Remarks on the two-dimensional power correction in the soft wall model

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Abstract

We present a direct derivation of the two-point correlation function of the vector current in the soft wall model by using the AdS/CFT dictionary. The resulting correlator is exactly the same as the one previously obtained from dispersion relation with the same spectral function as in this model. The coefficient $C_2$ of the two-dimensional power correction is found to be $C_2 = -c/2$ with $c$ the slope of the Regge trajectory, rather than $C_2 = -c/3$ derived from the strategy of first quantized string theory. Taking the slope of the $\rho$ trajectory $c \approx 0.9 \text{GeV}^2$ as input, we then get $C_2 \approx -0.45 \text{GeV}^2$. The gluon condensate is found to be $<\alpha_s G^2> \approx 0.064 \text{GeV}^4$, which is almost identical to the QCD sum rule estimation. By comparing these two equivalent derivation of the correlator of scalar glueball operator, we demonstrate that the two-dimensional correction can’t be eliminated by including the non-leading solution in the bulk-to-boundary propagator, as was done in \cite{1}. In other words, the two-dimensional correction does exist in the scalar glueball case. Also it is manifest by using the dispersion relation that the minus sign of gluon condensate and violation of the low energy theorem are related to the subtraction scheme.

PACS numbers: 11.25.Tq,11.55.Fv,12.38.Lg

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I. INTRODUCTION

The standard operator product expansion (OPE) for the two point correlator of the vector current $J_{\mu} = (\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d)/\sqrt{2}$ has the following form:

$$\Pi_{\mu\nu} = i \int d^4xe^{iQ \cdot x} <0|T\{J_\mu(x), J^\dagger_\nu(0)\}|0> = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(Q^2 = -q^2),$$ \hspace{1cm} (1)

with

$$NQ^2 d\Pi(Q^2) = C_0 + \frac{1}{Q^2} C_2 + \sum_{n \geq 2} \frac{n}{Q^{2n}} C_{2n} <\mathcal{O}_{2n}>,$$ \hspace{1cm} (2)

where $C_0$ is given as $C_0 = 1 + \sum_{n \geq 1} A_n \alpha_s^n(Q^2)$, and $C_4 <\mathcal{O}_4> = \frac{5}{3} <\alpha_s G^2>$ stands for the gluon condensate. The two-dimensional correction $\frac{1}{Q^2} C_2$ does not appear in QCD with massless quarks [2]. However, it was argued in Ref.[3] that a non-vanishing $C_2$ may be related to the string effect. It is also suggested that the origin of these corrections is due to nonlocal properties of the instanton vacuum [4]. In Ref.[5, 6], it was found that in the large-$N_c$ limit radial Regge trajectories naturally give rise to the presence of the two-dimensional correction, unless fine-tuning the parameters. Recently this term was found [7] to appear in the so called soft wall model [8], which can accomplish linear Regge trajectories very well based on gauge/string duality [9]. The coefficient $C_2$ is derived to be given by the Regge slope $c$, $C_2 = -c/3$, following the strategy of first quantized string theory. On the other hand, this correlator can be derived from the dispersion relation since the masses and decay constants have been given in this model. The result for $C_2$ seems to be a little different [10].

In order to determine the true correlator and consequently the value of $C_2$, we give a direct derivation based on the original AdS/CFT prescription. The resulting correlator is exactly the same as the one derived from the dispersion relation, showing the equivalence of these two approaches. The coefficient $C_2$ of the two-dimensional correction is found to be $C_2 = -c/2$, which has the same sign as in QCD if quark massed are taken into account. Taking the slope of the $\rho$ trajectory $c \approx 0.9\text{GeV}^2$ as input, we then get $C_2 \approx -0.45\text{GeV}^2$. The gluon condensate is also calculated and the numerical result is given by $<\alpha_s G^2> \approx 0.064\text{GeV}^4$, which is almost identical to the common QCD sum rule estimation [11].

The correlator of two-point scalar glueball operator in the same model was discussed in [1, 12]. With the standard choice for the bulk-to-boundary propagator, the correlator also contains a similar two-dimensional power correction (times a logarithm). Moreover, by expanding the correlator in large $Q^2$ limit and comparing to the OPE, one get a negative
gluon condensate contrast to the common value. It seems that by including another solution for the bulk-to-boundary and fine-tuning the coefficients, the two-dimensional correction can be eliminated and the common value of the gluon condensate can be obtained \cite{1}. However, since the scalar glueball spectrum and the corresponding residues have been given in this model, we can also get the correlator by using the dispersion relation. Again the result coincides with the one calculated in the standard way \cite{12}. Thus including of the other solution in the bulk-to-boundary propagator is redundant. Actually, with the normalizable modes in hand, using of the decomposition formula \cite{13} immediately results the bulk-to-boundary propagator with the standard choice, just as in the vector case \cite{14}. From the procedure of the dispersion relation it can also be found that the determination of gluon condensate is substraction scheme dependant.

II. THE TWO POINT VECTOR CORRELATOR IN THE SOFT MODEL

The soft-wall model can be given by introducing a non-constant dilaton field \cite{8} \( \Phi(z) = \kappa^2 z^2 \) to the original AdS metric

\[
ds^2 = e^{2A(z)}(-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)
\]

with \( A(z) = - \log z \). The relevant action for the vector field is given by:

\[
I = \int d^5xe^{-\Phi(z)} \sqrt{g} \mathrm{Tr} \left\{ -\frac{1}{g_5^2}(F_L^2 + F_R^2) \right\},
\]

where \( A_{L,R} = A_{L,R}^a t^a, F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N] \) and \( t^a = \frac{\sigma^a}{2} \). The vector current \( J_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/\sqrt{2} \) is considered to be dual to the vector combination \( V = (A_L + A_R)/2 \). We can use the gauge invariance of the action to go to the axial gauge \( V_z = 0 \), and derive the equation for the 4d-transverse components of \( V_\mu \). It is found to be

\[
\left[ \partial_z \left( \frac{e^{-\Phi}}{z} \partial_z V_\mu^a(q,z) \right) + \frac{e^{-\Phi}}{z} q^2 V_\mu^a(q,z) \right]_\perp = 0,
\]

where \( V_\mu^a(q,z) \) is the 4D Fourier transform of the original field \( V_\mu^a(x,z) \). Substituting the solution to Eq.(5) to the action, we get a surface term

\[
I = -\frac{1}{2g_5^2} \int d^4x \left( \frac{e^{-\Phi}}{z} V_\mu^a \partial_z V_\mu^a \right)_{z=\epsilon}.
\]

3
The non-normalizable solution of $V_\mu$ can be expressed by $V_\mu(q, z) = V(q, z)V_\mu^0(q)$, where $V_\mu^0(q)$ is the Fourier transform of the source coupled to $J_\mu$, and $V(q, z)$ is the bulk-to-boundary propagator. $V(q, z)$ must satisfy the boundary condition $V(q, z = 0) = 1$ to ensure the interpretation of $V_\mu^0(q)$ as the source. According to the AdS/CFT dictionary \cite{15, 16}, the partition function on the AdS side is equal to the generation function of the dual CFT. Thus we can obtain the two-point function by differentiating Eq.(6) twice with respect to the source $V_0$

$$\Pi_V(Q^2) = -\frac{1}{g_s^2 Q^2} \left[ \frac{e^{-\Phi}}{z} V(Q, z) \partial_z V(Q, z) \right]_{z=\epsilon}$$

(7)

with $Q^2 = -q^2$. The general solution of Eq.(5) is given by the confluent hypergeometric functions

$$V(Q, z) = A_V U\left(\frac{Q^2}{4\lambda^2}, 0, \lambda^2 z^2\right) + B_V \lambda^2 z^2 \binom{1}{1} F_1 \left(\frac{Q^2}{4\lambda^2} + 1, 2, \lambda^2 z^2\right).$$

(8)

Notice that in this case we should choose the regular solution $\binom{1}{1} F_1 \left(\frac{Q^2}{4\lambda^2} + 1, 2, \lambda^2 z^2\right)$, rather than $\binom{1}{1} F_1 \left(\frac{Q^2}{4\lambda^2}, 0, \lambda^2 z^2\right)$ \cite{14}, since the latter is not well-defined mathematically \cite{17}. Recently there is some controversy in choosing the solution of $V(Q, z)$ \cite{1, 12}, which we will discuss in the next section. Now we just follow the standard procedure and set $B_V = 0$. Then $A_V$ can be determined from the boundary condition to be $A_V = \Gamma(1 + \frac{Q^2}{4\lambda^2})$. Thus the correlator is given by

$$\Pi_V(Q^2) = -\frac{1}{g_s^2 Q^2} \Gamma\left(\frac{Q^2}{4\lambda^2} + 1\right) \lim_{z \to 0} \frac{e^{-\Phi}}{z} \partial_z U\left(\frac{Q^2}{4\lambda^2}, 0, \lambda^2 z^2\right)$$

(9)

Using the integral representation of the Tricomi function \cite{17}

$$U(a, c; x) = \frac{1}{\Gamma(a)} \int_0^1 du \frac{u^{a-1}}{(1-u)^c} \exp \left[ -\frac{u}{1-u} x \right]$$

(10)

Eq.(9) can be simplified to be

$$\Pi(Q^2) = \frac{1}{2g_s^5} \Gamma\left(\frac{Q^2}{4\lambda^2} + 1\right) \lim_{z \to 0} U\left(\frac{Q^2}{4\lambda^2} + 1, 1, \lambda^2 z^2\right).$$

(11)

The Tricomi function can be expanded as \cite{17}

$$U(a, n + 1; x) = \frac{(-1)^{n-1}}{n! \Gamma(a - n)} \{1 F_1 (a, n + 1; x) \log x \}
+ \sum_{r=0}^{\infty} \frac{(a)_r}{(n + 1)_r} \left[ \psi(a + r) - \psi(1 + r) - \psi(1 + n + r) \frac{x^r}{r!} \right]
+ \frac{(n - 1)!}{\Gamma(a)} \sum_{r=0}^{n-1} \frac{(a - n)_r x^{r-n}}{(1 - n)_r r!} \quad n = 0, 1, 2, ...,$$

(12)
where \((\lambda)_n = \Gamma(\lambda + n)/\Gamma(\lambda)(n \geq 1), (\lambda)_0 = 1\) and \(\psi(x)\) is the digamma function defined as

\[
\psi(x) = \frac{d}{dx} \log \Gamma(x). \tag{13}
\]

Taking the \(z \to 0\) limit in Eq. (11) we can easily get the expression for the correlator

\[
\Pi_V(Q^2) = -\frac{1}{2g_5^2} \psi \left( 1 + \frac{Q^2}{4\lambda^2} \right) \tag{14}
\]

up to one irrelevant (subtraction) constant which is actually infinite. This kind of correlator has been discussed extensively in the literature [5, 10], obtained from dispersion relation with a model for the spectral function. More specifically, the correlation function can be expressed by the spectral function by using the dispersion relation

\[
\Pi_V(q^2) = \frac{1}{\pi} \int_0^\infty \frac{\text{Im}\Pi(s)}{s - q^2} ds \tag{15}
\]

In the large-\(N_c\) limit, the spectral function is given by an infinite number of resonances

\[
\frac{1}{\pi} \text{Im}\Pi(s) = \sum_{n=0}^\infty F_n^2 \delta(s - m_n^2). \tag{16}
\]

Furthermore, one assumes that the resonances lie on linear Regge trajectory

\[
m_n^2 = m_0^2 + cn, \tag{17}
\]

and the residues are independent of the excitation number \(n\)

\[
F_n^2 = F^2. \tag{18}
\]

With these relations the correlator can be calculated explicitly, and is given by

\[
\Pi_V(Q^2) = -\frac{F^2}{c} \left[ \psi \left( \frac{Q^2 + m_0^2}{c} \right) + \text{Const} \right] \tag{19}
\]

Notice that in the soft wall model, Regge trajectory can be reproduced naturally [8]

\[
m_n^2 = 4\lambda^2(n + 1), \tag{20}
\]

and the residues are also constant

\[
F^2 = \frac{2\lambda^2}{g_5^2}. \tag{21}
\]

From these relations one can show that the correlator given in Eq.(14) and Eq.(19) are exactly the same up to the subtracted term. This in turn confirms our choice for the bulk-to-boundary propagator.
The OPE of the correlator \([19]\) can be obtained by using the properties of the digamma function \([10, 18]\)

\[
\Pi_{V}^{\text{OPE}}(Q^2) = -\frac{F^2}{c} \log \frac{Q^2}{\mu^2} + \sum_{k=1}^{\infty} \frac{c_{2k}}{Q^{2k}},
\]

where the condensates are given by

\[
c_{2k} = (-1)^k \frac{F^2}{k} B_k \left( \frac{m_0^2}{c} \right),
\]

and \(B_k(\xi)\) denotes the Berboulli polynomials. In our simpler case \([19]\) we can get the OPE directly since

\[
\psi(\xi + 1) = \psi(\xi) + \frac{1}{\xi},
\]

and the large-\(\xi\) behavior of the digamma function is

\[
\psi(\xi) = \log \xi - \frac{1}{2\xi} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2n\xi^{2n}}
\]

with the Bernoulli numbers \(B_{2n} = (-1)^{n-1}2(2n)!\zeta(2n)/(2\pi)^{2n}\) (\(\zeta(z)\) is Riemann’s zeta function). The result becomes

\[
\Pi_{V}^{\text{OPE}}(Q^2) = -\frac{1}{2g_5^2} \log \frac{Q^2}{4\lambda^2} - \frac{\lambda^2}{g_5^2 Q^2} + \frac{1}{2g_5^2} \sum_{n=1}^{\infty} \frac{B_{2n}(4\lambda^2)^{2n}}{2n(Q^2)^{2n}}.
\]

Notice that the leading behavior is the same as that of the hard wall model \([19]\). It shows that different deformation in the IR region do not affect the large \(Q^2\) behavior, which is governed by the essentially AdS metric in the UV. This was also true in the case of the scalar glueball \([12]\). Matching to the OPE in real QCD to leading order in \(\alpha_s\)

\[
\Pi_{V}^{\text{OPE}}(Q^2) = -\frac{2}{3} \frac{N_c}{(4\pi)^2} \log \frac{Q^2}{\mu^2} + ...,\]

we get the value \(g_5^2 = 12\pi^2/N_c\) as in Ref. \([19]\). Now applying the operator

\[
D(Q^2) = -\frac{12\pi^2}{N_c} Q^2 \frac{d}{dQ^2}
\]

on both sides of Eq.(26) and comparing with Eq.(2) we get

\[
C_0 = 1, \quad C_2 = -2\lambda^2 = -c/2, \quad <\alpha_s G^2> = \frac{1}{4\pi} c^2.
\]

This is in contradiction with the result \(C_2 = -c/3\) derived in Ref. \([7]\) by first quantizing string theory in the same background. From the coincidence of the correlator derived from
the two different approaches, it seems that $C_2 = -c/2$ is the right result. Then we get the value $C_2 \approx -0.45\text{GeV}^2$ if we take the slope of the $\rho$ trajectory $c \approx 0.9\text{GeV}^2$ as input. The gluon condensate can also be obtained, which is given by

$$< \alpha_s G^2 > \approx 0.064\text{GeV}^4.$$  

(30)

Surprisingly, this result is almost identical to the QCD sum rule estimation \[1\]

$$< \alpha_s G^2 > = (0.06 \pm 0.02)\text{GeV}^4.$$  

(31)

III. EXTENDING TO THE SCALAR GLUEBALL CASE

Recently the scalar glueball in the same model has been discussed in Ref. \[1, 12, 20\]. The scalar glueball operator $O_S (x) = G_{\mu \nu}^a (x) G^{a, \mu \nu} (x)$, which has dimension $\Delta = 4$, is considered to be dual to a massless scalar in the 5D setup. The action is given by

$$I = \frac{1}{2\kappa^2} \int d^5x \sqrt{g} e^{-\Phi(z)} g^{MN} \partial_M \varphi \partial_N \varphi$$  

(32)

The equation of motion for the Fourier transform $\varphi(q, z)$ reads

$$\left[ \partial_z \left( \frac{e^{-\Phi}}{z^3} \partial_z \varphi(q, z) \right) + \frac{e^{-\Phi}}{z^3} q^2 \varphi(q, z) \right] \perp = 0,$$  

(33)

For $q^2 = 4\lambda^2(n + 2)$, the solutions are normalizable \[1, 20\]

$$\varphi_n(z) = A_n \lambda^4 z^4 \mathbf{1} \mathbf{F}_1(-n, 3; \lambda^2 z^2)$$  

(34)

with $A_n^2 = (n + 1)(n + 2)/2\lambda^4$. From the AdS/CFT dictionary, these give the spectrum of the scalar glueball $m_n^2 = 4\lambda^2(n + 2)$. The general non-normalizable solution to Eq.(33) is given by

$$\varphi(Q, z) = A_G U \left( \frac{Q^2}{4\lambda^2}, -1; \lambda^2 z^2 \right) + B_G \lambda^4 z^4 \mathbf{1} \mathbf{F}_1 \left( \frac{Q^2}{4\lambda^2} + 2, 3; \lambda^2 z^2 \right)$$  

(35)

with $Q^2 = -q^2$.

\[1\] Notice in Ref.\[1\] the factor $\lambda$ is missing in $A_n^2$ due to an unusual normalization.
Now we again have to choose the right solution. In Ref. [12], \(B_G\) is simply set to be zero as in the standard way, and the corresponding correlator was obtained

\[
\Pi_G (Q^2) = i \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot x} \langle T \mathcal{O}_\Delta(x) \mathcal{O}_\Delta(0) \rangle
\]

\[
= -\frac{\lambda^4 Q^2}{\kappa^2 4\lambda^2} \left( \frac{Q^2}{4\lambda^2} + 1 \right) \Gamma \left( \frac{Q^2}{4\lambda^2} + 1 \right) \lim_{\varepsilon \to 0} U \left( \frac{Q^2}{4\lambda^2} + 2, 2, \lambda^2 \varepsilon^2 \right)
\]

\[
= -\frac{2\lambda^4}{\kappa^2} \left[ 1 + \frac{Q^2}{4\lambda^2} \left( 1 + \frac{Q^2}{4\lambda^2} \right) \psi \left( \frac{Q^2}{4\lambda^2} \right) \right]
\]

\[
= -\frac{1}{8\kappa^2 Q^4} \left[ \ln \frac{Q^2}{\mu^2} + \frac{4\lambda^2}{Q^2} \ln \frac{Q^2}{\mu^2} + \frac{225}{3} \frac{\lambda^4}{Q^4} + ... \right].
\]

where two contact terms proportional to \(Q^2\) and \(Q^4\) have been discarded. The corresponding OPE in QCD reads

\[
\Pi_G^{(OPE)} (Q^2) = A_0 Q^4 \ln \left( \frac{Q^2}{\mu^2} \right) + B_0 \langle G^2 \rangle + O \left( \frac{1}{Q^2} \right)
\]

with \(A_0 = - \left( N_c^2 - 1 \right) / (4\pi^2)\) and \(B_0 = 4 + 49\alpha_s / (3\pi)\) (for the number of colors (flavors) \(N_c (N_f) = 3\)). As in the vector case, a two-dimensional correction appeared here, just as in the vector case. Also the gluon condensate can be obtained

\[
\langle G^2 \rangle \simeq - \frac{10}{3\pi^2} \lambda^4,
\]

which is of the opposite sign to the common value \(\langle G^2 \rangle \sim 0.4 - 1.2\) GeV\(^4\). Actually, from Eq. (37) it can be seen that \(\hat{\Pi}(0) = 0\), thus a non-vanishing value of \(\langle G^2 \rangle\) imply the violation of the low energy theorem

\[
\Pi_G (0) = \frac{32\pi}{\alpha_s b_0} \langle G^2 \rangle + O (m_q).
\]

Later in Ref. [1] the full solution (35) was considered, with \(B_G\) non-vanishing and as a function of \(Q^2\). By fine-tuning the expansion coefficients of \(B_G\) in the large \(Q^2\) region, it was shown that the two-dimensional correction can be eliminated. Moreover, a acceptable value of \(\langle G^2 \rangle\) can be obtained and the low energy theorem can be resumed.

To clarify this issue, we can calculate the correlator using the dispersion relation. For this purpose we need the residues

\[
F_n \equiv \langle 0 | \mathcal{O}_S(0) | n \rangle.
\]

This can be easily calculated from the normalizable solution (34) and was given by \[1,12\]

\[
F_n^2 = 8(n + 1)(n + 2)\lambda^6 / \kappa^2.
\]
Then we can get the spectral function using the definition \((16)\)

\[
\frac{1}{\pi} \text{Im} \Pi_G(s) = \frac{8\lambda^6}{\kappa^2} \sum_{n=0}^{\infty} (n+1)(n+2) \delta \left( s - m_n^2 \right)
\]  

(44)

The behavior of \(F_n^2\) as \(n \to \infty\) suggests that we start from the subtracted correlator

\[
\hat{\Pi}_G(q^2) = \Pi_G(q^2) - \Pi_G(0) - q^2 \Pi_G'(0) - \frac{q^2}{2} \Pi_G''(0)
\]

\[
= \frac{q^6}{\pi} \int_{0}^{\infty} \text{Im} \hat{\Pi}(s) \frac{s^3}{s^3(s-q^2)}
\]

(45)

Now using the dispersion relation \((15)\) we can find

\[
\hat{\Pi}_G(Q^2) = -Q^6 \sum_{n=0}^{\infty} \frac{F_n^2}{m_n^6(m_n^2+Q^2)}
\]

\[
= -\frac{2\lambda^4}{\kappa^2} \left[ 1 + \frac{Q^2}{4\lambda^2} \left( 1 + \frac{Q^2}{4\lambda^2} \right) \psi \left( \frac{Q^2}{4\lambda^2} \right) + \eta_1 \frac{Q^2}{4\lambda^2} + \eta_2 \left( \frac{Q^2}{4\lambda^2} \right)^2 \right]
\]

(46)

with \(\eta_1 = 2 - \psi(2)\) and

\[
\eta_2 = -\psi(2) - \sum_{n=2}^{\infty} \frac{1}{n^2}.
\]

(47)

Thus we explicitly reproduce the correlator \((37)\), up to the subtracted terms. Actually, starting from the correlator \((37)\), the spectral function has been obtained \[12\] by using the causal pole definition

\[
\frac{1}{\pi} \text{Im} \Pi_G(s) = \frac{\lambda^2}{2\kappa^2} s \left( s - \frac{m_0^2}{2} \right) \sum_{n=0}^{\infty} \delta \left( s - m_n^2 \right).
\]

(48)

which is in fact the same as Eq.\((44)\).

The coincidence of the correlator calculated from two different approaches again tells us that we should discard the non-leading solution in the bulk-to-boundary propagator. This can be seen directly from another way, using the decomposition formula derived in Ref.\[13\]

\[
\varphi(q, z) = \kappa \sum_n \frac{F_n \varphi_n(z)}{-q^2 + m_n^2}
\]

(49)

where the factor \(\kappa\) is added since in Ref.\[13\] the ”canonical normalization" \(\varphi(q, 0) = 1/\kappa\) has been used. Starting from the normalizable solution \[(34)\], we then obtain the non-normalizable solution immediately

\[
\varphi(q, z) = \lambda^4 z^4 \sum_{n=0}^{\infty} \frac{L_n^2(\lambda^2 z^2)}{a + n + 2}
\]

\[
= \Gamma(a+2) U(a, -1; \lambda^2 z^2)
\]

(50)

(51)
where \( a = -q^2/4 \lambda^2 \) and \( L_n^\mu \) is the associated Lagurre polynomials. In deriving the last expression we have used the generation function of the Lagurre polynomial \[21\]

\[
\frac{e^{-zt}}{(1-t)^{\mu+1}} = \sum_{n=0}^{\infty} L_n^\mu(z)t^n \quad (|t| < 1).
\]

(52)

It can be confirmed that Eq.(51) gives the right boundary normalization. Similar representation in the vector case has been obtained in Ref.[14].

Therefore the two-dimensional correction can’t be eliminated by just including the non-leading part in the bulk-to-boundary propagator. Moreover, in our procedure an additional constant subtraction term \( \Pi_G(0) \) is needed but in Eq.(37) this term is identically zero. This confirms the conclusion in Ref.[12] that the determination of the gluon condensate, and correspondingly the low energy theorem, is dependant on the substraction procedure.

IV. SUMMARY

The two-point function of the vector operator in the soft wall model is derived by using the original AdS/CFT correspondence, which coincides with that derived from the dispersion relation. A non-vanishing two-dimensional correction is found in the OPE of this correlator, with the coefficient \( C_2 \) different from the result obtained from the strategy of first quantized string theory. The numerical estimation of the gluon condensate is found to be almost identical to the common value. From the equivalence of the two derivations of the correlator, we conclude that the non-leading part of the bulk-to-boundary propagator should be discarded.

Extending to the correlator of the scalar glueball operator, we shows that the two-dimensional correction can not be eliminated by just including the non-leading part. The odd sign of the gluon condensate and the violation of the low energy theorem is found to be related to the subtraction scheme.

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