Real-time Demodulation of Real Power Oscillations*

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Abstract: In this paper, a real-time demodulation of real power oscillations in a three phase electric power system is proposed. It is shown how demodulated real power oscillations can be used to formulate a low order state space model that models power oscillations. The procedure is illustrated on the measurements obtained from three phase Resistor-Inductor-Capacitor (RLC) network where the power oscillation frequency and model order is known and used for comparison and validation of the method.

Keywords: Micro-Grid; Real-Time Demodulation; Real Power Oscillations; Step-Based Realization Algorithm.

1. INTRODUCTION

As more renewable energy generation is added to the utility grid, less conventional generation will be required to meet the power demand. Photovoltaics (PV), the major way of converting sunlight into electricity, is a fast-growing technology doubling its worldwide installed capacity every couple of years due to its scalability from small, residential and commercial rooftop or building integrated installations, to large utility-scale solar plants. Typically, solar energy generation uses (3 phase) inverters that have fast dynamics and exhibit very little inertia in terms of power delivery onto the utility grid.

Utilizing more renewable energy generation leads to inherent variability in energy production. However, it also reduces the rotational inertia in the form of spinning rotational mass from conventional generation that tends to stabilize and maintain synchronous operation of the system (Elgerd, 1982). This could result in increasing instability and poorly damped oscillations in AC frequency and power, unless additional conventional generating sources are placed on-line or less renewable resources are installed.

Such circumstances have been detected in practice and installation of Phasor Measurement Units (PMU) facilitate real-time measurements of power quality and power oscillations in an electricity grid. An example of such power oscillations can be observed in Fig. 1, where oscillations in real power were observed in the 12kV connections at the University of California during a particular load switching (de Callafon and Wells, 2014).

In general, electric power systems are subjected to power oscillations due to the inherent inertia of generators and loads connected on the electric grid (Elgerd, 1982; Akagi et al., 2007). Such power oscillations are typically in the 0.2-3 Hz range, depending on the size of the (micro)grid and the characteristics of the interconnected power sys-

![Fig. 1. Measured real power oscillation on the main 3 phase interconnect of the UCSD Micro-Grid during a stepwise load demand change.](image_url)
systems (Kundur et al., 1994; Rogers, 2000). Detecting fluctuations in power flow in an electric grid has been an active field of study to improve the resiliency of electric networks. Power swing detectors that can detect unstable power swings in several milli-seconds are crucial for relay operation (Hemmingsson, 2003). In case of stable power oscillations, frequency and damping of electro-mechanical oscillations can be performed with a ring down analysis or a normal operation analysis. Assuming an unknown non-zero initial condition, eigenvalues or the frequency/damping of the observed power oscillations can be computed using the Pronys method for ring down analysis (Hauer et al., 1990; Pierre et al., 1992; Sanchez-Gasca and Chow, 1999) assuming the power oscillation is a sum of sinusoids (Trudnowski and Pierre, 2009) or more advanced methods using wavelet transforms (Rueda et al., 2011). In these methods, power oscillation dynamics is found by fitting models on the free response of an observed stable power oscillation.

The disturbance causing power oscillations can be a line switching, load switching, a fault or anything else that may have a large impact on the power flow through the power system. As these disturbances are typically step disturbances, explicit information on the shape of the input signal that caused the power oscillation will be beneficial, especially when multiple step signals occur in close proximity in time. Explicit use of input and observed output signals via a system identification procedure (Sanchez-Gasca and Chow, 1999; Ghasemi, 2006) will improve the quality of the models that capture the power oscillations. This paper shows how three phase real power oscillations can be measured in real-time by an appropriate demodulation and filtering of three phase AC voltages and currents.

The proposed real-time demodulation included in this paper ensures that three phase real power oscillations can be demodulated from transient effects of AC network. Subsequently, it is shown how a low order state space model can be realized on the basis of real-time measurements of three phase real power oscillations. The realization algorithm specifically uses the transient effects to formulate a low order model that accurately captures frequency and damping of the power oscillations. The approach is similar to the modal analysis approach in Rogers (2000) but allows the low order models to be formulated directly on the basis of real-time power oscillations. To verify the effectiveness of the approach, the methodology is illustrated on the measurements obtained from three phase RLC network where the power oscillation frequency and model order is known and used for comparison and validation of the method. The approach shows how the power oscillation frequency can be recovered from the real-time measurements.

2. EXPERIMENTAL SETUP

An experimental setup is required to verify the performance of three phase real power oscillations and possibly install a real-time damping control system. The experimental setup is used to repeat and initiate the scenario of an oscillatory three phase power disturbance similar to what could be observed on the real power grid. As DC power created by PV panels is exported to the grid via an inverter, a Grid-Tied Inverter (GTI) is used to synchronize the AC output with the grid. According to such circuit topology, an experimental setup is built as shown in Fig. 2. For testing purposes, the PV system is temporarily

Fig. 2. Diagram of experimental setup with DC power supply simulating the PhotoVoltaic (PV) power source, an EMI filter to reduce AC ground coupling and a Grid-Tied Inverter (GTI) to provide 3 phase AC power. The GTI is controlled by an external controller that can control the four quadrant power flow through the GTI, while the controller also digitally switches an auxiliary relay to switch in a three phase Resistor-Inductor-Capacitor (RLC) circuit to initiate three phase power oscillations in the circuit. Three phase voltage and current measurements (sensors) are processed by the controller to compute real-time power oscillation in the circuit.
replaced by a programmable DC power source. The GTI is a GT13100A6208/3652IR-PQ manufactured by One-Cycle Control Incorporation. It is a four-quadrant GTI, which is capable to accept external control signals for implementation of feedback control to control or damped power oscillations. Additional EMI filters FN2200B are placed between the DC source and the inverter to eliminate the effect of common AC mode currents due to the high frequency Pulse Width Modulation (PWM) of the GTI.

A three-phase RLC load circuit is designed and integrated into the testbed to act as a real power disturbance. As depicted in Fig. 2, each phase is composed by a bypass resistor of 100Ω that is in parallel with a series connection of a capacitor of 0.01F and an inductor of 0.1H. The Inductor-Capacitor (LC) circuit is to generate a resonance; the bypass resistor is to consume real power and also discharge the LC circuit while it is not energized. The circuit is connected to the output of the grid-tied inverter through an overload protection relay.

A controller with National Instruments (NI) myRIO is integrated into the testbed for data acquisition and controlling the grid-tied inverter. The three-phase AC voltage and current signal of grid-tied inverter is measured, conditioned, and sent into the controller. The controller can also send out control signals via signal conditioning circuit to drive the grid-tied inverter and moreover, to switch in the load circuit to the system by energizing the overload protection contactor via an auxiliary relay. The description of the testbed is completed by a photo as shown in Fig. 3. The parts are aligned and mounted in a cabinet for safety consideration. In the RLC load circuit, an array of AC capacitors is formed as a capacitive load.

The control diagram of the testbed is depicted in Fig. 4. The model $G$ represents the grid-tied inverter, while $H$ is the dynamic model of the RLC load circuit. In this paper, we are particularly interested in modeling the real power dynamics of the RLC load circuit by real-time real power demodulation.

3. REAL-TIME REAL POWER ANALYSIS

For control or mitigation of real or complex power oscillations, special care should be given to the time varying nature of the moving average values of the power signals. In the following discussion, the time varying behavior of the power signals can be derived as a multiplication of the AC grid frequency $\omega = 2\pi f$, $f = 60$Hz and the oscillations due to power fluctuations that may have a smaller oscillation frequency $f_d < f$. For real-time control, only the power oscillations with the frequency $f_d < f$ are of interest and detection of these power oscillations requires a demodulation of the power signals. The analysis in this section is in continuous-time cases and can be easily extended to the discrete-time case.

3.1 Analysis of Transient Effects

For the analysis of the transient effect, it is assumed that the three-phase voltage signals are time synchronized according to

$$v_A(t) = V \cos(\omega t)$$  \hspace{1cm} (1)
$$v_B(t) = V \cos(\omega t - \frac{2\pi}{3})$$ \hspace{1cm} (2)
$$v_C(t) = V \cos(\omega t - \frac{4\pi}{3})$$ \hspace{1cm} (3)

and higher order harmonics are ignored initially, to simplify the analysis. It will be shown that low pass filtering is used to reduce the effect of higher harmonics on the 3 phase AC voltage and current signals.

The three-phase symmetric RLC circuit used in this paper serves as a case study for the power oscillations and is used...
in the derivation of the results. Based on second order linear time-varying (LTV) dynamics of an RLC circuit, the transient effects in the current signals can be represented by

\[ i_A(t) = I_d \cos(\omega t) + I_d^A e^{\lambda t} \cos(\omega_d t - \beta) \]
\[ I_d^A = I_d \cos(\omega t) \]
\[ i_B(t) = I_d \cos(\omega t - \alpha - \frac{\pi}{3}) + I_d^B e^{\lambda t} \cos(\omega_d t - \beta) \]
\[ I_d^B = I_d \cos(\omega t - \frac{2\pi}{3}) \]
\[ i_C(t) = I_d \cos(\omega t - \alpha - \frac{4\pi}{3}) + I_d^C e^{\lambda t} \cos(\omega_d t - \beta) \]
\[ I_d^C = I_d \cos(\omega t - \frac{4\pi}{3}) \]

where \( \omega_d = 2\pi f_d < \omega \) is the (damped) oscillation frequency of the (power) transient with a phase shift of \( \beta \) and an exponential decay \( \lambda < 0 \). It should be noted that due to the three phase time synchronization, each current signal has a different initial condition \( I_d^A, I_d^B \) and \( I_d^C \).

Taking Phase A as an example, the instantaneous power \( p_A(t) = p_{dA}(t) + p_{fA}(t) \) can now be written as

\[ p_A(t) = VI \cos(\omega t) \cos(\omega t - \alpha) \]
\[ + \frac{V I_d}{2} \cos(2\omega t - \alpha) \cos(\omega t) \cos(\omega_d t - \beta) \]
\[ + \frac{V I_d}{2} \cos(\omega_d t)e^{\lambda t} \cos((\omega - \omega_d)t + \beta) \cos(\omega t) \]
\[ + \frac{V I_d}{2} \cos(\omega_d t)e^{\lambda t} \cos((\omega + \omega_d)t - \beta) \cos(\omega t) \]

From (6) it is clear that by computing a moving average over a single period of \( \frac{2\pi}{\omega} \), the first three cosine terms in (6) reduce to zero. Moving average filtering can be implemented in real-time using a discrete-time Finite Impulse Response (FIR) filter \( F_{FIR}(q) \). The last two terms have a frequency \( 2\omega \pm \omega_d \) and do not reduce to zero with moving average, but since \( 2\omega \pm \omega_d > \omega_d \), these terms can be reduced significantly by a discrete-time low pass filter \( F_{LP}(q) \) with a cut-off frequency just above \( \omega \). Low pass filtering will also reduce any higher harmonics that may be present on the 3 phase voltage and current signals. Hence, through filtering and modulation, a power signal \( p_{dA}(t) = F(q)p_A(t) \cos(\omega t) \) is obtained that can be approximated by

\[ A_F(\omega_d) \cdot \frac{V I_d}{2} \cos(\omega_d t - \beta + \phi(\omega_d)) \]

where \( \phi(\omega_d) \) is the discrete-time filter combination of the FIR filter and a low-pass filter as described above, \( A_F(\omega_d) \) and \( \phi(\omega_d) \) are the gain and the phase shift of filter \( F(q) \) at the frequency \( \omega_d \), respectively.

For the other two phases, the same procedure can be applied to obtain the modulated real power \( P_{dB}(t) \) for phase \( B \) given by

\[ A_F(\omega_d) \cdot \frac{V I_d}{2} \cos(\omega_d t - \frac{2\pi}{3})e^{\lambda t} \cos(\omega_d t - \beta + \phi(\omega_d)) \]

and the modulated real power \( P_{dC}(t) \) for phase \( C \) as

\[ A_F(\omega_d) \cdot \frac{V I_d}{2} \cos(\omega_d t - \frac{4\pi}{3})e^{\lambda t} \cos(\omega_d t - \beta + \phi(\omega_d)) \]

The modulated real power signals for each phase can now be used to compute the three phase real power oscillations.

3.3 Reconstruction of Three-Phase Real Power Oscillations

Applying the Clarke transformation to single-phase components obtained from (7), (8) and (9), the phasors are projected onto a decoupled coordinate \( \alpha - \beta \) given by

\[
\begin{bmatrix}
P_a(t)
\end{bmatrix} = 2 \frac{1}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
P_{dA}(t)
\end{bmatrix} = A_F(\omega_d) \cdot \frac{V I_d}{2} e^{\lambda t} \cos(\omega_d t - \beta + \phi(\omega_d)) \]

Then it can be seen that \( P_a(t)^2 + P_b(t)^2 \) satisfies

\[ \left( A_F(\omega_d) \cdot \frac{V I_d}{2} e^{\lambda t} \cos(\omega_d t - \beta + \phi(\omega_d)) \right)^2 \]

In practice, the direction of real power is usually a priori knowledge. As such, the three-phase real power oscillation can be reconstructed from the demodulated single-phase components.

4. CHARACTERIZING POWER OSCILLATION DYNAMICS BY STEP-BASED REALIZATION

A key assumption that could be made when a power oscillation occurs is to assume that the power oscillation is due to a step-wise change in load demand. The size of the load demand may not be known, but the a priori knowledge of the step-wise load demand can be exploited to formulate a low order state space model to model the dynamics of any observed power oscillations. In particular,
the low order state space model can be realized on the basis of a real-time measurements of three phase real power oscillations to accurately model frequency and damping of the power oscillations. Although the approach is similar to the modal analysis approach in Rogers (2000), the proposed realization method in this paper allows the low order models to be formulated directly on the basis of real-time measurements of power oscillations. More details on the step-based realization algorithms in included below.

4.1 The Step-Based Realization Algorithm

Let \( \{ y(0), y(1), \ldots, y(N) \} \) be a measured response of an LTI, single-input-multi-output (SIMO) system to a unit-step input applied at \( t = 0 \) that is corrupted by some possibly-colored measurement noise \( v(t) \). To estimate a state space model of the system

\[
\begin{align*}
    x(t+1) &= Ax(t) + Bu(t) \\
    y(t) &= Cx(t) + Du(t) + v(t),
\end{align*}
\]

one may follow the following steps:

- **Step I**
  Construct the block-Hankel data matrices
  \[
  Y = \begin{bmatrix}
    y(1) & y(2) & \cdots & y(l) \\
    y(2) & y(3) & \cdots & y(l+1) \\
    \vdots & \vdots & \ddots & \vdots \\
    y(r) & y(r+1) & \cdots & y(N-1) \\
    y(l+1) & y(l+2) & \cdots & y(N)
  \end{bmatrix},
  \]
  and matrices
  \[
  M = \begin{bmatrix}
    y(0) & y(0) & \cdots \\
    y(1) & y(1) & \cdots \\
    \vdots & \vdots & \ddots \\
    y(r-1) & y(r-1) & \cdots
  \end{bmatrix}, \quad M = \begin{bmatrix}
    y(1) & y(1) & \cdots \\
    y(2) & y(2) & \cdots \\
    \vdots & \vdots & \ddots \\
    y(r) & y(r) & \cdots
  \end{bmatrix}.
  \]

- **Step II**
  Construct matrices
  \[
  R = Y - M
  \]
  then take the singular value decomposition (SVD) of the matrix \( R \):
  \[
  R = \begin{bmatrix} U_n & U_s \end{bmatrix} \begin{bmatrix} \Sigma_n & 0 \\ 0 & \Sigma_s \end{bmatrix} \begin{bmatrix} V_n & V_s \end{bmatrix}
  \]
  An appropriate system order \( n \) may be found from the range of the singular values in (12).

- **Step III**
  Estimate \( A \) as
  \[
  \hat{A} = \Sigma_n^{-1/2} U_n^T \bar{R} V_n \Sigma_n^{-1/2}.
  \]
  \( C \) is estimated as
  \[
  \hat{C} = (U_n \Sigma_n^{1/2}) (U_n \Sigma_n v_n) .
  \]
  A possible estimate for \( B \) is
  \[
  \hat{B} = (\Sigma_n^{1/2} V_n^T) (U_n \Sigma_n v_n).
  \]
  then \( D \) is estimated as
  \[
  \hat{D} = y(0).
  \]

Improved estimates of \( B \) and \( D \) may also be found via a least-squares minimization. Given estimates \( \hat{A} \) and \( \hat{C} \), let \( \hat{B} \) and \( \hat{D} \) be the solution of

\[
\hat{B}, \hat{D} = \arg \min \| y - \hat{y} \|_2
\]

where

\[
\hat{y}(t) = \sum_{k=0}^{N-1} \hat{C} \hat{A}^{t-k-1} \hat{B}
\]

One is referred to Miller and de Callafon (2012) for additional details on the step realization method.

4.2 Identification of Real Power Oscillations

In the experimental verification of the real-time real power demodulation and application of the step-based realization algorithm, power oscillations are induced by step-wise excitation of the auxiliary relay depicted earlier in Fig. 2 to switch in a three phase Resistor-Inductor-Capacitor (RLC) circuit to initiate three phase power oscillations in the circuit. The input \( u(t) \) is used to denote the digital signal sent to the auxiliary relay; the output \( y(t) \) is the real-time demodulated real power calculated by the method proposed in the previous section.

In Fig. 5, \( u(t) \) stepped from 0 to 1 at \( t = 0 \), the upper plot shows the demodulated real power of each phase; the bottom plot shows the demodulated three-phase real power.

![Demodulated real power signal oscillations in each phase (top figure) and three phase (bottom figure) of the RLC circuit induced by a step-wise load change.](image)

Fig. 5. Demodulated real power signal oscillations in each phase (top figure) and three phase (bottom figure) of the RLC circuit induced by a step-wise load change.

The step-based realization algorithm is applied to verify the proposed method of real power demodulation. The RLC circuit depicted in Fig. 2 is a second-order system. With \( L = 0.1H, C = 0.01F \), we know that the (undamped) oscillation frequency of such an RLC circuit is given by

\[
f = \frac{1}{2\pi\sqrt{LC}} = 5.03Hz
\]

(13)
In practice, the contactor cannot be fully energized rapidly, thus it results in additional dynamics in the system. This can be observed by the irregular oscillation from $t = 0$ to $t = 0.15s$ in Fig. 5. To further verify this, the segment starting from $t = 0.15s$ is selected to estimate a model. With a second-order state space model, the step response of the RLC circuit can be reconstructed. By comparison with the raw demodulated real power as shown in Fig. 6, it is verified that the model captures very well the dynamics of the 3 phase RLC system. It also validates the proposed method of real-time demodulation of real power oscillation.

If the contactor dynamics is taken into account, a higher-order model can be used to capture this dynamics. As shown in Fig. 7, a third-order state space model is realized. The dynamics of the three phase RLC system including the contactor are both captured by the model.

5. CONCLUSIONS

In this paper, a real-time demodulation of real power oscillation in an electric three phase network is proposed and it is shown how a low order state space model of the three phase network can be realized on the basis of real-time measurements of real power oscillations. The realization algorithm formulates a low order model that can accurately capture frequency and damping of the power oscillations. The methodology is illustrated on the measurements obtained from three phase RLC network where the power oscillation frequency and model order is known and used for comparison and validation of the method. With an accurate low-order model, efficient control algorithms can be implemented to mitigate power oscillations.

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