Development of a steepest-descent control solution for Multi-Input Multi-Output swept sine testing in a virtual shaker environment

U Musella¹,²,³, S Waimer¹, B Peeters¹, F Marulo and P Guillaume²

¹ Siemens Industry Software NV, RTD Test Division, Interleuvenlaan 68, 3000 Leuven, BE
² Vrije Universiteit Brussel, Mechanical Engineering Department, Pleinlaan 2 1050 Elsene, BE
³ University of Naples "Federico II", Department of Industrial Engineering, Via Claudio 21, 80125, Napoli, IT
E-mail: umberto.musella.ext@siemens.com

Abstract. Multiple-Input Multiple-Output (MIMO) sine sweep is nowadays acknowledged to be one of the best excitation techniques for the modal testing of large aerospace structures, being an optimal solution in the tradeoff between high-quality FRFs testing time. Due to the test levels that can be reached during the sine sweeps and the increasing complexity and cost of the test articles, monitoring the recordings of multiple transducers translates in the actual need of guaranteeing that the responses are controlled on-line to a (safe) reference level. The development of a dedicated MIMO control strategy is very challenging, especially as a consequence of the transient behavior of the sweeps. This work proposes to investigate a solution for the on-line control of a MIMO continuous swept sine test combining the harmonic estimator with a steepest descent feedback control, typically adopted for the cancellation of tonal disturbances. The controller is developed in a virtual shaker testing environment, in order to tackle, already in the development stage, the effects that can be encountered during an actual test and that cannot be modeled with a linear system identification.

1. Introduction
Nowadays Multiple-Input Multiple-Output (MIMO) swept sine is one of the most popular excitation techniques for the modal testing of large aerospace structures. The success is related to the possibility of characterizing the dynamic response of the unit under test frequency-to-frequency, reaching high excitation levels, high signal to noise ratio and hence high quality FRFs [1, 2, 3, 4]. Some of these features were previously obtained running normal modes tests or combining normal modes with broadband or stepped sine excitations at the cost of extremely long testing times (normal modes and stepped sine) or poor excitation levels (broadband random) [5]. Even if these tests are critical, they are scheduled in periods of extreme time pressure, with the full ready-to-use assembly that still needs to undergo additional testing stages, for example acoustic and thermal testing of large satellites [6]. Another example is the Ground Vibration Testing (GVT) of large aircrafts, where the testing, required for flutter prediction analysis, is typically scheduled before the final painting and the maiden flight. Swept sine excitation is, in this sense, the best compromise between testing time and FRFs quality. Other advantages of swept sine testing are related to the deterministic nature of the excitation signals that guarantees
high repeatibility and the possibility of obtaining preliminar information about potential non- linearities [1, 2, 3, 4]. From this perspective, an appealing feature of the swept sine excitation is the possibility of using state-of-the-art feedforward control techniques to define reference levels for specific control channels, e.g. force transducers or accelerometers. This allows the pre-shaping of the voltages driving the exciters, by predicting the dynamic behaviour of the unit to be tested. Such prediction relies on a model of the system obtained with a low-level pre-test random system identification. Due to potential non-linear behaviour that can be triggered at the (generally high) test levels reached during the actual sine sweep runs, noise on the measurements and transient effects due to the sweeping itself (for instance beating phenomena [4], [7], [8]), the system can generally differ from the linear model estimated in the pre-test. The spectra of the control channels can therefore deviate from the references and closed loop feedback control strategies are sought to compensate for these differences.

The aim of this paper is to investigate the possibility of combining an amplitude and phase tracking technique to a steepest-descent correction algorithm in the frequency domain in order to simultaneously control the multiple responses to a specific reference amplitude value and guarantee that they will also keep a specific relative phase.

Since in this work most of the derivations are in the frequency domain, all the arrays are functions of the frequency $f$ (in Hz), if not specified otherwise. Vectors are denoted by lower case bold letters, e.g. $\mathbf{a}$, and matrices by upper case bold letters, e.g. $\mathbf{A}$. An over-bar $\overline{D}$ is used to indicate the complex conjugate operation and the Hermitian superscript $D^H$ to indicate the complex conjugate transpose of a matrix, e.g. $\overline{\mathbf{a}}$ and $\mathbf{A}^H$. The dagger symbol $D^\dagger$ is used to indicate the Moore-Penrose pseudo-inverse of a matrix, whereas the hat $\mathbf{D}$ is used to emphasize the estimation of a quantity, e.g. $\mathbf{\hat{A}}$ is an estimate of the matrix $\mathbf{A}$. Time domain derivatives are indicated with dots $\dot{D}$, for example $\dot{a}(t)$ and $\ddot{a}(t)$ are the first and second derivatives of the waveform $a(t)$, respectively.

2. MIMO swept sine controlled excitation

Fig. 1 is a schematic representation of a general MIMO vibration control test. A set of $m$ voltages is sent to multiple exciters. Typically these signals (the so-called drives) drive $m$ independent shakers or, possibly, the multiple degrees of freedom of a multi-axis shaking table [9], [10]. The unit under test responses are recorded in $\ell$ control channels (the so-called controls or pilots).

Figure 1. Schematic representation of a MIMO vibration control test.
During a MIMO test with swept sine excitation, the unit under test is excited with multiple swept sines over a user-defined frequency band. All the input signals are driven by the same instantaneous frequency.

The reasons to adopt a multi-input excitation for swept sine testing can be different, for example (i) the need for testing heavy or slender structures that cannot be accommodated on a single shaker, (ii) the possibility of setting a defined phase between the responses to better characterize specific mode shapes. (iii) Multi-axis exciters can be used to perform a true single-axis test, by setting as reference very low response spectra in the off-axis directions.

The control target for a MIMO swept sine vibration control test is therefore a set of reference spectra, characterized by amplitude and phase (relative to the phase of the first control channel). Current excitation techniques make use of a pre-test system identification to pre-shape the drives (feedforward control) in the attempt of matching the reference while continuously sweeping with a specific sweep mode (linear or logarithmic) and sweep rate (in Hz/s or Oct/min).

2.1. System Identification

For each frequency, in case the full dynamic system is linear and time invariant, it is possible to write the Input-Output relation

\[ y = Hu \]  (1)

where \( y \in \mathbb{C}^{\ell \times 1} \) and \( u \in \mathbb{C}^{m \times 1} \) are the spectra of the controls and the drives, respectively and \( H \in \mathbb{C}^{\ell \times m} \) is the matrix of FRFs. For the sake of simplicity, as most of the derivations are in the frequency domain, the terms drives, controls, responses and references are referred to the drives, controls, responses and reference spectra, respectively, if not specified differently.

In all the vibration control tests, a System Identification pre-test phase is always needed to estimate the system’s FRFs. This is usually performed by running a low-level random test and using the so called \( H_1 \) estimator \[ \hat{H} = \hat{S}_{yu} \hat{S}_{uu}^{-1} \]  (2)

where \( \hat{S}_{yu} \in \mathbb{C}^{\ell \times m} \) and \( \hat{S}_{uu}^{-1} \in \mathbb{C}^{m \times m} \) are spectral density matrices estimated via Welch’s averaged periodogram method [12] or other approaches.

Once the FRF is known, the system’s Mechanical Impedance Matrix \( Z \) can be calculated. Generally an operation of pseudo-inversion is always needed because the number of controls can exceed the number of drives, in what is known in the MIMO control literature as Rectangular Problem [13].

2.2. Drives pre-shape

As shown in [14], for each frequency line, independently, the control error

\[ e = r - y \]  (3)

where \( r \in \mathbb{C}^{\ell \times 1} \) are the reference spectra, is minimized in the least square sense by the optimal drives

\[ u_{opt} = (\hat{H}^H\hat{H})^{-1}\hat{H}^Hr \]  (4)

In order to calculate the drives, since the real plant is unknown, the best solution is to use information obtained from the system identification. To notice that the quantity \((\hat{H}^H\hat{H})^{-1}\hat{H}^H\) is, for the general case of a rectangular problem, the pseudo-inverse of the FRF matrix according to Moore and Penrose \((\hat{H}^H\hat{H})^{-1}\hat{H}^H = \hat{H}^H = \hat{Z} \). Hence, an estimate of the optimal drives can be calculated as
\[ \hat{u}_{opt} = \hat{Z}r \]  

With reference to the active control literature [15], the proposed approach is a traditional feedforward control, also called *drives pre-shape* in environmental testing [7], [8]. Unfortunately, due to the difference in levels between the low level random system identification and the actual test, the transient nature of the sweeps and potential noise in the measurements, the reference and the control spectra will generally differ, and an on-line feedback control is sought to keep the responses as close as possible to the desired spectra.

### 3. Sweeping and Estimate

#### Harmonic Estimator
The idea of the on-line frequency domain MIMO control for swept sine testing proposed in this paper is closely dependent on the capability of accurately tracking the amplitude and the phase of the response (to be corrected) and the drive (to be updated) waveforms, as shown in fig. 2. The idea of this paper is to use an on-line implementation of a traditional Harmonic Estimator, as currently adopted, for example, in the Siemens Simcenter Testlab Sine Control application [16].

For a general sinusoidal waveform with fundamental natural frequency \(2\pi \omega_f\)

\[ y(t) = |y(t)| \sin[\omega_f t + \phi(t)] = a_c \cos(\omega_f t) + a_s \sin(\omega_f t) \]  

the amplitude and phase

\[ |y(t)| = \sqrt{a_c^2 + a_s^2} \]  

\[ \phi(t) = \text{atan}\left(\frac{a_c}{a_s}\right) \]

(can be calculated assuming that, within \(p\) periods, eq. (6) holds, and therefore the parameters \(a_c\) and \(a_s\) can be estimated in a least square sense using the acquired data (left-hand-side of eq. (6)).

For the on-line estimation during a continuous sine sweep, the frequency of the signals, and therefore the argument \(\Phi(t) + \phi(t)\) of the sine wave in eq. (6), continuously varies with a specific sweep mode (linear or logarithmic).

#### Sweeping
Output of the frequency domain MIMO control is a set of corrected amplitudes and phases for the drive signals. This set of amplitude and phase can be used to generate the drives during the sweep

\[ u(t) = A(t) \odot \sin(\omega(t)t + \phi(t)) \]

where the \(\odot\) represent the element-wise multiplication, \(\omega(t)\) the instantaneous frequency at the time \(t\), and \(A(t)\) and \(\phi(t)\) vectors containing the drives amplitude and phase, update by the controller.

### 4. MIMO sine feedback control
The idea of this work is to use a classic feedback control for tonal disturbances in order to correct the potential differences between the reference and the control spectra. This is possible once the the amplitude and the phase of the swept sine waves are accurately estimated for each frequency line, as explained in sec. 3.

The proposed approach follows the theory largely discussed in [15] and is schematically represented in the block diagram of fig. 2.
4.1. Drives update

The cost function associated to the least square error (3) is

$$ J(u) = e^H e = u^H H^H H u - u^H H^H r = r^H H u + r^H r $$

(9)

The task of the controller is to iteratively adjust the drives at the $n$-th iteration in order to minimize the cost function (9). The drives can then be updated calculating the complex gradient

$$ g_n = \frac{\partial J_n}{\partial u_n} = 2(H^H H u_n - H^H r) = 2H^H e_n $$

(10)

and moving in the steepest descent direction

$$ u_{n+1} = u_n + 2\mu H^H e_n = u_n + \alpha H^H e_n $$

(11)

where $\mu$ is the step size or convergence factor and $\alpha = 2\mu$ is often referred to as convergence coefficient [15].

The convergence coefficient need to be chosen in order to guarantee the Bounded Input Bounded Output (BIBO) stability of the MIMO closed loop system.

4.2. Optimal convergence coefficient

In order for the Closed Loop system to be BIBO stable, it is necessary to guarantee, that the error will eventually converge. In order to analyze the stability of the system with the classic control theory, with reference to the fig. 2 and [15] it is convenient to write the error equation for the closed loop system in the $Z$-domain

$$ \tilde{\epsilon} = \tilde{r} - H C \tilde{\epsilon} = (I + H C)^{-1} \tilde{r} $$

(12)

In this section, a tilde $\tilde{D}$ is used to denote the quantities in the $Z$-domain, omitting the independent variable $z$. The Closed Loop system is BIBO stable if the error converges, hence if the poles of the complementary sensitivity function $T = H C (I + H C)^{-1}$ all lie inside the unity circle, as explained in details in [15]. The poles of the complementary sensitivity function are the roots of the characteristic equation

$$ det(I + H C) = 0 $$

(13)

Finding the roots of eq. (13) requires to find an expression for the controller parameters in function of the controller tunable parameters in the $Z$-domain. Rewriting the drives update (11) in the $Z$-domain ($\tilde{u} = \tilde{u}_{n+1}$) and considering that $u_n = \frac{1}{\bar{z}} \tilde{u}$, with reference to the fig. 2
\[ \hat{u} = (zI - I)^{-1} \alpha H^H \hat{e} \]  \hspace{1cm} (14)

In the general block scheme of the feedback control in fig. 2, the controller \( C \) relates the updated inputs to the error \( \hat{u} = C \hat{e} \) and therefore the feedback controller corresponding to the drives update (11), can be expressed as

\[ C = \frac{\alpha}{z - 1} H^H \] \hspace{1cm} (15)

With this expression it is possible to evaluate the determinant (13) in the \( Z \)-domain as explicit function of the convergence factor \( \alpha \)

\[ \det(I + HC) = \det \left( I + \frac{\alpha}{z - 1} HH^H \right) = \det(z - 1 + \alpha HH^H) = 0 \] \hspace{1cm} (16)

Since the determinant can be expressed as the product of the eigenvalues and noticing that the eigenvalues of the positive semi-definite matrix \( HH^H \) are equal to the ones of \( H^H H \), eq. (16) can be rearranged as

\[ \det(z - 1 + \alpha H^H H) = \prod_{i=1}^{M} (z - 1 + \alpha \lambda_i) = 0 \] \hspace{1cm} (17)

where \( \lambda_i \) is the i-th eigenvalue of the \( H^H H \) (which equals the square singular value \( \sigma_i^2 \) of the matrix \( H \)). The i-th pole can therefore be written as

\[ z_i = 1 - \alpha \lambda_i \] \hspace{1cm} (18)

For the BIBO stability of the closed loop system, all the poles need to lie within the unity circle, so that

\[ |z_i| \leq 1 \rightarrow |1 - \alpha \lambda_i| \leq 1 \hspace{1cm} \forall \lambda_i \] \hspace{1cm} (19)

This equation brings the general condition for the BIBO stability of the closed loop system, limiting the value of the convergence coefficient \( \alpha \)

\[ |1 - \alpha \lambda_{\text{max}}| \leq 1 \] \hspace{1cm} (20)

and finally returns the optimal value (with respect to the convergence speed) of the convergence coefficient

\[ 0 \leq \alpha \leq \frac{1}{\lambda_{\text{max}}} \rightarrow \alpha_{\text{opt}} = \frac{1}{\sigma_{\text{max}}^2} \] \hspace{1cm} (21)

It is worth to notice that

5. The challenges of MIMO swept sine control: a simulated example using a Virtual Shaker model

In this section an explanation of the potential challenges related to a MIMO swept sine controlled test with the proposed algorithm are supported by results obtained from simulations where the algorithm is used to control the responses of two uncoupled lumped mass parameters shakers. The idea is to build the controller in a simulation environment that includes also a full validated virtual model of the amplifiers and the vibration exciters. This approach allows, already in the development stage, to tackle potential issues that can be encountered during an actual test. It is therefore chosen, as excited system, a pair of virtual electro-mechanical lumped-parameters
vibration exciters in twin configuration, result of the research [7] and [8] and represented in fig. 3. Even though in the current work, for the sake of simplicity, no coupling structure is considered in the model and hence the two shakers are independently driven, the algorithm is generally developed for both coupled and uncoupled systems. The parameters used to model the shaker 1 are retrieved from the experimental test campaign performed to characterize a 6 kN shaker available at Siemens Industry Software N.V. HQ (Leuven, Belgium). Small differences, with respect to the parameters of the shaker 1, are introduced to obtain the ones adopted for the twin shaker 2. The selected control channels are the two virtual accelerometers located on the shaker heads (referring to fig. 3, the signals $\ddot{z}_{T1}$ and $\ddot{z}_{T2}$) whereas the drives are the voltages $V_1$ and $V_2$ sent to the virtual amplifiers and powering the electrical parts of the shakers. The FRFs from the voltages to the control accelerations of the identified system are represented by the solid blue lines in fig. 4. The dashed red lines represent the FRFs of the system excited during the sine sweep: to introduce a system identification error, the lumped parameters slightly differ from the ones of the system excited during the identification (coil masses and lumped payloads $m_i$ reduced of 1% for the shaker 1 and increased of the same amount for the shaker 2).

5.1. Challenges of MIMO swept sine feedback control
Following the theory of sec. 4, the feedback control of the responses for all the frequencies in a specific band of interest can be obtained considering the drives update (11) and the optimal convergence coefficient (21). First, the response amplitudes and phases need to be estimated using the procedure illustrated in sec. 3 so that the control error is a known quantity. The update equation and the estimate of the optimal convergence coefficient, require also the knowledge of the real plant, however unknown. To overcome this, the solution is to make use of the system’s FRFs obtained with a pre-test random system identification. Even though adopting the full drives update (corresponding to the optimal convergence coefficient) would return the fastest convergence rate, it is however a condition at the limit of stability, obtained considering that all the transients have died out and the responses at the control channels have reached their steady-state, which can be a strong assumption for a swept sine test, due to the nature of the sweep. It is hence a practical idea to scale down the convergence including a control gain $K$ (between 0 and 1)

Figure 3. Schematic representation of the two lumped-parameters Virtual Shakers in twin configuration.
\[ u_{n+1} = u_n + K \frac{1}{\hat{\sigma}^2_{max}} \hat{H} \hat{H}^H e_n \] (22)

The parameters set for the MIMO swept sine control runs are shown in Table 1.

| Table 1. Settings adopted for the MIMO swept sine test. |
|-------------------------------------------------------|
| Sampling frequency                                   | 6400 Hz |
| Bandwidth                                            | 10-2500 Hz |
| Sweep mode                                           | Linear |
| Sweep Rate                                           | 2 Hzs, sweep Up |
| Number of estimation period                          | 1 |

It is worth to underline that, once the parameters in Table 1 are set, the final test duration is automatically defined. For a sine sweep excitation, there is an exact correspondence between each time stamp and the instantaneous swept frequency, known a priori. This is fundamental as it allows to use at each time instant the frequency domain quantities needed for the drives update, i.e. the estimated values of the matrix $\hat{H}$ and the calculated optimal convergence coefficient. For the simulation example provided in this section, these quantities are illustrated in Figure 4 and 5, respectively.

Figure 4. FRFs matrix of the twin shaker. In blue and red the system excited during the MIMO sine control run and the identified system. In the details, it is highlighted the induced system identification error.

The main issue derived from the combination of the proposed control strategy with a continuous sweep is related to the continuous changes of the cost function resulting from the sweeping and the associated change of the system’s FRF matrix in the eq. (9). For a MIMO system this cost function is a multidimensional complex parabolic surface with as many dimensions as the number of drives and an eccentricity directly related to the condition number of the system. Figure 6, 7 and 8 represent the shape of the cost function, the cost function contours and the condition number at three different frequencies during the sweep. The cost function $J$ and its contours are plotted in function of $u - u_{opt}$, where $u_{opt}$ are the optimal drives that ideally minimize $J$ (known in a simulated scenario). These graphs are therefore centered in zero (the optimum, represented by the green dot in the plots). In the plot only the the real part of the drives is considered (indicated with an additional subscript $R$), but similar plots
Figure 5. Optimal convergence coefficient in the frequency range of interest is $2/\sigma_{max}^2$. For completeness, in the figure it is also reported the condition number of the system (right y axis).

Figure 6. Cost function and cost function’s contours in function of the real part of $u - u_{opt}$ (drive 1 on the x-axis, drive 2 on the y-axis) at frequency where the condition number of the system FRFs matrix is almost unitary. To notice, from the cost function contours plot, that the convergence occurs almost simultaneously for the two drives.

can be also obtained considering the imaginary part. The red dots illustrates the drives at each iteration and hence the convergence of the process.

From the figure it is possible to understand that the shape of the cost function, and hence the corrections needed for each drive to reach its optimal value, is strongly influenced by the system’s conditioning at a specific frequency. In case of a low condition number (at the limit unitary), the cost function has a regular (at the limit, semi-spherical) shape, indicating that the convergence will most likely occur simultaneously for the two different (transformed) drives, as shown in fig. 6. On the contrary, for the frequency lines where the system has an high condition number, the cost function assumes a stretched shape in the direction of one of the (transformed) drives and hence the convergence for this (transformed) drive would require more iterations than the ones needed by the other. It is worth to notice that, for this uncoupled example, the axes of the ellipsoidal cost function’s contours are well-aligned with the plane defined by the components of $u - u_{opt}$. The simplification of uncoupled system allows to draw considerations on the physical drives, but generally these considerations are strictly valid only in the transformed space defined...
Figure 7. Cost function and cost function’s contours in function of the real part of $\mathbf{u} - \mathbf{u}_{opt}$ (drive 1 on the x-axis, drive 2 on the y-axis) at a frequency where the condition number of the system FRFs matrix is relatively high. Because of the shape of cost function, the drive 1 needs more iterations than the drive 2 to converge at this specific frequency.

Figure 8. Cost function and cost function’s contours in function of the real part of $\mathbf{u} - \mathbf{u}_{opt}$ (drive 1 on the x-axis, drive 2 on the y-axis) at a frequency where the condition number of the system FRFs matrix is relatively high. Because of the shape of cost function, the drive 2 needs more iterations than the drive 1 to converge at this specific frequency.

by the principal coordinates, [15]. For the specific case of a swept excitation, this might represent a main issue because, during the sweep, the cost function can vary abruptly while the correction algorithm converges, as shown in figure 9, depending on the system’s conditioning and the sweep rate. This challenge combines with the re-known limitation of the steepest descent algorithm in the neighborhood of the solution (for a specific drive). To overstep this latter limitation, a stopping condition based on the number of iterations (or on the slope of the cost function) can
be included in the algorithm. At the current stage, however, no link has been theoretically made between the convergence and the sweep rate and a scheduling of the control gain $K$ can be used to overcome the deriving potential issue.

![Cost function](image)

**Figure 9.** During the sweep the cost function shifts and deforms.

A MIMO swept sine control simulation is run to show the applicability of the proposed procedure with twin shaker virtual model and the settings reported in tab. 1. Fig. 10 illustrates the comparison from the a classic feedforward approach (drives pre-shape) and the prototype solution of the MIMO swept sine control. Besides the system identification error introduced, as illustrated in fig. 4 also two step disturbances have been introduced on the responses at 400 Hz and 800 Hz for the responses 1 and 2, respectively. From the details in the figure, it is clearly shown the capability of the proposed feedback control strategy of attenuating the differences due to the system identification error and quickly responding to the introduced step disturbances (less than half a second for the response one and approximately one second for the response 2).

6. Conclusions and future directions

In this paper the details and the mathematical derivation of a newly developed MIMO swept sine control strategy are discussed. The algorithm makes use of a fast on-line tracking of amplitudes and phases between responses to correct in feedback potential mismatches between the spectra of a user-defined number of control channels and a specific reference, using a steepest descent algorithm. The details of the convergence challenges associated to the MIMO correction for swept sine excitation and the results of a MIMO closed loop controlled simulation are shown with a virtual model of a setup with shakers in twin configuration, continuously sweeping in a specific band of interest. The promising results motivates further investigations and a real time proof-of-concept, in order to study the feasibility of adopt the proposed methodology for MIMO swept sine environmental testing and acquisition.

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Figure 10. Control results for the simulations run with the pre-shaped drives (solid magenta) and the feedback control (solid blue). The induced errors to be corrected are the system identification error shown in figure 4 and two step disturbances. In the figure the alarm and abort (±3dB and ±6dB) are also reported for completeness.

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