HADRONIC AND ELEMENTARY MULTIPLICITY DISTRIBUTIONS IN A GEOMETRICAL APPROACH

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We construct the hadronic multiplicity distribution in terms of an elementary distribution (at a given impact parameter) and the inelastic overlap function characterized by the observed BEL (Blacker-Edgier-Larger) behaviour. With suitable parametrizations for the elementary quantities, based on some geometrical arguments and the most recent data on $e^+e^-$ annihilation, an excellent description of $pp$ and $\bar{p}p$ inelastic multiplicity distributions at the highest energies is obtained.

I. INTRODUCTION

It is well known that the underlying theory of hadronic interactions, the quantum chromodynamics (QCD) is not yet able, by its own, to describe the bulk of experimental data associated with soft (long range) processes, in particular elastic scattering. At present, phenomenological approaches are very important as a source of information for adequate theoretical developments. On the other hand, from the experimental point of view, a renewed interest on both $pp$ and $\bar{p}p$ elastic and inelastic scattering is expected with the advent of the next accelerator generation, the RHIC and LHC \cite{1}. At this stage, due to our limited theoretical understanding of elastic scattering and, on the contrary, the success of QCD in treating hard (inelastic) processes, it may be important to investigate possible connections between elastic and inelastic channels, even from a phenomenological point of view. In this work we shall treat this subject in the contexts of the impact parameter picture, unitarity and the eikonal approximation.

Following other authors \cite{2}, we express the “complex” (overall) hadron-$p$ multiplicity distributions (inelastic channel) in terms of an “elementary” distribution associated with an elementary process taking place at given impact parameter and the inelastic overlap function (which is constructed from elastic channel data). The novel aspects concern: (a) quantitative correlation between the violations of the KNO scaling \cite{3} (inelastic channel) and geometrical scaling \cite{4} (elastic channel); (b) introduction of suitable parametrizations for the elementary quantities, based on some geometrical arguments and the most recent data on $e^+e^-$ annihilation. With this formalism, the hadronic multiplicity distribution may be evaluated without any free parameter and an excellent reproduction of the experimental data on $pp$ (ISR) and $\bar{p}p$ ($Sp\bar{p}S$) inelastic multiplicities is achieved. A detailed discussion on all the results presented here and also a complete list of references to the experimental data may be found in a recent paper \cite{5}.

II. IMPACT PARAMETER FORMALISM

In the geometrical picture, unitarity correlates the elastic scattering amplitude in the the impact parameter $b$ space, $\Gamma(b, s)$, with the inelastic overlap function, $G_{in}(b, s)$, by $2\text{Re} \Gamma(b, s) = |\Gamma(b, s)|^2 + G_{in}(b, s)$. For a purely imaginary elastic amplitude in momentum transfer space the profile function $\Gamma(b, s)$ is real and in the eikonal approximation is expressed by $\Gamma(b, s) = 1 - e^{\Omega(b, s)}$, so that

$$G_{in}(b, s) = 1 - e^{\Omega(b, s)}$$

is the probability for an inelastic event to take place at $b$, namely, $\sigma_{in}(s) = \int d^2 b \ G_{in}(b, s)$. In this picture the topological cross section for producing an even number $N$ of charged particles at $\sqrt{s}$ may be expressed by $\sigma_N(s) = \int d^2 b \ G_N(b, s) = \int d^2 b \ G_{in}(b, s) \sigma_{in}(b, s) / \sigma_{in}(b, s)$.

In this context, the formal connection between hadronic and elementary multiplicity distributions is obtained as follows. Let $\varphi$ be the elementary multiplicity distribution, $< n > (b, s)$ the average number of particles produced at $b$
and \( s = n - (b, s) \) a KNO variable associated with the elementary process taking place at \( b \) (and \( s \)). Then, in general, \( \varphi = < n > (b, s) \sigma_{N}(b, s)/\sigma_{in}(b, s) = \varphi(z, s) \). Representing the overall multiplicity distribution by \( \Phi \) and the corresponding KNO variable by \( Z = N(s)/< N > (s) \), where \( < N > (s) \) is the average multiplicity at \( \sqrt{s} \), we have in general \( \Phi = < N > (s)\sigma_{N}(s)/\sigma_{in}(s) = \Phi(Z, s) \). Both distributions are normalized by the usual conditions

\[
\int_{0}^{\infty} \varphi(z)dz = 2 = \int_{0}^{\infty} \varphi(z)dz, \quad \int_{0}^{\infty} \Phi(Z)dZ = 2 = \int_{0}^{\infty} \Phi(Z)dZ.
\]  

(2)

Now, introducing a multiplicity function as the ratio

\[
< n > (b, s) \equiv m(b, s),
\]  

(3)

the relationship between \( \Phi \) and \( \varphi \) follows from the above equations:

\[
\Phi = \frac{\int d^{2}b \frac{G_{in}(b, s)}{m(b, s)} \varphi(\frac{Z}{m(b, s)})}{\int d^{2}b \ G_{in}(b, s)} = \Phi(Z, s).
\]  

(4)

This result means that, once one has parametrizations for \( G_{in}(b, s) \) and the elementary quantities \( \varphi \) (multiplicity function) and \( m(b, s) \) (multiplicity function) the overall hadronic multiplicity distribution may be evaluated. In this work we consider \( G_{in}(b, s) \) from analyses of elastic \( pp \) and \( p\bar{p} \) scattering data (taking account of geometrical scaling violation) and infer the elementary quantities based on geometric arguments and experimental data on \( e^{+}e^{-} \) annihilation, as explained in what follows.

In the elastic channel, the breaking of Geometrical scaling is quite well described by the BEL behaviour, analytically expressed by the Short Range Expansion \[3\], \( G_{in}(b, s) = P(s)exp(-b^{2}/4B(s))k(x, s) \), with \( k \) expanded in terms of a short-range variable \( x = b \ exp(-\gamma b^{2}/4B(s)) \). With suitable parametrizations for \( P(s) \) and \( B(s) \) an excellent agreement with experimental data on \( pp \) and \( p\bar{p} \) elastic scattering is achieved and we shall use this well known result \[3\] as the input from the elastic channel.

III. ELEMENTARY HADRONIC PROCESS

We now turn to the discussion of the elementary hadronic processes, characterized by \( \varphi \) and \( m \) in Eq. (4). At a given impact parameter, the most elementary process is an \( e^{+}e^{-} \) collision, which is a process occurring in a unique angular momentum state and therefore also at a given impact parameter (zero in this case). Although \( e^{+}e^{-} \) annihilations cannot be exactly the same as collisions between hadron constituents, it is reasonable, even from a geometric point of view, to think that some characteristics of both processes may be similar. The point is to find out or infer what they could be.

First, let us consider that the average multiplicity at given impact parameter depends on the center-of-mass energy in the form of a general power law \( < n > (b \ fixed, s) \propto E_{CM}^{\gamma} \). Now, from Eq. (1), \( exp(-2\Omega(b, s)) \) is the transmission coefficient, i.e. the probability of having no interaction at a given impact parameter, and therefore \( \Omega \) should be proportional to the thickness of the target, or the energy \( E_{CM} \) that can be deposited at \( b \) for particle production at a given \( s \). Then, we can express \( < n > (b, s) \propto \Omega^{\gamma}(b, s) \) and comparison with Eq. (3), allows us to infer the multiplicity function:

\[
m(b, s) = \xi \Omega^{\gamma}(b, s),
\]  

(5)

with \( \xi \) being calculated by the normalization condition of the overall multiplicity distribution, Eq. (2). With this, Eq. (4) becomes

\[
\Phi = \frac{\int d^{2}b \frac{G_{in}(b, s)}{\xi \Omega^{\gamma}(b, s)} \varphi(\frac{Z}{\xi \Omega^{\gamma}(b, s)})}{\int d^{2}b \ G_{in}(b, s)} = \Phi(Z, s),
\]  

(6)

where

\[
\xi = \frac{\int db^{2}G_{in}(b, s)}{\int db^{2} \ G_{in}(b, s)\Omega^{\gamma}(b, s)} = \xi(s).
\]  

(7)
We proceed with the determination of the elementary distribution \( \varphi \) and the power coefficient \( \gamma \), through quantitative analyses of \( e^+e^- \) data and under the following arguments. Because the elementary process occurs at a given impact parameter, its elementary structure suggests that it should scale in the KNO sense. Now, since experimental information on \( e^+e^- \) multiplicity distributions shows agreement with this scaling, we shall base our parametrization for \( \varphi \) just on these data. In particular, it is sufficient to assume a gamma distribution, normalized according to Eq. (2), \( \varphi(z) = 2K^K z^{K-1} \exp\{-Kz\}/\Gamma(K) \). Fit to the most recent data, covering the interval \( 22.0 \text{ GeV} \leq \sqrt{s} \leq 161 \text{ GeV} \) furnished \( K = 10.775 \pm 0.064 \) with \( \chi^2/N_{DF} = 508/195 \).

Finally, we consider fits to the \( e^+e^- \) average multiplicity through a general power law \( \langle n \rangle_{e^+e^-} = \alpha(\sqrt{s})^\gamma \). We collected the most recent experimental data at the highest energies, covering the interval \( 5.1 \text{ GeV} \leq \sqrt{s} \leq 183 \text{ GeV} \). Fitting to this equation yields \( \alpha = 2.06 \pm 0.02 \) and \( \gamma = 0.522 \pm 0.002 \), with \( \chi^2/N_{DF} = 354/45 \). The above parametrization deviates from the data above \( \sqrt{s} \sim 100 \text{ GeV} \) and this contributes to the high \( \chi^2 \) value. However, as commented before, we do not expect that \( e^+e^- \) annihilation exactly represent the collisions between hadrons constituents.

IV. RESULTS AND CONCLUSIONS

With the above results we are now able to predict the hadronic multiplicity distribution \( \Phi(Z, s) \), Eq. (6), without free parameters: \( G_{in}(b, s) \) (and \( \Omega(b, s) \)) comes from analysis of the elastic scattering data and \( \varphi(z) \) and \( \gamma \) from fits to \( e^+e^- \) data. We express \( \Phi \) in terms of the scaling variable \( Z' = N'/\langle N' \rangle \) where \( \langle N' \rangle = N(s) - N_0 \) with \( N_0 = 0.9 \) leading charges removed. The predictions for \( pp \) scattering at \( 52.6 \text{ GeV} \) and \( \bar{p}p \) at \( 546 \text{ GeV} \) are shown in Figs. 1 and 2, together with the experimental data. The behaviour of the normalization factor \( \xi(s) \), as determined from Eq. (7), is shown in Fig. (3) in a wide interval of energy. The multiplicity functions \( m(b, s) \), Eqs. (1) and (5), corresponding to the energy and reactions of Figs. 1 and 2 are displayed in Fig. 4.

From Figs. 1 and 2, we conclude that, in the context of the geometrical approach and based on the BEL behaviour for \( G_{in}(b, s) \), our guess for the elementary hadronic quantities (based on geometrical arguments and quantitative analysis of \( e^+e^- \) data) yields excellent predictions for the hadronic distribution \( \Phi(Z, s) \).

The fact that the average multiplicity associated with hadron constituents, increases faster then \( e^+e^- \) data above \( \sim 100 \text{ GeV} \) seems quite reasonable, since at these energies we expect additional contributions from gluons/quarks interactions which are not present in lepton-lepton collisions.

Anyway, despite the success of the geometrical approach in the phenomenological context, it is in general difficult to obtain direct connections with a microscopic theory. Some aspects of these connections have been recently discussed [3] and other results, along these lines, were also presented at this Conference [4].

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FIG. 1. Scaled multiplicity distribution for inelastic $pp$ data at ISR energies compared to theoretical expectations using Eq. (6).

FIG. 2. Same as Fig. 1 for inelastic $\bar{p}p$ data at collider energies.

FIG. 3. Normalization factor from Eq. (7) as function of the energy.

FIG. 4. Multiplicity function from Eq. (5) for $pp$ at 52.6 GeV and $\bar{p}p$ at 546 GeV.