Cosmic opacity: cosmological-model-independent tests and their impacts on cosmic acceleration

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Abstract

With assumptions that the violation of the distance-duality relation entirely arises from non-conservation of the photon number and the absorption is frequency independent in the observed frequency range, we perform cosmological-model-independent tests for the cosmic opacity. The observational data include the largest Union2.1 type Ia supernova sample, which is taken for observed $D_L$, and galaxy cluster samples compiled by De Filippis et al. and Bonamente et al., which are responsible for providing observed $D_A$. Two parameterizations, $\tau(z) = 2\epsilon z$ and $\tau(z) = (1 + z)^{2\epsilon} - 1$ are adopted for the optical depth associated to the cosmic absorption. We find that, an almost transparent universe is favored by Filippis et al. sample but it is only marginally accommodated by Bonamente et al. samples at 95.4% confidence level (C. L.) (even at 99.7% C. L. when the $r < 100$ kpc-cut spherical $\beta$ model is considered). Taking the possible cosmic absorption (in 68.3% C. L. range) constrained from the model-independent tests into consideration, we correct the distance modulus of SNe Ia and then use them to study their cosmological implications. The constraints on the $\Lambda$CDM show that a decelerating expanding universe with $\Omega_\Lambda = 0$ is only allowed at 99.7% C. L. by observations when the Bonamente et al. sample is considered. Therefore, our analysis suggests that an accelerated cosmic expansion is still needed to account for the dimming of SNe and the standard cosmological scenario remains being supported by current observations.

PACS numbers: 95.36.+x, 04.50.Kd, 98.80.-k
I. INTRODUCTION

The type Ia supernovae (SNe Ia) are observed to be fainter than expected from the luminosity-redshift relationship in a decelerating universe. This unanticipated dimming was first attributed to an accelerating expansion of the universe [1, 2]. Although the existence of cosmic acceleration has been verified by several other observations, initially, there had been some debates on the interpretation of underlying physical mechanism for the observed SNe Ia dimming. For example, dust in the Milk Way and oscillation of photons propagating in extragalactic magnetic fields into very light axions had been proposed to account for the dimming [3, 4]. Any kind of photon number violation, such as absorption, scattering or axion-photon mixing, sensibly imprints its influence on the Tolman test [5], which can be rewritten as a relationship among cosmological distance measurements known as the famous distance-duality (DD) relation [6–8],

\[
\frac{D_L}{D_A}(1 + z)^{-2} = 1, \tag{1}
\]

where \( z \) is the redshift, \( D_L \) and \( D_A \) are the luminosity distance and the angular diameter distance (ADD) respectively. This reciprocity relation holds for general metric theories of gravity in any background, not just in that of the Friedmann-Lemaître-Robertson-Walker background, and it is also valid for all cosmological models based on the Riemannian geometry. That is, its validity depends neither on Einstein field equations nor on the nature of the matter-energy content. The DD relation plays an important role in modern cosmology [9–12], and, in most cases, it has been applied, without any doubt, to analyze the cosmological observations. However, the reciprocity relation may be violated if photons do not travel on null geodesics or the universe is opaque.

Fortunately, it is in principle possible to perform a valid check on the DD relation by means of astronomical observations. The basic idea is to search for observational candidates with known intrinsic luminosities as well as intrinsic sizes, and then determine their \( D_L \) and \( D_A \) to test the Etherington relation directly. It is difficult for us to find objects of the same class with both known intrinsic luminosities and intrinsic sizes. Thus, a \( \Lambda \)CDM cosmological model is usually assumed when one performs tests by utilizing observed \( D_L \) or \( D_A \) [13–16] and the results show that there is no strong evidence for deviations from
the standard DD relation. Recently, a model-independent method has been proposed to test DD relation by considering two different classes of objects, for example, SNe Ia and galaxy clusters, from which $D_L$ and $D_A$ are determined separately \cite{17,20}. For a given ADD data, in order to obtain the corresponding $D_L$ from SNe Ia, a selection criteria $\Delta z = |z_{\text{Cluster}} - z_{\text{SNe Ia}}| \leq 0.005$ is adopted. Using the phenomenological parameterized forms

$$\frac{D_L}{D_A}(1 + z)^{-2} = \eta(z),$$

(2)

with $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \frac{z}{1+z} \eta_0$, and the data from Union2 SN Ia \cite{21} and galaxy clusters, it was found that the DD relation can be accommodated at 1$\sigma$ C. L. for the elliptical $\beta$ model \cite{22} and at 3$\sigma$ C. L. for the spherical $\beta$ model \cite{23}. More recently, in order to avoid the bias from the redshift difference between $D_L$ and ADD, methods, such as, the binning of SNe Ia \cite{24}, the interpolation \cite{25} and local regression \cite{26} from nearby SNe Ia points for a given galaxy cluster, were proposed and similar results were obtained. In addition, The DD relation tests by use of SNe Ia and gas mass function of galaxy clusters were carried out and similar results were also obtained \cite{27,29}. So, overall, all the tests performed so far show that there is no strong indication of the DD relation violation. However, let us note that systematic uncertainties resulting from the morphological models of galaxy clusters and the redshift difference between $D_L$ and ADD might exert influences on DD relation tests.

If one considers that the photon traveling along null geodesic is unassailable, the DD relation violation most likely implies non-conservation of the photon number which has a mundane origin (scattering from dust or free electron) or an exotic origin (photon decay or photon mixing with other light states such as the dark energy, dilaton or axion \cite{4,14}). In this case, the flux received by the observer will be reduced and so the universe is opaque. If we assume that the flux from the source is decreased by a factor $e^{-\tau(z)}$, then the inferred (observed) luminosity distance differs from the “true” one \cite{30,32}

$$D_{L,\text{obs}} = D_{L,\text{true}} \cdot e^{\tau/2},$$

(3)

where $\tau$ is the opacity parameter which denotes the optical depth associated to the cosmic absorption. Initially, More et al. \cite{32} studied the cosmic opacity by examining the differe-
ence of the opacity parameter at redshifts $z = 0.20$ and $z = 0.35$, $\Delta \tau = \tau(0.35) - \tau(0.2)$, where the difference of the observational luminosity distance ($\Delta D_{L, \text{obs}}$) at these two redshifts was estimated from two subsamples of ESSENCE SN Ia [33] and the corresponding $\Delta D_{L, \text{true}}$ was derived from the distance measurements of baryonic acoustic feature [34] in the context of $\Lambda$CDM. Assuming flat priors on $\Omega_\Lambda$ and $\Omega_M$ in the range of $0 < \Omega < 1$, and uniformly spaced values of $\Delta \tau \in [0, 0.5]$, they found that a transparent universe is favored (posterior probabilities of $\Delta \tau$ peaked at 0) and $\Delta \tau < 0.13$ at 95% C.L. This method has been applied to investigate the homogeneity of the cosmic opacity in different redshift regions and the results suggest that the cosmic opacity oscillates between zero and non-zero values as redshift varies [35, 36]. Later, Avgoustidis et al. [37, 38] carried out further studies by assuming an optical depth parameterization $\tau(z) = 2\epsilon z$ or $\tau(z) = (1 + z)^2 - 1$ for small $\epsilon$ and $z \leq 1$. There they took the standard luminosity distance in the spatially flat $\Lambda$CDM $((1 + z)^2 D_A(z, \Omega_M))$ and the Union SN Ia [39] for $D_{L, \text{true}}$ and $D_{L, \text{obs}}$ respectively. In addition to the SNe Ia data, they also used the measurements of the cosmic expansion $H(z)$ [40, 41]. By taking $\epsilon \in [-0.5, 0.5]$, $\Omega_M \in [0, 1]$ and $H_0 \in [74.2 - 3 \times 3.36, 74.2 + 3 \times 3.36]$ [41] all uniformly spaced over the relevant intervals in a flat $\Lambda$CDM model and performing a full Bayesian likelihood analysis, they obtained a result $\epsilon = -0.04^{+0.08}_{-0.07}$ (2σ C.L.), which corresponds to an opacity $\Delta \tau < 0.012$ (95% C.L.) for the redshift range between 0.2 and 0.35, almost a factor of 11 stronger than the constraint obtained in Ref. [32]. Recently, Lima et al. [42] reexamined this issue by confronting the luminosity distance which is dependent on two free parameters, i.e., the so-called cosmic absorption parameter ($\alpha_*$) and the matter density ($\Omega_M$), with observations, using a subsample of Union2 SN Ia obtained by selecting SNe Ia with redshifts greater than $cz = 7000 km/s$ in order to avoid effects from Hubble bubble, and they found that the Einstein-de Sitter model ($\Omega_M = 1$) could be allowed at 68.3% (95.4%) C.L. in the case of a constant (epoch-dependent) absorption and concluded that a cosmic absorption may be responsible for the dimming of the distant SNe Ia without the need of an accelerated expansion of the universe. However, all these studies concerning the cosmic opacity assume a (flat) $\Lambda$CDM model, and are thus model-dependent.

Here we propose another model-independent method to examine the cosmic opacity
and investigate its possible implications for the cosmic evolution. If we assume that the violation of the DD relation is purely caused by the photon number non-conservation, then we can find out whether the universe is opaque by checking the possible violation of the DD relation. It should be emphasized that the cosmic absorption not only affects the luminosity distance measurements of SNe Ia observations as shown in Eq. (3), but also exerts influences on the angular diameter distance measurements determined from SZE+X-ray surface brightness observations [43, 44],

$$D_A \propto \frac{\Delta T_{\text{CMB}}^2}{S_X}, \quad (4)$$

where $\Delta T_{\text{CMB}}$ is the temperature change due to the Sunyaev-Zel’dovich effect (SZE) when the cosmic microwave background (CMB) photons pass through the hot intra-cluster medium and $S_X$ is the X-ray surface brightness of galaxy clusters. The SZE spectra distortion of the CMB is determined by measuring the intensity decrements, $\Delta I$, which is sensitive to the cosmic absorption. Additionally, the surveys of X-ray surface brightness are also sensitive to the opacity of the universe. Supposing the absorption is frequency independent in the observed frequency range (from microwave band to X-ray band), the “true” ADD connects the observed one measured in an opaque universe with $D_{A,\text{true}} = D_{A,\text{cluster}} \cdot e^\tau$. Thus, in actual calculations, the DD relation takes the following form:

$$\frac{D_{L,\text{SN}}}{D_{A,\text{cluster}}} (1 + z)^{-2} = e^{3\tau/2}. \quad (5)$$

Now we will use the the largest Union2.1 SN Ia sample [46] and the ADD data from galaxy cluster samples [22, 23] to test, model-independently, the possible violation of the DD relation, which can be translated to a possible cosmic opacity. The observed $D_L$ and $D_A$ come from the latest Union2.1 SN Ia and galaxy clusters samples [22, 23], respectively. Actually, there are a number of inherent uncertainties in the selected astrophysical objects from which the observed $D_A$ are derived, e. g., the cluster asphericity [22] and the model for the cluster gas distribution [23]. In this paper, we consider the elliptical $\beta$ model galaxy clusters sample [22], spherical $\beta$ model, $r < 100$ kpc-cut spherical $\beta$ model and

1 See Ref. [45]
hydrostatic equilibrium model galaxy clusters samples [23] to investigate the impact of these inherent uncertainties on the cosmic opacity test.

II. DATA AND CONSTRAINT RESULTS

In order to place constraints on the cosmic opacity parameter \( \tau \), we first parameterize it with two monotonically increasing functions of redshift, i.e., \( \tau(z) = 2\epsilon z \) and \( \tau(z) = (1 + z)^{2\epsilon} - 1 \) [37]. These two parameterizations are basically similar for \( z \ll 1 \) but they differ when \( z \) is not very small. As the data applied in our following analysis discretely distribute in the redshift range \( 0.023 \leq z \leq 0.890 \), our analysis we are going to perform may tell us something about the possible dependence of the test results on the parametric forms for \( \tau \). To obtain \( \tau_{\text{obs}}(z) \) determined by the following expression:

\[
\tau_{\text{obs}}(z) = \frac{2}{3} \ln \left[ \frac{D_{\text{SN}}}{D_{\text{cluster}}(1 + z)^2} \right],
\]

the data pairs of observed \( D_L \) and \( D_A \) almost at the same redshift should be supplied. For the observed \( D_L \), the largest Union2.1 SN Ia is considered. Galaxy cluster samples, where the \( D_A \) are obtained by combining the SZE+X-ray surface brightness measurements [43, 44], are responsible for providing the observed \( D_A \). The first one, including a selection of 7 clusters compiled by Mason et al. [47] and a sample of 18 clusters collected by Reese et al. [48], was re-analyzed by Filippis et al. [22] by assuming an elliptical \( \beta \) model for the galaxy clusters. The second kind of samples are compiled by Bonamente et al. [23] with three different models for the cluster plasma and dark matter distribution, i.e., the spherical \( \beta \) model, spherical \( \beta \) model with \( r < 100 \text{ kpc} \) cut, and hydrostatic equilibrium model. Therefore, the data derived from these three different models are adopted to check whether the cosmic opacity tests are sensitive to the model for the cluster gas distribution. The observed \( D_L \) are binned from the data points of Union2.1 SN Ia with their redshifts satisfying the certain criteria \( \Delta z_{\text{max}} = |z_{\text{cluster}} - z_{\text{SNe Ia}}|_{\text{max}} \leq 0.005 \) to match the observational data of the ADD samples [24, 49]. This binning method can minimize the statistical errors originating from the redshift difference between \( D_L \) and \( D_A \). On the other hand, we alter \( \Delta z_{\text{max}} \) from 0.000 to 0.005 to ensure the number of the clusters that share the same SNe to be as few as possible, so as to reduce the dependence
of opacity tests on the correlation of redshift-matched SNe. After obtaining $\tau_{\text{obs}}(z)$ from these selected data pairs, we estimate the free parameters of a given parametric form by using the standard minimum $\chi^2$ route:

$$
\chi^2(z; \mathbf{p}) = \sum_i \frac{[\tau(z; \mathbf{p}) - \tau_{\text{obs}}(z)]^2}{\sigma^2_{\tau_{\text{obs}}}},
$$

where $\sigma_{\tau_{\text{obs}}}$ is the error of $\tau_{\text{obs}}$ associated with the observed $D_L$ and $D_A$, and $\mathbf{p}$ represents the free parameters to be constrained. The graphic representations and numerical results of the probability distribution of the opacity parameter $\epsilon$ constrained from the model-independent tests are shown in Figures 1 and Table I. These suggest that the dependence of test results on the above-chosen parameterizations for $\tau(z)$ is relatively weak. Similar to the results obtained by examining the cosmic opacity in a particular redshift range (0.20-0.35) [32, 37, 38] and deforming the DD relation in terms of the cosmic absorption parameter [42], we find, from Figure 1, that an almost transparent universe is also favored by the elliptical $\beta$ model galaxy clusters sample [22]. For the ADD samples given by Bonamente et al. [23], the results are shown in Figure 2. We find that the results are nearly not sensitive to the model of cluster gas distribution and a transparent universe can only be marginally accommodated at 95.4% C. L. (even at 99.7% C. L. when the $r < 100$ kpc-cut spherical $\beta$ model is considered). That is, all the constraints on the opacity parameter obtained from the Bonamente et al. samples prefer an opaque universe. These results are clearly different from what were obtained based on the $\Lambda$CDM in Refs. [32, 42], where a transparent universe is obviously supported. In fact, these cosmic opacity test results are very similar to the previous model-independent tests for the DD relation [17–19, 24, 25]. However, our objective here is the cosmic opacity test with an assumption that the violation of the DD relation entirely originates from the non-conservation of photon number, rather than the DD relation test itself.

In order to explore the implications of the cosmic opacity, let us transform the SNe Ia distance modulus in a transparent universe into that in an opaque one

$$
\mu_{\text{true}}(z) = \mu_{\text{obs}}(z) - 2.5[\log \epsilon]\tau(z),
$$

and study the cosmological constraints resulting from this correction. Since the high redshift galaxy cluster data are absent in our discussion of the cosmic opacity, the distance-
modulus-modified SNe Ia used to investigate the cosmological constraints are cut down from Union2.1 with the criteria \( z \leq 0.784 \) and \( z \leq 0.890 \) when clusters in the Filippis et al. sample and the Bonamente et al. samples are applied, respectively. Since the dependence of cosmic opacity tests on the above-chosen parametric forms of \( \tau(z) \) is relatively weak, we only consider results obtained from the linear parametrization \( \tau(z) = 2\epsilon z \) in the following cosmological implication analysis. For the flat \( \Lambda \)CDM model, different from the methods used in Refs. [37, 38, 42], we examine the probability distributions of \( \Omega_M \) by considering the possible (68.3% C. L. range) opacity parameter \( \epsilon \) constrained from the previous model-independent tests. The results are shown in Figures (3, 4). We find that the opacity parameter \( \epsilon \) constrained from previous cosmological-model-independent tests impact slightly on the likelihood distributions of \( \Omega_M \) and a universe with \( \Omega_\Lambda > 0 \) is required to account for the dimming of the SNe Ia. This differs from the results in Ref. [42], where the Einstein-de Sitter universe \( (\Omega_M = 1) \) can be easily accommodated at 68.3% and 95.4% C. L. for the constant and epoch-dependent absorptions, respectively.

Without a spatially flat universe prior, we also investigate the \( \Lambda \)CDM with the corrected distance modulus of Union2.1 SN Ia by taking the opacity parameter \( \epsilon \) in 68.3% C. L. range constrained from the previous model-independent tests into consideration. The linear parametrization for the cosmic opacity is also considered. The results projecting to the \( \Omega_M - \Omega_\Lambda \) plane are shown in Figure 5. Because of the “marginalization” of \( \epsilon \), which somewhat weakens the constraints on parameters \( \Omega_M \) and \( \Omega_\Lambda \), the statement that the expansion of universe is accelerating is less eloquent than the one in Ref. [1], which concludes that a currently accelerating universe is needed at 99.9% (3.9\( \sigma \)) C. L. to agree with their distance measurements of SNe Ia. However, we find that a decelerating universe with \( \Omega_\Lambda = 0 \) is only allowed at 99.7% C. L. by observations when the spherical \( \beta \) model is taken into account. So, the standard cosmological scenario is still supported by observations, although current data may favor a universe with non-zero opacity,
III. CONCLUSION AND DISCUSSION

In this paper, by considering the luminosity distances provided by the largest Union2.1 SN Ia sample together with the ADD given by galaxy clusters samples, we first examine the possible cosmic opacity in a cosmological-model-independent way. Two redshift-dependent parametric expressions: \( \tau(z) = 2\epsilon z \) and \( \tau(z) = (1 + z)^{2\epsilon} - 1 \) are considered to describe the optical depth associated to the cosmic absorption. The results suggest that the tests of cosmic opacity are not significantly sensitive to the parametrization for \( \tau(z) \). For the ADD sample compiled by Filippis et al. [22] with an elliptical \( \beta \) model, we obtain that a universe with little opacity (almost transparent) is favored. For the ADD samples given by Bonamente et al. [23], where three different cluster gas distribution models are applied, the test results suggest that a transparent universe can only be marginally consistent with observations at 95.4% C. L. (even at 99.7% C. L. as the \( r < 100 \) kpc-cut spherical \( \beta \) model is applied). These are fairly different from the conclusions in Refs. [32, 42]. By considering the possible cosmic opacity (68.3% C. L. range) constrained from the previous model-independent tests, we obtain the corrected distance modulus of SNe Ia and then use them to investigate its cosmological implications. In the context of a flat \( \Lambda \)CDM, the likelihood functions of \( \Omega_M \) are examined. The results are shown in Figures (3, 4). We find that the opacity parameter \( \epsilon \) constrained from previous cosmological-model-independent tests have slight influences on the probability distributions of \( \Omega_M \) and a universe with \( \Omega_\Lambda > 0 \) is required to account for the dimming of the SNe Ia. Discarding the condition of the spatial flatness, we display the corresponding plots in the \( \Omega_M - \Omega_\Lambda \) plane in Figure 5. We find that a decelerating expanding universe with \( \Omega_\Lambda = 0 \) is only accommodated by observations at 99.7% C. L. when the spherical \( \beta \) model is considered. That is, a positive cosmological constant is still needed to account for the dimming of SNe Ia and the standard cosmological scenario remains being supported by current observations.

Finally, it should be pointed out that, the presence of systematic uncertainties in observations, especially ADD measurements using SZE+X-ray surface brightness observations, and the assumption of the frequency independency of absorption in the observed frequency
range in our analysis might result in some biases of our test results. Moreover, as for the DD relation test, the morphological models of galaxy cluster may also exert a remarkable influence on the tests for cosmic opacity. In fact, any conclusion that current data may favor a non-zero opacity should be backed up with a thorough analysis of these systematics. Therefore, we may expect more vigorous and convincing constraints on the cosmic opacity within the coming years with more precise data, especially the ADD data, and a deeper understanding for the absorption in various wavelength bands and the intrinsic three dimensional shape of clusters of galaxies.

acknowledgments

We would like to thank A. Avgoustidis for helpful discussions. This work was supported by the National Natural Science Foundation of China under Grants Nos. 10935013, 11175093, 11075083 and 11222545, the Ministry of Science and Technology National Basic Science Program (Project 973) under Grant No.2012CB821804, Zhejiang Provincial Natural Science Foundation of China under Grants Nos. Z6100077 and R6110518, the FANEDD under Grant No. 200922, the National Basic Research Program of China under Grant No. 2010CB832803, the NCET under Grant No. 09-0144, the PCSIRT under Grant No. IRT0964, the Hunan Provincial Natural Science Foundation of China under Grant No. 11JJ7001, and the SRFDP under Grant No.20124306110001. ZL was partially supported by China Postdoc Grant No.2013M530541.

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FIG. 1: Probability distribution functions of opacity parameter $\epsilon$ obtained from the De Filippis et al. sample and Union2.1 SN Ia pairs for two parameterizations: $\tau(z) = 2\epsilon z$ (blue solid) and $\tau(z) = (1 + z)^2\epsilon - 1$ (red dashing).

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FIG. 2: Probability distribution functions of opacity parameter $\epsilon$ obtained from the Bonamente et al. samples and Union2.1 SN Ia pairs for two parameterizations: $\tau(z) = 2\epsilon z$ (blue solid) and $\tau(z) = (1 + z)^{2\epsilon} - 1$ (red dashing). The left, middle, and right panel represent results constrained from the isothermal $\beta$, $r < 100$ kpc-cut isothermal $\beta$ and hydrostatic equilibrium model respectively.

| gas distribution model                      | $\tau(z) = 2\epsilon z$ | $\tau(z) = (1 + z)^{2\epsilon} - 1$ |
|---------------------------------------------|--------------------------|--------------------------------------|
| Elliptical $\beta$ model                   | $\epsilon = 0.009^{+0.059+0.127+0.199}_{-0.055-0.110-0.160}$ | $\epsilon = 0.014^{+0.071+0.145+0.219}_{-0.069-0.138-0.203}$ |
| Spherical $\beta$ model                     | $\epsilon = 0.081^{+0.046+0.100+0.158}_{-0.042-0.085-0.124}$ | $\epsilon = 0.096^{+0.058+0.114+0.169}_{-0.056-0.107-0.154}$ |
| Spherical $\beta$ model ($r < 100$ kpc-cut) | $\epsilon = 0.120^{+0.047+0.101+0.143}_{-0.043-0.086-0.114}$ | $\epsilon = 0.140^{+0.054+0.110+0.148}_{-0.052-0.102-0.142}$ |
| hydrostatic equilibrium model               | $\epsilon = 0.066^{+0.037+0.079+0.123}_{-0.035-0.070-0.102}$ | $\epsilon = 0.080^{+0.046+0.090+0.135}_{-0.045-0.086-0.126}$ |

TABLE I: Summary of the results for different optical depth parameterizations and cluster gas distribution models.
FIG. 3: Upper: The probability distributions of $\Omega_M$ in the context of flat $\Lambda$CDM when the absorptions model-independently constrained from the combination of De Filippis et al. sample and Union2.1 SN Ia are considered. The green, blue and red zonal regions represent the spans of $\Omega_M$ at 68.3%, 95.4% and 99.7% C. L. respectively. As the results are not sensitive to the parametric form of $\tau(z)$, here the linear expression ($\tau(z) = 2\epsilon z$) is applied. Lower: The projections of the upper zonal regions in the $\Omega_M - \epsilon$ plane. The red dot ($\Omega_M$=0.285, $\epsilon$=0.009) represents the best fit case.
FIG. 4: Upper: The probability distributions of $\Omega_M$ in the context of flat $\Lambda$CDM when the absorptions model-independently constrained from the combination of Bonemente et al. sample and Union2.1 SN Ia are considered. The green, blue and red zonal regions represent the spans of $\Omega_M$ at 68.3%, 95.4% and 99.7% C. L. respectively. As the results are not sensitive to the parametric form of $\tau(z)$, here the linear expression ($\tau(z) = 2\epsilon z$) is applied. Lower: The projections of the upper zonal regions in the $\Omega_M - \epsilon$ plane. The red dot ($\Omega_M=0.397$, $\epsilon=0.081$) represents the best fit case.
FIG. 5: Marginalized regions at 68.3%, 95.4% and 99.7% C. L. in the $\Omega_M - \Omega_\Lambda$ plane for the corrected data of subsamples of Union2.1 SN Ia by considering the observational constrained cosmic absorptions. As the results are not sensitive to the parametric form of $\tau(z)$, here the linear expression ($\tau(z) = 2\epsilon z$) is applied. The red stars ($\Omega_M = 1.0, \Omega_\Lambda = 0.0$) represent the Einstein-de Sitter universe. The left(right) panel corresponds to the results obtained from the De Filippis et al. (Bonamente et al.) sample.