Internal consistency of the most sensitive electroweak measurements within the standard model framework is examined. Confirming an earlier observation on the separation of $Z$-pole asymmetry measurements into hadronisation-free and hadronisation-sensitive, the electroweak mixing angle derived using the former is in perfect agreement with the precision W mass. These two complimentary measurements of weak radiative corrections, when combined with the lower limit on Higgs mass, are incompatible with the measured top quark mass. To overcome this inconsistency, a scenario readily testable in Run-II at Tevatron is envisaged: an upward shift of the top quark mass by about 10 GeV ($\sim 2\sigma$). If, however, the improved top quark mass remains at its current value or the lower limit on Higgs mass moves up substantially, then abandoning the SM may become inevitable.
Over the last several years the Standard Model (SM) of electroweak interactions has been subjected to precision tests through a variety of measurements at LEP, SLC and Tevatron. While the measurements at Z pole are sensitive to the presence of top quark and Higgs boson via weak radiative corrections, the direct measurement of top quark mass at Tevatron and W boson mass at LEP2 and Tevatron, along with the lower bound on Higgs mass at LEP2, provide a detailed set to test the validity of SM. These measurements, which are now final or almost final, have reached a precision where it is possible to verify their self-consistency within the SM framework. In case of serious inconsistency either this framework collapses or the data have to be re-looked. In this paper we examine the self-consistency of these data and discuss their implications.

The important weak radiative corrections that affect \( Z \rightarrow f \bar{f} \) (\( f \): fermion) widths and asymmetries \(^1\) appear as propagator correction, \( \Delta \rho \), sensitive to both top and Higgs masses, and \( Z \rightarrow b \bar{b} \) width where there is an additional vertex correction, \( \delta_{vtb} \), sensitive only to top quark mass. All those measurements where ratio of widths are involved, like \( R_\ell \) and \( \sigma_{\text{peak}} \), the \( \Delta \rho \) sensitivity is practically lost and whatever top dependence one sees is due to indirect effect of \( \delta_{vtb} \) in \( Z \rightarrow b \bar{b} \) width. Thus the only sensitive measurements to test \( \Delta \rho \) are fermion asymmetries leading to effective weak mixing angle\(^2\) \( \sin^2 \theta_{\text{lept eff}} \), and the total Z decay width, \( \Gamma_Z \). The ratio \( R_b = (Z \rightarrow b\bar{b}/Z \rightarrow \text{hadrons}) \) is a clean measure of \( \delta_{vtb} \) and practically independent of the Higgs mass. We will restrict ourselves to only those measurements whose sensitivity for weak radiative effects is well above the measurement uncertainties. On the Z pole these are \( \sin^2 \theta_{\text{lept eff}} \), \( R_b \) and \( \Gamma_z \). The first two are free from \( \alpha_s \) uncertainty, the last one requires \( \alpha_s \) for its interpretation.

Observations on the measurements of asymmetries and hence \( \sin^2 \theta_{\text{lept eff}} \) over the years have shown a clear pattern and several years ago it was pointed out \(^2\) that all asymmetry measurements should be placed in two classes. Class A measurements where hadronisation effects are not relevant for the final result and class B measurements where hadronisation effects cannot be avoided and can only be corrected with whatever understanding of these phenomena we have. In class A measurements are forward-backward asymmetry of leptons \( e, \mu, \tau \) at LEP and left-right asymmetry at SLC. In class B are measurements of all quark asymmetries as well as \( \tau \) polarisation asymmetry measured through hadronic decays. The two classes of measurements were found to be more than 3 \( \sigma \) apart already in 1997 \(^2\). It was also pointed out that the measured W mass was in favour of class A measurements, though W mass was much less precise than now. With increased precision of the W mass with time, its consistency with class A asymmetry measurements and incompatibility with class B measurements has become even more pronounced.

Meanwhile the lower limit on Higgs mass from direct searches at LEP has increased considerably: it is now 114.1 GeV \(^3\) (with even a \( \sim 2\sigma \) hint of Higgs at \( \sim 115 \) GeV). This increase in lower bound on Higgs mass along with the improvement in W mass helps significantly in testing the self-consistency of data. The latest compilation of all the electroweak measurements from LEP, SLC and Tevatron is taken from \(^4\). With the above mentioned classification of asymmetry data, the average \( \sin^2 \theta_{\text{lept eff}} \) for the two classes of measurements from LEP and SLC are: \( 0.23098 \pm 0.00023 \) for class A (hadronisation-free: SLD-dominated) and \( 0.23206 \pm 0.00024 \) for class B (hadronisation-sensitive: LEP-

\(^1\)In terms of vector and axial vector couplings \( g_{Vf}, g_{Af} \), the definition of the effective weak mixing angle is \( \sin^2 \theta_{\text{lept eff}} = \frac{1}{4}(1 - g_V/g_A) \)
The simplest consistency test of data is provided by comparison of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ measurements with the $W$ boson mass, as both are affected by weak radiative effects arising from top and Higgs and provide a complementary measure of the same weak corrections. These measurements are compared in figure 1. This figure clearly illustrates the difference between the Class A and Class B measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ as well as a strong preference for Class A due to pretty precise $W$ mass. Thus, as already emphasised earlier\cite{2}, for

\cite{3} The discrepancy is mainly driven by two most precise measurements. The $b$ quark forward-backward asymmetry, $A_{\text{FB}}^b$, at LEP and the left-right asymmetry, $A_{\text{LR}}$, at SLC. Since $A_{\text{FB}}^b$ is pretty insensitive to vertex corrections, any new physics explanation is unlikely to resolve such large discrepancy. More practical approach would be to treat the discrepancy due to either systematic problem\cite{4} or strong statistical fluctuation\cite{4}.
As the next step we study the consistency of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ or $M_W$ with that of the measured top quark mass. This is shown in figure 2. One sees explicitly the effect of the recent LEP lower limit on the Higgs mass: it is clear that for $M_{\text{Higgs}} > 114.1$ GeV the present value of top quark mass is not sufficient to drive the radiative corrections to explain either $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ or $M_W$. It seems impossible to satisfy all the measurement constraints, $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, $M_W$, $M_{\text{top}}$ and $M_{\text{Higgs}} > 114.1$ GeV simultaneously within the SM framework. One has to make a choice: either one goes beyond the scope of the SM and formulates a model in which all the existing data can be explained, or one can examine the data critically and see if a reasonable change in one of the measurements would remove the discrepancy within the SM. There have been recent approaches using the first option [3]. In this paper we explore the second alternative. We note, firstly, that two precise and independent measurements, $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ and $M_W$, are self-consistent within the SM framework. Secondly, in relative terms $M_{\text{top}}$ is presently the least accurately determined data point, being determined to an accuracy of 2.9%, whereas $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ and $M_W$ are determined to 0.1% and 0.04% respectively. Thus the minimal change in data which may make it consistent within the SM is an upward movement of $M_{\text{top}}$. We now try to estimate how much change in it is required for everything to fit together.

To arrive at a quantitative estimate of required increase in $M_{\text{top}}$ to explain the data, we
have taken the most sensitive and clean measurements that are indicative of weak radiative effects. These are the $\sin^2 \theta_{\text{eff}} = 0.23098 \pm 0.00023$ from class A (hadronisation-free) measurements, $R_b = 0.21646 \pm 0.00065$ and $\Gamma_Z = 2.4952 \pm 0.0023$ GeV at the Z pole, $M_W = 80.451 \pm 0.033$ GeV and the lower bound of 114.1 GeV on the Higgs mass. All in association with the $M_Z = 91.1875 \pm 0.0021$ GeV, $\alpha(M_Z) = 1/(128.945 \pm 0.052)$ and $\alpha_s(M_Z) = 0.1181 \pm 0.0020$ \cite{8} as inputs and ZFITTER (version 6.36) \cite{9} as the SM package. A fit to these data is performed to extract top quark mass for various Higgs masses in an extended range, from 10 - 1000 GeV. Fit results for a few representative values of the Higgs mass are given in table \ref{table1} and in figure \ref{fig3} the variation of the top quark mass is shown as a function of the Higgs mass for which the $\chi^2$ is minimum. The band indicated as the Higgs-top SM band along with the central line in figure \ref{fig3} indicates the best line with 1σ fit error where weak radiative effects balance each-other to get the minimum $\chi^2$. The band with central line parallel to Higgs-axis indicates the measured top quark mass with its errors. The broad band parallel to top-axis is the lower bound on

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{$M_{\text{top}}$ as function of Higgs mass for which fit to data give minimum $\chi^2$. The band Higgs-top SM band corresponds to 1σ fit uncertainty and the central line as the best value. The measured top mass (174.3±5.1 GeV) is depicted as a band parallel to Higgs axis. The Higgs mass excluded region below 114.1 GeV is also indicated.}
\end{figure}

\footnote{This corresponds to the best measured value of $\Delta \alpha_{\text{had}}^{(5)}(M_Z) = 0.02761 \pm 0.00036$ as determined using BES collaboration data \cite{6,7}.}
Table 1: Result of the fit using data most sensitive to radiative corrections. The Higgs mass has been varied starting from the lower bound and top mass has been extracted.

| Fit Parameter | $M_{\text{Higgs}}$ (GeV) |
|---------------|--------------------------|
|               | 114.1 | 200 | 300 | 450 | 1000 |
| $M_{\text{top}}$ (GeV) | 184 ± 4 | 190 ± 4 | 195 ± 4 | 200 ± 4 | 211 ± 4 |
| $\chi^2/\text{DOF}$ (Prob %) | 4.3/4 | 6.0/4 | 7.5 | 9.2/4 | 13.8/4 |

Higgs from LEP2 searches [3]. Given the Higgs-bound of 114.1 GeV, the only reasonable option to gain self-consistency of data within the SM is to move top quark mass up by about 10 GeV to $\sim$184 GeV. This represents a $2\sigma$ shift upward from the current value of 174.3±5.1 GeV, which does not seem dramatic. However, this would suffice only if the Higgs is indeed around the corner. In case the lower bound on the Higgs mass increases substantially beyond the current value, the change in $M_{\text{top}}$ required for SM consistency of data would be much more and would become increasingly unrealistic. In such a scenario the SM would really have to be abandoned. Indeed, the variation of $\chi^2$ with Higgs mass in table 1 indicates that data favour a light Higgs.

Run-II at the Tevatron will play a crucial role in clarifying the situation: the error on the top mass is expected to be reduced considerably and on the W mass to some extent. The lower bound on Higgs mass may also be improved or, if nature is kind, it may be discovered.

We conclude that the present measurements on W mass, $\sin^2\theta_{\text{eff}}^{\text{lept}}$ and lower Higgs-bound are not compatible with the present value of the top quark mass within the standard model framework, the incompatibility being driven by the recent LEP2 result on the lower Higgs-bound. The simplest way to restore compatibility, without resorting to new physics, would be to move the top quark mass up from its present value of 174 GeV to a higher value of 184 GeV or so. If the Higgs lower bound does not move very much upward, this required increase of about 10 GeV is essentially a $2\sigma$ increase and can be tested soon in Run-II at the Tevatron. In case the top quark mass remains at its present value and the measurement uncertainty goes down to 3 GeV or better, or if the lower limit on the Higgs mass increases substantially, that will be the real beginning of the end of the standard model.

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