NLO Heavy Quark Energy Loss in Strongly-Coupled Quark-Gluon Plasmas

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Abstract. Improvements to heavy and light flavor energy loss models based on strong-coupling physics in quark-gluon plasma lead to qualitative agreement between the models and data from RHIC and LHC within the regimes of applicability of the calculations. Thus it is possible to describe self-consistently the dynamics of quark-gluon plasma from the lowest observed momentum modes to the highest from the AdS/CFT correspondence.

1. Introduction
In high-energy nuclear physics we seek a self-consistent theoretical picture of the dynamics of the quark-gluon plasma (QGP) that accurately describes the data observed from heavy ion collisions at facilities such as the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and the Large Hadron Collider (LHC) at CERN. With such a description in hand we hope to claim an understanding of the properties of the QGP and thus map out simultaneously theoretically and experimentally a major portion of the phase diagram of quantum chromodynamics (QCD) [1].

Two of the major sets of observables associated with heavy ion collisions are those associated with low-$p_T$ physics described by relativistic hydrodynamics with a very small viscosity-to-entropy ratio $\eta/s$ [2, 3, 4, 5] and those associated with high-$p_T$ probes such as the suppression of jets and open heavy flavor [6, 7]. The very small $\eta/s$ is most naturally explained by assuming the dynamics of low momentum modes in the QGP are best described by strong-coupling physics as derived using the methods of the anti–de-Sitter/conformal field theory (AdS/CFT) correspondence [8, 9, 10], but not by the methods of weakly-coupled plasma using the methods of perturbative QCD (pQCD) [11]. On the other hand, a huge variety of observables related to high-$p_T$ probes are accurately described using leading order methods of pQCD [6, 7, 12, 13], but are not described using the leading order methods of AdS/CFT [14, 15].

Since we seek a self-consistent description of QGP physics, we hope to either extend the weak-coupling description of the QGP down to low-momentum observables or, alternatively, extend the strong-coupling description up to high-momentum observables. We will demonstrate in this proceedings that it is possible to do the latter, to extend the AdS/CFT description of QGP physics up to the highest momentum probes of the medium [16, 13].
2. AdS/CFT and Hard Probes

It is natural to ask why one might hope that AdS/CFT could be used to describe the physics of hard probes of heavy ion collisions. Since QCD is an asymptotically free theory, one might naturally expect that the physics of hard probes is dominated by that of pQCD. However, it is not clear that the pQCD perspective is a self-consistent one. Despite the fact that the QGP exists at a temperature of a trillion degrees \[ T_{\text{QGP}} \] \cite{2, 3, 4, 5}, the scale set by the temperature of the plasma \[ T_{\text{QGP}} \] is not asymptotically large compared to the natural scale of high-energy nuclear physics, \( \Lambda_{\text{QCD}} \); in fact the two scales are the same order of magnitude, \( T_{\text{QGP}} \sim \Lambda_{\text{QCD}} \). In pQCD, the dominant energy loss mode at high momentum is that of radiation, gluon bremsstrahlung \cite{17}. This radiation process is necessarily third order in the strong coupling \( \alpha_s \), and it is not yet clear at what scale(s) these couplings run: while it is likely that large scales such as the momentum of the probe \( P \) is an important factor in the strength of the coupling of the parton to the medium, it is also likely that \( T_{\text{QGP}} \) is a critically important scale, too. In fact, as the coupling grows non-perturbatively large at low momenta, it is not unreasonable to think that energy loss physics is actually dominated by scales on the order of \( T_{\text{QGP}} \) rather than \( P \).

In the work that follows we will assume that the strong coupling physics dominates the energy loss processes associated with high-momentum probes of the QGP. One may then hope that the AdS/CFT correspondence will provide qualitative, perhaps even quantitative, insight into the properties of the QGP created in heavy ion collisions. We will show that under this assumption of the dominance of strong-coupling physics we can describe a number of observables from RHIC and LHC associated with hard probes of QGP \cite{16, 13}.

2.1. Light Flavor

At strong coupling the relevant degrees of freedom of the sQGP are not quasiparticles. Therefore it is difficult to connect AdS/CFT calculations to light flavor single particle observables. Rather, it is much more natural to compare to composite objects such as jets \cite{18}. Even so, it is still highly non-trivial to find the correct description in the dual string theory from AdS/CFT of the relevant object from the field theory. For jets, the original proposal was that the part of the string within some physical distance \( \Delta x \) to the string endpoint corresponded to a jet in the field theory \cite{18}. With this description, however, one finds that a fully stopped string corresponds to a jet that still has a sizable fraction \( \gtrsim 30\% \) of its original energy; i.e. the jet is not fully thermalized in the medium despite its complete lack of motion. Inspired by the scale separation used in thermal field theory calculations, an alternative jet prescription was proposed in \cite{16}: the part of a string above some \( \Delta u \) threshold in the fifth dimension corresponds to the jet while the part of the string below the threshold corresponds to thermalized, low-momentum medium modes. With this prescription and a reasonable separation scale of \( \sim 500 \) MeV a fully stopped jet is completely thermalized: the jet has no energy in the high-momentum modes; all its energy is in low-momentum modes associated with the medium.

We found that the new jet prescription led to the recovery of the Bragg peak in the energy loss of light flavor in QGP \cite{16}. Incorporating the full numerical jet energy loss calculation from AdS/CFT in a simple geometrical model for the medium produced in heavy ion collisions led to a theoretical prediction \cite{16}, shown in Fig. 1 (a), that was significantly oversuppressed compared to data\footnote{The observable shown in the figure is known as the nuclear modification factor, or \( R_{AA} \), which is defined as the differential yield of some observable in \( A + A \) collisions divided by the number observed in \( p + p \) collisions scaled by the expected number of binary (or \( p + p \)-like) collisions in the \( A + A \) collisions. An \( R_{AA} = 1 \) implies that the QGP medium has no effect on the investigated probe; null control measurements of known weakly-coupled probes from the electroweak sector have \( R_{AA} \sim 1 \).}. However, despite a better description of thermalized jets, one can see from Fig. 1 (a) that the proposed jet description still does not correspond perfectly to the physics in QCD: with the energy scale separation prescription, jets propagating in a \( T = 0 \) “vacuum” QGP still lose
a significant fraction of their energy; see Fig. 1 (a). Since $R_{AA}$ measures the difference between vacuum and medium physics, it is natural (and critical) to properly account for this “vacuum” energy loss by subtracting it in some way from the “in-medium” energy loss. Fig. 1 (b) shows the results when this renormalization was performed by taking the ratio of $R_{AA}$ to $R_{pp}$ [16]. (A similar result emerges when one defines a renormalized energy loss $\Delta E_{\text{renorm}} = \Delta E_{AA} - \Delta E_{pp}$ and then computes $R_{AA}$.)

**Figure 1:** (a) $R_{AA}(p_T)$ for light flavor jets in static (“AdS-Sch”), Bjorken expanding (“JP”), and $T_{QGP} = 0$ (“AdS5”) plasmas from AdS/CFT [16]. (b) The renormalized jet $R_{AA}(p_T)$ from AdS/CFT [16] compared to preliminary data from the CMS collaboration [19].

### 2.2. Heavy Flavor

It is much easier to directly connect the strong-coupling picture to heavy quark observables: the heavy quark can be treated theoretically as an external probe [20, 21, 22]. Early leading order derivations of the mean momentum loss rate of a massive quark in a strongly-coupled medium [20,21] implemented in an energy loss model showed a quantitative agreement with data measured at RHIC [23, 24]. However, the application of this same model to LHC led to a prediction in contradiction with data: the model predicted significant oversuppression compared to observation [15].

These early leading order energy loss derivations were improved by the calculation of the momentum fluctuations associated with energy loss. It was shown in [25] that the three-momentum $p^i$ of an on-shell heavy quark moving at constant velocity in a thermal bath evolves as:

$$ \frac{dp_i}{dt} = -\mu p_i + F^L_i + F^T_i, $$

where the drag coefficient [20, 21, 22] of a heavy quark of mass $M_Q$ in a plasma of temperature $T$ with ’t Hooft coupling $\lambda$ is

$$ \mu = \frac{\pi \sqrt{\lambda} T^2}{2M_Q} $$

and the longitudinal ($L$) and transverse ($T$) fluctuating momentum kicks are correlated as

$$ \langle F^L_i(t_1)F^L_j(t_1) \rangle = \kappa L \hat{p}_i \hat{p}_j g(t_2 - t_1) $$

$$ \langle F^T_i(t_1)F^T_j(t_1) \rangle = \kappa T (\delta_{ij} - \hat{p}_i \hat{p}_j) g(t_2 - t_1), $$

where $\hat{p}_i = p_i/|p|$, 

$$ \kappa_T = \pi \sqrt{\lambda} T^3 \gamma^{1/2} $$

$$ \kappa_L = \gamma^2 \kappa_T, $$
\(\gamma\) is the usual Lorentz boost factor, and \(g\) is a function known only numerically.

It is important to estimate the regime where it is appropriate to use the above formulae to compute the energy loss of a heavy quark in a strongly-coupled plasma. In the setup for the derivation of the above formulae, the heavy quark is assumed to move at an (approximately) constant velocity. There are two obvious possible speed limits for the heavy quark that then result. The first speed limit is well-known, and is associated with the calculation of the drag coefficient, which is to say the mean energy loss rate [25, 26]. For the string dual of a finite mass heavy quark to have a time-like string endpoint motion, the heavy quark velocity is restricted to

\[
\gamma < \gamma_{\text{crit}}^{sl} = \left(1 + \frac{2M_Q}{\sqrt{\lambda T}}\right)^2 \approx \frac{4M_Q^2}{\lambda T^2}.
\]

There is another speed limit, which comes from requiring that the accumulated momentum fluctuations for a particular heavy quark are not large compared to its natural momentum relaxation; i.e. by requiring that the momentum picked up via fluctuations over the time scale set by the drag coefficient is small in comparison to the total momentum of the heavy quark one finds an upper limit on the speed of the heavy quark given by

\[
\gamma \lesssim \gamma_{\text{crit}}^{\text{fluc}} = \frac{M_Q^2}{4T^2}.
\]

One can see that the speed limit set by the mean loss rate, Eq. (7) is parametrically smaller than that set by the fluctuations, Eq. (8). However, for finite \(\lambda \lesssim 12\), it turns out that the speed limit from fluctuations is numerically smaller than that set by the mean loss rate. At the temperatures reached by the medium at RHIC and LHC, the speed limit from fluctuations leads to a maximum momentum for which the calculations apply to charm quarks (of mass \(\sim 1.5\) GeV) of \(p_T \sim 10\) GeV/c. We will see explicitly this breakdown in the calculation in the model calculations that follow.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The \(p^x\)-differential (a) and \(p^z\)-differential (b) distribution of \(m_c = 1.5\) GeV/c\(^2\) charm quarks strongly-coupled to a strongly-coupled plasma in three spatial dimensions at \(T = 400\) MeV that is moving at 0.9c in the \(z\) direction. The effect of the thermal drag and fluctuations on the heavy quarks from strong coupling is found from implementing Eq. (1) using the Itô (pre-point), Stratonovich (mid-point), and Hänggi-Klimontovich (post-point) stochastic integrals. An implementation for which the fluctuation-dissipation theorem holds is also shown and compared to the result expected, the Jüttner distribution.}
\end{figure}

A major complication of the momentum fluctuation calculation of Eqs. (2)–(6) is that the momentum kicks are multiplicative; i.e. the strength of the fluctuations is a function of...
momentum itself. Stochastic integration in which the fluctuations are multiplicative have an irreducible ambiguity: the result depends explicitly on the time within each time step that the integrand is evaluated [27]. (Compare this ambiguity to usual Riemann integration for which the result converges to a unique result as timestep size goes to zero irrespective of the exact time within a timestep that the integrand is evaluated.) Fig. 2 illustrates the significant difference of result from stochastic integration for different (common) choices of time within a timestep to evaluate the integrand. The ambiguity in the stochastic integration procedure can be resolved with the use of the Wong-Zakai theorem [28]: the solution of any Langevin system with an autocorrelation time that goes to zero is given by the Stratonovich stochastic integral. For the problem at hand, the autocorrelation time is small compared to the natural time scale of the problem set by the drag coefficient so long as the heavy quark obeys the speed limit set by the mean energy loss calculation, Eq. (7).

In order to compare with data from $A + A$ collisions, we must first demonstrate that we have the production mechanism in $p + p$ collisions under control. Fig. 3 (a) compares the spectrum of electrons from the decay of heavy flavor mesons from RHIC and from FONLL [29, 30, 31]. The spectra of open heavy quarks is readily available from an online generator [32], and the FONLL $D$ and $B$ meson fragmentation functions are well documented [31]. The decay of the mesons to electrons in FONLL is not as well documented, and the results of this work [13] as shown in Fig. 3 (a) differ significantly from those shown in [33], with the results presented here diverging much more from the experimental results at low, $p_T \lesssim 3$ GeV/c. Thus the comparison of electrons from $A + A$ is restricted from above by the speed limit Eq. (7) and from below by the currently inaccurate production/fragmentation function mechanism.

Nevertheless, we show in Fig. 3 (b) the comparison of the strong-coupling energy loss model predictions for electrons from heavy flavor decay to RHIC data [33, 34], and then $D$ and $B$ meson suppression to LHC data [35, 36] in Fig. 4. The plots include both the leading and next-to-leading order predictions from AdS/CFT [13]. One of the subtleties involved in comparing the AdS/CFT calculations to data is the map between the parameters ($\alpha_s$, $N_c$, and $T$) in

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Figure 3: (a) FONLL predictions [29, 30, 31] for the spectrum of electrons from heavy flavor decay at RHIC compared to data from PHENIX [33]. (b) Strong-coupling energy loss predictions at leading (dashed) and next-to-leading order (solid) for two schemes (red, blue) of translating QCD parameters to $\mathcal{N} = 4$ SYM parameters [13] compared to data from PHENIX [33] and preliminary data from STAR [34].
QCD and the parameters in \( \mathcal{N} = 4 \) SYM. Two schemes for translating between the theories were proposed in [25]; we report the results of both schemes here as an attempt to provide a reasonable exploration of the parameter space. (One would hope that a theory dual to QCD would produce results somewhere between the predictions presented here.) In the range of applicability, \( 3 \text{ GeV/c} \lesssim p_T \lesssim 4 \text{ GeV/c} \), the theory and data agree for electron suppression. Similarly, in the range of \( p_T^D \lesssim 10 \text{ GeV/c} \) and for all current \( B \) meson measurements, the theory and data agree. In fact, the fast rise in \( R_{AA} \) for the observables for momenta above the scale set by the speed limit is precisely the sign of the calculation breaking down. A more correct treatment of the above equations would include autocorrelations, which would mitigate some of that spurious rapid rise. Additionally, a derivation of the energy loss that allowed the heavy quark to dynamically slow down (a highly non-trivial calculation) would include the Larmor-like energy loss experienced by accelerating probes in AdS/CFT [37, 38]. It is not yet known how much of a quantitative difference including this additional energy loss channel would make as a function of momentum, but an implementation of this improved energy loss theory would almost certainly compare more favorably with data.

3. Conclusions

We showed in this proceedings that strong-coupling energy loss models based derivations from the AdS/CFT correspondence—in which a high-momentum probe is strongly-coupled to a strongly-coupled medium—can qualitatively describe both the light and heavy flavor suppression data measured at RHIC and LHC. Thus one can view the physics of a wide range of observables related to the quark-gluon plasma produced in these colliders self-consistently within a single theoretical framework. Future theoretical work includes refining the treatment of energy loss of light flavors and increasing the regime of applicability of heavy quark energy loss. The latter will likely be accomplished by considering the fluctuations in energy loss of dynamical string solutions (as opposed to fluctuations on the static, approximately infinite mass solutions). Further comparisons with data are of course desired to more stringently test the strong-coupling energy loss theory. Of especial utility are observables related to \( b \) quarks for which the theory is better controlled. Additionally, as the fluctuations are predicted to increase significantly with the speed of the heavy quark, heavy flavor correlations measurements will likely provide a valuable discriminator between strong- and weak-coupling energy loss physics.
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