Observer-based region tracking control for underwater vehicles without velocity measurement

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Abstract This paper addresses the problem of region tracking control for underwater vehicles without velocity measurement in marine environment. For this case, an improved region tracking control strategy is proposed based on a Nussbaum-type state observer. In the proposed scheme, a Nussbaum-type state observer is developed to estimate the unmeasured velocity of the vehicle. And then an improved region tracking control strategy is presented by incorporating the estimated velocity of the state observer, such that the tracking errors satisfy the requirement of the prescribed boundaries. In addition, a RBF neural network is applied to approximate the unknown dynamics of the vehicle. It is verified that both the estimated error and the tracking error are uniformly ultimately bounded. Finally, the proposed observer-based region tracking control strategy is applied on an underwater vehicle to perform simulation studies and compared with a traditional backstepping controller and a traditional region tracking controller based on a high-gain observer, respectively. The simulation results verify the effectiveness of the proposed control strategy.

Keywords Underwater vehicles · Region tracking control · State observer · Nussbaum-type function

1 Introduction

Due to their special advantages, e.g., moving in complex marine environment, especially in deep-sea environment, underwater vehicles have wide applications, including underwater detection, underwater searching, and operation with manipulators [1–4]. Among these applications, a reliable control system is required for the successful completion of the given mission [5–8]. Due to the existence of ocean current, modeling uncertainty and nonlinear coupling, there are many difficulties in the control design of underwater vehicles, which is also a current research hot [9–11]. Recently, the concept of region tracking control has been proposed by considering the speciality of some applications [12], namely, the tracking precision is not their first priority in pipeline tracking. And for these special missions, the control signals are expected to be as smooth as possible on the premise that the tracking errors of the vehicle are always kept within the prescribed boundaries. It is also the original intention of region tracking control.

From the existing region tracking control schemes, most of them only focus on steady performance of the tracking error [13–16], i.e., the region tracking is achieved if the tracking error of the vehicle in the steady state is within the prescribed boundaries. As a representative of the design previously reported,
[12] proposed a potential energy function-based region tracking control scheme, where the tracking errors converged to the desired region in the steady state. Another representative is the region tracking control scheme based on the piecewise and differential Lyapunov function in [17], where the region tracking control law was derived in the framework of backstepping technique by the special Lyapunov functions with the desired region. However, if the requirement about the transient-state performance of the tracking error is made in some special missions, the aforementioned region tracking control schemes are not applicable.

Recently, prescribed performance control (PPC) has been proposed, where the tracking error can satisfy the prescribed requirements in both the transient state and the steady state by the constraint transformation [18–20]. The idea of the PPC schemes maybe provide a choice for the design of region tracking control with the added requirements in the transient state. However, the tracking error always converges to zero from the simulation results of the existing PPC schemes, which does not satisfy the original intention of region tracking control. To solve this problem, [21] proposed an adaptive region tracking control with the prescribed transient performance for underwater vehicles, where an error transformation inspired by PPC was combined with the piecewise and differential Lyapunov function to design the region tracking control law. In addition, the predefined-time control design was investigated in [22] for an underwater vehicle with three degrees of freedom, where a new sliding mode manifold was designed to achieve predefined-time stability.

It is worth noting that both position and velocity of the vehicle should be available in the existing region tracking control schemes. However, the sensors carried by an underwater vehicle sometimes cannot provide such enough states for the control system, due to the cost or other practical issues [23,24]. For example, the Doppler velocity log (DVL), always used to measure the velocity of the vehicle, requires that the vehicle’s attitude off the seafloor is less than a maximum range which is related with the DVL’s frequency and DVL is also susceptible to sporadic failures [25]. For this scenario where the velocity of the vehicle is not available, it is necessary to investigate the region tracking control problem for underwater vehicles with only position measurement.

For the control design without velocity measurement, observer-based control strategies have been considered to implement tracking control for underwater vehicles only equipped with position sensors [9,24,26], but they have focused on the tracking control system with high precision. It is known that state observers have the ability to well estimate the unmeasured state based on the known state information of the vehicle and its control input. So far, different forms of state observers have been presented. Among them, high-gain observer or variants thereof has been widely applied to estimate the unmeasured velocity, due to its simple structure [27,28]. For example, a high-gain observer-based linear-quadratic-integral controller was developed in [29] for a towed underwater vehicle with movable wings subject to uncertainties from towing cables, external disturbances, and parametric variations. In order to obtain more precise estimation results, other state observers with complex forms have been investigated. For example, adaptive sliding mode-based observers were developed in [23,24], where the finite-time convergence of the estimated errors was achieved and the estimation precision was improved. A trajectory tracking control scheme was investigated based on a generalized super-twisting algorithm for an underwater vehicle in [30], where the unmeasured velocity was estimated based on a higher-order sliding mode observer. Moreover, in [31], in order to achieve the fixed-time convergence, a fixed-time extended state observer was proposed to estimate the external disturbances and the unmeasurable velocity for an underwater vehicle. According to the existing forms of state observers, it is known that the estimation ability about the unmeasured state can be enhanced by introducing a nonlinear feedback in the state observer. Hence, this paper develops a new form of state observer by introducing Nussbaum-type functions.

Nussbaum-type functions have an important property about uniformly ultimately boundedness. For the compounded disturbance term in the general nonlinear system, [32] proposed a Nussbaum-type disturbance observer based on the complete states. On this basis, [33] gives an improved Nussbaum-type disturbance observer based on a RBF neural network. These Nussbaum-type observers in [32,33] have shown good estimation precision for the general disturbances. However, they all require the full states of the nonlinear systems, which means that these forms of the Nussbaum-type observers cannot be directly used to estimate the unmeasured state of the nonlinear system.
Different from these published researches, this paper investigates observer-based region tracking control problem for underwater vehicles with only position measurement. In this paper, the unknown dynamics is on-line approximated by a RBF neural network. The objective of this paper is to estimate the unmeasured velocity of the vehicle and guarantee the tracking error to be kept within the prescribed boundaries in the whole process. The main contributions of this paper are summarized as follows:

(1) In order to improve the estimation precision of the unmeasured velocity, Nussbaum-type functions are introduced into the conventional state observer. And the stability analysis of the observer is presented.

(2) In order to make full use of the predesigned region and reduce the chattering phenomenon of the control input, a new error transformation is developed. And then the region tracking control law is presented. A new error transformation is developed based on the developed error transformation and the prescribed performance control algorithm, where the tracking error can satisfy the prescribed performance control algorithm.

(3) To show the efficiency of the proposed region tracking control scheme, a case with parametric uncertainty and another case with measurement noise are considered in simulation verification.

The structure of this paper is organized as follows. Section 2 gives the problem statement, including the typical dynamic model of an underwater vehicle. Section 3 focuses on the design of the Nussbaum-type state observer, and then the region tracking controller is presented after the new error transformation. In Sect. 4, the proposed observer-based region tracking control scheme is applied on an underwater vehicle, and compared with the existing similar control schemes. Finally, the conclusions are drawn in Sect. 5.

2 Problem statement

2.1 Underwater vehicles’ dynamics

Consider the dynamics of an underwater vehicle with six degrees of freedom (DOFs) fully actuated by thrusters [7,34], given by

\[
\dot{\eta} = M^{-1}_\eta (\eta) J Bu - M^{-1}_\eta (\eta) \left(C_{RB\eta} (\dot{\eta}) \dot{\eta} + \left(C_{A\eta} (\dot{\eta}_r) \dot{\eta}_r + D_{\eta} (\dot{\eta}_r) \dot{\eta}_r + G_{\eta} (\eta)\right)\right),
\]

where \(\eta \in \mathbb{R}^6\) is the position and attitude vector, \(M_{\eta} (\eta) \in \mathbb{R}^{6\times 6}\) denotes the inertia matrix including the added matrix, \(C_{RB\eta} (\dot{\eta}) \in \mathbb{R}^{6\times 6}\) expresses the rigid-body centripetal and Coriolis matrix, \(C_{A\eta} (\dot{\eta}_r) \in \mathbb{R}^{6\times 6}\) shows the hydrodynamic centripetal and Coriolis matrix, \(D_{\eta} (\dot{\eta}_r) \in \mathbb{R}^{6\times 6}\) is the hydrodynamics damping matrix, \(G_{\eta} (\eta) \in \mathbb{R}^6\) is the gravity and buoyancy vector, \(\dot{\eta}_r = \dot{\eta} - \dot{\eta}_c\), \(\dot{\eta}_c\) is the speed of ocean current. \(J \in \mathbb{R}^{6\times 6}\) is the velocity transformation matrix from the body-fixed frame to the earth inertial frame. \(B \in \mathbb{R}^{6\times n}\) is the thruster distribution matrix, \(n \geq 6\) denotes the number of thrusters, \(u \in \mathbb{R}^n\) is the control input of thrusters.

From Eq.(1), the dynamics of an underwater vehicle is rewritten to simplify the control design, shown as

\[
\dot{\eta} = M^{-1}_\eta (\eta) J Bu + F(\eta, \dot{\eta}),
\]

where \(F(\eta, \dot{\eta}) = -M^{-1}_\eta (\eta) \left(C_{RB\eta} (\dot{\eta}) \dot{\eta} + C_{A\eta} (\dot{\eta}_r) \right) \dot{\eta}_r - M^{-1}_\eta (\eta) \left(D_{\eta} (\dot{\eta}_r) \dot{\eta}_r + G_{\eta} (\eta)\right)\) will be estimated by a neural network in the next section.

Let \(\dot{x}_1 = \eta \in \mathbb{R}^6\), \(\dot{x}_2 = \dot{\eta} \in \mathbb{R}^6\), and define \(\ddot{x} = \left[\dot{x}_1^T, \dot{x}_2^T\right]^T\). Then, introduce the following coordinate transformation:

\[
x = \left[\begin{array}{c}
i_0 x_1 \\
T_1 x_2 \\
I_{6\times 6}
\end{array}\right], \quad \ddot{x}, \quad \text{(3)}
\]

where \(x = \left[\begin{array}{c}x_1^T, x_2^T\end{array}\right]^T\) with \(x_1, x_2 \in \mathbb{R}^6\), \(I_{6\times 6}\) is a six-order identity matrix while \(0_{6\times 6}\) is a six-order null matrix. \(T_1 \in \mathbb{R}^{6\times 6}\) is a positive-definite diagonal matrix.

Eq.(2) can be rewritten as the following state-space form

\[
\begin{align*}
\dot{x}_1 &= T_1 x_1 + x_2 \\
\dot{x}_2 &= -T_1^2 x_1 + T_1 x_2 + \ddot{F} (x_1, x_2) + E (x_1) u,
\end{align*} \quad \text{(4)}
\]

where \(\ddot{F} (x_1, x_2) = F (\eta, \dot{\eta})\) and \(E (x_1) = M^{-1}_\eta (\eta) J B\).

Nussbaum-type functions have an important property about uniformly ultimately boundedness, which have shown good estimation precision for the general disturbances in disturbance observers [32,33]. In this paper, Nussbaum-type functions will be used to improve the estimation accuracy of the unmeasured velocity. Now, the definition of Nussbaum-type functions is given below. A continuous function \(N (\bullet)\) is
considered as a Nussbaum-type function if it has the following properties \[35,36\]:
\[
\lim_{k \to +\infty} \sup_{\chi} \frac{1}{k} \int_{0}^{k} N(\chi)\,d\chi = +\infty
\]
\[
\lim_{k \to +\infty} \inf_{\chi} \frac{1}{k} \int_{0}^{k} N(\chi)\,d\chi = -\infty.
\]  
(5)

Typically, the continuous function \(k^2 \cos(k), k^2 \sin(k)\) are selected as Nussbaum-type functions.

**Lemma 1** \[35,36\] Let \(V(t), \varphi_i(t), i = 1, 2, \ldots, N\) be smooth functions defined on \([0, T]\) with \(V(t) \geq 0\) and \(\varphi_i(0) = 0\). And \(N(\bullet)\) is Nussbaum-type functions. If the following inequality holds:
\[
V(t) \leq C + \exp(-c_\alpha t) \sum_{i=1}^{N} \int_{0}^{t} (\Xi(\tau)\,d\tau),
\]  
(6)

where \(\Xi(\tau) = (g(\tau)\,N(\varphi_i(\tau)) - 1)\,\dot{\varphi}_i(\tau)\,\exp(c_\alpha \tau)\). \(C\) and \(c_\alpha\) are positive constants. \(g(\tau)\) is a time-varying parameter that takes values in the unknown set \(I = [g^-, g^+]\) with \(0 \notin I\), then \(V(t), \varphi_i(t), i = 1, 2, \ldots, N,\)
\[
\sum_{i=1}^{N} \int_{0}^{t} (g(\tau)\,N(\varphi_i(\tau)) - 1)\,\dot{\varphi}_i(\tau)\,\exp(c_\alpha \tau)\,d\tau \text{ are bounded on } [0, T].
\]

2.2 Control objective

Considering special tasks of underwater vehicles, e.g., pipeline tracking or target searching, high control precision sometimes is not their first priority, while longer operation time and smoother control signals may be pursued, after all the cost of implementation for one task is much expensive. In addition, when implementing the state-feedback controller to underwater vehicles, the position/attitude \(\eta\) of an underwater vehicle can be measured and provided by position sensors, such as ultrashort baseline and compass module. However, due to the cost and other practical reasons, as discussed in Introduction, the velocity \(\dot{\eta}\) cannot be always available. In this scenario, this paper investigates the region tracking control problem for underwater vehicles without velocity measurement. The objective of this paper is twofold: 1) **to design a state observer to estimate the unmeasurable velocity \(\dot{\eta}\),** and 2) **to derive a region tracking control strategy such that the tracking errors are always kept within the prescribed boundaries but do not converge to zero.**

The next section will focus on the design of the observer-based region tracking control.

3 Observer-based region tracking control

According to the dynamic model and the control objective described in the above section, an adaptive region tracking control scheme is proposed based on a Nussbaum-type state observer for underwater vehicles in presence of ocean current disturbances and unmeasured velocity state in this section. The overall conceptual diagram of the proposed observer-based region tracking control strategy is presented in Fig. 1.

In Fig. 1, the RBF neural network is used to approximate the unknown nonlinear function, while the Nussbaum-type observer is developed to estimate the velocity of the vehicle based on the control signals and the position of the vehicle. Furthermore, the error transformation is designed to enlarge the tracking error but still kept within the prescribed boundaries. Next, it will elaborate the observer design and control design, respectively.

3.1 Nussbaum-type state observer

To estimate the unmeasured state \(x_2\) in (4), the Nussbaum-type state observer is designed as
\[
\dot{x}_1 = T_1 \dot{x}_1 + \dot{x}_2 + L_1 \Delta_1 + f_1(\Delta_1)
\]
\[
\dot{x}_2 = -T_1^2 \dot{x}_1 - T_1 \dot{x}_2 + \bar{F}^e (\dot{x}_1, \dot{x}_2) + E(x_1)\,u
\]
\[
\quad + L_2 \Delta_1 + f_2(\Delta_1),
\]  
(7)

where \(\Delta_1 = x_1 - \hat{x}_1, \hat{x}_1\) and \(\dot{x}_2\) is the estimated quantities of the \(x_1\) and \(x_2\), respectively. \(L_1\) and \(L_2\) are diagonal matrices, \(\bar{F}^e (\dot{x}_1, \dot{x}_2)\) is the output of a neural network given in the next subsection, and it is always assumed that \[24,37\]
\[
\| \tilde{F} (x_1, x_2) - \tilde{F}^\varepsilon (\hat{x}_1, \hat{x}_2) \| \leq \delta \| x_2 - \hat{x}_2 \| ,
\]

where \( \delta \) is a known positive constant, and \( \| \cdot \| \) denote the 2-norm of a vector. Then, \( f_1 (\Delta_1) \) and \( f_2 (\Delta_1) \) are given by

\[
f_1 (\Delta_1) = k_1 |\Delta_1|^\gamma_1 + k_2 \Delta_1 + k_3 |\Delta_1|^\gamma_2 - p_1 \text{col} ((a_1 N_{1i} (\xi_{1i}) - 1)) \]
\[
f_2 (\Delta_1) = k_2 \text{sign} (\Delta_1) - p_2 \text{col} ((a_2 N_{2i} (\xi_{2i}) - 1)),
\]

where in order to achieve the boundedness of the estimated error, \( k_j (j = 1, \ldots, 6), p_1, p_2, a_1 \) and \( a_2 \) are positive constants, \( \gamma_1 \in (0, 1), \gamma_2 > 1 \). The selection of the above parameters is discussed in simulation verification. \( |\Delta_1|^\gamma_1 = [(|\Delta_{11}|)^\gamma_1 \text{sign} (\Delta_{11}), \ldots, |\Delta_{16}|)^\gamma_1 \text{sign} (\Delta_{16})] \), \( \Delta_{1i} \) is the \( i \)th entry of \( \Delta_1, |\Delta_1|^\gamma_2 \) has the similar forms as \( |\Delta_1|^\gamma_1 \). The operation \( \text{"col"} \) means to combine elements to be a column vector and \( i = 1, \ldots, 6, \xi_{1i} \) and \( \xi_{2i} \) are the \( i \)th entry of vectors \( \xi_1 \) and \( \xi_2 \), respectively. \( N_{1i} (\xi_{1i}) \) and \( N_{2i} (\xi_{2i}) \) are Nussbaum-type functions.

And

\[
\xi_1 = k_4 \int_0^\tau |\Delta_1|^\alpha d\tau \\
\xi_2 = k_6 \left( \Delta_1 + \int_0^\tau (L_1 \Delta_1 - T_1 \Delta_1 + f_1 (\Delta_1)) d\tau \right).
\]

Remark 1 Different from the conventional high-gain observer, some nonlinear terms about the position estimation error, including Nussbaum-type functions, are introduced into the observer, which can improve the estimation accuracy.

Then, the following error dynamic equation is obtained by subtracting (7) from (4), given by

\[
\dot{\Delta}_1 = (T_1 - L_1) \Delta_1 + \Delta_2 - f_1 (\Delta_1) \\
\dot{\Delta}_2 = -T_1^2 \Delta_1 - L_2 \Delta_1 - T_1 \Delta_2 + \tilde{F} (x_1, x_2) - \tilde{F}^\varepsilon (\hat{x}_1, \hat{x}_2) - f_2 (\Delta_1),
\]

where \( \Delta_2 = x_2 - \hat{x}_2 \).

Lemma 2 Consider underwater vehicles, whose dynamic model is always described as Eq. (1), and suppose assumption (8) holds. After using the coordinate transformation (3) and applying the designed Nussbaum-type state observer (7) to estimate the unmeasured velocity, if the following inequality holds

\[
-\Psi = P A_0 + A_0^T P + \frac{2}{\xi_0} P P + \Psi < 0,
\]

where \( \Psi = \text{diag} (0_{6 \times 6}, \xi_0 \varepsilon_0^2 I_{6 \times 6}) \), \( A_0 = A - \left[ \begin{array}{c} T_1 \ I_{6 \times 6} \\ L_2 \ 0_{6 \times 6} \end{array} \right] \), and \( P = \text{diag} (P_1, P_2) \). \( P_1 \) and \( P_2 \) are positive-definite diagonal matrices, and the operation \( \text{"diag"} \) is to combine the elements into a diagonal matrix, then the estimated error is guaranteed to be uniformly ultimately bounded.

Proof Consider the following Lyapunov function

\[
V_1 = \frac{2}{1 + \alpha} \left[ |\Delta_1|^\alpha \right]^T P_1 \Delta_1 + \Delta_2^T P_2 \Delta_2.
\]

where \( \alpha \in (0, 1) \).

Since \( P_1 \) and \( P_2 \) are positive-definite diagonal matrices, it has the following equation:

\[
\left[ |\Delta_1|^\alpha \right]^T P_1 \Delta_1 = \sum_{i=1}^6 (|\Delta_{1i}|^\alpha \text{sign} (\Delta_{1i}) P_{1i} \Delta_{1i})
\]

\[
= \sum_{i=1}^6 \left( P_{1i} |\Delta_{1i}|^{\alpha+1} \right),
\]

where \( \Delta_{1i} \) is the \( i \)th element of \( \Delta_1 \); and \( P_{1i} \) is the \( i \)th diagonal element of the diagonal matrix \( P_1 \). Hence, \( \left[ |\Delta_1|^\alpha \right]^T P_1 \Delta_1 \) is nonnegative. Therefore, it can be concluded that the Lyapunov function \( V_1 \) is always nonnegative for any \( t \geq 0 \).

From Eq.(14), the derivative of \( \left[ |\Delta_1|^\alpha \right]^T P_1 \Delta_1 \) with respect to time can be described as follows:

\[
\frac{d}{dt} \left( \left[ |\Delta_1|^\alpha \right]^T P_1 \Delta_1 \right) = \frac{d}{dt} \left( \sum_{i=1}^6 \left( P_{1i} |\Delta_{1i}|^{\alpha+1} \right) \right)
\]

\[
= (\alpha + 1) \sum_{i=1}^6 P_{1i} |\Delta_{1i}|^{\alpha+1} \text{sign} (\Delta_{1i}) \Delta_{1i}
\]

\[
= (\alpha + 1) \left[ |\Delta_1|^\alpha \right]^T P_1 \Delta_1.
\]

Similarly, since \( P_2 \) is the diagonal matrix, it follows that \( \Delta_2^T P_2 \Delta_2 = \Delta_2^T P_2 \Delta_2 \).
Then, the time derivative of $V_1$ is given by
\[
\dot{V}_1 = 2\left[\Delta_1^{\alpha}\right]^T P_1 \dot{\Delta}_1 + 2\Delta_2^T P_2 \dot{\Delta}_2. \tag{16}
\]

Let $X = col\left(\Delta_1^{\alpha}, \Delta_2\right)$, and substitute (11) into (16). It follows that
\[
\dot{V}_1 = \Delta_1^T A_0^T PX + 2\Delta_2^T P_2 \left(\ddot{F}(x_1, x_2) - \ddot{\tilde{F}}(\tilde{x}_1, \tilde{x}_2)\right)
+ X^T P A_0 \Delta - 2\left[\Delta_1^{\alpha}\right]^T P_1 \left(k_1 \Delta_1^{\alpha} + k_2 \Delta_1\right)
- 2\left[\Delta_1^{\alpha}\right]^T P_1 \left(k_3 \Delta_1^{\alpha\alpha} - p_1 \text{col} \left(\left(\alpha_1 N_{1i}(\xi_{1i}) - 1\right)\right)\right)
- 2\Delta_2^T P_2 \left(k_5 \text{sign}(\Delta_1) - p_2 \text{col} \left(\left(\alpha_2 N_{2i}(\xi_{2i}) - 1\right)\right)\right). \tag{17}
\]

According to the Young’s inequality [38] and assumption (8), it follows that
\[
2\Delta_2^T P_2 \left(\ddot{F}(x_1, x_2) - \ddot{\tilde{F}}(\tilde{x}_1, \tilde{x}_2)\right) \leq \frac{1}{\varepsilon_0} X^T P P \Delta + \varepsilon_0 \delta^2 \Delta_2^T \Delta_2, \tag{18}
\]

where $\varepsilon_0$ is a positive constant.

Similarly, the following inequality is also obtained based on the Young’s inequality [38].
\[
2\Delta_2^T P_2 k_5 \text{sign}(\Delta_1) \leq \frac{1}{\varepsilon_0} \Delta_2^T P_2 \Delta_2 + \varepsilon_0 k_5^2
\leq \frac{1}{\varepsilon_0} X^T P P \Delta + \varepsilon_0 k_5^2. \tag{19}
\]

Substituting (18) and (19) into (17), it follows that
\[
\dot{V}_1 \leq X^T P A_0 \Delta + \Delta_1^T A_0^T PX + \frac{2}{\varepsilon_0} X^T P P \Delta + \varepsilon_0 \delta^2 \Delta_2^T \Delta_2
+ \varepsilon_0 k_5^2 - 2\left[\Delta_1^{\alpha}\right]^T P_1 \left(k_1 \Delta_1^{\alpha} + k_2 \Delta_1 + k_3 \Delta_1^{\alpha\alpha}\right)
+ 2p_1 \left[\Delta_1^{\alpha}\right]^T P_1 \left(\text{col} \left(\left(\alpha_1 N_{1i}(\xi_{1i}) - 1\right)\right)\right)
+ 2p_2 \Delta_2^T P_2 \left(\text{col} \left(\left(\alpha_2 N_{2i}(\xi_{2i}) - 1\right)\right)\right). \tag{20}
\]

where the definitions of $P$, $A_0$ and $\varepsilon_0$ are given as Eq. (12) as follows.

According to (10), Eq.(20) can be further simplified as
\[
\dot{V}_1 \leq X^T \left(P A_0 + \Delta_1^T P + \frac{2}{\varepsilon_0} P P + \Psi\right) \Delta + \varepsilon_0 k_5^2
+ 2p_1/k_4 \tilde{\epsilon}_i \xi_i P_1 \left(\text{col} \left(\left(\alpha_1 N_{1i}(\xi_{1i}) - 1\right)\right)\right)
+ 2p_2/k_6 \tilde{\xi}_2 / P_2 \left(\text{col} \left(\left(\alpha_2 N_{2i}(\xi_{2i}) - 1\right)\right)\right)
+ 2p_1/k_4 \tilde{\epsilon}_i \xi_i P_1 \left(\text{col} \left(\left(\alpha_1 N_{1i}(\xi_{1i}) - 1\right)\right)\right)
+ 2p_2/k_6 \tilde{\xi}_2 / P_2 \left(\text{col} \left(\left(\alpha_2 N_{2i}(\xi_{2i}) - 1\right)\right)\right)
\leq -X^T Q \Delta + \varepsilon_0 k_5^2 + 2p_1/k_4 \sum_{i=1}^{6} \tilde{\epsilon}_i \xi_i P_1 \left(\left(\alpha_1 N_{1i}(\xi_{1i}) - 1\right)\right)
+ 2p_2/k_6 \sum_{i=1}^{6} \tilde{\xi}_2 / P_2 \left(\left(\alpha_2 N_{2i}(\xi_{2i}) - 1\right)\right). \tag{21}
\]

From (12), one has that $Q$ is a symmetric matrix. And inequality (12) holds, i.e., the matrix $Q$ is positive-definite matrix, i.e., the minimum eigenvalue of the matrix $Q$ is positive, it follows that
\[
-X^T Q \Delta \leq -\frac{\lambda_{\min} (Q)}{\lambda_{\max} \left(\text{diag}\left(\frac{2}{1+\alpha} P_1, P_2\right)\right)} V_1, \tag{22}
\]

where $\lambda_{\min}$ and $\lambda_{\max}$, respectively, denote the minimum eigenvalue and the maximum eigenvalue of a matrix. Define $c_a = \frac{\lambda_{\min}(Q)}{\lambda_{\max} \left(\text{diag}\left(\frac{2}{1+\alpha} P_1, P_2\right)\right)}$. Then, multiplying both sides of Eq.(21) by $\exp(c_a t)$ gives
\[
\frac{d}{dt} \left(V_1 \exp(c_a t)\right) \leq \varepsilon_0 k_5^2 \exp(c_a t)
+ 2p_1/k_4 \sum_{i=1}^{6} \tilde{\epsilon}_i \xi_i \Omega_{1i} \exp(c_a t)
+ 2p_2/k_6 \sum_{i=1}^{6} \tilde{\xi}_2 / \Omega_{2i} \exp(c_a t), \tag{23}
\]

where $\Omega_{1i} = P_1 \left(\alpha_1 N_{1i}(\xi_{1i}) - 1\right)$ and $\Omega_{2i} = P_2 \left(\alpha_2 N_{2i}(\xi_{2i}) - 1\right)$.

Integrating Eq.(23) on $[0, t]$ yields
\[
V_1(t) \leq \varepsilon_0 k_5^2 \frac{e^{c_a t}}{c_a} + \left(V_1(0) - \varepsilon_0 k_5^2 \frac{e^{c_a t}}{c_a}\right) \exp(-c_a t)
+ 2p_1/k_4 \exp(-c_a t) \int_{0}^{t} \tilde{\epsilon}_i \xi_i \Omega_{1i} \exp(c_a \tau) d\tau
+ 2p_2/k_6 \exp(-c_a t) \int_{0}^{t} \tilde{\xi}_2 / \Omega_{2i} \exp(c_a \tau) d\tau
\leq C + 2p_1/k_4 \exp(-c_a t) \int_{0}^{t} \tilde{\epsilon}_i \xi_i \Omega_{1i} \exp(c_a \tau) d\tau
+ 2p_2/k_6 \exp(-c_a t) \int_{0}^{t} \tilde{\xi}_2 / \Omega_{2i} \exp(c_a \tau) d\tau. \tag{24}
\]
According to Lemma 1, it can be concluded that
\[
V_1(t, \xi_1, \xi_2, \int_0^t \sum_{i=1}^6 \xi_{1i} P_{1i} (a_1 N_{1i} (\xi_{1i}) - 1) \exp (c_{ai} t) d\tau + \int_0^t \sum_{i=1}^6 \xi_{2i} P_{2i} (a_2 N_{2i} (\xi_{2i}) - 1) \exp (c_{ai} t) d\tau \leq V(0, 0) + 1,
\]
for bounded on \([0, t_f]\). And from [35,36], it has that \(t_f = \infty\). Therefore, it is verified that the estimation error \(\Delta\) is uniformly ultimately bounded. \(\square\)

Next it needs to solve inequality (12). According to Schur complement lemma, (12) is equivalent to the following inequality. Referring to the existence of positive-definite matrix \(Q\), after determining the values of \(\varepsilon_0\) and \(\delta\), a proper \(T_1\) can be selected to calculate the following inequality according to linear matrix inequality technique in priori [39].

\[
\begin{bmatrix}
PA + A^T P - S^T K^T - KS + \Psi & P \\
& -\varepsilon^2 I_{12 \times 12}
\end{bmatrix} < 0,
\]

where \(S = \begin{bmatrix} I_{6 \times 6} & 0_{6 \times 6} \end{bmatrix}\), \(K = \begin{bmatrix} P_1 L_1 & P_2 L_2 \end{bmatrix}^T\).

It should be noted that \(T_1\) should be selected to guarantee that the solution of the above inequality exists. After obtaining the solution \(P\) and \(K\), the values of \(L_1\) and \(L_2\) are also found, i.e., \(L = [L_1, L_2] = P^{-1} K\).

3.2 Region tracking control design

After using the Nussbaum-type state observer to estimate the unmeasured state \(x_2\), this subsection will use these estimated states to design region tracking control law.

At first, define the tracking error \(e_1 = x_1 - x_{1d}\), where \(x_{1d}\) is the desired trajectory. As discussed in Sect. 1, the tracking error would converge to zero, if directly adopting prescribed performance control scheme, which is not the purpose of the region tracking. Therefore, a special transformation about the tracking error is designed to achieve region tracking, shown below.

\[
H_n(s, d) = \begin{cases} 
\frac{(|s| - d^n)}{n!}, & \text{if } |s| \geq d \\
0, & \text{otherwise},
\end{cases}
\]

where \(s\) is a variable, \(d\) is a positive scale, \(n\) is a positive integer.

And the partial derivative of \(H_n(s, d)\) with respect to \(s\) is given by \(\frac{d}{ds} H_n(s, d) = H_{n-1}(s, d) \text{sign}(s)\), for \(n \geq 2\).

Consider the following Lyapunov function:

\[
V_2 = \frac{1}{2} \sum_{i=1}^6 \ln \frac{\rho_i}{\rho_i - z_{ii}},
\]

where \(z_{ii} = H_3(e_{1i}, \varepsilon_{1i})\), \(z_{ii}, e_{1i}, \rho_i\) and \(\varepsilon_{1i}\) are the \(i\)th entry of \(z_1, e_1, \rho_i\) and \(\varepsilon_{1i}\), respectively. \(\varepsilon_1\) is a positive vector. \(\rho_i\) is a function about the prescribed performance, describing the requirements of the transient- and steady-state performances. Here it is set that \(\rho_{ai} = \left(\left(\rho_i - \rho_j\right) \exp (-\mu_i t) + \rho_j - \varepsilon_{1i}\right)^3/6\), where \(\rho_i, \rho_j\) are positive vectors (their entries are \(\rho_i\) and \(\rho_j\)) and \(\mu_i, \mu_j\) is positive constant with \(\rho_i > \rho_j, i = 1, 2, \ldots, 6\). The initial condition of \(\rho_i\) should be chosen to guarantee that \(\rho_{ai}(0) > z_{ii}(0)\). From Eq. (27), it is straightforward to verify that \(V_2\) is positive and continuous in the set that \(\rho_{ai} > z_{ii}, i = 1, 2, \ldots, 6\). One also has that \(V_2 \rightarrow \infty\) as \(z_{ii} \rightarrow \rho_{ai}\). Let \(V(z_1) = V_2(z_1) + U(\xi)\), where \(\gamma_a(\|\xi\|) \leq U(\xi) \leq \gamma_b(\|\xi\|)\) with class \(K\) functions \(\gamma_a\) and \(\gamma_b\). Based on [19,40,41], \(\rho_{ai}(0) > z_{ii}(0)\) and if the inequality \(V(z_1) \leq -c_2 V(z_1) + \vartheta_2\) holds with the positive constants \(c_2, \vartheta_2\), then \(z_1(t)\) remains in the set that \(\rho_{ai} > z_{ii}(i = 1, 2, \ldots, 6) \forall t \in [0, \infty)\).

Then, define \(e_2 = \hat{x}_2 - x_{2e}\), and \(x_{2e}\) is from the following first-order filter:

\[
\theta \dot{x}_{2e} + x_{2e} = \alpha_{ce},
\]

where \(\theta\) is a positive constant, less than one in general. \(\alpha_{ce}\) is the virtual control law designed later. And \(x_{2e}\) follows \(\alpha_{ce}\) with a bound, i.e., \(|x_{2e} - \alpha_{ce}| \leq \gamma_c\) with a positive constant \(\gamma_c\). And the time-derivative of \(z_1\) is expressed as the following equation, after substituting (4) and the estimation error of the Nussbaum-type state observer:

\[
\dot{z}_1 = \text{diag} \left( H_2(e_{11}, \varepsilon_{11}) \text{sign}(e_{11}) \right) \dot{e}_1 \\
= \text{diag} \left( H_2(e_{11}, \varepsilon_{11}) \text{sign}(e_{11}) \right) (\dot{x}_1 - \dot{x}_{1d}) \\
= \text{diag} \left( H_2(e_{11}, \varepsilon_{11}) \text{sign}(e_{11}) \right) \times (\hat{x}_2 + T_1 x_1 + \Delta_2 - \dot{x}_{1d}),
\]

where \(i = (1, \ldots, 6)\).
The virtual control law $\alpha_c$ considered in this paper has the following form:

$$
\alpha_c = -\frac{1}{3} \left( b_1 - \frac{\dot{\rho}_{ai}}{\rho_{ai}} \right) \text{col} \left( H_1 (e_{1i}, e_{1i}) \text{sign} \left( e_{1i} \right) \right)
- \frac{1}{3} b_2 \text{col} \left( H_1 (e_{1i}, e_{1i}) \text{sign} \left( e_{1i} \right) \frac{\dot{l}_i}{l_i} \right)
- \text{col} \left( H_2 (e_{1i}, e_{1i}) \text{sign} \left( e_{1i} \right) \frac{1}{\rho_{ai} - z_{1i}} \right) + \dot{x}_{1d} - T_1 x_1,
$$

(30)

where $b_1$ and $b_2$ are positive constants, $l_1 \in (0, 1)$.

Substituting (29) and (30) into the time derivative of $V_2$ yields

$$
\dot{V}_2 = \sum_{i=1}^{6} \frac{\dot{\rho}_{ai} z_{1i}}{\rho_{ai} (\rho_{ai} - z_{1i})} + \gamma_a \left( \dot{x}_2 + T_1 x_1 + \Delta_2 - \dot{x}_{1d} \right)
- \gamma_a (\Delta_2 + x_{2c} - \alpha_c)
= -b_1 \sum_{i=1}^{6} \frac{z_{1i}}{\rho_{ai} (\rho_{ai} - z_{1i})}
- b_2 \sum_{i=1}^{6} \left( \frac{z_{1i}}{\rho_{ai} (\rho_{ai} - z_{1i})} \right)^{1+1_i}
+ \sum_{i=1}^{6} \frac{H_2 (e_{1i}, e_{1i}) \text{sign} \left( e_{1i} \right) \epsilon_{2i}}{(\rho_{ai} - z_{1i})}
- \gamma_a (\Delta_2 + x_{2c} - \alpha_c),
$$

(31)

where $\gamma_a = \text{row} \left( \frac{1}{\rho_{ai} - z_{1i}} \right) \text{diag} \left( H_2 (e_{1i}, e_{1i}) \text{sign} \left( e_{1i} \right) \right)$, the operation “row” is to combine entries into a row vector, for example, $\text{row} \left( Y_i \right) = [Y_1, Y_2, \ldots, Y_N]$.

By employing the Young’s inequality [38], the following inequalities can be obtained

$$
\gamma_a \Delta_2 \leq \frac{1}{2} \sum_{i=1}^{6} \left( \frac{H_2 (e_{1i}, e_{1i})}{(\rho_{ai} - z_{1i})} \right)^2 + \frac{1}{2} \Delta_2^T \Delta_2
$$

(32)

$$
\gamma_a (x_{2c} - \alpha_c) \leq \frac{1}{2} \sum_{i=1}^{6} \left( \frac{H_2 (e_{1i}, e_{1i})}{(\rho_{ai} - z_{1i})} \right)^2 + \frac{1}{2} \epsilon_f^2.
$$

(33)

Consequently, substituting the above two inequalities into (31) gives

$$
\dot{V}_2 \leq -b_1 \frac{z_{1i}}{\rho_{ai} (\rho_{ai} - z_{1i})} - b_2 \left( \frac{z_{1i}}{\rho_{ai} - z_{1i}} \right)^{1+1_i}
+ \sum_{i=1}^{6} \frac{H_2 (e_{1i}, e_{1i}) \text{sign} \left( e_{1i} \right) \epsilon_{2i}}{(\rho_{ai} - z_{1i})}
+ \frac{1}{2} \Delta_2^T \Delta_2 + \frac{1}{2} \epsilon_f^2.
$$

(34)

Next, differentiating $\epsilon_2$ with respect to time, and then substituting the estimation error of the Nussbaum state observer and (4) obtains

$$
\dot{\epsilon}_2 = \ddot{x}_2 - \dot{x}_{2c} = \ddot{x}_2 - \frac{x_{2c} - \alpha_c}{\theta}
= \ddot{x}_2 - \frac{x_{2c} - \alpha_c}{\theta} - \dot{\Delta}_2
= -T_1^2 x_1 - T_1 x_2 + \tilde{F} (x_1, x_2) + E (x_1) u
- \frac{x_{2c} - \alpha_c}{\theta} - \dot{\Delta}_2.
$$

(35)

Neural network is always used to approximate the unknown function in control designs. Here, a RBF neural network is used to approximate the unknown function $\tilde{F} (x_1, x_2)$. Considering that the ocean current and the velocity of the vehicle are bounded, it is reasonable that the unknown function $\tilde{F} (x_1, x_2)$ can be approximated on a compact set. Define $\Gamma$ as the compact set. Due to the approximation property of the RBF neural network, there exists an ideal weighting matrix $W^* \in \mathbb{R}^{6 \times m}$, where $m$ denotes the number of neurons in the hidden layer. $W^*$ satisfies $\|W^*\| \leq \bar{W}$ with $\bar{W} > 0$, such that [42,43]

$$
\tilde{F} (x_1, x_2) = W^* \Phi \left( \dot{x}_1, \dot{x}_2 \right) + \epsilon_f,
$$

(36)

where $\Phi \left( \dot{x}_1, \dot{x}_2 \right) = [\Phi_1 (\dot{x}_1, \dot{x}_2), \ldots, \Phi_m (\dot{x}_1, \dot{x}_2)]$, $\Phi_i (\dot{x}_1, \dot{x}_2) = \exp \left( -\| (\dot{x}_2 + T_1 \dot{x}_1) - \mu_i \|^2/\sigma_i^2 \right)$, $\sigma_i$ is the width of the $i$th neuron, while $\mu_i$ is the center vector of the $i$th neuron. The input of the neural network is the estimated quantity of the vehicle’s velocity, i.e., $\dot{x}_2 + T_1 \dot{x}_1$, since $\tilde{F} (x_1, x_2)$ is mainly affected by the vehicle’s velocity from our previous experiences. And $\epsilon_f$ is the approximation error of the neural network, affected by the number of neurons and network structure. It is always assumed that the approximation error is bounded with a positive bound $\mathcal{E}$, i.e., $\| \epsilon_f \| \leq \mathcal{E}$.
The output of the RBF neural network is given by
\[ \hat{F}^e (\hat{x}_1, \hat{x}_2) = \hat{W} \Phi (\hat{x}_1, \hat{x}_2), \] (37)
where \( \hat{W} \) is the estimated weighting matrix of \( W^* \), and \( \hat{W} = W^* - \hat{W} \).

Therefore, the control law \( u \) is designed with the following form:

\[
\begin{align*}
    u &= u_0 + u_1 \\
    u_0 &= E(x_1)^+ \left( T_1^2 x_1 + T_1 \hat{x}_2 + \frac{(x_2 - a_\mu)}{\theta} - \hat{W} \Phi (\hat{x}_1, \hat{x}_2) \right) \\
    u_1 &= E(x_1)^+ \left( -b_3 e_2 - \hat{\mathcal{S}} \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) \right),
\end{align*}
\]
(38)
where \( E(x_1)^+ \) is the pseudo-inverse matrix of \( E(x_1) \). \( b_3 \) and \( \rho_b \) are the positive constants. \( \hat{\mathcal{S}} \) is the estimated quantity of \( \mathcal{S} \), given by

\[
\begin{align*}
    \dot{\hat{W}} &= \Gamma_1 \hat{e}_2^T \Phi (\hat{x}_1, \hat{x}_2) - \beta_1 \hat{W} \\
    \dot{\hat{\mathcal{S}}} &= \Gamma_2 \hat{e}_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) - \beta_2 \hat{\mathcal{S}},
\end{align*}
\]
(39)
where \( \Gamma_1, \Gamma_2, \beta_1 \) and \( \beta_2 \) are positive constants.

Then, consider the following Lyapunov function

\[
V_3 = \frac{1}{2} \hat{e}_2^T e_2 + \frac{1}{2 \Gamma_1} tr \left( \hat{W} \hat{W}^T \right) + \frac{1}{2 \Gamma_2} \hat{\mathcal{S}}^2,
\]
(40)
where \( \hat{\mathcal{S}} = \mathcal{S} - \hat{\mathcal{S}} \). \( tr \left( \hat{W} \hat{W}^T \right) \) means to take the trace of the matrix \( \hat{W} \hat{W}^T \). And differentiating both sides of (40) with respect to time yields

\[
\begin{align*}
    \dot{V}_3 &= e_2^T \left( -T_1^2 x_1 + T_1 \hat{x}_2 + \hat{F} (x_1, x_2) + E (x_1) u \right) \\
    &\quad - e_2^T \left( \frac{(x_2 - a_\mu)}{\theta} + \Delta_2 \right) - \frac{1}{\Gamma_1} tr \left( \hat{W} \hat{W}^T \right) - \frac{1}{\Gamma_2} \hat{\mathcal{S}} \hat{\mathcal{S}} \\
    &= e_2^T \left( -\text{col} \left( \frac{H_2 (e_{1i}, e_{1j}) \text{sign} (e_{1i})}{\rho_{ai} - z_{1i}} \right) + \hat{W} \Phi (\hat{x}_1, \hat{x}_2) \right) \\
    &\quad + e_2^T \left( \epsilon_f - \Delta_2 - b_3 e_2 - \hat{\mathcal{S}} \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) \right) \\
    &\quad - e_2^T \hat{W} \Phi (\hat{x}_1, \hat{x}_2) + \frac{\beta_1}{\Gamma_1} tr \left( \hat{W} \hat{W}^T \right) \\
    &\quad - \hat{\mathcal{S}} e_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) - \frac{\beta_2}{\Gamma_2} \hat{\mathcal{S}} \hat{\mathcal{S}} \\
    &= -e_2^T \left( \frac{H_2 (e_{1i}, e_{1j}) \text{sign} (e_{1i})}{\rho_{ai} - z_{1i}} \right) + e_2^T \left( \epsilon_f - \Delta_2 \right) \\
    &\quad + \frac{\beta_1}{\Gamma_1} tr \left( \hat{W} \hat{W}^T \right) - \hat{\mathcal{S}} e_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) - \frac{\beta_2}{\Gamma_2} \hat{\mathcal{S}} \hat{\mathcal{S}}. \quad (41)
\end{align*}
\]

By employing the Young’s inequality, the following inequalities can be obtained

\[
\begin{align*}
    e_2^T (\epsilon_f - \hat{\mathcal{S}} e_2) &\leq \hat{\mathcal{S}} e_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) + 6\xi \rho_b \mathcal{E} \quad (42) \\
    e_2^T \epsilon_f - \hat{\mathcal{S}} e_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) &\leq \hat{\mathcal{S}} e_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) + 6\xi \rho_b \mathcal{E} \quad (43) \\
    tr \left( \hat{W} \hat{W}^T \right) &\leq \frac{1}{2} \hat{\mathcal{S}} e_2^T \text{col} \left( \tanh \left( \frac{e_1}{\rho_b} \right) \right) + 6\xi \rho_b \mathcal{E} + \frac{1}{2} \| \Delta_2 \|^2 \quad (44) \\
    \hat{\mathcal{S}} \hat{\mathcal{S}} &\leq \frac{1}{2} \mathcal{S}^2 - \frac{1}{2} \hat{\mathcal{S}}^2. \quad (45)
\end{align*}
\]

Then, substituting the above four inequalities into (41) gives

\[
\begin{align*}
    \dot{V}_3 &\leq -e_2^T \left( \text{col} \left( \frac{H_2 (e_{1i}, e_{1j}) \text{sign} (e_{1i})}{\rho_{ai} - z_{1i}} \right) + b_3 e_2 \right) \\
    &\quad + \frac{\beta_1}{2\Gamma_1} tr \left( \hat{W} \hat{W}^T \right) - \frac{\beta_2}{2\Gamma_2} \hat{\mathcal{S}}^2 \\
    &\quad + \frac{\beta_1}{2\Gamma_1} tr \left( W^* \hat{W} \right) - \frac{\beta_2}{2\Gamma_2} \hat{\mathcal{S}}^2 \quad (46)
\end{align*}
\]

Now, the main result of this paper is given below.

**Theorem 1** Consider an underwater vehicle without velocity measurement (1), whose dynamic model is always described as Eq. (1). Given the performance function \( \rho_{ai}, i = 1, 2, \ldots, 6 \) with the constraint that \( \rho_{ai} > z_{1i} (0) \). If the state observer is designed as the form in (7) and (9), and the region tracking control law is presented as (38) together with the adaptive laws as (39), then the following properties always hold.

1. all the signals in the closed-loop system, including \( z_1, e_2, \hat{W} \) and \( \hat{\mathcal{S}} \), are uniformly ultimately bounded;
2. the tracking error \( e_{1i} \) is always kept within the prescribed boundaries \( [\bar{\rho}_i - \rho_i] \exp (-\mu_t) + \rho_i \) for \( i = 1, 2, \ldots, 6 \), i.e., the region tracking control is achieved.

\( \odot \) Springer
Proof To demonstrate the convergence of the signals in the closed-loop systems, the following Lyapunov function is considered

\[ V = V_2 + V_3. \]  

(47)

Then, differentiating both sides of (47), and substituting (34) and (46) yields

\[
\dot{V} \leq -b_1 \sum_{i=1}^{6} \frac{z_{ii}}{(\rho_{ai} - z_{ii})} - b_2 \sum_{i=1}^{6} \left( \frac{z_{ii}}{(\rho_{ai} - z_{ii})} \right)^{1+i_{li}}
\]

\[
- b_3 \varepsilon_2 - \frac{\beta_1}{2T_1} \text{tr} \left( \hat{W}^T \hat{W} \right) - \frac{\beta_2}{2T_2} \tilde{z}^2
\]

\[
+ \frac{1}{2} \Delta_2^2 \Delta_2 + \frac{1}{2} \varepsilon_1^2 + \frac{\beta_1}{2T_1} \text{tr} \left( W^* W^* \right) + \frac{\beta_2}{2T_2} \hat{z}^2
\]

\[
+ 6 \varepsilon_3 \rho_0 \tilde{z} + \frac{1}{2} \| \Delta_2 \|^2.
\]

(48)

According to [44], for all \( z_{ii} \) in the set \( z_{ii} < \rho_{ai} \), the following inequality holds

\[
\sum_{i=1}^{6} \ln \frac{\rho_{ai}}{\rho_{ai} - z_{ii}} \leq \sum_{i=1}^{6} \frac{z_{ii}}{(\rho_{ai} - z_{ii})}.
\]

(49)

Consequently, (48) can be further rewritten as

\[
\dot{V} \leq -b_1 V_2 - b_2 V_2^{1+i_{li}} - \lambda_1 V_3 + \tilde{C}_0
\]

\[
\leq -\lambda_2 V + \tilde{C}_0,
\]

(50)

where \( \lambda_1 = \min (2b_3, \beta_1, \beta_2) > 0 \), \( \lambda_2 = \min (\lambda_1, b_1) > 0 \). \( \tilde{C}_0 = \frac{1}{2} \Delta_2^2 \Delta_2 + \frac{1}{2} \varepsilon_1^2 + \frac{\beta_1}{2T_1} \text{tr} \left( W^* W^* \right) + \frac{\beta_2}{2T_2} \hat{z}^2 + 6 \varepsilon_3 \rho_0 \tilde{z} + \frac{1}{2} \| \Delta_2 \|^2 \). Since \( \Delta_2, \Delta_2, y_c, W^*, \tilde{z} \) are bounded, \( \tilde{C}_0 \) is also bounded.

Integrating Eq. (50) over \([0, t]\), one has

\[
0 \leq V(t) \leq \frac{\tilde{C}_0}{\lambda_2} + \left( V(0) - \frac{\tilde{C}_0}{\lambda_2} \right) \exp (-\lambda_2 t)
\]

\[
\leq V(0) \exp (-\lambda_2 t) + \frac{\tilde{C}_0}{\lambda_2}.
\]

(51)

Due to the constraint that \( \rho_{ai} > z_{ii} > 0 \), it is obvious that \( V(0) > 0 \). Therefore, one has that \( V(t) \) is uniformly ultimately bounded according to [45]. Then, in light of (47), all the signals of the closed-loop system, including \( z_1, e_2, \hat{W} \) and \( \tilde{z} \), are uniformly ultimately bounded. The proof of Result 1 is completed. Furthermore, one has that \( z_{ii} \leq \rho_{ai}, i = 1, 2, \ldots, 6 \) for any \( t > 0 \) when the initial condition is satisfied, according to the research in [19,40,41].

Then, according to the definition of the variable \( z_1 \) and \( \rho_{ai} \), for the case of \( |e_{ii}| \geq \varepsilon_{li} \), one has the following inequality:

\[
0 \leq \frac{\left( |e_{ii}| - \varepsilon_{li} \right)^3}{3!} < \rho_{ai}
\]

\[
\varepsilon_{li} \leq |e_{li}| < (\rho_{i} - \rho_{j}) \exp (-\mu \rho_{i} t) + \rho_{i}.
\]

(52)

From Eq.(52), it can be obtained that the tracking error \( e_{li} \) is always kept within the prescribed boundaries defined as Eq.(27) as follows. The proof of Result 2 is completed.

\( \square \)

Remark 2 The separation principle, extended to nonlinear systems by using high-gain observer in [46], is used for the stability proof of the closed-loop system. To further analyze the stability of the observer-based controller for the nonlinear system, another Lyapunov function is needed, i.e., \( V_T = V_1 + V \). Then, taking the derivative of the Lyapunov function \( V_T \) and substituting Eq.(21) and Eq.(50), the following inequality is obtained after some simple manipulations, i.e., \( V_T \leq -\lambda_{\min} (c_a, \lambda_2 \ v_T + C_b \), where \( C_b \) is a bounded positive constant. To some extent, the separation principle used here is also reasonable.

This section elaborates the design of the proposed observer-based region tracking control strategy. Then, simulation verification will be presented next.

4 Simulation studies

Section 3 has elaborated the design of the proposed observer-based region tracking control scheme. This section will present a series of simulation results to demonstrate the effectiveness and feasibility of the proposed control scheme for underwater vehicles without velocity measurement. In addition, two control schemes are used to conduct comparative studies. At first, some simulation preparation is provided. And then the test about the robustness to the parametric uncertainty is performed. Finally, measurement noise is also considered to test the efficiency of the control scheme.
4.1 Simulation preparation

Here, it provides some descriptions about simulation environment, control parameters, comparative control schemes and desired trajectory.

(1) Simulation environment

As usual, a typical representation of fully actuated underwater vehicles is considered. The vehicle’s dynamic parameters and thruster distribution are presented in [47]. The initial condition of the vehicle is set as \( \eta (0) = [1, 1, -1, 2\pi/9, 2\pi/9, 2\pi/9]^T \), and the velocity \( \dot{\eta} (0) = [0.04, 0.04, 0.04, 0.02, 0.02, 0.02]^T \).

To reflect the effectiveness of the proposed observer-based region tracking control scheme to reject ocean current disturbances, the following equation is used to generate ocean current \( \dot{\eta}_c \) [21,34]:

\[
\frac{d}{dt} \| \dot{\eta}_c \| + \mu_c \| \dot{\eta}_c \| = \omega_c, \tag{53}
\]

where \( \| \dot{\eta}_c \| \) denotes the magnitude of ocean current \( \dot{\eta}_c \); \( \mu_c \) is a fixed constant, set as 3; \( \omega_c \) is a Gaussian noise with mean 1.5 and variance 1. The orientation of ocean current is simulated by the sum of Gaussian noise with mean 0 and variance 50. According to the model, the mean value of ocean current used in simulations is 0.50m/s while its standard variance is 0.056.

(2) Control parameters

Now, it gives the selection of control parameters, the difficulty of which will go from the easy to the complicated slowly.

1) Approximation of neural network

This paper uses a RBF neural network to approximate the unknown function \( \tilde{F} (x_1, x_2) \). Based on the authors’ previous experiences in [48], in the approximation of the RBF neural network Eq. (37), only one hidden layer is considered, and the number of the neurons in the hidden layer is 54, \( \sigma_1 = 0.1 \) and the centers of radial basis functions are distributed evenly within the range of \([-0.5, 0.5] \times [-0.5, 0.5] \times [-0.5, 0.5] \times \ldots \times [-0, 0] \times [-0, 0] \times [-0.2, 0.2] \). Also, the initial entries of the weighting matrix \( \tilde{W} \) are zero.

2) Parameters for the state observer

At first, \( L_1 \) and \( L_2 \) in Eq. (7) can be determined by solving (25) using LMI technique after selecting the value of \( T_1 \) based on the previous experience in [24]. The details are as follows:

\[
T_1 = 2.5I_{6 \times 6}, \quad L_1 = 4.365I_{6 \times 6}, \quad L_2 = -5.25I_{6 \times 6}.
\]

Then, according to the definitions of variables \( \alpha, \gamma_1 \) and \( \gamma_2 \) in (11), the values of these variables are given as \( \alpha = 0.5, \gamma_1 = 1/3, \gamma_2 = 1.5 \). And Nussbaurn-type functions are used in the observer, including \( N_{1i} (\xi_{1i}) = \xi_{1i}^2 \cos (\xi_{1i}), N_{2i} (\xi_{2i}) = \xi_{2i}^2 \cos (\xi_{2i}) \).

Finally, in order not to introduce serious fluctuation in the estimated velocity, \( k_1, k_3, k_5, a_1, a_2 \) in (11) should be set relatively small values. Here, these variables are set as \( k_1 = 1, k_3 = 1, k_5 = 0.5, a_1 = 0.1, a_2 = 0.1 \). After determining these values of the above variables, the other parameters in (11) are selected by trial and error. Here, \( k_2 = 10, k_4 = 10, k_6 = 10, p_1 = 0.1 + \tanh (t/30), p_2 = 0.1 + \tanh (t/30) \).

3) Parameters for the controller

At first, the parameters of the prescribed performance function in Eq. (27) are determined based on the given mission. Here, taking the following example, if it is required that the steady accuracy is within 0.3 and the convergence speed of the tracking error is 0.1 at least, the prescribed performance function can be selected as \( \rho_{ai} = ((2 - 0.3) \exp (-0.1 t) + (0.3 - 0.1))^3 / 6, \) (i = 1, 2, …, 6), where \( \epsilon_{1i} \) is a small value designed by users and here \( \epsilon_{1i} = 0.1 \).

Then, after determining the above-mentioned parameters, the other control parameters in Eqs. (38) and (39) are selected by trial and error, shown as follows:

\[
b_1 = 0.5, \quad b_2 = 0.5, \quad \theta = 0.1, \quad l_1 = 1/3, \quad b_3 = 2, \quad \rho_b = 2.01, \quad \beta_1 = 0.6, \quad \Gamma_2 = 0.1, \quad \beta_2 = 0.05.
\]

(3) Comparative control schemes

Two comparative control algorithms are considered. Specifically, a traditional observer-based backstepping control algorithm is used to demonstrate the advantages of the region tracking control strategy, while a traditional region tracking control scheme based on high-gain observer is applied to verify the superior of the proposed region tracking control strategy.

The form of the traditional observer-based backstepping controller is shown as
\[ \alpha_c = -T_1 x_1 + \dot{x}_1 - b_1 e_1 - b_2 \left[ e_1 \right]^{\gamma_1} \]
\[ \dot{x}_c = (\alpha_c - x_c) / \theta \]
\[ e_2 = \dot{x}_2 - x_c \]
\[ u = u_0 + u_1 \]
\[ u_0 = E (x_1)^T \left( T^2_1 x_1 + T_1 \dot{x}_2 + \left( \frac{x_c - \alpha_c}{\theta} \right) \right) \]
\[ - E (x_1)^T \hat{W} \Phi (\hat{x}_1, \hat{x}_2) \]
\[ u_1 = E (x_1)^T \left( -b_3 e_2 b_4 \left[ e_2 \right]^2 - \hat{\Sigma} \hat{col} \left( \tanh \left( e_2 / \rho_b \right) \right) \right), \]

where \( T_1 = 2.5 I_{6 \times 6}, b_1 = 3, b_2 = 1, b_3 = 2, b_4 = 2, \gamma_1 = \gamma_2 = 1/3, \theta = 0.2, \rho_b = 2. \) In addition, the weighting matrix \( \hat{W} \) and the parameter \( \hat{\Sigma} \) are still from (39), and the estimated quantities \( \hat{x}_1 \) and \( \hat{x}_2 \) are from observer (7).

The traditional region tracking control scheme is from \([13,14]\). Since the control law is designed based on the full states of an underwater vehicle, high-gain observer is added to estimate the unmeasured velocity. Hence, the form of the traditional region tracking control scheme based on high-gain observer is given as follows:

\[ F_p (e_1) = \sum_{i=1}^{6} \frac{\kappa_i}{2} \left( \max (0, f p_i (e_{1i})) \right)^2 \]
\[ v_s = J^{-1} (\eta) \left( \hat{v}_d - \alpha_v \left( \frac{\partial F_p (e_1)}{\partial e_1} \right)^T \right) \]

\[ \hat{v} = \hat{\nu} - v_s, \hat{\nu} \]

is the estimated velocity of the vehicle with respect to the body-fixed frame. \( \nu = J^{-1} (\eta) \hat{x}_2 \) from the following high-gain observer:

**Fig. 2** Path considered in simulation

**Fig. 3** Tracking results based on the proposed observer-based region tracking control strategy with parametric uncertainty
\[
\dot{x}_1 = T_1 \dot{x}_1 + \dot{x}_2 + L_1 \Delta_1 \\
\dot{x}_2 = -T_1^2 \dot{x}_1 - T_1 \dot{x}_2 + M_\eta^{-1} (\eta) \dot{W} \Phi (0, \bar{v}) \\
+ E (x_1) u + L_2 \Delta_1, \\
\]

(57)

where \( T_1 = 0, L_1 = 100, L_2 = 2000. \)

(4) Desired trajectory

In simulations, the vehicle is commanded to track the following desired trajectory:

\[
\eta_d = [x_d, y_d, z_d, 0, 0, 0]^T, \]

(58)

where \( x_d = 4 (\sin (0.05t) + \sin (0.15t)), y_d = -2, z_d = 4 (\cos (0.05t) - \cos (0.15t)) \).

4.2 Simulation verification for the case with parametric uncertainty

In order to show the sensitivity of the proposed control strategy to parameter uncertainty, each parameter of the vehicle’s dynamics is expressed as follows: \( Y'' = (1 + 0.3 \text{rand}) Y \), where \( Y \) is the original dynamic parameter and \( Y'' \) is the one with parametric uncertainty, \text{rand} denotes a single uniformly distributed random number in the interval \((0, 1)\) inspired by [49].
Table 1  Comparative results of these control strategies for the case with parametric uncertainty

| Strategies       | Region reachability | Energy consumption ($N^2$) | Chattering (N) |
|------------------|---------------------|----------------------------|----------------|
| Proposed         | YES                 | $1.86 \times 10^8$        | $2.69 \times 10^4$ |
| Backstepping     | YES                 | $2.43 \times 10^8$        | $2.87 \times 10^5$ |
| Conventional     | NO                  | $2.07 \times 10^8$        | $6.84 \times 10^4$ |

Under the action of the proposed observer-based region tracking control strategy, the desired path and the real path are shown in Fig. 2, and the tracking results of the vehicle are shown in Fig. 3, where the dash line denotes the prescribed boundaries.

From Fig. 2, it is obvious that the vehicle did not accurately follow the desired path under the proposed observer-based region tracking control strategy. That is because that the high tracking precision is not the priority of the region tracking control, as long as the tracking errors are always kept within the prescribed boundaries as shown in Fig. 3a. The corresponding control inputs of each thrusters are presented in Fig. 3b. Figure 4 is the results of the conventional backstepping control scheme based on the Nussbaum state observer. It is shown from Figs. 3a and 4a that the tracking precision of the conventional backstepping control scheme is better than the proposed control scheme, while the chattering phenomena are much more serious. From Figs. 3 and 4, the results demonstrate that the control inputs become relatively smooth by sacrificing a certain tracking precision and the tracking errors still satisfy the requirement of the prescribed boundaries under the action of the proposed control strategy.

Figure 5 presents the tracking results, including tracking error and control input, based on the conventional region tracking control scheme with the high-gain observer, where the dash line denotes the prescribed boundaries. Compared with the proposed control strategy, the tracking errors cannot be maintained within the prescribed region under the action of the conventional region tracking control scheme, although the control inputs are relatively smooth in comparison with the conventional backstepping control scheme.

It is highlighted that the tracking accuracy is not the first priority in the region tracking control, as long as the tracking error satisfies the prescribed requirements/boundaries. Hence, the indexes about control accuracy in the normal tracking control are not used to evaluate the advantages of the proposed control strategy. Contrarily, on the premise that the tracking error is always kept within the prescribed boundaries, more attention should paid on control inputs to demonstrate the performance of the region tracking control strategy. To quantitatively evaluate the performances of the
proposed region tracking control strategy, two indexes about control inputs are used, including energy consumption (i.e., the integral of the square of the control input) and chattering (i.e., the integral of the variance of control input), while whether the tracking errors are always kept within the prescribed boundaries or not is considered as another index, after all high precision is not the original purpose of the region tracking control strategy.

From Figs. 3, 4 and 5 and Table 1, it can be concluded that under the action of the proposed region tracking control strategy, the tracking errors are within the prescribed boundaries all the time and meanwhile the control inputs are relatively smooth among these control strategies for underwater vehicles with parametric uncertainty.

4.3 Simulation verification for the case with measurement noise

For an underwater vehicle, it is inevitable that the vehicle’s states measured from sensors suffer from measurement noises. Hence, in this subsection, measurement noise is considered to verify the efficiency of the proposed control strategy. The measurement noise is simulated by the following process. Gaussian noise with mean 0 and variance 1 is filtered by a low-
Table 2 Comparative results of these control strategies for the case with measurement noise

| Strategies          | Region reachability | Energy consumption \((N^2)\) | Chattering \((N)\) |
|---------------------|---------------------|------------------------------|-------------------|
| Proposed            | YES                 | 2.04\(\times\)10^8           | 9.45\(\times\)10^4 |
| Backstepping        | YES                 | 2.76\(\times\)10^8           | 7.03\(\times\)10^5 |
| Conventional region | NO                  | 2.20\(\times\)10^8           | 8.07\(\times\)10^4 |

frequency filter and the result (change between −0.2 and 0.25) is added into the vehicle’s position/attitude states.

Under the simulation environment with the mentioned-above measurement noise and the parametric uncertainty presented in the above subsection, the tracking results of these control strategies are, respectively, presented in Figs. 6, 7 and 8. Among these control strategies, the control inputs from the conventional backstepping control scheme are most easily to be affected by measurement noises, which is caused by the fact that the goal of this control strategy is to obtain high tracking precision. Compared Fig. 6 with Fig. 8, the tracking errors of the vehicle are always kept within the prescribed boundaries based on the proposed control strategy while the conventional region tracking control scheme is failed in terms of the region reachability. Similar to the above case, the results about region reachability and evaluation about control inputs are tabulated in Table 2.

From Figs. 6, 7 and 8 and Table 2, it can be concluded that the proposed observer-based region tracking control strategy can guarantee the tracking errors to be always within the prescribed and its control inputs are relatively smooth among these control strategies even in presence of the measurement noise and parametric uncertainty.

5 Conclusions

In this paper, the problem of region tracking control design has been investigated for an underwater vehicle with unmeasurable velocity and external disturbances. The proposed region tracking control scheme has been designed by incorporating a Nussbaum-type state observer, error nonlinear transformation, backstepping control and RBF neural network. The advantages of using the Nussbaum-type functions in the state observer are that it can provide better estimation accuracy than the traditional high-gain observer. Then, the main feature of the proposed region tracking control is that the nonlinear transformation of the tracking error is introduced into the traditional form of the prescribed performance control to enlarge the tracking error but still kept within the prescribed boundaries and to obtain smoother control signals. Corresponding stability analysis demonstrates that both the estimation error of the Nussbaum-type state observer and the tracking error of the controller are uniformly ultimately bounded. The analysis of the simulation results on a fully actuated underwater vehicle confirms that the region tracking with superior performance in terms of the region reachability can be obtained based on the proposed control scheme, compared with the traditional region tracking control approach. Also, in comparison with the traditional backstepping control approach, the control signals under the proposed control are smoother, even if there exist parametric uncertainty and measurement noise. These comparative simulation results verify the effectiveness of the proposed control scheme in this paper. Future researches will include the separation principle for the observer-based controller for the underwater vehicle, it has to be admitted that the stability proof of the combined observer-based controller is not perfect.

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Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.
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