Abstract: We investigate the $Q^2$-evolution of the chiral-odd spin-dependent parton distribution $h_L(x,Q^2)$ relevant for the polarized Drell-Yan processes. The results are obtained in the leading logarithmic order in the framework of the renormalization group and the standard QCD perturbation theory. We calculate the anomalous dimension matrix for the twist-3 operators for $h_L$ in the one-loop order. The operator mixing among the relevant twist-3 operators including the operators proportional to the QCD equations of motion is treated properly in a consistent scheme. Implications for future experiments are also discussed.
1 Introduction

The EMC measurement of the $g_1$-structure function of the nucleon \[1\] has given rise to renewed interest in the spin effects in high energy processes. In particular, many authors have been attracted by the chiral-odd spin-dependent distribution functions $h_{1,L} \[2, 3, 4, 5\]$. Because of the chirality, they can not be measured in the totally inclusive deep inelastic lepton-nucleon scattering (except as a quark-mass effect), but reveal themselves in polarized hadron-hadron collisions (Direct photon production, Drell-Yan process, etc.) and semi-inclusive polarized electron scatterings. Thus they are expected to open up a new window to explore hadron structure.

The twist-2 parton distribution $h_1$ was first addressed by Ralston and Soper more than ten years ago \[6\] and was recently named as the “transversity” distribution in ref. \[4\]. It has a simple parton model interpretation like $f_1$ and $g_1$ \[1\] and appears as a leading contribution, for example, in the Drell-Yan process with the transversely polarized nucleon-nucleon collision \[4\].

The other chiral-odd distribution $h_L$ is twist-3. Physically higher twist distributions represent complicated quark-gluon correlations in hadrons. It is difficult to isolate them by experiments because they are usually hidden by the leading twist-2 contributions. However, the twist-3 distribution $h_L$ is somewhat immune to this difficulty: $h_L$ reveals itself as a leading contribution to the longitudinal-transverse asymmetry in the Drell-Yan process \[4\], although it appears with a factor $1/\sqrt{Q^2}$. This is similar to the circumstance of $g_2$ (the transverse spin structure function) whose contribution to the transversely polarized DIS becomes leading order but with the suppression factor $1/\sqrt{Q^2}$ \[4\]. Thus, the uniqueness of $h_L$ is twofold: it is a “measurable” higher-twist distribution and corresponds to chirality violating process. It is expected to provide new information about the hadron structure and the QCD dynamics beyond the conventional structure function data.

In addition to more precise measurements of the familiar distribution functions $f_1$ and $g_1$, these “new” distribution functions $h_1$, $h_L$, and $g_T \[4\]$ will be measured in the future collider experiments, such as HERMES, SMC \[8\] and RHIC. In view of this, it is especially important to develop theoretical study of these distribution functions as much as possible based on QCD. Among these efforts, the first step is the perturbative QCD prediction on the $Q^2$-evolution of the distribution functions: Owing to the factorization property of hard processes, the $Q^2$-evolution of the distribution functions can be predicted unambiguously in the framework of the renormalization group and the QCD perturbation theory. Its prediction is indispensable to extract physical information from the experimental data in the high-energy scale by comparing them with the prediction of hadron models at the low-energy scale. Furthermore, the comparison of the $Q^2$-evolution itself between theory and experiment will provide a deeper test of QCD beyond the conventional twist-2 level. The $Q^2$-evolution of the twist-2 distribution functions has been fully discussed since the first application of QCD to hard processes; for example, the $Q^2$-evolution of $h_1$ was studied in \[2\] by employing the Altarelli-Parisi equation \[3\]. As for the higher twist ones, there has been several works on the $Q^2$-evolution of $g_T$ by generalizing the Altarelli-Parisi equation to the higher twist

\[3\]We denote the twist-2 distributions corresponding to the structure functions $F_1$ ($F_2$) and $g_1$ by $f_1$ and $g_1$ following ref. \[4\].

\[4\]$g_T$ denotes the twist-3 distribution corresponding to the structure function $g_2$. 
distribution \cite{10,11} and by the anomalous dimension calculation \cite{12,13,14,15}. But there has been no discussion on the $Q^2$-evolution of another important twist-3 distribution $h_L$.

In this paper we study the $Q^2$-evolution of $h_L$. We shall calculate the anomalous dimension matrix for the twist-3 distribution $h_L$ based on the standard QCD perturbation theory. In general, the moments of a twist-3 distribution can be written in terms of the matrix elements of a set of twist-3 operators involving explicitly the gluon field strength tensor, and the mixing among them occurs through renormalization, as was emphasized in ref. \cite{12,14} in the context of the $g_2$-structure function. However, the operator mixing occurs not only among these twist-3 operators but also with the other twist-3 operators which vanish by the naive use of the QCD equation of motion, $(i\not{D} - m_q)\psi = 0$, (referred to as the “equation-of-motion (EOM) operators” from now on). This is due to the fact that the naive equations of motion and thus the vanishing of these operators are not correct as an operator statement because of quantum effects and renormalization. The use of the equations of motion is allowed only when their matrix elements are taken with respect to a physical state \cite{16,17}. On the other hand, the renormalization of composite operators has to be carried out in terms of general Green functions which imbed these composite operators. Therefore the mixing involving the EOM operators is essential to perform renormalization of the higher twist operators consistently, which was recently pointed out by Kodaira, Yasui, and Uematsu \cite{15} in the context of $g_2$. We shall pay particular attention to this mixing and we will find that it certainly plays a role also for the present case of $h_L$.

The outline of this paper is as follows: In section 2, we shall first introduce the chiral-odd distributions $h_1$ and $h_L$, following the procedure of ref. \cite{4}. We include this part to identify the manifestly interaction-dependent operators (“canonical basis” \cite{14,21}) as well as the EOM operators relevant for the twist-3 part of $h_L$. Readers familiar with ref. \cite{4} can skip this part by just noting the existence of the first term of eq.(2.10). Next we present the general procedure for the renormalization of the twist-3 operators. In section 3, we present the actual calculation of the anomalous dimension matrix for the twist-3 operators by employing standard QCD perturbation theory. The calculation is performed with the Feynman gauge. The loop integration is dimensionally regularized, and the minimal subtraction (MS) scheme is adopted. The details of the calculation will be discussed in the Appendices to make the discussion transparent. In section 4, we will discuss experimental implication of our result.

2 Basic formulation

2.1 Twist-three operators for $h_L$

In this section we shall briefly summarize general aspects of the chiral-odd spin-dependent distributions $h_1$ and $h_L$ relevant for our analysis. For the detail, we refer the readers to ref. \cite{4}. The QCD factorization theorem tells us that a cross section for an inclusive hard process can be decomposed into the perturbatively calculable hard cross section and the parton distribution function \cite{18,19}. The latter is known to be written as the light-cone Fourier transform of the quark (or gluon) correlation function in a hadron. The chiral-odd parton distribution functions (renormalized at the scale $\mu$) in our interest are defined as
follows:
\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\sigma_{\mu\nu}i\gamma_5\psi(\lambda n)|e\rangle = 2 \left[ h_1(x, \mu^2)(S_{\perp\mu}p_{\nu} - S_{\perp\nu}p_{\mu})/M + h_L(x, \mu^2)M(p_{\mu}n_{\nu} - p_{\nu}n_{\mu})(S \cdot n) + h_3(x, \mu^2)M(S_{\perp\mu}n_{\nu} - S_{\perp\nu}n_{\mu}) \right],
\]
(2.1)
where \(|PS\) is the nucleon (mass \(M\)) state with its momentum \(P\) and spin \(S\) \((P^2 = M^2, S^2 = -M^2, P\cdot S = 0\). We introduced the null vectors \(p\) and \(n\) by the relation \(P_{\mu} = p_{\mu} + \frac{M}{2}n_{\mu}, p^2 = n^2 = 0, p \cdot n = 1, n^+ = p^- = 0\), which specify the Lorentz frame of the system. The light-cone gauge \(n \cdot A = 0\) was employed in (2.1). \(h_1\) and \(h_L\) are directly accessible by measuring the proper asymmetries in the polarized Drell-Yan process \(D\). \((h_3\) is twist-4 and is irrelevant for the following discussion.) Taylor expanding the bilocal operator appearing in the correlation function, l.h.s. of (2.1), one can derive the relation between the moment of these parton distribution functions \(h_{1,L}\) and the local operator,
\[
\theta^{\mu\nu\mu_1\cdots\mu_n} = S_n\bar{\psi}i\gamma_5\sigma^{\mu\nu}iD^{\mu_1}\cdots iD^{\mu_n}\psi,
\]
(2.2)
where the covariant derivative \(D_\mu = \partial_\mu - igA_\mu\) restores the gauge invariance and the symbol \(S_n\) symmetrizes the indices \(\mu_1, \ldots, \mu_n\). Here and below, we often suppress the explicit dependence on the renormalization scale of the local operators and the parton distribution functions. For the study of the twist-2 and -3 distributions \(h_1\) and \(h_L\), it suffices to consider the piece of \(\theta^{\mu\nu\mu_1\cdots\mu_n}\) which is further symmetrized among \(\nu, \mu_1, \ldots, \mu_n\). We thus introduce an arbitrary light-like vector \(\Delta_n\) \((\Delta^2 = 0\) and consider
\[
\theta^{\mu}_n \cdot \Delta = \theta^{\mu\nu\mu_1\cdots\mu_n}_n \Delta_\nu \Delta_{\mu_1} \cdots \Delta_{\mu_n},
\]
(2.3)
\(\theta^{\mu}_n \cdot \Delta\) can be decomposed into the traceless part \(\bar{\theta}\) and the remainder \(T\),
\[
\theta^{\mu}_n \cdot \Delta = \bar{\theta}^{\mu}_n \cdot \Delta + T^{\mu}_n \cdot \Delta
\]
(2.4)
by the condition that
\[
\bar{\theta}^{\mu\nu\mu_1\cdots\mu_n} = g_{\mu\nu} g^{\mu\nu\mu_1\cdots\mu_n} = g_{\nu,\mu_1} g^{\nu,\mu_1\cdots\mu_n} = 0.
\]
(2.5)
\(\bar{\theta}^{\mu\nu\mu_1\cdots\mu_n}\) contains all the twist-2 effect in \(\theta^{\mu\nu\mu_1\cdots\mu_n}\) and it is related to the moments of \(h_1(x, \mu)\) as
\[
\mathcal{M}_n[h_1(\mu)] = a_n(\mu),
\]
(2.6)
\[
\langle PS|\bar{\theta}^{\mu}_n \cdot \Delta(\mu)|PS\rangle = \frac{2a_n(\mu)}{M} \left( S^{\mu} \hat{P}^{n+1} - P^{n} \hat{S} \hat{P}^{n} + \frac{n}{n+2}M^2 \Delta^{\mu} \hat{S} \hat{P}^{n-1} \right),
\]
(2.7)
where we introduced the shorthand notation \(\mathcal{M}_n[h(\mu)] \equiv \int dx x^n h(x, \mu)\) and \(\hat{k} \equiv k \cdot \Delta\) for an arbitrary four vector \(k_\mu\). Also, we obtain for the moments of \(h_L\):
\[
\mathcal{M}_n[h_L] = \frac{2}{n+2} \mathcal{M}_n[h_1] + \mathcal{M}_n[\tilde{h}_L].
\]
(2.8)
The first term shows that \( h_L \) receives a contribution from the twist-2 distribution \( h_1 \). This piece is an analogue of the Wandzura-Wilczek contribution \[20\] for \( g_2 \). \( \mathcal{M}_n[h_L] \) of (2.8) are directly related to the matrix elements of the twist-3 operator \( T^\mu_n \cdot \Delta \) of (2.4). Explicit calculation gives

\[
T^\mu_n \cdot \Delta = \frac{\Delta^\mu}{n+2} \sum_{j=0}^{n-1} \bar{\psi} i \gamma_5 \sigma^{\mu\nu} \Delta_\nu d^j i D^\mu d^{n-j-1} \psi,
\]

(2.9)

where \( d \equiv i D \cdot \Delta \). \( T^\mu_n \cdot \Delta \) can be recast into the following form by using the relation \([D_\mu, D_\nu] = -ig G_{\mu\nu}\) with \( G_{\mu\nu} \) the gluon field strength tensor

\[
T^\mu_n \cdot \Delta = \frac{n}{n+2} \Delta^\mu E_n \cdot \Delta + \frac{n}{n+2} \Delta^\mu N_n \cdot \Delta - \Delta^\mu \left( \sum_{l=2}^{\frac{n+1}{2}} \left( 1 - \frac{2l}{n+2} \right) \right) R_{n,l} \cdot \Delta,
\]

(2.10)

where \( O \cdot \Delta = \mathcal{O}^{\mu_1 \cdots \mu_n} \Delta_{\mu_1} \cdots \Delta_{\mu_n} \). Here the first term (the “EOM operator”) is defined as

\[
E^{\mu_1 \cdots \mu_n}_n = \frac{1}{2} S_n \left[ \bar{\psi} (i D - m_q) \gamma_5 \gamma^{\mu_1} i D^{\mu_2} \cdots i D^{\mu_n} \psi + \bar{\psi} i \gamma_5 \gamma^{\mu_1} i D^{\mu_2} \cdots i D^{\mu_n} (i D - m_q) \psi \right] - \text{traces}.
\]

(2.11)

This operator vanishes by the naive use of the QCD equation of motion \((i D - m_q) \psi = 0\). We can set it to zero when we take its matrix element with respect to a physical state (such as the nucleon state) \[10, 17\], which is why it is discarded in ref.\[4\]. However, this is not an operator identity and the mixing between \( E_n \) and the other twist-3 operators defined below should be taken into account during the course of renormalization. The second term is given by

\[
N^{\mu_1 \cdots \mu_n}_n = S_n m_q \bar{\psi} \gamma_5 \gamma^{\mu_1} i D^{\mu_2} \cdots i D^{\mu_n} \psi - \text{traces}.
\]

(2.12)

\( R^{\mu_1 \cdots \mu_n}_{n,l} \) in the third term of (2.10) is defined as

\[
R^{\mu_1 \cdots \mu_n}_{n,l} = \theta_{n-l+2}^{\mu_1 \cdots \mu_n} - \theta_l^{\mu_1 \cdots \mu_n}, \quad \left( l = 2, \ldots, \left[ \frac{n+1}{2} \right] \right)
\]

(2.13)

\[
\theta_l^{\mu_1 \cdots \mu_n} = \frac{1}{2} S_n \bar{\psi} \gamma^{\alpha \mu_1} i \gamma_5 i D^{\mu_2} \cdots i D^{\mu_n} \psi - \text{traces},
\]

(2.14)

which explicitly involves the gluon field strength tensor, suggesting that the twist-3 operators truly represents the effect of quark-gluon correlations. By the combination of \( \theta_l^{\mu_1 \cdots \mu_n} \) in the form of the r.h.s. of (2.13), \( R^{\mu_1 \cdots \mu_n}_{n,l} \) can have definite charge conjugation property.

With these definitions and the relation (2.10), the \( n \)-th moment of the genuine twist-3 piece of \( h_L \) can be written down in terms of the nucleon matrix elements of the twist-3 operators (see (2.8))

\[
\mathcal{M}_n[h_L] = \frac{n}{n+2} \frac{m_q}{M} \mathcal{M}_{n-1}[g_1] + \mathcal{M}_n[h_L^3].
\]

(2.15)

The first term is due to the contribution of the second term of eq.(2.11), and thus shows the quark mass effect; it is in fact the quark mass times the twist-2 operator corresponding to the \( g_1 \)-distribution:

\[
\langle PS|S_n \bar{\psi} \gamma^{\mu_1} \gamma_5 i D^{\mu_2} \cdots i D^{\mu_n} \psi|PS \rangle = 2 \mathcal{M}_{n-1}[g_1(\mu)] \mathcal{S}_n(S^{\mu_1} P^{\mu_2} \cdots P^{\mu_n} - \text{traces}).
\]

(2.16)
The second term designates the contribution from $R_{n,l}^{\mu_1 \cdots \mu_n}$:

$$
\mathcal{M}_n[h_L^3] = \sum_{l=2}^{[n+1]/2} \left( 1 - \frac{2l}{n+2} \right) b_{n,l}(\mu^2)
$$

with

$$
\langle PS|R_{n,l}^{\mu_1 \cdots \mu_n}(\mu^2)|PS \rangle = 2b_{n,l}(\mu^2)MS_n(S^{\mu_1}P^{\mu_2} \cdots P^{\mu_n} - \text{traces}).
$$

By inverting the moments, (2.8) and (2.15) give the relation between the structure functions themselves:

$$
h_L(x, \mu^2) = 2x \int_x^1 \frac{h_1(y, \mu^2)}{y^2} dy + \frac{m_q}{M} \left[ \frac{g_1(x, \mu^2)}{x} - 2x \int_x^1 \frac{g_1(y, \mu^2)}{y^3} dy \right] + h_L^3(x, \mu^2),
$$

for $x > 0$.

### 2.2 Renormalization of the twist-three operators

We now proceed to discuss the renormalization of the twist-3 operators. As discussed in section 2.1, $[(n+1)/2] + 1$ twist-3 operators, $R_{n,l} \cdot \Delta$ ($l = 2, 3, \ldots, \left[ \frac{n+1}{2} \right]$) $N_n \cdot \Delta$ and $E_n \cdot \Delta$ formally participate in the $n$-th moment of $\tilde{h}_L(x, Q^2)$. It has been known [21] that as a complete basis of higher twist operators one can always choose “canonical” ones which (1) are traceless and symmetric with respect to all the Lorentz indices and (2) have no contracted derivatives. Any noncanonical operators which could appear through radiative corrections can be transformed into the canonical ones modulo EOM operators by use of the relation $[D_\mu, D_\nu] = -igG_{\mu\nu}$, and the physical matrix elements of the EOM operators vanish. These canonical operators mix with each other and also with the EOM operators under renormalization. Therefore we can choose $R_{n,l} \cdot \Delta$, $N_n \cdot \Delta$, and $E_n \cdot \Delta$ as a basis for renormalization. We further recall that the renormalization of composite operators generally involves the mixing with gauge noninvariant EOM operators which do not exist in the original basis [21]. We will come back to this point in the next section.

The scale dependence of the physical matrix elements, e.g., $b_{n,l}(\mu)$ of (2.18) are determined by the anomalous dimensions of the corresponding composite operators. To see this, we write down the renormalization group equation for these operators. The bare- ($\mathcal{O}_i^B$) and the renormalized- ($\mathcal{O}_i$) composite operators are related by the renormalization constant matrix $Z_{ij}$:

$$
\mathcal{O}_i(\mu) = Z_{ij}^{-1} (\mu) \mathcal{O}_j^B,
$$

where $\mathcal{O}_i$ symbolically refer to $R_{n,l}^{\mu_1 \cdots \mu_n}$, $E_n^{\mu_1 \cdots \mu_n}$, and $N_n^{\mu_1 \cdots \mu_n}$. The renormalization group equation for $\mathcal{O}_i(\mu)$ is obtained by using the fact that the unrenormalized operators do not depend on the renormalization scale:

$$
\mu \frac{d \mathcal{O}_i(\mu)}{d\mu} + \gamma_{ij} (g(\mu)) \mathcal{O}_j(\mu) = 0,
$$

5$h_L(x)$ for $x < 0$ should be related to the “antiquark distribution” $\bar{h}_L(x)$ as $\bar{h}_L(x) = -h_L(-x)$ by charge conjugation.
where the anomalous dimension matrix $\gamma_{ij}$ for $\{R, E, N\}$ is defined as

$$\gamma_{ij}(\mu) = \mu \frac{dZ_{ij}(\mu)}{d\mu} Z_{ik}^{-1}(\mu).$$  \tag{2.22}$$

In the leading logarithmic approximation, this equation is solved to give

$$O_i(Q^2) = \sum_j \left[ \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)_{\gamma_0} \right]_{ij} O_j(\mu^2),$$  \tag{2.23}$$

where $\alpha(\mu^2)$ is the QCD running coupling constant, $\beta_0 = (11 - (2/3)N_f)/(4\pi)^2$ and $\gamma_0$ are the lowest order (1-loop) coefficient of the $\beta$-function and the anomalous dimension matrix

$$\gamma_{ij}(g(\mu)) = \gamma_{ij}^0 g(\mu)^2 + O(g(\mu)^4).$$  \tag{2.24}$$

In order to compute the $Z_{ij}$-factor of (2.20), one imbeds the composite operators $O_i$ into an appropriate Green functions with a convenient kinematics, and computes radiative corrections to this Green function. For a familiar case of the twist-2 operators, for example, the two-point functions with the on-shell external lines are usually considered. In the present case, however, the EOM operator $E_\mu^{n_1\cdots n_n}$ should be retained as a nonzero quantity and the mixing of them with the other operators should be consistently taken into account: This means that it is necessary to compute Green functions with the off-shell kinematics for the external lines.

Thus, in order to compute renormalization of $O_i = R_{n_1\cdots n_n}$, for example, we may consider the truncated three-point Green function $F_i(p, q, q - p)$ defined by

$$F_i(p, q, k)(2\pi)^4 \delta^4(p + k - q)G(p)G(q)D(k)$$

$$= \int d^4x d^4y d^4z \epsilon^{ipx} \epsilon^{-iqy} \epsilon^{ikz} \langle T \{O_i \psi(x) \bar{\psi}(y) A_\mu(z) \} \rangle,$$  \tag{2.25}$$

where $G$ and $D$ are the quark and gluon propagators, respectively. (We suppressed the Lorentz and the spinor indices for simplicity.) We consider the three-point Green function for the off-shell quark and gluon external lines (not a physical state), and therefore $F_i$ with $O_i = E_n$ do not vanish. In the next section, we present the one-loop calculation of this function to get $Z_{ij}$.

### 3 Anomalous Dimension Matrix for Twist-3 Operators

In this section we present the computation of the anomalous dimension matrix for the twist-3 operators for $h_L$. The calculation is performed up to the one-loop order. The loop integrals are dimensionally regularized and the MS scheme is employed for renormalization. Thus we keep only the simple dimensional pole proportional to $1/\varepsilon$ in the one-loop amplitudes ($\varepsilon = (4 - d)/2$ with $d$ the space-time dimension). We use the Feynman gauge for the gluon propagator, but the results should be independent of the gauge.

As we discussed in the last section, we imbed the relevant twist-three composite operators into the three-point function $F_i(p, q, k)$ of (2.25), assuming the off-shell kinematics. In
this case, the basic ingredients are the tree level vertices for the operators $R_{n,t}$, $E_n$, $N_n$
corresponding to the diagrams shown in Fig. 1, which we call the “basic vertices”.

The three-point basic vertex of $R_{n,t} \cdot \Delta$ shown in Fig. 1 (a) becomes

$$\mathcal{R}_{n,t,\mu}^{(3)} = \frac{g}{2} \sigma^\alpha \Delta \lambda \gamma_5 q^{n-l-2} (\hat{p}^{i-2} \hat{q}^{n-l}) (-\hat{k} g_{\alpha \mu} + k_{\alpha} \Delta_{\mu}) t^a,$$  \hspace{1cm} (3.1)

while those for $E_n \cdot \Delta$, $N_n \cdot \Delta$ are given by

$$\mathcal{E}_{n,\mu}^{(3)} = \frac{g}{2} \left[ \gamma_\mu \gamma_5 \Delta q^{n-1} + \gamma_5 \Delta \tilde{p}^{n-1} \gamma_\mu + \Delta_{\mu} \sum_{j=2}^{n} (q - m_q) \gamma_5 \Delta \tilde{p}^{j-2} \hat{q}^{n-j} \right] t^a,$$ \hspace{1cm} (3.2)

$$\mathcal{N}_{n,\mu}^{(3)} = m_q g \Delta_{\mu} \sum_{j=2}^{n} \tilde{p}^{j-2} \hat{q}^{n-j} t^a,$$ \hspace{1cm} (3.3)

where $k = q - p$ and $t^a$ ($a = 1, ..., N_c^2 - 1$) is the color matrix normalized as $\text{Tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$.

We first imbue $R_{n,t} \cdot \Delta$ into the three-point function $F_i$ of (2.23). The Feynman diagrams
which give the one-loop radiative corrections to the operator $R_{n,t} \cdot \Delta$ are the one-particle-
irreducible diagrams shown in Fig. 2. In order to write down those amplitudes, the vertex
for $R_{n,t} \cdot \Delta$ corresponding to Fig. 1(b) is necessary in addition to the usual Feynman rules:

$$\begin{align*}
\frac{g^2}{2} &\left[ i f^{abc} t^c \sigma^\alpha \Delta \lambda \gamma_5 \Delta_{\mu} g_{\alpha \nu} \hat{p}^{n-l-2} q^{l-2} \\
&+ \sum_{j=2}^{n-l+1} \sigma^\alpha \Delta \lambda \gamma_5 t^a t^b \tilde{p}^{j-2} \Delta_{\mu} (\hat{p} + \hat{k})^{n-l+1-j} (-\hat{k} g_{\alpha \nu} + k_{\alpha} \Delta_{\nu}) \hat{q}^{l-2} \\
&+ \sum_{j=n-l+3}^{n} \sigma^\alpha \Delta \lambda \gamma_5 t^a t^b \hat{p}^{n-l} \Delta_{\nu} (\hat{p} + \hat{k})^{j-n+l-3} (-\hat{k} g_{\alpha \mu} + k_{\alpha} \Delta_{\mu}) \hat{q}^{n-j} \\
&+ (\mu \leftrightarrow \nu, k \leftrightarrow k', a \leftrightarrow b) \right] - (l \rightarrow n - l + 2) \tag{3.4}
\end{align*}$$

with $k' = q - p - k$. We have to add the counter term contribution (see (2.20))

$$\left( Z_{l't'}^{-1} Z_2 \sqrt{Z_3 Z_g} - \delta_{l't'} \right) \mathcal{R}_{n,t',\mu}^{(3)} + Z_{lE}^{-1} Z_2 \sqrt{Z_3 Z_g} \left\{ \mathcal{E}_{n,\mu}^{(3)} + (Z_m - 1) \mathcal{N}_{n,\mu}^{(3)} \right\} + Z_{lN}^{-1} Z_2 \sqrt{Z_3 Z_g Z_m} \mathcal{N}_{n,\mu}^{(3)} \tag{3.5}
$$

to the sum of all the one-loop amplitudes of Fig. 2, and require that the total results be finite
as $\varepsilon \rightarrow 0$. In (3.3), $Z_2$ and $Z_3$ are the usual wave function renormalization constants for the
quark and gluon fields, while $Z_g$ and $Z_m$ are the coupling constant and mass renormalization
constants defined by

$$g = \frac{1}{Z_g} \mu^{1-\varepsilon} g^B, \quad m_q = \frac{1}{Z_m} m_q^B, \tag{3.6}$$

where “B” denotes the unrenormalized quantities similarly to (2.20). Note that $\mathcal{R}_{n,t,\mu}^{(3)}$, $\mathcal{E}_{n,\mu}^{(3)}$
and $\mathcal{N}_{n,\mu}^{(3)}$ are proportional to the coupling constant and thus each term of (3.5) involves the
factor $Z_g$ as a coefficient. Similarly, the factor $Z_m$ appears because $N_{n,µ}^{(3)}$ contains the quark mass $m$. In the MS scheme, (the finite part of) the renormalization constants are chosen so that the counter term contributions (3.3) precisely cancel out the terms proportional to the $1/ε$ pole from the dimensionally regularized one-loop amplitudes.

For the case of the three-point functions imbedding $E_n·∆$ or $N_n·∆$, we can proceed in a similar manner by interchanging the roles of $R_n,l·∆$ with $E_n·∆$ or $N_n·∆$. These results, together with the well known results for $Z_{2,3,g,m}$ (in the Feynman gauge),

$$Z_2 = 1 - \frac{g^2}{(4π)^2ε}C_F; \quad Z_g \sqrt{Z_3} = 1 - \frac{g^2}{(4π)^2ε}C_G; \quad Z_m = 1 - 3\frac{g^2}{(4π)^2ε}C_F \quad (3.7)$$

completely determine the relevant $Z_{ij}$-factor ($i,j = 2,...,\lfloor \frac{n+1}{2}\rfloor, E, N$). ($C_F = \frac{N^2_c-1}{2N_c}$ and $C_G = N_c$ are the Casimir operators of the color gauge group $SU(N_c)$.)

The actual computation of the Feynman amplitudes of Fig. 2 is rather cumbersome due to the complicated structure of the vertices, e.g., (3.1)-(3.4). Moreover, the participation of the gauge-noninvariant EOM operators would further complicate the computation (see below). In this light it is convenient to employ the following procedure (1)-(3) to make the computation tractable:

1. We introduce a vector $Ω_µ$, which satisfies the condition $\hat{Ω} = ∆^µΩ_µ = 0$. The contraction of the implicit Lorentz index $µ$ of $F_i((2.25))$ with $Ω_µ$ kills off many terms and simplifies the computation enormously: For example, the basic vertices in (3.1) and (3.2) become

$$R_{n,l}^{(3)}·Ω = -\frac{g}{2Ω_ασ^{αλ}∆_λiγ_5(\hat{q} - \hat{p})\left(p^{n-i}q^{l-2} - p^{l-2}q^{n-i}\right)t^a \quad (3.8)$$

and

$$E_n^{(3)}·Ω = \frac{g}{2Ω_ασ^{αλ}∆_λiγ_5\left(\hat{p}^{n-1} + \hat{q}^{n-1}\right)t^a \quad (3.9)$$

This contraction brings another favorable effect: As was exemplified in ref.[14] in the context of $gr$, the gauge-noninvariant EOM operators generally mix through renormalization in addition to the gauge-invariant EOM operator $E_n$ [17]. Typically, those gauge-noninvariant operators are obtained by replacing some of the uncontracted covariant derivatives contained in $E_n^{α_1...α_n}$ by the simple derivatives. Therefore, for large $n$, a large number of different gauge-noninvariant operators are expected to come into play. Due to this phenomenon, it is extremely difficult to identify the tensor structure of the one-loop amplitudes by the basic vertices (3.1)-(3.3) and those for the gauge-noninvariant EOM operators. On the other hand, after the contraction with $Ω_µ$, we do not distinguish $E_n·∆$ and the gauge-noninvariant EOM operators; i.e., the contracted basic vertex $E_n^{(3)}·Ω$ of (3.9) and the contracted basic vertices for the gauge-noninvariant EOM operators coincide due to the condition $\hat{Ω} = 0$. It is easy to see that such an identification between the gauge-invariant and gauge-noninvariant EOM operators does not affect the prediction of the $Q^2$-evolution of the moment sum rules, because of the property that the physical matrix elements of the EOM operators vanish.

2. For the computation of the three-point functions imbedding $R_{n,l}·∆$ and $E_n·∆$, we set $m_q = 0$. Taking this limit is legitimate for the present case employing the off-shell kinematics for the external lines, and amounts to neglecting the contribution of the basic
vertex $N_{n,i}^{(3)}$ of (3.3) in the preceding discussions of this section. Clearly, this procedure still gives the correct results for the renormalization mixing between $R_{n,l}$ and $E_n$, i.e., for $Z_{ij}$ with $i, j = 2, \cdots, [(n + 1)/2], E$. Actually, we need not compute the one-loop correction to the three-point functions imbedding $E_n \cdot \Delta$. The property $\langle PS | E_n \cdot \Delta | PS \rangle = 0$ immediately implies $Z_{Ei} = 0 \ (i = 2, \cdots, [(n + 1)/2], N)$.

(3) In order to obtain the other components of $Z_{ij}$, it is sufficient to consider the two-point functions shown in Fig. 3 with the insertion of the relevant operators. We again employ the off-shell kinematics for the external quark lines, but the computation is now performed for the nonzero quark mass. The basic vertices corresponding to the operators $E_n \cdot \Delta, N_n \cdot \Delta$ are given by

$$E_n^{(2)} = \frac{1}{2} \hat{\rho}^{n-1} (\Delta \phi - \hat{\rho} \Delta + 2m_q \Delta) \gamma_5,$$

$$N_n^{(2)} = m_q \gamma_5 \hat{\rho}^{n-1},$$

while the one for $R_{n,l} \cdot \Delta$ vanishes. Thus, the computation of the one-loop corrections to the two-point functions imbedding $R_{n,l}, E_n$ and $N_n$, by using the vertex given in (3.3) and by following the steps similar to the case of the three-point functions, gives the components $Z_{iE}, Z_{iN} \ (i = 2, \cdots, [(n + 1)/2], E, N)$. For the remaining components $Z_{Ni}$ ($i = 2, \cdots, [(n + 1)/2]$) we need not perform any actual computation: Those vanish because $N_n$ is a twist-2 operator multiplied by a quark mass.

We note that the several components of $Z_{ij}$ can be obtained by different methods, giving a consistency check of our procedure: $Z_{iE} \ (i = 2, \cdots, [(n + 1)/2])$ by calculating the two- as well as three-point functions; $Z_{EN}, Z_{NE}$ by the calculation of the two-point functions and by a special property of the operators $E_n, N_n$ discussed above. For all those cases, the different methods gave the identical results.

In the above discussion, we completely neglected the flavor structure of the operator. Even for the case of flavor-singlet combinations of $h_{1,L}$, there exist no gluon distributions which mix with them. In fact, if there would be any mixing between flavor-singlet $h_1$ and a gluon distribution, it would arise from the diagrams shown in Fig.4. But all of them are identically zero because of chirality. The situation for $h_L$ is completely the same.

Now we get all the $Z_{ij}$-factor for the twist-3 operators. (The contributions of the relevant three- and two-point Feynman amplitudes corresponding to the diagrams of Figs.2 and 3 are presented in Appendix A.) We summarize the final result in the following matrix form:

$$\begin{pmatrix}
Z_{lm}(\mu) & Z_{iE}(\mu) & Z_{iN}(\mu) \\
0 & Z_{EE}(\mu) & 0 \\
0 & 0 & Z_{NN}(\mu)
\end{pmatrix}
\begin{pmatrix}
R_{n,m}(\mu) \\
E_n(\mu) \\
N_n(\mu)
\end{pmatrix}, \quad (l, m = 2, \cdots, [(n + 1)/2]). \quad (3.12)
$$

If we express $Z_{ij}$ as

$$Z_{ij} = \delta_{ij} + \frac{g^2}{16\pi^2\varepsilon} X_{ij} \quad (i, j = 2, \cdots, [(n + 1)/2], E, N), \quad (3.13)$$

then $X_{ij}$ is given as follows:

$$X_{lm} = C_G \left[ \frac{m + 1}{2} \left( \frac{1}{[n - l + 1]_2} - \frac{1}{[l - 1]_2} \right) \right]$$

9
\[+2 \left( \frac{1}{n-l+2} - \frac{1}{l} \right) - \frac{1}{n-l+2-m} + \frac{1}{l-m}\]
\[+ \frac{1}{2} \left( \frac{2l-1}{[l-1]_2} - \frac{m(l-3)}{[l-1]_3} \right) - \frac{1}{2} \left( \frac{2n-2l+3}{[n-l+1]_2} - \frac{m(n-l-1)}{[n-l+1]_3} \right)\]
\[+ (C_G - 2C_F) \left[ \frac{(-1)^{n-l-m} \binom{n-l}{m-2} C_{m-2}}{(n-l+2-m) \binom{n-m+1}{l-1} C_{l-1}} - \frac{(-1)^{l-m} \binom{n-l}{m-1} C_{m-1}}{(l-m) \binom{n-m+1}{l-1} C_{l-1}} \right]\]
\[+ 2(-1)^m \left( \binom{l-1}{m-1} C_{m-1} - \binom{n-l+1}{m-1} C_{m-1} \right) \quad (2 \leq m \leq l-1), \quad (3.14)\]

\[X_{ll} = C_G \left[ -\frac{(l-1)(n+2)}{2(n-2l+2)[n-l+1]_2} - \frac{1}{2} (S_{n-l+1} + S_{l-1} + S_{n-l+2} + S_l) \right.\]
\[+ \frac{1}{2} \left( \frac{(2l-n-2)(n-l-1)}{[n-l+1]_3} + \frac{l+2}{l(l-1)} + \frac{l-3}{[l-1]_3} - \frac{1}{7} \right]\]
\[+ 2(C_G - 2C_F) \left[ (-1)^l \left( \frac{1}{[l-1]_3} - \frac{(-1)^n \circ \binom{n-l}{m-1} C_{l-1}}{[n-l+1]_3} \right) \right.\]
\[+ \left. (-1)^m \frac{l-1}{2(n-l+1)(n-2l+2)} \right] - C_F (2S_{l-1} + 2S_{n-l+1} - 3), \quad (3.15)\]

\[X_{lm} = C_G \left[ -\frac{1}{n-l+2-m} - \frac{1}{l-m} - \frac{n-2m+2}{2[n-l+1]_2} \right.\]
\[+ \frac{(n-l-1)(2m-n-2)}{2[n-l+1]_3} \left. \right] - (C_G - 2C_F) \left[ 2(-1)^m \frac{\binom{n-l+1}{m-1} C_{m-1} - (-1)^n \binom{n-l+1}{m-1} C_{m-1}}{[n-l+1]_3} \right.\]
\[+ \left. \binom{n-l-m}{m-l} \frac{\binom{n-l}{m-1} C_{m-1}}{\binom{n-l+2}{m-1} C_{m-1}} \right] \quad (l+1 \leq m \leq \left[ \frac{n+1}{2} \right]), \quad (3.16)\]

\[X_{lE} = 2C_F \left( \frac{1}{[l]_2} - \frac{1}{[n-l+2]_2} \right), \quad (3.17)\]

\[X_{lN} = -4C_F \left( \frac{1}{[l-1]_3} - \frac{1}{[n-l+1]_3} \right), \quad (3.18)\]
\[ X_{EE} = 2(1 - S_n)C_F, \]  
\[ X_{NN} = C_F \left( \frac{2}{n(n+1)} - 4S_n \right), \]

where \( S_n = \sum_{j=1}^{n} \frac{1}{j} \) and \([j]_k = j(j+1) \cdots (j+k-1)\). With these \( X_{ij} \) \( i, j = 2, 3, \ldots, \left[ \frac{n+1}{2} \right], E, N \), the anomalous dimension matrix for the twist-3 operators \( R_{n,l}, E_n \) and \( N_n \) take the form of the upper triangular matrix as

\[ \gamma_{ij} = -\frac{g^2}{8\pi^2} X_{ij}. \]  

From (2.23), the \( Q^2 \)-evolution of the nucleon matrix elements are given by

\[ b_{n,l}(Q^2) = \sum_{m=2}^{[n+1]} \left[ \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-X/16\pi^2\beta_0} \right]_{\text{mn}} b_{n,m}(\mu^2) + \left[ \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-X/16\pi^2\beta_0} \right]_{\text{IN}} d_n(\mu^2), \]  

\[ d_n(Q^2) = \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-X_{NN}/16\pi^2\beta_0} d_n(\mu^2), \]

where we set \( d_n(\mu^2) = (m_q/M) \mathcal{M}_{n-1}[g_1(\mu^2)] \) (see (2.16)). This is our main result of this paper.

Before leaving this section, we shall briefly discuss the \( Q^2 \)-evolution of the twist-2 distribution \( h_1(x, Q^2) \) because it appears in \( h_L \) as a Wandzura-Wilczek analogue ((2.8) and (2.19)). There is only one operator \( \bar{\theta}_n^\mu \cdot \Delta \) (eq.(2.4)) for each \( n \) in the twist-2 level, and thus there is no complication arising from operator mixing. The lowest order coefficient of the anomalous dimension can be obtained from the one-loop diagram of the two-point functions shown in Fig. 3. We note that the contribution from the diagram in Fig.3 (a) vanishes, and the contribution to the \( Z \)-factor from Figs. 3 (b) and (c) is the same as those for \( f_1 \) and \( g_1 \) from the same diagrams. By using a similar technique as above, we obtain for the \( Z \)-factor for the composite operator \( \bar{\theta}_n^\mu \cdot \Delta \), which is now a single constant and not a matrix:

\[ Z = 1 - \frac{g^2}{16\pi^2\varepsilon} C_F K_{n+1}, \]  

with

\[ K_n = 1 + 4 \sum_{j=2}^{n} \frac{1}{j}. \]

By substituting the result into (2.22), we get the anomalous dimension

\[ \gamma = \frac{g^2}{8\pi^2} C_F K_{n+1}. \]

This governs the \( Q^2 \)-evolution of the first term on the r.h.s. of (2.8) (see eqs. (2.8) and (2.7)). The same result was obtained in ref.[2] from the Altarelli-Parisi equation for \( h_1 \).
4 Examples of $Q^2$-Evolution

Here we present some examples of the $Q^2$-evolution of the twist-3 distribution $\tilde{h}_L(x, Q^2)$ by using the result obtained in the previous section. Since there is no mixing of $h_L$ with the gluonic distribution, we shall consider a distribution for one quark flavor. We also neglect the contribution from the quark mass operator $N_n$ in the following.

For the 3-rd and 4-th moments of $\tilde{h}_L$, only one twist-3 operator $R_{n,l}$ contributes. Their anomalous dimensions are (ignoring the common factor $g^2/8\pi^2$) 104/9 and 1099/90 for the 3-rd and 4-th moments, respectively. This gives for $\mathcal{M}_{3,4}[\tilde{h}_L(Q^2)]$ as

$$
\mathcal{M}_3[\tilde{h}_L(Q^2)] = \frac{1}{5} b_{3,2}(\mu) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{1.284} ; \quad \mathcal{M}_4[\tilde{h}_L(Q^2)] = \frac{1}{3} b_{4,2}(\mu) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{1.357} .
$$

These curves normalized at $\mu = 1$ GeV are shown in Fig. 5. Here and below we set $N_f = 3$ and $\Lambda_{QCD} = 0.5$ GeV. For comparison, we also plotted the moments of the twist-2 distributions $f_1$ and $h_1$:

$$
\begin{align*}
\frac{\mathcal{M}_3[f_1(Q^2)]}{\mathcal{M}_3[f_1(\mu^2)]} = \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{0.775} ; \\
\frac{\mathcal{M}_3[h_1(Q^2)]}{\mathcal{M}_3[h_1(\mu^2)]} = \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{0.790} .
\end{align*}
$$

From this figure, one can clearly see that the third moment of the twist-3 distribution evolves significantly faster than that of the twist-2 structure function.

For $n = 5$, the anomalous dimension becomes the $2 \times 2$ matrix:

$$
X = \begin{pmatrix} -\frac{202}{71} & \frac{191}{180} \\ \frac{60}{9} & -\frac{31}{549} \end{pmatrix} ,
$$

and the eigenvalues of $\tilde{\gamma} \equiv -X/(11 - \frac{2}{3}N_f)$ are 1.435 and 2.005. We thus get for $\mathcal{M}_5[\tilde{h}_L(Q^2)]$ as

$$
\begin{align*}
\mathcal{M}_5[\tilde{h}_L(Q^2)] &= \left( 0.416b_{5,2}(\mu) + 0.193b_{5,3}(\mu) \right) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{1.435} \\
&\quad + \left( 0.013b_{5,2}(\mu) - 0.050b_{5,3}(\mu) \right) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{2.005} .
\end{align*}
$$

In principle, if one measures $\mathcal{M}_5[\tilde{h}_L(Q^2)]$ at two different values of $Q^2$ with sufficient accuracy, one could fix the two matrix elements $b_{5,2}(\mu)$ and $b_{5,3}(\mu)$, and the measurement of $\mathcal{M}_5[\tilde{h}_L(Q^2)]$ at different $Q^2$ supplies a test of the QCD evolution. Since we do not have any physical insight on these matrix elements, we plotted $\mathcal{M}_5[\tilde{h}_L(Q^2)]$ normalized at $\mu = 1$ GeV in Fig. 6 with four moderate values of $\lambda(\mu) = b_{5,3}(\mu)/b_{5,2}(\mu) = -4.0, -2.0, 1.0, 4.0$ at $\mu = 1$ GeV. One can see from the figure that there is a large variation in the $Q^2$-evolution among different choices of $\lambda(\mu)$. This fact suggests that a nucleon model and a nonperturbative technique of QCD can be tested by comparison of their prediction on $\lambda(\mu)$ with future experiments.

As a measure of the large $Q^2$-behavior of the moments, we have plotted in Fig. 7 the lowest eigenvalues of the matrix $\tilde{\gamma}_{lm}$ $(l, m = 2, \ldots, [n+1]/2)$ with $N_f = 3$ for $\tilde{h}_L$, $\tilde{g}_T$ (defined as the twist-3 part of $g_T$), $h_1$, and $f_1$ as a function of $n$. For $\tilde{g}_T$, we used the result for $\tilde{\gamma}$.
obtained in ref. [12]. (The authors of ref.[12] calculated the anomalous dimension matrix of twist-3 operators only for the even moments of $g_T$, since they discussed it in the context of the deep inelastic scattering.) From this figure we expect that the moment of the twist-3 distributions evolves faster than that of the twist-2 distributions. This fact indicates that the measurement of $g_T$ and $h_L$ greatly serves as a new test of QCD at high energies. If one looks into more detail of fig. 7 one sees that the chiral-odd structure function, $h_1$ and $\tilde{h}_L$, evolves slightly faster than the chiral-even ones with the same twist, $f_1$, $g_1$ and $\tilde{g}_T$. In general, however, actual form of the $Q^2$-evolution in a finite $Q^2$ window strongly depends on the relative magnitude among $b_{n,l}(\mu)$ as we saw in the above example for $n = 5$. We thus should take the result shown in Fig. 7 only as a rough measure for the asymptotic behavior of the $Q^2$-evolution.

5 Summary and Conclusion

In this paper we have studied the $Q^2$-evolution of the moments of the chiral-odd twist-3 spin structure function $h_L(x, Q^2)$ in the standard QCD perturbation theory. For the $n$-th moment of the twist-3 part of $h_L(x, Q^2)$, $M_n[\tilde{h}_L(Q^2)]$, $\left[\frac{n+3}{2}\right]$ independent twist-3 operators, $R_{n,l}$ ($l = 2, \ldots, \left[\frac{n+1}{2}\right]$), $N_n$ and $E_n$ play roles. The $Q^2$-evolution of $M_n[\tilde{h}_L(Q^2)]$ is governed by the anomalous dimension matrix for the operators \{$R_{n,l}, N_n, E_n$\}. We thus have calculated the one-loop correction to the three-point Green function which imbeds these operators. Although the physical (on-shell) matrix element of the EOM (equation of motion) operator $E_n$ vanishes, $E_n$ mixes with $R_{n,l}$ and $N_n$ through renormalization and it is essential to take into account this operator mixing to determine the anomalous dimension matrix. In order to incorporate the mixing correctly, we employed the off-shell kinematics for the external lines. Using the minimal subtraction (MS) scheme we obtained the renormalization constants in the one-loop order in the form of the upper triangular matrix which really shows the mixing among \{\$R_{n,l}, N_n, E_n$\}. As an example of the $Q^2$-evolution of $h_L$, we have studied the 3-rd, 4-th and 5-th moments of $h_L(x, Q^2)$ in detail for $m_q = 0$, comparing them with the known twist-2 distributions, $f_1$, $g_1$ and $h_1$, and the other twist-3 distribution $g_T$. To consider the case $m_q = 0$ is a sufficiently good approximation for the $u$ and $d$-quark distributions, and is a reasonable one for the $s$-quark distribution, since the nucleon matrix element of $N_n$ is expected to be small for the $s$-quark. The notable features of the $Q^2$-evolution of these moments can be summarized as follows:

1. The $Q^2$-evolution of the 3-rd and 4-th moments can be predicted uniquely, since there is only one relevant operator $R_{n,2}$ ($n = 3, 4$). (Note the nucleon matrix element of $E_n$ vanishes.) Compared with the twist-2 distributions $f_1$, $g_1$ and $h_1$, the moments of $\tilde{h}_L(x, Q^2)$ evolves significantly faster.

2. The fifth or higher moments of $\tilde{h}_L$ receives contribution from two or more operators, and thus the $Q^2$-evolution depends on the ratio among these matrix elements at a reference scale $\mu$. They can, in principle, be determined by measuring $h_L(x, Q^2)$ at several values of $Q^2$, and the measurement at different values of $Q^2$ gives a test for the QCD evolution.
3. The lowest eigenvalues of the anomalous dimension matrix for each moment of $\tilde{h}_L$ and $\tilde{g}_T$ (the twist-3 part of $g_T$) are much larger than those of twist-2 distributions, which implies that the $Q^2$-evolution of these twist-3 distributions is significantly faster than the twist-2 distributions. These numbers for $\tilde{h}_L$ are slightly larger than those for $\tilde{g}_T$, which suggests that the chiral-odd distribution $\tilde{h}_L$ evolves faster than the chiral-even one $\tilde{g}_T$. The similar tendency has been known for the twist-2 distributions, $h_1$ and $g_1$.

The anomalous dimension matrix for the twist-3 operators obtained in this paper determines only the $Q^2$-evolution of the moments of $\tilde{h}_L(x, Q^2)$. In order to predict the $Q^2$-evolution of the whole $x$-dependent distribution $\tilde{h}_L(x, Q^2)$, we need to construct a generalized Altarelli-Parisi equation for a relevant multi-parton distribution function. This is because a higher-twist distribution essentially represents a correlated quark-gluon distribution and $\tilde{h}_L(x, Q^2)$ is only a particular projection of this generalized multi-parton distribution. This work is under way and will be published in a future publication. Nevertheless, we already found in this work some peculiar features in the $Q^2$-evolution of the moments of $h_L(x, Q^2)$, which we hope will be measured in the future collider experiments.

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Appendices

A. One-loop Feynman amplitudes

In this Appendix we present the expressions for the one-loop Feynman amplitudes for the truncated one-particle-irreducible Green functions with the insertion of the composite operators \( O_i = R_{n,l} \cdot \Delta \) \( (l = 2, \cdots [(n + 1)/2]) \), \( E_n \cdot \Delta \) and \( N_n \cdot \Delta \).

First we consider the three-point function with the insertion of \( R_{n,l} \cdot \Delta \). We set \( m_q = 0 \) following the discussion of (2) in sect. \( [3] \). The Feynman diagrams for the one-loop corrections are shown in Fig. 2. The Feynman amplitudes are contracted by \( \Omega \) and are grouped to possess a definite charge conjugation. (Note that, in the Feynman diagrams of Fig. 2, (d), (f), and (h) are connected by the charge conjugation to (c), (e), and (g), respectively.) Each group can be expanded by the basic vertices \( R_{n,m}^{(3)} \) and \( \mathcal{E}_n^{(3)} \). (Here and in the following, we denote \( R_{n,m}^{(3)} \cdot \Omega \) and \( \mathcal{E}_n^{(3)} \cdot \Omega \) of (3.8) and (3.9) simply by \( R_{n,m}^{(3)} \) and \( \mathcal{E}_n^{(3)} \). A sample calculation of the diagrams (e) and (f) are given in Appendix B.)

Fig. 2(a) gives 0.

Fig. 2(b) gives:

\[
\frac{g^2}{16 \pi^2 \varepsilon} C_G \left[ \sum_{m=2}^{l-1} \left( \frac{(l+m)(m-1)}{2(l-m)[l-1]} - \frac{(n-l+2+m)(m-1)}{2(n-l+2-m)[n-l+1]} \right) R_{n,m}^{(3)} \\
+ \left\{ 1 - S_l - S_{n-l+2} + \frac{1}{2l} + \frac{1}{2(n-l+2)} - \frac{(n+2)(l-1)}{2(n-2l+2)[n-l+1]} \right\} R_{n,l}^{(3)} \\
+ \sum_{m=l+1}^{[(n+1)/2]} \left( \frac{(2n-l-m+4)(n-m+1)}{2(n-l+1)} - \frac{(n-l+2+m)(m-1)}{2(n-l+2-m)[n-l+1]} \right) R_{n,m}^{(3)} \right] \tag{A.1}
\]

Fig. 2(c) + Fig. 2(d) gives:

\[
\frac{g^2}{16 \pi^2 \varepsilon} (2C_F - C_G) \left[ \sum_{m=2}^{l-1} 2(-1)^m \left( \frac{n-l+1C_{m-1}}{n-l+1} - \frac{1}{l-1} \right) R_{n,m}^{(3)} \\
+ 2(-1)^l \left( \frac{n-l+1C_{l-1}}{n-l+1} - \frac{1}{l-1} \right) R_{n,l}^{(3)} \\
+ \sum_{m=l+1}^{[(n+1)/2]} \left( \frac{2(-1)^m}{n-l+1} \right) (n-l+1C_{m-1} - (-1)^n n-l+1C_{m-l}) R_{n,m}^{(3)} \\
+ \left( \frac{1}{l+2} - \frac{1}{n-l+2} \right) \mathcal{E}_n^{(3)} \right]. \tag{A.2}
\]

Fig. 2(e) + Fig. 2(f) gives:

\[
\frac{g^2}{16 \pi^2 \varepsilon} \left[ \sum_{m=2}^{l-1} \left\{ (2C_F - C_G) \left( \frac{(-1)^{l-m} l-2C_{l-m}}{(l-m) n-m+1C_{l-m}} - \frac{(-1)^{n-l-m} n-lC_{m-2}}{(n-l+2-m) n-m+1C_{l-1}} \right) \right\} \right].
\]
\[- C_G \left( \frac{1}{l} - \frac{1}{n-l+2} \right) \mathcal{R}_{n,m}^{(3)} \]
\[ - \left\{ (2C_F - C_G) \frac{(-1)^n(l-1)}{(n-2l+2)(n-l+1)} + 2C_F(S_{l-1} + S_{n-l+1} - 2) + C_G \frac{1}{l} \right\} \mathcal{R}_{n,l}^{(3)} \]
\[ + \sum_{m=l+1}^{[n+1/2]} (2C_F - C_G) \left( \frac{(-1)^n-l_C}{(m-l)} \right) \mathcal{R}_{n,m}^{(3)} \]
\[ + C_G \left( \frac{1}{l} - \frac{1}{n-l+2} \right) \mathcal{E}_n^{(3)} \].

\[ (A.3) \]

Fig. 2(g) + Fig. 2(h) gives:
\[ \frac{g^2}{16\pi^2} C_G \left[ \sum_{m=2}^{l-1} \frac{1}{2} \left( \frac{(l-3)(l+1-m)}{[l-1]_2} + \frac{l+2}{[l-1]_2} \right) + \frac{1}{2} \left( \frac{(l-3)}{[l-1]_3} + \frac{l+2}{[l-1]_2} - \frac{(n-l-1)(n-2l+2)}{[n-l+1]_3} \right) \right] \mathcal{R}_{n,m}^{(3)} \]
\[ - \sum_{m=l+1}^{[n+1/2]} \frac{(n-l-1)(n-2m+2)}{2[n-l+1]_3} \mathcal{R}_{n,m}^{(3)} \]
\[ - \left( \frac{1}{l+1} - \frac{1}{n-l+3} \right) \mathcal{E}_n^{(3)} \].

\[ (A.4) \]

The coefficients in these expansions diverge as \( \varepsilon = (4-d)/2 \to 0 \) with \( d \) the space-time dimension. By adding the counter term contribution \( (3.5) \) to the sum of eqs. \( (A.1)-(A.4) \) and by requiring that those counter term contributions cancel the \( 1/\varepsilon \) pole terms, we obtain the renormalization constants given by \( (3.14)-(3.17) \) in the MS scheme.

Next we proceed to calculate the two-point function with the insertion of \( R_{n,l} \cdot \Delta, E_n \cdot \Delta \) and \( N_n \cdot \Delta \), following the discussion of (3) in sect. \( \emptyset \). We keep the quark mass \( m_q \) as a nonzero quantity. The computation can be performed in a similar manner as in the case of the three-point functions. The three-point vertices necessary to compute the one-loop diagrams of Fig. 3 are given by \( (3.1)-(3.3) \). Again the amplitudes combined appropriately can be expanded by the basic vertices \( (3.10) \) and \( (3.11) \).

The one-loop correction to the two-point function with \( R_{n,l} \cdot \Delta \) comes from Figs. 3 (b) and (c). It gives
\[ \frac{g^2}{16\pi^2} C_F \left[ \left( \frac{2}{[l]_2} - \frac{2}{[n-l+2]_2} \right) \mathcal{E}^{(2)} \right] - \left( \frac{4}{[l-1]_3} - \frac{4}{[n-l+1]_3} \right) \mathcal{N}^{(2)} \].

\[ (A.5) \]

For the one-loop correction to the one with \( E_n \cdot \Delta \), Fig. 3(a) gives
\[ \frac{g^2}{16\pi^2} C_F \frac{2}{n} \mathcal{N}^{(2)}, \]

\[ (A.6) \]
and Fig.3(b)+Fig.3(c) gives
\[
\frac{g^2}{16\pi^2\varepsilon} C_F \left[ \left( 1 - 2 \sum_{j=2}^{n} \frac{1}{j} \right) \mathcal{E}^{(2)} - \left( 3 + \frac{2}{n} \right) \mathcal{N}^{(2)} \right]. \tag{A.7}
\]

Finally, for the one-loop correction to the one with \( N_n \cdot \Delta \), Fig.3(a) gives
\[
\frac{g^2}{16\pi^2\varepsilon} C_F \left( \frac{2}{n(n+1)} \mathcal{N}^{(2)} \right), \tag{A.8}
\]
while each of Figs.3 (b) and (c) gives the same contribution,
\[
\frac{g^2}{16\pi^2\varepsilon} C_F \left( -2 \sum_{j=2}^{n} \frac{1}{j} \right) \mathcal{N}^{(2)}. \tag{A.9}
\]

By adding the appropriate counter term contributions, we can determine the counter terms in the MS scheme, which give the results (3.17)-(3.20). Note that (3.17) are obtained from the three-point functions as well as from the two-point functions, giving a consistency check of our methods. Also, \( Z_{EN} = 0 \) can be verified explicitly by using the results (A.6) and (A.7) (see the discussion of (3) in sect. 3).

### B. Sample calculation

In this appendix we describe the details of the calculation of the one-loop Feynman amplitudes. We choose the diagram (e) and (f) of Fig.2 with the insertion of \( R_n \cdot \Delta \) as an example; the other diagrams can be calculated using the similar technique.

First we write down the Feynman amplitude \( \mathcal{F}_{(e)} \) for the diagram (e) by using the vertex (3.4) and the usual Feynman rule (in the Feynman gauge):
\[
\mathcal{F}_{(e)} = \mu^{2\varepsilon} \int \frac{d^4 k}{(2\pi)^4} \frac{\hat{q}^\mu t^b}{(p - k)^2} \left( -\frac{g^2}{2} \Omega_{\mu}^{\mu\lambda} \Delta^{i} \gamma^{5} \Delta^{j} \right)
\]
\[
\times \left\{ (\hat{p} - \hat{k})^{n-l} \hat{q}^{l-2} - (\hat{p} - \hat{k})^{l-2} \hat{q}^{n-l} \right\} f^{abc} t^{c}
\]
\[
+ \sum_{j=2}^{n-l+1} (\hat{p} - \hat{k})^{j-2} \hat{p}^{n-l+1-j} \hat{q}^{l-2} - \sum_{j=2}^{l-1} (\hat{p} - \hat{k})^{j-2} \hat{p}^{l-1-j} \hat{q}^{n-l} \right\} (\hat{q} - \hat{p}) t^{b} t^{c}
\]
\[
+ \sum_{j=n-l+3}^{n} (\hat{p} - \hat{k})^{n-l} (\hat{q} - \hat{k})^{j-3-n+l} \hat{q}^{n-j}
\]
\[
- \sum_{j=l+1}^{n} (\hat{p} - \hat{k})^{l-2} (\hat{q} - \hat{k})^{j-1-l} \hat{q}^{n-j} \right\} (\hat{q} - \hat{p}) t^{a} t^{b} \right\} \frac{-i g^{\nu \rho}}{k^{2}}, \tag{B.1}
\]
where \( q \) and \( p \) are the incoming and the outgoing off-shell quark momenta, and \( k \) is the gluon loop momentum. We work in the massless quark limit \( (m_q = 0) \) following (2) of sect. 3. The Lorentz index \( \mu \) corresponding to the external gluon line is contracted by \( \Omega_{\mu} \), which
killing off the terms involving $\Delta_n$ in the vertex (B.4). Also, owing to this property, the factors involving the gamma matrices can be easily evaluated:

$$\gamma_0(p + k) \Omega_{\mu} \sigma^{\mu \lambda} \Delta_\lambda i \gamma_5 \Delta_\delta g^{\mu \rho} = 2(p + \hat{k}) \Omega_{\mu} \sigma^{\mu \lambda} \Delta_\lambda i \gamma_5.$$  

(B.2)

We follow the standard procedure: We use the Feynman parameterization to collect all the denominators of the quark and the gluon propagators. After shifting the $k$-integration, many terms can be dropped by the condition $\Delta^2 = 0$. We retain only the divergent contribution of the $k$-integration:

$$\mathcal{F}_{(e)} = g^3 \frac{1}{16\pi^2 \varepsilon} \Omega_{\mu} \sigma^{\mu \lambda} \Delta_\lambda i \gamma_5 t^a \int_0^1 dx \left\{ \right. $$

$$\left. \times \left\{ \frac{C_G}{2} \left( \hat{t}^{n-l+1} \hat{q}^{l-2} - \hat{t}^{l-1} \hat{q}^{n-l} \right) + C_F \left( \sum_{j=2}^{n-l+1} \hat{t}^{j-1} \hat{p}^{n-l+1-j} \hat{q}^{l-2} - \sum_{j=2}^{l-1} \hat{t}^{j-1} \hat{p}^{l-1-j} \hat{q}^{n-l} \right) \left( \hat{q} - \hat{p} \right) \right. $$

$$+ \left. \left( C_F - \frac{C_G}{2} \right) \left( \sum_{j=n-l+3}^{n} \hat{t}^{n-l+1} \hat{s}^{j-3-n+l} \hat{q}^{n-j} - \sum_{j=l+1}^{n} \hat{t}^{l-1} \hat{s}^{j-1-l} \hat{p}^{n-j} \right) \left( \hat{q} - \hat{p} \right) \right\}, \quad (B.3)$$

where $t = (1 - x)p$, $s = -xp + q$. We perform the Feynman parameter integral by

$$\int_0^1 dx \hat{t}^{n-l+1} \hat{s}^{l-1} \hat{t}^{l-1} \hat{q}^{n-l} = \frac{1}{l} \sum_{r=0}^{j-1-l} (-1)^r j \hat{S}_{l-r} \hat{C}_{l-r} \hat{p}^{l-1+r} \hat{q}^{l-1-r}. \quad (B.4)$$

The resulting formula of $\mathcal{F}_{(e)}$ involves double summation. These terms can be simplified into single summation by interchanging the order of summation and by using the relation $\sum_{n=m}^{n} r C_m = n+1 C_{m+1}$. Then, we obtain

$$\mathcal{F}_{(e)} = 2 \frac{g^2}{16\pi^2 \varepsilon} \Omega_{\mu} \sigma^{\mu \lambda} \Delta_\lambda i \gamma_5 t^a \left[ \frac{C_G}{2} \left( \frac{1}{l} \hat{p}^{l-1} \hat{q}^{n-l} - \frac{1}{n-l+2} \hat{p}^{n-l+1} \hat{q}^{l-2} \right) \right. $$

$$\left. + C_F \left( (S_{n-l+1} - 1) \hat{p}^{n-l} \hat{q}^{l-2} - (S_{l-1} - 1) \right) \hat{p}^{l-2} \hat{q}^{n-l} \right) \left( \hat{q} - \hat{p} \right) $$

$$+ \left( C_F - \frac{C_G}{2} \right) \left( \sum_{m=2}^{l-1} \frac{(-1)^{l-m} C_{l-m}}{(l-m) n-l+1 C_{l-m}} \hat{p}^{n-m} \hat{q}^{m-2} \right. $$

$$- \left. \sum_{m=l+1}^{n} \frac{(-1)^{m-l} n-l C_{m-l}}{(m-l) n-l+1 C_{m-l}} \hat{p}^{m-2} \hat{q}^{n-m} \right) \left( \hat{q} - \hat{p} \right) \right]. \quad (B.5)$$

The amplitude corresponding to the diagram (f) can be computed in a similar manner as above. The result should be related to (B.3) by charge conjugation. By adding this result to (B.3), we obtain as the total result:

$$\mathcal{F} \equiv \mathcal{F}_{(e)} + \mathcal{F}_{(f)} $$

$$= 2 \frac{g^2}{16\pi^2 \varepsilon} \Omega_{\mu} \sigma^{\mu \lambda} \Delta_\lambda i \gamma_5 t^a \left[ \frac{C_G}{2} \left( \frac{1}{l} \hat{p}^{l-1} \hat{q}^{n-l} + \hat{p}^{l-1} \hat{q}^{n-l} \right) - \frac{1}{n-l+2} \left( \hat{p}^{n-l+1} \hat{q}^{l-2} + \hat{p}^{l-2} \hat{q}^{n-l+1} \right) \right]$$

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\[ + C_F (S_{l-1} + S_{n-l+1} - 2) \left( \hat{p}^{n-l} \hat{q}^{l-1} - \hat{p}^{l-1} \hat{q}^{n-l} \right) (\hat{q} - \hat{p}) \]

\[- \left( C_F - \frac{C_G}{2} \right) \sum_{m=2}^{l-1} \frac{(-1)^{l-m} C_{l-m}}{(l-m) n-m+1 C_{l-m}} \left( \hat{p}^{n-m} \hat{q}^{m-2} - \hat{p}^{m-2} \hat{q}^{n-m} \right) (\hat{q} - \hat{p}) \]

\[ + \sum_{m=1}^{n} \frac{(-1)^{m-l}}{(m-l) n-m+1 C_{m-l}} \left( \hat{p}^{n-m} \hat{q}^{m-2} - \hat{p}^{m-2} \hat{q}^{n-m} \right) (\hat{q} - \hat{p}) \].

This should be expressed as a linear combination of the basic vertices (3.8) and (3.9), which we denote simply by \( R^{(3)}_{n,l} \) and \( E^{(3)}_n \). This is readily performed for the terms proportional to \( C_F \) or \( C_F - C_G/2 \). For the terms proportional to \( C_G \), we use the identity

\[ \hat{p}^{n-l} \hat{q}^{l-1} + \hat{p}^{l-1} \hat{q}^{n-l} = \hat{p}^{n-1} + \hat{q}^{n-1} + \sum_{m=2}^{l} \left( \hat{p}^{n-m} \hat{q}^{m-2} - \hat{p}^{m-2} \hat{q}^{n-m} \right) (\hat{q} - \hat{p}) \]

\[ = \hat{p}^{n-1} + \hat{q}^{n-1} - \sum_{m=l+1}^{n} \left( \hat{p}^{n-m} \hat{q}^{m-2} - \hat{p}^{m-2} \hat{q}^{n-m} \right) (\hat{q} - \hat{p}). \]

Now we obtain

\[ F = -2 \frac{g^2}{16 \pi^2 \varepsilon} \left\{ C_F (S_{l-1} + S_{n-l+1} - 2) R^{(3)}_{n,l} + \frac{C_G}{2} \left( \frac{1}{l} \sum_{m=2}^{l} R^{(3)}_{n,m} + \frac{1}{n-l+2} \sum_{m=l}^{n} R^{(3)}_{n,m} \right) \right. \]

\[- \left( C_F - \frac{C_G}{2} \right) \sum_{m=2}^{l-1} \frac{(-1)^{l-m} C_{l-m}}{(l-m) n-m+1 C_{l-m}} R^{(3)}_{n,m} + \sum_{m=l+1}^{n} \frac{(-1)^{m-l}}{(m-l) m-l C_{m-l}} R^{(3)}_{n,m} \]

\[ + \frac{C_G}{2} \left( \frac{1}{n-l+2} - \frac{1}{l} \right) E^{(3)}_n \}. \]

Here \( R^{(3)}_{n,m} \) with \( m = \lfloor (n+1)/2 \rfloor + 1, \ldots, n \) appear. These can be expressed by those for \( m = 2, \ldots, \lfloor (n+1)/2 \rfloor \) using \( R^{(3)}_{n,n-m+2} = -R^{(3)}_{n,m} \), and we obtain (A.3) of Appendix A.
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**Figure Captions**

Fig. 1 (a) Three-point basic vertex for $R_{n,l}$, $E_n$ and $N_n$. (b) Four-point basic vertex for $R_{n,l}$ necessary for the calculation of the diagrams shown in Fig. 2.

Fig. 2 One-particle-irreducible diagrams for the one-loop correction to the three-point Green function $F_i(p, q, k)$ (eq. (2.25)).

Fig. 3 One-loop corrections to the two-point function relevant for the calculation of $Z_{lE}$, $Z_{lN}$. These diagrams are also used for the calculation of the anomalous dimension of the twist-2 distributions.

Fig. 4 Diagrams which could cause mixing between a flavor-singlet quark distribution $h_1$ and a gluon distribution (if any). These diagrams are identically zero for the chiral-odd distribution.

Fig. 5 The $Q^2$-evolution of the 3-rd and 4-th moments of $\tilde{h}_L(x, Q^2)$ normalized at $\mu = 1$ GeV. The 3-rd moments of twist-2 distributions $f_1$ and $h_1$ are also plotted for comparison.

Fig. 6 The $Q^2$-evolution of the 5-th moment of $\tilde{h}_L(x, Q^2)$ normalized at $\mu = 1$ GeV for four moderate values of $\lambda(\mu) = -4.0, -2.0, 1.0, 4.0$.

Fig. 7 The smallest eigenvalues of $\tilde{\gamma} = -X/16\pi^2\beta_0$ as a function of the dimension of the moment, $n$. 

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