Energy Emission by Quantum Systems in an Expanding FRW Metric

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Abstract

Bound quantum mechanical systems not expanding with the comoving frame of an expanding, flat FRW metric are found to release energy at a rate linearly proportional to the local Hubble constant ($H_o$) and the systems’ binding energy ($E_b$); i.e., $\dot{E} = H_o E_b$. Three exemplary quantum systems are examined. For systems with early cosmological condensation times — notably hadrons — time-integrated energy release could have been significant and could account for an appreciable fraction of the dark matter inventory.

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Key Words: cosmology, metric expansion, dark matter, dark energy, energy conservation.
1 Introduction

A perennial unresolved issue intrinsic to the standard cosmological model is the scale length at which the expansion of the spacetime metric ceases. Concerns about this date back at least to McVittie [1], Einstein and Straus [2] and Noerdlinger and Petrosian [3]. These have been taken up with increasing earnest in recent years by Anderson [4], Bonner [5], Cooperstock, et al. [6], and others. Recent evidence for dark energy, which accelerates cosmic expansion and which presumably permeates the vacuum at all scale lengths, makes the metric expansion cut-off increasingly salient [7-9].

Most recent studies [4-6] have concentrated on issues of particle dynamics in the presence of cosmological expansion. These studies have found that multiparticle systems at size scales below the scale of the perceived Hubble flow should not appreciably expand along with the comoving metric. However, a facet of this problem that has not received adequate attention is the relationship of cosmic expansion to the behavior of quantum systems, especially with regard to energy radiation.

This paper explores the physical ramifications of extending the principle of cosmological metric expansion down to quantum mechanical scale lengths. We find that negative-energy, bound quantum systems which collapse in the comoving frame against the expanding cosmological metric should release energy at a rate \( \frac{dE}{dt} \equiv \dot{E} \) that is linearly proportional to the local Hubble constant \( (H_o) \) and to the system binding energy \( (E_b) \); that is: \( \dot{E} \sim H_o E_b \). Exemplary quantum systems are shown to follow this prescription. For quantum systems that condensed early out of the primordial fireball — particularly hadrons — the time-integrated contraction energy since condensation might have been sizable. In the present era, the Hubble constant is sufficiently small to make this energy release rate negligible. Hereafter, we will refer to this type of energy as metric contraction energy (MCE).

In this paper, a system will refer to any multiparticle assemblage, for instance, baryons, nuclei, atoms, molecules, planets, stars, or galaxies. We assume the Friedmann-Robertson-Walker line element, \( ds, \) for a spatially-flat, open universe: \( ds^2 = dt^2 + dl^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \), where \( t \) is time, \((x,y,z)\) are the spatial coordinates, and \( a(t) \) is the metric expansion parameter, which defines the Hubble parameter via \( H \equiv \frac{a'(t)}{a(t)} \).

The remainder of this paper is organized as follows. Primary assumptions for MCE emission are introduced and three archetypical quantum systems are considered for MCE emission. General expressions for instantaneous and time-integrated MCE emission are derived. The latter is applied to quantum systems that condensed from the primordial fireball. Finally, issues of global energy conservation are raised.

2 Model Assumptions

MCE emission follows directly from the following assumptions:

A) Metric expansion proceeds at all gravitationally classical scale lengths, meaning scale lengths greater than the Planck length \( (L_p = \sqrt{\frac{hG}{c^5}} \approx 10^{-35} \text{ m}) \).
B) Bound systems contract in the comoving frame in such a way that they appear spatially unaffected in the proper frame.

C) Energy emission is possible in quantum systems since they undergo transitions differently than classical systems.

We examine each of these assumptions for physical reasonableness.

Assumption A: It is normally assumed that cosmic metric expansion ceases at scale lengths below those of Hubble flow. This view has been championed by many, but this does not settle the issue. In view of the evidence from the supernova searches [7,8] and WMAP results [9] that suggest the universe is filled with an ubiquitous negative-pressure dark energy, expansion should proceed down to the smallest classically allowed gravitational scale length ($L_g$). This would be at scale lengths 20 orders of magnitude smaller than the proton, roughly 60 orders of magnitude smaller than where Hubble flow is typically observed, thereby subsuming all known physical systems. Even theoretically, this is not so untenable a position as it may seem since the general relativistic field equations, from which cosmological expansion is derived, presumably hold sway at all such gravitationally classical scales. The field equations contain the cosmological constant (as the simplest explanation for dark energy) which appears in the vacuum energy density that is the source for the long-range gravitational field in effective quantum field theories.

Metric expansion — as inferred from the non-expansion of material systems — is not observed to proceed in regions of space in which the mass-energy density exceeds the critical density, $\rho_{\text{critical}} \simeq \frac{H_o^2}{G}$, where $H_o$ is the local Hubble constant. This is equivalent, however, to having metric expansion in fact proceed identically on all gravitationally classical scale lengths, but having it not proceed if the local self-gravitational acceleration ($a_g$ radially inward) exceeds the Hubble acceleration ($a_H$ radially outward); that is, $a_g > a_H \sim rH_o^2$, where $r$ is the system scale length. (This criterion for non-expansion also applies to forces other than gravity.) This interpretation reproduces local physical behavior while remaining faithful to the strict FRW formalism. In summary, Assumption A — that metric expansion proceeds at all gravitationally classical scale lengths — is not contradicted by available physical evidence, and it is consistent with the standard general relativistic interpretation of the metric.

Assumption B: That bound systems contract in the comoving frame to counter the expansion of space follows naturally from assumption (A) and is consistent with observational evidence such as the recent investigations of the value of the fine structure constant in distant galaxies [10]. The contraction of bound systems (such as hydrogen atoms) proceeds in such a way that fundamental energy levels of quantum systems are unchanged over measurable time-averaged periods. Thus, the energy structure of the systems appears unaffected, in agreement with observation.

Assumption C: It is through the fundamental differences by which classical and quantum systems undergo energy state transitions that we propose the MCE originates. A classical system can spatially contract continually and continuously against the comoving metric expansion and, thereby, maintain a fixed size (i.e., shrink within an expanding comoving frame). Furthermore, its inertial response (follow-through) favors its continuous local contraction which
counteracts metric expansion. As a result, we predict little or no continuous MCE emission from classical systems [11].

In contrast to classical systems, quantum systems radiate energy intermittently and discontinuously, and they do not exhibit inertial follow-through. As a result, unlike classical systems, we posit that they can follow the expansion of the comoving metric during time intervals between radiating (Assumption A). If they expand along with the comoving metric between emission, but intermittently collapse back to their original fixed proper sizes, then they should emit MCE, analogously to classical systems (e.g., gas clouds) and more typical quantum systems (e.g., hydrogen atoms). We emphasize that quantum MCE emission is not due to change in quantum number, but is due to change in spatial scale only. Emission should continue as long as they are embedded in an expanding spacetime metric, but emission rate clearly depends on the metric expansion rate [12]. Assumption C is consistent with the standard physical interpretation of energy release by classical and quantum systems. It presumes, of course, the validity of Assumptions A and B.

Justification for the emission of radiation is provided by the interpretation of recent evidence for accelerated cosmic expansion and dark energy [7-9]. If every point in space is a locus of expansion and if expansion is indeed driven by intrinsic properties of the vacuum, then it is reasonable to assume that all interactions and bound systems described by Feynman diagrams — even at the tree level — should be modified (albeit slightly) by the expansion of space as the ”moving” vacuum passes through the system. This will be true in both the comoving and proper frames. In the comoving frame, particles in a shrinking bound system will slide through a vacuum at rest. On the other hand, in the bound system’s proper frame, a moving vacuum will slide past the bound particles. The exact mechanism producing the radiation remains obscure since we do not have a good theoretical model for describing how expansion (via dark energy?) affects the vacuum. There is a suggestion that the energy producing mechanism has similarity to the Unruh effect, where spatial expansion plays a role similar to acceleration in providing an energy source. A better model for the mechanism of spatial expansion is required before we can say more on this subject.

3 MCE in Quantum Systems

1-D Systems
In this section, several quantum systems are examined for MCE emission, subject to Assumptions A-C above. To begin, consider an archetypical quantum mechanical wave comoving in an expanding R-W metric, as described by Peebles [14]. (This model is commonly used to illustrate the observed redshifting of radiation from distant objects in an expanding universe. We adopt Peebles’ notation.) Let a normal mode wave form a one-dimensional closed circular loop (radius $a(t)$); that is, an integer number of wavelengths ($n$) span the loop circumference (Fig. 1). The loop is of cosmic proportion. The time evolution of the wavelength ($\lambda(t)$) follows the radial metric expansion parameter: $\lambda(t) \propto a(t)$. Since the expansion is adiabatic, there are no discontinuous changes in state or occupation numbers; thus the wavelength continuously increases, producing a redshift, and the mode energy continuously decreases.
If, from time to time, this mode were to decouple from the comoving metric and collapse down to a smaller, fixed, fiduciary radius \( a(t_o) \), then an energy change is expected. For a photon \( E = \frac{hc}{\lambda} = \frac{nhc}{a(t)} \), the photonic energy gain resulting from its intermittent contraction from \( a(t) \) to \( a(t_o) \equiv a_o \) (with \( t \geq t_o \)) would be:

\[
\Delta E = nhc \left[ \frac{1}{a_o} - \frac{1}{a(t)} \right] \tag{1}
\]

Returning to Fig. 1, now let the loop be opened, the wave straightened and placed between two endpoints. Metric expansion now corresponds to the separation of the endpoints. Wavelength can be written:

\[
n\lambda = a(t)
\]

with \( (n = 1, 2, 3...) \). If one applies the deBroglie relation for wavelength and energy, one obtains the bound state energies for a particle in a one-dimensional square well (with comoving walls):

\[
E_n(t) = \frac{n^2h^2}{8ma^2(t)} \tag{2}
\]

Thus, the redshift of radiation in an expanding spacetime metric can be seamlessly translated into the one-dimensional particle-in-box with expanding walls.

To estimate the MCE emission rate for this system, let the system expand with the comoving frame for time \( \delta t \), after which it undergoes a transition back to its original size. (Again, we emphasize that this transition does not involve a change in quantum number, but a change in spatial scale only.) Via Hubble’s law, one can write the metric expansion as

\[
a(t_o + \delta t) = a_o + v\delta t = a_o + H_o \mathcal{L} \delta t,
\]

where \( \mathcal{L} \) is the scale size over which expansion occurs. The energy of the comoving, expanding ground state \((n = 1)\) becomes:

\[
E_1(\delta t) = \frac{h^2}{8ma_o^2 + H_o \mathcal{L} \delta t} = \frac{h^2}{8ma_o^2[1 + \frac{H_o \mathcal{L} \delta t}{a_o}]^2}.
\]

Under the assumption that the system expansion during time interval \( \delta t \) is small compared with \( \mathcal{L} \) or \( a_o \), this can be expanded to first order \( (\frac{H_o \mathcal{L} \delta t}{a_o} \ll 1) \), rendering:

\[
E_1(\delta t) \simeq \frac{h^2}{8ma_o^2} \left[ 1 - \frac{2H_o \mathcal{L} \delta t}{a_o} \right].
\]

Letting the proper physical distance \( \mathcal{L} \) be the system scale size \( \mathcal{L} = \frac{a_o}{2} \), one has

\[
E_1(\delta t) \simeq \frac{h^2}{8ma_o^2} \left[ 1 - H_o \delta t \right] = E_1(0) \left[ 1 - H_o \delta t \right],
\]

from which the MCE increment is \( \delta E_{mce} \equiv E_1(\delta t) - E_1(0) = -E_1(0)H_o \delta t \). In the continuum limit \((\delta t \rightarrow dt \text{ and } \delta E_{mce} \rightarrow dE_{mce})\), recognizing that \( E_1(0) = E_o \) is the system’s binding energy, the MCE emission rate from the ground state particle-in-box can be written [15]
\[ \dot{E}_{\text{mce}} = -H_0 E_b. \] (7)

These first two examples are positive-energy bound states so the energy change for the universe (in the comoving frame) is negative (\( \dot{E}_{\text{mce}} < 0 \)); however, were these negative-energy states, typical of realistic bound systems, then \( \dot{E}_{\text{mce}} > 0 \). This latter result suggests that negative-energy bound quantum states should radiate MCE at a time-averaged rate which is linearly proportional to their bound state energies and the present epoch Hubble constant, \( H_0 \). MCE represents \textit{radiated} energy rather than simply an energy \textit{shift} such as the Lamb shift. Given the low value of \( H_0 \) in the present era (\( H_0 \sim 10^{-18} \text{ sec}^{-1} \)), the MCE emission rate can be shown to be small for standard bound systems; however, it will be shown that, for earlier epochs, \( \dot{E}_{\text{mce}} \) could have been considerable.

**Hydrogen Atom**

A more realistic case of cosmic expansion affecting a bound state is that of the hydrogen atom. This system was investigated for expansion by Bonner [5] who studied the ground state Bohr atom in a flat FRW metric. He concluded that cosmological expansion had negligible effect on the size of the atom. We, too, presume that the long-time average size and energy of the atom are essentially constant, but that these are due to the atom’s intermittent quantum mechanical contractions against the comoving frame. We take as the size of the ground state hydrogen atom the Bohr radius, \( b \). Unlike the case of the 1-D square well, with size \( a_o \), there is no specific expansion parameter for the hydrogen atom, so we will take a different route to an expression for MCE: time-dependent perturbation theory. This proceeds from the relations: \( i\hbar \frac{\partial}{\partial t} \psi = \mathcal{H} \psi \) and \( \mathcal{H}(t) = \mathcal{H}_0 + \mathcal{H}_1(t) \), where \( \mathcal{H}_1(t) \) is the first-order perturbation.

Since the perturbation arising from metric expansion is small and is slowly varying compared with the system transitions [16], we can adopt the adiabatic approximation; thus, we can consider the perturbation as being time-independent over the interval \( \delta t \) and time-independent perturbation theory can be validly employed. For the ground state hydrogen atom, we write

\[ \mathcal{H}\psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} \psi(r^1) - \frac{e^2}{4\pi \epsilon_o r^1} \psi(r^1) = E\psi(r^1) \] (8)

Inserting the Hubble-expanded radius, \( r^1(\delta t) = r + H_0 \mathcal{L} \delta t \), the potential energy term becomes

\[ V(r^1) = -\frac{e^2}{4\pi \epsilon_o [r + H_0 \mathcal{L} \delta t]} \approx -\frac{e^2}{4\pi \epsilon_o r} \left[ 1 - \frac{H_0 \mathcal{L} \delta t}{r} \right]. \] (9)

From this, the perturbation to the Hamiltonian for the hydrogen ground state is inferred to be:

\[ H^1(r) = \frac{e^2}{4\pi \epsilon_o r^2} (H_0 \mathcal{L} \delta t). \] (10)

The first-order energy correction to the unperturbed wavefunction \( \psi_n^o \) is \( \delta E^1 = \langle \psi_n^o | \mathcal{H}^1 | \psi_n^o \rangle \).

For the unperturbed ground state hydrogen wavefunction, given by \( \psi_1^o(r) = \frac{1}{\sqrt{\pi b}} \exp[-r/b] \), evaluating Eq.(10) and noting \( \langle r^{-2} \rangle = \frac{2}{b^2} \), one obtains

\[ \frac{\delta E^1}{\delta t} = \frac{e^2}{4\pi \epsilon_o} H_0 \mathcal{L} \left( \frac{2}{b^2} \right). \] (11)
If one now takes the proper distance $L$ to be $L = \langle \psi_0| r |\psi_0 \rangle = \langle r \rangle = b^2$, then in the continuum limit, one obtains for the first-order MCE emission rate from the ground state hydrogen atom

$$\dot{E}_{mce} = 2H_oE_b.$$  \(12\)

Within a factor of order unity, this agrees with the principal result of the square well.

**Homogeneous Potentials**

By invoking the virial theorem, these results can be generalized for arbitrary homogeneous, negative-energy potentials of the form $V(r) = \kappa r^n$. The total system energy is $E = \bar{T} + \bar{U}$, where $\bar{T}$ and $\bar{U}$ are the time-averaged kinetic and potential energies, and $E = (n^2 + 1)\bar{U}$. The metric-perturbed energy expectation value is

$$\langle E_1 \rangle \equiv \langle \psi| H_1 |\psi \rangle \equiv \langle \psi| r^n |\psi \rangle \approx \kappa(n^2 + 1)\langle \psi| r^n |\psi \rangle.$$  \(13\)

Taking $L_r \equiv \chi$ to be a constant, then to first order Eq. (13) becomes

$$\langle E_1 \rangle \approx \kappa(n^2 + 1) \left[ \langle \psi| r^n |\psi \rangle + nH_o\chi\delta t \langle \psi| r^n |\psi \rangle \right] = \langle E_b \rangle + n\chi H_o\delta t \langle E_b \rangle$$  \(14\)

The contraction energy release is $\delta E \equiv \langle E_1 \rangle - \langle E_b \rangle = n\chi H_o\langle E_b \rangle\delta t$. In the continuum limit, and with $n\chi \sim 1$, one has for the time-averaged MCE release rate

$$\dot{E}_{mce} \approx H_o\langle E_b \rangle,$$  \(15\)

as was found for the previous explicit cases.

**Time-Integrated MCE – Cosmological Implications**

The adiabatic approximation can be extended to longer times periods, for which $H_o$ is no longer a constant, but varies with time. In most expansion models, $H$ can be written $H(t) = \frac{\alpha}{t}$, where $0 \leq \alpha \leq 1$ and $t$ is the age of the universe. For a radiation-dominated universe, $\alpha_r = \frac{1}{2}$, while for a matter-dominated universe $\alpha_m = \frac{2}{3}$. The time of cross-over from radiation to matter dominance, $t_{eq}$, is roughly $t_{eq} \approx 10^{11}$ sec. Here $\Omega$ is assumed to be the critical value, $\Omega = 1$. (In this study, we do not consider the effects of accelerated expansion due to dark energy [7-9], but, in principle, these can be incorporated into this model.) As the universe expands and cools, multiparticle systems (e.g. baryons, nuclei, atoms) condense as the temperature falls below their rest masses and binding energies. From Eq. (15), the time-integrated MCE emission since the time of system condensation, $t_c$, will be:

$$E(t_c \rightarrow t) = \int_{t_c}^{t} \langle E_b \rangle \frac{\alpha}{t} dt = \langle E_b \rangle \left[ \int_{t_c}^{t_{eq}} \frac{\alpha_r}{t} dt + \int_{t_{eq}}^{t} \frac{\alpha_m}{t} dt \right] = \langle E_b \rangle \left\{ \frac{1}{2}ln(\frac{t_{eq}}{t_c}) + \frac{2}{3}ln(\frac{t}{t_{eq}}) \right\},$$  \(16\)

with the proviso, $t_{eq} \geq t_c$.

In Table I are given the approximate condensation times, binding energies, and net mass gains for several multiparticle quantum systems of cosmological importance. Condensation
times are taken from the standard cosmological model. The net mass gain, \( G \), is the average fractional mass-equivalent of energy released from the condensation time to the present era, as calculated from Eq. (16). As indicated in Table I, the dominant source of MCE derives from hadrons, specifically baryons. Emissions from nuclei, atoms and molecules are orders of magnitude less, both by virtue of their significantly lesser binding energies and also due to their more recent condensation times. For the present era \((t \sim 5 \times 10^{17} \text{ sec})\), the mass-energy of the baryonic MCE radiated since the hadron condensation should be about 3 times the baryonic mass in the universe; if one includes estimates of additional MCE generated near the quark-hadron condensation time due to thermal effects, the total time-integrated MCE could have been as much as roughly 10 times the baryonic mass-energy [13,17].

4 Discussion

The analysis above indicates that non-expanding, multiparticle systems in an expanding universe should release energy (MCE) continuously. In this study, we do not speculate as to what form MCE takes or whether it survived the effects of subsequent metric expansion [18]. We also make no account of the underlying source of MCE; we simply argue that it should be radiated. Clearly, it is tied to metric expansion, but we do not speculate as to whether MCE draws from global expansion energy, dark energy, or some other energy reservoir. In this model, the choice has been made to preserve the metric expansion for all gravitationally classical scale lengths even at the possible expense of global energy conservation. Since the former seems demanded by the standard cosmological model while the latter does not, this seems a defensible choice.

Energy conservation on universal distance scales remains an open question. It is well-known that in the standard model the cosmological redshift of the cosmic background radiation does not conserve energy globally. To resolve this paradox, it is usually pointed out that the general theory of relativity does not contain a general global energy conservation law. Schemes have been proposed for mining energy from the expanding universe, not all of which appear to satisfy global energy conservation. Davies [19] has proposed a simple means by which to mine energy from a deSitter universe, but concludes that this "second helping" is not a 'free lunch.' Harrison [20], on the other hand, proposes a mining scheme for Friedmann universes using a network of strings which he claims is a 'free lunch.' A consensus has not been reached as to the meaning of these gedanken experiments, however, it keeps at the fore unsettling and unsettled issues about energy conservation.

This MCE model bears strong resemblance to Harrison’s tethered body gedanken experiment with the Harrison tether replaced by force-mediating exchange particles. If so, many of the same issues Harrison raises concerning energy conservation might also apply here. This ambiguity seems traceable to the lack of global energy conservation in the standard cosmological model. It is unclear whether the ultimate source of the energy is found in cosmic expansion or whether it is "nascent" energy, as argued by Harrison [20]. Additional analysis or suitable experimental tests appear necessary to settle these issues.
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11. One may argue, however, that exceptions exist. For instance, the release of gravitational potential energy by a molecular cloud can be traced ultimately to metric expansion of the gas during an earlier epoch of the universe and therefore qualifies as classical (not quantum) MCE. This MCE release is neither continuous, nor sizable compared with what quantum systems should display, nor is it the result of contemporary metric expansion; rather, it is a relic energy release.

12. In order to plausibly qualify for collapse in the comoving frame, a quantum system should expand during the time between transitions by at least the minimum gravitational scale length: $L_{p}$. Additionally, to respect causality, the time between transitions should equal or exceed the light travel time across the system (i.e., $\tau_L = \frac{L_{QM}}{c}$). Causality-challenging EPR effects aside, this constraint is generally met by real systems. For systems subject to Hubble expansion, these two constraints can be shown to translate into a
single condition for MCE emission: \( L_{QM} > \sqrt{L_p L_u} \). Here \( L_u \) is the visible scale size of the universe. If the time interval between transitions (\( \tau_t \)) is longer than \( \tau_L \) — as is the case for many quantum systems — then the minimum \( L_{QM} \) will decrease accordingly; the condition for MCE emission can then be written: \( L_{QM} > \frac{L_p}{\tau_t H_0} \).

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15. Analysis of the 3-D spherical well bound states returns the same results as the 1-D case.

16. Under Assumption A, for an atom to ‘expand’ with the metric but remain ‘fixed’ in size, requires intermittent contractions on a time scale short compared with observational time scales.

17. This baryonic MCE mass gain roughly matches the currently estimated dark matter inventory.

18. If the MCE were in the form of relativistic particles (\( e.g. \), photons), subject themselves to metric expansion, then little relic MCE would be expected to survive to the present day.

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Table I: Sources of MCE. Notation: $t_c \equiv$ condensation time after Big Bang; $\frac{E_b}{mc^2} \equiv$ ratio of binding energy to rest mass energy; $C \equiv \left( \frac{1}{2} \ln\left(\frac{t_c}{t_e}\right) + \frac{2}{3} \ln\left(\frac{t}{t_{eq}}\right) \right)$; $G \equiv C \frac{E_b}{mc^2} \equiv$ average fractional gain in mass of composite system over the lifetime of the universe. ($t \simeq 5 \times 10^{17}$ sec, $t_{eq} \simeq 10^{11}$ sec).

| Particles      | System      | $t_c$(sec) | $\frac{E_b}{mc^2}$ | C   | G    |
|----------------|-------------|------------|--------------------|-----|------|
| quark          | hadron      | $10^{-6}$  | $10^{-1}$          | 30  | 3    |
| nucleon        | nucleus     | $10^{2}$   | $2 \times 10^{-3}$ | 20  | 0.04 |
| $e^-$/nuclei   | atom        | $10^{12}$  | $10^{-8}$          | 9   | $10^{-7}$ |
| atoms          | molecule    | $10^{13}$  | $2 \times 10^{-9}$ | 7   | $10^{-8}$ |
Figures

**Figure 1:** One dimensional normal mode wave (solid line) on adiabatically expanding closed loop (dotted line) representing cosmic metric. (After Peebles [14].)