Energy-momentum tensor form factors of the nucleon within a $\pi$-$\rho$-$\omega$ soliton model

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Abstract. We investigate the energy-momentum tensor form factors of the nucleon within the framework of a chiral soliton model, including the $\rho$ and $\omega$ vector mesons. We examine the role of each meson degrees of freedom in these form factors. It is explicitly shown that the pion provides strong attraction whereas the $\rho$ and $\omega$ yield repulsion in such a way that the soliton becomes stabilized. The results are discussed in comparison with those of other models.

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1. Introduction

Nucleon form factors are essential quantities in understanding the internal structure of the nucleon. For example, the electromagnetic form factors reveal how the charge and the magnetization of quarks are distributed inside a nucleon. The scalar and axial-vector form factors also provide information on certain aspects of the nucleon structure such as chiral and flavor symmetries and breakdown of them. Because of these reasons, a great deal of investigations has been performed extensively over decades. On the other hand, the energy-momentum tensor form factors (EMTFFs) of the nucleon, even though they were proposed in 1966 [1], has attracted attention only very recently, since there is no probe to measure them directly. However, the EMTFFs are as equally important as the electromagnetic FFs, since they also provide crucial information on how the internal structure of the nucleon. The generalized parton distributions (GPDs) enable one to extract them from hard exclusive reactions [2, 3, 4, 5]. The Melin transforms of certain GPDs can be identified as the EMTFFs that expose how the mass and the spin are distributed inside a nucleon. Moreover, the EMTFFs can be regarded as the touchstone of checking the validity of any model for the nucleon: they provide strong constraints on the model in such a way that the pressure should be zero. Moreover, the $D$-term, one of the EMTFFs, was deeply related to the spontaneous breakdown of chiral symmetry [6, 7, 8]. Thus, the EMTFFs give us a whole new perspective on the structure of the nucleon.
The EMTFFs are defined as a nucleon matrix element of the totally symmetric Energy-momentum tensor (EMT) operator as follows \[9, 10\]:

\[
\langle p' | \hat{T}_{\mu\nu}(0) | p \rangle = \bar{u}(p', s') \left[ M_2(t) \frac{P_\mu P_\nu}{M_N} + J(t) \frac{i (P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\rho\mu}) \Delta^\rho}{2M_N} + d_1(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{5M_N} \right] u(p, s),
\]

where \( P = (p + p')/2, \Delta = (p' - p) \) and \( t = \Delta^2 \). The \( M_N \) and \( u(p, s) \) denote the nucleon mass and spinor, respectively. The form factor \( M_2(t) \) gives information about the ratio of the momenta carried by constituents of a nucleon. In particular, \( M_2(t) \) at the zero-momentum transfer shows that about 1/2 of the momentum of a fast moving nucleon is carried by quarks, and the other half by gluons. The other form factor \( J(t) \) reveals information on the total angular momentum of the quark and gluons, though it is not much known experimentally. It is less trivial to understand the physical meaning of the last form factor \( d_1(t) \) in Eq.(1) but is equally important, since it explains how the strong forces are distributed and stabilized in the nucleon \[10, 11\]. It can be extracted from the beam charge asymmetry in deeply virtual Compton scattering \[7\].

The EMT form factors of the nucleon have been investigated in various approaches, for example, in lattice QCD \[12, 13, 14, 15, 16, 17, 18, 19\], in chiral perturbation theory \[20, 21, 22, 23, 24\], in the chiral quark-soliton model (χQSM) \[25, 26, 27, 28, 29, 30, 31, 32\] as well as in the Skyrme model \[33\]. Those of nuclei have also been studied \[10, 34, 35, 36\]. The study of the nucleon EMTFFs was also extended to nuclear matter \[37\]. In the present work, we want to examine the EMTFFs of the nucleon, based on a chiral soliton model with vector mesons in the minimal form \[38\]. The model is based on the fact that the nonlinear sigma model has a hidden local gauge symmetry SU(2)_V \[43\], in which the ρ meson is identified as a gauge boson. Extending this symmetry to SU(2)_V ⊗ U(1), the ω meson can be also regarded as a gauge boson \[38\]. In this way, parameters of the model are completely determined in the mesonic sector, so that we can investigate properties of the nucleon in the solitonic sector unambiguously. This model has a certain virtue in studying the EMTFFs, since one can study the role of the ρ and ω mesons in describing the nucleon. It is known that the vector mesons provide short-range repulsion in the one-boson exchange model for the nucleon-nucleon interaction, while the pion dominates the long-range interaction \[44\]. We will soon show explicitly that the pion gives attraction to form a soliton whereas the vector mesons become repulsive to stabilize it, which is analogous to the nucleon-nucleon interaction. This is in line with what was found in the Skyrme model \[33\], where it was shown for the Skyrme term to stabilize obviously the soliton.

The present work is organized as follows: In Section II, we briefly introduce the chiral soliton model with the ρ and ω mesons. In Section III, we explain how to compute

\[\ddagger\] In the present work, we consider one of the simplest ones among various soliton models \[38, 39, 40, 41\] with vector mesons. For the detailed development of chiral solitons with vector mesons, we refer to a recent review \[42\].
the EMTFFs of the nucleon within this model. In Section IV, we discuss the results of the EMTFFs. The final Section is devoted to summary and conclusion.

2. Chiral soliton model with vector mesons

We start with the effective Lagrangian with the \( \pi, \rho, \) and \( \omega \) meson degrees of freedom, from which the nucleon arises as a topological solution [38, 45, 46]. In fact, almost all formulae presented in this section can be found in Ref. [46]. We briefly recapitulate here the pertinent ones for the EMTffs. The Lagrangian has the following form

\[
L = L_\pi + L_{\text{kin}} + L_V + L_{WZ},
\]

\[
L_\pi = \frac{f_\pi^2}{4} \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right) + \frac{f_\pi^2 m_\pi^2}{2} \text{Tr} \left( U - 1 \right),
\]

\[
L_{\text{kin}} = -\frac{1}{2} g^2 \text{Tr} \left( \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu] \right)^2,
\]

\[
L_V = \frac{a}{4} f_\pi^2 \text{Tr} \left[ D_\mu \xi \cdot \xi^\dagger + D_\mu \xi^\dagger \cdot \xi \right]^2,
\]

\[
L_{WZ} = \left( \frac{N_c}{2} g \right) \omega^\mu \epsilon_{\mu\nu\alpha\beta} \text{Tr} \left\{ \left( U^\dagger \partial_\nu U \right) \left( U^\dagger \partial_\alpha U \right) \left( U^\dagger \partial_\beta U \right) \right\},
\]

where \( U = \xi_L^\dagger \xi_R \) in unitary gauge, and the covariant derivative is defined as

\[
D_\mu \xi_{L(R)} = \partial_\mu \xi_{L(R)} - i V_\mu \xi_{L(R)}.
\]

The field \( V_\mu \) consists of the \( \rho \) and \( \omega \) fields, i.e. \( \bar{\rho}_\mu \) and \( \omega_\mu \), respectively, being expressed as

\[
V_\mu = \frac{g}{2} (\vec{r} \cdot \bar{\rho}_\mu + \omega_\mu).
\]

The pion decay constant \( f_\pi \) and the pion mass \( m_\pi \), are fixed by experimental data, i.e. \( f_\pi = 93 \text{ MeV} \) and \( m_\pi = 135 \text{ MeV} \) (the neutral pion mass). The number of colors is taken to be \( N_c = 3 \) and the coupling constant \( g \) is related to the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [47, 48] \( m_\rho^2 = m_\omega^2 = a g^2 f_\pi^2 \) with \( a = 2 \) such that we have the \( \rho \pi \pi \) coupling \( g_{\rho \pi \pi} = ag/2 \) and \( g = 5.85 \). Note that the \( g_{\rho \pi \pi} \) is taken to be close to its empirical value \( g_{\rho \pi \pi} = 6.11 \).

Assuming the following Ansätze for the pseudoscalar and vector mesons

\[
U = \exp \left\{ \frac{i \vec{r} \cdot \vec{r}}{r} F(r) \right\}, \quad \rho^a_\mu = \frac{\varepsilon_{iak} r_k}{gr^2} G(r) \delta_{\mu i}, \quad \omega_\mu = \omega(r) \delta_{\mu 0},
\]

one can derive the static energy functional from the Lagrangian. It is identified as the classical soliton mass

\[
M_{\text{sol}} = 4\pi \int_0^\infty dr \left\{ \frac{f_\pi^2}{2} \left( r^2 F'^2 + 2 \sin^2 F \right) + r^2 f_\pi^2 m_\pi^2 (1 - \cos F) \
+ 2 f_\pi^2 (G + 1 - \cos F)^2 + \frac{1}{g^2} \left[ G'^2 + \frac{G^2 (G + 2)^2}{2r^2} \right] \right\} \
- r^2 \left( f_\pi^2 g^2 \omega^2 + \frac{1}{2} \omega'^2 \right) + \frac{3g}{4\pi^2} \omega F' \sin^2 F \right\},
\]

(10)
where \( f' = \partial f / \partial r \), generically. Minimizing the classical soliton mass can be achieved by solving the equations of motion, which are given as the coupled nonlinear differential equations

\[
F'' = -\frac{2}{r} F' + \frac{1}{r^2} [4(G + 1) \sin F - \sin 2F] + m_\pi^2 \sin F - \frac{3g \omega'}{4\pi^2 f_\pi^2} \frac{\sin^2 F}{r^2},
\]

\[
G'' = 2g^2 f_\pi^2 \left[ G + 2\sin^2 F \frac{F}{2} + \frac{G(G + 1)(G + 2)}{r^2} \right],
\]

\[
\omega'' = -\frac{2}{r} \omega' + 2f_\pi^2 g^2 \omega - \frac{3g}{4\pi^2 r^2} F' \sin^2 F
\]

with the boundary conditions

\[
F(0) = \pi, \quad G(0) = -2, \quad F(\infty) = G(\infty) = \omega(\infty) = \omega'(0) = 0. \quad (11)
\]

The collective quantization brings out the following relations

\[
U(\vec{r}, t) = A(t)U(\vec{r})A^+(t),
\]

\[
\omega_i(\vec{r}, t) = \frac{\phi_i(r)}{r} \left( \vec{K} \times \frac{\vec{r}}{r} \right)_i,
\]

\[
\vec{r} \cdot \vec{\rho}_0(\vec{r}, t) = \frac{2}{g} A(t) \vec{r} \cdot \left[ \vec{K} \xi_1(r) + \frac{\vec{r}}{r} \left( \vec{K} \cdot \frac{\vec{r}}{r} \right) \xi_2(r) \right] A^+(t),
\]

\[
\vec{r} \cdot \vec{\rho}_i(\vec{r}, t) = A(t) \vec{r} \cdot \vec{\rho}_i(\vec{r})A^+(t), \quad (13)
\]

where \( 2\vec{K} \) denotes the angular velocity of the soliton with the relation \( i\vec{r} \cdot \vec{K} = A^+ \dot{A} \).

This leads to the time-dependent collective Hamiltonian

\[
H(t) = M_{\text{sol}} + \Lambda \text{Tr}(\dot{A}^+ \dot{A}), \quad (14)
\]

where \( \Lambda \) denotes the moment of inertia of the rotating soliton

\[
\Lambda = 4\pi \int_0^\infty dr \left\{ \frac{2}{3} f_\pi^2 r^2 \left( \sin^2 F + 8 \sin^1 F \frac{F}{2} - 8 \xi_1 \sin^2 \frac{F}{2} + 3\xi_1^2 + 2\xi_1 \xi_2 + \xi_2^2 \right) \right. \\
+ \frac{1}{3g^2} \left[ 4G^2 \left( \xi_1^2 + \xi_1 \xi_2 - 2\xi_1 + \xi_2 + 1 \right) \\
+ 2 \left( G^2 + 2G + 2 \right) \xi_2^2 + r^2 \left( 3\xi_1^2 + \xi_2^2 + 2\xi_1 \xi_2 \right) \right] \\
- \frac{1}{6} \left[ \Phi'^2 + \frac{2\Phi^2}{r^2} + 2 \left( g f_\pi \right)^2 \Phi^2 \right] + \frac{g \Phi F'}{2\pi^2} \sin^2 F \right\}. \quad (15)
\]

In the large \( N_c \) expansion, one extremizes the moment of inertia and gets the coupled nonlinear differential equations for the next-order profile functions \( \xi_1, \xi_2, \phi \) in the presence of the leading-order profile functions \( F, G \) and \( \omega \)

\[
\xi_1'' = 2f_\pi^2 g^2 (\xi_1 - 1 + \cos F) - \frac{2\xi_1'}{r} + \frac{G^2 (\xi_1 - 1) + 2(G + 1) \xi_2}{r^2},
\]

\[
\xi_2'' = 2f_\pi^2 g^2 (\xi_2 + 1 - \cos F) - \frac{2\xi_2'}{r} + \frac{G^2 (\xi_1 - 1) + 2(G^2 + 3G + 3) \xi_2}{r^2},
\]

\[
\phi'' = 2f_\pi^2 g^2 \phi - \frac{3g F' \sin^2 F}{2\pi^2} + \frac{2\phi'}{r^2}
\]

with the boundary conditions

\[
\phi(0) = \phi(\infty) = \xi_1'(0) = \xi_1'(\infty) = \xi_2'(0) = \xi_2'(\infty) = 0. \quad (17)
\]
The boundary conditions for $\xi_1$ and $\xi_2$ satisfy the relation $2\xi_1(0) + \xi_2(0) = 2$.

Finally, the effective masses of the nucleon and the $\Delta$ isobar are expressed in terms of the hedgehog mass $M_{\text{sol}}$ and the moment of inertia $\Lambda$:

$$M_N = M_{\text{sol}} + \frac{3}{8\Lambda}, \quad M_{\Delta} = M_{\text{sol}} + \frac{15}{8\Lambda}. \tag{18}$$

In the present model, the hedgehog mass is $M_{\text{sol}} \simeq 1473 \text{ MeV}$ and nucleon mass is $M_N \simeq 1562 \text{ MeV}$. Small differences between these values and the values presented in Ref. [46] are due to the different values of the pion mass, $m_{\pi} = 135 \text{ MeV}$ in the present work and $m_{\pi} = 138 \text{ MeV}$ in Ref. [46].

### 3. EMT form factors

Using the Lagrangian in Eq. (2), one can calculate each component of the EMT as follows:

$$T^{00}(r) = \frac{f_\pi^2}{2} \left(2 \frac{\sin^2 F}{r^2} + F' r^2 \right) + f_\pi^2 m_\pi^2 \left(1 - \cos F\right)$$

$$+ \frac{2 f_\pi^2}{r^2} \left(1 - \cos F + G\right)^2 + \frac{1}{2 g_\pi^2 r^2} \left\{2 r^2 G'^2 + G^2 (G + 2)^2 \right\}$$

$$- g_\pi^2 f_\pi^2 \omega^2 - \left[\frac{3}{2} \omega \right] \frac{1}{2 \pi^2 r^2} \omega F' \sin^2 F,$$

$$T^{0i}(r, \vec{s}) = e^{i m r \cdot \vec{s}} \rho_J(r),$$

$$T^{ij}(r) = s(r) \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij}\right) + p(r) \delta^{ij}, \tag{19}$$

where $T_{00}(r)$ is called the energy density. The vector $\vec{s}$ denotes the direction of the quantization axis for the spin and coincides with the space part of the polarization vector of the nucleon in the rest frame. The density of angular momentum is given by $\rho_J(r)$ while $p(r)$ and $s(r)$ are pressure and shear force densities, respectively. Their explicit forms are given as

$$\rho_J(r) = \frac{f_\pi^2}{3\Lambda} \left[\sin^2 F + 8 \sin^4 F \frac{F'}{2} + 4 \sin^2 F \frac{G}{2} - 4 \sin^2 F \frac{F}{2} \xi_1 - 2 \xi_1 G\right]$$

$$+ \frac{1}{3 g_\pi^2 r^2 \Lambda} \left[-r^2 \xi_1' G' - (\xi_1 G - G - \xi_2) \left(2G + G' \right) \right]$$

$$+ \frac{g}{8\pi^2 \Lambda} \Phi \sin^2 F F', \tag{20}$$

$$p(r) = -\frac{1}{6} f_\pi^2 \left(F'^2 + 2 \frac{\sin^2 F'}{r^2}\right) - f_\pi^2 m_\pi^2 \left(1 - \cos F\right)$$

$$- \frac{2}{3 r^2} f_\pi^2 \left(1 - \cos F + G\right)^2 + f_\pi^2 g_\pi^2 \omega^2$$

$$+ \frac{1}{6 g_\pi^2 r^2} \left\{2 r^2 G'^2 + G^2 (G + 2)^2 \right\} + \frac{1}{6} \omega^2, \tag{21}$$

$$s(r) = \frac{f_\pi^2}{r^2} \left(F'^2 - \frac{\sin^2 F}{r^2}\right) - \frac{2 f_\pi^2}{r^2} \left(1 - \cos F + G\right)^2$$

$$+ \frac{1}{g_\pi^2 r^2} \left\{r^2 G'^2 - G^2 (G + 2)^2 \right\} - \omega^2. \tag{22}$$
The corresponding three form factors in Eq. (11) are finally obtained in the large $N_c$ limit as

\[ M_2(t) = \frac{1}{M_{\text{sol}}} \int d^3r \, T_{00}(r) \, j_0(r\sqrt{-t}) - \frac{t}{5M_{\text{sol}}^2} \, d_1(t), \]
\[ d_1(t) = \frac{15M_{\text{sol}}}{2} \int d^3r \, p(r) \, \frac{j_0(r\sqrt{-t})}{t}, \]
\[ J(t) = 3 \int d^3r \, \rho_J(r) \, \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}, \]

(23)

where $j_0(z)$ and $j_1(z)$ represent the spherical Bessel functions of order 0 and 1, respectively. At the zero momentum transfer $t = 0$, $M_2(0)$ and $J(0)$ are normalized as

\[ M_2(0) = \frac{1}{M_{\text{sol}}} \int d^3r \, T_{00}(r) = 1, \quad J(0) = \int d^3r \, \rho_J(r) = \frac{1}{2}. \]

(24)

These relations are very important, since the integration of $T_{00}$ should be the same as the nucleon mass, and the spin of the nucleon should be 1/2. The first condition in Eq. (24) is obviously seen from the comparison of the integrand in the Hedgehog mass and the expression for $T_{00}$. To prove the second condition in Eq. (24) we integrate by part the terms of the bilinear combinations in derivatives (e.g. $r^2\xi_1^2$) in the expression of moment of inertia and use the equations of motion. Then the moment of inertia takes the form

\[ \Lambda = 4\pi \int_0^\infty dr \, r^2 \left\{ \frac{2f_1^2}{3} \left( \sin^2 F + 8\sin^4 \frac{F}{2} - 4\sin^2 \frac{F}{2}\xi_1 \right) \right. \\
\left. + \frac{2}{3g_1^2r^2} \left\{ (2 - 2\xi_1 - \xi_2) G^2 \right\} + \frac{g}{4\pi^2} \phi F' \sin^2 F \right\}. \]

(25)

Analogously, integrating by part the term proportional to $r^2\xi_1' G'$ in the expression of the angular density $\rho_J(r)$, one can show that the second condition is also satisfied.

Furthermore, the conservation of the EMT leads to the following stability condition

\[ \int_0^\infty dr \, r^2 \, p(r) = 0. \]

(26)

We can easily prove analytically that the stability condition (26) is satisfied within the present model.

\[ r^2 \, p(r) = \frac{\partial}{\partial r} \left[ r^3 p - 2r^3 \left( -\frac{1}{6} f_\pi^2 F' r^2 + \frac{1}{6} \omega^2 \right) \right. \]
\[ \left. - \frac{2}{3} r^3 \left\{ f_\pi^2 g_\omega^2 + \frac{1}{3g_1^2r^2} \left\{ r^2 G'^2 \right\} - f_\pi^2 m_\pi^2 (1 - \cos F) \right\} \right] \]
\[ - \frac{f_1^2 r F'}{3} \times (\text{equations of motion}) \]
\[ - \frac{1}{3g_1^2} r G' \times (\text{equations of motion}) \]
\[ - r \omega' \times (\text{equations of motion}). \]

(27)

Any reasonable model for the nucleon should satisfy Eq. (26). Moreover, the pressure density exhibits how each contribution of the mesons contribute to the shape of the nucleon.
4. Results and discussion

In this section, we now discuss the results of the EMTFFs obtained from the $\pi$-$\rho$-$\omega$ soliton model. In Table 1, the relevant observables to the EMTFFs are listed in comparison with the Skyrme model [33] and the $\chi$QSM [31]. Note that the Skyrme model takes the value of the pion decay constant $f_\pi = 54$ MeV such that the nucleon mass can be fitted to the experimental data. On the other hand, the present model and the $\chi$QSM fix it to be the experimental value $f_\pi = 93$ MeV. Because of this, the present

| Model                  | $\langle r_{00}^2 \rangle$ [fm$^2$] | $\langle r_J^2 \rangle$ [fm$^2$] | $r_0$ [fm] | $d_1(0)$ | $p_0(0)$ [GeV/fm$^3$] | $T_{00}(0)$ [GeV/fm$^3$] |
|------------------------|----------------------------------|----------------------------------|------------|----------|----------------------|------------------------|
| $\pi\rho\omega$ soliton model | 0.78                             | 0.74                             | 0.55       | -5.03    | 0.58                 | 3.56                   |
| Skyrme model [33]     | 0.54                             | 0.92                             | 0.64       | -4.48    | 0.48                 | 2.28                   |
| $\chi$QSM [31]       | 0.67                             | 1.32                             | 0.57       | -2.35    | 0.23                 | 1.70                   |

work and the $\chi$QSM overestimate the nucleon mass. The result of the $\langle r_{00}^2 \rangle$ is similar to those from the other two models, while that of $\langle r_J^2 \rangle$ turns out to be smaller than those from the other models. It already indicates that the form factor $J(t)$ will fall off slower than those from the other models, which we will discuss later. The $D$ term, i.e. $d_1(0)$, is yielded to be larger, compared to those of the Skyrme model and the $\chi$QSM. We find that the values of the pressure and the energy density at the origin are larger in comparison with the results of the other two models.

Figure 1 shows the three densities of the energy, the angular momentum and pressure, respectively. In general, the present results are more shifted to the center, compared with those of the Skyrme model and the $\chi$QSM. The pressure density becomes the most interesting one, since it takes a picture of the nucleon internal structure. As shown in the lower panel of Fig. 1, the pressure density turns out to be positive in the inner part of the nucleon but is changed to be negative as $r$ increases. However, it should comply with the stability condition given in Eq. (26). So does the present result.

Figure 2 reveals a salient feature of the pressure density. The pion provides a strong attraction together with the long-range tail. As in the case of the Skyrmion, the soliton is never stabilized with the pion only. The Skyrme term provides a repulsive force enough to stabilize it. In the present model, the $\rho$ and $\omega$ do play the same role as the Skyrme term. As shown in Fig. 2, the $\rho$ and $\omega$ mesons yield repulsive interactions, in particular, in the inner part of the nucleon. Thus, the pressure density becomes positive in the inner part while it turns out to be negative in the outer region with the
Figure 1. The energy densities normalized by the nucleon mass are drawn in the upper panel. In the middle panel, the angular-momentum densities of the nucleon normalized by the nucleon spin are presented. The lower panel depicts the pressure densities of the nucleon. The solid curve represents the result of the present model, while the dashed and dot-dashed ones stand for those of the Skyrme model and the chiral quark-soliton model, respectively.

long-range pion tail. This feature is in line with the one-boson exchange picture of the
nucleon-nucleon interaction [44], as mentioned in Introduction.

Finally, we discuss the results of the form factors in Fig. 3. The upper panel of Fig. 3 depicts the result of the normalized mass form factor $M_2(t)$. As expected from that of $\langle r_{00}^2 \rangle$, the present result shows a very similar $t$ dependence to those from the Skyrme model and the $\chi$QSM. On the other hand, the form factor of the angular momentum $J(t)$ and the $D$-term form factor $d_1(t)$ fall off rather slowly in comparison with those of the other two models.

Form factors of the proton are often parameterized with the dipole-type form factor, $F(t) = F(0)/(1 - t/M_{dipole}^2)^2$. For example, the nucleon electric form factor is well described with this parameterization. Because of this fact, Refs. [33, 31] fitted the EMFFFs with the dipole-type parameterization and derived the dipole mass for each form factor. However, we find that though the dipole-type parameterization describes approximately well the EMTFFs, there are still discrepancies. Thus, we use rather the
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\[ F(t) = \frac{F(0)}{(1 - t/(pM_p^2))^p}, \quad p \geq 1, \]  

(28)

which parameterizes the EMTFFs quantitatively. Note that this parameterization is often employed in lattice QCD [50]. Using Eq. (28), we find that the \( p \)-pole mass

Figure 3. The dependence of form factors \( M_2(t) \), \( J(t) \) and \( d_1(t) \) on the momentum transfer \( t \). The notation is the same as in Fig. 1.

\( p \)-pole form factor

\[ F(t) = \frac{F(0)}{(1 - t/(pM_p^2))^p}, \quad p \geq 1, \]  

(28)
\[ M_{M_2} = 0.724 \text{ GeV} \] for the mass form factor \( M_2(t) \) with \( p = 2.17 \), \( M_J = 0.786 \text{ GeV} \) for \( J(t) \) with \( p \approx 1 \), and \( M_{d_1} = 0.510 \text{ GeV} \) for \( d_1(t) \) with \( p = 1.57 \). Note that the \( p \)-pole parametrization is defined for \( p \geq 1 \) to satisfy an analytic behavior at \( t = 0 \). Because of this, the value of \( p \) for \( J(t) \) is approximately fitted to \( p \approx 1 \).

5. Summary and outlook

In the present work, we aimed at investigating the energy-momentum tensor form factors of the nucleon, based on the \( \pi - \rho - \omega \) soliton model. Having fixed all the relevant parameters in the mesonic sector, we were able to derive the densities for the form factors. We discussed the results of the densities in comparison with the two different solitonic model, i.e., the Skyrme model and the chiral quark-soliton model. The results were in general more shifted to the inner part of the nucleon, compared with these two models. The present model was shown to satisfy the stability condition. the result of the pressure density exhibited each role of the \( \pi \) and the vector mesons: While the pion provides the strong attraction, the \( \rho \) and \( \omega \) yield the repulsive force that balances in such a way that the stability condition is satisfied. We finally discussed the results of the three form factors: the mass form factor, the angular-momentum form factor, and the \( D \)-term formfactor. While that of the mass form factor was quite similar to those of the other models, the results of the angular-momentum and \( D \)-term form factors turned out to fall off more slowly than those of the Skyrme model and the chiral quark-soliton model.

The energy-momentum tensor form factors provide important information on how the nucleon undergoes changes in nuclear matter \[37\]. It is in particular of great interest to study them within the present framework, since the medium-modified \( \pi - \rho - \omega \) soliton model connects the change of the vector meson in nuclear matter with the medium modification of the nucleon \[49\]. Thus, the energy-momentum tensor form factors in nuclear matter within the medium-modified \( \pi - \rho - \omega \) soliton model will shed light on the physics of the nucleon in medium. The corresponding investigation is under way \[51\]. Last but least, it is also interesting to examine the quantum corrections, which might be of great importance in the context of the energy-momentum tensor form factors \[52\]. This can be considered as a future work.

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