Effect of TD viscosity and Coriolis force on thermohaline convection of ferrofluid in Darcy porous matrix: A Galerkin numerical technique

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Abstract: The effect of TD viscosity (Temperature Dependent viscosity) and Coriolis force in horizontal ferromagnetic fluid layer with Darcy porous matrix is investigated theoretically. This fluid layer is heated from below and it is salted from above. The non-linear behavior is omitted by use of a linear stability theory. The normal mode technique is taken to account to investigate the convective system with free-free boundaries. To investigate the entire system the exact solution is obtained. The stationary instability is obtained. But, oscillatory instability can not occur in the study. The critical thermal magnetic Rayleigh number \( N_{SC} \) is calculated for sufficient large values of \( M_1 \). In the analysis, we have attempted to analyze \( T_\alpha \), \( V \) and \( Da \) on the system in many situations.

Keywords: Coriolis force, TD viscosity, Darcy porous matrix, Galerkin numerical technique, thermohaline convection, ferrofluid.

1 Introduction

The double diffusion related to the thermohaline convective effects were first discovered an outstanding results of the various investigations by Stern [1], Turner [2], Huppert and Turner [3] - [4] and Turner and Stommel [5]. The investigation is made on sugar-salt system by Turner and Chen [6] and they have taken the shadowgraph photography is to analyse excellent studies of motions. Rubin and Roth [7] have been given the thermohaline convection in flowing groundwater. In this excremental investigation, the three different mechanism are done. Erland Kallen and Xiang-Yu Huang [8] have analysed the large-scale thermohaline convective instability cells by time integrating and 2D Boussinesq model is considered. Veronis [9] introduced the thermohaline convective system of fluid layer. This thermohaline convective instability is taken on Boussinesq approximation with 2D non-linearity by Veronis [10].

The non-linearity effects is introduced on the double diffusive convection by Huppert and Moore [11]. In this, they have been obtained a various results. Later, Knobloch et al. [12] used a five mode truncation originally suggested by Veronis [9] to get the solution that were in excellent qualitative agreement with the numeral results of Huppert and Moore [11]. The two illustrations of 2D non-linear double diffusive convective instability has been studied by Knobloch and Proctor [13]. The unstable mode has been studied in the most cases on thermohaline convective instability subjected to linear gradients by Baines and Gill [14] and they have been introduced salinity Rayleigh number. The two component ferroconvective system is taken to investigate on thermohaline convective instability by Vaidyanathan et
The fluid flow phenomena with porous medium is a wonderful investigation because of its natural occurrence. The stability analysis of flow of fluid in a pores effect was taken by Lapwood [17] and Wooding [16] with the Darcy resistance. Moreover, the porous behavior on double diffusive convection in a fluid is of interesting study in engineering sciences. Nield and Bejan [18] scrutinised the double diffusive convection in porous medium. One of the many fascinating features of the ferrofluid is the prospect of influencing flow by the magnetic field and vice-versa (Feynman et al. [19], Shliomis [20]. High quality review on the convection of ferrofluid has been done by Rosensweig [21]. Finlayson [22] opened the convective instability work on ferrofluid layer with uniform vertical magnetic field. Vaidyanathan et al.[23] investigated the porous behavior on Finlayson [22] using Brinkman number. Then, this analysis was taken for investigate with the anisotropy effect on Brinkman pores by Sekar et al. [24]. An anisotropic porous effect is introduced on the thermohaline convection with Soret effect has been given by Sekar et al [25]. Straughen [26] given the energy method, stability and non-linear convection and the thermorheological effect on magnet convection in week electrically fluids under $1g$ or $\mu g$ has been investigated by Siddheshwar [27].

In this present work, effect of rotation on infinite horizontal ferrofluid layer with the effect of TD viscosity and coriolis force and $d$ is the thickness $d$ fluid layer. The temperature and salinity at the bottom and top surfaces $z = \mp d/2$ are $T_0 \pm (\Delta T)/2$ and $S_0 \mp (\Delta S)/2$, respectively and $\beta_t(= |dT/dz|)$ and $\beta_s(= |dS/dz|)$ are maintained. The viscosity of fluid is assumed to be temperature dependent is $\mu(T) = \mu_1(1 - \delta(T - T_a)^2)$ (Straughan [26] and Siddheshwar [27]).

Figure 1: Geometrical Configuration

2 Mathematical formulation of problem

The continuity equation is

$$\nabla \cdot \mathbf{q} = 0$$

The momentum equation is

$$\rho_0 \left( \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) \mathbf{q}$$

$$= \rho \mathbf{g} - \nabla p + \nabla \cdot (\mathbf{HB}) + \nabla (\mu(T)(\nabla \mathbf{q} + \nabla \mathbf{q}^T))$$

$$2(\mathbf{Q} + \mathbf{q}) + \frac{\rho_0}{2} \nabla |\mathbf{Q} \times \mathbf{r}|^2 - \frac{\mu(T)}{k_0} \mathbf{q},$$

The temperature equation is
The mass flux equation is
\[ \rho_0 \left( \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) S = K_S \nabla^2 S, \] (2.4)

Assuming the magnetization by use of Maxwell’s equation is
\[ \mathbf{M} = \frac{\mathbf{H}}{H} M(H,T,S), \] (2.5)

The linearized magnetic equation in term of \( H_0, T_a \) and \( S_a \) is
\[ M = M_0 + \chi (H - H_0) - K(T - T_a) + K_2(S - S_a), \] (2.6)

The density equation is
\[ \rho = \rho_0 [1 - \alpha_s (T - T_a) + \alpha_s (S - S_a)], \] (2.7)
where \( \mathbf{q} \) - velocity of fluid, \( p \) - pressure, \( \rho_0 \) - mean density of the clean fluid, \( \mathbf{B} \) - magnetic field, \( C_{v,H} \) - effective heat capacity at constant volume, \( C_s \) - specific heat solid material, \( \mathbf{M} \) - magnetization, \( K_1 \) - thermal diffusivity, \( \mu_0 \) - viscosity of the fluid when the applied magnetic field is absent, \( S \) - solute concentration, \( T \) - temperature, \( K_x \) - concentration diffusivity, \( T_a \) - average temperature, \( S_a \) - average salinity, \( H_0 \) - uniform magnetic field, \( \alpha_s \) - analogous solvent coefficient and \( \alpha_t \) - thermal expansion coefficient.

Further investigation is carried out by use of Ramanathan and Muchikel [28]. We undertake the perturbation quantities by use of normal modes are
\[ f(x, y, z, t) = f(z, t) \exp[i k_x x + i k_y y], \] (2.8)
where \( f(z, t) \) represents \( w(z, t) \), \( T(z, t) \), \( \phi(z, t) \), \( \zeta(z, t) \) and \( S(z, t) \).

The vertical component of Eq. (2.2) is
\[ \rho_0 \frac{\partial}{\partial t} \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w = \mu_0 \chi T_0 \frac{\partial}{\partial z} \phi + \frac{\mu_0 K_0^2 T_0^2 \phi^2}{1 + \chi} \theta - \rho_0 g \alpha_s k_0^2 \theta. \] (2.9)

Eq. (2.3) can be calculated as
\[ \rho_0 \frac{\partial}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial z} \phi = \left[ \rho C_\beta - \left( \frac{\mu_0 K_0^2 T_0 \beta_S}{1 + \chi} \right) \right] w + K_1 \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \theta \] (2.10)

The Salinity equation is
\[ \left( \frac{\partial S}{\partial t} + \beta_S w = K_S \frac{\partial^2}{\partial z^2} - k_0^2 \right) S, \] (2.11)

Using the analysis similar to Vaidyanathan et al. [15], one gets
\[ (1 + \chi) \frac{\partial^2 \phi}{\partial z^2} - \left( 1 + \frac{M_0}{\mu_0} \right) k_0^2 \phi - K \frac{\partial^2 \theta}{\partial z^2} + K_2 \frac{\partial S}{\partial z^2} \] (2.12)

From the \( x \) and \( y \) components of momentum equation, one can gets
\[ \rho_0 \frac{\partial \zeta}{\partial t} = \mu_b (T) \left( \frac{\partial^2}{\partial z^2} \right) \zeta + 2 \Omega \frac{\partial w}{\partial z} \] (2.13)

Then, Eqs. (2.9)-(2.12) become non-dimensional equations.
where the dimensionless quantities used are

\[
 w^* = \frac{w}{v}, \quad t^* = \frac{t}{a^2}, \quad T^* = \left( \frac{K_1 a R^{1/2}}{\rho_0 c_{v,H} \beta \nu d} \right) \theta, 
\]

\[
 z^* = \frac{z}{a}, \quad a = k_0 d, \quad D = \frac{D}{a^2}, \quad S^* = \left( \frac{K_S a R^{1/2} \beta_s }{\rho_0 c_{v,H} \beta \nu d} \right) S, 
\]

\[
 M_1 = \left( \frac{\mu_0 R^{2/3}}{\mu_0 C_{v,H}^{4/3}} \right), \quad P_r = \frac{v}{K_1} \rho C, 
\]

\[
 M_2 = \frac{1+\chi}{1+\chi} M_0, \quad M_3 = \frac{K_S a R^{1/2} \beta_s }{K_S a R^{1/2} \beta_s }, \quad M_5 = \frac{K_2 a R^{1/2} \beta_s }{K_1}, 
\]

\[
 M_6 = \frac{\rho_0 c_{v,H} \beta_s}{K_1}, \quad \tau = \frac{\rho_0 c_{v,H} K_S a R^{1/2} \beta_s}{K_1}, 
\]

\[
 R_S = \frac{\rho_0 c_{v,H} \beta_s a_4 d^4}{vK_S}, \quad R = \frac{\rho_0 c_{v,H} \beta_s a_4 d^4}{vK_1}, 
\]

\[
 \phi^* = \left( \frac{\mu_0 R^{2/3}}{\rho_0 c_{v,H} \beta \nu d} \right)^{1/2}, \quad \nu = \frac{\mu_0}{\rho_0}, 
\]

\[
 k^* = \frac{k}{d^2}, \quad \zeta = \frac{\nu \zeta}{d^2}, \quad T_a = \frac{16d^4}{\nu^2}, 
\]

In the analysis, we can scrutinise the stationary instability. But, oscillatory instability can not occur (Ramanathan and Muchikel [28]). Moreover, the linear theory is used in this moment. The boundary conditions on velocity, temperature and salinity are

\[
 w^* = D^2 w^* = T^* = D\phi^* = S^* = 0 \quad \text{at} \quad z = \pm 1/2, 
\]

The exact solutions satisfying above Eq. (2.18) are

\[
 w^* = A_1 e^{\sigma t} w_1(z) \cos \pi z^*, \quad T^* = B_1 e^{\sigma t} T_1(z) \cos \pi z^*, 
\]

\[
 S^* = C_1 e^{\sigma t} S_1(z) \cos \pi z^*, \quad D\phi^* = D_1 e^{\sigma t} \phi_1(z) \cos \pi z^*, 
\]

\[
 \phi^* = \frac{D_1 \phi_1(z)}{\pi} \sin \pi z^*, 
\]

where \( A_1, B_1, C_1 \) and \( D_1 \) are constants.

In this portion, with the use of Eq. (2.18) and (2.19) all the partial derivatives are removed to obtain the solution of Eqs. (2.14)-(2.17). Using Eq. (2.19) in Eqs. (2.14)-(2.17), we get

\[
 \frac{\partial}{\partial \tau^*} \left( D^2 - a^2 \right) w^* 
= a R^{1/2} M_1 (1 - M_5) D\phi^* - a R^{1/2} M_1 (1 - M_5) 
+ a R^{1/2} T + (1 - Vz^*) (D^2 - a^2)^2 w^* 
- 2V (D^2 - a^2) w^* + T a^{1/2} D \zeta^*, 
\]

\[
 (2.14)
\]

\[
 \frac{P_r}{P} \frac{\partial T^*}{\partial \tau} = \frac{P_t}{P_2} \frac{\partial (D\phi^*)}{\partial \tau} 
= (D^2 - a^2) T^* 
+ a R^{1/2} (1 - M_2 - M_3) w^*, 
\]

\[
 (2.15)
\]

\[
 P_r \frac{\partial S^*}{\partial \tau} = \tau (D^2 - a^2) S^* - a R^{1/2} M_5 w^*/M_6, 
\]

\[
 (2.16)
\]

\[
 D^2 \phi^* - M_3 a^2 \phi^* - DT^* + \frac{M_5}{\tau} \left( \frac{R}{R_S} \right)^{1/2} DS^* = 0, 
\]

\[
 (2.17)
\]
We use the analysis similar to Vaidyanathan et al. [15] to obtain the Eigen function. Application of the Galerkin technique (Ramanathan and Muchikek [28]) yields the eigenvalue for the stationary convection in the form

\[
-\alpha R^{1/2}(1 + M_4 + M_4 M_5)S_1(z)C_1 + R^{1/2} M_1(1 + M_5)\phi_1(z)D_1, \tag{2.20}
\]

\[
a R^{1/2}(1 - M_2 - M_2 M_5)w_1(z)A_1 - (\pi^2 + \alpha^2 + P_r \sigma)T_1(z)B_1 + P_r \sigma \phi_1(z)D_1 = 0, \tag{2.21}
\]

\[
a R^{1/2} S_1(z)w_1(z)A_1 + [\pi(\pi^2 + \alpha^2) + \sigma P_r]S_1(z)C_1 = 0, \tag{2.22}
\]

\[
-R^{1/2} S_1(z)B_1 + R^{1/2} \pi^2 (M_5/M_6)S_1(z)C_1 + R^{1/2} (\pi^2 + \alpha^2 M_3)\phi_1(z)D_1 = 0, \tag{2.23}
\]

We use the analysis similar to Vaidyanathan et al. [15] to obtain the Eigen function. Application of the Galerkin technique (Ramanathan and Muchikek [28]) yields the eigenvalue $R_{sc}$ for the stationary convection in the form

\[
R_{sc} = \frac{\alpha^2 y_2 V R_s + y_2 T_s [(y_3 + y_5)V - y_4)V]d_\alpha}{\alpha^2 V \tau(y_6 + y_8 + y_7)}, \tag{2.24}
\]

When $M_1$ is very large, we get

\[
N_{sc} = \frac{\alpha^2 y_2 V R_s + y_2 T_s [(y_3 + y_5)V - y_4)V]d_\alpha}{\alpha^2 V \tau(x_1 + x_3 + x_3)}, \tag{2.25}
\]

where

\[
y_1 = b_0 b_1 b_2 M_6 < T_1 < w_1 S_1 > \phi_1 > \]

\[
y_2 = b_0 b_1 b_2 \pi^2 < T_1 < w_1 S_1 > \phi_1 > \]

\[
y_3 = 2b_0 b_1 < S_1 \phi_1 z > \]

\[
y_4 = b_0 b_1 b_2 < S_1 \phi_1 z > \]

\[
y_5 = b_0 b_2^2 < S_1 \phi_1 z > \]

\[
y_6 = b_0 b_1 \pi^2 < S_1 < \phi_1 T_1 > w_1 z > \]

\[
y_7 = b_1 b_4 b_6 \pi^2 < T_1 < w_1 z \phi_1 > S_1 > \]

\[
y_8 = b_0 b_2 b_3 < T_1 < \phi_1 w_1 z > S_1 > \]

\[
x_1 = b_1 b_2 b_3 < S_1 < \phi_1 T_1 > w_1 z > \]

\[
x_2 = b_0 b_2 b_7 < \phi_1 < T_1 w_1 z > S_1 > \]

\[
x_3 = b_1 b_2 b_7 < S_1 < w_1 z p_1 > S_1 > \]

\[
b_0 = \pi^2 + \alpha^2 M_3 \ b_1 = \pi^2 + \alpha^2, \]

\[
b_2 = 1 + M_1 (1 + M_5), \ b_3 = 1 + M_4 + M_4 / M_5 \]

\[
b_4 = M_1 (1 + M_5), \ b_5 = 1 - M_2 - M_2 M_5 \]

\[
b_6 = M_5 / M_6, \ b_7 = 1 + M_5 \]

where $< f g > = \int_{-1/2}^{1/2} f g dz$ and $w_1, \phi_1, T_1$ and $S_1$ are trail functions. They are satisfying the boundary conditions. The salinity, temperature, velocity and magnetic potential trial functions and they are satisfying the boundary conditions in Eq.(2.18) are $w_1 = cospix, T_1 = cospix, S_1 = cospix$ and $\phi_1 = sinpix$.

3 Results and Discussion

The classical linear stability analysis has been taken over to investigate the onset of temperature dependent viscosity on thermohaline convective instability on ferrofluid with
uniform angular velocity. The coriolis force is introduced on the convective system. The thermal perturbation and normal mode techniques are taken to get the solution. The stationary instability is calculated using free-free boundary conditions. But, oscillatory instability cannot occur in the study. Furthermore, we attempted to investigate the effect of coriolis force on thermohaline convective instability in ferrofluid. Before we analyze the various physical quantities, we first form some physical comments on these like $M_1$ is assumed to be 1000 and $M_2$ is taken as zero, $M_3$ is taken from 5 to 25 and $\tau$ is taken as 0.05 (0.02) 0.11 (Vaidyanathan et al.[15]). $R_s$ is taken from 0 to 100 and $M_4$ and $M_6$ are taken to be 0.1 and $M_5 = 0.5$. The Darcy number $Da$ is taken from 10 to 100 and the TD viscosity parameter $V$ is ranges from 0.1 to 0.5 (Ramanathan and Muchikel [28]).

Fig. 2 shows the variation of $N_{SC}$ versus $V$ for various $Da$, $R_s = 0$, $T_a = 100$ and $\tau = 0.03$. This gives that $Da$ has a destabilizing effect which is less pronounced. This is because, as Darcy porous matrix $Da$, $N_{SC}$ gets the same values for different values of $Da$. The effect of isotropic and anisotropic porous medium have a destabilizing effect which has investigated by (Sekar et al. [26]). But, in the presence of Darcy porous matrix $Da$, the convective system gets same thermal energy. In other words, the system is on an equilibrium state. Fig. 3 displays the variation of $N_{SC}$ versus $V$ for various $Da$, $R_s = 100$, $T_a = 100$ and $\tau = 0.03$. It is obvious that this figure gives the same effect which is analyzed in Fig. 2.

Fig. 4 represents the plot of $N_{SC}$ versus $V$ for different $M_3$. This figure shows clearly that $V$ has destabilizing effect for increasing of $M_3$. This destabilizing behavior is not much given. When $M_3 = 25$, the system gets high energy and observed that the increasing of $M_3$ from 10 to 25, the system gets low energy.

Figs. 5 shows the variation of $N_{SC}$ with respect to $\tau$. In this figure, it is very clear that $\tau$ has destabilizing behavior. It gives that $N_{SC}$ decreases with increasing of porous matrix $Da$. This is depicted by Ramanathan and Muchikel [28] for various values $V$. They analyzed the destabilizing effect only.

In Fig. 6, $N_{SC}$ versus $\tau$ for various $M_3$ is analyzed. This figure shows that the increase in $M_3$ is found to cause large destabilization due to both magnetic and thermal mechanism favour destabilization.

Figs. 7 gives the variation of $N_{SC}$ with respect to $V$ for different $T_a$. This figure shows that $N_{SC}$ is increased when $V$ and $T_a$ increases. Hence, $T_a$ has always a stabilizing flow. Further, it is observed that Fig. 7 has exponential increases.

Fig. 8 analyzed for $\tau$, and $V$. The nature of the destabilizing effect is made for presence of TD viscosity $V$. In this situation, when $\tau$ is increasing from 0.03 to 0.11, $N_{SC}$ is decreased.

4 Conclusion

In this analysis, the results of thermohaline convective instability in a ferrofluid in temperature dependent viscosity and rotation is considered with free-free boundary conditions. These free-free boundary conditions are most significant because we can calculate an exact solution. The critical thermal magnetic Rayleigh number is calculated graphically for sufficient large values of $M_4$. The destabilizing behavior of $M_3$, $V$, $\tau$ and $Da$ is investigated which are depicted in all figures except Fig. 7. The stabilizing effect is investigated in Fig. 7 for different values of Taylor number $T_a$. 
Figure 2: Variation of $N_{sc}$ versus $V$ for various $Da$, $M_3 = 5$, $R_S = 0$, $T_a = 100$ and $\tau = 0.03$.

Figure 3: Variation of $N_{sc}$ versus $V$ for various $Da$, $M_3 = 5$, $R_S = 100$, $T_a = 100$ and $\tau = 0.03$. 
Figure 4: Variation of $N_{sc}$ versus $V$ for various $M_3$, $R_S = 100$, $\tau = 0.03$, $T_a = 100$ and $Da = 10$.

Figure 5: Variation of $N_{sc}$ versus $\tau$ for various $V = 0.1$, $Da$, $R_S = 100$, $T_a = 10$ and $M_3 = 5$. 

Figure 6: Variation of $N_{ac}$ versus $\tau$ for various $M_3$, $R_s = 100$, $T_\alpha = 100$ and $V = 0.1$

Figure 7: Variation of $R_{sc}$ versus $V$ for various $T_\alpha$, $R_s = 100$, $M_3 = 5$ and $\tau = 0.03$. 
Figure 8: Variation of $N_{sc}$ versus $\tau$ for various $V$, $R_S = 1000$, $M_3 = 5$, and $T_a = 10$.

References

[1] Stern, M. E. The salt-Fountain and thermohaline convection, *Tellus*, 12, 72-175 (1960)
[2] Turner, J. S. The behavior of a stable salinity gradient heated from below, *Journal of Fluid Mechanics*, 33, 183-200 (1968)
[3] Huppert, H. E., Turner, J. S. Ice blocks melting into a salinity gradient, *Journal of Fluid Mechanics*, 100, 367-384 (1980)
[4] Huppert, H. E., Turner, J. S. Double-Diffusive convection, *Journal of Fluid Mechanics*, 106, 299-329 (1981)
[5] Turner, J. S., Stommel H. A new case of convection in the presence of combined vertical salinity and temperature gradients, *Proceedings of National Academy of Science, U.S.A*, 55, 49-53 (1964)
[6] Turner, J. S., Chen, C. F. Two-dimensional effects in double diffusive convection, *Journal of Fluid Mechanics*, 63, 577-592 (1974)
[7] Rubin, H., Roth, C. Thermohaline convection in flowing groundwater, *Advances in Water Resources*, 6, 146-156 (1983).
[8] Erland Kallen, Xiang-Yu, A simple model for large-scale thermohaline convection, *Dynamics of Atmosphere and Oceans*, 11, 153-173 (1987)
[9] Veronis, G. On finite amplitude instability in thermohaline convection, *Journal of Marine Research*, 23, 1-17 (1965)
[10] Veronis, G. Effect of a stabilizing gradient of solute on thermal convection, *Journal of Fluid Mechanics*, 34, 315-336 (1968)
[11] Huppert, H. E., Moore, D. R. Non-linearity double diffusion convection, *Journal of Fluid Mechanics*, 78, 821-854 (1976)
[12] Knobloch, E., Weiss, N. O., Da Dosta, L. N. Oscillatory and steady convection in a magnetic field, *Journal of Fluid Mechanics*, 113, 153-186 (1981)
[13] Knobloch, E., Proctor, M. R. E. Non-linear periodic convection in double diffusive system, *Journal of Fluid Mechanics*, 108, 291-316 (1981)
[14] Baines, P. G., Gill, A. E. On thermohaline convection with linear gradients, *Journal of Fluid Mechanics*, 37, 289-306 (1969)
[15] Vaidyanathan, G., Sekar, R., Ramanathan, A. Ferro thermohaline convection, *Journal of Magnetism and Magnetic Materials*, 176, 321-330 (1997)
[16] Wooding, R. A. Rayleigh instability of a thermal boundary layer in flow through a porous medium, *Journal of Fluid Mechanics*, 9, 183-192 (1960)
[17] Lapwood, E. R. Convection in a porous medium, *Proc. Comb. Phil. Soc.*, 44, 508-521 (1948)
[18] Nield, D. A., Bejan, A. Convection in a porous medium, *Springer, New York (Third Edition)* (2006)
[19] Feynman, R. P., Leighton, R. B., Sands, M. Lecturers on physics, *Addition-Wesley, Reading*, (1963)
[20] Shliomis, M. I. Ferrofluids as thermal Ratchets, *Physical Review Letters*, 92, Article ID: 188901 (2004)
[21] Rosensweig, R. E. Ferrohydrodynamics, *Cambridge University Press, Cambridge* (1985)
[22] Finlayson, B. A. Convective instability of ferromagnetic fluids, *International Journal of Fluid Mechanics*, 40, 753-767 (1970)
[23] Vaidyanathan, G., Sekar, R., Balasubramanian, R. Ferroconvective instability of fluids saturating a porous medium, *International Journal of Engineering Sciences*, 29, 1259-1267 (1991)
[24] Sekar, R., Vaidyanathan, G., Ramanathan, A. Ferroconvection in an anisotropic porous medium, *International Journal of Engineering Sciences*, 34, 399-405 (1996)
[25] Sekar R., Raju K., Vasanthakumari R. A linear analytical study of Soret-driven ferrothermohaline convection in an anisotropic porous medium, *Journal of Magnetism and Magnetic Materials*, 331, 122-128 (2013)
[26] Straughan., B. The Energy Method, Stability and Nonlinear Convection. New York: Springer (2004).
[27] Siddheshwar, P.G. Thermorheological effect on magnetoconvection in weak electrically conducting fluids under $1g$ or $\mu g$, *Pramana J. Phys.*, 62 (2004) 61-68.
[28] Ramanathan, A., Muchikel, N. Effect of temperature dependent viscosity on ferroconvection in a porous medium, *Int. J. of Applied Mechanics and Engineering*, 11, 93-104 (2006).