Radiative Decays of Heavy Hadrons From Light-Cone QCD Sum Rules in the Leading Order of HQET

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Abstract

The radiative decays of heavy baryons and lowest three doublets of heavy mesons are studied with the light cone QCD sum rules in the leading order of heavy quark effective theory.

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1 Introduction

Heavy quark effective theory (HQET) [1] provides a framework to study the heavy hadron spectra and transition amplitude with the systematic expansion in terms of $1/m_Q$. Using the light cone photon wave function (PWF), radiative decay processes like $B \rightarrow l\nu\gamma$, $B \rightarrow \rho\gamma$ have been studied [2, 3, 4, 5, 6] with QCD sum rules (QSR) [7]. Recently similar approach was employed to analyze the couplings of pions with heavy hadrons [8]. In this work we will study the radiative decays of heavy baryons and lowest three doublets of heavy mesons using the light cone QSR (LCQSR) [9] in the leading order of HQET. With LCQSR the continuum and excited states contribution is subtracted more cleanly, which is in contrast with the analysis of meson radiative decays using the external field method in QSR [10]. The LCQSRs for the radiative decays of heavy baryons are presented in section 2. Section 3 discusses radiative decays of lowest three doublets of heavy mesons. The following section is a discussion of the parameters and the photon wave functions. The last section is the numerical analysis and a short summary.

2 Radiative decays of heavy baryons

We first introduce the interpolating currents for the heavy baryons:

$$\eta_\Lambda(x) = \epsilon_{abc} [u^a T(x) C \gamma_5 d^b(x)] h^c_{\nu}(x), \quad (1)$$

$$\eta_{\Sigma^+}(x) = \epsilon_{abc} [u^a T(x) C \gamma_\mu d^b(x)] \gamma^\mu \gamma_5 h^c_{\nu}(x), \quad (2)$$

...
\( \eta_{\Sigma^+}^\mu(x) = \epsilon_{abc}[u^a(x)C\gamma_\nu u^b(x)](-g_{\mu\nu}^t + \frac{1}{3}\gamma_t^\mu\gamma_\nu)h^c_v(x), \) 

where \( a, b, c \) is the color index, \( u(x), d(x), h_v(x) \) is the up, down and heavy quark fields, \( T \) denotes the transpose, \( C \) is the charge conjugate matrix, \( g_{\mu\nu}^t = g_{\mu\nu} - v^\mu v^\nu, \gamma_t^\mu = \gamma_\mu - \hat{v}v^\mu, \) and \( v^\mu \) is the velocity of the heavy hadron.

The overlap amplitudes of the interpolating currents with the heavy baryons is defined as:

\( \langle 0|\eta_\Lambda|\Lambda \rangle = f_\Lambda u_\Lambda, \) 
\( \langle 0|\eta_{\Sigma}|\Sigma \rangle = f_\Sigma u_\Sigma, \) 
\( \langle 0|\eta_{\Sigma^+}|\Sigma^+ \rangle = \frac{f_{\Sigma^+}}{\sqrt{3}}u_{\Sigma^+}, \)

where \( u_{\Sigma^+} \) is the Rarita-Schwinger spinor in HQET. In the leading order of HQET, \( f_\Sigma = f_{\Sigma^+}. \)

The coupling constants \( \eta_k \) are defined through the following amplitudes:

\( M(\Sigma_c \to \Lambda_c\gamma) = ie\eta_1\bar{u}_{\Lambda_c}\sigma^{\mu\nu}q_\mu e_\nu u_{\Sigma_c}, \)
\( M(\Sigma_c^\ast \to \Lambda_c\gamma) = ie\eta_2\epsilon_{\mu\nu\alpha\beta}\bar{u}_{\Lambda_c}\gamma^\nu q^\alpha e^\beta u_{\Sigma^\ast_c}, \)
\( M(\Sigma_c^\ast \to \Sigma_c\gamma) = ie\eta_3\epsilon_{\mu\nu\alpha\beta}\bar{u}_{\Sigma_c}\gamma^\nu q^\alpha e^\beta u_{\Sigma^\ast_c}, \)

where \( e_\mu \) and \( q_\mu \) are the photon polarization vector and momentum respectively, \( e \) is the charge unit.

In order to derive the sum rules for the coupling constants we consider the correlator

\( \int d^4x \ e^{-ik \cdot x}\langle \gamma(q)|T(\eta_{\Sigma}(0)\bar{\eta}_{\Lambda}(x))|0\rangle = e\frac{1 + \hat{v}^\mu}{2}\gamma^\mu\gamma_5\epsilon_{\mu\nu\alpha\beta}\epsilon^{\alpha\beta\gamma\delta}q^\gamma v^\delta G_{\Sigma,\Lambda}(\omega,\omega'), \)

\( \int d^4x \ e^{-ik \cdot x}\langle \gamma(q)|T(\eta_{\Sigma^+}(0)\bar{\eta}_{\Lambda}(x))|0\rangle = e(e_\alpha q_\nu - e_\nu q_\alpha)\frac{1 + \hat{v}^\mu}{2}(-g_{\mu\nu}^t + \frac{1}{3}\gamma_t^\mu\gamma_\nu)\epsilon_{\nu\alpha\beta\sigma}e^{\alpha\beta\gamma\delta}q^\gamma v^\delta G_{\Sigma^+,\Lambda}(\omega,\omega'), \)

\( \int d^4x \ e^{-ik \cdot x}\langle \gamma(q)|T(\eta_{\Sigma^+}(0)\bar{\eta}_{\Sigma}(x))|0\rangle = e\frac{1 + \hat{v}}{2}\gamma^\alpha\gamma_5(-g_{\mu\nu}^t + \frac{1}{3}\gamma_t^\mu\gamma_\nu)(e_\alpha q_\nu - e_\nu q_\alpha)G_{\Sigma^+,\Sigma}(\omega,\omega'), \)

where \( k' = k - q, q'_\mu = q_\mu - (q \cdot v)v_\mu, \omega = 2v \cdot k, \omega' = 2v \cdot k' \) and \( q^2 = 0. \)

Let us first consider the functions \( G_{\Sigma,\Lambda}(\omega,\omega') \) etc in (10)-(12). As functions of two variables, they have the following pole terms from double dispersion relation

\( -4i\eta_1 f_{\Sigma} f_{\Lambda}/(2\Lambda_{\Sigma} - \omega')(2\Lambda_{\Lambda} - \omega) + c/2\Lambda_{\Sigma} - \omega' + c'/2\Lambda_{\Lambda} - \omega, \)

\( -4i\eta_2 f_{\Sigma^+} f_{\Lambda}/\sqrt{3}/(2\Lambda_{\Sigma^+} - \omega')(2\Lambda_{\Lambda} - \omega) + c/2\Lambda_{\Sigma^+} - \omega' + c'/2\Lambda_{\Lambda} - \omega, \)
\[
\frac{4\eta_3}{\sqrt{3}} \frac{f_{\Sigma^*} f_{\Sigma}}{(2\Lambda_{\Sigma^*} - \omega')(2\Lambda_{\Sigma} - \omega)} + \frac{c}{2\Lambda_{\Sigma^*} - \omega'} + \frac{c'}{2\Lambda_{\Sigma} - \omega},
\]

where \(f_{\Sigma^*}\) etc. are constants defined in (4)-(3), \(\bar{\Lambda}_{\Sigma^*} = m_{\Sigma^*} - m_Q\).

Keeping the two particle component of the photon wave function, the expression for \(G_{\Sigma^*,A}(\omega, \omega')\) with the tensor structure reads

\[
-2 \int_0^\infty dt \int dx e^{-ikx} \delta(-x-vt) \text{Tr}\{[C'\langle \gamma(\omega) | u(0) \bar{u}(x) | 0 \rangle \gamma^T C_{\mu} iS(-x) \gamma_5] \\
+ [CiS^T(-x)C_{\mu}]\langle \gamma(\omega) | u(0) \bar{u}(x) | 0 \rangle \gamma_5\},
\]

where \(iS(-x)\) is the full light quark propagator with both perturbative term and contribution from vacuum fields.

\[
iS(x) = \langle 0 | T[q(x), \bar{q}(0)] | 0 \rangle = \frac{i\hat{x}}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} \langle \bar{q}g_s \sigma \cdot Gq \rangle \\
-ig_s \frac{1}{16\pi^2} \int_0^1 du \left\{ \frac{\hat{x}}{x^2} \sigma \cdot G(ux) - 4i \frac{u e^{\mu \nu}}{x^2} G^{\mu \nu}(ux) \gamma_\nu \right\} + \cdots.
\]

The light cone two-particle photon wave functions are [4]:

\[
\langle \gamma(\omega) | \bar{q}(x) \gamma_{\mu} q(0) | 0 \rangle = \langle \bar{q}q \rangle \int_0^1 du e^{iuxq} \left\{ (e_{\mu} q_{\nu} - e_{\nu} q_{\mu})[\chi\phi(u) + x^2 g_1(u)] \\
+ [\langle q(x) (e_{\mu} x_{\nu} - e_{\nu} x_{\mu}) + (e x)(x_{\mu} q_{\nu} - x_{\nu} q_{\mu}) - x^2 (e_{\mu} q_{\nu} - e_{\nu} q_{\mu}) g_2(u) \} \right\},
\]

\[
\langle \gamma(\omega) | \bar{q}(x) \gamma_{\mu} \gamma_5 q(0) | 0 \rangle = \frac{f}{4} e_q e_{\nu} e_{\rho} e_{\sigma} e^{\nu} q^\rho x^\sigma \int_0^1 du e^{iuxq} \psi(u).
\]

Due to the choice of the gauge \(x^\mu A_\mu(x) = 0\), the path-ordered gauge factor \(P \exp(i g_s \int_0^1 du x^\mu A_\mu(ux))\) has been omitted. The \(\phi(u), \psi(u)\) is associated with the leading twist two photon wave function, while \(g_1(u)\) and \(g_2(u)\) are twist-4 PWFs. All these PWFs are normalized to unity, \(\int_0^1 du f(u) = 1\).

Expressing (13) with the photon wave functions, we arrive at:

\[
G_{\Sigma^*,A}(\omega, \omega') = -(e_u - e_d) \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{\omega}{2}} e^{iuxq} \left\{ \frac{1}{\pi^2 f_3} \chi\phi(u) \\
+ \frac{1}{\pi^2 t} (g_1(u) - g_2(u))] + \frac{f}{24} \psi(u) t(\langle \bar{q}q \rangle + \frac{t^2}{16} \langle \bar{q}g_s \sigma \cdot Gq \rangle) \} + \cdots.
\]

Similarly we have,

\[
G_{\Sigma,A}(\omega, \omega') = G_{\Sigma^*,A}(\omega, \omega'),
\]

\[
G_{\Sigma,\Sigma}(\omega, \omega') = (e_u + e_d) \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{\omega}{2}} e^{iuxq} \left\{ \frac{f}{4\pi^2 t^2} \psi(u) \\
+ \frac{\langle \bar{q}q \rangle}{6} (\langle \bar{q}q \rangle + \frac{t^2}{16} \langle \bar{q}g_s \sigma \cdot Gq \rangle)[\chi\phi(u) + t^2 (g_1(u) - g_2(u))] + \cdots \right\},
\]

where \(\langle \bar{q}q \rangle = -(225\text{MeV})^3\), \(\langle \bar{q}g_s \sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle\), \(m_0^2 = 0.8\text{GeV}^2\). For large Euclidean values of \(\omega\) and \(\omega'\) this integral is dominated by the region of small \(t\), therefore it can be approximated by the first a few terms.
After Wick rotations and making double Borel transformation with the variables $\omega$ and $\omega'$ the single-pole terms in (13)-(17) are eliminated. Subtracting the continuum contribution which is modeled by the dispersion integral in the region $\omega, \omega' \geq \omega_c$, we arrive at:

\[
\eta_1 f_\Sigma f_\Lambda = -\frac{1}{64\pi^2}(e_u - e_d)a e^{\frac{\Delta_S + \Delta_A}{T}} \{\chi\phi(u_0)T^4 f_3(\frac{\omega}{T})
- 4[g_1(u_0) - g_2(u_0)]T^2 f_1(\frac{\omega}{T}) + \frac{2\pi^2}{3} f \psi(u_0)(1 - \frac{m_3^2}{4T^2})\} ,
\]

\[
\eta_2 f_\Sigma^* f_\Lambda = -\sqrt{3}\frac{\eta}{64\pi^2}(e_u - e_d)a e^{\frac{\Delta_{S^*} + \Delta_A}{T}} \{\chi\phi(u_0)T^4 f_3(\frac{\omega}{T})
- 4[g_1(u_0) - g_2(u_0)]T^2 f_1(\frac{\omega}{T}) + \frac{2\pi^2}{3} f \psi(u_0)(1 - \frac{m_3^2}{4T^2})\} ,
\]

\[
\eta_3 f_\Sigma^* f_\Sigma = -\frac{\sqrt{3}}{32\pi^2}(e_u + e_d)a e^{\frac{\Delta_{S^*} + \Delta_A}{T}} \{f \psi(u_0)T^3 f_2(\frac{\omega}{T})
- \frac{a^2}{64\pi^2}(1 - \frac{m_3^2}{4T^2})[\chi\phi(u_0)TF_0(\frac{\omega}{T}) - \frac{1}{T}[g_1(u_0) - g_2(u_0)])\} ,
\]

where $f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}$ is the factor used to subtract the continuum, $\omega_c$ is the continuum threshold. $u_0 = \frac{T_1 T_2}{T_1 + T_2}$, $T \equiv \frac{T_1 T_2}{T_1 + T_2}$, $T_1$, $T_2$ are the Borel parameters $a = -(2\pi)^2 \langle \bar{q}q \rangle$. We have used the Borel transformation formula: $\hat{B}_\alpha e^{\alpha \omega} = \delta(\alpha - \frac{1}{T})$.

Due to the heavy quark symmetry, $\Lambda_{S^*} = \Lambda_S$ and $f_{S^*} = f_S$ in the limit $m_Q \to \infty$. So from (23) and (24) we have $\eta_2 = \sqrt{3}\eta_1$. For the decays $\Sigma_{c^0} \to \Sigma_0 \gamma$ and $\Sigma_{c^+} \to \Sigma_{c^0} \gamma$, we need make replacement $(e_u + e_d) \to 2e_d, 2e_u$ in (25).

### 3 Radiative decays of heavy mesons

We shall confine ourselves to the lowest lying three doublets and consider all possible radiative decay processes among them in the leading order of $1/m_Q$ expansion. Denote the doublet $(1^+, 2^+)$ with $j_6 = 3/2$ by $(B_1, B_2)$, the doublet $(0^+, 1^+)$ with $j_6 = 1/2$ by $(B_0', B_1')$ and the doublet $(0^-, 1^-)$ by $(B, B^*)$.

The interpolating currents are given in [12] as

\[
J_{1^+, \frac{3}{2}}^\alpha = \sqrt{\frac{3}{4}} \bar{h}_v \gamma^5 (-i) \left( D_t^\alpha - \frac{1}{3} \gamma_t^\alpha D_t \right) q ,
\]

\[
J_{2^+, \frac{3}{2}}^{\alpha_1, \alpha_2} = \sqrt{\frac{1}{2}} \bar{h}_v \frac{(-i)}{2} \left( \gamma_t^{\alpha_1} D_t^\alpha + \gamma_t^{\alpha_2} D_t^\alpha - \frac{2}{3} g_t^{\alpha_1 \alpha_2} D_t \right) q ,
\]

\[
J_{1^-, \frac{1}{2}}^\alpha = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_t^\alpha q ,
\]

\[
J_{1^+, \frac{3}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v q ,
\]

\[
J_{1^+, \frac{3}{2}}^{\alpha_1} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma_t^{\alpha_1} q .
\]

4
\( (1^+, 2^+) \rightarrow (0^-, 1^-) + \gamma \)

The decay amplitudes are

\[
M(B_1 \rightarrow B^*\gamma) = e_q e^\mu e^\nu e^\sigma e^\beta \eta_\alpha \{[\epsilon_{\mu\nu\beta\sigma} q_1^\alpha_1 g_1^\beta_1 g_1^\sigma_1 g_1^\gamma_1] (B_1, B^*) + [\epsilon_{\mu\nu\beta\sigma} q_1^\alpha_1 g_1^\beta_1 g_1^\sigma_1 g_1^\gamma_1] (B_1, B^*) + [\epsilon_{\mu\nu\beta\sigma} g_2^\beta_2 g_2^\sigma_2 g_2^\gamma_2] (B_1, B^*) + [\epsilon_{\mu\nu\beta\sigma} g_2^\beta_2 g_2^\sigma_2 g_2^\gamma_2] (B_1, B^*) \}
\]

where the tensor structure associated with \( g_D^\beta_1 (B_1, B^*) \) and \( g_2^\beta_2 (B_1, B^*) \) is symmetric and antisymmetric under the exchange of \( (\alpha \leftrightarrow \beta) \) respectively.

\[
M(B_1 \rightarrow B\gamma) = e_q e^\mu e^\nu e^\sigma e^\beta \eta_\alpha \{[\epsilon_{\mu\nu\beta\sigma} q_1^\alpha_1 g_1^\beta_1 g_1^\sigma_1 g_1^\gamma_1] (B_1, B) + g_1^\alpha_1 g_2^\beta_2 g_2^\sigma_2 g_2^\gamma_2] (B_1, B) \}
\]

\[
M(B_2^* \rightarrow B\gamma) = e_q e^\mu e^\nu e^\sigma e^\beta \eta_\alpha \{[\epsilon_{\mu\nu\beta\sigma} q_1^\alpha_1 g_1^\beta_1 g_1^\sigma_1 g_1^\gamma_1] (B_2^*, B) + g_1^\alpha_1 g_2^\beta_2 g_2^\sigma_2 g_2^\gamma_2] (B_2^*, B) \}
\]

\[
M(B_2^* \rightarrow B^*\gamma) = e_q e^\mu e^\nu e^\sigma e^\beta \eta_\alpha \{[\epsilon_{\mu\nu\beta\sigma} q_1^\alpha_1 g_1^\beta_1 g_1^\sigma_1 g_1^\gamma_1] (B_2^*, B^*) + g_1^\alpha_1 g_2^\beta_2 g_2^\sigma_2 g_2^\gamma_2] (B_2^*, B^*) \}
\]

Due to heavy quark symmetry, there exist only two independent coupling constants for the D-wave and S-wave decay respectively. Let \( g_d \equiv g_D(B_2^*, B) \) and \( g_s \equiv -g_S(B_2^*, B^*) \). Then we have:

\[
\frac{\sqrt{6}}{2} g_s (B_1, B^*) = \frac{\sqrt{6}}{4} g_s (B_1, B) = g_s ,
\]

\[
\frac{\sqrt{6}}{3} g_D^1 (B_1, B^*) = \frac{\sqrt{6}}{2} g_D^2 (B_1, B^*) = \frac{\sqrt{6}}{2} g_D (B_1, B)
\]

\[
= -\frac{1}{2} g_D^1 (B_2^*, B^*) = g_D^2 (B_2^*, B^*) = g_d .
\]

The above relation is confirmed by our detailed calculation.

For deriving the sum rules for the coupling constants we consider the correlator

\[
\int d^4x \ e^{-ik\cdot x} \langle \gamma(q) | T \left( J_{0,-,\frac{1}{2}}(0) J^\dagger_{1,+,\frac{1}{2}}(x) \right) | 0 \rangle
\]

\[
= e_q e^\epsilon q^\beta q^\sigma q^\gamma \{[\epsilon_{\mu\nu\beta\sigma} q_1^\alpha_1 g_1^\beta_1 g_1^\sigma_1 g_1^\gamma_1] (B_2^*, B^*) + e^\epsilon q^\alpha q^\beta q^\gamma \}
\]

The functions \( G_{B_1B}^{D,S}(\omega, \omega') \) in (32) have the following double dispersion relation

\[
\frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} g_{D,S} (B_1 B)}{(2\Lambda_{-\frac{1}{2}} - \omega')(2\Lambda_{+\frac{1}{2}} - \omega')} + \frac{c}{2(2\Lambda_{-\frac{1}{2}} - \omega)} + \frac{c'}{2(2\Lambda_{+\frac{1}{2}} - \omega)} .
\]
where \( \Lambda_{P,\ell} = m_{P,\ell} - m_Q \) and \( f_{P,\ell} \) are constants defined as:

\[
\langle 0| J_{j,P,\ell}^{\alpha_1 \cdots \alpha_j}(0) | j', P', \ell' \rangle = f_{P,\ell} \delta_{jj'} \delta_{PP'} \delta_{\ell \ell'} \eta_r^{\alpha_1 \cdots \alpha_j}.
\] (37)

Applying the same procedure as in section 2 we obtain

\[
G_{B_1B}(\omega, \omega') = -\frac{\sqrt{6}}{24} \int_0^\infty dt \int_0^1 du e^{i(1-u)q_\mu} T^\mu u \left\{ \frac{-it}{4} f_\psi(u) + \langle \bar{q}q \rangle [\chi \phi(u) + t^2 (g_1(u) - g_2(u))] \right\} + \cdots ,
\] (38)

\[
G_{B_1B}^S(\omega, \omega') = -\frac{2}{3} g_1^2 G_{B_1B}^D(\omega, \omega') = \frac{1}{6} (\omega - \omega')^2 G_{B_1B}^D(\omega, \omega') .
\] (39)

Finally we have:

\[
g_d f_{-\frac{1}{2}} f_{+\frac{1}{2}} = \frac{1}{8} e^{\frac{\lambda}{\sqrt{2}} + \frac{\lambda_+}{2}} \left\{ f u_0 \psi(u_0) + \frac{a}{2\pi^2} u_0 [\chi \phi(u_0) T f_0(\omega_c) - \frac{4}{T} (g_1(u_0) - g_2(u_0))] \right\},
\] (40)

\[
g_s f_{-\frac{1}{2}} f_{+\frac{1}{2}} = -\frac{1}{96} e^{\frac{\lambda}{\sqrt{2}} + \frac{\lambda_+}{2}} \left\{ f \frac{d^2}{du^2} \left( u \psi(u) \right) T^2 f_1(\omega_c) + \frac{a}{2\pi^2} \frac{d^2}{du^2} (u \phi(u)) T^3 f_2(\omega_c) \right\}
\]

\[
-4 \frac{d^2}{du^2} \left( u g_1(u) - u g_2(u) \right) T f_0(\omega_c) \right\} \bigg|_{u=u_0}
\] (41)

Here we have used integration by parts to absorb the factor \((q \cdot v)^2\), which leads to the second derivatives in (31). In this way we arrive at the simple form after double Borel transformation.

- \((0^+, 1^+) \to (0^-, 1^-) + \gamma\)

There exists only one independent coupling constant, corresponding to S-wave decay. The decay amplitudes are:

\[
M(B_1' \to B^* \gamma) = e_\epsilon e^{i\epsilon \sigma \alpha} \epsilon_\mu v_\sigma \eta_{\alpha}^r g_s(B_1', B^*),
\] (42)

where \( \eta_{\alpha}^r \) is the polarization vector of \( B_1' \).

\[
M(B_1' \to B \gamma) = e_\epsilon e^{i\epsilon \sigma \alpha} \eta_{\alpha}^r g_s(B_1', B),
\] (43)

\[
M(B_0' \to B^* \gamma) = e_\epsilon e^{i\epsilon \sigma \alpha} \eta_{\alpha}^r g_s(B_0', B^*).
\] (44)

The process \( B_0' \to B \gamma \) is forbidden due to parity and angular momentum conservation. Due to heavy quark symmetry, we have

\[
g_s(B_1', B^*) = g_s(B_1', B) = -g_s(B_0', B^*) \equiv g_1.
\] (45)

We consider the correlator

\[
\int d^4 x \ e^{-ik \cdot x} \langle \gamma(q)| T \left( J_{0,+\frac{1}{2}}(0)J_{1,-\frac{1}{2}}^\beta(x) \right) | 0 \rangle = e_\epsilon e^{i\epsilon \sigma \alpha} G_{B_0B}(\omega, \omega'),
\] (46)
\[ G_{B_0^* B} (\omega, \omega') = -\frac{1}{4} \langle \bar{q} q \rangle (q \cdot v) \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{q\cdot v}{T}} e^{iu\frac{\omega'}{T}} \{ \chi \phi(u) + t^2 g_1(u) \} + \cdots . \]  

(47)

\[ g_1 f_+ f_+ = \frac{a}{16\pi^2} \frac{\lambda^+_{\frac{1}{2}} + \lambda^+_{\frac{3}{2}} + \frac{1}{2}}{\tau} \{ \chi \frac{d\phi(u)}{du} T^2 f_1(\frac{\omega_c}{T}) - 4 \frac{dg_1(u)}{du} \} |_{u=u_0} . \]  

(48)

• \((1^+, 2^+) \rightarrow (0^+, 1^+) + \gamma\)

There exists only one independent coupling constant, corresponding to P-wave decay. The decay amplitudes are:

\[ M(B_1 \rightarrow B'_1 \gamma) = e_q e \eta_\alpha \eta^\beta e^\mu_\nu \{ [\eta_\alpha^\beta \eta^\mu_\nu - \frac{1}{3} \eta_\mu_\nu \eta^\alpha \eta^\beta] g^1_p(B_1, B'_1) + (\alpha \leftrightarrow \beta) g^2_p(B_1, B'_1) \} , \]  

(49)

\[ M(B_1 \rightarrow B'_0 \gamma) = e_q e e^{\mu_\sigma} \eta_\alpha \eta^\beta e^\mu_\nu q^\rho g_p(B_1, B'_0) , \]  

(50)

\[ M(B^*_2 \rightarrow B'_0 \gamma) = e_q e e^{\mu_\sigma} \eta_\alpha \eta^\beta e^\mu_\nu \{ [\eta_\alpha^\beta \eta^\mu_\nu - \frac{1}{3} \eta_\mu_\nu \eta^\alpha \eta^\beta] g^2_p(B^*_2, B'_0) \} , \]  

(51)

\[ M(B^*_2 \rightarrow B'_1 \gamma) = e_q e e^{\mu_\sigma} \eta_\alpha \eta^\beta e^\mu_\nu \{ [\eta_\alpha^\beta \eta^\mu_\nu - \frac{1}{3} \eta_\mu_\nu \eta^\alpha \eta^\beta] g^2_p(B^*_2, B'_1) \} . \]  

(52)

Due to heavy quark symmetry we have

\[ \frac{\sqrt{6}}{3} g^1_p(B_1, B'_1) = \sqrt{6} g^2_p(B_1, B'_1) = \sqrt{6} g_p(B_1, B'_0) = g_p(B^*_2, B'_0) = g_p(B^*_2, B'_1) \equiv g_2 . \]  

(53)

We consider the correlator

\[ \int d^4x \ e^{-ik\cdot x} \langle \pi(q) | T \left( \bar{J}_{0,0}^\alpha (0), J_{2,2}^\alpha (x) \right) | 0 \rangle = e_q e e^{\mu_\nu} \{ [\eta_\alpha^\beta \eta^\mu_\nu - \frac{1}{3} \eta_\mu_\nu \eta^\alpha \eta^\beta] g_{B^*_2 B'_0} (\omega, \omega') \} , \]  

(54)

where

\[ g_{B^*_2 B'_0} (\omega, \omega') = -\frac{1}{8} \langle \bar{q} q \rangle (q \cdot v) \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{q\cdot v}{T}} e^{iu\frac{\omega'}{T}} u \{ \chi \phi(u) + t^2 g_1(u) \} + \cdots . \]  

(55)

\[ g_2 f_+ f_+ = \frac{a}{32\pi^2} \frac{\lambda^+_{\frac{1}{2}} + \lambda^+_{\frac{3}{2}} + \frac{1}{2}}{\tau} \{ \chi \frac{d(u\phi(u))}{du} T^2 f_1(\frac{\omega_c}{T}) - 4 \frac{d(u g_1(u))}{du} \} |_{u=u_0} . \]  

(56)

• \(B'_1 \rightarrow B'_0 \gamma\)

\[ M(B'_1 \rightarrow B'_0 \gamma) = e_q e e^{\alpha\mu\sigma} \eta_\alpha^\beta e^\mu_\nu g_1 v_\tau g_3 . \]  

(57)
In order to derive $g_3$, we consider the correlator

$$
\int d^4 x \ e^{-ik \cdot x} \langle \gamma(q) | T \left( J_{0, \pm \frac{1}{2}}(0) J_{1, -\frac{1}{2}}^{\alpha}(x) \right) | 0 \rangle = e_q e e^{\alpha \mu \nu \sigma} e_\mu q_\nu v_\sigma G_{B'B_0}(\omega, \omega') , \quad (58)
$$

where

$$
G_{B'B_0}(\omega, \omega') = \frac{i}{4} \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{\omega}{T}} e^{iu\frac{\omega'}{T}} \left\{ \frac{4}{t} f\psi(u) + \langle \bar{q}q \rangle [\chi\phi(u) + t^2 (g_1(u) - g_2(u))] \right\} . \quad (59)
$$

$$
g_3 f_{+, \frac{1}{2}}^2 = -\frac{1}{4} e^\frac{2\lambda + 1}{2} \left\{ -f\psi(u_0) + \frac{a}{2\pi^2} [\chi\phi(u_0) T f_0(\omega_c) + \frac{4}{T} (g_1(u_0) - g_2(u_0))] \right\} . \quad (60)
$$

- $B^* \rightarrow B\gamma$

$$
M(B^* \rightarrow B\gamma) = e_q e e^{\alpha \mu \nu \sigma} e_\alpha e_\mu q_\nu v_\sigma g_4 . \quad (61)
$$

In order to derive $g_4$, we consider the correlator

$$
\int d^4 x \ e^{-ik \cdot x} \langle \gamma(q) | T \left( J_{0, -\frac{1}{2}}(0) J_{1, \frac{1}{2}}^{\alpha}(x) \right) | 0 \rangle = e_q e e^{\alpha \mu \nu \sigma} e_\mu q_\nu v_\sigma G_{BB}(\omega, \omega') , \quad (62)
$$

where

$$
G_{BB}(\omega, \omega') = \frac{i}{4} \int_0^\infty dt \int_0^1 du e^{i(1-u)\frac{\omega}{T}} e^{iu\frac{\omega'}{T}} \left\{ \frac{4}{t} f\psi(u) + \langle \bar{q}q \rangle [\chi\phi(u) + t^2 (g_1(u) - g_2(u))] \right\} . \quad (63)
$$

Note this coupling was calculated in [3] using LCQSR. But there the contribution from the photon wave function $\psi(u)$ has not been taken into account.

$$
g_4 f_{-, \frac{1}{2}}^2 = -\frac{1}{4} e^\frac{2\lambda - 1}{2} \left\{ f\psi(u_0) + \frac{a}{2\pi^2} [\chi\phi(u_0) T f_0(\omega_c) - \frac{4}{T} (g_1(u_0) - g_2(u_0))] \right\} . \quad (64)
$$

- $B_2^* \rightarrow B_1\gamma$

$$
M(B_2^* \rightarrow B_1\gamma) = e_q e e^{\alpha \mu \nu \sigma} e_\mu q_\nu v_\sigma q_\beta q_\alpha_1 q_\alpha_2 (g_1^{\alpha_1 \rho} q_1^{\alpha_2} + g_1^{\alpha_2 \rho} q_1^{\alpha_1} - \frac{2}{3} g_1^{\alpha_1 \alpha_2} q_1^{\rho}) (2q_2^{\beta} g_2^{\rho \alpha} + g_2^{\beta \rho}) g_5 . \quad (65)
$$

In order to derive $g_5$, we consider the correlator

$$
\int d^4 x \ e^{-ik \cdot x} \langle \gamma(q) | T \left( J_{2, \pm \frac{1}{2}}(0) J_{1, \frac{1}{2}}^{\alpha_1 \alpha_2}(x) \right) | 0 \rangle = e_q e e^{\alpha \mu \nu \sigma} e_\mu q_\nu v_\sigma (g_1^{\alpha_1 \rho} q_1^{\alpha_2} + g_1^{\alpha_2 \rho} q_1^{\alpha_1} - \frac{2}{3} g_1^{\alpha_1 \alpha_2} q_1^{\rho}) (2q_2^{\beta} g_2^{\rho \alpha} + g_2^{\beta \rho}) G_{B_2'B_1}(\omega, \omega') . \quad (66)
$$
where
\[ G_{B_2B_1}(\omega, \omega') = -i\frac{\sqrt{6}}{16} \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega_0} \omega' u(1-u) \left\{ -\frac{it}{4} f\psi(u) + \langle q\bar{q} \rangle \chi\phi(u) + t^2 (g_1(u) - g_2(u)) \right\} \cdots \].

(67)
\[ g_5 f_{e+}^2 = \frac{\sqrt{6}}{16} e^{2\Lambda c/\Phi} u_0(1-u_0) \left\{ f\psi(u_0) + \frac{a}{2\pi^2} \chi\phi(u_0) T f_0(\omega c) - \frac{4}{T} (g_1(u_0) - g_2(u_0)) \right\}. \]

(68)

4 Determination of the parameters

The leading photon wave functions receive only small corrections from the higher conformal spins [9] so they do not deviate much from the asymptotic form. We shall use [4]
\[ \phi(u) = 6u\bar{u}, \]
\[ \psi(u) = 1, \]
\[ g_1(u) = -\frac{1}{8} \bar{u}(3-u), \]
\[ g_2(u) = -\frac{1}{4} \bar{u}^2. \]

(69) \quad (70) \quad (71) \quad (72)

with \( f = 0.028 \) GeV\(^2\) and \( \chi = -4.4 \) GeV\(^2\) [13] at the scale \( \mu = 1 \) GeV. Using this value of \( \chi \), the octet, decuplet and heavy baryon magnetic moments have been calculated to a good accuracy [14, 15, 16].

We need the mass parameters \( \tilde{\Lambda} \)'s and the coupling constants \( f \)'s of the corresponding interpolating currents in the leading order of \( \alpha_s \) as input. The results are [11, 8]
\[ \tilde{\Lambda}_\Lambda = 0.8 \text{ GeV} \quad f_\Lambda = (0.018 \pm 0.002) \text{ GeV}^3, \]
\[ \tilde{\Lambda}_\Sigma = 1.0 \text{ GeV} \quad f_\Sigma = (0.04 \pm 0.004) \text{ GeV}^3. \]

(73)

\[ \tilde{\Lambda}_{+3/2} = 0.82 \text{ GeV} \quad f_{+3/2} = 0.19 \pm 0.03 \text{ GeV}^{5/2}, \]
\[ \tilde{\Lambda}_{+1/2} = 1.1 \text{ GeV} \quad f_{+1/2} = 0.40 \pm 0.06 \text{ GeV}^{3/2}, \]
\[ \tilde{\Lambda}_{-1/2} = 0.5 \text{ GeV} \quad f_{-1/2} = 0.25 \text{ GeV}^{3/2}. \]

(74)

We choose to work at the symmetric point \( T_1 = T_2 = 2T \), i.e., \( u_0 = \frac{1}{2} \). Such a choice is very reasonable for the symmetric sum rules (23), (60), (64) and (68) since \( \Sigma^*_c \) and \( \Sigma_c \), and the three meson doublets are degenerate in the leading order of HQET. Moreover, the mass difference between \( \Sigma^*_c \) and \( \Lambda_c \) is only about 0.2 GeV. The \((0^+, 1^+)\) doublet lies only slightly below \((1^+, 2^+)\) doublet. Due to the large values of \( T_1, T_2 \) used below, the choice of \( T_1 = T_2 \) is also reasonable for sum rules (23) and (56).

Note the choice \( T_1 = T_2 \) is not unique for the asymmetric sum rules (10), (11) and (18) since the initial and final mesons have different masses. But the choice \( T_1 = T_2 \) will enable
the clean subtraction of the continuum contribution, which is crucial for the numerical analysis of the sum rules. In our case the sum rules are stable with reasonable variations of the Borel parameter $T_1$ and $T_2$. Such a choice does not alter significantly the numerical results. Based on these considerations we adopt $u_0 = \frac{1}{2}$ for the sum rules (10), (11) and (13) too.

5 Numerical results and discussion

5.1 Numerical analysis of the baryon sum rules

We now turn to the numerical evaluation of the sum rules for the coupling constants. Since the spectral density of the sum rule (23)-(25) $\rho(s)$ is either proportional to $s^2$ or $s^3$, the continuum has to be subtracted carefully. We use the value of the continuum threshold $\omega_c$ determined from the corresponding mass sum rule at the leading order of $\alpha_s$ and $1/m_Q$ [11].

The lower limit of $T$ is determined by the requirement that the terms of higher twists in the operator expansion is reasonably smaller than the leading twist, say $\leq 1/3$ of the latter. This leads to $T > 1.3$ GeV for the sum rules (23)-(25). In fact the twist-four terms contribute only a few percent to the sum rules. The upper limit of $T$ is constrained by the requirement that the continuum contribution is less than 50%. This corresponds to $T < 2.2$ GeV.

The variation of $\eta_{1,3}$ with the Borel parameter $T$ and $\omega_c$ is presented in FIG. 1 and FIG. 2. The curves correspond to $\omega_c = 2.4, 2.5, 2.6$ GeV from bottom to top respectively. Stability develops for the sum rules (23) and (25) in the region $1.3$ GeV $< T < 2.2$ GeV, we get:

$$\eta_1 f_{\Sigma} f_{\Lambda} = (7.0 \pm 0.9) \times 10^{-4} \text{GeV}^5,$$

$$\eta_3 f_{\Sigma^*} f_{\Sigma} = (3.9 \pm 0.5) \times 10^{-4} \text{GeV}^5,$$

where the errors refer to the variations with $T$ and $\omega_c$ in this region. And the central value corresponds to $T = 1.6$ GeV and $\omega_c = 2.5$ GeV.

Combining (73) we arrive at

$$\eta_1 = (1.0 \pm 0.2) \text{GeV}^{-1},$$

$$\eta_3 = (0.24 \pm 0.05) \text{GeV}^{-1}.$$

5.2 Numerical analysis of the meson sum rules

We now turn to the numerical evaluation of the sum rules for the coupling constants. The lower limit of $T$ is determined by the requirement that the terms of higher twists in the operator expansion is less than one third of the whole sum rule. This leads to $T > 1.0$ GeV for the sum rules (14), (15), (23), (24), (25) and (27). In fact the twist-four terms contribute only a few percent to the sum rules for such $T$ values. The upper limit of $T$ is constrained by the requirement that the continuum contribution is less than 30%. This corresponds to $T < 2.5$ GeV. With the values of photon wave functions at $u_0 = \frac{1}{2}$ we
obtain the left hand side of these sum rules as functions of $T$. The continuum threshold is $\omega_c = 3.0 \pm 0.2\,\text{GeV}$ except $\omega_c = 2.4 \pm 0.2\,\text{GeV}$ for the sum rule (64). Stability develops for the sum rules in the region $1.0\,\text{GeV} < T < 2.5\,\text{GeV}$. The results are shown in FIG. 3-9. Numerically we have:

$$
g_d f_{-\frac{1}{2}\frac{1}{2}} f_{+\frac{3}{2}} = -(3.0 \pm 0.2) \times 10^{-2} \ \text{GeV}^2, \quad (79)$$

$$
g_s f_{-\frac{1}{2}\frac{1}{2}} f_{+\frac{3}{2}} = -(1.9 \pm 0.2) \times 10^{-2} \ \text{GeV}^4, \quad (80)$$

$$
g_1 f_{-\frac{1}{2}\frac{1}{2}} f_{+\frac{3}{2}} = -(1.5 \pm 0.5) \times 10^{-2} \ \text{GeV}^3, \quad (81)$$

$$
g_2 f_{+\frac{1}{2}\frac{3}{2}} f_{+\frac{3}{2}} = -(5.5 \pm 0.4) \times 10^{-2} \ \text{GeV}^3, \quad (82)$$

$$
g_3 f_{+\frac{1}{2}\frac{3}{2}} = (0.28 \pm 0.04) \ \text{GeV}^2, \quad (83)$$

$$
g_4 f_{-\frac{1}{2}\frac{1}{2}} = (8.9 \pm 0.5) \times 10^{-2} \ \text{GeV}^2, \quad (84)$$

$$
g_5 f_{+\frac{3}{2}\frac{3}{2}} = -(2.3 \pm 0.3) \times 10^{-2} \ \text{GeV}^2, \quad (85)$$

where the errors refer to the variations with $T$ in this region and the uncertainty in $\omega_c$. And the central value corresponds to $T = 1.5\,\text{GeV}$ and $\omega_c = 3.0 \pm 0.2\,\text{GeV}$ except that we use $\omega_c = 2.4 \pm 0.2\,\text{GeV}$ for the sum rule (64).

With the central values of f’s in (74) we get the absolute value of the coupling constants:

$$
g_d = -(0.63 \pm 0.10) \ \text{GeV}^{-2}, \quad (86)$$

$$
g_s = -(0.40 \pm 0.05), \quad (87)$$

$$
g_1 = -(0.20 \pm 0.06), \quad (88)$$

$$
g_2 = -(0.72 \pm 0.07) \ \text{GeV}^{-1}, \quad (89)$$

$$
g_3 = (1.8 \pm 0.3) \ \text{GeV}^{-1}, \quad (90)$$

$$
g_4 = (1.4 \pm 0.2) \ \text{GeV}^{-1}, \quad (91)$$

$$
g_5 = -(0.64 \pm 0.08) \ \text{GeV}^{-3}. \quad (92)$$

Note we have only considered the uncertainty due to the variations of the Borel parameter and the continuum threshold in the above expressions. There are other sources of uncertainty. The input parameters $\chi$ and $f$ are associated with the photon distribution amplitude. Especially the value of $\chi$ has been estimated with QCD sum rules [13] and with the octet baryon magnetic moments as inputs using the external field method [14]. Both approaches yield consistent results $\chi \approx -4.4\,\text{GeV}$. With this value the octet, decuplet and heavy baryon magnetic moments derived using the external field method are in good agreement with the experimental data. So we expect its accuracy is better than 30%. The value of $f$ has been estimated with the vector meson dominance model also with an accuracy of 30% [2].

The light cone sum rules for the coupling constants $g_i$ and the mass sum rules for the heavy hadrons in HQET both receive large perturbative QCD corrections. But their ratio does not depend on radiative corrections strongly because of large cancellation [17]. In the present case, the uncertainty of the coupling constants $g_i$ due to radiative corrections is expected to around 10% while the couplings $f_A$ etc are affected significantly.

Another possible source of error is the truncation of the light cone expansion at twist four. We take the sum rules for $\eta_1$ for example. At $T = 1.5\,\text{GeV}$, the twist-four term
involved with \( g_1, g_2 \) is only \(-2.5\%\) of the leading twist term after making double Borel transformation to (20). Even after the subtraction of the continuum and excited states contribution the twist-four term is only \(-15\%\) of the twist-two one in (23). So the light cone expansion converges quickly. We expect the contribution of higher twist distribution amplitudes to be small.

We have calculated the coupling constant in the leading order of HQET. The \( 1/m_Q \) correction is sizable for the charm system. But for the bottom system the \( 1/m_Q \) correction is typically around \( 5\% \sim 10\% \) for the pionic coupling constants \( [8] \). We expect the \( 1/m_Q \) correction to the electromagnetic coupling constants is of the same order. The inherent uncertainty of the method of QCD sum rules is not included, which is typically about \( 10\% \).

### 5.3 Decay widths of heavy hadrons

With these coupling constants we can calculate the decay widths of heavy hadrons.

The decay width formulas in the leading order of HQET are

\[
\Gamma(\Sigma_b \to \Lambda_b \gamma) = 4\eta_1^2 \alpha |\vec{q}|^3, \\
\Gamma(\Sigma_b^* \to \Lambda_b \gamma) = \eta_2^2 \alpha |\vec{q}|^3 \frac{3m_i^2 + m_f^2}{3m_i^2}, \\
\Gamma(\Sigma_b^* \to \Sigma_b \gamma) = \eta_3^2 \alpha |\vec{q}|^3 \frac{3m_i^2 + m_f^2}{3m_i^2}, \\
\Gamma(B_1 \to B^* \gamma) = \frac{2}{3} e_2^2 \alpha (\frac{14}{9} g_d^2 |\vec{q}|^5 + g_s^2 |\vec{q}|^4), \\
\Gamma(B_1 \to B \gamma) = \frac{4}{3} e_2^2 \alpha (\frac{1}{18} g_d^2 |\vec{q}|^5 + g_s^2 |\vec{q}|^4), \\
\Gamma(B_2^* \to B \gamma) = \frac{2}{5} e_2^2 \alpha g_d^2 |\vec{q}|^5, \\
\Gamma(B_2^* \to B^* \gamma) = e_2^2 \alpha (\frac{64}{45} g_d^2 |\vec{q}|^5 + 4g_s^2 |\vec{q}|^4), \\
\Gamma(B_1' \to B^* \gamma) = e_2^2 \alpha g_1^2 |\vec{q}|, \\
\Gamma(B_1' \to B \gamma) = \frac{1}{2} e_2^2 \alpha g_1^2 |\vec{q}|, \\
\Gamma(B_0' \to B^* \gamma) = \frac{3}{2} e_2^2 \alpha g_1^2 |\vec{q}|, \\
\Gamma(B^* \to B \gamma) = \frac{1}{3} e_2^2 \alpha g_1^2 |\vec{q}|^3. 
\]  

(93)

where \( |\vec{q}| = \frac{m_i^2 - m_f^2}{2m_i} \), \( m_i, m_f \) is the parent and decay heavy hadron mass.

We apply the leading order formulas obtained above to the excited states of bottomed hadrons using the central values of the coupling constants in the previous section.

\[
\Gamma(\Sigma_b \to \Lambda_b \gamma) = 131 \times \left( \frac{|\vec{q}|}{165\text{MeV}} \right)^3 \text{keV}, \\
\Gamma(\Sigma_b^* \to \Lambda_b \gamma) = 313 \times \left( \frac{|\vec{q}|}{224\text{MeV}} \right)^3 \text{keV}, \\
\]
\[ \Gamma(\Sigma_b^{*+} \to \Sigma_b^{+}\gamma) = 2.2 \times \left( \frac{|q|}{63.4\text{MeV}} \right)^3 \text{keV}, \]
\[ \Gamma(\Sigma_b^{*0} \to \Sigma_b^{0}\gamma) = 0.14 \times \left( \frac{|q|}{63.4\text{MeV}} \right)^3 \text{keV}, \]
\[ \Gamma(\Sigma_b^{*-} \to \Sigma_b^{-}\gamma) = 0.56 \times \left( \frac{|q|}{63.4\text{MeV}} \right)^3 \text{keV}, \]
\[ \Gamma(B^{*0} \to B^{0}\gamma) = 1.4 \times \left( \frac{|q|}{137\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B^{*+} \to B^{+}\gamma) = 5.5 \times \left( \frac{|q|}{137\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_1^0 \to B^{0}\gamma) = 84.4 \times \left( \frac{|q|}{490\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_1^+ \to B^{+}\gamma) = 338 \times \left( \frac{|q|}{490\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_1^0 \to B^{0*}\gamma) = 42.2 \times \left( \frac{|q|}{377\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_1^+ \to B^{*+}\gamma) = 169 \times \left( \frac{|q|}{377\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_2^{*0} \to B^{0}\gamma) = 5.8 \times \left( \frac{|q|}{537\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_2^{*+} \to B^{+}\gamma) = 23 \times \left( \frac{|q|}{537\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_2^{*0} \to B^{*0}\gamma) = 211 \times \left( \frac{|q|}{408\text{MeV}} \right) \text{keV}, \]
\[ \Gamma(B_2^{*+} \to B^{*+}\gamma) = 844 \times \left( \frac{|q|}{408\text{MeV}} \right) \text{keV}. \] (94)

The uncertainty of the decay width is typically about 30%.

We do not present numerical results for the radiative decay widths for the charmed hadrons since \(1/m_Q\) corrections are sizable for the charm system while such corrections are only a few percent of the leading order term for the bottom system [8].

In summary we have calculated the coupling constants of photons with the heavy baryons and the lowest three heavy meson doublets using the light cone QCD sum rules with the photon wave functions in the leading order of HQET. We hope these calculations will be tested in the future experiments.

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Figure Captions

FIG. 1. Dependence of $f_{\Sigma} f_{\Lambda}\eta_1$ on the Borel parameter $T$ for different values of the continuum threshold $\omega_c$. From top to bottom the curves correspond to $\omega_c = 2.6, 2.5, 2.4$ GeV.

FIG. 2. Dependence of $f_{\Sigma} f_{3\Sigma}\eta_3$ on $T$ with $\omega_c = 2.6, 2.5, 2.4$ GeV.

FIG. 3. Dependence of $g_d f_{-\frac{1}{2}} f_{+\frac{1}{2}}$ on $T$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

FIG. 4. Dependence of $g_s f_{-\frac{1}{2}} f_{+\frac{1}{2}}$ on $T$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

FIG. 5. Dependence of $g_{1} f_{-\frac{1}{2}} f_{+\frac{1}{2}}$ on $T$.

FIG. 6. Dependence of $g_{2} f_{+\frac{5}{2}} f_{+\frac{3}{2}}$ on $T$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

FIG. 7. Dependence of $g_{3} f_{2\frac{3}{2}}^2$ on $T$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

FIG. 8. Dependence of $g_{4} f_{2\frac{3}{2}}$ on $T$ with $\omega_c = 2.6, 2.4, 2.2$ GeV.

FIG. 9. Dependence of $g_{5} f_{2\frac{3}{2}}$ on $T$ with $\omega_c = 3.2, 3.0, 2.8$ GeV.

References

[1] B. Grinstein, Nucl. Phys. B339, 253 (1990);
E. Eichten and B. Hill, Phys. Lett. B234, 511 (1990);
A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. B343, 1 (1990);
F. Hussain, J. G. Körner, K. Schilcher, G. Thompson and Y. L. Wu, Phys. Lett. B249, 295 (1990);
H. Georgi, Phys. Lett. B240, 447 (1990);
J. G. Körner and G. Thompson, Phys. Lett. B264, 185 (1991).

[2] G. Eilam, I. Halperin and R. R. Mendel, Phys. Lett. B 361, 137 (1995).

[3] T. M. Aliev, D. A. Demir, E. Iltan and N. K. Pak, Phys. Rev. D 54, 857 (1996).

[4] A. Ali and V. M. Braun, Phys. Lett. B 359, 223 (1995).

[5] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B 358, 129 (1995).

[6] P. Ball and V. M. Braun, hep-ph/9805422.

[7] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 174, 385, 448, 519 (1979).

[8] Yuan-Ben Dai and Shi-Lin Zhu, Phys. Rev. D 58, 074009 (1998); Euro. Phys. J. C 6, 307 (1999); Shi-Lin Zhu and Yuan-Ben Dai, Phys. Lett. B 429, 72 (1998).
[9] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989); V. M. Braun and I. E. Filyanov, Z. Phys. C 44, 157 (1989); V. M. Braun and I. E. Filyanov, Z. Phys. C 48, 239 (1990); V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D 51, 6177 (1995).

[10] Shi-Lin Zhu, W-Y. P. Hwang and Ze-sen Yang, Phys. Lett. B 420, 8 (1998); Mod. Phys. Lett. A 12, 3027 (1997).

[11] A. G. Grozin and O. I. Yakovlev, Phys. Lett. B285, 254 (1992); S. Groote, J. G. Korner, and O. I. Yakovlev, Phys. Rev. D54, 3447 (1996); D55, 3016 (1997).

[12] Y. B. Dai, C. S. Huang, M. Q. Huang and C. Liu, Phys. Lett. B390, 350 (1997).

[13] V. M. Belyaev and Y. I. Kogan, Yad. Fiz. 40, 1035 (1984); I. I. Balitsky and A. V. Kolesnichenko, Yad. Fiz. 41, 282 (1985).

[14] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232, 109 (1984); I.I.Balitsky and A. V. Yung, Phys. Lett. B 129, 328 (1983).

[15] C. B. Chiu, J. Pasupathy and S. J. Wilson, Phys. Rev. D 33, 1961 (1986).

[16] Shi-Lin Zhu, W-Y. P. Hwang and Ze-sen Yang, Phys. Rev. D 57, 1527 (1998); Phys. Rev. D 56, 7273 (1997).

[17] M. Neubert, Phys. Rep. 245, 259 (1994).
\[ f(T) \]

- \( \omega_c = 2.6 \text{GeV} \)
- \( \omega_c = 2.5 \text{GeV} \)
- \( \omega_c = 2.4 \text{GeV} \)
$g_{f,-1/2f,+3/2}$

\begin{align*}
\omega_c &= 2.8\text{GeV} \\
\omega_c &= 3.0\text{GeV} \\
\omega_c &= 3.2\text{GeV}
\end{align*}
\( c = 2.8 \text{GeV} \)
\( c = 3.0 \text{GeV} \)
\( c = 3.2 \text{GeV} \)
