Broadband large-ellipticity harmonic generation with polar molecules

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Abstract: We investigate the polarization properties of high harmonic generation from polar molecules with a linearly polarized field. It is found that elliptically polarized harmonics are observed in a wide spectral range from the plateau to the cutoff. Further analyses show that the nonsymmetric structure of the highest occupied molecular orbital is the origin of ellipticity of the harmonics. The results provide a method for generation of large-ellipticity XUV pulses, which will benefit the application of HHG as a tool of detection in materials and biology science.

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References and links

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High harmonic generation (HHG) has attracted significant attention for producing coherent attosecond pulses in the XUV regime \[1\] to \[5\] and detecting the molecular structure as well as attosecond electronic dynamics \[6\], \[8\]. The power spectrum of high harmonic emission has been extensively studied in previous experimental and theoretical investigations \[9\] to \[11\]. Recently, the polarization characteristics of the harmonic emission are also actively studied due to the potential applications of elliptically polarized harmonics. It is found that information about the molecular system participating in the harmonic generation can be imprinted in the polarization characteristics of harmonics \[12\] to \[14\]. Therefore elliptically polarized harmonics can provide a sensitive probe of the molecular system. For example, Y. Mairesse et al. \[15\] both theoretically and experimentally demonstrated that elliptically polarized harmonics can be applied in HHG spectroscopy of oriented N\(_2\) to track multichannel dynamics during strong-field ionization. In
addition, the search for ellipticity in HHG is also motivated by the generation of elliptically
(even circularly) polarized attosecond pulses in the xuv regime [10].

Several attempts have been made to obtain large ellipticity in the harmonics. It has been
shown that the nonlinearly polarized harmonics can be obtained using an elliptically polarized
driving laser [17]. In such case, the harmonic efficiency drops significantly with increasing el-
lipticity of the driving laser [18]. The elliptically polarized harmonics are also observed when
the oriented molecule is driven by a linearly polarized laser [19–23]. Previous works have
proposed several origins of the nonzero ellipticity of the harmonics generated from oriented
molecules. For example, both two-center [20, 21] and channel [16] interference, which can
result in structural and dynamical minima respectively, are responsible for the elliptically po-
larized harmonics. In detail, the structural and dynamical minima in the parallel component of
the harmonics make the amplitude of the parallel and perpendicular components comparable.
Meanwhile, a phase jump by $\sim \pi$ of the parallel component is accompanied by the interfer-
ence minimum position, leading to the phase difference between the two components passing
through $\pi/2$. Therefore the harmonics in the vicinity of the interference minima are elliptically
polarized. Recently, Ramakrishna et al. [14] demonstrated that the molecular potential also
plays an important role in determining the polarization properties of the harmonics. In previous
investigations, the targets are either atoms or non-polar molecules such as H$_2$, CO$_2$, O$_2$, and
N$_2$. However, the polarization characteristics of the harmonics generated from polar molecules
are seldom studied to the best of our knowledge. The orbital symmetry is one of the basic prop-
erties of molecules and can embody the difference between the polar and non-polar molecules.
Nonetheless, its influence on the elliptical polarization is also seldom investigated.

In this paper, we investigate the polarization characteristics of HHG from the polar molecules
with a linearly polarized laser and analyze the influence of the structure of the highest occupied
molecular orbital (HOMO). Our results show that for oriented polar molecules strongly ellip-
tically polarized harmonics are observed in a spectral range from the plateau to the cutoff at
the orientation angles ranging from 30° to 80°. Further analyses reveal that the nonsymmetric
structure of the HOMO is responsible for those polarization characteristics of the harmonics in
the case of oriented polar molecules.

2. Theoretical model

In this paper, we use the example of CO to investigate the polarization characteristics of the
harmonic emission from oriented polar molecules driven by a linearly polarized laser. Our
coordinate system, depicted in Fig. 1(a), defines the polarization vector of the driving pulse
as the space-fixed $z$ axis, with the molecular axis $z_1$ lying in the y-z plane. The orientation
angle between the molecular axis ($z_1$) and the driving laser polarization axis ($z$) is denoted as
$\theta$. Note that we only need to consider the $y$, $z$ components of the harmonic emission, since
the $x$ component along the propagation direction of the driving laser cannot be phase matched
[19,22]. In Fig. 1(b), an illustration of the HHG ellipse is presented. The major, minor axes of
the ellipse are denoted as $a$, $b$ respectively. $\phi$ represents the rotation angle of the major axis ($a$)
of the HHG ellipse with respect to the polarization axis ($z$) of the driving laser field. Ellipticity
of HHG $\varepsilon$ is defined as the ratio of the length between the minor and major axes, i.e. $\varepsilon = b/a$.

The simulations are carried out using the extended strong field approximation (SFA) model
[24]. With its analytical and fully quantum-mechanical formulations for harmonics, one can
explicitly and directly analyze the influence of the orbital symmetry of the HOMO. In present
work, the main focus is the ellipticity of the harmonics, which is dominantly influenced by
the electronic dynamics. The rotational dynamics are not taken into account by assuming a
perfect orientation, which is analogous to [20,21]. More sophisticated theory that takes both
the rotations and the molecular potential into account is presented in recent works [14,23,25].
Within the SFA model, the $y, z$ components of the laser-induced dipole moments responsible for harmonic emission are expressed as (in atomic units)

\[
\begin{align*}
    r_y(t; \theta) &= \ii \int_{-\infty}^{t} dt' \left[ \frac{\pi}{\zeta + \ii (t-t')} \right]^{3/2} E(t') g(t'; \theta) d_z \left[ p_{st}(t', t) + A(t') ; \theta \right] \\
    &\times \exp \left[ -i S_{st}(t', t) \right] g^*(t; \theta) d_z^* \left[ p_{st}(t', t) + A(t) ; \theta \right] + c.c.
    \tag{1}
\end{align*}
\]

\[
\begin{align*}
    r_z(t; \theta) &= \ii \int_{-\infty}^{t} dt' \left[ \frac{\pi}{\zeta + \ii (t-t')} \right]^{3/2} E(t') g(t'; \theta) d_z \left[ p_{st}(t', t) + A(t') ; \theta \right] \\
    &\times \exp \left[ -i S_{st}(t', t) \right] g^*(t; \theta) d_z^* \left[ p_{st}(t', t) + A(t) ; \theta \right] + c.c.
    \tag{2}
\end{align*}
\]

In these equations, $E(t)$ refers to the electric field of the driving laser pulse, $A(t)$ is its associated vector potential, $\zeta$ is a positive constant, $t'$ and $t$ correspond to the moments of electronic ionization and recombination respectively, $p_{st}$ and $S_{st}$ are the stationary momentum and the quasi-classical action, which are given by

\[
p_{st}(t', t) = \frac{1}{t-t'} \int_{t'}^{t} A(t'') dt'' ,
\]

\[
S_{st}(t', t) = (t-t') I_p - \frac{1}{2} p_{st}^2(t', t)(t-t') + \frac{1}{2} \int_{t'}^{t} A^2(t'') dt'' ,
\]

where $I_p$ is the ionization energy of the target molecule. $g(t; \theta)$ represents the ground state amplitude at the time $t$, and can be expressed as \( \text{(26, 27)} \):

\[
g(t; \theta) = \exp \left[ - \int_{-\infty}^{t} w(t'; \theta) dt' \right] .
\]

$w(t'; \theta)$ is the ionization rate, which is calculated by Molecular Ammosov-Delone-Krainov (MO-ADK) model for aligned molecules \( \text{(28)} \). $\vec{d}(p)$ is the field-free dipole matrix element for
transition from the ground state to the continuum state, where \( \mathbf{p} \) stands for the momentum of the electron. \( d_y, d_z \) correspond to its \( y, z \) components, respectively. Within the single active electron approximation (SAE), this transition dipole moment is given by \(^{13}\)

\[
\tilde{d}(p; \theta) = \langle \psi_0(x, y, z; \theta) | \mathbf{r} | \psi_p \rangle. \tag{6}
\]

Here \( \psi_0(x, y, z; \theta) \) represents the ground state of the target molecule, i.e. the highest occupied molecular orbital (HOMO). The HOMO is obtained using the Gaussian 03 ab initio code \(^{29}\) with the 3-21G basis set. \( \psi_p = \exp(ipz) \) refers to the electronic continuum state with a momentum \( p \). The harmonic spectrum is then obtained by Fourier transforming the time-dependent dipole acceleration \( \tilde{d}(t; \theta) \). The equations for the two components are:

\[
a_y(q; \theta) = \frac{1}{T} \int_0^T a_y(t; \theta) \exp(-iq \omega t) dt, \tag{7}
\]

\[
a_z(q; \theta) = \frac{1}{T} \int_0^T a_z(t; \theta) \exp(-iq \omega t) dt, \tag{8}
\]

where \( \tilde{d}(t; \theta) = \tilde{r}(t; \theta) \), \( T \) and \( \omega \) are the duration and frequency of the driving pulse, \( q \) corresponds to the harmonic order. The spectral intensity \( (I) \) and phase \( (\phi) \) of harmonic components are given by

\[
I_j(q) = |a_j(q)|^2, \quad \phi_j(q) = \arg[a_j(q)], \quad j = y, z.
\]

The ellipticity \( \varepsilon \) is determined by the amplitude ratio and the phase difference of the two components \(^{21}\):

\[
\varepsilon = \sqrt{\frac{1 + r^2 - \sqrt{1 + 2r^2 \cos 2\delta + r^4}}{1 + r^2 + \sqrt{1 + 2r^2 \cos 2\delta + r^4}}} \tag{9}
\]

where \( r = \sqrt{T/\pi} \) and \( \delta = \phi_y - \phi_z \). The range of the ellipticity is \( 0 \leq \varepsilon \leq 1 \). The linear, elliptical, and circular polarization correspond to \( \varepsilon = 0 \), \( 0 < \varepsilon < 1 \), and \( \varepsilon = 1 \) respectively. If \( \delta \) is \( 0 \) or \( \pi \), the HHG will be linearly polarized. For other values of \( \delta \) between \( 0 \) and \( \pi \), the HHG will be elliptically polarized. In order to attain high ellipticity, the phase difference must be around \( \pi/2 \).

### 3. Result and discussion

We study the ellipticity of HHG from oriented polar CO molecules driven by a linearly polarized laser. A 30-fs 800-nm driving pulse with an intensity of \( 2 \times 10^{14} \) W/cm\(^2\) is used in our simulation. The electric field of the laser pulse is expressed as

\[
\tilde{E}(t) = E_0 \cos(\omega t) \sin^2 \left( \frac{\pi t}{T} \right) \mathbf{\hat{z}}, \tag{10}
\]

where \( E_0 \) is the amplitude of the driving field. Figure 2(a) presents the ellipticity of the harmonic emission at two orientation angles 30° (red line) and 45° (black line). One can see that for the two orientation angles, strongly elliptically polarized harmonics are observed in a wide spectral range covering the plateau to the cutoff. In Fig. 2(b), the phase difference (blue line) and the ellipticity (black line) of the recombination dipole moment \(^{13},^{30}\) for CO oriented at 45° are also presented. Here the phase difference is shifted into \( [-\pi, 0] \). Interestingly, the relative phase between the \( y, z \) components of the dipole moment is close to \( -\pi/2 \) for all the electronic momenta. As a result, the ellipticity of the dipole moment is quite large, as shown by the black curve. Furthermore, it is also shown that the ellipticity curve of the dipole moment in panel (b) is similar to that of the harmonics at the same orientation angle (black line) in panel (a). The electronic momentum is transformed into the harmonic order by the dispersion relation.
\[ \vec{\text{d}}(p) = E_p + I_p, \] wherein \( E_p \) is the kinetic energy of the free electron with the momentum \( p \). This phenomenon will be discussed later.

The ellipticity of the harmonics for CO oriented at other angles from 0° to 90° is also studied. The map of the harmonic ellipticity is shown in Fig. 3(a) versus the harmonic order and the orientation angle. As shown in Fig. 3(a), strongly elliptically polarized harmonics are also obtained at other orientation angles ranging from 30° to 80°. In Fig. 3(b), the ellipticity of the dipole moment as a function of the orientation angle and the electronic momentum is presented as well. Here the electronic momentum is transformed into the harmonic order using the same dispersion relation. One can see that the phenomenon observed in Fig. 2 is not specific to the case when the molecule is oriented at angle 45°. As shown in Fig. 3, the ellipticity map of the harmonics in panel (a) also exhibits a quite similar shape to that of the dipole moment in panel (b). To explain this phenomenon, a theoretical analysis is performed based on the three-step model \([24,31]\) for high harmonic generation from oriented molecules. In this model, an electron first tunnels from the ground state, then oscillates in the field, finally recombines with the parent ion. The recombination step leads to the emission of a high-energy photon with two polarization components, parallel (\( z \)) and perpendicular (\( y \)) to the laser polarization. Since the first two steps are identical for the \( y, z \) components of the harmonics, the phase difference and amplitude ratio between the two components of the harmonics is mainly contributed by those of the recombination dipole moment \( \vec{d}(p) \). Hence the ellipticity curves between harmonics and the recombination dipole moment exhibit a similar shape at the same orientation angle, as shown in Fig. 2 and Fig. 3. In other words, the recombination dipole moment \( \vec{d}(p) \) is responsible for the large ellipticity of the harmonics over a wide spectral range for CO.

From the formula \( \vec{d}(p; \theta) = \langle \exp(i p z) \vec{r} | \psi_0(r; \theta) \rangle \) for the recombination dipole moment, one can see that the phase difference \( \arg(d_y) - \arg(d_z) \) and the amplitude ratio \( |d_y/d_z| \) of the recombination dipole moment are directly related to the spatial structure of the HOMO. In the following, we analyze the influence of the HOMO structure on the phase difference of \( \vec{d}(p) \) in detail to grasp a deeper insight into the large ellipticity of the harmonics generated from the polar CO molecules. For comparison, we also discuss the case of non-polar N\(_2\) molecule, which has a symmetric HOMO with a similar geometry (\( \sigma_g \)) to that of CO. Sectional views of...
the HOMO for N₂ and CO at the orientation angle 45° are presented in Fig. 4. As shown in this figure, the section of the non-polar N₂ molecule in panel (a) exhibits a centro-symmetric structure. Then the wavefunction satisfies \( \psi_0(x, y, z; \theta) = \psi_0(x, -y, -z; \theta) \) (for arbitrary value of \( x \)). However, for the nonsymmetric HOMO of CO this condition is not satisfied, as shown in panel (b). With Eq. (6), one can obtain \( \text{Re}\{d_j^*\} = 0 \) and \( \text{Im}\{d_j^*\} \neq 0 \) for N₂, \( \text{Re}\{d_j^*\} \neq 0 \) and \( \text{Im}|d_j^*| \neq 0 \) (\( j = y, z \)) for CO, respectively. As a result the phase difference of \( \vec{d}^*(p) \) for N₂ is either 0 or \( \pm \pi \), as shown by the blue curve in Fig. 5(b). Whereas for CO, they can take other values including \( \pm \pi/2 \) for all the electronic momenta, as demonstrated in Fig. 2(b). The phase difference of the harmonics is dominated by that of the recombination dipole moment, as illustrated above. Therefore large ellipticity of the harmonics is obtained in a wide spectral spectrum, as presented in Fig. 2(a) by the black curve. Finally, it can be concluded from the above discussion that the nonsymmetric structure of the HOMO is the major origin of the ellipticity observed in the case of the oriented polar CO molecules.

Figure 5(a) shows the ellipticity of the harmonics generated from N₂ at the same orientation angles with Fig. 2(a). In contrast to CO, the nonzero ellipticity of the harmonics are only obtained in the vicinity of the structure minima in the HHG spectra, where the positions of the minima coincide with those in [30]. This nonzero ellipticity of the harmonics is well explained by the two-center interference effects, which is analogous to [16][20][21] for non-polar molecules. Comparison between the cases of CO and N₂ also shows that the ellipticity of the harmonics of CO is larger than that of N₂ for most of the orientation angles, as demonstrated in Fig. 2 and Fig. 5 for the orientation angles at 30° and 45°. Therefore our results suggest a potential application of HHG from polar CO molecules in generating the broadband large-ellipticity harmonics, which will benefit the application of HHG as a tool of detection in biology and materials science in the future.
4. Conclusion

We use the example of CO to investigate the polarization characteristics of HHG from the oriented polar molecules driven by a linearly polarized laser. It is found that the harmonics over a wide spectral range are strongly elliptically polarized for the orientation angles ranging from 30° to 80°. The spectral range of large-ellipticity harmonics is covered from the plateau to the cutoff, while for non-polar molecules the harmonics only in the vicinity of the interference minimum are elliptically polarized. Further analyses show that the nonsymmetric structure of the HOMO is the major origin of the ellipticity of the harmonics observed in oriented CO molecules. Since the nonsymmetric structure is a general characteristic of the HOMO for polar molecules, the conclusions made from the results of CO can be extended to other polar molecules. Thus our investigations provide a method for generation of large-ellipticity XUV
pulses, which will benefit the application of HHG as a tool of detection in materials and biology science.

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