Irreducible specific energy of new surfaces creation in materials with crack-type macro defects under pulse action

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Abstract. The study of destruction of samples with crack-type macro defects in shockwave microsecond duration range mode with amplitude up to 1 GPa was carried out using the magnetic pulse method of pressure pulse creation. The result analysis held on the basis of computer modeling of stressed condition and thermodynamic approach. The relation between the surface fracture energy and the material parameter, such as the energy accumulation time required for destruction, was revealed.

1. Introduction
Obviously, the generation and the growth of cracks is one of the major mechanisms of materials failure. The study of this phenomenon is the subject of many experimental and computational and analytical works. The structure of the material, the type of the stress state, the loading rate and other factors lead to the variety of observed features of the fracture process.

Choice of certain stressed state can significantly reduce the number of interacting factors and facilitate the analysis of the failure process. The results of the study of brittle materials fracture process under shock-wave loading mode and the generalization of these results are given in this work.

Tests, carried out with various materials, exhibit the threshold nature of fracture and increase of failure amplitude in response to loading pulse contraction during destruction of both defect-free samples, and those with crack-type macro-defects [1–5]. The results of experimental research and analysis of material destruction during the exposure of magnetic pressure pulse action are represented below.

2. Testing of samples with crack-type macro-defects
In addition to spallation loading schemes, the magnetic pulse technique also enables to conduct pulse loading while studying the destruction of samples with crack-type macro-defects [6, 7] under shock-wave stress state conditions. This is valid for a number of applied technologies in the sphere of finely ground material production and ore mineral concentration, as well as in the sphere of natural and man-induced earthquakes [8].

Fracture of samples, made of various materials, was studied, following the magnetic pulse loading technique, described in [7]. Samples with typical sizes, given in figure 1, were used for
these experiments. By exposing such samples to loading at micro-second loading duration, the magnetic pulse shock conditions are formed, as opposed to experimental conditions, described in [9, 10]. Due to the symmetric nature of loading and sample symmetry, propagation of the main crack (MC) is nearly unidirectional [1].

Shown in figure 2—typical dependences of the propagated crack length in samples with crack-type macro-defects, loaded along the macro fracture zones in accordance with mode by uniformly applied pulse pressure, on pulse amplitude and loading duration. As illustrated by polymethylmethacrylate (PMMA) fracture (curves 7–9 in figure 2), one can see the growth of threshold fracture loads, concurrently with the exposure time decrease. Threshold fracture loads $P_{tr}$ can be specified by projecting the propagated crack length $L_{ct}$ into the area $L_{ct} \rightarrow 0$, as illustrated by curve 4.

General in the dependencies $L_{ct}(P_{m})$ are the threshold nature of the destruction and the linear growth of the crack length when the load increases.

3. Experimental data analysis
The analysis of experimental data, related to crippling tests under mechanic action, can be reduced to detection of the stress-strain state of a sample and choosing a relevant fracture criterion.

3.1. Destruction criterion
Under dynamic (pulse) loading the potential energy is defined by time-dependent parameters $P(t)$ and $V(t)$. Given that the system transition from one state into another state takes time, we shall reckon that the system transition into the destroyed state has been completed, once the potential energy achieves the ultimate value $P_{m}V_{m}$ within a certain time interval $\tau_{L}$. This statement can take the form

$$\frac{1}{\tau_{L}} \int_{0}^{t} P(t)V(t)\,ds \leq P_{m}V_{m},$$

which, when violated, corresponds to system destruction.
Figure 2. Dependences of the propagated crack length on the single loading pulse amplitude at loading duration $T_p$: 1—limestone, 4.4 $\mu$s; 2—marble, 3.6 $\mu$s; 3—gabbro-diabase, 3.6 $\mu$s; 4—sandstone, 3.6 $\mu$s; 5—granite, 3.6 $\mu$s; 6—polymer compound—1.5 $\mu$s [11]; PMMA: 7—2 $\mu$s; 8—4 $\mu$s; 9—8.6 $\mu$s [6]. I—pulse waveform.

To describe the system behavior in conditions of one-dimensional elongation, implemented during material testing in accordance with spallation destruction pattern, the expression (1) can take the form:

$$\frac{1}{\tau_L} \max_x \int_0^t \varepsilon(s,x)\sigma(s,x)ds \leq \varepsilon_m \sigma_0,$$

(2)

where $\varepsilon(s,x)$ and $\sigma(s,x)$—actual values of strain and stress at a point with coordinate $x$, $\varepsilon_m$—strain value that corresponds to material breaking strength $\sigma_0$ under static conditions.

The justification of the applicability of this approach is given in [12]. In [13] to describe fracture processes in metals, exposed to pulse loading, the chosen criterion, conceptually close to (2), is based on specification of the material threshold energy, released in critical area within a certain time interval, or energy transfer time.

3.2. Threshold destruction loading

In [14, 15] one can find data for some rocks from testing, performed under spallation loading conditions, with the use of the magnetic pulse loading technique.

Simple stress condition, occurring at spallation area under known parameters of the load pulse, allows us to calculate the parameter $\tau_L$, while taking into account (2), that is listed in table 1.

In the event of testing samples with crack-type macro-defects, their results given in figure 2, the stress condition at the crack apex is largely predetermined by its design. The experiments, described above, were set with the use of standard samples, their general view given in figure 1. Numerical analysis of wave loading of samples, conducted in ANSYS [16] environment, consistently with actual sizes of pre-crack and loading pulse time parameters, shows that actual sizes of the fracture apex have immense influence on potential energy variation, traced at a maximum stress point. Concurrently, there can be observed a significant difference between
calculation data for defects with actual sizes and those to be calculated while taking into account an approach described in [17] for a crack tip. In figure 3 are given the values of the parameter

\[ \text{PE}(t) = \int_0^t \varepsilon(s, x) \sigma(s, x) \, ds \] (3)

at a point M (see figure 1), which corresponds to maximum stresses.

Based on the analysis of numerical simulation of stressed state at the apex of the crack with actual sizes, it is possible both to describe the observed in [18] delay of fracture process \( \tau_d \) of PMMA samples, and to find the parameter \( \tau_L \), its value coinciding with the one, based on the results of the spallation loading pattern sample testing, with an accuracy of ±5%. As one can conclude from the simulation results, represented in figure 3, the fulfillment of the condition (2) would occur at different moments of time, depending on the nature of the macro defect and loading duration.

Basic mechanical properties of the materials, tested with application of the magnetic pulse technique, are given in table 1.

3.3. Fracture energy at the pulse action

Let us consider the behavior of the dependencies \( L_{cr}(P_m) \), obtained for different materials in similar loading time modes, represented in figure 2. In each case these dependencies are characterized by threshold behavior. When \( P_m > P_{tr} \) the dependence of the propagated crack length on the amplitude of a pressure pulse with certain duration is expressed by:

\[ L_{cr}(P_m) \approx \frac{dL_{cr}(P_m)}{dP_m} \bigg|_{P_{tr}} (P_m - P_{tr}) = k_{LP}(P_m - P_{tr}). \] (4)

Functional parameters, represented in figure 4, \( A = P_{cr}/\tau_L \) and \( B = k_{LP}/\tau_L \), can be explained as constant values \( A = \text{const}_A(\tau_L) \) and \( B = \text{const}_B(\tau_L) \) at a given value of exposure time.
Table 1. Properties of the materials, tested with application of the magnetic pulse technique.

| No | Materials       | Static tensile strength $\sigma_0$, MPa | Longitudinal wave velocity $c_1$, m/s | Density $\rho$, kg/m$^3$ | Young’s modulus $E$, GPa | Energy accumulation time $\tau_L$, $\mu$s |
|----|----------------|--------------------------------------|--------------------------------------|--------------------------|--------------------------|-----------------------------------------|
| 1  | Limestone      | 12.4                                 | 3780                                 | 2570                     | 48                       | 1.75                                    |
| 2  | Marble         | 6.2                                  | 3790                                 | 2550                     | 67                       | 2.75                                    |
| 3  | PMMA           | 72                                   | 2450                                 | 1140                     | 3.97                     | 4.2                                     |
| 4  | Gabbro diabase | 17.5                                 | 5600                                 | 3286                     | 76                       | 11                                      |
| 5  | Granite        | 5.2                                  | 4250                                 | 2670                     | 35                       | 12.4                                    |
| 6  | Sandstone      | 4.12                                 | 5100                                 | 2520                     | 39                       | 14.7                                    |

Figure 4. Fracture process parameters: numbers 1–6 correspond to materials represented in respective lines in table 1.

The distinctive features were observed in loading mode with characteristic pulse duration of the order of 4 microseconds. Extra tests with other values of loading pulse duration were set for material No. 3 (see figure 4). The shorter the fracture pulse is, the steeper is the parameter $A(\tau_L)$ and the parameter $B(\tau_L)$, while the longer fracture pulse duration causes an opposite effect. Test points, corresponding to the fast loading mode, are marked as $3f$, while those, corresponding to the slow loading mode, as $3s$.

The dependence $C(\tau_L) = (dL_{cr}/dP_m)P_{cr}$ (figure 4) illustrates the lack of response to exposure time. Functional parameter $C(\tau_L)$ can be explained as spatial scale of accumulation area of the energy, needed to initiate the destruction process (crack propagation start), or the critical zone.

All studied samples are characterized by efficient exposure to shock-wave loading and generic stress state, when values of critical threshold loading at the crack edge $P_{cr}$ and the stress value at the crack apex are related by parameter $k_{Pr}$, which depends on loading duration.

The results of numerical simulation in ANSYS environment [16] of the stressed state at the crack apex with account of various values of loading pulse duration for typical samples (see
figure 1) show that, for a pulse with duration \( T = 4 \mu s \), the parameter \( k_{P\sigma} = 1.38 \), and it increases nearly linearly with the growth of loading pulse duration within the analyzed range \( (k_{P\sigma} = 1.92 \text{ at } T = 6 \mu s; k_{P\sigma} = 2.42 \text{ at } T = 8 \mu s) \).

According to Griffith [19], for crack propagation the stored energy should suffice to produce new surfaces. The surface energy \( \gamma \), released in process of elastic medium fracturing together with the occurrence of new surfaces, is related to the energy, stored in the medium, by expression
\[
\gamma = \pi \sigma^2 l/E,
\]
where \( E \) — Young’s modulus, \( l \) — crack length.

This expression, with the account of simulation results, can be written as \( \gamma(P_m) = (\pi/E)k_{P\sigma}^2P_m^2L \). Assuming the continuity of functions \( \gamma(P_{cr}) \) and \( P_{cr}(\tau_L) \), the derivative of surface energy can be expressed as
\[
\frac{d\gamma}{dP_m} = \frac{\pi}{E}k_{P\sigma}^2 \left( 2P_m \frac{dP_m}{d\tau_L} L_{cr} + P_m^2 \frac{dL_{cr}}{dP_m} \frac{dP_m}{d\tau_L} \right).
\]

The first summand represents the surface energy change in process of crack propagation, the second—represents the energy, needed for this process, and accumulated in the area of crack propagation. At a point, corresponding to \( P_{tr} \) the crack length is \( L_{cr}(P_m) = 0 \), therefore, the first summand of expression (6) becomes zero. Thus, when critical value is achieved at the area, corresponding to maximum value of the criterion (3), the accumulated energy becomes sufficient for a crack to start. Dividing both sides of the expression (6) by \( \tau_L \) we get
\[
\frac{1}{\tau_L} \frac{d\gamma}{dP_m} = \frac{\pi}{E}k_{P\sigma}^2 \frac{P_m^2}{P_m^2} \frac{dL_{cr}}{d\tau_L} \frac{dP_m}{d\tau_L} \frac{1}{\tau_L}.
\]

With respect to experimentally revealed dependencies \( P_{tr}/\tau_L = \text{const}_A(\tau_L) = A \) and \( k_{LP}/\tau_L = \text{const}_B(\tau_L) = B \) we get expression:
\[
\frac{d\gamma}{dP_m} = \frac{\pi}{E}k_{P\sigma}^2 A^2 B \tau_L^2.
\]

Upon integrating (8), the relation between the surface fracture energy, actualized at the crack starting moment, and the parameter of material \( \tau_L \) is revealed:
\[
\gamma|_{P_{tr}}(\tau_L) = \frac{1}{4} \frac{\pi}{E}k_{P\sigma}^2 A^3 B \tau_L^4 + C_\gamma.
\]

The obtained dependence, accurate to the constant of integration \( C_\gamma \), can be seen in figure 5, and represents the increase of energy, needed for a crack to start, as related to the increase of \( \tau_L \). Such behavior of the dependence \( \gamma|_{P_{tr}}(\tau_L) \) can also be related to the growth of critical zone, the scale of which \( C(\tau_L) \) also grows together with the increase of energy accumulation time \( \tau_L \).

Analysis of the relation (9), exemplified by material 3 (figure 4), shows that the starting surface energy increases together with the increase of the loading rate. This observation is consistent with the data in [12], where it is shown that the surface energy of fracture increases with the load growth rate. Furthermore, the increase of the surface fracture energy at the growth of loading rate is observed in [20] and in experiments for metal destruction by spallation [21].
4. Brief conclusion
Spatial scale of fracture process determines the dimensions of the area, where energy, necessary for the main crack to start, is being accumulated. When critical energy, introduced into this area, exceeds the threshold, defined by properties of this material within the time interval, or energy accumulation time, specific for this material, it leads to occurrence of dissipative processes and their development ends with the main crack formation.

As in order to elaborate the criterion we considered material properties, corresponding to complete failure of functionality, while these very properties (threshold strength, critical strain) can be established upon destruction of standard samples, this criteria approach is justified when describing destruction processes occurring at macro-level. However, as one can see in [13, 22, 23], in order to analyze the scale hierarchy of the destruction, the thermodynamic approach can also prove to be efficient.

We demonstrate that the use of the criterial approach to describe material fracture test data, obtained by application of different loading patterns, and based on estimation of the threshold potential energy, released in critical area during a specific time interval, allows us to reveal the relation between the principle fracture parameters and the energy accumulation time. This parameter can be regarded as a material property, being part of its basic characteristics, responsible for behavior of the mechanical system when exposed to pulse action.

Acknowledgments
This work is supported by the Russian Federal Targeted Program for the years 2014–2020, grant No. 14.584.21.0019.

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