In this letter, hadronic decay channels of light pseudoscalar mesons are realized in Yangian algebra. In the framework of Yangian, we find that these decay channels can be formulated by acting transition operators, composed of the generators of Yangian, on the corresponding pseudoscalar mesons. This new description of decays allows us to present a possible interpretation of the new unknown particle $X$ in the decay $K^0_L \rightarrow \pi^0\pi^0X$: it is an entangled state of $\pi^0$ and $\eta$.

As is known, mesons can be looked upon as entanglement states of quarks, and the high energy quantum teleportation related to kaons has already been under investigation. As a co-product, the entanglement degrees of initial and final states of the transition processes are also presented in this letter.

The Yangian related to the Lie algebra $su(3)$ is denoted as $Y(su(3))$ \cite{Epstein}. It is generated by the generators $\{I_\alpha, J_\alpha\}$ which are usually defined as follows,

$$I^a = \sum_i F^a_i,$$

$$J^a = \mu I^a_1 + \nu I^a_2 + \frac{i}{2} \lambda f_{abc} \sum_{i \not= j} \omega_{ij} I^b_i I^c_j$$  \hspace{1cm} (1)

with $i, j = 1, 2$. Here

$$\omega_{ij} = \begin{cases} 1 & \text{if} \ i > j \\ -1 & \text{if} \ i < j \\ 0 & \text{if} \ i = j \end{cases}$$  \hspace{1cm} (2)

$\mu, \nu, \lambda$ are parameters or Casimir operators, and $f_{abc}$ ($a, b, c = 1, 2, 3$) are the structure constants of $su(3)$ algebra. $\{F^a, a = 1, 2, \ldots, 8\}$ form a local $su(3)$ on the site, and are equal to half of the corresponding Gell-Mann matrices.

For brevity, transformations are introduced as follows: $\hat{F}^\pm = J^1 \pm i J^2, \hat{U}^\pm = J^6 \pm i J^7, \hat{V}^\pm = $
$J^4 \pm iJ^5, \bar{J}^3 = J^3, \bar{J}^8 = \frac{2}{\sqrt{3}}J^8$. Different combinations of these operators will provide us with different transition operators. We will show that hadronic decay channels of pseudoscalar mesons can be formulated by acting properly chosen transition operators on the corresponding initial states of mesons. In order to illustrate this clearly, in the following, we will discuss different cases one by one.

Yangian $Y(su(3))$ in $\eta$ decay channels – The $\eta$ and $\eta'$ mesons play a special role in understanding low energy QCD. They are isoscalar members of the nonet of the lightest pseudoscalar mesons, and $\eta$-$\eta'$ mixing system is one of the most attractive problems all along $[12, 13, 14, 15, 16, 17]$. Moreover, the decays of $\eta$ provide information about the pseudoscalar form factors $[18]$.

We choose $\eta$, which is superposition of singlet and octet of $su(3)$, as the initial state

$$|\eta\rangle_{ini} = \alpha_1|\eta'\rangle + \alpha_2|\eta^0\rangle,$$  \hspace{1cm} (3)

where $\alpha_1$ and $\alpha_2$ are the normalized real amplitudes and they satisfy $\alpha_1^2 + \alpha_2^2 = 1$. $|\eta'\rangle = \frac{\sqrt{2}}{2}(|u\bar{u}| + |d\bar{d}|)$ and $|\eta^0\rangle = \frac{\sqrt{2}}{2}(-|u\bar{u}| - |d\bar{d}| + 2|s\bar{s}|)$.

As is known, the entanglement degree of the genuine N-particle qutrit pure state $[19]$ is measured by the mean entropy as follows $[20]$

$$C_{\Phi}^{(N)} = \left\{ \begin{array}{ll}
\frac{1}{N} \sum_{i=1}^{n} S_{i} & \text{if } S_{i} \neq 0 \forall i \\
0 & \text{otherwise}
\end{array} \right.,$$  \hspace{1cm} (4)

where $S_{i} = -Tr((\rho_{p_{i}})_{i}Log_{3}(\rho_{p_{i}}))$ is the reduced partial Von Neumann entropy for the $i$th particle only, with the other N-1 particles traced out, and $(\rho_{p_{i}})$ is the corresponding reduced density matrix. The system we discuss here is bipartite qutrit, $N=2$, thus the entanglement degree of the initial state can be gotten by applying the Eq. (1),

$$C_{ini} = -2\left(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{6}}{6}\alpha_2\right)^2Log_{3}\left(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{6}}{6}\alpha_2\right)^2$$

$$-\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{6}\alpha_2\right)^2Log_{3}\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{6}\alpha_2\right)^2.$$  \hspace{1cm} (5)

Fig. 1 shows the variation of entanglement degree of initial state depending on the amplitude of the singlet state.

Now, let us take the transition operator $P = 3\bar{J}^3$ and act it on the initial state $|\eta\rangle_{ini}$, we then get the final state

$$|\eta\rangle_{fin} = P|\eta\rangle_{ini} = |\pi^0\rangle + |\pi^0\rangle + |\pi^0\rangle,$$  \hspace{1cm} (6)

with the normalization condition $(\mu + \nu)(\alpha_1 - \frac{\sqrt{6}}{\sqrt{2}}\alpha_2) = \pm \sqrt{3}$. Calculation shows that the entanglement degree of the final state $C_{fin}$ is a constant $C_{fin} = 0.631$, independent with the amplitude $\alpha_i$ of the initial state.

From the above formula, to our surprise, we find that this transition is exactly corresponding to the decay channel of $\eta \rightarrow \pi^0\pi^0\pi^0$.

Other hadronic decay channels of $\eta$ can be also obtained with the same method in the framework of Yangian. For instance, after tedious calculation, we find that, by taking $P = \sqrt{6}\bar{J}^3 - \sqrt{3}(\bar{I}^+ - \bar{I}^-)$ with $\nu = -\frac{1}{2}$ and $(\mu - \frac{1}{2})(\alpha_1 - \frac{\sqrt{2}}{\sqrt{3}}\alpha_2) = \pm 1$ or $P = \sqrt{6}\bar{J}^3 + \sqrt{3}(\bar{I}^+ - \bar{I}^-)$ with $\mu = \frac{1}{2}$ and $(\nu + \frac{1}{2})(\alpha_1 - \frac{\sqrt{2}}{\sqrt{3}}\alpha_2) = \pm 1$, the initial state $|\eta\rangle_{ini}$ transits to

$$|\eta\rangle_{fin} = |\pi^0\rangle + |\pi^+\rangle + |\pi^-\rangle$$  \hspace{1cm} (7)

with entanglement degree of $C_{fin} = 0.118$. This corresponds to the decay channel of $\eta \rightarrow \pi^0\pi^+\pi^-$. Moreover, the final state becomes

$$|\eta\rangle_{fin} = |\pi^+\rangle + |\pi^-\rangle,$$  \hspace{1cm} (8)

when the transition operator takes the form of $P = \sqrt{3}\bar{J}^3$ with normalizing condition $(\mu + \nu)(\alpha_1 - \frac{\sqrt{6}}{\sqrt{2}}\alpha_2) = \pm \sqrt{3}$. Apparently, this is just the decay channel of $\eta \rightarrow \pi^+\pi^-$. The results mentioned above are summarized in table II.

Yangian $Y(su(3))$ in $\eta'$ decay channels – $\eta'$ is the most esoteric meson of the pseudoscalar nonet, closely related to the axial U(1) anomaly $[18]$. We take the initial state $|\eta\rangle_{ini}$ which is also the superposition of $su(3)$ singlet and octet

$$|\eta\rangle_{ini} = \frac{1}{3}|\eta'\rangle - \frac{2\sqrt{2}}{3}|\eta^0\rangle,$$  \hspace{1cm} (9)

with entanglement degree $C_{ini} = 0.790$.

Here we choose the transition operators as $P_{\pm} = \sqrt{3}\bar{J}^8 \pm \frac{2}{\sqrt{3}}\bar{J}^3(\bar{I}^+ - \bar{I}^-)$ and act them on the initial state $|\eta\rangle_{ini}$, respectively, we get the same final state

$$|\eta\rangle_{fin} = |\eta\rangle + |\pi^+\rangle + |\pi^-\rangle,$$  \hspace{1cm} (10)

Its entanglement degree is $C_{fin} = 0.973$. Here the normalization condition reads $\mu + \nu = \pm \frac{3\sqrt{3}}{2}$. Clearly,
we find that this transition corresponds to the decay channel of \( \eta' \to \pi^+\pi^-\eta \).

Acting the operators \( P_\pm = \sqrt{\frac{3}{2}} F^3 \pm \sqrt{\frac{1}{2}} F^8 \) on \( |\eta'\rangle_{\text{ini}} \), we get correspondingly the final state as

\[
|\eta'\rangle_{\text{fin}} = |\eta\rangle + |\pi^0\rangle + |\pi^0\rangle
\]

with the normalization condition \( \mu + \nu = \pm \frac{2\sqrt{2}}{\sqrt{3}} \). And the entanglement degree of the final state is \( C_{\text{fin}} = 0.786 \). This provides us with another decay channel \( \eta' \to \pi^0\pi^0\eta \).

TABLE II: Yangian \( su(3) \) in \( \eta' \) decay channels

| \( |\eta'\rangle_{\text{ini}} \) | \( P_\pm \) | normalizing condition | \( |\eta'\rangle_{\text{fin}} \) | \( C_{\text{fin}} \) | decay |
|---|---|---|---|---|---|
| \( 2\sqrt{2} (I^+ - I^-) \) | \( \mu + \nu \) | \( |\eta\rangle + |\pi^0\rangle + |\pi^0\rangle \) | 0.973 | \( \eta \) | \( \pi^+\pi^- \) |
| \( \frac{\sqrt{5}}{3} (I^+ - I^-) \) | \( \mu + \nu \) | \( |\eta\rangle + |\pi^0\rangle + |\pi^0\rangle \) | 0.786 | \( \eta \) | \( \pi^0\pi^0 \) |

TABLE III: Yangian \( su(3) \) in \( \pi^\pm \) and \( \kappa^0_{L(S)} \) decay channels

| \( |\varphi\rangle_{\text{ini}} \) | \( P \) | normalizing condition | \( |\varphi\rangle_{\text{fin}} \) | \( C_{\text{fin}} \) | decay |
|---|---|---|---|---|---|
| \( \sqrt{3} U^+ \) | \( \lambda = \pm \frac{2}{\sqrt{3}} \) | \( |\pi^+\rangle + |\pi^-\rangle \) | 0.455 | \( \kappa^+ \) | \( \pi^+ \) |
| \( \sqrt{3} V^+ \) | \( \lambda = \pm \sqrt{\frac{2}{3}} \) | \( |\pi^+\rangle + |\pi^-\rangle \) | 0.951 | \( \kappa^+ \) | \( \pi^+ \) |

By comparing this formula with Eq. (12), it is obvious that the unknown particle \( X \) could be interpreted as an entangled state of \( \pi^0 \) and \( \eta \) with a certain proportion. With the masses of \( \pi^0 \) and \( \eta \) being \( M_{\pi^0} \approx 135MeV/c^2 \) and \( M_{\eta} \approx 548MeV/c^2 \), we have

\[
|\kappa^0_{L(S)}\rangle_{\text{fin}} = \alpha_1 |\pi^0\rangle + \alpha_2 |\kappa^0\rangle.
\]
immediately the mass of the particle $X$ being $M_X = (0.899^2 \ast 135 + 0.438^2 \ast 548)MeV/c^2 \approx 214.3MeV/c^2$ which is in agreement with the experimental observation \([1,4]\). In addition, both $\pi^0$ and $\eta$ can decay to $2\gamma$, this is again coincident with experiments.

In summary, we find that the hadronic decay channels of pseudo-scalar mesons can be reformulated under the framework of Yangian by acting properly chosen transition operators - consisting of generators of Yangian $Y(su(3))$ - on corresponding initial states.

Besides conventional theories and methods in investigating the hadronic decay channels of pseudo-scalar mesons, Yangian shows us another way to look insight into the phenomena of meson decay, and provides us with an alternative method to investigate exotic effects observed in experiments. One of the examples is the puzzling new particle $X$ in a new decay channel of $\kappa^0_0 \rightarrow \pi^0\pi^0X$, our result indicates that this new particle $X$ might be interpreted as an entangled state of $\pi^0$ and $\eta$.

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