Detection of Stacked Filament Lensing Between SDSS Luminous Red Galaxies

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ABSTRACT

We search for the lensing signal of massive filaments between 220,000 pairs of Luminous Red Galaxies (LRGs) from the Sloan Digital Sky Survey. We use a nulling technique to remove the contribution of the LRG halos, resulting in a $10\sigma$ detection of the filament lensing signal. We compare the measurements with halo model predictions based on a calculation of 3-point halo-halo-mass correlations. Comparing the “thick” halo model filament to a “thin” string of halos, thick filaments larger than a Mpc in width are clearly preferred by the data.

Key words: filaments, weak lensing

1 INTRODUCTION

One of the most striking features of N-body simulations is the network of filaments into which dark matter particles arrange themselves. Some attempts to quantify this network have been made (Sousbie 2011). Other work has attempted to study the largest filaments, those between close pairs of large dark matter halos (Colberg et al. 2005). Such filaments are likely the easiest to identify in data, e.g., Zhang et al. (2013) look for overdensities in the galaxy distribution between close pairs of galaxy clusters. However, since filaments include both dark and luminous matter, weak lensing techniques are useful to understand the entire structure: Diétrich et al. (2012) identifies a large filament by focusing on a weak lensing analysis of a single cluster pair.

In this study we measure the weak lensing signal of filaments between stacked Luminous Red Galaxy (LRG) pairs in Sloan Digital Sky Survey (SDSS) data. The mass distribution and therefore weak lensing shear in the neighborhood of LRG pairs is dominated by the massive halos themselves. Methods which aim at filament detection, e.g., Maturi & Merten (2013), may have large degeneracy with the signal from these nearby halos. In the face of this degeneracy, we construct an estimator of the lensing signal which removes the shear due to these halos, assuming only that they are spherically symmetric. We will show that this technique is sufficient to obtain a detection, and some physical implications on filament size and shape can be extracted by comparison to filament models. Systematic errors which are expected to be spherically symmetric with respect to the halos, such as intrinsic alignments, are nulled simultaneously.

Other work has attempted to estimate the feasibility of weak lensing stacked filament detection. Maturi & Merten (2013) make optimistic choices for survey parameters and find that $\sim 2 - 4\sigma$ detections are possible for single clusters but state that their method has difficulties in application to stacked filament detection. In another study (Mead et al. 2010) use lens and source redshifts that make their lensing strength a factor of 2 greater than ours, and a galaxy number density at least a factor of 30 higher. The lower mass limit of their stacked clusters is $M_{200} = 4 \times 10^{14} M_\odot/h$, much larger than the dark matter halos associated with our LRGs. With these parameters, they estimate that $\sim 20$ cluster pairs are necessary to obtain a detection. We have $\sim 200,000$ pairs of LRG halos, and have been able to obtain a detection without new ground or space data.

Filaments can also be characterized using the language of higher-order correlations. In this case, one would describe the filament as the part of the matter-matter-matter three point function in the neighborhood of the halos forming a cluster pair. A detection of the halo-halo-matter three point function around such cluster pairs was made using the Red Cluster Survey (Simon et al. 2008). More recently Simon et al. (2013) used CFHTLen_s survey to measure the galaxy-galaxy-shear correlation function and attempted to measure the average mass distribution around galaxies. This measurement was done by subtracting off the two point contribution of the lensing signal. As these authors discovered, the three-point signal peaks at the cluster locations. However, for our purposes of identifying filaments, such a location of the three point function’s peak makes the technique of two-point subtraction unsatisfactory. Just as our nulling estimator removes two-point contributions which are spher-
2 MEASUREMENT TECHNIQUE

In this section, we describe the nulling technique for spherically symmetric components, which includes most of the two-point signal and the peak of the three-point signal. We also describe an additional subtraction which removes contributions from constant shear biases.

2.1 Nulling spherical components

We bin the data in such a way as to null the shear signal from any spherically symmetric source at the location of either member of the halo pair. To first order, such halos are expected to follow a spherically-symmetric NFW density distribution (Navarro et al. 1997) when stacked. However our technique is not dependent on the precise shape of the halo profile, only on its spherical symmetry. We note that halo anisotropy which is preferentially aligned with the inter-pair direction would not be nulled by the following procedure, but its small contribution is treated in Appendix A.

First consider just one spherically symmetric halo, $h_1$, as pictured in Fig. 1. Pick any point $p_1$ nearby. Draw another point $p_2$ which is (i) 90 degrees away from $p_1$ with respect to the halo, and (ii) at the same distance from the halo as $p_1$. The tangential shears $\gamma_1$ from these points add, while the cross component $\gamma_2$ is zero. This is the standard galaxy-galaxy lensing measurement. But if the shear components at $p_1$ and $p_2$ are measured with respect to a fixed coordinate system on the sky, they average out to zero. We denote the shear components relative to this fixed Cartesian coordinate system $\gamma_1$ and $\gamma_2$. As shown in Fig. 1, we choose this coordinate system such that $\gamma_2 = 0$ is perpendicular to the x-axis, and $\gamma_1 > 0$ is parallel.

Now add a second halo, $h_2$. We need to null the $h_2$ shear signal in both $p_1$ and $p_2$ as well. To do so, rotate both points by 90 degrees about $h_2$ to make points $p_3$ and $p_4$. By construction, the average $\gamma_1$ and $\gamma_2$ shear signal measured at these four points has no contribution from a spherical halo at $h_2$. Furthermore, one can check that rotating $p_3$ by 90 degrees about $h_1$ brings it into $p_4$, so that this set of four points is null with respect to both halos.

Such sets of four points are the building blocks for a number of possible binning schemes which attempt to null the spherically-symmetric halo signal. Note that any set of bins which exploit this property will necessarily mix scales relative to the hypothesized filament. However, since the most likely location for an inter-halo filament is on the line connecting the halo pair, we choose bins which will minimize this mixing of scales. The background shears are separated into bands that run parallel or perpendicular to the filament direction: these are marked as the “Filament region” on the left side of Fig. 2. The first two such bins are numbered on the figure. This binning scheme also exploits the expected symmetries about the center of the filament, in both horizontal and vertical directions. To verify that a bin does indeed fulfill the conditions for nulling the spherical signal mentioned above, imagine rotating the part of the bin above the $R_{\text{pair}}$ line about either halo, and see that it goes into the same colored bin in the region below the line. Note also that each background source is counted twice due to the overlap between different bins. This means a naive shape noise accounting of errors would underestimate the noise by a factor $\sqrt{2}$.

In what follows, we describe our measurement procedures of filament lensing. Following the method in Mandelbaum et al. (2013), we use, as the lensing observable, the stacked surface mass density field at the pixel $(x,y)$ in the region around each LRG pair (see Fig. 2), estimated from the measured shapes of background galaxies as

$$\Delta \Sigma_k(x,y; z_L) = \frac{\sum_j w_j \left( \langle \Sigma_{\text{crit}}^{-1} \rangle_j(z_L) \right)^{-1} \gamma_k(z_j)}{\sum_j w_j} \tag{1}$$

where the summation $\sum_j$ runs over all the background galaxies in the pixel $(x,y)$, around all the LRG pairs, the indices $k = 1, 2$ denote the two components of shear, and the weight for the $j$-th galaxy is given by

$$w_j = \left( \frac{\langle \Sigma_{\text{crit}}^{-1} \rangle_j(z_L)}{\sigma_{\text{shape}}^2 + \sigma_{\text{meas},j}^2} \right)^2 \tag{2}$$

We use $\sigma_{\text{shape}} = 0.32$ for the typical intrinsic ellipticities and $\sigma_{\text{meas},j}$ denotes measurement noise on each background galaxy. Again notice that, when computing the average shear field, we use the same coordinate system for each LRG pair; take one LRG at the coordinate origin and take the x-axis along the line connecting two LRGs as pictured in Fig. 1. $\langle \Sigma_{\text{crit}}^{-1} \rangle_j$ is the lensing critical density for the $j$-th source galaxy, computed by taking into account the photometric redshift uncertainty:

$$\langle \Sigma_{\text{crit}}^{-1} \rangle_j(z_L) = \int_0^\infty dz_s \Sigma_{\text{crit}}^{-1}(z_L, z_s) P_j(z_s) \tag{3}$$

where $z_s$ is the redshift of the LRG pair and $P_j(z_s)$ is the probability distribution of photometric redshift for the $j$-th galaxy. Note that $\Sigma_{\text{crit}}^{-1}(z_L, z_s)$ is computed as a function of lens and source redshifts for the assumed cosmology as

$$\Sigma_{\text{crit}}^{-1}(z_L, z_s) = \frac{c^2}{4\pi G} \frac{D_A(z_s)}{D_A(z_L) D_L(z_L, z_s)} \tag{4}$$

and we set $\Sigma_{\text{crit}}^{-1}(z_L, z_s) = 0$ for $z_s < z_L$ in the computation.

To increase statistics, we will measure the stacked weak lensing signal of filaments as a function of distance $R$ from...
the line connecting the two LRGs, rather than the two-dimensional mass distribution (see Fig. 2). Based on our nulling method in Fig. 1, each “p1” point at distance R has its counterparts with coordinate values

\[
p1(x, R) \rightarrow \{p2(R, -x), p3(1 - x, 1 - R), p4(1 - R, x - 1)\},
\]

where we set the first LRG position “h1” as the coordinate center. The first and second lines on Fig. 1, respectively. This nulling method only works when all shears are measured relative to the fixed Cartesian coordinate system on the sky and we again note that we set the “h1” (the first LRG) position as the coordinate center. The first and second lines on Fig. 1, respectively. This nulling method only works when all shears are measured relative to the fixed Cartesian coordinate system on the sky

The standard \(g-g\) lensing measurement of tangential shears about halos is immune to some effects which are worrisome for our method. Constant spurious shear on scales larger than the halo automatically cancels out in such tangential shear measurements. The logic is very similar to that used above to null the spherically-symmetric signal: a constant shear which is present at two points rotated by 90 degrees about the halo relative to each other is cancelled when those two points are averaged in a single bin.

Since we are not measuring the tangential shear \(\gamma_1\) relative to some center, another way of mitigating spurious constant shears is needed. We do this by repeating the measurement in the “systematic region” surrounding the filament region, as pictured on the right and lower sides of Fig. 2. Note that we use, but do not picture, identical systematic regions on the left and top of the filament region. The layout of these systematic regions was chosen such that they also null the spherically symmetric signal from both halos.

Similarly to the estimator for the signal region (Eq. 6), we can define the estimator for the systematic regions as

\[
\Delta \Sigma^\text{sys}_k (R_a) \equiv \sum_{x_b; 0 < x_b < 0.5} \left[ \Delta \Sigma_k (x_b, R_a) + \Delta \Sigma_k (R_a, -x_b) \right]
\]

\[
+ \Delta \Sigma_k (1 - x_b, 1 - R_a) + \Delta \Sigma_k (1 - R_a, x_a - 1) + \Delta \Sigma_k (x_b, -R_a) + \Delta \Sigma_k (-R_a, -x_b)
\]

\[
+ \Delta \Sigma_k (1 - x_b, 1 + R_a) + \Delta \Sigma_k (1 + R_a, x_a - 1),
\]

where \(\Delta \Sigma_k (x,y)\) denotes the \(k\)-th component of projected mass density at the position \((x, y)\) (see Eq. [1]) but note that the sum in the denominator of Eq. [1] runs over all lens-source pairs in the bin when plugged into Eq. [3]. Alternatively, the summation is over the \(x\)-axis bins, and the summation range is confined to \(0 < x_b < 0.5\) in order to avoid a double counting of the same background galaxies in the different quads of points p1, . . . , p4. Note however that the above binning does not put each galaxy in two different bins. The third and fourth lines of Eq. [6] exploit the symmetry about the line joining the LRG pair, by letting \(R_a \rightarrow -R_a\). Putting each galaxy in two bins in this way does add to our covariance between bins, but even so there is a gain in information. This is because when a galaxy is put in, say, bin 1 it is averaged together with a different set of galaxies compared to when it is placed in bin 2.

### 2.2 Systematic and halo ellipticity subtraction

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\[
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\]

\[
+ \Delta \Sigma_k (1 - x_b, 1 - R_a) + \Delta \Sigma_k (1 - R_a, x_a - 1) + \Delta \Sigma_k (x_b, -R_a) + \Delta \Sigma_k (-R_a, -x_b)
\]

\[
+ \Delta \Sigma_k (1 - x_b, 1 + R_a) + \Delta \Sigma_k (1 + R_a, x_a - 1) + \Delta \Sigma_k (x_b, R_a) + \Delta \Sigma_k (R_a, -x_b)
\]

\[
+ \Delta \Sigma_k (1 - x_b, 1 - R_a) + \Delta \Sigma_k (1 - R_a, x_a - 1) + \Delta \Sigma_k (x_b, -R_a) + \Delta \Sigma_k (-R_a, -x_b)
\]

where we again note that we set the “h1” (the first LRG) position as the coordinate center. The first and second lines on Fig. 2 show a layout of the systematic regions surrounding the filament region.
Figure 2. The lensing measurement (cross-component null test) is performed by combining all background shears’ $\gamma_1 (\gamma_2)$ components in bins, such as the pictured bins 1 and 2. We call the region including the LRG pairs the “Filament” region, where we expect the filament exists along the line connecting the two LRGs denoted by bold points. We also use the regions surrounding the Filament region, called “Systematic” regions, in order to estimate a possible coherent shear signal. We will estimate the filament lensing signal by subtracting the shear signal of Systematic regions from the shear of Filament region, as described in the text. (Note that the left and top Systematic regions of Eq. (7) are not pictured.)

Hence our estimator of the filament lensing is

$$\Delta \Sigma_{\text{fil}}^k (R_a) = \sum_{\text{all LRG pairs}} \left[ \Delta \Sigma_{\text{signal}}^k (R_a) - \Delta \Sigma_{\text{sys}}^k (R_a) \right]$$

(8)

Note that using these regions automatically assures that our systematic regions will have the same distribution in redshift $z$, pair separation $R_{\text{pair}}$, and pair orientation $\phi$ as the halo pairs themselves.

The nulling technique and systematic subtraction have the extra benefit of mostly removing contributions from halo ellipticity, expected to point along the line joining the LRG pair. The ellipticity-direction cross-correlation of Lee et al. (2008) has shown that simulated dark matter halos tend to point towards other halos in their vicinity. While the intrinsic alignment of LRGs has been measured at a less significant level, the smallness of the intrinsic alignment of the galaxy ellipticity is more likely due to misalignment of the light and mass profiles (Okumura et al. 2009), rather than the lack of alignment between neighboring massive halos. But if we let the virial radii of these halos be $\Delta \lesssim 1 \text{ Mpc}/h$ and the pair separation be $R_{\text{pair}} \gtrsim 6 \text{ Mpc}/h$, then the ratio of these $\Delta/R_{\text{pair}}$ is a small quantity, and we show in Appendix A that contributions to the signal are highly suppressed as this ratio gets smaller.

2.3 Jackknife Realizations

We perform the measurement and all null tests by first dividing up the survey area of 8,000 sq. deg. into 32 approximately equal area regions, as shown in Fig. 3. We then measure each quantity multiple times with each region omitted in turn to make $N = 32$ jackknife realizations. The covariance of the
measurement (Norberg et al. 2009) is given by
\[
C[\Delta \Sigma_i^{fil}, \Delta \Sigma_j^{fil}] = \frac{(N - 1)}{N} \times \left[ \sum_{k=1}^{N} \frac{(\Delta \Sigma_i^{fil})^k (\Delta \Sigma_j^{fil})^k}{N} - \frac{\Delta \Sigma_i^{fil} \Delta \Sigma_j^{fil}}{N} \right],
\]
where the mean value is
\[
\overline{\Delta \Sigma_i^{fil}} = \frac{1}{N} \sum_{k=1}^{N} (\Delta \Sigma_i^{fil})^k,
\]
and \((\Delta \Sigma_i^{fil})^k\) denotes the measurement from the \(k\)-th realization and the \(i\)-th spatial bin. The covariance is measured for both components of shear; for clarity we do not denote the separate components in Eqs. [9] and [10].

3 DATA

3.1 Pair catalog

We use the SDSS DR7-Full LRG catalog of Kazin et al. (2010), which contains 105,831 LRGs between 0.16 < \(z\) < 0.47. The sky coverage is approximately 8,000 sq. deg. The pair catalog is constructed by choosing each LRG in turn, and finding all neighboring LRGs within a cylinder of proper radius 18 Mpc/h and proper line-of-sight distance ± 6 Mpc/h. The redshift distribution of our pairs is in the left panel of Fig. 3. The distribution in line-of-sight distance is roughly uniform, as shown in the right panel of Fig. 4. The cut-off of \(\Delta z_{los} < 6\) Mpc/h corresponds roughly to a redshift separation of \(\Delta z < 0.004\) between pairs. Note that this line-of-sight separation assumes the LRG velocity is only due to Hubble flow; in other words, the redshift difference can arise from the difference of line-of-sight peculiar velocities in the LRG proper radius \(R_{vir}/2\), even if the two LRGs are in the same distance. This is the so-called redshift space distortion (RSD), and we will discuss the effect of RSD on our lensing measurements.

Each LRG can be a member of multiple pairs, we obtain \(\sim 220,000\) pairs with the separation cutoffs given above, about twice the number of the original LRG catalog. With \(R_{pair}\) defined to be the physical (or proper) projected separation between the LRGs, for pairs between 6 Mpc/h < \(R_{pair}\) < 18 Mpc/h we have a distribution \(P(R_{pair})\) which grows very slightly with \(R_{pair}\) (Fig. 3 right panel). The virial radii of these halos are \(\sim 0.5 - 1.0\) Mpc/h, so our selection of objects with \(R_{pair} \geq 6\) Mpc/h ensures that these LRGs live in different dark matter halos. We have checked that the measurement is insensitive to the choice of physical vs. comoving distances.

In Fig. 3 we show the stacked shear whiskers for the smallest \(R_{pair}\) bin, obtained from the average Eq. 1. The tangential shear signal around each member of the LRG pair is clearly visible. Each lens-source pair is optimized weighted in Eq. 2, and we convert back to \(\gamma\) by assuming fiducial redshifts \(z_i = 0.25\) and \(z_s = 0.4\). The nearest whisker to each LRG has magnitude \(\approx 0.003\). Note that due to the large distance between whiskers (0.1 \(R_{pair}\) \(\sim 1\) Mpc/h) even the closest ones to each halo are far from the center at \(\sim R_{vir}/2\). The dominance of the LRG halos in these fields motivates our use of the nulling scheme to isolate the relatively tiny filament lensing signal.

3.2 Background source catalog

The shear catalog is composed of 34.5 million sources, and is nearly identical to that used in Sheldon et al. (2009). The source redshift distribution is shown in the left panel of Fig. 3 and is obtained by stacking the posterior probability distribution of photometric redshift for each source, \(P(z_s)\). While the peak of this source catalog is approximately at the same redshift as the peak of our LRG pairs, \(z \sim 0.35\), the source distribution has a substantial tail extending out to higher redshifts. For further details of the shear catalog, see Sheldon et al. (2009).

4 THEORY: THICK- AND THIN-FILAMENT MODELS

We compare the measurement to the following two models, which generally predict “thick” or “thin” filaments:

- the three-point halo model prediction using the halo-halo-matter bispectrum based on the perturbation theory of structure formation;
- a one-dimensional string of less massive NFW halos (a collection of NFW halos along the 1D filament).

4.1 Thick-filament from the halo model

Here we use the halo model (Cooray & Sheth 2002; Takada & Jain 2003) to make a prediction for the size and shape of filaments between LRG pairs. We first obtain the projected mass density map around the pair of halos, based on the halo-halo-matter three-point correlation function, and then Fourier-transform the mass map to compute the shear field in order to compare with the measurements.

4.1.1 Surface Density Map from three-point correlations

We are interested in the three-point correlation among halos at \(\theta_1, \theta_2\) and \(\kappa\) at \(\theta_3\),
\[
\zeta_{hh} \equiv \langle \delta_h(\theta_1) \delta_h(\theta_2) \kappa(\theta_3) \rangle,
\]
where the 2D halo overdensity \(\delta_h\) and convergence field \(\kappa\) can be written in terms of the matter overdensity \(\delta_m\) as follows
\[
\delta_h = \int d\chi \ p(\chi) \ \delta^{(3D)}(\chi) = \int d\chi \ p(\chi) \ b \delta_m(\chi),
\]
and
\[
\kappa = \int d\chi \ \Sigma_{\kappa}^{-1}(\chi) \ \bar{\rho}_{m,0} \ \delta_m(\chi),
\]
with \(\bar{\chi}_s\) taken to be a fixed source plane. Here the halo bias \(b \sim 2\) for the large host halos of LRGs, and \(p(\chi)\) is the line-of-sight probability distribution of our LRG halos.

Under the flat-sky approximation, the projected correlation function \(\zeta_{hh}\) is given in terms of the 3D matter three-point correlation function as
\[
\zeta_{hh}(\theta_1, \theta_2, \theta_3) = \int d\chi_1 d\chi_2 d\chi_3 p_1(\chi_1) p_2(\chi_2) \times \Sigma_{\kappa}^{-1}(\chi_1, \chi_2, \chi_3) \bar{\rho}_{m,0} b^2 \langle \delta_m(\chi_3 \theta_1) \delta_m(\chi_2 \theta_2) \ \delta_m(\chi_3 \theta_3) \rangle.
\]
Figure 3. The SDSS footprint covered by our LRG catalog, and corresponding background sources. Total area is approximately 8,000 sq. deg. We divide the area into 33 jackknife regions as pictured, repeating the measurement 33 times with each region omitted once, giving an estimate of the covariance matrix.

Figure 4. (left panel): The redshift distribution of LRG pairs used as lenses (solid line) and background sources (dashed line). (top right panel): The distribution of proper projected distances between the two members of each galaxy pair. Each LRG can form multiple pairs, and pair separation grows linearly with distance. (lower right panel): The distribution of differences in line-of-sight distance for our LRG pairs. Note that a correction for RSD does not enter in these distances.

with

\[
(\delta_m(\chi_1 \vec{\theta}_1) \delta_m(\chi_2 \vec{\theta}_2) \delta_m(\chi_3 \vec{\theta}_3))
= \int \frac{d^3 \vec{k}_A d^3 \vec{k}_B d^3 \vec{k}_C}{(2\pi)^3(2\pi)^3(2\pi)^3} B^T_{\text{mmm}}(\vec{k}_A, \vec{k}_B, \vec{k}_C)
\times (2\pi)^2 \delta_D^3(\vec{k}_A + \vec{k}_B + \vec{k}_C) e^{i(\vec{E}_A \cdot \vec{x}_1 + \vec{E}_B \cdot \vec{x}_2 + \vec{E}_C \cdot \vec{x}_3)},
\]

where \( \vec{x}_i \equiv \chi_i \vec{\theta}_i \). We choose the line-of-sight LRG distributions to closely follow the measurement method. Since the measurement involves one LRG at essentially known line-of-sight comoving distance \( \chi \), we set the first distribution \( p_1 \) to a delta function. The second LRG also has known redshift, which is fixed to be nearby the first LRG, but has some finite width due to the uncertainty of RSD. Thus we use the
where $\sigma$ denotes the line-of-sight width of the distribution of the second LRG around the first.

Since we are interested in weak lensing due to filaments that arises from the matter distribution in the weakly non-linear regime, we employ perturbation theory (Goroff et al. 1986; Bernardeau et al. 2002) to compute the matter bispectrum:

\[ B_{\text{linm}}(k_A, k_B, -k_{AB}) = P(k_A) \times \]
\[ \left\{ \left[ \frac{10}{7} - \left( \frac{k_A}{k_B} + \frac{k_B}{k_A} \right) \left( \frac{3}{7} k_A + \frac{k_{AB}}{k_A} \right) \right] P(k_A) \right. \]
\[ + \left. \left[ \frac{10}{7} - \left( \frac{k_B}{k_{AB}} + \frac{k_{AB}}{k_B} \right) \left( \frac{3}{7} k_B + \frac{k_{AB}}{k_B} \right) \right] P(k_B) \right\} \]
\[ + \left[ \frac{10}{7} + \mu \left( \frac{k_A}{k_B} + \frac{k_B}{k_A} \right) + \frac{4}{7} \right] P(k_A) P(k_B) , \]

where $\mu \equiv \cos \phi$, $k_{AB} = \sqrt{k_A^2 + k_B^2 + 2k_A k_B \mu}$, and $P(k)$ is the linear matter power spectrum.

Employing Limber’s approximation sets $\chi_3 \approx \chi_2 \approx \chi_1$, and the three-point function can be simplified as

\[ \zeta_{hh} = \int d\chi_3 \, p_1(\chi_3) \, p_2(\chi_3) \, \sum_{ij} (\chi_3, \chi_3) \, \rho_{m,0} b^2 \]
\[ \times \int \frac{d^2 \vec{k}_A d^2 \vec{k}_B}{(2\pi)^2} \left[ B_{\text{linm}}(k_A, k_B, -k_{AB}) \right] \times e^{i \chi_3 (\vec{k}_A \cdot \vec{h}_1 + \vec{k}_B \cdot (\vec{h}_2 - \vec{h}_1))} , \]

where $k_A \equiv |\vec{k}_A|$, and the vectors $\vec{k}_A, \vec{k}_B$ are now two-dimensional, lying in the plane of the sky. The line-of-sight LRG distributions are now

\[ p_1(\chi_3) = \delta_D(\chi_3 - \chi) \]
\[ p_2(\chi_3) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\chi_3^2/(2\sigma^2)} \]

The delta function $p_1(\chi_3)$ thus removes the last $\chi$ integral, leaving

\[ \zeta_{hh} = \zeta_{h}\sum_{ij} (\chi_3, \chi_3) \, \rho_{m,0} b^2 \int \frac{d^2 \vec{k}_A d^2 \vec{k}_B}{(2\pi)^2} \left[ B_{\text{linm}}(k_A, k_B, -k_{AB}) \right] \]
\[ \times e^{i \chi_3 (\vec{k}_A \cdot (\vec{h}_1 - \vec{h}_3) + \vec{k}_B \cdot (\vec{h}_2 - \vec{h}_3))} , \]

with again, $k_{AB} \equiv |\vec{k}_A + \vec{k}_B|$. Choose the shear point to be at the origin, $\vec{h}_1 = 0$, and use comoving distances in the lens plane, $R_i = \chi \theta_i$, as in Fig. 6. Then the two $d^2 \vec{k}$ integrals can be written in terms of the magnitude of the wavevectors $k_A$ and $k_B$, the angle...
where $\Delta r_{\text{los}}$ is the rms separation inferred from the redshift difference of LRG pairs, $\Delta z / H(z_{\text{LRG}})$, and $\sigma_{\text{RSD}}$ is the width due to RSD. For our fiducial choice of LRG pair selection, we employ $\Delta r_{\text{los}} \lesssim 6 \text{ Mpc}/h$. However, the RSD is not a direct observable, and causes an uncertainty in the model prediction. The RSD has two contributions: bulk motions of halos in large-scale structure and virial motions of LRG within its host halo, where the latter is the so-called Finger-of-God (FoG) effect. The RSD due to halo bulk motions is estimated as $\sigma \sim 4 \text{ Mpc}/h$ from $N$-body simulations of $\Lambda$CDM model (e.g., Nishimichi & Taruya 2011). For the virial motion contribution, recently Hikage et al. (2013) used the DR7 LRG catalog to measure the g-g weak lensing and clustering measurements in order to study the FoG. For multiple LRG systems, which are massive halos (with $\sim 10^{14} h^{-1} M_{\odot}$) hosting multiple LRGs inside, the FoG effect is estimated as $\sigma \sim 9 \text{ Mpc}/h$ for LRGs at $z \approx 0.35$. For other LRGs residing in less massive host halos, the virial motions are smaller.

Summing up these effects, the line-of-sight spread of LRGs in the pairs can be as large as $\sigma \sim 10 \text{ Mpc}/h$. However, since the majority (above 90%) of the LRGs are only single-LRG systems, for which $\sigma_{\text{RSD}} \sim 6$, our best estimate is $\sigma \sim 8 \text{ Mpc}/h$. However, for any reasonable estimate of the RSD effect, the amplitude of the theory prediction is significantly larger than our measurement from the data. With the choice of $\sigma = 8 \text{ Mpc}/h$, the magnitude of the offset is a factor of 10. This requires further investigation with simulated lensing maps on which the measurement procedure is applied. In all plots involving the halo model prediction, we scale the amplitude to match the data.

In Fig. 7, we show the perturbation theory prediction for the kappa maps around hypothetical halo pairs hosting LRGs, for various choices of $R_{\text{pair}}$. In these figures, we employ $b = 2$ for linear bias of LRGs, and $z_{\text{L}} = 0.25$ and $z_{\text{r}} = 0.4$ for LRG redshift and source redshift, respectively. As described above, the amplitude is scaled to match the data.

4.1.2 Shear Map

To compare the two, we bin the prediction in the same way as the data. We begin by transforming the predicted kappa maps into shear maps. Due to symmetry, the resulting map of the cross-component $\gamma_2$, when binned in the same way as our data, gives identically zero signal. This provides one of our null tests. However, in order to visualize the shear...
map resulting from the three-point function, we go ahead and calculate the shear map for both \( \gamma_1 \) and \( \gamma_2 \).

The shear and convergence fields are related in Fourier space by

\[
\begin{align*}
\tilde{\gamma}_1(\vec{l}) &= \tilde{\kappa}(\vec{l}) \cos 2\phi'_l, \\
\tilde{\gamma}_2(\vec{l}) &= \tilde{\kappa}(\vec{l}) \sin 2\phi'_l, 
\end{align*}
\]

where \( \phi'_l \) is the angle between the wavevector \( \vec{l} \) and the \( x \)-axis of the coordinate system. We zero-pad the \( \zeta_{hhx} \) map out to a spatial scale 5 times larger than the map itself. This ensures that there is no spurious shear due to the periodic boundaries assumed in an FFT. After zero padding, we perform the FFT, then apply Eqs. (27, 28), and finally carry out the inverse FFT.

Note that for close pairs, such as in the top left panel of Fig. 4, the surface density is still significant at the edge of the pictured region. If such a map is zero-padded and the above process is applied to obtain \( \gamma \) maps, they will contain spurious shear due to the steep fall in density. We find that the FFT converges for \( R_{\text{pair}} \gtrsim 6 \text{ Mpc}/h \) as long as the \( \zeta_{hhx} \) map is calculated out to \( \pm 16 R_{\text{pair}} \) from the center of the line joining the pair of halos. We also check convergence of the FFT as a function of resolution, and find that spacing between grid points of 0.1 \( R_{\text{pair}} \) is sufficient.

An example of the resulting whisker plot is shown in Fig. 5 for \( R_{\text{pair}} = 10 \text{ Mpc}/h \). The largest magnitude shears of \( \gamma_1 \sim -0.001 \) lie between the two peaks of the three-point function. At radial distances beyond 0.5 \( R_{\text{pair}} \) from the line connecting two halos, the shear direction is parallel to the line \( (\gamma_1 > 0) \) for our definition as given in Fig. 4, as expected in the thin filament. Moving closer to the midpoint of the halos, the shear vanishes at about 0.5 \( R_{\text{pair}} \), and the direction then becomes flipped, now perpendicular to the connecting line \( (\gamma_1 < 0) \), which looks like “tangential shear” with respect to each halo. However, the width of the perpendicular shear region is about 0.5 \( R_{\text{pair}} \sim 5 \text{ Mpc}/h \), wider than the virial radius of the halos. Hence we call this model the “thick-filament” model. These features are from the perturbation theory matter bispectrum, thus reflecting the nature of large-scale structure in the weakly nonlinear regime. The shear pattern is qualitatively the same for other \( R_{\text{pair}} \) values, being well described by a decreasing amplitude for larger \( R_{\text{pair}} \). The green box outlines the filament region of Fig. 4.

4.1.3 Averaging over \( R_{\text{pair}}, z_L \) and \( z_s \) distributions

The PT 3pt function has a trivial redshift dependence according to the linear growth rate, \( \propto D_\text{lin}^4(z_L) \). Therefore we only have to do the time-consuming \( k_A, k_B \) and \( \phi \) integrals in Eq. (25) once for some arbitrary redshift values (here \( z_L = 0.25, z_s = 0.40 \)).

The measurement is of \( \Sigma_\text{crit} \gamma_1 \) and to this point we are still working with the dimensionless \( \zeta_{hhx} \). We should rather compare the data with \( \zeta_{hhx} \Sigma_\text{crit,eff} \), therefore we next obtain the effective lensing strength from the data. It is a redshift
weighting
\[ \Sigma_{\text{crit, eff}} \equiv \int dz_l \rho(z_l) \int dz_s \rho(z_s) \Sigma_{\text{crit}}(z_l, z_s), \] (29)
over the lens and source redshift distributions shown in Fig. 4.

The \( R_{\text{pair}} \) distribution within a given bin is essentially flat (see Fig. 4). The combination of different \( R_{\text{pair}} \) predictions is therefore easily modeled by a geometric factor accounting for the relative number of source galaxies which enter the measurement for each pair separation. This difference in area sampled by each pair goes as \( R_{\text{pair}}^2 \), so the prediction for a given bin is
\[ \langle \hat{\zeta}_{\text{hh}} \Sigma_{\text{crit}} \rangle_{R_{\text{pair}}} = \frac{\int_{R_{\text{min}}}^{R_{\text{max}}} dR_{\text{pair}} R_{\text{pair}}^2 \hat{\zeta}_{\text{hh}} \Sigma_{\text{crit}}}{\int_{R_{\text{min}}}^{R_{\text{max}}} dR_{\text{pair}} R_{\text{pair}}^2}. \] (30)

We apply this weighting when comparing data and theory in Sec. 5.

The last step in obtaining predictions for our binning scheme (Fig. 2) involves generating random points within the pictured area to imitate source galaxies. Then we interpolate over the \( \gamma_1 \) grid as pictured in Fig. 8 to obtain the shear for each random point. Finally, the shears are binned together in the same way as the data.

### 4.2 Thin-filament model

Here we consider the “string of halos” model as an independent model from the halo model. For this simple model, we use a string of NFW halos as in [Maturi & Merten] (2013). The shear induced by an NFW profile has an exact solution given by [Wright & Brainerd] (2000):
\[ \Delta \Sigma_{\text{NFW}} = r_s \delta_c \rho_c g(x) \] (31)
where \( g(x) \) is given by
\[ g(x) = \begin{cases} g_< & \text{for } x < 1 \\ (10/3) + 4 \ln (1/2) & \text{for } x = 1 \\ g_> & \text{for } x > 1 \end{cases} \] (32)
with
\[ g_< (x) = \frac{8 \arctanh \sqrt{(1-x)/(1+x)}}{x^2 \sqrt{1-x^2}} + \frac{4}{x^2} \ln (x/2) - \frac{2}{x^2 - 1} + 4 \arctanh \sqrt{(1-x)/(1+x)} \] (33)
\[ g_> (x) = \frac{8 \arctan \sqrt{(x-1)/(1+x)}}{x^2 \sqrt{x^2 - 1}} + \frac{4}{x^2} \ln (x/2) - \frac{2}{x^2 - 1} + 4 \arctan \sqrt{(x-1)/(1+x)} \] (34)

The model has just two parameters: \( M_{\text{fil}} \), the total mass in the string of halos, and \( N_{\text{fil}} \), the number of halos in the string. Each halo is given a mass \( M_{\text{halo}} = M_{\text{fil}}/N_{\text{fil}} \), and different halos are equally-spaced along the string between two LRGs. To generate predictions for this model, we calculate the shear profile at any given point by adding up the contribution for each halo in the string, with each halo’s contribution calculated according to Eq. (31). The overall shear amplitude depends on the total mass \( M_{\text{fil}} \). This model generally predicts the shear pattern that is parallel to the string (i.e. \( \gamma_1 > 0 \)), at the distance \( R \gtrsim 1 \) Mpc/h. Hence we call this the “thin-filament” model.

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5 RESULTS

We have detected at $10\sigma$ a stacked filament lensing signal by comparing the measurement to the null hypothesis that there is no excess mass extending between the LRGs. Under that hypothesis, we expect a lensing signal consistent with zero. The null has an expected chi-square of

$$\langle \chi^2 \rangle = N - n \pm 2N + 2n = 18 \pm 6 \quad (35)$$

where $N = 18$ is the number of bins and $n = 0$ the number of model parameters. To validate our detection, we first show four separate null tests which are consistent with the null hypothesis, before moving on to show the measurement itself and comparison to theory.

For all null tests, we repeat the measurement of our Eq. (8) estimator for $\Delta \Sigma_{ij}^{fil}$ using the same jackknife regions. The difference is that rather than using close LRG pairs to define the measurement regions of Fig. 2, we choose the “pair center,” $R_{pair}$, and angle on the sky $\phi$ in such a way that the result should be consistent with the null hypothesis of no excess mass lying along the center of the Fig. 2 filament region. The summary of all chi-square results for our null tests and the measurement itself is shown in Table 1.

5.1 Null tests: Unpaired LRG, Separated pairs, and Cross-component

Our first three null tests pass straightforwardly. First, the unpaired LRG test involves removing one LRG of the pair. In other words, we use the entire catalog of LRGs, assign each one a random $R_{pair}$ and orientation angle, then calculate $\Delta \Sigma_{ij}^{fil}$ as if it has a partner LRG at that $R_{pair}$ and angle. While we expect many LRGs to have filaments, the random orientations used in this test should stack individual filaments such that the final mass distribution is isotropic, and thus nulled by our procedure. The result is shown in Fig. 9 (red triangles), and with a $\chi^2 = 19.8$ is consistent with zero. The detection significance of $0.3\sigma$ shown in Table 1 is calculated as

$$\text{significance} = \frac{\chi^2 - \langle \chi^2 \rangle_{null}}{\sigma_{null}} = \frac{19.8 - 18}{6} = 0.3\sigma,$$

and is well under $1\sigma$.

The separated pair test involves using two LRGs as in the measurement, but with line-of-sight separation $100 \, \text{Mpc}/h < \Delta r_{los} < 120 \, \text{Mpc}/h$. The 3D distance of such pairs is so large that we expect no excess mass to build up between them. For the lens redshift $z_L$, needed in this test, we simply use the average of the two LRG redshifts. The result is shown in Fig. 11 (green diamonds) and is consistent with zero, with $\chi^2 = 19.5$ and significance $0.2\sigma$. Furthermore, this test shows that the spherically symmetric shear signal from both LRGs in the measurement is truly nulled, as claimed.

As in tangential shear measurements, where the cross-component of shear rotated by $45^\circ$ has no first-order contribution from gravitational lensing, our cross-component (the $\Delta \Sigma_{21}^{fil}$ component of Eq. 3) has no contribution from a filament. This statement holds as long as the stacked mass distribution around the LRG pairs has reflection symmetry about the line joining the pairs. For such a mass distribution, in the Cartesian coordinate system of Fig. L

$$\gamma_2(y) = -\gamma_2(-y).$$

Since background sources at $y$ are always put in the same bin with sources at $-y$, (see Fig. 2), $\Delta \Sigma_{21}^{fil} = 0$ on average. This is what we find in Fig. 10, where the green points show the result of this null test. The $\chi^2 = 16.3$ for a significance of $0.3\sigma$, consistent with the null hypothesis.

5.2 Null test: Random points

Finally, for the random points test, we repeat the measurement on $\sim 10$ times as many random points with the same distribution in $\phi$, $z$, and $R_{pair}$ as the pair catalog. The result shown in Fig. 9 (blue circles) has a small magnitude $\sim 0.1 \, M_\odot h^2/pc^2$, but with a $\chi^2 = 37.3$ it is $3.2\sigma$ inconsistent with zero. We assume that this inconsistency is the result of some unknown systematic error(s) in the measurement. We account for this systematic error by adding to each error bar a constant $\sigma_{syst} = 0.039 \, M_\odot h^2/pc^2$, which is smaller than the jackknife error on any individual bin of the random points measurement, and corresponds on average to an increase of $36\%$ on each error bar. (In other words, we assume that this systematic acts only on the diagonal of the covariance matrix, and so we add $\sigma_{syst}^2$ to each diagonal element of the covariance matrix.) The total error on any given bin becomes $\sqrt{\sigma_{JK}^2 + \sigma_{syst}^2}$, yielding a $\chi^2 = 23.6$ for the random points test, within $1\sigma$ of the null hypothesis.

5.3 Measurement

Now we turn to the filament measurement itself, using LRG pairs which are likely to have excess mass in between. Our initial measurement with covariance from jackknife alone yields a $\chi^2 = 89.7$ and corresponding significance $12.0\sigma$. However, we need to take account of the fact that the random points measurement was inconsistent with zero until the errors were increased to account for unknown systematics. Thus, we add in quadrature the same constant $\sigma_{syst} = 0.039 \, M_\odot h^2/pc^2$ to the measurement error bars. This addition made the random points test consistent with zero, but since the magnitude of the error bars on the measurement itself is much larger, this constant diagonal error only increases the uncertainty on each data point by $<2\%$. The resulting $\chi^2$ is then $78.0$, decreasing the detection significance to $10.0\sigma$, still a robust detection of filament lensing. The black points of Fig. 10 show the measurement with these larger error bars.

In Fig. 11 we show the normalized covariance matrix of $\Delta \Sigma_{ij}^{fil}$, $r_{ij} \equiv C_{ij} / \sqrt{C_{ii}C_{jj}}$. Most off-diagonal elements are near zero, with a scattered few of magnitude $r_{ij} \sim 0.5$. The highest covariance with $r_{ij} > 0.5$ is found in the top right corner of the matrix, corresponding to pairs with $14 \, \text{Mpc}/h < R_{pair} < 18 \, \text{Mpc}/h$.

5.4 Comparison to theory

In Fig. 11 we compare the halo-model and thin-filament models to the data. With a sign-flip relative to the data, the thin-filament model (blue dashed line) is difficult to support, but the thicker filament predicted by a halo model calculation (blue solid line) is more accurate. For the halo model, our best estimate of the contribution from redshift
Figure 9. The results of three null tests (labelled in the legend) for our closest set of pairs (left panel) and more widely separated pairs (middle and right panels). The unpaired LRG test and separated pair test are both consistent with the null hypothesis, showing that our estimator does null the spherically symmetric signal from the LRG halos.

Figure 10. Same as Fig. 9 but showing the cross-component null test $\Delta \Sigma_{\text{fil}}^2$ (green points) and measurement $\Delta \Sigma_{\text{fil}}^1$ (black points). The cross-component is consistent with zero, while the measurement deviates by 10$\sigma$. We compare the measurement to two theoretical models, the halo model (solid blue line) and NFW string (dashed blue), both of which have an amplitude adjusted to match the measurement. The shape of the halo model prediction is supported by the data, while the NFW string is clearly ruled out.

Figure 11. (left panel): The normalized covariance matrix of $\Delta \Sigma_{\text{fil}}^1$. The pairs separated by the largest distance of 14 Mpc/$h < R_{\text{pair}} < 18$ Mpc/$h$ (top right corner of $r_{ij}$) show the strongest correlations between bins. (right panel): The same, but for $\Delta \Sigma_{\text{fil}}^2$. We use the full covariance matrices when calculating the significance of the deviation from the null hypothesis for the measurement and each null test.
space distortions (with a dispersion of $\sigma_{\text{RSD}} = 5$ Mpc/$h$) is $\sigma = 8$ Mpc/$h$. Even with the resulting RSD dilution, we need to scale the halo model amplitude down by a factor of ten. Thus, the combination of bias in the measurement, dilution of the signal, and error in the halo model prediction leads to a large offset between theory and measurement. As for the NFW string prediction, the magnitude is controlled by the total filament mass, $M_{\text{fil}}$. For this plot, it was adjusted to $M_{\text{fil}} = 2 \times 10^{14} M_\odot/h$, giving a magnitude roughly equal to the halo model prediction. (Although clearly the shape is still wrong.)

We calculate the average mass and density in the region between the halos using the halo model mass maps of Fig. 7. However, note that these results should be taken with caution, as again the amplitude has been scaled to match the measurement. The result is shown as a function of $R_{\text{pair}}$ in Fig. 12. The different curves show the results for different choices of $y_{\text{max}}$, the maximum distance which we include in the average. The averaging along the x-axis includes all mass which is both between the two halos, and at least 1 Mpc/$h$ from either halo center. In other words, we do not count mass that would be within either halos’ virial radius in the estimate of the filament mass.

6 DISCUSSION

We have presented a technique for the statistical measurement of properties of dark matter filaments between LRG halos separated by $\sim 10$ Mpc. We use an empirical approach to cancel out the contribution of spherical halos and constant shear patterns in the data. The residual shear patterns are attributed to filamentary structures and the mass and thickness of the filament are estimated. We find the data prefer thick filaments that contain at least twice as much mass as the halos that set at their end points.

There are several approximations and sources of error in our analysis.

- The stacking of hundreds of thousands of LRG pairs leads to a smearing of the mass distribution. This means that we cannot make definitive statements about the typical filament structures in the universe, in particular the limits we obtain on the thickness of the filament only apply to the stacked profile.
- The binning scheme we use to null out the contribution of spherical halos and other considerations mixes scales. It also preserves the signal only from perfect cylindrical symmetry. So even genuine structures beyond spherical mass distributions are nulled out.
- Errors in shear estimates from the intrinsic ellipticities of background galaxies dominate the statistical error in our measurement. The other major source of statistical error is the variations in the mass distribution between different LRG pairs – this cannot be quantified from the data, so we intend to study it with simulations.
- The calibration of the shear, which relies on a correction for the smearing due to the PSF, introduces a redshift dependent bias that propagates to the filament mass estimate. Uncertainties in the photometric redshifts of background galaxies have a similar effect.
- Redshift space distortions: the line of sight separation of the LRG’s is uncertain owing to their relative peculiar velocity. We have attempted to account for it in our discussion above.
- The inevitable contamination of the LRG sample with other galaxies and stars leads to a dilution of the signal. This should be controlled to better than the 10% level.
- Finally, the theoretical model is based on halo-halo-mass correlations in the halo model. This model is known to have limitations, with the amplitude being consistent with N-body simulations only at the 30% level. In particular, the halo model tends to overestimate the clustering amplitude over a range of 1-10Mpc, the transition regime between the weakly and strongly nonlinear regimes. The regime also involves theoretical approximations such as the linear halo bias assumption.

In future work several improvements can be made that address nearly all the above points. In addition forward modeling of the measurement can be done using simulations and the halo model, so that comparisons can be made without use of our nulling technique. Such an approach may allow for more detailed tests of the halo model and of filamentary properties, though care will need to be exercised to distinguish systematic errors. Finally, an obvious complement to our study is to compare the mass distribution inferred from lensing shears with the distribution of foreground galaxies and hot gas.

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APPENDIX A: HALO ELLIPTICITY

In order to show that the contribution from halo ellipticity is small, we consider a very simple model which is even less spherical than an elliptical halo. Thus, if the shear from this model is negligible, then so is shear from elliptical halos. We take two point masses labelled E1 and E2 on Fig. A1. These are each separated from the halo center by $\Delta \lesssim R_{\text{vir}}$. The outermost square region pictured corresponds to the top part of Fig. 1 with side length $R_{\text{pair}}$.

On the left panel of Fig. A1 we extend two lines from E1 which are both 45 degrees from the horizontal axis. With our shear sign convention (Fig. 1), these lines describe points where the shear from E1 is purely $\gamma_2$, i.e., these lines are the zeros of $\gamma_1$. Thus, points which are opposite sides and equidistant from these lines have a net contribution of $\gamma_1 = 0$. As a result, the net $\gamma_1$ shear when summed over all galaxies in regions A and A' is zero. In the same way, regions B and B' sum to zero.

Likewise, on the right panel we draw a line from E2 which is 45 degrees from the vertical, and the net $\gamma_1$ shear in C and C' is zero. A final cancellation occurs in regions D and D', where the positive $\gamma_1$ shear from E1 in D cancels the negative shear from E2 in D'. The net shear from these two point masses is then given by the remaining regions, labelled $\gamma_1^+$ and $\gamma_1^-$. These two regions do not cancel perfectly, but it is clear that (i) these regions nearly cancel: while the $\gamma_1^+$ region is slightly closer to E1 than the $\gamma_1^-$ region is to E2, in area, the $\gamma_1^+$ region is slightly smaller; (ii) the size of these imperfectly cancelled regions shrinks rapidly as $\Delta / R_{\text{pair}}$ gets smaller. The upper bound is

$$\Delta / R_{\text{pair}} \leq \frac{R_{\text{vir}}}{R_{\text{pair}}} = \frac{1}{6} \frac{\text{Mpc}/h}{\text{Mpc}/h},$$

but most of our LRG pairs have smaller virial radii and larger pair separation. Furthermore, the density profile of

**Figure 12.** Estimates of the mass density (upper panel) and the enclosed mass (lower) in the filament region, using the best-fit halo model predictions to the measurements in Fig. 10. To estimate these quantities, we integrate the projected mass density over the rectangular area that is defined by the separation distance of paired LRGs (x-axis) and the distance from the line connecting the two LRGs ($y_{\text{max}}$ denoted by the legend). The estimated volume in the top panel is taken to be a cylinder of radius $y_{\text{max}}$, with four choices indicated in the legend. With density contrasts $\rho / \rho_m \sim 10$, the component of matter we measure falls between the regimes of high density halos and the low density linear regime.
halos falls off quickly, so that relatively little of the mass is
displaced an entire virial radius from the center.

Finally, note two more points concerning the contribution
of halo ellipticity to the systematic regions of Fig. 2.
First, subtracting the signal in the left and right systematic
regions, which have the same shape as the filament region,
partially removes the very small ellipticity contribution de-
scribed above. Second, halo ellipticity could also contribute
to the top and bottom systematic regions of Fig. 2. However,
being offset by an additional distance of $R_{\text{pair}}$, the con-
tribution in these regions will be even smaller than the closer
regions which we have just considered.

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Figure A1. As an extreme model of halo ellipticity, we consider the shear from point masses E1 and E2. The two panels show the same region twice: the left panel highlights the contribution from E1, and the right that from E2. The net $\gamma_1$ shear (with the sign convention of Fig. 1) cancels in regions A and A', B and B', etc. (See the text for the details.) The size of the uncancelled regions, $+\gamma$ and $-\gamma$, shrinks rapidly with the small number $\Delta/R_{\text{pair}} \leq 1/6$, showing that contributions from halo ellipticity are highly suppressed in our measurement.