Multiarmed spiral waves generated by periodic stimuli in excitable systems

Jiraporn Luengviriya¹, Malee Sutthiopad²*, Jarin Kanchanawarin², and Chaiya Luengviriya²

¹ Lasers and Optics Research Group, Department of Industrial Physics and Medical Instrumentation, King Mongkut’s University of Technology North Bangkok, 1518 Pibulsongkram Road, Bangkok 10800, Thailand.
² Department of Physics, Kasetsart University, 50 Phaholyothin Road, Jatujak, Bangkok 10900, Thailand

fscimls@ku.ac.th

Abstract. Rotating spiral waves occur in different excitable systems. They also play an important role in the human health since the spiral waves of electrical signals correspond to some cardiac arrhythmia like tachycardia and fibrillations. Pinning of a spiral wave is found to stabilize extreme conditions such as the superconductivity and the superfluidity and it also causes longer cardiac arrhythmia. Multiarmed spiral waves are multiple spiral arms rotating in the same direction around a common center. In heart, they are expected to be seriously dangerous since the wave frequency increases with the number of spiral arms. In this article, we present a partition method with periodic stimuli to generate multiarmed spiral waves pinned to a circular obstacle. Periodic stimuli applied on one side of a thin line partition ignite multiple waves with large distances between them so only a few wave fronts can be distributed around the obstacle and subsequently result in multiarmed spiral waves with a few arms after the removal of the partition. In contrast, Periodic stimuli on both two sides of the partition create waves with smaller distances between them and more wave fronts can fit around the obstacle boundary. Therefore, multiarmed spiral waves with more arms can be generated using the latter method.

1. Introduction

Rotating spiral waves [1], often observed in excitable systems, are characterized by three states: excitable, excited, and refractory states. These dynamical patterns stay in the systems longer when their organizing centers so-called spiral tips [2] are pinned to unexcitable areas which act as obstacles for the waves. The spiral waves pinned to inhomogeneities are found to maintain the superconductor state [3] and the superfluidity [4]. For cardiac systems, the spiral waves of electrical action potential correspond to some cardiac arrhythmia like tachycardia and fibrillations, which potentially lead to sudden cardiac death [5]. Such pathological conditions will occur longer when the spiral waves are pinned to veins or scars [6]. Therefore, it is important to elucidate their dynamics so that their effective elimination from the heart can be realized.

Multiarmed spiral waves, which are composed of multiple spiral arms rotating in the same direction around an unexcitable area, are firstly demonstrated in excitable chemical media [7]. When
temporary obstacles are removed, all arms are separated and rotate around individual cores [8]. Therefore, the obstacles stabilize the structure of multiarmed spiral waves. Furthermore, it is shown that multiarmed spiral waves in cardiac cell culture samples, constructed by a short and rapid train of electrical stimuli, have a higher frequency in comparison to free spiral waves (without pinning to obstacles) [9]. This implies that the multiarmed spiral waves are more dangerous for human health. Prior an investigation of their dynamics and elimination, a construction of such wave structure is necessary.

In this article, we demonstrate the generation of multiarmed spiral waves pinned to a circular obstacle by a so-call partition method [10] together with periodic stimuli in excitable systems.

2. Methods

We use the two-variable Oregonator model [11], a well-known model for excitable media, to describe two variables: the activator $u$ and the control $v$, whose dynamics are governed by the reaction-diffusion:

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon} \left( u - u^3 - f v \frac{u-q}{u+q} \right) + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = u - v + D_v \nabla^2 v$$

(1)

The parameter values are set as in a study [12]: $q = 0.002$, $f = 1.4$, diffusion coefficients $D_u = 1.0$ and $D_v = 0.6$ and $\varepsilon = 100$. In the absence of obstacles, the spiral tip rotates around a circle of 0.9 s.u. diameter. The Laplace operator in Eq. (1) is approximated using a 9-point discretization [13]. We use a uniform grid space of $\Delta x = 0.1$ s.u. with a time step $\Delta t = 3.0 \times 10^{-3}$ time unit (t.u.) and no-flux boundary conditions for both the boundaries of the system and a circular obstacle as described in [14].

Multiarmed spiral waves pinned to the obstacles (12.0 s.u. in diameter) are initiated by using a partition method with multiple planar stimuli. A vertical thin no-flux partition is placed between the top boundary and the obstacle (see figure 1). Periodic stimuli of planar waves are applied by setting a 5-grid-point strip to an excited state ($u = 1.0$) close to either one or two sides of the partition.

3. Results and Discussion

We now demonstrate a generation of multiarmed spiral waves pinned to an obstacle using single-site stimuli and discuss the limitation of the method. In figure 1(a), a single stimulation is applied close to the right of the partition and it creates a single-armed spiral wave in figure 1(a'). Actually, each vertical line stimulation will result in two planar waves propagating in opposite directions (to the left and the right) but the wave propagating to the left (i.e., to the partition) is blocked by the partition, while the one propagating to the right is survived. The partition is removed before this wave completes one rotation around the obstacle. In the course of time, the wave curls in to form a spiral wave pinned to the obstacle. In figure 1(b), two subsequent stimuli with a delay time of 1.35 t.u. provide two planar wave fronts (moving to the right) which soon adopt a two-arm spiral structure as in figure 1(b'). Note that multiple stimuli with too short delay time (e.g. 1.26 t.u.) fails to ignite propagation waves since the system need more time to recover to the excitable state.

Then, we apply more stimuli with the same delay time to obtain three ignited waves as in figure 1(c) and the three-armed spiral waves as in figure 1(c'). The limitation of this method is shown in figures 1(d) and 1(d'). For the used parameter set and the obstacle size, we found that four initiated
Figure 1. Initiation of pinned multiarmed spiral waves by single-site stimuli. (a) – (d) One to four planar waves are subsequently ignited on the right of the partition (the thin vertical lines). Shortly after initiation, the wave fronts propagate to the right [arrows in (a)], while their ends trace the obstacle boundary. (a’) – (d’) After the partition is removed, the waves continuously rotate clockwise around the obstacle (black circles). The numeric labels indicate the spiral arms.

Wave fronts are the maximal waves which can fit around the obstacle, as in figure 1(d). After the partition is removed, the leading wave front (closest to the left of the partition) propagates across the partition location into the wake of the newest ignited wave. Since the wake area is in refractory so the leading wave is quickly detached from the obstacle while the other three waves are still pinned. As in figure 1(d’), all waves form three-armed spiral waves (labeled 1, 2, and 3) pinned to the obstacle and one free spiral wave (labeled 4). It is worth noting that the structure of the multiarmed spiral waves is disturbed by the free spiral wave [compare figures 1(c’) and 1(d’)]. Therefore, for the studied conditions this single-site stimulation method can generate multiarmed spiral waves with three arms at most. This limitation comes from the well-known dispersion relation [15] where the later initiated waves propagate slower so that a few initiated waves at the beginning have long distances to each other and they cover the entire obstacle boundary.

In the following, we will illustrate the generation of pinned multiarmed spiral waves using two-site stimuli. This method overcomes the limitation found in the single-site stimulation. The two-site stimuli initiate periodic waves with a less distance and create the corresponding pinned spiral waves with more arms. In figure 2a, the first two planar waves are simultaneously stimulated close to the left and the right of the partition and then they propagate in opposite directions. The two wave ends move down clockwise and counterclockwise along the obstacle boundary until they meet at the bottom of the obstacle as in figure 2b. Note that the two waves subsequently merge to one front which is detached from the obstacle (e.g., the wave close to the bottom of figure 2c). The later stimuli from both sites are performed periodically with two different delay times: 1.80 and 1.35 t.u. for those at the left and the right of the partition, respectively. As the results, the merging point of the waves moves clockwise along the obstacle and the amount of waves tracing clockwise the obstacle boundary increases in time, as in figures 2c and 2d. It can be clearly seen that the clockwise waves in figure 2d have shorter distances between adjacent waves in comparison to those in figure 1d.
Figure 2. Two-site stimulation method for initiating pinned multiarmed spiral waves. (a) At the beginning, two planar waves are ignited at both the left and the right of the partition (the thin vertical line) so they propagate in opposite directions (blue arrows). (b) Then, their ends tracing the obstacle meet (the red arrow) and merge each other. (c) and (d) Periodic stimuli at the left (delay time = 1.80 t.u.) and the right (delay time = 1.35 t.u.) cause the merging location (red arrows) to move to the left and there are more wave fronts to the right.

Figure 3. Pinned multiarmed spiral waves generated by two-site stimuli: 4 – 7 spiral arms (a) – (d) before and (a´) – (d´) after the partition is removed. The numeric labels indicate the spiral arms.

Using the two-site stimuli, we obtain pinned multiarmed spiral waves with different number of arms. As shown in figure 3(a), the two-site stimuli are continually applied until four clockwise rotating waves appear (the merging wave is excluded). Then the stimuli are stopped and the partition is removed. The pinned waves clockwise rotate with their ends attached to the obstacle. As reported earlier [16], the structure is self-organized during a transient interval before the four-armed spiral waves are evenly distributed around the obstacle as in figure 3(a´). The same procedure is applied to obtain five and six waves as in figures 3(b´) and 3(c´) and the corresponding multiarmed spiral waves appear as in figures 3(b´) and 3(c´). Actually, this method can provide seven waves fitting around the obstacle boundary before the partition removal as in figure 3(d). However, after the partition is removed, one wave is detached from the obstacle and subsequently there are 6 pinned spiral arms (labeled 1 – 6) and one single free spiral wave (labeled 7) as in figure 3(d´). These results show that the number of pinned spiral arms is limited to six for the given system and the obstacle size.
4. Conclusion
We have presented the generation of pinned multiarmed spiral waves in a simulated excitable system by two different methods. The single-site stimuli create multiple waves with large distances to each other so a few waves can fit around the obstacle boundary before the partition is removed. This results in multiarmed spiral waves with a few arms after the partition removal. The two-site stimuli create periodic waves with shorter distances of adjacent waves so more waves can be distributed around the obstacle prior the partition removal. Therefore, the latter method can generate of pinned multiarmed spiral waves with more arms.

Acknowledgments
We thank the Faculty of Science, the Research and Development Institute (KURDI), the Center for Advanced Studies of Industrial Technology, Kasetsart University; and the Faculty of Applied Science, King Mongkut’s University of Technology North Bangkok for financial support.

References
[1] Winfree A T 1972 Science 175 634
[2] Ross J, Müller S C and Vidal C 1988 Science 240 460
[3] De Gennes P G 1966 Superconductivity of Metals and Alloys (New York: W.A. Benjamin)
[4] Blaauwgeers R, Eitsov V B, Krusius M, Ruohlo J J, Schanen R and Volovik G E 2000 Nature 404 471
[5] Gray R A, Pertsov A M and Jalife J 1998 Nature 392 75
[6] Davidenko J M, Pertsov A V, Salomonsz R, Baxter W and Jalife J 1992 Nature 355 349
[7] Agladze K I and Krinsky V I 1982 Nature 296 424
[8] Steinbock O and Müller SC 1993 Int. J. Bif. Chaos 3 437
[9] Bursac N, Aguel F and Tung L 2004 Proc. Natl. Acad. Sci. USA 101 15530
[10] Ponboonjaroenchai B, Sutthiopad M, Kanchanawarin J, Luengviriya C, Müller S C and Luengviriya J 2019 11th Biomedical Engineering International Conference 8609978
[11] Field R J and Noyes R M 1974 J. Chem. Phys. 60 1877
[12] Jahnke W and Winfree A T 1991 Int. J. Bif. Chaos 1 445
[13] Dowle M, Mantel R M and Barkley D 1997 Int. J. Bif. Chaos 7 2529
[14] Luengviriya J, Sutthiopad M, Phantu M, Porjai P, Kanchanawarin J, Müller S C, and Luengviriya C 2014 Phys. Rev. E 90 052919
[15] Flesselles J M, Belmonte A, and Gaspar V 1998 J. Chem. Soc. Faraday trans. 94, 851
[16] Ponboonjaroenchai B, Luengviriya J, Sutthiopad M, Wungmool P, Kumchaiseemak N, Müller S C and Luengviriya C 2019 Phys. Rev. E 100 042203