Multi-stream portrait of the Cosmic web

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ABSTRACT
We report the results of the first study of the multi-stream environment of dark matter halos in cosmological N-body simulations in the ΛCDM cosmology. The full dynamical state of dark matter can be described as a three-dimensional submanifold in six-dimensional phase space - the dark matter sheet. In our study we use a Lagrangian submanifold $x = x(q, t)$ (where $x$ and $q$ are comoving Eulerian and Lagrangian coordinates respectively) which is dynamically equivalent to the dark matter sheet but is more convenient for numerical analysis. Its convenience is two-fold. Firstly, $x$ is a single-valued function of $q$ at any stage including highly non-linear stages while the phase space sheet in any set of three of six phase space axes is not. And secondly, storing the Lagrangian submanifold does not require additional space for Lagrangian coordinates if the uniform state of the simulation is represented by a uniform three-dimensional mesh. Our major results can be summarized as follows: At the resolution of the simulation i.e. without additional smoothing the cosmic web represents a hierarchical structure: each halo is embedded in the filamentary framework of the web at the filament crossings, and each filament is embedded in the wall like fabric of the web at the wall crossings. Locally, the halos are the regions of highest number of streams, the number of streams in the neighboring filaments is higher than in the neighboring walls, and walls are regions where number of streams is greater or equal to three. Voids are uniquely defined by the local condition requiring to be a single-stream flow region. The shells of streams around halos are quite thin and the closest void region is typically within roughly one and a half of FOF radii of the halo.

Key words: methods: numerical – cosmology: theory – dark matter – large-scale structure of Universe

1 INTRODUCTION
The problem of objective identification and quantitative characterization of anisotropic structures in the distribution of galaxies in space emerged after the first evidences of their existence (see the review by Oort 1983 and the references therein). The first theoretical model predicting highly anisotropic concentrations in the mass distribution coming into existence at non-linear stage of gravitational instability is known as the Zeldovich Approximation (ZA) (Zel’dovich 1970 for further developments see also Shandarin & Zeldovich 1989 and the references therein). ZA predicted the formation of ‘pancakes’ also known as walls in the currently popular jargon. The later development of the model by Arnold, Shandarin & Zeldovich (1982) predicted the formation of filaments along with the pancakes. Klypin & Shandarin (1983) and Shandarin & Klypin (1984) demonstrated that the filaments emerge in cosmological N-body simulation in three-dimensional space. However, they failed to identify the pancakes. Both the existence of filaments connecting compact clumps of matter and absence of pancakes were confirmed by Frenk, White & Davis (1983). Puzzled by the absence of the pancakes Klypin & Shandarin (1983) speculated that insufficient mass resolution of the simulation was the cause of the negative outcome. This has been unambiguously confirmed by recent simulations using a better numerical technique of computing the density field from the particle coordinates in cosmological N-body simulations (Shandarin, Habib & Heitmann 2012 and Abel, Hahn & Kaehler 2012). Klypin & Shandarin (1983) also stressed that the most of filaments are incorporated in ‘a single three-dimensional web structure’.

Although the four archetypical elements of the cosmic web: voids, walls/pancakes, filaments and halos were predicted by ZA and confirmed in cosmological N-body simulation their identification and quantitative characterization

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remains under vigorous debate (see e.g. [Colberg et al. 2008; Elahi et al. 2013; Knebe et al. 2013; Hoffmann et al. 2014]). The dark matter halos are arguably the easiest objects to identify in N-body simulations. They are also can be reliably associated with observed objects like galaxies and clusters of galaxies. But even in this case [Knebe et al. (2013)] refer to almost thirty different halo finders suggested after 2000. Identifying filaments and pancakes/walls is even much more controversial in both N-body simulations and galaxy catalogs. For instance, even estimating the global parameters of the web in N-body simulation such as the fractions of volume and mass in voids, walls/pancakes, filaments and halos produced quite different results. For example, the estimates of volume fractions of voids range from 13 to 86%. [Cautun et al. 2014; Falck & Neyrinck 2014; Porero-Romero et al. 2009, Hahn et al. 2007; Aragon-Calvo, van de Weygaert & Jones 2010]. Similar estimates for walls/pancakes, filaments and halos are respectively 5-56%, 2-26% and 0.1-1%. Estimates of the mass content vary in large ranges as well.

Large differences in estimates of the volume and mass fractions made by different groups are not surprising if we recognize considerable differences in the definitions of the components of the cosmic web and numerical methods used in the estimates. Not trying to provide an exhaustive review of all definitions and techniques used for the quantitative morphological analysis of the web we just briefly describe a few approaches in order to illustrate how different they could be. Some groups study the web morphology using only coordinates of simulation particles others use also the particle velocities. It is useful to transform the data from particle representations to the fields on a grid because fields allow to use a variety of mathematical techniques not available for the particle sets. This step can be done by a variety of methods some as simple as CIC, or more complicated as SPH, or using Voronoi and Delaunay tessellations as in the Delaunay Tessellation Field Estimator (DTFE) method [van de Weygaert & Schaap 2009; Cautun et al. 2014]. This by itself often result in different outcomes in the studies of the web.

However a new method called a discrete persistent structure extractor (DisPerSE [Sousbie 2011; Sousbie, Pichon & Kawahara 2011]) allowing to identify halos and other components of the web directly from the particles has been designed. The method can be applied to the galaxy catalogs, for instance [Sousbie, Pichon & Kawahara 2011] applied it to SDSS catalog and extracted the filaments which are available online.

An obvious advantage of methods based on particle coordinates, both based on the density fields and directly particle sets themselves, is the applicability to redshift catalogs. However, the redshift catalogues like SDSS and 2dF provide only two angular coordinates and distances in redshift space. Cosmological N-body dark matter simulation provide the full dynamical information in six-dimensional phase space. This additional information provides a much greater opportunity for understanding the web.

Dark matter occupies a three-dimensional sub-manifold in six-dimensional phase space because it is cold. Therefore three dimensions of a six-dimensional fluid particle can be ignored in most of calculations. In the linear regime the dark matter submanifold is a single-valued function of Eulerian coordinates which means that at each point the dark matter flow is described by one stream in all respects. As the density perturbations in dark matter grow with time the number of streams jumps to three then to five and to many however remaining odd in generic points. The corresponding parts of the three-dimensional dark matter submanifold experience complicated foldings in six-dimensional phase space.

The regions with multi-stream flow constitute the web while the regions with only one stream form voids [Shandarin 2011; Shandarin, Habib & Heitmann 2012; Abel, Hahn & Kaehler 2012]. The first three-stream flow regions are similar to the pancakes in ZA. They quickly grow and merge into a complicated three-dimensional structure: filaments making the framework of the web manifest themselves at the pancake crossings, and halos emerge at the filament crossings. At later times different parts of the web participating in the large-scale motion overlap which increases the web complexity even more.

Using the full six-dimensional information allows to generate new fields which provide additional useful information about the evolution and morphology of the web. One of them is a multi stream field in Eulerian space, which we will focus on in this paper. Another example is the flip-flop field in Lagrangian space. In cosmological context it was firstly used in ZA. [Vogelsberger & White 2011] used it in a study of multi-stream structure of galaxy size halos, Shandarin & Medvedev (2014) applied it for identifying subhalos in dark matter halos. A similar although somewhat simplistic realization of this idea has been revealed in the ORIGAMI method used for the analysis of the web [Falck, Neyrinck & Szalay 2012; Falck & Neyrinck 2014] Although these methods cannot be used directly on observational data because the initial state is not known, they provide a deeper understanding of the non-linear clustering of collisionless dark matter and reveal new features of the web.

We do not discuss the relation between the multi-stream and density fields here because this issue was discussed in detail in previous publications (Shandarin, Habib & Heitmann 2012 and Abel, Hahn & Kaehler 2012).

In order to compute the multi-stream field we will use the tessellation scheme described in Shandarin, Habib & Heitmann (2012) and it is briefly discussed in Section 3. Using this methodology on the entire simulation box, we discuss the global behavior of the number of streams in the cosmic web in Section 4. The tessellation technique we have utilized can be used to find multi-stream fields in smaller Eulerian boxes with very high resolution too. In Section 5 we study the local behavior of multi-streams flows in regions around halos detected using friends-of-friends (FOF) technique.

2 THE SIMULATION

We have utilized Gadget-2 cosmological simulation data [Springel 2005] for 100 h$^{-1}$ Mpc and 200 h$^{-1}$ Mpc box sizes with 128$^3$, 256$^3$ and 512$^3$ grids. Each particle is between $10^9 - 10^{12} \, h^{-1} M_\odot$. The initial conditions and cosmological parameters are consistent with Planck cosmology. We utilize the initial Lagrangian box and do a 3D mapping onto corresponding evolved simulations. In addition, for local multi-stream analyses around halos, we have utilized halo catalogue for each of these simulation boxes. These halos are
detected using FOF method considering objects with more than 20 particles found at linking length, $b = 0.2$.

3 MULTI-STREAM FIELD CALCULATION

Phase space tessellation considers the dynamics of the particles similar to standard N-body code. However the particles are nodes of the tessellation, and are just massless tracers of the flow. Assuming that the uniform state is modeled by a simple rectangular grid, the particles are the nodes of the grid. Each elementary cube of the grid is tessellated by five tetrahedra (Shandarin, Habib & Heitmann 2012) if which vertices are the vertices of the cube. Mass is assumed to be uniformly distributed within each tetrahedron and the tessellation remain in tact at all times. The tetrahedra of the tessellation change their shapes and volumes, the latter are used for computing the densities of the tetrahedra.

Despite the complicated deformations experienced by the three-dimensional submanifold tessellated by the tetrahedra it remains continuous. Projected on three-dimensional configuration space, the tetrahedra may form a complicated structures. The number of streams at a chosen point is simply the number of tetrahedra that contain the point. Number of streams are odd-valued in the entire configuration space, except in a set of points of measure zero where caustics are formed. A single-stream flow implies that the tetrahedra do not overlap in corresponding region and thus it is defined as a void region. The web is defined as a set of non-void regions, i.e. the set of regions where the number of stream is more or equal to three. Level of non-linearity in the web can be characterized by using number of streams as a parameter.

4 GLOBAL STATISTICS OF COSMIC WEB

The 3-dimensional multi-stream field for the entire simulation box exhibits cosmic web structure with void, walls, filaments and halos. We propose the number of streams, $n_{str}$ as the parameter for characterizing and distinguishing structures in the universe. This is different from Falck & Neyrinck (2014), where the authors have identified voids, walls, filaments and halos by particles which have undergone any number of flip-flops along 0, 1, 2 and 3 orthogonal axes respectively. Their description of voids is close to ours except that some particles that have experienced no flip-flops might be in the region of multi-stream flow formed by other particles. Falck & Neyrinck (2014) studied this effect and concluded that it is small. This description is entirely physically motivated, hence the unambiguous definition.

Whereas for non-linear structures, our parameter space has more freedom in terms of number of streams. Similar to the density threshold, the number of streams - used as a local parameter - cannot distinguish unambiguously whether a point is in a wall, filament or halo. Only some parts of walls where there are only three streams can be identified locally without confusion. This is because the formation of a filament require at least five streams: a flip-flop along one axis would produce a pancake and then another flip-flop along the other axis in one of the streams from previous stage transforms it into a three-stream flow. Thus the total becomes five. However if the second flip-flop happens along the same axis the resultant structure will remain a wall. Therefore some points in the five-stream flows can be within walls while the other in filaments. Thus we rely on visual impressions initially to understand the phase space behavior of walls, filaments and halos. By inspection, we have identified regions with three streams as walls. Unfortunately, walls are difficult to display on paper since they essentially blocks the view in two-dimensional projection. Nevertheless, we have demonstrated and analyzed walls on a smaller Eulerian box around halos in Section 5 using a simple and effective technique.

With the increase of number of streams, more and more points belong to filaments until at the level $n_{str} \gtrsim 17$ we decide that the number of wall points become negligible.

The filamentary structure of regions with 17 or more streams (denoted as 17+) is shown for the simulation box of size 100 Mpc and 128$^3$ particles in Figure 1. It has to be noted that all the regions with 17+ streams are a subset of regions with 3 streams. Thus, the filaments are just segments of walls with higher streams. These exist at the intersections of walls. Further, at the intersections of multiple filaments, there are regions with locally maximum number of streams, signifying the most dense regions in the simulations; the dark matter halos. By superimposing the positions from the FOF-halo catalogue, it is visually confirmed that all the halos coincide with these high-streaming intersections.

4.1 Volume and mass fractions

The single-stream flow, which corresponds to the void, occupies around 90% of the volume of the simulation box (Figure 2). As mentioned in Section 4, higher multi-streaming flow regions are nested inside lower streaming regions. Thus the volume occupied by higher number of streams decreases
with the number of streams. This relation is found to be a power law. For the box of size $100 \ Mpc$ and $128^3$ particles, the volume fraction $f_{\text{vol}}(n_{\text{str}})$ of each stream in the multi-stream field calculated with refinement factor of 8 is

$$f_{\text{vol}}(n_{\text{str}}) = 0.93 n_{\text{str}}^{-2.80} \quad (1)$$

Due to lower occupancy of volume, the higher streams have less mass particles. being more non-linear regions, the number of mass particles is more in high multi-streams. These two opposing contributions result in a mass fraction for the same simulation box,

$$f_{\text{mass}}(n_{\text{str}}) = 0.59 n_{\text{str}}^{-1.27} \quad (2)$$

This is a good fit for the range of number of streams $n_{\text{str}} \geq 5$. The mean density of each stream, given by the the ratio of mass and volume fractions of the corresponding stream, increases as expected. Note that, this parameter of mean density is different from density of streams, which can be directly calculated from the volume and number of intersecting tetrahedra in the phase-space tessellation field.

$$\bar{\rho}(n_{\text{str}}) = 0.92 n_{\text{str}}^{-1.21} \quad (3)$$

This also quantifies our previous claim that very high multi-streams correspond to the most dense areas in the Universe, i.e. the condensed halos. The common overdensity threshold of 200 using virial equilibrium corresponds to roughly 90 streams in Figure 2.

Comparing the volume fraction of various simulation boxes in Figure 3, we find that the profile is similar for boxes with same inter-particle resolution; i.e., equal box length to grid size ratio. For e.g., $L/N = 0.78 \ Mpc$ for the simulation box of $100 \ Mpc - 128^3$ particles and $200 \ Mpc - 256^3$ particles. The box with minimum inter-particle resolution in the data, hence the best raw resolution ($L/N = 0.20 \ Mpc$ for $100 \ Mpc - 512^3$ particles), has higher volume fraction for each multi-stream compared to lower resolution boxes. Additionally, it has higher streams because of the inherent larger number of initial perturbations evolved into highly non-linear structure over time. Box with least raw inter-particle resolution ($L/N = 1.56 \ Mpc$ for $200 \ Mpc - 128^3$ particles), occupies lower volumes than other boxes for every number of stream. It is also prone to noise at very high streaming regions.

One of the advantages of using tessellation is the freedom to compute densities at very high resolutions. We use the parameter ‘refinement factor’ to denote the ratio of grid size of the multi-stream field to that of raw simulation data. High refinement factors are extensively used in understanding stream behavior not only in the halo environment, but inside the halo too (Section 5). The volume fraction of resulting number of streams normalized with resolution of the multi-stream field is identical for all refinement factors as shown in bottom of the Figure 3. Hence our triangulation algorithm is universal in the sense that it is independent of the refinement factors used in tessellating the box.

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5 STREAM ENVIRONMENT AROUND HALOS

Multi-stream field can be computed for a small Eulerian box with arbitrary refinement factor. This can be utilized to analyze the phase-space behavior inside and around halos. In this section, we have used the halo coordinates identified by the FOF method, and select Eulerian boxes around it. An excellent correspondence between FOF halo centers and local maxima of multi-stream field is visually examined in Figure 1.

Since each multi-stream region is surrounded by lower number of streams, the walls sandwich filaments inside themselves (Figure 4). The filaments are embedded with halos at various intersections. These high-streaming halos are completely covered by relatively low-stream filaments and hence surrounded by walls too. This result differs considerably from the several void finder methods, which find existence of halos within void regions (See Colberg et al. 2008 and references therein). By our classification, we distinguish configuration space of the simulation box as void and non-void or web regions. Further, the web can be classified into walls, filaments and halos based on thresholds.

Visual inspection of Figure 4 reveals that the halo environment is a highly intricate. Though the halos are surrounded by filaments and walls, it can be surprisingly close to the voids in a few directions. Filaments are roughly cylindrical in shape, with branching and intersections. Finally, the condensed halos are not roughly spherical or ellipsoidal.

We use a simple technique of projecting the number of streams onto a diagnostic spherical surface around a halo to understand this halo environment.

5.1 Technique

Motivated by the complicated morphology of multi-stream field around a dark matter halo, we have a devised an empirical methodology to quantify the random number of points on an diagnostic spherical surface that are selected and the number of streams passing through these points are identified. By coinciding the center of this sphere with center of FOF halo and increasing the size of the sphere from inside the halo to several times the halo size, the scaling of streams around the halo is determined.

The geometry of a filament can be crudely approximated to a cylinder and that of a wall to a sheet with a small constant thickness ‘d’. Upon intersecting with a spherical surface, these geometries occupy certain cross-sectional area, $\text{Area}_{c/s}$, on the sphere (See Figure 5). The ratio of this area to the surface area of the sphere is given by,

\begin{align}
    f_{\text{wall}}(r) &= \frac{\text{Area}_{c/s}}{4\pi r^2} = \frac{2\pi rd}{4\pi r^2} \propto r^{-1} \\
    f_{\text{fil}}(r) &= \frac{\text{Area}_{c/s}}{4\pi r^2} = \text{const} \cdot \frac{1}{4\pi r^2} \propto r^{-2}
\end{align}

The fractions of points on the surface of the sphere by multiple number of intersecting sheet-like walls or cylindrical filaments also scale proportional to $r^{-1}$ and $r^{-2}$ respectively.

For the diagnostic spheres of different radii, the location and scaling of multi-streams at the intersections are calculated. By checking the variation in the fraction of area occupied, we associate the number of streams with wall or halo.

Figure 4. Multi-stream flow regions in a small box of the simulation. Top left: regions with more than 3-stream flow are identified as walls (brown). Intersection of multiple walls have higher stream regions (green, 17+ streams) Single-streaming voids (white) occupy large volume and are very close to the filaments in some directions. Top right: 17+ streams (green) form filamental structures with nodes at the intersections (red, 93+ streams) Bottom left and right: Closer look at the highly non-linear region reveals that a filament is sandwiched between the walls (brown). The 93+ stream region (red) forms a virialized structure and is entirely contained within the filament. The black dots show the particles around FOF halos within linking length of 0.2.

Figure 5. Modelling a wall and a filament. A diagnostic spherical surface is intersected by a cylinder and plane.
Each of the Mollweide projections in Figures 6 - 9 show the projections of multi-stream field on the spherical surface, and provide useful insight of the phase space properties around a halo.

5.2 Voids, filaments and walls around halos

From the technique mentioned above, we arrive at quantitative cut-offs for non-void structures. For the non-linear regions, the fraction of total points on the sphere with distance from halo center.

The deviation from the exact slope is expected, since the assumption is made for uniform cylinders and planes is rather crude. The filaments and walls have a more complicated structure, where they branch out, and do not correspond to regular geometrical shapes. Detailed explanations for deviations are illustrated using Mollweide projections in the next section.

Variation of multi-streams regions of 5+ to 17+ streams slowly changes from $r^{-1}$ to $r^{-2}$. At 17+ stream regions scaling is closest to $r^{-2}$, the approximate filamentary geometry. Our seemingly arbitrary choice filament definition of regions more than 17+ streams (in Section 4, Figure 1 and Figure 4) was motivated by this observation. Thus projections on an diagnostic sphere is a convenient tool for classifying regions in the simulation as belonging to void, wall, filament or a halo.

We have picked 4 halos from different mass ranges: $3.7 \times 10^{14}\odot$, $5.0 \times 10^{13}\odot$, $7.0 \times 10^{12}\odot$ and $1.1 \times 10^{12}\odot$ from the simulation box of $100 \ h^{-1}$ Mpc length and $128^3$ particles. Multi-stream field with a high refinement factor of 8 is calculated for a greater resolution on scales of the halo volume.

Diagonal spheres of radii $0.1 \ h^{-1}$ Mpc to $5 \ h^{-1}$ Mpc are drawn for each of these halos (Figures 6 - 9 bottom figures), with the multi-stream field projected onto the surface. In the Mollweide projections of these spheres, the white space refers to single stream voids. For the largest halo ( Figure 6) with FOF radii 1.2 $h^{-1}$ Mpc, the voids already appear in sphere of radius 1.5 $h^{-1}$ Mpc and in the smaller halos (Figure 9) it appears as early as 0.5 $h^{-1}$ Mpc.

Upto 1 $h^{-1}$ Mpc from halo center of the largest halo, the surfaces are uniformly covered with high number of streams (red, 17+). This shows that the most non-linear regions are close to centers of Halos. A similar trend is seen for the halo of radius 0.7 $h^{-1}$ Mpc (Figure 7). For smaller halos (Figures 8 and 9) lower number of streams (even the wall forming 3+ streams; blue) start occupying the spherical surface at radii lesser than FOF-radius. In the case of the smallest halo of $1.1 \times 10^{12}\odot$ mass, 7+ streams are seen at scales as low as 0.1 $h^{-1}$ Mpc. The distribution of multi-streams on the surface of the sphere is not symmetrical about any axes, signifying a complex morphology of phase space around highly non-linear structures. Regions with 5+ to 15+ streams form structures intermediary to filament-like and wall-like behavior, as seen by scaling of fraction of total points on the space with distance from halo center.

Halo environment at distance over twice the FOF radius reveals interesting morphological features. The walls intersect the sphere, and in the projections, appear like a thin strip. We also note that a filament along the surface shall appear as a strip too (like in Figure 7) see the corresponding discrepancy in fraction of streams), but upon inspecting the spheres at various radii, we can clearly identify the persisting line-like structures, and they correspond to a wall. Similarly, a filament is projected as a dot-like structure, which occurs by an intersection of a cylinder-like geometry with the spherical surfaces. It is clearly observed at the distance of 4 - 5 $h^{-1}$ Mpc in Figure 6 and in between 0.5 - 5.0 $h^{-1}$ Mpc in Figure 5.

Hence we conclude that 3+ stream regions constitute walls and the regions with 17+ streams correspond to filaments. The higher streams are surrounded by regions with lower streams. Thus, the filaments are within the walls, and do not exist independently. We remind that the each spheres have radius varying from $0.1 - 5 \ h^{-1}$ Mpc, whereas the projections shown here are of same size, hence the walls and filaments appear more narrow and smaller in larger spheres due to zooming-out effects. In a few cases ( Figure 7), the
largest diagnostic sphere shows the large scale cosmic web structure through the surface of the sphere.

Discrepancy in variation of fraction of points with distance from halo center reflect complicated structures near halos. In Figure 9, where the line corresponding to higher streams (17+) has a peak, is mostly due to the presence of another halo nearby (seen at bottom of Mollweide projection of 1 - 2 \(h^{-1}\) Mpc). Figure 9 also has some deviation, and this is due to the intricate shape of 17+ stream filament, which appears to be branching out after 1 \(h^{-1}\) Mpc.

Generally the transitions from halos to filaments then to walls and finally to voids appear to be rather smooth. However occasionally sharp features as one seen in Figure 9 may emerge when the diagnostic sphere hits a neighboring halo.

Number of streams and their fraction of points on the diagnostic sphere of radius given by FOF method reveal the multi-stream properties around halos of different masses (See Figure 10). Considering samples of 10 halos around each of the 4 mass ranges, we see that the large halos have very high number of streams on their radii (Over 200 streams in the case of 2.1 \(\times 10^{14} - 3.9 \times 10^{15} \odot\) ), whereas, for less massive halos, there are negligible 50+ streams (note that a large number of halos are detected by FOF for lesser particles, hence the mass range is narrow. The large halos are fewer in the simulation, contribution the higher variance in mass range). The number of streams corresponding to maximum contribution of fraction of points is less for smaller halos. In the case of large halos, the maximum contribution is reduced due to presence of larger range of multi-streams around it.

6 SUMMARY AND CONCLUSION

In this paper we explore the multi-stream environment of FOF haloes compute in an N-body simulation in \(\Lambda\)CDM cosmology at \(z = 0\). Then using the tessellation of the three-dimensional Lagrangian submanifold \(x = x(q, t)\) in six-dimensional \((x, q)\) space, Shandarin, Habib & Heitmann (2012) we compute the multi-stream field i.e. the number of stream on a regular grid in the configuration x-space, \(n_{str}(x)\). The multi-stream field takes odd whole numbers everywhere except at a set of points of measure zero where it takes positive even whole numbers. The multi-stream field allows to define physical voids as the regions with \(n_{str} = 1\).
Fraction of streams

The rest of space with \(n_{str} \geq 3\) can be called the non-void or web. This division of the space into two parts is unique and physically motivated: no object can form before shell crossing happens. It is worth emphasizing that the division of space into voids and web is based on the local parameter, the number of streams at a single point. Shandarin, Habib & Heitmann (2012) Abel, Hahn & Kuehler (2012), Falck, Neyrinck & Szalay (2012) and Falck & Neyrinck (2014) used similar physical idea in the ORIGAMI method, but their numerical method is different. It is approximate and somewhat less accurate.

The further division of the web into walls, filaments and haloes is not straightforward although halos can be well defined using dynamical parameters related to the requirement of virialization of haloes. One of the simplest is the famous density threshold \(\rho/\bar{\rho} \approx 200\). Identifying filaments and walls is significantly more tricky (see e.g. Hahn et al. 2007 Forero-Romero et al. 2009, Aragon-Calvo, van de Weylaert & Jones 2010, Cautun et al. 2011, Falck & Neyrinck 2014).

In this study we introduced an empirical statistical criteria for identifying filaments and walls. Defining a relatively large part of walls can also be done locally since the regions where \(n_{str} = 3\) can be only walls. Unfortunately, some regions with five, seven and more also can be walls. However, due to quite steep decline of function \(f_{vol}(n_{str})\) the regions with \(n_{str} = 3\) make a substantial fraction of the walls; according to eq. 1 \(f_{vol}(n_{str} = 3) \approx 4.3\%\) and \(f_{vol}(5 \leq n_{str} \leq 15) \approx 1.8\%\). The latter by itself makes only about 40% of the former while it contains some input from filaments. Unfortunately we are not able to make better estimates of the volume fraction of walls.

We have found empirically that in the studied simulation the transition from wall points to filament points takes place approximately at \(5 \leq n_{str} \leq 15\). We also estimated that the haloes correspond to the regions with \(n_{str} \geq 90\). Thus, the transition from filament to haloes takes place in the range \(17 \leq n_{str} \leq 90\).

Using our prescription, for the simulation box size of least inter-particle resolution \(L/N = 1.56 h^{-1}\) Mpc (200 \(h^{-1}\) Mpc and 128\(^3\) particles), the volume fraction of voids, walls, filaments and halos are approximately 95.6%, 4.2%, 0.07% and 0.003% respectively. For the highest resolution inter-particle resolution box \(L/N = 0.2 h^{-1}\) Mpc (100 \(h^{-1}\) Mpc and 512\(^3\) particles), more nonlinear structures are formed, and corresponding fractions are approximately 87.8%, 10.7%, 1.2% and 0.1% respectively.

Concluding we would like to emphasize that locally the thickness of the walls is essentially similar to that of filaments. However it vary significantly within the whole volume. Roughly the thickness of the web corresponds to the radii of the halos. The voids appear on the diagnostic sphere with radius about 1.5 of FOF radius of the halo as Figures 6 - 9 illustrate.

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