Pair production: de Sitter vs Schwinger

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We consider the difference between the radiation of particles in the de Sitter spacetime and the Schwinger pair creation in the electric field. We use the stationary Painleve-Gullstrand metric for the de Sitter spacetime, where the particles are created by Hawking radiation from the cosmological horizon, and time independent gauge for the electric field. In these stationary frames the Hamiltonians and the energy spectra of massive particles look rather similar. However, the final results are essentially different. In case of Schwinger pair production the number density of the created pairs grows with time, while in the de Sitter vacuum the number density of the created pairs is finite. The latter suggests that Hawking radiation from the cosmological horizon does not lead to instability of the de Sitter vacuum, and the other mechanisms of instability are required for the dynamical solution of the cosmological constant problem.

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I. INTRODUCTION

The issue of the stability of the de-Sitter vacuum is still an unsolved problem. In particular, in discussion of the stability problem, the close analogy between the Schwinger creation of charged particle in a uniform electric field and the Hawking radiation in the de Sitter vacuum has been used.\textsuperscript{1,2} Here we compare these two processes using the approach of semiclassical tunneling.\textsuperscript{3–7} As distinct from Ref.\textsuperscript{1,2}, we consider the time independent gauge for the electric field and the stationary Painleve-Gullstrand metric\textsuperscript{8,9} for the de Sitter spacetime, where the particles are created by Hawking radiation from the cosmological horizon. In these stationary frames the two systems look rather similar. However, the final results are essentially different. In case of Schwinger pair production the number density of the created pairs grows with time, while in the de Sitter vacuum the number density of the created pairs is finite. The latter suggests that Hawking radiation from the cosmological horizon does not lead to instability of the de Sitter vacuum.

II. SEMICLASSICAL ENERGY SPECTRUM OF PARTICLE IN ELECTRIC FIELD AND IN DE SITTER SPACETIME

Let us compare the particle production in de Sitter vacuum and in constant electric field in the tunneling approximation. In this semiclassical approximation there is no difference between fermions and bosons, except for the integer number (number of species, spin, polarization, etc.). In both cases we use the description in terms of the stationary metric and fields.

We use the Painleve-Guustrand metric known in condensed matter as acoustic metric\textsuperscript{10}:

\begin{equation}
    ds^2 = -dt^2(1 - v^2) - 2dt \, dr \cdot v + dr^2 ,
\end{equation}

where $v(r)$ is the velocity of the free-falling observer, who crosses the horizon, and $c = 1$. In the de Sitter case the velocity $v(r) = Hr$, where $H$ is Hubble parameter, and the cosmological horizon is at $r_{\text{hor}} = 1/H$.

In the Painleve-Guustrand metric, the energy spectrum of particles is Doppler shifted:

\begin{equation}
    E(p, r) = \pm \sqrt{M^2 + p^2} + H p \cdot r .
\end{equation}

This can be compared with the spectrum of a charged particle in a constant electric field:

\begin{equation}
    E(p, r) = \pm \sqrt{M^2 + p^2 + E \cdot r } ,
\end{equation}

where electric charge $q = 1$ is assumed.

The main difference is that in the de Sitter case the electric field $E$ is substituted by $H p$.

Let us now neglect the curvature of cosmological horizon at $Hr = 1$. Then introducing the coordinate $z$ across the cosmological horizon:

\begin{equation}
    E(p, z) = \pm \sqrt{M^2 + p_\perp^2 + p_z^2 + H p_z} .
\end{equation}

Introducing correspondingly the coordinate $z$ along the electric field, one obtains for Schwinger:

\begin{equation}
    E(p, z) = \pm \sqrt{M^2 + p_\perp^2 + p_z^2 + E z} .
\end{equation}

III. TUNNELING APPROXIMATION

In the semiclassical approximation the probability of the particle creation is given by the tunneling exponent $2\text{Im} \int dz p_z(z)$:

\begin{equation}
    W = \sum_p w_p = \sum_p \exp \left( -2\text{Im} \int dz p_z(z) \right) .
\end{equation}
where the tunneling trajectories along $z$ are given by $E(p, z) = E$. In case of Schwinger pair production the tunneling exponent depends only on the transverse momentum $p_\perp$:

$$w^{\text{Schwinger}}_p = \exp \left( -\frac{\pi \tilde{M}^2}{\mathcal{E}} \right) , \quad \tilde{M}^2 = M^2 + p_\perp^2 , \quad (7)$$

while in the de Sitter case the tunneling exponent depends also on the longitudinal momentum:

$$w^{\text{ds}}_p = \exp \left( -\frac{2\pi E}{H} \right) , \quad E^2 = M^2 + p_\perp^2 + p^{2\perp} , \quad (8)$$

The semiclassical approximation is valid for correspondingly $M \gg H$ and $M^2 \gg \mathcal{E}$.

IV. DE SITTER VS SCHWINGER

Let us first consider the Schwinger pair production. Integration over transverse momenta gives

$$\int \frac{d^2 p_\perp}{(2\pi)^2} \exp \left( -\frac{\pi \tilde{M}^2}{\mathcal{E}} \right) = \frac{\mathcal{E}}{(2\pi)^2} \exp \left( -\frac{\pi M^2}{\mathcal{E}} \right) . \quad (9)$$

Integral over $dp_z/2\pi$ diverges because there is no dependence on $p_z$. Due to the motion equation $dp_z = \mathcal{E}dt$, one obtains the known Schwinger pair creation per unit volume per unit time (the integer factor for polarization and spin of particles is ignored):

$$\Gamma^{\text{Schwinger}} = \frac{dW^{\text{Schwinger}}}{dt} = \frac{\mathcal{E}^2}{(2\pi)^3} \exp \left( -\frac{\pi M^2}{\mathcal{E}} \right) . \quad (10)$$

On the other hand, for the de Sitter the integral over $p_z$ is finite, and one obtains the total probability of the particle production per unit volume:

$$W^{\text{ds}} = \int \frac{d^3 p}{(2\pi)^3} \exp \left( -\frac{2\pi E}{H} \right) = \frac{\sqrt{2}}{\pi^3} (MH)^{3/2} \exp \left( -\frac{2\pi M}{H} \right) . \quad (11)$$

That is why in the limit of infinite time the creation of particles per unit time is zero, $\Gamma^{\text{ds}} = dW^{\text{ds}}/dt = 0$. This is in contrast to $\Gamma^{\text{ds}} \sim M^3 H \exp \left( -\frac{2\pi M}{H} \right)$ obtained in Ref.\textsuperscript{12}.

V. CONCLUSION

Using the stationary spacetime for the de Sitter expansion and the semiclassical approximation for quantum tunneling, we obtained that the Hawking radiation across the cosmological horizon is described in the same way as the Schwinger pair production in the uniform electric field. However, the final results are different. For the Schwinger pair production the semiclassical approximations reproduces the known result for the intensity of the pair production $\Gamma^{\text{Schwinger}}$. For the de Sitter case the production of pairs per unit volume, $W^{\text{ds}}$, is finite and thus the intensity (the production per unit volume per unit time) is $\Gamma^{\text{ds}} = dW^{\text{ds}}/dt = 0$.

This does not mean that the de Sitter vacuum is stable: this only means that the Hawking radiation alone does not lead to instability, i.e. the de Sitter vacuum is stable with respect to the decay via the Hawking radiation\textsuperscript{11}. The Hawking radiation alone does not lead to the change of the vacuum energy density, which generates the de Sitter expansion. This means that the pair creation takes place, but de Sitter expansion immediately dilutes the produced particles, and thus there is no vacuum decay in de Sitter.

In principle, there can be the other mechanisms, not related to the Hawking radiation, which could lead to the decay of the de Sitter spacetime\textsuperscript{12–16} including the infrared instability, instability due to the dynamic effects of a certain type of quantum fields, instability towards spontaneous breaking of the symmetry of the de Sitter spacetime or the instability towards the first order phase transition in the vacuum, etc. But all this does not close the fact that the de Sitter vacuum is very special. A particular example is provided by the $q$-theory, which describes the dynamics of the dark energy in terms of the nonlinear 4-form field\textsuperscript{17}. In this theory, the initial state with the large dark energy relaxes either to Minkowski or to de Sitter vacuum, which demonstrates that both Minkowski and de Sitter vacua serve as the attractors in the vacuum dynamics. To exclude the de Sitter attractor some mechanism of the decay of the de Sitter vacuum is necessary. In other words, the decay of the de Sitter vacuum is the necessary condition for the dynamical solution of the cosmological constant problem\textsuperscript{18}.

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