Constraints on general second-order scalar-tensor models from gravitational Cherenkov radiation

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Abstract. We demonstrate that the general second-order scalar-tensor theories, which have attracted attention as possible modified gravity models to explain the late time cosmic acceleration, could be strongly constrained from the argument of the gravitational Cherenkov radiation. To this end, we consider the purely kinetic coupled gravity and the extended galileon model on a cosmological background. In these models, the propagation speed of tensor mode could be less than the speed of light, which puts very strong constraints from the gravitational Cherenkov radiation.

Keywords: modified gravity, gravitational waves / theory, cosmological perturbation theory

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1 Introduction

Cosmological observations of type Ia supernovae [1–3], the cosmic microwave background [4, 5], and the large scale structures [6–9] indicate that the universe undergoes a phase of accelerated expansion, and this discovery opened up a new field in cosmology. A number of attempts to explain the origin of the present cosmic acceleration have been proposed over the past decade. The Einstein’s cosmological constant might be a possible solution, however, the smallness of value of the cosmological constant cannot be explained naturally [10, 11].

The possible solution for an accelerated expansion of the universe at the present time is an alternative theory of gravity. So far various modified gravity models have been proposed such as the scalar-tensor theories [12–16], \( f(R) \) gravity [17–20], Dvali-Gabadazde-Porrati (DGP) braneworld model [21, 22], and Galileon gravity [23–32]. In these models, additional degrees of freedom can mimic the cosmological constant and lead to cosmic acceleration today. Most of these theories are a subclass of the most general second-order scalar-tensor theory, which was first constructed by Horndeski [33] and also independently derived by Deffayet et al. [34] as an extension of galileon theory. The most general second-order scalar-tensor theory is applied to the late-time accelerated expansion [35] as well as the inflationary models [36–39].

The Lagrangian in the most general second-order scalar-tensor theory contains the coupling of the scalar field \( \phi \) and its kinetic term \( X \equiv -g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi/2 \) with gravity, such as \( G_4(\phi, X)R \) and \( G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \), where \( G_4(\phi, X) \) and \( G_5(\phi, X) \) are arbitrary functions of \( \phi \) and \( X \). This theory is covariant, but in the presence of a cosmological background Lorentz invariance could be broken due to these coupling terms. As a result, the propagation speed

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The propagation speed of gravitational waves differs from the speed of light and also depends on the cosmological background.

When the propagation speed of gravitational waves is less than the speed of light, the gravitons could be emitted through a similar process to the Cherenkov radiation \[40–42\]. The observation of the high energy cosmic rays puts constraints on this process, i.e., the speed of the gravitational waves. Assuming a galactic origin for the high energy cosmic rays, the lower bound on the propagation speed of gravity from gravitational Cherenkov radiation is given by \[41, 42\]

\[
c - c_T < 2 \times 10^{-15} \text{c},
\]

where \(c_T\) is the propagation speed of gravity. When the origin of the high energy cosmic rays is located at a cosmological distance, the constraint is four orders of magnitude tighter than (1.1).

In the present paper, we show that the argument of the gravitational Cherenkov radiation puts a tight constraint on general second-order scalar-tensor models on a cosmological background with a time-varying propagation speed of gravitational waves. As a demonstration, we consider two models: the purely kinetic coupled gravity \[43\] and the extended galileon model \[44\], which are a subclass of the most general second-order scalar-tensor theory.

This paper is organized as follows. In section 2, we briefly review the most general second-order scalar-tensor theory and the tensor perturbations. In section 3, we derive the gravitational Cherenkov radiation on a cosmological background. In section 4, we explicitly show that gravitational Cherenkov radiation reject the purely kinetic coupled gravity model. In section 5, we briefly review the extended galileon model and see how gravitational Cherenkov radiation can tightly constrain model parameters. Section 6 is devoted to conclusions. In appendix A–C, we summarize the scalar perturbations, derived in \[36\], and the coefficients of the scalar and tensor perturbations in various regimes in the extended galileon model, derived in \[44\]. In appendix D, a useful constraint on parameters in the extended galileon model is demonstrated.

Throughout the paper, we use units in which the speed of light and the Planck constant are unity, \(c = \hbar = 1\), and \(M_{\text{Pl}}\) is the reduced Planck mass related with Newton’s constant by \(M_{\text{Pl}} = 1/\sqrt{8\pi G}\). We follow the metric signature convention (−,+,+,+).

\section{The most general second-order scalar-tensor theory}

The most general second-order scalar-tensor theory is described by the action,

\[
S = \int d^4x \sqrt{-g} \left( \sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_m \right),
\]

where

\[\begin{align*}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X) \Box \phi, \\
\mathcal{L}_4 &= G_4(\phi, X) R + G_{4,X}[(\Box \phi)^2 - (\nabla_{\mu} \nabla_{\nu} \phi)(\nabla^{\mu} \nabla^{\nu} \phi)], \\
\mathcal{L}_5 &= G_5(\phi, X) G_{\mu\nu}(\nabla^{\mu} \nabla^{\nu} \phi) - \frac{1}{6} G_5,X [(\Box \phi)^3 - 3(\Box \phi)(\nabla_{\mu} \nabla_{\nu} \phi)(\nabla^{\mu} \nabla^{\nu} \phi) \\
&+ 2(\nabla^{\mu} \nabla_{\alpha} \phi)(\nabla^{\alpha} \nabla_{\beta} \phi)(\nabla^{\beta} \nabla_{\mu} \phi)],
\end{align*}\]

(2.2)
where $K$, $G_3$, $G_4$, and $G_5$ are arbitrary functions of the scalar field $\phi$ and the kinetic term $X \equiv -g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi/2$, $G_\phi$ and $G_X$ stands for $\partial G_i/\partial \phi$ and $\partial G_i/\partial X$, respectively, and $\mathcal{L}_m$ is the matter Lagrangian. We assume that matter is minimally coupled to gravity. Note that for the case, $G_4 = M^2_{\text{Pl}}/2$, the Lagrangian $\mathcal{L}_4$ reproduces the Einstein-Hilbert term.

We consider the tensor perturbations in the most general second-order scalar-tensor theory on a cosmological background, and briefly review the results in derived in [36]. We briefly review the tensor perturbations in the most general second-order scalar-tensor theory, derived in [36]. The quadratic action for the tensor perturbations can be written as

$$S^{(2)}_T = \frac{1}{8} \int dt d^3x a^3 \left[ G_T \dot{h}_{ij}^2 - \frac{F_T}{a^2} (\nabla h_{ij})^2 \right],$$

where

$$F_T \equiv 2 \left[ G_4 - X \left( \dot{\phi} G_{5X} + G_{5\phi} \right) \right],$$

$$G_T \equiv 2 \left[ G_4 - 2XG_{4X} - X \left( \dot{H} G_{5X} - G_{5\phi} \right) \right].$$

Here an overdot denotes differentiation with respect to $t$, and $H = \dot{a}/a$ is the Hubble parameter. We find the propagation speed of the tensor perturbations,

$$c^2_T \equiv \frac{F_T}{G_T}. \quad (2.6)$$

When $G_4 = G_4(\phi)$ and $G_5 = 0$, the propagation speed of gravitational waves is equal to the speed of light. On the other hand, the propagation speed of gravitational waves depends on the cosmological background in the presence of $G_5$ or $G_4$ being dependent on $X$. If the propagation speed of gravitational waves is less than the speed of light, it is tightly constrained from gravitational Cherenkov radiation.

### 3 Gravitational Cherenkov radiation in an expanding universe

In this section, we derive the gravitational Cherenkov radiation in a cosmological background. For simplicity, we consider a complex scalar field with the action

$$S_m = \int d^4x \sqrt{-g} \left[ -g^{\mu\nu}\partial_\mu\Psi^* \partial_\nu \Psi - m^2 \Psi^* \Psi - \xi R \Psi^* \Psi \right]. \quad (3.1)$$

Here we assume the conformal coupling with spacetime curvature $\xi = 1/6$, for simplicity, but this term can be neglected as long as we focus on the subhorizon scales, $p/a, m \gg H$, where $p$ is the comoving momentum. The free part of $\Psi$ can be quantized as

$$\hat{\Psi}(\eta, \mathbf{x}) = \frac{1}{a} \int \frac{d^3p}{(2\pi)^{3/2}} \left[ b_p \psi_p(\eta)e^{ip\cdot x} + c_\mathbf{p}^\dagger \psi_p^\ast(\eta)e^{-ip\cdot x} \right],$$

where $\eta$ is the conformal time, $b_p$ and $c_\mathbf{p}^\dagger$ are the annihilation and creation operators of the particle and anti-particle, respectively, which satisfy the commutation relations $[b_p, b_\mathbf{p}^\dagger] = \delta(\mathbf{p} - \mathbf{p}'), [c_\mathbf{p}, c_\mathbf{p'}^\dagger] = \delta(\mathbf{p} - \mathbf{p}')$, and the mode function obeys

$$\left( \frac{d^2}{d\eta^2} + p^2 + m^2 a^2 \right) \psi_p(\eta) = 0. \quad (3.3)$$
The WKB approximate solution is given by (e.g., [45])

$$\psi_p(\eta) = \frac{1}{\sqrt{2\Omega_p}} \exp \left[ -i \int_{\eta_n}^{\eta} \Omega_p(\eta') d\eta' \right]$$  \hspace{1cm} (3.4)

with $\Omega_p(\eta) = \sqrt{p^2 + m^2 a^2}$. The WKB approximation is valid for

$$\Omega_p^2 \gg \left| \int_{\eta_n}^{\eta} \Omega_p(\eta') d\eta' \right|^2,$$  \hspace{1cm} (3.5)

which can be satisfied as long as $p/a, m \gg H$.

On the other hand, the action of the graviton is given by eq. (2.3), then, we have the quantized graviton field

$$\hat{h}_{\mu\nu} = \frac{1}{a \sqrt{2G_T}} \sum_{\lambda} \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \varepsilon_{\mu\nu}^{(\lambda)} \hat{a}_k(\eta) e^{ik \cdot x} + \varepsilon_{\mu\nu}^{(\lambda)} \hat{a}_k^\dagger(\eta) e^{-ik \cdot x} \right],$$  \hspace{1cm} (3.6)

where $\varepsilon_{\mu\nu}^{(\lambda)}$ is the polarization tensor, $\hat{a}_k$ and $\hat{a}^\dagger_k$ are the creation and annihilation operators, which satisfy the commutation relation $[\hat{a}_k, \hat{a}^\dagger_{k'}] = \delta(k - k')$, and the mode function satisfies

$$\left( \frac{d^2}{d\eta^2} + c_s^2 k^2 - \frac{a''}{a} \right) h_k(\eta) = 0.$$  \hspace{1cm} (3.7)

For the case $c_s \sim O(1)$ and $c_s k/a \gg H$, we may write

$$h_k(\eta) = \frac{1}{\sqrt{2\omega_k}} \exp \left[ -i \int_{\eta_n}^{\eta} \omega_k(\eta') d\eta' \right],$$  \hspace{1cm} (3.8)

where we defined $\omega_k = c_s k$, and the approximate solution is valid as long as $c_s k/a \gg H$. The interaction part of the action (3.1) is given by

$$S_I = -\int dt d^3x h_{ij} \partial_i \Psi \partial_j \Psi^*$$
$$= -\int d\eta d^3x h_{ij} \partial_i \psi \partial_j \psi^*,$$  \hspace{1cm} (3.9)

where we defined $\psi = a \Psi$, and the interaction Hamiltonian is

$$H_I = a \int d^3x h_{ij} \partial_i \Psi \partial_j \Psi^*.$$  \hspace{1cm} (3.10)

In order to evaluate the gravitational Cherenkov radiation, we adopt the method developed in [46, 47]. Based on the in-in formalism [48], the lowest order contribution is given by

$$\langle Q(t) \rangle = i^2 \int_{t_n}^{t} dt_2 \int_{t_n}^{t_2} dt_1 \langle [H_I(t_1), [H_I(t_2), Q]] \rangle.$$  \hspace{1cm} (3.11)

We consider the expectation value of the number operator and the initial state with the one particle state with the initial momentum, i.e., $\hat{b}^\dagger_{p_0} |0\rangle$. Then the lowest-order contribution of
Figure 1. Feynman diagram for the process

The process so that one graviton with the momentum \( k \) is emitted from the massive particle with the initial momentum \( \mathbf{p}_{in} \), as shown in fig. 1, is written as

\[
\langle \hat{a}_{k}^\dagger(\lambda) \hat{a}_{k}^{(\lambda)} \rangle = 2 \Re \int_{t_{in}}^{t} dt_{2} \int_{t_{in}}^{t_{2}} dt_{1} \left\langle H_{f}(t_{1}) \hat{a}_{k}^\dagger(\lambda) \hat{a}_{k}^{(\lambda)} H_{f}(t_{2}) \right\rangle. \tag{3.12}
\]

Then, the total radiation energy from the scalar particle can be estimated as

\[
E = \sum_{\lambda} \sum_{k} (\omega_{k}/a) \langle \hat{a}_{k}^\dagger(\lambda) \hat{a}_{k}^{(\lambda)} \rangle,
\]

which leads to

\[
E = \int \frac{d^{3}k}{(2\pi)^{3}} \frac{\omega_{k}}{a} \int_{\eta_{in}}^{\eta} d\eta_{1} \frac{1}{a(\eta_{1})} \sqrt{\frac{2}{G_{T}}} h_{k}(\eta_{1}) \psi_{p_{in}}(\eta_{1}) \psi_{p_{in}}^{*}(\eta_{1}) \epsilon_{ij} p_{in}^{i} p_{in}^{j} \right|^{2}, \tag{3.13}
\]

where \( \mathbf{p}_{f} + \mathbf{k} = \mathbf{p}_{in} \) (\( p_{f}^{i} + k^{i} = p_{in}^{i} \)). With the use of the relation \( \sum_{\lambda} \epsilon_{ij} p_{in}^{i} p_{in}^{j} = p_{in}^{4} \sin^{4} \theta \), we have

\[
E = \int \frac{d^{3}k}{(2\pi)^{3}} \frac{\omega_{k}}{a} p_{in}^{4} \sin^{4} \theta \int_{\eta_{in}}^{\eta} d\eta_{1} \frac{1}{a(\eta_{1})} \sqrt{\frac{2}{G_{T}}} h_{k}(\eta_{1}) \psi_{p_{f}}(\eta_{1}) \psi_{p_{in}}^{*}(\eta_{1}) \right|^{2}.
\tag{3.14}
\]

We are now interested in the subhorizon scales, \( k/a, p/a, m, c_{s}k/a \gg H \), and the situation so that the scale factor \( a \) is constant, then we can approximate as

\[
\int_{\eta_{in}}^{\eta} d\eta_{1} \frac{1}{a(\eta_{1})} \sqrt{\frac{2}{G_{T}}} h_{k}(\eta_{1}) \psi_{p_{f}}(\eta_{1}) \psi_{p_{in}}^{*}(\eta_{1})
\]

\[
\simeq \frac{1}{a} \sqrt{\frac{2}{G_{T}}} \frac{1}{\sqrt{2\omega_{k}}} \frac{1}{\sqrt{2\Omega_{p_{in}}}} \frac{1}{\sqrt{2\Omega_{p_{f}}}} \int_{\eta_{in}}^{\eta} d\eta_{1} \exp \left[ i(\Omega_{in} - \Omega_{f} - \omega_{k})(\eta_{1} - \eta_{in}) \right]. \tag{3.15}
\]

Then the total radiation energy eq. (3.14) reduces to

\[
E \simeq \frac{1}{4G_{T}a^{3}} \int \frac{d^{3}k}{(2\pi)^{3}} \frac{p_{in}^{4} \sin^{4} \theta}{\Omega_{f}\Omega_{in}} 2\pi T \frac{\delta(\Omega_{in} - \Omega_{f} - \omega_{k})}{a}.
\tag{3.16}
\]

Here we assumed the long time duration of the integration,

\[
\int_{\eta_{in}}^{\eta} d\eta_{1} \exp \left[ i(\Omega_{in} - \Omega_{f} - \omega_{k})(\eta_{1} - \eta_{in}) \right] \right|^{2} \simeq \frac{2\pi T}{a} \delta(\Omega_{in} - \Omega_{f} - \omega_{k}). \tag{3.17}
\]
where \( T/a = \eta - \eta_{in} \). Then, we have the expression in the relativistic limit of the massive particle, \( p_{in}/a \gg m \),

\[
\frac{dE}{dt} = \frac{p_{in}^2}{4G_T a^4} \int_0^\infty \frac{dk}{2\pi} \int_{-1}^1 d(\cos \theta) \sin^4 \theta \delta(\Omega_{in} - \Omega_f - \omega_k). \tag{3.18}
\]

Now consider the delta-function, which can be written as

\[
\delta(\Omega_{in} - \Omega_f - \omega_k) = 2\Omega_f \delta(\Omega_f^2 - (\Omega_{in} - \omega_k)^2) \tag{3.19}
\]

where \( \omega_k = c_s k, \Omega_{in} = \sqrt{p_{in}^2 + a^2 m^2} \), and \( \Omega_f = \sqrt{(p_{in} - k)^2 + a^2 m^2} \). With the use of the fact

\[
\Omega_f^2 - (\Omega_{in} - \omega_k)^2 = -2p_{in}k \left( \cos \theta - \frac{c_s}{\beta} - \frac{(1 - c_s^2)k}{2p_{in}} \right), \tag{3.20}
\]

where we defined \( \beta = p_{in}/\sqrt{p_{in}^2 + m^2 a^2} \) and \( p_{in}^2 = |p_{in}|^2 \), we find (cf. eq.(3.2) in reference by Moore and Nelson [41])

\[
\frac{dE}{dt} = \frac{p_{in}^2}{4G_T a^4} \int_0^{k_{max}} \frac{dk}{2\pi} \sin^4 \theta \tag{3.21}
\]

with

\[
\cos \theta = \frac{c_s}{\beta} + \frac{(1 - c_s^2)k}{2p_{in}} \tag{3.22}
\]

and

\[
k_{max} = \frac{2p_{in}}{1 - c_s} \left( 1 - \frac{c_s}{\beta} \right). \tag{3.23}
\]

Assuming \( \beta \sim 1 \), we have \( k_{max} \simeq 2p_{in}/(1 + c_s) \) and

\[
\frac{dE}{dt} \simeq \frac{p_{in}^2}{8\pi G_T a^4} 4(1 - c_s)^2 \int_0^{k_{max}} dk \left( 1 - \frac{k}{k_{max}} \right)^2, \tag{3.24}
\]

which yields (cf.[41])

\[
\frac{dE}{dt} \simeq \frac{G_N p_{in}^4}{a^4} \frac{4(1 - c_s)^2}{3(1 + c_s)^2}, \tag{3.25}
\]

where we introduce the Newtonian gravity constant by \( G_N = 1/16\pi G_T \). One may notice that this definition of the Newton’s constant is slightly different from that in the most general second-order scalar-tensor theory (cf. [50]), however, it does not affect the constraints significantly. Our results are consistent with those in Ref. [41]. Then, a particle with momentum \( p \) cannot possibly have been traveling for longer than

\[
t \sim \frac{a^4}{G_N} \frac{(1 + c_s)^2}{4(1 - c_s)^2} \frac{1}{p^3}. \tag{3.26}
\]
Therefore, the highest energy cosmic ray put the constraint on the sound speed of the graviton
\[(1 - c_s) \lesssim 2 \times 10^{-17} \left( \frac{10^{11} \text{GeV}}{p} \right)^{3/2} \left( \frac{1 \text{Mpc}}{ct} \right)^{1/2} . \] (3.27)

Since we are considering the theory on a cosmological background, the sound speed of the graviton is determined by the cosmological evolution of the background field. This situation is slightly different from that in ref. [41]. However, as we have shown in this section, the theory on a cosmological background can be constrained from the gravitational Cherenkov radiation when the speed of the graviton is smaller than that of light. Also, there are no higher order nonlinear interaction terms of the graviton like the galileon cubic term that becomes important at short distance [51], which suggests that the nonlinear interactions of the gravitons can be ignored.

4 Purely kinetic coupled gravity

We first consider the modified gravity model, whose action contains a nonminimal derivative coupling to gravity. The action proposed by Gubitosi and Linder [43] is given by
\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R + X + \frac{\lambda}{M_{Pl}^2} G^\mu\nu \nabla_\mu \phi \nabla_\nu \phi \right] , \] (4.1)
where \( \lambda \) is a dimensionless constant. In this model, the arbitrary functions in eq. (2.2) correspond to \( K = X \), \( G_3 = 0 \), \( G_4 = M_{Pl}^2/2 \), and \( G_5 = -\lambda \phi/M_{Pl}^2 \). Using the matter density parameter \( \Omega_m = \rho_m/3M_{Pl}^2H^2 \), the modified Friedmann equation can be written as
\[ 1 = \Omega_m + \Omega_{\phi} , \] (4.2)
where
\[ \Omega_{\phi} = \frac{X}{3M_{Pl}^2H^2} (1 + 18C) . \]
Here we defined the key parameter,
\[ C \equiv \frac{\lambda H^2}{M_{Pl}^2} > C_* , \] (4.3)
where \( C_* = -1/18 \). The second inequality is the condition which ensures the positivity of the energy density of the scalar field, \( \Omega_{\phi} > 0 \). Using the gravity equations and the energy density \( \rho_{\phi} \) and the pressure \( p_{\phi} \) for the scalar field, the effective equation of state, \( w_{\text{eff}} \equiv p_{\phi}/\rho_{\phi} \), can be written as
\[ w_{\text{eff}} = \frac{1 + 30C}{1 + (24 - 6\Omega_{\phi})C + 108(1 + \Omega_{\phi})C^2} . \] (4.4)
Gubitosi and Linder showed that if the deviation parameter at the present time, \( \delta \equiv (C_* - C)/C_* |_{z=0} \), satisfies \( \delta < 2/5 \), corresponding to the condition for negative pressure \( w_{\text{eff}} < 0 \), the kinetic term \( X \) behaves as the cosmological constant around the present time.

The propagation speed of gravitational waves (2.6) can be written as
\[ c_T^2 = \frac{M_{Pl}^2 + 2\lambda X/M_{Pl}^2}{M_{Pl}^2 - 2\lambda X/M_{Pl}^2} . \] (4.5)
The condition for avoiding ghosts of the tensor perturbations, $G_T > 0$, is $\delta > \Omega_\phi (\Omega_\phi - 3)$, which is automatically satisfied, while the condition for avoiding instability $c_T^2 \geq 0$ is

$$\delta \geq \frac{\Omega_\phi}{\Omega_\phi + 3}.$$  \hfill (4.6)

Therefore, $\delta > 0$ is required for avoiding ghost-instability. Thus the theoretically allowed parameter range is

$$0 < \delta < \frac{2}{5},$$

which is equivalent with

$$-\frac{1}{18} < C(z = 0) < -\frac{1}{30}. \hfill (4.8)$$

The propagation speed of gravitational waves in terms of $\Omega_\phi$ is rephrased as

$$c_T^2 = \frac{(3 + \Omega_\phi)\delta - \Omega_\phi}{(3 - \Omega_\phi)\delta + \Omega_\phi}. \hfill (4.9)$$

The constraints from gravitational Cherenkov radiation $c_T > 1 - \epsilon$, where $\epsilon = 2 \times 10^{-15}$, reads $\delta > 1 - \mathcal{O}(\epsilon)$ from eq. (4.9), which contradicts with the condition (4.7). Equivalently, from eqs. (4.3) and (4.8), $\lambda$ is always negative, therefore, the propagation speed of gravitational waves is always smaller than unity from eq. (4.5). Thus this purely kinetic coupled gravity is inconsistent with the constraint from the gravitational Cherenkov radiation for any theoretically allowed parameter $\lambda$.

5 Extended galileon model

In this section, we consider the model proposed by De Felice and Tsujikawa [44], which is an extension of the covariant galileon model [52]. In this model, the arbitrary functions has the following form,

$$K = -c_2 M_2^{1-(1-p_2)} X^{p_2},$$

$$G_3 = c_3 M_3^{-4p_3} X^{p_3},$$

$$G_4 = \frac{1}{2} M_{pl}^2 - c_4 M_4^{2-4p_4} X^{p_4},$$

$$G_5 = 3c_5 M_5^{-(1+4p_5)} X^{p_5}, \hfill (5.1)$$

where $c_i$ and $p_i$ are the model parameters and $M_i$ are constants with dimensions of mass. We impose the conditions that the tracker solution is characterized by $H \dot{\phi}^{2q} = \text{const}$ and the energy density of the scalar field is proportional to $\dot{\phi}^{2p}$. These conditions enable us to reduce the model parameters, which is given by $p_2 = p$, $p_3 = p + (2q - 1)/2$, $p_4 = p + 2q$, and $p_5 = p + (6q - 1)/2 \, ^1$. Note that the covariant Galileon model corresponds to $p = 1$ and $q = 1/2$.

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\(^1\)Kimura and Yamamoto considered the case : $p = 1$, $q = n - 1/2$, $c_4 = 0$, and $c_5 = 0$ [53].
5.1 Cosmological Dynamics

In this subsection, we briefly review the background dynamics in the extended galileon model. For convenience, we write the mass dimension constants as

\[
M_2 \equiv \left( H_{\text{dS}} M_{\text{Pl}} \right)^{1/2},
\]
\[
M_3 \equiv \left( \frac{M_{\text{Pl}} - 2}{H_{\text{dS}}^2 M_{\text{Pl}}} \right)^{1/(1-4p_3)},
\]
\[
M_4 \equiv \left( \frac{M_{\text{Pl}} - 2}{H_{\text{dS}}^2 M_{\text{Pl}}} \right)^{1/(2-4p_4)},
\]
\[
M_5 \equiv \left( \frac{H_{\text{dS}}^2 - 2}{M_{\text{Pl}} - 2} \right)^{1/(1+4p_5)}.
\] (5.2)

where \( H_{\text{dS}} \) is the hubble parameter at the de-Sitter point. At the de Sitter point \( \dot{H} = 0 \) and \( \ddot{\phi} = 0 \), we obtain the following relations from the gravitational and scalar field equations

\[
c_2 = \frac{3(3\alpha - 4\beta + 2)}{2} \left( \frac{2}{x^2_{\text{dS}}} \right)^p,
\]
\[
c_3 = \frac{\sqrt{2}}{2p + q - 1} \left[ 3(p + q)(\alpha - \beta) + p \right] \left( \frac{2}{x^2_{\text{dS}}} \right)^{p+q}.
\] (5.3)

where \( x \equiv \dot{\phi}/H_{\text{dS}} M_{\text{Pl}} \) and

\[
\alpha \equiv \frac{4(2p_4 - 1)}{3} \left( \frac{x_{\text{dS}}^2}{2} \right)^{p_4} c_4,
\]
\[
\beta \equiv 2\sqrt{2} p_5 \left( \frac{x_{\text{dS}}^2}{2} \right)^{p_5+1/2} c_5.
\] (5.4)

Thus this model is characterized by only four parameters \( p, q, \alpha, \) and \( \beta \). In order to simplify the analysis, we introduce the following variables,

\[
r_1 \equiv \left( \frac{x_{\text{dS}}}{x} \right)^{2q} \left( \frac{H_{\text{dS}}}{H} \right)^{1+2q},
\]
\[
r_2 \equiv \left[ \left( \frac{x}{x_{\text{dS}}} \right)^2 \right]^{\frac{p+2q}{2+2q}} \frac{1}{r_1^2},
\] (5.5)

and the radiation density parameter \( \Omega_r \equiv \rho_r/3H^2 M_{\text{Pl}}^2 \). Note that the de Sitter fixed point corresponds to \((r_1, r_2, \Omega_r) = (1, 1, 0)\).

Along the tracker \( r_1 = 1 \), the evolution of \( r_2 \) and \( \Omega_r \) are governed by the following differential equations,

\[
r_2' = \frac{(1 + s)(\Omega_r + 3 - 3r_2)}{sr_2 + 1} r_2,
\] (5.6)
\[
\Omega_r' = \frac{\Omega_r - 1 - 3r_2 - 4sr_2}{sr_2 + 1} \Omega_r,
\] (5.7)

where a prime denotes a derivative with respect to \( N = \ln a \) and only one parameter \( s = p/2q \) determines the background dynamics in the case of the tracker solution. In this case, the
density parameter of the scalar field is simply given by $\Omega_\phi = r_2$, satisfying the constraint $1 = \Omega_\phi + \Omega_m + \Omega_r$. Integrating these equations yields the following algebraic equations,

\begin{align}
    r_2 &= b_1 a^{4(1+s)} \Omega_r^{1+s}, \\
    b_1 a^{4(1+s)} \Omega_r^{1+s} &= 1 - \Omega_r(1 - b_2 a),
\end{align}

(5.8) (5.9)

where the integration constants are given by

\begin{align}
    b_1 &= \frac{1 - \Omega_{m0} - \Omega_{r0}}{\Omega_r^{1+s}}, \\
    b_2 &= \frac{\Omega_{m0}}{\Omega_{r0}},
\end{align}

(5.10)

and $\Omega_{m0}$ and $\Omega_{r0}$ are the matter and radiation density parameter at present, respectively. To see how the Friedmann equation is modified, we rewrite the algebraic equation (5.9) in terms of the hubble parameter $H$, then we find

\begin{equation}
    \left(\frac{H}{H_0}\right)^2 = (1 - \Omega_{m0} - \Omega_{r0}) \left(\frac{H}{H_0}\right)^{-2s} + \Omega_{m0} a^{-3} + \Omega_{r0} a^{-4}.
\end{equation}

(5.11)

This modified Friedmann equation is known as the Dvali-Turner model [54]. The authors in [53] placed the observational constraints on this modified Friedmann equation (5.11) in the special case $p = 1$ using type Ia supernovae and the CMB shift parameter and showed that the model parameter $s$ has to be small, $s \ll 1$, in order to be consistent with cosmological observations.

5.2 Conditions

In this subsection, we summarize the theoretically allowed parameter space in the extended galileon model, discussed in [44], and show that the constraint from gravitational Cherenkov radiation is crucial. To avoid ghost-instabilities, we must impose the conditions, $G_T > 0$, $c_T^2 > 0$, $G_S > 0$, and $c_S^2 > 0$ in the history of the universe. The coefficients in the tensor and scalar perturbation equations in terms of $r_1$, $r_2$, $\Omega_r$, and the model parameters are listed in appendix B. We find that the propagation speed of gravitational waves along the tracker $r_1 = 1$ is written

\begin{equation}
    c_T^2 = \frac{2(1 - 2p - 4q)(2q + pr_2) + 3\alpha(2q + pr_2)r_2 - 3\beta(1 - 2p - 4q)(3 - 3r_2 + \Omega_r)r_2}{(1 - 2p - 4q)[2 + 3(\alpha - 2\beta)r_2](2q + pr_2)}.
\end{equation}

(5.12)

Note that eq. (5.12) reduces $c_T^2 = 1$ when $\alpha = \beta = 0$, which correspond to $G_4 = M_{Pl}^2/2$ and $G_5 = 0$. We further impose no-instability condition at $r_2 = r_{2,\text{min}}$, where a minimum of propagation speed of gravitational waves $c_T^2$ is located. Setting $r_1 = 1$ and $\Omega_r \simeq 0$, the minimum of $c_T^2$ is given by eq.(5.12) at $r_2 = r_{2,\text{min}}$,

\begin{equation}
    r_{2,\text{min}} = \left[\frac{2(3 + 2p)(1 - 2p - 4q)q \beta - 8p q(p + 2q)\alpha \pm \sqrt{3\Gamma_1}}{\Gamma_2}\right]/\Gamma_2.
\end{equation}

(5.13)

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2 Observational constraints on eq. (5.11) from type Ia supernovae, cosmic microwave background, and baryon acoustic oscillations including the cosmic curvature $K$ in the context of the extended galileon model has been recently studied by De Felice and Tsujikawa [55]. They found that the parameter $s$ is constrained to be $s = 0.034^{+0.327}_{-0.034}$ (95% CL) in the flat case $K = 0$. 

---
where
\begin{equation}
\Gamma_1 = (1 - 2p - 4q)(p + 2q)q \beta \times [4(p + 2q)(p - 3q)\alpha] \\
+ 2(1 - 2p - 4q)(3 + 2p - 3(3 - 4q)\beta) + 3[3 - 16q(1 - 2q) - 2p(3 - 8q)\alpha \beta], \\
\Gamma_2 = 4p^2(p + 2q)\alpha - 18(p + 2q)(1 - 2p - 4q)\beta^2 + (1 - 2p - 4q)[2p(3 + 2p) + 9(p + 2q)\alpha \beta].
\end{equation}
(5.14)

The conditions for avoiding ghost-instabilities in the regimes along the tracker are given by
\begin{align}
\mathcal{G}_S|_{r_1=1, r_2<1} &> 0, & \mathcal{G}_S|_{\text{de Sitter}} &> 0, \\
c_S^2|_{r_1=1, r_2<1} &\geq 0, & c_S^2|_{\text{de Sitter}} &\geq 0, \\
\mathcal{G}_T|_{r_1=1, r_2<1} &> 0, & \mathcal{G}_T|_{\text{de Sitter}} &> 0, \\
c_T^2|_{r_1=1, r_2<1} &\geq 0, & c_T^2|_{\text{de Sitter}} &\geq 0, \\
c_T^2|_{r_2, \text{min}} &> 0. &
\end{align}
(5.15)

If the initial condition of \( r_1 \) is \( r_1 \ll 1 \), we then must impose the conditions for avoiding ghost-instabilities in the regime \( r_1 \ll 1 \) and \( r_2 \ll 1 \), which is given by
\begin{align}
\mathcal{G}_S|_{r_1<1, r_2<1} &> 0, & c_S^2|_{r_1<1, r_2<1} &\geq 0, \\
\mathcal{G}_T|_{r_1<1, r_2<1} &> 0, & c_T^2|_{r_1<1, r_2<1} &\geq 0. &
\end{align}
(5.16)

We also impose the condition that the other fixed points \( r_a \) and \( r_b \) (see appendix C) is not real or outside the interval \( 0 < r_1 \leq 1 \), which is given by
\begin{equation}
\Delta < 0 \quad \text{or} \quad r_{a,b} < 0 \quad \text{or} \quad r_{a,b} > 1.
\end{equation}
(5.17)

Note that as long as the initial condition of \( r_1 \) is near \( r_1 = 1 \) and the scalar field follows the tracker from early stage, these conditions (5.16) and (5.17) do not have to be imposed.

Let us classify the constraints into four classes: (a) the constraint from the gravitational Cherenkov radiation, which is given by eq. (1.1) and eq. (5.12) with setting \( c_T = c_T|_{z=0} \), (b) the theoretical constraint (5.15) to avoid the ghost-instabilities when the scalar field follows the tracker solution from early stage, assuming that the tracker is near \( r_1 = 1 \) initially, (c) the theoretical constraint (5.16) and (5.17) in addition to (5.15) to avoid the ghost-instabilities when the scalar field does not follow the tracker solution initially, assuming that the initial condition of \( r_1 \) is sufficiently small, (d) the other constraint from the cosmological observations, type Ia supernovae, the shift parameter from the cosmic microwave background, and the baryon acoustic oscillations.

Figure 2 shows the allowed regions to satisfy the constraint (a) and the constraint (c) for \( p = 1 \) and \( q = 1/2 \) (left panel) and \( p = 1 \) and \( 5/2 \) (right panel), where we adopt \( \Omega_{\text{m0}}h^2 = 0.1344 \) and \( \Omega_{\text{r0}}h^2 = 4.17 \times 10^{-5} \) with \( h = 0.7 \). In this case, we see that there is no overlap region except for \( \alpha = 0 \) and \( \beta = 0 \). Thus, the constraint from the gravitational Cherenkov radiation is crucial. Figure 3 is the same as figure 2, but for the constraint (a) and the constraint (b). We see that the allowed region in parameter space is significantly reduced, by combining with the constraint from gravitational Cherenkov radiation (a). Especially, there is no overlap region with the positive values of \( \alpha \) and \( \beta \), in figure 3. In general one can show that both the constraints (a) and (b) impose \( \alpha \) and \( \beta \) to be negative or zero for any values of \( p \geq 1 \) and \( q \geq 0 \) (see appendix D).
Figure 2. The allowed parameter space which satisfies the constraint (a) from the gravitational Cherenkov radiation (1.1) and the constraint (c) from (5.15), (5.16), and (5.17). The left panel assumes $p = 1$ and $q = 1/2$, while the right panel does $p = 1$ and $q = 5/2$.

Figure 3. The allowed parameter space which satisfies the constraint (a) from the gravitational Cherenkov radiation (1.1) and the constraint (b) from (5.15). The left panel assumes $p = 1$ and $q = 1/2$, while the right panel does $p = 1$ and $q = 5/2$.

We must further include the constraint from cosmological observations (d). The authors in ref. [56] investigated the constraint on the covariant galileon model ($p = 1$ and $q = 1/2$) from the observational data of type Ia supernovae, the shift parameter from the cosmic microwave background, and the baryon acoustic oscillations. They showed the early tracking solution, corresponding to the case of (b), is disfavored by the cosmological constraint (d). On the other hand the solutions that approach the tracker solution only at late times, corresponding to the case of (c), are favored (also see [55]) taking small spatial curvature into account. However, the latter case is significantly constrained by combining the constraint (a), though we do not take the spatial curvature into account.

Thus, the constraint from the gravitational Cherenkov radiation plays a very important
role to reduce the allowed parameter-space of the extended galileon model. In ref. [57], it is demonstrated that the integrated Sachs Wolfe effect derives a stringent constraint on a subclass of the galileon model. Further tight constraint could be obtained by combining these constraints.

6 Conclusion

In this paper, we studied constraints on the general scalar-tensor theories on a cosmological background, whose propagation speed of gravitational waves differs from the speed of light, using the survival of high energy cosmic ray against the gravitational Cherenkov radiation. In these theories, the coupling of the scalar field $\phi$ and its kinetic term $X$ with gravity causes the violation of Lorentz invariance in a cosmological background, leading to a time-dependent propagation speed of gravitational waves. We demonstrated that such a model can be constrained using the survival of high energy cosmic ray against the gravitational Cherenkov radiation.

We first considered constraints on the purely kinetic coupled gravity and found that the conditions for the existence of a desired late-time solution and avoiding ghost-instability is $0 < \delta < 2/5$ while the constraint from the gravitational Cherenkov radiation gives $\delta > 1 - \mathcal{O}(\epsilon)$, where $\epsilon = 2 \times 10^{-15}$. Thus the purely kinetic coupled gravity is inconsistent with the argument of the gravitational Cherenkov radiation.

We also focused our investigation on the extended galileon model, which is a generalization of the covariant galileon model in the framework of the most general second-order scalar-tensor theory. We showed that there is no allowed parameter space except for $\alpha = \beta = 0$ by combining the condition for avoiding ghost-instabilities and the constraints from the gravitational Cherenkov radiation if the initial condition of $r_1$ is sufficiently small. Even if the initial condition of $r_1$ is placed near the tracker $r_1 = 1$, the allowed parameter space is tightly constrained by combining the gravitational Cherenkov radiation and cosmological constraint such as type Ia supernovae, the shift parameter from cosmic microwave background, baryon acoustic oscillations.

Thus the constraint from the gravitational Cherenkov radiation is important to constrain the general second-order scalar-tensor theories on a cosmological background, whose propagation speed of gravitational waves is less than the speed of light.

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A Scalar perturbations

Here we summarize the scalar perturbations including coefficients in the most general second-order scalar-tensor theory derived in [36]. For the unitary gauge $\phi = \phi(t)$ with the line
The propagation speed of the scalar perturbations is defined as

\[ S_S^{(2)} = \int dt d^3x a^3 \left[ G_S \zeta^2 - \frac{F_S}{a^2} (\bar{\nabla} \zeta)^2 \right], \quad (A.1) \]

where

\[ F_S = \frac{1}{a} \frac{d}{dt} \left( \frac{a}{\Theta} \frac{\partial G}{\partial \Theta} \right) - \mathcal{F}_T, \quad (A.2) \]
\[ G_S = \sum \frac{\Theta^2}{\Theta^2} G_T^2 + 3G_T, \quad (A.3) \]

and

\[ \Theta \equiv -\dot{\phi} X G_{3X} + 2H G_4 - 8H X G_{4X} - 8H X^2 G_{4XX} + \dot{\phi} G_{4\phi} + 2\dot{\phi} G_{4\phi X} \]
\[ - H^2 \dot{\phi} \left( 5X G_{5X} + 2X^2 G_{5XX} \right) + 2H X \left( 3G_{5\phi} + 2X G_{5\phi X} \right), \quad (A.4) \]
\[ \Sigma \equiv X K_X + 2X^2 K_{XX} + 12H \dot{\phi} X G_{3X} + 6H \dot{\phi} X^2 G_{3XX} - 2X G_{3\phi} - 2X^2 G_{3\phi X} \]
\[ - 6H^2 G_4 + 6 \left[ H^2 \left( 7X G_{4X} + 16X^2 G_{4XX} + 4X^3 G_{4XXX} \right) \right] \]
\[ - H \dot{\phi} \left( 5G_{4\phi} + 7X G_{4\phi X} + 2X^2 G_{4\phi XX} \right) + 30H^3 \dot{\phi} X G_{5X} + 26H^3 \dot{\phi} X^2 G_{5XX} \]
\[ + 4H^3 \dot{\phi} X^3 G_{5XX} - 6H^2 X (6G_{5\phi} + 9X G_{5\phi X} + 2X^2 G_{5\phi XX}). \quad (A.5) \]

The propagation speed of the scalar perturbations is defined as

\[ c_S^2 \equiv \frac{F_S}{G_S}. \quad (A.6) \]

## B Coefficients and propagation speed in various regimes

In this appendix, we summarize the coefficients and propagation speed in the tensor and scalar perturbation equations in the extended galileon model in various regimes, derived in [44].

In the regime, \( r_1 = 1 \) and \( r_2 \ll 1 \), the coefficients (2.5), (2.6), (A.3), and (A.6) are given by

\[ G_S|_{r_1=1, r_2\ll1} \simeq 6q \left[ p - 3(\alpha - 2\beta)q \right] r_2, \quad (B.1) \]
\[ c_S^2|_{r_1=1, r_2\ll1} \simeq \left\{ 4p^3(q_r + 3) - 2p^2 \left( (q_r + 3) (6\beta - 3\alpha + 2) \right. \right. \]
\[ \left. - 2q [3q_r + 11 - 3(q_r - 2\beta)(q_r + 3)] \right\} - 3\left\{ q_r (q_r + 3) \right. \]
\[ + 8q^2(q_r + 5)(q_r - 2\beta - 2q^2(7q_r + 27)(q_r - 2\beta) \]
\[ + q [3q_r (q_r + 3) - 2\beta (5q_r + 17)] \left. \right\} \]
\[ - p \{ (q_r + 3)(3q_r - 12\beta - 1) + 4q^2 [(q_r - 2\beta)(9q_r + 33) - 2(q_r + 5)] \}
\[ + q [12(2\beta - \alpha)(3q_r + 10) + 6q_r + 22] \left. \right\} \]
\[ \times 1/\left[ 24q^2 (2p + 4q - 1) \right] \left\{ p - 3(\alpha - 2\beta)q \right\}], \quad (B.2) \]
\[ G_T|_{r_1=1, r_2\ll1} \simeq \frac{1}{2} \left[ 2 + 3(\alpha - 2\beta) r_2 \right], \quad (B.3) \]
\[ c_T^2|_{r_1=1, r_2\ll1} \simeq 1 - \left\{ 6[2(\alpha - 2\beta)q + 3\beta]p + 24(\alpha - 2\beta)q^2 \right. \]
\[ + 3(16q - 3) + 3\beta (2p + 4q - 1) q_r \left. \right\} / 4q^2 (2p + 4q - 1) r_2. \quad (B.4) \]
At the de Sitter point, \( r_1 = r_2 = 1 \), the coefficients \((2.5), (2.6), (A.3),\) and \((A.6)\) are given by

\[
G_S|_{\text{de Sitter}} = \frac{6(p + 2q)(3\alpha - 6\beta + 2)[p - 3(\alpha - 2\beta)q]}{[2p - 6(\alpha - 2\beta)q - 3\alpha + 6\beta - 2]^2},
\]

\[
c_S^2|_{\text{de Sitter}} = \frac{(6\beta + 4p^2 + p)[9(\alpha - 2\beta)^2 + 3\alpha - 12\beta + 4q(6\beta - 3\alpha + 2) - 2] + 3(\alpha - 2\beta)[3\beta + q(9\alpha - 12\beta - 8q + 6)]}{3(2\beta - \alpha)(2q + 1) + 2p - 2} \times \frac{6(6\beta - 3\alpha - 2)(p + 2q)(2p + 4q - 1)(p - 3\alpha q + 6\beta q)}{6(6\beta - 3\alpha - 2)(p + 2q)(2p + 4q - 1)(p - 3\alpha q + 6\beta q)},
\]

\[
G_T|_{\text{de Sitter}} = \frac{1}{2} \frac{3(3\alpha - 6\beta + 2)}{(3\alpha - 6\beta + 2)},
\]

\[
c_T^2|_{\text{de Sitter}} = \frac{2(2p + 4q - 1) - 3\alpha}{(2p + 4q - 1)(3\alpha - 6\beta + 2)}.
\]

In the regime, \( r_1 \ll 1 \) and \( r_2 \ll 1 \), the coefficients \((2.5), (2.6), (A.3),\) and \((A.6)\) are given by

\[
G_S|_{r_1, r_2 \ll 1} \simeq 3(p + 3q)(2p + 6q - 1)\beta r_1^{(p-1)/(2q+1)} r_2,
\]

\[
c_S^2|_{r_1, r_2 \ll 1} \simeq \frac{p + 3q - 2}{2(p + 3q)(2p + 6q - 1)} (1 + \Omega_r),
\]

\[
G_T|_{r_1, r_2 \ll 1} \simeq 1 - 3\beta r_2 r_1^{(p-1)/(2q+1)},
\]

\[
c_T^2|_{r_1, r_2 \ll 1} \simeq 1 + \frac{3(4p + 12q - 5 - 3\Omega_r)}{4p + 12q - 2} \beta r_1^{(p-1)/(2q+1)} r_2.
\]

### C Other fixed points

There also exist the other fixed points found by De Felice and Tsujikawa [44], which is characterized by the equation,

\[
p(3\alpha - 4\beta + 2)r_i^2 + [2\beta(p + 3q) - 3\alpha(p + 2q)]r_i + 2\beta(p + 3q) = 0,
\]

where \( r_i = r_a \) and \( r_b \), and

\[
\begin{align*}
& r_{a,b} = \frac{3\alpha(p + 2q) - 2\beta(p + 3q) \pm \sqrt{\Delta}}{2p(3\alpha - 4\beta + 2)}, \\
& \Delta = [2\beta(p + 3q) - 3\alpha(p + 2q)]^2 - 8\beta p(3\alpha - 4\beta + 2)(p + 3q).
\end{align*}
\]

### D Constraint on the values of \( \alpha \) and \( \beta \)

In this appendix, we assume \( p \geq 1, q \geq 0 \), and \( r_2 = 1 - \Omega_{m0} - \Omega_{r0} \), where \( \Omega_{m0} h^2 = 0.1344 \) and \( \Omega_{r0} h^2 = 4.17 \times 10^{-5} \) with \( h = 0.7 \). One can show that \( \alpha \) and \( \beta \) must be negative or zero to satisfy both the constraints (a) and (b) for any values of \( p \geq 1 \) and \( q \geq 0 \). This can be proved as follows. The upper bound of the constraint (b) is determined by the straight line in the plane of \( \alpha \) and \( \beta \),

\[
\beta = \frac{1}{2} \alpha - \frac{p - 1}{3(2q + 1)},
\]

which comes from \( c_S^2|_{\text{de Sitter}} \geq 0 \) (see e.g., the left panel of figure 3). On the other hand, the constraint (a) is characterized by the two straight lines,

\[
\beta = \frac{1}{2} \alpha + \frac{1}{3r_2},
\]

\[-15-\]
and

\[
\beta = \frac{2(p + 2q)(2q + pr_2)}{2(p + 4q - 1)(4q - 3 + 3r_2 + 2pr_2 - \Omega_r)} \alpha. \tag{D.3}
\]

Note that the lines of eqs. (D.2) and (D.1) are parallel each other. Since we have

\[
\frac{2(p + 2q)(2q + pr_2)}{2(p + 4q - 1)(4q - 3 + 3r_2 + 2pr_2 - \Omega_r)} > \frac{1}{2}, \tag{D.4}
\]

at present epoch, the lines of eqs. (D.2) and (D.3) intersect at a point with \( \alpha > 0 \) and \( \beta > 0 \). (D.4) means the slope of the line (D.3) is larger than that of the line (D.1), then the lines of eqs. (D.1) and (D.3) intersect at a point with \( \alpha \leq 0 \) and \( \beta \leq 0 \), Therefore, \( \alpha \) and \( \beta \) must be negative or zero to satisfy the constraints (a) and (b) in the case of \( p \geq 1 \) and \( q \geq 0 \).

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