Research Article

Asymptotic Behavior of a Predator-Prey Model with Allee Threshold Applied to Online Social Network Users’ Data Forwarding

Yaming Zhang, Yaya Hamadou Koura, and Yanyuan Su

School of Economics and Management, Yanshan University, Qinhuangdao 066004, China

Correspondence should be addressed to Yaming Zhang; yaming99@ysu.edu.cn and Yaya Hamadou Koura; 2689395039@qq.com

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We consider a predator-prey relationship in a fair system in which interacting species have different needs of resources to survive. We analyzed qualitatively the outcome of interaction using a modified logistic predator-prey model with Allee threshold in both predator and prey equations. We showed that the system had very rich dynamical behavior as stability around fixed points and periodic solutions could be obtained at certain conditions. Interaction outcome is highly submitted to initial conditions, species behavior, and the threshold applied. Numerical results suggested adapting resource allocation and the threshold value to optimize ecosystem sustainability.

1. Introduction

In an ecosystem where resource accessibility for interactive species is fair, competition intensity, which can be assimilated to each species ability to harvest, is a key parameter in controlling system stability and optimizing ecosystem sustainability. In addition, when shared resource, for instance, niche or habitat is extremely limited, one needs to monitor species birth rate, death rate, and overall population size to avoid overdomination and extinction that can lead to chaos [1–4].

In applied mathematics and engineering sciences, predator-prey models based on Lotka–Volterra are commonly used to study such systems. There exist numerous models adjusted to formalize the particular relationship between individuals of the same or different species sharing one or many types of resources. Models with logistic growth, which implies limiting capacity, are of interest as they are admitted to be more realistic in studying underlying relationship between entities. However, classical Lotka–Volterra systems by supposing unlimited resources for interacting species and focusing mainly on interaction outcome fail to formalize small variations happening in species intrinsic growth rate due to resource abundance, competition, or delay in assimilation of harvested resource, for example. As many research studies pointed out, in nature and in complex systems, such as social networks, most of interactions occur in limited resource environment [5–9]. Thus, models that incorporate threshold or refuge terms in predator or prey equations present special interest for implementing better control mechanism. Such models are widely spread and intensively studied in related literature. Particularly, models with Allee-threshold extinction control mechanism present the advantage of capturing such variations of species growth rate by setting a maximum value to reach for population size to not go extinct [10–15].

Furthermore, systems that exhibit rich dynamical behavior, including periodic solutions, stable limit cycles, bifurcation, and chaos have many applications in economics, management sciences, engineering sciences, and so forth.

In this article, we are interested in analyzing an ecosystem with limited resources in which two interacting species with different needs and resource accessibility are competing for their growth and survival. We assimilate this type of dynamic and relationship to online social network users characterized by their traffic profile during peak hours.
Their generated data packets travel through network segment and queue at accommodating segment buffer space, considered as the limiting carrying capacity of the environment. This investigation is motivated by current network congestion control issues and aimed to provide a new angle of analysis for decision-making process by incorporating users’ behavior in the traffic shaping and priority to give to certain flows to avoid latency and congestion at peak hours [16–28].

By applying stability theory and solving model differential equations, we found that, at certain conditions, the proposed system could be stable and species coexistence could be obtained when resource accessibility is fair. Competition outcome depends on initial conditions, the Allee threshold, and species ability to harvest and consume available resources. Results of numerical simulations suggested adopting hybrid resource allocation and priority at certain saturated nodes to avoid latency and poor quality of service by enhancing segment reliability.

2. Model

Consider $U_1$ and $U_2$, respectively, as prey and predator population size at time $t > 0$ and $K$, the buffering capacity or carrying capacity of the environment. Prey population size represents the number of individuals of this species present in the system. Here, it is assimilated to amount of data packets generated by $U_1$ during time interval $[t − 1, t]$. This value is proportional to this user’s behavior and type and nature of his online activities. Particularly, variation over time of $U_1$ amount of packets is proportional to system configuration in terms of amount of packets generated per unit time and the proportion of packets that have left the system (corrupted, dropped, and transmitted). This analysis holds for the second user $U_2$. We assume the maximum to reach for both species is $K$.

Predator-prey principle formalizes variation over time of population density due to the presence or absence of other species in the ecosystem, representing the effect of increase or decrease via interaction coefficients. Based on this principle, $U_1$ and $U_2$ amount of packets traveling the segment will vary in function of interaction intensity and overall $U_1$ or $U_2$ amount of packets queuing at the buffer space. Mathematically, we have

$$\frac{dU_1}{U_1} = a(1 - \frac{U_1}{K}) - a_1U_2, \quad a, a_1, K > 0, \quad (1)$$

$$\frac{dU_2}{U_2} = \beta(1 - \frac{U_2}{K}) + a_1b_1U_1, \quad \beta, b_1 > 0.$$

In this configuration, when interaction occurs, $U_2$ benefits from competition by occupying more space in the buffer, while $U_1$ decreases density as system will give priority to $U_2$ packets to be accommodated first. This scenario will lead to over-domination of low priority users who will experience very poor quality of service. To be more realistic, let us incorporate in system (1) a threshold to control extinction. It follows

$$\frac{dU_1}{U_1} = a\left(1 - \frac{U_1}{K}\right)\left(A - \frac{U_1}{K}\right) - a_1U_2, \quad (2)$$

$$\frac{dU_2}{U_2} = \beta\left(1 - \frac{U_2}{K}\right) + a_1b_1U_1\left(A - \frac{U_2}{K}\right), \quad K > A > 0.$$

It is clear that $U_1$ and $U_2$ will increase density as far as $A$ is not reached. However, as users’ behavior is random and network segment reliability and availability to respond may vary from time to time, it would be more realistic to consider analyzing this particular dynamic incorporating a decay factor and consider $U_1(t)$ and $U_2(t)$ as functions of time. The competition model can then be expressed as

$$\frac{dU_1}{dt} = U_1\left[a\left(1 - \frac{U_1}{K}\right) - a_1U_2\right] = f(U_1, U_2),$$

$$\frac{dU_2}{dt} = U_2\left[a_1b_1U_1\left(A - \frac{U_2}{K}\right) - b_2\right] = g(U_1, U_2),$$

where $a, b$ represent, respectively, intrinsic growth factor of $U_1$ and $U_2$ in the absence of interaction. These parameters can be explained by the ratio of each user’s generated packets per unit time. In the next section of this article, we will consider $a_1, b$ as positive constant number for analysis purpose.

$a_1$ represents interaction coefficient of $U_3$, denoting the fact that when meeting with $U_1$, system will give $U_2$ packets priority, while $U_1$ packets will have to queue longer in the buffer if there are still rooms.

$b_1$ represents $U_2$ harvesting or competition efficiency. $a_2$ and $b_2$ are, respectively, $U_1$ and $U_2$ decay factors formalizing rate of decrease in density when respective packets leave the system for any reason related to delay, latency, congestion, successful transmission, and so forth.

Coefficient $A$ is the threshold setup to control extinction of species. This parameter has to be fixed smaller than segment maximum carrying capacity such that, at any $t > 0$, users’ generated amount of packets never exceed this value.

3. Model Analysis

3.1. Uniqueness of Solutions. As all parameters of system (3) are positive constant numbers, when interaction is occurring at peak hours, we can consider $f$ and $g$ as continuous and differentiable functions in $\mathbb{R}_+^2 \setminus \{0\}$. If $U_1 < A$, then, $(dU_1/dt) > 0$, signifying $U_1$ will increase density only if $(K - U_1 - a_1KU_2)U_1(A - U_1) > (a_2K/a > 0)$. 

Similarly for \( U_2 \), we have \( U_2 < A \), \((dU_2/dt) > 0 \) and \( U_1 (A - U_2) + b_1 (K - U_2) > (b_2 K/a_1 \beta) > 0 \).

It follows that, for resources not to be exhausted and for both users to increase density, the following condition must be satisfied:

\[
K > U_1 + a_1 K U_2. \tag{4}
\]

Graphical solution space of inequality \((4)\) is illustrated in Figure \(1\). We can conclude that coexistence of \( U_1 \) and \( U_2 \) is peaceful and its evolution over time is submitted to resource availability and \( U_2 \) interaction coefficient.

In function of time, \( 0 < U_1(t); U_2(t) < A \) for \( t \to \infty \) and \( 0 < U_1 \leq K, 0 < U_2 \leq (1/a_1) \).

For solutions to be unique, the following Lipschitz condition must hold:

\[
\left| \frac{\partial f(t, U_1)}{\partial U_1} \right| \leq \varepsilon_1, \varepsilon_1 \in \mathbb{R}^2, \tag{5}
\]

\[
\left| \frac{\partial g(t, U_2)}{\partial U_2} \right| \leq \varepsilon_2, \varepsilon_2 \in \mathbb{R}^2.
\]

It follows

\[
\frac{\partial f(t, U_1)}{\partial U_1} = \left| \frac{\alpha A}{K} \left( 1 - \frac{1}{K} \right) + \frac{aa_1 U_2}{K} (2U_1 - A) - a_2 \right| \leq \frac{\alpha}{K} \left| A - a_1 U_2 (2U_1 - A) - \frac{a_2 K}{\alpha} \right|,
\]

\[
\frac{\partial g(t, U_2)}{\partial U_2} = \frac{\alpha A^2 K (1 - 1/K) + (aa_1 U_2/K) (2U_1 - A) - a_2}{(A - a_1 U_2 (2U_1 - A)) - (a_2 K/a)} \leq \frac{\alpha}{K}.
\]

Similarly, for the predator, we have

\[
\frac{\partial f(t, U_1)}{\partial U_1} = \left| \frac{(a b_1 A U_1/K) + \beta} - \left( \frac{2a b_1 U_1 U_2}{K^2} + \frac{2 \beta U_2}{K} + b_2 \right) \right| \leq \left| \frac{a b_1 A U_1}{K} + \beta \right|,
\]

\[
\frac{\partial g(t, U_2)}{\partial U_2} = \left| \left( (a b_1 A U_1/K) + \beta \right) - \left( (2a b_1 U_1 U_2/K) + (2 \beta U_2/K) + b_2 \right) \right| \leq \frac{\beta}{K}.
\]

We can conclude that \( \forall t > 0 \) and \( U_1, U_2 < A < K \), and solutions of system \((3)\) are unique according to Lipschitz condition.

3.2. Periodicity. If users interact continuously during time interval \([t - 1, t]\), then \( f \) and \( g \) are doubly continuously differentiable functions in \( \mathbb{R}^2 \), and we have then

\[
\frac{\partial f}{\partial U_1} = \frac{a A}{K} + 3a U_1^2 - \frac{2 a a_1 U_1^2 U_2}{K^2} + \left( \frac{2 a U_1^2}{K} + \frac{2 a a_1 U_1 U_2}{K} - a_2 \right),
\]

\[
\frac{\partial^2 f}{\partial U_1^2} = \frac{2 a}{K} - \frac{2 a A}{K} + \frac{6 U_1^2}{K^2} + \frac{2 a a_1 U_2}{K},
\]

\[
\frac{\partial g}{\partial U_2} = \frac{a b_1 A U_1}{K} - \frac{2 a b_1 U_1 U_2}{K} - \frac{2 \beta U_2}{K} + \beta - b_2,
\]

\[
\frac{\partial^2 g}{\partial U_2^2} = \frac{2 a b_1 U_1}{K} - \frac{2 \beta}{K^2}.
\]
where \( X = (K + AK)^2 - (4K(aA - a_2)/(a)) > 0 \). To stay in the positive quadrant, the following conditions must be satisfied: \((K + AK)^2 > (4K(aA-a_2)/a)\) and \((K + AK) > \sqrt{X}\).

(iii) System admits \( P_2 \) on y-axis corresponding to the case \( U_1 \) is extinct and only \( U_2 \) is present in the system. We have then

\[
P_2 = (0; (K(\beta - b_1)/\beta)), \beta > b_1. \tag{11}
\]

(iv) When both users are simultaneously sending traffic, \( U_1 \) and \( U_2 \) zero-growth isoclines may intersect at more than one point, depending on parameters value and accommodating segment state, making system (3) to admit several positive solutions in the positive quadrant as portrayed in Figure 2. We will restrict our analysis on the most interesting case when system zero-growth isoclines intersect at \( P_3 \), corresponding to the unique stable coexistence of species.

System (3) zero-growth isoclines are given:

\[
F: aU_1^2 - a(1 + A)U_1 - aa_1AU_2 + aa_1U_1U_2 + aA - a_2K = 0,
\]

\[
G: a_1b_1AU_1 - \beta U_2 - a_1b_1U_1U_2 + K(\beta - b_2) = 0. \tag{12}
\]

Phase portrait of the system is illustrated in Figure 3 where it is shown \( P_3 \) has asymptotical stable behavior. Using Newton–Raphson iterative method, one can approximate positive solutions of system (3) in case of coexistence of species for a given set of parameters value and initial conditions. We have then

\[
U_1^{(k+1)} = U_1^{(k)} - \frac{F}{\partial F/\partial U_1^{(k)}},
\]

\[
U_2^{(k+1)} = U_2^{(k)} - \frac{G}{\partial G/\partial U_2^{(k)}} \tag{13}
\]

### 3.3. Existence of Equilibrium Points.

System (3) critical or fixed points can be determined by solving the differential equations. We have found four equilibrium points lying in \( \mathbb{R}^2_+ \):

(i) The origin \( O = (0; 0) \), corresponding to the situation where no traffic is traveling the segment or both users are offline.

(ii) System may admit two positive points lying on x-axis when only \( U_1 \) is sending traffic and \( U_2 \) is extinct. We have then

\[
P_1 = \left( K + AK \pm \sqrt{X}; 0 \right), \tag{10}
\]

where \( X = (K + AK)^2 - (4K(aA - a_2)/(a)) > 0 \). To stay in the positive quadrant, the following conditions must be satisfied: \((K + AK)^2 > (4K(aA-a_2)/a)\) and \((K + AK) > \sqrt{X}\).

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\[
U_1^{(k+1)} = U_1^{(k)} - \frac{F}{\partial F/\partial U_1^{(k)}},
\]

\[
U_2^{(k+1)} = U_2^{(k)} - \frac{G}{\partial G/\partial U_2^{(k)}} \tag{13}
\]
\[ J(U_1; U_2) = \begin{pmatrix} B & \frac{a_1 U_1^*}{K^2} (3aU_1^* - A) \\ \frac{a_1 b_1 U_1^*}{K} (A - 2U_2^*) + \frac{a_1 b_1 A U_1^*}{K} & \frac{2a_1 b_1 U_1^* U_2^*}{K} + \frac{2\beta U_2^*}{K} + b_2 \end{pmatrix}, \]

\[ B = \frac{aA}{K} + \frac{3aU_1^*}{K^2} + \frac{2aa_1 U_1^* U_2^*}{K^2} \left( \frac{2aU_1^*}{K} + \frac{2\alpha A U_1^*}{K} + \frac{aa_2 U_2^*}{K} + a_2 \right). \]
(i) At $O = (0; 0)$, we have

\[ J(0; 0) = \begin{pmatrix} \frac{\alpha A}{K} - a_2 & 0 \\ 0 & \beta - b_2 \end{pmatrix}, \]

\[ T = \frac{\alpha A + \beta K - K(a_2 + b_2)}{K}, \]

\[ D = \frac{(\beta - b_2)(\alpha A - a_2K)}{K}. \]

Roots of the characteristic polynomial equation are given as follows:

\[ \lambda_{1,2} = \frac{T \pm \sqrt{T^2 - 4D}}{2}. \]

Case 1: $\beta = b_2$, $T = (\alpha A/K) - a_2$. The origin is a saddle unstable node if $\alpha A > a_2K$. This point will behave as a nodal saddle stable if $\alpha A < a_2K$.

Case 2: $\beta > b_2$, $D, T > 0$, then if $T^2 \geq 4D$, the origin is a source repelling node for all closer enough trajectories if $\alpha A > a_2K$. If $T^2 < 4D$, then the origin is a spiral source node, and all nearby trajectories will orbit around this equilibrium point in clockwise direction.

Case 3: $\beta < b_2$, $\alpha A > a_2K$, the origin is a saddle. For $\alpha A < a_2K$, this equilibrium point is behaving as a sink node sucking all trajectories. For $\alpha A = a_2K$, this equilibrium point is a saddle stable node of the system.

Case 4: $\alpha A + \beta K = K(a_1 + b_2)$, system bifurcates at the origin, and roots of the polynomial equation are complex conjugate purely imaginary numbers expressed as

\[ \lambda_{1,2} = \pm \sqrt{-D} = \pm i(\beta - b_2). \]

$O$ is a center for the system if $\beta \neq b_2$, trajectories are closed curves and solutions are periodic.

(ii) At $P_1$, suppose $(K + AK)^2 > (4K(\alpha A - a_2)/\alpha)$ holds; then, we have

\[ J\left(0; \frac{K(\beta - b_1)}{\beta} \right) = \begin{pmatrix} \frac{\alpha A}{K} - \frac{aa_1A(\beta - b_1)}{\beta} - a_2 & 0 \\ \frac{a_1b_1A(\beta - b_1)}{\beta} - \frac{2a_1b_1K(\beta - b_1)^2}{\beta^2} & b_2 - b_1 \end{pmatrix}. \]

where

\[ B = m_1 - m_2, \]

\[ C = m_3 - m_4, \]

\[ E = 0, \]

\[ F = m_5 - b_2, \]

\[ m_1 = \frac{3\alpha}{K} \left[ (1 + A) + \sqrt{X} \right]^2 + \frac{\alpha A}{K}, \]

\[ m_2 = \frac{\alpha}{K} \left[ (1 + A) \left( (1 + A) + \sqrt{X} \right) \right] + a_2, \]

\[ m_3 = \frac{2aa_1}{K} \left[ (1 + A) + \sqrt{X} \right]^2, \]

\[ m_4 = \frac{aa_1A}{K} \left[ (1 + A) + \sqrt{X} \right], \]

\[ m_5 = \beta + \frac{a_1b_1A}{K} \left[ (1 + A) + \sqrt{X} \right]. \]

System is stable at the neighborhood of $P_1$ only if $m_1 + m_5 < m_2 + b_2$ and $(m_1 - m_2)(m_5 - b_2) > 0$. This could be fulfilled if $m_1 > m_2$ and $m_5 > b_2$ or $m_1 < m_2$ and $m_5 < b_2$.

System will bifurcate at $P_1$ only if $m_1 + m_5 = m_2 + b_2$ and $(m_1 - m_2)(m_5 - b_2) > 0$. This implies $m_5 = m_1 + b_2 - m_2$. It follows

\[ \lambda_{1,2} = \pm (m_1 - m_2). \]

System will undergo saddle node bifurcation at $P_1$ if (20) holds.

(iii) At $P_2$, we have
Setting
\[
m_6 = \frac{\alpha A}{K} + \frac{a a_1 b_2 A}{\beta} + b_2, \\
m_7 = \frac{a a_1 b_2 A}{\beta} + \beta + a_2,
\]
we can write
\[
T = m_6 - m_7, \\
D = (b_2 - \beta) \left[ (m_6 - b_2) - (m_7 - \beta) \right].
\]

System is stable at the neighborhood of \( P_2 \) only if \((m_6 - b_2) > (m_7 - \beta), b_2 > \beta \) and \( m_6 < m_7 \), which cannot be fulfilled; therefore, \( P_2 \) is unstable.

Small perturbations can be applied to invert or modify this dynamic and make system bifurcates at \( P_2 \). For that to happen, the following condition must be satisfied: \( m_6 = m_7 \). This implies
\[
\lambda_{12} = \pm (\beta - b_2).
\]

System will undergo saddle node bifurcation at \( P_2 \) as far as (24) holds.

(iv) At \( P_3 \), we have
\[
J(p; s) = \begin{pmatrix} V_1 & V_2 \\ V_3 & V_4 \end{pmatrix} = \begin{pmatrix} B & \frac{a a_1 p}{K^2} (3p - \lambda A) \\ \frac{a_1 b_1 s}{K} (A - 2s) & \beta + \frac{a_1 b_1 A p}{K} - \left( \frac{2a_1 b_1 p s}{K^2} + \frac{2\beta s}{K} + b_2 \right) \end{pmatrix},
\]
\[
B = \frac{\alpha A}{K} + \frac{\alpha p}{K^2} (3p + 2a_1 s) - \frac{\alpha}{K} \left( 2p + 2A p + a_1 A s \right) - a_2,
\]
where \( p, s > 0 \) represent coordinates of the stable lower positive intersection point of the zero-growth isoclines in the positive quadrant.

We have
\[
T = \mu_1 - \mu_2, \\
D = (\mu_3 - \mu_4) (\mu_5 - \mu_6) - V_2 V_3,
\]
where
\[
\mu_1 = \frac{\alpha A}{K} + \frac{\alpha p}{K^2} (3p + 2a_1 s) + \beta + \frac{a_1 b_1 A p}{K},
\mu_2 = \frac{\alpha}{K} \left( 2p + 2A p + a_1 A s \right) + \frac{2a_1 b_1 ps}{K} + \frac{2\beta s}{K} + a_2 + b_2,
\mu_3 = \frac{\alpha A}{K} + \frac{\alpha p}{K^2} (3p + 2a_1 s),
\mu_4 = \frac{\alpha}{K} \left( 2p + 2A p + a_1 A s \right) + a_2
\mu_5 = \beta + \frac{a_1 b_1 A p}{K},
\mu_6 = \frac{2a_1 b_1 p s}{K} + \frac{2\beta s}{K} + b_2.
\]

System is stable, and all nearby trajectories will be attracted by \( P_3 \) only if \( \mu_1 < \mu_2 \) and \( (\mu_3 - \mu_4) (\mu_5 - \mu_6) > V_2 V_3 \). This can happen if \( \mu_3 > \mu_4, \mu_5 > \mu_6 \) or \( \mu_3 < \mu_4, \mu_5 < \mu_6 \). Both \( U_1 \) and \( U_2 \) will grow at relative speed depending on available resources and parameters value. This dynamic is highly sensitive to initial conditions, interaction intensity, respective amount of packets traveling the segment, users' behavior, and whether system undergoes weaker or stronger Allee effects.

5. Numerical Results

In this section, we test the proposed model applicability and predictability by simulating two interacting social network users defined by their online activities in terms of amount of data packets generated per unit time. We assume \( U_1 \) generates less packets compared with \( U_2 \), respectively, considered as prey and predator. Generated packets are handled and travel through the accommodating segment. Based on system configuration, when the accommodating segment is heavily loaded and there is no room in the buffer space, all incoming packets will be discarded. In this simulation, we consider the case all active sources are sending traffic continuously during peak hours. Packets may be corrupted during their travel due to factors related to congestion, latency, errors, and so forth. If that happens, considered packets will be dropped and leave the segment, decreasing respective users' density.

Buffering capacity, queuing discipline, and queuing time are important in maintaining system stability. In heavy load situation, as shown in Figure 4 (with threshold applied) and Figure 5 (no threshold applied), optimizing resource allocation is crucial to reduce latency. When the system has a threshold mechanism, prey abundance may affect its overall population size as self-competition will become intensive.
Figure 4: Dynamic of interaction (a) and phase space (b) when threshold is applied, coexistence is peaceful, and system is stable for $\alpha = 1.18; \ a_1 = 0.2; \ a_2 = 0.13; \ \beta = 1.15; \ b_1 = 0.18; \ A = 2; \ K = 3; \ b_2 = 0.25$.

Figure 5: Dynamic of interaction (a) and phase space (b) when no threshold is applied, system is stable, and $U_1$ shrinks and dies out for $\alpha = 2.18; \ a_1 = 0.2; \ a_2 = 0.13; \ \beta = 3.15; \ b_1 = 0.18; \ K = 30; \ b_2 = 0.25$.

Figure 6: Dynamic of interaction (a) and phase space (b) when system is stable accommodating both users packets and $U_1$ dominates for $\alpha = 3.18; \ a_1 = 0.2; \ a_2 = 0.6; \ \beta = 1.15; \ b_1 = 0.18; \ A = 2.5; \ K = 3; \ b_2 = 0.8$. 
For predator, more prey individuals signify more resources to harvest. This dynamic is submitted to initial conditions, the threshold value, interaction parameters, and system state with respect to congestion control, queuing discipline, and segment saturation.

When resources are limited, adjusting the output link transmission rate to free up buffer space may be necessary to stabilize the system and optimize throughput. In Figure 6, when the threshold is fixed at more than 80 percent of the maximum carrying capacity, $U_1$ stands the competition and gets its packets accommodated at relatively high speed benefiting from $U_2$ significantly larger decay factor ($b_2 = 0.8$). Allocating more resources to users with lower priority has an impact on interaction outcome and system dynamic. This phenomenon is identifiable to weaker Allee effect that results in relatively lower extinction occurrences.

Periodic trajectories appear if no threshold is applied, and solutions are periodic, and system is dynamically stable for $\alpha = 1.6; \ a_1 = 0.28; \ a_2 = 0.15; \ \beta = 0.028; \ b_1 = 0.19; \ K = 80; \ b_2 = 0.159$.

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Periodic trajectories appear if no threshold is applied, and solutions are periodic, and system is dynamically stable for $\alpha = 1.6; \ a_1 = 0.28; \ a_2 = 0.15; \ \beta = 0.028; \ b_1 = 0.19; \ K = 80; \ b_2 = 0.159$.

6. Conclusion
A two-species modified Lotka–Volterra predator system with threshold in both prey and predator equations has been proposed in this paper. We have analyzed the interaction phenomenon occurring by focusing on resource availability and the Allee threshold applied to control extinction of species. We applied the proposed model to online social network users sending traffic at peak hours and characterized by their traffic profile in terms of amount of packets generated per unit time. System can be stable at certain
conditions if chosen some right values for model parameters. System dynamic is highly submitted to interaction intensity, available resources, system state, initial conditions, threshold value, and users’ behavior. Results suggested adapting resource allocation in function of users’ behavior by optimizing accessibility to guarantee quality of service.

Data Availability
No data have been used.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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