LOOKING FOR $Z'$ BOSONS IN SUPERSYMMETRIC $E_6$ MODELS THROUGH ELECTROWEAK PRECISION DATA

GI-CHOL CHO
Scuola Normale Superiore,
Piazza dei Cavalieri 7, Pisa 56126, Italy

We review constraints on additional $Z'$ bosons predicted in supersymmetric (SUSY) $E_6$ models from electroweak experiments – $Z$-pole experiments, $m_W$ measurements and the low-energy neutral current (LENC) experiments. Four representative models – $\chi, \psi, \eta, \nu$ models – are studied in some detail. We find that the improved data of parity violation in cesium atom, which is 2.2-$\sigma$ away from the Standard Model (SM) prediction, could be explained by the exchange of the heavy mass eigenstate $Z_2$ in the intermediate state. The improvement over the SM can be found in $\chi, \eta, \nu$ models, where the total $\chi^2$ of the fit to the 26 data points decreases by about five units, owing to the better fit to the atomic parity violation. Impacts of the kinetic mixing between the $U(1)_Y$ and $U(1)'$ gauge bosons on the $\chi^2$-analysis are studied. We find that the $Z'$ model with $(\beta_E, \delta) = (-\pi/4, 0.2)$, where $\beta_E$ is the mixing angle between $Z_\chi$ and $Z_\psi$ bosons and $\delta$ denotes the kinetic mixing, shows the most excellent fit to the data: the total $\chi^2$ decreases by about seven units as compared to the SM. We introduce the effective mixing parameter $\zeta$, a combination of the mass and the kinetic mixing parameters. The 95% CL lower mass bound of $Z_2$ can be shown as a function of $\zeta$. A theoretical prediction on $\zeta$ and the $U(1)'$ gauge coupling $g_{E'}$ is studied for the $\chi, \psi, \eta$ and $\nu$ models by assuming the minimal particle content of the SUSY $E_6$ models.

1. Introduction

The presence of an additional $Z'$ boson is predicted in a certain class of grand unified theories (GUT) with a gauge group whose rank is higher than that of the Standard Model (SM). The supersymmetric (SUSY) $E_6$ models are the promising candidates which predict the additional $Z'$-boson at the weak scale. Because $E_6$ is a rank-six group, it can have two extra $U(1)$ factors besides the SM gauge group. A superposition of the two extra $U(1)$ groups may survive as the $U(1)'$ gauge symmetry at the GUT scale. The $U(1)'$ symmetry may break spontaneously at the weak scale through the radiative corrections to the mass term of the SM singlet scalar field.

In general, the additional $U(1)'$ gauge boson $Z'$ can mix with the hypercharge $U(1)_Y$ gauge boson through the kinetic term at above the electroweak scale, and also it can mix with the SM $Z$ boson after the electroweak symmetry is spontaneously broken. Through those mixings, the $Z'$ boson can affect the electroweak observables at the $Z$-pole and the $W$-boson mass $m_W$. Both the $Z-Z'$ mixing and the direct $Z'$ contribution can affect neutral current experiments off the $Z$ pole. The presence of...
an additional $Z'$ boson can be explored directly at $p\bar{p}$ collider experiments.

In this review article, we report constraints on $Z'$ bosons in the SUSY $E_6$ models from electroweak experiments based on the formalism in Refs. 3, 4. Constraints on the $Z'$ bosons from electroweak experiments have been studied by several authors 5, 6, 7, 3, 8. Especially, a special attention has been paid to this subject 10 after the new analysis of parity violation in cesium atom has led to the improved data of the weak charge $Q_W(\frac{131}{55}\text{Cs})$ 9, which is 2.2-$\sigma$ away from the SM prediction (see Table 2). The analysis given in this article updates their studies by allowing for an arbitrary kinetic mixing 11, 12, 13 between the $Z'$ boson and the hypercharge $B$ boson. The constraints on the $Z'$ bosons can be found by using the results of $Z$-pole experiments at LEP1 and SLC, and the $m_W$ measurements at Tevatron and LEP2. Also the low-energy neutral current (LENC) experiments – lepton-quark, lepton-lepton scattering experiments and atomic parity violation (APV) measurements – constrain the direct exchange of $Z'$ boson.

It has been found 3, 4 that the lower mass limit of the heavier mass eigenstate $Z_2$ is obtained as a function of the effective $Z$-$Z'$ mixing term $\zeta$, which is a combination of the mass and kinetic mixings. In principle, $\zeta$ is calculable, together with the gauge coupling $g_E$, once the particle spectrum of the $E_6$ model is specified. We show the theoretical prediction for $\zeta$ and $g_E$ in the SUSY $E_6$ models by assuming the minimal particle content which satisfies the anomaly free condition and the gauge coupling unification a.

This paper is organized as follows. In the next section, we review the additional $Z'$ boson in the SUSY $E_6$ models and the generic feature of $Z$-$Z'$ mixing in order to fix our notation. We show that the effects of $Z$-$Z'$ mixing and direct $Z'$ boson contribution are parametrized by the following three terms: (i) a tree-level contribution to the $T$ parameter 15, $T_{new}$, (ii) the effective $Z$-$Z'$ mass mixing angle $\bar{\xi}$ and (iii) a contact term $g_E^2/c_\chi^2m_{Z_2}^2$ which appears in the low-energy processes. In Sec. 3, we collect the data of electroweak experiments which will be used in our analysis. We also present the theoretical framework to calculate the electroweak observables. In Sec. 4, we show constraints on the $Z'$ bosons from the electroweak data. The presence of non-zero kinetic mixing between the $U(1)_Y$ and $U(1)_'$ gauge bosons modifies the couplings between the $Z'$ boson and the SM fermions. We discuss impacts of the kinetic mixing term on the $\chi^2$-analysis. The 95% CL lower mass limit of the heavier mass eigenstate $Z_2$ in four representative models – $\chi, \psi, \eta, \nu$ models – is given as a function of the effective $Z$-$Z'$ mixing parameter $\zeta$. The $\zeta$-independent constraints from the low-energy experiments and those from the direct search experiments at Tevatron are also discussed. In Sec. 5, we find the theoretical prediction for $\zeta$ in $\chi, \psi, \eta, \nu$ models by assuming the minimal particle contents. Stringent $Z_2$ boson mass bounds are found for most models. Sec. 6 is devoted to summarize this paper.

2. $Z$-$Z'$ mixing in supersymmetric $E_6$ model

a Consequence in the case of the maximal particle content which preserve the perturbative unification of the gauge couplings has been studied in Ref. 14.
2.1. $Z'$ boson in supersymmetric $E_6$ model

Since the rank of $E_6$ is six, it has two U(1) factors besides the SM gauge group which arise from the following decompositions:

$$E_6 \supseteq \text{SO}(10) \times \text{U}(1)_\psi$$
$$\supseteq \text{SU}(5) \times \text{U}(1)_\chi \times \text{U}(1)_\psi. \quad (1)$$

An additional $Z'$ boson in the electroweak scale can be parametrized as a linear combination of the U(1)$_\psi$ gauge boson $Z_\psi$ and the U(1)$_\chi$ gauge boson $Z_\chi$ as

$$Z' = Z_\chi \cos \beta_E + Z_\psi \sin \beta_E. \quad (2)$$

In this paper, the following $Z'$ models are studied in some detail:

| Model | $\beta_E$ | $\chi$ | $\psi$ | $\eta$ | $\nu$ |
|-------|-----------|--------|--------|-------|-------|
| $\nu$ | 0         | $\pi/2$| $\tan^{-1}(-\sqrt{5}/3)$| $\tan^{-1}(\sqrt{15})$| $\tan^{-1}(\sqrt{5}/3)$| $\tan^{-1}(\sqrt{15})$| $\tan^{-1}(-\sqrt{5}/3)$| $\tan^{-1}(\sqrt{15})$|

In the SUSY-$E_6$ models, each generation of the SM quarks and leptons is embedded into a 27 representation. In Table 1, we show all the matter fields contained in a 27 and their classification in SO(10) and SU(5). The U(1)' charge assignment on the matter fields for each model is also given in the same table. The normalization of the U(1)' charge follows that of the hypercharge. Besides the SM quarks and leptons, there are two SM singlets $\nu^c$ and $S$, a pair of weak doublets $H_u$ and $H_d$, a pair of color triplets $D$ and $\overline{D}$ in each generation. The $\eta$ model arises when $E_6$ breaks into a rank-5 group directly in a specific compactification of the heterotic string theory $\xi$. In the $\nu$ model, the right-handed neutrinos $\nu^c$ are gauge singlet $\xi$ and can have large Majorana masses to realize the see-saw mechanism $\xi$.

The U(1)' symmetry breaking occurs if the scalar component of the SM singlet field develops the vacuum expectation value (VEV). It can be achieved at near the weak scale via radiative corrections to the mass term of the SM singlet scalar field. Recent studies of the radiative U(1)' symmetry breaking can be found, e.g., in Ref. $\xi$. $\xi$. $\xi$.$\xi$

Several problems may arise in the $E_6$ models from view of low-energy phenomenology. For example, the presence of the baryon number violating operators give rise to too fast proton decay, or the absence of the Majorana neutrino mass terms (except for the $\nu$ model) requires a fine-tuning of the Dirac neutrino mass in order to satisfy experimentally observed neutrino mass relations. Some approaches to these problems are summarized in Ref. $\xi$. $\xi$. In the following, we assume that these problems are solved by an unknown mechanism. Moreover we assume that all the super-partners of the SM particles and the exotic matters do not affect the radiative corrections to the electroweak observables significantly, i.e., they are assumed to be heavy enough to decouple from the weak boson mass scale.

2.2. Phenomenological consequences of $Z$-$Z'$ mixing
Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data

Table 1. The hypercharge $Y$ and the U(1)' charge $Q_E$ of all the matter fields in a $27$ for the $\chi, \psi, \eta$ and $\nu$ models. The classification of the fields in the SO(10) and the SU(5) groups is also shown. The value of U(1)' charge follows the hypercharge normalization.

| SO(10) | SU(5) | field | $Y$ | $\sqrt{6}Q_\chi$ | $\sqrt{72/5}Q_\psi$ | $Q_\eta$ | $Q_\nu$ |
|--------|-------|-------|-----|-----------------|-------------------|---------|---------|
| 16     | 10    | $Q$   | $+\frac{1}{6}$ | $-1$ | $+1$ | $-\frac{1}{3}$ | $+\sqrt{\frac{1}{24}}$ |
|        |       | $u^c$ | $-\frac{2}{3}$ | $-1$ | $+1$ | $-\frac{1}{3}$ | $+\sqrt{\frac{1}{24}}$ |
|        |       | $e^c$ | $+1$ | $-1$ | $+1$ | $-\frac{1}{3}$ | $+\sqrt{\frac{1}{24}}$ |
| 5      |       | $L$   | $-\frac{1}{2}$ | $+3$ | $+1$ | $+\frac{1}{6}$ | $+\sqrt{\frac{1}{6}}$ |
|        |       | $d^c$ | $+\frac{1}{3}$ | $+3$ | $+1$ | $+\frac{1}{6}$ | $+\sqrt{\frac{1}{6}}$ |
| 1      |       | $\nu^c$ | 0 | $-5$ | $+1$ | $-\frac{5}{6}$ | 0 |
| 10     | 5     | $H_u$ | $+\frac{1}{2}$ | $+2$ | $-2$ | $+\frac{2}{3}$ | $-\sqrt{\frac{1}{6}}$ |
|        |       | $D$   | $-\frac{1}{3}$ | $+2$ | $-2$ | $+\frac{2}{3}$ | $-\sqrt{\frac{1}{6}}$ |
| 5      |       | $H_d$ | $-\frac{1}{2}$ | $-2$ | $-2$ | $+\frac{1}{6}$ | $-\sqrt{\frac{3}{8}}$ |
|        |       | $\mathbf{D}$ | $+\frac{1}{3}$ | $-2$ | $-2$ | $+\frac{1}{6}$ | $-\sqrt{\frac{3}{8}}$ |
| 1      | 1     | $S$   | 0 | 0 | 4 | $-\frac{5}{6}$ | $\sqrt{\frac{25}{24}}$ |

If the SM Higgs field carries a non-trivial U(1)' charge, its VEV induces the $Z-Z'$ mass mixing. On the other hand, the kinetic mixing between the hypercharge gauge boson $B$ and the U(1)' gauge boson $Z'$ can occur through the quantum effects below the GUT scale. After the electroweak symmetry is broken, the effective Lagrangian for the neutral gauge bosons in the SU(2)$_L \times$ U(1)$_Y \times$ U(1)' theory is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} Z^{\mu \nu} Z_{\mu \nu} - \frac{1}{4} Z^{\mu \nu} Z'_{\mu \nu} - \frac{\sin \chi}{2} B^{\mu \nu} Z'_{\mu \nu} - \frac{1}{4} A^{0 \mu \nu} A^0_{\mu \nu}$$
$$+ m_{Z'}^2 Z^{\mu \nu} Z'_{\mu \nu} + \frac{1}{2} m_{Z'}^2 Z^{\mu} Z_{\mu} + \frac{1}{2} m_{Z'}^2 Z'^{\mu} Z'_\mu,$$

(4)

where $F^{\mu \nu} (F = Z, Z', A^0, B)$ represents the gauge field strength. The $Z-Z'$ mass mixing and the kinetic mixing are characterized by $m_{Z'}^2$ and $\sin \chi$, respectively. In this basis, the interaction Lagrangian for the neutral current process is given as

$$\mathcal{L}_{\text{NC}} = - \sum_{f, \alpha} \left( e Q_f Q_{f, \alpha} \overline{f}_{\alpha} A^0_{\mu} f_{\alpha} + g Z \overline{f}_{\alpha} A^0_{\mu} (I^3_f - Q_{f, \alpha} \sin^2 \theta_W) f_{\alpha} Z_{\mu} \right)$$
$$+ g_E Q_{E, \alpha} f_{\alpha} Z'_{\mu} \right),$$

(5)
where \( g_Z = g / \cos \theta_W = g_Y / \sin \theta_W \). The U(1)\(^{'}\) gauge coupling constant is denoted by \( g_E \) in the hypercharge normalization. The symbol \( f_\alpha \) denotes the quarks or leptons with the chirality \( \alpha \) (\( \alpha = L \) or \( R \)). The third component of the weak isospin, the electric charge and the U(1)\(^{'}\) charge of \( f_\alpha \) are given by \( I_{f_\alpha}^3, Q_{f_\alpha} \) and \( Q_{f_\alpha}^{E'}, \) respectively. The U(1)\(^{'}\) charge of the quarks and leptons listed in Table 1 should be read as

\[
Q_{f}^{E'} = Q_{f}^{L'} = Q_{f}^{U'}, \quad Q_{E}^{L'} = Q_{E}^{U'}, \quad Q_{E}^{c'} = -Q_{E}^{f_\alpha} \quad (f = e, u, d). \tag{6}
\]

The mass eigenstates \((Z_1, Z_2, A)\) is obtained by the following transformation:

\[
\begin{pmatrix}
Z \\
Z' \\
A^0
\end{pmatrix}
= \begin{pmatrix}
\cos \xi & \sin \xi \sin \theta_W \tan \chi & -\sin \xi \sin \theta_W \tan \chi \\
-\sin \xi / \cos \chi & \cos \xi / \cos \chi & 0 \\
-\sin \xi \cos \theta_W \tan \chi & -\cos \xi \cos \theta_W \tan \chi & 1
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_2 \\
A
\end{pmatrix} \tag{7}
\]

Here the mixing angle \( \xi \) is given by

\[
\tan 2\xi = \frac{-2\cos \xi (m_Z^2 + s_W s_\chi m_Z^2)}{m_Z^2 - (c_W^2 - s_W^2) s_\chi^2 m_Z^2 + 2s_W s_\chi m_Z^2}, \tag{8}
\]

with the short-hand notation, \( c_\chi = \cos \chi, \ s_\chi = \sin \chi \) and \( s_W = \sin \theta_W \). The physical masses \( m_{Z_1} \) and \( m_{Z_2} \) (\( m_{Z_1} < m_{Z_2} \)) are given as follows:

\[
m_{Z_1}^2 = m_Z^2 (c_\chi + s_W s_\chi t_\chi)^2 + m_{Z_2}^2 \left( \frac{s_\chi}{c_\chi} \right)^2 + 2m_{Z_2}^2 \frac{s_\chi}{c_\chi} (c_\chi + s_W s_\chi t_\chi), \tag{9a}
\]

\[
m_{Z_2}^2 = m_Z^2 (c_\chi s_W t_\chi - s_\chi)^2 + m_{Z_2}^2 \left( \frac{c_\chi}{s_\chi} \right)^2 + 2m_{Z_2}^2 \frac{c_\chi}{s_\chi} (c_\chi s_W t_\chi - s_\chi), \tag{9b}
\]

where \( c_\xi = \cos \xi, s_\xi = \sin \xi \) and \( t_\chi = \tan \chi \). The lighter mass eigenstate \( Z_1 \) should be identified with the observed \( Z \) boson at LEP1 or SLC. The excellent agreement between the current experimental results and the SM predictions at the quantum level implies that the mixing angle \( \xi \) has to be small. In the limit of small \( \xi \), the interaction Lagrangians for the processes \( Z_{1,2} \to f_\alpha f_\alpha \) are expressed as

\[
\mathcal{L}_{Z_1} = -\sum_{f, \alpha} g_Z f_\alpha \gamma^\mu \left[ (I_{f_\alpha}^3 - Q_{f_\alpha} \sin^2 \theta_W) + Q_{f_\alpha}^{L'} \xi \right] f_\alpha Z_{1\mu}, \tag{10a}
\]

\[
\mathcal{L}_{Z_2} = -\sum_{f, \alpha} g_E f_\alpha \gamma^\mu \left[ Q_{f_\alpha}^{E'} - (I_{f_\alpha}^3 - Q_{f_\alpha} \sin^2 \theta_W) \frac{g_Z s_\chi}{g_E} \xi \right] f_\alpha Z_{2\mu}, \tag{10b}
\]

where the effective mixing angle \( \tilde{\xi} \) in Eq. \((10a)\) is given as

\[
\tilde{\xi} = \frac{g_E}{g_Z \cos \chi} \xi. \tag{11}
\]

In Eq. \((10)\), the effective U(1)\(^{'}\) charge \( \tilde{Q}_{f_\alpha}^{E'} \) is introduced as a combination of \( Q_{f_\alpha}^{E'} \) and the hypercharge \( Y_{f_\alpha} \):

\[
\tilde{Q}_{f_\alpha}^{E'} = Q_{f_\alpha}^{E'} + Y_{f_\alpha} \delta, \tag{12a}
\]

\[
\delta = -\frac{g_Z}{g_E} s_W s_\chi, \tag{12b}
\]
where the hypercharge $Y_{f\alpha}$ should be read from Table 1 in the same manner with $Q_{E_6}^{f\alpha}$ (see, Eq. (12a)). As a notable example, one can see from Table 1 that the effective charge $\tilde{Q}_{E_6}^{f\alpha}$ of the leptons ($L$ and $e^c$) disappears in the $\eta$ model if $\delta$ is taken to be $1/3$ [4].

Now, due to the $Z-Z'$ mixing, the observed $Z$ boson mass $m_{Z_1}$ at LEP1 or SLC is shifted from the SM $Z$ boson mass $m_{Z}$:

$$\Delta m^2 \equiv m_{Z_1}^2 - m_{Z}^2 \leq 0.$$  

(13)

The presence of the mass shift affects the $T$-parameter [15] at tree level. Following the notation of Ref. [20], the $T$-parameter is expressed in terms of the effective form factors $\bar{g}_Z^2(0), \bar{g}_W^2(0)$ and the fine structure constant $\alpha$:

$$\alpha T \equiv 1 - \frac{\bar{g}_W^2(0)}{m_W^2} \frac{m_{Z_1}^2}{\bar{g}_Z^2(0)},$$

(14a)

$$\alpha T_{SM} = 1 - \frac{\bar{g}_W^2(0)}{m_W^2} \frac{m_{Z_1}^2}{\bar{g}_Z^2(0)},$$

(15a)

$$\alpha T_{new} = -\frac{\Delta m^2}{m_{Z_1}^2} \geq 0.$$  

(15b)

It is worth noting that the sign of $T_{new}$ is always positive. The effects of the $Z-Z'$ mixing in the $Z$-pole experiments have hence been parametrized by the effective mixing angle $\xi$ and the positive parameter $T_{new}$.

We note here that we retain the kinetic mixing term $\delta$ as a part of the effective $Z_1$ coupling $\tilde{Q}_{E_6}^{f\alpha}$ in Eq. (12a). As shown in Refs. [12,13,21], the kinetic mixing term $\delta$ can be absorbed into a further redefinition of $S$ and $T$. Such re-parametrization may be useful if the term $Y_{f\alpha}\delta$ in Eq. (12a) is much larger than the $Z'$ charge $Q_{E_6}^{f\alpha}$. In the $E_6$ models studied in this paper, we find no merit in absorbing the $Y_{f}\delta$ term because, the remaining $Q_{E_6}^{f\alpha}$ term is always significant. We therefore adopt $\tilde{Q}_{E_6}^{f\alpha}$ as the effective $Z_1$ couplings and $T_{new}$ accounts only for the mass shift (13). All physical consequences such as the bounds on $\xi$ and $m_{Z_2}$ are of course independent of our choice of the parametrization.

The two parameters $T_{new}$ and $\xi$ are complicated functions of the parameters of the effective Lagrangian [4]. In the small mixing limit, we find the following useful expressions

$$\xi = -\left(\frac{g_E}{g_Z} \frac{m_Z}{m_{Z'}}\right)^2 \bar{\xi} \left[1 + O\left(\frac{m_{Z_1}^2}{m_{Z'}^2}\right)\right],$$

(16a)

$$\alpha T_{new} = \alpha \left(\frac{g_E}{g_Z} \frac{m_Z}{m_{Z'}}\right)^2 \bar{\xi}^2 \left[1 + O\left(\frac{m_{Z_1}^2}{m_{Z'}^2}\right)\right],$$

(16b)
where we introduced an effective mixing parameter $\zeta$

$$\zeta = \frac{g_Z}{g_E} \frac{m_{Z'}^2}{m_Z^2} - \delta. \quad (17)$$

The $Z$-$Z'$ mixing effect disappears at $\zeta = 0$. Stringent limits on $m_{Z'}$ and hence on $m_{Z_2}$ can be obtained through the mixing effect if $\zeta$ is $O(1)$. We will show in Sec. 5 that $\zeta$ is calculable once the particle spectrum of the model is specified. The parameter $\zeta$ plays an essential role in the analysis of $Z'$ models.

In the low-energy neutral current processes, effects of the exchange of the heavier mass eigenstate $Z_2$ can be detected. In the small $\bar{\xi}$ limit, they constrain the contact term $g_E^2/c^2 Z_2^2$.

3. Electroweak observables in the $Z'$ model

In this section, we briefly discuss the theoretical framework \[3\] to calculate the electroweak observables which are used in our analysis. The experimental data of the $Z$-pole experiments, the $W$-boson mass measurement and the low-energy experiments used in this paper are summarized in Table 2.

The pseudo-observables of the $Z$-pole experiments are expressed in terms of the effective coupling $g_f^{\alpha}$, where $f$ denotes all the SM fermions except for the top-quark, and $\alpha$ being their chirality, $L$ or $R$. Following our parametrization of the $Z$-$Z'$ mixing \[10a\], the effective coupling $g_f^{\alpha}$ in the $Z'$ models can be expressed as

$$g_f^{\alpha} = (g_f^{\alpha})_{SM} + \tilde{Q}_f^{\alpha} \bar{\xi}. \quad (18)$$

The SM prediction for the effective coupling $(g_f^{\alpha})_{SM}$ can be expanded in terms of the gauge boson propagator corrections $\Delta g_Z^2$ and $\Delta s^2$:

$$(g_f^{\alpha})_{SM} = a + b \Delta g_Z^2 + c \Delta s^2, \quad (19)$$

where the numerical coefficients $a$, $b$ and $c$ are given in Refs. \[3\]. Two parameters $\Delta g_Z^2$ and $\Delta s^2$ in Eq. (19) are defined as the shift in the effective couplings $g_Z^2(m_{Z_2}^2)$ and $s^2(m_{Z_2}^2)$ from their SM reference values at $m_t = 175$ GeV and $m_H = 100$ GeV. They can be expressed in terms of the $S$ and $T$ parameters \[3\] as

$$\Delta g_Z^2 = g_Z^2(m_{Z_2}^2) - 0.55635 = 0.00412 \Delta T + 0.00005[1 - (100 \text{ GeV}/m_H)^2], \quad (20a)$$

$$\Delta s^2 = s^2(m_{Z_2}^2) - 0.23035 = 0.00360 \Delta S - 0.00241 \Delta T - 0.00023 x_\alpha, \quad (20b)$$

where the expansion parameter $x_\alpha$ is introduced to estimate the uncertainty of the hadronic contribution to the QED coupling $1/\alpha(m_{Z_2}^2) = 128.75 \pm 0.09$ \[3\].

$$x_\alpha \equiv \frac{1/\alpha(m_{Z_2}^2) - 128.75}{0.09}. \quad (21)$$

Here, $\Delta S$, $\Delta T$, $\Delta U$ parameters are also measured from their SM reference values and they are given as the sum of the SM and the new physics contributions

$$\Delta S = \Delta S_{SM} + S_{\text{new}}, \quad \Delta T = \Delta T_{SM} + T_{\text{new}}, \quad \Delta U = \Delta U_{SM} + U_{\text{new}}. \quad (22)$$
Table 2. Electroweak measurements at LEP, SLC, Tevatron and LENC experiments. The average W-boson mass is found in Ref. [2]. Except for the weak charge of cesium atom the data of LENC experiments given in this table, which have been reduced from the original data, is summarized in Refs. [3]. The pull factors are given at the best fit point of the SM and four $Z'$ models at $m_H = 100$ GeV and a constraint $T_{new} \geq 0$. The best fit values of parameters in both the SM and $Z'$ models are shown in Table [4]. Correlation matrix elements of the $Z$ line-shape parameters and those for the heavy-quark parameters are found in Ref. [4]. The data $R_c$ and $A_{FB}^{\nu,\ell}$ are obtained by assuming the $\epsilon, \mu, \tau$ universality.

| Parameters | data | SM | $\chi$ | $\psi$ | $\eta$ | $\nu$ |
|------------|------|----|--------|--------|--------|--------|
| $m_Z$ (GeV) | 91.1867 ± 0.0021 | | | | | |
| $\Gamma_Z$ (GeV) | 2.4939 ± 0.0024 | | | | | |
| $\sigma^0_R$ (nb) | 41.491 ± 0.058 | | | | | |
| $R_t$ | 20.765 ± 0.026 | | | | | |
| $A_{FB}^{\nu,\ell}$ | 0.01683 ± 0.00096 | | | | | |
| $A_\tau$ | 0.1431 ± 0.0045 | | | | | |
| $A_e$ | 0.1479 ± 0.0051 | | | | | |
| $R_0$ | 0.21656 ± 0.00074 | | | | | |
| $R_c$ | 0.1735 ± 0.0044 | | | | | |
| $A_{FB}^{0,\ell}$ | 0.0990 ± 0.0021 | | | | | |
| $A_{FB}^{\nu,\ell}$ | 0.0709 ± 0.0044 | | | | | |
| $A_{FB}^{0,\ell}$ | 0.1510 ± 0.0025 | | | | | |
| $A_{FB}^{\nu,\ell}$ | 0.867 ± 0.035 | | | | | |
| $A_e$ | 0.647 ± 0.040 | | | | | |

| $m_{W'}$ (GeV) | 80.410 ± 0.044 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |

| LENC exp. | | | | | | |
|-----------|----|----|----|----|----|----|
| $A_{SLAC}$ | 0.80 ± 0.058 | 1.0 | 1.0 | 1.0 | 1.1 | 1.0 |
| $A_{CERN}$ | −1.57 ± 0.38 | −0.4 | −0.4 | −0.4 | −0.4 | −0.4 |
| $A_{Bates}$ | −0.137 ± 0.033 | 0.5 | 0.4 | 0.5 | 0.4 | 0.4 |
| $A_{Main}$ | −0.94 ± 0.19 | −0.3 | −0.3 | −0.3 | −0.4 | −0.3 |
| $Q_{W'^{0,\ell}}(C_{S_f})$ | −72.06 ± 0.44 | 2.2 | −0.1 | 2.2 | 0.1 | 0.0 |
| $K_{FH}$ | 0.3247 ± 0.0040 | −1.5 | −1.4 | −1.5 | −1.5 | −1.4 |
| $K_{CCFR}$ | 0.5820 ± 0.0049 | −0.5 | −0.4 | −0.4 | −0.5 | −0.4 |
| $g_{\chi^{\nu,\ell}}$ | −0.269 ± 0.011 | 0.4 | 0.1 | 0.1 | 0.4 | 0.1 |
| $g_{\chi^{\nu,\ell}}$ | 0.234 ± 0.011 | 0.1 | 0.0 | 0.4 | 0.2 | 0.2 |
| $\chi^2_{min}$ | 23.8 | 18.3 | 23.5 | 19.2 | 18.4 |
A convenient parametrization of $\Delta S_{\text{SM}}, \Delta T_{\text{SM}}$ and $\Delta U_{\text{SM}}$ in terms of $m_t$ and $m_H$ has been given in Ref. 32. The formulae of the $Z$-pole observables listed in Table 2 in terms of $g^f_{\nu}$ can be found in Refs. 33, 34.

The theoretical prediction of $m_W$ is given as

$$m_W (\text{GeV}) = 80.402 - 0.288 \Delta S + 0.418 \Delta T + 0.337 \Delta U + 0.012 x_\alpha,$$

by using the same parameters, $\Delta S, \Delta T, \Delta U$ (22) and $x_\alpha$ (21).

The observables in the LENC experiments which are used in our analysis are as follows – (i) polarization asymmetry of the charged lepton scattering off nucleus target, (ii) parity violation in cesium atom, (iii) inelastic $\nu_\mu$-scattering off nucleus target and (iv) neutrino-electron scattering. Theoretical expressions for the observables of (i) and (ii) are conveniently given in terms of the model-independent parameters $C_{1q}, C_{2q}$ and $C_{3q}$. The $\nu_\mu$-scattering data (iii) and (iv) are expressed in terms of the parameters $g^\nu_f_{\nu}$. In the $Z'$ models, these model-independent parameters can be written as follows:

$$C_{iq} = (C_{iq})_{\text{SM}} + \Delta C_{iq},$$

$$g^{\nu_f}_{\nu} = (g^{\nu_f}_{\nu})_{\text{SM}} + \Delta g^{\nu_f}_{\nu},$$

where the first term in each equation is the SM contribution which is parametrized conveniently by $\Delta S$ and $\Delta T$. The second terms $\Delta C_{iq}$ and $\Delta g^{\nu_f}_{\nu}$ represent the additional contributions from the $Z-Z'$ mixing and the direct $Z_2$ exchange, which are proportional to $\xi$ and $g^2_E/c^2_\xi m^2_{Z_2}$, respectively. The theoretical prediction of the LENC observables in terms of $\Delta C_{iq}$ and $\Delta g^{\nu_f}_{\nu}$ can be found in Ref. 33.

### 4. Constraints on $Z'$ bosons from electroweak experiments

#### 4.1. $\chi^2$-analysis on the $Z'$ models

There are six free parameters in the $Z'$ models – the tree level contribution to the $T$ parameter $T_{\text{new}}$, the $Z-Z'$ mass mixing angle $\xi$, the direct $Z_2$-boson contribution to the low-energy processes $g^2_E/c^2_\xi m^2_{Z_2}$, and the three SM parameters, $m_t, \alpha_s(m_{Z_1})$ and $1/\alpha_s(m_{Z_1})$. Throughout our analysis, we use $m_t = 173.8 \pm 5.2$ (GeV)[34], $\alpha_s(m_{Z_1}) = 0.119 \pm 0.002$[35], and $1/\alpha_s(m_{Z_1}) = 128.75 \pm 0.09$ as constraints on the SM parameters. The Higgs mass dependence of the results are parametrized by $x_H \equiv \ln(m_H/100 \text{ GeV})$ in the range $90 \text{ GeV} < m_H < 150 \text{ GeV}$. The lower bound is obtained at the LEP2 experiment.[36] The upper bound is the theoretical limit on the lightest Higgs boson mass in any supersymmetric models that accommodate perturbative unification of the gauge couplings.[37]

We summarize the results of the fit for the $\psi, \chi, \eta$ and $\nu$ models:

$$\begin{align*}
(1) \ & \chi \text{ model } (\delta = 0) \\
& T_{\text{new}} = -0.059 + 0.14 x_H \pm 0.098 \\
& \xi(10^{-4}) = 0.02 - 9 x_H \pm 4.05 \\
& g^2_E/c^2_\xi m^2_{Z_2} = 0.237 + 0.0032 x_H \pm 0.107
\end{align*}$$

$$\chi^2_{\text{min}} = 17.8 + 1.2 x_H,$$
Table 3. Summary of the best fit values of the parameters in the SM and four Z’ models (δ = 0) for m_H = 100 GeV and T_{new} ≥ 0. The mixing parameter ξ and the contact term \( g_E^2 / c_\chi^2 m_{Z_2}^2 \) are given in units of 10^{-4} and TeV^{-2}, respectively. The constraints on m_t and \( \alpha_{s}(m_{Z_1}) \) are found in Ref. [3] while that on \( \tilde{\alpha}(m_{Z_1}) \) is given in Ref. [3].

| Parameters | Constraints | SM | χ | η | ν |
|------------|-------------|----|----|----|----|
| m (GeV)    | Constraints | SM | χ | η | ν |
| α_s(m_{Z_1}) | 0.119 ± 0.002 | 170.5 | 171.3 | 170.8 | 170.9 | 171.3 |
| 1/\tilde{\alpha}(m_{Z_1})^2 | 128.75 ± 0.09 | 128.76 | 128.74 | 128.75 | 128.73 | 128.74 |
| T_{new}    | —           | — | 0 | 0 | 0 |
| ξ(10^{-4}) | —           | — | 0.32 | 1.63 | −2.66 | 0.68 |
| \( g_E^2 / c_\chi^2 m_{Z_2}^2 \) | —           | — | 0.245 | 1.41 | −0.839 | 0.635 |

(2) ψ model (δ = 0)

\[
\begin{align*}
T_{new} &= -0.075 + 0.14x_H + 0.097 \\
ξ(10^{-4}) &= 1.2 - 1.3x_H ± 4.8 \\
g_E^2 / c_\chi^2 m_{Z_2}^2 &= 1.31 + 0.16x_H + 2.97
\end{align*}
\]

\( \chi_{\text{min}}^2 = 22.9 + 1.4x_H \) \hspace{1cm} (25b)

(3) η model (δ = 0)

\[
\begin{align*}
T_{new} &= -0.062 + 0.14x_H + 0.097 \\
ξ(10^{-4}) &= -2.7 - 6.7x_H ± 9.4 \\
g_E^2 / c_\chi^2 m_{Z_2}^2 &= -0.814 + 0.089x_H + 0.449
\end{align*}
\]

\( \chi_{\text{min}}^2 = 18.7 + 0.7x_H \) \hspace{1cm} (25c)

(4) ν model (δ = 0)

\[
\begin{align*}
T_{new} &= -0.057 + 0.14x_H + 0.098 \\
ξ(10^{-4}) &= -0.42 + 0.6x_H ± 3.9 \\
g_E^2 / c_\chi^2 m_{Z_2}^2 &= 0.619 + 0.024x_H ± 0.275
\end{align*}
\]

\( \chi_{\text{min}}^2 = 18.0 + 1.1x_H \) \hspace{1cm} (25d)

where d.o.f. = 20. The mixing angle ξ and the contact term \( g_E^2 / c_\chi^2 m_{Z_2}^2 \) are given in units of 10^{-4} and TeV^{-2}, respectively. The best fit value of T_{new} falls into the unphysical region (T_{new} < 0) for all Z’ models even if the Higgs boson mass is its upper limit (∼ 150 GeV). It should be noticed that T_{new} and ξ are consistent with zero in all models while \( g_E^2 / c_\chi^2 m_{Z_2}^2 \) shows the deviation from zero in the 1-σ level for χ, η, ν models.

The best fit values of the six-parameters for m_H = 100 GeV under the condition T_{new} ≥ 0 are shown in Table 3, together with the SM best fit result at m_H = 100 GeV. Only the best fit value of \( g_E^2 / c_\chi^2 m_{Z_2}^2 \) in the η model is found in the unphysical region (\( g_E^2 / c_\chi^2 m_{Z_2}^2 < 0 \)). The pull factors of the electroweak observables at the best fit point are also shown in Table 3. We learn from the table that almost no improvement of the fit over the SM is found for the Z-pole and m_W measurements. However, the χ, η, ν models show the excellent fit to the weak charge of cesium atom \( Q_W^{\text{133Cs}} \): the pull factor is reduced from 2.2 (SM) to less than 0.1. This may imply that more than 2-σ deviation of the APV data from the SM prediction could be explained by the direct exchange of the Z_2 boson in the low-energy processes. On the other hand, the ψ model does not show the reduction
Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data  

\[ \Delta \chi^2 \equiv \chi^2_{\text{min}}(\beta_E, \delta) - \chi^2_{\text{min}}(\text{SM}) \]

Figure 1: Contour plot of $\Delta \chi^2 \equiv \chi^2_{\text{min}}(\beta_E, \delta) - \chi^2_{\text{min}}(\text{SM})$ for $m_H = 100$ GeV. The mixing angle $\beta_E$ for the $\chi, \psi, \eta$ and $\nu$ models are shown by vertical dotted lines. The step of each contour is 1.

of the pull factor in $Q_W(133C_s)$. Since all the SM matter fields in the $\psi$ model have the same U(1)$'$ charge (see Table 1), the couplings of contact interactions are parity conserving, which makes the contact term useless in the fit to the APV.

We introduce a parameter

\[ \Delta \chi^2 \equiv \chi^2_{\text{min}}(\beta_E, \delta) - \chi^2_{\text{min}}(\text{SM}), \]  

(26)

to measure the goodness of the fit in the $Z'$ models compared to the SM. We can see from Table 2 that the $\chi, \eta$ and $\nu$ models lead to $\Delta \chi^2 = -5.5(\chi), -4.6(\eta)$ and $-5.4(\nu)$, respectively while the $\psi$ model shows no improvement of the fit, $\Delta \chi^2 = -0.3$.

In order to see the impact of kinetic mixing on the fit, we show the contour plot of $\Delta \chi^2$ from the electroweak data under the conditions $T_{\text{new}}, g_E^2/c_\chi^2 m_{Z_2}^2 \geq 0$ on the $(\beta_E, \delta)$ plane in Fig. 1. We can see from the figure that the fit of the $\eta$ model at $\delta = 0$ is rather worsen ($\Delta \chi^2 \sim -3$) as compared to that given in Table 2 ($\Delta \chi^2 = -4.6$) because the fit in the table has been found without imposing the constraint $g_E^2/c_\chi^2 m_{Z_2}^2 \geq 0$. The leptophobic $\eta$ model ($\delta = 1/3$) does not improve the fit because, due to the leptophobity, the model does not have the contact term $g_E^2/c_\chi^2 m_{Z_2}^2$ which is used to make the fit to the LENC (essentially the APV) data.
Figure 2: Contour plot of the 95% CL lower mass limit of the $Z_2$ boson obtained from the LENC experiments for $g_E = g_Y$ and $m_H = 100$ GeV. The vertical dotted lines correspond to the $\chi, \psi, \eta$ and $\nu$ models. The limits are given in unit of GeV.

better. We find that the $Z'$ model with $(\beta_E, \delta) \approx (-\pi/4, 0.2)$ shows the most excellent fit over the SM where $\Delta \chi^2 \lesssim -7$.

4.2. Lower mass bound on $Z'$ bosons

As we expected from the formulae for $T_{\text{new}}$ and $\bar{\xi}$ in the small mixing limit (16), the $Z_2$ mass is unbounded from the $Z$-pole data at $\zeta = 0$. For models with very small $\zeta$, the lower bound of the heavier mass eigenstate $Z_2$ in the $Z'$ models, therefore, comes from the LENC experiments. In Fig. 2, we show the contour plot of the 95% CL lower mass limit of $Z_2$ boson from the LENC experiments on the $(\beta_E, \delta)$ plane by setting $g_E = g_Y$ and $m_H = 100$ GeV under the condition $m_{Z_2} \geq 0$. In practice, we obtain the 95% CL lower limit of the $Z_2$ boson mass $m_{95}$ in the following way:

$$0.05 = \frac{\int_{g_E^2/m_{Z_2}^2}^{\infty} d\left(\frac{g_E^2}{m_{Z_2}^2}\right) P\left(\frac{g_E^2}{m_{Z_2}^2}\right)}{\int_{0}^{\infty} d\left(\frac{g_E^2}{m_{Z_2}^2}\right) P\left(\frac{g_E^2}{m_{Z_2}^2}\right)},$$

(27)

where we assume that the probability density function $P(g_E^2/m_{Z_2}^2)$ is proportional
to \( \exp(-\chi^2(g_2^E/m_{Z_2}^2)/2) \).

We can read off from Fig. 2 that the lower mass bound of the \( Z_2 \) boson in the \( \psi \) model at \( \delta = 0 \) is much weaker than those of the other \( Z' \) models. This is because, as we mentioned before, the \( U(1)' \) charge assignment on the SM matter fields in the model makes the constraint from the APV measurement useless. We also find in Fig. 2 that the lower mass bound of the \( Z_2 \) boson disappears near the leptonphobic \( \eta \)-model (\( \beta_E = \tan^{-1}(\sqrt{5}/3) \) and \( \delta = 1/3 \)) \cite{13}. Furthermore the lower mass bound tend to be small at the “best fit” point which we found in Fig. 1 (\( \beta_E, \delta = (\pi/4, 0.2) \)).

We summarize the 95% CL lower bound on \( m_{Z_2} \) for the \( \chi, \psi, \eta \) and \( \nu \) models (\( \delta = 0 \)) in Table 4. For comparison, those in Ref. 3 are given in the same table. It should be noticed that the bounds on \( Z_\chi, Z_\eta \) and \( Z_\nu \) masses are more severely constrained as compared to Ref. 3 due to the improved value of \( Q_W(136) \) C while the bound on the \( Z_\psi \) mass is almost unchanged.

We have found that the \( Z \)-pole, \( m_W \) and the LENC data constrain \( (T_{new}, \bar{\xi}) \), \( T_{new} \) and \( g_2^E/c_\chi^2m_{Z_2}^2 \), respectively. We can see from Eq. 19 that, for a given \( \zeta \), constraints on \( T_{new}, \bar{\xi} \) and \( g_2^E/c_\chi^2m_{Z_2}^2 \) can be interpreted as the bound on \( m_{Z_2} \). We show the 95% CL lower mass bound of the \( Z_2 \) boson for \( m_H = 100 \text{ GeV} \) in four \( Z' \) models as a function of \( \zeta \). The bound is again found under the condition \( m_{Z_2} \geq 0 \). Results are shown in Fig. 3(a) to 3(d) for the \( \chi, \psi, \eta, \nu \) models, respectively. The lower bound from the \( Z \)-pole and \( m_W \) data, and that from the LENC data are separately plotted in the same figure. Shown in the figure is the lower bound of \( m_{Z_2}g_\chi/g_E \) for \( g_E = g_\chi \). The bound on \( m_{Z_2}g_\chi/g_E \) is approximately independent of \( g_E \) for \( g_E/g_\chi = 0.5 \sim 2.0 \) in each model. The \( Z_2 \) mass is unbounded from the \( Z \)-pole data at \( \zeta = 0 \) because the data constrain \( T_{new} \) and \( \bar{\xi} \) which are proportional to \( \zeta^2 \) and \( \zeta \), respectively. Then, the lower bound on \( m_{Z_2} \) at very small \( \zeta \) is obtained from the LENC experiments and the direct search experiment at Tevatron. For comparison, we plot the 95% CL lower bound on \( m_{Z_2} \) obtained from the direct search experiment in Fig. 2. In the direct search experiment, the \( Z' \) decays into the exotic particles, \( e.g. \), the decays into the light right-handed neutrinos which are expected for some models, are not taken into account. We summarize the 95% CL lower bound on \( m_{Z_2} \) for the \( \chi, \psi, \eta \) and \( \nu \) models (\( \delta = 0 \)) obtained from the low-energy data and the direct search experiment in Table 4. The lower bound of \( m_{Z_2} \) in the \( \eta \) model from the LENC experiments is competitive the bound from the direct search experiment.

The lower bound of \( m_{Z_2} \) is affected by the Higgs boson mass through the \( T \)
Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data

Fig. 3. The 95% CL lower mass limit of $Z_2$ in the $\chi$, $\psi$, $\eta$ and $\nu$ models for $m_H = 100$ GeV. The $Z_2$ boson mass is normalized by $g_E/g_Y$. Constraints from $Z$-pole experiments and LENC experiments are separately shown. The results of the direct search at Tevatron for the $\chi$, $\psi$ and $\eta$ models are also shown.

parameter. As we mentioned previously, $T_{\text{new}}$ tends to be in the physical region ($T_{\text{new}} \geq 0$) for large $m_H$ ($x_H$). Then, we find that the large Higgs boson mass decreases the lower bound of $m_{Z_2}$. For $\zeta = 1$, the lower $m_{Z_2}$ bound in the $\chi$, $\psi$, $\nu$ ($\eta$) models for $m_H = 150$ GeV is weaker than that for $m_H = 100$ GeV about 7% (11%). On the other hand, the Higgs boson with $m_H = 80$ GeV makes the lower $m_{Z_2}$ bound in all the $Z'$ models severe about 5% as compared to the case for $m_H = 100$ GeV. Because $T_{\text{new}}$ and $\bar{\xi}$ are proportional to $\zeta^2$ and $\zeta$, respectively (see Eq. (16)), and it is unbounded at $|\zeta| \approx 0$, the lower bound of $m_{Z_2}$ may be independent of $m_H$ in the small $|\zeta|$ region. The $m_H$-dependence of the lower mass bound obtained from the LENC data is safely negligible.

It has been discussed that the presence of $Z_2$ boson whose mass is much heavier than the SM Z boson mass, say 1 TeV, may lead to a find-tuning problem to stabilize the electroweak scale against the $U(1)'$ scale. The $Z_2$ boson with $m_{Z_2} \leq 1$ TeV for $g_E = g_Y$ is allowed by the electroweak data only if $\zeta$ satisfies the following condition:

$$-0.5 \lesssim \zeta \lesssim +0.4 \quad \text{for the } \chi, \psi, \nu \text{ models,}$$

$$-0.6 \lesssim \zeta \lesssim +0.6 \quad \text{for the } \eta \text{ model.} \quad (28)$$

5. Light $Z'$ boson in minimal SUSY $E_6$-models
Table 5. Coefficients of the 1-loop $\beta$-functions for the gauge couplings in the MSSM and the minimal $E_6$ models. The model $\chi(16)$ has three generations of $16$ and a pair $2 + \bar{2}$. The model $\chi(27)$ and $\psi, \eta, \nu$ have three generations of $27$ and a pair $2 + \bar{2}$.

|      | MSSM | $\chi(16)$ | $\chi(27)$ | $\psi$ | $\eta$ | $\nu$ |
|------|------|------------|------------|--------|--------|--------|
| $b_1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $b_2$ | 1 | 1 | 4 | 4 | 4 | 4 |
| $b_3$ | $-3$ | $-3$ | 0 | 0 | 0 | 0 |
| $b_E$ | $-\frac{6}{a}^2 - \frac{9}{a}^2 + \frac{9}{a}^2$ | $9 + \frac{9}{a}^2$ | $9 + \frac{9}{a}^2$ | $9 + \frac{9}{a}^2$ |
| $b_{1E}$ | $-\frac{1}{3b_1} \sqrt{\frac{3}{5a}} a$ | $-\frac{1}{3b_1} \sqrt{\frac{3}{5a}} a$ | $-\frac{1}{3b_1} \sqrt{\frac{3}{5a}} a$ | $-\frac{1}{3b_1} \sqrt{\frac{3}{5a}} a$ |

It is known that the gauge couplings are not unified in the $E_6$ models with three generations of $27$. In order to guarantee the gauge coupling unification, a pair of weak-doublets, $H'$ and $\bar{H}'$, should be added into the particle spectrum at the electroweak scale. They could be taken from $27 + \bar{27}$ or the adjoint representation $78$. The $U(1)'$ charges of the additional weak doublets should have the same magnitude and opposite sign, $a$ and $-a$, to cancel the $U(1)'$ anomaly. In addition, a pair of the complete SU(5) multiplet such as $5 + \bar{5}$ can be added without spoiling the unification of the gauge couplings.

The minimal $E_6$ model which has three generations of $27$ and a pair $2 + \bar{2}$ depends in principle on the three cases; $H'$ has the same quantum number as $L$ or $H_d$ of $27$, or $\bar{H}_u$ of $27$. In the following we represent the hypercharge and the $U(1)'$ quantum numbers of the additional pair as $(-1/2, a)$ for $H'$ and $(+1/2, -a)$ for $\bar{H}'$, where the $U(1)'$ quantum number $a$ in each $Z'$ model follows the same normalization in Table 1.

In the minimal model, the following eight scalar-doublets can develop VEV to give the mass terms $m_Z^2$ and $m_{ZZ'}^2$ in Eq. (3): three generations of $H_u, H_d$, and an extra pair, $H'$ and $\bar{H}'$. We express their VEVs as follows:

$$
\sum_{i=1}^{3} \langle H_u^i \rangle^2 = \frac{v_u^2}{2}, \quad \sum_{i=1}^{3} \langle H_d^i \rangle^2 = \frac{v_d^2}{2}, \quad \langle H' \rangle^2 = \frac{v_{H'}^2}{2}, \quad \langle \bar{H}' \rangle^2 = \frac{v_{\bar{H}'}^2}{2},
$$

where $i$ is the generation index. The sum of these VEVs gives the observed $\mu$-decay constant: $v_u^2 + v_d^2 + v_{H'}^2 + v_{\bar{H}'}^2 \equiv v^2 = \frac{1}{\sqrt{2}G_F} \approx (246 \text{ GeV})^2$. By further introducing the notation

$$
\tan \beta = \frac{v_u}{v_d}, \quad x^2 = \frac{v_{H'}^2 + v_{\bar{H}'}^2}{v^2},
$$

we can express $\zeta$ in Eq. (17) as

$$
\zeta = 2 \left\{ -Q_{E}^{H'}(1 - x^2) \sin^2 \beta + Q_{E}^{H_d}(1 - x^2) \cos^2 \beta + Q_{E}^{H'} x^2 \right\} - \delta.
$$

Because $H'$ and $\bar{H}'$ are taken from $27 + \bar{27}$, the $U(1)'$ charge of $H'$, $Q_{E}^{H'}$, is identified with that of $L$, $H_d$ or $\bar{H}_u$. 

---

**Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data**

---
Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data

Table 6. Predictions for $g_E$ and $\delta$ at $\mu = m_Z$ in the minimal models and the $\eta_{\text{BKM}}$ model.

| model   | $a$ | $g_E$ | $g_E/g_Y$ | $\delta$ |
|---------|-----|-------|-----------|----------|
| $\chi(16)$ | 3   | 0.353 | 0.989     | 0.066    |
|         | −2  | 0.361 | 1.010     | −0.044   |
| $\chi(27)$ | 3   | 0.353 | 0.989     | 0.066    |
|         | −2  | 0.361 | 1.010     | −0.044   |
| $\psi$   | 1   | 0.364 | 1.020     | 0.028    |
|         | 2   | 0.356 | 0.999     | 0.056    |
|         | −2  | 0.356 | 0.999     | −0.056   |
| $\eta$   | 1/6 | 0.366 | 1.025     | 0.018    |
|         | −2/3| 0.351 | 0.982     | −0.071   |
| $\nu$    | $\sqrt{1/6}$ | 0.361 | 1.010     | 0.044    |
|         | $\sqrt{3/8}$ | 0.353 | 0.989     | −0.066   |
| $\eta_{\text{BKM}}$ | —   | 0.308 | 0.862     | 0.286    |

Let us remind the reader that, in the $\chi$ model, three generations of the matter fields $16$ and a pair of Higgs doublets make the model anomaly free. In this case, $\zeta$ is found to be independent of $\tan \beta$:

$$\zeta = 2 Q^H_d - \delta.$$

We can now examine the kinetic mixing parameter $\delta$ in each model. The boundary condition of $\delta$ at the GUT scale is $\delta = 0$. The non-zero kinetic mixing term can arise at low-energy scale through the following RGEs:

$$\frac{d}{dt} \alpha_1 = \frac{1}{2\pi} b_1 \alpha_1^2,$$

$$\frac{d}{dt} \alpha_4 = \frac{1}{2\pi} (b_E + 2b_{1E} \delta + b_1 \delta^2) \alpha_4^2,$$

$$\frac{d}{dt} \delta = \frac{1}{2\pi} (b_{1E} + b_1 \delta) \alpha_1,$$

where $i = 1, 2, 3$ and $t = \ln \mu$. We define $\alpha_1$ and $\alpha_4$ as

$$\alpha_1 = \frac{5 g_Y^2}{3 4\pi}, \quad \alpha_4 = \frac{5 g_E^2}{3 4\pi}.$$

The coefficients of the $\beta$-functions for $\alpha_1, \alpha_4$ and $\delta$ are:

$$b_1 = \frac{3}{5} \text{Tr}(Y^2), \quad b_E = \frac{3}{5} \text{Tr}(Q_E^2), \quad b_{1E} = \frac{3}{5} \text{Tr}(Y Q_E).$$

From Eq. (33a), we can clearly see that the non-zero $\delta$ is generated at the weak scale if $b_{1E} \neq 0$ holds. In Table 5, we list $b_1, b_E$ and $b_{1E}$ in the minimal $\chi, \psi, \eta$ and $\nu$ models. As explained above, the $\chi(16)$ model has three generations of $16$, and the $\chi(27)$ model has three generations of $27$. We can see from Table 5 that
the magnitudes of the differences $b_1 - b_2$ and $b_2 - b_3$ are common among all the models including the minimal supersymmetric SM (MSSM). This guarantees the gauge coupling unification at $\mu = m_{\text{GUT}} \approx 10^{16}$ GeV. It is straightforward to obtain $g_E(m_{Z_1})$ and $\delta(m_{Z_1})$ for each model. The analytical solutions of Eqs. (33a)–(33d) are given in Ref. 3. In our calculation, $\alpha_3(m_{Z_1}) = 0.118$ and $\alpha(m_{Z_1}) = e^2(m_{Z_1})/4\pi = 1/128$ are used as example. These numbers give $g_Y(m_{Z_1}) = 0.357$. We summarize the predictions for $g_E$ and $\delta$ at $\mu = m_{Z_1}$ in the minimal $E_6$ models in Table 6. In all the minimal models, the ratio $g_E/g_Y$ is approximately unity and $|\delta|$ is smaller than about 0.07. Some further extra fields, therefore, may be needed to give $\delta = 0.2$ which leads to the “minimal $\Delta\chi^2$” when $\beta_E = -\pi/4$, which we found in Fig. 4. We also show the result of the quasi leptophobic $\eta$ model ($\eta_{\text{BKM}}$) proposed by Babu et al. 26 in the same table. The $\eta_{\text{BKM}}$ has, besides three generation of $27$, two pairs of $2 + \overline{2}$ from $78$ and a pair of $3 + \overline{3}$ from $27 + 27$ in order to achieve the leptophobity ($\delta \sim 1/3$) at the weak scale through the quantum corrections. We find that the $\eta_{\text{BKM}}$ model predicts $g_E/g_Y \sim 0.86$ and $\delta \sim 0.29$, which is rather close to the leptophoby, $\delta = 1/3$.

Next we estimate the parameter $\zeta$ for several sets of $\tan \beta$ and $x$. In Table 7, we show the predictions for $\zeta$ in the minimal $\chi, \psi, \eta$ and $\nu$ models. The results are shown for $\tan \beta = 2$ and $30$, and $x^2 = 0$ and $0.5$. We find from the table that the parameter $\zeta$ is in the range $|\zeta| \lesssim 1.35$. It is shown in Fig. 5 that $m_{Z_2}g_Y/g_E$ is approximately independent of $g_E/g_Y$. Actually, we find in Table 5 and Table 6 that the predicted $|\delta|$ is smaller than about 0.1 and $g_E/g_Y$ is quite close to unity in all the minimal models. We can, therefore, read off from Fig. 5 the lower bound of $m_{Z_2}$ in the minimal models at $g_E = g_Y$. In Table 8, we summarize the 95% CL lower $m_{Z_2}$ bound for the minimal $\chi, \psi, \eta$ and $\nu$ models which correspond to the predicted $\zeta$ in Table 7. Most of the lower mass bounds in Table 8 exceed 1 TeV. The $Z_2$ boson with $m_{Z_2} \sim O(1 \text{ TeV})$ should be explored at the future collider such
Table 8: Summary of the 95% CL lower bound of \( m_{Z_2} \) (GeV) which corresponds to the predicted \( \zeta \) in Table 7.

| \( x^2 = 0 \) | \( x^2 = 0.5 \) |
|----------------|-----------------|
| \( \tan \beta \) | \( \tan \beta \) |
| \( \alpha \) | \( \alpha \) |
| 2 | 30 | 2 | 30 |
| \( \chi \) | \( \chi \) |
| 3 | 1490 | 620 |
| \( \psi \) | \( \psi \) |
| 1 | 1270 | 1780 | 1200 | 1470 |
| 2 | 1250 | 1760 | 1510 | 1760 |
| \( \eta \) | \( \eta \) |
| +1/6 | 1440 | 1840 | 660 | 860 |
| -2/3 | 1330 | 1730 | 1550 | 1730 |
| \( \nu \) | \( \nu \) |
| +\sqrt{1/6} | 1030 | 1600 | 1340 | 1600 |
| -\sqrt{3/8} | 1200 | 1730 | 880 | 580 |

as LHC. The discovery limit of the \( Z' \) boson in the \( E_6 \) models at LHC is expected as (in unit of GeV) \[ (36) \]

\[
\begin{array}{cccc}
\chi & \psi & \eta & \nu \\
3040 & 2910 & 2980 & ** *
\end{array}
\]

All the lower bounds of \( m_{Z_2} \) listed in Table 8 are smaller than 2 TeV and they are, therefore, in the detectable range of LHC. But, it should be noticed that most of them (1 TeV \( \lesssim m_{Z_2} \)) may require the fine-tuning to stabilize the electroweak scale against the U(1)' scale.

6. Summary

In this review article, we have studied constraints on \( Z' \) bosons in the SUSY \( E_6 \) models. Four \( Z' \) models — the \( \chi, \psi, \eta \) and \( \nu \) models are studied in detail. The presence of the \( Z' \) boson affects the electroweak processes through the effective \( Z-Z' \) mass mixing angle \( \xi \), a tree level contribution \( T_{\text{new}} \) and the contact term \( g_E^2/c_\chi^2 m_{Z_2}^2 \), where the latter two parameters are positive definite quantities. The \( Z \)-pole, \( m_W \) and LENC data constrain \( (T_{\text{new}}, \xi) \), \( T_{\text{new}} \) and \( g_E^2/c_\chi^2 m_{Z_2}^2 \), respectively. From the updated electroweak data, we find that three \( Z' \) models \( (\chi, \eta, \nu) \) improve the fit over the SM where the total \( \chi^2 \) decrease about five units, owing to the excellent fit mainly to the improved data of parity violation in cesium atom which is expressed by the weak charge \( Q_W(\frac{133}{55}C_s) \). The more than 2-\( \sigma \) deviation of \( Q_W(\frac{133}{55}C_s) \) from the SM prediction could be explained in these three \( Z' \) models. Due to its parity conserving property of the U(1)' charge assignment on the SM matter fields, the \( \psi \) model does not improve the fit to the \( Q_W(\frac{133}{55}C_s) \) data. The impact of the kinetic mixing (\( \delta \neq 0 \)) on the fit is also examined on the \( (\beta_E, \delta) \) plane. The \( Z' \) model with \( (\beta_E, \delta) = (-\pi/4, 0.2) \) shows the most excellent fit to the data among the SUSY \( E_6 \) models where the total \( \chi^2 \) decreases by about seven units as compared to the
Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data

SM best fit. The 95% CL lower mass bound of the heavier mass eigenstate $Z_2$ is shown as a function of the effective $Z$-$Z'$ mixing parameter $\zeta$ together with the result of direct search experiment. By assuming the minimal particle content of the $E_6$ model, we have found the theoretical predictions for $\zeta$. It is shown that the $E_6$ models with minimal particle content which is consistent with the gauge coupling unification predict the non-zero kinetic mixing term $\delta$ and the effective mixing parameter $\zeta$ of order one. The present electroweak experiments lead to the lower mass bound of order 1 TeV or larger for those models.

Acknowledgements

The author would like to thank K. Hagiwara and Y. Umeda for fruitful collaborations which this report is based upon. He is also grateful to R. Barbieri for reading manuscript.

References

1. J. Hewett and T. Rizzo, Phys. Rep. 183 (1989) 193.
2. M. Cvetić et al., Phys. Rev. D56 (1997) 2861; P. Langacker and J. Wang, Phys. Rev. D58 (1998) 115010.
3. G.C. Cho, K. Hagiwara and Y. Umeda, Nucl. Phys. B531 (1998) 65; B555 (1999) 651 (E).
4. Y. Umeda, G.C. Cho and K. Hagiwara, Phys. Rev. D58 (1998) 115008.
5. G. Altarelli et al., Phys. Lett. B263 (1991) 459; F. del Aguila, W. Hollik, J.M. Moreno and M. Quiros, Nucl. Phys. B372 (1992) 3; P. Langacker and M. Luo, Phys. Rev. D45 (1992) 278; G. Altarelli et al., Phys. Lett. B318 (1993) 139; P. Langacker, in Precision Tests of the Standard Electroweak Model, (ed) P. Langacker, World Scientific, (1995) 883; T. Gherghetta, T.A. Kaeding and G.L. Kane, Phys. Rev. D57 (1998) 3178.
6. G.C. Cho, K. Hagiwara and S. Matsumoto, Eur. Phys. J. C5 (1998) 155.
7. L3 Collaboration, Phys. Lett. B306 (1993) 187; ALEPH Collaboration, Z. Phys. C62 (1994) 539.
8. J. Erler and P. Langacker, Phys. Lett. B456 (1999) 68.
9. S.C. Bannett and C.E. Wieman, Phys. Rev. Lett. 82 (1999) 2484.
10. R. Casalbuoni et al., Phys. Lett. B460 (1999) 135; J.L. Rosner, Phys. Rev. D61 (2000) 016006; J. Erler and P. Langacker, Phys. Rev. Lett. 84 (2000) 212.
11. B. Holdom, Phys. Lett. B166 (1986) 196.
12. K.S. Babu, C. Kolda and J. March-Russell, Phys. Rev. D54 (1996) 4635.
13. K.S. Babu, C. Kolda and J.March-Russell, Phys. Rev. D57 (1998) 6788.
14. T. Rizzo, Phys. Rev. D59 (1999) 015020; Y. Umeda, Ph.D. thesis, Hokkaido University, Japan (1999).
15. M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; Phys. Rev. D46 (1992) 381.
16. Review of Particle Physics, C. Casas et al., Eur. Phys. J. C3 (1998) 1
17. J. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A1 (1986) 57; Nucl. Phys. B276 (1986) 14.
18. L.E. Ibáñez and J. Mas, Nucl. Phys. B286 (1987) 107; T. Matsuoka, H. Mino, D. Suematsu and S. Watanebe, Prog. Theor. Phys. 76 (1986) 915.
19. T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the
Looking for $Z'$ bosons in Supersymmetric $E_6$ Models through Electroweak Precision Data

Baryon Number in the Universe, (ed) O. Sawada and A. Sugamoto, KEK (1979); M. Gell-Man, P. Ramond and R. Slansky, in Supergravity, (ed) P. von Nienhuizen and D.Z. Freedman, North Holland (1979).

20. K. Hagiwara, D. Haidt, C.S. Kim and S. Matsumoto, Z. Phys. C64 (1994) 559; C68 (1995) 352 (E).

21. B. Holdom, Phys. Lett. B259 (1991) 329.

22. The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavor Group, CERN-EP/99-15.

23. I. Riu, talk given at the XXXIVth Rencontres de Moriond, March 13-20, 1999.

24. C.Y. Prescott et al., Phys. Lett. B84 (1979) 524.

25. A. Argento et al., Phys. Lett. B120 (1983) 245.

26. P.A. Souder et al., Phys. Rev. Lett. 65 (1990) 694.

27. W. Heil et al., Nucl. Phys. B327 (1989) 1.

28. G.L. Fogli and D. Haidt, Z. Phys. C40 (1988) 379.

29. K. McFarland et al., Eur. Phys. J. C1 (1998) 500.

30. CHARM-II Collaboration, Phys. Lett. B281 (1992) 150.

31. S. Eidelman and F. Jegerlehner, Z. Phys. C67 (95) 585.

32. K. Hagiwara, D. Haidt and S. Matsumoto, Eur. Phys. J. C2 (1998) 95.

33. J.E. Kim, P. Langacker, M. Levine and H.H. Williams, Rev. Mod. Phys. 53 (1981) 211.

34. M. Felcini, talk given at the XXXIVth Rencontres de Moriond, March 13-20, 1999.

35. G.L. Kane, C. Kolda and J.D. Wells, Phys. Rev. Lett. 70 (1993) 2686; D. Comelli and C. Verzegnassi, Phys. Lett. B303 (1993) 277.

36. CDF Collaboration, Phys. Rev. Lett. 79 (1997) 2192.

37. M. Drees, N.K. Falek and M. Glück, Phys. Lett. B167 (1986) 187.

38. K. Dienes, Phys. Rep. 287 (1997) 447.

39. M. Cvetić and S. Godfrey, summary of the Working Subgroup on Extra Gauge Bosons of the DPF long-range planning study, in Electro-weak Symmetry Breaking and Beyond the Standard Model, (ed). T. Barklow et al., World Scientific (1995) [hep-ph/9504214].