Axion core–halo mass and the black hole–halo mass relation: constraints on a few parsec scales

Vincent Desjacques⋆ and Adi Nusser

Physics department and Asher Space Science Institute, Technion, Haifa 3200003, Israel

ABSTRACT
If the dark matter is made of ultra-light axions, stable solitonic cores form at the centers of virialized halos. In some range for the mass $m$ of the axion particle, these cores are sufficiently compact and can mimic supermassive black holes (SMBH) residing at galactic nuclei. We use the solitonic core–halo mass relation, validated in numerical simulations, to constrain a new range of allowed axion mass from measurements of the SMBH mass in (pseudo)bulge and bulgeless galaxies. These limits are based on observations of galactic nuclei on scales smaller than 10 pc. Our analysis strongly suggests that $m \lesssim 10^{-18}$ eV is ruled out by the data. We briefly discuss whether an attractive self-interaction among axions could alleviate this constraint.

Key words: cosmology: theory, large-scale structure of Universe, dark matter

1 INTRODUCTION
In the fuzzy dark matter (FDM) scenario (see, e.g., Baldeschi et al. 1983; Khlopov et al. 1985; Sin 1994; Hu et al. 2000; Syrcek & Witten 2006; Amendola & Barbieri 2006; Chavanis 2011; Marsh & Silk 2014; Hlozek et al. 2015; Hui et al. 2017, and references therein), a halo is made of a solitonic core engulfed by a haze of fluctuating density granules resulting from the interference of (classical) waves. When the FDM is ultra-light axions (see, e.g., Marsh 2016, for a recent review), the solitonic core is dubbed “axion star” or, simply, an axion core. Numerical simulations of the Gross-Pitaevskii-Poisson (GPP) system have established that the mass of the axion core $M_c$ increases with the FDM halo mass $M_h$ (Schive et al. 2014; Schive et al. 2014; Schwabe et al. 2016). Furthermore, simulations have robustly demonstrated the existence of a haze of fluctuating granules extending much farther than the embedded solitonic core (Schive et al. 2014). This quasi-particle picture has been explored further in Hui et al. (2017); Bar-Or et al. (2018) in the context of dynamical friction. It can also be used to understand the properties of the axion cores.

Measurements from the Lyman-α forest power spectrum set a lower bound on the axion mass of $m \gtrsim 2 \times 10^{-21}$ eV at 95% C.L. (Iršič et al. 2017; Armengaud et al. 2017). While our own galaxy could still harbour a solitonic core for a axion mass as low as $m \sim 10^{-22}$ eV (De Martino et al. 2018; Broadhurst et al. 2019), this is quite unlikely in light of the large scale structure constraints (see, however, Zhang et al. 2017). The existence of solitonic cores and, thereby, FDM scenarios can be further constrained using different astrophysical observables (see Hui et al. 2017, for a detailed overview), such as galactic rotation curves (Bar et al. 2018; Robles et al. 2018), the survival of star clusters in dwarf galaxies (Marsh & Niemeyer 2018) or, even, the absence of black-hole superradiance in M87 (Davoudiasl & Denton 2019).

Here, we assess the extent to which the presence or absence of supermassive black holes (SMBHs) constrain FDM scenarios. The paper is organized as follows. After a brief digression on the origin of the axion core – halo mass relation ($\S$2), we demonstrate that measurements of SMBH and host halo mass in bulge and, in particular, bulgeless galaxies yield constraints at least as as competitive as rotation curves ($\S$3). We conclude in $\S$4.

In all illustrations, we assume a concordance ΛCDM cosmology with Hubble parameter $h = 0.7$ and present-day matter density $\Omega_m = 0.3$.

2 AXION CORE - HALO MASS RELATION
For sake of completeness, we shall discuss briefly the origin of the axion core–halo mass relation in the context of virial equilibrium, and illustrate how it can be extended to a non-vanishing (attractive) self-interaction. More thorough discussions can be found in Chavanis (2011); Schive et al. (2014); Marsh & Pop (2015); Hui et al. (2017).

We use natural units $c = \hbar = 1$ throughout and write Newton’s gravitational constant as $G = 1/m_P^2$, where $m_P = 1.22 \times 10^{19}$ GeV is the Planck mass. Furthermore, we
parametrize the axion mass $m$ and decay constant $f$ as

$$m = 10^{-22} m_{22} \text{ eV} \quad (1)$$

$$f = 10^{17} f_{17} \text{ GeV}. \quad (2)$$

As a rule of thumb, the present-day axion energy density is given by $\Omega \sim 0.1 f_{17}^{1/3} m_{22}^{1/3}$ (Marsh 2016). Note also that $f_{17}$ quantifies the strength of the axion self-interaction, which we assume attractive. Since we will consider $f < m_{\text{Pl}}$ always, we are in the “strong regime” of axion self-interactions (Chavanis 2018b).

2.1 Virial equilibrium considerations

Equilibrium configurations of FDM halos with a density and velocity profile $(\rho, u)$ can be obtained by means of minimizing the total energy (Chavanis 2011)

$$E = W + K + Q + U, \quad (3)$$

where $K$ and $Q$ are the potential and “quantum pressure” contributions, $W$ is the gravitational binding energy of the self-gravitating FDM halo, and $U$ is the “internal energy” arising from the self-interaction. Equilibrium configurations also satisfy the quantum analog of the classical virial theorem (Chavanis 2011; Hui et al. 2017) implying, in the steady state limit,

$$0 = W + 2K + 2Q + 2U. \quad (4)$$

Since $K \geq 0$, this yields the inequality $U + Q \leq |W|/2$, which is saturated in the axion core where $K = 0$. By contrast, in the gaseous atmosphere, the quantum pressure and the self-interaction can be neglected, and we recover the usual virial theorem $W + 2K = 0$.

For virialized FDM halos, the velocity dispersion of the gaseous atmosphere of quasi-particles surrounding the core is (see Hui et al. 2017, and Appendix A for additional details)

$$\langle v^2 \rangle \approx \frac{GM_h}{R_h}. \quad (5)$$

We shall see below that the core properties are determined through the requirement that the quasi-particles are barely bound to the core, that is

$$\langle v^2 \rangle \approx v_{\text{esc}}^2. \quad (6)$$

2.2 Without self-interaction

The axion core is characterized by an approximately Gaussian density profile, which reaches a constant central density $\rho_c$ on scales less than the core radius $R_c$. In the absence of a self-interaction, $U = 0$ and the virial equilibrium condition $W + 2Q = 0$ inside the solitonic core yields $R_c \propto M_c^{-1}$. This scaling arises from $W \propto M_c^2/R_c$ and $Q \propto M_c/R_c^2$. A more detailed analysis gives (Chavanis 2011)

$$R_c = \frac{3\sqrt{\pi}}{2M_c} \left( \frac{m_{\text{Pl}}}{m} \right)^2 \quad (7)$$

$$\simeq 227 m_{22}^{-2} \left( \frac{10^9 M_\odot}{M_c} \right) \text{ pc}.$$

As a result, the escape velocity $v_{\text{esc}}$ at the surface of the solitonic core is given by

$$v_{\text{esc}} = \sqrt{\frac{GM_c}{R_c}} = \sqrt{\frac{2}{3\sqrt{\pi} m_{\text{Pl}}^2}} M_c \quad (8)$$

$$\simeq 138 m_{22} \left( \frac{M_c}{10^9 M_\odot} \right) \text{ km s}^{-1}.$$

This relation reproduces the empirical scaling $M_c \propto ((E_h/M_h)^{1/2})$, where $E_h$ is the energy of the halo. This can also be understood in terms of a wave-like uncertainty principle (Schive et al. 2014), or diffusive equilibrium (Bar et al. 2018).

The axion core–halo mass relation follows immediately from Eq.(6):

$$M_c = \mathcal{N} M_{c,\text{min}}^{2/3} M_h^{1/3}. \quad (9)$$

Here, $M_{c,\text{min}}$ is a minimum core mass,

$$M_{c,\text{min}} = \frac{1}{2} 3^{3/4} 4^{3/8} \pi^{3/4} M_\odot \frac{\Delta_{\text{vir}}^{1/4} m_{22}^2 H_0^{1/2}}{m_{\text{Pl}}^{3/2}} \quad (10)$$

and $\mathcal{N} \approx 0.25$ is an empirical normalization factor which accounts for the fact that the mass assigned to an axion core in numerical simulations is computed from the central region with $R \lessapprox R_c$ only. The virial overdensity $\Delta_{\text{vir}}(z)$ is defined relative to the average matter density $\bar{\rho}_m(z)$. We ignore the mild redshift dependence of $\Delta_{\text{vir}}(z)$ and assume $\Delta_{\text{vir}}(z) = 200$ throughout. Finally, $a$ is the scale factor. Since all the data considered here is at redshift $z < 1$, we will simply set $a = 1$ in all subsequent illustrations.

For a present-day MW-size galaxy with $M_h = 4 \times 10^{12} M_\odot$, the axion core mass would be $M_c \sim 5.4 \times 10^8 m_{22} m_{22} M_\odot$. The minimum core mass $M_{c,\text{min}}$ originates from the fact that a solitonic core with mass $M_c = M_{c,\text{min}}$ would have the average density of the Universe (Marsh & Pop 2015; Hui et al. 2017). In principle, there is a maximum stable core mass reached when $R_c$ equals the Schwarzschild radius $R_s = 2GM_c$. For realistic CDM cosmologies however, there is not enough time by $z = 0$ to form virialized structures which could host axion cores with $M_c \lessapprox M_c(R_c = R_s)$.

2.3 With attractive self-interaction

In the presence of an attractive interaction, $U \neq 0$ and the relation between $R_c$ and $M_c$ is more involved. One finds (Chavanis 2011)

$$R_c = \frac{3\sqrt{\pi}}{4M_c} \left( \frac{m_{\text{Pl}}}{m} \right)^2 \left[ 1 \pm \frac{1}{12\pi^2} \left( \frac{m}{m_{\text{Pl}}} \right)^2 M_c^2 \right] \quad (11)$$

The stable branch corresponds to the plus sign. In this case, the core radius monotonically decreases with increasing $M_c$ to reach $\frac{3\sqrt{\pi}}{4M_c} \left( \frac{m_{\text{Pl}}}{m} \right)^2$ at the maximum core mass

$$M_{c,\text{max}} = 2\sqrt{3\pi} \left( \frac{m_{\text{Pl}}}{m} \right) \quad (12)$$

$$\simeq 1.19 \times 10^{11} f_{17} m_{22} M_\odot,$$

above which there is no stable solution.
In Appendix A, we show that the quasiparticle approach discussed above also holds in the presence of a self-interaction. Applying the same hydrostatic considerations yield a core–halo mass relation given by

$$M_c = N \frac{M_{c,\text{min}}^{4/3} M_h^{2/3}}{2 M_{c,\text{max}}} \sqrt{\frac{4 M_{c,\text{max}}}{M_{c,\text{min}}^{4/3} M_h^{2/3}}} - 1. \quad (13)$$

For the normalisation, we shall adopt again $N = 0.25$. The axion core mass reaches its maximum $M_c = M_{c,\text{max}}$ for a halo mass

$$M_h = \left(\frac{3/2}{M_{c,\text{max}}^{3/2} f_{17}^3 M_{c,\text{min}}} \right)$$

independently of the axion mass $m_{22}$. For $M_h \geq M_h$, hydrostatic equilibrium cannot be satisfied.

An attractive self-interaction lowers the minimum core mass obtained upon setting $M_c = M_h$. However, for values of $f_{17} \geq 0.01$ compatible with all axions being the dark matter, this is at most a factor of 2 smaller than $M_{c,\text{min}}$: the axion self-interaction scale as $\propto \rho^2$ and, thus, is very weak at low densities.

### 2.4 Mergers and the persistence of axion cores

The equilibrium considerations above do not take into account the evolution of $M$ and $M_h$ through mergers and smooth accretion, which is an essential aspect of hierarchical structure formation. A related issue is the persistence of the axion core–halo mass relation Eq.9 through the assembly history of the host FDM halos (see, e.g. Schwabe et al. 2016; Du et al. 2017).

Although numerical simulations indicate that cores are ubiquitous inside FDM halos (Schive et al. 2014; Veltmaat et al. 2018), the fate of solitonic cores during the merger of two FDM halos is unclear. Therefore, a lack of evidence for a central core does not necessarily translate into a constraint on the axion mass, unless the characteristic timescale for the formation of a new core following a merger event is shorter than the age of the galaxy.

The cores of the progenitor FDM halos may i) remain intact, or momentarily ii) disappear during the merging process. To determine whether a core forms in the central region of the descendant FDM halo, one should consider either i) the dynamical friction timescale on which they sink to the center of the merged halo, or ii) the relaxation timescale of FDM quasi-particles, which defines the region within which virial equilibrium can be established. Furthermore, all this could depend on the axion mass since the solitonic cores become more compact as $m_{22}$ increases. For simplicity however, we will assume that scenario ii) is the relevant picture for the range of axion masses considered in Fig.1. This scenario likely applies to major mergers during which the gravitational potential fluctuates significantly on a short timescale and, thereby, destroys the coherence of the axion core.

Under this assumption, the relevant timescale is the two-body relaxation timescale between the FDM quasiparticles. As discussed in Hui et al. (2017) and Bar-Or et al. 2018, in further details), this reads

$$t_{\text{relax}} = \frac{10^{10} \text{ yr}}{f_{\text{relax}} m_{22}^2 \left(\frac{v}{100 \text{ km s}^{-1}}\right)^2 \left(\frac{r}{5 \text{ Kpc}}\right)^4}. \quad (15)$$

An FDM halo will develop a compact solitonic core from the mass bound to the descendant halo within a radius $R_c$ if the condition $t_{\text{relax}}(R_c) = t_{\text{mg}}$ is satisfied. Here, $t_{\text{mg}}$ is the time elapsed since the merger. Setting $v = V_{\text{vir}}$ in the above expression, and using the core–halo mass relation Eq.(9), the newly merged halo will develop an axion core of mass $M_c \propto M_h^{-1/3}$ provided that

$$M_c \gtrsim 3.5 \times 10^4 \, M_\odot \left(\frac{\alpha^{1/2} m_{22}^{-3/2}}{f_{\text{relax}}} \frac{(10^{10} \text{ yr})^{1/2}}{t_{\text{mg}}}\right). \quad (16)$$

Although $t_{\text{relax}}$ increases with the axion mass, the minimum core mass scales like $M_c \propto m_{22}^{-2/3}$ because of the core radius $R_c \propto m_{22}^{-2}$ shrinks rapidly as the axion mass is increased. Assuming $f_{\text{relax}} \sim 1$ and $t_{\text{mg}} = H_0^{-1}$ for illustration, where $H_0$ is the Hubble constant today, this condition is satisfied for the whole range of circular velocities and mass shown in Fig.1.

Requiring that the whole descendant FDM halo be in virial equilibrium (which amounts to setting $v = V_{\text{vir}}$ and $r = R_{\text{vir}}$ in Eq.(15)) would ensure that the axion core mass of the merged halo precisely falls on the relation Eq.(9).

However, we found that such a condition cannot be satisfied unless the core mass is close to $M_{c,\text{min}}$ (so that the FDM atmosphere is tenuous). Therefore, one should expect some scatter in $M_c$ at fixed halo mass.

Note that the sum of the progenitor core mass is always larger than the core mass expected if the final descendant halo reaches hydrostatic equilibrium. To see this, let $M_{h,1}$, $M_{h,2}$ be the mass of the progenitor halos, with corresponding core mass $M_{c,1}$ and $M_{c,2}$; and $M_h = M_{c,1} + M_{c,2}$ be the mass of the merged halo. Let us also define $M_c = M_{c,1} + M_{c,2}$. Assuming that the core–atmosphere of the progenitors is in hydrostatic equilibrium, so that Eq.(9) initially holds, we have

$$M_c = \frac{M_{c,\text{min}}^{4/3} M_h^{2/3}}{2 M_{c,\text{max}}} \sqrt{\frac{4 M_{c,\text{max}}}{M_{c,\text{min}}^{4/3} M_h^{2/3}}} - 1. \quad (13)$$

which shows that $M_{c,1} + M_{c,2} > M_{c,\text{min}}^{4/3} M_h^{2/3}$. The difference is maximum for a major merger with $M_{h,1} \approx M_{h,2}$, in which case $M_c \approx 1.6 M_{c,\text{min}} M_h^{1/3}$.

### 3 CONSTRAINTS ON AXION MASS FROM $M_\bullet - V_{\text{circ}}$ MEASUREMENTS

We discuss now constraints on the axion mass $m$ that arise from measurements of the mass, $M_\bullet$, of SMBH residing at galactic nuclei, and from the galactic (asymptotic) circular velocity $V_{\text{circ}}$ at larger radii. The asymptotic circular velocity is used as a proxy for the host halo mass $M_h$. The full rotation curve is irrelevant for the constraints derived here.

For sufficiently small $R_c$, the axion core could masquerade as a galactic SMBH. Hui et al. (2017) briefly discussed this possibility for large galaxies. Here, we will show that small galaxies actually give the strongest limits on $m$. 
3.1 Strategy

Observational constraints on galactic SMBH masses are mainly obtained from studying the stellar kinematics within small distances ($R_{\ast} < 10$ pc) of a few times the radius of influence of the SMBH. When an estimate of the host halo mass $M_h$ is available, the core mass radius, $R_c(M_c, m)$ and mass $M_c(M_h, m)$ can be obtained from the relations (7) and (9), respectively. More precisely, taking into account the dependence $M_{c, \min} \propto m^{-4}$, cf. Eq.(10), we find

$$M_c \propto \frac{M_h^{1/3}}{m}, \quad R_c \propto \frac{1}{m M_h^2}. \quad (18)$$

On the one hand, too low values for $m$ imply large core masses $M_c$, yet constraints cannot be obtained because the core is too diffuse. On the other hand, too high $m$ cannot be ruled out either since they yield $M_c \ll M_\bullet$. Therefore, this technique can constrain a limited, albeit interesting range of $m$ where the core is sufficiently compact and massive.

To compare the data to theoretical expectations based on the axion core – halo mass relation, we need to associate the observed circular velocity $V_{\text{circ}}$ to the halo mass $M_h$. We adopt the following relation

$$V_{\text{circ}} \approx 144 \text{ km s}^{-1} \left(\frac{M_h}{10^{12} M_\odot}\right)^{1/3}, \quad (19)$$

which assumes an overdensity threshold $\Delta_{\text{vir}} = 200$ (in unit of the critical density $\rho_c$). This allows us to convert $M_c(M_h, m_{\text{ax}})$ into a relation $M_c - V_{\text{circ}}$ once an axion mass is assumed.

3.2 Data

The analysis requires a sample of measured black hole masses and circular velocities of the respective host halos. Kormendy & Ho (2013) provides an excellent review of the relevant techniques for measuring SMBH masses, as well as a discussion of the correlations between the inferred masses and properties of their host galaxies. The tightest correlation is between $M_h$ and the velocity dispersion $\sigma$ of the central stellar component. Fortunately, Kormendy & Ho (2013) also list the circular velocities $V_{\text{circ}}$ of many of the host galaxies given in their paper. For spiral galaxies, $V_{\text{circ}}$ is derived from the rotation curves while, for ellipticals, it is simply $\sqrt{2}\sigma$. At a given galaxy mass, the least massive SMBH are found in spirals with pseudobulges or no bulge at all. Thus, we expect that the strongest constraints will be obtained using these galaxies, rather than ellipticals or galaxies with classical bulges.

In Fig.1, we display measurements from classical and pseudo-bulges as (filled) red and (empty) blue circles, respectively, along with the empirical relation

$$M_\bullet \approx 0.32 \times 10^8 M_\odot \left(\frac{V_{\text{circ}}}{200 \text{ km s}^{-1}}\right)^{5.1}, \quad (20)$$

as the thick black line. Furthermore, we overlay measurements from bulgeless galaxies as the green squares. Except for NGC 4395, for which a reverberation-mapping measurement gives $M_\bullet = (3.6 \pm 1.1) \times 10^5 M_\odot$ (Peterson et al. 2005), all these measurements provide an upper limit on the mass of the SMBH. For NGC 300, 3423, 7424 and 7793, the limits on $M_\bullet$ are from Neumayer & Walcher (2012). $V_{\text{circ}}$ for NGC 7424 is from Sorgho et al. (2019) whereas, for NGC 7793, $V_{\text{circ}}$ is from de Blok et al. (2008). Finally, M33 (a nearby spiral galaxy embedded in a dark matter halo) has an asymptotic circular velocity of $V_{\text{circ}} \approx 125 \text{ km s}^{-1}$ (Mayall & Aller 1942) and the tightest upper limit on the SMBH mass: $M_\bullet < 1.5 \times 10^5 M_\odot$ (Gebhardt et al. 2001). We have thus labelled the corresponding data point on the figure. The data of the bulgeless galaxies is all summarized in Table 1.

The $M_\bullet - V_{\text{circ}}$ relations, shown as the dashed lines in Fig.1, assume there is no self-interaction ($\sigma_{\text{chi}} = 0$). The axion mass increases in steps of an order of magnitude, from $m_{\text{ax}} = 1$ until $m_{\text{ax}} = 10^5$ (from top to bottom). By contrast, the two dotted curves are for a fixed axion mass $m_{\text{ax}} = 10^2$, but a decay constant $f_{\text{chi}} = 0.005$ and 0.01 (from left to right). Numerical simulations indicate that, although the correlation between $M_h$ and $M_\bullet$ is tight, one should expect a scatter of order 0.3 dex, presumably arising from the imperfect relaxation after merger events etc. outlined in §2.

Axion cores could mimic a point source like a SMBH provided their radius is smaller than the radius $R_c$ of the central nuclear star cluster, the velocity dispersion of which constrains the SMBH mass. Typical values of $R_c$ are in the range $R_c \sim 1 - 10$ pc. For illustration, the thick orange line shows the locus $M_c(m_{\text{ax}})$ for which the core radius is $R_c = 10$ pc, so that the shaded upper half of Fig.1 corresponds to axion cores with a radius $R_c > 10$ pc. Such axion cores cannot be approximated as a central point source similar to a SMBH.

3.3 Constraints

We now turn on the constraints that can be set on the axion mass $m$.

Measurements of the Lyman-$\alpha$ forest rule out the range $m < 2 \times 10^{-21}$ eV (i.e. $m_{\text{ax}} \lesssim 20$) at 95% C.L. (Irsic et al. 2017; Armengaud et al. 2017). As can be seen from the figure, such low values of $m$ typically correspond to large core radii $R_c$, which cannot mimic a central point source. For larger axion masses, essentially all the classical bulges must correspond to SMBH. If the axion is self-interacting, with a decay constant $0.005 \lesssim f_{\text{chi}} \lesssim 0.01$, then the low-$V_{\text{circ}}$ pseudobulges could correspond to axion cores, while the cores in the large-$V_{\text{circ}}$ pseudobulges would have exceeded the threshold Eq.(12) and collapsed to form black holes. However, this does not take into account the low-$V_{\text{circ}}$ bulgeless galaxies, which yield the strongest constraints on $m$ as we shall see next.

To exclude a range of axion mass from bulgeless galaxies, we require that the core mass within the radius of the nuclear cluster be less than the maximum black hole mass inferred from the nuclear star cluster. Let $R_e$ be the radius of the central stellar cluster. There are two possibilities depending on whether $R_e$ is larger or smaller than $R_c$. If $R_e > R_c$, we demand

$$M_c \left(\frac{R_e}{R_c}\right)^3 < M_{\text{max}}, \quad (21)$$

where $M_{\text{max}}$ is the upper limit on the SMBH mass as given in Table 1. Assuming the core is in hydrostatic equilibrium so that relation Eq.(9) holds, this translates into an upper
Figure 1. Measurements of central SMBH mass $M_\bullet$ vs circular velocity $V_{\text{circ}}$ for different types of galaxies. Filled red and empty blue circles designate the dynamical measurement of $M_\bullet$ in classical and pseudobulges, respectively, while the green squares represent (mostly) upper limits for bulgeless galaxies (from the compilation of Kormendy & Ho 2013). The galaxy with the tightest black hole mass upper limit is M33 as indicated on the figure. The thick black line is the empirical $M_\bullet - V_{\text{circ}}$ relation, whereas the dashed and dotted curves indicate the axion core mass $M_c$ vs. $V_{\text{circ}}$ expected for ultra-light axions w/o self-interactions (see text for details). The shaded orange area shows the region in which the core radius $R_c$ is larger than 10 pc.

Figure 1. Measurements of central SMBH mass $M_\bullet$ vs circular velocity $V_{\text{circ}}$ for different types of galaxies. Filled red and empty blue circles designate the dynamical measurement of $M_\bullet$ in classical and pseudobulges, respectively, while the green squares represent (mostly) upper limits for bulgeless galaxies (from the compilation of Kormendy & Ho 2013). The galaxy with the tightest black hole mass upper limit is M33 as indicated on the figure. The thick black line is the empirical $M_\bullet - V_{\text{circ}}$ relation, whereas the dashed and dotted curves indicate the axion core mass $M_c$ vs. $V_{\text{circ}}$ expected for ultra-light axions w/o self-interactions (see text for details). The shaded orange area shows the region in which the core radius $R_c$ is larger than 10 pc.

limit on the allowed axion mass of

$$m_{22} \lesssim 9.7 \frac{a}{\sqrt{N}} \left( \frac{M_{\bullet,\text{max}}}{10^3 M_\odot} \right)^{1/2} \left( \frac{1 \text{ pc}}{R_e} \right)^{3/2} \times \left( \frac{100 \text{ km s}^{-1}}{V_{\text{circ}}} \right)^2,$$

in which we set $a = 1$ and $N = 0.25$ as advocated above. The multiplicative term $\sqrt{N}$ arises from the fact that, in Eq. (21), $M_c$ comes with one normalisation factor $N$ (since we consider the mass enclosed in $R_c$), while $R_e$ does not.

If $R_c < R_e$, then the core mass must satisfy

$$m_{22} \gtrsim 1.5 \times 10^4 \left( \frac{10^3 M_\odot}{M_{\bullet,\text{max}}} \right)^{1/2} \left( \frac{1 \text{ pc}}{R_e} \right)^{1/2} \times \left( \frac{100 \text{ km s}^{-1}}{V_{\text{circ}}} \right)^2,$$

independent of the host halo mass.

Values of $R_e$ are obtained from Gebhardt et al. (2001) for M33, and from Neumayer & Walcher (2012) for the remaining galaxies. They are all summarized in Table 1, along with the constraints on $m$. Taking into account the finite extent of the nuclear cluster, the actual limits on the axion mass are different from those directly inferred from Fig. 1. Notwithstanding, the range of low axion masses allowed by this data, $m \lesssim 10^{-21}$ eV, is incompatible with the Lyman-$\alpha$ forest constraints. Therefore, if dark matter is an ultra-light axion, then its mass must exceed the lower limits given in the last column of Table 1 in order to satisfy the constraints from bulgeless galaxies. The absence of a compact object at the center of M33 gives the strongest constraint: $m > 1.2 \times 10^{-18}$ eV (or, equivalently, $m_{22} > 1.2 \times 10^4$).

4 CONCLUSIONS

We have assessed the extent to which measurements of the SMBH mass $M_\bullet$, and the halo mass $M_h$ for bulge and bulgeless galaxies can constrain the mass $m$ of ultra-light axion dark matter. This data can constrain an interesting range of axion mass $10^{-20} - 10^{-18}$ eV for which the axion cores are neither too diffuse nor too massive.

While we have used the compilation of Kormendy & Ho (2013) for (pseudo)bulge galaxies, small bulgeless galaxies actually give the strongest constraints on $m$, with $m \gtrsim 1.2 \times 10^{-18}$ eV for M33, a regular spiral galaxy. This range of mass is not easily accessible to measurements from rotation curves, which typically probe scales $r \gg 1$ pc (e.g. Slepian...
Table 1. Constraint on axion mass from bulgeless galaxies. $M_*$ is the mass of the central SMBH (in $M_\odot$), $R_e$ is the radius of the nuclear star cluster (in pc), $V_{\text{circ}}$ is the asymptotic circular velocity (in km s$^{-1}$), and the axion mass $m_{22}$ is in unit of $10^{-22}$ eV. The constraints on the axion mass assume that the core radius $R_c$ is either larger (left column) or smaller (right column) than $R_e$. See text for details.

| $M_*$          | $R_e$       | $V_{\text{circ}}$ | constraint on $m_{22}$ |
|---------------|------------|-------------------|------------------------|
| M33           | $<1.5 \times 10^3$ | 1.0              | 125 $< 15$             | $> 1.2 \times 10^4$ |
| NGC 300       | $< 10^2$    | 2.9               | 90 $< 48$              | $> 880$ |
| NGC 3423      | $< 7 \times 10^5$ | 4.18             | 127 $< 36$             | $> 280$ |
| NGC 4395 (3.6 $\pm 1.1$) | $\times 10^5$ | $-$              | $-$                  | $-$ |
| NGC 7424      | $< 4 \times 10^5$ | 7.4              | 145 $< 9.0$            | $> 270$ |
| NGC 7793      | $< 8 \times 10^5$ | 7.7              | 86 $< 34$             | $> 190$ |

& Goodman 2012; Bar et al. 2018). Our constraints also improve on those inferred by Marsh & Niemeyer (2018) from the presence of old star clusters in Eridanus II.

Instability of the axion core owing to an attractive self-interaction (e.g., Chavanis 2011; Visinelli et al. 2018) with $f_{17} \lesssim 0.01$ (possibly amplified by external perturbers, cf. Eby et al. 2018), could help relaxing the constraints on $m$ if, during the collapse to a black hole, a significant fraction of the axion core mass can be expelled. In this regards, self-similar solutions to the "wave collapse" indicate that interactions near the center create an outgoing stream of particles which can carry away a large fraction of the axion core before the formation of a black hole remnant (see, e.g., Levkov et al. 2017). It would be interesting to investigate whether this effect can produce a range of remnant SMBH masses broad enough to explain the non-detection of a SMBH in M33, together with the detection of a $\sim 10^5 M_\odot$ SMBH in NGC 4395.

5 ACKNOWLEDGMENTS

V.D. acknowledges support by the ISRAEL SCIENCE FOUNDATION (grant no. 1395/16). A.N. acknowledges support by the I-CORE Program of the Planning and Budgeting Committee, the ISRAEL SCIENCE FOUNDATION (grants No. 1829/12 and No. 936/18) and the Asher Space Research Institute.

APPENDIX A: ON THE QUASI-PARTICLE DESCRIPTION OF FDM HALOS

Motivated by numerical simulations, Hui et al. (2017) suggested that the atmosphere of FDM halos can be approximated as a gas of quasi-particles of characteristic size $\lambda_{\text{dB}}$, where $\lambda_{\text{dB}} = (mv)^{-1}$ is the de Broglie wavelength of the axion particle. For a typical velocity $v \sim 10^{-4}$, $\lambda_{\text{dB}} \sim m_{22}^{-1} h^{-1}$Kpc is on galactic scales.

Numerical simulations show that the binding energy of these quasi-particles is negligible compared to their kinetic energy (Veltmaat et al. 2018). In other words, their self-gravity can be neglected so that their dispersion relation is that of a free particle, $\omega(k) = k^2/2m$. As a result, a quasi-particle of initial width $\lambda_{\text{dB}}$ gradually spreads over with an rms width given by $\sqrt{(\lambda_{\text{dB}}^2 + t/m^2)/\lambda_{\text{dB}}}$. Quasi-particles thus disperse after a time $\tau \sim \lambda_{\text{dB}}^2/m$, that is,

$$\tau \sim 2.1 \times 10^7 m_{22}^{-1} \left( \frac{v}{10^{-4}} \right)^2 \text{yr}, \quad (A1)$$

in agreement with the findings of Veltmaat et al. (2018). The number $N$ of quasi-particles populating a FDM halo is, therefore, not conserved. Nevertheless, we expect the average number $\langle N \rangle$ to be conserved for FDM halos in virial equilibrium.

Turning on the self-interaction should not affect this picture noticeably. To see this, we assume that the quasi-particles are described by Gaussian wave packets of size $\lambda_{\text{dB}}$ and mass $m_{\text{eff}} = (2\pi)^{3/2}\rho \lambda_{\text{dB}}^3$ as in Hui et al. (2017). Here, $\rho$ is the density in the FDM atmosphere surrounding the axion core. The various energy contributions straightforwardly follow from the Gaussian ansatz used by Chavanis (2011). We find:

$$K = \frac{1}{2m^2} \frac{m_{\text{eff}}}{\lambda_{\text{dB}}^2}, \quad Q = \frac{\sigma_3}{\rho m^2} \frac{m_{\text{eff}}}{\lambda_{\text{dB}}^3}, \quad (A2)$$

$$U = \frac{\zeta_3}{m^2 f^2} \frac{m_{\text{eff}}^2}{\lambda_{\text{dB}}^4}, \quad W = \frac{\nu_3}{m^2} \frac{m_{\text{eff}}^2}{\lambda_{\text{dB}}^2},$$

where

$$\sigma_3 = \frac{3}{8}, \quad \zeta_3 = -\frac{1}{128\pi^{3/2}}, \quad \nu_3 = -\frac{1}{2\sqrt{\pi}} \quad (A3)$$

This gives

$$\left| \frac{K}{W} \right| \approx 5.7 \times 10^{-2} m_{22}^2 \left( \frac{\rho}{M_\odot \text{pc}^{-3}} \right)^{-1} \left( \frac{v}{10^{-7}} \right)^4, \quad (A4)$$

and

$$\left| \frac{K}{U} \right| \approx 2.4 m_{22}^2 f_{17}^2 \left( \frac{\rho}{M_\odot \text{pc}^{-3}} \right)^{-1} \left( \frac{v}{10^{-7}} \right)^2. \quad (A5)$$

Note that a relic axion dark matter density $\Omega$ today implies $m_{22}^{1/2} f_{17}^{-1} \sim \Omega/0.1$. Therefore, at fixed $\Omega$, $|K/U|$ only weakly depends on $f_{17}$.

The wave packet behaves as a transient quasi-particle if its kinetic energy is much larger than both its potential and internal energy, $W$ and $U$. For an axion mass $m_{22} \gtrsim 0.1$, the conditions $|K| \gg |W|$ and $|K| \gg |U|$ are satisfied provided that $\rho \lesssim 1 M_\odot \text{pc}^{-3}$. For comparison, the dark matter density in the neighborhood of the solar system is $\rho_\odot \approx 0.5 - 1 \times 10^{-2} M_\odot \text{pc}^{-3}$.

To conclude, note that there is an interesting similarity between the properties of halos in axion dark matter, and in repulsive BEC (Bose-Einstein condensate) dark matter.

© 0000 RAS, MNRAS 000, 1–7
cosmologies. In the latter case, the mass of the dark matter particle is orders of magnitude larger than $10^{-22}$ eV, so that the delocalization arising from the de Broglie wavelength is irrelevant. What provides the "pressure" support is a repulsive interaction, rather than the "quantum pressure". Interestingly, dense cores also form at the centers of virialized halos, which can affect rotation curves (Slepian & Goodman 2012). This suggests it should be possible to adapt the approach of Chavanis (2018a) to describe the core and atmosphere of FDM halos.

REFERENCES

Amendola L., Barbieri R., 2006, Phys. Lett., B642, 192
Armengaud E., Palanque-Delabrouille N., Marsh D. J. E., Baur J., Yéche C., 2017, Mon. Not. Roy. Astron. Soc., 471, 4606
Baldeschi M. R., Ruffini R., Gelmini G. B., 1983, Phys. Lett., 122B, 221
Bar N., Blas D., Blum K., Sibiryakov S., 2018, Phys. Rev., D98, 083027
Bar-Or B., Fouvré J.-B., Tremaine S., 2018
Broadhurst T., de Martino I., Luu H. N., Smoot G. F., Tye S. H. H., 2019
Chavanis P.-H., 2011, Phys. Rev., D84, 043531
Chavanis P.-H., 2018a
Chavanis P.-H., 2018b, Phys. Rev., D98, 023009
Davoudiasl H., Denton P. B., 2019, arXiv e-prints
de Blok W. J. G., Walter F., Brinks E., Trachternach C., Oh S.-H., Kennicutt Jr. R. C., 2008, The Astronomical Journal, 136, 2648
De Martino I., Broadhurst T., Tye S. H. H., Chiuhe T., Schive H.-Y., 2018
Du X., Behrens C., Niemeyer J. C., Schwabe B., 2017, Phys. Rev. D., 95, 043519
Eby J., Leembruggen M., Suranyi P., Wijewardhana L. C. R., 2018, JCAP, 1810, 058
Gebhardt K., Lauer T. R., Kormendy J., Pinkney J., Bower G. A., Green R., Gull T., Hutchings J. B., Kaiser M. E., Nelson C. H., Richstone D., Weistrop D., 2001, The Astronomical Journal, 122, 2469
Hlozek R., Grin D., Marsh D. J. E., Ferreira P. G., 2015, Phys. Rev., D91, 103512
Hu W., Barkana R., Gruzinov A., 2000, Phys. Rev. Lett., 85, 1158
Hui L., Ostriker J. P., Tremaine S., Witten E., 2017, Phys. Rev., D95, 043541
Iršič V., Viel M., Haehnelt M. G., Bolton J. S., Becker G. D., 2017, Phys. Rev. Lett., 119, 031302
Khlopov M., Malomed B. A., Zeldovich I. B., 1985, Mon. Not. Roy. Astron. Soc., 215, 575
Kormendy J., Ho L. C., 2013, ARA&A, 51, 511
Levkov D. G., Panin A. G., Tkachev I. I., 2017
Levkov D. G., Panin A. G., Tkachev I. I., 2017, Phys. Rev. Lett., 118, 011301
Marsh D. J. E., 2016, Phys. Rept., 643, 1
Marsh D. J. E., Niemeyer J. C., 2018, arXiv e-prints
Marsh D. J. E., Pop A.-R., 2015, Mon. Not. Roy. Astron. Soc., 451, 2479
Marsh D. J. E., Silk J., 2014, Mon. Not. Roy. Astron. Soc., 437, 2652
Mayall N. U., Aller L. H., 1942, Astrophys. J., 95, 5
Neumayer N., Walcher C. J., 2012, Advances in Astronomy, 2012, 709038
Petersen B. M., Bentz M. C., Desroches L.-B., Filippenko A. V., Ho L. C., Kaspi S., Lair A., Maoz D., Moran E. C., Pogge R. W., Quillen A. C., 2005, Astrophys. J., 632, 799
Robles V. H., Bullock J. S., Boylan-Kolchin M., 2018
Schive H.-Y., Chiuhe T., Broadhurst T., 2014, Nature Phys., 10, 496
Schive H.-Y., Liao M.-H., Woo T.-P., Wong S.-K., Chiuhe T., Broadhurst T., Hwang W.-Y. P., 2014, Physical Review Letters, 113, 261302
Schwabe B., Niemeyer J. C., Engels J. F., 2016, Phys. Rev. D., 94, 043513
Sin S.-J., 1994, Phys. Rev., D50, 3570
Slepian Z., Goodman J., 2012, Mon. Not. Roy. Astron. Soc., 427, 839
Sorgho A., Carignan C., Pisano D. J., Oosterloo T., de Blok W. J. G., Korsaga M., Pingel N. M., Sardone A., Goodhart S., Passmoor S., Dikgale A., Sirotlho S. K., 2019, Mon. Not. Roy. Astron. Soc., 482, 1248
Svrcek P., Witten E., 2006, JHEP, 06, 051
Veltmaat J., Niemeyer J. C., Schwabe B., 2018, Phys. Rev. D., 98, 043509
Visinelli L., Baum S., Redondo J., Freese K., Wilczek F., 2018, Phys. Lett., B777, 64
Zhang J., Kuo J.-L., Liu H., Tsai Y.-L. S., Cheung K., Chu M.-C., 2017