The near threshold \( \pi^- p \rightarrow \eta n \) reaction in an effective Lagrangian approach

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The near threshold \( \pi^- p \rightarrow \eta n \) reaction is studied within an effective Lagrangian approach and the isobar model. By considering the contributions from \( s \) - and \( u \)-channel nucleon pole and \( N^*(1535) \) resonance, the total and differential cross sections of the \( \pi^- p \rightarrow \eta n \) reaction near threshold are calculated. Our theoretical results can fairly reproduce the current experimental data. It is also shown that while the center-of-mass energy lies in the range from the reaction threshold up to 1.65 GeV, \( s \)-channel \( N^*(1535) \) resonance plays the dominant role. The effect from nucleon pole is found to be small but the interference terms between the \( N^*(1535) \) resonance and the nucleon pole are significant. The contributions from \( t \)-channel processes are negligible in the present calculation.

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I. INTRODUCTION

The study of the nucleon and its excited states is an interesting topic in hadron physics. In classical constituent quark models (CQM), proton is described as a three-quark \((uud)\) state. This picture is very successful in explaining the properties of the spatial ground states, but not for the case of the excited states. For example, the \( N^*(1535) \) resonance, four-star state in the Particle Data Review Book (PDG) \[1\], with spin-parity \( J^P = 1/2^- \), is expected to be the lowest \( L = 1 \) orbital excited nucleon state \[2,3\] according to the CQM. However, the \( N^*(1535) \) resonance is heavier than the spatial excited nucleon state, \( N^*(1440) \) \((J^P = 1/2^+)\). This is the long standing inverse mass problem for the low-lying excited nucleon states. Furthermore, it is well known that \( N^*(1535) \) resonance couples strongly to the final states with strangeness, such as, \( \eta N \) channel \[1,4,5\], \( K\Lambda \) channel \[6,7\], \( \eta'N \) \[10,11\], and \( \phi N \) channel \[12,13\], which implies a considerable amount of \( s\bar{s} \) component in the \( N^*(1535) \) wave function \[2,15,16\].

On the other hand, the \( \pi^- p \rightarrow \eta n \) reaction is of particular interest in studying the structure of the \( N^*(1535) \) resonance, of which the properties still bore a lot of controversies. Since there are no isospin-3/2 \( \Delta \) baryons contributing here, this reaction gives us a rather clean platform to study the isospin 1/2 nucleon resonances, especially for studying the \( N^*(1535) \) resonance because it couples strongly to the \( \eta N \) channel. This reaction has been theoretically studied by using a chiral quark-model approach in Refs. \[5,6\], and also in Ref. \[2\] with an updated coupled-channel method. They all found that the contributions from the \( N^*(1535) \) resonance dominate the reaction near threshold. However, the present theoretical calculations are still far from being as accurate as the experiment \[4\]. Thus, more theoretical studies are welcome.

Along this line, with the near threshold experimental data \[17–20\], we reanalysis the \( \pi^- p \rightarrow \eta n \) reaction from the production threshold to the center-of-mass energy \( W \simeq 1.65 \) GeV by using the effective Lagrangian approach and the isobar model. We payed especial attention to the role of the \( N^*(1535) \) resonance, while the contribution of nucleon pole is also considered in the present calculation, and we find that the interference terms between the \( N^*(1535) \) resonance and the nucleon pole are significant. Moreover, in those previous works, they all take a constant total decay width, 150 MeV, for \( N^*(1535) \) resonance. In this work, both the energy-dependent total width and the constant total width for the \( N^*(1535) \) resonance are used.

This paper is organized as follows. In Sect. II we shall discuss the formalism and the main ingredients of the model. The numerical results and discussions are presented in Sect. III. Finally, a short summary is given in the last section.

II. FORMALISM AND INGREDIENTS

As shown in Refs \[5,6,12,21,29\], the combination of the effective Lagrangian approach and the isobar model is a good method to study the hadron resonances production in the \( \pi N, N N, \) and \( K N \) scattering. In this work, we will use this approach to study the near threshold \( \pi^- p \rightarrow \eta n \) reaction. The basic tree level Feynman diagrams for \( \pi^- p \rightarrow \eta n \) reaction are depicted in Fig. 1.
where contributions from the $s$-channel and $u$-channel nucleon pole and $N^*(1535)$ ($\equiv N^*$) resonance are considered. The contributions from the $t$-channel $a_0(980)$ exchange are ignored, because the information of $a_0NN$ vertex is scarce and the mass of $a_0(980)$ is heavy, which will suppress its contribution.

To compute those terms shown in Fig. 1 we take the effective Lagrangian densities as

\begin{align}
\mathcal{L}_{\pi NN} &= ig\pi NN\bar{N}\gamma_5\\
\mathcal{L}_{\eta NN} &= ig\eta NN\bar{N}\gamma_5N, \quad \mathcal{L}_{\pi NN^*} = -ig\pi NN^*\bar{N}\gamma_5N^* + h.c., \\
\mathcal{L}_{\eta NN^*} &= -ig\eta NN^*\bar{N}\gamma_5N^* + h.c.,
\end{align}

with $g_{\pi NN}^2/4\pi = 14.4$ and $g_{\eta NN}^2/4\pi = 0.4$ as used in Ref. [12]. The values of coupling constants $g_{\pi NN^*}$ and $g_{\eta NN^*}$ can be determined from the $N^*(1535)$ partial decay widths,

\begin{align}
\Gamma_{N^*\rightarrow N\pi} &= \frac{3g_{\pi NN^*}^2(m_N + E_N^*)\rho_{c.m.}}{4\pi M_{N^*}}, \quad \Gamma_{N^*\rightarrow \eta\pi} = \frac{g_{\eta NN^*}^2(m_N + E_N^*)\rho_{c.m.}}{4\pi M_{N^*}},
\end{align}

where $m_N, m_{\pi}, m_{\eta}$, and $M_{N^*}$ are the masses of proton, $\pi$ meson, $\eta$ meson, and $N^*(1535)$ resonance, respectively. The $\rho_{c.m.}$ is the magnitude of the 3-momentum of $\pi/\eta$ meson that was measured in the $N^*(1535)$ resonance rest frame, and $E_{N^*}^\pi/\eta$ is the energy of proton in the $N\pi$ or $N\eta$ decay. The $p_{\pi/\eta}^{c.m.}$ and $E_{N^*}^{\pi/\eta}$ have the following forms,

\begin{align}
p_{\pi/\eta}^{c.m.} &= \frac{\lambda^{1/2}(M_{N^*}, m_N, m_{\pi/\eta})}{2M_{N^*}}, \\
E_{N^*}^{\pi/\eta} &= \sqrt{(p_{\pi/\eta}^{c.m.})^2 + m_{N^*}^2},
\end{align}

where $\lambda$ is the Källen function with $\lambda(x, y, z) = (x - y - z)^2 - 4yz$.

With experimental mass (1535 MeV), total decay width (150 MeV), and the branch ratios of $N^*(1535)$ resonance quoted in the PDG [1], we obtain $g_{\pi NN}^2/4\pi = 0.037$ and $g_{\eta NN^*}^2/4\pi = 0.28$.

Next, we pay attention to the total scattering amplitude $\mathcal{M}$ of $\pi^- p \rightarrow \eta n$ reaction,

\begin{align}
\mathcal{M} &= \mathcal{M}_s + \mathcal{M}_u \\
&= \mathcal{M}_s^N + \mathcal{M}_s^{N^*} + \mathcal{M}_u^N + \mathcal{M}_u^{N^*},
\end{align}

Each amplitude can be obtained straightforwardly with the effective Lagrangian densities shown above. Here we give explicitly the amplitudes, $\mathcal{M}_s^N$ and $\mathcal{M}_s^{N^*}$, as an example,

\begin{align}
\mathcal{M}_s^N &= -\sqrt{2}g_{\pi NN\pi}g_{\eta NN\eta}F_N(s) \times \\
&\quad \bar{u}(p_n, s_n)\gamma_5G_N(s)\gamma_5u(p_p, s_p), \\
\mathcal{M}_s^{N^*} &= \sqrt{2}g_{\pi NN^*\pi}g_{\eta NN^*\eta}F_{N^*}(s) \times \\
&\quad \bar{u}(p_n, s_n)\gamma_5G_{N^*}(s)u(p_p, s_p),
\end{align}

with $s = W^2 = (p_p + p_e)^2$, the invariant mass square of the $\pi^- p$ system. The $F_N(s)$ and $G_N(s)$ [$F_{N^*}(s)$ and $G_{N^*}(s)$] are respectively the form factor and propagator for the nucleon pole [$N^*(1535)$ resonance].

The form factors for nucleon pole and $N^*(1535)$ resonance, $F_N(s)$ and $F_{N^*}(s)$, are introduced to describe the off-shell properties of the amplitude, and we choose the forms of them as,

\begin{align}
F_N(s) &= \frac{\Lambda_N^4}{\Lambda_N^4 + (s - m_N^2)^2}, \\
F_{N^*}(s) &= \frac{\Lambda_{N^*}^4}{\Lambda_{N^*}^4 + (s - M_{N^*}^2)^2},
\end{align}

with $\Lambda_N = 0.6$ GeV and $\Lambda_{N^*} = 2.0$ GeV as used in Ref. [12] for the $\pi^ - p \rightarrow \phi n$ reaction.

For the propagators $G_N(s)$ and $G_{N^*}(s)$ of the nucleon pole and the $N^*(1535)$ resonance in the $s$–channel, we take them as [30],

\begin{align}
G_N(s) &= \frac{i(\not{s} + m_N)}{s - m_N^2}, \\
G_{N^*}(s) &= \frac{i(\not{s} + M_{N^*})}{s - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}(s)},
\end{align}

where $\Gamma_{N^*}(s)$ is the $N^*(1535)$ energy-dependent total decay width. Since the main decay channels of $N^*(1535)$ resonance are the $\pi N$ and $\eta N$, we take the commonly
used phase space dependent width for the $N^*(1535)$ resonance as \[7,9\],

$$\Gamma_{N^*}(s) = \Gamma_{N^*\to N\pi} \frac{\rho_{\pi N}(s)}{\sqrt{s}} \left( \frac{M^2_{N^*} + m_N^2 + m_{\pi}^2}{2s} \right) + \Gamma_{N^*\to N\eta} \frac{\rho_{\eta N}(s)}{\sqrt{s}} \left( \frac{M^2_{N^*} + m_N^2 + m_\eta^2}{2s} \right),$$ \(16\)

where $\Gamma_{N^*\to N\pi} = 75$ MeV and $\Gamma_{N^*\to N\eta} = 75$ MeV are the $N^*(1535)$ partial decay widths, the $\rho_{\pi N}(s)$ and $\rho_{\eta N}(s)$ are the phase space factors for $\pi N$ and $\eta N$ final states, respectively, for example,

$$\rho_{\pi N}(s) = \frac{\sqrt{(s - (m_N + m_{\pi})^2)(s - (m_N - m_{\pi})^2)}}{\sqrt{s}}.$$

From the scattering amplitudes given above, we can calculate the total and differential cross sections for $\pi^- p \to \eta n$ reaction. At the center of mass (c.m.) frame, the differential cross section for $\pi^- p \to \eta n$ reaction can be expressed as,

$$\frac{d\sigma}{dcos\theta} = \frac{m_N^2 |p_\eta|^2}{8\pi s |p_\pi|^2} \left( \sum_{s_p, s_n} |M|^2 \right),$$ \(18\)

where $s_p$ and $s_n$ are the spin polarization variables of initial proton and final neutron, respectively. The $\theta$ is the angle of the outgoing $\eta$ meson relative to the beam direction in the c.m. frame, while $p_\pi$ and $p_\eta$ are the 3-momentum of the initial $\pi^-$ and the final $\eta$ mesons, respectively.

III. NUMERICAL RESULTS AND DISCUSSION

Firstly, with Eq. \(18\), we calculate the total and differential cross sections of $\pi^- p \to \eta n$ reaction from the production threshold up to the center-of-mass energy $W = 1.65$ GeV by using the $N^*(1535)$ energy-dependent width as in Eq. \(16\). The corresponding theoretical results as well as the experimental data from Refs. \[17,18\] are shown in Fig. 2, where the dotted and dashed curves stand for the contributions from the nucleon pole and the $N^*(1535)$ resonance, respectively, while the solid line stands for the total contributions, which can describes well the experimental data with $A_N = 0.6$ GeV and $A_{N^*} = 2.0$ GeV.

From Fig. 2 we can see that the contributions from $N^*(1535)$ resonance are dominant, but not enough to reproduce the experimental data, whereas the contributions from the nucleon pole are minor, but, the interference terms of them are significant. We also find that the results are sensitive to the cutoff parameter, $A_N$, but not to the cutoff parameter, $A_{N^*}$, which is because the form factor for $N^*(1535)$ is close to 1 with the invariant mass $W$ around 1535 MeV in the present calculation.

On the other hand, we also calculate the total cross section by using a constant total decay width $\Gamma_{N^*} = 150$ MeV for $N^*(1535)$ resonance, the results are shown in Fig. 3. In this case, the contributions from the $N^*(1535)$ resonance are absolutely dominant, and the experimental data can be reasonably reproduced by only considering the $N^*(1535)$ resonance. The contributions from nucleon pole are minor and the interference terms are significant, which is the same as the case with the energy-dependent total decay width $\Gamma_{N^*}(s)$. This is in agreement with the previous calculations \[4,6\].

From the results, dashed lines in Fig. 2 and Fig. 3 we find that the Breit-Wigner mass of $N^*(1535)$ resonance will be pushed down if we use the energy-dependent width, which is similar to the case of $\Lambda(1405)$ state that we found in Ref. \[31\]. This will have important implications on various model calculations on the mass of $N^*(1535)$ resonance \[7,9\].

Furthermore, the differential cross section of the $\pi^- p \to \eta n$ reaction with the energy-dependent total decay width $\Gamma_{N^*}(s)$ for the $N^*(1535)$ resonance \[1\] is calculated and shown in Fig. 4. We can see that our theoretical results can reasonably describe the experimental data, especially for those energy points near reaction threshold thanks to the main contributions from the $N^*(1535)$ resonance and the significant interference between the $N^*(1535)$ resonance and the nucleon pole.

IV. SUMMARY

The $\pi^- p \to \eta n$ reaction near-threshold is studied in the frame of the effective Lagrangian method and the isobar model, which have been extensively used to deal with

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1 We do not show our theoretical results by using a constant total decay width since the similar results have been shown in the previous works \[4,6\].
hadron collisions. We calculate the total and differential cross sections for this reaction by considering the contributions from the \(N^*(1535)\) resonance and the nucleon pole. The energy-dependent total width and the constant total width for the \(N^*(1535)\) resonance are used. In both cases, the contributions from the \(N^*(1535)\) resonance are absolutely dominant. In the case of the \(N^*(1535)\) total decay width being 150 MeV, the experimental data can be reasonably accounted for by only considering the \(N^*(1535)\) resonance, which is in agreement with the previous calculations\[4,5\].

From our results, it is shown that the \(s\)-channel \(N^*(1535)\) resonance exchange plays the dominated role from the reaction threshold to the center-of-mass energy \(W \simeq 1.65 \text{ GeV}\). The effect from nucleon pole is found to be small but the interference terms between the \(N^*(1535)\) resonance and the nucleon pole are significant. We also find that the Breit-Wigner mass of \(N^*(1535)\) resonance will be pushed down if we use the energy-dependent total width, which will have important implications on various model calculations on its mass.

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FIG. 4: Differential cross sections for $\pi^-p \rightarrow \eta n$ reaction as a function of $\cos\theta$ at different center-of-mass energy, $W$, in the presence of the $N^*(1535)$ total width being energy-dependent. The dotted and dashed curves stand for the contributions from the nucleon pole and the $N^*(1535)$ resonance, respectively; the solid line stands for the total contributions. The experimental data are from Refs. [17](open square), Ref. [19](dot), and Ref. [20](triangle).