Extended TOPSIS method for multi-criteria group decision-making problems under cubic intuitionistic fuzzy environment

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Abstract

The objective of this work is to present a novel multi-criteria group decision making (MCGDM) method under cubic intuitionistic fuzzy (CIF) environment by integrating extended TOPSIS method. In the existing studies, the uncertainties which are present in the data are handled either with an interval-valued intuitionistic fuzzy sets (IVIFS) or with an intuitionistic fuzzy set (IFS) information, which may lose some useful information of alternatives. On the other hand, CIF set (CIFS) handles the uncertainties by considering both the IVIFS and IFS simultaneously. Thus, motivated by this, in the present work, we presented some series of distance measures between the pairs of CIFSs and investigated their various relationship. Further, under this environment, a group decision-making method based on the proposed measure is presented by taking the different priority pairs of the decision makers. A practical example is provided to verify the developed approach and to demonstrate its practicality and feasibility, we compared their results with the several existing approaches results.

Keywords: Cubic intuitionistic fuzzy sets; IVIFS; TOPSIS method; distance measures; closeness coefficients; multicriteria group decision-making.

1. Introduction

Multicriteria group decision-making (MCGDM) plays a pivotal role in our day-to-day living environment. In this era of cut-throat competition, our target is to select the best alternative from a set of alternatives which has to be assessed against the multiple influential criteria. However, selecting only the best alternative doesn’t compile up our problem, but a suitable ranking of the all the available options is needed to be done so as to understand their nature and hence proceed with the further analysis. In such areas, decision-making (DM) approaches acts as a boon for the person who has to reach some conclusion, by keeping all the favorable as well as unfavorable conditions in their mind. Traditionally, DM information had been assumed to be determinable and clear; however, these properties have not been observed. In practice, due to an increasing complexity of the socioeconomic environment and the problem itself, inaccuracies and cognitive limitations of the human mind can cause decision makers difficulty in utilizing crisp numbers to express their information [1, 2]. Thus, the traditional MCGDM method is more limited in real applications. To deal with it, the theory of fuzzy set (FS) [3] or extended fuzzy sets such as intuitionistic fuzzy (IF) set (IFS) [4], interval-valued IFS (IVIFS) [5] are the most successful ones, which characterize the criterion values in terms of membership degrees. Under these environments, various researchers

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presented different kinds of algorithms based on aggregation operator and information measures to solve the MCGDM problems [6–16].

However, apart from that technique for order preference with respect to the similarity to the ideal solution (TOPSIS), developed by Hwang and Yoon [17], is a well-known multicriteria decision-making (MCDM) method. The aim of this method is to choose the best alternative whose distance from its positive ideal solution is shortest. After their existence, numerous attempts are made by the researchers to apply the TOPSIS method under the fuzzy and IFS environment. For instance, Szmidt and Kacprzyk [18] defined the concept of distance measure between the IFSs. Hung and Yang [19] presented the similarity measures between the two different IFSs based on Hausdorff distance. Boran et al. [20] applied the TOPSIS method to solve the problem of Human Resource Personnel selection. Dugenci [21] presented a distance measure for interval-valued IF (IVIF) set and their applications to MCDM with incomplete weight information. Garg [22] presented a generalized improved score function for IVIFSs and their based TOPSIS method for solving the decision-making problems. Mohammadi et al. [23] presented a gray relational analysis and TOPSIS approach to solving the DM problems. Garg et al. [24] presented a generalized entropy measure of order $\alpha$ and degree $\beta$ under the IFS environment and applied to solve the decision-making problems. Biswas and Kumar [25] presented an integrated TOPSIS approach for solving the DM problems with IVIFS environment. Vommi [26] presented a TOPSIS method using statistical distances to solve DM problems. Singh and Garg [27] developed the distance measures between the type-2 intuitionistic fuzzy sets. Li [28] presented a nonlinear programming methodology based TOPSIS method for solving MADM problems under IVIFS environment. Garg and Arora [29] extended the Li [28] approach to the interval-valued intuitionistic fuzzy soft set environment. Lu and Ye [30] developed logarithm similarity measures to solve the problems under interval-valued fuzzy set environment. Garg and Kumar [31] presented new similarity measures for IFSs based on the connection number of the set pair analysis. Askarifar et al. [32] presented an approach to studying the framework of Iran’s seashores using TOPSIS and best-worst MCDM methods. In [33, 34], authors developed a group decision-making method under IVIF environment by integrating extended TOPSIS and linear programming methods. Kumar and Garg [35, 36] presented TOPSIS approach for solving the DM problems by using connection number of the set pair analysis theory.

Since all these facilitate the uncertainties to a great extent, but still they cannot withstand the situations where the decision-maker has to consider the falsity corresponding to the truth value ranging over an interval. But, cubic fuzzy set (CFS) corroborated by Jun et al. [37] is an efficient tool in handling possible disagreement of the agreed interval values and vice-versa. In this set, the degree of agreement/disagreement corresponding to the truth interval has been added to the analysis. Under this environment, Khan et al. [38] and Mahmood et al. [39] presented some aggregation operators under the cubic and cubic hesitant fuzzy set environment. Fahmi et al. [40] worked on grey relational analysis method using cubic information and developed an approach to solving the DM problems under CFS environment.

As the cubic fuzzy sets take into account only the membership intervals and do not stress on the non-membership portion of the data entities. However, in the real world, it is regularly hard to express the estimation of membership degree by an exact value in a fuzzy set. In such cases, it might be easier to depict vagueness and uncertainty in the real world using an interval value and an exact value, instead of unique interval/exact values. Consequently, the hybrid form of an interval value might be extremely valuable to depict the uncertainties because of his/her reluctant judgment in complex decision-making problems. For this reason, Kaur and Garg [41, 42] introduced the idea of the cubic intuitionistic fuzzy set (CIFS) which is described by two parts simultaneously, where one represents the membership degrees by an IVIF number (IVIFN) and the other represents the membership degrees by IF number (IFN). Henceforth, a CIFS is the hybrid set joined by both an
IVIFN and an IFN. Clearly, the advantage of the CIFS is that it can contain substantially more data to express the IVIFN and IFN at the same time. For instance, suppose a manager has to evaluate the work of his teammates. The teammate provides him with his self-analyzed report saying that he has completed 20% - 30% and simultaneously has not accomplished 50% - 60% of the work assigned to him. After analyzing his report by the manager, he gives their judgment under IFS environment by saying that he disagrees with the completed work by 20% and agrees to the incomplete work by 10%. Then, in that case, CIFS is formulated as R-order given by $\langle [0.20, 0.30], [0.50, 0.60], (0.20, 0.10) \rangle$. On the other hand, if the manager agrees by 40% and disagree to the incomplete work by 50% then P-order CIFS is formed as $\langle [0.20, 0.30], [0.50, 0.60], (0.40, 0.50) \rangle$. Therefore, this environment increases the level of precision by enhancing the scope of the membership interval by considering a fuzzy set membership value corresponding to it. Hence, it is a useful tool for handling the imprecise and ambiguous information during the decision-making process under the uncertain environment.

Keeping the advantages of the CIFS, in this paper, we study the MCGDM problem under CIF setting and propose a methodology that utilizes extended TOPSIS method where each of the element is characterized by CIF numbers (CIFNs). CIFNs combine the advantages of both IVIFNs and IFNs. Furthermore, we propose some new weighted and generalized weighted distance measures in order to signify the level of resemblance between two CIF values based upon the decision values and both the CIF positive ideal alternative (CIF-PIA) and CIF negative ideal alternative (CIF-NIA). Several desirable properties of the proposed distance and weighted distance measures are investigated. Multiple decision makers have been included in the decision-making process, highlighting the impetus of different perspectives which makes the proposed approach more realistic for an MCGDM process. The presented approach has been illustrated with a numerical example to verify its feasibility and effectiveness. Finally, the computed results obtained by the presented approach are compared with several existing approaches results to show the superiority of the approach.

The rest of the paper is organized as follows: In Section 2, some basic concepts related to IFSs, IVIFSs, CFSs and CIFSs over the universal set $X$.

**Definition 2.1.** [4, 11] An intuitionistic fuzzy set (IFS) in a set $X$ is an ordered pair defined as:

$$A = \{(x, \zeta_A(x), \vartheta_A(x)) \mid x \in X\},$$

where $\zeta_A$ and $\vartheta_A$ are the mappings from $X$ to $[0,1]$ such that $0 \leq \zeta_A(x) \leq 1$, $0 \leq \vartheta_A(x) \leq 1$ and $0 \leq \zeta_A(x) + \vartheta_A(x) \leq 1$ for all $x \in X$. We denote this pair as $A = \langle \zeta_A, \vartheta_A \rangle$ and called as an IF number (IFN).

After that, Atanassov and Gargov [5] extend its concept to interval-valued numbers and hence define an IVIFS as

$$A = \{\langle x, [\zeta_A^L(x), \zeta_A^U(x)], [\vartheta_A^L(x), \vartheta_A^U(x)] \rangle \mid x \in X\},$$

where $0 \leq \zeta_A^L(x) \leq \zeta_A^U(x) \leq 1$, $0 \leq \vartheta_A^L(x) \leq \vartheta_A^U(x) \leq 1$ and $\zeta_A^L(x) + \vartheta_A^U(x) \leq 1$ for all $x$. This pair is often called as interval-valued intuitionistic fuzzy number (IVIFN).
Definition 2.2. Let $A = \langle \zeta_A, \vartheta_A \rangle$ and $B = \langle \zeta_B, \vartheta_B \rangle$ be two IFNs. Then the following expressions are defined as [4, 11]

(i) $A \subseteq B$ if $\zeta_A(x) \leq \zeta_B(x)$ and $\vartheta_A(x) \geq \vartheta_B(x)$ for all $x$ in $X$;
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(iii) $A^c = \{x, \langle \vartheta_A(x), \zeta_A(x) \rangle \mid x \in U \}$;
(iv) $A \cap B = \{x, \langle \min(\zeta_A(x), \zeta_B(x)), \max(\vartheta_A(x), \vartheta_B(x)) \rangle \mid x \in U \}$;
(v) $A \cup B = \{x, \langle \max(\zeta_A(x), \zeta_B(x)), \min(\vartheta_A(x), \vartheta_B(x)) \rangle \mid x \in U \}$.

Definition 2.3. [37] A cubic set $A$ defined in $X$ is given by

$$A = \{x, A_F(x), \lambda_F(x) \mid x \in X \},$$

where $A_F(x) = [A^L(x), A^U(x)]$ and $\lambda_F(x)$, respectively, represents the interval-valued FS and FS in $x \in X$. We denote these pairs as $A = \langle A_F, \lambda_F \rangle$ and called as cubic fuzzy numbers.

Definition 2.4. [37] For $A_i = \langle A_i, \lambda_i \rangle$ where $i \in A$, we have

(i) P-union: $\cup_{i \in A} A_i = \langle \cup_i \zeta_i, \cup_i \lambda_i \rangle$;
(ii) P-intersection: $\cap_{i \in A} A_i = \langle \cap_i \zeta_i, \cap_i \lambda_i \rangle$;
(iii) R-union: $\cup_{i \in A} A_i = \langle \cup_i \zeta_i, \cup_i \lambda_i \rangle$;
(iv) R-intersection: $\cap_{i \in A} A_i = \langle \cap_i \zeta_i, \cap_i \lambda_i \rangle$.

Definition 2.5. [41, 42] A CIFS $A$ defined over the universal set $X$ is an ordered pair which is defined as follows

$$A = \{x, A(x), \lambda(x) \mid x \in X \},$$

where $A = \{x, \langle [\zeta_1^L(x), \zeta_1^U(x)], [\vartheta_1^L(x), \vartheta_1^U(x)] \rangle \mid x \in X \}$ represents the IVIFS defined on $X$ while $\lambda(x) = \{x, \langle \zeta_A(x), \vartheta_A(x) \rangle \mid x \in X \}$ represents an IFS such that $0 \leq \zeta_A(x) \leq 1$, $0 \leq \vartheta_A(x) \leq 1$ and $\zeta_A(x) + \vartheta_A(x) = 1$. Also, $0 \leq A(x), 0 \leq A^L(x), 0 \leq A^U(x)$ and $\lambda(x) = \langle \zeta_A, \vartheta_A \rangle$ and called as cubic intuitionistic fuzzy number (CIFN).

Definition 2.6. [41, 42] Let $A_i = \langle [\zeta_i^L, \zeta_i^U], [\vartheta_i^L, \vartheta_i^U] \rangle, \langle \zeta_i, \vartheta_i \rangle$, $i = 1, 2$ be two CIFNs in $X$. Then, we define:

(i) (Equality) $A_1 = A_2 \Leftrightarrow [\zeta_1^L, \zeta_1^U] = [\zeta_2^L, \zeta_2^U], [\vartheta_1^L, \vartheta_1^U] = [\vartheta_2^L, \vartheta_2^U], \zeta_1 = \zeta_2$ and $\vartheta_1 = \vartheta_2$;
(ii) (P-order) $A_1 \subseteq_P A_2$ if $[\zeta_1^L, \zeta_1^U] \subseteq [\zeta_2^L, \zeta_2^U], [\vartheta_1^L, \vartheta_1^U] \supseteq [\vartheta_2^L, \vartheta_2^U], \zeta_1 \leq \zeta_2$ and $\vartheta_1 \geq \vartheta_2$;
(iii) (R-order) $A_1 \subseteq_R A_2$ if $[\zeta_1^L, \zeta_1^U] \subseteq [\zeta_2^L, \zeta_2^U], [\vartheta_1^L, \vartheta_1^U] \supseteq [\vartheta_2^L, \vartheta_2^U], \zeta_1 \geq \zeta_2$ and $\vartheta_1 \leq \vartheta_2$.

3. Distance measures for CIFS

In this section, we propose some new distance measures for the non-zero CIFN over the finite universal set $X = \{x_1, x_2, \ldots, x_n\}$. For it, we consider $\phi(X)$ to be family of CIFSs over the set $X$.

Definition 3.1. A real-valued function $d: \phi(X) \times \phi(X) \rightarrow [0, 1]$ is called the distance measure if it satisfies the following properties for $A, B, C \in \phi(X)$:

(P1) $0 \leq d(A, B) \leq 1$;
(P2) $d(A, B) = 0$ if and only if $A = B$;
(P3) $d(A, B) = d(B, A)$;
(P4) If \( \mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{C} \), then \( d(\mathcal{A}, \mathcal{B}) \leq d(\mathcal{A}, \mathcal{C}) \) and \( d(\mathcal{B}, \mathcal{C}) \leq d(\mathcal{A}, \mathcal{C}) \).

where \( \phi(\cdot) \) represents the set of all CIFNs.

**Definition 3.2.** Let \( \mathcal{A} = (\{\zeta^{L}_{A}(x), \zeta^{U}_{A}(x)\}, [\vartheta^{L}_{A}(x), \vartheta^{U}_{A}(x)], \{\zeta_{A}(x), \vartheta_{A}(x)\}) \) and \( \mathcal{B} = (\{\zeta^{L}_{B}(x), \zeta^{U}_{B}(x)\}, [\vartheta^{L}_{B}(x), \vartheta^{U}_{B}(x)], \{\zeta_{B}(x), \vartheta_{B}(x)\}) \) be two CIFNs, then, for \( q \geq 1 \), the following distance measures are defined as

(i) **Distance measures**:

\[
d_{q}^{6}(\mathcal{A}, \mathcal{B}) = \left( \frac{1}{6} \sum_{i=1}^{n} \left| \zeta^{L}_{A}(x_{i}) - \zeta^{L}_{B}(x_{i}) \right|^{q} + \left| \zeta^{U}_{A}(x_{i}) - \zeta^{U}_{B}(x_{i}) \right|^{q} + \left| \vartheta^{L}_{A}(x_{i}) - \vartheta^{L}_{B}(x_{i}) \right|^{q} + \left| \vartheta^{U}_{A}(x_{i}) - \vartheta^{U}_{B}(x_{i}) \right|^{q} \right)^{1/q} \tag{5}
\]

(ii) **Normalized distance measures**:

\[
d_{q}^{4}(\mathcal{A}, \mathcal{B}) = \left( \frac{1}{6n} \sum_{i=1}^{n} \left| \zeta^{L}_{A}(x_{i}) - \zeta^{L}_{B}(x_{i}) \right|^{q} + \left| \zeta^{U}_{A}(x_{i}) - \zeta^{U}_{B}(x_{i}) \right|^{q} + \left| \vartheta^{L}_{A}(x_{i}) - \vartheta^{L}_{B}(x_{i}) \right|^{q} + \left| \vartheta^{U}_{A}(x_{i}) - \vartheta^{U}_{B}(x_{i}) \right|^{q} \right)^{1/q} \tag{6}
\]

Next, we validate that the above defined measures are valid distance measures.

**Theorem 3.1.** The measure \( d_{q}^{6} \) between two CIFNs \( \mathcal{A} \) and \( \mathcal{B} \) satisfies the properties (P1)–(P4) as defined in Definition 3.1.

**Proof.** In order to prove that measure defined in Eq. (5) is a valid distance measure, we shall prove that it satisfies the properties (P1) - (P4) as defined in Definition 3.1 for a collection of CIFNs \( \mathcal{A} = (\{\zeta^{L}_{A}(x), \zeta^{U}_{A}(x)\}, [\vartheta^{L}_{A}(x), \vartheta^{U}_{A}(x)], \{\zeta_{A}(x), \vartheta_{A}(x)\}) \) and \( \mathcal{B} = (\{\zeta^{L}_{B}(x), \zeta^{U}_{B}(x)\}, [\vartheta^{L}_{B}(x), \vartheta^{U}_{B}(x)], \{\zeta_{B}(x), \vartheta_{B}(x)\}) \), (\{\zeta_{A}(x), \vartheta_{A}(x)\})

(P1) By definition of \( d_{q}^{6} \), we have \( d_{q}^{6}(\mathcal{A}, \mathcal{B}) \geq 0 \), so for an arbitrary CIFNs \( \mathcal{A} \) and \( \mathcal{B} \), it is enough to show that \( d_{q}^{6}(\mathcal{A}, \mathcal{B}) \leq 1 \). Since \( \mathcal{A} \) and \( \mathcal{B} \) are two CIFNs, so we have, \( 0 \leq \zeta^{L}_{A}(x_{i}), \zeta^{U}_{A}(x_{i}), \vartheta^{L}_{A}(x_{i}), \vartheta^{U}_{A}(x_{i}), \zeta^{L}_{B}(x_{i}), \zeta^{U}_{B}(x_{i}), \vartheta^{L}_{B}(x_{i}), \vartheta^{U}_{B}(x_{i}) \leq 1 \) and \( 0 \leq \zeta_{A}(x_{i}), \vartheta_{A}(x_{i}) \leq 1 \). This implies that \( 0 \leq \zeta^{L}_{A}(x_{i}) - \zeta^{L}_{B}(x_{i}) \leq 1 \) and \( 0 \leq \zeta^{U}_{A}(x_{i}) - \zeta^{U}_{B}(x_{i}) \leq 1 \). Similarly, \( 0 \leq \vartheta^{L}_{A}(x_{i}) - \vartheta^{L}_{B}(x_{i}) \leq 1 \) and \( 0 \leq \vartheta^{U}_{A}(x_{i}) - \vartheta^{U}_{B}(x_{i}) \leq 1 \). Thus, it follows that \( 0 \leq d_{q}^{6}(\mathcal{A}, \mathcal{B}) \leq 1 \).

(P2) For any two CIFNs \( \mathcal{A} \) and \( \mathcal{B} \),

\[
d_{q}^{6}(\mathcal{A}, \mathcal{B}) = 0 \iff \frac{1}{6n} \sum_{i=1}^{n} \left| \zeta^{L}_{A}(x_{i}) - \zeta^{L}_{B}(x_{i}) \right|^{q} + \left| \zeta^{U}_{A}(x_{i}) - \zeta^{U}_{B}(x_{i}) \right|^{q} + \left| \vartheta^{L}_{A}(x_{i}) - \vartheta^{L}_{B}(x_{i}) \right|^{q} + \left| \vartheta^{U}_{A}(x_{i}) - \vartheta^{U}_{B}(x_{i}) \right|^{q} = 0 \]

\[
\iff \left| \zeta^{L}_{A}(x_{i}) - \zeta^{L}_{B}(x_{i}) \right|^{q} = 0, \left| \zeta^{U}_{A}(x_{i}) - \zeta^{U}_{B}(x_{i}) \right|^{q} = 0, \left| \vartheta^{L}_{A}(x_{i}) - \vartheta^{L}_{B}(x_{i}) \right|^{q} = 0, \left| \vartheta^{U}_{A}(x_{i}) - \vartheta^{U}_{B}(x_{i}) \right|^{q} = 0, \text{ for all } i \]

\[
\iff \zeta_{A}(x_{i}) = \zeta_{B}(x_{i}), \vartheta_{A}(x_{i}) = \vartheta_{B}(x_{i}), \text{ for all } i \]

\[
\iff \mathcal{A} = \mathcal{B}
\]

(P3) For any two real numbers \( a \) and \( b \), we have \( |a - b| = |b - a| \). Thus, we have \( d_{q}^{6}(\mathcal{A}, \mathcal{B}) = d_{q}^{6}(\mathcal{B}, \mathcal{A}) \).
(P4) If $A \subseteq B \subseteq C$ are R-order CIFNs then for all $i$, we have $[\zeta^L_A(x_i), \zeta^U_A(x_i)] \subseteq [\zeta^L_B(x_i), \zeta^U_B(x_i)] \subseteq [\zeta^L_C(x_i), \zeta^U_C(x_i)]$, $[\vartheta^L_A(x_i), \vartheta^U_A(x_i)] \supseteq [\vartheta^L_B(x_i), \vartheta^U_B(x_i)] \supseteq [\vartheta^L_C(x_i), \vartheta^U_C(x_i)]$. Therefore,

$$
|\zeta^L_A(x_i) - \zeta^L_B(x_i)|^q \leq |\zeta^L_A(x_i) - \zeta^L_C(x_i)|^q, \quad |\zeta^U_A(x_i) - \zeta^U_B(x_i)|^q \leq |\zeta^U_A(x_i) - \zeta^U_C(x_i)|^q,
$$

$$
|\vartheta^L_A(x_i) - \vartheta^L_B(x_i)|^q \geq |\vartheta^L_A(x_i) - \vartheta^L_C(x_i)|^q, \quad |\vartheta^U_A(x_i) - \vartheta^U_B(x_i)|^q \geq |\vartheta^U_A(x_i) - \vartheta^U_C(x_i)|^q,
$$

$$
|\zeta_A(x_i) - \zeta_B(x_i)|^q \geq |\zeta_A(x_i) - \zeta_C(x_i)|^q, \quad \text{and} \quad |\vartheta_A(x_i) - \vartheta_B(x_i)|^q \leq |\vartheta_A(x_i) - \vartheta_C(x_i)|^q.
$$

Thus,

$$
d_q'(A, C) = \left[ \frac{1}{6n} \sum_{i=1}^{n} \left\{ |\zeta^L_A(x_i) - \zeta^L_C(x_i)|^q + |\zeta^U_A(x_i) - \zeta^U_C(x_i)|^q + |\vartheta^L_A(x_i) - \vartheta^L_C(x_i)|^q \right. \right.
$$

$$
\left. + |\vartheta^U_A(x_i) - \vartheta^U_C(x_i)|^q \right\} \right]^{1/q}
$$

$$
\geq \left[ \frac{1}{6n} \sum_{i=1}^{n} \left\{ |\zeta^L_A(x_i) - \zeta^L_B(x_i)|^q + |\zeta^U_A(x_i) - \zeta^U_B(x_i)|^q + |\vartheta^L_A(x_i) - \vartheta^L_B(x_i)|^q \right. \right.
$$

$$
\left. + |\vartheta^U_A(x_i) - \vartheta^U_B(x_i)|^q \right\} \right]^{1/q}.
$$

Hence, $d_q'(A, C) \geq d_q'(A, B)$. Similarly, $d_q'(A, C) \geq d_q'(B, C)$. Similarly, we can prove it for P-order CIFNs.

Hence, $d_q'(q \geq 1)$ is a valid distance measure.

\[ \square \]

Theorem 3.2. The measures $d_q'$ satisfies the inequality $d_q'' \leq n^{1/q}$.

Proof. For any real number $q \geq 1$, and for two CIFNs $A$ and $B$, we have $|\zeta^L_A(x_i) - \zeta^L_B(x_i)|^q \leq 1, |\zeta^U_A(x_i) - \zeta^U_B(x_i)|^q \leq 1$ and so on. Therefore, we get

$$
d_q''(A, B) = \left( \frac{1}{6} \sum_{i=1}^{n} \left\{ |\zeta^L_A(x_i) - \zeta^L_B(x_i)|^q + |\zeta^U_A(x_i) - \zeta^U_B(x_i)|^q + |\vartheta^L_A(x_i) - \vartheta^L_B(x_i)|^q \right. \right.
$$

$$
\left. + |\vartheta^U_A(x_i) - \vartheta^U_B(x_i)|^q + |\zeta_A(x_i) - \zeta_B(x_i)|^q + |\vartheta_A(x_i) - \vartheta_B(x_i)|^q \right\} \right]^{1/q}
$$

$$
\leq \left( \frac{1}{6} \sum_{i=1}^{n} (1 + 1 + 1 + 1 + 1) \right)^{1/q}
$$

$$
\leq n^{1/q}
$$

Hence, the result.

\[ \square \]

Theorem 3.3. The measures $d_q'$ and $d_q''$ satisfy the inequality $d_q' \leq \sqrt[q]{d_1}$ and $d_q'' \leq \sqrt[q]{d'_1}$.

Proof. For any real number $q \geq 1$, and for two CIFNs $A$ and $B$, we have $|\zeta^L_A(x_i) - \zeta^L_B(x_i)|^q \leq |\zeta^L_A(x_i) - \zeta^L_B(x_i)|$, $|\zeta^U_A(x_i) - \zeta^U_B(x_i)|^q \leq |\zeta^U_A(x_i) - \zeta^U_B(x_i)|$ and so on. Therefore, we get

$$
d_q'(A, B) = \left( \frac{1}{6n} \sum_{i=1}^{n} \left\{ |\zeta^L_A(x_i) - \zeta^L_B(x_i)|^q + |\zeta^U_A(x_i) - \zeta^U_B(x_i)|^q + |\vartheta^L_A(x_i) - \vartheta^L_B(x_i)|^q \right. \right.
$$

$$
\left. + |\vartheta^U_A(x_i) - \vartheta^U_B(x_i)|^q + |\zeta_A(x_i) - \zeta_B(x_i)|^q + |\vartheta_A(x_i) - \vartheta_B(x_i)|^q \right\} \right]^{1/q}
$$

$$
\leq \left( \frac{1}{6n} \sum_{i=1}^{n} \left\{ |\zeta^L_A(x_i) - \zeta^L_B(x_i)| + |\zeta^U_A(x_i) - \zeta^U_B(x_i)| + |\vartheta^L_A(x_i) - \vartheta^L_B(x_i)| \right. \right.
$$

$$
\left. + |\vartheta^U_A(x_i) - \vartheta^U_B(x_i)| + |\zeta_A(x_i) - \zeta_B(x_i)| + |\vartheta_A(x_i) - \vartheta_B(x_i)| \right\} \right]^{1/q}
$$

$$
\leq \left( d_1'(A, B) \right)^{1/q}
$$

6
Similarly, we can prove $d''_q \leq \sqrt[d'_q]{m}$.

**Theorem 3.4.** The measures $d''_q$ and $d'_q$ satisfy the equation $d''_q = n^{1/q}d'_q$.

**Proof.** Easily follows from the definitions of $d'_q$ and $d''_q$. □

**Remark 3.1.** From the proposed measure, it has been observed that

(i) When $q = 1$, Eq. (6) reduces to the normalized hamming distance measure, and

(ii) When $q = 2$, Eq. (6) reduces to the normalized Euclidean distance measure.

As in practical situations, many times we have to deal with such situations in which various CIFSs may have weights assigned to them. So, taking into account weights $\omega_i (i = 1, 2, \ldots, n)$, where each $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$, we define generalized weighted distances between two CIFSs $A$ and $B$ as follows:

$$d_q(A, B) = \left(\frac{1}{6} \sum_{i=1}^{n} \omega_i \left\{ |\zeta_L^L(x_i) - \zeta_L^B(x_i)|^q + |\zeta_U^L(x_i) - \zeta_U^B(x_i)|^q + |\vartheta_L^L(x_i) - \vartheta_L^B(x_i)|^q + |\vartheta_U^L(x_i) - \vartheta_U^B(x_i)|^q + |\zeta_A(x_i) - \zeta_B(x_i)|^q + |\vartheta_A(x_i) - \vartheta_B(x_i)|^q \right\} \right)^{1/q} \quad (7)$$

**Theorem 3.5.** The weighted distance measure $d_q$, $(1 \leq q < \infty)$, defined in Eq. (7), satisfies the following properties:

(P1) $0 \leq d_q(A, B) \leq 1$;

(P2) $d_q(A, B) = 0 \iff A = B$;

(P3) $d_q(A, B) = d_q(B, A)$;

(P4) If $A \subseteq B \subseteq C$ then $d_q(A, B) \leq d_q(A, C)$ and $d_q(B, C) \leq d_q(A, C)$.

**Proof.** The proof is similar to the Theorem 3.1, so we omit here. □

**Theorem 3.6.** The measures $d'_q$, $d''_q$ and $d_q$ satisfy the following inequalities:

(i) $d'_q \leq d''_q \leq \sqrt[d'_q]{d'_q}$;

(ii) $d'_q \leq d''_q \leq \sqrt[d'_q]{d'_q}$.

**Proof.** Since $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$ and hence we follows the results from their definitions. □

**Remark 3.2.** From this weighted measure, it has been observed that

(i) If $q = 1$, then Eq. (7) reduces to weighted Hamming distance,

(ii) If $q = 2$, then Eq. (7) is called as weighted Euclidean distance.

(iii) Especially, when $\omega_i = 1/n$, for $i = 1, 2, \ldots, n$, then Eq. (7) reduces to (6).

4. An extended TOPSIS approach based on the proposed distance

In this section, we present a TOPSIS approach under the CIFNs environment for solving MCGDM problems based on the proposed distance measure.
4.1. Description of the problem

Assume that there is set of \( m \) alternatives \( A = \{A_1, A_2, \ldots, A_m\} \) which are evaluated under the set of \( n \) different criteria \( C = \{C_1, C_2, \ldots, C_n\} \) such that their rating values are summarized in the form of CIFNs \( \alpha_{ij} = (A_{ij}, \lambda_{ij}) \) where \( A_{ij} = ([\zeta_{ij}^L, \zeta_{ij}^U], [\vartheta_{ij}^L, \vartheta_{ij}^U]) \) represent the IVIFNs and \( \lambda_{ij} = (\xi_{ij}, \vartheta_{ij}) \) represent the IFNs. Here, the components \([\zeta_{ij}^L, \zeta_{ij}^U]\) and \(\vartheta_{ij}\) represent the degree up to which the given alternative \( A_i \) satisfies the criterion \( C_j \) whereas the components \([\vartheta_{ij}^L, \vartheta_{ij}^U]\) and \(\zeta_{ij}\) indicate the dissatisfaction degree of alternative \( A_i \) regarding the criterion \( C_j \). Thus, the overall representation of these rating values can be framed into the CIFN environment and hence the collective decision matrix is summarized as \(D = (\alpha_{ij})_{m \times n}\).

4.2. Compute the CIF-PIA and CIF-NIA

As all the rating values of the alternatives are CIFNs, so the CIF-positive ideal alternative (CIF-PIA), and CIF-negative ideal alternative (CIF-NIA) on the alternative \( A_i (i = 1, 2, \ldots, m) \) may be chosen as 1, and 0 respectively. Thus, rating values of CIF-PIA and CIF-NIA are expressed as \(\zeta^+ = ([1, 1], [0, 0]); (0, 1)\) and \(\alpha^- = ([0, 0], [1, 1], (1, 0)]\). From these, it has been seen that \(\alpha^+\) and \(\alpha^-\) are complement to each other.

However, if we take the fixed a priori CIF-PIA and CIF-NIA reference points, then the overall performance value and hence the ranking order of the alternatives could not change if the alternatives are changed. Instead of it, if decision-maker wants to define these references points as \(\alpha^+ = \left( ([1, 1], [0, 0]), (0, 1) \right)\) and \(\alpha^- = \left( ([0, 0], [1, 1]), (1, 0) \right)\). Assume that there is set of \( n \) criteria \( \{\omega_1, \omega_2, \ldots, \omega_n\}^T \) along with the CIF-PIA \(\alpha^+\) and CIF-NIA \(\alpha^-\), we compute the weighted distances between the alternative \( A_i \) and \(\alpha^+\) as well as \(\alpha^-\) as:

\[
d_q(A_i, \alpha^+) = \left( \frac{1}{6} \sum_{j=1}^{n} \omega_j \left\{ \left| g_j^L - \zeta^L \right|^q + \left| g_j^U - \zeta^U \right|^q + \left| \vartheta^L - h_j^L \right|^q \right\} \right)^{\frac{1}{q}}
\]

and

\[
d_q(A_i, \alpha^-) = \left( \frac{1}{6} \sum_{j=1}^{n} \omega_j \left\{ \left| \zeta^L - g_j^L \right|^q + \left| \zeta^U - g_j^U \right|^q + \left| h_j^L - \vartheta^L \right|^q \right\} \right)^{\frac{1}{q}}
\]

where \( q \geq 1 \) be a real number.

Based on these weighted distances, the relative closeness coefficient of alternative \( A_i (i = 1, 2, \ldots, n) \) with respect to CIF-PIA \(\alpha^+\) is given as follows:

\[
C_i = \frac{d_q(A_i, \alpha^-)}{d_q(A_i, \alpha^+) + d_q(A_i, \alpha^-)}; \quad d_q(A_i, \alpha^+) \neq 0.
\]

Further, it has been seen that, \(0 \leq d_q(A_i, \alpha^-) \leq d_q(A_i, \alpha^-) + d_q(A_i, \alpha^+)\) and hence \(0 \leq C_i \leq 1.\)
4.4. Proposed group decision making TOPSIS approach

Based on the above analysis, we have presented an approach for solving the group decision making problems under the CIFN environment. For it, consider that there are ‘\(K\)’ decision makers \(\{D^{(1)}, D^{(2)}, \ldots, D^{(K)}\}\) which are evaluating the given set of ‘\(m\)’ alternatives \(A_i (i = 1, 2, \ldots, m)\) under the set of ‘\(n\)’ criteria \(C_j (j = 1, 2, \ldots, n)\). These decision makers give their preferences in terms of CIFNs \((\alpha_{ij})^{(k)} = \left(\left\{\left[\left(\zeta_{ij}^{(k)}(r), \eta_{ij}^{(k)}(r)\right]\right], \left[\left(\varphi_{ij}^{(k)}(r), \varphi_{ij}^{(k)}(r)\right]\right]\right\}\right)\) where \(k = 1, 2, \ldots, K\).

Further, assume that \(\omega^{(k)} = (\omega_1^{(k)}, \omega_2^{(k)}, \ldots, \omega_n^{(k)})^T\) such that each \(\omega_j^{(k)} > 0\) and \(\sum_{j=1}^{n} \omega_j^{(k)} = 1\) be the weight vector of the criteria. Also, in order to overcome the diverse judgements by different experts, their opinion is prioritized in accordance to the weight vector \(\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)\) such that \(\lambda_k > 0\) and \(\sum_{k=1}^{K} \lambda_k = 1\). Then the following steps of the proposed approach has been summarized as follows:

Step 1: Arrange the rating values of the alternative given by each decision maker in the matrix form as

\[
D^{(k)} = \begin{pmatrix}
C_1 & C_2 & \cdots & C_n \\
A_1 & (\alpha_{11}^{(k)}, \alpha_{12}^{(k)}, \cdots, \alpha_{1n}^{(k)}) \\
A_2 & (\alpha_{21}^{(k)}, \alpha_{22}^{(k)}, \cdots, \alpha_{2n}^{(k)}) \\
\vdots & \vdots & \ddots & \vdots \\
A_m & (\alpha_{m1}^{(k)}, \alpha_{m2}^{(k)}, \cdots, \alpha_{mn}^{(k)})
\end{pmatrix}
\]

Step 2: For each decision maker \(D^{(k)}, k = 1, 2, \ldots, K\), compute CIF-PIA and CIF-NIA corresponding to alternative \(A_i ; i = 1, 2, \ldots, m\) by using Eqs. (8) and (9) respectively and are defined as

\[
(\alpha^+)^{(k)} = \left(\left\{\left[\left(g_{ij}^{L+} (r), \eta_{ij}^{U+} (r)\right]\right], \left[\left(h_{ij}^{L+} (r), \eta_{ij}^{U+} (r)\right]\right]\right\}\right) \quad (13)
\]

and

\[
(\alpha^-)^{(k)} = \left(\left\{\left[\left(g_{ij}^{L-} (r), \eta_{ij}^{U-} (r)\right]\right], \left[\left(h_{ij}^{L-} (r), \eta_{ij}^{U-} (r)\right]\right]\right\}\right) \quad (14)
\]

where \((g_{ij}^{L+} (r)) = \max_j \{(\zeta_{ij}^{(k)}), (\eta_{ij}^{(k)})\}, (g_{ij}^{U+} (r)) = \max_j \{(\varphi_{ij}^{(k)}), (\varphi_{ij}^{(k)})\}, (h_{ij}^{L+} (r)) = \min_j \{(\eta_{ij}^{(k)}), (\varphi_{ij}^{(k)})\}, (h_{ij}^{U+} (r)) = \min_j \{(\varphi_{ij}^{(k)}), (\eta_{ij}^{(k)})\}, (g_{ij}^{L-} (r)) = \min_j \{(\zeta_{ij}^{(k)}), (\eta_{ij}^{(k)})\}, (g_{ij}^{U-} (r)) = \min_j \{(\varphi_{ij}^{(k)}), (\eta_{ij}^{(k)})\}, (h_{ij}^{L-} (r)) = \max_j \{(\eta_{ij}^{(k)}), (\varphi_{ij}^{(k)})\}, (h_{ij}^{U-} (r)) = \max_j \{(\varphi_{ij}^{(k)}), (\eta_{ij}^{(k)})\}, (r^+)^{(k)} = \min_j \{(\zeta_{ij}^{(k)}), (\varphi_{ij}^{(k)})\}, (s^+)^{(k)} = \max_j \{(\eta_{ij}^{(k)}), (\varphi_{ij}^{(k)})\}, (r^-)^{(k)} = \max_j \{(\zeta_{ij}^{(k)}), (\eta_{ij}^{(k)})\}, (s^-)^{(k)} = \min_j \{(\eta_{ij}^{(k)}), (\varphi_{ij}^{(k)})\}.

Step 3: For each decision maker, compute the separation measures between the alternatives \(A_i\) from its CIF-PIA and CIF-NIA and are denoted by \(d_q\left((A_i)^{(k)}, (\alpha^+)^{(k)}\right)\) and \(d_q\left((A_i)^{(k)}, (\alpha^-)^{(k)}\right)\) respectively.

Step 4: For each decision maker, the relative closeness coefficient is determined as

\[
C_i^{(k)} = \frac{d_q\left((A_i)^{(k)}, (\alpha^+)^{(k)}\right)}{d_q\left((A_i)^{(k)}, (\alpha^+)^{(k)}\right) + d_q\left((A_i)^{(k)}, (\alpha^-)^{(k)}\right)} \quad ; \quad k = 1, 2, \ldots, K.
\]

where \(d_q\left((A_i)^{(k)}, (\alpha^+)^{(k)}\right) \neq 0\).
Step 5: Since each decision maker may have obtained the different ranking towards the alternatives and hence the overall finding of the best alternative remain unclear. In order to overcome these variable rankings, different values of the experts are aggregated by assigning a priority value, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)^T$ such that $\lambda_k > 0$ and $\sum_{k=1}^{K} \lambda_k = 1$, to each expert. The separation measures of each expert are aggregated by using these weight vector, and get the overall measurement values of the alternative as follows

$$D_i^+ = \sum_{k=1}^{K} \lambda_k \ d_q \left( (A_i)^{(k)}, (\alpha^+)^{(k)} \right) \quad \text{and} \quad D_i^- = \sum_{k=1}^{K} \lambda_k \ d_q \left( (A_i)^{(k)}, (\alpha^-)^{(k)} \right)$$  \hspace{1cm} (16)

Step 6: Based on these values, $D_i^+$ and $D_i^-$, the closeness coefficient for an alternative $A_i(i = 1, 2, \ldots, m)$ is determined as

$$\mathcal{C}_i = \frac{D_i^-}{D_i^+ + D_i^-}; \quad D_i^+ \neq 0$$  \hspace{1cm} (17)

Step 7: Rank the alternative(s) based on the descending values of $\mathcal{C}_i$’s.

5. Illustrative example

In order to demonstrate the above mentioned approach, an illustrative example has been taken as below:

5.1. Case study

A multinational company has started its recruitment process for selecting the best candidate for the new project. For this, a company has published notification in the newspaper and based on it, different candidates have applied for it. Out of that, four candidates $A_i; i = 1, 2, 3, 4$ are to be selected for the interview. To evaluate the candidates, company manager has invited four decision-makers $D^{(1)}, D^{(2)}, D^{(3)}$ and $D^{(4)}$ and give them responsibilities to find the best candidate for the company. The panel has decided to evaluate the candidates $A_i; i = 1, 2, 3, 4$ on the basis of four criteria namely, $C_1 : ‘Educational qualification’; C_2 : ‘Technical knowledge’; C_3 : ‘Communication skills’; C_4 : ‘Work experience’. For it, they firstly conducted group discussions (GDs) with all the candidates and the results for each candidate are formulated by a panel in the form of IVIFNs. Among the pool of applicants appearing for GD, four candidates were shortlisted for personal interview and the results for this stage of the recruitment process are recorded in the form of IFNs. Then the following steps of the proposed approach are executed in order to find the best candidate(s) for the required post.

Step 1: The rating values of each decision maker towards the evaluation of the given alternatives are summarized in Table 1. In this table, rating values under both the recruitment stages are clubbed, that is the previously obtained IVIFNs (from GD sessions) and IFNs (from the personal interview round), in the form of CIFNs.

Insert Table 1 here

Step 2: By using Eq. (13) and Eq. (14), the ideal alternatives namely CIF-PIA and CIF-NIA are determined for each decision maker. The corresponding values are summarized in Table 2.

Insert Table 2 here

Step 3: Without loss of generality, we choose $q = 2$, compute the distance measure values by using Eq. (7) for each decision maker and their results are summarized in Table 3.
Step 4: Utilize Eq. (15) to compute the closeness coefficients with respect to each decision maker. The results and the corresponding ranking order of the alternatives are summarized in Table 4 and observed that A_3 is the best candidate for the decision maker D^{(1)} and D^{(2)} while A_1 for the other decision makers.

Step 5: To overcome the ambiguity about the best alternatives w.r.t. the decision makers, aggregate the ideal distance measurement values, as given in Table 3, of every decision-maker by using Eq. (16) corresponding to the five different priority pairs (λ_1,λ_2,λ_3,λ_4) of decision makers. The results are summarized in the fourth and fifth column of the Table 5.

Step 6: For each priority pair, the values of C_i’s are computed by using Eq. (17) and their results are summarized in sixth column of the Table 5.

Step 7: Based on the values of C_i’s, the ranking order of the alternatives is summarized in the last column of the Table 5. From this table, we can see that corresponding to the different pairs, the best alternative is either A_1 or A_3.

5.2. Validity test

The following test criteria are presented by Wang and Triantaphyllou [9] to validate the approach. 

Test criterion 1: “If we replace the rating values of non-optimal alternative with worse alternative then the best alternative should not change, provided the relative weighted criteria remains unchanged.”

Test criterion 2: “Method should possess transitive nature.”

Test criterion 3: “When a given problem is decomposed into smaller ones and the same MCDM method has been applied, then the combined ranking of the alternatives should be identical to the ranking of un-decomposed one.”

Below, we have validated these test criteria on our proposed method.

5.2.1. Validity test by test criterion 1

Without loss of generality, we have considered the case 5 of the above-discussed analysis (similarly for the other cases) where the priority level of the decision makers has been taken as 0.42, 0.36, 0.12, 0.10 respectively. The original ranking order for the case is A_3 ≻ A_2 ≻ A_1 ≻ A_4. Now, in order to validate it with respect to criterion 1, the following decision makers, given in Table 6, are obtained from the original matrices after replacing the non-optimal alternative (A_1) with an arbitrary worst alternative (A_4).

Then, by applying the proposed approach to this data closeness coefficients C_i’s of each candidate A_i(i = 1,2,3,4) are obtained as 0.3601, 0.5246, 0.5465 and 0.4111. Thus, the ranking order of the candidate is A_3 ≻ A_2 ≻ A_4 ≻ A_1, which shows that the best alternative remains the same i.e., A_3.

5.2.2. Validity test by test criteria 2 and 3

Under this test, if we decomposed the given problem into a sub-problems, namely \{A_2,A_3,A_1\}, \{A_2,A_3,A_4\} and \{A_3,A_1,A_4\} and the same procedure steps of the approach has been applied, then we get the ranking orders of these sub-problems as A_3 ≻ A_1 ≻ A_2, A_3 ≻ A_2 ≻ A_4 and A_3 ≻ A_1 ≻ A_4, respectively. Therefore, by combining these, we get the overall ranking order of the alternative is A_3 ≻ A_1 ≻ A_2 ≻ A_4 which is same as that of the original ranking order, hence it beholds the transitive property. Thus, the proposed approach is valid under the test criteria 2 and 3.
5.3. Comparative Studies:

In order to compare the performance of the proposed approach with respect to the existing approaches [21, 25, 30, 33, 34, 40] under the CFSs, IVIFSs, IFSs, interval-valued FSs environment, an analysis has been conducted. To apply these existing approaches on to the considered data, we first convert the rating values of CIFNs into these numbers by taking the rating corresponding to IFNs be zero. Further, without loss of generality, we take the case by taking the weight vector of the decision makers as \( \lambda = (0.42, 0.36, 0.12, 0.10)^T \) and hence the existing approaches are applied to the considered data. The results computed by these different approaches are summarized in Table 7 and conclude that the ranking order of the given alternatives is \( A_3 \succ A_2 \succ A_4 \succ A_1 \) and hence the best alternative is \( A_3 \) which coincides with the proposed approach results given in Table 5, which validates the stability of our approach.

Compared with these existing approaches with general intuitionistic sets (IVIFSs or IFSs), the proposed decision-making method under cubic intuitionistic fuzzy set environment contains much more evaluation information on the alternatives by considering both the IVIFSs and IFSs simultaneously, while the existing approaches contain either IFS or IVIFS information. Therefore, the approaches under the IVIFSs or IFSs may lose some useful information, either IVIFNs or IFNs, of alternatives which may affect the decision results. Furthermore, it is noted from the study that the computational procedure of the proposed approach is different from the existing approaches under the different environment, but the proposed result in this paper is more rational to reality in the decision process due to the consideration of the consistent priority degree between the pairs of the arguments as well as between different experts. Also, the corresponding studies under the IVIFS or IFS environment can be considered as a special case of the proposed operators. Finally, the existing decision-making methods under IVIFSs or IFSs cannot deal with the decision-making problem with CIFS. Therefore, the proposed approach is more generalized and suitable to capture the real-life fuzziness more accurately than the existing ones.

In addition to these, we give some characteristics comparison of our proposed method and the aforementioned methods, which are listed in Table 8.

6. Conclusion

CIFS is one of the successful extensions of the IFS in which a degree of the disagreement (in the form of IFS values) corresponding to the agreed interval region (in form of IVIFS) has been used to represent the data. By taking the advantages of it, in this paper, we present an extended TOPSIS approach to solve the group decision-making problems under the CIFS environment. For it, we propose some generalized distance measures between the pairs of the CIF numbers. The prominent characteristic of these distance measures is also studied. Then, based on these measures, we present an extended TOPSIS group decision-making approach for solving MCGDM problem under CIFS environment. The proposed approach has been illustrated with a numerical example and their results have been compared with some of the existing approaches. In addition to these, the characteristics comparison of the proposed approach with the existing approaches is summarized. From the computed study, it is obtained that the several approaches under CFSs, IVIFSs and/or IFSs are the special case of the proposed approach. Thus, the proposed approach is more generalized and suitable to capture the real-life fuzziness more accurately than the existing ones. In the future, the result of this paper can be extended to the Pythagorean fuzzy environment and other uncertain and fuzzy environments [12, 35, 43–45].
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### Table 1: Rating values of each decision maker in terms of CIFNs

| Decision maker | Candidates | $A_1$ | $A_2$ | $A_3$ | $A_4$ | Weights |
|----------------|-----------|-------|-------|-------|-------|----------|
| $D^{(1)}$      |           | $[0.15, 0.30]$, $[0.35, 0.40]$, $[0.20, 0.65]$ | $[0.13, 0.25]$, $[0.40, 0.45]$, $[0.30, 0.60]$ | $[0.30, 0.45]$, $[0.25, 0.30]$, $[0.55, 0.33]$ | $[0.10, 0.30]$, $[0.25, 0.35]$, $[0.11, 0.26]$ | 0.17 |
| $D^{(2)}$      |           | $[0.10, 0.15]$, $[0.35, 0.40]$, $[0.40, 0.17]$ | $[0.15, 0.22]$, $[0.27, 0.30]$, $[0.15, 0.29]$ | $[0.40, 0.45]$, $[0.21, 0.33]$, $[0.16, 0.35]$ | $[0.10, 0.50]$, $[0.15, 0.20]$, $[0.35, 0.19]$ | 0.13 |
| $D^{(3)}$      |           | $[0.14, 0.25]$, $[0.35, 0.65]$, $[0.10, 0.40]$ | $[0.35, 0.45]$, $[0.15, 0.20]$, $[0.00, 0.50]$ | $[0.45, 0.55]$, $[0.15, 0.25]$, $[0.20, 0.80]$ | $[0.30, 0.50]$, $[0.10, 0.30]$, $[0.20, 0.35]$ | 0.40 |
| $D^{(4)}$      |           | $[0.30, 0.35]$, $[0.25, 0.45]$, $[0.20, 0.30]$ | $[0.20, 0.55]$, $[0.40, 0.45]$, $[0.20, 0.45]$ | $[0.15, 0.25]$, $[0.20, 0.35]$, $[0.60, 0.20]$ | $[0.10, 0.29]$, $[0.04, 0.50]$, $[0.30, 0.40]$, $[0.10, 0.30]$ | 0.13 |

### Table 2: Positive and Negative ideals for each decision maker

| Decision maker | PIA NIA | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|----------------|---------|-------|-------|-------|-------|
| $D^{(1)}$      | $A_1$   | $[0.30, 0.35]$, $[0.25, 0.40]$, $[0.10, 0.65]$ | $[0.30, 0.55]$, $[0.15, 0.20]$, $[0.10, 0.60]$ | $[0.45, 0.55]$, $[0.15, 0.25]$, $[0.16, 0.80]$ | $[0.50, 0.60]$, $[0.10, 0.20]$, $[0.11, 0.40]$ |
| $D^{(2)}$      | $A_2$   | $[0.20, 0.45]$, $[0.25, 0.30]$, $[0.10, 0.60]$ | $[0.30, 0.40]$, $[0.40, 0.45]$, $[0.10, 0.25]$, $[0.10, 0.40]$ | $[0.40, 0.45]$, $[0.15, 0.25]$, $[0.60, 0.20]$ | $[0.10, 0.29]$, $[0.04, 0.50]$, $[0.35, 0.19]$ |
| $D^{(3)}$      | $A_3$   | $[0.10, 0.20]$, $[0.20, 0.50]$, $[0.30, 0.10]$ | $[0.25, 0.29]$, $[0.32, 0.45]$, $[0.60, 0.10]$, $[0.10, 0.10]$ | $[0.40, 0.45]$, $[0.15, 0.25]$, $[0.70, 0.50]$ | $[0.10, 0.15]$, $[0.20, 0.25]$, $[0.30, 0.50]$ |
| $D^{(4)}$      | $A_4$   | $[0.30, 0.40]$, $[0.25, 0.30]$, $[0.20, 0.60]$ | $[0.18, 0.30]$, $[0.52, 0.50]$, $[0.10, 0.40]$ | $[0.23, 0.32]$, $[0.40, 0.45]$, $[0.30, 0.60]$ | $[0.16, 0.32]$, $[0.17, 0.34]$, $[0.30, 0.40]$, $[0.10, 0.30]$ |

### Table 3: Separation measures from ideal solutions corresponding to each decision maker

| Alternatives | $D^{(1)}$ | $D^{(2)}$ | $D^{(3)}$ | $D^{(4)}$ |
|--------------|-----------|-----------|-----------|-----------|
| $D^{(1)}$    | $D^{(1+)}$ | $D^{(1-)}$ | $D^{(2+)}$ | $D^{(2-)}$ | $D^{(3+)}$ | $D^{(3-)}$ | $D^{(4+)}$ | $D^{(4-)}$ |
| $A_1$        | 0.2167    | 0.1539    | 0.2344    | 0.1852    | 0.1361    | 0.1998    | 0.1359    | 0.2214    |
| $A_2$        | 0.1921    | 0.1977    | 0.2166    | 0.1889    | 0.1899    | 0.1625    | 0.2378    | 0.1203    |
| $A_3$        | 0.1082    | 0.2258    | 0.1966    | 0.2173    | 0.1658    | 0.1689    | 0.1909    | 0.1609    |
| $A_4$        | 0.2451    | 0.1262    | 0.1970    | 0.1983    | 0.1780    | 0.1819    | 0.1868    | 0.1803    |

### Table 4: Closeness coefficients and ranking order with respect to decision maker

| Alternatives | $C_i^{(1)}$ | Ranking | $C_i^{(2)}$ | Ranking | $C_i^{(3)}$ | Ranking | $C_i^{(4)}$ | Ranking |
|--------------|-------------|---------|-------------|---------|-------------|---------|-------------|---------|
| $A_1$        | 0.4152      | 3       | 0.4414      | 4       | 0.5948      | 1       | 0.6196      | 1       |
| $A_2$        | 0.5071      | 2       | 0.4659      | 3       | 0.4612      | 4       | 0.3359      | 4       |
| $A_3$        | 0.6761      | 1       | 0.5250      | 1       | 0.5047      | 3       | 0.4696      | 3       |
| $A_4$        | 0.3398      | 4       | 0.5016      | 2       | 0.5054      | 2       | 0.4912      | 2       |
### Table 5: Aggregated closeness coefficient and ranking for each candidate

| Case 1 | Weights | Candidates | distance measures | \(C_i\) | Ranking | Selected Candidate |
|--------|---------|------------|-------------------|--------|---------|-------------------|
|        | \(D^{(1)}\) | \(A_1\) | \(D_i^1\) | \(D_i^2\) |
|        | 0.20    | 0.1817    | 0.1884            | 0.5090 | 2       | \(A_3\)           |
|        | \(D^{(2)}\) | \(A_2\) | 0.2031            | 0.1732 | 4       |                   |
|        | 0.30    | 0.1948    | 0.5399            |        |         |                   |
|        | \(D^{(3)}\) | \(A_3\) | 0.1980            | 0.1755 | 3       |                   |
|        | 0.40    | 0.1718    | 0.5333            |        |         |                   |
|        | \(D^{(4)}\) | \(A_4\) | 0.1988            | 0.4699 |         |                   |

| Case 2 | Weights | Candidates | distance measures | \(C_i\) | Ranking | Selected Candidate |
|--------|---------|------------|-------------------|--------|---------|-------------------|
|        | \(D^{(1)}\) | \(A_1\) | \(D_i^1\) | \(D_i^2\) |
|        | 0.20    | 0.1718    | 0.1963            | 0.5333 | 1       | \(A_1\)           |
|        | \(D^{(2)}\) | \(A_2\) | 0.2148            | 0.1579 | 4       |                   |
|        | 0.20    | 0.1900    | 0.5271            |        |         |                   |
|        | \(D^{(3)}\) | \(A_3\) | 0.1705            | 0.4237 | 3       |                   |
|        | 0.40    | 0.1988    | 0.4660            |        |         |                   |

| Case 3 | Weights | Candidates | distance measures | \(C_i\) | Ranking | Selected Candidate |
|--------|---------|------------|-------------------|--------|---------|-------------------|
|        | \(D^{(1)}\) | \(A_1\) | \(D_i^1\) | \(D_i^2\) |
|        | 0.13    | 0.1612    | 0.2007            | 0.5545 | 1       | \(A_1\)           |
|        | \(D^{(2)}\) | \(A_2\) | 0.2143            | 0.1533 | 4       |                   |
|        | 0.15    | 0.1836    | 0.5142            |        |         |                   |
|        | \(D^{(3)}\) | \(A_3\) | 0.1735            | 0.4170 | 2       |                   |
|        | 0.30    | 0.1765    | 0.4773            |        |         |                   |

| Case 4 | Weights | Candidates | distance measures | \(C_i\) | Ranking | Selected Candidate |
|--------|---------|------------|-------------------|--------|---------|-------------------|
|        | \(D^{(1)}\) | \(A_1\) | \(D_i^1\) | \(D_i^2\) |
|        | 0.35    | 0.1957    | 0.1827            | 0.4828 | 2       | \(A_3\)           |
|        | \(D^{(2)}\) | \(A_2\) | 0.2074            | 0.1761 | 3       |                   |
|        | 0.32    | 0.2043    | 0.5612            |        |         |                   |
|        | \(D^{(3)}\) | \(A_3\) | 0.1598            | 0.4592 | 3       |                   |
|        | 0.16    | 0.1674    | 0.4446            |        |         |                   |

| Case 5 | Weights | Candidates | distance measures | \(C_i\) | Ranking | Selected Candidate |
|--------|---------|------------|-------------------|--------|---------|-------------------|
|        | \(D^{(1)}\) | \(A_1\) | \(D_i^1\) | \(D_i^2\) |
|        | 0.42    | 0.2053    | 0.1774            | 0.4635 | 3       | \(A_3\)           |
|        | \(D^{(2)}\) | \(A_2\) | 0.2052            | 0.1826 | 2       |                   |
|        | 0.36    | 0.2102    | 0.5753            |        |         |                   |
|        | \(D^{(3)}\) | \(A_3\) | 0.1552            | 0.4708 | 2       |                   |
|        | 0.12    | 0.1643    | 0.4343            |        |         |                   |

### Table 6: Rating values of the worse alternative \(A_1\) for each decision maker

| Decision Maker | \(C_1\) | \(C_2\) | \(C_3\) | \(C_4\) |
|----------------|--------|--------|--------|--------|
| \(D^{(1)}\)    | \([0.15, 0.20],[0.30, 0.45]\) | \([0.13, 0.20],[0.40, 0.48]\) | \([0.30, 0.35]\) | \([0.60, 0.30]\) |
| \(D^{(2)}\)    | \([0.10, 0.15],[0.30, 0.45]\) | \([0.15, 0.18],[0.25, 0.35]\) | \([0.44, 0.48],[0.20, 0.35]\) | \([0.20, 0.30]\) |
| \(D^{(3)}\)    | \([0.20, 0.25],[0.25, 0.42]\) | \([0.15, 0.18],[0.25, 0.35]\) | \([0.44, 0.48],[0.20, 0.35]\) | \([0.60, 0.30]\) |
| \(D^{(4)}\)    | \([0.30, 0.35],[0.20, 0.35]\) | \([0.18, 0.25],[0.19, 0.39]\) | \([0.50, 0.30]\) | \([0.40, 0.20]\) |

### Table 7: Comparison analysis with some of the existing approaches

| Existing approaches | Aggregated closeness coefficients | Ranking |
|---------------------|----------------------------------|---------|
| Fahmi et al. [40]   | \(\mathcal{C}_1\) | \(\mathcal{C}_2\) | \(\mathcal{C}_3\) | \(\mathcal{C}_4\) | \(A_3 \succ A_2 \succ A_4 \succ A_1\) |
| Lu and Ye [30]      | 0.4350 | 0.5103 | 0.5618 | 0.4693 | \(A_3 \succ A_2 \succ A_4 \succ A_1\) |
| Biswas and Kumar [25]| 0.5471 | 0.5729 | 0.5867 | 0.5553 | \(A_3 \succ A_2 \succ A_4 \succ A_1\) |
| Gupta et al. [34]   | 0.5648 | 0.5021 | 0.4453 | 0.5356 | \(A_3 \succ A_2 \succ A_4 \succ A_1\) |
| Dugenci [21]        | 0.3510 | 0.5396 | 0.5803 | 0.4187 | \(A_3 \succ A_2 \succ A_4 \succ A_1\) |
| Wang and Chen [33]  | 0.5300 | 0.4917 | 0.4865 | 0.5161 | \(A_3 \succ A_2 \succ A_4 \succ A_1\) |
Table 8: The characteristic comparisons of different methods

| Methods                  | Whether flexible to express a wider range of information | Whether consider more than one decision-maker | Whether describe hybrid information at same level | Whether have the characteristic of generalization |
|--------------------------|----------------------------------------------------------|----------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| Lu and Ye [30]           | ✓                                                        | ×                                            | ×                                                | ×                                                |
| Wang and Chen [33]       | ✓                                                        | ×                                            | ×                                                | ×                                                |
| Gupta et al. [34]        | ✓                                                        | ✓                                            | ×                                                | ×                                                |
| Dugenci [21]             | ✓                                                        | ✓                                            | ×                                                | ×                                                |
| Biswas and Kumar [25]    | ✓                                                        | ×                                            | ×                                                | ×                                                |
| Fahmi et al. [40]        | ✓                                                        | ✓                                            | ×                                                | ×                                                |
| The proposed method      | ✓                                                        | ✓                                            | ✓                                                | ✓                                                |