Sub-eV scalar dark matter through the super-renormalizable Higgs portal

Federico Piazza\textsuperscript{1, 2} and Maxim Pospelov\textsuperscript{1, 3}

\textsuperscript{1}Perimeter Institute for Theoretical Physics, Waterloo, ON, N2L 2Y5, Canada
\textsuperscript{2}Canadian Institute for Theoretical Astrophysics (CITA), Toronto, Canada
\textsuperscript{3}Department of Physics and Astronomy, University of Victoria, Victoria, BC, V8P 1A1, Canada

(Dated: April 8, 2010)

The Higgs portal of the Standard Model provides the opportunity for coupling to a very light scalar field $\phi$ via the super-renormalizable operator $\phi (H^\dagger H)$. This allows for the existence of a very light scalar dark matter that has coherent interaction with the Standard Model particles and yet has its mass protected against radiative corrections. We analyze ensuing constraints from the fifth-force measurements, along with the cosmological requirements. We find that the detectable level of the fifth-force can be achieved in models with low inflationary scales, and certain amount of fine-tuning in the nonrenormalizable (NR) case.

I. INTRODUCTION

About 95\% of the energy budget of the Universe consists of "dark" – and unknown – components. This is a strong motivation for considering and studying hidden sectors beyond the Standard Model (SM). Gravitational effects of dark matter cannot reveal the mass of its constituents, and indeed a wide variety of mass ranges, from the inverse galactic size to the super-Planckian scales, is conceivable. While many models that possess stable particles with masses comparable to the SM energy scales have been a subject of incessant theoretical and experimental activity, models with light sub-eV mass scale dark matter received far less attention.

Below the eV mass scale the dark matter would have to be of integer spin, and be produced non-thermally. The only chance of detecting such dark matter non-gravitationally would occur if such particles are converted into electromagnetic radiation in the external fields or they modify the interaction strength of SM particles. But if light dark matter interacts with the SM, then immediately its lightness comes to question as the quantum loops with SM particle may easily destabilize the mass scale. A prominent particle in this category is the QCD axion \cite{Goldfisch:1983nd} that interacts with the SM currents derivatively, $J_\mu \partial_\mu a$, and has its tiny mass generated by the non-perturbative QCD effects protected at any loop level. Because of the pseudoscalar nature of $a$ and its derivative couplings, it does not generate a long-range attractive force.

A very natural question to ask is whether SM allows for couplings to other types of sub-eV dark matter fields that lead to additional observable effects. For a recent review of the light sector phenomenology see, e.g. \cite{redietal}. Real scalar field $\phi$ and the vector field $V_\mu$ provide such opportunities with their couplings to the SM fields via the so-called Higgs and vector portals:

\begin{equation}
(A\phi + \lambda \phi^2) H^\dagger H \quad \text{Higgs portal} \end{equation}

\begin{equation}
J_\mu V_\mu; \quad \partial_\mu J_\mu = 0 \quad \text{Vector portal,} \end{equation}

where $H$ is the Higgs doublet, $A$ and $\lambda$ are parameters and $J_\mu$ is some locally conserved SM current, such as hypercharge of baryon current. If there is some initial value for $\phi$ or $V_\mu$ fields with respect to their zero energy configurations, one can source part/all of the Universe’s energy density from the coherent oscillations around the minimum.

The perils of low mass scale stabilization are immediately apparent in Eq. (1). Indeed, any loops of the SM fields would tend to induce the correction to the mass of $\phi$ field $\sim \lambda A_{UV}^2$, where $A_{UV}$ is the highest energy scale in the problem serving as the ultra-violet cutoff. Therefore, $\lambda$ should be taken to incredibly small values, making this portal irrelevant for the phenomenology of sub-eV dark matter. In contrast, the vector portals and the super-renormalizable Higgs portal, $A\phi H^\dagger H$, allow to avoid problems with technical naturalness. In the latter case loop corrections scale only as $A^2 \log A_{UV}$, while the quadratic divergences affect only the term linear in $\phi$, which can typically be absorbed in an overall field shift. In this paper we examine generic consequences of this coupling for the sub-eV scalar dark matter, leaving vector dark matter to future studies.

II. SUPER-RENORMALIZABLE PORTAL TO THE SCALAR DARK MATTER

The specific case of a singlet scalar $\phi$ coupled via a super-renormalizable term of the type $\phi H^\dagger H$, (see e.g. \cite{1} and references therein), has been mostly studied in connection with electroweak and GeV-scale phenomenology, with a notable exception of \cite{2}, where a possibility of super-weakly interacting Higgs-coupled dark matter was pointed out. The scalar potential in the model of interest reads as:

\begin{equation}
V = -\frac{m_h^2}{2} H^\dagger H + \lambda (H^\dagger H)^2 + AH^\dagger H\phi + \frac{m_\phi^2}{2} \phi^2. \end{equation}

This model is explicitly renormalizable and does not require any additional UV completion (if one is willing to...
tolerate the usual fine-tuning problem with $m_{1/2}^2$ itself. We chose to redefine away possible linear terms in $\phi$ by shifting the field, and absorbing $A\Delta \phi$ into $m_{1/2}^2$.

After spontaneous symmetry breaking, the two fields acquire a vacuum expectation value, $\langle H^\dagger H \rangle = v^2/2$, $\langle \phi \rangle = \phi_0$, where

$$v^2 = \frac{m_{h}^2}{2\lambda - A^2/m_{\phi}^2}, \quad \phi_0 = -\frac{Av^2}{2m_{\phi}^2},$$

and $v = 246$ GeV. The potential $V$ has a stable minimum only if $A^2/m_{\phi}^2 < 2\lambda$, which is what we assume in the following; otherwise, it develops a runaway direction in the $(\phi, H^\dagger H)$ plane unless additional nonlinear $\phi^4$ terms are introduced. The low energy dynamics is encoded in the two physical fields $h$ and $\varphi$, defined as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad \varphi = \phi_0 + \varphi$$

and with Lagrangian

$$\mathcal{L} = \frac{1}{2} \left( (\partial h)^2 - (\partial \varphi)^2 - m_{h}^2 h^2 - m_{\varphi}^2 \varphi^2 \right) - \frac{1}{2} Av \varphi - \frac{A}{2} h^2 \varphi + \ldots$$

As already noted, Higgs loops give only logarithmically divergent corrections to $m_{\varphi}$. Therefore, the requirement of technical naturalness bounds the scale of $m_{\varphi}$ from below by the coupling $A$. In summary, by defining the dimensionless ratio $x = A/m_{\varphi}$, we assume $x \lesssim 1$ and $x < \sqrt{2\lambda}$, although also values $x \ll 1$ will be considered.

## III. FIFTH FORCE AND EQUIVALENCE

### PRINCIPLE VIOLATION

The singlet $\varphi$ couples to SM particles through the mixing with the Higgs field. Depending on the mass $m_{\varphi}$ and coupling $A$, the $\varphi$-mediated attractive force can produce testable deviations from $1/r^2$-gravitational force as well as composition dependence, thus violating the Equivalence Principle (EP). The leading contributions to $\varphi$-couplings mediated by the $\varphi$-Higgs propagator is shown in Fig. 1. As a rule of thumb, the $\varphi$-couplings are suppressed with respect to the Higgs couplings by a factor of $A v / m_{h}^2$:

$$g_{\varphi xx} = \frac{A v}{m_{h}^2} g_{h xx},$$

where $g_{h xx}$ is the effective dimensionless coupling of the Higgs to $x$-particle at very low momentum transier. Therefore, the effective Lagrangian describing the interactions with the SM gauge and fermion fields takes the following form:

$$\mathcal{L}_{\text{eff}} = \frac{A v}{m_{h}^2} \left( g_{h f f} \bar{f} f + \frac{g_{h \gamma \gamma}}{v} F_{\mu \nu} F^{\mu \nu} + \ldots \right) \varphi.$$
to at most 10% correction. Including these fermion contributions gives \( g_{h\gamma\gamma}(q^2 = m_{\gamma}^2) \approx \alpha_{\text{EM}}/(8\pi) \). For the purpose of calculating the coupling of \( \varphi \) to nuclei when the EM fraction of energy is taken into account, electrons should not be included in the sum, and muon contribution should include a form-factor. We are not going to pursue this calculation, because it turns out that \( g_{h\gamma\gamma} \) provides a subleading contribution to the EP violation.

Field \( \varphi \) mediates a fifth force of range \( \sim m_{\varphi}^{-1} \). More precisely, at the Newtonian level of approximation, the total effective gravitational potential between two bodies \( A \) and \( B \) at relative distance \( r \), presents a Yukawa contribution due to the interaction of the long range field \( \varphi \).

\[
V(r) = -G \frac{m_A m_B}{r} (1 + \alpha_A \alpha_B e^{-m_{\varphi} r}).
\] (12)

The scalar couplings \( \alpha \) can be expressed in terms of the log-derivative of the masses as

\[
\frac{\alpha_A}{\sqrt{2} M_P} = \frac{d \ln m_A(\varphi)}{d \varphi},
\] (13)

where \( M_P \) is the reduced Planck mass and \( m_A(\varphi) \) includes terms in the Lagrangian that are bilinear in the fields and couple to \( \varphi \), such as those in eq. [8]. When calculating \( \alpha_A \), one should consider the leading universal contribution from the nucleons and all the corrections that are specific to the element \( A \) (See e.g. [13]). The main, species-independent part of the nuclear mass is given by \( m_{\text{nuc}}(N_A + Z_A) \), and the universal coupling \( \alpha \) is obtained from eqs. [8], [9] and [13]:

\[
\alpha = g_{hNN} \frac{\sqrt{2} M_P Av}{m_{\text{nuc}} m_K^2} \] (14)

\[
\simeq 10^{-3} \left( \frac{m_h}{115 \text{ GeV}} \right)^{-2} \frac{A}{10^{-3} \text{eV}}.
\]

In the limit of a very long range force, the value of \( \alpha \) is bounded by post-Newtonian tests of General Relativity to \( \alpha^2 \lesssim 10^{-5} \) [14]. However, one can easily see that for mass range of \( m_{\varphi} \) below \( 10^{-12} \) eV, the relative strength of the \( \varphi \)-induced force drops below \( 10^{-14} \) from the gravitational field strength, which would make it extremely challenging for experimental detection and immune to the Solar System tests. Thus, it is more interesting to consider intermediate-range forces. Tests of gravitational inverse-square law limit the Yukawa component of the gravitational potential [15] [16]. By means of equation [14], such tests give a bound on \( A \). This is shown in Fig. 2. The two panels are elaborations of plots taken from Refs. [15] and [16]. A force with similar values of \( m_{\varphi} \) and \( A (\approx 1) \) is excluded in the range of masses \( m_{\varphi} \sim 10^{-8} \text{eV} - 10^{-3} \) eV.

The calculations of the EP-violating part of the scalar exchange is a far more delicate exercise. One should recognize that the equivalence principle is violated already at the level of nucleons, that is \( g_{hnn}/m_n \neq g_{hpp}/m_p \). As is well-known, the neutron and proton mass difference comes about because of the unequal quark masses, and electromagnetic contribution to the nucleon mass. One can estimate \( (m_n - m_p)|_{m_n \neq m_d} \sim 2.1 \text{ MeV} \) and \( (m_n - m_p)|_{m_d} \sim -0.8 \text{ MeV} \), so that together both contributions combine to the observable mass difference \( \Delta m_{np} = 1.3 \) MeV. The \( \varphi \)-dependence of both pieces is completely different. Because of the loop smallness of \( g_{h\gamma\gamma} \), the electromagnetic fraction of nucleon mass is far less dependent on \( \varphi \): \( \varphi \partial (m_n - m_p)|_{m_d} / \partial \varphi \ll \varphi (m_n - m_p)|_{m_n \neq m_d} / \partial \varphi \). Therefore, when we estimate the mass of an atom, we add to the universal term proportional to the baryon number a
correction proportional the the nucleon mass difference:

\[ m = (N + Z) m_{\text{nuc}}(\varphi) + \frac{N - Z}{2} \Delta_{np} m(\varphi) + \ldots \]  

The first term in (15) produces the universal coupling \( \alpha \) calculated in (14). The composition-dependent correction reads

\[ \alpha^{\text{EPV}} \simeq \alpha \frac{N - Z}{2(2N + Z)} \frac{\Delta m_{np}}{m_N} \left( \frac{m_{np}}{g_{hNN} \Delta m_{np} / \partial h} - 1 \right) \simeq \alpha \frac{N - Z}{2(2N + Z)} \times 3 \times 10^{-5} \]  

This may lead to a sizable variation of acceleration \( \Delta a \) between light atoms with \( Z = N \) and heavy atoms with \( N - Z \simeq 0.1 \),

\[ \frac{\Delta a}{a} \simeq \alpha^2 \times \mathcal{O}(10^{-3} - 10^{-4}). \]  

Other important effect should be related to the dependence of the nuclear binding energy on \( \varphi \), that can easily reach a level comparable to $10^6$. More detailed considerations of nuclear mass dependence on \( \varphi \) go outside the scope of the present paper.

As long as we adhere to our naturalness condition \( A \simeq m_\varphi \), the present bounds on composition dependent EP violations \( (\Delta a/a \lesssim 10^{-13}) \) are easily evaded. When the Earth is the common attractor of the two free-falling bodies, the relevant range \( m_\varphi \simeq 10^4 \) km turns into extremely tiny values for the coupling \( A \). Still, if we were to consider more fine-tuned scenarios \( (m_\varphi \ll A) \), it is interesting to note that a fifth force attached to the Higgs portal displays a peculiar relation between composition independent and composition dependent effects, as clearly follows from eq. (16). In principle, this allows to distinguish between the Higgs portal and, e.g., the string-inspired scenarios [13][18][19].

### IV. COSMOLOGICAL CONSTRAINTS

Since all couplings to SM particles are suppressed by a factor \( A v / m_h^2 \), the scalar field is sufficiently stable to be a non-thermal relic (for earlier studies of scalar dark matter see, e.g., [20][21]). Its decay rate into photons is smaller than the current Hubble rate as long as the mass is under a keV:

\[ \Gamma_\varphi = \frac{m_\varphi^3 A^2 g_{h\gamma}^2}{4\pi m_h^2} \simeq 10^{-37} \text{eV} \times x^2 \left( \frac{m_\varphi}{1\text{keV}} \right)^5 \left( \frac{100 \text{GeV}}{m_h} \right)^4. \]  

However, as emphasized in [9], the constraints from the gamma ray background would provide much tighter constraints, and in what follows we will concentrate on the sub-eV range.

The abundance of \( \varphi \)-particles today can be estimated [22][23] in terms of the initial misalignment \( \varphi_0 \) of the field from its minimum at the time \( t_* \) when \( m_\varphi \sim 3H \). At that moment, the field starts oscillating around the minimum of its potential and behaves like non-relativistic matter. The number of particles in a comoving volume is conserved, so that \( n_\varphi / s = \text{const} \), where \( s = 0.44g_0T^3 \) is the entropy density, \( n_\varphi \) is the number density of \( \varphi \) particles and \( g_0 \) is the number of effective degrees of freedom in equilibrium with the photons. The (average) energy density of \( \varphi \) today is thus given by \( \rho_0 = 0.4g_0n_\varphi s_0 (m_\varphi)^3 / 2 \), where \( s_* \) and \( s \) should be taken at \( t_* \). Uniting the relations \( m_\varphi n_\varphi = m_\varphi^2 \varphi_0^2 / 2 \) and \( s_0 / s_* = (2/g_0)(T_0^4 / T_*)^3 \) together with the Hubble rate

\[ H \simeq \frac{g_*^{1/2}}{3M_P} T^2, \]  

we express \( T_* \) in terms of the parameters of the model to obtain

\[ \Omega_\varphi h^2 = 0.4 \frac{g_*^{3/4}}{g_*} \left( \frac{m_\varphi}{10^{-3} \text{eV}} \right)^{1/2} \left( \frac{\varphi_*}{10^{14} \text{GeV}} \right)^2, \]  

where \( g_* \) is the number of degrees of freedom relevant for estimating \( H \). Since \( T_* \sim 10^9 (m_\varphi / \text{eV})^{1/2} \text{GeV} \), in the mass range of interest \( g_* = g_0 \) and \( 0.4 \frac{g_*^{3/4}}{g_*} \simeq O(0.1) \).

An important constraint on the model comes from the smallest allowed mass for a dark matter particle. The observations of smallest halos show their size to be comparable to 1 kpc [24], which means that the Compton wavelength of \( \varphi \) field would have to be comparable or smaller that this scale. This in turn imposes the constraint on \( \varphi_* \):

\[ m_\varphi > 10^{-26} \text{eV} \Rightarrow \varphi_* < 2 \times 10^{18} \text{GeV} \times \left( \frac{\Omega_\varphi h^2}{0.1} \right)^{1/2}. \]  

Notice that at the boundary of the allowed value, \( m_\varphi \sim 10^{-26} \text{eV} \), the oscillations start around 10 eV, that is just before the matter-radiation equality.

By eq. (20) it is clear that the abundance of \( \varphi \)-particles depends on the VEV of the field at the moment when the Hubble parameter becomes of the order of its mass. This is ultimately a matter of initial conditions. However, it is interesting to study the preceding evolution of \( \varphi \) up to electroweak symmetry breaking. During inflation, while the field would classically stay constant, its vacuum expectation value gets random kicks of order \( H/2\pi \) every Hubble time due to quantum fluctuations (see e.g. [25]). Its behavior can be described formally with a Langevin type equation [18]

\[ \frac{d\phi}{dp} = \frac{H(p)}{2\pi} \zeta(p). \]  

In the above \( p = \ln a \) is the number of e-folds and \( \zeta \) is a Gaussian random variable. Its \( p \)-averages are \( \langle \zeta(p) \zeta(p') \rangle = \delta(p - p') \). It is straightforward to estimate the expected shift in the field during inflation:

\[ |\Delta\phi_{\text{inf}}| \equiv \sqrt{\langle (\phi_{\text{end}} - \phi_{\text{ini}})^2 \rangle} = \frac{1}{2\pi} \left( \int_{\phi_{\text{ini}}}^{\phi_{\text{end}}} dp' H^2(p') \right)^{1/2}. \]  

(23)
As a rough order of magnitude, this gives $\Delta \phi_{\text{inf}} \gtrsim 10H_{\text{CMB}}$, where $H_{\text{CMB}}$ is the Hubble parameter at the epoch where the scales relevant for the CMB left the horizon. However, in scenarios with a long epoch of self-regenerating inflation, $\Delta \phi_{\text{inf}}$ can be much larger.

At the onset of radiation domination quantum fluctuations become irrelevant and the field is governed by the classical equation

$$\ddot{\phi} + 3H\dot{\phi} + m^2 \phi + A(H^\dagger H) = 0.$$  \hspace{1cm} (24)\]

The behavior of $\phi$ up to electroweak phase transition is obtained by neglecting the second derivative and the mass term from the above equation. While the Higgs field is in thermal equilibrium we have, with good approximation $23$, $(H^\dagger H) = 3T^2$. By using (19) we thus get

$$\dot{\phi} = -3AM_P.$$ \hspace{1cm} (25)

To a good approximation, the field has a constant velocity, which justifies neglecting the second derivative term in (24). Every time a relativistic species leaves the thermal bath, $\dot{\phi}$ decreases, giving a little “kick” to the field’s velocity. Thus, the details of this mechanism depend on the physics beyond the Standard Model. By making the minimal assumption $\dot{\phi}_0 \approx 100$ we count only for the SM degrees of freedom and obtain a shift in field space.

$$\frac{\phi_{\text{EW}} - \phi_{\text{end}}}{M_P} = -0.4 \ A t_{\text{EW}} = -2 x 10^{-6} \ m_\phi / 10^{-10} \text{eV},$$ \hspace{1cm} (26)

where the subscript EW indicates quantities at electro-weak phase transition.

Finally, at the onset of EW phase transition, the field finds itself displaced from its true minimum by an amount $\phi_{\text{EW}} \equiv \phi_{\text{EW}} - \phi_0$, where $\phi_0$ is given in eq. (3).

$$\phi_0 \approx -3 x 10^{23} \left( \frac{m_\phi}{10^{-10} \text{eV}} \right)^{-1} \text{GeV}.$$ \hspace{1cm} (27)

A potential disparity between (21) and (27) signifies possible fine-tuning problem. The starting point for the $\phi$ field at the end of inflation would have to be reasonably close to $\phi_0$. Therefore, if we start, say, with $\phi_{\text{end}} = 0$ at the end of inflation, the field starts running towards its true minimum, $\phi_0$ thanks to the coupling to the Higgs [26]. However, for masses $m_\phi \lesssim 10^{-5} \text{eV}$, the shift during radiation domination (26) is irrelevant with respect to the scale set by (27). If $\phi$ mediates long/intermediate range ($\lambda \gtrsim cm$) forces, its initial value $\phi_*$ has to be fine tuned ($\phi_* \ll |\phi_0|$), or otherwise $\phi$-particles are overproduced.

Any super-cold dark matter, such as axion or $\varphi$ field discussed in this paper, are prone to the CMB constraints on the amount of isocurvature perturbations (For the recent discussions of the axion isocurvature perturbations see e.g. [26, 27]). Before going to implications of these constraints for the model, we would like to comment that the scalar field with the quadratic potential is less susceptible to the isocurvature constraints than axions. In case of the quadratic potential the increase in the homogeneous displacement from the minimum, $\varphi_*$, over the fluctuating value $\delta \phi$ leads to the $\delta \varphi/\varphi_*$ suppression of the isocurvature perturbations, that in principle can be made arbitrarily small by the increase of $\varphi_*$. In contrast, the increase in the homogenous value of the axion field due to the periodicity of the potential $V_\alpha(a) = V(a + 2\pi f_a)$ can lead to at most $\delta a/f_a$ suppression of the isocurvature perturbations.

During inflation, the field $\varphi$ undergoes fluctuations of order $\delta \varphi = H/2\pi$ as any other light field, $H$ being the Hubble rate at the time when the fluctuation exits the horizon. The produced perturbation are of isocurvature type. Following the standard treatment that also applies to axions [22, 28], we can estimate the power spectrum of entropy perturbations $P_\Sigma(k)$ and compare it to that of curvature perturbations $P_R(k)$. The ratio of the two defines a parameter

$$\frac{\alpha(k)}{1 - \alpha(k)} \equiv \frac{P_\Sigma(k)}{P_R(k)} = \frac{8\epsilon_\alpha}{\Omega_\phi^2 M_P^2} \left( \frac{m_\phi}{10^{-9} \text{eV}} \right)^{1/2},$$ \hspace{1cm} (28)

where $\Omega_\phi$ is proportional to the total energy density in dark matter, $\epsilon$ is the usual inflationary slow-roll parameter and $c_\alpha$ the speed of sound of the adiabatic fluctuations during inflation [29].

By using (20) and $\Omega_\phi h^2 \approx 0.1$ we get, for small $\alpha(k_0)$,

$$\alpha(k) = 4.7 \times 10^9 \epsilon_\alpha \frac{\Omega_\phi}{\Omega_\text{c}} \left( \frac{m_\phi}{10^{-9} \text{eV}} \right)^{1/2}.$$ \hspace{1cm} (29)

The above result can nicely be re-expressed in terms of the tensor-to-scalar ratio $r = 16\epsilon_\alpha$. At the pivot wavenumber $k_0 = 0.002 \text{Mpc}^{-1}$ the limit set by WMAP+BAO+SN is $\alpha(k_0) < 0.067$ [28]. This gives the rather strict constraint

$$\frac{\Omega_\phi}{\Omega_\text{c}} \left( \frac{m_\phi}{10^{-9} \text{eV}} \right)^{1/2} \lesssim 2.3 \times 10^{-10},$$ \hspace{1cm} (30)

which is very similar to the conclusions reached for the axion cosmology [26, 27]. If we insist on $\Omega_\phi$ making most of the cold dark matter density, this result shows that the detectable level of inflationary gravitational waves ($r > 10^{-2}$) implies a very light scalar, close to the bound [24], which would make $\varphi$-mediated fifth force totally negligible. Conversely, a detectable level of the fifth force ($m_\phi > A > 10^{-9} \text{eV}$) would imply tiny $r$ on the order of $10^{-10}$ favoring some intermediate scale inflationary scenarios, $H \sim O(r^{1/2} \times 10^{14}) \text{eV}$. Given that in some models of inflation (see e.g. [30]) the Hubble parameter can be as low as $H \sim \text{GeV}$, producing a tensor to scalar ratio $r \approx 10^{-28}$, constructing an inflationary model with the Hubble parameter at some intermediate scale does not pause any model-building challenge.
V. DISCUSSION

The model we considered in this work is very similar to the linearized version of the Brans-Dicke (BD) theory when the scalar field is supplied with the mass term. Indeed, the transformation from the Jordan to the Einstein frame puts the BD scalar in front of any dimensionful parameter. Therefore, the $A$ parameter from the model considered here can be identified with $A \sim m_{\phi}^2/(\omega^{1/2} M_P)$, where $\omega$ is the BD parameter. It is very important to keep in mind, however, one crucial difference. In the BD theory, the $\phi$-field also couples to all massive states that may exist beyond the SM states, and therefore, even at the electroweak scale one should expect the extension of Eq. (2) by additional higher-dimensional operators. Such terms alter the couplings of BD scalar to matter, and make couplings to gauge bosons, e.g. $g_{\phi\gamma\gamma}$, different from the values in the model considered here. Moreover, the BD theory requires explicit UV completion, while the model with coupling to the SM and the mass of the vector fields do not have to follow the strength $\times$ range = const constraint of the scalar case. This could open more room for the fifth-force mediated by the vector-like sub-eV dark matter.

The model considered here falls into the class of the “super-cool” dark matter models, such as axion dark matter. Another example, worth of investigation is the vector dark matter. There, the coupling of vector fields to the SM and the mass of the vector fields do not have to follow the strength $\times$ range = const constraint of the scalar case. This could open more room for the fifth-force mediated by the vector-like sub-eV dark matter.

We would like to thank N. Afshordi, N. Barnaby, J. Bond, C. Burgess, A. Erickcek and A. Nicolis for useful discussions. Research at the Perimeter Institute supported in part by the Government of Canada through NSERC and by the Province of Ontario through MEDT.

[1] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.
[2] J. Jaeckel and A. Ringwald, arXiv:1002.0329 [hep-ph].
[3] B. Patt and W. Fischler, arXiv:hep-ph/0605188.
[4] D. O’Connell, M. J. Ramsey-Musolf and M. B. Wise, Phys. Rev. D 75, 037701 (2007) [arXiv:hep-ph/0611014].
[5] V. Barger, P. Langacker, M. McCaskey, J. H. Gundelach and E. G. Adelberger, Phys. Rev. D 83, 075005 (2008) [arXiv:0706.4311 [hep-ph]].
[6] O. Bertolami and R. Rosenfeld, Int. J. Mod. Phys. A 23, 4817 (2008) [arXiv:0708.1784 [hep-ph]].
[7] B. Batell, M. Pospelov and A. Ritz, arXiv:0911.4938.
[8] M. Ahlers, J. Jaeckel, J. Redondo and A. Ringwald, Phys. Rev. D 78, 075005 (2008) [arXiv:0807.4143 [hep-ph]].
[9] M. Pospelov, A. Ritz and M. B. Voloshin, Phys. Rev. D 78, 115012 (2008) [arXiv:0807.3279 [hep-ph]].
[10] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. B 78, 443 (1978).
[11] T. P. Cheng, Phys. Rev. D 38, 2869 (1988).
[12] C. Burgess, J. Jaeckel and M. Pospelov, Int. J. Mod. Phys. A 17, 1841 (2002) [arXiv:hep-ph/0012249].
[13] T. Damour and A. M. Polyakov, Nucl. Phys. B 423, 532 (1994); Gen. Rel. Grav. 26, 1171 (1994).
[14] B. Bertotti, L. Iess and P. Tortora, Nature 425, 374 (2003).
[15] S. Schlamminger, K. Y. Choi, T. A. Wagner, J. H. Gundlach and E. G. Adelberger, Phys. Rev. Lett. 100, 041101 (2008) [arXiv:0712.0607 [gr-qc]].
[16] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, Phys. Rev. Lett. 98, 021101 (2007) [arXiv:hep-ph/0611184].
[17] S. Schlamminger, K. Y. Choi, T. A. Wagner, J. H. Gundlach and E. G. Adelberger, Phys. Rev. Lett. 100, 041101 (2008) [arXiv:0712.0607 [gr-qc]].
[18] T. Damour, F. Piazza and G. Veneziano, Phys. Rev. D 66, 046007 (2002) [arXiv:hep-th/0205111].
[19] T. Damour, F. Piazza and G. Veneziano, Phys. Rev. Lett. 89, 081601 (2002) [arXiv:gr-qc/0204094].
[20] J. w. Lee and I. g. Koh, Phys. Rev. D 53, 2236 (1996).
[21] T. Matos, J. A. Vazquez and J. Magana, Mon. Not. Roy. Astron. Soc. 389, 1359 (2009) [arXiv:0806.0683 [astro-ph]].
[22] E. W. Kolb and M. S. Turner, “The Early Universe” (New York: Addison-Wesley (1990))
[23] V. Mukhanov, “Physical foundations of cosmology,” Cambridge, UK: Univ. Pr. (2005)
[24] B. Willman, arXiv:0907.4758 [astro-ph.CO].
[25] A. D. Linde, arXiv:hep-th/0503203.
[26] M. P. Hertzberg, M. Tegmark and F. Wilczek, Phys. Rev. D 78, 083507 (2008) [arXiv:0807.1726 [astro-ph]]
[27] J. Hamann, S. Hannestad, G. G. Raffelt and Y. Y. W. Wong, JCAP 0906, 022 (2009) [arXiv:0904.0647 [hep-ph]].
[28] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180, 330 (2009) [arXiv:0803.0547 [astro-ph]].
[29] J. Khoury and F. Piazza, JCAP 0907, 026 (2009) [arXiv:0811.3633 [hep-th]].
[30] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. 97, 191304 (2006) [arXiv:hep-ph/0605035].