Decentralized Predictor Output Feedback for Large-scale Systems with Large Delays

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Abstract: A majority of existing literature on time-delay systems focus on the robust stability of a single plant with respect to a “small” delay. This paper proposes a decentralized predictor-based feedback to compensate large delays for large-scale interconnected systems. The full-state of each subsystem is assumed to be unmeasurable and the observer-based output feedback is employed. Two methods are used to tackle the large delays: the backstepping-based partial differential equation (PDE) method is employed for continuous-time control, which derives simpler linear matrix inequality (LMI) conditions and manages with larger delays, whereas the reduction-based ordinary differential equation (ODE) method is applied to sampled-data implementation under continuous-time measurement. Instead of treating the large-scale systems as a whole, a decentralized Lyapunov-Krasovskii method is presented to guarantee the exponential stability of the large-scale systems under decentralized predictors.

Keywords: Decentralized, Predictor, Output Feedback, Large-scale Systems, Delay.

1. INTRODUCTION

By virtue of rapidly-developed communication and digital technologies, networked control systems (NCSs) show great potential in modern control. However, the development of NCSs is also full of challenges. Among many technical difficulties, an important and popular topic is the time-delay, which render the controlled system unstable when disregarded. A large body of existing literature on NCSs concentrate on the robust stability analysis with respect to “small” delays in the feedback loop via communication network. In other words, the delays are not compensated and the largest values of the delays that preserve the performance are investigated in terms of LMI condition Fridman (2014); Freirich (2016); Liu et al. (2012).

To compensate large delays, a key tool is the predictor feedback, which has found a widespread application in practice since it was developed 60 years ago Smith (1959). However, most results assume a single plant with a centralized controller Artstein (1982); Selivanov et al. (2016a,b). The recent paper Liu et al. (2018) considers predictor-based stabilization for two interconnected systems, but the results are based on state feedback and restricted to continuous-time control.

This paper extends the predictor feedback to decentralized control for large-scale interconnected systems with large input delays. Here the large delays denote such delays that do not preserve the stability of the control system (which is stable without the delays), and need compensation. Otherwise, the delays are called small. Different from our preliminary work Zhu et al. (2020) where the state-feedback is considered, this paper addresses a more challenging problem where the full-state of each subsystem is assumed to be unmeasured. The observer-based output feedback is important in implementation. Accordingly, the closed-loop system is more complicated.

The proposed Lyapunov-Krasovskii functional (LKF) and the resulting LMI are more sophisticated than those of state feedback. We propose two approaches for the delay compensation: the backstepping-based PDE method and the reduction-based ODE method. The PDE-based predictor is capable to derive simpler LMI conditions and withstand larger delays, whereas the ODE-based method is applicable to both continuous-time and sampled-data stabilization.

Instead of analyzing the large-scale systems as a global system, a decentralized Lyapunov-Krasovskii method is presented for the exponential stability analysis of the large-scale systems under decentralized predictors, in which the delays and sampling instants of each subsystem may be distinct from each other. One of the main challenges of decentralized analysis is to deal with the distributed delay terms from the neighbors. To address the distinct delay terms in the closed-loop system, various inequality techniques like Jensen, Wirtinger, Halanay and S-procedure Fridman (2014) are employed.

2. CONTINUOUS-TIME FEEDBACK

2.1 PDE Framework

Consider large-scale interconnected linear systems with input delays as follows:

\[ \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t - r_i) + \sum_{i \neq j} F_{ij} x_j(t) \]  

\[ y_i(t) = C_i x_i(t) \]

where \( i = 1, 2, \cdots, M \) is the subsystem index, \( x_i(t) \in \mathbb{R}^m \), \( y_i(t) \in \mathbb{R}^n \) and \( u_i(t) \in \mathbb{R}^m \) are the state, output and local control input of the \( i \)-th plant, respectively, \( x_j(t) \in \mathbb{R}^m \) are coupling terms. The control input is subject to a large constant and known input delay \( r_i > 0 \). We assume that the plant state \( x_i(t) \) is unmeasurable, the pair \((A_i, B_i)\) is stabilizable and \((A_i, C_i)\) is detectable, which means there exist matrices of appropriate
dimensions $K_i$ and $L_i$ such that $A_i + B_i K_i$ and $A_i - L_i C_i$ are Hurwitz.

In this section, we deal with the case of continuous-time feedback by the PDE-based framework Krstic (2009).

We introduce a multi-variable function

$$v_i(\sigma, t) = u_i(t + \sigma) - r_i, \quad \sigma \in [0, r_i]$$

(3)

to represent the control input $u_i(\theta)$ over the time interval $\theta \in [t - r_i, t]$. With (3), the system (1)-(2) is represented by the ODE-PDE cascade as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i v_i(0, t) + \sum_{i \neq j} F_{ij} x_j(t)$$

(4)

$$y_i(t) = C_i x_i(t)$$

(5)

$$\partial_\sigma v_i(\sigma, t) = \partial_\sigma v_i(\sigma, t), \quad \sigma \in [0, r_i]$$

(6)

$$v_i(r_i, t) = u_i(t)$$

(7)

It is apparent that (3) is a solution of the transport PDE (6)-(7). We denote $\hat{x}_i(t)$ to be an estimate of the unmeasured state $x_i(t)$ and the observer is designed as

$$\dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + B_i v_i(0, t) + L_i (y_i(t) - C_i \hat{x}_i(t))$$

(8)

with the estimation error $\dot{e}_i(t) = x_i(t) - \hat{x}_i(t)$ satisfying

$$\dot{e}_i(t) = (A_i - L_i C_i) e_i(t) + \sum_{i \neq j} F_{ij} e_j(t)$$

(9)

The predictor-based multi-variable controller is designed as

$$u_i(t) = v_i(r_i, t) = K_i \left( e^{A_i r_i} \hat{x}_i(t) + \int_0^{r_i} e^{A_i (r_i - \delta)} B_i v_i(\delta, t) d\delta \right)$$

(10)

For convenience of stability analysis, we bring in the invertible backstepping transformation

$$w_i(\sigma, t) = v_i(\sigma, t) - K_i e^{A_i \sigma} \hat{x}_i(t)$$

$$- K_i \int_0^{\sigma} e^{A_i (\sigma - \delta)} B_i v_i(\delta, t) d\delta$$

(11)

$$v_i(\sigma, t) = w_i(\sigma, t) + K_i e^{A_i + B_i K_i} \sigma \hat{x}_i(t)$$

$$+ K_i \int_0^{\sigma} e^{A_i + B_i K_i} (\sigma - \delta) B_i w_i(\delta, t) d\delta$$

(12)

through which the transport PDE (6)-(7), the observer and its error (8)-(9) are converted into the close-loop target system as follows:

$$\dot{\hat{x}}_i(t) = (A_i + B_i K_i) \hat{x}_i(t) + B_i v_i(0, t) + L_i C_i \hat{x}_i(t)$$

(13)

$$\dot{\hat{x}}_i(t) = (A_i - L_i C_i) \hat{x}_i(t) + \sum_{i \neq j} F_{ij} (\dot{e}_j(t) + \hat{x}_j(t))$$

(14)

$$\partial_\sigma w_i(\sigma, t) = \partial_\sigma w_i(\sigma, t) - K_i e^{A_i \sigma} L_i C_i \hat{x}_i(t)$$

$$- v_i(\sigma, t) d\sigma$$

(15)

$$w_i(r_i, t) = K_i \left( e^{A_i r_i} \hat{x}_i(t) - \hat{x}_i(t) \right) + \int_0^{r_i} e^{A_i (r_i - \delta)} B_i (v_i(\delta, t) - v_i(\delta, t)) d\delta \neq 0$$

Thus it is difficult to apply the PDE-based method to sampled-data control.

Proof: The Lyapunov-Krasovskii functional (LKF) is selected as $V_i(t) = V_i(t) + V_i(t) + V_i(t)$ where

$$V_i(t) = \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) U_i w_i(\sigma, t) d\sigma$$

(19)

$$V_i(t) = \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) R_i w_i(\sigma, t) d\sigma$$

(20)

$$V_i(t) = \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) U_i w_i(\sigma, t) d\sigma$$

(21)

Taking the time derivative of (19) along (13), we have

$$\dot{V}_i(t) + 2 \alpha V_i(t)$$

$$= \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) \partial_\sigma w_i(\sigma, t) d\sigma$$

$$= \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) \partial_\sigma w_i(\sigma, t) d\sigma$$

(22)

Taking the time derivative of (20) along (14), we get

$$\dot{V}_i(t) + 2 \alpha V_i(t)$$

$$= \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) \partial_\sigma w_i(\sigma, t) d\sigma$$

$$- \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) U_i w_i(\sigma, t) d\sigma$$

(23)

Taking the time derivative of (21) along (15)-(16) and using the integration by parts in $\sigma$, we obtain

$$\dot{V}_i(t) + 2 \alpha V_i(t)$$

$$= \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) \partial_\sigma w_i(\sigma, t) d\sigma$$

$$- \int_0^{r_i} e^{(1 + 2 \alpha) \sigma} w_i^T(\sigma, t) U_i w_i(\sigma, t) d\sigma$$

(24)

where $|\lambda_i| = \lambda_{max} (A_i^T A_i)$ and

$$\phi_i^{(1)} = (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + 2 \alpha P_i$$

$$\phi_i^{(2)} = (A_i - L_i C_i)^T R_i + R_i (A_i - L_i C_i) + 2 \alpha R_i$$

$$\phi_i^{(3)} = \text{diag}_{j=1\cdots M} \left\{ \frac{2 \alpha}{M - 1} P_i, j \neq i \right\}$$

(17)
Using Jensen’s inequality, $\xi^T_i(t)$ satisfies
$$\left|\xi^T_i(t)\right|^2 \leq n \int_0^t e^{(1+2\alpha)\sigma} w^T_i(\sigma,t) U_i K_i e^{\alpha \sigma} d\sigma \leq n e^{2(1+2\alpha)\tau_i/2} \int_0^t |w^T_i(\sigma,t) U_i K_i|^2 d\sigma$$

From (22)-(25), we have
$$V_i(t) + 2\alpha V_i(t) - \frac{2\epsilon}{M - 1} \sum_{j \neq i} V_j(t) + \frac{1}{\lambda_i} \left[ \int_0^t w^T_i(\sigma,t) U_i K_i e^{\alpha \sigma} d\sigma - \left|\xi^T_i(t)\right|^2 \right]$$
$$\leq - \int_0^t w^T_i(\sigma,t) \left[ U_i - \frac{1}{\lambda_i} U_i K_i K_i U_i^T \right] w_i(\sigma,t) d\sigma + \eta_i(t) \text{diag} \left( I, I, \frac{1}{\lambda_i}, I \right) \Phi_i \text{diag} \left( I, I, \frac{1}{\lambda_i}, I \right) \eta_i(t) \leq 0$$

where $\lambda_i > 0$ and $\eta_i(t) = \text{col} \left\{ \tilde{x}_i(t), \tilde{\theta}_i(t), w_i(0,t), \tilde{\xi}_i(t) \right\}$, $\text{col}_{j=1,\ldots,M} \left\{ \tilde{x}_i(t), j \neq i \right\}$ and $I$ is a unit matrix of appropriate dimension.

Applying Schur complement lemma in Section 3.2.3 of Fridman (2014), the inequality (26) is implied by LMI-condition (17)-(18). From (26), we conclude the LKF candidate along the solution of closed-loop system (13)-(16) satisfies $V_i(t) + 2\alpha V_i(t) \leq \frac{2\epsilon}{M - 1} \sum_{j \neq i} V_j(t)$, then we have $V_i(t) + 2(\alpha - \epsilon) V_i(t) \leq 0$ where $V_i(t) = \sum_{j \neq i} V_j(t)$, which implies the exponential stability of the closed-loop system by the comparison principle.

**Remark 2**: In our preliminary work Zhu et al. (2020) where the full-state of each subsystem is assumed to be measurable, we compare the conventionally centralized analysis with the decentralized analysis which is similar to Theorem 1. In the centralized analysis, we treat the large-scale system as a global system and apply a full-order LKF to stability analysis. It is revealed that the LMIs via the decentralized method have essentially less decision variables and are of smaller order comparatively to the LMI resulting from the centralized one. In the sampled-data case with asynchronous sampling, the decentralized analysis leads to essentially simpler results than the centralized one, where multiple integral terms should be inserted into LKF to take care of multiple samplings. This advantage should be more apparent in the observer-based output feedback considered in this paper since the closed-loop system (13)-(16) is higher-order.

### 2.2 ODE Framework

In this section, as shown in Fig. 1, we still address the case of continuous-time control. To lay a foundation for the sampled-data implementation in later sections, we employ the ODE scheme here.

Concentrating on the system (1)-(2), the variable $\tilde{x}_i(t)$ is used to denote the estimate of the unmeasured state $x_i(t)$ with the estimation error $\tilde{e}_i = x_i - \tilde{x}_i$. The observer is designed as
$$\dot{\tilde{x}}_i(t) = A_i \tilde{x}_i(t) + B_i u_i(t - r_i) + L_i (y_i(t) - C_i \tilde{x}_i(t)) \tag{27}$$

We introduce the change of variable
$$\dot{\tilde{z}}_i(t) = \hat{e}^{A_i} \tilde{x}_i(t) + \int_{t-r_i}^t \hat{e}^{A_i(s)} B_i u_i(s) ds$$

If the term of estimation error $L_i(y_i(t) - C_i \tilde{x}_i(t)) = 0$ in (27), it is evident that (28) is an exact prediction of the future state $\tilde{z}_i(t) = \tilde{x}_i(t + r_i)$.

The predictor control law is selected as
$$u_i(t) = K_i \tilde{z}_i(t)$$

For stability analysis, taking the time derivative of (28) along (27), the dynamics of $\dot{\tilde{z}}_i(t)$ is calculated as
$$\dot{\tilde{z}}_i(t) = A_i \tilde{z}_i(t) + B_i u_i(t) + \hat{e}^{A_i} L_i (y_i(t) - C_i \tilde{x}_i(t)) = (A_i + B_i K_i) \tilde{z}_i(t) + \hat{e}^{A_i} L_i C_i \tilde{x}_i(t)$$

Making use of (29), the inverse conversion of (28) is brought in as
$$\dot{\tilde{z}}_i(t) = e^{-A_i \theta} \tilde{z}_i(t) - \int_{t-r_i}^t e^{A_i(s)} B_i u_i(s) ds$$

**Theorem 2**: Consider the closed-loop system consisting of the plant (1)-(2), observer (27) and controller (30). Given tuning parameters $0 < \epsilon < \alpha$, let matrices $P_i, R_j, W_j \in \mathbb{R}^{n_i \times n_i} > 0$, $P_j, R_j, W_j \in \mathbb{R}^{n_j \times n_j} > 0$, for $j = 1, \ldots, M$ and $j \neq i$, satisfy the LMIs:

$$\Phi_i = \begin{bmatrix} \phi^T_{11} & P_i \hat{e}^{A_i} L_i C_i & 0 & 0 & 0 \\ \phi^T_{22} & R_i & R_i & R_i & -R_i \\ \phi^T_{33} & * & * & * & 0 \\ \phi^T_{44} & * & * & * & 0 \\ \phi^T_{55} & * & * & * & 0 \end{bmatrix} < 0 \tag{34}$$

where $\Phi_i$ is a symmetric matrix, and
Fig. 1. Continuous-time Control for Large-scale Systems with Input Delays

\[ \phi_{i1} = (A_i + B_iK_i)^T P_i + P_i (A_i + B_iK_i) + 2\alpha P_i + \tilde{W}_i, \]

\[ W_i = r_i K_i^T B_i^T \left( \int_{-\tau_i}^{0} e^{-A_i (\theta + \tau_i)} W_i e^{-A_i (\theta + \tau_i)} d\theta \right) B_i K_i, \]

\[ \phi_{i2} = (A_i - L_i C_i)^T R_i + R_i (A_i - L_i C_i) + 2\alpha R_i, \]

\[ \phi_{i3} = \text{diag}_{i=1,\ldots,M} \left\{ \frac{2\epsilon}{M-1} P_i, j \neq i \right\}, \]

\[ \phi_{i4} = \text{diag}_{i=1,\ldots,M} \left\{ \frac{2\epsilon}{M-1} P_i, j \neq i \right\}, \]

\[ \phi_{i5} = \text{diag}_{i=1,\ldots,M} \left\{ \frac{1}{M-1} e^{-2\alpha r_j} W_j, j \neq i \right\}, \]

\[ \mathcal{F}_i = \text{row}_{j=1,\ldots,M} \{ F_{ij}, j \neq i \}, \]

\[ \mathcal{R}_i = \text{row}_{j=1,\ldots,M} \{ F_{ij} e^{A_i r_j}, j \neq i \}. \]

Then the closed-loop system is exponentially stable with a decay rate \( \rho = \alpha + \epsilon. \)

**proof:** The LKF is constructed as

\[ V_i(t) = V_P(t) + V_R(t) + V_W(t) \]

where

\[ V_P(t) = \zeta_i^T(t) P_i \zeta_i(t), \quad P_i > 0 \]

(35)

\[ V_R(t) = \tilde{x}_i^T(t) R_i \tilde{x}_i(t), \quad R_i > 0 \]

(36)

\[ V_W(t) = r_i \int_{-\tau_i}^{0} \int_{s+j}^{s} e^{2\alpha (t-s)} \zeta_i^T(s) K_i^T B_i^T e^{-A_i (\theta + \tau_i)} W_i \]

\[ \times e^{-A_i (\theta + \tau_i)} B_i K_i \tilde{z}_i(s) d\theta d\theta, \quad W_i > 0 \]

(37)

Please note that \( V_W(t) \) is used to handle the distributed delay \( \tilde{z}_j(t) \) in (33).

Taking the time derivative of (35) along (31), we have

\[ V_P(t) + 2\alpha V_P(t) = \zeta_i^T(t) (2P_i (A_i + B_iK_i) + 2\alpha P_i) \tilde{z}_i(t) + 2\zeta_i^T(t) P_i e^{A_i r_i} L_i C_i \tilde{x}_i(t) \]

(38)

Taking the time derivative of (36) along (33), we get

\[ V_R(t) + 2\alpha V_R(t) = \tilde{x}_i^T(t) (2R_i (A_i - L_i C_i) + 2\alpha R_i) \tilde{x}_i(t) + 2\tilde{x}_i^T(t) R_i \sum_{s \neq j} F_{ij} (\tilde{z}_j(t) + e^{-A_i r_j} \tilde{z}_j(t) - \zeta_j(t)) \]

(39)

Taking the time derivative of (37) and using Jensen’s inequality, we have

\[ V_W(t) + 2\alpha V_W(t) = r_i \zeta_i^T(t) K_i^T B_i^T \left( \int_{-\tau_i}^{0} e^{-A_i (\theta + \tau_i)} W_i e^{-A_i (\theta + \tau_i)} d\theta \right) B_i K_i \tilde{z}_i(t) \]

\[ - r_i \int_{-\tau_i}^{0} e^{2\alpha r_j} \zeta_i^T(t + \theta) K_i^T B_i^T e^{-A_i (\theta + \tau_i)} \]

\[ \times W_i e^{-A_i (\theta + \tau_i)} B_i K_i \tilde{z}_i(t + \theta) d\theta \]

\[ \leq \bar{\xi}_i^T(t) \tilde{W}_i \tilde{z}_i(t) \]

\[ - e^{-2\alpha r_j} \left( \int_{-\tau_i}^{0} \zeta_i^T(t + \theta) K_i^T B_i^T e^{-A_i (\theta + \tau_i)} d\theta \right) \]

\[ \times W_i \left( \int_{-\tau_i}^{0} e^{-A_i (\theta + \tau_i)} B_i K_i \tilde{z}_i(t + \theta) d\theta \right) \]

\[ = \bar{\xi}_i^T(t) \tilde{W}_i \tilde{z}_i(t) - e^{-2\alpha r_j} \bar{\xi}_i^T(t) \tilde{W}_i \tilde{z}_i(t) \]

(40)

where \( \tilde{W}_i \) has been given underneath (34). From (38)-(40), we get

\[ V_i(t) + 2\alpha V_i(t) - \frac{2\epsilon}{M-1} \sum_{j \neq i} V_j(t) + e^{-2\alpha r_j} \bar{\xi}_j^T(t) W_L \bar{\xi}_j(t) \]

\[ - \frac{1}{M-1} \sum_{j \neq i} e^{-2\alpha r_j} \bar{\xi}_j^T(t) W_j \bar{\xi}_j(t) \]

\[ \leq \eta_i(t) \Phi \eta_i(t) \leq 0 \]

(41)

where \( \eta_i(t) = \text{col} \{ \zeta_i(t), \tilde{x}_i(t), \text{col}_{j=1,\ldots,M} \{ \tilde{z}_j(t), j \neq i \}, \text{col}_{j=1,\ldots,M} \{ \xi_j(t), j \neq i \} \} \). It is apparent that inequality (41) is suggested by LMI-condition (34). Thus we derive \( V(t) + 2(\alpha - \epsilon) V(t) \leq 0 \) from (41) where \( V(t) = \sum_{i=1}^{M} V_i(t) \), which implies the exponential stability of the closed-loop system.

**3. SAMPLED-DATA FEEDBACK WITH CONTINUOUS-TIME MEASUREMENT**

In this section, as revealed in Fig. 2, we consider a more complicated case where the system is with a controller-to-actuator network subject to a large transmission delay \( r_i \) and is able to continuously measure the plant output \( y_i(t) \). The continuous-time control signal \( u_i(t) \) is sampled at the time instants \( \tau_i^j \) and sent to the zero-order hold (ZOH) through the delayed network. The sampling series \( \{ \tau_i^j \} \) satisfy

\[ 0 = \tau_i^0 < \tau_i^1 < \tau_i^2 < \cdots, \quad \lim_{k \to \infty} \tau_i^k = \infty, \quad \tau_i^{k+1} - \tau_i^k \leq h_i \]

(42)

The ZOH is assumed to be event-driven so that it updates its output once it receives new data. Thus the updating instants of the ZOH satisfies \( \tau_i^k = \tau_i^{k+1} < \tau_i^{k+1} \), \( k \in \mathbb{Z}_0^+ \). As analyzed in Remark 1, when the control signals are sampled, the PDE-based method is not trivially applicable to NCSs so that the ODE-based approach is employed.
Fig. 2. Sampled-data Control with Continuous-time Measurement for Large-scale Systems with Delays

Under the controller-to-actuator network with delay, it is evident that the system (1)-(2) becomes
\[
\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j \neq i} F_{ij} x_j(t), \quad t \in [t_k, t_{k+1})
\]
(43)
\[
y_i(t) = C_i x_i(t)
\]
(44)

Based on the system (43)-(44), we denote \( \hat{x}_i(t) \) to be the estimate of the unmeasured state \( x_i(t) \), and \( \hat{x}_i(t) = x_i(t) - \hat{x}_i(t) \) to be the estimation error. We design the observer as
\[
\dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + B_i u_i(t) + L_i(y_i(t) - C_i \hat{x}_i(t)), \quad t \in [t_k, t_{k+1})
\]
(45)

We select the observer-based predictor as
\[
\dot{z}_i(t) = e^{A_i t} \hat{x}_i(t) + \int_{t_r}^{t} e^{A_i(t-s)} B_i u_i(s) ds
\]
and choose the predictor-based control law as
\[
u_i(t) = K_i \dot{z}_i(t)
\]
(46)

For stability analysis, the dynamics of \( \dot{z}_i(t) \) along (45) and (47) is of the form
\[
\dot{z}_i(t) = A_i \dot{z}_i(t) + B_i u_i(t) + e^{A_i t} B_i (u_i(t - r_i) - u_i(t - r_i)) + e^{A_i t} L_i (y_i(t) - C_i \hat{x}_i(t))
\]
\[
= (A_i + B_i K_i) \dot{z}_i(t) + e^{A_i t} B_i K_i \dot{x}_i(t) + e^{A_i t} L_i C_i \hat{x}_i(t),
\]
t \in [t_k, t_{k+1})
(49)

where \( r_i = t_i - r_i = \hat{x}_i(t_k - r_k) - \hat{x}_i(t - r_i) \).

Subtracting (43) from (49) and utilizing the inverse transformation of (46), the estimation error \( \dot{x}_i(t) = x_i(t) - \dot{x}_i(t) \) is governed by
\[
\dot{x}_i(t) = A_i \dot{x}_i(t) - L_i (y_i(t) - C_i \hat{x}_i(t)) + \sum_{j \neq i} F_{ij} x_j(t)
\]
\[
= (A_i - L_i C_i) \dot{x}_i(t) + \sum_{j \neq i} F_{ij} (\dot{x}_i(t) + e^{-A_i t} \dot{z}_j(t) - \dot{z}_j(t)),
\]
t \in [t_k, t_{k+1})
(50)

where \( \dot{z}_j(t) = \int_{t_r}^{t} e^{-A_i(t-s)} B_i K_i \dot{z}_j(t + \theta) d\theta \).

**Remark 3:** In Selivanov et al. (2016b) where a single plant is considered, besides the observer predictor (46), the plant predictor is also introduced such that \( z_i(t) = e^{A_i t} x_i(t) + \int_{t_r}^{t} e^{A_i(t-s)} B_i u_i(s) ds \). If the method of Selivanov et al. (2016b) is applied to large-scale systems, an alternative version of the closed-loop system (49)-(50) is of the form:
\[
\dot{x}_i(t) = (A_i + B_i K_i) \dot{x}_i(t) + e^{A_i t} L_i C_i \dot{x}_i(t) + e^{A_i t} B_i K_i \dot{x}_i(t) = (A_i - e^{A_i t} L_i C_i e^{-A_i t} \dot{x}_i(t) + e^{A_i t} \sum_{j \neq i} F_{ij} (e^{-A_i t} \dot{z}_j(t) + e^{-A_i t} \dot{z}_j(t) - \dot{z}_j(t)))
\]
t \in [t_k, t_{k+1})
(51)

where \( \dot{z}_i(t) = z_i(t) - \dot{z}_i(t) \). It is apparent that (49)-(50) proposed in the present paper is simpler and the redundant change of variable \( z_i(t) \) is avoided.

**Theorem 3:** Consider the closed-loop system consisting of the plant (43)-(44), observer (45) and controller (48). Given tuning parameters \( 0 < \varepsilon < \alpha \), let matrices \( P, \Phi_i, W \in \mathbb{R}^{n \times n}, P_1, P_2, P_3, P_{ij} \in \mathbb{R}^{n \times n} \) and \( P_1, P_2, P_{ij} \in \mathbb{R}^{n \times n} \) for \( j = 1, \ldots, M \) and \( j \neq i \), satisfy the LMI:
\[
\begin{bmatrix}
\Phi_i & \Psi_i \\
\ast & -H_i
\end{bmatrix} < 0
\]
(52)

where
\[
\Psi_i = \begin{bmatrix}
P_1 e^{A_i T_i} L_i C_i \Phi_i & P_1 e^{A_i T_i} B_i K_i & 0 & 0 & 0 \\
* & 0 & R_i \bar{F}_i & R_i \bar{F}_i & -R_i \bar{F}_i \\
* & * & * & \Phi_{i4} & 0 \\
* & * & * & * & \Phi_{i5} \\
* & * & * & * & * & \Phi_{i6}
\end{bmatrix}
\]
(53)

and \( \Phi_i \) is a symmetric matrix such that
\[
\Phi_i = \begin{bmatrix}
\Phi_{i1} & \Phi_{i2} & \Phi_{i3} & \Phi_{i4} & \Phi_{i5} & \Phi_{i6}
\end{bmatrix}
\]
(54)

where \( \Phi_i \) is a symmetric matrix, and \( \Phi_{i1}, \Phi_{i2}, \Phi_{i3}, \Phi_{i4}, \Phi_{i5}, \Phi_{i6} \) in (53) are exactly the same with \( \Phi_{i1}, \Phi_{i2}, \Phi_{i3}, \Phi_{i4}, \Phi_{i5}, \Phi_{i6} \) in (34), \( \Phi_{i3} = -\varepsilon^2 2 a r_i U_i \), and \( \Phi_{i4}, \Phi_{i5}, \Phi_{i6} \) in (53) are exactly the same with \( \Phi_{i4}, \Phi_{i5}, \Phi_{i6} \) in (34). Then the closed-loop system is exponentially stable with a decay rate \( \rho = \alpha - \varepsilon \).

**Proof:** The LKF is built as \( V(t) = V_R(t) + V_U(t) + V_W(t) \) where \( V_R(t), V_U(t), V_W(t) \) are exactly the same as (35)-(37), and
\[
V_U(t) = h_i^2 \varepsilon 2 a r_i \int_{t_r}^{t} e^{2 a r_i \varepsilon T_3} (U_i \dot{z}_i(t)) ds - \varepsilon^2 4 \int_{t_r}^{t} e^{2 a r_i \varepsilon T_3} (\dot{z}_i(t) - \dot{z}_i(t)) T U_i x_i(t) - \dot{z}_i(t) - \dot{z}_i(t)) T U_i x_i(t)
\]
(54)

Please note that \( V_U(t) \) is bounded and \( \lim_{t \to \infty} V_U(t) \geq V_U(t) \) by Wirtinger’s inequality in Liu et al. (2012), Selivanov et al. (2016b) and Section 7.4 of Fridman (2014). The term \( V_W(t) \) is employed to compensate \( \dot{z}_i(t) \) in (50), whereas \( V_U(t) \) is utilized.
to compensate $v_i(t)$ in (49). The remaining is similar to the step of proof of Theorem 2.

4. SIMULATION

Fig. 3. Three Interconnected Subsystems

Fig. 4. Predictor-free Feedback with Small Delays $r_1 = r_2 = r_3 = 0.1s$

Fig. 5. Predictor-based Feedback with Large Delays $r_1 = r_2 = r_3 = 0.13s$

In this section, we use an example of two coupled inverted pendulums on two carts from Borgers et al. (2014) under the decentralized control scheme.

The system matrices are $A_1 = A_2 = A_3 = \begin{bmatrix} -0.956 & 0 & 0 & 0 \\ 0 & -0.0042 & 0 & 0 \\ 0 & 0 & -0.0042 & 0 \\ 0 & 0 & 0 & -0.0002 \end{bmatrix}$.

$B_1 = B_2 = B_3 = \begin{bmatrix} -0.0042 \\ 0.0167 \end{bmatrix}$, $C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $F_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

$F_{12} = F_{23} = F_{31} = F_{13} = \begin{bmatrix} 0 & 0.0011 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0003 & 0 & 0 & 0 \end{bmatrix}$. The control gains are selected as $K_1 = \begin{bmatrix} 11396 & 7196.2 & 357.96 & 1199.0 \end{bmatrix}$, $K_2 = \begin{bmatrix} 29241 & 18135 & 2875.3 & 3693.9 \end{bmatrix}$, The observer gains are selected as $L_1 = L_2 = L_3 = \begin{bmatrix} 11.7 & -1.2 & 37 & -8.9 \\ -1.2 & 11 & -7.9 & 36 \end{bmatrix}$.

The simulation results are shown in Figs. 3-5. It is evident that the predictor-based controller promises a larger delay than the predictor-free controller.

REFERENCES

O. J. M. Smith, A controller to overcome dead time, ISA, vol. 6, pp. 28-33, 1959.

Z. Artstein, Linear systems with delayed controls: A reduction, IEEE Trans. Autom. Control, vol. 27, no. 4, pp. 869-879, 1982.

M. Krstic. Delay Compensation for Nonlinear, Adaptive, and PDE Systems. Berlin: Birkhauser, 2009.

K-Z. Liu, X-M. Sun, M. Krstic, Distributed predictor-based stabilization of continuous interconnected systems with input delays, Automatica, vol. 91, pp.69-78, 2018.

E. Fridman. Introduction to Time-Delay Systems: Analysis and Control, Birkhauser, 2014.

D. Freirich, E. Fridman, Decentralized networked control of systems with local networks: a time-delay approach, Automatica, vol. 69, pp. 201-209, 2016.

K Liu, E Fridman, Wirtinger’s inequality and Lyapunov-based sampled-data stabilization, Automatica, vol. 48, pp. 102-108, 2012.

A. Selivanov, E. Fridman, Predictor-based networked control under uncertain transmission delays, Automatica, vol. 70, pp. 101-108, 2016.

A. Selivanov, E. Fridman, Observer-based input-to-state stabilization of networked control systems with large uncertain delays, Automatica, vol. 74, pp. 63-70, 2016.

D. Borgers, W. Heemels, Stability analysis of large-scale networked control systems with local networks: A hybrid small-gain approach. CST report, 2014.025.

Y. Zhu, E. Fridman, Predictor methods for decentralized control of large-scale systems with input delays, Automatica, vol. 116, 108903, 2020.