The observation of $\beta$-decay has furnished us with the evidence for many fundamental ingredients of what is known nowadays as the Standard Model. The universality of the weak interaction and conservation of the vector current (CVC) led to the introduction of the quark mixing matrix CKM which has to obey the constraint of unitarity $\sum_k V_{ik} V_{jk}^* = \delta_{ij}$. More specifically, the test of unitarity of its first row, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$ [1] is one of the most stringent constraints on the parameters of the Standard Model and its extensions [2].

The top left corner element $|V_{ud}| = 0.97420(21)$ [1] dominates both the value and the uncertainty of the unitarity constraint, and is obtained almost exclusively from the global analysis of a number of superallowed $0^+ \rightarrow 0^+$ $\beta$-decays [3,4]. One of the cornerstones of this analysis is an adequate calculation of one-loop radiative corrections which have been studied for over 6 decades, and the formalism has been worked out, e.g., in Refs. [5,6]. The very accurate extraction of $V_{ud}$ from superallowed nuclear decays is empowered by the following formula,

$$|V_{ud}|^2 = 2984.43 s / [\mathcal{F} t(1 + \Delta R^0)].$$  \hspace{1cm} (1)

The radiative correction $\Delta R^0$ is evaluated on a free neutron [6], and is conventionally singled out also for nuclear decays. The universal and very precise value $\mathcal{F} t = 3072.07(63) s$ is an average of 14 reduced half-lives [3,4] $\mathcal{F} t = ft(1 + \delta_R^0)(1 + \delta_{NS} - \delta_C)$, which are obtained from the measured reduced half-lives $ft$, and should be independent of the particular decay as a consequence of CVC. Here, $\delta_R^0$ is the “outer” correction which depends on the emitted electron energy and the charge of the daughter nucleus. The nuclear structure dependence resides in the energy-independent “inner” corrections $\delta_C$ and $\delta_{NS}$: the former stems from isospin-breaking corrections to the tree-level matrix element of the Fermi operator, and the latter from the nuclear effects in the $\gamma W$-box, defined with respect to the free-neutron result for $\gamma W$-box entering $\Delta R_0^0$.

The $\gamma W$-box correction plays a central role in the uncertainty of $V_{ud}$. Recently, it was re-examined in the framework of dispersion relations (DR) [7,8]. Ref. [7] addressed hadronic contributions to the universal correction $\Delta V_0$, and found a substantial shift in the extracted value of $V_{ud}$ with a reduced hadronic uncertainty, $|V_{ud}| = 0.97370(14)$, raising tension with the unitarity, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9894(4)$. Consequently, Ref. [8] investigated the robustness of the procedure of splitting the $\gamma W$-box on a nucleus into the universal, free-neutron $\Delta V_0$ and the nucleus-specific $\delta_{NS}$. In particular, the “quenching” of the free-nucleon elastic box contribution was addressed, and a dispersive evaluation suggested that this effect previously calculated in Ref. [9] and included in all subsequent analyses of the superallowed nuclear decays was underestimated. A proper account of the quasielastic contribution led to a reduction in the reduced half-life, $\mathcal{F} t = 3072.07(63) s \rightarrow \mathcal{F} t_{\text{new}} = 3070.65(69) s$, bringing the value of $V_{ud}$ closer to the old value, $|V_{ud}| = 0.97392(15)$ and improving the agreement with unitarity somewhat, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9988(4)$.

This Letter is dedicated to a critical assessment of yet another ingredient of Eqs. (1,2), the splitting of the full radiative correction into “inner” and “outer”. The logic behind this splitting uses the fact that while the energy released in superallowed decays is few MeV, the scale that governs the strong interaction is the pion mass $\approx 140$ MeV. Following this line of reasoning, energy-dependent effects which at the same time depend on the details of strong interaction will only start to show up at $\alpha/\pi \times E/m_\pi \sim 10^{-5}$ (with $\alpha \approx 1/137$ the fine structure constant and $E$ the energy of the lepton), negligible at the present level of precision.

All references that have dealt with nuclear structure contributions to the $\gamma W$-box in the past have assumed
The correctness of this argument. However, the presence of the nuclear excitation spectrum that is separated from the ground state by only a few MeV provides a more natural energy scale \( \Lambda_{\text{Nucl}} \ll M_\pi \) with which the decay electron energy should be compared. It is then possible that energy- and nuclear structure-dependent corrections may scale as \( (\alpha/\pi)(E/\Lambda_{\text{Nucl}}) \sim 10^{-3} \) instead. This would mean that, at the current precision level, the conventional splitting of the \( \gamma W \)-box into inner and outer contributions is not warranted (the inner correction leaks into the outer one), and along with the pure QED outer correction \( \delta_\pi \), a new energy-dependent nuclear polarizability correction \( \Delta_{\text{NS}}^{\gamma W} \) has to be included in the universal \( \mathcal{F} \) value. I investigate this scenario below.

The \( \gamma W \)-box correction depicted in Fig. 1 is defined as

\[
T_{\gamma W} = \frac{\alpha G_F V_{ud}}{\sqrt{2}} \times \int \frac{d^4k_1}{\pi^2} \frac{\bar{u}_e \gamma^{\nu}(k_1+m_e)\gamma^{\mu}(1-\gamma_5)v_\nu}{(k-k_1)^2[k_1^2-m_\pi^2]} M_W^2 W_{\mu\nu},
\]

with the forward Compton tensor

\[
W_{\mu\nu}^{\gamma W} = \frac{i}{4\pi} \int dx e^{iqx} \langle A'|T[J_{\text{em}}^\nu(x), J_{\text{em}}^{\mu}(0)]|A \rangle,
\]

with the isospin index \( a = \pm \). This amplitude evaluated at zero momentum transfer (at the needed precision level) but finite energy of the emitted electron is normalized to the tree-level \( W \)-exchange. I define the \( \Box_{\gamma W} \)-correction per active nucleon as

\[
T_W + T_{\gamma W} = -N \sqrt{2} G_F V_{ud} [1 + \Box_{\gamma W}] \bar{u}_e \gamma(1-\gamma_5)v_\nu, \tag{5}
\]

where \( N \) stands for the number of neutrons (protons) for \( \beta^- (\beta^+) \) process, respectively. The spin-independent Compton tensor is given by,

\[
W_{\mu\nu}^{\gamma W} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{p^\mu q^\nu}{(p \cdot q)} T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p \cdot q)} T_3,
\]

with \( q^\mu = p^\mu - (p \cdot q)/q^2 q^\mu \), and the forward amplitudes are defined such that \( \text{Im} T_i = F_i(\nu, Q^2) \), with \( F_i \) the structure functions that are functions of the photon energy \( \nu = (p \cdot q)/M \) and photon virtuality \( Q^2 \).

The imaginary part of the box diagram is easily obtained using definitions in Eqs. 356, tensor contraction and some simple algebra as

\[
\text{Im} \Box_{\gamma W}(E) = \frac{\alpha}{\pi N} \int_0^{E_{\text{max}}} \frac{dE_1}{E} \int_0^{Q_{\text{max}}} dQ^2 \left| F_3 \right|^2 \frac{1}{\nu Q^2 - 1 - 4E_1/E} F_2 \right|, \tag{7}
\]

where I neglected the electron mass. The \((-+)\) sign in front of the contributions of \( F_1, F_2 \) corresponds to \( \beta^- (\beta^+) \) decay, respectively. The limits of the \( E_1 \)-integration is \( E_{\text{max}} = E - \epsilon \), with \( \epsilon \) the threshold for the photo-breakup of the target nucleus. The \( Q^2 \)-integral runs up to \( Q_{\text{max}} = (s - W^2)(s - M^2)/s \), and \( W \in [M + \epsilon, \sqrt{s}] \) is the invariant mass of the excited intermediate nuclear or hadronic state. Finally, \( s = (p + k)^2 = M^2 + 2ME \) and \( E \) the energy of the leptons in the lab system where the target is at rest. The real part of the box correction is obtained from the forward dispersion relation of the form

\[
\text{Re} \Box_{\gamma W}(E) = \frac{1}{\pi} \int \frac{dE'}{E' - E + \pm dE'} \left( \text{Im} \Box_{\gamma W}(E') \right), \tag{8}
\]

with the first term in the square bracket originating from the discontinuity of the direct graph, and the second term from that with the boson legs crossed. To determine the sign between the two contributions the properties of the Compton amplitudes need be to discussed. I start with considering the isospin structure of amplitudes \( T_i \). Electromagnetic interaction does not conserve isospin and contains both isoscalar and isovector components, \( T_i^{(0)} \gamma^a + T_i^{(-)} (\frac{1}{2}[\tau^a, \tau^a]) \). The two isospin amplitudes have a different behavior under the boson crossing \( q \to -q, \mu \leftrightarrow \nu, (p \cdot q) \to -(p \cdot q) \),

\[
T_i^{(0)}(-\nu, Q^2) = \xi_i^{(0)}(-\nu, Q^2), \tag{9}
\]

with \( \xi_i^{(0)} = 1 \) for \( T_1 \) and \( \xi_i^{(0)} = -1 \) for \( T_2, T_3 \), and \( \xi_i^{(-)} = -\xi_i^{(0)} \). As a result, the \( \gamma W \)-box will contain both even and odd powers of energy. I account for the leading \( E \)-dependence: \( \mathcal{O}(E^0) \) in the \( E \)-even and \( \mathcal{O}(E) \) \( E \)-odd pieces, respectively. The hadronic structure-dependent part of the \( E \)-even piece that is due to the weak vector current (contribution of \( F_{1,2}^{(-)} \) cancels against other 1-loop corrections [5] and is omitted. To reflect this subtraction I use an overscore for the \( E \)-even correction \( \Box_{\gamma W}^{\text{even}} \). Changing the order of integration and assuming that the energy released in the \( \beta^- \)-decay process is smaller than the nuclear excitations, I obtain the dispersion representation for the leading \( E \)-behavior of the \( \gamma W \)-box:

\[
\text{Re} \Box_{\gamma W}^{\text{even}} = \frac{\alpha}{\pi N} \int dQ^2 \int_{\nu_{\text{thr}}}^{\infty} \frac{d\nu}{\nu_{\text{thr}}} F_3^{(0)} M\nu \left( \frac{1}{E_{\text{min}}} - \frac{\nu}{4E_{\text{min}}^2} \right),
\]
the contribution of the nuclear form factor of polarizabilities is available, I obtain for the \( \alpha \) and that of the nuclear electric dipole response polarizability and charge radius, and a more microscopic methods: dimension analysis with the nuclear dipole structure functions \( F \) and \( e \), their vector charged current - electromagnetic current interaction.

This correction leads to an \( E \)-dependent correction to the differential decay rate, which is roughly independent of the nucleus since \( A/N \approx 2 \) for all nuclei relevant for the superallowed decays,

\[
\Delta_R(E) = 2 \text{Re} \Box^{\text{odd}}_{\gamma W}(E) = 2 \times 10^{-4} \left( \frac{E}{5 \text{MeV}} \right),
\]

the scale 5 MeV represents an average \( Q \)-value across the 14 superallowed decays used for the \( V_{ud} \) extraction.

**Estimate in the free Fermi gas model**

In a microscopic picture, a large part of the nuclear polarizability can be explained by the quasiparticle mechanism. The (generalized) Compton reaction on a nucleus proceeds via the knockout of a single active nucleon by the initial electroweak probe, leaving the remaining part of the nucleus unaffected, and the reabsorption of the nucleon back into the nucleus accompanied by the emission of the final photon, see Fig. 2. The finite gap between

![FIG. 2: Quasielastic contribution to the nuclear \( \gamma W \)-box.](image)

The observed approximate scaling of nuclear radii with the atomic number \( R_{\text{Ch}} \sim R_0 A^{1/3} \) with \( R_0 \approx 1.2 \text{ fm} \) [10], and that of the nuclear electric dipole response \( \alpha_E \sim (2.2 \times 10^{-3}) A^{2/3} \text{ fm}^3 \) [11], leads to the estimate

\[
\text{Re} \Box^{\text{odd}}_{\gamma W} \sim 5 \times 10^{-5} \left( \frac{E}{5 \text{MeV}} \right) \left( \frac{A}{N} \right).
\]

This correction leads to an \( E \)-dependent correction to the differential decay rate, which is roughly independent of the nucleus since \( A/N \approx 2 \) for all nuclei relevant for the superallowed decays,

\[
\Delta_R(E) = 2 \text{Re} \Box^{\text{odd}}_{\gamma W}(E) = 2 \times 10^{-4} \left( \frac{E}{5 \text{MeV}} \right),
\]

the bound state and the continuum, the removal energy, is one relevant parameter that governs the size of the nuclear polarizability. The other parameter is the Fermi momentum \( k_F \), the typical momentum of a nucleon inside the nucleus, which defines the initial kinematics from which the knockout process results. In the case of a decay process, the initial and final states are not identical due to the \( n \rightarrow p \) conversion for the \( \beta^- \) process, and \( p \rightarrow n \) for \( \beta^+ \) process. Apart from the change of the nucleon species and thus the change of the charge of the nucleus in the initial (parent) and final (daughter) state, the mass of the daughter is slightly smaller, which is a prerequisite of the decay to take place. For the quasielastic process \( W^\pm + A \rightarrow n(p) + A'' \rightarrow \gamma + A' \), with \( A'' \) a spectator nuclear state, there are two distinct removal energies at the first and the second stage of the reaction. Specifically for the \( \beta^+ \) process, \( \epsilon_1 = M_{A''} + M_n - M_A \) and \( \epsilon_2 = M_{A'} + M_n - M_A' \) obeying \( \epsilon_2 > \epsilon_1 \). In the recent work [8] it was proposed to use an effective removal energy defined as \( \bar{\epsilon} = \sqrt{\epsilon_1 \epsilon_2} \). For the 20 superallowed \( \beta^+ \) decays listed in [3] the effective removal energies fall within a narrow range, \( \bar{\epsilon} = 7.5 \pm 1.5 \text{ MeV} \) [8]. In the free Fermi gas (FFG) model the structure functions entering Re\( \Box_{\gamma W} \) have a generic form

\[
\frac{1}{N} F_2(\nu, Q^2) = f_2^B(Q^2) S(\nu, Q^2, \bar{\epsilon}, k_F),
\]

with the spectral function

\[
S = F_2(|q|, k_F) \int d^3k |\phi(k)|^2 \delta((k + q)^2 - M^2).
\]
3/(4\pi k_F^2)\theta(k_F-k) normalized as \( \int d^3k|\phi(k)|^2 = 1 \). Pauli blocking is described by the Pauli function

\[ F_P(|q|, k_F) = \frac{3|q|}{4k_F} \left[ 1 - \frac{q^2}{12k_F^2} \right] \text{ for } |q| \leq 2k_F, \quad (17) \]

and \( F_P = 1 \) otherwise, and \( |q| = \sqrt{q^2 + Q^2} \) stands for the 3-momentum of the virtual photon (\( W^\pm \) boson). The \( \delta \)-function reflects the knock-out nucleon on shell. The integral in Eq. (16) can be carried out analytically after which the dependence of the spectral function \( S \) on the breakup threshold becomes explicit. Finally, the residues \( f_i \) corresponding to the coefficient in front of the \( \delta \) function in the nucleon Born contribution read

\[ f_1^{(0)} = \frac{Q^2}{8} G_M W G_S, \quad f_2^{(0)} = \frac{Q^2}{4} G_E G_S + \frac{\tau G_V G_S}{1 + \tau}, \]
\[ f_3^{(-)} = -\frac{Q^2}{4} G_A G_M, \quad (18) \]

with \( G_{E,M}^{S,V} = G_{E,M}^{v}(Q^2) \pm G_{E,M}^{n}(Q^2) \) the nucleon isoscalar and isovector electromagnetic form factors, the axial form factor \( G_A \) with \( G_A(0) = -1.2755 \), and the nucleon recoil \( \tau = Q^2/4M_p^2 \). A numerical evaluation with the effective separation energy \( \bar{E} = 7.5 \pm 1.5 \text{ MeV} \) and Pauli momentum \( k_F = 235 \pm 10 \text{ MeV} \) leads to the FFG estimate

\[ \Delta_R(E) = (1.4 \pm 0.2) \times 10^{-3} \left( \frac{E}{5 \text{ MeV}} \right). \quad (19) \]

This estimate is one order of magnitude larger than the naive estimate with the nucleon electric dipole polarizability and the nuclear size. It is well known that QE cross sections with slightly virtual photons are much larger than with real photons, so the estimate \( a_E(Q^2) \sim a_E(0)e^{-R^2/4Q^2/\hbar^2} \) used in the previous section is likely to underestimate the actual effect. On the other hand, the FFG model is known to overestimate the quasielastic response at very low values of \( Q^2 \) where meson exchange currents tend to lead to a suppression. So the realistic size of the effect is likely to lie between those two extremes. Note that the contribution of \( F_3^{(-)} \) dominates over the other two terms in Eq. (10) in FFG due to the large nucleon isovector magnetic moment.

**Numerical results and the effect on the \( Ft \)-values**

Above, I obtained an estimate of the energy-dependent correction in two different models which give a rough idea of the lower and upper bound of the size of the effect. For numerical estimates I will use the average of the two estimates with a 100% uncertainty,

\[ \Delta_R(E) \sim (8 \pm 8) \times 10^{-4} \left( \frac{E}{5 \text{ MeV}} \right), \quad (20) \]

and this result is independent of the nucleus and directly measurable if the \( \beta \)-spectrum can be determined to the necessary precision. If the spectrum is not measured and only the total rate is observed, the respective correction to the \( Ft \)-value is obtained by integrating \( \Delta_R(E) \) over the \( \beta \)-spectrum. This correction will be decay-specific via the decay-specific \( Q \)-value, \( Q = M_A - M_{A'} \), with \( M_A (M_{A'}) \) the mass of the parent (daughter) nucleus, respectively. It is defined as

\[ \Delta_R^{NS} = \frac{\int_{E_m}^{E_m} dEEp(E_m - E)^2 \Delta_R(E)}{\int_{E_m}^{E_m} dEEp(E_m - E)^2}, \quad (21) \]

where \( p = \sqrt{E^2 - m_e^2} \) is the electron 3-momentum, \( m_e \) the electron mass, and \( E_m = (M_A^2 - M_{A'}^2 + m_e^2)/2M_A \approx Q \) the maximal electron energy available in a given decay. The integration with the estimate of Eq. (20) leads to

\[ \Delta_R^{NS} \approx (8 \pm 8) \times 10^{-5} \frac{Q}{\text{MeV}}, \quad (22) \]

which modifies the \( Ft \) values according to

\[ Ft' = Ft(1 + \Delta_R^{NS}). \quad (23) \]

The absolute shift in the \( Ft \) values due to the nuclear polarizability contribution obtained as \( \delta Ft = Ft \times \Delta_R^{NS} \) is shown for the 14 most accurately measured superallowed decays in Table I along with the central values and the respective uncertainties of the original analysis of Ref. [3]. It is seen that for the seven most precise \( Ft \)

| Decay | \( Q \) (MeV) | \( \Delta_R^{NS} \times 10^{-4} \) | \( \delta Ft(s) \) | \( Ft(s) \) [3] |
|-------|-------------|-----------------|-----------------|-------------|
| \( ^{10}C \) | 1.91 | 1.5 | 0.5 | 3078.0(4.5) |
| \( ^{14}O \) | 2.83 | 2.3 | 0.7 | 3071.3(3.2) |
| \( ^{22}Mg \) | 4.12 | 3.3 | 1.0 | 3077.9(7.3) |
| \( ^{34}Ar \) | 6.06 | 4.8 | 1.5 | 3065.6(8.4) |
| \( ^{38}Ca \) | 6.61 | 5.3 | 1.6 | 3076.4(7.2) |
| \( ^{26m}Al \) | 4.23 | 3.4 | 1.0 | 3072.9(1.0) |
| \( ^{34}Cl \) | 5.49 | 4.4 | 1.4 | 3070.7(1.7) |
| \( ^{38m}K \) | 6.04 | 4.8 | 1.5 | 3071.6(2.0) |
| \( ^{42}Sc \) | 6.43 | 5.1 | 1.6 | 3072.4(2.3) |
| \( ^{46}V \) | 7.05 | 5.6 | 1.7 | 3074.1(2.0) |
| \( ^{50}Mn \) | 7.63 | 6.1 | 1.9 | 3071.2(2.1) |
| \( ^{54}Co \) | 8.24 | 6.6 | 2.0 | 3069.8(2.4) |
| \( ^{62}Ga \) | 9.18 | 7.3 | 2.2 | 3071.5(6.7) |
| \( ^{74}Rb \) | 10.42 | 8.3 | 2.6 | 3076(11) |

**TABLE I:** For 14 superallowed decay channels, the respective \( Q \)-value, the fractional effect on the decay rate obtained from the energy-dependent correction integrated over the electron spectrum, the respective shift in the \( Ft \) value, in comparison with the \( Ft \) values and respective uncertainties taken from [3] are displayed.
where I used the average of $\delta F t$’s for the seven most precise measurements that dominate the $F t$ fit ($^{26}$Al through $^{54}$Co) resulting in $\delta F t = (1.5 \pm 1.5) s$, at the level of 2.2 standard deviations of the analysis of Ref. [3]. This effect has been neglected in all analyses of nuclear beta decay because it was believed to be negligible, as it is in the free neutron decay. Present analysis shows that that assumption is not justified, and if the relative precision of $2 \times 10^{-4}$ for the $F t$ value and its constancy as a test of CVC and a constraint of non-standard scalar interactions is to be maintained, a robust estimate of this novel effect at the relative 20-30% or better is necessary.

The extraction of $V_{ud}$ from nuclear decays is based on the $F t$ value, so the inclusion of this energy-dependent nuclear polarizability effect will also induce a shift in $V_{ud}$. Recently we re-evaluated the nuclear modification of the energy-independent correction $\text{Re} C_{1W}^{\gamma W}$ [8] and found that this modification was underestimated in the literature [3,9]. Our analysis of the inner correction suggested a shift of a size similar to that obtained in this work but in the opposite direction,

$$F t = 3072.07(63)s \rightarrow F t = 3070.65(63)(28)s. \quad (25)$$

The physics underlying both corrections, energy-independent and energy-dependent, is the same, so the two effects should be considered jointly. When added together, the positive energy-dependent correction tends to cancel the energy-independent correction, leaving the central value of $F t$ roughly unchanged. This cancellation supports the conclusion of Ref. [2] that the correct value of $V_{ud}$ extracted from the superallowed nuclear decays is lower than previously thought. While individual shifts are substantial, no firm conclusion on the shift of the central value of $F t$ with respect to the analysis of Ref. [3] can be made at this stage. The size of the additional nuclear corrections found in this work and in [7] can be used to estimate the additional uncertainty, and the deficit of the CKM first-row unitarity becomes $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0016(6)$. In summary, I considered a novel effect of a distortion of the electron spectrum in superallowed nuclear $\beta$-decays due to nuclear polarizabilities. This effect has been completely neglected in the literature based on dimensional arguments adopted originally in the neutron decay. I showed that these arguments are not applicable to nuclear decays where the $Q$-values and nuclear separation energies are of the similar size, leading to a slightly higher probability for emitting the electron at the upper end of the spectrum, than at the lower end. I estimated the size of the correction to be applied to the $F t$ values using a naive dimensional analysis operating with the dipole nuclear polarizability, and in the free Fermi gas model, and demonstrated that the effect is sizable and shifts the resulting average $F t$ value towards larger values. On the other hand, the free Fermi gas estimate of the energy-independent nuclear polarizability correction of Ref. [8] led to a shift of the average $F t$ value in the opposite direction and of the similar size. Upon incorporating both contributions the $F t$ value remains roughly unaffected.

The exact extent of this cancellation and the size of both effects should be assessed in a more precise way, and I seize the opportunity to prompt the nuclear theory community to address the issues raised in this work. The distortion of the energy spectrum of positrons from nuclear $\beta^+$ decay is a measurable effect. One of the motivations of high-precision measurements of the $\beta$-decay spectra is the search for new scalar and tensor interactions [2]. The presence of new scalar interactions, e.g., would lead to a distortion of the lower part of the spectrum, $\sim b^\pm r$. The energy-dependent effect considered here would mostly enhance the higher part of the spectrum, so the planned experiments may help confirming or constraining this novel effect. Conversely, experimental searches for Fierz interference in beta decays may crucially depend on its inclusion: if both distortions go in the same direction, their joint effect on the spectrum may not change its shape, erroneously returning a null result for the Fierz interference if the distortion due to nuclear polarizabilities is not properly taken into account.

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