A new class of composite indicators: the penalized power means

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Abstract In this paper we propose a new aggregation method for constructing composite indicators that is based on a penalization of the power means. The idea underlying this approach consists in multiplying the power mean by a factor that takes into account for the horizontal heterogeneity among indicators with the aim of penalizing the units with larger heterogeneity. In order to measure this heterogeneity, we scale the vector of normalized indicators by their power means, we compute the variance of the scaled normalized indicators transformed by means of the appropriate Box-Cox function, and we measure the heterogeneity as the counter image of this variance through the Box-Cox function. The resulting penalization factor can be interpreted as the relative error, or the loss of information, that we obtain substituting the vector of the normalized indicators with their power mean. This penalization approach has the advantage to be fully data-driven and to be coherent with the same principle underlying the power mean approach, that is the minimum loss of information principle as well as to allow for a more refined rankings. The penalized power mean of order one coincides with the Mazziotta Pareto Index.

Keywords Composite indicator; Aggregation method; Minimum loss of information principle

1 Introduction

The construction of composite indicators consists in reducing a multidimensional phenomenon into a one-dimensional phenomenon aggregating the multiple dimensions of the phenomenon, namely the indicators, into a single one, namely the composite indicator. The resulting composite indicator, although more simple and

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manageable to interpret and understand, is less informative with respect the vector of the indicators. That is, necessary, the aggregation process involves a loss of information. Despite the effort of many authors to develop objective measures of information loss (see, among others, [13], [12], [14]), choosing effective measures of the information loss is a crucial task and depends on the subjective preference of the decision maker.

A good aggregation should conjugate the two-fold opposite objectives of reducing the dimension of the phenomenon under investigation as well as of generating a reasonable loss of information. The majority of the aggregation functions are based on minimizing the loss for deviation of individual indicators from the aggregated value. In their paper Calvo and Beliakov [5] systemize the various type of aggregation function, proving that any averaging aggregation functions are penalty-based functions, that is functions obtained minimizing the penalty due to substitute the vectors of indicators with the aggregated value. Specifically, the power means are found choosing as penalty (loss) function the Euclidean distance from the vector of indicators transformed through a Box-Cox function (for more detail we refer to Berger and Casella [1]). In other words, the power means can be seen as the least squares estimate of the vector of the indicators in the Box-Cox transformed space. Therefore, in the transformed space the power mean suffers from the same troubles of the arithmetic mean, that is the compensability and the substitutability. These issues, that are consequences of the loss of information, should be taken into account in the aggregation phase in order to differentiate units with same value of power mean.

The idea of penalizing units in different way is shared by Mauro et al. [8] and Biggeri et al. [2] that develop and apply in the well-being context the Multidimensional Synthesis of Indicators (MSI) approach, that aggregate the indicators relative to different units with a power mean with different order. The order of the power mean is a function of the arithmetic mean of the indicators, in a way such that units with lower indicator arithmetic mean have associated a lower order. Moreover, Rogge [10] applies a Benefit of Doubt (BoD)-based weighted version of the power means for constructing an HDI index. The weights associated to the indicators depend on the unit in order to take into account for the difference in the indicator unbalance. In line with the works of Mauro et al. [8], Biggeri et al. [2] and Rogge [10] we propose a new aggregation approach that penalizes the power mean associated to a unit with a factor that takes into account for the horizontal heterogeneity among indicators with the aim of penalizing the units with larger heterogeneity. In order to measure this heterogeneity, we scale the vector of normalized indicators by their power means, we compute the variance of the scaled normalized indicators transformed by means of the appropriate Box-Cox function, and we measure the heterogeneity as the counter image of this variance through the Box-Cox function. The resulting penalization factor can be interpreted as the relative error, or the loss of information, that we obtain substituting the vector of the normalized indicators with their power mean. The penalized power mean of order one coincides with the Mazziotta Pareto Index [9].

The penalization approach proposed in this paper is fully data-driven and can be effectively applied in many other fields, such as environmental indices [11], fuzzy rule based systems, pattern recognition, decision making problems [7], weighted voting systems [4].

The paper is organized as follows: in Section 1 we introduce the penalized
2 A new class of composite indicators

Let $x_{ij}$ and $I_{ij}$ be, respectively, the value and the normalized value of the indicator $j$, $j = 1, 2, \ldots, m$, relative to unit $i$, $i = 1, 2, \ldots, n$, such that $I_{ij} \in [0, 1]$. Let $\mathbf{I}_i = [I_{i1} I_{i2} \cdots I_{im}]$ be vector of indicators relative to the $i$–th unit, $i = 1, 2, \ldots, n$, the power mean of order $p$ associated to $\mathbf{I}_i$ is defined by:

$$M_{p,i} = M_p(\mathbf{I}_i) = \begin{cases} \left( \frac{1}{m} \sum_{j=1}^{m} I_{ij}^p \right)^{\frac{1}{p}}, & p \neq 0, \\ \prod_{j=1}^{m} I_{ij}, & p = 0. \end{cases} \quad (1)$$

The arithmetic mean, the geometric mean and the harmonic mean are special cases of the power mean for $p = 1, p = 0$ and $p = -1$, respectively.

For $i = 1, 2, \ldots, n$ the composite indicator $M_{p,i}$ can be read as solution of the following optimization problem [1]:

$$\min_{c \in \mathcal{A}_p} F_p(c; \mathbf{I}_i), \quad (2)$$

where:

$$F_p(c; \mathbf{I}_i) = \frac{1}{m} \sum_{j=1}^{m} (h_p(I_{ij}) - h_p(c))^2 \quad (3)$$

is the (information) loss function (or, the penalty function, to use the nomenclature of Calvo and Beliakov [5]) and $h_p(x)$ is the Box-Cox transformation [3]:

$$h_p(x) = \begin{cases} x^p - \frac{1}{p}, & p \neq 0, \\ \ln x, & p = 0. \end{cases} \quad (4)$$

In [2] $\mathcal{A}_1 = \mathbb{R}$ and $\mathcal{A}_p = \{x \in \mathbb{R} : x > 0\}$ denote, respectively, the domain of the function $h_p$ for $p = 1$ and $p \neq 1$. Note that the function $h_p(1) = 0$ and $h_p(x)$ is a strictly increasing function of $x$.

The function $F_p$ in [3] measures the sample (biased) variance of vector $h_p(\mathbf{I}_i) = [h_p(I_{i1}) h_p(I_{i2}) \cdots h_p(I_{im})]'$ from its arithmetic mean:

$$h_p(M_{p,i}) = M_1(h_p(\mathbf{I}_i)) = \frac{1}{m} \sum_{j=1}^{m} h_p(I_{ij}), \quad (5)$$

that is in the $p$–transformed space (the space obtained transformed the m dimensional vectors via the Box-Cox function of order $p$) the $p$-order generalized mean plays the role of arithmetic mean.

Moreover, for any unit $i$ we can measure the error (loss of information) caused
from substituting the vector of indicators \( h_p(\mathbf{I}_i) \) with \( h_p(M_{p,i}) \) evaluating the objective function \( F_p \) at its optimum:

\[
F_p(M_{p,i}; \mathbf{I}_i) = \frac{1}{m} \sum_{j=1}^{m} (h_p(I_{ij}) - h_p(M_{p,i}))^2.
\]

(6)

It is easy to see that this error coincides with the (biased) sample variance of the vector \( h_p(\mathbf{I}_i) \), that here and in the rest of paper we denote by \( S^2_{p,i} \). Therefore the quantity \( h_p^{-1}(S^2_{p,i}) \) is a measure of the information loss caused from substituting \( \mathbf{I}_i \) with \( M_{p,i} \). Note that the size of \( h_p^{-1}(S^2_{p,i}) \) depends strongly on \( M_{p,i} \), then the variances associated to units with different power means are not comparable.

In order to measure the relative information loss caused from substituting \( \mathbf{I}_i \) with \( M_{p,i} \), it is necessary to scale the indicators referring to a same unit by a specific criterion that removes the dependence from the power mean. To this purpose, let us consider the vector of scaled normalized indicators, \( \tilde{\mathbf{I}}_i = [\tilde{I}_{i1}, \tilde{I}_{i2}, \ldots, \tilde{I}_{in}]^\top \), where:

\[
\tilde{I}_{ij} = \frac{I_{ij}}{M_{p,i}}, \quad i = 1, 2, \ldots, n.
\]

(7)

Note that from the homogeneity property of the \( p \)-order power means it follows that \( M_{p}(\tilde{\mathbf{I}}_i) = 1 \), for \( i = 1, 2, \ldots, n \), then the error (loss of information) caused from substituting the vector of indicators \( h_p(\tilde{\mathbf{I}}_i) \) with \( h_p(M_{p}(\tilde{\mathbf{I}}_i)) = h_p(1) = 0 \) is given by:

\[
\tilde{S}^2_{p,i} = \frac{1}{m} \sum_{j=1}^{m} (h_p(\tilde{I}_{ij}) - h_p(1))^2 = \frac{1}{m} \sum_{j=1}^{m} (h_p(\tilde{I}_{ij}))^2, \quad i = 1, 2, \ldots, n.
\]

(8)

For \( i = 1, 2, \ldots, n \), the quantity

\[
h_p^{-1}(\tilde{S}^2_{p,i}) = \begin{cases} 
(1 + p \tilde{S}^2_{p,i})^{\frac{1}{p}}, & p \neq 0, \\
\exp(\tilde{S}^2_{0,i}), & p = 0,
\end{cases}
\]

(9)

is independent from the size of \( M_{p,i} \) and measures the relative information loss caused from substituting \( \mathbf{I}_i \) with \( M_{p,i} \). The higher the value of \( h_p^{-1}(\tilde{S}^2_{p,i}) \), the higher the loss of information caused from considering \( M_{p,i} \) instead of the sub-indicator vector \( \mathbf{I}_i \), no matter the value of \( M_{p,i} \).

We use \( h_p^{-1}(\pm \tilde{S}^2_{p,i}) \) to penalize the power mean of order \( p \). In particular, for \( i = 1, 2, \ldots, n \), the penalized power mean of order \( p \) associated to the normalized indicator vector \( \tilde{\mathbf{I}}_i \) is defined by:

\[
PM_{p,i}^{\pm} = M_{p,i}h_p^{-1}(\pm \tilde{S}^2_{p,i}) = \begin{cases} 
M_{p,i} \left(1 \pm p \tilde{S}^2_{p,i}\right)^{\frac{1}{p}}, & p \neq 0, \\
M_{0,i} \exp(\pm \tilde{S}^2_{0,i}), & p = 0.
\end{cases}
\]

(10)

Note that the \( \pm \) sign in (10) depends on the type of phenomenon considered, if increasing variations of the indicator correspond to positive variations of the
phenomenon (positive polarity) we choose the sign $-$, otherwise (negative polarity) we choose the sign $+$.

The scaling done in (7) ensures that the term $h_{p,i}^{-1}(\pm \tilde{S}_2^p(I))$ penalizes the score of each unit (the $p$-order power mean of the normalized indicators) independently from the value of the power mean itself with a quantity that is directly proportional to the “horizontal variability” of the indicators. The aim of the penalization is to favour the units that, power mean being equal, have a greater balance among the indicators. This is the idea underlying the “Method of Penalties by Coefficient of Variation”, introduced by Mazziotta and Pareto [9], that adjust the arithmetic mean by a penalization coefficient that is function, for each unit, of the indicator coefficient of variation to define the Mazziotta Pareto Index (MPI).

**Proposition 1** The penalized power mean of order one, $PM^\pm_{1,i}$, coincides with the MPI.

**Proof** For the proof see the Appendix.

**Proposition 2** The penalized power mean defined in (10) satisfies the following properties:

1. $PM^+_{p,i} \geq M_{p,i} \geq PM^-_{p,i}$.
2. $PM^+_{p,i} = PM^-_{p,i}$, if and only if $\tilde{S}_{p,i} = 0$.
3. $(PM^+_{p,i})^p = 2(M_{p,i})^p - (PM^-_{p,i})^p$ for $p \neq 0$.
4. $PM^+_{0,i} = PM^-_{0,i} \exp \{2\tilde{S}_0^2\}$.
5. Given two units $k$ and $h$ ($k \neq h$) with $M_{p,k} = M_{p,h}$, we have:
   - $PM^+_{p,k} > PM^-_{p,h}$ iff $\tilde{S}_{p,h} > \tilde{S}_{p,k}$,
   - $PM^-_{p,k} > PM^+_{p,h}$ iff $\tilde{S}_{p,k} > \tilde{S}_{p,h}$.
6. Given two units $k$ and $h$ ($k \neq h$) with $M_{p,k} > M_{p,h}$, for $p \neq 0$, we have:
   - $PM^+_{p,k} > PM^+_{p,h}$ iff $M_{p,k}^p - M_{p,h}^p > p \left(M_{p,k}^p \tilde{S}_{p,k}^2 - M_{p,h}^p \tilde{S}_{p,h}^2\right)$,
   - $PM^-_{p,k} > PM^-_{p,h}$ iff $M_{p,k}^p - M_{p,h}^p > p \left(M_{p,h}^p \tilde{S}_{p,h}^2 - M_{p,k}^p \tilde{S}_{p,k}^2\right)$.
7. Given two units $k$ and $h$ ($k \neq h$) with $M_{0,k} > M_{0,h}$, we have:
   - $PM^+_{0,k} > PM^+_{0,h}$ iff $M_{0,k} \exp \{2\tilde{S}_{0,k}^2 - \tilde{S}_{0,h}^2\}$,
   - $PM^-_{0,k} > PM^-_{0,h}$ iff $M_{0,h} \exp \{2\tilde{S}_{0,h}^2 - \tilde{S}_{0,k}^2\}$.
8. $\lim_{p \to -\infty} PM^\pm_{p,i} = \min_{j=1,2,\ldots,m} I_{ij}$.
9. $\lim_{p \to +\infty} PM^\pm_{p,i} = \max_{j=1,2,\ldots,m} I_{ij}$.

**Proof** For the proof of Properties 8 and 9 see the Appendix.
Note that Properties 8 and 9 in Proposition 2 imply that the penalization has not effect when the power mean of order $-\infty$ (i.e. the minimum function) or $+\infty$ (i.e. the maximum function) is considered. In fact the minimum and the maximum functions realize already, respectively, in the case of positive polarity and in the case of negative polarity, the maximum penalization for unbalanced values of the indicators, therefore they do not need further penalizations.

**Proposition 3** The penalization factor in (10):

$$g_{p,i}^\pm = (1 \pm \bar{S}^2_{p,i})^{\frac{1}{p}}$$

satisfies the following properties:

1. $\frac{\partial g_{p,i}^+}{\partial p} < 0$ for $p > 0$ and $\frac{\partial g_{p,i}^+}{\partial p} > 0$ for $p < 0$.
2. $\frac{\partial g_{p,i}^+}{\partial p} > 0$ for $p > 0$ and $\frac{\partial g_{p,i}^+}{\partial p} < 0$ for $p < 0$.
3. $\lim_{p \to 0} g_{p,i}^\pm = \exp \left\{ \pm \bar{S}^2_{0,i} \right\}$.
4. $\lim_{p \to \pm \infty} g_{p,i}^\pm = 1$.

**Proof** See the Appendix.

**Proposition 4** For the penalized power mean (10) the Marginal Rate of Compensation (MRC) [6] between variables $z_{ik}$, $z_{ih}$:

$$MRC_{kh,i} = \frac{\partial PM_{p,i}^+}{\partial I_{ik}} / \frac{\partial PM_{p,i}^+}{\partial I_{ih}} = \begin{cases} \left( \frac{I_{ik}}{I_{ih}} \right)^{p-1}, & p \neq 0, \\ \frac{I_{ik}}{I_{ih}}, & p = 0. \end{cases}$$

**Proof** See the Appendix.
3 Appendix

Proof of Proposition 1 Substituting (4) for \( p = 1 \) in (8) we have:

\[
\tilde{S}_{1,i}^2 = \frac{1}{m} \sum_{j=1}^{m} (\tilde{I}_{ij} - 1)^2 = \frac{1}{m} \sum_{j=1}^{m} \left( \frac{I_{ij}}{M_{1,i}} - 1 \right)^2 = \frac{1}{m} \sum_{j=1}^{m} (I_{ij} - M_{1,i})^2 \]

\[
= \frac{S_{1,i}^2}{M_{1,i}^2}, \quad i = 1, 2, \ldots, n, \tag{13}
\]

where:

\[
S_{1,i}^2 = \frac{1}{m} \sum_{j=1}^{m} (I_{ij} - M_{1,i})^2, \quad i = 1, 2, \ldots, n, \tag{14}
\]

is the (biased) sample variance of vector \( \underline{I}_i \).

Substituting (13) into (11) for \( p = 1 \) we have:

\[
PM_{1,i}^\pm = M_{1,i} \left( 1 \pm \frac{S_{1,i}^2}{M_{1,i}^2} \right), \quad i = 1, 2, \ldots, n, \tag{15}
\]

that is the MPI.

Proof of Proposition 2 (Properties 8 and 9) The (biased) sample variance \( \tilde{S}_{p,i}^2 \) in (14) can be rewritten as follows:

\[
\tilde{S}_{p,i}^2 = \frac{1}{p^2} \frac{1}{m} \sum_{j=1}^{m} \left( \frac{I_{ij}}{M_{p,i}} \right)^p - 1 \tag{16}
\]

then, taking the limit of (16) for \( p \to +\infty \) and recalling that \( M_{p,i} \to \max(I_{i1}, I_{i2}, \ldots, I_{im}) \geq I_{ij}, \ j = 1, 2, \ldots, m, \) we obtain:

\[
\lim_{p \to +\infty} p \tilde{S}_{p,i}^2 = \lim_{p \to +\infty} \frac{1}{p} = 0^+. \tag{17}
\]

Substituting (17) into (16) we obtain Property 7.

Analogously, recalling that \( M_{p,i} \to \min(I_{i1}, I_{i2}, \ldots, I_{im}) \leq I_{ij}, \ j = 1, 2, \ldots, m, \) we have:

\[
\lim_{p \to -\infty} p \tilde{S}_{p,i}^2 = \lim_{p \to -\infty} \frac{1}{p} = 0^- \tag{18}
\]

Substituting (18) into (16) we prove Property 8.

Proof of Proposition 3 The derivative of \( g_{p,i}^\pm \) with respect to \( p \) is:

\[
\frac{\partial g_{p,i}^\pm}{\partial p} = g_{p,i} \left[ -\frac{1}{p^2} \ln \left( 1 \pm p \tilde{S}_{p,i}^2 \right) + \frac{1}{p} \frac{1}{1 \pm p \tilde{S}_{p,i}^2} \left( \tilde{S}_{p,i}^2 + p \frac{\partial \tilde{S}_{p,i}^2}{\partial p} \right) \right]. \tag{19}
\]
The derivative of $S^2_{p,i}$ with respect to $p$ is:
\[
\frac{\partial \tilde{S}^2_{p,i}}{\partial p} = -\frac{2}{p} \tilde{S}^2_{p,i} + \frac{2}{p^2} \sum_{j=1}^{m} \left( \frac{I_{ij}}{M_{p,i}} \right)^p - 1 \right) \frac{\partial}{\partial p} \left( \frac{I_{ij}}{M_{p,i}} \right)^p \right) = -\frac{2}{p} \tilde{S}^2_{p,i}. \tag{20}
\]

Substituting (20) into (19), we obtain:
\[
\frac{\partial g^\pm_{p,i}}{\partial p} = g_{p,i} \left[ -\frac{1}{p^2} \ln \left( 1 \pm p \tilde{S}^2_{p,i} \right) + \frac{1}{p} \frac{\tilde{S}^2_{p,i}}{1 \pm p \tilde{S}^2_{p,i}} \right]. \tag{21}
\]

Finally, observing that $\ln(1 + p \tilde{S}^2_{p,i}) > 0$ and $\ln(1 - p \tilde{S}^2_{p,i}) < 0$, we obtain prove Properties 1 and 2.

Property 2 has been proved in the proof of Proposition 1 (see Equations (22), (23)).

Taking the limit for $p \to 0$ of $g^\pm_{p,i}$ we obtain:
\[
\lim_{p \to 0} g^\pm_{p,i} = \exp \left\{ \pm \tilde{S}^2_{0,i} \right\}. \tag{22}
\]

This concludes the proof.

**Proof of Proposition 4** The derivative of $PM_{p,i}$ with respect to $I_{ik}$ is:
\[
\frac{\partial PM_{p,i}}{\partial I_{ik}} = \frac{\partial M_{p,i}}{\partial I_{ik}} g^\pm_{p,i} + M_{p,i} \frac{\partial g^\pm_{p,i}}{\partial I_{ik}}. \tag{24}
\]

The derivative of $M_{p,i}$ with respect to $I_{ik}$ is:
\[
\frac{\partial M_{p,i}}{\partial I_{ik}} = \begin{cases} 
\frac{1}{m} p I_{ik}^{p-1}, & p \neq 0, \\
\frac{1}{m} I_{ik}, & p = 0.
\end{cases} \tag{25}
\]

The derivative of $g_{p,i}$ with respect to $I_{ik}$ is:
\[
\frac{\partial g^\pm_{p,i}}{\partial I_{ik}} = \begin{cases} 
\frac{1}{p} \left( 1 \pm p \tilde{S}^2_{p,i} \right)^{p-1} \frac{\partial \tilde{S}^2_{p,i}}{\partial I_{ik}}, & p \neq 0, \\
0, & p = 0.
\end{cases} \tag{26}
\]
The derivative of $S^2_{p,i}$ with respect to $I_{ik}$ is:

$$
\frac{\partial S^2_{p,i}}{\partial I_{ik}} = 2 \frac{p^2}{m} \sum_{j=1}^{m} \left( \left( \frac{I_{ij}}{M_{p,i}} \right)^p - 1 \right) \frac{\partial \left( \frac{I_{ik}}{M_{p,i}} \right)^p}{\partial I_{ik}} = 0. \tag{27}
$$

Substituting (25), (26), (27) into (24) we obtain:

$$
\frac{\partial PM_{p,i}}{\partial I_{ik}} = \begin{cases} 
\frac{1}{m} \frac{p}{I_{ik}} \frac{p^{p-1}}{m} g^{\pm}_{p,i}, & p \neq 0, \\
\frac{1}{m} \frac{1}{I_{ik}} g^{\pm}_{p,i}, & p = 0.
\end{cases} \tag{28}
$$

From (28) it follows easily (12).
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