ALIGNED ELECTROMAGNETIC EXCITATIONS OF THE
KERR-SCHILD SOLUTIONS∗

ALEXANDER BURINSKII
Gravity Research Group, NSI Russian Academy of Sciences,
B. Tulskaya 52, Moscow 115191, Russia, bur@ibrae.ac.ru

Aligned to the Kerr-Schild geometry electromagnetic excitations are investigated, and asymptotically exact solutions are obtained for the low-frequency limit.

1. In this paper we consider the aligned electromagnetic excitations of the Kerr-Schild geometry, taking into account the back reaction of the excitations on metric. To our knowledge, it is the first attempt to get in the Kerr-Schild formalism a self-consistent solution for the case \( \gamma \neq 0 \).

Electromagnetic field of the exact Kerr-Schild solutions has to be aligned to the Kerr null congruence which is generated by tangent vector \( k^\mu(x) \). Aligned e.m. excitations on the Kerr background were investigated.

Contrary to the usual ‘quasi-normal’ modes, the aligned excitations are compatible with the Kerr congruence and type D of the metric. On the other hand they have very specific exhibition in the form of semi-infinite ‘axial’ singular lines producing narrow beams which can lead to some new astrophysical effects like the holes in the horizons and jet formation.

2. The vector field \( k^\mu \) is determined by the Kerr Theorem via a complex function \( Y(x) \),

\[
k^\mu dx^\mu = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - \bar{Y}du),
\]

where \( P \) is a normalizing factor, providing \( k^0 = 1 \). For the geodesic and shear-free congruences, satisfying to \( Y \), the Einstein-Maxwell field equations were integrated out in a general form and reduced to the system of equations for electromagnetic field

\[
A_{1,2} - 2Z^{-1}Z_{,13}A = 0, \quad A_{1,4} = 0,
\]

\[
\mathcal{D}A + Z^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0, \quad \gamma_{,4} = 0,
\]

where \( \mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - Z^{-1}\bar{Y}_{,3}\partial_2 \) and for gravitational field, which will be discussed below.

Electromagnetic Sector, was discussed in. The first equation has the general solution

\[
A = \psi/P^2,
\]

where \( \psi_{,2} = \psi_{,4} = 0 \). Therefore \( \psi \) has to be a holomorphic function of variable \( Y \), since \( Y_{,2} = Y_{,4} = 0 \). Function \( Y \) is a projective (complex) angular coordinate \( \theta \) in \( \mathbb{CP}^1 = S^2 \), \( Y = e^{i\theta} \tan \frac{\theta}{2} \). A holomorphic function may be represented as an infinite Laurent series \( \psi(Y) = \sum_{n=-\infty}^{\infty} Y^n \). If the function \( Y \in S^2 \) is not constant, it has to contain at least one pole which may also be at \( Y = \infty \) (or \( \theta = \pi \)). So, for exclusion of the Kerr-Newman solution having \( Y = e = \text{const.} \), we have to consider solutions \( \psi(Y) = \sum_i \frac{q_i}{Y-Y_i} \) which are singular at angular directions \( Y_i = e^{i\theta_i} \tan \frac{\theta_i}{2} \), and represent a narrow beams in there angular directions. Note, that for \( q_i = \text{const.} \) these solutions are exact self-consistent solutions of the full system of Kerr-Schild equations.

A wave excitation propagating in the direction \( Y_i \) will be described by the func-

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tion \( \psi(Y, \tau) = q(\tau) \exp\{i\omega\tau\} + \frac{1}{T - Y} \), where \( \tau \) is a retarded time. For the rotating Kerr source the retarded time is complex. In the nonstationary case, solution for \( A \) has the only difference that the function \( \psi \) acquires extra dependence from the ‘left’ retarded time \( \tau_L \). In the rest frame the function \( P \) has the form \( P = 2^{-1/2}(1 + Y\bar{Y}) \). The real operator \( \mathcal{D} \) acts on the real slice as follows \( \mathcal{D}Y = \mathcal{D}Y = 0 \), and \( \mathcal{D}P = 0 \).

The explicit form of the retarded time is \( \tau_L = t - r + ia\cos \theta \). Since \( \cos \theta = \frac{1 - 2Y^2}{1 + 2Y^2} \), we have \( \mathcal{D}\cos \theta = 0 \), and \( \mathcal{D}\tau = \mathcal{D}P = \frac{1}{P} \).

The second e.m. equation takes the form \( \dot{A} = -(\gamma P)\psi \). Integration yields

\[
\gamma = \frac{2^{1/2}\psi}{P^2Y^2} + \phi(Y, \tau)/P, \tag{1}
\]

where we neglected recoil and \( \phi \) is an arbitrary analytic function of \( Y \) and \( \tau \).

3. Gravitational sector is:

\[
\begin{align*}
M_{12} - 3Z^{-1}\bar{Z}Y_{3} M &= A\bar{\gamma}\bar{Z}, \\
\mathcal{D}M &= \frac{1}{2}\gamma\bar{\gamma},
\end{align*} \tag{2}
\]

Solutions of this system were given for stationary case, corresponding to \( \gamma = 0 \). We assume that the energy of electromagnetic wave excitation is much lower then the mass of rotating object \( m \), and does not affect on the motion of the center of mass of the solution. However, influence of the electromagnetic field on the metric occurs also via the function \( H = \frac{mr - \psi/2}{\sqrt{r^2 + a^2}\cos \theta} \), in the K-S metric form \( g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_{\mu}k_{\nu} \), where \( \eta_{\mu\nu} \) is the metric of auxiliary Minkowski space-time. This is a more thin effect, leading to a deformation of the metric tensor around rotating black hole by electromagnetic excitations. The poles in function \( \psi \) which cause the ‘axial’ singular electromagnetic beams deform strongly the function \( H \).

The equation (2) acquires the form \( (MP^3)_{12} = A\bar{\gamma}\bar{Z} \). The equation (3) takes the form \( \ddot{m} = \frac{1}{2}P^4\gamma\bar{\gamma} \). It is known that it determines the loss of mass by radiation. The right sides of (2) and (3) will be small for the small (low-frequency) aligned wave excitations, since the functions \( \psi \) and \( \gamma \) will be of order \( \sim i\omega\psi \). In this sense the aligned excitations will be asymptotically exact solutions in the low-frequency limit. However, since \( \psi \) contains the singular poles in \( Y \), the limit \( \gamma \to 0 \) is not uniform one, and an extra trick is necessary - a regularization. Such a regularization may be performed by the free function \( \phi(Y, \tau) \) in (1). The function \( \gamma \) is represented as a sum of simple poles \( \sum \frac{a_i(Y_i, \tau)}{P_i(Y - Y_i)} \), where the coefficients \( a_i \) are determined by function \( \psi \), and coefficients \( b_i \) are chosen from free function \( \phi \) to provide cancelling of the poles. It allows us to perform regularization of the most of poles in \( \gamma \). If all the poles in the function \( \gamma \) will be cancelled, the result of integration will be a stochastic radiation which will reduce to zero for weak excitations, and solutions of

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\footnote{The Kerr solution is described by a complex ‘point-like’ source propagating along a complex world line. There are different ‘left’ and ‘right’ complex conjugate world lines and corresponding ‘left’ and ‘right’ retarded times \( \tau_L \) and \( \tau_R \).}
and (3) will be asymptotically exact. However, the pole at \( Y = \infty \) can not be regularized by this method and demands especial treatment.

4. Structure of the solutions near the beams (pp-waves) is discussed in.\(^2\)\(^,\)\(^4\) It was shown that such beams pierce the horizons forming the tube-like holes connecting internal and external regions. So the classical structure of black hole turns out to be destroyed.

Our solution turns out to be exact in the asymptotic limit \( \gamma \to 0 \), which corresponds to the weak and slowly changed electromagnetic field. In particular, it shall tend to exact one for a black hole immersed into the zero point field of virtual photons. In this case we have a sum of excitations in diverse directions

\[
\psi(Y,\tau) = \sum_i q_i(\tau) Y^{-Y_i}
\]

which leads to a flow and migration of many singular beams leading to an instantaneous appearance and disappearance of the holes in horizon, as it is shown on fig.1. One can assume that it may be a mechanism of BH evaporation.

![Fig. 1. The vacuum flow of virtual photons pierces the black hole horizon.](image)

Note, that this picture is reminiscent of the haired black hole which was suggested by the approach from the loop quantum gravity, where singular hairs were formed from the horizon contrary to the appearance of the holes in horizon.\(^9\)

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