Anatomy of a Bounce

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Abstract

Holographic considerations are used in the scrutiny of a special class of brane-world cosmologies. Inherently to this class, the brane typically bounces, at a finite size, as a consequence of a charged black hole in the bulk. Whereas a prior treatment [1] emphasized a brane that is void of standard-model matter, the analysis is now extended to include an intrinsic (radiation-dominated) matter source. An interesting feature of this generalized model is that a bounce is no longer guaranteed but, rather, depends on the initial conditions. Ultimately, we demonstrate that compliance with an appropriate holographic bound is a sufficient prerequisite for a bounce to occur.

I. INTRODUCTION

In recent years, there has been dramatic progress in our understanding of cosmology. Nonetheless, many questions remain unanswered and this will, in all likelihood, continue to be the case until a fundamental theory of quantum gravity can be realized. (For some relevant discussion, see [2].) For instance, the very early stages of the universe, where transplanckian effects must certainly be accounted for, can not yet properly be addressed.

One of the more notorious issues of the transplanckian universe is that, from at least a classical perspective, a big bang/crunch type of singularity seems to be an inevitable feature of most cosmological models. It is often hoped that some (yet unknown) effects of quantum gravity can smooth out this singular behavior. An alternative means of circumvention is the notion of a universe that smoothly “bounces” (rather than collapsing) at some finite value of the cosmological scale factor. Recent directions along this line have often employed a brane-world setting [3]: colliding branes can be used to effectively describe the so-called cyclic theories (e.g., [4]), and a bouncing brane world can be induced by, for instance, a suitably compactified string-theory background [5], more than one large extra dimension [6] and an electrostatic charge in the bulk [7]. (For a more generic discussion on bounce cosmologies and earlier references, see [8].)

In spite of the obvious virtues of a bounce cosmology, there is (at least) one significant concern that must be addressed in such scenarios. Namely, bounce cosmologies are generally not possible unless the null energy condition (which, given causality, requires non-negative
energy densities [9]) has somewhere been violated [8].\(^1\) Hence, the pertinent question be-
comes whether such a violation should necessarily be regarded as a catastrophic event. On
one hand, violations of the null condition are known to be commonplace at the quantum
level and can even occur in purely classical circumstances [10]. On the other hand, it is
pretty much accepted that there must be some modest limit to the degree of violation (e.g.,
[11]), although it is often quite difficult to express this limit in quantitative terms.

In view of the above discussion, an important issue becomes the discrimination of \textit{exotic}
cosmologies (\textit{i.e.}, cosmologies containing negative-energy matter). That is to say, we require,
ideally speaking, some means of testing the validity of an exotic spacetime, without resorting
to the rather dubious energy conditions of general relativity [10]. Notably, the holographic
paradigm [12–14] can serve (and has served) nicely in the proposed manner [15,16]. To
briefly elaborate, the holographic principle provides a natural bound on the amount of
information and, hence, entropy that can be stored in a given region of spacetime (When
this “holographic limit” is saturated, a black hole can be expected to form.) It stands to
reason that any cosmology that leads to a holographic violation (either directly by exceeding
an upper bound on the entropy or indirectly by contradicting a manifestation of the principle)
can \textit{not} be viewed as a realistic one.

Although the holographic principle is, in a sense, conceptually quite simple, its appli-
cation in a given context can often be far from straightforward. This has especially been
the case in cosmological situations, where there has been considerable debate as to how the
principle should best be utilized [17–21] (also, [14] and references therein). Such matters
are further complicated for exotic cosmologies, as it is clear (intuitively speaking) that all
entropy bounds will inevitably be threatened by negative-enough energies. A formulation
of the principle that is especially well-suited for generic (including exotic!) cosmological
scenarios is the \textit{causal entropy bound} [22,23,15]. We will elaborate on the logistics of this
bound at an appropriate juncture in the paper.

Not long ago, the current author used the causal entropy bound to test the holographic
viability of a certain class of exotic cosmologies [1]. More specifically, brane-world cosmolo-
gies (\textit{a la} Randall and Sundrum [24]) such that a four-dimensional brane moves through
the five-dimensional anti-de Sitter background of a static and charged (“Reissner-Nordstrom-
like”) black hole.\(^2\) The effective cosmology of the brane (which is regarded as “our universe”)
can be shown to translate into a Friedmann-Robertson-Walker (FRW) universe with \textit{holographically induced} matter.\(^3\) Along with a vacuum energy, this induced matter includes both
radiation and exotic stiff matter; with the latter being directly related to the bulk charge
and responsible for the occurrence of a non-singular bounce. It was ultimately shown that
such cosmologies are indeed holographically viable (\textit{i.e.}, satisfy the causal entropy bound

\(^{1}\)We say “generally” because this need not be the case for a universe with closed spacelike slices.

\(^{2}\)For other studies on brane cosmologies with a charged black hole bulk, see [25–32,7,33–36].

\(^{3}\)This matter is “holographic” in the sense that it is essentially a projection of the bulk geometry
onto the lower-dimensional brane. This notion of holography should not be confused with the
holographic principle \textit{per se}. 

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at the bounce\(^4\)) provided that the bulk charge satisfies a *lower* bound. This is, perhaps, a counter-intuitive outcome but can be attributed to the exotic energy density (as calculated at the bounce) going as an inverse power of the charge.

In the prior work of note \([1]\), the brane was, for the sake of simplicity, regarded as being empty *modulo* the holographically induced matter. The current paper will, however, extend considerations to the more realistic case of a brane that also contains an intrinsic matter source; presumably, the contributions from the standard model. This may seem, at first glance, to be a somewhat trivial extension. Nevertheless, it is well known (see, *e.g.*, \([37]\)) that such an inclusion leads to an unorthodox energy-density-squared term in the brane-world cosmological equation. As an immediate consequence of this “addition”, there is now a distinct possibility that the brane universe no longer bounces. (Note that a bounce is an inevitable feature of the empty-brane version, as long as the bulk charge is non-vanishing \([7,33]\).) In fact, it has been recently shown that, for a radiation-dominated brane universe (which is also the interest of the current paper), the bounce can only occur if the bulk charge satisfies a finite lower bound \([36]\). We will eventually be able to show that this particular bound has a clear holographic interpretation.

Before proceeding, we should point out that a recent paper \([36]\) (also see \([7]\)) has also considered this same brane-world model when it contains standard-model (radiation-dominated) matter. Nonetheless, much of the emphasis has been, up to now, on the phenomenological viability, whereas the current treatment stresses the holographic implications. For further discussion on holography in the context of brane worlds, see \([38]\).

The rest of the paper is organized as follows. The next section discusses the forementioned brane-world cosmologies, with special attention paid to the solution near the bounce (which is the most interesting region from a holographic perspective). In Section III, after a brief explanation of the causal entropy bound, we consider the implications of this holographic bound on the cosmologies of interest. The final section contains a summary.

### II. BRANE-WORLD COSMOLOGIES WITH RADIATIVE MATTER

As in our prior related work \([1]\), the model of interest is a certain class of brane-world cosmologies for which a non-singular bounce is known to occur \([7,33]\). However, unlike in the preceding study, we will now incorporate the effects of intrinsic brane matter; that is, a “non-holographic” source that lives strictly on the brane (*i.e.*, standard-model matter). This inclusion is, of course, necessary for a physically realistic treatment of the problem. It should, however, be kept in mind that the presence of intrinsic brane matter can potentially jeopardize the existence of the bounce.

To be precise, let us consider the scenario of a 3+1-dimensional brane (possibly but not necessarily curved\(^5\)) moving in an (otherwise static) 4+1-dimensional anti-de Sitter

\(^4\)The bounce hypersurface - that is, the spacelike surface for which the cosmological scale factor reaches its minimal size - is precisely where such a cosmology would be most susceptible to a holographic violation. This point will be clarified later in the paper.

\(^5\)Empirical evidence suggests that our universe and, hence, the brane should be positively curved
bulk spacetime. Without loss of generality, the geometry of this bulk can be described by an anti-de Sitter black hole with a constant-curvature horizon and an electrostatic charge [26]. (Such black holes may be regarded as Reissner-Nordstrom-like but having an arbitrary horizon topology [40].) It is, in particular, the presence of a non-vanishing charge that can induce the desirable feature of a non-singular bounce. We will therefore assume, for the duration, that a charge is always present and, consequently, a bounce will generally occur (and will always occur if the brane is empty).

Further commentary on these bouncing brane cosmologies can be found in [7,33,34,1,36]. Typically, after a suitable tuning of the (effective) brane cosmological constant, one obtains a universe that, far from the bounce, tends asymptotically toward de Sitter space. However, as our current interest is near the bounce, such details, although important in their own right, are not of relevance to the current analysis.

As is well documented, this type of moving-brane scenario leads to an effective cosmology that resembles a FRW universe [42]. (Keep in mind that an observer on the brane perceives the motion through the bulk as either a cosmological expansion or contraction.) In general terms, it is the nature of the bulk solution that determines what types of matter will be holographically induced on the brane world. Furthermore, the current analysis will, as alluded to above, also allow for matter that lives strictly on the brane.

We will now briefly outline the steps that lead up to the cosmological equation of motion. More rigorous discussions can be found in (e.g.) [42–44,38,33,34] and references therein.

It is, first of all, necessary to explicitly formalize the bulk solution; in this case, a five-dimensional anti-de Sitter spacetime with a Reissner-Nordstrom-like geometry. That is,

\[ ds^2_5 = -f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2_{k,3}, \]

where

\[ f(r) = \frac{r^2}{L^2} + k - \frac{\omega M}{r^2} + \frac{3\omega^2 Q^2}{16r^4}, \]

\[ \omega \equiv \frac{16\pi G_5}{3V_3}, \]

and where the following definitions have been employed: \( L \) is the curvature radius of the anti-de Sitter bulk, \( G_5 \) is the five-dimensional Newton constant (which will be related to the usual four-dimensional constant, \( G_4 \), later on) and \( V_3 \) is the dimensionless volume element [39]. Nonetheless, the distinction between a flat and “de Sitter” brane is inconsequential to the regime of current interest; namely, near the bounce.

\(^6\)Even if such a black hole is classically neutral, there are compelling reasons to believe that quantum fluctuations would still induce an effective charge of significant magnitude [41].

\(^7\)Note that, here and throughout, all fundamental constants, besides \( G_4 \) and \( G_5 \), are set equal to unity.
associated with $d\Omega^2_{k,3}$ (a three-dimensional spacelike hypersurface of constant curvature). Also in evidence are three constants of integration, $k$, $M$ and $Q$, which have the following interpretations: (i) $k$ is a discrete parameter that describes the horizon topology, whereby $k = +1$, 0 and -1 for a spherical, flat and hyperbolic geometry (respectively), (ii) $M$ is the conserved mass of the bulk black hole and, for our purposes, can be regarded as a strictly positive quantity (however, see [40]), (iii) $Q$ is the electrostatic charge of the black hole, which - assuming the existence of a pair of positive and real horizons (i.e., assuming cosmic censorship) - must be bounded in accordance with $Q^2 < 4M^2/3$. For this reason, we will usually work with the following dimensionless measure of charge:

$$\epsilon^2 \equiv 3Q^2/4M^2 < 1.$$  

\begin{equation}
\end{equation}

The next topic, on the agenda, is the dynamical behavior of the brane world. After some suitable identifications, it can be shown that the induced metric on the brane takes on a FRW form,

$$ds^2 = -dt^2 + a^2(t)d\Omega^2_{k,3},$$  \hspace{3cm} (5)

where $t$ is the physical time as measured by a brane observer and $a \equiv r(t)$ is the cosmological scale factor. Note that a dot will always denote a differentiation with respect to $t$.

After further manipulations, the corresponding Friedmann-like equation of motion is found to be as follows:

$$H^2 = \frac{1}{L^2} - \frac{k}{a^2} + \frac{\omega M}{a^4} - \frac{\omega^2 M^2 \epsilon^2}{4a^6} + \left(\frac{8\pi G_5}{3}\right)^2 \rho_{br}^2,$$  \hspace{3cm} (6)

where $H \equiv \dot{a}/a$ is the Hubble parameter (constant to some) and $\rho_{br}$ is the total energy density of any matter living on the brane, including the contribution from the brane tension. (Note that $\rho_{br}$ enters the equation of motion by application of the well-known Israel junction conditions [45]).

It is convenient to make a clear distinction between the vacuum-energy contribution from the tension and the "conventional" matter sources on the brane. Hence, let us write $\rho_{br} = \rho_0 + \rho_m$, where $\rho_0$ is the vacuum energy density and $\rho_m$ is the remainder. It follows that the cosmological equation (6) can be recast in the following manner:

$$H^2 = \frac{\Lambda_4}{3} - \frac{k}{a^2} + \frac{8\pi G_4}{3}\rho_m + \frac{\omega M}{a^4} - \frac{\omega^2 M^2 \epsilon^2}{4a^6} + \frac{4\pi G_4}{3\rho_0} \rho_m^2,$$  \hspace{3cm} (7)

where we have identified an effective cosmological constant on the brane,

$$\Lambda_4 \equiv 3 \left[ \left(\frac{8\pi G_5}{3}\right)^2 \rho_0^2 - \frac{1}{L^2} \right],$$  \hspace{3cm} (8)

and used standard expectations to relate the bulk and brane-world gravitational constants,

$$\frac{8\pi G_4}{3} \equiv 2 \left(\frac{8\pi G_5}{3}\right) \rho_0.$$  \hspace{3cm} (9)

Note that, for the very special case of a flat or critical brane, one must finely tune $\Lambda_4$ to be a vanishing quantity. Otherwise, in general, one obtains a non-critical brane.
Remarkably, Eq. (7) is, up to three extra terms, just the standard (four-dimensional) Friedmann equation for arbitrary matter and curvature. Meanwhile, the three extra terms include a holographic radiative-matter contribution (the $a^{-4}$ term), a holographic stiff-matter source (the $a^{-6}$ term) and a quadratic energy-density term. The quadratic term, although quite unorthodox, happens to be a typical feature of many brane-world cosmologies [37]. Let us also take note of the negative-energy stiff matter. It is this exotic source - which is a direct manifestation of the bulk charge - that can create a significant enough repulsive force (when the universe is small) so that a singular collapse may be avoided.

Let us now, for the sake of definiteness, specialize to brane matter that is dominated by radiation; hence, $\rho_m \sim a^{-4}$. The reason for this choice is twofold. Firstly, from a phenomenological standpoint, the early stages of the observable universe are certainly radiation dominated; suggesting that radiation should similarly be the dominant form in a near-bounce regime. Secondly, in view of our holographic interest, radiation is clearly the most entropic form of conventional matter and, therefore, would provide the sternest test for any proposed entropy bound.

With the above in mind, let us adopt the notation

$$\rho_m = \rho_r = \frac{\hat{\rho}_r}{a^4}$$

and then appropriately re-express the cosmological equation (7) as follows:

$$H^2 = \frac{\Lambda_4}{3} - \frac{k}{a^2} + \left[ \frac{8\pi G_4}{3} \frac{\hat{\rho}_r + \omega M}{a^4} \right] - \frac{\omega^2 M^2 c^2}{4 a^6} + \frac{4\pi G_4 a^2}{3a^8}.$$  

In this form, one can clearly see that the problematic terms from a phenomenological viewpoint - namely, the exotic stiff matter ($\sim a^{-6}$) and the unorthodox quadratic term ($\sim a^{-8}$) - will rapidly dilute with the expansion of the brane universe. Hence, this brane-world model remains quite plausible in spite of the conspicuous deviations from the standard picture.

An analytic solution for the above equation is not generally obtainable. Nonetheless, our current interest is in the near-bounce solution, which is indeed extractable by way of some appropriate simplifications. More specifically, since the scale factor attains its minimal value at the bounce, we can safely disregard the constant vacuum ($\Lambda_4/3$) and “near-constant” curvature ($k/a^2$) terms relative to the other contributors. After defining

$$A \equiv \frac{8\pi G_4}{3} \left[ \frac{\hat{\rho}_r}{8\pi G_4} + \omega M \right],$$

$$B \equiv \frac{\omega^2 M^2 c^2}{4}$$

and

Note that this brane universe can be open, closed or flat (respectively, $k = -1, +1$ or 0) depending on the horizon topology of the bulk solution.
\[ C \equiv \frac{4\pi G_4}{3\rho_0} \hat{\rho}_r^2 \]  

(14)

(all of which are manifestly positive), we are then left with a comparatively simple expression,

\[ H^2 = \frac{A}{a^4} - \frac{B}{a^6} + \frac{C}{a^8} . \]  

(15)

With the assumption that a bounce does indeed take place (that is, \( H \) vanishes for some finite value of \( a \)), the above equation is readily solvable, as recently demonstrated in [36]. Following the cited paper, we first introduce a pair of new variables, \( x \equiv a^2 \) and \( dt \equiv a^2 d\eta \), and then inspect the resultant form,

\[ \frac{1}{4} (x')^2 = Ax^2 - Bx + C \]  

(16)

(with a prime indicating a differentiation with respect to \( \eta \)). The solution is then expressible as

\[ a^2 = x = \frac{B}{2A} + \sqrt{\Delta} \cosh \left[ 2\sqrt{A}\eta \right] \]  

(17)

and

\[ t = \frac{B}{2A} \eta + \sqrt{\frac{\Delta}{4A}} \sinh \left[ 2\sqrt{A}\eta \right] , \]  

(18)

with

\[ \Delta \equiv \left( \frac{B}{2A} \right)^2 - \frac{C}{A} > 0 . \]  

(19)

Here, the (implied) integration constants have been conveniently fixed so that \( t \) and \( \eta \) both vanish at the bounce; that is, when \( a^2 = a_{\text{min}}^2 \equiv \frac{B}{2A} + \sqrt{\Delta} \).

The inequality in the above equation (19) is precisely what we obtained by constraining \( H \) to vanish at finite \( a \). This important condition can also be expressed as a lower bound on the charge \( (\epsilon^2 \sim Q^2) \),

\[ \left( \frac{\omega M \epsilon}{2} \right)^4 > 2\alpha \frac{\hat{\rho}_r^2}{\rho_0} \left( \frac{8\pi G_4}{3} \right)^2 , \]  

(20)

where,

\[ \alpha \equiv 1 + \frac{3\omega M}{8\pi G_4 \hat{\rho}_r} . \]  

(21)

Note that constraints from nucleosynthesis [46,47] naturally limit the energy density of holographic radiation such that, at the very most, \( \omega M / G_4 \sim \mathcal{O}[\rho_r] \); and so \( \alpha \) can safely be regarded as a constant of the order unity.

It should be emphasized that the above constraint (20), along with the upper bound \( \epsilon^2 < 1 \) (cf, Eq.(4)), severely restricts the allowed range of values for the bulk charge. This
is somewhat different than the case of an empty (modulo induced matter) brane, in which there is, a priori, only an upper bound on the charge. That is, for an empty brane, a bounce will be obtained for any non-vanishing value of the charge, regardless of how small \([7,33]\). (Note that the difference between the two scenarios can be attributed to the presence, or lack thereof, of a positive term in the Friedmann equation that goes as \(a^{-8}\).) It is, however, interesting that, even for an empty brane, holographic considerations still imply the necessity for a finite lower bound on the charge \([1]\). In the next section, we will find out what the holographic principle can tell us about the current (intrinsic matter) case.

III. CAUSAL ENTROPY BOUND AT THE BOUNCE

In this section, we will test the holographic viability of our bounce cosmologies by way of the causal entropy bound \([22,23,15]\). Firstly, a brief explanation of this holographic bound is in order. (For a more detailed account as relevant to the present context, see \([1]\).)

The causal entropy bound can best be viewed as a covariant generalization of its predecessor, the Hubble entropy bound \([18]\) (also, \([19–21]\)). The Hubble bound, itself, follows from a pair of intuitive notions: (i) the entropy is maximized, in a given region of space, by filling up the volume with maximal-sized black holes and (ii) in a cosmological background, the maximal size that can be achieved by a stable black hole is, roughly, dictated by the Hubble horizon (i.e., \(R_s \sim H^{-1}\), where \(R_s\) is the maximal Schwarzschild radius).

Given the above considerations and the black hole area law \([48,49]\), the following bound can be deduced on the entropy, \(S\), contained in a region of volume \(V\):

\[
S < \frac{V}{H^{-3}} \times \frac{H^{-2}}{G_4} = \frac{V H}{G_4},
\]

(22)

where a four-dimensional spacetime has been assumed and numerical factors of the order unity have been ignored.

The premise of the causal entropy bound is to replace the Hubble horizon with a “causal connection scale”, \(R_{CC}\), that can be interpreted as the length scale above which spacetime perturbations are causally disconnected. (Presumably, a black hole could not maintain its stability over any greater distance than this.) Although a highly technical process (see \([22]\) for the gory details), the scale \(R_{CC}\) can, as it so happens, be expressed in an explicitly covariant form. For our purposes, however, it is sufficient to consider the expression as appropriate for a spacelike slice of a FRW spacetime \([22]\):

\[
R_{CC}^2 = \text{Max} \left[ \dot{H} + 2H^2 + \frac{k}{a^2}, \quad -\dot{H} + \frac{k}{a^2} \right].
\]

(23)

It is reassuring that, for a slowly evolving and flat spacetime, one obtains the intuitive expectation, \(R_{CC} \sim H^{-1}\).

For future reference, the causal entropy bound (in four dimensions) takes on the form

\[
S < S_{CB} \equiv \beta \frac{VR_{CC}^{-1}}{G_4},
\]

(24)

where \(\beta\) is a “fudge factor” of the order unity (reflecting any neglected constants and the inherent ambiguity in bounds of this nature). There is significant evidence that \(S_{CB}\) does,
indeed, serve as a true holographic bound for physical spacetimes. (See [22,23,15,1] for an elaboration.) Moreover, the formulation of the causal bound does not assume, a priori, any of the energy conditions of general relativity.\(^9\) In this sense, the causal bound can play a particularly useful role in the holographic discrimination of exotic cosmologies.

As advertised, we will now proceed to test, via the above entropy bound (24), the exotic bounce cosmologies of Section II. Let us first point out that the causal bound is known to persist, automatically, for any spacetime that contains only non-exotic, causal matter at non-Planckian temperatures [15]. Now consider that, for our bounce cosmologies in particular, both the exotic stiff matter and the (presumably acausal) quadratic-density term are rapidly diluted by the spacetime expansion. Hence, for current purposes, it will be sufficient to focus on the spacelike surface that describes the bounce \((t = \eta = 0)\).

As an initial step, let us calculate \(R_{CC}\) by way of Eq.(23). Since \(H = 0\) at the bounce and the curvature term can be neglected, we have

\[
R_{CC}^{-2} = |\dot{H}| \quad \text{at} \quad t = \eta = 0 .
\]

We can calculate \(\dot{H}\) by differentiating Eq.(15). Also employing Eq.(17) for \(a^2\) and Eq.(19) for \(\Delta\), we obtain

\[
R_{CC}^{-2} = \frac{A}{a^4} \left| 1 - \frac{C}{Aa^4} \right| \quad \text{at} \quad t = \eta = 0 .
\]

It will prove to be convenient if the scale factor is re-expressed in terms of the “total” energy density of radiation,

\[
\rho_R \equiv \frac{1}{a^4} \left[ \dot{\rho}_r + \frac{3}{8\pi G_4} \omega M \right] ,
\]

from which it follows that (cf, Eqs.(12,21))

\[
\rho_R = \frac{\alpha}{a^4} \dot{\rho}_r = \frac{3}{8\pi G_4} \frac{A}{a^4} ,
\]

and so,

\[
R_{CC}^{-2} = \frac{8\pi G_4}{3} \rho_R \left[ 1 - \frac{1}{2\alpha^2} \frac{\rho_R}{\rho_0} \right] \quad \text{at} \quad t = \eta = 0 ,
\]

where we have also applied Eq.(14).

Next, we will evaluate the entropy contained in this spacelike bounce surface. As the entropy of radiation can be expected to dominate over any other type of matter, it should be sufficient to consider the contribution from just the radiative sources. To further simplify matters, let us assume that the effective number of particle species is roughly the same for both the standard-model and holographic forms of radiation. (As can be seen in the subsequent analysis, any discrepancy of \(10^4\) or smaller is inconsequential up to factors of the

\(^9\)For an elaboration on the energy conditions in a brane-world context, see [38].
order unity.) In this way, we can define both a total entropy, \( S_R \), and entropy density, \( s_R \), and be able to relate these to \( \rho_R \) (27) in a straightforward manner.

It is helpful, at this stage, to recall the Stephan-Boltzmann thermodynamic relations for radiative matter (in thermal equilibrium at temperature \( T \)):

\[
s_R = \frac{S_R}{V} = \mathcal{N}T^3 ,
\]

\[
\rho_R = \mathcal{N}T^4 ,
\]

where the missing numerical factors (obviously of the order unity) have been absorbed into the effective number of particle species, \( \mathcal{N} \). Eliminating \( T \) from this pair of equations, we find that

\[
S_R = V\mathcal{N}^{1/4}\rho_R^{3/4} .
\]

Substituting the above outcome into the left-hand side of Eq.(24) and our prior result for \( R_{CC} \) (29) into the right-hand side, we are able to establish the following inequality (up to overall factors of the order unity\(^\text{10}\))

\[
G_4\sqrt{\rho_R} < \left| 1 - \frac{1}{2\alpha^2} \frac{\rho_R}{\rho_0} \right| \quad \text{at} \quad t = \eta = 0 .
\]

Hence, we have obtained, by virtue of the holographic principle, an upper bound on the energy density of radiation at the bounce. What may not be immediately clear is the existence of a lower bound as well. To elaborate, if we regard the unscaled charge \((Q^2)\) as a fixed quantity, then \( M \) (and, hence, the total radiation density via Eq.(27)) must remain sufficiently large to ensure that the dimensionless charge \((\epsilon^2)\) is always less than unity; \( \text{cf} \), Eq.(4). Moreover, phenomenologically speaking, any lower bound on \( M \) will translate directly into a lower bound on \( \rho_r \) (i.e., the energy density of standard-model radiation); inasmuch as nucleosynthesis considerations must limit the relative contribution of the holographic radiation [46,47]. (Alternatively, \( \rho_R = \alpha\rho_r \), where compliance with observations suggests that \( \alpha \sim \mathcal{O}(1) \).) Putting this altogether, we see that holographic considerations help to significantly constrain the allowable values of \( \rho_r \).

To get a better feel for the derived bound (33), it is instructive to consider the “natural” limiting case, \( \rho_R/\rho_0 << 1 \). (That is to say, it is quite natural to take, at the very least, \( \rho_R/\rho_0 < 1 \); as this ensures that the quadratic energy-density term is subdominant to the linear term at the time of nucleosynthesis [7,36].) In this limit, the situation essentially simplifies to the one studied in our prior paper [1] (except that the model in [1] contained purely holographic radiation) and one can show that

\(^{10}\text{Here, we are assuming that } \mathcal{N} \text{ is not significantly greater than } 10^4, \text{ which would seem to be a reasonable enough constraint. This number also agrees, coincidentally, with an upper limit that ensures the stability of a Minkowski vacuum [50].}\)
\[ \epsilon^2 > \frac{G_4^4 [\alpha \dot{\rho}_r]^{3/2}}{\omega^2 M^2} > \left[ \alpha \dot{\rho}_r \right]^{-1/2}. \]  

(As usual, up to numerical factors and also note that the second inequality assumes the induced radiation to be the subdominant source.) Which is to say, the upper bound on the energy density of radiation can also be interpreted as a lower bound on the bulk charge. Interestingly, a lower bound on the charge is also necessitated by the existence of a bounce; cf. Eq.(20).

The deep implications of the holographic principle on these brane worlds can be further illuminated in the following way. First of all, let us rewrite our prior inequality (33) in terms of the parameters \( A \) and \( C \) (12,14):

\[ G_4^4 A < \left[ \frac{c_A}{a^2 A} \right]^2 \text{ at } t = \eta = 0, \tag{35} \]

where we have also utilized Eq.(28) for \( \rho_R \), and note that, although overall numerical factors have been neglected, the expression inside the square brackets is exact.

Next, we can apply the relation for \( a^2 \) at the bounce (cf, Eq.(17)) to obtain (up to overall factors)

\[ G_4^4 A < \frac{1}{A^2} \left[ \frac{(B^2)^2 + AB\sqrt{\Delta} + A^2 \Delta - AC}{B + 2A\sqrt{\Delta}} \right]^2. \tag{36} \]

Finally, the definition of \( \Delta \) (19) (and some simplification) can be employed to yield a remarkably concise form,

\[ G_4^4 A < \Delta. \tag{37} \]

Since \( A \) is a manifestly positive quantity, one has, as an immediate consequence of the holographic paradigm, that \( \Delta \) must be strictly positive. However, this is just the constraint (19) that we previously *assumed* so as to ensure that a bounce does indeed take place. That is to say, when a brane is moving in the background of a charged black hole, then compliance with the causal entropy bound is a sufficient prerequisite for the existence of a bounce.

**IV. CONCLUSION**

To summarize, we have been considering the implications of holography on a certain class of brane-world models; more specifically, a four-dimensional brane world moving in the five-dimensional anti-de Sitter background of a charged black hole. Such models are of significant interest because they allow for the possibility of a non-singular bounce (as opposed to a big bang/crunch), although at the expense of (holographically induced) exotic matter.

In a previous related paper [1], such cosmologies were studied, from a holographic perspective, for the very special case of a brane that is void of any intrinsic matter sources.
Meanwhile, in the current treatment, we took a step in the direction of realism and generalized considerations to a brane that does indeed contain standard-model matter. In particular, radiative matter was incorporated, as this form complies with empirical expectations (for the small-scale universe) and provides the most challenging test for any holographic bound.

For the purposes of testing holographic viability, we called upon the so-called causal entropy bound [22]. Significantly, this holographic bound does not assume, \textit{a priori}, any of the usual energy conditions, and so is particularly well suited for the discrimination of exotic cosmologies. Ultimately, we found that the causal bound implies a lower limit on the (bulk) black hole charge or, equivalently, an upper limit on the energy density of the radiative matter (including both standard-model and holographically induced contributions). Moreover, we have demonstrated that compliance with the causal bound is a sufficient condition for a brane universe to avoid a singular collapse.

Finally, it is interesting to recall our earlier related study [1] in light of these new findings. Even when the brane is void of intrinsic matter, the causal bound implies a finite lower limit on the magnitude of the bulk charge. However, somewhat paradoxically, a bounce is always realized in this (empty-brane) class of models. That is to say, the “motivation” for a lower bound on the bulk charge seems to be missing in the empty-brane scenario. It may be of interest to see if similar patterns of behavior show up in other types of bounce cosmologies.

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