Two-loop QCD corrections to charged-Higgs-mediated $b \to s\gamma$ decay

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Abstract

The charged-Higgs-mediated contribution to the Wilson coefficient of the $b \to s\gamma$ magnetic penguin is expected to be one of the more promising candidates for a supersymmetric effect in $B$ physics, probably the only one in gauge-mediated models. We compute the two-loop QCD correction to it. With naïve dimensional regularization and $\overline{MS}$ subtraction, for reasonable values of the charged Higgs mass and for $\bar{\mu} = m_t$, we find a $(10 \div 20)\%$ reduction of the corresponding one-loop effect.

1 Introduction

In supersymmetric models with sparticle masses mediated by supergravity [1], physics at the unification scale leaves its imprint in the supersymmetry breaking terms. As a consequence we expect that the sfermion mass matrices contain new sources of flavour and CP violation, either due to unification physics [2] or related to the generation of the flavour structure itself [3]. In this scenario it is quite possible that the effects due to virtual sparticle exchanges (mostly $\mu \to e\gamma$, $\mu \to e$ conversion in atoms, electric dipoles of the electron and of the neutron, CP-violation in the $K$-system and $B$-systems), will be discovered even before than the sparticles themselves.

Alternatively, in supersymmetric models, like Gauge-Mediated (GM) models, where the supersymmetry-breaking soft terms are generated at a lower scale where non-MSSM physics has decoupled, we expect that the only flavour violation present at low energy be the supersymmetrized extension of the standard CKM matrix. In this scenario the new supersymmetric flavour violating interactions (mainly the ones at charged Higgs and higgsino vertices, present in any realistic supersymmetric extension of the SM) are essentially unrelated to supersymmetry breaking and their flavour structure is controlled by the same CKM matrix. Consequently they do not introduce any new CP-violating phase nor they affect leptons, so that supersymmetry gives contributions only to ‘standard’ flavour and/or CP violating effects. These effects can nevertheless lead to detectable deviations from the expectations of the standard model in a few crucial observables in flavour physics, mainly in the $b \to s\gamma$ and $b \to s\ell^+\ell^-$ decays [4, 5]. Supersymmetric effects in other processes are less interesting, because the SM background is larger and/or plagued by larger QCD uncertainties.

From the point of view of the effective theory below the Fermi scale, in this scenario supersymmetry is expected to give a detectable correction to the Wilson coefficient, $C_7$, of the $b \to s\gamma$ magnetic penguin operator, usually named $O_7$. This coefficient can be extracted from B.R.$(b \to s\gamma)$ with a $\sim 10\%$ theoretical uncertainty [6]. Alternatively it can be deduced from the spectrum of $b \to s\ell^+\ell^-$ decays away from resonances. These decays have a lower branching ratio (around $6 \cdot 10^{-6}$ for $\ell = e, \mu$ and smaller for $\ell = \tau$) than $b \to s\gamma$. Consequently, for a precise determination of $C_7(\bar{\mu} \approx m_t)$ one needs to wait for sufficient statistics. However, with data from $5 \cdot 10^8 BB$ pairs, the $1\sigma$ error on $C_7$ will again be slightly larger than $\pm 0.01$ [7].
In the SM the perturbative QCD uncertainties in B.R.$(b \to s\gamma)$ have been reduced from $\sim 30\%$ to $\sim 5\%$ computing the full Next-to-Leading-Order (NLO) QCD corrections. This computation can be divided in three steps: (i) at some scale $\mu$ close to the electroweak scale, the couplings of the effective theory (containing only the light degrees of freedom) are determined up to $\mathcal{O}(\alpha_3)$ \cite{4,5} in such a way that the full and effective theories describe equivalent physics; (ii) the effective theory is evolved via RGE techniques at NLO from the electroweak scale down to the $B$ scale \cite{4}; (iii) at the $B$ scale the matrix elements for the $b \to s\gamma$ process are computed with $\mathcal{O}(\alpha_3)$ precision \cite{11}. With $\overline{\text{MS}}$ subtraction, and in the ‘naïve dimensional regularization’ (NDR) scheme commonly employed, these three parts are of comparable numerical importance \cite{10}.

In the scenario where supersymmetry affects the low energy theory only giving extra contributions to the same Wilson coefficients that describe the SM physics, ‘only’ the step (i) is missing for a complete computation of supersymmetric corrections at NLO precision. We do this computation in the case of the charged Higgs mediated correction to $C_7$. In the next section we explain why this correction is of particular interest. In section 3 we collect the ingredients for the computation. The computation in the full theory is presented in section 4, and the one in the effective theory in section 5. The final result is given in section 6. For a reasonable Higgs mass, $m_H \sim 2m_t$, we find that in the NDR scheme the one-loop charged-Higgs contribution, evaluated at $\bar{\mu} = m_t$, is reduced by two-loop QCD corrections by 15%.

2 Relevant supersymmetric effects

Once that $C_7$ will be measured with sufficient precision, the accurate SM computations could say that some new physics is required. Needless to say, however, as always in the case of radiative corrections effects, the discovery of non SM effect would not allow an immediate identification of its physical origin. The supersymmetric effects, in particular, depend on many unknown parameters\cite{2,5} the charged Higgs contribution depend on $m_H$ and $\tan \beta$; the chargino contribution (at one loop) depend on the $\mu$-parameter, on the weak gaugino masses, on the trilinear term of stop squark, $A_t$, and on the up-type squark masses. So, it does not seem useful to replace an approximate function of many unknown parameters with a more precise function (the NLO chargino contribution depends also on the masses of various supersymmetric particles in a non-decoupling way, if the couplings $\lambda_t, \lambda_b, g_2$ at chargino vertices are expressed in terms of measured SM parameters).

To discuss this point and to assess the relative importance between the different contributions we need to look in more detail at the one-loop predictions. In the simplifying limit $m_H \gg m_t, m_\tilde{e} \gg m_\tilde{\chi} \gg \mu \gg M_W$ the relevant contributions from Standard Model (SM), charged Higgs ($H$) and charginos ($\chi$) to the Wilson coefficients $C_7$ of the $b \to s\gamma$ magnetic penguin, and $C_8$ of the $b \to s\gamma$ chromo-magnetic penguin are

\[
C_7^{\text{SM}} \approx -0.2 \\
C_7^H \approx - \frac{m_t^2}{2m_H^2} \left( \frac{7}{36} \tan^2 \beta + \frac{2}{3} \ln \frac{m_H^2}{m_t^2} - \frac{1}{2} \right) \\
C_7^\chi \approx \frac{m_t^2}{2m_t^2 \sin^2 \beta} \left( \frac{2}{9} + \frac{\mu (A_t \tan \beta + \mu)}{m_t^2} \ln \frac{m_t^2}{\mu^2} - \frac{13}{6} \right) \\
C_8^{\text{SM}} \approx -0.1 \\
C_8^H \approx - \frac{m_t^2}{2m_H^2} \left( \frac{1}{6} \tan^2 \beta + \ln \frac{m_H^2}{m_t^2} - \frac{3}{2} \right) \\
C_8^\chi \approx \frac{m_t^2}{2m_t^2 \sin^2 \beta} \left( \frac{1}{12} - \frac{1}{2} \frac{\mu (A_t \tan \beta + \mu)}{m_t^2} \right)
\]

We see that the charged Higgs gives an important correction, with the same sign of the SM contribution. For example, even a heavy charged Higgs with mass $m_H = 700$ GeV gives a 10% enhancement of B.R.$(b \to s\gamma)$ over the SM prediction. The chargino contribution is relevant if a stop state is relatively light and the other up-squarks are heavier. A possible gluino/squark contribution is at most few % than the SM result, even in presence of new CKM-like mixing matrices with elements of the order of the CKM-ones \cite{4,5}. A possible neutralino/squark contribution is even more negligible.

Moreover, the scenario where $b \to s\gamma$ seems the most interesting candidate for a detectable supersymmetric effect can naturally be realized in gauge-mediation models \cite{3}. In these quite predictive models the chargino-up-squark loops do not give large contributions, unless $\tan \beta$ is large \cite{4}. This happens because in a typical gauge mediated spectrum the squarks are rather heavy (their mass terms are mediated by strong gauge interactions) and ‘rather’ degenerate among them (gauge interactions are generation universal), so that chargino/squark contributions are small and GIM-suppressed. Consequently, in gauge-mediation scenarios, the most (and, probably, only) interesting supersymmetric effect in $B$ physics is given by a charged-Higgs/top correction to the $b \to s\gamma$ magnetic penguin. This extra contribution depends only on $m_H$ and $\tan \beta$, and more precisely:

\footnote{We use standard notations for the supersymmetric parameters; in particular $m_H$ is the charged Higgs mass, $\tan \beta$ is the ratio between the vacuum expectation values of the two Higgs fields, and $\mu$ is the ‘$\mu$-parameter’, not to be confused with the MS scale $\bar{\mu}$.}
• The dependence on \( \tan \beta \) of the charged-Higgs-mediated magnetic penguin is very weak. As clear from [4], the term suppressed by \( 1/\tan^2 \beta \) is very small already for \( \tan \beta \gg 2 \). We remember that smaller values of \( \tan \beta \), close to one, are unnatural in the successful scenario of radiative breaking of the electroweak symmetry [4], and make difficult to accommodate the large value of the top mass.

• The charged Higgs mass \( m_H \) is not predicted by gauge-mediation scenarios; however the mass parameters of the Higgs potential determine the electroweak scale, so that (unless \( \tan \beta \) is large), barring unnatural accidental cancellations, the charged Higgs should not be much heavier than the Z boson.

So, the charged-Higgs mediated contribution depends almost uniquely on the charged Higgs mass and has defined sign. For these reasons we think that it is interesting to have a more accurate determination of the term not suppressed by \( 1/\tan^2 \beta \), that arise from graphs that contain one vertex \( \lambda_b \, b_R \, L \, H_- \).

More generally this computation also applies to two-Higgs doublet models, where the charged Higgs mediated contribution can be the only correction to the SM prediction.

3 Preparation for the computation

To be more precise, and to conform to the standard notations, we recall the effective Hamiltonian for the \( b \to s \gamma \) decay

\[
\mathcal{H}_{\text{eff}} = -\frac{g_2^2}{2 M_W^2} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i \frac{O_i}{4}.
\]

We have approximated \( V_{tb} V_{ts}^* \approx -V_{tb} V_{ts}^* \) and, in the limit \( m_s \to 0 \),

\[
\begin{align*}
O_1 & = 4 \bar{s} \gamma_\mu P_L c_i \bar{b} \gamma^\mu P_L b_j \\
O_2 & = 4 \bar{s} \gamma_\mu P_L c_i \bar{b} \gamma^\mu P_L b_j \\
O_3 & = 4 \bar{s} \gamma_\mu P_L b_i \bar{b} \gamma^\mu P_L c_j \\
O_4 & = 4 \bar{s} \gamma_\mu P_L b_i \bar{b} \gamma^\mu P_L c_j \\
O_5 & = 4 \bar{s} \gamma_\mu P_L b_i \bar{b} \gamma^\mu P_R c_j \\
O_6 & = 4 \bar{s} \gamma_\mu P_L b_i \bar{b} \gamma^\mu P_R c_j \\
O_7 & = 4 \frac{g_3}{(4\pi)^2} m_b \bar{s} \gamma_\mu \gamma^\nu P_R b \gamma^\nu P_L c \\
O_8 & = 4 \frac{g_3}{(4\pi)^2} m_b \bar{s} \gamma_\mu \gamma^\nu P_R b \gamma^\nu P_L c
\end{align*}
\]

where \( i, j \) are colour indexes, and the sum is over all light quarks \( q = \{ b, c, s, u, d \} \). The relevant Wilson coefficients at the electroweak scale, in LO approximation, are

\[
\begin{align*}
C_{2}^{\text{SM}} & = 1 \\
C_{7}^{\text{SM}} & = \frac{3}{2} \left( q_u P_{FE} + q_W P_{BE} \right) \\
C_{8}^{\text{SM}} & = \frac{3}{2} P_{FE}
\end{align*}
\]

\[
\begin{align*}
C_{2}^{H} & = 0 \\
C_{7}^{H} & = \frac{1}{2} \left( q_u P_{FI} + q_W P_{BI} \right) + \frac{(q_u P_{FE} + q_W P_{BE})}{\tan^2 \beta} \\
C_{8}^{H} & = \frac{1}{2} \left( P_{FI} + \frac{P_{FE}}{\tan^2 \beta} \right)
\end{align*}
\]

At leading order the magnetic penguin coefficients \( C_7 \) and \( C_8 \) are given by the value of the corresponding penguin diagrams, without corrections from the matching procedure. The magnetic penguin loop-functions \( P_{F/E,B,I/E}(r) \), that we will also employ to write the divergent part of our two-loop Feynman graphs, are listed in the appendix. The indices \( F \) or \( B \) indicate if the external photon (or gluon) is attached to a Fermion or a Boson. The indices \( E \) or \( I \) indicate if the helicity flip factor, \( m_b \), comes from the External b leg, or from an Internal vertex of the graph. The parameter \( r \equiv m_B^2/m_F^2 \) is the ratio between the (squared) masses of the boson \( B \) and of the fermion \( F \) in the loop (\( r = M_W^2/m_t^2 \) in the SM case and \( r = m_H^2/m_t^2 \) in the charged Higgs case).

We will not compute the NLO corrections to the \( b \to s q \) chromo-magnetic penguin operator \( O_8 \). This would be formally inconsistent in an expansion where no small parameter is present in the QCD RGE corrections so that the RGE mixing between \( O_7 \) and \( O_8 \) is considered of \( O(1) \). However the relevant RGE-loop factor is of order \( \ell \equiv (a_3/4\pi) \ln M_W^2/m_t^2 \approx 0.1 \), smaller than 1. More in detail, at leading order, the coefficient \( C_7 \) at the \( B \) scale is obtained in terms of the coefficients at the electroweak scale as

\[
C_7(m_b) \approx U_{72} + U_{77} C_7 + U_{78} C_8 \approx -0.155 + 0.7(-0.2 + C_7^{\text{SUSY}}) + 0.085(-0.1 + C_8^{\text{SUSY}}) \approx -0.30 + 0.7 C_7^{\text{SUSY}}.
\]

Here \( U_{ij} \) is the evolution matrix from the weak scale to the \( B \) scale for the Wilson coefficients \( C_i \). This confirms that it is not necessary to compute \( C_8 \) at NLO.

More generally, the RGE-loop factors of order one, that surely need to be exactly resummed via the RGE techniques, are the ones that determine the running of the strong coupling constant. This happens because the
QCD $\beta$ function receives contributions from all ‘active’ quarks, while only some specific flavours contribute, i.e., to the mixing between the $b \to s$ penguins $O_7$ and $O_8$.

Consequently, when studying the decay $b \to s\ell^+\ell^-$, it is not inconsistent to add the correction to $C_7$ we are going to compute (together with the corresponding low energy part), even if not required by a ‘formal’ expansion at NLO [15].

This correction has already been computed in [18] with effective theory techniques, but only in the limit $m_H \gg m_t$ where the effect is negligible. The complete NLO computation has instead been done in the case of charged Higgs corrections to the $B_0\bar{B}_0$ mixing [17].

We can now pass to the NLO computation of the part of the charged-Higgs mediated contribution to $C_7$ not suppressed by $1/\tan^2 \beta$. This computation requires to match the NLO (two-loop) $b \to s\gamma$ amplitude in the full theory with the one of the effective theory. We choose to employ the equations of motion $(i\partial b = m_b b)$ so that $O_7$ is the only relevant operator. In this way we are forced to match on-shell $b \to s\gamma$ amplitudes, plagued by infrared divergences that must be properly treated.

In both versions of the theory we choose the Feynman gauge for the gluon propagator. Since we never need to define traces like $\text{Tr} \gamma_5 \gamma_5 \cdots \gamma_5$, nor the completely antisymmetric tensor, we can employ naïve dimensional regularization (i.e. anticommuting $\gamma_5$) with MS renormalization scale $\bar{\mu}$. This regularization is more convenient than the supersymmetry-preserving ‘dimensional reduction’. Infact the contribution we want to compute does not depend on supersymmetric parameters (like gaugino and sfermion masses, . . . ) that can be related among them making assumptions about physics at high energy, giving rise to predictions usually computed employing dimensional reduction.

For simplicity in the following we denote with $C_7$, $C_8$, . . . , only their charged-Higgs mediated part that we want to compute.

### 4 $b \to s\gamma$ in the full theory

As said we are interested in diagrams that contain one $\lambda_b \bar{b}rtL H_-$ vertex. We have to compute the 16 two-loop diagrams shown in figure 1.

We write the various contributions $\Gamma$ to the two-loop $b \to s\gamma$ effective Hamiltonian in the full theory as

$$H_{b \to s\gamma} = -\frac{g_2^2}{2 M_W^2} \cdot V_{tb} V_{t*} \cdot (\hat{C}_{70} + \frac{\alpha_3}{4\pi} c_3 \hat{M}) \frac{\hat{O}_7}{4}, \quad \hat{M} = \sum \hat{M}_\Gamma$$

where $c_3 = 4/3$ is the quadratic Casimir for the fundamental representation of SU(3)$_c$, and the overall factor has been defined in such a way that the $c_3 \hat{M}$ is normalized as $C_{70}$, the NLO term of the Wilson coefficient $C_7$ of the operator $O_7$ in the effective theory:

$$C_7 = C_{70} + \frac{\alpha_3}{4\pi} C_{71} + O(\alpha_3^2).$$

The LO coefficient of $O_7$ in the full theory, $\hat{C}_{70}$, differs from the corresponding one in the effective theory, $C_{70}$, by terms that vanish as $d \to 4$, as described in eq. (A.1) in the appendix. Since the parameters of the theory receive divergent corrections, these terms will give a finite contribution to $C_{71}$.

#### 4.1 Two loop diagrams

A given Feynman diagram $\Gamma$ gives a contribution $\hat{M}_\Gamma = \int F_\Gamma$ (where $\int$ denotes the standard two-loop integration over internal loop momenta) that depends on the heavy masses, $M = \{m_t, m_H\}$ and on the light masses and momenta, $m = \{m_b, m_s; p_b, p_s\}$, of the $b$ and $s$ quarks. The full expression $\hat{M}_\Gamma(M, m)$ contains negligible terms suppressed by powers of $m/M$. The natural trick that allows to avoid computing all the unnecessary terms, without loosing terms like $\ln m/M$, consists in adding and subtracting to the loop-integrand $F_\Gamma(M, m)$ some appropriately chosen simpler term $\Delta F_\Gamma(M, m)$ that has the same low energy structure of the full $F_\Gamma(M, m)$. The result without irrelevant powers of $m/M$ is given by the simpler integral $\int [F_\Gamma(M, 0) - \Delta F_\Gamma(M, 0)] + \Delta F_\Gamma(M, m)$. In practice simpler means that $\Delta F_\Gamma$ can be chosen as a product of two one-loop integrals, instead of a two-loop integral, so that, in dimensional regularization, $\int \Delta F(M, 0) = 0$.

It is convenient to employ a standard technique [13], usually named ‘Heavy Mass Expansion’ (HME), that generalises and systemizes this procedure. It is possible to show that, in dimensional regularization, the expansion of a Feynman diagram $\Gamma$, that depends on ‘heavy’ masses and momenta $M$ and on ‘light’ masses and
Figure 1: The 16 two-loop Feynman graphs (we show on a single diagrams all possible attachment of the photon and write near to it the name we give to the diagram) that we have to compute. The symbol $\otimes$ denotes the vertex $\bar{b}_R t_L H_-$.

Moments $m$, in the limit $M \gg m$ can be done at the level of integrands as

$$F_\Gamma(M,m) \simeq \sum_{\gamma \subseteq \Gamma} \frac{T}{\gamma}(m) \frac{F_\Gamma(q,\gamma, m, M)}{\gamma}\left(\frac{q}{m}\right)$$

where $T_x$ indicates Taylor expansion in the variables $x$ up to the desired order. The sum is performed over all the “asymptotically irreducible” subgraphs $\gamma$ of $\Gamma$, i.e. those which satisfy the following two conditions:

1. $\gamma$ contains all the lines with heavy masses;
2. $\gamma$ consists of connectivity components that are one-particle-irreducible with respect to lines with small masses.

Finally $q_\gamma$ denotes the set of momenta external to the subgraph $\gamma$, $F_\gamma$ is its Feynman integrand, and $F_{\Gamma/\gamma}$ is the Feynman integrand of the reduced graph $\Gamma/\gamma$. We refer the reader to [10, 18] for explanations and examples. Here we discuss how this technique works in our case, and its relation with the natural trick previously discussed.

- The case $\gamma = \Gamma$ is always present and gives a ‘naïve’ Taylor expansion in $m$. This contribution corresponds to the term $F_\Gamma(M,0)$ of our previous example.

- There is one non-vanishing contribution from the ‘non naïve’ part of the HME expansion from each one of our graphs in fig. 1 (except the “GI” diagrams with an ‘internal’ gluon loop). The “asymptotically irreducible” subgraph $\gamma$ coincides with the Higgs/top loop that gives the LO result. This term of the HME expansion can be seen, at a diagrammatic level, as the contribution of the full diagram with the heavy propagators contracted to a point, and corresponds, in our previous example, to the term $\Delta F_\Gamma(M,0)$. When computing it we neglect powers of $\rho \equiv m_s^2/m_b^2$.

- Terms analogous to $-\Delta F_\Gamma(M,0)$ are not included in this form of the HME, since they vanish in dimensional regularization. For this reason some fake infrared (IR) divergences can (and will) appear in the ‘naïve part’ where $m = 0$. These divergences are cancelled diagram by diagram by fake ultraviolet (UV) divergences that appear in the non naïve part of the HME expansion.

The ‘naïve part’ of the expansion also contains the usual UV divergences that cancel upon renormalization. Similarly, the ‘non naïve part’ also contains IR divergences, cancelled in the final physical result after phase-space integration and inclusion of QCD bremsstrahlung $b \to s\gamma g$.

In order to check in detail all these cancellations, we have separated IR from UV divergences. It has been possible to do it in dimensional regularization, since our graphs have only single poles $1/\varepsilon$. In the following we will denote as $1/\varepsilon_{uv}$ an ultraviolet pole, and as $1/\varepsilon_{ir}$ a pole of infrared origin.
| graph and charge | coefficient | naive part of the HME expansion \(1/\varepsilon_{uv}\) | non naive part of the HME expansion \(1/\varepsilon_{ir}\) | \(\ln(m_B^2/m_b^2)/\varepsilon_{ir}\) |
|-----------------|-------------|-----------------|-----------------|-----------------|
| BIGE \(q_H\)    | 0           | -\(P_{B1}\)     | +\(P_{B1}\)     | -\(\frac{1}{2}P_{B1}\) |
| BIGI \(q_H\)    | -2\(P_{B1} - 3rP'_{B1}\) | 0               | 0               | 0               |
| BIGL, BIGR \(q_H\) | 2\(P_{B1}\) | 0               | -\(\frac{1}{2}P_{B1}\) | 0               |
| FIGI \(q_u\)    | +\(\frac{1}{2}P_{F1}\) | 0               | 0               | 0               |
| FLIGI, FRIGI \(q_u\) | -\(\frac{5}{4}P_{F1} - \frac{3}{4}rP'_{F1}\) | 0               | 0               | 0               |
| FLIGR, FRIGL \(q_u\) | 2\(P_{F1}\) | +\(\frac{1}{2}P_{F1}\) | -\(\frac{1}{4}P_{F1}\) | 0               |
| FIGL, FIGR \(q_u\) | 0           | -\(\frac{1}{4}P_{F1} + \frac{1}{2}P_{B1}\) | +\(\frac{1}{4}P_{F1} - \frac{1}{2}P_{B1}\) | 0               |
| FIGE \(q_u\)    | 0           | -\(P_{B1}\)     | 0               | -\(\frac{1}{2}P_{F1}\) |
| FRIGR, FRIGL \(q_d\) | 0           | \(P_{F1} - \frac{1}{2}P_{B1}\) | \(-P_{F1} + \frac{1}{2}P_{B1}\) | 0               |
| FLIGE, FRIGE \(q_d\) | 0           | \(\frac{1}{2}P_{B1}\) | -\(\frac{1}{2}P_{B1}\) | 0               |

| all graphs \(\propto q_H\) | 2\(P_{B1} - 3rP'_{B1}\) | 0               | 0               | -\(\frac{1}{2}P_{B1}\) |
| all graphs \(\propto q_u\) | 2\(P_{F1} - 3rP'_{F1}\) | 0               | 0               | -\(\frac{1}{2}P_{F1}\) |
| all graphs \(\propto q_d\) | 0           | 2\(P_{F1}\)     | -2\(P_{F1}\)   | 0               |

Table 1: Coefficients of the divergent parts of the single graphs. The loop functions \(P_{B1}\) and \(P_{F1}\) are defined in the appendix; the ‘names’ of the graphs are defined by fig. 1.

In our case, since we have already a \(m_b\) factor from the vertex, the Taylor expansions have to be performed up to first order (up to second order in the ‘non-naive part’ of the diagrams ‘FLIGE’ and ‘FRIGE’ with four light propagators). The ‘naive’ parts of diagrams that differ by the exchange of \(b \leftrightarrow s\) (like the diagrams ‘FLIGE’ and ‘FRIGE’) are equal because the Dirac equation, that sees the difference between the \(b\) and \(s\) masses, is never used.

We have written a Mathematica code that simplifies the spinor algebra, expands the ‘light’ factors in the loop integrands in Taylor expansion up to the appropriate order, reducing the full expression to a sum of one and two-loop scalar integrals that have been separately computed. This technique is less efficient than the alternative one based on Feynman parameters; however it is easy to teach it to a computer. The non-naive part of the HME expansion is computed in the same way, performing the Taylor expansion also in one (appropriately chosen) loop momentum. In this way the whole computation is done in few minutes by a normal ‘personal’ computer. We show the separate result of the two parts of the expansion (\(\mathcal{M}_{2\text{ loop}}\) from the ‘naive’ (high-energy) part, and \(\mathcal{M}_{\text{HME}}\) from the ‘non naive’ (low energy) term of the HME expansion) in eqs [12]. We have listed in table 1 all the divergences present in the single graphs.

### 4.2 Renormalization

It is necessary to renormalize the top mass, \(m_t\), the bottom mass, \(m_b\), and to express the result in terms of canonically normalized \(b\) and \(s\) quark fields.

The simplest way of renormalizing the top mass consists in expressing the bare top mass, \(m_{t0}\), in term of the renormalized ‘running’ mass, \(m_t\),

\[
m_t = m_{t0} + \delta m_t, \quad \frac{\delta m_t}{m_t} = c_3 \frac{3}{4\pi \varepsilon_{uv}}
\]

\((c_3 \equiv 4/3)\) in the one loop contribution, \(\tilde{C}_{70}(r)\). The corresponding contribution \(\mathcal{M}_t\) to the amplitude \(\mathcal{M}\) is

\[
\mathcal{M}_t = \frac{3}{\varepsilon_{uv}} 2r \tilde{C}'_{70} \tag{7a}
\]

where ′ denotes derivation with respect to \(r = m_{t0}^2/m_t^2\). The pole \(b\)-quark mass \(m_b\) is expressed in terms of the bare \(b\) mass, \(m_{b0}\), as

\[
m_b = m_{b0} + \delta m_b, \quad \frac{\delta m_b}{m_b} = c_3 \frac{3}{4\pi (\varepsilon_{uv})} + 3 \ln \left(\frac{m_t^2}{m_b^2}\right) + 4.
\]

Since in the one-loop graphs the \(m_{b0}\) factor comes only from the vertex (we never use the Dirac equation to reduce the operator basis), the contribution \(\mathcal{M}_b\) to \(\mathcal{M}\) from the renormalization of \(m_b\) is simply given by
Figure 2: The Feynman graph that accounts for the renormalization of the mass and kinetic terms that mix the b and s quarks.

\[ \hat{\mathcal{M}}_b = -\hat{C}_{70}\left(\frac{3}{\varepsilon} + 3 \ln \frac{\mu^2}{m_b^2} + 4\right). \]  

(7b)

The counterterms for on-shell renormalization of the wave-function of the b and s quarks are

\[ Z_q = 1 - \frac{\alpha_3}{4\pi} \left( \frac{1}{\varepsilon_{\text{uv}}} + \frac{2}{\varepsilon_{\text{ir}}} + 3 \ln \frac{\mu^2}{m_q^2} + 4 \right), \quad q = \{s, b\}. \]

Expressing the one-loop result in terms of canonically normalized b and s quark fields, we get the following correction \( \hat{\mathcal{M}}_Z \) to \( \hat{\mathcal{M}} \):

\[ \hat{\mathcal{M}}_Z = -\hat{C}_{70} \left( \frac{1}{\varepsilon_{\text{ir}}} + \frac{2}{\varepsilon_{\text{ir}}} + 3 \ln \frac{\mu^2}{m_b m_s} + 4 \right). \]  

(7c)

Finally, top/Higgs loops generate non diagonal kinetic and mass terms between the b and s quarks that is necessary to eliminate via appropriate flavour rotations of the quark fields. This gives rise to a contribution to \( \hat{\mathcal{M}} \) when a magnetic \( bb\gamma \) dipole operator is rotated into a \( bs\gamma \) dipole. However, it is more convenient to take into account this particular correction adding the (one+one)-loop diagram of figure 2 (the similar graph with \( b \leftrightarrow s \) is suppressed by powers of \( m_s/m_b \)) to the list of the two-loop diagrams.

We denote the renormalization contributions as \( \hat{\mathcal{M}}_{\text{ctr}} = \hat{\mathcal{M}}_t + \hat{\mathcal{M}}_b + \hat{\mathcal{M}}_Z \).

5 \( b \to s\gamma \) in the effective theory

Since all the supersymmetric contributions (if \( R \)-parity is conserved) arise first at one loop level, their mixing structure is simpler than the one of the SM contributions. In particular all the LO supersymmetric contributions to the Wilson coefficients \( C_i \) are given by the corresponding penguin diagrams without extra matching contributions. We list here all the various contributions to the \( b \to s\gamma \) decay amplitude in the effective theory relevant for our computation (we are neglecting terms suppressed by \( \tan^{-2}\beta \)). We denote by \( \mathcal{M} \) the value of the on-shell \( b \to s\gamma \) amplitude in the effective theory, normalized in the same way as \( \hat{\mathcal{M}} \) (the corresponding amplitude in the full theory). The non zero contributions to \( \mathcal{M} \) are, in our case:

- The contribution \( \mathcal{M}_7 \) given by the matrix element of \( \mathcal{O}_7 \) itself at order \( \alpha_3 \)
  \[ \mathcal{M}_7 = \frac{1}{\varepsilon_3} C_{71} - C_{70} \ln \rho \left[ \frac{1}{\varepsilon_{\text{ir}}} + \ln \frac{\mu^2}{m_b^2} + 2 - \frac{1}{2} \ln \rho \right]. \]  

(8)

The first term is the tree-level matrix element of the term we ultimately wish to extract. The other terms are obtained by the one-loop QCD correction to the leading order magnetic penguin \( \mathcal{O}_7 \).

- The order \( \alpha_3 \) matrix element of the chromo-magnetic penguin operator \( \mathcal{O}_8 \)
  \[ \mathcal{M}_8 = q_d C_{80} \left[ -\frac{4}{\varepsilon_{\text{uv}}} - 4 \ln \frac{\mu^2}{m_b^2} - 11 + \frac{2\pi^2}{3} - 2i\pi \right]. \]  

(9)

- As the operators mix under renormalization, we have to consider counterterm contributions induced by operators of the form \( C_i \delta Z_{ij} \langle s|\mathcal{O}_j|b \rangle \). Using the known renormalization constants [19], the non-vanishing contributions to the \( b \to s\gamma \) amplitude \( \mathcal{M} \) are
  \[ \mathcal{M}_{77} = \frac{4}{\varepsilon_{\text{uv}}} C_{70}, \quad \text{and} \quad \mathcal{M}_{87} = q_d \frac{4}{\varepsilon_{\text{uv}}} C_{80}. \]

‘Evanescent’ operators do not give contributions.
• Like in the full theory, is of course necessary to renormalize also the parameters and fields of the theory, that in this case are the $b$-quark mass and the $b$ and $s$ wave-functions. The counterterm due to the $b$-quark mass renormalization is

$$\mathcal{M}_b = -C_70 \left[ \frac{3}{\varepsilon_{uv}} + 3 \ln \frac{\bar{\mu}^2}{m_b^2} + 4 \right]$$

(10)

when using the pole $b$-quark mass as in the full theory, while the renormalization of the $b$ and $s$ fields gives a contribution $\mathcal{M}_Z$

$$\mathcal{M}_Z = -C_70 \left[ \frac{1}{\varepsilon_{uv}} + \frac{2}{\varepsilon_{ir}} + 3 \ln \frac{\bar{\mu}^2}{m_b m_s} + 4 \right].$$

(11)

The amplitude $\hat{\mathcal{M}}$ in the effective theory is UV finite and has an IR divergence $-(2 + \ln \rho)C_70/\varepsilon_{ir}$.

### 6 Matching and final result

We now collect all the necessary terms and illustrate the matching procedure. The amplitude in the effective theory is

$$\hat{\mathcal{M}} = \frac{1}{C_3}C_{71} + q_d C_{80} \left[ \frac{2\pi^2}{3} - 11 - 2i\pi - 4 \ln \frac{\bar{\mu}^2}{m_b^2} \right] - C_70 \left[ (2 + \ln \rho) \frac{(\bar{\mu}^2/m_b^2)^\varepsilon}{\varepsilon_{ir}} + 8 + 4 \ln \frac{\bar{\mu}^2}{m_b^2} + \frac{1}{2} \ln \rho - \frac{1}{2} \ln^2 \rho \right].$$

The amplitude $\hat{\mathcal{M}}$ in the full theory is the sum of the contribution $\hat{\mathcal{M}}_\text{HME}$, defined as the ‘naive’ part of the two-loop diagrams, of the ‘non-naive’ part, $\hat{\mathcal{M}}_\text{HME}$, and of the counterterms, $\hat{\mathcal{M}}_\text{ctr}$

$$\hat{\mathcal{M}}_\text{loop} = \left(-\frac{6r\tilde{C}_70 - 4\tilde{C}_70}{\varepsilon_{uv}} + 4q_d \tilde{C}_{80} \right) \left( \frac{\bar{\mu}^2}{m_b^2} \right)^\varepsilon_{ir} + f(r)$$

(12a)

$$\hat{\mathcal{M}}_\text{HME} = -\tilde{C}_70 \left( \frac{\bar{\mu}^2/m_b^2}{\varepsilon_{ir}} \right) \ln \rho - \frac{1}{2} \ln^2 \rho + 2 \ln \rho + q_d \tilde{C}_{80} \left[ \frac{2\pi^2}{3} - 11 - 2i\pi - \frac{4(\bar{\mu}^2/m_b^2)^\varepsilon}{\varepsilon_{uv}} \right]$$

(12b)

$$\hat{\mathcal{M}}_\text{ctr} = 6r\tilde{C}_70 - 4\tilde{C}_70 - \tilde{C}_70 \left( \frac{(\bar{\mu}^2/m_b^2)^\varepsilon}{\varepsilon_{ir}} + 8 + 4 \ln \frac{\bar{\mu}^2}{m_b^2} - \frac{3}{2} \ln \rho \right)$$

(12c)

where the two-loop function $f(r)$ is, for arbitrary electric charges $q_d = q_H + q_u$.

$$f(r) = q_H \left[ \frac{17r - 3 - 2r^2}{(r-1)^3} + \frac{1 - 20r - 5r^2}{2(r-1)^4} r \ln r + \frac{1 - 4r - r^2}{(r-1)^3} \operatorname{Li}_2(1-r) \right] +$$

$$+ q_u \left[ \frac{3r - 1 - 5r^2}{2(r-1)^3} + \frac{r - 1 - 12r^2}{(r-1)^4} r \ln r + \frac{2r - 1 - 9r^2}{(r-1)^3} \operatorname{Li}_2(1-r) \right].$$

(13)

We have neglected higher powers in $\varepsilon$ and $\rho \equiv m_s^2/m_b^2$, and we have included in $\hat{\mathcal{M}}_\text{HME}$ the diagram in figure 2. The full amplitude $\hat{\mathcal{M}}$ is

$$\hat{\mathcal{M}} = f(r) + (4C_70 - 6rC_70 + 4q_d C_{80}) \ln \frac{\bar{\mu}^2}{m_b^2} + q_d \tilde{C}_{80} \left[ \frac{2\pi^2}{3} - 11 - 2i\pi - 4 \ln \frac{\bar{\mu}^2}{m_b^2} \right] +$$

$$-\tilde{C}_70 \left[ (2 + \ln \rho) \frac{(\bar{\mu}^2/m_b^2)^\varepsilon}{\varepsilon_{ir}} + 8 + 4 \ln \frac{\bar{\mu}^2}{m_b^2} + \frac{1}{2} \ln \rho - \frac{1}{2} \ln^2 \rho \right] + \mathcal{O}(\varepsilon).$$

The on-shell $b \to s\gamma$ amplitudes ($\hat{\mathcal{M}}$ in the full theory and $\mathcal{M}$ in the effective theory) are infrared-divergent. The correct matching is obtained requiring that the infrared safe decay rate obtained including QCD bremsstrahlung $b \to s\gamma g$ be the same in both (full and effective) descriptions. This gives

$$\hat{\mathcal{M}} + \hat{\mathcal{M}}_{\text{ir}} = \mathcal{M} + \mathcal{M}_{\text{ir}}$$

where

$$\hat{\mathcal{M}}_{\text{ir}} = (2 + \ln \rho)\tilde{C}_{70} \left( \frac{\bar{\mu}/m_b}{\varepsilon_{ir}} \right)^2, \quad \mathcal{M}_{\text{ir}} = (2 + \ln \rho)C_70 \left( \frac{\bar{\mu}/m_b}{\varepsilon_{ir}} \right)^2.$$
So, the final result for the NLO charged Higgs contribution is (setting $q_H = -1$ and $q_u = 2/3$)

$$
C_{71} = c_3 \left[ f(r) + (4C_{70} - 6rC'_{70} + 4q_u C_{80}) \ln \frac{\bar{\mu}^2}{m_t^2} \right] = \frac{42}{3} - 13r + 7r^2 - \frac{2r}{9} \left( 7 - 64r + 33r^2 \right) \ln r + \frac{16}{9} - 7r + 3r^2 \ln 2(1 - r) + \left( \frac{28 - 47r + 21r^2}{9} \right) + \frac{4r}{9} \left( 3 + 14r - 8r^2 \right) \ln r \ln \frac{\bar{\mu}^2}{m_t^2}.
$$

(15a)

In the limits $m_H = m_t$ and $m_H \gg m_t$ the charged Higgs contribution becomes

$$
C_7(r \to 1) = -\frac{7}{36} + \frac{\alpha_s}{4\pi} \left[ \frac{181}{81} - \frac{35}{27} \ln \frac{\bar{\mu}^2}{m_t^2} \right],
$$

(15b)

$$
C_7(r \to \infty) = \frac{1}{4r} - \frac{\ln r}{3r} + \frac{\alpha_s}{4\pi} \left[ \frac{2(42 + 4\pi^2 - 33 \ln r + 12 \ln^2 r) + (42 - 32 \ln r) \ln \frac{\bar{\mu}^2}{m_t^2}}{9} \right].
$$

(15c)

For $m_H = 500\;\text{GeV}$ and $\bar{\mu} = m_t$ the term we have computed gives a $-17\%$ correction to the LO value.

### 7 Conclusion

In a class of supersymmetric scenarios, i.e. in gauge mediated models, the charged Higgs mediated contribution to the $b \to s\gamma$ magnetic penguin is expected to be the most interesting supersymmetric effect in B-physics. The NLO QCD correction to its Wilson coefficient $C_7(\bar{\mu})$, that we have computed employing naïve dimensional regularization with $\overline{\text{MS}}$ subtraction, gives a $(10 \div 20)\%$ correction to the corresponding LO result (for $\bar{\mu} = m_t$ and realistic charged Higgs masses).

Note added: Together with our article, another one containing the same computation \cite{20} has recently appeared. Our results are in perfect agreement, even if the conclusions are apparently different. In particular the authors of \cite{20} claim that, in the usual two-Higgs doublet model, the inclusion of NLO effects enhances the B.R.($b \to s\gamma$) with respect to its LO value. This is due to the already known NLO enhancement in the SM contribution \cite{15,16,17}. On the contrary, the new correction to $C_7(\bar{\mu} \approx m_t)$ that we have computed tends to reduce the charged-Higgs mediated correction. As discussed in the text, we consider $C_7(\bar{\mu} \approx m_t)$ a more useful phenomenological quantity than B.R.($b \to s\gamma$).

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### A Useful functions

The penguin one-loop functions $P(r)$ employed in eq.s (13) are given by $P = \hat{P}_{\varepsilon \to 0}$, where

$$
\hat{P}_{BI} = (1 + \varepsilon \ln \frac{\bar{\mu}^2}{m_t^2}) \left[ \frac{r^2 - 1 - 2r \ln r}{2(r - 1)^3} + \frac{3(r^2 - 1) - 2r(2 + r) \ln r + 2r \ln^2 r}{4(r - 1)^3} \right],
$$

$$
\hat{P}_{FI} = (1 + \varepsilon \ln \frac{\bar{\mu}^2}{m_t^2}) \left[ \frac{3 - 4r - 4r^2}{2(r - 1)^3} + \frac{1 - 8r + 7r^2 - 6r \ln r + 2r \ln^2 r}{4(r - 1)^3} \right],
$$

$$
\hat{P}_{BE} = (1 + \varepsilon \ln \frac{\bar{\mu}^2}{m_t^2}) \left[ \frac{3 - 2r - 3r^2 + 6r \ln r}{12(r - 1)^4} + \frac{22 + 27r - 54r^2 + 6r^3 - 6r(-6 - 6r + r^2) \ln r - 18r \ln^2 r}{72(r - 1)^4} \right],
$$

$$
\hat{P}_{FE} = (1 + \varepsilon \ln \frac{\bar{\mu}^2}{m_t^2}) \left[ \frac{6r - 1 - 3r^2 + 6r \ln r}{12(r - 1)^4} - \frac{5 - 54r + 27r^2 + 22r \ln r + 18r \ln^2 r}{72(r - 1)^4} \right].
$$

The charged-Higgs mediated one-loop result for the coefficients $\hat{C}_{70}$ of the $b \to s\gamma$ magnetic penguin, and $\hat{C}_{80}$ of the $b \to sg$ chromo-magnetic penguin are

$$
\hat{C}_{70} = \frac{1}{2} (q_H \hat{P}_{BI} + q_u \hat{P}_{FI}) + \mathcal{O}(\varepsilon^2), \quad \hat{C}_{80} = \frac{1}{2} \hat{P}_{FI} + \mathcal{O}(\varepsilon^2).
$$

(A.1)
The bi-logarithmic function, \( Li_2(x) \), employed in eq. (13) is defined as

\[
Li_2(x) \equiv -\int_0^x \frac{\ln(1 - \xi)}{\xi} d\xi.
\]

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