Investigating the flow field dynamics of transonic shock buffet using particle image velocimetry

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Abstract
An experimental investigation was performed in order to better understand the transonic shock buffet phenomenon and determine the dominant flow interactions at specific flow conditions. A rigid wing in the shape of an OAT15A airfoil was placed in the Trisonic Wind Tunnel Munich, where both the Mach number and the angle of attack were varied between $0.65 < M_\infty < 0.77$ and $3.8 < \alpha < 6.3^\circ$ respectively. With the use of high-speed imaging, high-quality optics and state-of-the-art laser equipment, highly resolved velocity field measurements were obtained via particle image velocimetry, where the streamwise and vertical velocity components were computed over the suction side of the wing center plane. It was shown that sustained buffet first occurs at $M_\infty \geq 0.74$ when maintaining the angle of attack constant at $\alpha = 5.8^\circ$. Similarly, an increase in $\alpha$ for a fixed $M_\infty = 0.74$ also led to the development of shock buffet. Instantaneous snapshots confirmed the presence of a recirculation region downstream of the moving shock, where an increase in the wake size was confirmed when the shock was located most upstream. Streamwise correlations were also computed near the airfoil’s upper surface in order to extract the characteristic convective velocity of flow structures. The convective velocity appeared to increase with streamwise distance, ranging on average between 50 and 150 m/s. Overall, these time-resolved velocity field measurements allow for the investigation of the flow dynamics during shock buffet and highlight the independent effect of Mach number and angle of attack on this complex phenomenon.

1 Introduction

The theory of flight has matured from the first powered flight in the early twentieth century to what is now known as modern day aerodynamics (Anderson Jr 1998; Von Mises 1959). The fundamental principles associated with bird flight and air travel are among the well-understood aerodynamics phenomena. Less understood, however, are the self-sustained, large-amplitude and low-frequency shock wave oscillations which occur on wings travelling at transonic speeds. Coined shock buffet, this undesirable phenomenon occurs under very specific flow conditions and can lead to significant fluid-structure interaction. Since the dominant flow frequency is on the order of the natural frequency of the structure, this coupling can in turn damage, or worse destroy, the airfoil surface (Jacquin et al. 2009). For this reason, it is of paramount importance to understand the conditions under which shock buffet occurs.

Both Lee (2001) and Giannelis et al. (2017) compiled relevant scientific findings into extensive reviews of the shock buffet phenomenon. Shock buffet can either occur on both surfaces of biconvex airfoils at zero incidence (“Type I”) or on the suction side of supercritical airfoils at a given angle of attack (“Type II”) (Giannelis et al. 2017). It must be noted that Type II shock buffet will be the focus of this investigation due to its complex nature and its relevance for modern airplanes.

For a given wing geometry, both the freestream Mach number, $M_\infty$, and the angle of attack, $\alpha$, were found to be tuning parameters which affect this flow instability (Lee 2001). McDevitt and Okuno (1985) investigated the effect of $\alpha$ and $M_\infty$ experimentally and observed that, as $M_\infty$ increases, the minimum value of $\alpha$ leading to shock buffet decreases. This was also found by Crouch et al. (2009) who combined global stability theory and unsteady Reynolds-averaged Navier-Stokes simulations and compared results.
with McDevitt and Okuno (1985). Both data sets agreed well with one another for \( M < 0.8 \). Giannelis et al. (2018) also performed a thorough numerical investigation of the influence of \( M_{\infty} \) and \( \alpha \) on the transonic shock buffet phenomenon. Reynolds-averaged Navier-Stokes simulations were performed and compared to available experimental data obtained at ONERA. They observed an increase in fundamental buffet frequency with both \( M_{\infty} \) and \( \alpha \). Regarding the effect of Reynolds number, investigations performed at ONERA showed that its influence is negligible when Re is close to \( 3 \times 10^6 \) (Jacquin et al. 2016). Therefore, although not directly investigated herein, it is assumed that the boundary layer in this study quickly becomes fully turbulent and that the results do not significantly vary with slightly different Reynolds numbers. The effect of Reynolds number on transonic shock buffet can also be found in both McDevitt and Okuno (1985) and Dandois (2016).

For a Mach number value in the transonic regime, the flow is accelerated over the suction side of the airfoil and a local region of supersonic flow appears, which initially causes a steady shock wave to form. With an increase in \( \alpha \), the shock wave travels downstream and generates a stronger pressure rise. This allows for meeting both the equality condition (the top and bottom static pressures must be equal at the trailing edge) and the compatibility condition (the difference between the pressure at the trailing edge and the pressure in the freestream must be attributed to the viscous wake) (Pearcey 1955). An additional increase in \( \alpha \) can lead to the formation of a separation bubble or trailing edge separation, both of which hinder pressure recovery (Pearcey 1955). Tijdeman (1977) observed that shock-induced boundary layer separation occurs when the Mach number immediately upstream of the shock ranges between 1.25 and 1.3. In order to enhance pressure recovery and still satisfy both the equality and the compatibility conditions, the shock must travel upstream (Pearcey 1955). This upstream travel, or “inversion of shock motion”, has been shown to be an indication of boundary layer separation (Pearcey 1955; Lee 2001) and, more recently, a necessary condition for buffet onset (Accorinti et al. 2022). In the case of constant \( \alpha \) and increasing \( M_{\infty} \), it was shown in Pearcey (1955) that the inversion of shock motion may not be necessary to satisfy the equality and compatibility conditions. Instead, a reduced downstream displacement rate of the shock with \( M_{\infty} \) may suffice. A reasonable explanation for that was again presented in Pearcey (1955); by increasing the Mach number, not only does the pressure in front of the shock drop, but also the freestream pressure.

Once shock buffet occurs, Tijdeman (1977) defined three different shock motions, from one of which Lee (2001) derived a working model. Lee proposed that the shock motion is modulated by the interaction of pressure waves travelling up and downstream (Lee et al. 1994). Indeed, disturbances generated from the shock wave travel downstream through the separated flow region and, when reaching the sharp trailing edge, generate sound waves travelling upstream. Such waves were named “Kutta waves” due to their association with the Kutta condition (Tijdeman 1977; Lee 2001). This interaction leads to a feedback loop which modulates the shock motion. Hartmann et al. (2013) and later Feldhusen-Hoffmann et al. (2018) conducted experiments to further probe and confirm Lee’s model, while slightly adapting the characteristic length for the upstream wave propagation; an artificial sound source was placed downstream of the model and its effect on the shock motion was investigated, in an attempt to simulate the sound waves generated at the trailing edge. They showed that the generated sound waves downstream of the shock impacted its oscillation. Xiao et al. (2006) also supported Lee’s model by performing an unsteady Reynolds-averaged Navier-Stokes simulation of the Bauer-Garabedian-Korn (BGK) supercritical airfoil. However, several studies hypothesized a slightly different modulation between wave propagation and shock motion, where the flow travelling upstream via the pressure side of the wing must be considered (Jacquin et al. 2009; Crouch et al. 2009; Deck 2005; Garnier and Deck 2010). More recently, Hartmann et al. (2013) neglected the waves travelling on the lower surface due to the longer path to reach the shock (compared to the path on the upper surface) and the decay of pressure waves with distance.

Many studies attempted to pinpoint the dominant flow features attributed to shock buffet by employing dynamic mode decomposition, proper orthogonal decomposition (POD) and global stability theory (Crouch et al. 2009; Kou et al. 2018; Ohmichi et al. 2018; Poplingher et al. 2019; Feldhusen-Hoffmann et al. 2021; D’Aguanno et al. 2021). The buffet onset predicted by Crouch et al. (2009a) had fairly good agreement with the experimental results obtained by McDevitt and Okuno (1985) and Jacquin et al. (2009). Although significant progress was made in enhancing our overall understanding of the shock buffet phenomenon, there appears to be a wide gamut of results regarding buffet features (frequency and amplitude of the oscillations, mean shock location as well as onset). Accorinti et al. (2022) compiled results found in the literature and attributed the marked range of results to the different numerical and experimental boundary conditions. Therefore, the onset conditions of Accorinti et al. (2022) were used as a benchmark given the use of the same experimental facility. For a detailed explanation of the differences between results, the reader is referred to Accorinti et al. (2022). Although both this manuscript and Accorinti et al. (2022) extract statistics and frequency content of the shock location, results are obtained by different means and are complimentary; this manuscript focuses on the buffet features which can more suitably be obtained via PIV (such as wake extent and correlation velocity), while
Accorinti et al. (2022) focused on a detailed description and detection of the flow development from a stable shock to buffet flow using background-oriented schlieren.

Given the fluid-structure interaction associated with the shock buffet phenomenon, it is particularly interesting to study the shock motion on a wing with an activated pitching degree of freedom (Gao et al. 2018). Although the experimental setup was designed to allow for such tests to be performed, this particular study focuses on a fully fixed wing (zero degrees of freedom). For results regarding shock buffet on a supercritical airfoil with a pitching degree of freedom, the reader is referred to Scharnowski et al. (2022).

Similar to Jacquin et al. (2009), the following manuscript attempts to provide the community with highly reliable experimental transonic shock buffet data. Precisely, the effects of both the Mach number and the angle of attack are examined on the buffet frequency, mean shock motion and shock amplitude. The range of interest was selected based on the results of Accorinti et al. (2022); with the use of deformation and force measurements as well as background-oriented schlieren, the particularly interesting cases regarding buffet flow features were chosen. A description of characteristic flow features is first provided, which includes shock statistics from time-resolved PIV and an analysis of the boundary layer evolution. A spectral investigation follows, where the dominant frequencies obtained via PIV are compared to those obtained via pressure measurements. The final subsection describes the method for extracting the downstream convective velocity of the large-scale turbulent structures propagating towards the wing’s trailing edge.

2 Experimental setup

In order to understand the independent effect of aerodynamic parameters on shock buffet, both the freestream Mach number and the model’s angle of attack were incrementally and independently increased from $0.65 < M_{\infty} < 0.77$ and $3.8 < \alpha < 6.3^\circ$. For all tests, the total pressure, $p_0$, was maintained constant at 1.5 bar, yielding a chord-based Reynolds number of $2.8 \times 10^6 < Re_c < 3.1 \times 10^6$ for the given $M_{\infty}$ range. Particle image velocimetry was performed to obtain a planar representation of the velocity field above the suction side of the OAT15A model. The test facility and the experimental setup are described in detail in this section.

2.1 Trisonic Wind Tunnel Munich

All tests were performed in the Trisonic Wind Tunnel Munich (TWM) located at Bundeswehr University Munich. This two-throat blowdown facility allows for tests in the subsonic, transonic and supersonic regimes to be carried out successfully, with a Mach number range of $0.2 < M_{\infty} < 3.0$.

Fed by two pressurized tanks with a total volume capability of 356 m$^3$, compressed dry air is flown through the 300 mm wide, 675 mm high and 1700 mm long test section. An adjustable de Laval nozzle upstream as well as an adaptive diffuser downstream of the test section allow for $M_{\infty}$ to vary throughout wind tunnel operation. Figure 1 depicts the main components of the TWM.

Ranging from $p_0 = 1.2$ to 5 bar, the total pressure can also be modified by adjusting the regulator valve accordingly. The maximum operational Reynolds number per unit length is $Re = 8 \times 10^7$ m$^{-1}$. As for the turbulence level of the facility, it ranges between 1.1 and 1.4% for the tested Mach number range (Scharnowski et al. 2019). The TWM is equipped with sensors in both the test section and the settling chamber in order to accurately measure the stagnation temperature, the stagnation pressure as well as the static pressure. Typical wind tunnel run times are on the order of 100 s (Accorinti et al. 2021), whereas in this study, the runs lasted approximately 20 s due to the memory capacity of the high-speed cameras. Recordings were started only after confirming that the set pressure and Mach number values were reached and stable. Additional information regarding the TWM can be found in Scharnowski et al. (2019).

2.2 OAT15A model

The model was designed according to the OAT15A supercritical airfoil geometry. This model was selected due to its substantial investigation both experimentally (Jacquin et al. 2009; Accorinti et al. 2021) and numerically (Deck 2005; Thiery and Coustols 2005). With a chord length of $c = 150$ mm and a span of $s = 298$ mm, the model’s aspect ratio is $AR \approx 2$. The wing’s thickness increases up to 12.3% of the chord, before decreasing to 0.5%. This specific cut-off was selected in order to avoid a very thin
trailing edge which would hinder manufacturing. The wing was manufactured out of carbon fiber reinforced polymer and was painted white to reduce surface heating. A speckle pattern was later added to the wing’s suction side in order to better evaluate its deformation (Accorinti et al. 2021). A thin strip, located in the middle of the model and along its chord, was painted with chrome in order to render the surface which would be exposed to the laser reflective. Prior to its application on the model, chrome spray paint was applied to a sample and tests were performed with full laser power to confirm its reflective capability. A spanwise dot pattern, located at \( x/c = 0.07 \), was also applied on both sides of the model to trip the boundary layer. Spaced 6 mm apart, these dots are approximately 70 \( \mu \)m in height and 3 mm in diameter. Figure 2 illustrates the model used in this measurement campaign. The light green shaded area represents the region exposed to the laser, which was positioned downstream of the test section and shining upstream (marked by a dark green arrow in Fig. 2).

Since the mount was not perfectly rigid, deformations occurred during a wind tunnel run which led to a difference between the set and actual angles of attack of approximately \( \alpha_{set} - \alpha = 0.7^\circ \). The values of \( \alpha \) reported herein represent the actual angles of attack during the experiment, with an approximate uncertainty of \( \pm 0.2^\circ \). Although present, blockage due to the wing model was not deemed significant; at the highest set angle of attack of \( \alpha = 7^\circ \), the blockage ratio, defined as the ratio between the projected solid area and the cross-sectional area of the test section, was below 4%.

2.3 Particle image velocimetry

Planar PIV was performed on the suction side of the OAT15A model. Upstream of the settling chamber, two PIVTEC seeding generators seeded the flow with Di-Ethyl-Hexyl-Sebacat (DEHS) tracer particles via Laskin nozzles. The DEHS tracer particles have a mean diameter of \( d_p = 0.4 \mu m \) and an approximate density of \( \rho_p = 910 \text{ kg/m}^3 \) (Paredes et al. 2012). If the fluid’s dynamic viscosity is considered to be \( \mu_f = 1.655 \times 10^{-5} \text{ kg/s} \), and its density \( \rho_f = 1.37 \text{ kg/m}^3 \), the response time of the tracer particles is approximately (Raffel et al. 2018):

\[
\tau_p = \frac{d_p^2 (\rho_p - \rho_f)}{18 \mu_f} = 0.5 \text{ ms},
\]

which is deemed appropriate for the current study apart from a small region downstream of the shock. A dual-pulse Nd:YAG (532 nm wavelength) laser from Photonics Industries was used for all measurements. A combination of mirrors and lenses placed outside of the test section was used to form a light sheet and shine the laser tangentially along the top surface of the model in order to minimize surface reflections (Kähler et al. 2006). The illuminated region is visualized in Fig. 2. At a repetition rate of 10 kHz, each individual head of the laser system can produce an average power of 150 W, a pulse energy of 15 mJ and a pulse width of 200 ns. A 5 µs pulse separation was selected in order to reach a particle image displacement of approximately 13 pixel for the freestream velocity which is considered to be well-suited for high accuracy and low uncertainty, as discussed in Scharnowski and Kähler (2020). The repetition rate during data acquisition was set to 4 kHz. A DataRay laser beam profiler was used in order to estimate the laser sheet thickness. For the laser power used during the measurement campaign, the laser sheet thickness was approximately 0.5 mm. The 50 mm thick test section windows were manufactured from Poly(methyl methacrylate), commonly known as PMMA or acrylic, for mounting purposes. Therefore, care was taken to position the camera perpendicular to the window in order to reduce the adverse effects of refraction. The overall specifications of the experiment are summarized in Table 1.

For a single run, 10,000 double images were captured using the Phantom V2640 high-speed camera, for a total of 2.5 s. Since the buffer frequency was expected to be on the order of 100 Hz (Accorinti et al. 2021), several buffet cycles were captured during this time. LaVision’s DaVis software was used to process the images upon completion of data acquisition. A sliding background was subtracted with a kernel length of \( L = 6 \) pixel to pre-process the images. For all processing performed, a multi-pass interrogation algorithm was used; an interrogation window of 96 × 96 pixel was used for the three initial passes, followed by a

![Fig. 2 OAT15A model mounted in the TWM’s test section. The random dot pattern on the surface of the model allows for deformation measurements (Accorinti et al. 2021). Shown in light green is the area exposed to the laser, located downstream of the model and shining light in the direction of the dark green arrow](image-url)
refined interrogation window of 24 × 24 pixel for the last three passes. A 50% window overlap was used to highlight otherwise imperceptible small-scale features (Raffel et al. 2018). A two-dimensional median filter was also applied to remove outliers in the post-processing phase.

The measurement uncertainty of the estimated velocity data is influenced by many parameters (Sciacchitano and Wienke 2016). While the freestream velocity is believed to be accurate within 1% according to the findings in Scharnowski et al. (2019), regions of higher fluctuations are subject to larger uncertainties. The three-dimensional nature of the turbulent flow causes loss-of-pairs due to out-of-plane motion that cause a weakened correlation signal and thus higher noise for the shift vector detection. Additionally, small scale turbulent structures cause inhomogeneous motion of the traces within an interrogation window, which broadens and weakens the correlation signal. The chosen magnification and particle image displacement are a compromise between accuracy, spatial resolution and dynamic spatial range, as discussed in (Scharnowski and Kähler 2020). The uncertainty is assumed to be on the order of 0.1 pixel (corresponding to ≈ 3 m/s) for regions with highly turbulent flow. However, the spatial low-pass filtering by the interrogation windows, the lack of seeding in the near wall region and distortion due to the density changes can cause other significantly enhanced errors.

### 3 Results

#### 3.1 Description of flow features

As mentioned, a range of Mach numbers and angles of attack were tested to shed light on the buffet phenomenon. More specifically, the buffet frequency as well as its variation with aerodynamic parameters were investigated. Runs were first performed at a constant $\alpha = 5.8^\circ$ and varying $M_{\infty}$, followed by a constant $M_{\infty} = 0.74$ and varying $\alpha$ investigation. It must be noted that the $\alpha$ variation induced a slight variation in $M_{\infty}$; this was accounted for, however, by varying the diffusor cross section accordingly.

As an example, Fig. 3 displays two instances of the instantaneous velocity field at $M_{\infty} = 0.77$ and $\alpha = 5.8^\circ$. In the top image, the shock is located at its upstream turning point, while in the bottom image, it is at its downstream turning point. For each vertical distance above the upper surface, the shock location was identified corresponding to the point where $u = 340$ m/s. The detected shock is clearly visible in cyan, nearing the surface at $x/c \approx 0.25$ and 0.4 in the top and bottom instances, respectively. The shock curvature is most pronounced when the shock is located most upstream; far from the surface of the wing ($y/c \approx 0.4$), the shock resembles a normal shock, but becomes oblique as it reaches the surface of the wing. This oblique shock is a sign that the flow separates at the shock foot, which is also clearly observed as of $x/c \approx 0.6$. In the second instance, reverse

![Fig. 3](image-url)
flow cannot be observed. The shock remains relatively vertical at $x/c \approx 0.5$ for $y/c > 0.2$, before shifting slightly upstream at $x/c \approx 0.4$. It must be noted that the wind tunnel’s reference frame was selected in this figure, where the freestream velocity is exactly horizontal while the angle is pitched. The rotated coordinate system along the chord line of the airfoil is denoted as $(\hat{x}/c, \hat{y}/c)$ and will also be used throughout this manuscript.

Broadening the analysis, the shock position, $\hat{x}/c$, can be seen in Fig. 4, as a function of time for different normalized heights, $\hat{y}/c$. The two dotted lines correspond to the two instances highlighted in Fig. 3. Although not perfectly periodic, the different buffet cycles can clearly be seen, having an approximate period of 10 ms.

The shock position statistics are shown in Fig. 5 for a vertical distance above the upper surface equivalent to $\approx 10\%$ of the chord length; this corresponds to $\hat{y}/c = 0.17$. This value was chosen to easily compare results to Accorinti et al. (2022). The shock position is plotted as a function of both $M_\infty$ (left) and $\alpha$ (right). In order to highlight the amplitude of the shock motion, the 0.25 and 0.75 quantiles are displayed as well as the full range of all positions. In the case of a constant angle of attack (left plot), as the freestream Mach number is increased from $M_\infty = 0.72$ to 0.74, the median shock position moves downstream. Starting from $M_\infty = 0.74$, the range of the shock motion signal becomes significantly larger, a sign that buffet onset has already taken place. Moreover, the inversion of shock motion occurs, which confirms the presence of large separation in the flow. Interestingly, the median shock position does not seem to vary between $M_\infty = 0.75$ and 0.77. In the case of a constant Mach number (right plot), strong shock oscillations are visible as of $\alpha = 5.7^\circ$. This angle is higher than the one corresponding to the inversion of shock motion ($\alpha \approx 4.9^\circ$). This confirms the findings in Accorinti et al. (2022), according to which the inversion of shock motion is a necessary condition for buffet onset in the case of increasing angle of attack. The results in Fig. 5 highlight differences with respect to the relevant literature in terms of buffet onset and offset. Experimentally, Jacquin et al. (2009) found that buffet onset occurred at $\alpha = 3.2^\circ$ for $M_\infty = 0.73$, whereas in the present study, it is delayed by approximately $2^\circ$ for similar Mach numbers. Moreover, in the numerical simulations of Giannelis et al. (2018), shock buffet is quenched for $M_\infty \geq 0.75$, while in the present work, shock oscillations can still be detected for $M_\infty \geq 0.77$. These differences were already addressed in Accorinti et al. (2022). Potential causes, such as three-dimensional effects and wind tunnel characteristics, were suggested. In the same study, a detailed comparison of numerous literature results revealed a general sensitivity of the buffet features (frequency and amplitude of the oscillations, mean shock location as well as onset) to the numerical and experimental boundary conditions.

Taking a closer look at the temporal evolution of the shock location, Fig. 6 helps visualize the shock location and its motion for a variety of $M_\infty$ (top) and $\alpha$ (bottom), in the wing’s reference frame. Similar to what was displayed in Fig. 5 for one wall-normal distance, 20 time steps are visually depicted in different colors for a variety of wall-normal distances. The dashed line represents the median shock location, while the dotted lines indicate 99% of the full shock motion. Only the upstream travel of the shock was plotted for readability.

At $t = 0$ ms, the shock is positioned most downstream, while it completes half its full cycle at $t = 5$ ms; this remains true for all flow conditions. As $M_\infty$ increases, the shock travels upstream and its amplitude increases. At $M_\infty = 0.77$, the
oblique shock foot is most pronounced, spanning a larger streamwise distance. This suggests the presence of a stronger boundary layer separation. That being said, the shock motion remains larger at $M_\infty = 0.75$ than at 0.77.

Similarly, the shock location appears to move upstream with an increase in angle of attack. The different time steps almost collapse at $M_\infty = 0.74$ and $\alpha = 5.3^\circ$, indicating the initial stage of shock buffet. As $\alpha$ increases, the shock curvature increases and the oblique shock foot is present for a wider $\hat{y}/c$ range; at $\alpha = 5.3^\circ$, the oblique shock ends around $\hat{y}/c = 0.2$, while it ends at $\hat{y}/c = 0.3$ when $\alpha = 6.3^\circ$. This indicates an increase in the extent of boundary layer separation.

The velocity data were further analyzed in a limited time interval ($\Delta t = 100\text{ms}$) to give a qualitative overview of the boundary layer evolution and the variation in wake size. In Fig. 7, the streamwise velocity, $u$, is plotted as a function of both time, $t$, and non-dimensional streamwise position, $\hat{x}/c$, for both pre-buffet and buffet cases. The data shown here were extracted at a fixed $\hat{y}/c = 0.1$ and $\alpha = 5.8^\circ$. In the top image, $M_\infty = 0.72$, while it is 0.77 in the bottom figure. In both scenarios, the angle of attack is kept constant at $5.8^\circ$ and 100 ms of data are shown, equivalent to approximately 11 buffet cycles. This allows for a much longer visualization of the temporal evolution of the shock wave compared to Fig. 6. The dashed line represents the median shock position, while the dotted lines indicate 99% of the full shock motion. The shock position ranges from $\hat{x}/c = 0.35$ to 0.43 and from 0.28 to 0.46 in the pre-buffet and buffet cases, respectively. At $M_\infty = 0.72$, the shock buffet phenomenon has not yet reached a fully developed state as large-scale amplitudes and reverse flow downstream of the shock are absent. At $M_\infty = 0.77$, however, developed shock buffet can be observed. As expected, strong boundary layer separation
is observed when the shock is most upstream, at \( \hat{x}/c \approx 0.3 \). As the shock moves downstream, the boundary layer reattaches and a subsequent cycle is initiated.

Probing further into the wake structure, the streamwise velocity in the wake can be seen in Fig. 8 as a function of both time and normalized vertical distance. The horizontal position was, in this case, set to \( \hat{x}/c = 1.3 \), the angle of attack was again kept constant at \( \alpha = 5.8^\circ \) and the Mach number was increased from \( M_\infty = 0.72 \) (top) to 0.77 (bottom). The two dotted lines in the bottom image correspond to the snapshots shown in Fig. 3.

In Fig. 8 represent the two instances shown in Fig. 3. The width of the separated region is larger for the first instance (shock at its upstream turning point) than for the second (shock at its downstream turning point), reaching a maximal vertical distance of \( \hat{y}/c \approx 0.3 \). Figures 7 and 8 corroborate the findings of Jacquin et al. (2009), who phase averaged the flow field’s Mach number distribution and found a maximal thickening of the wake when the shock was at its most upstream location.

### 3.2 Shock dynamics

A spectral analysis of the shock location was carried out in order to extract the dominant frequency of the shock motion. Figure 9 displays the spectra at various aerodynamic parameters. On the left, the spectra at \( M_\infty = 0.74, 0.75 \) and 0.77 are plotted at a constant \( \alpha = 5.8^\circ \). On the right, \( M_\infty \) is kept constant at 0.74, while the angle of attack is varied from \( \alpha = 5.3 \) to 6.3°. The normalized power spectral densities are
The normalized power spectral density of the pressure signal, \( \phi_{p,c} \), was obtained from XEQ-062 pressure transducers located on the upper surface of the wing model and mounted along the chord direction. Simultaneous pressure measurements were obtained from a second pressure tap as a comparative measure. This second tap captured the frequency content of the boundary layer more and hence recorded lower power spectral density. For each pressure tap, the variation in both the freestream Mach number (left plots) and the angle of attack (right plots) was investigated, while maintaining the other variable constant. Focusing on the left plots, as \( M_\infty \) increases, the value of \( k \) corresponding to a peak in \( \phi_{p,c} \) also increases. Focusing particularly on the data obtained at \( \frac{x}{c} = 0.42 \), the peak is smaller for the \( M_\infty = 0.77 \) case than for the other two cases. Recalling Fig. 5, given that the shock is rarely positioned at \( \frac{x}{c} = 0.42 \) although the shock motion remains large, this might lead to a reduced signal at \( M_\infty = 0.77 \). However, at the same Mach number but slightly downstream at \( \frac{x}{c} = 0.57 \), there does not appear to be a pronounced peak at all. Three phenomena could cause this flattening, notably the strong boundary layer separation dampening the peak, the buffet phenomenon moving towards its offset and a stronger fluid structure interaction with heave. The power spectral density peak also seems to shift towards greater values of \( k \) as \( \alpha \) increases. This agrees well with the results shown in Fig. 9 as well as the findings of McDevitt and Okuno (1985). For instance, at \( \frac{x}{c} = 0.42 \) and \( \alpha = 5.8^\circ \), the dominant reduced frequency increases from \( k = 0.19 \) at \( M_\infty = 0.74 \) to \( k = 0.23 \) at \( M_\infty = 0.77 \), while it ranged from \( k = 0.20 \) to \( 0.22 \) in Fig. 9. Similarly, a positive shift occurs with an increase in \( \alpha \); again at \( \frac{x}{c} = 0.42 \), the dominant reduced frequency increases from \( k = 0.17 \) to \( 0.24 \) as \( \alpha \) increases from \( 5.3 \) to \( 6.3^\circ \), whereas it varied from \( k = 0.14 \) to \( 0.25 \) in Fig. 9. The same peaks are found with the pressure tap slightly downstream, at \( \frac{x}{c} = 0.57 \).

Looking closely at the power spectral density obtained via the pressure sensor at \( \frac{x}{c} = 0.42 \), two smaller peaks can be observed at \( k \approx 1.8 \) and \( 3.3 \) (or, equivalently, 920 and 1690 Hz). The peak at \( k \approx 1.8 \) can also be seen in the PIV data, for the case of \( M_\infty = 0.74 \) and \( \alpha = 5.3^\circ \). The origin of the peaks at higher frequency than the fundamental buffet frequency could be upstream travelling waves (UTW), although this hypothesis remains to be further investigated. D’Aguanno et al. (2021) observed UTW at a tenfold higher frequency than the buffet frequency.

![Fig. 9](image_url)  
**Fig. 9** Power spectral density of the shock location, \( \phi_{p,c} \), as a function of reduced frequency, \( k \), for three values of \( M_\infty \) at a constant \( \alpha = 5.8^\circ \) (left) and three values of \( \alpha \) at a constant \( M_\infty = 0.74 \) (right). The different colors represent different runs.
(in their case 2000 Hz, or equivalently, \( k = 2.76 \)), which compares well with both the findings of Hartmann et al. (2013) and the current study. The slightly smaller peak at \( k \approx 3.3 \) could potentially be a higher harmonic of the peak indicative of UTW. It must be noted that both peaks are approximately two orders of magnitude smaller than the one indicative of shock buffet.

Additionally, the power spectral density was computed for each streamwise velocity fluctuation obtained via PIV as a comparative measure to the two previous spectral analyses. A field average was later computed, from which the dominant frequency of \( f_{\text{dom}} = 98\text{Hz} \) or, equivalently, \( k_{\text{dom}} = 0.19 \), was extracted for the case where \( \alpha = 5.8^\circ \) while \( M_\infty = 0.74 \). This frequency closely matches the results obtained previously via shock location and surface pressure measurements. Figure 11 highlights the regions exhibiting this dominant frequency; the normalized power spectral density at \( k_{\text{dom}} \) is plotted within the field of view. Interestingly, this dominant frequency appears mostly at the shock and the wake regions, showcasing a hundred-fold increase in energy compared to the remaining flow field. This result closely matches that of Crouch et al. (2007).

### 3.3 Extraction of downstream convective velocity

According to the model derived by Lee (2001), shock buffet is sustained by a feedback mechanism between downstream and upstream travelling waves. The velocity of the downstream travelling waves was estimated in Lee (2001), Xiao et al. (2006), Deck (2005), Jacquin et al. (2009), Garnier and Deck (2010) as well as Hartmann et al. (2013) by correlating the pressure signals on the upper surface. In Hartmann...
et al. (2013), the convection velocity was also computed by correlating the absolute velocity fluctuations at various streamwise positions along a time-averaged streamline above the region of separated flow, yielding a good agreement with the one obtained via pressure sensors. Determining the convective velocity at which the downstream travelling waves travel is an important part of the validation of Lee’s model.

Due to the available highly resolved field information obtained via PIV, a similar analysis was performed herein, where the streamwise velocity correlation, $R_{uw}$, was computed at various streamwise and wall-normal positions close to the model’s upper surface. $R_{uw}$ is defined as follows:

$$R_{uw}(n, m, \tau) = \frac{u'_n(t)u'_m(t + \tau)}{\sqrt{u'_n(t)^2u'_m(t)^2}}, \quad (2)$$

where $u'_n(t)$ and $u'_m(t + \tau)$ represent streamwise velocity fluctuation signals at positions $(x_n/c, y_n/c)$ and $(x_m/c, y_m/c)$ respectively, the second data set having been shifted by $\tau$. Their product is then averaged over all time steps, $t$. As for $\sqrt{u'_n(t)^2}$ and $\sqrt{u'_m(t)^2}$, they represent the standard deviation of both fluctuating velocity signals.

The correlation of the streamwise velocity was computed for different values of $(\hat{x}_n/c, \hat{y}_n/c)$, ranging from $0.6 \leq \frac{\hat{x}_n}{c} \leq 1.1$ and $0.10 \leq \frac{\hat{y}_n}{c} \leq 0.20$, in order to examine how flow structures convect within and slightly above the wake.

Color-coded between $-1$ and $1$, Fig. 12 displays $R_{uw}$ for three consecutive time steps, where $\tau = -0.25$ and $\tau = 0.25$ms are both associated with a negative and positive shift of one snapshot, respectively. The x-shaped marker represents the $(\hat{x}_n/c, \hat{y}_n/c)$ position, while the circular marker represents the maximum $R_{uw}$ value along $\hat{y}_n/c$ for a given $\tau$. That being said, the x-shaped marker position remains unchanged as $\tau$ varies, while the circular marker detects the streamwise position which is most correlated to the origin for a specific $\tau$ and therefore varies in time. Detecting this maximum enables the computation of the convective velocity, $u_c$, defined as:

$$u_c(\tau) = \frac{\hat{x}_{m,\text{max}} - \hat{x}_n}{\tau}. \quad (3)$$

Since data with two non-zero values of $\tau$ were computed, an average velocity was extracted:

$$u_{c,\text{avg}} = \frac{u_c|_{\tau=-0.25} + u_c|_{\tau=0.25}}{2}. \quad (4)$$

Figure 13 is a compilation of the average convective velocity for different values of $\frac{\hat{x}_n}{c}$ and $\frac{\hat{y}_n}{c}$, where three compelling aspects can be highlighted. Firstly, the convection velocity exhibits an increase as it moves downstream. Focusing on the pink data points representing $\frac{\hat{y}_n}{c} = 0.19$, $u_{c,\text{avg}} = 79$ m/s ($\approx 0.33u_\infty$) at $\frac{\hat{x}_n}{c} = 0.6$, while it is rather $\approx 150$ m/s ($\approx 0.63u_\infty$) at $\frac{\hat{x}_n}{c} = 1.1$. A similar trend can be recognized for the other wall-normal positions. A streamwise increase of the downstream convection velocity was also reported in Lee (2001) for some aerodynamic cases. However, in Xiao et al. (2006), Deck (2005), Jacquin et al. (2009), Garnier and Deck (2010) as well as Hartmann et al. (2013), constant downstream convection velocities were found: $0.12u_\infty$, $0.05u_\infty$, $0.07u_\infty$, $0.06u_\infty$, and $0.08u_\infty$, respectively.

Secondly, the values of convection velocity measured in the mentioned literature are significantly smaller than the ones in Fig. 13. Nonetheless, it should be recalled that in the present work, the convection velocity was not...
estimated at the model’s surface but at a certain distance from it (between $\frac{\hat{y}_n}{c} = 0.1$ and $\frac{\hat{y}_n}{c} = 0.2$). Therefore, the so-computed velocity could correspond to vortical structures travelling within the mixing layer. In Jacquin et al. (2009), it was stressed that if one associated the downstream travelling waves of Lee’s model to Kelvin-Helmholtz type structures in the mixing layer instead of to perturbations at the surface, a value on the order of $0.5u_{\infty}^{\circ}$ should be expected for the convection velocity. Neither in the present work nor in Xiao et al. (2006), Deck (2005), Jacquin et al. (2009) or in Garnier and Deck (2010) was the estimated shedding frequency of the vortical structures associated with the computed convection velocities. Therefore, no proof was provided that these very downstream travelling structures are responsible for the formation of upstream travelling waves. Only in Hartmann et al. (2013) was a model proposed, which connects downstream travelling vortical structures detected via PIV to the generation of acoustic waves. The characteristic length of the vortices was extracted from temporal snapshots. The velocity of the vortices was not directly measured. Instead, the hypothesis was made that they travel at the convection velocity ($0.08u_{\infty}^{\circ}$) estimated via cross-correlation of the pressure signal at the surface. The shedding frequency of the vortices, computed by dividing their velocity by their characteristic length, agreed well with the one of the UTW. Based on the equality between these two shedding frequencies, Hartmann et al. (2013) gave a detailed description of the physical mechanism of shock buffet. The variation in strength of the vortical structures passing by the trailing edge and the pulsation of the separated boundary layer modulated the strength of acoustic UTW. The variation in intensity of the acoustic waves sustained the shock oscillation and completed the feedback. However, the vortices detected via PIV lay at a distance from the upper surface that is comparable to the ones investigated in the present study. Therefore, it cannot be excluded that their convection velocity should be considered higher. This would result in a mismatch between the shedding frequencies of the downstream travelling vortices and of the upstream travelling waves and would weaken the model proposed by Hartmann et al. (2013). This mismatch was found in D’Aguanno et al. (2021), where downstream travelling vortical structures were detected in the shear layer via autocorrelation of POD-reconstructed PIV snapshots. Their characteristic length was also extracted from time snapshots and their velocity was assumed to be equal to the average of the streamwise velocity in the shear layer region where the correlation was performed ($\approx 0.5u_{\infty}^{\circ}$). The resulting shedding frequency of the vortices was approximately three times higher than the one computed for the UTW. It should be stressed that both in D’Aguanno et al. (2021) and Hartmann et al. (2013) was an assumption made for the velocity of the vortices, which led to two remarkably different results. Further research is needed to understand which of the downstream travelling structures are actually responsible for the formation of the upstream travelling waves at the trailing edge.

Finally, a general sensitivity of the convection velocity to the wall-normal distance can also be observed in Fig. 13. Although the trend of the convection velocity appears to converge irrespective of $\frac{\hat{y}_n}{c}$ for a given $\frac{x_n}{c} > 0.7$, differences on the order of 10 m/s can be spotted at several streamwise positions. A closer look was taken to understand the spread around $\frac{x_n}{c} = 0.6$. The lower values of $u_{c,avg}$ occur at $\frac{\hat{y}_n}{c} = 0.16$ and 0.17, which correspond to the wake-freestream interface, where a distinct correlation peak is not present. Additionally, the sudden drops in $u_{c,avg}$, which sometimes occur for $\frac{\hat{y}_n}{c} > 0.17$, are a result of the wake-freestream interface’s motion. An upward shift in the interface would lead to a sudden uncorrelation between the origin and the structures downstream. Uncertainties in estimating the location of maximal $R_{nu}$ might also lead to the discontinuous increase in $u_{c,avg}$. The correlating flow structures are of different size and shape and, consequently, the correlation shows a rather broad and weak peak. This makes it difficult to estimate its maximum location accurately. The uncertainty of the maximum locations for $\tau = \pm 0.25$ ms is estimated to be on the order of 5% of the chord length. This leads to the spread of the estimated convection velocity shown in Fig. 13.
4 Conclusions and outlook

The shock buffet phenomenon was investigated for a range of freestream Mach numbers ($0.65 < M_\infty < 0.77$) and angles of attack ($3.8 < \alpha < 6.3^\circ$) with the use of particle image velocimetry. The objectives of this study included obtaining highly resolved measurements of this complex phenomenon and drawing conclusions regarding the effect of aerodynamic parameters on the mean shock position, shock amplitude and buffet frequency. An increase in both $M_\infty$ and $\alpha$ initially resulted in a downstream shock motion, followed by an upstream travel of the shock, i.e. towards the wing’s leading edge, once large separation occurred. Additionally, a general increase in shock oscillation amplitude was observed, reaching a maximum for the case of $M_\infty = 0.75$ and $\alpha = 5.8^\circ$. The shock curvature also varied with both an increase in $M_\infty$ and $\alpha$; the oblique shock section appeared to cover a larger vertical distance as $M_\infty$ increased from 0.74 to 0.77 and as $\alpha$ increased from 5.3 to 6.3°. In addition to the shock statistics, the evolution of the boundary layer and wake downstream of the shock was investigated. Similar to the findings of Jacquin et al. (2009), a large separation region was observed when the shock was at its most upstream position. Extracting the streamwise velocity for various streamwise and vertical distances clearly showcased the wide separated region at regular intervals for the buffet case, spanning a maximal vertical distance of $\Delta y/c \approx 0.3$ when the shock was located most upstream.

A spectral investigation of the shock motion confirmed an increase in dominant frequency with an increase in both $M_\infty$ and $\alpha$, an observation in support of Giannelis et al. (2018) and Accorinti et al. (2022). The dominant shock motion frequencies were compared to that obtained via surface pressure measurements and similar peaks were found. The fundamental frequency obtained via PIV ranged from $k = 0.20$ to 0.22 when varying $M_\infty$ from 0.74 to 0.77 (for $\alpha = 5.8^\circ$), while it varied from $k = 0.19$ to 0.23 for the pressure signal obtained at $\hat{x}/c = 0.42$. Similarly, an increase from $k = 0.14$ to 0.25 was observed in the PIV data as $\alpha$ increased from 5.3 to 6.3° (at $M_\infty = 0.74$), while it was rather from $k = 0.17$ to 0.24 for the pressure signal obtained at $\hat{x}/c = 0.42$. Additionally, the power spectral density was computed for every velocity vector, from which a peak frequency of $k_{dom} = 0.19$ was extracted. The regions exhibiting this peak frequency were the moving shock as well as the downstream wake; this result was expected given the known coupling between both regions.

The propagation properties of coherent flow structures travelling downstream of the shock were also examined. The flow structures’ convective velocity consistently increased with downstream position, ranging on average between $u_{c,avg} \approx 50 m/s$ at $\hat{x} = 0.6$ and $150 m/s$ at $\hat{x} = 1.1$. These velocity values are significantly higher than those obtained by other groups via cross-correlation of the pressure signals at the surface. This could be due to the fact that the convection velocity in the present study was estimated by correlating the velocity fluctuations at a certain distance from the surface and may correspond to structures travelling within the mixing layer. This variation in propagating speed in both streamwise and normal direction should be accounted for in the modeling of the shock wave-wake modulation. In fact, by carefully comparing the literature results, it emerged that the shedding frequency of the downstream travelling vortices significantly changes depending on the assumed convection velocity (in the mixing layer or at the surface). Therefore, it is probably necessary to directly measure the velocity of the vortices in order to correctly compute their shedding frequency and verify whether it coincides with the one of the UTW.

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