Nielsen, M.; Navarra, F. S.; Bracco, M. E.
Investigating the Tetraquark Structure of the New Mesons
Brazilian Journal of Physics, vol. 37, núm. 1A, março, 2007, pp. 56-58
Sociedade Brasileira de Física
São Paulo, Brasil

Available in: http://www.redalyc.org/articulo.oa?id=46437118
Investigating the Tetraquark Structure of the New Mesons

M. Nielsen¹, F. S. Navarra¹, and M. E. Bracco²

¹Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil
²Instituto de Física, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, 20550-900 Rio de Janeiro, RJ, Brazil

Received on 29 September, 2006

Using the QCD sum rule approach we investigate the vertex associated with the decay \(D^0(0^+) \to D^+ \pi^-\), where the scalar meson \(D^0(0^+)\) is considered as a four-quark state \(|\epsilon d(\bar{u}d)|\). Although our results for the mass and partial decay width are smaller than the mass and the total decay width of the broad scalar meson \(D^0_b(2308)\) reported by BELLE Collaboration, we can not discard the possibility that the BELLE’s resonance can be interpreted as the four-quark state studied here.

Keywords: QCD sum rules; Four-quark states; Scalar mesons

The recent observations of the very narrow resonances \(D^+_s(2317)\) by BaBar [1], \(D^+_s(2460)\) by CLEO [2], and \(X(3872)\) by BELLE [3], all of them with masses below quark model predictions, have stimulated a renewed interest in the spectroscopy of open charm and charmonium states. Due to their narrowness and small masses, these new mesons were considered as good candidates for four-quark states by many authors [4]. The idea of mesons as four-quark states is not new. Indeed, even Gell-Mann in his first work about quarks had mentioned that mesons could be made out of \((q\bar{q})\), \((qq\bar{q}\bar{q}) etc. [5]. The best known example of applying the idea of four-quark states for mesons is for the light scalar mesons (the isoscalars \(\sigma(500)\), \(f_0(980)\), the isodoublet \(\kappa(800)\) and the isovector \(a_0(980)\)) [6, 7]. In a four-quark scenario, the mass degeneracy of \(f_0(980)\) and \(a_0(980)\) is natural, the mass hierarchy pattern of the nonet is understandable, and it is easy to explain why \(\sigma\) and \(\kappa\) are broader than \(f_0(980)\) and \(a_0(980)\).

In refs. [8, 9] the method of QCD sum rules (QCDSR) [10–12] was used to study the two-point functions for the mesons \(D^+_s(2317)\) and \(X(3872)\) considering them as four-quark states in a diquark-antidiquark configuration. The results obtained for their masses are given in Table I.

Comparing the results in Table I with the resonance masses given by: \(D^+_s(2317)\) and \(X(3872)\), we see that it is possible to reproduce the experimental value of the masses using a four-quark representation for these states.

The study of the three-point functions related to the decay widths \(D^+_s(2317) \to D^+_s \pi^0\) and \(X(3872) \to J/\psi \pi^+ \pi^−\), using the diquark-antidiquark configuration for \(D_{sJ}\) and \(X\), was done in refs. [13, 14]. The results obtained for their partial decay widths are given in Table II.

From Table II we see that the partial decay width obtained in ref. [13], supposing that the mesons \(D^+_s(2317)\) is a four-quark state, is consistent with the experimental upper limit. However, in the case of the meson \(X(3872)\), the partial decay width obtained in ref. [14] is much bigger than the experimental upper limit to the total width.

In ref. [14] some arguments were presented to reduce the value of this decay width, by imposing that the initial four-quark state needs to have a non-trivial color structure. In this case, its partial decay width can be reduced to \(\Gamma(X \to J/\psi \pi^+ \pi^-) = (0.7 \pm 0.2)\) MeV. However, that procedure may appear somewhat unjustified and, therefore, more study is needed until one can arrive at a definitive conclusion about the structure of the meson \(X(3872)\).

In ref. [8], besides the four-quark state \((cq)(\bar{q}\bar{q})\) representing the meson \(D^+_s(2317)\), it was also studied the configuration \((cq)(\bar{u}\bar{d})\) associated with a possible scalar meson that we will call \(D(0^+)\) (the \(0^+\) stands for \(J^P\)). The mass obtained for this state is: \(m_{D(0^+)} = (2.22 \pm 0.21)\) MeV, in a very good agreement with the prediction made in ref. [15] for the \(D(0^+)\) scalar meson: \(m_{D(0^+)} = (2.215 \pm 0.002)\) MeV. This value was obtained in ref. [15] by supposing that the meson \(D(0^+)\) is the chiral partner of the meson \(D_s\), with the same mass difference as the chiral pair \(D^+_s(2317) - D_s\). The authors of ref. [15] have also evaluated the decay widths \(D^+_s \to D_s^* \pi^0\) and \(D(0^+) \to D \pi^\pm\) obtaining: \(\Gamma(D^+_s \to D_s^* \pi^0) = 21.5 G_A^2\) keV and \(\Gamma(D(0^+) \to D \pi^\pm) = 326 G_A^2\) MeV, where they expect \(G_A \sim 1\).

Here, we extend the calculation done in refs. [8, 13] to study the vertex associated with the decay \(D^0(0^+) \to D^+ \pi^-\). The QCDSR calculation for the vertex, \(D^0(0^+) \to D^+ \pi^-\), centers around the three-point function given by

\[
T_{\mu}(p, p', q) = \int d^4 x d^4 y \, e^{ip' \cdot x} e^{iq \cdot y} (0) T [j_D(x) j_{SP}(y) j_A^0(0)] (0),
\]

where \(j_0\) is the interpolating field for the scalar \(D^0(0^+)\) meson.

**TABLE I: Numerical results for the resonance masses**

| resonance | \(D_{sJ}\) | \(X\) |
|-----------|------------|-------|
| mass (GeV) | 2.32 ± 0.13 | 3.93 ± 0.15 |

**TABLE II: Numerical results for the resonance decay widths**

| decay | \(D^+_s \to D_s^* \pi^0\) | \(X \to J/\psi \pi^+ \pi^-\) |
|-------|--------------------------|--------------------------|
| \(\Gamma\) (MeV) | \((6.2 \pm 3)\) | \((50 \pm 15)\) |
| \(\Gamma_{tot}\) (MeV) | \(< 5\) | \(< 2.3\) |
The calculation of the phenomenological side proceeds by inserting intermediate states for $D, \pi$ and $D(0^+)$, and by using the definitions: 
\[ \langle 0 | j_5 | p(q) \rangle = iq_μ F_5, \quad \langle 0 | j_0 | D(p') \rangle = \frac{m_0^2 f_D}{m_0} , \quad \langle 0 | j_0 | D(0^+)(p) \rangle = \lambda_0. \]
We obtain the following relation: 
\[ T_\mu^{\text{phen}}(p, p', q) = \frac{\lambda_0 m_D^2 f_D F_5 g_{D(0^+)} \delta(0^+)}{m_c (m_0^2 - m_D^2)} \left( e^{-m_0^2/M^2} - e^{-m_0^2/D(0^+)^2/M^2} \right) + A e^{-m_0^2/M^2} + \int_0^\infty \rho_{cc}(u) e^{-u/M^2} du, \]
where $A$ and $\rho_{cc}(u)$ stands for the pole-continuum transitions and pure continuum contributions, with $s_0$ and $u_0$ being the continuum thresholds for $D(0^+)$ and $D$ respectively. For simplicity, one assumes that the pure continuum contribution to the spectral density, $\rho_{cc}(u)$, is given by the result obtained in the OPE side. Therefore, one uses the ansatz: $\rho_{cc}(u) = \rho_{\text{OPE}}(u)$. In Eq.(5), $A$ is a parameter which, together with $g_{D(0^+)} \delta(0^+)$, has to be determined by the sum rule.

In the OPE side we single out the leading terms proportional to $q_μ/q^2$. Transferring the pure continuum contribution to the OPE side, the sum rule for the coupling constant, up to dimension 7, is given by:
\[ C \left( e^{-m_D^2/M^2} - e^{-m_D^2/D(0^+)^2/M^2} \right) + A e^{-m_0^2/M^2} + 2 \langle \bar{q}q \rangle \left[ \frac{1}{2\pi^2} \int_{m_0^2}^{u_0} du e^{-u/M^2} u \left( 1 - \frac{u^2}{m_0^2} \right)^2 - \frac{1}{6} \langle \bar{q}q \rangle e^{-m_0^2/M^2} \right], \]
with
\[ C = \frac{\lambda_0 m_D^2 F_5 g_{D(0^+)} \delta(0^+)}{m_c (m_0^2 - m_D^2)} \]

In the numerical analysis of the sum rules, the values used for the meson masses, quark masses and condensates are: $m_{D(0^+)} = 2.2$ GeV, $m_D = 1.87$ GeV, $m_c = 1.2$ GeV, $\langle \bar{q}q \rangle = -(0.23)^3$ GeV$^3$. For the meson decay constants we use $F_5 = \sqrt{2} \ 93$ MeV and $F_0 = 0.20$ GeV [19]. We use $u_0 = 6$ GeV$^2$ and for the current meson coupling, $\lambda_0$, we are going to use the result obtained from the two-point function in ref. [8]. Considering $2.6 < \sqrt{s_0} < 2.8$ GeV we get $\lambda_0 = (3.3 \pm 0.3) \times 10^{-3}$ GeV$^3$.

In Fig. 1 we show, through the dots, the right-hand side (RHS) of Eq.(6) as a function of the Borel mass. We use the same Borel window as defined in ref.[8]. To determine $g_{D(0^+)} \delta(0^+)$ we fit the QCD sum rules with the analytical expression in the left-hand side (LHS) of Eq.(6). Using $\sqrt{s_0} = 2.7$ GeV we get: $C = 1.25 \times 10^{-3}$ GeV$^2$ and $A = 1.47 \times 10^{-3}$ GeV$^2$. Using the definition of $C$ in Eq.(7) and $\lambda_0 = 3.3 \times 10^{-3}$ GeV$^3$ (the value obtained for $\sqrt{s_0} = 2.7$ GeV we get $g_{D(0^+)} \delta(0^+) = 6.94$ GeV. Allowing so to vary in the interval $2.6 < \sqrt{s_0} < 2.8$ GeV, the corresponding variation obtained for the coupling constant is $5$ GeV $\leq g_{D(0^+)} \delta(0^+) \leq 7.5$ GeV.

The coupling constant, $g_{D(0^+)} \delta(0^+)$, is related to the partial decay width through the relation:
\[ \Gamma(D^0(0^+) \rightarrow D^+ \pi^-) \]
In Table III we show the partial decay width obtained in ref. [15], in ref. [13] and here for different decays. From the results in Table III we see that if one uses $G_A = 0.6$, the result presented here and the result in ref. [13] are consistent with the results presented in ref. [15] for both decays.

Table III: Numerical results for the resonance partial decay widths from different approaches

| decay                  | ref. [15]  | ref. [13]  | this work |
|------------------------|------------|------------|-----------|
| $D_{sJ}^0 \rightarrow D^+_s \pi^0$ | 21.5 $G_A^2$ keV ($6\pm 2$ keV) |
| $D^0(0^+) \rightarrow D^+ \pi^-$ | 326 $G_A^2$ MeV | 120 $\pm$ 20 MeV |

It is important to notice that the BELLE Collaboration [20] has reported the observation of a rather broad scalar meson $D_{sJ}^0$($2308$) in the decay mode $D_{sJ}^0(2308) \rightarrow D^+ \pi^-$ with a total width $\Gamma \sim 270$ MeV. Although both, the mass and the total decay width reported in [20], are bigger than the values found for the meson $D(0^+)$ studied here, we can not discard the possibility that the BELLE’s resonance can be interpreted as a four-quark state.

We have presented a QCD sum rule study of the vertex function associated with the strong decay $D^0(0^+) \rightarrow D^+ \pi^-$, where the scalar $D(0^+)$ meson was considered as diquark-antidiquark state. We get for the partial decay width: $\Gamma(D^0(0^+) \rightarrow D^+ \pi^-) = (120 \pm 20)$ MeV.

Acknowledgements

This work has been supported by CNPq and FAPESP.