Gravitational wave constraints on the observable inflation

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Gravitational waves (GW) produced in the early Universe contribute to the number of relativistic degrees of freedom, $N_{\text{eff}}$, during Big Bang Nucleosynthesis (BBN). By using the constraints on $N_{\text{eff}}$, we present a new bound on how much the observable Universe could have expanded during cosmic inflation. The new bound is roughly four orders of magnitude more stringent than previous bounds.

We also discuss the sensitivities of the current and planned GW observatories such as LIGO and LISA, and show that the constraints they could impose are always less stringent than the BBN bound.

Cosmic inflation, an early period of accelerated expansion, is the current paradigm for explaining the origins and properties of temperature fluctuations in the Cosmic Microwave Background radiation (CMB) and the large scale structure of the Universe $^1$ $^2$. Cosmic inflation is also successful in explaining why the Universe is spatially flat, homogeneous, and isotropic to a high degree, and why the abundance of topological defects predicted by grand unified theories of particle physics within our observable Universe is unobservably low.

Among the parameters that are relevant to the curvature perturbations that seed the large scale structure formation, two have already been measured to a high precision: the magnitude of scalar perturbations at large scales (the amplitude $P_{\zeta}(k_\ast)$) $\simeq 2.1 \times 10^{-9}$ at the reference scale $k_\ast = 0.05 \text{ Mpc}^{-1}$, corresponding to CMB temperature fluctuations $\delta T/T \sim 10^{-5}$ and how the perturbations change with scale (the spectral tilt, $n_{\zeta} = 1 + \frac{\text{d} \ln P_{\zeta}}{\text{d} \ln k} \simeq 0.965$, as measured by the Planck satellite $^3$$^4$. The Planck and BICEP2/Keck Array collaborations have also placed strong constraints on the magnitude of primordial gravitational waves (GWs), usually expressed in terms of the tensor-to-scalar ratio: $r \equiv P_{\zeta}(k_\ast)/P_T(k_\ast) < 0.06$, where $P_T(k)$ represents the tensor power spectrum $^5$.

Yet, the duration of inflation is unknown. Hence, also the amount by which our observable Universe expanded during inflation is unknown. This uncertainty makes it difficult to link a given model of inflation with the properties of observable cosmological perturbations that it supposedly explains (see e.g. Ref. $^1$). The expansion during inflation is usually characterized by the number of e-folds $N(k) \equiv \ln(a_{\text{inf}}/a_k)$ between the scale factor $a_k = k/H_{\text{inf}}$ at which the mode $k$ of interest exited the inflationary horizon $H_{\text{inf}}^{-1}$ and that at the end of inflation $a_{\text{inf}}$. While the value $N(k = a_0H_0) \sim 60$ is usually assumed for the mode corresponding to the size of our observable Universe, the actual amount can differ considerably from this. Assuming that the post-inflationary expansion of the Universe is controlled by a set of perfect fluids, such as radiation and cold dark matter, transitions between different epochs are prompt, and the Hubble rate did not decrease much during inflation $^1$, the available range for the number of e-folds is $N = 18 - 77$ $^6$$^7$, which corresponds to inflationary expansion of the currently observable Universe $^8$ by a factor $a_{\text{inf}}/a_0H_0 = 10^8 - 10^{53}$.

In this paper we utilize the constraints on the number of relativistic degrees of freedom, $N_{\text{eff}}$, during Big Bang Nucleosynthesis (BBN) to derive a new bound on the expansion of the observable Universe during inflation. The new bound is almost four orders of magnitude more stringent than previous bounds and valid within the standard assumptions of the inflationary dynamics and post-inflationary expansion history. We also discuss the sensitivities of the current and planned gravitational wave observatories such as LIGO, LISA, Einstein Telescope, and BBO, and show that the constraints they could impose are always less stringent than the BBN bound.

We begin by presenting the number of e-folds between the horizon exit of a scale $k$ and the end of inflation. The scale is related to the present Hubble scale $H_0$ as

$$
\frac{k}{a_0H_0} = \frac{a_kH_{\text{inf}}}{a_0H_0} = e^{-N} \frac{a_{\text{inf}}}{a_{\text{RD}}} \frac{a_{\text{RD}}}{a_0} \frac{H_{\text{inf}}}{H_0},
$$

where $H_{\text{inf}}$ is the Hubble parameter during inflation, $a_0$ is the scale factor at present, and $a_{\text{RD}}$ is the scale factor at the time when the radiation-dominated epoch (RD) began after inflation, i.e. we are not assuming that the Universe entered into the usual Hot Big Bang (HBB)

$^1$ Because the first slow-roll parameter is $\epsilon = \dot{H}/H^2 \ll 1$, it is usually a very good approximation that the Hubble rate did not decrease much during inflation for the range of e-folds that is of interest here. In particular, this is the case for plateau models that give the best fit to the CMB data $^3$, and therefore in this paper we maintain this assumption throughout the paper.

$^2$ How much the whole Universe that continues beyond our current horizon expanded during inflation is something we cannot, unfortunately, answer with confidence. See, however, Ref. $^8$ for model-dependent discussion on this aspect.
epoch immediately after inflation but allow for an intermediary period of non-radiation dominated expansion between inflation and HBB. This gives the number of e-folds as

\[ N = \ln \left( \frac{a_{\text{inf}}}{a_{\text{RD}}} \right) + \ln \left( \frac{a_{\text{RD}}}{a_0} \right) - \ln \left( \frac{k}{a_0 H_{\text{inf}}} \right), \]

(2)

which shows that the amount of expansion \(N\) of the observable Universe during inflation is completely determined by the post-inflationary expansion history. The question we would like to ask then is: given all observational constraints, what is the maximum value of \(N\)?

To answer this question, we have to find an expression for \(N(k)\) in terms of observables and quantities one can hope to be able to compute from the underlying particle physics theory, such as the energy scale when the RD commenced after inflation. First, we assume that between inflation and RD the total energy density scaled as \(\rho(a) = \rho(a_{\text{inf}}) \left( a_{\text{inf}} / a \right)^3(1+w)\), where \(w \equiv p/\rho\) is the effective equation of state (EoS) parameter of the dominant fluid which is characterized by its energy density \(\rho\) and pressure \(p\), and the scaling of \(\rho\) in terms of \(a\) follows in the usual way from the continuity equation \(\dot{\rho} = -3H(1+w)\rho\), where \(H = \dot{a}/a\) and the overdot denotes derivative with respect to cosmic time \(t\). Therefore, \(w = 0\) and \(w = 1/3\) correspond to an effectively matter-dominated post-inflation pre-RD epoch and an instant reheating into RD, respectively, whereas scenarios with \(1/3 < w < 1\) are encountered in models where the total energy density of the Universe after inflation is dominated by the kinetic energy of a scalar field, either through oscillations in a steep potential (e.g. \(V(\phi) \propto \phi^p\) with \(p > 4\)), or by an abrupt drop in the potential \([9]\). This is the case in e.g. quintessential inflation \([11]\), where the inflaton field makes a transition from potential energy domination to kinetic energy domination at the end of inflation, reaching values of \(w\) close to unity. The bound \(w \leq 1\) comes from the requirement that the sound speed of the dominant fluid does not exceed the speed of light. On the other hand, for \(w > -1/3\) the Universe does not inflate. A plausible range for post-inflationary EoS parameter is therefore between these two values, and in the following we will maintain the dependence on \(w\) explicitly in our calculations. Therefore, for the first term in Eq. (2), we obtain

\[ \ln \left( \frac{a_{\text{inf}}}{a_{\text{RD}}} \right) = \frac{1}{3(1+w)} \ln \left( \frac{\rho_{\text{RD}}}{\rho_{\text{inf}}} \right), \]

(3)

where \(\rho_{\text{inf}} \equiv \rho(a_{\text{inf}})\) and \(\rho_{\text{RD}} \equiv \rho(a_{\text{RD}})\) is the radiation energy density at the time the RD epoch began.

Assuming entropy conservation between RD and the present day and that the Universe thermalized quickly at the start of RD, we can write

\[ \frac{a_{\text{RD}}}{a_0} \simeq \left( \frac{\pi^2}{30} \right)^{1/4} \frac{g_*^{1/3} (a_0)}{g_*^{1/3}(a_{\text{RD}})} \frac{T_0}{\rho_{\text{RD}}^{1/4}}, \]

(4)

where \(g_*\) is the number of effective relativistic degrees of freedom (assumed to be the same for entropy and energy density), and \(T_0 = 2.725\) K is the present-day CMB temperature \([4]\). The second term in Eq. (2) thus becomes

\[ \ln \left( \frac{a_{\text{RD}}}{a_0} \right) \simeq -66.1 - \ln \left( \frac{\rho_{\text{RD}}^{1/4}}{10^{16}\text{ GeV}} \right), \]

(5)

where we have taken \(g_* (a_0) = 3.909\) and \(g_* (a_{\text{RD}}) = 106.75\). If \(g_* (a_{\text{RD}})\) was e.g. an order of magnitude larger or smaller, the first term above would change by only \(O(0.1)\), and so we will henceforth neglect the \(g_* (a_{\text{RD}})\) dependence.

Next, we can express the tensor-to-scalar ratio \(r\) in the single-field slow-roll approximation (see e.g. Ref. \([11]\)) as

\[ r \simeq \frac{8}{M_P^2} \frac{P_\zeta (k_*)}{\zeta (k_*)^2}, \]

(6)

where \(M_P\) is the reduced Planck mass and we assumed that the Hubble scale and the amplitude of perturbations did not change between the horizon exits of the scale \(k\) and the pivot scale \(k_*\), where \(r\) is measured. With this, the last term in Eq. (2) becomes

\[ - \ln \left( \frac{k}{a_0 H_{\text{inf}}} \right) \simeq 128.3 + \frac{1}{2} \ln \left( \frac{r}{0.1} \right) - \ln \left( \frac{k}{a_0 H_0} \right), \]

(7)

where we have used the measured value of \(P_\zeta (k_*)\) and normalized \(k\) to the present horizon \(a_0 H_0\).

Finally, by assuming that the total energy density did not decrease much during the final \(N\) e-folds so that

\[ \rho_{\text{inf}} = 3H_{\text{inf}}^2 M_P^2 \simeq (10^{16}\text{ GeV})^4 \left( \frac{r}{0.1} \right), \]

(8)

as given by Eq. (6), and substituting Eqs. (3), (5), and (7) into Eq. (2), we obtain for \(k = a_0 H_0\) the result

\[ N \simeq 62 + \frac{1 + 3w}{6(1+w)} \ln \left( \frac{r}{0.1} \right) + \frac{1 - 3w}{3(1+w)} \ln \left( \frac{\rho_{\text{RD}}^{1/4}}{10^{16}\text{ GeV}} \right). \]

(9)

This is our final result for the number of e-folds between horizon exit of the largest observable scale today and end of inflation. By assuming that before the usual Hot Big Bang epoch the Universe was effectively matter-dominated, i.e. by setting \(w = 0\), we recover the usual result discussed in e.g. Ref. \([6]\).

We will now determine the maximum possible value of \(N\). From Eqs. (2), (3), (5), (7), it is clear that for given \(r\) and \(\rho_{\text{RD}}\) (smaller than \(\rho_{\text{inf}}\)), \(N\) is largest when the equation of state \(w\) during the intermediary epoch is maximized, \(w \approx 1\). Furthermore, if \(w\) is stiff, i.e. \(1/3 < w < 1\), then Eq. (9) tells us that \(N\) is maximized when \(\rho_{\text{RD}}\) is minimized. At the very least, radiation domination must
commence before the onset of BBN, so \( \rho_{\text{RD}}^{1/4} \gtrsim T_{\text{BBN}} \sim 1 \text{ MeV} \) or, equivalently,
\[
\ln \left( \frac{\rho_{\text{RD}}^{1/4}}{10^{16} \text{ GeV}} \right) \gtrsim -44 .
\]  

(10)

By setting \( \rho_{\text{RD}} \) in such a way that the above condition is saturated and \( w \approx 1 \), we find an upper bound on \( N \):
\[
N \lesssim 77 + \frac{1}{3} \ln \left( \frac{r}{0.1} \right) - \ln \left( \frac{k}{a_0 H_0} \right) .
\]

(11)

Thus, for the maximum allowed value of the tensor-to-scalar ratio, \( r = 0.06 \), we find \( N_{\text{max}} \approx 77 \) for the largest observable scale \( k = a_0 H_0 \), in agreement with Ref. [6]. This is the maximum value of \( N \), i.e., the maximum amount of inflationary expansion of the observable Universe one can obtain within the standard assumptions discussed above. However, this result does not take into account the constraints the lack of observation of a stochastic GW background imposes on \( w \) and \( \rho_{\text{RD}} \).

Let us therefore consider gravitational waves. During a stiff epoch, the energy density parameter \( \Omega_{\text{GW}} \) of gravitational waves gets amplified as the universe expands [12, 13]. The lower the RD scale \( \rho_{\text{RD}} \), the more \( \epsilon \)-folds the stiff epoch lasts, and hence the more amplification \( \Omega_{\text{GW}} \) receives. This means \( \rho_{\text{RD}} \) is not allowed be too low since the BBN bound on the number of extra relativistic degrees of freedom imposes an upper limit on the amount of gravitational waves present during BBN.

In the presence of a stiff epoch between the end of inflation and the beginning of radiation domination, the present-day GW energy density spectrum\(^3\) \( h^2 \Omega_{\text{GW}}^{(0)}(f) \) originated from inflation is enhanced relative to that in the absence of a stiff epoch, \( h^2 \Omega_{\text{GW, plat.}}^{(0)} \approx 1 \times 10^{-16} (r/0.1) \), as [14]
\[
h^2 \Omega_{\text{GW}}^{(0)}(f) \approx C(w) h^2 \Omega_{\text{GW, plat.}}^{(0)} \left( \frac{f}{f_{\text{RD}}} \right)^{2(\frac{3w-1}{2})} ,
\]

(12)

where the subscript “plat.” refers to “plateau” (no tilt), \( f_{\text{RD}} \equiv k_{\text{RD}}/(2\pi a_0) \) is the present-day frequency of the mode \( k_{\text{RD}} \equiv a_{\text{RD}} H_{\text{RD}} \) that matches the horizon size at the onset of RD, the expression applies for \( f > f_{\text{RD}} \), and \( C(w) \) is an \( O(1) \) factor that depends on \( w \) and how abruptly the universe transitions from stiff-fluid domination to RD. We will set, without losing much accuracy, \( C(w) = 1 \) as it ranges from 1 to 1.3 (1.8) for instantaneous (smooth) transition. It should be noted that Eq. (12) does not account for the slight red spectral tilt

\(^3\) The GW energy density spectrum is defined as the GW energy density \( \rho_{\text{GW}} \) per unit logarithm of frequency normalized to the critical density \( \rho_{\text{crit}} \equiv 3M_p^2 H^2 \); \( \Omega_{\text{GW}} \equiv (d\rho_{\text{GW}}/dln f)/\rho_{\text{crit}} \).

\( n_t \equiv d \ln \mathcal{P}_t / d \ln k \) of the tensor power spectrum expected in slow-roll inflationary scenarios, which has been constrained down to \( -n_t \lesssim 0.008 \) at around the CMB pivot scale [3, 5]. In the absence of running of the spectral index, the spectral tilt can reduce the GW energy density by at most a factor of \( (e^{77})^{0.008} \approx 1.85 \), in which the end weakens our constraint on \( N \) by only \( \Delta N = O(0.1) \). We will therefore neglect this effect.

The constraint on the number of extra relativistic degrees of freedom \( \Delta N_{\text{eff}} \lesssim 0.2 \) during BBN [15] sets an upper bound on the the GW energy density today
\[
\int_{f_{\text{BBN}}}^{f_{\text{inf}}} h^2 \Omega_{\text{GW}}^{(0)}(f) d \ln f < 1 \times 10^{-6} ,
\]

(13)

where \( f_{\text{BBN}} \) and \( f_{\text{inf}} \) are the present-day frequencies of the modes that match the horizon size at BBN and the end of inflation, respectively. Evaluating the integral above with the help of Eq. (12) and \( f_{\text{inf}} \gg f_{\text{BBN}} \), we arrive at
\[
h^2 \Omega_{\text{GW}}^{(0)}(f_{\text{inf}}) \lesssim 2 \times 10^{-6} \left( \frac{3w-1}{3w+1} \right) .
\]

(14)

Next, using Eqs. [3], [8], [12], and
\[
\frac{f_{\text{inf}}}{f_{\text{RD}}} = \frac{a_{\text{inf}} H_{\text{inf}}}{a_{\text{RD}} H_{\text{RD}}} \approx \left( \frac{a_{\text{inf}}}{a_{\text{RD}}} \right)^{-(3w+1)/2} ,
\]

(15)

we can rewrite Eq. (14) as
\[
\ln \left( \frac{\rho_{\text{RD}}^{1/4}}{10^{16} \text{ GeV}} \right) \gtrsim \Theta_{\text{BBN}}(w, r) ,
\]

(16)

with
\[
\Theta_{\text{BBN}}(w, r) \equiv \frac{3(1 + w)}{4(3w - 1)} \left[ 24 + \ln \left( \frac{3w - 1}{3w + 1} \right) \right] + \frac{3w + 1}{2(3w - 1)} \ln \frac{r}{0.1} .
\]

(17)

We thus need
\[
\ln \left( \frac{\rho_{\text{RD}}^{1/4}}{10^{16} \text{ GeV}} \right) \gtrsim \max [\Theta_{\text{BBN}}(w, r), -44] ,
\]

(18)

where we have included also the previous bound from Eq. (10). If \( r \lesssim 10^{-13} \), it is always the case that \( \Theta_{\text{BBN}} \lesssim -44 \), meaning that the condition (18) reduces to (10) and the upper limit (11) on \( N \) remains applicable. On the other hand, if \( r \gtrsim 10^{-13} \), there are values of \( w \), including \( w = 1 \), for which (18) is a stricter constraint than (10). However, it turns out that \( N \) remains to be maximized at \( w = 1 \). Substituting \( w = 1 \) and the value of \( \rho_{\text{RD}} \) that saturates (18) into (9), we find
\[
N \lesssim N_{\text{BBN}} \approx 68 - \ln \left( \frac{k}{a_0 H_0} \right) ,
\]

(19)
TABLE I. Optimum frequencies and best sensitivities of current and planned gravitational wave detectors, together with their $A_{\text{det}}$ and $B_{\text{det}}$ values as defined in Eq. (24). Here we have computed the values for LIGO O2/O5 and LISA from the results presented in Ref. [13], and used the limits presented in Refs. [13] (Einstein Telescope, ET) and [14] (BBO).

| Detector | $f_{\text{det}}$ | $h^2\Omega_{GW}^{(\text{det})}$ | $A_{\text{det}}$ | $B_{\text{det}}$ |
|----------|-----------------|-------------------------------|------------------|------------------|
| LIGO O2  | 30 Hz           | $5 \times 10^{-9}$            | -15              | -18              |
| LIGO O5  | 30 Hz           | $6 \times 10^{-10}$           | -15              | -16              |
| LISA     | 3 mHz           | $2 \times 10^{-14}$           | -25              | -5               |
| ET       | 30 Hz           | $5 \times 10^{-12}$           | -15              | -11              |
| BBO      | 0.2 Hz          | $1 \times 10^{-17}$           | -20              | 2                |

where “det” denotes different GW detectors, and the values of $f_{\text{det}}$ and $\Omega_{GW}^{(\text{det})}$ are listed in Table I for different experiments. Then, using the conversion [13]

$$f_{\text{RD}} \text{ Hz} \approx 1.5 \times 10^8 \left(\frac{\rho_{\text{RD}}}{10^{16} \text{ GeV}}\right)^{1/4},$$

and assuming $f_{\text{RD}} < f_{\text{det}}$ and $h^2\Omega_{GW, \text{plat}}^{(0)} < h^2\Omega_{GW}^{(\text{det})}$, we can rewrite Eq. (20) as

$$\ln \left(\frac{\rho_{\text{RD}}}{10^{16} \text{ GeV}}\right)^{1/4} \gtrsim \Theta_{\text{det}}(w, r)$$

with

$$\Theta_{\text{det}}(w, r) \equiv A_{\text{det}} + \frac{1}{2} \left(\frac{3w + 1}{3w - 1}\right) B_{\text{det}} + \ln \left(\frac{r}{0.1}\right)$$

and

$$A_{\text{det}} \equiv \ln \left(\frac{f_{\text{det}}}{1.5 \times 10^8 \text{ Hz}}\right), \quad B_{\text{det}} \equiv -\ln \left(\frac{h^2\Omega_{GW}^{(\text{det})}}{10^{-16}}\right).$$

The values of $A_{\text{det}}$, $B_{\text{det}}$ are listed in Table I for different experiments. However, the bound [22] does not change our result [1] because for $w = 1$ it is always the case that

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4. We note that there is some variation in the numbers found in the literature. However, as long as the resulting constraints from these detectors are less stringent than the BBN bound, their exact values are not important for our purposes. This is clearly true for all the detectors listed in Table 1, apart from (the most optimistic version of) BBO for which the difference is marginal.

5. If $0.01 \lesssim r \lesssim 0.06$, then Eq. (24) does not apply for BBO because then $h^2\Omega_{GW, \text{plat}}^{(0)} > h^2\Omega_{GW}^{(\text{det})}$. Instead, BBO would simply rule out the aforementioned range of $r$. This would not affect the upper bound on $N$, Eq. (19), from the BBN bound since it is independent of $r$. 

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Independently of the value of $r$. This is a new bound and our most important result. As can be seen in Figure 1 for the maximum allowed value of tensor-to-scalar ratio $r \approx 0.1$, the new bound is more stringent by $\Delta N \approx -9$, which corresponds to $\Delta(a_{\text{inf}}/a_{k=a_0H_0}) = 1 \times 10^{-4}$, i.e. the new bound places a constraint on the amount of inflationary expansion which is roughly four orders of magnitude more stringent than the previous bound.

Let us then discuss what are the prospects of future experiments for making the bound even more stringent. First, if the number of the extra relativistic degrees of freedom $N_{\text{eff}}$ was measured to an accuracy better than the current bound $\Delta N_{\text{eff}} \lesssim 0.2$, the maximum value [19] would go down by $1/4 \ln (0.2/\Delta N_{\text{eff}})$. For instance, an order of magnitude improvement in the $\Delta N_{\text{eff}}$ upper bound could be obtained from the constraints on the Hubble rate at the CMB decoupling [16, 17], nevertheless this would only lower the maximum $N$ by roughly one e-fold.

On the other hand, the lack of detection of a stochastic GW background places a constraint on the ($\rho_{\text{RD}}, w, r$) parameter space which can be written in terms of a lower bound on $\rho_{\text{RD}}$ as follows. Judging from the steepness of the sensitivity curves of the current or future GW experiments such as LIGO or LISA compared to that of the $h^2\Omega_{GW}^{(0)}(f)$ curve, as we vary the parameters ($\rho_{\text{RD}}, w, r$), the GW energy spectrum $h^2\Omega_{GW}^{(0)}(f)$ first intersects with the detector sensitivity curves close to the frequencies $f_{\text{det}}$ where the detectors are at their best sensitivities, $h^2\Omega_{GW}^{(\text{det})}$. Hence, if a detector fails to detect a primordial GW background, the following approximate constraint can be placed [14]

$$h^2\Omega_{GW}^{(0)}(f_{\text{det}}) \lesssim h^2\Omega_{GW}^{(\text{det})},$$

FIG. 1. Maximum allowed $N$ for $k = a_0H_0$ before and after imposing the BBN bound, Eq. (13). The shaded region has been ruled out by the non-observation of primordial B-mode polarization on the CMB [5].
$\Theta_{\text{BBN}} > \Theta_{\text{det}}$, as one can check by substituting the values in Table I into Eq. (23) and comparing with Eq. (17).

Finally, let us discuss bounds on other observables. The new bound (19) is only valid for $r \gtrsim 10^{-13}$, and if $r$ was smaller than this, the result (11) remains as a valid upper limit. However, as the next generation CMB B-mode polarization experiments such as BICEP3 [20], LiteBIRD [21], and the Simons Observatory [22] aim at detecting or constraining $r$ only at the level $O(10^{-3})$, it seems unlikely that the constraint on $r$ could be improved by more than 10 orders of magnitude in any foreseeable future. Thus, we conclude that the limit (19) is a robust upper limit on the amount of inflationary expansion of the observable Universe.

To summarize, we have used constraints on the number of relativistic degrees of freedom during BBN to derive a new, robust upper limit on the amount of expansion of the observable Universe during inflation, Eq. (19). By comparing this result to the previous bound, Eq. (11), one can see that for the maximum allowed value of $r$, the new bound is more stringent by $\Delta N \approx -9$, which corresponds to $\Delta (\alpha_{\text{end}}/a_{k=0} H_0) = 1 \times 10^{-4}$. The bound on the energy density ofgravitational waves at the time of BBN therefore places a constraint on the amount of inflationary expansion which is roughly four orders of magnitude more stringent than any previous bound.

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