The Custodial Symmetry

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Abstract

In the present work, we elucidate the meaning of the custodial symmetry and its importance at the phenomenological level in the framework of the standard model of the electroweak interactions and its possible extensions.

1 Introduction

The Standard Model (SM) of the strong and electroweak interactions which has the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group symmetry[1] has been quite successful. Its predictions are in excellent agreement with the experimental data. Moreover, the model has predicted new particles like the $Z$ gauge boson which is the quantum of the weak interaction with zero electric charge. It has also predicted the charm quark which was introduced to avoid the flavor changing neutral currents at tree level[2], and top quark. However, the SM has some problems. It does not unify the strong and electroweak interactions and has three coupling constants associated to each gauge group. Currently there are some theories which unify these interactions and they only contain one coupling constant and a simple Lie group; however, they predict a proton decay faster than the experimental limit. Some of these groups are: $SU(5)$, $SU(6)$, $SO(10)$, $E_6$ etc[3].

One of the most interesting problems of the SM is that there is no explanation of the origin of masses, mixing of the fermions and the number of families appears as an identical pattern, although there are some models which introduce horizontal symmetries to explain the relation of the fermion masses by spontaneous symmetry breaking (SSB). The physicists expect to find a symmetry which relates the fermion generations; the general aim is
to distinguish among them by breaking this symmetry and generating hierarchical mass scales that give different radiative corrections to the fermions. This symmetry would commute with the SM or a simple group that contains it. Some of the simple gauge groups that unify the SM with horizontal symmetries are: $SU(11)$, $SO(14)$, $SO(18)$, $E_8$, etc[4]. At the present time there is not a realistic model that can explain satisfactorily the problem of the fermion masses.

To give mass to the fermions and the gauge bosons in the SM, it is necessary to introduce a multiplet of scalar fields which has some components that are 'eaten' by the gauge bosons, the would-be Goldstone bosons, and another component which is called the Higgs particle. This is a physical degree of freedom, but the theory cannot predict its mass. There are some energy regions to look for the Higgs according to the production and decay channels. Up to now there is not evidence of it.

The left (right) quark/lepton fermions in the SM transform according to the global symmetry $SU(2)_L \otimes U(1)_Y$ and the Higgs doublet, four real fields, is a bidoublet under this global symmetry. Before the breaking of $SU(2)_L \otimes U(1)_Y$, the Higgs potential has a global $SU(2)_L \otimes SU(2)_R$ symmetry which reduces to $SU(2)_V$ when the symmetry is broken. This remanent global symmetry is called the "custodial symmetry"[5]. Additionally, it is possible to write down the Yukawa Lagrangian to see the same $SU(2)_L \times SU(2)_R$ global symmetry explicitly.

In the present work, we explain the meaning of the custodial symmetry specially when dealing with Higgs multiplets appearing in extensions of the SM. We also explain that if the gauge symmetry of the electroweak model is broken by the Higgs doublet, there is a custodial symmetry which protects the mass relation of the $W$ and $Z$ gauge fields, i.e.,

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta} = 1.$$  \hspace{1cm} (1)

This relation is also valid after the radiative corrections if the custodial symmetry is not broken. Experimentally this relation is satisfied at the 1% level, which restricts the new physics beyond the SM and allows us to distinguish among different models.

2 The gauge field masses

The kinetic energy parts of the Lagrangian of the scalar fields produce the mass of the gauge fields after the SSB. If we have an arbitrary scalar multi-
plet of $SU(2)_L \otimes U(1)_Y$, the gauge field masses arise from

$$D_\mu < \phi >_o \rightarrow i g \bar{A}_\mu \cdot \left[ \bar{T}, < \phi >_o \right] + i \frac{g'}{2} B_\mu \left[ Y, < \phi >_o \right]$$

where $< \phi >_o$ is the vacuum expectation value (v.e.v.) of the scalar multiplet, i.e., $< \phi >_o = < 0 | \phi | 0 >$. The covariant derivative for a general representation of $SU(2)_L \otimes U(1)_Y$ can be written as

$$D_\mu < \phi >_o \rightarrow i \frac{g}{\sqrt{2}} W^+_\mu \left[ T^+, < \phi >_o \right] + i \frac{g}{\sqrt{2}} W^-_\mu \left[ T^-, < \phi >_o \right] + i g A_\mu^3 \left[ T^3_L, < \phi >_o \right] + i g' B_\mu \left[ Y, < \phi >_o \right]. \quad (2)$$

where we have defined $W^\pm_\mu = (A^1_\mu \mp i A^2_\mu)/\sqrt{2}$.

The SM has a $U(1)_Q$ remanent symmetry and $< \phi >_o$ is different from zero because of the neutral component, so that we get

$$T^3_L < \phi >_o = -\frac{1}{2} Y < \phi >_o. \quad (3)$$

Using the Lie algebra, the mass terms of the gauge fields can be written as [6]

$$\left( \frac{g^2}{2} \left[ t(t+1) - t_L^2 \right] \left( A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu} \right) + t^2_L \left( g A_3^\mu - g' B_\mu \right)^2 \right) < \phi >^2_o,$$  

(4)

where $t(t+1)$ and $t_L$ are the eigenvalues of the $\sum_{i=1}^3 T^2_{iL}$ and $T^3_L$ operators, respectively. Under the assumption that $g' = 0$ or equivalently $\sin \theta_W = 0$, and that the $SU(2)_L$ gauge fields transform as a triplet of the $SU(2)$ global symmetry, and the kinetic energy Lagrangian is invariant under these transformation, the following relation is satisfied

$$t(t+1) - t^2_L = 2t^2_L. \quad (5)$$

This symmetry is only exact when the $B_\mu$ fields is zero because it breaks the custodial symmetry.

From the above relations and the definition of the mass terms

$$M^2_W = g^2 \left( t(t+1) - t^2_L \phi >^2_o,$$  

$$M^2_Z = 2 (g^2 + g'^2) t^2_L < \phi >^2_o,$$  

(6)
for the $\rho$ parameter we get

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{t(t+1) - t_{3L}^2}{2t_{3L}^2} \frac{g^2}{(g^2 + g'^2) \cos^2 \theta_W} = 1$$

(7)

where $\tan \theta_W = g'/g$ and $Z_\mu = \cos \theta_W A_{3\mu} - \sin \theta_W B_\mu$. Taking into account the eq.(5), we can get different values for $t$ when $\rho = 1$

$$t = \frac{1}{2} \left( -1 + \sqrt{1 + 12t_{3L}^2} \right)$$

$$= 0, \frac{1}{2}, 3, \ldots$$

(8)

The first and second solutions correspond to singlets and doublets, respectively. The other solutions are not important in the SM because they do not couple to the fermions in the Yukawa Lagrangian and they do not produce fermion mass terms. However, for other fermion representations, physics beyond the SM, Higgs fields with other values of $t$ are important to construct Yukawa terms. One interesting example is the see-saw mechanism implemented with new fermions and triplet Higgs fields. In the minimal SM with only left handed neutrino, it is neccesary a Higgs triplet to get a massive neutrino; however the vev of the new Higgs representation is restricted by the experimental value of the $\rho$ parameter. With the above arguments we can say that if we use doublets to break the SM, the $SU(2)_L$ gauge fields will be a triplet of the $SU(2)$ global symmetry or the custodial symmetry and $\rho$ is equal to one. Radiative corrections to the $\rho$ parameter by gauge fields violate softly this symmetry.

3 The Higgs and fermions Lagrangian

The most general Higgs potential which is renormalizable and invariant under $SU(2)_L \otimes U(1)_Y$ gauge transformations have the form

$$V = \lambda \left( \phi^\dagger \phi - \mu^2 \right)^2 ,$$

(9)

where

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} ,$$

4
The potential as function of the \( \phi_i \) scalar fields can be written as

\[
V = \frac{\lambda}{4} \left( \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - 2\mu^2 \right)^2.
\] (10)

It is invariant under rotations of the four fields which lead to \( SO(4) \) as the global symmetry group. This group is isomorphic to \( SU(2)_L \otimes SU(2)_R \) because both have the same Lie algebra. This symmetry is global and it does not necessary to introduce gauge fields.

When the symmetry is broken, the scalar field \( \phi_4 \) get a v.e.v different from zero, and it can be redefined as follows

\[
\phi_4 = H + v
\] (11)

where \( H \) gets its mass and is called the Higgs particle. Moreover, it has a v.e.v equal to zero. This field is a physical degree of freedom and its mass is proportional to the \( \lambda \) parameter which is unknown in the model. The other scalar fields remain massless. They are the would-be Goldstone bosons and correspond to the degrees that the gauge fields ‘eat’ in order to get mass or longitudinal component.

The Higgs potential can be written as a function of the new fields as follows

\[
V = \frac{\lambda}{4} \left( \phi_1^2 + \phi_2^2 + \phi_3^2 + H^2 + 2Hv + v^2 - 2\mu^2 \right)^2.
\] (12)

In this new potential the global symmetry is broken to \( SO(3) \), which only rotates three scalar fields. It is isomorphic to \( SU(2)_V \), the diagonal part of \( SU(2)_L \otimes SU(2)_R \). We can say that the symmetry was broken according to the following scheme

\[
SU(2)_L \otimes SU(2)_R \longrightarrow SU(2)_V.
\] (13)

To evidentiate the isomorphism between \( SO(4) \) and \( SU(2)_L \otimes SU(2)_R \), we will express the fields according to

\[
\phi' = U_L \phi U_R^\dagger,
\]

\[
\phi = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_4 - i\phi_3 & \phi_1 + i\phi_2 \\ -(\phi_1 - i\phi_2) & \phi_4 + i\phi_3 \end{bmatrix},
\] (14)

where \( U_{L(R)} \in SU(2)_{L(R)} \). The elements of the \( SU(2)_L \) group act over the columns and the elements of the \( SU(2)_R \) group act over the rows. These transformations leave invariant the following quadratic form

\[
Tr(\phi^\dagger \phi),
\] (15)
and, obviously, leave invariant the Higgs potential.

If \( \theta_L = \theta_R \) the group of transformations becomes \( SU(2)_V \), and if \( \theta_L = -\theta_R \) the group of transformations reduces to \( SU(2)_A \). Under the \( SU(2)_V \) infinitesimal transformations the fields of the multiplet transform as follows

\[
\phi' = e^{i\vec{\theta} \cdot \vec{T}} \phi e^{-i\vec{\theta} \cdot \vec{T}},
\]

\[
\delta \phi_i \simeq \epsilon_{ijk} \theta_j T_k, \quad i, j, k = 1, 2, 3,
\]

\[
\delta \phi_4 = \delta H = 0.
\] (16)

After the symmetry breaking, the quadratic form still invariant under \( SU(2)_V \) is

\[
\phi_1^2 + \phi_2^2 + \phi_3^2 = \phi^+ \phi^- + \phi^- \phi^+ + \phi_3^2,
\] (17)

The interactions between fermions and the Higgs fields are given by the Yukawa Lagrangian. It also gives mass to the fermions when the vacuum is aligned. The renormalizable Yukawa Lagrangian is giving by

\[
L_Y = h_u Q_L \bar{\tilde{\phi}} u_R + h_d Q_L \phi d_R + h.c.,
\] (18)

where \( Q_L^T = (u_L \ d_L) \), \( m_u = h_u < \phi_o > \) and \( m_d = h_d < \phi_o > \). If we assume that the quarks have the same masses, \( h_u = h_d \), then the Yukawa Lagrangian can be written as

\[
h( \bar{u}_L \ \bar{d}_L ) \begin{bmatrix}
\phi_4 - i\phi_3 & \phi_1 + i\phi_2 \\
-(\phi_1 - i\phi_2) & \phi_4 + i\phi_3
\end{bmatrix} \begin{bmatrix}
u_R \\
d_R
\end{bmatrix} + h.c..
\]

To get \( L_Y \) invariant under the global \( SU(2)_L \otimes SU(2)_R \) symmetry, the quarks have to transform according to

\[
Q_L^{(R)} = U^{(R)} \cdot Q_L^{(R)}.
\] (19)

After SSB the quarks get mass and the global symmetry is broken down to \( SU(2)_V \). The mass terms are given by

\[
L_m = h(\bar{u}_R \ u_L + \bar{d}_R \ d_L) + h.c.,
\] (20)

which are invariant under \( U_L = U_R = SU(2)_V \), i.e., we have the isospin symmetry. It is the same custodial symmetry.
4 Custodial symmetry and radiative corrections

Considering the contribution of the first doublet of fermions \((u, d)\) to the radiative corrections of the \(\rho\) parameter, we have

\[
\Delta \rho = \Pi_{ZZ}(0) - \Pi_{WW}(0),
\]

\[
\Pi_{ZZ}(0) = -\frac{3G_F}{8\sqrt{2\pi}^2} (2m_u^2 \log m_u^2 - 2m_d^2 \log m_d^2),
\]

\[
\Pi_{WW}(0) = -\frac{3G_F}{8\sqrt{2\pi}^2} \left( m_u^2 + m_d^2 + \frac{2m_u^4 \log m_u^2 - 2m_d^4 \log m_d^2}{m_u^2 - m_d^2} \right),
\]

where \(\Pi_{ii}(0) = \sum_{ii}(0)/M_i^2\) and \(\sum_{ii}(0)\) is the diagonal contribution to the self energy. Summarizing all terms yielded \([7]\) we get

\[
\Delta \rho = \frac{3G_F}{8\sqrt{2\pi}^2} F(m_u, m_d),
\]

\[
F(m_u, m_d) = m_u^2 + m_d^2 - \frac{2m_u^2 m_d^2}{m_u^2 - m_d^2} \log \frac{m_u^2}{m_d^2}.
\]

In the limit \(m_u = m_d\), i.e., when the isospin symmetry or custodial symmetry is valid the function \(F(m_u, m_d)\) is equal to zero. The radiative corrections to \(\rho\) are different from zero when the custodial symmetry is broken. For example, the third family of the SM where the up and down quarks are replaced by the top and the bottom quarks, respectively, the global symmetry is broken because \(m_t \gg m_b\) and the correction to \(\rho\) is proportional to \(m_t^2\)

\[
\Delta \rho = \frac{3G_F m_t^2}{8\sqrt{2\pi}^2}.
\]

5 Conclusions

When the SM symmetry is broken by Higgs doublets, there is a global symmetry that protects the mass relation of the \(W\) and \(Z\) fields, which transforms as the components of a triplet. This relation is violated when this symmetry is not exact. If the isospin symmetry or custodial symmetry is broken, then \(\rho\) parameter may get radiative corrections different from zero. To look for new physics beyond the SM it is important to know if this symmetry is still exact.

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References

[1] S. Glashow, Nucl. Phys. 22 (1961) 579; S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; A. Salam, in Elementary Particle theory (nobel Symposium No. 8), edited by N. Svartholm (Almquist and Wiksell, Stockholm, 1968).

[2] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. rev. D 2 (1970) 1285.

[3] H. Fritzsch and P. Minkowsky, Ann. Phys. 93 (1975) 193; P. Langacker, Phys. Rep. C 72 (1981) 1; H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438; R. W. Robinet and J. Rosner, Phys. Rev. D 30 (1984) 1470.

[4] H. Georgi, Nucl. Phys. B 156 (1979) 126; F. Wilczek and A. Zee, Phys. Rev. D 25 1982 553.

[5] M. Veltman, Nucl. Phys. 123 (1977) 89; P. Sikivie et. al. Nucl. Phys. B 173 (1980) 189; M. S. Chanowitz et. al. Phys. Lett. B 78 (1980) 28.

[6] T. P. Cheng and L. F. Li, Gauge Theory of Elementary Particle Physics. New York: Oxford University Press, 1984.

[7] M. B. Einborn, D. Jones and M. Veltman, Nucl. Phys. B 191 (1981) 146.