Initial state radiation effects in inclusive $J/\psi$ production at B factories

Hua-Sheng Shao

*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

*E-mail: erdissshaw@gmail.com*

**Abstract:** Based on Monte Carlo techniques, we analyze the initial state radiation (ISR) effects in prompt $J/\psi$ inclusive production at B factories. ISR enhances cross section $\sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ by about $15 - 25\%$, which is almost the same size as the QCD and relativistic correction. Moreover, ISR slightly changes $\sigma(e^-e^+ \rightarrow J/\psi + c\bar{c} + X)$. The $J/\psi$ momentum spectrum in $e^-e^+ \rightarrow J/\psi + gg + X$ and in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ is softer after the photon showering from the initial $e^\pm$ beam radiation. After combining the QCD, relativistic, and ISR corrections, a more precise theoretical result is obtained. The new result provides a more stringent constraint of the color-octet contribution to $\sigma(e^-e^+ \rightarrow J/\psi + X_{\text{non-}c\bar{c}})$.

**Keywords:** QCD Phenomenology, Monte Carlo Simulations.
1. Introduction

A suitable method for studying the interplay between perturbative QCD and non-perturbative QCD is to investigate the quarkonium production at various colliders. The prompt $\Upsilon$ production in hadronic collisions indicates that color-octet mechanism plays a crucial role in reducing the discrepancies between theory and experiments [1, 2, 3, 4, 5, 6, 7]. Although color-singlet contribution may receive large QCD corrections in several examples [8, 9, 10, 11, 12, 13], difficulty in explaining the simultaneous yields and polarization of prompt $\Upsilon$ in hadronic collisions is still encountered. A significant color-octet component is important to explain the high-transverse momentum hadronic data in the current framework. By contrast, the color-singlet one in a relatively small-scale physics regime is sufficient to explain the experimental data at hadron colliders [14, 15] and B factories [16, 17, 18, 19, 20, 21, 22]. Hence, the physics of these two scales should be studied further.

In this research, we consider the prompt $\Upsilon$ inclusive production in $e^-e^+$ collisions at center-of-mass energy $\sqrt{s} = 10.6$ GeV. Over a decade ago, BABAR collaboration and BELLE collaboration reported the cross section $\sigma(e^-e^+ \rightarrow \Upsilon + X)$ is $2.52 \pm 0.21 \pm 0.21$ pb [23] and $1.47 \pm 0.10 \pm 0.13$ pb [24] respectively. By contrast, the leading order (LO) color-singlet theoretical prediction [25, 26, 27, 28, 29, 30] that mainly includes the contributions from the processes $e^-e^+ \rightarrow \Upsilon + c\bar{c} + X$ and $e^-e^+ \rightarrow \Upsilon + gg + X$, was at least 3–5 times lower than the obtained measurements. This finding suggests a substantial color-octet contribution that can be generated at lower $\alpha_s$ power via $e^-e^+ \rightarrow \Upsilon(\Upsilon_3, \Upsilon_5^0) + g$. Moreover, the BELLE collaboration measured the associated production cross section $\sigma(e^-e^+ \rightarrow \Upsilon + c\bar{c} + X) = 0.87^{+0.21}_{-0.19} \pm 0.17$ pb and the ratio $R_{c\bar{c}} = \frac{\sigma(e^-e^+ \rightarrow \Upsilon + c\bar{c} + X)}{\sigma(e^-e^+ \rightarrow \Upsilon + X)} = 0.59^{+0.15}_{-0.13} \pm 0.12$ [31]. The cross section for $\Upsilon + c\bar{c} + X$ is at least a factor of five and higher than LO color-singlet [26, 27, 28, 29, 30, 32] and color-octet [32] theoretical estimations. Several
Theoretical improvements are formulated subsequently to decrease the large discrepancies [33, 34, 35]. Specifically, next-to-leading order (NLO) QCD correction is used to obtain a large enhancement [16] of the cross section for $J/\psi + c\bar{c} + X$, which has been confirmed by other authors [17]. The result is comparable with the experimental data in some specific parameter choices. Moreover, the QCD [18, 19] and relativistic correction [20, 21] enhance the color-singlet cross section $\sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ by about $20 - 30\%$ respectively. The QCD correction [18, 19] to $e^-e^+ \rightarrow J/\psi + gg + X$ significantly changes the $J/\psi$ momentum distribution, particularly near the kinematic endpoint. This change is important because the color-octet part [36] enhances the cross section at the endpoint prior to resummation [33]1. Thus, the $J/\psi$ momentum spectrum might provide a constraint to color-octet matrix elements. A comparison of the theoretical result with the current BELLE measurements [37] was performed in ref.[22]. A strong constraint on the non-perturbative color-octet matrix elements was extracted as [22]

$$\langle O_{J/\psi}(S_0^{[8]}) \rangle + 4.0 \frac{\langle O_{J/\psi}(P_0^{[8]}) \rangle}{m_c^2} < (2.0 \pm 0.6) \times 10^{-2} \text{GeV}^3, \quad (1.1)$$

which apparently contradicts the high $p_T$ hadronic results in non-relativistic QCD (NRQCD) theory [38] at NLO level [3, 4, 5, 7, 39, 40]2.

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1Resummation is heavily based on a phenomenological shape function [33].

2In refs.[1, 2, 6], the authors obtained a color-octet matrix elements set satisfying this constraint by using the world yields data. However, in their fit, they included many small transverse momentum data. Hence, they predicted a completely wrong $J/\psi$ polarization at Tevatron and LHC [6].
In this article, we examine the initial state radiation (ISR) effect to prompt $J/\psi$ inclusive production at B factories. ISR is an extremely important feature that should be considered in investigating the physics in electron-positron collisions. After including the ISR effect in the inclusive $J/\psi$ production at B factories, we obtain a more precise theoretical result. In general, the detailed studies of photon-radiative corrections from the initial $e^\pm$ beams require Monte Carlo (MC) generators [41, 42, 43, 44, 45, 46, 47, 48, 49]. We interface a general-purpose matrix element and events generator HELAC-Onia [50, 51, 52, 53, 54] to the general photon shower program QEDPS [47, 48, 49] to include the ISR in various $e^-e^+$ annihilation processes. In the two processes $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ and $e^-e^+ \rightarrow J/\psi + gg + X$, two representative Feynman diagrams with ISR are shown in figure 1. After the photon shower, the annihilating $e^-$ and $e^+$ do not produce head-on collisions because they might deviate from the beam axis through the radiation. The center-of-mass energy after showering $\sqrt{s}$ is lower than $\sqrt{s} = 10.6$ GeV. As shown in figure 2, the LO cross section for $e^-e^+ \rightarrow J/\psi + gg + X$ increases as $\sqrt{s}$ decreases near 10.6 GeV, whereas that for $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ changes minimally as $\sqrt{s}$ decreases near 10.6 GeV. Therefore, the ISR effect should be significant in $J/\psi + gg + X$, whereas it might not be that important in $J/\psi + c\bar{c} + X$. Considering that ISR and $J/\psi$ production are factorizable, ISR should not change the QCD correction and relativistic correction unless the K factors of these two corrections considerably vary with $\sqrt{s}$ near 10.6 GeV. We

\[ m_c = 1.4 \text{ GeV} \]
\[ \mu = 2m_c \]
\[ \alpha_s = 0.267 \]
\[ |R(0)|^2 = 1.01 \text{ GeV}^3 \]
Figure 3: Cross section as functions of number of photon and $\sqrt{\hat{s}}/\sqrt{s}$ in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$.

will study the ISR effect in detail by including the QCD correction and relativistic correction.

The article is organized as follows: we will study the ISR effects in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ in section 2 and in $e^-e^+ \rightarrow J/\psi + gg + X$ in section 3. We then formulate our conclusion in section 4.

2. ISR in $J/\psi c\bar{c} + X$

The inclusive double charm production at B factories is one of the most interesting processes for probing heavy quarkonium physics. The most precise measurement of its cross section performed by BELLE collaboration is \[ \sigma_{\text{prompt}}(e^-e^+ \rightarrow J/\psi + c\bar{c} + X) = 0.74 \pm 0.08^{+0.09}_{-0.08} \text{ pb}. \] (2.1)

The color-singlet cross section with NLO QCD correction is $0.33(0.47)$ pb when $m_c = 1.5(1.4)$ GeV and $\mu = 2m_c|\mathcal{R}(0)|^2 = 1.01$ GeV$^3$. The QED and double photon contributions enhance the cross section by $8 + 29$ fb \[22\]. The feed-down contribution from $\psi(2S)$ increases the cross section by a factor of 1.355, whereas that from $\chi_{cJ}$ is 21 fb \[22, 32\]. The small color-octet contribution is 11 fb \[32\]. After these combinations are completed, the prompt cross section becomes $0.51(0.71)$ pb \[22\]. The relativistic correction was performed in ref.\[55\], but its effect was negligible.

However, the physical cross section should always include ISR at $e^-e^+$ annihilation. Figure 3 shows the cross sections with a number of radiated photons for $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$. A substantial probability exists to radiate at least a photon. The average number of ISR photon in each event is about 0.88, which can be presumed because besides an $\alpha$ suppression for radiating a photon, there is also an extra
Figure 4: Cross sections and $K^{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ as functions of the $J/\psi$ momentum $p^*_J$ in the rest frame of initial $e^-e^+$. We take the parameter set as $m_c = 1.4$ GeV, $\mu = 2m_c$ in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$.

Figure 5: Comparisons of the theoretical predictions and BELLE measurement [37] with $\mu = 2m_c$ (left-panel) and $\mu = \sqrt{\hat{s}}/2$ (right-panel) respectively in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$. We have multiplied a factor 1.355 to account for the feed-down contribution from $\psi(2S) \rightarrow J/\psi + X$.

$\log(m_e/\sqrt{s})$ enhancement. To determine the changes of the center-of-mass energy $\sqrt{\hat{s}}$ after showering, we also plot the $\sqrt{\hat{s}}/\sqrt{s}$ distribution in figure 3. The average value for $\sqrt{\hat{s}}/\sqrt{s}$ is about 0.98. The value is quite close to 1, which indicates that ISR correction is small.

Figure 4 shows the ISR effect in the $J/\psi$ momentum spectrum. We obtain the curves for NLO and NLO + ISR by normalizing the corresponding LO and LO + ISR\(^3\) results by a NLO K factor from ref.[22], because the K factor changes slightly in the

\(^3\)LO + ISR refers to LO with ISR effects. LO result already includes the QCD and QED diagrams.
### Table 1: Cross sections of $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ in different parameter sets.

| Parameter sets | LO (pb) | LO+ISR (pb) | NLO (pb) | NLO+ISR (pb) |
|----------------|---------|-------------|----------|--------------|
| $m_c = 1.4$ GeV, $\mu = 2m_c$ | 0.45    | 0.44        | 0.77     | 0.75         |
| $m_c = 1.5$ GeV, $\mu = 2m_c$ | 0.31    | 0.30        | 0.54     | 0.53         |
| $m_c = 1.4$ GeV, $\mu = \sqrt{\hat{s}}/2$ | 0.31    | 0.31        | 0.59     | 0.59         |
| $m_c = 1.5$ GeV, $\mu = \sqrt{\hat{s}}/2$ | 0.23    | 0.22        | 0.42     | 0.42         |

$p_{J/\psi}^4$ spectrum and with $\sqrt{\hat{s}}$ [17]. We take $m_c = 1.4$ GeV, $\mu = 2m_c, |R(0)|^2 = 1.01$ GeV$^3$ here. As expected, ISR minimally corrects the momentum spectrum. We also present $K_{\text{ISR}} = \sigma_{\text{LO+ISR}}/\sigma_{\text{LO}}$ as a function of $p_{J/\psi}^4$ in the right panel of figure 4 for further clarification. ISR causes the $J/\psi$ momentum spectrum to become slightly softer. We also compare the BELLE measurement [37] with the theoretical prompt results in figure 5. Considering the uncertainties in the input parameters, such as $m_c, \mu$, we compare the experimental with the theoretical results in different parameter sets. It is shown that $m_c = 1.4$ GeV, $\mu = 2m_c$ is the closest set to the BELLE data [37], although large uncertainties remain in the experimental data. Finally, the prompt total cross sections are summarized in table 1. ISR decreases the cross section by an extremely minimal amount.

### 3. ISR in $J/\psi gg + X$

In this section, we will study the process $e^-e^+ \rightarrow J/\psi + gg + X$. BELLE collaboration has measured the cross section for $J/\psi + X_{\text{non-}\bar{c}c}$ in ref.[37] as

$$\sigma_{\text{prompt}}(e^+e^- \rightarrow J/\psi + X_{\text{non-}\bar{c}c}) = 0.43 \pm 0.09 \pm 0.09 \text{ pb.} \quad (3.1)$$

Theoretically, the NLO color-singlet cross section for the prompt $J/\psi + gg + X$ is 0.67(0.53) pb when $m_c = 1.4$ GeV, $\mu = 2.8(5.3)$ GeV, $|R(0)|^2 = 1.01$ GeV$^3$ [18, 19], which enhances the LO cross section by about 20 – 30%. Subsequently, the authors of ref.[20] found that the relativistic correction also contributes a factor of 20 – 30% to the color-singlet $\sigma(e^+e^- \rightarrow J/\psi + gg + X)$, which is comparable to that of the QCD correction. The relativistic correction was also confirmed by other authors [21]. Based on their calculations, the color-singlet result is already saturating the experimental data upon the assumption that the whole partonic cross section contributes to $J/\psi + X_{\text{non-}\bar{c}c}$ final states.

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$p_{J/\psi}^4$ refers to the momentum of $J/\psi$ in the rest frame of the initial $e^-e^+$ before showering.
Figure 6: Cross section as functions of number of photon and $\sqrt{s}/\sqrt{s}$ in $e^-e^+ \rightarrow J/\psi + gg + X$.

Figure 7: Cross sections and $K_{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}$ as functions of the $J/\psi$ momentum $p^*_{J/\psi}$ in the rest frame of initial $e^-e^+$. We take the parameter set as $m_c = 1.4$ GeV, $\mu = 2m_c$ in $e^-e^+ \rightarrow J/\psi + gg + X$.

As discussed in the first section, ISR should significantly change the cross section. We plot the number of ISR photon distribution and $\sqrt{s}/\sqrt{s}$ in figure 6. Compared with $J/\psi + c\bar{c} + X$, a slightly higher probability to radiate a photon is observed in $J/\psi + gg + X$. The reason mainly relies on the cross section $\sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ that increases as $\sqrt{s}$ decreases. The average number of photon per event improves to 1.04. Moreover, the average $\sqrt{s}/\sqrt{s}$ is 0.93. Therefore, the ISR effects are more important in $e^-e^+ \rightarrow J/\psi + gg + X$.

Next, we intend to include the QCD correction, relativistic correction and ISR in the color-singlet results. By contrast to the case in $J/\psi + c\bar{c}$, the QCD correction to $J/\psi + gg$ results in $J/\psi$ softer momentum spectrum than the LO one [18]. At the
endpoint, the LO result suffers from the large logarithms \(\log(1 - E_{J/\psi}/E_{J/\psi}^{\text{max}})\) because of kinematic reasons. The LO spectrum significantly changes at the endpoint with the resummation of such logarithms, whereas resummation has limited effects on the NLO spectrum [18]. Moreover, the relativistic correction should not change the LO spectrum but enhance it by a simple K factor. We use the formula

\[
\frac{d\sigma^{\text{NLO}}}{dp_{J/\psi}^*} = (K_{\text{NLO}}(\alpha_s) + K_{\text{NLO}}(v^2) - 1) \frac{d\sigma^{\text{LO}}}{dp_{J/\psi}^*},
\]

(3.2)

to obtain the fixed-order result by considering the QCD and relativistic corrections. A similar formula can be applied to the result with ISR

\[
\frac{d\sigma^{\text{NLO+ISR}}}{dp_{J/\psi}^*} = (K_{\text{ISR}}(\alpha_s) + K_{\text{NLO}}(v^2) - 1) \frac{d\sigma^{\text{LO+ISR}}}{dp_{J/\psi}^*}.
\]

(3.3)

This treatment is justified because the ISR and QCD/relativistic correction can be factorized. Moreover, the K factor of the QCD/relativistic correction slightly changes with \(\sqrt{s}\) [19, 20]. The result is shown in figure 7. ISR results in softer \(J/\psi\) momentum spectrum, which is apparent from the \(K_{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}\) shown in the right panel of figure 7. Another interesting aspect that we want to determine is the sensitivity of the K factors to \(m_c\) values. We establish two plots in figure 8. As shown in the figure, the values of \(K_{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}\) and \(K_{\text{NLO}} = K_{\text{NLO}}(\alpha_s) + K_{\text{NLO}}(v^2) - 1\) are insensitive to \(m_c\).

**Figure 8:** \(K_{\text{ISR}} = \sigma^{\text{LO+ISR}}/\sigma^{\text{LO}}\) (left) and \(K_{\text{NLO}} = \sigma^{\text{NLO}}/\sigma^{\text{LO}}\) (right) as functions of \(J/\psi\) momentum \(p_{J/\psi}^*\) with \(m_c = 1.5\) GeV and \(m_c = 1.4\) GeV and \(\mu = 2m_c\) in \(e^-e^+ \rightarrow J/\psi + gg + X\) than in \(e^-e^+ \rightarrow J/\psi + c\bar{c} + X\).
For comparison with the BELLE measurement, we adapt the same bin size that they used. The $J/\psi$ momentum spectrum is shown in figure 9. We obtain four different input parameter sets. The color-singlet result already saturates the experimental data. With all of the three corrections (i.e. QCD, relativistic and ISR corrections), more stringent room is left for color-octet contribution in $J/\psi + X_{\text{non-cc}}$. The total theoretical cross sections for $e^-e^+ \to J/\psi + gg + X$ in various parameter sets are summarized in table 2. ISR enlarges the cross section by $15-25\%$. Although the cross sections are slightly higher than the experimental data [37] and considering the large theoretical uncertainties, the theoretical result can still be lower. For example,  

\footnote{Resummation also has a very minimal effect on the total cross section [18].}

![Figure 9: Comparisons of the theoretical predictions and BELLE measurement [37] with $\mu = 2m_c$ (left-panel) and $\mu = \sqrt{s}/2$ (right-panel) respectively in $e^-e^+ \to J/\psi + gg + X$. We have multiplied a factor 1.355 to account for the feed-down contribution from $\psi(2S) \to J/\psi + X$.](image)

| parameter sets | LO (pb) | LO+ISR (pb) | NLO (pb) | NLO+ISR (pb) |
|----------------|---------|-------------|----------|--------------|
| $m_c = 1.4 \text{ GeV, } \mu = 2m_c$ | 0.57    | 0.65        | 0.79     | 0.91         |
| $m_c = 1.5 \text{ GeV, } \mu = 2m_c$ | 0.45    | 0.50        | 0.63     | 0.72         |
| $m_c = 1.4 \text{ GeV, } \mu = \sqrt{s}/2$ | 0.35    | 0.45        | 0.60     | 0.77         |
| $m_c = 1.5 \text{ GeV, } \mu = \sqrt{s}/2$ | 0.30    | 0.37        | 0.51     | 0.64         |

Table 2: Cross sections of $e^-e^+ \to J/\psi + gg + X$ in different parameter sets.

| parameter sets | LO | LO+ISR | NLO | NLO+ISR | BELLE |
|----------------|----|--------|-----|---------|-------|
| $m_c = 1.4 \text{ GeV, } \mu = 2m_c$ | 0.44 | 0.41    | 0.49 | 0.45    | 0.63 ± 0.11 |
| $m_c = 1.5 \text{ GeV, } \mu = 2m_c$ | 0.41 | 0.38    | 0.46 | 0.42    | 0.63 ± 0.11 |
| $m_c = 1.4 \text{ GeV, } \mu = \sqrt{s}/2$ | 0.47 | 0.41    | 0.50 | 0.44    | 0.63 ± 0.11 |
| $m_c = 1.5 \text{ GeV, } \mu = \sqrt{s}/2$ | 0.43 | 0.38    | 0.45 | 0.40    | 0.63 ± 0.11 |

Table 3: Comparisons of $R_{c\bar{c}}$ in different parameter sets with BELLE measurement [37].
respectively, and table 2 we also list the total cross sections. We expect that a more precise measurement could further elucidate the situation. Finally, we included for comparison with the experimental data. The total cross sections for $J/\psi + gg + X$ and ISR corrections. The feed-down contributions (mainly from $\psi$) are different theoretical results for these two processes in combination with the QCD, relativistic, and hadronic data and small scale $e^\pm$ data motivate us to reconsider the theoretical results. In this article, we focus on the issue about the prompt $J/\psi$ inclusive production at B factories. Considering that the cross sections in $e^-e^+ \rightarrow J/\psi + X$ are presented in table 1 and table 2 respectively, whereas the $J/\psi$ momentum spectra in the BELLE bin size [37] are shown in figure 5 and figure 9. Considering the large experimental and theoretical uncertainties, we are still unable to draw strong conclusions. However, the corrections in $J/\psi + gg + X$ considerably constrain the color-octet contribution. Finally, we also present $R_{c\bar{c}}$

| parameter sets | LO (pb) | LO+ISR (pb) | NLO (pb) | NLO+ISR (pb) |
|---------------|---------|-------------|----------|--------------|
| $m_c = 1.4$ GeV, $\mu = 2m_c$ | 1.02 | 1.09 | 1.55 | 1.66 |
| $m_c = 1.5$ GeV, $\mu = 2m_c$ | 0.76 | 0.80 | 1.17 | 1.25 |
| $m_c = 1.4$ GeV, $\mu = \sqrt{s}/2$ | 0.66 | 0.76 | 1.19 | 1.36 |
| $m_c = 1.5$ GeV, $\mu = \sqrt{s}/2$ | 0.52 | 0.59 | 0.93 | 1.05 |

Table 4: Cross sections of $e^-e^+ \rightarrow J/\psi + X$ in different parameter sets.

we can use a lower value of $|R(0)|^2$ as performed in ref.[19] or from the potential model estimation [56]. In principle, the ratio $R_{c\bar{c}}$ should be independent of the value of $|R(0)|^2$ in color-singlet case. We present the theoretical $R_{c\bar{c}}$ in table 3. We use a same parameter set in $J/\psi + c\bar{c} + X$ and $J/\psi + gg + X$ and assume that $\sigma(e^-e^+ \rightarrow J/\psi + gg + X) = \sigma(e^-e^+ \rightarrow J/\psi + X_{non-c\bar{c}})$. The theoretical result is slightly lower than BELLE measurement, but still within the 2 standard deviation. Therefore, we expect that a more precise measurement could further elucidate the situation. Finally, we also list the total cross sections $\sigma(e^-e^+ \rightarrow J/\psi + X) = \sigma(e^-e^+ \rightarrow J/\psi + c\bar{c} + X) + \sigma(e^-e^+ \rightarrow J/\psi + gg + X)$ in table 4, which is compatible with the experimental [37] value $\sigma_{\text{prompt}}(e^-e^+ \rightarrow J/\psi + X) = 1.17 \pm 0.02 \pm 0.07 \text{ pb}$.

4. Summary

The different conclusions formulated from the large $p_T$ hadronic data and small scale $e^\pm$ data motivate us to reconsider the theoretical results. In this article, we focus on the issue about the prompt $J/\psi$ inclusive production at B factories. Considering that the cross sections in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ and $J/\psi + gg + X$. ISR enhances the total cross section of $e^-e^+ \rightarrow J/\psi + gg + X$ by a factor of $15 - 25\%$, which depends on the input values of the parameters. Moreover, ISR causes the $J/\psi$ momentum spectrum to be softer in $e^-e^+ \rightarrow J/\psi + c\bar{c} + X$ and $e^-e^+ \rightarrow J/\psi + gg + X$. This result is important because it provides a good perspective to the color-octet contribution. We present the theoretical results for these two processes in combination with the QCD, relativistic, and ISR corrections. The feed-down contributions (mainly from $\psi(2S)$) are also included for comparison with the experimental data. The total cross sections for $J/\psi + c\bar{c} + X$ and $J/\psi + gg + X$ are presented in table 1 and table 2 respectively, whereas the $J/\psi$ momentum spectra in the BELLE bin size [37] are shown in figure 5 and figure 9. Considering the large experimental and theoretical uncertainties, we are still unable to draw strong conclusions. However, the corrections in $J/\psi + gg + X$ considerably constrain the color-octet contribution. Finally, we also present $R_{c\bar{c}}$.
with the assumption $\sigma(e^-e^+ \to J/\psi + gg + X) = \sigma(e^-e^+ \to J/\psi + X_{\text{non-cc}})$, which should be more precise than the cross section alone. The theoretical result is lower than BELLE measurement [37], but still within $2\sigma$. More careful theoretical and experimental analyses are necessary in the future.

Acknowledgments

We are thankful to Prof. Kuang-Ta Chao, Dr. Yu-Jie Zhang and Dr. Zhi-Guo He for helpful discussions. This work was supported in part by the National Natural Science Foundation of China (No 11075002, No 11021092), and the Ministry of Science and Technology of China (2009CB825200).

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