Gravitino and Goldstino at Colliders*

Wilfried Buchmüller\textsuperscript{(a)}, Koichi Hamaguchi\textsuperscript{(a)},
Michael Ratz\textsuperscript{(a)} and Tsutomu Yanagida\textsuperscript{(b)(c)}

\textsuperscript{(a)}Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany
\textsuperscript{(b)}Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
\textsuperscript{(c)}Research Center for the Early Universe, University of Tokyo, Japan

Abstract

We consider theories with spontaneously broken global or local supersymmetry
where the pseudo-goldstino or the gravitino is the lightest superparticle (LSP).
Assuming that the long-lived next-to-lightest superparticle (NSP) is a charged
slepton, we study several supergravity predictions: the NSP lifetime, angular and
energy distributions in 3-body NSP decays. The characteristic couplings of the
gravitino, or goldstino, can be tested even for very small masses.

* Contribution to the LHC / LC study group report, eds. G. Weiglein et al.
Introduction

The discovery of supersymmetry at the Tevatron, the LHC or a future Linear Collider would raise the question how supersymmetry is realized in nature. Clearly, supersymmetry is broken. Spontaneously broken global supersymmetry would predict the existence of a spin-1/2 goldstino ($\chi$) whereas the theoretically favoured case of local supersymmetry requires a massive spin-3/2 gravitino ($\psi_{3/2}$).

In a recent paper [1] we have studied how a massive gravitino, if it is the lightest superparticle (LSP), may be discovered in decays of $\tilde{\tau}$, the scalar $\tau$ lepton, which is naturally the next-to-lightest superparticle (NSP). The determination of gravitino mass and spin appears feasible for gravitino masses in the range from about 1 GeV to 100 GeV. As we shall discuss in this note, evidence for the characteristic couplings of a pseudo-goldstino, which corresponds to the spin-1/2 part of the gravitino, can be obtained even for masses much smaller than 1 GeV.

The gravitino mass depends on the mechanism of supersymmetry breaking. It can be of the same order as other superparticle masses, like in gaugino mediation [2] or gravity mediation [3]. But it might also be much smaller as in gauge mediation scenarios [4]. As LSP, the gravitino is an attractive dark matter candidate [5].

The $\tilde{\tau}$ NSP has generally a long lifetime because of the small, Planck scale suppressed coupling to the gravitino LSP. The production of charged long-lived heavy particles at colliders is an exciting possibility [6]. They can be directly produced in pairs or in cascade decays of heavier superparticles. In the context of models with gauge mediated supersymmetry breaking the production of slepton NSPs has previously studied for the Tevatron [7], for the LHC [8] and for a Linear Collider [9].

The dominant $\tilde{\tau}$ NSP decay channel is $\tilde{\tau} \rightarrow \tau + \text{missing energy}$. In the following we shall study how to identify the gravitino or goldstino as carrier of the missing energy. First, one will measure the NSP lifetime. Since the gravitino couplings are fixed by symmetry, the NSP lifetime is predicted by supergravity given the gravitino mass, which can be inferred from kinematics. Second, one can make use of the 3-body NSP decay $\tilde{\tau} \rightarrow \tau + \gamma + X$ where $X = \psi_{3/2}$ or $X = \chi$. The angular and energy distributions and the polarizations of the final state photon and lepton carry the information on the spin and couplings of gravitino or goldstino.

For gravitino masses in the range from about 10 keV to 100 GeV, the NSP is essentially stable for collider experiments, and one has to accumulate the NSPs to study their decay. Sufficiently slow, strongly ionizing sleptons will be stopped within the detector. One may also be able to collect faster sleptons in a storage ring. For gravitino masses less than $O(10 \text{ keV})$ the $\tilde{\tau}$ can decay inside the detector, which may be advantageous
from the experimental point of view.

At LHC one expects $\mathcal{O}(10^6)$ NSPs per year which are mainly produced in cascade decays of squarks and gluinos \[10\]. The NSPs are mostly produced in the forward direction \[11\] which should make it easier to accumulate $\tilde{\tau}$s in a storage ring. In a Linear Collider an integrated luminosity of 500 fb$^{-1}$ will yield $\mathcal{O}(10^5) \tilde{\tau}$s \[12\]. Note that, in a Linear Collider, one can also tune the velocity of the produced $\tilde{\tau}$s by adjusting the $e^+e^-$ center-of-mass energy. A detailed study of the possibilities to accumulate $\tilde{\tau}$ NSPs is beyond the scope of this note. In the following we shall assume that a sufficiently large number of $\tilde{\tau}$s can be produced and collected.

This study is strongly based on Ref. \[1\]. Here, we discuss in more detail the case of a very light gravitino, or pseudo-goldstino, for which the $\tilde{\tau}$ NSP can decay inside the detector. Although in this case it is difficult to determine mass and spin of the gravitino, one can still see the characteristic coupling of the gravitino, which is essentially the goldstino coupling, via the 3-body decay $\tilde{\tau} \rightarrow \tau + \gamma + X$ with $X = \psi_{3/2}$ or $X = \chi$.

**Planck mass from $\tilde{\tau}$ decays**

The $\tilde{\tau}$ decay rate is dominated by the two-body decay into $\tau$ and gravitino,

$$\Gamma_{\tilde{\tau}}^{2\text{-body}} = \left(\frac{m_\tau^2 - m_{3/2}^2 - m_\tau^2}{48\pi m_{3/2}^2 M_P^2 m_\tau^2}\right)^4 \left[1 - \frac{4m_{3/2}^2 m_\tau^2}{(m_\tau^2 - m_{3/2}^2 - m_\tau^2)^2}\right]^{3/2}, \quad (1)$$

where $M_P = (8\pi G_N)^{-1/2}$ denotes the reduced Planck mass, $m_\tau = 1.78$ GeV is the $\tau$ mass, $m_\tau$ is the $\tilde{\tau}$ mass, and $m_{3/2}$ is the gravitino mass. For instance, $m_\tau = 150$ GeV and $m_{3/2} = 10$ keV leads to a lifetime of $\Gamma_{\tilde{\tau}}^{-1} \simeq 7.8 \times 10^{-7}$ s, and $m_\tau = 150$ GeV and $m_{3/2} = 75$ GeV results in $\Gamma_{\tilde{\tau}}^{-1} \simeq 4.4$ y.

Since the decay rate depends only on two unknown masses $m_\tau$ and $m_{3/2}$, independently of other SUSY parameters, gauge and Yukawa couplings, it is possible to test the prediction of the supergravity if one can measure these masses. The mass $m_\tau$ of the NSP will be measured in the process of accumulation. Although the outgoing gravitino is not directly measurable, its mass can also be inferred kinematically unless it is too small,

$$m_{3/2}^2 = m_\tau^2 + m_\tau^2 - 2m_\tau E_\tau. \quad (2)$$

The gravitino mass can be determined with the same accuracy as $E_\tau$ and $m_\tau$, i.e. with an uncertainty of a few GeV.

Once the masses $m_\tau$ and $m_{3/2}$ are measured, one can compare the predicted decay rate \[1\] with the observed decay rate, thereby testing an important supergravity predic-
tion. In other words, one can determine the ‘supergravity Planck scale’ from the NSP decay rate which yields, up to $O(\alpha)$ corrections,

$$M^2_P(\text{supergravity}) = \left( \frac{m^2_\tilde{\tau} - m^2_{3/2} - m^2_\tau}{48\pi m^2_{3/2} m^2_\tilde{\tau} \Gamma_\tilde{\tau}} \right)^4 \left[ 1 - \frac{4m^2_{3/2} m^2_\tau}{m^2_\tilde{\tau} - m^2_{3/2} - m^2_\tau} \right]^{3/2}. \quad (3)$$

The result can be compared with the Planck scale of Einstein gravity, i.e. Newton’s constant determined by macroscopic measurements, $G_N = 6.707(10) \cdot 10^{-39} \text{ GeV}^{-2}$ \cite{13},

$$M^2_P(\text{gravity}) = (8\pi G_N)^{-1} = (2.436(2) \cdot 10^{18} \text{ GeV})^2. \quad (4)$$

The consistency of the microscopic and macroscopic determinations of the Planck scale is an unequivocal test of supergravity.

Note that the measurement of the gravitino mass yields another important quantity in supergravity, the mass scale of spontaneous supersymmetry breaking $M_{\text{SUSY}} = \sqrt{3} M_P m_{3/2}$. This is the analogue of the Higgs vacuum expectation value $v$ in the electroweak theory, where $v = \sqrt{2} m_W / g = (2\sqrt{2} G_F)^{-1/2}$.

**Gravitino and goldstino versus neutralino**

If the measured decay rate and the kinematically determined mass of the invisible particle are consistent with Eq. (1), one already has strong evidence for supergravity and the gravitino LSP. To analyze the couplings of the invisible particle, one can study the 3-body decay $\tilde{\tau} \rightarrow \tau + \gamma + X$ for the gravitino $X = \psi_{3/2}$ and compare it with the case where $X$ is a hypothetical spin-1/2 neutralino. This is of particular importance if the mass of the invisible particle is so small that the supergravity prediction for the NSP lifetime, as described in the previous section, cannot be tested.

The NSP $\tilde{\tau}$ is in general a linear combination of $\tilde{\tau}_R$ and $\tilde{\tau}_L$, the superpartners of the right-handed and left-handed $\tau$ leptons $\tau_R$ and $\tau_L$, respectively. The interaction of the gravitino $\psi_{3/2}$ with scalar and fermionic $\tau$ leptons is described by the lagrangian \cite{14},

$$\mathcal{L}_{3/2} = -\frac{1}{\sqrt{2} M_P} \left[ (D_\nu \tilde{\tau}_R)^* \psi^{\mu*} \gamma^\nu \gamma_\mu P_R \tau + (D_\nu \tilde{\tau}_R) \tau P_L \gamma_\mu \gamma^\nu \psi^{\mu*} \right], \quad (5)$$

where $D_\nu \tilde{\tau}_R = (\partial_\nu + i e A_\nu) \tilde{\tau}_R$ and $A_\nu$ denotes the gauge boson. The interaction lagrangian of $\tilde{\tau}_L$ has an analogous form.

As an example for the coupling of a hypothetical spin-1/2 neutralino to $\tilde{\tau}$ and $\tau$, we
consider the Yukawa interaction\(^1\),
\[
\mathcal{L}_{\text{Yukawa}} = h \left( \bar{\tau}^* R P R \tau + \bar{\tau}^* L P L \tau \right) + \text{h.c.} \tag{6}
\]

Note that for very small coupling \(h\), the \(\bar{\tau}\) decay rate could accidentally be consistent with the supergravity prediction Eq. (1).

Also the goldstino \(\chi\) has Yukawa couplings of the type given in Eq. (6). The full interaction lagrangian is obtained by performing the substitution \(\psi_\mu \rightarrow \sqrt{\frac{2}{3}} \frac{1}{m_{3/2}} \partial_\mu \chi\) in the supergravity lagrangian. The non-derivative form of the effective lagrangian for \(\chi\) is given by \[15\],
\[
\mathcal{L}_{\text{eff}} = \frac{m_{\tilde{\tau}}^2}{\sqrt{3} M_P m_{3/2}} \left( \bar{\tau}^* R P R \tau + \bar{\tau}^* R P L \chi \right) - \frac{m_{\tilde{\gamma}}}{4\sqrt{6} M_P m_{3/2}} \chi[\gamma^\mu, \gamma^\nu] \tilde{\gamma} F_{\mu\nu}, \tag{7}
\]
where we have neglected a quartic interaction term which is irrelevant for our discussion. In the following, we consider a massive pseudo-goldstino \(\chi\), in order to compare it with the massive gravitino and neutralino. Like a pseudo-Goldstone boson, the pseudo-goldstino has goldstino couplings and a mass which explicitly breaks global supersymmetry.

Note that the goldstino coupling to the photon supermultiplet is proportional to the photino mass \(m_{\tilde{\gamma}}\). As a consequence, the contribution to 3-body \(\bar{\tau}\)-decay with intermediate photino (cf. Fig. 1(c)) is not suppressed for very large photino masses. As we shall see, this leads to significant differences between the angular distributions for pure Yukawa and goldstino couplings, even when \(\chi\) and \(\lambda\) are very light.

In \(\bar{\tau}\) decays both, photon and \(\tau\) lepton will mostly be very energetic. Hence the photon energy \(E_\gamma\) and the angle \(\theta\) between \(\tau\) and \(\gamma\) can be well measured (cf. Fig. 2(a)). We can then compare the differential decay rate
\[
\Delta(E_\gamma, \cos \theta) = \frac{1}{\alpha \Gamma_{\bar{\tau}}} \frac{d^2 \Gamma(\bar{\tau} \rightarrow \tau + \gamma + X)}{dE_\gamma d \cos \theta}, \tag{8}
\]
for the gravitino LSP \((X = \psi_{3/2})\), the pseudo-goldstino \((X = \chi)\) and the hypothetical neutralino \((X = \lambda)\). Details of the calculation are given in Ref. \[1\]. The differences between \(\psi_{3/2}, \chi\) and \(\lambda\) become significant in the backward direction \((\cos \theta < 0)\) as demonstrated by Fig. 2(b)-(d), where \(m_{\tilde{\tau}} = 150\) GeV and \(m_X = 75\) GeV \((X = \psi_{3/2}, \chi, \lambda)\). The three differential distributions are qualitatively different and should allow to distinguish experimentally gravitino, goldstino and neutralino.

Let us now consider the case of small \(m_X\). Then the goldstino lagrangian \([7]\) effectively describes the gravitino interactions. Therefore, one can no longer distinguish
\[^1\]This interaction would arise from gauging the anomaly free U(1) symmetry \(L_\tau - L_\mu\), the difference of \(\tau\)- and \(\mu\)-number, in the MSSM, with \(\lambda\) being the gaugino.
between gravitino and goldstino in this case. However, even for small $m_X$ one can discriminate the gravitino or goldstino from the neutralino. The difference between goldstino $\chi$ and neutralino $\lambda$ stems from the photino contribution (cf. Fig. 1(c)) which does not decouple for large photino mass $m_\gamma$. This is different from the gravitino case where the analogous diagram becomes irrelevant in the limit $m_\gamma \gg m_\tau$ which is employed throughout this study.

The arising discrepancy between gravitino or goldstino and neutralino is demonstrated by Fig. 3. It clearly shows that even for very small masses $m_{3/2}$ and $m_\lambda$, the differential decay rates $\Delta$ for gravitino $\psi_{3/2}$ and neutralino $\lambda$ are distinguishable. This makes it possible to discriminate gravitino and goldstino from a hypothetical neutralino even for very small masses. Note that the plots of Fig. 3 remain essentially the same as long as $r = m_X^2/m_\tau^2 \ll 1$. 

Figure 1: Diagrams contributing to the 3-body decay $\bar{\tau} \to \tau + \gamma + X$ where $X = \psi_{3/2}, \chi, \lambda$. In the limit of very large $m_\gamma$, diagram (c) becomes irrelevant for the gravitino, but it always contributes for the goldstino where it leads effectively to a 4-point interaction.
Figure 2: (a) shows the kinematical configuration of the 3-body decay. The others are contour plots of the differential decay rates for (b) gravitino $\psi_{3/2}$, (c) pseudo-goldstino $\chi$ and (d) neutralino $\lambda$. $m_{\tilde{\tau}} = 150$ GeV and $m_X = 75$ GeV ($X = \psi_{3/2}, \lambda, \chi$). The boundaries of the different gray shaded regions (from bottom to top) correspond to $\Delta(E_\gamma, \cos \theta) [\text{GeV}^{-1}] = 10^{-3}, 2 \times 10^{-3}, 3 \times 10^{-3}, 4 \times 10^{-3}, 5 \times 10^{-3}$. Darker shading implies larger rate.
Let us finally comment on the experimental feasibility to determine gravitino or goldstino couplings. The angular distribution of the 3-body decay is peaked in forward direction ($\theta = 0$). Compared to the 2-body decay, backward ($\cos \theta < 0$) 3-body decays are suppressed by $\sim 10^{-1} \times \alpha \simeq 10^{-3}$. Requiring $10 \ldots 100$ events for a signal one therefore needs $10^4$ to $10^5 \tilde{\tau}$s, which appears possible at the LHC and also at a Linear Collider according to the above discussion.

**Gravitino spin**

A third test of supergravity is intuitively more straightforward though experimentally even more challenging than the previous ones. It is again based on 3-body decays. We now take into account also the polarizations of the visible particles, $\gamma$ and $\tau$. The main point is obvious from Fig. 4(a) where a left-handed photon and a right-handed $\tau$ move in opposite directions.\(^2\) Clearly, this configuration is allowed for an invisible spin-3/2 gravitino but it is forbidden for a spin-1/2 goldstino or neutralino. Unfortunately, measuring the polarizations is a difficult task.

\(^2\)For simplicity, we here restrict ourselves to the case of a right-handed $\tilde{\tau}$ LSP, leaving finite left-right mixing angles for future investigations.
Figure 4: (a) illustrates the characteristic spin-$3/2$ process: photon and $\tau$ lepton move in opposite directions and the spins add up to $3/2$, so the invisible particle also has spin $3/2$. The other figures show angular asymmetries for gravitino $\psi_{3/2}$ (solid curve), goldstino $\chi$ (dashed curve) and neutralino $\lambda$ (dotted curve). $m_{\tilde{\tau}} = 150$ GeV. The photon energy is larger than 10% of the maximal kinematically allowed energy (cf. Ref. [1]). Note that the asymmetries only depend on the ratio $r = m_X^2/m_{\tilde{\tau}}^2$ ($X = \psi_{3/2}, \chi, \lambda$).
As Fig. 4(a) illustrates, the spin of the invisible particle influences the angular distribution of final states with polarized photons and τ leptons. An appropriate observable is the angular asymmetry

$$A_{RL}(\cos \theta) = \frac{\frac{d\Gamma}{d\cos \theta}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + X)}{\frac{d\Gamma}{d\cos \theta}(\tilde{\tau}_R \rightarrow \tau_R + \gamma_R + X) + \frac{d\Gamma}{d\cos \theta}(\tilde{\tau}_L \rightarrow \tau_R + \gamma_L + X)},$$

(9)

where $X$ denotes gravitino ($X = \psi_{3/2}$), goldstino ($X = \chi$) or neutralino ($X = \lambda$). Note that, as discussed before, the photino does not decouple in the case $X = \chi$.

The three angular asymmetries are shown in Fig. 4 for $m_{\tilde{\tau}} = 150$ GeV and different masses of the invisible particle. As expected, the decay into right-handed τ and left-handed photon at $\theta = \pi$ is forbidden for spin-1/2 invisible particles ($\chi$ and $\lambda$), whereas it is allowed for the spin-3/2 gravitino. This is clearly visible in Figs. 4(c) and (d); for small gravitino masses the goldstino component dominates the gravitino interaction as illustrated by Fig. 4(b). The discrepancy between gravitino and goldstino compared to a hypothetical neutralino persists for arbitrarily small $m_X$, which is analogous to the double differential distribution discussed in the previous section.

Conclusions

We have discussed how one may discover a massive gravitino, and thereby supergravity, at the LHC or a future Linear Collider, if the gravitino is the LSP and a charged slepton is the NSP. With the gravitino mass inferred from kinematics, the measurement of the NSP lifetime will test an unequivocal prediction of supergravity. The analysis of 3-body NSP decays will reveal the couplings of the gravitino or the goldstino. For very small masses, one can distinguish the gravitino from the neutralino but not from the goldstino. For masses larger than about 1 GeV, the determination of gravitino mass and spin appears feasible.

Acknowledgements

We would like to thank T. Plehn, P. Schleper and P. M. Zerwas for valuable discussions.

References

[1] W. Buchmüller, K. Hamaguchi, M. Ratz, T. Yanagida, hep-ph/0402179
[2] D. E. Kaplan, G. D. Kirits, M. Schmaltz, Phys. Rev. D62 (2000) 035010;
    Z. Chacko, M. A. Luty, A. E. Nelson, E. Pontón, JHEP 01 (2000) 003;
    K. Inoue, M. Kawasaki, M. Yamaguchi, T. Yanagida, Phys. Rev. D45 (1992) 328.

[3] H. P. Nilles, Phys. Rept. 110 (1984) 1.

[4] G. F. Giudice, R. Rattazzi, Phys. Rept. 322 (1999) 419.

[5] M. Bolz, W. Buchmüller, M. Plümacher, Phys. Lett. B443 (1998) 209;
    M. Fujii, T. Yanagida, Phys. Lett. B549 (2002) 273;
    J. L. Feng, A. Rajaraman, F. Takayama, Phys. Rev. D68 (2003) 063504;
    W. Buchmüller, K. Hamaguchi, M. Ratz, Phys. Lett. B574 (2003) 156;
    M. Fujii, M. Ibe, T. Yanagida, Phys. Rev. D69 (2004) 015006;
    J. Ellis, K. A. Olive, Y. Santoso, V. C. Spanos, hep-ph/0312262.

[6] M. Drees, X. Tata, Phys. Lett. B252 (1990) 695.

[7] J. L. Feng, T. Moroi, Phys. Rev. D58 (1998) 035001;
    S. P. Martin, J. D. Wells, Phys. Rev. D59 (1999) 035008.

[8] S. Ambrosanio, B. Mele, S. Petrarca, G. Polesello, A. Rimoldi, JHEP 01 (2001) 014.

[9] S. Ambrosanio, G. A. Blair, Eur. Phys. J. C12 (2000) 287;
    P. G. Mercadante, J. K. Mizukoshi, H. Yamamoto, Phys. Rev. D64 (2001) 015005.

[10] W. Beenakker, R. Hopker, M. Spira, P. M. Zerwas, Nucl. Phys. B492 (1997) 51.

[11] K. Maki, S. Orito, Phys. Rev. D57 (1998) 554.

[12] TESLA, Technical Design Report Part III, eds. R.-D. Heuer, D. Miller, F. Richard,
    P. Zerwas, DESY 01-011 (2001), hep-ph/0106315;
    ACFA LC Working Group, K. Abe et al., KEK-REPORT-2001-11, hep-ph/0109166;
    American LC Working Group, T. Abe et al., SLAC-R-570 (2001), hep-ex/0106055.

[13] Particle Data Group, K. Hagiwara et al., Phys. Rev. D66 (2002) 010001.

[14] J. Wess, J. Bagger, Supersymmetry and Supergravity, Princeton University Press,
    Princeton, New Jersey, 1992.

[15] T. Lee, G.-H. Wu, Phys. Lett. B447 (1999) 83.