String Scale Gauge Coupling Unification with Vector-like Exotics and Non-Canonical $U(1)_Y$ Normalization

V. Barger, Jing Jiang, Paul Langacker, and Tianjun Li

1 Department of Physics, University of Wisconsin, Madison, WI 53706, USA
2 Institute of Theoretical Science, University of Oregon, Eugene, OR 97403, USA
3 School of Natural Sciences, Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA
4 George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA
5 Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China

(Dated: March 26, 2022)

Abstract

We use a new approach to study string scale gauge coupling unification systematically, allowing both the possibility of non-canonical $U(1)_Y$ normalization and the existence of vector-like particles whose quantum numbers are the same as those of the Standard Model (SM) fermions and their Hermitian conjugates and the SM adjoint particles. We first give all the independent sets $(Y_i)$ of particles that can be employed to achieve $SU(3)_C \times SU(2)_L$ string scale gauge coupling unification and calculate their masses. Second, for a non-canonical $U(1)_Y$ normalization, we obtain string scale $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge coupling unification by choosing suitable $U(1)_Y$ normalizations for each of the $Y_i$ sets. Alternatively, for the canonical $U(1)_Y$ normalization, we achieve string scale gauge coupling unification by considering suitable combinations of the $Y_i$ sets or by introducing additional independent sets $(Z_i)$, that do not affect the $SU(3)_C \times SU(2)_L$ unification at tree level, and then choosing suitable combinations, one from the $Y_i$ sets and one from the $Z_i$ sets. We also briefly discuss string scale gauge coupling unification in models with higher Kac-Moody levels for $SU(2)_L$ or $SU(3)_C$.

PACS numbers: 11.25.Mj, 12.10.Kt, 12.10.-g
I. INTRODUCTION

It is well known that the three gauge couplings in the Standard Model (SM) do not unify for the canonical normalization of $U(1)_Y$ (Gauge coupling unification in the SM can be realized via non-canonical $U(1)_Y$ normalization [1]). With supersymmetry (SUSY), which provides an elegant solution to the gauge hierarchy problem, gauge coupling unification can be approximately achieved in the Minimal Supersymmetric Standard Model (MSSM) with unification scale $M_U$ around $2 \times 10^{16}$ GeV [2, 3, 4]. This unification is based on two implicit assumptions: (1) the $U(1)_Y$ normalization is canonical; (2) there are no intermediate threshold corrections.

However, the string scale $M_{\text{string}}$ in weakly coupled heterotic string theory is [5]

$$M_{\text{string}} = g_{\text{string}} \times 5.3 \times 10^{17} \text{ GeV} ,$$

where $g_{\text{string}}$ is the string coupling constant. Since $g_{\text{string}} \sim O(1)$, we have

$$M_{\text{string}} \approx 5 \times 10^{17} \text{ GeV} .$$

Thus, there exists a factor of approximately 20 to 25 between the MSSM unification scale and the string scale. In the strongly coupled heterotic string theory or M-theory on $S^1/Z_2$ [6], the eleven dimensional Planck scale can be the MSSM unification scale [7]. However, our focus in this paper is on the weakly coupled heterotic string theory. The discrepancy between $M_U$ and $M_{\text{string}}$ implies that the weakly coupled heterotic string theory naively predicts the wrong values for the electroweak mixing angle ($\sin^2 \theta_W$) and strong coupling ($\alpha_3$) at the weak scale. Because the weakly coupled heterotic string theory is one of the leading candidates for a unified theory of the fundamental particles and interactions in nature, how to achieve string scale gauge coupling unification is an important question in string phenomenology [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

In addition, there exist intermediate scales in many supersymmetric theories, for example, the invisible axion models with an intermediate Peccei-Quinn (PQ) symmetry breaking scale around $10^{11}$ GeV [19, 20], the see-saw neutrino models with intermediate right-handed neutrino mass scale around $10^{14}$ GeV [21], string models with gaugino condensation scale around

---

1 The unification is not perfect: The $SU(3)$ and $SU(2)$ couplings unify at around $2 \times 10^{16}$ GeV, while the $SU(2)$ and $U(1)$ unification occurs around $3 \times 10^{16}$ GeV.
$10^{13}$ GeV \cite{22}, or string constructions leading to new vector-like matter not associated with any particular motivation. Thus, there could exist intermediate threshold corrections. In addition, there may exist threshold corrections close to the string scale, for example, around $10^{16}$ GeV. Similarly, heterotic constructions often involve non-canonical $U(1)_Y$ embeddings (and normalizations) \cite{5}.

In this paper, assuming a low scale (TeV) supersymmetry, we systematically study string scale gauge coupling unification by introducing intermediate scale extra particles. We introduce vector-like particles whose quantum numbers are the same as those of the SM fermions and their Hermitian conjugates, and SM adjoint particles. We do not consider particles which form complete (or equivalent) $SU(5)$ multiplets because they do not change the relative running among the gauge couplings at one-loop.

Our approach is different from previous approaches. We first list all the independent sets $Y_i$ (defined in Section III) of particles that can be employed to achieve $SU(3)_C$ and $SU(2)_L$ string scale gauge coupling unification. Second, for the case of non-canonical $U(1)_Y$ normalization, we achieve string scale $SU(3)_C \times SU(2)_L \times U(1)_Y$ unification by choosing suitable $U(1)_Y$ normalizations. Alternatively, for canonical $U(1)_Y$ normalization, string scale unification can be realized by considering suitable combinations of $Y_i$ sets. We also introduce the independent sets $Z_i$ (defined in subsection B in Section IV) of particles which do not affect the relative running between the $SU(3)_C$ and $SU(2)_L$ gauge couplings at one-loop. Then, string scale gauge coupling unification can also be achieved by choosing suitable combinations of one set of particles from $Y_i$ and one from $Z_i$. In quite a few cases the masses for all the extra particles are roughly the same at the intermediate scale of about $10^{15}$ GeV, or else one set is around $10^{17}$ GeV, which can be considered as string scale threshold corrections. In some cases there may exist extra particles with masses around hundreds of GeV, which can be produced at the LHC or other future colliders. One can easily use our approach to discuss more general and complicated cases of gauge coupling unification at the string scale. Any set of additional particles that can be employed to achieve the string scale gauge coupling unification can be decomposed as a combination of the $Y_i$ and $Z_i$ sets of particles, plus complete $SU(5)$ multiplets (these could merely have the quantum numbers of $SU(5)$ complete multiplets; the full $SU(5)$ structure is unnecessary). With complete $SU(5)$ multiplet (or the equivalent), we can shift the mass scales of the extra particles by splitting their masses. Furthermore, we briefly discuss string scale gauge coupling unification in
models with higher Kac-Moody levels for $SU(2)_L$ or $SU(3)_C$. A specific model with higher $SU(3)_C$ Kac-Moody level and no-canonical $U(1)_Y$ normalization can be found in [23].

This paper is organized as follows: in Section II we present our conventions and input data for the renormalization group equations (RGEs). We also list the SM vector-like and adjoint particles and their contributions to the beta functions. We study the string scale $SU(3)_C$ and $SU(2)_L$ gauge coupling unification in Section III, and the string scale $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge coupling unification in Section IV. In Section V, we briefly discuss string scale gauge coupling unification in models with higher Kac-Moody levels for $SU(2)_L$ or $SU(3)_C$. Discussion and conclusions are presented in Section VI.

II. RGES AND EXTRA PARTICLES

The relation between the string scale $M_{\text{string}}$ and the string coupling $g_{\text{string}}$ is given in Eq. (1). At the string scale, the gauge couplings satisfy

$$g_1 = g_2 = g_3 = g_{\text{string}},$$

where $g_i^2 \equiv k_Y g_Y^2$ with $k_Y = 5/3$ for canonical normalization, and $g_Y$, $g_2$ and $g_3$ are the gauge couplings for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. In addition, there exist threshold corrections to the gauge coupling running in string models due to the massive string states [5] and in orbifold models due to the massive Kaluza-Klein states [27]. Although these threshold corrections could be important in general, we will not consider them in this paper because we would like to give generic discussions which are model independent.

We define $\alpha_i = g_i^2/4\pi$ and denote the $Z$ boson mass as $M_Z$. The one-loop renormalization group equations are given by

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \log \frac{\mu}{M_Z},$$

with $b \equiv (b_1, b_2, b_3) = (41/6k_Y, -19/6, -7)$ for the SM and $b = (11/k_Y, 1, -3)$ for the MSSM. The two-loop RGE equations and beta-functions can be found in the Appendix of [18]. For the numerical calculations, we use the central values of $\alpha_3(M_Z) = 0.1189 \pm 0.0010$ [24], and $\sin^2 \theta_W(M_Z) = 0.23122 \pm 0.00015$ [25]. For simplicity, we usually assume a supersymmetry breaking scale of 300 GeV. This would be appropriate if all of the sparticles had that value as a common mass. However, in more realistic scenarios the mass splittings, e.g., between
squarks and sleptons lead to an effective scale that is often much lower than the physical masses \[3, 4\]. Hence, we will also consider an effective scale of 50 GeV.

To achieve string scale gauge coupling unification, we only introduce vector-like particles whose quantum numbers are the same as those of the SM fermions and their Hermitian conjugates, and SM adjoint particles. The quantum numbers for additional particle multiplets under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry and their contributions to the one-loop beta functions ($\Delta b$) are

\[XQ + \overline{XQ} = (3, 2, \frac{1}{6}) + (\bar{3}, 2, -\frac{1}{6}), \quad \Delta b = (\frac{1}{5}, 3, 2); \quad (5)\]

\[XU + \overline{XU} = (3, 1, -\frac{2}{3}) + (3, 1, \frac{2}{3}), \quad \Delta b = (\frac{8}{5}, 0, 1); \quad (6)\]

\[XD + \overline{XD} = (\bar{3}, 1, \frac{1}{3}) + (3, 1, -\frac{1}{3}), \quad \Delta b = (\frac{2}{5}, 0, 1); \quad (7)\]

\[XL + \overline{XL} = (1, 2, \frac{1}{2}) + (1, 2, -\frac{1}{2}), \quad \Delta b = (\frac{3}{5}, 1, 0); \quad (8)\]

\[XE + \overline{XE} = (1, 1, 1) + (1, 1, -1), \quad \Delta b = (\frac{6}{5}, 0, 0); \quad (9)\]

\[XG = (8, 1, 0), \quad \Delta b = (0, 0, 3); \quad (10)\]

\[XW = (1, 3, 0), \quad \Delta b = (0, 2, 0). \quad (11)\]

Their two-loop beta functions are presented in \[18\]. We include two-loop running for the gauge couplings and one-loop running for the Yukawa coupling in the RG evolution. The Yukawa couplings of the vector-like particles are not included. We do not consider particles with the quantum number of complete $SU(5)$ multiplets because they do not change the relative running among the gauge couplings at one-loop, although they can be employed to shift the mass scales of the extra particles by splitting the masses of the particles in the $SU(5)$ multiplets.

III. $SU(3)_C \times SU(2)_L$ GAUGE COUPLING UNIFICATION

In supersymmetric models with low scale supersymmetry breaking, without introducing additional particles, the $SU(3)_C$ and $SU(2)_L$ gauge couplings unify at about $2 \times 10^{16}$ GeV. To achieve $SU(3)_C \times SU(2)_L$ unification at the string scale, we need to introduce sets of particles with $\Delta b_2 < \Delta b_3$. The independent sets of Yi particles, constructed from the extra
particles in Section II, that satisfy $\Delta b_2 < \Delta b_3$ are

$$
Y_1 : \quad XU + \overline{XU}, \quad \Delta b = (\frac{8}{5}, 0, 1); \tag{12}
$$
$$
Y_2 : \quad XD + \overline{XD}, \quad \Delta b = (\frac{2}{5}, 0, 1); \tag{13}
$$
$$
Y_3 : \quad XG + XW, \quad \Delta b = (0, 2, 3); \tag{14}
$$
$$
Y_4 : \quad XG + k(XQ + \overline{XQ}) + l(XL + \overline{XL}), \quad \Delta b = (\frac{k}{5} + \frac{3l}{5}, 3k + l, 2k + 3), \tag{15}
$$

where $k = 0, 1, 2$, and $l = 0, 1, 2$ with $k + l \leq 2$.

The $SU(3)_C$ and $SU(2)_L$ gauge coupling unification at the string scale can be realized by introducing any $Y_i$ set of particles, or any combinations of the $Y_i$ sets. Assuming that the new particles are degenerate, their command mass can be determined from Eqs. (11) and (13). For simplicity, we will not study the cases with general combinations of the $Y_i$ sets here. In the cases $Y_1, Y_2, Y_3$, and $Y_4$ with $k + l = 2$, we have $\Delta b_2 = \Delta b_3 - 1$. At one-loop level, the gauge coupling unification only depends on the differences between the one-loop beta functions. Hence, to achieve string scale $SU(3)_C \times SU(2)_L$ unification we estimate that the mass scales ($\Lambda_Y$) at which these extra particles are introduced are all approximately

$$
\Lambda_Y = 1.6 \times 10^{13} \text{ GeV}. \tag{16}
$$

For the other $Y_4$ cases, the mass scales of the extra particles depend on $k$ and $l$

$$
k = 0, \ l = 0 : \quad \Lambda_Y = 1.6 \times 10^{16} \text{ GeV};
$$
$$
k = 0, \ l = 1 : \quad \Lambda_Y = 2.9 \times 10^{15} \text{ GeV};
$$
$$
k = 1, \ l = 0 : \quad \Lambda_Y = 2.9 \times 10^{15} \text{ GeV}. \tag{17}
$$

The particles in the set $Y_4$ with $k = 0$ and $l = 0$ can be considered as string scale threshold corrections. For the two-loop predictions, the actual values of the beta functions matter and these values are shifted, as can be seen in the Table. The mass scales of the extra particles can be shifted by introducing complete $SU(5)$ multiplets but splitting their masses. For example, consider the $Y_2$ set $XD + \overline{XD}$. We can give them vector-like mass $M_V$ with $M_V < 1.6 \times 10^{13}$, provide that we also introduce vector-like particles $XL + \overline{XL}$ with mass $M'_V \simeq M_V \times M_{\text{string}}/(1.6 \times 10^{13} \text{ GeV})$. Above $M'_V$, we have complete $5 + \overline{5}$ contributions to the gauge coupling RGE running, so that
the $SU(3)_C \times SU(2)_L$ gauge couplings can still be unified at the string scale. However, this approach introduces two mass scales for the extra particles, so for simplicity we do not consider it further.

IV. $SU(3)_C \times SU(2)_L \times U(1)_Y$ GAUGE COUPLING UNIFICATION

At the string scale, where the $SU(3)_C$ and $SU(2)_L$ gauge couplings unify, the $U(1)_Y$ gauge coupling in general does not coincide with the other two couplings for a canonical $U(1)_Y$ normalization. To achieve string scale $SU(3)_C \times SU(2)_L \times U(1)_Y$ unification, the simplest possibilities are to consider suitable non-canonical $U(1)_Y$ normalizations, add another $Y_i$ set at a different scale, or introduce additional sets with $\Delta b_2 = \Delta b_3$ at an intermediate scale.

A. Non-Canonical $U(1)_Y$ Normalization

The $U(1)_Y$ normalization need not be canonical in string model building, orbifold Grand Unified Theories (GUTs) and their deconstruction, and in 4D GUTs with product gauge groups. Similar to heterotic string models, we assume that $k_Y$ is a rational number.

Once the $SU(3)_C \times SU(2)_L$ gauge coupling unification at the string scale is realized, we can unify the $U(1)_Y$ gauge coupling by choosing a suitable $U(1)_Y$ normalization. The non-canonical $U(1)_Y$ normalizations required for the $Y_i$ sets of the particles are given in Table II. The corresponding string scales $M_{\text{string}}$ can be obtained from Eq. (1). The two-loop prediction of the scale at which the $Y_i$ particles are introduced is shown as $\Lambda_Y$, together with gauge coupling $g_{\text{string}}$ at the string scale $M_{\text{string}}$. We also show the percentage deviation of $\alpha^{-1}(M_{\text{string}})$ from $\alpha_U^{-1}(M_{\text{string}})$, $\Delta = |\alpha^{-1}(M_{\text{string}}) - \alpha_U^{-1}(M_{\text{string}})|/\alpha_U^{-1}(M_{\text{string}})$, where $\alpha_U^{-1}(M_{\text{string}})$ is the unified gauge coupling for $SU(3)_C \times SU(2)_L$ at the string scale. The choice of $k_Y$ is not unique. We present the fractional number with the smallest possible denominator, while requiring $\Delta$ defined above to be less than 5%. In Fig. 1 we show the two-loop gauge couplings for the cases $Y_3$ with $k_Y = 9/5$, and $Y_4$ ($k = 0$, $l = 0$) with $k_Y = 3/2$. It is interesting to point out that although the canonical gauge coupling unification can be realized at one-loop level for the $Y_3$ set of particles, this is no longer true at two-loop level.
TABLE I: The required mass scales (for $SU(3)_C \times SU(2)_L$ unification) and $U(1)_Y$ normalization of the Yi sets (for $U(1)_Y$ unification). The rows with an asterisk are for an effective SUSY breaking scale of 50 GeV.

| Yi's | $\Lambda_Y$ | $g_{\text{string}}$ | $k_Y$ | $k_Y/(5/3)$ | $\Delta$ (%) |
|------|-------------|----------------------|------|-------------|--------------|
| $Y_1$ | $1.8 \times 10^{12}$ | 0.725 | 9/7 | 0.771 | 3.1 |
| $Y_2$ | $1.8 \times 10^{12}$ | 0.726 | 29/20 | 0.870 | 2.1 |
| $Y_3$ | $3.4 \times 10^{12}$ | 0.794 | 9/5 | 1.080 | 3.2 |
| $Y_4$ ($k=0$, $l=0$) | $6.9 \times 10^{15}$ | 0.725 | 3/2 | 0.900 | 0.9 |
| $Y_4$ ($k=0$, $l=1$) | $1.0 \times 10^{15}$ | 0.741 | 29/19 | 0.916 | 1.7 |
| $Y_4$ ($k=0$, $l=2$) | $4.7 \times 10^{12}$ | 0.791 | 8/5 | 0.960 | 2.3 |
| $Y_4^*$ ($k=0$, $l=2$) | $4.3 \times 10^{12}$ | 0.818 | 18/11 | 0.982 | 0.0 |
| $Y_4$ ($k=1$, $l=0$) | $9.9 \times 10^{14}$ | 0.776 | 17/10 | 1.020 | 2.1 |
| $Y_4^*$ ($k=1$, $l=0$) | $9.3 \times 10^{14}$ | 0.803 | 7/4 | 1.050 | 0.7 |
| $Y_4$ ($k=1$, $l=1$) | $3.8 \times 10^{12}$ | 0.887 | 31/15 | 1.240 | 2.2 |
| $Y_4$ ($k=2$, $l=0$) | $3.0 \times 10^{12}$ | 1.051 | 3 | 1.800 | 3.2 |

FIG. 1: Two-loop gauge coupling unification with non-canonical $U(1)_Y$ normalizations for the Yi sets. Left: $Y_3$ with $k_Y = 9/5$. Right: $Y_4$ ($k=0$, $l=0$) with $k_Y = 3/2$.

The above Yi sets can be categorized according to their $k_Y$ values, depending on whether $k_Y$ is smaller than (AY) or greater than (BY) 5/3:

Case AY : $Y_1$, $Y_2$, $Y_4$ ($k = 0$, $l = 0$), $Y_4$ ($k = 0$, $l = 1$), $Y_4$ ($k = 0$, $l = 2$);
Case BY : Y3, Y4 \((k = 1, l = 0)\), Y4 \((k = 1, l = 1)\), Y4 \((k = 2, l = 0)\).

Using the effective SUSY scale of 50 GeV instead of 300 GeV could potentially change the \(k_Y\) value and \(\Lambda_Y\) scale for each case. In Table II we show the effect of using 50 GeV for two cases with \(k_Y\) close to \(5/3\), Y4 \((k = 0, l = 2)\) and Y4 \((k = 1, l = 0)\). We see from these examples that the change is less than 10\% for \(\Lambda_Y\) and even smaller for \(k_Y\).

B. Canonical \(U(1)_Y\) Normalization

Two ways to achieve string scale \(SU(3)_C \times SU(2)_L \times U(1)_Y\) gauge coupling unification for canonical \(U(1)_Y\) normalization are by combining Case AY and Case BY sets, or by introducing another set of particles with \(\Delta b_2 = \Delta b_3\) at an intermediate scale.

(1) If we introduce one set of particles at mass scale \(\Lambda_1\) from the AY sets and another set at scale \(\Lambda_2\) from BY, we are able to realize gauge coupling unification at the string scale, by adjusting \(\Lambda_1\) and \(\Lambda_2\) to satisfy Eqs. (1) and (3). In Table II we present the mass scales in GeV and the corresponding unified gauge couplings \(g_{\text{string}}\). For the cases Y4 \((k = 0, l = 1)\) with Y4 \((k = 1, l = 1)\); and Y4 \((k = 0, l = 2)\) with Y4 \((k = 1, l = 0)\), we have \(\Lambda_1 \sim \Lambda_2\), i.e., almost a common mass scale for all the extra particles. For the cases Y1 with Y4 \((k = 1, l = 0)\); Y4 \((k = 0, l = 0)\) with Y4 \((k = 1, l = 0)\); and Y4 \((k = 0, l = 2)\) with Y4 \((k = 2, l = 0)\), one set of the extra particles could be considered as string scale threshold correction because \(\Lambda \sim 10^{17}\) GeV. We show the couplings for the cases Y4 \((k = 0, l = 1)\) with Y4 \((k = 1, l = 1)\), and Y4 \((k = 0, l = 2)\) with Y4 \((k = 2, l = 0)\) in Fig. 2. For the case Y4 \((k = 0, l = 1)\) with Y4 \((k = 1, l = 0)\), and the cases Y4 \((k = 0, l = 0)\) with Y3, Y4 \((k = 1, l = 1)\), or Y4 \((k = 2, l = 0)\), we cannot achieve string scale gauge coupling unification.

One could, of course, consider more complicated cases involving more sets of particles.

(2) As another way to achieve \(SU(3)_C \times SU(2)_L \times U(1)_Y\) unification, we introduce the following particle sets \(Z_i\) with \(\Delta b_2 = \Delta b_3\) at another scale:

\[
Z1 : XE + \overline{XE}, \Delta b = \left(\frac{6}{5}, 0, 0\right);
\]
\[
Z2 : XQ + \overline{XQ} + XU + \overline{XU}, \Delta b = \left(\frac{9}{5}, 3, 3\right);
\]
\[
Z3 : XQ + \overline{XQ} + XD + \overline{XD}, \Delta b = \left(\frac{3}{5}, 3, 3\right);
\]
\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
& Y_3 & & Y_4 (k=1, l=0) & & \\
& \Lambda_{Y_3} & \Lambda_Y & g_{\text{string}} & \Lambda_{Y_4(1,0)} & \Lambda_Y & g_{\text{string}} \\
\hline
Y_1 & 4.1 \times 10^{13} & 2.9 \times 10^{16} & 0.777 & 1.5 \times 10^{15} & 1.6 \times 10^{17} & 0.772 \\
Y_2 & 1.5 \times 10^{14} & 7.0 \times 10^{15} & 0.769 & 2.1 \times 10^{15} & 8.8 \times 10^{16} & 0.769 \\
Y_4 (k=0, l=0) & - & - & - & 2.5 \times 10^{15} & 2.1 \times 10^{17} & 0.768 \\
Y_4 (k=0, l=1) & 5.6 \times 10^{14} & 2.9 \times 10^{16} & 0.769 & - & - & - \\
Y_4 (k=0, l=2) & 6.3 \times 10^{15} & 2.5 \times 10^{14} & 0.792 & 7.8 \times 10^{15} & 7.9 \times 10^{15} & 0.781 \\
\hline
& Y_4 (k=1, l=1) & & Y_4 (k=2, l=0) & & \\
& \Lambda_{Y_4(1,1)} & \Lambda_Y & g_{\text{string}} & \Lambda_{Y_4(2,0)} & \Lambda_Y & g_{\text{string}} \\
\hline
Y_1 & 4.1 \times 10^{14} & 2.8 \times 10^{15} & 0.809 & 4.0 \times 10^{15} & 2.2 \times 10^{14} & 0.810 \\
Y_2 & 2.1 \times 10^{15} & 4.7 \times 10^{14} & 0.786 & 1.7 \times 10^{16} & 4.7 \times 10^{13} & 0.779 \\
Y_4 (k=0, l=0) & - & - & - & - & - & - \\
Y_4 (k=0, l=1) & 7.9 \times 10^{15} & 7.9 \times 10^{15} & 0.781 & 4.6 \times 10^{16} & 3.1 \times 10^{15} & 0.776 \\
Y_4 (k=0, l=2) & 5.0 \times 10^{16} & 3.6 \times 10^{13} & 0.806 & 1.5 \times 10^{17} & 1.3 \times 10^{13} & 0.806 \\
\hline
\end{array}
\]

TABLE II: The mass scales in GeV for the particles in the Yi sets and the corresponding unified gauge couplings \( g_{\text{string}} \).

\[
Z_4 : XL + X\bar{L} + XU + \bar{X}U, \quad \Delta b = (\frac{11}{5}, 1, 1); \quad (21)
\]
\[
Z_5 : XG + XW + XL + X\bar{L}, \quad \Delta b = (\frac{3}{5}, 3, 3); \quad (22)
\]
\[
Z_6 : XG + XW + XQ + \bar{X}Q, \quad \Delta b = (\frac{1}{5}, 5, 5); \quad (23)
\]
\[
Z_7 : XG + n(XQ + \bar{X}Q) + (3 - n)(XL + X\bar{L}), \quad \Delta b = (\frac{9}{5} - \frac{2}{5}n, 3 + 2n, 3 + 2n); \quad (24)
\]
\[
Z_8 : XW + m(XU + \bar{X}U) + (2 - m)(XD + \bar{X}D), \quad \Delta b = (\frac{4}{5} + \frac{6}{5}m, 2, 2), \quad (25)
\]

where \( m = 0, 1, 2 \), and \( n = 0, 1, 2, 3 \). The \( Z_3 \) and \( Z_5 \) sets of particles give the same contribution to \( \Delta b \), so we only show the results for the \( Z_3 \) set\(^2\). The \( Z_8 \) \(( m = 1 \)\) set satisfies

\(^2\) If we only introduce one \( Z_i \) set and no \( Y_i \) sets, gauge coupling unification can be achieved at about \( 2 \times 10^{16} \) GeV at one-loop level by considering non-canonical \( U(1)_Y \) normalizations. \( k_Y > 5/3 \) for the sets \( Z_2, Z_3, \)
FIG. 2: Two-loop gauge coupling unification for one set of particles from Case AY and one from BY. Left: \( Y_4 (k = 0, l = 1) \) and \( Y_4 (k = 1, l = 1) \) at \( 7.9 \times 10^{15} \) GeV. Right: \( Y_4 (k = 1, l = 0) \) at \( 2.5 \times 10^{15} \) GeV and \( Y_4 (k = 0, l = 0) \) at \( 2.1 \times 10^{17} \) GeV.

\( \Delta b_1 = \Delta b_2 = \Delta b_3 \), so we do not consider it here. The sets \( Z_1 + Z_2 \) and \( Z_3 + Z_4 - Z_2 \) form complete \( SU(5) \) multiplets and would not contribute if they are degenerate.

We can introduce one combination of \( Y_i \) sets and another from the \( Z_i \). For simplicity, we only consider the cases with one set from \( Y_i \) and one from \( Z_i \). The \( SU(3)_C \times SU(2)_L \times U(1)_Y \) unification can be achieved by the following combinations:

Case AYZ: \( Y_1, Y_2, Y_4 \) \((k = 0, l = 0, 1, 2)\), \( Y_4 \) \((k = 1, l = 0)\) with \( Z_2, Z_3, Z_5, Z_6, Z_7, Z_8 \) \((m = 0)\);

Case BYZ: \( Y_3, Y_4 \) \((k = 1, l = 1)\), \( Y_4 \) \((k = 2, l = 0)\) with \( Z_1, Z_4, Z_8 \) \((m = 2)\).

In all cases, the \( Y_i \) sets guarantee the unification of \( SU(3)_C \) and \( SU(2)_L \) at the string scale, and the \( Z_i \) sets of particles ensure the unification of the \( U(1)_Y \) gauge coupling.

We denote \( \Lambda_Y \) and \( \Lambda_Z \) as the mass scales for the \( Y_i \) and \( Z_i \) sets of particles, respectively. The energy scales \( \Lambda_Y \) and \( \Lambda_Z \) as well as the corresponding unified gauge couplings \( g_{\text{string}} \) for Case AYZ are listed in Table III. There are several cases with \( \Lambda_Y \sim \Lambda_Z \). There are also several cases with \( \Lambda_Z \sim 10^{17} \) GeV, which can be considered as string scale threshold corrections. In Fig. 3 we plot the two-loop gauge coupling unification for Case AYZ, \( Y_4(k = 0, l = 0) \) with \( Z_7 \) \((n = 1)\) as an example of the case with \( \Lambda_Y \sim \Lambda_Z \), and \( Y_4 \) \((k = 0, l = 2)\) with \( Z_7 \) \((n = 3)\) as an example of the case with \( \Lambda_Z \sim 10^{17} \).

---

\( Z_5, Z_6, Z_7, Z_8 \) \((m=0)\) and \( k_Y < 5/3 \) for \( Z_1, Z_4, Z_8 \) \((m=2)\).
FIG. 3: Two-loop gauge coupling unification for Cases AYZ. Left: Y4 \((k=0, l=1)\) at \(2.9 \times 10^{15}\) GeV and Z3 at \(1.1 \times 10^{15}\) GeV. Right: Y4 \((k=0, l=2)\) at \(6.4 \times 10^{12}\) GeV and Z7 \((n=3)\) at \(2.2 \times 10^{17}\) GeV.

We show the mass scales and the corresponding unified gauge couplings for Case BYZ in Table IV. For the cases Y4 \((k=2, l=0)\) with Z4, we find that \(\Lambda_Z < M_Z\) and \(\Lambda_Y \sim 10^{13}\) GeV. The low \(\Lambda_Z\) value is very sensitive to the supersymmetry and the string scale threshold corrections, and should be considered an order of magnitude estimate only. In particular, for a higher effective supersymmetric mass scale, one can raise the \(\Lambda_Z\) above 200 GeV, which is the Tevatron bound [30]. Then, Z4 set of particles may be observable at the LHC. In practice, it is difficult to obtain a higher effective scale in the SUSY breaking schemes with physical masses not too much higher than the TeV scale. That is because the effective scale is very sensitive to the mass splittings, and those schemes (such as most SUGRA and gauge mediated models) in which the colored sparticles are typically heavier than the uncolored ones tend to give a low effective mass [3, 4].

V. MODELS WITH HIGHER KAC-MOODY LEVELS FOR \(SU(2)_L\) OR \(SU(3)_C\)

We assume that at the string scale \(M_{\text{string}} \approx 5 \times 10^{17}\) GeV the gauge couplings satisfy

\[ g_1^2 = g_2^2 = g_3^2 , \]

(26)

where

\[ g_1^2 \equiv k_Y g_Y^2 , \; g_2^2 \equiv k_2 g_Z^2 , \; g_3^2 \equiv k_3 g_3^2 . \]

(27)
|       | Y1                                      | Y2                                      |
|-------|-----------------------------------------|-----------------------------------------|
|       | $\Lambda_Z$ | $\Lambda_Y$ | $g_{\text{string}}$ | $\Lambda_Z$ | $\Lambda_Y$ | $g_{\text{string}}$ |
| Z2    | $2.0 \times 10^5$ | $9.2 \times 10^{11}$ | 1.213            | $6.2 \times 10^{10}$ | $1.3 \times 10^{12}$ | 0.893            |
| Z3    | $2.8 \times 10^{11}$ | $1.4 \times 10^{12}$ | 0.870            | $1.5 \times 10^{14}$ | $1.5 \times 10^{12}$ | 0.796            |
| Z6    | $4.0 \times 10^{14}$ | $2.3 \times 10^{12}$ | 0.836            | $7.8 \times 10^{15}$ | $2.0 \times 10^{12}$ | 0.782            |
| Z7 (n=1) | $4.7 \times 10^{13}$ | $3.5 \times 10^{12}$ | 0.882            | $2.1 \times 10^{15}$ | $2.5 \times 10^{12}$ | 0.804            |
| Z7 (n=2) | $1.6 \times 10^{15}$ | $2.3 \times 10^{12}$ | 0.853            | $1.7 \times 10^{16}$ | $2.0 \times 10^{12}$ | 0.790            |
| Z7 (n=3) | $7.9 \times 10^{15}$ | $1.9 \times 10^{12}$ | 0.841            | $4.3 \times 10^{16}$ | $1.8 \times 10^{12}$ | 0.784            |
| Z8 (m=0) | $1.8 \times 10^{5}$ | $1.6 \times 10^{12}$ | 0.946            | $5.1 \times 10^{10}$ | $1.4 \times 10^{12}$ | 0.824            |

|       | Y4 (k=0, l=0)                                      | Y4 (k=0, l=1)                                      |
|-------|-----------------------------------------|-----------------------------------------|
|       | $\Lambda_Z$ | $\Lambda_Y$ | $g_{\text{string}}$ | $\Lambda_Z$ | $\Lambda_Y$ | $g_{\text{string}}$ |
| Z2    | $3.6 \times 10^{12}$ | $7.3 \times 10^{15}$ | 0.837            | $2.3 \times 10^{13}$ | $1.1 \times 10^{15}$ | 0.838            |
| Z3    | $1.1 \times 10^{15}$ | $7.0 \times 10^{15}$ | 0.775            | $1.1 \times 10^{15}$ | $2.9 \times 10^{15}$ | 0.785            |
| Z6    | $2.1 \times 10^{16}$ | $7.4 \times 10^{15}$ | 0.766            | $3.4 \times 10^{16}$ | $1.1 \times 10^{15}$ | 0.777            |
| Z7 (n=1) | $7.9 \times 10^{15}$ | $7.9 \times 10^{15}$ | 0.781            | $1.5 \times 10^{16}$ | $1.2 \times 10^{15}$ | 0.790            |
| Z7 (n=2) | $3.8 \times 10^{16}$ | $7.4 \times 10^{15}$ | 0.771            | $5.6 \times 10^{16}$ | $1.1 \times 10^{15}$ | 0.781            |
| Z7 (n=3) | $7.5 \times 10^{16}$ | $7.2 \times 10^{15}$ | 0.767            | $9.9 \times 10^{16}$ | $1.0 \times 10^{15}$ | 0.778            |
| Z8 (m=0) | $3.2 \times 10^{12}$ | $6.9 \times 10^{15}$ | 0.794            | $2.1 \times 10^{13}$ | $1.0 \times 10^{15}$ | 0.801            |

|       | Y4 (k=0, l=2)                                      |
|-------|-----------------------------------------|
|       | $\Lambda_Z$ | $\Lambda_Y$ | $g_{\text{string}}$ |
| Z2    | $3.9 \times 10^{15}$ | $6.0 \times 10^{12}$ | 0.839            |
| Z3    | $3.9 \times 10^{16}$ | $6.1 \times 10^{12}$ | 0.812            |
| Z6    | $1.3 \times 10^{17}$ | $6.6 \times 10^{12}$ | 0.808            |
| Z7 (n=1) | $8.4 \times 10^{16}$ | $7.0 \times 10^{12}$ | 0.815            |
| Z7 (n=2) | $1.6 \times 10^{17}$ | $6.6 \times 10^{12}$ | 0.810            |
| Z7 (n=3) | $2.2 \times 10^{17}$ | $6.4 \times 10^{12}$ | 0.809            |
| Z8 (m=0) | $3.8 \times 10^{15}$ | $5.9 \times 10^{12}$ | 0.821            |

**TABLE III:** The mass scales in GeV for the particles in the Yi and Zi sets and the corresponding unified gauge couplings $g_{\text{string}}$ for case AYZ.
Then, 

\[ b_1 = \frac{b_Y}{k_Y}, \quad b'_2 = \frac{b_2}{k_2}, \quad b'_3 = \frac{b_3}{k_3}. \quad (28) \]

It is very difficult to construct string models with \( k_2 \) or \( k_3 \) larger than 2, and the discussions in models with \( k_2 = k_3 = 2 \) are similar to those in Section IV by rescaling \( k_Y \). Therefore, we only study the models with \((k_2, k_3) = (1, 2) \) or \((2, 1) \). The canonical \( U(1)_Y \) normalization, \( k_Y = 5/3 \), is not very interesting in these cases. For brevity we will not consider it here although the discussions would be similar to those in Section IV.

### A. Models with \( k_2 = 1 \) and \( k_3 = 2 \)

To achieve string scale \( SU(3)_C \times SU(2)_L \) unification, we need to introduce sets of particles with \( \Delta b_2 > \Delta b_3 \). With the extra particles in Section II, we have the following independent Ti sets

\[
\begin{align*}
T1 & : \quad XQ + \overline{XQ}, \quad \Delta b = \left( \frac{1}{5}, 3, 2 \right); \\
T2 & : \quad XL + \overline{XL}, \quad \Delta b = \left( \frac{3}{5}, 1, 0 \right); \\
T3 & : \quad XW, \quad \Delta b = (0, 2, 0); \\
T4 & : \quad XW + XU + \overline{XU}, \quad \Delta b = \left( \frac{8}{5}, 2, 1 \right);
\end{align*}
\]
\[ T5: \quad XW + XD + \overline{XD}, \quad \Delta b = \left(\frac{2}{5}, 2, 1\right); \quad (33) \]
\[ T6: \quad 2XW + XG, \quad \Delta b = (0, 4, 3). \quad (34) \]

For simplicity, we only consider the cases with a single type of particle set \( T_i \) but allow multiple copies. In Table V, we list the numbers of the \( T_i \) sets necessary to ensure \( SU(3)_C \times SU(2)_L \) unification at the string scale, their mass scales, and the corresponding non-canonical \( U(1)_Y \) normalizations. We can employ the non-canonical normalizations as shown in the Table V.

| \( T_i \)'s | \( \Lambda_T \) | \( k_Y \) | \( k_Y/(5/3) \) | \( g_{\text{string}} \) |
|---|---|---|---|---|
| 1\( \times \) T1 | \( 6.1 \times 10^6 \) | 58/15 | 2.320 | 1.227 |
| 2\( \times \) T2 | \( 3.1 \times 10^4 \) | 36/19 | 1.137 | 0.984 |
| 1\( \times \) T3 | \( 5.2 \times 10^4 \) | 8/3 | 1.600 | 0.984 |
| 2\( \times \) T4 | \( 2.1 \times 10^9 \) | 12/7 | 1.029 | 1.136 |
| 2\( \times \) T5 | \( 2.1 \times 10^9 \) | 52/17 | 1.835 | 1.136 |
| 1\( \times \) T6 | \( 7.2 \times 10^7 \) | 29/6 | 2.900 | 1.344 |

**TABLE V:** The mass scales and the corresponding \( U(1)_Y \) normalizations for \( T_i \) with \((k_2, k_3) = (1, 2)\).

**B. Models with \( k_2 = 2 \) and \( k_3 = 1 \)**

In this case we need to introduce \( Y_i \) sets of particles. The numbers of the \( Y_i \) sets, the mass scales at which they are introduced and the appropriate \( k_Y \) are shown in Table VI.

The \( Y_3 \) and \( Y_4 \) \((k = 0, \ l = 2)\) cases imply small \( \Lambda_Y \). The values are sensitive to the supersymmetric and string scale threshold corrections, but the new particles may be observable at the LHC. For the case with \( Y_4 \) \((k = 0, \ l = 0)\), \( Y_4 \) \((k = 1, \ l = 1)\) and \( Y_4 \) \((k = 2, \ l = 0)\), we cannot achieve string scale gauge coupling unification.

**VI. DISCUSSION AND CONCLUSIONS**

Gauge coupling unification in the MSSM implies a unification scale \( M_U \) around \( 2 \times 10^{16} \) GeV, while in weakly coupled heterotic string theory the string scale \( M_{\text{string}} \) is about \( 5 \times \)
TABLE VI: Same as Table V, only for \((k_2, k_3) = (2, 1)\). The rows with an asterisk are for an effective SUSY scale of 50 GeV.

10^{17} \text{ GeV}. Because the latter is still one of the leading candidates for a unified theory of the fundamental particles and interactions, we studied the string scale gauge coupling unification systematically by introducing vector-like particles whose quantum numbers are the same as those of the SM fermions and their Hermitian conjugates, and SM adjoint particles, and also allowed for the possibility of non-canonical \(U(1)_Y\) normalization.

We proposed a new approach. We first considered the independent sets \(Y_i\) of particles that can be employed to achieve the \(SU(3)_C\) and \(SU(2)_L\) string scale unification, and calculated the needed mass scales. We were then able to achieved string scale \(SU(3)_C\), \(SU(2)_L\) and \(U(1)_Y\) unification by choosing suitable \(U(1)_Y\) normalizations for the \(Y_i\) sets. Alternatively, for canonical \(U(1)_Y\) normalization, we considered suitable combinations of \(Y_i\) sets or introduced independent sets \(Z_i\) with \(\Delta b_2 = \Delta b_3\) in addition to the \(Y_i\) sets. In a few cases the masses for all the extra particles are roughly the same at the scale around \(10^{15}\) GeV, or else with one set around \(10^{17}\) GeV, which can be considered as string scale threshold corrections. In some cases there exist additional particles with masses around hundreds of GeV with large uncertainty from the supersymmetric and string thresholds, which may be observable at the LHC. Our approach can be easily generalized to more complicated cases. We also briefly discussed string scale unification in models with higher Kac-Moody levels for \(SU(2)_L\) or \(SU(3)_C\).
Acknowledgments

This research was supported in part by the U.S. Department of Energy under Grants No. DE-FG02-95ER40896 and DE-FG02-96ER40969, by the friends of the IAS, by the Cambridge-Mitchell Collaboration in Theoretical Cosmology, and by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

[1] V. Barger, J. Jiang, P. Langacker and T. Li, Phys. Lett. B 624, 233 (2005); Nucl. Phys. B 726, 149 (2005).

[2] P. Langacker and M. X. Luo, Phys. Rev. D 44, 817 (1991); J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 260, 131 (1991); U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260, 447 (1991).

[3] P. Langacker and N. Polonsky, Phys. Rev. D 47, 4028 (1993) [arXiv:hep-ph/9210235]; M. Carena, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B 406, 59 (1993) [arXiv:hep-ph/9303202].

[4] P. Langacker and N. Polonsky, Phys. Rev. D 52, 3081 (1995) [arXiv:hep-ph/9503214]; J. Bagger, K. T. Matchev and D. Pierce, Phys. Lett. B 348, 443 (1995) [arXiv:hep-ph/9501277].

[5] For a review, see K. R. Dienes, Phys. Rept. 287, 447 (1997), and references therein.

[6] P. Horava and E. Witten, Nucl. Phys. B 460, 506 (1996).

[7] E. Witten, Nucl. Phys. B 471, 135 (1996).

[8] V. S. Kaplunovsky, Nucl. Phys. B 307, 145 (1988) [Erratum-ibid. B 382, 436 (1992)].

[9] L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B 355, 649 (1991).

[10] L. E. Ibanez, Phys. Lett. B 318, 73 (1993).

[11] K. R. Dienes and A. E. Faraggi, Phys. Rev. Lett. 75, 2646 (1995); Nucl. Phys. B 457, 409 (1995); K. R. Dienes, A. E. Faraggi and J. March-Russell, Nucl. Phys. B 467, 44 (1996).

[12] P. Mayr and S. Stieberger, Phys. Lett. B 355, 107 (1995).

[13] H. P. Nilles and S. Stieberger, Phys. Lett. B 367, 126 (1996).

[14] S. P. Martin and P. Ramond, Phys. Rev. D 51, 6515 (1995).
[15] C. Bachas, C. Fabre and T. Yanagida, Phys. Lett. B 370, 49 (1996).
[16] J. Giedt, Mod. Phys. Lett. A 18, 1625 (2003).
[17] D. Emmanuel-Costa and R. Gonzalez Felipe, Phys. Lett. B 623, 111 (2005).
[18] J. Jiang, T. Li and D. V. Nanopoulos, arXiv:hep-ph/0610054.
[19] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38 1440 (1977); Phys. Rev. D16 1791 (1977); J. E. Kim, Phys. Rev. Lett. 43 103 (1979); M. Shifman, A. Vainshtein, V. Zakharov, Nucl. Phys. B166 493 (1980); A. R. Zhitnitskii, Sov. J. Nucl. Phys. 31 260 (1980); M. Dine, W. Fischler, M. Srednicki, Phys. Lett. B104 199 (1981).
[20] For reviews, see J. E. Kim, Phys. Rep. 150 1 (1987); H. Y. Cheng, Phys. Rep. 158 1 (1988); M. S. Turner, Phys. Rep. 197 67 (1991); G. G. Raffelt, Phys. Rep. 333 593 (2000); G. Gabadadze and M. Shifman, Int. J. Mod. Phys. A17 3689 (2002).
[21] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, ed. F. van Nieuwenhuizen and D. Freedman, (North Holland, Amsterdam, 1979) p. 315; T. Yanagida, Proc. of the Workshop on Unified Theory and the Baryon Number of the Universe, KEK, Japan, 1979; S. L. Glashow, in Quarks and Leptons, Cargese, eds. M. Levy, et al., July 9-29, 1979; S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[22] For examples, see P. Binetruy, M. K. Gaillard and Y. Y. Wu, Nucl. Phys. B 481, 109 (1996); Nucl. Phys. B 493, 27 (1997); M. K. Gaillard and B. D. Nelson, Nucl. Phys. B 571, 3 (2000).
[23] D. Emmanuel-Costa and R. G. Felipe, arXiv:hep-ph/0606012.
[24] S. Bethke, arXiv:hep-ex/0606035.
[25] W. M. Yao et al. [Particle Data Group], J. Phys. G 33, 1 (2006).
[26] G. Aldazabal, L. E. Ibanez and F. Quevedo, JHEP 0002, 015 (2000); for a review, see R. Bluemenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005).
[27] Y. Kawamura, Prog. Theor. Phys. 103, 613 (2000); G. Altarelli and F. Feruglio, Phys. Lett. B 511, 257 (2001); L. Hall and Y. Nomura, Phys. Rev. D 64, 055003 (2001); A. Hebecker and J. March-Russell, Nucl. Phys. B 613, 3 (2001); T. Li, Phys. Lett. B 520, 377 (2001); Nucl. Phys. B 619, 75 (2001).
[28] T. Li, Nucl. Phys. B 633, 83 (2002).
[29] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. 86, 4757 (2001); C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. D 64, 105005 (2001).
[30] T. Affolder et al. [CDF Collaboration], Phys. Rev. Lett. 84, 835 (2000); D. E. Morrissey and C. E. M. Wagner, Phys. Rev. D 69, 053001 (2004).