Analyses of multi-pion Hanbury-Brown-Twiss correlations for the pion-emitting sources with Bose-Einstein condensation

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Abstract

We calculate the three- and four-particle correlations of identical pions in an evolving pion gas (EPG) model with Bose-Einstein condensation. The multi-pion correlation functions in the EPG model are analyzed in different momentum intervals and compared with the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. It is found that the multi-pion correlation functions and cumulant correlation functions are sensitive to the condensation fraction of the EPG sources in the low average transverse-momentum intervals of the three and four pions. The model results of the multi-pion correlations are consistent with the experimental data in a considerable degree, which gives a source condensation fraction between 16 – 47%.

Keywords: multi-pion correlations, HBT interferometry, femtoscopy, Bose-Einstein condensation, partially coherent source

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I. INTRODUCTION

Two-pion Hanbury Brown–Twiss (HBT) interferometry, also known as two-pion femtoscopy, has been widely applied in high-energy heavy-ion collisions to study the space-time structure of particle-emitting sources by measuring the intensity correlations of two identical pions [1–6]. Because the intensity correlations occur for chaotic particle emission and disappear for coherent particle emission, HBT interferometry can also be used to study the source coherence [1–6]. The intercept of the two-pion correlation function at zero relative momentum is related to the source coherence degree, although many other effects may affect the measurement value of the intercept [1–6]. As extension of two-pion interferometry, multipion correlation analyses are developed and carried out in high-energy heavy-ion collisions [7–27]. Recently, the ALICE collaboration analyzes the three- and four-pion correlations in pp, p-Pb, and Pb-Pb collisions at the Large Hadron Collider (LHC) [28]. A significant suppression of three- and four-pion correlations observed in Pb-Pb collisions may arise from a considerable coherence degree of the particle-emitting sources, which is consistent with the previous measurements of three-pion correlations in the collisions [25]. It is of interest to explain the experimental observations of multi-pion correlations.

In Ref. [29], C. Y. Wong and W. N. Zhang studied the pion Bose-Einstein condensation and the chaoticity parameter \( \lambda \) in two-pion HBT interferometry for a static boson gas source within a mean-field with harmonic oscillator potential in high-energy heavy-ion collisions. The model of the non-relativistic boson gas within harmonic oscillator potential can be solved analytically [29] and be used in atomic HBT correlation analyses [30, 31]. In Ref. [32], the chaoticity parameter \( \lambda \) was investigated in an evolving pion gas (EPG) model with Bose-Einstein condensation. The pion gas in this model was considered within a harmonic oscillator mean-field and expanding in relativistic hydrodynamics [32]. The investigations [32] indicate that the pion sources produced in the Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV at the LHC is partially coherent, perhaps due to a degree of Bose-Einstein condensation. The finite condensation decreases the chaoticity parameter \( \lambda \) in the two-pion interferometry measurements in low momentum interval of pion pair, and influences very slightly the \( \lambda \) value for the pion pair with high momenta [32]. In this work, we shall investigate three- and four-pion HBT correlations in the EPG model [32]. We shall examine the relationship between the condensation and the strength of the multi-pion correlations in different momentum...
intervals. We shall compare the model results of multi-pion correlation functions with the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC \[28\]. It is speculated that the coherent fraction of the particle-emitting sources is between 16 – 47%, consistent with the analysis result for the four-pion correlations measured in the collisions \[28\].

The rest of this paper is organized as follows. In Sec. II, we review the EPG model and present the calculations of the three- and four-pion correlation functions in the EPG model with Bose-Einstein condensation. In Sec III, we examine the multi-pion correlation functions in different momentum intervals. In Sec. IV, we compare the model results of the multi-pion correlation functions with experimental data. Finally, we give the summary and conclusion in Sec V.

II. MODEL AND MULTI-PION CORRELATION FORMULAS

A. EPG model

As in Ref. \[32\], we consider a pion-emitting source as a relativistic boson gas of identical pions within the time-dependent harmonic oscillator potential that arises approximately from the mean field of the hadronic medium in high-energy heavy-ion collisions \[29, 32, 33\],

$$V(r, t) = \frac{1}{2} m \omega^2(t) r^2 = \frac{1}{2} \hbar \omega(t) \frac{r^2}{a^2(t)},$$

where $\hbar \omega(t)$ measures the potential strength and $a(t) = \sqrt{\hbar / m \omega(t)}$ is the characteristic length of harmonic oscillator.

Assuming that the relaxation time of the system is smaller than the source evolution time, the expansion of the pion gas may approximately deal with a quasi-static adiabatic process \[32\]. In this case, the temperature $T$ and volume $V$ have the relationship $TV^{\gamma-1} = \text{constant}$, where $\gamma$ is the ratio of the specific heats at constant pressure and volume. For example, $\gamma = \frac{5}{3}$ for non-relativistic monatomic gas. We assume the characteristic length $a$ is proportional to a parameterized source radius as in Ref. \[32\],

$$a(t) = C_1 R = C_1 (R_0 + \alpha t),$$

where the proportional parameter $C_1$ can be determined by the source root-mean-squared radius, $R_0$ is initial radius of the source and $\alpha$ is a parameter related to the source average expansion velocity. With a hydrodynamical calculation for $R_0 = 6$ fm, $T_0 = 170$ MeV, the model parameters $\gamma$ and $\alpha$ are fixed to be 1.627 and 0.62 \[32\], respectively. And, the
parameter $C_1$ in the model calculations in this paper is taken to be 0.35 and 0.40 as in Ref. [32].

For the identical boson gas with a fixed number of particles, $N$, and at a given temperature $T = 1/\beta$, one has

$$N = N_0 + N_T,$$

where, $N_0$ is the number of particles in $n = 0$ state,

$$N_0 = \frac{Z}{1 - Z},$$

and $N_T$ is the number of the particles in $n > 0$ states,

$$N_T = \sum_{n>0}^{\infty} \frac{g_n Z e^{-\beta \tilde{E}_n}}{1 - Z e^{-\beta \tilde{E}_n}},$$

where $g_n$ is the degeneracy of the $n$-th energy level, $Z$ is the fugacity parameter which includes the factor for the lowest energy $\varepsilon_0$, and $\tilde{E}_n$ is the relative energy levels to $\varepsilon_0$ [29, 30]. Because $N_0 \geq 0$, the values of $Z$ are between zero and one. From Eqs. (2) — (4) and with the energy levels of harmonic oscillator, we can calculate $Z$ numerically for fixed $N$ [29, 32], and then obtain the condensation fraction,

$$f_0 = \frac{N_0}{N} = \frac{Z}{(1 - Z)N}.$$  

![FIG. 1: (Color online) Condensation fraction as a function of temperature.](image_url)

In Fig. 1 we show the condensation fractions as a function of temperature for the sources with $N = 1000, 1500, \text{ and } 2000$. Here, the left and right panels are for the parameter $C_1 =$
0.35 and 0.40, respectively. One can see that the condensation fraction $f_0$ increases with the particle number $N$ and decreases with increasing temperature. For fixed $N$ and $T$, the condensation fraction for $C_1 = 0.35$ is higher than that for $C_1 = 0.40$ because the condensation is significant for the system with a small characteristic length $a$.

As we know the density matrix of a generic quantum ensemble can be written as

$$\hat{\rho} = \sum_{N=0}^{\infty} P_N \hat{\rho}_N,$$  

(6)

where the set $\{ P_N \}_{N=0}^{\infty}$ is normalized multiplicity distribution, $\hat{\rho}_N$ denotes the density matrix of the ensemble in which the systems with a fixed particle number $N$, and then an observable is given by

$$\langle \langle \hat{A} \rangle \rangle = \text{Tr}(\hat{A} \hat{\rho}) = \sum_{N=0}^{\infty} P_N \langle \langle \hat{A} \rangle \rangle_N = \sum_{N=0}^{\infty} P_N \text{Tr}(\hat{A} \hat{\rho}_N),$$

(7)

where $\langle \langle \cdot \cdot \cdot \rangle \rangle$ denotes the double average over the quantum states of system and ensemble systems. Quantities $\langle \langle \hat{A} \rangle \rangle$ and $\langle \langle \hat{A} \rangle \rangle_N$ may also be referred to as the “inclusive” and “exclusive” quantities with respect to the multiplicity of event.

In Refs. [15], T. Csörgő and J. Zimányi solve analytically the multiplicity distribution, single-particle momentum spectra, and two-particle HBT correlations using the particle-wave-packet technique, for the static identical pion system with all order Bose-Einstein symmetrizations. Because of the symmetrization, the emission of pion encourages the emission of more identical pions when the particle density is sufficiently high, which is referred to as a “pion laser” first introduced by S. Pratt [13].

Compared to the pion-laser model (PLM) [13, 15], the EPG model describes an evolving pion-emitting source. It deals with the canonical ensemble in which the systems of pion gas have a fixed particle number $N$ and assumed to have certain temperature and volume at each hydrodynamically evolving state [32]. Obviously, the EPG model is an approximate description for the sources produced in high-energy heavy-ion collisions after chemical freeze-out, and cannot be used to investigate the multiplicity distribution in the collisions.

In the EPG model, the one- and two-particle density matrices in momentum space are

$$G^{(1)}(p_1, p_2) = \sum_n u_n^*(p_1)u_n(p_2) \langle \hat{a}_n^\dagger \hat{a}_n \rangle$$

$$= \sum_n u_n^*(p_1)u_n(p_2) \frac{g_n Z e^{-\beta E_n}}{1 - Z e^{-\beta E_n}},$$

(8)
\[ G^{(2)}(p_1, p_2; p_1, p_2) = \sum_{klmn} u^*_k(p_1) a^+_l p_2) u_m(p_2) a_n p_1) \langle \hat{a}^+_k \hat{a}^+_l \hat{a}^+_m \hat{a}^+_n \rangle, \tag{9} \]

where \( u_n(p) \) is the wave function of single-particle for the \( n \)-th state, \( \hat{a}_n \) (\( \hat{a}_n^\dagger \)) is the annihilation (creation) operator of particle, and \( \langle \cdots \rangle \) denotes the ensemble average. The invariant single-pion momentum distribution is

\[ E \frac{dN}{dp} = \sqrt{p^2 + m^2} G^{(1)}(p, p), \tag{10} \]

and the two-pion correlation function is defined as

\[ C_2(p_1, p_2) = \frac{G^{(2)}(p_1, p_2; p_1, p_2)}{G^{(1)}(p_1, p_1) G^{(1)}(p_2, p_2)}. \tag{11} \]

In the limit of a large number of particles, \( N(N - 1) \sim N^2 (\gg N_T, N_0) \), the numerator in Eq. (11) can be written as

\[ G^{(2)}(p_1, p_2; p_1, p_2) = G^{(1)}(p_1, p_1) G^{(1)}(p_2, p_2) + G^{(1)}(p_1, p_2) G^{(1)}(p_2, p_1) - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2. \tag{12} \]

Then, the two-pion correlation function is

\[ C_2(p_1, p_2) = 1 + \frac{|G^{(1)}(p_1, p_2)|^2 - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2}{G^{(1)}(p_1, p_1) G^{(1)}(p_2, p_2)}. \tag{13} \]

In the nearly completely coherent case with almost all particles in the ground condensate state, \( N_0 \rightarrow N \), the two terms in the numerator approximately cancel each other and the correlation function approaches 1. For the other extreme of a completely chaotic source with \( N_0 \ll N \), the second term in the numerator can be neglected, and we have

\[ C_2(p_1, p_2) = 1 + \frac{|G^{(1)}(p_1, p_2)|^2}{G^{(1)}(p_1, p_1) G^{(1)}(p_2, p_2)}. \tag{14} \]

In Fig. 2(a), the thick solid and dashed curves show the invariant single-pion momentum distributions in the EPG model with the parameters \( C_1 = 0.40 \) and 0.35, respectively. The particle number \( N \) and temperature \( T \) are taken to be 1200 and 100 MeV. It can be seen that the momentum distribution for \( C_1 = 0.35 \) has a more obvious enhancement in low momentum region compared to that for \( C_1 = 0.40 \). It is because the source size for \( C_1 = 0.35 \) is small than that for \( C_1 = 0.40 \), and the higher condensation for the smaller source leads to more pions condensed in the ground state and with small momenta. The thin solid and dashed curves in Fig. 2(a) represent the exclusive invariant single-pion momentum
distributions, calculated with the formulas in Ref. [15], in the PLM for identical particle number $N = 1200$ and with the source radii $R = 11$ and 13 fm, respectively. The other parameters in the calculations are taken to be $\sigma_x = 2$ fm and $T = 120$ MeV as in Ref. [15]. It also can be seen that the momentum distribution for the smaller source has a more obvious enhancement in low momentum region than that for the larger source. Because the PLM calculation formulas are non-relativistic [15], the invariant momentum distributions for the PLM sources decrease more rapidly in high momentum region than those for the EPG sources.

![Diagram](image)

**FIG. 2:** (Color online) The invariant single-pion momentum distributions (a) and the two-pion correlation functions (b) in the EPG model and PLM [13, 15].

In Fig. 2(b) we show the two-pion correlation functions for the EPG and PLM sources as in Fig. 2(a). Here, $q_{12}$ is invariant relative momentum of the two pions, $q_{12} = \sqrt{-(p_1 - p_2)^\mu(p_1 - p_2)_\mu}$. For the EPG sources, $K1$ and $K2$ denote the results calculated in the momentum intervals $|p_1 + p_2|/2 < 150$ MeV/c and $|p_1 + p_2|/2 > 150$ MeV/c, respectively. The results for the PLM sources are calculated with the formulas in Ref. [15]. Here, $K_{12}^{(1)}$ and $K_{12}^{(2)}$ denote the results calculated for $|p_1 + p_2|/2 = 100$ MeV/c and $|p_1 + p_2|/2 = 250$ MeV/c, respectively. For the EPG sources, the intercepts of two-pion correlation functions decrease with decreasing source size because the condensation is significant in small system. Also, the intercepts of two-pion correlation functions calculated in the lower momentum interval are smaller than those calculated in the higher momentum interval because of the condensation. For the PLM sources, the intercept of two-pion correlation function approaches to two [15],
except for the result in the case of the small radius and momentum.

**B. Calculations of multi-pion correlation functions in EPG model**

Generalizing Eq. (11), the three- and four-pion correlation functions are defined as,

\[
C_3(p_1, p_2, p_3) = \frac{G(3)(p_1, p_2, p_3; p_1, p_2, p_3)}{G(1)(p_1; p_1) G(1)(p_2; p_2) G(1)(p_3; p_3)},
\]

\[
C_4(p_1, p_2, p_3, p_4) = \frac{G(4)(p_1, p_2, p_3, p_4; p_1, p_2, p_3, p_4)}{G(1)(p_1; p_1) G(1)(p_2; p_2) G(1)(p_3; p_3) G(1)(p_4; p_4)},
\]

where

\[
G^{(n)}(p_1, \ldots, p_n; p_1, \ldots, p_n) = \sum_{k_1, \ldots, k_n, l_1, \ldots, l_n} u_{k_1}^*(p_1) \cdots u_{k_n}^*(p_n) u_{l_1}(p_1) \cdots u_{l_n}(p_n)
\times \langle \hat{a}_{k_1}^\dagger \cdots \hat{a}_{k_n}^\dagger \hat{a}_{l_1} \cdots \hat{a}_{l_n} \rangle
\]

is the \(n\)-particle density matrix in momentum space.

For the EPG source with Bose-Einstein condensation, the multi-pion correlation functions can be written as,

\[
C_3(p_1, p_2, p_3) = 1 + R(1, 2) + R(1, 3) + R(2, 3) + R(1, 2, 3),
\]

\[
C_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2) + R(1, 3) + R(1, 4) + R(2, 3) + R(2, 4) + R(3, 4)
+ R(1, 2, 3) + R(1, 2, 4) + R(1, 3, 4) + R(2, 3, 4)
+ R(1, 2)R(3, 4) + R(1, 3)R(2, 4) + R(1, 4)R(2, 3)
+ R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4),
\]

where

\[
R(i, j) = \frac{|G(1)(p_i; p_j)|^2 - N_0^2|u_0(p_i)|^2|u_0(p_j)|^2}{G(1)(p_i; p_i) G(1)(p_j; p_j)}
\]

\[
R(i, j, k) = \frac{2 \text{Re}[G(1)(p_i; p_j)G(1)(p_j; p_k)G(1)(p_k; p_i) - N_0^2 f_3(p_i; p_j; p_k)]}{G(1)(p_i; p_i) G(1)(p_j; p_j) G(1)(p_k; p_k)},
\]

\[
R(i, j, k, l) = \frac{2 \text{Re}[G(1)(p_i; p_j)G(1)(p_j; p_k)G(1)(p_k; p_l)G(1)(p_l; p_i) - N_0^4 f_4(p_i; p_j; p_k; p_l)]}{G(1)(p_i; p_i) G(1)(p_j; p_j) G(1)(p_k; p_k) G(1)(p_l; p_l)}.
\]
Here, \(R(i, j)\), \([R(i, j)R(k, l)]\), \(R(i, j, k)\), and \(R(i, j, k, l)\) denote the correlations of single pion pair, double pion pair, pure pion-triplet interference or true three-pion correlator \([7, 17]\), and pure pion-quadruplet interference, respectively. The functions \(f_3(p_i, p_j, p_k)\) and \(f_4(p_i, p_j, p_k, p_l)\) in Eqs. (21) and (22) are given by

\[
f_3(p_i, p_j, p_k) = G^{(1)}(p_i, p_j)u_0(p_i)u_0^*(p_j)|u_0(p_k)|^2/N_0
+ G^{(1)}(p_j, p_k)u_0(p_j)u_0^*(p_k)|u_0(p_i)|^2/N_0
+ G^{(1)}(p_k, p_i)u_0(p_k)u_0^*(p_i)|u_0(p_j)|^2/N_0
- 2|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2,
\]

(23)

\[
f_4(p_i, p_j, p_k, p_l) = G^{(1)}(p_i, p_j)G^{(1)}(p_j, p_k)u_0(p_i)u_0^*(p_j)u_0^*(p_k)|u_0(p_l)|^2/N_0^2
+ G^{(1)}(p_i, p_j)G^{(1)}(p_i, p_l)u_0^*(p_j)u_0(p_l)|u_0(p_k)|^2/N_0^2
+ G^{(1)}(p_j, p_k)G^{(1)}(p_k, p_l)u_0(p_j)u_0^*(p_l)|u_0(p_i)|^2/N_0^2
+ G^{(1)}(p_i, p_k)G^{(1)}(p_i, p_l)u_0^*(p_k)u_0(p_l)|u_0(p_j)|^2/N_0^2
+ G^{(1)}(p_j, p_k)u_0^*(p_j)u_0(p_k)u_0^*(p_i)u_0(p_l)/N_0^2
+ G^{(1)}(p_k, p_i)u_0^*(p_k)u_0(p_i)u_0^*(p_j)u_0(p_l)/N_0^2
- 2G^{(1)}(p_i, p_j)u_0(p_i)u_0^*(p_j)u_0^*(p_k)u_0(p_l)/N_0
- 2G^{(1)}(p_j, p_k)u_0(p_j)u_0^*(p_k)u_0^*(p_i)u_0(p_l)/N_0
- 2G^{(1)}(p_k, p_i)u_0(p_k)u_0^*(p_i)u_0(p_j)u_0(p_l)/N_0
- 2G^{(1)}(p_i, p_l)u_0(p_i)u_0^*(p_l)u_0(p_j)u_0(p_k)/N_0
+ 3|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2|u_0(p_l)|^2.
\]

(24)

In the nearly completely coherent case, almost all particles are in the ground condensate state, functions \(f_3(p_i, p_j, p_k)\) → \(|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2\) and \(f_4(p_i, p_j, p_k, p_l)\) → \(|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2u_0(p_l)|^2\), and the two terms in the numerators in Eqs. (20), (21) and (22) cancel each other approximately. So, the two-pion, three-pion, and four-pion correlation functions approaches 1 in the completely coherent case.

In EPG model, we can calculate density matrices \(G^{(1)}(p_i, p_j)\) and wave function \(u_0(p)\) \([32]\), and then obtain three- and four-pion correlation functions with Eqs. (18) — (22). In Fig. 3 we plot the three-pion correlations as a function of \(Q_3\) for the EPG sources with different temperatures and particle numbers. Here, the Lorentz-invariant momentum of the
three pions with four-dimension momenta \( p_i = (E_i, \mathbf{p}_i) \) \((i = 1, 2, 3)\) is defined as

\[
Q_3 = \sqrt{q_{12}^2 + q_{13}^2 + q_{23}^2},
\]

(25)

where

\[
q_{ij} = \sqrt{-(p_i - p_j)^\mu(p_i - p_j)_\mu}.
\]

(26)

FIG. 3: (Color online) Three-pion correlation functions for the EPG sources with different temperatures and particle numbers.

The three-pion correlation functions for the sources with small particle numbers \((N = 400 \text{ for } C_1 = 0.35 \text{ and } N = 800 \text{ for } C_1 = 0.40)\) are high. They decrease with increasing \(N\) because the source with large particle number has significant condensation. For fixed source particle number \(N\), the three-pion correlation function increases with increasing temperature because the condensation fraction is low at high temperatures. For fixed \(N\) and \(T\), the three-pion correlation functions for the sources with \(C_1 = 0.35\) are lower than those for the sources with \(C_1 = 0.40\) because the source with a small \(C_1\) has small characteristic length and high condensation fraction.
FIG. 4: (Color online) Four-pion correlation functions for the EPG sources with different temperatures and particle numbers.

In Fig. 4 we plot the four-pion correlations as a function of $Q_4$,

$$Q_4 = \sqrt{q_{12}^2 + q_{13}^2 + q_{23}^2 + q_{14}^2 + q_{24}^2 + q_{34}^2}, \quad (27)$$

for the EPG sources with different temperatures and the particle numbers. The four-pion correlation functions exhibit the similar variations with source particle number, temperature, and parameter $C_1$ as those of the three-pion correlation functions. However, the four-pion correlation functions are higher than the corresponding three-pion correlation functions because there are more contributions of the correlations of single pion pair, double pion pair, pure pion-triplet interference, and the contribution of the correlations of pure pion-quadruplet interference in four-pion correlation functions.
III. ANALYSES OF MULTI-PION CORRELATIONS IN EPG MODEL

In Ref. [28], the ALICE collaboration measured the three- and four-pion correlation functions in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, in the average transverse-momentum intervals $0.16 < K_{T3,T4} < 0.3$ GeV/$c$ and $0.3 < K_{T3,T4} < 1$ GeV/$c$, where

\[ K_{T3} = \frac{|p_{T1} + p_{T2} + p_{T3}|}{3}, \quad K_{T4} = \frac{|p_{T1} + p_{T2} + p_{T3} + p_{T4}|}{4}. \]  

(28)

In this section we shall investigate the three- and four-pion correlation functions in the EPG model in different transverse-momentum intervals in order to compare the model results with experimental data.

A. Three-pion correlations in EPG model

In the EPG model considered, the average momentum of the particles emitted from the ground state (coherent emission) is smaller than that of the particles emitted from the excited states (chaotic emission). So, the multi-pion correlation functions for the EPG source with a finite condensation fraction are momentum dependent.

We plot in Fig. 5 the three-pion correlation functions for the EPG sources with $C_1 = 0.35$ and the particle numbers $N = 400, 800$, and $1200$. The transverse-momentum cuts $K_{T3} < 0.16$ GeV/$c$, $0.16 < K_{T3} < 0.3$ GeV/$c$, and $K_{T3} > 0.3$ GeV/$c$ are applied in the simulated calculations of the correlation functions shown in Figs. 5(a)–(c), Figs. 5(d)–(f), and Figs. 5(g)–(i), respectively. In the lowest momentum interval $K_{T3} < 0.16$ GeV/$c$ [Figs. 5(a)–(c)], the correlation functions increase with source temperature $T$ and decrease with increasing particle number $N$ in the source. The reasons are that the source has a lower condensation fraction at higher temperature than that at lower temperature, and the condensation fraction increases with increasing particle number in the source. For $N = 400$, the results in Fig. 5(a) show that the intercepts of the three-pion correlation functions for the sources with the temperatures higher than 80 MeV approach the maximum 6 when being extrapolated to $Q_3 = 0$. This indicates that the sources with the higher temperatures are almost completely chaotic. The result of the three-pion correlation function for $T = 80$ MeV shown in Fig. 5(a) indicates that there is a finite fraction of coherent emission when the source has a temperature of $T = 80$ MeV and particle number $N = 400$. However, the results in Fig. 5(c) indicate that all the sources with the three temperatures have high condensation
fractions when $N = 1200$. On the other hand, in the highest momentum interval $K_{T3} > 0.3$ GeV/c [Figs. 5(g)–(i)], the high intercepts of correlation functions indicate that most of the pions with high momenta are emitted chaotically from excited states, even if the sources with high condensation fractions (with large $N$) \[32\]. The widths of the correlation functions in the highest momentum interval are narrower than those in the lowest momentum interval because the source has a wider spatial distribution for the pions emitted from excited states than that from ground state \[32\]. The correlation functions for the sources with $T = 80$ MeV are slightly higher than those for the sources with the higher temperatures in the highest momentum interval because the source spatial distribution is narrow at low temperature for the chaotic emission from excited states \[32\]. In the middle momentum interval $0.16 < K_{T3} < 0.3$ GeV/c [Figs. 5(d)–(f)], the condensation effect on the correlation functions is weaker than that in the lowest momentum interval $K_{T3} < 0.16$ GeV/c, because the number of the pions emitted from excited states is averagely larger in the middle momentum interval.
than that in the lowest momentum interval. Meanwhile, there is also the influence of source spatial distributions at different temperatures on the correlation functions in the middle momentum interval.

We plot in Fig. 6 the three-pion correlation functions for the EPG sources with $C_1 = 0.40$ and the particle numbers $N = 800, 1200, \text{and } 1600$. The transverse-momentum cuts $K_{T3} < 0.16 \text{ GeV/c}$, $0.16 < K_{T3} < 0.3 \text{ GeV/c}$, and $K_{T3} > 0.3 \text{ GeV/c}$ are applied in the simulated calculations of the correlation functions shown in Figs. (a)–(c), Figs. (d)–(f), and Figs. (g)–(i), respectively. One can see that the correlation functions in Fig. 6 exhibit the similar variations with source temperature and particle number in the transverse-momentum intervals as those in Fig. 5. We further show the comparisons of the three-pion correlation functions for the sources with $C_1 = 0.35$ and 0.40 in Fig. 7. Here, the particle numbers of both the sources with $C_1 = 0.35$ and 0.40 are 1200. In the lowest transverse-momentum interval $K_{T3} < 0.16 \text{ GeV/c}$, the three-pion correlation functions for the sources
with $C_1 = 0.35$ are lower than those for the sources with $C_1 = 0.40$ at all the temperatures. It is because the condensation fraction is high for the source with small $C_1$ and therefore with small characteristic length $a$. The differences between the correlation functions for the sources with $C_1 = 0.35$ and 0.40 become small in the middle transverse-momentum interval $0.16 < K_{T3} < 0.3$ GeV/c and almost zero in the highest transverse-momentum interval $K_{T3} > 0.3$ GeV/c. This indicates that the condensation effect on the correlation functions decreases with the increasing average transverse momentum $K_{T3}$ because the pions emitted chaotically from excited states have high average momentum.

We plot in Fig. 7 the three-pion cumulant correlation functions, $c_3(Q_3) = 1 + R(1,2,3)$, for the EPG sources with $C_1 = 0.35$ and 0.40 and in the low and high transverse-momentum intervals $K_{T3} < 0.3$ GeV/c and $K_{T3} > 0.3$ GeV/c. The particle numbers of the sources with $C_1 = 0.35$ are 400, 800, and 1200, and the particle numbers of the sources with $C_1 = 0.40$ are 800, 1200, and 1600, respectively. In the low transverse-momentum interval, $c_3$ decreases

FIG. 7: (Color online) Three-pion correlation functions for the EPG sources with $C_1 = 0.35$ and 0.40, in the transverse momentum intervals $K_{T3} < 0.16$ GeV/c [(a)–(c)], $0.16 < K_{T3} < 0.3$ GeV/c [(d)–(f)], and $K_{T3} > 0.3$ GeV/c [(g)–(i)]. The particle number is 1200.
with increasing $N$ because the condensation fraction of source increases with increasing $N$. As the correlation from the pure pion-triplet interference, $R(1,2,3)$, approaches zero when any pion pair among the three pions is uncorrelated, $c_3$ is sensitive to the source condensation in the low momentum interval. In the high transverse-momentum interval, $c_3$ is almost independent of the source particle number $N$. This indicates that most of the pions with high momenta are emitted chaotically from excited states even if the source with a considerable condensation fraction (for large $N$). The correlation function for $T = 80$ MeV is wider than that for the higher temperatures because the source spatial distribution is narrower at lower temperature than that at higher temperature for the chaotic emission from excited states.

![Figure 8](image_url)

**FIG. 8:** (Color online) Three-pion cumulant correlation functions for the EPG sources with $C_1 = 0.35$ and 0.40, in the transverse momentum intervals $K_{T3} < 0.3$ GeV/c and $K_{T3} > 0.3$ GeV/c.

### B. Four-pion correlations in EPG model

We plot in Fig. the four-pion correlation functions for the EPG sources with $C_1 = 0.35$ and 0.40 and in the low and high transverse-momentum intervals $K_{T4} < 0.3$ GeV/c.
FIG. 9: (Color online) Four-pion correlation functions $C_4(Q_4)$ for the EPG sources with $C_1 = 0.35$ and 0.40 in the transverse-momentum intervals $K_T^4 < 0.3$ GeV/$c$ and $K_T^4 > 0.3$ GeV/$c$. The particle numbers of the sources with $C_1 = 0.35$ are 400, 800, and 1200, and the particle numbers of the sources with $C_1 = 0.40$ are 800, 1200, and 1600, respectively. In the low transverse-momentum interval, the results of $C_4(Q_4)$ are sensitive to the source condensation. They increase with source temperature $T$ and decrease with increasing particle number $N$ in the source, because the source has a low condensation fraction at high temperature and the condensation fraction increases with increasing $N$. In the high transverse-momentum interval, the correlation functions have poor statistics in small $Q_4$ bins. They behave almost independent of source temperature and particle number. This indicates that they are insensitive to the source condensation because most of the pions with high momenta are emitted chaotically from excited states. The four-pion correlation functions for the sources with $T = 80$ MeV are slightly higher than those for the sources with the higher temperatures in the high transverse-momentum interval, as the three-pion correlation functions behaved. Because the pions emitted from excited states have wider spatial distribution than that emitted from ground state, the widths of the correlation functions become narrower in the high transverse-momentum interval than those in the low
transverse-momentum interval.

FIG. 10: (Color online) Four-pion cumulant correlation function $a_4(Q_4)$ for the EPG sources with $C_1 = 0.35$ and 0.40 in the transverse-momentum intervals $K_{T4} < 0.3$ GeV/$c$ and $K_{T4} > 0.3$ GeV/$c$.

In four-pion correlation function $C_4$, there are the contributions of the correlations of pion pair $R(i, j)$, double pion pair $[R(i, j)R(k, l)]$, pure pion-triplet interference $R(i, j, k)$, and pure pion-quadruplet interference $R(i, j, k, l)$. We use $a_4$, $b_4$, and $c_4$ to denote the four-pion cumulant correlations as 

\[ a_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2, 3) + R(1, 2, 4) + R(1, 3, 4) + R(2, 3, 4) + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4) + R(1, 2)R(3, 4) + R(1, 4)R(2, 3) + R(1, 3)R(2, 4); \]  

(29)

\[ b_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2, 3) + R(1, 2, 4) + R(1, 3, 4) + R(2, 3, 4) + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4); \]  

(30)

\[ c_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4). \]  

(31)
We plot in Figs. 10 and 11 the four-pion cumulant correlations $a_4(Q_4)$ and $b_4(Q_4)$ respectively, in the low and high transverse-momentum intervals $K_{T4} < 0.3$ GeV/c and $K_{T4} > 0.3$ GeV/c. In $a_4$ the correlations of single pair are removed. So, the results of $a_4(Q_4)$ are lower than those of $C_4(Q_4)$ (see Fig. 9). In the low transverse-momentum interval, $a_4(Q_4)$ is sensitive to the source condensation as $C_4(Q_4)$. It increases with source temperature $T$ and decrease with increasing particle number $N$ in the source. However, $a_4(Q_4)$ is also insensitive to source condensation in the high transverse-momentum interval as $C_4(Q_4)$. In $b_4$, the correlations of single and double pair are removed. One can see from Figs. 10 and 11 that $b_4(Q_4)$ is slightly lower than $a_4(Q_4)$ and they have the similar variations with source temperature and particle number in the low and high transverse-momentum intervals.

We plot in Fig. 12 the four-pion cumulant correlation $c_4(Q_4)$ for the EPG sources with $C_1 = 0.35$ and $0.40$ and in the low and high transverse-momentum intervals $K_{T4} < 0.3$ GeV/c and $K_{T4} > 0.3$ GeV/c. As $c_4$ contains only the correlations of pure pion-quadruplet inter-
FIG. 12: (Color online) Four-pion cumulant correlation function $c_4(Q_4)$ for the EPG sources with $C_1 = 0.35$ and 0.40 in the transverse-momentum intervals $K_{T4} < 0.3$ GeV/c and $K_{T4} > 0.3$ GeV/c.

REFERENCES, $c_4(Q_4)$ results are lower than those of $a_4(Q_4)$ and $b_4(Q_4)$. In the low transverse-momentum interval, $c_4(Q_4)$ decreases with increasing $N$ rapidly. It drops to 1 for all the temperatures when $N = 1200$ for $C_1 = 0.35$ and $N = 1600$ for $C_1 = 0.40$ [see Figs. 12(c) and Fig. 12(f)]. In the low transverse-momentum interval, $c_4(Q_4)$ is more sensitive to source condensation compared to $a_4(Q_4)$ and $b_4(Q_4)$.

IV. COMPARISON WITH EXPERIMENTAL DATA

In Ref. [28], the ALICE collaboration analyzed the three- and four-pion correlation functions in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. They observed a significant and centrality-independent suppression of the three- and four-pion correlations. In this section we shall compare the calculated three- and four-pion correlation functions in the EPG model with the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28], and further obtain the information of source condensation fraction.
A. Three-pion correlations

In Fig. 13 we show the comparison of the three-pion correlation functions $C_3(Q_3)$ in the EPG model with the experimental data for central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28], in the transverse-momentum intervals $0.3 < K_{T3} < 1$ GeV/c and $0.16 < K_{T3} < 0.3$ GeV/c. We first examine the three-pion correlation functions in the high transverse-momentum interval as shown in Figs. 13(a) and 13(b). In this momentum interval, the correlation function $C_3(Q_3)$ for the EPG source is almost independent of source particle number $N$. It is insensitive to source condensation because most of the pions with high momenta are emitted chaotically from exited states. As discussed in the last section, the strength of the multi-pion correlations for the EPG source varies with temperature in the high momentum interval due to the variation of the source spatial distribution at different temperatures [32]. We determine the temperature $T = 100$ and 90 MeV for the sources with $C_1 = 0.35$ and 0.40, respectively, by comparing the calculated three-pion correlation functions with the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC [28]. Then, we examine the three-pion correlation functions in the low transverse-momentum interval as shown in Figs. 13(c) and 13(d). In this momentum interval, the correlation function $C_3(Q_3)$ for the EPG source is sensitive to the source condensation. Its strength decreases with
increasing $N$ because the condensation fraction of source increases with $N$. One can see from Fig. 13(d) that the experimental data are almost between the results for $N = 1200$ and 1600 for the EPG sources with $C_1 = 0.40$, although they are slightly lower than the model results in large $Q_3$ region. However, the results for the EPG sources with $C_1 = 0.35$ [Fig. 13(c)] are higher than the experimental data in large $Q_3$ region. This may be because the average longitudinal momentum of the three pions, $K_{L3} = |p_{L1} + p_{L2} + p_{L3}|/3$, in the spherical EPG model is smaller than that in the experiment in the low transverse-momentum interval. The values of the three-pion correlation functions would decrease if we only increase the longitudinal momenta of the pions by a factor and let all other aspects remain the same.

FIG. 14: (Color online) Comparison of the three-pion cumulant correlation function $c_3(Q_3)$ for the EPG sources and the experimental data for central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28].

In Fig. 14 we show the comparison of the three-pion cumulant correlation functions $c_3(Q_3)$ in the EPG model with the experimental data for central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28], in the transverse-momentum intervals $0.3 < K_{T3} < 1$ GeV/$c$ and $0.16 < K_{T3} < 0.3$ GeV/$c$. One can see that the $c_3(Q_3)$ results for the EPG sources are almost consistent with the experimental data in the high and low transverse-momentum intervals except for those for the EPG source with $C_1 = 0.35$ and $N = 800$ in the low transverse-momentum interval [Fig. 14(c)].
B. Four-pion correlations

We show in Fig. 15 the comparisons of the four-pion correlation functions $C_4(Q_4)$ in the EPG model with the experimental data for central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28]. The temperatures of the EPG sources with $C_1 = 0.35$ and 0.40 are taken to be 100 and 90 MeV, respectively. One can see from Figs. 15(a) and 15(b) that the four-pion correlation functions for the EPG sources are almost independent of the source particle number $N$ because they are insensitive to source condensation in the high momentum interval as discussed in the last section. The model results are consistent with the experimental data in the high transverse-momentum interval as the three-pion correlation functions. From Figs. 15(c) and 15(d) one can see that the four-pion correlation function for the EPG source with larger $N$ is lower than that for the source with smaller $N$ in the low transverse-momentum interval. It is because the condensation fraction is higher for the source with larger $N$. In small $Q_4$ region, the experimental data are between the model results for the sources with the small and large $N$. In large $Q_4$ region, the model results are slight higher than the experimental data. This may be because the average longitudinal momentum of the four pions, $K_{L4} = |p_{L1} + p_{L2} + p_{L3} + p_{L4}|/4$, in the spherical EPG model is smaller than that in experiment in the low transverse-momentum interval.
FIG. 16: (Color online) Comparison of the four-pion cumulant correlation function $a_4(Q_4)$ for the EPG sources and the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28].

FIG. 17: (Color online) Comparison of the four-pion cumulant correlation function $b_4(Q_4)$ for the EPG sources and the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28].

We show in Figs. 16 and 17 the comparisons of the four-pion cumulant correlation functions $a_4(Q_4)$ and $b_4(Q_4)$ in the EPG model with the experimental data for central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28], respectively. Because the correlations of single pair...
are removed from $a_4$ and the correlations of single and double pair are removed from $b_4$, the results of $a_4(Q_4)$ are lower than those of $C_4(Q_4)$, and the results of $b_4(Q_4)$ are further lower than those of $a_4(Q_4)$. One can see that the model results of $a_4(Q_4)$ and $b_4(Q_4)$ in the high transverse-momentum interval are almost independent of the source particle number $N$. They are consistent with the experimental data in the high momentum interval. However, the model results are sensitive to the source particle number $N$ in the low transverse-momentum interval. The experimental data are almost between the model results for $N = 1200$ and 1600 for the sources with $C_1 = 0.40$ and more consistent with the model results for $N = 1200$ for the source with $C_1 = 0.35$.

![Diagram](image)

**FIG. 18:** (Color online) Comparison of the four-pion cumulant correlation function $c_4(Q_4)$ for the EPG sources and the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28].

We show in Fig. 18 the comparisons of the four-pion cumulant correlation functions $c_4(Q_4)$ in the EPG model with the experimental data for central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28]. It should be mentioned that the error bars of the experimental data shown in the figures in this paper are statistic error plus the system error, which is large for $c_4(Q_4)$ in the small $Q_4$ region [28]. The error bars of the model results shown in the figures in this paper are statistic error. One can see that the model results of $c_4(Q_4)$ are independent of the particle number of the sources and consistent with the experimental data in the high transverse-momentum interval. However, the model results of $c_4(Q_4)$ in the low transverse-
momentum interval are particle-number dependent in the low $Q_4$ region. The experimental data of $c_4(Q_4)$ in the low transverse-momentum interval are between the model results for the low and high $N$ in the small $Q_4$ region, and can be reproduced by the EPG model in the large $Q_4$ region.

C. Condensation fraction

We find in the last two subsections that the three- and four-pion correlation functions in the EPG model can reproduce in some degree the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28]. By comparing with the experimental data, we determine that the most suitable temperatures for the EPG sources with $C_1 = 0.35$ and 0.40 are 100 and 90 MeV, and the particle numbers are perhaps in the regions $[800, 1200]$ for the source with $C_1 = 0.35$ and $[1200, 1600]$ for the source with $C_1 = 0.40$. With these source parameters, we further determine the condensation fractions between 0.22 – 0.47% for the source with $C_1 = 0.35$ and 0.16 – 0.37% for the source with $C_1 = 0.40$, as shown in Fig. 19.

![Condensation fractions for the EPG sources](image)

FIG. 19: (Color online) Condensation fractions for the EPG sources with $T = 100$ MeV for $C_1 = 0.35$ and $T = 90$ MeV for $C_1 = 0.40$.

In Ref. [28], the ALICE collaboration extracted the coherent fraction, $32\% \pm 3\%$(stat) $\pm 9\%$(syst), by analyzing the suppression of four-pion correlations in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, and pointed out that this coherent fraction value cannot explain the suppression of three-pion correlations observed. In the EPG model, the pion emission from ground state is coherent and the condensation fraction defined by Eq. (5) is coherent fraction.
The values of condensation fraction are determined by the comparisons of the model results and experimental data of three- and four-pion correlations. They are consistent with the value of coherent fraction extracted by the ALICE collaboration \[28\]. Also, in the EPG model the source size for a smaller $C_1$ parameter is smaller than that for a larger $C_1$ parameter. Considering the source size is larger for central collisions than that for peripheral collisions in experiments, the determined condensation fraction 0.22 – 0.47% for $C_1 = 0.35$ and 0.16 – 0.37% for $C_1 = 0.40$ are also consistent with the conclusion of experimental analyses that “There does not appear to be a significant centrality dependence to the extracted coherent fractions.” \[28\] According to the EPG model, the condensation not only depends on the particle number which is smaller in peripheral collisions than in central collisions, but also depends on the source size which is also smaller in peripheral collisions than in central collisions. The condensation degree increases with increasing particle number and decreases with increasing source size. So, the comprehensive effect of particle number and source size may lead to the result that the condensation fraction or coherent fraction is independent of collision centrality.

Finally, it should be mentioned that the EPG model deals with the canonical ensemble in which the systems of pion gas have a fixed particle number $N$. So, the two- and multi-pion correlation functions calculated in the EPG model are the so-called “exclusive correlation functions” \[15\]. They should be compared with the corresponding experimental correlation functions obtained from the events with the same multiplicity. However, because of data statistics the correlation functions obtained experimentally are from many events in some multiplicity intervals. In this case, a strict comparison should be between the experimental data in a multiplicity interval and the averaged EPG exclusive results over the same multiplicity interval with the weights of multiplicity obtained experimentally. On the other hand, it is also meaningful to make a comparison between the experimental correlation functions in a multiplicity interval and the EPG exclusive results with the particle number consistent with the average multiplicity in the multiplicity interval, if the differences between the exclusive and inclusive correlation functions are negligible approximately. In fact, the the difference between the inclusive and exclusive correlation functions is from the effects of higher-order correlations \[15\], the residual correlation effects in single- two- and multi-pion samples \[11, 34, 35\]. In a $m$-pion sample, the leading-order effect of multi-pion correlations is approximately proportional to $[ m \cdot \int |\tilde{\rho}(\mathbf{p}, E(\mathbf{p}))| d^3p ]$, where $\tilde{\rho}(p)$ is the on-shell Fourier
transform of source density, which is very small in high-energy heavy-ion collisions where the source radius and lifetime are about 10 fm and 10 fm/c [1, 15]. More detailed investigations of the difference between the exclusive and inclusive correlation functions and the comparison between the EPG correlation functions and the experimental data will be of great interest. Additionally, the intercepts of pion HBT correlation functions can be affected by long-lived resonance decays, their effects on pion transverse-momentum spectra are discussed in the chemical nonequilibrium thermal model [36, 37]. It will be of considerable interest to estimate the influence of long-lived resonance decay and remove the influence in the coherence analyses of multi-pion interferometry.

V. SUMMARY AND CONCLUSION

We have calculated the three- and four-pion correlations in the EPG model with Bose-Einstein condensation. The relationship between the multi-pion correlations and the source condensation fraction is investigated. It is found that the multi-pion correlation functions and cumulant correlation functions are sensitive to the condensation fraction of the EPG source in the low transverse-momentum intervals of the three and four pions, $K_{T3, T4} < 0.3$ GeV/c. These correlation functions exhibit significant decreases with decreasing source temperature and increasing source particle number in the low transverse-momentum intervals, because the condensation fraction of the EPG source is high at a low temperature and large particle number. On the other hand, the multi-pion correlation functions and cumulant correlation functions are insensitive to the source condensation in the high transverse-momentum intervals $K_{T3, T4} > 0.3$ GeV/c. They are almost independent of the source particle number in the high transverse-momentum intervals, because most of the pions with high momenta are emitted chaotically from excited states in the EPG model even if with a considerable condensation fraction. We have compared the model results of three- and four-pion correlation functions and cumulant correlation functions with the experimental data for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC. It is found that the multi-pion correlation functions and cumulant correlation functions in the EPG model may reproduce the experimental results in a considerable degree. The source condensation fraction determined by the comparisons is between 16 – 47%. Further investigations of the comparison between the EPG correlation functions and the experimental data are of great
interest.

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