Singlet-doublet Higgs mixing and its implications on the Higgs mass in the PQ-NMSSM

Kwang Sik Jeong*, Yutaro Shoji†, Masahiro Yamaguchi‡

Department of Physics, Tohoku University, Sendai 980-8578, Japan

Abstract

We examine the implications of singlet-doublet Higgs mixing on the properties of a Standard Model (SM)-like Higgs boson within the Peccei-Quinn invariant extension of the NMSSM (PQ-NMSSM). The SM singlet added to the Higgs sector connects the PQ and visible sectors through a PQ-invariant non-renormalizable Kähler potential term, making the model free from the tadpole and domain-wall problems. For the case that the lightest Higgs boson is dominated by the singlet scalar, the Higgs mixing increases the mass of a SM-like Higgs boson while reducing its signal rate at collider experiments compared to the SM case. The Higgs mixing is important also in the region of parameter space where the NMSSM contribution to the Higgs mass is small, but its size is limited by the experimental constraints on the singlet-like Higgs boson and on the lightest neutralino constituted mainly by the singlino whose Majorana mass term is forbidden by the PQ symmetry. Nonetheless the Higgs mixing can increase the SM-like Higgs boson mass by a few GeV or more even when the Higgs signal rate is close to the SM prediction, and thus may be crucial for achieving a 125 GeV Higgs mass, as hinted by the recent ATLAS and CMS data. Such an effect can reduce the role of stop mixing.

PACS numbers:

*email: ksjeong@tuhep.phys.tohoku.ac.jp
†email: yshoji@tuhep.phys.tohoku.ac.jp
‡email: yama@tuhep.phys.tohoku.ac.jp
I. INTRODUCTION

A natural way to explain the smallness of the higgsino mass parameter in the minimal supersymmetric standard model (MSSM) is to promote it to a SM singlet $S$ so that the superpotential includes a coupling $\lambda S H_u H_d$. This leads to the next-to-MSSM (NMSSM) [1], which however generally suffers from the tadpole [2, 3] and domain-wall [4, 5] problems once one adds self-interactions of $S$ to avoid a visible axion. In particular, the tadpole problem makes it difficult to embed the NMSSM into a grand unified theory.

We have recently pointed out in Ref. [6] that all the problems arising due to the SM singlet $S$ can be avoided when the NMSSM is extended to incorporate the Peccei-Quinn (PQ) symmetry solving the strong CP problem [7, 8]. The PQ symmetry protects $S$ from acquiring large tadpoles. Furthermore, the domain-wall problem is resolved by introducing an appropriate number of PQ messengers or considering the situation where the saxion is displaced far from the origin during inflation. In the PQ-invariant extension of the NMSSM (PQ-NMSSM), $S$ plays the role of a messenger that connects the PQ sector with the visible sector. This is done through its non-renormalizable coupling with a SM singlet responsible for spontaneous PQ symmetry breaking at a scale much higher than the electroweak scale. Such non-renormalizable coupling generates a small effective tadpole for $S$. As a result, the electroweak scale originates from the SUSY breaking scale and the axion decay constant. Another interesting property is the existence of a relatively light neutralino dominated by the singlino.

The inclusion of $S$ modifies the Higgs and neutralino sectors, and opens the possibility to have singlet-doublet Higgs mixing. Such mixing increases the mass of the SM-like Higgs boson when the lightest Higgs boson is singlet-like [9]. It should be noted that the mixing effect is important even in the region with large $\tan \beta$ and small $\lambda$, where the NMSSM tree-level contribution to the Higgs quartic coupling and thus to the Higgs boson mass is negligible. The PQ-NMSSM, in which quadratic and cubic terms in $S$ are forbidden by the PQ symmetry, includes a light singlino-like neutralino. This results in that the Higgs sector has a different phenomenology from other NMSSM models because, if kinematically allowed, the Higgs bosons and the $Z$ boson invisibly decay into a pair of neutralinos through couplings proportional to $\lambda$. Furthermore, the Higgs quartic coupling can receive additional sizable radiative corrections involving the Yukawa interaction, Higgs-higgsino-singlino [6]. In this
paper, we examine the implications of the Higgs mixing on the properties of a SM-like Higgs boson by combining the experimental constraints placed on the singlino-like neutralino and the lightest Higgs boson.

The recent ATLAS and CMS data hint the existence of a SM-like Higgs boson with mass around 125 GeV [10]. To account for this, one needs a particular mechanism of SUSY breaking giving large stop mixing or an extension of the MSSM for superparticles having masses around a TeV as suggested by the gauge hierarchy problem [11].

Thus, the singlet-doublet Higgs mixing may be crucial for achieving a 125 GeV Higgs mass without invoking large stop mixing. The Higgs mixing needs to be small in order for the Higgs signal rate at collider experiments to be close to the SM prediction. Nonetheless, we find that it can still increase the Higgs boson mass by more than a few GeV while avoiding the experimental constraints.

In the next section, we briefly discuss the properties of the PQ-NMSSM and the effects of the singlet-doublet Higgs mixing. To see the impacts of the Higgs mixing, we construct a low energy effective theory assuming that the MSSM superparticles except the higgsinos have masses around a TeV. In section 3, we explore the properties of the singlino-like light neutralino, on which the LEP experiments put constraints. Finally, in section 4, we discuss how large the Higgs mixing can be in the PQ-NMSSM and how much it can increase the mass of a SM-like Higgs boson. Section 5 is devoted to discussion and conclusions.

II. PQ-INvariant Extension of the NMSSM

The PQ-NMSSM is obtained by extending the NMSSM to incorporate the PQ solution to the strong CP problem, and the SM singlet $S$ added to the Higgs sector connects the PQ and visible sectors through the operators,

$$\int d^4 \theta \kappa \frac{X^2}{M_{Pl}} S + \int d^2 \theta \lambda S H_u H_d, \quad (1)$$

where the PQ symmetry is assumed to be spontaneously broken by another SM singlet $X$ at a scale much higher than the electroweak scale. The $\kappa$ term induces an effective tadpole for $S$ at $\sim F_a^2/M_{Pl}$ after PQ symmetry breaking and drives $S$ to acquire a vacuum expectation

---

1 See Refs. [12–17] for a recent discussion of singlet extensions of the MSSM.
value. Here $F_a$ is the axion decay constant, and we drop a PQ-invariant superpotential term $X^2 H_u H_d / M_{Pl}$ since it can always be absorbed into the $\kappa$ term by redefining $S$. It should be noted that, protected by the PQ symmetry, $S$ does not have a large tadpole.\footnote{The tadpole problem can be avoided also by imposing a discrete R symmetry as discussed in Refs. \cite{18, 21}. Then, the SM singlet $S$ added to the Higgs sector only has a superpotential term $S H_u H_d$ and a small tadpole term induced after SUSY breaking. See also Ref. \cite{22}, where the NMSSM is extended to incorporate the PQ symmetry and local R symmetry.} On the other hand, as in other NMSSM models, the $\lambda$ term replaces a supersymmetric $\mu$ term of the MSSM. Having non-negligible coupling $\lambda$ to the Higgs doublets, $S$ modifies the Higgs and neutralino sectors in a significant way.

A. Low energy effective action

The scalar potential for the extended Higgs sector depends on the effective tadpole term of $S$ and the SUSY breaking parameters of $S$ and $H_{u,d}$,

$$
V = \frac{g_2^2 + g'_2^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2} |H_u^\dagger H_u|^2 + |\lambda H_u H_d + m_0^2|^2 + |\lambda S|^2 (|H_u|^2 + |H_d|^2) \\
+ m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + (A_\lambda \lambda SH_u H_d - B_\kappa m_0^2 S + h.c.),
$$

(2)

where we take the same notation as in our previous paper \cite{6}, with the exception that we will use $B$ and $\mu$ instead of $B_{\text{eff}}$ and $\mu_{\text{eff}}$, respectively. It should be noted that $m_t^2$, $A_\lambda$ and $B_\kappa$ are set by the SUSY breaking scale $m_{\text{soft}}$, while $m_0^2 \sim \kappa m_{\text{soft}} \langle |X| \rangle^2 / M_{Pl}$ is determined by the axion decay constant. Assuming for simplicity that the phases of $A_\lambda$ and $B_\kappa$ are aligned, one can always make $A_\lambda$, $B_\kappa$, $\lambda$ and $m_0^2$ real positive by rotating the phases of $S$ and $H_{u,d}$. We take such a field basis throughout this paper. Using the extremum conditions, one can replace $(B_\kappa, m_{H_u}^2, m_{H_d}^2, m_0^2)$ by $(B, \mu, \tan \beta, v)$, where $\tan \beta$ is the ratio between Higgs vacuum expectation values with $v \approx 174$ GeV, and $B$ is the soft parameter associated with the effective $\mu$ term, $\mu = \lambda \langle |S| \rangle$. Then, the Higgs sector is parameterized by

$$(\lambda, m_S^2, A_\lambda, B, \mu, \tan \beta),
$$

(3)

in terms of which $m_0^2$ is written $m_0^2 = (B - A_\lambda) \mu / \lambda$, implying that $B > A_\lambda$ in the model. See also the appendix for the relations between other SUSY breaking terms and the above parameters.
The region of parameter space that we will investigate is such that the singlet scalar is relatively light, around the electroweak scale. To see the impacts of the singlet-doublet Higgs mixing, we further assume that heavy Higgs scalars and MSSM superparticles except the higgsinos obtain TeV masses. Integrating out the heavy fields, one obtains a low energy effective theory, which reduces to the SM with additional fields: the singlet complex scalar $S$, singlino $\tilde{S}$ and higgsinos $\tilde{H}_{u,d}$. The effective scalar interactions read

$$V_{\text{eff}} = \frac{\lambda_H}{2} (|H|^2 - v^2)^2 + \left( \frac{m_S^2}{\lambda^2} + v^2 \right) |\lambda S - \mu|^2 + (|H|^2 - v^2)|\lambda S - \mu|^2$$

$$+ \frac{1}{2} (2\mu - A_\lambda \sin 2\beta) \left\{ (|H|^2 - v^2)(\lambda S - \mu) + \text{h.c.} \right\}$$

$$- \frac{\sin 2\beta}{2B\mu} \left| f_{\text{mix}}^* \sin^2 \beta - f_{\text{mix}} \cos^2 \beta \right|^2 |H|^2,$$

(4)

where $H$ denotes the light Higgs doublet scalar in the decoupling limit. The last term in the potential results from the tree-level exchange of the heavy Higgs scalars having mass $\simeq (2B\mu/\sin 2\beta)^{1/2} \gg m_W$:

$$f_{\text{mix}} = A_\lambda (\lambda S - \mu) + \frac{\sin 2\beta}{4} (g^2 + g'^2 - 2\lambda^2) (|H|^2 - v^2),$$

(5)

which vanishes at the vacuum. The derivation of $f_{\text{mix}}$ is presented in the appendix. As in the conventional NMSSM, the Higgs quartic coupling is given by $\lambda_H(m_{\text{soft}}) = \frac{g^2 + g'^2}{4} \cos^2 2\beta + \frac{\lambda^2}{2} \sin^2 2\beta$ at the tree-level, and receives threshold corrections coming from stop loops. The Yukawa interactions relevant to our discussion are

$$- L_{\text{eff}} = y_t \bar{t}_R Q H^c + y_b \bar{b}_R Q H + y_s' S \tilde{H}_u \tilde{H}_d + y_u' H \tilde{H}_u \tilde{S} + y_d' H^c \tilde{H}_d \tilde{S} + \text{h.c.},$$

(6)

where $H^c = i\sigma_2 H^*$, and the couplings are given by $y_s' = \lambda$, $y_u' = \lambda \cos \beta$ and $y_d' = \lambda \sin \beta$ at the scale $m_{\text{soft}}$.

**B. Higgs mixing**

In this subsection, we briefly review the effects of Higgs mixing in an extension of the SM where the SM Higgs boson mixes with a SM singlet scalar. The low energy effective theory (4) belongs to this class of models. The CP-even neutral scalars $h$ and $s$, coming from $H$ and $S$ respectively, compose the mass eigenstates $H_{1,2}$:

$$H_1 = s \cos \theta - h \sin \theta,$$

$$H_2 = s \sin \theta + h \cos \theta,$$

(7)
which have mass $m_{H_1}^2 = M_{ss}^2 - M_{hs}^2 \tan \theta$ and $m_{H_2}^2 = M_{hh}^2 + M_{hs}^2 \tan \theta$ for the Higgs mixing angle fixed by

$$\tan 2\theta = \frac{2M_{hs}^2}{M_{hh}^2 - M_{ss}^2},$$

(8)

where $M_{ij}^2 = \langle \partial_i \partial_j V \rangle$ is the mass squared matrix element for $(h, s)$, and should satisfy the stability conditions $M_{hh,ss}^2 > 0$ and $(M_{hs}^2)^2 < M_{hh}^2 M_{ss}^2$.

Let us consider the case where the lightest Higgs boson $H_1$ originates mainly from the singlet $s$. Then, the mass of a SM-like Higgs $H_2$

$$m_{H_2}^2 = M_{hh}^2 + (M_{hh}^2 - m_{H_1}^2)^2 \tan^2 \theta$$

(9)

receives a positive contribution from the mixing. To see how much it can increase $m_{H_2}$, one should take into account the experimental results on the Higgs search. For a singlet-like $H_1$ lighter than 114 GeV, LEP places stringent constraints on $e^+e^- \rightarrow Z H_1 \rightarrow Zb\bar{b}$, i.e. on the effective coupling [23],

$$R_{H_1}^{b\bar{b}} = \frac{g_{ZZH_1}^2}{g_{ZZh}^2} \text{Br}(H_1 \rightarrow b\bar{b}) = \text{Br}(H_1 \rightarrow b\bar{b}) \sin^2 \theta,$$

(10)

where $\text{Br}(H_1 \rightarrow b\bar{b})$ is the branching ratio of the corresponding process. On the other hand, the production of $H_2$ at hadron colliders proceeds essentially through the same processes as those in the SM. However, the Higgs mixing modifies the discovery reach:

$$R_{H_2}^{SM} \equiv \frac{\sigma(H_2)}{\sigma_{SM}(h)} \text{Br}(H_2 \rightarrow \text{SM}) = \text{Br}(H_2 \rightarrow \text{SM}) \cos^2 \theta,$$

(11)

which provides the signal rate for the decays of $H_2$ to SM particles in comparison with the SM case. Here $\sigma(H_2)$ is the cross section for the $H_2$ production, while $\sigma_{SM}(h)$ is the Higgs production cross section in the SM. If the Higgs signal rate is measured to be close to the SM prediction, only small mixing is allowed.

The contribution to $m_{H_2}$ from the singlet-doublet Higgs mixing is estimated by

$$\Delta m_{H_2} \equiv m_{H_2} - M_{hh} = \frac{M_{hh}^2 - m_{H_1}^2}{2M_{hh}} \tan^2 \theta + O \left( \frac{\Delta m_{H_2}^2}{M_{hh}} \right),$$

(12)

in which $\tan \theta$ should be small enough to satisfy the LEP limits on $R_{H_1}^{b\bar{b}}$, and will be further constrained if one requires the $H_2$ signal rate to be close to what the SM predicts. For $H_2$ having mass around 125 GeV, if for instance $R_{H_2}^{SM} > 0.7$ is imposed, the contribution to
m_{H_2}$ from the Higgs mixing can be as large as about 7 GeV when $H_1$ has mass in the range between about 90 and 100 GeV. Here we have used that the LEP limits on $R_{H_1}^{HH}$ require $\sin^2 \theta$ less than about 0.25 for $H_1$ having mass in the indicated range, and that $R_{H_2}^{SM} > 0.7$ translates into $\sin^2 \theta < 0.3$ assuming that $H_2$ dominantly decays into SM particles. Even if $R_{H_2}^{SM}$ is to be more close to unity, for instance larger than 0.9, $\Delta m_{H_2}$ can be about 3 GeV at $m_{H_1}$ around 90 GeV. The contribution from the Higgs mixing can thus be crucial for achieving a 125 GeV Higgs mass in the supersymmetric SM. In the PQ-NMSSM, the situation changes a bit due to invisible Higgs decays into neutralinos, but not significantly if one requires $R_{H_2}^{SM} \gtrsim 0.7$. We will return to this point in section [IV].

III. SINGLINO-LIKE NEUTRALINO LSP

Having no Majorana mass term, the singlino obtains a small mass through mixing with the neutral higgsinos, and becomes the dominant component of the lightest neutralino $\chi_1^0$. This opens the possibility that invisible decays of the $Z$ and Higgs bosons into neutralinos are kinematically accessible. Here we assume $R$-parity conservation. Then, the size of the higgsino component in $\chi_1^0$ is severely constrained by the LEP limit on the invisible $Z$ decay rate. Also, the Higgs production rate at collider experiments is modified.

Let us examine the properties of the lightest neutralino before moving to discuss the Higgs properties. We note that the neutralino properties discussed here are independent of the details of the Higgs sector. In the low energy effective theory, the neutralino sector is comprised of two neutral higgsinos and the singlino,

$$
\tilde{H}_d = N_{3i} \chi_i^0, \quad \tilde{H}_u = N_{4i} \chi_i^0, \quad \tilde{S} = N_{5i} \chi_i^0,
$$

(13)

where $i = (1, 2, 3)$, and we are assuming that gauginos are heavy and decouple below the SUSY breaking scale. The lightest neutralino couples to the SM particles through its higgsino component, whose size depends on $\epsilon \equiv \lambda v / \mu$ and $\tan \beta$. The relevant neutralino interactions are

$$
- \mathcal{L} = \frac{1}{2} y_{ij} \bar{\psi}_i^0 \psi_j^0 s + \frac{1}{2} y_{ij} \bar{\psi}_i^0 \psi_j^0 h + \frac{1}{2} g_{ij} \bar{\psi}_i^0 \gamma^\mu \gamma^5 \psi_j^0 Z_\mu,
$$

(14)

where $s$ and $h$ are the CP-even neutral Higgs boson coming from $S$ and $H$, respectively, and
\((\psi^0_i)^T = (\chi^0_i, \bar{\chi}^0_i)\) is the four-component spinor. The neutralino couplings read \[11\],
\[
\begin{align*}
y_{ij}^a &= \frac{\lambda}{\sqrt{2}} N_{3i} N_{4j} + (i \leftrightarrow j), \\
y_{ij}^b &= \frac{\lambda}{\sqrt{2}} (N_{3i} N_{5j} \sin \beta + N_{4i} N_{5j} \cos \beta) + (i \leftrightarrow j), \\
g_{ij} &= \frac{g}{2 \cos \theta_W} (N_{3i} N_{3j}^* - N_{4i} N_{4j}^*) + (i \leftrightarrow j), \\
\end{align*}
\]
(15)
at the scale \(m_{\text{soft}}\).

For \(\epsilon = \lambda v / \mu \ll 1\), we find the neutralinos to be
\[
\begin{pmatrix}
\chi_0^0 \\
\chi_2^0 \\
\chi_1^0
\end{pmatrix}
\approx
\begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{1-\epsilon^2} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{1-\epsilon^2} & \frac{1}{\sqrt{2}} \\
-\epsilon \cos \beta & -\epsilon \sin \beta & 1 - \frac{\epsilon^2}{2}
\end{pmatrix}
\begin{pmatrix}
\tilde{H}^0_d \\
\tilde{H}^0_u \\
\tilde{S}
\end{pmatrix},
\]
(16)
with masses given by
\[
\begin{align*}
m_{\chi_1^0} &\simeq \epsilon^2 \mu (1 - \epsilon^2) \sin 2\beta, \\
m_{\chi_2^0} &\simeq \mu \left(1 + \frac{1 - \sin 2\beta}{2} \epsilon^2\right), \\
m_{\chi_3^0} &\simeq m_{\chi_1^0} + m_{\chi_2^0},
\end{align*}
\]
(17)
where we have ignored higher order terms in \(\epsilon\) and small mixing with the gauginos. Thus, \(\chi_1^0\) is the lightest superparticle (LSP) for small \(\epsilon\), and becomes lighter as \(\tan \beta\) grows.

For gauginos with mass much larger than \(\mu\), the lightest chargino \(\chi_1^\pm\) originates mainly from the charged higgsino. Thus, the LEP bound on the chargino mass demands \(\mu \gtrsim 100\) GeV. In the case under consideration, the next-to-LSP (NLSP) is either \(\chi_2^0\) or the higgsino-like chargino \(\chi_1^\pm\). The 1-loop correction to the chargino mass is dominated by that from gauge boson loops, which is about a few hundred MeV \[24\]. As being higgsino-like, the chargino dominantly decays through the interaction \(W^- \chi_1^\pm \chi_1^0\) for \(\mu > m_W + m_{\chi_1^0}\). On the other hand, the main decay modes of \(\chi_2^0\) are into \(Z \chi_1^0\) and into a Higgs boson plus \(\chi_1^0\), depending on the size of \(\mu\). For \(\epsilon \ll 1\), the singlino-like neutralino couples very weakly to other superparticles, and therefore supersymmetric cascade decays will proceed first into the NLSP through the same interactions as in the MSSM \[25\]. The NLSP then promptly decays into the LSP.

Now we look into the constraints placed on the neutralino sector. If \(\chi_1^0\) has a mass smaller than half of the \(Z\)-boson mass, the coupling \(g_{11}\) should be sufficiently small in order not to
FIG. 1: Neutralino properties for \( \tan \beta = 5 \) (left) and \( \tan \beta = 10 \) (right). The shaded region is excluded by the LEP bound on the invisible \( Z \) decay rate. Red lines represent the contours of the cross section for neutralino pair production \( \sigma(e^+e^- \rightarrow \chi^0_2\chi^0_1) \) at \( \sqrt{s} = 209 \) GeV in the \( (\mu, \lambda) \) plane. In the right side of the dashed red line, \( m_{\chi^0_2} + m_{\chi^0_1} \) is larger than 209 GeV. We also show the contours for the mass of \( \chi^0_1 \) in the dashed blue line.

Exceed the experimental bound on the invisible decay rate of \( Z \) \([26, 27]\). This requires

\[
\Gamma_{Z \rightarrow \chi^0_2\chi^0_1} = \frac{g_{11}^2}{96\pi} m_Z \left( 1 - \frac{4m_{\chi^0_1}^2}{m_Z^2} \right)^{3/2} \lesssim 2 \text{ MeV}, \tag{18}
\]

which translates into

\[
\left( \frac{\epsilon}{0.3} \right)^4 \cos^2 2\beta \lesssim 1.5, \tag{19}
\]

for \( \epsilon \ll 1 \). The above constraint becomes strong at large \( \tan \beta \), where we thus need a small \( \lambda \) or large \( \mu \). For \( \mu \gtrsim 100 \) GeV as required by the chargino mass bound, the decay \( Z \rightarrow \chi^0_2\chi^0_1 \) is kinematically forbidden.

LEP has placed constraints also on the neutralino production rate \([28]\). Having higgsino components, neutralinos are produced at \( e^+e^- \) colliders through \( Z \) exchange in the \( s \) channel. For \( \mu \) much smaller than the neutral gaugino masses, production processes through the exchange of selectron in the \( t \) and \( u \) channels are negligible because of a small coupling for the higgsino-electron-selection Yukawa interaction. These processes are further suppressed when the sleptons are heavy. The LEP bound on \( e^+e^- \rightarrow \chi^0_2\chi^0_1 \) requires

\[
\sigma(e^+e^- \rightarrow \chi^0_2\chi^0_1) \times \text{Br}(\chi^0_2 \rightarrow Z\chi^0_1) \lesssim 70 \text{ fb}, \tag{20}
\]
putting constraint on the size of $g_{11,12}$. If $\mu$ is large enough to kinematically allow the decays of $\chi^0_2$ into a Higgs boson and $\chi^0_1$, the branching fraction for $\chi^0_2 \to Z \chi^0_1$ is reduced.

Fig. 1 illustrates the properties of the neutralino sector, which depend on $(\lambda, \mu, \tan \beta)$. The shaded region, where $\Gamma_{Z \to \chi^0_1 \chi^0_1}$ is larger than 2 MeV, is excluded by the LEP bound on the invisible $Z$ decay rate. In the allowed region with $\tan \beta \gtrsim 5$ and $100 \text{ GeV} \lesssim \mu \lesssim 200 \text{ GeV}$, we find the following properties. The lightest neutralino has mass less than a few GeV, and the neutralino production cross section $\sigma(e^+e^- \to \chi^0_2 \chi^0_1)$ is less than 70 fb in the non-shaded region. The constraint (20) is thus evaded in that region regardless of how strong the process $\chi^0_2 \to$ Higgs boson and $\chi^0_1$ is. On the other hand, if one takes $\tan \beta \lesssim 3$ and $\lambda \gtrsim 0.6$, it is possible to render $\chi^0_1$ to have a mass larger than $m_Z/2$.

We close this section by briefly discussing the LSP relic abundance in the case with $\epsilon \ll 1$ and $\mu$ around a few hundred GeV. Since $\chi^0_1$ is then much lighter than the $Z$ and Higgs bosons and interacts very weakly with them, its annihilation processes mediated by the exchange of $Z$ or Higgs bosons are too weak to avoid overclosing the universe. In order to resolve this cosmological difficulty, one may consider a superparticle lighter than $\chi^0_1$, such as the axino or the gravitino, into which $\chi^0_1$ decays. For those two, the relevant interactions include

$$\mathcal{L} = C_\tilde{a} \bar{\tilde{a}} \sigma^\mu \chi^0_1 (\partial_\mu a) + C_{3/2} \tilde{G}_{1/2} \sigma^\mu \chi^0_1 (\partial_\mu a) + \text{h.c.},$$

(21)

with couplings given by

$$C_\tilde{a} \sim \frac{\langle K_{\tilde{X}_S} \rangle}{F_{\tilde{a}}} \sim \frac{\kappa}{M_{Pl}},$$

$$C_{3/2} \sim \frac{m_{\chi^0_1}}{m_{3/2} M_{Pl}},$$

(22)

where $\tilde{a}$ is the axino, and $a$ is the axion. The $C_\tilde{a}$ interaction comes from the $\kappa$ term, while the other is the effective interaction for the goldstino component $\tilde{G}_{1/2}$ of the gravitino [31] for the case that the gravitino mass $m_{3/2}$ is smaller than $m_{\chi^0_1}$. If the axino or the gravitino is the LSP, $\chi^0_1$ will decay with a very long lifetime. Such late decay of $\chi^0_1$ would cause another cosmological problem because the produced LSP becomes a hot dark matter component, whose energy density is severely constrained by the CMBR and structure formation [32, 33].

---

[3] See Refs. [29, 30] for the case with not so small $\epsilon$. In such case, $\chi^0_1$ obtains a mass larger than a few ten GeV, and its thermal relic abundance can be consistent with the observed amount of dark matter.
It would be possible to avoid the bound on the hot dark matter abundance for a tiny LSP mass. Then, the LSP, produced not by the late decay of $\chi_1^0$, and the axion can constitute the cold dark matter of the universe.

On the other hand, if $R$-parity is conserved and $\chi_1^0$ is the LSP, to avoid the LSP overproduction, one needs a sufficiently low reheating temperature or a mechanism of late entropy production. Indeed, in the PQ-NMSSM, it is plausible that the saxion potential energy dominates the universe as it has a very flat potential lifted by SUSY breaking. Let us assume such a case and examine the LSP production processes. In the absence of the $\kappa$ term, the main decay mode of the saxion $\sigma$ is generally $\sigma \to aa$. However, the $\kappa$ term induces the interactions of $\sigma$, as well as of $\tilde{a}$, with the NMSSM particles through kinetic mixing of the scalars and fermions in $X$ and $S$:

$$\langle K_{XS} \rangle \sim \kappa \frac{F_a}{M_{Pl}} \sim \frac{m_0^2}{m_{\text{soft}} F_a}. \quad (23)$$

The saxion couplings to SM particles are $\sim \lambda m_{\text{soft}}/F_a$ for $m_0^2 \sim m_{\text{soft}}^2$, and are crucial for suppressing the branching ratio of the saxion decay into axions [34]. The LSP is produced by thermal and non-thermal processes. For thermally produced LSPs not to overclose the universe, the saxion decay temperature should be low enough. On the other hand, non-thermal production proceeds through the interaction $\sigma \chi_1^0 \chi_1^0$ having coupling $\sim y_{11}^a \langle K_{XS} \rangle$ where $y_{11}^a \sim \lambda \epsilon^2 \sin 2\beta$. Since the involved coupling is further suppressed by $\epsilon^2$, it would not be difficult to make the produced LSP energy density small. Meanwhile, if the saxion decay into the axino is kinematically allowed, one should pay attention also to axinos directly produced by saxion decays. The axino decays via the $C_{\tilde{a}}$ coupling in (21), but mainly through the Higgs-$\tilde{a}$-$\chi_{1,2}^0$ interactions if kinematically accessible. If lighter than the Higgs bosons, the axino will dominantly decay to $\chi_1^0$ with a very long lifetime. Since the produced LSP then behaves like a hot dark matter, its energy density should be small. A more detailed analysis will be given elsewhere.

IV. HIGGS SECTOR

In this section, we explore quantitatively the implications of singlet-doublet Higgs mixing on the mass and the production rate of a SM-like Higgs boson in the case where the lightest Higgs boson is dominated by the singlet scalar.
A. Higgs properties

Let us first examine the Higgs properties within the effective low energy theory below $m_{\text{soft}}$. From the Higgs potential (4), it is straightforward to get

$$M_{hh}^2 = 2\lambda_H v^2,$$
$$M_{ss}^2 = m_S^2 + \lambda^2 v^2 \left( 1 - \frac{A^2}{2B\mu} \sin 2\beta \cos^2 2\beta \right),$$
$$M_{hs}^2 = \lambda v (2\mu - A \sin 2\beta).$$  \hspace{1cm} (24)

Here radiative contributions to $M_{hh}^2$ are easily evaluated by solving the renormalization group (RG) running equation of the Higgs quartic coupling $\lambda_H$ in the effective theory \[35, 36\]. The RG running is affected by the singlet and neutralinos when they are light. On the other hand, the CP-odd neutral Higgs boson $A$ originates from the singlet scalar and obtains mass,

$$m_A^2 = m_S^2 + \lambda^2 v^2 \left( 1 - \frac{A^2}{2B\mu} \sin 2\beta \right),$$
$$k_{H_1,2} \approx \frac{1}{2}, \quad k_{H_1,2} \approx \left(1 - \frac{m_{\chi_0}^2}{m_{H_1,2}^2}\right)^2,$$

which differs from $M_{ss}^2$ due to the contribution from $f_{\text{mix}}$.

The main purpose of this work is to explore if the model can accommodate $H_2$ around 125 GeV for a singlet-like Higgs $H_1$ having mass above about 80 GeV, below which, as we will see, only small mixing is allowed. In such a case, the decays of $H_{1,2}$ into SM particles are dominated by those into $b\bar{b}$ and $VV^*$ where $V = (W, Z)$. Since the processes are mediated via the $h$ component, one finds the decay rates to be

$$\Gamma_{H_2\rightarrow\text{SM}} = \Gamma_{\text{SM}}(m_{H_2}) \cos^2 \theta,$$
$$\Gamma_{H_1\rightarrow\text{SM}} = \Gamma_{\text{SM}}(m_{H_1}) \sin^2 \theta,$$  \hspace{1cm} (26)

in which $\Gamma_{\text{SM}}(m_{H_i})$ is the decay rate of $h \rightarrow \text{SM}$ obtained for a SM Higgs boson $h$ having mass $m_{H_i}$, in the limit of no mixing with $s$. In addition, because the singlino-like neutralino is very light, invisible Higgs decays are possible:

$$\Gamma_{H_2\rightarrow\chi_i^0\chi_j^0} = \frac{k_{ij}^{H_2}}{8\pi} (y_{ij}^h \cos \theta + y_{ij}^s \sin \theta)^2 m_{H_2},$$
$$\Gamma_{H_1\rightarrow\chi_i^0\chi_j^0} = \frac{k_{ij}^{H_1}}{8\pi} (y_{ij}^h \sin \theta - y_{ij}^s \cos \theta)^2 m_{H_1},$$  \hspace{1cm} (27)

for $i \geq j$, where neglecting the mass of $\chi_1^0$ we have $k_{11}^{H_1} \simeq 1/2$ and $k_{21}^{H_1} \simeq (1 - m_{\chi_0}^2/m_{H_1,2}^2)^2$ if the corresponding process is kinematically allowed. The Yukawa couplings responsible for
the Higgs decays into neutralinos read
\[ y_{11}^h = \sqrt{2\lambda(1 + O(\epsilon))}\epsilon \sin 2\beta, \quad y_{11}^s = -\frac{\lambda}{\sqrt{2}}(1 + O(\epsilon))\epsilon^2 \sin 2\beta, \]
\[ y_{21}^h = \frac{\lambda}{2}(1 + O(\epsilon))(\cos \beta - \sin \beta), \quad y_{21}^s = -\frac{\lambda}{2}(1 + O(\epsilon))(\cos \beta - \sin \beta)\epsilon, \]
(28)

at the scale \( m_{\text{soft}} \).

The neutralino Yukawa couplings to \( H_{1,2} \) show that the invisible Higgs decay \( H_{1,2} \to \chi_1^0\chi_1^0 \) becomes weak at large \( \tan \beta \). We also note that \( y_{21}^h \) is stronger than the bottom Yukawa coupling as long as \( \lambda \) is larger than about 0.1 and \( \tan \beta \) is not close to unity. Hence, if kinematically open, the mode \( H_i \to \chi_2^0\chi_1^0 \), followed by the decay of \( \chi_2^0 \) into \( Z \) (or a Higgs boson) and \( \chi_1^0 \), will dominate the Higgs decays while making Higgs searches much more difficult. Meanwhile, from the couplings \( y_{11}^{h,s} \), it is straightforward to see
\[ \frac{\Gamma_{H_i \to \chi_1^0\chi_1^0}}{\Gamma_{H_i \to bb}} = \frac{c_{H_1}^2}{6} \left( \frac{v}{m_b} \lambda \epsilon \sin 2\beta \right)^2, \]
(29)

where \( c_{H_1} \simeq 2 + \epsilon \cot \theta \) and \( c_{H_2} \simeq 2 - \epsilon \tan \theta \) for \( \epsilon \ll 1 \), neglecting the masses of the final states. The branching ratio for the invisible Higgs decay is essentially determined by the above quantity, which increases as \( \tan \beta \) decreases and \( \epsilon \) grows for given \( \tan \theta \). The invisible channel of \( H_1 \) relaxes the LEP constraints on it, but that of \( H_2 \) reduces the Higgs signal rate compared to the SM prediction.

### B. Effects of Higgs mixing

Taking into account various constraints on the model parameters, we investigate how much the Higgs mixing can contribute to the mass of the SM-like Higgs \( H_2 \), which is assumed to be around 125 GeV as hinted by the recent ATLAS and CMS data \[10\]. The Higgs mixing should be small in order for the signal rate of \( H_2 \) decays into SM particles to be near what is predicted by the SM.\(^4\) Here we focus on the region with \( R_{H_2}^{SM} \) larger than 0.7. In the analysis, it is convenient to use
\[ (\lambda, \mu, \tan \beta, m_{H_1}, M_{hh}, \sin^2 \theta) \]
(30)

\(^4\) Large stop mixing can enhance the gluon-Higgs coupling mediated by squarks, altering the rate of the gluon fusion for the Higgs production \[37\]. However, such effects are non-negligible only when stops are relatively light. For sfermions having masses around a TeV or higher, one can take \( \sigma(H_2)/\sigma_{SM}(h) \simeq \cos^2 \theta \) in \[11\].
instead of the parameters \((\lambda, \mu, \tan \beta, B, A_\lambda, m_2^2)\). In the limit of \(\lambda = 0\), which corresponds to the MSSM case, large stop mixing is required to realize \(M_{hh}\) around 125 GeV for stops having a TeV mass. The NMSSM contribution to the Higgs quartic coupling improves the situation, but only in the low \(\tan \beta\) regime if \(\lambda\) is less than about 0.7 as required by the perturbativity constraint. It is thus interesting to consider the increase of \(m_{H_2}\) by the Higgs mixing.

Before proceeding further, we summarize the region of parameter space under investigation here:

\[
0.1 \lesssim \lambda \lesssim 0.4, \quad 120 \text{ GeV} \lesssim \mu \lesssim 300 \text{ GeV}, \quad 5 \lesssim \tan \beta \lesssim 20,
\]

\[
80 \text{ GeV} \lesssim m_{H_1} \lesssim 110 \text{ GeV}, \quad 115 \text{ GeV} \lesssim M_{hh} \lesssim 120 \text{ GeV}, \quad \sin^2 \theta \lesssim 0.3, \quad (31)
\]

where the MSSM superparticles except the higgsinos are assumed to have masses around a TeV. It has been taken into account that the constraint \((18)\) requires a rather small \(\lambda\) unless \(Z \to \chi_1^0 \chi_1^0\) is kinematically forbidden, and that \(\sin^2 \theta\) less than 0.3 is needed to get \(R^{SM}_{H_2} > 0.7\). Furthermore, to ensure the stability of the electroweak vacuum \((M_{hs}^2)^2 < M_{hh}^2 M_{ss}^2\), one needs \(A_\lambda \sim \mu \tan \beta\) unless \(\lambda\) is very small. This leads us to consider \(A_\lambda\) around or above a TeV for \(\mu\) at a few hundred GeV, which is consistent with the assumption of the decoupling limit \((2B\mu/\sin 2\beta)^{1/2} \gg m_W\) because \(B > A_\lambda\) in the model.

Let us explain a bit more on the above parameter range. We consider \(\mu\) larger than about 120 GeV so that \(H_2 \to \chi_2^0 \chi_1^0\) is kinematically closed, which would otherwise dominate the Higgs decay and reduce the identification capability of \(H_2\) signals at collider experiments. For \(\mu \gtrsim 120\) GeV and \(\lambda \lesssim 0.4\), \(\chi_1^0\) is lighter than about 10 GeV, and the invisible channel \(H_2 \to \chi_1^0 \chi_1^0\) becomes strong at low \(\tan \beta\) while making Higgs searches difficult. To avoid such a situation, we consider \(\tan \beta\) larger than 5. Finally, we take \(\tan \theta > 0\), which is the case with \(2\mu > A_\lambda \sin 2\beta\), because in order to make the Higgs mixing effect sizable it is preferred that the branching fraction of the invisible decay is large for \(H_1\) but small for \(H_2\) as much as possible. This requires \(\tan \theta > 0\) as can be seen from \((29)\).

In Fig. 2 we illustrate the effects of the singlet-doublet Higgs mixing for two cases. For the case with \((\lambda, \mu, \tan \beta, M_{hh}) = (0.2, 180 \text{ GeV}, 10, 120 \text{ GeV})\), the neutralinos have masses \(m_{\chi_2^0} \simeq 182.7\) GeV and \(m_{\chi_1^0} \simeq 1.3\) GeV, and the invisible \(Z\) decay width into neutralinos is \(\Gamma_{Z \to \chi_1^0 \chi_1^0} \simeq 0.21\) MeV. In this case, the Higgs mixing can increase the mass of \(H_2\) up to about 7 GeV in the range \(R^{SM}_{H_2} > 0.7\) for \(H_1\) having mass around 95 GeV. On the other hand,
FIG. 2: Higgs mixing in two cases: \((\lambda, \mu, \tan \beta, M_{hh}) = (0.2, 180 \text{ GeV}, 10, 120 \text{ GeV})\) for the left panel, while \((\lambda, \mu, \tan \beta, M_{hh}) = (0.1, 130 \text{ GeV}, 15, 122 \text{ GeV})\) for the right one. We plot the constant contours of \(\Delta m_{H_2} = m_{H_2} - M_{hh}\) in the blue line, and also those of the mass of the CP-odd Higgs boson in the dotted green line on the \((m_{H_1}, \sin^2 \theta)\) plane. The dashed black line represents the \(H_2\) signal rate \(R_{H_2}^{\text{SM}}\), compared to the SM case. The shaded region is excluded by the LEP limits on the coupling of \(H_1\) to \(\bar{t}t\). This constraint is a bit relaxed due to the invisible mode \(H_1 \rightarrow \chi_0^0 \chi_1^0\). For the comparison, we show the bound on \(R_{H_1}^{\bar{t}}\) in the dashed red line, assuming that \(H_1\) decays only into \(\bar{t}t\). For the other case with \((\lambda, \mu, \tan \beta, M_{hh}) = (0.1, 130 \text{ GeV}, 15, 122 \text{ GeV})\), we have \(m_{\chi_1^0} \approx 131\) GeV and \(m_{\chi_1^0} \approx 0.3\) GeV. The invisible \(Z\) decay width into neutralinos is \(\Gamma_{Z \rightarrow \chi_1^0 \chi_1^0} \approx 0.05\) MeV. For this case, \(\Delta m_{H_2}\) can be as large as 3 GeV at \(m_{H_1}\) around 90 GeV even when one requires \(R_{H_2}^{\text{SM}}\) larger than 0.9. Therefore the Higgs mixing can considerably reduce the role of stop mixing in achieving a 125 GeV Higgs boson mass. Meanwhile, in both cases, the branching ratio for \(H_2 \rightarrow \chi_1^0 \chi_1^0\) is smaller than 0.1, and thus below the detectable level at the LHC \([38, 39]\).

In the low energy effective theory below \(m_{\text{soft}}\), \(M_{hh}\) is determined by the relation \(M_{hh}^2 = 2\lambda_H v^2\) for \(\lambda_H\) renormalized at the electroweak scale. The RG running of \(\lambda_H\) is affected by the higgsinos for \(\mu\) less than \(m_{\text{soft}}\), but only slightly for small \(\lambda\). Note also that \(M_{ss}^2\) and \(M_{hs}^2\) explicitly depend on the SUSY breaking parameters. The left panel of Fig. \(\Box\) shows the value of \(M_{hh}\) for different values of the stop mixing parameter \(X_t \equiv (A_t - \mu \cot \beta)/m_{\text{soft}}\). 

15
in the case with $m_{\text{soft}} = 1.5$ TeV, $\mu = 180$ GeV and $\lambda = 0.2$. Here $A_t$ is the trilinear soft parameter for $H_u\tilde{t}_R\tilde{Q}_L$, and the contribution from stop loops is maximized at $X_t = \sqrt{6}$. As one can see, $M_{hh}$ around 125 GeV requires the almost maximal stop mixing. The situation does not change much at $\tan \beta \gtrsim 5$ if $\lambda$ is less than about 0.7 as required for the interaction $SH_uH_d$ to remain perturbative up to $M_{\text{GUT}} \sim 10^{16}$ GeV. In the right panel, the blue band represents the Higgs boson mass $m_{H_2}$ for the case with $X_t = 1$ and the same values of $m_{\text{soft}}$, $\mu$ and $\lambda$. Here we have taken $m_{H_1} = 95$ GeV and $0.18 \leq \sin^2 \theta \leq 0.26$, which are obtained for $m_S \sim 90$ GeV and $A_\lambda \sim \mu \tan \beta$ (a TeV). In the indicated range of model parameters, $R_{H_2}^{\text{SM}}$ has a value between 0.73 and 0.8. If one requires $R_{H_2}^{\text{SM}}$ more close to unity, the amount of mass increase by the mixing will be lowered.

For simplicity, we use (24) to calculate $M_{hh}^2$ with $\lambda_H$ evaluated by the use of one-loop RG equations, where we assume the squarks/sleptons and gauginos to have a universal mass $m_{\text{soft}}$. This is sufficient for the purpose of our discussion on how much the singlet-doublet Higgs mixing can increase the mass of the SM-like Higgs boson. Note that higher-loop corrections to $M_{hh}$, which involve the strong gauge coupling and the Yukawa couplings of the third generation fermions, can induce a shift of a few GeV on the Higgs boson mass

---

5 For simplicity, we use [23] to calculate $M_{hh}^2$ with $\lambda_H$ evaluated by the use of one-loop RG equations, where we assume the squarks/sleptons and gauginos to have a universal mass $m_{\text{soft}}$. This is sufficient for the purpose of our discussion on how much the singlet-doublet Higgs mixing can increase the mass of the SM-like Higgs boson. Note that higher-loop corrections to $M_{hh}$, which involve the strong gauge coupling and the Yukawa couplings of the third generation fermions, can induce a shift of a few GeV on the Higgs boson mass [40].
On the other hand, the mass of the CP-odd Higgs boson $A$ reads

$$m_A^2 = M_{ss}^2 - \lambda^2 v^2 \frac{A_1^2}{2B \mu} \sin^2 2\beta = m_{H_1}^2 + m_{H_2}^2 - M_{hh}^2 - \lambda^2 v^2 \frac{A_1^2}{2B \mu} \sin^2 2\beta,$$  

(32)

where we have used the relations (24) and (25). In the parameter space under consideration, if the singlet-doublet mixing yields $\Delta m_{H_2}$ of a few GeV or larger, $A$ has mass,

$$m_A \simeq m_{H_1} + \frac{m_{H_2}}{m_{H_1}} \Delta m_{H_2} - \mathcal{O}(1) \times \frac{\lambda^2 v^2}{m_{H_1} \tan^2 \beta},$$

(33)

and thus is heavier than $H_1$ by about $\Delta m_{H_2}$, but always lighter than $H_2$ for $m_{H_1} < M_{hh}$. Fig. 2 shows the dependence of $m_A$ on $m_{H_1}$ and the singlet-doublet mixing. Note that $H_2$ with mass around 125 GeV is forbidden to decay on-shell into $H_1 H_1$ or $A A$ for $80 \text{ GeV} \lesssim m_{H_1} \lesssim 110 \text{ GeV}$, where $\Delta m_{H_2}$ can be sizable. Meanwhile, in the decoupling regime, $A$ is mostly singlet-like and contains only a small doublet component fixed by the mixing angle,

$$\theta' \simeq \frac{\lambda v A}{2 |B \mu|} \sin 2\beta,$$

(34)

to which all the couplings of $A$ to the SM fermions and gauge bosons are proportional. The $A \bar{b} b$ coupling $\simeq (y_b \tan \beta) \theta'$ is at most about $\lambda y_b$ for $B > A$, but can be similar to or larger than the $A \chi_1^0 \chi_1^0$ coupling $\simeq \lambda v^2 \sin 2\beta$ in the parameter space (31). There are no $A ZZ$ and $A WW$ couplings at tree-level, while the $ZH_i A$ ($i = 1, 2$) coupling is highly suppressed by small factors since it is generated from the $Z h$-(doublet CP-odd Higgs boson) interaction whose coupling itself vanishes in the decoupling limit. One thus finds that $A$ mainly decays into $b \bar{b}$ and $\chi_1^0 \chi_1^0$, but evades the LEP constraints from $e^+ e^- \rightarrow Z A \rightarrow Z b \bar{b}$ due to the suppressed production cross section. The process $e^+ e^- \rightarrow Z^* \rightarrow H_1 A$ (and $H_2 A$ if kinematically allowed) also produces $A$, but only with small rates due to the suppressed $Z H_1 A$ coupling. In addition, the decay of the SM-like Higgs boson via $H_2 \rightarrow A A^* \rightarrow 4b$ is suppressed compared to $H_2 \rightarrow Z Z^* \rightarrow 4b$ because the $A \bar{b} b$ coupling is smaller than $\lambda y_b$ and $H_2$ couples to $AA$ with coupling $\simeq \lambda^2 v$.

Finally, we discuss the effect of mixing between the doublet Higgs bosons, which we have neglected since it is small when the heavier doublet Higgs obtains a mass much larger than the electroweak scale. In the MSSM, the light CP-even Higgs boson is composed out of $H_{a,d}^0$,

$$h = (\text{Re}(H_{d}^0) \sin \alpha + \text{Re}(H_{a}^0) \cos \alpha)/\sqrt{2},$$

and thus couples to the SM particles with

$$g_{h_H}^{\text{SM}} = \frac{\cos \alpha}{\sin \beta}, \quad g_{h_{bb}}^{\text{SM}} = -\frac{\sin \alpha}{\cos \beta}, \quad g_{h_Z Z}^{\text{SM}} = g_{h_W W}^{\text{SM}} = \sin(\beta - \alpha),$$

(35)
where $g_i^{SM}$ denotes the Higgs coupling in the SM case. The production of a SM-like Higgs boson at the LHC is dominated by the gluon-gluon fusion, to which the top quark loop gives the dominant contribution. Using this property, one can estimate the signal rate for each decay channel of $H_2 = s \sin \theta + h \cos \theta$ compared to the SM prediction:

$$R_{H_2}^i = \frac{\sigma(H_2)\text{Br}(H_2 \to i)}{\sigma_{\text{SM}}(h)\text{Br}(h \to i)|_\text{SM}} \approx \frac{\text{Br}(H_2 \to i)}{\text{Br}(h \to i)|_\text{SM}} \left(\frac{\cos \alpha}{\sin \beta}\right)^2 \cos^2 \theta,$$

for small values of $\theta$ and $\delta \equiv (\alpha - \beta + \pi/2)$. Here the quantities with the subscript, SM, refer to those for the SM case. For a Higgs boson at 125 GeV, the SM predicts $\text{Br}(h \to b\bar{b}) \approx 0.58$, $\text{Br}(h \to WW^*) \approx 0.22$, $\text{Br}(h \to gg) \approx 0.09$ and $\text{Br}(h \to \gamma\gamma) \approx 2.3 \times 10^{-3}$ with the total decay width $\Gamma_h \approx 4.03$ MeV and the production cross section $\sigma(pp \to h) \approx 15.3$ pb [41]. Hence, for $m_{H_2} = 125$ GeV and a moderately large $\tan \beta$, one finds

$$R_{H_2}^{bb} \approx (1 - 0.6 \delta \tan \beta) (1 - \text{Br}(H_2 \to \chi_1^0)) \cos^2 \theta,$$

$$R_{H_2}^{WW^*} \approx (1 + 1.4 \delta \tan \beta) (1 - \text{Br}(H_2 \to \chi_1^0)) \cos^2 \theta,$$

$$R_{H_2}^{\gamma\gamma} \approx (1 + 1.4 \delta \tan \beta) (1 - \text{Br}(H_2 \to \chi_1^0)) \cos^2 \theta,$$

assuming that the Higgs invisible decay is weak and the mixing angles $\theta$ and $\delta$ are small. Though being a naive estimation, the above shows the general property of the NMSSM that, if $\delta$ is positive, the mixing between doublet Higgs bosons enhances the signal rate for $\gamma\gamma$ and $WW^*$ while reducing that for $b\bar{b}$ [42]. In the case that the lightest Higgs boson is singlet-like, $\delta$ is determined mainly by the mass mixing between doublet Higgs bosons $h$ and $h'$,

$$\delta \approx \frac{m_{h'}^2 - \lambda^2 v^2}{4|B_\mu|} \sin 2\beta \sin 4\beta \approx \frac{-2(m_{h'}^2 - \lambda^2 v^2)}{|B_\mu|} \frac{1}{\tan^2 \beta},$$

for $m_{h'}^2 \approx 2|B_\mu|/\sin 2\beta \gg m_W^2$, because the contribution from the $s^2$-term to $\delta$ requires both $s-h$ and $s-h'$ mixings and is further suppressed by the mass ratio, $m_{h'}^2/(2|B_\mu|/\sin 2\beta)$. The above indicates that the NMSSM contribution can make $\delta$ positive when $\lambda^2 v^2 > m_Z^2$, i.e. when $\lambda$ is larger than about 0.52. Such effect can be sizable if $h'$ has a mass not far above the weak scale. Meanwhile, doublet mixing decreases the mass of $H_2$ by a small amount,

$$\Delta m_{H_2}|_{\delta} \approx -\frac{(m_Z^2 - \lambda^2 v^2)\sin 4\beta}{4m_{H_2}} \delta,$$

for $|\delta| \ll 1$. However, the tree-level relation $M_{hh}^2|_{\text{tree}} = m_Z^2 + (\lambda^2 v^2 - m_Z^2) \sin^2 2\beta$ implies that $M_{hh}$ is larger than $m_Z$ at the tree-level for $\lambda^2 v^2 > m_Z^2$. Thus, when $\delta$ is positive, the decrease
of the Higgs mass by the doublet mixing can be compensated by the NMSSM contribution to the Higgs quartic coupling.

Let us now see the situation in the PQ-NMSSM, where \( \lambda \gtrsim 0.52 \) requires \( \mu \gtrsim 260 \text{ GeV} \) for a moderately large \( \tan \beta \) in order to satisfy the constraint on the invisible \( Z \) decay rate. Provided \( \arg(A_\lambda) = \arg(B_\kappa) \) or \( \arg(A_\lambda) = \arg(B_\kappa) \pm \pi \), all the couplings in the Higgs potential (2) can be made real without loss of generality through an appropriate field redefinition. In the discussion so far, we have for simplicity assumed \( \arg(A_\lambda) = \arg(B_\kappa) \), for which CP is not spontaneously broken in the Higgs sector because one can always take a basis where all the couplings involved are real and positive. Also note that, in this case, \( B \) is larger than \( A_\lambda \) as follows from the minimization condition, and \( A_\lambda \sim \mu \tan \beta \) is needed to avoid too large singlet-doublet mixing. Hence, when \( H_1 \) is singlet-like, the heavier doublet Higgs will be very heavy for \( \lambda \gtrsim 0.52 \) and \( \mu \gtrsim 260 \text{ GeV} \), making the doublet mixing effect small. However, the situation changes for \( \arg(A_\lambda) = \arg(B_\kappa) \pm \pi \). In this case, depending on the model parameters, the minimum of the Higgs potential can still lie on a point preserving CP. Then, since \( B \) is not necessarily larger than \( A_\lambda \), it is possible for the heavier doublet Higgs to get a relatively small mass so that the doublet mixing can yield a sizable positive \( \delta \).

V. CONCLUSIONS

We have examined the implications of singlet-doublet Higgs mixing on the properties of a SM-like Higgs boson in the PQ-NMSSM, which incorporates the PQ symmetry solving the strong CP problem and does not suffer from the tadpole and domain-wall problems. For the case where the lightest Higgs boson is dominated by the singlet scalar, the Higgs mixing increases the mass of the SM-like Higgs boson while reducing its couplings to SM particles. Such mixing effect can be sizable also for large \( \tan \beta \) and small \( \lambda \), where the NMSSM direct contribution to the Higgs mass is negligible. However, the amount of mass increase is limited by the LEP constraints on the properties of the singlet-like Higgs boson and the lightest neutralino constituted mainly by the singlino. In addition, in order for the Higgs signal rate to be close to what the SM predicts, the mixing should be small. Combining these, we find that the mixing can enhance the Higgs boson mass by a few GeV or more even when the ratio for the Higgs signal rate compared to the SM case is to be 0.9. For the ratio around 0.7, the
amount of increase can be as large as about 7 GeV for the singlet-like Higgs boson around 95 GeV. Thus, the singlet-doublet Higgs mixing may be crucial for achieving a 125 GeV Higgs mass, as hinted by the recent ATLAS and CMS data, within the supersymmetric SM with superparticles having masses around a TeV. Once the Higgs production cross section is measured, we will learn the bound on how much the Higgs mixing can modify the properties of the SM-like Higgs boson.

Acknowledgments

We thank Marek Olechowski for pointing out a sign error in the doublet mixing parameter in an earlier version of the paper, which has been corrected as given by eq. (38) of the present version. This work was supported by Grants-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, No. 23104008 and No. 23540283.

Low energy effective Higgs potential

In this appendix, we present the relations between \((\lambda, m_S^2, A_\lambda, B, \mu, \tan \beta)\) and the SUSY breaking parameters involved in the Higgs potential. We also discuss how to integrate out the heavy Higgs scalars in the decoupling limit.

The SUSY breaking parameters are written

\[
B_\kappa = \frac{1}{B - A_\lambda} \left( m_S^2 + \left( 1 - \frac{\tan \beta}{\tan^2 \beta + 1} \frac{A_\lambda}{\mu} \right) \lambda^2 u^2 \right),
\]
\[
m_{H_u}^2 = \frac{B \mu}{\tan \beta} - \frac{2 \mu^2 + \lambda^2 u^2}{\tan^2 \beta + 1} - \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} \left( \frac{m_Z^2}{2} + \mu^2 \right),
\]
\[
m_{H_d}^2 = \frac{B \mu}{\cot \beta} - \frac{2 \mu^2 + \lambda^2 u^2}{\cot^2 \beta + 1} - \frac{\cot^2 \beta - 1}{\cot^2 \beta + 1} \left( \frac{m_Z^2}{2} + \mu^2 \right),
\]

which are obtained by using the extremum conditions.

Let us construct an effective Higgs potential by integrating out the heavy Higgs scalars in the decoupling limit \((2B\mu/\sin 2\beta)^{1/2} \gg m_W\). In the field basis \(H = -H_d \sin \alpha + H_u^c \cos \alpha\) and \(H' = H_d \cos \alpha + H_u^c \sin \alpha\) with \(H_u^c = i \sigma_2 H_u^*\), the Higgs potential reads

\[
V = V_0(S, |H|^2) + f_2 |H'|^2 - \left\{ (f_1 \cos^2 \alpha - f_1^* \sin^2 \alpha) H^H H' + \text{h.c.} \right\} + \cdots,
\]
where $f_{1,2}$ are given by

$$
\begin{align*}
    f_2 &= m_{H_u}^2 \cos^2 \alpha + m_{H_d}^2 \sin^2 \alpha + \lambda^2 |S|^2 - \frac{1}{2} \lambda (A_\lambda S + m_0^2 + \text{h.c.}) \sin 2\alpha, \\
    f_1 &= \frac{1}{2} (m_{H_d}^2 - m_{H_u}^2) \tan 2\alpha + \lambda (A_\lambda S + m_0^2) - \frac{1}{4} (g^2 + g'^2 - 2\lambda^2) |H|^2 \sin 2\alpha,
\end{align*}
$$

and the ellipsis indicates Higgs quartic terms with two or more powers of $H'$. It is straightforward to find

$$
\begin{align*}
    \langle f_2 \rangle &= \frac{2B\mu}{\sin 2\beta} \sin^2 (\alpha - \beta) + \mathcal{O}(v^2), \\
    \langle f_1 \rangle &\propto \cos (\alpha - \beta).
\end{align*}
$$

Therefore, $\langle f_1 \rangle$ vanishes at $\alpha = \beta - \pi/2$, for which $\langle H^0 \rangle = v$ and $\langle H' \rangle = 0$. In this case, if $2B\mu/\sin 2\beta \gg v^2$, the heavy Higgs doublet $H'$ can be integrated out by solving $\partial_{H'} V = 0$:

$$
H' = \frac{\sin 2\beta}{2B\mu} (f_1^* \sin^2 \beta - f_1 \cos^2 \beta) (1 + \cdots) H,
$$

where the ellipsis in the bracket includes terms depending on $S$ and $|H|^2$, which are irrelevant to the low energy physics because $f_1$ vanishes at the vacuum. The relevant interaction terms arise only through the dependence of $f_1$ on $S$ and $|H|^2$. Using the extremum conditions, one can rewrite $f_1$,

$$
\begin{align*}
    f_1 &= A_\lambda (\lambda S - \mu) + \frac{\sin 2\beta}{4} (g^2 + g'^2 - 2\lambda^2) (|H|^2 - v^2)^2 \equiv f_{\text{mix}}.
\end{align*}
$$

Finally, substituting $H'$ by the solution (44) leads to the effective potential (4).

---

[1] For a review, see M. Maniatis, “The Next-to-Minimal Supersymmetric extension of the Standard Model reviewed,” Int. J. Mod. Phys. A 25, 3505-3602 (2010). [arXiv:0906.0777 [hep-ph]]; U. Ellwanger, C. Hugonie, A. M. Teixeira, “The Next-to-Minimal Supersymmetric Standard Model,” Phys. Rept. 496, 1-77 (2010). [arXiv:0910.1785 [hep-ph]].

[2] H. P. Nilles, M. Srednicki and D. Wyler, “Constraints On The Stability Of Mass Hierarchies In Supergravity,” Phys. Lett. B 124, 337 (1983); A. B. Lahanas, “Light Singlet, Gauge Hierarchy And Supergravity,” Phys. Lett. B 124, 341 (1983); U. Ellwanger, “Nonrenormalizable Interactions From Supergravity, Quantum Corrections And Effective Low-Energy Theories,” Phys. Lett. B 133, 187 (1983); H. P. Nilles and N. Polonsky, “Gravitational divergences as a mediator of supersymmetry breaking,” Phys. Lett. B 412, 69 (1997) [hep-ph/9707249].
[3] J. Bagger, E. Poppitz, “Destabilizing divergences in supergravity coupled supersymmetric theories,” Phys. Rev. Lett. 71, 2380-2382 (1993). [hep-ph/9307317]; V. Jain, “On destabilizing divergences in supergravity models,” Phys. Lett. B351, 481-486 (1995). [arXiv:hep-ph/9407382 [hep-ph]]; J. Bagger, E. Poppitz, L. Randall, “Destabilizing divergences in supergravity theories at two loops,” Nucl. Phys. B455, 59-82 (1995). [hep-ph/9505244].

[4] A. Vilenkin, “Cosmic Strings And Domain Walls,” Phys. Rept. 121, 263 (1985).

[5] S. A. Abel, S. Sarkar, P. L. White, “On the cosmological domain wall problem for the minimally extended supersymmetric standard model,” Nucl. Phys. B454, 663-684 (1995). [hep-ph/9506359].

[6] K. S. Jeong, Y. Shoji and M. Yamaguchi, “Peccei-Quinn invariant extension of the NMSSM,” JHEP 1204, 022 (2012) [arXiv:1112.1014 [hep-ph]].

[7] R. D. Peccei and H. R. Quinn, “CP Conservation In The Presence Of Instantons,” Phys. Rev. Lett. 38, 1440 (1977); “Constraints Imposed By CP Conservation In The Presence Of Instantons,” Phys. Rev. D 16, 1791 (1977).

[8] For a review, see J. E. Kim, “Light Pseudoscalars, Particle Physics and Cosmology,” Phys. Rept. 150, 1 (1987); H. Y. Cheng, “The Strong CP Problem Revisited,” Phys. Rept. 158, 1 (1988); J. E. Kim and G. Carosi, “Axions and the Strong CP Problem,” Rev. Mod. Phys. 82, 557 (2010) [arXiv:0807.3125 [hep-ph]].

[9] U. Ellwanger, “Higgs Bosons in the Next-to-Minimal Supersymmetric Standard Model at the LHC,” Eur. Phys. J. C 71, 1782 (2011) [arXiv:1108.0157 [hep-ph]].

[10] The ATLAS and CMS collaborations, ATLAS-CONF-2011-163 and CMS-PAS-HIG-11-032 (December, 2011).

[11] H. P. Nilles, “Supersymmetry, Supergravity And Particle Physics,” Phys. Rept. 110, 1 (1984); H. E. Haber and G. L. Kane, “The Search For Supersymmetry: Probing Physics Beyond The Standard Model,” Phys. Rept. 117, 75 (1985).

[12] U. Ellwanger, G. Espitalier-Noel and C. Hugonie, “Naturalness and Fine Tuning in the NMSSM: Implications of Early LHC Results,” arXiv:1107.2472 [hep-ph]; G. G. Ross and K. Schmidt-Hoberg, “The fine-tuning and phenomenology of the generalised NMSSM,” arXiv:1108.1284 [hep-ph]; A. Delgado, C. Kolda and A. de la Puente, “Solving the Hierarchy Problem with a Light Singlet and Supersymmetric Mass Terms,” arXiv:1111.4008 [hep-ph]; Z. Kang, J. Li and T. Li, “On Naturalness of the (N)MSSM,” arXiv:1201.5305 [hep-ph].
[13] T. Kobayashi, T. Shimomura and T. Takahashi, “Constraining the Higgs sector from False Vacua in the Next-to-Minimal Supersymmetric Standard Model,” arXiv:1203.4328 [hep-ph].

[14] L. J. Hall, D. Pinner and J. T. Ruderman, “A Natural SUSY Higgs Near 126 GeV,” JHEP 1204, 131 (2012) [arXiv:1112.2703 [hep-ph]]; J. F. Gunion, Y. Jiang and S. Kraml, “The Constrained NMSSM and Higgs near 125 GeV,” Phys. Lett. B 710, 454 (2012) [arXiv:1201.0982 [hep-ph]]; S. F. King, M. Muhlleitner and R. Nevzorov, “NMSSM Higgs Benchmarks Near 125 GeV,” Nucl. Phys. B 860, 207 (2012) [arXiv:1201.2671 [hep-ph]]; J. Cao, Z. Heng, J. M. Yang, Y. Zhang and J. Zhu, “A SM-like Higgs near 125 GeV in low energy SUSY: a comparative study for MSSM and NMSSM,” JHEP 1203, 086 (2012) [arXiv:1202.5821 [hep-ph]]; D. A. Vasquez, G. Belanger, C. Boehm, J. Da Silva, P. Richardson and C. Wyman, “The 125 GeV Higgs in the NMSSM in light of LHC results and astrophysics constraints,” [arXiv:1203.3446 [hep-ph]]; U. Ellwanger and C. Hugonie, “Higgs bosons near 125 GeV in the NMSSM with constraints at the GUT scale,” arXiv:1203.5048 [hep-ph].

[15] E. Bertuzzo and M. Farina, “Higgs boson signals in lambda-SUSY with a Scale Invariant Superpotential,” arXiv:1105.5389 [hep-ph]; M. Almarashi and S. Moretti, “Reinforcing the no-lose theorem for NMSSM Higgs discovery at the LHC,” Phys. Rev. D 84, 035009 (2011) [arXiv:1106.1599 [hep-ph]]; M. M. Almarashi and S. Moretti, “LHC Signals of a Heavy CP-even Higgs Boson in the NMSSM via Decays into a Z and a Light CP-odd Higgs State,” arXiv:1109.1735 [hep-ph].

[16] M. Asano and T. Higaki, “Natural supersymmetric spectrum in mirage mediation,” arXiv:1204.0508 [hep-ph]; T. Kobayashi, H. Makino, K. -i. Okumura, T. Shimomura and T. Takahashi, “TeV scale mirage mediation in NMSSM,” arXiv:1204.3561 [hep-ph].

[17] K. Hamaguchi, K. Nakayama and N. Yokozaki, “NMSSM in gauge-mediated SUSY breaking without domain wall problem,” arXiv:1107.4760 [hep-ph]; K. Nakayama, N. Yokozaki and K. Yonekura, “Relaxing the Higgs mass bound in singlet extensions of the MSSM,” JHEP 1111, 021 (2011) [arXiv:1108.4338 [hep-ph]]; B. Kyae and J. -C. Park, “Hidden Sector Assisted 125 GeV Higgs,” arXiv:1203.1656 [hep-ph].

[18] C. Panagiotakopoulos and K. Tamvakis, “New minimal extension of MSSM,” Phys. Lett. B 469, 145 (1999) [hep-ph/9908351].

[19] C. Panagiotakopoulos, A. Pilaftsis, “Higgs scalars in the minimal nonminimal supersymmetric standard model,” Phys. Rev. D63, 055003 (2001). [hep-ph/0008268].
[20] A. Dedes, C. Hugonie, S. Moretti and K. Tamvakis, “Phenomenology of a new minimal supersymmetric extension of the standard model,” Phys. Rev. D 63, 055009 (2001) [arXiv:hep-ph/0009125].

[21] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. K. S. Vandervange, “Discrete R symmetries for the MSSM and its singlet extensions,” Nucl. Phys. B 850 (2011) 1 [arXiv:1102.3595 [hep-ph]].

[22] J. E. Kim, H. P. Nilles and M. -S. Seo, “The mu-problem, the NPQMSSM, and a light pseudoscalar Higgs boson for the LHC,” [arXiv:1201.6547 [hep-ph]].

[23] R. Barate et al. [LEP Working Group for Higgs boson searches and ALEPH and DELPHI and L3 and OPAL Collaborations], “Search for the standard model Higgs boson at LEP,” Phys. Lett. B 565, 61 (2003) [hep-ex/0306033].

[24] S. Mizuta, D. Ng and M. Yamaguchi, “Phenomenological aspects of supersymmetric standard models without grand unification,” Phys. Lett. B 300, 96 (1993) [arXiv:hep-ph/9210241].

[25] D. Das, U. Ellwanger and A. M. Teixeira, “Modified Signals for Supersymmetry in the NMSSM with a Singlino-like LSP,” JHEP 1204, 067 (2012) [arXiv:1202.5244 [hep-ph]].

[26] [ALEPH and DELPHI and L3 and OPAL and SLD and LEP Electroweak Working Group and SLD Electroweak Group and SLD Heavy Flavour Group Collaborations], “Precision electroweak measurements on the Z resonance,” Phys. Rept. 427, 257 (2006) [hep-ex/0509008].

[27] K. Nakamura et al. [Particle Data Group], “Review of particle physics,” J. Phys. G 37, 075021 (2010).

[28] H. K. Dreiner, S. Heinemeyer, O. Kittel, U. Langenfeld, A. M. Weber and G. Weiglein, “Mass Bounds on a Very Light Neutralino,” Eur. Phys. J. C 62, 547 (2009) [arXiv:0901.3485 [hep-ph]].

[29] A. Menon, D. E. Morrissey and C. E. M. Wagner, “Electroweak baryogenesis and dark matter in the nMSSM,” Phys. Rev. D 70 (2004) 035005 [arXiv:hep-ph/0404184].

[30] C. Balazs, M. S. Carena, A. Freitas and C. E. M. Wagner, “Phenomenology of the nMSSM from colliders to cosmology,” JHEP 0706, 066 (2007) [arXiv:0705.0431 [hep-ph]]; J. Cao, H. E. Logan and J. M. Yang, “Experimental constraints on nMSSM and implications on its phenomenology,” Phys. Rev. D 79, 091701 (2009) [arXiv:0901.1437 [hep-ph]]; P. Draper, T. Liu, C. E. M. Wagner, L. -T. Wang and H. Zhang, “Dark Light Higgs,” Phys. Rev. Lett. 106, 121805 (2011) [arXiv:1009.3963 [hep-ph]]; M. Carena, N. R. Shah and C. E. M. Wagner,
“Light Dark Matter and the Electroweak Phase Transition in the NMSSM,” Phys. Rev. D 85, 036003 (2012) [arXiv:1110.4378 [hep-ph]].

[31] P. Fayet, “Lower Limit on the Mass of a Light Gravitino from e+ e- Annihilation Experiments,” Phys. Lett. B 175, 471 (1986).

[32] S. Bashinsky and U. Seljak, “Signatures of relativistic neutrinos in CMB anisotropy and matter clustering,” Phys. Rev. D 69, 083002 (2004) [arXiv:astro-ph/0310198].

[33] S. Hannestad and G. Raffelt, “Cosmological mass limits on neutrinos, axions, and other light particles,” JCAP 0404, 008 (2004) [arXiv:hep-ph/0312154]; P. Crotty, J. Lesgourgues and S. Pastor, “Current cosmological bounds on neutrino masses and relativistic relics,” Phys. Rev. D 69, 123007 (2004) [arXiv:hep-ph/0402049]; S. Hannestad, A. Mirizzi, G. G. Raffelt and Y. Y. Y. Wong, “Cosmological constraints on neutrino plus axion hot dark matter: Update after WMAP-5,” JCAP 0804, 019 (2008) [arXiv:0803.1585 [astro-ph]].

[34] S. Kim, W. I. Park and E. D. Stewart, “Thermal inflation, baryogenesis and axions,” JHEP 0901, 015 (2009) [arXiv:0807.3607 [hep-ph]]; K. Choi, K. S. Jeong, W. I. Park and C. S. Shin, “Thermal inflation and baryogenesis in heavy gravitino scenario,” JCAP 0911, 018 (2009) [arXiv:0908.2154 [hep-ph]]; K. Choi, E. J. Chun, H. D. Kim, W. I. Park and C. S. Shin, “The µ-problem and axion in gauge mediation,” Phys. Rev. D 83, 123503 (2011) [arXiv:1102.2900 [hep-ph]]; K. S. Jeong and M. Yamaguchi, “Axion model in gauge-mediated supersymmetry breaking and a solution to the µ/Bµ problem,” JHEP 1107, 124 (2011) [arXiv:1102.3301 [hep-ph]].

[35] Y. Okada, M. Yamaguchi and T. Yanagida, “Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model,” Prog. Theor. Phys. 85, 1 (1991); “Renormalization Group Analysis On The Higgs Mass In The Softly Broken Supersymmetric Standard Model,” Phys. Lett. B 262, 54 (1991).

[36] J. R. Ellis, G. Ridolfi and F. Zwirner, “Radiative corrections to the masses of supersymmetric Higgs bosons,” Phys. Lett. B 257, 83 (1991); H. E. Haber and R. Hempfling, “Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than m(Z)?,” Phys. Rev. Lett. 66, 1815 (1991).

[37] R. V. Harlander and M. Steinhauser, “Supersymmetric Higgs production in gluon fusion at next-to-leading order,” JHEP 0409, 066 (2004) [hep-ph/0409010].

[38] N. Desai, B. Mukhopadhyaya and S. Niyogi, “Constraints on invisible Higgs decay in MSSM
in the light of diphoton rates from the LHC,” arXiv:1202.5190 [hep-ph].

[39] J. Cao, Z. Heng, J. M. Yang and J. Zhu, “Higgs decay to dark matter in low energy SUSY: is it detectable at the LHC ?,” arXiv:1203.0694 [hep-ph].

[40] B. C. Allanach, A. Djouadi, J. L. Kneur, W. Porod and P. Slavich, “Precise determination of the neutral Higgs boson masses in the MSSM,” JHEP 0409, 044 (2004) [hep-ph/0406166].

[41] For a review, see A. Djouadi, “The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model,” Phys. Rept. 457, 1 (2008) [hep-ph/0503172].

[42] M. Carena, S. Gori, N. R. Shah and C. E. M. Wagner, “A 125 GeV SM-like Higgs in the MSSM and the $\gamma\gamma$ rate,” JHEP 1203, 014 (2012) [arXiv:1112.3336 [hep-ph]]; U. Ellwanger, “A Higgs boson near 125 GeV with enhanced di-photon signal in the NMSSM,” JHEP 1203, 044 (2012) [arXiv:1112.3548 [hep-ph]]; A. Arvanitaki and G. Villadoro, “A Non Standard Model Higgs at the LHC as a Sign of Naturalness,” JHEP 1202, 144 (2012) [arXiv:1112.4835 [hep-ph]]; P. P. Giardino, K. Kannike, M. Raidal and A. Strumia, “Reconstructing Higgs boson properties from the LHC and Tevatron data,” arXiv:1203.4254 [hep-ph].