Direct and indirect grouping strategies for a multi-item probabilistic inventory model

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Abstract. In this paper we develop joint replenishment strategies, namely direct and indirect grouping to find the optimal solution for a multi-item probabilistic inventory problem. Basically, direct grouping is a strategy to group items according to a predetermined ratio of holding cost and minor ordering cost. On the other hand, in indirect grouping strategy, items are ordered every predetermined basic cycle time with a possibility that not all items are ordered. We assume that demands are normally distributed and all shortages are backordered. In developing our model, we also develop an algorithm for direct and indirect grouping. In the numerical experiments, we consider six items with their own parameters, ordered from one supplier, and compare the total inventory cost between direct and indirect grouping strategies. We found that in our examples that indirect grouping strategies gives lower total inventory cost than direct grouping strategy.

1. Introduction
There are some relevant costs that should be considered when managing inventory such as ordering cost, holding cost and shortage cost \cite{2}. Ordering cost relates to a setup cost in a production process, ordering or delivering cost for a retailer and usually a fixed amount for every order. Holding cost occurs within a period of time and proportional to the average of stored goods. Shortage cost occurs when the goods are not available when an order comes. There are two types of shortage cost, lost sales and backorder. Lost sales cost occurs when customers cancel their order due to the unavailability of the goods they order while in the backorder case, customers are willing to wait until goods are available.

In managing inventory there are two big concerns, namely determination of the optimal order quantity and time between replenishment to minimize the total inventory cost. One of the problem in determining the time between replenishment is the joint replenishment problem (JRP). JRP coordinates the replenishment on a group of item ordered from one supplier \cite{5}. There are two ordering costs involved in JRP, major and minor ordering cost. Major ordering cost does not depend on the quantity of item ordered, such as the freight cost while minor ordering cost depends on the number of item ordered such as inspection cost.

There are two strategies to determine the replenishment policy in JRP, viz. indirect grouping and direct grouping. In indirect grouping, replenishment is performed every predetermined basic cycle time but possibly not all items are ordered. Some items may be ordered every two basic cycles time, other items at three basic cycles time and so on. Grouping in the indirect grouping strategy is based on the multiple of basic cycle time. For example, suppose the basic cycle time for ordering five items is one week. Item 1 and 3 are ordered every week, item 2 and 4 are ordered every two basic cycle time (two...
weeks) and item 5 is ordered every three basic cycle time (three weeks). Thus, in this case we have three
groups, i.e group 1 for item 1 and 3, group 2 for item 2 and 4 and group 3 for item 5.

On the other hand, in direct grouping strategy, items are grouped based on the predetermined ratio
between holding cost and minor ordering cost. [6]. Items that have a quite similar ratio are put on the
same group. In every group, then the time between replenishment is determined. In this paper, we will
employ those two strategies in the multi-item probabilistic inventory problem. The aim of this model is
to determine the grouping strategy and the optimal time between replenishment that minimize the total
inventory cost.

2. Literature Review
Over the last few decades, the JRP has become an interesting topic in inventory control literature. It has
a real-world application such as ordering several items to a single supplier, transportation sharing for
some items and packaging items in different quantities [4]. The two different grouping strategies in JRP
have also been analyzed by many authors and algorithms in solving the JRP have also been proposed.
The analysis and comparison of direct and indirect grouping strategies for multi-item inventory costs
for deterministic demand have been employed by Van Eijs, et.al [6]. Since then, some authors suggested
different approaches in building algorithms or heuristics methods to solve the JRP. Olsen [5] proposed
an approach called evolutionary algorithm to solve JRP using direct grouping strategy. Another
approach of heuristic method using a spreadsheet technique to solve JRP was introduced by Nilsson,
et.al. [4]. Earlier in 2012 Wang, et al [8] proposed a differential evolution algorithm to solve JRP using
direct grouping strategy. The same method proposed by Wang, et.al [9] in 2012 under the condition of
interdependent minor ordering costs involved in the JRP.

However, all the algorithms proposed by the abovementioned papers are for deterministic and
constant demand. In this paper, we consider the JRP under the multi-item probabilistic inventory
problem. Demands are assumed to be normally distributed and we will compare the direct and indirect
grouping strategies in terms of their total inventory cost. This is the main contribution of this paper. The
organization of the rest of this paper is as follows. In section 3, we propose our model under direct
grouping and indirect grouping strategies along with our proposed algorithm to solve the JRP. Section 4
is devoted for numerical experiments to give illustration for our model and algorithm while conclusion
and further research are relegated in the last section.

3. The Model
In this paper, we develop a probabilistic inventory model using direct and indirect grouping strategy,
where annual demand is normally distributed. There are some assumptions in developing our model.
1. Ordering and backorder costs are assumed to be known and constant.
2. There is no *quantity discount*.
3. There is no storage constraint.
4. There is no cost constraint in the amount ordered.
5. *Lead time* is four days and one week consists of five days.
6. All shortages are backordered.

Some notations used in this paper are:
- \( n \) : The number of items.
- \( A \) : Major ordering cost.
- \( a_i \) : Minor ordering cost for item \( i \).
- \( D_i \) : Annual demand of item \( i \).
- \( h_i \) : Holding cost per unit of item \( i \) per period.
- \( T \) : Time between replenishment.
- \( T_j \) : Time between replenishment from group \( j \).
- \( M \) : The number of group.
- \( S_j \) : The \( j^{th} \) group.
- \( k_i \) : Integer multiple of \( T \) for item \( i \).
\[ r_i : \text{Reorder point for item } i \]
\[ \mu_{Li} : \text{Average demand during lead time.} \]
\[ \pi_i : \text{Backorder cost for item } i. \]
\[ X_i : \text{Random variable for annual demand of item } i. \]
\[ \int_{r_i}^{\infty} f(x_i) \, dx_i : \text{Probability of shortage for item } i. \]

\[ TC : \text{Annual total inventory cost.} \]

For this model, there are three costs involved, namely ordering cost, holding cost and shortage cost. For each item, ordering cost is the sum of major ordering cost and minor ordering cost divided by the time between replenishment.

Ordering cost = \( (A + a)/T \).

Holding cost is the holding cost per unit of items per year times the average items held in inventory. The average items held in inventory is \( DT/2 + r - \mu_L \), so the holding cost is

\[ \text{Holding cost} = h( DT/2 + r - \mu_L ). \]

Shortage cost occurs when there are unmet demands. The probability of shortage is given by \( \int_r^{\infty} f(x) \, dx \), therefore the shortage cost is given by

\[ \text{Shortage cost} = (\pi/T) \cdot (\int_r^{\infty} f(x) \, dx). \]

Total inventory cost for one item is the sum of ordering, holding and shortage costs.

Total inventory cost = \( (A + a)/T + h( DT/2 + r - \mu_L ) + (\pi/T) \cdot (\int_r^{\infty} f(x) \, dx). \)

In this section we will develop probabilistic JRP problem by considering direct grouping and indirect grouping strategies.

### 3.1. Direct Grouping

By assuming that there is no backorder cost, the total inventory cost for direct grouping strategy is given by ([5])

\[ TC = \sum_{j=1}^{M} \left( A + \sum_{i \in S_j} a_i \frac{1}{T_j} + \frac{T_j}{2} \sum_{i \in S_j} D_i h_i \right). \]  

(1)

When there is backorder, total inventory cost for direct grouping strategy is

\[ TC = \sum_{j=1}^{M} \left( A + \sum_{i \in S_j} a_i \frac{1}{T_j} + \sum_{i \in S_j} h_i \left( \frac{D_i T_j}{2} + r_i - \mu_{Li} \right) + \sum_{i \in S_j} \pi_i \frac{1}{T_j} \int_{r_i}^{\infty} f(x_i) \, dx_i \right). \]  

(2)

Grouping in the direct grouping strategy is based on the ratio between holding cost and minor ordering cost, i.e, \( \frac{D_i h_i}{a_i} \) ([6]). To minimize the total inventory cost in (2), we need to find the value of \( T_j \) for \( j = 1, 2, ..., M \) that satisfies \( \frac{\partial TC}{\partial T_j} = 0 \). Notice that

\[ \frac{\partial TC}{\partial T_j} = -A + \sum_{i \in S_j} \frac{a_i}{T_j^2} + \sum_{i \in S_j} \frac{D_i h_i}{2} - \sum_{i \in S_j} \sum_{i \in S_j} \pi_i \frac{1}{T_j} \int_{r_i}^{\infty} f(x_i) \, dx_i \]

\[ T_j = \sqrt{\frac{2A + 2 \sum_{i \in S_j} (a_i + \pi_i \int_{r_i}^{\infty} f(x_i) \, dx_i)}{\sum_{i \in S_j} D_i h_i}}. \]

(3)

The time between replenishment for each group can be determined by using (3). The value of \( r_i \) can be obtained by minimizing the total inventory cost for individual order. The total inventory cost for individual order is given by
\[ TC_i = \frac{A}{T_i} + h_i \left( \frac{D_i T_i}{2} - r_i + \mu_{Li} \right) + \frac{\pi_i}{T_i} \int_{r_i}^{\infty} f(x_i) \, dx_i. \] (4)

The total inventory cost in (4) will be minimum when the values of \( T_i \) and \( r_i \) satisfy \( \frac{\partial TC_i}{\partial T_i} = 0 \) and \( \frac{\partial TC_i}{\partial r_i} = 0 \). The first partial derivative of \( TC_i \) with respect to \( T_i \) is

\[ \frac{\partial TC_i}{\partial T_i} = - \frac{A + a_i}{T_i^2} + \frac{h_i D_i}{T_i^2} - \frac{\pi_i}{T_i} \int_{r_i}^{\infty} f(x_i) \, dx_i. \]

\[ 0 = - \frac{A + a_i + \pi_i \int_{r_i}^{\infty} f(x_i) \, dx_i}{T_i^2} + \frac{h_i D_i}{2} \]

\[ T_i = \frac{2 \left( A + a_i + \pi_i \int_{r_i}^{\infty} f(x_i) \, dx_i \right)}{h_i D_i}. \] (5)

The first partial derivative of \( TC \) with respect to \( r_i \) is

\[ \frac{\partial TC_i}{\partial r_i} = h_i + \frac{\pi_i}{T_i} \frac{\partial}{\partial r_i} \int_{r_i}^{\infty} f(x_i) \, dx_i. \]

\[ 0 = h_i + \frac{\pi_i}{T_i} \left( 0 - f(r_i) \right) \]

\[ f(\eta_i) = \frac{h_i T_i}{\pi_i} \] (7)

Since we assume that demand follows normal distribution, then we have

\[ \frac{1}{\sigma_i \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{\eta_i - \mu_i}{\sigma_i} \right)^2 \right) = \frac{h_i T_i}{\pi_i} \]

\[ -\frac{1}{2} \left( \frac{\eta_i - \mu_i}{\sigma_i} \right)^2 = \ln \left( \frac{\sigma_i \sqrt{2\pi h_i T_i}}{\pi_i} \right) \]

\[ \eta_i = \mu_i + \sigma_i \sqrt{\ln \left( \frac{\pi_i}{\sigma_i \sqrt{2\pi h_i T_i}} \right)^2} \] (8)

To find the optimal values of \( r_i \) and \( T_i \) we employ the Hadley-Whitin algorithm as follows ([3]):

1. Find the value of \( T_i \) using formula Wilson (EOQ) by employing (3) with no backorder cost.
2. Find the value of \( r_i \) using (8).
3. By using (5), calculate the new value of \( T_i \).
4. If the new value of \( T_i \) is relatively the same as the old one, then the algorithm stops. Otherwise, go back to step 2.

### 3.2. Indirect Grouping

In the JRP MODEL with no backorder, the total inventory cost for indirect grouping strategy is given by

\[ TC = \frac{1}{T} \left( A + \sum_{i=1}^{n} \frac{s_i}{k_i} \right) + \frac{T}{2} \sum_{i=1}^{n} k_i D_i h_i \] (9)

If there is a backorder, then the total inventory cost for the indirect grouping strategy is

\[ TC = \frac{1}{T} \left( A + \sum_{i=1}^{n} \frac{a_i}{k_i} \right) + \sum_{i=1}^{n} h_i \left( \frac{k_i T D_i}{2} + r_i - \mu_{Li} \right) + \sum_{i=1}^{n} \frac{\pi_i}{k_i T} \int_{r_i}^{\infty} f(x_i) \, dx_i \] (10)

To find the minimum value of total inventory cost in (10), we need to find the values of \( T \) and \( k_i \) for \( i = 1, 2, \ldots, n \) that satisfy \( \frac{\partial TC}{\partial T} = 0 \) and \( \frac{\partial TC}{\partial k_i} = 0 \). Notice that
\[
\frac{\partial TC}{\partial T} = -\frac{A + \sum_{i=1}^{n} \frac{a_i}{k_i^2}}{T^2} + \sum_{i=1}^{n} \frac{k_i h_i D_i}{2} - \frac{1}{T^{2}} \sum_{i=1}^{n} \pi_i \int_{r_i}^{\infty} f(x_i) \, dx_i
\]

\[
0 = -\frac{1}{T^{2}} \left( A + \sum_{i=1}^{n} \frac{a_i}{k_i} + \sum_{i=1}^{n} \frac{\pi_i}{k_i} \int_{r_i}^{\infty} f(x_i) \, dx_i \right) + \sum_{i=1}^{n} \frac{k_i h_i D_i}{2}
\]

\[
2A + 2 \sum_{i=1}^{n} \left( \frac{a_i}{k_i} + \frac{\pi_i}{k_i} \int_{r_i}^{\infty} f(x_i) \, dx_i \right) = T^2 \sum_{i=1}^{n} k_i h_i D_i
\]

\[
T = \frac{2A + 2 \sum_{i=1}^{n} \left( \frac{a_i}{k_i} + \frac{\pi_i}{k_i} \int_{r_i}^{\infty} f(x_i) \, dx_i \right)}{\sum_{i=1}^{n} k_i h_i D_i}
\]

\[
\frac{\partial TC}{\partial k_i} = \frac{1}{T} \left( \frac{a_i}{k_i^2} - \frac{h_i T D_i}{2} \right) - \frac{\pi_i}{k_i^2} \int_{r_i}^{\infty} f(x_i) \, dx_i
\]

\[
0 = -\frac{a_i}{T k_i^2} + \frac{h_i T D_i}{2} - \frac{\pi_i}{k_i^2} \int_{r_i}^{\infty} f(x_i) \, dx_i
\]

\[
\frac{1}{k_i} \left( \frac{a_i}{T} + \frac{\pi_i}{T} \int_{r_i}^{\infty} f(x_i) \, dx_i \right) = \frac{h_i T D_i}{2}
\]

\[
k_i = \frac{1}{T} \sqrt{\frac{2 \left( \frac{a_i}{T} + \frac{\pi_i}{T} \int_{r_i}^{\infty} f(x_i) \, dx_i \right)}{h_i D_i}}
\]

Since the values of \( k_i \) are integers, then we need an algorithm to determine the values of \( k_i \) and \( T \). Nilsson et al. [4] has developed an algorithm to find the values of \( k_i \) and \( T \) for deterministic case. For probabilistic case, we need to modify the Nilsson algorithm especially in determining the values of the ratio \( r a_i \). With this modification, the Nilsson algorithm becomes:

1. Choose \( k_i = 1 \) for each \( i \) and calculate the values of \( T \) and \( TC \).
2. Calculate the ratio \( r a_i \) that is between minor ordering cost plus backorder cost divided by holding cost.

\[
r a_i = \frac{\frac{a_i}{T k_i} + \frac{\pi_i}{T k_i} \int_{r_i}^{\infty} f(x_i) \, dx_i}{h_i \left( \frac{k_i T D_i}{2} + r_i - \mu_{Li} \right)}
\]

3. Rank each item according to the ratio obtained in step 2.
4. If all the ratios are less or equal 1.4 then the optimal solution has been obtained.
5. Otherwise, increase the values of \( k_i \) whose ratio is greater than 1.4 and recalculate the basic cycle time \( TC \) then repeat step 2 until 4.
6. If the \( TC \) increases, then we will do iteration from the iteration result with the best solution. Choose the highest ranking of \( r a_i \) and add 1 to the value of \( k_i \) with the highest ranking.
7. Recalculate the value of \( T \), \( TC \), and ratio for each item. Repeat step 3 and 4.

4. Numerical Experiments

Data in this numerical experiment are secondary data taken from Aritonang et al. [1], that consist of major ordering cost of Rp 118.682 and minor ordering cost of Rp 6.836. Demand data, holding and backorder costs are given in Table 1 and Table 2 below.
| Item | Mean (unit/week) | Standard deviation (unit/week) |
|------|-----------------|-----------------------------|
| 1    | 39.61           | 13.67                       |
| 2    | 8.96            | 3.55                        |
| 3    | 4.01            | 1.21                        |
| 4    | 17.88           | 6.9                         |
| 5    | 8.92            | 3.96                        |
| 6    | 2.34            | 1.04                        |

**Table 2.** Holding and backorder costs

| Item | Holding cost (Rp) | Backorder cost (Rp) |
|------|-------------------|---------------------|
| 1    | 139.142           | 247.000             |
| 2    | 127.679           | 213.200             |
| 3    | 95.654            | 143.000             |
| 4    | 121.196           | 148.200             |
| 5    | 107.868           | 138.840             |
| 6    | 81.021            | 85.800              |

In this section we will compare the total inventory cost from the direct grouping strategy and indirect grouping strategy using data in Table 1 and Table 2. We first calculate the value of $r_i$ for each item using equation (5), (8) and Hadley-Within algorithm. Step 1 of the algorithm calculates the values of $T_i$ and $\mu_{ti}$ for $i = 1, 2, \ldots, 6$ using Wilson formula (EOQ). The values of $T_i$ are given in Table 3 and $\mu_{ti}$ can be calculated using $0.8 \times$ average weekly demand.

**Table 3.** Values of $T_i$ using Wilson formula

| Item | $T_i$ (years) |
|------|---------------|
| 1    | 0.0287        |
| 2    | 0.0631        |
| 3    | 0.1091        |
| 4    | 0.0459        |
| 5    | 0.0688        |
| 6    | 0.1551        |

Results from the Hadley-Whitin algorithm are given in Table 4.

**Table 4.** Values of $T_i$ using Hadley-Whitin algorithm

| Item | $r_i$ (unit) | $T_i$ (years) | $\int_{r_i}^{\infty} f(x_i) \, dx_i$ |
|------|-------------|---------------|-------------------------------------|
| 1    | 45          | 0.0336        | 0.1486                              |
| 2    | 12          | 0.0686        | 0.0674                              |
| 3    | 6           | 0.1146        | 0.0381                              |
| 4    | 20          | 0.0519        | 0.1799                              |
| 5    | 12          | 0.0756        | 0.1270                              |
| 6    | 4           | 0.1637        | 0.0765                              |

Grouping on direct grouping strategy is performed based on the ratio $\frac{D_i}{a_i} h_i$. The ratio and grouping under this strategy are given in Table 5.
From Table 5, we can see that direct grouping strategy yields three groups. The first group consists of item 1 and item 4 ($S_1 = \{1, 4\}$), second group with item 2 and item 5 ($S_2 = \{2, 5\}$), and the third group consists of item 3 and item 6 ($S_3 = \{3, 6\}$). We can then determine the time between replenishment for each group using equation (3) and find that the time between replenishment for the first, second and third group are 0.0336 years, 0.0584 years, and 0.0984 years, respectively. The total inventory cost for this strategy can be calculated using equation (2) and that is Rp 25.590.949.

In the indirect grouping strategy, the values of $r_i$ for each item is obtained from Table 4. To determine the basic cycle time, we need to employ the algorithm discussed in the previous section. Based on step 1 of the algorithm, we pick $k_i = 1$ for $i = 1, 2, ..., n$. We found the basic cycle time of 0.0315 years using equation (11) with total inventory cost of Rp 21.085.337. The values of $k_i$ for each $i$ is 1, meaning that every item will be ordered at the same time. In this case indirect grouping strategy is the same as the joint order or joint replenishment. Based on step 2, we can find the values of $ra_i$ for $i = 1, 2, ..., n$ from equation (13). Table 6 gives the values of $ra_i$ for each item (first iteration).

Since the values of $ra_i$ for each item are less than 1.4, the optimal solution has been obtained with the total inventory cost of Rp 21.085.337. Since the values of $k_i = 1$ for each $i$, then indirect grouping strategy will order all items at the same time. Thus, the total inventory cost will be the same as the total inventory cost using joint order strategy. Based on the data used, we found that the total inventory cost for indirect grouping strategy is less than direct grouping strategy. Indirect grouping strategy in our case is the same with joint order strategy.

**5. Conclusions and Further Research**

We consider a JRP in a multi-item probabilistic inventory model by employing direct and indirect grouping strategies. These strategies are crucial in inventory management especially when dealing with multi-item inventory and a single supplier. We have also developed and modify the algorithm to find the optimal solution for multi-item probabilistic inventory problem. Numerical examples based on the six items data of tire vulcanizing showed that the total inventory cost in the indirect grouping strategy is less than in the direct grouping strategy. Further direction of this research includes the possibility of using general distribution for the demand and finding another robust algorithm for direct and indirect grouping strategy.
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