Clustering local laws of the dynamics of complex living systems

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Abstract. In this paper, we generalize the studies of the local laws clustering for dynamics of non-Hamiltonian live systems. We analyze the human heart-rate variability. We use the localization procedure to extract the local properties about the space-time structure of the $R-R$ interval time series. The window-time behavior of the memory functions power spectra reflects the periodic properties in studied dynamics. The age-related alterations in heart-rate variability for young and old testees are manifested in velocity of relaxation. They may be derived by localizing the kinetic and relaxation parameters. These parameters can be considered as the specific predictors of the dynamic intermittency in the initial time signal. The proposed window-time representation of the spectral characteristics, kinetic and relaxation parameters is useful for medical physics and physics of live systems.

1. Introduction

At present time the various approaches of time series analysis are used to study the dynamic properties of open complex systems (physical, chemical, biological, economical) [1–3]. They are based, for example, on statistical theoretical physics [1], semiphenomenological methods and non-linear dynamics [3]. To carry out the good-quality result they need huge as possible set of experimental data. It means that the larger length of time series leads to more detailed results derived by these methods. Generally, the study consists in analyzing either the bifurcation properties related to dynamical phase transitions or the global characteristics related to averaging procedures for large time intervals revealing the intermittency, fractality, self-organized criticality and other properties of dynamical and statistical systems.

Each dynamical process is characterized by alternation of consecutive states. To derive the global process parameters we need to consider it entirely. However if we are interested by the separate states, then we use the local studying methods: localization procedures for time series. Localization means considering the separate part of the process, i.e. its time series.

2. Localization procedures for experimental data of non-Hamiltonian live systems

In this paper, we generalize the studies of local laws in dynamics of the live systems from analyzing the human heart rate variability [4]. To study the space-time $R-R$ interval structure for young and old testees we develop the localization procedures. They allow us to extract the information about correlations, relaxation properties, memory and intermittency effects from local time series samples. These methods are based on Memory Functions Formalism and on the technique of Zwanzig-Mori projection operators, which are generalized to analyze the discrete dynamics of complex non-Hamiltonian systems [1, 5, 6]. The first procedure consists of window-time representation of the memory functions power spectra and the frequency-dependent measures of the statistical memory. The second method is based on deriving the time dependencies of local kinetic and relaxation parameters, reflecting the dynamical intermittency. Time dependencies are calculated by the step-by-step shift of local sample along the initial time series. Using the localization procedures involves to selection the optimal length of local sample [7]. The short sample length leads to incorrect results as far as the statistics is insufficient. In the opposite case the parameters may lose their sensitivity. To solve this problem we have developed the optimization procedure for the sample length. Clustering the initial time series leads to regularizing the information from local samples by deriving the local characteristics and parameters. It allows us to study the separate processes in different time intervals in the time series and also predict the critical phenomena in system [8].

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3. Basic relations of the Memory Functions Formalism

The variety of structures and the nature of complex system dynamics necessitate us to use the variety of approaches to their study. Besides, the distributed objects require the consideration of their interaction between the individual parts, which leads to the complication of applied mathematical approaches. The methods which allow “to disengage” from the internal structure of a studied system, make it possible to simplify the task of its evolution study. The use of such techniques allows to perform parameterization – the quantitative description of distributed system evolution based on the extraction of information from the signals produced by them.

The Memory Functions Formalism – the theoretical approach to the study of auto and cross correlations generated by complex systems with discrete time.

We consider the stochastic dynamics of the studied system as a sequence $x_i$ of random value $X$:

$$X = \{x(T), x(T + \tau), x(T + 2\tau), ..., x(T + (N - 1)\tau)\}.$$  

(1)

Where $T$ is the initial time point, $(N - 1)\tau$ is the set of time period of signal registration, $\tau = \Delta t$ is the time interval of signal discretization.

Mean value, fluctuation and dispersion for a set of random value (1) can be written as follows:

$$\langle X \rangle = \frac{1}{N} \sum_{j=0}^{N-1} x(T + j\tau), \quad \delta x_j = x_j - \langle X \rangle, \quad \sigma^2 = \frac{1}{N} \sum_{j=0}^{N-1} \delta x_j^2.$$  

(2)

To describe the dynamic properties of the studied live system we use the normalized time correlation function (TCF):

$$a(t) = \frac{1}{(N - m)\sigma^2} \sum_{j=0}^{N-m-1} \delta x_j \delta x_{j+m} = \frac{1}{(N - m)\sigma^2} \sum_{j=0}^{N-m-1} \delta x(T + j\tau) \delta x(T + (j + m)\tau).$$  

(3)

Here $x_j, x_{j+m}$ – the values of $X$ in $j, j + m$ steps, $\delta x_j, \delta x_{j+m}$ – fluctuations of $x_j, x_{j+m}$, $\sigma^2$ – dispersion of $X$.

Using the Zwanzig-Mori technique of projection operators [7] we will derive a chain of non-Markov finite-difference discrete equations for the initial TCF $a(t)$ and for the memory functions of higher order $M_i(t) (i=1,2,\ldots,n)$:

$$\Delta a(t) = \lambda_i a(t) - \tau \Lambda_i \sum_{j=0}^{m-1} M_i(j\tau)a(t - j\tau), \ldots,$$

$$\Delta M_{n-1}(t) = \lambda_n M_{n-1}(t) - \tau \Lambda_n \sum_{j=0}^{m-1} M_n(j\tau)M_{n-1}(t - j\tau).$$  

(4)

Here $\lambda_i$ are the kinetic parameters and $\Lambda_i$ are the relaxation parameters:

$$\lambda_i = i \left( \frac{W_{n-1}}{\left| W_{n-1} \right|^2} \right), \quad \Lambda_n = i \left( \frac{W_{n-1} \hat{L} W_n}{\left| W_{n-1} \right|^2} \right).$$  

(5)

Dynamic orthogonal variables $W_n$ in equation (5) are derived by the Gram-Schmidt orthogonalization procedure:

$$\langle W_n, W_m \rangle = \delta_{n,m} \left| W_n \right|^2.$$  

(6)

where $\delta_{n,m}$ is Kronecker’s symbol.

The method has been successfully used to study the complex non-Hamiltonian systems of various nature [1, 5-10].

4. Clusterization of local laws of heart rate variability

4.1. Window-time representation for spectral characteristics

The essence of this procedure is given below. Initially, we need to choose the optimal length of the window (sample). With a small sample length, the accumulated information will be insufficient for
sufficient analysis of the time signal. It is tied with large errors and noise effects. The large sample length leads to losing the necessary “sensitivity” of localized parameters. After choosing the optimal window length, we carry out the window-time procedure for spectral characteristics and parameters. From the initial array of experimental data, we “select” the first \( N \) points. Thus we derive the first window of \( N \) points. Then we obtain the spectral characteristic for this sample. Further we select the new window of \( N \) points (from \( N+1 \) to \( 2N+1 \)) and calculate the parameter again. We need to repeat this procedure to the end of time series.

We used the time series of \( R-R \) intervals for young and old testees as the experimental data [4]. Two groups of healthy human subjects: 10 young (mean age 27 yr, range 21–34 yr) and 10 elderly (mean age 74 yr, range 68–81 yr), participated in this study.

Figure 1 demonstrates the power spectrum of the memory function of the first order \( \mu_1(\nu) \) and its window-time representation for the \( R-R \) interval dynamics of young and elderly subjects. The spectrum \( \mu_1(\nu) \) for heart rate variability of elderly testee has the smaller dynamical peaks in low frequency range. The high frequency peaks related to the respiratory arrhythmia. This dynamic splashes remind of the well known shape of the Suyumbike Tower in Kazan [9]. Value increasing the power spectrum in these frequencies reflects the age-related alterations in human cardiovascular system. The respiratory arrhythmia dynamic peaks for young testee are in range 0.25 f.u.<\nu<0.45 f.u. For elderly testee they are in range 0.4 f.u.<\nu<0.55 f.u. Thus age-related alterations lead to shifting the dynamic peaks to higher frequencies. The elderly testee has the higher variability of the respiratory arrhythmia dynamic peaks. The more detailed information about the time signal structure contains in window-time representation of the spectral characteristic (sample 256 points).

Figure 1. Power spectrum (in log-log scale) of the memory function of the first order \( \mu_1(\nu) \) and its window-time representation for the \( R-R \) interval dynamics of young and elderly testees (upper figure for young subject; under figure for elderly one). We mark the peaks connected to respiratory arrhythmia. Age-related alterations lead to shifting the dynamic peaks to higher frequencies.
4.2. Local kinetic and relaxation parameters of the R–R interval dynamics

Using the second procedure we can obtain the local kinetic $\lambda_i$ and relaxation $\Lambda_i$ parameters. The method essence is calculating these parameters for each local window with further step-by-step shifting up to end of time series. Thus we derive the sets of parameters $\lambda_i$ and $\Lambda_i$. These sets are very sensitive to local effects of dynamical intermittency and non-stationarity.

Figure 2 shows the time dependences for the group averaged local parameters – kinetic $\lambda_i$ and relaxation $\Lambda_i$ ($a,c$ for young subjects and $b,d$ for elderly ones). The group-averaged $|\lambda_i|$ has range $0.1719 \tau^{-1} < |\lambda_i| < 0.8522 \tau^{-1}$ for young subjects and $0.0187 \tau^{-1} < |\lambda_i| < 0.358 \tau^{-1}$ for elderly ones.

Ratio of root-mean-square amplitudes $\langle A \rangle = \left( \frac{1}{N} \sum_{j=0}^{N-1} x_j^2 \right)^{1/2}$ for young and old people is 3.3 times. It means that the relaxation velocity in dynamics of the human cardiovascular system is decreased with age. For comparing the root-mean-square amplitude for group-averaged parameter $\Lambda_i$ is different in 1.7 times.

![Figure 2](image)

**Figure 2.** Time dependences of the group-averaged local kinetic $\lambda_i(t)$ and relaxation $\Lambda_i(t)$ parameters ($a,c$ – for young and $b,d$ – for elderly).

5. Conclusions

The complex systems occupy a special place in the diversity of the world objects which were the subjects of scientific research. They have a set of unique properties, primarily non-equilibrium, nonstationarity and nonlinearity, which are fundamentally different from other objects. These properties, on the one hand, provide an extraordinary “flexibility” to complex systems, and make the task of their description quite a difficult one on the other.

The proposed localization technique allows to reveal some properties of complex live systems. Analyzing the heart rate variability by the localization procedure allows to obtain the quantitative characteristics of relaxation velocity. We reveal the significant differences in relaxation processes of the R–R intervals for the considering age groups. The cardiovascular system of young people has the higher relaxation velocity and thus it is more stable for heart rate abnormalities. The relaxation velocity is decreased with age. These findings are consistent with results obtained by other authors [4].

The window-time procedure for considering frequency dependences allows to obtain additional information about the age-related alterations in studied system. We observe the increasing the variability of respiratory arrhythmia with age, because the corresponding dynamic peaks are shifted to higher frequency range.
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