Moving Target Tracking Using TDOA and FDOA Measurements from Two UAVs with Varying Baseline

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Abstract. This paper considers the problem of tracking a moving target using the time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurements obtained at two unmanned aerial vehicles (UAVs) with varying baseline. Accumulation time is necessary because the emitter position cannot be estimated at each emission when there are only two UAVs as sensors. And the target position and velocity estimation accuracy often suffer from low convergence in conventional formation flight mode no matter what nonlinear filtering algorithm is used. Based on the analysis of the influence of different flight modes on the positioning performance, a moving target tracking system from two UAVs with varying baseline is proposed. The performance of unscented Kalman filter (UKF) under proposed system is analysed and compared with the Cramer-Rao lower bound (CRLB). Simulation results show that the proposed system can speed up the convergence.

1. Introduction

Moving target tracking using TDOA or FDOA measurements is a highly nonlinear problem [1]. When the number of sensors is sufficient for locating the emitter at a single emission [2]-[5], the emitter can be tracked using the position estimate calculated at each emission as measurements with smoothing techniques such as linear Kalman filter [6]. Another approach is using TDOA or FDOA measurements obtained at a single emission as input of nonlinear filter. Unlike techniques such as least squares [7], processing all measurements at once, which is not suitable when the emitter is mobile, Kalman filter is a kind of recursive estimation by combing the current time measurements with the estimated results at the previous moment without affecting performance. Many nonlinear filter algorithms have been studied using TDOA or FDOA measurements, but more than two sensors are needed [8]-[11].

When there are only two UAVs as sensors, however, the emitter position cannot be estimated at each emission and therefore sufficient measurements obtained over a period of time are necessary. The emitter position can be estimated using a sequence of TDOA measurements with two UAVs [12]-[13]. The combination of TDOA and FDOA measurements will cover a wider range of emitters that may be tracked [1] and performs better than using only one of them [4], thus causing much concern. Icki [1]
proposes a GMM-EKF algorithm, which achieves recursive tracking of one mobile emitter using a sequence of TDOA and FDOA measurement pairs obtained by one pair of sensors, but suffers from low convergence for the target 100km away. In fact, it is possible to accelerate the convergence rate only by changing the motion state of sensors when the target motion state and measurement accuracy are certain. How to design the motion state of sensors to achieve short-time moving target tracking in the case of only two UAVs is the problem to be discussed.

The paper is organised as follows. The problem is described in Section 2. Section 3 shows the evaluation of CRLB when the target moves in a constant speed. Section 4 shows performance results and application scene under proposed system. Section 5 concludes the paper.

2. Problem Description

Consider an emitter at position $\mathbf{p}(t) = (x(t), y(t), z(t))^T$ which emits signal with carrier frequency $f_c$, where $t$ indicates time. There are $M$ sensors receive the signal from the emitter, where sensor $i$ is located at position $\mathbf{s}_i(t) = (x_i(t), y_i(t), z_i(t))^T$.

The distance vector between the emitter and sensor $i$ at time $t$ is $\mathbf{r}_i(t) = \mathbf{p}(t) - \mathbf{s}_i(t)$, and the time delay of signal received at sensor $i$ is

$$\tau_i(t) = \frac{\|\mathbf{r}_i(t)\|}{c}$$

where $c$ denotes the signal propagation speed.

Assuming the velocity of emitter is $\dot{\mathbf{p}}(t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t))^T$ and of sensor $i$ is $\dot{\mathbf{s}}_i(t) = (\dot{x}_i(t), \dot{y}_i(t), \dot{z}_i(t))^T$. The Doppler shift of the signal received by sensor $i$ at time $t$ is defined by

$$f_i(t) = \frac{f_c}{c} \left( \frac{\mathbf{p}(t) - \dot{\mathbf{s}}_i(t)}{\|\mathbf{r}_i(t)\|} \right) \frac{\mathbf{r}_i(t)}{\|\mathbf{r}_i(t)\|^2}$$

The TDOA and FDOA is given by

$$\tau_{ii}(t) = \tau_i(t) - \tau_i(t) + n_{i1}$$

$$f_{ii}(t) = f_i(t) - f_i(t) + \hat{n}_{i1}$$

with additive zero-mean white Gaussian noise $n_{i1}$ with covariance $\sigma_{i1}^2$ and $\hat{n}_{i1}$ with covariance $\sigma_{i1}^2$.

For a two dimensional position noise, at least three sensors are required to estimate target position at a single time. Now that the number of sensors is two, a sequence of TDOA and FDOA measurement pairs are necessary to estimate target state by solving an over-determined problem. To clarify the idea clearly, only the target of uniform motion is considered below.

3. CRLB

It is assumed that the target of current time should be located by using the measured data of the current time $t_k$ and the preceding $K$ moments. The initial time is $t_0$ and the $k$-th time is $t_k, 0 \leq k \leq K$.

Assuming the velocity of emitter is constant and written as $\dot{\mathbf{p}}(t_k) = (\dot{x}, \dot{y}, \dot{z})^T$, the position of emitter of current time is $\mathbf{p}(t_k) = (x, y, z)^T$, then the position of emitter at $k$-th time can be represented by the position of the current time and is given by

$$\mathbf{p}(t_k) = (x(t_k), y(t_k), z(t_k))^T$$

$$= (x - (K - k)x, y - (K - k)y, z - (K - k)z)^T$$

$$\mathbf{p}(t_k)$$
The TDOA and FDOA measurements vector of the current time $t_k$ and the preceding $K$ moments is defined by

$$ F = F' + n $$

$$ F = \begin{bmatrix} \tau_{21}(t_0) & \tau_{21}(t_1) & \cdots & \tau_{21}(t_k) \\ f_{21}(t_0) & f_{21}(t_1) & \cdots & f_{21}(t_k) \end{bmatrix}^{(M-1)(K+1)} $$

with $(\ast)'$ represents the true value without noise and $n$ is measurements noise with covariance

$$ Q = \text{diag}\left[ \sigma_i^2, \cdots, \sigma_i^2 \right]^{(M-1)(K+1)} $$

Let target state vector $x = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$. Hence the conditional probability density function of $F$ is

$$ p(F|x) = \frac{1}{(2\pi)^{(M-1)(K+1)/2}} |Q|^{1/2} \exp\left\{ -\frac{1}{2} (F-F')^T Q^{-1} (F-F') \right\} $$

And the CRLB of $x$ at time $t_k$ is given by

$$ \Phi = \left( A^T Q^{-1} A \right)^{-1} $$

where $A$ is the derivative of $F'$ with respect to $x$ [14].

4. Simulation

By setting up multiple scenarios, this section analyses the factors affecting the positioning accuracy, and proposes a moving target tracking system from two UAVs with varying baseline. The performance of the UKF and application scene under proposed system are analysed by simulation.

4.1. Scenario

Set up three scenarios as shown in Table 1 to Table 3. In scenario 1, two planes move in a uniform straight line at a certain speed side by side. In scenario 2, two planes fly side by side, with one plane moving in a uniform straight line, and the other plane moving in a uniform acceleration straight line for 15 seconds before doing a uniform straight line motion. In scenario 3, two planes fly by column, with one plane doing a uniform linear motion, and the other doing a uniform acceleration linear motion for 15 seconds before doing a uniform linear motion.

Assuming an emitter is moving in a uniform straight line at a speed of 36m/s on the ground, note that only two-dimensional position is discussed here and three-dimensional positioning can be similarly extended. The distance between the target and the centre of the baseline is about 200 km at the initial time. The specific initial settings of emitter are shown in Table 4.

After 200 seconds of relative motion, the changes of TDOA and FDOA in different scenarios are shown in Figure 1. Figure 1(b) shows that the FDOA varies greatly in scenario 2 and scenario 3 when
the aircraft is uniformly accelerated, but the TDOA is almost unaffected by the flight trajectory of the UAVs.

| Table 1. Initial state of UAV in scenario 1 |
|-------------------------------------------|
| x(km) | y(km) | z(km) | Vx(m/s) | Vy(m/s) | Vz(m/s) |
|-------|-------|-------|---------|---------|---------|
| 1     | -10.000 | 0.000 | 10.000  | 0.000   | 100.000 | 0.000   |
| 2     | 10.000  | 0.000 | 10.000  | 0.000   | 200.000 | 0.000   |

| Table 2. Initial state of UAV in scenario 2. |
|---------------------------------------------|
| X(km) | y(km) | z(km) | Vx(m/s) | Vy(m/s) | Vz(m/s) |
|-------|-------|-------|---------|---------|---------|
| 1     | -10.000 | 0.000 | 10.000  | 0.000   | 100.000 | 0.000   |
| 2     | 10.000  | 0.000 | 10.000  | 0.000   | 50.000  | 0.000   |

| Table 3. Initial state of UAV in scenario 3. |
|---------------------------------------------|
| X(km) | y(km) | z(km) | Vx(m/s) | Vy(m/s) | Vz(m/s) |
|-------|-------|-------|---------|---------|---------|
| 1     | 0.000  | -10.000 | 10.000 | 0.000   | 100.000 | 0.000   |
| 2     | 0.000  | 10.000  | 10.000 | 0.000   | 50.000  | 0.000   |

| Table 4. Initial state of target. |
|-----------------------------------|
| x(km) | y(km) | z(km) | Vx(m/s) | Vy(m/s) | Vz(m/s) | fc(GHz) |
|-------|-------|-------|---------|---------|---------|---------|
| 1     | 0.000  | -10.000 | 10.000 | 20.000  | 30.000  | 0.000   | 2       |

![Figure 1](image1.png)

**Figure 1.** (a) The change of TDOA; (b) The change of FDOA.

### 4.2. Result

Setting the initial estimate of target state and covariance of UKF as below.

\[ \hat{x}_{10} = \begin{bmatrix} 100 \text{(km)} \\ 100 \text{(km)} \\ 10 \text{(m/s)} \\ 10 \text{(m/s)} \end{bmatrix} \]

\[ P_{0} = \begin{bmatrix} 50 \text{(km)}^{2} & 0 \\ 0 & 20 \text{(m/s)}^{2} \\ 50 \text{(km)}^{2} & 20 \text{(m/s)}^{2} \\ 0 & 20 \text{(m/s)}^{2} \end{bmatrix} \]

(12)

(13)

The performance of UKF in in different scenarios are shown in Figure 2 and Figure 3. RMSE curve and CRLB keep the same downward trend and tend to be consistent when the accumulation time is long enough. There is an abnormal phenomenon that UKF has better tracking results than CRLB when the accumulation time is short, which results from the initial estimation of target state that utilizes some prior information. As can be seen from Figure 4, using only TDOA measurements can hardly
achieve moving target tracking because the target is far from the baseline centre, and the baseline length is much smaller than the distance from the target to the baseline so that the TDOA measurements changes very little. Thus we can learn that the uniformly accelerated linear motion of the UAVs in scenario 2 and 3 improves the change rate of FDOA and the FDOA measurements plays a major role in locating emitter 100km away, which explains why the convergence speed of RMSE in scenarios 2 and 3 are superior to scenario 1 in the same initial conditions.

In scenario 3, when target moving at a speed of (20, 30) m/s and the accumulated time is 30 s, GDOP with $\sigma_f = 60ns$ and $\sigma_f = 0.5Hz$ is presented in Figure 5. It can be seen from the graph that the targets within 250 km except for the blind area in the baseline direction, the corresponding position RMSE is no more than 5 Km under this flying mode.

Figure 2. The performance of UKF for target location estimation in different scenarios and compared with CRLB.

Figure 3. The performance of UKF for target velocity estimation in different scenarios and compared with CRLB.

Figure 4. Comparison of location CRLB in scenario 3 between using only TDOA/FDOA measurements and using both of them simultaneously.

Figure 5. GDOP in scenario 3 when accumulation time is 30s and target moves at a speed of (20, 30) m/s.

5. Conclusion
For slowly moving target 100 km away, the low-speed of the UAVs causes little variation in the TDOA measurements of the received signal at the different time, thence the higher the change rate of FDOA under the same accumulation time, the higher the positioning accuracy. Because the change of FDOA is mainly affected by the velocity change of UAVs when the moving state of the target is certain, which accounts for low convergence in conventional formation flight mode. Our proposed
system from two UAVs with varying baseline is easy to perform under engineering conditions and simulation results show that the accumulation time of target tracking can be shortened effectively.

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