Pure Rolling Steering System Design and Research for Wheeled Mobile Robot

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Abstract: In order to solve the problem of the side slip of the wheeled mobile robot adopting the steering trapezium, a novel pure rolling steering system for wheeled mobile robot is designed. It takes advantage of synchronous belt characteristic with long-distance, variable-direction and precise transmission and is equipped with a dual power differential drive axle. This paper details the design points of the whole mechanism; sets up the kinematic models of four-wheeled Ackerman steering, crab movement, and pivot turn modes; and then the steering drive control strategies of various modes of the robot is established. A prototype equipped with a control system consisting of a control center, four steering motors, two drive motors, an electromagnetic clutch and four angle sensors is constructed. Finally, the test results for pivot turn and four-wheel minimum radius steering on the hard road shows that the trajectory slip error are less than 0.2% and 1.7%, respectively. This verifies clearly the correctness and effectiveness of the proposed pure rolling steering system for wheeled mobile robot.

1. Introduction

In the field of mobile robot, wheeled mobile robot has always been the most common. Compared with legged robot or crawler robot, it has higher efficiency, better traction ability and stability [1]. Some outdoor wheeled mobile robots equipping trapezoidal steering mechanism has poor omnidirectional mobility. Most of the other schemes with distributed drive and electronic slip steering also have some shortcomings, such as tire wear, poor calculated position accuracy and difficulty of control. At the same time, the dynamic models of these schemes are very complex, involving the characteristics of the robot body, tire dynamics and the interaction between the road and tire. So it is difficult to build the model with a large number of uncertain parameters [2]. In addition, there are some representative omni-directional driving mechanisms, such as Mecanum-wheel [3], Universal-wheel, Ball-wheel [4], etc. Those simple structures are widely used in soccer robot games, service robots and other indoor places, however, they are not suitable for outdoor roads. Therefore, the realization of pure rolling steering without side slip and accurately completing the scheduled trajectory is important research topic.

Based on these problems, a novel rolling and non-sliding wheeled omnidirectional mobile robot and its key designing points is proposed. By establishing the steering dynamic models of various steering modes, the corresponding control strategies are established. These designs and analyses provide references for the innovative design of the outdoor wheeled mobile robot steering transmission system. The prototype and experiments verify the correctness and effectiveness of the robot’s pure rolling steering system and steering control strategies.
2. System Design of Robot

As shown in figure 1, the robot mainly consists of a dual power differential drive axle, two synchronous belt transmission and steering mechanisms.

2.1. Dual Power Differential Drive Axle

In this section, a novel dual power drive axle is proposed what mainly consists of ordinary bevel gear differential, electromagnetic clutch and two brushless DC motors, as shown in Figure 2. Both power sources are composed of planetary reducer (gear ratio is $i_0=6$), brushless DC motor (24V, 300W), electromagnetic brake and incremental photoelectric encoder (1000 pulses per cycle).

The power of the main-motor is transmitted to the belt pulley of the differential shell through the synchronous belt. And the power of the sub-motor is transmitted to the pulley of the electromagnetic clutch. Transmission ratio between small pulley and large pulley is $i_1=1.5$.

The dual power differential drive axle has three cases as shown in figure 6. The yellow line shows the power flow in different situations. The control strategies for the three cases are as follows.

Figure 1. Main mechanism of the robot

Figure 2. Dual power drive axle.

Figure 3. Power Flow of Three Driving Strategies.
Figure 3 (a) is single-motor driving. The main-motor transmits power to the belt pulley of the differential shell. Meanwhile, the electromagnetic clutch is separated and the sub-motor does not work. The difference of speed on the left and right is adjusted by the action of road and tires on both sides, which makes four wheels pure rolling without sideslip.

Figure 3 (b) is dual-motor driving. The electromagnetic brakes at the tail of two motors are opened after closing the electromagnetic clutch. Then, the output speed of the main and subsidiary motors are controlled according to the determined relationship. In this case, the subsidiary motor is equivalent to the backup power to improve the dynamic performance of the robot. For example, it could make robot climb more sloping road or carry more load.

Figure 3 (c) is a case of pivot turn. After closing the electromagnetic clutch, the subsidiary motor transmits power to the belt pulley of the clutch. Meanwhile, the electromagnetic brake of the main motor is closed so that the speed on the differential axle housing is always 0. The differential has the following kinematic principle:

\[ 2n_{ms} = n_l + n_r \]  

Therefore, when the sub-motor on right side input power \( n_r = n_s \), the left side will get the same speed but in the opposite direction \( n_l = -n_l = -n_r \). The power on both sides is transmitted to four wheels through synchronous belt transmission mechanism, the mobile robot obtaining a driving rotary torque around the center of the robot. Then the robot will performs clockwise or counterclockwise rotation around the center of the robot.

2.2. Synchronous Belt Transmission and Steering Mechanism

In order to cooperate with H-type single drive axle with balanced rocker arm, a synchronous belt transmission and steering mechanism is designed, as shown in figure 4.

![Figure 4. Synchronous belt transmission mechanism](image1)

![Figure 5. Top view of steering process](image2)

The power from the main and subsidiary drive motor is transmitted to the belt pulley 1 at both ends of the drive axle. Then, the belt pulley 1 transmits power to the front and rear wheels on the same side through two-stage synchronous belt transmission. The synchronous belt transmission mechanism which is pure rolling without slipping is composed of duplex belt pulley (2,3), guide pulley 1, belt pulley 2 and belt pulley 4. Power and motion are transmitted by twisted synchronous-belt between belt pulleys. Here, \( i_2 \) is the transmission ratio of belt pulley 1 to belt pulley 2, and \( i_3 \) is the transmission ratio of belt pulley 3 to belt pulley 4. The steering mechanism consist of L-type steering leg, steering gear motor and angle sensor. The wheels and steering legs are driven directly by a gear motor rotates around a central axis of the belt pulley 2.

Considering the kinematic interference between synchronous belt transmission and steering, the relationship between the horizontal length of steering leg \( s \) and the radius of wheel \( r \) is analyzed as follows:
Figure 5 shows a top view of the steering process. The steering leg rotates from position $A$ to $A_1$ and the rotation angle is $\beta$. The distance of the synchronous belt rotating along the belt pulley 4 is as:

$$L_4 = \beta d_3 / 2$$  \hspace{1cm} (2)

From equation (1), we get rotation angle of belt pulley 4:

$$\alpha = \frac{L_4}{d_4} = \frac{\beta d_3}{d_4}$$  \hspace{1cm} (3)

The motion distance of the wheel driven by belt pulley 4 is as follows:

$$L_2 = \alpha r$$  \hspace{1cm} (4)

According to figure 5, we derive the distance of the wheel moving along the circular trajectory with radius $s$:

$$L_3 = \beta s$$  \hspace{1cm} (5)

From equation (2), (3), (4) and $L_3 = L_2$, we derive:

$$i_4 = \frac{d_4}{d_3} = \frac{r}{s}$$  \hspace{1cm} (6)

In those equations, $d_3$ is the reference diameter of the stationary pulley 3; $d_4$ is reference diameter of the rotating pulley 4; $r$ is the wheel radius; $s$ is the length of steering leg (the horizontal distance from the center of the pulley 3 to the center of the wheel).

Therefore, it is necessary to ensure that the ratio of the diameter of the pulley 3 to the diameter of the pulley 4 is equal to the ratio of the length of the steering leg to the radius of the wheel. This design avoids "Jumping over Teeth" of synchronous belt or wheel slide slip as a result of the kinematic interference [5].

Due to the precise transmission of synchronous belt and pure rolling steering of wheels, the system has many advantages in practical applications, such as small error of trajectory tracking, accurate kinematic equation and no tire wear during steering.

3. Steering Mode Analysis and Kinematic Model

According to the structure and transmission of the mobile robot, it’s function is decomposed into the three modes: ackerman steering mode, crab driving mode and pivot turn mode.

3.1. Ackerman Steering Mode and Kinematic Model

In this paper, an optimal four wheel steering mode is adopted after several possible Ackerman steering models are analyzed. And its kinematic model also is analyzed in 3.1.2.

3.1.1. Front-Wheel Steering
In figure 6, 7, and 8, \( l \) is the distance between the front and back rotation centers of the robot. \( s \) is the distance between the center of left and right rotation of the robot. The centers of the four wheels are recorded as \( A_1A_2A_3A_4 \). The rotation centers of the four steering legs are recorded as \( O_1O_2O_3O_4 \). \( P \) is the instantaneous turning center of robot. \( \delta_i \) is the inner turning angle. \( \delta_o \) is the outer turning angle. The turning angles of four wheels around the rotation centers are \( \beta_1 \beta_2 \beta_3 \beta_4 \).

As shown in figure 9, we analyze the possibility of front wheel steering. According to Ackerman steering geometry, the relationship of inside and outside steering angles can be expressed as follow:

\[
\delta_s = \arctan \frac{l}{l/\tan \delta_i + h}
\]

Instantaneous steering radius of four wheels be derived.

\[
\begin{align*}
R_1 &= PA_1 = l/\sin \delta_o + s \\
R_2 &= PA_2 = l/\sin \delta_i - s \\
R_3 &= PA_3 = l/\cos \delta_o + s \\
R_4 &= PA_4 = l/\cos \delta_i - s
\end{align*}
\]

From equation (8), we derive that \( R_1 \neq R_2 \) and \( R_3 \neq R_4 \). But figure 2 and figure 4 show that the speed of the front and rear wheels on the same side is equal. In this mode, wheels will certainly slip or the synchronous belt will “Jumping over Teeth”, causing damage to the belt and inaccuracy of transmission. Therefore, we don’t adopt front wheel steering mode.

### 3.1.2 Four Wheel Steering

In four-wheel steering mode, two cases are analyzed as follows.

1. **Unequal steering angles of front and rear wheels.**

   Figure 7 shows the kinematic relationship of this case. We can draw the same conclusion as the front wheel steering: \( R_1 \neq R_3 \) and \( R_2 \neq R_4 \); the speed of the front and rear wheels on the same side is equal. Accordingly, we don’t adopt this mode when formulating control strategies.

2. **Equal steering angles of front and rear wheels.**

   Figure 8 shows the kinematic relationship of this case. According to Ackerman steering geometry, the relationship of inside and outside steering angles can be expressed as follow:

\[
\frac{l/2}{\tan \delta_o} = \frac{l/2}{\tan \delta_i} + h
\]

Instantaneous steering radius of four wheels can be derived:

\[
\begin{align*}
R_1 &= R_3 = PA_1 = 0.5l/\sin \delta_o + s \\
R_2 &= R_4 = PA_2 = 0.5l/\sin \delta_i - s
\end{align*}
\]
Combined with the above analysis, there is no doubt that this case is the best Ackerman steering mode. This mode achieves minimum turning radius without slip.

Considering the kinematic interference of the mobile robot in the actual steering process, the rotation angles of the four steering legs are limited. The maximum rotation angle of each steering leg to the inside is 30 degree by kinematic simulation of the designed robot.

Figure 9 shows the relationship between the rotation angles of the inside and outside steering legs and the turning radius of robot during four-wheel steering.

Figure 10. Model of four wheel steering.

The kinematic model of four-wheel steering is established in order to explore the relationship among the speed of two motors, the wheel speed on both sides and the navigation parameters. Figure 10 establishes a global coordinate system and a local coordinate system at the center of the robot. Conversion relations between the two reference systems can be derived [6]:

$$\dot{\xi}_O = \mathbf{R}(\theta) \dot{\xi}_C$$

Where, $\xi_O = [X, Y, \theta]^T$ is parameter of global coordinate system. $\xi_C = [\dot{x}, \dot{y}, \dot{\theta}]^T$ is parameter of local coordinate system. $\mathbf{R} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is defined as rotation matrix.

When both main and subsidiary motors provide power, from equation (1) and transmission parameters, wheel speed on outside and inside can be derived as:

$$v_o = \frac{\pi r (2n_m - n_s)}{30i_{12}i_{10}}$$

$$v_i = \frac{\pi r n_s}{30i_{12}i_{10}}$$

The navigation parameters for the robot in Figure 10 is derived as

$$\dot{x} = v_c = \frac{v_o + v_i}{2} = \frac{\pi r n_m}{30i_{12}i_{10}} = f(n_m)$$

$$\dot{y} = 0$$

$$\dot{\theta} = \frac{\dot{x}}{R_c} = \frac{v_c}{0.5h + 0.5l / \tan \delta} = \frac{\pi r n_m}{15i_{12}i_{10}(h + l / \tan \delta)} = f(n_m, \delta)$$

The speed of the main-motor and the sub-motor is constrained by the following equation:

$$2n_m = n_r + n_s; n_s = n_r$$

$$n_m = \frac{K(\delta) + 1}{2}$$
Where, \( K(\delta) = \frac{n_l}{n_r} = \frac{PA_i}{PA_2} \frac{0.5l / \sin \delta + s}{0.5l / \tan \delta + h} = \frac{0.5l / \sin(\arctan \frac{0.5l}{0.5l / \tan \delta + h}) + s}{0.5l / \sin \delta + s} \).

Then, the kinematics model of four-wheel steering is greatly simplified as,

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
f(n_m) \\
0 \\
f(n_m, \delta)
\end{bmatrix}
\] (19)

3.2. Crab driving mode

The robot can run like a crab by turning the steering leg to make four wheels move in one direction. If the turning limit of the steering leg is designed to be greater than 90 degree, it can even move horizontally left or right. Figure 11 shows an example of crab driving mode.

The relationship between wheel speed and motor speed:

\[
v = v_{\text{left}} = v_{\text{right}} = \frac{\pi rn_s}{30l_i^2 i_1 i_0}
\] (20)

In this case, the robot can be powered by two motors or only main motor. When two motors are applied, the speed constraint of the main-motor and the sub-motor is \(2n_m = n_1 + n_r\) and \(n_m = n_s\).

3.3. Pivot turn mode.

Figure 12 shows pivot turn mode of steering mechanism. The robot will rotate around their center by closing the electromagnetic brake of the main-motor and controlling the power output of the sub-motor. The speed of wheel rotation and radius of the trajectory are as follow:

\[
v = \frac{\pi rn_s}{30l_i^2 i_1 i_0}
\] (21)

\[R_p = s + 0.5(l^2 + h^2)^{1/2}
\] (22)

The relationship between the course angle (\(\phi\)) of the robot and the output speed of the sub-motor in trajectory tracking control is described as,

\[
\dot{\phi} = \frac{v}{R_p} = \frac{\pi rn_s}{15l_i^2 i_1 i_0 (2s + (l^2 + h^2)^{1/2})}
\] (23)

\[\beta_1 = \beta_2 = \beta_3 = \beta_4 = \arctan \frac{l}{h}
\] (24)
Therefore, the omnidirectional motion of robot can be decomposed into the rotation around the center of robot and point-to-point linear motion. This mode is very useful in narrow space, such as office buildings, hospital corridors, warehouse corridors, factory floors, etc.

4. Prototyping

4.1. Prototype Mechanism
In order to confirm the proposed mechanism and analyze the problems in real road, a prototype was developed. Specifications of this prototype are shown in Table 1. Figure 13 and 14 show top view and side view of the prototype. Table 1 lists some specifications of the prototype.

![Figure 13. Top view of the prototype.](image)

![Figure 14. Side view of the prototype.](image)

| Parameters                  | Symbol | Value(unit)     |
|-----------------------------|--------|-----------------|
| Wheel radius                | $r$    | 90(mm)          |
| Distance of rotation center | $l \times h$ | 500 × 326 (mm) |
| Length of steering leg      | $s$    | 90(mm)          |
| Weight of the robot         | $G$    | 55(kg)          |

4.2. Control System for Prototype Robot
From the above analysis, the robot is driven by synchronous belt, motors, clutches and brakes according to different modes. Figure 15 shows the control system of the prototype. The steering control system adopts four position closed-loop based on PID controller. The speed control system adopts two speed closed-loop on PID controller, and coordinates the output speed of the two driving motors by establishing the kinematic models of different modes. According to the kinematic model of different modes, the controller coordinates the output speed of the main-motor and sub-motor. As shown in Figure15, the robot can be commanded by teleoperation system and computer connected via WiFi.

![Figure 15. Block diagram of the control system.](image)
5. Tests and Results
In order to test the various functions and mobility of the designed prototype, several tests were designed. Since it is impossible to directly measure the rotational motion and direction of each part, we recorded the trajectory of the four wheels by smearing black paint on the surface of the tire to verify whether the mobile robot complete the designed function and measure its error.

Figure 16(a) shows a pure rotation motion test (pivot turn mode). the prototype rotates at least 20 laps around the center. As shown in Figure 16(b), the robot has almost no slip error by measuring the trajectory of the wheel, and the measured actual trajectory radius is approximately equal to the radius obtained by equation (22). Error of radius is about \( e_1 = (688.5 - 687) / 687 \approx 0.002 \).

Figure 16. Experimental results of the prototype.

Figure 16(c) and 16 (d) show four-wheel steering test in which the prototype turns not less than one circle around an instantaneous center. According to the actual measurement results, the minimum turning radius is 560mm (steering angle is 30 degree). The error of radius is about 10mm \( e_2 \approx 0.017 \).

These experiments verify that the proposed robot has excellent mobility and small slip error.

6. Conclusion
This paper introduces a novel pure rolling steering system for wheeled mobile robot and key designing points. The mobile robot equipped with the synchronous belt transmission and pure rolling steering mechanism has simpler control algorithm and smaller slip than the four-wheel sliding steering robot or the steering trapezium robot. Due to the existence of single-motor mode and dual-motor mode, the robot can save electricity consumption to extend battery operating time.

In addition, the prototype and test are also introduced in the above chapters, verifying the effectiveness of the proposed robot.

Future works will be considered as follows:
• Research on real-time coordinated control algorithm of four steering legs.
• Analysis of the relationship between the elongation of synchronous belt and the transmission distance in case of bending.
• Application to autonomous navigation on outdoor road.

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