Introductions about Microstrip Couplers and Power Splitters

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Abstract. Microstrip couplers and power splitters are widely used electrical components in the field of microwave technology, which has the function of dividing one wave into multiple waves with same magnitudes but different phases. In this essay, 5 typical types of couplers and power splitters are introduced, with their applications, layout designs, effect on the wave, advantages and weaknesses. Couplers and power splitters are labelled with their output wave phase differences and their voltage magnitude transfer ratio. In this essay, all the couplers introduced are 3dB couplers, which means that the output voltage is decreased by 3dB. The Wilkinson power splitter has an output phase difference of 0 degree, the monolithic multilayer 2-line microstrip directional coupler, the Lange coupler, the microstrip branch-line coupler have output phase difference of 90 degrees, and the planar Marchand balun has output phase difference of 180 degrees. The microstrip rat-race coupler has 2 divider modes and 1 combiner mode, where in the divider mode the phase difference could be set as 0 degree or 180 degrees. Therefore, in the design of microwave circuits, the type of coupler should be carefully chosen according to the working characteristics, the structure features and the working conditions of couplers.

1. Introduction
Microstrip couplers and power splitters are electrical components in microwave technology. They have wide applications in Monolithic Microwave Integrated Circuit (MMIC) circuits, such as balanced amplifiers, power amplifiers, phase shifters and mixers [1].

The main function of a coupler, or power splitter, is to transfer power from one input to one or more different outputs. The power (normally in the form of plane wave) could be divided in different portions or different phases when transfers through the coupler, where the portion and the phase is determined by the design of the coupler. Therefore, in order to establish circuits with different purposes, the type of the microstrip coupler or power splitters should be carefully chosen.

In this essay, some typical types of microstrip couplers and power splitters including the Wilkinson power splitter, the 2-line microstrip directional coupler, the Lange coupler, the branch-line coupler and the rat-race coupler, would be introduced with their layout design, their influence on the wave, their condition for normal working, their drawbacks, advantages and applications.

2. Wilkinson power splitter
The Wilkinson power splitter is a type of microwave circuit with 3 ports (1 input and 2 outputs). The design of the Wilkinson power splitter on the substrate and its equivalent circuit diagram are shown below in Figure 1a and Figure 1b.
There is one impedance of magnitude $Z_0$ at the input terminal and two impedances of magnitude $Z_0$ at the output terminals [3]. The power of the input wave, is divided by the two transformers, where each one has an impedance are $\sqrt{2}Z_0$ and length of $\frac{\lambda}{4}$ ($\frac{\lambda}{4}$ of the wave length $\lambda$) [3]. Notice that there is a resistor of $2Z_0$ connected between the two output terminals, which provides an isolation between the two output ports, preventing the reflection of the wave [3].

The effect that the Wilkinson power splitter would perform on the wave can be derived by assuming the input voltage signal to be $1e^{j\theta}$:

$$V_{in} = 1e^{j\theta} \quad (1)$$

Where 1 represents the amplitude of the voltage signal, and $e^{j\theta}$ represents the signal phase of 0 degree in exponential form. In following sections, the input voltage signals may also be assumed in the form equation (1) shows, which would not be explained repetitively. The wave transmits through the upper $\frac{\lambda}{4}$ transformer, reaching the upper output terminal with a lag of $\frac{\lambda}{4}$, corresponding to the phase of $\theta_1 = 2\pi \times \frac{\lambda/4}{\lambda} = \frac{\pi}{2}$. Thus, the upper output voltage wave could be represented as:

$$V_{out1} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} \quad (2)$$

Similarly, since the length of the lower transformer is also $\frac{\lambda}{4}$, the lower output could be represented as:

$$V_{out2} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} \quad (3)$$

By comparing the two output waves, they are found to be of the same phase and the same magnitude, so the phase difference between the two outputs is 0 degree.

The power of the wave is proportional to the square of the voltage magnitude:

$$P_{out} \propto (V_{out})^2 \quad (4)$$

Since the voltage magnitude $|V_{out}| = \frac{1}{\sqrt{2}} |V_{in}|$, there is

$$P_{out} \propto \frac{1}{2} (V_{in})^2 \quad (5)$$
Therefore, the power is equally divided by half to either of the outputs. If the voltage magnitude transferring ratio \( \frac{V_{\text{out}}}{V_{\text{in}}} \) is transferred into dB, there is

\[
20 \log_{10} \left( \frac{1}{\sqrt{2}} \right) = -3 \text{dB}
\]

Therefore, the Wilkinson power splitter could be labelled as 0 degree, 3dB coupler.

There are two main advantages of the Wilkinson power splitter, which is its wide bandwidth and its low loss. The bandwidth of a Wilkinson power splitter is normally more than an octave, making it able to be working under a wide range of frequencies [1]. For the loss, the Wilkinson power splitter could have a loss of only 0.25dB at the center frequency (when the two output terminals are matched) [3, 4]. However, there is an obvious drawback of the Wilkinson power splitter, where it is necessary to connect a ballast resistor between the two output terminals, which restricts the design of the structure.

3. The monolithic multilayer 2-line microstrip directional coupler

The monolithic multilayer 2-line microstrip directional coupler is a typical design of directional couplers. The directional coupler is a type of microstrip components which combines or divides the microwave with a phase difference of 90 degrees [5]. The structure of monolithic multilayer microstrip directional coupler consists of conductors and dielectric films on substrate, such as a GaAs wafer surface [5].

The design of monolithic multilayer 2-line microstrip directional coupler is illustrated in Figure 2:

![Figure 2. The basic design of the monolithic multilayer 2-line directional coupler](image)

It is shown that there are 4 ports, where the input port (labelled as port 1 in the figure) is connected with the direct port (labelled as port 2 in the diagram) by a transformer of length \( \frac{\lambda}{4} \). The coupled port (labelled as port 3 in the figure) is connected with the isolated port (labelled as port 4 in the figure) by a transformer of length \( \frac{\lambda}{4} \). Additionally, an impedance of \( Z_0 \) is normally connected to the isolated port (port 4) and then connected to the ground, to provide isolation for port 4. The effect this coupler would make on the microwave transmits through could be analyzed by assuming the input voltage signal to be \( 1e^{j\theta} \):

\[
V_{\text{in}} = 1e^{j\theta}
\]

The power of the input wave will be dissipated uniformly by half, to the direct port and the coupled port respectively. By equation (5), there is \( V_{\text{out}} = \frac{1}{\sqrt{2}} e^{-j\theta} \), where \( \theta \) is the phase of the output voltage signal.
For the phase of the output wave, it has transmitted through the upper \( \frac{\lambda}{4} \) transformer. Thus, the wave outputs with a lag of \( \frac{\lambda}{4} \), corresponding to the phase of \( \theta_{\text{direct}} = 2\pi \times \frac{\lambda/4}{\lambda} = \frac{\pi}{2} \). Thus, the upper output voltage wave could be represented as: 

\[
V_{\text{out - direct}} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}} \quad (8)
\]

The wave generated by coupling effect at coupled port (port 3) can be expressed by: 

\[
V_{\text{out - coupled}} = \frac{1}{\sqrt{2}} e^{-j0} \quad (9)
\]

Note that there is no phase difference between the coupled wave and the input wave, since the wave is generated by coupling effect at the input port with no lag. The isolated port (port 4) is isolated with the \( Z_0 \) impedance, so there is no output at port 4.

By comparing the two output waves, they are same in magnitude but different in phase, where the phase difference is \( \frac{\pi}{2} \) (90 degrees). The voltage transferring magnitude ratio \( \frac{|V_{\text{out}}|}{|V_{\text{in}}|} \) in dB is:

\[
20\log_{10}(\frac{1}{\sqrt{2}}) = -3\text{dB} \quad (10)
\]

Therefore, the monolithic multilayer 2-line microstrip directional coupler could be labelled as 90 degrees, 3dB coupler.

The cross-sectional figure of the monolithic multilayer 2-line microstrip directional coupler is shown in Figure 3a, for a detailed analysis of its structure design. An example picture is shown in Figure 4a:

**Figure 3a.** The cross-sectional structure of the monolithic multilayer 2-line microstrip directional coupler [1]

**Figure 4a.** A example of the monolithic multilayer 2-line microstrip directional coupler [1]

In Figure 3a, there are two conductor tracks on the GaAs substrate, separated by a dielectric material, where there is a horizontally overlap region of these two tracks. The design of overlap
region is for achieving a better coupling effect between the two lines. The overlap region is
marked out in Figure 3b.

![Track 1 and Track 2](image)

**Figure 3b.** The overlap region (marked by red circle)

In Figure 4b, the two tracks and the four ports of the monolithic multilayer 2-line microstrip
directional coupler are marked out.

![4 ports and 2 tracks](image)

**Figure 4b.** The 4 ports and 2 tracks

The advantage of the monolithic multilayer 2-line microstrip directional coupler is that it has a
very wide operational bandwidth [6]. However, it has a drawback of lacking symmetry in the
design, which is solved by the Lange coupler.

4. **The lange coupler**

The Lange coupler is another typical type of directional coupler, which is derived from the
monolithic multilayer 2-line microstrip directional coupler [7]. Its structure design is illustrated
below in Figure 5:

![Layout design of the Lange coupler](image)

**Figure 5.** The layout design of the Lange coupler [1]

Two example pictures of the Lange coupler are demonstrated below in Figure 6a and Figure 6b:
The derivation procedure from a 2-line microstrip directional coupler to a Lange coupler is shown below:

In step (a) to (b), both the two strip lines in the original 2-line directional coupler is divided by half, forming 4 new strip lines, where the positions of the strip lines are rearranged for a better coupling effect. In step (b) to (c), the lowest strip line is cut by half, and the right half is placed above the highest strip line, where the final design of the Lange coupler is formed. This design is both electrically and mechanically symmetrical, and the extent for isolation between the input port, the isolated port and the two output ports is very high [8]. The effect the Lange coupler would perform on the wave can be analyzed by assuming the input voltage signal to be $1e^{j\theta}$ and the isolated port shorted.

$$V_{in} = 1e^{j\theta} \quad (11)$$

The effect on the wave by the Lange coupler is similar to that of the monolithic multilayer 2-line directional microstrip coupler. The power of the input wave is uniformly distributed into the two output ports. For the phases, the wave at the direct port would have a phase lag of 90 degrees, while at the coupled port, there is no phase lag [8]. There is no output at the isolated port, because it is shorted.

$$V_{out\_direct} = \frac{1}{\sqrt{2}} e^{-j\pi/2} \quad (12)$$

$$V_{out\_coupled} = \frac{1}{\sqrt{2}} e^{-j\theta} \quad (13)$$
By comparing the two output waves, they share the same magnitude, but differ in phase by $\frac{\pi}{2}$ (90 degrees). The voltage transferring magnitude ratio $\frac{V_{\text{out}}}{V_{\text{in}}}$ in dB is:

$$20 \log_{10} \left( \frac{1}{\sqrt{2}} \right) = -3dB$$  \hspace{1cm} (14)

Therefore, the Lange coupler can be labelled as 90 degrees, 3dB coupler.

The Lange coupler has several advantages. The first one is its broadband, which can be at least 4 GHz, making it able to work with a wide range of frequencies [7]. The second one is its coupling effect of high degree, which is proportional to the capacitance in the coupler. By comparing the Lange coupler with the 2-line directional coupler, the capacitance has doubled in the Lange coupler, providing the Lange coupler with a better coupling effect [1]. The third advantage is its safety of avoiding problems that might be caused by reflection waves. When the Lange coupler is working properly, the reflection waves back into the output from the circuit connected to the output port will flow to the isolated port, or cancel at the input port, preventing the source (connected to the input port) from the harm of the reflection waves [8]. Nevertheless, there are several drawbacks existing in the manufacture of the Lange coupler. The high loss of the Lange coupler has always been a problem. The figure of the power loss against a variation of frequency from 5GHz to 20 GHz is illustrated in Figure 8:

![Figure 8. The power loss against a variation of frequency [10]](image)

Where in this figure, it is shown that the loss at the coupling and the direct port is approximately -4dB, and that of the isolated port is smaller than -20dB for most frequencies. Therefore, the loss at the two output ports of the Lange coupler is relatively high.

There is also difficulty in the fabrication process of the Lange coupler. For a single Lange coupler, there are at least 4 pieces of bond wires necessarily required, whose fabrication process is of high difficulty [1]. Additionally, the distances between the tracks are very narrow, which is required to be highly precise, resulting in the high difficulty of patterning the tracks [1]. By comparison with the manufacture of the Wilkinson power splitter and the monolithic multilayer 2-line directional microstrip coupler, the demand of the fabrication of the Lange coupler is much higher.

5. The planar marchand balun
The planar Marchand balun is a further design based on the Lange coupler, which is a 3-port element which converts one unbalanced input signal into two balanced output signals [11]. It has applications in microwave integrated circuits and monolithic microwave integrated circuits, such as balanced mixer, antenna-feed networks and push-pull amplifier [12-14]. The structure of the planar Marchand balun is demonstrated below in Figure 9.

![Figure 9. The structure of Marchand balun [15]](image)

For a planar Marchand balun, the coupling structure between port 1 (the input port) and port 2 is a Lange coupler, and the coupling structure between port 1 (the input port) and port 3 is also a Lange coupler. The port on the upper-right corner of Figure 9 is left open circuit, marked as O.C.

An example picture of a planar Marchand balun, as well as the same picture with the ports marked, are illustrated below in Figure 10a and Figure 10b:

![Figure 10a. The planar Marchand balun](image)

![Figure 10b. The labeled ports and the feature of the Lange couplers (in red circle)](image)

The effect on the transmission wave could be analyzed by assuming the input voltage signal to be $V_{in} = 1e^{j\theta}$. 

$$V_{in} = 1e^{j\theta} \quad (15)$$

For the output waves, the power of the input wave is uniformly distributed into the two output ports, so by equation (5), the amplitude for each output wave is $\frac{1}{\sqrt{2}}$. In terms of the phase for port 2, the wave transmits by the transformer whose length is $\frac{\lambda}{4}$ and then outputs at port 2 with a lag of $\frac{\lambda}{4}$, corresponding to the phase of $\theta_2 = 2\pi \times \frac{\lambda}{4} = \frac{\pi}{2}$. Thus, the output voltage signal at port 2 could be represented as:

$$V_{out2} = \frac{1}{\sqrt{2}} e^{-j\pi/2} \quad (16)$$
For the wave outputs at port 3, it transmits by the transformer whose length is $\frac{\lambda}{4}$ and outputs with a lag of $\frac{\lambda}{4}$, corresponding to the phase of $\theta_3 = 2\pi \times \frac{\lambda/4}{\lambda} = \frac{\pi}{2}$. However, notice that the propagation direction of the coupled wave at port 3 is opposite to the original input wave, so the phase in the expression of the voltage signal has an opposite sign. Thus, the output voltage signal at port 3 can be represented as:

$$V_{out3} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}}$$ (17)

By comparing $V_{out2}$ and $V_{out3}$, these two voltages are same in magnitude but different in phase by 180 degrees. The voltage transferring magnitude ratio $\frac{V_{out}}{V_{in}}$ in dB is:

$$20\log_{10}(\frac{1}{\sqrt{2}}) = -3dB$$ (18)

Therefore, the planar Marchand balun can be labelled as 180 degrees, 3dB coupler. The planar Marchand balun has a broadband, which means that it is able to be working within a wide operating frequency range [16]. Moreover, the stable balance property of the planar Marchand balun could also be regarded as a strength in function [16]. Nonetheless, there exist problems of high loss in the Marchand balun, mainly due to the high loss of the coupled line section [11]. The curve of loss against the frequency is demonstrated below in Figure 11.

![Figure 11. The return loss of a Marchand balun with different air-cavity [15]](image)

The loss of the Marchand balun is larger than -10dB for a wide range of frequencies, reaching the peak value of approximately -1dB, which is very high loss.

6. The microstrip branch-line coupler

The microstrip branch-line coupler is a 4-port coupler which works on the principle of interference [17]. It has applications in balanced mixers, phase shifters, power splitters and frequency discriminators [17]. The layout of the microstrip branch-line coupler is illustrated below:
The microstrip branch-line coupler has 4 ports, including the input port (port 1), the direct output (port 2), the coupled port (port 3) and the isolated port (port 4). The impedances $z_0$ in the layout is usually around the value of 200Ω [17]. Some example pictures of the are demonstrated below:

The effect on the transmission wave could be analyzed by assuming the input voltage signal to be $V_{in} = 1e^{j0}$. 

$$V_{in} = 1e^{j0} \quad (19)$$

The power of the input wave is equally divided in magnitude by half to either of the outputs. Therefore, by equation (5), the amplitude for each output wave is $\frac{1}{\sqrt{2}}$. For the phase of the output at port 2, it has transmitted through the upper $\frac{\lambda}{4}$ transformer. Thus, the wave outputs with a lag of $\frac{\lambda}{4}$, corresponding to the phase of $\theta_{-direct} = 2\pi \times \frac{\lambda/4}{\lambda} = \frac{\pi}{2}$. Thus, the output voltage signal is:

$$V_{out\ -direct} = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{2}} \quad (20)$$

For the output at port 3, it is actually the sum of two waves transmitted by two different paths. The two paths are shown in Figure 15. Therefore, the output wave at port 3 is the sum of the waves propagated in the red path and in the green path, where there exists interference between these two waves. Since both the two

**Figure 12.** The layout of the microstrip branch-line coupler [1]

**Figure 13.** The microstrip branch-line coupler example 1 [1]

**Figure 14.** The microstrip branch-line coupler example 2 [18]
waves have transmitted through the transformer of length \( \frac{\lambda}{4} \times 2 = \frac{\lambda}{2} \), the wave outputs with a lag of \( \frac{\lambda}{2} \), corresponding to the phase of \( \theta_{\text{coupled}} = 2\pi \times \frac{\lambda/2}{\lambda} = \pi \). Thus, the output voltage signal at coupled port is:

\[
V_{\text{out - coupled}} = \frac{1}{\sqrt{2}} e^{-j\pi} \quad (21)
\]

The wave at the isolated port (port 4) is also the sum of two waves transmitted by two different paths. The two paths are shown below in Figure 16.

The wave which propagates by the green path transmits through the transformer of length \( \frac{\lambda}{4} \), corresponding to the phase of \( \theta_{\text{iso - 1}} = 2\pi \times \frac{\lambda/4}{\lambda} = \frac{\pi}{2} \). The wave propagates by the red path transmits through the transformer of length \( \frac{\lambda}{4} \times 3 = \frac{3\lambda}{4} \), corresponding to the phase of \( \theta_{\text{iso - 2}} = 2\pi \times \frac{3\lambda/4}{\lambda} = \frac{3\pi}{2} \). Therefore, these two waves always have a phase difference of \( \frac{3\pi}{2} - \frac{\pi}{2} = \pi \). Assume this coupler to be ideal, these two waves should be same in magnitude. Therefore, by the phase difference of \( \pi \), these two waves always cancel each other, resulting in the output at the isolated port to be 0. By comparing equations (20) and (21), the phase difference is 90 degrees.

The voltage transferring magnitude ratio \( \frac{V_{\text{out}}}{V_{\text{in}}} \) in dB is:

\[
20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = \sim 3dB \quad (22)
\]

Therefore, the microstrip branch-line coupler can be labelled as 90 degrees, 3dB coupler. In comparison with the Wilkinson power splitter, the microstrip branch-line coupler is not restricted by the isolation impedance in the design of structure. Compared to the directional couplers, including the monolithic multilayer 2-line microstrip directional coupler, the Lange coupler and the planar Marchand balun, the microstrip branch-line coupler is not required for
patterning the tracks or bond wires. Therefore, the structure design of the microstrip branch-line coupler can be more flexible. The impedances could be designed in a twisted path, such as the one Figure 13 shows, which is hard for directional couplers [1]. Another advantage of the microstrip branch-line coupler is that current flows through the coupler can be larger than directional couplers. To put it differently, it can be applied for higher power applications, and its loss is relatively small [1]. The microstrip branch-line coupler has wider tracks, compared to the Lange coupler, making the fabrication process easier than that of the Lange coupler. Nonetheless, the wider tracks could also be the weakness of the microstrip branch-line coupler. Simple design and wider tracks lead to larger component area, which is contrary to the design principle of integrated circuits that the area of the circuit should be as small as possible. In addition, there exists serious narrow band phenomena in the working of microstrip branch-line coupler. In other words, it is only suitable for narrow-band applications. The bandwidth for normal working of the microstrip branch-line coupler is only around 15%, limited in the range of 10%-20% [1, 17].

7. The microstrip rat-race coupler

The microstrip rat-race coupler is a 4-port coupler designed based on the branch-line coupler, with an extra length $\frac{\lambda}{2}$ of the line [1]. It has a wide application in active circuit design where a high symmetry of circuit is required [19]. The layout of the microstrip rat-race coupler is illustrated below in Figure 17.

Comparing the structure of the microstrip rat-race coupler with the 4-port branch-line coupler, it can be found that the length of the line connecting the isolated port and the coupled port is $\frac{3\lambda}{4}$ instead of $\frac{\lambda}{4}$. Two example pictures of the rat-race coupler are illustrated below in Figure 18 and Figure 19.
The highlight design of the microstrip rat-race coupler is that it has two operational modes, including the divider mode and the combiner mode, which is determined by the input waves and ports [1]. The divider mode can also be used in two methods, where the first one is named as the sum input. In this method, use port 1 as the input port, port 2 as the coupled port, port 3 as the direct port and port 4 as the isolated port (the numbers marked on the ports are in reference to Figure 17). In this mode it is a 1-input and 2-output coupler. The effect this mode of the coupler that would perform on the input wave could be analyzed by assuming the input voltage signal to be $1e^{j\theta}$.

$$V_{in} = 1e^{j\theta}$$  \hspace{1cm} (23)

The power of the input wave is equally divided in magnitude by half to either of the outputs. Therefore, by equation (5), the amplitude for each output wave is $\frac{1}{\sqrt{2}}$. For the phases of the outputs at the direct port (port 3) and the coupled port (port 2), they have both transmitted through $\frac{\lambda}{4}$ transformers. Thus, the wave outputs with a lag of $\frac{\lambda}{4}$, corresponding to the phase of $\frac{\lambda}{4} \times \frac{\lambda}{4} = \frac{\pi}{2}$. Thus, the output voltage signal at the direct output and the coupled port could be represented as:

$$V_{out \text{- direct}} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}}$$  \hspace{1cm} (24)

$$V_{out \text{- coupled}} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}}$$  \hspace{1cm} (25)

For the isolated port, it is the sum of two waves transmitted by two different paths of lengths $\frac{\lambda}{4}$ and $\frac{\lambda}{2}$ respectively, corresponding to a phase difference of $\pi$. Therefore, the waves cancel with each other at the isolated port, leaving the output 0. By comparing equations (24) and (25), there is no phase difference.

The voltage transferring magnitude ratio $\frac{|V_{out}|}{|V_{in}|}$ in dB is:

$$20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3dB$$  \hspace{1cm} (26)
Therefore, the microstrip rat-race coupler in sum input divider mode can be labelled as 0 degree, 3dB coupler.

The second method of the divider mode is named as the difference input. In this method, use port 4 as the input port, port 2 as the coupled port, port 3 as the direct port and port 1 as the isolated port (the numbers marked on the ports are in reference to Figure 17). In this mode it is a 1-input and 2-output coupler. The effect this mode of the coupler that would perform on the input wave could be analyzed by assuming the input voltage signal to be $1e^{j0}$.

$$V_i = 1e^{j0} \quad (27)$$

The power of the input wave is equally divided in magnitude by half to either of the outputs. Therefore, by equation (5), the amplitude for each output wave is $1/\sqrt{2}$. For the phase of the output at the direct port (port 3), it has transmitted through a $\frac{\lambda}{4}$ transformer, then outputs with a lag of $\frac{\lambda}{4}$, corresponding to the phase of $\theta_d = 2\pi \times \frac{\lambda/4}{\lambda} = \frac{\pi}{2}$. Thus, the output voltage signal at the direct output could be represented as:

$$V_{out\cdot direct} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}} \quad (28)$$

For the output at the coupled port (port 2), it has transmitted through $\frac{3\lambda}{4}$ transformers. Thus, the wave outputs with a lag of $\frac{3\lambda}{4}$, corresponding to the phase of $\theta_c = 2\pi \times \frac{3\lambda/4}{\lambda} = \frac{3\pi}{2}$. Thus, the output voltage signal at the coupled port could be represented as:

$$V_{out\cdot coupled} = \frac{1}{\sqrt{2}} e^{-j\frac{3\pi}{2}} or \frac{1}{\sqrt{2}} e^{j\frac{\pi}{2}} \quad (29)$$

For the isolated port, it is the sum of two waves transmitted by two different paths of lengths $\frac{\lambda}{2}$ and $\lambda$ respectively, corresponding to a phase difference of $\pi$. Therefore, the waves cancel with each other at the isolated port, leaving the output 0. By comparing the two output waves, there is a phase difference of 180 degrees. The voltage transferring magnitude ratio $\frac{|V_{out}|}{|V_i|}$ in dB is:

$$20\log_{10}(\frac{1}{\sqrt{2}}) = -3dB \quad (30)$$

Therefore, the microstrip rat-race coupler in difference input divider mode can be labelled as 180 degrees, 3dB coupler.

The other mode of the microstrip rat-race coupler is the combiner mode. In this mode, use port 2 as the input A port, port 3 as the input B port, port 1 as the sum output port and port 4 as the difference output port (the numbers marked on the ports are in reference to Figure 17). In this mode it is a 2-input and 2-output coupler. The effect this mode of the coupler that would perform on the input wave could be analyzed by assuming the two input voltage signals to be:

$$V_{in\cdot A} = |V_{in\cdot A}|e^{j0} \quad (31)$$
$$V_{in\cdot B} = |V_{in\cdot B}|e^{j0} \quad (32)$$
For the output at the sum output port (port 1), both of the two waves have transmitted through \( \frac{\lambda}{4} \) transformers. Thus, the wave outputs with a lag of \( \frac{\lambda}{4} \), corresponding to the phase of \( \frac{\pi}{2} \). Thus, the output voltage signals could be represented as:

\[
V_{\text{out}_{\text{sum}}} = V_{\text{out}_A} + V_{\text{out}_B} = \frac{1}{\sqrt{2}} (|V_{\text{in}_A}| e^{j\frac{\pi}{2}} + |V_{\text{in}_B}| e^{-j\frac{\pi}{2}})
\]  

(33)

For the output at the output port (port 4), the wave from input A has transmitted through \( \frac{3\lambda}{4} \) transformers. Thus, the wave outputs with a lag of \( \frac{3\lambda}{4} \), corresponding to the phase of \( \frac{3\pi}{2} \). The wave from input B has transmitted through \( \frac{\lambda}{4} \) transformers, outputs with a lag of \( \frac{\lambda}{4} \), corresponding to the phase of \( \frac{\pi}{2} \). Thus, the output voltage signals could be represented as:

\[
V_{\text{out}_{\text{difference}}} = V_{\text{out}_A} - V_{\text{out}_B} = \frac{1}{\sqrt{2}} (|V_{\text{in}_A}| e^{j\frac{\lambda}{2}} + |V_{\text{in}_B}| e^{-j\frac{\lambda}{2}})
\]  

(34)

The capability of switching modes with different operational effects on the waves is the highlight design of the microstrip coupler. In addition, similar to the branch-line coupler, the structure design of the microstrip coupler is not restricted with the isolation resistance or bond wires, making the design much more flexible [1].

The disadvantage of the microstrip coupler is also similar to the branch-line coupler, which is its narrow fractional bandwidth. The bandwidth is limited by a maximum of 15%, making the microstrip coupler only able to work within a very small range of frequencies [1]. There is also a drawback in the manufacture of the microstrip coupler. The tracks of the microstrip coupler are narrower those that of the branch-line coupler, whose fabrication process can be of high difficulty. In addition, the extra \( \frac{\lambda}{2} \) length on the line would occupy a large area of the circuit.

However, this problem could be solved by replacing the \( \frac{3\lambda}{4} \) line with the combination of a \( \frac{\lambda}{4} \) line and a 180 degrees inverter [20].

8. Conclusion

In conclusion, this dissertation mainly introduces 5 classic types of couplers (or power splitters) with their applications, layout structures, effect on the wave, advantages and weaknesses. All the couplers are labelled by the phase difference between outputs and the magnitude. Summarizing all the couplers introduced in this paper, the Wilkinson power splitter is a 0 degree, 3dB coupler, broad in bandwidth but restricted in design due to the need for isolation impedance; the monolithic multilayer 2-line microstrip directional coupler is a 90 degrees, 3dB coupler, broad in bandwidth but lack in symmetry in design; the Lange coupler is a 90 degrees, 3dB coupler, broad in bandwidth and good in coupling effect, but very lossy and hard for manufacturing; the planar Marchand balun is a 180 degrees, 3dB coupler, broad in bandwidth and good in coupling effect, but very lossy and hard for manufacturing; the microstrip branch-line coupler is a 90 degree, 3dB coupler, flexible in structure design, easy for
fabrication, low loss but very small bandwidth and too large area; and the microstrip rat-race
coupler can be used as a 0 degree, 3dB divider coupler, or a 180 degrees, 3dB divider coupler,
or a combiner with a sum output and a difference output, highlight for its various modes but
small in bandwidth and hard to fabricate. In the microwave circuit design, the application of
couplers should be carefully chosen, according to their performances, structure features as well
as working conditions.

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