Finite-element simulations of hysteretic ac losses in a magnetically coated superconducting tubular wire subject to an oscillating transverse magnetic field

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Numerical simulations of hysteretic ac losses in a tubular superconductor/paramagnet heterostructure subject to an oscillating transverse magnetic field are performed within the quasistatic approach, calling upon the COMSOL finite-element software package and exploiting magnetostatic-electrostatic analogues. It is shown that one-sided magnetic shielding of a thin, type-II superconducting tube by a coaxial paramagnetic support results in a slight increase of hysteretic ac losses as compared to those for a vacuum environment, when the support is placed inside; a spectacular shielding effect with a possible reduction of hysteretic ac losses by orders of magnitude, however, ensues, depending on the magnetic permeability and the amplitude of the applied magnetic field, when the support is placed outside.

I. INTRODUCTION

Cylindrical heterostructures made up of superconductor and paramagnet constituents find use in various technological applications, power transmission cables involving second-generation high-temperature superconductor multistrand wires and electromagnetic coils based on coated conductors[1] or bulk MgB$_2$/Fe filaments[2] playing a prominent role thereby. The combined magnetic shielding properties of these kinds of constituents render beneficial effects like a reduction of hysteretic ac losses[2, 3] or an enhancement of the (field-dependent) critical current,[4–7] to name just a few. Given specific material characteristics and geometries, coaxial superconductor/paramagnet heterostructures may even disclose the exceptional hallmarks of magnetic cloaks.[8–12]

Hysteretic ac losses in unshielded superconductors of cylindrical shapes have been established for the cases of an oscillating transport current imposed or an oscillating transverse magnetic field applied,[13, 14] as well as for scenarios with both types of excitations in operation at the same time.[15, 16] By comparison, hysteretic ac losses in superconductor strips on magnetic supports were until recently discussed controversially, some experiments demonstrating a monotonic rise of hysteretic ac losses with increasing permeability of soft-magnetic constituents,[17] others showing a contingent reduction of such losses due to soft-magnetic supports.[18, 19] In fact, hysteretic ac losses may either be depressed or enhanced depending on the geometrical and material characteristics of the magnetic environment[20–22] and the amplitude of the applied magnetic field.[23, 24] Taking advantage of magnetostatic-electrostatic analogues,[25] satisfactory agreement between numerically simulated[26] and experimentally determined[27] hysteretic ac losses in a planar bilayer superconductor/paramagnet heterostructure subject to an oscillating transverse magnetic field could be achieved, with proper dependences on the permeability of the magnetic support $\mu$ and the amplitude of the applied magnetic field $H_0$. Thorough accounts of both analytical and numerical methods for modelling electromagnetic properties of high-temperature superconductors, including hysteretic ac losses in the presence of paramagnetic supports, offer two up-to-date reviews.[28, 29]

Regarding tubular superconductor/paramagnet heterostructures forged by polygonal assemblies of coated superconductor strips, great effort has been spent on finite-element simulations of hysteretic ac losses for an oscillating transport current imposed.[30–32] Accordingly, when assuming a nonlinear current-voltage characteristic of the superconductors and a constant permeability $\mu$ of the magnetic constituents, the transport current hysteretic ac loss with the magnetic supports placed inside turns out to be far less than with these supports placed outside.[31] Increasing $\mu$ in configurations with inner magnetic supports leads to enhancements of the hysteretic ac loss;[30] an effect which saturates for a relative magnetic permeability near $\mu/\mu_0 \approx 100$, as anticipated from previous analysis.[20, 33] Incorporation of a nonlinear, but reversible, field-dependence of the magnetic permeability $\mu$ has merely little bearing on the saturated magnetic property, as comparative modellings using fixed values of $\mu$ show.[30] Very much like for a single magnetically shielded superconductor strip,[34] advanced studies of a power transmission cable involving two coaxial sets of coated superconductor strips—with hysteretic magnetization losses suffered by the magnetic constituents allowed for too—unfold that these losses control the dissipation of electromagnetic energy of the entire heterostructure.[32]

Although multistrand wires and electromagnetic coils...
are obviously exposed to transverse magnetic fields, only few works seem to have addressed the electromagnetic behaviour of tubular superconductor/paramagnet heterostructures in oscillating applied magnetic fields. An elaborate numerical study of the shielding properties of such heterostructures, for example, assumes a nonlinear current-voltage characteristic of the superconductors and a reversible field-dependence of the permeability of the magnetic constituents \( \mu \), apart from including the field-effect on the critical current density \( J_c \).\[35\] Hence, configurations with the paramagnet layers placed outside the superconductor constituents exhibit much stronger shieldings than those with the respective layers placed inside. Based on an elegant analytical approach,\[23, 36\] recent work probes the effect of outer magnetic supports on the hysteretic ac loss in a power transmission cable assembled from curved superconductor/soft-magnet tapes conforming to a cylindrical shape.\[37\] Both types of constituents are deemed infinitesimally thin, the magnetic supports being characterized by an infinite permeability, \( \mu \rightarrow \infty \). A realistic enhancement of the hysteretic ac loss due to the magnetic supports obtains for a transport current imposed, consistent with investigations before:\[30, 31\] results derived for an applied magnetic field of radial symmetry, however, scarcely hit considering the fact that an external magnetic field must be source free. Significant progress exemplifies a theoretical analysis of the electromagnetic response of a cylindrical tubular wire represented by an infinitesimally thin superconductor constituent subject to an oscillating transverse magnetic field.\[38\] In a description where the superconductor is delineated by the sheet current \( J \), with a (field-independent) critical value \( J_c \), the profiles of the magnetic field, the field of first penetration of magnetic flux and the hysteretic ac losses that ensue pave the way towards research on the electromagnetic behaviour of tubular heterostructures embracing superconductor as well as paramagnet constituents. We here extend this ansatz for a coated superconducting tubular wire, with a coaxial paramagnetic support, by making recourse to Bean’s model of the critical state.\[22, 23, 25, 26, 40\] In conformity with Bean’s model of the critical state duly adapted to the geometry of the tube for a polar orientation of the applied magnetic field,\[38, 40\] magnetic flux penetrates from both equatorial sides of the tube into two cylindrical segments, of angle \( 2\gamma \), where the sheet current \( J \) equals the constant \( J_c \); flux-free regions prevail in the polar segments of the tube, where the normal component of the magnetic field \( H_n \) disappears.

As shown elsewhere,\[25\] the physical state of the tube in the corresponding electrostatic problem then is characterized by the surface charge density \( \sigma \) alone, invoking two central, dielectric parts of the tube, of angle \( 2\gamma \), where the surface charge density \( \sigma \) adopts the (field-independent) critical value \( \sigma_c \), contiguous to polar, metallic parts of the tube at zero electrostatic potential \( \varphi \); the support is represented by a dielectric with a finite permittivity \( \varepsilon \). The heterostructure itself is subject to an electric field perpendicular to the applied magnetic field, with strength \( E_a \), generated by finite potential values \( \varphi = \pm \varphi_a \) at opposite (left and right) sides of the quadratic computation frame well encompassing the domain covered by a cross section of the heterostructure.

Therefore, the following analogues between the magnetostatic problem and the electrostatic problem hold:\[25, 26\] \( J_c = a\sigma_c \) with a free constant \( a \), relating the critical sheet current to the critical surface charge density, \( \mu / \mu_0 = \varepsilon_0 / \varepsilon \) with the vacuum permeability \( \mu_0 \) and the vacuum permittivity \( \varepsilon_0 \), linking the permeability of the paramagnetic support to the permittivity of the dielectric support, and \( H_a = a\varepsilon_0 E_a \), relating the strength of the applied magnetic field to the strength of the applied electric field. It is thus clear that lines of the magnetic field and equipotentials of the electric field coincide. Hence, for an oscillating applied magnetic field with amplitude \( H_a \), the penetration of magnetic flux and the consequential dissipation of energy, per cycle and unit length of the tube, \( U_{ac} \) can be ascertained, resorting to the quasistatic approach.\[22, 25, 26, 40\]

II. THEORETICAL MODEL

Let us first define the magnetostatic problem by considering a cylindrical superconductor/paramagnet heterostructure of bilayer geometry, viz., an infinitely extended type-II superconducting tubular wire of radius \( R \) and thickness \( d \) on an inner, or outer, paramagnetic support of respective thickness \( D \), buffered by an infinitesimally thin non-magnetic layer in between, and subject to a transverse magnetic field with strength \( H_a \). We choose dimensions that second-generation coated conductors typically display,\[17, 27\] i.e., \( R = 5 \text{ mm}, d = 2 \mu \text{ m}, \text{ and } D = 250 \mu \text{ m}, \) understanding that the paramagnetic support is delineated by a finite permeability \( \mu \). Since \( d \ll R \), we ignore spatial variations of the induced current on a length scale less than \( d \) and, for mathematical convenience, regard the superconducting tube as infinitesimally thin, so that its physical state can be characterized by the sheet current \( J \) alone.\[22, 23, 25, 26, 40\] Computation run as follows: given \( \varepsilon \) and \( \gamma \) for an arbitrarily chosen non-zero potential value \( \varphi_a \), the critical value \( \sigma_c \) is varied until a continuous profile of the surface charge density \( \sigma \) over the circumference of the tube is reached; procedure here performed with COMSOL, a commercial finite-element software package not originally designed for modelling superconducting states. On introducing the characteristic magnetic field \( H_c = J_c / \pi \), the half-angle of flux penetration \( \gamma \) then is given in terms of the ratio \( H_a / H_c = \pi \varepsilon_0 E_a / \sigma_c \), independent of the constant \( a \) and the potential value \( \varphi_a \). The normalized hys-
Hysteretic ac loss appears as \[ U_{ac}/H_a^2 = (16\mu_0\sigma_c R^2/\varepsilon_0 E_a^2) \lim_{d \to 0} \int_0^\gamma d\phi \int_0^\gamma d\phi' \hat{E}_t(\phi'), \] independent of \( \alpha \) and \( \varphi_a \) too, since the electric quantities \( \sigma_c, E_a, \) and \( \hat{E}_t \) in this equation, where \( \hat{E}_t \) denotes the average tangential component of the electric field on the surface of the tube, all scale with the magnitude of \( \varphi_a \). We comment that the above analysis implies a neglect of the (small) contribution to the hysteretic ac loss arising from the tangential component of the magnetic field \( H_t \), consistent with a threshold for the half-angle of flux penetration \( \gamma \).

III. NUMERICAL RESULTS

The following numerical results address supports placed on either side of the superconducting tube.

A. Inner paramagnetic support

Referring to the case of an inner paramagnetic support, Fig. 1 illustrates the distribution of the magnetic field around the magnetically coated superconducting tubular wire for a fixed permeability and three progressive values of the amplitude of the applied magnetic field. At \( H_a/H_c = 1.5 \), the interior of the wire is com-

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{(Colour online) Lines of the magnetic field around the superconducting tubular wire coated with an inner paramagnetic support of relative permeability \( \mu/\mu_0 = 10 \), when the normalized amplitude of the magnetic field (a) \( H_a/H_c = 1.5 \), (b) \( H_a/H_c = 2.0 \), and (c) \( H_a/H_c = 2.5 \). The support together with the tube is indicated by black contour lines.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{(Colour online) Dependence of (a) the half-angle of flux penetration \( \gamma \) and (b) the normalized hysteretic ac loss \( U_{ac}/H_a^2 \) on \( H_a/H_c \), the normalized amplitude of the magnetic field applied to the magnetically coated superconducting tubular wire, in the computationally accessible regimes, for four different values of the relative permeability of the inner paramagnetic support \( \mu/\mu_0 \) identified on the curves. The dashed lines represent analytical results for the isolated superconducting tube.}
\end{figure}
pletely shielded from the applied magnetic field by the superconducting tube on top of the paramagnetic support: the lines of the magnetic field pass the wire outside (Fig. 1(a)). As the amplitude of the applied magnetic field is increased to $H_a/H_c = 2.0$, magnetic flux starts to enter the superconductor constituent tangentially from both equatorial sides, threading two cylindrical segments of the tube and permeating the paramagnetic support, with refraction of the lines of the magnetic field at the interfaces therein, before the interior of the wire accommodates the flux (Fig. 1(b)); an effect which, for $H_a/H_c = 2.5$, gets more extended still, demonstrating intensified refraction towards the poles (Fig. 1(c)). Clearly, when penetration of magnetic flux occurs, the paramagnetic support is bound to exert an influence on the distribution of the magnetic field. The overall impact, however, remains weak due to the protecting action of the tube.

The variation of the half-angle of flux penetration with the amplitude of the applied magnetic field, depicted in Fig. 2(a) for a range of values of the permeability, confirms these traits: penetration of magnetic flux sets in at $H_a/H_c = \pi/2$, like for an isolated superconducting tube;\[38\] the half-angle of flux penetration rises monotonically with a tendency to saturation in a fully flux-filled state, as the amplitude of the applied magnetic field augments. The dependence on the permeability, by comparison, is non-monotonic and rather weak. The variation of the hysteretic ac loss with the amplitude of the applied magnetic field shown in Fig. 2(b) corroborates the threshold for the onset of this loss followed by a sharp monotonic rise, with a turn to a shallow maximum. A slight, but distinctly monotonic, increase occurs when the permeability is raised; even signs of saturation at higher values of the permeability exist, as predicted before.\[20, 33\]

**B. Outer paramagnetic support**

Evidently, matters look different in the case of an outer paramagnetic support. This first concerns the distribution of the magnetic field around the magnetically coated superconducting tubular wire portrayied in Fig. 3, again for the permeability set and three progressive values of the amplitude of the applied magnetic field. At $H_a/H_c = 2.0$, the interior of the wire is still completely shielded from the applied magnetic field by the paramagnetic support on top of the superconducting tube: the lines of the magnetic field are refracted at the wire’s outer surface and guided around inside the paramagnetic support (Fig. 3(a)). As the amplitude of the applied magnetic field is increased to $H_a/H_c = 3.0$, magnetic flux starts to enter the superconductor constituent tangentially from both equatorial sides, threading two cylindrical segments of the tube, with refraction of the lines of the magnetic field at the wire’s inner surface too, before the interior of the wire accommodates the flux (Fig. 3(b)); an effect which, for $H_a/H_c = 4.0$, gets more pronounced still, exhibiting intensified refraction towards the poles (Fig. 3(c)). The paramagnetic support here always plays a prominent role in the distribution of the magnetic field. Its shielding capacity defines an effective, reduced field $H_{\mu}$ that acts on the wire’s superconductor constituent.

The dissimilarity of behaviour carries over to the variation of the half-angle of flux penetration with the amplitude of the applied magnetic field, depicted in Fig. 4(a) for a range of values of the permeability: assuming the permeability of a vacuum, penetration of magnetic flux starts at $H_a/H_c = \pi/2$, again like for an isolated superconducting tube,\[38\] with a monotonic rise and a tendency towards saturation in a fully flux-filled state, as the amplitude of the applied magnetic field augments. Increasing the permeability to account for a paramag-
FIG. 4. (Colour online) Dependence of (a) the half-angle of flux penetration $\gamma$ and (b) the normalized hysteretic ac loss $U_{ac}/H_f^2$ on $H_a/H_c$, the normalized amplitude of the magnetic field inside the tube, like for a massive cylindrical set, the inverse of its reduced strength $H_a/H_\mu$ unfolding a nonlinear dependence on the permeability $\mu$. The magnetic field in the interior of the tubular wire configuration discussed here, however, proves to be inhomogeneous and strongly redistributed due to the presence of the superconductor constituent, the inverse of the effective, reduced field $H_a/H_\mu$ extracted from the numerical simulations demonstrating a perfectly linear dependence on $\mu$, as eqn. (3) states. The variation of the shielding capacity of the outer paramagnetic support with $\mu$ in turn admits tuning the hysteretic ac loss by a judicious choice of $\mu$.

V. DISCUSSION AND CONCLUSION

There are two sources bringing about modifications of the hysteretic ac loss in the superconducting tube for a given paramagnetic support: changes of the amplitude of the applied magnetic field and adjustments of the orientation of the local magnetic field which acts on the superconducting tube, a steep angle with the surface of the tube resulting in an enhancement of the loss. The case of an inner paramagnetic support clearly testifies to the latter fact. On the other hand, both sources favour a reduction of the hysteretic ac loss in the case of an outer paramagnetic support by depressing and guiding the magnetic flux, consistent with the findings of a previous analysis, which noted the factorization of the shielding effects of the superconducting and paramagnetic constituents before. The decrease of the hysteretic ac loss in this configuration certainly involves a more subtle interaction between the two types of constituents. Shielding of the transverse applied magnetic field by the paramagnetic support alone would generate a homogeneous magnetic field inside the tube, like for a massive cylindrical set, the inverse of its reduced strength $H_a/H_\mu$ unfolding a nonlinear dependence on the permeability $\mu$. The magnetic field in the interior of the tubular wire configuration discussed here, however, proves to be inhomogeneous and strongly redistributed due to the presence of the superconductor constituent, the inverse of the effective, reduced field $H_a/H_\mu$ extracted from the numerical simulations demonstrating a perfectly linear dependence on $\mu$, as eqn. (3) states. The variation of the shielding capacity of the outer paramagnetic support with $\mu$ in turn admits tuning the hysteretic ac loss by a judicious choice of $\mu$. 

\[ \gamma = \gamma (H_\mu/H_c) \] (2) with 
\[ H_a/H_\mu = 1 + 0.048 (\mu/\mu_0 - 1) \] (3) and 
\[ U_{ac}/H_a^2 = 8\pi\mu_0 R^2 \lambda_\mu f (H_\mu/H_c) \] (4) with 
\[ \log \lambda_\mu = -0.325 \log^2 (\mu/\mu_0) \], (5) the chosen values of the geometrical parameters of the wire implied, making recourse to the functional dependences $\gamma (H_a/H_c)$ and $f (H_a/H_c)$ deduced for the isolated superconducting tube, either from analytical theory or numerical analysis.
Attention should be paid to the circumstance that, for a transverse applied magnetic field, the shielding effect of an outer paramagnetic support in the superconducting tubular wire configuration is usually strong and strictly advantageous regarding the hysteretic ac loss, unlike that of a paramagnetic support in a planar bilayer superconductor/paramagnet heterostructure which is comparatively weak and either beneficial or detrimental, depending on the geometrical and material characteristics of the magnetic environment and the amplitude of the applied magnetic field.\[24, 26\] The reason for this significant difference is of geometrical sort. The component of the magnetic field penetrating into the flat bilayer heterostructure is oriented normal to the plane of the structure; due to the paramagnetic support guiding the magnetic flux, it may either be enhanced or reduced. By contrast, the component of the magnetic field penetrating into the superconducting tube at its equatorial line is oriented tangential to the surface of the tube; an outer paramagnetic support guiding the magnetic flux can only promote this grazing orientation, thus reducing the field-effect on the superconductor constituent.

In conclusion, our finite-element simulations reveal effects of a paramagnetic support on hysteretic ac losses in a tubular superconductor/paramagnet heterostructure subject to an oscillating transverse magnetic field. Accordingly, one-sided magnetic shielding of the superconducting tube by a coaxial paramagnetic support gives rise to a slight increase of hysteretic ac losses as compared to those for a vacuum environment, when the support is placed inside; a spectacular shielding effect with a possible reduction of hysteric ac losses by orders of magnitude, however, ensues, depending on the magnetic permeability and the amplitude of the applied magnetic field, when the support is placed outside.
[40] E. H. Brandt and M. Indenbom, Phys. Rev. B 48, 12893 (1993).