Superfluid and insulator of dirty Bose-Fermi mixture

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Abstract. One-dimensional Bose-Fermi mixture systems in a random potential are numerically investigated with focus on their superfluidity and localization properties. It is known that one-dimensional bosons and fermions behave differently in a weak random potential: Interacting bosons in a weak random potential exhibit superfluidity if the interactions are repulsive and not too strong, while interacting fermions with repulsive forces are localized in the presence of randomness. So the question is what happens if weakly interacting bosons and fermions are mixed in a random potential, which might be actually realized by the technique of trapping atoms on a random optical lattice. To answer this question we performed quantum Monte Carlo simulations of bosons and fermions on a one-dimensional lattice with random site energies. The result showed that when the boson-fermion interactions were attractive, both bosons and fermions tended to be localized. However when the boson-fermion interactions were repulsive and relatively weak, the superfluidity of bosons was enhanced and the fermions were delocalized.

1. Introduction
The realization of Bose-Einstein condensation in a single component dilute gas of alkali atoms has led to a variety of significant achievements, including trapping and cooling of fermions, formation of periodic potential with laser technology, and even production of random potentials. The comparison between pure boson and pure fermion systems has shed light on the role of quantum statistics, revealing quite distinctive properties of bosons and fermions. On the other hand, mixing of bosons and fermions in a confinement potential has opened up a new perspective of quantum liquids. In particular the interactions between the two species play a crucial role in the onset of new phenomena of mixed quantum liquids. We present in this article an example of such phenomena.

Introduction of random potential has further extended the possibility of cold atom systems[1] and has attracted attention in connection to solid state physics, where disorder effect has been extensively studied as one of the major topics. Disorder plays a critical role especially in low dimensional systems. The wave functions of free fermions in one dimension, for example, are known to be all localized in a random potential and the system is insulating. The localization property of free or interacting particles in one dimension was already well investigated by using the bosonization and renormalization group techniques[2]. Bosons and fermions respond to random potential differently. In particular the interactions between the two species play a crucial role in the onset of new phenomena of mixed quantum liquids. We present in this article an example of such phenomena.
to realize such a system in cold bosonic and fermionic atoms trapped in a wide one-dimensional confinement potential plus a random potential.

Focusing on the above problem, we studied the localization property of bosons and fermions mixed on a one-dimensional disordered lattice with repulsive interatomic interactions. Suppose that we have a one-dimensional lattice with random site energy and distribute bosons and fermions over it. Bosons tend to be delocalized by the boson-boson repulsive interactions, while fermions tend to be localized. Application of the boson-fermion interaction would change the situation drastically. To observe the behavior of this mixture system, we performed quantum Monte Carlo simulations of the boson-fermion mixture system and measured the localization property for different interparticle interactions. The simulation demonstrated delocalization of the fermions and enhancement of the superfluidity of the bosons. The paper is organized as follows. Section 2 is devoted to the detail of the model that we used for the simulation. In Sec. 3 we present the result of the numerical calculation and Sec. 4 is given for conclusions.

2. Model

We assume that \( N_f \) spinless fermions and \( N_b \) bosons are interacting on a one-dimensional \( N \)-site lattice with random site energy. The Hamiltonian that we employed is given by

\[
H = -t_f \sum_{\langle i,j \rangle} f_i^\dagger f_j - t_b \sum_{\langle i,j \rangle} b_i^\dagger b_j + \sum_i \epsilon_i (n_{fi} + n_{bi}) + U_{bb} \sum_i n_{bi}(n_{bi} - 1) + U_{ff} \sum_{\langle i,j \rangle} (n_{fi} - 1/2)(n_{fj} - 1/2) + U_{fb} \sum_i n_{fi}n_{bi},
\]

where \( b \) and \( f \) are boson and fermion operators respectively, and \( n_{bi} = b_i^\dagger b_i \) and \( n_{fi} = f_i^\dagger f_i \). In the following result we set the hopping energies to \( t_b = t_f = 1 \) to use as energy unit. With this Hamiltonian we performed quantum Monte Carlo simulations\([3, 4]\), fixing the inverse temperature to \( \beta = 6 \), the number of lattice sites to \( N = 16 \), and the number of the bosons to \( N_b = 10 \). For the random site energy \( \epsilon_i \) we assumed uniform probability in the range from -2.0 to 2.0. The fermion-fermion interaction \( U_{ff} \) is chosen to be 3.0 to realize moderate localization of the fermions. (Note that the repulsive fermion-fermion interaction enhances the localization tendency of fermions in one dimension.\([2]\))

In order to see the transport property of the particles, we measured the current-current correlation function defined by \( C_f(\omega) = \langle J(\omega)J(-\omega) \rangle \), where \( J(\omega) \) is the Fourier transform of current operator \( J(\tau) \) of the bosons or fermions with \( \tau \) being imaginary time in the path integral formalism. From the zero frequency limit of the correlation function, \( C_f(\omega \to 0) \), we obtain Drude weight for the fermions and superfluid density for the bosons\([4]\).

3. Result

Figures 1 and 2 show the superfluid density of the bosons and the Drude weight of the fermions, respectively, as functions of the fermion-boson interaction \( U_{fb} \). The circles present the data of 5 fermions, the crosses 6 fermions and the squares 10 fermions. The boson-boson interaction \( U_{bb} \) was fixed to 1.5.

The bosons are all Anderson localized in the presence of disorder when they are free. If weak boson-boson interactions are turned on, the bosons are promoted out of the localized states into extended states. This delocalization effect is exhibited in the finite superfluid density of the bosons at \( U_{fb} = 0 \) in Fig. 1. (With stronger boson-boson interactions, the bosons go into Bose glass phase\([4, 5]\).) The Drude weight of the fermions is zero in the absence of the fermion-boson interaction due to the localization of the wave functions. So we have delocalized bosons and localized fermions in the mixture before switching on the fermion-boson interactions.
The mixing effect is expected to arise when interactions are turned on between the bosons and the fermions. As we see in the figures, both the superfluid density and the Drude weight increase first and then decrease as the fermion-boson interactions increase. If we take a look at the detail, the superfluid density is enhanced but the Drude weight remains to be zero as far as $U_{fb}$ is small. The Drude weight becomes finite and increases as $U_{fb}$ gets larger. In the strong interaction regime, on the other hand, the increase of the fermion-boson interaction makes both the superfluid density of the bosons and the Drude weight of the fermions decrease to zero.

This behavior could be explained by considering the following scenario. When $U_{fb} = 0$ the fermions are localized in deep potential wells. As we turn on the fermion-boson interactions, the bosons see the potential wells shallower since the positive interaction energy with the localized fermions in the wells compensate the negative potential energy. This results in the up-rise of the superfluid density in the weak $U_{fb}$ region in Fig. 1. As the fermion-boson interactions become stronger, the bosons would kick the fermions out of the wells and the Drude weight of the fermions becomes finite as seen in Fig. 2. However, too strong $U_{fb}$ drives the system into a demixing phase, where the fermions and bosons reside in a separate areas to decrease the interaction energy, which was already pointed out in [6]. The demixing effect is the cause for the decrease of the superfluid density and the Drude weight in the large fermion-boson interaction region of Fig.2.

Figures 3 and 4 show the superfluid density of the bosons and the Drude weight of the fermions for different boson-boson interactions $U_{bb}$ (= 1.5 for the circles and 2.0 for the crosses), where the parameters are the same as those used in Figs. 1 and 2 except the number of the fermions which is fixed to 5 here. The overall behaviors in the figures are similar to what we have seen in Figs. 1 and 2. Namely, the superfluid density and the Drude weight increase with the fermion-boson interactions as far as the interactions are relatively weak, and decrease due to the demixing effect when the interactions become strong. The increase of the boson-boson interactions make the peak sharper, i.e. make the bosons and the fermions more sensitive to $U_{fb}$.

4. Conclusions
We numerically studied localization property of one-dimensional Bose-Fermi mixture in a weak random potential. It is known for one-dimensional systems that repulsive interboson
interactions work competitively against random potential to delocalize the bosons while repulsive interfermion interactions work cooperatively with random potential to strengthen the fermion localization. We mixed one-dimensional bosons and fermions in a random potential, turning on repulsive interaction between the bosons and the fermions, and performed quantum Monte Carlo simulations to see how the fermion-boson interactions affected the boson superfluidity of the bosons and the fermion localization.

The result of the simulation showed that, as we switched on the fermion-boson interactions, the superfluidity of the bosons was enhanced and the fermions remained in the Anderson insulator state. When the interactions increased further, the fermions began to be delocalized. The enhancement of the superfluidity of the bosons is caused by the fermions localized in the deep potential wells that make the potential energy for the bosons effectively weaker by the fermion-boson interaction. With strong fermion-boson interactions, the enhanced boson flow pushes the fermions out of the wells and the fermions are delocalized. The repulsive fermion-boson interactions thus have strong delocalization effect for the bosons and fermions and makes the system totally conductive, although too strong $U_{fb}$ demixes the fermions and bosons.

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