Gravitino production in hybrid inflationary models

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It has been recently shown that it is possible to excite gravitinos in an expanding background due to time-varying scalar field oscillating at the bottom of the inflationary potential. The two components of the gravitino, namely helicity 1/2 and helicity 3/2 are excited differently due to the presence of different time-varying mass scales in the problem. In this paper we analyse the production of both the helicities in multi-chiral scenario, in particular focusing on a general model of hybrid inflation. Fermion production in hybrid models is very much different from that of the chaotic models discussed so far in the literature. In this paper we give a full account of gravitino production analytically and numerically. It is noticed that the creation of gravitinos does not take place in the first few oscillations of the inflaton field, rather the production is a gradual and delayed process. It takes roughly 30 – 40 oscillations to build up the production and for the saturation to take place it can even take longer time, depending on the model parameters. We give an estimation of the reheat temperature and a brief discussion upon back-reaction on the gravitino production which could change its abundance.

I. INTRODUCTION

Low energy effective $N = 1$ supergravity is a predictive theory [1], which could predict an inflationary potential flat enough to provide adequate density perturbations [2]. So far, such viable inflationary models were constrained from observations by fixing the height of the potential, which essentially determines the amplitude of the COBE normalization. The first and the second derivative of the potential determines the tilt in the power spectrum, and the Yukawa couplings of the inflaton to other particles determines the reheat temperature of the Universe. The higher the coupling constant is, the higher is the temperature of the thermal bath and so the creation of the gravitinos from the collisions or the decay of other particles. The gravitinos decay very late and depending on their mass their life time could be long enough to disrupt the synthesis of light elements, via hadronic shower or by altering the entropy density of the baryons. Due to these reasons there is a strong constraint on the reheat temperature, for a review see Ref. [3].

However, there is also a non-thermal phase of the Universe, just after the end of slow-roll inflation when the scalar field begins to oscillate coherently at the bottom of the potential. During this era an explosive production of particles, both bosons [4] and fermions [5], may take place due to the non-perturbative decay of the inflaton to other fields. It has also been shown that it is possible to create super-massive bosons and fermions. However, fermionic production is always saturated by the Pauli blocking. As a matter of fact creation of heavy non-thermal bosons can be a good candidate for weakly interacting massive particles, known as WIMPS [6]. The decay of super-massive bosons can explain the ultra-high energy cosmic rays [7]. Super-massive fermions can be used in leptogenesis, mainly from the decay of right-handed neutrinos into Higgs and leptons, which explicitly violates the CP conserving phase [8]. It is worth mentioning that the non-perturbative technique of decaying inflaton to other particles has given a new paradigm shift in understanding the hot big bang universe from the ultra-cold inflationary regime.

Recently preheating in the context of global supersymmetric theories has been considered [9] and, as a natural extension it has been necessary to consider a local version of the supersymmetric theory and discuss the non-perturbative aspects of particle production and their consequences to nucleosynthesis. The local version of supersymmetry, known as supergravity, naturally accommodates the graviton and its superpartner the gravitino, a spin 3/2 particle. Quantization of spin 3/2 particles in the presence of an external background is plagued by consistency problems. It has been known for a long time that quantization of spin 3/2 particles in scalar, electromagnetic, or gravitational backgrounds can give rise to acausal behaviour [10]. Supergravity is the only set-up where such problems do not occur, provided the background fields also satisfy the corresponding equations of motion [11]. Nevertheless, the complicated form of the Rarita-Schwinger equation makes it extremely difficult to extract any explicit results even in a simple background [12]. The problem was first addressed in Ref. [13], where the authors quantized spin 3/2 particles in a non-vanishing cosmological background, almost two decades
The slightest generalization of quantizing spin 3/2 has been done very recently in literature, in Ref. [12]. The authors have extended the calculation of quantizing spin 3/2 in presence of a time-varying homogeneously oscillating scalar field in a cosmological background. This is the first result where the non-perturbative decay of the inflaton to gravitinos during preheating has been taken into account. The authors have explicitly shown the production of a particular helicity, the ±3/2 components of gravitino, in a particular new-inflationary type model [14]. It has been noticed that the non-perturbative result can give rise to a larger abundance compared to the perturbative decay. The gravitino to photon number density has been found to be n_{3/2}/n_γ ∼ 10^{-12} [12]. This abundance is 3 orders of magnitude larger than the thermal abundance for a reheating temperature 10^5 GeV [8]. Such over production of gravitinos has been the first proof of the non-thermal production of gravitinos with helicity ±3/2, which demands constraining the reheating temperature in any supergravity motivated inflationary model.

However, a massive gravitino has 4 degrees of freedom, the other two degrees of freedom due to the helicity ±1/2 components of the gravitino. The production of helicity ±1/2 gravitinos is directly related to the problem of super-Higgs mechanism in supergravity models, which was studied in Ref. [14] in the context of a non-vanishing cosmological constant, and in Ref. [17] with a vanishing cosmological constant. In the presence of a time-varying scalar field there is an additional source of supersymmetry breaking via the non-vanishing time derivative of the homogeneous scalar field. This plays an important role in the context of cosmology when the scalar field is recognized as the inflaton, oscillating coherently at the bottom of the scalar potential. Due to the presence of such a field, supersymmetry is always broken at the minima of the potential and the initially massless gravitino which possesses only the helicity ±3/2 components “eats” the goldstino to gain the other ±1/2 components. At this point, one may wonder how to generalize the super-Higgs mechanism in such a scenario. In fact the problem turns out to be quite complicated and it has been addressed in two seminal papers [18,14]. These papers also study for the first time the production mechanism of the helicity ±1/2 components of gravitino (see other papers in the similar context [20]). In all these papers the authors have assumed the existence of a unitary gauge, where the physical Lagrangian is free from the goldstino field. We will explicitly show that it is possible to choose such a gauge, where the gravitino equation of motion is free from the goldstino field. Our calculation is valid for more than one chiral field as well. Though we shall give a proof for the F-type supersymmetry breaking, this can be extended to D-type supersymmetry breaking also.

It has been noticed in Refs. [13,14] that the production of the two helicity states are completely different. The helicity ±1/2 components are produced copiously compared to the helicity ±3/2. In the case of helicity ±3/2, conformal invariance is broken due to the presence of a time-varying gravitino mass, which is usually Planck mass suppressed, whereas for helicity ±1/2 the breaking of conformal invariance is related to the presence of a massive Goldstone fermion [13,14], whose time-varying mass is not suppressed by the Planck mass. Moreover in the high momentum limit the helicity ±1/2 gravitino behaves like a fermion, the goldstino, as it is stated by the equivalence principle [20,21]. This has been studied in a single-chiral field scenario, where the source of conformal breaking can be directly related to the mass of Goldstino. Unfortunately in the multi-chiral field scenario the quantization scheme becomes more involved, and the relation between conformal breaking and Goldstino mass is not so straightforward. The situation has been briefly discussed in Ref. [21], and an attempt of a perturbative scheme has been suggested. However, it would be nice to discuss a non-perturbative scheme. Our paper fills that gap and, as we shall see, we can discuss non-perturbative production of the helicity ±1/2 gravitinos in the multi-chiral case.

The best example to study the multi-chiral field scenario is in the context of a general class of supersymmetric hybrid inflation model [13,14]. There are essentially two scalar fields, one is responsible for inflation, and the other field is responsible for the phase transition which results in terminating the inflationary era. Unlike the non-supersymmetric version of hybrid inflation model [29], the supersymmetric version considered in this paper has only one coupling constant in the potential. This leads to a single natural frequency of oscillations. This gives us an ample opportunity to use techniques to explore gravitino production in a similar spirit as in the case of a single-chiral field. Fermionic production in this case is very much different compared to that of a quadratic inflationary potential. In the hybrid models the effective mass term for the fermions is always positive and as a result the production can never be completed in just a few oscillations, rather the occupation number gradually increases and depending on the model parameters the rate of production could be slowed down significantly. For example, in order to reach the Fermi saturation it may be necessary more than 100 oscillations. Due to such a slow rate of production, the issue of back-reaction becomes important. It is very likely that simultaneous non-perturbative production of bosons can change the picture quite significantly. In some sense, hybrid model can be considered to be the safest of all the supergravity motivated inflationary models, because the gravitino production can stop due to back-reaction effects coming from other newly created bosons or fermions. In other models, such as in the chaotic models, where most of the particle creation takes place in the very first few oscillations, the issue of back-reaction hardly plays any significant role.

The layout of the paper is as follows; in section 2, we establish the supergravity Lagrangian and discuss the gauge fixing mechanism for the multi-chiral field scenario; this is a mere generalization of the $R^2_{\zeta}$ gauge used...
in pure gauge theories as well as an alternative way of explaining the existence of a unitary gauge in supergravity theories \cite{23,25,22}. In section 3, we discuss the quantization procedure for the gravitinos. In section 4, we briefly describe a general class of supersymmetric hybrid inflationary models we use. Analytical results and discussions based on our numerical results are presented in section 5. The implications of these results on the reheat temperature are discussed in section 6. We give a detailed discussion on fermionic creation in hybrid model in the appendix.

II. GENERAL SUPERGRAVITY LAGRANGIAN AND GAUGE-FIXING

In this section we describe the supergravity Lagrangian. For the sake of brevity, and for our purpose, we concentrate upon the chiral-supermultiplet which contains the bosonic part, fermionic part and the interaction terms between the fermions. We consider minimal Kähler potential with \( K = \phi^i \phi_i \), where the scalar fields \( \phi_i \) are taken to be real. The superpotential is denoted \( W = G(W) \) and the Kähler function is given as usual by \( K = K + \ln |W|^2 \). The choice of minimal Kähler potential also ensures \( G = G(W) \) is the bosonic part, fermionic part and the interaction, where \( \partial \) contains the bosonic part, fermionic part and the interaction terms between the fermions. We consider minimal Kähler potential with \( K = \phi^i \phi_i \), where the scalar fields \( \phi_i \) are taken to be real. The superpotential is denoted \( W = G(W) \) and the Kähler function is given as usual by \( K = K + \ln |W|^2 \). The choice of minimal Kähler potential also ensures \( G = G(W) \) is the bosonic part, fermionic part and the interaction, where \( \partial \) contains the bosonic part, fermionic part and the interaction terms between the fermions.

The above Lagrangian is invariant under local supersymmetric transformation laws \cite{26}. For spontaneous supersymmetry breaking to occur at least one of the field vacuum expectation values should be non-zero. In particular, for \( F \)-term type breaking of local supersymmetry one requires

\[
\langle 0 | \phi^i \chi_i | 0 \rangle = (-i \phi^i \xi - e^{G/2} G_i \xi) \neq 0, \tag{2}
\]

where \( \xi \) is the infinitesimal Grassmann-odd parameter. The right-hand side of Eq. (4) has two explicit terms which can break local supersymmetry. The second term is the usual \( F \)-term of the scalar field whose non-vanishing vacuum expectation value induces supersymmetry breaking. The first term will give a non-zero contribution in case of a time-varying scalar background field. This will be the situation if we identify the background fields \( \phi_i \) with the oscillating inflaton fields. In this case local supersymmetry is always broken during the oscillations.

The goldstino can be identified as usual from Eq. (2):

\[
\eta = \theta_i \chi_i, \tag{3}
\]

where,

\[
\theta_i = i \phi^i \phi_i - e^{G/2} G_i, \tag{4}
\]

and we follow the notations introduced in Ref. \cite{21}. We are only interested in homogeneous scalar fields \( \phi_i \) which are solely function of time. The goldstino is then “eaten” by the gravitino in local supersymmetric theories. This also ensures that the gravitino gains the helicity \( \pm 1/2 \) components other than the \( \pm 3/2 \) components, and becomes massive. In the high energy limit it is possible to relate the helicity \( \pm 1/2 \) components of the gravitino to the goldstino via the equivalence principle \cite{22,23,21}. In the limit when \( M_P \to \infty \), the helicity \( \pm 1/2 \) components retain the memory of the goldstino contribution and this is the reason why the two helicities behave differently and have different production rates \cite{18,19}.

As it can be realized by inspecting Eq. (4), the gravitino is coupled to the fermions through the following mixing terms

\[
\frac{1}{\sqrt{2}} \bar{\psi}_\mu \left( i e^{G/2} G_i + \phi \partial_i \right) \gamma^\mu \chi_i. \tag{5}
\]

However, it is possible to get rid of these mixing terms by adding a gauge-fixing term to the Lagrangian as it has already been discussed in various places \cite{22,21,23}. In this section we mainly concentrate on the mere existence of such a gauge-fixing term in supersymmetric theories. Regarding this, we extend the previous calculation made on gauge-fixing for a single-chiral field \cite{4,22}.

\*In Ref. \cite{22}, the authors have only considered the equation of motion for the goldstino, because they were more interested in establishing the high-energy equivalence between helicity \( \pm 1/2 \) gravitinos and goldstinos.
Before we move onto specifying the gauge, let us introduce the projection operators, which we need to use later on.

\[ \mathcal{P}^\perp_{ij} = \delta_{ij} - \frac{\theta_i^\dagger}{\theta^\dagger \theta} \theta_j, \quad (6) \]

\[ \mathcal{P}^\parallel_{ij} = \frac{\theta_i^\dagger}{\theta^\dagger \theta} \theta_j. \quad (7) \]

The modulus of \( \theta_i \) is given by

\[ \theta^\dagger \theta = e^{2G}G^iG_i + \dot{\phi}(t)\dot{\bar{\phi}}(t) = \rho + 3e^G, \quad (8) \]

where derivative with respect to time is denoted by dot, \( \rho = \phi \dot{\phi} + V \) is the energy density, and the scalar potential is \( V = e^{2G}(G^2G_i - 3) \). Accordingly, the fermion \( \chi_i \) can be split into two components by using the projection operators Eqs. (6) and (7)

\[ \chi_i = \chi_i^\perp + \theta_i^\dagger \theta \chi_i^\parallel, \quad (9) \]

\[ \chi_i^\perp = \mathcal{P}^\perp_{ij} \chi^j. \quad (10) \]

Now, with the help of Eqs. (3) and (5) the mixing terms between the gravitino and the chiral fermions can be recast as

\[ - \frac{i}{\sqrt{2}} \psi_i \gamma^\mu \theta^\dagger \theta \eta, \quad (11) \]

where we have used

\[ \theta_i \gamma^\mu \chi_i^\parallel = 0. \quad (12) \]

In a similar way, one can also reduce the complex conjugate part of the mixing terms. Eq. (4) tells us about the direct coupling of the gravitino to the goldstino. The equation of motion for the gravitino \( \psi_\mu \) that follows from the Lagrangian Eq. (4) is then

\[ e^{2G}e^{\mu \nu \rho \sigma} \gamma_5 \gamma_\nu D_\rho \psi_\sigma + \frac{1}{2} e^{G/2}[\gamma_\mu, \gamma_\nu] \psi_\nu \]

\[ - \frac{i}{\sqrt{2}} \theta_\eta \gamma^\mu \theta^\dagger \theta \eta = 0, \quad (13) \]

with an explicit term depending on the goldstino field due to the mixing. However, we notice that the contribution of the mixing term in Eq. (13) can be canceled by adding to the Lagrangian the following gauge-fixing term

\[ i\zeta \bar{F} \not D F, \quad (14) \]

where \( \not D = \gamma^\lambda D_\lambda \), \( \bar{F} = F^\dagger \gamma_0 \), and the gauge-fixing function is given by

\[ F(\psi, \eta) = \frac{\theta_i \gamma^\mu \theta^\dagger}{\theta^\dagger \theta} \psi_i + \frac{1}{\sqrt{2} \zeta} \frac{1}{D} \eta, \quad (15) \]

This is a mere generalization of the \( R_\zeta \) gauge used in pure gauge theories. Similar gauge-fixing has been initially introduced in Ref. [23] in the static case, where the scalar field has been taken to its value at the minimum of the potential. Therefore, once we fix the gauge, the equation of motion for the gravitino is completely free from the goldstino. The limit \( \zeta \to 0 \) corresponds to the unitary gauge, and it implies \( \eta \to 0 \). This is equivalent to demand that no goldstino be present in the physical spectrum. From here onwards we will work in the unitary gauge.

Using the gauge-fixing condition, \( F(\psi, \eta) = 0 \), and demanding \( \zeta \to 0 \), the final equation of motion for the gravitino, namely Eq. (13) in the unitary gauge can be written as

\[ e^{-1} e^{\mu \nu \rho \sigma} \gamma_5 \gamma_\nu D_\rho \psi_\sigma + \frac{1}{2} e^{G/2}[\gamma_\mu, \gamma_\nu] \psi_\nu = 0. \quad (16) \]

Here the mass term for the gravitino \( e^{G/2} \) depends on time, due to presence of oscillating background scalar field \( \phi(t) \), whose dynamics we shall discuss later on. From now onwards we express the mass term as \( m(t) \)

\[ m(t) = e^{G/2} \equiv e^{G/2} |W|, \quad (17) \]

where \( W \) is the model dependent superpotential. Detailed discussion will be given in the coming sections. The above demonstration of removing the goldstino dependence from the gravitino equation of motion in the unitary gauge suggests that it is possible to generalize \( R_\zeta \) gauge for a multi-chiral time-varying scalar background. Imposing the unitary gauge simplifies the gravitino field equation of motion in general and now we will be interested in quantizing gravitino field in a cosmological background.

III. DIFFERENCE BETWEEN HELICITY 1/2 AND 3/2

We have seen in the preceding section that we can recognize the goldstino component which couples to the gravitino field in a scenario when the dynamics of the background scalar field is also taken into consideration. By using the gauge-fixing term in the Lagrangian, we have shown that it is possible to cancel the gravitino-goldstino coupling term appropriately. The final equation of motion for the gravitino field, thus free from the goldstino, can be used to study the gravitino production in a dynamical background dominated by an oscillating scalar field. In this section we do not attempt to re-derive the equations of motion, which have already been discussed in several papers [15,16,17,18], rather we discuss few issues and the main equations. By studying the equation of motion for the Rarita-Schwinger field, one notices that there is a free index left, which in principle can be contracted by at best two possible ways, say \( \gamma_\mu \) or \( D_\mu \), thus giving rise to two constraint equations for the whole system. In presence of a cosmological constant, the equations of motion for both the helicities look alike, with two simple constraint equations, namely...
\( \gamma^\mu \psi_\mu = 0, D^\mu \psi_\mu = 0 \). However, this is not correct in any arbitrary gravitational background. These constraints do not hold true for the helicity \pm 1/2 case in an oscillating scalar background \(^{18,19}\), even though these constraints continue to hold for the helicity \pm 3/2 case in the same oscillating background \(^{13}\). The helicity \pm 1/2 components gain an effective mass during the oscillations of the inflaton, but the helicity \pm 3/2 do not seem to see the effect of curvature at all. This gain in mass is purely due to presence of a non-trivial background curvature, which suggests that the different helicities couple to gravity differently.

We follow the notations of Ref. \(^{19}\) in order to write down the equations of motions for helicity 1 and 3 components in terms of the Kähler potential \( K \). Here \( \psi_{1/2} \) and \( \psi_{3/2} \) are Majorana spinors which would correspond in the flat limit case to the 1 and 3/2 helicity states respectively (for details see Ref. \(^{14}\)). The matrix \( G \) in Eq. \(^{13}\) can be expressed in terms of \( A \) and \( B \) functions \(^{18,19}\)

\[
G = A + i\gamma^0 B = \frac{p - 3m^2}{\rho + 3m^2} + i\gamma^0 \frac{2m' a^{-1}}{\rho + 3m^2},
\]

where \( \rho \) and \( p \) are denoted by

\[
\rho = \sum_i |\phi_i|^2 + V(\phi_i),
\]

\[
p = \sum_i |\phi_i|^2 - V(\phi_i),
\]

with

\[
V(\phi_i) = e^K (|\partial_i W + \phi_i W|^2 - 3|W|^2).
\]

It is important to point out that in general \(|G|^2 = A^2 + B^2 \) is time dependent, and \(|G| \neq 1 \). Only in the case of a single-chiral field \(|G| = 1 \). For multi-chiral field scenario, in particular in the case of a supersymmetric hybrid model, \(|G| \) departs from 1. This makes the quantization scheme slightly more involved than simple scenarios where \(|G| = 1 \). To proceed with the quantization we redefine \( G \) in terms of the conformal time

\[
G = A(\tau) + i\gamma^0 B(\tau) = e^{\int \alpha d\tau} e^{2icr} e^{i\gamma^0 \int \mu d\tau},
\]

where the coefficient in front of the overall phase represents \(|G| \). We concentrate upon the helicity 1/2 case, helicity 3/2 being a simpler generalization of that. We expand \( \psi_{1/2} \) in terms of the mode functions

\[
\psi_{1/2} = a^{-5/2} \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{x}} e^{-i\gamma^0 \int \mu d\tau} \times \sum_{r=1,2} (u^r(\tau, \vec{k}) a^\dagger_k + v^r(\tau, \vec{k}) b^\dagger_{-k}),
\]

where, \( v^r(\tau, \vec{k}) = u^r(\tau, -\vec{k}) \). The spinor \( u^r(\tau, \vec{k}) \) satisfies the following equations of motion

\[
u_r = \mp im_{\text{eff}} u_r \pm i k |G| u_\pm,
\]

\[
m_{\text{eff}} = m + \mu,
\]

\[
u_\pm + (k^2 |G|^2 + \Omega^2 \mp i\Omega') u_\pm = 0,
\]

\[
\Omega = m_{\text{eff}} + i \frac{\alpha}{2},
\]

or, by redefining a new time in Eq. \(^{30}\) \( \frac{d}{d\tau} = |G| \frac{d}{dz} \),

\[
\frac{d^2 u_\pm}{dz^2} + \left(k^2 + \frac{m_{\text{eff}}}{|G|} \right) \Omega u_\pm = 0,
\]

where \( m_{\text{eff}} \) is defined in Eq. \(^{28}\), and the equation is formally analogous to the evolution equation for a spin-1/2 fermion in a time-varying background. It is important to notice that all the three equations Eq. \(^{31}\), Eq. \(^{32}\) and Eq. \(^{33}\) are equivalent, expressed in different forms. For our numerical results we have used Eq. \(^{30}\), and for our analytical treatment we can consider any of these three
In which the momentum \( k \) can be defined as annihilation operators which diagonalize the Hamiltonian. However, from Eq. (32) it is clear that this requires a non-vanishing effective mass term \( m_{\text{eff}} \), even if it is merely a constant. Particle production takes place due to the breaking of the conformal invariance. In our case, the violation of conformal invariance is not only due to presence of the gravitino mass but also due to \( \mu \), which can be related to the goldstino mass \([13,19,21]\).

In order to evaluate the occupation number we first evaluate the hamiltonian

\[
H(\tau) = \int d^3k \sum_r E_k(\tau) (a^+_k a_r - b^+_k b_r) + F_k(\tau) b_r a_r
+ F_k^*(\tau) a^+_r b^+_k ,
\]

(33)
in which the momentum \( k \) is along the third axis, and

\[
E_k = 2k|G| \text{Re}(u^*_k u_-) + m_{\text{eff}}(|u_-|^2 - |u_+|^2) ,
F_k = 2m_{\text{eff}} u_+ u_- + k|G|(u^*_k - u^*_k) ,
E^2_k + |F_k|^2 = m^2_{\text{eff}} + |G|^2 k^2 .
\]

The Hamiltonian can be diagonalized with the help of a Bogolyubov transformation. The new set of creation and annihilation operators which diagonalize the Hamiltonian can be defined as

\[
\hat{a}(k, \tau) = \alpha_k(\tau) a(k) + \beta_k(\tau) b^+(\tau) ,
\hat{b}^+(\tau) = -\beta^*_k(\tau) a(k) + \alpha^*_k(\tau) b^+(\tau) ,
\]

(35)
where \( \alpha_k, \beta_k \) are the normalized Bogolyubov coefficients,

\[
\frac{\alpha_k}{\beta} = \frac{E_k + \omega}{F^*_k} ,
|\beta_k|^2 = \frac{|F_k|^2}{2\omega(\omega + E_k)} = \frac{\omega - E_k}{2\omega} ,
\omega^2 = m^2_{\text{eff}} + |G|^2 k^2 .
\]

(36)
Now, the time dependent occupation number can be written in terms of the vacuum expectation value of the number operator

\[
n(\tau) = \langle 0|N|0 \rangle = \frac{1}{\pi^2 a^3(\tau)} \int dk k^2 |\beta_k|^2 .
\]

(37)
In order to solve Eq. (31), one needs to specify the boundary conditions. Usually they are defined such that at the beginning, when \( \tau \rightarrow 0 \), we have \( |\beta_k|^2 = 0 \), and the occupation number \( n(0) = 0 \), suggesting that there is no particle density at the initial time:

\[
u_{\pm}(0) = \sqrt{\frac{\omega \pm m_{\text{eff}}}{\omega}} ,
u'_{\pm}(0) = \mp im_{\text{eff}} u_{\pm}(0) + ik|G(0)|u_{\mp}(0) .
\]

(38)
Now, we have all the tools necessary to study the production of gravitinos in the hybrid inflationary model. Next we describe the hybrid model and how to estimate the occupation number.

**IV. HYBRID INFLATION**

The hybrid inflation potential can be derived from the following superpotential

\[
W = \lambda \phi(N^2 - N^2_0) ,
\]

(39)
where \( \phi \) plays the role of inflaton. During inflation the other field \( N \) is trapped in its false vacuum, \( N = 0 \), while the \( \phi \) field rolls down towards the critical value determined by the value of \( N \) at the global minimum, \( N_0 \), and the coupling constant \( \lambda \) between \( \phi \) and \( N \). At this point the \( N \) field rolls down from its zero value towards the global minima and begins to oscillate around \( N_0 \), while \( \phi \) oscillates around zero. This will enable the preheating phase of the Universe. During this period an effective potential for the fields \( \phi \) and \( N \) can be derived

\[
V = |W_{\phi}|^2 + |W_N|^2 ,
= \lambda^2 (N^2 - N^2_0)^2 + 4\lambda^2 \phi^2 N^2 ,
\]

(40)
where the subscripts denote the derivative of \( W \) with respect to the fields. The superpotential in Eq. (34) also ensures a non-vanishing constant vacuum energy during inflation \( V(0) = \lambda^2 N^2_0 \). We should also mention that \( \lambda \) and \( N_0 \) act as free parameters of the model, but they are constrained to some extent from the COBE normalization and the tilt in the power spectrum \([10]\)

\[
\lambda N_0 \approx 1.27 \times 10^{15} |\eta_*| \, \text{GeV} ,
\]

(41)
where we have taken \( |\eta_*| \approx 0.01 \) in our analysis, which is a reasonable assumption in order not to generate a sharp tilt in the power spectrum of the density perturbation during inflation. The present constraint on the spectral index is \( |n - 1| < 0.2 \) \([3]\). Hybrid inflationary model derived from such a superpotential is known as F-term hybrid inflation. Slightly different version of hybrid inflation popularly known as D-term inflation \([31]\) can be derived from the Fayet-Illiopoulos term appearing from an anomalous \( U(1) \) symmetry, which could provide the necessary potential energy during inflation. Whosoever be the cause of such a potential, our argument for the

\[1\] \( \eta_* \) is one of the two slow-roll parameters, usually defined in inflationary cosmology as \( \eta_* = (m^2_{\phi}/(8\pi))(V''/V(0)) \); the “star” index means that it is evaluated at least 60 e-foldings before the end of inflation. This should not be confused with the goldstino; we have already defined goldstino with the same notation earlier.
gravitino production is quite generic and will not depend upon a particular origin of the vacuum energy.

Due to presence of a single mass scale $\lambda N_0$ in the model, which is related to the supersymmetric breaking scale during the inflationary era, we will have a single frequency of oscillations during the preheating phase, and effectively a single scalar field oscillating. In our case this can be taken as $N$, related to the other field $\phi$ by

$$\phi = \frac{N_0 - N(t)}{\sqrt{2}}.$$  \hfill (42)

By solving the equation of motion for $N(t)$ we get the approximate solution when the field is around the bottom of the potential

$$\frac{N(t)}{N_0} \approx 1 + \Sigma(t) \cos(\nu_\phi t),$$  \hfill (43)

where $\nu_\phi = 2\lambda N_0$ provides the natural frequency of the oscillations, and $\Sigma(t)$ is the amplitude of the oscillations decreasing in time as $\sim 1/t$. The above expression is valid when $\frac{N(t)}{N_0} - 1 \ll 1/3$, with $\Sigma(0) \sim 1/3$. What makes the hybrid scenario interesting is that $N(t)$ never vanishes and that causes the fermionic creation to be completely different than the usual chaotic type inflationary potentials.

Now, with the present knowledge we can evaluate $|G|$ with the help of Eq. (23)

$$|G|^2 = \frac{(\rho - 2V)^2}{\rho^2} + \frac{4|\dot{W}|^2}{\rho^2},$$  \hfill (44)

$$= 1 - \frac{4|W_\phi \dot{N} - W_N \phi|^2}{\rho^2},$$  \hfill (45)

$$= 1 - \frac{4\lambda^2 N_0^2 |\dot{N}|^2(N - N_0)^2}{\rho^2},$$  \hfill (46)

where the subscript in $W_\phi$, $W_N$ means derivative with respect to the corresponding field, and dot denotes as usual physical time derivative. The last equation has been written using Eq. (22), knowing that $|\dot{\phi}|^2 + |N|^2 = 3/2|\dot{N}|^2$. Here it is important to notice that the departure from 1 is quite obvious, even though there is effectively a single scalar field oscillating. In fact, we can easily generalize Eq. (12) to any arbitrary number of scalar fields $\phi_i$ contributing to the energy density. Using Eqs. (24-25), without assuming $\phi_i \ll 1$, we get

$$|G|^2 = 1 - \frac{4e^{k\rho}}{(\rho + 3m^2)^2} \sum_i \sum_j |D_i W_\phi j - D_j W_\phi i|^2,$$  \hfill (47)

where $D_i W = W_i + K_i W$, and the subscript $i$ denotes derivative with respect to the homogeneous scalar field $\phi_i$. This shows explicitly that $|G| \leq 1$.

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\footnote{The actual decrease in amplitude over an oscillation period depends on the ratio $H/\nu_\phi$, $H$ being the Hubble parameter.}

V. ANALYTICAL ESTIMATION OF OCCUPATION NUMBER

A. Helicity 1/2

It is possible to analytically estimate the occupation number, and the number density $n(t)$ for both the helicities. Assuming that $|\beta| \approx 1$, for a given momentum $k$, our task reduces to estimate the cut-off momentum $k$. For this we need to solve Eq. (11), which will then lead us to study the occupation number for the helicity 1/2 gravitinos in hybrid model. In the case of helicity 1/2 states, the contribution of $|G|^2$ can be important, and we need to evaluate it before we could estimate the abundance of helicity 1/2 states. Later on we will discuss the abundance of helicity 3/2 states, where $|G|^2 = 1$. To carry out our calculation we need to know the dominant contribution to $\Omega$ appearing in Eq. (33).

$$\Omega = ma + \mu + i\frac{\alpha}{2},$$  \hfill (48)

where

$$\mu = -\frac{A'}{2B} + \frac{A}{2B} \frac{|G|^2}{|G|},$$  \hfill (49)

$$\alpha = \frac{|G|^2}{|G|}.$$  \hfill (50)

With the help of Eq. (43) and Eq. (44) it is possible to evaluate all the terms in Eq. (48). Here we simply quote the final results in terms of the physical time:

$$m(t) = -\frac{\nu_\phi^2 \Sigma^2(t)}{4\sqrt{2} \lambda^2 M_p^2} (1 + \cos(2\nu_\phi t)) + \mathcal{O}(1/t^2),$$  \hfill (51)

$$|G|^2 = 1 - \frac{1}{9} \sin^2(2\nu_\phi t) + \mathcal{O}(1/t),$$  \hfill (52)

$$\frac{|\dot{G}|}{|G|} = -\frac{\nu_\phi}{9} \sin(4\nu_\phi t) \left(1 + \frac{\sin^2(2\nu_\phi t)}{9}\right) + \mathcal{O}(1/t^2),$$  \hfill (53)

$$\frac{\dot{A}}{2B} = \frac{3\nu_\phi}{2\sqrt{2}} \left(1 + \frac{5\Sigma(t)}{4} \cos(\nu_\phi t)\right) + \mathcal{O}(1/t^2),$$  \hfill (54)

$$\frac{A}{2B} \frac{|\dot{G}|}{|G|} = \frac{\nu_\phi}{6\sqrt{2}} \cos^2(2\nu_\phi t) \left(1 + \frac{\sin^2(2\nu_\phi t)}{9}\right) + \mathcal{O}(1/t).$$  \hfill (55)

It is apparent that the only term which dominates $\Omega(t)$ in Eq. (33) is due to Eq. (48). The mass term $m(t)$ is subdominant due to the Planck suppression (here we have explicitly written the Planck mass). The amplitude of the oscillations from Eq. (53) is one order of magnitude smaller compared to that in Eq. (54). As a first approximation we can consider $\dot{A}/(2B)$ to be the effective mass term for the helicity 1/2 states.

A general discussion of production of massive fermions in an oscillatory background is given in the Appendix. We will use the results quoted there in order to estimate the occupation number for the helicity 1/2 gravitinos. By taking $\Omega(t) \approx -\dot{A}/(2B)$, and $|G|^2 \approx 1$, we
notice that Eq. (B1) mimics Eq. (A1) in the appendix for a time-varying mass which is denoted by $\Omega(t)$ in our present situation. First of all we notice that the effective mass term $A/(2\nu)$ never vanishes at any point of the scalar field oscillations. With $\Sigma(t) \sim 1/3$, the effective mass is always positive. This situation is typical of the scalar field oscillations. With $\Sigma(t) \sim 1/3$, the effective mass term $\dot{\nu} = \nu_0/(2\sqrt{2})$ and $g\phi(0) = 5\nu_0/(8\sqrt{2})$. It is evident that $\nu_0 > g\phi(0)$, and as a result $\nu(t)$ is always positive, quite similar to the case of bosonic production, where the mass term appears always as squared. However, when studying fermionic creation for a chaotic type potential there is a possibility of having $\nu(t)$ vanishing at some point during the oscillations [8]. In this case fermion production takes place in the first few oscillations, since the adiabatic condition is violated maximally when the inflaton field passes through the point where the effective mass vanishes, and it is possible to create very heavy fermions in the process. In hybrid models and for the helicity $1/2$ gravitinos, because the effective mass remains always positive, the adiabatic condition is broken when the effective mass reaches its minimal (non-zero) value, and the mass of the fermions created never exceeds the mass of inflaton. In this case it is also important to note that the amplitude of the oscillations decays slowly and it takes roughly $20-30$ oscillations to make any significant change in the initial amplitude. As a result the production of fermions takes place at each and every oscillation, and since the degree of violation of the adiabaticity is much weaker compared to the former, the production process takes a longer time to saturate the Fermi band. This is quite evident from our numerical results, see Fig. (1) and Fig. (2). The choice of model parameters in Fig. (1) leads to a higher inflationary scale compared to that of Fig. (2). However, the rate of production is exactly the same in both the cases. The reason is that the occupation number essentially depends on $\nu_0$, which is exactly the same in both the models we have considered. This can be vividly seen by comparing the number of peaks in the occupation number.

In Fig. (2) it is easy to see the particle creation taking place in bursts and in regular intervals. The peaks remind us the violation of the adiabatic condition. This is a perfect example of particle creation in a broad resonance regime. In the narrow resonance, particles are always created throughout the evolution of the scalar field [1], they are not produced in bursts like in Fig. (2). It is visible that the particle number remains constant for some time and then there is a sudden jump in the occupation number. Physically one can understand the situation from the comparison with an interacting quantum field theory. Usually in field theory, at extreme past/future we consider the plane wave solution of the free equation of motion. However, in between extreme past and future the interaction is switched on. Here also as we can see that the occupation number remains constant for a while, which represents the ingoing wave and then the interaction switches on, which is depicted by a jump in the occupation number. The outgoing wave in this picture is the newly filled occupation number. In fact the Bugolyubov coefficients can be calculated by estimating the reflection and the transmission coefficients of the plane wave passing through a barrier. Most of the important information concerning the production of helicity $1/2$ gravitinos can be extracted from Fig. (2). The production of helicity $1/2$ gravitinos builds up in each and every oscillation. The production does not saturate the Fermi level in the first few oscillations, but it takes several oscillations to reach the Fermi level. This behaviour is in stark contrast to the fermionic creation in a quadratic potential [8], where the Fermi level is reached in a few oscillations. It is also noticeable that there is no stochastic behaviour in the occupation number, and it is always increasing until it reaches the Fermi level. The main reason is that the effect of expansion is felt very slowly in hybrid models, when compared to the typical period of oscillation, that is, $H_0/\nu_0 \approx N_0/M_P \ll 1$, where $H_0$ is the value of the Hubble constant at the end of inflation. Therefore, the amplitude decreases very slowly in most of the cases, almost adiabatically. There is also a slight change in frequency, as it can be noticed from the plot (for details see Ref. [10]). However this small change in frequency is not going to affect our analytical estimation.

We have plotted the spectrum for the helicity $1/2$ gravitinos in Figs. (3) and (4), for two different set of model parameters. In both the cases Fermi-level is saturated.
for $k_{\text{max}} \approx \nu_0$. This confirms our analytical study in the preheating section, see Eq. (A14), which reduces to numerically observed $k_{\text{max}}$ for $m_X \sim m_\phi$. The number density of helicity 1/2 can be obtained from Eq. (57)

$$n_{1/2} \approx \frac{1}{4\pi^3} \int d^3k n(k) \approx \frac{k_{\text{max}}^3}{3\pi^2} \approx \frac{\nu_0^3}{3\pi^2}. \quad (56)$$

Here $k_{\text{max}}$ has been taken to be a comoving momentum. It is evident from Fig. (3) and Fig. (4) that the occupation number grows gradually and saturates after many oscillations depending on the choice of $\lambda$ and $N_0$.

### B. Helicity 3/2

We can follow similar arguments to evaluate the occupation number for helicity 3/2 gravitinos. Notice that Eq. (18) for helicity 3/2 reduces exactly to Eq. (31) with $|G| = 1$ and $\Omega = m_{\text{eff}} = ma$. [15, 18, 19, 21]. Hence, the single time-varying mass scale appearing in the problem is now given by Eq. (A3). Comparing Eq. (52) with Eq. (A3), it becomes clear that the bare mass $m_X$ itself is time-varying for the helicity 3/2 case, but as we have noted before, the amplitude of the oscillations is almost constant, especially in the hybrid model we are interested in. The parameters $m_X$ and $g\phi(0)$ are equal to each other, and both are Planck mass suppressed. But this does not mean that we cannot excite helicity 3/2 gravitinos. The main point is that the effective mass always vanishes at some point during each and every oscillation. As a matter of fact we can excite them precisely due to this. What matters is the violation of adiabaticity, and that takes place precisely at those points where $m(t)$ vanishes. Production of helicity 3/2 mimics the first scenario we have discussed in the appendix, but now $m_X \ll m_\phi = 2\nu_0$. However, since the effective mass $m(t)$ vanishes in each and every oscillation, so the gravitino production takes place in each and every oscillation. The spectrum of helicity 3/2 gravitinos is plotted in Fig. (3), for the model parameters given in Fig. (4). The spectrum preserves the essential features obtained before, but most importantly Fermi level never gets saturated and the production is extremely subdominant compared to helicity 1/2 case. Nevertheless, the helicity 3/2 gravitino abundance may also pose a strong bound on the model parameters and the reheating temperature, and thus it is necessary to study them as well [13]. Our task is to estimate $k_{\text{max}}$, and with the help of Eq. (A8) we get

$$k_{\text{max},3/2}^3 \approx \frac{\nu_0^3 \Sigma^2(t)}{2\sqrt{2\pi^2} M_p^2}. \quad (57)$$

By taking $M_p \approx 10^{18}$ GeV, and $\nu_0 \approx 10^{14}$ GeV, we get $k_{\text{max}} \approx 0.1\nu_0$, which matches very well with our numerical result. In Fig. (3), the spectrum peaks around $0.3\nu_0$. It is straightforward to estimate the occupation number for the helicity 3/2 gravitinos

$$n_{3/2} \approx \frac{\beta^2 \nu_0^3 \Sigma^2(t)}{6\sqrt{2\pi^2} \lambda^2 M_p^2}. \quad (58)$$
Here we have assumed $\beta_k$ in Eq. (57) to be a constant, 0 $\leq |\beta|^2 \leq 1$, for the cut-off momentum $k_{\text{max}}$. The actual value of $\beta$ is difficult to estimate analytically, and we do not attempt to analyse it here. We can also estimate the abundance ratio for the two helicities

$$\frac{n_{1/2}}{n_{3/2}} = \frac{2\sqrt{2}\lambda^2 M_p^2}{|\beta|^2 \nu_0^2 \Sigma^2(t)} .$$

We should mention that in Eqs. (57-59), $\Sigma(t)$ can be taken to be $1/3$, since the amplitude of the oscillations remains unchanged for many oscillations. For the numerical values we have considered, we would get the production of helicity $1/2$ gravitinos to be roughly 4 orders of magnitude larger than that of the helicity $3/2$ states, if we naively assumed that helicity $3/2$ saturated the Fermi-level ($|\beta|^2 = 1$, which is an over estimated production). It is evident from the numerical example, that the production is suppressed by at least 8 orders of magnitude compared to that of helicity $1/2$, see Fig. (4) and Fig. (5).

\[ \text{FIG. 4. The spectrum of helicity 1/2, when the number of oscillations is } N_{\text{osc}} = 60, \text{ for the model parameters } \lambda = 1.0 \text{ and } N_0 = 2 \times 10^{13} \text{ GeV. This choice of model parameters leads to } \eta_\nu \sim 0.01. \text{ It is evident that the Fermi level is saturated for a cut-off momentum } k \sim \nu_0 \sim 2A N_0. \text{ Important point to notice that this choice of model parameters leads to low inflationary oscillations is } \lambda N \text{ and this gives a different spectrum than noted earlier in Fig. (3).} \]

\[ \text{FIG. 5. Spectrum of helicity 3/2, when the number of oscillations is } N_{\text{osc}} = 60, \text{ for the model parameters } \lambda = 10^{-3} \text{ and } N_0 = 2 \times 10^{10} \text{ GeV. This choice of model parameters leads to } \eta_\nu \sim 0.01. \text{ It is evident that the Fermi level is not saturated in this case. Even though the production is small compared to helicity 1/2, it is certainly not negligible. Important thing to notice that the spectrum preserves its shape and peaks around } \eta_\nu. \]

VI. IMPLICATIONS ON $T_{\text{reh}}$

Through parametric resonance, other particles can also be created, and especially if their couplings are not suppressed by the Planck mass, they are perhaps produced more abundantly than gravitinos. Assuming that the end of reheating gives rise to a thermal bath with a final temperature $T_{\text{reh}}$, it is possible to estimate the ratio $n/s$, where $s$ is the entropy density, and $n$ represents the number density of gravitinos after the end of reheating. Both $n$ and $s$ scales like $a^{-3}$, and we can also assume that the background scalar field density behaves on average like matter, $\rho_\phi \propto a^{-3}$, during the oscillatory period. It is possible then to estimate the final ratio by noticing that $\rho_i \propto \lambda^2 N^3_0$ and $n(t_i) \propto \lambda^3 N^3_0$, where the subscript denotes the initial values

$$\frac{n}{s} = \frac{n(t_i)}{\rho_i} T_{\text{reh}} \approx \frac{\lambda}{N_0} T_{\text{reh}} .$$

Here the left hand side represents the final abundance of gravitinos during nucleosynthesis. Gravitinos are weakly coupled to gauge bosons and its gaugino partners and their lifetime, $\tau_{\text{decay}} \approx M_p^2/m_1^2/2$, is very long. For a TeV mass gravitino it could be around $10^4 - 10^5$ seconds, and it would pose a genuine threat to nucleosynthesis. However, this statement is strictly correct only for the helicity 3/2 component, since they can decay to gauge bosons and its gaugino partner through a dimension 5 operator. At high energies the interaction channels are governed by 3/2 component rather than 1/2 component gravitinos. In particular, helicity 3/2 may be produced with a mass close to a TeV range, so they decay very late and they are the ones which survive till late to cause problems for nucleosynthesis. However, the same can not be
said with confidence about the helicity 1/2 components. As we have seen, the violation of conformal invariance is not the same for both the helicities, and the helicity 1/2 gains an effective mass which is of the order of \(\nu_\phi \gg m_{3/2}\). Essentially, helicity 1/2 gravitinos are in an oscillatory scalar background with a frequency similar to their effective mass. There is no good reason to believe that the decay rate of helicity 1/2 to gauge bosons and to gauge fermions would mimic the decay rate similar to that in a flat background. So far, a detailed study is lacking in this area, but there is sufficient hint that the decay rate of helicity 1/2 is much smaller than the Hubble parameter. We do not repeat the argument, rather we refer the reader to Ref. \([13]\). The detailed calculation of the decay rate seems to be quite involved and we leave that for our future investigation. The important point to realize is that once the Universe reheats and thermalizes, the effective mass of the helicity 1/2 gravitinos becomes similar to that of the helicity 3/2, and as a result the decay rate would essentially be given by the usual decay rate in a flat background limit. whatsoever be the detailed analysis, we must mention that while deriving Eq. \((60)\), we have implicitly assumed that the initial abundance of gravitinos produced remain frozen until thermalization. Since the decay rate is smaller than the Hubble rate, the gravitinos produced during preheating will be able to survive until the end of reheating. The final abundance will mainly depend on the model parameters such as \(\lambda\) and \(N_0\), see Eq. \((64)\). Therefore, the constraint on the reheating temperature derived from Eq. \((60)\) is clearly model dependent.

We also mention that we have not included the effect of back-reaction coming from newly created bosons and fermions. In hybrid models, the production of the quanta associated with \(N\) and \(\phi\) is very efficient and can take place just in the first few oscillations \([10]\), much before the gravitinos have saturated the Fermi level. Therefore, back-reaction effects due to these quanta will quickly change the frequency and also the amplitude of the oscillating fields. We strongly suspect that especially in the hybrid scenario gravitino production will be affected due to such considerations and hence our current estimation of gravitino abundance will not hold anymore. However, even though the non-thermal production of gravitinos may stop after a while, there is a possibility to create them through scattering processes. This is beyond the scope of the present discussion and we shall hope to come back to these issues elsewhere \([31]\).

VII. DISCUSSION AND CONCLUSIONS

We have carried out the calculation for the gravitino production in multi-chiral field scenario, in particular in the context of hybrid model. As we have shown, it is possible to add a gauge-fixing term to the supergravity Lagrangian to get rid of the mixing between the goldstino and the gravitino field. Our method is a generalization of \(R_c\) gauge studied in various contexts. We choose to work in a unitary gauge, in which the goldstino is completely removed from the physical spectrum. Our study emphasizes major points in the non-perturbative production mechanism of gravitinos, analytically and numerically for the multi-chiral field models. We also give detailed analysis of fermionic creation in general. We have observed that the fermionic creation in hybrid model is quite different from other chaotic inflationary models. The effective fermionic mass in hybrid models never vanishes, and as a result the particle production does not take place in first few oscillations, rather it builds up gradually. This makes it more interesting as far as the gravitino production is concerned. If we really want nucleosynthesis to be preserved in the context of supergravity inflationary models, we believe models based on hybrid inflation with low scales are probably going to be the only saviour. The reason is very simple; in other models there is no way we can argue the back-reaction due to the creation of other particles would stop creating gravitinos, but in hybrid models there is a scope where the back-reaction due to non-perturbative creation of bosons could affect the coherent oscillations of the inflaton and halt the particle production completely. This gives us a new hope to understand the abundance of gravitinos during nucleosynthesis and we leave these important issues to be investigated in near future.

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APPENDIX: ESTIMATION OF \(K_{\text{MAX}}\)

In this section we briefly discuss non-perturbative production of massive fermions. This discussion is general and self-sufficient. We begin with the second order Dirac equation represented in terms of the two mode functions. It reads in conformal time

\[
 u''_\pm + (k^2 + (ma)^2 \pm i(ma)') u_\pm = 0 , \tag{A1}
\]

where \(u_\pm\) are the mode functions, \(k\) is the momentum, prime denotes the derivative with respect to the conformal time, \(a\) denotes the scale factor, and \(m\) is an effective time-dependent mass. This mass can be written in terms of the physical time as

\[
 m(t) = m_X + g\phi(t) , \tag{A2}
\]

\[
 \phi(t) = \Phi(t)\cos(m\phi t) , \tag{A3}
\]
where $m_X$ is the mass of the fermion we are interested in, $\phi(t)$ is the scalar field oscillating with some initial amplitude $\Phi(0)$, and $m_\phi$ is its mass. The Yukawa coupling between the fermion and the scalar is determined by $g$. Inspecting the mode equation Eq. (A1), it is obvious that it mimics harmonic oscillator with a time-varying imaginary frequency

$$\omega^2 = k^2 + (ma)^2 \pm i(ma)' \, .$$

(A4)

Particle creation occurs due to non-adiabatic evolution of the total frequency $\omega(t)$, that is, whenever

$$\frac{d\omega(t)}{dt} \geq \omega^2 \, ,$$

(A5)

where we have expressed the total frequency in physical time.

We shall explore here two possible scenarios for fermionic production, depending on whether or not the effective mass for the fermion vanishes at some point of the scalar field oscillations. To start our discussion, we consider first the situation when the amplitude of the scalar field initially satisfies $|\Phi(0)| \geq m_X/g$, so that the effective mass goes through zero at some point during the oscillations. Since the particle production takes place within a time $\Delta t \ll H^{-1}$, we can safely neglect the effect of expansion during the production of particles, especially in a broad resonance regime. However, we have to keep in mind that the amplitude is gradually decreasing due to the effect of expansion. Once the amplitude drops below a critical value, the production of the fermions will soon stop and their number density will be frozen in time.

The critical amplitude is given approximately by equating Eq. (A2) to zero

$$\Phi_* \sim \frac{m_X}{g} \, .$$

(A6)

Below this value, the effective mass remains always positive. It is also important to mention that the production enhances near the regime when the effective mass vanishes, or, in other words, when there is a maximum violation of adiabaticity condition. With the above information it can be possible to estimate the typical momentum $k$ required to violate the adiabatic condition when the amplitude of the field is close to $\Phi_*$. This will occur when $\cos(m_\phi t_*) \sim -m_X/(g\Phi_*)$, or $t_* \sim \pi/m_\phi$. Our condition Eq. (A5) implies

$$2g^2\dot{\phi}(t)^2 (m_X + g\phi(t))^2 + \frac{1}{2}g^2\dot{\phi}(t)^2 \geq \left((k^2 + (m_X + g\phi(t))^2)^2 + g^2\dot{\phi}(t)^2\right)^{3/2} \, .$$

(A7)

Since $\dot{\phi}(t_*) = 0$, and $\ddot{\phi}(t_*) \approx m_\phi^2 m_X/g$, we can estimate the left-hand side of Eq. (A7) around $\phi(t_*)$, and the final condition translates to a simpler form,

$$\frac{1}{2}m_X m_\phi^2 \geq k_{\text{max}}^3 \, .$$

(A8)

We have assumed $\omega \approx k_{\text{max}}$ in the final derivation. This result confirms similar result already obtained in Ref. [8]. We can express Eq. (A8) in terms of an effective $q$ parameter

$$k_{\text{max}} \sim \left(\frac{m_\phi^4}{m_X} q_*\right)^{1/3} \, ,$$

(A9)

where $q(t) = g^2\Phi^2(t)/m_X^2$. In the above equation the value of $q$ is evaluated at $\Phi \sim \Phi_*$. The $q$ dependence in $k_{\text{max}}$ is quite different from the bosonic production. This is mainly due to the presence of the imaginary part of the frequency, which has a significant contribution to the violation of the adiabatic condition. At this point one may be able to estimate the maximum mass $m_X$ allowed by the violation of the adiabatic condition. With the help of the $q$ parameter, it is possible to re-express $m(t)$ as

$$m(t) = m_X + \sqrt{q(0)/t} \cos(m_\phi t) \, ,$$

(A10)

where we have written explicitly the time-dependence of the amplitude. Hence, the maximum mass is achieved when $t_* = \pi/m_\phi$, and this gives

$$m_X \leq \frac{m_\phi}{\pi} \sqrt{q(0)} \, .$$

(A11)

For reasonable values of the coupling constant $g$, it is possible to achieve very high values of the fermion mass $m_X \gg m_\phi$. This suggests that such production of supermassive fermions is indeed non-thermal and non-perturbative in nature. It is also important to notice that for values of $q$ which are of the order of tens, the maximum fermionic mass obtained is of the order of the mass of the oscillating field.

The above analysis suggests that creating supermassive fermions is possible because the effective mass $m(t)$ vanishes around $\Phi_*$, and this is where the adiabatic condition is violated maximally. This happens quite naturally in quadratic inflationary potentials because the initial amplitude of the oscillations for the inflaton is large enough to pass through the point where the effective mass $m(t)$ vanishes. A similar situation arises in the case of the helicity $3/2$ states for the gravitino, and we find that its mass term Eq. (B1) vanishes in each and every oscillation of the $N$ field, defined earlier. This means that
the adiabatic condition is violated maximally at those points. However, in the hybrid case the amplitude of the oscillations and the mass of the helicity 3/2 states are exactly the same $m_X = g\phi(t)$. This suggests that we can not create very massive 3/2 states, although we can create them with mass $m_X \ll m_\phi$. Therefore, the hybrid situation is slightly different from the chaotic inflationary scenario with quadratic potential.

Next, we study a scenario where the effective mass of the fermion never vanishes at any point, that is $g\phi(0) < m_X$. This is the situation which arises in the hybrid inflationary scenario for the helicity 1/2 gravitino states. In this case the adiabatic condition is violated maximally near the point where $m(t)$ is minimal but non zero (then $|g\phi(t)|$ is maximal), and this again happens when $\phi(t) \approx 0$. Using Eq. (A13), we can then easily estimate the upper limit on $|\phi(t)|$

$$\left( \frac{m_X^2 g\phi(t)}{\sqrt{2}} \right)^{2/3} - (m_X - |g\phi(t)|)^2 \geq k^2, \quad (A12)$$

where we have replaced $\phi(t) = -m_X^2 \phi(t)$. Notice that we have explicitly taken the negative sign for $\phi(t)$, in order to extremise the left-hand side of the above equation. We would like to see the range of the violation of the adiabatic condition for small $k$,

$$\frac{m_X}{3} \leq |g\phi(t)| \leq m_X. \quad (A13)$$

Hence, in general the production could continue as long as the field amplitude follows the above condition Eq. (A13).

The maximal range of momenta for which the fermions are produced is obtained when $|g\phi(t)| \approx m_X$, and is given by

$$\left( \frac{m_X^2 m_X}{\sqrt{2}} \right)^{1/3} \geq k_{\text{max}}, \quad (A14)$$

and for $m_X \approx m_\phi$, the above expression reduces to $k_{\text{max}} \leq m_X$. This result could have been easily derived from Eq. (A8) by taking the masses to be almost equal.

In the hybrid model, which we have considered, the amplitude of the oscillations is small, never exceeding $m_X / g$. Hence we do not expect to produce super-massive fermions. The above result can be directly applied to the production mechanism of helicity 1/2 gravitinos. We have mentioned in the main section that the effective time-varying mass for the helicity 1/2 states does not vanish. The effective mass of the 1/2 states is almost equal to that of the oscillating frequency, which corresponds to having $m_X \approx m_\phi$ in our present discussion. In addition the adiabatic condition is broken for a narrow range of field values, see Eq. (A13). This means that the occupation number for the helicity 1/2 builds up in each and every oscillation and the gravitinos are produced in bursts.

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