Avalanche of entanglement and correlations at quantum phase transitions

We study the ground-state entanglement in the quantum Ising model with nearest neighbor ferromagnetic coupling $J$ and find a sequential increase of entanglement depth $d$ with growing $J$. This entanglement avalanche starts with two-point entanglement, as measured by the concurrence, and continues via the three-tangle and four-tangle, until finally, deep in the ferromagnetic phase for $J = \infty$, arriving at a pure $L$-partite (GHZ type) entanglement of all $L$ spins. Comparison with the two, three, and four-point correlations reveals a similar sequence and shows strong ties to the above entanglement measures for small $J$. However, we also find a partial inversion of the hierarchy, where the four-point correlation exceeds the three- and two-point correlations, well before the critical point is reached. Qualitatively similar behavior is also found for the Bose-Hubbard model, suggesting that this is a general feature of a quantum phase transition. This should be taken into account in the approximations starting from a mean-field limit.