Stability of the laminar boundary layer on the surface of the Mach-6 contoured nozzle

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Abstract. The stability of the laminar boundary layer on the surface of the Mach 6 contoured nozzle designed for short-duration wind tunnel Transit-M is investigated. The numerical simulation of the laminar flow inside the nozzle is performed using Ansys Fluent software. The obtained boundary layers profiles on the nozzle surface are used to calculate the growth rate of the first and second Mack mode and Görtler vortices in the framework of the linear stability theory. It has been shown that, for the Mach 6 nozzle, the Görtler vortices grow most rapidly, the first Mack mode grows slower, and the second mode is insignificant.

1. Introduction
Investigations of the stability and laminar-turbulent transition of the boundary layer in wind tunnels are important for understanding the fundamental nature of the transition, as well as for the validation of theoretical and numerical models. For this purpose, wind tunnels must create a uniform flow with a low noise level. As for the supersonic and hypersonic wind tunnels, the flow uniformity is achieved by special contouring of a nozzle surface [1]. The noise in the flow is created due to design future (throttles, joints of parts, measurement equipment and others) and turbulent boundary layer on the nozzle wall [2]. Special honeycombs and screens are used to decrease the fluctuations level just before the convergent section of a nozzle. At the low noise level of a flow and small roughness of a nozzle surface, the laminar to turbulent boundary layer on the nozzle wall takes place due to growing Görtler vortices or/and first and second Mack modes. The linear stability theory predicts well the growth rate of the small disturbances in the laminar boundary layers [3]. In this work, the linear stability theory is used to determine the most unstable disturbances of the boundary layer on the surface of the Mach 6 nozzle designed for short-duration wind tunnel Transit-M. Using this nozzle, many important experimental data were obtained on the stability, laminar-turbulent transition, and stability control of hypersonic boundary layers (for example, in recent works [4–7]). It was shown that the level of pressure pulsations is below 3\%, and the level of pulsations of the heat flux is below 8\% in the range of free stream unit Reynolds numbers $\text{Re}_1 = (4–24) \times 10^6$ m\textsuperscript{-1} at the nozzle exit [8]. Therefore, the research of the causes and methods of reducing high-level flow fluctuations for a certain nozzle is an urgent task.
2. Calculations method

2.1. Mean flow

The calculations are performed for the Mach 6 nozzle designed for the Transit-M hot-shot wind tunnel based at ITAM SB RAS. The mean flow in the nozzle is obtained by the direct numerical simulation of axisymmetric Navier-Stokes equations with the Ansys Fluent software. The Mach 6 nozzle has an exit diameter 0.3 m and nozzle expanding section length of 1.46 m. Figure 1 shows the nozzle geometry and contours of Mach number. The boundary condition on the inlet of the convergent section of the nozzle is the pressure inlet with the stagnation temperature \( T_0 = 380 \) K and different cases of the stagnation pressure \( P_0 = 1.5 \) MPa and 3.75 MPa, which corresponds to the unit Reynolds number at the nozzle exit \( Re_{1x} = 19.4 \times 10^6 \) m\(^{-1}\) and \( 48.1 \times 10^6 \) m\(^{-1}\). The exit from the nozzle is the pressure outlet with the static pressure \( P = 0 \) and static temperature \( T = 300 \). The influence of the outlet condition on the boundary layer is checked by the nonphysical change of the wall shear stress, and the affected zone \((x > 1.42)\) is not used for further research. The nozzle surface is the wall with no slip condition and temperature \( T_w = 295 \) K.

![Figure 1. The nozzle geometry and Mach number contours.](image)

The working gas is air with the perfect-gas law and the laminar viscosity. The viscosity and the thermal-conductivity are determined by the three coefficients Sutherland law and the kinetic theory, respectively.

The structured mesh with quadrilaterals elements is used. The size of the mesh elements decreases toward the surface in the normal direction and to the nozzle throat in the axis direction. The total grid size is 1700 × 200.

2.2. Stability

The linear stability theory is used to analyze the stability of the laminar boundary layer on the nozzle expanding section surface. The Görtler vortices and first and second Mack modes are significant for the supersonic and hypersonic boundary layers on the concave surfaces. Calculations of the Mack modes involve the linearized Navier-Stokes equations for propagation of disturbances from [9]. The growth rates of Mack modes are found for the fixed dimension transverse wave number \( \beta \) and frequency \( f \). The system of equations for the Görtler vortices is taken from [10]. The Görtler vortices with only \( f = 0 \) Hz are taken into account because the steady Görtler vortices grow faster than the unsteady ones [11]. The Görtler vortices growth rates are found for the fixed number of waves around the nozzle periphery:

\[
n = \frac{2\pi r}{\lambda},
\]

where \( r \) and \( \lambda \) are the local nozzle radius and spanwise wavelength, respectively.

The collocation method is used to solve the systems of equations for the Mack modes and Görtler vortices similarly to [9]. The amplitudes of the disturbances on the wall and on the upper bound are set to zero. The upper bound height \( y_{\text{max}} \) is set as twenty thicknesses of the boundary layer. The mesh points for stability equations defined by the equation:

\[
k = \cos \left( \frac{2\pi j}{L} \right), j = 0, 1, ... L,
\]
\[ y = C_1 \frac{1+k}{C_2-k'} \]

where \( L \) is total number of mesh points, \( y_\delta \) is the boundary layer thickness, and \( C_1 = y_\delta y_{\text{max}}/(y_{\text{max}} - 2y_\delta) \), \( C_2 = 1 + 2C_1/y_{\text{max}} \). So, the half of the total numbers of collocation points is concentrated in the boundary layer.

3. Results

The stability analysis of the laminar boundary layer on the Mach 6 nozzle surface is performed for the nozzle exit unit Reynolds numbers \( Re_{1\infty} = 19.4 \times 10^6 \) m\(^{-1}\) and \( 48.1 \times 10^6 \) m\(^{-1}\). The example of the boundary layer profile is show in figure 2. The mesh nodes number in the boundary layer is from 10 to 80. Then these profiles are interpolated to the mesh points for stability equations with the cubic interpolation.

![Figure 2. Boundary layer profile at \( x = 1 \) m for the \( Re_{1\infty} = 19.4 \times 10^6 \) m\(^{-1}\).](image)

![Figure 3. N-factor of the Görler vortices along the nozzle surface for \( Re_{1\infty} = 19.4 \times 10^6 \) m\(^{-1}\) (a) and for \( Re_{1\infty} = 48.1 \times 10^6 \) m\(^{-1}\) (b).](image)

To show the results of the stability calculations, the N-factor is obtained as

\[ N = \int_{x_0}^{x} -\alpha_1(x) \, dx \]
where \(-\alpha_i\) is the grow rate of disturbances, \(x\) is the coordinate along the nozzle surface, \(x_0\) is neutral point of disturbances.

Görtler vortices N-factors are shown in figure 3 for \(Re_{1x} = 19.4 \times 10^6\) m\(^{-1}\) (a) and for \(Re_{1x} = 48.1 \times 10^6\) m\(^{-1}\) (b). Evident that the Görtler vortices grow fast, and the higher is unit Reynolds number, the faster they grow. The number of waves around the nozzle periphery \(n\) of the most unstable Görtler vortices increases with the increase of the unit Reynolds number.

First Mack mode disturbances are shown in figure 4 for \(Re_{1x} = 19.4 \times 10^6\) m\(^{-1}\) (a) and for \(Re_{1x} = 48.1 \times 10^6\) m\(^{-1}\) (b). The growth of the N-factor of disturbances starts from \(x \approx 0.2\) m. The Mack first mode grows much slower than the Görtler vortices. The maximum N-factor for the first mode is \(N_{max} = 5.3\) whereas for the Görtler vortices is \(N_{max} = 15.6\) at the unit Reynolds number \(Re_{1x} = 19.4 \times 10^6\) m\(^{-1}\). The transverse wave number \(\beta\) and the frequency \(f\) of the most unstable Mack first mode increase as the unit Reynold number increases.

The second Mack mode instability is typical for the hypersonic boundary layers with Mach number on the edge above 5. For this reason, the second mode (figure 5) starts to grow from the \(x > 0.4\) were \(M > 5\). Evident that the amplification of the second mode disturbances is negligible compared to the...
amplification the Görtler vortices and thirst Mack mode. The frequency of the most unstable second mode disturbances increases along with unit Reynolds number.

4. Conclusions
The numerical study of the stability of the boundary layer on the Mach-6 contoured nozzle surface is carried out in the framework of the linear stability theory. The calculation is carried out for the nozzle designed for the wind tunnel Transit-M of ITAM SB RAS. Numerical simulation of the steady laminar flow in the nozzle is performed using the Ansys fluent software. N-factor of the Görtler vortices and first and second Mack mode are obtained.

It is shown that for this nozzle, the most unstable are Görtler vortices are for unit Reynolds numbers at the nozzle exit $Re_1 = 19.4 \times 10^6 \text{ m}^{-1}$ and $48.1 \times 10^6 \text{ m}^{-1}$. The first Mack mode grows slower than the Goertler vortices, and the first mode is negligible for this nozzle.

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