OSTROWSKI’S TYPE INEQUALITIES FOR STRONGLY–CONVEX FUNCTIONS

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Abstract. In this paper, we establish Ostrowski’s type inequalities for strongly–convex functions where $c > 0$ by using some classical inequalities and elementary analysis. We also give some results for product of two strongly–convex functions.

1. INTRODUCTION

Let $f : I \subset [0, \infty] \to \mathbb{R}$ be a differentiable mapping on $I^o$, the interior of the interval $I$, such that $f' \in L[a, b]$ where $a, b \in I$ with $a < b$. If $|f'(x)| \leq M$, then the following inequality holds (see [3]).

\begin{equation}
|f(x) - \frac{1}{b-a} \int_a^b f(u)du| \leq \frac{M}{b-a} \left[ \frac{(x-a)^2 + (b-x)^2}{2} \right]
\end{equation}

This inequality is well known in the literature as the Ostrowski inequality. For some results which generalize, improve and extend the inequality (1.1) see ([1], [9]) and the references therein.

Let us recall some known definitions and results which we will use in this paper. A function $f : I \to \mathbb{R}$, $I \subseteq \mathbb{R}$ is an interval, is said to be a convex function on $I$ if

\begin{equation}
f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)
\end{equation}

holds for all $x, y \in I$ and $t \in [0,1]$. If the reversed inequality in (1.2) holds, then $f$ is concave.

Definition of strongly–convex functions was given by Polyak in 1966 as following:

Definition 1. (See [2]) $f : I \to \mathbb{R}$ is called strongly–convex with modulus $c > 0$, if

\begin{equation}
f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) - ct(1-t)(x-y)^2
\end{equation}

for all $x, y \in I$ and $t \in (0,1)$.

Strongly convex functions have been introduced by Polyak in [2] and they play an important role in optimization theory and mathematical economics. Various properties and applications of them can be found in the literature see ([2]–[7]) and the references cited therein.

In [1], Alomari et al. proved following result:

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Corollary 1. Let \( f : I \subset [0, \infty) \to \mathbb{R} \) be a differentiable mapping on \( I^0 \) such that \( f' \in L[a, b] \), where \( a, b \in I \) with \( a < b \). If \( |f'|^p \) is convex on \([a, b], p > 1\) and \(|f'| \leq M\), then the following inequality holds:

\[
(1.3) \quad \left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{M}{b-a} \left[ \frac{(x-a)^2 + (b-x)^2}{(p+1)^2} \right]
\]

for each \( x \in [a, b] \).

The main purpose of this paper is to prove some new Ostrowski-type inequality for strongly-convex functions and to give new results under some special conditions of our Theorems. We also establish several integral inequalities which involving product of strongly-convex and convex functions.

2. MAIN RESULTS

To prove our main results we need the following lemma (see [1]):

Lemma 1. Let \( f : I \subset \mathbb{R} \to \mathbb{R} \) be a differentiable mapping on \( I^0 \) where \( a, b \in I \) with \( a < b \). If \( f' \in L[a, b] \), then the following equality holds:

\[
f(x) - \frac{1}{b-a} \int_a^b f(u) \, du = \frac{(x-a)^2}{b-a} \int_0^1 tf'(tx + (1-t) a) \, dt - \frac{(b-x)^2}{b-a} \int_0^1 tf'(tx + (1-t) b) \, dt
\]

for each \( x \in [a, b] \).

Theorem 1. Let \( f : I \subset \mathbb{R} \to \mathbb{R} \) be a differentiable mapping on \( I^0 \) such that \( f' \in L[a, b] \), where \( a, b \in I \) with \( a < b \). If \( |f'| \) is strongly-convex on \([a, b] \) with respect to \( c > 0 \), \( |f'| \leq M \) and \( M \geq \max \left\{ \frac{c(x-a)^2}{6}, \frac{c(b-x)^2}{6} \right\} \), then the following inequality holds:

\[
(2.1) \quad \left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{2(b-a)} \left( M - \frac{c(x-a)^2}{6} \right) \\
+ \frac{(b-x)^2}{2(b-a)} \left( M - \frac{c(b-x)^2}{6} \right).
\]

for all \( x, y \in [a, b] \) and \( t \in (0, 1) \).

Proof. From Lemma 1 and by using the property of modulus, we have

\[
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{b-a} \int_0^1 t |f'(tx + (1-t) a)| \, dt \\
+ \frac{(b-x)^2}{b-a} \int_0^1 t |f'(tx + (1-t) b)| \, dt.
\]
Since $|f'|$ is strongly−convex on $[a, b]$ and $|f'| \leq M$, we get
\[
\int_{0}^{1} t |f' (tx + (1-t)a)| \, dt \leq \int_{0}^{1} \left[ t^2 |f'(x)| + t (1-t) |f'(a)| - ct^2 (1-t) (x-a)^2 \right] \, dt \\
\leq \frac{M}{2} - \frac{c (x-a)^2}{12}
\]
and
\[
\int_{0}^{1} t |f' (tx + (1-t)b)| \, dt \leq \int_{0}^{1} \left[ t^2 |f'(x)| + t (1-t) |f'(b)| - ct^2 (1-t) (b-x)^2 \right] \, dt \\
\leq \frac{M}{2} - \frac{c (b-x)^2}{12}.
\]
We can easily deduce
\[
\left| f(x) - \frac{1}{b-a} \int_{a}^{b} f(u) \, du \right| \leq \frac{(x-a)^2}{2 (b-a)} \left( M - \frac{c (x-a)^2}{6} \right) \\
+ \frac{(b-x)^2}{2 (b-a)} \left( M - \frac{c (b-x)^2}{6} \right).
\]
which completes the proof. \(\square\)

**Remark 1.** If we take $c \to 0^+$ in the inequality (2.1), we obtain the inequality (1.1).

**Corollary 2.** If we choose $x = \frac{a+b}{2}$ in the inequality (2.1), we obtain the following inequality:
\[
\left| f \left( \frac{a+b}{2} \right) - \frac{1}{b-a} \int_{a}^{b} f(u) \, du \right| \leq M \frac{(b-a)}{4} - \frac{c (b-a)^3}{96}.
\]

**Theorem 2.** Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on $I^0$ such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$. If $|f'|^q$ is strongly−convex on $[a, b]$ with respect to $c > 0$, $|f'| \leq M$ and $M^q \geq \max \left\{ \frac{c(x-a)^2}{6}, \frac{c(b-x)^2}{6} \right\}$ then the following inequality holds;
\[
(2.2) \left| f(x) - \frac{1}{b-a} \int_{a}^{b} f(u) \, du \right| \leq \frac{(x-a)^2}{b-a} \left( \frac{1}{p+1} \right)^\frac{1}{p} \left( M^q - \frac{c (x-a)^2}{6} \right)^\frac{1}{q} \\
+ \frac{(b-x)^2}{b-a} \left( \frac{1}{p+1} \right)^\frac{1}{p} \left( M^q - \frac{c (b-x)^2}{6} \right)^\frac{1}{q}
\]
for all $x, y \in [a, b], t \in (0, 1), q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$. 

Proof. From Lemma 1 and by using the Hölder’s inequality for \( q > 1 \), we have
\[
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{b-a} \left( \int_0^1 t^p \, dt \right)^{\frac{1}{p}} \left( \int_0^1 |f'(tx + (1-t)a)|^q \, dt \right)^{\frac{1}{q}} + \frac{(b-x)^2}{b-a} \left( \int_0^1 t^p \, dt \right)^{\frac{1}{p}} \left( \int_0^1 |f'(tx + (1-t)b)|^q \, dt \right)^{\frac{1}{q}}.
\]

Since \(|f'|^q\) is strongly-convex on \([a,b]\) and \(|f'|^q \leq M\), we get
\[
\int_0^1 |f'(tx + (1-t)a)|^q \, dt \leq \int_0^1 \left[ t |f'(x)|^q + (1-t) |f'(a)|^q - ct (1-t) (x-a)^2 \right] \, dt \leq M^q - \frac{c(x-a)^2}{6}
\]
and
\[
\int_0^1 |f'(tx + (1-t)b)|^q \, dt \leq \int_0^1 \left[ t |f'(x)|^q + (1-t) |f'(b)|^q - ct (1-t) (b-x)^2 \right] \, dt \leq M^q - \frac{c(b-x)^2}{6}.
\]
Therefore, we obtain
\[
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{b-a} \left( \frac{1}{p+1} \right)^{\frac{1}{p}} \left( M^q - \frac{c(x-a)^2}{6} \right)^{\frac{1}{q}} + \frac{(b-x)^2}{b-a} \left( \frac{1}{p+1} \right)^{\frac{1}{p}} \left( M^q - \frac{c(b-x)^2}{6} \right)^{\frac{1}{q}}.
\]
which completes the proof. \(\square\)

Remark 2. If we take \( c \to 0^+ \) in the inequality (2.3), we obtain the inequality (1.3).

Corollary 3. If we choose \( x = \frac{a+b}{2} \) in the inequality (2.3), we obtain the following inequality:
\[
\left| f\left( \frac{a+b}{2} \right) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(b-a)^2}{2(b-a)} \left( \frac{1}{p+1} \right)^{\frac{1}{p}} \left( M^q - \frac{c(b-a)^2}{24} \right)^{\frac{1}{q}}.
\]

Theorem 3. Let \( f : I \subset \mathbb{R} \to \mathbb{R} \) be a differentiable mapping on \( I^0 \) such that \( f' \in L[a,b] \), where \( a, b \in I \) with \( a < b \). If \(|f'|^q\) is strongly-convex on \([a,b]\) with respect to \( b,c > 0, q \geq 1, |f'| \leq M \) and \( M^q \geq \max \left\{ \frac{c(b-a)^2}{6}, \frac{c(b-x)^2}{6} \right\} \) then the
following inequality holds:

\begin{equation}
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{2(b-a)} \left( M^q - \frac{c(x-a)^2}{6} \right)^{\frac{1}{q}} + \frac{(b-x)^2}{2(b-a)} \left( M^q - \frac{c(b-x)^2}{6} \right)^{\frac{1}{q}}
\end{equation}

for all \( x, y \in [a, b] \) and \( t \in (0,1) \).

**Proof.** From Lemma 1 and applying the Power mean inequality for \( q \geq 1 \), we have

\[
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{2(b-a)} \left( \int_0^1 t \, dt \right)^{1 - \frac{1}{q}} \left( \int_0^1 t |f'(tx + (1-t)a)|^q \, dt \right)^{\frac{1}{q}} + \frac{(b-x)^2}{2(b-a)} \left( \int_0^1 t |f'(tx + (1-t)b)|^q \, dt \right)^{\frac{1}{q}}.
\]

Since \(|f'|^q \) is strongly-convex on \([a, b]\) and \(|f'|^q \leq M\), we get

\[
\int_0^1 t |f'(tx + (1-t)a)|^q \, dt \leq \int_0^1 \left[ t^2 |f'(x)|^q + t(1-t) |f'(a)|^q - ct^2 (1-t) (x-a)^2 \right] \, dt
\]

\[
\leq \frac{M^q - c(x-a)^2}{12}
\]

and

\[
\int_0^1 t |f'(tx + (1-t)b)|^q \, dt \leq \int_0^1 \left[ t^2 |f'(x)|^q + t(1-t) |f'(b)|^q - ct^2 (1-t) (b-x)^2 \right] \, dt
\]

\[
\leq \frac{M^q}{2} - \frac{c(b-x)^2}{12}.
\]

Hence, we deduce

\[
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(x-a)^2}{2(b-a)} \left( M^q - \frac{c(x-a)^2}{6} \right)^{\frac{1}{q}} + \frac{(b-x)^2}{2(b-a)} \left( M^q - \frac{c(b-x)^2}{6} \right)^{\frac{1}{q}}.
\]

which completes the proof. \(\square\)

**Remark 3.** If we take \( c \to 0^+ \) in the inequality (2.3), we obtain the inequality (1.1).

**Corollary 4.** If we choose \( x = \frac{a+b}{2} \) in the inequality (2.3), we obtain the following inequality:

\[
\left| f\left( \frac{a+b}{2} \right) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \frac{(b-a)}{4} \left( M^q - \frac{c(b-a)^2}{24} \right)^{\frac{1}{q}}.
\]
Theorem 4. Suppose that \( f, g : I \subset \mathbb{R} \to [0, \infty) \) are strongly-convex functions on \( I^0 \) with respect to \( c > 0 \) such that \( fg \in L[a, b] \), where \( a, b \in I \) with \( a < b \). Then the following inequality holds:

\[
\frac{1}{b-a} \int_a^b f(x)g(x) \, dx \\
\leq \frac{1}{3} [f(a)g(a) + f(b)g(b)] + \frac{1}{6} [f(a)g(b) + f(b)g(a)] \\
- \frac{c(b-a)^2}{12} [f(a) + f(b) + g(a) + g(b)] + \frac{c(b-a)^4}{30}.
\]

Proof. From strongly-convexity of \( f \) and \( g \), we can write

\[
f(tb + (1-t)a) \leq tf(b) + (1-t)f(a) - ct(1-t)(b-a)^2
\]

and

\[
g(tb + (1-t)a) \leq tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2
\]

Since \( f, g \) are non-negative, we have

\[
f(tb + (1-t)a)g(tb + (1-t)a) \\
\leq \left[ tf(b) + (1-t)f(a) - ct(1-t)(b-a)^2 \right] \\
\times \left[ tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2 \right].
\]

By integrating the resulting inequality with respect to \( t \) over \([0, 1]\), we get

\[
\int_0^1 f(tb + (1-t)a)g(tb + (1-t)a) \, dt \\
\leq \frac{1}{3} [f(a)g(a) + f(b)g(b)] + \frac{1}{6} [f(a)g(b) + f(b)g(a)] \\
- \frac{c(b-a)^2}{12} [f(a) + f(b) + g(a) + g(b)] + \frac{c(b-a)^4}{30}.
\]

Hence, by taking into account the change of the variable \( tb + (1-t)a = x, (b-a)dt = dx \), we obtain the required result. \( \square \)

Corollary 5. If we choose \( g(x) = 1 \) in (2.4), we obtain the following inequality:

\[
\frac{1}{b-a} \int_a^b f(x) \, dx \leq \frac{f(a) + f(b)}{2} - \frac{c(b-a)^2}{12} [f(a) + f(b) + 2] + \frac{c(b-a)^4}{30}.
\]

Theorem 5. Suppose that \( f, g : I \subset \mathbb{R} \to [0, \infty) \) are strongly-convex functions on \( I^0 \) with respect to \( c > 0 \) such that \( fg \in L[a, b] \), where \( a, b \in I \) with \( a < b \). Then the
By integrating this inequality with respect to \( t \) functions, respectively, on 

\[ \int_a^b (x-a) f(x) dx \] 

and 

\[ \int_a^b (b-x) g(x) dx \] 

we obtain

**Proof.** Since \( f \) and \( g \) are strongly-convex functions, we can write

\[ f(tb + (1-t)a) \leq tf(b) + (1-t)f(a) - ct(1-t)(b-a)^2 \]

and

\[ g(tb + (1-t)a) \leq tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2 \]

By using the elementary inequality, \( e \leq f \) and \( p \leq r \), then \( er + fp \leq ep + fr \) for \( e, f, p, r \in \mathbb{R} \), then we get

\[ f(tb + (1-t)a) \left[ tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2 \right] + g(tb + (1-t)a) \left[ tf(b) + (1-t)f(a) - ct(1-t)(b-a)^2 \right] \leq f(tb + (1-t)a)g(tb + (1-t)a) + [tf(b) + (1-t)f(a) - ct(1-t)(b-a)^2] \left[ tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2 \right]. \]

So, we obtain

\[ tf(tb + (1-t)a)g(b) + (1-t)f(tb + (1-t)a)g(a) - ct(1-t)f(tb + (1-t)a)(b-a)^2 + tf(b)g(tb + (1-t)a) + (1-t)f(a)g(tb + (1-t)a) - ct(1-t)g(tb + (1-t)a)(b-a)^2 \]

\[ \leq f(tb + (1-t)a)g(tb + (1-t)a) + t^2 f(b) g(b) + tf(b)g(a) + t(1-t)f(a)g(b) + (1-t)^2 f(a)g(a) - ct(1-t)(b-a)^2 \left[ tf(b) + g(b) + (1-t)[f(a) + g(a)] - ct^2(1-t)^2(b-a)^4. \right. \]

By integrating this inequality with respect to \( t \) over \([0,1]\) and by using the change of the variable \( tb + (1-t)a = x \), \( (b-a)dt = dx \), the proof is completed. \( \square \)

**Theorem 6.** Suppose that \( f, g : I \subset \mathbb{R} \rightarrow [0, \infty) \) are convex and strongly-convex functions, respectively, on \( I^0 \) with respect to \( c > 0 \) such that \( fg \in L[a,b] \), where
a, b ∈ I with a < b. Then the following inequality holds:

\[
\frac{1}{b-a} \int_a^b f(x) g(x) \, dx + \frac{c(b-a)^2}{6} \left[ \frac{f(a) + f(b)}{2} \right]
\leq \frac{1}{3} [f(a) g(a) + f(b) g(b)] + \frac{1}{6} [f(a) g(b) + f(b) g(a)].
\]

**Proof.** Since f is convex and g is strongly-convex function, we can write

\[
f(tb + (1-t)a) \leq tf(b) + (1-t)f(a)
\]

and

\[
g(tb + (1-t)a) \leq tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2
\]

By multiplying the above inequalities side by side, we have

\[
f(tb + (1-t)a) g(tb + (1-t)a) \leq [tf(b) + (1-t)f(a)] [tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2].
\]

By integrating the resulting inequality with respect to t over [0, 1], we get

\[
\int_0^1 f(tb + (1-t)a) g(tb + (1-t)a) \, dt 
\leq \frac{1}{3} [f(a) g(a) + f(b) g(b)] + \frac{1}{6} [f(a) g(b) + f(b) g(a)]
- \frac{c(b-a)^2}{6} \left[ \frac{f(a) + f(b)}{2} \right].
\]

Hence, by taking into account the change of the variable tb+(1-t)a = x, (b-a)dt = dx, we obtain the required result. □

**Corollary 6.** If we choose g(x) = 1 in (2.7), we obtain the following inequality:

\[
\frac{1}{b-a} \int_a^b f(x) \, dx \leq \left[ 1 - \frac{c(b-a)^2}{6} \right] \frac{f(a) + f(b)}{2}.
\]

**Theorem 7.** Suppose that f, g : I ⊂ R → [0, ∞) are convex and strongly-convex functions, respectively, on I0 with respect to c > 0 such that fg ∈ L[a,b], where
a, b ∈ I with a < b. Then the following inequality holds:

\[
\frac{g(b)}{(b-a)^2} \int_a^b (x-a)f(x) dx + \frac{g(a)}{(b-a)^2} \int_a^b (b-x) f(x) dx \\
+ \frac{f(b)}{(b-a)^2} \int_a^b (x-a)g(x) dx + \frac{f(a)}{(b-a)^2} \int_a^b (b-x) g(x) dx \\
- \frac{c}{(b-a)^2} \int_a^b (x-a)(b-x) f(x) dx \\
\leq \frac{1}{b-a} \int_a^b f(x) dx + \frac{1}{3} [f(a)g(a) + f(b)g(b)] \\
+ \frac{1}{6} [f(a)g(b) + f(b)g(a)] - \frac{c(b-a)^2}{6} \left[ \frac{f(a) + f(b)}{2} \right].
\]

Proof. Since f and g are convex and strongly-convex functions, respectively, we can write

\[
f(tb + (1-t)a) \leq tf(b) + (1-t)f(a)
\]

and

\[
g(tb + (1-t)a) \leq tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2
\]

By using the elementary inequality, e ≤ f and p ≤ r, then er + fp ≤ ep + fr for e, f, p, r ∈ ℝ, then we get

\[
f(tb + (1-t)a) \left[ tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2 \right] \\
+ g(tb + (1-t)a) \left[ tf(b) + (1-t)f(a) \right] \\
\leq f(tb + (1-t)a) g(tb + (1-t)a) \\
+ \left[ tf(b) + (1-t)f(a) \right] \left[ tg(b) + (1-t)g(a) - ct(1-t)(b-a)^2 \right].
\]

So, we obtain

\[
 tf(tb + (1-t)a) g(b) + (1-t) f(tb + (1-t)a) g(a) - ct(1-t) f(tb + (1-t)a)(b-a)^2 \\
+ tf(b) g(tb + (1-t)a) + (1-t) f(a) g(tb + (1-t)a) \\
\leq f(tb + (1-t)a) g(tb + (1-t)a) + t^2 f(b) g(b) \\
+ t(1-t) f(b) g(a) + t(1-t) f(a) g(b) + (1-t)^2 f(a) g(a) \\
- ct^2 (1-t)(b-a)^2 f(b) - ct(1-t)^2(b-a)^2 f(a).
\]

By integrating this inequality with respect to t over [0, 1] and by using the change of the variable \( tb + (1-t)a = x \), \( (b-a)dt = dx \), the proof is completed. □

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