Towards quantitative magnetic force microscopy: theory and experiment

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Abstract. We introduce a simple and effective model of a commercial magnetic thin-film sensor for magnetic force microscopy (MFM), and we test the model employing buried magnetic dipoles. The model can be solved analytically in the half-space in front of the sensor tip, leading to a simple 1/R dependence of the magnetic stray field projected to the symmetry axis. The model resolves the earlier issue as to why the magnetic sensors cannot be described reasonably by a restricted multipole expansion as in the point pole approximation: the point pole model must be extended to incorporate a ‘lower-order’ pole, which we term ‘pseudo-pole’. The near-field dependence (\(\propto R^{-1}\)) turns into the well-known and frequently used dipole behavior (\(\propto R^{-3}\)) if the separation, R, exceeds the height of the sensor. Using magnetic nanoparticles (average diameter 18 nm) embedded in a SiO cover as dipolar point probes, we show that the force gradient–distance curves and magnetic images fit almost perfectly to the proposed model. The easy axis of magnetization of single nanoparticles is successfully deduced from these magnetic images. Our model paves the way for quantitative MFM, at least if the sensor and the sample are independent.

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1. Introduction

In the late 1980s, magnetic force microscopy (MFM) [1, 2] was invented as a derivative of atomic force microscopy (AFM), promising a resolution of magnetic properties on the nanometer scale. Shortly after its invention, the first attempts were made to understand these measurements quantitatively. It is immediately clear that, similarly to the case of scanning tunneling microscopy, the quantitative description of the measurement is hooked to the description of the magnetic properties of the sensor tip. However, satisfactory models explaining MFM data quantitatively were not available at that time; neither are they today. Hartmann [3, 4] introduced a multipole-type expansion to reduce the complex geometric/magnetic structure of the sensor tip to a magnetic monopole and/or a magnetic dipole. The beauty of that so-called point pole or point probe approximation is that it is simple, despite reducing the complex sensor structure to two low-order and intuitive terms of the multipole expansion. This would facilitate image interpretation and simulation as well as their intuitive understanding.

Apart from this more theoretical approach, attempts have been made to calibrate the magnetic sensors experimentally. These attempts rest on the Biot–Savart law, creating a well-defined magnetic field by an electric current through a well-defined micro-machined arrangement of wires [5–8]. Kebe and Carl [9] presented a similar analysis employing parallel wires. It turned out that in the point pole model, the magnitudes and positions of the poles would be dependent on the specific length scale in an experiment, i.e. the tip–sample separation and the magnetic domain structure of the sample. With such a behavior the point pole model would be of questionable avail. Vergara et al [10] and Vock et al [11] obtained connatural results by analyzing the magnetic field of well-defined micro-structures. They found that the point pole model is unsuitable for describing the field of a typical commercially available MFM tip, i.e. a non-magnetic pyramid covered by a thin layer of magnetic material. Schendel et al [12] calibrated a magnetic thin film sensor using a transfer function method and found that the resulting tip transfer function fit neither to that of a monopole nor to that of a dipole (see also [13]). This conclusion has been substantiated by Lorentz microscopy [14, 15] and electron holography (compare also [16]) taken from those pyramidal sensors. In contrast, Vock et al [11] presented measurements for a long Fe-filled carbon nanotube that is not commercially available. In that case, the only one presented in the literature, the point pole approximation is successful with the sensor behaving effectively like a single monopole.

For lack of a satisfactory theoretical model for the stray field of these tips, numerous numerical calculations have been performed employing geometric models and the extended charge model. In these simulations, the basic geometry of the tip ranged from simple
rectangular [17], cones with sharp, blunt or rounded tips with different magnetization directions [3, 18–21], pyramidal shapes [15] and single-sided prisms [12], to even more complex geometric structures [22–24]. Some of these authors compared their results with the point pole model determining the strength and position of the poles, in the end corroborating the conclusion drawn above.

As a consequence, it currently appears to be very difficult to achieve a quantitative result by MFM. One possibility would be using nanoscale imaging magnetometry where maps of the stray field could be generated in three-dimensional (3D) space [25]. The sensors had to undergo an extensive calibration procedure and the simulation of images was time-consuming.

In this paper, we return to the basic question: how can we describe a thin-film sensor in a convenient mathematical way? We will introduce a very simple model, a cone covered homogeneously with magnetic dipoles representing the ferromagnetic coating, and we will arrive at a surprisingly simple analytic solution for the stray field of that tip. Since the stray field originates mathematically from double integration of a dipolar field over the surface of an infinite cone, it cannot be similar to that of a monopole or a dipole; we term the resulting object ‘pseudo-pole’, with the separation dependence of the magnetic field, $B_z$, along the symmetry axis being $B_z \sim R^{-1}$. We apply this model to non-interacting, single-domain magnetic nanoparticles acting as independent point dipoles and show that it fits nicely to experimental force gradient–distance curves and MFM images. In the limit of the independent tip and sample, the ‘pseudo-pole’ model facilitates the simulation of distance curves and images of complex magnetic structures [26].

2. Theory

The theoretical model of the MFM sensor is a cone, the surface of which is covered by magnetic dipoles all pointing towards the tip of the cone. The magnitude of the dipole density in the surface of the cone, $\tilde{m}_p$, shall be constant and represents the constant magnetization of the ferromagnetic coating projected onto the cone surface. We neglect possible demagnetization effects that may occur at the very tip. In a simple calculation we consider the magnetic field along the z-axis ($r = 0$; see figure 1(a) for the geometry) by performing the integration over the surface of the cone, $S$. The differential dipole moment is given by

$$d\tilde{m}_p = \tilde{m}_p \frac{dS}{\cos \alpha} = \tilde{m}_p \frac{r}{\cos \alpha} d\phi dh$$

and the magnetic field component along the z-axis is

$$dB_z (z) = \frac{\mu_0}{4\pi |p|^3} \left( 2 \cos \theta \hat{r} + \sin \theta \hat{\varphi} \right),$$

with the vector, $p$, pointing from the dipole moment, $m_p$, to the point $P = (r, z)$ and the unit vectors, $\hat{r}$ and $\hat{\varphi}$, in the spherical coordinate system of that dipole moment. For $z > 0$ and $0 < \alpha < \pi/2$, integration over $S$ results in (see figure 1(a))

$$B_z|_{x,y=0} = \frac{1}{2} \frac{\mu_0 \tilde{m}_p \sin \alpha}{z},$$

with the vacuum permeability, $\mu_0$, and the opening half-angle of the cone, $\alpha$, as defined in figure 1(a). Accordingly, the magnetic field decays in proportion to $z^{-1}$, in contrast to $z^{-2}$ or $z^{-3}$ as for a monopole or a dipole, respectively. Note that the infinite cone possesses no length
Figure 1. (a) Sketch of the cone model of the MFM sensor (left) and sketch of the tip–sample geometry for the test measurement (right). (b) $\Delta f$–$z$ curve measured on the same sample as figure 3 but in a flat area with no particles in the neighborhood. The solid lines are fits to data according to the vdW interaction for different tip geometries. For the paraboloidal geometry we obtain $B_{\text{parabol}} = (1173 \pm 5) \text{ Hz nm}^3$. The best correspondence is clearly given by the paraboloidal geometry. Note that $\Delta f$ is negative since the vdW interaction is attractive.

To introduce a length scale, one could, e.g., carry out integration over the surface of a finite cone, which leads to a crossover to the behavior of a dipole on a length scale greater than the height of the cone. Since the height of coated pyramidal tips is typically larger than 10 $\mu$m, most experiments would be in the limit of the pseudo-pole considered here.

In a more general and lengthy calculation, one finds that the magnetic field components are

$$B_z = \frac{1}{2} \mu_0 \tilde{m}_p \sin \alpha \cdot \frac{1}{R}, \quad B_r = \frac{1}{2} \mu_0 \tilde{m}_p \sin \alpha \cdot \frac{R - z}{R \cdot r},$$

with $r = \sqrt{x^2 + y^2}$, $R = \sqrt{x^2 + y^2 + z^2}$, and the in-plane component of the magnetic field, $B_r$. With these analytic solutions one could even calculate analytically the transfer functions employed in [12] to simulate MFM images of a given sample.
The force experienced in that field by a probing point dipole, \( m_s \), in the attractive/parallel configuration is given by

\[
F_z|_{x,y=0} = m_s \frac{\partial B_z}{\partial z} = -\frac{1}{2} \mu_0 m_s \tilde{m}_p \sin \alpha \cdot \frac{1}{z^2} \tag{3}
\]

and the frequency shift, \( \Delta f \), detected in a dynamic measurement as in MFM is given by [27]

\[
\Delta f|_{x,y=0} \approx -\frac{f_0}{2k} \frac{\partial F}{\partial z} = -\frac{f_0}{2k} m_s \frac{\partial^2 B_z}{\partial z^2} = -\frac{f_0}{2k} \mu_0 m_s \tilde{m}_p \sin \alpha \cdot \frac{1}{z^3}, \tag{4}
\]

with the oscillation frequency, \( f_0 \), and the spring constant, \( k \), of the cantilever. Due to finite modulation amplitude, \( z_m \), employed in dynamic measurements, equation (4) changes to [28, 29]

\[
\Delta f|_{x,y=0} \approx -\frac{A}{\left[ (z + z_0 + z_m)^2 - z_m^2 \right]^{\frac{3}{2}}} \tag{5}
\]

We introduced here an offset in the tip–sample separation, \( z_0 \), which accounts for (i) an offset between the apex of a real MFM tip and the magnetic center of the pseudo-pole (compare figure 1(a), left) and (ii) a variable depth of the dipole underneath the sample surface (compare figure 1(a), right). The constant \( A = \frac{f_0}{2k} \mu_0 m_s \tilde{m}_p \sin \alpha \) combines the conversion factor of the measurement, the geometry of the cone and the magnitude of the magnetic interaction between tip and sample.

According to Giessibl [29], the van der Waals (vdW) interaction leads to the same separation dependence of the force \( (F \sim z^{-2}) \) if the sensor has a cross-sectional area, \( A_s \), behaving like \( A_s \sim h \) with the height above the apex of the tip, \( h \) (e.g. a paraboloid), interacting with a semi-infinite sample. This is the behavior we found experimentally for the magnetic tips interacting with a diamagnetic sample (SiO\textsubscript{x}/Si), as shown in figure 1(b). For a cone as well as for a rod, the separation dependence would be significantly different as compared to the measurement. We understand this if we accept that all tips have a finite radius of curvature and the vdW interaction, \( F_{\text{vdW}} \sim z^{-7} \), is restricted mainly to that curved apex (compare [28]). The magnetic interaction between dipoles, \( F_{\text{mag}} \sim z^{-4} \), however, exhibits a longer interaction length and, hence, is governed by the pyramidal shape of the sensor tip. Accordingly, with a magnetic dipole, it is difficult in MFM to distinguish between the separation dependence of the vdW and the magnetic interaction unless the separations are different as for magnetic dipoles buried underneath the sample surface. In the vdW case the separation is measured between the apex of the tip and the surface of the sample. In the magnetic case the separation is larger and measured between the centers of the magnetic dipoles and the pseudo-pole, i.e. there is a constant offset. Then, the vdW interaction dominates very close to the surface and the magnetic interaction overcomes the vdW interaction further away from the sample only if its amplitude significantly exceeds that of the vdW interaction. The total frequency shift measured is then the sum of vdW and magnetic interactions

\[
\Delta f|_{x,y=0} \approx -\frac{A}{\left[ (z + z_0 + z_m)^2 - z_m^2 \right]^{\frac{3}{2}}} - \frac{B_{\text{parabol}}}{\left[ (z + z_m)^2 - z_m^2 \right]^{\frac{3}{2}}}, \tag{6}
\]

with \( B_{\text{parabol}} \) representing the amplitude of the vdW interaction of a parabolic tip. An example of how these two contributions fit together is presented in figure 3(a).
3. Experiment

To verify that the above model describes the thin film sensors (Bruker, MESP), we may probe the field of the tip with a test dipole (magnetic nanoparticle). The particles shall be distributed sparsely over the sample such that we may assume that the magnetic interaction between the tip and the sample is due to a single dipole. First we determine the separation dependence of the interaction and then we evaluate the lateral profile of single nanoparticles with a random orientation of the magnetization.

The test dipoles were produced as follows. Ferromagnetic Co nanoparticles of diameter $(18 \pm 5)$ nm were deposited on Si substrates with native oxide layer by inert gas condensation. The base pressure of the preparation chamber was below $1 \times 10^{-8}$ mbar. The details of the preparation process can be found in [30]. After deposition, samples were transferred at ambient to another vacuum system to perform x-ray photoelectron spectroscopy (XPS) and to cover the particles with a protective layer. Prior to XPS, the partially oxidized particles were reduced by hydrogen radio-frequency-plasma treatment followed by an annealing step at 600 °C for 10 min to expel hydrogen from the particles, which could lead to a reduced magnetization otherwise. The efficiency of plasma treatment in reducing metal nanoparticles has been shown earlier [31, 32]. XPS was used to prove that pure Co nanoparticles had formed (data not shown). Next, the particles were covered in situ by a SiO layer of nominal thickness $(23 \pm 3)$ nm as a protective coating against oxidation. In addition, the homogeneous cover avoids material contrast at the sample surface, which could have an impact on AFM/MFM investigations. Also, it introduces an increased offset, $z_0$, between the sample surface (origin of vdW interaction) and the centers of the magnetic particles (origin of magnetic interaction). MFM image data shown below have been obtained from pristine samples to observe a random orientation of magnetic moments. The force gradient–distance curves shown were measured after magnetizing the nanoparticles in an external magnetic field of $\sim 1$ T perpendicular to the sample surface in order to favor a magnetization direction, leading to attractive interaction with the sensor.

The required properties of the sample are: (i) the particles behave as static point dipoles and (ii) the particle diameter can be measured as particle height by AFM at the surface due to a homogeneous SiO coverage. With respect to (i), we note that the critical diameter for superparamagnetic Co particles is $d_{C1} \sim 7$ nm using the hexagonal close-packed (HCP) Co anisotropy constant of $4.1 \times 10^5$ J m$^{-3}$ at ambient temperature. Since we cannot expect to prepare single-crystalline HCP Co particles using the preparation route described above, the effective anisotropy might be significantly lowered. This in turn increases the critical diameter for blocking at ambient temperature. Integral magnetometry data (not shown), however, have proven that the ensemble behaves similarly to the non-interacting uniaxial Stoner–Wohlfarth particles with random distribution of the anisotropy axes and a coercive field of $\mu_0H_C = 50$ mT at $T = 10$ K. At ambient, thermal agitation sets in and we find that $\mu_0H_C = 25$ mT and a slightly reduced remnant magnetization. For Co particles, the upper limit for the single-domain state is $d_{C2} \sim 70$ nm [33]. Consequently, we may assume that the Co nanoparticles with diameter $(18 \pm 5)$ nm are magnetically blocked at ambient temperature and carry a magnetization close to the saturation magnetization of Co [30]. The orientation of the easy axis of magnetization is randomly distributed in the particle ensemble and for the majority of particles their coercive field is sufficient to keep their magnetization direction in the presence of the tip’s magnetic field.

The MFM is a home-built atomic force microscope exploiting beam deflection and achieving a resolution of 0.02 nm$_{rms}$ along the vertical direction at ambient [34]. The oscillation
frequency of the cantilever is measured by a home-built phase-locked loop delivering a frequency resolution of \( \sim 0.1 \, \text{Hz}_{\text{rms}} \) at a signal bandwidth of 1 kHz and an oscillation amplitude of \( \sim 9 \, \text{nm} \). The canting angle of the microscope is 10° with respect to the sample normal. In the following analysis we neglect the canting angle since \( \cos(10°) \approx 1 \) within experimental error.

### 4. Experimental results and discussion

Figure 2 displays the topography of the sample measured in tapping mode together with the magnetic signal, i.e. the frequency shift, \( \Delta f \), of the lever's resonance frequency. The magnetic image has been recorded consecutively to the topographic image in a plane parallel to the median topographic plane 5 nm above the maximum of topography. We use this ‘plane mode’ because it is more intuitive and less susceptible to artifacts which may occur at steep structures of the sample otherwise. The average particle height is \( (18 \pm 5) \, \text{nm} \). Due to the 23 nm SiO cover and the finite tip radius, the lateral extent of the particles appears much larger than the particles are. The maximum frequency shift amounts to less than \( \sim 0.5 \, \text{Hz} \) in the center of the particle. In figure 2(b), the particles exhibit preferentially repulsive interaction (bright contrast), indicating an anti-parallel alignment of the sample dipole with the tip moment. In particular, the particle at the top of figure 2(a) shows broken circular symmetry with attractive interaction on the right reminiscent of a dipole with in-plane orientation. Overall, the orientation of dipoles is broadly distributed as expected for a pristine sample with non-interacting magnetic particles.

Figure 3(a) shows the average of fifty \( \Delta f - z \) curves measured consecutively above the center of a collinear particle (dark contrast in the magnetic image). Since smaller particles produce at moderate separation a frequency shift at the limit of resolution \( \sim 10^{-6} \), repetition of measurements is mandatory in order to allow for a meaningful evaluation of frequency data. The collinear configuration was stable over the measurements, while the anti-parallel configuration tended to switch to the collinear arrangement. This limits the number of repetitions and, hence,
Figure 3. (a) Δf–z curve above a Co particle of height (21 ± 1) nm with its magnetization being anti-parallel to the z-axis. The red curve is a fit according to the pseudo-pole model with its vdW (blue) and magnetic (green) components. The fit parameters are $A = (25700 ± 4300) \text{ Hz nm}^3$, $B_{\text{parabol}} = (282 ± 16) \text{ Hz nm}^3$ and $z_0 = (32.1 ± 2.8) \text{ nm}$. Note that Δf is negative since both components of interaction (vdW and magnetic) are attractive. (b, c) Extracted prefactors $A$ (b) and $B_{\text{parabol}}$ (c), determined for particles of different sizes plotted against their size/height. The solid curve in (b) is a fitted power law $A \sim d^p$ with $p = (2.91 ± 0.23)$ and in (c) the linear regression for $B_{\text{parabol}} = a \times d + c$ with $a = (-0.9 ± 3.6) \text{ Hz nm}^2$ and $c = (302 ± 65) \text{ Hz nm}^3$. (d) The offset, $z_0$, plotted against the particle size. The solid line is a linear regression $z_0 = a \times d + c$ with $a = (0.72 ± 0.24)$ and $c = (14.9 ± 4.5) \text{ nm}$. The red points have been ignored in the fits. In (c) and (d), red points correspond to the same particles.

restricts frequency data to an insufficient accuracy. The origin of the z-axis in figure 3(a) has been chosen such that $z = 0$ in the topographic mode. For an oscillation amplitude of 9 nm and a damping of 20%, the sensor tip would just touch the surface at the turning point of the oscillation at $z = 1.8 \text{ nm}$. The origin of the frequency shift is such that Δf = 0 with the tip withdrawn from the surface ($\sim 3 \mu\text{m}$). The red curve is a fit of equation (6) to data in the range 4–83 nm, resulting in $A = (25750 ± 4250) \text{ Hz nm}^3$, $B_{\text{parabol}} = (282 ± 16) \text{ Hz nm}^3$ and $z_0 = (32.1 ± 1.8) \text{ nm}$. The blue and green curves indicate the vdW and magnetic contributions to the red curve, respectively. The offset, $z_0$, is to a good approximation the sum of the particle radius ($10.5 ± 0.5 \text{ nm}$) and the thickness of the oxide (nominally $23 ± 3 \text{ nm}$), which means that the pseudo-pole would be located approximately in the tip apex. From the geometric model (figure 1(a) with $R_t \cong 10 \text{ nm}$ and $z \cong 15^\circ$), one would expect a value of approximately 10 nm.
It should be stressed that the errors of the fit parameters are impressively small and, hence, support our model. One might expect that introducing an offset into the separation of the vdW component, \( z_{0,\text{vdW}} \), could even improve the presented result. However, the adjusted offset would be in the range of \((-0.5 \pm 0.5)\) nm, leaving the other parameters and corresponding errors essentially unchanged. Accordingly, the effective vdW surface would be at \( z \approx 0 \), which includes a damping of the oscillation amplitude by 1.8 nm in the present case. A possible explanation is that for the given parameters the damping on the particles is due to an adsorbed water film of thickness \( \sim 1.5 \) nm, which introduces the kink in the \( \Delta f - z \) curve at \( z \approx 1.5 \) nm (see figure 3(a)). To exclude such details of the contact area from our considerations, we fit the model functions to frequency data for \( z > 4 \) nm only.

In figures 3(b)–(d), we show the extracted fit parameters from measurements on differently sized particles magnetized collinearly with the tip. All measurements have been carried out with a single tip. The amplitude of the magnetic interaction, \( A \), (figure 3(b)) is shown together with a power law fit to the data. Here we omitted three data points (red) because those appear to satisfy a different relation. The resulting power is 2.91 \( \pm \) 0.23 and confirms a dipole moment of the particles being linear in volume as expected for spherical particles. The amplitude of the vdW interaction, however, does not vary too much (figure 3(c)). Skipping the largest particle, a linear regression delivers a constant within accuracy, as expected for the vdW interaction. Finally, the offset, \( z_0 \), is on display in figure 3(d). Skipping the same data points as in (b), a linear regression delivers a slope of 0.72 \( \pm \) 0.24. The expected value of 0.5 is close to the boundary of the error interval, which may indicate that larger particles are covered by a slightly thicker oxide. The intercept of the linear regression is \((15 \pm 5)\) nm, which should be the difference of the oxide thickness \((23 \pm 3)\) nm and the position of the pseudo-pole in front of the tip apex \((8 \pm 8)\) nm. Also these values are within experimental error.

The three data points skipped in figures 3(b) and (d) correlate and are approximately linear in both plots. We suppose that a second type of particles develops during preparation, e.g. by two particles that have landed close to each other and partial sintering during the annealing step, leading to variations of the magnetic character. Alternatively, those particles exhibit a larger Hamaker constant as compared with SiO or an electrostatic interaction with a force \( F_{el} \sim z^{-2} \).

As an important outcome of our considerations the magnetization direction of single nanoparticles can be determined. In figure 4, we show a cross-section as indicated in the magnetic image of figure 2 where we determined manually the mirror plane of the magnetic signal. We included a fit which corresponds to the expected signal of a dipole within the pseudo-pole model (green curve). Neglecting the modulation amplitude and the vdW interaction, the expected frequency shift can be calculated by starting with equation (2) and following the same ideas that led to equation (5), resulting in

\[
\Delta f = \frac{f_0}{4k} \mu_0 m_S \tilde{m}_p \sin(\alpha) \left[ \frac{3z (z \sin(\beta) + x \cos(\beta))}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} - \frac{\sin(\beta)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \right],
\]

with \( \beta \) being the tilt angle of the dipole with respect to the surface plane, and \( x, y \) are the coordinates within the plane perpendicular to the tip–sample separation, \( z \). Using \( y = 0 \) for the cross-section, the pseudo-pole model describes data very well (see figure 4). For the particle at the upper border of the image (figure 2), the tilt angle with respect to the surface plane is \((11.0 \pm 2.2)\)°. The tilt angle is relatively small as expected from the symmetric appearance in the magnetic image, and the error of the tilt angle is surprisingly small. With increasing the angle...
β towards 90° (perpendicular to the sample plane) the error increases to ∼8° for β ≈ 57° (lower left particle in figure 2). The extracted pseudo-pole–dipole separation $z = (68.5 \pm 2.0)$ nm is a bit too large as compared with our expectation (58.5 ± 7) nm (particle diameter $d \approx 23$ nm). The large value of the prefactor, $A$, may be ascribed to the large value of $z$ and/or neglect of the vdW interaction. Note that we used here a cross-section extracted manually from 2D data to obtain the spatial orientation of the nanoparticle’s magnetization. Using a 2D fit routine would result in a strongly enhanced accuracy since the number of data points contributing to the result increased by a factor of about 100.

So far the pseudo-pole model seems to describe MFM data very accurately. However, we assumed that the tip and the sample have static magnetic properties. If we assume further that the Co nanoparticles show properties of single-domain Co and form an ideal sphere, the magnetization of the Co particles would be close to the saturation magnetization of Co ($\sim 1.5$ T/$\mu_0$). Combining equations (2) and (4), solving for $B_z$ and using experimental data ($\Delta f = 0.5$ Hz, $z = 40$ nm), we obtain $B_z \approx 47$ mT$ \cdot \frac{10 \text{nm}}{z}$. This value corresponds nicely to values given in the literature (see, e.g., [25]). The maximum field at the center of a particle of diameter 18 nm produced by the tip can be calculated to be $B_{z, \text{max}} \approx 12$ mT with the sensor tip touching the surface. This is well below the coercive field as determined by integral magnetometry. So it appears quite reasonable to assume for the experiment that the particles are not strongly influenced by the magnetic field of the tip. Conversely, the field of a particle of diameter 18 nm would be $\sim 15$ mT $\mu_0^{-1}$ at the tip apex, which is well below the coercive field of the sensor tip (supplier information: 40 mT$ \mu_0^{-1}$). Therefore, our initial assumption of the independent sensor and sample appears to be justified.

It is important to note that our experiments benefit in several ways from the $\sim 20$ nm thick SiO cover. On the one hand, the oxide protects the Co particles from oxidation and prevents a material contrast at the surface. On the other hand, it reduces the magnetic interaction...
between the tip and the sample such that they can be considered independent. The oxide also restricts measurements to sufficiently large separations ($z > 20\,\text{nm}$) where the pseudo-pole model obviously applies. Closer to the pseudo-pole we may expect deviations due to finite thickness of the magnetic film, finite radius of curvature of the tip, rearrangement of the magnetization in the magnetic film and due to grains in the magnetic film.

All experimental results shown here have been obtained with one sample and with one sensor tip within two consecutive days. This way we ensured equal conditions for all measurements. A sequence with a different tip yielded similar results with respect to estimated relations as $A \sim d^{3\pm0.3}$, $B_{\text{parabol}} \approx \text{const}$ and $\partial z_0/\partial d = 0.8 \pm 0.3$. However, the absolute values of the prefactors $A$ and $B_{\text{parabol}}$ may vary by a factor of three.

5. Conclusion

We introduced the pseudo-pole model for describing magnetic properties of a magnetic thin-film sensor. Accordingly, the axial component of the magnetic stray field shows strikingly simple $1/R$ dependence with $R$ being the separation from the tip apex. In practice, this separation dependence is expected to hold for separations being small compared to the height of the magnetic sensor. A lower limit may be imposed by details of the tip geometry such as the thickness of the magnetic coating and the radius of curvature of the tip. We tested the proposed model by using buried Co nanoparticles as dipolar point probes and found an accurate correspondence between the model and the experiment in the separation dependence as well as in the lateral dependence of the magnetic signal. To image the spatial distribution of the magnetic field of the sample, we employed the plane or linear lift mode of MFM scanning the magnetic image of the sample in a plane above the sample. It appears to us that the plane mode is more intuitive than the ‘lift’ mode and it is less sensitive to artifacts. From magnetic images the spatial orientation of the magnetization of a single nanoparticle with sizes ranging from 12 to 26 nm has been deduced with accuracy better than $\pm10^\circ$.

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