What Can We Learn from Reaction Zone in Relativistic Nucleus-Nucleus Collisions?

D. Anchishkin$^a$, A. Muskeyev$^b$, S. Yezhov$^b$

$^a$ Bogolyubov Institute for Theoretical Physics, 03680 Kiev, Ukraine
$^b$ Taras Shevchenko Kiev National University, 03022 Kiev, Ukraine

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Abstract

We apply the “zone of reactions” as a tool in studying the interacting system formed in a collision of relativistic nuclei. With the use of the intensity of collisions of particles (the number of collisions in unit volume per unit time), we study the space-time structure of a fireball. In this approach, three basic regions for the evolution of a system are separated by the scale of the intensity of collisions: the zone of a hot fireball, the zone of a cold fireball, and the zone of residual interaction. It is shown that the conception of a zone of reactions can be used for the determination of the hypersurfaces of a chemical freeze-out and a sharp kinetic freeze-out.

1 Introduction

In the collision of nuclei at high energies, a strongly excited system of interacting particles, a fireball, is formed. In fact, a fireball is identified with the zone of reactions by such a definition, i.e. with a space-time region, in which the collisions of particles occur. Hence, the zone of reactions must reflect the space-time characteristics of a fireball, and its study gives information about the evolution of the interacting system.

While studying the evolution of a fireball, it is important to know the size of regions, where the majority of various processes is running. Depending on a model describing the system, we can distinguish the regions of the formation of a fireball, its isotropization and thermalization, the creation of particles, the regions of a chemical freeze-out and a kinetic one, etc. This allows one to conditionally select the stages of evolution of the system and, hence, to obtain the limits of validity of simple phenomenological models used for the description of the complicated physical phenomenon, as well as to describe separate stages of the development of the system in more details. In particular, the stages of formation ($\tau \sim 0.1 \text{ fm/c}$) and thermalization ($\tau \sim 1 \text{ fm/c}$) are most often described with the use of microscopic models based on the processes of interaction of quarks and gluons $[1, 2, 3]$. To describe the stage of expansion of a dense medium ($1 \leq \tau \leq 7 \text{ fm/c}$), the relativistic hydrodynamics is most often in use, see $[4, 5, 6, 7, 8]$ and references therein. The further evolution of a hadron gas and the process of kinetic freeze-out are covered by kinetic models $[9, 10, 11]$. As a parameter for the determination of stages of the evolution of a system, one can take the energy density, mean free path, frequency of collisions of particles, etc.
In the present work, we use the number of collisions in a given four-dimensional region of the space-time as a parameter of the spatial evolution of the interacting system. Such a quantitative estimate allows one to define the reaction zone, whose study gives a possibility to establish the space-time structure of a fireball from the viewpoint of the interaction intensity at every point of the space-time. The regions of a fireball can be distinguished by the interaction intensity which can be characterized by the number of collisions in a unit volume of the space-time.

Another important question which can be clarified by the study of the zone of reactions is how the space-time boundary of a fireball is related to the so-called sharp kinetic freeze-out hypersurface. Since the kinetic freeze-out is the process of establishment of a final distribution of particles in the momentum space, the sharp kinetic freeze-out hypersurface is an imaginary hypersurface, outside of which there are no collisions between particles of the system. As a rule, the sharp kinetic freeze-out hypersurface is defined as such, on which a parameter of the system \( L(t, x) \) takes the critical value \( L_c \). That is, the equation of the hypersurface has form

\[
L(t, x) = L_c.
\]

As such a parameter, we may choose the energy density \( \epsilon(t, x) \) \([7, 12]\), temperature \( T(t, x) \) \([13, 14]\), density of particles \( n(t, x) \) \([15]\), etc. In the present work, we will determine the sharp kinetic freeze-out hypersurface as a surface bounding the space-time region, in which almost all collisions between hadrons of the system happened. That is, we identify the reaction zone boundary and the sharp kinetic freeze-out hypersurface.

## 2 Zone of reactions

The number of reactions in the given space-time region can be determined with the use of the distribution function \( f(x, p) \). For example, in the approximation of two-particle collisions, this function satisfies the Boltzmann equation \([16]\)

\[
p^\mu_1 \partial_\mu f_1 = \int_2 \int_3 \int_4 W_{12 \rightarrow 34} (f_3 f_4 - f_1 f_2),
\]

where the right-hand side contains the collision integral. The quantity \( W_{12 \rightarrow 34} \) is the transition rate which involves the reaction cross-section and the conservation laws of energy and momentum, \( f_i \equiv \int \frac{d^3 p_i}{(2\pi)^3} E_i \), \( f_i \equiv f(x, p_i) \) are one-particle distribution functions, and \( E_i = \sqrt{m_i^2 + p_i^2} \) is the energy of a particle with momentum \( p_i \).

The probability of a collision of two particles with momenta \( p_1 \) and \( p_2 \) corresponding to the distribution functions \( f_1 \) and \( f_2 \), respectively, is determined at a space-time point \( x \) as \( \int_3 \int_4 W_{12 \rightarrow 34} f(x, p_1) f(x, p_2) \). By integrating over the momenta of particles, we obtain the frequency of collisions at the point \( x \):

\[
\Gamma(x) = \int_1 \int_2 \int_3 \int_4 W_{12 \rightarrow 34} f(x, p_1) f(x, p_2).
\]

Then the number of collisions in the space-time region \( \Omega \) is

\[
N_{\text{coll}}(\Omega) = \int_\Omega d^4 x \Gamma(x) = \int_\Omega d^4 x \int_1 \int_2 \int_3 \int_4 W_{12 \rightarrow 34} f(x, p_1) f(x, p_2).
\]

It is seen that the number of collisions in the given space-time region depends on the frequency of collisions \( \Gamma(x) \) which can be determined in a certain model approximation, e.g., like that in \([17, 18, 19]\). In particular, \( \Gamma(x) \) can be determined with the use of transport models.
Let us consider a large space-time region containing a fireball at its center. Let this region be so large that the dominant part of all collisions of hadrons of the system occurs in it. For example, let this part contain 99.99% of all two-particle reactions and decays of resonances related to this event, i.e., to the given fireball. So we obtain a 4-cube of reactions \( C_R \) with edges \( L_i \), where \( i = t, x, y, z \). In order to determine the zone of reactions, we divide the cube into separate equal parts (pixels), i.e., elements of the 4-space. Let \( \Omega = \Omega(t, x) \) be the 4-volume of a pixel with coordinates of the center of this volume \((t, x)\). Totally, we have \( N_{\text{pix}} = L_t L_x L_y L_z / \Omega \) pixels. Then, for each four numbers \((t, x)\), we can calculate the absolute number of collisions and decays of resonances in the given pixel \( \Omega(t, x) \) (see Fig. 1 left panel) by using, e.g., formula (3). We will determine the absolute number of reactions in each pixel and place the pixels from left to right by the following hierarchy: from a pair of pixels, the left pixel is that, in which a larger number of reactions has occurred. (In the programming, such a problem is called the “sorting problem” and has a standard algorithm of its solution.)

The arrangement of pixels is shown in Fig. 1 (right panel). The total area of the whole histogram (area covered by all bins) is equal to the total number of collisions of all hadrons and decays of resonances \( N_{\text{tot}} \) in the 4-cube of reactions \( C_R \).

It is obvious that the problem of the determination of a space-time region, where all collisions have occurred, has a statistical character. Therefore, it is reasonable to search for this region with a certain precision. We define “the zone of reactions” as a part of the 4-cube of reactions \( C_R \), in which a certain share (e.g., 90% or 99%) of all possible processes in the system is running. A small part of unaccounted reactions, to which we can refer also the decays of long-lived resonances, happens on such time intervals and at such distances which exceed the sizes of the 4-cube of reactions \( C_R \).

We can calculate a sum of some part of bins depicted in Fig. 1 (right panel). Namely, we can sum the areas of bins, by beginning from the left according to the obtained hierarchy. We recall that the area of each bin gives the number of reactions in the corresponding pixel. In such a way, we can reach the value of the sum equal to 90% of the absolute number of all reactions \( N_{\text{tot}} \) (see Fig. 1). The hyperspace region which is occupied by the pixels contributing to this sum gives the zone with the most intense interaction; i.e., this region covers 90% of

![Diagram showing the algorithm of selection of pixels for the determination of the zone of reactions.](image-url)
all hadronic reactions $N_{\text{tot}}$. We call this zone a “hot fireball”. By this algorithm, we will find a region, where else additionally 9% of collisions have occurred, and call it a “cold fireball”. We also determine a region including else 0.9% of collisions and call it conditionally a “fireball halo”. This region corresponds to the residual interaction and is mainly formed by decays of resonances. We note that the names of the zones of reactions are conditional and have nothing in common with a real temperature.

Totally, the three zones include 99.9% of all hadronic reactions $N_{\text{tot}}$ (see Fig. 1). As for the available long-lived resonances, they decay in the large temporal limits and at distances of the order of hundreds of fm. It is obvious that such reactions are improbable and are not contained in the basic region of the evolution of the interacting system. We note that the above-presented specific numbers (90, 99, and 99.9%) are quite conventional and can be replaced by other ones.

2.1 The results of calculations

To realize the above-presented algorithm, we use the transport model UrQMD v2.3 \[20] \[21] which allows one to calculate the density of reactions at every point of the space-time and to select reactions for the given kind of particles. In the present calculations, we took a 4-cube of reactions $C_R$ with the size of edges $L_i = 200$ fm, where $i = t, x, y, z$.

In Figs. 2 and 3, we show the results of calculations for AGS experiments ($Au+Au$) at 10.8A GeV and for SPS ones ($Pb+Pb$) at 158A GeV in the case of central collisions. The density of collisions is represented in the coordinates $z-t$, which is the corresponding projection of the reaction zone. In order to construct this projection of the three-dimensional spatial pattern onto the $z$ axis, we sum firstly all collisions along the transverse direction at the fixed coordinates $(t, z)$, namely (see (3)),

$$\tilde{N}_{\text{coll}}(\tilde{\Omega}(t, z)) = \int dx \, dy \, N_{\text{coll}}(\Omega(t, x, y, z)).$$

(4)

That is, we put a number of collisions $\tilde{N}_{\text{coll}}$ in correspondence to the pixel $\tilde{\Omega}(t, z)$ with the coordinates $(t, z)$. Then we construct the hierarchy of pixels $\tilde{\Omega}(t, z)$ according to the sorting algorithm, where the basic quantity is now the number of collisions $\tilde{N}_{\text{coll}}$ (see Fig. 1). Further, like the general case where we dealt with the 4-cube of reactions $C_R$, we sum, in succession, the areas of bins from left to right, i.e., we determine the total number of reactions. When we obtain the value of the sum equal to 90% of the absolute number of all reactions $N_{\text{tot}}$, we arrange the pixels contributing to this sum on the plane in the correspondence to their coordinates $(t, z)$. We mark the zone occupied by these pixels with red color and call it a “hot fireball” in the $z-t$ projection (see Figs. 2 and 3). Analogously, we gather additionally a sum of the areas of bins which gives 9% of the total number of all reactions $N_{\text{tot}}$ and name this zone a “cold fireball” in the $z-t$ projection. The zone of a “cold fireball” in Figs. 2 and 3 is drawn with blue color. By continuing to move along the hierarchical structure of pixels from left to right and by gathering a sum of the areas of their bins, i.e. the numbers of collisions corresponding to pixels, we determine those pixels which give, in total, else 0.9% of the total number of all reactions $N_{\text{tot}}$. By arranging them on the same plane in the correspondence to the coordinates $(t, z)$ of each pixel, we obtain a zone, name it a “fireball halo” in the $z-t$ projection, and mark it with cyan-blue color. That is, the three zones cover 99.9% of the total number $N_{\text{tot}}$ of all hadronic reactions.

Two competing processes coexist in the system arising in a collision of relativistic ions. On the one hand, as a result of the collision of nuclei, their initial kinetic energy of a longitudinal
motion is transformed into the energy of created secondary particles, e.g. $\pi$-mesons, and into the energy of transverse motion. The density of particles increases; as a result, the probability of collisions of particles grows. On the other hand, it is the rapidly expanding system, which
is accompanied by a drop of the density of particles. Thus, we determine the region of intense interaction, in which the process of creation of new particles compensates a decrease in the density due to the spreading of the system.

The analysis performed with the use of UrQMD indicates that the zone of a hot fireball contains all inelastic reactions running in the system, 98% and 99% of all inelastic reactions for AGS and SPS, respectively. Moreover, the numbers of decays and fusions of particles in this zone are practically the same. If we accept the viewpoint that the chemical freeze-out happens with the termination of inelastic reactions [22], then, according to this definition, the chemical freeze-out hypersurface can be associated with the boundary of the zone of a hot fireball.

The zone of a cold fireball includes 9% of all reactions in the system. Their greater part consists of decays of resonances, 49% and 56% of all reactions in this zone for AGS and SPS, respectively. In the residual interaction zone (a fireball halo) marked with cyan-blue color, else 0.9% of reactions run. The decays of resonances in the residual interaction zone contribute more than those in the zone of a cold fireball, 51% and 64% of all reactions in this zone for AGS and SPS, respectively. Together the zone of a cold fireball and the fireball halo form a space-time region occupied by a hadron-resonance gas. In this region, the reactions are represented by the processes of decays of resonances, fusion of particles, and elastic scattering. In this case, the number of the decays of resonances exceeds the total number of other processes. Since the processes of fusion run earlier than the processes of decay, which dominate in these two regions, we may assert that the greater part of particles in these regions leaves the system as a result of the decays of resonances.

In Figs. 2 and 3, we present the projections of the reaction zones on the z-t plane. It is seen that the interacting system has a comparatively high lifetime – a hot fireball (in the on-line representation, the region is marked by red color) decays completely only in the time intervals of the order of 16 fm/c for AGS and 50 fm/c for SPS. A cold fireball (in the on-line representation, the region is marked by blue color) lives for 32 ÷ 34 fm/c for AGS and 90 ÷ 100 fm/c for SPS, respectively. Black lines show the approximation of the boundaries of the reaction zones.

By definition, the zones of hot and cold fireballs together contain 99% of all reactions. Moreover, the greater part of reactions outside of these zones consists of decays of resonances. Thus, the hypersurface which bounds the region of a cold fireball (in the on-line representation, it is the boundary between the blue and cyan-blue regions) can be taken as the sharp freeze-out hypersurface. For both collision energies of nuclei under consideration, the upper part of the boundary of the reaction zone, i.e. \( t = t_B(z) \), is a space-like hyperbola and takes the form of a constant proper-time surface to within some factor, namely \( t_B(z) = A\sqrt{\tau^2 + z^2} \), where \( A=0.75 \), \( \tau=64 \text{ fm/c for AGS energies and } A=0.95, \tau=38 \text{ fm/c for SPS energies, respectively (see Figs. 2 and 3).} \) The lower time-like surface bounding a cold fireball has the form of a straight line \( t_B(z) = t_0 + \frac{1}{v}z \), where \( t_0 \) is close to zero, and \( v=0.8 \) for AGS energies and \( v=0.98 \) for for SPS energies. We note that, for SPS energies \( E_{\text{kin}} = 158A \text{ GeV} \), the time-like surfaces bounding all three zones of a fireball practically coincide with one another and are close to the light cone. This does not occur for AGS energies \( E_{\text{kin}} = 10.8A \text{ GeV} \), where the time-like boundaries of the reaction zones differ significantly from one another and the light cone (see Figs. 2, 3).

It is useful to represent the reaction zones in the coordinates \( \eta \) and \( \tau \),

\[
\eta = \frac{1}{2} \ln \left( \frac{t + z}{t - z} \right), \quad \tau = \sqrt{t^2 - z^2},
\]

where \( \eta \) is the longitudinal coordinate rapidity, and \( \tau \) is the proper time. In Figs. 4 and 5, we present the reaction zones in these coordinates, respectively, for the collision energies of nuclei.
In order to construct projections of the reaction zone in the coordinates $\eta - \tau$, we determine the number of reactions $\tilde{N}_{\text{coll}}(\tilde{\Omega}(\tau, \eta))$ in each pixel $\tilde{\Omega}(\tau, \eta)$. Then, by using the above-presented algorithm, we construct the reaction zones. In correspondence with the previous figures 2 and 3, we mark the zone of a hot fireball, where 90% of all reactions run, by red color. The zone of a cold fireball and the fireball halo are indicated by blue and cyan-blue colors, respectively. In Figs. 2 and 3, black lines show the upper boundaries of the reaction zones in their representation in the coordinates $\eta - \tau$. That is, these lines are a result of the transformation $(t, z) \rightarrow (\tau, \eta)$ of the space-like hyperbolas $t = A\sqrt{\tau^2 + z^2}$ which correspond to boundaries of all three stages of the evolution of the fireball.

![Figure 4: Projection of the reaction zone in the $\eta-\tau$ coordinates.](image)

The presentation of the reaction zone in the coordinates $\eta - \tau$ allows one to see the rapidity region, where the processes of collisions of particles run. In particular, Figs. 4 and 5 show that the maximum intervals of the coordinate longitudinal rapidity $\Delta \eta$ coincide with the intervals of the momentum rapidity $\Delta y = 3.2$ for the energy $E_{\text{kin}} = 10.8A$ GeV on AGS and $\Delta y = 5.8$ for the energy $E_{\text{kin}} = 158A$ GeV on SPS, respectively. For both energies, the reaction zones differ significantly along the $\eta$ axis, but their sizes differ slightly along the $\tau$ axis. Indeed, the size of the zone of a hot fireball near $\eta = 0$ for reactions with participation of all hadrons on SPS is by 10% larger than that of the corresponding region on AGS ($\Delta \tau_{\text{hot}} = 17$ fm/c for AGS and $\Delta \tau_{\text{hot}} = 19$ fm/c for SPS, respectively). The spreading of the reaction zone along the coordinate rapidity axis at the transition from the AGS- to SPS-energies is a result of the increase in the initial energy of particles. As was shown in [23, 24], the edges of the momentum rapidity distribution $dN/dy$ of nucleons are formed by those nucleons which participated only in several collisions. We may assume that the initial direction of motion and the initial velocity are almost invariable after the first collision at the eikonal scattering, and, therefore, the momentum rapidity of a scattered particle is close to the initial one. Just such particles scatter then
elastically on one another and form the reaction zone at great values of $\eta$ (what corresponds to big values of $t$ and $z$).

At the same time, the size of a reaction zone along the $\tau$ axis at $\eta = 0$ remains approximately invariable, as the kinetic energy increases by more than 15 times. This can be related to peculiarities of the behavior of the system in the central region of the momentum rapidity, which corresponds, undoubtedly, to the $\eta = 0$ region of the coordinate rapidity. For example, it was shown in [23, 24] that the spectrum of nucleons in the region near $y = 0$ is determined by the thermal distribution or a distribution close to it irrespective of the initial collision energy. Therefore, the course of reactions in this region for identical nuclei and the same centrality will be invariable, i.e., it will slightly depend on the energy.

### 3 Conclusions

Thus, the proposed algorithm of the determination of the reaction zone allows one to get information about the space-time structure of an interacting system which is created in the collision of relativistic ions.

By taking the number of collisions which have occurred in unit volume of a space-time region as the interaction intensity degree, we can separate the following parts of a fireball which characterize its evolution (see Figs. 2–5): 1) Region of a “hot” fireball, where 90% of all hadronic reactions, i.e. $0.9N_{\text{tot}}$, have occurred; as has been shown, this region of intense interaction contains all inelastic collisions (in the on-line presentation, this zone is marked by red color); 2) Region of a “cold” fireball, where 9% of all hadronic reactions, i.e. $0.09N_{\text{tot}}$, have occurred (in the on-line presentation, this zone is marked by blue color); 3) Region of a fireball halo, where 0.9% of all hadronic reactions, i.e. $0.009N_{\text{tot}}$, have occurred (in the on-line presentation, this zone is marked by cyan-blue color). Two last zones together are a space-
time region containing the hadron-resonance gas, and the reactions in this region are mainly presented by decays of resonances.

The hypersurface separating the zones of a hot fireball and a cold one can be associated with the chemical freeze-out hypersurface (we assume that the chemical freeze-out occurs, when the inelastic reactions are completed [22]).

Let us define that the kinetic freeze-out hypersurface separates the region, where the collisions between particles of the fireball are running, from the region, where there are no collisions. Then the hypersurface including the first two zones, i.e. the boundary between the zone of a cold fireball and the fireball halo, can be interpreted as a sharp kinetic freeze-out hypersurface. In the coordinates \((t, z)\), the space-time part of this hypersurface is a hyperbola and has form \(t = A\sqrt{\tau^2 + z^2}\), where \(A=0.75, \tau=64 \text{ fm/c}\) for the AGS-energy \(E_{\text{kin}} = 10.8A\text{ GeV}\) and \(A=0.95, \tau=38 \text{ fm/c}\) for the SPS-energy \(E_{\text{kin}} = 158A\text{ GeV}\).

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**References**

[1] R. Baier, A.H. Mueller, et al., Phys. Lett. B502, 51 (2001); [arXiv:hep-ph/0009237].
[2] P. Arnold, J. Lenaghan, et al., Phys. Rev. Lett. 94, 072302 (2005); [arXiv:nucl-th/0409068].
[3] A. Rebhan, P. Romatschke, M. Strickland, Phys. Rev. Lett. 94, 102303 (2005); [arXiv:hep-ph/0412016].
[4] H. Stoecker and W. Greiner, Phys. Rept. 137, 277 (1986).
[5] D.H. Rischke, [arXiv:nucl-th/9809044].
[6] P. Huovinen, P.V. Ruuskanen, Ann. Rev. Nucl. Part. Sci. 56, 163 (2006); [arXiv:nucl-th/0605008].
[7] V. Russkikh, Yu. Ivanov, Phys. Rev. C76, 054907 (2007); [arXiv:nucl-th/0611094].
[8] P. Romatschke, [arXiv:0902.3663 [hep-ph]].
[9] D. Molnar, M. Gyulassy, Phys. Rev. C62, 054907, 2000; [arXiv:nucl-th/0005051].
[10] A. Kisiel, W. Florkowski, W. Broniowski, J. Pluta, Phys. Rev. C73, 064902 (2006); [arXiv:nucl-th/0602039].
[11] V.K. Magas, L.P. Csernai, Phys. Lett. B663, 191 (2008); [arXiv:0804.3520 [nucl-th]].
[12] J. Sollfrank, P. Huovinen, P.V. Ruuskanen, Eur. Phys. J. C6, 525 (1999); [arXiv:nucl-th/9801023].
[13] H. von Gersdorff, L. McLerran, M. Kataja, and V. Ruuskane, Phys. Rev. D**34**, 794 (1986).

[14] P. Huovinen, Eur. Phys. J. A**37**, 121 (2008); arXiv:0710.4379.

[15] D.Adamova et al. (CERES Collaboration), Phys. Rev. Lett. **90**, 022301 (2003); arXiv:nucl-ex/0207008.

[16] S.R. de Groot, W.A. van Leeuwen, Ch.G. van Weert, Relativistic Kinetic Theory, Amsterdam, 1980.

[17] K.J. Eskola, H. Niemi, P.V. Ruuskanen, Phys. Rev. C**77**, 044907 (2008); arXiv:0710.4476.

[18] B. Tomasik, U. Wiedeman, Phys. Rev. C**68**, 034905 (2003); arXiv:nucl-th/0207074.

[19] C.M. Hung, E. Shuryak, Phys. Rev. C**57**, 1891 (1998); arXiv:nucl-ph/9709264.

[20] S.A. Bass, M. Belkacem, M. Bleicher et al., Prog. Part. Nucl. Phys. **41**, 225 (1998); arXiv:nucl-th/9803035.

[21] M. Bleicher, E. Zabrodin, C. Spieles et al., J. Phys. G: Nucl. Part. Phys. **25**, 1859 (1999).

[22] U. Heinz, Nucl. Phys. A**661** 349 (1999) and Nucl. Phys. A**685**, 414 (2001).

[23] D. Anchishkin, A. Muskeyev, S. Yezhov, Nucl. Phys. A**820**, 307C (2009); arXiv:0902.0999 [nucl-th].

[24] D. Anchishkin, A. Muskeyev, S. Yezhov, arXiv:0902.4171 [nucl-th], to be published in the Int. J. Mod. Phys.