Bouncing Universes with Varying Constants

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Abstract. We investigate the behaviour of exact closed bouncing Friedmann universes in theories with varying constants. We show that the simplest BSBM varying-alpha theory leads to a bouncing universe. The value of alpha increases monotonically, remaining approximately constant during most of each cycle, but increasing significantly around each bounce. When dissipation is introduced we show that in each new cycle the universe expands for longer and to a larger size. We find a similar effect for closed bouncing universes in Brans-Dicke theory, where $G$ also varies monotonically in time from cycle to cycle. Similar behaviour occurs also in varying speed of light theories.
The "bouncing universe" is a modern cosmological reincarnation of an ancient fascination with the cyclic patterns of nature and the myth of the "eternal return" [1, 2, 3]. Gravitation theories like general relativity allow us to make precise models of this popular conception of a "phoenix" cosmology, in which a universe periodically collapses to a Big Crunch, only to rebound into a new state of expansion, as if emerging from a unique Big Bang [4]. Multiple bounces are possible but each cycle lasts longer and expands to a larger maximum size than the previous one, a consequence of a simple application of the second law of thermodynamics [4, 5, 6, 7], unless there is a finite positive cosmological constant, in which case the oscillations must eventually cease [8] and are replaced by eternal de Sitter expansion. A sequence of many oscillations will drive the bouncing closed universe closer and closer to flatness.

"Quantum gravity" effects are invariably invoked to justify the bounce; possible detailed calculations, however, have only recently emerged. In loop quantum gravity, the semi-classical Friedmann equations receive corrections that produce a bounce [9, 10]. The ekpyrotic model of the universe, inspired by string/M-theory, is another possible realization of phoenix cosmology [11]. It is also possible that ghost fields – fields endowed with negative energy – are capable of producing a classical bounce (this idea has been often rediscovered; see [12] for a good review). Classical bounces produced by conventional scalar fields with potentials which only violate the strong energy condition are difficult to produce in universes that grow large enough to be realistic: typically the probability of bounce is of order the ratio of the minimum to the maximum expansion size [13, 14].

It has been speculated that whatever causes a collapsing universe to bounce can reprocesses some aspects of physics, either randomly [15], or systematically [16], by changing the particle spectrum or resetting the dimensionless "constants" of Nature. Both of these options are severely constrained by anthropic requirements but it is interesting to ask whether there are monotonic or asymptotic trends in the values of some quantities, as seems to be the case for the degree of flatness of the universe, over many bounces. This matter is clearly of great importance in the context of varying-constant theories [17, 18]. Here, quantities which are traditionally constants become space-time variables and if singularities are avoided in the bounce then their evolution from cycle to cycle is predictable by the field equations rather than the outcome of effectively random reprocessing. The values of any dimensionless 'constants' of Nature could evolve towards asymptotic attractors if they are allowed to be variables in a self-consistent theory. Studies of these theories are also important in assessing the stability and level of fluctuations in a bouncing universe. It has been suggested that thermal fluctuations in bouncing models could be the origin of the cosmic structure [19, 18]; this would provide a distinct alternative to an origin from vacuum quantum fluctuations in a de Sitter phase of cosmological expansion.

Although these claims are intriguing, it is difficult to evaluate them in the absence of a concrete model for the bounce, which is usually viewed as a black box from which anything can emerge [15]. In this paper we examine some exactly soluble
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examples. First, we consider a simple bouncing cosmology following from the simplest BSBM varying-constant theory we developed [20] from Bekenstein’s varying-alpha model [21]. We derive the result that for suitable couplings (indeed those favoured by observations [22, 23]; see however [24]) the theory leads to a bouncing universe. We derive similar behaviour in the Brans-Dicke theory of a varying gravitational constant $G$ [25], and in a class of varying speed of light theories [26, 27, 28, 29, 30]. In each case we are able to find exact solutions in the absence of dissipation and compute the evolutionary trends in the fine structure constant and $G$ from cycle to cycle when dissipation occurs in accord with the Second Law. We find the interesting result that the varying constants in these theories change monotonically from cycle to cycle when the scale factor oscillates: the scalar fields determining the constants in each cycle do not oscillate.

Crucial to our models is the idea that cosmological fields may have a negative energy. Such fields are called ghosts [12], and are far from new, having found widespread application in the study of steady-state cosmology [31], phantom dark matter [32, 33], and $\kappa$-essence [34]. Bekenstein’s theory [21] is rooted in the use of a real scalar dielectric field $\psi$, representing the allowed variation of the electron charge according to $e = e_0 e^\psi$, where $e_0$ is the present-day value of the electron charge, and so the fine structure ‘constant’ evolves with respect to its present-day value, $\alpha_0$, as $\alpha = \alpha_0 e^{2\psi}$. By redefining the electromagnetic gauge field [20], $A_\mu$, as $a_\mu = e^\psi A_\mu$, and the electromagnetic field tensor, $F_{\mu\nu}$, as $f_{\mu\nu} = e^\psi F_{\mu\nu}$, the action may be written as

$$S = \int d^4x \sqrt{-g} \left( L_g + L_{\text{mat}} + L_\psi + L_{\text{em}} e^{-2\psi} \right),$$

where $L_\psi = -\frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi$, $L_{\text{em}} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu}$, and $L_{\text{mat}}$ does not depend on $\psi$. The gravitational Lagrangian is the usual $L_g = \frac{1}{16\pi G} R$, with $R$ being the curvature scalar. If the scalar coupling satisfies $\omega < 0$, then $\psi$ has a negative kinetic energy term, and is thus a ghost field.

This theory can fit the observational constraints on varying alpha reported by [22, 23] as well as others [20, 35]. However for this to be possible in the ghost-free case with $\omega > 0$ one has to choose a very special type of dark matter, in which the magnetostatic energy $B^2$ dominates over the electrostatic energy $E^2$. Even though a dark-matter candidate was found satisfying this condition (superconducting cosmic strings), most types of matter, including baryonic matter, are $E^2$ dominated, for which observational data implies $\omega < 0$, so that $\psi$ is a ghost field.

Ghosts have been criticized on a variety of grounds. Classically they are a source of instabilities if coupled to other forms of matter, since they will try to off-load an infinite amount of positive energy into them. This is not necessarily cataclysmic if the rate of these processes is sufficiently slow. For instance, in steady-state cosmology to negative probabilities. At the quantum level, ghosts lead to negative norm states and so negative probabilities. The quantum instabilities are also much more severe and are present even without direct coupling to matter, for example in runaway particle production via the graviton vertex. Hence, at the quantum level ghosts are pathological.
However, we know that quantisation of the field $\psi$ is pathological even for $\omega > 0$. For one thing, the theory is non-renormalisable. The attitude to $\psi$ should therefore be similar to that with regards to gravity: “don’t quantise”. General relativity is also, at face value, non-renormalisable: for instance, the quantum corrections to the relativistic precession of the perihelion of Mercury are infinite. This doesn’t stop the classical theory from being very successful. It may be that the quantisation of ghosts is simply more subtle; ghosts have been found as type $II^*$ string theories [36].

Non-relativistic matter and the cosmological constant may be neglected near a bounce, so let us consider a Friedmann-Robertson-Walker (FRW) universe filled with radiation and a dielectric field, $\psi$. The cosmological equations for BSBM varying-$\alpha$ theory [1] are

$$H^2 = \frac{1}{3} \left( \rho_r e^{-2\psi} + \rho_\psi \right) - \frac{K}{a^2}, \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6} \left( 2 \rho_r e^{-2\psi} + 4 \rho_\psi \right), \quad (3)$$

where we have set $8\pi G = 1$, $H \equiv \dot{a}/a$ is the Hubble expansion rate, $K$ is the 3-curvature constant, and $\rho_\psi = \omega \dot{\psi}^2 / 2$. For the scalar field, in the absence of non-relativistic matter, we have

$$\ddot{\psi} + 3H \dot{\psi} = 0. \quad (4)$$

so $\dot{\psi} \propto a^{-3}$. We should not ignore the possibility that at high curvatures quantum processes may allow the conversion of $\psi$ energy into radiation. We take this into account by introducing variable $\tilde{\rho}_r = \rho_r e^{-2\psi}$ and rewriting the conservation equations as:

$$\dot{\rho}_\psi + 6H \rho_\psi = -s(\tilde{\rho}_r, \dot{\psi}, a), \quad (5)$$

$$\dot{\tilde{\rho}}_r + 4H \tilde{\rho}_r = s(\tilde{\rho}_r, \dot{\psi}, a). \quad (6)$$

In this case, the equation of motion for $\psi$ will contain an additional $s$ term which models energy transfer between the $\psi$ field and the radiation sea in accord with the second law of thermodynamics. We shall consider the implications of such a process, in all its generality, below, but let us look first at the equations neglecting this coupling function.

Consider first a model $s = 0$ bouncing universe which is exactly soluble. Taking $K = +1$ and $\omega < 0$, Eqn. (2) is

$$\frac{\ddot{a}}{a^2} = -\frac{S}{a^6} + \frac{\Gamma}{a^4} - \frac{1}{a^2}, \quad (7)$$

where $S$ and $\Gamma$ are positive constants. In terms of conformal time $d\eta = a^{-1}dt$, this can be integrated to give

$$a^2(\eta) = \frac{1}{2} \left[ \Gamma + \sqrt{\Gamma^2 - 4S} \sin \{2(\eta + \eta_0)\} \right] \quad (8)$$

when $\Gamma^2 > 4S$. Identifying the expansion maximum and minimum, we see that $a(\eta)$ is given by

$$a^2 = \frac{1}{2} \left[ a_{\text{max}}^2 + a_{\text{min}}^2 + (a_{\text{max}}^2 - a_{\text{min}}^2) \sin \{2(\eta + \eta_0)\} \right], \quad (9)$$
where $a_{\text{max}}$ is global expansion maximum and $a_{\text{min}}$ is the global minimum of $a(\eta)$, defined by

$$
a_{\text{max}}^2/a_{\text{min}}^2 \equiv \frac{\Gamma \pm \sqrt{\Gamma^2 - 4S}}{2} \quad (10)
$$

Since $\dot{\psi} = Ca^{-3}$ we have, for $\omega < 0$, that $S = -\omega C^2/2$ and the scalar field driving time-variation of the fine structure constant is given by

$$
\psi = \pm \frac{2}{|\omega|} \tan^{-1} \left( \frac{\Gamma \tan(\eta + \eta_0) + \sqrt{\Gamma^2 - 4S}}{2\sqrt{S}} \right).
$$

In Fig. 1 we plot these solutions and show $a(t)$ and $\alpha(t)$ (recall that $\alpha \propto e^{2\psi}$), as functions of proper time, $t$. We note the steady increase of $\alpha$ with time despite the oscillatory behaviour of the expansion scale factor.

When $s = 0$ we have a variety of oscillating solutions whose characteristics depend on the initial conditions. For bouncing solutions $\alpha$ remains nearly constant during each cycle but changes sharply, but still monotonically, at the bounce. There is no significant change of behaviour at the expansion maximum which also implies that there should be no gross difference in evolution inside and outside spherical overdensities far from the bounce. With $a_{\text{max}} \gg a_{\text{min}}$, and setting $\Gamma = \tilde{\rho}_n a^4/3$ and $S = -\rho_\psi a^6/3$, we have $a_{\text{min}} = \sqrt{S/\Gamma}$ and $a_{\text{max}} = \sqrt{\Gamma}$. We can then see that the bounce duration is $\Delta t \sim a_{\text{min}}^2/a_{\text{max}}$. Since $\dot{\psi} \sim \sqrt{6\Gamma^{3/2}}/(S|\omega|^{1/2})$ near the bounce, we find that $\Delta \psi \sim \sqrt{6}/|\omega|$, independently of initial conditions, during each bounce.
Figure 2. A bouncing universe with $\omega = -0.01$ and $s = 100\rho_\psi$. Other couplings lead to qualitatively identical scenarios. The growth in $a(t)$ and total period from cycle to cycle is accompanied by increase in $\alpha$ which occurs primarily in the vicinity of each bounce.

The extreme case is a stable static universe. Setting $\dot{a} = 0$ and $\ddot{a} = 0$, we can see that this case is realized when $\rho_\psi = -\tilde{\rho}_r/2$, giving $a = \sqrt{6/\tilde{\rho}_r}$. For such a universe $\psi$ evolves linearly in $t$, and since we have $\alpha \propto e^{2\psi}$ there is exponentially rapid increase [37]. Even though such a universe is static, the rulers and clocks of observers change as alpha changes, so that they actually observe a Milne universe. We can see that the solution is stable because homogeneous and isotropic perturbations lead to a universe with regular sinusoidal oscillations as can be seen in Fig. 1. Such solutions are described by [31] in the case where $a_{\text{max}} \approx a_{\text{min}}$. This situation differs from that found in general relativity in the absence of ghost fields [38].

If $s$ is a non-vanishing then, regardless of its exact functional form, there are two type of solutions. If $s \neq 0$ at all times, then sooner or later the universe enters a steady-state evolution with exponential expansion and constant overall energy density ensured by the appropriate transfer of energy between the $\psi$ field and radiation. However, we expect that these energy-transport processes will switch off at low curvatures, when the universe expands to a sufficiently large size ($a \gg a_{\text{min}}$) and transport processes become collisionless and far slower than the expansion rate. Then, the typical evolution is as plotted in Fig. 2. Again, $\psi$ is approximately constant during each cycle and changes dramatically at the bounce. In addition, each cycle is now bigger than the previous one, because $\Gamma$ increases at each bounce. This is an interesting realisation of the standard Tolman scenario. Cycles get bigger (and entropy is generated near the bounce) specifically because radiation is produced from the scalar field close to each bounce. In producing Fig. 2 we have used $s \propto \rho_\psi$ ($\psi$ decays into radiation), but other
Figure 3. A bouncing universe for a Brans-Dicke theory with $\omega_{BD} < -3/2$, in the Jordan frame. In this case the field $\phi$ and matter are decoupled: they never exchange energy. The amplitude of oscillations in the scale factor grows monotonically in time and $G$ increases in the Jordan frame. The increases in $G$ occur primarily near the bounce.

functional forms may be used with similar effects.

We now examine similar solutions for the Brans-Dicke (BD) theory of varying $G$\textsuperscript{25}. In the Einstein frame, if the matter content is pure radiation, we recover the same equations. All we need to do, then, is convert the above results into the Jordan frame: the results found for $a$, $\rho$, and $\psi$ should then be translated into variables $a_J = a/\sqrt{\phi}$, $\rho_J = \tilde{\rho}\phi^2$, and $\phi = e^\psi$ (the latter corresponding roughly to $1/G$). Under this transformation we obtain $\omega = \omega_{BD} + \frac{3}{2}$, where $\omega_{BD}$ in the Brand-Dicke coupling parameter. Thus we need $\omega_{BD} < -3/2$ for the Brans-Dicke field to behave like a proper ghost in the Einstein frame. The resulting dynamics is plotted in Fig. 3.

These results may be understood analytically. The essential BD field equations in the Jordan frame are

$$\ddot{\phi} + 3H\dot{\phi} = 0$$

$$H^2 = \frac{8\pi\rho}{3\phi} - \frac{H\dot{\phi}}{\phi} + \frac{w_{BD}\dot{\phi}^2}{6\phi^2} - \frac{K}{a_J^4}$$

where overdots now refer to derivatives with respect to $t_J$ and $H = \dot{a}_J/a_J$. Hence

$$\dot{\phi} = \frac{A}{a_J^3}$$

$$\frac{\dot{a}_J^2}{a_J^2} = \lambda - \frac{\dot{a}_J}{a_J}\dot{\phi} + \frac{w_{BD}\dot{\phi}^2}{6\phi^2} - \frac{K}{a_J^4}$$

with $A$ and $\lambda > 0$ constants. We are interested in negative $\omega$ solutions with $K = 1$, which give oscillating closed universes (note that $\omega_{BD} < 0$ is not sufficient for an
expansion minimum, we need $\omega_{BD} < -3/2)$. Following the techniques of \[39\] we put $\frac{4\omega}{3}(2\omega + 3) \equiv -C$, and in the bouncing case $\lambda^2 - C > 0$, we have simple exact solution in terms of conformal Jordan time:

$$\phi a^2_J = \frac{\lambda}{2} + \frac{1}{2} \sqrt{\lambda^2 - C} \sin\{2(\eta + \eta_0)\}. \quad (12)$$

We see the same behaviour as displayed by the radiation-scalar universe given above with $s = 0$. The minimum value of $\phi a^2_J$ is $\lambda - \sqrt{\lambda^2 - C}$ and the maximum is $\lambda + \sqrt{\lambda^2 - C}$. So in conformal time $\phi a^2_J \approx a^2_J/G$ undergoes oscillations of increasing amplitude as the entropy increases (ie if $\lambda$ increases in value to model increasing radiation density); that is, the horizon area ('entropy') in Planck units increases from cycle to cycle. The full solution for $a(\eta)$ and $\phi(\eta)$ is then obtained using eqns. (11) and (12):

$$\phi = \phi_1 \exp \left[ \frac{2A}{\sqrt{C}} \arctan \left( \frac{\lambda \tan\{\eta + \eta_0\} + \sqrt{\lambda^2 - C}}{\sqrt{C}} \right) \right], \quad (13)$$

with $\phi_1$ constant and

$$a^2_J = \phi_1^{-1} \left[ \frac{\lambda}{2} + \frac{1}{2} \sqrt{\lambda^2 - C} \sin\{2(\eta + \eta_0)\} \right] \times \exp \left[ \frac{2A}{\sqrt{C}} \arctan \left( \frac{\lambda \tan\{\eta + \eta_0\} + \sqrt{\lambda^2 - C}}{\sqrt{C}} \right) \right]. \quad (14)$$

The effect of increasing radiation entropy can be seen by increasing the constant $\lambda$ in these expressions. In this model the field $\phi$ and radiation are fully decoupled. Although the size of the universe increases in each successive cycle, its size with respect to Planck units remains the same, unless of course we consider a model in which the field $\phi$ and radiation may exchange energy.

We found similar solutions in the BSBM and BD theories because the dynamics in the Einstein frame is very similar in the absence of non-relativistic matter. Likewise, one can find identical solutions for the covariant varying speed of light (VSL) theories described in Refs. \[26, 30\]. This does not imply that the VSL, Brans-Dicke and BSBM theories are equivalent; merely that one needs to add more general matter (even if only as “test matter” in a radiation-dominated universe) for their differences to become obvious. Specifically, the coupling to $L_{em}$ in (11) is replaced in Brans-Dicke theory by non-minimally coupling the matter fields to the metric.

In summary, we have considered some simple exactly soluble models for closed bouncing universes in theories with varying $\alpha$ and varying $G$ and examined the effects of simple non-equilibrium behaviour. Even though the expansion scale factor of the universes undergoes periodic oscillations about a finite non-singular expansion minimum, we find steady monotonic change in the values of $\alpha$ and $G$ from cycle to cycle both in the presence and absence of non-equilibrium behaviour. In the non-equilibrium case the oscillations of the expansion scale factor grow monotonically in amplitude and period in accord with the Second Law, and the expansion dynamics approaches flatness.

Acknowledgements We would like to thank B. Carr and C. Hull for helpful comments.
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