Emergent Symmetry and Tricritical Points near the deconfined Quantum Critical Point

Chao-Ming Jian,1,2 Alex Rasmussen,3 Yi-Zhuang You,4 and Cenke Xu

1Kavli Institute of Theoretical Physics, Santa Barbara, CA 93106, USA
2Station Q, Microsoft Research, Santa Barbara, California 93106-6105, USA
3Department of Physics, University of California, Santa Barbara, CA 93106, USA
4Department of Physics, Harvard University, Cambridge, MA 02138, USA

(Dated: August 11, 2017)

Recent proposal of the duality between the $N = 2$ noncompact QED$_3$ and the easy-plane noncompact CP$^1$ (NCCP$^1$) model suggests that the deconfined quantum critical point (dQCP) between the easy-plane antiferromagnet and the VBS order on the square lattice may have an emergent O(4) symmetry, due to the self-duality of the $N = 2$ noncompact QED$_3$. Recent numerical progresses suggest that this easy-plane dQCP does exist and it has an emergent O(4) symmetry. But for the O(4) symmetry to really emerge at the dQCP, certain O(4) symmetry breaking perturbations need to be irrelevant at the putative O(4) fixed point. It is more convenient to study these symmetry breaking perturbations in the $N = 2$ noncompact QED$_3$. We demonstrate that a natural large-$N$ generalization and a controlled 1/N expansion supports the stability of the O(4) fixed point against the symmetry breaking perturbations. We also develop the theory for two tricritical points close to the easy-plane dQCP. One tricritical point is between the dQCP and a self-dual Z$_2$ topological order; the other is the tricritical point that connects the continuous dQCP and a first order Néel-VBS transition, motivated by recent numerical results.

PACS numbers:

Recent progress of $(2 + 1)d$ conformal field theories (CFT) has led us to expect that different Lagrangians at their quantum critical points may correspond to the same CFT, e.g. a property called “duality”. Within these proposed dualities, one is of great importance to condensed matter theory, which is the duality between the $N = 2$ noncompact QED$_3$ and the easy-plane NCCP$^1$ model at the critical point [1][3]. These two field theories can be written as

\[ \mathcal{L}_{\text{QED}} = \sum_{j=1}^{2} \bar{\psi}_j \gamma \cdot (\partial - i a) \psi_j + m \bar{\psi}_j \psi_j + M \bar{\psi} \sigma^3 \psi \]  

(1a)

\[ \mathcal{L}_{\text{CP}^1} = \sum_{j=1}^{2} (|\partial - i b| z_j)^2 + g |z_j|^4 + r |z_j|^2 + h \sigma^z z \]  

(1b)

where $\psi_j$ and $z_j$ are two-component Dirac fermions (with an extra flavor index $j$) and complex boson fields coupled to non-compact U(1) gauge fields, $a_\mu$ and $b_\mu$, respectively. The duality maps the variables $(m, M)$ to $(h, r)$.

When realized in terms of lattice quantum many-body systems, the tuning parameter $r$ of Eq. [1] drives a phase transition between the easy-plane Néel order and a valence bond solid (VBS) order, and it is called the deconfined quantum critical point (dQCP) [4][5]. Despite the earlier numerics which suggest a first order easy-plane Néel-to-VBS transition [4][8], most recently a modified lattice model was found which did show a continuous easy-plane dQCP [9] (there were more numerical evidences for the continuous dQCP with isotropic SO(3) spin symmetry [10][20]).

On the other hand, theoretically the tuning parameter $m$ in Eq. [1] drives the phase transition between the bosonic symmetry protected topological phase and the trivial phase [21][22], and it was shown numerically that such transition is also second order, as long as the system has high enough symmetries [23][24].

Before the more recent proposal of duality Eq. [1], it was first shown in Ref. [25] that Eq. [1] is self-dual at its critical point $r = 0$ and $h = 0$. This self-duality can be derived by performing the particle-vortex duality for each flavor of $z_2$ [26][28], followed by integrating out the gauge field $b_\mu$. Thus at the critical point $r = 0$, $h = 0$, the field theory has an explicit symmetry $[O(2)_s \times O(2)_v] \times Z_2$. The $O(2)_s = U(1)_s \times Z_2^s$ symmetry is the inplane spin rotation symmetry that acts on the CP$^1$ field $(z_1, z_2)$ as

\[ U(1)_s : z \rightarrow e^{i \frac{\pi}{2} \sigma^3}, \quad Z_2^s : z \rightarrow \sigma^z z. \]  

(2)

The $O(2)_v = U(1)_v \times Z_2^v$ symmetry corresponds to the conservation and particle-hole symmetry of the gauge flux of $b_\mu$:

\[ U(1)_v : M_\theta \rightarrow e^{i \theta} M_\theta, \quad Z_2^v : M_\theta \rightarrow M_\theta^\dagger, \quad z \rightarrow i \sigma^2 z^\dagger. \]  

(3)

where $M_\theta$ is the monopole operator ($2\pi$-gauge flux annihilation operator) of the gauge field $b_\mu$. The last $Z_2^v$ corresponds to the self-duality transformation which interchanges the two $O(2)$ symmetries, and it precludes the $r$ term in Eq. [1] if $Z_2^v$ is imposed as an actual symmetry. The $h$ term is excluded by the $Z_2^s$ symmetry, which is the improper rotation subgroup of $O(2)_s$.

Eq. [1a] was shown to be also self-dual in Ref. [29][31], by performing the fermionic version of the particle-vortex
FIG. 1: (a) Our RG equation Eq. 8 suggests that the perturbation $\lambda$ in Eq. 7 that breaks the O(4) symmetry down to O(2)$_s \times O(2)_v \times Z_2^d$ at the self-dual dQCP is irrelevant, which supports the emergence of O(4) symmetry at the infrared limit of the dQCP, and is consistent with recent numerical results; (b) The sketched phase diagram of Eq. 10 plus the tuning parameter $r$, or $M$ from Eq. 11. Especially, across a tricritical point, the system enters a self-dual Z$_2$ topological order where the self-dual symmetry $Z_2^d$ exchanges the e and m anyons.

The $Z_2^d$ and $Z_2^v$ symmetries involve the self-duality transformation of the $N = 2$ QED$_3$, while their product $Z_2^d \times Z_2^v$ flips the charge of both $U(1)_s$ and $U(1)_v$, and it acts as $Z_2^d \times Z_2^v$: $\psi \rightarrow e^{i\theta} \psi$, $a_\mu \rightarrow -a_\mu$. The self-dual $Z_2^d$ transformation of Eq. 10 corresponds to the “flavor flipping” symmetry $\psi \rightarrow \sigma^\dagger \psi$. (For more details of how the symmetries act on the $N = 2$ noncompact QED$_3$, please refer to Ref. 2.)

It appears that another four-fermion term $\sum_{\alpha \beta} \langle \bar{\psi} \sigma^3 \gamma_4 \psi \rangle^2$ is allowed once we break the symmetry of the $N = 2$ QED$_3$ down to O(2)$_s \times O(2)_v \times Z_2$. But this term is not linearly independent from the term in Eq. 5 and two other SU(2) symmetric terms when $N = 2$. To analyze whether the symmetry breaking term is relevant or not at the $N = 2$ noncompact QED$_3$ fixed point, we need a controlled calculation of its scaling dimension. And like many previous studies of (2 + 1)d QED$_3$, a large-$N$ generalization and a 1/N expansion is very helpful for this purpose.

In Ref. 38–40, a large-$N$ generalization of Eq. 1a was taken, and a 1/N-expansion calculation of the scaling dimensions of SU(N) invariant four fermion interaction perturbations suggest that these SU(N) invariant four fermion terms are likely always irrelevant even for small $N$. However, in Ref. 38–39 it was also shown that once we break the SU(N) flavor symmetry of the QED$_3$, some four fermion interaction may become relevant for small enough $N$, and lead to instability of the QED$_3$. Thus, we need to test whether the symmetry breaking term Eq. 5 causes this potential concern. But to evaluate this we need a large-$N$ generalization of Eq. 5. For this purpose, we change the basis and consider the interaction $\lambda(\bar{\psi} \sigma^3 \psi)^2$, which then has a natural large-$N$ generalization:

$$\frac{g}{N} \sum_{i,j} (\bar{\psi}_i \psi_j)(\bar{\psi}_i \psi_j).$$

This term breaks the global symmetry of noncompact QED$_3$ with $N$ flavors of Dirac fermions down to
$O(N) \times O(2)$, where the $O(2)$ corresponds to the conservation and particle-hole symmetry of the gauge flux of $a_\mu$. The advantage of this large-$N$ generalization is that, there is also only one term that breaks the symmetry down to $O(N) \times O(2)$, for arbitrary $N$. Another seemingly $O(N) \times O(2)$ invariant four-fermion term $\sum_\mu \sum_\gamma (\bar{\psi}_i \gamma_\mu \psi_j) (\bar{\psi}_j \gamma_\gamma \psi_i)$ is a multiple of Eq. 7 after using the Fierz identity of $\gamma_\mu$.

Unfortunately, the self-duality of the original $N = 2$ noncompact QED$_3$ no longer holds in this large-$N$ generalization. Despite the disadvantage of losing the self-duality, the same method as Ref. 39 leads to the following RG equation for $g$ at the leading order of the $1/N$ expansion:

$$\frac{dg}{dt} = \left( -1 - \frac{64}{3N\pi^2} \right) g + O(g^2). \tag{8}$$

This means that the first order $1/N$ correction to $g$ makes it even more irrelevant. This calculation is consistent with the recent numerical observation that an easy-plane J-Q model 39, a model that has a continuous transition between the easy-plane Néel and VBS order has the same set of critical exponents as another model with an exact microscopic SO(4) symmetry, hence both models are supposed to have an emergent O(4) symmetry at the critical point, meaning the perturbations that break the O(4) to $O(2)$ symmetry down to $O(2)$ are irrelevant.

The four fermion term Eq. 7 is perturbatively irrelevant at the $N = 2$ noncompact QED$_3$ fixed point, but when the microscopic perturbation that leads to Eq. 7 is strong enough, it can lead to new physics. For example, when $\lambda$ is negative, its effect can be captured by the following Lagrangian:

$$\mathcal{L}_{\text{QED-Yukawa}} = \sum_{j=1}^{2} \bar{\psi}_j \gamma \cdot (\partial - i\alpha) \psi_j + u \bar{\psi} \sigma^3 \psi \phi + (\partial_\mu \phi)^2 + r|\phi|^2 + g|\phi|^4. \tag{9}$$

$\phi$ is a real scalar field. When $\tilde{r} > 0$, $\phi$ is in its disordered phase, and integrating out $\phi$ will generate a short range four fermion interaction term Eq. 7, which as we evaluated above is an irrelevant perturbation at the $N = 2$ noncompact QED$_3$ fixed point. When $\tilde{r} < 0$, $\phi$ will be ordered, and the system spontaneously generates an expectation value of $\phi$. Recalling that the mass term $M \bar{\psi} \sigma^3 \psi$ is the tuning parameter of the Néel-VBS phase transition, thus when $\tilde{r} < 0$, the system spontaneously breaks the “self-dual” symmetry of the easy-plane NCCP$^1$ model, and the Néel-VBS phase transition becomes first order. Thus $\tilde{r} = 0$ is a tricritical point between the continuous easy-plane deconfined QCP and a first order Néel-VBS transition, which is an analogue of the tricritical Ising fixed point. This tricritical point between a continuous and discontinuous easy-plane Néel-VBS transition was first discussed in Ref. 11 in the formalism of NCCP$^1$ field theory, and our Lagrangian Eq. 9 can be viewed as the dual description of this tricritical point. Recent numerical simulation of one particular class of easy-plane spin-1/2 model on the square lattice also suggests the existence of this tricritical point 39.

Another tricritical point near the dQCP can be described by the following QED-Yukawa-Higgs type of Lagrangian:

$$\mathcal{L}_{\text{QED-Yukawa}} = \sum_{j=1}^{2} \bar{\psi}_j \gamma \cdot (\partial - i\alpha) \psi_j + u \left( \sigma_1^j \bar{\psi}_j \epsilon_{\alpha\beta} \psi_{j\beta} \right) \phi + H.c. + |(\partial - i2\alpha)|^2 + \tilde{r}|\phi|^2 + g|\phi|^4. \tag{10}$$

Now $\phi$ is a complex scalar field instead of a real scalar. Again, when $\tilde{r}$ is positive, $\phi$ is disordered, and system is described by Eq. 10 plus irrelevant short range four-fermion interaction Eq. 7 while when $\tilde{r} < 0$, $\phi$ forms a Cooper pair condensate, and the U(1) gauge field $a_\mu$ is Higgsed and broken down to a Z$_2$ gauge field.

To understand exactly the phase with $\tilde{r} < 0$, let us first analyze its symmetry. The Cooper pair $(\sigma_1^j \bar{\psi}_j \epsilon_{\alpha\beta} \psi_{j\beta})$ preserves the $U(1)_A$ flavor symmetry of $\psi_j$ generated by $\sigma^1$, and since the photon is Higgsed and gapped, the $U(1)_B$ symmetry which corresponds to the conservation of the gauge flux is also preserved. Since $U(1)_s$ and $U(1)_v$ are combinations of these two U(1) symmetries, both $U(1)_s$ and $U(1)_v$ are preserved. The $Z_2^{s,v} = Z_2^s \times Z_2^v$ symmetry is also obviously preserved even in the condensate, because in the condensate of $\phi$, the particle-hole transformation of the expectation value $\langle \phi \rangle$ can be cancelled by a gauge transformation.

Obviously the condensate of $\phi$ will gap out all the fermions, and the photon $a_\mu$ acquires a Higgs mass, thus this phase is fully gapped. The gapped excitations of this phase include a fermion $\psi$, which carries a Z$_2$ gauge charge, and the $U(1)_A$ flavor symmetry, which is a combination of $U(1)_s$ and $U(1)_v$. The $\pi$-flux of $a_\mu$ (the so-called vison) which is bound with a vortex of $\phi$ is another gapped excitation. The quantum number of the vison can be extracted by solving the Dirac equation with a background vortex of $\phi$, and we can see that there is one complex fermion zero mode at the vortex core. Each vortex core will carry the $U(1)_B$ quantum number of $\pi$-flux of $a_\mu$, and $\pm 1/2$ quantum number of the flavor $U(1)_A$ charge carried by the fermion zero mode. Thus these two visons with filled and unfilled fermion zero modes will carry half charge under $U(1)_s$ and half charge under $U(1)_v$ respectively. Because these two types of visons differ by a fermion, they will have mutual semion statistics caused by the Aharonov-Bohm effect between the fermion and the $\pi$-flux. For the same reason, the two types of visons also carry the same topological spins. This is because their difference in topological spins is the sum of the topological spin of the extra fermion and the Aharonov-Bohm phase between the extra fermion and the $\pi$-flux, which cancel each other.
Now let us label the $\pi$–flux carrying half $U(1)_e$ charge as the $e$ particle, and label the $\pi$–flux carrying half $U(1)_m$ charge as the $m$ particle. Usually, different topological excitations have different energies. However, in this case, since the $Z_2^d$ self-dual symmetry is unbroken, the $e$ and $m$ particles transform into each other under the $Z_2^d$ symmetry, and hence, are degenerate. A more concrete way of showing this degeneracy is to understand the previously mentioned complex fermion zero mode more carefully. When $\phi$ condenses, each of the Dirac fermions with $\sigma^1 = \pm 1$ eigenvalues, denoted as $\psi_\pm$ respectively, forms a copy of the Fu-Kane superconductor [42], while both are coupled to the same $Z_2$ gauge field. Therefore, the vortex of $\phi$ will carry two Majorana zero modes $\gamma_\pm$ from the two copies of the Fu-Kane superconductors. These two Majorana zero modes form the previously mentioned complex fermion zero mode. Generically, the two Majorana zero modes can hybridize and, as a consequence, lift the complex fermion zero mode. Such hybridization can, for example, be induced by a finite $N \psi \sigma^3 \bar{\psi}$ term. However, when the $Z_2^d$ symmetry is preserved, any hybridization of the two Majorana zero modes are prohibited because they carry different charges under $\sigma^\dagger$ or equivalently under $Z_2^d$:

$$Z_2^d : \gamma_+ \rightarrow \gamma_+, \quad \gamma_- \rightarrow -\gamma_-.$$ (11)

Therefore, the degeneracy between the $e$ and $m$ particle is ensured by $Z_2^d$. Also, the occupation number $|\gamma_+ - \gamma_-|$ of the complex fermion zero modes (constructed from the two Majorana zero modes), which distinguishes the $e$ and $m$ particles, changes under the action of $Z_2^d$. Therefore, we can conclude that the $Z_2^d$ symmetry exchanges the $e$ and $m$ particles.

Since the $Z_2^d$ self-dual symmetry is unbroken, $e$ and $m$ particles are transformed into each other under the $Z_2^d$ symmetry. Also, since the $Z_2^{x\times v}$ is unbroken, each $e$ and $m$ are a doublet, because $Z_2^{x\times v}$ perform a particle-hole transformation on both $U(1)_e$ and $U(1)_m$, and $e$ and $m$ can both carry +1/2 or −1/2 of their respective $U(1)$ symmetry. Or in other words, $e$ and $m$ carry projective representation of $O(2)_e$ and $O(2)_m$ respectively.

We can also understand the condensate of $\phi$ in another way. In a single slab geometry of a 3d TI, when we consider a Fu-Kane superconductor on its top surface and a time-reversal breaking bottom surface, this slab can be identified with a $p_x + ip_y$ or $p_x - ip_y$ superconductors depending on the time-reversal breaking pattern on the bottom surface. Now, in the condensate phase of $\phi$ in theory Eq. 10 we are effectively dealing with two copies of the (gauged) Fu-Kane superconductors. We can equivalently think of this phase as a phase hosted by two copies of the TI slabs. On the top surfaces of both slabs we consider the F-K superconductors as we did before. But on the bottom surfaces of two TI slabs, we demand them have opposite time-reversal breaking patterns. When the fermions are coupled to the same gauge field $a_\mu$, the Chern-Simons terms of the gauge field $a_\mu$ coming from the two bottom surfaces cancel each other, leaving the total topological order of the two slabs exactly that of two gauged Fu-Kane superconductors on the top surfaces of the two TI slabs.

This picture is very helpful for the understanding of the topological order of the $\phi$ condensate. Suppose we couple the fermions in the two TI slabs to two independent $Z_2$ gauge fields, we would get a $\text{Ising} \times \text{Ising}$ topological order. An $\text{Ising}$ topological order have anyon $1, \sigma, f$, which are vacuum, non-abelian anyon, and a fermion respectively, while the anyon content, labelled by $1, \sigma, f$ is similar in the $\text{Ising}$ topological order. To recover the condensed phase of $\phi$ in Eq. 10, we need to set the gauge fields in the two TI slabs equal (and identify them with the gauge field $a_\mu$ in Eq. 10), which can be enforced by condensing the $ff$ particle in the $\text{Ising} \times \text{Ising}$ topological order. The topological order induced by the condensate of $ff$ is exactly a $Z_2$ topological order [43]. In the condensate, the fermions $f$ and $\bar{f}$ are identified with each other and also with the fermion excitation $\psi$. The $\sigma\bar{\sigma}$ particle is also deconfined in the $ff$ condensate. In fact, it splits into two Abelian particles, and these two Abelian anyons are exactly the two types of visons $e$ and $m$ introduced before. Also, the topological spins of the two visors are inherited from that of the $\sigma\bar{\sigma}$ particle, which is trivial (bosonic). All other topological excitations, $\sigma_\pm$ for instance, in the $\text{Ising} \times \text{Ising}$ topological order are confined and hence will not appear the condensate of $ff$.

Now let us summarize our results: when tuning $\tilde{r}$ in Eq. 10 from positive to negative, the system enters a $Z_2$ topological order, with bosonic and mutual semionic $e$ and $m$ particles carrying projective representation of $O(2)_e$ and $O(2)_m$ symmetries respectively. The $Z_2^d$ duality symmetry interchanges the $e, m$ particles.

The $N = 2$ noncompact QED$_3$ was proposed as the boundary state of the 3d bosonic symmetry protected topological (SPT) state [29]. And it has been known that the boundary of many 3d bosonic SPT states could be a 2d $Z_2$ topological order with $e$ and $m$ particles carrying anomalous quantum numbers [44, 45] (the symmetry of $O(2)_e \times O(2)_m \times Z_2^d$ of Eq. 13 is anomalous if viewed as an on-site symmetry, and it is a subgroup of the SO(5) symmetry which also supports a 3d bosonic SPT state [2]). The $Z_2$ spin liquid for spin-1/2 systems on the square lattice that preserve the square lattice symmetry has also been discussed recently [46, 47], thus it is conceivable that this $Z_2$ spin liquid is not so far away from the easy-plane dQCP, and in that system $e$ is the standard bosonic spinon, while $m$ will carry lattice momentum hence its condensate will lead to the VBS order [48, 51].

We can also build exactly the same $Z_2$ topological order using the dual theory of Eq. 13 by condensing the Cooper pair of the dual Dirac fermions. Starting with Eq. 11, a more standard way to enter this $Z_2$ topological order, is by first breaking the $Z_2^d$ symmetry and spon-
taneously breaking the $U(1)_s$ or $U(1)_v$ symmetry, and then condense the double vortex of the $U(1)$ order parameter to restore the symmetries. This is equivalent to condensing the singlet pair of $z_1$ in Eq. (11), which was discussed in detail in Ref. [10]. The phase transition between the $Z_2$ topological order and the standard spontaneous $U(1)$ symmetry breaking phase (superfluid) is the so-called 3d XY$^*$ transition [52, 50]. In this procedure, the final topological order has the $Z_2^d$ symmetry (or the self-duality of Eq. (1a)), but eventually we need to adjust the energy gap for $e$ and $m$ to restore the $Z_2^d$ self-dual symmetry of Eq. (1b). Eq. (10) shows how to connect the easy-plane dQCP to the $Z_2$ spin liquid, while preserving the self-dual $Z_2^d$ symmetry. Our results are summarized in the sketched phase diagram Fig. 1.

While finishing the current paper, the authors became aware of two independent upcoming works that partially relate to our current paper (Ref. [57, 58]). Cenke Xu and Alex Rasmussen are supported by the David and Lucile Packard Foundation and NSF Grant No. DMR-1151208. Chao-Ming Jian is funded by the Gordon and Betty Moore Foundation’s EPiQS Initiative through Grant GBMF4304.

[1] A. C. Potter, C. Wang, M. A. Metlitski, and A. Vishwanath, ArXiv e-prints (2016), 1609.08618.
[2] C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, ArXiv e-prints (2017), 1703.02426.
[3] A. Karch and D. Tong, Phys. Rev. X 6, 031043 (2016), URL https://link.aps.org/doi/10.1103/PhysRevX.6.031043.
[4] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science 303, 1490 (2004), ISSN 0036-8075, URL http://science.sciencemag.org/content/303/5663/1490.
[5] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Phys. Rev. B 70, 144407 (2004), URL http://link.aps.org/doi/10.1103/PhysRevB.70.144407.
[6] S. D. Geraedts and O. I. Motrunich, Phys. Rev. B 85, 045114 (2012), URL https://link.aps.org/doi/10.1103/PhysRevB.85.045114.
[7] J. D’Emidio and R. K. Kaul, Phys. Rev. B 93, 054406 (2016), URL https://link.aps.org/doi/10.1103/PhysRevB.93.054406.
[8] J. D’Emidio and R. K. Kaul, Phys. Rev. Lett. 118, 187202 (2017), URL https://link.aps.org/doi/10.1103/PhysRevLett.118.187202.
[9] Y.-Q. Qin, Y.-Y. He, Y.-Z. You, Z.-Y. Lu, A. Sen, A. W. Sandvik, C. Xu, and Z.-Y. Meng, arXiv:1705.10670 (2017).
[10] A. W. Sandvik, Phys. Rev. Lett. 98, 227202 (2007), URL https://link.aps.org/doi/10.1103/PhysRevLett.98.227202.
[11] H. Shao, W. Guo, and A. W. Sandvik, Science 352, 213 (2016), ISSN 0036-8075, http://science.sciencemag.org/content/352/6282/213.full.pdf.
[12] J. Lou, A. W. Sandvik, and N. Kawashima, Phys. Rev. B 80, 180414 (2009), URL https://link.aps.org/doi/10.1103/PhysRevB.80.180414.
[13] A. W. Sandvik, Phys. Rev. Lett. 104, 177201 (2010), URL https://link.aps.org/doi/10.1103/PhysRevLett.104.177201.
[14] A. Nahum, J. T. Chalker, P. Serna, M. Ortúñ, and A. M. Somoza, Phys. Rev. Lett. 107, 110601 (2011), URL https://link.aps.org/doi/10.1103/PhysRevLett.107.110601.
[15] K. Harada, T. Suzuki, T. Okubo, H. Matsuo, J. Lou, H. Watanabe, S.Todo, and N. Kawashima, Phys. Rev. B 88, 220408 (2013), URL https://link.aps.org/doi/10.1103/PhysRevB.88.220408.
[16] S. Pujari, K. Danile, and F. Alet, Phys. Rev. Lett. 111, 087203 (2013), URL https://link.aps.org/doi/10.1103/PhysRevLett.111.087203.
[17] S. Pujari, F. Alet, and K. Danile, Phys. Rev. B 91, 104411 (2015), URL https://link.aps.org/doi/10.1103/PhysRevB.91.104411.
[18] A. Nahum, J. T. Chalker, P. Serna, M. Ortúñ, and A. M. Somoza, Phys. Rev. X 5, 041048 (2015), URL https://link.aps.org/doi/10.1103/PhysRevX.5.041048.
[19] A. Nahum, P. Serna, J. T. Chalker, M. Ortúñ, and A. M. Somoza, Phys. Rev. Lett. 115, 267203 (2015), URL https://link.aps.org/doi/10.1103/PhysRevLett.115.267203.
[20] R. K. Kaul and A. W. Sandvik, Phys. Rev. Lett. 108, 137201 (2012), URL https://link.aps.org/doi/10.1103/PhysRevLett.108.137201.
[21] T. Grover and A. Vishwanath, Phys. Rev. B 87, 045129 (2013).
[22] Y.-M. Lu and D.-H. Lee, Phys. Rev. B 89, 195143 (2014), URL http://link.aps.org/doi/10.1103/PhysRevB.89.195143.
[23] K. Slagle, Y.-Z. You, and C. Xu, Phys. Rev. B 91, 115121 (2015).
[24] Y.-Y. He, H.-Q. Wu, Y.-Z. You, C. Xu, Z. Y. Meng, and Z.-Y. Lu, Phys. Rev. B 93, 115150 (2016), URL http://link.aps.org/doi/10.1103/PhysRevB.93.115150.
[25] O. I. Motrunich and A. Vishwanath, Phys. Rev. B 70, 075104 (2004), URL http://link.aps.org/doi/10.1103/PhysRevB.70.075104.
[26] M. E. Peskin, Annals of Physics 113, 122 (1978), ISSN 0003-4916, URL http://www.sciencedirect.com/science/article/pii/000349167890252X.
[27] C. Dasgupta and B. I. Halperin, Phys. Rev. Lett. 47, 1556 (1981), URL http://link.aps.org/doi/10.1103/PhysRevLett.47.1556.
[28] M. P. A. Fisher and D. H. Lee, Phys. Rev. B 39, 2756 (1989), URL http://link.aps.org/doi/10.1103/PhysRevB.39.2756.
[29] C. Xu and Y.-Z. You, Phys. Rev. B 92, 220416 (2015), URL http://link.aps.org/doi/10.1103/PhysRevB.92.220416.
[30] D. F. Mross, J. Alicea, and O. I. Motrunich, Phys. Rev. Lett. 117, 016802 (2016).
[31] P.-S. Hsin and N. Seiberg, Journal of High Energy Physics 2016, 95 (2016), ISSN 1029-8479, URL http://dx.doi.org/10.1007/JHEP09(2016)095.
[32] N. Seiberg, T. Senthil, C. Wang, and E. Witten, Annals of Physics 374, 395 (2016), ISSN 0003-
4916. URL http://www.sciencedirect.com/science/article/pii/S0003491616301531

[33] D. T. Son, Phys. Rev. X 5, 031027 (2015).
[34] C. Wang and T. Senthil, Phys. Rev. X 5, 041031 (2015), URL http://link.aps.org/doi/10.1103/PhysRevX.5.041031.
[35] C. Wang and T. Senthil, Phys. Rev. B 93, 085110 (2016), URL http://link.aps.org/doi/10.1103/PhysRevB.93.085110.

[36] M. A. Metlitski and A. Vishwanath, Phys. Rev. B 93, 245151 (2016), URL http://link.aps.org/doi/10.1103/PhysRevB.93.245151.
[37] N. Karthik and R. Narayanan, Phys. Rev. D 94, 065026 (2016), URL http://link.aps.org/doi/10.1103/PhysRevD.94.065026.

[38] K. Kaveh and I. F. Herbut, Phys. Rev. B 71, 184519 (2005), URL https://link.aps.org/doi/10.1103/PhysRevB.71.184519.

[39] C. Xu and S. Sachdev, Phys. Rev. Lett. 100, 137201 (2008).

[40] S. M. Chester and S. S. Pufu, Journal of High Energy Physics 2016, 69 (2016), ISSN 1029-8479, URL https://doi.org/10.1007/JHEP08(2016)069.

[41] J. D’Emidio and R. K. Kaul, Phys. Rev. Lett. 118, 187202 (2017), URL https://link.aps.org/doi/10.1103/PhysRevLett.118.187202.

[42] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. 98, 106803 (2008).

[43] F. A. Bais and J. K. Slingerland, Phys. Rev. B 79, 045316 (2009), URL https://link.aps.org/doi/10.1103/PhysRevB.79.045316.

[44] A. Vishwanath and T. Senthil, Phys. Rev. X 3, 011016 (2013).

[45] Z. Bi, A. Rasmussen, and C. Xu, Phys. Rev. B 91, 134404 (2015).

[46] S. Chatterjee, S. Sachdev, and M. Scheurer, arXiv:1705.06289 (2017).

[47] X. Yang and F. Wang, Phys. Rev. B 94, 035160 (2016), URL https://link.aps.org/doi/10.1103/PhysRevB.94.035160.

[48] D. Blankschtein, M. Ma, and A. N. Berker, Phys. Rev. B 30, 1362 (1984), URL https://link.aps.org/doi/10.1103/PhysRevB.30.1362.

[49] R. A. Jalalbert and S. Sachdev, Phys. Rev. B 44, 686 (1991), URL https://link.aps.org/doi/10.1103/PhysRevB.44.686.

[50] R. Moessner and S. L. Sondhi, Phys. Rev. B 63, 224401 (2001), URL https://link.aps.org/doi/10.1103/PhysRevB.63.224401.

[51] C. Xu and S. Sachdev, Phys. Rev. B 79, 064405 (2009), URL https://link.aps.org/doi/10.1103/PhysRevB.79.064405.

[52] L. Balents, M. P. A. Fisher, and S. M. Girvin, Phys. Rev. B 65, 224412 (2002), URL https://link.aps.org/doi/10.1103/PhysRevB.65.224412.

[53] T. Senthil and O. Motrunich, Phys. Rev. B 66, 205104 (2002), URL https://link.aps.org/doi/10.1103/PhysRevB.66.205104.

[54] S. V. Isakov, Y. B. Kim, and A. Paramekanti, Phys. Rev. Lett. 97, 207204 (2006), URL https://link.aps.org/doi/10.1103/PhysRevLett.97.207204.

[55] S. V. Isakov, M. B. Hastings, and R. G. Melko, Nature Physics 7, 772 (2011).

[56] S. V. Isakov, M. B. Hastings, and R. G. Melko, Science 335, 193 (2012).

[57] M. Dan, Y. Qi, and T. Senthil, upcoming (2017).

[58] A. Thomson and S. Sachdev, upcoming (2017).

[59] We will take the Euclidean space-time, and choose the following convention for the \( \gamma \mu \) matrices throughout: \((\gamma_0, \gamma_1, \gamma_2) = (\sigma^2, \sigma^3, \sigma^1)\).