Open inflation without anthropic principle

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Abstract

We propose the mechanism of quantum creation of the open Universe in the observable range of values of Ω. This mechanism is based on the no-boundary quantum state with the Hawking-Turok instanton in the model with nonminimally coupled inflaton field and does not use any anthropic considerations. Rather, the probability distribution peak with necessary parameters of the inflation stage is generated on this instanton due to quantum loop effects. In contrast with a similar mechanism for closed models, existing only for the tunneling quantum state of the Universe, open inflation originates from the no-boundary cosmological wavefunction.

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1. Introduction

Hawking and Turok have recently suggested the mechanism of quantum creation of an open Universe from the no-boundary cosmological state [1]. Motivated by the observational evidence for inflationary models with Ω < 1 they constructed a singular gravitational instanton capable of generating expanding universes with open spatially homogeneous sections. The prior quantum probability of such universes weighted by the anthropic probability of galaxy formation was shown to be peaked at Ω ∼ 0.01. This idea, despite its extremely attractive nature, was criticized from various sides. In order to increase the amount of inflation to larger values of Ω and avoid anthropic considerations Linde [2] proposed to replace the no-boundary quantum state [1] by the tunneling one [2]. The singularity of the Hawking-Turok instanton raised a number of objections both in the Euclidean theory [3, 4, 5] and from the viewpoint of the resulting timelike singularity in the expanding Universe [6, 7]. The criticism of singular instantons was followed by attempts of their justification [8, 9] which still leave their issue open.

In any case it seems that the practical goal of quantum cosmology – generating the open Universe with observationally justified modern value of Ω, not very close to one or zero, –
has not yet been reached. The use of anthropic principle, as was recognized by the authors of [1], is certainly a retreat in theory, because by and large this principle has such a disadvantage that it can explain practically everything without being able to predict anything. The tunneling state advocated by Linde [2] (and strongly criticized in [15]) requires special supergravity induced potentials and takes place at energies beyond reliable perturbative domain with the resulting $\Omega \simeq 1$. Other works in the above series discuss conceptual issues of the Hawking-Turok proposal without offering the concrete mechanism of generating the needed $\Omega$.

On the other hand, in spatially closed context there exists a mechanism of generating the probability peak in the cosmological wavefunction at a low (typically GUT) energy scale. It does not appeal to anthropic considerations. Rather it is based on quantum loop effects [10, 17] in the model of chaotic inflation with large negative nonminimal coupling of the inflaton [18, 19, 20]. In the quantum gravitational domain the conventional expression for the no-boundary and tunneling probability distributions of the inflaton field $\rho_{NB,T}(\phi) \sim \exp[\mp I(\phi)]$ is replaced by

$$\rho_{NB,T}(\phi) \sim \exp[\mp I(\phi) - \Gamma(\phi)], \quad (1.1)$$

where the classical Euclidean action $I(\phi)$ on the quasi-DeSitter instanton with the inflaton value $\phi$ is amended by the loop effective action $\Gamma(\phi)$ calculated on the same instanton [10, 17]. The contribution of the latter can qualitatively change predictions of the tree-level theory due to the dominant anomalous scaling part of the effective action. On the instanton of the size $1/H(\phi)$ – the inverse of the Hubble constant, it looks like $\Gamma(\phi) \sim Z \ln H(\phi)$ where $Z$ is the total anomalous scaling of all quantum fields in the model. For the model of [18, 19]

$$L(g_{\mu\nu}, \phi) = g^{1/2}\left\{\frac{m_p^2}{16\pi} R(g_{\mu\nu}) - \frac{1}{2}\xi \phi^2 R(g_{\mu\nu}) - \frac{1}{2}(\nabla \phi)^2 - \frac{1}{2}m^2 \phi^2 - \frac{\lambda}{4} \phi^4\right\}, \quad (1.2)$$

with a big negative constant $-\xi = |\xi| \gg 1$ of nonminimal curvature coupling, and generic GUT sector of Higgs $\chi$, vector gauge $A_\mu$ and spinor fields $\psi$ coupled to the inflaton via the interaction term

$$L_{int} = \sum_\chi \frac{\lambda_\chi}{4} \chi^2 \phi^2 + \sum_A \frac{1}{2} g_A^2 A_\mu^2 \phi^2 + \sum_\psi f_\psi \phi \bar{\psi} \psi + \text{derivative coupling}, \quad (1.3)$$

this parameter can be very big, because it is quadratic in $|\xi|$, $Z = 6|\xi|^2 A/\lambda$ with a universal combination of the coupling constants above

$$A = \frac{1}{2\lambda} \left(\sum_\chi \lambda_\chi^2 + 16 \sum_A g_A^4 - 16 \sum_\psi f_\psi^4\right). \quad (1.4)$$

Thus, the probability peak in this model reduces to the extremum of the function

$$\ln \rho_{NB,T}(\phi) \simeq \mp I(\phi) - 3 \frac{|\xi|^2}{\lambda} A \ln \frac{\phi^2}{\mu^2}. \quad (1.5)$$
in which the $\varphi$-dependent part of the classical instanton action

$$I(\varphi) = -\frac{96\pi^2|\xi|^2}{\lambda} - \frac{24\pi(1+\delta)|\xi|m_p^2}{\varphi_0^2} + O\left(\frac{m_p^4}{\varphi_0^4}\right),$$

(1.6)

$$\delta \equiv -\frac{8\pi|\xi|m_p^2}{\lambda m_p^2},$$

(1.7)

should be balanced by the anomalous scaling term provided the signs of $(1+\delta)$ and $A$ are properly correlated with the $(\mp)$ signs of the no-boundary (tunneling) proposals. As a result the probability peak exists with parameters – mean values of the inflaton and Hubble constants and relative width

$$\varphi_I^2 = m_p^2 \frac{8\pi|1+\delta|}{|\xi|A}, \quad H^2(\varphi_I) = m_p^2 \frac{2\pi|1+\delta|}{3A},$$

(1.8)

$$\frac{\Delta \varphi}{\varphi_I} \sim \frac{\Delta H}{H} \sim \frac{1}{\sqrt{12A}} \frac{\sqrt{\lambda}}{|\xi|},$$

(1.9)

which are strongly suppressed by a small ratio $\sqrt{\lambda}/|\xi|$ known from the COBE normalization for $\Delta T/T \sim 10^{-5}$ [21, 22] (because the CMBR anisotropy in this model is proportional to this ratio [23]). This GUT scale peak gives rise to the finite inflationary epoch with the e-folding number

$$N \approx \frac{48\pi^2}{A},$$

(1.10)

only for $1+\delta > 0$ and, therefore, only for the tunneling quantum state (plus sign in (1.5)). Comparison with $N \geq 60$ necessary for $\Omega > 1$ immediately yields the bound on $A \sim 5.5$ [20] which can be regarded as a selection criterion for particle physics models [18]. This conclusions on the nature of the inflation dynamics from the initial probability peak remain true also at the quantum level – with the effective equations replacing the classical equations of motion [20].

For the proponents of the no-boundary vs tunneling quantum states this situation might seem unacceptable. According to this result the no-boundary proposal does not generate realistic inflationary scenario, while the tunneling state does not satisfy important aesthetic criterion – the universal formulation of both the initial conditions and dynamical aspects in one concept – spacetime covariant path integral over geometries. The criticism of the tunneling state in [15] is not completely justified, because as a solution of the Wheeler-DeWitt equation this state can be constructed as a normalizable (gaussian) vacuum of linearized inhomogeneous modes, see [24, 25]. But this construction, apparently, cannot be achieved by a sort of Wick rotation in the spacetime covariant path integral without breaking important locality properties [15].

In this paper we show that for the open Universe the situation qualitatively reverses: the probability peak of the quantum (one-loop) distribution of the open inflationary models exists for the no-boundary state based on the Hawking-Turok instanton. Similarly to (1.8) - (1.10) it has GUT scale parameters and the value of $N$ easily adjustable (without fine tuning
of initial conditions and anthropic considerations) for observationally justified values of $\Omega$. For this purpose in the next section we develop the slow-roll approximation technique for the Hawking-Turok instanton with a minimal inflaton. In Sect. 3 we extend this technique to the nonminimally coupled inflaton and obtain the instanton action to the subleading order in the slow-roll parameter. Remarkably, this tree-level action features large positive contribution logarithmic in the inflaton field structurally analogous to loop corrections. In Sect. 4 we discuss the difficulties with the quantum effective action due to the singularity of the Hawking-Turok instanton and establish that the dominant scaling behaviour is robust against this singularity. Finally, in Sect. 5 we calculate the most probable $H$, $N$ and $\Omega$ of the open inflation generated within the no-boundary Hawking-Turok paradigm.

2. Minimal inflaton

We assume that the reader is familiar with the construction of the Hawking-Turok instanton in a simple model with the minimally coupled inflaton field \[ I[g_{\mu\nu}, \varphi] = \int_M d^4x \sqrt{g} \left\{ -\frac{m_p^2}{16\pi} R + \frac{1}{2}(\nabla \varphi)^2 + V(\varphi) \right\} + \frac{m_p^2}{8\pi} \int_{\partial M} d^3x (3g)^{1/2} K. \] (2.1)

Here $V(\varphi)$ is the inflaton potential, $K$ is the extrinsic curvature of the spacetime boundary at the point of the singularity in the Euclidean spacetime domain. With the spatially homogeneous ansatz for the Euclidean metric ($d\Omega^2_{(3)}$ is the metric of the 3-dimensional sphere of unit radius and $a(\sigma)$ is the scale factor),

$$ds^2 = d\sigma^2 + a^2(\sigma) d\Omega^2_{(3)},$$

the Euclidean equations of motion take the form

\begin{align*}
\ddot{\varphi} + 3 \frac{\dot{a}}{a} \dot{\varphi} - V' &= 0, \\
a^3V - \frac{3m_p^2}{8\pi}(a - aa') - \frac{1}{2}a^3\varphi^2 &= 0,
\end{align*}

(2.3) (2.4)

where dots denote the derivatives with respect to the coordinate $\sigma$ and the prime denotes the derivative with respect to the inflaton scalar field. In the vicinity of the point $\sigma = 0$ where $\dot{\varphi}(0) = 0$ and $a(\sigma) \sim \sigma$ the solution of these equations can be obtained by the slow-roll expansion in powers of the gradient of the inflaton potential. In the lowest order approximation this is the constant inflaton field $\varphi_0 = \varphi(0)$ and the Euclidean DeSitter geometry

$$a = \frac{1}{H_0} \sin \theta + \delta a, \quad \varphi = \varphi_0 + \delta \varphi,$$

(2.5)

$$H_0^2 = \frac{8\pi V(\varphi_0)}{3m_p^2}, \quad \theta = H_0 \sigma,$$

(2.6)

with the effective Hubble constant $H_0$ given in terms of the constant potential $V(\varphi_0)$. 

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When the slope of the inflaton potential is not too steep one can apply the slow roll expansion in powers of the parameter

$$\varepsilon = \frac{1}{\sqrt{3\pi}} \frac{V'(\varphi_0)}{V(\varphi_0)}$$

\((|\dot{V}/HV| \simeq 3\varepsilon^2/8)\) to find the first-order approximation (see \[13\] for details)

$$\delta\varphi(\theta) = \sqrt{\frac{3}{16\pi}} m_P \varepsilon \left( \frac{1}{4} \tan^2 \frac{\theta}{2} - \ln \cos \frac{\theta}{2} \right),$$

$$\delta a(\theta) = \frac{1}{H_0} O \left( \varepsilon^2 \right).$$

As shown in \[1\], for monotonically growing potentials the scale factor of the solution, starting at \(\sigma = 0\) with the initial conditions of the above type, vanishes at some \(\sigma = \sigma_f\), \(a(\sigma_f) = 0\), and the behaviour of fields near this point have the form

$$a \simeq A(\sigma_f - \sigma)^{1/3}, \; \varphi \simeq -\frac{m_P}{\sqrt{12\pi}} \ln(\sigma_f - \sigma) + \Phi_0, \; \sigma \to \sigma_f.$$  \hfill (2.10)

It is important that the coefficient of the logarithmic singularity of the scalar field is unambiguously defined from the equations of motion, whereas the coefficients \(A\) and \(\Phi_0\) nontrivially depend on the initial condition at \(\sigma = 0\), that is on \(\varphi_0\). To find them as functions of \(\varphi_0\) we develop the perturbation expansion of the solution near \(\sigma_f\). In contrast with the slow roll expansion near \(\sigma = 0\) this is the expansion in powers of the potential \(V(\varphi)\) itself rather than its gradient, and \(\varphi\)-derivatives give the dominant contribution at this asymptotics. Then we match the both asymptotic expansions in the domain of \(\sigma\) where they are both valid (it turns out that such a domain really exists and corresponds to the range of the angular coordinate \(\theta\) in eqs. (2.3)-(2.9), \(1 \gg \pi - \theta \gg \varepsilon^{1/2}\), where the corrections (2.8)-(2.9) are small). From this match one easily finds all the unknown parameters \(A, \Phi_0, \sigma_f\) as functions of \(\varphi_0\). Omitting the details which will be published elsewhere we give here the result in the lowest order of the slow roll expansion

$$\theta_f \equiv H_0 \sigma_f \simeq \pi - \frac{2\pi^{3/2}}{\Gamma^2(1/4)} \varepsilon^{1/2},$$

$$A \simeq \left( \frac{3\varepsilon}{H_0^2} \right)^{1/3},$$

$$\Phi_0 \simeq \varphi_0 - \frac{1}{2} \frac{m_P}{\sqrt{12\pi}} \ln \left[ \frac{9H_0^2}{8\varepsilon} \right] = \varphi_0 + \frac{1}{2} \frac{m_P}{2 \sqrt{12\pi}} \ln \frac{V'(\varphi_0)}{V_0} + \text{const.}$$

The knowledge of \(A(\varphi_0)\) and \(\Phi_0(\varphi_0)\) allows one to obtain the action of the Hawking-Turok instanton. Its classical Euclidean action

$$I(\varphi_0) = 2\pi^2 \int_0^{\sigma_f} d\sigma \left\{ a^3 V - \frac{3m_P^2}{8\pi} a(1 + \dot{a}^2) + \frac{1}{2} a^3 \dot{\varphi}^2 \right\}_{\varphi(\sigma, \varphi_0), a(\sigma, \varphi_0)}$$

\hfill (2.14)
depends on $\varphi_0$ and the mechanism of this dependence originates from the behaviour of fields at the singularity. Indeed, differentiating (2.14) with respect to $\varphi_0$ one finds that the volume part vanishes in virtue of equations of motion, while integrations by parts give a typical surface term involving the Lagrangian of (2.14) and its derivatives with respect to $(\dot{\varphi}, \dot{a})$, which does not vanish at $\sigma_f$.

For $\sigma$ close to $\sigma_f$ the functional dependence of variables (2.10) on $\varphi_0$ enters through the coefficients $A, \Phi_0$ as well as through $\sigma_f$. Since $\sigma_f$ enters the fields in the combination $\sigma - \sigma_f$, the total derivative of the field takes the form $d\varphi/d\varphi_0 = \partial\varphi/\partial\varphi_0 - \dot{\varphi}(\partial\sigma_f/\partial\varphi_0)$, where partial derivative with respect to $\varphi_0$ acts only on $\Phi_0$ (and coefficients of higher powers in $(\sigma - \sigma_f)$). Similar relation holds also for the scale factor. Thus, the surface term at $\sigma_f$ equals

\[
\frac{dI(\varphi_0)}{d\varphi_0} = \left( L - \dot{\varphi} \frac{\partial L}{\partial \dot{\varphi}} - \dot{a} \frac{\partial L}{\partial \dot{a}} \right) \frac{\partial \sigma_f}{\partial \varphi_0}
+ \left[ \frac{\partial L}{\partial \dot{a}} \frac{\partial A}{\partial \varphi_0} (\sigma_f - \sigma)^{1/3} + \frac{\partial L}{\partial \varphi} \frac{\partial \Phi_0}{\partial \varphi_0} \right]_{\sigma \rightarrow \sigma_f},
\]

(2.15)

where $L$ is the Lagrangian of the Euclidean action (2.14). The first term here identically vanishes, because it is proportional to the Hamiltonian constraint (in terms of velocities). On using (2.10) - (2.13) one then finds

\[
\frac{dI(\varphi_0)}{d\varphi_0} \simeq \left( \frac{d}{d\varphi_0} + \sqrt{\frac{3}{16\pi}} \frac{d^2}{d\varphi_0^2} \right) \left( -\frac{3m_P^4}{8V(\varphi_0)} \right),
\]

whence in the first order of the slow roll expansion

\[
I(\varphi_0) \simeq \left( 1 + \sqrt{\frac{3}{16\pi}} \frac{d}{d\varphi_0} \right) \left( -\frac{3m_P^4}{8V(\varphi_0)} \right).
\]

(2.17)

This expression reproduces the result of ref.[13] obtained by indirect and less rigorous method\footnote{The calculations of [13] do not take into account the slow roll corrections to the volume part of the action.}. One can check that the second term corresponds to the contribution of the extrinsic curvature surface part of the action (2.1). A remarkable property of the obtained algorithm is that it is universal for a wide class of inflaton potentials $V(\varphi_0)$ ($V(\varphi)$ should only satisfy typical restrictions imposed by the slow roll expansion) and in a closed form expresses the result in terms of $V(\varphi)$ and its derivative.

3. Nonminimal coupling

We shall be interested in the action with the nonminimal inflaton field coupled to curvature via the $\varphi$-dependent Planck “mass” $16\pi U(\varphi)$

\[
I = \int_M d^4x \sqrt{g}^{1/2} \left\{ V(\varphi) - U(\varphi) R + \frac{1}{2} (\nabla \varphi)^2 \right\} + 2 \int_{\partial M} d^3x (\sqrt{g})^{1/2} U(\varphi) K.
\]

(3.1)
It is well known that this action can be reparametrized to the Einstein frame by special
conformal transformation and reparametrization of the inflaton field \((g_{\mu\nu}, \varphi) \rightarrow (G_{\mu\nu}, \phi)\). These transformations are implicitly given by equations \([26]\)

\[
G_{\mu\nu} = \frac{16\pi U(\varphi)}{m_P^2} g_{\mu\nu},
\]

\[
\left( \frac{d\phi}{d\varphi} \right)^2 = \frac{m_P^2}{16\pi} \left( \frac{U + 3U'^2}{U^2} \right).
\]

The action in terms of new fields

\[
\bar{I} = \int_M d^4x G^{1/2} \left\{ \bar{V}(\phi) - \frac{m_P^2}{16\pi} R(G_{\mu\nu}) + \frac{1}{2} (\nabla \phi)^2 \right\} + \frac{m_P^2}{8\pi} \int_{\partial M} d^3x (G)^{1/2} \bar{K}
\]

has a minimal coupling and the new inflaton potential

\[
\bar{V}(\phi) = \left( \frac{m_P^2}{16\pi} \right)^2 \left. \frac{V(\varphi)}{U^2(\varphi)} \right|_{\varphi=\varphi(\phi)}. \tag{3.5}
\]

The bar indicates here that the corresponding quantity is calculated in the Einstein frame of fields \((G_{\mu\nu}, \phi)\).

The above transition to the Einstein frame allows one to find the Hawking-Turok instanton for the model \((3.1)\) by transforming the results of the previous section to the nonminimal frame. Here we shall do it in the case of a big negative nonminimal coupling \(|\xi| \gg 1\):

\[
U(\varphi) = \frac{m_P^2}{16\pi} + \frac{1}{2} |\xi| \varphi^2 \tag{3.6}
\]

and quartic potential of \((1.2)\). The integration of eq.(3.3) for large values of the inflaton field, \(|\xi| \varphi^2/m_P^2 \gg 1\), expresses \(\varphi\) in terms of Einstein frame field \(\phi\)

\[
\varphi(\phi) \simeq \frac{m_P}{|\xi|^{1/2}} \exp \left[ \sqrt{4\pi/3} \left( 1 + \frac{1}{6 |\xi|} \right)^{-1/2} \frac{\phi}{m_P} \right], \tag{3.7}
\]

where the integration constant is chosen so that the above range of \(\varphi\) corresponds to \(\phi \gg m_P\). The potential in the Einstein frame equals

\[
\bar{V}(\phi) = \frac{\lambda m_P^4}{256\pi^2 |\xi|^2} \left[ 1 - \frac{1 + \delta}{4\pi} \frac{m_P^2}{|\xi| \varphi^2} + ... \right]_{\varphi=\varphi(\phi)}, \tag{3.8}
\]

where we have retained only the first order term in \(m_P^2/|\xi| \varphi^2\). In view of (3.7), for large \(\phi\) this potential exponentially approaches a constant and satisfies a slow roll approximation with the expansion parameter

\[
\varepsilon = \frac{m_P}{\sqrt{3\pi}} \frac{\bar{V}'(\phi_0)}{\bar{V}(\phi_0)} \simeq \frac{1 + \delta}{3\pi} \left( 1 + \frac{1}{6 |\xi|} \right)^{-1/2} \frac{m_P^2}{|\xi| \varphi_0^2} \ll 1. \tag{3.9}
\]

This justifies the above choice of range for the values of the inflaton field. In this range the Hawking-Turok instanton is described by the equations of the previous section for the
Einstein frame fields $\bar{a}(\sigma)$ and inflaton $\phi(\bar{\sigma})$. Here $\bar{\sigma}$ is the coordinate in the spacetime interval $d\bar{s}^2 = d\sigma^2 + \bar{a}^2(\bar{\sigma}) d\Omega_{(3)}^2$ of the Einstein frame metric. In view of (3.2) these intervals are related by the equation $d\bar{s}^2 = (16\pi U/m_P^2) ds^2$, so that the coordinates and scale factors of both frames are related by $d\sigma = \sqrt{16\pi U/m_P^2} ds$ and $\bar{a} = \sqrt{16\pi U/m_P^2} a$. Combining these equations with the asymptotic behaviour of the Einstein frame fields at $\bar{\sigma} \to \bar{\sigma}_f$ (eqs. (2.10)-(2.13) rewritten for $\bar{a}(\bar{\sigma}), \phi(\bar{\sigma})$ with the potential $V(\phi)$) one can easily find the behaviour of fields in the nonminimal frame. We give it in the limit of large $|\xi|$:}

$$a(\sigma) \simeq \frac{4}{m_P} \left( \frac{m_P}{\varphi_0} \right)^{1+2\epsilon} \left( \frac{1 + \delta}{4\pi\lambda} \right)^{1/4 + \epsilon/2} \left[ m_P(\sigma_f - \sigma) \right]^{1/2 - \epsilon}, \quad (3.10)$$

$$\varphi(\sigma) \simeq m_P \left[ \frac{\varphi_0}{m_P} \right]^{1/2 + 3\epsilon} \left( \frac{1 + \delta}{4\pi\lambda} \right)^{1/8 - 3\epsilon/4} \left[ m_P(\sigma_f - \sigma) \right]^{-1/4 + 3\epsilon/2}, \quad (3.11)$$

$$\epsilon \equiv \frac{1}{2} \sqrt{1 + 1/6 |\xi|} - \frac{1}{96 |\xi|} \ll 1. \quad (3.12)$$

In contrast with the minimal coupling we now have the power singularities for both fields. For large $|\xi| \gg 1$, in particular, they look like $a \sim (\sigma_f - \sigma)^{1/2}$ and $\varphi \sim (\sigma_f - \sigma)^{-1/4}$. The inflaton singularity is thus stronger than the logarithmic one in the minimal case, while that of the scale factor is softer ($1/2 - \epsilon \geq 1/3$). Note, by the way, that the coefficient of strongest singularity of the scalar curvature is also suppressed by $1/|\xi|$, $R \sim (1/|\xi|)(\sigma - \sigma_f)^{-2}$. This property can be qualitatively explained by the fact that the effective gravitational constant $(m_P^2 + 8\pi|\xi|\varphi_0^2)^{-1}$ tends to zero at the singularity.

The classical action can also be easily calculated in the Einstein frame $I(\varphi_0) = I(\phi_0)$ with the aid of eq.(2.17). Taking into account that

$$\frac{3m_P^4}{8V(\varphi_0)} \simeq 96\pi^2|\xi|^2 \frac{\lambda}{\lambda} + \frac{24\pi(1 + \delta)|\xi| m_P^2}{\varphi_0^2} + \frac{3}{2} \frac{(1 + 2\delta)^2}{\lambda} \left( \frac{m_P^2}{\varphi_0^2} \right)^2 + \ldots \quad (3.13)$$

and using the relation

$$\sqrt{\frac{3}{16\pi}} m_P \frac{d}{d\phi} \simeq \frac{1}{2} \left( \frac{1}{6 |\xi|} \right)^{-1/2} \varphi \frac{d}{d\varphi}, \quad (3.14)$$

one finds that the surface term at the singularity (the term with the derivative in (2.17)) almost cancels the first subleading term in the expansion (3.13) and inverts the sign of the second order term

$$\left( 1 + \sqrt{\frac{3}{16\pi}} m_P \frac{d}{d\phi_0} \right) \left( - \frac{3m_P^4}{8V(\varphi_0)} \right) \approx -\frac{96\pi^2|\xi|^2}{\lambda} - \frac{2\pi(1 + \delta) m_P^2}{\varphi_0^2}$$

$$+ \frac{3}{2} \frac{(1 + 2\delta)^2}{\lambda} \left( \frac{m_P^2}{\varphi_0^2} \right)^2 + \ldots \quad (3.15)$$

$^2$Note that the relation (3.2) holds in one coordinate system covering the both conformally related spacetimes, while the coordinates $\bar{\sigma}$ and $\sigma$ are essentially different.
Thus, in contrast with the closed model, these terms (of different powers in $m_P^2/\varphi^2$) are of the same order of magnitude in $1/|\xi|$. Later we shall see that at the probability maximum $m_P^2/\varphi^2 \gg 1$ (although $m_P^2/|\xi|\varphi^2 \sim \varepsilon \ll 1$) which means that the dominant effect comes from the third term of (3.15). This term is however not reliable unless we take into account the complete subleading order of the slow roll expansion in $\varepsilon$. Obtaining it is rather cumbersome and is not so universal as in the lowest order, because the result depends on a particular form of the inflaton potential. Without going into details which will be published elsewhere we present the result for our model:

$$\theta_f \equiv H_0 \sigma_f \simeq \pi - \frac{2\pi^{3/2}}{\Gamma^2(1/4)} \varepsilon^{1/2} - \frac{5\Gamma^2(1/4)}{48\pi^{1/2}} \varepsilon^{3/2},$$

$$A^3 \simeq \frac{3\varepsilon}{H_0^2} - \frac{11\varepsilon^2}{4H_0^2},$$

$$\Phi_0 \simeq \varphi_0 - \frac{1}{2\sqrt{12\pi}} \ln \left[ \frac{9H_0^2}{8\varepsilon} \right] - \frac{1}{2} \sqrt{\frac{3}{16\pi}} m_P \varepsilon \left( \ln \frac{\varepsilon}{8} + \frac{35}{18} \right),$$

where $\varepsilon = [(1 + \delta)/3\pi]m_P^2/\varphi_0^2$. Using these expressions for $A(\varphi_0)$ and $\Phi(\varphi_0)$ one obtains the second order approximation for the Hawking-Turok action

$$I_{HT}(\varphi) = -\frac{96\pi^2|\xi|^2}{\lambda} - \frac{2\pi(1 + \delta) m_P^2}{\lambda} \frac{1}{\varphi_0^2}$$

$$+ \frac{(1 + \delta)^2}{\lambda} \left( \frac{m_P^2}{\varphi_0^2} \right)^2 \left[ \frac{3(1 + 2\delta)}{2(1 + \delta)^2} - \frac{22}{3} + 2\ln \left( \frac{24\pi|\xi|\varphi_0^2}{m_P^2(1 + \delta)} \right) \right] + O \left( \frac{m_P^6}{|\xi|\varphi_0^6} \right).$$

Due to big $|\xi|$ it contains a large but slowly varying (in $\varphi$) logarithmic coefficient. The positive coefficient of this logarithmic term actually follows from the sign of $\ln \cos(\theta/2)$ in the equation (2.8) for $\delta \varphi$ above and, thus, it is pretty well fixed. This sign will have important consequences for quantum creation of the open Universe. Note, by the way, that the logarithmic structure of the result resembles the behaviour of Coleman-Weinberg loop effective potentials (up to inversion of $\varphi$), even though this term is entirely of a tree-level origin. Thus, the classical theory somehow feels quantum structures when probing Planckian scales near the singularity. This sounds rather coherent with recent results on holographic principle in string theory when the tree-level theory in the bulk of spacetime generates quantum theory on the boundary surface [27].

4. Quantum corrections

The most vulnerable point of the Hawking-Turok instanton is the construction of quantum corrections on its singular background. Although the classical Euclidean action is finite, the quantum part of the effective action involving the higher order curvature invariants is infinite, because their spacetime integrals are not convergent at the singularity. At least
naively, this means that the whole amplitude is either suppressed to zero or infinitely diverges indicating strong instability. Clearly, a self-consistent treatment should regularize the arising infinities due to the back reaction of the infinitely growing quantum stress tensor.

The result of such a self-consistent treatment is hardly predictable because we do not yet have for it an exhaustive theoretical framework. This framework might include fundamental stringy structures underlying our local field theory, which are probed by Planckian curvatures near the singularity. However, even without the knowledge of this fundamental framework it is worth considering usual quantum corrections due to local fields on a given singular background. This might help revealing those dominant mechanisms that are robust against the presence of singularities and their regulation due to back reaction and fundamental strings.

These quantum corrections can be devided into two main categories – nonlocal contributions due to massless or light degrees of freedom and local contributions due to heavy massive fields [28]. The effects from the first category can be exactly calculable when they are due to the conformal anomaly of the conformal invariant fields. For the Hawking-Turok instanton this calculation can be based on the (singular) conformal transformation mapping its geometry to the regular metric $\tilde{d}s^2$ of the half-tube $R^+ \times S^3$,

$$ds^2 = a^2(\sigma(X)) d\tilde{s}^2, \quad d\tilde{s}^2 = dX^2 + d\Omega^2_{(3)},$$

with the conformal coordinate $X = \int_0^{\sigma'} d\sigma'/a(\sigma')$, $0 \leq X < \infty$. With this conformal decomposition of the metric the effective action $\Gamma[g_{\mu\nu}]$ can be represented as a sum of the finite effective action of $\tilde{g}_{\mu\nu}$, $\Gamma[\tilde{g}_{\mu\nu}]$, and $\Delta \Gamma[\tilde{g}_{\mu\nu}, a]$ – the anomalous action obtained by integrating the known conformal anomaly along the orbit of the local conformal group joining $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. The anomalous action $\Delta \Gamma[\tilde{g}_{\mu\nu}, a]$ is known for problems without boundaries [23, 30, 31]. For a singular conformal factor $a^2(\sigma(X))$ the bulk part of this action is divergent, but the question of its finiteness is still open, because in problems with boundaries the conformal anomaly should have surface (simple and double layer) contributions that might lead to finite anomalous action on the Hawking-Turok instanton [32].

Fortunately, the problem with large nonminimal coupling of the inflaton falls into the second category of problems – local effective action of heavy massive fields. Due to the Higgs mechanism for all matter fields interacting with inflaton by (1.3) their particles acquire masses $m^2 \sim \varphi^2$ strongly exceeding the spacetime curvature $R \sim \lambda \varphi^2/|\xi|$ [18, 19, 20]. The renormalized effective action expanded in powers of the curvature to mass squared ratio $R/m^2 \sim \lambda/|\xi| \ll 1$ for generic spacetime background has the following form of the local Schwinger-DeWitt expansion [33, 35, 20]

$$\Gamma_{1{-}\text{loop}} = -\frac{1}{32\pi^2} \int d^4x g^{1/2} \text{tr} \left\{ \frac{1}{2} \left( \frac{3}{2} - \ln \frac{m^2}{\mu^2} \right) m^4 \hat{1} + \left( 1 - \ln \frac{m^2}{\mu^2} \right) m^2 \hat{a}_1(x, x) - \ln \frac{m^2}{\mu^2} \hat{a}_2(x, x) + \sum_{n=1}^{\infty} \frac{(n-1)!}{m^{2n}} \hat{a}_{n+2}(x, x) \right\}.$$  

(4.2)
Here tr denotes the trace over isotopic field indices, hats denote the corresponding matrix structures in vector space of quantum fields and $\hat{a}_n(x, x)$ are the Schwinger-DeWitt coefficients. The latter can be systematically calculated for generic theory as spacetime invariants of growing power in spacetime and fibre bundle curvatures [33, 34, 35, 36].

The situation with this expansion on a singular instanton of Hawking and Turok is also not satisfactory – all integrals of curvature invariants starting with $\hat{a}_2(x, x)$ (quadratic in the curvature and higher) diverge at the singularity. Fortunately, the lessons from the asymptotic theory of semiclassical expansion teach us that the lowest order terms can be trusted as long as they are well defined. In our case this is the term quartic in masses with the logarithm. But this term is exactly responsible for the dominant contribution to the anomalous scaling (1.5) quadratic in $|\xi|$. This term is dominating the quantum part of effective action, while the others, although being divergent at the singularity, are strongly suppressed by powers of $1/|\xi|$. With the assumption that the back reaction of quantum stress tensor regulates these divergences, one can conjecture that the quantum effective action on the Hawking-Turok instanton in our model is still dominated by the anomalous scaling term of eq.(1.5). In the next section we analyze the consequences of this conjecture.

5. Energy scale, $N$ and $\Omega$ of open inflation

Consider the no-boundary and tunneling distribution functions (1.5) with the Hawking-Turok action (3.19) replacing $I(\varphi)$ and the anomalous scaling term. A crucial difference from the case of closed cosmology is that the $\varphi$-dependence of $I_{HT}(\varphi)$ is dominated now by $|\xi|^0$-term which contains due to a big slowly varying logarithm a large positive contribution quartic in $m_P/\varphi$. Therefore, for large $|\xi| \gg 1$ the maximum of this distribution exists only for the no-boundary state. The corresponding peak is located at $\varphi = \varphi_I$, where $\varphi_I$ solves the equation

$$\varphi_I^2 \approx 2 \frac{m_P^2}{|\xi|} \left[ \frac{1}{3A} \ln \frac{2\pi|\xi|\varphi_I^2}{m_P^2} \right]^{1/2}, \ |\delta| \ll 1. \ 	ag{5.1}$$

(To simplify equations we consider here and in what follows a small value of $|\delta|$ and use approximate value of the numerical combination $3/2 - 20/3 \simeq -2 \ln(18)$.) The solution of (5.1) for small $A$ corresponds to the following parameters of the probability peak – mean value, the Hubble constant $H(\varphi) \simeq \sqrt{\lambda/12|\xi|}$ and quantum dispersion $\Delta \varphi \equiv [-d^2 \ln \rho(\varphi_I)/d\varphi_I^2]^{-1/2}$

$$\varphi_I^2 \approx 2 \frac{m_P^2}{|\xi|} \left[ \frac{1}{3A} \ln \frac{8\pi}{\sqrt{54A}} \right]^{1/2}, \quad H^2(\varphi_I) \simeq m_P^2 \frac{\lambda}{|\xi|^2} \frac{1}{6} \left[ \frac{1}{3A} \ln \frac{8\pi}{\sqrt{54A}} \right]^{1/2}, \quad \Delta \varphi / \varphi_I \sim \Delta H / H \sim \frac{1}{\sqrt{27A}} \sqrt{\lambda} / |\xi|. \tag{5.2}$$

Similarly to the closed model, these parameters are suppressed relative to the Planck scale by a small dimensionless ratio $\sqrt{\lambda}/|\xi|$ known from the COBE normalization $\sqrt{\lambda}/|\xi| \sim \Delta T/T \sim $
10^{-5}. However, in contrast with the closed model, this peak has a more complicated dependence on the parameter $A$.

To analyze the inflationary scenario generated by this peak we first use the classical equations of motion. For the model (3.1) they were considered in much detail in [20]. The slope of the potential (3.8) is positive for $\delta > -1$ which implies the finite duration of the inflationary epoch with slowly decreasing inflaton only in this range of $\delta$. The inflationary e-folding number in this case is approximately given by the equation

$$N \simeq \int_{\phi_i}^{\phi_f} d\phi \frac{3H^2(\phi)}{|F(\phi)|},$$

(5.3)

where $F(\phi)$ is the rolling force in the inflaton equation of motion $\ddot{\phi} + 3H\dot{\phi} - F(\phi) = 0$, $F(\phi) = (2VU' - UV')/(U + 3U'^2) \sim -\lambda m_p^2 (1 + \delta)\phi/48\pi \xi^2$. The integration in (5.3) gives

$$N \simeq 12\pi \left[ \frac{1}{3A} \ln \frac{8\pi}{\sqrt{3}\sqrt{A}} \right]^{1/2}.\quad (5.4)$$

Comparison of this result with the e-folding number, $N \sim 60$, necessary for generating the observable density $\Omega$, $0 < \Omega < 1$, not very close to one or zero, immediately gives the bound on $A$

$$A \sim \frac{48\pi^2}{N^2} \ln \frac{2N}{9} \sim 0.3.\quad (5.5)$$

This bound justifies the validity of the slow roll approximation – the expansion in powers of $m_p^2/4\pi |\xi|^2 \sim \ln N/N$ (see eqs.(3.8) and (3.17)).

These conclusions are based on classical equations of inflationary dynamics. The latter should certainly be replaced by effective equations for mean fields to have reliable answers within the same accuracy as the calculation of the one-loop distribution function for the initial value of the inflaton. Since quantum effects qualitatively change the tree-level initial conditions, one should expect that they might strongly influence the dynamics as well. Effective equations of motion for the model (3.1) were obtained in our recent paper [20], but according to the discussion of the previous section they are not, strictly speaking, applicable here. This is because the Hawking-Turok instanton background does not satisfy the condition of the local Schwinger-DeWitt expansion\(^3\). It does not satisfy this condition globally, in the vicinity of the singularity, but the open inflationary Universe lies inside the light cone originating from the instanton pole antipodal to the singularity. Therefore, if we restrict ourselves with the local part of effective equations, then for this part we can use our old results [20]. The influence of the spatially remote singularity domain is mediated by nonlocal terms. These terms strongly depend on the boundary conditions at the singular boundary and are beyond the control of the local Schwinger-DeWitt approximation. Within the same reservations as those concerning the finiteness of quantum effects on this instanton

\(^3\)For closed cosmology with no-boundary or tunneling quantum states the slow roll approximation guarantees the validity of the local Schwinger-DeWitt expansion that was used in [20] for the derivation of the effective equations.
(and the validity of the Hawking-Turok model as a whole) we can neglect nonlocal terms and use only the local part of effective equations.

There are two arguments in favour of this approximation. Firstly, it is very likely that the nonlocal contribution of the singularity is suppressed by inverse powers of $|\xi| \gg 1$. At least naively, these effects are inverse to the size of the Universe given by the Hubble constant in (5.2) and also can be damped by $1/|\xi|$ in the same way as for the singular part of the scalar curvature. Secondly, if we restrict ourselves with a limited spatial domain of the very early open Universe (close to the tip of the light cone originating from the regular pole of the Hawking-Turok instanton), then these nonlocal terms do not contribute at all in view of causality of effective equations, because at early moments of time this domain is causally disconnected from singularity.

Local quantum corrections in effective equations depend only on local geometry of the quasi-DeSitter open Universe. They boil down to the replacement of the classical coefficient functions $(V(\varphi), U(\varphi))$ of the model (3.1) by the effective ones calculated in [20] for a wide class of quantum fields coupled to the inflaton in the limit of big $|\xi| \gg 1$. It remains to use these functions in classical equations and study the inflation dynamics starting from the initial value of the inflaton (5.1). It turns out that unlike in the closed model (where the quantum terms were of the same order of magnitude as classical ones) the quantum corrections are strongly suppressed by the slow-roll parameter $\sqrt{A}/4\pi \sim \ln N/N$ already at the start of inflation. In particular, the effective rolling force differs from the classical one above by negligible correction

$$F_{\text{eff}}(\varphi) \simeq -\frac{\lambda m_\varphi^2 (1 + \delta)}{48\pi \xi^2} \varphi \left(1 + \left[\frac{A}{48\pi^2} \ln \frac{8\pi}{\sqrt{54A}}\right]^{1/2} \frac{\varphi^2}{\varphi_f^2}\right),$$

(5.6)

For comparison, in the closed model the second (quantum) term in curly brackets enters with a unit coefficient of $\varphi^2/\varphi_f^2$, see eqs.(6.7) and (6.10) of [20]). This quantum correction gives inessential contribution to the duration of the inflationary stage (5.4) and thus does not qualitatively change the above predictions and bounds. The smallness of quantum corrections (roughly proportional to $A/32\pi^2$) can be explained by stronger bound on $A \sim 0.3$ (cf. $A \leq 5.5$ in the closed model [20]) and another dependence of $\varphi_f$ on $A$.

Thus, the no-boundary quantum state on the Hawking-Turok instanton in the model with large nonminimal curvature coupling generates open inflationary scenario compatible with observations and, in particular, capable of producing the needed value of $\Omega$. No anthropic considerations or fine tuning of initial conditions is necessary to reach such a final state of the Universe. The only fine tuning we get is the bound on the parameter $A$ of the matter field sector (5.3) and the estimate of the ratio $\sqrt{\lambda}/|\xi| \sim 10^{-5}$ based on the normalization from COBE which looks as a natural determination of coupling constants of Nature from the experiment. The mechanism of such quantum birth of the Universe is based on quantum effects on the Hawking-Turok instanton, treated within semiclassical loop expansion. The

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4This argument is rigorous but somewhat macabre for the inhabitants of this domain, because after a while they will suffer a fatal influence of fields propagating from the singularity.
validity of this expansion is in its turn justified by the energy scale of the phenomenon (5.2) which belongs to the GUT domain rather than the Planckian one.

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