Mutual consideration of $b \to s\gamma$ and $\mu \to e\gamma$ in supersymmetric SO(10) grand unification

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Abstract

We compare the branching ratios for $b \to s\gamma$ and $\mu \to e\gamma$ in terms of constraining the parameter space in supersymmetric SO(10) grand unification models where supersymmetry is broken softly near the Planck scale by generationally symmetric operators. We observe two general cases. One with small $\tan\beta = 2$ and the other one with large $\tan\beta$ having third generation Yukawa coupling unification at the GUT scale.

We show that for small $\tan\beta$ the branching ratio constraints allow only a smaller region of parameter space for $\mu > 0$ compared to $\mu < 0$ for gluino mass $\lesssim 500$ GeV. With large $\tan\beta$, we find acceptable regions of parameter space with $|\mu| \lesssim 1$ TeV only for $\mu < 0$. The dominant constraint on large $\tan\beta$ with $\mu > 0$ parameter space is found to be given by the $b \to s\gamma$ branching ratio, while for large $\tan\beta$ with $\mu < 0$ it is found to be given by the $\mu \to e\gamma$ branching ratio. In many of these acceptable regions, we find that the $\mu \to e\gamma$ branching ratio is predicted to be within one order of magnitude
of its current experimental bound. We also show that the usually neglected gluino mediated diagrams in $b \rightarrow s\gamma$ can not be ignored in some regions of parameter space, especially for large tan $\beta$ scenarios when the gluino mass is near its lower experimental bound.
Recently it has been demonstrated [1–4] that supersymmetric grand unified theories (GUTs) with soft supersymmetry (SUSY) breaking terms which are generated near the Planck scale can predict lepton flavor violating processes with highly significant rates. Due to the present experimental limits, this is shown to be especially true for $\mu \rightarrow e\gamma$. The $\mu \rightarrow e\gamma$ decay has been studied for both SO(10) and SU(5) grand unification. It is found that SO(10) grand unification [5] predicts the greater rates of the two as a result of its unifying all fields of a particular generation into one multiplet [2]. The decay has been studied for both small [1,2] and large $\tan\beta$ [4], and found in both cases to sometimes rule out parameter space and sometimes predict rates only one or two orders of magnitude beneath the current experimental limit. In this paper, we examine regions of parameter space for which the $\mu \rightarrow e\gamma$ branching ratio has interesting values by which it could be a signal for supersymmetric grand unification, and test to see if those regions are reduced by consideration of the quark flavor violating process $b \rightarrow s\gamma$. It has also recently been shown [6] in the case of small $\tan\beta$ SU(5) grand unification that with the present accuracy in the determination of the $b \rightarrow s\gamma$ branching ratio, one can not neglect the effects of running the soft SUSY breaking parameters above the GUT breaking scale and should include the gluino mediated decay amplitude when the gluino mass is not particuarly large. This will be found to be true with SO(10) grand unification as well, and especially true for the large $\tan\beta$ case where sometimes the gluino mediated amplitude can be even larger than the standard model (SM) amplitude. We will also find that applying the constraints from both decays simultaneously, as must be done, limits available parameter space much more severely than one of the two constraints alone would. We will be primarilly interested in regions for which the magnitude of $\mu$, the coefficient of the Higgs superpotential interaction $\mu H_1 H_2$, is $\lesssim$1 TeV, so as to avoid models requiring a fine tuning of their parameters [7].

The prediction of significant rates for lepton flavor decay processes and the increased importance of the gluino mediated $b \rightarrow s\gamma$ decay amplitude are the result of the fact that in GUTs the top is unified with other third generation fields which then also feel the radiative effects of the relatively large top Yukawa coupling. If a universal soft supersymmetry break-
ing condition exists at a scale $M_X > M_G$, then the radiative corrections in the masses of these third generation fields' soft SUSY breaking scalar masses are manifest as $\ln M_X/M_G$ rather than suppressed as powers of $1/M_G$, where $M_G$ is the GUT breaking scale. Renormalization causes the soft SUSY breaking masses for the fields unified with the top fields to become lighter than the other two generations of those fields, and leads to a suppression of the GIM mechanism in processes mediated by these scalar fields. The degree to which this occurs depends, of course, on the size of the top Yukawa coupling and $M_X$ i.e. the larger values these have the greater the suppression of the GIM mechanism is. To understand the subtle points of these effects and how they lead to a substantial enhancement of the $\mu \to e\gamma$ branching ratio in the case of SO(10) over the case of SU(5) grand unification, one must examine the flavor changing amplitudes in the interaction basis as discussed in detail in Ref. [2]. We will take $M_X = 2.4 \cdot 10^{18}$ GeV throughout this paper. Above the GUT scale we will use the one loop renormalization group equations (RGEs) as for example appear in Ref. [2], although we use the convention gaugino mass $M \to -M$ in the RGEs of that reference for the tri-linear scalar soft breaking SUSY terms $A_i$ so as to be consistent with the convention we will use for our s-particle mass matrices [9]. We will take the SO(10) gauge coupling beta function to be $b_G = -3$ as an ad hoc choice since our calculation is not very sensitive to its value and we do not know the complete field content of the GUT model. We will also use $\alpha_s(M_Z) = 0.121$ and $M_G = 2 \cdot 10^{16}$ GeV with the $M_G$ scale coupling $\alpha_G = 1/23.9$. We will calculate the parameter $\mu$ at the tree level.

Below the GUT scale we will use the one loop RGEs in matrix form in the $3 \times 3$ generation space for the Yukawa couplings and soft SUSY breaking parameters as found in Ref. [3,10] rather than just running the eigenvalues of these matrices as is often done. Although doing this does not provide any new information when $\tan \beta = 2$, when $\tan \beta$ is large it allows one to know the relative rotation of squarks to quarks and sleptons to leptons. When $\tan \beta$ is small, one can diagonalize both the $3 \times 3$ up Yukawa matrix $\lambda_U$ and all of the soft SUSY breaking mass matrices at the scale $M_X$ of the universal boundary condition, and they will then always remain diagonal. In this case, for example, the mixing between a down squark
soft breaking mass matrix and the down quark mass basis is determined by the KM matrix, which diagonalizes $\lambda_D$ when in the basis where $\lambda_U$ is diagonal. On the contrary when $\tan \beta$ is large, both the top and bottom Yukawa couplings are large and have important effects in the RGEs. Hence, if one chooses the soft breaking mass matrices to be diagonal at the scale $M_X$ they will no longer be diagonal at the weak scale since both $\lambda_U$ and $\lambda_D$ can not be diagonalized simultaneously.

The SM expression for the $b \rightarrow s\gamma$ amplitude has been derived in Refs. [11,12], and the expressions for the additional MSSM amplitudes have been derived in Refs. [10,13,14]. We will use the expressions given in Ref. [10] because those expressions use the squark mass eigenstate basis derived from the full $6 \times 6$ mass matrices, as will be discussed below, and automatically incorporates mixing between “right-handed” down squarks and right-handed down quarks as is inevitable with either large $\tan \beta$ or SO(10) grand unification. We will include the W-boson, charged Higgs, chargino, and gluino mediated amplitudes, however we will neglect the neutralino mediated amplitude since we find this to be inconsequential in all viable regions of parameter space. The calculation is performed in the basis where $\lambda_D$ is diagonal at the weak scale. In terms of the mass eigenstates $q$ and current eigenstates $q^0$ for quarks, we use $d_{L,R} = d^0_{L,R}$ and $u_{L,R} = V u^0_{L,R} V^T_R$, where $V$ is the KM matrix and $V_R$ is the analogous mixing matrix for the relative right-handed rotation between $d$ and $u$. In the SM $V_R$ is not of any physical significance, however here it will effect the chargino-quark-squark vertex.

Neglecting the amplitude for $b_L \rightarrow s_R \gamma$, the leading-order QCD corrected branching ratio $B$ for $b \rightarrow s \gamma$ is given by

$$B (b \rightarrow s \gamma) = \frac{\Gamma (b \rightarrow s \gamma)}{\Gamma (b \rightarrow ce\bar{\nu})} B (b \rightarrow ce\bar{\nu}) \right),$$

where $B (b \rightarrow ce\bar{\nu}) = 0.107$ is the experimentally determined value, and $\Gamma (b \rightarrow ce\bar{\nu}) = G_F m_b^5/192\pi^3 |V_{cb}|^2 [g(m_c/m_b) g(m_c/m_b)]$ with $g(m_c/m_b)$ being the phase space factor and $m_c/m_b = 0.316$. The inclusive width for the $b \rightarrow s \gamma$ is given by

$$\Gamma (b \rightarrow s \gamma) = \frac{m_b^5}{16\pi} |c_\gamma (m_b)|^2$$

5
The QCD corrected amplitude $c_7(m_b)$ is given as
\begin{equation}
    c_7(m_b) = \eta^{16/23} \left[ c_7(M_W) - \frac{8}{3} c_8(M_W) \left( 1 - \eta^{-2/23} \right) \right] + \sum_{i=1}^{8} a_i \eta^{b_i},
\end{equation}
with $a_i$ and $b_i$ being given in ref. \cite{15}, $\eta = \alpha_s(M_W)/\alpha_s(m_b)$, for which we will use $\eta = 0.548$.

The present experimentally accepted range for this branching ratio is $(1 - 4.2) \cdot 10^{-4}$ \cite{16} at the 95% C.L.. The terms $c_7(M_W)$ and $c_8(M_W)$ are respectively $A_\gamma$, the amplitude for $b_R \to s_L \gamma$ evaluated at the scale $M_W$ and divided by the b-mass $m_b$ and $A_g$, the amplitude for $b_R \to s_L g$ also given in Ref. \cite{16} divided by the factor $m_b \sqrt{\alpha_s/\alpha}$. The effective interactions for $b \to s \gamma$ and $b \to s g$ are given by
\begin{equation}
    L_{\text{eff}} = \frac{m_b}{2} (A_\gamma \bar{s} \sigma^{\mu\nu} P_R b F_{\mu\nu} + A_g \bar{s} \sigma^{\mu\nu} P_R b G_{\mu\nu}) + \text{h.c.}. \tag{4}
\end{equation}

In calculating $c_7(M_W)$ and $c_8(M_W)$, we will use the conventional approximation of taking the complete MSSM to be the correct effective field theory all the way from the scale $M_G$ down to $M_W$, and hence ignore threshold corrections.

We acknowledge that since we are working with SO(10) grand unification with $M_X > M_G$, that there will necessarily be a gluino mediated contribution to $b_L \to s_R \gamma$, however we find this contribution to not be of much significance in any of the scenarios which we consider. This is despite the fact that the gluino mediated contribution to $b_R \to s_L \gamma$ is often important.

Three major differences exist between these two contributions to the branching ratio. First of all, the the gluino mediated $b_L \to s_R \gamma$ contribution does not have an interference term with an appreciable SM amplitude or any other appreciable MSSM amplitude. Secondly, even though the $\tilde{g} - \tilde{b}_R - s_R$ and the $\tilde{g} - \tilde{b}_L - s_L$ vertexes have the same mixing angles at the GUT scale due to the symmetric nature of the 10-dimensional Higgs, the “right-handed” mixing angle is smaller than the “left-handed” mixing angle at the weak scale. To understand this one can observe the case of small $\tan \beta$, where the $\tilde{g} - \tilde{b}_R - s_R$ vertex’s mixing angle is given by $|V_{ts}(M_G)| \approx 0.03$ since right-handed quark mixing angles $V_{ts}^R$ essentially do not run in the MSSM:
\begin{equation}
    16\pi^2 \frac{d \ln |V_{ts}^R|}{dt} = -2\lambda_t^2 \frac{\lambda_s}{\lambda_b} - 2\lambda_b^2 \frac{\lambda_c}{\lambda_t}, \tag{5}
\end{equation}
as can be surmised from Ref. [17], while the $\tilde{g} - \tilde{b}_L - s_L$ vertex’s mixing angle is given by $|V_{ts}(M_W)| > |V_{ts}(M_G)|$. In fact for small $\tan \beta$ if one uses $1 - V_{tb}^* V_{tb} \approx 0$, the ratio of the two gluino mediated amplitudes may be estimated as follows:

$$\frac{|A_{L\rightarrow R}|}{|A_{R\rightarrow L}|} \approx \left| \frac{V_{ts}^R(M_W)}{V_{ts}(M_W)} \cdot \frac{G_2(\tilde{b}_L, \tilde{b}_R) - G_2(\tilde{b}_L, \tilde{d}_R)}{G_2(\tilde{b}_R, \tilde{b}_L) - G_2(\tilde{d}_R, \tilde{d}_L)} \right|,$$

(6)

where the functions $G_2(x_1, x_2, \ldots)$ are given in Ref. [2] and used for the case of gluino mediated decay in Ref. [3]. Thirdly, even though $\tilde{b}_R$ and $\tilde{b}_L$ have the same mass terms at the GUT scale, this is not true at the weak scale. With low $\tan \beta$, $\tilde{b}_L$’s mass is driven lower than that of $\tilde{b}_R$. Although the opposite is true for large $\tan \beta$, there the mixing angle for $\tilde{g} - \tilde{b}_R - s_R$ is further suppressed, due in part to the nature of the boundary condition at $M_G$. In none of the examples that we look at do we find $|A_{L\rightarrow R}/A_{R\rightarrow L}|$ to be greater than about 0.65, and further in the regions where $A_{L\rightarrow R}/A_{R\rightarrow L}$ is greater than 0.5 the parameter space is ruled out by either the $\mu \rightarrow e \gamma$ or the $b \rightarrow s \gamma$ branching ratio being too large. On the other hand, we find the contribution to the branching ratio of $\mu \rightarrow e \gamma$ from the widths of $\mu_L \rightarrow e_R \gamma$ and $\mu_R \rightarrow e_L \gamma$ to be virtually the same [2].

Using notation analogous to that used for $b \rightarrow s \gamma$ in Ref. [10], we now give the expressions we use to calculate the $\mu \rightarrow e \gamma$ branching ratio. $\mu \rightarrow e \gamma$ has the following effective Lagrangian:

$$L_{\text{eff}} = \frac{m_\mu}{2} (A_{1\gamma} \overline{\sigma}^{\mu\nu} P_R \mu F_{\mu\nu} + A_{2\gamma} \overline{\sigma}^{\mu\nu} P_L \mu F_{\mu\nu}) + \text{h.c.},$$

(7)

where we have included helicities. $A_{1\gamma}$ and $A_{2\gamma}$ are given by

$$A_{1\gamma} = -\frac{\alpha \sqrt{\alpha}}{2 \cos^2 \theta_W \sqrt{\pi}} \sum_{j=1}^{4} \sum_{k=1}^{6} \frac{1}{M_{l_k}^2} \times$$

$$\left\{ \left( \sqrt{2} G_{0lL}^{j\mu} \right) \left( \sqrt{2} G_{0lL}^{jke} \right) F_2 \left( x_{\chi_0^0 l_k} \right) - \left( \sqrt{2} G_{0lR}^{j\mu} - H_{0lL}^{j\mu} \right) \left( \sqrt{2} G_{0lL}^{jke} \right) \frac{m_{\chi_0^0}}{m_\mu} F_4 \left( x_{\chi_0^0 l_k} \right) \right\},$$

$$A_{2\gamma} = -\frac{\alpha \sqrt{\alpha}}{2 \cos^2 \theta_W \sqrt{\pi}} \sum_{j=1}^{4} \sum_{k=1}^{6} \frac{1}{M_{l_k}^2} \times$$

$$\left\{ \left( \sqrt{2} G_{0lR}^{j\mu} \right) \left( \sqrt{2} G_{0lR}^{jke} \right) F_2 \left( x_{\chi_0^0 l_k} \right) - \left( \sqrt{2} G_{0lL}^{j\mu} - H_{0lR}^{j\mu} \right) \left( \sqrt{2} G_{0lR}^{jke} \right) \frac{m_{\chi_0^0}}{m_\mu} F_4 \left( x_{\chi_0^0 l_k} \right) \right\},$$

(8)
\[
\left( \sqrt{2} G_{0L}^{jk\mu} + H_{0R}^{jk\mu} \right) \left( \sqrt{2} G^{*jke}_{0R} \right) \frac{m_{\tilde{\chi}_0}}{m_\mu} F_4 \left( x_{\tilde{\chi}_0} \right) \right) \]

where the convention \( x_{ab} = m_a^2/m_b^2 \) has been adopted and \( \tilde{M}_k \) are the slepton mass eigenstates. The functions \( F_2(x) \) and \( F_4(x) \) are given in Ref. [10] and

\[
G_{0L}^{jki} = -1/2 [Z_{j1} + \cot \theta_W Z_{j2}] \Gamma_{iL}^{ki} \\
G_{0R}^{jki} = -Z_{j1} \Gamma_{iR}^{ki} \\
H_{0L}^{jki} = Z_{j3} \left( \Gamma_{iL} \tilde{Y}_i \right)^{ki} \\
H_{0R}^{jki} = Z_{j3} \left( \Gamma_{iR} \tilde{Y}_i \right)^{ki},
\]

where \( \tilde{Y}_i \equiv \text{diag}(\lambda_e, \lambda_\mu, \lambda_\tau)/(g') \), \( Z \) is the 4 × 4 neutralino mixing matrix on the \((\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)\) basis, the fact the 4 × 4 neutralino mass matrix can have negative eigenvalues must be taken into account [19], and \( \tilde{t}_{L,R} = \Gamma_{iL,R}^{\dagger} \tilde{t}_i \). As explained in Ref. [10], the terms proportional to the function \( F_2 \) are small compared to the terms proportional to \( F_4 \) in SO(10) grand unification. For small \( \tan \beta \), the terms proportional to \( F_2 \) provide about a 5-percent correction to the branching ratio. As one would expect, when \( \tan \beta \) is large we find very little difference between the branching ratios predicted for the two possible signs of \( \mu \). The Higgsino-gaugino mediated terms are also found to provide a small correction, about 5-percent, when \( \tan \beta \) is large. The width is then given by:

\[
\Gamma (\mu \rightarrow e\gamma) = \frac{m_\mu^5}{16\pi^2} \left( |A_{1\gamma}|^2 + |A_{2\gamma}|^2 \right). \tag{10}
\]

The experimental upper limit on the \( \mu \rightarrow e\gamma \) branching ratio is \( 4.9 \cdot 10^{-11} \) at the 90 % C.L. [18].

The 6 × 6 slepton or squark mass matrix can be written in the 3 × 3 forms having the submatrices \( M_{LL}, M_{LR} \) and \( M_{RR}^2 \) as follows:

\[
\begin{pmatrix} M_{IL} M_{IL}^\dagger & M_{ILR}^2 & M_{ILR}^2 \\ M_{ILR}^2 & M_{iLR}^\dagger M_{iLR} \end{pmatrix}.
\tag{11}
\]
At a scale $M^2_X$ near the Planck scale, the $3 \times 3$ blocks acquire the form:

$$M_{IL}M_{IL}^\dagger = m_0^2 1, \quad M_{IR}M_{IR} = m_0^2 1, \quad M_{ILR}^2 = 0,$$

(12)

where we have taken the $M_X$ scale tri-linear soft SUSY breaking scalar coupling $A^0 = 0$. For simplicity, we will take $A^0 = 0$ in all of the cases that we consider. These $3 \times 3$ mass matrices must be run down to the weak scale where one must add the mass terms that arise from weak scale gauge symmetry breaking including the weak scale $D$ terms. The weak scale soft SUSY breaking $3 \times 3$ mass matrices may be expressed in terms of the universal soft breaking parameters. Hence, the weak scale $3 \times 3$ matrices may be written as:

$$
\begin{align*}
(M_{IL}M_{IL}^\dagger)_{ij} &= a_{ij}M_{10}^2 + b_{ij}m_0^2 + (M_iM_i^\dagger)_{ij} + \\
&+ M_Z^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right) \delta_{ij}, \\
(M_{ILR}^2)_{ij} &= (c_{ij}M_{10} + \mu \tan \beta) (M_{i})_{ij}, \\
(M_{IR}M_{IR}^\dagger)_{ij} &= d_{ij}M_{10}^2 + e_{ij}m_0^2 + (M_iM_i^\dagger)_{ij} + \\
&+ M_Z^2 \cos 2\beta \left( -1 \sin^2 \theta_W \right) \delta_{ij},
\end{align*}
$$

(13)

where $M_{10}$ is the gaugino mass at the $M_G$ scale and the dimensionless coefficients are determined by the numerical solutions of the RGEs [10]. We can write the up and the down squark sectors in the same $6 \times 6$ matrix form with the $3 \times 3$ blocks looking like those shown above.

Since we are working with the SO(10) unification group, the above equations for the low energy mass matrices may be modified by additional D-terms which are generated at the GUT breaking scale $M_G$. A D-term contribution may appear when the rank of the group is reduced due to the gauge symmetry breaking [20–22]. In the case of SO(10) breaking to the SM gauge group, the rank is reduced by one and the broken generators constitute a $U(1)$ subgroup. In this case, the D term contribution can be described by one additional parameter, which we will refer to as $m_D^2$. The scalar masses get modified [4] at the unification scale as follows:
\[ m_{H_1,H_2}^2 = m_{10}^2 + \begin{pmatrix} 2 \\ -2 \end{pmatrix} m_D^2, \]  

(14)

\[ m_{Q,U,E,D,L}^2 = m_{16}^2 + \begin{pmatrix} 1 \\ 1 \\ 1 \\ -3 \\ -3 \end{pmatrix} m_D^2. \]  

(15)

These additional terms lead to an additional term proportional to \( m_D^2 \) in the expansion of each 3 \( \times \) 3 squark or slepton mass matrices \( M_L M_L^\dagger \) or \( M_R^\dagger M_R \) at the low scale. One must remember that when \( X_Y \equiv \sum_i Y_i m_i^2 \neq 0 \) at the \( M_G \) scale one must add an additional term to the soft breaking scalar mass RGEs which scales in a simple fashion (see Ref. [4]). In SO(10) models, \( X_Y(M_G) = M_{H_2}^2(M_G) - M_{H_1}^2(M_G) \).

In large tan \( \beta \) scenarios due to the effect of the bottom Yukawa coupling now being large as well as the top Yukawa coupling, it is very difficult to make \( (m_{H_2}^2 + \mu^2) \) negative so as to break the electroweak symmetry and yet keep the pseudoscalar mass-squared \( m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \) positive, however the introduction of the additional GUT scale D-terms can solve that problem. The requirement to keep the psuedoscalar mass-squared positive gives a lower bound on the possible size of \( m_D^2 \) for a choice of all of the other parameters. An upper bound is imposed on \( m_D^2 \) through the requirement that all of the down squark and slepton mass eigenstates remain positive. For small tan \( \beta \) since only the top Yukawa coupling is large, we do not have the above mentioned problem. In this case, the lower bound on \( m_D^2 \) is 0. For simplicity and because the presence of a nonzero \( m_D^2 \) raises the value of \(|\mu|\), we will take \( m_D^2 = 0 \) for tan \( \beta = 2 \), but we will invoke the parameter in the full allowed range for large tan \( \beta \).

In the high tan \( \beta \) cases, we assume the complete unification of the third generation Yukawa couplings. This means that both MSSM Higgs doublets come from the same 10-dimensional representation Higgs field. This would seem to indicate that all of the Yukawa
coupling matrices are identical at the GUT scale, which can not produce a realistic fermion mass spectrum and quark mixing parameters at the low scale. However if one assumes that only the \((3, 3)\) entries of the Yukawa matrices are direct couplings to the low mass Higgs field and the other couplings arise from non-renormalizable operators involving super heavy fields, then it is possible. Maximally predictive Yukawa textures have been developed for this case which use only four operators as follows [23]:

\[
\lambda_i = \begin{pmatrix}
0 & \dot{z}_i C_i & 0 \\
z_i C_i & y_i E e^{i\phi} & \dot{x}_i B \\
0 & x_i B & A
\end{pmatrix},
\]

(16)

where \(x_i, y_i, z_i\) and \(\dot{x}_i, \dot{y}_i, \dot{z}_i\) are Clebsch factors. To determine the four magnitudes, the phase, and \(\tan \beta\), one should use the six best determined low energy parameters as discussed in Ref. [23]. From this reference, we choose to use for our example the ansatz which is its Model 6, which appears to give the most comfortable fit to the low energy data amongst its models. With that ansatz we will look at two scenarios corresponding to two different values of \(A \approx \lambda_t(M_G)\). In the first example, case (i), \(A = 1\) and \(\tan \beta = 57.15\) and this gives us the pole mass \(m_t = 182\) GeV and the running mass \(m_b = 4.43\) GeV. In the second example, case (ii), \(A = 1.18\) and \(\tan \beta = 58.86\) and this gives us the pole mass \(m_t = 184\) GeV and the running mass \(m_b = 4.35\) GeV. In running the single third generation Yukawa coupling \(A\) and soft SUSY breaking parameters between \(M_X\) and \(M_G\), we make the simplifying assumption that \(\lambda_t\) is the only large Yukawa coupling in the GUT scale model, and that hence the soft breaking matrices remain diagonal as one runs them down from \(M_X\) to \(M_G\).

We also assume for low \(\tan \beta\) that \(\lambda_t\) is the only large tri-linear coupling in the GUT scale superpotential, and at the scale \(M_G\) we take \(\lambda_U\) and the scalar matrices to be diagonal, and take \(\lambda_D = \lambda_E = V^* \lambda_D^{\text{diag}} V^\dagger\) with all parameters evaluated at \(M_G\). We also always take the tri-linear soft breaking parameter matrices \(A_{U,D,E}(M_G)\) to be diagonal, and take the tri-linear couplings to be given by the symmetric combination \({A_{U,D,E}, \lambda_{U,D,E}}/2\) at \(M_G\). (See Ref. [2]) In the low \(\tan \beta\) scenario as well as in both of the large \(\tan \beta\) examples, our KM matrix is typically described by the following four parameters: \(|V_{us}| = 0.22\), \(|V_{cb}| = 0.44\),
\[|V_{ub}/V_{cb}| = 0.7, \text{ and the Jarlskog CP violation parameter } 2.8 \cdot 10^{-5}. \] This provides the ratio, \[|V_{ts}^* V_{tb}/V_{cb}|^2 = 0.96, \] relevant to the \(b \to s\gamma\) branching ratio.

Now beginning with low \(\tan \beta = 2\), we will discuss the results. We use \(\lambda_t(M_G) = 1.25\), which gives the top quark the pole mass \(m_t = 176\text{ GeV}\). We forego the \(M_G\) scale constraint \(m_0 = m_{\tau}\) and give the b-quark a \(\overline{\text{MS}}\) running mass of 4.35 GeV. We display the \(\mu \to e\gamma\) rate as \(\log_{10}(B/4.9 \cdot 10^{-11})\). We plot both branching ratios as functions of the \(M_X\) scale parameter \(m_0\) for fixed values of the \(M_G\) scale gaugino mass \(M_{10}\), and also as a function of \(M_{10}\) for fixed values of \(m_0\), in the Figs 1 and 2. The significance of \(M_{10}\) for the weak scale gaugino masses is that the gaugino mass \(M_i = \alpha_i M_{10}/\alpha_G\). As for the other parameter, the universal soft SUSY breaking scalar mass \(m_0\) is close to the low scale “right-handed” s-electron mass, and differs from it only by the renormalization effects of GUT scale gaugino loops and bino loops below the GUT scale.

For negative values of \(\mu\) (Fig.1), \(\mu \to e\gamma\) (Figs 1b and 1e) rules out the smaller values of the gaugino masses although they are allowed by the \(b \to s\gamma\) branching ratio (Figs 1a, 1d). For any value of \(m_0\), \(M_{10}\) needs to be around 100 GeV for the allowed regions. Also large values of gaugino masses and \(m_0\) tend to increase the amount of fine-tuning needed in the model, and invite larger values of \(|\mu|\). We note that, roughly \(|\mu| \simeq 500\text{ GeV}\) when \(M_{10}\) is \(\simeq 240\text{ GeV}\). We also plot, \(r_{A_g} \equiv |A_g/A_{SM}|\), the absolute value of the ratio of the gluino mediated contribution to the \(b_R \to s_L\gamma\) amplitude to the SM amplitude, and find that in the regions allowed by both decays it could be as large as 10%. For example when \(m_0=700\text{ GeV}\) and \(M_{10}=130\text{ GeV}\), the ratio is about 0.09. One also observes that, in the \(\mu \to e\gamma\) plot there are two allowed regions of \(m_0\) for each value of \(M_{10}\). For example when \(M_{10}=225\text{ GeV}\), we find that values of \(m_0\) between 0-200 GeV as well as values greater than 600 GeV are allowed.

For positive values of \(\mu\), the lower values of the gaugino masses are disfavored by the \(b \to s\gamma\) branching ratio (Figs 2a, 2d). The branching ratio reduces to more acceptable values at higher values of gaugino and scalar masses. For values of \(|\mu| < 800\text{ GeV}\), one also finds that the gluino diagram is never found to contribute more than 13% than that of the SM.
amplitude. The $\mu \to e\gamma$ branching ratio plots (Figs 2b. and 2e) have two regions of allowed $m_0$ values for a particular value of $M_{10}$. For example when $M_{10}=150$ GeV, we find that values of $m_0$ between 0-160 GeV and also values greater than 450 GeV are allowed.

Now we look at the results for the two previously mentioned high tan $\beta$ cases: case (i) with $A = 1$, and case (ii) with $A = 1.18$. For neither case do we find parameter space with acceptable values of the $b \to s\gamma$ rate for positive $\mu \lesssim 1$ TeV. In fact, roughly $\mu$ needs to be at least 1300 GeV to find an acceptable rate. For this reason, we only show plots for the cases with negative values of $\mu$, where we find the predominant constraint to come from the $\mu \to e\gamma$ rate. When tan $\beta$ is large, the values obtained for the $\mu \to e\gamma$ branching ratio and $r \equiv |A_{\tilde{g}}/A_{SM}|$ as a function of $|\mu|$ when $\mu > 0$ are virtually identical to those obtained when $\mu < 0$. The plots we show are parametric plots with the parameter $m_2^D$ varying over its allowed range. As previously stated, we have chosen to use Model 6 of Ref. [23]. If we had chosen a different model of the type given in that reference, the amount of leptonic flavor violation would differ in a predictable way, as discussed in Ref. [4]. In particular, the $\mu \to e\gamma$ branching ratio is found to be proportional to $(\chi_L\chi_R/3)^2$, where $\chi_L \equiv \hat{x}_e/ (\hat{x}_d - \hat{x}_u)$ and $\chi_R \equiv x_e/(\hat{x}_d - \hat{x}_u)$. In Model 6, $(\chi_L\chi_R/3)^2 = 0.36$, however in all nine models the factor ranges from 0.1 to 10. Hence, in our plots of $\log_{10}(BR(\mu \to e\gamma)/4.9 \cdot 10^{-11})$ although the value of this function being 0 corresponds to the experimental limit in the model we use, for all nine models the experimental limit on our plots could correspond to values as low as $-0.56$ or as high as 1.44.

In Fig. 3a we plot the $b \to s\gamma$ branching ratio as a function of $\mu$ with $m_0=1000$ GeV for case (i). The curves are for different values of $M_{10}$. In Fig 3b, one notes that values of $M_{10}$ greater than 145 GeV are disfavored by $\mu \to e\gamma$ rate though they are allowed by the $b \to s\gamma$ rate. To find the interesting regions for the signal of $\mu \to e\gamma$, we need to look for the part of the curve which projects on the same $\mu$ space as that done by the $b \to s\gamma$ curve having the same value of $M_{10}$ and where the $\mu \to e\gamma$ branching ratio is within an order of magnitude beneath the experimental value. To illustrate the gluino contribution we also plot $r_{A_g}$ as a function of $\mu$ for different values of the gaugino masses in Fig. 3c. In the
regions allowed by the both the decay processes the gluino diagram amplitude is found to be same or sometimes larger than the SM amplitude. In fact in the interesting regions, the gluino diagram contribution can be as much four times greater than that of the SM diagram.

For case (ii) we need to raise $m_0$ to about 2200 GeV in order to find any appreciable regions allowed by the $\mu \rightarrow e\gamma$ branching ratio. For this example, in the interesting region allowed by both decays we have that $\mu$ lies between about -136 and -250 GeV as shown in figs 4a and 4b. The gluino diagram contribution (Fig. 4c) is as high as only 8 percent of the SM contribution in this region. The requirement of large gaugino mass for the existence of this parameter space is the cause for a relatively small gluino contribution.

We now summarize the constraints on the parameter space given by considering both decays simultaneously. For small tan $\beta$ with $\mu < 0$ one finds that, a large range of gaugino masses for any value of scalar mass is allowed by $b \rightarrow s\gamma$ branching ratio, but the $\mu \rightarrow e\gamma$ rate is found in general to be more restrictive on the lighter gaugino masses. In the case of $\mu > 0$ with small tan $\beta$, it is the $b \rightarrow s\gamma$ branching ratio which provides the more stringent constraint and in general forbids $M_{10}$ to be less than about 180 GeV. For large tan $\beta$ with $\mu < 0$, $b \rightarrow s\gamma$ allows a larger range of gaugino masses than $\mu \rightarrow e\gamma$ does, which tends to rule out larger values of $|\mu|$. On the other hand for large tan $\beta$ with $\mu > 0$, $b \rightarrow s\gamma$ rules out all the parameter space that has $\mu \lesssim 1$ TeV.

In this letter, we have given first complete calculation of $b \rightarrow s\gamma$ branching ratio in SO(10) grand unification with flavor uniform soft SUSY breaking terms introduced at the reduced Planck scale, although with this boundary condition the branching ratio for $b \rightarrow s\gamma$ has been calculated for SU(5) grand unification with low tan $\beta$ in the literature [4]. We have performed the calcualtion for both large tan $\beta$ with $\lambda_t = \lambda_b$ at the grand unification scale and for low tan $\beta = 2$. For the purpose of comparison with the constraints provided by muon flavor violation, we have also plotted the $\mu \rightarrow e\gamma$ rate over exactly the same parameter space. As discussed previously in our third and fourth paragraphs and unlike previous SO(10) calculations from the reduced Planck scale, our calculations use the complete $6 \times 6$ squark and slepton mass matrices and include all flavor mixing effects induced through the
one-loop RGEs by the MSSM yukawa couplings, which are important for the large \( \tan \beta \) calculation since \( \lambda_t = \lambda_b = \lambda_\tau \) at the grand unification scale.

In conclusion, if one is to calculate the decay rate for the flavor changing processes in a SUSY GUT with SUSY breaking communicated by gravity above the GUT breaking scale in the form of soft breaking mass terms, it is essential to include the GUT scale renormalization group effects. The mutual consideration of experimental limits on hadronic and leptonic flavor violating decays can give strong constraints on parameter space, and gives some preference to negative values of \( \mu \). One then finds there are some regions of parameter space which are allowed by both the \( b \to s\gamma \) and the \( \mu \to e\gamma \) decay rates, and where it may soon be possible to search for the signal for new physics in the form of \( \mu \to e\gamma \).

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Figure captions

Fig. 1: Plots for tan $\beta = 2$ and $\mu < 0$.

a) $b \to s\gamma$ branching ratio as a function of $M_{10}$.

b) $\log_{10} Br(\mu \to e\gamma)/4.9 \cdot 10^{-11}$ as a function of $M_{10}$.

c) $r_{A_g} \equiv A_g/A_{SM}$ as a function of $M_{10}$.

The five curves in each of a), b), and c) correspond to $m_0=0$, 100, 200, 300, and 400 GeV, and are labeled with their values of $m_0$.

d) $b \to s\gamma$ branching ratio as a function of $m_0$.

e) $\log_{10} Br(\mu \to e\gamma)/4.9 \cdot 10^{-11}$ as a function of $m_0$.

The three curves in both of d) and e) correspond to $M_{10}=100$, 150, and 225 GeV, and are labeled with their values of $M_{10}$.

Fig. 2: Same as Fig. 1 caption except $\mu > 0$.

Fig. 3: Plots for the large tan $\beta$ case (i) with $A=1$ and $\mu < 0$. For all plots, $m_0 = 1000$ GeV.

a) $b \to s\gamma$ branching ratio as a function of $\mu$.

b) $\log_{10} Br(\mu \to e\gamma)/4.9 \cdot 10^{-11}$ as a function of $\mu$.

c) $r_{A_g}$ as a function of $\mu$.

The curves corresponds to the gaugino masses $M_{10}=65$, 105, 145, 185, 225, and 265 GeV, and are labeled their values of $M_{10}$/GeV.

Fig. 4: 4a. Plots for the large tan $\beta$ case (ii) with $A=1.18$ and $\mu < 0$. For all plots, $m_0 = 2200$ GeV.

a) $b \to s\gamma$ branching ratio as a function of $\mu$.

b) $\log_{10} Br(\mu \to e\gamma)/4.9 \cdot 10^{-11}$ as a function of $\mu$.

c) $r_{A_g}$ as a function of $\mu$.

The curves corresponds to the gaugino masses $M_{10}=250$, 275, 300, and 325 GeV, and are labeled by their values of $M_{10}$/GeV.
Fig. 1
$B_{b \to s} \left[ 10^{-4} \right]$

\begin{align*}
\text{Log}_{10}(B/B_{\text{exp}}) &\to e \\
\end{align*}

\textbf{Fig. 1}
Fig. 2
**Fig. 2**
Fig. 3

\[ m_0 = 1000 \text{ GeV} \]
Fig. 4 \[ {^{25}m_0 = 2200 \text{ GeV}} \]