Estimation of optimized timely system matrix with improved image quality in iterative reconstruction algorithm: A simulation study

Vahid Moslemi a, Vahid Erfanian b, Mansour Ashoor a, *

a Radiation Applications Research School, Nuclear Science and Technology Research Institute, P.O. Box: 113653486, Tehran, Iran
b Mechanical Engineering Department, Amirkabir University of Technology, P.O. Box: 158754413, Tehran, Iran

ARTICLE INFO
Keywords:
Biophysics
Nuclear engineering
Biomedical devices
Medical imaging
System matrix
Subdividing common regions (SCR) algorithm
MLEM
MCNP5

ABSTRACT
The system matrix (SM) being a main part of statistical image reconstruction algorithms establishes relationship between the object and projection space. The aim was to determine it in a short duration time, towards obtaining the best quality of contrast images. In this study, a new analytical method based on Cavalieri's principle as subdividing common regions has been proposed in which the precision of the amounts of estimated areas was improved by increasing the number of divisions (NOD), and consequently the total SM's time was increased. An important issue is the tradeoff between the NODs and computational time. For this purpose, a Monte Carlo simulated Jaszczak phantom study was performed by the Monte Carlo N-Particle transport code version 5 (MCNP5) in which the tomographic images of resolution and contrast phantoms were reconstructed by maximum likelihood expectation maximization (MLEM) algorithm, and the influence of NODs variations was investigated.

The results show that the lowest and best quality have been obtained at the NODs of 0 and 8, respectively and in the optimum case, the SM's total time at NOD of 8 was 925 s, which was much lower than those of the conventional Monte Carlo simulations and experimental test.

1. Introduction

Image reconstruction algorithms play a key role to obtain the anatomical and functional maps towards better recognition in the medical fields. The shorter the time of algorithm, the better temporal resolution of images will be. This is an important aim in nuclear medical imaging because making internal functionality of the organs more apparent, and projections are converted to transaxial images by these algorithms, which are categorized in analytical and iterative methods [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The maximum likelihood expectation maximization (MLEM) algorithm is used to reconstruct tomographic images by iteratively maximizing a likelihood function. The slow convergence rate and high computational cost are major shortcomings of the MLEM [12, 13]. The thresholding technique is a solution to speed up the convergence rate of MLEM algorithms [14, 15, 16].

The system matrix (SM) indicating relationship between the object and projection space is a main part of statistical image reconstruction algorithms. It is calculated by three methods as; (i) analytical method in which the common regions (CR) considering as interacting probability of emitted gamma photons with detector elements are calculated by geometrical relations [17, 18, 19, 20, 21]; (ii) Monte Carlo (MC) simulations by modeling a digital phantom and estimating projections, which the ratio of counts in i-th element of detector for the originated photons from j-th pixel of object matrix is considered as ai,j element of the SM [22, 23, 24]; and (iii) experimental method where the SM is obtained by measuring projections around the phantoms [25, 26].

In this study, a new analytical method based on Cavalieri's principle namely as subdividing common regions (SCR) algorithm have been proposed to estimate the optimized timely SM. In the proposed method, due to intersection between detector element path and pixel border, the common region (CR) was divided into even-smaller parts and the total CR area was calculated using trapezoidal integration rule. Finally, the influence of number of divisions (NOD) on the precision of the amount of CR area and the quality of tomographic images using a Monte Carlo simulated Jaszczak phantom by the Monte Carlo N-Particle version 5 (MCNP5) code was investigated and an optimum NOD was introduced.
2. Materials and methods

2.1. Theory

The SM has been calculated by the probability of recording emitted gamma photons from a voxel of object by a detector element at a distinct detector angle towards the generation of projections. In other words, the interacting probability of the photons from the \( g \)-pixel of object matrix with \( k \)-element of detector depends on the position of the element related to the corresponding pixel. In emission computed tomography, the correlation between the photons from the object matrix (\( f \)) and generated projection on the detector plane (\( g \)) is as follows:

\[
g = A \times f. \tag{1}
\]

where \( A \) is the SM. An \( 4 \times 4 \) object matrix and a 4-element detector array are shown in Figure 1. The CR between pixel borders and detector element virtual paths have been made by covering the detector element (g2) upon F6 pixel, indicating as \( a_{2,6} \) of the SM. The CRs go on values between 0 and 1. In contrast, while the detector element completely covers the pixel, it does not interact with \( g \) to estimate the area. The polygonal of \( aocabc \) in Figure 3(a) has a lower area in compared to that of the desired CR. In the second step, a line from the middle of CR causes to divide into two new equivalent parts to have a better estimation. By placing linear segments of \( aa \) and \( oo \) with the distance of \( \frac{1}{2} \) and similarly \( cc \) with distance of \( \frac{1}{2} \) a new polygonal of \( aocabc \) was made, which its area determined by trapezoidal integration is closer to the desired CR, as shown in Figure 3(b). In the third step, the CR was divided into four parts and new polygon was constructed by five linear segments, as shown in Figure 3(c). The distance between \( aa \) and next line segment is the \( \frac{1}{2} \). The area of obtained polygonal in this step equals to that of the desired CR. Thus, the estimated area approaches to the desired value in four divisions. However, the area of CR was estimated here correctly by four divisions, but for other situations more divisions might be required. The precision of estimations was improved by increasing divisions.

The length of passing line segments through the CR are changed by the variation of the detector angle. By defining the centre of image matrix as the origin of a polar coordinate system at detector angle of zero (\( \theta = 0^\circ \)), the passing lines from the detector with k elements through the image pixels can be characterized as:

\[
\begin{bmatrix}
g_1 \\
g_2 \\
g_3 \\
g_4
\end{bmatrix} = \begin{bmatrix}
a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a_{3,7} & a_{3,8} & a_{3,9} & a_{3,10} & a_{3,11} & a_{3,12} & a_{3,13} & a_{3,14} & a_{3,15} & a_{3,16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{4,11} & a_{4,12} & a_{4,13} & a_{4,14} & a_{4,15} & a_{4,16}
\end{bmatrix} \times \begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
f_4 \\
f_5 \\
f_6
\end{bmatrix}
\tag{2}
\]

The CRs based on the Cavalieri’s principle were divided as smaller parts, and the entire area of CR was determined through summation of the area of these parts by trapezoidal integration rule. Figure 2 exhibits the implementation of SCR method to calculate the CR on the pixel.

Figure 1. Covering the image matrix (F6) by the detector element (g2), as yellow cross section area.

Figure 2. Implementation of the SCR to calculate the area of CR on the pixel.
where the $g_k$ equals to the width from the origin. By increasing the detector angle on the range from 0 to 45°, the passing lines through image may be formulated as follows:

$$y = (\tan \theta)x + \frac{1}{\cos \theta}g_k$$  \hspace{1cm} (4)

Assuming the $F_{i,j}$ as unitary pixel in an $n \times n$ image matrix, as shown in Figure 4, the passing (red) lines through the pixel have three situations that one may obtain; (a) the length of (1) segment placed in the up-left side of pixel ($\Delta l$) as:

$$\Delta l = \left\lfloor 1 \tan \theta \left(\frac{n}{2} - i - \frac{g_i}{\cos \theta} + 1\right) - \left( j - \frac{n}{2} - 1\right) \right\rfloor \frac{1}{\cos \theta}$$  \hspace{1cm} (5)

(b) the length of (2) segment in the left-right side of pixel as:

$$\Delta l = \left\lfloor 1 \cos \theta \right\rfloor$$  \hspace{1cm} (6)

and, (c) the length of (3) segment in the right-down side of pixel as:

$$\Delta l = \left\lfloor \left( j - \frac{n}{2} \right) - \frac{1}{\tan \theta} \left( \frac{n}{2} - i - \frac{g_i}{\cos \theta} \right) \right\rfloor \frac{1}{\cos \theta}$$  \hspace{1cm} (7)

In general, the passing line segments through pixel sides are categorized in one of the above mentioned scenarios and there is no other situation.

The geometric symmetry principle was used to calculate the length of line segments in the other degrees. By mirroring procedure, all data from 45° to 90° were obtained using the angles from 0° to 45° related to the axis of 45°. This procedure was continued and all data of the SM in 360° was calculated to reduce the computational time. Our analytical algorithm was implemented for 1/8 of the SM and the rest of the SM data were completed by repeating and mirroring technique.

By increasing the NODs in a detector element, the precision of estimated CR area is increased due to intersection between pixel areas and detector element in a distinct angle, and as the NODs approaches infinity, the difference between the estimated and desired areas goes to zero. Since increasing the NODs increases the computational time, thus the

Figure 3. The illustration of modeling CR (a) without SCR method, (b) twofold and (c) fourfold subdivision, using Cavalieri’s principle.

Figure 4. Passage of line segments through a pixel (three red situations, 1-2-3).

Figure 5. Intersection between the detector elements and the pixels of $F_{2,5}$, $F_{4,4}$, $F_{5,2}$ and $F_{6,8}$ in the object matrix to evaluate the precision of the estimated area for different NODs.
and activity concentration of 140-keV and 800-MBq/cc respectively as hot lesions, and the 25.4 and 31.8-mm spheres without any activity concentrations were modeled as cold lesions. Since the activity concentrations of surrounding volume the spheres were 100 MBq/cc, the real contrast for hot and cold spheres to background became 8/1 and 0/1, respectively.

In addition, an standard LEHR lead collimator with hexagonal hole size, hole length and septa thickness of 1.5, 35 and 0.25mm, respectively was modeled and simulated. The NaI detector was divided into separate rows with 128 cubic cells and dimensions of $1.75 \times 1.75 \times 9.525 \text{ mm}$ to calculate the simulated projections.

To reconstruct the tomographic images of phantom, the 120 projections were obtained in a $360^\circ$ rotation around the phantom in 3-step. The distance of collimator to the center of phantom and distance between collimator to detector were 150 and 5 mm, respectively. To calculate each projection, $2 \times 10^{10}$ photons were run and signal intensity in detector cells were calculated by F8 tally in which the interacting photons in a cell is considered as pulse height distribution. The statistical error of simulations was fewer than 5%.

2.3. MLEM image reconstruction algorithm and image analysis

Expectation maximization algorithm is an approach to iterative computation of maximum likelihood estimates when the observations can be viewed as incomplete data. In this study, the MLEM algorithm was developed by the MATLAB software to reconstruct tomographic images of resolution and contrast phantoms in which its process is given as follows [28]:

$$
\bar{p}_{ij}^{(k+1)} = \frac{\bar{p}_{ij}^{(k)}}{\sum_{j=1}^{N-1} p_{ij}^{(k)}} \frac{g_i}{a_k} \prod_{j=1}^{N-1} \bar{b}_j
$$

(8)

where $\bar{p}_{ij}^{(k+1)}$, $\bar{p}_{ij}^{(0)}$, $g_i$ and $a_k$ are the latest estimated image from k-th iteration, the first estimation as a uniform disc, the projection value in the $i$-th element of detector and the SM, respectively. The data of projections (g) was arranged in a $128 \times 128$ matrix and the SMs were calculated for various NODs of 2, 4, 8, 16, 32 and 1024. Tomographic images of resolution and contrast phantoms were reconstructed for 100 iterations in the MLEM.

To assess the tomographic image quality of contrast phantom, the contrast recovery coefficient (CRC%) may be defined as follows [29]:

$$
\text{CRC\%} = \frac{\bar{a}_h - \bar{a}_b}{\bar{a}_c - \bar{a}_b} \times 100
$$

(9)

Over the lesions (hot and cold) and background as shown in Figure 9(e), the average intensity on each pixel was calculated as $\bar{a}_h$ and $\bar{a}_b$ respectively. C is the real contrast, which was 8 and 0 for hot and cold lesions, respectively. The noise coefficient (NC%) in images was evaluated by calculating standard deviation of pixel values in ROI at the central region of contrast image ($\sigma_b$) as follows [29]:

$$
\text{NC\%} = \frac{\sigma_b}{\bar{a}_b} \times 100
$$

(10)

The contrast to noise ratio (CNR) as a criterion for evaluating image quality was calculated for all lesions as follows [29]:

$$
\text{CNR} = \frac{\text{CRC}}{\text{NC\%}}
$$

(11)

3. Results and discussion

Figure 7 shows the values of estimated CR area of $F_{2,5}$, $F_{4,6}$, $F_{6,2}$ and $F_{6,8}$ pixels at detector angles of $2^\circ$, $99^\circ$, $192^\circ$ and $285^\circ$ at various NODs. It is found that these values depend on the detector position at various...
angles with respect to the CR changes in Figure 7. For instance, the pixel of F5,2 at detector angles of 2°, 99°, 192° and 285° was intersected with detector elements of g4, g5, g7 and g2, respectively. Estimated areas in low divisions had intense fluctuations. They converged to their desired values by increasing divisions. For the NODs less than 10, the estimated areas in four pixels were changed for different divisions. The estimated areas tend to their desired values by increasing NOD more than 10, and the curves became smoother. In other words, the fluctuations were almost removed, and the difference with desired values tend to become zero.

The reconstructed tomographic images of resolution phantom for the SMs with the NODs of 0, 2, 4, 8, 16 and 1024 had the same quality and the number of division did not affect the quality of images, as shown in Figure 8.

Figure 9 exhibits the tomographic images of contrast phantom obtained for the SMs with the NODs of 0, 2, 4, 8, 16 and 1024. The quality of tomographic images for different NODs was assessed quantitatively through the contrast phantom images. The maximum image distortion was observed in contrast image obtained for NOD of 0, as shown in Figure 9(a). Additionally, the maximum noise and the minimum CNR was found in this image. Since the CNR is a criterion to evaluate the quality of images, the obtained image for 0 divisions had the lowest quality.

The NC% was determined by calculating the average and standard deviation of pixel values of ROI (white dashed square in Figure 9(e)) over the central part of contrast phantom images. Figure 10 shows the changes of NC% for NODs of 0, 2, 4, 8, 16 and 1024. Variations of NC% indicates
The real applications of this method in the medical world are while the images have been acquired from the single photon emission computed tomography (SPECT) system as well as the positron emission tomography (PET) and computed tomography (CT) ones in which the implementation of the iterative reconstruction algorithm is time-consuming due to the computing system matrix being a very important part in obtaining reconstructed images should be done. In general, the SM needs to be recomputed while there are any geometrical changes of the images have been acquired from the single photon emission computed tomography (SPECT) system as well as the positron emission tomography (PET) and computed tomography (CT) ones in which the implementation of the iterative reconstruction algorithm is time-consuming due to the computing system matrix being a very important part in obtaining reconstructed images should be done. In general, the SM needs to be recomputed while there are any geometrical changes of detection systems in medical devices. In this study, the theoretical concepts of computational algorithms were first explained and then SM was calculated and by using of calculated projections of Jaszczak phantom obtained by MCNP simulations, transaxial images for different NODs were reconstructed. The presented algorithm is able to improve the time limitation of the iterative algorithm as well as may be used for image reconstruction of industrial CT scans. Despite of medical CT scans which usually have the same geometrical specifications, the industrial CTs are constructed with various geometrical specifications because the industrial facilities need to use CTs with different dimensions along with different detector sizes and types [31, 32]. Its corresponding system matrix for each collimator type may be computed and the images are reconstructed exclusively in a short time.

4. Conclusion

In this study, a novel analytical method was proposed to estimate the SM in a short duration time based on Cavalieri's principle towards the best quality of images. By increasing the number of divisions, the precision of estimated CR area was improved, and consequently the total SM's time was increased. The results show that the lowest and best quality have been obtained at the NODs of 0 and 8, respectively and in the optimum case, the SM's total time at NOD of 8 was 925 s, which was

Table 1. Computational times at various NODs. The SM with the NOD of 8 and total computational time of 925 s was selected as the optimum case.

| NOD  | 0    | 2    | 4    | 8    | 32   | 1024 |
|------|------|------|------|------|------|------|
| time (sec) | 575  | 604  | 684  | 925  | 4534 | 62028 |
much lower than those of the conventional Monte Carlo simulations and experimental test. Thus, the proposed method can expedite the deploying iterative reconstruction in clinical uses along with the best image quality.

Declarations

Author contribution statement

Mansour Ashoor: Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Vahid Moslemi: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data.

Vahid Erfanian: Performed the experiments; Analyzed and interpreted the data.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

References

[1] B.C. Tai, et al., Delimitated strike artifacts from FBP using a robust morphological structure operation, Radiat. Phys. Chem. 97 (April 2014) 31–37.
[2] N.E. Protonotarios, G.M. Spyrou, A. Kastis, Automatic cumulative sums contour detection of FBP- reconstructed multi-object nuclear medicine images, Comput. Biol. Med. 85 (June 2017) 43–52.
[3] R.A. Brooks, G. Di Chiro, Principles of computer assisted tomography (CAT) in radiographic and radiosotopic imaging, Phys. Med. Biol. 21 (1976) 689–732.
[4] D.B. Kay, J.W. Keyes, W. Simon, Radionuclide tomographic image reconstruction using Fourier transform techniques, J. Nucl. Med. 15 (1974) 981–986.
[5] A.C. Kakand, M. Slaney, Principles of Computed Tomographic Imaging, 49–112, IEEE Press, New York, NY, USA, 1988.
[6] P.R. Edholm, G.T. Herman, Linograms in image reconstruction from projections, IEEE Trans. Med. Imaging 6 (1987) 301–307.
[7] P.R. Edholm, G.T. Herman, D.A. Roberts, Image reconstruction from linograms: implementation and evaluation, IEEE Trans. Med. Imaging 7 (1988) 239–246.
[8] T. Borfeldt, U. Oellke, Fast and exact 2D image reconstruction by means of Chebyshev decomposition and backprojection, Phys. Med. Biol. 44 (1999) 1105–1120.
[9] B.I. Andia, K.D. Bauer, C.A. Bouman, Nonlinear backprojection for tomographic reconstruction, IEEE Trans. Nucl. Sci. 49 (2002) 61–68.
[10] V. Moslemi, M. Ashoor, Introducing a novel parallel hole collimator: the theoretical and Monte Carlo investigations, IEEE Trans. Nucl. Sci. 64 (9) (2017) 2578–2587.
[11] P.F. Brayant, Analytic and iterative reconstruction algorithms in SPECT, J. Nucl. Med. 43 (10) (Oct. 2002) 1343–1358.
[12] A.J. Rockmore, A. Mackowski, A maximum likelihood approach to emission image reconstruction from projections, IEEE Trans. Nucl. Sci. 23 (4) (1976) 1428–1432.
[13] R.G. Wells, et al., Maximizing the detection and localization of Ga-67 tumors in thoracic SPECT MLEM (OSEM) reconstructions, IEEE Trans. Nucl. Sci. 46 (4) (1999) 1191–1198.
[14] K.S. Chuang, et al., A maximum likelihood expectation maximization algorithm with thresholding, Comput. Med. Imag. Graph. 29 (7) (2005) 571–578.
[15] S.C. Keh, L.J. Meei, W. Jay, C. Sharon, C.N. Yu, K.F. Ying, The thresholding MLEM algorithm, J. Med. Biol. Eng. 24 (2) (2004) 85–91.
[16] H. Wisczorek, The image quality of FBP and MLEM reconstruction, Phys. Med. Biol. 55 (11) (2010) 3161–3176.
[17] A. Iriarte, R. Maraelementi, S. Matij, C.O.S. Sorzano, R.M. Lewitt, System models for PET statistical iterative reconstruction: a review, Comput. Med. Imag. Graph. 48 (Mar. 2016) 30–48.
[18] V. Maxim, et al., Probabilistic models and numerical calculation of system matrix sensitivity in list-mode MLEM 3D reconstruction of Compton camera images, Phys. Med. Biol. 61 (1) (2016) 243–264.
[19] G.K. Loudos, An efficient analytical calculation of probability matrix in 2D SPECT, Comput. Med. Imag. Graph. 32 (2) (2008) 83–94.
[20] A. Rahimn, et al., Analytic system matrix resolution modeling in PET: an application to Rb-82 cardiac imaging, Phys. Med. Biol. 53 (2008) 5947–5965.
[21] S. Moebrs, et al., Multi-ray-based system matrix generation for 3D PET reconstruction, Phys. Med. Biol. 53 (2008) 6925–6945.
[22] K. Li, et al., A new virtual ring-based system matrix generator for iterative image reconstruction in high resolution small volume PET systems, Phys. Med. Biol. 60 (17) (2015) 6945–6973.
[23] K. Saha, K.J. Strauss, Y. Chen, S.J. Glick, Iterative reconstruction using a Monte Carlo based system transfer matrix for dedicated breast positron emission tomography, J. Appl. Phys. 116 (8) (2014) 1–11.
[24] M. Rafecas, et al., Use of a Monte Carlo-based probability matrix for 3D iterative reconstruction of MADPET-II data, IEEE Trans. Nucl. Sci. 51 (5) (2004) 2597–2605.
[25] V.V. Panin, F. Kehren, C. Michel, M. Casey, Fully 3-D PET reconstruction with system matrix derived from point source measurements, IEEE Trans. Med. Imaging 25 (7) (2006) 907–921.
[26] D. Borsy, K. Szczuka, K. Gorczewski, System matrix computation for iterative reconstruction algorithms in SPECT based on direct measurement, Int. J. Appl. Math. Comput. Sci. 21 (1) (2011) 193–202.
[27] F. Brown, J. Sweezy, J. Bull, J.T. Goorlay, A. Sood, MCNP - A General Monte Carlo N-Particle Transport Code Version 5, Los Alamos National laboratory, 2009.
[28] V. Moslemi, M. Ashoor, Evaluation of tomographic image quality of extended and conventional parallel hole collimators using maximum likelihood expectation maximization algorithm by Monte Carlo simulations, Nucl. Med. Commun. 38 (10) (2017) 843–853.
[29] V. Moslemi, M. Ashoor, Design and performance evaluation of a new high energy parallel hole collimator for radioiodine imaging by gamma cameras: Monte Carlo simulation study, Ann. Nucl. Med. 31 (4) (2017) 324–334.
[30] E.N. Gimenez, E. Nacher, M. Gimenez, J.M. Benlloch, M. Rafecas, Comparison of different approaches based on Monte Carlo methods to calculate the system matrix for small animal PET, Nucl. Instrum. Methods A. 606 (3) (2009) 755–761.