The lepton flavour violating charged lepton decays $\mu \rightarrow e + \gamma$ and thermal leptogenesis are analysed in the minimal supersymmetric standard model with see-saw mechanism of neutrino mass generation and soft supersymmetry breaking terms with universal boundary conditions. Hierarchical spectrum of heavy Majorana neutrino masses, $M_1 \ll M_2 \ll M_3$, is considered. In this scenario, the requirement of successful thermal leptogenesis implies a lower bound on $M_1$. For the natural GUT values of the heaviest right-handed Majorana neutrino mass, $M_3 \gtrsim 5 \times 10^{13}$ GeV, and supersymmetry particle masses in the few $\times 100$ GeV range, the predicted $\mu \rightarrow e + \gamma$ decay rate exceeds by few order of magnitude the experimental upper limit. This problem is avoided if the matrix of neutrino Yukawa couplings has a specific structure. The latter leads to a correlation between the baryon asymmetry of the Universe predicted by leptogenesis, $\text{BR}(\mu \rightarrow e + \gamma)$ and the effective Majorana mass in neutrinoless double beta decay.

1 Introduction

The supersymmetric (SUSY) extension of the Standard Model (SM) is a widely discussed candidate of a theory beyond the SM. If the SUSY particles have masses in the few $\times 100$ GeV range, they will be observed in the large hadron collider (LHC) experiments. In this case lepton flavour violating (LFV) processes, like $\mu \rightarrow e + \gamma$ decay, etc., are also predicted to take place with rates which can be close to the existing experimental upper limits.

We focus on the minimal supersymmetric standard model with right-handed heavy Majorana neutrinos (MSSMRN). In this model one can implement the seesaw mechanism, which provides a natural explanation of the smallness of neutrino masses. The seesaw mechanism predicts the light massive neutrinos to be Majorana particles. In this case the process of neutrinoless
double beta (0ν2β) decay, (A, Z) → (A, Z + 2) + e− + e−, can occur. The seesaw mechanism provides also, through the leptogenesis scenario\textsuperscript{2}, an attractive explanation of the observed baryon asymmetry of the Universe (BAU).

In MSSMRN\textsuperscript{3}, the neutrino Yukawa couplings which do not conserve the lepton flavour, affect the predictions of LFV processes\textsuperscript{5}. Even when the soft SUSY breaking terms present at the cutoff scale (\(M_X\)) are flavour blind, non-zero flavour mixing in the slepton sector is induced through the renormalization group (GR) running between \(M_X\) and the seesaw scale \(M_R < M_X\).

For \(M_R \gtrsim 5 \times 10^{13}\) GeV one typically gets in SUSY GUT\textsuperscript{7}, the prediction for the \(\mu \rightarrow e + \gamma\) decay branching ratio, \(\text{BR}(\mu \rightarrow e + \gamma)\), in MSSMRN with SUSY particles masses in the few \(\times 100\) GeV range, is in conflict with the present experimental upper bound\textsuperscript{6}, \(\text{BR}(\mu \rightarrow e + \gamma) < 1.2 \times 10^{-11}\). Thus, the leading contribution(s) to \(\text{BR}(\mu \rightarrow e + \gamma)\) has to be suppressed.

In this article, we consider a specific form of the matrix of neutrino Yukawa couplings, \(Y_N\), for which \(\text{BR}(\mu \rightarrow e + \gamma)\) is suppressed, but still can be within the sensitivity of the ongoing MEG experiment. The form thus chosen of \(Y_N\) implies a correlation between the predicted values of the effective Majorana mass in 0ν2β decay, of BAU, and of \(\text{BR}(\mu \rightarrow e + \gamma)\).

2 Seesaw model and a parametrisation neutrino Yukawa coupling matrix

We consider MSSMRN. The presence of the RH neutrinos, \(N_i^c\), in the theory makes it possible to introduce neutrino Yukawa couplings and Majorana mass term for \(N_i^c\) in the superpotential. In the framework of MSSMRN, a basis in which both the charged lepton mass matrix, \(Y_E\), and the heavy Majorana neutrino mass matrix, \(M_N\), are real and diagonal, can always be chosen without loss of generality. Henceforth, we will work in this basis and denote the diagonal RH neutrino mass matrix by \(D_N \equiv \text{diag}(M_1, M_2, M_3)\).

Below the seesaw scale, \(M_R = \text{min}(M_j)\), the heavy RH neutrino fields \(N_j\) are integrated out, and as a result of the electroweak symmetry breaking, Majorana mass term for the left-handed flavour neutrinos is generated. The corresponding mass matrix can be expressed in terms of the matrix of neutrino Yukawa couplings, \(Y_N\), and \(M_N\) as

\[
(m_\nu)^{ij} = (U^*)^{ik} m_k (U^\dagger)_{kj} = v_u^2 (Y_N^T)^{ik} (M_N^{-1})^{kl} (Y_N)^{lj}.
\]

Here \(v_u = v \sin \beta\), where \(v = 174\) GeV, \(\tan \beta\) is the ratio of the vacuum expectation values of up-type and down-type Higgs fields, and \(U\) is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix\textsuperscript{8}.

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}s_{13}e^{-i\delta} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13}
\end{pmatrix} \text{diag}(1, e^{i\frac{\alpha}{2}}, e^{i\frac{\beta M}{2}}). \tag{2}
\]

Eq. (1) can be “solved” as\textsuperscript{9}

\[
Y_N = \frac{1}{v_u} \sqrt{D_N R} \sqrt{D_\nu} U^\dagger, \tag{3}
\]

where \(D_\nu = \text{diag}(m_1, m_2, m_3)\) and \(R\) is a complex orthogonal matrix, \(R^T R = 1\). The CP phases in \(R\) are directly related to the CP asymmetry parameter in leptogenesis. In addition, these phases can affect significantly the predicted rates of the LFV processes\textsuperscript{10,11,12}.

3 Models with texture zero for the inverted hierarchical light neutrinos

Hereafter we focus on the case of light neutrino mass spectrum of inverted hierarchical (IHI) type\textsuperscript{d}, \(m_1 \sim m_2 \simeq \sqrt{\Delta m^2_{31}} \simeq 0.05\) eV, and \(m_3 \simeq 0\). For IHI light neutrinos, the effective

\footnote{In the SM with massive neutrinos, \(\text{BR}(\mu \rightarrow e + \gamma)\) is suppressed by the factor\textsuperscript{31} \((m_3/M_W)^4 < 6.7 \times 10^{-43}\).}

\footnote{The cases of normal hierarchical and of quasi-degenerate light neutrinos are discussed in Ref. 14.}
Majorana mass in $0\nu 2\beta$ decay, $\langle m \rangle$, is given by
\[
|\langle m \rangle| = |m_{12}e^{2i\theta_{12}} + m_{23}e^{2i\theta_{13}}| \, ,
\]  
$\theta_{12}$ and $\theta_{13}$ being the solar and CHOOZ mixing angles, respectively. Thus, $\sqrt{\Delta m^2_{31}} \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m^2_{31}}$, the two limits corresponding to the CP-conserving values of $\alpha = 0; \pi$.

Even when we consider MSSMRRN with flavour universal soft scalar masses, $m_0$, GUT gaugino mass, $m_{1/2}$, and universal $A$-term coefficient, $A_0$, at the cutoff scale $M_X$, slepton flavour leading to LFV decays such as $\mu \rightarrow e + \gamma$, is induced by $Y_N$ which does not conserve lepton flavour, through the RG running between $M_R$ and $M_X$. The branching ratio $\text{BR}(\mu \rightarrow e + \gamma)$ in this framework is predicted as
\[
\text{BR}(\mu \rightarrow e + \gamma) \simeq \frac{\alpha^2}{G_F^2} \frac{\langle M_{L}^2 \rangle_{21}}{m_S^8} \frac{\langle h_d \rangle^2}{\langle h_u \rangle^2} \times |(Y_N^\dagger L Y_N)_{21}|^2 .
\]  
Here $L = \text{diag}(\ln \frac{M_1}{M_X}, \ln \frac{M_2}{M_X}, \ln \frac{M_3}{M_X})$ and $m_S$ denotes the SUSY particle mass scale which can be approximately estimated as
\[
m_S^8 \simeq 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2 .
\]  
Using Eq. (3), one can decompose $(Y_N^\dagger L Y_N)_{21}$ as
\[
(Y_N^\dagger L Y_N)_{21} = \frac{M_3}{v_u^2} \ln \frac{M_3}{M_X} (\mathbf{R} \sqrt{D_u U^\dagger})_{32}^* (\mathbf{R} \sqrt{D_u U^\dagger})_{31} + \frac{M_2}{v_u^2} \ln \frac{M_2}{M_X} (\mathbf{R} \sqrt{D_u U^\dagger})_{22}^* (\mathbf{R} \sqrt{D_u U^\dagger})_{21} + \frac{M_1}{v_u^2} \ln \frac{M_1}{M_X} (\mathbf{R} \sqrt{D_u U^\dagger})_{12}^* (\mathbf{R} \sqrt{D_u U^\dagger})_{11} .
\]  
If RH neutrinos have hierarchical mass spectrum, $M_1 \ll M_2 \ll M_3$, the term proportional to $M_3$ gives the dominant contribution. Suppressing this term is the most natural way to avoid too large SUSY contribution to $\text{BR}(\mu \rightarrow e + \gamma)$. This is realised by taking matrix $\mathbf{R}$ in the form
\[
\mathbf{R} = \begin{pmatrix}
\cos \omega & \sin \omega & 0 \\
-\sin \omega & \cos \omega & 0 \\
0 & 0 & 1
\end{pmatrix},
\]  
where $\omega$ is a complex angle – the leptogenesis CPV parameter. However, in this case, the requirement of successful leptogenesis leads to a very strong constraint on $M_1$: $M_1 \geq 6.7 \times 10^{12}$ GeV, which in turn makes the contribution of the term $\propto M_2$ in $\text{BR}(\mu \rightarrow e + \gamma)$ bigger than the experimental upper bound by $\sim 3$ orders of magnitude in the case the SUSY particles have masses in the few $\times 100$ GeV range. Thus, we consider a scheme in which the $M_2$ contribution to $\text{BR}(\mu \rightarrow e + \gamma)$ is also suppressed. There are two possibilities:

A: $(Y_N)_{21} = 0$, which is satisfied for $\tan \omega = \tan \theta_{12} e^{-i\alpha/2}$.

B: $(Y_N)_{22} = 0$, which holds if $\tan \omega = -\cot \theta_{12} e^{-i\alpha/2} + \mathcal{O}(s_{13})$.

Since in the cases A and B the complex parameter $\omega$ is completely determined by the low energy neutrino mixing parameters, $\theta_{12}$ and the Majorana phase $\alpha$, the quantities $Y_B$, $\text{BR}(\mu \rightarrow e + \gamma)$, and $\langle m \rangle$ are related to each other through the Majorana CP phase $\alpha$. The correlation between these three observables for Case A is illustrated in Fig. 4 (for further details, see Ref. 14.).
4 Summary

The experiments at LHC are planned to start in 2007 and are the only experiments which could provide direct evidence for new physics beyond the SM. If low energy scale SUSY is realised in nature, supersymmetry (SUSY) particles might be observed in the few × 100 GeV mass range. In the SUSY extension of the SM incorporating the seesaw mechanism of neutrino mass generation and SUSY particles masses in the few × 100 GeV range, the existing experimental limit on BR(μ → e + γ) and the requirement of successful leptogenesis imply rather stringent constraint on the matrix of neutrino Yukawa couplings, Y_N. For hierarchical heavy Majorana neutrino spectrum and inverted hierarchical light neutrino masses, these constraints lead to a specific rather simple form of Y_N. The specific form of Y_N thus found implies that the leptogenesis CPV parameter is given by the Majorana phase in the PMNS matrix U. Thus, the predicted values of the baryon asymmetry of the Universe, of BR(μ → e + γ), and of the effective Majorana mass in the neutrinoless double beta decay, are all correlated.

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