Exotic particles below the TeV from low scale flavour theories

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Abstract

A flavour gauge theory is observable only if the symmetry is broken at relatively low energies. The intrinsic parity-violation of the fermion representations in a flavour theory describing quark, lepton and higgsino masses and mixings generically requires anomaly cancellation by new fermions. Benchmark supersymmetric flavour models are built and studied to argue that: i) the flavour symmetry breaking should be about three orders of magnitude above the higgsino mass, enough also to efficiently suppress FCNC and CP violation coming from higher-dimensional operators; ii) new fermions with exotic decays into lighter particles are typically required at scales of the order of the higgsino mass.
1 Introduction

Notwithstanding our fair knowledge of quark masses, mixings and CP phases and strong constraints on neutrino ones, and the profusion of models in various frameworks, we have no cogent explanation for their origins. Even worse, most of the acceptable models are not directly testable as they do not predict any low energy energy effect but the fermion mass spectra they were designed for - some nice relations are encouraging but cannot quite prove a model. In this paper we focus on 4D perturbative supersymmetric gauged flavour theories - these five assumptions being relevant in our analysis - and claim that, under some circumstances, these models might predict new characteristic states within the reach of the LHC.

Indeed, flavour symmetries are chiral, \textit{i.e.}, the parity conjugated states in the small mass operators of quarks, leptons, higgsinos (\(\mu\)-term) and neutrinos have different flavour charges so that the masses are controlled by the amount(s) of flavour symmetry breaking(s) and the charge differences between parity conjugated states, which we call flavour-chiralities herein. The contributions of all these states to the anomalous coupling of flavour gauge bosons to photons and gluons are proportional to their flavour-chiralities as introduced to explain their masses. We argue that, in low energy abelian flavour models, anomaly cancellation requires a few extra charged and coloured particles whose flavour chiralities are typically close to the higgsino one, resulting into heavy states of mass \(O(\mu)\), while the flavour symmetry breaking scale is quite higher. They have peculiar decays into light states.

In a matter-of-fact approach, it is not necessary to impose anomaly compensation within the Standard Model (SM) or the Minimal Supersymmetric SM (MSSM) fermion field content. Just as some of their masses are reduced by the flavour symmetry, so could some states that are parity-symmetric with respect to the electroweak interactions, be flavour-chiral, get their masses suppressed with respect to the cutoff scale and contribute to anomaly-compensation below it. This is the generic case. If the cutoff is high enough, they can be integrated out together with the other flavour theory components, but it is not quite so when the cutoff occurs at relatively low energies.

Let us first recollect the seven main questions to be addressed by a supersymmetric flavour theory: 1) the hierarchy among the SM fermion masses, the hierarchy among the entries of the CKM matrix and the value of the CP violation phase, \(\delta\); 2) the different pattern in the effective neutrino mass, with (at least two) less hierarchical eigenvalues and two large mixing angles - the smallness of the eigenvalues can be ascribed to a large scale suppression and/or tiny couplings; 3) flavour mixings and \(CP\)-violating phases in the soft parameters of the MSSM, some of them having tight upper bounds from the investigation of flavour changing neutral currents (FCNCs) and CP violation; 4) renormalisable \(R\)-parity violating superpotential operators that cause the emergence of lepton and/or baryon number violating terms and, in particular, destabilize the proton; 5) non-renormalisable \(R\)-parity conserving superpotential operators (like \(QQQL\)) giving rise to lepton and baryon number violation as well; 6) non-renormalisable superpotential operators (like \(UQDQ\)) leading to yet further contributions to FCNC and CP violation; 7) the \(\mu\)-problem, the higgsino mass must be suppressed from the cutoff scale down to the level of the supersymmetry breaking scalar masses.

Fermion masses are protected by chiral symmetries, and suppressed by broken chiral symmetries. Theories based on flavour symmetries are characterized by a cutoff scale \(\Lambda\)
and the scales where the flavour symmetries are broken down, $\epsilon\Lambda$, $\epsilon'\Lambda$, ... In the spirit of the Frogatt-Nielsen (FN) idea \cite{1, 2}, non-abelian flavour symmetries more naturally explain empirical relations between masses and mixings\footnote{There is a large variety of those models in the literature; we can only quote a small part here\cite{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}. They are not all consistent with the present data on fermion masses and mixings\cite{15}.}, while abelian symmetries are suitable to deal with hierarchies. Here we consider gauged continuous symmetries - in particular to avoid Nambu Goldstone bosons - but also discrete symmetries that can result from symmetry breaking of the continuous flavour symmetry.

The textures in the quark and lepton (including neutrino) couplings to the Higgses are controlled by the dimensionless parameters $\epsilon$, $\epsilon'$, ..., while the higgsino mass, or $\mu$-term, and the effective neutrino masses also depend on the cutoff $\Lambda$. The soft masses depend on the supersymmetry breaking mechanism and mediation; we implicitly assume that they satisfy the experimental bounds and do not address the issue in this paper\footnote{In some cases, the squark and slepton mass differences could provide tests for the flavour model (see, e.g., \cite{16} but since they are already tightly constrained by FCNC experiments, they would be difficult to measure.}. Baryon and lepton number violations can be alleviated by continuous or discrete symmetries, and large $\Lambda$ for higher dimension terms. Some dangerous operators can be eliminated by exact discrete symmetries like matter parity\cite{17}, baryon triality \cite{18, 19} or proton hexality \cite{20}, that survive as relics of the flavour symmetry breaking.

In general, $R$-parity preserving, flavour violating, non-renormalisable operators like $UQDQ$ or $UQLE$ are not all suppressed by the symmetries alone to the level imposed by experiments. This puts a - not so low - lower limit on the cutoff $\Lambda$, somewhat above those directly obtained through generic SM effective operators. Thus, the action of the flavour symmetry on higgsinos must be such that the $\mu$-term is reduced by the FN mechanism to the effective supersymmetry breaking scale $O$(TeV). We follow here another path to fix $\Lambda$, based on anomaly cancellation, and check its consistency with FCNC/CP experimental tests, as discussed later on.

We simplify our approach by picking a single $U(1)_X$ flavour group\footnote{If the flavour symmetry is non-abelian, the sequential breaking of symmetries is to be treated along the same lines as here, their hierarchy being mostly fixed by mass matrix textures. The case with several abelian flavour symmetries is similar. They could be less predictive than our all-in-one approach but we may argue as follows: the cancellations of the many flavour-chiralities involved in the anomalies are an exception and even after some optimization, the needed extra particles will generically have flavour-chiralities similar to the larger ones (or associated to the small scales), typically higgsinos, neutrinos or the lighter quarks and leptons. In most instances the new particles would lie quite below the cutoff, although not necessarily within the LHC reach.}. We further require that a combination of the flavour and the weak hypercharge transformations contains the exact discrete symmetries that survive at low energies. This $U(1)_X$ should take care of all the seven tasks above - the third one is not addressed in this paper, though - a sort of “all-in-one” model. If it is anomalous, one must rely on the Green-Schwarz cancellation mechanism \cite{22}, which assumes an underlying string theory. Then, the Dine-Seiberg-Witten mechanism \cite{23, 24, 25} ensures the breaking of $U(1)_X$ and defines the scale $\epsilon\Lambda$ a bit below the Planck scale\cite{26, 27, 28, 29, 30, 31}. However, this makes the search for direct signals of this $U(1)_X$ moot.

Only if the $U(1)_X$ is non-anomalous, one can adjust the flavour theory such that $\epsilon\Lambda$ is much lower than the Planck scale. It is known that anomaly cancellation within the MSSM field content in abelian flavour models is tightly constrained by the quark and
lepton masses and mixings \cite{28}. However, these constraints, retrieved in section 3, are inconsistent with our purpose in this work which is precisely to investigate how low $\Lambda$ can be before running into contradiction with experiment. Our way to try and tackle the seven supersymmetric flavour tasks above and to single out the new particles needed to compensate the flavour anomalies is presented in the next section while the explicit models and their analysis are displayed in section 3. From the balance among the value for $\Lambda$, the types and the masses of the newly introduced heavy particles, we find, under (presumably) reasonable assumptions, that $\Lambda$ should be $O(10^4)$ TeV, while some new state get much lower masses are, plausibly within the LHC reach.

Naïvely, the natural signal for a low-energy abelian flavour model would be a neutral gauge boson with family dependent couplings and mass $M_X = \sqrt{2}g_X \epsilon \Lambda$, where $g_X$ is the $U(1)_X$ coupling with our normalization of $X$. As long as many chiralities, hence many charges, are not small, the $\beta$-function is large and perturbativity through the cutoff\footnote{The Landau pole only has to be above $\Lambda$ if, \textit{e.g.} the flavour $U(1)_X$ integrates into a non-Abelian symmetry; for the pole to be at Planck mass, the bound on $g_X$ is further reduced by a factor $O(3)$.} implies a (model dependent) upper limit $g_X < O(.05)$, so that $M_X < O(20$ TeV$)$. However, the effective range of this $Z'$ interactions would be $(\epsilon \Lambda)^{-2} < O(10^{-5} G_F)$, leading to very low production rates at the LHC.

The new exotic states are dealt with in section 4, where their masses are estimated and their decay modes established, for abelian flavour benchmark models. To avoid stable heavy “quarks” or “leptons”, the models are also selected by the condition that heavy states can decay into MSSM states, which is naturally implemented by the exact residual discrete symmetries. The new uncoloured weak doublets, are produced like heavy (s)leptons, but decay into three (s)quarks, one of each family! Actually, the “easier” signal at the LHC would be the production of a heavy coloured weak-isosinglet “squark” with more model dependent signatures: two quarks (in which case they could show up as resonances) or one lepton plus one or two (s)quarks! Since the precise signatures are quite model dependent, they are not discussed in detail in the present paper.

This paper is organized so that in the next section, we further discuss the approach in the framework of a single flavon charge, and we state our assumptions and the main results. The details and calculations are presented in section 3 where six of the seven tasks above are addressed and the cutoff is fixed. An appendix completes the solution of the anomaly cancellation therein. Hence the hurried reader can skip section 3 (up to a few definitions therein) and move to section 4 where the properties of the exotic states are specified. The closing section presents a few conclusions, as well as comments on shortcomings and generalizations.

2 All-in-U(1) model

Before displaying and discussing explicit models in the next sections, we briefly describe the main steps in the construction of abelian flavour models at low energies. The basic parameters are $\Lambda$, $\epsilon$ and, rather than the $X$-charges, the $X$-chiralities defined by the differences between the $X$ eigenvalues of fermions with the same electric charge and colour, but opposite chiralities, that we denote by its CP equivalent, $\mathcal{X}_f = X(f) + X(f^c)$. Indeed - with the exception of the effective neutrino masses - the observed flavour physics involve $B$ and $L$ conserving operators and thus constrains (flavour dependent) quark and lepton
X-chiralities. In particular, the lepton-higgsino X-chiralities, controlling both R-parity and neutrino masses, are defined similarly to the higgsino one that is in charge of the $\mu$-term.

Within the Froggatt-Nielsen mechanism, a fermion mass $m$ is suppressed by a factor $\epsilon^{|X|}$, where the chirality $X$ is the $X$ value associated to $m$ to compensate the $X$ imbalance among the fields, since the lowest dimension invariant operator has $|X|$ flavon fields. In order to engender acceptable fermion masses and mixings for all the MSSM fermions, the various $X$-chiralities are multiply constrained and more or less fixed (see next section for details):

- For conjugated states under $SU(2)_W \otimes U(1)_{Y}$, like the two higgsinos, $X$ is the fermion $X$-chirality defined above. Thus, flavour symmetry must reduce the higgsino mass $\mu$ from its natural scale $\Lambda$ to the supersymmetry breaking scale $\mu \sim \epsilon^{|X_{\mu}|}\Lambda \sim \text{TeV} \ll \Lambda$. Hence low energy flavour means non-vanishing higgsino chirality $X_{\mu}$, implying that the Higgses cannot have vanishing flavour charges.

- For the Higgs generated quark and lepton masses, the appropriate Higgs charge must be added and the suppression exponent is $|X^f_{ij}| = |X(f_i) + X(f^c_j) + X(h)|$ where $i, j$ are generation indices, $f = u, d, e$, and the reference scale is the Higgs v.e.v.. The large hierarchies and small mixing angles suggest: (i) to fix $\epsilon \sim \theta_C$, and (ii) to take all $X^u_{ij}$ and $X^d_{ij}$ to be of same sign, which can be chosen positive.

- Neutrino masses are inhibited with respect to their reference scale $(174 \text{ GeV})^2/\Lambda$ by a factor $\epsilon^{2|X_{he_i}|}$. Notice that low energy flavour implies large $|X_{he_i}|$ from neutrino masses and suggests half-integer $|X_{he_i}|$ to implement matter parity or R-parity conservation.

Cancellations of anomalies corresponding to the vector-like SM symmetries, electromagnetic and colour, require $X$-chiralities of both signs, hence both positive and negative $X$ eigenvalues\(^\text{[28]}\). Therefore, low energy flavour symmetry needs flavons, and flavon v.e.v.’s, with $X$ of both signs. Of course, in the absence of a Fayet-Iliopoulos term, this is also a requisite to write a $X$-invariant superpotential and to break the flavour symmetry. Here we concentrate on the case of two flavons with $X = \pm 1$, (so defining the normalization of $X$). With an ad hoc flavon mass $\epsilon \Lambda$ this is realized by a generic effective superpotential.

Furthermore, because the lightnesses of most of the quarks and leptons (including neutrino masses) imply large $X$-chiralities, hence large contributions to anomalies, their cancelation within the MSSM field content implies a large (fixed) $X$-chirality between the two Higgs doublets\(^\text{[28]}\). In turn, this implies a very small ratio $\mu/\Lambda$, hence a lower bound $O(10^4 \text{ TeV})$ on $\Lambda$ and no flavour theory signal at the LHC. Thus, lower values of $\Lambda$ require anomaly compensation through the addition of $U(1)_X$-chiral heavy matter fields in conjugate representations of the SM gauge group whose mass matrices are related to their $X$-chiralities à la Froggatt-Nielsen. The relatively large anomalies within the MSSM-sector are to be compensated by one (SM vector-like) heavy pair with a large $X$-chirality or several states with smaller ones.

Asymptotic freedom of the SM gauge sector imposes a strong limit on the number and quantum numbers of heavy matter, while the preservation of the unification of gauge

\(\text{[28]}\) Since $U(1)_X$ does not specify $O(1)$ coefficients, we use the notation $A \sim B$ for $A = O(B)$, associated to this freedom.
couplings prefers matter in complete $SU(5)$ multiplets, even if we cannot achieve $SU(5) \otimes U(1)_X$ compatible solutions. With one $(10 + \bar{10})$ or three $(\bar{5} + \bar{\bar{5}})$ the QCD $\beta$-function vanishes, leaving only two choices consistent with asymptotic freedom: one or two $(\bar{5} + \bar{\bar{5}})$'s. As shown in the next section, the better choice turns out to be with two flavons and with heavy matter characterized by two heavy vector-like down quarks and squarks and their lepton and slepton weak-doublet partners in two $(\bar{5} + \bar{\bar{5}})$'s.

Then, anomaly cancellation will almost fix the higgsino $X$—chirality, hence the $\Lambda/\mu$ ratio so that the cutoff must be $O(500 - 2000)$ TeV. Indeed, for lower cutoffs, the new states turn out to be too light to have escaped observation. The solutions to these conditions are often consistent with heavy “quarks” and “leptons” at the scales to be investigated with the LHC in spite of the fact that the flavour symmetry breaking scale, $\epsilon \Lambda$, is much higher, $\epsilon \Lambda = O(100 - 400)$ TeV.

Flavour and CP issues are generic obstacles for new physics just above the SM scale. The constraints on higher dimension supersymmetric operators from FCNC processes and CPV transitions put a lower bound on the cutoff $\Lambda$ when they are not suppressed enough by their flavour-chirality. Typical ones are $Q_3 U_3 L_i E_j$ that contribute to lepton flavour violations and lepton EDM through a stop loop, enhanced by a factor of the top mass as compared to the lepton masses that appear in more usual supersymmetric contributions. When estimated, these constraints generically requires $\Lambda \gtrsim 500$ TeV, quite in agreement with the estimates from anomaly cancellation. By an order of magnitude comparison between the usual MSSM contributions and the ones arising from the higher dimension terms in the case of electromagnetic dipole transitions, we show that this agreement is not fortuitous. Indeed it is basically a consequence of the values allowed for the $X$—chiralities from the study of fermion masses and anomalies (see next section).

With such values for $\Lambda$, it seems hopeless to reduce proton decay down below the experimental bound. The solution is to assume an exact $Z_3$ symmetry (baryon triality, $B_3$), that excludes supersymmetric operators like $QQQL$ or $UDD$. Lepton number conservation can be introduced through a $Z_3$ (matter parity, $M_p$), which combines with baryon triality into a $Z_6$ (proton hexality). This exact (gauged) discrete symmetries should result from the breaking of a continuous gauged anomaly-free symmetry and we make the economical and elegant choice that it coincides with $U(1)_X$. More precisely, in general, it is a discrete subgroup of $U(1)_X \otimes U(1)_Y$ that leaves the Higgses invariant. This solution has a price: this $Z_6$ does not commute with $SU(5)$, but, in practice, Abelian flavour models are only marginally consistent with grand-unification anyway. Its implementation is presented in the next section.

As a bonus, one can fulfill another needed selection rule: the mixing between new and MSSM states must be minimized while the new charged and coloured particles are to be unstable, hence to decay into MSSM particles. Indeed, the $Z_6$ charges of the new particles can be chosen differently to those associated to quarks and leptons so to forbid the mixings and allow the decays into - exotic - final states. Reversely, states that mix in the SM identically transform under $Z_6$.

Therefore the discrete symmetry dictates the selection rules that define the effective Lagrangian beneath the flavour symmetry breaking scale $\epsilon \Lambda$, including the terms containing the new fields to be added in section 4. The hierarchy of the allowed couplings in the effective Lagrangian are then fixed à la Froggatt–Nielsen. Notice that only the integer part of $X$ is relevant in this second step because the fractional part is (related to) a preserved symmetry.
3 Effective supersymmetric model

Let us now construct and analyse models based on the suggested scenario. We assume the anomaly-free $U(1)_X$ flavour symmetry to be broken by a vectorlike pair of flavon chiral superfields $(A, B)$ with $X$-charges $\mp 1$ into a residual discrete $Z_6$ symmetry $P_6 = M_p \otimes B_3$ acting on the matter superfields so that dangerous $B$ and $L$ violating operators are excluded. The breaking scale is given by the $v.e.v's$

$$\epsilon \equiv \frac{\langle A \rangle}{\Lambda} = \frac{\langle B \rangle}{\Lambda},$$

that result from a generic superpotential $W(A, B) = \Lambda AB(\epsilon + f(AB/\Lambda^2)$ with the small parameter $\epsilon$ fixed by phenomenology.

The first step is to define the action of the anomaly-free $Z_6$ symmetry on the MSSM fields and then write the invariant MSSM effective model. The charges must be consistent with the presence of several operators in the superpotential, whose invariance under $P_6$ means that the corresponding charge combinations must be integers. Of course, they must be family-independent to allow for family mixing. The appropriate choice of the $Z_6$ charges can be written as:

$$Z_Q = 0, \quad Z_U = Z_E = Z_{H_d} = 1/6, \quad Z_L = -2/6, \quad Z_D = Z_{H_u} = -1/6. \quad (1)$$

The $X$-charges are given by $X_i = \text{integer} + Z_i$. This $Z_6$ is broken by the Higgs $v.e.v's$ but the combination $X' = X + Y/3 = \text{integer} + Z_i'$ is such that $Z_i'H_i = 0$ and so defines the exact abelian discrete symmetry that imposes the needed selection rules. The charges are simply $Z' = 1/18 = B/6$ for any quark, $Z' = 1/2 = L/2$ for any lepton, and the opposite ones for the $C$-conjugated states. For completeness, this is explained in the Appendix.

Then the general superpotential of the SSM superfields with operators up to dimension five consistent with the $Z_6$ charges in (1) is:

$$W = \mu H_d H_u + Y^{u}_{ij} Q^i H_u U^j + Y^{d}_{ij} Q^i H_d D^j + Y^{e}_{ij} L^i H_d E^j + C^{ijk}_{ij} U^i Q^j D^k + C^{ijk}_{ij} U^i Q^j E^k L^j + C^{ijk}_{ij} (H_d H_u)^2 + C^{ijk}_{ij} L^i H_u L^j H_u. \quad (2)$$

The orders of magnitude of the coefficients of the bilinear ($\mu$-term), trilinear (Yukawa couplings to the Higgses) and quadrilinear couplings are given by powers of the parameter $\epsilon$ defined by the modulus of the sum of charges of the corresponding superfields (because of the symmetry $X \to -X$). These charge combinations are fixed by the phenomenology of the corresponding operators that we now turn to discuss.

3.1 SM fermion masses and mixings

The trilinear terms $Q^i H_d D^j$, $Q^i H_u U^j$ and $L^i H_d E^j$, yield the fermion mass and mixing hierarchies, so, with

$$q_i + h_u + \bar{u}_j = X^u_{ij}, \quad q_i + h_d + \bar{d}_j = X^d_{ij}, \quad l_i + h_d + \bar{e}_j = X^e_{ij}. \quad (6)$$

\footnote{Notations are quite standard MSSM ones. As usual the $X$-charges are denoted by the same symbol as the fermions of the corresponding chiral multiplets, $i, j = 1, 2, 3$ are family indices.}

\footnote{Possible $D=5$ operators (trilinear terms) in the Kähler potential can be transposed into the superpotential by an analytic field redefinition in the effective theory}
then $X_{u,d,e}^{i,j} \in \mathbb{Z}$, and

$$Y_{u}^{i,j} \sim e^{iX_{u}^{i,j}}, \quad Y_{d}^{i,j} \sim e^{iX_{d}^{i,j}}, \quad Y_{e}^{i,j} \sim e^{iX_{e}^{i,j}}.$$  

(3)

Many of these $X$’s can be specified from the known fermion masses and mixings. Because of the symmetry in the flavon sector, the results are invariant under $X \to -X$, so we choose the value of the $X_{u}^{i,j}$ and $X_{d}^{i,j}$ to be positive. The fact that all of them have the same sign comes from the strong hierarchies in quarks masses and mixings and the well-known strong correlations among them (thus only one flavon is relevant for their masses). Instead, for leptons, one must keep free the signs in the matrix elements of $X_{e}^{i,j}$ as we shall prove later on. The dependence on $\tan \beta$ is taken into account by the parameter $x$, defined by $\tan \beta \sim \frac{e^{2} - x}{1 + x}$. We also introduce two “fuzzy factors”, $y$ and $z$ taking values 0 or 1, to account for some freedom in the relations. Then, with $e \sim \theta_{C}$, the Cabbibbo angle, the charged fermion masses lead to the following choices:

$$X_{u} = \begin{pmatrix} 8 & 5 + y & 3 + y \\ 7 - y & 4 & 2 \\ 5 - y & 2 & 0 \end{pmatrix} \quad (4)$$

and

$$X_{d} = \begin{pmatrix} 4 + x & 3 + x + y & 3 + x + y \\ 3 + x - y & 2 + x & 2 + x \\ 1 + x - y & x & x \end{pmatrix}.$$

(5)

We assume a hierarchical structure in $Y^{e}$ that reproduces the charged lepton mass ratios,

$$\text{diag} \, X_{e} = \{ \pm(4 + x + z) , \pm(2 + x) , \pm x \}, \quad (6)$$

since the diagonal terms (or the trace) mostly appear in the relations herebelow.

### 3.2 Effective neutrino masses and mixings

The quadrilinear term $L^{i}H_{u}L^{j}H_{u}$ gives rise to the effective neutrino mass matrix, so, with

$$l_{i} + h_{u} + l_{j} + h_{u} = X_{i,j}^{\nu}, \quad (7)$$

the $X_{i,j}^{\nu} \in \mathbb{Z}$ must be large enough to suppress $(174 \text{ GeV})^{2}/\Lambda$ down to the typical neutrino mass eigenvalues, $M_{\nu_{i,j}} \sim e^{iX_{i,j}^{\nu}}/\Lambda$. Within the indeterminacy inherent to the model, we take a texture consistent with the small hierarchy and large mixings of the MNS matrix

$$\text{diag} \, X_{\nu} = \pm(X_{\nu} + 2v , X_{\nu} , X_{\nu}) , \quad (v = 0,1) \quad (8)$$

As already mentioned, the mass parameter of atmospheric neutrino oscillations must satisfy

$$e^{X_{\nu}} \sim m_{\text{atm}}\Lambda/(174 \text{ GeV})^{2} \sim e^{17}\Lambda/(1000 \text{ TeV}). \quad (9)$$

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8Note that this ensures only a nice contribution from the neutrino sector to the MNS matrix – it is not guaranteed that the contribution from the charged lepton sector is as nice, so once $X$-charges have been established, one must check how $Y^{e}$ and $M_{\nu_{i,j}}$ fit together. As discussed later, $X_{\nu}$ is odd.
3.3 $\mu$-parameter

The bilinear term, $H_dH_u$, has a charge $h_d + h_u = X_\mu \in \mathbb{Z}$, so, the effective higgsino mass is related to the cutoff by:

$$\mu \sim e^{[h_d + h_u]} \cdot \Lambda = e^{[X_\mu]} \cdot \Lambda$$

(10)

Of course the cutoff has to be well above the MSSM scale, to avoid FCNC and CP flavour problems as further discussed below and, anyway, for the superpotential in (2) to be meaningful. The study of anomaly cancellation below requires $|X_\mu| = 3$, 4 or 5, while $|X_\mu| = 6$ corresponds to $\Lambda$ too large for our purposes - observable new states at the LHC. In this sense the approach is consistent and flavour problems related to higher dimension operator are fairly under control.

Of course, this is not quite a solution to the $\mu$-problem since it does not relate the $\mu$ scale to the supersymmetry breaking one. But if one assumes a solution to the $\mu$-problem, its consistency requires that the contribution (10) to the final higgsino mass be subdominant. Then, it would seem that one could lower $\Lambda$, but in the models discussed below this freedom is basically ineffective. One cannot allow for a small contribution from (10) and invoke a straightforward Giudice-Masiero mechanism (32) because the flavour symmetry would imply a similar suppression factor, but with respect to the effective supersymmetric breaking scale and, in particular, to the Higgs soft mass parameters.

Finally, note that since $H_dH_u$ exists, then so does $H_dH_uH_dH_u$, with $C_h \sim \mu^2/\Lambda^2$, which turns out to be very small and negligible to affect the electroweak symmetry breaking.

3.4 Anomaly cancellation

The next step is to fulfill the no-anomaly requirements

$$A_C = A_W = A_Y = A'_Y = 0,$$

(11)

corresponding to the vanishing of the strong, weak isospin, and the the two weak hypercharge anomalies, respectively. Since $Q_{em} = Y + T_3$, has vector-like representations, it is convenient to replace $A_Y$ and $A'_Y$ by the corresponding $A_{em}$, more directly related to the $X$-chiralities fitted to fermion masses, and $A'_{em}$ (linear in $Q_{em}$). As already anticipated in (3), and as we now prove, anomaly cancellation without extra-states require lepton $X$-chiralities of both signs, hence at least two flavons to generate lepton masses. But it leads to models with too large cutoff values to yield any specific physics at the LHC scale.

More generally, we must introduce $X$-chiral strongly and weakly interacting heavy matter to compensate the anomalies generated in the MSSM sector, which has to be vector-like under the SM symmetries, to lie above the weak scale. Our choice here is to preserve the nice MSSM gauge coupling unification and asymptotic freedom. Thus, we can only add SM vector-like matter associated to quarks and leptons filling one or two $\mathbf{5} + \mathbf{5}$ representations of $SU(5)$: quarks, ($\mathcal{Q}_i, \bar{\mathcal{Q}}_i$), and leptons ($\mathcal{L}_i, \bar{\mathcal{L}}_i$), $i = 1, 2$ ($\mathcal{Q}_i$ and $\mathcal{L}_i$ have the same SM charges as $D$’s and $L$’s, respectively). Their total $X$-chiralities, are the traces of the matrices (lowercase letters are the corresponding $X$-charges):

$$X^0_{ij} = (\delta_i + \bar{\delta}_j) \quad X^i_{ij} = (l_i + \bar{l}_j).$$

(12)

Correspondingly, their mass matrix elements are $m^0_{ij} \sim e^{[X^0_{ij}]} \Lambda$ and $m^i_{ij} \sim e^{[X^i_{ij}]} \Lambda$, respectively.
Here we focus on anomalies quadratic in the SM vector-like charges, namely, colour and $Q_{em}$, directly related to the Yukawa matrices through the $X$-chiralities defined in (3) and (4). Gathering the contributions from the MSSM states as well as the possible new heavy states, the anomalies to be cancelled are:

\begin{align}
A_C &= \text{Tr} \left[ X^u + X^d + X^e \right] - 3X_\mu, \\
A_{em} - \frac{4}{3}A_C &= \text{Tr} \left[ X^e - X^d - X^e + X^l \right] + X_\mu.
\end{align}

Hence anomaly cancellation means:

\begin{align}
\text{Tr} X^d &= -\text{Tr} \left[ X^u + X^d \right] + 3X_\mu, \\
\text{Tr} X^l &= \text{Tr} X^d + \text{Tr} \left[ X^d - X^e \right] - X_\mu.
\end{align}

Since $X^u$ and $X^d$ are non-negative matrices, we can replace (4) and (5) into (14) to get

\begin{equation}
\text{Tr} X^d = 3 \left( X_\mu - 6 - x \right),
\end{equation}

First note that (15) excludes $X_\mu \leq 3$ which leads to $m^2_i \ll \mu$, too low to have escaped present experiments - and, anyhow, a cutoff too low to suppress FCNC/CP effects, see below. Without $X$-chiral heavy matter, $A_C = 0$ implies $X_\mu = 6 + x$, hence a cutoff $\Lambda \gtrsim e^{-6}\mu \sim 10^4$ TeV. Any direct evidence for the model would show up far beyond the LHC reach. In order to have observable TeV-scale phenomena we need to introduce appropriate heavy states and $X_\mu = 4$ or 5.

Now, let is define the difference $w = \text{Tr} X^e - \text{Tr} X^l$ and replace the fit to the fermions masses into the second relation in (14) to obtain,

\begin{equation}
\text{Tr} \left[ |X^e| - X^e \right] = X_\mu + z - w.
\end{equation}

Notice that without extra-matter, it follows that at least one of $X^e_i$ must be negative, hence that anomaly cancellation within the MSSM field content cannot be realized by imposing only one flavon. Therefore, anomaly compensation by new fermions (which always requires two opposite-sign flavons) is the generic case. However the existence of one negative eigenvalue $X^e_i$ is still true for $w < X_\mu$.

The vanishing of the other two anomalies (as well as the pure $U(1)_X$ anomalies) are not so simply related to the fermion mass eigenvalues and $X$-chiralities and will further constrain the charges. Since the latter are fractional, we study in the Appendix the cancellation of the fractional part of the anomalies. The weak anomaly, $A_W$, imposes the choice of the $Z_6$ as in (1), while $A'_{em}$, involving $X^2$, just requires $w = 3n$. As discussed later on, $w \geq 3$ badly spoils gauge coupling unification, and we keep only $w = 0$ hereafter. The integer part of the $X$-charges are not uniquely defined by the cancellations of $A_W$ and $A'_{em}$, the neutrino masses and some constraints from the other mass matrices. They are important for the decay properties of the heavy states, but this is not discussed in this paper to such a level.

Finally, for the relevant values, $X_\mu = 4, 5, w = 0$, one gets the solutions in Table 1, where the only negative $X^e_i$ in each case is also displayed.
### 3.5 New sources of FCNC and CPV

The quadrilinear matter interactions in (2) are also strongly bounded from the experimental limits on FCNC and CP violations so setting a lower limit on the flavour symmetry breaking scale. Bounds on these operators from their FCNC/CP violating effects were numerically studied, e.g., in [?]. For our purposes here, we would rather present an analysis on an order of magnitude footing that looks appropriate to models that only predict orders of magnitude. Since $QH_uD$, $QH_dD$, $LH_dE$ and $H_dH_u$ exist, then so do $UQEL$ and $UQDQ$ with couplings

$$C_{ijkl}^{fe} \sim \epsilon^{X_{ij}^u + X_{kl}^d - X_{ji}^u}$$

and

$$C_{ijkl}^{qq} \sim \epsilon^{X_{ij}^u + X_{kl}^d - X_{ji}^u}. \quad (17)$$

Let us concentrate on the contributions from the operators $UQEL$ and $UQDQ$ to FCNC and CP violating electromagnetic transitions of leptons and quarks: $\ell_j \to \ell_k \gamma$ and $d_j \to d_k \gamma$ through the flavour changing magnetic moments $\mu_{fjk}$ and $\mu_{fjk}$ and the electric dipole moments, $d_{fj}$ and $d_{jk}$. The two-loop diagrams are the supersymmetric analogous to the Barr-Zee one – in the artificial limit where the higgsino mass is very large – and presupposes the needed supersymmetry breaking insertions to contribute to these dipole-type transitions. As usual it corresponds to different interfering contributions that might partially compensate each other according to the choice of the soft mass parameters. For the sake of a generic order of magnitude estimate, we drastically assume those to be the same order of magnitude and replace all by the $\mu$ parameter. Then, up to several $O(1)$ factors, the magnetic and electric dipole moments are roughly given by

$$\mu + i d_{jk} \sim \sum_i C_{ijkl}^{ef} \epsilon^{\alpha_w m_{ui} \mu} \frac{\mu m_{ui}}{\Lambda} \quad (f = e, d) \quad (18)$$

where: the quark mass $m_{ui}$ keeps track of the chirality/weak isospin change, $2\pi$ factors stand for loop integrations, $\alpha_w$ represents the gauge field coupling and, as stressed, $\mu$ stays for the overall dependence on supersymmetry breaking. An estimate of the traditional (one-loop) supersymmetric contributions due to the textures in the so-called $A$-terms to $(\mu + i d)_{jk}$ along the same lines gives $O(e\alpha_w m_{fjk} \tan \beta / 4\pi \mu)$, where the mass matrix elements represent the isospin, flavour and CP violations (of course this choice is only indicative). We are basically considering the Now, let us require that the higher dimension contributions, (18), are at most of the same order of magnitude as those traditional one-loop ones, namely,

$$C_{ijkl}^{ef} \frac{\mu m_{ui}}{\Lambda m_{fjk} \tan \beta} \lesssim O(2\pi) \quad (f = e, d) \quad (19)$$

---

| $w$ | $X_\mu$ | $z$ | $x$ | $X_\varepsilon < 0$ |
|-----|--------|-----|-----|-------------------|
| 0   | 4      | 0   | 0   | $X_\mu^e = -2$   |
| 0   | 4      | 0   | 2   | $X_\varepsilon = -2$ |
| 0   | 5      | 1   | 1   | $X_\mu = -3$     |

Table 1: Solutions to the anomaly conditions, see text.

---

9 Actually we do not know the charged lepton mixing angles and CP phases, we are assuming they are similar in both scalar and fermion masses.
and after replacing (3) and (17) we obtain the constraints,
\[
\Delta \chi^f_{jk} = \left| \chi^u_{ii} + \chi^f_{jk} - \chi^u \right| + \chi^u + \chi^u_{ii} - \left| \chi^f_{jk} \right| \geq -1,
\]
(20)
With the allowed values for the \(X\)-chiralities, this condition is always satisfied. The worst case is for \(i = 3\) and \(\chi^f_{jk} \geq \chi^u\) when \(\Delta \chi^f_{kl} = 0\) from the stop loops. Therefore, the only cases in the balance are \(s \rightarrow d \gamma\), \(b \rightarrow s \gamma\), possibly \(\mu \rightarrow d_k \gamma\), as well as to \(d_e\) and \(d_d\) for some choices of \(X\)-chiralities.

The conclusion of this order of magnitude analysis is that the flavour/CP issues related to the \(UQEL\) and \(UQDQ\) terms in (2) are not worse than the standard MSSM \(A\)-term contributions. Notice that the 1-loop and the 2-loop contributions are inversely proportional to \(\mu\) and \(\Lambda\), respectively, so that their ratio gets a factor \(\epsilon^{X^u}\). The explicit calculation of the bounds on \(\Lambda\) in [33] agree with our rough estimate within the many uncertainties\(^{10}\). Therefore, the models discussed here will be typically as sensitive to the next round of FCNC/CP experiments as the renormalizable MSSM, even for unflavoured real soft terms.

Of course, by integrating out the \(U(1)_{X}\) gauge sector and flavon fields one generates further contributions to \(C_{ijkl}^{qe}\), \(C_{ijkl}^{qq}\), as well as to \(C_h\). The term generated from the later can be written
\[
\frac{1}{8\epsilon^2\Lambda^3} \left( \frac{\partial W_{\text{MSSM}}}{\partial \epsilon} \right)^2
\]
(21)
where \(W_{\text{MSSM}}\) is the superpotential (2) with the couplings replaced by the corresponding power of \(\epsilon\). Because of a factor \(v^2/\Lambda^2\) the contributions to \(C_{ijkl}^{qe}\) and \(C_{ijkl}^{qq}\) are sub-leading while the flavon exchange is \(\epsilon^{-2}\) larger than the original \(C_h\), but still too small to be relevant.

Integrating out the gauge sector to define the supersymmetric Fermi approximation, one obtains a four-fermion current-current interaction with a cutoff (equivalent to \(G_F\)) \(\epsilon^{-2} \Lambda^{-2} < O(10^{-5}) G_F\). The flavour diagonal components of these operators are therefore reduced by a factor \(O(10^{-10})\) with respect to the similar weak interaction transition rates in the Fermion limit. The FCNC components associated to a transition between the fermions \(f\) and \(f'\) are reduced by their mixing angle, \(\sim \epsilon |f' - f|\) in the notation we adopt here. That is enough to lessen the FCNC effects and fulfill the experimental limits, but for the lowest values of \(\Lambda\) the results are sometimes critical and will be tested by future experimental improvements.

### 4 Exotic matter below the TeV

Several properties of the new heavy states are fixed from the conditions and results stated in the previous sections. We now turn to show how they point to their masses being around the TeV and their couplings to the known quarks and leptons being exotic. For this sake we impose approximate gauge coupling unification and ask the discrete symmetry to forbid the heavy states to mix to SM ones in the mass matrices but without making them stable. In this sense, the new matter hold exotic baryon and lepton numbers. We also simplify the analysis by considering more generic cases and skipping more peculiar issues since our aim is to define a series of robust benchmark models.

\(^{10}\)In comparing the results one must rescale the couplings, taken \(O(1)\) in [33], by the corresponding power of \(\epsilon\), so reducing the limit on the cutoff by many orders of magnitude.
4.1 Masses

If one wants to preserve gauge coupling unification at a level close to that of the MSSM, the masses of the heavy leptons, \(m_{\text{L}_i}\), and heavy quarks \(m_{\text{D}_i}\) cannot differ too much. Indeed, their (one-loop) contribution to the difference between the strong and weak couplings at \(m_Z\) are given in terms of their mass matrices by

\[
\Delta \left( \alpha_s^{-1} - \alpha_2^{-1} \right) = \frac{1}{2\pi} \ln \det \frac{m_\Sigma}{m_\Delta}, \tag{22}
\]

The experimental uncertainties on this difference is \(O(0.12)\) and for the new contributions not to be larger than this uncertainty, we should impose \(0.5 \lesssim \det(m_\Sigma/m_\Delta) \lesssim 2\). To translates it into a condition on charges, we have to fix the ambiguity in the pairing of the indices in the \(X\)-chiralities defined in (12).

We notice that, in the absence of fine-tuning, there is always a choice - not necessarily the one adopted later on - such that \(\ln \det m_\Sigma \simeq \text{Tr} |X_l| \ln \epsilon\), and similarlly for \(m_D\). With these choices we get

\[-0.5 \leq \text{Tr}|X^l| - \text{Tr}|X^0| \leq 0.5 \tag{23}\]

Since \(\text{Tr}|X^l| \geq |\text{Tr}|X^d|\) and \(\text{Tr}|X^0| \geq |\text{Tr}|X^0|\), one cannot obtain a definite limit on the difference \(w\) defined above, but the limits in (23) suggest to take the matrix \(X^l\) as close as possible to \(X^0\), namely, \(l_i \simeq \bar{d}_i\) and \(\bar{l}_j \simeq \bar{d}_j\) unless (23) is checked case by case.

Basically, the LHC could detect heavy quarks and, possibly, leptons whose masses are \(O(\mu)\). To discuss this condition, it is convenient to redefine the indices in such a way that \(|d_2 + \bar{d}_2| = \min |d_i + \bar{d}_j|\), so that the mass eigenvalues satisfy:

\[m_{\text{D}_2} \sim e^{|d_2 + \bar{d}_2|}, \quad m_{\text{D}_1} \lesssim e^{|d_1 + \bar{d}_1|}. \tag{24}\]

From the QCD anomaly condition (15), and the condition that the lightest heavy quark mass must be at least \(O(\epsilon\mu)\), we have

\[X_\mu + 1 \geq |d_1 + \bar{d}_1| \geq \frac{3}{2} (6 + x - X_\mu) \tag{25}\]

This implies \(X_\mu > 3\) to avoid conflict with experimental limits on heavy quarks, leaving only two possibilities, \(X_\mu = 4, 5\). The solutions to (25) are displayed in the Table 2, where the masses are given by their ratios to \(\mu\) in units of \(\epsilon\).

Therefore, after the Higgs \(X\)-chirality is chosen to allow for low energy flavour symmetry, and to fulfil the anomaly cancellation relations without states too light to have escaped observation, one ends with: \(X_\mu = 4\ or 5\), corresponding to a cutoff \(\Lambda \sim 400\mu\), and \(\Lambda \sim 2000\mu\) respectively; and several possibilities for the masses of heavy “quarks and leptons”. Notice that many of these solutions correspond to the presence of a heavy quark within the LHC reach, namely, with masses \(O(\mu)\) or even \(O(\epsilon\mu)\) for large enough \(\mu\). Of course, their detection depends also on the decay properties, which we turn do discuss.

4.2 Decays

Fields with the same SM and \(Z_6\) quantum numbers can mix in the mass matrices. We do not want \(\Sigma H_u/\Sigma H_d\), \(\Sigma L\) and \(\Sigma D\) mass couplings that might destabilize the assumed

\[\text{Indeed, in general, the C-conjugated states defined by the mass eigenstates are not eigenstates of the broken charge } X, \text{ unless these states differ by their transformation under the discrete symmetry}\]
Table 2: Solutions to the anomaly conditions: $X_μ$ is the higgsino $X$-chirality, $x$ is related to $\tan β$ as defined in the text, $δ_1 + δ_i$ are the $X$-chiralities of the heavy antiquarks, $TrX^0$ is their contribution to the anomalies. The (orders of magnitude of the) masses of the heavy “quarks/anti-quarks” corresponding to each solution are given in units of the higgsino mass as powers of the Cabbibbo angle, $ε$. The symbols in the last row denote one of the following situation with respect to the heavy quark range to be scanned at the LHC: within (√), already excluded or within (!), above or within (?) and much above (~).

| $X_μ$  | 4 | 4 | 4 | 5 | 5 | 5 | 5 |
|--------|---|---|---|---|---|---|---|
| $x$    | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $TrX^0$| 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| $δ_1 + δ_1$ | -5 | -4 | -3 | -6 | -5 | -4 | -3 |
| $δ_2 + δ_2$ | -1 | -2 | -3 | 0 | -1 | -2 | -3 |
| $m_{δ_1/μ}$ | $ε$ | $ε^0$ | $ε^{-1}$ | $ε$ | $ε^0$ | $ε^{-1}$ | $ε^2$ |
| $m_{δ_2/μ}$ | $ε^{-3}$ | $ε^{-2}$ | $ε^{-1}$ | $ε^{-5}$ | $ε^{-4}$ | $ε^{-3}$ | $ε^{-2}$ |
| N.B.   | ! | √ | ? | ! | √ | ? | ~ |

light mass matrices (though this might be an interesting alternative in some cases) and we naturally implement it by the choice of the $Z_6$ charges, $Z_i$. From Eq. (1), this amounts to choose: $Z_6 ≠ 1/6, -2/6$ and $Z_6 ≠ -1/6$.

For these states to be unstable and have at least one decay channel into MSSM states one ask such a coupling with dimension four or five (up to quadrilinear in the superpotential, trilinear in the Kähler potential). From Eq. (1) one selects the $SU(3) ⊗ SU(2) ⊗ U(1) ⊗ Z_6$ invariant operators according to $Z_6$ and $Z_6$. The solution $Z_6 = 0$ is unique and leads to the operator $QQQQL$ while there are three solutions for $Z_6$ which we list below together with the respective allowed exotic superpotential operators:

- $Z_6 = 3/6, Z_6 = 0$: $QQQQL, UUDL, QDLD, LDLDL$; the first two cause the decay of heavies into three MSSM particles.
- $Z_6 = 2/6, Z_6 = 0$: $QQQQL, QLDL, EUDDL, LDHUDDL, DDLDL$; the first causes the decay of $L$ into three quarks; the decay of $D$ into a quark plus a lepton happens mainly due to the second and the third Yukawa couplings.
- $Z_6 = 0, Z_6 = 0$: $QQQQL, QDD, UDDL, QLDDL, QADDL, DDLDH, DDLH, DUHDL$; the first causes the decay of $L$ into three (s)quarks; the decay of $D$ into a quark and a squark happens due to the second and the third term.

The quantum numbers and main decays are collected in Table 3 for the two allowed values of the higgsino chirality. They can be retrieved from the $Z_6$ quantum numbers of the $SU(3) ⊗ SU(2) ⊗ U(1)$ operators, or equivalently the exact “proton hexality” defined by the $Z'$ quantum numbers above. The change in the phase of an operator is:

$$\frac{2πi}{18}(n_{\text{light quarks}} + 9 \cdot n_{\text{light leptons}} + δ \cdot n_{\text{heavy quarks}} - 3 \cdot n_{\text{heavy leptons}}),$$

where $n_{\text{light quarks}} + n_{\text{heavy quarks}}$ is a multiple of 3, heavy “quarks” (“antiquarks”) are associated to the $\bar{U}$, $(\bar{D})$, resp.) fields in view of their charge and colour assignments, and $δ = 7, -8, -2$, for $Z_6 = 3/6, 2/6$ and 0, respectively.
Table 3: Main decays of exotic quarks and leptons.

| $Z_D$ | 0 | 1/3 | 1/2 |
|-------|---|-----|-----|
| $Z_D$ |   |     |     |
| $D$   | UD| QL | UUE |
| $D$   | QQ| UE |     |
| $L$   |   | QQQ|

The two $D_i$’s may have different $Z_6$ charges and the heavier states could also only decay by cascading.

Lifetimes and flavour structures of the decay products are fixed by further defining the different charges in each model, consistently with the conditions below. This would be imperative to distinguish these exotic particles in the busy environment of a LHC events, but this study is not pursued here.

4.3 Further constraints

Now that the $X$-chiralities are basically fixed on a rather generic basis, one has to proceed and determine the $X$ charges. First of all, the additional anomaly cancellation conditions have to be satisfied. Of course the system is yet underdetermined but there are several crosschecks that are needed.

For some of the $Z_D$ choices, integrating out the heavy quark introduce new contributions of the operators discussed in ?? 17. The relative change in the couplings $C$ in 17 is easily checked to satisfy, in all the solutions obtained so far, the inequality:

$$
\Delta C_{ijkl}^{qq} \lesssim \epsilon |X^u_{ij} + X^d_{kl} - X^\mu| - |X^q| = \epsilon X^u_{ij} + X^d_{kl} - X^\mu,
$$

which is dangerous for $X^f_{jk} \geq X^\mu$, much the same as for (??). The actual condition in this case depends on the explicit charges. Thus, the selection criterion that the models comply with FCNC/CPV bounds could marginalise some of the solutions to the anomaly conditions.

Finally, the mass matrices for leptons and for the heavy particles must be checked against large deviations from the assumed pattern. In the case of the heavy states, the choice of the $Z_6$ quantum numbers might be helpful in model building.

Of course all these issues must be addressed together with some choice of the supersymmetry breaking parameters to define complete benchmark models that can be compared to experiments. But this goes beyond the scope of the paper.

5 Conclusions

In this paper, we argue that gauged flavour theories generically require new states to compensate for anomalies from quarks and leptons in chiral representations of the gauged flavour group and that QCD asymptotic freedom and some level of gauge coupling unification favour their masses being close to the higgsino mass, or $\mu$-term, of supersymmetric theories. This has been explicitly shown in supersymmetric models with a single $U(1)$ flavour group which, after its breaking, delivers discrete baryon and lepton symmetries.
that forbid dangerous processes such as proton decay as well as mixings between the new and the MSSM states.

As these new particles are often predicted to lie around the TeV scale, they provide a test for the flavour theories, which are hardly testable otherwise. They have exotic discrete baryon and lepton numbers, hence peculiar decay modes, although their signatures are model dependent and not necessarily distinguishing in the busy LHC environment. On the other hand, the flavour gauge boson could also have few TeV mass if its coupling is small enough, but its production rate is too low for its detection at the LHC.

The higher dimension dimension operators that are sources of FCNC/CPV supersymmetric operators cannot be all suppressed enough in these models, designed to explain fermion masses, so that the cutoff $\Lambda$, cannot be below 500 TeV, although relatively model dependent. In the models studied here, from the requirement that the small $\mu/\Lambda$ ratio is explained in terms of flavour symmetry: (i) there is a similar lower bound if asymptotic freedom is imposed to limit the number of heavy states; (ii) these theories are not testable for a cutoff beyond $10^4$ TeV. Actually the anomalies cannot be reasonably compensated between, say, $10^6$ TeV and the Planck mass where the Dine-Witten mechanism becomes effective, in this class of models.

Concerning the fact that these models do not solve the conceptual $\mu$-problem, it is worth recalling its situation in the basic mechanisms of gauge mediation (GMSB) and supergravity mediation (mSUGRA). The family independence of the effective supersymmetry breaking Lagrangian is implemented by the assumption that the breaking occurs in a hidden sector, such that the Kähler potential is separable as $K_{\text{MSSM}} + K_{\text{hidden}}$, and so is the superpotential as $W_{\text{MSSM}} + W_{\text{hidden}}$. The mediation in GMSB models comes from a common gauge-invariance which is family independent while in mSUGRA it comes from the Kähler-invariance which is universal for terms with the same dimensionality in $W_{\text{MSSM}}$.

Once the potential $\mu$-term is suppressed by some symmetry, a new one can be engendered from supersymmetry breaking but at the price of an ad hoc violation of the separability principle or by the addition of a singlet with the appropriate charge (NMSSM) whose v.e.v. has also to be protected. In this sense, the $\mu$-term discussed here could indeed be considered as just the suppressed supersymmetric higgsino mass. Therefore it should be at most comparable to the effective $\mu$-term and all the results remain valid as upper-bounds, and the new states could be even a bit lower in mass! Since the Giudice-Masiero contribution would be suppressed as already noticed, the NMSSM singlet mechanism seems the natural solution to produce most of the effective $\mu$-term.

Of course the charges in the models developed here are quite confusing although they are largely dictated by the known quark and lepton masses and mixings, and it seems difficult to conceive a UV completion yielding such a structure. These models are then consistent but not quite convincing at least for this reason. Also, they do not predict precise empirical properties of the mass matrices. These shortcomings could be remedied by introducing non-abelian flavour symmetries (or, at least, several abelian ones) so to replace large charges by small symmetry breaking scales, should it seem more satisfactory. As already stated in the introduction, the arguments of this paper could be transposed to these more complex cases. In the present all-in-$U(1)$ model the MSSM contributions to the anomalies are dominated by the large charge-chiralities associated to light quark, neutrino or the higgsino masses asking for new states with similar flavour-chiralities. Analogously,

\footnote{But note that, in the simplest case, they are all even and reduce to 4, 2, 1 or 0 by taking $\epsilon \sim \theta_C^2$.}
the lighter heavy states will be associated to the anomalies of the symmetries broken at the lowest scale, presumably in correspondence with the same fermions.

Finally, let us comment on the non-supersymmetric counterpart of these flavour theories with a cutoff bounded by neutrino masses and FCNC/CPV restrictions as above, the last being a bit less constraining because of the higher dimension of the dangerous operators. In this case, one needs only one Higgs doublet and one flavon field and, assuming that the Higgs mass can be fixed, the analysis is quite similar to the supersymmetric version, but for the absence of the $\mu$-term and the corresponding higgsino chirality. This increases the SM anomalies to be compensated but, because of the change in the $\beta$-functions, one can take advantage of a larger number of new fermions consistent with asymptotic freedom to do it and even to implement gauge coupling unification.

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A Appendix: Other anomalies

We study here the cancellation of the other two anomalies, $A_W$ and $A'_{em}$ and, in particular the vanishing of the fractional contributions related to the conserved discrete symmetry. While the previously discussed anomalies involve only the $X$-chiralities, these two additional ones constrain the $X$-charges themselves. We shall just cancel the fractional part of the anomalies, $\text{frac}(A_W)$ and $\text{frac}(A'_{em})$ since the integer part can be eliminated by two combinations of the various (integer parts of) the charges, $\text{int}(X_i)$ or $\text{int}(X'_i)$, with many solutions that we do not discuss here, although they are relevant for the properties of the heavy state decays.

For this purpose, notice that the conserved symmetry correspond to charges $Z'_i$ such that: (i) they change sign under charge conjugation, hence all $X$-chiralities are integers, (ii) the experimental flavour mixing for quarks and leptons require the $Z'$ to be generation independent. Therefore one has $Z'_Q = Z'_Q = Z'_L = Z'_L = Z'_E = Z'_E = Z'_D = Z'_D = Z'_L = Z'_L$. Furthermore, the neutrino mass imposes $\text{frac}(2Z'_L) = 0$. At the exotic side, the phenomenological constraints in section (??) gives $Z'_{\xi_i} = 0$ and $Z'_{\bar{D}_i} = \delta/18$, with $\delta = 2, 8, -7$.

First consider the weak isospin anomaly, which we write for convenience in terms of $X'$, as

$$A_W = \text{Tr} X' T^2_3 = \lambda'_\mu + \sum_{i=1}^3 (3q_i + l_i) + \text{Tr} X'd = 0,$$

$$\text{frac}(A_W) = \text{frac}(9Z'_Q + 3Z'_L) = 0,$$
and notice, besides the well-known solution, \( Z' \propto B - L \), which allows for the proton decay, the choice \( Z' = (B - 3L)/6 \), which forbids it and is chosen here, when applied to the MSSM states.

The \( A'_{em} = \text{Tr}X'Q^2_{em} \) anomaly reads,

\[
A'_{em} = h_u^2 - h_d^2 + \sum_i \left[ 2 \left( q_i^2 - \bar{u}_i^2 \right) - \left( q_i^2 - \bar{d}_i^2 \right) - \left( l_i^2 + \bar{e}_i^2 \right) \right] + \sum_i \left[ \left( \delta_i^2 - \delta_i^2 \right) - \left( \delta_i^2 - \delta_i^2 \right) \right] .
\]

and its fractional part is then,

\[
\text{frac} \left[ 2 \left( 2\text{Tr}X^u - \text{Tr}X^d - 6h_u + 3h_d \right) Z'_Q - 2 \left( \text{Tr}X^e - 3h_d \right) Z'_L + 2\text{Tr}X^a Z'_D - 2\text{Tr}X^d Z'_L \right]
\]

Interestingly enough, when the \( Z'_i \), the traces of the matrices given by (4, 5, 6) in section (3.1) and the solutions to the anomaly cancelation conditions (14), (15) and (16) of section (3.4) are all replaced in this expression, we get a very simple result for its cancellation for any of the three values of \( \delta_i \), namely,

\[
A'_{em} = -\frac{w}{3} + \text{integer} = 0
\]

This requires \( w = 0 \) corresponding to the approximate equality between the products of masses of the exotic heavy quarks and of the exotic heavy leptons, otherwise the gauge coupling unification would be badly violated for \( |w| = 3 \) or larger, as previously discussed.

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