Attitude Stabilization of Hypersonic Vehicle Under Actuator Faults and Saturation

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Abstract. The paper focuses on fault tolerant attitude stabilization control for hypersonic vehicles (HSV) in the presence of loss of actuator effectiveness, input saturation and modeling uncertainties simultaneously. Firstly, an adaptive fuzzy observer was constructed to estimate the actuator faults and lumped disturbance. Then, the auxiliary system is introduced to compensate for saturation effect with the loss fault. Accordingly, a fault tolerant control law based on backstepping method is reconfigured under the reconstructed fault information. Finally based on the Lyapunov theory analysis, tracking error converges to arbitrary neighborhood around zero. It is shown that the proposed controller guarantees all the signals in the closed-loop system to be uniformly ultimately bounded. Finally, simulation results are given to illustrate the effectiveness of the proposed controller.

Introduction

In recent years, various strategies are utilized to design fault tolerant controller for nonlinear HSV systems, such as sliding mode control, predictive control, backstepping control, adaptive control, intelligent control and so on[1-4]. In the study of [5], an active FTC approach is proposed for the T–S fuzzy models of near space vehicle attitude dynamics using adaptive sliding mode techniques, but only the actuator loss of effectiveness fault was considered. In [6], an adaptive dynamic sliding mode fault-tolerant control methodology based on hierarchical inner-outer loop architecture is developed for near space vehicle to stable systems and tracks the desired signals accurately. An adaptive fault estimation algorithm using neural network was proposed in [7] to enhance the performance of fault estimation, and then the observer-based fault tolerant tracking controller is then designed to guarantee tracking performance.

In addition, actuator saturation is an important issue in practical application, which should be taken into account in the design of the fault tolerant control [8]. Particularly, when the faults with loss of effectiveness happen, the remaining controller may potentially output a magnified signal to compensate for the loss. Because of the physical limitations of the actuator, any efforts to increase the actuator output would keep no variation. As a result, it may potentially lead to performance degradation or even instability. Most of the FDI-FTC techniques above-mentioned can only counteract one or several types of faults, but few methods are able to cope with actuator faults and saturation simultaneously [9-11].

In this paper, a fault diagnosis system based on adaptive fuzzy observer was constructed to estimate the partial losses in control effectiveness and generalized disturbances. Then, an active fault torrent controller using the adaptive backstepping technique was proposed to stabilize the HSV attitude dynamic when the fault happens. In addition to the normal controller, an additional item supplied by the auxiliary system which includes the estimation of faults is added to compensate for the effect of both faults and actuator saturation. Based on the Lyapunov theory, it is analyzed and demonstrated that the proposed method can protect the control effort from actuator saturation and guarantees all the signals in the closed-loop system to be uniformly ultimately bounded.
The paper is organized as follows. Section 2, we firstly present the HSV reentry model with parameter uncertainty and external disturbance and actuator fault model whose behavior is characterized by affine nonlinear forms. Section 3, an adaptive fuzzy observer is proposed for HSV actuator loss-of-effectiveness failures and lumped disturbances. Next, we proposed a fault tolerant controller by means of an auxiliary system to endure effects of actuator saturation. Simultaneously, the stability of the FTC system is proved in Section 4. In Section 5, simulation results are presented to show the effectiveness of the proposed method.

Problem Formulation

Hypersonic Flight Vehicle Reentry Mode

The reentry attitude dynamics for a hypersonic vehicle with parameter uncertainty and external disturbance is described by [12]

\[
\begin{align*}
\dot{\omega} &= -J^{-1}\omega J\omega + J^{-1}\psi \delta + \eta(\omega, d) \\
\dot{\Omega} &= \Xi(\Omega)\omega + \Delta f
\end{align*}
\]

(1)

Where \( J \in \mathbb{R}^{3\times3} \) is the symmetric, positive definite moment of inertia tensor, \( \omega = [p, q, r]^T \) is the known angular rate (roll, pitch, and yaw rate, respectively) vector, \( u = [u_1, u_2, u_3] \) is the control torque vector. The operator \( \omega \times \) denote a skew-symmetric matrix acting on the vector \( \omega \). \( \psi \in \mathbb{R}^{3\times3} \) is the control command distribution matrix. \( \delta = \delta_1, \delta_2, \delta_3 \) denote desired surface deflections by means of elevator, rudder and aileron. \( \eta(\omega, d) = -J^{-1}\omega J\omega - J^{-1}\Delta J\omega + J^{-1}d \) is the lumped disturbance which represents the combination of parameter uncertainty and external disturbance. Specifically, it is assumed that \( \|\eta\| \leq \eta \). \( \Omega = [\alpha, \beta, \mu] \) represents the angle of attack, and the sideslip angle, the bank angle respectively. \( \Xi(\Omega) \) represents transform matrix with angular rate vector \( \omega \). \( \Delta f = [\dot{\alpha} \sin \beta, 0, 0] \) represents the uncertainties induced due to model simplification and it is can be ignored by when HSV operates in small sideslip condition.

Attitude Faulty Description

To formulate the fault tolerant control problem, the attitude dynamics with faults mode of hypersonic vehicle must be established, the type of actuator fault considered in this study is loss of control effectiveness. The hypersonic vehicle dynamics under actuator fault case can be rewritten as strict feedback form as follows:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)\Sigma u + \eta(x, t)
\end{align*}
\]

(2)

where \( x_1 = \Omega = [\alpha, \beta, \mu] \in \mathbb{R}^3 \), \( x_2 = \omega = [p, q, r]^T \in \mathbb{R}^3 \), and \( f_1(x_1) = 0 \), \( g_1(x_1) = \Xi(\gamma) \), and \( f_2(x_1, x_2) = -J^{-1}\omega J\omega \), \( g_2(x_1, x_2) = -J^{-1}\psi \). \( \Sigma(t) = \text{diag}\{\rho_1, \rho_2, \rho_3\} \in \mathbb{R}^{3\times3} \) is factor diagonal matrix which represents the maximum percentage of the admissible loss of control effectiveness, satisfying \( 0 < \rho_i(t) \leq 1 \) for \( i = 1, 2, 3 \). The case when \( \rho_i(t) = 1 \), the \( i \)-th control surface deflection \( u_i(t) \) is in a fault-free condition. The coupling matrix \( g_1(x_1) \) and \( g_2(x_1, x_2) \) can be satisfied by: \( 0 \leq \|g_1(x_1)\| \leq \bar{J}_1 \), \( 0 \leq \|g_2(x_1, x_2)\| \leq \bar{J}_2 \), \( \forall x \in \mathbb{R}^1 \) where \( \bar{J}_1 \) and \( \bar{J}_2 \) are some known positive constants and denotes the Euclidean norm of vectors.

Active Fault-tolerant Attitude Controller Design

In this section, a FTC law based on backstepping technology is designed using the reconstructed information from the FDD observer to achieve the attitude stabilization when the actuators are in the presence of the faults and saturation.
Fuzzy Logic Systems Theory

Fuzzy logic systems are usually used to approximate the unknown functions and construct a robust controller. Therefore, some useful Lemmas are first introduced as follows.

**Lemma 1** [13]: Let \( f(x) \) be a continuous function defined on compact set \( \Omega \). Then for any constant \( \varepsilon > 0 \), there exists a fuzzy logic system \( \hat{f}(x) = \theta^T \phi(x) \) such as \( \sup_{x \in \Omega} \left| f(x) - \theta^T \phi(x) \right| \leq \varepsilon \), where \( \theta = [\theta_1, \theta_2, \ldots, \theta_N]^T \) is the ideal constant optimal vector and \( \phi(x) = [\phi_1(x), \ldots, \phi_N(x)]^T \) is the basis function vector with \( N > 1 \) being the number of the fuzzy rules and \( \phi_i \) are the basis functions defined as

\[
\phi_i = \prod_{j=1}^n \mu_{F_{i,j}}(x_j) / \sum_{i=1}^N \left( \prod_{j=1}^n \mu_{F_{i,j}}(x_j) \right)
\]

where \( \mu_{F_{i,j}}(x_j) \) is the membership function value of the fuzzy variable.

**Fuzzy Observer-based FDD Mechanism Design**

The purpose of the observer-based FDD is reconstructed the fault information through the estimation information. Here, we design an adaptive Fuzzy observer for reconstructed the fault with control surface damage and disturbance information. The second equation in the model (2) can be transformed into:

\[
\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)U \rho + \eta(x, t)
\]

where \( \rho(t) = [\rho_1, \rho_2, \rho_3]^T \in \mathbb{R}^3 \), \( U = \text{diag} \{ u_i, u_i \} \in \mathbb{R}^{3 \times 3} \).

From Lemma 1, the unknown continuous uncertainty function \( \eta \) can be approximated by the fuzzy logic systems \( \hat{\eta} = \theta^T \phi(x) \), and the fuzzy approximation errors \( \varepsilon \) defined by

\[
\eta = \theta^T \phi(x) + \varepsilon
\]

where \( \theta_i \) are optimal parameter vectors bounded by a compact set. \( \varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]^T \) satisfies \( \| \varepsilon \| \leq \varepsilon^* \).

Based on adaptation and the fuzzy logic system, we design an observer for possible faults as follows:

\[
\dot{x}_2 = A e + f_2(x_1, x_2) + g_2(x_1, x_2)U \hat{\rho}(t) + \hat{\theta}^T \phi(x)
\]

where \( \hat{x}_2 \) is the state estimation of \( x_2 \), and \( e = \hat{x}_2 - x_2 \) denotes the errors between \( \hat{x}_2 \) and \( x_2 \), and \( \hat{\rho}(t) = [\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3]^T \) denotes the estimation of the control surface damage factor.

Let \( \hat{\theta} = \hat{\theta} - \theta \), \( \hat{\rho} = \hat{\rho} - \rho \). By using the aforementioned adaptive fuzzy observer (5), the resulting state estimation error dynamic is

\[
\dot{e} = A e + g_2(x_1, x_2)U \hat{\rho}(t) + \hat{\theta}^T \phi(x)
\]

where \( P = P^T > 0 \) is a solution to Lyapunov equation \( A^T P + PA^T = -Q \) (matrix \( A \) is Hurwitz), with \( Q = Q^T > 0 \). In addition, the adaptive fuzzy logic parameter updating law is designed as

\[
\dot{\theta} = -2 \Gamma \phi(x) e^T P
\]

\[
\dot{\rho}(t) \text{ is designed as the updating law,}
\]

\[
\dot{\rho} = -2 \gamma U^T g_2(x_1, x_2)^T Pe
\]

Where \( \gamma \in \mathbb{R} \) is the positive adaptive gains. Thus, we have the following theorem.

**Theorem 1**: Considering the adaptive fuzzy observer in (5), the state estimation error dynamic given by (6) is globally asymptotically stable, i.e., for any initial conditions \( \hat{x}_2(0) \) and, we have \( \lim_{t \to \infty} e(t) = 0 \).
**Proof:** Consider the following Lyapunov function.

\[
V = e^T Pe + \frac{1}{2\gamma} \rho^T \dot{\rho} + \frac{1}{2} \text{tr}(\theta^T \Gamma^{-1} \theta)
\]

The derivative of \( V \) along the trajectory of the augmented state error dynamic can be written as

\[
\dot{V} = e^T Pe + e^T P \dot{e} + \frac{1}{2\gamma} \rho^T \dot{\rho} + \text{tr}(\theta^T \Gamma^{-1} \theta)
\]

\[
= [Ae + g_2(x_1, x_2) U \rho + \theta^T \Phi(x) - \varepsilon] e^T Pe + e^T P[Ae + g_2(x_1, x_2) U \rho + \theta^T \Phi(x) - \varepsilon] + \frac{1}{\gamma} \rho^T \dot{\rho} + \text{tr}(\theta^T \Gamma^{-1} \theta)
\]

\[
= e^T (A^T P + PA)e + \theta^T \left[ \frac{1}{\gamma} \dot{\rho} - 2U^T g_1(x_1, x_2)^T Pe + 2e^T P\left( \theta^T \Phi(x) - \varepsilon \right) + \text{tr}(\theta^T \Gamma^{-1} \theta) \right]
\]

\[
\leq -e^T Qe + \text{tr}(\theta^T \left( \Gamma^{-1} \theta + 2 \Phi(x) \dot{e}^T P \right)) - 2e^T Pe
\]

\[
= -e^T Qe - 2e^T Pe
\]

By selecting proper fuzzy vector, we can make \( e \to 0 \), as \( t \to 0 \), asymptotically and have a proper convergence performance. Then it can prove that \( e \to 0 \) as \( t \to 0 \) by Barbalat’s lemma [14]. Then a measurable reconstruction signal \( \eta(x, t) \) can be computed online by (4) asymptotically. The proof is complete.

**Robust Fault Tolerant Controller Design under Actuator Faults and Saturation**

In this section, an auxiliary system is integrated into the nominal backstepping controller to compensate for the effect of actuator faults and saturation.

Based on the reconstructed faults \( \hat{\eta} \) and \( \hat{\rho} \) from the observer-based FDD given by (4) and (8), we rewrite HSV attitude system (6) to

\[
\begin{cases}
\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)P(t)u + \tilde{P}(t)u + \eta(x, t) \\
y = x_1
\end{cases}
\]

where \( \tilde{P}(t) = \text{diag}(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{\rho}_3) \) is a diagonal matrix, and \( \tilde{P}(t) = \text{diag}(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{\rho}_3) \cdot u \) is the actual input which is constrained by physical limit. To simplify the prove process, we define \( g_1(x_1), g_2(x_1, x_2) \) as \( g_1, g_2 \).

The control objective is to force the output \( y \) to asymptotically track the reference signal \( y_d \). The tracking error of altitude and attitude rate is defined as

\[
\begin{cases}
z_1 = y - y_d \\
z_2 = x_2 - x_{2d}
\end{cases}
\]

where \( y_d \in \mathbb{R}^1 \), \( x_{2d} \in \mathbb{R}^3 \) are the desired attitude commands and attitude rate commands respectively. From (12), we have

\[
\begin{align*}
z_1 &= \dot{y} - \dot{y}_d \\
 &= f_1 + g_1 x_2 - \dot{y}_d \\
 &= f_1 + g_1 z_2 + g_1 x_{2d} - \dot{y}_d \\
&= f_1 + g_1 z_2 + g_1 x_{2d} - \dot{y}_d \\
&= f_1 + g_1 z_2 + g_1 x_{2d} - \dot{y}_d
\end{align*}
\]

where \( k \) is a positive constant. Clearly, if \( z_2 = 0 \), then \( \dot{V}_1 = -k z_1^T z_1 < 0 \) and \( z_1 \) is guaranteed to converge to zero asymptotically.

The second error \( z_2 \) is differentiated from (12).
\[
\dot{z}_i = \dot{x}_i - \dot{x}_{id}
\]
\[
= f_1(x_1, x_2) + g_1 \hat{p}(t)u - \hat{p}(t)u_e + \eta(x_1, x_2, t) - \dot{x}_{id}
\]

The second Lyapunov function is constructed as
\[
V_2 = V_1 + \frac{1}{2} z_i^T z_i
\]

The time derivative of (15) is written as
\[
\dot{V}_2 = \dot{V}_1 + \frac{1}{2} z_i^T \dot{z}_i
\]
\[
= -k_i z_i^T g_j z_j + \dot{z}_i^T \left[ f_j(x_1, x_2) + g_j(x_1, x_2) \hat{p}(t)u_e - \hat{p}(t)u_e + \eta(x_1, x_2, t) - \dot{x}_{id} \right]
\]

The following FTC law is designed:
\[
v = u_{\text{sat}} + u_e
\]

Where \(u_{\text{sat}} = -(g_2 \hat{p}(t))^T (g_1^T X + g_2 (x_1, x_2) - \dot{x}_{id} - \eta(x_1, x_2, t))\), and \(u_e = -(g_2 \hat{p}(t))^T \left[ \frac{z_i^T z_i}{\eta + \exp(-\beta)} \right] \), which is the robustness term to compensate the bounded disturbance, where \(\epsilon\) is a sufficiently small positive scalar and \(k_2, \beta\) are positive control parameters to be designed. In the meanwhile, the output of controller should satisfy the constraint \(u_{\text{sat}}\),

\[
u = \text{sat}(v, u_{\text{sat}})
\]

Where \(u_e\) is actual control signal which meet the constraint requirement and \(v\) is the designed by the normal backstepping technology when the actuator set free. Thus, there is a difference \(\Delta u\) between the actual control input and the desired control input, which is described as \(\Delta u = u_e - v\).

In order to compensate the effect of actuator saturation, we construct an auxiliary system

\[
\begin{align*}
\dot{X}_1 &= -k_1 X_1 + g_1 X_2 + g_1 (x_{id} - X_{id}) \\
\dot{X}_2 &= -k_2 X_2 + g_2 X_3 + g_2 (x_{id} - X_{id}) \\
&\vdots \\
\dot{X}_{n-1} &= -k_{n-1} X_{n-1} + g_{n-1} X_n + g_{n-1} (x_{id} - X_{id}) \\
\dot{X}_n &= -k_n X_e + g_n \hat{p}(t) \Delta u
\end{align*}
\]

where \(k_i (i=1\cdots n)\) are positive constant parameter which were designed in the backstepping process, \(x_{id} (i=1\cdots n)\) are the actual control signals acting on the hierarchy system which satisfy the requirement of limits in practice. In according to the order of HSV attitude dynamic, it follows \(n=2\).

**Assumption 1.** All the control signals \(x_i (i=2, \cdots n)\) in the HSV attitude dynamic system are piecewise continuous and bounded and it is such as to satisfy the bounded input and bounded output when the actuators with free.

**Assumption 2.** \(\left\| (g_1^T X + g_2 (x_1, x_2) - \dot{x}_{id} - \eta(x_1, x_2, t)) \right\| \leq \zeta_{s_{\text{sat}}}, \zeta_{s_{\text{sat}}}\) is a unknown constant

**Theorem 2** Consider a nonlinear attitude dynamic system, which involves three fading actuation under saturation is governed by Eq. (18). If Assumptions 1 and 2 are satisfied, the control scheme (17) can guarantee that all the signals in the closed-loop system are uniformly ultimately bounded (UUB) by incorporating an auxiliary system and the observer-based FDD.

**Proof.** We revised the augmented error systems considering the effect brought out from input saturation and actuator fault

\[
\begin{align*}
\dot{z}_1 &= \dot{z}_1 - X_i \\
\dot{z}_2 &= \dot{z}_2 - X_2
\end{align*}
\]

The time derivative of \(z_i\) is then given as
\[ \ddot{z}_i = \dot{z}_i - \chi_i \]
\[ = f_i + g_i x_i - \dot{y}_d + k_i X_i - g_i x_i - g_i (x_{2d} - x_{2d}^0) \]
\[ = f_i + g_i z_i - \dot{y}_d + k_i X_i - g_i x_i + g_i x_{2d}^0 \]  

(21)

It is obvious from (27) that
\[ \ddot{z}_i = f_i + g_i z_i - \dot{y}_d + k_i X_i + g_i x_{2d}^0 \]  

(22)

Then, choose the candidate Lyapunov function \( V_i = \frac{1}{2} \dot{z}_i^T \dot{z}_i \). Taking time derivative gives
\[ \dot{V}_i = \dot{z}_i^T \dot{z}_i \]
\[ = \dot{z}_i^T (f_i + g_i z_i - \dot{y}_d + k_i X_i + g_i x_{2d}^0) \]  

(23)

Choose the virtual control law \( x_{2d}^0 = -g_i^{-1}(k_i z_i + f_i - \dot{y}_d) \)  

(24)

According to (28), \( k_i z_i - k_i X_i = k_i \dot{z}_i \), it is straightforward to show that
\[ \ddot{z}_i = -k_i \dot{z}_i + g_i \dot{z}_2 \]  

(25)

Clearly, if \( \ddot{z}_i \to 0 \), then \( \dot{V}_i = -k_i \| z \|^2 \) and \( z \) is guaranteed to converge to zero asymptotically.

By using \( g_i \dot{z}_i = g_i z_i - g_i X_i \), we have the time derivative of \( \dot{z}_2 \)
\[ \ddot{z}_2 = \dot{z}_2 - \dot{X}_2 \]
\[ = f_2 + g_2 \dot{P}(t)(v + \Delta u) - g_2 \dot{P}(t) u + \eta - \dot{x}_{2d} + k_2 X_2 - (g_2 \dot{P}(t) \Delta u) \]
\[ = f_2 + g_2 \dot{P}(t)(v) - g_2 \dot{P}(t) u + \eta - \dot{x}_{2d} + k_2 X_2 \]  

(26)

Consider another candidate Lyapunov function as
\[ V_2 = V_1 + \frac{1}{2} \dot{z}_2^T \dot{z}_2 \]  

(27)

Then the derivative of \( V_2 \) along with time is given by
\[ \dot{V}_2 = -k_i \| \dot{z}_2 \|^2 + \dot{z}_2^T g_i \dot{z}_i + \dot{z}_i^T \tau_i \]  

(28)

To stabilize the system (31), the actual control input \( v \) is selected to remove the residual term and make \( V_2 \leq 0 \), the control input \( v \) to the system in Eq.(28) is

Hence
\[ \dot{V}_2 = -k_i \| \dot{z}_2 \|^2 - k_i \| \dot{z}_2 \|^2 + \dot{z}_2^T [f_2 + g_2 \dot{P}(t) (v + \Delta u) - g_2 \dot{P}(t) u + \eta - \dot{x}_{2d} + k_2 X_2] \]
\[ = -k_i \| \dot{z}_2 \|^2 - k_i \| \dot{z}_2 \|^2 + \dot{z}_2^T [f_2 + g_2 \dot{P}(t)(v) - g_2 \dot{P}(t) u + \eta - \dot{x}_{2d} + k_2 X_2] \]  

(29)

Therefore, from the Assumption 2, using inequality \( \| g_2 (x_1, x_2) \dot{P}(t) u + \eta (x_1, x_2, t) \| \leq \zeta_{2d} \), it follows that
\[ \dot{V}_2 \leq -k_i \| \dot{z}_2 \|^2 - k_i \| \dot{z}_2 \|^2 + \| \dot{z}_2 \| \zeta_{2d} - \frac{\| \dot{z}_2 \| \zeta_{2d}^2}{\| \dot{z}_2 \| \zeta_{2d} + \varepsilon \exp(-\beta t)} \]
\[ \leq -k_i \| \dot{z}_2 \|^2 - k_i \| \dot{z}_2 \|^2 + \varepsilon \exp(-\beta t) \]
\[ \leq -k_i \| \dot{z}_2 \|^2 + \varepsilon \exp(-\beta t) \]  

(30)
where $K' = \min\{k, k_z\} > 0$, and all terms in the definition of $K'$ are positive. Using Theorem 4.18 in [14], it can be proved that $V_z$ is uniformly ultimately bounded with the states $\xi$, when $K' \geq \varepsilon I / (2V_z(0))$.

It means there exist a finite time $T' > 0$ such that for all $\varepsilon > \sqrt{\frac{\varepsilon}{K'}}$ and for all $t > T'$, and this shows that

$$\xi \leq e^T (i = 1, 2) \text{ as } t \geq T'$$

So the tracking error converges to arbitrary neighborhood around zero. It’s worth noting that the bound of $\|y(t) - \gamma_i(t)\|$ depends on the bound of $\|e^{\hat{P}(t)} \Delta u\|$, the effects of which on system transient performance can be decreased by increasing parameter $k$. If the actuator has no saturation, so $\Delta u \rightarrow 0$ as $t \rightarrow \infty$, we have $\xi_i \rightarrow 0$, then $\lim_{t \to \infty} \|y(t) - \gamma_i(t)\| = 0$.

**Simulated Example**

To demonstrate the effectiveness and performance of the proposed controller, simulation results are presented in this section. Consider the HSV reentry model which is considered in [5]. The nominal flight of the HSV is at a trimmed reentry conditions: $H=40\text{km}$, $V=2500\text{m/s}$, and HSV attitude tracking command $y_d$ (angle of attack, bank angle and sideslip angle) are 4deg, 5deg and, 0deg respectively.

The initial attitude angles $x_0 = [0, 0, 0]^T$. The initial angular rate $\omega_0 = [p_0, q_0, r_0] = [0, 0, 0]^T\text{deg/s}$. We assume the model parameter uncertainty as $\Delta J \in [(1-10\%) J, (1+10\%) J]$ and external disturbance as $\eta = [5\sin(0.1t), \sin(t), 5\cos(0.1t)]\text{Nm}$.

To verify the performance of the proposed approach, the following simulation comparisons are given. It is assumed that occurs a time varying fault at $1s$, and loss of actuator effectiveness is namely by $\rho_1=0.5, \rho_2=0.8, \rho_3=0.5$. The parameters of the observer and controller are chosen as $A = \text{diag}(-1, -1, -1)$ and $P = \text{diag}(10, 10, 10)$, $\lambda = 15$, $k_1=1$, $k_2=1.6$. The deflection angle of control surfaces was bounded in the intervals [-25º, 15º].

For the purpose of comparison for the effectiveness of the method, active fault tolerant control (AFTC) approach proposed in this paper and normal backstepping without FTC in [8] is compared under the afore-mentioned actuator faults and constraints, the corresponding simulation results are depicted in Figs. 1-2.

![Figure 1. The attitude angle responses in actuator faulty case.](image)

Referring to Figure 1, the normal backstepping without FTC method cannot meet system attitude tracking performance requirements and even cannot be stable when system occurs fault. From Figure 2, it can be seen that the method cannot tolerate the actuator fault, and the controller output cannot maintain its stability after 1s. However, it shows that the active fault tolerant controller can compensate for the effects of the faults within 5 seconds. In the meanwhile, the constrained fault tolerant back stepping controller (18) can be constructed based on the estimated fault information and satisfy the actuator constraints.
In summary, the simulation results demonstrate that our proposed scheme with input constraints is rather tolerant to faults, which can obtain better dynamic performances in the event of system faults.

![Graphs showing control surface deflection angles](image)

Figure 2. Control surface deflection angle in actuator faulty case.

**Conclusion**

A fault tolerant attitude stabilization control scheme was presented for a hypersonic vehicle which may experience uncertainties in inertia matrix, partial loss of actuator effectiveness and input saturation simultaneously. In this method, lumped disturbances and actuator fault are estimated by the adaptive fuzzy observer. The proposed controller based on backstepping strategy consists of nominal and auxiliary control actions; the former is utilized to stabilize the system with actuator faults while the latter compensates for the effect of input limits. Then, the system stability has been proved in the presence of the proposed controller based on Lyapunov stability theorem and all the signals of controlled systems are ensured to be semi-global uniformly ultimately bounded.

Finally, attitude control performance has been investigated by simulation in the presence of external disturbances, actuator faults and saturation. The controller designs are evaluated using numerical simulation to compare the proposed approach with other existing schemes. The results show that the proposed control scheme can successfully handle those above-mentioned issues simultaneously.

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