Maximal Independent Sets in Clique-free Graphs

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Abstract

An independent set $I$ of a graph $G$ is said to be a maximal independent set (MIS) if it is maximal with respect to set inclusion. Nielsen proved that the maximum number of MIS’s of size $k$ in an $n$-vertex graph is asymptotic to $(n/k)^k$, with the extremal construction being a disjoint union of $k$ cliques with sizes as close to $n/k$ as possible. In this talk we study how many MIS’s of size $k$ an $n$-vertex graph $G$ can have if $G$ does not contain a clique $K_t$. We prove for all fixed $k$ and $t$ that there exist such graphs with $n^{\left\lfloor \frac{(t-2)k}{t-3} \right\rfloor - o(1)}$ MIS’s of size $k$ by utilizing recent work of Gowers and B. Janzer on a generalization of the Ruzsa-Szemerédi problem. We prove that this bound is essentially best possible for triangle-free graphs when $k \leq 4$.

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