Symmetry of Longitudinal Magneto-resistance in the Presence of Multiple Domain Walls in Ferromagnetic Thin Films with Perpendicular Magnetic Anisotropy

Gang Xiang¹, Xi Zhang
Department of Physics, Sichuan University
Chengdu, Sichuan, People’s Republic of China, 610064
E-mail: gxiang@scu.edu.cn

Abstract Symmetry of the longitudinal magneto-resistance (MR) of ferromagnetic thin films with perpendicular magnetic anisotropy (PMA) in the presence of multiple domain walls (DWs) has been studied. Based on exact solutions, schematic representations of electric fields and MR have been established considering anomalous Hall effect and the effect from the eddy currents in the proximity of domain walls in the ferromagnetic samples with PMA. Symmetry analysis shows that MR with single DW in opposite sweeps is antisymmetric with respect to $H=0$, MR with two or $2n$ ($n$ is integer, $n > 0$) evenly distributed DWs is symmetric, and MR with the three or $2n+1$ ($n$ is integer, $n > 0$) evenly distributed DWs is antisymmetric.

1. Introduction
Contemporary developments in spintronics [1] provide a strong motivation for understanding the interplay between electrical transport and magnetic domain walls (DWs) in both ferromagnetic metals [2,3] and in ferromagnetic semiconductors [4-6]. Although the mechanisms of various magnetoresistance (MR) effects are different, most of the intrinsic MR effects, such as the ordinary MR in nonmagnetic metals, the anisotropic magnetoresistance in ferromagnetic metals [7], the giant magnetoresistance in metallic multilayers [8], and spin valve effect in ferromagnetic semiconductor multilayers [9], share the common symmetry obeying Onsager reciprocity, i.e., field dependence of longitudinal MR should be symmetric with respect to the sign of the magnetic field $H$, i.e., $\Delta R_{xx}(H) = \Delta R_{xx}(-H)$. However, the presence of a single domain wall, both in metallic ferromagnets and ferromagnetic semiconductors, changes this symmetry [3,5]. The physical origin of the antisymmetric term in the ferromagnets with perpendicular magnetic anisotropy (PMA) can be understood as follows [5,6]. When a DW is formed, creating a spatially inhomogeneous magnetization, the current density is perturbed by circulating currents that flow in the vicinity of the DW. This current is accompanied by an electric field essentially parallel to it, resulting in an antisymmetric Hall resistivity contribution to the longitudinal MR. However, the study on the interplay of DW and electrical transport have been focused on the effect of a single domain wall [3,5,6,10], while the effect of multiple domain walls on the longitudinal MR is rarely discussed.

In this Letter, we will discuss the symmetry of the longitudinal MR in the presence of multiple domain walls in ferromagnetic thin films with PMA. The external magnetic field is swept along easy axis [3], swept along hard axis [5] of the sample, or rotated $360^0$ around one of the crystalline directions in the
sample plane [5]. The electrical current flows from the left to the right, and the longitudinal MR is obtained by measuring the voltage between the electrodes \( \alpha \) and \( \beta \), as shown in Fig. 1 (a).

In our model, we treat the device as a rectangular Hall bar of width \( w \) and thickness \( t \). Starting from the case with a single domain wall in the sample with PMA, the longitudinal resistance can be written as [6]

\[
R_{xx} = R_S + R_{xy} \left[ \frac{4}{\pi} \sum \theta \left( \frac{l}{2} - |x_{DW}| \right) \right],
\]

\[
R_S = \frac{\rho l}{wt};
\]

\[
R_{xy} = \frac{\rho_H}{t};
\]

\[
\sum = \sum_{n=odd} \exp \left( -\frac{n \pi}{w} \left[ \frac{l}{2} + x_{DW} \right] \right) - \text{sgn} \left( x_{DW} - \frac{l}{2} \right) \exp \left( -\frac{n \pi}{w} \left[ \frac{l}{2} - x_{DW} \right] \right);
\]

where \( l \) is the distance between the two electrodes, \( x_{DW} \) is the domain wall location.

---

**Figure 1.** The schematics of a thin film with 180° domain wall at \( x = 0 \). (a) The width and the thickness of the film are \( w \) and \( t \). The distance between the two electrodes \( \alpha \) and \( \beta \) is \( l \). The magnetization in the left region points upward, while that in the right region downward. (b) 2-dimensional top view of the thin film sample. The direction of the red arrows represents the direction of the electric fields between different points. The dashed lines represent the electric fields around the proximity of the DW considering the effect of eddy currents. \( V_{C1}, V_{C2}, V_{C3}, \) and \( V_{C4} \) are the potentials of the points along the middle line of the width of the sample. \( V_{C2} \) and \( V_{C3} \), the potentials of the points in the proximity of the DW. \( V_{DW} \), the potential of the DW on the upper side of the sample. The magnetization in the left region points outward, while that in the right region inward.
When the domain wall is located outside the two electrodes, but not in the proximity of the DW, we get the trivial solution, i.e., \( R_{xx} = R_S \). When the single domain wall is located between the two electrodes, i.e., \( x_{DW} < l/2 \), we get the longitudinal resistance between two electrodes \( \alpha \) and \( \beta \) from Eqn (1)-(4)

\[
R_{\alpha\beta} = R_S + \left( -\frac{1}{2} \right) R_{xy} + C_L R_{xy} + C_R R_{xy} + \left( -\frac{1}{2} \right) R_{xy};
\]

where

\[
C_L = \left\{ \frac{4}{\pi^2} \sum_{n=odd} \exp \left( -\frac{n\pi}{w} \frac{l}{2} + x_{DW} \right) \right\} > 0; \quad (6)
\]

\[
C_R = \left\{ \frac{4}{\pi^2} \sum_{n=odd} \exp \left( -\frac{n\pi}{w} \frac{l}{2} - x_{DW} \right) \right\} > 0; \quad (6)
\]

Note that the \( \sum(x) \) function expresses the perturbation of electrical current in the domain wall proximity, while the term \( -\frac{1}{2} R_{xy} \) in the equation above represents the contribution to the longitudinal MR from the anomalous Hall effect due to magnetization in each domain.

We now develop a strategy to schematically represent the longitudinal resistance terms in Equation (5) in the presence of the domain wall in the ferromagnetic samples with PMA, via connecting the exact solutions to the effective electric fields between different points in the sample.

Starting from Ohm’s law, we get

\[
R_{\alpha\beta} = \frac{V_\alpha - V_\beta}{I} = \frac{(V_a - V_{C1}) + (V_{C1} - V_{C2}) + (V_{C2} - V_{DW}) + (V_{DW} - V_{C3}) + (V_{C3} - V_{C4}) + (V_{C4} - V_\beta)}{I}; \quad (7)
\]

where \( V_{C1}, V_{C2}, V_{C3}, \) and \( V_{C4} \) are the potentials of the points along the middle line of the width of the sample, as shown in Figure 1. \( V_{C2} \) and \( V_{C3} \) represent the potentials of the points in the proximity of the DW. \( V_{DW} \) is the potential of the DW on the upper side of the sample.

Note that

\[
R_S = \frac{(V_{C1} - V_{C2}) + (V_{C3} - V_{C4})}{I}; \quad (8)
\]

Anomalous Hall effect requires that

\[
-\frac{1}{2} R_{xy} = \frac{(V_a - V_{C1})}{I} = \frac{(V_{C4} - V_\beta)}{I}; \quad (9)
\]

By symmetry,

\[
C_L R_{xy} = \frac{V_{DW} - V_{C2}}{I}; \quad (10)
\]
Combining equations (5) – (11), we can establish the relationships between the resistance terms in the Equation (5) and the effective electric fields between different points in the sample. Since longitudinal MR is \( \Delta R_{xx} = \Delta R_{ab} = (R_{ab} - R_s)/R_s \), and the value of \( R_s \) is constant as long as the distance \( l \) between points \( \alpha \) and \( \beta \) is chosen, we’ll focus on the other 4 terms in equation (5) only. As shown in Fig. 1 (b), the arrows are used to represent the directions of effective electric fields between different points, while the magnitude of the fields is marked by the coefficients besides. The magnitude of the electric current \( I \) is omitted in the coefficients for simplicity.

Specifically, for the situation that the DW is located at the middle of the two electrodes \( \alpha \) and \( \beta \), the diagram is shown in Fig 1(b). By symmetry, we get

\[
(C_L)_a = (C_R)_a \tag{12}
\]

This is exactly what equation (4) shows when the domain wall is located at the middle, i.e., \( x_{DW} = 0 \).

2. Single Domain Wall

We have established the schematic representation of the electric fields and longitudinal MR in different parts of the sample considering the anomalous Hall effect and the effect from the eddy currents in the proximity of the DWs. Now we can draw similar diagrams when the DW is located at other places.

**Figure 2.** The schematic of a thin film with PMA in the presence of a single domain wall. (a) The DW is located between the two electrodes, closer to the left electrodes \( \alpha \). The magnetization in the left region is outward, while that in the right region inward. (b) The counterpart of (a). The DW is located between the two electrodes, closer to the right electrodes \( \beta \). The magnetization in the left region is inward, while that in the right region outward. (c) The DW is located outside the electrodes, but in the proximity of the left electrodes \( \alpha \). (d) The counterpart of (c). The domain wall is located outside the electrodes, but in the proximity of the right electrodes \( \beta \).

Fig 2 (a) shows the schematics when the DW is located between the two electrodes, closer to, but not in the proximity of the left electrode \( \alpha \). The DW configuration in Fig. 2 (b) is the counterpart of that in Fig. 2 (a): the distance between the DW and the electrode \( \beta \) is the same as that between the DW and the electrode \( \alpha \) in Fig. 2(a), while the magnetization in the left and right region in Fig. (b) is opposite.
to that in Fig. 2(a), respectively. The configuration in Fig. 1(b) is obtained in an external \( \mathbf{H} \) field sweep opposite to that in Fig. (a). Note that the DW configuration can always find its counterpart when the external magnetic field sweep is reversed. The value of anomalous Hall effect coefficient \( R_{xy} \) is omitted in the coefficients for simplicity.

By symmetry, the coefficients are related to each other as follows,

\[
(C_L)_a = (C_R)_b \tag{13}
\]

\[
(C_R)_b = (C_L)_a \tag{14}
\]

The coefficients \( C_L \) and \( C_R \) express the perturbation of electrical currents in the domain wall proximity.

Since we have already connect the exact solutions to the effective electric fields between different points in the sample, now we can just go along the electric lines from \( V_\alpha \) to \( V_{DW} \) to \( V_\beta \) to calculate the resistance of the system. Then for the situation in Fig. 2(a) and (b)

\[
(R_{a\beta} - R_S)_a = -\frac{1}{2} R_{xy} + (C_L)_a R_{xy} + (C_R)_a R_{xy} - \frac{1}{2} R_{xy} = R_{xy} \left( -1 + (C_L)_a + (C_R)_a \right) \tag{15}
\]

\[
(R_{a\beta} - R_S)_b = \frac{1}{2} R_{xy} - (C_R)_b R_{xy} - (C_L)_b R_{xy} + \frac{1}{2} R_{xy} = R_{xy} \left( 1 - (C_L)_a - (C_R)_a \right) \tag{16}
\]

Thus,

\[
(R_{a\beta} - R_S)_a = -(R_{a\beta} - R_S)_b \tag{17}
\]

As a result, the longitudinal MR in this situation is antisymmetric with respect to \( H = 0 \).

When the DW is located outside the electrodes, but in the proximity of one of the electrodes, the diagrams are shown in Fig. 2 (c) and (d). Similarly, we have

\[
(C_R)_c = (C_L)_d \tag{18}
\]

\[
(R_{a\beta} - R_S)_c = -(C_R)_c R_{xy} - \frac{1}{2} R_{xy} \tag{19}
\]

\[
(R_{a\beta} - R_S)_d = \frac{1}{2} R_{xy} + (C_L)_d R_{xy} \tag{20}
\]

Therefore,

\[
(R_{a\beta} - R_S)_c = -(R_{a\beta} - R_S)_d \tag{21}
\]

As a result, the longitudinal MR in the reverse sweeps in this situation is antisymmetric with respect to \( H = 0 \).

One can do the similar symmetry analysis when the DW is located inside the electrodes but in the proximity of one of the electrodes. The same symmetry holds. When the DW is located outside the electrodes, but far away from both of them, such that the circulating current around the DW is negligible, we get \((R_{a\beta} - R_S) = 0\). This is a trivial case same as a single domain sample without any DW.

In summary, in the presence of a single domain wall in the ferromagnetic sample with PMA, the analysis of symmetry in the diagram shows that when the \( \mathbf{H} \) sweep reverse its direction, the magnetization in each domain reverses too, and the electric potential difference between difference points changes sign. This results in an antisymmetric longitudinal MR with respect to \( H=0 \) when the \( \mathbf{H} \) sweeps reverse its direction, which has been observed experimentally in both metallic ferromagnets and ferromagnetic semiconductors. The similar analysis can also clearly explain the puzzling
antisymmetry in $\Delta R_{xx}$ when the external field was rotated $360^0$ around one of the crystalline direction in the sample plane with the same magnitude [5], but the explanation of multiple peaks in some $\Delta R_{xx}$ will have to addressed section 4. This analysis gives a detailed validation of the schematic analysis of the symmetry in MR in the presence of DW in the ferromagnetic samples with PMA.

3. Two Domain Walls

Using the similar schematic analysis shown in the Fig.3, one can do the symmetry analysis in the situation that there are two domain walls in the ferromagnetic thin films with PMA.

When the DWs are located between $\alpha$ and $\beta$, the diagrams are shown in Fig 3 (a) and (b) with the assumption that the distance $d$ between the 2DWs is much shorter than the distance $l$ between the two electrodes. Note that the DW location can always find its symmetric counterpart when the external magnetic field is swept in an opposite direc tion. Similarly, we use the coefficients $C_{Li}$ and $C_{Ri}$ to express the perturbation of electrical currents in the $i$th domain wall proximity. To get the expressions of $C_{Li}$ and $C_{Ri}$, one needs to solve the problem exactly in the presence of two DWs, which is very difficult when there is more than one DW in the system. However, no matter what the expressions are, symmetry analysis shows the coefficients are related to each other as follows,

$$\left( C_{Li} \right)_a = \left( C_{Ri} \right)_b$$  \hspace{1cm} (22)

$$\left( C_{Ri} \right)_a = \left( C_{Li} \right)_b$$  \hspace{1cm} (23)

where $i = 1, 2$. The coefficients $C_{Li}$ and $C_{Ri}$ expresses the perturbation of electrical currents in the $i$th domain wall proximity.

Now we can just go along the electric line from $V_\alpha$ to $V_{DW1}$ to $V_{DW2}$ to $V_\beta$ to calculate the resistance of the system. Then for the situation in Fig. 2(a) and (b)
Thus,

$$\left( R_{\alpha\beta} - R_S \right)_a = R_{xy} \left( \frac{1}{2} + (C_{L_1})_a + (C_{R_1})_a - (C_{L_2})_a - (C_{R_2})_a + \frac{1}{2} \right) \quad (24)$$

$$\left( R_{\alpha\beta} - R_S \right)_b = R_{xy} \left( (C_{L_1})_a + (C_{R_1})_a - (C_{L_2})_a - (C_{R_2})_a \right) \quad (25)$$

Thus,

$$\left( R_{\alpha\beta} - R_S \right)_a = \left( R_{\alpha\beta} - R_S \right)_b \quad (26)$$

As a result, the longitudinal MR in this situation is symmetric with respect to $H=0$.

Specifically, when the two domain walls are located equally away from the middle point of the two electrodes, the perturbation effects of the two domain walls cancel each other based on symmetry analysis, and we get

$$R_{\alpha\beta} = R_S \quad (27)$$

When one of the DWs is located outside the electrodes, and is in the proximity one of the electrodes, the diagrams are shown in Fig. 3 (c) and (d). Similarly, we have

$$(C_{R_1})_c = (C_{L_1})_d \quad (28)$$

$$(C_{L_2})_c = (C_{R_2})_d \quad (29)$$

$$(C_{R_2})_c = (C_{L_2})_d \quad (30)$$

Then

$$\left( R_{\alpha\beta} - R_S \right)_c = R_{xy} \left( (C_{R_1})_c - (C_{L_2})_c - (C_{R_2})_c + \frac{1}{2} \right) \quad (31)$$

$$= R_{xy} \left( \frac{1}{2} + (C_{R_1})_c - (C_{L_2})_c - (C_{R_2})_c \right)$$

$$\left( R_{\alpha\beta} - R_S \right)_d = R_{xy} \left( \frac{1}{2} - (C_{L_2})_d - (C_{R_2})_d + (C_{L_1})_d \right) \quad (32)$$

$$= R_{xy} \left( \frac{1}{2} + (C_{R_1})_c - (C_{L_2})_c - (C_{R_2})_c \right)$$

Therefore,

$$\left( R_{\alpha\beta} - R_S \right)_c = \left( R_{\alpha\beta} - R_S \right)_d \quad (33)$$

As a result, the longitudinal MR in this situation is symmetric with respect to $H=0$.

One can do similar analysis for the other cases, such as or both DWs are located inside the electrodes but one of them is in the proximity of one of the electrodes, one DW is located outside the electrodes but not in the proximity of either electrodes, both DWs both are located outside the electrodes and so on, same symmetry for the MR still holds.

In summary, in the presence of two domain walls in the ferromagnetic sample with PMA, the analysis of symmetry based on the diagram shows the longitudinal MR is an even function of $H$ and $M$ during the opposite $H$ sweeps. Specifically, in the special configuration in that the two domain walls are located between the two electrodes $\alpha$ and $\beta$ and the $x=0$ plane is at the middle of the two electrodes, the longitudinal resistance $R_{\alpha\beta} = R_S$. More generally, if there are $2n$ (n is integer, n>0) domain walls and all the domain walls are separated with the same distance, the longitudinal MR is still symmetric.
with respect to $H=0$ during the opposite $H$ sweeps. The symmetry of MR in the presence of 2 DWs and MR without DWs share the same symmetry, but MR in the presence of 2 DWs should show extra peaks in the feature due to the resistance change when each DW passes the electrodes.

4. **Three Domain Walls**

In the situation that there are three domain walls in the ferromagnetic thin films with PMA, one can do the symmetry analysis using the similar schematic analysis shown in the Fig.4.

To grab the physical nature and simplify the discussion, let’s start with the simplest case that the three domain walls are equally distributed. We also assume that the distance $d$ between two nearest 2DWs is much shorter than the distance $l$ between the two electrodes. When the DWs are located between $\alpha$ and $\beta$, the diagrams are shown in Fig 3 (a) and (b). Note that the DW location can always find its symmetric counterpart when the external magnetic field is swept. Similarly, we use the coefficients $C_{Li}$ and $C_{Ri}$ to express the perturbation of electrical currents in the $i$th domain wall proximity. To get the expressions of $C_{Li}$ and $C_{Ri}$, one needs to solve the problem quantitatively in the presence of three DWs. By symmetry, the coefficients are related to each other as follows,

$$(C_{Li})_a = (C_{Ri})_b$$

$$(C_{Ri})_a = (C_{Li})_b$$

where $i = 1, 2, 3$.

**Figure 4.** The schematic of a thin film with PMA in the presence of three DWs. (a) all the DWs are located between the two electrodes, closer to the left electrodes $\alpha$. The magnetization in the left region is outward, while that in the right region inward. (b) The counterpart of the case in (a). all the DWs are located between the two electrodes, closer to the right electrodes $\beta$. The magnetization in the left region is inward, while that in the right region outward. (c) One DW is located outside the electrodes, but in the proximity of the left electrodes $\alpha$. (d) The counterpart of the case in (c). One domain wall is located outside the electrodes, but in the proximity of the right electrodes $\beta$.

Then for the situation in Fig. 3(a) and (b)
\[
(R_{a\beta} - R_S)_a = R_{xy} \left( -\frac{1}{2} + (C_{L_1})_a + (C_{R_1})_a - (C_{L_2})_a - (C_{R_2})_a + (C_{L_3})_a + (C_{R_3})_a - \frac{1}{2} \right) \\
= R_{xy} \left( -1 + (C_{L_1})_a + (C_{R_1})_a - (C_{L_2})_a - (C_{R_2})_a + + (C_{L_3})_a + (C_{R_3})_a \right) \\
(R_{a\beta} - R_S)_b = R_{xy} \left( \frac{1}{2} - (C_{L_1})_b - (C_{R_1})_b - (C_{L_2})_b - (C_{R_2})_b + (C_{L_3})_b - (C_{R_3})_b + \frac{1}{2} \right) \\
= R_{xy} \left( 1 - (C_{L_1})_a - (C_{R_1})_a + (C_{L_2})_a + (C_{R_2})_a - (C_{L_3})_b - (C_{R_3})_b \right)
\]

Thus,
\[
(R_{a\beta} - R_S)_a = -(R_{a\beta} - R_S)_b
\]

As a result, the longitudinal MR in this situation is antisymmetric with respect to \(H=0\).

When one of the DWs is located outside the electrodes, and is in the proximity of one of the electrodes, the diagrams are shown in Fig. 3 (c) and (d). Similarly, we have
\[
(C_{R_i})_c = (C_{L_i})_d
\]
\[
(C_{L_i})_c = (C_{R_i})_d
\]
\[
(C_{R_i})_c = (C_{L_i})_d
\]
where \(i = 2, 3\). Then by going along the electric line from \(V_a\) to \(V_b\), we get
\[
(R_{a\beta} - R_S)_c = R_{xy} \left( (C_{R_1})_c - (C_{L_2})_c - (C_{R_2})_c + (C_{L_3})_c + (C_{R_3})_c - \frac{1}{2} \right)
\]
\[
(R_{a\beta} - R_S)_d = R_{xy} \left( \frac{1}{2} - (C_{L_1})_d - (C_{R_1})_d - (C_{L_2})_d + (C_{R_2})_d - (C_{L_3})_d \right)
\]
\[
= -R_{xy} \left( (C_{R_1})_c - (C_{L_2})_c - (C_{R_2})_c + (C_{L_3})_c + (C_{R_3})_c - \frac{1}{2} \right)
\]
Therefore,
\[
(R_{a\beta} - R_S)_c = -(R_{a\beta} - R_S)_d
\]
As a result, the longitudinal MR in this situation is symmetric with respect to \(H=0\).

One can do similar analysis for the other cases, such as all the DWs are located inside the electrodes but one of the DWs is in the proximity of one of the electrodes, two of the DWs located outside the electrodes, three DWs located outside the electrodes and so on, same symmetry for the MR still holds.

In summary, when there are three domain walls in the ferromagnetic thin films with perpendicular anisotropy, detailed symmetry analysis based on the diagram shows that that the longitudinal MR is antisymmetric with respect to \(H=0\) during the opposite \(H\) sweeps. To get the exact expressions of longitudinal MR one needs the exact solution of the problem in the three-domain-wall system. More generally, if there are \(2n+1\) (\(n\) is integer, \(n > 0\)) domain walls and all the domain walls are separated evenly, the longitudinal MR is still antisymmetric with respect to \(H=0\) during the opposite \(H\) sweeps.

Although the symmetry of MR in the presence of single DW and MR in the presence of odd numbers of DWs share the same symmetry, there is one major difference: the MR in the presence of multiple DWs should have multiple peaks in the MR feature due to the abrupt changes of MR when each DW passes the electrodes. The results here could be used to explain the multiple peaks in the antisymmetry of \(\Delta R_{xx}\) spikes when the external field was rotated \(360^\circ\) around one of the crystalline direction in the sample plane with the same magnitude [5].
5. Conclusion
The effect of multiple domain walls on the longitudinal MR of the ferromagnetic samples with PMA has been studied. By connecting the electric fields in the sample to the anomalous Hall effect and the effect from the eddy currents in the proximity of the DWs, the schematic representation of the electric fields and longitudinal MR in different parts of the sample has been established. Based on the symmetry analysis of the schematics, the symmetry of the ferromagnetic thin films with PMA with multiple domain walls are discussed. The results could be useful to fundamental studies of MR in ferromagnetic samples, and be potentially used in the magnetic devices that incorporate domain wall generation, manipulation and detection.

Acknowledgement. This work was financially supported by National Natural Science Foundation of China (Grant 11004141 and Grant 11004142).

References
[1] S. A. Wolf, D. D.Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnar, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Science 294 , 1488 (2001)
[2] D. A. Allwood et al., Science 296 , 2003 (2002)
[3] X. M. Cheng, S. Urazhdin, O. Tchernyshyov, C. L. Chien, V. I. Nikitenko, A. J. Shapiro, and R. D. Shull, Phys. Rev. Lett. 94 , 017203 (2005)
[4] H. X. Tang, S. Masmanidis, R. K. Kawakami, D. D. Awschalom, and M. L. Roukes, Nature 431 , 52 (2004).
[5] G. Xiang, A. W. Holleitner, B. L. Sheu, F. Mendoza, O. Maksimov, M. B. Stone, D. D. Awschalom, P. Schiffer and N. Samarth, Phys Rev. B Rapid Comm. 71,241307 (R)(2005)
[6] G. Xiang and N. Samarth, Phys. Rev. B 76 , 054440 (2007)
[7] R. I. Potter, Phys. Rev. B 10 , 4626, (1974).
[8] M. N. Baibich et al., Phys. Rev. Lett. 61 , 2472 (1988).
[9] G. Xiang, B. Sheu, M. Zhu, P. Schiffer and N. Samarth, Phys. Rev. B 76 , 035324 (2007)
[10] W. Desrat, S. Kamara1, F. Terki, S. Charar, J. Sadowski and D. K. Maude, Semicond. Sci. Technol. 24 , 065011 (2009)