1 SUPPLEMENTARY METHODS

1.1 Neuron model

For all our simulations we use the leaky integrate-and-fire neuron model (Stein (1967); Gerstner and Kistler (2002)). In this model, the membrane potential of a neuron is described by

\[ \tau_m \frac{du}{dt} = - (u - u_{rest}) + RI(t), \]  

(1)

where \( u_{rest} \) denotes the membrane voltage at rest, \( R \) denotes the membrane resistance, \( I(t) \) denotes the external current and \( \tau_m \) denotes the membrane time constant. If the membrane potential reaches a threshold \( u_{th} \) at time \( t_f \), the membrane potential is reset to \( u_{reset} \) and we call \( t_f \) the firing time. After being reset, the membrane potential follows equation (1) again.

The term \( I(t) \) takes into account all of the current being injected into a neuron; these can be from an external source (e.g. an electrode) or from other neurons. When a neuron fires, it propagates a current to all other connected neurons. In order to model this current, we assume that the conductance between a presynaptic neuron \( j \) and a postsynaptic neuron \( i \) increases instantaneously every time the presynaptic neuron fires, and decays exponentially otherwise:

\[ g_j \rightarrow g_j + 1 \quad \text{if } j \text{ fires} \]  

(2)

\[ \frac{dg_j}{dt} = - \frac{g_j}{\tau_{syn}} \quad \text{otherwise}, \]  

(3)

where \( \tau_{syn} \) is the synaptic time constant. The synaptic current is then calculated through

\[ I^{syn}(t) = - w_{ij}g_j(u - E^{syn}), \]  

(4)

where \( w_{ij} \) is the synaptic weight from neuron \( j \) to neuron \( i \) and \( E^{syn} \) is the synaptic reversal potential.

1.2 Input signals

Each stimulus is represented by a filtered gaussian noise, with a time constant of \( \tau_{filt} = 50 \) ms, from which we subtract a constant \( c \), then rectify all negative values to zero and multiply by a constant \( \nu_j \) (to
control the mean), resulting in a function $s_j(t)$ (a signal, or stimulus). In summary, 

$$s_j(t) = \nu_j[f_j(t) - c]_+ ,$$  
(5)

$$\tau_{filt} \frac{df_j(t)}{dt} = f_j(t) - \xi_j(t) ,$$  
(6)

where $\xi_j(t)$ is a Gaussian white noise. The constant $c$ controls the lifetime sparseness of the signals (Franco et al. (2007)), controlling how well the postsynaptic neuron can differentiate between two presynaptic neurons. When all the stimuli are equally represented, $\nu_j = m$ for all $j$. When one of the stimuli is over-represented, the corresponding value of $\nu_j$ is increased.

We assume that all input neurons received contributions from all stimuli with different intensities. For each neuron, we associated a weighted sum of the stimuli, 

$$p_i(t) = \sum_j T_{ij} s_j(t) ,$$

where $T_{ij}$ is a tuning strength defined from a Gaussian distribution: $T_{ij} \propto \exp\left[-(i - j)^2 / 2\sigma^2\right]$ and it is normalized such that $\sum_j T_{ij} = 1$. The parameter $\sigma = 1$ is the tuning width. The function $p_i(t)$ is then defined as the firing probability of input neuron $i$.

See diagram in supplementary figure[1]

1.3 Synaptic plasticity model (pair-based)

According to the STDP rule we use, the synaptic weight between a presynaptic neuron $j$ and a postsynaptic neuron $i$ evolves following (Kistler and van Hemmen (2000); Gerstner and Kistler (2002))

$$\frac{d}{dt}w_{ij}(t) = A_- S_j(t) \int_0^{\infty} e^{-s/\tau_-} S_i(t - s)ds + A_+ S_i(t) \int_0^{\infty} e^{-s/\tau_+} S_j(t - s)ds ,$$  
(7)

where $S_j = \sum_f \delta\left(t - t_j^{(f)}\right)$ and $S_i = \sum_f \delta\left(t - t_i^{(f)}\right)$ are pre- and postsynaptic spike trains, respectively, $A_-$ is the depression amplitude, $A_+$ is the potentiation amplitude, $\tau_-$ is the depression time constant and $\tau_+$ is the potentiation time constant. This rule takes into account only pairs of pre-post or post-pre activity and therefore can be summarized by the effect of only one pair (known as a learning window),

$$\Delta w_{ij}(t) = \begin{cases} 
A_+ \exp(-t/\tau_+) & \text{if } t < 0 \\
A_- \exp(-t/\tau_-) & \text{if } t > 0 
\end{cases} ,$$  
(8)

where $t = t_j^{(f)} - t_i^{(f)}$ is the difference between the postsynaptic and the presynaptic firing times. In order to implement this rule, we define a presynaptic trace $\bar{x}_j$ and a postsynaptic trace $\bar{y}_i$ that is incremented by 1 for each pre or postsynaptic spike, respectively, and decay otherwise following

$$\bar{x}_j \rightarrow \bar{x}_j + 1 \quad \text{if presynaptic neuron } j \text{ fires and}$$

$$\tau_+ \frac{d\bar{x}_j}{dt} = -\bar{x}_j \quad \text{otherwise},$$  
(9)

and

$$\bar{y}_i \rightarrow \bar{y}_i + 1 \quad \text{if postsynaptic neuron } i \text{ fires and}$$

$$\tau_- \frac{d\bar{y}_i}{dt} = -\bar{y}_i \quad \text{otherwise}.$$  
(10)
The synaptic weight $w_{ij}$ is then updated by the following:

$$
\begin{align*}
\text{if } t = t^{\text{pre}}, & \quad w_{ij}(t) \rightarrow w_{ij}(t) + A_{-} \tilde{y}(t) \\
\text{if } t = t^{\text{post}}, & \quad w_{ij}(t) \rightarrow w_{ij}(t) + A_{+} \tilde{x}(t)
\end{align*}
$$

(11)

Synaptic weights were also bounded between 0 and 2: $0 < w < 2$.

1.4 Synaptic plasticity model (triplet)

In supplementary figure 3, we use the triplet model (Pfister and Gerstner (2006)) to update the synaptic weights. We first consider two detectors of presynaptic events, $r_1$ and $r_2$, and two detectors of postsynaptic events, $o_1$ and $o_2$. These variables evolve according to the following:

$$
\begin{align*}
\frac{dr_1(t)}{dt} &= -r_1(t) \tau_+ \quad \text{and if } t = t^{\text{pre}} \text{ then } r_1 \rightarrow r_1 + 1 \\
\frac{dr_2(t)}{dt} &= -r_2(t) \tau_x \quad \text{and if } t = t^{\text{pre}} \text{ then } r_2 \rightarrow r_2 + 1 \\
\frac{do_1(t)}{dt} &= -o_1(t) \tau_- \quad \text{and if } t = t^{\text{post}} \text{ then } o_1 \rightarrow o_1 + 1 \\
\frac{do_2(t)}{dt} &= -o_2(t) \tau_y \quad \text{and if } t = t^{\text{post}} \text{ then } o_2 \rightarrow o_2 + 1
\end{align*}
$$

(12)

where $\tau_+$ and $\tau_x$ are time constants for presynaptic events and $\tau_-$ and $\tau_y$ are time constants for postsynaptic events. The change in the synaptic weight $w_{ij}$ is then calculated through

$$
\begin{align*}
\text{if } t = t^{\text{pre}}, & \quad w_{ij}(t) \rightarrow w_{ij}(t) + o_1(t) \left[ A_{-}^2 + A_{-}^- r_2(t - \epsilon) \right] \\
\text{if } t = t^{\text{post}}, & \quad w_{ij}(t) \rightarrow w_{ij}(t) + r_1(t) \left[ A_{+}^2 + A_{+}^+ o_2(t - \epsilon) \right]
\end{align*}
$$

(13)

where $A_{-}^2$ and $A_{+}^2$ denote the amplitude of synaptic weight change for post-pre and pre-post events, respectively. $A_{-}^-$ and $A_{+}^+$ denote the amplitude of depression and potentiation, respectively, for the triplet term.

1.5 Parameters and simulations

1.5.1 Figures 1C - 1E

Our feedforward network consisted of 100 presynaptic neurons and one postsynaptic neuron. All the weights were initialized to 0.2 and the network was run for 1000 s with a time step of 1 ms. All the numerical parameters can be seen in supplementary table 1. In figure 1C, rule 1 was modified by increasing the amplitude for depression by 2%.

1.5.2 Figures 1F - 1K

The feedforward network consisted of 10 presynaptic neurons and one postsynaptic neuron. The weights were initialized by assuming an initial receptive field tuned to input 7. All the curves are calculated from an average over 200 trials. The shading areas represent one standard deviation from the mean. For figures 1F and 1G, the network runs for 20 s. For figures 1I and 1J, the network runs for 80 s. All other numerical parameters can be found in supplementary table 1.
1.5.3 Figures 2A, 2B, 2D and 2E

The feedforward network consisted of one presynaptic neuron and one postsynaptic neuron. All the parameters used in these simulations can be found in supplementary table 2. The network was simulated for 100 s. For each value of the synaptic weight, \( w \), the total change in synaptic weight was calculated as the sum of synaptic weight changes in each time step (1 ms). However, the changes were not applied to the weights to ensure that the calculated synaptic change was specific to each value of \( w \). Therefore, the calculated value of \( \Delta w \) is the effective change in synaptic weight. The curves show an average over 200 trials.

1.5.4 Figure 2C

The feedforward network consisted of 10 presynaptic neurons and one postsynaptic neuron. All the parameters used in these simulations can be found in supplementary table 2. Weights were initialized assuming an initial receptive field tuned to inputs 5 and 6. The final receptive field was calculated for \( \nu = 1 \) Hz and two values of the learning rate: \( \alpha = 0.02 \) (large) and \( \alpha = 0.01 \) (small). The initial and final receptive fields were re-scaled by dividing all the tuning curves by their respective maximum weights. Curves show an average over 50 trials.

1.5.5 Figure 2F

The feedforward network consisted of 10 presynaptic neurons and one postsynaptic neuron. All the parameters used in these simulations can be found in supplementary table 2. Weights were initialized assuming an initial receptive field tuned to inputs 5 and 6. The final receptive field was calculated for \( \alpha = 0.02 \) Hz and two values of presynaptic activity: \( \nu = 10 \) Hz (large) and \( \nu = 1 \) Hz (small). The initial and final receptive fields were re-scaled by dividing all the tuning curves by their respective maximum weights. Curves show an average over 50 trials.

1.5.6 Supplementary figure 2

The feedforward network consisted of 10 presynaptic neurons and one postsynaptic neuron. All the parameters used in these simulations can be found in supplementary table 2. Weights were initialized assuming an initial receptive field tuned to inputs 5 and 6. The final receptive field was calculated for \( \alpha \downarrow = 0.01, \alpha \uparrow = 0.02, \nu \downarrow = 1 \) Hz, \( \nu \uparrow = 10 \) Hz. The initial and final receptive fields were re-scaled by dividing all the tuning curves by their respective maximum weights. Curves show an average over 50 trials.

1.5.7 Supplementary figure 3A

The synaptic weights were updated following a triplet rule. The feedforward network consisted of 10 presynaptic neurons and one postsynaptic neuron. All the parameters used in these simulations can be found in supplementary table 2. Weights were initialized assuming an initial receptive field tuned to inputs 5 and 6. The final receptive field was calculated for \( \nu = 1 \) Hz and two values of the learning rate: \( \alpha = 0.02 \) (large) and \( \alpha = 0.01 \) (small). The initial and final receptive fields were re-scaled by dividing all the tuning curves by their respective maximum weights. Curves show an average over 50 trials.

1.5.8 Supplementary figure 3B

The synaptic weights were updated following a triplet rule. The feedforward network consisted of 10 presynaptic neurons and one postsynaptic neuron. All the parameters used in these simulations can be found in supplementary table 2. Weights were initialized assuming an initial receptive field tuned to inputs
5 and 6. The final receptive field was calculated for $\alpha = 0.02$ Hz and two values of presynaptic activity: $\nu = 5$ Hz (large) and $\nu = 1$ Hz (small). The initial and final receptive fields were re-scaled by dividing all the tuning curves by their respective maximum weights. Curves show an average over 50 trials.

2 SUPPLEMENTARY TABLES AND FIGURES

**Supplementary Figure 1. Generation of presynaptic spike trains.** The procedure used to generate the firing probability of each input neuron can be split into two steps: (i) generation of $N$ independent rectified time-filtered Gaussian white noise (left box); (ii) weighted sum of the $N$ signals generated in the first step (right box). The set of tuning strengths for each neuron is defined such that input neuron $i$ is tuned to signal $i$. 

\[
\xi(t) \quad \text{Gaussian Noise} \quad \frac{df}{dt} = f - \xi \quad \text{Filter} \quad s = \nu[f - c]_+ \quad \text{Rectify}
\]

\[
\text{For each input neuron} \quad T_j \quad \text{Tuning strength} \quad p(t) = \sum_j T_j s_j(t) \quad \text{Weighted sum} \quad p(t) \quad \text{Firing probability}
\]
Supplementary Figure 2. Receptive field plasticity under neuronal activity and/or learning rate modulation. Synaptic weights for a feedforward network with 10 presynaptic neurons and one postsynaptic neuron. The final synaptic weights were calculated for the following values of learning rate ($\alpha$) and presynaptic activity ($\nu$): $\alpha \downarrow = 0.01$ (dashed lines), $\alpha \uparrow = 0.02$ (solid lines), $\nu \downarrow = 1$ Hz (red curves), $\nu \uparrow = 10$ Hz (blue curves). The initial and final receptive fields were rescaled by dividing all the tuning curves by their respective maximum weights. The effects of both types of modulations are independent.

Supplementary Figure 3. Modulation of activity vs modulation of learning rate, using the triplet model (Pfister and Gerstner [2006]). Synaptic weights for a feedforward network with 10 presynaptic neurons and one postsynaptic neuron. The final synaptic weights were simulated for the following values of learning rate ($\alpha$) and presynaptic activity ($\nu$): A $\nu = 1$ Hz, small $\alpha = 0.01$ (red curve), large $\alpha = 0.02$ (blue curve); B $\alpha = 0.01$, small $\nu = 1$ Hz (red curve), large $\nu = 5$ Hz (blue curve). In all the figures, the initial and final receptive fields were rescaled by dividing all the tuning curves by their respective maximum weights. The results observed with the triplet model are qualitatively identical to the results observed with the standard pair-based STDP model.
**Supplementary Table 1.** Parameter summary for simulations in figure 1.

| Name         | Value     | Description                                      |
|--------------|-----------|--------------------------------------------------|
| \( \tau_m \) | 50 ms     | Membrane time constant                           |
| \( u_{th} \) | 10 mV     | Spiking threshold                                |
| \( u_{rest} \) | 0 mV     | Resting potential                                |
| \( E^{syn} \) | 30 mV     | Synaptic reversal potential                      |
| \( u_{reset} \) | 0 mV     | Value at which the potential is reset after a spike |

| Name         | Value     | Description                                      |
|--------------|-----------|--------------------------------------------------|
| \( N_E \) (C-E) | 100      | Size of presynaptic population for figure 1C-E    |
| \( N_E \) (F-K) | 10       | Size of presynaptic population for figure 1F-K    |
| \( \tau_E \) | 15 ms     | Decay constant of excitatory conductance         |
| \( \bar{g}_E \) | 1 nS     | Peak synaptic conductance                        |

| Name         | Value     | Description                                      |
|--------------|-----------|--------------------------------------------------|
| \( \tau_{STDP} \) | 8 ms     | Decay constant of pre- and post synaptic traces  |
| \( A_+ \)    | \( \gamma_+ \times 10^{-4} \) | Amplitude of learning rate for pre-post events   |
| \( A_- \)    | \( \gamma_- \times 10^{-4} \) | Amplitude of learning rate for post-pre events   |
| \( \gamma_+ \) | 1, 1, 1, 0 | For rules 1, 2, 3 and 4, respectively            |
| \( \gamma_- \) | -1, 1, 0, -1 | For rules 1, 2, 3 and 4, respectively           |
**Supplementary Table 2.** Parameter summary for simulations in figure 2.

### Neuron Model

| Name   | Value | Description |
|--------|-------|-------------|
| $\tau_m$ | 10 ms | Membrane time constant |
| $u_{th}$ | 10 mV | Spiking threshold |
| $u_{rest}$ | 0 mV | Resting potential |
| $E_{syn}$ | 30 mV | Synaptic reversal potential |
| $u_{reset}$ | 0 mV | Value at which the potential is reset after a spike |

### Network and Synapse Model

| Name | Value | Description |
|------|-------|-------------|
| $N_E$ | 1 | Size of presynaptic population for figure 2 |
| $N_E$ | 10 | Size of presynaptic population for figure 2 |
| $\tau_E$ | 10 ms | Decay constant of excitatory conductance |
| $g_E^\infty$ | 0.1 nS | Peak synaptic conductance |
| $\nu$ | $\nu_0$ | Presynaptic firing rate ($\nu_0 = 10$ Hz) |

### Plasticity Model (pair-based)

| Name | Value | Description |
|------|-------|-------------|
| $\tau_{STDP}$ | 8 ms | Decay constant of pre- and post synaptic traces |
| $A_+$ | $+\alpha$ | Amplitude of learning rate for pre-post events ($\alpha_0 = 5 \times 10^{-4}$) |
| $A_-$ | $-\alpha$ | Amplitude of learning rate for post-pre events ($\alpha_0 = 5 \times 10^{-4}$) |

### Plasticity Model (triplet)

| Name | Value | Description |
|------|-------|-------------|
| $\tau_{+/−}$ | 8 ms | Decay constant of pre- and post synaptic traces ($r_1$ and $o_1$) |
| $\tau_{x/y}$ | 10 ms | Decay constant of pre- and post synaptic traces ($r_2$ and $o_2$) |
| $A_+^x$ | $+\alpha$ | Amplitude of learning rate for pre-post events |
| $A_−^x$ | $−\alpha$ | Amplitude of learning rate for post-pre events |
| $A_+^y$ | $+\alpha$ | Amplitude of the triplet term for potentiation |
| $A_−^y$ | $−\alpha$ | Amplitude of the triplet term for depression |
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