Investigating problem-solving process of pre-service mathematics teachers who are truth-seekers

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Abstract. The present research aims to investigate the problem-solving process of pre-service Mathematics teachers who are truth-seekers. The process of problem-solving is the stages in solving two types of problems, namely the problems with contradictory information and problems with no universal set. The participants are four Mathematics education students in the University of Jember who perform truth-seeking. The research stages comprised of (1) giving four problems and recording the participant's behavior during solving the problems, (2) analyzing the answer sheets and recordings, (3) doing triangulation through interviews, and (4) summarizing the tendency of the problem-solving process of pre-service mathematics teachers who are truth-seekers. The results of this research are (1) when solving the problems with contradictory information, participants tend not to proceed to the process of determining the solution of the problem, and (2) when solving the problems with no universal set is given, participants tend to determine the universal set before solving the problems. Therefore, the trends of the solving-problem process of the pre-service Mathematics teachers who are truth-seekers are not to determine the solution if the problems have wrong information and will add the universal set if the informations in the problems have not the universal set.

1. Introduction
Critical thinking is one of the important skills Indonesian students need to master so as to face the era of 4.0 Industrial Revolution and 21st century learning [1], [2]. Indonesian universities need to develop students' cognitive capacity, i.e. higher order thinking skill, critical thinking skill, and systematic skill as an endeavour to help them survive in the era of 4.0 Industrial Revolution [1]. Furthermore, learning in the 21st century also focuses on the development of four abilities encompassing creative thinking, critical thinking, communicative competence, and collaborative competence, commonly known as 4C's [2]. In addition, critical thinking is a means of making a better future for everyone [3] and critical thinking is an essential skill to keep up with the era of reform. [4]

Critical thinking is reflective and rational thinking that is focused on the process of deciding what to do or to believe [5]. This is in line with the opinion of other experts who contend that critical thinking is a skilful and responsible thinking in making the best decisions [6]. Based on the definition of critical thinking, every critical thinker always performs the best decision making process when working on a non-routine problem. The decision-making process is also performed by a mathematician who is a critical thinker when dealing with mathematical problems.

Critical thinking is divided into two components, namely the ability to think critically and critical thinking dispositions [5], [7]. The critical thinking disposition, especially the truth-seeking component, strongly influences a person’s reflective thinking in the process of making decisions to
solve problems [8], [9]. Truth-seeking is a habit of someone who always wants the best understanding of a particular situation, stressing the evidence and reasoning even on the things that have been acknowledged to be true, questioning an established person's belief, and not ignoring the important details [10].

There are few Mathematics teachers in Indonesia who are accustomed to performing truth-seeking, for example in East Java, particularly in the University of Jember and the State University of Malang [11], [12]. Therefore, a very unique Mathematics teacher candidate is accustomed to performing truth-seeking because many mathematician candidates are not truth-seekers. However, there has been no research observing the problem solving process of Mathematics teacher candidates who are passionate about being truth-seekers. It is therefore necessary to conduct research to investigate the problem-solving process as a decision-making process by Mathematics teacher candidate capable of becoming truth-seekers. This denotes the attempt to familiarize and develop truth-seeking competence on other mathematics teacher candidates. The first process of solving a mathematical problem is is to find a solution between difficulties, between obstacles, find solutions to unknown mathematical problems and clarify information from mathematical problems [13].

Not all types of problems can be employed to track the problem solving process of Mathematics teacher candidates who are accustomed to performing truth-seeking. This is because prospective teachers capable of truth-seeking can be traced if there is incorrect or contrary information or conditions in a problem. Such problem requires a deep analysis before working on the problem. Through this type of problem-solving, mathematician candidates are able to show the truth-seeking indicators through their behaviour or action when working on the given problem. In addition, problems that do not have complete information will also accrue the truth-seekers habit of Mathematics teacher candidates. This is because the Mathematics teacher candidates with truth-seekers tendency will determine any missing information in a problem beforehand. Thus, the type of problem that can be used to determine whether Mathematics teacher candidates possess the habit of truth-seekers is the problem with contradictory information and problems with no universal set.

Based on the explanation, the purpose of this study is to describe and examine the problem solving process of Mathematics teacher candidate possessing the habit of truth-seekers when solving problems with contradictory information and problems with no universal set. The benefits the present study attempt to gain are (1) developing the basis for designing problem-based learning to familiarize prospective Mathematics teachers with truth-seeking, (2) setting the foundation for composing problems and Mathematics tasks that can be used as an instrument for tracing the truth-seeking Mathematics teacher candidate, and (3) setting the basis for determining the stages of decision-making process Mathematics teachers capable of truth-seekers have to go through.

2. Literature Review
A person with a critical thinking disposition is one who bases his action on critical thinking. Before carrying out the action, he or she tends to think about everything that has to do with the action beforehand [13]. The disposition of critical thinking is divided into seven components, which include truth-seeking, open-mindedness, inquisitiveness, self-confidence, analyticity, systematicity, and maturity [10]. Of the seven components of critical thinking disposition, at least one component must be mastered by a person with a critical thinking position, that is truth-seeking [8]. There are seven traits of truth-seeking people, inter alia, (1) seeking a clear statement of theory or question, (2) seeking or scrutinizing argument, (3) trying to find the best information, (4) using credible source and (5) seeking as much precision as possible for justification of a material, (6) attempting to remain relevant to the main points, (7) keeping in mind the original and/or basic problems [5]. Accordingly, truth-seeking is a habit of a person who (1) always wants the best understanding of a particular situation, (2) places great emphasis on proofs and reasoning even on things that are already acknowledged, (3) questions one’s firm belief, and (4) never ignores important details [10]. In this study, the participants, candidates for Mathematics teachers who do truth-seeking,
are determined based on the four traits of truth-seekers proposed by Insight Assessment and Facione. The Mathematics teacher candidates with truth-seeking habit master diverse types of problem-solving process, but they have some commonality, that is always focusing on the best understanding of problem. There are some understandings of problem solving according to the expert. Solving mathematical problems such as finding a way between difficulties, between obstacles and finding solutions to unknown mathematical problems [10]. Furthermore, problem solving is the ability that a person has in responding or giving action when faced with a problem. A good problem solver will identify exactly what the problem is, what might be an obstacle to solving it, and what solutions might be expected to work [4], [14]. In this study, the indicators of problem solving process which Mathematics teacher candidates capable of truth-seeking deal, when solving the problems with contradictory information and problems with no universal set, are shown in Table 1.

Table 1. Process indicators of problem solving for truth-seekers mathematics teachers.

| No | The Indicators of Truth-Problem-solving Process |
|----|-----------------------------------------------|
| 1  | The habits of always wanting question the best understanding of a particular situation | a. Trying to understand a problem from different point of view  
b. Reexamining logical truth of the information given in the question  
c. Always looking at any particular situation words, symbols, or terms shown in problem  
d. Always paying attention to the logical truth of the information provided and the universe of speech |
| 2  | Emphasizing the evidence and reasoning even to the things that have been confirmed | a. Using his logic and reasoning to check the truth of information provided  
b. Always questioning whether every step taken is a logical consequence of the previous step based on logical reasoning and logic  
c. Checking whether the generated response is appropriate or not to the requested and logical and whether there is any other answer than the one generated |
| 3  | Questioning one’s firm belief | a. questioning the universe of a problem beforehand  
b. considering all possible universe of speech from a problem |
| 4  | Not ignoring important details | a. always checking the truth of every word, symbol, and term in question.  
b. checking the truth of the statement by setting its limits |

3. Methods

3.1 Participants

On April 17, 2018, a trial was conducted to 50 students of 6th semester of Mathematics Education Study Program at the Universitas of Jember by giving 6 non-routine questions. Based on the results of the trials, there were four students who always performed truth-seeking when working on the six problems. Therefore, the four students were referred to as participants in this study. In addition, the four students also mastered good communication skills. The purpose of selecting four truth-seekers students was to know the tendency of problem-solving process that they performed when working on four non-routine problems, i.e. the problems with contradictory information and problems with no universal set.
3.2 Instruments
The instruments in this study were (1) four non-routine mathematical problems with two problems containing contradictory information and two problems with no universal set, (2) observation sheets to observe the behaviour and problem-solving process of the participants when solving four problems, 3) semi-structured interview sheets used to confirm behavioural trends and problem-solving processes. All three instruments were tested for validity and reliability before being used in this study. Content validation of questions, observation sheets, and interview sheets were conducted by Mathematics education experts and critical thinking disposition specialists. All three research instruments were revised based on experts’ advice. Instrument reliability was examined using SPSS, and the Cronbach alpha was 0.8.

3.3 Procedures and Data Analysis
This research was analyzed and described qualitatively by representing the tendency of problem-solving process of Mathematics teacher candidate performing truth-seeking when solving problem with contradictory information and problems with no universal set. The data in this study were analyzed by six stages, which included data collection, data collection, encoding data, descriptive analysis, sustainability analysis, and interpretation of findings, and validation. There were four procedures employed to analyze the data in this study. First, giving four problems and recording the participant’s behaviour during solving the problems. The four questions are as follows.

a. Given an isosceles triangle EAB with AB = EB, \( \angle E = 70^\circ \) and \( \angle B = 45^\circ \). BD lines are created parallel to the AE line, then determine the angle of the DBC. (The problems with contradictory information)
b. Given \( x, y, z \in \mathbb{R} \), with \( x = 5 \) and \( y = -6 \) and \( z = 4 \). Determine the value \( x^3 + y^4 - z^3 \). (The problems with contradictory information)
c. Determine the range of \( f(x) = x^2 \). (The problems with no universal set)
d. Find the solution of the quadratic equation \( 3x^2 - 12 = 5 \). (The problems with no universal set)

Second, the study also dealt with analyzing the answer sheets and video recordings of participants during the problem. The focus of analysis was the tendency of problem-solving process when solving two types of non-routine questions. Third, the researchers also conducted triangulation through interviews. Triangulation was conducted to confirm the findings of the analysis of answer sheets and observations based on the video recording to the participants. The interview was done until the data obtained was saturated, which meant that the data regarding the problem-solving process indicated the same tendency of all the truth-seekers teacher candidates. The code in the interview was \( R = \text{Researcher} \) and \( P = \text{Participant} \). Fourth, the researchers summarized the tendency of the problem-solving process of pre-service Mathematics teachers with truth-seeking capability. The conclusions obtained in this study were based on analytical results of answer sheets, video recordings and interviews.

4. Result and Discussion

4.1 Problem solving process by a truth-seekers mathematics teacher candidate when completing the problems with contradictory information
Participants in this study completed two problems with contradictory information. The first question is set as follow: Given an isosceles triangle EAB with AB = EB, \( \angle E = 70^\circ \) and \( \angle B = 45^\circ \). BD lines are created parallel to the AE line, then determine the angle of the DBC. The second question is set as follow: Given with \( x, y, z \in \mathbb{R} \), with \( x = 5 \) and \( y = -6 \) and \( z = 4 \). Determine the value \( x^3 + y^4 - z^3 \).

The four participants completed the two questions given by writing down all the problem-solving processes. For question number 1, the four participants conducted analysis by writing the steps to determine the angle of A by using angular properties in an AEB triangle. Furthermore, the four
participants also determined the angle $A$ by using the equilateral triangle trait of the side. Based on two ways of determining the angle of $A$, which was by using angular properties in triangle and triangle of foot, the analysis indicated that the angle $A$ was different, $65^\circ$ and $70^\circ$. Therefore, the four participants concluded that the problem given was wrong because there were two contradictory information. The result of information on the problem related to mutual contradiction was the illogical conclusion. Furthermore, the four participants did not continue the process of resolving the first question because the problem was an illogical one and would lead to a false conclusion. This was in line with the interview conducted on the same date after the participants completed the question number 1. Hereunder is the interview excerpt.

R: In your opinion, what can you conclude about number 1?

P: Problem number 1 is a wrong question because there is information that contradicts to each other.

R: Why do you say that question number 1 is a wrong question?

P: Because after I set the angle $A$ in two ways, the result is different, the one producing the angle $A = 65^\circ$ and the other way producing the angle $A = 70^\circ$.

R: What do you do when you find the contradictory information?

P: I am not continuing the process of solving the problem because the problem does not have a solution because the problem is wrong.

One example of the participant’s answer to question 1 can be seen in Figure 1.

Transcribed Version

\[ \triangle AEB, \angle E = 70^\circ, \angle B = 45^\circ, \text{ then } \angle A = 180^\circ - (70^\circ + 45^\circ) = 180^\circ - 115^\circ = 65^\circ \]

Because $AB = EB$ and $\triangle AEB$ is the isosceles triangle, so $\angle A = \angle E$. But $\angle A = 65^\circ \neq \angle E$.

So, the problem is wrong such that I can’t find $\angle DBC$.

Figure 1. One of four Participants’ answers for question number 1.

For question number 2, the four participants had the same answer stating that there was misinformation in the problem, i.e. $y^4 = -1$.

6. Since it was not true that the real number raised 4 would produce a real negative number, it would generate a positive real number. Therefore, the four participants did not continue the problem-solving process to determine the solution. In this case, by referring to the recording of the behavior of the four participants, it was found that the four participants always analyzed the information in the question before they wrote it on the answer sheet beforehand. Therefore, the four participants always performed truth-seeking process before working on the given questions. In addition, they also pondered all the information in the matter carefully and correctly with reference to the logical reasoning and evidence applicable in Mathematics. One example of the participant's answer to question 2 can be seen in Figure 2.
1. **Problem solving process by a truth-seekers mathematics teacher candidate when completing the problems with no universal set**

Participants in this study completed two problems with no universal sets. On the first question, they had to determine the range of functions \( f(x) = x^2 \). On the second one, they had to find the solution of the quasiating equation \( 3s^2 - 12 = 5 \).

The four participants when solving the problem number 3 wrote that the given problem was incomplete because the universal set was not specified. Therefore, the range of function varied, depending on the domain specified. In this case, participants took note of important matters supported by logical reasoning and evidence relating to function. Also, based on the recording, participants said that question number 3 was not determined by the universal set. They knew it before writing down the problem solving process of number 3. One of the participants wrote three domains: (1) if the domain is a real number then the range is a positive real number, (2) if the domain is a real negative number then the range is a positive real number, and (3) if the domain is an integer then the range is an integer. The example of participants’ answers when working on question number 3 can be seen in Figure 3.

**Figure 2.** One of the participants’ answers on question number 2.

**Translated Version**

\[ x, y, z \in \mathbb{R}, x = 5, \]
\[ y^3 = -6, z = 4 \]
\[ y^4 = -6 \text{ is wrong. If } R \text{ are the real numbers, then } R^* \text{ always the positive real numbers.} \]

So, I can’t find the value of \( X^2 + Y^3 - Z^2 \).

**Figure 3.** One of the participants’ answers on question number 3.

This is also in line with the results of interviews conducted on the same day after working on the problem. Hereunder is the interview with one of the participants.

R: What can you conclude from question number 3?

P: Problem number 3 has incomplete information that is universal set, so before working on question 3, I must define universal set.

R: Will the determination of universal sets affect the problem solving process? Please explain your answer.

P: Yes, for sure. Because when the domain of the problem is different then the range is also different. Suppose the domain is a real number, then the range is a positive real number. If the
If \( s \in \mathbb{R} \), then there are no values of \( s \) satisfying \( 3s^2 - 12 = 5 \).

If \( s \in \mathbb{Z} \), then \( s = \pm \frac{\sqrt{17}}{3} \).

Because in the task, there is no value of \( s \) so the values of \( s \) that satisfying \( 3s^2 - 12 = 5 \) are following.

\[ s = \pm \frac{\sqrt{17}}{3} \]

The one of the participants’ answer in solving the problem number is presented in figure 4.

**Figure 4.** One of the participants’ answers on question number 4.

The findings of this study indicate that (1) when solving the problems with contradictory information, participants tend not to proceed to the process of determining the solution of the problem, and (2) when solving the problems with no universal set is given, participants tend to determine the universal set before solving the problems. The tendency of Mathematics teacher candidates when solving the problems with contradictory information and problems with no universal set is to always check the truth and completeness of the information given in the problem. This is in line with the results of studies which reveal that a truth-seeker person always questions all the information in the question and does not necessarily trust information given [7], [10], [12], [15], [16]. However, Mathematics teacher candidates who are truth-seekers are always cautious when solving mathematical problems. They are accustomed to their past experiences and who gives the problems. Thus, the factor that influences Mathematics teacher candidate to perform truth-seeking is the desire to behave truth-seeking [17]. Based on the finding, familiarizing the Mathematics teacher candidate to do truth-seeking requires (1) familiarizing them to work on non-routine problems on regular basis and (2) applying learning strategy that focuses on developing the desire to behave that is infusion approach [18].

5. Conclusion

The study has concluded that the Mathematics teacher candidates do not continue the process of problem solving and determine the solution of the problem with contradictory information. That is because they always emphasize on logical evidence and reasoning, so they claim that problems with contradictory information is an illogical problem because there are several contradictory statements. Furthermore, when solving problems with no universal set, they tend to determine the universal set first before working on the problem. This is because the Mathematics teacher candidates capable of truth-seeking are always careful when solving mathematical problem by not ignoring the important details of a question. To conclude, the study has concluded that the prospective Mathematics teachers with truth-seeking behaviour always check the truth and completeness of the information in the question before working on the problems with contradictory information and problems with no universal set.
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