Research Article

Approximate and Closed-Form Solutions of Newell-Whitehead-Segel Equations via Modified Conformable Shehu Transform Decomposition Method

Muhammad Imran Liaqat¹, ¹ Adnan Khan ¹, ¹ Md. Ashraful Alam ², ² M. K. Pandit ², ² Sina Etemad ³, ³ and Shahram Rezapour ³, ⁴

¹National College of Business Administration & Economics, Lahore, Pakistan
²Department of Mathematics, Jahangirnagar University, Savar, Dhaka, Bangladesh
³Department of Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran
⁴Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan

Correspondence should be addressed to Md. Ashraful Alam; ashraf_math20@juniv.edu

Received 12 January 2022; Revised 15 February 2022; Accepted 19 February 2022; Published 11 April 2022

Academic Editor: Aida Mustapha

Copyright © 2022 Muhammad Imran Liaqat et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this study, we introduced a novel scheme to attain approximate and closed-form solutions of conformable Newell-Whitehead-Segel (NWS) equations, which belong to the most consequential amplitude equations in physics. The conformable Shehu transform (CST) and the Adomian decomposition method (ADM) are combined in the proposed method. We call it the conformable Shehu decomposition method (CSDM). To assess the efficiency and consistency of the recommended method, we demonstrate 2D and 3D graphs as well as numerical simulations of the derived solutions. As a result, CSDM demonstrates that it is a useful and simple mathematical tool for getting approximate and exact analytical solutions to linear-nonlinear fractional partial differential equations (PDEs) of the given kind. The convergence and absolute error analysis of the series solutions is also offered.

1. Introduction

While the most well-known fractional derivatives, particularly Caputo and Riemann–Liouville (R-L), are used by many mathematicians [1–4], there is some evidence that these operators have limitations, as shown in the following [5, 6]. Khalil et al. in 2014 recommended a novel fractional derivative operator, so-called conformable fractional derivative (CFD) [7], that contains variety of the features in an operator that are impossible to be satisfied with the prevailing operators. This operator is relatively similar to the characterization of derivatives in the traditional limit method, and it is relatively effortless to manage. As a result, it was quickly recognized and became the subject of various scientific studies [8–12].

In the context of science and engineering, we derive logical and physical processes that, when viewed through the lens of mathematical representations, occur as differential equations (DEs). For example, the equation of simple harmonic motion, the equation of motion, and the deflection of the beam, and as a consequence, all are categorized through DEs. Subsequently, the explanations of DEs are necessitated. The various DEs emitted by applications become so complicated as a result that it is frequently unreasonable to participate in closed-form results. Numerical methods afford an appreciable alternate means for explaining the DEs under specified early circumstances. Transforms are interesting in their sense, and they make it easier for researchers to solve mathematical problems such as ordinary, partial, and fractional-order DEs. The Laplace transform and the Fourier transform are two well-known transforms that were used to solve ordinary and partial DEs in the beginning. Later, in the domain of fractional calculus, these transformations were applied to fractional-order DEs [13–15]. Many other transformations have been proposed by
researchers in recent years, addressing a wide range of mathematical problems. To handle fractional-order DEs, these transformations are combined with additional analytical, numerical, or homotopy-based approaches. The Laplace transform [16], fractional complex transform [17], travelling wave transform [18], Elzaki transform [19], Sumudu transforms [20], and ZZ transforms [21], among others, are still used to solve fractional-order DEs. Recently, Shehu and Zhao [22] introduced a Laplace-type integral transform called the Shehu transform, which is used to solve DEs in the time domain. The proposed integral transform is successfully derived from the classical Fourier integral transform and is applied to both ordinary and partial DEs. Numerous mathematicians have lately become interested in the Shehu transform, which has been employed by many researchers for fractional-order DEs [23–26].

The main advantages of the recommended transform can be summarized as follows:

(i) The natural, the Sumudu, and the Elzaki integral transforms are all more difficult to comprehend than the suggested transform.

(ii) When variable \( v = 1 \) is used, the recommended transform becomes a Laplace transform, and when the variable \( \mu = 1 \) is used, it becomes a Yang integral transform.

(iii) It is a generalization of the Sumudu and the Laplace transforms.

(iv) It can be used to solve exactly and numerically fractional-order DEs easily and efficiently.

During the past years, abundant procedures have been put forward to explain fractional-order DEs, including the Elzaki residual power series method [27], natural decomposition method [28], Legendre wavelet method [29], Laplace variational iteration scheme [30], differential transform method [31], homotopy perturbation with the Sumudu transformation approach [32], operational matrix method [33], Aboodh decomposition method [34], residual power series method [35], and the rational symmetric contraction mappings approach [36].

The variations in the sand, the appearances of the seashells, and various further striped shapes happen in many spatial structures that are possibly demonstrated through the amplitude model. The NWS model is one of the most important amplitude models in the practical sciences, and it explains how stripe patterns appear in two-dimensional structures [37, 38].

In the current effort, the CSDM is utilized to acquire the approximate and closed-form results of the linear and nonlinear fractional NWS equations of the procedure specified as follows [39]:

\[
T^\alpha_x \Phi (x, \tau) = qD^\alpha_x \Phi (x, \tau) + k\Phi (x, \tau) - g\Phi^n (x, \tau),
\]

where \( T^\alpha_x \) is CFD, \( n \) is a positive integer, and \( q, k, \) and \( g \in R \) \( q > 0, \tau \geq 0, 0 < k \leq 1, \) and \( x \in R. \) Here, \( \Phi (x, \tau) \) possibly engaged employing the velocity of a fluid or temperature distribution in a thin and infinitely long pipe.

The solution of the time-fractional Newell-Whitehead-Segel (NWS) in the sense of Caputo and conformable derivative has attracted attention in recent years. Prakash et al. [40] have used the fractional variation iteration method to obtain approximate analytical solutions for the fractional NWS equations in the sense of the Caputo derivative. The variational iteration approach was used by Nadeem et al. [41] to develop approximate and precise solutions to the Caputo time-fractional NWS problem. Using the fractional power series methodology, Ali et al. [42] solved the Caputo time-fractional NWS equation. To solve NWS equations, Benattia and Belghaba [39] employed the conformable Sumudu transform and the Adomian decomposition approach. The Sumudu decomposition approach was utilized by Ahmed and Elbadri [43] to determine approximate and definite solutions to the Caputo time-fractional NWS equations. Saadeh et al. [44] obtained the approximate analytical solutions to the fractional-order NWS equations in the sense of Caputo by using the residual power series method. In the work of Prakash and Verma [45], they used the Adomian decomposition method to solve time-fractional NWS equations in the sense of Caputo.

For the first time in research, we established a new procedure by using the conformable Shehu transform (CST) and an Adomian decomposition method (ADM) for solving fractional-order NWS equations in the sense of conformable fractional derivative (CFD). The results obtained by the recommended method are in very good agreement with the results already existing in the literature. The advantage of the CSDM is that it significantly reduces the amount of numerical computation required to find approximate and exact solutions to these types of equations compared to the current methods, especially when compared to the Caputo base methods.

This paper is schematized as it goes along with Section 2, where we recall considerable elementary explanations and significant proceedings utilizing the CST and CFD. The essential suggestion beyond the CSDM and convergence consideration for the conformable time-fractional NWS model is established in Section 3. In Section 4, we verified numerical illustrations of NWS models to illustrate the capability, potential, and uncomplicated nature of the adapted method. To end, in Section 5, significance is congregated in the conclusions.

2. Fundamental Concepts

Fractional calculus (FC) is a branch of mathematical analysis that studies the integral and derivative of any real or complex order. FC is correspondingly recognized utilizing non-Newtonian calculus and generalized calculus. The FC has developed influential contrivances in numerous domains of physics, engineering, biology, chemistry, image processing, solid-state, stochastic-based finance, control theory, economics, signal, viscoelasticity, and fiber optics by interpreting these problems into mathematical representations [46, 47].

In this part, we will elucidate substantial expressive compatible explanations and theorems in the Shehu
transform (ST) and conformable fractional derivative (CFD) senses, which we will utilize in this paper.

Definition 1 (see [8]). A function $\Phi: [0, \infty) \rightarrow R$ was specified as well the CFD of $\Phi$ order $\alpha$ is demarcated by

$$T^\alpha_\tau \Phi (\tau) = \lim_{\epsilon \to 0} \Phi (\tau + \epsilon \tau^{-\alpha}) - \Phi (\tau)$$

(2)

For $\tau > 0$, and $\alpha \in (0, 1]$, if $\Phi$ is $\alpha$-differentiable in some $(0, \theta)$, $\theta > 0$, and $\lim_{\tau \to 0^+} (T^\alpha_\tau \Phi) (\tau)$ occurs subsequently, it is demarcated as

$$(T^\alpha_\tau \Phi) (0) = \lim_{\tau \to 0^+} (T^\alpha_\tau \Phi) (\tau).$$

(3)

Definition 2 (see [22]). The Shehu transform (ST) is a novel integral transformation that is expedient for the function of exponential order. We employ a function belonging to the set $Y$ explicated through $Y = \{ \Phi (\tau): \exists \xi_1, \xi_2 > 0, |\Phi (\tau)| < e^{\xi_1 |\tau|}, \forall \tau \in [0, \infty) \}$.

The ST which is characterized through $S(.)$ for a function $\Phi (\tau)$ is demarcated by way of

$$S[\Phi (\tau)] = Y (u, v) = \int_0^\infty \Phi (\tau) e^{-uv\tau} d\tau, \quad \tau > 0.$$  

(4)

The ST of a function $\Phi (\tau)$ is $Y (u, v)$, then $\Phi (\tau)$ is termed the inverse of $Y (u, v)$ that is stated as $S^{-1} [Y (u, v)] = \Phi (\tau)$, for $\tau \geq 0$, $S^{-1}$ which is called inverse ST.

Definition 3. The Laplace transform of a function $\Phi (\tau)$ $\tau > 0$ is defined as

$$L[\Phi (\tau)] = F (u) = \int_0^\infty e^{-uv\tau} \Phi (\tau) d\tau,$$  

(5)

where $F (u)$ is the Laplace transform of $\Phi (\tau)$.

Definition 4. The Sumudu transform of a function $\Phi (\tau)$ is defined as

$$S[\Phi (\tau)] = P (v) = \int_0^\infty e^{-v\tau} \Phi (\tau) d\tau,$$  

(6)

where $P (v)$ is the Sumudu transform of $\Phi (\tau)$.

Definition 5. Let $0 < \alpha \leq 1$, and $\Phi: [0, \infty) \rightarrow R$ be a real value function. Subsequently, the CST of order $\alpha$ is defined by

$$S_\alpha [\Phi] = Y (u, v) = \int_0^\infty e^{-(u/v) (r^{\alpha}/\alpha)} \Phi r^{\alpha-1} dr.$$  

(7)

Definition 6. The Mittag-Leffler function is defined as follows:

$$E_\alpha [r] = \sum_{j=0}^\infty \frac{r^j}{\Gamma (\alpha j + 1)}, \quad \alpha \in C, \quad \text{Re} (\alpha) > 0.$$  

(8)

Theorem 1 (see [8]). Let $0 < \alpha \leq 1$, $\Phi_1$, and $\Phi_2$ be $\alpha$-differentiable at a point $\tau > 0$ subsequently.

(i) $T^\alpha_\tau (\Phi_1 (r)) = cr^{\alpha - \alpha} , \forall c \in R.$

(ii) $T^\alpha_\tau (c) = 0$, where $c \in R.$

(iii) $T^\alpha_\tau (\Phi_1 (\Phi_2)) = \Phi_1 T^\alpha_\tau (\Phi_2) + \Phi_1 T^\alpha_\tau (\Phi_1).$

(iv) $T^\alpha_\tau (\partial /\partial \tau \Phi_1) = \Phi_1 T^\alpha_\tau (\Phi_1) - \Phi_1 T^\alpha_\tau (\Phi_1)/\Phi_2^2.$

(v) $T^\alpha_\tau (c_1 \Phi_1 + c_2 \Phi_2) = c_1 T^\alpha_\tau (\Phi_1) + c_2 T^\alpha_\tau (\Phi_2), \forall c_1, c_2 \in R.$

Theorem 2 (see [48]). Let $\Phi: [0, \infty) \rightarrow R$ be a real value function and $0 < \alpha \leq 1$, then we have

$$S_\alpha [\Phi (\tau)] = \int_0^\infty \frac{u}{v} S_\alpha [\Phi (x, \tau)] - \Phi (x, 0).$$  

(9)

Theorem 3 (see [48]). Supposing $a$ and $c \in R$ and $0 < \alpha \leq 1$, we have the following:

(i) $S_\alpha [c] = c/vu.$

(ii) $S_\alpha [\exp (\alpha u^2/\alpha)] = Y (u, v) = u/v - u/v > 0.$

(iii) $S_\alpha [\sin (\alpha u^2/\alpha)] = Y (u, v) = uv/v^2 + u^2v^2, u/v > 0.$

(iv) $S_\alpha [\cos (\alpha u^2/\alpha)] = Y (u, v) = uv/v^2 + u^2v^2, u/v < 0.$

(v) $S_\alpha [\sinh (\alpha u^2/\alpha)] = Y (u, v) = u/v^2 - u^2v^2, u/v < [a].$

(vi) $S_\alpha [\cosh (\alpha u^2/\alpha)] = Y (u, v) = uv/v^2 + u^2v^2, u/v > [a].$

(vii) $S_\alpha [\tau^2] = Y (u, v) = a^\alpha/v (v/\alpha)^{\alpha+1}/(1 + c/\alpha).$

In the succeeding part, we generate the foremost suggestion of the CSDM to obtain the results for linear and nonlinear NWS equations and deliberate on the convergence of the series solution and maximum error analysis.

3. The Algorithm of the CSDM with Convergence and Error Analysis

We discuss the following conformable PDEs in common operator systems to demonstrate the fundamental conceptualization of CSDM:

$$T^\alpha_\tau \Phi (x, \tau) + D^\alpha_\tau \Phi (x, \tau) + L (\Phi (x, \tau)) + N (\Phi (x, \tau)) = M (x, \tau), \quad \tau > 0, x > 0, 0 < \alpha \leq 1,$$  

(10)

Subject to the initial conditions,

$$\Phi (x, 0) = f (x),$$  

(11)

where $T^\alpha_\tau$ represents the CFD of order $\alpha$ in $\tau$, $D^\alpha_\tau$ is the uppermost order linear classical derivative in $x$, $L$ represents further linear expression through lesser derivatives, $N$ demonstrates nonlinear expression, and $M (x, \tau)$ is the nonhomogenous part.

Now, applying the CST to (10), we have
\[ S_a \left[ T^0_a \Phi(x, t) + D^0_a \Phi(x, t) + L(\Phi(x, t)) + N(\Phi(x, t)) \right] = S_a [M(x, t)]. \]  

(12)

We get the following by utilizing Theorem 2 in equation (13):

\[ S_a [\Phi(x, t)] = \frac{V}{u} S_a [M(x, t)] + \frac{V}{u} \Phi(x, 0) - \frac{V}{u} S_a [D^0_a \Phi(x, t)] \]

\[ - S_a \left[ \frac{V}{u} S_a [L(\Phi(x, t)) + N(\Phi(x, t))] \right] \]  

(15)

When inverse CST is implemented, (14) becomes as follows:

\[ \Phi(x, t) = S^{-1}_a \left[ \frac{V}{u} S_a [M(x, t)] + \frac{V}{u} \Phi(x, 0) \right] \]

\[ - S^{-1}_a \left[ \frac{V}{u} S_a [D^0_a \Phi(x, t)] \right] \]

\[ - S^{-1}_a \left[ \frac{V}{u} S_a [L(\Phi(x, t)) + N(\Phi(x, t))] \right] \]  

(16)

By using the linear property of CST, (12) becomes as follows:

\[ \Phi_0(x, t) = S^{-1}_a \left[ \frac{V}{u} S_a [M(x, t)] + \frac{V}{u} \Phi(x, 0) \right] \]  

(20)

Also,

\[ \Phi_{i+1}(x, t) = - S^{-1}_a \left[ \frac{V}{u} S_a [D^0_a \sum_{j=0}^{\infty} \Phi_j(x, t)] \right] \]

\[ - S^{-1}_a \left[ \frac{V}{u} S_a [L(\sum_{j=0}^{\infty} \Phi_j(x, t)) + N(\sum_{j=0}^{\infty} A_j)] \right] \]  

(21)

where \( i = 0, 1, 2, \ldots \)

The following theorem clarifies and governs the condition for convergence of the expansion solution, (16).

**Theorem 4.** Let \( B \) represent a Banach space; subsequently, the expansion result of \( \Phi(x, t) \) converges uncertainty; there occurs \( z, 0 < z < 1 \), so that \( \| \Phi_i(x, t) \| \leq z \| \Phi_{i-1}(x, t) \| \); for all \( i \in \mathbb{N} \).

**Proof.** Deliberate the subsequent succession

\[ H_i(x, t) = \Phi_0(x, t) + \Phi_1(x, t) + \Phi_2(x, t) + \cdots + \Phi_i(x, t). \]

(22)

It is essential to validate that successions of ith partial sums \( \{ H_i(x, t) \} \) are a Cauchy series in Banach space. Intended for this, we contemplate the following:

\[ \sum_{i=0}^{\infty} \| H_i(x, t) \| \leq \sum_{i=0}^{\infty} \left( z^i \| \Phi_0(x, t) \| \right) \]

\[ \leq z \| \Phi_0(x, t) \| \prod_{i=0}^{\infty} (1 - z), \quad 0 < z < 1 \]  

(23)

For every \( i, j \in \mathbb{N}, i \leq j \), we get as follows:

\[ \| H_i(x, t) - H_j(x, t) \| \leq \sum_{i=0}^{\infty} \| \Phi_i(x, t) - \Phi_{i+1}(x, t) \| \]

\[ \leq z \| \Phi_i(x, t) \| \]  

(24)

becomes as follows when the triangle inequality is used:

\[ \| H_i(x, t) - H_j(x, t) \| \leq \sum_{i=0}^{\infty} \| \Phi_i(x, t) - \Phi_j(x, t) \| \]

\[ + \| \Phi_j(x, t) - \Phi_{j+1}(x, t) \| \]

\[ + \cdots + \| \Phi_{j-i}(x, t) - \Phi_{j-i+1}(x, t) \| \]

\[ + \| H_i(x, t) - H_{i+1}(x, t) \| \]  

(25)

Inequality (25) can be expressed in the following way:
\[ \| H_i(x, r) - H_j(x, r) \| \leq z^{i+1} \left\| \Phi_0(x, r) \right\| + z^{j+2} \left\| \Phi_0(x, r) \right\| + \cdots + z^j \left\| \Phi_0(x, r) \right\|. \]  

(26)

By simple calculation, inequality (26) can be written as

\[ \| H_i(x, r) - H_j(x, r) \| \leq z^{i+1} \left( 1 + z + z^2 + \cdots + z^{i-j} \right) \left\| \Phi_0(x, r) \right\|. \]  

(27)

as \((1 - z^{i-j}/1 - z) = 1 + z + z^2 + \cdots + z^{i-j}\). 
As a result, inequality (27) is as follows:

\[ \| H_i(x, r) - H_j(x, r) \| \leq z^{i+1} \left( \frac{1 - z^{i-j}}{1 - z} \right) \left\| \Phi_0(x, r) \right\|. \]  

(28)

So, \(0 < z < 1\), and \(1 - z^{i-j} \leq 1\). 
Therefore, from inequality (28), we get as

\[ \| H_i(x, r) - H_j(x, r) \| \leq z^{i+1} \left\| \Phi_0(x, r) \right\|. \]  

(29)

As a result, \(\Phi_0(x, r)\) is bounded, and we have

\[ \lim_{i,j \to \infty} \| H_i(x, r) - H_j(x, r) \| = 0. \]  

(30)

Therefore, \(\{H_i\}\) is a Cauchy series in Banach space, so the expansion solution (16) converges.

In the subsequent theorem, we offer an absolute error investigation of the proposed procedure.

**Theorem 5.** Let \(\Phi(x, r)\) be the approximate solution of the truncated finite series \(\sum_{i=0}^{\kappa} \Phi_i(x, r)\). Assume it is attainable to acquire a real number \(z \in (0, 1)\) in order that \(\|\Phi_{i+1}(x, r)\| \leq z \|\Phi_0(x, r)\| \forall i \in \mathbb{N}\). Furthermore, the utmost absolute error is

\[ \left\| \Phi(x, r) - \sum_{i=0}^{\kappa} \Phi_i(x, r) \right\| \leq \frac{z^{\kappa+1}}{1 - z} \left\| \Phi_0(x, r) \right\|. \]  

(31)

**Proof.** Let the series \(\sum_{i=0}^{\kappa} \Phi_i(x, r)\) be finite, then

\[ \left\| \Phi(x, r) - \sum_{i=0}^{\kappa} \Phi_i(x, r) \right\| = \sum_{i=0}^{\infty} \| \Phi_i(x, r) \| \leq \sum_{i=0}^{\infty} \left\| \Phi_0(x, r) \right\| \]

\[ \leq \sum_{i=0}^{\infty} z^i \left\| \Phi_0(x, r) \right\| \]

\[ \leq z^{\kappa+1} \left( 1 + z + z^2 + z^3 + \cdots \right) \]

\[ \left\| \Phi_0(x, r) \right\| \leq \left( \frac{z^{\kappa+1}}{1 - z} \right) \left\| \Phi_0(x, r) \right\|. \]  

(32)

This proof is complete.

In the next section, we determine the appropriateness of the recommended method.

**4. Numerical Examples and Concluding Remarks**

In this section, three problems with conformable NWS equations are recognized to illustrate the performance and appropriateness of the suggested method.

**Example 1.** Consider the linear conformable fractional NWS as follows:

\[ T_{\tau}^\alpha \Phi(x, r) = D_x^2 \Phi(x, r) - 2 \Phi(x, r), \quad 0 < \alpha \leq 1. \]  

(33)

Subject to the initial condition,

\[ \Phi(x, 0) = f(x) = e^x. \]  

(34)

Using the conformable Shehu transform on (30), we get

\[ S_a[\tau^\alpha T \Phi(x, r)] = S_a[D_x^2 \Phi(x, r) - 2 \Phi(x, r)]. \]  

(35)

Using Theorem 2, equation (32) is transformed as follows:

\[ S_a[\Phi(x, r)] = \left( \frac{v}{u + 2v} \right) e^x + \left( \frac{v}{u + 2v} \right) S_a[D_x^2 \Phi(x, r)]. \]  

(36)

By using the inverse conformable Shehu transform, (33) becomes as

\[ \Phi(x, r) = S_a^{-1} \left( \frac{v}{u + 2v} \right) e^x + S_a^{-1} \left( \frac{v}{u + 2v} \right) S_a[D_x^2 \Phi(x, r)]. \]  

(37)

By using the procedure of CSDM, as explained in Section 3, the expansion solution of (34) can be represented by the expansion form as follows:

\[ \Phi(x, r) = \sum_{i=0}^{\infty} \Phi_i(x, r). \]  

(38)

We get as by substituting (35) into (34),

\[ \sum_{i=0}^{\infty} \Phi_i(x, r) = S_a^{-1} \left( \frac{v}{2v + u} \right) e^x \]

\[ + \left( \frac{v}{2v + u} \right) S_a[D_x^2 \sum_{i=0}^{\infty} \Phi_i(x, r)]. \]  

(39)

Using the approach outlined in Section 3, we can get the following from:

\[ \Phi_0(x, r) = S_a^{-1} \left( \frac{v}{u + 2v} \right) e^x. \]  

(40)

We obtain as a result of applying the 2nd part of Theorem 3,

\[ \Phi_0(x, r) = e^{-2r^{\alpha/2}} e^x. \]  

(41)

We can also extract the following from (36) using the methodology discussed in Section 3:
\[ \Phi_{i+1}(x, \tau) = S^{-1}_{\alpha} \left[ \left( \frac{v}{2v+u} \right) S_{\alpha} \left[ D^2_{\alpha} \Phi_i(x, \tau) \right] \right], \quad i = 0, 1, 2, \ldots \]

By repeating the iteration process in (39), we obtain the following results:

\[ \Phi(x, \tau) = \Phi_0(x, \tau) + \Phi_1(x, \tau) + \Phi_2(x, \tau) + \cdots. \]

\[ \Phi(x, \tau) = e^{x^{-2}r^\alpha} + \frac{\tau^\alpha}{\alpha} e^{x^{-2}r^\alpha} + \frac{\tau^{2\alpha}}{2\alpha^2} e^{x^{-2}r^\alpha} + \frac{\tau^{3\alpha}}{6\alpha^3} e^{x^{-2}r^\alpha} + \frac{\tau^{4\alpha}}{24\alpha^4} e^{x^{-2}r^\alpha} + \frac{\tau^{5\alpha}}{120\alpha^5} e^{x^{-2}r^\alpha} + \cdots. \]

As a result, the series solution can be found as

\[ \Phi(x, \tau) = e^{x^{-2}r^\alpha} e^{\tau^\alpha}. \]

When \( \alpha = 1 \), we get a closed-form solution of (30) in the following form:

\[ \Phi(x, \tau) = e^{x^{-\tau}}. \]

Figure 1(a) shows the behavior of the 5th term approximate and exact solutions of (30) at several values of \( \alpha \), the approximate result corresponds with the precise result at \( \alpha = 1 \), and this admits the effectiveness and precision of the suggested method. Figures 1(b) and 1(c) demonstrate 3D and 2D graphs of absolute errors in the intervals \( \tau, x \in [0, 1] \) and \( r \in [0, 1] \) over the 5th approximate and accurate solutions of (30) at \( \alpha = 1 \), respectively. As of the figures, the approximate solution is in a preeminently compact with the precise solution.

Error functions are available to distinguish the precision and capability of the scheme. To indicate the accuracy and capability of CSDM, we selected residual, recurrence, and absolute errors functions.

Table 1 displays absolute and relative errors at reasonable nominated grid points in the interval \( \tau \in [0, 1] \) amongst the 5th approximate and precise solutions of (30) at \( \alpha = 1 \), when \( x = 1 \), attained using CSDM. From Table 1, it can be perceived that the approximate solutions are in eminent contact with the precise solution, and this sanctions the efficiency of the recommended method. The convergence of the CSDM approximate solution to the exact solution for (30) has been shown numerically, as in Table 2. From the obtained results, it is evident that the present technique is an effective and convenient algorithm to solve certain classes of fractional order DEs with fewer calculations and iteration steps.

**Example 2.** Consider the conformable nonlinear fractional NWSE as follows:

\[ T^\alpha \Phi(x, r) = 5D^2_x \Phi(x, r) + 2\Phi(x, r) + \Phi^2(x, r), \]

\[ 0 < \alpha \leq 1. \]

With the initial condition,

\[ \Phi(x, 0) = \phi. \]

Using the CST on (43),

\[ S_{\alpha} [T^\alpha \Phi(x, r)] = S_{\alpha} \left[ 5D^2_x \Phi(x, r) + 2\Phi(x, r) + \Phi^2(x, r) \right]. \]

By using Theorem 2 and making some simple calculations, we get the following from equation (45):
Table 2: The recurrence errors $|\Phi^5(x, \tau) - \Phi^4(x, \tau)|$ of the fifth approximate solution with different values of $\alpha$, when $x = 1$ for Example 1.

| $\tau$ | $\alpha = 0.7$ | $\alpha = 0.8$ | $\alpha = 0.9$ | $\alpha = 1.0$ |
|--------|----------------|----------------|----------------|----------------|
| 0.001  | 0.0009099960543213 | 0.000554584599332872 | 0.000162858810296945 | 0.00000485898346770802 |
| 0.002  | 0.0005532428816266587 | 0.000344700848745971 | 0.0001422636869633455 | 0.0000010422636869633455 |
| 0.003  | 0.000353735944994873 | 0.0002466155529428013 | 0.000053053752799453220 | 0.0000014986216409607550 |
| 0.004  | 0.00005384300024572984 | 0.000028911804842350 | 0.000005941846488085602 | 0.00000306566201027972830 |
Using inverse CST on (46) yields the following:

\[
\Phi (x, \tau) = S_a^{-1}\left[\frac{\nu}{u-2\nu}\phi + \frac{\nu}{u-2\nu}S_a[5D_a^2\Phi (x, \tau)] \right] \]

Suppose that (47) has the following expansion solution by using the procedure discussed in Section 3:

\[
\Phi (x, \tau) = \sum_{i=0}^{\infty} \Phi_i (x, \tau). \tag{51}
\]

Using (48), in (47), we get as follows:

\[
\sum_{i=0}^{\infty} \Phi_i (x, \tau) = S_a^{-1}\left[\frac{\nu}{u-2\nu}\phi \right]
+ S_a^{-1}\left[\frac{\nu}{u-2\nu}S_a[5D_a^2 \sum_{i=0}^{\infty} \Phi_i (x, \tau)] \right] \tag{52}
+ S_a^{-1}\left[\frac{\nu}{u-2\nu}S_a[5D_a^2 \sum_{i=0}^{\infty} A_i] \right].
\]

The nonlinear term is represented by the following Adomian polynomials:

\[
A_i = \frac{1}{(i)!} \frac{d^i}{d\lambda^i} \left[ N \left( \sum_{k=0}^{\lambda} \lambda^k \phi_k (x, \tau) \right) \right]_{\lambda=0}, \quad i = 0, 1, 2, \ldots \tag{53}
\]

A few Adomian polynomial components obtained from (50) are as follows:

\[
\begin{align*}
A_0 &= \Phi_0^2 (x, \tau), \\
A_1 &= 2\Phi_0 (x, \tau)\Phi_1 (x, \tau), \\
A_2 &= 2\Phi_0 (x, \tau)\Phi_2 (x, \tau) + \Phi_1^2 (x, \tau), \\
A_3 &= 2\Phi_0 (x, \tau)\Phi_3 (x, \tau) + 2\Phi_1 (x, \tau)\Phi_2 (x, \tau).
\end{align*} \tag{54}
\]

Utilizing these Adomian polynomials in (49) and employing the iterative process of CSDM, we get as follows:

\[
\Phi_0 (x, \tau) = \phi e^{2\tau/\alpha},
\Phi_1 (x, \tau) = \frac{1}{2}\phi^2 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1), \\
\Phi_2 (x, \tau) = \left(\frac{1}{2}\right)^2 \phi^3 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^2, \\
\Phi_3 (x, \tau) = \left(\frac{1}{2}\right)^3 \phi^4 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^3, \\
\Phi_4 (x, \tau) = \left(\frac{1}{2}\right)^4 \phi^5 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^4, \\
\Phi_5 (x, \tau) = \left(\frac{1}{2}\right)^5 \phi^6 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^5.
\]

As a result, the series solution is as follows:

\[
\Phi (x, \tau) = \Phi_0 (x, \tau) + \Phi_1 (x, \tau) + \Phi_2 (x, \tau) + \cdots,
\]

\[
\Phi (x, \tau) = \phi e^{2\tau/\alpha} + \frac{1}{2}\phi^2 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1) \\
+ \left(\frac{1}{2}\right)^2 \phi^3 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^2 \\
+ \left(\frac{1}{2}\right)^3 \phi^4 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^3 \\
+ \left(\frac{1}{2}\right)^4 \phi^5 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^4 \\
+ \left(\frac{1}{2}\right)^5 \phi^6 e^{2\tau/\alpha} (e^{2\tau/\alpha} - 1)^5 + \cdots.
\]

(53) can be written as follows:

\[
\Phi (x, \tau) = \phi e^{2\tau/\alpha} (1 + \left(\frac{1}{2}\phi (e^{2\tau/\alpha} - 1) + \left(\frac{1}{2}\phi (e^{2\tau/\alpha} - 1) \right)^2 \\
+ \left(\frac{1}{2}\phi (e^{2\tau/\alpha} - 1) \right)^3 + \left(\frac{1}{2}\phi (e^{2\tau/\alpha} - 1) \right)^4 \\
+ \left(\frac{1}{2}\phi (e^{2\tau/\alpha} - 1) \right)^5 + \cdots).
\]

The (54) can be written as follows:

\[
\Phi (x, \tau) = \phi e^{2\tau/\alpha} \left( \frac{1}{1-1/2\phi (e^{2\tau/\alpha} - 1)} \right) \tag{58}
\]

For $\alpha = 1$, we get the closed-form solution of (43) in the following form:
\[ \Phi(x, \tau) = \frac{2\Phi e^{2\tau}}{2 + \Phi(1 - e^{2\tau})}. \]  
(59)

Figure 2(a) illustrates the performance of the 5th approximate and exact CSDM solutions of (43) for diverse values of \( \alpha \) when \( \Phi = 0.10 \) in the interval \( \tau \in [0, 1] \). Indisputably, results in instances of fractional values of \( \alpha \) converge to the results in case of \( \alpha = 1 \). As well, the imprecise results correspond with the precise solution at \( \alpha = 1 \) and this supports the efficiency and exactness of the recommended method. Figures 2(b) and 2(c) describe the 3D and 2D graphs of absolute errors in the intervals \( \tau, x \in [0, 1] \) and \( \tau \in [0, 1] \) when \( \Phi = 0.10 \), respectively, over the 5th approximate and accurate solutions of (43) at \( \alpha = 1 \), attained utilizing CSDM. As of the figure, it can be predicted that the approximate solution is very close to the exact solutions which show the efficiency of the CSDM.

Table 3 displays absolute and relative errors at suitable chosen grid points in the interval \( \tau \in [0, 1] \) between the 5th approximate and precise solutions of (43) at \( \alpha = 1 \) when \( \Phi = 0.10 \) accomplished employing CSDM. Moreover, numerical comparisons are made to validate the accuracy of our method by calculating the recurrence errors for the approximate solution of (43) obtained for various values of when, as shown in Table 4. From Tables 3 and 4, it can be perceived that the approximate solution is in distinguished contract with the precise solution, and this is in agreement with the efficacy of the CSDM.

**Example 3.** Consider the confomal nonlinear fractional NWSE as follows:

\[ T^\alpha \Phi(x, \tau) = D^2_x \Phi(x, \tau) + 2\Phi(x, \tau) - 3\Phi^2(x, \tau), \]

\[ 0 < \alpha \leq 1. \]  
(60)

Subject to the initial conditions,

\[ \Phi(x, 0) = \beta. \]  
(61)

By utilizing the CST on (57), we get as

\[ S_a[T^\alpha \Phi(x, \tau)] = S_a[D^2_x \Phi(x, \tau) + 2\Phi(x, \tau) - 3\Phi^2(x, \tau)]. \]  
(62)

Using Theorem 2 and some calculations based on equation (59), we get to the following:

\[ S_a[2\Phi(x, \tau)] = \left(\frac{v}{u - 2v}\right)\beta + \left(\frac{v}{u - 2v}\right)S_a[-3\Phi^2(x, \tau)]. \]  
(63)

By applying inverse CST to (60), we get as

\[ \Phi(x, \tau) = S^{-1}_a\left[\left(\frac{v}{u - 2v}\right)\beta\right] + S^{-1}_a\left[\left(\frac{v}{u - 2v}\right)S_a[D^2_x \Phi(x, \tau)]\right] - S^{-1}_a\left[3\left(\frac{v}{u - 2v}\right)S_a[\Phi^2(x, \tau)]\right]. \]  
(64)

Assume that (61), using the approach outlined in Section 3, yields the following expansion solution:

\[ \Phi(x, \tau) = \sum_{i=0}^{\infty} \Phi_i(x, \tau). \]  
(65)

By using (62), in (61),

\[ \sum_{i=0}^{\infty} \Phi_i(x, \tau) = S^{-1}_a\left[\left(\frac{v}{u - 2v}\right)\beta\right] + S^{-1}_a\left[\left(\frac{v}{u - 2v}\right)S_a[D^2_x \sum_{i=0}^{\infty} \Phi_i(x, \tau)]\right] - S^{-1}_a\left[3\left(\frac{v}{u - 2v}\right)S_a[\sum_{i=0}^{\infty} \Phi_i(x, \tau)]\right]. \]  
(66)

The Adomian polynomials for nonlinear terms are given as

\[ A_i = \frac{1}{(i)!} \frac{d^i}{d\lambda^i} \left[N\left(\sum_{x=0}^{\infty} \Phi^x(x, \tau)\right)\right]_{\lambda=0}, \quad i = 0, 1, 2, \ldots. \]  
(67)

The following are a few components of Adomian polynomials calculated from (64):

\[ A_0 = \Phi_0^1(x, \tau), \]

\[ A_1 = 2\Phi_0(x, \tau)\Phi_1(x, \tau), \]

\[ A_2 = 2\Phi_0(x, \tau)\Phi_2(x, \tau) + \Phi_1^2(x, \tau). \]  
(68)

If we put the Adomian polynomials into (63), and use the iterative scheme of CSDM, we get as follows:

\[ \phi_0(x, \tau) = \beta e^{2^{1/\alpha}}, \]

\[ \phi_1(x, \tau) = \frac{3}{2}\beta^2 e^{2^{1/\alpha}}\left(1 - e^{2^{1/\alpha}}\right), \]

\[ \phi_2(x, \tau) = \left(\frac{3}{2}\right)^2 \beta^3 e^{2^{1/\alpha}}\left(1 - e^{2^{1/\alpha}}\right)^2, \]  
(69)

\[ \phi_3(x, \tau) = \left(\frac{3}{2}\right)^3 \beta^4 e^{2^{1/\alpha}}\left(1 - e^{2^{1/\alpha}}\right)^3, \]

\[ \phi_4(x, \tau) = \left(\frac{3}{2}\right)^4 \beta^5 e^{2^{1/\alpha}}\left(1 - e^{2^{1/\alpha}}\right)^4, \]

\[ \phi_5(x, \tau) = \left(\frac{3}{2}\right)^5 \beta^6 e^{2^{1/\alpha}}\left(1 - e^{2^{1/\alpha}}\right)^5. \]

Hence, the solution to (57) is given by
\( \Phi(x, \tau) \)

\[ \begin{array}{cccc}
0.2 & 0.000000000000000022730420 & 0.000000000000000022730420 & 0.000000000000000022730420 \\
0.4 & 0.000000000000000022730420 & 0.000000000000000022730420 & 0.000000000000000022730420 \\
0.6 & 0.000000000000000022730420 & 0.000000000000000022730420 & 0.000000000000000022730420 \\
0.8 & 0.000000000000000022730420 & 0.000000000000000022730420 & 0.000000000000000022730420 \\
1.0 & 0.000000000000000022730420 & 0.000000000000000022730420 & 0.000000000000000022730420 \\
\end{array} \]

Table 3: The absolute and relative errors of Example 2.

| \( \tau \) | \( \Phi(x, \tau) \) | \( \Phi^{(5)}(x, \tau) \) | Rel. error = \( |\Phi - \Phi^{(5)}|/|\Phi| \) | Abs. error = \( |\Phi - \Phi^{(5)}| \) |
|---|---|---|---|---|
| 0.1 | 0.123507521107351640 | 0.12350752110724330 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.2 | 0.152943540287019501 | 0.152943540287196420 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.3 | 0.190022952110852800 | 0.190022951194339400 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.4 | 0.237081763050999310 | 0.237081750498903810 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.5 | 0.297377063025791400 | 0.297376943436063500 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.6 | 0.375581321236932700 | 0.375580405895244661 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.7 | 0.478636509006893034 | 0.478635682241948922 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.8 | 0.617316935559198600 | 0.617316693559198600 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 0.9 | 0.809298302499914500 | 0.809298302499914500 | 0.000000000000000022730420 | 0.000000000000000022730420 |
| 1.0 | 1.085752193622681500 | 1.085752193622681500 | 0.000000000000000022730420 | 0.000000000000000022730420 |

Table 4: The recurrence errors \( |\Phi^{(5)}(x, \tau) - \Phi^{(4)}(x, \tau)| \) of the fifth approximate solution with different values of \( \alpha \), when \( \phi = 0.10 \) for Example 2.

| \( \tau \) | \( \alpha = 0.7 \) | \( \alpha = 0.8 \) | \( \alpha = 0.9 \) | \( \alpha = 1.0 \) |
|---|---|---|---|---|
| 0.2 | 0.00000006990485380113 | 0.00000006280379060576 | 0.000000069719001178 | 0.000000069719001178 |
| 0.4 | 0.000007405633646659506 | 0.00000709378538814585 | 0.0000010936107976550 | 0.0000010936107976550 |
| 0.6 | 0.00023263737930772630 | 0.00023263737930772630 | 0.000345709827019152 | 0.000345709827019152 |
| 0.8 | 0.00439490288212737600 | 0.00439490288212737600 | 0.00070327418635774600 | 0.00070327418635774600 |
| 1.0 | 0.6478288929555560000 | 0.6478288929555560000 | 0.0108735483546798000 | 0.0108735483546798000 |
Figure 3: (a) The behavior of the 5th approximate and exact solutions. (b) The absolute errors of Example 3. (c) The absolute errors of Example 3.

Table 5: The absolute and relative errors of Example 3.

| τ  | Φ(α, τ) | Φ(5)(α, τ) | Rel.error = |Φ − Φ(5)|/|Φ| | Abs.error = |Φ − Φ(5)| |
|----|---------|-------------|-------------|-----------------|-----------------|
| 0.1| 0.06210134356430267 | 0.062101343563000811 | 0.0000000000,9633862 | 0.0000000001,30185321 |
| 0.2| 0.07744804965229965 | 0.077448049457207372 | 0.0000000002,5190082451 | 0.0000000001,9509227562 |
| 0.3| 0.09790256153577280 | 0.097902562004466070 | 0.0000000005,4950931746 | 0.0000000005,3353267220 |
| 0.4| 0.12254041882441845 | 0.12254034499261740 | 0.0000000006,0302386283 | 0.0000000007,3894796712 |
| 0.5| 0.1560206500563053 | 0.156019935318359921 | 0.0000000004,5807229918 | 0.0000000007,4687270614 |
| 0.6| 0.20097773432424765 | 0.200972155087929043 | 0.0000000002,7760469772 | 0.0000000005,5579236316 |
| 0.7| 0.26303089755360004 | 0.262992824983832251 | 0.00001447456178034378 | 0.0000380725697788611 |
| 0.8| 0.35201659822202610 | 0.351777534882588801 | 0.0000027760469772 | 0.0000026727636186 |
| 0.9| 0.48687243497975200 | 0.485435791111232004 | 0.00002950760330947573 | 0.0000143664386852 |
| 1.0| 0.70936646579762000 | 0.7007791300052878502 | 0.00121056096103100001 | 0.000857335744741464005 |

Table 6: The recurrence errors |Φ(5)(x, τ) − Φ(4)(x, τ)| of the fifth approximate solution with different values of α, when β = 0.05 for Example 3.

| τ  | α = 0.7 | α = 0.8 | α = 0.9 | α = 1.0 |
|----|---------|---------|---------|---------|
| 0.2| 0.0000022887706424562 | 0.00000003026830825991 | 0.0000000005,9384566260 | 0.0000000000,30185321 |
| 0.4| 0.0000262764038956162 | 0.000038324174973465 | 0.0000000007,3004730996 | 0.0000000001,9509227562 |
| 0.6| 0.00088329509495590930 | 0.0013126169994320686 | 0.000264837243261806 | 0.0000000005,9384566260 |
| 0.8| 0.0172722875681390300 | 0.00267024619055417560 | 0.000567284587143230 | 0.0000000000,30185321 |
| 1.0| 0.01727992875681390300 | 0.00268024619055417560 | 0.00093351565483440000 | 0.0000000000,30185321 |
\[ \Phi(x, \tau) = \Phi_0(x, \tau) + \Phi_1(x, \tau) + \Phi_2(x, \tau) + \ldots. \]

\[ \Phi(x, \tau) = \beta e^{2r \tau / \alpha} + \frac{3}{2} \beta^2 e^{2r \tau / \alpha} \left( 1 - e^{2r \tau / \alpha} \right) + \left( \frac{3}{2} \right)^2 \beta^3 e^{2r \tau / \alpha} \left( 1 - e^{2r \tau / \alpha} \right)^2 + \left( \frac{3}{2} \right)^3 \beta^4 e^{2r \tau / \alpha} \left( 1 - e^{2r \tau / \alpha} \right)^3 \frac{2^{2r \tau}}{3 \beta \left( 1 - e^{2r \tau / \alpha} \right)} \]  

(70)

\[ \Phi(x, \tau) = \beta e^{2r \tau / \alpha} \left( 1 + \left( \frac{3}{2} \right)^2 \beta \left( 1 - e^{2r \tau / \alpha} \right) \right) + \left( \frac{3}{2} \right)^3 \beta^3 e^{2r \tau / \alpha} \left( 1 - e^{2r \tau / \alpha} \right)^3 + \left( \frac{3}{2} \right)^4 \beta^4 e^{2r \tau / \alpha} \left( 1 - e^{2r \tau / \alpha} \right)^5 + \ldots. \]  

(71)

(68) can be represented by the closed-form as follows:

\[ \Phi(x, \tau) = \beta e^{2r \tau / \alpha} \left( \frac{1}{1 - \left( 3/2 \beta \left( 1 - e^{2r \tau / \alpha} \right) \right)} \right). \]  

(72)

\[ \Phi(x, \tau) = \frac{2 \beta e^{2r \tau / \alpha}}{2 - 3 \beta (1 - e^{2r \tau / \alpha})}. \]  

(73)

For, \( \alpha = 1 \), we get a closed-form solution in the following form of (57):

The authors declare that they have no conflicts of interest.

**Results**

The linear and nonlinear conformable PDEs of the presumed kind.

Furthermore, to evaluate the effectiveness and consistency of the proposed method for conformable PDEs, absolute, recurrence, and relative errors of three complications are considered graphically and numerically. The implications demonstrate that the CSDM is more effective and accurate with fewer calculations than existing schemes. Subsequently, we further demonstrate that this procedure can be applied to solve more linear and nonlinear DEs.

**Data Availability**

No data were used to support this study.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**References**

[1] S. Hasan, A. Al-Zoubi, A. Freihet, M. Al-Smad, and S. Momani, “Solution of fractional SIR epidemic model using residual power series method,” *Applied Mathematics & Information Sciences*, vol. 13, no. 2, pp. 153–161, 2019.

[2] M. Li and J. Wang, “Representation of solution of a Riemann-Liouville fractional differential equation with pure delay,” *Applied Mathematics Letters*, vol. 85, pp. 118–124, 2018.

[3] D. Baleanu, G. C. Wu, and S. D. Zeng, “Chaos analysis and asymptotic stability of generalized Caputo fractional differential equations,” *Chaos, Solitons & Fractals*, vol. 102, pp. 99–105, 2017.

[4] M. Houas and M. Bezzou, “Existence and stability results for fractional differential equations with two Caputo fractional derivatives,” *Facta Universitatis – Series: Mathematics and Informatics*, vol. 34, no. 2, pp. 341–357, 2019.

[5] N. Heymans and I. Podlubny, “Physical interpretation of initial conditions for fractional differential equations with Riemann-Liouville fractional derivatives,” *Rheologica Acta*, vol. 45, no. 5, pp. 765–771, 2006.

[6] A. Atangana, *Derivative with a New Parameter: Theory, Methods, and Applications*, Academic Press, Massachusetts, MA, USA, 2015.

[7] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh, “A new definition of fractional derivative,” *Journal of Computational and Applied Mathematics*, vol. 264, pp. 65–70, 2014.
[8] Z. Al-Zhour, N. Al-Mutairi, F. Alrawajeh, and R. Alkhasawneh, "Series solutions for the Lagueurre and Lane-Emden fractional differential equations in the sense of conformable fractional derivative," *Alexandria Engineering Journal*, vol. 58, no. 4, pp. 1413–1420, 2019.

[9] O. S. Iyiola and E. R. Nwaeze, "Some new results on the new conformable fractional calculus with application using D’alambert approach," *Progress in Fractional Differential and Applications*, vol. 2, no. 2, pp. 115–122, 2016.

[10] M. Eslami, F. S. Khodadad, F. Nazari, and H. Rezazadeh, "The first integral method applied to the Bogoyavlenskii equations by means of conformable fractional derivative," *Optical and Quantum Electronics*, vol. 49, no. 12, pp. 1–18, 2017.

[11] E. Ünal and A. Gökdoğan, "Solution of conformable fractional ordinary differential equations via differential transform method," *Optik*, vol. 128, pp. 264–273, 2017.

[12] W. Z. Wu, L. Zeng, C. Liu, W. Xie, and M. Goh, "A time power-based grey model with conformable fractional derivative and its applications," *Chaos, Solitons & Fractals*, vol. 155, Article ID 111657, 2021.

[13] V. Namias, "The fractional order Fourier transform and its application to quantum mechanics," *IMA Journal of Applied Mathematics*, vol. 25, no. 3, pp. 241–265, 1980.

[14] F. Haq, K. Shah, A. Khan, and M. Shahzad, "Numerical solution of fractional order epidemic model of a vector born disease by Laplace Adomian decomposition method," *Punjab University Journal of Mathematics*, vol. 49, no. 2, 2020.

[15] F. Haq, K. Shah, G. ur Rahman, and M. Shahzad, "Numerical solution of fractional order smoking model via Laplace Adomian decomposition method," *Alexandria Engineering Journal*, vol. 57, no. 2, pp. 1061–1069, 2018.

[16] S. J. Johnston, H. Jafari, S. P. Moshokoa, V. M. Ariyan, and D. Baleanu, "Laplace homotopy perturbation method for Burgers equation with space- and time-fractional order," *Open Physics*, vol. 14, no. 1, pp. 247–252, 2016.

[17] J. Manafian and M. Lakestani, "A new analytical approach to solve some of the fractional-order partial differential equations," *Indian Journal of Physics*, vol. 91, no. 3, pp. 243–258, 2017.

[18] A. Neamty, B. Agheli, and R. Darzi, "Applications of homotopy perturbation method and Elzaki transform for solving nonlinear partial differential equations of fractional order," *J. Nonlin. Evolut. Equat. Appl.*, vol. 2015, no. 6, pp. 91–104, 2016.

[19] Q. D. Katatbeh and F. B. M. Belgacem, "Applications of the Sumudu transform to fractional differential equations," *Nonlinear Studies*, vol. 18, no. 1, pp. 99–112, 2011.

[20] X. Dong, W. Kong, and T. Cui, “Fault classification and faulted-phase selection based on the initial current traveling wave,” IEEE Transactions on Power Delivery, vol. 24, no. 2, pp. 552–559, 2008.

[21] R. I. Nuruddeen, L. Muhammad, A. M. Nass, and T. A. Sulaiman, "A review of the integral transforms-based decomposition methods and their applications in solving nonlinear PDEs," *Palestine Journal of Mathematics*, vol. 7, no. 1, pp. 262–280, 2018.

[22] S. Maitama and W. Zhao, "New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations," 2019, https://arxiv.org/abs/1904.11370.

[23] L. Akinbode and O. S. Iyiola, "Exact and approximate solutions of time-fractional models arising from physics via Shehu transform," *Mathematical Methods in the Applied Sciences*, vol. 43, no. 12, pp. 7442–7464, 2020.

[24] A. Khalouta and A. Kadem, "A comparative study of Shehu variational iteration method and Shehu decomposition method for solving nonlinear Caputo time-fractional wave-like equations with variable coefficients," *Applications and Applied Mathematics: International Journal*, vol. 15, no. 1, p. 24, 2020.

[25] S. Maitama and W. Zhao, "Homotopy perturbation Shehu transform method for solving fractional models arising in applied sciences," *Journal of Applied Mathematics and Computational Mechanics*, vol. 20, no. 1, pp. 71–82, 2021.

[26] N. A. Shah, I. Dassios, E. R. El-Zahar, J. D. Chung, and S. Taherifar, "The variational iteration transform method for solving the time-fractional fornberg-whitham equation and comparison with decomposition transform method," *Mathematics*, vol. 9, no. 2, 2021.

[27] A. Khan, M. I. Uqail, M. Younis, and A. Alam, "Approximate and exact solutions to fractional order Cauchy reaction-diffusion equations by new combine techniques," *Journal of Mathematics*, vol. 2021, Article ID 5337255, 15 pages, 2021.

[28] K. Shah, H. Khalil, and R. A. Khan, "Analytical solutions of fractional order diffusion equations by natural transform method," *Iranian Journal of Science and Technology Transaction A-Science*, vol. 42, no. 3, pp. 1479–1490, 2018.

[29] M. R. M. Shabestari, R. Ezzati, and T. Allahviranloo, "Numerical solution of fuzzy fractional integro-differential equation via two-dimensional Legendre wavelet method," *Journal of Intelligent and Fuzzy Systems*, vol. 34, no. 4, pp. 2453–2465, 2018.

[30] M. Nadeem, F. Li, and H. Ahmad, "Modified Laplace variational iteration method for solving fourth-order parabolic partial differential equation with variable coefficients," *Computers & Mathematics with Applications*, vol. 78, no. 6, pp. 2052–2062, 2019.

[31] F. Ayaz, "On the two-dimensional differential transform method," *Applied Mathematics and Computation*, vol. 143, no. 2-3, pp. 361–374, 2003.

[32] J. Singh, D. Kumar, and D. Sushila, "Homotopy perturbation Sumudu transform method for nonlinear equations," *Advances in Theoretical and Applied Mathematics*, vol. 4, no. 4, pp. 165–175, 2011.

[33] M. Yi and J. Huang, "Wavelet operational matrix method for solving fractional differential equations with variable coefficients," *Applied Mathematics and Computation*, vol. 230, pp. 383–394, 2014.

[34] R. I. Nuruddeen and A. M. Nass, "Exact solutions of wave-type equations by the Aboodh decomposition method," *Stochastic Modelling and Applications*, vol. 21, no. 1, pp. 23–30, 2017.

[35] M. Alquran, K. Al-Khaled, and J. Chattopadhyay, "Analytical solutions of fractional population diffusion model: residual power series," *Nonlinear Studies*, vol. 22, no. 1, pp. 31–39, 2015.

[36] H. A. Hammad, P. Agarwal, S. Momani, and F. Alsharari, "Solving a fractional-order differential equation using rational symmetric contraction mappings," *Fractal and Fractional*, vol. 5, no. 4, 2021.

[37] S. Kumar, A. Kumar, S. Momani, M. Alshaiwallah, and K. S. Nisar, "Numerical solutions of nonlinear fractional model arising in the appearance of the strip patterns in two-dimensional systems," *Advances in Difference Equations*, vol. 413, no. 1, pp. 1–19, 2019.

[38] H. Gandhi, A. Tomar, and D. Singh, "The comparative study of time fractional linear and nonlinear newell-whitehead-
segel equation,” in Soft Computing: Theories and Applications, pp. 419–431, Springer, Singapore, 2022.
[39] M. E. Benattia and K. Belghaba, “Solution of newel-whitehead-segel equation using conformable fractional sumudu decomposition method,” Journal of Science and Arts, vol. 21, no. 2, pp. 479–486, 2021.
[40] A. Prakash, M. Goyal, and S. Gupta, “Fractional variational iteration method for solving time-fractional Newell-Whitehead-Segel equation,” Nonlinear Engineering, vol. 8, no. 1, pp. 164–171, 2019.
[41] M. Nadeem, J. H. He, and A. Islam, “The homotopy perturbation method for fractional differential equations: part 1 Mohan transform,” International Journal of Numerical Methods for Heat and Fluid Flow, vol. 31, 2021.
[42] M. Ali, M. Alquran, and I. Jaradat, “Asymptotic-sequentially solution style for the generalized Caputo time-fractional Newell–Whitehead–Segel system,” Advances in Difference Equations, vol. 2019, no. 1, pp. 1–9, 2019.
[43] S. A. Ahmed and M. Elbadri, “Solution of newell–whitehead–segel equation of fractional order by using sumudu decomposition method,” Order, vol. 8, 2020.
[44] R. Saadeh, M. Alaroud, M. Al-Smadi, R. Ahmad, and U. Salma Din, “Application of fractional residual power series algorithm to solve newell-whitehead-segel equation of fractional order,” Symmetry, vol. 11, no. 12, 2019.
[45] A. Prakash and V. Verma, “Numerical method for fractional model of Newell-Whitehead-Segel equation,” Frontiers in Physics, vol. 7, 2019.
[46] M. Saqib, I. Khan, and S. Shafie, “Application of fractional differential equations to heat transfer in hybrid nanofluid: modeling and solution via integral transforms,” Advances in Difference Equations, vol. 2019, no. 1, pp. 1–18, 2019.
[47] C. D. Constantinescu, J. M. Ramirez, and W. R. Zhu, “An application of fractional differential equations to risk theory,” Finance and Stochastics, vol. 23, no. 4, pp. 1001–1024, 2019.
[48] M. E. Benattia and K. Belghaba, “Shehu conformable fractional transform, theories and applications,” Cankaya University Journal of Science and Engineering, vol. 18, no. 1, pp. 24–32, 2021.