Optimization of Safety Stock under Controllable Production Rate and Energy Consumption in an Automated Smart Production Management

Mitali Sarkar and Biswajit Sarkar

Department of Industrial Engineering, Yonsei University, 50 Yonsei-ro, Sinchon-dong, Seodaemun-gu, Seoul 03722, Korea; mitalisarkar.ms@gmail.com
Department of Industrial & Management Engineering, Hanyang University, Ansan, Gyeonggi-do 155 88, Korea
* Correspondence: bsbiswajitsarkar@gmail.com; Tel.: +82-10-7498-1981

Abstract: A smart production system is essential to produce complex products under the consumption of efficient energy. The main ramification of controllable production rate, amount of production size, and safety stocks is simultaneously optimized under proper utilization of energy within a smart production system with a random breakdown of spare parts. Due to the random breakdown, a greater amount of energy may be used. For this purpose, this study is concerned about the optimum safety stock level under the exact amount of energy utilization. For random breakdown, there are three cases as production inventory meets the demand without utilization of the safety stock, with using of the safety stock, and consumed the total safety stock amount and facing shortages. After the random breakdown time, the smart production system may move to an out-of-control state and may produce defective items, where the production rate of defective items is a random variable, which follows an exponential distribution. The total cost is highly nonlinear and cannot be solved by any classical optimization technique. A mathematical optimization tool is utilized to test the model. Numerical study proves that the effect of energy plays an important role for the smart manufacturing system even though random breakdowns are there. It is found that the controllable production rate under the effect of the optimum energy consumption really effects significantly in the minimization cost. It saves cost regarding the corrective and preventive maintenance cost. The amount of safety stock can have more support under the effect of optimum energy utilization. The energy can be replaced by the solar energy.

Keywords: renewable energy; smart production system; random breakdown; safety stock; controllable production rate

1. Introduction

Recently, the main development in the production sector is the inspection of smart manufacturing within the production context. The effect of energy is another critical milestone for the smart manufacturing system (see for reference Sarkar et al. [1]). Cárdenas-Barrón [2] introduced an optimum production strategy for those products, which have special property like as fixed lifetime but that model did not consider the machine breakdown during making the production policies. Thus, there was no assumption for using the extra energies during machine breakdown and how to reduce those energies by using renewable energy. This proposed model would like to consider this research gap within the literature to obtain the minimum total cost for the smart production system. Talizadeh et al. [3] introduced the outsourcing reworked strategy for defective products within a traditional production system, but there is not a single assumption regarding energy reduction or uses of renewable energy.
That research gap is also fulfilled by this proposed research. Wee et al. [4] introduced renewable energy within the supply chain management for the performance, limitations for their uses, and further improvement for different policies. But, there is no indication how to transfer non-renewable to renewable energy within a smart production system. Therefore, this study introduces the strategy for reduced energy consumption through renewable energy resources.

The main aim of this smart production is to make a controllable production for making more smart products than the existing one and to reduce human labor’s efforts. But the main difference between an additive manufacturing and a smart manufacturing is that the additive manufacturing prefers more machinery controls than human labor, whereas the main theme of smart manufacturing systems is to make a human-factor related complex controllable management such that human-machine interactions can be used in the proper way to obtain the best complex products Kang et al. ([5]) through skilled nature of labors. Kang et al. [5] proved that the human interaction system must be controlled by a smart manufacturing system where the main performance of inspection is conducted by the three specified labors like skilled, unskilled, and semi-skilled labors. But the main issue is that they did not consider anything in the direction of energy. However, in any smart manufacturing system, the main base is smart machines and human labors and the interactions between them. Within the smart production system, there are two policies for breakdown case as no-resumption (NR) and abort/resume (AR). NR means the production may start again after repair of defective machine provided all inventories are utilized and AR means the production will start just after the repair of breakdown machine with the condition that the existing inventory should deplete to a certain amount of safety stock. Therefore, to maintain the breakdown of the system, the preventive and corrective maintenance are used. Even though both the maintenance are used, the inventory can touch the zero level without facing shortages, it may happen that inventory can go below the zero level but, it still lies in between zero and safety stock level, or sometimes it may happen that the inventory may reach to the downside of the safety stock and facing the shortages. Therefore, it can be understood that the safety stock plays an important role, whose decision is generally taken by the management sector of the smart manufacturing system. Porteus [6] first introduced the concept of improvement for the production system’s quality and reduction of setup cost by using some investments. Khouja and Mehrraj [7] first introduced the controllable production rate, where the production rate is variable within a certain interval of production capacity. They used that the unit production cost is dependent on a variable production rate. This is the basic idea for the controllable production rate within a traditional production model, which is the major turning of the smart production without having any consideration of energy consumption. Chung and Hou [8] extended the basic production model Khouja and Mehrraj [7] with shortages, imperfect production, and the elapsed time duration of the process distribution without anything related with energy. Giri and Dohi [9] first introduced the random breakdown within a production context, where the production lot size is a decision variable and the failure rate is considered as random exponential distribution. Avancini et al. [10] defined several matrices for energy evaluation in any smart system. They explained how the intelligent network supports for energy efficiency but there is a lack of specification any production system. Because, each production system is different with each other for the purpose of energy consumption issue. They just described about the control, communication, and display with the efficient energy. Chen and Lo [11] developed warranty policy for defective products in an imperfect production system where shortages appear due to random defective products. Even though the defective products are produced, there is no concept of the optimum energy consumption. Nižetić et al. [12] introduced several smart technologies to manage wastes through the efficient energies. They used sustainable resources to reduce the global energy but it has lack of concept within the smart production system. Specifically, the effect of efficient energy was not considered by any researcher yet. An economic production model for imperfect production system was developed by Sarkar and Moon [13]. They considered defective products when the system moves to out-of-control state from the in-control state. They did not consider any type of preventive or corrective maintenance. They just considered a traditional production system with out-of-control
movement without any concept of energy. A two-echelon supply chain management is explained with probabilistic deterioration in Sarkar [14]. The aim of the supply chain is to reduce the total cost but with a traditional production system. Any energy consumption is not considered within the whole supply chain model. For any smart production system, smart optimum energy and finally smart optimum energy system are very much essential to make any grid system with the help of grid system, the smart machine can be worked properly within several smart production systems, but Lund et al. [15] described only the explanation of grid system without modelling any specific smart production system. Generally, the smart manufacturing utilizes smart integrating intelligent manufacturing machines to produce smart products. The process is totally controlled though automated machines Edgar and Pistikopoulos [16]. Sana et al. [17] discussed that those defective products can be sold by reduced price. However, none of the authors did consider energy consumption in any production context. Louly and Dolgui [18] derived the optimum safety stock level in an assembly production system but without any concept of energy. Jaber et al. [19] were the pioneer researcher about the incorporation the entropy/heat effect in any inventory model but no energy effect. Sarkar et al. [20] explained the joint effect of failure rate, safety stock, and production lot in a basic production mode under a corrective maintenance and preventive maintenance. They proved that safety stock production model converges always over the non-safety stock mode but without any concept of energy or heat effect. Sana [21] developed a production model with machine breakdown under both types of maintenance where the shifting mode for machine follows an exponential distribution. Sarkar et al. [22] developed a closed-loop supply chain model with environmental issues. In this model remanufacturing of returned products are considered. But, any energy issue were not take into consideration. A supply chain model for complementary products was proposed by Sarkar et al. [23] without any energy effect, but the variable production rate and time dependent holding costs are applied. A logistic model was developed by Sarkar et al. [24], where the optimal cash flow for a smart production system is described. In this model carbon foot print and carbon emission costs were considered witout any energy consideration. Cárdenas-Barrón et al. [25] developed the benefit of multi-shipment in an economic manufacturing model without having any concept of energy matter. Giotitsa et al. [26] strives an appropriate approach for production of energy and its distribution. Tayyab and Sarkar [27] were the pioneer about incoporation random backorder rate within a multi-stage production system. Omair and Sarkar [28] introduced some sustainability issues within the production context without having any concept of energy consideration. Kim and Sarkar [29] explained a multi-stage cleaner production model, but they did not consider any idea about renewable energy effect within the multi-stage production system. They consider production system moves to out-of-control state even though they did not consider machine breakdown only, they just assumed the production system moves out-of-control state from in-control state due to labor strickes and machine breakdown without calculating breakdowns within inventory calculation. In this same direction, Moon et al. [30] approached a continous review model with service level and variable lead time without any smart production consideration. A two-echelon supply chain model with quality improvement and setup cost reduction in a tradition production system was considered by Sarkar et al. [31], but production rate is constant though it moves to out-of-control state. A supply chain model Kim and Sarkar [32] was developed with stochastic lead time and transportation discount policy without any maintainence policy. A study on setup cost reduction and quality improvement of manufacturing system was proposed by Majumder et al. [33] within a traditional production system. For an imprfect production system, lost sales reduction and quality improvement was taken account in fuzzy environment by Soni et al. [34], but still they did not think about the smart production system. Sarkar et al. [35] explained about a two-echelon supply chain model with setup time reduction and effect of safety factor in a traditional system. An integrated inventory model was developed by Dey et al. [36] on variable safety factor and setup cost reduction. But for all of these models, maintainence costs along with the energy costs are not considered for any stage of the supply chain or the production. Sarkar et al. [37] explained only out-of-control machines but did not consider the breakdown effects on the
inventory of the production system. Biel and Glock [38] discussed an efficient way to use waste energy within a two-stage controllable production system. Kim et al. [39] developed a cleaner multi-stage production system with a random backorder rate without any sustainable issue or energy issue. Unver and Kara [40] developed the efficiency of energy consumption within a traditional process with the optimum energy consumption. They did not consider any maintenance or any controllable smart production system. Kumar et al. [41] maintained the efficient balance level within a production system. They calculated the amount of wastes in a downstream production system and they obtained several strategies to reduce those wastes. Morato et al. [42] did the reverse way of energy issue. They calculated the amount of energy during the production system for the agricultural products. Ahmed and Sarkar [43] introduced sustainable framework and an energy issue for the case of biofuel production. Darom et al. [44] developed two serial supply chain disruption recoveries with controlled safety-stock and constant production rate. Assid et al. [45] introduced an unreliable hybrid manufacturing for production and setup cost controlling. They used demand of new and remanufacturing products, which are reworked but they did not mention the time of rework. [46] introduced an unique way to decide the exact time of rework within a multi-stage single cycle and multi-stage multi-cycle production system even though Sarkar [46] did not take into account of energy cost and any controllable production system. Utilizing photovoltaic technology, the optimum amount of energy consumption within a production buffer stock was developed by Caro-Ruiz et al. [47]. They proved that the coordination is possible for production system with a energy factory with the optimum consumption. Finally, one can obtain the idea about the automation policy in a smart production system, where machinery systems are automatic and they are controlled by several automated system of energy (Dincer and Ezzat [48], Dincer and Al-Zareer [49], Dincer and Rosen [50], Dincer and Rosen [51], and Dincer and Bicer [52]). However, if the automated system is utilized in several research models (Lu et al. [53], Kazemi et al. [54], Kulczek [55], Harris et al. [56], Bruni et al. [57], Dehning et al. [58], and Nordborg et al. [59]), the optimum renewable energy under production maintenance is not considered. The main research gap regarding maintenance is not considered as it was assumed that smart production system always gives perfect smart products without any breakdown and without any maintenance ( Khalil et al. [60], Keen et al. [61], Chen et al. [62], and Liang et al. [63]).

Still, there is a research gap in the direction of smart production direction with random breakdown under the proper application of corrective and preventive maintenance where the main task is to control used energy within the smart production system. This proposed model has solved this research gap for this matter. The Table 1 shows the research gaps of the model.

| Author(s)          | Energy   | Smart Production System | Breakdown | Safety Stock | Production Rate | Maintenance |
|--------------------|----------|-------------------------|-----------|--------------|----------------|-------------|
| Sarkar et al. [1]  | Consumption | Smart                  | Random    | NA           | Variable       | NA          |
| Kang et al. [2]    | NA       | Smart                   | NA        | NA           | Constant       | NA          |
| Porteus [3]        | NA       | Traditional             | NA        | NA           | Constant       | NA          |
| Khouja and Mehraj [4]| NA    | Traditional             | NA        | NA           | Variable       | NA          |
| Giri and Dohi [6]  | NA       | Traditional             | Random    | NA           | Constant       | CM and PM   |
| Chenand Lo [7]     | NA       | Traditional             | NA        | NA           | Constant       | NA          |
| Sana et al. [8]    | NA       | Traditional             | Random    | NA           | Constant       | CM and PM   |
| Louly and Dolgui [9]| NA    | Traditional             | NA        | Variable     | Constant       | CM and PM   |
| Sarkar et al. [11] | NA       | Traditional             | Random    | Variable     | Constant       | CM and PM   |
| This paper         | Consumption | Smart                  | Random    | Variable     | Variable       | CM and PM   |

NA indicates that is not applicable for that paper. CM and PM stand for corrective and preventive maintenance, respectively.
2. Problem Definition, Notation, and Assumptions

In this section, problem definition, assumptions and notation are given.

2.1. Problem Definition

It is a smart production system, random breakdown and defective production exists under controllable energy consumption and production rate, then how safety stock can be maintained within the production with the minimum cost. The unit production cost is dependent on controllable production rate and efficient amount of energy. Due to the random breakdown, the corrective and preservative maintenance are considered. As random breakdown is an unusual event and similarly amount of energy is used as extra amount of energy during corrective maintenance of smart machines. There are several cases, where the existing inventory’s position is before safety stock or after safety stock amount. The controllable production rate must vary within proper limit of production capacity of the machines such that the optimum amount of energy can be used. Finally, the aim is to minimize the optimum cost under the optimum production lot, safety stock and the production rate.

2.2. Assumptions

The following assumptions are considered to develop this model.

1. A smart production management is considered for single-type of products production under a controllable production rate and the optimum energy consumption.
2. During a long-run production system, the random breakdown may occur. Based on it, the model considers three cases as (i) the inventory just reaches to zero but no shortage appears (ii) the inventory may exceed the zero axis level but less than the safety stock level (iii) the inventory may cross the safety stock level and faces shortages.
3. Regular and emergency maintenance are considered under the effect of energy with two specified probability distribution functions.
4. Random breakdown may be at a random time, which follows an exponential distribution.
5. The effect of energy is considered for all possible position with the optimum amount of energy consumption.
6. Safety stock, production rate, and production quality are considered as decision variable where production rate may vary within the range of \([p_{min}, p_{max}]\). The optimum value of production rate must lie within the interval.
7. Shortages appear if the inventory crosses the safety stock level. If shortages appear, it is considered fully backordered.

2.3. Notation

Table 2 represents the notation of this model.
Table 2. Notation for parameters and variables.

| Decision variables                             | Description                                                                 |
|------------------------------------------------|-----------------------------------------------------------------------------|
| $P$                                           | production rate (units/time)                                                |
| $Q$                                           | production lot size (units)                                                 |
| $M_s$                                         | safety stock (units)                                                       |

| Random variable                               | Description                                                                 |
|------------------------------------------------|-----------------------------------------------------------------------------|
| $\rho$                                        | random variable of shifting from in-control-state to an out-of-control state |

| Parameters                                     | Description                                                                 |
|------------------------------------------------|-----------------------------------------------------------------------------|
| $D$                                           | demand rate (units)                                                         |
| $X$                                           | non-negative random variable denoting time                                  |
| $F_X(t)$                                      | failure time-distribution of $X$ with probability density function $f_X(t) = \frac{d}{dt} (F_X(t))$ |
| $A_1(S_1)$                                    | corrective(efficiency) repair time distribution with probability density function $a_1(S_1)$ with finite mean $1/\mu_1(>0)$ |
| $A_2(S_2)$                                    | preventive(regular) repair time distribution with probability density function $a_2(S_2)$ with finite mean $1/\mu_2(>0)$ |
| $A_0$                                         | setup cost per setup for the smart manufacturing system                    |
| $A'_0$                                        | energy cost for setup the production system                                |
| $C_{CRC}$                                     | corrective repair cost per unit time                                        |
| $C'_{CRC}$                                    | energy cost for corrective maintenance per unit time                        |
| $C_{PRC}$                                     | preventive repair cost per unit time                                        |
| $C'_{PRC}$                                    | preventive maintenance energy cost per unit time                            |
| $C_{hold}$                                    | holding cost per unit per unit time                                         |
| $C_{hold}$                                    | energy cost to hold all products per unit per unit time                     |
| $C_{short}$                                   | shortage cost per unit product                                              |
| $C_{re}$                                      | rework cost per unit of defective item                                     |
| $C_{re}$                                      | energy cost per unit to rework defective items                             |
| $h(\rho)$                                     | probability distribution function of the shift time distribution            |
| $\beta$                                       | proportion of defective items produced in the out-of control state where $0 < \beta < 1$ |
| $T$                                           | cycle length of production-inventory system                                |

3. Mathematical Model

A basic smart production is taken to produce a single-type of smart products. Even though, it is a smart production system, the machine failure exists and defective products produced. During long-run, the smart production system may move to out-of-control state from in-control state due to semi-skilled laborer’s issues or machinery problems. The smart production begins to produce smart products at time $t = 0$ with a controllable production rate $\rho$ and continues until the time $t_1$, when the inventory touches the maximum holding if there does not exist any smart machine breakdown. But the random breakdown occurs within this smart production system. Due to the random breakdown there are some possibilities like the preventive or corrective maintenance’s time with efficient energy is less than or greater than expected time. Thus, shortages may occur. The following cases under efficient time can be founded as follows:

Case I: Inventory position due to preventive maintenance

If the preventive maintenance time under the efficient energy is less than or equal to $\frac{Q(P-D)}{PD}$, then amount of inventory is (see Figure 1)

$$I_1 = \int_0^{Q(P-D)/(PD)} \left[ \frac{(P-D)Q^2}{2PD} + \frac{M_sQ}{D} \right] dA_1(S_1). \quad (1)$$
Figure 1. Inventory when the preventive maintenance time interval is in $\left[0, \frac{Q(P-D)}{PD}\right]$.

But, if the preventive maintenance time belongs to $\left[\frac{Q(P-D)}{PD}, \frac{M_s + \frac{Q(P-D)}{P}}{D}\right]$, then the inventory may reach lower than the safety stock level but still there is no shortage as the inventory level is above the zero level, then the amount of inventory is (refer to Figure 2)

$$I_2 = \int_{\frac{Q(P-D)/PD}{(M_s + \frac{Q(P-D)}{P})/D}}^{(M_s + Q(P-D)/P)/D} \left[ \frac{(P-D)Q^2}{2PD} + \frac{PM_sS_1}{P(P-D)} - \frac{P(S_1D - (P-D)Q/P)^2}{2D(P-D)} \right] dA_1(S_1). \quad (2)$$

Finally, if the preventive maintenance time belongs to $\left[\frac{M_s + \frac{Q(P-D)}{P}}{D}, \infty\right)$, then the shortage arises due to the inventory level reaches below the zero level, then the total amount of inventory is (see Figure 3)

$$I_3 = \int_{(M_s + Q(P-D)/P)/D}^{\infty} \left[ \frac{(P-D)Q^2}{2PD} + \frac{M_sQ}{P} + \frac{PM_s^2}{2D(P-D)} \right] dA_s(S_1), \quad (3)$$

and the inventory due to shortage for the preventive maintenance is

$$I_4 = \int_{(M_s + Q(P-D)/P)/D}^{\infty} \left[ S_1 - (M_s + Q(P-D)/P)/D \right] dA_s(S_1). \quad (4)$$

Figure 2. Inventory when the preventive maintenance time interval is in $\left[\frac{Q(P-D)}{PD}, \frac{M_s + \frac{Q(P-D)}{P}}{D}\right]$. 
Now, the inventory related with corrected maintenance can be done in case II.

Case II: Inventory position due to the corrective maintenance

The corrective maintenance is conducted when there is any urgent mishap and to save the production system, it is needed to maintain with cost and more energy investment. Thus, one can calculate the amount of inventory position based on random time $t$ for corrective maintenance as at random time $t = \rho$, the process moves to out-of-control state from in-control state. Similar to Case I, there are three cases when the corrective maintenance time belongs to $[0, \frac{t(P - D)}{D}]$, the inventory position is within the general range of inventory above zero level, or the maintenance time belongs to $[\frac{t(P - D)}{D}, \frac{(M_s + t(P - D))}{D}]$, the position of inventory should lie in between zero level and safety stock level without having shortages, but if the time for maintenance belongs to $[(M_s + t(P - D))/D, \infty)$, then inventory level reaches below the safety stock level, with having shortages, then the inventory positions, respectively are (see Figures 4–6)

$$I_5 = \int_0^{(P - D)/D} \left[ \frac{(P - D)P t^2}{2D} + \frac{M_s P t}{D} \right] dA_2(S_2),$$

$$I_6 = \int_{(P - D)/D}^{(M_s + t(P - D))/D} \left[ \frac{(P - D)P t^2}{2D} + \frac{PM_sS_1}{(P - D)} - \frac{P(S_1D - (P - D)t)^2}{2D(P - D)} \right] dA_2(S_2),$$

$$I_7 = \int_{(M_s + t(P - D))/D}^{\infty} \left[ \frac{(P - D)P t^2}{2D} + \frac{M_s P t}{D} + \frac{PM_s^2}{2D(P - D)} \right] dA_2(S_2),$$

And

$$I_8 = \int_{(M_s + t(P - D))/D}^{\infty} \left[ S_2 - \frac{(M_s + (P - D)t)}{D} \right] dA_2(S_2).$$

Now, one can find the expected number of defective items as the production system at random $t = \rho$ moves to out-of-control state. Therefore, the expected number of defective items are

$$I_9 = P\beta \left[ \int_0^{Q/P} \int_0^{t(P - \rho)} h(\rho) dPdF_x(t) + \int_0^{Q/P} \left( \frac{Q}{P} - \rho \right) h(\rho) dPdF_x \left( \frac{Q}{P} \right) \right].$$
where $h(\rho)$ is the probability distribution of the shifting of production system from in-control state to out-of-control state and it is given by an exponential function as follows:

$$h(\rho) = ke^{-k\rho}, \quad (10)$$

where $\frac{1}{k}$ is the mean. Now other production distribution functions are given as follows:

$$F(X) = 1 - e^{-t}, \quad (11)$$

which gives

$$\bar{F}(X) = e^{-t}. \quad (12)$$

Figure 4. Inventory when the corrective maintenance time interval is in $[0, \frac{t(P-D)}{D}]$.

Figure 5. Inventory when the corrective maintenance time interval is in $[\frac{t(P-D)}{D}, \frac{(M_s + t(P-D))}{D}]$.

Figure 6. Inventory when the corrective maintenance time interval is in $[\frac{(M_s + t(P-D))}{D}, \infty]$.
The corrective and preventive repair time are given by

\[
A_1(S_1) = 1 - e^{-\mu_1 S_1}, \quad (13)
\]

\[
A_2(S_2) = 1 - e^{-\mu_2 S_2}. \quad (14)
\]

As it is a smart production system, thus the controllable unit production cost is considered and is as follows:

\[
C_P = \frac{A + A'}{P} + (\gamma + \gamma')P, \quad (15)
\]

where \(A\) and \(\gamma\) are the scaling parameters related with production cost and \(A'\) and \(\gamma'\) are the scaling parameters related with the energy cost for the production.

Now, the different types of costs can be calculated for the production system as follows:

For two types of maintenance different costs are considered here. For corrective and preventive both maintenance, energy cost is essential cost. Thus, the cost corrective and preventive maintenance can be written respectively, as

\[
C_{\text{correct}} = (C_{\text{CRC}} + C'_{\text{CRC}}) \left[ \int_0^{Q/P} \left[ \int_0^\infty S_2 dA_s(S_2) dF_X(t) \right] \right], \quad (17)
\]

and

\[
C_{\text{preven}} = (C_{\text{PRC}} + C'_{\text{PRC}}) \left[ \int_0^\infty S_1 dA_s(S_1) dF_X(Q/P) \right]. \quad (18)
\]

The total inventory for both corrective and preventive maintenance are already shown. Here holding cost for all types of inventory is considered as same. To hold any inventory, its relevant energy costs are also needed. Thus, the total holding cost and its energy cost can be written as

\[
(C_{\text{hold}} + C'_{\text{hold}}) \left[ \int_0^{Q/P} (I_1 + I_2 + I_3) dF_X(t) \right] + (C_{\text{hold}} + C'_{\text{hold}}) \left[ \int_0^{Q/P} (I_5 + I_6 + I_7) dF_X(t) \right]. \quad (19)
\]

Rework is allowed for defective items. The inventory for rework is \(I_9\). The rework cost and its energy cost can be written as

\[
(C_{\text{re}} + C'_{\text{re}}) I_9. \quad (20)
\]

Therefore, the expected total cost is given by
4.1. Numerical Example

The revised data for numerical experiment is taken from Sarkar et al. [20] and provided in Table 3 as follows:

| Parameter | Value |
|-----------|-------|
| $D$ | 400 units |
| $T$ | $3$/unit time |
| $C_{te}$ | $1.8$/item |
| $C_{rc}$ | $250$ |
| $C_{c}$ | $0.2$/item |
| $k$ | $1.2$ |
| $A$ | 60 |
| $B$ | 0.2 |
| $C_{c}$ | $0.018$ |
| $C_{th}$ | $15$/item |
| $C_{p}$ | $2$/unit time |
| $C_{s}$ | $0.01$/unit/unit time |
| $C_{th}$ | $0.1$/item |
| $C_{s}$ | $0.1$/unit/unit time |
| $C_{th}$ | $0.2$/item |
| $C_{s}$ | $0.002$ |

The optimal solutions of the numerical example can be written as $P = 6108.89$ units/time; $Q = 2202.34$ units; $M_s = 26.34$ units; $ETC = $343.63.
4.2. Sensitivity Analysis

This subsection consists of the sensitivity analysis of the key parameters of the model. From Table 4 the sensitivity analysis of key parameters can be described as follows:

(1) If the setup increased, the total cost of the smart production system increased. But with the increases of 50% setup cost, the total cost increased only 8.62%. Thus, one can conclude that the setup cost was not effecting significantly the total cost of the production system. However as it was a smart production system, there were several smart technology and smart machinery systems and those were involved within this setup cost. Therefore, through the total cost only increased 8.62% for a 50% increase in the setup cost, still, it was a valuable and significant cost for any traditional or the smart production system.

(2) The defective cost was most sensitive within all costs of the smart production system. For negative and positive charge of the defective cost, the total cost changed 19.46% and in both directions, it was the same change. Thus, it can be concluded that the value of the rework cost follows an equilibrium position, which was the main theme for steady state of any production system. As this cost effects more, the automation policy can be used where the defective productions are inspected by a machine not by a human labor. Thus, the probability of defective products reduced and along with the defective cost and corresponding rework cost was reduced.

(3) The cost of corrective and preventive maintenance were much less sensitive with the total cost compared to the other costs. As the breakdown was random, thus the preventive and corrective maintenance should be least sensitive among all cost parameters. The main reason behind it is that the production system is a smart production system. Thus, the probability of breakdown is very less. Therefore, these costs were less sensitive. If there was a breakdown then the importance of corrective maintenance increased, but still the effect of the breakdown was not more than the regular breakdown event as the production rate was controllable and before moving to out-of-control, the management reduced the production rate. Thus, the amount of loss was much less. But, still the corrective maintenance played an important role on that time. Thus, if a breakdown occurs, then the setup cost, defective cost, corrective maintenance cost, holding cost, and shortage cost will increase significantly, whereas the preventive maintenance cost will be reduced in the next cycle more as the huge corrective maintenance is already used during maintenance.

(4) The holding cost of the smart production is very much sensitive with the total cost of the system. For negative change of the holding cost, the total cost is changed more than the positive change of it. Therefore, it can be concluded that the holding cost does not follow the equilibrium positions like the rework cost. However, the industry should try to reduce the holding cost for maintaining the minimum cost of the system. For the long term strategy to reduce holding cost, several policies should be adopted by the management such that it can reduce the total cost again.

(5) As it is a smart manufacturing system and due to the random breakdown pattern, any time the inventory level can go below the safety stock level. Therefore, the shortage cost is almost equally sensitive like other costs. Just like the holding cost, the shortage cost is also the most sensitive in the negative change than the positive change of the total cost.
Table 4. Sensitivity analysis for expected total cost.

| Parameters | Changes (in %) | Changes of ETC (in %) | Parameters | Changes (in %) | Changes of ETC (in %) |
|------------|----------------|-----------------------|------------|----------------|-----------------------|
| $A_0$      | $-50$          | $-10.66$              | $-50$      | $-0.0024$      |                       |
|            | $-25$          | $-4.97$               | $-25$      | $-0.0012$      |                       |
|            | $+25$          | $+4.49$               | $+25$      | $+0.0012$      |                       |
|            | $+50$          | $+8.62$               | $+50$      | $+0.0024$      |                       |
| $C_{re}$   | $-50$          | $-19.46$              | $-50$      | $-17.08$       |                       |
|            | $-25$          | $-9.73$               | $-25$      | $-7.78$        |                       |
|            | $+25$          | $+9.73$               | $+25$      | $+6.74$        |                       |
|            | $+50$          | $+19.46$              | $+50$      | $+12.68$       |                       |
| $C_{PRC}$  | $-50$          | $-0.0109$             | $-50$      | $-3.99$        |                       |
|            | $-25$          | $-0.0055$             | $-25$      | $-1.66$        |                       |
|            | $+25$          | $+0.0055$             | $+25$      | $+1.29$        |                       |
|            | $+50$          | $+0.0109$             | $+50$      | $+2.35$        |                       |

4.3. Managerial Insights

This study reveals a strong recommendation how the industry will manage the situation of the breakdown within the framework of smart production system, where generally breakdown is not expected. However, the management can decide the optimum production rate easily, based on the situation of huge or less products necessity. They can increase or decrease the production rate based on system movement from in-control to out-of-control movement. For that case, the chances of defective item production will be reduced.

A smart production system was considered for smart products. The effectiveness of energy should be taken care as major tasks are done by smart machines. Thus, the management can get the proper amount of safety stock of products with the optimum energy level. As this was a smart system, thus the rate of breakdown generally less than the traditional production system. Therefore, the wastage of energy can be reduced easily due to less number of breakdown. The management should take care about the corrective and preventive maintenance always.

As this is a smart production system with all smart machines, it is always needed more skilled labors than unskilled labors. The skilled labors will get benefits as their workloads will be reduced. Therefore, all of production and maintenance staff will be happy to accommodate this type of production system.

5. Conclusions

The study was conducted to obtain the optimum controllable production rate, the amount of the production quality and finally the amount of safety stock during the random machine breakdown under optimum energy consumption within the framework of smart production management. The total cost was minimized under the optimum energy consumption for both maintenance, corrective and preventive under the random breakdown. The variable production rate was varied within the interval of minimum and maximum range of production rate. Due to random breakdown the amount of energy was used in high rate but due to controllable production it was optimized. For a controllable production rate, the optimum safety stock is needed from the management as demand rate is constant and holding cost is comparatively low. But, due to high shortage cost, the management aimed to fulfill the demand by the optimum production quantity immediately. Thus, the controllable production rate was the best fit strategy for any smart production system. The main finding of this model was with respect to the optimum energy, the safety stock, production quantity, and production rate were optimized to obtain the minimum total cost. Sarkar et al. [1], Talizadeh et al. [3], and Sana et al. [17] considered only fixed traditional production system for complex products. But in reality, the complex products or smart technological products are easy to produce in a smart production system with a
controllable production rate and proper safety stock under the optimum energy. This study fulfilled this specific research gap. Sarkar et al. [20] explained the similar breakdown in a traditional production system but this model extended that in a smart production system with the effect of controllable production rate and smart energy. Finally, the proposed model obtained the optimum cost at the optimum values of the decision variables. The main limitation of the model, the setup cost, is constant even though the random breakdowns are there. Therefore, the model can be extended with variable setup cost of the model under the similar conditions. The model can be further extended for multi-stage smart production system with random breakdowns.

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