Mathematical model for studying the braking distance of a car equipped with ABS in winter conditions

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Abstract. The article aims to develop a mathematical model for analytical studies of the braking process of a car equipped with an anti-lock braking system in order to identify the relationship between the stopping and braking distances of a car and pedestrian visibility in dark winter conditions, when headlights are contaminated with anti-icing materials. The mathematical model of the braking process of a car equipped with ABS was analyzed. Experimental studies of the braking process of a car with running and disabled ABSs were carried out on winter roads. The experimental studies of headlight contamination with chemical anti-icing materials (CAIM) and changes in luminous intensity were carried out in real road conditions in winter. The results are presented in the article. The results of the experimental studies of pedestrian visibility in dark conditions with contaminated headlights are presented. The adequacy of the mathematical model of the braking distance of a car with running and disabled ABSs in winter conditions was assessed. The trends characterizing the effect of headlight contamination on the pedestrian visibility in dark winter conditions are described. The results of the study are presented in tables, diagrams and equations. The values of the stopping distance were determined at different initial braking speed rates. The visibility distance for a car with contaminated headlights was determined in dark winter conditions.

1. Introduction
The task of improving vehicle safety on winter roads is crucial. According to the official statistics of the State Traffic Safety Inspectorate of the Russian Federation, every third traffic accident occurs at night [1].

The climate of many Russian regions is a factor of slippery roads. In winter, most of the roads are covered with snow and ice. One of the methods for eliminating snow-ice deposits (SID) is the use of CAIM [2]. At low temperatures, a “sandwich” from ice, snow, anti-icing materials and salt brine is formed on the roads [3].

In conditions of heavy traffic, anti-icing materials mixed with snow, salt brine and dirt are risen into the air by wheels and form an aerosol suspension contaminating car headlights. In dark conditions, the road illumination and pedestrian visibility are reduced.

The car has no systems informing drivers about the level of headlight contamination. Reduced visibility can be assessed intuitively and subjectively. This problem is aggravated by a low coefficient of tire grip to the road covered with CAIM which increases the braking distance [4,5]. These factors reduce vehicle safety on winter roads covered with CAIM in dark conditions. This problem is
especially acute on winter roads of Siberia with a sharply continental climate and a daily temperature difference of 20-25 °C.

Using the developed mathematical model and experimental methods, the article aims to study the braking distance of cars on winter roads. The trends of the influence of headlight contamination on pedestrian and road infrastructure visibility are presented. The stopping distance of a car equipped with an anti-lock braking system is analyzed at varying initial braking speed rates.

2. Materials and methods

An analysis of papers studying the influence of CAIM on vehicle safety and operational properties shows that the effect of CAIM on the headlight contamination is not taken into account. Their influence on the road illumination, road infrastructure visibility and vehicle safety on dark winter roads is not taken into account [3, 6].

The results of 2018 and 2019 experimental studies conducted by the authors on winter Irkutsk roads covered with CAIM were published earlier. TOYOTA FUN CARGO and NISSAN QASHQAI were used in the experiments [7].

The results formed the basis of the method which can be used to study the effect of headlight contamination with CAIM on the possibility of stopping the car in front of an obstacle.

It should be noted that the braking distance is a component of the stopping distance along with the distance of the driver’s response and the response time of the brake system (Fig. 1).

![Figure 1. Components of the stopping distance](image.png)

An experimental study of the braking performance was carried out on winter roads with a low grip coefficient, both with running and disabled ABS using TOYOTA FUN CARGO with a set of winter non-studded TOYO STUDLESS GARIT G4, 185/65 R14 tires. The studies were carried out on packed and not packed snow at a temperature of -20-25 °C, using the Effect device at initial speed rates of 10, 20, 40, 60 km/h [8]. The results are presented in Figure 2.
Figure 2. The braking distance graphs for Toyota Fun Cargo car with 185/65 R14 TOYO STUDLESS GARIT G4 tires on packed snow roads at -25 °C: The upper graph - with the working ABS; the bottom graph - with the ABS disabled; ◆ experiment; —— calculation

An approximating dependence of the stopping distance of a car with a running ABS on the initial braking speed $V_0$ was determined for the packed snow surface.

$$S_{T, ABS} = 0,0099 \cdot V_0^2 + 0,1587 \cdot V_0$$

(1)

where $S_{b, ABS}$ – the braking distance of a car equipped with ABS.

The approximating dependence (1) was obtained at the confidence approximation coefficient $R^2 = 0,99$.

When a car with a disabled ABS is braking, the length of its stopping distance $S_T$ can be described by approximating expression

$$S_{b, ABS} = 0,0083 \cdot V_0^2 + 0,1183 \cdot V_0$$

(2)

The approximating dependence (2) was obtained at the confidence approximation coefficient $R^2 = 0,99$.

The studies were conducted to analyze the braking performance of vehicles on winter roads covered with CAIM. For this purpose, the Automobile Transport department of Irkutsk National Research Technical University developed a mathematical model which presents the moving mass of the car as an oscillating system with a sprung mass in the form of a solid body with six degrees of freedom (Fig. 3, 4, 5).

Figure 3. Design diagram of the car braking process (XOZ)
The equations describing the movement in space are built in relation to the moving (X, Y, Z) and fixed (X', Y', Z') coordinate systems. The fixed coordinate system X', Y', Z' is rigidly fixed to the road. The plane X, O, Y of the moving coordinate system lies in plane X', O, Y' and moves along it together with the car. The position of the moving system X, Y, Z with respect to the fixed system X', Y', Z' is determined by coordinates x', y' and angle γ of its rotation relative to the OZ axis.

The car body is connected with the moving coordinate system X, Y, Z so that its center of mass can perform only linear movements along the OZ axis, and the sprung mass M can rotate relative to the axes OX - (by the roll angle) and OY - (by the differential angle). Moreover, the roll center coincides with the mass center.

The position of the car body relative to the moving coordinate system (X, Y, Z) is determined by the coordinates:

α – the angle of rotation of the car body relative to the OY axis - (trim);

β – the angle of rotation of the car body relative to the OX axis - (roll);

z – the coordinate of the center of mass relative to the OZ axis.

The car suspension is shown in the form of stiffness C and shock absorbers having asymmetric compression and rebound characteristics. Damping of shock absorbers was determined by coefficients $K_c$ – compression and $K_r$ - rebound.

The differential equations of movement of the body mass written in relation to the moving X, Y...
and Z axes are as follows:

\[
d^2x\over dt^2 = \frac{R_{11} \cdot \cos \theta_1 + R_{12} \cdot \cos \theta_2 + R_{21} + R_{22} + R_{y_{11}} \cdot \sin \theta_1 + R_{y_{12}} \cdot \sin \theta_2}{M} \cdot \omega_z^2
\]

\[
d^2y\over dt^2 = \frac{R_{11} \cdot \cos \theta_1 + R_{12} \cdot \cos \theta_2 + R_{y_{11}} \cdot \sin \theta_1 - R_{y_{12}} \cdot \sin \theta_2}{M} \cdot \omega_z^2
\]

\[
d^2z\over dt^2 = \frac{F_{Z11} + F_{Z12} + F_{Z21} + F_{Z22} - M \cdot g}{M}
\]

where:  
- \( R_{11}, R_{12} \) and \( R_{y_{11}}, R_{y_{12}} \) – projections of the longitudinal reactions of the right and left front wheels on the X and Y axes;  
- \( R_{21}, R_{22} \) and \( R_{y_{21}}, R_{y_{22}} \) – projections of the longitudinal reactions of the right and left rear wheels on the X and Y axes;  
- \( F_{Z11}, F_{Z12}, F_{Z21}, F_{Z22} \) – forces in shock absorbers and elastic suspension elements of each wheel;  
- \( M \) – mass;  
- \( \theta_1 \) and \( \theta_2 \) – angles of rotation of the right and left wheels.  

The differential equations of the angular movement of the mass relative to X, Y, and Z axes after solving them with respect to the highest derivatives:

\[
\frac{d^2 \alpha}{dt^2} = \frac{\{(F_{Z22}+F_{Z21}) \cdot b \cdot (F_{Z12}+F_{Z11}) \cdot a + (R_{y_{11}} \cdot \cos \theta_1) + R_{y_{12}} \cdot \cos \theta_2 + R_{y_{21}} + R_{y_{22}} + R_{y_{11}} \cdot \sin \theta_1 + + R_{y_{12}} \cdot \sin \theta_2 \cdot (h_g + Z) \}/J_y}{2}
\]

\[
\frac{d^2 \beta}{dt^2} = \frac{\{(F_{Z12} - F_{Z11}) \cdot S_x/2 + (F_{Z22} - F_{Z21}) \cdot S_y/2 + (R_{y_{11}} \cdot \cos \theta_1) + R_{y_{12}} \cdot \cos \theta_2 + R_{y_{21}} + R_{y_{22}} + R_{y_{11}} \cdot \sin \theta_1 - R_{y_{12}} \cdot \sin \theta_2 \cdot (h_g + Z) \}/J_x}{2}
\]

\[
\frac{d^2 \gamma}{dt^2} = \frac{\{(R_{y_{11}} \cdot \cos \theta_1) + R_{y_{12}} \cdot \cos \theta_2 - R_{y_{21}} \cdot \sin \theta_1 - R_{y_{12}} \cdot \sin \theta_2 \cdot a + (R_{y_{21}} \cdot \sin \theta_2 - R_{y_{11}} \cdot \sin \theta_1 + R_{y_{12}} \cdot \cos \theta_2 - R_{y_{11}} \cdot \cos \theta_1) \cdot S_z/2 - (R_{y_{21}} + R_{y_{22}}) \cdot b + (R_{y_{22}} - R_{y_{21}}) \cdot S_z/2 \}/J_z}{2}
\]

where:  
- \( J_x, J_y, J_z \) – axial moments of inertia relative to the x, y, z axes of the sprung mass (body);  
- \( a, b \) – the distance from the centers of the front and rear axles to the center of mass;  
- \( S_x, S_y \) – tracks of the front and rear wheels;  
- \( h_g \) – the distance from the center of mass to the road surface.  

The forces in the elastic elements of the suspension and in the shock absorbers were determined through the deformation rates of the elastic elements. The deformation rates of each elastic element are represented by equations taking into account the angular speed of the roll and defenter with respect to x and y axes, as well as the speed of movement of the center of mass along the Z axis:

\[
V_{11} = \frac{\alpha}{2} + \frac{\beta}{2} - \frac{dz}{dt}
\]

\[
V_{12} = \frac{\alpha}{2} - \frac{\beta}{2} + \frac{dz}{dt}
\]
The deformations of the elastic elements of the suspension were determined by equation:

\[ Z_{ij} = Z_{ij-1} + V_{ij} \Delta t \]  

(6)

where: \( Z_{ij} \) - deformations of elastic elements at a time \( t \);
\( Z_{ij-1} \) - deformations of elastic elements at a previous time \( t-1 \);
\( i \) - vehicle axis index: 1 - front, 2 - rear;
\( j \) - wheel axis index: 1 - front, 2 - rear;
\( \Delta t = 0.001 \) s - step of integrating differential equations.

The forces in the shock absorbers and elastic elements of the suspension were determined taking into account the speed rates and deformations of the suspension and equations (5) and (6):

\[ F_{ij} = C_{ij} \ddot{z}_{ij} + K_{ij} \dot{z}_{ij} \]  

(7)

where: \( C_{ij} \) - stiffness of the elastic elements of the car suspension;
\( K_{ij} \) - shock absorber damping coefficients; they change their values depending on the suspension path (compression \( K_{cij} \) or rebound \( K_{rij} \)).

The moving (X, Y, Z) and fixed (X', Y', Z') coordinate systems are connected by equations

\[ V_x = \dot{x} \cdot \cos \omega_z - \dot{y} \cdot \sin \omega_z \]

\[ V_y = \dot{x} \cdot \sin \omega_z - \dot{y} \cdot \cos \omega_z \]  

(8)

where: \( V_x \) - absolute speed of the center of mass relative to the X' axis;
\( V_y \) - absolute velocity of the center of mass relative to the Y' axis.

The wheel slip angles were determined by equations (9). The front wheel slip angles were determined taking into account the rotation angles of the wheels \( \theta_1 \) and \( \theta_2 \).

\[ \delta_{11} = -\arctan \left[ \frac{\dot{y} + \omega_z \cdot a}{\dot{x} + \omega_z \cdot S_1 / 2} \right] + \theta_1 \]

\[ \delta_{12} = -\arctan \left[ \frac{\dot{y} + \omega_z \cdot a}{\dot{x} - \omega_z \cdot S_1 / 2} \right] + \theta_2 \]

\[ \delta_{21} = -\arctan \left[ \frac{\dot{y} - \omega_z \cdot b}{\dot{x} + \omega_z \cdot S_2 / 2} \right] \]

\[ \delta_{22} = -\arctan \left[ \frac{\dot{y} - \omega_z \cdot b}{\dot{x} - \omega_z \cdot S_2 / 2} \right] \]  

(9)

When choosing a mathematical model of the elastic tire, its peculiarity was taken into account - the shift (during braking) of the maximum of the longitudinal reaction \( R_x \) with an increase in the slip angle \( \delta \) to the greater slippage \( S \) (Fig. 6). The operation of the anti-lock system was simulated. The ABS searches for maximum braking forces during the critical tire slippage and maintains the braking process at their level. Figure 6 shows the shift of critical slip \( S_c \) at which the flat spot is disrupted, if the wheel moves with a slip angle. It can vary from 0.1 to 1.0.

Therefore, to describe the output characteristics of the elastic tire, the mathematical model developed by A.B. Dick [4,5] was applied. In the projection model of \( R_c \) and \( R_y \), the longitudinal reaction of the braking wheel was determined using the normed slip function [4,5]:

\[ f(s) = \sin \{ a_1 \cdot \arctan(b_1 \cdot S) \} \]  

(10)

The normed function accurately describes stationary characteristics (Fig. 5) of car tires determined by processing the experimental grip characteristics.
Figure 6. Dick’s graphs of stationery characteristics of the 185/65 R14 TOYO STUDLESS GARIT G4 tire ($R_z = 3000\text{H}$), 1 – slip angle $\delta = 0^\circ$; 2 – slip angle $\delta = 5^\circ$; 3 – slip angle $\delta = 10^\circ$; 4 – slip angle $\delta = 15^\circ$; 5 – slip angle $\delta = 20^\circ$; 6 – slip angle $\delta = 25^\circ$; 7 – slip angle $\delta = 30^\circ$; 8 – slip angle $\delta = 35^\circ$.

The power radius of the wheel (the rolling radius in the driven mode) $r_w$ was determined taking into account the normal reaction $R_z$ acting on the tire and the radius $r_c$ of the unloaded wheel [4]:

$$R_w = r_c - (R_zC_1 + \sqrt{R_zC_1C_2}),$$

(11)

where $C_1$ and $C_2$ – coefficients adjusting the force radius function.

The total longitudinal reaction of the brake wheel in the contact spot was calculated by formula [4]:

$$R_\Sigma = R_z\phi_{\text{max}}f(s)$$

(12)

The projections of the longitudinal reaction of $R_x$ and $R_y$ on the x and y axes were determined as [4]:

$$R_x = R_\Sigma m_{\text{br}} S_x / S$$

$$R_y = R_\Sigma m_{\text{br}} S_y / S$$

(13)

The angular acceleration of the wheel was determined from the dynamic equilibrium equation:

$$\frac{d\omega_k}{dt} = \frac{R_x \cdot r_w - M_f - M_f}{J_k}$$

(14)

where: $M_f$ - the braking torque;
$J_k$ - the moment of inertia.

To describe the operation of the brake mechanism, we used a mathematical model that takes into account

- hysteresis losses in the brake;
- its inertia (phase delay operation);
- deadband of the brake depending on the gap in the friction pair;
- changes in the braking torque depending on the relative speed of movement of the elements of the friction pair.

The model is simple and accurate.

The model describes the inertia (phase response delays) of the brake using the first order dynamic link equation:

$$T_{\text{mm}} \cdot \Delta P_{\text{mm}} = P_m - P_{\text{mm}},$$

(15)

where: $T_{\text{mm}}$ – a disc brake time constant;
$P_{\text{mm}}$ – fluid pressure in the brake;
$P_m$ – fluid pressure at the brake inlet;
$\Delta P_{\text{mm}}$ – the rate of changes in fluid pressure pressure in the brake.
The nonlinear characteristics (dependences $M_i = f (P_{mu})$) were described by the piecewise linear approximation method. It takes into account the zone of insensitivity of the brake mechanism to fluid pressure and the braking torque decreased to the grip moment when the wheel is blocked.

$$M_i = \begin{cases} 0, & \text{if } \Delta P_{mu} > 0 \text{ and } \Delta P_{mu} > \Delta_0 \text{ or } \Delta P_{mu} \leq 0 \\ K_1 \cdot (P_{mu} - \Delta_0), & \text{if } \Delta P_{mu} > 0 \text{ and } K_1 \cdot (P_{mu} - \Delta_0) - M_{mp} > 0 \\ K_2 \cdot P_{mu}, & \text{if } \Delta P_{mu} \leq 0 \text{ and } K_2 \cdot P_{mu} - M_{mp} < 0 \\ M_{mu}, & \text{if } \Delta P_{mu} > 0 \text{ and } K_1 \cdot (P_{mu} - \Delta_0) - M_{mp} < 0 \end{cases} \quad (16)$$

where $\Delta_0$ – the zone of insensitivity of the brake mechanism to the increasing fluid pressure;

$M_{mu}$ and $M_{mp}$ – the braking torque during changes in a sign of the first pressure derivative in the actuator;

$K_1$ and $K_2$ – the rates of increase and decrease in the braking torque.

Rates $K_1$ and $K_2$ of increase and decrease in the braking torque are adjusted by changing the speed of the relative movement of the friction pair, or depending on the angular speed of wheel rotation:

$$K_1 = K_{10} - K_{\omega_k} \cdot \omega_k \quad (17)$$

$$K_2 = K_{20} - K_{\omega_2} \cdot \omega_k \quad (18)$$

where $K_{10}$ and $K_{20}$ – the coefficients of increase and decrease in the rate of changes in the braking torque at $\omega_k \to 0$;

$K_{\omega_k}$ and $K_{\omega_2}$ – the coefficients of adjustment of the rate of changes in the braking torque from the angular velocity of the wheel $\omega_k$.

The mathematical model simulates executive commands to trigger the ABS pressure modulator, depending on slip $S$ and angular acceleration $\omega_k$ of the elastic tire of the braking wheel.

The electronic control unit (ECU) opens and closes the solenoid shutdown and drain valves of the modulator using commands A and B. This changes fluid pressure in the brake drive. Initially, the pressure shutdown valve A is open, and the drain valve B is closed.

With a three-phase ABS operation algorithm, the mathematical description of the ECU operation is as follows:

- an increase in pressure: $\frac{dP_{mu}}{dt} > 0 \text{ at } \begin{cases} A = 0 \\ B = 0 \end{cases}$, if $S < S_y$ and $\omega_k < 0$, (19)
- a decrease in pressure: $\frac{dP_{mu}}{dt} < 0 \text{ at } \begin{cases} A = 1 \\ B = 1 \end{cases}$, if $S \geq S_y$ and $\omega_k < 0$, (20)
- pressure holding: $\frac{dP_{mu}}{dt} = 0 \text{ at } \begin{cases} A = 1 \\ B = 0 \end{cases}$, if $S \geq S_y$ and $\omega_k \geq 0$, or if $S < S_y$ and $\omega_k \geq 0$, (21)

where $\omega_k$ – angular acceleration.

The EC determines values $S, S_y, \omega_k$ with errors and time delays whose values depend on the type of sensors, characteristics of the ABS computer, etc. All these time delays take into account the mathematical description of the modulator. At the same time, the operation of the computer was modeled without delays.
Thus, the mathematical model of fluid pressure pressure in the braking mechanism drive calculates pressure changes with time delays. These delays have a significant impact on the efficiency of the braking process of a car equipped with a running ABS.

The time delays are calculated by equations

\[
\begin{align*}
\tau_m &= \tau_m' + \tau_m'', \\
\tau_c &= \tau_c' + \tau_c'', \\
\tau_a &= \tau_a' + \tau_a'',
\end{align*}
\] (22)

where \( \tau_m, \tau_c, \tau_a \) – total time delays of the computer and the modulator for an increase, reset and pressure stabilization;

\( \tau_m', \tau_c', \tau_a' \) – ECU-related delays;

\( \tau_m'', \tau_c'', \tau_a'' \) – modulator-related delays.

Figure 7 is a diagram illustrating the functioning of fluid pressure modulator by ABS commands and pressure delays in after the computer commands have been issued. The mathematical description of the pressure modulator was developed.

Figure 7. The pressure modulator schedule for three-phase ECU commands

An increase in \( K_t \) and a pressure release \( K_c \) were taken constant. They can be changed in the initial conditions of the calculation program. The mathematical description of fluid pressure pressure modulator is as follows:

\[
P_m = \begin{cases} 
P_{m_1} + K_m \cdot t_m, & \text{if } A = 0, \ B = 0, \ t_m > 0 \\
P_{m_1} - K_c \cdot t_c, & \text{if } A = 1, \ B = 1, \ t_p > 0 \\
P_{m_1}, & \text{if } A = 1, \ B = 0, \ t_p > 0 \\
P_{m_{\text{max}}}, & \text{if } P_m \geq P_{m_{\text{max}}} \\
P_{m_{\text{max}}}, & \text{if } P_m \leq P_{m_{\text{max}}}
\end{cases}
\] (23)

\[
\begin{align*}
t_m &= t_i - t_{nm} - \tau_m' \\
t_p &= t_i - t_{np} - \tau_c' \\
t_c &= t_i - t_{nc} - \tau_b'
\end{align*}
\] (24)

where \( P_{m_1} \) and \( P_{m_2} \) – pressure values when changing the sign of its first derivative;
$P_{\text{max}}$ and $P_{\text{min}}$ – maximum and minimum pressure values in the actuator when $\frac{dP_{\text{nu}}}{dt} = 0$;

$K_\text{m}$ and $K_\text{c}$ – the rates of pressure increase and release;

$t_\text{m}$, $t_\text{p}$, $t_\text{c}$ – current values of time of pressure increase, release and holding;

$t_\text{i}$ – the time coordinate of changes in pressure in the fluid drive of the braking mechanism;

$t_{\text{it}}, t_{\text{ips}}, t_{\text{ic}}$ – moments of time for issuing computer control commands to dump, lift and stabilize pressure.

The mathematical description of the braking process for a car equipped with ABS has a large number of nonlinear differential equations solved by computer numerical methods. The method of numerical integration of Euler differential equations was chosen.

At the next stage, the experimental results of the pedestrian visibility on night roads with contaminated headlights were processed in Microsoft Excel.

Figure 8 shows graphs of dependence of the pedestrian visibility distance on the headlight contamination level in dark winter conditions.

The graph shows how the pedestrian visibility can be reduced if headlights are contaminated with CAIM.

The change in the TOYOTA FUN CARGO headlight contamination level from 2.5% to 83.5% decreases the pedestrian visibility:

- in the low beam mode: from 39.5 m. to 29 m.;
- in the high beam mode: from 69.5 m. to 50 m.

NISSAN QASHQAI headlight contamination reduces the pedestrian visibility from 1.7% to 67.6%:

- in the low beam mode: from 52.6 m to 35.6 m;
- in the high beam mode: from 95 m to 57.5 m

The largest braking distance on the packed snow surface for a car equipped with a running ABS at an initial braking speed of 60 km/h was 45 meters (Figure 2) [8].

According to formula (27), the stopping distance was calculated at a grip coefficient of 0.3 [4].

$$S_{\text{stop}} = \left( t_1 + t_2 + 0.5 \cdot t_3 \right) \cdot \frac{V_a}{3.6} + \frac{V_a^2}{26 \cdot g \cdot \varphi}$$  \hspace{1cm} (27)
The grip coefficient of 0.3 is characteristic of the winter roads of Siberia covered with CAIM. The results of the experimental and analytical studies of the stopping distance are presented in Table 1.

Table 1. The results of the experimental and analytical studies of the stopping distance

| Speed [km/h] | Calculated by formula (27) | According to the road tests | According to the mathematical model |
|--------------|-----------------------------|-----------------------------|-------------------------------------|
|              | With ABS                    | Without ABS                 | With ABS                            | Without ABS                        |
| 0            | 0                           | 0                           | 0                                   | 0                                   |
| 10           | 5,19                        | 5,3                         | 5,3                                 | 5                                   |
| 20           | 13,0                        | 13,0                        | 12,51                               | 12,2                               |
| 30           | 23,4                        | 21,9                        | 22,79                               | 19,7                               |
| 40           | 36,4                        | 33,1                        | 36,2                                | 30,2                               |
| 50           | 52,1                        | 46,2                        | 50,83                               | 43,5                               |
| 60           | 70,4                        | 67,3                        | 68,71                               | 63,2                               |
| 70           | 91,2                        | 82,9                        | 90,12                               | 79,1                               |
| 80           | 114,7                       | 106,7                       | 113,1                               | 101,3                              |
| 90           | 140,8                       | 128,1                       | 138,5                               | 122,2                              |

The results presented in Table 1 made it possible to build graphs of dependence of the stopping distance on the speed rate for packed snow roads (Fig. 9-11).

Figure 9. The graphs of dependence of the stopping distance of a Toyota Fun Cargo with winter 185/65 R14 TOYO STUDLESS GARIT G4 tires at the initial braking speed on the packed snow road (experiment): The upper graph - braking without an ABS; the bottom graph - braking with ABS
Figure 10. The graphs of dependence of the stopping distance of a Toyota Fun Cargo with winter 185/65 R14 TOYO STUDLESS GARIT G4 tires at the initial braking speed on the packed snow road (braking without ABS): The upper graph was built using the mathematical model; the middle graph was built using formula (27); the bottom graph was built using the experimental results.

Figure 11. The graphs of dependence of the stopping distance of a Toyota Fun Cargo with winter 185/65 R14 TOYO STUDLESS GARIT G4 tires at the initial braking speed on packed snow road (braking with ABS): The upper graph is based on the experimental results; the bottom graph is based on the calculation results.

3. Conclusion
The results show that the mathematical model can calculate the stopping and braking distances for winter roads with an error not exceeding 3 - 5%. It can analyze the braking process of the vehicle equipped with ABS on winter roads.

The calculation results allow us to identify patterns of the influence of the initial braking speed on the stopping and braking distances of a car equipped with ABS on winter roads with a low grip coefficient.

The results make it possible to determine a safe speed rate of the vehicle under limited pedestrian visibility conditions due to the headlight contamination with chemical anti-icing materials.

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