Spectroscopy of Superradiant scattering from an array of Bose-Einstein Condensates

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We theoretically study the superradiant gain and the direction of this coherent radiative for an array of Bose-Einstein condensates in an optical lattice. We find that the density grating is formed to amplify the scattering light within the phase match condition. The scattering spectroscopy in the momentum space can provide a method for measuring the overlap of wavefunction between the neighboring sites, which is related to their inner-site and inter-site coherence.

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I. INTRODUCTION

Superradiance from a Bose-Einstein Condensation (BEC) offers the possibility to study the novel physics associated with cooperative scattering of light in ultracold atomic systems. A series of experiments [1, 2, 3, 4, 5, 6, 7, 8] and related theories [9, 10, 11, 12, 13, 14] have sparked the related interest in the quantum information [5], collective instability [12, 13, 14], high precision measurement [7], and coherent atom optics [15, 16].

In a typical BEC superradiant experiment, the pattern of recoiling atoms by absorption image method reflects the atomic momentum spectroscopy or momentum distribution [2, 3, 4, 5, 6], where the moving atoms and the static BEC form a matter wave grating. At the same time, the scattering optical spectroscopy shows the gain process with time [4]. To enhance the scattering light signal, the optical cavity is applied in the similar experimental setting which usually called collective atomic recoil lasing (CARL) [15, 16, 19], where the atoms are forced to maintain in the density grating by the optical lattice in the cavity. Different from the above case, here we consider the scattering superradiance from an array of BECs.

Optical lattices (OL), created by pairs of off-resonance counter-propagating laser beams, offer new opportunities to investigate quantum information processing and strongly correlated quantum matter [20]. The periodical potential in an optical lattice forms an atomic density grating, hence to study the superradiance in this array the coherence of atom both inner site and between sites need to be considered. Therefore superradiance has the potential to become a method to detect the coherence of atoms in an optical potential. This is different from the interaction of light and BEC in an OL trap without the atoms’ recoiling [21], where the inter sites atomic coherence was considered and the inner site coherence is neglected.

To study the superradiance in an optical lattice, there are several problems to be considered with regard to the theory about the superradiance from BEC [6]. First the frequency of optical lattice, usually several $kH_z$, is much larger than that of the magnetic trap (tens to hundreds $Hz$) where the effects of trap is usually neglected for its frequency is much smaller than the recoil frequency. Secondly, we need to calculate the gain in the special emission angle. In the case of a magnetic trap, the atomic cloud experiences the maximum gain when the mode is along its long axis. And it can also be understood by that the direction is selected for the least width of momentum to get the maximum gain [6]. The long axis and the least width of momentum, these two directions are the same in a magnetic trap, but not necessarily true in an OL trap. Lastly but important, since the light scattering depends on the coherence of different sites, the interference of scattered light results in amplification at some specific frequency and suppression at the others. It could provide us a method to obtain the information of the atoms in OL. The density grating formed by optical lattice and the grating formed by moving and static atoms give two criterions for the optical amplification. Hence, similar to the coherent-enhanced imaging where Raman superradiance is used to probe the spatial coherence of BEC in a magnetic trap [18], the scattering spectroscopy reflects the cooperative radiation of atoms inner-site and inter-sites.

II. GAIN FOR THE CONDENSATE IN THE TRAP

We consider the model that the atomic cloud is prepared in an optical lattice, as shown in Fig.1. The optical lattice is placed along the x-axis with $M$-sites centered at origin. The lattice constant is $a_0 = \lambda / 2$, the length of condensate is $L = M a_0$, and the pumping laser incident with wave vector $k_0$ propagates along its short axis $y$.

In order to investigate the superradiant gain, we adiabatically eliminate the excited state for the far-off resonant pump laser and use the rotating wave approximation. Therefore the effective Hamiltonian about the coupling between the atomic and electromagnetic fields can be written as [6]

$$\hat{H} = \hat{H}_a + \hat{H}_p + \hat{H}_i, \quad (1)$$

where the $\hat{H}_a = \int d^3r \hat{\Psi}^\dagger(r) \left[ \hat{\mathbf{p}}^2 / 2m + \hat{V}(r) \right] \hat{\Psi}(r)$ is the...
and we define the coefficient
\[ \rho_q(k) = \int d^3r |\psi_0(r)|^2 \exp[-i(k - k_0 + q) \cdot r] \]  \tag{7}

is the Fourier transform of the ground state density distribution centered at \( k_0 - q \).

As the experiment shows that there are just several side modes dominate the whole scattering process, and in order to simplify this problem, we just take one side mode \( q \) into consideration. Under the Born-Markov approximation, the optical field could be obtained that
\[ \hat{b}(t) = \hat{b}(0)e^{-i\omega_k t} + \sum_{n \neq 0} g(k)\rho_q(k)A_n \hat{c}_n e^{i\delta_0(k \cdot \omega_k)} \]  \tag{8}

Inserting (8) into the dynamic equation of atomic field, \( \hat{c}_n = [H, c_n]/i\hbar \), we obtain its evolution equation
\[ \frac{d}{dt} \hat{c}_n = A_n G_q \frac{\hat{c}_0}{N} \sum_{m \neq 0} A_m \hat{c}_m + \hat{f}(t) - i\omega_n\hat{c}_n \]  \tag{9}

with the BEC’s gain with the \( q \)-th mode as
\[ G_q = N g^2/\hbar^2 \int d^3k |\rho_q(k)|^2 \delta(|k| - k_0) \]  \tag{10}

where we assumed that \( g(k) \) is isotropic in the \( k_x - k_z \) plane. The first term on the r.h.s. of Eq. (9) is the gain from the condensate. The second term is the quantum fluctuation, which has been discussed in [8] and does not affect the superradiant behavior in long time, and we just take it as an initial seed. The last term on the r.h.s. of Eq. (9) is the energy term.

In order to compare the two cases of pumping the condensate with and without the external potential, we need to discuss equation (9) by the mean-field approximation, replacing the field operator \( \hat{c}_n \) by a c-number \( c_n \). By transformation \( \tilde{c}_n = c_n \exp(-i\omega_n t) \), the equation (9) becomes
\[ \frac{d}{dt} \tilde{c}_n = A_n G_q \frac{\tilde{c}_0}{N} \sum_{m \neq 0} A_m \tilde{c}_m e^{i(n-m)\omega_T t} \]  \tag{11}

For the case that potential is turned off when pumping the condensate by the laser, we assume that there is just only one level \( n \) satisfying the condition \( A_n = 1 \), with eigenenergy \( \omega_n = p^2/2m = \omega_r \), and for the else \( A_n = 0 \). Thus on the r.h.s. of the equation (11) the phase factor \( e^{i(n-m)\omega_T t} \) is always equal to one. And this form is consistent with the equation (8) in [9]. The gain of atomic number is therefore \( G_q \). For the case that...
the external potential exists, the parameters $A_n$ are centered at $\omega_r/\omega_T$ and have a standard deviation $\sqrt{\omega_r/\omega_T}$, and we just need to consider these $2\sqrt{\omega_r/\omega_T}$ trap levels that make $A_n \neq 0$. And the phase factor $e^{i(n-m)\omega_T t}$ is different for different levels. These factors could be approximated by calculating the average phase difference

$$\omega_r\sqrt{\omega_r/\omega_T}dt = \sqrt{\omega_r/\omega_T}dt$$

in a during time $dt$. Thus the different phases in different levels lead to that the sum in the r.h.s. of equation (11) is smaller than the case that they are the same phase, in the previous case. This results in the gain loss, given by

$$G' - G \propto -\frac{\sqrt{\omega_r/\omega_T}}{N}.$$  

(12)

A typical value of the gain is $G_q = 4 \times 10^4$ for $I = 100\text{mW/cm}^2$ and $N = 10^6$, which is close to the frequency of the atom in the optical lattice trap. Thus when the $\omega_T$ is large enough to dephase the coherence of condensate and side mode, the gain of light is suppressed. Since the effect of the trap is just a shift in the gain, in the below we mainly discuss the gain without the trap $G_q$.

III. GAIN FROM AN ARRAY OF CONDENSATES IN RELEASED TRAP

Now we consider the case that the pump beam immediately incidents after switching off the potential, so the external trapping potential could be neglected. The wave function of ground state for the $i$-th site is the Wannier function $w_i(r)$, which is approximated by the gaussian function $\exp(-\sum_{j=1}^M r_j^2/2\sigma_j^2)$ with the half width of the wave function $\sigma_j$ in $j$ direction $(j = x, y, z)$. Thus the ground-state wave-function in the optical lattice is given by $\psi_0(r) = C_{nor} \sum w_i(x, y, z)$, where the $C_{nor} = (\int d^3r \sum_{i=1}^M w_i(r)^2)^{1/2}$ is the normalization factor. For one site, we assume $\sigma_x \gg \sigma_y, \sigma_z$, the maximum gain is the $z$ direction because the photon could experience the most atomic amplification \[3\]. Here, we need to carefully discuss the gain for the whole atomic cloud. Since the $M$ sites are placed along $x$-axis and centered at origin with lattice spacing $a_0$, then the atomic density can be expressed as

$$|\psi_0(r)|^2 = C_{nor}^2 \sum_{i=1}^M |w_i(r)|^2 + 2 \sum_{i=1}^{M-1} w_i(r)w_{i+1}(r),$$  

(13)

where the first term on the r.h.s. describes the atomic density of site $i$. The second term is the overlapping between neighboring sites, which is considered only when the wavefunction of one site is wide enough to overlap its neighbors. The Fourier transformation of the density at $q = k_0$, is

$$\rho_{k_0}(k) = C_{nor}^2 \exp(-\sum_{j=1}^M \frac{\sigma_j^2 k_j^2}{4}) \left[ \sin(Ma_0 k_x)/\sin(\frac{a_0}{2} k_x) \right] \left[ 1 + \exp(-\frac{a_0^2}{4\sigma_z^2}) \right].$$  

(14)

We proceed to calculate the superradiant gain in \[10\]. The factor in $\rho_q(k)$, $\sin(Ma_0 k_x)/\sin(\frac{a_0}{2} k_x)$, as shown in solid line of Fig. 2 gives the density profile a sampling, which means that the $\rho_q(k)$ is not zero in regions with half width $2\pi/Ma_0$ and these regions separating $2\pi/a_0$ from each other. Moreover, the factor of $\rho_q(k)$, $\exp(-\sum_{j=1}^M \frac{\sigma_j^2 k_j^2}{4})$, is centered at $k_0 - q$ and spreads in $k_1$ direction, which is significant in the region $|k_1| < 1/\sigma_z$. The gaussian factor of $\rho_q(k)$ is the dashed and dot-dashed line in Fig. 2 which shows that it is wider when the width in the single site wave function is smaller. We assume that $k_0 \approx 1/\sigma_j, k_j \approx 1/a_0$, which means that the atomic momentum is narrow enough that all its components could contribute to the optical amplification\[8\]. And in the region of $\rho_q(k)$ where the gaussian factor is significant, we can approximate the surface of the sphere $|k| = |k_0|$ as a plane tangent to the sphere, and the integral in \[10\] is equivalent to be the integral on this plane. So the integral when $q = k_0 + |k_0|\hat{\theta}$, where $\hat{\theta} = \cos \theta \hat{k}_x + \sin \theta \hat{k}_z$, is a unit direction in the $k_x - k_z$ plane.

Here we mainly consider the maximum gain of light in two extreme directions, $\theta = 0$ corresponding to the $z$ direction, and $\theta = \pi/2$ for $x$ direction. For $\theta = \pi/2$, the gain is

$$G_x = G_0 \frac{M^2}{\sigma_z},$$  

(15)

where

$$G_0 = \frac{g^2}{k_0^2} C_{nor}^2 \frac{2\pi}{\sigma_y} \left[ 1 + \exp(-\frac{a_0^2}{4\sigma_z^2}) \right],$$  

(16)

is related to the normalization factor $C_{nor}$, the width
in y-direction $\sigma_y$ and the factor $[1 + \exp(-\frac{\sigma_y^2}{a_0^2})]$ which reflects the coherence of neighboring sites. Eq. (10) shows that the gain $G_x$ is proportional to the number of site squared $M^2$, which is the result of cooperative radiation.

Because the non-zero region of $\delta$-function in Fig. 1 is a plane parallel to the $k_x - k_y$ plane when $\theta = 0$, the profile width of $\rho_q(k)$ will affect the result of $G_x$. Hence, we need to discuss two condition $\sigma_x < a_0$ and $\sigma_x \geq a_0$.

For the case that $\sigma_x < a_0$, $\rho_q(k)$ has $a_0/\sigma_x$ side bands, after summing all these side bands whose half width is $1/Ma_0$, we get the superradiant gain given by

$$G_z = G_0 M^2 a_0 - \frac{1}{\sigma_x M a_0} = G_0 \frac{M}{\sigma_x}.$$  

(17)

For the case that $\sigma_x \geq a_0$, there is just one non-zero region of $\rho_q(k)$, thus, the superradiant gain is

$$G_z = G_0 M^2 a_0 - \frac{1}{M a_0} = G_0 \frac{M}{a_0}.$$  

(18)

In both cases, the gain $G_z$ is proportional to $M$, due to the incoherent sum of different sites.

When $\sigma_x < a_0$, the gain ratio for the two extreme direction is that $\frac{G_z}{G_0} = \frac{M a_0}{\sigma_x a_0} = L/\sigma_x$ which is the aspect ratio, consistent with the theory without OL trap. When $\sigma_x > a_0$, the gain ratio becomes $\frac{G_z}{G_0} = \frac{M a_0}{\sigma_x a_0}$ which is the effective length ratio in this two directions. It should be noted that this theory is sound under the condition that $k_0 \gg 1/\sigma_x$. A typical value is $k_0 = 2\pi/780nm$, hence $\sigma_x \gg 780nm$. If we need the gain in $z$ direction larger than that in $x$ direction, we need $\sigma_z \gg M \cdot 780nm$, which is hard to be realized in experiment. Thus the radiation usually takes place in the $x$ direction.

IV. SPECTROSCOPY OF SUPERRADIANT SCATTERING

In the previous section, we understand that the gain of light is usually propagating along the $x$-axis. Considering $\mathbf{q} = \delta \mathbf{x}$ in the $x$-direction, for the different $\mathbf{q}$ the gain can be expressed as

$$G_{q} = \frac{G_0}{\sigma_x} \exp[-\frac{\sigma_x^2 (k_0 + q)^2}{2}] \sin^2 \frac{M a_0 (k_0 + q)}{2} \sin^2 \frac{a_0 (k_0 + q)}{2}.$$  

(19)

In this equation, we know that the maximum gains emerge at $\frac{a_0 (k_0 + q)}{2} = n\pi$. In other words, the gain has maximum around $k_0 + 2n\pi/a_0$ with separation $2\pi/a_0$.

The spectroscopy of different width of single site wave function is plotted in Fig. 3. As shown in subfigure (a), when $\sigma_x \geq a_0$, there is only one peak in the gain. Subfigure (b) shows that when the $\sigma_x \ll a_0$, there are side bands. The radiant light has side bands when the wave functions of neighboring sites are not overlapped. The reason is that atoms in different sites are pumping by the same phase light and become to the same phase dipole. The radiant light which is propagating along the lattice has a phase difference in neighboring sites, $(\mathbf{k} - \mathbf{k}_0) \cdot a_0 \mathbf{x}$. Thus radiant lights with different frequencies will have different gains by the averaging over the whole lattice. The constructive interference will single out the frequency component satisfying the condition $(\mathbf{k} - \mathbf{k}_0) \cdot a_0 \mathbf{x} = 2n\pi$ to amplify, and other components will be suppressed due to the destructive interference of $M$ sites.

For the larger width of single site wave function, the gain of side bands is smaller. Thus by measuring the side band gain could give us a method to obtain the information about the width $\sigma_x$. Fig. 4 shows the ratio of maximum gain to second maximum gain versus $\sigma_x$. By the spectroscopical measuring, we could obtain the information of the width of wave packet, which is relevant to the potential quantum phase transition.
V. DISCUSSION AND CONCLUSIONS

In the BEC superradiant experiment [1], a photon is scattered by one atom in the BEC, and this atom acquires the recoil momentum. The moving atoms and the static BEC form a matter wave grating, which enhances the same direction scattering. Due to the mode competition, the highly directional emissions of light travel along the long axis of the condensate. The stability of the relative phase between different atomic matter waves determines the coherence time of the matter wave. On the other hand, in the coherent atomic recoil lasing (CARL), the situation is changed by the presence of the cavity, the coherence is preserved as the relative phase of the cavity light-fields, and it is independent of the atomic motion while given by the cavity linewidth.

Different to these experimental scheme, here we extend the theory of superradiance of BEC [10] to the case of in OL trap. In this trap, an array of atoms from the optical lattice form a density grating, and the superradiance gain are calculated in the quantum theory. In this theory, we consider inner-site and inter-site coherence of atoms. Only the scattering light satisfy the condition that \((k - k_0) \cdot a_0 \hat{x} = 2n\pi\) will be singled out to be amplified and the other components will be suppressed in different extent. Together with the grating formed by the static and moving condensate, both gratings give frequency selection rules. It is similar to a diode laser with internal and external cavities. Only the light is resonant to both cavities will be amplified.

The motion of recoiling atom in the high-frequency OL trap will dissipate the coherence resulting in a loss of the optical gain proportional to \(\sqrt{\omega_T\omega_r}\). In the magnetic trap, the trap frequency is smaller, and the loss can be neglected. It can inhibit the collective radiation when the trap frequency is high enough. By calculating the ratio of optical gain in the two extreme direction, we show that the gain is proportional to the length that the light travels in the condensate.

Depending on the lattice depth, the wave function for one site overlap differently with its neighboring sites. When the OL potential is low enough, the wave functions of neighboring sites fully overlap, just like a condensate in the magnetic trap. When the OL potential is high enough, the wave functions of neighboring sites are separated. The different overlap results in the different scattering spectroscopy. Thus the spectroscopy provides us with a new method to detect the coherence of different sites. This spectroscopy method offers much more precision than the absorption image method. Moreover, unlike the time-of-flight method, which is usually used to detect quantum phase transition [21], the spectroscopy method is a non-destructive method. More understanding of this mechanism can help to understand the self-organization, especially how the long-range order arise in the self-synchronization process. Superradiance may be helpful to detect the phase transition between the superfluid (SF) and Mott-insulate (MI).

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