Probing Dark Energy with Lensing Magnification in Photometric Surveys

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I present an estimator for the angular cross-correlation of two tracers of the cosmological large-scale structure that utilizes redshift information to isolate separate physical contributions. The estimator is derived by solving the Limber equation for a re-weighting of the foreground tracer that nulls either clustering or lensing contributions to the cross-correlation function. Applied to future photometric surveys, the estimator can enhance the measurement of gravitational lensing magnification effects to provide a competitive independent constraint on the dark energy equation of state.

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\textbf{Introduction}  

The two-point correlation functions of galaxies and quasars have proven to be powerful probes of cosmological models [e.g. 1]. Over the next two decades, wide-field astronomical surveys [28] will rely on two-point correlation functions to self-calibrate several types of systematic error [e.g. 2–5] and constrain models of dark energy [e.g. 5–7].

Precise interpretation of correlation function measurements requires knowledge of the line-of-sight distribution of the sources. Astronomical surveys obtain this information either with a spectrograph, which requires long integration times and therefore limits the total number of sources observed, or with broad-band photometry, which allows improved statistical precision at the expense of larger errors in the line-of-sight source distribution.

In this letter, I derive an estimator for cosmological cross-correlation functions that simultaneously removes the ambiguities in theoretical interpretation when the line-of-sight source distributions have large observational errors and maximizes the signal-to-noise ratio (SNR) of shot-noise limited measurements through optimal sample selection. The method of Ref. [8] is complimentary to this letter in optimizing the SNR for sample variance dominated measurements.

While the cross-correlation estimator is broadly applicable, I will focus on the measurement of gravitational lensing magnification, where an optimal estimator promises particularly large scientific gains. Gravitational lensing by cosmological large-scale structure causes the images of background sources to be both magnified and sheared. Lensing magnification alters the apparent number density of background sources by two competing effects, 1) magnifying the area in a given patch of sky thereby reducing the source number density, and 2) increasing the apparent brightness of sources otherwise just below a survey detection limit thereby increasing the observed source number density. The dominant effect is determined by the slope of the differential source number counts as a function of magnitude, with steep slopes yielding increases in the apparent number density.

Most detections of lensing magnification to-date have measured the angular cross-correlation function of two source samples widely separated in redshift [9–11] (although see a novel method in Ref. [12]). In the absence of lensing, such a correlation would be zero, yielding a clean measurement of lensing effects.

The cross-correlation of galaxy shapes to detect lensing shears (cosmic shear) is well established as an important cosmological probe [e.g. 13]. Current and next generation surveys targeted at measuring dark energy properties via cosmic shear will rely on ‘photometric redshifts’ (photo-z’ s) that measure a galaxy spectrum in a few broadband optical filters. While photo-z’s show great promise for statistical measurements of large-scale structure, the large photo-z errors pose several challenges for analysis. The detection of lensing magnification through cross-correlation of photo-z bins is contaminated by the intrinsic clustering of galaxies that are co-located in redshift and poorly separated due to the photo-z errors. This difficulty, along with the intrinsically lower-amplitude correlations, have made lensing magnification a less attractive probe of dark energy properties than cosmic shear in photometric surveys, although magnification has been appreciated as a means of self-calibrating systematic errors [4, 14–16].

Consider then the two-point cross-correlation function of two tracers of the cosmological mass density that are separated, but may partially overlap, in redshift. The observed cross-correlation is the sum of those from intrinsic clustering where the samples overlap in redshift ($\mu_{gg}$), lensing magnification ($\mu$) of background sources by the mass around foreground galaxies ($w_{g\mu}$), and the lensing two-point correlation from line-of-sight structures that magnify both tracer samples ($w_{\mu\mu}$) [17].

The angular cross-correlation function of two catalogs can be estimated by summing the number of source pairings in angular bins ($\theta$) normalized to the expected pairings for catalogs with uniform positions over the sky [18],...
\[ \hat{w}(\theta) = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} w_{i,j} \Delta_\theta (|\theta_i - \theta_j|) / N_{\text{rand}-\text{pairs}}(\theta; u) \]  

where \( \Delta_\theta \) is one if the magnitude of the angular separation vector of source \( i \) and source \( j \) is in the bin centered at \( \theta \) and \( w_{i,j} \) are arbitrary weights for each pair of sources.

We seek a set of weights, \( w_{i,j} \), in the sum over pairs in the cross-correlation function estimator that will minimize or maximize the amplitude of either the clustering (\( w_{gg} \)) or lensing contributions (\( w_{gm} \)) to the observed cross-correlation. Ref. [19] pursued a related goal in attempting to minimize both \( w_{gg} \) and \( w_{gm} \) to measure \( w_{mm} \). This latter term is typically an order-of-magnitude smaller than even \( w_{gg} \), which would require unrealistically large galaxy samples for a statistically significant measurement. Ref. [20] showed that the clustering (\( w_{gg} \)) and magnification (\( w_{gm} \)) terms can be separated using the strong luminosity dependence of the lensing magnification, but assuming sources can be cleanly separated into non-overlapping redshift bins. I take a more general approach for arbitrary cross-correlations that may include large uncertainties in the redshift distributions. I expect luminosity information will only further improve the method in this letter for the measurement of lensing magnification.

**Derivation of the optimal estimator** Each term contributing to the angular cross-correlation of cosmological tracers can be written as a projection of the 3D matter or galaxy correlation function, assuming Limber’s approximation [21], flat-sky approximation, and zero spatial curvature,

\[ w_{XY}(\theta) = \int_0^\infty d\chi W_X(\chi) K(\chi, \theta), \]  

where \( W_X(\chi) \) is either a redshift distribution or lensing kernel [see the appendix of 17] for a given sample, \( K(\chi, \theta) \equiv W_Y(\chi) \bar{\xi}(\chi, \theta) \) is the product of the weight function for the background sample and the 3D galaxy correlation function,

\[ \xi(r) = \int \frac{k dk}{2\pi} P(k) J_0(kr), \]

\( P(k) \) is the 3D matter or galaxy power spectrum, \( J_0 \) is the zeroth order Bessel function and we have neglected redshift space distortions.

Eqn. (2) is a Fredholm integral equation of the first kind and can be solved for \( W_X(\chi) \) to yield a minimum or maximum angular correlation \( w_{XY}(\theta) \). At least for the standard cosmological model the source kernel \( K(\chi, \theta) \) has a non-trivial null space such that there are non-trivial \( W_X(\chi) \) that satisfy \( \int d\chi W_X(\chi) K(\chi, \theta) = 0 \). We will use functions in the null space of \( K(\chi, \theta) \), assuming \( W_Y(\chi) \) is either a lensing or clustering window function, to construct foreground weights \( W_X(\chi) \) that minimize the amplitude of \( w_{XY}(\theta) \) for all \( \theta \).

The eigenvectors of the source kernel provide a convenient means to define an orthogonal basis in the range of the kernel operator. The kernel \( K(\chi, \theta) \) as defined by Eq. 2 is not Hermitian (\( K(\chi, \theta) \neq K(\theta, \chi) \)) due to the presence of \( W_Y(\chi) \) and we cannot be guaranteed to have real eigenvalues and orthogonal eigenvectors. A better kernel is found by considering the square of \( K(\chi, \theta) \),

\[ C(\chi, \chi') \equiv \int d\theta K(\chi, \theta) K(\chi', \theta), \]

with eigenvalue equation,

\[ \int d\chi C(\chi, \chi') \psi(\chi) = \lambda \psi(\chi'). \]

The kernel \( C(\chi, \chi') \) is symmetric and normalizable, yielding orthogonal and non-trivial eigenvectors that can be used to construct a desirable solution for \( W_X(\chi) \).

With the aid of the auxiliary symmetric kernel eigenvectors,

\[ C_{\text{aux}}(\theta, \theta') \equiv \int d\chi K(\chi, \theta) K(\chi', \theta'), \]

\[ \int d\chi C_{\text{aux}}(\theta, \theta') \eta(\theta) = \lambda \eta(\theta'), \]

the original source kernel can be reconstructed [22],

\[ K(\chi, \theta) = \sum_{i=1}^\infty \sqrt{\lambda_i} \hat{\psi}_i(\chi) \hat{\eta}_i(\theta), \]

where hats denote unit normalized eigenvectors with an L2 norm. So, solutions of the symmetric kernel eigenvalue equation also provide solutions for the original source kernel eigenvalue equation.

The algorithm for optimizing the cross-correlation estimator has three steps:

1. Solve for the eigenfunctions of the lensing and clustering symmetric source kernels and define which physical effect is to be maximized (e.g. lensing) and which is to be nulled (e.g. clustering).
2. Find components of the eigenfunctions of the source kernel to be maximized (e.g. lensing) that are in the null space of the source kernel to be nulled (e.g. clustering) using Gram-Schmidt orthogonalization. Construct a basis set from these components for the final weight functions.
3. Solve for a combination of basis functions that optimizes the signal-to-noise ratio to construct the pair weights for the cross-correlation function estimator in Eqn. 1.

The dominant Poisson contribution to the covariance of the angular correlation function is nearly diagonal, yielding a simple expression for the signal-to-noise ratio to
optimize when we neglect sample variance,

$$\text{SNR} = \frac{1}{N_{X,\text{eff}}(w) N_Y} \sum_i w(\theta_i; w)^2 \Delta \theta_i,$$

(8)

where $\Delta \theta_i$ is the width of angular bin $i$, $N_Y$ is the number of sources in catalog $Y$, and the effective number of foreground sources in the catalog $X$ after re-weighting is,

$$N_{X,\text{eff}}(w) = \int dX w^2(X) \frac{dN_X}{dX},$$

(9)

with the convention $-1 \leq w \leq 1$.

**Implementation and example**  
I will describe the implementation of the optimal cross-correlation estimator by means of an example lensing magnification measurement that could be made with the Large Synoptic Survey Telescope (LSST) [29]. The LSST will measure $\sim 10^9$ galaxies with photo-$z$ rms error of 0.05($1+z$) [23]. I consider 4 tomographic bins with centers linearly spaced between $z = 0.5$ and $z = 1.4$ (where the time-dependence of dark energy may be most easily detected). The redshift distributions of the first 2 tomographic bins are depicted by the dashed lines in Fig. 1. This bin spacing is a tradeoff between clean separation in redshift and informative sampling in redshift. I consider only linear clustering predictions for the dark matter, with a maximum wavenumber in the matter power spectrum of $0.1 h$Mpc$^{-1}$ and a multiplicative linear galaxy clustering bias relating the galaxy and matter power spectra, $P_g(k; z_1, z_2) \equiv b_g(z_1)b_g(z_2)P_{mm}(k; z_1, z_2)$.

The optimal pair weights need to be recomputed for every pair of tomographic redshift bins. For two distinct bins in redshift with source distributions $n_1(z)$ and $n_2(z)$, we first solve for the eigenfunctions $\psi_i^{\text{lens}}(\chi)$ of the symmetric kernel $C_{\text{lens}}(\chi, \chi') \equiv W_{\text{Y, lens}}(\chi)W_{\text{Y, lens}}(\chi') \int d\theta \xi(\chi \theta)\xi(\chi' \theta)$ as well as the eigenfunctions $\psi_i^{\text{clust}}(\chi)$ for the analogous clustering kernel.

Then, to down-weight the intrinsic clustering contribution to the final cross-correlation we select the components of $\psi_i^{\text{lens}}(\chi)$ that are orthogonal (with an L2 norm) to all the $\psi_i^{\text{clust}}(\chi)$ for all $i$ up to a pre-specified numerical tolerance in the rank-ordered eigenvalue spectrum. After projection onto the null space of the clustering kernel the eigenfunctions are no longer orthogonal. Let $\hat{\phi}_i^{\text{opt}}(\chi)$ denote the orthonormal basis constructed from the projected eigenfunctions via Gram-Schmidt orthogonalization.

We then define the optimal weights as functions of the line-of-sight distance, $\chi$, as,

$$W_X(\chi) = \sum_{i=1}^{n_{\text{basis-func}}} w_i \hat{\phi}_i^{\text{opt}}(\chi) + \phi(\chi),$$

(10)

where $\phi(\chi)$ is an arbitrary function in the null space of both the lensing and clustering kernels that can be used to minimize the shot noise and $w_i$ are parameters that we will set to optimize the signal-to-noise ratio of the correlation function measurement.

Most solutions for the optimal window will increase the shot noise by down-weighting redshift ranges where the galaxy number density is large. Intuitively, the shot noise should be minimized when the optimal window is similar to the observed redshift distribution of the foreground sample (so that all galaxies have nearly equal magnitude weights). A useful guess for $\phi(\chi)$ in Eq. 10 could then be the original foreground redshift distribution projected into the null space of the lensing and clustering kernels.

I assume in Eq. 10 that the basis functions are sorted in order of decreasing eigenvalue of the kernel to be maximized (e.g. lensing). The total number of basis functions in Eq. 10 is limited in practice by numerical errors in the discretization of the continuous eigenvalue Eq. 5. I use 20 Gauss quadrature weights for the example pair of bins in Fig. 1. By calculating basis vectors of the kernel null spaces and then applying the kernel to these basis functions (which should yield zeros for all $\chi$) I find the first 4 eigenfunctions yield numerical errors at least 3 orders of magnitude below the expected amplitudes of the lensing cross-covariances. The solid red line in Fig. 1 shows the solution from Eq. 10 for the 2-bin example. For this optimal weighting, 9 $\times 10^5$ galaxies are needed in the foreground redshift bin to obtain a signal-to-noise ratio of one.

In Fig. 2 I show the predicted components of the angular cross-correlation function for the 2 redshift bins shown in Fig. 1 assuming the uniform foreground weighting (dashed) and the optimal weighting (solid). Using the optimal window reduces the amplitude of the intrinsic galaxy clustering correlation (red) (due to the partial overlap of the redshift bins) by an order of magnitude and increases the amplitude of the magnification-clustering cross-correlation (blue) over all scales consid-
Angular correlation

FIG. 2: Cross-correlation functions between two photometric redshift bins with (solid) and without (dashed) optimal weighting. The contributions to the cross-correlation include galaxy clustering ('Gal') and lensing magnification ('Mag'). The optimal weights both reduce the intrinsic clustering amplitude and increase the amplitude of the lensing magnification cross-correlation.

FIG. 3: Forecasted constraints on dark energy equation of state parameters from galaxy angular auto- and cross-correlation functions in 4 tomographic bins limited to the linear clustering regime. Inset panel: no systematic biases. Main panel: assuming an uncorrected 1% systematic bias in the linear galaxy clustering bias. The largest (red) contour shows the constraints using uniform weighting for each galaxy pair in the correlation function estimator. The smaller contours (blue) show the constraints when using the optimal redshift weighting in this letter. For comparison, the smallest contours (green) show the constraints with a cosmic shear measurement (correlated galaxy shapes). In each case, I marginalize over uncertainties in the photo-z distributions in each bin (mean, variance, and outlier fraction) as well as the linear clustering bias in each bin, with 30% priors. The optimal weighting in the magnification cross-correlations increases the dark energy Figure of Merit by 25% over uniform galaxy weighting and reduces the systematic parameter biases to less than the 1-σ uncertainties.

Conclusions  Figure 3 demonstrates that, with optimization, the lensing magnification measured via cross-correlations of photo-z bins may yield a dark energy Figure of Merit (defined as the inverse of the $w_0 - w_a$ ellipse area) [26] up to 80% that from cosmic shear with the same set of galaxies and tomographic binning. Because the lensing magnification only depends on the clustering bias of the foreground sample, the optimally weighted cross-correlations are able to partially self-calibrate an uncorrected systematic error in the bias, further improving the dark energy measurement relative to the standard redshift weighting. By down-weighting the lensing contributions to cosmological angular cross-correlations with the same source catalog, we can also improve the measurement of the linear clustering bias with respect to the dark matter and the calibration of photometric redshift uncertainties [27].

For partially overlapping redshift bins such as those modeled in Fig. 1, the optimal weighting effectively discards a large number of galaxies. We therefore require...
large surveys such as the LSST to reduce the shot noise. Typically, at least $10^6$ sources are required in each foreground redshift bin to detect lensing magnification via cross-correlations with our optimal redshift weighting. But LSST will yield at least 10 times more galaxies per bin near the peak of the redshift distribution, with the exact usable number depending on the accuracy of the photometric calibration.

Other possible sources of systematic errors in the optimal redshift weighting include unknown variations of the source redshift distributions or luminosity functions due to photometric calibration errors over a survey footprint on the sky and uncertainties in the probability distribution for the true redshift of each foreground object. Because the optimal redshift weights are sensitive to the source redshift distributions or luminosity functions due to photometric calibration as well, other possible sources of systematic errors in the optimal redshift weighting include unknown variations of the source redshift distributions or luminosity functions due to photometric calibration errors over a survey footprint on the sky and uncertainties in the probability distribution for the true redshift of each foreground object. Because the optimal redshift weights are sensitive to the model photo-z distributions, an iterative solution for the redshift weights may yield important information on the photo-z calibration as well.

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