Universal Seesaw Mass Matrix Model
with Three Light Pseudo-Dirac Neutrinos

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Abstract

A universal seesaw mass matrix model, which gives successful description of
quark mass matrix in terms of lepton masses, yields three “sterile” neutrinos
$\nu^s_i$, which compose pseudo-Dirac neutrinos $\nu^p_{i\pm} \simeq (\nu_i \pm \nu^s_i)/\sqrt{2}$ together with
the active neutrinos $\nu_i$ ($i = e, \mu, \tau$). The solar and atmospheric neutrino
data are explained by the mixings $\nu_e \leftrightarrow \nu^s_e$ and $\nu_\mu \leftrightarrow \nu^s_\mu$, respectively. In
spite of such observations of the large mixing $\sin^2 \theta \simeq 1$ in the disappearance
experiments, effective mixing parameters $\sin^2 2\theta^{\alpha\beta}$ in appearance experiments
$\nu_\alpha \to \nu_\beta$ ($\alpha, \beta = e, \mu, \tau$) are highly suppressed.

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It has, for many years, been a longed-for goal in the particle physics to give a unified description of masses and mixings of quarks and leptons. The most crucial clue to the unified description is in the investigation of the neutrino mass matrix. Recent remarkable progress in the experimental studies of the solar, atmospheric and terrestrial neutrinos has put the realistic investigation of the neutrino mass matrix model within our reach.

As a promising model which gives a unified description of quark and lepton mass matrices, the so-called “universal seesaw” mass matrix model [1] has recently revived. The model has hypothetical fermions $F_i = U_i, D_i, N_i$ and $E_i$ ($i = 1, 2, 3$) in addition to the conventional quarks and leptons $f_i = u_i, d_i, \nu_i, e_i$, where these fermions belong to $f_L = (2, 1), f_R = (1, 2), F_L = (1, 1)$ and $F_R = (1, 1)$ of SU(2)$_L \times$SU(2)$_R$. Note that the neutral leptons $N_i$ are the so-called “sterile” fermions because they are singlets of SU(2)$_L \times$SU(2)$_R$ and do not have U(1)-charges. It has recently been pointed out [2] by one of the authors (YK) that such a model can yield three massless sterile neutrinos $\nu_s^i \equiv (N_{Li} - N_{Ri})/\sqrt{2} [N^c_R \equiv (N_R)^c \equiv C N^T_R]$ in a limit, and the sterile neutrinos $\nu^s_i$ compose pseudo-Dirac neutrinos $\nu^{ps}_{i \pm} \simeq (\nu_i \pm \nu_s^i)/\sqrt{2}$ with the masses $m(\nu^{ps}_{i \pm}) \simeq m(\nu^{ps}_{i -})$ together with the conventional left-handed neutrinos $\nu_i \equiv \nu_{Li}$.

The purposes of the present paper are to investigate an explicit model with three sterile neutrinos within the framework of the universal seesaw mass matrix model and to give numerical predictions for neutrino masses and mixings. The solar [4] and atmospheric [5] data will be explained by the mixings $\nu_e \leftrightarrow \nu^s_e$ and $\nu_\mu \leftrightarrow \nu^s_\mu$, respectively. A similar idea has recently been proposed by Geiser [6], who has introduced three sterile neutrinos phenomenologically. However, in the present model, our sterile neutrinos $\nu^s_i$ come from the sterile fermions $N_i$ in the universal seesaw mass matrix model, so that their masses and mixings are strongly constrained by the parameters determined at the charged lepton masses and quark masses.

The seesaw mechanism was first proposed [7] in order to answer the question of why neutrino masses are so invisibly small. Then, in order to understand that the observed quark and lepton masses are considerably smaller than the electroweak scale $\Lambda_L$, the mechanism
was applied to the quarks [1]: A would-be seesaw mass matrix for \((f, F)\) is expressed as
\[
M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z_L \\ \kappa Z_R & \lambda Y_f \end{pmatrix},
\]
where the matrices \(Z_L, Z_R\) and \(Y_f\) are of the order one. The matrices \(m_L\) and \(m_R\) take universal structures for quarks and leptons. Only the heavy fermion matrix \(M_F\) takes a structure dependent on \(f = u, d, \nu, e\). For the case \(\lambda \gg \kappa \gg 1\), the mass matrix (1) leads to the well-known seesaw expression of the \(3 \times 3\) mass matrix for the fermions \(f\):
\[
M_f \simeq -m_L M_F^{-1} m_R.
\]

However, the observation of the top quark of 1994 [8] once aroused a doubt that the observed fact \(m_t \simeq 180\text{ GeV} \sim \Lambda_L = O(m_L)\) was not able to be understood from the universal seesaw mass matrix scenario, because \(m_t \sim O(m_L)\) means \(M_F^{-1} m_R \sim O(1)\). For this question, a recent study [9,10] has given the answer that the universal seesaw scenario is still applicable to top quark if we put an additional constraint
\[
\det M_F = 0,
\]
on the up-quark sector \((F = U)\). The light quarks \((q_{Li}, q_{Ri})\) acquire the well-known seesaw masses of the order of \(\Lambda_L\Lambda_R/\Lambda_S\) though the heavy fermion masses \(m(Q_{Li}, Q_{Ri}) \sim \Lambda_S [\Lambda_L \equiv m_0 = O(m_L), \Lambda_R \equiv \kappa m_0 = O(m_R), \text{and } \Lambda_S \equiv \lambda m_0 = O(M_F)]\), while the third up-quark \(u_{L3}\) acquires a mass of the order of \(\Lambda_L\) together with the partner \(U_{R3}\) since the seesaw mechanism does not work for the fermions \((u_{L3}, u_{R3})\) because of \(m(U_{L3}, U_{R3}) = 0\). Thus, by regarding the fermion state \((u_{L3}, U_{R3})\) [not \((u_{L3}, u_{R3})\)] as the top quark, we will easily be able to understand why only top quark \(t\) acquires the mass \(m_t \sim O(m_L)\). A suitable choice [1] of the matrix forms of \(Z_L, Z_R\) and \(Y_f\) can give reasonable values of the quark masses and Cabibbo-Kobayashi-Maskawa [11] (CKM) matrix parameters in terms of charged lepton masses.

On the other hand, in the conventional universal seesaw mass matrix model [1], the light neutrino mass matrix \(M_\nu\) is given by [12]
so that the smallness of the neutrino masses \( m_\nu \) is understood by a large value of \( \kappa \equiv \Lambda_R/\Lambda_L \) (\( \kappa \gg 1 \)) because of \( m_\nu \sim \Lambda_L^2/\Lambda_S \sim (1/\kappa)m_{\text{charged lepton}} \). The case (4) can be brought, for example, by assuming that the neutral leptons \( N_{Li} \) and \( N_{Ri} \) which are singlets of \( SU(2)_L \times SU(2)_R \) and have no \( U(1) \) charges acquire Majorana masses \( M_M \sim \Lambda_S \) in addition to the Dirac masses \( M_D \equiv M_N \sim \Lambda_S \) and assuming \( M_M = M_D \) [13]:
\[
M_\nu \simeq -m_L M_N^{-1} m_L^T, \quad (4)
\]

where \( M_\nu \) is the mass matrix which is sandwiched between \((\nu_L^c, \nu_R, N_L^c, N_R)\) and \((\nu_L^c, \nu_R, N_L^c, N_R)^T\), and \( \nu_L^c \equiv (\nu_L)^c \equiv C\nu_L^T \) and so on. The model given in Ref. [13] can readily lead to a large mixing between \( \nu_\mu \) and \( \nu_\tau \) [or \( \nu_e \) and \( \nu_\mu \)] with \( m_1^\nu \ll m_2^\nu \simeq m_3^\nu \) [or \( m_1^\nu \simeq m_2^\nu \ll m_3^\nu \)], so that the model is favorable to the explanation of the atmospheric neutrino data [5] which have suggested a large neutrino mixing \( \nu_\mu \leftrightarrow \nu_\tau \). However, the model is hard to give a simultaneous explanation of the atmospheric [5] and solar [4] neutrino data.

We want to give a simultaneous explanation of the atmospheric and solar neutrino data. The essential idea is as follows [2]: In the present model, there is no quantum number which distinguishes \( N_L \) from \( N_R \), because both fields are \( SU(2)_L \times SU(2)_R \) singlets and do not have \( U(1) \) charges. Therefore, if \( \nu_L \) (\( \nu_R \)) acquire masses \( m_L \) (\( m_R \)) together with the partners \( N_R \) (\( N_L \)), they may also acquire masses \( m'_L \) (\( m'_R \)) together with the partners \( N_L^c \) (\( N_R^c \)). Then, the mass matrix for the neutrino sector is given by
\[
M = \begin{pmatrix}
0 & 0 & m'_L & m_L \\
0 & 0 & m'_R^T & m'_R \\
m'_L^T & m_R & M_M & M_D \\
m'_L^T & m'_R & M_D^T & M_M
\end{pmatrix}, \quad (6)
\]

By rotating the fields \((\nu_L^c, \nu_R, N_L^c, N_R)\) by
\[
R_{34} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}, \tag{7}
\]

the 12 × 12 mass matrix (6) is expressed as
\[
M' = R_{34} M R_{34}^T = \begin{pmatrix}
0 & 0 & \sqrt{2} \Delta_L & \sqrt{2} \bar{m}_L \\
0 & 0 & \sqrt{2} \Delta_R^T & \sqrt{2} \bar{m}_R \\
\sqrt{2} \Delta_L^T & \sqrt{2} \Delta_R & M_M - M_D & 0 \\
\sqrt{2} \bar{m}_L^T & \sqrt{2} \bar{m}_R & 0 & M_M + M_D
\end{pmatrix}, \tag{8}
\]

where we have used \( M_D^T = M_D \), and
\[
2 \Delta_L = m'_L - m_L, \quad 2 \Delta_R = m_R - m'_R, \tag{9}
\]
\[
2 \bar{m}_L = m'_L + m_L, \quad 2 \bar{m}_R = m'_R + m_R. \tag{10}
\]

In the \( N_L \leftrightarrow N_R^c \) symmetric limit, i.e., in the limit of \( m'_L = m_L, m'_R = m_R \) and \( M_M = M_D = M_N \), the mass matrices for the neutrinos \( \nu^s, N^s \) and \( \nu_R \) are given by
\[
M(\nu^s) = 0, \tag{11}
\]
\[
M(N^s) \simeq 2 M_N, \tag{12}
\]
\[
M(\nu_R) \simeq -m_R^T M_N^{-1} m_R, \tag{13}
\]

where \( \nu^s \) and \( N^s \) are defined by
\[
\nu^s_i = \frac{1}{\sqrt{2}} (N_{Li} - N_{Ri}^c), \quad N^s_i = \frac{1}{\sqrt{2}} (N_{Li} + N_{Ri}^c). \tag{14}
\]

Furthermore, for a model with the relation \( m_R \propto m_L \) such as a model given in Ref. [9], we obtain
\[ M(\nu_L) = 0 \, . \quad (15) \]

Only when we assume a sizable difference between \( m'_L \) and \( m_L \) (and also between \( m'_R \) and \( m_R \)), we can obtain visible neutrino masses \( m(\nu_L) \neq 0 \) and \( m(\nu^s) \neq 0 \).

For \( \Delta_L \neq 0 \) and \( \Delta_R \neq 0 \), by using a unitary matrix \( U_{12} \)

\[
U_{12} = \begin{pmatrix}
    C & -S & 0 & 0 \\
    S^\dagger & C^\dagger & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}, \quad (16)
\]

\[ [CC^\dagger + SS^\dagger = 1] \], we can express the mass matrix \( (8) \) as

\[
M'' = U_{12}M'U_{12}^T = \begin{pmatrix}
    0 & 0 & A & 0 \\
    0 & 0 & B & G \\
    A^T & B^T & 0 & 0 \\
    0 & G^T & 0 & 2M_N
\end{pmatrix}, \quad (17)
\]

where we have put \( M_M = M_D \equiv M_N \) and

\[
A = \sqrt{2}(C\Delta_L - S\Delta_T^R), \quad (18)
\]

\[
B = \sqrt{2}(S^\dagger\Delta_L + C^\dagger\Delta_T^R), \quad (19)
\]

\[
G = \sqrt{2}(S^\dagger m_L + C^\dagger m_T^R), \quad (20)
\]

\[
CS^{-1} = m_T^R(m_T^L)^{-1}. \quad (21)
\]

Since \( M_N \sim \Lambda_S \), \( G \sim \Lambda_R \), \( O(B) \ll \Lambda_R \) and \( O(A) \ll \Lambda_L \), by using the seesaw approximation, we obtain the \( 9 \times 9 \) mass matrix for states \((\nu_L, \nu_R, \nu^s)\)

\[
M^{(9)} \approx \begin{pmatrix}
    0 & 0 & A \\
    0 & -G(2M_N)^{-1}G^T & B \\
    A^T & B^T & 0
\end{pmatrix}, \quad (22)
\]
together with the $3 \times 3$ mass matrix (12) for the neutral leptons $N^s$. Furthermore, for $O(GM_N^{-1}G^T) = \Lambda^2_R/\Lambda_S = (\kappa^2/\lambda)m_0 \gg O(A), O(B)$, we obtain the $6 \times 6$ mass matrix for $(\nu_L, \nu^s)$

\[
M^{(6)} \simeq \begin{pmatrix}
0 & A \\
A^T & 2B(G^T)^{-1}M_NG^{-1}B^T
\end{pmatrix},
\]

(23)
together with the $3 \times 3$ mass matrix for the right-handed neutrinos $\nu_R$

\[
M(\nu_R) \simeq -G(2M_N)^{-1}G^T .
\]

(24)

If we choose $O(A) \gg O(B(G^T)^{-1}M_NG^{-1}B^T)$, we obtain three sets of the pseudo-Dirac neutrinos $(\nu_{i+}^{ps}, \nu_{i-}^{ps})$ ($i = e, \mu, \tau$) which are large mixing states between $\nu_i$ and $\nu_i^s$, i.e.,

\[
\nu_{i\pm}^{ps} \simeq \frac{1}{\sqrt{2}}(\nu_i \pm \nu_i^s),
\]

(25)

and whose masses are almost degenerate.

So far, we have not assumed explicit structures of the matrices $Z_L$, $Z_R$ and $Y_f$. Here, in order to give a realistic numerical example, we adopt a model with special forms of $Z_L$, $Z_R$ and $Y_f$ which can lead to successful quark masses and mixings. The model is based on the following working hypotheses [9]:

(i) The matrices $Z_L$ and $Z_R$, which are universal for quarks and leptons, have the same structure:

\[
Z_L = Z_R \equiv Z = \text{diag}(z_1, z_2, z_3),
\]

(26)

with $z_1^2 + z_2^2 + z_3^2 = 1$, where, for convenience, we have taken a basis on which the matrix $Z$ is diagonal.

(ii) The matrices $Y_f$, which have structures dependent on the fermion sectors $f = u, d, \nu, e$, take a simple form $[(\text{unit matrix}) + (\text{a rank one matrix})]$

\[
Y_f = 1 + 3b_fX .
\]

(27)

(iii) The rank one matrix $X$ is given by a democratic form
\[
X = \frac{1}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix},
\] (28)

on the family-basis where the matrix \(Z\) is diagonal.

(iv) In order to fix the parameters \(z_i\), we tentatively take \(b_e = 0\) for the charged lepton sector, so that the parameters \(z_i\) are given by

\[
\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}},
\] (29)

from \(M_e \simeq m_L M_E^{-1} m_R = (\kappa/\lambda) m_0 Z \cdot 1 \cdot Z\).

The mass spectra are essentially characterized by the parameter \(b_f\). We take \(b_u = -1/3\) for up-quark sector, because, at \(b_u = -1/3\), we can obtain the maximal top-quark mass enhancement \(m_t \simeq (1/\sqrt{3}) m_0\), and a successful relation \(m_u/m_e \simeq (3/4)(m_e/m_\mu)\) independently of the value of \(\kappa/\lambda\). The value of \(\kappa/\lambda\) is determined from the observed ratio \(m_c/m_t\) as \(\kappa/\lambda = 0.0198\). Considering the successful relation \(m_d m_s/m_\tau^2 \simeq 4 m_e m_\mu/m_\tau^2\) for \(b_d \simeq -1\), we seek for the best fit point of \(b_d = -e^{i\beta_d} (\beta_d^2 \ll 1)\). The observed ratio \(m_d/m_s\) fixes the value \(\beta_d\) as \(\beta_d = 18^\circ\). Then we can obtain the reasonable quark mass ratios, not only \(m_u^a/m_j^a\), \(m_t^d/m_j^d\), but also \(m_u^a/m_j^d\): \(m_u = 0.00234\) GeV, \(m_c = 0.610\) GeV, \(m_t = 0.181\) GeV, \(m_d = 0.00475\) GeV, \(m_s = 0.0923\) GeV, \(m_b = 3.01\) GeV, where we have taken \((m_0 \kappa/\lambda) q/(m_0 \kappa/\lambda) e = 3.02\) in order to fit the observed quark mass values at \(\mu = m_Z\) [14]. We also obtain the reasonable values of the CKM matrix parameters: \(|V_{us}| = 0.220\), \(|V_{cb}| = 0.0598\), \(|V_{ub}| = 0.00330\), \(|V_{td}| = 0.0155\). (The value of \(|V_{cb}|\) is somewhat larger than the observed value. For the improvement of the numerical value, see Ref. [15].)

Stimulated by this phenomenological success in the present model for the quark masses and mixings, we apply the model with (26) - (28) to the neutrino mass matrix (6). For simplicity, we consider the case

\[
m_L' = (1 - \varepsilon_L)m_L, \quad m_R' = (1 - \varepsilon_R)m_R.
\] (30)

Then, we obtain
A \simeq \frac{1}{\sqrt{2}} m_0 (\varepsilon_R + \varepsilon_L - \varepsilon_L \varepsilon_R) Z , \quad (31)

B \simeq \frac{1}{\sqrt{2}} m_0 \kappa \left( \varepsilon_R - \frac{1}{\kappa^2} \varepsilon_L \right) Z , \quad (32)

G \simeq \sqrt{2} m_0 \kappa \left( 1 - \frac{1}{2} \varepsilon_R + \frac{1}{\kappa^2} \right) Z , \quad (33)

M_N = m_0 \lambda (1 + 3 b_\nu X) . \quad (34)

What is of great interest to us is to evaluate the masses of the pseudo-Dirac neutrino states, whose mass matrix is given by (23):

\[ M^{(6)} \simeq \frac{1}{\sqrt{2}} m_0 (\varepsilon_R + \varepsilon_L) \begin{pmatrix} 0 & Z \\ Z & \rho Y_\nu \end{pmatrix} , \quad (35) \]

where

\[ \rho \simeq \frac{1}{\sqrt{2}} \frac{\varepsilon_R^2}{\varepsilon_R + \varepsilon_L} \lambda . \quad (36) \]

For \( Z \gg |\rho Y_\nu| \), i.e., \( z_i \gg |\rho (1 + b_\nu)| \), we obtain the masses \( m(\nu^{ps}_{i\pm}) \) of the pseudo-Dirac neutrinos \( \nu^{ps}_{i\pm} \)

\[ m(\nu^{ps}_{i\pm}) \simeq \frac{1}{\sqrt{2}} m_0 (\varepsilon_R + \varepsilon_L) \left( z_i \pm \rho \frac{1 + b_\nu}{2} \right) . \quad (37) \]

Here, the double sign, \( \pm \), in the right-hand side of (37) does not always correspond to the double sign of \( \nu^{ps}_{i\pm} \) in the left-hand side. For the mass eigenstates \( \nu^{ps}_{i\pm} \) defined by (25), the order of the magnitudes of the mass eigenvalues are \( m(\nu^{ps}_{e-}) < m(\nu^{ps}_{e+}) < m(\nu^{ps}_{\mu+}) < m(\nu^{ps}_{\mu-}) < m(\nu^{ps}_{\tau+}) < m(\nu^{ps}_{\tau-}) \) (see Fig. 1). On the other hand, the squared mass differences \( \Delta m^2(\nu^{ps}_{i\pm}) = |m^2(\nu^{ps}_{i+}) - m^2(\nu^{ps}_{i-})| \) are given by

\[ \Delta m^2(\nu^{ps}_{i\pm}) \simeq m_0^2 (\varepsilon_R + \varepsilon_L)^2 \rho |z_i| \left[ (1 + b_\nu) + \rho^2 b_\nu^2 2b_\nu z_i^2 + (1 + b_\nu) (2z_i^2 - z_j^2 - z_k^2) \right] \left( z_i^2 - z_j^2 \right) \left( z_i^2 - z_k^2 \right) , \quad (38) \]

where \((i, j, k)\) is a cyclic permutation of \((i, j, k) = (1, 2, 3)\).

As the numerical input, we use
\( \left( m_0 \frac{\kappa}{\lambda} \right)_\ell \equiv \left( m_0 \frac{\kappa}{\lambda} \right)_e = \text{Tr} M_e = (m_\tau + m_\mu + m_e)_{\mu=m_Z} = 1.8499 \text{ GeV} \), \hspace{1cm} (39)

\( (\kappa/\lambda)_\ell \equiv (\kappa/\lambda)_q = 0.02 \), \hspace{1cm} (40)

so that

\[ m_0 = 0.925 \times 10^{11} \text{ eV} . \] \hspace{1cm} (41)

For simplicity, we seek for the solutions for the case \( \varepsilon_L = \varepsilon_R \equiv \varepsilon \), so that the parameters in the neutrino masses and mixings are three, i.e., \( \varepsilon, b_\nu \) and \( \lambda \). We tentatively choose \( m(\nu_{\tau \pm}^{ps}) \simeq 5 \text{ eV} \), which is suggested from a cosmological model with cold+hot dark matter (CHDM) \[13\]. Then, for \( b_\nu \simeq -1 \), the parameter value \( \varepsilon \) is given from (37) as

\[ \varepsilon \equiv \varepsilon_L = \varepsilon_R \simeq 3.8 \times 10^{-11} . \] \hspace{1cm} (42)

The recent atmospheric data from Super-Kamiokande \[17\] have suggested the neutrino oscillation \( \nu_\mu \to \nu_\tau \) with \( \Delta m^2 \simeq 2.2 \times 10^{-3} \text{ eV}^2 \) and \( \sin^2 2\theta \simeq 1 \). Since the value of \( \Delta m^2 \) in the case of \( \nu_\mu \to \nu_\mu^s \) is given by \( \Delta m^2(\nu_\mu \to \nu_\mu^s) \simeq 2 \times \Delta m^2(\nu_\mu \to \nu_\tau) \) \[18\], we put \( \Delta m^2(\nu_\mu^{ps}) = \Delta m^2_{\text{atm}} \simeq 5 \times 10^{-3} \text{ eV}^2 \). Then, from (38), for \( b_\nu \simeq -1 \), we obtain

\[ \rho \simeq 0.057 , \quad \lambda \simeq 4.3 \times 10^9 \left( \kappa \simeq 8.5 \times 10^7 \right) . \] \hspace{1cm} (43)

The numerical results of the masses \( m(\nu_i^{ps}) \equiv (m(\nu_i^{ps}) + m(\nu_i^{ps}))/2 \) for the case of \( b_\nu \simeq -1 \) are as follows:

\[ m(\nu_e^{ps}) \simeq 0.078 \text{ eV} , \quad m(\nu_\mu^{ps}) \simeq 1.2 \text{ eV} , \quad m(\nu_\tau^{ps}) \simeq 4.8 \text{ eV} . \] \hspace{1cm} (44)

The value of \( \Delta m^2(\nu_e^{ps}) \) is highly sensitive to the parameter \( b_\nu \). The large-angle Mikheyev-Smirnov-Wolfenstein (MSW) solutions \[18\] of solar neutrino problem, \( (\Delta m^2, \sin^2 2\theta) \simeq (10^{-5} \text{ eV}^2, 0.6) \), is ruled out for the case \( \nu_e \to \nu_e^s \) \[20\]. Therefore, we use the vacuum solution \( (\Delta m^2, \sin^2 2\theta) \simeq (0.6 - 0.8 \times 10^{-10} \text{ eV}^2, 1) \) \[21\] in order to fix the parameter \( b_\nu \). For example, the value \( b_\nu + 1 = 3.078 \times 10^{-5} \) gives \( (\Delta m^2, \sin^2 2\theta) \simeq (0.605 \times 10^{-5} \text{ eV}^2, 0.9997) \), which is safely in the allowed region obtained by Faid et al. \[22\] by using the solar neutrino
data from Super-Kamiokande and Borexino. In Table I, the neutrino oscillation parameters $\Delta m_{ij}^2$ and $\sin^2 2\theta_{ij}^{\alpha\beta}$ are listed. Here, the neutrino oscillation $P(\nu_\alpha \rightarrow \nu_\beta)$ is given by

$$P(\nu_\alpha \rightarrow \nu_\beta(\nu_\beta^\dagger)) \simeq \sum_{i=e,\mu,\tau} \sin^2 2\theta_{ii}^{\alpha\beta} \sin^2 \frac{L\Delta m_{ii}^2}{4E} + \sum_{i>j} \sin^2 2\theta_{ij}^{\alpha\beta} \sin^2 \frac{L\Delta m_{ij}^2}{4E},$$

(45)

where $\Delta m^2(\nu_i^{\pm}) = |m^2(\nu_i^{\pm}) - m^2(\nu_j^{\pm})|$, $\Delta m_{ij}^2 = m^2(\nu_i^{\pm}) - m^2(\nu_j^{\pm})$,

$$\sin^2 2\theta_{ii}^{\alpha\beta} = -4\text{Re}(U_{\alpha,i}^* U_{\alpha,i}^* U_{\beta,i+} U_{\beta,i-})$$

and we have used the approximation $m^2(\nu_i^{\pm}) - m^2(\nu_j^{\pm}) \simeq m^2(\nu_i^{\pm}) - m^2(\nu_j^{\pm}) \simeq m^2(\nu_i^{\pm}) - m^2(\nu_j^{\pm}) \simeq m^2(\nu_i^{\pm}) - m^2(\nu_j^{\pm}) \simeq \Delta m_{ij}^2 (i, j = e, \mu, \tau)$. Also, since $\Delta m_{\tau\mu}^2 \gg \Delta m_{\mu e}^2$, i.e., $\Delta m_{\tau\mu}^2 \simeq \Delta m_{\tau e}^2$, in Table I, we have denoted the sum $\sin^2 2\theta_{\tau\beta}^{\alpha\beta} + \sin^2 2\theta_{\tau\mu}^{\alpha\beta}$ instead of $\sin^2 2\theta_{\tau\mu}^{\alpha\beta}$ and $\sin^2 2\theta_{\tau e}^{\alpha\beta}$ as the effective mixing parameter $\sin^2 2\theta$ for $\Delta m_{\tau\mu}^2$.

As seen in Table I, at $\Delta m^2 \simeq 0.605 \times 10^{-10}$ eV$^2$, the oscillation $\nu_e \rightarrow \nu_x$ is caused not only by $\nu_e \rightarrow \nu^s_e$ ($\sin^2 2\theta_{ee}^{\mu e} = 0.941$) but also by $\nu_e \rightarrow \nu_\mu$ ($\sin^2 2\theta_{ee}^{\mu e} = 0.055$). The small parameter $b_\nu + 1$ is insensitive to the numerical results except for $\Delta m^2(\nu^s_e)$. For example, by taking $b_\nu + 1 = 2.8 \times 10^{-4}$, we obtain $\Delta m^2(\nu^s_e) = 1.0 \times 10^{-5}$ eV$^2$, while the other numerical results, $\Delta m^2$ and $\sin^2 2\theta$, are almost unchanged. The value $\sin^2 2\theta_{ee}^{\mu e} = 0.055$ at $\Delta m^2 \simeq 10^{-5}$ eV$^2$ is too large compared with the small angle solution $\sin^2 2\theta \simeq 7 \times 10^{-3}$ in the MSW solutions [19,20], we cannot accommodate the present model to the MSW solutions.

Thus, the model can explain the solar and atmospheric neutrino data by the large mixings $\nu_e \leftrightarrow \nu^s_e$ and $\nu_\mu \leftrightarrow \nu^s_\mu$, respectively. On the other hand, in the appearance experiments $\nu_\alpha \rightarrow \nu_\beta$ ($\alpha, \beta = e, \mu, \tau$), the effective mixing parameters $\sin^2 2\theta_{ij}^{\alpha\beta}$ defined by (47) are highly suppressed because of the cancellation between $\nu_{ij}^{ps}$ and $\nu_{ij}^{ps}$. Therefore, in spite of a suitable value of $\Delta m^2$, $\Delta m_{ee}^2 \simeq 1.4$ eV$^2$, the present model cannot explain the neutrino oscillation $\nu_\mu \rightarrow \nu_e$ experiment by the liquid scintillator neutrino detector (LSND) [21] at Los Alamos because our prediction is $\sin^2 2\theta_{ee}^{\mu e} = 1.7 \times 10^{-7}$, although Geiser has explained the LSND data by the oscillation $\nu_e \leftrightarrow \nu_\mu$ in his model with three sterile neutrinos [1]. The effective
mixing angles $\sin^2 2\theta^{\mu\tau} = 3 \times 10^{-6}$ at $\Delta m^2 \simeq 22$ eV$^2$ and $\sin^2 2\theta^{\mu\tau} = 0.0008$ at $\Delta m^2 \simeq 1.4$ eV$^2$ are too small to be detected by CHORUS [24] and NOMAD [25] experiments at CERN. The effective electron neutrino mass $\langle m_\nu \rangle$ for the case in Table I is also suppressed:

$$\langle m_\nu \rangle = \left| \sum_{i=1}^{6} m_i^\nu U^2_{ei} \right| = 2.8 \times 10^{-6} \text{ eV}$$

which is safely small compared with the upper bound on $\langle m_\nu \rangle$ from the neutrino double $\beta$ decays $\langle m_\nu \rangle < 0.68$ eV [26].

The numerical results in Table I should not be taken rigidly. These values are sensitive to the parameter values used here, especially the value $\Delta m^2(\nu^p_e)$ is to the parameter $(b_\nu + 1)$. The values are only an example. We can give any values as far as these values are within the similar orders. Furthermore, we can also obtain similar results by using another parametrization

$$m'_L = m_{L} e^{i\varepsilon_L}, \quad m'_R = m_{R} e^{i\varepsilon_R},$$

instead of (30) (the numerical values of $\varepsilon_L$ and $\varepsilon_R$ are different from those in the parametrization (30)).

Generally, a model with sterile neutrinos is stringently constrained by the big bang nucleosynthesis (BBN) scenario. However, it has recently been pointed out by Foot and Volkas [27] that the stringent bounds can considerably be reduced by the lepton number asymmetry in the early Universe. Our pseudo-Dirac neutrinos $\nu^p_{\tau\pm}$ and $\nu^p_{\mu\pm}$ have masses greater than 1 eV, and the squared mass difference $\Delta m^2_{\tau\mu}$ is $\Delta m^2_{\tau\mu} \simeq 22$ eV$^2$. The mixing angles except those for $\nu_i \leftrightarrow \nu_i^s$ are sufficiently small. Therefore, the present model does not spoil the BBN scenario.

In conclusion, we have discussed a possible neutrino-mass-generation scenario within the framework of the universal seesaw mechanism, especially, on the basis of a model [2] with special matrix forms of $m_L$, $m_R$ and $M_F$, which can answer the question why only top quark $t$ acquires the mass of the order of the electroweak scale $\Lambda_L = O(m_L)$, and can give reasonable quark masses and mixings in terms of the charged lepton masses. The model
provides three sets of the light pseudo-Dirac neutrinos \((\nu^p_{i\pm}, \nu^p_{i-})\) \((i = e, \mu, \tau)\) are mixing states \(\nu^p_{i\pm} \simeq (\nu_i \pm \nu^s_i)/\sqrt{2}\) between the conventional left-handed neutrinos \(\nu_i\) and sterile neutrinos \(\nu^s_i \equiv (N^R_i - N^L_i)/\sqrt{2}\). The mixings \(\nu_e \leftrightarrow \nu^s_e\) and \(\nu_\mu \leftrightarrow \nu^s_\mu\) can give reasonable explanations of the solar and atmospheric neutrino data, respectively, with \(m(\nu^p_{\tau\pm}) \simeq 5\ eV\). Note that in spite of such a large mixing in a disappearance experiment, the effective mixing parameters \(\sin^2 2\theta^{ij}_{13}\) in the appearance experiments \(\nu_\alpha \rightarrow \nu_\beta\) are highly suppressed.

This is the most remarkable feature in the model with the pseudo-Dirac neutrinos. If future appearance experiments \(\nu_\mu \rightarrow \nu_\tau\) at MINOS [28], ICARUS [29] and K2K [30] rule out the region \((\Delta m^2, \sin^2 2\theta) \simeq (10^{-3} - 10^{-2} \ eV^2, 0.8 - 1)\) which is suggested by the atmospheric data, the present scenario with the three sterile neutrinos \(\nu^s_i\) will become an promising candidate of the possible interpretation of the discrepancy.

At present, except for the LSND data, there is no positive motivation for such a model with three sterile neutrinos. Since the recent KARMEN data [31] seem to disfavor the LSND solution \((\Delta m^2, \sin^2 2\theta) = (0.3 \ eV^2, 0.04) - (2 \ eV^2, 0.002)\), the motivation of the model with the sterile neutrinos seems to disappear. However, the present model with three sterile neutrinos is motivated not on the phenomenological interests in the neutrino data, but on the phenomenological success [4,13] of the universal seesaw mass matrix model for quarks. As far as the model is based on the SU(2)_L × SU(2)_R × U(1)_Y symmetry, the model includes the sterile fermions \(N_i\), so that we inevitably have three sterile neutrinos \(\nu^s_i\) in the model. If we do not want such the light sterile neutrinos \(\nu^s_i\), we must consider an additional mechanism which guarantees \(m'_{L} = m'_{R} = 0\).

The present scenario can bring fruitful phenomenology into the neutrino physics, but there are many adjustable parameters in the present model, and those parameters have been determined by hand and by way of trial. In the present scenario, the active neutrinos \(\nu_i\) and sterile neutrinos \(\nu^s_i\) are massless in the limit of \(N^L_i \leftrightarrow N^R_i\) symmetry, and the small neutrino masses are induced by the violation of the \(N^L_i \leftrightarrow N^R_i\) symmetry, i.e., by the parameters \(\varepsilon_L \neq 0\) and \(\varepsilon_R \neq 0\). In other words, the neutrino masses are originated in “non-seesaw” mechanism in spite of the scenario in the “seesaw” model. We consider that the deviation between
$m'_L$ ($m'_R$) and $m_L$ ($m_R$) can come from an asymmetric evolution of the Yukawa coupling constants from a unification scale $\mu = \Lambda_X$ to the electroweak scale $\mu = \Lambda_L$. However, whether this conjecture is reasonable or not is open question at present.

Note added. After completion of this manuscript the authors became aware of a paper by Boweres and Volkas (hep-ph/9804310), in which a neutrino mass matrix model with three pseudo-Dirac neutrinos has also been proposed within the framework of the universal seesaw mass matrix model. They have derived the light pseudo-Dirac neutrinos in their model with $m'_L = m'_R = 0$. Their mass matrix resembles the model (5), but they assumed that $O(M_M) \ll O(M_D)$. Even if we accept the assumption $O(M_M) \ll O(M_D)$, their idea is not applicable to our model, because their light pseudo-Dirac neutrinos have masses of the order of $m_L M_D^{-1} m_R$ in our scheme, so that the pseudo-Dirac neutrinos will acquire the masses of the same order as the charged leptons and light quarks have. However, it is noticeable that the model with $m'_L = m'_R = 0$ can also lead to three pseudo-Dirac neutrinos.

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TABLE I. Neutrino oscillation parameters for the case: $b_\nu + 1 = 3.078 \times 10^{-5}$, $\varepsilon_L = \varepsilon_R = 3.79 \times 10^{-11}$, and $\lambda = 4.25 \times 10^9$ ($\kappa = 8.50 \times 10^7$).

| $\nu_\alpha \rightarrow \nu_\beta$ | $\Delta m^2(\nu^e_e) = 0.605 \times 10^{-10}$ eV$^2$ | $\Delta m^2(\nu^s_\mu) = 4.97 \times 10^{-3}$ eV$^2$ | $\Delta m^2(\nu^s_\tau) = 1.98 \times 10^{-2}$ eV$^2$ | $\Delta m^2_{\mu e} = 1.43$ eV$^2$ | $\Delta m^2_{\tau \mu} = 21.9$ eV$^2$ |
|---------------------------------|------------------|------------------|------------------|------------------|------------------|
| $\nu_e \rightarrow \nu_e$       | $\sin^2 2\theta^e_{ee} = 0.941$ | $\sin^2 2\theta^e_{\mu\mu} = 1.4 \times 10^{-5}$ | $\sin^2 2\theta^e_{\tau\tau} = 3.2 \times 10^{-9}$ | $\sin^2 2\theta^e_{\mu\tau} = 6.1 \times 10^{-8}$ | $\sin^2 2\theta^e_{e\tau} = -8.6 \times 10^{-10}$ |
| $\nu_\mu \rightarrow \nu_\mu$   | $1.4 \times 10^{-5}$ | $0.940$ | $6.7 \times 10^{-7}$ | $-3.4 \times 10^{-9}$ | $5.9 \times 10^{-8}$ |
| $\nu_\tau \rightarrow \nu_\tau$ | $2.7 \times 10^{-9}$ | $8.2 \times 10^{-7}$ | $0.992$ | $3.1 \times 10^{-9}$ | $8.9 \times 10^{-7}$ |
| $\nu_e \rightarrow \nu_\mu$     | $0.055$ | $0.00023$ | $-2.1 \times 10^{-10}$ | $1.7 \times 10^{-7}$ | $1.1 \times 10^{-9}$ |
| $\nu_\mu \rightarrow \nu_\tau$  | $-1.8 \times 10^{-5}$ | $0.0034$ | $0.00022$ | $0.00076$ | $3.0 \times 10^{-6}$ |
| $\nu_\tau \rightarrow \nu_\tau$ | $0.0032$ | $-9.0 \times 10^{-7}$ | $9.1 \times 10^{-7}$ | $3.6 \times 10^{-6}$ | $1.4 \times 10^{-8}$ |
| $\nu_e \rightarrow \nu_e$       | $0.9997$ | $6.2 \times 10^{-8}$ | $9.0 \times 10^{-13}$ | $0.00099$ | $3.8 \times 10^{-6}$ |
| $\nu_\mu \rightarrow \nu_\mu$   | $0.0031$ | $0.892$ | $5.1 \times 10^{-8}$ | $0.210$ | $0.00091$ |
| $\nu_\tau \rightarrow \nu_\tau$ | $1.0 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $0.986$ | $5.3 \times 10^{-5}$ | $0.029$ |
FIGURES

FIG. 1. Mass spectrum of the pseudo-Dirac neutrinos $\nu_{i\pm}^{ps}$. The parameter values are the same as those in Table I. Boxes correspond to different mass eigenstates. The sizes of different regions in the boxes determine flavors of mass eigenstates, $|U_{ai}|^2$. White and black regions correspond to the active and sterile flavors, respectively.

\[
\begin{align*}
\nu_6 &= \nu_{\tau+}^{ps} & m(\nu_{i\pm}^{ps}) & \Delta m_{ij}^2 \\
\nu_5 &= \nu_{\tau-}^{ps} & 4.84 \text{ eV} & 0.020 \text{ eV}^2 \\
\nu_4 &= \nu_{\mu-}^{ps} & 21.9 \text{ eV}^2 & \nu_3 = \nu_{\mu+}^{ps} & 1.20 \text{ eV} & 0.005 \text{ eV}^2 \\
\nu_2 &= \nu_{e+}^{ps} & 1.43 \text{ eV}^2 & \nu_1 = \nu_{e-}^{ps} & 0.078 \text{ eV} & 6 \times 10^{-11} \text{ eV}^2
\end{align*}
\]