The crucial role of CLEO-c in the measurement of $\gamma$

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Abstract. The most sensitive method to measure the CKM angle $\gamma$ is to exploit interference in $B^\pm \to D K^\pm$ decays, with the $D$-meson decaying to a hadronic final state. The analysis of quantum-correlated decays of the $\psi(3770)$ at CLEO-c provides invaluable information on the strong-phase difference between the $D_0$ and $\bar{D}_0$ across the Dalitz plane. Results from analyses of the decays $D \to K^0\pi^+\pi^-$ and $D \to K^0K^+K^-$ will be presented.

1. Introduction

The angle $\gamma$ is currently the least precisely known of the CKM angles of the unitarity triangle. The average value of $\gamma$ obtained from direct measurements is $70^{+27}_{-29}$° [1]. A theoretically clean strategy to extract $\gamma$ is to exploit the interference between $B^\pm \to D_0 K^\pm$ and $B^\pm \to \bar{D}_0 K^\pm$ decays in which the $D_0$ and $\bar{D}_0$ decay to the same final state $F$ [2]. This method requires precise knowledge of how the strong-phase difference between the $D_0$ and $\bar{D}_0$ varies across the Dalitz plane. Quantum-correlated data from CLEO-c provide a unique opportunity to measure this variation. This paper describes the procedure used when $F$ is either $K^0\pi^+\pi^-$ or $K^0K^+K^-$, collectively denoted $K^0h^+h^-$. The strong phase distribution for $D_1 \to K^0\pi^+\pi^-$ is in general different to that for $D \to K^0K^+K^-$, but the formalism is the same.

2. Determination of $\gamma$ with $B^\pm \to D(K^0h^+h^-)K^\pm$

The expression for the decay amplitude of $B^\pm \to D(K^0h^+h^-)K^\pm$ is:

$$A(B^\pm \to D(K^0h^+h^-)K^\pm) \propto f_D(x,y) + r_B e^{i(\delta_B \pm \gamma)} f_D(y,x)$$  \hspace{1cm} (1)

where $r_B \sim 0.1$ is the ratio of the magnitudes of the amplitudes of suppressed and favoured $B^\pm \to DK^\pm$ decays [1], $\delta_B$ is the CP-invariant strong-phase difference between interfering $B^\pm$ decays, $x$ and $y$ are the squared invariant masses $m^2(K^0h^+)$ and $m^2(K^0h^-)$ respectively and $f_D(x,y) \equiv |f_D(x,y)|e^{i\delta_D(x,y)}$ is the $D$-decay amplitude. Neglecting CP violation, the square of $A$ contains the strong-phase difference term $\Delta\delta_D \equiv \delta_D(x,y) - \delta_D(y,x)$. In order to extract $\gamma$ from the difference in $B^\pm$ decay rates across the Dalitz plane, $\Delta\delta_D$ must be determined.

Previous studies have modelled the $D$ decay using several intermediate two-body resonances [3,4]. This introduces a model systematic uncertainty of $7 - 9$° to the value of $\gamma$ which is likely to be the main limitation at future $b$-physics experiments.

$^1$ Henceforth, $D$ indicates either $D_0$ or $\bar{D}_0$
An alternative approach, which is the subject of the remainder of this paper, is to use external information on the strong-phase difference in a binned fit to the distribution of events across the Dalitz plane [2]. A good choice of binning is to divide the Dalitz plane into regions of similar $\Delta\delta_D$ [5] based on a model of the $D$-decay. An incorrect model will not bias the measurement of $\gamma$, but will reduce the statistical precision. An example of a particular binning, for $K_S^0\pi^+\pi^-$, is given in Figure 1. This uses a model from Ref. [6].

The Dalitz plot is binned symmetrically about $y = x$; bins below this axis are numbered $i$ and those above $-i$. The number of events in the $i^{th}$ bin of the $D\to K_S^0 h^+ h^-$ Dalitz plot in the decay $B^{\pm}\to D(K_S^0 h^+ h^-)K^\pm$ is dependent upon $c_i$ and $s_i$ which are the average cosine and sine of the strong-phase difference across the $i^{th}$ bin [2].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Figure1.png}
\caption{The strong-phase difference for $K_S^0\pi^+\pi^-$, binned uniformly in eight bins of $\Delta\delta_D$}
\end{figure}

3. Determination of $\Delta\delta_D$ with quantum-correlated decays of the $\psi(3770)$

The $D$-mesons are quantum-correlated with an overall CP of $-1$, so knowledge of the CP state of one of the pair reveals the CP state of the other. When one $D$ decays to $K_S^0 h^+ h^-$, the decay product of the other $D$ is denoted the opposite-side tag.

Define $K_i$ as the number of events in the $i^{th}$ bin of the flavour-tagged $K_S^0 h^+ h^-$ Dalitz plot. $c_i$ and $s_i$ can then be expressed as:

$$c_i \equiv \frac{a_D^2}{\sqrt{K_i}K_{-i}} \int |f_D(x,y)||f_D(y,x)|\cos[\Delta\delta_D(x,y)]dxdy$$

and

$$s_i \equiv \frac{a_D^2}{\sqrt{K_i}K_{-i}} \int |f_D(x,y)||f_D(y,x)|\sin[\Delta\delta_D(x,y)]dxdy$$

where $a_D$ is a normalization factor. Analogous quantities, denoted $K'_i$, $c'_i$ and $s'_i$, exist for $K_L^0 h^+ h^-$. To first order, $c'_i = c_i$ and $s'_i = s_i$, but there are second-order differences due to doubly Cabibbo suppressed contributions to the $D\to K_L^0 h^+ h^-$ decay amplitude.

$c_i^{(o)}$ can be determined from CP-tagged $D$ decays to $K^0 h^+ h^-$. The number of events in the $i^{th}$ bin of a $K_{S(L)}^0 h^+ h^-$ Dalitz plot, where the opposite-side tag is a CP eigenstate, is given by:

$$M_i^{(o)} = h_{CP\pm}^{(o)}(K_i^{(o)} \pm (-1)^p 2c_i^{(o)}\sqrt{K_i^{(o)}K_{-i}^{(o)} + K_{-i}^{(o)}})$$

where $h_{CP\pm}^{(o)}$ is a normalization factor, $p = 0$ for $K^0 h^+ h^-$ and 1 for $K^0 h^+ h^-$.  

In order to determine $s_i^{(o)}$, decays in which both $D$-mesons decay to $K^0 h^+ h^-$ are required. The number of events in the $i^{th}$ Dalitz plot bin of $D\to K_S^0 h^+ h^-$ and the $j^{th}$ of $D\to K_{S(L)}^0 h^+ h^-$ is:

$$M_{i,j}^{(o)} = h_{corr}^{(o)}(K_i K_{-j}^{(o)} + K_{-i} K_j^{(o)}) - (-1)^p 2\sqrt{K_i K_{-i} K_j^{(o)} K_{-j}^{(o)}} (c_i c_j^{(o)} + s_i s_j^{(o)})$$

where $h_{corr}^{(o)}$ is a normalization factor.
The parameters $c_i, s_i, c_i'$ and $s_i'$ are extracted by minimizing a maximum likelihood function based on the expected and observed values of $M_i^{(l)}$ and $M_{i,j}^{(l)}$. The quantities $(c_i - c_i')$ and $(s_i - s_i')$, predicted by the model, are used as a constraint in the fit. The consequences of variations in the predicted values are evaluated when assigning systematic errors.

4. Event Selection at CLEO-c

An integrated luminosity of $(818 \pm 8) \text{pb}^{-1}$ of $\psi(3770) \rightarrow D^0 \bar{D}^0$ data were recorded at CLEO-c. The tags selected for $K^0\bar{K}^0$ and $K^0\pi^+\pi^-$ are shown in Tables 1 and 2 respectively. In order to maximize statistics, a large number of tags were selected. Approximately 23,000 events were selected for $K^0\pi^+\pi^-$ and approximately 1,900 for $K^0K^0$. The $\Delta\delta_D$ bins mentioned earlier were appropriately populated using selected event yields.

$K^\pm, \pi^\pm, \pi^0$ and $\eta$ particles were selected using kinematic and reconstruction quality criteria [7]. Suitable invariant mass cuts were used to select composite particles; for example, for the $\omega \rightarrow \pi^+\pi^-\pi^0$ decay, the invariant mass of the $\pi^+\pi^-\pi^0$ system was constrained to lie within 20 MeV of the nominal $\omega$ mass.

For tags containing a $K^0_L$, a different approach was used, because $\sim 97\%$ of $K^0_L$ particles escape the CLEO-c detector. All other particles in the $K^0_L$ tag were reconstructed and the square of the missing mass, $m_{\text{miss}}^2$, was computed. Events could then be accepted or rejected depending on where they lay on the $m_{\text{miss}}^2$ plane.

All raw yields were efficiency-corrected and background-subtracted. Flat backgrounds were estimated using sidebands and peaking backgrounds were estimated from Monte Carlo. The CLEO-c environment is clean; most backgrounds were low, between 1 and 10%.

| Tag Group       | Opposite-Side Tags                                                                 |
|-----------------|------------------------------------------------------------------------------------|
| $K^+_0K^-_0$    | $K^-_0, \pi^+\pi^-, K^-_0\pi^0\pi^0, K^+_0\pi^0\pi^0, K^-_0\omega(\pi^+\pi^-\pi^0), K^+_0\omega(\pi^+\pi^-\pi^0), K^-_0\eta'(\pi^+\pi^-\eta)$ |
| $K^+_0K^-_0$    | $K^-_0, \pi^+\pi^-, K^-_0\pi^0\pi^0, K^+_0\pi^0\pi^0, K^-_0\omega(\pi^+\pi^-\pi^0), K^+_0\omega(\pi^+\pi^-\pi^0), K^-_0\eta'(\pi^+\pi^-\eta)$ |
| $K^+_0K^-_0$    | $K^-_0, \pi^+\pi^-, K^-_0\pi^0\pi^0, K^+_0\pi^0\pi^0, K^-_0\omega(\pi^+\pi^-\pi^0), K^+_0\omega(\pi^+\pi^-\pi^0), K^-_0\eta'(\pi^+\pi^-\eta)$ |
| $K^0_SK^0_S$    | $K^0_S\pi^+\pi^-, K^0_S\pi^0\pi^0, K^0_S\omega(\pi^+\pi^-\pi^0), K^0_S\omega(\pi^+\pi^-\pi^0), K^0_S\eta'(\pi^+\pi^-\eta)$ |
| $K^0_SK^0_S$    | $K^0_S\pi^+\pi^-, K^0_S\pi^0\pi^0, K^0_S\omega(\pi^+\pi^-\pi^0), K^0_S\omega(\pi^+\pi^-\pi^0), K^0_S\eta'(\pi^+\pi^-\eta)$ |
| $K^0_SK^0_L$    | $K^0_S\pi^+\pi^-, K^0_S\pi^0\pi^0, K^0_S\omega(\pi^+\pi^-\pi^0), K^0_S\omega(\pi^+\pi^-\pi^0), K^0_S\eta'(\pi^+\pi^-\eta)$ |

| Tag Group       | Opposite-Side Tags                                                                 |
|-----------------|------------------------------------------------------------------------------------|
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |
| $K^0_S\pi^+\pi^-$ | $K^+K^-\pi^+, K^0\pi^0\pi^0, K^0\pi^0\pi^0$                                     |

Table 1. Tags selected for $K^0K^0$

Table 2. Tags selected for $K^0\pi^+\pi^-$
5. Results for $D \to K^0\pi^+\pi^-$

Fit and predicted values of $c_i$ and $s_i$ for $D^0 \to K_S^0\pi^+\pi^-$ are shown in Figure 2. Systematic errors, including those taking into account variations in the model, are relatively small.

![Figure 2](image)

In order to understand the impact these results have on the $\gamma$ measurement, a toy Monte Carlo study of $B^\pm \to D K^\pm$ was performed, with enough data so the statistical uncertainty associated with $B$ decays was minimal. $r_B$, $\delta_B$ and $\gamma$ were fit, with initial values respectively of 0.1, 130° and 60°. The uncertainty on $\gamma$ which propagates through from the uncertainty on $c_i$ and $s_i$ was found to be $1.7^\circ$ [7]. This is a large improvement on the current model-dependent determinations.

6. Preliminary Results for $D \to K^0K^+K^-$

Studies of $D \to K^0K^+K^-$ are ongoing. Combined $K^0_{(S,L)}K^+K^-$ CP-tagged Dalitz plots are shown in Figure 3 and exhibit striking differences; this is a consequence of the entanglement of the $D^0\bar{D}^0$ system. $K_S^0K^+K^-$ against CP+ tags is in a CP- state, hence decays predominantly via $K_S\phi$. Similarly, $K_S^0K^+K^-$ against CP- tags mostly decays via $K^0_L\phi$. The $\phi$ resonance is narrow so most points lie close to $m_{K^+K^-}^2 = m_{\phi}^2$. This effect is not seen for the other CP-tagged data because the $K^0\phi$ resonance is not present.

![Figure 3](image)

References

[1] Charles J et al. (CKMfitter Group) 2005 Eur. Phys. J. C 41 1
[2] Giri A, Grossman Y, Soffer A and Zupan J 2003 Phys. Rev. D 68 054018
[3] Aubert B et al. (BaBar Collaboration) 2008 Phys. Rev. D 78 034023
[4] Abe K et al. (Belle Collaboration) 2008 (Preprint hep-ex/0803.3375)
[5] Bondar A et al. 2006 Eur. Phys. J. C 47 347
[6] Aubert B et al. (BaBar Collaboration) 2005 Phys. Rev. Lett. D 95 121802
[7] Briere R A et al. (CLEO Collaboration) 2008 (Preprint hep-ex/0903.1681)