of the free layer is given by

$$E = \frac{J_1}{d} \mathbf{m} \cdot \mathbf{p} + \frac{J_2}{d} (\mathbf{m} \cdot \mathbf{p})^2 - \frac{M_s^2}{2} [H_k - 4\pi M_s] (\mathbf{m} \cdot \mathbf{e}_z)^2$$

(2)

and the effective magnetic field $\mathbf{H}_{eff} = -\partial E/\partial (M_s \mathbf{m})$ of the free layer is

$$\mathbf{H}_{eff} = (H_k - 4\pi M_s) m_z \mathbf{e}_z - \frac{1}{dM_s} (J_1 + 2J_2 m_x) \mathbf{e}_x,$$

(3)

where $J_1$ and $J_2$ are the bilinear and biquadratic exchange coupling constants, respectively. The effective field $\mathbf{H}_{eff}$ includes the field due to bilinear and biquadratic couplings, magneto-crystalline anisotropy field $H_k$ and demagnetization field $4\pi M_s$. In Eqs. (2) and (3), $d$ is the thickness of the free layer. The material parameters are considered as $M_s = 1210$ emu/c.c., $H_k = 3471$ Oe, $\eta = 0.54$, $\lambda = \eta^2$, $d = 2$ nm, $A = \pi \times 60 \times 60$ nm$^2$, $V = Ad$, $\alpha = 0.005$ and $\gamma = 17.64$ Mrad/(Oe s)$^{9–11,21,22}$. Here $A$ is the cross sectional area of the STNO. In this paper, $J_1$ is restricted to be positive, corresponding to the Fe concentration below 75% in the spacer, since oscillations are not observed for negative $J_1$.

III. RESULTS AND DISCUSSION

Before applying any current the magnetization would settle in the $xy$-plane at $(m_{x0}, m_{y0}, m_{z0}) = (-1, 0, 0)$ when $J_1 \geq 2J_2$ or at $(J_1/2J_2, \sqrt{1 - J_1^2/4J_2^2}, 0)$ when $J_1 < 2J_2$. This is confirmed in Fig. 1(a), where the time evolution of $m_x$ is plotted for $J_1 = 4.97$ mJ/m$^2$ and $J_2 = 0.919$ mJ/m$^2$ for two different initial conditions and we can see that finally, $m_x$ reaches the state $(\mathbf{1}, 0, 0)$ (see the inset of Fig.1(a)) irrespective of the initial condition. Similarly, the polar angle $\phi_0(= \cos^{-1}(m_{x0}))$, corresponding to $(m_{x0}, m_{y0}, m_{z0})$ is plotted for different sets of $J_1$ and $J_2$ in the absence of current and marked by downhill black triangles in Fig. 1(b). The values of $\phi_0$ can also be determined using microwave field [see Appendix A]. The red upward triangles are the experimental values measured by Nunn et al corresponding to the spacer layer Ru$_{30}$Fe$_{70}$ which confirms the validation of the macrospin simulation.$^{21}$

In the presence of current, the magnetization leaves the $(m_{x0}, m_{y0}, m_{z0})$ state and exhibits three kinds of dynamics depending upon the values of $I$ and $J_1$ (or $J_2$). The magnetization $\mathbf{m}$ may continue in the same initial state,
or switch from one to another state or oscillate. The magnetization $\mathbf{m}$ switches to $(1, 0, 0)$ when $|I| > |I_{c,\text{max}}|$ or oscillates when $|I_{c,\text{min}}| < |I| < |I_{c,\text{max}}|$. When $|I| < |I_{c,\text{min}}|$ it continues to exist in $(-1, 0, 0)$ for $J_1 \geq 2J_2$ or switches to $(m_{x1}, m_{y1}, m_{z1}) \approx (m_{x0}, m_{y0}, m_{z0})$ for $J_1 < 2J_2$. Here $I_{c,\text{min}}$ and $I_{c,\text{max}}$ are the critical currents between which oscillations are possible, which can be derived as [see Appendix B for more details]

$$I_{c,\text{max}} = \frac{-\alpha e A (1 + \lambda)}{h \eta} [2(J_1 + 2J_2) + (H_k - 4\pi M_s) d M_s],$$

$$I_{c,\text{min}} = \begin{cases} \frac{-\alpha e A (1 - \lambda)}{h \eta} [2(J_1 - 2J_2) - (H_k - 4\pi M_s) d M_s], & \text{for } J_1 \geq 2J_2, \\ \frac{-\alpha e A (2J_2 - J_1) \lambda^2}{h \eta J_2} \left\{ \frac{2J_2 d M_s (H_k - 4\pi M_s - (4J_2 - J_1)^2 \sqrt{\lambda(J_2 - J_1)^2 - 2J_2 (2J_2 - J_1) \lambda})}{4J_2 (2J_2 - J_1) \lambda} \right\}, & \text{for } J_1 < 2J_2. \end{cases}$$

In Figs. 2(a-d), the time evolution and trajectories of $\mathbf{m}$ corresponding to the oscillations and switching are plotted for $J_1 = 4.97 \text{ mJ/m}^2$ and $J_2 = 0.919 \text{ mJ/m}^2$. The values of $I_{c,\text{max}}$ and $I_{c,\text{min}}$ are calculated as -2.214 mA and -1.026 mA from Eqs. (1) and (5), respectively. When the current is below -1.026 mA the magnetization continues to get settled at (-1,0,0) as shown in the inset of Fig. 2(a). When $I = -1.5$ mA, the magnetization exhibits oscillations as shown in Figs. 2(a-b) and when $I = -5$ mA it switches from $m_x = -1$ to 1 as shown in Figs. 2(c-d). Note that the oscillations (inset of Fig. 2(a)) are achieved due to the balance between the energy supplied by STT and the energy dissipated by damping. The switching time, that is the time for $m_x$ to cross from -1 to +1, is plotted against current for different values of $J_1$ and $J_2$ in Fig. 2(e). The switching current, that is the current above which the switching takes place, can be obtained from Eq. (4). This equation implies that the switching current increases with $J_1$ and $J_2$. Also, we can see that at a given value of the current the switching time decreases with a decrease of $J_1$ or $J_2$.

As we discussed before, oscillations are exhibited for the currents between $I_{c,\text{min}}$ and $I_{c,\text{max}}$. To discuss the impact of the current on the frequency for different values of $J_1$ and $J_2$, current versus frequency is plotted in Fig. 2(a) for $J_1 < 2J_2$. The oscillations of $m_x$ is shown as inset of Fig. 2(a) when $I = -3.5$ mA, $J_1 = 5 \text{ mJ/m}^2$ and $J_2 = 4.5 \text{ mJ/m}^2$. From Fig. 2(a) we can understand that as the value of $J_1$ decreases the minimum current required to achieve the oscillations increases while when $J_2$ increases the maximum frequency attainable by the current also increases. Also, we can observe that the frequency can be increased by the current up to or above 300 GHz, which is the largest value achieved to date for the nanopillar shaped STNOs. To investigate the influence of $J_1$ on the frequency, Fig. 2(b) is plotted for the frequency when $J_2 = 5 \text{ mJ/m}^2$, where the nonoscillatory regions are represented by white color which includes two steady states $(m_{x1}, m_{y1}, m_{z1})$ for $J_1 < 2J_2$ and $(-1, 0, 0)$ without any oscillations. Inset in Fig. 2(b) is plotted for the frequency $(a-b)$ and when $J_2$ is plotted as open circles and triangles in Fig. 2(b), respectively, match well with the numerical results. From Fig. 2(b) we can observe that there is a critical value for $J_1$ where $I_{c,\text{max}} = I_{c,\text{min}}$, namely $J_1^c$, below which there occur no oscillations due to the current. The value of $J_1^c$ can be determined by equating the Eqs. (4) and (5) as

$$J_1^c = \frac{1}{3\lambda^2} \left( 2J_2 \lambda (3 + 2\lambda) - (Q + \sqrt{S})^{1/3} + \frac{J_2 \lambda^3 R}{(Q + \sqrt{S})^{1/3}} \right),$$

where $P = (\lambda - 1) d M_s (H_k - 4\pi M_s), Q = J_2^3 \lambda^3 (-9P\lambda(3+\lambda)-4J_2(-27+\lambda(-27+\lambda(27+43\lambda))))$, $R = 3P - 16J_2\lambda$ and $S = J_2^3 R^3 + Q^2$. As confirmed in Fig. 2(b), the value of $J_1^c$ is 4.27 mJ/m² for $J_2 = 5 \text{ mJ/m}^2$. For $J_1 < J_1^c$, the increase in current drives the magnetization to switch from $(m_{x1}, m_{y1}, m_{z1})$ to (1, 0, 0) without any oscillations. For $J_1^c < J_1 < 2J_2$, the bilinear coupling decreases the value of $I_{c,\text{min}}$ and increases the value of $I_{c,\text{max}}$, implying that the tunability range of the frequency increases. The output power of the STNO corresponding to $I = -4$ mA, $d$.

FIG. 2. Time evolution of $m_x$ and trajectory of $\mathbf{m}$ for (a-b) $I = -1.5$ mA and (c-d) $I = -5$ mA. Here, $J_1 = 4.97 \text{ mJ/m}^2$ and $J_2 = 0.919 \text{ mJ/m}^2$. (e) Variation of switching time versus current for different values of $J_1$ and $J_2$. Inset in (a) corresponds to the oscillations for the time between 19.95 ns and 20 ns.
We have shown that in the presence of bilinear and biquadratic couplings the frequency can be enhanced above 300 GHz by the current. We believe that the experimental achievements on noncollinear alignments in STNO may provide greater avenue for applications related to microwave generation and current induced switching.

ACKNOWLEDGMENTS

The works of V.K.C. and R. G are supported by the DST-SERB-CRG Grant No. CRG/2020/004353 and they wish to thank DST, New Delhi for computational facilities under the DST-FIST programme (SR/FST/PS1/2020/135) to the Department of Physics. M.L. wishes to thank the Department of Science and Technology for the award of a DST-SERB National Science Chair under Grant No. NSC/2020/00029 in which R. Arun is supported by a Research Associateship.

Appendix A: Identification of the orientation of m in the absence of current by using microwave field

Before applying any current, the magnetization would settle in the xy-plane at \( (\mathbf{m}_{x0}, \mathbf{m}_{y0}, \mathbf{m}_{z0}) = (-1, 0, 0) \) when \( J_1 \geq 2J_2 \) or at \( (-J_1/2J_2, \sqrt{1 - J_1^2/4J_2^2}, 0) \) when \( J_1 < 2J_2 \). The spherical polar coordinates corresponding to \( (\mathbf{m}_{x0}, \mathbf{m}_{y0}, \mathbf{m}_{z0}) \) are given by \( (\phi_0, \theta_0) = (\cos^{-1}(\mathbf{m}_{x0}), \pi/2) \). In Fig.1(b) of the main text, the values of \( \phi_0 \) for different values of \( J_1 \) and \( J_2 \) have been plotted (black downward triangles) from the time evolution of \( m_x \) as shown in Fig.1(a). The polar angle \( \phi_0 \) where the magnetization would settle before applying any current can also be determined by applying a microwave field perpendicular (along z-axis) to the plane of the free layer. Let the effective field that includes the microwave field along positive z-direction be

\[
\mathbf{H}_{\text{eff}} = (\mathbf{H}_k - 4\pi M_s \mathbf{m}_z + H_a \sin(2\pi ft)) \mathbf{e}_z - \frac{1}{dM_s}(J_1 + 2J_2m_x) \mathbf{e}_x, \quad (A1)
\]

where the amplitude of the perpendicular microwave field \( H_a = 100 \text{ Oe} \) and the frequency of the microwave field \( f = 1 \text{ GHz} \). When the microwave field is applied perpendicular to the plane, the magnetization \( \mathbf{m} \) will get perturbed about \( \phi_0 \) from \( \phi_0 - (\Delta \phi/2) \) to \( \phi_0 + (\Delta \phi/2) \), where \( \Delta \phi \) is the angular range of the perturbation. This perturbation of \( \mathbf{m} \) can be projected along any unit vector \( \mathbf{m}' = \cos \psi \mathbf{e}_x + \sin \psi \mathbf{e}_y \) along the direction \( \psi \) in the xy-plane. The component of the projected \( \mathbf{m} \) along \( \mathbf{m}' \) is given by \( m_y = \mathbf{m} \cdot \mathbf{m}' = m_x \cos \psi + m_y \sin \psi \). The range of the projected \( \mathbf{m} \) along \( \mathbf{m}' \) can be determined as \( \Delta m_p = m_{p \text{ max}} - m_{p \text{ min}} \), where the \( m_{p \text{ max}} \) and \( m_{p \text{ min}} \) are the maximum and minimum values of the projection \( m_p \), respectively. Since the magnetization will get perturbed in the perpendicular direction to its initial orientation, the
value of $\Delta m_p$ will be minimum when $\psi = \phi_0$ and maximum when $\psi = \phi_0 - 90^\circ$. The values of $\Delta m_p$ against $\psi$ for different pair of bilinear and biquadratic exchange coupling constants $(J_1, J_2)$ mJ/m$^2$ are plotted in Fig. 4.

The minimum values of $\Delta m_p$ indicates the angle of orientation of the magnetization $\phi_0$ in the absence of current. From Fig. 4, we can identify the values of $\phi_0$ corresponding to $(J_1 = 2.56, J_2 = 3.51)$ mJ/m$^2$, $(4.1,2.74)$ and $(3.87,2.03)$ as $111^\circ$, $138^\circ$ and $162^\circ$, respectively. The plot corresponding to $(0.948,0.38)$ indicates two minimum values of $\phi_0$ as $0^\circ$ and $180^\circ$. Since the positive value of $J_1$ leads to antiferromagnetic coupling as we mentioned in the main text, $\phi_0$ is $180^\circ$ for $J_1 = 0.948$ mJ/m$^2$ and $J_2 = 0.38$ mJ/m$^2$. These values of $\phi_0$ exactly match with the experimental and numerical values of $\phi_0$ plotted in Fig.1(b) of the main text.

Appendix B: Determination of critical currents

In this appendix, we briefly explain the procedure to determine the minimum and maximum critical currents between which the STNO exhibits the magnetization oscillations. Let us consider the LLGS equation (Eq.(1)) transformed into the spherical polar co-ordinates by using the transformation equations $m_x = \sin \theta \cos \phi$, $m_y = \sin \theta \sin \phi$, $m_z = \cos \theta$:

$$\frac{d\theta}{dt} = \frac{\gamma}{1 + \alpha^2} \left[ -P(\alpha \cos \theta \cos \phi - \sin \phi) - \alpha(H_k - 4\pi M_s) \sin \theta \cos \theta - H_{SO}(\alpha \sin \phi + \cos \theta \cos \phi) \right] / (1 + \lambda \sin \theta \cos \phi) = M(\theta, \phi),$$

$$\frac{d\phi}{dt} = \frac{\gamma \csc \theta}{1 + \alpha^2} \left[ P(\cos \theta \cos \phi + \alpha \sin \phi) + (H_k - 4\pi M_s) \sin \theta \cos \phi + H_{SO}(\sin \phi - \alpha \cos \theta \cos \phi) \right] / (1 + \lambda \sin \theta \cos \phi) = N(\theta, \phi).$$

Here $\theta$ and $\phi$ are the polar and azimuthal angles, respectively, $H_{SO} = \hbar n J/(2eM_V)$ and $P = (J_1 + 2J_2 \sin \theta \cos \phi)/dM_s$. The nature of the stability of a fixed point $(\theta^*, \phi^*)$, where $M(\theta^*, \phi^*) = N(\theta^*, \phi^*) = 0$, can be identified from the eigen values of the Jacobian matrix,

$$J = \begin{pmatrix} \frac{dM}{d\theta} |_{(\theta^*, \phi^*)} & \frac{dM}{d\phi} |_{(\theta^*, \phi^*)} \\ \frac{dN}{d\theta} |_{(\theta^*, \phi^*)} & \frac{dN}{d\phi} |_{(\theta^*, \phi^*)} \end{pmatrix},$$

associated with Eqs. (B1) and (B2) corresponding to $(\theta^*, \phi^*)$. The fixed point $(\theta^*, \phi^*)$ will be stable only when the real parts of both the eigen values are negative. According to Routh-Hurwitz’s criterion the real parts of both the eigen values will be negative if and only if the trace of the matrix $J$ becomes negative,

$$\text{Tr}(J) < 0.$$ 

In the absence of the current $(\theta^*, \phi^*) = (\pi/2, \cos^{-1}(-J_1/2J_2))$ for $J_1 < 2J_2$ or $(\pi/2, \pi)$ for $J_1 \geq 2J_2$.

When $|I| < |I_{c_{\text{min}}}^0|$ and $J_1 < 2J_2$ the $(\theta^*, \phi^*)$ is approximately equal to $(\pi/2, \cos^{-1}(-J_1/2J_2))$ and becomes stable where the magnetization settles. The trace of the matrix corresponding to $(\pi/2, \cos^{-1}(-J_1/2J_2))$ is given by

$$\text{Tr}(J) |_{(\theta^*, \phi^*)} = \frac{\gamma}{1 + \alpha^2} \left[ \alpha(H_k - 4\pi M_s) - \frac{\alpha(4J_2^2 - J_1^2)}{2J_2dM_s} \right. + \left. \frac{H_{SO}}{2J_2 - J_1 \lambda} \left( \frac{4J_2^2 - J_1^2}{2J_2 - J_1 \lambda} - 2H_{SO}J_1 \right) \right].$$

The minimum critical current $I_{c_{\text{min}}}^0$ (for $J_1 < 2J_2$), below which the fixed point $(\pi/2, \cos^{-1}(-J_1/2J_2))$ is stable, will be derived from the condition of stability (B4).
and by using the Eq. (B5) as

$$I_{e}^{\text{min}} = -\frac{eA\alpha(2J_2 - J_1\lambda)}{\hbar\eta J_2} \left[ 2J_2dM_s(H_k - 4\pi M_s) - (4J_2^2 - J_1^2) \right] \left\{ \frac{2J_2dM_s(H_k - 4\pi M_s)}{\lambda(4J_2^2 - J_1^2) - 2J_2(2J_2 - J_1\lambda)} \right\}. \tag{B6}$$

Similarly, when $|I| < |I_{e}^{\text{min}}|$ and $J_1 \geq 2J_2$ the state $(\theta^*, \phi^*) = (\pi/2, \pi)$ becomes stable and the magnetization settles into $(-1,0,0)$. The trace of the matrix $J$ corresponding to the $(\pi/2, \pi)$ is derived as

$$\text{Tr}(J)_{(\pi/2, \pi)} = -\gamma \left\{ \frac{2H_{SU}}{1 - \lambda} - \frac{(H_k - 4\pi M_s)\alpha}{dM_s} + \frac{2(J_1 - 2J_2)\alpha}{dM_s} \right\}. \tag{B7}$$

From the condition (B4) and Eq. (B7), we can derive the minimum critical current for $J_1 \geq 2J_2$ below which the $(\pi/2, \pi)$ is stable as

$$I_{e}^{\text{min}} = -\frac{eA\alpha(1 - \lambda)}{\hbar\eta} \left\{ 2(J_1 - 2J_2) + dM_s(H_k - 4\pi M_s) \right\}. \tag{B8}$$

When $|I| > |I_{e}^{\text{max}}|$ the magnetization state $(\theta^*, \phi^*) = (\pi/2, 0)$ becomes stable and the magnetization settles into $(1,0,0)$. The trace of the matrix $J$ corresponding to $(\pi/2, 0)$ is

$$\text{Tr}(J)_{(\pi/2, 0)} = \gamma \left\{ \frac{2(J_1 + 2J_2)\alpha}{dM_s} + \frac{2H_{SU}}{1 + \lambda} \right\} + (H_k - 4\pi M_s)\alpha. \tag{B9}$$

The maximum critical current $I_{e}^{\text{max}}$, above which $(\pi/2, 0)$ becomes stable, can be derived from the condition (B4) and Eq. (B9) as

$$I_{e}^{\text{max}} = -\frac{eA\alpha(1 + \lambda)}{\hbar\eta} \left\{ 2(J_1 + 2J_2) + (H_k - 4\pi M_s)dM_s \right\}. \tag{B10}$$

### Appendix C: Output power and its enhancement

The output power $P$ corresponding to the output voltage $V$ of an STNO is given by

$$P = \frac{V^2}{2R_L} = \frac{1}{2R_L} \left( \frac{I \Delta R R_L (\mathbf{m} \cdot \mathbf{p})^2}{2(R_{avg} + R_L)} \right)^2 = \frac{I^2 \Delta R^2 R_L m_x^2}{8(R_{avg} + R_L)^2}. \tag{C1}$$

where $\Delta R = R_{AP} - R_P$ and $R_{avg} = (R_{AP} + R_P)/2$. The quantities $R_{AP}$ and $R_P$ are the resistances while the STNO in antiparallel and parallel configuration, respectively. $R_L$ is the load resistance across which the output power $< P >_{\text{time}}$ is detected. The time averaged power is delivered as

$$< P >_{\text{time}} = \frac{I^2 \Delta R^2 R_L}{8(R_{avg} + R_L)^2} < m_x^2 >_{\text{time}}. \tag{C2}$$

The expression for $R_{AP}$ can be calculated from the GMR ratio $\Delta R/R_P$ as $R_{AP} = R_P + (\Delta R/R_P)R_P$. Since the magnetoresistance for Co[RuFe]Co is not available in the literature, we consider the GMR ratio 0.004 corresponding to Co(3.2nm)|Ru(0.5nm)|Co(3.2nm) to find the approximated value of $< P >_{\text{time}}$ for our system. For $R_P = 100 \, \Omega$, $R_L = 50 \, \Omega$ and $I = -4 \, mA$, $< P >_{\text{time}}$ is determined from $< m_x^2 >_{\text{time}}$ corresponding to the $m_x(t)$ between 950 ns and 1000 ns as 0.41 nW. The $< P >_{\text{time}}$ is plotted for the different currents in Fig. 5(a) for $d = 2 \, nm$ and different thicknesses of the free layer in Fig. 5(b) for $I = -4 \, mA$ when $J_1 = 5 \, mJ/m^2$ and $J_2 = 4.5 \, mJ/m^2$, which confirms the tunability and enhancement of power by the current and the thickness of the free layer. In general, the output power can be enhanced further by connecting the STNOs parallelly or serially and phase locking them by microwave field and current phases.

### AUTHOR DECLARATIONS

#### Conflict of Interest

The authors have no conflicts to disclose.

#### DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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