Shrinkage Deformation’s Simulation of Injection Molding Product
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Abstract. In the text, study effects of process parameters on average volumetric shrinkage of aspherical plastic optical lens. By using Moldflow and Taguchi experiment design method with seven factors three levels, including mold temperature, melt temperature, packing pressure, filling time, packing time, cooling time and gate size, analyze average volumetric shrinkage of aspherical plastic optical lens. According to analysis of simulation results, optimize process parameters and gain the minimum average volumetric shrinkage, it can be concluded that effects of packing pressure, injection time, melt temperature and gate size are remarkable. Thus select these four process parameters to conduct Taguchi experiment considering each two’s interaction. In the end, obtain a conclusion that these interaction effects are small and can be neglected.

Introduction
Shrinkage behavior of injection product plays an important part in the final determination of product’s size and shape [1]. Factors that have effects on shrinkage of injection product mainly from three aspects: properties of injection molding material, injection molding process conditions and mold structure [2]. Process parameters directly affect melt state in the mold and final quality of plastic products, hence obtain optimized process parameters is precondition of molding high-quality plastic products.

Surface quality of aspherical plastic optical lens is highly demanded, requires smaller shrinkage deformation. Therefore, combining Moldflow with Taguchi experiment technology, effects of seven process parameters including mold temperature, melt temperature, packing pressure, filling time, packing time, cooling time and gate size on the quality of plastic part are studied, process parameters are optimized, and principal effect factors’ interactions are discussed.

Simulation of Volumetric Shrinkage Deformation

Quality Character Selection
There are many effect factors on lens performance, and main factor is shrinkage which can lead to lens deformation, and main purpose of this paper is to reduce lens volumetric shrinkage deformation, which will be used as Taguchi quality character.

Since quality character is the overall direction of volumetric shrinkage deformation, the smaller the deformation the better the quality is, so smaller-the-better is selected as measurement method of this quality, whose S/N ratio can be calculated as follows:

\[ S / N = h = -10 \log \frac{\sum y_i^2}{n}. \]  

where \( y_i \) is the first \( i \) quality character; \( \bar{y} \) is the average quality character and \( n \) is number of experiments; \( S \) is standard deviation, which can be expressed by Eq. 2,

\[ S = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n}}. \]
Experiment Design and Arrangement

Volumetric shrinkage deformation analyzed by using Taguchi experiment Process parameters affect quality of the final product, use Taguchi experiment design method and arrange process parameters at different levels reasonably to conduct experiment, obtain optimum process parameters after analysis of results to achieve the purpose of parameter optimization [3].

Analysis Model

Analysis pre-treatment Surface quality of aspherical plastic optical lens is highly demanded, light transmittance requires well, there must be no residual traces on both top and bottom surface, therefore, material PMMA is chosen, rectangular edge gate and four cavities in one mold are adopted, Fig.1 shows sizes of aspherical plastic lens and Fig.2 shows model gating system and cooling system optimized.

Experiment Arrangement

Level settings of process parameters and simulation analysis use prior simulation analysis results to determine appropriate scope of process parameters, seven factors including mold temperature, melt temperature, packing pressure, filling time, packing time, cooling time and gate size are arranged with three levels, all factors’ level configuration can be shown in Table 1, of which gate size is size of its cross-section that is width × depth. Where A represents mold temperature, B melt temperature, C packing pressure, D filling time, E packing time, F cooling time and G gate size.

![Figure 1. Sizes of aspherical plastic optical lens.](image1)

![Figure 2. Injection molding product picture.](image2)

|        | A[℃] | B[℃] | C[MPa] | D[s] | E[s] | F[s] | G[mm×mm] |
|--------|------|------|--------|------|------|------|----------|
| level 1| 45   | 250  | 15     | 4.2  | 11   | 37   | 2×1      |
| level 2| 55   | 260  | 20     | 5.2  | 12   | 38   | 2×2      |
| level 3| 65   | 270  | 25     | 6.2  | 13   | 39   | 3×2      |

Adopt seven factors with three levels to arrange orthogonal array L_{18}(3^7) to conduct experiment, use Moldflow simulation to replace the real experiments, results of plastic product’s average volumetric shrinkage deformation at different injection conditions are obtained, as shown in Table 2, where ε represents average volumetric shrinkage deformation.

|        | A[℃] | B[℃] | C[MPa] | D[s] | E[s] | F[s] | G[mm×mm] |
|--------|------|------|--------|------|------|------|----------|
| level 1| 45   | 250  | 15     | 4.2  | 11   | 37   | 2×1      |
| level 2| 55   | 260  | 20     | 5.2  | 12   | 38   | 2×2      |
| level 3| 65   | 270  | 25     | 6.2  | 13   | 39   | 3×2      |

Results and Discussion

Signal-noise-ratio Values Calculation

Analysis of simulation results Mean square deviation (MSD) and signal-noise-ratio values (S/N) can be calculated by simulation analysis results in Table 2, and MSD is defined as:

\[
MSD = \frac{1}{n} \sum_{i=1}^{n} y_i^2 = \bar{y}^2 + s^2.
\]

the S/N value can be calculated as follows:
S/N=−10log[MSD].
For example, to L1:

\[ MSD=6.2401^2=35.9388 \]
\[ S/N=10\log[35.9388]=-15.9038 \text{(db)} \]

The results calculated finally can be shown in Table 2.

Table 2. Analysis result and S/N value.

| serial number | A | B | C | D | E | F | G | ε(%) | S/N   |
|---------------|---|---|---|---|---|---|---|------|-------|
| 1             | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 6.2401 | -15.9038 |
| 2             | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 5.7035 | -15.1228 |
| 3             | 2 | 1 | 1 | 2 | 3 | 3 | 3 | 5.3564 | -14.5775 |
| 4             | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 5.7372 | -15.1740 |
| 5             | 2 | 3 | 3 | 1 | 2 | 2 | 2 | 5.7224 | -15.1516 |
| 6             | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 5.6853 | -15.0951 |
| 7             | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 5.5770 | -14.9280 |
| 8             | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 6.0294 | -15.6055 |
| 9             | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 5.2880 | -14.4658 |
| 10            | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 6.2007 | -15.8488 |
| 11            | 1 | 3 | 2 | 1 | 1 | 3 | 1 | 5.8032 | -15.2734 |
| 12            | 1 | 4 | 5 | 6 | 13 | 14 | 15 | 5.4212 | -14.6819 |
| 13            | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 5.5251 | -14.8468 |
| 14            | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 6.3614 | -16.0711 |
| 15            | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 5.2665 | -14.4304 |
| 16            | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 5.8121 | -15.2867 |
| 17            | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 6.1995 | -15.8471 |
| 18            | 3 | 3 | 2 | 1 | 3 | 1 | 2 | 5.5770 | -14.9280 |

Factor’s Effect Analysis

Model reaction table After S/N is obtained, each factor’s effect can be understood by calculation, the algorithm is to get factor’s S/N value of all the same level averaged.

For example, A factor’s average of level 1, 2 and 3 can be calculated as follows:

\[ h_{A1} = \frac{1}{6}(h_1 + h_2 + h_3 + h_{10} + h_{11} + h_{12}) = -15.1987 \text{(db)} \]

\[ h_{A2} = \frac{1}{6}(h_1 + h_2 + h_3 + h_{13} + h_{14} + h_{15}) = -15.2114 \text{(db)} \]

\[ h_{A3} = \frac{1}{6}(h_1 + h_3 + h_5 + h_{16} + h_{17} + h_{18}) = -15.1988 \text{(db)} \]

Table 3. Model S/N ratio table.

| A    | B    | C    | D    | E    | F    | G    | ε(%) | S/N   |
|------|------|------|------|------|------|------|------|-------|
| level 1 | -15.1987 | -14.9867 | -15.6765 | -15.4489 | -15.2042 | -15.2057 | -15.3983 |
| level 2 | -15.2114 | -15.2012 | -15.1991 | -15.1948 | -15.2052 | -15.1989 | -15.1402 |
| level 3 | -15.1988 | -15.4210 | -14.7334 | -14.9652 | -15.1995 | -15.2044 | -15.0704 |
| range  | 0.0127 | 0.4343 | 0.9431 | 0.4837 | 0.0057 | 0.0068 | 0.3279 |
| order  | 5     | 3     | 1     | 2     | 7     | 6     | 4     |

Table 3 is reaction table of control factor on the S/N ratio, the greater the S/N value, the smaller the variant is, hence the greater the S/N value the better it is, where factor effect is evaluated by calculating the difference value of factor’s S/N value between maximum and minimum, that is range in the reaction table, and according to range value importance of each factor can be got.

Through analysis above the following conclusions can be drawn:

(1) By range value, the order of process parameters’ effect on average volumetric shrinkage can be deduced as follows which is from great to small: packing pressure, filling time, melt temperature, gate size, mold temperature, cooling time and packing time, among which effects of packing
pressure, filling time, melt temperature and gate size are remarkable, and other factors’ are small, hence these four factors are selected to be discussed in the following study of investigating different factors’ interactions.

(2) After process parameter level is studied, the optimal combination of process parameters are that: mold temperature is 45 °C, melt temperature is 250 °C, packing pressure is 25 MPa, filling time is 6.2 s, packing time is 13 s, cooling time is 38 s and gate size is 3 mm × 2 mm. Under this process parameter condition, the average volumetric shrinkage simulated by Moldflow is 5.0616%, compared to the experiments conducted anteriorly, the average volumetric shrinkage under this process parameter condition is certainly the smallest.

Variation Analysis

By using variation analysis, degree of different injection process parameters’ effects on a spherical lens average volumetric shrinkage deformation is analyzed more accurately.

1) Calculation of factor variation deviation $SS$

$$SS_{total} = \left(\sum_{i=1}^{n} \sum_{j=1}^{r} y_{ij}^2 \right) - n \times r \times \bar{y}^2.$$  

$$SS_{Factor} = \frac{n \times r}{L} \sum_{k=1}^{L} (\bar{y}_k - \bar{y})^2.$$  

$$SS_{Error} = SS_{Total} - SS_{Factor}.$$  

2) Calculation of degree-of-freedom $DOF$

$$DOF_{Total} = n \times r - 1.$$  

$$DOF_{Factor} = L - 1.$$  

$$DOF_{Error} = DOF_{Total} - DOF_{Factor}.$$  

3) Calculation of average variation deviation $Var$

$$Var_{Factor} = \frac{SS_{Factor}}{DOF_{Factor}}.$$  

$$Var_{Error} = \frac{SS_{Error}}{DOF_{Error}}.$$  

4) Calculation of pure variation deviation $SS'$

$$SS'_{Factor} = SS_{Factor} \times (Var_{Factor}) (Var_{Error}).$$  

where its contribution percentage $\rho$ can be expressed as follows

$$r_{Factor} = \frac{SS'_{Factor}}{SS_{Total}} \times 100\%.$$  

5) Calculation of $F$-Test ratio (F Ratio)

$F$-Test is also known as variance ratio, the greater the value, the larger the contribution of factor to $\eta$ or that the smaller the effect of errors to $\eta$.

$$F_{Factor} = \frac{Var_{Factor}}{Var_{Error}}.$$
where $n \times r$ represents that there are $n$ groups of experiments and $r$ repeat experiments datum in each group of experiment; $L$ is the number of level; $\bar{y}$ is average value; $\bar{y}_k$ stands for reaction value of this factor at the level of $K$. After calculation, $SS_{Total} = 4.2975$, $DOF_{Total} = 17$.

Variation analysis results calculated in accordance with formula mentioned above can be shown in Table 4. It can be concluded from the table, those factors’ contribution percentage to plastic part average volume shrinkage including packing pressure, filling time, melt temperature and gate size is more than 99%, it can be seen that effect of these four factors on plastic part average volumetric shrinkage are greater. This is because packing pressure is useful to compensate the amount of plastic melt for plastic part shrinkage caused by cooling. Fill time is related to fill rate, while plastic part shrinkage differences of molecular orientation character caused by flowing is decided by speed of filling rate. And value of gate size is related to flow instance of plastic part and whether plastic parts can be fully packed.

**Table 4. Data table of variance analysis.**

| ANOVA | A    | B    | C    | D    | E    | F    | G    | Error |
|-------|------|------|------|------|------|------|------|-------|
| SS    | 0.0006401 | 0.5659 | 2.6684 | 0.7025 | 0.0001112 | 0.0001564 | 0.3580 | 0.001792 |
| DOF   | 2    | 2    | 2    | 2    | 2    | 2    | 2    | 3     |
| Var   | 0.0003201 | 0.2830 | 1.3342 | 0.3513 | 0.0000556 | 0.0000782 | 0.1790 | 0.0005973 |
| F     | 0.5359 | 473.7988 | 2233.7184 | 588.1467 | 0.09309 | 0.1309 | 299.6819 | 1     |
| SS'   | 0.0006399 | 0.5657 | 2.6676 | 0.7023 | 0.00011117 | 0.00015635 | 0.3579 | 1     |
| $\rho$(%) | 0.01489 | 13.1635 | 62.0733 | 16.3421 | 0.002587 | 0.003638 | 8.3281 | 0.071885 |

**Simulation Analysis of Interaction**

Select packing pressure, filling time, melt temperature and gate size to arrange Taguchi experiment considering interactions to carry out experiment, each factor level arrangement is shown in Table 5, of which gate size is size of its cross-section that is width $\times$ depth. A represents melt temperature, B packing pressure, C filling time, D gate size.

**Table 5. Arrangement of process parameter level.**

| A[℃] | B[MPa] | C[s] | D[mm×mm] |
|------|--------|------|----------|
| level 1 | 250 | 15 | 4.2 | 2×1 |
| level 2 | 260 | 20 | 5.2 | 2×2 |

**Table 6. Analysis results.**

| serial number | A | B | A×B | C | A×C | B×C | D | A×D | B×D | C×D | $\delta$[%] |
|---------------|---|---|-----|---|-----|-----|---|-----|-----|-----|-----|
| 1             | 1 | 1 | 1   | 1 | 1   | 1   | 1 | 1   | 1   | 6.2327 |
| 2             | 1 | 1 | 1   | 1 | 2   | 2   | 2 | 2   | 2   | 6.1913 |
| 3             | 1 | 1 | 1   | 2 | 2   | 2   | 2 | 1   | 1   | 6.0659 |
| 4             | 1 | 1 | 1   | 2 | 2   | 2   | 2 | 2   | 1   | 5.9076 |
| 5             | 1 | 2 | 2   | 1 | 1   | 2   | 1 | 2   | 1   | 5.7334 |
| 6             | 1 | 2 | 2   | 1 | 2   | 2   | 1 | 2   | 1   | 5.7403 |
| 7             | 1 | 2 | 2   | 2 | 1   | 1   | 1 | 2   | 2   | 5.7500 |
| 8             | 1 | 2 | 2   | 2 | 1   | 2   | 2 | 1   | 1   | 5.7300 |
| 9             | 2 | 1 | 2   | 1 | 2   | 1   | 1 | 1   | 1   | 6.3742 |
| 10            | 2 | 1 | 2   | 1 | 2   | 1   | 2 | 1   | 2   | 6.2991 |
| 11            | 2 | 1 | 2   | 2 | 1   | 2   | 1 | 2   | 1   | 6.4007 |
| 12            | 2 | 1 | 2   | 2 | 1   | 2   | 2 | 1   | 2   | 6.0312 |
| 13            | 2 | 2 | 1   | 1 | 2   | 1   | 2 | 2   | 1   | 6.0405 |
| 14            | 2 | 2 | 1   | 1 | 2   | 1   | 2 | 1   | 2   | 5.8786 |
| 15            | 2 | 2 | 1   | 2 | 1   | 1   | 2 | 2   | 2   | 5.8602 |
| 16            | 2 | 2 | 1   | 2 | 1   | 1   | 2 | 1   | 1   | 5.6999 |

Considering four factors including their interactions there are 10 factors in total, so adopt fifteen factors with two levels to arrange orthogonal array $L_{16}(2^{15})$ to conduct experiment, colligate table.
head design of L$_{16}$ (2$^{15}$) and orthogonal array L$_{16}$ (2$^{15}$) [4], the final form of orthogonal table and simulation analysis results can be shown in Table 6, where $\varepsilon$ represents average volumetric shrinkage deformation.

Direct analysis can be carried out on experiment results, which is getting each process parameter’s average volumetric shrinkage averaged in three levels, its calculation formula can be seen as Eq. 16:

$$m = \frac{1}{n} \sum_{i=1}^{n} x_i.$$  \hspace{1cm} (16)

where $m$ is the average value of process parameter at a certain level, $n$ is the number occurring at this level, $x_i$ is process parameter’s average volumetric shrinkage at this level. Then each process parameter’s range can be calculated by the difference between the largest and the smallest average value, results can be shown in Table 7.

|    | A   | B   | A×B | C   | A×C | B×C | D   | A×D | B×D | C×D |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| level 1 | 5.9178 | 6.1492 | 5.9860 | 6.0697 | 5.9846 | 5.9863 | 6.0553 | 5.9694 | 5.9698 | 5.9697 |
| level 2 | 6.0381 | 5.8067 | 5.9699 | 5.8862 | 5.9713 | 5.9696 | 5.9006 | 5.9865 | 5.9861 | 5.9862 |
| range | 0.1203 | 0.3425 | 0.0161 | 0.1835 | 0.0133 | 0.0167 | 0.1547 | 0.0171 | 0.0163 | 0.0165 |
| order | 4   | 1   | 9   | 2   | 10  | 6   | 3   | 5   | 8   | 7   |

Through analysis above some conclusions can be obtained as follows:

By the range value in Table 7, the order of process parameters’ effect on average volumetric shrinkage can be educed directly as follows which is from great to small: B > C > D > A > A × D > B × C > C × D > B × D > A × B > A × C, effect of four factor’s interaction on plastic part average volume shrinkage is not very obvious, hence in the course of analyzing volumetric shrinkage deformation, there is no need to consider these factors’ interaction.

**Summary**

(1) Moldflow software combined with Taguchi experiment design method, it is useful to reduce the number of experiments, by using Taguchi experiment design method, experiment can be quickly arranged and the optimum process parameters can be analyzed.

(2) In this paper, for the PMMA material, in the range of process parameters studied, effects of packing pressure, filling time, melt temperature and gate size on injection part average volumetric shrinkage are more significant, and their effect degrees reduce in turn, while effects of mold temperature, packing time and cooling time are small and can be neglected.

(3) Effects of the interaction between packing pressure, filling time, melt temperature and gate size on average volumetric shrinkage of injection molding part are much smaller.

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**References**

[1] Wang Li-xia, Yang Yang, Wang Bei and Shen Chang-yu: Polymer Materials Science & Engineering. 2 (2004) 173-176.

[2] Zhu Tie-li and Wang Min-jie: Chinese Journal of Mechanical Engineering. 9 (2002) 145-149.

[3] Montgomery Douglas C: *Design and Analysis of Experiments*, John Wiley & Sons, New York, 1997.

[4] Tian Sheng-Yuan and Xiao Yue-rong: *Experiment Design and Data Processing*, China Building Industry Press, Beijing, 2002.