Mass spectra of dimesonic states in light flavour sector

D P Rathaud* and A K Rai

Department of Applied Physics, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat 395007, India

Received: 17 February 2016 / Accepted: 07 April 2016 / Published online: 12 May 2016

Abstract: Masses of the dimesonic (meson–antimeson) molecular states are calculated using Yukawa like and meson exchange potential in semi-relativistic approach. The digamma decay width of the dimesonic systems are evaluated using the spectroscopic parameters. Many states such as $f_0(980)$, $h_1(1380)$, $a_0(1450)$, $f_0(1500)$, $f_2^*(1525)$, $f_2(1565)$, $a_2(1700)$, $f_0(1710)$, $f_2(1810)$, $f_2(2010)$ have shown the signature of dimesonic molecules. Their masses and digamma widths are in good agreement with other theoretical results as well as experimental measurements.

Keywords: Mass spectra; Digamma width; Dimesonic states; Exotic states

PACS Nos.: 12.39.Pn

1. Introduction

The exotic states are the states which could not be explained by conventional mesonic ($q\bar{q}$) and baryonic ($qqq$) scheme. The huge amount of data (states) generated by modern experimental facilitie available across the world need extra theoretical attention [1–5]. All these states carry the information of the fundamental behaviour of the QCD. As per the underlying theory, only the color neutral states are possible, just like mesons and baryons. The theory predicts more complicated color neutral states like tetraquark, pentaquark, hexaquark, hybrid, glueball and molecule like structures [6–10]. The explanation of structures and dynamics of such states are still open questions.

Recently, some experimental groups like, BES III-collaboration [11–13], GAMS-group at CERN and BNLE852-collaboration [14–17] have been observed few states (below 2.5 GeV), may have exotic structure and quantum numbers, required theoretical attentions. In such aspect, the spectroscopy of mesons are playing a dominant role in the study of the QCD, especially, the light meson sector which carries the information of nonperturbative regime. The scalar mesons in the light mass regime may give the clue for the symmetry breaking, as they have the same quantum numbers as of vacuum. The difficulties with the study of light mesons sector are their large decay widths as well as their appearance does not come with narrow isolated peaks.

In this paper, we have studied the molecule like structure of mesons in the light flavour (u, d, s) sector. We have listed many possible candidates for the exotic states in Table 1 (for reference). The multiquark states have been studied and predicted previously in Bag model and in nonrelativistic potential models [6–10, 18–22]. The molecular like structures have been studied in various theoretical approaches like potential models [18–26], gauge invariant model [27], QCD sum rules [28, 29], Bethe–Salpeter equation approach [30] and field theory approach [31]. Recently, the molecular-like states are predicted in the framework of chiral SU(3) quark model [32], Ref. [33] have studied the photoproduction of the tensor states.

We have studied the molecular dimesonic (meson-antimeson) systems in the potential model framework. The multiquark structure (four quark state) could be formed by two ways, (1) no subcomponent of system could make color neutral, probably known as tetraquark. (2) the quark-antiquark subcomponent (meson) is being color neutral and same for the other subcomponent (antimeson). In the second case the both color neutral subcomponent makes bound state, say dimesonic state. The dimesonic system consists the meson–antimeson bound state, just like deuteron (bound state of the proton and neutron). By taking this approximation, we have employed various interaction potentials like Yukawa like potential, one pion exchange
and sigma exchange potential. The meson-antimeson system have been studied previously with the same approximation by Törnqvist [24, 25] and introduced it as a ‘dimesons’. The mass of the dimesonic state should be less than the sum of the mass of the two respective mesons and the binding energy of the system should be small. The theoretical calculation of this binding energy of dimesonic states is very complex. As such, it deals with the delicate theoretical calculation of this binding energy of dimesonic states. The binding energy the digamma width have been found very close to the experimental results [1]. Thus, we have fixed this parameter for all other dimesonic combinations for their mass spectra. We have calculated the digamma decay width by using the formula given by Ref. [35]. In the dimesonic states, we have used meson and anti-meson constituents which are colour singlets, so confinement potential is not required as required in quark anti-quark system [36]. In this molecular system the kinetic energy of the constituent mesons in the bound state is also small, so that the relativistic effects can be neglected and the present method represents the same [36]. By considering the Pseudoscalar and Vector mesons for dimesonic states, we have three combinations (1) Pseudoscalar–Pseudoscalar (PP-states) (2) Pseudoscalar–Vector (PV-states) (3) Vector–Vector (VV-states).

2. Theoretical framework

The variational scheme is employed to solve the Schrödinger equation and to get the value of variational parameter for each state, the virial theorem is used. The Hamiltonian of the dimesonic (meson–antimeson) system is given by [37–40]

$$H = \sqrt{P^2 + m_{h1}^2} + \sqrt{P^2 + m_{h2}^2} + V(r)$$  

(1)

where \(m_{h1}\) and \(m_{h2}\) are masses of mesons, \(P\) is the relative momentum of two mesons and \(V(r)\) is the molecular interaction potential of the dimesonic system. In the present study, we have dealt with the light flavour mesons (except pions). To incorporate the relativistic correction to the kinetic energy, we have expanded the kinetic energy term of the Hamiltonian up to \(O(P^8)\)

| Table 1 | Experimentally observed states (in MeV) [1] |
|---------|---------------------------------------------|
| States  | Mass (MeV) | Possible \(J^{PC}\) | Compared dimesonic states |
| \(f_0(980)\) | 990 ± 20 | \(^0(0^{++})\) | \(K - \bar{K}\) |
| \(h_1(1380)\) | 1386 ± 19 | \(^0(1^{++})\) | \(K - \bar{K}^*\) |
| \(a_0(1450)\) | 1474 ± 19 | \(^1(0^{++})\) | \(\rho - \bar{\rho}\) |
| \(f_0(1500)\) | 1505 ± 06 | \(^0(1^{++})\) | \(\rho - \bar{\rho}\) |
| \(f_2(1525)\) | 1525 ± 05 | \(^0(2^{++})\) | \(\rho - \bar{\rho}\) |
| \(f_0(1565)\) | 1562 ± 13 | \(^0(2^{++})\) | \(\omega - \bar{\omega}\) |
| \(a_2(1700)\) | 1722 ± 16 | \(^1(2^{++})\) | \(K^* - \bar{K}^*\) |
| \(f_0(1710)\) | 1722 ± 05 | \(^0(0^{++})\) | \(K^* - \bar{K}^*\) |
| \(f_2(1810)\) | 1815 ± 12 | \(^0(2^{++})\) | \(\phi - \bar{\phi}\) |
| \(f_2(2010)\) | 2011 ± 66 | \(^0(2^{++})\) | \(\phi - \bar{\phi}\) |

| Table 2 | Masses of mesons (in MeV) [1] |
|---------|-----------------|
| Meson   | \(K\) | \(\eta\) | \(\eta'\) | \(\rho\) | \(\omega\) | \(K^*\) | \(\phi\) |
| Mass    | 497.6 | 547.8 | 957.7 | 775.4 | 782.6 | 895.9 | 1019.4 |
The interaction potential of meson-antimeson system is consisted the Yukawa like potential, one pion exchange potential (OPE) \( V_\pi \) and sigma exchange potential \( V_\sigma \). Whereas, the spin dependent potential \( V_{SD} \) added perturbatively.

\[
V(r) = V(r_{12}) + V_\sigma + V_\pi
\]

\[
V(r_{12}) = \frac{k_{mol} e^{-\frac{C_{mol}}{r_{12}}}}{r_{12}}
\]

\( r_{12} \) is the relative co-ordinates of meson–antimeson system, \( k_{mol} \) is the residual strength of strong running coupling constant and \( C \) is the effective color screening parameter of the confined gluon \([18, 19]\). The dimesonic molecular interaction is consist with the complicated interaction of short range repulsion and long range attraction. The Yukawa like potential and sigma exchange potential cares for short and mid range interaction while one pion exchange potential take care for long range part. The value of the \( k_{mol} \) (running coupling constant) is determined through,

\[
k_{mol} = \frac{4\pi}{(1 - \frac{2}{3} n_f) \ln \frac{M^2 + M_B^2}{\Lambda^2}}
\]

where \( M = 2m_{h_1}m_{h_2}/(m_{h_1} + m_{h_2}), M_B = 0.95 \text{ GeV} \) and \( \Lambda = 0.413 \text{ GeV} \) \([41]\) whereas \( n_f \) is number of flavours.

One pion exchange potential (OPE) and \( \sigma \) exchange potential \([42–44]\) is used, based on the assumption that molecular like structure of dimesonic system is being deuteron like structure of nucleon \([23]\). The conventional OPE potential is consist with the tensor interaction term which plays a dominant role in NN scattering \([45, 46]\). If two hadrons are in \( L = 0 \) (ground state) state, then the matrix element of tensor operator becomes zero \([47]\). The sigma exchange includes spin-orbital interactions and as we deal with the system of S-wave \( (L = 0) \), the term with spin-orbital coupling vanishes. The nonrelativistic form of the pion and sigma exchange potential is used here \([42–44]\).

\[
V_\pi = \frac{g_\pi^2}{34\pi} \left( \frac{m_{\pi}^2}{4m_1m_2} \right) (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2)
\]

\[
\times \left( \frac{e^{-m_{\pi}r_{12}}}{r_{12}} - \left( \frac{\Lambda_{\pi}}{m_{\pi}} \right)^2 e^{-\Lambda_{\pi}r_{12}} \right)
\]

\[
V_\sigma = \frac{g_\sigma^2}{8\pi} \left( \frac{e^{-m_{\sigma}r_{12}}}{r_{12}} - e^{-\Lambda_{\sigma}r_{12}} \right)
\]

where \( g = g_8 \) is a quark-meson coupling constant for \( \pi, K, \) and \( \eta \). The coupling constant for sigma meson is taken to be the same as for pseudoscalar meson, \( g_8 = 0.69 \), is taken from \([42–44]\). The \( \tau \) and \( \sigma \) are isospin, spin matrices respectively. \( \Lambda_M = km_M \) (\( m_M \) -exchange meson mass) is the form factor, appears due to the dressing of quarks and assume to be depended on the exchange boson masses. Where \( m_1 \) and \( m_j \) are the constituent masses in GeV, \( m_\pi = 0.1349 \text{ GeV}, m_\eta = 0.675 \text{ GeV} \) and \( k = 2.2 \). For PV states the values of spin-isospin factors have been taken as \( (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_j) = -3, 1 \) for \( I = 0, 1 \). While for VV states values taken as \( (\tau_1 \cdot \tau_j)(\sigma_i \cdot \sigma_j) = -6, -3, 3 \) for isospin \( I = 0 \) and spin \( S = 0, 1, 2 \) respectively. Whereas for isospin \( I = 1 \) and spin \( S = 0, 1, 2 \) it takes values \( (\tau_1 \cdot \tau_j)(\sigma_i \cdot \sigma_j) = 2, 1, -1 \). The values of spin-isospin factor are from Ref. \([24, 25]\) and the other parameters used in pion exchange potential are from Ref. \([42–44, 47]\).

The spin dependent interaction potential added separately \([18, 19]\)

\[
V_{SD} = \frac{8}{9m_{h_1}m_{h_2}} S_1 \cdot S_2 |\psi(0)|^2
\]

where the spin factor \( S_1 \cdot S_2 \) can be found by general formula \( S_1 \cdot S_2 = \frac{1}{2}(S_1 + S_2)^2 - S_1^2 - S_2^2 \) \([18]\). We have used the hydrogenic like trial wave function such as,
Table 4  Binding energy and mass spectra of dimesonic \((q\overline{q} - \overline{q}q)\) systems with their \(J^{PC}\) values

| System   | \(I^G(J^{PC})\) | \(\mu\) (GeV) | B.E. (MeV) | Mass (GeV) | Exp. [1] (GeV) | State  |
|----------|-----------------|---------------|------------|------------|----------------|--------|
| PP-states|                 |               |            |            |                |        |
| \(K - \overline{K}\) | 0\(^+\)(0\(^++\)) | 0.2823 | -0.979 | 0.985 | 0.990 ± 0.020 | \(f_0(980)\) |
| \(\eta - \overline{\eta}\) | 0\(^+\)(0\(^++\)) | 0.3065 | -1.257 | 1.083 |                |        |
| \(\eta - \overline{\eta}\) | 0\(^+\)(0\(^++\)) | 0.3623 | -1.901 | 1.486 |                |        |
| \(\eta' - \overline{\eta}'\) | 0\(^+\)(0\(^++\)) | 0.4239 | -2.484 | 1.890 |                |        |
| PV-states|                 |               |            |            |                |        |
| \(\eta - \overline{\eta}\) | 1\(^+\)(1\(^++\)) | 0.3410 | -1.657 | 1.306 |                |        |
| \(\eta - \overline{\eta}\) | 0\(^+\)(0\(^++\)) | 0.3539 | -1.820 | 1.312 |                |        |
| \(K - \overline{K}\) | 0\(^-\)(1\(^--\)) | 0.3509 | -1.784 | 1.371 | 1.386 ± 0.019 | \(h_1(1380)\) |
| \(\eta' - \overline{\eta}'\) | 1\(^+\)(1\(^++\)) | 0.3394 | -1.638 | 1.373 |                |        |
| \(\eta' - \overline{\eta}'\) | 0\(^-\)(1\(^--\)) | 0.4070 | -2.690 | 1.540 |                |        |
| \(\eta' - \overline{\eta}'\) | 0\(^-\)(1\(^--\)) | 0.4016 | -2.291 | 1.710 |                |        |
| \(\eta' - \overline{\eta}'\) | 0\(^-\)(1\(^--\)) | 0.4117 | -2.420 | 1.716 |                |        |
| \(\eta' - \overline{\eta}'\) | 0\(^-\)(1\(^--\)) | 0.4712 | -3.296 | 1.944 |                |        |
| VV-states|                 |               |            |            |                |        |
| \(\rho - \overline{\rho}\) | 0\(^+\)(0\(^++\)) | 0.4000 | -2.344 | 1.472 | 1.505 ± 0.006 | \(f_0(1500)\) |
| \(\rho - \overline{\rho}\) | 0\(^-\)(1\(^--\)) | 0.3921 | -2.237 | 1.502 |                |        |
| \(\rho - \overline{\rho}\) | 0\(^+\)(2\(^++\)) | 0.3768 | -2.036 | 1.553 | 1.525 ± 0.005 | \(f_2^\prime(1525)\) |
| \(\rho - \overline{\rho}\) | 1\(^+\)(0\(^++\)) | 0.3793 | -2.068 | 1.483 |                |        |
| \(\rho - \overline{\rho}\) | 1\(^+\)(1\(^++\)) | 0.3816 | -2.101 | 1.506 |                |        |
| \(\rho - \overline{\rho}\) | 1\(^+\)(2\(^++\)) | 0.3869 | -2.168 | 1.554 |                |        |
| \(\omega - \overline{\omega}\) | 0\(^+\)(0\(^++\)) | 0.4017 | -2.360 | 1.487 |                |        |
| \(\omega - \overline{\omega}\) | 0\(^-\)(1\(^--\)) | 0.3939 | -2.255 | 1.517 |                |        |
| \(\omega - \overline{\omega}\) | 0\(^+\)(2\(^++\)) | 0.3787 | -2.055 | 1.567 | 1.562 ± 0.013 | \(f_2(1565)\) |
| \(K^- - \overline{K}^0\) | 0\(^+\)(0\(^++\)) | 0.4256 | -2.570 | 1.719 | 1.722\(^{+0.06}_{-0.08}\) | \(f_0(1710)\) |
| \(K^- - \overline{K}^0\) | 0\(^-\)(1\(^--\)) | 0.4138 | -2.479 | 1.744 |                |        |
| \(K^- - \overline{K}^0\) | 0\(^+\)(2\(^++\)) | 0.4055 | -2.036 | 1.789 |                |        |
| \(K^- - \overline{K}^0\) | 1\(^+\)(0\(^++\)) | 0.4077 | -2.334 | 1.727 |                |        |
| \(K^- - \overline{K}^0\) | 1\(^+\)(1\(^++\)) | 0.4099 | -2.263 | 1.747 |                |        |
| \(K^- - \overline{K}^0\) | 1\(^+\)(2\(^++\)) | 0.4143 | -2.420 | 1.789 | 1.722 ± 0.016 | \(a_2(1700)\) |
| \(\phi - \overline{\phi}\) | 0\(^+\)(0\(^++\)) | 0.4462 | -2.708 | 1.972 |                |        |
| \(\phi - \overline{\phi}\) | 0\(^-\)(1\(^--\)) | 0.4402 | -2.631 | 1.993 |                |        |
| \(\phi - \overline{\phi}\) | 0\(^+\)(2\(^++\)) | 0.4285 | -2.482 | 2.031 | 2.011\(^{+0.08}_{-0.09}\) | \(f_2(2010)\) |
| \(\rho - \overline{\rho}\) | 1\(^-\)(0\(^--\)) | 0.3803 | -2.078 | 1.490 | 1.474 ± 0.019 | \(a_0(1450)\) |
| \(\rho - \overline{\rho}\) | 1\(^+\)(1\(^++\)) | 0.3828 | -2.101 | 1.513 |                |        |
| \(\rho - \overline{\rho}\) | 1\(^+\)(2\(^++\)) | 0.3879 | -2.177 | 1.561 |                |        |
| \(\phi - \overline{\phi}\) | 1\(^-\)(0\(^--\)) | 0.4388 | -2.967 | 1.708 |                |        |
| \(\phi - \overline{\phi}\) | 1\(^+\)(0\(^++\)) | 0.4411 | -3.000 | 1.736 |                |        |
| \(\phi - \overline{\phi}\) | 1\(^+\)(1\(^++\)) | 0.4457 | -3.068 | 1.793 |                |        |
| \(\phi - \overline{\phi}\) | 0\(^+\)(0\(^++\)) | 0.4584 | -3.253 | 1.705 |                |        |
| \(\phi - \overline{\phi}\) | 0\(^-\)(1\(^--\)) | 0.4514 | -3.146 | 1.740 |                |        |
| \(\phi - \overline{\phi}\) | 0\(^+\)(2\(^++\)) | 0.4376 | -2.943 | 1.800 | 1.815 ± 0.012 | \(f_2(1810)\) |

\[ R_{nl}(r) = \left( \frac{\mu^2(n-l+1)!}{2^{2n}(n+l)!} \right)^{\frac{1}{2}} \langle \mu \rangle \langle \mu \rangle \left( e^{\mu^2 L^2_{n-l-1}(\mu)} \right) \]  

(9)

where \(\mu\) is the variational parameter and \(L^2_{n-l-1}(\mu)\) is the Laguerre polynomial. We have fixed the color screening parameter \(C = 0.15\) GeV for all the dimesonic combinations. The experimental (PDG) masses of the mesons are used for the present study, tabulated in Table 2. We have obtained the expectation value of Hamiltonian as

\[ H\psi = E\psi \]  

(10)

by employing the Ritz variational scheme for mass calculation and \(\mu\) is determined for each state using the virial theorem [37, 39].
Table 5 Root mean square radius and digamma decay width of dimesonic \((q\overline{q} - \overline{q}q)\) systems

| System | \(I^G(J^P)\) | \(R(0)\ (\text{GeV}^2)\) | \(\sqrt{<r^2>}\ (\text{fm})\) | \(\Gamma_{\gamma}\)\text{ KeV}\ | Exp. [1]\text{ KeV}\ | State |
|--------|---|---|---|---|---|---|
| PP-states | | | | | | |
| \(K - \overline{K}\) | 0\(^+(0^{++})\) | 0.1060 | 0.242 | 0.2260 | \(0.29^{+0.07}_{-0.06}\) | \(f_0(980)\) |
| \(\eta - \overline{\eta}\) | 0\(^+(0^{++})\) | 0.1200 | 0.233 | 0.2882 | | |
| \(\eta' - \overline{\eta}'\) | 0\(^+(0^{++})\) | 0.1542 | 0.188 | 0.2556 | | |
| \(\eta' - \overline{\eta}'\) | 0\(^+(0^{++})\) | 0.1951 | 0.161 | 0.3274 | | |
| PV-states | | | | | | |
| \(\eta - \overline{\pi}\) | 1\(^+(1^{--})\) | 0.1408 | 0.200 | 0.5082 | | |
| \(\eta' - \overline{\pi}'\) | 0\(^+(1^{--})\) | 0.1489 | 0.193 | 0.5340 | | |
| \(K - \overline{K}'\) | 0\(^-(1^{++})\) | 0.1469 | 0.194 | 0.1676 | | \(h_1(1380)\) |
| \(K - \overline{K}'\) | 1\(^+(1^{++})\) | 0.1398 | 0.020 | 0.1548 | | |
| \(\eta' - \overline{\eta}\) | 0\(^-(1^{--})\) | 0.1836 | 0.167 | 0.2480 | | |
| \(\eta' - \overline{\eta}\) | 1\(^+(1^{--})\) | 0.1800 | 0.170 | 1.5308 | | |
| \(\eta' - \overline{\eta}\) | 0\(^-(1^{--})\) | 0.1868 | 0.165 | 1.5159 | | |
| \(\eta' - \overline{\eta}\) | 0\(^-(1^{++})\) | 0.2287 | 0.145 | 1.5617 | | |
| VV-states | | | | | | |
| \(\rho - \overline{\pi}\) | 0\(^+(0^{++})\) | 0.1789 | 0.170 | 0.4233 | | \(f_0(1500)\) |
| \(\rho - \overline{\pi}\) | 0\(^+(1^{++})\) | 0.1736 | 0.174 | 0.5396 | | |
| \(\rho - \overline{\pi}\) | 0\(^+(2^{++})\) | 0.1635 | 0.181 | 0.7067 | \(0.081 \pm 0.009\) | \(f_2(1525)\) |
| \(\rho - \overline{\pi}\) | 1\(^+(0^{++})\) | 0.1652 | 0.180 | 0.4344 | | |
| \(\rho - \overline{\pi}\) | 1\(^+(1^{++})\) | 0.1668 | 0.179 | 0.5413 | | |
| \(\omega - \overline{\pi}\) | 0\(^+(0^{++})\) | 0.1800 | 0.170 | 0.2120 | | |
| \(\omega - \overline{\pi}\) | 0\(^+(1^{++})\) | 0.1748 | 0.173 | 0.2739 | | |
| \(K' - \overline{K}\) | 0\(^+(0^{++})\) | 0.1963 | 0.160 | 0.2690 | | \(f_0(1710)\) |
| \(K' - \overline{K}\) | 0\(^-(1^{++})\) | 0.1916 | 0.163 | 0.3320 | | |
| \(K' - \overline{K}\) | 0\(^+(2^{++})\) | 0.1826 | 0.168 | 0.6618 | | |
| \(K' - \overline{K}\) | 1\(^-(0^{++})\) | 0.1840 | 0.167 | 0.2740 | | |
| \(K' - \overline{K}\) | 1\(^-(1^{++})\) | 0.1855 | 0.166 | 0.3326 | | |
| \(K' - \overline{K}\) | 1\(^-(2^{++})\) | 0.1885 | 0.164 | 0.6843 | \(0.30 \pm 0.05\) | \(a_2(1700)\) |
| \(\phi - \overline{\phi}\) | 0\(^+(0^{++})\) | 0.2107 | 0.153 | 0.3319 | | |
| \(\phi - \overline{\phi}\) | 0\(^+(1^{++})\) | 0.2065 | 0.155 | 0.3939 | | |
| \(\omega - \overline{\phi}\) | 0\(^+(2^{++})\) | 0.1984 | 0.159 | 0.6486 | | \(f_2(2010)\) |
| \(\rho - \overline{\sigma}\) | 1\(^+(0^{++})\) | 0.1658 | 0.179 | 0.2170 | | \(a_0(1450)\) |
| \(\rho - \overline{\sigma}\) | 1\(^+(1^{++})\) | 0.1674 | 0.178 | 0.2734 | | |
| \(\rho - \overline{\sigma}\) | 1\(^+(2^{++})\) | 0.1708 | 0.176 | 0.7287 | | |
| \(\phi - \overline{\sigma}\) | 1\(^+(0^{++})\) | 0.2056 | 0.155 | 0.2942 | | |
| \(\phi - \overline{\sigma}\) | 1\(^+(1^{++})\) | 0.2072 | 0.154 | 0.3825 | | |
| \(\phi - \overline{\sigma}\) | 1\(^+(2^{++})\) | 0.2104 | 0.153 | 0.8025 | | |
| \(\phi - \overline{\sigma}\) | 0\(^+(0^{++})\) | 0.2194 | 0.149 | 0.2835 | | |
| \(\phi - \overline{\sigma}\) | 0\(^+(1^{++})\) | 0.2144 | 0.151 | 0.3767 | | |
| \(\phi - \overline{\sigma}\) | 0\(^+(2^{++})\) | 0.2047 | 0.156 | 0.8043 | | \(f_2(1810)\) |

\[\langle KE.\rangle = \frac{1}{2} \int r^2 V(r) dr\]  \( (11)\)

The one pion exchange potential for the meson–antimeson system in Eq. (6) is spin-isospin dependent. The spin-isospin dependency of one pion exchange have been discussed by Törnqvist for 'deusons' \[24, 25\]. We get attractive channel for \((S,I) = (1,1)\) for PV-states while repulsive for \((S,I) = (1,0)\) channel in VV-states, we get an attractive channel for \((S,I) = (0,0)\) \((1,0)\) \((2,0)\) while repulsive for \((S,I) = (0,1)\) \((1,1)\) \((2,0)\). The two Pseudoscalar could not be bound by Pseudoscalar due to parity violation \[23–25\], for such PP-states, we have not considered the one pion exchange potential. For the PV- states, the pion as a constituent is too light and gives large kinetic energy which is difficult to be bound by potential to bind a molecule \[25\].
we have not considered the pion for dimesonic system, as it requires relativistic treatment. The calculated masses are close to experimental measurements (PDG), tabulated in Table 4.

Since angular momentum, spin, isospin, parity all are conserved in strong interaction, be a good quantum number for dimesonic system. Thus, we can apply a usual coupling rule to dimesonic states. The orbital angular momentum of meson and antimeson are $L_1$ and $L_2$ respectively. In same way spin denotes by $S_1$, $S_2$ and isospin $I_1$, $I_2$. Employing the coupling rules, one has the relative orbital momentum and total spin of system $L_{12}$ and $S_{12}$. Whereas, the total angular momentum to be $J = L_{12} + S_{12}$. In the similar way, we have $I = I_1 + I_2$. The parity of the two particle system (meson–antimeson) is given by $P = P_1 P_2 (-1)^{L_{12}}$ [21, 22] while the charge conjugation be $C = (-1)^{I_{12} + S_{12}}$ [48] and $G$-parity define as $G = (-1)^{L_{12} + S_{12} + I}$. While one can refer ref. [21, 22, 48] for more detailed discussion about quantum numbers for normal mesons as well as for exotic states. The spin, isospin, angular momentum for some states are listed in Table 3.

2.1. Digamma width

The digamma decay width have been calculated on hadron loops employing an expansion in the range for force while the leading term assumes to a point like vertex. The formula with leading order correction [35] is being

$$\Gamma_\gamma = \frac{\zeta^2 \sqrt{m c}}{\pi} \left( \frac{2m}{m_{sa}} \right)^2 \arcsin \left( \frac{m_{sa}}{2m} - 1 \right)^2$$

(12)

$$\Gamma_\gamma(\beta) = \Gamma_\gamma \left( 1 + \frac{4m_{m1}m_{m2} - m_{sa}^2}{\beta^2} \right)$$

(13)

$\Gamma_\gamma$ is given by Eq. (12)

$$m_{sa} = 2m - \xi$$

(14)

where $\alpha = e^2/4\pi$ indicate fine-structure constant and the factor $\zeta = \frac{1}{\sqrt{2}}$, the $\beta = 0.8 \text{ GeV}$, $m$ is the mass of constituent meson, $m_{sa}$ is the mass of molecular meson and $\xi$ is binding energy. The digamma decay widths are tabulated in Table 5.

3. Results and discussion

We have employed the semirelativistic approach to calculate the masses and digamma decay width of the proposed dimesonic molecular states. Here, we have considered the three combination PP-states, PV-states and VV-states. Our results are tabulated in Tables 4 and 5. The Yukawa-like potential with sigma exchange and one pion exchange potential are used as an interaction potential for dimesonic systems. We have found that the contribution from Yukawa-like potential being approximately 80–85% (attractive), sigma exchange 12–15% (repulsive) and one pion exchange up to 3% (attractive or repulsive depending upon spin-isospin channel) to the net potential strength.

We have applied our model to $f_0(980)$ which is widely believed to be a $K - \bar{K}$ molecule in literatures [1]. The color screening parameter $C$ is the only the parameter which has fitted once, to get the approximate mass of the state $f_0(980)$. The calculated mass is found 985 MeV with approximately 9.8 MeV binding energy and both are close to experimental measurements [1]. In the Ref. [34], the binding energy is found nearly 20 MeV which is almost twice then in the present study (see Table 3, Ref. [34]), same with other dimesonic states. The digamma width and mass is being found close in agreement with experimental results [1]. Thus, we have used this model to study other dimesonic combinations. The various dimesonic states are very close to the experimentally observed states in accordance to their masses.

The molecular state should have small binding and large radius compared to the size of their constituents. We have found radius of all combinations near ~2 fm to ~4 fm with small binding energy, see Table 4 (negative sign shows the bound state). The various states like $h_1(1380)$ in PV-state, $f_0(1500), f^*_2(1525), f_2(1565), f_0(1710), a_2(1700), a_0(1450), f_2(1810), f_0(2010)$ in VV-states are compared with dimesonic states. The masses of the states $f_0(980), h_1(1380), f_2(1565), f_0(1710), a_0(1450), f_2(2010), a_2(1810)$ are in excellent agreement with the PDG listing [1]. Moreover, some states like $f_0(1500)$ and $a_2(1700)$ compared with $\rho - \pi$ and $K^- - \bar{K}^0$ having 20 to 50 MeV difference in mass compared to experimental measurement [1]. The quantum numbers for the dimesonic system are assigned as discussed in the previous section. The digamma decay width for the states $f_0(980)$ and $f_2(1565)$ are in agreement with experimental values, while for the state $a_2(1700)$ has been overestimated, see Table 5.

4. Conclusions

On the basis of mass spectra and quantum numbers, we are able to identify states having promising non $q\bar{q}$ structures. The calculated masses and digamma widths of the compared states are close to the experimental measurements [1] except few states like $f_0(1500), f^*_2(1525)$ and $a_2(1700)$ (see Tables 3 and 4). Moreover, the states $f_0(1500), f^*_2(1525)$ and $f_0(1710)$ have been found in the literature with the possibilities of other structures like glueball or tetraquark.
Whereas, in the Ref. [1, 34, 34] the state $f_2(1525)$ and $f_0(1710)$ have been predicted as a promising VV molecule. Our calculated mass spectra of the state $f_2(1525)$ and $f_0(1710)$ are in agreement with [1, 33, 34] and suggest it as VV-molecule. Even more, some states like $a_0(1450)$ and $f_2(1810)$ have not confirmed place in the PDG list [1].

In the framework of the present study, we have only incorporated the pion and sigma exchange as SU(2) symmetry. The tensor and spin-orbit coupling may have significant contribution which are very useful for $L \neq 0$ as well as for mixed states. To confirm the molecular structures of all these states, one has to study other properties like decay width, possible coupling, etc. Our aim of the present study is to give the attention to all these states which may have a possible molecular internal structure and the predicted mass spectra will provide reference for future study of molecular dimesonic states. We will take this study one step further ahead by including other mesons contributions with possible S-D wave mixing, as our future study.

Finally, in this paper, we are able to calculate the S-wave masses and digamma decay widths of the dimesonic states in the light meson sector. We would like to improve and implement this model to calculate heavy flavour mesons as well as for higher excited states.

Acknowledgments A. K. Rai would like to thanks Prof. Atsushi Hosaka from Osaka University for the useful discussion during the Hadron Spectroscopy 2013, and also acknowledge the financial support extended by D.S.T., Government of India under SERB fast track scheme SR/FTP/PS-152/2012.

References

[1] K A Olive et al. Chin. Phys. C 38 090001 (2014)
[2] A Ali PoS BEAUTY 2011 002 (2011) arXiv:1108.2197 [hep-ph]
[3] M Gersabeck Mod. Phys. Lett. A 27 1230026 (2012)
[4] S L Olsen Prog. Theor. Phys. Suppl. 193 38 (2012)
[5] L Li Nucl. Phys. Proc. Suppl. 225 107 (2012)
[6] R L Jaffe Phys. Rev. D 15 281 (1977)
[7] J Weinstein and N Isgur Phys. Rev. Lett. 48 659 (1982)
[8] J Weinstein and N Isgur Phys. Rev. D 27 588 (1983)
[9] J Weinstein and N Isgur Phys. Rev. D 41 2236 (1990)
[10] Y R Liu and Zong-Ye Zhang Phys. Rev. C 80 015208 (2009)
[11] M Ablikim et al. Phys. Rev. D 87 092009 (2013)
[12] H-B Li (BESIII collaboration) EPJ Web of Conf. 72 00011 (2014)
[13] L Dong arXiv:1310.4409v1 [hep-ph] (2013)
[14] G S Adams et al. Phys. Rev. Lett. 81 5760 (1998)
[15] E I Ivanov et al. Phys. Rev. Lett. 86 3977 (2001)
[16] J Kuhn et al. Phys. Lett. B 595 109 (2004)
[17] D S Carman Jour. of Phys.: Conf. Series 69 012011 (2007)
[18] A K Rai, J N Pandya and P C Vinodkumar Indian. Phys. A 782 406 (2007)
[19] A K Rai, J N Pandya and P C Vinodkumar Indian. Phys. A 80 378 (2006)
[20] S Patel, M Shah and P C Vinodkumar Eur. Phys. J. A 50 131 (2014)
[21] B Silvestre-Brac and C. Semay Z. Phys. C 57 273–282 (1993)
[22] B Silvestre-Brac and C. Semay Z. Phys. C 59 457–470 (1993)
[23] C Amsler and N A Törnqvist Phys. Report 389 61 (2004)
[24] N A Törnqvist Phys. Rev. Lett. 67 556 (1991)
[25] N A Törnqvist Z. Phys. C 61 525 (1994)
[26] G-J Ding, J-F Liu and M-L Yan Phys. Rev. D 79 054005 (2009)
[27] T Branz, T Gutsche and V E Lyubovitskij Phys. Rev. D 78 114004 (2008)
[28] J-R Zhang and G-F Chen Phys. Rev. D 86 116006 (2012)
[29] J-R Zhang, L-F Gan and M-Q Huang Phys. Rev. D 85 0116007 (2012)
[30] X L Ren, Li S Geng, E Oset and J Meng Eur. Phys. J. A 50 133 (2014)
[31] C Hidalgo-Duque, J Nieves and M P Valderrama Phys. Rev. D 87 076006 (2013)
[32] W L Wang and Z Y Zhang Phys. Rev. C 84 054006 (2011)
[33] J-J Xie, E Oset and L-S Geng Phys. Rev. C 93 025020 (2016)
[34] A K Rai and D P Rathaud Eur. Phys. J. C 75 462 (2015)
[35] C Hanhart, Y’ S Kalashnikova, A E Kudryavtsev and A V Nefediev Phys. Rev. D 75 074015 (2007)
[36] R Z, Y-B Ding, X-Q Li and P R Page Phys. Rev. D 65 096005 (2002)
[37] N Devlani and A K Rai Eur. Phys. J. A 48 104 (2012)
[38] N Devlani and A K Rai Eur. Phys. J. A 50 154 (2014)
[39] N Devlani and A K Rai Phys. Rev. D 84 074030(2011)
[40] N Devlani and A K Rai Int. J. Theor. Phys. 52 2196 (2013)
[41] D Ebert, R N Faustov and V O Galkin Phys. Rev. D 79 114029 (2009)
[42] S Takeuchi, V E Lyubovitskij, T Gutsche and A Faessler Nucl. Phys. A 790 502c–505c (2007)
[43] M Furuiuchi, K Shimizu and S Takeuchi Phys. Rev. C 65 025201 (2002)
[44] M Furuiuchi, K Shimizu and S Takeuchi Phys. Rev. C 68 034001 (2003)
[45] M W Paris and V R Pandharipande Phys. Rev. C 62 015201 (2000)
[46] V R Pandharipande, S C Pieper and R Schiavilla (their notes on nuclear forces on light nuclei) http://minimafisica.biodec.com/ Members/k/Schiavilla-notes
[47] T E O Ericson and G Karl Phys. Lett. B 309 426 (1993)
[48] C A Meyer and Y Van haarlem Phys. Rev. C 82 025208 (2010)
[49] F Brunner D Parganlija and A Rebhan Phys. Rev. D 91 106002 (2015); arXiv:1504.05815 [hep-ph]