Exploring CPT violation in $B \to J/\psi K$ decays using kaon regeneration

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Abstract

We present an analysis of CPT violation in $B \to J/\psi K$ decays. A method of kaon regeneration is proposed to increase the $K_{S(L)}$ interference effects. With the time evolution of both neutral $B$ and $K$ mesons, we show that it is possible to determine the CPT violating parameter in $B^0 - \bar{B}^0$ mixing or constrain it.

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With the running of two B factories and the proceeding of the future B physics projects (BTeV and LHCb etc.), a large number of B mesons will be accumulated to test the standard model and explore new physics beyond it. The CPT invariance is an important theorem of local, relativistic field theory, but its validity must be checked by precise experiment. Except kaon decays, B decays can provide another ideal place to perform such test. The exploration of CPT violation in B decays has been got many theoretical interests recently [1-8]. In experiment, only OPAL collaborations measured the the CPT violation in neutral B meson oscillation [9].

Besides the uncertainties in the standard model, CPT violation is a new physics effect and may be entangled with other new physics effects. In [2], authors point out that the CPT violation violation and the violation of $\Delta B = \Delta Q$ rule can not be distinguished by the opposite-sign dilepton asymmetry of neutral B decays. These new physics effects increase the difficulty of extracting the information of CPT violation. The study of CPT violation is unavoidable to consider some complex systems.

$B \rightarrow J/\psi K$ decay is an interesting decay process. It is well known that $B \rightarrow J/\psi K_S$ decay is a ”gold- plated” mode to determine clean $\sin2\beta$ through time dependent rate asymmetries between $B^0$ and $\bar{B}^0$. Recent preliminary results from the BABAR and BELLE collaborations stimulates theoretical interests about this decay. Another feature of $B \rightarrow J/\psi K_S$ decay is ”cascade decay”. In the decay chain $B \rightarrow J/\psi K \rightarrow J/\psi |f|_K$ where the initial $B^0 - \bar{B}^0$ mixing is followed by neutral K mixing, this ”cascade mixing” process contains more information than the ordinary decays. Azimov [10] first pointed out this unique feature. In [11], Kayser shows that the interference of $K_{S(L)} \rightarrow \pi l \nu$ can be used to explore $\cos2\beta$. But this method suffers from the small branching ratio of $K_S \rightarrow \pi l \nu$. One procedure to solve this problem is proposed in [12] by using kaon regeneration. This procedure utilizes the fact that kaon mass eigenstates are not same in matter and in vacuum. With the kaon regeneration, the interference between the $K_S$ and $K_L$ decays to $\pi \pi$ provides information to determine $\cos2\beta$. Here we want to point out this interference provides enough knowledge to extract the information of CPT violation.
Kaon regeneration is an important experimental tool to study the discrete symmetry violation. The direct CP violation \( \text{Re}(\epsilon'/\epsilon) \) is measured through the kaon regeneration. In \[7\], cascade mixing is used to explore CPT violation where CPT violation comes from the \( B^0 - B^0\) mixing. This study discuss the kaon decays in vacuum and has the disadvantage discussed above. In this paper, our purpose is to use the kaon regeneration to discuss the CPT violation in cascade decay \( B \rightarrow J/\psi K \rightarrow J/\psi[2\pi]K \).

Let us first discuss \( B^0 - B^0\) mixing including CPT violation. The oscillation of \( B^0\) and \( B^0\) caused by weak interaction leads to the mass eigenstates are not the flavor states but their superpositions. For neutral \( B^0\) system, the two eigenstates can be generally given by

\[
|B_1> = \frac{1}{\sqrt{|p_1|^2 + |q_1|^2}}[p_1|B^0> + q_1|\bar{B}^0>]
\]

\[
|B_2> = \frac{1}{\sqrt{|p_2|^2 + |q_2|^2}}[p_2|B^0> - q_2|\bar{B}^0>]
\]

(1)

and their eigenvalues are

\[
\mu_1 = m_B - \frac{\Delta m_B}{2} - \frac{i}{2}(\Gamma_B + \frac{\Delta \Gamma_B}{2}) = m_B - \frac{i}{2}\Gamma_B - \frac{\Delta m_B}{2} - \frac{i}{2}y\Gamma_B
\]

\[
\mu_2 = m_B + \frac{\Delta m_B}{2} - \frac{i}{2}(\Gamma_B - \frac{\Delta \Gamma_B}{2}) = m_B - \frac{i}{2}\Gamma_B + \frac{\Delta m_B}{2} + \frac{i}{2}y\Gamma_B
\]

(2)

The quantities \( p_i\), \( q_i\) are mixing parameters. In the standard model, mixing parameters \( p_i\), \( q_i\) are usually parameterized by small quantities \( \epsilon\), \( \delta\) which represents CP, T violation and CPT violation respectively. As we have known from the standard model, CP violation in B decays is expected to be large. So it is convenient to introduce the exponential parameterization for \( p_i\), \( q_i\) as

\[
\frac{q_1}{p_1} = tg\frac{\theta}{2}e^{i\phi}, \quad \frac{q_2}{p_2} = ctg\frac{\theta}{2}e^{i\phi}
\]

(3)

where \( \theta\) and \( \phi\) are complex phases in general. In the standard model, \( \theta = 0, \phi \approx -2\beta\).

Define \( (\frac{q}{p})_B = \sqrt{\frac{p_1 p_2}{q_1 q_2}}\) and CPT violating parameter \( \theta' \equiv \theta - \frac{\theta}{2}\), then \( \theta' \neq 0\) represents indirect CPT violation in \( B^0 - B^0\) mixing.

From Eq.(1) and Eq.(3), one can obtain the evolution of the initially pure \( B^0\) or \( \bar{B}^0\) state
after proper time $t$ as

$$|B^0(t) > = g_+ (t)|B^0 > + \bar{g}_+ (t)|\bar{B}^0 >$$

$$|\bar{B}^0 (t) > = g_- (t)|\bar{B}^0 > + \bar{g}_- (t)|B^0 >$$

(4)

where

$$g_+ (t) = f_+ (t) + \cos \theta f_-(t), \quad g_- (t) = f_+ (t) - \cos \theta f_- (t)$$

$$\bar{g}_+ (t) = \sin \theta e^{i \phi} f_-(t), \quad \bar{g}_- (t) = \sin \theta e^{-i \phi} f_-(t)$$

(5)

and

$$f_+ (t) = \frac{1}{2} (e^{-i \mu_L t} + e^{-i \mu_B t}) = e^{-im_B t - \frac{1}{2} \Gamma_B t \text{ch} \left( \frac{ix - y}{2} \Gamma_B t \right)}$$

$$f_- (t) = \frac{1}{2} (e^{-i \mu_L t} - e^{-i \mu_B t}) = e^{-im_B t - \frac{1}{2} \Gamma_B t \text{sh} \left( \frac{ix - y}{2} \Gamma_B t \right)}$$

(6)

Next we turn to the $K^0 - \bar{K}^0$ mixing. Up to now, the experiment in neutral $K$ systems have gained a high level and find no new physics effects. Moreover, it is difficult to explore new physics effects of $K$ system from $B$ decays. So it is reasonable to neglect the new physics effects of $K$ system when studying $B$ decays. We take the formulae for $K^0 - \bar{K}^0$ mixing within the standard model. The eigenstates of the neutral $K$ mesons can be represented by

$$|K_S > = \frac{1}{\sqrt{|p_K^2| + |q_K|^2}} [p_K |K^0 > + q_K |\bar{K}^0 >]$$

$$|K_L > = \frac{1}{\sqrt{|p_K^2| + |q_K|^2}} [p_K |K^0 > - q_K |\bar{K}^0 >]$$

(7)

and their eigenvalues are

$$\mu_{S(L)} = m_K \frac{\Delta m_K}{2} - i \Gamma_{S(L)} \frac{\Delta m_K}{2}$$

(8)

where $m_K$ is the average of the $K_S$ and $K_L$ masses, $\Gamma_{S,L}$ are the $K_{S,L}$ widths.

The time evolution of $K^0_{S(L)}$ state in vacuum obeys the simple exponential law

$$K_S (t) = e^{-i \mu_S t} K_S (0), \quad K_L (t) = e^{-i \mu_L t} K_L (0)$$

(9)

Due to the scattering of $K^0$ and $\bar{K}^0$ with the nuclei in the matter, the eigenstates are not $K^0_{S(L)}$ but their mixture. The relation between them is: $K^0_S \sim K_S - r K_L$ and $K^0_L \sim K_L + r K_S$.  


where $r$ is the regeneration parameter. The difference of the eigenstates $K'_S$, $K'_L$ in matter and the eigenstates $K_S$, $K_L$ in vacuum can be used as an effective tool to convert the coherent mixture of $K_S$ and $K_L$. We use the regeneration formulae given in [12]. Considering neutral kaon hits one regenerator with length $L$ at time $t = 0$, after time $t$, it passes through the regenerator. The kaon components $\alpha_{S(L)}$ in neutral kaons is written as [12]

$$\alpha_i(t) = e^{-N(\sigma_T+\bar{\sigma}_T)L/4}e^{-i\mu_St} \sum_{j=S,L} m_{ij} \alpha_j(0)$$

(10)

where $\sigma_T(\bar{\sigma}_T)$ is the total cross section of $K^0(\bar{K}^0)$ forward scattering, the regeneration parameters $m_{ij}$ is taken to linear order in $r$, $m_{SS} \sim 1$, $m_{SL} \sim m_{LS} \sim r[e^{-i(\mu_L-\mu_S)t} - 1]$, and $m_{LL} \sim e^{-i(\mu_L-\mu_S)t}$.

Now consider the decay chain $B \rightarrow J/\psi K \rightarrow J/\psi f$ where $f$ refers to $2\pi$ in our paper. At time $t_B$, the neutral B decays to $J/\psi K$. Then after time $t_1$, the neutral kaons hits a kaon regenerator. At time $t_2$, the kaon passes through the regenerator and again evolves in vacuum until at time $t_K$ it decays to $f_K$. In the standard model, $B^0$ can only decay to $J/\psi K^0$ while $\bar{B}^0$ decays to $J/\psi \bar{K}^0$. The decay of $B^0 \rightarrow J/\psi \bar{K}^0$ and $\bar{B}^0 \rightarrow J/\psi K^0$ occurs due to higher order effects which are estimated to be very small and negligible. The possible new physics effects can cause $B^0 \rightarrow J/\psi K^0$ and $\bar{B}^0 \rightarrow J/\psi K^0$ and these effects are similar to the violation of $\Delta B = \Delta Q$ rule. When considering the CPT violation, these effects should be considered.

Define the nonrephasing variables

$$\lambda \equiv \left( \begin{array}{c} q \\ p \end{array} \right)_B \frac{\bar{A}_f}{A_f} \frac{(p)}{q} K,$$

$$y' \equiv \frac{\bar{A}_f}{A_f} \frac{(p)}{q} K,$$

$$\bar{y}' \equiv \left( \begin{array}{c} q \\ p \end{array} \right)_B \frac{\bar{A}_f}{A_f}$$

(11)

where

$$A_f = A(B^0 \rightarrow J/\psi K^0), \quad \bar{A}_f = A(B^0 \rightarrow J/\psi \bar{K}^0),$$

$$\bar{A}_f = A(\bar{B}^0 \rightarrow J/\psi K^0), \quad \bar{A}_f = A(\bar{B}^0 \rightarrow J/\psi \bar{K}^0).$$

(12)
The components of $K_{S(L)}$ in $B^0$ to $J/\psi K$ decay are

$$\alpha_S = \sqrt{\frac{p_K^2 + q_K^2}{2p_K}} A_f [f_+(t_B) + \lambda f_-(t_B) - \theta' f_-(t_B) + y' f_+(t_B) + \bar{y}' f_-(t_B)]$$

$$\alpha_L = \sqrt{\frac{p_K^2 + q_K^2}{2p_K}} A_f [f_+(t_B) - \lambda f_-(t_B) - \theta' f_-(t_B) - y' f_+(t_B) + \bar{y}' f_-(t_B)]$$

(13)

So the decay rate of $B^0$ to $J/\psi f$ is

$$\Gamma(B^0 \to J/\psi f) = \frac{1}{2} e^{-\Gamma_{B^0}} \Gamma(B^0 \to J/\psi K)$$

$$\times e^{-N_0(t_B)} e^{-t_B K} \Gamma(K \to f)$$

$$\times \left\{ |a_{SS} + \eta a_{LS}|^2 |\alpha_S|^2 + |a_{SL} + \eta a_{LL}|^2 |\alpha_L|^2 + 2 \text{Im}((a_{SS} + \eta a_{LS})(a_{SL} + \eta a_{LL})^*) \text{Im}(\alpha_S \alpha_L^*) + 2 \text{Re}((a_{SS} + \eta a_{LS})(a_{SL} + \eta a_{LL})^*) \text{Re}(\alpha_S \alpha_L^*) \right\}$$

(14)

where $\eta = \frac{N_K}{N_{K_0}}$, $a_{SS} = m_{SS}$, $a_{SL} = m_{SL}e^{-i(\lambda_L - \lambda_S)t_1}$, $a_{LS} = m_{LS}e^{-i(\lambda_L - \lambda_S)(t_K - t_2)}$, $a_{LL} = m_{LL}e^{-i(\lambda_L - \lambda_S)t_1}$.

The decay rate of $\bar{B}^0 \to J/\psi f$ can be similarly obtained. The four terms in Eq.(14) corresponding to different passes from the initial B meson to the final state. The first and second terms in the first line arise from $B^0 \to J/\psi K_S$ and $B^0 \to J/\psi K_L$ respectively. The second and third lines arise from the interference of $B^0 \to J/\psi K_{S(L)}$ to $J/\psi f$. In the standard model, the second line gives the information of $\cos^2 \beta$. This is the Kayser’s original proposal to determine $\cos^2 \beta$ [11].

From Eq.(14), one can see that cascade decay has rich time evolution behavior. The time behaviors of neutral kaon decay are contained in the coefficients of $a_{ij}$ which can be determined from the experiment. $\alpha_S$ and $\alpha_L$ represents the time behavior of neural B meson decay. Their explicit formulae with new physics of CPT violation are listed in the Appendix. Without new physics, this formulae will go back to the form within the standard model [12].

From the known parameter of neutral K system, we can fit the unknown parameter in B system. There are 8 unknown parameters in total: the real and imaginary part of $\lambda, \theta', y', \bar{y}'$ in total. Combining decay rate of $\bar{B}^0 \to J/\psi K \to J/\psi f$ with kaon evolves in matter and in
vacuum, we have enough information to determine these parameters in principle. However, it is difficult to determine all the unknown parameters in practice. If the small \( y' \) and \( \bar{y}' \) can be neglected, only the unknown \( \lambda \) and \( \theta' \) are left. The multiparameter fit can be used to determine them. At least experiment can give a constraint on CPT violating parameter \( \theta' \).

In our proposal, the B meson corresponds the uncorrelated case which B meson is tagged by its flavor. Our method of extracting the CPT violating parameter \( \theta' \) can be extended to the correlated case. In such case, the odd function of B meson decay time \( \sin \Delta m_{Bt_B} \) term can be cancelled by the decay rate asymmetry \[1\]. This can lead to a further simplification.

In summary, it is possible to extract the CPT violation in \( B \to J/\psi K \) decays using kaon regeneration.

Appendix

Only considering the new physics of CPT violation

\[
|\alpha_S|^2 = \cos^2 \frac{\Delta m_{Bt_B}}{2} + |\lambda|^2 \sin^2 \frac{\Delta m_{Bt_B}}{2} + (\text{Im}\theta' - \text{Im}\lambda)\text{sin}\Delta m_{Bt_B} - (\text{Re}\theta'\text{Re}\lambda + \text{Im}\theta'\text{Im}\lambda)(1 - \cos\Delta m_{Bt_B})
\]

\[
|\alpha_L|^2 = \cos^2 \frac{\Delta m_{Bt_B}}{2} + |\lambda|^2 \sin^2 \frac{\Delta m_{Bt_B}}{2} + (\text{Im}\theta' + \text{Im}\lambda)\text{sin}\Delta m_{Bt_B} + (\text{Re}\theta'\text{Re}\lambda + \text{Im}\theta'\text{Im}\lambda)(1 - \cos\Delta m_{Bt_B})
\]

\[
\text{Im}(\alpha_S\alpha_L^*) = \text{Re}\lambda\text{sin}\Delta m_{Bt_B} + (\text{Im}\theta'\text{Re}\lambda - \text{Re}\theta'\text{Im}\lambda)(1 - \cos\Delta m_{Bt_B})
\]

\[
\text{Re}(\alpha_S\alpha_L^*) = \cos^2 \frac{\Delta m_{Bt_B}}{2} - |\lambda|^2 \sin^2 \frac{\Delta m_{Bt_B}}{2} + \text{Im}\theta'\text{sin}\Delta m_{Bt_B}
\]

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