Electron Quasielastic Scattering at High Energy from $^{56}\text{Fe}$, What Suppression?

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Abstract

Quasielastic electron scattering ($e,e'$) from $^{56}\text{Fe}$ is calculated at large electron energies (2-4 GeV) and large three momentum transfer (0.5-1.5 GeV/c). We use a relativistic mean-field single particle model for the bound and continuum nucleon wavefunctions based on the $\sigma-\omega$ model and we include the effects of electron Coulomb distortion in the calculation. The calculations are compared to high energy data from SLAC and more recent data from Jefferson Laboratory, particularly for kinematics where the energy transfer is less than 500 to 600 MeV and the quasielastic process is expected to dominate the cross section. The effects of the predicted weakening of the strong scalar and vector potentials of the $\sigma-\omega$ model at high energy are investigated. Possible evidence for ‘longitudinal suppression’ or modifications of nucleon form factors in the medium is considered, but neither is necessary to explain the quasielastic data for four momentum transfers less than 1 (GeV/c)$^2$.

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Medium and high energy electron scattering is one of the most useful tools to study nucleon properties inside nuclei, especially in the quasielastic region using the inclusive reaction \((e, e')\). There have been many experiments \([1-8]\) on medium and heavy nuclei at incident electron energies less than 1 GeV, and a number of theoretical attempts \([9-15]\) to fit the measured cross section and to separate the longitudinal and transverse structure functions. The Fermi gas model in the impulse approximation provides a rough description of the inclusive \((e, e')\) cross sections but fails in describing the details. This is not too surprising since the Fermi gas model does not include the nuclear spatial geometry correctly. In some cases, there appeared to be large suppression (about 30-40%) of the longitudinal structure functions (as compared to theoretical calculations of \((e, e')\) using non-relativistic wavefunctions and current operators) which implied missing strength in the Coulomb sum rule \([1]\). There was also disagreement between the transverse structure functions extracted from the experimental data and the predictions of the Fermi gas model, but this was expected since exchange currents, pion production, and other processes induced by transverse photons were not included in the model. Many attempts were made to explain this purported missing strength of the longitudinal structure function by improving the nuclear bound states, modifying the nucleon form factors in the nuclear medium, including final state interactions, and relativistic dynamics effects.

Two ingredients enter the comparison of experimental \((e, e')\) data from medium and heavy nuclei to theory. One of these is the inclusion of electron Coulomb distortion effects and the second is the model used to calculate the nuclear transition current. In the early 90’s, Coulomb distortion for the reactions \((e, e')\) and \((e, e'p)\) in the quasielastic region was treated exactly by the Ohio University group \([7,9,16-18]\) using partial wave expansions of the electron wavefunctions. While such Distorted Wave Born Approximation (DWBA) calculations permit the comparison of different nuclear models against measured cross sections and provide an invaluable check on various approximate techniques of including Coulomb distortion effects, they are numerically challenging and computation time increases rapidly with higher incident electron energies. Jourdan \([19]\) used the Ohio group’s calculation of
Coulomb distortion effects to investigate the Coulomb sum rule. His conclusion was that the sum rule appeared to be obeyed and thus there was no ‘longitudinal suppression’.

In order to avoid the difficulties associated with DWBA analyzes at higher electron energies and to look for a way to still define structure functions, Kim and Wright developed an approximate treatment of the Coulomb distortion. The essence of the approximation is to include Coulomb distortion in the four potential arising from the electron current by letting the magnitude of the electron momentum include the effect of the static Coulomb potential. This leads to an $r$-dependent momentum transfer. This approximation allows the separation of the cross section into a ‘longitudinal’ term and a ‘transverse’ term which is not formally possible in a full DWBA calculation. For medium and heavy nuclei at moderate incident electron energies, a good treatment of Coulomb distortion effects is necessary in order to extract the ‘longitudinal’ and ‘transverse’ structure functions.

It should be noted that not all investigators found a longitudinal suppression. For example, the Ohio group analyzed the Bates data for $^{40}\text{Ca}(e,e^\prime)$ using the relativistic $\sigma-\omega$ model mean field potential for the bound and continuum nucleons along with the relativistic current operator coupled with a good description of Coulomb distortion. Their results were in good agreement with the data with no evidence of longitudinal suppression. This same model, coupled with our approximate treatment of Coulomb distortion, was compared to the Saclay quasielastic data on $^{208}\text{Pb}$ taken with both electrons and positrons. The DWBA and approximate calculations of Coulomb distortion by positrons and electrons were not consistent with the data quoted by Saclay, so it was not possible to extract a ‘longitudinal’ structure function. In addition, we investigated the approximation used by the Saclay group for Coulomb corrections and found it not be a good approximation.

The absence or presence of ‘longitudinal suppression’ has been argued vigorously at various conferences—partially because of different theoretical treatments, but also because of some experimental discrepancies among various laboratories. There is now new $(e,e^\prime)$ data at much higher energies and momentum transfer on a number of nuclei including $^{56}\text{Fe}$ from JLAB which is similar kinematically to some older data from SLAC. To our
knowledge, no one has attempted to calculate the quasielastic contributions within a nuclear model to these cross sections, probably because of the numerical difficulties in calculating the \((e, e')\) process with a good nuclear model at such high energies. We have extended the capabilities of our codes to handle these kinematics and in this paper will compare our simple relativistic mean field model to the available high energy data on \(^{56}\text{Fe}\) from both SLAC and JLAB. In particular, we will look for kinematic regions at relatively low energy transfer where quasielastic scattering is expected to dominate the cross section. Our results may be useful in examining various scaling studies \([20,21]\) of \((e, e')\) at large \(Q^2\) and in separating the quasielastic process from inelastic contributions.

In the plane wave Born approximation (PWBA), where electrons or positrons are described as Dirac plane waves, the cross section for the inclusive quasielastic \((e, e')\) processes can be written as

\[
d\sigma/d\Omega d\omega = \sigma_M \left\{ \frac{q^4}{q^4} S_L(q, \omega) + \frac{q^2}{2q^2} S_T(q, \omega) \right\},
\]

where \(q^2 = \omega^2 - q^2\) is the four-momentum transfer, \(\sigma_M\) is the Mott cross section and \(S_L\) and \(S_T\) are the longitudinal and transverse structure functions which depend only on the three-momentum transfer \(q\) and the energy transfer \(\omega\). Explicitly, the structure functions for a given bound state with angular momentum \(j_b\) are given by

\[
S_L(q, \omega) = \sum_{\mu_b \sigma_p} \rho_p \frac{1}{2(2j_b + 1)} \int |N_0|^2 d\Omega_p
\]

\[
S_T(q, \omega) = \sum_{\mu_b \sigma_p} \rho_p \frac{1}{2(2j_b + 1)} \int (|N_x|^2 + |N_y|^2) d\Omega_p
\]

with the outgoing nucleon density of states \(\rho_p = \frac{p_{E_p}}{(2\pi)^3}\). The \(\hat{z}\)-axis is taken to be along the momentum transfer \(\mathbf{q}\) and \(\mu_b\) and \(s_p\) are the \(z\)-components of the angular momentum of the bound and continuum state nucleons. The Fourier transform of the nuclear current \(J^\mu(\mathbf{r})\) is simply

\[
N^\mu = \int J^\mu(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3r,
\]

where \(J^\mu(\mathbf{r})\) denotes the nucleon transition current. The continuity equation has been used to eliminate the \(z\)-component \((N_z)\) via the equation \(N_z = -\frac{\omega}{q} N_0\) which is valid if current
is conserved. Note that if we do not have nuclear current conservation (i.e., a different Hamiltonian in the initial and final state), we need to calculate \( N_z \) directly.

The *ad hoc* expressions for the longitudinal and transverse structure functions with inclusion of the electron Coulomb distortion (see Ref. [15] for details) are similar to above, but the Fourier operators are modified by Coulomb distortion. We include the Coulomb distortion effects in our results, but unlike the medium energy cases, Coulomb effects on the cross section at these high electron energies are quite small.

The nucleon transition current in the relativistic single particle model is given by

\[
J^\mu(r) = e\bar{\psi}_p(r)\hat{J}^\mu\psi_b(r),
\]

where \( \hat{J}^\mu \) is a free nucleon current operator, and \( \psi_p \) and \( \psi_b \) are the wave functions of the knocked out nucleon and the bound state, respectively. For a free nucleon, the operator consists of the Dirac contribution and the contribution of an anomalous magnetic moment \( \mu_T \) given by

\[
\hat{J}^\mu = F_1(q^2_\mu)\gamma^\mu + F_2(q^2_\mu)\frac{\mu_T}{2M}\sigma^{\mu\nu}q_\nu.
\]

The form factors \( F_1 \) and \( F_2 \) are related to the electric and magnetic Sachs form factors \( G_E \) and \( G_M \) by

\[
G_E = F_1 + \frac{\mu_T q^2}{4M^2} F_2 \quad \text{and} \quad G_M = F_1 + \mu_T F_2
\]

where \( \mu_T \) is assumed to take the following standard form [24]:

\[
G_E = \frac{1}{(1 - \frac{q^2}{\Lambda^2})^2} = \frac{G_M}{(\mu_T + 1)},
\]

where the standard value for \( \Lambda^2 \) is 0.71 (GeV/c)^2. Several investigators [25,26] have suggested that nuclear medium effects may affect the value of \( \Lambda^2 \) and the value of \( \mu_T \).

Four \((e,e')\) data sets on \(^{56}\text{Fe}\) taken at SLAC and JLAB correspond to significant cross sections for energy transfers less than 500-600 MeV. Three of these data sets were taken at SLAC [21] with the following initial electron energy and scattering angle: \( E_i = 2.02 \text{ GeV}, \theta = 15^\circ \), \( E_i = 2.02 \text{ GeV}, \theta = 20^\circ \), and \( E_i = 3.595 \text{ GeV}, \theta = 16^\circ \) while the fourth was taken at JLAB [20] with: \( E_i = 3.595 \text{ GeV}, \theta = 16^\circ \). In Figs. 1-4 we show the experimental data as compared to three theoretical results, with the longitudinal and transverse contributions to the cross section being shown for the results labeled “Lorentz”.

Our standard calculation at lower energies is to use a current conserving model where the bound and continuum nucleons move in the scalar \( S(r) \) and vector \( V(r) \) potentials generated

\[ \text{equation}\]

\[ \text{equation}\]
by the TIMORA code \[27\]. This result is labeled “Const. S & V” in the four figures. However, the Ohio State group \[28\] found in their global fits to proton-nucleus scattering from a range of nuclei that the strengths of the scalar and vector potentials decreased as the proton energy increased. We chose to investigate this effect, also considered in an approximate way in Ref. \[29\], by using a parametrization of S and V strength as a function of proton kinetic energy consistent with the results that Cooper et al. \[28\] found. In particular, we calculated \((e,e')\) with \(S(r)\) and \(V(r)\) for protons and neutrons scaled by the functions, 

\[
\begin{align*}
    f_S &= 0.97 - 0.66x + 0.28x^2 \\
    f_V &= 0.97 - 0.91x + 0.30x^2
\end{align*}
\]

respectively where \(x\) is the outgoing nucleon kinetic energy divided by the nucleon mass \((x = T_p/M)\). For 500 MeV nucleons these factors are 0.70 and 0.59. The results using these weakened potentials for the outgoing nucleons are labeled, “\(T_p\)-dep. S & V”. As noted earlier, changing the potentials for the bound and continuum nucleons results in current non-conservation. To get an estimate of the size of this effect, rather than using the relation \(N_z = -\omega/qN_o\) from current conservation we evaluated \(N_z\) directly. This result is labeled “Lorentz” since it is calculated in the Lorentz gauge.

The peak of the quasielastic peak in Fig. 1 occurs at an energy transfer of approximately 150 MeV and hence contains very little pion production or other inelastic processes. Our relativistic-mean field calculation with the energy dependent scalar and vector potentials fit the data very well. Furthermore, since the longitudinal contribution is a significant fraction of the total, there is no evidence for any kind of longitudinal suppression. One very interesting consequence of using the energy dependent scalar and vector potentials is the much more rapid fall off of the quasielastic peak at higher energy transfer as compared to the energy independent potentials. Since the fit at the peak is much better with the energy dependent potentials, this result suggests that most of the cross section above the quasielastic peak is due to inelastic processes.

In Figs. 2-4, the kinematics lead to the peak of the quasielastic peak being well above pion production threshold and hence, it is not surprising that the theoretical model falls 10-20\% below the data on the low energy side of the quasielastic peak. The most troubling case
for our model is the 2.02 GeV data from SLAC at 20° shown in Fig. 2 where our curve falls below the data on the low energy side of the quasielastic peak for energy transfers between 150 and 200 MeV. However, due to the electron kinematics, most of the cross section is expected to be transverse. Again, the energy dependent potentials predict a rapid fall off of the quasielastic peak on the high energy side. Scaling models of these data sets should take this result into account.

Various authors [25,26] have suggested that the nucleon form factors, $G_E$ and $G_M$ are modified in the nuclear medium. For example, in Ref. [25], the $\Lambda^2$ factor in Eq. 6 is predicted to be 0.41 (Gev/c)$^2$ for $G_E$ and 0.58 (GeV/c)$^2$ for $G_M$ for the proton while the proton anomalous magnetic moment is predicted to increase from 1.71 to 1.91 while the neutron anomalous magnetic moment changes from -1.92 to -2.21. Since the longitudinal and transverse cross sections are roughly proportional to the squares of these form factors we can estimate the effect at $Q^2=0.25$ (GeV/c)$^2$ ($G_E$ drops by 11%, while $G_M$ increases by 3%) and at 1.0 (GeV/c)$^2$ ($G_E$ drops by 51% and $G_M$ decreases by 16%). Such decreases are not needed in our fit in Fig. 1 and they would imply much greater inelastic contributions to the results shown in Figs. 2-4.

A recent experiment [30] from JLAB has shown that the ratio of $G_E/G_M$ for the proton begins to decrease for $Q^2$ values over about 0.50 (GeV/c)$^2$. Assuming that $G_E$ is the form factor that is decreasing, this would show up in the longitudinal contribution to the cross section. To see this in the data sets shown in Figs. 2-4 would require a good theoretical modeling of pion production which, at this low energy transfer, is the primary inelastic process.

In conclusion, we find no evidence of any kind of ‘longitudinal suppression’ in quasielastic scattering from $^{56}$Fe at high momentum transfer. This agrees with our previous analyses of quasielastic scattering from $^{40}$Ca at much lower energies and momentum transfer. That is, the relativistic mean field of the $\sigma - \omega$ model with energy dependent scalar and vector potentials provides an excellent description of quasielastic scattering from medium and heavy nuclei. While it is certainly possible to use inadequate nuclear models coupled
with approximate current operators to deduce the need for additional effects, Occam’s razor would suggest that the relativistic mean field description is superior. In addition, our results suggest that there is no evidence of modification of the nucleon form factors in the nuclear medium.

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FIGURES

FIG. 1. Theoretical quasielastic scattering cross sections for $^{56}$Fe with incident electron energy of $E_i = 2.02$ GeV and electron scattering angle $\theta = 15^\circ$ as a function on energy transfer compared to experimental data from SLAC. The cross sections labeled $\sigma_T$ and $\sigma_L$ add up to the model cross section labeled ‘Lorentz’. See text for details of the models.

FIG. 2. Same as in Fig. 1, except $\theta = 20^\circ$.

FIG. 3. Same as Fig. 1, except the electron energy $E_i = 3.593$ GeV and the scattering angle $\theta = 16^\circ$.

FIG. 4. Same as Fig. 1, except that the electron energy $E_i = 4.045$ GeV, the scattering angle $\theta = 15^\circ$, and the experimental data is from JLAB.
$1.1 \text{ (GeV/c)}^2 > Q^2 \text{ (GeV/c)}^2 > 0.84 \text{ (GeV/c)}^2$

- **Const. S & V**
- **$T_p$-dep. S & V**
- **Lorentz**
- **JLAB Data**

$^{56}\text{Fe}(e,e')$

$E_i = 4.045 \text{ GeV}$

$\theta = 15^\circ$

$d^2\sigma/d\omega d\Omega \ (\text{nb/sr MeV}^{-1})$

$\omega \ (\text{MeV})$

$0$ $200$ $400$ $600$ $800$ $1000$
$^{56}$Fe(e,e')

$E_i = 2.02$ GeV

$\theta = 15^\circ$

$0.27 \ (GeV/c)^2 > Q^2 \ (GeV/c)^2 > 0.23 \ (GeV/c)^2$

$\frac{d^2\sigma}{d\omega d\Omega}$ (nb/sr MeV$^{-1}$)

$\sigma_T$

$\sigma_L$

Const. S & V

$T_\rho$-dep. S & V

Lorentz

SLAC Data
$0.48 \ (GeV/c)^2 > Q^2 \ (GeV/c)^2 > 0.38 \ (GeV/c)^2$

- Const. S & V
- $^{68}_{Fe}(e,e')$
- $T_p$-dep. S & V
- $E_i=2.02 \ GeV$
- $\theta=20^\circ$
- Lorentz
- SLAC Data

$d^2\sigma/d\omega d\Omega \ (nb/sr \ MeV^{-1})$

$\omega \ (MeV)$