Indirect Collider Signals for Extra Dimensions

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Abstract

A recent suggestion that quantum gravity may become strong near the weak scale has several testable consequences. In addition to probing for the new large (submillimeter) extra dimensions associated with these theories via gravitational experiments, one could search for the Kaluza Klein towers of massive gravitons which are predicted in these models and which can interact with the fields of the Standard Model. Here we examine the indirect effects of these massive gravitons being exchanged in fermion pair production in $e^+e^-$ annihilation and Drell-Yan production at hadron colliders. In the latter case, we examine a novel feature of this theory, which is the contribution of gluon gluon initiated processes to lepton pair production. We find that these processes provide strong bounds, up to several TeV, on the string scale which are essentially independent of the number of extra dimensions. In addition, we analyze the angular distributions for fermion pair production with spin-2 graviton exchanges and demonstrate that they provide a smoking gun signal for low-scale quantum gravity which cannot be mimicked by other new physics scenarios.

*Work supported by the Department of Energy, Contract DE-AC03-76SF00515
It has recently been suggested\[1\] that the hierarchy problem, \textit{i.e.}, the smallness of the ratio of the weak scale to the Planck scale \((M_{Pl})\), may be avoided by simply removing the hierarchy. In this case, gravitational interactions become strong near the weak scale and take place mainly in \(n\) new large spatial dimensions, known as the bulk. Due to experimental constraints, \textit{e.g.}, the width of the Z-boson, Standard Model (SM) fields cannot propagate into the bulk and are forced to lie on a wall, or 3-dimensional brane, in the higher-dimensional space. Gravity thus only appears to be weak in ordinary 4-dimensional space-time as we only observe its projection onto the wall. The relation between the scales where gravity becomes strong in the \(4+n\) and 4-dimensional theories can be derived from Gauss’ Law and is given by

\[
M_{Pl}^2 \sim r^n M_{eff}^{2+n},
\]

where \(r\) is the size of the additional dimensions and \(M_{eff}\) is the effective Planck scale in the bulk. The hierarchy dilemma is thus resolved by taking \(M_{eff}\) to be near a TeV, which yields \(r \sim 10^{30/n-19}\) meters. In this scenario, \(n = 1\) theories are automatically excluded as \(r\) would be too large, while the case of \(n = 2\) with \(r\) at a sub-millimeter will be probed by future gravitational experiments\[2\]. In addition, it has been recently shown\[3\] that this framework can be embedded into string models, where the effective Planck scale can be identified with the string scale \(M_s\). While we concentrate on this particular scenario, we note that there have been other interesting suggestions\[4\] for a low effective Planck, or string, scale and for larger extra dimensions arising from string theory and Kaluza Klein models.

While this is a fascinating concept, what makes this theory really interesting is that it has testable consequences. One manifestation of these theories is the existence of a Kaluza Klein (KK) tower of massive gravitons which can interact with the SM fields on the wall. Here we examine the indirect effects of these massive gravitons being exchanged in fermion pair production in \(e^+e^-\) annihilation and Drell-Yan production at hadron colliders. As we
will see below, these processes provide strong bounds on the effective Planck scale which are essentially independent of the number of extra dimensions. In addition, we quantify the extent to which the spin-2 nature of the graviton exchange is distinguishable from other potential new physics contributions to $e^+e^- \rightarrow f\bar{f}$. In the case of Drell-Yan production, we examine a novel feature of this theory, which is the contribution of gluon-gluon initiated processes to lepton pair production. The direct production of KK excitations of the gauge bosons and Higgs fields at colliders has been studied in [5].

The effective theory below $M_{eff}$ consists of the SM fields and y-states on the wall (the y-states are infinitely massive if the wall is rigid, or they can be Nambu-Goldstone bosons if the translational invariance of the wall in the extra dimensions is broken spontaneously), and gravity which propagates in the full $4 + n$ bulk. The interactions of the y-modes are dependent on the specific dynamics of the brane and we will not consider them here. The bulk metric can be written as

$$G_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}} + \frac{h_{\hat{\mu}\hat{\nu}}(x^\mu, x^a)}{M_{eff}^{n/2+1}},$$

(2)

where the indices $\hat{\mu}$ extend over the full $4 + n$ dimensions, $\mu$ over the $3 + 1$ dimensions on the wall, and $a$ over the $n$ bulk dimensions. The graviton field-strength tensor, $h_{\hat{\mu}\hat{\nu}}$, can be decomposed into spin-2, 1, and 0 fields. The interactions of these fields are given by

$$\int d^{4+n}x \, T^{\hat{\mu}\hat{\nu}} \frac{h_{\hat{\mu}\hat{\nu}}(x^\mu, x^a)}{M_{eff}^{n/2+1}},$$

(3)

where $T^{\hat{\mu}\hat{\nu}}$ is the symmetric, conserved stress-energy tensor in the bulk. The induced metric on the wall is given by $G_{\mu\nu}(x^\mu, x^a = 0)$ and the interactions with the SM matter fields are obtained by decomposing (3) into the 4-dimensional states. The bulk fields $h_{\hat{\mu}\hat{\nu}}$ appear as Kaluza-Klein towers in the 4-dimensional space arising from a Fourier analysis over the
cyclic boundary conditions of the compactified dimensions. Performing this decomposition, we immediately see that $T_{\mu a} = 0$ and hence the spin-1 KK states don’t interact with the wall fields. The scalar, or dilaton, states couple proportionally to the trace of the stress-energy tensor. For interactions with fermions, this trace is linear in the fermion mass, while for gauge bosons it is quadratic in the boson mass. Hence, the dilaton does not contribute to the processes under consideration here.

We thus only have to consider the interactions of the KK spin-2 gravitons with the SM fields. All the gravitons in the KK tower, including the massless state, couple in an identical manner. Hence we may use the couplings to matter as obtained in the case of linearized general relativity\[7\]. In this linearized theory, the matrix element for $e^+e^- \rightarrow f \bar{f}$ generalized for the case of $n$ massive graviton exchanges can be written as

$$\mathcal{M} = \frac{1}{M_{Pl}^2} \sum_n \frac{T^e_{\mu\nu}P^{\mu\nu\lambda\sigma}T^f_{\lambda\sigma}}{s - m_{gr}^2[n]},$$  \hspace{1cm} (4)$$

where the sum extends over the KK modes. $P_{\mu\nu\lambda\sigma}$ represents the polarization sum of the product of two graviton fields and is given in \[7\]. The terms in the polarization sum that are quadratic and quartic in the transferred momentum do not contribute to the above matrix element since $T_{\mu\nu}$ is conserved. Likewise, the terms which go as $\eta_{\mu\nu}\eta_{\lambda\sigma}$ lead to terms proportional to $T^e_{\mu\nu}T^f_{\lambda\nu}$ which vanish in the limit of zero electron mass. The remaining terms are $P_{\mu\nu\lambda\sigma} = \frac{1}{2}[\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}]$ and are exactly those present in the massless graviton case; they are thus universally applicable to all of the states in the KK tower. Since the spacing of the KK states is given by $\sim 1/r$, the sum over the states in (4) above can be approximated by an integral which is log divergent for $n = 2$ and power divergent for $n > 2$. A cut-off must then be applied to regulate these ultraviolet divergences, and is generally taken to be the scale of the new physics. For $n > 2$ it can be shown\[1, 8\] that the dominant
contribution to this integral is of order $\sim M_P^2/M_s^4$, where we have taken the cut-off to be the string scale, while for $n = 2$ this result is multiplied by a factor of order $\log(M_s^2/E^2)$, where $E$ is the center-of-mass energy of the process under consideration. The exact computation of this integral can only be performed with some knowledge of the full underlying theory. Combining these results yields the matrix element

$$
\mathcal{M} = \frac{\lambda}{M_s^4} \left\{ \bar{e}(p_1)\gamma_\mu e(p_2)\bar{f}(p_3)\gamma^\mu f(p_4)(p_2 - p_1) \cdot (p_4 - p_3) \right\}
$$

(5)

$$
\bar{e}(p_1)\gamma_\mu e(p_2)\bar{f}(p_3)\gamma_\nu f(p_4)(p_2 - p_1)^\nu (p_4 - p_3)^\mu \right\} .
$$

Here, the momentum flow is defined with $p_{1,2}$ into the vertex and $p_{3,4}$ outgoing. Note that graviton exchange is $C$ and $P$ conserving, and is independent of the flavor of the final state. The coefficient $\lambda$ is of $\mathcal{O}(1)$ and cannot be explicitly calculated without knowledge of the full quantum gravity theory. It is dependent on the number of extra dimensions, how they are compactified, and is in principle a power series in $s/M_s^2$. However, we neglect this possible energy dependence in $\lambda$ and note that the limits obtained here, which go as $|\lambda|^{1/4}$, are only very weakly dependent on its precise value and hence on the specific model realization. In principle the sign of $\lambda$ is undetermined and we examine the constraints that can be placed on $M_s$ with either choice of signs.

The angular distribution for $e^+e^- \rightarrow f \bar{f}$ with massive fermions is then calculated to be

$$
\frac{d\sigma}{dz} = N_c \frac{\pi \alpha^2}{2s} \beta \left[ P_{ij} \left[ A_{ij}^e A_{ij}^f (1 + \beta^2 z^2) + 2\beta B_{ij}^e B_{ij}^f z + A_{ij}^e C_{ij}^f (1 - \beta^2) \right] 
\right. 
$$

$$
- \frac{\lambda s^2}{2\pi \alpha M_s^4} P_i \left[ 2\beta^3 z^3 v_i^e v_i^f - \beta^2 (1 - 3z^2) a_i^e a_i^f \right] 
$$

(6)

$$
+ \frac{\lambda^2 s^4}{16\pi^2 \alpha^2 M_s^8} \left[ 1 - 3\beta^2 z^2 + 4\beta^4 z^4 - (1 - \beta^2)(1 - 4\beta^2 z^2) \right] \right\} .
$$
where the indices $i, j$ are summed over $\gamma$ and $Z$ exchange, $z = \cos \theta$, $P_{ij}$ and $P_i$ are the usual propagator factors (defined in e.g., $[9]$), $P_{ij} = (v_i^f v_j^f + a_i^f a_j^f)$, $B_{ij}^f = (v_i^f a_j^f + v_j^f a_i^f)$, $C_{ij}^f = (v_i^f v_j^f - a_i^f a_j^f)$, and $N_c$ represents the number of colors of the final state. In the case of Bhabha scattering, $t$- and $u$-channel graviton exchanges will also be present. If polarized beams are available a $z$-dependent Left-Right asymmetry can also be formed:

$$A_{LR}(z) = P_{ij} \left[ B_{ij}^e A_{ij}^f (1 + \beta^2 z^2) + 2 \beta A_{ij}^e B_{ij}^f z + B_{ij}^e C_{ij}^f (1 - \beta^2) \right] / D$$

$$- \frac{\lambda s^2}{2 \pi M^4} P_i \left[ 2 \beta^3 z^2 a_i^f v_i^f - \beta^2 (1 - 3 z^2) v_i^e a_i^f \right] / D,$$

where $D$ is given by the curly bracket in $[8]$ above. Note that the total cross section and integrated left-right asymmetry are unaltered by graviton exchanges, independently of fermion flavor up to terms of order $s^4/M^8$, and hence only the angular distributions for these quantities will be sensitive to these new exchanges. This is not the case for other new physics scenarios $[10]$ which also have indirect contributions to $e^+ e^- \rightarrow f \bar{f}$ via new particle exchange, such as those with additional neutral gauge bosons or with scalar exchange in the $s/t$-channels as in e.g., supersymmetry with R-parity violation. In general, these other scenarios affect the total integrated quantities as well as the angular distributions in a flavor dependent manner. In addition, the shape of the angular distributions for spin-2 exchange is unique and provides a smoking gun signature for graviton exchange.

The bin integrated angular distributions are displayed in Fig. 1 for $\mu^+ \mu^-$, $b\bar{b}$, and $c\bar{c}$ final states with $\sqrt{s} = 500$ GeV. Here the solid histogram corresponds to the SM expectations and the ‘data’ points represent the case with graviton exchanges with $M_s = 1.5$ TeV. The two sets of data points (squares and x’s) correspond to the two choices of sign for $\lambda$. The errors on the data points represent the statistics in each bin for an integrated luminosity of $75 fb^{-1}$. Here, we have assumed a 60% heavy quark tagging efficiency for $b$ and $c$ corresponding to
what is expected[1] to be achieved at high energy linear colliders, we have taken the electron beam polarization to be 90%, employed a $10^\circ$ angular cut around the beam pipe (to remove backgrounds from the interaction region), and included the effects of initial state radiation. We see that each of these distributions, with the exception of $A_{LR}(z)$ for $\mu$’s, provides a statistically significant signal for the graviton exchanges. Note that the deviations from the SM for the $b$-quark final state are particularly outstanding. From [3] we see that the expected shape of the angular distribution for the SM, (or for any spin-1 exchange) goes as $\sim (1 + z^2)$ and it is clear by eye that the spectrum with the graviton exchanges do not have this parabolic shape. Unfortunately $A_{LR}(z)$ for leptonic final states is numerically small and hence relatively poorly determined and will carry little statistical weight in our analysis. Summing over $e, \mu, \tau, b, c$ and $t$ final states (employing a 60% reconstruction efficiency for the case of top-quarks), including the $\tau$ polarization asymmetry, and performing the usual $\chi^2$ analysis[12] results in the 95% C.L. search reach shown in Fig. 2(a) as a function of luminosity with center-of-mass energies as indicated. Note that the effects of string scales up to $6\sqrt{s}$ (for canonical luminosity values at linear colliders) are discernable. Performing this same procedure for LEP II, but excluding the $A_{LR}(z)$ observable since polarized beams are not available, excluding top final states, and using heavy quark tagging efficiencies applicable for the LEP II detectors, we find that string scales up to $M_s = 985$ GeV are excluded from present data (taken to be 200pb$^{-1}$ per detector at $\sqrt{s} = 189$ GeV), and that this reach may be extended to $M_s = 1.14$ TeV with 2.5fb$^{-1}$ summed over all 4 detectors with $\sqrt{s} = 195$ GeV. The 95% C.L. search reaches for design luminosity at these machines are constrained in Table 1. Note that these constraints are actually placed on the quantity $|\lambda|^{-1/4}M_s$. We find that the difference in the search reach due to the sign ambiguity in $\lambda$ is only a few GeV.

We note that the reaction $e^+e^- \rightarrow gg$ can also be mediated by graviton exchanges
and will affect 2-jet production in $e^+e^-$ annihilation. However, since this reaction does not interfere with the four-fermion matrix element, it is suppressed by $1/M_s^8$ and hence will have little effect on the overall search reach in $e^+e^-$ collisions.

Next, we quantify the extent to which these spin-2 exchanges are distinguishable from other new physics sources. As an example, we perform a fit to the ‘data’ shown in Fig. 1 assuming that the unpolarized and polarized angular distributions take the forms $A(1 + z^2) + Bz$ and $[C(1 + z^2) + Dz]/[A(1 + z^2) + Bz]$, respectively, where $A, B, C$, and $D$ represent arbitrary constants to be determined from the fit. These forms are what would be expected in the case of new vector boson exchange. For the angular distribution, we include $e, \mu, \tau, b$ and $c$ final states (the top-quark is excluded as its mass effects would alter the constants $A$ and $B$), while for $A_{LR}(z)$ we only include $b\bar{b}$ and $c\bar{c}$ production as the leptonic final states carry no statistical weight for this observable. This corresponds to 88 degrees of freedom in the fit. The value of $\chi^2$ per degree of freedom is computed and the confidence level of the fit is presented in Fig. 2(b) as a function of the string scale. We see that the quality of the fit is quite poor for string scales up to $\sim 5\sqrt{s}$, which is almost up to the discovery limit. This demonstrates that spin-2 graviton exchanges are easily separated from that of new vector bosons. Similar studies can also be performed for comparison with new scalar exchange [10].

We now examine the case of lepton pair production in hadronic collisions. The subprocess contribution of the graviton exchanges to ordinary Drell-Yan production is essentially given by Eq. (3) in the massless limit. However, as noted above, gravitons can also mediate gluon-gluon contributions to lepton pair production via $s$-channel exchange. Such gluon initiated processes are a remarkable consequence of this theory and have the potential to modify the Drell-Yan spectrum in a unique manner. Following an analogous procedure as
Figure 1: Bin integrated angular distribution and $z$-dependent Left-Right asymmetry for $e^+e^- \rightarrow \mu^+\mu^-, bb, cc$. In each case, the solid histogram represents the SM, while the ‘data’ points are for $M_\chi = 1.5$ TeV with $\lambda = \pm 1$. The error bars correspond to the statistics in each bin.
Figure 2: Left: 95% C.L. search reach for the string scale as a function of integrated luminosity at $e^+e^-$ colliders with center-of-mass energy as labeled. Right: The percentage confidence level as a function of the string scale for a fit to the ‘data’ of Fig. 1 assuming the angular distributions take the form expected in the case of new gauge boson exchange. The assumed center-of-mass energy and luminosity is as labeled, and the dashed and solid curves in each case correspond to the choice $\pm \lambda$.

outlined above for the four-fermion case, the matrix element for $gg \to \ell^+\ell^-$ via graviton exchanges is found to be

$$
\mathcal{M} = -\frac{4\lambda}{M_g^2} \bar{f}(p')[(p' - p)_\mu \gamma^\nu + (p' - p)_\nu \gamma^\mu] f(p) \left\{ k'_\alpha (k_\mu \eta_{\beta\nu} + k_\nu \eta_{\beta\mu}) + k_\beta (k'_\mu \eta_{\alpha\nu} + k'_\nu \eta_{\alpha\mu}) - \eta_{\alpha\beta} (k'_\mu k_\nu + k_\mu k'_\nu) + \eta_{\mu\nu} (k' \cdot k_\alpha \eta_{\beta\lambda} - k_\beta k'_\alpha) - k' \cdot k (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha}) \right\} \epsilon^\alpha_g(k') \epsilon^\beta_g(k),
$$

(8)

where the momentum flow is defined with both $k, k'$ flowing into the vertex and $p, p'$ being outgoing and $\epsilon$ represents the gluon polarization vector. Because the graviton couplings and the summation over the KK tower of states for $2 \rightarrow 2$ processes are universal, $\lambda$ is the same $\mathcal{O}(1)$ coefficient as in Eq. (4). This matrix element yields the $gg \to \ell^+\ell^-$ differential cross
\[ \frac{d\sigma}{dz} = \frac{\lambda^2 s^3}{64\pi M_s^8}(1 - z^2)(1 + z^2), \]  

which has a remarkably simple form. The large parton luminosity for gluons at higher energy colliders may also compensate for the \( M_s^{-8} \) dependence. Since this cross section is also even in \( \cos \theta \), the gluon-gluon contributions will only affect the total cross section and not the forward-backward asymmetry. Also note that the ambiguity in the sign of \( \lambda \) does not affect the gluon-gluon contributions as they do not interfere with the \( q\bar{q} \) initiated process.

The bin integrated lepton pair invariant mass distribution and forward-backward asymmetry \( A_{FB} \) is presented in Fig. 3 for the Tevatron Main Injector and the LHC. In each case the solid histogram represents the SM expectations, and the ‘data’ points include the graviton exchanges with the error bars representing the statistics in each bin. The rapidity cuts, parton density parameterizations, and assumed integrated luminosity are as labeled.

|                  | \( \sqrt{s} \) (TeV) | \( \mathcal{L} \) (fb\(^{-1}\)) | \( \lambda = \pm 1 \) |
|------------------|----------------------|-------------------------------|------------------|
| LEP II           | 0.189                | 0.8                           | 0.98             |
|                  | 0.195                | 2.5                           | 1.1              |
| Linear Collider  | 0.5                  | 75                            | 3.4              |
|                  | 0.5                  | 500                           | 4.1              |
|                  | 1.0                  | 200                           | 6.6              |
| Tevatron         | 1.8                  | 0.11                          | 0.99             |
|                  | 2.0                  | 2                             | 1.3              |
|                  | 2.0                  | 30                            | 1.7              |
| LHC              | 14                   | 10                            | 5.2              |
|                  | 14                   | 100                           | 6.0              |

Table 1: 95% C.L. search reach for the string scale in TeV for various colliders with center-of-mass energies and integrated luminosities as indicated.
and we have summed over electron and muon final states. For the Tevatron we show the sample case of $M_s = 800$ GeV and the sign ambiguity in $\lambda$ is visible in the forward-backward asymmetry. For the LHC we display the effects of a $M_s = 2.5$ and 4 TeV string scale on the lepton pair invariant mass spectrum (with the smaller string scale having the larger effect), and again show $A_{FB}$ for both signs of the coefficient taking $M_s = 2.5$ TeV. Since the graviton exchanges only affect the invariant mass distribution at order $\lambda^2/M_s^8$, we would expect only minor modifications to this spectrum. We see that this holds true for the Tevatron, however, large string scales do have a sizable effect on the $M_{\ell\ell}$ spectrum at the LHC; this is due to the large gluon luminosity at these center-of-mass energies. The deviations in $A_{FB}$, however, are not as pronounced at the LHC, whereas even the two cases $\lambda = \pm 1$ are statistically distinguishable from each other at the Tevatron for this sample case. The resulting 95\% C.L. search reaches are given in Fig. and and Table for both machines. Here we see the effect of the sign difference in the forward-backward asymmetry at the Tevatron, while the LHC limits, which arise mainly from the $M_{\ell\ell}$ spectrum, are independent of the sign. We also find that present Tevatron data from Run I with 110pb$^{-1}$ of integrated luminosity excludes a string scale up to 990 (930) GeV at 95\% C.L. for $\lambda = -1(+1)$.

In conclusion, we have studied the indirect effects at high energy colliders of a TeV string scale resulting from new large extra dimensions. One prediction of these theories is the existence of a Kaluza Klein tower of massive gravitons, which can interact with the SM fields. We derived the form of these interactions and examined their effect in the $2 \to 2$ processes $e^+e^- \to f\bar{f}, q\bar{q} \to \ell^+\ell^-$, and $gg \to \ell^+\ell^-$ and found that present colliders can exclude a string scale up to $\sim 1$ TeV and that future colliders can extend this reach up to several TeV. In addition, these constraints are essentially independent of the number of extra dimensions as well as the details of the full underlying theory. Furthermore, we demonstrated that the angular distributions in $e^+e^-$ collisions uniquely reveal the spin-2 nature of the graviton
exchanges and can be distinguished from other sources of new physics for string scales close to the discovery limit and at a high confidence level.

These recent theories of low-scale quantum gravity are exciting, precisely because they have numerous experimentally testable consequences. The phenomenology of these models is just beginning to be explored and we look forward to the continued theoretical, phenomenological, and experimental investigations of these theories.

**Acknowledgements** The author would like to thank Tao Han, Joe Lykken, and Tom Rizzo for discussions related to this work, and Nima Arkani-Hamed, Savas Dimopoulos, and John Conway for their enthusiasm about this work. After this work was completed related material by Giudice *et al.*[13] appeared.

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Figure 3: Bin integrated lepton pair invariant mass distribution and forward-backward asymmetry for Drell-Yan production at the Main Injector and the LHC. The SM is represented by the solid histogram. The data points represent graviton exchanges with (a) $M_s = 800$ GeV and $\lambda = +1$ or $-1$, (b) $M_s = 800$ GeV and $\lambda = +1$ and $-1$, (c) $M_s = 2.5$ and 4.0 TeV and $\lambda = +1$ or $-1$, (d) $M_s = 2.5$ TeV and $\lambda = +1$ and $-1$. 
Figure 4: 95% C.L. search reach for the string scale as a function of integrated luminosity at the (a) Tevatron with the sign of $\lambda$ as labeled and (b) LHC for either sign.