Influence of orthogonalization procedure on astrophysical S-factor for the direct $\alpha + d \rightarrow ^6\text{Li} + \gamma$ capture process in a three-body model

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Abstract

The astrophysical S-factor for the direct $\alpha(d, \gamma)^6\text{Li}$ capture reaction is calculated in a three-body model based on the hyperspherical Lagrange-mesh method. A sensitivity of the E1 and E2 astrophysical S-factors to the orthogonalization method of Pauli forbidden states in the three-body system is studied. It is found that the method of orthogonalising pseudopotentials (OPP) yields larger isotriplet ($T = 1$) components than the supersymmetric transformation (SUSY) procedure. The E1 astrophysical S-factor shows the same energy dependence in both cases, but strongly different absolute values. At the same time, the E2 S-factor does not depend on the orthogonalization procedure. As a result, the OPP method yields a very good description of the direct data of the LUNA collaboration at low energies, while the SUSY transformation strongly underestimates the LUNA data.

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I. INTRODUCTION

A consistent realistic estimation of the primordial abundance ratio $^6\text{Li}/^7\text{Li}$ of the lithium isotopes is one of the open problems in nuclear astrophysics. For this ratio the BBN model \[1\] yields a value about three orders of magnitude less than the astrophysical data \[2\]. One input parameter for the estimation of the abundance ratio is the reaction rates of the direct radiative capture process

$$\alpha + d \rightarrow ^6\text{Li} + \gamma$$  \hspace{1cm} (1)

at low energies within the range $30 \leq E_{\text{cm}} \leq 400$ keV \[1\].

The reaction rate evaluations are based on the straightforward calculations starting from the astrophysical S-factor. The data set of the LUNA collaboration available at energies $E=94$ keV and $E=134$ keV \[3\] was recently renewed with additional data at $E=80$ keV and $E=120$ keV \[4\]. The results of the LUNA collaboration for the reaction rates turn out to be even lower than previously reported. This further increases the discrepancy between prediction of the BBN model and the astronomical observations for the primordial abundance of the $^6\text{Li}$ element in the Universe \[4\].

From a theoretical point of view, an important step toward the solution of the lithium abundance problem has been taken within the three-body model \[5 - 7\]. As was shown in Ref. \[6\] in detail, the so-called exact mass prescription, used in the literature for the estimation of the E1 astrophysical S-factor during a long period \[8 - 12\], has no microscopic background at all. Some models even neglect this important contribution to the S-factor \[13, 14\]. As was shown within the frame of three-body model based on the hyperspherical Lagrange mesh method \[6, 7\], the E1 S-factor is dominant in the low energy region $E < 100$ keV, while the E2 S-factor is mostly important at higher energies. As a result, the new data of the LUNA collaboration for the astrophysical S-factor at low energies have been reproduced with a good accuracy. The estimated $^6\text{Li}/\text{H}$ abundance ratio of $(0.67 \pm 0.01) \times 10^{-14}$ was in a very good agreement with the experimental value of $(0.80 \pm 0.18) \times 10^{-14}$ from the LUNA collaboration.

On the other hand, the final three-body $\alpha + p + n$ hyperspherical wave function of $^6\text{Li}$ was calculated with the Voronchev et al. $\alpha N$ -potentials \[15\] with a forbidden state in the S-wave. The forbidden states in the three-body system have been treated within the method
of orthogonalising pseudopotentials (OPP) [16]. The wave function has a small is triplet component of about 0.5 percent. This important is triplet part of the three-body wave function [5–7] enables one to estimate the forbidden E1 S-factor in a consistent way without using any exact mass prescription. An important question is, how sensitive are the results for the estimated astrophysical S-factor on the projecting method used in the variational calculations of the three-body wave function. The aim of present study is to answer this important question. To this end we estimate the astrophysical S-factor with the three-body wave function of the $^6$Li ground state, calculated using the supersymmetric transformation (SUSY) method [17] and compare with the results of the OPP approach.

II. THEORETICAL MODEL

The cross sections of the radiative capture process reads as

$$\sigma_E(\lambda) = \sum_{J_f T_f \pi_f} \sum_{J_i T_i \pi_i} \sum_{\Omega \lambda} \frac{(2J_f + 1)}{[I_1][I_2]} \frac{32\pi^2(\lambda + 1)}{\hbar \lambda ([\lambda]!!)^2} k^{2\lambda + 1}_\gamma \times \sum_{I_\omega I_\omega} \frac{1}{k^{2}_{\omega} v_{\omega}} | \langle \Psi^{J_f T_f \pi_f} || M^\Omega || \Psi^{J_i T_i \pi_i} \rangle |^2, \quad (2)$$

where $\Omega = E$ or M (electric or magnetic transition), $\omega$ denotes the entrance channel, $k_{\omega}$, $v_{\omega}$, $I_{\omega}$ are the wave number, velocity of the $\alpha - d$ relative motion and the spin of the entrance channel, respectively, $J_f$, $T_f$, $\pi_f$ ($J_i$, $T_i$, $\pi_i$) are the spin, isospin and parity of the final (initial) state, $I_1$, $I_2$ are channel spins, $k_\gamma = E_\gamma/\hbar c$ is the wave number of the photon corresponding to the energy $E_\gamma = E_{th} + E$ with the threshold energy $E_{th} = 1.474$ MeV. The wave functions $\Psi^{J_f T_f \pi_f}_{I_\omega I_\omega}$ and $\Psi^{J_i T_i \pi_i}$ represent the initial and final states, respectively. The reduced matrix elements of the transition operators are evaluated between the initial and final states. We also use short-hand notations $[I] = 2I + 1$ and $[\lambda]!! = (2\lambda + 1)!!$. Details of the matrix-element calculations have been given in Ref. [5].

The astrophysical S-factor of the process is expressed in terms of the cross section as [18]

$$S(E) = E \sigma_E(\lambda) \exp(2\pi\eta), \quad (3)$$

where $\eta$ is the Coulomb parameter.
III. NUMERICAL RESULTS

Calculations of the cross section and astrophysical S-factor have been performed under the same conditions as in Refs.\[6, 7\]. The radial wave function of the deuteron is the solution of the bound-state Schrödinger equation with the central Minnesota potential $V_{NN}$ \[19, 20\] with $\hbar^2/2m_N = 20.7343$ MeV fm$^2$. The Schrödinger equation is solved using a highly accurate Lagrange-Laguerre mesh method \[21\]. It yields $E_d = -2.202$ MeV for the deuteron ground-state energy with the number of mesh points $N = 40$ and a scaling parameter $h_d = 0.40$.

The scattering wave function of the $\alpha - d$ relative motion is calculated with a deep potential of Dubovichenko \[22\] with a small modification in the $S$-wave \[10\]: $V_d^{(S)}(R) = -92.44 \exp(-0.25R^2)$ MeV. The potential parameters in the $3P_0$, $3P_1$, $3P_2$ and $3D_1$, $3D_2$, $3D_3$ partial waves are the same as in Ref. \[22\]. The potential contains additional states in the $S$- and $P$-waves forbidden by the Pauli principle. The above modification of the $S$-wave potential reproduces the empirical value $C_{\alpha d} = 2.31$ fm$^{-1/2}$ of the asymptotic normalization coefficient (ANC) of the $^6\text{Li}(1+)$ ground state derived from $\alpha - d$ elastic scattering data \[23\].

The final $^6\text{Li}(1+)$ ground-state wave function was calculated using the hyperspherical Lagrange-mesh method \[24\] with the same Minnesota NN-potential. For the $\alpha - N$ nuclear interaction the potential of Voronchev et al. \[15\] was employed, which contains a deep Pauli-forbidden state in the $S$-wave. The potential was slightly renormalized by a scaling factors 1.014 to reproduce the experimental binding energy of $E_b = 3.70$ MeV. The Coulomb interaction between $\alpha$ and proton is taken as $2e^2 \text{erf}(0.83 R)/R$ \[20\]. The coupled hyperradial equations are solved with the Lagrange-mesh method \[21\]. The hypermomentum expansion includes terms up to a large value of $K_{\text{max}}$, which ensures a good convergence of the energy and of the $T = 1$ component of $^6\text{Li}$. In Refs. \[6, 7\] the OPP method was used for the treatment of the Pauli forbidden states in the three-body model. Here we also examine the SUSY transformation \[17\] of the initial $\alpha - N$ nuclear interaction potential. This operation yields a shallow potential which gives the same phase shift, but removes unphysical forbidden state from the $S$-wave $\alpha + N$ spectrum.

Firstly, the energy convergence in the three-body $\alpha + p + n$ system for the SUSY and OPP methods shows the same behavior. The energy of the $^6\text{Li}$ ground state $E = -3.70$ MeV converges already at a maximal hypermomentum $K_{\text{max}} = 24$ in the both cases. However,
the structure of the $^6$Li g.s. wave function in these two versions of the projection yields different pictures. The important isotriplet ($T = 1$) component of the $^6$Li g.s. wave function used in the OPP method has a norm square of about $5.27 \times 10^{-3}$, while in the case of the SUSY method it is $1.10 \times 10^{-4}$. As we noted above, the important isotriplet component of the final $^6$Li ground state is responsible for the E1 astrophysical S-factor in the $\alpha(d, \gamma)^6$Li direct capture reaction. Therefore, the above difference should yield the same effect for the E1 S-factor. Additionally, it is important to check, whether the energy dependence of the E1 S-factor is the same in both cases.

![FIG. 1: Astrophysical E1 S-factor of the direct $\alpha(d, \gamma)^6$Li capture process.](image)

In Fig. [1] we show the E1 astrophysical S-factor of the direct $\alpha(d, \gamma)^6$Li capture process estimated with the OPP and SUSY three-body wave functions. As we can see from the figure, the two methods yield the same energy behavior. However, the SUSY method yields too small S-factor in the entire energy region, more than one order of magnitude smaller than the OPP method. This result indicates that the E1 astrophysical S-factor is highly sensitive to the orthogonalization method. A similar effect was found in the beta decay of the $^6$He halo nucleus [25, 26] and M1-transition of the $^6$Li(0+) isobar-analog state to the $\alpha + d$ continuum [24]. In fact, the OPP method yields scattering and bound state wave functions with a node at short distances, while this nodal behavior disappears in the SUSY method. In the present study, a nodal behavior of the S-wave $\alpha + N$ wave function yields a strong contribution to the important isotriplet component of the total $^6$Li ground state.
FIG. 2: Astrophysical E2 S-factor of the direct $\alpha(d, \gamma)^6\text{Li}$ capture process.

FIG. 3: Astrophysical S-factor of the direct $\alpha(d, \gamma)^6\text{Li}$ capture process.

wave function.

The E2 astrophysical S-factor is displayed in Fig. 2. As can be seen, the OPP and SUSY methods give the same theoretical estimations. This means that the E2 S-factor is not sensitive to the orthogonalization procedure in the wave function of the $^6\text{Li}$ ground state.

The total theoretical astrophysical S-factor for the process is shown in Fig. 3 in comparison with the direct data of the LUNA collaboration [3, 4] and old data from Refs. [27–29].
Due to a strong effect of the orthogonalization method we have a big difference in the SUSY and OPP results. While the OPP method yields a good description of the direct data of the LUNA collaboration, the SUSY transformation gives a strong underestimation.

IV. CONCLUSION

A sensitivity of the theoretical astrophysical S-factor for the direct $\alpha(d,\gamma)^6\text{Li}$ capture reaction to the orthogonalization procedure has been examined within the hyperspherical Lagrange-mesh method. It was found that the E1 astrophysical S-factor is very sensitive to the orthogonalization method, however the E2 S-factor does not depend on the orthogonalization procedure. As a result, the OPP method yields a very good description of the direct data of the LUNA collaboration at low energies, while the SUSY transformation significantly underestimates the LUNA data. On the other hand, both methods show the same energy dependence for the E1 S-factor.

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