Stability of horizon with pressure and volume of d-dimensional charged AdS black holes with cloud of strings and quintessence

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Abstract

In this paper, the thermodynamics and the stability of horizon of the charged AdS black hole surrounded by quintessence and cloud of strings in d-dimensional spacetime are studied via the scalar field scattering and the charged particle absorption. The cosmological constant is interpreted as a thermodynamics variable. During the study, we consider the case where the energy of the particle(scalar field) is related to the internal energy of the black hole. Furthermore, we also consider another assumption, which is proposed in [Phys. Rev. D 100, no.10, 104022 (2019)]. This assumption considers that the energy of the particle(scalar field) is related to the internal energy of the black hole. In addition, we compare and discuss the results obtained under these two assumptions. At the same time, we also considered the effect of the dimension. The thermodynamics of black holes in different dimensions has also been studied and compared.
I. INTRODUCTION

Usually, black holes are defined as one of the compact objects formed by the concentration of matter in a small space, which exhibits various features of gravity. A significant feature of a black hole is its event horizon, through which no particle can escape from its gravity, even if the particles are photons. Therefore, the event horizon plays a foremost role in preventing the observer from viewing the inside of a black hole. A particle going through the outside region horizon cannot be seen, but its physical quantities affect the black hole through a
back-reaction. The recent theoretical developments are in favor of a scenario that represents black holes energies were divided into two parts: irreducible mass, reducible energy \([1, 2]\). The irreducible mass increases in an irreversible process, even if a Penrose process extracts energy from the black hole. As energy, the irreducible mass is considered to be distributed on the horizon’s surface area and is proportional to the square-root of the horizon surface area. However, the mass of a black hole can decrease, such as the Penrose process, and the reduced mass is the reducible energy among the energy of a black hole. This reducible energy includes electric and rotational energies, and external fields or particles can reduce it. In thermodynamics, the irreducible property of entropy is similar to that of irreducible mass. The Bekenstein-Hawking entropy of a black hole is proportional to the square of the irreducible mass \([3, 4]\). According to these definitions of the temperature and entropy of a black hole, the laws of thermodynamics are defined. Furthermore, black holes can be regarded as thermodynamic systems with the Hawking temperature \([5, 6]\), for the reason that there is an energy radiated from the black hole that do with the quantum effects near the horizon \([5]\).

High precision astronomical observations have shown that the universe is undergoing a phase of accelerated expansion \([7, 8]\). Formation of a singularity with infinite matter density is inevitable during the gravitational collapse \([9]\). The existence of a singularity will destroy the deterministic nature of general relativity. Since a naked singularity without a horizon causes problems in terms of causality, the weak cosmic censorship conjecture states that the singularity should be hidden to an observer in the spacetime of a black hole owing to the horizon. Hence, the horizon should be stable. There is not a concrete proof of the weak cosmic censorship conjecture, whose validity should be checked in different spacetimes. Wald proposed firstly a gedanken experiment to check this conjecture by examining whether the black hole horizon can be destroyed by absorbing a point particle. Until now, there are some debates on the test particle mode. When it comes to the higher order terms in the energy, angular momentum, and charge of the test particle are taken into account, the weak cosmic censorship conjecture was found to be violated too even for an extremal Kerr-Newman black hole \([10]\). Later, it was claimed that in all of these situations, the test particle assumption was not perfect since they did not take into account the self force \([11, 13]\) and back reaction effects \([14, 15]\). As these effects were considered, the weak cosmic censorship conjecture was found to be valid for both the extremal and near-extremal black holes. Especially, by
applying the Wald formalism rather than matter, a new version of Gedanken experiment has been designed recently. Over the years, the validity of the weak universe censorship conjecture has received extensive attention and a lot of research work has been carried out under particle absorption \[16-39\]. The weak cosmic censorship conjecture was found to be valid for the non-extremal black holes. In this framework, the second order variation of the mass of the black hole emerges, which somehow incorporates both the self force and back reaction effects. Then, this study was also generalised to scalar field \[40, 41\]. Semiz first proposed a way of destroying the event horizon of a black hole to test the validity of the weak cosmic censorship conjecture, which is the scattering of a classical test field. Others have extended this approach further \[42-44\]. Recently, Gwak divided the scattering process into a series of in-finitesimal time interval and considered an infinitesimal process only, the result shows that Kerr-(anti) de Sitter black holes cannot be overspun by a test scalar field \[45\]. It is important that the time interval for particles crossing the event horizon for the weak cosmic censorship conjecture \[46-48\]. And further developed by others \[49-62\].

In addition, various investigations have been conducted on the conjecture for not only black holes of Einstein’s theory of gravity, but also anti-de Sitter (AdS), lower-dimensional, and higher-dimensional black holes \[63-71\]. From the research results, this conjecture and the laws of thermodynamics have great relevance. If the entropy of the black hole increases, as ensured by the second law for an irreversible process, the horizon can cover the inside of a black hole, and the variation of a black hole is consistent with the first law of thermodynamics under particle absorption \[16, 72\]. In recent years, black hole thermodynamics related issues received much attention with the discoveries of the Bekenstein Hawking entropy and Hawking radiation. This has changed our understanding of black holes ever since, opening up vast areas of research including phase transitions and holography \[73\]. For an black hole, the usual first law of black hole thermodynamics takes the form

\[
dM = TdS + \phi dQ. \tag{1}
\]

where \(M\) denotes the Arnowitt-Deser-Misner (ADM) mass of the black hole, \(T\) is the Hawking temperature, \(S\) is the Bekenstein-Hawking entropy, \(\phi\) is the electrostatic potential and \(Q\) is the electrical charge. Compared with ordinary thermodynamics, there is no an absence of a \(VdP\) term. This term in the context of black hole spacetime was eventually introduced and requires an anti-de Sitter (AdS) background \[74, 75\]. Then the first law is generalized
to

\[ dM = TdS + VdP + \phi dQ. \]  \hspace{1cm} (2)

Where \( M \) is now reinterpreted as the enthalpy, \( V \) is the volume of the black hole and is defined as the thermodynamic conjugate to the pressure. The relationship between \( M \), the internal energy \( U \) and \( PV \) of the black hole is

\[ M = U + PV. \]  \hspace{1cm} (3)

The d-precision observations confirmed the existence of a gravitationally repulsive interaction at a global scale (cosmic dark energy) recently [76]. It is founded that one type of dark energy models produces some gravitational effect when it surrounds black holes. For this type of dark energy, the equation of state parameters is in the interval \([-1, -\frac{1}{3}]\) [77]. This type of dark energy models is called quintessence dark energy or quintessence for short. In this case, the first law of thermodynamics is given by [78]

\[ dM = TdS + VdP + \phi dQ - \frac{1}{2r_h^2} d\alpha, \]  \hspace{1cm} (4)

where \( \alpha \) is a positive normalization factor. There has been much interest in studying the physics of black holes surrounded by quintessence [79–96].

According to string theory, nature can be represented by a set of extended objects (such as one-dimensional strings) rather than point particles. Therefore, understanding the gravitational effects caused by a set of strings is necessary. This can be achieved by solving Einstein’s equations with a finite number of strings. The results obtained by the Letelier show that the existence of cloud of strings will produce a global origin effect that related to a solid deficit angle. Moreover, the solid deficit angle depends on the parameters that determine the existence of the cloud [97]. Therefore, the existence of cloud of strings will have an impact on black holes. When we consider the existence of cloud of strings, the first law of thermodynamics takes on the form as

\[ dM = TdS + VdP + \phi dQ - \frac{r_h}{2} da, \]  \hspace{1cm} (5)

where \( a \) is the state parameter of cloud of strings. The effect of cloud of strings on black holes have been explored for various black holes [98–103]. As noted in [104], considered that the parameters related to the cloud of string and quintessence are extensive thermodynamic
parameters, the first law of thermodynamics of black hole is modified as
\[
dM = TdS + VdP + \phi dQ - \frac{1}{2r_h^3} d\alpha - \frac{r_h}{2} da.
\] (6)

There has been much interest in deducing and discussing the physical properties of various black holes when they are surrounded by cloud of strings and quintessence \[105\text{--}108\].

The rest is organized as follows. In section II we present a generalized solution corresponding to charged AdS black holes surrounded by quintessence and cloud of strings in higher dimensional spacetime. In section II we have to study the problem from four aspects of the black hole with particle absorption. In section III A we investigate the absorptions of the scalar particle and fermion by the black hole. The relation between energy and charge of the particle is gotten. In section III C the thermodynamics in the extended phase space are investigated by the absorptions of the particles. In section III D the overcharging problem is tested by throwing a particle in the near-extremal and extremal black holes. In section III E the first and second laws of thermodynamics and the stability of the horizon are discussed under a new assumption. In section IV We also describe the related problems of black holes Under scalar field scattering from four aspects. In section IV A we get the changes in the internal energy and charge of the black hole during the time interval \(dt\). In section IV B the laws of thermodynamics through scalar field scattering are discussed. In section IV C we tested the stability of horizon by evaluating the minimum of function \(f\) in the final state. In section IV D We use the scattering of a scalar field to investigate thermodynamics and the stability of horizon under a new assumption. The last section is devoted to our discussion and conclusion.

II. QUINTESSENCE SURROUNDING D-DIMENSIONAL RN-ADS BLACK HOLES WITH A CLOUD OF STRINGS

It was recently considered a metric for AdS asymptotically spacetime in \(d\)-dimension, which generated by a charged static black hole and surrounded by cloud of strings and quintessence. Assuming that the cloud of strings and quintessence do not interact \[109\], the energy momentum tensor of the two sources can be seen as a linear superposition. The solution corresponding to a black hole immersed in quintessence with cloud of strings, in a
$d$-dimensional spacetime, is given by the general form \[105\]

$$dS^2_d = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2_{d-2}.$$ \hfill (7)

Where $d\Omega^2_{d-2}$ denotes the metric on unit $(d-2)$-sphere, which can eliminate $\rho_q$ from the Einstein equation. The following equations can be obtained from the metric ansatza above and the Einstein equation

$$G_{\mu}^\nu + \Lambda g_{\mu}^\nu = \sum T_{\mu}^\nu,$$ \hfill (8)

$$-\frac{d-2}{2r}f'(r) - \frac{(d-2)(d-3)}{2r^2}(f(r) - 1) - \Lambda = \sum T_t^t = \sum T_r^r,$$ \hfill (9)

$$-\frac{f''(r)}{2} - \frac{d-3}{2r}f'(r) - \frac{(d-3)(d-4)}{2r^2}(f(r) - 1) - \Lambda = \sum T_{\theta t}^\theta = \sum T_{\theta_d}^{\theta_{d-2}},$$ \hfill (10)

which gives the following master equation

$$r^2f''(r) + F_1rf'(r) + F_2(f(r) - 1) + F_3r^2 + F_4r^{d-2} + F_5r^{-(d-4)} = 0,$$ \hfill (11)

with

$$F_1 = ((d-1)\omega_q + 2d - 5),$$

$$F_2 = (d-3)((d-1)\omega_q + d - 3),$$

$$F_3 = \Lambda \frac{2(d-1)(\omega_q + 1)}{d - 2},$$

$$F_4 = q^2(d-3)((d-1)\omega_q - d + 3),$$

$$F_5 = \frac{2((d-1)\omega_q + 1)a}{d - 2}.$$ \hfill (12)

It is important to note that the required advertising space cosmological constant $\Lambda$ is negative, and then we use Maxwell equations ($\nabla_\nu(\sqrt{-g}F^{\mu\nu}) = 0$) to evaluate the potential

$$A = -\sqrt{\frac{d-2}{2(d-3)}} \frac{q}{r^{d-3}} dt.$$ \hfill (13)

The solution of the main equation is given by

$$f(r) = 1 - \frac{m}{r^{d-3}} + \frac{q^2}{r^{2(d-3)}} - \frac{2\Lambda r^2}{(d-2)(d-1)} - \frac{\alpha}{r^{(d-1)\omega_q + d - 3}} - \frac{2a}{(d - 2)r^{d-4}},$$ \hfill (14)

where $m$ is the integral constant proportional to the mass, and $q$ is proportional to the integral constant black holes, which are given by the following equation \[110, 111\]

$$M = \frac{(d-2)}{16\pi} \Omega_{d-2} m, Q = \sqrt{\frac{2(d-2)(d-3)\Omega_{d-2} q}{8\pi}},$$ \hfill (15)
where $\Omega_{d-2}$ is the volume of unit $(d-2)$-sphere, $\alpha$ is a positive normalization factor related to the quintessence, whose relationship with density $\rho_q$ is \[112\],

$$\rho_q = -\frac{\alpha \omega_q (d-1)(d-2)}{4(\omega_q+1)}.$$ \(16\)

In addition, the asymptotic effect of the quintessence term may be different due to the existence of power $\frac{\alpha r}{\omega_q}$ in Eq. \(14\). When only the quintessential contribution is considered, the above formula can be modified as

$$f_\alpha(r) = 1 - \frac{m}{r^{d-3}} - \frac{\alpha}{r^{(d-1)\omega_q+d-3}}.$$ \(17\)

Where the spacetime becomes asymptotically dS-like for $\omega_q < -\frac{d-3}{d-1}$, otherwise it becomes asymptotically flat. In this paper we consider only the asymptotically dS behavior and set $\omega_q$ to the value $\omega_q = -\frac{d-2}{d-1}$ in numerical analysis.

In Fig. 1, the graphs of the function $f(r)$ are shown for different values of the parameters $a$, $\alpha$ when $d = 5$. In Fig. 2, the graphs of the function $f(r)$ are shown for different values of the parameters $a$, $\alpha$ and $d$, when it is the non-extremal black hole, the equation $f(r) = 0$ has two positive real roots $r_h$ and $r_-$. The $r_h$ represents the radius of the event horizon. When it is the extremal black hole, $f(r) = 0$ has only one root. The mass of the black hole is

$$M = \frac{(d-2)\Omega_{d-2}r_h^{d-3}}{16\pi} + \frac{(d-2)\Omega_{d-2}q^2}{16\pi r_h^{d-3}} + \frac{\Omega_{d-2}Pr_h^{d-1}}{(d-1)} - \frac{\alpha(d-2)\Omega_{d-2}}{16\pi r_h^{(d-1)\omega_q}} - \frac{ar_h\Omega_{d-2}}{8\pi}.$$ \(18\)

Where the mass of the black hole $M$ is defined as its enthalpy. Therefore, the relationship between enthalpy, internal energy and pressure can be expressed by the following equation

$$M = U + PV.$$ \(19\)

### III. PARTICLE ABSORPTION

**A. Scalar particle’s absorption**

In this subsection, we discuss the absorption of the scalar particle in the d-dimensional spacetime and the motion of scattered particles satisfy the Klein-Gordon equation \[113\] of curved spacetime, which is

$$-\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^\mu} - \frac{iq}{\hbar} A_\mu\right)[\sqrt{-g}g^{\mu\nu}(\frac{\partial}{\partial x^\nu} - \frac{iq}{\hbar} A_\nu)]\phi - \frac{m^2}{\hbar^2}\phi = 0.$$ \(20\)
Fig. 1: The relationship between \( f(r) \), \( Q \) and \( r_h \) for different values of \( a \) and \( \alpha \). We choose \( M = 1, l = 1, \omega_q = -\frac{d-2}{d-1}, d = 5 \) and \( \Omega_{d-2} = 1 \).

Where \( m \) and \( q \) are the particle’s mass and charge, respectively, \( \phi \) is the scalar field, and \( A_\mu \) is the electro magnetic potential. Assuming the WKB ansatz for \( \phi \)

\[
\phi = A \exp\left(\frac{iI}{\hbar}\right). \tag{21}
\]

Where \( A \) is a slowly varying amplitude. In a semiclassical approximation, the Hamilton-Jacobi equation for a scalar particle is the lowest order of the WKB expansion of the corresponding Klein-Gordon equation. We can expand Eq. (20) in powers of \( \hbar \) and find that the lowest order term is

\[
g^{\mu\nu}(p_\mu - qA_\mu)(p_\nu - qA_\nu) + m^2 = 0, \tag{22}
\]
Fig. 2: The relationship between $f(r)$ and $r_h$ with parameter values $M = 2, l = 1, \omega_q = -\frac{d-2}{d-1}, Q = 0.7, a = 0.08, \alpha = 0.01$ and $\Omega_{d-2} = 1$.

with

$$p_\mu = \partial_\mu I.$$  \hfill (23)

Which is the Hamilton-Jacobi equation. Where $p_\mu$ is the momentum of the particle, and $I$ is the Hamilton action of the particle. Considering the symmetry of space and time, the role of the Hamiltonian motion of the particles can be divided into

$$I = -\omega t + I_r(r) + \sum_{i=1}^{d-3} I_{\theta_i}(\theta_i) + L\Psi.$$  \hfill (24)

And where the conserved quantities $\omega$ and $L$ are energy and angular momentum of particle, based on the formula (22) of symmetry and translational regulatory moderate, which is the amount of time and space conservation in gravitational systems. In addition, $I_r(r)$ and $I_{\theta_i}(\theta_i)$ are the radial directional component and $\theta$-directional component of the action respectively. The black hole includes a $(d-2)$ -dimensional sphere $\Omega_{d-2}$ because of $d$-dimensional solution, whose the translation symmetry of the last angle coordinate corresponding to the angular momentum $L$. Then, the $(d-2)$ -dimensional sphere can be written as

$$h_{ij}dx^i dx^j = \sum_{i=1}^{d-2} (\prod_{j=1}^{i} \sin^2 \theta_{j-1}) d\theta_i^2, \theta_{d-2} = \Psi.$$  \hfill (25)

To solve the Hamilton-Jacobi equation, we inserting the above ansatz and the contravariant metric of the black hole into the Klein-Gordon equation and yields

$$g^\mu\nu \partial_\mu \partial_\nu = -f(r)^{-1}(\partial_t)^2 + f(r)(\partial_r)^2 + r^{-2} \sum_{i=1}^{d-2} (\prod_{j=1}^{i} \sin^2 \theta_{j-1})(\partial_{\theta_i})^2.$$  \hfill (26)
Substituting the above equations into Eq. (22), we obtain

$$-m^2 = - f(r)^{-1} r^2 (-\omega - qA_t)^2 + f(r)(\partial_r I(r))^2 + r^{-2} \sum_{i=1}^{d-3} (\prod_{j=1}^{i} \sin^{-2}\theta_{j-1}) (\partial_\theta_i I(\theta_i))^2$$

$$+ r^{-2} (\prod_{j=1}^{d-2} \sin^{-2}\theta_{j-1}) L^2. \quad (27)$$

We carry out the separation of variables by introducing a variable $\kappa$ and $R_i$, Therefore, the radial and angular components are

$$\kappa = -m^2 r^2 + \frac{r^2}{f(r)} (-\omega - qA_t)^2 - r^2 f(r)(\partial_r I(r))^2,$$

with

$$R_i = \sum_{i=1}^{d-3} (\prod_{j=1}^{i} \sin^{-2}\theta_{j-1}) (\partial_\theta_i I(\theta_i))^2 + (\prod_{j=1}^{d-2} \sin^{-2}\theta_{j-1}) L^2. \quad (29)$$

Then, we can determine entire equations of motion. The radial and $\theta$-directional are sufficient to obtain the relationship between the equations and the energy of the charged particles. The momenta of the particle are

$$p^r \equiv g^{rt} \partial_t I(r) = f(r) \sqrt{-m^2 r^2 - \kappa} \frac{1}{r^2 f(r)} (-\omega - qA_t)^2 + \frac{1}{f(r)^2} (-\omega - qA_t)^2. \quad (30)$$

We take into account the case of the absorbed particle near the event horizon. This implies $f(r) \to 0$ and the above equation is simplified to

$$p^r = \omega - qA_t = \omega - q\phi,$$

where $\phi = \sqrt{\frac{d-2}{2(d-3)}} \frac{q}{r_h}$ represents the electric potential at the event horizon. The condition of the super radiation is that the boundary condition of the scalar field should be in the asymptotic region and $\omega < q\phi$. Then, at the limit of the outer horizon, the energy relation between conserved quantities and momenta is obtained as

$$E = \sqrt{\frac{d-2}{2(d-3)} \frac{q^2}{r_h^{d-3}}} + |p^r|. \quad (32)$$

The particle enters the black hole in the positive flow of time. At this moment, the energy of the particle should be defined as a positive value thus that the signs of $E$ and $|p^r|$ are both positive. Therefore a positive sign is required in front of the $|p^r|$ term

$$E = \sqrt{\frac{d-2}{2(d-3)} \frac{q^2}{r_h^{d-3}}} + |p^r|,$$

in which various dependencies between variables are reduced to this simple relation.
B. Fermion absorption

In curved spacetime, a spin-1/2 fermion of the mass $m$ and the charge $q$ obeys the Dirac equation

$$i\gamma_\mu(\partial^\mu + \Omega^\mu - \frac{iqA^\mu}{\hbar})\Psi - \frac{m}{\hbar}\Psi = 0. \quad (34)$$

where $\Omega^\mu \equiv \frac{i}{2}\omega^\mu_{ab}\Sigma^{ab}$ is the Lorentz spinor generator, $\Sigma^{ab}$ is the Lorentz spinor generator, $\omega^\mu_{ab}$ is the spin connection and $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$. The Greek indices are raised and lowered by the curved metric $g_{\mu\nu}$, while the Latin indices are governed by the flat metric $\eta^{ab}$. In order to obtain the fermions Hamilton-Jacobi equation, assuming that the WKB ansatz $\Psi$ is

$$\Psi = \exp\left(\frac{iI}{\hbar}\right)u, \quad (35)$$

where $u$ is a slowly varying spinor amplitude. Substituting Eq. (34) into Eq. (35), we find that the lowest order term of $\hbar$ is

$$\gamma_\mu(\partial^\mu I - qA^\mu)u = -mu, \quad (36)$$

which is the Hamilton-Jacobi equation for the fermion. Multiplying both sides of Eq. (36) from the left by $\gamma_\nu(\partial^\nu I + qA^\nu)$ and then using Eq. (36) to simplify the RHS, one obtains

$$\gamma_\nu(\partial^\nu I - qA^\nu)\gamma_\mu(\partial^\mu I - qA^\mu)u = m^2 u. \quad (37)$$

Using $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, we have

$$\left[(\partial^\mu I - qA^\mu)(\partial_\mu I - qA_\mu) - m^2\right] u = 0. \quad (38)$$

Since $u$ is nonzero, the Hamilton-Jacobi equation reduces to

$$(\partial^\mu I - qA^\mu)(\partial_\mu I - qA_\mu) = m^2. \quad (39)$$

which is the same as the Hamilton-Jacobi equation for a scalar. And then, the following formula can be obtained by using the same way

$$E = \sqrt{\frac{d-2}{2(d-3)}\frac{q^2}{r_h^{d-3}} + |p^r|^2}. \quad (40)$$
Table I: The relation between $dS$, $Q$ and $r_h$ for $d = 5$ in the extended phase space via particle absorption.

|     | $a = 0.01$ |     | $a = 10$ |     | $a = 20$ |
|-----|-----------|-----|----------|-----|----------|
| $Q$ | $r_h$     | $dS$| $Q$      | $r_h$| $dS$     | $Q$ | $r_h$ | $dS$     |
| 0.640747 | 1.43509 | 1.720080 | 0.965855 | 1.75208 | 3.231130 | 1.33654 | 2.01130 | 5.074160 |
| 0.64 | 1.40537 | 1.562300 | 0.96 | 1.67481 | 2.512370 | 0.99 | 1.32019 | 0.726566 |
| 0.6 | 1.19173 | 0.797021 | 0.9 | 1.46858 | 1.367870 | 0.9 | 1.20661 | 0.539207 |
| 0.55 | 1.04937 | 0.504545 | 0.8 | 1.26515 | 0.773806 | 0.8 | 1.08407 | 0.384662 |
| 0.5 | 0.93213 | 0.338204 | 0.7 | 1.09707 | 0.476048 | 0.7 | 0.96294 | 0.268491 |
| 0.45 | 0.82613 | 0.228518 | 0.6 | 0.94171 | 0.292462 | 0.6 | 0.84135 | 0.180201 |
| 0.4 | 0.72648 | 0.152164 | 0.5 | 0.79128 | 0.171589 | 0.5 | 0.71771 | 0.113574 |
| 0.35 | 0.63078 | 0.098106 | 0.4 | 0.64168 | 0.091637 | 0.4 | 0.59042 | 0.064739 |
| 0.3 | 0.53766 | 0.060084 | 0.3 | 0.49007 | 0.041264 | 0.3 | 0.45777 | 0.031139 |
| 0.2 | 0.35598 | 0.017191 | 0.2 | 0.33413 | 0.013315 | 0.2 | 0.31756 | 0.010813 |
| 0.1 | 0.17741 | 0.002109 | 0.1 | 0.17163 | 0.001847 | 0.1 | 0.16667 | 0.001639 |

C. The first and second laws of Thermodynamics

In this section, we will discuss the thermodynamics-related issues of a $d$-dimensional charged AdS black hole surrounded by quintessence and cloud of strings. As usual, we consider the cosmological constant as the dynamical pressure of a black hole.

$$P = \frac{-\Lambda}{8\pi} = \frac{(d - 1)(d - 2)}{16\pi l^2}.$$  \hspace{1cm} (41)

The Hawking temperature of the black hole is expressed as

$$T = \frac{f(r)'}{4\pi} \bigg|_{r = r_h} = \frac{m(d - 3)}{4\pi r_h^{d-2}} + \frac{q^2(3-d)}{2\pi r_h^{2d-5}} + \frac{8Pr_h}{(d-2)(d-1)}$$  \hspace{1cm} (42)

$$+ \frac{[(d-1)\omega_q + d-3]\alpha}{4\pi r_h^{(d-1)\omega_q + d-2}} + \frac{(d-4)a}{2\pi(d-2)r_h^{d-3}}.$$  

With the help of the Bekenstein-Hawking formula [5], Entropy can be obtained

$$S = \frac{A_{d-2}}{4} = \frac{\Omega_{d-2}}{4} r_h^{d-2}.$$  \hspace{1cm} (43)

After the black hole absorbs a particle, the change in the enthalpy is connected to the
Table II: The relation between $dS$, $Q$ and $r_h$ for $d = 6$ in the extended phase space via particle absorption.

|   | $a = 0.01$ |       |       | $a = 10$ |       |       | $a = 20$ |       |       |
|---|------------|-------|-------|----------|-------|-------|----------|-------|-------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.751095 | 1.26380 | 1.339280 | 1.09831 | 1.43654 | 2.198080 | 1.48119 | 1.57562 | 2.624600 |
| 0.75 | 1.24297 | 1.210610 | 0.99 | 1.23344 | 0.813316 | 0.99 | 1.07695 | 0.34108 |
| 0.7 | 1.10288 | 0.631245 | 0.9 | 1.12473 | 0.517795 | 0.9 | 1.01152 | 0.26210 |
| 0.65 | 1.02777 | 0.437442 | 0.8 | 1.02853 | 0.345687 | 0.8 | 0.93810 | 0.19275 |
| 0.6 | 0.95369 | 0.316940 | 0.7 | 0.93622 | 0.231018 | 0.7 | 0.86278 | 0.13814 |
| 0.55 | 0.88937 | 0.232461 | 0.6 | 0.84395 | 0.150458 | 0.6 | 0.78436 | 0.09518 |
| 0.5 | 0.82727 | 0.170035 | 0.5 | 0.74900 | 0.092995 | 0.5 | 0.70140 | 0.06181 |
| 0.4 | 0.70437 | 0.086572 | 0.4 | 0.64873 | 0.052578 | 0.4 | 0.61198 | 0.03664 |
| 0.3 | 0.57726 | 0.038301 | 0.3 | 0.53976 | 0.025506 | 0.3 | 0.51316 | 0.01867 |
| 0.2 | 0.43870 | 0.012623 | 0.2 | 0.41655 | 0.009229 | 0.2 | 0.39967 | 0.00715 |
| 0.1 | 0.27580 | 0.001955 | 0.1 | 0.26666 | 0.001595 | 0.1 | 0.25905 | 0.00133 |

The change in internal energy as

$$E = dU = d(M - PV).$$

with

$$dU = dM - PdV - VdP = \frac{8\pi Q}{2(d - 3) r_h^{d-3} \Omega_{d-2}} dQ + |p'|. \quad (45)$$

The initial state of the black hole is represented by $(M, Q, P, a, \alpha, r_h)$, and the final state is represented by $(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h)$. The functions $f(M, Q, P, a, \alpha, r_h)$ and $f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h)$ satisfy

$$f(M, Q, P, a, \alpha, r_h) = f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) = 0. \quad (46)$$

The relation between the functions $f(M, Q, P, a, \alpha, r_h)$ and $f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h)$ is

$$f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) = f(M, Q, P, a, \alpha, r_h) + \frac{\partial f}{\partial M} |_{r=r_h} dM + \frac{\partial f}{\partial Q} |_{r=r_h} dQ + \frac{\partial f}{\partial r} |_{r=r_h} dr_h + \frac{\partial f}{\partial P} |_{r=r_h} dP + \frac{\partial f}{\partial a} |_{r=r_h} da + \frac{\partial f}{\partial \alpha} |_{r=r_h} d\alpha. \quad (47)$$
Table III: The relation between $dS$, $Q$ and $r_h$ for $d = 7$ in the extended phase space via particle absorption.

| $a = 0.01$ | $a = 10$ | $a = 20$ |
|------------|---------|---------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.818145 | 1.16885 | 1.0788 | 1.17528 | 1.28263 | 1.459720 | 1.56137 | 1.37345 | 1.795660 |
| 0.8 | 1.10675 | 0.728925 | 0.99 | 1.07803 | 0.443659 | 0.99 | 0.99049 | 0.219661 |
| 0.75 | 1.03766 | 0.480032 | 0.9 | 1.01703 | 0.316912 | 0.9 | 0.94394 | 0.170971 |
| 0.7 | 0.98535 | 0.351321 | 0.8 | 0.95202 | 0.220464 | 0.8 | 0.89072 | 0.127292 |
| 0.65 | 0.93833 | 0.264973 | 0.7 | 0.88699 | 0.151683 | 0.7 | 0.83506 | 0.092296 |
| 0.6 | 0.89360 | 0.201726 | 0.6 | 0.81988 | 0.101251 | 0.6 | 0.77593 | 0.064373 |
| 0.5 | 0.85079 | 0.115536 | 0.5 | 0.74876 | 0.064123 | 0.5 | 0.71197 | 0.042409 |
| 0.4 | 0.71509 | 0.061968 | 0.4 | 0.67125 | 0.037275 | 0.4 | 0.64122 | 0.025614 |
| 0.3 | 0.61620 | 0.028949 | 0.3 | 0.58382 | 0.018751 | 0.3 | 0.56042 | 0.013418 |
| 0.2 | 0.50162 | 0.010235 | 0.2 | 0.47991 | 0.007168 | 0.2 | 0.46334 | 0.005381 |
| 0.1 | 0.35417 | 0.001781 | 0.1 | 0.34299 | 0.001376 | 0.1 | 0.33386 | 0.001105 |

Where

\[
\frac{\partial f}{\partial M} \bigg|_{r=r_h} = -\frac{16\pi}{r_h^{d-3}(d-2)\Omega_{d-2}},
\]

\[
\frac{\partial f}{\partial Q} \bigg|_{r=r_h} = \frac{16\pi q}{r_h^{2(d-3)}\Omega_{d-2}\sqrt{2(d-3)(d-2)}},
\]

\[
\frac{\partial f}{\partial P} \bigg|_{r=r_h} = \frac{16\pi r_h^2}{(d-2)(d-1)},
\]

\[
\frac{\partial f}{\partial r} \bigg|_{r=r_h} = 4\pi T,
\]

\[
\frac{\partial f}{\partial \alpha} \bigg|_{r=r_h} = -\frac{1}{r_h^{(d-1)\omega+q+d-3}},
\]

\[
\frac{\partial f}{\partial a} \bigg|_{r=r_h} = \frac{-2}{(d-2)r^{d-4}}.
\]

Combining Eq. (45) with Eq. (47), we get

\[
dr_h = \frac{2(r_h^{d-3}da + \frac{1}{r_h^{(d-1)\omega+q+d-3}}d\alpha + \frac{16\pi}{r_h^{(d-2)\Omega_{d-2}}}|p'|}{(d-2)r^{d-4}}.
\]

Then the variations of entropy and thermodynamic volume of the black hole are obtained.
Table IV: The relation between $dS$, $Q$ and $r_h$ for $d = 8$ in the extended phase space via particle absorption.

| $Q$    | $r_h$ | $dS$  | $Q$    | $r_h$ | $dS$  | $Q$    | $r_h$ | $dS$  |
|--------|-------|-------|--------|-------|-------|--------|-------|-------|
| 0.861699 | 1.11151 | 0.900437 | 1.22409 | 1.19494 | 1.15260 | 1.611 | 1.26111 | 1.369570 |
| 0.8     | 1.01782 | 0.438211 | 0.99    | 1.01482 | 0.31313 | 0.99   | 0.95131 | 0.162462 |
| 0.75    | 0.97711 | 0.324878 | 0.9     | 0.96949 | 0.22958 | 0.9    | 0.91486 | 0.127323 |
| 0.7     | 0.94082 | 0.249153 | 0.8     | 0.91979 | 0.16287 | 0.8    | 0.87272 | 0.095452 |
| 0.65    | 0.90635 | 0.193416 | 0.7     | 0.86901 | 0.11381 | 0.7    | 0.82812 | 0.069675 |
| 0.6     | 0.87253 | 0.150415 | 0.6     | 0.81565 | 0.07703 | 0.6    | 0.78013 | 0.048944 |
| 0.5     | 0.80409 | 0.089039 | 0.5     | 0.75808 | 0.04948 | 0.5    | 0.72747 | 0.032514 |
| 0.4     | 0.73116 | 0.049179 | 0.4     | 0.69412 | 0.02924 | 0.4    | 0.66824 | 0.019851 |
| 0.3     | 0.64919 | 0.023723 | 0.3     | 0.62028 | 0.01502 | 0.3    | 0.59919 | 0.010559 |
| 0.2     | 0.55070 | 0.008747 | 0.2     | 0.52978 | 0.00593 | 0.2    | 0.51382 | 0.004342 |
| 0.1     | 0.41685 | 0.001632 | 0.1     | 0.40457 | 0.00121 | 0.1    | 0.39465 | 0.000939 |

Table V: The relation between $dS$, $Q$ and $r_h$ for $d = 5$ in the extended phase space via particle absorption.

| $Q$    | $r_h$ | $dS$  | $Q$    | $r_h$ | $dS$  | $Q$    | $r_h$ | $dS$  |
|--------|-------|-------|--------|-------|-------|--------|-------|-------|
| 0.640747 | 1.43509 | 1.72008 | 0.99    | 1.25816 | 3.276210 | 0.99   | 1.11358 | 6.888790 |
| 0.6     | 1.19173 | 0.797021 | 0.9     | 1.18899 | 1.840720 | 0.9    | 1.05811 | 2.220120 |
| 0.55    | 1.04937 | 0.504545 | 0.8     | 1.10560 | 1.105600 | 0.8    | 0.99118 | 1.087460 |
| 0.5     | 0.93212 | 0.338204 | 0.7     | 1.01377 | 0.671353 | 0.7    | 0.91733 | 0.619633 |
| 0.45    | 0.82613 | 0.228518 | 0.6     | 0.91142 | 0.411186 | 0.6    | 0.83458 | 0.367834 |
| 0.4     | 0.72648 | 0.152164 | 0.5     | 0.79599 | 0.240074 | 0.5    | 0.74015 | 0.214234 |
| 0.35    | 0.63078 | 0.098105 | 0.4     | 0.66474 | 0.126627 | 0.4    | 0.63015 | 0.115212 |
| 0.3     | 0.53766 | 0.060086 | 0.3     | 0.51596 | 0.055265 | 0.3    | 0.50004 | 0.052067 |
| 0.2     | 0.35598 | 0.017191 | 0.2     | 0.35138 | 0.016747 | 0.2    | 0.34726 | 0.016361 |
| 0.1     | 0.17741 | 0.002109 | 0.1     | 0.17712 | 0.002103 | 0.1    | 0.17682 | 0.002096 |
Table VI: The relation between $dS$, $Q$ and $r_h$ for $d = 6$ in the extended phase space via particle absorption.

| $\alpha = 0.01$ | $\alpha = 10$ | $\alpha = 20$ |
|-----------------|----------------|----------------|
| $Q$             | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.751095        | 1.2638 | 1.339280 | 0.99 | 1.04990 | 0.778412 | 0.99 | 0.96842 | 0.689644 |
| 0.7             | 1.10288 | 0.631251 | 0.9 | 1.00783 | 0.585157 | 0.9 | 0.93329 | 0.506712 |
| 0.6             | 0.95369 | 0.316940 | 0.8 | 0.95630 | 0.422232 | 0.8 | 0.89028 | 0.361227 |
| 0.55            | 0.88937 | 0.232461 | 0.7 | 0.89852 | 0.297847 | 0.7 | 0.84200 | 0.254615 |
| 0.5             | 0.82727 | 0.170035 | 0.6 | 0.83281 | 0.201932 | 0.6 | 0.78685 | 0.174284 |
| 0.45            | 0.76594 | 0.122736 | 0.5 | 0.75694 | 0.128363 | 0.5 | 0.72250 | 0.113016 |
| 0.4             | 0.70437 | 0.086573 | 0.4 | 0.66796 | 0.073522 | 0.4 | 0.64545 | 0.066696 |
| 0.3             | 0.57726 | 0.038301 | 0.3 | 0.56214 | 0.035340 | 0.3 | 0.55061 | 0.033263 |
| 0.2             | 0.43871 | 0.012623 | 0.2 | 0.43457 | 0.012268 | 0.2 | 0.43085 | 0.011959 |
| 0.1             | 0.27580 | 0.001955 | 0.1 | 0.27538 | 0.001946 | 0.1 | 0.27496 | 0.001938 |

Table VII: The relation between $dS$, $Q$ and $r_h$ for $d = 7$ in the extended phase space via particle absorption.

| $\alpha = 0.01$ | $\alpha = 10$ | $\alpha = 20$ |
|-----------------|----------------|----------------|
| $Q$             | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.818145        | 1.16885 | 1.078800 | 0.99 | 0.97520 | 0.441467 | 0.99 | 0.918038 | 0.366732 |
| 0.8             | 1.12844 | 0.833796 | 0.9 | 0.94455 | 0.348482 | 0.9 | 0.891935 | 0.289174 |
| 0.7             | 0.98534 | 0.351322 | 0.8 | 0.90670 | 0.263045 | 0.8 | 0.859722 | 0.219253 |
| 0.65            | 0.93833 | 0.264973 | 0.7 | 0.86385 | 0.192861 | 0.7 | 0.823226 | 0.823226 |
| 0.6             | 0.89360 | 0.20173 | 0.6 | 0.81459 | 0.135414 | 0.6 | 0.781100 | 0.116055 |
| 0.5             | 0.80578 | 0.115536 | 0.5 | 0.75695 | 0.089096 | 0.5 | 0.731328 | 0.078253 |
| 0.4             | 0.71509 | 0.061968 | 0.4 | 0.68812 | 0.053003 | 0.4 | 0.67080 | 0.048053 |
| 0.3             | 0.61620 | 0.028949 | 0.3 | 0.60401 | 0.026718 | 0.3 | 0.594569 | 0.025125 |
| 0.2             | 0.50162 | 0.010235 | 0.2 | 0.49782 | 0.009929 | 0.2 | 0.494408 | 0.009664 |
| 0.1             | 0.35416 | 0.001786 | 0.1 | 0.35368 | 0.001771 | 0.1 | 0.353200 | 0.001762 |
as
\[
dS = \frac{\Omega_{d-2} r_h}{2} da + \frac{\Omega_{d-2}}{4 r_{(d-1)q}^2} d\alpha + 4\pi |p^r| \frac{1}{4\pi T - \frac{16\pi Pr_h}{d-2}},
\]
(50)
and
\[
dV = \frac{2\Omega_{d-2} r_h^2}{(d-2)} da + \frac{\Omega_{d-2}}{r_{(d-1)q}^2} d\alpha + \frac{16\pi r_h}{(d-2)} |p^r| \frac{1}{4\pi T - \frac{16\pi Pr_h}{d-2}}.
\]
(51)

Using Eqs. (50) and (51) yields
\[
TdS - PdV = \frac{\frac{T\Omega_{d-2} r_h}{2} - 2\Omega_{d-2} r_h^2}{4\pi T - \frac{16\pi Pr_h}{d-2}} da + \frac{\frac{T\Omega_{d-2} (d-2)}{4 r_{(d-1)q}^2} - \frac{P\Omega_{d-2}}{r_{(d-1)q}^2} - 1}{4\pi T - \frac{16\pi Pr_h}{d-2}} d\alpha + 4\pi T \frac{16\pi r_h P}{(d-2)} |p^r|.
\]
(52)

The generalized first law in the extended phase space which account for the cosmological constant effect, the cloud of strings and the quintessence contributions is then expressed as
\[
dM = TdS + VdP + \phi dQ + A da + Q d\alpha.
\]
(53)
Table VIII: The relation between $dS$, $Q$ and $r_h$ for $d = 8$ in the extended phase space via particle absorption.

| $\alpha = 0.01$ | $\alpha = 10$ | $\alpha = 20$ |
|-----------------|---------------|---------------|
| $Q$             | $r_h$         | $dS$          | $Q$             | $r_h$         | $dS$          | $Q$             | $r_h$         | $dS$          |
| 0.861699        | 1.11151       | 0.900437      | 0.99           | 0.914176       | 0.314099      | 0.99           | 0.89711       | 0.256923      |
| 0.8             | 1.01782       | 0.438211      | 0.99           | 0.91718       | 0.178334      | 0.99           | 0.87616       | 0.207588      |
| 0.75            | 0.97711       | 0.324878      | 0.89           | 0.88701       | 0.194437      | 0.89           | 0.85019       | 0.161139      |
| 0.7             | 0.94082       | 0.249153      | 0.89           | 0.85267       | 0.145112      | 0.89           | 0.82059       | 0.121929      |
| 0.6             | 0.87253       | 0.150415      | 0.89           | 0.81291       | 0.103680      | 0.89           | 0.78620       | 0.088816      |
| 0.5             | 0.80409       | 0.089039      | 0.89           | 0.76597       | 0.069489      | 0.89           | 0.74525       | 0.061066      |
| 0.4             | 0.73116       | 0.049179      | 0.89           | 0.70928       | 0.042241      | 0.89           | 0.69494       | 0.038315      |
| 0.3             | 0.64919       | 0.023723      | 0.89           | 0.63877       | 0.021899      | 0.89           | 0.63064       | 0.020586      |
| 0.2             | 0.55071       | 0.0087473     | 0.89           | 0.54719       | 0.008477      | 0.89           | 0.54403       | 0.008241      |
| 0.1             | 0.41685       | 0.0016326     | 0.89           | 0.41633       | 0.001623      | 0.89           | 0.41583       | 0.001613      |

Where $Q$ and $A$ are the physical quantity conjugated to the parameter $\alpha$ and $a$ respectively, they satisfy

$$Q = \left( \frac{\partial M}{\partial \alpha} \right)_{S,P} = \frac{(2-d)\Omega_{d-2}}{16\pi r_h^{(d-1)\omega_q}}, \quad A = \left( \frac{\partial M}{\partial a} \right)_{S,P} = \frac{-\Omega_{d-2}r_h}{8\pi}. \quad (54)$$

According to the above, the first law of thermodynamics proved to be satisfied. However, the effectiveness of the first law does not mean that the second law is also effective. The second law of thermodynamics needs to be tested in extended phase space, which states that the entropy of the black hole never decreases. In other words, as the particle is absorbed, the entropy of the final state is always greater than the initial state according to the second law of thermodynamics.

When it is the extremal black hole, the temperature is zero. Then Eq. (50) is modified as

$$dS = \frac{\Omega_{d-2}r_h}{2} da + \frac{\Omega_{d-2}r_h}{4\pi r_h^{(d-1)\omega_q}} d\alpha + 4\pi |p^r| \left[ p^r \right]_{l=1}.$$  \quad (55)

It is negative, which means that the second law is invalid for the extremal black hole. Next, we focus on investigating the non-extremal black hole by analyzing Eq. (50) numerically to represent the changes of entropy intuitively. We set $M = 1, \Omega_{d-2} = 1, |p^r| = 1, l = 1$
to discuss the influence of other parameters on the change of entropy for given values of \( d \).

First, the object of our explore is the behaviour of the function (50), for different values of \( a \) in the case of \( da = 0.9, d\alpha = 0.6, \alpha = 0.01 \), which are represented by Fig. 3 and Table II, Table III and Table IV. When the charge is less than the extremal charge, it can be obtained that the event horizon of the black hole and the variation of entropy decreases when the charge of the black hole decreases. While for \( dS \), there is a divergent point, which divides the variation of entropy into the positive and negative region. We also find that as the values of \( a \) decrease, the values of the critical horizon where \( dS \) is divergent become smaller. And as the values of \( d \) decrease, the values of the divergent point become greater.

So the second law of thermodynamics is violated in extended phase space. This conclusion is independent of the values of \( d \) and \( a \).

We also can set \( a = 0.01 \) investigate how \( d \) and \( \alpha \) affect the values of \( dS \). For different values of \( \alpha \) and \( d \), the Eq. (50) is represented in Fig. 4 and Table V, Table VI, Table VII.
Table IX: The relation between $dS$, $Q$ and $r_h$ for $d = 5$ in the extended phase space via particle absorption.

| $d\alpha = 0.6$, $da = 0.9$ | $d\alpha = 0.6$, $da = -0.9$ | $d\alpha = -0.6$, $da = 0.9$ |
|---------------------------|---------------------------|---------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.796654 | 1.697710 | 5.7962600 | 0.796654 | 1.697710 | 5.2260700 | 0.796654 | 1.697710 | 4.152850 |
| 0.7 | 1.299390 | 1.1726300 | 0.7 | 1.299390 | 1.0756400 | 0.7 | 1.299390 | 1.008870 |
| 0.6 | 1.081420 | 0.5704530 | 0.6 | 1.081420 | 0.5296950 | 0.6 | 1.081420 | 0.522788 |
| 0.5 | 0.889426 | 0.2910120 | 0.5 | 0.889426 | 0.2734750 | 0.5 | 0.889426 | 0.277139 |
| 0.4 | 0.707405 | 0.1395270 | 0.4 | 0.707405 | 0.1327160 | 0.4 | 0.707405 | 0.136119 |
| 0.3 | 0.529490 | 0.0569941 | 0.3 | 0.529490 | 0.0548840 | 0.3 | 0.529490 | 0.056402 |
| 0.2 | 0.353027 | 0.0166699 | 0.2 | 0.353027 | 0.0162543 | 0.2 | 0.353027 | 0.016618 |
| 0.1 | 0.176765 | 0.0020799 | 0.1 | 0.176765 | 0.0020537 | 0.1 | 0.176765 | 0.002079 |

From these tables, it can be seen that the event horizon of the black hole and the variation of entropy decrease when the charge of the black hole decreases. From Figs above, it is evident that there exists a phase transition point that divides the value of $dS$ into positive and negative regions. The values of the divergent point decreases as $d$ increases. At the same time, the value of divergence point also decreases with the increase of $\alpha$. The invalidity of the second law for the near-extremal black holes thus is universal, independent of the values of $\alpha$ and $d$.

From Ref. [114] we know that the value of the state parameter of the cloud of strings or quintessence affects the second law of thermodynamics. Still, the parameters do not determine whether the second law of thermodynamics is ultimately violated, which is consistent with our conclusion.

In fact, the relation between $dS$ and $r_h$ also can be effected by $da$ and $d\alpha$. We fix $a = 0.001$ and $\alpha = 0.001$ to investigate entropy in different dimensions. From Fig. 3, there is a phase transition point which divides $dS$ into two branches. By comparing the data in the Table IX, Table X, Table XI and Table XII, we find that $d\alpha$ has more obvious influence on the change of entropy. While for the values of the divergent point, it decreases as $d$ increases. In order to explore the difference of entropy change in high and low dimensional cases, the function graph is used to express the relationship between $dS$, $Q$ and $r_h$ in different situations, which
Table X: The relation between $dS$, $Q$ and $r_h$ for $d = 6$ in the extended phase space via particle absorption.

| $d\alpha = 0.6, da = 0.9$ | $d\alpha = 0.6, da = -0.9$ | $d\alpha = -0.6, da = 0.9$ |
|---------------------------|---------------------------|---------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.978391 | 1.465170 | 5.184920 | 0.978391 | 1.465170 | 4.757350 | 0.978391 | 1.465170 | 3.391810 |
| 0.9 | 1.267230 | 1.430560 | 0.9 | 1.267230 | 1.319450 | 0.9 | 1.267230 | 1.129080 |
| 0.8 | 1.137970 | 0.754340 | 0.8 | 1.137970 | 0.699487 | 0.8 | 1.137970 | 0.646563 |
| 0.7 | 1.025040 | 0.443271 | 0.7 | 1.025040 | 0.413400 | 0.7 | 1.025040 | 0.400375 |
| 0.6 | 0.916339 | 0.263653 | 0.6 | 0.916339 | 0.247428 | 0.6 | 0.916339 | 0.247008 |
| 0.5 | 0.806853 | 0.151338 | 0.5 | 0.806853 | 0.143002 | 0.5 | 0.806853 | 0.145500 |
| 0.4 | 0.693037 | 0.079978 | 0.4 | 0.693037 | 0.076146 | 0.4 | 0.693037 | 0.078278 |
| 0.3 | 0.571182 | 0.036246 | 0.3 | 0.571182 | 0.034801 | 0.3 | 0.571182 | 0.035887 |
| 0.2 | 0.435772 | 0.012159 | 0.2 | 0.435772 | 0.011786 | 0.2 | 0.435772 | 0.012118 |
| 0.1 | 0.274769 | 0.001913 | 0.1 | 0.274769 | 0.001875 | 0.1 | 0.274769 | 0.001916 |

Table XI: The relation between $dS$, $Q$ and $r_h$ for $d = 7$ in the extended phase space via particle absorption.

| $d\alpha = 0.6, da = 0.9$ | $d\alpha = 0.6, da = -0.9$ | $d\alpha = -0.6, da = 0.9$ |
|---------------------------|---------------------------|---------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 1.09785 | 1.329700 | 4.421750 | 1.09785 | 1.329700 | 4.09676 | 1.09785 | 1.329700 | 2.728440 |
| 0.9 | 1.094730 | 0.684067 | 0.9 | 1.094730 | 0.636731 | 0.9 | 1.094730 | 0.570757 |
| 0.8 | 1.019380 | 0.429625 | 0.8 | 1.019380 | 0.401167 | 0.8 | 1.019380 | 0.378410 |
| 0.7 | 0.945735 | 0.274614 | 0.7 | 0.945735 | 0.257376 | 0.7 | 0.945735 | 0.251631 |
| 0.6 | 0.870696 | 0.172710 | 0.6 | 0.870696 | 0.162559 | 0.6 | 0.870696 | 0.162987 |
| 0.5 | 0.791846 | 0.103746 | 0.5 | 0.791846 | 0.098126 | 0.5 | 0.791846 | 0.100064 |
| 0.4 | 0.706553 | 0.057305 | 0.4 | 0.706553 | 0.054505 | 0.4 | 0.706553 | 0.056142 |
| 0.3 | 0.611095 | 0.027317 | 0.3 | 0.611095 | 0.026153 | 0.3 | 0.611095 | 0.027046 |
| 0.2 | 0.498774 | 0.009809 | 0.2 | 0.498774 | 0.009465 | 0.2 | 0.498774 | 0.009773 |
| 0.1 | 0.352893 | 0.001732 | 0.1 | 0.352893 | 0.001688 | 0.1 | 0.352893 | 0.001731 |
Table XII: The relation between $dS$, $Q$ and $r_h$ for $d = 8$ in the extended phase space via particle absorption.

| $d\alpha = 0.6, da = 0.9$ | $d\alpha = 0.6, da = -0.9$ | $d\alpha = -0.6, da = 0.9$ |
|--------------------------|--------------------------|--------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 1.17861 | 1.244110 | 3.755540 | 1.17861 | 1.244110 | 3.500120 | 1.17861 | 1.244110 | 2.232980 |
| 0.99 | 1.075100 | 0.681459 | 0.99 | 1.075100 | 0.635796 | 0.99 | 1.075100 | 0.550287 |
| 0.9 | 1.024280 | 0.459031 | 0.9 | 1.024280 | 0.428949 | 0.9 | 1.024280 | 0.391199 |
| 0.8 | 0.969409 | 0.304303 | 0.8 | 0.969409 | 0.284994 | 0.8 | 0.969409 | 0.271241 |
| 0.7 | 0.913819 | 0.201539 | 0.7 | 0.913819 | 0.189263 | 0.7 | 0.913819 | 0.185893 |
| 0.6 | 0.855703 | 0.130299 | 0.6 | 0.855703 | 0.122757 | 0.6 | 0.855703 | 0.123379 |
| 0.5 | 0.793252 | 0.080237 | 0.5 | 0.793252 | 0.075880 | 0.5 | 0.793252 | 0.077500 |
| 0.4 | 0.724135 | 0.045465 | 0.4 | 0.724135 | 0.043189 | 0.4 | 0.724135 | 0.044559 |
| 0.3 | 0.644708 | 0.022331 | 0.3 | 0.644708 | 0.021328 | 0.3 | 0.644708 | 0.022108 |
| 0.2 | 0.547972 | 0.008357 | 0.2 | 0.547972 | 0.008029 | 0.2 | 0.547972 | 0.008319 |
| 0.1 | 0.415445 | 0.001579 | 0.1 | 0.415445 | 0.001533 | 0.1 | 0.415445 | 0.001578 |

is shown in Fig. (6) and Fig. (7). It is clear that there is indeed a phase change point that divides $dS$ into positive and negative values. This conclusion is independent of dimension $d$. From the above discussion, it can be concluded that the second law of thermodynamics is not always valid for near-extremal black holes in the extended phase space.

D. Stability of horizon

In this section, we consider whether the horizons continue exist in the final state because the horizons are significant in defining a black hole. The outer horizon not only divides the inside and outside of the black hole, but is also the location where the thermodynamic variables are defined. We will examine the stability of horizon. The mass and charge of the black hole will change after absorbing particles, which will lead to changes in the field of horizon inside and outside the black hole. The event horizon is determined by the metric component $f(r)$. If the minimal value of $f(r)$ is negative or zero, the horizon exists. Otherwise, the horizon does not exist.
Fig. 5: The relationship between $dS, Q$ and $r_h$ for $da$ and $d\alpha$.

The sign of the minimum value in the final of $f(r)$ state can be obtained in term of the initial state. Assuming $(M, Q, P, r_0, a, \alpha)$ and $(M + dM, Q + dQ, P + dP, r_0 + dr_0, a + da, \alpha + d\alpha)$ represent the initial state and the final state, respectively. At $r = r_0 + dr_0$, $f(M + dM, Q + dQ, P + dP, r_0 + dr_0, a + da, \alpha + d\alpha)$ is written as

$$
f(M + dM, Q + dQ, P + dP, r_0 + dr_0, a + da, \alpha + d\alpha) = \delta + \frac{\partial f}{\partial M}|_{r=r_0}dM + \frac{\partial f}{\partial Q}|_{r=r_0}dQ + \frac{\partial f}{\partial P}|_{r=r_0}dP$$

$$+ \frac{\partial f}{\partial a}|_{r=r_0}da + \frac{\partial f}{\partial \alpha}|_{r=r_0}d\alpha + \frac{\partial f}{\partial r}|_{r=r_0}dr,$$

(56)
\[
\begin{align*}
\frac{\partial f}{\partial r} & \bigg|_{r=r_0} = 0, \\
\frac{\partial f}{\partial M} & \bigg|_{r=r_0} = -\frac{16\pi}{r_0^{d-3}(d-2)\Omega_{d-2}}, \\
\frac{\partial f}{\partial Q} & \bigg|_{r=r_0} = \frac{16\pi q}{r_0^{2(d-3)}\Omega_{d-2}\sqrt{2}(d-3)(d-2)}, \\
\frac{\partial f}{\partial P} & \bigg|_{r=r_0} = \frac{16\pi r_0^2}{(d-2)(d-1)}, \\
\frac{\partial f}{\partial \alpha} & \bigg|_{r=r_0} = -\frac{1}{r_0^{(d-1)\omega_q+d-3}}, \\
\frac{\partial f}{\partial a} & \bigg|_{r=r_0} = -\frac{2}{(d-2)r_0^{d-4}}.
\end{align*}
\]

Fig. 6: The relationship between $dS, Q$ and $r_h$.

where
Therefore, we have
\[ f(M, Q, P, a, \alpha, r_0) \equiv f_0 = \delta \leq 0, \quad (58) \]
and
\[ \partial_r f(M, Q, P, a, \alpha, r_0) \equiv f_{\min}' = 0. \quad (59) \]
From Eqs. (56), (57), (58) and (59) we obtain
\[
\begin{align*}
  f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) = & \\
  \delta + \frac{16\pi q}{\Omega_{d-2}r_0^{d-3}} \sqrt{2(d - 2)(d - 3)} \left( \frac{1}{r_0^{d-3}} - \frac{1}{r_h^{d-3}} \right) dQ & \\
  - \frac{16\pi}{r_0^{d-3} \Omega_{d-2}} \left[ \frac{16\pi Pr_h}{(d - 2)(4\pi T - \frac{16\pi Pr_h}{d - 2})} + 1 \right] |p^r|^2 & \\
  - \frac{2}{(d - 2)r_0^{d-3}} \left[ r_0 + \frac{16\pi Pr_h^2}{(d - 2)4\pi T - 16\pi Pr_h} \right] da & \\
  - r_0^{3-d} \left[ \frac{16\pi Pr_h}{r_h^{(d-1)\omega_q} [4\pi T(d - 2) - 16\pi Pr_h]} + \frac{1}{r_0^{(d-1)\omega_q}} \right] d\alpha.
\end{align*} \quad (60)
\]
When the initial black hole is the extremal black hole, \( r_0 = r_h \), \( T = 0 \) and \( \delta = 0 \). Then we can obtain \( f_{\min} = \delta = 0 \) and \( f_{\min}' = 0 \). Hence, Eq. (60) is written as
\[
\begin{align*}
  f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, dr_0 + r_0) = & \\
  0. \quad (61)
\end{align*} \]
This implies that the horizon of the extremal black hole is still exists at the final state. When the initial black hole is the near-extremal black hole, \( r_0 \) and \( r_h \) do not coincide. Two
locations \((r_h, r_0)\) are very close for the near-extremal black holes. Thus, we assume the condition \(r_h = r_0 + \epsilon\). Using this condition, the Eq. (60) can be expand at the location \(r_0\). To the first order, it yields

\[
\delta \left[ f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \right] = \\
\delta + \frac{16\pi q}{\Omega_{d-2} \sqrt{2(d-2)(d-3)}} \left[ \frac{(d-3)\epsilon}{r_0^{d-2}} + O(\epsilon)^2 \right] dQ \\
- \frac{16\pi}{\Omega_{d-2} r_0^{d-3}} \left[ \frac{1}{16\pi P(r_0 + \epsilon)} \right] dP + \frac{(d-2)(4\pi T) - 16\pi P(r_0 + \epsilon)}{16\pi P(r_0 + \epsilon)} d\alpha \quad \text{(62)}
\]

where \(\delta\) and \(\epsilon\) are all the very small quantity. We set \(dQ \sim \epsilon, d\alpha \sim \epsilon, da \sim \epsilon\). Thus, Eq. (62) is modified as

\[
\delta \left[ f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \right] = \\
\delta + 16\pi q \left[ \frac{1}{\Omega_{d-2} \sqrt{2(d-2)(d-3)}} \right] \left[ \frac{(d-3)\epsilon}{r_0^{d-2}} + O(\epsilon)^2 \right] dQ \\
- \frac{16\pi}{\Omega_{d-2} r_0^{d-3}} \left[ \frac{1}{16\pi P(r_0 + \epsilon)} \right] dP + \frac{(d-2)(4\pi T) - 16\pi P(r_0 + \epsilon)}{16\pi P(r_0 + \epsilon)} d\alpha \quad \text{(63)}
\]

Therefore, at the minimum point, we have

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \leq 0. \quad \text{(64)}
\]

Where the term is negative, which implies that the minimum value is always negative. Hence, the stability of horizons exists in spacetime. The near-extremal black hole can not be overcharged, which stays near-extremal after absorbing a particle. The WCCC is satisfied for both the extremal and near-extremal black holes in the extended phase space.

**E. A new assumption: \(E = dM\)**

In this section, by dropping particles into the black hole, we have employed the recently new assumption \([30, 115]\) that the change of the black hole mass(enthalpy) should be the
same amount as the energy of an infalling particle \( E = dM \), to test the laws of thermodynamics and stability of horizon of a black hole in extended phase spaces under this assumption. Where the energy-momentum relation near the event horizon can be simplified as

\[
E = \phi dQ + |p^r|, \tag{65}
\]

In Eq. (65), we choose the positive sign in front of the \( |p^r| \) term to ensure the positive flow of time direction of a particle when it fell into the black hole. If it is assumed that the changes in the black hole parameters are not lost in this process, the changes in the black hole parameters should be same as the changes in the falling particles. In this sense, the relationship between the infalling particle changes the enthalpy of the black hole is

\[
E = dM, \tag{66}
\]

In this case, Eq. (65) change into

\[
dM = \phi dQ + p^r. \tag{67}
\]

As a charged particle dropped into the black hole, the configurations of the black hole will be changed. This progress will lead to a shift for the horizon. The relation between the functions \( f(M, Q, P, a, \alpha, r_h) \) and \( f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) \) is

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) = f(M, Q, P, a, \alpha, r_h) + \frac{\partial f}{\partial M}|_{r=r_h} dM + \frac{\partial f}{\partial Q}|_{r=r_h} dQ + \frac{\partial f}{\partial r}|_{r=r_h} dr_h \tag{68}
\]

\[
+ \frac{\partial f}{\partial P}|_{r=r_h} dP + \frac{\partial f}{\partial a}|_{r=r_h} da + \frac{\partial f}{\partial \alpha}|_{r=r_h} d\alpha.
\]

By substituting Eq. (67) into Eq. (68), we can obtain the value of the \( dr_h \), which is

\[
dr_h = \frac{2}{(d-2)r_h^{d-2}} da + \frac{1}{r_h^{(d-1)-q+1}} d\alpha + \frac{16\pi}{r_h^{d-3}(d-2)\Omega_{d-2}} p^r - \frac{16\pi r_h^{d-2}}{(d-2)(d-1)} dP. \tag{69}
\]

With the aid of Eq. \( (dS = \frac{\Omega_{d-2}(d-2)r^{d-3}}{4} dr_h) \), the variation of entropy is given by

\[
dS = \frac{\Omega_{d-2}r_h^{d-2} da + \Omega_{d-2}(d-2) dr_h^{(d-1)-q} d\alpha + 4\pi p^r - 4\pi r_h^{d-3}\Omega_{d-2} dP}{4\pi T}. \tag{70}
\]

Using Eq. (70), it is easy to get

\[
TdS - PdV = \frac{T \Omega_{d-2}r_h^{d-2} da + \frac{T \Omega_{d-2}}{(d-1)-q} d\alpha}{4\pi T} + \frac{16\pi r_h^{d-2}}{(d-2)(d-1)} p^r - \frac{16\pi r_h^{d-1}\Omega_{d-2}}{(d-2)(d-1)} dP. \tag{71}
\]
Then, the Eq. (67) is rewritten as

\[ dM = TdS + VdP + \phi dQ + A\, da + Q\, d\alpha. \quad (72) \]

Obviously, the Eq. (72) is the same as Eq. (53). It means that the first law of black hole thermodynamics still holds. Next, we will continue to check the second law of black hole thermodynamics when a charged particle is captured by the black hole. As the black hole entropy increase in a clockwise direction will not be less than zero, we can examine the second law of thermodynamics of the black hole by studying the change in entropy. For the extremal black hole where its temperature is zero. Then, combining this condition and the black hole mass, the variation of entropy finally reads

\[ dS_{\text{extremal}} \rightarrow \infty. \quad (73) \]

Therefore the second law of black hole thermodynamics is still valid for the extremal black holes. Besides, the non-extremal black holes have temperatures greater than zero, so the variation of entropy \( dS \) always has a positive value under certain conditions, which means the second law of black hole thermodynamics does not violate for the non-extremal black holes.

Next, we will further check the stability of horizon black hole with particle’s absorption. In a similar way, Eq. (60) is rewritten as

\[
\begin{align*}
&f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \\
&= \delta + \frac{(r_0^{3-d} - r_h^{3-d})16\pi q}{r_0^{d-3}\Omega_{d-2}\sqrt{2(d-3)(d-2)}}dQ \\
&\quad - \frac{16\pi p^r}{r_0^{d-3}(d-2)\Omega_{d-2}} + \frac{16\pi r_0^2}{(d-2)(d-1)}dP \\
&\quad - \frac{1}{r_0^{(d-1)\omega_y + d-3}}d\alpha - \frac{2}{(d-2)r_0^{d-4}}da.
\end{align*}
\]

Using Eq. (41), it is easy to get

\[
\begin{align*}
&f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \\
&= \delta + \frac{(r_0^{3-d} - r_h^{3-d})16\pi q}{r_0^{d-3}\Omega_{d-2}\sqrt{2(d-3)(d-2)}}dQ \\
&\quad - \frac{16\pi p^r}{r_0^{d-3}(d-2)\Omega_{d-2}} - \frac{2r_0^2}{l^3}dl \\
&\quad - \frac{1}{r_0^{(d-1)\omega_y + d-3}}d\alpha - \frac{2}{(d-2)r_0^{d-4}}da.
\end{align*}
\]

29
When the initial black hole is the extremal black hole, \( r_0 = r_h \), \( T = 0 \) and \( \delta = 0 \). Then we can obtain \( f_{\text{min}} = \delta = 0 \) and \( f'_{\text{min}} = 0 \). Hence, Eq. (75) is written as

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, dr_0 + r_0) < 0.
\]

(76)

When the initial black hole is the near-extremal black hole, \( r_0 \) and \( r_h \) do not coincide. In a similar way, the Eq. (74) can be expanded near the minimum point by using the relation

\[
r_h = r_0 + \epsilon,
\]

which is

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0)
\]

\[
= \delta_{\epsilon} + \frac{[(d-3)\epsilon + O(\epsilon)^2]}{r_0^{d-3} \Omega_{d-2} \sqrt{2(d-3)(d-2)}} dQ
\]

\[
- \frac{16\pi \rho'}{r_0^{d-3}(d-2) \Omega_{d-2}} - \frac{2r_0^2}{l^3} dl
\]

\[
- \frac{1}{r_0^{(d-1)\omega_q + d-3}} d\alpha - \frac{2}{(d-2)r_0^{d-4}} da.
\]

(77)

We set \( dQ \sim \epsilon, \ d\alpha \sim \epsilon, \ da \sim \epsilon \). Thus, Eq. (77) is modified as

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0)
\]

\[
= \delta_{\epsilon} - \frac{16\pi \rho'}{r_0^{d-3}(d-2) \Omega_{d-2}} - \frac{2r_0^2 \epsilon}{l^3} - \frac{\epsilon}{r_0^{(d-1)\omega_q + d-3}} - \frac{2\epsilon}{(d-2)r_0^{d-4}} + O(\epsilon^2).
\]

(78)

Correspondingly, at the minimum point, we have

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \leq 0.
\]

(79)

Therefore, it is obviously that the horizon stably exists at the final state of the near-extremal black hole.

IV. THE SCALAR FIELD

A. Solution to Charged Scalar Field Equation

In order to investigate the scattering of the nonminimally coupled massive scalar field with RN-AdS black hole with a cloud of strings in d-dimensional spacetime, the amount of conserved quintessence taken into the black hole is given as the fluxes of the scattered external field. The action of the charged scalar field in the fixed gravitational and electromagnetic
fields is
\[ S_\Psi = -\frac{1}{2} \int d^Dx \sqrt{-g} (D_\mu \Psi D^\mu \Psi^* + (\mu^2 + \zeta R)\Psi \Psi^*), \]  
where the spacetime dimension is assumed to be \( D \geq 4 \). Owing to a scalar field with electric charge \( q \), we consider the covariant derivative \( D_\mu = \partial_\mu - iqA_\mu \). The scalar field has the mass \( \mu \) and nonminimal coupling \( \zeta \) with the curvature. There are two field equations, including the complex conjugate
\[
\frac{1}{\sqrt{-g}} D_\mu (\sqrt{-g} g^{\mu\nu} D_\nu \phi) - (\mu^2 + \zeta R)\Psi = 0, \quad \frac{1}{\sqrt{-g}} D_\mu^* (\sqrt{-g} g^{\mu\nu} D_\nu^* \phi^*) - (\mu^2 + \zeta R)\Psi^* = 0. \tag{81}
\]

The determinant of the metric is simply noted as
\[
\sqrt{-g} = r^{d-2} \prod_{j=0}^{d-3} \sin^{d-2-j} \theta_j. \tag{82}
\]

Then the separable equation with respect to \( \Psi \) is obtained as
\[
\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) - 2iqA_0 g^{00} \partial_0 \Psi - q^2 g^{00}(A_0)^2 \Psi - (\mu^2 + \zeta R)\Psi = 0. \tag{83}
\]

The solution to the scalar field is
\[ \Psi(t, r, \phi, \Theta) = e^{-i\omega t} R(r) Y_{lm}(\Theta_1, \Theta_2, \ldots \Theta_{d-2}), \tag{84} \]
where \( Y_{lm}(\Theta_1, \Theta_2, \ldots \Theta_{d-2}) \) is the hyperspherical harmonics on a \((d-2)\)-dimensional sphere. At the outer horizon, the radial solution of the scalar field is \[ R(r) = e^{\pm i(\omega_{\eta} - q\phi)r}. \tag{85} \]
The negative sign in the Eq. \[ \text{(85)} \] selected to represent the scalar field entering the outer horizon under scalar field scattering. Thus two solutions of the scalar field is represented as
\[ \Psi = e^{-i\omega t} e^{-i(\omega_{\eta} - q\phi)r} Y_{lm}(\Theta_1, \Theta_2, \ldots \Theta_{d-2}), \Psi^* = e^{i\omega t} e^{i(\omega_{\eta} - q\phi)r} Y_{lm}^*(\Theta_1, \Theta_2, \ldots \Theta_{d-2}). \tag{86} \]

According to these solutions, it can be deduced that the relationship between the black hole conserved and the scalar field, considering the \( PV \) term. When entering a black hole, the energy and charge of the scalar field change as much as the changes in the black hole. These transfer fluxes entering the black hole can be obtained by the energy of the momentum tensor of the scalar field
\[
T^\mu_\nu = \frac{1}{2} D^\mu \partial_\nu \Psi^* + \frac{1}{2} \Psi D^\mu D^\nu \Psi^* - \delta^\mu_\nu (\frac{1}{2} D^\mu \Psi D^\mu \Psi^* - \frac{1}{2} (\mu^2 + \zeta R)\Psi \Psi^*). \tag{87}
\]
The energy flux is the component $T^\mu_{\nu}$ integrated by a solid angle on an $S^{d-2}$ sphere at the outer horizon. Then, fluxes of energy and electric charge are

$$\frac{dE}{dt} = \int T^r_r \sqrt{-g} d\Omega_{d-2} = \omega_q (\omega_q - \sqrt{\frac{d-2}{2(d-3)} \frac{q^2}{r_h^{d-3}}} r_h^{d-2}),$$

$$\frac{de}{dt} = \frac{q}{\omega_q} \frac{dE}{dt} = q (\omega_q - \sqrt{\frac{d-2}{2(d-3)} \frac{q^2}{r_h^{d-3}}} r_h^{d-2}).$$

The fluxes in the above formulas will change the corresponding properties of the black hole during the infinitesimal time interval $dt$.

**B. The first and second laws of thermodynamics**

In this section, we will discuss issues related to thermodynamics under scalar field scattering. When the change in the enthalpy is connected to the change in internal energy [116], the charge flux corresponds to the change in that of the black hole. Moreover, the changes in internal energy and charge are given as

$$dU = (\frac{dE}{dt}) dt, \quad dQ = (\frac{de}{dt}) dt.$$  \hspace{1cm} (89)

with

$$dU = d(M - PV) = \omega_q (\omega_q - \sqrt{\frac{d-2}{2(d-3)} \frac{q^2}{r_h^{d-3}}} r_h^{d-2}) dt,$$

$$dQ = q (\omega_q - \sqrt{\frac{d-2}{2(d-3)} \frac{q^2}{r_h^{d-3}}} r_h^{d-2}).$$  \hspace{1cm} (90)

The location of the outer horizon is of great significance in the analysis process, and the outer horizon $r_h$ is located at the point satisfying $f(M, Q, P, a, \alpha, r_h) = 0$. Assuming that the initial state of the black hole is represented by $(M, Q, P, a, \alpha, r_h)$, and the final state is represented by $(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h)$. The functions $f(M, Q, P, a, \alpha, r_h)$ and $f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h)$ satisfy

$$f(M, Q, P, a, \alpha, r_h) = f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) = 0.$$  \hspace{1cm} (91)
The relation between the functions $f(M, Q, P, a, \alpha, r_h)$ and $f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h)$ is

$$f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) = f(r)$$

$$+ \frac{\partial f}{\partial M} \big|_{r=r_h} dM + \frac{\partial f}{\partial Q} \big|_{r=r_h} dQ + \frac{\partial f}{\partial r} \big|_{r=r_h} dr_h +$$

$$\frac{\partial f}{\partial P} \big|_{r=r_h} dP + \frac{\partial f}{\partial a} \big|_{r=r_h} da + \frac{\partial f}{\partial \alpha} \big|_{r=r_h} d\alpha,$$

where

$$\frac{\partial f}{\partial M} \big|_{r=r_h} = -\frac{16\pi r_h^d}{r_h^{d-3}(d-2)\Omega_{d-2}},$$

$$\frac{\partial f}{\partial Q} \big|_{r=r_h} = \frac{16\pi q}{r_h^{2(d-3)}\Omega_{d-2}\sqrt{2(d-3)(d-2)}},$$

$$\frac{\partial f}{\partial P} \big|_{r=r_h} = \frac{16\pi r_h^2}{(d-2)(d-1)},$$

$$\frac{\partial f}{\partial r} \big|_{r=r_h} = 4\pi T,$$

$$\frac{\partial f}{\partial a} \big|_{r=r_h} = -\frac{1}{r_h^{(d-1)\omega_q+d-3}},$$

$$\frac{\partial f}{\partial \alpha} \big|_{r=r_h} = -\frac{2}{(d-2)r_h^{d-4}}.$$

Then, we can figure out

$$dr_h = \frac{16\pi r_h}{(16\pi P_{r_h} - 4\pi T)} \int \frac{2q^2\omega_q}{(d-3)\sqrt{2(d-2)(d-3)}} - \frac{\omega^2_q}{d-2} - \frac{q^4}{2(d-3)r_h^{2(d-3)}} dt$$

$$- \frac{1}{(16\pi P_{r_h} - 4\pi T)(d-2)r_h^{d-4}} da - \frac{16\pi P_{r_h}}{d-2} - 4\pi T.$$

Then the variation of the entropy and volume are obtained

$$dS = \frac{4\pi r_h^{d-2}(d-2)[\frac{2q^2\omega_q}{(d-3)\sqrt{2(d-2)(d-3)}} - \frac{\omega^2_q}{d-2} - \frac{q^4}{2(d-3)r_h^{2(d-3)}}] dt$$

$$- \frac{r_h\omega_{d-2}}{2} da + \frac{\Omega_{d-2}(d-2)}{4r_h^{(d-1)\omega_q-1}} d\alpha,$$

and

$$dV = \frac{16\pi r_h^{d-1}[\frac{2q^2\omega_q}{(d-3)\sqrt{2(d-2)(d-3)}} - \frac{\omega^2_q}{d-2} - \frac{q^4}{2(d-3)r_h^{2(d-3)}}] dt$$

$$- \frac{2\pi \omega_{d-2}}{(d-2)} da + \frac{\Omega_{d-2}(d-2)}{4r_h^{(d-1)\omega_q-1}} d\alpha.$$

Using Eqs. (95) and (96), we yield

$$[4\pi r_h^{d-2}T(d-2) - 16\pi r_h^{d-1}P][\frac{2q^2\omega_q}{(d-3)\sqrt{2(d-2)(d-3)}} - \frac{\omega^2_q}{d-2} - \frac{q^4}{2(d-3)r_h^{2(d-3)}}] dt$$

$$- \frac{T(d-2)r_h\omega_{d-2}}{(d-2)(\frac{32\pi P_{r_h}}{d-2} - 8\pi T)} da - \frac{T\omega_{d-2}(d-2)}{(16\pi P_{r_h} - 4\pi T)} r_h^{(d-1)\omega_q-1} d\alpha.$$
Table XIII: The relation between $dS$, $Q$ and $r_h$ for $d = 5$ in the extended phase space via scalar field scattering.

|       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|
| $a = 0.01$ |       |       |       |       |       |       |
| $Q$    | $r_h$ | $dS$  | $Q$    | $r_h$ | $dS$  | $Q$    | $r_h$ | $dS$  |
| 0.640747 | 1.43509 | 0.2762150 | 0.965855 | 1.752080 | 0.9524490 | 1.33654 | 2.01130 | 2.6726000 |
| 0.64    | 1.40537 | 0.2427220 | 0.96    | 1.674810 | 0.7045120 | 0.99    | 1.32019 | 0.1864200 |
| 0.6     | 1.19173 | 0.0931272 | 0.70    | 1.097070 | 0.0594952 | 0.7     | 0.96294 | 0.0306009 |
| 0.55    | 1.04937 | 0.0464416 | 0.8     | 1.295200 | 0.0135020 | 0.8     | 1.08407 | 0.0595115 |
| 0.5     | 0.93212 | 0.0248162 | 0.7     | 1.097070 | 0.0594952 | 0.7     | 0.96294 | 0.0306009 |
| 0.45    | 0.82613 | 0.0133363 | 0.6     | 0.941706 | 0.0260095 | 0.6     | 0.84134 | 0.0146652 |
| 0.4     | 0.72648 | 0.0070078 | 0.5     | 0.791284 | 0.0104770 | 0.5     | 0.71771 | 0.0063719 |
| 0.35    | 0.63079 | 0.0035247 | 0.4     | 0.641677 | 0.0036787 | 0.4     | 0.59042 | 0.0024033 |
| 0.3     | 0.53766 | 0.0016586 | 0.3     | 0.490067 | 0.0010301 | 0.3     | 0.45777 | 0.0007265 |
| 0.2     | 0.35598 | 0.0002610 | 0.2     | 0.334134 | 0.0001891 | 0.2     | 0.31756 | 0.0001458 |
| 0.1     | 0.17741 | 0.0000140 | 0.1     | 0.171631 | 0.0000119 | 0.1     | 0.16667 | 0.0000102 |

The generalized first law in the extended phase space which account for the cosmological constant effect, cloud of strings and the quintessence contributions is then expressed as

$$dM = TdS + VdP + \phi dQ + A da + Qd\alpha. \quad (98)$$

Thus, the first law of thermodynamics is recovered by the scattering of the scalar field. Then the second law of thermodynamics is validated in the extended phase space. When it is the extremal black hole, the temperature is zero. Then Eq. (95) is modified as

$$dS = \frac{4\pi r_h^{d-2}(d-2)[\frac{q^2\omega q}{r_h^{d-3}\sqrt{2(d-2)(d-3)}} - \frac{\omega q^4}{d-2} - \frac{q^4}{2(d-2)r_h^{2(d-3)}}]dt}{16\pi P r_h^{d-2}} - \frac{r_h^d \Omega_d - 2 da}{4 r_h^{(d-1)\omega q}} d\alpha. \quad (99)$$

which is positive when $d\alpha > 0$ and $da > 0$, and the reverse is uncertain. Therefore, the second law of thermodynamics can be indefinite for the extremal black hole in the extended phase space.

Then we focus on the near-extremal black hole. In the process of exploring the change of entropy, we set $M = 1$, $l = 1$, $\Omega_d = 1$, $P = 1$, $\omega_q = \frac{d-1}{d-2}$ and $dt = 0.0001$, using different charge values in various dimensions to investigate. We analysed the effect of parameter $a$
Table XIV: The relation between $dS$, $Q$ and $r_h$ for $d = 6$ in the extended phase space via scalar field scattering.

| $a = 0.01$ | $a = 10$ | $a = 20$ |
|------------|----------|----------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.751095 | 1.26380 | 0.205270 | 1.09831 | 1.43654 | 0.4682340 | 1.48119 | 1.57562 | 0.8792900 |
| 0.75 | 1.24297 | 0.1781660 | 0.99 | 1.23444 | 0.1334240 | 0.99 | 1.07695 | 0.0447711 |
| 0.7 | 1.10288 | 0.0689191 | 0.9 | 1.12473 | 0.0667108 | 0.9 | 1.01152 | 0.0286431 |
| 0.65 | 1.02277 | 0.0394930 | 0.8 | 1.02853 | 0.0350302 | 0.8 | 0.93810 | 0.0169569 |
| 0.6 | 0.95369 | 0.0240479 | 0.7 | 0.93622 | 0.0182371 | 0.7 | 0.86279 | 0.0096341 |
| 0.55 | 0.88937 | 0.0148948 | 0.6 | 0.84395 | 0.0091214 | 0.6 | 0.78436 | 0.0051753 |
| 0.5 | 0.82727 | 0.0092044 | 0.5 | 0.74901 | 0.0042539 | 0.5 | 0.70140 | 0.0025733 |
| 0.4 | 0.70438 | 0.0033197 | 0.4 | 0.64873 | 0.0017768 | 0.4 | 0.61198 | 0.0011440 |
| 0.3 | 0.57726 | 0.0010185 | 0.3 | 0.53976 | 0.0006191 | 0.3 | 0.51317 | 0.0004255 |
| 0.2 | 0.43871 | 0.0002224 | 0.2 | 0.41656 | 0.0001529 | 0.2 | 0.39967 | 0.0001132 |
| 0.1 | 0.27580 | 0.0001980 | 0.1 | 0.26667 | 0.0001560 | 0.1 | 0.25906 | 0.0000127 |

Table XV: The relation between $dS$, $Q$ and $r_h$ for $d = 7$ in the extended phase space via scalar field scattering.

| $a = 0.01$ | $a = 10$ | $a = 20$ |
|------------|----------|----------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.818145 | 1.16885 | 0.1640370 | 1.17528 | 1.28623 | 0.3158390 | 1.56137 | 1.373450 | 0.5141460 |
| 0.8 | 1.10675 | 0.0928402 | 0.99 | 1.07803 | 0.0550888 | 0.99 | 0.990496 | 0.0220919 |
| 0.75 | 1.03766 | 0.0496371 | 0.9 | 1.01703 | 0.0324783 | 0.9 | 0.943946 | 0.0146366 |
| 0.7 | 0.98534 | 0.0308256 | 0.8 | 0.95202 | 0.0182544 | 0.8 | 0.890724 | 0.0090351 |
| 0.65 | 0.93833 | 0.0199918 | 0.7 | 0.88699 | 0.0100934 | 0.7 | 0.835066 | 0.0053799 |
| 0.6 | 0.89360 | 0.0131606 | 0.6 | 0.81988 | 0.0053710 | 0.6 | 0.775936 | 0.0030504 |
| 0.5 | 0.80578 | 0.0056642 | 0.5 | 0.74876 | 0.0026852 | 0.5 | 0.71979 | 0.0016162 |
| 0.4 | 0.71509 | 0.0022778 | 0.4 | 0.67125 | 0.0012179 | 0.4 | 0.641222 | 0.0007755 |
| 0.3 | 0.61620 | 0.0007898 | 0.3 | 0.58382 | 0.0004702 | 0.3 | 0.560427 | 0.0003172 |
| 0.2 | 0.50162 | 0.0002017 | 0.2 | 0.47991 | 0.0001335 | 0.2 | 0.463348 | 0.0000959 |
| 0.1 | 0.35416 | 0.0000229 | 0.1 | 0.34299 | 0.0000172 | 0.1 | 0.333862 | 0.0000134 |
Table XVI: The relation between $dS$, $Q$ and $r_h$ for $d = 8$ in the extended phase space via scalar field scattering.

| $a = 0.01$ | $a = 10$ | $a = 20$ |
|-----------|----------|----------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.861699 | 1.111510 | 0.1368470 | 1.22409 | 1.194940 | 0.2407210 | 1.6107 | 1.261110 | 0.3655200 |
| 0.8 | 1.017820 | 0.0469615 | 0.99 | 1.014820 | 0.0342930 | 0.99 | 0.951314 | 0.01439230 |
| 0.75 | 0.977116 | 0.0297433 | 0.9 | 0.969496 | 0.0210418 | 0.9 | 0.914867 | 0.00972157 |
| 0.7 | 0.940827 | 0.0198000 | 0.8 | 0.919793 | 0.0122545 | 0.8 | 0.872725 | 0.00614304 |
| 0.65 | 0.906358 | 0.0134389 | 0.7 | 0.869011 | 0.0070074 | 0.7 | 0.828122 | 0.00375529 |
| 0.6 | 0.872532 | 0.0091714 | 0.6 | 0.815649 | 0.0038632 | 0.6 | 0.780130 | 0.00219467 |
| 0.5 | 0.804095 | 0.0042006 | 0.5 | 0.758080 | 0.0020099 | 0.5 | 0.727477 | 0.00120488 |
| 0.4 | 0.731161 | 0.0017970 | 0.4 | 0.694122 | 0.0009557 | 0.4 | 0.668245 | 0.00060337 |
| 0.3 | 0.649196 | 0.0006688 | 0.3 | 0.620283 | 0.0003914 | 0.3 | 0.599186 | 0.00026037 |
| 0.2 | 0.550705 | 0.0001872 | 0.2 | 0.529782 | 0.0001204 | 0.2 | 0.513820 | 0.00008474 |
| 0.1 | 0.416850 | 0.0000247 | 0.1 | 0.404567 | 0.0000176 | 0.1 | 0.394645 | 0.00001337 |

Table XVII: The relation between $dS$, $Q$ and $r_h$ for $d = 5$ in the extended phase space via scalar field scattering.

| $\alpha = 0.01$ | $\alpha = 10$ | $\alpha = 20$ |
|----------------|----------------|----------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.640747 | 1.435090 | 0.2762150 | 0.99 | 1.25816 | 0.8322130 | 0.99 | 1.113580 | 1.7461700 |
| 0.6 | 1.191730 | 0.0931272 | 0.9 | 1.18899 | 0.3740940 | 0.9 | 1.058110 | 0.4420350 |
| 0.55 | 1.049370 | 0.0464415 | 0.8 | 1.10560 | 0.1700950 | 0.8 | 0.991187 | 0.4420350 |
| 0.5 | 0.932126 | 0.0248162 | 0.7 | 1.01377 | 0.0789687 | 0.7 | 0.917336 | 0.0689460 |
| 0.45 | 0.826131 | 0.0133363 | 0.6 | 0.91142 | 0.0355355 | 0.6 | 0.834588 | 0.0297781 |
| 0.4 | 0.726485 | 0.0070078 | 0.5 | 0.79599 | 0.0147432 | 0.5 | 0.740149 | 0.0123187 |
| 0.35 | 0.630789 | 0.0035247 | 0.4 | 0.66473 | 0.0052727 | 0.4 | 0.630155 | 0.0045427 |
| 0.3 | 0.537666 | 0.0016586 | 0.3 | 0.51596 | 0.0014571 | 0.3 | 0.500041 | 0.0013274 |
| 0.2 | 0.355981 | 0.0002609 | 0.2 | 0.35138 | 0.0002507 | 0.2 | 0.347263 | 0.0002419 |
| 0.1 | 0.177411 | 0.0000140 | 0.1 | 0.17711 | 0.0000139 | 0.1 | 0.176826 | 0.0000138 |
Table XVIII: The relation between $dS$, $Q$ and $r_h$ for $d = 6$ in the extended phase space via scalar field scattering.

| $\alpha$ | $Q$   | $r_h$ | $dS$   | $\alpha$ | $Q$   | $r_h$ | $dS$   | $\alpha$ | $Q$   | $r_h$ | $dS$   |
|---------|-------|-------|--------|---------|-------|-------|--------|---------|-------|-------|--------|
| 0.01    | 0.751095 | 1.26380 | 0.2052720 | 0.99    | 1.04990 | 0.0990892 | 0.99    | 0.968419 | 0.0815352 |
| 0.1     | 0.7    | 1.10288 | 0.0689191 | 0.9     | 1.00783 | 0.0636439 | 0.9     | 0.933293 | 0.0507251 |
| 0.2     | 0.6    | 0.95369 | 0.0240479 | 0.8     | 0.95630 | 0.0381356 | 0.8     | 0.890286 | 0.0298474 |
| 0.3     | 0.55   | 0.88937 | 0.0148948 | 0.7     | 0.89852 | 0.0220132 | 0.7     | 0.842006 | 0.0172073 |
| 0.4     | 0.5    | 0.82727 | 0.0092043 | 0.6     | 0.83282 | 0.0119845 | 0.6     | 0.786858 | 0.0095171 |
| 0.5     | 0.45   | 0.76594 | 0.0056019 | 0.5     | 0.75695 | 0.0059689 | 0.5     | 0.722502 | 0.0049009 |
| 0.6     | 0.4    | 0.70438 | 0.0033196 | 0.4     | 0.66796 | 0.0025935 | 0.4     | 0.645455 | 0.0022377 |
| 0.7     | 0.3    | 0.57726 | 0.0010184 | 0.3     | 0.56214 | 0.0009055 | 0.3     | 0.550610 | 0.0008287 |
| 0.8     | 0.2    | 0.43871 | 0.0002224 | 0.2     | 0.43457 | 0.0002136 | 0.2     | 0.430850 | 0.0002062 |
| 0.9     | 0.1    | 0.27580 | 0.0000198 | 0.1     | 0.27538 | 0.0000196 | 0.1     | 0.274968 | 0.0000195 |

Table XIX: The relation between $dS$, $Q$ and $r_h$ for $d = 7$ in the extended phase space via scalar field scattering.

| $\alpha$ | $Q$   | $r_h$ | $dS$   | $\alpha$ | $Q$   | $r_h$ | $dS$   | $\alpha$ | $Q$   | $r_h$ | $dS$   |
|---------|-------|-------|--------|---------|-------|-------|--------|---------|-------|-------|--------|
| 0.01    | 0.818145 | 1.16885 | 0.1640370 | 0.99    | 0.975201 | 0.0429284 | 0.99    | 0.918038 | 0.0318422 |
| 0.1     | 0.8    | 1.12844 | 0.1131090 | 0.9     | 0.944555 | 0.0298752 | 0.9     | 0.891935 | 0.0221392 |
| 0.2     | 0.75   | 1.03766 | 0.0496371 | 0.8     | 0.906704 | 0.0194035 | 0.8     | 0.859722 | 0.0145037 |
| 0.3     | 0.7    | 0.98534 | 0.0308256 | 0.7     | 0.863856 | 0.0120788 | 0.7     | 0.823226 | 0.0092066 |
| 0.4     | 0.6    | 0.89360 | 0.0131606 | 0.6     | 0.814597 | 0.0070827 | 0.6     | 0.781100 | 0.0055698 |
| 0.5     | 0.5    | 0.80578 | 0.0056642 | 0.5     | 0.756955 | 0.0038133 | 0.5     | 0.731328 | 0.0031311 |
| 0.6     | 0.4    | 0.71509 | 0.0022778 | 0.4     | 0.688128 | 0.0018105 | 0.4     | 0.670800 | 0.0015682 |
| 0.7     | 0.3    | 0.61620 | 0.0007898 | 0.3     | 0.604011 | 0.0007059 | 0.3     | 0.594569 | 0.0006477 |
| 0.8     | 0.2    | 0.50162 | 0.0002017 | 0.2     | 0.497826 | 0.0001937 | 0.2     | 0.494408 | 0.0001869 |
| 0.9     | 0.1    | 0.35416 | 0.0000230 | 0.1     | 0.353678 | 0.3536780 | 0.1     | 0.353200 | 0.0000226 |
on $dS$ in the beginning. From Table XIII, Table XIV, Table XIV, and Table XVI, it is evident that the event horizon of the black hole and the variation of entropy decreases when the charge of the black hole decreases, which is the same as the conclusion obtained at the section 9. Besides, as the value of $a$ decreases, the value of the critical horizon become smaller. And as the values of $d$ decrease, the values of the divergent point become greater. From Fig. (8), we conclude that there are regions of $dS$ which are positive and negative. Similar to that in the particle absorption section, we will also investigate how $d$ and $\alpha$ affect the value of $dS_h$. By observing Table XVII, Table XVIII, Table XIX, and Table XX, we found the event horizon of the black hole and the variation of entropy decrease when the charge of the black hole decreases. Fig. (9) has shown that the values of the divergent point decreases as $d$ or $\alpha$ increases. At the same time, there’s always a region where entropy is less than zero. So far, the second law of thermodynamics has been violated, irrespective of the values of $a$ and $\alpha$. In order to intuitively understand the changes in entropy associated
Table XX: The relation between $dS$, $Q$ and $r_h$ for $d = 8$ in the extended phase space via scalar field scattering.

| $Q$  | $r_h$  | $dS$  | $Q$  | $r_h$  | $dS$  | $Q$  | $r_h$  | $dS$  |
|------|--------|-------|------|--------|-------|------|--------|-------|
| 0.861699 | 1.111510 | 0.1368470 | 0.99 | 0.941476 | 0.0269757 | 0.99 | 0.897107 | 0.0193602 |
| 0.8  | 1.017820 | 0.0469615 | 0.9  | 0.917179 | 0.0194393 | 0.9  | 0.876162 | 0.0140900 |
| 0.75 | 0.977116 | 0.0297433 | 0.8  | 0.887017 | 0.0131023 | 0.8  | 0.850188 | 0.0096751 |
| 0.7  | 0.940827 | 0.0198000 | 0.7  | 0.852669 | 0.0084608 | 0.7  | 0.820590 | 0.0064206 |
| 0.6  | 0.872532 | 0.0091714 | 0.6  | 0.812908 | 0.0051536 | 0.6  | 0.786200 | 0.0040569 |
| 0.5  | 0.804095 | 0.0042005 | 0.5  | 0.765975 | 0.0028935 | 0.5  | 0.745248 | 0.0023849 |
| 0.4  | 0.731161 | 0.0017969 | 0.4  | 0.709280 | 0.0014438 | 0.4  | 0.694940 | 0.0012554 |
| 0.3  | 0.649196 | 0.0006688 | 0.3  | 0.638772 | 0.0005998 | 0.3  | 0.630644 | 0.0005514 |
| 0.2  | 0.550705 | 0.0001872 | 0.2  | 0.547191 | 0.0001798 | 0.2  | 0.544033 | 0.0001734 |
| 0.1  | 0.416850 | 0.0000246 | 0.1  | 0.416333 | 0.0000245 | 0.1  | 0.415826 | 0.0000243 |

with $dS$ and $d\alpha$, we list different tables and graphs of functions. In the Table XXII, Table XXIII and Table XXIV, the influence of $d\alpha$ on the change of entropy is more obvious. From Fig. (10), it is obviously that there is indeed a phase change point causes a positive or negative change in the value of $dS$. It’s worth mentioning that the change of $d\alpha$ has a different effect on the changes of entropy than the section III. In order to explore the difference of entropy change in high and low dimensional cases, we plot Fig. (11), and compare which with Fig. (12), then the conclusion obtained is the same as when the particle is absorbed. That is indeed a phase change point that divides $dS$ into positive and negative values independent of dimension $d$. From the above discussion, it can be concluded that the second law of thermodynamics is not always valid for the near-extremal black hole in the extended phase space.

C. Stability of horizon

In the extended phase space, the stability of horizon also tests through checking the sign of the minimum value of the function $f(r)$ in the initial state. Assuming that there is a
minimum value of $f(r)$ and the minimum value is less than zero. For the extremal black hole, $\delta = 0$. For the near-extremal black hole, $\delta$ is a small quantity. After the flux of the scalar field enters the black hole, the sign of the minimum value in the final state can be obtained in term of the initial state. Assuming $(M, Q, P, r_0, a, \alpha)$ and $(M + dM, Q + dQ, P + dP, r_0 + dr_0, a + da, \alpha + d\alpha)$ represent the initial state and the final state, respectively. At $r = r_0 + dr_0$, \( f(M + dM, Q + dQ, P + dP, r_0 + dr_0, a + da, \alpha + d\alpha) \) is written as

\[
\delta + \frac{\partial f}{\partial M} \bigg|_{r=r_0} dM + \frac{\partial f}{\partial Q} \bigg|_{r=r_0} dQ + \frac{\partial f}{\partial P} \bigg|_{r=r_0} dP \\
+ \frac{\partial f}{\partial a} \bigg|_{r=r_0} da + \frac{\partial f}{\partial \alpha} \bigg|_{r=r_0} d\alpha + \frac{\partial f}{\partial r} \bigg|_{r=r_0} dr
\]

\( \delta = \delta_1 + \delta_2. \)
Table XXI: The relation between $dS$, $Q$ and $r_h$ for $d = 5$ in the extended phase space via scalar field scattering.

| $d\alpha = 0.6$, $da = 0.9$ | $d\alpha = 0.6$, $da = -0.9$ | $d\alpha = -0.6$, $da = 0.9$ |
|---|---|---|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 0.796654 | 1.69771 | 1.373840 | 0.796654 | 1.69771 | 0.803655 | 0.796654 | 1.69771 | -0.26956 |
| 0.7 | 1.29939 | 0.175067 | 0.7 | 1.29939 | 0.078073 | 0.7 | 1.29939 | 0.011301 |
| 0.6 | 1.08142 | 0.020155 | 0.6 | 1.08142 | 0.006282 | 0.6 | 1.08142 | 0.002819 |
| 0.5 | 0.88942 | 0.001546 | 0.5 | 0.88942 | 0.000955 | 0.5 | 0.88942 | 0.000199 |
| 0.4 | 0.70741 | 0.000251 | 0.4 | 0.70741 | 0.000199 | 0.4 | 0.70741 | 0.000013 |
| 0.3 | 0.52949 | 0.000013 | 0.3 | 0.52949 | 0.000013 | 0.3 | 0.52949 | 0.000013 |

where

\[
\frac{\partial f}{\partial r} \bigg|_{r=r_0} = 0, \\
\frac{\partial f}{\partial M} \bigg|_{r=r_0} = -\frac{16\pi}{r_0^{d-3}(d-2)\Omega_{d-2}}, \\
\frac{\partial f}{\partial Q} \bigg|_{r=r_0} = \frac{16\pi q}{r_0^{2(d-3)}\Omega_{d-2}\sqrt{2(d-3)(d-2)}}, \\
\frac{\partial f}{\partial P} \bigg|_{r=r_0} = \frac{16\pi r_0^2}{(d-2)(d-1)}, \\
\frac{\partial f}{\partial \alpha} \bigg|_{r=r_0} = -\frac{1}{r_0^{(d-1)\omega_2+d-3}}, \\
\frac{\partial f}{\partial a} \bigg|_{r=r_0} = -\frac{1}{(d-2)r_0^{d-4}}.
\]

Inserting Eq. (100) into Eq. (101) yields

\[
f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0)
\]

\[
= \delta + \frac{16\pi r_0^{d-2}P}{r_0^{d-3}} \left[ \omega_q - \sqrt{2(d-3)} \frac{q^2}{r_0^{d-3}} \right] \left[ \frac{16\pi Pr_h}{(d-2)4\pi T - 16\pi Pr_h} \right] \left[ \frac{q^2}{r_h^{d-3}} - \omega_q - \frac{2}{d-2} \frac{\omega_q}{r_0^{d-3}} \right] \\
+ \frac{\left( \frac{q^2}{r_0^{d-3}} - \frac{\omega_q}{d-2} \right) dt}{(d-2)(d-1)} \\
+ \frac{16\pi}{(d-2)(d-1)} \left( \frac{r_0^{d-1}}{r_0^{d-3}} - r_h^{d-4} - r_0^{d-4} \right) dP \\
+ \frac{2}{d-2} \left( \frac{16\pi Pr_h}{(d-2)4\pi T} \right) \left( \frac{r_0^{d-3}}{r_h^{d-3}} - r_h^{d-4} - r_0^{d-4} \right) da \\
+ \frac{16\pi Pr_h}{(d-2)4\pi T} \left( \frac{r_0^{d-3}}{r_h^{d-3}} \right) \left( \frac{1}{r_h^{(d-1)\omega_2+d-3}} - \frac{1}{r_0^{(d-1)\omega_2+d-3}} \right) d\alpha.
\]
Table XXII: The relation between $dS$, $Q$ and $r_h$ for $d = 6$ in the extended phase space via scalar field scattering.

| $d\alpha = 0.6, da = 0.9$ | $d\alpha = 0.6, da = -0.9$ | $d\alpha = -0.6, da = 0.9$ |
|---------------------------|---------------------------|---------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
|------|-------|------|------|-------|------|------|-------|------|
| 0.978391 | 1.46517 | 2.1220000 | 0.978391 | 1.46517 | 1.6944400 | 0.978391 | 1.465170 | 0.328897 |
| 0.9 | 1.26723 | 0.4825120 | 0.9 | 1.26723 | 0.3713990 | 0.9 | 1.267230 | 0.181025 |
| 0.8 | 1.13797 | 0.1971970 | 0.8 | 1.13797 | 0.1423440 | 0.8 | 1.137970 | 0.0894198 |
| 0.7 | 1.02504 | 0.0866350 | 0.7 | 1.02504 | 0.0567645 | 0.7 | 1.025040 | 0.0437401 |
| 0.6 | 0.91634 | 0.0049729 | 0.6 | 0.91634 | 0.0020746 | 0.6 | 0.916339 | 0.0203260 |
| 0.5 | 0.80685 | 0.0013726 | 0.5 | 0.80685 | 0.0006185 | 0.5 | 0.806850 | 0.008836 |
| 0.4 | 0.69303 | 0.00049729 | 0.4 | 0.693037 | 0.0001140 | 0.4 | 0.693037 | 0.0032719 |
| 0.3 | 0.57118 | 0.00013726 | 0.3 | 0.571182 | -0.000073 | 0.3 | 0.571182 | 0.0010134 |
| 0.2 | 0.43577 | 0.00002610 | 0.2 | 0.435772 | -0.000112 | 0.2 | 0.435772 | 0.0002198 |
| 0.1 | 0.27477 | 0.0000205 | 0.1 | 0.274769 | -0.000017 | 0.1 | 0.274769 | 0.0000194 |

Table XXIII: The relation between $dS$, $Q$ and $r_h$ for $d = 7$ in the extended phase space via scalar field scattering.

| $d\alpha = 0.6, da = 0.9$ | $d\alpha = 0.6, da = -0.9$ | $d\alpha = -0.6, da = 0.9$ |
|---------------------------|---------------------------|---------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
|------|-------|------|------|-------|------|------|-------|------|
| 1.09785 | 1.329700 | 1.0543800 | 1.09785 | 1.329700 | 0.72938600 | 1.09785 | 1.329700 | -0.6389280 |
| 0.9 | 1.094730 | 0.0862086 | 0.9 | 1.094730 | 0.03887250 | 0.9 | 1.094730 | -0.0271017 |
| 0.8 | 1.019380 | 0.0427440 | 0.8 | 1.019380 | 0.01428640 | 0.8 | 1.019380 | -0.0084702 |
| 0.7 | 0.945735 | 0.0214957 | 0.7 | 0.945735 | 0.00425739 | 0.7 | 0.945735 | -0.0014881 |
| 0.6 | 0.870696 | 0.0105465 | 0.6 | 0.870696 | 0.00039618 | 0.6 | 0.870696 | 0.00083262 |
| 0.5 | 0.791846 | 0.0048885 | 0.5 | 0.791846 | -0.0007313 | 0.5 | 0.791846 | 0.00126008 |
| 0.4 | 0.706553 | 0.0020574 | 0.4 | 0.706553 | -0.0007421 | 0.4 | 0.706553 | 0.00089463 |
| 0.3 | 0.611095 | 0.0007353 | 0.3 | 0.611095 | -0.0004288 | 0.3 | 0.611095 | 0.00046471 |
| 0.2 | 0.498774 | 0.0001918 | 0.2 | 0.498774 | -0.0001517 | 0.2 | 0.498774 | 0.00015644 |
| 0.1 | 0.352893 | 0.0000222 | 0.1 | 0.352893 | -0.0000209 | 0.1 | 0.352893 | 0.00002114 |
Table XXIV: The relation between $dS$, $Q$ and $r_h$ for $d = 8$ in the extended phase space via scalar field scattering.

| $d\alpha = 0.6, da = 0.9$ | $d\alpha = 0.6, da = -0.9$ | $d\alpha = -0.6, da = 0.9$ |
|--------------------------|--------------------------|--------------------------|
| $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ | $Q$ | $r_h$ | $dS$ |
| 1.17861 | 1.244110 | 0.9120490 | 1.17861 | 1.244110 | 0.65663100 | 1.17861 | 1.244110 | -0.61051 |
| 0.99 | 1.075100 | 0.0921938 | 0.99 | 1.075100 | 0.04653060 | 0.99 | 1.075100 | -0.0389781 |
| 0.9 | 1.024280 | 0.0510932 | 0.9 | 1.024280 | 0.02101080 | 0.9 | 1.024280 | 0.04653060 |
| 0.8 | 0.969409 | 0.0273018 | 0.8 | 0.969409 | 0.00799241 | 0.8 | 0.969409 | 0.04653060 |
| 0.7 | 0.913819 | 0.0074858 | 0.7 | 0.913819 | 0.00005640 | 0.7 | 0.913819 | -0.00005640 |
| 0.6 | 0.855703 | 0.0036501 | 0.6 | 0.855703 | -0.00005640 | 0.6 | 0.855703 | 0.000056517 |
| 0.5 | 0.793252 | 0.0016255 | 0.5 | 0.793252 | -0.00005640 | 0.5 | 0.793252 | 0.00091316 |
| 0.4 | 0.724135 | 0.0006217 | 0.4 | 0.724135 | -0.00005640 | 0.4 | 0.724135 | 0.00071939 |
| 0.3 | 0.644708 | 0.0006217 | 0.3 | 0.644708 | -0.00005640 | 0.3 | 0.644708 | 0.00039836 |
| 0.2 | 0.547972 | 0.0001775 | 0.2 | 0.547972 | -0.00005640 | 0.2 | 0.547972 | 0.00014579 |
| 0.1 | 0.415445 | 0.0000238 | 0.1 | 0.415445 | -0.00005640 | 0.1 | 0.415445 | 0.00002262 |

Then, we have

\[
\delta = 0,
\]

\[
\delta_1 = \frac{16\pi r_h^{d-2} P}{r_0^{d-3}} [\omega_q - \frac{d-2}{2(d-3)} q^2 r_h^{d-3}] \left\{ \frac{16\pi P r_h}{(d-2)4\pi T - 16\pi P r_h} \right\} q^2 r_h^{d-3} - \frac{\omega_q}{d-2} dt
\]

\[
+ \left[ \frac{q^2}{r_0^{d-3}} - \frac{\omega_q}{d-2} \right] dt
\]

\[
\delta_2 = \frac{2}{d-2} \frac{16\pi P r_h}{(d-2)4\pi T r_h^{d-3} r_0^{d-3} - r_0^{d-3}} \frac{r_h^{d-3}}{r_0^{d-3}} r_0^{d-3} r_h^{d-3} r_0^{d-3} - \frac{1}{r_h^{d-3}} + \frac{1}{r_0^{d-3}} \frac{r_h^{d-3}}{r_0^{d-3}} dP
\]

\[
+ (\frac{16\pi P r_h}{16\pi P r_h - (d-2)4\pi T r_h^{d-3} r_0^{d-3} r_h^{d-3} r_0^{d-3} \omega_q + d-3} - \frac{1}{r_h^{d-3} \omega_q + d-3}) d\alpha.
\]

(103)

In the extremal black hole, $r_0 = r_h, T = 0$ and $df_{\min} = 0$. Hence, Eq. [100] is written as

\[
\delta = 0,
\]

\[
\delta_1 = 0,
\]

(104)

\[
\delta_2 = 0.
\]

43
Thus, we have

\[ f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, dr_0 + r_0) = \delta + \delta_1 + \delta_2 = 0. \]  

(105)

Therefore, the scattering of the scalar field doesn’t cause the horizon changes in the minimum value of \( f(r) \). This proves that the extremal black hole is still hold and the horizon is still exists at the final state. For the near-extremal black hole, \( r_0 \) and \( r_h \) are very close. To calculate the value of Eq. (103), we can suppose that \( r_h = r_0 + \epsilon \), where \( 0 < \epsilon \ll 1 \). In this
situation, Eq. (103) is written as

$$
\delta < 0,
$$

$$
\delta_1 = \left\{ 16\pi Pr_0(\omega_q - q^2 \sqrt{\frac{d - 2}{2(d - 3)}} \frac{1}{r_0^{d-3}})(\frac{q^2 r_0 + q^2}{r_0^{d-3}} - \frac{\omega_q + \omega_q r_0}{d - 2}) + O(\epsilon) + O(\epsilon)^2 \right\} dt,
$$

$$
\delta_2 = \frac{2}{d - 2} \left( \frac{16\pi P(r_0 + \epsilon)}{16\pi P(r_0 + \epsilon) - (d - 2)4\pi T} \right) \frac{r_0 + (d - 3)\epsilon + O(\epsilon)^2}{r_0^{d-3}} da
$$

$$
- \frac{16\pi}{(d - 2)(d - 1)} r_0^2 (d - 1)\epsilon dP
$$

$$
+ \left\{ \frac{16\pi P}{16\pi P(r_0 + \epsilon) - (d - 2)4\pi T} \left[ \frac{1}{r_0^{(d-1)\omega_q d-4}} + \frac{(2 - d)\omega_q \epsilon}{r_0^{(d-1)\omega_q + d-3}} + O(\epsilon)^2 \right] - \frac{1}{r_0^{(d-1)\omega_q + d-3}} \right\} d\alpha,
$$

where $dt$ is an infinitesimal scale and is set as $dt \sim \epsilon$. If the initial black hole is near extremal,
we have \( dP \sim \epsilon, d\alpha \sim \epsilon, da \sim \epsilon \). So we have

\[
\delta < 0, \delta_1 + \delta_2 \ll \delta, \tag{107}
\]

and

\[
f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, dr_0 + r_0) \approx \delta < 0. \tag{108}
\]

Therefore, the event horizon exists and the black hole isn’t overcharged in the finial state. The weak cosmic censorship conjecture is valid in the near-extremal black hole.

**D. A new assumption: \( dE = dM \)**

In the previous subsection, we found that the second law of thermodynamics may be violated. It is believed that this assumption of violation of the second law is not physical but is an absurd conclusion of a false assumption that scalar field scattering changes the internal energy of a black hole. In this subsection, we assume that after the scalar field scattering, the black hole’s enthalpy changes. When the energy flux is assumed to the enthalpy of the black hole

\[
dE = dM, \tag{109}
\]

where the variation of the charge of the black hole \( dQ \) is the same as the variation of the electric charge flux of the scalar field \( de \)

\[
dQ = (\frac{de}{dt})dt. \tag{110}
\]
Thus, we obtain
\[ dM = \omega_q(\omega_q + q\phi)r_h^{d-2}dt, \quad dQ = q(\omega_q + q\phi)r_h^{d-2}dt. \]  

(111)

As a charged particle dropped into the black hole, the configurations of the black hole will be changed. This progress will lead to a shift for the horizon. The relation between the functions \( f(r) \) and \( f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) \) is

\[ f(M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_h + dr_h) = f(r) \]

+ \( \frac{\partial f}{\partial M} \bigg|_{r=r_h} dM + \frac{\partial f}{\partial Q} \bigg|_{r=r_h} dQ + \frac{\partial f}{\partial r} \bigg|_{r=r_h} dr + \]

\( \frac{\partial f}{\partial P} \bigg|_{r=r_h} dP + \frac{\partial f}{\partial a} \bigg|_{r=r_h} da + \frac{\partial f}{\partial \alpha} \bigg|_{r=r_h} d\alpha. \)  

(112)

Substituting Eq. (111) into Eq. (112), we can obtain the value of the \( dr_h \), which is

\[ dr_h = \frac{-4r_h}{T\Omega_{d-2}} \left[ \frac{2q^2\omega_q}{r_h^{d-3}} \right] \] 

\[ - \frac{1}{2(d-2)(d-1)} \] 

\[ - \frac{2}{4\pi T(d-2)r_h^{d-4}} da. \]  

(113)

With the aid of \([dS = \frac{\Omega_{d-2}(d-2)r_h^{d-3}}{4} dr_h] \), the variation of entropy is given by

\[ dS = \frac{(d-2)r_h^{d-2}}{-T} \] 

\[ \frac{2q^2\omega_q}{r_h^{d-3}} \] 

\[ - \frac{\omega_q^2}{d-2} - \frac{q^4}{2(d-3)r_h^{2(d-3)}} dt \] 

\[ - \frac{r_h^{d-1}\Omega_{d-2}}{T(d-1)} dP + \frac{\Omega_{d-2}(d-2)}{16\pi T r_{h}^{(d-1)\omega_q}} da + \frac{\Omega_{d-2}(d-2)r_h}{8\pi T(d-2)} da. \]  

(114)

Using Eq. (114), it is easy to get

\[ TdS - VdP \]

\[ = \frac{4Pr_h^{d-1} - (d-2)Tr_h^{d-2}}{T} \] 

\[ - \frac{2q^2\omega_q}{r_h^{d-3}} \] 

\[ - \frac{\omega_q^2}{d-2} - \frac{q^4}{2(d-3)r_h^{2(d-3)}} dt \]

\[ + \] 

\[ \frac{\Omega_{d-2}[4Pr_h^{d-2} - (d-2)Tr_h^{d-3}]}{T(d-2)(d-1)} dP - \frac{\Omega_{d-2}[4Pr_h^{d-2} - (d-2)Tr_h^{d-3}]}{16\pi T r_{h}^{(d-1)\omega_q+d-3}} da \]

\[ + \frac{\Omega_{d-2}[4Pr_h^{d-2} - (d-2)Tr_h^{d-3}]}{8\pi T(d-2)r_h^{d-4}} da. \]  

(115)

Then, the Eq. (111) reduces to

\[ dM =TdS + VdP + \phi dQ + \mathcal{A} da + Q d\alpha. \]  

(116)

Obviously, the Eq. (116) is exactly same as Eq. (98). This means that the first law of black hole thermodynamics still holds. Next, we will continue to check the second law
of black hole thermodynamics when a charged particle is captured by the black hole. As the black hole entropy increases in a clockwise direction will not be less than zero, we can examine the second law of thermodynamics black hole by studying the change in entropy. For the extremal black hole, the temperature is zero. Then, combining this condition and the black hole mass and the variation of entropy finally reads

$$dS_{\text{extremal}} \rightarrow \infty.$$  

(117)

It is true from Eq. (117) that the second law of black hole thermodynamics is still hold for the extremal black holes. In addition, the temperatures of the non-extremal black hole is greater than zero, so the variation of entropy $dS$ always has a positive value under certain conditions, which means the second law of black hole thermodynamics dose not violate for the non-extremal black holes. Next, we will further check the stability of horizon of the black hole. In a similar way, $f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0)$ is rewritten as

$$f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0)$$

$$= \delta + \frac{16r_h^{d-2}}{r_0^{d-3}\Omega_{d-2}} \left[ \frac{q^2\omega_q}{r_0^{d-3}\sqrt{2(d-2)(d-3)}} - \frac{\omega_q^2}{d-2} - \frac{q^4}{r_0^{d-3}r_h^{d-3}2(d-3)} + \frac{\omega_qq^2}{r_h^{d-3}\sqrt{2(d-3)(d-2)}} \right]dt$$

$$- \frac{16\pi p^r}{r_0^{d-3}(d-2)\Omega_{d-2}} + \frac{16\pi r_0^2}{(d-2)(d-1)}dP$$

$$- \frac{1}{r_0^{(d-1)\omega_q+d-3}}d\alpha - \frac{2}{(d-2)r_0^{d-4}}da. \quad (118)$$

Therefore, at the minimum point, we have

$$\delta = 0,$$

$$\delta_1 = + \frac{16r_h^{d-2}}{r_0^{d-3}\Omega_{d-2}} \left[ \frac{q^2\omega_q}{r_0^{d-3}\sqrt{2(d-2)(d-3)}} - \frac{\omega_q^2}{d-2} - \frac{q^4}{r_0^{d-3}r_h^{d-3}2(d-3)} + \frac{\omega_qq^2}{r_h^{d-3}\sqrt{2(d-3)(d-2)}} \right]dt.$$

$$\delta_2 = - \frac{16\pi p^r}{r_0^{d-3}(d-2)\Omega_{d-2}} - \frac{2r_0^2}{F^3}dl - \frac{1}{r_0^{(d-1)\omega_q+d-3}}d\alpha - \frac{2}{(d-2)r_0^{d-4}}da. \quad (119)$$

In the extremal black hole, $r_0 = r_h$, $T = 0$, and $df_{\text{min}} = 0$. Hence,

$$f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) < 0.$$  

(120)

Therefore, the event horizon exists in the extremal black hole. For the near-extremal black hole, the location $r_0$ is no longer equal to the event horizon $r_h$, which leads to that the
condition is not available. To calculate the value of Eq. (119), we can suppose that 
\( r_h = r_0 + \epsilon \), where \( 0 < \epsilon \ll 1 \). Using the same method above we can get

\[
\begin{align*}
\delta &= 0, \\
\delta_1 &= \left\{ \frac{16\pi}{\Omega_{d-2}} \left[ \frac{q^2r_0\omega_q + q^2\omega_q}{r_0^{d-3}\sqrt{2(d-3)(d-2)}} - \frac{\omega_q^2r_0}{d-2} - \frac{q^4}{2r_0^{d-3}(d-3)} \right] + O(\epsilon) + O(\epsilon^2) \right\} dt, \\
\delta_2 &= -\frac{16\pi p^r}{r_0^{d-3}(d-2)\Omega_{d-2}} \left[ \frac{2r_0^{d-3} dl}{l^3} - \frac{1}{r_0^{(d-1)\omega_q+d-3}} d\alpha - \frac{2}{(d-2)r_0^{d-4}} da. \right]
\end{align*}
\]

If the initial black hole is near extremal, we have \( dl \sim \epsilon, d\alpha \sim \epsilon, da \sim \epsilon, dt \sim \epsilon \). So we have

\[
\delta < 0, \delta_1 + \delta_2 \ll \delta.
\]

and

\[
f (M + dM, Q + dQ, P + dP, a + da, \alpha + d\alpha, r_0 + dr_0) \approx \delta < 0.
\]

Therefore, the event horizon exists and the black hole isn't overcharged in the final state. The weak cosmic censorship conjecture is valid in the near-extremal black hole.

V. DISCUSSION AND CONCLUSION

This paper investigated the first and second laws of thermodynamics and the stability of the horizon of a charged AdS black hole with cloud of strings and quintessence present in d-dimensional spacetime via particle absorption and scalar field scattering in the extended phase space. Our research was based on two assumptions in two cases, i.e., the energy of the particle is related to the internal energy or enthalpy of the black hole in the case of particle absorption, and the energy flux of the scalar field is combined with the internal energy or enthalpy of the black hole under the scalar field scattering.

At first, we reviewed the thermodynamics of the black hole by considering the cosmological constant as the function of thermodynamic pressure \( P \), and treating the state parameters of cloud of strings and quintessence as variables. Then we studied the absorption of scalar particle and fermion, and found they finally simplified to the same relation \( p^r = \omega - q\phi \) by deriving the Hamilton Jacobi equation. Furthermore, we tested the validity of the first and second laws of thermodynamics and the stability of the horizon under the assumption that the energy of particle \( E \) changes the internal energy of the black hole \( dU \). The first law
of thermodynamics is recovered, and the second law of thermodynamics is indefinite. The WCCC is valid all the time for extremal and near-extremal black holes, which means the horizons stable exist.

During the discussion of the second law, we mainly studied the change of the black hole entropy under different circumstances dimensions after fixing the variables. With the variation of the charge of the black hole, we found that there was always a phase transition point, which divides the variation of entropy into positive and negative region. The variation of entropy is negative for the extremal and near-extremal black holes, while positive for the far-extremal black holes. Therefore, it is concluded that in the extended phase space, the second law is violated for the extremal and near-extremal black holes. In addition, we compared the entropy changes of black holes in high and low dimensions, and found that the value of the phase change point increases with the decreases of dimension. While for the stability of horizons, we checked the sign of the minimum value of $f(r)$, and found it never greater than zero. Therefore, neither extremal black holes nor near-extremal black holes will be overcharged.

Furthermore, another assumption was considered, namely the energy of particle $E$ changes the enthalpy of the black hole $dM$. In this case, we found that the first law of thermodynamics and the stability of horizons results were same with the results obtained by the former $E = dU$. Moreover, the increment of the black hole’s entropy is always positive after particle absorption. Therefore, the second law of thermodynamics holds. The results are concluding in Table XXV.

In the section IV, at first the variations of the energy and charge of the black hole in an infinitesimal time interval after scalar field scattering were calculated. Then we recovered the first law of thermodynamics and discussed the validity of the second law of thermodynamics. Using the same research methods as the particle absorption part, we also found that there was always a phase transition point. Then, we further calculated and discussed the stability of the horizon via checking the sign of the minimum value of $f(r)$. Moreover, the thermodynamics and the stability of the horizon were also discussed under two assumptions, i.e., the energy flux of the scalar field $dE$ changes the internal energy of the black hole $dU$ and the energy flux of the scalar field $dE$ changes the enthalpy of the black hole $dM$. Our results are summarized in Table XXVI.

As shown in Refs. [115, 116], the RN-AdS black hole is studied in d-dimensional space via
|                      | Particle absorption |                      |
|----------------------|---------------------|----------------------|
| 1st law              | $\Delta M = TdS + VdP + \phi dQ + A\Delta a + Q\Delta \alpha$ | $\Delta M = TdS + VdP + \phi dQ + A\Delta a + Q\Delta \alpha$ |
| 2nd law              | Indefinite.         | Satisfied.           |
| The stability of horizon | The horizon still exists for the extremal and near-extremal black holes. | The horizon still exists for the extremal and near-extremal black holes. |

**Table XXV:** Results for the first and second laws of thermodynamics and the stability of horizons, which are tested for d-dimensional charged AdS black holes with cloud of strings and quintessence via particle absorption.

|                      | Scalar field scattering |                      |
|----------------------|-------------------------|----------------------|
| 1st law              | $\Delta M = TdS + VdP + \phi dQ + A\Delta a + Q\Delta \alpha$ | $\Delta M = TdS + VdP + \phi dQ + A\Delta a + Q\Delta \alpha$ |
| 2nd law              | Indefinite.             | Satisfied.           |
| The stability of horizon | Satisfied for the extremal and near-extremal black holes. The extremal/near-extremal black hole stays extremal/near-extremal after the scalar field scattering. | Satisfied for the extremal and near-extremal black holes. The extremal/near-extremal black hole stays extremal/near-extremal after the scalar field scattering. |

**Table XXVI:** Results for the first and second laws of thermodynamics and the stability of horizons, which are tested for d-dimensional charged AdS black holes with cloud of strings and quintessence via scalar field scattering.

Scalar field scattering and particle absorption, respectively. When the dimension is reduced to four, the laws of thermodynamics and the overcharging problem of the charged AdS black hole with cloud of strings and quintessence are investigated by particle absorption in [114], and studied under scalar field in [106]. When only quintessence is considered without cloud of strings, the thermodynamics and the stability of horizon for RN-AdS black hole with quintessence are investigated by particle absorption in Ref. [39], and are studied under scalar field scattering in Ref. [82]. The results of the first thermodynamic law under different
Types of black holes | 1st law
---|---
RN-AdS BH | $dM = TdS + VdP + \phi dQ$
RN-AdS BH with cloud of strings | $dM = TdS + VdP + \phi dQ - \frac{r_h}{2} da$
RN-AdS BH with quintessence | $dM = TdS + VdP + \phi dQ - \frac{1}{2r_h^{\omega_q}} d\alpha$
RN-AdS BH with cloud of strings and quintessence | $dM = TdS + VdP + \phi dQ - \frac{1}{2r_h^{\omega_q}} d\alpha - \frac{r_h}{2} da$
d-dimensional RN-AdS BH with cloud of strings and quintessence | $dM = TdS + VdP + \phi dQ - \frac{\Omega_d - 2\pi}{8\pi r_h^{\omega_q}} da + \frac{(2-d)\Omega_d - 2\pi}{16\pi r_h^{\omega_q}} d\alpha$

Table XXVII: Results for the first law of thermodynamic under different conditions.

conditions are summarized in Table XXVII.

In Refs. [39, 114, 115], the energy of the particle is assumed to correspond to internal energy of the black hole, i.e., $E = dU$ in the extended phase space. In Refs. [82, 106, 116], the energy flux of the field is assumed to correspond to internal energy of the black hole, i.e., $dE = dU$ in the extended phase space. Under this assumption, the second law of thermodynamics for black holes is violated in extended phase space. In Refs. [22, 117, 118], another assumption is proposed. In this assumption, the energy(energy flux) is assumed to change the enthalpy of the black hole instead of the internal energy of the black hole, i.e., $E = dM(dE = dM)$. Under this assumption, the second law of thermodynamics of the black hole is valid. Besides, the first law of thermodynamics and the stability of the horizon under this assumption have the same results as the previous one. The results of the black hole under two assumptions are same in normal phase space, since the mass can be regarded as the internal energy, i.e., $M = U$.

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