Quantum correlations between separated particles

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Abstract

Long-range quantum correlations between particles are usually formulated by assuming the persistence of an entangled state after the particles have separated. Here this approach is re-examined based upon studying the types of correlations present in a pair of EPR spins. One type, due to the quantum interference terms, is characterized by parity. Second is correlation due to conservation of angular momentum. The two contributions are equal and have the same functional form. When entanglement is present, Bell’s Inequalities are violated but when parity is destroyed by disentanglement, Bell’s inequalities are satisfied. It is shown that some experiments which have been interpreted assuming entangled states can also be described by disentanglement. Implications for quantum non-locality are discussed.

Keywords: Entanglement, disentanglement, quantum correlations, EPR paradox, Bell’s inequalities, quantum non-locality and locality, coincidence detection, detection loophole

1. Introduction

In 1935 Schrödinger\textsuperscript{1} first introduced the term “entanglement”. In the same paper he also described a process he termed “disentanglement”. This refers specifically to the process of measurement. Furry\textsuperscript{2} in a similar discussion showed that upon measurement the interference terms, fundamental to quantum mechanics, are lost.

Both Schrödinger\textsuperscript{1} and Furry\textsuperscript{2} were concerned with the process of measurement. However Schrödinger\textsuperscript{3} in his penultimate paragraph on page 451 states that another form of disentanglement is possible. There he specifically refers to the process of separation as a means of decoherence of phase relationships between separated particles. His exact words in reference to the

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EPR$^4$ paradox are:

“It seems worth noticing that the paradox could be avoided by a
very simple assumption, namely if the situation after separating
were described by the expression (12), but with the additional
statement that the knowledge of the *phase relations* between the
complex constants $a_k$ has been entirely lost in consequence of the
process of separation. This would mean that not only the parts, but
the whole system, would be in the situation of a mixture, not of a
pure state. It would not preclude the possibility of determining the
state of the first system by suitable measurement in the second one
or vice versa. But it would utterly eliminate the experimenters
influence on the state of that system which he does not touch.”

For completeness, Schrödinger$^3$ equation (12) is

$$\Psi(x, y) = \sum_k a_k g_k(x) f_k(y)$$

The purpose of this paper is to study the types of correlations that exist
between an EPR entangled pair of spins after they have separated. Specifically,
two types of correlation are identified. One arises from the preservation of parity
between the separated spins. The other arises from the conservation of angular
momentum between separated EPR spins due to the same quantization axis for
both. The former (parity) must be preserved for entanglement to survive and
leads to a violation of Bell’s Inequalities. The latter (conservation of angular
momentum), resolves the EPR paradox and satisfies Bell’s Inequalities.

In this context, Bell’s Inequalities are useful to distinguish one type of
correlation from another; one arising from the quantum interference terms and the
other from the classical contributions. Since correlations due to both parity and
conservation of angular momentum are determined at separation, and these
properties are carried by the particles as they move apart, the approach is local.
This leaves as debatable the question as to whether violation of Bell’s inequalities
necessarily leads to quantum non-locality in the ensemble treatment presented
here. Moreover, disentanglement is a process that destroys parity while
conserving angular momentum as the particles separate. That is, the process of
disentanglement causes the interference terms to decohere and therefore destroys
entanglement between the EPR pair.

The term “disentanglement” is used here to define such a separation
process that causes entanglement to be lost thereby leaving the system in a mixed state characterized by different quantization axes. This ensemble of quantization axes must be averaged in order to obtain the final result. Although disentanglement can occur as particles separate, it is also possible that the phase coherence needed to maintain entanglement is lost between the source of entangled pairs and the filters that resolve them.

If entangled states are reduced to mixed states, then the classical correlation is weaker than the correlation due to entanglement. However, results from current coincidence detection techniques, after ensemble averaging, are indistinguishable from those results that retain entanglement. Disentanglement, however, leads to the prediction of random coincidences and low detection rates. Low detection rates are commonly referred to as the detection loophole. The presence, however, of such a loophole makes it impossible to distinguish entanglement from disentanglement because the mathematical functionality is the same for both. In particular, correlation due to entanglement of an EPR spin pair is $-\cos \theta_{ab}$, where $\theta_{ab}$ is the angle between the two Stern-Gerlach filters oriented in directions $a$ and $b$. When entanglement is lost, the correlation drops to $-\frac{1}{2} \cos \theta_{ab}$.

In the CHSH form of Bell's inequalities, the former leads to violation $\left(2\sqrt{2} < 2\right)$, while in the latter there is no violation, $\left(\sqrt{2} < 2\right)$. Since it is not yet possible to distinguish the prefactor of $\frac{1}{2}$ between the two approaches, the experiments to date cannot be used to support the non-local nature of a quantum mechanics. Experimental confirmation of quantum “teleportation” has been reported. If disentanglement occurs and these experiments agree with the predictions, then they give support for the ensemble treatment here.

2. The EPR Density Operator and Parity

An EPR pair comprised of two spins with angular momentum of one half are entangled when they can be represented by a singlet state $^{\Psi_{12}^-}$ where

$$\left|\Psi_{12}^-ight> = \frac{1}{\sqrt{2}} \left[ |+\rangle^1_z |\rangle^2_z - \langle |\rangle^1_z |\rangle^2_z \right].$$

(2.1)

Superscripts label the two spins and subscripts indicate that the two spins are quantized along an arbitrary $z$-axis of a coordinate frame located on the pair. In general, the state in Eq. (2.1) can be represented along any direction defined by a
unit vector $\hat{P}$ in the same coordinate frame resting on the pair,

$$\sigma_{\hat{P}}^i |\pm_i\rangle_P = \pm |\pm_i\rangle_P$$  \hspace{1cm} (2.2)

where the kets have their usual representation

$$|+\rangle_P = \begin{pmatrix} \cos \frac{\theta}{2} \\ +\sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} \quad \text{and} \quad |-\rangle_P = \begin{pmatrix} -\sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix}$$  \hspace{1cm} (2.3)

The polar angles $\theta$ and $\phi$ orient $\hat{P}$ that defines the axis of quantization and $\hat{P} \cdot \sigma^i = \sigma_{\hat{P}}^i$ is in terms of the Pauli spin vector, $\sigma^i$. From Eq.(2.1), the EPR density operator is isotropic and describes a pure state of zero spin angular momentum,

$$\rho_{\text{EPR}}^{12} = \langle \Psi^-_{12} | \Psi^-_{12} \rangle = \frac{1}{4} (I^1 I^2 - \sigma^1 \cdot \sigma^2)$$  \hspace{1cm} (2.4)

This can be used to calculate the expectation value of the correlation operator $a \cdot \sigma^1 \sigma^2 \cdot b$ that depends on the orientation of two Stern-Gerlach filters in the $a$ and $b$ the directions. The result is

$$E(a,b)_E = \langle a \cdot \sigma^1 \sigma^2 \cdot b \rangle_E = a \cdot \left[ Tr_{12} \rho_{\text{EPR}}^{12} \sigma^1 \sigma^2 \right] \cdot b = -\cos \theta_{ab}.$$  \hspace{1cm} (2.5)

where $\theta_{ab}$ is the angle between the two vectors, $a$ and $b$. Equation (2.5) is given the subscript $E$ for “Entangled” to distinguish it from the results that follow for disentanglement. A significant property of entangled EPR pairs is that all possible quantization axes lead to the same pure state, Eq.(2.4).

Since photons are used in the majority of experiments, conservation of angular momentum and parity are summarized here for completeness. For circularly polarized light the helicity is defined along the direction of propagation ($z$ axis), and the states are traditionally denoted by $|R\rangle$ and $|L\rangle$ for the right and left components respectively. Conservation of angular momentum when the two photons propagate in opposite directions requires the pair state to be $|R,R_L\rangle$ and
\[ \ket{L_1L_2}, \text{ or their superposition. Figure 1a gives a classical depiction of an entangled pair of photons. Parity is the inversion through the source and this changes } \ket{R_1R_2} \text{ to } \ket{L_1L_2} \text{ and vice versa. If parity is to be conserved then the resulting states must be entangled,} \]
\[ \ket{\Phi_{12}^\pm} = \frac{1}{\sqrt{2}} \left( \ket{R_1R_2} \pm \ket{L_1L_2} \right) \tag{2.6} \]

Unless parity is maintained between the two photons as they propagate, then they are not entangled. If parity is preserved as the particles separate, then they remain entangled until some interaction disrupts one or the other particles. The singlet state, \[ \ket{\Psi_{12}^-} = \frac{1}{\sqrt{2}} \left( \ket{R_1L_2} - \ket{L_1R_2} \right) \]
is even to parity.

3. Disentanglement upon Particle Separation.

Here disentanglement is defined as a partial conditional reduction of the state that takes place during the process of separation or at some time after the particles move apart (see Eqs.(3.1) and (3.2) below). This leads to a decoherence of the quantum interference terms between the two separated spins. As applied to a singlet state, as the two spins separate, they are quantized along a specific direction \( \hat{P} \) (Figure 1). That is, if the second spin is in a definite quantum state, say the “minus” state at the point of separation, and so the first spin must be in the “plus” quantum state with respect to the same \( \hat{P} \) axis. The EPR pair density operator is then traced over the departed particle to give a single spin density operator that retains correlation from its separated partner. Thus the separation process maintains a correlation between the two spins as they separate leading to a density operator for spin 1 of,
\[ \rho^1_{\hat{P}}(+) = \frac{1}{4} \left[ I^1 + \sigma^1 \cdot \hat{P} \right] = \frac{1}{2} \ket{+}^1\bra{+}_{\hat{P}} \]
\[ \rho^2_{\hat{P}}(-) = \frac{1}{4} \left[ I^2 - \sigma^2 \cdot \hat{P} \right] = \frac{1}{2} \ket{-}^2\bra{-}_{\hat{P}} \]

The single spin density operator for spin 2 is obtained in an identical manner. The result is,
\[ \rho^2_{\hat{P}}(-) = \frac{1}{4} \left[ I^2 - \sigma^2 \cdot \hat{P} \right] = \frac{1}{2} \ket{-}^2\bra{-}_{\hat{P}} \]

Likewise \( \rho^1_{\hat{P}}(-) \) and \( \rho^2_{\hat{P}}(+) \) are defined.
After the two particles have disentangled, the two single-spin density operators can be used for measurements on either spin. Evaluation of the expectation value in Eq.(2.5) for the disentangled EPR pair is therefore given by

\[
\left\langle \mathbf{a} \cdot \mathbf{\sigma}^1 \mathbf{\sigma}^2 \cdot \mathbf{b} \right\rangle_{\mathbf{P},\mathbf{D}} = \left\langle \mathbf{a} \cdot \mathbf{\sigma}^1 \right\rangle_{\mathbf{P}} \left\langle \mathbf{\sigma}^2 \cdot \mathbf{b} \right\rangle_{\mathbf{P}} = \text{Tr} \left\{ \mathbf{a} \cdot \mathbf{\rho}_P^1(+) \right\} \text{Tr} \left\{ \mathbf{b} \cdot \mathbf{\rho}_P^2(-) \right\}
\]

The two expectation values are

\[
\left\langle \mathbf{a} \cdot \mathbf{\sigma}^1 \right\rangle_{\mathbf{P}} = \frac{1}{2} \mathbf{a} \cdot \hat{\mathbf{P}} = +\frac{1}{2} \cos \theta_a
\]

and

\[
\left\langle \mathbf{b} \cdot \mathbf{\sigma}^2 \right\rangle_{\mathbf{P}} = -\frac{1}{2} \mathbf{b} \cdot \hat{\mathbf{P}} = -\frac{1}{2} \cos \theta_b
\]

The combined pair is therefore
where the two angles are those defined relative to the specific quantization axis by \( \mathbf{a} \cdot \hat{\mathbf{P}} \) and \( \mathbf{b} \cdot \hat{\mathbf{P}} \). From Equations (3.1)-(3.2) and (3.4)-(3.6), it is apparent that the axis of quantization is carried with each spin as it leaves the singlet state. Hence each spin carries with it specific information of its partner so that the results of the two measurements at remote locations, when combined in Eq.(3.6), are correlated even though measurement of one can have no influence on the other. The common quantization axis ensures the conservation of angular momentum as the two particles separate. No information need be transported between the two measuring devices in order to arrive at Eq.(3.6). The two spins, however, must be from either the same EPR pair or an ensemble of spins with the same axis of quantization.

Besides the difference between the numerical factors Eq.(2.5) and (3.6), it is clear that phase information is lost. The relationship for the angle between two vectors defined relative to a coordinate system, here \( \hat{\mathbf{P}} \), is

\[
\cos \theta_{ab} = \cos \theta_a \cos \theta_b + \sin \theta_a \sin \theta_b \cos(\phi_a - \phi_b) \tag{3.7}
\]

where the angles on the RHS orient the filter vectors, \( \mathbf{a} \) and \( \mathbf{b} \), relative to the quantization axis. The first RHS term in Eq.(3.7) is the contribution from correlation that is left over after disentanglement, Eq.(3.6), while the second right hand term gives the extra correlation from entanglement that is present before disentanglement has occurred. Since the former arises from a mixed state that has no quantum interference terms, it can be deduced that the second RHS contribution is due to the presence of quantum interference terms. It is just these terms that cause violation of Bell’s inequalities.

In general, because many EPR pairs separate, the direction \( \hat{\mathbf{P}} \) is random and Eqs.(3.4) to (3.6) must be ensemble averaged. If the ensemble average is taken over individual spins that in general do not come from the same EPR pair, then there is no correlation. Denoting ensemble averaging by a bar gives, therefore

\[
\langle \mathbf{a} \cdot \sigma^1 \rangle_{\bar{P}} = 0 \quad \text{and} \quad \langle \mathbf{b} \cdot \sigma^2 \rangle_{\bar{P}} = 0 \tag{3.8}
\]

The correlation is likewise zero for non-EPR pairs since, in general, \( \hat{\mathbf{P}} \neq \hat{\mathbf{P}}' \) and
\[
\langle a \cdot \sigma^i \sigma^j \cdot b \rangle_{\text{non-EPR pairs}} = -\frac{1}{4} a \cdot \hat{P} \hat{P} \cdot b = 0
\]  

(3.9)

In contrast, the ensemble average of disentangled EPR pairs is not zero. If it is assumed that all the spins are produced isotropically, then the ensemble average is

\[E(a, b)_D \equiv \langle a \cdot \sigma^i \sigma^j \cdot b \rangle_{ P, D} = -\frac{1}{4} a \cdot \hat{P} \hat{P} \cdot b = -\frac{1}{12} a \cdot b = -\frac{1}{12} \cos \theta_{ab} .\]  

(3.10)

The factor of 1/3 arises from the isotropy of three dimensional space and the factor of 1/4 arises from the normalization of Eqs. (3.1) and (3.2).

4. The EPR paradox and disentanglement

The correlation carried by the two separating spins is sufficient to account for the conservation of angular momentum as required by the EPR paradox. Figure 3 shows a schematic of the two separating spins and the probabilities associated with each of the four detections. These can be calculated for each EPR pair when the Stern-Gerlach filter is oriented in the direction \( \mathbf{a} \) on the left and in direction \( \mathbf{b} \) on the right. The results are

\[P^1_+ (+, a) \equiv Tr_{+} \left| + \rangle_a \langle + |_a \rho_p^1 (+) \right| = \frac{1}{2} \cos^2 \left( \theta_a / 2 \right) \]  

(4.1)

\[P^1_- (-, a) \equiv Tr_{-} \left| - \rangle_a \langle - |_a \rho_p^1 (+) \right| = \frac{1}{2} \sin^2 \left( \theta_a / 2 \right) \]  

(4.2)

\[P^2_+ (+, b) \equiv Tr_{+} \left| + \rangle_b \langle + |_b \rho_p^2 (-) \right| = \frac{1}{2} \sin^2 \left( \theta_b / 2 \right) \]  

(4.3)

\[P^2_- (-, b) \equiv Tr_{-} \left| - \rangle_b \langle - |_b \rho_p^2 (-) \right| = \frac{1}{2} \cos^2 \left( \theta_b / 2 \right) \]  

(4.4)

The sum of these four probabilities is unity. These equations show conservation of angular momentum between the two locations. This is maintained not by the collapse of the singlet wave function, but rather by the spins belonging to a sub-ensemble characterized by one quantization axis.
5. Aspect experiments

The correlation function, \( E(a, b) = a \cdot \left\langle \sigma^x \sigma^y \right\rangle \cdot b \) can be expressed in terms of the four probabilities \(^{19} \),

\[
E(a, b) = P_{++}(a, b) - P_{--}(a, b) - P_{+-}(a, b) + P_{-+}(a, b)
\]

Using entanglement, the density operator, \( \rho_{EPR}^{12} \), Eq.(2.4), can be used to evaluate the coincidence probabilities that are defined by

\[
P^E_{++} (a, b) = \text{Tr} \left\{ \left( |\pm \rangle_a \langle \pm \rangle_b \right)^2 \rho_{EPR}^{12} \right\} = \left| \left\langle \pm \right|_a \langle \pm \right|_b \Psi^{-}_{12} \right|^2
\]

\[
P^E_{+-} (a, b) = \text{Tr} \left\{ \left( |\pm \rangle_a \langle \mp \rangle_b \right)^2 \rho_{EPR}^{12} \right\} = \left| \left\langle \pm \right|_a \langle \mp \right|_b \Psi^{-}_{12} \right|^2
\]

The results are

\[
P^E_{++} (a, b) = \frac{1}{2} \sin^2 \frac{\theta_{ab}}{2} = \frac{1}{4} (1 - \cos \theta_{ab})
\]

\[
P^E_{+-} (a, b) = \frac{1}{2} \cos^2 \frac{\theta_{ab}}{2} = \frac{1}{4} (1 + \cos \theta_{ab})
\]

These are consistent with the quantum mechanical prediction for entanglement using Eq.(5.1) and as given by Eq.(2.5).

6. Aspect experiments assuming disentanglement.

Equations (4.1) to (4.4) assume that all the spins moving left are in the “plus” state and all those moving right are in the “minus” state, see Figure 3. The experiment, however, has both “plus” and “minus” states moving both left and right. Such a situation is described by a mixed state of the two possibilities that is expressed as

\[
\rho_{P,D}^{12} = \frac{1}{2} \left( \rho_p^1 (+) \rho_p^2 (-) + \rho_p^1 (-) \rho_p^2 (+) \right)
\]

Using the same definitions as Eqs. (5.2) and (5.3) gives
\[ P^D_{\pm}(a,b) = Tr_{1,2} \left\{ \left( |\pm\rangle_a \langle\pm|_b \right)^2 \left( \pm|_a \langle\pm|_b \right) \rho_{\text{P,disentangled}}^{12} \right\} \] (6.2)
and
\[ P^D_{\mp}(a,b) = Tr_{1,2} \left\{ \left( |\pm\rangle_a \langle\pm|_b \right)^2 \left( |\mp\rangle_a \langle\mp|_b \right) \rho_{\text{P,disentangled}}^{12} \right\} \] (6.3)

and upon evaluation are equal to
\[ P^D_{\pm}(a,b) = \frac{1}{16} \left( 1 - \cos\theta_a \cos\theta_b \right) \] (6.4)
\[ P^D_{\mp}(a,b) = \frac{1}{16} \left( 1 + \cos\theta_a \cos\theta_b \right) \] (6.5)

The superscript D denotes disentanglement. For photons, the direction of motion gives the axis about which the helicities are defined. For an ensemble of pairs, it is therefore necessary to average over the components of \( \hat{P} \) in the plane perpendicular to the \( z \) axis. (cf. Eq.(3.10))

\[ \overline{P}^D_{\pm}(a,b) = \frac{1}{16} \left( 1 - a \cdot \hat{P} \cdot b \right) = \frac{1}{16} \left( 1 - \frac{1}{2} \left( \frac{\hat{x} \hat{x} + \hat{y} \hat{y}}{2} \right) \cdot b \right) \]
\[ = \frac{1}{16} \left( 1 - \frac{1}{2} a \cdot b + \frac{1}{2} \hat{z} \cdot b \right) = \frac{1}{16} \left( 1 - \frac{1}{2} \cos\theta_{ab} \right) \] (6.6)

Since the two polarizers are coplanar and are perpendicular to the beam of photons, the term \( a \cdot \hat{z} \cdot b = 0 \). The ensemble averaging leads, therefore, to
\[ \overline{P}^D_{\pm}(a,b) = \frac{1}{16} \left( 1 - \frac{1}{2} \cos\theta_{ab} \right) = \frac{1}{16} \left( 1 - \frac{1}{2} \cos\theta_{ab} \right) \] (6.7)
\[ \overline{P}^D_{\mp}(a,b) = \frac{1}{16} \left( 1 + \frac{1}{2} \cos\theta_{ab} \right) = \frac{1}{16} \left( 1 + \frac{1}{2} \cos\theta_{ab} \right) \] (6.8)

If the two spins are not from the same EPR pair, the ensemble average is zero \( (a \cdot \hat{P}' \cdot b = 0) \) for \( \hat{P} \neq \hat{P}' \), cf. Eq.(3.9). Using Eq.(5.1), the correlation from
disentanglement is found to be,

\[ E(a, b)_{\text{disentanglement}} = -\frac{1}{8} \cos \theta_{ab} \quad (6.9) \]

Apart from the factor of 1/8 the results from entanglement and disentanglement have the same functional form, i.e. \( \cos \theta_{ab} \), reflecting both the a difference in normalization (1/4) and a reduced level of correlation, (1/2).

The results from the entangled states, Eq.(5.4) and (5.5), can be directly compared with if the normalization is changed to be consistent between the single state probabilities and the pair state probabilities. The sum of each set of four probabilities is presently different because the former is normalized with respect to the total number of spins while the latter is normalized with respect to the total number of spin pairs. Changing the normalization to reflect normalization with respect to spin pairs leads to the following change in Eqs.(6.7) and (6.8),

\[ P^{D}_{\pm\pm}(a, b)_{\text{normalized to pairs}} \rightarrow \frac{1}{8} + \frac{1}{8} (1 - \cos \theta_{ab}) \quad (6.10) \]

\[ P^{D}_{\pm\mp}(a, b)_{\text{normalized to pairs}} \rightarrow \frac{1}{8} + \frac{1}{8} (1 + \cos \theta_{ab}) \quad (6.11) \]

With this, the sum of the four coincidence probabilities is unity, rather than ¼.

The first term of 1/8 in the above equations accounts for the random coincidences and is constant due from the destruction of the correlation from the quantum interference terms. The second term accounts for correlated coincidences arising from the conservation of angular momentum. The functional form from disentanglement is indistinguishable from that from entanglement.

7. Discussion

The correlation function, \( E(a, b) \) defined by Bell is\(^{13}\),

\[ E(a, b) = \int P(\lambda) A(a, \lambda) B(b, \lambda) d\lambda \quad (7.1) \]
where $A(a, \lambda)$ and $B(b, \lambda)$ are local functions that can take any values from $-1$ to $+1$. These spin functions are averaged over a distribution, $P(\lambda)$, of hidden variables $\lambda$. In the ensemble treatment, each EPR pair is characterized by its own axis of quantization $\hat{P}$, therefore accounting for all possible spin states. The quantities $A(a, \lambda)$ and $B(b, \lambda)$ can be associated with the expectation values of the two spins, Eqs.(3.4) and (3.5), (re-normalized to 1), with $\lambda$ replaced by $\hat{P}$.

$$A(a, \hat{P}) = \langle a \cdot \sigma^1 \rangle_{\hat{P}} = +a \cdot \hat{P} \quad (7.2)$$

$$B(b, \hat{P}') = \langle b \cdot \sigma^2 \rangle_{\hat{P}'} = -b \cdot \hat{P}' \quad (7.3)$$

In order to arrive at the expression for the correlation, Eq.(3.6), the distribution function, $P(\lambda)$, is replaced by

$$P(\hat{P}, \hat{P}') = \delta(\hat{P} - \hat{P}') \quad (7.4)$$

Bell’s inequalities are satisfied, as they must be for product states. On the other hand, if parity is conserved, and the EPR pair remains entangled up to detection, then Bell’s inequalities can be violated. Bell’s Inequalities can be interpreted as a way of distinguishing between the cases when the quantum interference terms are present or absent. In other words, if a sub-ensemble is characterized by a specific quantization axis $\hat{P}$ this is enough to conserve angular momentum between the separated EPR pairs without recourse to non-local arguments.

The above discussion does not take into account ensemble averaging which must be performed if all orientations of $\hat{P}$ are possible. Ensemble averaging has no effects on the pure state entangled EPR pairs. For disentangled EPR pairs, in Section 6, it is shown how the ensemble average results in two contributions to the probabilities. One, an oscillatory term, arises from the correlation that exists locally as the particles separate. This is carried by each separated EPR pair and is detected as correlated coincidences between Alice and Bob. In addition there is a constant term that arises from random coincidences at both Alice’s and Bob’s detectors. Since the ensemble averaging is performed in the plane perpendicular to the direction of propagation, the different helicity phases are randomised. Although a different prefactor occur between entanglement and disentanglement correlation expressions, experiments to date cannot distinguished between the two. It can be concluded, therefore, that such
experiments are consistent with quantum mechanics but no other conclusions regarding quantum non-locality can be drawn.

Using the coincidence probabilities from disentanglement, Eq.(6.7) and (6.8), gives a correlation from entanglement that is \(-\frac{1}{16} \cos^2 \theta_{ab}\), or a detection rate of only 6.25%. Typical experiments detect only about 5% of photons produced. Hence the prefactor from disentanglement underscores the necessity of performing more experiments to resolve the detection loophole.

Entanglement assumes that the same zero angular momentum pure state exists both before and after the particles have moved apart. Some process that results in the total pair density operator becoming a product of two density operators, however, is usually needed to bring about disentanglement. Whether this occurs at the time of separation or at some later time, in many experiments performed to date disentanglement can account for the results\(^{28}\). This means that decoherence has occurred that destroys the interference terms and, in the case of EPR-pairs their entanglement.

Generalizing from EPR pairs to any disentanglement process can be expressed as,

\[
\rho^n = Tr_{N-n} \{ |N-n\rangle \langle N-n| \rho^N \}
\]  

That is, disentanglement of \(n\) interacting particles from a system of \(N\) interacting particles involves a reduction of the \(N\)-particle density operator \(\rho^N\) to an \(n\)-particle density operator, \(\rho^n\). The projector \(|N-n\rangle \langle N-n|\) represents the state of the remaining particles at the instant of separation. Such a reduction destroys the quantum interference terms, and any symmetry associated with them, between the two separated parts.

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