Higgs Sector Radiative Corrections and \(s\)-Channel Production

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Higgs boson mass sum rules of supersymmetric models offer attractive targets for precision tests at future muon colliders. These sum rules involve the gauge boson masses as well as the masses of the Higgs boson states which can be precisely measured in the \(s\)-channel production process at a muon collider. These measurements can sensitively probe radiative corrections to the Higgs boson masses as well as test for CP-violation and nonminimality of the Higgs sector.

Introduction: In recent years the electroweak precision measurements have played a large role in establishing the validity of the Standard Model as well as constraining the possibilities of new physics. In particular the precision measurements narrowed the allowed values for the top quark mass, and the top quark mass was observed directly and its mass is consistent with radiative corrections. Now the smaller corrections of the Higgs boson are being constrained, and there are tantalizing hints that the first direct evidence for a Higgs boson has been seen at LEP. The mass of this Higgs boson is in agreement with the predictions from the precision measurements and is in a range that is consistent with supersymmetry.

This strategy of testing the consistency of theories will continue after a future discovery of supersymmetry and the required Higgs sector. The purpose of the present note is to emphasize that in an era following the discovery of a supersymmetric Higgs sector, there are some sets of observables for which precision measurements will be particularly powerful. In the minimal supersymmetric standard model (MSSM), supersymmetry together with gauge invariance impose constraints on the Higgs sector that gives rise to mass sum rules. The Higgs sector of the MSSM contains three neutral Higgs bosons, \(h\), \(H\), and \(A\) as well as two charged Higgs bosons, \(H^{\pm}\). The sum rules relate certain combinations of mass-squares of the Higgs and gauge boson masses. The gauge boson (\(W\) and \(Z\)) masses are now known very precisely. A future muon collider can produce neutral Higgs bosons in the \(s\)-channel. By adjusting the energy of the machine so that one is sitting on the Higgs boson resonances, the muon collider can produce thousands of Higgs bosons per year, and the mass and total width determined very precisely. Hence it will be possible to do precision tests of the sum rules.

Higgs Boson Mass Sum Rule: At tree-level, the mass sum rule for the neutral states of the MSSM is

\[
M_h^2 + M_H^2 = M_A^2 + M_Z^2 ,
\]

This is a natural relation in that it is satisfied at tree-level without a tuning of parameters. At tree-level it can be shown that \(M_H \geq M_A\) so that one has the constraint \(M_A \leq M_Z\). The sum rule does not depend on any parameters like mixing angles or couplings; only the physical Higgs boson masses need to be measured to test the sum rule at the tree-level. The sum rule receives a non-zero but finite and calculable correction from loop diagrams. The correction can be summarized as a contribution \(\Delta\), so that

\[
M_h^2 + M_H^2 = M_A^2 + M_Z^2 + \Delta .
\]

One can solve this equation for the difference in the heavy Higgs boson masses,

\[
M_H - M_A = \frac{M_Z^2 - M_h^2 + \Delta}{M_A + M_H} .
\]

This form is instructive as it is clear that in the decoupling limit, \(M_A, M_H \to \infty\), the mass difference \(M_H - M_A\) becomes small. The mass difference is positive for most of supersymmetric parameter space, but it can take either sign depending on the details of the spectrum and couplings of the supersymmetric particles. There are theoretical reasons to believe that the absolute value of this mass difference is small. In the MSSM, large \(M_A\) and large \(\tan \beta\) give highly degenerate heavy Higgs states separated by a few GeV or less. The ever increasing lower bound on Higgs masses from the LEP experiments is gradually increasing the lower bound on \(\tan \beta\) that is allowed in the MSSM making it more likely that \(\tan \beta\) is large.

The leading contribution to \(\Delta\) was first calculated in Ref. \[1\] and is

\[
\Delta = \frac{3g^2 m_t^4}{16\pi^2 m_W^2 \sin^2 \beta} \log \frac{m_t^2}{m_t^2} ,
\]

where \(\tilde{t}_1\) and \(\tilde{t}_2\) are the top quark mass eigenstates. There are smaller corrections from diagrams involving the lighter quarks, gauge bosons, and their superpartners, and there are corrections from two and higher loops. Following the renormalization of the sum rule, the radiative corrections to the light Higgs boson \(h\) were isolated.
and the tree level upper bound, $M_h \leq M_Z$, was shown to no longer be satisfied. In fact, for most of parameter space, $\Delta$ contributes largely to the renormalization of the lightest Higgs ($h$) mass for fixed $M_A$. Therefore a measurement of $M_h$ will provide the first test of radiative corrections in Higgs sector the MSSM. A subsequent measurement of $\Delta$ as described below would constitute a precision test of these radiative corrections.

It should be emphasized that a precise measurement of $\Delta$ does not isolate any single supersymmetric mass or parameter, but rather picks out a slice of parameter space. If the leading correction shown in Eq. (3) was the only contribution, then a measurement of $\Delta$ would provide a measurement of the quantity $m_l^2$, since the other quantities have already been experimentally measured. The value of $\Delta$ can be calculated theoretically for any choice of parameters and compared to the measured value. The size of $\Delta$ is generally of order a few times 10$^4$ GeV$^2$. The theoretical calculations for the radiative corrections have reached a high level of sophistication; see Ref. [8] and references therein for the present status. One of the advantages of the sum rule is that it is a radiative correction to the tree-level relation does not involve the supersymmetry parameter $\tan \beta$ which enters into the particle couplings. Any dependence on $\tan \beta$ enters only in the radiative correction $\Delta$, so the precision measurement discussed in this note can be carried out solely by measuring Higgs boson masses very precisely.

**Precision Test of a Supersymmetric Higgs Sector:** In the context of the MSSM, the correction $\Delta$ arises exclusively from loop diagrams involving all the particles that couple to the Higgs bosons. But in a more general model, the correction $\Delta$ might involve corrections from some heavier Higgs boson states. So an experimental test of the sum rules probes radiative corrections in the MSSM, and probes for the presence of heavier undetected Higgs bosons. As a concrete example, consider a multi-Higgs doublet supersymmetric model. Then the sum rule is generalized to be

$$\sum_{\text{CP-even}} M^2_{H_i} = \sum_{\text{CP-odd}} M^2_{A_i} + M^2_Z + \Delta. \quad (5)$$

where $M_{H_i}$ and $M_{A_i}$ represent the masses of CP-even and CP-odd Higgs bosons respectively. In this model where there are $2N$ Higgs doublets, there are $2N$ CP-even mass eigenstates and there are $2N - 1$ CP-odd mass eigenstates. The mass difference between the lightest CP-odd Higgs boson and the second-lightest CP-even Higgs boson gets contributions not only from the radiative correction $\Delta$ but also from possibly small mass-squared differences in the heavier Higgs boson states, $M^2_{H_i} - M^2_{A_i}$.

The presence of other electroweak representations of Higgs bosons can also contribute an effective contribution to $\Delta$. For example, a small amount of mixing with a singlet Higgs boson will add a contribution [10] that can be detected by accurately measuring $\Delta$. The most important feature of supersymmetric models with Higgs sectors more complicated than the MSSM is that the modifications to the mass sum rule in Eq. (1) appear already at the tree level.

In addition to the neutral Higgs boson mass sum rule in Eq. (2), there is a sum rule involving the charged Higgs boson,

$$M^2_{H^\pm} = M^2_{A} + M^2_{W} + \Delta, \quad (6)$$

where $\Delta$ is the calculable correction to the tree-level sum rule. The measurement of the radiative correction $\Delta$ is not as interesting as the measurement of $\Delta$ we are highlighting in this note, since the mass of the charged Higgs boson can not be measured in $s$-channel production. However, a precise measurement of $M_{H^\pm}$ by other means might prove useful as another probe of radiative corrections to the Higgs sector.

It has been shown [12,13] that the loop contributions $\Delta$ and $\tilde{\Delta}$ are given exclusively by self-energy diagrams. All contributions involving the loop corrections to the Higgs sector mixing angles ($\alpha$ and $\beta$) conveniently cancel out in the radiative corrections, so that measuring the couplings is not necessary to obtain the experimental inputs to the highest order part (tree-level) of the sum rule.

**CP-violation:** CP-violation can also be probed just by accurately measuring the Higgs boson masses. Loop-induced CP-violation can mix [14,15] the heavy Higgs CP-eigenstates, $H$ and $A$, which generally leads to a shift in the relative positions of the mass-eigenstates (which are no longer the same as the CP-eigenstates). Higgs bosons that are highly degenerate in the absence of CP-violation can be split when a CP-violating phase is nonzero [13,15,16]. Since this splitting can be greater than a GeV, this constitutes another very interesting physical effect that can be probed by accurately measuring $\Delta$. On the other hand the mass splitting is a single parameter and ultimately one would want to exploit beam polarization to obtain more information. If one has polarized beams available at the muon collider, then there are many more observables [19] that can be exploited to separate and measure the CP-mixing. In fact, even if CP is conserved in the Higgs sector and the heavy Higgs bosons are highly degenerate, one can use polarization of the muon beams to separate the two resonances [20].

**An Example:** In this section we present an example of the level of precision for the mass and total width of the heavy Higgs bosons that can be achieved through $s$-channel production. We take as an example the following parameters: $M_A = 350$ GeV, $\tan \beta = 5$, and take all supersymmetry breaking mass and mixing parameters to be 1 TeV, e.g. $m_{q_{u,r}} = A_t = A_b = 1$ TeV. We also assume that CP-mixing between the heavy Higgs bosons is negligible. We use the program HDECAY [21] to calculate the radiatively corrected masses, decay widths, and branching ratios of the Higgs bosons. While it has been demonstrated that a muon collider is the optimal place
to measure the light Higgs boson mass, $M_h$, a muon collider can also measure the heavy Higgs boson masses, $M_A$ and $M_H$, very well in the $s$-channel production process. In fact, the muon collider may be the only possible machine that can separate two highly degenerate heavy Higgs and measure the mass difference, $M_H - M_A$.

We consider the process $\mu^+\mu^- \rightarrow A, H \rightarrow bb$. A scan over the Higgs resonances devoting $0.01 \text{ fb}^{-1}$ of integrated luminosity to a sequence of center-of-mass energy values is employed to determine the Higgs boson masses and total widths. The measurement is a counting experiment and does not require a precise energy determination of the $b$ jets; rather at a muon collider the energy of the beams is expected to be known very well. The result of such a scan is shown in Fig. (1) for 11 scan energies separated by 0.1 GeV around the Higgs resonance. We have assumed a Gaussian energy spectrum of each muon beam with an rms deviation $R = 0.01\%$, and that the $b$'s are tagged with a 50% efficiency. One can simply multiply the ellipse in Fig. (1) by an overall factor if one assumes a different tagging efficiency. The partial widths have not been allowed to vary in this scan, but relaxing this condition does not substantially change the accuracy with which the Higgs boson mass can be determined.

One sees from Fig. (1) in particular that one can determine the Higgs boson mass with a $1\sigma$ error of just 15 MeV. The $1\sigma$ error on the width is roughly 20 MeV, which is about a 10% measurement. A very similar determination can be attained for the $H$ boson since its total width and couplings to $\mu^+\mu^-$ are similar to those of the $A$ boson. The determined error generally shrinks as one goes to larger values of tan $\beta$ as the couplings of the Higgs to $\mu^+\mu^-$ increases. The Higgs boson widths also increase as tan $\beta$ increases, and ultimately the Higgs resonances overlap; when this happens, determining the mass difference is problematic (see below).

Most of the discriminating power occurs for the luminosity devoted in the interval $M_A - \Gamma_A < \sqrt{s} < M_A + \Gamma_A$, but how much luminosity must be wasted on scan points outside this range depends on how well the Higgs boson mass is known prior to the scan. The masses of the heavy Higgs bosons must be known to less than or about 1 GeV before this type of scanning can be done, since it must be guaranteed that the Higgs peak cross section is within the scan energy range. Strategies for obtaining this precision have been discussed previously and could take place at a future linear collider or at a higher energy muon collider. The scenario in which things play out is not known, so it is not clear how well the heavy Higgs boson masses will be known prior to the scan. The light Higgs boson mass does give us some information on the radiative corrections, and if some of the radiative correction parameters were known (by a priori discovery of supersymmetry and measurement of the particle masses and couplings), one could obtain a rough indirect measurement of $M_A$ to 20%.

One can also discover the heavy Higgs bosons directly in the bremsstrahlung tail at a muon or linear collider operating at an energy above the Higgs masses. These rough determinations of the heavy Higgs boson masses could be followed by a rough scan that could pin down the mass(es) of $H$ or $A$ to a GeV.

The example in this section shows that the muon collider with a reasonable amount of integrated luminosity can measure the heavy Higgs bosons of supersymmetric models to tens of MeV. This represents an extraordinary probe of radiative corrections in the Higgs sector. The expected measurement of the mass of the light Higgs $h$ is of the order of 100's of keV and is limited by the precision (for the expected integrated luminosity of 0.2 $\text{fb}^{-1}$) with which the beam energy can be measured through the spin precession of muons around the ring. $Z$-boson mass is currently known with an error of 2.2 MeV from the LEP experiments. So the dominant error on the measurement of $\Delta$ will come from the errors on the mass measurements of the heavy Higgs bosons, $H$ and $A$. In the example, the contributions to the error on $\Delta$ are

$$\delta(M_H^2) \sim \delta(M_A^2) \sim 10 \text{ GeV}^2.$$  

This then results in a measurement of the radiative correction $\Delta$ of the order of one part in $10^4$.

**Higgs Boson Mass Degeneracy:** When the heavy Higgs bosons become very degenerate, it will be much harder to determine the mass difference $M_A - M_H$. A rough rule for when the scan as described in this note will fail to be adequate at resolving the two Higgs resonances occurs for mass differences less than the one third of the sum of the total widths of the heavy Higgs bosons, i.e.

$$|M_H - M_A| < \frac{1}{3} (\Gamma_A^{\text{tot}} + \Gamma_H^{\text{tot}}).$$  

This condition assumes that the rms deviation $R$ of the energy spectrum of each muon beam is sufficiently small so as to not smear two peaks together. It is adequate to have $R = 0.01\%$ for these heavy bosons, but a larger value such as $R = 0.06\%$ would smear two partially overlapping resonances together. In cases where the Higgs...
bosons are sufficiently separated in mass, the rms deviation can be increased with a resulting increase in luminosity because the heavy Higgs bosons $H$ and $A$ are very much broader than the light Higgs boson $h$ (a light Standard Model-like Higgs might require $R$ less than 0.01% to fully exploit the very narrow resonance). The condition in Eq. (3) occurs typically for the larger values of both $M_A$ and $\tan \beta$. Figure 2 shows this region for a particular choice of supersymmetric parameters ($M_3 = A_0 = 1$ TeV). It should be kept in mind that different values for the supersymmetric masses and mixing, or the presence of CP-violation, can produce heavy Higgs bosons that are shifted relative to each other, and would qualitatively change the contours in Fig. 2.

Even when one cannot experimentally distinguish the overlapping heavy Higgs bosons, one can still derive an upper bound on their mass difference if one makes the hypothesis that the one resonance peak that is being observed is two overlapping Higgs bosons. Furthermore, techniques exploiting any possible polarization of the muon collider can be used to unravel the CP-even $H$ boson from the CP-odd $A$ boson.

**Summary:** We have demonstrated that the study of the heavy Higgs bosons of the MSSM in the $s$-channel at a future muon collider can be combined with the mass measurement of the light Higgs boson to sensitively probe radiative corrections to the MSSM Higgs sector. Very accurately measuring the mass difference of the heavy neutral Higgs bosons of the MSSM can probe possible CP-violation or nonminimality of the Higgs sector. Comparison of the experimentally measured Higgs boson masses with calculations of the virtual effects of Standard Model and supersymmetric particles can give a precise test of the MSSM or a definite prediction that must be satisfied by the supersymmetric spectrum. An important question is whether the theoretical calculations will progress far enough to make full use of the possible experimental determination of the radiative corrections as suggested in this note. This would probably require the calculation of the subleading two-loop contributions of $O(M_Z^2 \alpha \alpha_s)$ and $O(M_Z^2 \alpha^2)$ as well as calculations of the self-energy diagrams without making the zero-momentum approximation on the external legs.

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