Study of similarity and dissimilarity between $^{10}$Be and $^9$Li nuclei with microscopic structure and reaction models

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Abstract. We apply the microscopic nuclear structure and reaction models to the elastic scattering of the $^{10}$Be and $^9$Li nuclei. The states of the $^{10}$Be and $^9$Li nuclei are described as an $\alpha + \alpha(t) + n + n$ four-body system based on the cluster wave function, respectively. The elastic scattering cross section is obtained by the microscopic coupled channel (MCC) calculation with the MPa interaction, which is the latest version of the complex $G$-matrix interaction derived from the ESC $NN$ interaction model. We compare the channel coupling (CC) effect of the $^{10}$Be nucleus with that of the $^9$Li nucleus in the elastic scattering cross sections calculated with the same incident energy per nucleon. At the backward angles, the CC effect on the elastic scattering cross section for the incident $^{10}$Be nucleus is clearly larger than that for the incident $^9$Li nucleus. We investigate the dissimilarity for the CC effect through the dynamical polarization potential.

1. Introduction

The cluster features of the $^{10}$Be and $^9$Li nuclei have been frequently investigated from both nuclear structure and reaction viewpoints. In terms of the cluster model, the $^{10}$Be ($^9$Li) nucleus is described as the $\alpha + \alpha(t) + n + n$ four-body systems, respectively [1, 2, 3, 4]. Recently, their similarity has been discussed from the view point of the nuclear structure as in Refs. [5, 6, 4, 7]. However, their dissimilarity is not clarified in those works. In addition, the similarity and/or dissimilarity are not discussed from the view point of the nuclear reaction, although numerous observations are obtained through the nuclear reaction experiments.

In this paper, we investigate the similarity and/or dissimilarity of the $^{10}$Be and $^9$Li nuclei with the microscopic structure and reaction models. The model consists of the combination of the microscopic cluster model and the microscopic coupled channel (MCC) method. In the microscopic cluster model, the total wave function is composed of superposition of various cluster configurations. The transition densities are derived from the total wave function, and we use them for the elastic cross section calculations in the MCC framework. The channel coupling (CC) effects with the excited states are clearly shown for the $^{10}$Be and $^9$Li scatterings. Here we focus on the dissimilarity of the CC effect in the $^{10}$Be and $^9$Li nuclei.
We first perform structure calculation and construct the wave functions of $^{10}$Be and $^9$Li based on the stochastic multi-configuration mixing (SMCM) method. In the SMCM method, the total wave function is obtained by superposing many $\alpha$ wave functions on the stochastic multi-configuration mixing (SMCM) method. In the SMCM method, the total wave function is obtained by superposing many $\alpha$ wave functions. In the SMCM method, the total wave function is obtained by superposing many $\alpha$ wave functions. In the SMCM method, the total wave function is obtained by superposing many $\alpha$ wave functions. In the SMCM method, the total wave function is obtained by superposing many $\alpha$ wave functions to form the wave function of the system. The obtained wave function is then used to calculate the differential cross section. The differential cross section is obtained by the microscopic coupled-channel (MCC) calculation. Here the coupling potential is obtained by folding the densities of the projectile and target nuclei with the effective $NN$ interaction. The detail of the calculation is presented in Ref. [4] while the $NN$ interaction is replaced by the MPa interaction [8], which is the latest version of the complex $G$-matrix interaction.

**2. Formalism**

For the $^{10}$Be nucleus, the $0^+_1$, $2^+_1$, and $2^+_2$ states are obtained as $-62.74$, $-59.49$ ($E_x = 3.247$) and $-57.46$ ($E_x = 5.278$) MeV, respectively. The obtained radius for the ground state is 2.559 fm. The transition strengths from the $2^+_1$ and $2^+_2$ states to the ground state (B(IS2)) are 43.84 and 0.3551 fm$^4$, respectively. The obtained values for the $^{10}$Be nucleus are comparable to the experimental data [10, 11, 12, 13]. The binding energies, radius of the ground state and transition strengths of the $^9$Li nucleus are obtained in Ref. [4].

Here, we compare the $^{10}$Be nucleus with the $^9$Li nucleus in the elastic scattering by the $^{12}$C target at $E/A = 59.4$ MeV. In the MCC calculation, we take into account the $0^+_1$, $2^+_1$, and $2^+_2$ states for the $^{10}$Be nucleus. On the other hand, the $3/2^+_1$, $1/2^+_1$, $5/2^+_1$, and $7/2^+_1$ states are included in our calculation for the $^9$Li elastic scattering. The transition density for the $^{12}$C nucleus is taken from Ref. [9]. For the $^{12}$C nucleus, we take into account the excitation of the $2^+_1$ ($E_x = 4.44$ MeV), $0^+_2$ ($E_x = 7.65$ MeV), $3^+_1$ ($E_x = 9.64$ MeV) states. In this paper, the result including the full combination of projectile and target excitations is called as full-CC calculation.

The renormalization factor is often introduced to reproduce the data by increasing/reducing the real and imaginary parts of the folding model potential. In this paper, we apply the renormalization factor ($N_W$) only to the imaginary part of the folding model potential with the complex $G$-matrix interaction.

![Figure 1.](image1.png) **Figure 1.** The elastic scattering cross sections for the $^{10}$Be + $^{12}$C and $^9$Li + $^{12}$C systems at $E/A = 59.4$ MeV. The experimental data is taken from Ref. [14, 15]

![Figure 2.](image2.png) **Figure 2.** The dynamical polarization potential (DPP) for the $^{10}$Be + $^{12}$C and $^9$Li + $^{12}$C systems at $E/A = 59.4$ MeV.

Figure 1 shows the calculated elastic cross section for the $^{10}$Be + $^{12}$C and $^9$Li + $^{12}$C systems at $E/A = 59.4$ MeV. The filled circles are the experimental data for the $^{10}$Be + $^{12}$C elastic
cross section at $E/A = 59.4$ MeV. The dotted and solid curves are the results by the 1-ch and full-CC calculations, respectively. The dashed curves include the excitation effect of the target nucleus ($^{12}$C) only. The thin and bold curves are the result of the calculated $^{10}$Be and $^9$Li elastic scatterings, respectively. When we take into account no projectile ($^{10}$Be and $^9$Li) excitation effect, the calculated elastic cross sections are closed to each other. The CC effect by the $^{10}$Be and $^9$Li nuclei is clearly seen on the elastic cross section. Their dissimilarity by the CC effect is also clearly seen, although the transition strength of them is not so different.

In order to investigate the reason why the $^{10}$Be and $^9$Li nuclei show the different CC effect, we show the dynamical polarization potential (DPP) in Fig. 2. In the tail region ($R = 3–6$ fm), it is clearly seen that the DPP for the $^{10}$Be nucleus is larger than that for the $^9$Li nucleus. The DPP is known to depend on the coupling potential, elastic wave function, inelastic wave function and the angular momentum algebra as shown in Ref. [16]. Then, we check all of the components of the DPP to clarify the cause of the different CC effect. Finally, we find that the angular momentum algebra has an important role in all of the components which are the elastic wave function, inelastic wave function and DPP. Here, we note that the angular momentum algebra contributes to the DPP not only directly but also through the elastic and inelastic wave functions indirectly.

4. Conclusion
The $^{10}$Be ($^9$Li) nucleus is constructed by the $\alpha + \alpha(t) + n + n$ four body cluster model with the SMCM method, respectively. With the wave functions, we obtain the $^{10}$Be and $^9$Li elastic cross sections by the $^{12}$C target nucleus in the MCC calculation. The $^{10}$Be and $^9$Li elastic cross sections are closed to each other without the CC effect by the $^{10}$Be or $^9$Li nuclei. However, their elastic cross sections give the different CC effect when their excitation effects are taken into account in the MCC calculation. By investigating the DPP, it is found that the different CC effect is caused by the angular momentum algebra.

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