Experimental verification of statistical correlation for bosons: Another kind of Hong-Ou-Mandel interference

Wei-Tao Liu\textsuperscript{1}, Wei Wu\textsuperscript{1}, Ping-Xing Chen\textsuperscript{1,2}, Cheng-Zu Li\textsuperscript{1}, and Jian-Min Yuan\textsuperscript{1}

\textsuperscript{1}Department of Physics, National University of Defense Technology, Changsha, 410073, China
\textsuperscript{2}State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China

(Dated: December 7, 2009)

According to the identity principle in quantum theory, states of a system consisted of identical particles should maintain unchanged under interchanging between two of the particles. The whole wavefunction should be symmetrized or antisymmetrized. This leads to statistical correlations between particles, which exhibit observable effects. We design an experiment to directly observe such effects for bosons. The experiment is performed with two photons. The effect of statistical correlations is clearly observed when the wavepackets of two photons are completely overlapped, and this effect varies with the degree of overlapping. The results of our experiment substantiate the statistical correlation in a simple way. Experiment reported here can also be regarded as another kind of two-photon Hong-Ou-Mandel interference, occurs in the polarization degree of freedom of photon.

PACS numbers: 03.65.-w, 05.30.-d, 42.50.Dv

One of the basic principles consolidating the foundation of quantum mechanics is identity principle, which indicates that there will be no observable change in a system consisted of identical particles when two of the particles are interchanged with each other\textsuperscript{1}. To meet this request, the whole wavefunction of the system should be symmetrical for bosons, while antisymmetrical for fermions. This leads to a quite different effect from classical physics. In a system containing multiple identical particles, there should be correlations between particles since only symmetrical or antisymmetrical state are permitted. It is known as statistical correlation. Many kinds of physical phenomena illuminate the existence of statistical correlation, such as superconductivity\textsuperscript{2, 3} and Bose-Einstein condensate\textsuperscript{4, 6}. That kind of correlation is closely related with indistinguishability of quantum states, which also excited many interests in fundamental problems in quantum mechanics and applications in quantum information processing\textsuperscript{6, 7}.

Statistical correlations result in observable effects. We demonstrate an experiment with photons to directly observe the effect of statistical correlation in boson system, by engineering the overlapping between the wavepackets of two photons and performing proper measurements. The results verify the correlations between two photons when their wavepackets are completely overlapped. The effect of correlation also varies with the degree of overlapping between two photons. By comparison, we find out the connection between our experiment and that of Hong-Ou-Mandel (HOM) interference\textsuperscript{8, 9}: our experiment can be regarded as another kind of HOM interference which occurs in the polarization degree of freedom of photon, while HOM interference can also be explained in the way similar to our experiment.

The main idea of the experiment is shown as follow-

\begin{equation}
|\psi_o\rangle = |H\rangle|V\rangle\text{ or }|V\rangle|H\rangle,
\end{equation}

with $|H\rangle(|V\rangle)$ represents the horizontally(vertically) polarized state. What is the two-photon polarization state when their wavepackets are overlapped in temporal-spatial space? Since they are indistinguishable when they overlap, statistical correlation between photons should be considered according to identity principle. For photons which are known as bosons, the whole wavefunction should be symmetrical under interchanging between two photons. Since the temporal-spatial part of the whole wavefunction is symmetrical, the polarization part of the wavefunction should be symmetrized to ensure the symmetry of the whole wavefunction. Thus we obtain the polarization state

\begin{equation}
|\psi_o\rangle = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle).
\end{equation}

It turns out to be a polarization entangled state.

To verify the effect of statistical correlation, i.e., to verify that two-photon polarization state does be the entangled state shown in Eq.(2) instead of a product state when two photons are indistinguishable, what we need to do is to find out the difference between the product state and the entangled state. Toward this, quantum state tomography\textsuperscript{10} can not work well here since those processes are designed for separated photons. Here, we perform direct projection measurement on the two-photon state. Projection on the state $|D\rangle|D\rangle$ are considered, where $|D\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$. When two photons are
FIG. 1: Experimental arrangements for directly observing the effect of statistical correlations with photons. Two photons are created via SPDC and firstly prepared in a product state of \(|HV\rangle\). Then they are combined into a single beam with a beam splitter (BS1). A polarizer oriented at +45° is employed to perform the projection measurement on the state \(|D\rangle|D\rangle\).

The number of photon pairs is counted with another beam splitter (BS2) and two single photon detectors. The degree of overlapping between two photons is engineered by changing the length difference between two arms via scanning a prism located on a PC-controlled motor.

separated and their polarization state is shown in Eq. (1), the probability of the state to be projected on \(|D\rangle|D\rangle\) is

\[ P_s = |\langle H|\langle V|D\rangle|D\rangle|^2 = \frac{1}{4} \]  

(3)

While two photons are completely overlapped, the state shown in Eq. (2) can be rewritten in the basis \(|D\rangle, |A\rangle = (|H\rangle - |V\rangle)/\sqrt{2}\) as

\[ |\psi_o\rangle = \frac{1}{\sqrt{2}}(|D\rangle|D\rangle - |A\rangle|A\rangle) \]  

(4)

Thus the probability of projection on the state \(|D\rangle|D\rangle\) will be

\[ P_s = |\langle \psi_o|D\rangle|D\rangle|^2 = \frac{1}{2} \]  

(5)

Therefore, the effect of statistical correlations can be observed directly by measuring the probability of the state projection on the state \(|D\rangle|D\rangle\).

In the experiments, two photons are created via spontaneous parametric down-conversion (SPDC). The experimental setup is shown in Fig. 1. A continuous wave laser (Coherent, MBR110 and MBD200) operated at a wavelength of 397 nm with a power of 500 mW serves as the pump source. A 0.59-mm-thick \(\beta\)-barium borate (BBO) crystal cut for type-I degenerate noncollinear phase matching is used as the down-converter. The down-converted photons are detected with single photon detectors (PerkinElmer, SPCM-AQR-16). An interference spectral filter centered at 794 nm with 10 nm bandwidth precedes each detector, which result in a coherence time of \(\tau_c = 210\) fs. The polarizer set at 45° serves for the projection measurement on the state \(|D\rangle|D\rangle\).

Both photons are initially created in horizontal polarization state and one of them is transformed into vertical polarization with a half wave plate (HWP), the fast axis of which is set at an angle of +45° with respect to the vertical direction. Two polarizers respectively set to transmit horizontal and vertical polarization states are employed to ensure the purity of the polarization state of two photons.

Then two photons are combined with a polarization-independent 50:50 beam splitter (BS1) into a single beam, as shown in Fig. 1. There are four cases for two photons flying from two outputs of BS1. Only those cases that two photons leave BS1 via the same one of the outputs are considered, the probability of which is 1/4. The degree of overlapping between the wavepackets of two photons is controlled by changing the path length of one photon, which is performed with a prism located on a computer-controlled motor. In the experiments, path length difference between two photons are scanned from \(-160\) \(\mu m\) to \(160\) \(\mu m\), thus the difference of arrival time between two photons varies from \(-533\) fs to 533 fs. When the path difference is large enough, say \(160\) \(\mu m\), two photons are temporally separated since the difference of arrival time is larger than coherence time of the photons. When the prism is located at the position of 0, two paths are in the same length and two photons are completely overlapped.

Firstly, we count the number of the photon pairs, without projection measurement. The photon pair counting is performed with another beam splitter (BS2) and two detectors, as shown in the dashed box in Fig. 1. Only those cases that both detectors click are recorded. Since detectors only perform exist-or-not measurements and no polarization information of the photons is revealed after photons going through BS2, the post-selection of detection does not causing any interference. The ideal efficiency of pair counting is 50% regardless of the efficiency of the detectors, whether there is correlation between two photons or not. The results are shown as dark dots show the counts with projection measurement. Dark squares show the counts without projection measurement (Polarizer@45° removed). The round dots show the counts with projection measurement.

FIG. 2: Coincident counts between two detectors with the prism scanning.
squares in Fig.2. The measured count of the photon pairs is \( N = 20777 \pm 308 \), which shows no change whether two photons are overlapped or not. The coincidence window is chosen to be 10 ns for all the measurements.

Then the projection measurement is performed with a polarizer set at +45° to transmit photons in the state \( |D\rangle \), and photon pairs transmit through the polarizer are also counted with the photon-pair-counter shown in the dashed box in Fig.1. When two photons are temporally separated, they are in product state since they are distinguishable. For product state, the number of photon pairs in state \( |D\rangle \langle D| \) should be \( N/4 \) according to Eq.(3), while it should be \( N/2 \) when two photons overlap due to statistical correlation as shown in Eq.(5). The results of the measurement varies with the change of path length difference between two photons, shown as round dots in Fig.4. When two photons temporally separated, the measured count is 4867 ± 96, almost one fourth of \( N \); including the loss costed by the polarizer. Then a maximal count of 9489 is obtained when two photons totally overlapped, which is almost a half of \( N \).

Therefore, the data obtained in the experiments agree well with Eq.(3) and Eq.(5), which verifies the effect of statistical correlation.

Statistical correlations are taking place not only when photons are completely overlapped. From Fig.2, the coincidence counts continuously increase to the maximum when the difference of arrival time between photons continuously decreasing to zero. That is, when two photons are partially overlapped, correlations are also observable, what’s different is the degree of correlation. In our experiments, only two photons are considered for convenience. For the cases that more photons involved, it’s possible to exhibit more considerable effects.

To ensure that two photons traveling in the same beam, Hong-Ou-Mandel interference is observed in the experiments. Two photons are transformed into the same polarization state with HWP and two polarizers shown in Fig.1 and Hong-Ou-Mandel interference shows up with the prism scanning. The interference visibility is observed to be higher than 97%. Then the two photons are set back to be orthogonally polarized again for subsequent projection measurement and Hong-Ou-Mandel interference disappears under this situation.

In addition, the state of the two photons is an entangled state when wavepackets of two photons are overlapped. This entanglement does not exist before two photons being combined on BS1. To make sure that, we perform quantum state tomography on the polarization state of two photons before they light on BS1. The results show that it indeed is a product state. The density matrix reconstructed from the experimental data with a maximum likelihood technique is shown in Fig.3. The fidelity between the measured density matrix and that of the state \( |H\rangle |V\rangle \) is 0.996.

That is, the state of two photons changes from a product state into an entangled state when wavepackets of two photons are overlapped. Generally speaking, entanglement can not be created without interaction. However, there is no interaction between photons as is well known. What makes two photons entangled? It is the equivalent interaction of statistical correlations. When two photons are overlapped with each other, the wavefunction has to be symmetrized, which makes two photons correlated with each other. Therefore they become entangled. Unfortunately, no applications on this kind of entanglement and interaction has been discussed so far.

Now let’s recall the experiment of Hong-Ou-Mandel interference and compare with our experiment. Following the above reasoning used for our experiment, HOM interference can also be well explained. In HOM experiment, two photons are prepared in different path states, as shown in Fig.1, one photon being in the state of \( |a\rangle \) while the other in \( |b\rangle \), with \( \langle a|b \rangle = 0 \). So the initial state of two-photon should be \( |\psi_i\rangle = |a\rangle |b\rangle + |b\rangle |a\rangle \). When two photons overlap on the beam splitter, the two-photon state becomes to be \( |\psi_o\rangle = (|a\rangle |b\rangle + |b\rangle |a\rangle )/\sqrt{2} \). According to statistical correlation between two photons. Represented in another basis \( \{|a', b'\}\rangle \), which contains the two output states of the beam splitter, with \( |a'\rangle = (|a\rangle + |b\rangle )/\sqrt{2} \) and \( |b'\rangle = (|b\rangle - |a\rangle )/\sqrt{2} \). The two-photon state can be

![FIG. 3: The reconstructed density matrix of the two-photon polarization state before they light on BS1, with the real part of the matrix on the left and the imaginary part on the right.](image369x659 to 377x689)

![FIG. 4: The diagram of Hong-Ou-Mandel interference experiment.](image398x703 to 416x725)
rewritten as $|\psi_\tau\rangle = (|a\rangle|a\rangle - |b\rangle|b\rangle)/\sqrt{2}$. Therefore when two photons leave the beam splitter, they will always travel out of the same output of BS. Coincidence detection between two outputs of BS is equivalent to the projection on the state of $|a\rangle|b\rangle$ with a result of zero, which agree with the results of Hong-Ou-Mandel interference. Similarly, if we project two-photon state on the state $|a\rangle|a\rangle$ or $|b\rangle|b\rangle$, that is, detecting two photons at the same output of BS, the coincidence count rate will increase when two photons overlap on BS.$^{12}$

On the other hand, our experiment can also be regarded as two-photon interference, similar to HOM interference. When two photons leave BS1 in the same output, the two-photon state can be written as $\hat{a}_H^\dagger(t)\hat{a}_V^\dagger(t + \tau)|00\rangle$, with $\hat{a}_H^\dagger(t)$ being the creation operator of a horizontal polarized photon at time $t$, and $\tau$ being the time delay between two photons. For photons in the states $|D\rangle$ or $|A\rangle$, the corresponding creation operators are

$$\hat{a}_D = \frac{1}{\sqrt{2}}(\hat{a}_H + \hat{a}_V),$$
$$\hat{a}_A = \frac{1}{\sqrt{2}}(\hat{a}_H - \hat{a}_V).$$

(6)

Therefore, the two-photon state becomes

$$|\psi_\tau\rangle = \frac{1}{2} \left[ \hat{a}_D^\dagger(t) + \hat{a}_A^\dagger(t) \right] \left[ \hat{a}_D^\dagger(t + \tau) - \hat{a}_A^\dagger(t + \tau) \right] |00\rangle$$
$$= \frac{1}{2} \left[ \hat{a}_D^\dagger(t)\hat{a}_D^\dagger(t + \tau) - \hat{a}_A^\dagger(t)\hat{a}_A^\dagger(t + \tau) + \hat{a}_D^\dagger(t)\hat{a}_A^\dagger(t + \tau) - \hat{a}_A^\dagger(t)\hat{a}_D^\dagger(t + \tau) \right] |00\rangle.$$

(7)

When two photons completely overlap, $\tau = 0$ and the last two terms in the above equation can not be distinguished which lead to destructive interference. The renormalized state is

$$|\psi_\tau\rangle = \frac{1}{\sqrt{2}} (\hat{a}_D^\dagger\hat{a}_D^\dagger - \hat{a}_A^\dagger\hat{a}_A^\dagger) |00\rangle$$

(8)

which is the same as the state shown in Eq. $^{[4]}$.

In this sense, our experiment can be treated as another kind of HOM interference. The original HOM interference is occurs in the path degree of freedom while our experiment show two-photon interference in the polarization degree of freedom. Although these experiments can be explained in both ways, this two kinds of explanation do not contradict with each other. When the state is represented with creation operators acting on vacuum state, the effects of statistic correlations are involved. As shown in Eq. $^{[7]}$, the operators $\hat{a}_D^\dagger$ and $\hat{a}_A^\dagger$ commute with each other when $\tau = 0$, which makes the last two terms indistinguishable and leads to completely destructive interference. For the cases that more photons involved, there will be more possible states involved and explaining with statistical correlations among photons will be more concise. In addition, when two photons in different polarization states from two different path overlap on a beam splitter, the whole wavefunction should be symmetrized according to identity principle, which can be used for Bell state analyzing for two-photon polarization states.$^{12}$ $^{[14]}$.

In conclusion, we designed an experiment to directly observe the effect of statistical correlations for bosons. Two photons in orthogonal polarization states were considered. By engineering the degree of overlapping between two photons and performing projection measurement on the photons, the effect of statistical correlations is substantiated. The experiment can also be regarded as another kind of two-photon Hong-Ou-Mandel interference, which occurs in the polarization degree of freedom of photon.

The authors thank Prof. Zhe-Yu Jeff Ou and Prof. Guo-Xiang Huang for useful discussions. This work is supported by National Natural Science Foundation of China (Grant No. 10774192), Fund of Innovation (No. B0602040) from Graduate School of NUDT, and the opening research foundation of State Key Laboratory of Precision Spectroscopy.

* Electronic address: mughual@hotmail.com
† Electronic address: pxchen@nudt.edu.cn

[1] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, 1947).
[2] H. K. Onnes, Comm. Phys. Lab. Univ. Leiden, Nos. 119, 120, 122(1911).
[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev., 108, 1175(1957).
[4] S. N. Bose, Zeitschrift für Physik, 26, 178(1924).
[5] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science, 269, 198(1995).
[6] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2001).
[7] N. Paunkovic, *The Role of Indistinguishability of Identical Particles in Quantum Information Processing* (Ph.D. thesis, University of Oxford, 2004).
[8] C. K. Hong, Z. Y. Ou and L. Mandel, Phys. Rev. Lett., 59, 2044 (1987).
[9] R. Kaltenbaek, B. Blauensteiner, M. Żukowski, M. Aspelmeyer, and A. Zeilinger, Phys. Rev. Lett., 96, 240502(2006).
[10] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A., 64, 052312 (2001).
[11] R. Jozsa, J. Mod. Opt., 41, 2315 (1994).
[12] J. G. Rarity and P. R. Tapster, J. Opt. Soc. Am. B, 6, 1221(1989).
[13] K. Matté, H. Weinfurter, P. G. Kwiat and A. Zeilinger, Phys. Rev. Lett., 76, 4656(1996).
[14] J. W. Pan, PhD thesis, Durchgeführt am Institut für Experimentalphysik der Universität Wien, 1999.