Sensitivity of quark matter EoS parameters and quark star properties.

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Abstract. Understanding of inter-quark interactions is very important not only from high energy physics but also from one of the most enigmatic problem of astrophysics: “The existence of deconfined quark matter inside the neutron star”. Presently, equation of state (EoS) based on phenomenological models for quark interactions are employed to describe deconfined quark matter. In the work presented in this paper we consider two of the popularly employed models for quark matter EoS. These models have certain parameters, known only in a range of values. This leads to a situation in which we get, set of EoS for a given model. It is interesting to study, qualitatively and quantitatively, the influence of these parameters on EoS and strange stars properties obtained using them. We define a dimensionless quantity “sensitivity” and study the influence of these parameters on EoS and on properties of strange star. The important conclusions regarding the influence of EoS parameters, in various range is presented.

1. Introduction
Study of true nature of ultra dense compact object called neutron star is of great importance not only in astronomy but also in the field of high energy physics. It is widely accepted that hadronic matter can undergo a phase transformation to quark matter at very high density. The core of a massive neutron star is one of the likely place where deconfined phase of the quark matter could be found. Witten, in 1984, proposed that strange matter could be the true ground state, and if true then no barrier would exist for the conversion of the entire star, once core has converted to strange matter \cite{1}. In this way neutron stars would be converted to strange stars. Such possibilities have renewed interest in study of compact stars like neutron stars and strange stars.

Since we do not have a rigorous theory for quark matter, one has to employ various phenomenological models to study strange star. We have considered for the present study, the popular models like MIT-Bag model (MIT-Bag) \cite{2} and Effective Bag (E-Bag) model \cite{3} in the study of strange stars.

2. MIT-Bag Model
Bag model \cite{4} is one of the earliest relativistic confinement models. In the MIT-Bag model, strange matter in bulk is parameterized by bag constant $B$, quark mass $m_f$ and the QCD coupling $\alpha_s$, renormalized at a mass scale $\sigma$ \cite{2}. The thermodynamic potential ($\Omega_f$) to first
order in the strong interaction coupling constant ($\alpha_s$) can be given as [2]

$$\Omega_f = -\frac{\gamma_f}{24\pi^2} \left\{ \frac{\mu_f k_f}{m_f} \left( \frac{\mu_f^2}{2 m_f^2} - \frac{5}{2 m_f^2} \right) + \frac{3}{2} m_f^4 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right. $$

$$ - \frac{2\alpha_s}{\pi} \left\{ 3 \left( \frac{\mu_f k_f}{m_f^2} - m_f^2 \ln \left( \frac{\mu_f + k_f}{\mu} \right) \right)^2 - 2k_f^4 - 3m_f^4 \ln^2 \left( \frac{m_f}{\mu_f} \right) \right\} + 6 \ln \left( \frac{\sigma}{\mu_f} \right) \left( \frac{\mu_f k_f m_f^2}{m_f^2} - m_f^2 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right) \right\}$$

(1)

where $\gamma_f = 2 \times 3_{\text{color}}$ is the degeneracy, $\mu_f$ & $m_f$ is chemical potential and mass for quark of flavor $f$ ($f = u, d, s$) and $k_f = \sqrt{\mu_f^2 - m_f^2}$. Here $\sigma$ is take as $\sigma = 300$ MeV [2].

3. Effective Bag Model

In effective bag model (E-Bag), Schertler et al [3] treated cold strange quark matter as an ideal gas of quasiparticles with effective mass. They considered up, down, and strange quarks with finite chemical potential at zero temperature instead of gluons at finite temperature. The effective mass ($m_f^*$) for quark of flavor $f$ is obtained from the zero momentum limit of quark dispersion relation which is given by [3].

$$m_f^* = \frac{m_f}{2} + \sqrt{\frac{m_f^2}{4} + \frac{g^2 \mu_f^2}{6\pi^2}}$$

(2)

where $g$ is a medium parameter.

4. Calculations and Results

In both the models, MIT-Bag and E-Bag, employed in the present study, apart from parameters like $m_f, \sigma, \alpha_s, g...$, one more parameter, bag constant ($B$) also appears in EoS [2, 3]. Since, there are allowed range for the numerical values for these parameters, it is interesting to study effects (sensitivity) of these parameters on EoS as well as on strange star properties obtained using them. For this we define the relative sensitivity of a physical property, say $X$ (which could be pressure in an EoS or mass/radius of neutron star,...) to variation of parameters of the EoS, say $Y$ (which could be Bag constant, coupling constant, medium parameter etc.) as

$$\text{Sensitivity} = \frac{\Delta X / X}{\Delta Y / Y}$$

(3)

The sensitivity defined as above will not only give the idea of influence of these parameters, but can also serve as means to compare different sensitivity as it is a dimensionless number (eg. sensitivity of neutron star mass to variation in bag constant can be compared with sensitivity of radius of neutron star to variation in coupling constant). In calculation of sensitivity, the choice of $\Delta Y$ range must be taken carefully. We consider the choice of $\Delta Y$ based on two criterions. One, that it should not be very small so that the error blows up in calculation of $\Delta X / X$. Secondly, it should not be that large where the linearity breaks.

In this paper, we study the influence of model parameters $B, \alpha_s,$ and $g$ as their value have large uncertainties and present our study of sensitivity, as defined in eq (3). In figure 1, we plot sensitivity of pressure (property of EoS, at energy density 1000 MeV fm$^{-3}$), mass and radius (properties of strange star with central energy density 1000 MeV fm$^{-3}$). The solid line shows the sensitivity of pressure to variation in parameter $\alpha_s$ of MIT-Bag and dash-dot line shows variation with $g$ of E-Bag. While calculating sensitivities to these parameters ($\alpha_s$ & $g$) other
parameters are kept constant. We observe very strong dependence of pressure on \( \alpha_s \), pressure in MIT-Bag, increases with increase in \( \alpha_s \). While as \( g \) increases the pressure in E-Bag model decreases and the dependence is not that strong. It is further observed that the mass of strange star is more sensitive than it’s radius in both the cases. In figure 2, we plot sensitivity of pressure (property of EoS, at energy density 1000 MeV fm\(^{-3}\)), mass and radius (properties of strange star with central energy density 1000 MeV fm\(^{-3}\))to variation in bag constant \( B \). It is observed that both the models behave similarly with E-Bag being little more sensitive than MIT-Bag. Here again mass is more sensitive than radius to variation in \( B \) in both the models. Sensitivity to \( B \) is negative as expected. With increasing \( B \) both EoS becomes softer, and the sensitivity of both the EoS show similar behavior.

We conclude from our observations that the sensitivity of pressure and strange star properties like mass and radius with respect to the E-Bag model parameter, \( g \) is very less compared to MIT-Bag model parameter \( \alpha_s \). Thus results of E-bag model will not change much due to uncertainty in the value of its parameters.

![Figure 1](image1.png)

**Figure 1.** Sensitivity of pressure, mass and radius to variation in \( \alpha_s \) (MIT-Bag) and with \( g \) (E-Bag).

![Figure 2](image2.png)

**Figure 2.** Sensitivity of pressure, mass and radius to variation in bag constant \( B \) for MIT-Bag and E-Bag.

**References**

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