Determination of the structural efficiency of short bridges according to their physical and economic characteristics

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Abstract. Bridges are structures designed and built to overcome natural and artificial obstacles. The analysis and design of bridges is carried out following principles of physics applied to engineering. The aim is to estimate the structural efficiency of short bridges according to their physical and economic characteristics through equations that measure the cost tendency, identifying which ones have the highest structural efficiency for a given length between supports. The design of short bridges was carried out by means of a mathematical modeling in a software based on finite elements. A comparative cost analysis was carried out for various types of short vehicular bridges designed with reinforced concrete decks supported by reinforced concrete beams; post-tensioned concrete beams; and structural steel beams. It was determined that the bridge with a reinforced concrete deck is the alternative with the best structural efficiency according to its physical and economic characteristics with spans of 12 meters and 18 meters. According to the estimated cost equations, it is concluded that, from 18.49 meters post-tensioned concrete bridges are the most efficient physically and economically. Structural steel bridges are the least structurally efficient for short bridges. Therefore, the construction of short vehicular bridges with structural steel is not feasible.

1. Introduction
Bridges are constructions of road infrastructure used to cross natural obstacles (e.g., passage of a water tributary, valleys, canyons, a geographical feature) and artificial (e.g., passage of railways and roads). Bridges are designed according to their function, length, and nature of the terrain. There are numerous types of structural systems that have been applied throughout history and in which different materials have been used [1]. Structural design of a bridge must consider the most economical structure without prejudice to the functional purposes for which it is being designed [2]. Reinforced and post-tensioned concrete, and structural steel are the most common materials with which bridges are built. Being the construction of reinforced concrete bridges and mixed steel beams, they are widely used as an economic structural system, although this depends on the length of the bridge itself and the amount of supports it has [3,4].

There are various standards and standard procedures that are intended to ensure the safety of bridges during their useful life [5]. Among the most common standards are: American Association of State Highway and Transportation Officials (AASHTO) with the methodology load and resistance factor design (LRFD) in bridge design specifications [6]; American Concrete Institute (ACI) in building code requirements for structural concrete [7]; “Asociación Colombiana de Ingeniería Sísmica (AIS)” bridge design standard [8]; and Colombian regulation of earthquake resistant construction (NSR-10) [9].
Various studies have been carried out in relation to the study of costs, design, and construction of bridges. Haas investigated the cost effectiveness of steel girders compared to conventional reinforced concrete girders used in bridge construction in South Africa [10]. Delgado-Gil and Zuñiga-Jurado made a cost comparison between bridges with steel girders, reinforced concrete and post-tensioned considering the variation of the free span [11]. Almeida-Hernández and Armas-Herrera studied the economic behavior of two types of bridges by comparing the superstructure of bridges of 30 meters span without intermediate supports, considering the variants of slab on post-tensioned concrete beams and slab-beams-frame, with designs based on the AASHTO LRFD 2017, ACI-14, NEC-15, and NEVI-12 standards [12]. As well as Alvarado-Meza and Suescun-Garcia studied the economic behavior for three types of bridges, reinforced concrete, post-tensioned concrete, and metal beams, all designed with reinforced concrete boards [13].

The aim of this work is to study the behavior of three types of simply supported bridges. Designed with reinforced concrete boards, supported on reinforced concrete beams, post-tensioned concrete, and structural steel. The purpose of the research is to identify the type of bridge that has the best economic viability for short spans less than 26 meters in length between supports, so that structural engineers have a starting point for the selection of the best design alternative, which is functionally and economically feasible.

2. Methodology
This work contemplated a quantitative approach of descriptive scope. The structural design of short vehicular bridges was carried out, on which the materials of each of the elements were dimensioned, schematized, and quantified. The bridges were designed with lengths of 12 m, 18 m, and 26 m, between longitudinal supports. The bridge course decks were designed of reinforced concrete in all cases for two lanes with a width of 8 m. The bridge decks are supported on four beams, which were varied the type of construction material in order to identify the one with the lowest cost. The construction materials for the beams were as follows: reinforced concrete, post-tensioned concrete, and structural steel. The bridges were designed with the help of software that uses the stiffness matrix method to calculate the schematic mathematical models.

2.1. Cost of construction materials for the year 2020 in Colombian pesos
The cost of construction materials was calculated according to commercial prices for the year 2020 in Colombia. Within the unit price budgets, in addition to the cost of building material, the price of labor and the equipment and tools used to manufacture it was considered. Table 1 shows the price in thousands of Colombian pesos (COP) involved in manufacturing each of the construction materials required for the design of each of the bridges.

| Building materials     | Unit of measurement | Cost in thousands of COP |
|------------------------|---------------------|--------------------------|
| Concrete 28MPa         | M³                  | 528.0                    |
| Concrete 35MPa         | M³                  | 566.0                    |
| Reinforcing steel 420MPa| KG                  | 5.2                      |
| Prestressing steel     | KG                  | 33.3                     |
| Steel A572-01 G-50     | KG                  | 10.2                     |
| Steel profiles C 12x20.7| KG                  | 8.8                      |
| Steel profiles         | KG                  | 8.8                      |
| Safety railing         | ML                  | 374.0                    |

* M³ means “cubic meters”; ** KG means “kilograms”; *** ML means “linear meters”

2.2. Work quantities of the designed bridges
The calculation of the amounts of work of each of the schematized vehicular bridges was carried out. Figure 1 shows the diagram of a vehicular bridge supported on rectangular reinforced concrete beams.
The size of the beams varies according to the need of each of the lengths of the free span between supports. The 12 m bridges have beams of $B_1 = 0.40$ m and $H_1 = 0.72$ m. The 18 m bridges have beams of $B_1 = 0.40$ m and $H_1 = 1.12$ m. The 26 m bridges have beams of $B_1 = 0.40$ m and $H_1 = 1.62$ m.

Table 2 shows the quantities of work for each of the construction materials (concrete, reinforcing steel and safety railing) taken into account in the design of each of the bridges.

Table 3 shows the quantities of work for each of the construction materials (concrete, reinforcing steel, prestressing steel and safety railing) taken into account in the design of each of the bridges.

Figure 2 shows the diagram of a vehicular bridge supported on post-tensioned concrete beams. The size of the beams varies according to the need of each of the lengths of the free span between supports. The 12 m bridges have beams of $B_2 = 0.50$ m and $H_2 = 1.00$ m. The 18 m bridges have beams of $B_2 = 0.60$ m and $H_2 = 1.30$ m. The 26 m bridges have beams of $B_2 = 0.70$ m and $H_2 = 1.65$ m.

Table 3 shows the quantities of work for each of the construction materials (concrete, reinforcing steel, prestressing steel and safety railing) taken into account in the design of each of the bridges.
The 12 m bridges have beams of $B = 0.40 \text{ m}$ and $H = 0.60 \text{ m}$. The 18 m bridges have beams of $B = 0.40 \text{ m}$ and $H = 1.00 \text{ m}$. The 26 m bridges have beams of $B = 0.40 \text{ m}$ and $H = 1.30 \text{ m}$.

Figure 3. Schematic of the vehicular bridge deck supported on structural steel beams.

Table 4 shows the quantities of work for each of the construction materials (concrete, A572-01 G-50 steel, C 12x20.7 profiles, steel profiles, reinforcing steel and safety railing) taken into account in the design of each of the bridges.

**Table 4. Bridges with structural steel beams.**

| Building material                                      | Bridges 12 m | Bridges 18 m | Bridges 26 m |
|--------------------------------------------------------|--------------|--------------|--------------|
| Concrete 28MPa for beams M³                            | 19.38        | 29.07        | 41.99        |
| Steel A572-01 G-50 Kg                                  | 8890.02      | 23793.22     | 53841.77     |
| Profiles C 12x20.7 for braces KG                       | 1641.91      | 1641.91      | 1641.91      |
| Steel profiles for shear connectors KG                 | 415.42       | 563.13       | 775.45       |
| Reinforcing steel 420MPa KG                            | 2608.71      | 3153.36      | 3477.32      |
| Safety railing ML                                      | 24.00        | 36.00        | 52.00        |

2.3. Estimation of the trend line equation

Starting from the least squares method, the equation that describes the trend in the cost of short vehicular bridges designed with different lengths between supports and construction material of the support beams was estimated. The least squares method calculates from the N pairs of experimental data $(x, y)$, the m and b values that best fit the data to a line. The expression is based on the equation of the line $y = mx + b$ (where m is the slope of the line and b is the cut-off point. Equation (1) and Equation (2) express the slope of the line and the short point of the trend line, respectively. Which, when replaced in the basic equation of the line, Equation (3) is obtained.

$$m = \frac{n \sum x y - \sum x \sum y}{n \sum x^2 - (\sum x)^2},$$  \hspace{1cm} (1)$$

$$b = \frac{\sum y \sum x^2 - \sum x \sum (x \cdot y)}{n \sum x^2 - (\sum x)^2},$$  \hspace{1cm} (2)$$

$$y = \left(\frac{n \sum x y - \sum x \sum y}{n \sum x^2 - (\sum x)^2}\right) x + \left(\frac{\sum y \sum x^2 - \sum x \sum (x \cdot y)}{n \sum x^2 - (\sum x)^2}\right).$$  \hspace{1cm} (3)$$

3. Results and discussion

The study of structures implements principles of physics when analyzing the transfer of service loads from the point where they are applied to the point where they are transferred to the ground. In the development of this research, two comparison parameters were studied in order to define the structural efficiency of short bridges with reinforced concrete decks. The first comparison parameter was determined according to the physical and structural functionality of the bridges, by means of the relative stiffness analysis. Since the materials of the proposed bridges differ in density, weight,
elasticity, stiffness, and shape, it was necessary to analyze that the relative stiffness of each of the materials allowed each of the different bridges to present the same structural conditions of deformation. The second point of comparison is related to the cost-benefit ratio of the analyzed and designed bridges.

The analysis and design of the bridges was carried out in software that uses the principles of finite elements for calculations. The software analyzes and compares the moment distribution factor produced by the load generated by the passage of the design vehicle. Also, the bending and shear stresses produced by the action of gravitational loads were analyzed, applying increased factors as safety factors. The design of the reinforced concrete decks considered the physical and structural steel requirements necessary to obtain the appropriate ductility of the reinforced concrete. The design of the reinforced concrete, post-tensioned concrete and structural steel beams was carried out by means of the ultimate strength methodology, where the maximum bending and shear stresses were taken into account to determine the sections and steel requirements necessary for guarantee adequate structural functionality in each of the short study bridges.

It was proceeded to carry out the budget calculation of the different vehicular bridges designed with the information presented in Table 2, Table 3, and Table 4, on the amounts of work necessary for construction. Table 5 shows the cost involved in the construction of each of the bridges in thousands of COP. It is evident that short vehicular bridges designed with structural steel beams present the highest costs, being up to 3 times more expensive than short vehicular bridges designed with reinforced and post-tensioned concrete beams. On the other hand, it is evident that the cost of bridges designed with reinforced and post-tensioned concrete beams have similar costs for each of the analyzed lengths.

Table 5. Comparison of costs in thousands of COP of the different types of bridges analyzed.

| Building material | Bridges with reinforced concrete beams | Bridges with post-tensioned concrete beams | Bridges with structural steel beams |
|------------------|---------------------------------------|------------------------------------------|-----------------------------------|
|                  | Bridges 12 m | Bridges 18 m | Bridges 26 m | Bridges 12 m | Bridges 18 m | Bridges 26 m | Bridges 12 m | Bridges 18 m | Bridges 26 m |
| Concrete 28 MPa  | 17313        | 34663        | 63772        | 11458        | 17250        | 24811        | 10233        | 15349        | 22171        |
| Concrete 35 MPa  | –            | –            | –            | 9475         | 18701        | 41244        | –            | –            | –            |
| Reinforcing steel 420 MPa | 23661    | 54816        | 113066       | 29392        | 44434        | 63607        | –            | –            | –            |
| Prestressing steel | –            | –            | –            | 4845         | 12032        | 28810        | 13565        | 16397        | 18082        |
| Steel A572-01 G-50 | –            | –            | –            | –            | –            | –            | 90678        | 242691       | 549186       |
| Steel profiles C 12x20.7 | –            | –            | –            | –            | –            | –            | 14449        | 14449        | 14449        |
| Steel profiles   | –            | –            | –            | –            | –            | –            | 3656         | 4956         | 6824         |
| Safety railing   | 8976         | 13464        | 19448        | 8976         | 13464        | 19448        | 8976         | 13464        | 19448        |
| Total cost       | 49950        | 102943       | 196286       | 64146        | 105881       | 177920       | 141557       | 307306       | 630160       |

Figure 4 shows the behavior and trend of each of the cost vs. length graphs of the evaluated bridges. There is an intersection between the lines that describe the trend of short vehicular bridges designed with reinforced and post-tensioned concrete beams. This indicates that although post-tensioned concrete is initially more expensive, it becomes more economical as the length between supports of the short vehicular bridge becomes larger.

Using Equation (1), the equation that describes the trend of each of the graphs shown in Figure 4 was estimated. Table 6 shows the equation and the coefficient of determination (R²) for each of the lines. tendency of short vehicular bridges designed and analyzed.
The tendency of the increase in the cost of bridges when the length of the free span is increased is governed by the equations shown in Table 6. The equation of the variation of costs of the bridge with reinforced concrete beam presents a slope of 10.518, that of the bridge with post-tensioned concrete beam has a slope of 8.174, which indicates that although reinforced concrete is initially the lowest cost, the tendency to increase in cost is higher than that of the bridge designed with post-tensioned concrete. The breaking point at which the two materials have the same cost occurs when the free span between supports is 18.49 m. Therefore, it becomes unfeasible to build reinforced concrete bridges from 18.49 m of free length between supports. On the other hand, the slope of 35.195 in the cost variance equation of the metal girder bridge denotes that it is not feasible to be used in short distance bridges.

Figure 4. Cost comparison of short vehicular bridges less than 26 meters when they are design with different building materials.

| Bridge beam material          | Curve trend equation     | \( R^2 \) |
|------------------------------|--------------------------|-----------|
| Reinforced concrete          | \( Y = 10.518X - 79.947 \) | 0.9943    |
| Post-tensioned concrete      | \( Y = 8.174X - 36.603 \) | 0.9943    |
| Structural steel             | \( Y = 35.195X - 297.3 \) | 0.9898    |

\( Y \) is the cost of the bridge in millions of COP

\( X \) is the length of the bridge in meters in the interval (12 \( \leq X \) \( \leq 26 \))

4. Conclusions
The analysis and design of short bridges requires the principle of physics in the structural functionality of the different buildings. The analysis considered parameters such as density, weight, stiffness, and elasticity of the materials of the study beams. It was verified that all the mathematical models carried out in the finite element software presented the same relative stiffness, guaranteeing similar deformations in order to identify the short bridge as greater structural efficiency. The design of each of the bridges (decks and beams) guaranteed the size and reinforcement requirements to meet the minimum resistance needs of bending and shear stresses.

Short vehicular bridges designed with structural steel girders are unfeasible for construction. This is due to the high cost of construction materials, which includes skilled labor. When compared with reinforced and post-tensioned concrete bridges, it was obtained that structural steel bridges for spans between 12 m and 26 m can triple their cost. While the structural steel bridge can cost 630 millions of COP by 2020, a bridge of the same length in reinforced and post-tensioned concrete has a cost between 170 millions and 196 millions of COP for the same year. When comparing bridges designed with reinforced concrete beams with bridges designed with post-tensioned concrete beams, it was
found that although their cost is similar, from 8.49 meters, post-tensioned concrete is a more economical option.

The estimated equations show that the trend between the reinforced concrete bridge and the post-tensioned concrete bridge is similar. However, it is evident that the slope of the trend line is lower for the post-tensioned bridge. This indicates that the greater the length between the posts of the post-tensioned bridge, the lower its cost, compared to the same bridge designed with reinforced concrete.

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