Gravity Mediation of Supersymmetry Breaking with Dynamical Metastability

Izawa K.-I.\textsuperscript{1,2}, Fuminobu Takahashi\textsuperscript{2}, T.T. Yanagida\textsuperscript{2,3}, and Kazuya Yonekura\textsuperscript{3}

\textsuperscript{1}Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{2}Institute for the Physics and Mathematics of the Universe, University of Tokyo, Chiba 277-8568, Japan
\textsuperscript{3}Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

Abstract

We argue that the Polonyi problem can be avoided when our supersymmetry-breaking vacuum is surrounded by many supersymmetric vacua. We construct a dynamical class of supersymmetry-breaking models to demonstrate our point. These models naturally predict a small deviation from the standard big-bang nucleosynthesis.
1 Introduction

Theoretical landscape such as in string/M theory is expected to possess an enormous number of metastable vacua in addition to supersymmetric (SUSY) ones. With such vacua sufficiently long-lived in hand, we are led to suspect that our SUSY-breaking state is indeed metastable.

The possibility we pursue in this paper is that the Polonyi problem in gravity mediation of SUSY breaking can be avoided when our SUSY-breaking vacuum is surrounded by many SUSY vacua. Then, the initial value of the Polonyi field $S$ should be chosen not too far away from our vacuum, since otherwise, the $S$ would roll down to the SUSY vacua.

We provide a class of dynamical models to demonstrate the above point, which may be of some interest in its own right.

2 Dynamical Models

Let us consider supersymmetric QCD-like theory of the gauge group $SU(N_C)$ and $N_F$ quark flavors $Q_i, \tilde{Q}_i \ (i = 1, \cdots, N_F$; the gauge indices omitted) with a superpotential

$$W = \lambda S \sum_i Q_i \tilde{Q}_i,$$

where $S$ is a singlet chiral superfield and $\lambda, N_C, N_F$ are positive constants of order one.

With the other fields integrated out, the effective superpotential of $S$ is given by

$$W_{\text{eff}} = \frac{\lambda^3}{16\pi^2} \left( \frac{\lambda S}{\Lambda} \right)^{\frac{N_F}{N_C}},$$

where $\Lambda$ denotes a dynamical scale of the gauge interaction and we adopt a naive dimensional analysis [1].

The effective Kähler potential may be expressed as

$$K_{\text{eff}} = |S|^2 - \sum_{n=1}^N \frac{k_n}{16\pi^2(n+1)^2 \Lambda^2} |\lambda S|^{2(n+1)},$$

for $|\lambda S| \lesssim \Lambda$ with $k_n$ of order one[1].

[1] We abuse the notation of $S$ for its lowest component.
On the other hand, the perturbative Kähler potential is given by

\[ K_{\text{eff}} = |S|^2 - A|\lambda S|^2 \ln \left| \frac{\lambda S}{\mu} \right|^2 + \cdots, \quad (4) \]

for \(|\lambda S| \gg \Lambda\), where \(\mu\) is a renormalization point and \(A = N_C N_F / 16\pi^2\).

### 3 Metastable Supersymmetry Breaking

For an illustrative purpose, we only keep one term in the summation of Eq.(3) and assume its \(k_n\) to be positive. The effective scalar potential for \(|\lambda S| \lesssim \Lambda\) is then given by

\[ V_{\text{eff}} \simeq \frac{\lambda^2 A^4}{(16\pi^2)^2 N_C^2} \left( 1 + \frac{k_n \lambda^2}{16\pi^2} \left| \frac{\lambda S}{\Lambda} \right|^{2n} \right) \left| \frac{\lambda S}{\Lambda} \right|^{-a}, \quad (5) \]

where \(a = 2(1 - N_F/N_C)\). For \(N_F < N_C\), this gives a local minimum at

\[ |\lambda S_{\text{min}}| \simeq \Lambda \left( \frac{16\pi^2 a}{k_n \lambda^2 (2n - a)} \right)^{\frac{1}{2n}}. \quad (6) \]

The mass around the local minimum is

\[ m_S \simeq \beta \frac{\lambda^2 \Lambda}{16\pi^2}, \quad (7) \]

with \(\beta\) defined by

\[ \beta \equiv \sqrt{na} \left( \frac{16\pi^2 a}{k_n \lambda^2 (2n - a)} \right)^{-\frac{a+2}{4n}} \frac{N_F}{N_C}. \quad (8) \]

The potential height at the minimum is

\[ V_{\text{eff}}(S_{\text{min}}) \simeq \eta \frac{\lambda^2 \Lambda^4}{(16\pi^2)^2} \simeq 3m_{3/2}^2 M_P^2, \quad (9) \]

with

\[ \eta \equiv \left( \frac{2n}{2n - a} \right) \left( \frac{16\pi^2 a}{k_n \lambda^2 (2n - a)} \right)^{-\frac{2n}{4n}} \frac{N_F^2}{N_C^2}, \quad (10) \]

where \(M_P \simeq 2.4 \times 10^{18}\) GeV is the reduced Planck mass. Here we have used a fact that the potential height at the local minimum is related to the gravitino mass \(m_{3/2}\) provided that the cosmological constant vanishes in supergravity.
On the other hand, the effective scalar potential for $|\lambda S| \gg \Lambda$ is obtained as

$$V_{\text{eff}} \simeq \frac{\lambda^2 \Lambda^4}{(16\pi^2)^2 N_F^2} \left(1 + A\lambda^2 \ln \left|\frac{\lambda S}{\mu}\right|^2\right) \frac{|\lambda S|^{-a}}{\Lambda}.$$  \hspace{1cm} (11)

For $a$ of order one, this is monotonically decreasing within the perturbative validity. We see that there are runaway directions with $|S| \to \infty$ (as well as directions $S = 0$ with $Q_i\tilde{Q}_i$ running away for $N_F \neq 1$). The tunneling into such directions from the metastable state at $S = S_{\text{min}}$ may well be sufficiently suppressed for small $\lambda$, which we assume in the following analyses.

### 4 Polonyi Cosmology

Let us here explain the cosmological problem of the the Polonyi field $S$. In the gravity mediation, $S$ must be a singlet under any symmetries in order to generate SUSY standard-model gaugino masses. Since a singlet does not have any special point in its field space, there is no reason to expect that the initial position $S_{\text{ini}}$ happens to be very close to the SUSY-breaking (local) minimum. Generically, we expect that the deviation from the SUSY-breaking (local) minimum is of $O(M_P)$.

After inflation, $S$ rolls down to the nearest minimum when the Hubble parameter becomes comparable to its mass. Suppose that the $S$ starts to oscillate about the SUSY-breaking minimum with an initial amplitude of $O(M_P)$. Then $S$ decays mainly into a pair of the gravitinos\(^2\), whose decay produces energetic particles and significantly changes the light-element abundances\(^3\). Thus the abundance of the gravitinos produced by the $S$ decay will be in conflict with BBN. This is the notorious Polonyi problem\(^4\).

In the following, we point out a possibility to avoid the Polonyi problem based on a class of the SUSY-breaking models presented in the previous section.

---

\(^2\) This Polonyi decay is not the exclusive source of the gravitino production. In addition to the usual thermal production processes, the gravitinos can be produced non-thermally by the inflaton decay. In particular, the singlet SUSY-breaking field can couple to the inflaton sector so strongly that prohibitively large amount of gravitinos may be produced\(^2\), though this depends on the details of the inflaton sector. For instance, the chaotic inflation model with a $Z_2$ symmetry causes no problem in this regard.

\(^3\) Stable gravitinos may result in overclosure of the universe. Moreover, the next-to-lightest SUSY particle may significantly affect the light-element abundances, making the situation even worse.

\(^4\) See Ref.\(^3\). Several solutions have been proposed so far (see Ref.\(^4\)).
4.1 Inflationary stage

Let us first consider how the initial position $S_{\text{ini}}$ of the $S$ field is determined. The Kähler potential $K$ generically contains a linear term of $S$,

$$K = c M_P S + c^* M_P S^\dagger + \cdots ,$$

(12)

where $c$ is a numerical coefficient. Although the effects of this term on the local minimum $|S_{\text{min}}| \ll M_P$ may be neglected due to Planck suppression in supergravity, they possibly affect the behavior of $S$ during and after inflation.

The effective potential for $S$ after inflation before the reheating is given by

$$V(S) \simeq e^{K/M_P^2} (3H^2 M_P^2) + V_{\text{eff}}(S)$$
$$\simeq 3H^2 \left(|S|^2 + c M_P S + c^* M_P S^\dagger \right) + V_{\text{eff}}(S).$$

(13)

When the Hubble parameter during the inflation satisfies $H_{\text{inf}} \gg m_S$, the initial deviation $\Delta S$ is given by

$$\Delta S \equiv |S_{\text{ini}} - S_{\text{min}}| \simeq |c| M_P,$$

(14)

for $|S_{\text{ini}}| > |S_{\text{min}}|$. The $S$ remains at $S_{\text{ini}}$ until it starts to oscillate when $H \simeq m_S$.

For a low scale inflation with $H_{\text{inf}} \ll m_S$, the fate of the evolution of $S$ depends on the initial condition. If $S$ happens to be close to the SUSY-breaking vacua, $|S| \lesssim \Lambda/\lambda$, the typical deviation is given by

$$\Delta S = |S_{\text{ini}} - S_{\text{min}}| \sim |c| \frac{H^2}{m_S^2} M_P,$$

(15)

which is time-dependent. After inflation, $S$ gradually approaches $S_{\text{min}}$ as the Hubble parameter decreases, and no sizable coherent oscillations of $S$ are induced.

For completeness let us also consider a case that there is no Hubble-induced contribution to the Polonyi potential during and after inflation. In this case the $S$ field acquires

---

5 It should be noted that $V_{\text{eff}}(S)$ is absent until $H$ becomes smaller than or equal to $\Lambda$. However, this does not change our conclusion.

6 The discussion here can be applied to a class of the $D$-term inflation models in which the Hubble-induced mass of the $S$ field appears after inflation ends. This is indeed the case in the known $D$-term inflation models.
quantum fluctuations during inflation with an amplitude given by $\delta S = H_{\text{inf}}/2\pi$, which are not damped until $S$ starts oscillating. Therefore, the inflationary scale must satisfy $H_{\text{inf}}/2\pi \lesssim \Lambda/\lambda$ for the $S$ field in order not to fall into one of the SUSY minima.

4.2 Reheating stage

Suppose that the initial displacement from the local SUSY-breaking minimum is determined in an anthropic way. That is to say, there are many sub-universes where $S_{\text{ini}}$ takes randomly chosen values, and our observable universe is contained in one of them.

If a value of $S_{\text{ini}}$ in a sub-universe is such that the $S$ falls into one of the SUSY vacua in the end, we can simply discard such a sub-universe, since no life like us will be formed. Therefore, only the sub-universes with the value of $S_{\text{ini}}$ much smaller than $M_P$ are selected anthropically, and the deviation from the local minimum is necessarily smaller than $\Lambda/\lambda$ in such sub-universes.

We first consider the inflation with $H_{\text{inf}} \gg m_S$ in the following. Since there is no anthropic argument for suppressing the initial deviation to be much smaller than $\Lambda/\lambda$, we expect that the typical deviation leading to the habitable universe is of order $\Lambda/\lambda$.

Let us now estimate the cosmological abundance of $S$. The timing of the reheating is crucial for determining the $S$ abundance. As we will see later, if the reheating occurs before the $S$ starts to oscillate, too many gravitinos would be produced by thermal scatterings. Therefore we assume that the reheating occurs after the $S$ starts to oscillate when the Hubble parameter becomes comparable to its mass, $H \simeq m_S$. Then the abundance of $S$ is

$$\rho_S^s \simeq \frac{\rho_{\text{inf}}}{s} \left| \frac{\rho_S}{\rho_{\text{inf}}^\text{osc}} \right| \simeq \frac{3T_R}{4} \frac{m_S^2 \Delta S^2}{3m_S^2 M_P^2} = \frac{T_R}{4} \frac{\Lambda^2}{\lambda^2 M_P^2} \left( \frac{\Delta S}{\Lambda/\lambda} \right)^2,$$

where $\rho_S$ and $\rho_{\text{inf}}$ denote the energy densities of $S$ and the inflaton, $s$ the entropy density, $T_R$ the reheating temperature, $m_S$ the mass of $S$, and the subscripts “$R$” and “$\text{osc}$” mean that the variables are evaluated at the reheating and at the commencement of the oscillations, respectively.

---

7When the initial displacement is given by (14), this may be the case if $c$ takes different values in different sub-universes that are far apart from each other.
The condensate of $S$ will dominantly decay into a pair of the gravitinos. The abundance of the gravitinos produced from the $S$ decay is therefore related to the $S$ abundance as

$$Y_{3/2}^{(S)} \approx \frac{2 \rho_S}{m_S s}. \tag{17}$$

Eqs. (7) and (9) result in

$$Y_{3/2}^{(S)} \approx 1 \times 10^{-17} \beta^{-1} \eta^{-\frac{3}{4}} \lambda^{-\frac{9}{2}} \left(\frac{|S_{ini}|}{\Lambda/\lambda}\right)^2 \left(\frac{m_{3/2}}{1 \text{ TeV}}\right)^{\frac{1}{2}} \left(\frac{T_R}{10^6 \text{ GeV}}\right). \tag{18}$$

Note that $Y_{3/2}^{(S)}$ gets enhanced both for a heavier gravitino mass and for a smaller $\lambda$.

In addition to the production from the $S$ decay, the gravitinos are produced by particle scatterings in thermal plasma. The abundance is approximately given by

$$Y_{3/2}^{(th)} \approx 2 \times 10^{-16} \left(\frac{T_R}{10^6 \text{ GeV}}\right), \tag{19}$$

where we have neglected the contributions from the longitudinal mode. Thus, unless $\lambda$ is much smaller than unity, the gravitinos from the $S$ decay tends to be smaller than or comparable to the thermally produced gravitinos. In this respect our scenario makes the Polonyi problem less severe than the cosmological problem of the thermally produced gravitinos.

The gravitino abundance is tightly constrained by BBN. According to the latest analysis [6], the constraint is given by

$$Y_{3/2} \lesssim \begin{cases} 1 \times 10^{-16} - 6 \times 10^{-16} & \text{for } m_{3/2} \approx 0.1 - 0.2 \text{ TeV} \\ 4 \times 10^{-17} - 6 \times 10^{-16} & \text{for } m_{3/2} \approx 0.2 - 2 \text{ TeV} \\ 7 \times 10^{-17} - 2 \times 10^{-14} & \text{for } m_{3/2} \approx 2 - 10 \text{ TeV} \end{cases}. \tag{20}$$

We require that the reheating temperature should be low enough for the thermally produced gravitinos (19) to be compatible with the BBN bounds. Then we can see that the abundance of the gravitinos produced by the $S$ decay can satisfy the BBN bound for $|S_{ini}| \lesssim \Lambda/\lambda$, unless $\lambda$ is much smaller than unity. Note also that the gravitino abundance (19) will be in conflict with the BBN bounds if the reheating occurs before the $S$ starts oscillating, since $T_R$ would exceed $10^{10}$ GeV. This justifies our assumption in deriving (16).
It may be worth mentioning that the abundance \[^{18}\] is relatively close to the upper bound. For some choices of the \( \mathcal{O}(1) \) parameters, the gravitino abundance from \( S \) decay may become inconsistent with the BBN constraints for \( T_R \gtrsim 10^6 \text{GeV} \). For a lower reheating temperature or a heavier gravitino mass, the gravitino abundance from the \( S \) decay is well below the BBN constraints, and the Polonyi problem is indeed absent.

Lastly we comment on a case of the low-scale inflation with \( H_{\text{inf}} \ll m_S \). As mentioned before, the cosmological abundance of \( S \) is negligibly small since no sizable coherent oscillations are produced \[^{5}\]. Thus the Polonyi problem is absent in this case. On the other hand, the above discussion can be applied to a case that there is no Hubble-induced contribution to the \( S \) field during and after inflation, whereas a non-trivial constraint on the inflationary scale, \( H_{\text{inf}}/2\pi \ll \Lambda/\lambda \), must be imposed.

### 5 Conclusion

We proposed a class of dynamical models with SUSY-breaking metastable vacua which are surrounded by SUSY ones.

The Polonyi problem in gravity mediation of SUSY breaking can be avoided when our SUSY-breaking vacuum is surrounded by such SUSY vacua. Then, the initial value of the Polonyi field \( S \) should be chosen not too far away from our vacuum, since otherwise, the \( S \) would roll down to the SUSY vacua.

We emphasize that the concrete models are mere examples and the above circumstances might be generic enough. The absence of the Polonyi problem may be a consequence of the fact that our SUSY-breaking vacuum is metastable if gravity mediation is realized in Nature.

### Acknowledgements

This work was supported by the Grant-in-Aid for Yukawa International Program for Quark-Hadron Sciences, the Grant-in-Aid for the Global COE Program “The Next Gen-

---

[^18]: This is especially the case if \( T_R \) is close to \( 10^6 \text{GeV} \), which is the lower bound coming from the non-thermal leptogenesis to work. Thus the Polonyi problem with the non-thermal leptogenesis scenario is marginally consistent with BBN.
eration of Physics, Spun from Universality and Emergence”, and World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

References

[1] M.A. Luty, arXiv:hep-ph/9706235; A.G. Cohen, D.B. Kaplan, and A.E. Nelson, arXiv:hep-ph/9706275.

[2] M. Kawasaki, F. Takahashi, and T.T. Yanagida, arXiv:hep-ph/0603265, arXiv:hep-ph/0605297;
See M. Endo, F. Takahashi, and T.T. Yanagida, arXiv:0706.0986.

[3] M. Ibe, Y. Shinbara, and T.T. Yanagida, arXiv:hep-ph/0605252.

[4] L. Randall and S.D. Thomas, Nucl. Phys. B 449, 229 (1995);
D.H. Lyth and E.D. Stewart, Phys. Rev. D 53, 1784 (1996);
A.D. Linde, Phys. Rev. D 53, 4129 (1996);
M. Dine, Y. Nir, and Y. Shadmi, Phys. Lett. B 438, 61 (1998);
M. Kawasaki and F. Takahashi, Phys. Lett. B 618, 1 (2005).

[5] A.D. Linde, in Ref. [4].

[6] M. Kawasaki, K. Kohri, and T. Moroi, Phys. Lett. B 625, 7 (2005); Phys. Rev. D 71, 083502 (2005).