Formulation of parameters of three-layer thermal protective clothing based on Fourier’s law

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Abstract. With the rapid development of metallurgy, construction, machinery manufacturing and petrochemical industry, safety accidents often occur under high-temperature working environment. Therefore, this paper analyses the heat conduction process of thermal protective clothing, according to the analysis of unstable heat conduction process with Conservation of Energy and Fourier’s law, so as to get the distribution of temperatures with the thickness variation of the material layer. Furthermore, we shall fit the relation between temperature and the thickness of the material layer when the heat transfer reaches steady state, and then determine the optimal range of the thickness of the thermal protective clothing material layer with the upper bound being the highest temperature which human can tolerate. And it can be promoted to the fields of fire protection, metallurgy, machinery manufacturing and other high temperature work.

1. Introduction
According to statistics, 80% of construction site accidents in China occurred in the hot summer environment from June to August [1]. In the US, there were 423 workers died due to long-term work in high temperature environments between 1992 and 2006, while in Japan, an average of 13.8% of workers died from heat-related diseases every year from 1991 to 2001 [2]. Without good protection measures, workers who work in high temperature environment for a long time will have a continuous thermal stress response, which will damage the thermal balance of the body, and even lead to the occurrence of casualty accidents in serious cases. This problem has always been taken seriously [3-12]. The following model is designed for the optimal adjustment thickness of the thermal protective clothing material. The thermal protective clothing material layer and air layer can be simplified into uniform and flat material with same area. After entering the high temperature working environment, we considered that the material is uniformly heated and the heat transfer direction is from the higher side to the lower one. And the direction is perpendicular to the material plane, when the temperature outside the skin does not reach the stable value, the process can be simplified as the unsteady heat conduction problem of multilayer flat wall in the one-dimensional direction, see Figure 1.

The following assumptions are made for this one dimensional multi-layer flat wall unsteady heat conduction model:

(1) It is assumed that the materials of I, II, and III are uniform in texture.
(2) It is assumed that the density, thermal conductivity, and specific heat of each layer of material are constant values, they do not alter with changes in temperature.

From conservation of energy and the Fourier’s law, we can obtain partial differential equation for heat transfer of layers I, II, III and IV, respectively.
The first layer is
\[ \frac{\partial}{\partial x} \left( \lambda_1 \frac{\partial T}{\partial x} \right) + \frac{\partial F_L}{\partial x} = \rho_1 c_1 \frac{\partial T}{\partial t}, \quad (x, t) \in L_1, \] (1)

The second layer is
\[ \frac{\partial}{\partial x} \left( \lambda_2 \frac{\partial T}{\partial x} \right) = \rho_2 c_2 \frac{\partial T}{\partial t}, \quad (x, t) \in L_2, \] (2)

The third layer is
\[ \frac{\partial}{\partial x} \left( \lambda_3 \frac{\partial T}{\partial x} \right) - \frac{\partial F_R}{\partial x} = \rho_3 c_3 \frac{\partial T}{\partial t}, \quad (x, t) \in L_3, \] (3)

The air layer is
\[ \frac{\partial}{\partial x} \left( \lambda_4 \frac{\partial T}{\partial x} \right) - \frac{\partial q_{air}}{\partial x} = \rho_4 c_4 \frac{\partial T}{\partial t}, \quad (x, t) \in L_4, \] (4)

where \( \lambda_i \) is the pyroconductivity of the \( i \)th layer materials, \( L_i \) is the domain of the thickness of the \( i \)th layer materials, \( t \) is the temperature and \( q_{air} \) is the heat flux of air flow, and \( \frac{\partial F_L}{\partial x} \) is the left radial heat flux satisfying
\[ \frac{\partial F_L}{\partial x} = \beta F_L - \beta \delta t^A, \quad (x, t) \in L_1. \] (5)

And \( \frac{\partial F_R}{\partial x} \) is the right radial heat flux satisfying
\[ \frac{\partial F_R}{\partial x} = \beta F_R + \beta \delta t^A, \quad (x, t) \in L_3, \] (6)

where \( \beta \) is the radiation absorption constant with its unit being \( K^{-1} \), \( \delta \) is the Stefan-Boltzmann constant with value taking \( 5.670 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4) \).

2. Solution of the model

In the model, the outside of the layer I material is in contact with the environment, and the temperature inside the model is continuous. Therefore, we use a flat and uniform curve to describe the change of temperature with the thickness of the material. The initial condition of the material layer is set as:
\[ T(x, 0) = T_i(x), \quad x \in (0, L), \] (7)

where \( L \) is the sum of the thicknesses of the material layers I, II, III, IV, \( x_i \) is the density of materials of the \( i \)th layer.

The entire process considers the effects of heat transfer and heat radiation, so the left and right boundary conditions of the first layer of material are set to:
\[ -\rho_1 c_1 \frac{\partial T}{\partial x} \bigg|_{x=0} = (q_{conv} + q_{rad}) \bigg|_{x=0}, \] (8)
The left and right boundary conditions of the second layer of material are set to:

\[-\rho_2 c_2 \frac{\partial T}{\partial x} \Big|_{x=x_1} = q_{rad} \Big|_{x=x_1} ,
\]

\[-\rho_2 c_2 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2} = q_{rad} \Big|_{x=x_1+x_2} .
\]

(10)

(11)

The left and right boundary conditions of the third layer of material are set to:

\[-\rho_3 c_3 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2} = q_{rad} \Big|_{x=x_1+x_2} ,
\]

\[-\rho_3 c_3 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2+x_3} = q_{air} - \rho_4 c_4 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2+x_3} .
\]

(12)

(13)

where \( x' = x_1 + x_2 + x_3 \). The left and right boundary conditions of the air layer of material are set to:

\[-\rho_4 c_4 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2+x_3} = q_{air} - \rho_4 c_4 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2+x_3} ,
\]

\[-\rho_4 c_4 \frac{\partial T}{\partial x} \Big|_{x=x_1+x_2+x_3+x_4} = T_{skin} .
\]

(14)

(15)

where \( \rho_i \) is the density of the \( i \)-th layer materials, \( c_i \) is the spectral heat capacity of the \( i \)-th layer materials, \( x_i \) is the thickness of the \( i \)-th layer materials, \( T \) is the temperature, \( q_{conv} \) is the convective heat flux from the environment to the materials, \( q_{rad} \) is the radial heat flux, \( T_{skin} \) is the skin surface temperature.

2.1. Numerical solution of the equations

Thus the heat transfer model can be solved by using the classical display forward difference method [13, 14], with the scheme displayed in Figure 2. Let us take the second layer material as an example.

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\[ u_t = u_{xx} , \]

(16)

where

Figure 2. The scheme of classical forward difference method.

- Domain segmentation:
  - \( \Delta x \): grid constant
  - \( \Delta t \): time step
  - \((x_j, t_n)\): grid node
  - \( x_j = j\Delta x, j = 0,1,2,...,J \),
  - \( t_n = n\Delta t, n = 0,1,..., \)

- Discrete the differential equation:
  - The heat transfer equation can be written as

\[ u_t = u_{xx} , \]

(16)
\[ u_t = \rho c \frac{\partial T}{\partial t}, \quad u_{xx} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right), \]  
\[ (17) \]

Partial derivatives can be approximated by

\[ \frac{\partial u}{\partial t} (x_j, t_n) \approx \frac{u(x_j, t_{n+1}) - u(x_j, t_n)}{\Delta t}, \]
\[ (18) \]

using forward difference method.

Recursive calculation:

Calculate the value of \( t_{n+1} \) time layer from the value of \( t_n \) time layer, then the results are shown in Figure 3.

![Temperature distribution](image)

**Figure 3.** Temperature distribution.

2.2. Convergence analysis

For any point \((x^*, t^*) \in (0,1) \times (0, t_0)\), when \(\Delta x, \Delta t \rightarrow 0\) and \(x_j \rightarrow x^*, \ t_n \rightarrow t^*\), i.e., \(U^n_j \rightarrow u(x^*, t^*)\), the numerical schemes for those equations are convergent if [15, 16]

\[ \mu = \frac{\Delta t}{(\Delta x)^2} \leq 0.5. \]

Table 1 illustrates that using the classical finite difference method to solve the partial differential equations of the heat conduction converges.

| \( \mu_1 \) | \( \mu_2 \) | \( \mu_3 \) | \( \mu_4 \) |
|---|---|---|---|
| 0.0147 | 0.0151 | 0.0260 | 0.0175 |
3. Material optimal thickness model
We then focus on the optimal thickness for the material.

3.1. Relation between temperature and thickness of material layers
Consider the relation between temperature and thickness of materials when the heat transfer reaches the steady state. According to the solution of heat equation using Fourier series, the relation can be fitted by

\[ T(x, t) = ae^{-k^2x} \cos kx. \]

Taking environment temperature is 75 ℃ as an example, fitting material I, II, III function between temperature and thickness. The relationship between the temperature of the material layer I and the distance is shown in Figure 4.

![Figure 4](image)

**Figure 4.** The relationship between the temperature of the material layer I and the distance.

The relationship between the temperature of the material layer II and the distance is simulated by

\[ T = 75 \cos(-0.09292x) \times e^{-0.004537x} \]

thus the relationship between the temperature of the material layer II and the distance is shown in Figure 5. And the relationship between the temperature of the material layer III and the distance is modelled by

\[ T = 75 \cos(-0.07978x) \times e^{-0.01236x} \]

thus the relationship between the temperature of the material layer III and the distance is shown in Figure 6.

3.2. Adjustment range of material layer thickness
Human skin is very sensitive to temperature. When the human skin temperature reaches 45 °C, people will have a burning sensation [2]. After reaching a steady state, the temperature outside the skin will not change, so we use 45 °C as the upper boundary of the skin outer temperature. Using the function of temperature and thickness, the optimal range of material thickness of each layer can be obtained: \( T(x, t) = ae^{-k^2x} \cos kL < 45 \), L is solved by the above formula, which is the optimal range of thickness of I, II, III, and IV. In actual life, the cost of clothing materials and the flexibility of personnel operation can also be considered, and then the reasonable thickness of the thermal protective clothing material can be given.
4. Future work

The transmission of heat is very common in life. The insulation model of protective clothing can be extended to various scenes of heat transfer in real life, such as the choice of insulation materials of fire disaster and the choice of winter clothing and so on. This model has strong universality.

**Figure 5.** The relationship between the temperature of the material layer II and the distance.

**Figure 6.** The relationship between the temperature of the material layer III and the distance.
In this paper, we focus on the case of constant heat conductivity, which changes with temperature in real world [4]. To make results more accurate, one should consider variable heat conductivity with temperature changing.

Furthermore, the thickness of the four-layer material can be considered to be adjustable. For example, the thickness of the air layer can be adjusted by adding an inflatable interface to the protective clothing. And when the fabric layer temperature reaches a critical value, we can use automatic chemical reaction inside the protective clothing to make the temperature drop.

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