Mathematical modeling of the thermally stressed state of elliptical-section concrete structures

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Abstract. At present, there is an urgent need to determine the level of temperature stresses in super-large concrete structural elements that occur during the manufacture of such products and contain a huge amount of concrete at the same time, which is in fact a heat source that is inaccessible to the control of the exothermic process of the entire mass of concrete. The main task of this work is the mathematical modeling of temperature stresses in concrete structures of elliptical cross-section in order to reduce them. Based on mathematical modeling, it is established that the level of thermal stresses that occur during the transition to an elliptical cross-section decreases.

1. Introduction
When constructing concrete structures, their design schemes are diverse. The level of temperature stresses in concrete structures depends on both the geometry and the intensity of heat release of the solution of the concrete structures being built [1]. Therefore, an integral task is to determine the intensity of heat release and determine the residual stresses during the early solidification of concrete in order to reduce cracking.

The correct reasonable choice of the geometry of the cross-sections of the structural elements reduces cracking and the level of temperature stresses during the maturation of concrete. This applies especially to the study of the temperature stress state of large-sized concrete shaped support walls, box-shaped tunnel profiles, the correct choice of configuration and design of which determines the best crack resistance and the minimum number of dangerous and temperature cracks that are formed during the solidification of concrete.

The study of concrete is inextricably linked with the study of its components, processes occurring during its hardening, as well as the structure of the material, the influence of various factors on strength, durability, etc.

Some concrete and reinforced concrete structures are subjected to humidification, drying, aggressive environment, alternating freezing and thawing during their operation. To ensure the durability of concrete and gelatinous concrete structures operated in any conditions, it is necessary to solve many issues, in particular, to choose the design class of concrete in terms of strength, density, frost resistance, corrosion resistance, heat generation intensity. An insignificant level of temperature stresses occurs during the production of concrete products in the field [2; 3].

Currently, there is no unified theory of concrete destruction during cyclic freezing and thawing in water and in aggressive environments. There are only hypotheses. Most authors who study the
durability of concrete are convinced that the decrease in the strength of concrete subjected to cyclic freezing and thawing is due to the accumulation of mechanical damage under the influence of stresses resulting from the freezing of water in the pores of cement stone, the difference in the coefficients of temperature deformations of concrete components and other factors [4, 5, 6]. In addition, it is noted that the degree of destruction of concrete and reinforced concrete structures subjected to alternate freezing and thawing, largely depends on the mineralogical composition of the cement [7, 8, 9, 10].

Thus, when studying the durability of concrete and reinforced concrete structures operated under these conditions, a comprehensive approach is needed to address these issues. This will allow us to find out the mechanism of destruction of the material and give the prerequisites for the development of a unified theory of concrete destruction.

2. Materials and methods
The paper uses the method of mathematical modeling.

3. Results
The magnitude of the resulting thermal stresses depends on the temperature difference. Therefore, when changing the shape of concrete structures (while maintaining the area or volume), the level of temperature stresses decreases.

When determining the thermal stresses in a long circular or elliptical cylinder, we use the corresponding solutions to the plane problem of thermoelasticity. The stress function $F$ for a circular cross section is found from the solution of the equations

$$
\Delta \Delta F = \frac{\alpha E q \rho}{\lambda (1 - \nu)}, \quad F = \frac{\partial F}{\partial n} = 0
$$

(1)

where $\lambda$ is the coefficient of thermal conductivity of concrete, $\alpha$ is the coefficient of linear expansion, $E$ is the Young's modulus, and $\nu$ is the Poisson's ratio.

In a circular cylinder, the components of the stress tensor in the Cartesian coordinate system have the form

$$
\sigma_{xx} = \frac{\alpha E q R^2}{64 \lambda (1 - \nu)} \left( \frac{x^2 + 3y^2}{R^2} - 1 \right)
$$

(2)

$$
\sigma_{yy} = \frac{\alpha E q R^2}{64 \lambda (1 - \nu)} \left( \frac{3x^2 + y^2}{R^2} - 1 \right)
$$

(3)

In an elliptical cylinder, the components of the stress tensor have the form

$$
\sigma_{xx} = \frac{4 A}{b^2} \left( \frac{x^2}{a^2} + \frac{3y^2}{b^2} - 1 \right)
$$

(4)

$$
\sigma_{yy} = \frac{4 A}{a^2} \left( \frac{3x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)
$$

(5)

where
\[
A = \frac{\alpha E q_v}{\lambda (1-\nu) \left(\frac{a^4+b^4}{a^2b^2}+2\right)} \frac{a^2b^2}{8}
\]

(6)

the axial stress itself depends on the temperature distribution

\[
\sigma_{xx} = \nu(\sigma_{xx} + \sigma_{yy}) - \alpha ET
\]

(7)

We perform a comparative analysis of the corresponding components of the thermal stress tensor for circular or elliptical cylinders, taking into account \(R^2=ab\) (equality of cross-sectional areas). In this case, it is sufficient to consider one of the components of the thermal stress tensor (for example \(\sigma_{xx}\)) for relations (2) and (4). After simple transformations, we obtain the ratio of the considered thermal stresses

\[
\frac{\sigma_{xx}^{\text{ell}}}{\sigma_{xx}^{\text{cir}}} = \frac{8 \left(\frac{x^2}{a^2} + \frac{3y^2}{b^2} - 1\right)}{\left(3 \left(\frac{b}{a}\right)^3 + 2 \left(\frac{b}{a}\right) + 3 \left(\frac{a}{b}\right) \left(\frac{x^2+3y^2}{R^2} - 1\right)\right)}
\]

(8)

where \(\sigma_{xx}^{\text{ell}}\) - thermal stresses in a cylindrical body of elliptical cross-section, \(\sigma_{xx}^{\text{cir}}\) - thermal stresses in a cylindrical body of circular cross-section.

Using the relation (8), it can be shown that the ratio of the considered thermal stresses takes the maximum value when the half-axes are equal \((a=b)\).

To determine the temperature field inside concrete structures, we will use the temperature distribution in an infinitely long body, the cross section of which is an ellipse with semi-axes \(a\) and \(b\). The body in question is located in an environment with an ambient temperature of \(T_0\). The same heat source \(q_v\) operates inside the body.

To find the temperature distribution, you need to solve the Poisson equation

\[
\Delta T + \frac{q_v}{\lambda} = 0
\]

(9)

In this case, the boundary condition on the surface has the form

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \bigg|_{r=r_t}
\]

(10)

To obtain a formula describing the temperature field, we use the system of elliptic coordinates \(\alpha, \beta, 0 \leq \alpha < \infty, -\pi \leq \beta \leq \pi\). If \(\alpha=a_0\) is the equation of the body surface, then the Poisson equation in elliptic coordinates will take the form

\[
\frac{1}{c^2 (\sin^2 \alpha + \cos^2 \beta)} \left(\frac{\partial^2 T}{\partial \alpha^2} + \frac{\partial^2 T}{\partial \beta^2}\right) = -\frac{q_v}{\lambda}
\]

(11)

Finally, the desired temperature distribution is described by the equation [11]

\[
T = T_t + \frac{q_v c^2}{4\lambda} \left(\sin^2 \alpha_0 - \sin^2 \alpha\right)
\]

(12)
We transform the expression (12), bringing it to the dimensionless form:

\[
\frac{4\lambda(T - T_c)}{q_r c^2 \sh^2 \alpha_0} = 1 - \left( \frac{\sh \alpha}{\sh \alpha_0} \right)^2
\]

(13)

Thus, the temperature distribution in an elliptical body obeys a parabolic law, as shown in figure 1.

![Figure 1. Dimensionless field of body temperature in the section.](image)

We will conduct a comparative analysis of the temperature difference for circular or elliptical cylinders. As you know the temperature difference between the center of the cylinder and its surface is equal to

\[
T_0 - T_R = \frac{q_r R^2}{4\lambda}
\]

(14)

From formula (12), we find the temperature difference between the center of an elliptical body and its surface

\[
T_0 - T_c = \frac{q_r c^2 \sh^2 \alpha_0}{4\lambda}
\]

(15)

their relationship takes the form

\[
\frac{T_0 - T_c}{T_0 - T_R} = \frac{c^2 \sh^2 \alpha_0}{R^2}
\]

(16)

4. Discussion

Based on mathematical modeling, it is obtained that when the geometric shape of the cross-section changes (when moving to an elliptical cross-section), the level of thermal stresses that occur decreases. The last relation (16) shows that the temperature difference between the center and the surface decreases with the transition to an elliptical cross-section. This interesting result can be explained from the point of view of physics. With an increase in the length of the external boundary, the heat exchange surface also increases, which affects the intensity of heat removal to the environment. This leads to a decrease in the temperature difference between any center of the body and its surface. Thus,
by varying the geometric shape of the cross-section, it is possible to reduce the level of thermal stresses that occur, and, consequently, to increase the durability of concrete and reinforced concrete structures.

5. Conclusion
In this paper, an analysis of publications devoted to the study of the basic physical and mechanical properties of concrete, directly related to the mechanics of a deformable solid, mass transfer, increase in strength over time, and other concepts that connect all these properties with temperature stresses that occur at an early stage of maturation of concrete and reinforced concrete structures, is carried out. To determine the temperature fields and thermal stresses of concrete products, the method of mathematical modeling was used. On its basis, it was shown that it is necessary to determine the level of temperature stresses in super-large concrete structural elements that occur during the manufacture of such products and contain a huge amount of concrete at the same time. Based on mathematical modeling, it is established that the transition to an elliptical cross-section of concrete structures reduces the level of thermal stresses that occur. This confirms the choice of the section of concrete structures in the form of an ellipse.

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