QCD evolution with longitudinal fields and heavy quarks

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QCD evolution equations that naturally include longitudinal (non-propagating) fields and heavy quarks are derived. We start with the integral equations of quantum field kinetics and obtain the master equations, similar to DGLAP evolution equations after several consecutive approximations. We demonstrate that in their primary form, the evolution equations include a new element, feedback via longitudinal fields, leading to a low-$x$ enhancement in the $e-p$ DIS cross section. We show that the structure function $F_L$ is very sensitive to the dynamics of the longitudinal fields and that the heavy quarks in evolution equations make this effect even more pronounced.

I. INTRODUCTION

In this paper we address the dynamics of longitudinal fields and heavy quark fields in high-energy processes, keeping in mind practical importance of these questions for the theory of ultra-relativistic heavy ion collisions. One of the main difficulties of this (not yet existing) theory is the impossibility of describing the initial state (two nuclei), the transient process (which, hopefully, includes the QGP stage) and the hadrons of the final state using the standard language of scattering theory. In fact, even in the much simpler problem of the $pp$-collisions, one has to make certain model assumptions (like, $i.g.$, the parton model) in order to put the problem in the form required by the scattering theory. Unfortunately, for nuclear collisions, the situation is much worse, since one cannot even think about consistent decomposition of the whole process into the hard and the soft parts; the factorization scheme is inapplicable. The most unfortunate circumstance is that the structure functions of the protons, the partonic substitutes for the unknown wave functions, cannot be used any more.

Two possible ways to cure the problem were suggested recently. McLerran and Venugopolan have started with a classical model of a large nucleus in the infinite momentum frame (IMF) [1]. This model has been gradually improved by Jalilian-Marian, Kovner, McLerran, Venugopolan and Weigert by accounting for the small quantum fluctuations in the strong "external" classical field [2,3]. A remarkable yield of this ongoing study is an understanding that the so-called low-$x$ enhancement may be an effect of the classical field itself [1].

The idea of a geometrical and dynamical similarity between all deeply inelastic high energy processes, including the inelastic high energy $ep$-scattering, is the guide for our study [5,6]. The method of Quantum Field Kinetics (QFK) [5] allows one to show that the dynamics of all inclusive processes is sequential on the real time scale and ends at the moment of measurement. Consequently, one can avoid factorization in the derivation of the evolution equations [6]. The best prospect of this approach is connected with the hope of extracting from the DIS data more rich information ($e.g.$, information about the dynamics of the static fields,) than is usually done with the aid of the DGLAP evolution equations of the parton model [6,7]. The causal character of the QCD evolution which is clearly visible due to the QFK technique does imply such an opportunity. As a result, one may treat the hadronic collision not as the scattering of partons, but as the interaction of the fields from the colliding hadrons or nuclei. In this paper, we continue the study of Ref. [6] and show that the presence of the static (not propagating) fields in the evolution equations leads to a steep power-like behavior of the inelastic $ep$-cross section at low $x$.

The QFK approach to deep inelastic processes allows us to address the question of the accuracy of the evolution equations as well. The complexity of this problem in the calculations based on the operators product expansion (OPE) is well understood. Ranking the operator functions by their twists and the coefficient functions by the order of their perturbative expansion, the OPE employs two essentially different expansions, and their interplay is difficult to keep under the control ($e.g.$, Ref. [10]). Our approach is different; we descend from the Schwinger-Dyson equations all the way to the DGLAP equations and study what of physical importance is lost on this way. We show that the dynamics of the classical (longitudinal) fields is an unavoidable partner of the propagating (transverse) fields in the QCD evolution. As a result, the equations for the transverse fields acquire a kind of feed-back via the inherently
inseparable longitudinal fields. By examination, this new element may alone lead to the low-\(x\) enhancement of the inclusive cross-section. One does not even need to have a classical source like the valence quarks of the McLerran-Venugopalan model. In order to obtain the master equations, like DGLAP, one has to exclude the longitudinal fields from the evolution equations by the brute force. We argue that this step is equivalent to the introduction of the parton picture. It may be consistent only if the so-called factorization scale, \(\mu_0^2\), is a measured parameter.

In their full form, the evolution equations are nonlocal in the transverse direction. In conjunction with the collinear problems of null-plane dynamics, this leads to severe infrared singularities, which are easily regulated physically, but at the price of abandoning the null-plane dynamics as a technical tool. In order to obtain the master equations, any evidence of the non-locality has to be eliminated, once again, by a brute force. Thus, we solve (or at least address) the problem of the accuracy of the DGLAP equations by the extension of the system of evolution equations beyond their standard mathematical form and by a revision of their physical content.

The way we derive and study the evolution equations allows us to incorporate the massive quark fields into the QCD evolution. This is a very real problem since a vast set of the current and forthcoming experiments requires a theoretical understanding of the role of heavy quarks in high energy processes. At HERA energies, the range of the accessible momentum transfer between the electron and the proton is very wide and the \(c\) and \(b\)-mesons are quite frequently met among the secondaries, affecting the measured cross section of deep inelastic scattering (DIS). A significant multiplicity of charmed quarks is expected in heavy-ion collisions at RHIC energies. Their evolution may carry important signatures if the QGP was a part of the entire scenario. Here, the theoretical calculations rely heavily on the structure functions which are taken from DIS data. The definition of the heavy flavor DIS structure functions (or sources) should be identical to those of gluons and light sea quarks. Only in this way can one consistently introduce the notion of intrinsic charm and beauty, and describe their excitations unambiguously. The contribution of heavy flavors to the \(ep\)-DIS cross section should naturally die out when we move in the direction of lower \(Q^2\) along the evolution scale. However, the status of heavy quarks in QCD evolution equations has remained uncertain for over two decades. To handle this problem, one may wish to rely on a minimal extension of the renormalization group method and account for the heavy quarks only inside the fermion loop of the gluon self-energy, as is done in Ref. \[1\].

In these calculations, various regions (below and above the threshold) were treated in a different manner: with different numbers of active flavors, and different renormalization prescriptions. Special subtraction schemes were introduced at the threshold. However, this method does not eliminate the problem itself: special separate treatment of the intrinsic charm and beauty, the wee parton inside the proton, and the extrinsic charm and beauty, the heavy quarks created in \(\gamma^*g\)-interaction. As it will be seen later, both these treatments of the massive quark fields in QCD evolution are incomplete and it is expedient to consider this problem together with the problem of the evolution of static fields.

II. DEEP INELASTIC ELECTRON-PROTON SCATTERING

Our approach to high-energy collisions amounts to the elaboration of a way to incorporate (to extract and to use) the dynamical information about the transient processes which take place during the collision. Therefore, we start with the simplest example of inelastic \(ep\)-scattering where at least one of the participants is structureless and pay special attention to the procedure of measurement. We divide this section into the three parts. For the sake of completeness we begin with a brief definition of the DIS cross-section in terms of quantum field kinetics and define the null-plane variables which will be used for all the following calculations. Next, we discuss the temporal sequence of dynamical processes during inclusive measurements. Finally, we derive the explicit formula of the measurement which does not imply the parton decomposition of the proton and accounts for the static component of the proton’s wave function.

A. Observables for DIS

Let us consider a collision process with the parameters of only one final-state particle explicitly measured. Let this particle be the electron with momentum \(\mathbf{k}'\) and spin \(\sigma'\). The deep inelastic electron-proton scattering is an example of such an experiment. All the vectors of the final states which are accepted into the data ensemble are of the form, \(a_{\nu'}(\mathbf{k}')|X\rangle\), where the vectors \(|X\rangle\) form a full set. The initial state consists of an electron with momentum \(\mathbf{k}\) and spin \(\sigma\) and some other particle or composite system carrying quantum numbers \(P\). Thus, the initial state vector is \(a_{\nu}(\mathbf{k})|P\rangle\).

The inclusive transition amplitude reads as \(\langle X|a_{\nu}(\mathbf{k}')S\ a_{\nu}(\mathbf{k})|P\rangle\) and the inclusive momentum distribution of the final state electron is the sum of the squared moduli of these amplitudes over the full set of non-controlled states \(|X\rangle\). Therefore, we obtain the following formula,
which is just an expectation value of the Heisenberg operator of the number of final state electrons over the initial state. Since the state $|P\rangle$ contains no electrons, one may commute electron creation and annihilation operators with the $S$-matrix and its conjugate, $S^\dagger$, pulling the Fock operators $a$ and $a^\dagger$ to the right and to the left, respectively. Let $\psi_{k\sigma}(x)$ be the one-particle wave function of the electron. Then the procedure results in the following formula,

$$\frac{dN_e}{dk'} = \frac{1}{2} \sum_{\sigma\sigma'} \int dx dx' dy dy' \psi_{k\sigma}(x) \psi_{k'\sigma'}(x') (P| \frac{\delta^2}{\delta \Psi(x) \delta \Psi(y)} (\frac{\delta S}{\delta \Psi(y')}, \frac{\delta S}{\delta \Psi(x')}) |P) \psi_{k\sigma}(y) \psi_{k'\sigma'}(y') \right).$$  \hspace{5cm} (2.2)$$

Introducing the Keldysh convention about the contour ordering \[\llbracket\rrbracket\], we may rewrite this expression as

$$\frac{dN_e}{dk'} = \frac{1}{2} \sum_{\sigma\sigma'} \sum_{AB} (-1)^{A+B} \int dx dx' dy dy' \psi_{k\sigma}(x) \psi_{k'\sigma'}(x') (P| \frac{\delta^4 S_{\epsilon}}{\delta \Psi(x) \delta \Psi(y) \delta \Psi(x') \delta \Psi(y')}, |P) \psi_{k\sigma}(y) \psi_{k'\sigma'}(y') \right).$$  \hspace{5cm} (2.3)$$

where $S_{\epsilon} = S^\dagger S$. The electron couples only to the electromagnetic field. Therefore, to the lowest order in electromagnetic coupling $e$,  \begin{align*}
\frac{dN_e}{dk'} &= e^2 \frac{1}{2} \sum_{\sigma\sigma'} \int dx dy \psi_{k\sigma}(y) \psi_{k'\sigma'}(y) (P| A(x) A(y) |P) \psi_{k\sigma}(y) \psi_{k'\sigma'}(x) \right),
\end{align*}

where $A(x)$ is the Heisenberg operator of the electromagnetic field. Already at this very early stage of the calculations, the answer has a clear physical interpretation. Since only the final state electron is measured, the probability of the electron scattering is entirely defined by the electromagnetic field produced by the rest of the system. The final answer can be obtained either by using the equations of QFK \[\llbracket\rrbracket\], or one may take short cut and iterate Eq. (2.4) by means of the Yang-Feldman equation, $A(x) = \int d^4 y \Delta_{ret}(x,y) j(y), \text{ where } j(y)$ is the Heisenberg operator of the electromagnetic current and $\Delta_{ret}(x,y)$ is the retarded propagator of the photon.

Summation over the electrons spins brings in the leptonic tensor $L_{\mu\nu}(k,k')$. If $q = k - k'$ is the space-like momentum transfer, then the DIS cross section is given by

$$\epsilon_0 \frac{d\sigma}{dk'} = \frac{i e^2 \alpha}{4\pi} \frac{L_{\mu\nu}(k,k')}{(kP)} \Delta_{\mu\nu}(q) W_{\alpha\beta}(q) \Delta_{\alpha\beta}(q),$$  \hspace{5cm} (2.5)$$

where $W_{\mu\nu}(q)$ is the standard Bjorken notation for the correlator of two electromagnetic currents,

$$W_{\mu\nu}(q) = \frac{2V_{tq}P_0}{4\pi} [-i \pi_{10}(q)] .$$  \hspace{5cm} (2.6)$$

normalized in a way which provides correspondence to the parton model, and $\pi_{10}^{\mu\nu}(x,y) = -i(P|j^\mu(x)j^\nu(y)|P)$ is the electromagnetic polarization tensor, correlator of the currents, which are the sources of the field which has scattered the electron. We accept the standard decomposition of the correlator $W_{\mu\nu}$ in momentum space,

$$W_{\mu\nu}(q) = -e^{\mu\nu} W_L \frac{2x_{BJ}}{2x_{BJ} M^2} \right),$$  \hspace{5cm} (2.7)$$

where $\nu = qP$, $Q^2 = -q^2 > 0$, $x_{BJ} = Q^2/2\nu$ and

$$e^{\mu\nu} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}; \quad \zeta^{\mu\nu} = -g^{\mu\nu} + \frac{P^\mu q^\nu + q^\mu P^\nu}{\nu} - q^2 \frac{P^\mu P^\nu}{\nu^2} .$$  \hspace{5cm} (2.8)$$

We shall perform all computations using the infinite momentum frame fixed by the null-plane vector $n^\mu, n^\nu = (1,0,0,-1)$, $n^2 = 0$. It defines the “$+$”-components of the Lorentz vectors, $a^+ = 2a_+ = a_0 + a_3$; $a^- = 2a_- = a_0 - a_3$. In the infinite momentum frame, the 4-vector of the proton’s momentum has components $P^+ = (P^+/2, 0, P^+/2), P^- = P^0 - P^3 = 0$. The vector of the momentum transfer has the components, $q^\mu = (\nu/P^+, q_0, -\nu/P^+), q^+ = 0, q^- = 2\nu/P^+.$

Instead of the invariant $W_2$, we shall use the mass-independent structure function $F_2(x_{BJ}, Q^2) = \nu W_2/M^2$, which is calculated via the equation

$$c_2 = W^{\mu\nu} n^\mu_n n^\nu = \frac{(P^+)^2 F_2}{\nu} .$$  \hspace{5cm} (2.9)$$

The longitudinal structure function, $F_L(x_{BJ}, Q^2) = W_L$, should be calculated in accordance with

$$3F_L = 2x_{BJ} c_1 + 2F_2; \quad c_1 = W^{\mu\nu} g_{\mu\nu} .$$  \hspace{5cm} (2.10)$$
B. Causality and temporal order in DIS

The position of the two photon propagators, \( \Delta_{\text{ret}}(q) \) and \( \Delta_{\text{adv}}(q) \) (since \( \Delta_{\text{adv}} \) enters with inverted space-time arguments, both are retarded propagators) in Eq. (2.3) reflects the two major aspects of causality in relativistic quantum mechanics. First, any statement concerning the time ordering is in manifest agreement with the light cone boundaries. Second, the dynamical information about the quantum-mechanical evolution is read out only after the evolution is interrupted by the measurement (the wave function has collapsed). Therefore, the standard inclusive e-p DIS delivers information about quantum fluctuations associated with the proton. The space-like photon which scatters the electron “belongs” to the proton and is, in fact, its part. The method of quantum field kinetics allows one to continue this line of reasoning by employing the basic definition of the electromagnetic polarization tensor. Then, neglecting any corrections to the electromagnetic vertex, we may rewrite Eq. (2.6) in the following way,

\[
W^{\mu\nu}(q) = \sum_f e_f^2 \frac{2V_{\text{lab}}P_{\text{lab}}}{4\pi} \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \gamma^\mu G_{\text{ret}}^f(0)(p + q) \gamma^\nu G_{\text{adv}}^f(01)(p) .
\] (2.11)

In what follows we shall calculate only the non-singlet functions of a given flavor and omit the flavor label \( f \) when it causes no confusion. The quark field correlators in this equation form a \( 2 \times 2 \) matrix and obey a matrix integral equation of the Schwinger-Dyson type,

\[
G_{AB} = G_{AB} + G_{AB} \Sigma_{RS} G_{SB} ,
\] (2.12)

which allows for a symbolic solution. This solution expresses the entire matrix of field correlators in terms of the full retarded and advanced propagators and the sources \( \Sigma_{AB} \) (the “current” correlators),

\[
G_{AB} = G_{\text{ret}} G_{(0)}^{-1} G_{AB} G_{(0)}^{-1} G_{\text{adv}} + (-1)^{A+B} G_{\text{ret}} \Sigma_{AB} G_{\text{adv}} ,
\] (2.13)

where \( G_{(0)}^{-1} \) and \( G_{(0)}^{-1} \) are the differential Dirac operators acting on the right and on the left, respectively. The retarded and advanced Green’s functions obey more familiar equations,

\[
G_{\text{ret}} = G_{\text{ret}} + G_{\text{ret}} \Sigma_{\text{ret}} G_{\text{ret}} ,
\] (2.14)

which allow for the symbolic solutions also,

\[
G_{\text{ret}}^{-1} = G_{\text{adv}}^{-1} - \Sigma_{\text{ret}} .
\] (2.15)

A simple examination of Eqs. (2.13) shows that all four dressed correlators \( G_{AB} \) can be found as formal solution of the retarded Cauchy problem, with the bare correlators \( G_{AB} \) as the initial data and the self-energies as the sources. Indeed, integrating the first term of each of these equations twice by parts, we find for all four elements of \( G_{AB} \):

\[
G_{01}(x, y) = \int d\Sigma_{(\xi)} d\Sigma_{(\eta)} G_{\text{ret}}(x, \xi) \gamma^\mu G_{01}(\xi, \eta) \gamma^\nu G_{\text{adv}}(\eta, y) - \int d\xi d\eta G_{\text{ret}}(x, \xi) \Sigma_{01}(\xi, \eta) G_{\text{adv}}(\eta, y),
\] (2.16)

\[
G_{10}(x, y) = \int d\Sigma_{(\xi)} d\Sigma_{(\eta)} G_{\text{ret}}(x, \xi) \gamma^\mu G_{10}(\xi, \eta) \gamma^\nu G_{\text{adv}}(\eta, y) - \int d\xi d\eta G_{\text{ret}}(x, \xi) [\pm G_{0}^{-1} + \Sigma_{00}(\xi, \eta)] G_{\text{adv}}(\eta, y). \] (2.17)

Though in the original derivation of these equations we employed the “bare” fields of the interaction picture, the last two formulae allow one to make a step further and to replace the correlators \( G_{AB} \) of the bare fields with the actual values of correlators \( G_{AB} \) on the hypersurface of the initial data which, in their turn, were created in the course of the preceding evolution. After this modification, Eqs. (2.13)–(2.17) become an excellent tool for the study of all inclusive inelastic processes. Similar equations can be derived for the correlators of the gauge boson fields and their self-energies.

Viewed in the context of the QFK, the physical picture of the QCD-evolution looks like the virtual assembling of the source of a proper electromagnetic fluctuation, one which is capable of providing a given momentum transfer to the electron. In the event such a transfer is detected, the hadronic system has to recoil and, provided the transferred momentum is high enough, to emit a backward quark jet. This jet is a successor of the free quark in the perturbative vacuum. It is a general principle of the quantum mechanics of radiation that the emission is the excitation of the field mode. To be excited, the mode itself should exist. The normal modes of QCD are hadrons and they have a true
physical nonperturbative vacuum as the ground state. The process of the emission of a free quark is impossible without creation a new ground state. In the real world of asymptotically stable states, the words “perturbative vacuum” mean a special highly excited state of the hadronic system. The QCD evolution includes the description of its creation by definition. Thus, we are unavoidably led to a physical picture based on the long standing observation by Shuryak [14] that the energy scale of confinement is weak, while the energy density of the QCD vacuum is very high. From this point of view, multiparticle production in hadronic collisions looks like a signature of a “cavitation” of the physical QCD vacuum and the creation of a domain with a perturbative dynamics of quarks and gluons. To switch it on, one needs extreme Lorentz contraction which resolve fluctuations of much shorter scale than those providing confinement in hadrons. The entire process is strongly localized in space and time [17]. To describe this process quantitatively, we have to include a special highly excited state of the hadronic system. The QCD evolution includes the description of its creation by a new ground state. In the real world of asymptotically stable states, the words “perturbative vacuum” mean the physical nonperturbative vacuum as the ground state. The entire process is strongly localized in space and time [17]. To describe this process quantitatively, we have to include a special highly excited state of the hadronic system. The QCD evolution includes the description of its creation by

\[ G_{10}^{a} = G_{10}^{a} - G_{ret} \Sigma_{10} G_{adv} , \quad D_{10} = D_{10}^{a} - D_{ret} \Pi_{10} D_{adv} , \]  

(2.18)

where the superscript “#” is assigned to all the states in the continuum of the free on-mass-shell fields,

\[ G_{10}^{a} (p) = -2 \pi i \delta_{ij} (p + m \theta \delta (p^2 - m^2) , \]  

(2.19)

\[ D_{10}^{a} (p) = -2 \pi i \delta_{ab} d^{uv} (p) \theta (\pm p_0) \delta (p^2) , \quad d^{uv} (p) = -g^{uv} \frac{p^\mu n^\nu + n^\mu p^\nu}{p^2} . \]  

(2.20)

(Projector \( d^{uv} (p) \) corresponds to the null-plane gauge \( A^+ = 0 \).) These states are initially empty and the vacuum correlators \( G_{10}^{a} (p) \) and \( D_{10}^{a} (p) \) represent only on-mass-shell quarks and gluons in the final states. The second terms in Eqs. (2.18) include the off-diagonal self-energies \( \Sigma_{10} \) and \( \Pi_{10} \), the sources, which are the remainders of the classical configuration of fields in the hadron and in the physical vacuum at some moment of the dynamical evolution before the final interaction (the measurement). Before this interaction happens, the decomposition of the field configuration into the radiation field and the classical remainders is virtual; the phases of the partial waves are balanced to form a hadronic correlator propagating through the condensates of the physical vacuum.

C. Kinematics of the inclusive DIS measurement

We do not employ the method of operator product expansion product where \( x_{Bj} \) immediately appears as the natural variable corresponding to the light-cone momentum. Moreover, we are going beyond the parton model and intend to include new dynamical elements, e.g., the static fields, in the evolution equations. Therefore, we have to pay special attention to the formula of measurement. In particular, we have to establish a connection between the kinematic variable \( x_{Bj} \) and the Feynman variable \( x_F \) without any reference to the parton model. The \( ep \)-DIS measurements rely on a set of rare events where the space-like photon \( \gamma^* \) is created by a strong change of the state of one quark only. This quark almost instantaneously decouples from the bulk of the proton and initiates the jet. Thus, we have to require that one on-mass-shell quark appears in the final state in the perturbative vacuum, and, therefore, Eq. (2.11) can be rewritten as

\[ W^{\mu \nu} (q) = \sum_f e_f^2 \frac{2 V_{lab} P_{lab}}{4 \pi} \int \frac{d^4p}{(2 \pi)^3} \text{Tr} \left[ \gamma^\mu \left( p^+ - m_f \right) \gamma^\nu G_{ret} (p) \Sigma_{10} (p) G_{adv} (p) \right] \left[ 2 \pi i \delta (p + q)^2 - m^2 \right] . \]  

(2.21)

By virtue of Eq. (2.9), and introducing \( x \equiv x_{F} = p^+ / P^+ \), we obtain the expression,

\[ F_2 (x_{Bj}, Q^2) = \sum_f e_f^2 \frac{V_{lab} P^+}{2 (2 \pi)^3} \int_0^1 x dx \int d^2p_t dp^- \delta [p^+ p^- + 2 \nu x - (\vec{p}_t - \vec{q}_t)^2] i p^+ F^{(f)} (p) , \]  

(2.22)

where \( F^{(f)} \) is the coefficient in the following decomposition of the quark source,

\[ G_{ret} \Sigma_{10} G_{adv} = p F^{(\mu)} (p) + \bar{p} p^+ B^{(\mu)} (p) + m C^{(\mu)} (p) + H^{(\mu)} (p) (\bar{p} \not \! q - \not \! q \bar{p} - \not \! q \bar{p}) . \]  

(2.23)

Following Ref. [3], we can express the invariant \( F (p) \) via the “unintegrated structure function,”

\[ V_{lab} P^+ \int \frac{d q_f (x, p_t^2)}{d p_t^2} , \]  

(2.24)
where $\nu = Pq$. [As it will be seen later, in a certain approximation, $q_f(x, Q^2)$ becomes a structure function of the deep inelastic scattering of a quark of flavor $f$.] Introduction of the $\delta$-function $\delta(p^-)$ at this step of calculations is motivated physically. According to Eq. (2.5), the field fragment with the momentum $p^\mu$ belongs to the yet undestroyed proton. Let $f(x^+, x^-) \equiv f(x^0 + x^3, x^0 - x^3)$ be some proton-related field quantity, e.g., the proper electromagnetic field $A^\mu$ of the proton. If the proton, before its interaction with the electron, is viewed as a kind of a wave packet that propagates without dispersion at the speed of light, then the function $f$ should depend only on a single argument $x^-$. In the momentum representation, we have

$$f(x^+, x^-) = \int dp^+ dp^- f(p^+, p^-) e^{-\frac{i}{\hbar}(p^+ x^+ + p^- x^-)}.$$  

Thus, in order to eliminate the dependence on $x^+$ and to keep the proton as a stable wave packet which propagates along the light cone, one should require that $f(p^+, p^-) = f(p^+) \delta(p^-)$. Admitting the opposite, we would run into a conflict with the Lorentz contraction of the initial state proton, viz., the radiative corrections to its ‘valence structure’ would increase its size. The dynamics of the quark and gluon fields behind this condition is totally nonperturbative.

In the same way, following Eq. (2.10), we obtain the expression for the longitudinal structure function,

$$3F_L(x_{BJ}, Q^2) = -\frac{ie^2}{\pi} \frac{V_{ij} P^+}{(2\pi)^3} \int_0^1 dx \int d^2 \vec{p}_i dp^- \left[ \delta(p^+ p^- + 2\nu x - (\vec{p}_i - \vec{q}_i)^2) \right] F_1(p),$$ (2.25)

where

$$F_1 = [2\nu(x_F^2 - x_{BJ}^2) - x_{BJ}(p^2 + m^2)] F - 2(p^+)^2 x_{BJ} B + 4m^2 x_{BJ} C.$$  

The next calculations are as follows. We integrate over the angle between the vectors $\vec{p}_i$ and $\vec{q}_i$ and re-scale $p_i^2$ in the remaining integrals by $2\nu$, $y = p_i^2 / 2\nu$. Denoting $\mu = m^2 / 2\nu$, we obtain,

$$F_2(x_{BJ}, Q^2) = \frac{e^2}{4\pi} \int_0^1 dx \int_{(\sqrt{x_F^2} + \sqrt{\mu})^2}^{(\sqrt{x_F^2} - \sqrt{\mu})^2} dy \left[ \frac{dq_f(x, 2\nu y)}{dy} \right] \left[ (y - (\sqrt{x_{BJ}^2} - \sqrt{\mu})^2) \left( (\sqrt{x_{BJ}^2} + \sqrt{\mu})^2 - y \right) \right]^{1/2}.$$ (2.26)

Here, the denominator is the quadrupled area of the triangle with the sides $p_i$, $q_i$, and $\sqrt{2\nu x - m^2}$ which are rescaled by $2\nu$. The limits of integration in the Eq. (2.26) come from the triangle inequality. Now, we should take the limit $\nu \rightarrow \infty$ keeping the $x_{BJ}$ finite. In the new variables, the parameter $\nu$ enters only the argument of the function $dq_f(x, 2\nu y)/dy$. This function vanishes in the limit of $\nu \rightarrow \infty$. The integral (2.26) vanishes also unless the variable $y$ is allowed to take arbitrary small values. Therefore the lower limit of the integration over $y$ should be set equal to zero, i.e.,

$$x_F = x_{BJ} + \mu = x_{BJ}(1 + m^2 / Q^2).$$ (2.27)

For massless quarks, we obtain the usual condition, $x_F = x_{BJ}$. [If we were doing the same analysis in the natural momentum variables, then the lower limit of the integration over $p_i$ becomes infinite when $\nu \rightarrow \infty$ at fixed $x_{BJ}$. In fact, the finite interval of integration over $p_i$ would keep its length $2Q$ and slide as a whole to infinitely high values of $p_i$. However, if Eq. (2.27) is satisfied, then both limits become independent of $\nu$.] The condition (2.27) defines the upper limit of the $p_i$-integration integration also, and the final result reads as

$$F_2(x_{BJ}, Q^2) = e^2 \left( x_{BJ} + \frac{m^2}{2\nu} \right) \int_0^{Q^2} dp_i^2 \left[ dq_f(x_{BJ} + m^2 / 2\nu, p_i^2) \right].$$ (2.28)

Here, the shift of $x_{BJ}$ by $m^2 / 2\nu$ is a standard quark mass correction [13]. Change of the upper limit from the familiar $Q^2$ to $4Q^2$ is unusual but not unexpected. Indeed, of the three sides of the triangle, $p_i$, $q_i$ and $\sqrt{2\nu x - m^2}$, the third one, by virtue of Eq. (2.27), is equal to $q_i$. Therefore, the length of the vector $p_i$, the transverse momentum of the incoming off-mass-shell field, varies from 0 (at $\vec{p}_i = \vec{q}_i$) to $2q_i$ (at $\vec{p}_i = -\vec{q}_i$). Furthermore, with logarithmic accuracy there is no theoretical difference between $log Q^2$ and $log 4Q^2$ when $Q^2 \rightarrow \infty$ and no conflict with the leading-log approximation (LLA), the AP-equations, is anticipated. [For the usual on-mass-shell parton, we put $\vec{p}_i = 0$ and $p^- = 0$ and obtain Eq. (2.27) directly from the delta-function in the integrand of Eq. (2.22).]

As it was discussed in Ref. [1], the derivatives of the structure functions like $dq_f(x, Q^2)/dQ^2$ (the sources), are similar to the densities of states, $\rho(E)$, of the condensed matter or scattering theories, and the structure functions themselves are similar to the number of states $n(E)$ below the energy $E$. In an experiment we measure $n(E)$
The scalar invariant functions of this decomposition can be easily found from the three Dirac traces, 
\( \text{AB} \) fields are polarized. Thus, all the self-energies \( \Sigma \)
Eq. (2.21). Indeed, we have two vectors, exceeds the accuracy of approximations used in this paper. The complexity comes from the product
for the gluon field. However, the measure will never appear for those components of the fields which do not have the
property of propagation and are not independent variables in the Hamiltonian formulation of the system’s dynamics. These are the static components of the fermion field and longitudinal gluon field.

The full expressions for the invariants \( \mathcal{F} \) and \( \mathcal{F}_1 \) in Eqs. (2.22) and (2.23) are very long and their exact form exceeds the accuracy of approximations used in this paper. The complexity comes from the product
in Eq. (2.21). Indeed, we have two vectors, \( p^\mu \) and \( n^\mu \), at our disposal. The fermion field is massive and none of the fields are polarized. Thus, all the self-energies \( \Sigma_{AB} \) have three terms in their spinor decomposition,
\[
\Sigma(p) = (\not{p} - m)\sigma_2(p) + \not{p}n^+\sigma_3(p) + m\sigma_m(p) . \tag{2.29}
\]
The scalar invariant functions of this decomposition can be easily found from the three Dirac traces,
\[
\text{Tr}\Sigma = 4m(\sigma_m(p) - \sigma_2(p)), \quad \text{Tr}[\not{p}\Sigma] = 4p^+\sigma_2(p), \quad \text{Tr}[\not{p}\Sigma] = 4[\not{p}^2\sigma_2(p) + (p^+)^2\sigma_3(p)] . \tag{2.30}
\]
By virtue of Eqs. (2.15) and (2.29), the retarded and advanced propagators are readily found as
\[
G_{ret}^{(\not{p} - m)(1 - \sigma_2^{(\not{A})}) - \not{n}p^+\sigma_3^{(\not{A})} - m\sigma_m^{(\not{A})}} = G_{ret}^{(\not{A})}(1 - \sigma_2^{(\not{A})})(\not{p} + n) - \not{n}p^+\sigma_3^{(\not{A})} + m\sigma_m^{(\not{A})} , \tag{2.31}
\]
where
\[
G_{ret}^{(\not{A})} = (p^2 - m^2 - \sigma_0^{(\not{A})})(1 - \sigma_2^{(\not{A})}) - m^2(\sigma_m^{(\not{A})})^2 = S_0^{(\not{A})}(p)S_2^{(\not{A})} - [S_m^{(\not{A})}]^2 , \tag{2.32}
\]
and the following short-hand notation is introduced: \( \sigma_0 = (p^2 - m^2)\sigma_2 + 2(p^+)^2\sigma_3 + 2m^2\sigma_m \),
\[
S_0^{(\not{A})}(p) = p^2 - m^2 - \sigma_0^{(\not{A})} , \quad S_2^{(\not{A})}(p) = 1 - \sigma_2^{(\not{A})} , \quad S_m^{(\not{A})}(p) = m\sigma_m^{(\not{A})} .
\]
For the massless quark field, we can proceed without approximations and obtain a reasonably simple expression,
\[
G_{ret}(p)\Sigma_{\mu\nu}(\not{p})G_{adv}(p) = \frac{[\not{p} - \not{p}(p^2/2p^+)][\sigma_0^{(\not{A})}(p)] + \not{p} - \not{n}p^+\sigma_3^{(\not{A})}(p)}{S_0^{(\not{A})}(p)S_2^{(\not{A})}(p)} , \quad ( m = 0 ) \tag{2.33}
\]
which explicitly exhibits separation between the dynamical and the constrained \( (x^+ - \text{instantaneous or static}) \) parts of the fermion field. The constrained part (the second term in Eq. (2.33)) is easily recognized since (after the string \( G_{ret}\Sigma G_{adv} \) is assembled) it has no pole corresponding to the propagation. A reasonable approximation for the massive quark field will be to neglect the radiative corrections in the retarded and advanced propagators. Then
\[
\mathcal{F}(\not{p}) = \frac{\sigma_0^{(f)}}{p^2 - m_f^2} , \quad \mathcal{B}(\not{p}) = -\frac{\sigma_3^{(f)}}{p^2 - m_f^2} , \quad \mathcal{C}(\not{p}) = \frac{\sigma_0^{(f)} + \sigma_3^{(f)}}{p^2 - m_f^2} , \tag{2.34}
\]
and the expressions for the functions \( F_2 \) and \( F_L \) become significantly simpler. The factor \( [p^2 - m_f^2]^{-2} \) in the above formulae is never singular; it originates from the product of the retarded and advanced Green functions and should be read with the principal value prescription. With this simplified form of the invariants, the integrand of the structure function \( F_2 \), which is defined by Eq. (2.28), takes the following form,
\[
\delta(p^-) \frac{dg_{f}(x, p_f^2)}{dp_f^2} = \frac{V_{ub}P^+}{2(2\pi)^3}ip^+\frac{\sigma_0^{(f)}(p)}{p^2 - m_f^2} . \tag{2.35}
\]
The integrand of the longitudinal structure function \( F_L \) becomes relatively simple also,
\[
\mathcal{F}_L^{(f)}(p) = \frac{2m_f^2}{p^2 - m_f^2}\sigma_0^{(f)}(p) + \frac{2m_f^2}{p^2 - m_f^2}\sigma_3^{(f)}(p) - \sigma_2^{(f)}(p) . \tag{2.36}
\]
and the final expression can be conveniently written down in the following form,

\[
3Q^2 F_L(x_{BJ}, Q^2) = \frac{2m_f^2 Q^2(4Q^2 + m_f^2)}{(Q^2 + m_f^2)^2} F_2(x_{BJ}, Q^2)
+ \frac{2e_f^2 x_{BJ} Q^2}{Q^2 + m_f^2} \int_{0}^{4Q^2} dp_1^2 \left[ \frac{dM_f(x_{BJ}(1 + m_f^2/Q^2), p_1^2)}{dp_1^2} + Q_f(x_{BJ}(1 + m_f^2/Q^2), p_1^2) \right],
\]

(2.37)

where we have introduced the following notation,

\[
\frac{V_{lab}}{2(2\pi)^3} i p^+ \frac{\sigma_m{(f)}(p)}{p_i^2} = \delta(p^-) \frac{dM_f(x, p_i^2)}{dp_i^2}, \quad \frac{V_{lab}}{2(2\pi)^3} i p^+ \sigma_2{(f)}(p) = \delta(p^-) Q_f(x, p_i^2).
\]

(2.38)

The first term in Eq. (2.37) is in direct proportion to \(F_2\). It is entirely due to the finite quark mass \(m_f\). It shows that at \(Q^2 \sim m_f^2\) the non-singlet structure functions \(F_2\) and \(F_L\) of the heavy quark may be of the same order.

### III. EVOLUTION OF THE SOURCES

Our next step is to find evolution equations for the sources of the electromagnetic fluctuation which scatters the electron. These sources are given by the off-diagonal self-energies \(\Sigma_{01}\) (for the quarks) and \(\Sigma_{10}\) (for the anti-quarks). Their general expressions were derived in Refs. [5,6] and, as in Ref. [6], we shall use approximation of the bare vertices. Furthermore, we shall use simplified equations (2.18) which define evolution of the quark sources and make similar simplifications for the gluons (hereafter, the gauge \(A^+ = 0\) for the gluon field is assumed).

The general expression for the gluon sources, \(\Pi_{(\alpha)}^{(01)}\), were derived in Refs. [5,6] also and an approximation of the bare vertices will be used here as well,

\[
\Sigma_{(01)}^{(\alpha)}(p) = ig_F^2 C_F \int \frac{d^4 k}{(2\pi)^4} \{ \gamma_\mu G_{\mu\alpha}(k) \Sigma_{(01)}^{(\alpha)}(k) G_{\alpha\nu}(k) \gamma_\nu D_{(01)}^{\nu}(k - p)
+ \gamma_\mu G_{\mu\alpha}^{\#}(k + p) \gamma_\nu \left[ D_{\mu\alpha}(k) \Pi_{(\alpha)}^{(01)}(k) D_{\alpha\nu}(k) \right]^{\nu}\}
\]

(3.1)

\[
\Pi_{01}^{\nu}(p) = -ig_F^2 \{- \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \{ \gamma^\mu G_{\mu\alpha}(k) \Sigma_{01}(k) G_{\alpha\nu}(k) \gamma^\nu G_{\nu\alpha}^{\#}(k - p) + \gamma^\mu G_{\mu\alpha}^{\#}(k + p) \gamma^\nu G_{\nu\alpha}(k) \Sigma_{01}(k) G_{\alpha\nu}(k) \}
+ \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} V_{\alpha\beta\sigma}(p, k - p, k) \left[ D_{\alpha\beta}(k) \Pi_{01}(k) D_{\alpha\nu}(k) \right]^{\nu}\}
\]

(3.2)

The renormalization of these equations as well as the study of their infra-red behavior is a special problem. The idea of renormalization in the QFK-based calculations has been discussed in Ref. [6] for the case of pure glue-dynamics. In this simplest case the coupling constant should be replaced by the “running coupling,” \(g_F^2 \rightarrow 4\pi\alpha_s(p_F^2)\). However, we still have not obtained satisfactory balance of the collinear singularities except for the limiting case when the evolution is described by the DGLAP equations. The status of this problem will be discussed separately [20].

### A. Splitting of the evolution equations

The polarization tensor \(\Pi^{\mu\nu}\) only appears between the retarded and advanced propagators, i.e., in the combination \([D_{\mu\nu}(p)\Pi(p)D_{\mu\nu}(p)]^{\mu\nu}\). Both propagators contain the same projector, \(d^{\mu\nu}(p)\) which (by the gauge condition) is orthogonal to the 4-vector \(n^\nu\). So, of the general tensor form, only two terms survive,

\[
\Pi^{\mu\nu}(p) = gd^{\mu\nu} p^2 w_1(p) + p^\mu p^\nu w_2(p).
\]

(3.3)

The others, like \(p^\mu n^\nu + n^\mu p^\nu\) or \(n^\mu n^\nu\), will cancel out. Introducing one more projector,

\[
d^{\mu\nu}(p) = -d^{\mu\nu}(p)d^\rho_2(p) = -g^{\mu\nu} + \frac{p^{\mu} n^\nu + n^\mu p^\nu}{(np)} - p^2 \frac{n^\mu n^\nu}{(p^2)^2}.
\]

(3.4)
which is orthogonal to both vectors $n^\nu$ and $p^\mu$, we find that the invariants $w_1$ and $w_2$ can be found from two convolutions,

$$-\bar{\Pi}_{\mu\nu}(p)\Pi^{\mu\nu}(p) = 2p^2 w_1(p), \quad \text{and} \quad n_\mu n_\nu\Pi^{\mu\nu}(p) = (p^+)^2 w_2(p), \quad (3.5)$$

independently of the other invariants accompanying the missing tensor structures. The new projector, which includes only two transversal gluon modes, naturally appears in the tensor with a gluon source. Indeed, solution of the Schwinger-Dyson equation for the retarded gluon propagator,

$$D_{ret} = D_{ret} + D_{ret}\Pi_{ret}D_{ret}. \quad (3.6)$$
can be cast in the form,

$$D_{ret}^{\mu\nu}(p) = \frac{d^{\mu\nu}(p)}{p^2 - p^2 w_1^{\mu\nu}(p)} + \frac{1}{(p^+)^2} \frac{n^\mu n^\nu}{1 - w_2^{\mu\nu}(p)}. \quad (3.7)$$

With the shorthand notation, $W_{1\nu}(p) = p^2 - p^2 w_1^{\nu\nu}(p)$, and $W_{2\nu}(p) = 1 - w_2^{\nu\nu}(p)$, we easily obtain,

$$[D_{ret}(k)\Pi_{\nu\lambda}(k)D_{adv}(k)]^{\mu\nu} = \frac{d^{\mu\nu}(p)p^2 w_1^{(0)\nu}(p)}{W_1^2(p)W_2^2(p)} + \frac{w_2^{(0)\nu}(p)n^\mu n^\nu}{(p^+)^2 W_2^2(p)W_2^2(p)}. \quad (3.8)$$

Once again, similar to the case of the fermion field, we encounter the longitudinal $(x^+\text{-instantaneous})$ part $w_2$ of the gluon source which is not accompanied by the propagation poles in the string $D_{ret}\Pi_{adv}$.

The role of the tree propagators $S^{(3)}$ and $V^{(3)}$ which include the radiative corrections is clearly understood. These corrections keep memory about the balance of the phases between various fields in the yet undestroyed proton. In fact, they nonperturbatively maintain the $\delta(p^-)$-prescription, the requirement that the proton does not fall apart in the course of the QCD evolution. Keeping this important qualitative observation in mind we shall limit ourselves, in what follows, to the bare retarded and advanced (tree) propagators of the quark and gluon fields.

Our next goal is to find the evolution equations for the three invariants, $\sigma_0^{(f)}(x,p_1^2)$, $\sigma_1^{(f)}(x,p_1^2)$ and $\sigma_m^{(f)}(x,p_1^2)$. Using the Eqs. (2.30) and (3.5) we easily extract evolution equations for various invariants from the tensor evolution equations (3.1) and (3.2). In sequence, we obtain three equations for three invariants of the fermion source,

$$\sigma_0^{(f)}(p) = \frac{2g^2(p_1^2)}{(2\pi)^3} \int d^4k \left\{ C_F\delta_+[(k-p)^2] \frac{1}{z} \frac{\sigma_0^{(f)}(k)}{|k^2 - m_f^2|^2} + \delta_+[(k-p)^2 - m_f^2] \frac{1}{2z} \frac{k^2 w_1^{(10)}(-k)}{|k^2|^2} \right\}, \quad (3.9)$$

$$\sigma_1^{(f)}(p) = \frac{2g^2(p_1^2)}{(2\pi)^3} \int d^4k \left\{ C_F\delta_+[(k-p)^2] \frac{1}{z} \frac{\sigma_0^{(f)}(k)}{|k^2 - m_f^2|^2} + \delta_+[(k-p)^2 - m_f^2] \frac{1}{2z} \frac{k^2 w_1^{(10)}(-k)}{|k^2|^2} \right\}, \quad (3.10)$$

$$\sigma_m^{(f)}(p) = \frac{2g^2(p_1^2)}{(2\pi)^3} \int d^4k \left\{ C_F\delta_+[(k-p)^2] \frac{1}{z} \frac{\sigma_0^{(f)}(k)}{|k^2 - m_f^2|^2} + \delta_+[(k-p)^2 - m_f^2] \frac{1}{2z} \frac{k^2 w_1^{(10)}(-k)}{|k^2|^2} \right\}, \quad (3.11)$$

and two equations for the invariants of the gluon source,

$$p^2 w_1^{(0)}(p) = -\frac{2g^2(p_1^2)}{(2\pi)^3} \int d^4k \sum_f \left\{ T_f \left( \delta_+[(k-p)^2 - m_f^2] \frac{k^2 w_1^{(10)}(-k)}{|k^2|^2} \right. \right.$$}

$$\left. \left. \times \frac{(p^2 + k^2 - m_f^2)(z-1)^2 + 1 - 2m_f^2}{z} \sigma_0^{(f)}(k) \right) + (1 - z)\sigma_2^{(f)}(k) - 2m_f^2z\sigma_m^{(f)}(k) \right\}, \quad (3.12)$$
Here, the sum over $(f)$ runs over all quark flavors and anti-flavors. The longitudinal fields appear in the evolution equations (2.13)–(2.18) in an alternating regime. This remarkable feature of the evolution equations has a very clear physical explanation. Indeed, if the static source has emitted a propagating wave, at least one additional emission is necessary to create a new static field configuration.

Deriving the field equations (2.13)–(2.18) we were relying upon the most straightforward implementation of the idea of quantum mechanical evolution; viz., the Heisenberg picture of the evolution of observables of DIS with the quark and gluon fields of a stable hadron in physical vacuum as the initial data. Eqs. (3.9)–(3.13) are the mathematical expression of this physical picture. The initial data are not given explicitly, and if they were, the range of the incorporated physical information should be the same as in the local operators of the OPE. However, we do not view the perturbative part of the evolution as the scattering problem and this results in the new feature of the above equations; the invariants $\sigma_2$ and $w_2$ which correspond to the sources of longitudinal fields participate in the evolution equations on the same footing as the sources $\sigma_0$ and $w_1$ of the transverse fields. Therefore, we have a reason to suspect that the entire dynamics of the QCD evolution carries a classical component. This is in compliance with the qualitative understanding of the QCD evolution as the virtual decomposition of the hadrons, which are the genuine fundamental modes of QCD, in terms of the alien set of modes defined as excitations above an artificial perturbative vacuum. The parton picture should emerge after certain approximations, different from the formal twist classification of the local operators of the OPE expansion.

B. Feed-back via longitudinal fields

In what follows, we shall make several approximations and try to estimate their accuracy. In the OPE-based calculations, it is a formidable task. Indeed, the hierarchy of scales in OPE is two-fold. On the one hand, the operator functions are ordered by their twist which, would we wish to treat it statistically, corresponds to the order of the irreducible correlation function in condensed matter physics. On the other hand, the coefficient functions are ranked by the order of their expansion in powers of the coupling constant. Because of the capricious interplay of these two essentially different expansions it is difficult to obtain a reliable estimate of the accuracy even for the widely used DGLAP equations.

We shall pose the problem in a different way and try to understand what elements of the physical picture are lost when the field equations (3.9)–(3.13) are approximated by the master equations of the parton evolution. Let us consider the transverse components of the sources, $\sigma_0$ and $w_1$, as granted and use the connection to the observables, equations (3.36)–(3.38),

$$c \, i p^+ \frac{\sigma_0^{(f)}(p)}{[p^2 - m_f^2]^2} = \delta(p^-) \frac{d q_f(x, p_f^2)}{d p_f^2}, \quad c \, i p^+ \frac{\sigma_2^{(f)}(p)}{[p^2 - m_f^2]^2} = \delta(p^-) Q_f(x, p_f^2),$$

(3.14)

for the fermion field, and introduce the similar links for the gluon field,

$$c \, i p^+ \frac{w_1(p)}{[p^2]^2} = \delta(p^-) \frac{d G(x, p^2)}{d p^2}, \quad c \, i p^+ \quad w_2(p) = \delta(p^-) G(x, p^2),$$

(3.15)

where $c$ is an insignificant common normalization constant. Integrating Eqs. (3.10) and (3.13) over $p^-$ and using the delta-functions to integrate the variable $k^-$ out, we obtain,

$$Q_f(x, p_f^2) = c \int d p^- \quad i p^+ \frac{\sigma_0^{(f)}(p)}{[p^2 - m_f^2]^2} \left( C_F \int_{p^+}^{p^+} \frac{d k^+}{k^+ - p^+} q_f(k^+) + \frac{1}{2} \int_{p^+}^{p^+} \frac{d k^+}{k^+} G(k^+) \right),$$

(3.16)

$$G(x, p^2) = c \int d p^- \quad i p^+ \quad w_2(p) = g^2(p^2) \left[ 4 T_f \sum_f \frac{d k^+}{p^+} q_f(k^+) - N_c \int_{p^+}^{p^+} \frac{d k^+}{k^+ - p^+} \frac{(2 p^+ - k^2)^2}{4 p^+ k^+} G(k^+) \right],$$

(3.17)

where
\[ q_\perp(p^+) = \int dp^- d\eta_1 d\eta_2 \frac{g_2^2}{p^2 - m^2} \] and \[ G(p^+) = \int dp^- dG(x, p^2) \],

(3.18)

are the \( x \)-fractions of the quarks and the glue converted into the radiation, integrated over all transverse momenta. The requirements for the convergence of these integrals are exactly the same as in the proof of the resonant condition for the measurement, Eq. (2.23). The physical meaning of these equations can be uncovered by examining, e.g., the first term in Eq. (3.17), the rate of the depleting of the initial reservoir of static glue is proportional to the total number of quarks radiated above a given value \( x \). The collinear divergence at \( k^+ = p^+ \) in Eqs. (3.16) and (3.17) is not due to the spurious pole of the gluon propagator. It appears as a consequence of the singularity of the integration \( dp^- \) of the isolated mass shell delta function, \( \delta([k^+ - p^+)(k^- - p^-)] - (k_l - p_l)^2) \), at the point \( k^+ = p^+ \). This singularity appears only for the longitudinal modes. It is shielded when the delta-function is accompanied by the propagators of the transverse field with the off-mass-shell space-like momentum, i.e., when the field is slowed down by its virtuality. In coordinate space, this singularity corresponds to the integration of the \( x^+ \)-independent function in the infinite limits, and it is unavoidable when the wave packet is moving without dispersion in the light-like direction, thus depending only on \( x^- \). To shield this singularity, one has either to decelerate the proton (to take \( P^+ \sim \sqrt{s} \) finite), or to establish a lower limit of resolution for the transverse momentum. In the second case, the natural scale of hadronic confinement, \( \Lambda_{QCD}^{-1} \), has to be taken as the cut-off. Both cut-offs play the same role; they allow for the geometrical separation of the fields between the near and far zones. Physically, in QCD one cannot separate two waves if their momenta differ by less than the width of the first peak of diffraction on an object with the size \( \Lambda_{QCD}^{-1} \).

The next step is to examine the feed-back which appears when \( \sigma \) for our immediate purpose, it is enough to consider only one component of the on-mass-shell gluon correlator,

\[
D_{10}(x_1, x_2) = \int \frac{dk^0 d\theta}{2(2\pi)^3} \frac{4 \sinh \eta_1 \sinh \eta_2 \pi e^{-i k_i (x_{1i} - x_{2i})}}{(k^+ e^{-\eta_1} + k^- e^{\eta_1})(k^+ e^{-\eta_2} + k^- e^{\eta_2})} \approx \int \frac{dk^0 d\theta}{2(2\pi)^3} \frac{k^+ e^{-i k_i (x_{1i} - x_{2i})}}{k^+ + (e^{2\eta_1} + e^{2\eta_2})(k_t^2/k^+)}
\]

(3.19)

where \( k^+ = k_i e^\theta, k^- = k_i e^{-\theta} \) and \( k^+ k^- = k_i^2 \). Now, it is easy to see that the pole \( 1/k^+ \) is shielded. The cut-off is defined by the location of the interaction region; for the lowest \( k^+ \) one should take \( \eta_1, \eta_2 \sim -Y/2 \approx -\ln(2E_{max}/m_{had}) \approx \ln(\sqrt{s}/\Lambda_{QCD}) \). The invariant energy of the collision, \( \sqrt{s} \), or its equivalent, the full width \( Y \) of the hadronic rapidity plateau, is the measure of the resolution in longitudinal direction of the interaction which initiate the deeply inelastic process. Any extension of the theory beyond the parton model will include them explicitly.

The next step is to examine the feed-back which appears when \( \sigma_2 \) and \( w_2 \) from Eqs. (3.16) and (3.17) are inserted into the right hand side of the evolution equations (3.3) and (3.12) for the propagating (transverse) components of the quark and gluon fields. Since these terms in the integrand depend on \( k_t \) only via the coupling constant, all integrations except the one over \( k^+ \) can be carried out, viz.,

\[
I_{q\rightarrow q} = \int dp^- d\eta_1 d\eta_2 \frac{g_2^2}{p^2 - m^2} \int \frac{g_2^2(k^+ - p^2)^2 d\eta}{(p^2 - m^2)^2} = \int \frac{g_2^2(k^+ + p^2)^2 d\eta}{|k^2 + (1 - z)(p^2 + m^2)|^2} \approx \frac{\pi g_2^2(p_t^2)k^+}{p^2(p_t^2 + m^2)},
\]

\[
I_{q\rightarrow g} = \frac{\pi g_2^2(p_t^2)k^+}{p^2 p_t^2} \frac{1 - z}{m^2 + (1 - z)p_t^2}, \quad I_{g\rightarrow q} = \frac{\pi g_2^2(p_t^2)k^+}{p^2 p_t^2} \frac{1 - z}{m^2 + (1 - z)p_t^2}, \quad I_{g\rightarrow g} = \frac{\pi g_2^2(p_t^2)k^+}{p^2 p_t^2} \frac{1 - z}{m^2 + (1 - z)p_t^2},
\]

(3.20)

for the “feed-back” for the propagating fields via the longitudinal modes from quarks to quarks, quarks to glue, glue to glue and glue to quarks, respectively. At \( m = 0 \), all these terms yield a simple and remarkable function,

\[
\frac{g_2^2(p_t^2)}{p^2 p_t^2} \int k^+ f(k^+ \) \, dk^+,
\]

which contains a universal Weizsacker-Williams denominator corresponding to the static field of the ultra-relativistic charge. This induced static source is given, in its turn, by the integral over the light-cone momenta of the transverse fields above the currently probed momentum \( p^+ \). Almost mysteriously, this dependence appears for the fermion field also; it corresponds to the static pattern of the fermion wave function!
The intensities of the induced static sources are almost independent of the transverse momenta. It may be tempting to treat these sources as the equivalents of the valence quarks and gluons. However, there is a major difference. Evolution of the valence partons has to begin through splitting kernels, and this is the key point in derivation of the master AP equation; while in the field approach advocated here, the longitudinal modes are coupled to the transverse ones in a special way which regenerate the distribution of the field inherent to the ultra-relativistic classical source! Moreover, in this approach, the evolution ladder is built from the above and only from the above, solely from the requirement of the given momentum transfer in the measurement and in agreement with the causal picture of the inclusive measurement. This points us to the most important observation that the classical field pattern is inherent to the light front evolution, regardless what were the initial data.

The feed-back fragment of the field evolution equations can be cast in the following form, 

$$\left\{ \frac{dq_f(x, p_t^2)}{dp_t^2} \right\}_{f.b.} = \frac{\pi g^2(p_t^2)}{(2\pi)^3} \int_{p^+} \frac{dk^+}{k^+} \left[ C_F \frac{z}{p_t^2 + m_f^2} Q_f(k^+) - \frac{z(1-z)^2}{m_f^2 + (1-z)p_t^2} G(k^+) \right], \quad (3.21)$$

$$\left\{ \frac{dG(x, p_t^2)}{dp_t^2} \right\}_{f.b.} = \frac{\pi g^2(p_t^2)}{(2\pi)^3} \int_{p^+} \frac{dk^+}{k^+} \left[ -N_c \frac{(z-1/2)^2}{p_t^2} G(k^+) - T_f \sum_f \frac{(1-z)^2}{zm_f^2 + (1-z)p_t^2} Q_f(k^+) \right]. \quad (3.22)$$

where the subscript $f.b.$ stands for the additional contribution to the rate of evolution of the transverse fields from the feed-back via the longitudinal ones. $Q$ and $G$ stand for the left side of Eqs. (3.16) and (3.17) which connect longitudinal sources to the intensity of the radiated transverse fields. This contribution can be estimated in the following way. [From here to the end of this section we shall neglect the quark masses.]

The data, parameterized via DGLAP equations, clearly indicate that the quantity $xq(x, Q^2)$ for the sea quarks experiences a certain growth at small $x$. Let us start with the most conservative probe function, $q_f(x) = \kappa/x$, and calculate the feed-back along the string $q_f(x) \to G(x) \to G(x)$. Using Eqs. (3.16) and (3.21) we obtain the first estimate,

$$\left\{ \frac{p_t^2}{g^2(p_t^2)} \frac{dG(x, p_t^2)}{dp_t^2} \right\}_{f.b.} \Big|_{q \to G} = -4\pi T_f N_c g^2 \left( \frac{2\pi)^6}{(2\pi)^6} \right) \int_x^1 \frac{dy}{y} \left[ \frac{1}{4} - \frac{x}{y} + \frac{x^2}{y^2} \right] \int_y^1 \frac{du}{u} \kappa \frac{\lambda^2 + \lambda + 2}{\lambda(\lambda+1)(\lambda+1)(\lambda+3)} \frac{\kappa}{x^{1+\lambda}} + O(x^{-1}) \right \}, \quad (3.23)$$

which clearly indicates that our probe function is bad. The feed-back response, $\sim (1/x) \ln(1/x)$, is much bigger than the probe signal, $\sim 1/x$! This is exactly what happens in the calculations of the first order quantum correction to the Weizsacker-Williams field [3]. Let us take another probe function, one which exhibits more realistic behavior, e.g., $q_f(x) = \kappa x^{-1-\lambda}$. Repeating the same calculations, we obtain,

$$\left\{ \frac{p_t^2}{g^2(p_t^2)} \frac{dG(x, p_t^2)}{dp_t^2} \right\}_{f.b.} \Big|_{q \to G} = -4\pi T_f N_c g^2 \left( \frac{2\pi)^6}{(2\pi)^6} \right) \int_x^1 \frac{dy}{y} \left[ \frac{1}{4} - \frac{x}{y} + \frac{x^2}{y^2} \right] \int_y^1 \frac{du}{u^{1+\lambda}} \left( \frac{1}{4y} - \frac{1}{u} + \frac{1}{4(u-y)} \right). \quad (3.24)$$

Thus, our second probe is very good. The low-$x$ dependence $\sim x^{-1-\lambda}$ appears to be an eigen-function of the feed-back via the longitudinal fields! To confirm this statement, let us examine the feedback along the string $G(x) \to G(x) \to G(x)$, taking $G(x) = \gamma x^{-1-\lambda}$ as the probe function:

$$\left\{ \frac{p_t^2}{g^2(p_t^2)} \frac{dG(x, p_t^2)}{dp_t^2} \right\}_{f.b.} \Big|_{G \to G} = +\pi N_c^2 g^2 \gamma \left( \frac{2\pi)^6}{(2\pi)^6} \right) \int_x^1 \frac{dy}{y} \left[ \frac{1}{4} - \frac{x}{y} + \frac{x^2}{y^2} \right] \int_y^1 \frac{du}{u^{1+\lambda}} \left( \frac{1}{4y} - \frac{1}{u} + \frac{1}{4(u-y)} \right). \quad (3.25)$$

Retaining the terms which are most singular at low $x$, we obtain the estimate,

$$\left\{ \frac{p_t^2}{g^2(p_t^2)} \frac{dG(x, p_t^2)}{dp_t^2} \right\}_{f.b.} \Big|_{G \to G} = +\pi N_c^2 g^2 \gamma \left( \frac{2\pi)^6}{(2\pi)^6} \right) \left[ 1 - 3\lambda \right] \frac{\lambda^2 + \lambda + 2}{\lambda(\lambda+1)(\lambda+2)(\lambda+3)} \frac{\gamma}{x^{1+\lambda}}, \quad (3.26)$$

which supports the self-similarity of the power-like enhancement at low $x$ with respect to the feed-back. This dependence holds both for quarks and gluons. If $\ln(s/\Lambda^2)$ is considered to be parametrically large, then the leading terms are as follows,
\[
\left\{ \frac{p_t^2}{g^2(p_t^2)} \frac{d\xi(x, p_t^2)}{dp_t^2} \right\}_{f.b.} = \frac{\pi g^2(p_t^2)}{(2\pi)^6} \frac{1}{x^{1+\lambda}} \ln \frac{s}{\Lambda^2} \left[ \frac{C_F^2}{\lambda + 2} \kappa + \frac{N_c}{4(\lambda + 2)(\lambda + 3)} \gamma \right], \quad (3.27)
\]

\[
\left\{ \frac{p_t^2}{g^2(p_t^2)} \frac{dG(x, p_t^2)}{dp_t^2} \right\}_{f.b.} = \frac{\pi g^2(p_t^2)}{(2\pi)^6} \frac{1}{x^{1+\lambda}} \ln \frac{s}{\Lambda^2} \left[ - \frac{C_F T_1 n_f}{(\lambda + 1)(\lambda + 2)} \kappa + \frac{N_c^2(\lambda^2 + \lambda + 2)}{4(\lambda + 1)(\lambda + 2)(\lambda + 3)} \gamma \right]. \quad (3.28)
\]

The feed-back is positive along the string \( G \rightarrow \mathcal{G} \rightarrow G \) and negative along the strings involving quarks, like \( q \rightarrow \mathcal{G} \rightarrow G, \ q \rightarrow Q \rightarrow G, \ etc. \) The gluons tend to boost the rate of their own evolution, while the quarks slow the gluon evolution down. Evolution of quarks is always boosted by the feed-back. Though Eqs. (3.27), (3.28) contain an infra-red regulator, \( \ln s/\Lambda^2 \), which is inconsistent both with the philosophy and the technical design of the infinite momentum frame, it looks as though the potential source of the low-\( x \) enhancement in deep inelastic processes is found correctly. The QCD evolution of observables in high-energy inclusive processes includes the evolution of classical (longitudinal) quark and gluon fields which can be responsible for the enhancement. The power-like behavior at low \( x \) has been predicted long ago by Balitsky, Fadin, Kuraev, and Lipatov from the solution of the BFKL equation \[4\] and the exponent \( \lambda \) was explicitly found in the case of pure glue-dynamics. Considerable work is needed to find this exponent in our approach. Furthermore, it is not yet clear if these two approaches rely on exactly the same physical input.

Our conclusion about the power-like enhancement at low \( x \) is very close to that of McLerran et al. \[3\] but it is motivated in a different way. First, we neither employ nor even need the valence quarks as the classical source of the gluon field. In fact, we keep in mind that the strongest gluon fields are due to the vacuum condensates which exist even in the absence of hadrons. In the state of confinement, neither hadrons nor nuclei drag the glue; they propagate through it. Second, the coupling between the static and the propagating fields only weakly depends on transverse momentum, only via \( \alpha_s(p_t^2) \). Stronger dependence might come from the propagators, but the longitudinal modes do not propagate. The dynamics of the measurement always leads to induced static fields which have a steep behavior at low \( x \) but are integrated over all \( p_t \). For this reason, accurate measurement of the longitudinal structure function \( F_L \), which is directly connected to the static component of the quark field, is of extreme importance.

The classical pattern of the light-front evolution poses a severe problem. It is well known that energy-momentum conservation can be obtained either in quantum or in classical theory of radiation. Quantum theory in the presence of classical external field does not allow for the consistent formulation of the momentum conservation since even the transverse classical field can be presented at most as a superposition of states with various numbers of quanta in every mode (coherent states). Longitudinal fields are always “external” and they are not even a subject for quantization. Therefore, we have to admit that the equations of the light front QCD evolution cannot have the first integral of the light-cone momentum. However, momentum conservation plays an important role in the derivation of the AP evolution equations, being solely responsible for the so-called + prescription which regulates the collinear singularities of the splitting kernels. With momentum conservation the parton model is well motivated. Otherwise, we face the problem of identifying the subject of the QCD evolution itself.

### C. Evolution of classical fields and the structure function \( F_L \)

To leading order, the standard evolution equations completely disregard the dynamics of the longitudinal fields in the QCD evolution, thus missing a physically important part of the dynamical process. As it follows from the expression (3.37) for the longitudinal structure function \( F_L \), the function \( Q \) is solely responsible for the scaling violation of \( F_L \) when the quarks are massless. Eq. (3.16) indicates that \( Q \) depends on \( p_t \) only via the running coupling \( g_r(p_t^2) \sim 1/\ln(p_t^2/\Lambda^2) \). Therefore, it is easy to estimate the \( Q^2 \)-dependence of the non-singlet longitudinal structure function \( F_L \) for the case of massless quarks,

\[
Q^2 F_L \sim \int_0^{4Q^2} \frac{dp_t^2}{\ln(p_t^2/\Lambda^2)} = \Lambda^2 \text{li}(\frac{4Q^2}{\Lambda^2}) = -\Lambda^2 E_1\left(-\ln\left(\frac{4Q^2}{\Lambda^2}\right)\right) \approx \frac{4Q^2}{\ln(4Q^2/\Lambda^2)} \sim Q^2 \alpha_s(Q^2),
\]

where \( \text{li}(x) \) is the integral logarithm. Therefore, \( F_L \) is not strongly suppressed (i.e., by the powers of \( \Lambda^2/Q^2 \)) with respect to \( F_2 \). In the standard OPE approach, the non-singlet \( F_L \) appears only in the next-to-leading order. Indeed, if the longitudinal fields were eliminated from the initial data by putting the partons on mass shell in a perturbative vacuum, at least one extra emission is needed to regenerate the longitudinal component at the level of the coefficient functions. For this reason, the OPE-based result contains additional \( Q^2 \)-dependence due to the scaling violation of the \( F_2(x, Q^2) \) and \( G(x, Q^2) \). The OPE-based predictions for large \( x \), when one might expect the strongest effect from the longitudinal fields, systematically lie below the data. In our case, according to Eqs. (3.14) – (3.18), the geometry of
the static fields depends on the overall radiation above the currently probed value of the \( x_p \); the entire range of \( Q^2 \) is integrated. An active involvement of the longitudinal (classical) fields in the QCD evolution explains the enhancement of the longitudinal structure function \( F_L \) due to the quark masses, the first two terms in Eq. (2.37); the heavier the quark, the more static is its field.

### IV. MASTER EQUATIONS OF QCD EVOLUTION

Equations of motion of the quark and gluon fields do not allow one to eliminate longitudinal fields from the evolution equations by fiat. At most, one can use a renormalization procedure which absorbs their effects into the definition of some composite objects. The simplest objects of this kind are known as the particles in asymptotic states. Once this normalization is done, the only consistent way to proceed is to treat the entire problem as a problem of scattering. To comply with quantum mechanics, this parameter has to be explicitly measured. Moreover, this measured value has to be used in theoretical calculations! The recently discovered HERA events with rapidity gaps are good candidates to allow for this kind of measurements.

This qualitative analysis prompts further approximations. In order to obtain the master equations, \( \sigma_0 \) and \( w_2 \) must be eliminated from the equations (3.9)–(3.13) without any discussion of the accuracy. After that, one can express the static fields depends on the overall radiation above the currently probed value of the \( x_p \); the entire range of \( Q^2 \) is integrated. An active involvement of the longitudinal (classical) fields in the QCD evolution explains the enhancement of the longitudinal structure function \( F_L \) due to the quark masses, the first two terms in Eq. (2.37); the heavier the quark, the more static is its field.

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This qualitative analysis prompts further approximations. In order to obtain the master equations, \( \sigma_0 \) and \( w_2 \) must be eliminated from the equations (3.9)–(3.13) without any discussion of the accuracy. After that, one can express \( \sigma_0 \), \( \sigma_m \), and \( w_1 \) via the observables of DIS, and integrate the equations over \( p^− \). We end up with two integral equations for quarks,

\[
\left\{ \frac{dq_f(x, p_z^2)}{dp_z^2} \right\}_{LL} = \int \frac{dy}{y} \int d^2k \left\{ C_R \left( \frac{1}{H(\vec{p}, \vec{k}, z) + (1 - z)m_f^2} \right) \right. \\
\left. \frac{y^2(z^2 + 1)}{2z} \frac{(k_z^2 + m_f^2)(1 + z^2) + 2m_f^2(1 - z)}{[H(\vec{p}, \vec{k}, z) + (1 - z)m_f^2]^2} \right. \\
\left. \frac{dq_f(y, k_z^2)}{dk_z^2} - \frac{2m_f^2z(1 - z)^2}{[H(\vec{p}, \vec{k}, z) + (1 - z)m_f^2]^2} \frac{dM_f(y, k_z^2)}{dk_z^2} \right. \\
\left. + \left[ (1 - z)^2 + z^2 \right] \frac{1}{H(\vec{p}, \vec{k}, z) + m_f^2} \frac{z(1 - z) k_z^2}{[H(\vec{p}, \vec{k}, z) + m_f^2]^2} \right) \\
\left. \frac{1}{2} \frac{m_f^2 z(1 - z)}{[H(\vec{p}, \vec{k}, z) + m_f^2]^2} \frac{dG(y, k_z^2)}{dk_z^2} \right) \\
\right\}, \quad (4.1)
\]

and one equation for gluons,

\[
\left\{ \frac{dG(x, p_z^2)}{dp_z^2} \right\}_{LL} = \int \frac{dy}{y} \int d^2k \left\{ T_f \sum_{j} \left[ \left( \frac{-1}{H(\vec{p}, \vec{k}, z) + zm_f^2} \right) \left( \frac{1 - z)^2 + 1}{H(\vec{p}, \vec{k}, z) + zm_f^2 + 1} \right) \right. \\
\left. + \frac{2z(1 - z)m_f^2}{[H(\vec{p}, \vec{k}, z) + zm_f^2]^2} \frac{dq_f(y, k_z^2)}{dk_z^2} \right. \\
\left. + \frac{2z^2(1 - z)m_f^2}{[H(\vec{p}, \vec{k}, z) + zm_f^2]^2} \frac{dM_f(y, k_z^2)}{dk_z^2} \right. \\
\left. + N_c \left( \frac{1 - z}{z} + \frac{z}{1 - z} + (1 - z) \right) \frac{1}{H(\vec{p}, \vec{k}, z)} \frac{dG(y, k_z^2)}{dk_z^2} \right) \\
\right\}. \quad (4.2)
\]

where \( H(\vec{p}, \vec{k}, z) = (z\vec{p} + \vec{k})^2 + (1 - z)p_0^2 \). The subscript LL stands for the Leading Logarithms, the destination point of our study. It indicates that the rate of the evolution is considered with the switched off static fields. The further simplification of these equations will result in the DGLAP equations. One can easily see that Eq. (4.2) describes evolution of the dynamical quark mass (in fact, \( M \) is the imaginary part of the “pole mass” which is defined by
the retarded self-energy of the quark). Negative signs on the right side of Eq. (4.2) indicate that evolution of the effective mass leads to a faster decrease, the higher the $Q^2$ which is probed. This trend clearly supports our intuitive understanding of the interplay between the quark mass and the transverse momentum transfer. By examination, Eq. (4.2) is of the same type as the other two evolution equations and the solution for $M_f$ must be the standard logarithmic exponent. There is no reason to neglect $M_f(x, p_t^2)$, unless $m_f^2 \ll p_t^2$.

Equations (4.1)–(4.3) are still more complicated than the DGLAP equations. Unlike the DGLAP equations, they are the integral equations for the rates of evolution and exhibit non-locality in transverse directions. The kernels of these equations are singular; except for the familiar spurious poles, $z = 1$, of the splitting functions, the nonlocal part of the kernels contains overlapping infrared singularities at $z = 1$ and $k = p$. As discussed above, this behavior is quite consistent with the crude picture of the infinite momentum frame. Its smoothing requires a physical cut-off, which can be natural only in a more gentle picture. The cut-off brings into play new large logarithms, like $\log(s/\Lambda^2)$, which are alien to the dynamics of the infinite momentum frame and cannot really be estimated by its internal means. Even $x_F$, the main variable of the parton model, has to be sacrificed in order to incorporate the energy of the collision as a physical parameter of the theory. In fact, this parameter is vitally needed in order to bring into the theory the Lorentz contraction of the colliding hadrons as the measure of the initial resolution in the longitudinal direction.

To eliminate the need for the infrared cut-off, once again, by ordinance, one should follow the strategy of hunting the leading logarithms, either $\log(Q^2/\Lambda^2)$, or $\log(1/x)$. As it was shown in the previous section, it is inconsistent to address the low-$x$ behavior without explicit account of the longitudinal fields (however, they are already neglected in Eqs. (4.1)–(4.3)). In order to keep track of the leading logarithms $\log(Q^2/\Lambda^2)$, one should find a solution to the scattering problem in the Born’s approximation and exponentiate it. This is known also as ordering by angles or exponentiation of the kernels contains overlapping infrared singularities at $z = 1$ and $k = p$. As discussed above, this behavior is quite consistent with the crude picture of the infinite momentum frame. Its smoothing requires a physical cut-off, which can be natural only in a more gentle picture. The cut-off brings into play new large logarithms, like $\log(s/\Lambda^2)$, which are alien to the dynamics of the infinite momentum frame and cannot really be estimated by its internal means. Even $x_F$, the main variable of the parton model, has to be sacrificed in order to incorporate the energy of the collision as a physical parameter of the theory. In fact, this parameter is vitally needed in order to bring into the theory the Lorentz contraction of the colliding hadrons as the measure of the initial resolution in the longitudinal direction.

Thus, the angular ordering results in drastic simplification of the evolution equations. Only the splitting kernels $P_{qg}$ and $P_{gg}$ are singular now. However, we cannot regularize these singularities by means of the $(+)$-prescription as is done in AP equations. Indeed, in the AP approach the $(+)$-prescription maintains two conservation laws, conservation of the light cone charge, $j^+$, and conservation of the light-cone component of the momentum, $p^+$, in the process of sequential splitting of the parton. The parton is supposed to be free and no interaction terms are included into the operators $j^+$ and $p^+$. Though the dynamical mass $M_f$ of the fermion does not enter these operators, they are independent partners of the parton densities $q$ and $G$ in the evolution equations. Unless $m_f = 0$, the conservation laws do not follow from the $(+)$-prescription. This fact has a very clear physical interpretation. Though the wave equation for the massive quark is of hyperbolic type and allows for the propagation of signals along the light-cone characteristics, these signals cannot correspond to the on-mass-shell states of heavy quarks. Only dynamical polarization waves which include massive fermion fields can propagate at speed of light. Even though we can include the effect of quark masses into the gluon propagator and, eventually, into the running coupling, this will lead to an excess of accuracy if the dynamical mass term $M_f$ is not included into the evolution equations.

Our result for the running coupling (21) coincide with the one obtained by Dokshitser and Shirkov [1],

$$\frac{1}{g_f^2(\mu^2)} - \frac{1}{g_f^2(M^2)} = \frac{1}{16\pi^2} \left[ -N_c \frac{11}{6} \ln \frac{M^2}{\mu^2} + \sum_f T_f \frac{1}{3} \left[ \ln \frac{M^2}{\mu^2} + f(\frac{\mu^2}{m_f^2}) - f(\frac{M^2}{m_f^2}) \right] \right].$$

(4.7)
where the function \( f \) is obtained from the one-loop gluon self-energy with the subtraction at an arbitrary space-like momentum \( p^2 < 0 \),

\[
f\left( \frac{p^2}{m^2} \right) = \ln \frac{-p^2}{2m^2} - \frac{4m^2}{-p^2} + (1 - \frac{2m^2}{-p^2}) \sqrt{1 + \frac{4m^2}{-p^2}} \ln \frac{\sqrt{1 + \frac{4m^2}{-p^2}} - 1}{\sqrt{1 + \frac{4m^2}{-p^2}} + 1} ,
\]

and has the following limiting behavior in different domains of the transverse momentum,

\[
f\left( \frac{p^2}{m^2} \right) \sim \mathcal{O}\left( \frac{m^2}{p^2} \right) , \quad \left| \frac{p^2}{m^2} \right| \gg 1; \quad f\left( \frac{p^2}{m^2} \right) \rightarrow \ln \frac{-p^2}{2m^2} , \quad \left| \frac{p^2}{m^2} \right| \ll 1.
\]

These formulae smoothly interpolate running coupling between the domains with different transverse momenta and provide different numbers of “active flavors”. However, it is important to emphasize that with the quark masses retained, the dynamical quark mass should be kept in the evolution equations and we still face problem of the infrared regularization of the evolution equations.

V. SUMMARY

The method of QFK allows one to extend the standard definition of the evolution equations beyond the scope of the parton model. The extended equations include new elements, the static (longitudinal) gluon field and the static (non-propagating) quark fields. The quark masses are accounted for also and the dynamical mass term in the evolution equations is sensitive to the change of scale in the same way as the quark and gluon structure functions. Though the role of the new terms is perfectly understood physically, the extended set of equations suffers from the infrared-singular terms which cannot be regulated in the standard scheme where the proton is analyzed in the infinite momentum frame.

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