Research Article

Mathematical Modelling of the Spatial Epidemiology of COVID-19 with Different Diffusion Coefficients

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This paper addresses the discrepancy between model findings and field data obtained and how it is minimized using the binning smoothing techniques: means, medians, and boundaries. Employing both the quantitative and the qualitative methods to examine the complex pattern involved in COVID-19 transmission dynamics reveals model variation and provides a boundary signature for the potential of the disease's future spread across the country. To better understand the main underlying factor responsible for the epidemiology of COVID-19 infection in Ghana, the continuous inflow of foreigners, both with and without the disease, was incorporated into the classical Susceptible-Exposed-Quarantined-Recovered (SEIQR) model, which revealed the spread of the COVID-19 by these foreigners. Also, the diffusion model provided therein gives a threshold condition for the spatial spread of the COVID-19 infection in Ghana. Following the introduction of a new method for the construction of the Lyapunov function for global stability of the nonlinear system of ODEs was observed, overcoming the problem of guessing for the Lyapunov function.

1. Introduction

In Wuhan, China, in December 2019, a new type of coronavirus which is a member of severe acute respiratory syndrome (SARS) has been identified. The pandemic disease has since spread to other countries, including Ghana. Realizing the COVID-19 pandemic is not only necessary but also imperative given the increasing death toll, labour force decrease, productivity decrease, declining trend in the economy of the country, etc. There have been 6,418,381 deaths and 581,305,772 reported cases of the disease since it originally spread throughout the world. In order of size, the USA reported 93,054,184 cases and 1,055,020 deaths, India reported 44,000,138 cases and 526,312 deaths, and Brazil reported 33,795,192 cases and 678,375 deaths. Ghana, which is rated 123rd in the statistics table, is hardly an exception with 168,007 cases, 1457 deaths [1].

It is impossible to look down upon the current state of the economies of the world and those under the productivity of their labour forces as many nations, particularly those in sub-Saharan Africa that are developing, look for long-term solutions to the spread of the COVID-19 pandemic. There are continuous interventions and control efforts everywhere in the world, including the provision of the Johnson and Johnson vaccine and the AstraZeneca vaccine to population subjects. In Ghana, the government has implemented a number of measures to prevent the spread of the COVID-19 infection from one person to another, including lockdown declarations, school closures, wearing of face mask, social withdrawal, limiting attendance at social gatherings, frequent hand washing, and the use of hand sanitizers. With all these measures put in place by the government, people are not adhering to the COVID-19 protocols as they still go by with their normal activities without a sense of fear of the
pandemic of COVID-19. People typically attend weddings with their close friends and family in jest, hoping to catch a glimpse of the celebration. Funerals are events that people simply cannot do away with. It is customary for family members and friends to say their final farewell to the cherished (deceased) individual without considering their proximity to one another or, in the worst-case scenario, without wearing a face mask [2].

Mathematical models are used to predict the extent of COVID-19 infection outbreaks, the rate at which the disease spreads globally, and the effectiveness of control measures in limiting the spatial spread of the disease. It cannot be overemphasized how important it is to employ mathematical models to help government officials and policymakers make informed decisions about how changes in the geographical mixing of individuals have affected the COVID-19 transmission dynamics in the country. A mathematical model provides a framework for analyzing causes, connections and concepts. A mathematical model is used to evaluate quantitative assertions. As a result, the effectiveness of the measures for preventing the spatial spread of COVID-19 in the country depends heavily on how well the model parameters capture the many characteristics of the disease. Even the trends of COVID-19 cases could be appreciated with a minimal error by the use of reliable or sufficient parameters in the model. Researchers from all across the world have investigated and evaluated the outcomes of various formulations and assumptions. The results of examining various COVID-19 infection formulations offer policymakers insights and are undoubtedly helpful in selecting models for the epidemiology of COVID-19 infection. Without taking into account the major spreaders and the influx of foreigners into the country, the authors in [3–7] modeled the epidemiology of COVID-19 in their respective countries. The COVID-19 pandemic in a country other than China, which involves the important factor responsible for the spatial spread of disease, requires a robust use of mathematics in order to explain and predict the major underlying mechanism and forecast trends. Since they are unable to identify the cause of the epidemiology and calculate the number of people who acquired the COVID-19 infections during the disease outbreak, their conclusions cast doubt in the eyes of the general public.

The key to any mathematical model is to formulate it as simple as possible but being adequate for the subject under consideration. Identifying the primary underlying factor, the influx of immigrants into the country, the suitable collection of data, and a mathematical model whose analyses result in the solution constitute the most challenging issues in modeling the epidemiology of COVID-19 infections. The spatial spread of COVID-19 has attracted interest in this period as it involves the main underlying factor and threshold condition for the onset of the disease. The authors of [8] have used the reaction-diffusion equation to describe the COVID-19 epidemiology in Spain and Greece. They included diffusion terms for each class in the susceptible-exposed-asymptomatic-infected-hospitalization-recovered model to account for the discrepancy between predictions and observed data. It is interesting to note that the diffusion model for the spatial spread of COVID-19 developed in [9] is very similar to the one developed in [8], with the exception that diffusion terms were added to the susceptible, asymptomatic, infected, and recovered humans instead of the six subgroups of the population size. Similar to [8, 9], the study in [10] applied the susceptible-exposed-infected-recovered-deceased model to describe the spatial spread of COVID-19 in Italy. The authors in [11] considered the diffused susceptible, asymptomatic, and symptomatic infected people. Unfortunately, their model did not take into account the isolated COVID-19 infected individuals or the patients who had received treatment or had recovered from their illnesses. All of these research works have applied the diffusion concept to a group of people from various countries who have recovered from the disease. An individual who has recovered from COVID-19 is frequently not driven to travel to another country because he or she is afraid of contracting the illness again due to the control measures in place there. In terms of the spatial spread of COVID-19, this assumption is not acceptable. Nevertheless, their diffusion models did not account for the quarantined class, a crucial subgroup of the population size for describing the pandemic of COVID-19 infections. This is a major shortcoming that needs immediate attention from the scientific community and has to be addressed immediately.

Nevertheless, research works from many settings have developed the Lyapunov function, which is used to assess the overall stability of the system of ODEs. A significant flaw in the global stability of nonlinear systems of ODEs, such as the Lyapunov method, is that there is no systematic method for determining the Lyapunov function to aid in establishing the global stability of the nonlinear system of ODEs. As a result, the Lyapunov method for determining the global stability of a nonlinear system of ODEs is purely. A significant flaw in the global stability of nonlinear systems of ODEs, such as the Lyapunov method, is that there is no systematic method for determining the Lyapunov function to aid in establishing the global stability of the nonlinear system of ODEs. As a result, the Lyapunov method for determining the global stability of a nonlinear system of ODEs is purely a guess of function properties that meet the requirements. For examples, the study [12] observed the following:

$$L(u, v) = a \left( u - u^* - u^* \ln \left( \frac{u}{u^*} \right) \right) + \left( v - v^* - v^* \ln \left( \frac{v}{v^*} \right) \right),$$

(1)

where $a \in \mathbb{R}^+$ is a parameter and $(u, v) = (u^*, v^*)$ is a fixed point of the system of ordinary differential equations (ODEs), as a Lyapunov function for the temporal-spatial variations in the interaction of the predator-prey model, and the study in [13] observed...
where \((s, e, i) = (s^*, e^*, i^*)\) is a fixed point of the system of ODEs, as the Lyapunov function for determining the global stability for the susceptible-exposed-infected model describing the epidemiology of COVID-19 infection.

According to the theory that the continuous inflow of immigrants into the country constitutes the spatial effects, the diffusion terms are only incorporated in the susceptible, exposed, and infected subgroups of the susceptible-exposed-infected-quarantined-recovered (SEIQR) model in this paper. Thus, the major underlying cause of the spatial spread of the COVID-19 infection is the continuous inflow of foreigners into the country. The reliability and robustness of this mathematical model are established through field data assessments of its trend. The main factors that would support robustness when the discrepancy between the model values and the data get reduced over time are the model parameters that have been evaluated. In fact, the validity of the mathematical model was assessed using monthly data on COVID-19 infections in Ghana. The behavior of the immediate nonstationary points around the fixed point of the nonlinear system of ODEs is given by the local stability of the fixed point, but it does not provide complete information on all nonstationary points in the domain of the nonlinear system of ODEs. For establishing the stability of a nonlinear system of ODEs, the Lyapunov criterion provides information on all nonstationary points in the nonlinear system. This method of determining global stability requires the construction of the Lyapunov function. A systematic method for constructing the Lyapunov function for determining the global stability of a nonlinear system of ODEs is presented in this paper.

Section 1 of this paper comprises an introduction, Section 2 contains analysis and results, and Section 3 contains the key conclusions from the analyses.

2. Main Results

In this section, we provide the mathematical model for describing the epidemiology of COVID-19 in Ghana.

2.1. Preliminary Results

**Theorem 1** (Lyapunov stability). Let \(X^*\) be an equilibrium point for \(X^* = F(X)\). Let \(L: \mathbb{R} \rightarrow \mathbb{R}\) be a differentiable function defined on an open set \(\Theta\) containing \(X^*\). Suppose further that

(i) \(L(X^*) = 0\) and \(L(X) > 0\) if \(X \neq X^*\)

(ii) \(\dot{L} \leq 0\) in \(\Theta - X^*\)

Then, \(X^*\) is stable. Furthermore, if \(L\) also satisfies

(iii) \(\dot{L} < 0\) in \(\Theta - X^*\), then \(X^*\) is asymptotically stable.

A function \(L\) satisfying (a) and (b) is called a Lyapunov function for \(X^*\). If (c) also holds, we call \(L\) a strict Lyapunov function [14].

2.2. Development of the Mathematical Model and Its Analyses

The population of Ghana, \(N\), is split into five distinct classes based on their epidemiological status: the susceptible class \((S(t))\), the exposed class \((E(t))\), the infected class \((I(t))\), the quarantined class \((Q(t))\), and the recovered class \((R(t))\). Due to the continuous inflow of immigrants into the country, including those with and without COVID-19 disease, the susceptible, exposed, and infected classes have changed over time and space. The classifications of quarantined and recovered, however, change over time since they include people who have either been isolated or who have been treated for their sickness after contracting the COVID-19 infection by the Ghana Health Service (GHS). Partial differential equations (PDEs) are made up of the susceptible, exposed, and infected classes, whereas ordinary differential equations (ODEs) are made up of the quarantined and recovered classes.

Assumptions of the mathematical model are as follows:

1. Newborn babies are recruited into the susceptible class.
2. New COVID-19 infection arises as the susceptible comes in contact with the COVID-19 patient (infected or quarantined).
3. Renewal of susceptible people through recovery from transient immunity is not taken into consideration.
4. The foreigner (invader) is categorized as either susceptible, exposed, or infective. Only these groups of people have different diffusion coefficients in one dimension because the COVID-19 infection was brought into the country through their immigration.

Based on Figure 1, the following equations are obtained to describe the epidemiology of COVID-19 infection in the country:

\[
\begin{align*}
\frac{dS}{dt} & = \alpha N - \mu S - \frac{bS(I + Q)}{N} + D_1 \frac{\partial^3 S}{\partial X^3}, \\
\frac{dE}{dt} & = \frac{bS(I + Q)}{N} - \left(\mu + \kappa + \theta_1 + \delta_1\right) E + D_2 \frac{\partial^3 E}{\partial X^3}, \\
\frac{dI}{dt} & = \kappa E - \left(\mu + \gamma_1 + \theta_2 + \delta_2\right) I + D_3 \frac{\partial^2 I}{\partial X^2}, \\
\frac{dQ}{dt} & = \theta_1 E + \theta_2 I - \left(\delta_3 + \mu + \gamma_2\right) Q, \\
\frac{dR}{dt} & = \delta_3 E + \delta_2 I + \delta_4 Q - \mu R.
\end{align*}
\]

Thus, the total population size of Ghana is represented by following equation:

\[
N(t) = S(t) + E(t) + I(t) + Q(t) + R(t).
\]
Together with $S(X,0) = S_0$, $E(X,0) = E_0$, $I(X,0) = I_0$, $Q(0) = Q_0$, and $R(0) = 0$, where $\alpha$ is the birth rate. That is, the rate at which newborn babies are recruited into the susceptible class. The transmission rate, $\beta$, is the rate at which a susceptible person contracts the COVID-19 virus, $\kappa$ is the rate at which exposed people become infectious while the average incubation period is $1/\kappa$. The disease-induced death rate, $\gamma_1$, is the rate at which an infected person dies from the COVID-19 infection, while the disease-induced death rate, $\gamma_2$, is the rate at which a quarantined person dies from the disease. $\theta_1$ is the rate at which exposed people are quarantined, whereas $\theta_2$ is the rate at which an infectious person is quarantined. $\delta_1$ is the rate at which exposed people recover from the virus, $\delta_2$ is the rate at which infectious people recover from the disease, $\delta_3$ is the rate at which quarantined people recover from the disease, and $\mu$ is the natural death rate. $D_1$ represents the constant diffusion coefficient of susceptible travelers into the country, $D_2$ is the constant diffusion coefficient of exposed travelers into the country, and $D_3$ is the constant diffusion coefficient of infectious travelers into the country.

Setting $m_1 = \mu + \kappa + \theta_1 + \delta_1$, $m_2 = \mu + \gamma_1 + \theta_2 + \delta_2$ and $m_3 = \delta_3 + \mu + \gamma_2$, the system of (3) becomes

$$\frac{\partial S}{\partial t} = \alpha N + D_1 \frac{\partial^2 S}{\partial X^2} - \mu S - \frac{\beta S (I + Q)}{N},$$

$$\frac{\partial E}{\partial t} = \frac{\beta S (I + Q)}{N} + D_2 \frac{\partial^2 E}{\partial X^2} - m_1 E,$$

$$\frac{\partial I}{\partial t} = \kappa E + D_3 \frac{\partial^2 I}{\partial X^2} - m_2 I,$$  \hspace{1cm} (5)

$$\frac{dQ}{dt} = \theta_1 E + \theta_2 I - m_3 Q,$$

$$\frac{dR}{dt} = \delta_1 E + \delta_2 I + \delta_3 Q - \mu R,$$

together with $S(X,0) = S_0$, $E(X,0) = E_0$, $I(X,0) = I_0$, $Q(0) = Q_0$, and $R(0) = 0$.

Nondimensionalizing of (5) is done by using the following system of equations:

$$s = \frac{S}{N}, \quad e = \frac{E}{N}, \quad i = \frac{I}{N}, \quad q = \frac{Q}{N}, \quad r = \frac{R}{N}$$

$$\tau = a t, \quad x = X \left(\frac{a}{D_1}\right)^{1/2}, \quad a = \frac{\mu}{\alpha}, \quad b = \frac{\beta}{\alpha}, \quad d_1 = \frac{D_2}{D_1}, \quad n_1 = \frac{m_1}{\alpha}, \quad p_1 = \frac{\theta_1}{\alpha},$$

$$p_2 = \frac{\theta_2}{\alpha}, \quad f_1 = \frac{\delta_1}{\alpha}, \quad f_2 = \frac{\delta_2}{\alpha}, \quad f_3 = \frac{\delta_3}{\alpha}, \quad n_3 = \frac{m_3}{\alpha}.$$  \hspace{1cm} (6)

Using the system of (6), we obtain the following system of equations:

$$\frac{\partial s}{\partial \tau} = 1 + \frac{\partial^2 s}{\partial x^2} - as - bs(i + q),$$

$$\frac{\partial e}{\partial \tau} = bs(i + q) + d_1 \frac{\partial^2 e}{\partial x^2} - n_1 e,$$

$$\frac{\partial i}{\partial \tau} = ke + d_1 \frac{\partial^2 i}{\partial x^2} - n_2 i,$$  \hspace{1cm} (7)

$$\frac{dq}{d\tau} = p_1 e + p_2 i - n_3 q,$$

$$\frac{dr}{d\tau} = f_1 e + f_2 i + f_3 q - a r,$$

together with $s(\tau,0) = s_0$, $e(\tau,0) = e_0$, $i(\tau,0) = i_0$, $q(\tau) = q_0$, and $r(\tau) = 0$.  \hspace{1cm} \text{Figure 1: Compartmental model for COVID-19 transmission dynamics.}
2.2.1. Boundedness of the Solution of the System of ODEs. The boundedness of the system of (7) is obtained by setting the diffusion coefficients of the susceptible, exposed, and infective subgroups to zero which yields

\[
\begin{align*}
\frac{ds}{dt} &= 1 - as - bs(i + q), \\
\frac{de}{dt} &= bs(i + q) - n_1e, \\
\frac{di}{dt} &= \kappa e - n_2i, \\
\frac{dq}{dt} &= p_1e + p_2i - n_3q, \\
\frac{dr}{dt} &= f_1e + f_2i + f_3q - ar.
\end{align*}
\]  

(8)

Together with \( s(x, 0) = s_0 \), \( e(x, 0) = e_0 \), \( i(x, 0) = i_0 \), \( q(0) = q_0 \), and \( r(0) = 0 \).

Summing the ordinary derivatives on the left-hand side of the system of equations yields

\[
\begin{align*}
\frac{\partial N}{\partial t} &= \alpha N - \mu (S + E + I + Q + R) - \gamma_1I - \gamma_2Q \\
\frac{\partial N}{\partial t} &= (\alpha - \mu)N - \gamma_1I - \gamma_2Q \\
\frac{\partial N}{\partial t} &\leq (\alpha - \mu)N \\
N(t) &\leq N_0e^{(\alpha - \mu)t}.
\end{align*}
\]  

Setting \( N(0) = N_0 \),

\[
N(t) \leq N_0e^{(\alpha - \mu)t}.
\]

(9)

The asymptotic behavior of the scaled population size is

\[
\lim_{t \to \infty} N(t) = \lim_{t \to \infty} N_0e^{(\alpha - \mu)t} \lim_{t \to \infty} N(t) = N_0, \forall \|\mu\| > \|\alpha\|.
\]

(10)

The result in inequality (7) implies that the solution set is invariant. Thus, the population size of Ghana approaches a fixed finite number.

2.2.2. Existence and Uniqueness of the Solution of the System of Nondimensionalized PDEs. In this section, we show that the system of (7) has a unique solution in a Banach space. Consider the first three equations of the system of (7), which are given by

\[
\begin{align*}
\frac{\partial s}{\partial t} &= 1 + \frac{\partial^2 s}{\partial x^2} - as - bs(i + q), \\
\frac{\partial e}{\partial t} &= bs(i + q) + d_1\frac{\partial^2 e}{\partial x^2} - n_1e, \\
\frac{\partial i}{\partial t} &= \kappa e + d_2\frac{\partial^2 i}{\partial x^2} - n_2i.
\end{align*}
\]  

(11)

The existence and uniqueness of solutions of the system of nondimensionalized PDEs (11) with homogeneous Neumann boundary conditions

\[
\begin{align*}
\frac{\partial s}{\partial t} &= \frac{\partial e}{\partial t} = \frac{\partial i}{\partial t} = 0, \text{ on } \partial \Omega \times (0, +\infty)
\end{align*}
\]

(12)

and initial conditions

\[
\begin{align*}
s(x, 0) &= \phi_1(x) \geq 0, \\
e(x, 0) &= \phi_2(x) \geq 0, \\
i(x, 0) &= \phi_3(x) \geq 0, \forall x \in \overline{\Omega},
\end{align*}
\]

(13)

where \( \Omega \) is a bounded domain in \( \mathbb{R}^3 \) with a smooth boundary \( \partial \Omega \).

\( \frac{\partial s}{\partial t}, \frac{\partial e}{\partial t}, \text{ and } \frac{\partial i}{\partial t}, \text{ respectively, are the normal derivatives of } s(x, t), e(x, t), \text{ and } i(x, t) \text{ on the boundary of } \partial \Omega. \text{ Moreover, } 0 < s^* < 1, \ 0 < e^* < a/n_1, \text{ and } 0 < r^* < b. \)

We set \( L_M = \{ u_0 = (\phi_1(x), \phi_2(x), \phi_3(x)) \in \mathbb{R}^3: 0 \leq u_0 \leq M, \forall x \in \overline{\Omega} \} \) with \( M = (1, a/n_1, b) \).

Setting a map \( F = (F_1, F_2, F_3) : L_M \longrightarrow L \), the system of (11) is written as

\[
\begin{align*}
F_1(u_0)(x) &= 1 - a\phi_1(x) - b\phi_1(x)(\phi_3(x) + q), \\
F_2(u_0)(x) &= b\phi_1(x)(\phi_3(x) + q) - n_1\phi_2(x) \\
F_3(u_0)(x) &= k\phi_3(x) - n_2\phi_3(x),
\end{align*}
\]

(14)

where \( q \in \mathbb{R}^+ \) is a parameter. Thus, \( F \) satisfies the Lipschitz condition in the subspace of \( L_M \), and the system of (11) can be written in the Banach space \( L = C(\overline{\Omega}) \), where \( \overline{\Omega} \subset \mathbb{R}^3 \).

\[
\begin{align*}
u(r) &= Au(r) + Fu(r),
\end{align*}
\]

(15)

where \( u(., r) = \col(s(., r), e(., r), i(., r)), \text{ Au: } \col(\Delta s(., .), d_1\Delta e(., .), d_2\Delta i(., .)), u_0 = \col(\phi_1, \phi_2, \phi_3), \text{ and } \Delta \text{ is the Laplace-type operator in one dimension. This result implies that the system of (11) has a solution in a Banach space.}

Theorem 2. Setting the initial data \( u_0 = (\phi_1(x), \phi_2(x), \phi_3(x)) \in L_M \), there exists a unique nonnegative solution \( u(x, r) \) of the system of equations (8) defined on \( [0, \infty) \times \overline{\Omega} \) with \( u(0, ., \phi) = \phi \). Moreover, \( u(r, s(x, r), e(x, r), i(x, r)) \in L_M \) for \( r \geq 0 \) and \( u(s(x, r), e(x, r), i(x, r)), \forall (r, s, e, i) \in [0, \infty) \times \overline{\Omega} \), is a classical solution of the system of equations (8).
Proof 1. In this proof, the Euclidean norm is used to achieve the results. Setting $0 \leq h < \min \{1/a + b, 1/n_1, 1/n_2 \}$ and for any sufficiently small $h \geq 0$, we obtain

$$\frac{\partial s}{\partial t} - \Delta s = 1 - as,$$

$$\frac{\partial s}{\partial y} = 0,$$  \hspace{1cm} (18)

By solving $ds/dt = 1 - as_1$ and $s_1(0) = \|\phi_1\|_{\infty}$, using the method of integrating factors yields

$$s_1(t) = \frac{1}{a} + ce^{-at}.$$  \hspace{1cm} (19)

At $s(x,0) = \phi_1(x)$, it implies

$$c = \phi_1(x) - \frac{1}{a},$$

$$\Rightarrow s_1(t) = \phi_1(x)e^{at} + \frac{1}{a}(1 - e^{-at}).$$

$$s(x,t) \leq s_1(x,t) \leq \max \{1/\|\phi_1\|_{\infty} \}, \text{ for } t \in [0, \infty).$$

To establish the $L^\infty$ uniformly boundedness of $e(x,t)$ and $i(x,t)$, we have

$$\frac{\partial s}{\partial y} = \frac{\partial e}{\partial y} = \frac{\partial i}{\partial y} = 0.$$  \hspace{1cm} (22)

Then,
\[
\frac{d}{dt} (s + e) - \Delta (s + d_2e) = 1 - a(s + i)
\]

\[
\Rightarrow \frac{d}{dt} (s + e) \leq 1 - a(s + i).
\]

Solving the equation \( (d/dt)(s_1 + i_1) \leq 1 - a(s_1 + i_1) \) yields

\[
s_1(t) + i_1(t) = \phi_1(x)e^{-at} + \phi_3(x)e^{-at} + \frac{1}{a} (1 - e^{-at})
\]

\[
\Rightarrow \int_\Omega (s + i)dx
\]

\[
\leq \text{mes}(\Omega)\max\left\{\frac{1}{a} \|\phi_1 + \phi_3\|_\infty\right\}
\]

\[
= \text{mes}(\Omega)\max\left\{\frac{1}{a} \|\phi_1 + \phi_3\|_\infty\right\},
\]

where \( k_1 \) is a finite positive constant. Also,

\[
\frac{d}{dt} (s + e) - \Delta (s + d_2e) = 1 - as - bs(i + q) + bs(i + q) - ne,
\]

\[
\Rightarrow \frac{d}{dt} (s + e) \leq 1 - a(s + i).
\]

From the above, it has been proved that \( s(x, t), e(x, t) \), and \( i(x, t) \) are \( L^\infty \) uniform bounded on \( \bar{\Omega} \times [0, T_{\text{max}}] \). Therefore, it follows from the standard theory of semilinear parabolic systems that \( T_{\text{max}} = +\infty \) is the maximal existence time for the solution of the system of equations (11). This completes \( g \) of the theorem.
The eigenvalues of the linearized system of (27) are obtained as

\[
\begin{vmatrix}
(-a - \lambda) & 0 & \frac{b}{a} & \frac{b}{a} & 0 \\
0 & (-n_1 - \lambda) & \frac{b}{a} & \frac{b}{a} & 0 \\
0 & k & (-n_2 - \lambda) & 0 & 0 \\
0 & p_1 & p_2 & -n_3 - \lambda & 0 \\
0 & f_1 & f_2 & f_3 & (-a - \lambda)
\end{vmatrix} = 0,
\]

where \( C_1 = n_1 + n_2 + n_3, \ C_2 = n_1 n_2 + n_2 n_3 + n_1 n_3 - \frac{bk}{a} - p_1 b/a, \) and \( C_3 = n_1 n_2 n_3 - n_3 b k/a - p_2 k - n_2 p_1. \)

We observe that \( C_1 > 0, \ C_2 > 0, \) and \( C_1 C_2 - C_3 > 0, \) so all the eigenvalues of the system of linearized ODEs in (27) without the diffusion term are all negative for all positive parameter values, this system of (27) is a linearly asymptotically stable spatial-homogeneous solution at \( (s^*, e^*, i^*, q^*, r^*) = (1/a, 0, 0, 0, 0). \) This implies that there would not have been COVID-19 in Ghana without an influx of foreigners in the country.

Again, a situation where there is an influx of foreigners in the country is considered as follows. The system of equation (7) can be written as

\[
\frac{\partial u}{\partial t} = J u + D \Delta u.
\]

We also have \( u(x, t) = [s(x, t), e(x, t), i(x, t), q(t), r(t)]. \)

Here, \( J \) is a Jacobian matrix of the linear terms, \( \Delta \) is the Laplacian operator in one dimension, and \( D \) is a matrix that contains the coefficients of diffusion, that is,

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & d_1 & 0 & 0 & 0 \\
0 & 0 & d_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

together with \( s(x, 0) = s_o, e(x, 0) = e_o, i(x, 0) = i_o, q(0) = 0, r(0) = 0, \bar{n} \cdot \nabla s(., t) = 0, \bar{n} \cdot \nabla e(., t) = 0, \) and \( \bar{n} \cdot \nabla i(., t) = 0. \)

The solution of the system of (29) is of the form

\[
u(x, t) = \sum_k c_k u_k(r) e^{\lambda t}.
\]

For the time-independent component, \( u_k(r) \) satisfies the following equation with the same boundary conditions as the system of (29):

\[
\Delta u + k^2 u = 0
\]

with \( (u \cdot \nabla) u = 0 \) on \( \partial \mathbb{R} \) or \( \mathbb{R}. \)

\[
\Rightarrow \left( \Delta + k^2 \right) u = 0,
\]

\[
\Rightarrow u \neq 0 \text{ or } \Delta = -k^2.
\]
Substituting (31) and (32) into (29) yields

\[
J(s, e, i, q, r) = \begin{bmatrix}
-a - bi - bq - k^2 & 0 & -bs & -bs & 0 \\
bs & b(i + q) & -n_1 - d_1k^2 & bs & bs & 0 \\
0 & k & -n_2 - d_2k^2 & 0 & 0 \\
0 & p_1 & p_2 & -n_3 & 0 \\
0 & f_1 & f_2 & f_3 & -a
\end{bmatrix}.
\]

(33)

By the Hartman–Grobman theorem, the system of (7) at \((s^*, e^*, i^*, q^*, r^*) = (1/a, 0, 0, 0, 0)\) is unstable. At \((a^{-1}, e^*, i^*, q^*, r^*) = (a^{-1}, 0, 0, 0, 0)\),

\[
|J(s^*, e^*, i^*, q^*, r^*) - \lambda I| = \begin{vmatrix}
(-a - k^2) - \lambda & 0 & \frac{b}{a} & \frac{b}{a} & 0 \\
0 & (-n_1 - d_1k^2) - \lambda & \frac{b}{a} & \frac{b}{a} & 0 \\
0 & k & (-n_2 - d_2k^2) - \lambda & 0 & 0 \\
0 & p_1 & p_2 & -n_3 - \lambda & 0 \\
0 & f_1 & f_2 & f_3 & -(a + k)
\end{vmatrix} = 0
\]

(34)

\[
\Rightarrow \lambda_1 = -(a + k^2) \\
\lambda_2 = -a, \\
\lambda_3 + C_1\lambda^2 + C_2\lambda + C_3 = 0,
\]

where,

\[
C_1 = n_1 + n_2 + n_3 + (d_1 + d_2)k^2 \\
C_2 = n_3(n_1 + n_2) + n_3(d_1 + d_2)k^2 + (n_1 + d_1k^2)(n_2 + d_2k^2) + b(k - 1)(\frac{1}{a}), \\
C_3 = n_3(n_1 + d_1k^2)(n_2 + d_2k^2) - bk(n_3 + p_2) + bp_1(n_2 + d_2k^2)(\frac{1}{a}).
\]

(35)

Using the Routh–Hurwitz criterion for determining the sign of the roots of the characteristic polynomial in (35), the system of (7) at \((s^*, e^*, i^*, q^*, r^*) = (1/a, 0, 0, 0, 0)\) is unstable if at least one of the following conditions is not met: \(C_1 > 0, C_2 > 0, \) and \(C_1C_2 - C_1 > 0\). Since all the parameters are positive real numbers if \(|(bk(n_3 + p_2) + bp_1(n_2 + d_2k^2)) / a| \
\geq |n_3(n_1 + d_1k^2)(n_2 + d_2k^2)|\), then the characteristic polynomial in (34) has at least one positive root. This implies that the system of (7) is unstable at \((s^*, e^*, i^*, q^*, r^*) = (1/a, 0, 0, 0, 0)\). We observed that \(C_3 \geq 0\) is not met which implies that there is at least one of the eigenvalues has a positive real part.

The stability of the system of (7) revealed that without arrival of foreigners with the COVID-19 virus (SARS virus), there had not been the epidemiology of COVID-19 in the country. On the other hand, the arrival of foreigners, for example, COVID-19 patients from Norway and Turkey, into the country brought about the epidemiology of COVID-19.
infection in Ghana. This finding is confirmed by the instability of the system of (7) at the endemic equilibrium point. The instability of the system of (7) is attributed to the assertion that the pandemic COVID-19 infection in the country is brought by the influx of foreigners especially two people: one from Norway and the other from Turkey.

2.3. Global Stability of the System of ODEs. In this subsection, we construct a Lyapunov function to assess the overall stability of the system of ODEs. A comprehensive review of related studies indicates that neither a systematic method nor a special function is utilized to assess the overall stability of system of ODEs in the Lyapunov sense. Due to this issue, one must make an informed guess for a function whose properties satisfy the requirements in the Lyapunov criterion for global stability of system of ODEs. In most cases, the guessing is not feasible since there are no guidelines in arriving at this function. In order to overcome this difficulty, we introduce a new function that can be used in determining the global stability of the system of ODEs in the Lyapunov sense. This function serves as a Lyapunov function for any system of ODEs and the result is given in Theorem 3.

**Theorem 3** (Lyapunov function for the system of ODEs). Suppose a system of ordinary differential equations,

\[
\frac{dx_i}{dt} = F_i(x_i), \forall x_i \in \mathbb{R}^n,
\]

is an increasing function on \( \mathbb{R}^n \). Then,

\[
V(x_i(t)) = \sum_{i=1}^{n} \frac{1}{x_i(t)} \left( x_i(t) - x_i^* + x_i^* \ln \left( \frac{x_i^*}{x_i(t)} \right) \right),
\]

where \( x^* = (x_1^*, \ldots, x_n^*) \) is the fixed point of \( F(x_i) \), is the Lyapunov function for constructing the stability of the system of ODEs (19).

**Proof 2.** We can see that \( V(x_i(t)) \) is a monotone increasing function on \( \mathbb{R}^n \) for \( x_i(t) \neq x_i^* \). At \( x_i = x^* \), we observe that \( V(x_i) \) has a minimum value at \( x_i = x^* \). Therefore, \( x^* \) is a fixed point of \( V(x_i(t)) \).

Differentiating \( V(x_i(t)) \) with respect to \( t \), we have

\[
\frac{dV}{dt} = \frac{\partial V}{\partial x_1} \frac{dx_1}{dt} + \ldots + \frac{\partial V}{\partial x_n} \frac{dx_n}{dt},
\]

\[
\frac{dV}{dt} = \left[ \frac{1}{x_1} \left( x_1 - x_1^* + x_1^* \ln \left( \frac{x_1^*}{x_1} \right) \right) + \frac{1}{x_1} \left( 1 - \frac{x_1^*}{x_1} \right) \right] \times F_1(x_1) + \ldots + \left[ \frac{1}{x_n} \left( x_n - x_n^* + x_n^* \ln \left( \frac{x_n^*}{x_n} \right) \right) + \frac{1}{x_n} \left( 1 - \frac{x_n^*}{x_n} \right) \right] \times F_n(x_n),
\]

\[
\frac{dV}{dt} = -\sum_{i=1}^{n} \frac{(1)x_i^*}{x_i(t)} \ln \left( \frac{x_i^*}{x_i(t)} \right) F_i(x_i) \frac{dV}{dt} < 0.
\]

Since the gradient of \( V(x_i(t)) \) is decreasing on \([0, \infty)\), it implies that \( V(x_i(t)) = \sum_{i=1}^{n} 1/x_i(t) (x_i(t) - x_i^* + x_i^* \ln (x_i^* / x_i(t))) \) is a Lyapunov function for the system of (3). This completes the proof.

This implies that \( V(x_i) \) has a minimum value at \( x_i = x^* \).

**2.3.1. Application of Theorem 2 in Determining the Global Stability of the System of ODEs.** In this subsection, the global stability of the system of equation is obtained by setting \( \partial s/\partial x = 0, \partial e/\partial x = 0, \) and \( \partial i/\partial x = 0 \). By Theorem 2, the Lyapunov function for the system of equation (8) is given by
\[ V(s, e, i, q, r) = \frac{1}{s}(s - s^* + s^* \ln\left(\frac{s^*}{s}\right)) + \frac{1}{e}(e - e^* + e^* \ln\left(\frac{e^*}{e}\right)) + \frac{1}{i}(i - i^* + i^* \ln\left(\frac{i^*}{i}\right)) + \frac{1}{q}\left(q - q^* + q^* \ln\left(\frac{q^*}{q}\right)\right) \]

We can see that
\[ V(s, e, i, q, r) > 0, \forall s, e, i, q, r > 0. \tag{41} \]

Again,

\[
\frac{dV}{dt} = \frac{\partial V}{\partial s} \frac{ds}{dt} + \ldots + \frac{\partial V}{\partial r} \frac{dr}{dt}.
\]

\[
\frac{dV}{dt} = \left[ -\frac{1}{s}(s - s^* + s^* \ln\left(\frac{s^*}{s}\right)) + \frac{1}{s}(1 - \frac{s^*}{s}) \right] \times [1 - as - bs(i + q)] + \left[ -\frac{1}{e}(e - e^* + e^* \ln\left(\frac{e^*}{e}\right)) + \frac{1}{e}(1 - \frac{e^*}{e}) \right] \times [bs(i + q) - n_1 e] + \ldots,
\]

\[
\frac{dV}{dt} = -\frac{s^*}{s} \ln\left(\frac{s^*}{s}\right)(1 - as - bs(i + q)) - e^* \ln\left(\frac{e^*}{e}\right)(bs(i + q) - n_1 e) - \frac{i^*}{i} \ln\left(\frac{i^*}{i}\right)(ke - n_3 i) - \frac{q^*}{q} \ln\left(\frac{q^*}{q}\right)(p_1 e + p_2 i - n_3 q) - \frac{r^*}{r} \ln\left(\frac{r^*}{r}\right)(f_1 e + f_2 i + f_3 q - ar).
\]

Since \(\|1 - as - bs(i + q)\| > 0, \|bs(i + q) - n_1 e\| > 0, \|(ke - n_3 i)\| > 0, \|(p_1 e + p_2 i - n_3 q)\| > 0, \|f_1 e + f_2 i + f_3 q - ar\| > 0,\) we have
\[
\frac{dV}{dt} < 0. \tag{44}
\]

Since all conditions of the Lyapunov function are met, it implies that the system of (8) is globally asymptotically stable.

2.4. Effects of the COVID-19 Infection in the Country. In this subsection, the travelling wave solution of the influx of foreigners into the country is analysed. Thus, the analytic method via Homotopy Perturbation Method (HPM) is used to obtain the solutions for the proportions of susceptible, exposed, and infective people.

Setting \(s(x, r) = s(z), e(x, r) = e(z),\) and \(i(x, r) = i(z)\) where \(z = x - cr, c\) is the constant wave speed of the foreigners into the country; the first equation in the system of equation (4) becomes
\[
\frac{d^2 s}{dz^2} + \frac{c}{d} \frac{ds}{dz} \text{ as } 1 - bs(i + q) = 0. \tag{45}
\]

\[ q(t) = 0, \text{ since the movement of a quarantined person is restricted so he or she cannot travel to another country.} \]

\[
\frac{d^2 s}{dz^2} + \frac{c}{d} \frac{ds}{dz} \text{ as } 1 - bs(i + q) = 0. \tag{46}
\]

Similarly, we have
\[
\frac{d^2 e}{dz^2} + \frac{c}{d} \frac{de}{dz} + \frac{bs(i + q)}{d_1} \frac{de}{dz} - \frac{n_1}{d_1} e = 0. \tag{47}
\]

Setting \(q(t) = 0,\) we have
\[
\frac{d^2 e}{dz^2} + \frac{c}{d} \frac{de}{dz} - \frac{n_1}{d_1} e + b \frac{si}{d_1} = 0, \tag{48}
\]

and
\[
\frac{d^2 i}{dz^2} + \frac{c}{d} \frac{di}{dz} \frac{k}{d_2} \frac{ni}{d_2} = 0. \tag{49}
\]

For the travelling wave solution, that is, the influx of foreigners into the country, we consider the proportion of the susceptible, exposed, and infective people.

Using the HPM, the series approximation solutions of the system of equations (46), (48), and (49) are of the following form:
\[ (1 - p) \text{ (linear terms of the system of equations) } + p \text{ (linear + nonlinear terms of the system of equations) } = 0, \]
\[
\Rightarrow (1 - p) \left( \frac{d^2 s}{dz^2} + c \frac{ds}{dz} - as + 1 \right) + p \left( \frac{d^2 s}{dz^2} + c \frac{ds}{dz} - as + 1 - bsi \right) = 0, \tag{50}
\]
\[
(1 - p) \left( \frac{d^2 e}{dz^2} + \frac{de}{dz} - \frac{n_1}{d_1} e \right) + p \left( \frac{d^2 e}{dz^2} + \frac{de}{dz} + \frac{n_1}{d_1} e + \frac{b}{d_1} s i \right) = 0,
\]
\[
(1 - p) \left( \frac{d^2 i}{dz^2} + \frac{di}{dz} - \frac{n_2}{d_2} i \right) + p \left( \frac{d^2 i}{dz^2} + \frac{di}{dz} + \frac{k}{d_2} e - \frac{n_2}{d_2} i \right) = 0.
\]

We use the HPM to find the solution of the linearized system of (7) as follows.

Equating \( p^0 = 0 \), the following equation is obtained from equation (50).
\[
\frac{d^2 s}{dz^2} + c \frac{ds}{dz} - as = 1, \tag{51}
\]
where the homogeneous part of (51) is given by
\[
\frac{d^2 s}{dz^2} + c \frac{ds}{dz} - as = 0. \tag{52}
\]

Setting \( s(z) = e^{\lambda z} \) into (52) yields
\[
\Rightarrow \lambda^2 + c \lambda - a = 0, \forall e^{\lambda z} \neq 0
\]
\[
\lambda_{1,2} = -\frac{c \pm \sqrt{c^2 + 4a}}{2}, \forall c^2 + 4a > 0.
\tag{53}
\]

The complementary function of (51) is
\[
s(z) = D_1 e^{\lambda_1 z} + D_2 e^{\lambda_2 z} = D_1 e^{\frac{c + \sqrt{c^2 + 4a}}{2} z} + D_2 e^{\frac{c - \sqrt{c^2 + 4a}}{2} z}, \tag{54}
\]
with \( s(0) = s_0 \) and \( s(1) = s_1 \).

Setting the particular solution \( s_p(z) = A z^2 + B z + C \), finding the first and second derivative and substituting the results into (51) yield the particular solution \( s_p(z) = -\frac{1}{a} \). Thus, the general solution of the proportion of susceptible is given by
\[
s(z) = D_1 e^{\lambda_1 z} + D_2 e^{\lambda_2 z} = D_1 e^{\frac{c + \sqrt{c^2 + 4a}}{2} z} + D_2 e^{\frac{c - \sqrt{c^2 + 4a}}{2} z} - \frac{1}{a} \tag{55}
\]

Similarly, the proportion for exposed people and infected people is given by
\[
e(z) = D_3 e^{\lambda_1 z} + D_4 e^{\lambda_2 z} - \frac{c}{d_1} u_1 - \frac{n_1}{d_1} u_5 - \frac{b}{d_1} u_6,
\]
\[
i(z) = D_5 e^{\lambda_1 z} + D_6 e^{\lambda_2 z} - \frac{b}{d_2} u_7 - \frac{n_2}{d_2} u_8,
\]
respectively.

2.5. Conversion of a System of Mixture of PDEs and ODEs into a System of ODEs. Now, all the PDEs are expressed in their respective ODEs in order to have solutions which are of the same state.
The equilibrium point of the system of ODEs in (58) is obtained by setting the ordinary derivatives on the right hand side of (58) to zero and solving the resulting equations simultaneously which yields the disease-free equilibrium point as:

\[ u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*, u_8^* \]  

\[ a, 0, 0, 0, 0, 0, 0, 0 \]  

Linearizing the system of ODEs (30) yields the following results:

Table 1: Exposed and infected people from March 2020 to December 2020.

| Month      | Exposed people | Infected people |
|------------|----------------|-----------------|
| March      | 155            | 6               |
| April      | 1438           | 475             |
| May        | 2335           | 3661            |
| June       | 5777           | 3894            |
| July       | 9376           | 8384            |
| August     | 1766           | 7031            |
| September  | 971            | 1357            |
| October    | 882            | 547             |
| November   | 1544           | 2068            |
| December   | 13385          | 1719            |

Table 2: Exposed and infected people from January 2021 to January 2022.

| Month      | Exposed people | Infected people |
|------------|----------------|-----------------|
| January    | 10029          | 2210            |
| February   | 8187           | 8826            |
| March      | 2355           | 4205            |
| April      | 1017           | 962             |
| May        | 655            | 681             |
| June       | 1215           | 801             |
| July       | 4905           | 2200            |
| August     | 8204           | 8213            |
| September  | 3608           | 4438            |
| October    | 819            | 1776            |
| November   | 210            | 633             |
| December   | 11075          | 991             |
| January (2022) | 3786    | 9743            |

Table 3: List of parameters for the model.

| Parameter | Value          |
|-----------|----------------|
| \( \alpha \) | 0.2000000000000 |
| \( \beta \) | 0.8000000000000 |
| \( \kappa \) | 0.0714285714300 |
| \( \gamma_1 \) | 0.7500000000000 |
| \( \gamma_2 \) | 0.00008889880194 |
| \( \theta_1 \) | 0.0140014195900 |
| \( \theta_2 \) | 0.7800000000000 |
| \( \delta_1 \) | 0.2937464950000 |
| \( \delta_2 \) | 0.0195830996700 |
| \( \delta_3 \) | 0.5874929900000 |
| \( \mu \) | 0.0000195371580 |

Table 4: Initial conditions for the five subgroups of the population size of Ghana.

| Compartment | Value | Proportion |
|-------------|-------|------------|
| Susceptible | 30417756 | 0.99999966467 |
| Exposed     | 100   | 3.287542469 × 10^{-6} |
| Infectious  | 2     | 6.575084939 × 10^{-8} |
| Quarantined | 0     | 0           |
| Recovered   | 0     | 0           |

Table 5: Varied coefficients of diffusion for the proportion of susceptible, exposed, and infective.

| Case Parameter | Value (km²/hr) |
|----------------|---------------|
| \( d_1 \)     | 0.01          |
| \( d_2 \)     | 0.001         |
Figure 4: A 3-dimensional plot for the proportion of infective in space and time.

Figure 5: Proportion of the infective curve.

Figure 6: Proportion of the infective from March 2020 to January 2022.
Figure 7: Proportion of infective curves from normalized data and the model.

Table 6: The relative error between the normalized data and the model values.

| Time (month) | Normalized data value | Model data value | Relative error |
|--------------|-----------------------|------------------|----------------|
| 0            | 0.0000                | 0.15             | 249            |
| 1            | 0.0481                | 0.0941           | 0.9563409563   |
| 2            | 0.3753                | 0.1328           | 0.6461497469   |
| 3            | 0.3993                | 0.3358           | 0.1590282995   |
| 4            | 0.8604                | 0.7497           | 0.1286610879   |
| 5            | 0.7214                | 0.9153           | 0.2687829221   |
| 6            | 0.1387                | 0.6329           | 3.5630857970   |
| 7            | 0.0555                | 0.2683           | 3.8342342340   |
| 8            | 0.2117                | 0.07764          | 0.6332546056   |
| 9            | 0.1759                | 0.1419           | 0.1932916430   |
| 10           | 0.2263                | 0.3218           | 0.4220061865   |
| 11           | 0.9058                | 0.5008           | 0.447185696    |
| 12           | 0.4312                | 0.6783           | 0.5730519481   |
| 13           | 0.0981                | 0.5994           | 5.1100917430   |
| 14           | 0.0693                | 0.3916           | 4.6507936510   |
| 15           | 0.0816                | 0.2395           | 1.9350490200   |
| 16           | 0.2253                | 0.2235           | 0.0079834753   |
| 17           | 0.8428                | 0.3281           | 0.6107024205   |
| 18           | 0.4551                | 0.4535           | 0.0035157108   |
| 19           | 0.1817                | 0.5411           | 1.9779856910   |
| 20           | 0.0643                | 0.5188           | 7.0684292380   |
| 21           | 0.1011                | 0.4131           | 3.0860534120   |
| 22           | 1.000                 | 0.3238           | 0.6762000000   |
Figure 8: Proportion of the exposed people from March 2020 to January 2022.

Figure 9: Proportion of the exposed curve from the model.

Figure 10: Proportion of exposed curves from normalized data and the model.
Figure 11: The proportion of infective curves from smoothing by bin boundaries data and the model.

Figure 12: Proportion of the infective from March 2020 to January 2022.

Figure 13: The proportion of the infective from March 2020 to January 2022.
Table 7: The relative error between the bin boundaries data and the model values.

| Time (month) | Bin boundaries data value | Model data value | Relative error |
|-------------|---------------------------|-----------------|----------------|
| 0           | 0.0000                    | 0.15            |                |
| 1           | 0.0000                    | 0.0941          |                |
| 2           | 0.3753                    | 0.1328          | 0.6461497469   |
| 3           | 0.3993                    | 0.3358          | 0.1590282995   |
| 4           | 0.7214                    | 0.7497          | 0.0392292764   |
| 5           | 0.7214                    | 0.9153          | 0.2687829221   |
| 6           | 0.1387                    | 0.6329          | 3.5630857970   |
| 7           | 0.1387                    | 0.2683          | 0.9343907714   |
| 8           | 0.2117                    | 0.07764         | 0.6332546056   |
| 9           | 0.1759                    | 0.1419          | 0.1932916430   |
| 10          | 0.1759                    | 0.3218          | 0.829485503    |
| 11          | 0.9058                    | 0.5008          | 0.447185696    |
| 12          | 0.4312                    | 0.6783          | 0.5730519481   |
| 13          | 0.0693                    | 0.5994          | 7.6493506490   |
| 14          | 0.0693                    | 0.3916          | 4.6507936510   |
| 15          | 0.0816                    | 0.2395          | 1.9350492000   |
| 16          | 0.0816                    | 0.2235          | 1.1738970588   |
| 17          | 0.8428                    | 0.3281          | 0.6107024205   |
| 18          | 0.4551                    | 0.4535          | 0.0035157108   |
| 19          | 0.0643                    | 0.5411          | 7.4152410580   |
| 20          | 0.0643                    | 0.5188          | 7.0684292380   |
| 21          | 0.1011                    | 0.4131          | 3.0860534120   |
| 22          | 1.000                     | 0.3238          | 0.6762000000   |

**Figure 14:** Proportion of the infective from March 2020 to January 2022.
\[ f(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha + b(u_1 + u_5) - c & 0 & b u_1 & 0 & b u_1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-b(u_5 + u_7) & 0 & \frac{n_5}{d_1} & \frac{c}{d_1} & \frac{bu_1}{d_1} & 0 & -\frac{bu_1}{d_1} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & \frac{k}{d_2} & 0 & \frac{n_7}{d_2} & \frac{c}{d_2} & 0 & 0 \\
0 & 0 & p_1 & p_2 & 0 & -n_5 & 0 & 0 \\
0 & 0 & f_1 & f_2 & 0 & f_3 & -\alpha & 0 \\
\end{bmatrix}. \] (60)

At \((u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*, u_8^*) = (1/\alpha, 0, 0, 0, 0, 0, 0, 0)\),
the eigenvalues of the Jacobian matrix in (60) are obtained as

\[ |f(u^*_1, 0, 0, 0, 0, 0, 0, 0) - \lambda I| = \begin{bmatrix}
-\lambda & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha & -c - \lambda & 0 & 0 & b/\alpha & 0 & b/\alpha & 0 \\
0 & 0 & 1 - \lambda & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{n_1}{d_1} & \frac{c}{d_1} - \lambda & -\frac{b}{\alpha d_1} & 0 & -\frac{b}{\alpha d_1} & 0 \\
0 & 0 & 0 & 0 & \lambda & 1 & 0 & 0 \\
0 & 0 & \frac{k}{d_2} & 0 & \frac{n_7}{d_2} & \frac{c}{d_2} - \lambda & 0 & 0 \\
0 & 0 & p_1 & p_2 & 0 & -n_5 - \lambda & 0 & 0 \\
0 & 0 & f_1 & f_2 & 0 & f_3 & -\alpha - \lambda & 0 \\
\end{bmatrix} \] (61)

\[ \lambda_1 = -\alpha < 0, \]
\[ \lambda_2 = \frac{-c + \sqrt{c^2 + 4\alpha}}{2} > 0, \]
\[ \lambda_3 = \frac{-c - \sqrt{c^2 + 4\alpha}}{2} < 0, \]
\[ \lambda_4 = 1 > 0, \]
\[ \lambda_5 = \frac{c}{d_1} < 0, \]
\[ \lambda_6 = -n_5 < 0, \]
\[ \lambda_7 = \frac{-c/d_2 + \sqrt{(c/d_2)^2 + 4(n_7/d_2)}}{2} > 0, \]
\[ \lambda_8 = \frac{-c/d_2 - \sqrt{(c/d_2)^2 + 4(n_7/d_2)}}{2} < 0. \]
Since $\lambda_2$, $\lambda_4$, and $\lambda_7$ are all greater than zero, it implies that the COVID-19 infection at the disease-free equilibrium point,

Figure 15: The proportion of infective curves from smoothing by bin means data and the model.

Figure 16: Proportion of infective curves from smoothing by bin medians data and the model.

Figure 17: Proportion of the exposed people from March 2020 to January 2022.
Figure 18: Proportion of exposed curves from smoothing by bin boundaries data and the model.

Figure 19: Proportion of the exposed people from March 2020 to January 2022.

Figure 20: Proportion of exposed curves from smoothing by bin means data and the model.
In the system of (8), the basic reproductive number, \( R_0 \), is obtained from the spectral radius of \( FV^{-1} \). Thus, \[ R_0 = \rho(FV^{-1}), \] where
\[
F = \begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{-b}{ad_1} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]
\[
V^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{d_1}{c} & 0 & 0 \\
0 & 0 & \frac{d_2}{n_2} & \frac{d_2}{n_2} \\
0 & 0 & -1 & 0
\end{bmatrix},
\]
\[ R_0 \text{ is obtained as } R_0 = \frac{bc}{ad_1n_2} \Rightarrow R_0 = \frac{a\delta D_1 c}{\mu(\mu + \gamma_1 + \theta_2 + \delta_2)D_2}. \]

The basic reproductive number, \( R_0 \), for the system of ODEs without diffusion is given by
\[ R_0 = \rho(FV^{-1}), \]
where
\[
F = \begin{bmatrix}
0 & \frac{b}{a} & \frac{b}{a} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]
\[
V = \begin{bmatrix}
\eta_1 & 0 & 0 \\
-\kappa & \eta_2 & 0 \\
-p_1 & -p_2 & n_3
\end{bmatrix},
\]
\[ R_0 = \frac{a\delta \theta_1 (\gamma_1 + \delta_2 + \theta_2 + \theta_3 + \mu) + \kappa (\gamma_2 + \delta_1 + \mu + \theta_3)}{\mu (\gamma_1 + \delta_2 + \mu) (\gamma_1 + \delta_2 + \theta_2 + \mu) (\delta_1 + \theta_1 + \kappa + \mu)}. \]

We observe that the basic reproductive number, \( R_0 \), in the environment of inflow of foreigners into the country is more realistic than the basic reproductive number in a diffusive-free environment since \( R_0 \) involves the coefficient of diffusion of the susceptible, as well as the speed of the influx of the travellers into the country. The coefficient of diffusion of the susceptible and the wave speed is in direct relationship with \( R_0 \) which are inflows for the infective so as to result in transmission of the COVID-19 infection.

2.6. Numerical Results. The data on the COVID-19 patients in Ghana were obtained from [18] and then segregated into exposed people and infected people by summing the counts of COVID-19 cases of the first fourteen days (incubation period) in the month as the number of exposed people in that month and the remaining counts of COVID-19 patients as the number of infective in the month. The results on these exposed and infective people are summarized in Tables 1 and 2. The normalization of the data points was done using
\[ X^* = \frac{\{(X - \min(X))\}}{(\max(X) - \min(X))} (\text{new max}(X) - \text{new min}(X)) + \text{new min}(X). \]

Table 3 shows the estimates for the parameter in the model. Initial COVID-19 records in Ghana are shown in Table 4.

2.6.1. Numerical Results of the Nondimensionalized System of Mixture of PDEs and ODEs. This section discusses the effects of the movement of proportions of susceptible, exposed, and infectious individuals at a given time. Four cases are examined. Firstly, the same diffusion coefficient of the proportion of the exposed and the infective people are considered, i.e., \( d_1 \neq d_2 \).

The diffusion coefficients \( d_1 \) and \( d_2 \) are provided in Table 5 and the initial conditions for the first three compartments of system (4) are as follows:
\[ s(x,0) = \begin{cases} 1 - 0.6x^2, & 0 \leq x < 0.8, \\ 0.616(0.8 - x), & 0.8 \leq x \leq 1. \end{cases} \]

Figure 2 is plotted using the initial condition from (68). We can see that the proportion of the susceptible curve decreases as the scaled time and scaled distance start to rise from zero at the beginning. The proportion of the susceptibility curve decreases quickly as more susceptible are infected with the SARS virus (COVID-19 disease) since there are more susceptible people who are not adhering to the COVID-19 protocols by both public and private organizations in Ghana. The sharp rises in Figures 3 and 4 illustrate how the proportions of exposed and infected individuals rise when more susceptible individuals become infected with the SARS virus. The susceptible proportion continues to monotonically decrease until stabilizing at a specific point (adequately proportion of susceptible not able to contract SARS virus). The behavior of the susceptible curve shown in Figure 2 is concurrently supported by the decreasing patterns after the peaks in Figures 3 and 4.
2.6.2. Numerical Results on the Trends of the COVID-19 Patients. The quantitative solution of the proportion of the infective subgroup system of ODEs in (40) as depicted in Figure 5. Figure 5 reveals a sinusoidal wave of the COVID-19 infection in the country with an amplitude of approximately 0.92.

The time in Figure 6 begins at zero, which corresponds to March 2020, with no data on January and February 2020, since COVID-19 erupted in Ghana on March 14, 2020. After the onset of COVID-19 pandemic in the country, the proportion of infective fluctuates roughly every eight months. As a result, the proportion infective curve increases until its peak and then decreases as the month progresses. Even the peak values are inconsistent, with July 2020 having a peak value of roughly 0.9058, February 2021 recording a peak value of roughly 0.8604, and January 2022 recording a peak value of almost 1.000. However, the proportion of the infective by the model (theoretical results) in Figure 5 also fluctuates at regular interval of almost every 8 months, but the peaked values were recorded on August 2020, April 2021, and October 2021, respectively. There are variations of the peak values of the proportions of the infective on the data as in Figure 6 and the peak values of the proportion of infective from the model (see Figure 5). These variations are attributed to the noise in the data, such as outliers (peak values), incomplete data on the COVID-19 patients on the grounds that not every person who contracted COVID-19 reported the issue at the nearby medical facility, or the health professionals were unable to contact trace all the COVID-19 patients. One of the major deterrents for suspected COVID-19 patients not reporting their illness to the nearest medical facility is the fear of panic brought on by the SARS virus (COVID-19) spreading to the susceptible population. However, errors in instrumentation devices used for testing COVID-19 cases across the nation as well as human error in data entry caused by the healthcare personnel cannot be ignored when it comes to the data on COVID-19 patients. Before it can be identified whether someone has COVID-19 (caused by the SARS virus), they frequently need to undergo a number of tests. In Figure 7, the quantitative result (curve) from the model is placed on the observed data curve for comparison. It was revealed that the variations of the model curve and observed data curve decrease as time passes and then fluctuate for the remaining of the period. This disparity is attributed to the unavoidable inaccuracies in the data. This necessitates data cleansing via bin smoothing techniques.

Due to all the issues observed about data on COVID-19 patients in the country, inevitably the data cleaning was carried out to remove noise from the data. In Table 6, the relative errors between the model values and data values are summarized. The relative errors are not different from the observations in Figures 8–10.

2.6.3. Smoothing of the Normalized Data. In smoothing the normalized data from the Ghana Health Service (GHS), discrepancies in the data were detected, such as human error in the data entry of the number of infectives, because daily cases of COVID-19 patients and accumulated monthly cases of COVID-19 patients were not segregated into the number of infective and the number of exposed people. The two subgroups, the number of infective and the number of exposed people, of the population size of Ghana cannot be lumped together as they play different roles in the epidemiology of COVID-19 pandemic in the country.

To remove noise from the data obtained from GHS, the binning smoothing methods, smoothing by bin means, smoothing by bin medians, and smoothing by bin boundaries, were employed. The proportion of the infective curve superimposed on each normalized smoothed set of data is depicted in Figures 12, 14, and 16. These graphs show that the normalized smooth data by bin boundaries are consistent and that its values are robust against noise, especially when there are outliers. As a result, in Figure 11, the variation between the infectious curve (model) and the smoothed data is smaller as the time (month) goes up to 4, then it starts to go up to 0.2687829221 (see relative error in Table 7). It is imperative to note that this peculiar trend could be caused by the unsorting of the normalized data:

1. Comparison of the smoothed normalized data by bin boundaries, bin means, and bin medians and the normalized model for infective
2. Comparison of the smoothed normalized data by bin boundaries, bin means, and bin medians and the normalized model for exposed

We can see from Figure 12 that the smoothing by bin boundaries maintains most of the features of the original normalized data (see Figure 8). Thus, there are peaks in Figure 12 as in Figure 8, whereas in Figure 13 and Figure 14, they were flattened at the top. The curve of the model superimposed on the smoothed (normalized) data by the bin boundaries, bin means, and bin medians in Figures 12, 15, and 16, respectively, exhibited variations between the two curves in each figure. Comparatively, in Figure 11, the variation between the curve and smoothed (normalized) data by the bin boundaries was minimal and more consistent as time (month) increases than the smoothed by bin...
means and bin medians in Figures 15 and 16. The smoothing normalized data by the bin means using the frequency of three data points assumes the mean value for all the three data points like smoothing by the bin medians. Similar comparison study of the normalized equations (model) and the smoothened data, on the proportion of exposed people, by bin boundaries, bin means, and bin medians are observed in Figures 17–22. Undoubtedly, the variations between the smoothened data by bin boundaries and the model were least as compared with the variations between the smoothened data by the bin means and the model and the smoothened (normalized) data by bin medians and the model. The relative errors for the three smoothing techniques are summarized in Tables 7–9 for easy comparison.

Table 8: The relative error between the bin means data and the model values.

| Time (month) | Bin means data value | Model data value | Relative error |
|-------------|----------------------|------------------|---------------|
| 0           | 0.1411               | 0.15             | 0.06307583274 |
| 1           | 0.1411               | 0.0941           | 0.33309709430 |
| 2           | 0.1411               | 0.1328           | 0.05882352941 |
| 3           | 0.6603               | 0.3358           | 0.49144328340 |
| 4           | 0.6603               | 0.7497           | 0.13539300320 |
| 5           | 0.6603               | 0.9153           | 0.38618809630 |
| 6           | 0.1353               | 0.6329           | 3.67775314100 |
| 7           | 0.1353               | 0.2683           | 0.98300073910 |
| 8           | 0.1353               | 0.07764          | 0.42616407980 |
| 9           | 0.4360               | 0.1419           | 0.67454128440 |
| 10          | 0.4360               | 0.3218           | 0.26192660500 |
| 11          | 0.4360               | 0.5008           | 0.14862385320 |
| 12          | 0.1995               | 0.6783           | 2.40000000000 |
| 13          | 0.1995               | 0.5994           | 2.00451127819 |
| 14          | 0.1995               | 0.3916           | 0.96290726817 |
| 15          | 0.3832               | 0.2395           | 0.23408141962 |
| 16          | 0.3832               | 0.2235           | 0.41675365344 |
| 17          | 0.3832               | 0.3281           | 0.14378914405 |
| 18          | 0.2337               | 0.4535           | 0.94052203679 |
| 19          | 0.2337               | 0.5411           | 1.31536157460 |
| 20          | 0.2337               | 0.5188           | 1.21994009413 |
| 21          | 0.5505               | 0.4131           | 0.24959128065 |
| 22          | 0.5505               | 0.3238           | 0.41180744777 |

Figure 22: Proportion of expose curves from smoothing by bin medians data and the model.

Table 8: The relative error curves from smoothing by bin medians data and the model.
3. Conclusion

The epidemiology of COVID-19 has been modelled by taking into account the main geographical factor, such as the continuous inflow of immigrants into the country, which is the primary underlying factor for so-called pandemic disease, as well as disease-related factors, such as the infectious agent, mode of transmission, latent period, infectious period, susceptibility, and resistance to the disease. The validity and robustness of this mathematical model are established through field data assessments of its trend. The model parameters that have been tested are the major aspects that will support robustness when the disparity between the model values and the data decreases over time. In reality, the validity of the mathematical model was examined using monthly data on COVID-19 infections in Ghana. A newly proposed function for constructing the global stability of the system of ODEs in the Lyapunov sense overcomes the disadvantage of guessing the function for calculating the global stability of a nonlinear ODE system. Furthermore, the Lyapunov function provides information on the magnitude of an asymptotically stable fixed point of a nonlinear system of ODEs. Furthermore, the basic reproductive number in the presence of a continuous inflow of foreigners into the country is more realistic and robust to data noise than the basic reproductive number in the absence of a continuous inflow of foreigners into the country.

Data Availability

The data for parameterizing the model are freely available at [18] https://ourworldindata.org/coronavirus.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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