Students’ mathematical ability and spatial reasoning in solving geometric problem

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Abstract. Students have many difficulties in geometry such as disability to visualize two dimensional to three dimensional objects. That is indicated by lower spatial reasoning. This study aims at describing the students’ spatial reasoning in solving geometric problem. This qualitative research involved three elementary students that consist of highest, middle, and lowest student’s ability. The results pointed out the highest and middle recognized geometry object and they could explain some information which is known and asked correctly. The highest student also could imagine cube nets into cube and describing the surface of the cube when it viewed from the right, left, top, bottom, front, and back. The middle student couldn’t draw surface of the cube appropriately when it viewed from the right, left, top and bottom. The lowest student couldn’t recognize geometric object, nor could mention the information. Finally, we suggest teachers to teach middle and low students to deepen the introduction of geometric objects.

1. Introduction
Geometry is a mathematical subject that addresses the nature and relationship of point, line, shape, and space. According to Fabiyi geometry provides a rich source of visualization to understand numbers, algebra, and statistical concepts [1]. In addition, geometry is also seen as a place to develop students’ reasoning and justification skills [2]. Hauptman and Cohen explained that geometry is a material that is considered difficult by students due to weak in reasoning [3]. The reasoning associated with geometry is called spatial reasoning. Students in Indonesia have low spatial reasoning because of inability to visualize three dimensional objects into two dimensional objects [4]. Empirical data show that many students make mistakes in drawing three dimensions into two dimensional objects or vice versa from two dimensions to three dimensional objects.

Spatial reasoning is the ability to visualize images [5]. Mulligan explains that spatial reasoning refers to the process of recognizing and manipulating mentally the spatial nature of objects and the spatial relationships between objects [6]. According to Pollitt, spatial reasoning is a set of cognitive, process and skill functions that enable us to understand and describe the representation and spatial relationships between objects, ourselves and our environment [7]. Pollitt explains that spatial reasoning indicators are 1) drawn or constructing two and three dimensional objects from memory; 2) mentally rotate and describe the transformation of the object by thinking of the outcome first; 3) compile and decipher the form to create a new shape.
Students’ spatial reasoning can be seen when solving problems. Dominowski argues that reasoning is a special kind of problem solving or a specific part of problem solving [8]. Dominowski explains that the problem-solving experience will strengthen mathematical understanding and reasoning that can then be used to solve new problems or more complicated problems.

Mathematical ability is also thought to affect students’ spatial reasoning in solving problems. Mathematical ability according to Sumarmo [9] is the skill of doing math to accomplish math task. An example of doing a simple math is counting, while proving to be a complex or high performing math. Nizoloman [10] explains that mathematical ability is a skill in using or manipulating numbers effectively in the field of administration, scientific, and application. In this study, mathematical ability is divided into 3 that is highest, middle, and lowest.

Based on research background, the purpose of this study is to describe the students’ mathematical ability and spatial reasoning in solving geometric problem. In this study, researchers took elementary school students because according to Piaget’s psychological theory [11], they should be at a stage formal operations so that students are expected to think abstractly.

2. Method
This study used descriptive qualitative method. This method aims to describe mathematical ability and spatial reasoning in solving geometric problem. Participant of this study is three elementary students that consist of highest, middle, and lowest student’s ability in the 6th grade. Participant was selected based on purposive sampling. The choice of subject of 6th grade of elementary school is because according to Piaget Theory students at this stage have been at the formal operational stage. At this stage, students are able to reason logically. The students were given a test focused on geometric problem. This is a problem given to students. “Dhani will make the dice on the net that he created (Look at Figure 1). Draw Dhani’s dice and explain the appearance of the front, back, top, bottom, right, and left!”

![Figure 1. Dhani dice net.](image)

After they complete the test, they are given an interview based on semi-structured interviews. Interviews were conducted to identify students’ spatial reasoning in solving geometrical problem. Researchers provide digital audio recorders. The recorder captures all the information provided by the participants during the interview, so no information is lost.

Student test results and interview results are used as data sources. All data collected were analyzed using Miles and Huberman [12] descriptive methods. First, the task-based interview data categorized based on indicators of spatial reasoning. During the task-based interviews, data were obtained that did not appropriate the research objectives. The data should be reduced so the data becomes simpler. Then, the data has been reduced delivered in the narrative text. Then, the data is interpreted to make a conclusion.

3. Results and discussion
First, the participants were given a test consisting of one question about the geometry problem. Researchers oversee the participants during their tests. After completion, they are given an interview based on test results. Participants in this study were Tommy (T) as a student who has highest
mathematical ability, Michael (M) as a student who has middle mathematical ability and Liem (L) as students who have lowest mathematical ability. The three participants use a pseudonym.

3.1. Student’s spatial reasoning of highest mathematical ability

Figure 2 shows the student’s answer in answering the given problem. At the time of the interview, students with highest mathematical ability mentioned the information that was known and asked the question correctly. Student know that the nets that exist in the problem is the nets of the cube. Student can also answer the problem given correctly. However, he found no other solution to the problem. Here is an interview snippet with the researcher (R) when the student solves the given problem.

Figure 2. Tommy’s answer.

R: Look at the picture (Figure 1)! What do you think about it?
T: I think it is a cube nets
R: What is the information from this problem?
T: I was given cube nets and I must arrange it
R: Explain how you solved the problem!
T: First, I imagine the nets. I made the number 5 die as the base. After that, I fold the cube and make the dice number 4 as a lid.
R: Then, from the dice you form, what is the dice that looks in front, back, right, and left?
T: The front of the dice is numbered 2, the back of the dice numbered 1, the lid numbered 4, the right side and the left number 6 and number 3.
R: Are you sure of the answer of the given problem?
T: Yes, I am. I have imagined it many times.
R: Do you have another solution from this problem?
T: No, I don’t

Student with highest mathematical ability can mention the known information correctly. The steps taken by the student in solving the problem are as follows. Student construct cube nets into cubes. Then he determined the 5th dice to be the base of the cube. Furthermore, he imagined the collapsible nets on the sides. He got the front of the dice is numbered 2, the back of the dice numbered 1, the lid numbered 4, the right side and the left number 6 and number 3.

3.2. Student’s spatial reasoning of middle mathematical ability

Figure 3 shows the student’s answer in answering the given problem. Just like a high-ability student, a student with middle mathematical ability is naming known information and asking the question correctly. Student know that the nets in the problem are cube nets. He composed a cube that was given from the front of the dice, numbered 3, then compiled it as shown in Figure 3. Here is an excerpt of researchers (R) with the student.
Figure 3. Michael’s answer.

R: What is the information from this problem?
M: This is a picture of cube nets and I was asked to construct it to cube
R: Explain how you solved the above problem!
M: I specify the front of the cube, I select the dice numbered 3. After that, I was folding the net in my brain. Then, I draw the dice exist in my brain.
R: Then, from the dice you form, the number dice eyes on the back, top, bottom, right, and left?
M: The back of the dice is numbered 6, the top of the dice numbered 4, the bottom of the dice numbered 5, the right and left numbered 2 and 1.
R: Are you sure of the answer of the given problem?
M: Yes, I am sure.
R: Do you have another solution to the problem?
M: No, I don’t

Student with middle mathematical ability can mention the known information correctly but the answer is less precise in solving the problem. The steps taken by the student in solving the problem are as follows. He determined the front of the cube, he chose the dice numbered 3. After that, he folds the cube nets in his mind. Then he drew the dice eyes, he had previously thought of. He inaccurate in determining the top, bottom, right, and left of the cube.

3.3. Student’s spatial reasoning of lowest mathematical ability
Figure 4 shows the student's answer in answering the given problem. Student with lowest mathematical ability couldn’t recognize geometry object, nor could mention the information. Here is an excerpt of researchers (R) with the student.

Figure 4. Lim’s answer.

R: Based on picture (Figure 1), what do you think about it?
L: I don’t know
R: What information is known in the question?
L: Sorry, I do not understand the meaning of this problem
R: Then, how you can write an answer on your answer sheet?
L: I do not know
R: Well, will you explain me how you got the answers you wrote?
L: I do not know how to solve it. This answer I am reef. Sorry, I cannot finish it.
R: Okay, thank you Liem.
L: You’re welcome
Student with lowest mathematical ability couldn’t recognize geometry object, nor could mention the information. From the interview excerpts shown that students do not understand the meaning of a given problem. Students always say that he could not when given questions by the researcher.

3.4. Differences in students’ spatial reasoning
Based on the results of spatial reasoning test and interview, the researcher obtained some information that showed the difference in spatial reasoning between students with highest, middle, and lowest ability. The results of this study also show that mathematical ability is directly proportional to spatial reasoning. This is also in line with Garderen’s research indicating that students with high math skills have high spatial reasoning and vice versa [13]. Tikhomirova also described students with high spatial reasoning indicating a higher level of mathematical fluency, compared to students with low levels of mathematical fluency [14]. In addition, three students show different ways of solving geometry problem. This is in line with opinion of Guzel and Sener’s research which shows the differences in mathematical ability influencing the spatial reasoning [15]. Here is the difference in spatial reasoning between students with highest, middle, and lowest ability.

Students with highest mathematical ability construct the cube starting from the base. In answering the problem, the student draws a cube, then explains the eyes of the dice next to the image he made. After determining the dice for the base, he then imagined folding the nets of cubes into cubes. He got the front of the dice is numbered 2, the back of the dice numbered 1, the lid numbered 4, the right side and the left number 6 and number 3. The answer of him is correct.

Student with middle mathematical ability constructed the cube starting from the front. He answered the problem with a picture along with the eyes of the dice on the parts of the cube. After he determined the front of the cube, he folded the cube nets in his mind. Then he drew the dice eyes, he had previously thought of. He inaccurate in determining the top, bottom, right, and left of the cube.

Student with lowest mathematical ability could not understand the meaning of a given problem. Students always say that he could not when given questions by the researcher. He couldn’t recognize geometry object, nor could mention the information.

4. Conclusion
The results showed that there was a difference of students’ spatial reasoning in solving geometric problem. The results pointed out the highest and middle recognized geometry object and they could explain some information which is known and asked correctly. The highest student also could imagine cube nets into cubes and describing the surface of the cube when it viewed from the right, left, top, bottom, front, and back. The middle student couldn’t drawing surface of the cube appropriately when it viewed from the right, left, top and bottom. The lowest student couldn’t recognize geometry object, nor could mention the information. Finally, we suggest teachers to teach middle and low students to deepen the introduction of geometric objects.

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