A Study on Connectivity Concepts in Intuitionistic Fuzzy Graphs

WAEL AHMAD ALZOUBI, AS’AD MAHMOUD AS’AD ALNASER
Department of Applied Science, Ajloun University College,
Balqa Applied University
JORDAN

Abstract:- In this paper, we introduced some concepts of connectivity in an intuitionistic fuzzy graphs, also we study intuitionistic fuzzy cut vertices and intuitionistic fuzzy bridges in fuzzy graph. Connectivity in complete intuitionistic fuzzy graphs is also studied.

Key words:- fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy graphs, intuitionistic fuzzy cut vertex, intuitionistic fuzzy bridge.

1 Introduction
Zadeh in [1] introduced the concept of fuzzy sets in 1965. Fuzzy sets paved the way for new philosophical fuzzy logic thinking. This logic is used in the large number productions in electronics. Fuzzy sets and fuzzy logic theory have been applied widely in areas like database theory, robotics, expert systems, control theory, information theory, pattern recognition and nano-technology. In 1975, Rosenfeld [2], studied fuzzy graphs. Fuzzy graphs are useful to represent relationships which deals with uncertainty and its greatly different from graphs. Massa’deh et al [3, 4] studied more properties for fuzzy graph such as degree of vertices and isomorphism. Atanassov [5, 6] introduced the intuitionistic fuzzy set concepts, after that Karunambigai & Kalaivani [7] introduced the matrix representation of intuitionistic fuzzy graphs, while Mishra & Pal [8, 9] in 2013 and 2017 respectively discussed the product of interval valued intuitionistic fuzzy graph and regular interval valued, and in 2014 Yahya & Jahir [10] studied isomorphism on irregular fuzzy graph. Pathinathan & Rosline [11] gave the concept of vertex degree of cartesian product of intuitionistic fuzzy graph. In 2018, Sunny & Jose [12] introduced the notion of modular and homomorphic products on interval intuitionistic fuzzy graph. in 2020, Fallatah et. al.[13] and Alnaser et. al. [14] added new concepts which are intuitionistic fuzzy soft graph and bipolar intuitionistic fuzzy graphs. In this paper, we introduced some concepts of connectivity in an intuitionistic fuzzy graphs, also we studied intuitionistic fuzzy cut-vertices and intuitionistic fuzzy bridges in fuzzy graphs, on the other hand, some properties and concepts are added.

2 Preliminaries
In this section we review and recollect some concept for undirected graphs [2]. A graph is an ordered pair \( G = (V, E) \), where \( V \) is the set of vertices of \( G \) and \( E \) is the set of edges of \( G \), while a sub graph of \( G = (V, E) \) is a graph \( S = (W, F) \) such that \( W \leq V \) and \( F \leq E \). An undirected graphs that has no loops and not more than one edge between any two different vertices is a simple graph. A trivial graph is a simple graph with a single vertex. Two vertices \( i \) and \( j \) in an undirected graph \( G \) are said to be adjacent in \( G \) if \((i, j)\) is an edge of \( G \). \( i \ j \) or \( j \ i \) represented to edge. The set of all vertices adjacent to a vertex \( i \) in \( G \) is called the neighbor set of \( i \) and is denoted by \( N(i) \). A \( i_0, i_1, \ldots, i_n \) path \( P \) in \( G \) is an alternating sequence of vertices and edges \( i_0, e_1, i_1, e_2, \ldots, e_n, i_n \) such that \( i_k \ ne_{k+1} \) is an edge for \( i = 0, 1, \ldots, n - 1 \) The number of edges in the path is called the path length and \( P \) is called closed path or a cycle if \( i_0 = i_n \).
A graph $G$ is said to be connected if there is a path joining any two vertices in $G$ while, a graph $G$ is said to be a tree if it is connected and acyclic. The connected components number in $G$ is denoted by by $\omega(G)$. A vertex $i$ of $G$ is called cut vertex if $\omega(G - \{i\}) > \omega(G)$. Also, an edge $e$ of $G$ is said to be a cut edge if $\omega(G - \{e\}) > \omega(G)$.

A graph $G$ is called complete graph if all the vertices in $G$ are pairwise adjacent.

**Definition 2.1:** [1] a fuzzy set $\lambda$ on $G$ is just a map $\lambda: G \rightarrow [0,1]$.

**Definition 2.2:** [2] A map $\theta: G \times G \rightarrow [0,1]$ is said to be fuzzy relation on $\lambda$ if $\theta(i,j) \leq \min\{\mu(i),\mu(j)\} \forall i, j \in G$ on the other hand, a fuzzy relation $\theta$ is called reflexive if $\theta(i,j) = \lambda(i) \forall i \in G$, while $\theta$ is called symmetric if $\theta(i,j) = \theta(j,i) \forall i, j \in G$.

**Definition 2.3:** [2] a fuzzy graph is a path $G = (\delta, \lambda)$ such that $\delta$ is a fuzzy subset on a set $V$ and $\lambda$ is a fuzzy relation on $\delta$. Since $V$ is non empty and finite, $\lambda$ is symmetric and reflexive. Therefore, if $G = (\delta, \lambda)$ is a fuzzy graph, then $\delta: V \rightarrow [0,1]$ and $\lambda: V \times V \rightarrow [0,1]$ such that $\lambda(i,j) \leq \min\{\delta(i),\delta(j)\} \forall i, j \in V$.

**Remark 2.4:** [1] we represent the underlying graph of $G = (\delta, \lambda)$ by $G^+ = (\delta^+, \lambda^+)$ such that $

\delta^+ = \{i \in V; \delta(i) > 0\}$ and $\lambda^+ = \{(i,j) \in E \times V; \lambda(i,j) > 0\}$. $G = (\delta, \lambda)$ is a trivial fuzzy graph, if $G^+ = (\delta^+, \lambda^+)$ is trivial.

### 3 Connectivity in intuitionistic fuzzy graph

**Definition 3.1:** [7] an intuitionistic fuzzy graph $G = (V, E)$ such that $V = \{i_1, i_2, ..., i_n\}$ where $\lambda_1: V \rightarrow [0,1]$ and $\varphi_1: V \rightarrow [0,1]$ represent the membership degree of non-membership of $i_k \in V$ respectively, also $0 \leq \lambda_1(i_k) + \varphi_1(i_k) \leq 1$ for every $i_k \in V(k = 1, \ldots, n)$.

1) $\varphi_1 \leq V \times V$ such that $\lambda_2: V \times V \rightarrow [0,1]$ and $\varphi_2: V \times V \rightarrow [0,1]$ such that $\lambda_2(i_k + i_n) \leq \min\{\lambda_1(i_k), \lambda_1(i_n)\}$ and $\varphi_2(i_k + i_n) \leq \max\{\varphi_1(i_k), \varphi_1(i_n)\}$ and $0 \leq \lambda_2(i_k, i_n) + \varphi_2(i_k, i_n) \leq 1$ for every $(i_k, i_n) \in E$.

**Definition 3.2:** [7] the underlying crisp graph of an intuitionistic fuzzy graph $G = (\delta, \lambda)$ is the graph $G = (V', E')$ such that $V' = \{i \in V; y_\delta(i) > 0 \text{ or } \mu_\delta(i) > 0\}$ and $E' = \{(i,j); y_\lambda(i,j) > 0 \text{ or } \varphi_\lambda(i,j) > 0\}$

**Definition 3.3:** an intuitionistic fuzzy graph $G = (\delta, \lambda)$ is connected if the underlying crisp graph is connected.

**Definition 3.4:** A partial an intuitionistic fuzzy subgraph of an intuitionistic fuzzy graph $G$ is an intuitionistic fuzzy graph $\delta = (\delta', \lambda')$ such that $y_{\delta'}(i_k) \leq y_\delta(i_k)$ and $\varphi_{\delta'}(i_k) \geq \varphi_\delta(i_k) \forall i_k \in$
\[ V \text{ and } \chi(i_k, i_n) < \lambda(i_k, i_n) \text{ and } \chi(i_k, i_n) > \lambda(i_k, i_n) \text{ for every } i, j \in V. \]

**Definition 3.5:** an intuitionistic fuzzy subgraph of \( G \) is an intuitionistic fuzzy graph \( \delta = (\delta', \lambda') \) such that \( \chi(\delta')(i_k) < \chi(\delta)(i_k) \) and \( \lambda(\delta')(i_k) = \lambda(\delta)(i_k) \) \( \forall i_k \). Also, in the vertex set of \( \delta \) and \( \chi(\lambda')(i_k, i_n) = \chi(\lambda)(i_k, i_n) \) and \( \lambda(\lambda')(i_k, i_n) = \lambda(\lambda)(i_k, i_n) \) \( \forall i_k, i_n \) edge in \( \delta \).

**Example 3.6:** \( G_1 \) in fig 3.6.1 is an intuitionistic fuzzy graph \( \delta_1 \) in fig 3.6.2 is a partial an intuitionistic fuzzy subgraph and \( \delta_2 \) in fig 3.6.3 is an intuitionistic fuzzy subgraph \( G_1 \).

**Fig. 3.6.1**

**Fig. 3.6.2**

**Fig. 3.6.3**

**Definition 3.7:** an intuitionistic fuzzy graph \( G \) is called strong if \( \mu_{2kn} = \min\{\mu_{1k}, \mu_{1n}\} \) and \( \varphi_{2kn} = \max\{\varphi_{1k}, \varphi_{1n}\} \) for every edge \( i_k, i_n \in E \).

**Definition 3.8:** an intuitionistic fuzzy graph \( G \) is called complete if \( \lambda_{2kn} = \min\{\lambda_{1k}, \lambda_{1n}\} \) and \( \varphi_{2kn} = \max\{\varphi_{1k}, \varphi_{1n}\} \) \( \forall i_k, i_n \in V \).

**Definition 3.9:** A path \( P \) in an intuitionistic fuzzy graph \( G \) is a sequence of distinct vertices \( i_1, i_2, \ldots, i_n \) such that either one of the following axioms is hold

1) \( \lambda_{2kn} > 0 \) and \( \varphi_{2kn} = 0 \) for some \( k, n \)
2) \( \lambda_{2kn} = 0 \) and \( \varphi_{2kn} > 0 \) for some \( k, k \)

**Example 3.10:** an intuitionistic fuzzy graph in fig. 3.10.1 is not a path

**Fig. 3.10.1**

**Definition 3.11:** a sequence of vertices \( i_1, i_2, \ldots, i_k \) not necessarily distinct is said to be walk or \( k - \) walk if at least one of \( \lambda_{2kn}(k + 1) \) is different from zero for \( k = 1, \ldots, n - 1 \)

**Corollary 3.12:** an intuitionistic fuzzy graph is connected iff every pair of vertices is joined by a \( k - \) path.

**Proof:** Straightforward.

**4 An Intuitionistic Fuzzy Cut Vertices And Bridges**
**Definition 4.1:** If graph $G$ is an intuitionistic fuzzy graph with intuitionistic function, $\gamma_1$ and $\gamma_2$. A vertex $i \in V$ is called an intuitionistic fuzzy cut vertex if there exist two vertices $i, j \in V; i \neq j = x$, such that $G_{G-x}(i, j) < G_G(i, j)$ and $L_{G-x}(i, j) > L_G(i, j)$ where $G(i, j)$ represent the gain of $i$ and $j$ and $L(i, j)$ represent the loss of $i$ and $j$.

**Theorem 4.2:** A vertex $i$ in an intuitionistic fuzzy graph $G$ is an intuitionistic fuzzy cut vertex if and only if $i$ is a vertex in every $\max(k-m)$ gain path and $i$ in is in every $\min(k-m)$ loss path for some $k, m$ in $V$.

**Proof:** Suppose that $G$ is an intuitionistic fuzzy graph with intuitionistic function, $\mu_1(1) & \mu_2$ and let $X$ be an intuitionistic fuzzy cut vertex, then there exist some vertices $i, j$ in $G$ such that $i \neq j \neq X$ & $G(G-X)(i, j) < G_G(i, j)$ & $L(G-X)(i, j) > L_G(i, j)$.

If we removed $X$ from $G$ removes all $\max(i-j)$ gain paths and if we removed $X$ from $G$ removes all $\min(i-j)$ loss paths, then $X$ is in every $\max(i-j)$ and in every $\min(i-j)$.

Let $X$ is in any $\max(i-j)$ and in any $\min(i-j)$. Thus, if we removed $X$ from $G$ then we remove all $\max(i-j)$ and all $\min(i-j)$. Therefore, the gain will decrease and the loss will increase between $i$ and $j$. Hence $G(G-X)(i, j) < G_G(i, j)$ & $L(G-X)(i, j) > L_G(i, j)$ and we get $X$ is an intuitionistic fuzzy cut vertex.

**Corollary 4.3:** If $G$ is an intuitionistic fuzzy graph. A vertex $X$ is cut vertex of gain if and only if $X$ is in every $\max(i-j)$ gain path, where $i \neq j \neq X$ and is a cut vertex of loss if and only if $X$ is in every $\min(z-w)$ loss path, where $z \neq w \neq X$.

**Proof:** Straightforward.

**Corollary 4.4:** An edge $e \in E$ of an intuitionistic fuzzy graph $G$ is intuitionistic fuzzy bridge if and only if it is in every $\max(i-j)$ gain path and every $\min(i-j)$ loss path.

**Proof:** Suppose that $G$ is an intuitionistic fuzzy graph and $i, j$ is an edge in $G$ where $G_{G-i}(i, j) \leq \mu_{2\gamma}(i, j)$ and $L_{G-i}(i, j) > \theta_{2\gamma}(i, j)$.

**Theory 4.3:** An edge $i j$ is an intuitionistic fuzzy bridge if and only if $G_{G-i}(i, j) \leq \mu_{2\gamma}(i, j)$ and $L_{G-i}(i, j) > \theta_{2\gamma}(i, j)$.

**Proof:** Suppose that $G$ is an intuitionistic fuzzy graph and $i j$ is an edge in $G$ where $G_{G-i}(i, j) \leq \mu_{2\gamma}(i, j)$ and $L_{G-i}(i, j) > \theta_{2\gamma}(i, j)$, hence $\mu_{2\gamma}(i, j) < G_G(i, j)$ and $\theta_{2\gamma}(i, j) > L_G(i, j)$ then we get $G_{G-i}(i, j) < G_G(i, j)$ & $L_{G-i}(i, j) > L_G(i, j)$, if follow $i j$ is an intuitionistic fuzzy bridge.

Suppose that $i j$ is an intuitionistic fuzzy bridge, by corollary 4.4 there exist a vertex pair $v, w \in V; i j$ is present on every $\max(v-w)$ gain path and on every $\min(v-w)$ loss path. Now, let $G_{G-i}(i, j) \geq \mu_{2\gamma}(i, j)$, thus $G_{G-i}(i, j) = G_G(i, j)$, then there is a $\max(i-j)$ gain path in $G$ (say $Q$) such as different from $i j$. If $R$ is a $\max(v-w)$ gain path in $G$, put $i j$ in $R$ by $Q$ to obtain an $v-w$ walk, since this walk contains an $v-w$ path, the path gain is greater than or equal to $G_G(u, v)$ which is not possible hence $G_{G-i}(i, j) < \mu_{2\gamma}(i, j)$.
5 Conclusion

In this paper, connectivity intuitionistic fuzzy graph, intuitionistic fuzzy cut-vertices and intuitionistic fuzzy bridges and some properties of these concepts are discussed. Our future plan is to extend our research to some other properties of path, trees in intuitionistic fuzzy graphs. Also we will study this concept to generalize some important ideas on many papers such that (see [15, 16, 17,18,19]).

References:

[1] L. A. Zadeh. Fuzzy Sets. Information and Control, Volume 8, Issue 3, June 1965, pp 338 – 353.
[2] Rosenfeld. Fuzzy graphs, fuzzy sets and their applications. Academic Press, New York, 1975, pp 77 – 95.
[3] M. O. Massa’deh & N. Gharaiibeh. Some Properties on Fuzzy Graphs. Advances in Fuzzy Mathematics, Volume 6, Number 2, 2011 , pp. 245-252.
[4] M. O. Massa’deh & A. K. Baareh. Some Contribution on Isomorphic fuzzy Graph. Advances and Applications in Discrete Mathematics. Volume 11, Issue 2, April 2013, pp 199 - 206.
[5] K. T. Atanassov. Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems. Volume 20, Issue 1, August 1986, pp 87 – 96.
[6] K. T. Atanassov & G. Gargov. Interval Valued Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, Volume 31, Issue 3, 1989, pp 343- 34.
[7] M. G. Karunambigai, O. K. Kalaivani. International Journal of Scientific and Research Publications. Volume 6, Issue 6, June 2016, pp 520 – 537.
[8] S. N. Mishra & A. Pal. Product of Interval Valued Intuitionistic Fuzzy Graphs. Annals of Pure and Applied Mathematics. Vol. 5, No.1, 2013, pp 37 – 46.

[9] S. N. Mishra & A. Pal. Regular Interval-Valued Intuitionistic Fuzzy Graphs. Journal of Informatics and Mathematical Sciences, Vol. 9, No. 3, 2017, pp 609 – 621.
[10] S. Yahya Mohamed and R. Jahir Hussain. Isomorphic on irregular Intuitionistic Fuzzy Graphs and Its Complements. IOSR Journal of Mathematics, Volume 10, Issue 2 Ver. II, (Mar-Apr. 2014), pp 149 – 154.
[11] T. Pathinathan , J. Jesintha Rosline. Vertex Degree of Cartesian Product of Intuitionistic Fuzzy Graphs. International Journal of Scientific & Engineering Research, Volume 6, Issue 3, March-2015, pp 224 – 227.
[12] Tintumol Sunny and Dr. Sr. Magie Jose. Modular and Homomorphic product on interval valued Intuitionistic Fuzzy Graphs. International Journal of Pure and Applied Mathematics. Volume 118 No. 10 2018, pp 257-276.
[13] A. Fallatah, M. O. Massa’deh, A. Alnaser. Some Contributions on Operations and Connectivity Notations in Intuitionistic Fuzzy Soft Graphs. Advances and Applications in Discrete Mathematics. Volume 23, Number 2, 2020, pp 117-138.
[14] Asad M. A. Alnaser , Wael A. AlZoubi and Mourad O. Massa’deh. Bipolar Intuitionistic Fuzzy Graphs and Its Matrices. Applied Mathematics & Information Sciences, 14, No. 2, (2020) pp 205-214.
[15] A. M. Alnaser, Y. O. Kulakov. Reliable Multipath Secure Routing In Mobile Computer Networks. Computer Engineering and Intelligent Systems, 4, (2013)pp. 8-15
[16] A. M. Alnaser. Set-theoretic Foundations of the Modern Relational Databases: Representations of Table Algebras Operations. British Journal of Mathematics & Computer Science, 4, (2014), pp 3286-3293.
[17] A. M. Alnaser. Streaming Algorithm For Multi-path Secure Routing in Mobile Networks. IJCSI International Journal of Computer Science Issues, 11, (2014), pp 112-114.

[18] Xiaoyong Zhu, Hua Zhang, A Lean Green Implementation Evaluation Method based on Fuzzy Analytic Net Process and Fuzzy Complex Proportional Assessment, International Journal of Circuits, Systems and Signal Processing, pp.646-655, Volume 14, 2020.

[19] B. Khayut, L. Fabri, M. Avikhana, The Reasonable and Conscious Understanding System of reality Under Uncertainty, International Journal of Circuits, Systems and Signal Processing, pp. 296-308, Volume 14, 2020.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0
https://creativecommons.org/licenses/by/4.0/deed.en_US