THEORETICAL PROGRESS IN K AND B DECAYS

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Abstract

We review several aspects of the recent theoretical progress in K and B decays including the impact of the top quark discovery on rare and CP violating decays. In particular we summarize the present status of next-to-leading QCD calculations in this field stressing their importance in the determination of the parameters in the Cabibbo-Kobayashi-Maskawa matrix.

1 Introduction

An important target of particle physics is the determination of the unitary $3 \times 3$ Cabibbo-Kobayashi-Maskawa matrix \cite{1, 2} which parametrizes the charged current interactions of quarks:

$$J_{\mu}^{cc} = (\bar{u}, \bar{c}, \bar{t}) L \gamma_{\mu} \left( \begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{array} \right) \left( \begin{array}{c} d \\ s \\ b \end{array} \right)_L$$

(1)

The CP violation in the standard model is supposed to arise from a single phase in this matrix. It is customary these days to express the CKM-matrix in terms of four Wolfenstein parameters \cite{3} $(\lambda, A, \varrho, \eta)$ with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and $\eta$ representing the CP violating phase:

$$V_{CKM} = \left( \begin{array}{ccc} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{array} \right) + O(\lambda^4)$$

(2)

Following \cite{4} one can define the parameters $(\lambda, A, \varrho, \eta)$ through

$$s_{12} \equiv \lambda \quad s_{23} \equiv A\lambda^2 \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\varrho - i\eta)$$

(3)

where $s_{ij}$ and $\delta$ enter the standard exact parametrization \cite{5} of the CKM matrix. This specifies the higher orders terms in (2). With the definitions in (3),

$$V_{us} = \lambda \quad V_{cb} = A\lambda^2$$

(4)
\[ V_{ub} = A\lambda^3 (\bar{\rho} - i\bar{\eta}) \quad V_{td} = A\lambda^3 (1 - \bar{\rho} - i\bar{\eta}) \] (5)

where
\[ \bar{\rho} = \rho (1 - \frac{\lambda^2}{2}) \quad \bar{\eta} = \eta (1 - \frac{\lambda^2}{2}) \] (6)

turn out [4] to be excellent approximations to the exact expressions.

A useful geometrical representation of the CKM matrix is the unitarity triangle obtained by using the unitarity relation \( V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \), rescaling it by \( |V_{cd}V_{cb}^*| = A\lambda^3 \) and depicting the result in the complex \((\bar{\rho}, \bar{\eta})\) plane as shown in fig. 1. The lengths CB, CA and BA are equal respectively to 1,
\[ R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \left| \frac{V_{ub}}{V_{cb}} \right| \quad \text{and} \quad R_t = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|. \] (7)

The triangle in fig. 1, \( |V_{us}| \) and \( |V_{cb}| \) give the full description of the CKM matrix. Looking at \( R_b \) and \( R_t \) we observe that within the standard model the measurements of four CP conservative decays sensitive to \( |V_{us}|, |V_{ub}|, |V_{cb}| \) and \( |V_{td}| \) can tell us whether CP violation (\( \eta \neq 0 \)) is predicted in the standard model. This is a very remarkable property of the Kobayashi-Maskawa picture of CP violation: quark mixing and CP violation are closely related to each other.

There is of course the very important question whether the KM picture of CP violation is correct and more generally whether the standard model offers a correct description of weak decays of hadrons. In order to answer these important questions it is essential to calculate as many branching ratios as possible, measure them experimentally and check if they all can be described by the same set of the parameters \((\lambda, A, \rho, \eta)\). In the language of the unitarity triangle this means that various curves in the \((\bar{\rho}, \bar{\eta})\) plane extracted from different decays should cross each other at a single point which determines the apex of the unitarity triangle in fig. 1. Moreover the angles \((\alpha, \beta, \gamma)\) in the resulting triangle should agree with those extracted one day from CP-asymmetries in B-decays.

There is a common belief that during the coming fifteen years we will certainly witness a dramatic improvement in the determination of the CKM-parameters. To this end, however, it is essential not only to perform difficult experiments but also to have accurate formulae which would allow a confident and precise extraction of the CKM-parameters from the existing and future data. We will review what progress has been done in this direction.

Clearly the discovery of the top quark [5, 6] and its mass measurement had an important impact on the field of rare decays and CP violation reducing considerably one potential uncertainty. In loop induced K and B decays the relevant mass parameter is the running
current quark mass. With the pole mass measurement of CDF, \(m_t^{pole} = 176 \pm 13 \text{ GeV}\), one has \(m_t^* = \bar{m}_t(m_t) \approx 168 \pm 13 \text{ GeV}\). Similarly the D0 value \(m_t^{pole} = 199 \pm 30 \text{ GeV}\) corresponds to \(m_t^* \approx 190 \pm 30 \text{ GeV}\). In this review we will simply denote \(m_t^*\) by \(m_t\).

2 Basic Framework

2.1 OPE and Renormalization Group

The basic framework for weak decays of hadrons containing u, d, s, c and b quarks consists of the Operator Product Expansion (OPE) combined with the renormalization group techniques. In this framework the amplitude for a decay \(M \to F\) is written as

\[
A(M \to F) = \langle F | \mathcal{H}_{eff} | M \rangle = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) \langle F | Q_i(\mu) | M \rangle
\]

where \(\mathcal{H}_{eff}\) is an effective hamiltonian relevant for a given decay, \(M\) stands for the decaying meson, \(F\) for a given final state and \(V_{CKM}\) denotes the relevant \(CKM\) factor. \(Q_i(\mu)\) denote the local operators generated by QCD and electroweak interactions. \(C_i(\mu)\) stand for the Wilson coefficient functions. The scale \(\mu\) separates the physics contributions in the “short distance” contributions (corresponding to scales higher than \(\mu\)) contained in \(C_i(\mu)\) and the “long distance” contributions (scales lower than \(\mu\)) contained in \(\langle F | Q_i(\mu) | M \rangle\). Since physical amplitudes cannot depend on \(\mu\), the \(\mu\)-dependence of \(C_i(\mu)\) must be cancelled by the one present in \(\langle Q_i(\mu) \rangle\). It should be stressed, however, that this cancellation generally involves many operators due to the operator mixing under renormalization.

The \(\mu\) dependence of \(C_i(\mu)\) is given by:

\[
\hat{C}(\mu) = \hat{U}(\mu, M_W) \hat{C}(M_W)
\]

where \(\hat{C}\) is a column vector built out of \(C_i\)’s. \(\hat{C}(M_W)\) are the initial conditions which depend on the short distance physics at high energy scales. In particular they depend on \(m_t\). \(\hat{U}(\mu, M_W)\), the renormalization group evolution matrix, is given as follows

\[
\hat{U}(\mu, M_W) = T_g \exp \left( \int_{g(M_W)}^{g(\mu)} dg' \frac{\gamma(g')}{\beta(g')} \right)
\]

with \(g\) denoting QCD effective coupling constant. \(\beta(g)\) governs the evolution of \(g\) and \(\gamma\) is the anomalous dimension matrix of the operators involved. The structure of this equation makes it clear that the renormalization group approach goes beyond the usual perturbation theory. Indeed \(\hat{U}(\mu, M_W)\) sums automatically large logarithms \(\log M_W/\mu\) which appear for \(\mu << M_W\). In the so called leading logarithmic approximation (LO) terms \((g^2 \log M_W/\mu)^n\) are summed. The next-to-leading logarithmic correction (NLO) to this result involves summation of terms \((g^2)^n (\log M_W/\mu)^{n-1}\) and so on. This hierarchic structure gives the renormalization group improved perturbation theory. For instance in the case of a single operator one has including NLO corrections:

\[
U(\mu, M_W) = \left[ 1 + \frac{\alpha_{QCD}(\mu)}{4 \pi} J \right] \left[ \frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(\mu)} \right]^P \left[ 1 - \frac{\alpha_{QCD}(M_W)}{4 \pi} J \right]
\]

where \(P\) and \(J\) are given in terms of the coefficients in the perturbative expansions for \(\gamma(g)\) and \(\beta(g)\). General formulae for \(U(\mu, M_W)\) in the case of operator mixing and valid also for electroweak effects can be found in ref. [2]. The leading logarithmic approximation corresponds to setting \(J = 0\) in (11).
2.2 Classification of Operators

The most important operators are given as follows:

**Current–Current:**

\[ Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \quad Q_2 = (\bar{s}_u)_{V-A} (\bar{u}d)_{V-A} \] (12)

**QCD–Penguins:**

\[ Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A} \quad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A} \] (13)
\[ Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V+A} \quad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A} \] (14)

**Electroweak–Penguins:**

\[ Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A} \quad Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A} \] (15)
\[ Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A} \quad Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A} \] (16)

**Magnetic–Penguins:**

\[ Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha F_{\mu\nu} \quad Q_{8\gamma} = \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta} b_\beta G_{\mu\nu} \] (17)

\( \Delta S = 2 \) and \( \Delta B = 2 \) Operators:

\[ Q(\Delta S = 2) = (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} \quad Q(\Delta B = 2) = (\bar{b}d)_{V-A} (\bar{b}d)_{V-A} \] (18)

**Semi–Leptonic Operators:**

\[ Q_{0V} = (\bar{b}s)_{V-A} (\bar{e}e)_V \quad Q_{10A} = (\bar{b}s)_{V-A} (\bar{e}e)_A \] (19)
\[ Q(\nu\bar{\nu}) = (\bar{s}d)_{V-A} (\nu\bar{\nu})_{V-A} \quad Q(\mu\bar{\mu}) = (\bar{s}d)_{V-A} (\mu\bar{\mu})_{V-A} \] (20)

2.3 Towards Phenomenology

The rather formal expression for the decay amplitudes given in \[8\] can always be cast in a more useful form \[8\]:

\[ A(M \to F) = \sum_i B_i V_{CKM}^i \eta_{QCD}^i F_i(m_t, m_c) \] (21)

In writing \[21\] we have generalized \[8\] to include several CKM factors, \( F_i(m_t, m_c) \), the Inami-Lim functions, result from the evaluation of loop diagrams with internal top and charm exchanges and may also depend solely on \( m_t \) or \( m_c \). In the case of current-current operators \( F_i \) are mass independent. The factors \( \eta_{QCD}^i \) summarize short distance QCD corrections which can be calculated by formal methods discussed above. Finally \( B_i \) stand for nonperturbative factors related to the hadronic matrix elements of the contributing operators: the main theoretical uncertainty in the whole enterprise. In semi-leptonic decays such as \( K \to \pi \nu\bar{\nu} \), the nonperturbative \( B \)-factors can fortunately be determined from leading tree level decays such as \( K^+ \to \pi^0 e^+\nu \) reducing or removing the non-perturbative uncertainty. In non-leptonic decays this is generally not possible and we have to rely on existing non-perturbative methods. A well known example of a \( B_i \)-factor is the renormalization group invariant parameter \( B_K \) \[1\] defined by

\[ B_K = B_K(\mu) [\alpha_s(\mu)]^{-2/3} \quad \langle \bar{K}^o \mid Q(\Delta S = 2) \mid K^o \rangle = \frac{8}{3} B_K(\mu) F_K^2 m_K^2 \] (22)
2.4 Inclusive Decays

Sofar we have discussed only exclusive decays. During the recent years considerable progress has been made for inclusive decays of heavy mesons. The starting point is again the effective hamiltonian in (8) which includes the short distance QCD effects in $C_i(\mu)$. The actual decay described by the operators $Q_i$ is then calculated in the spectator model corrected for additional virtual and real gluon corrections. Support for this approximation comes from the $1/m_b$ expansions. Indeed the spectator model has been shown to correspond to the leading order approximation in the $1/m_b$ expansion. The next corrections appear at the $O(1/m_b^2)$ level. The latter terms have been studied by several authors [10, 11, 12] with the result that they affect various branching ratios by less than 10% and often by only a few percent. There is a vast literature on this subject and I can only refer here to recent reviews [12, 13] where further references can be found. Of particular importance for this field was also the issue of the renormalons which are nicely discussed in [14, 15].

3 Theoretical Progress in K and B Decays

It is impossible to review adequately the full theoretical progress here. Let me then list only a few achievements of the last five years which in my opinion should be considered as important contributions to the field of weak decays.

- Calculation of NLO corrections to the Wilson coefficients for nearly all decays (ordinary, rare and CP-violating) [16].
- Applications of heavy quark effective theory to exclusive decays which resulted in particular in an improved determination of $V_{cb}$ [17].
- Heavy Quark Expansions for inclusive decays (see reviews in [12, 13]), which by putting the spectator model on a firmer ground allow for an improved determination of $V_{cb}$ in agreement with the exclusive determination [12, 15].
- Some progress in the calculations of non-perturbative parameters such as $B_K$ and $F_B$.
- Identification of decays nearly without any hadronic uncertainties.

In this review I will mainly discuss the first and the last item on this list, incorporating in this discussion the achievements related to the remaining three items.

4 Weak Decays Beyond Leading Logarithms

4.1 General Remarks

Until 1989 all the calculations in the field of weak decays were done in the leading logarithmic approximation except for [18] where NLO QCD corrections to the Wilson coefficients of the current-current operators have been calculated. Today the effective hamiltonians for weak decays are available at the next-to-leading level for the most important and interesting cases due to a series of publications listed in table 1. We will discuss this list briefly below. An extended version of this discussion appeared recently [16]. A very detailed review of the existing NLO calculations will appear soon [19].

Let us recall why NLO calculations are important for the phenomenology of weak decays.

- The NLO is first of all necessary to test the validity of the renormalization group improved perturbation theory.
Without going to NLO the QCD scale $\Lambda_{\overline{MS}}$ extracted from various high energy processes cannot be used meaningfully in weak decays.

Due to renormalization group invariance the physical amplitudes do not depend on the scales $\mu$ present in $\alpha_s$ or in the running quark masses, in particular $m_t(\mu)$, $m_b(\mu)$ and $m_c(\mu)$. However in perturbation theory this property is broken through the truncation of the perturbative series. Consequently one finds sizable scale ambiguities in the leading order, which can be reduced considerably by going to NLO.

The central issue of the top quark mass dependence is often a NLO effect.

### 4.2 Current-Current Operators

The NLO corrections to the coefficients of $Q_1$ and $Q_2$ have been first calculated by Altarelli et al. using the Dimension Reduction Scheme (DRED) for $\gamma_5$. In 1989 these coefficients have been calculated in DRED, NDR and HV schemes for $\gamma_5$ by Peter Weisz and myself. The result for DRED obtained by the Italian group has been confirmed. The coefficients $C_1$ and $C_2$ show a rather strong renormalization scheme dependence which in physical quantities should be cancelled by the one present in the matrix elements of $Q_1$ and $Q_2$. This cancellation has been shown explicitly in demonstrating thereby the compatibility of the results for $C_1$ and $C_2$ in DRED, NDR and HV schemes. A recent discussion of $C_1(\mu)$ and $C_2(\mu)$ in these schemes can be found in.

### 4.3 NLO Corrections to $B_{SL}$

A direct physical application of the NLO corrections to $C_1$ and $C_2$ is the calculation of the non-leptonic width for B-Mesons which is relevant for the theoretical prediction of the inclusive semileptonic branching ratio $B_{SL}$ in B-decays. This calculation can be done within the spectator model corrected for small non-perturbative corrections and more important...
gluon bremsstrahlung and virtual gluon corrections. The calculation of \( B_{SL} \) for massless final quarks has been done by Altarelli et al.\( ^{[18]} \) in the DRED scheme and by Buchalla \( ^{[27]} \) in the HV scheme. The results of these papers agree with each other.

Unfortunately the theoretical branching ratio based on the QCD calculation of refs. \( ^{[18, 27]} \) give typically \( B_{SL} = 12.5 \pm 13.5\% \) \( ^{[4]} \) whereas the experimental world average \( ^{[3]} \) is \( B_{SL} = (10.43 \pm 0.24)\% \). The inclusion of the leading non-perturbative correction \( \mathcal{O}(1/m_b^2) \) lowers slightly the theoretical prediction but gives only \( \Delta_NPB_{SL} = -0.2\% \) \( ^{[12]} \). On the other hand Bagan et al. \( ^{[28]} \) have demonstrated that including mass effects in the QCD calculations of NP slightly the theoretical prediction but gives only \( \Delta_NPB_{SL} = -0.2\% \) \( ^{[12]} \). 

4.4 \( \Delta S = 2 \) and \( \Delta B = 2 \) Transitions

The \( M_{12} \) amplitude describing the \( K^0 - \bar{K}^0 \) mixing is given as follows

\[
M_{12}(\Delta S = 2) = \frac{G_F^2}{12\pi^2} F_K^2 B_K m_K M_W^2 \left[ \lambda_1^2 \eta_1 S(x_c) + \lambda_2^2 \eta_2 S(x_t) + 2\lambda_1^* \lambda_2^* \eta_3 S(x_c, x_t) \right]
\]

with \( x_i = m_i^2/M_W^2 \), \( \lambda_i = V_{td} V_{ts}^* \), \( S(x_i) \) denoting the Inami-Lim functions resulting from box diagrams and \( \eta_i \) representing QCD corrections. The parameter \( B_K \) is defined in (22). The corresponding amplitude for the \( B_s^0 - \bar{B}_s^0 \) mixing is dominated by the box diagrams with top quark exchanges and given by

\[
| M_{12}(\Delta B = 2) | = \frac{G_F^2}{12\pi^2} F_B^2 B_B m_B M_W^2 \left| V_{td} \right|^2 \eta_B S(x_s)
\]

where we have set \( V_{tb} = 1 \). A similar formula exists for \( B_s^0 - \bar{B}_s^0 \). In the leading order \( \eta_i \) are given roughly \( ^{[12]} \) as follows: \( \eta_1 = 0.85 \), \( \eta_2 = 0.62 \), \( \eta_3 = 0.36 \), \( \eta_B = 0.60 \). As of 1995 the coefficients \( \eta_i \) and \( \eta_B \) are known including NLO corrections \( ^{[13]} \). It has been stressed in these papers that the LO results for \( \eta_i \) suffer from sizable scale uncertainties, as large as \( \pm 20\% \) for \( \eta_2 \) and \( \pm 10\% \) for the remaining \( \eta_i \). As demonstrated in \( ^{[29]} \) these uncertainties are considerably reduced in the products like \( \eta_1 S(x_c) \), \( \eta_2 S(x_t) \), \( \eta_3 S(x_c, x_t) \) and \( \eta_B S(x_s) \) provided NLO corrections are taken into account. For \( m_c = \bar{m}_c(m_c) = 1.3 \pm 0.1 \text{ GeV} \) and \( m_t = \bar{m}_t(m_t) = 170 \pm 15 \text{ GeV} \) one finds:

\[
\eta_1 = 1.3 \pm 0.2 \quad \eta_2 = 0.57 \pm 0.01 \quad \eta_3 = 0. \quad \eta_B = 0.55 \pm 0.01
\]

where the "***" in \( \eta_3 \) will be public soon \( ^{[11]} \). It should be stressed that \( \eta_i \) given here are so defined that the relevant \( B_K \) and \( B_B \) non-perturbative factors (see (22)) are renormalization group invariant. Let us list the main implications of these results:

- The enhancement of \( \eta_1 \) implies the enhancing of the short distance contribution to the \( K_L - K_S \) mass difference so that for \( B_K = 3/4 \) as much as 80\% of the experimental value can be attributed to this contribution \( ^{[26]} \).

- The improved calculations of \( \eta_2 \) and \( \eta_3 \) combined with the analysis of the CP violating parameter \( \varepsilon_K \) allow an improved determination of the parameters \( \eta \) and \( \varrho \) in the CKM matrix \( ^{[1, 11]} \).

- Similarly the improved calculation of \( \eta_B \) combined with the analysis of \( B_d^0 - \bar{B}_d^0 \) mixing allows an improved determination of the element \( | V_{td} | \):

\[
| V_{td} | = 8.7 \cdot 10^{-3} \left[ \frac{200 \text{ MeV}}{\sqrt{B_B F_B}} \right] \left[ \frac{170 \text{ GeV}}{\bar{m}_t(m_t)} \right]^{0.76} \left[ x_d \right]^{0.5} \left[ \frac{1.50 \text{ ps}}{\tau_B} \right]^{0.5}
\]
This using all uncertainties (see below) gives:

\[ |V_{td}| = (9.6 \pm 3.0) \cdot 10^{-3} \implies (9.3 \pm 2.5) \cdot 10^{-3} \]  

(27)

with the last number obtained after the inclusion of the \( \varepsilon_K \)-analysis \[44\].

Concerning the parameter \( B_K \), the most recent analyses using the lattice methods \[17, 48\] \((B_K = 0.83 \pm 0.03)\) and the \( 1/N \) approach of \[19\] give results in the ball park of the \( 1/N \) result \( B_K = 0.70 \pm 0.10 \) obtained long time ago \[19\]. In particular the analysis of Bijnens and Prades \[50\] seems to have explained the difference between these values for \( B_K \) and the lower values obtained using the QCD Hadronic Duality approach \[51\] \((B_K = 0.39 \pm 0.10)\) or using SU(3) symmetry and PCAC \((B_K = 1/3)\) \[52\]. This is gratifying because such low values for \( B_K \) would require \( m_t > 250 \) GeV in order to explain the experimental value of \( \varepsilon \) \[53, 54\] \[55\].

There is a vast literature on the lattice calculations of \( F_B \). Based on a review by Chris Sachrajda \[42\], the recent extensive study by Duncan et al. \[55\] and the analyses in \[56\] we conclude: \( F_{B_d} = (180 \pm 40) \) MeV. This together with the earlier result of the European Collaboration for \( B_B \), gives \( F_{B_d}\sqrt{\frac{B_{B_d}}{B_{B_s}}} = 195 \pm 45 \) MeV. The reduction of the error in this important quantity is desirable. These results for \( F_B \) are compatible with the results obtained using QCD sum rules (eg. \[57\]). An interesting upper bound \( F_{B_d} < 195 \) MeV using QCD dispersion relations has also recently been obtained \[58\].

4.5 \( \Delta S = 1 \) Hamiltonian and \( \varepsilon'/\varepsilon \)

The effective Hamiltonian for \( \Delta S = 1 \) transitions is given as follows:

\[ \mathcal{H}_{\text{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} \left[ z_i(\mu) + \tau y_i(\mu) \right] Q_i \]  

(28)

where \( \tau = -(V_{ud}V_{us}^*)/(V_{us}V_{ub}^*) \). The coefficients of all ten operators are known including NLO QCD and QED effects in NDR and HV schemes due to the independent work of Munich and Rome groups \[21, 22, 23, 24, 25\]. A direct application of these results is the calculation of \( \text{Re}(\varepsilon'/\varepsilon) \) which measures the ratio of direct to indirect CP violation in \( K \to \pi\pi \) decays. In the standard model \( \varepsilon'/\varepsilon \) is governed by QCD penguins and electroweak (EW) penguins \[59\]. With increasing \( m_t \) the EW-penguins become increasingly important \[44, 48\] and entering \( \varepsilon'/\varepsilon \) with the opposite sign to QCD-penguins suppress this ratio for large \( m_t \). For \( m_t \approx 200 \) GeV the ratio can even be zero \[44\]. This strong cancellations between these two contributions was one of the prime motivations for the NLO calculations performed in Munich and Rome. Although these calculations can be regarded as an important step towards a reliable theoretical prediction for \( \varepsilon'/\varepsilon \) the situation is clearly not satisfactory at present. Indeed \( \varepsilon'/\varepsilon \) is plagued with uncertainties related to non-perturbative B-factors which multiply \( m_t \) dependent functions in a formula like \[21\]. Several of these B-factors can be extracted from leading CP-conserving \( K \to \pi\pi \) decays \[23\]. Two important B-factors \((B_6 = \text{the dominant QCD penguin (Q6)} \) and \( B_8 = \text{the dominant electroweak penguin (Q8)} \) cannot be determined this way and one has to use lattice or 1/N methods to predict \( \text{Re}(\varepsilon'/\varepsilon) \).

An analytic formula for \( \text{Re}(\varepsilon'/\varepsilon) \) as a function of \( m_t, \Lambda_{\overline{\text{MS}}}^4, B_6, B_8, m_s \) and \( V_{CKM} \) can be found in \[42\]. A very simplified version of this formula is given as follows

\[ \text{Re}(\varepsilon'/\varepsilon) = 12 \cdot 10^{-4} \left[ \frac{\eta \lambda^4 A^2}{1.7 \cdot 10^{-4}} \right] \left[ \frac{150 \text{ MeV}}{m_s(m_c)} \right]^2 \left[ \frac{\Lambda_{\overline{\text{MS}}}^4}{300 \text{ MeV}} \right]^{0.8} \left[ B_6 - Z(x_t)B_8 \right] \]  

(29)

where \( Z(x_t) = 0.175 \cdot 0.93 \). Note the strong dependence on \( \Lambda_{\overline{\text{MS}}}^4 \) pointed out in \[23\]. For \( m_t = 170 \pm 13 \) GeV, \( m_s(m_c) = 150 \pm 20 \) MeV \[62\] and using \( \varepsilon_K \)-analysis to determine \( \eta \) one
finds using the formulae in \cite{23,21} roughly

\[-1 \cdot 10^{-4} \leq Re(\varepsilon') \leq 15 \cdot 10^{-4}\]  \hfill (30)

if \(B_s = 1.0 \pm 0.2\) and \(B_d = 1.0 \pm 0.2\) are used. Such values are found in the \(1/N\) approach \cite{44} and using lattice methods: \cite{65} and \cite{65,66} for \(B_s\) and \(B_d\) respectively. A very recent analysis of the Rome group \cite{63} gives a smaller range, \(Re(\varepsilon'/\varepsilon) = (3.1 \pm 2.5) \cdot 10^{-4}\), which is however compatible with \cite{39}. Similar results are found with hadronic matrix elements calculated in the chiral quark model \cite{67}. However \(\varepsilon'/\varepsilon\) obtained in \cite{68} is substantially larger and about \(2 \cdot 10^{-3}\).

The experimental situation on \(Re(\varepsilon'/\varepsilon)\) is unclear at present. While the result of NA31 collaboration at CERN with \(Re(\varepsilon'/\varepsilon) = (23 \pm 7) \cdot 10^{-4}\) \cite{41} clearly indicates direct CP violation, the value of E731 at Fermilab, \(Re(\varepsilon'/\varepsilon) = (7.4 \pm 5.9) \cdot 10^{-4}\) \cite{70} is compatible with superweak theories \cite{72} in which \(\varepsilon'/\varepsilon = 0\). The E731 result is in the ballpark of the theoretical estimates. The NA31 value appears a bit high compared to the range given in \cite{39}.

Hopefully, in about five years the experimental situation concerning \(\varepsilon'/\varepsilon\) will be clarified through the improved measurements by the two collaborations at the \(10^{-3}\) level and by experiments at the \(\Phi\) factory in Frascati. One should also hope that the theoretical situation of \(\varepsilon'/\varepsilon\) will improve by then to confront the new data.

### 4.6 \(\Delta B = 1\) Effective Hamiltonian

The effective hamiltonian for \(\Delta B = 1\) transitions involving operators \(Q_1,..Q_{10}\) (with corresponding changes of flavours) is also known including NLO corrections \cite{23}. It has been used in the study of CP asymmetries in B-decays \cite{72}.

#### 4.7 \(K \to \pi^0 e^+ e^-\)

The effective Hamiltonian for \(K \to \pi^0 e^+ e^-\) is given as follows:

\[
H_{eff} (K \to \pi^0 e^+ e^-) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left[ \sum_{i=1}^{6,9} \left[ z_i(\mu) + \tau y_i(\mu) \right] Q_i + \tau y_{10A} (M_W) Q_{10A} \right] \tag{31}
\]

where \(Q_{9V}\) and \(Q_{10A}\) are given by \cite{19} with \(b\) replaced by \(d\).

Whereas in \(K \to \pi\pi\) decays the CP violating contribution is a tiny part of the full amplitude and the direct CP violation is expected to be at least by three orders of magnitude smaller than the indirect CP violation, the corresponding hierarchies are very different for the rare decay \(K_L \to \pi^0 e^+ e^-\). At lowest order in electroweak interactions this decay takes place only if CP symmetry is violated \cite{72}. Moreover, the direct CP violating contribution is predicted to be larger than the indirect one. The CP conserving contribution to the amplitude comes from a two photon exchange. The studies in \cite{73,74} indicate that it is smaller than the direct CP violating contribution.

The size of the indirect CP violating contribution will be known once the CP conserving decay \(K_S \to \pi^0 e^+ e^-\) has been measured \cite{74}. On the other hand the direct CP violating contribution can be fully calculated as a function of \(m_t\), CKM parameters and the QCD coupling constant \(\alpha_s\). There are practically no theoretical uncertainties related to hadronic matrix elements in this part, because the relevant matrix elements of the operators \(Q_{9V}\) and \(Q_{10A}\) can be extracted from the well-measured decay \(K^+ \to \pi^0 e^+ \nu\). The NLO QCD corrections to the direct CP violating part have been calculated in \cite{40} reducing certain ambiguities present in leading order analyses \cite{74} and enhancing somewhat the theoretical prediction. For \(m_t = 170 \pm 10\) GeV one finds \cite{30}

\[
Br(K_L \to \pi^0 e^+ e^-)_{\text{dir}} = (5. \pm 2.) \cdot 10^{-12} \tag{32}
\]
where the error comes dominantly from the uncertainties in the CKM parameters. This
should be compared with the present estimates of the other two contributions: $Br(K_L \to π^0 e^+ e^-)_{\text{indir}} \leq 1.6 \cdot 10^{-12}$ and $Br(K_L \to π^0 e^+ e^-)_{\text{cons}} \approx (0.3 - 1.8) \cdot 10^{-12}$ for the indirect
CP violating and the CP conserving contributions respectively [73]. Thus direct CP violation
is expected to dominate this decay. The present experimental bounds

$$Br(K_L \to π^0 e^+ e^-) \leq \begin{cases} 
4.3 \cdot 10^{-9} \\
5.5 \cdot 10^{-9} 
\end{cases} [78] [79] \tag{33}$$

are still by three orders of magnitude away from the theoretical expectations in the Standard
Model. Yet the prospects of getting the required sensitivity of order $10^{-11} - 10^{-12}$ in five years
are encouraging [50].

4.8 $B \to X_s γ$

The effective Hamiltonian for $B \to X_s γ$ at scales $\mu = O(m_b)$ is given by

$$\mathcal{H}_{\text{eff}}(b \to sγ) = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} \left[ \sum_{i=1}^{6} C_i(\mu)Q_i + C_{7γ}(\mu)Q_{7γ} + C_{8G}(\mu)Q_{8G} \right] \tag{34}$$

The perturbative QCD effects are very important in this decay. They are known [31] [32] to enhance $B \to X_s γ$ in the SM by 2–3 times, depending on the top quark mass. Since the first
analyses in [31] [32], a lot of progress has been made in calculating the QCD effects beginning
with the work in [33] [34].

A peculiar feature of the renormalization group analysis in $B \to X_s γ$ is that the mixing un-
der infinite renormalization between the set $(Q_1...Q_6)$ and the operators $(Q_{7γ}, Q_{8G})$ vanishes
at the one-loop level. Consequently in order to calculate the coefficients $C_{7γ}(\mu)$ and $C_{8G}(\mu)$
in the leading logarithmic approximation, two-loop calculations of $O(eg_s^2)$ and $O(g_s^3)$ are nec-

essary. The corresponding NLO analysis requires the evaluation of the mixing in question at the three-loop level.

At present, the coefficients $C_{7γ}$ and $C_{8G}$ are only known in the leading logarithmic approx-

imation. However the peculiar feature of this decay mentioned above caused that the first
fully correct calculation of the leading anomalous dimension matrix has been obtained only in 1993 [35] [36]. It has been confirmed subsequently in [37] [38] [39]. The NLO corrections are
only partially known. The two-loop mixing involving the operators $Q_1,...,Q_6$ is the same as
in section 4.5. The two-loop mixing in the sector $(Q_{7γ}, Q_{8G})$ has been calculated last year [24]. The three loop mixing between the set $(Q_1...Q_6)$ and the operators $(Q_{7γ}, Q_{8G})$ has not
be done. The $O(α_s)$ corrections to $C_{7γ}(M_W)$ and $C_{8G}(M_W)$ have been considered in [40].

Gluon corrections to the matrix elements of magnetic penguin operators have been calculated in [41] [42].

The leading logarithmic calculations of $Br(B \to X_s γ)$ [33] [36] [37] [38] [40] [41] [42] are based on the spectator model corrected for short-distance QCD effects discussed above. As we
have stressed previously support for this approximation comes from the $1/m_b$ expansions. A critical analysis of theoretical and experimental uncertainties present in the LO prediction for
$Br(B \to X_s γ)$ has been made in [42] giving

$$Br(B \to X_s γ)_{TH} = (2.8 \pm 0.8) \times 10^{-4}. \tag{35}$$

where the error is dominated by the uncertainty in choice of the renormalization scale $m_b/2 < μ < 2m_b$ as first stressed by Ali and Greub [41] and confirmed in [42]. Since $B \to X_s γ$ is
dominated by QCD effects, it is not surprising that this scale-uncertainty in the leading order
is particularly large.
The $B \to X_s \gamma$ decay has already been measured. In 1993 CLEO reported \cite{33} $\text{Br}(B \to K^* \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. In 1994 first measurement of the inclusive rate has been presented by CLEO \cite{34}:
\[ \text{Br}(B \to X_s \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \]
where the first error is statistical and the second is systematic. This result agrees with \cite{33} very well although the theoretical and experimental errors should be decreased in the future in order to reach a definite conclusion and to see whether some contributions beyond the standard model are required. In any case the agreement of the theory with data is consistent with the large QCD enhancement of $B \to X_s \gamma$. Without this enhancement the theoretical prediction would be at least by a factor of 2 below the data. The partial inclusion of NLO corrections done in \cite{33} lowers the theoretical branching ratio down to $\text{Br}(B \to X_s \gamma) = (1.9 \pm 0.2 \pm 0.5) \times 10^{-4}$.

We have to wait however for the final complete NLO calculation which should considerably reduce theoretical uncertainties in the leading order as formally demonstrated in \cite{32}.

### 4.9 $B \to X_s e^+e^-$ Beyond Leading Logarithms

The effective hamiltonian for $B \to X_s e^+e^-$ at scales $\mu = O(m_b)$ is given by
\[ \mathcal{H}_{\text{eff}}(b \to s e^+e^-) = \mathcal{H}_{\text{eff}}(b \to s \gamma) = \frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} [C_{9V}(\mu)Q_{9V} + C_{10A}(M_W)Q_{10A}] \] (37)
where $\mathcal{H}_{\text{eff}}(b \to s \gamma)$ is given in \cite{14}. In addition to the operators relevant for $B \to X_s \gamma$, there are two new operators $Q_{9V}$ and $Q_{10A}$ which appeared already in the decay $K_L \to \pi^0 e^+e^-$ except for an appropriate change of quark flavours and the fact that now $\mu = O(m_b)$ instead of $\mu = O(1 \text{ GeV})$ should be considered. There is a large literature on this decay. In particular Hou et al \cite{96} stressed the strong dependence of $B \to X_s e^+e^-$ on $m_t$. Further references to phenomenology can be found in \cite{38}.

The QCD corrections to this decay have been calculated over the last years with increasing precision by several groups \cite{77, 78, 79, 77} culminating in two complete next-to-leading QCD calculations \cite{14, 38} which agree with each other. An extensive numerical analysis of the differential decay rate including NLO corrections has been presented in \cite{38}. The NLO corrections enhance the leading order results by roughly 15%. The differential decay rate normalized to $\Gamma(B \to X_s e\nu \bar{\nu})$, varies for $0.1 < (p_{e+} + p_{e-})/m_b^2 < 0.8$ between $1 \cdot 10^{-4}$ and $1 \cdot 10^{-5}$ when $m_t = 170 \text{ GeV}$ and $\Lambda_{\text{MS}} = 225 \text{ MeV}$ are chosen. Similar result has been obtained by Misiak \cite{73}. The $1/m_b^2$ corrections calculated in \cite{100} enhance these results by roughly 10%.

### 4.10 $K_L \to \pi^0 \nu \bar{\nu}$, $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \mu \bar{\nu}$, $B \to \mu \bar{\nu}$ and $B \to X_s \nu \bar{\nu}$

$K_L \to \pi^0 \nu \bar{\nu}$, $K^+ \to \pi^+ \nu \bar{\nu}$, $B \to \mu \bar{\nu}$ and $B \to X_s \nu \bar{\nu}$ are the theoretically cleanest decays in the field of rare decays. $K_L \to \pi^0 \nu \bar{\nu}$, $B \to \mu \bar{\nu}$ and $B \to X_s \nu \bar{\nu}$ are dominated by short distance loop diagrams involving the top quark. $K^+ \to \pi^+ \nu \bar{\nu}$ receives additional sizable contributions from internal charm exchanges. The decay $K_L \to \mu \bar{\nu}$ receives substantial long distance contributions and consequently suffers from large theoretical uncertainties. This is very unfortunate because with the existing data this decay could offer a good determination of the parameter $\phi$ in the CKM matrix. The most accurate is the measurement from Brookhaven \cite{101}: $\text{Br}(K_L \to \mu \bar{\nu}) = (6.86 \pm 0.37) \cdot 10^{-9}$, which is somewhat lower than the KEK-137 result: $(7.9 \pm 0.6 \pm 0.3) \cdot 10^{-9}$ \cite{102}. For the short distance contribution I find using the formulae of \cite{24}: $\text{Br}(K_L \to \mu \bar{\nu})_{SD} = (1.5 \pm 0.8) \cdot 10^{-9}$. Details on this decay can be found in \cite{101, 34}. More promising from theoretical point of view is the parity-violating asymmetry in $K^+ \to \pi^+ \mu^+ \mu^-$.
The NLO QCD corrections to all these decays have been calculated in a series of papers by Buchalla and myself [32, 33, 34, 35]. These calculations considerably reduced the theoretical uncertainties due to the choice of the renormalization scales present in the leading order expressions [104]. Since the relevant hadronic matrix elements of the weak currents entering \( K \to \pi \nu \bar{\nu} \) can be measured in the leading decay \( K^+ \to \pi^0 e^+ \nu \), the resulting theoretical expressions for \( Br( K_L \to \pi^0 e^+ \nu) \) and \( Br( K^+ \to \pi^+ \nu \bar{\nu}) \) are only functions of the CKM parameters, the QCD scale \( \Lambda_{\overline{MS}} \) and the quark masses \( m_t \) and \( m_c \). The long distance contributions to \( K \to \pi \nu \bar{\nu} \) have been considered in [107] and found to be very small. Similar comments apply to \( B \to \mu \bar{\mu} \) and \( B \to X_s \nu \bar{\nu} \) except that \( B \to \mu \bar{\mu} \) depends on the B-meson decay constant \( F_B \) which brings in the main theoretical uncertainty.

The explicit expressions for \( Br( K^+ \to \pi^+ \nu \bar{\nu}) \) and \( Br( K_L \to \pi^0 \nu \bar{\nu}) \) are given as follows

\[
Br( K^+ \to \pi^+ \nu \bar{\nu}) = 4.57 \cdot 10^{-11} A^4 X^2(x_t) \cdot [\eta^2 + (\varrho_0 - \varrho)^2]
\]

\[
Br( K_L \to \pi^0 \nu \bar{\nu}) = 1.91 \cdot 10^{-10} \eta^2 A^4 X^2(x_t)
\]

Here

\[
\varrho_0 = 1 + \frac{P_0(K^+)}{A^2 X(x_t)} \quad X(x_t) = 0.65 \cdot x_t^{0.575}
\]

where the NLO correction calculated in \([33]\) is included in \( X(x_t) \) if \( m_t \equiv \bar{m}_t(m_t) \). Next \( P_0(K^+) = 0.40 \pm 0.09 \) \([34, 117]\) is a function of \( m_c \) and \( \Lambda_{\overline{MS}} \) and includes the residual uncertainty due to the renormalization scale \( \mu \). The absence of \( P_0 \) in \([39]\) makes \( K_L \to \pi^0 \nu \bar{\nu} \) theoretically even cleaner than \( K^+ \to \pi^+ \nu \bar{\nu} \). We should remark that \([38]\) is an approximation. A more accurate formula is given in \([14]\).

Similarly for \( B_s \to \mu \bar{\mu} \) one has \([33]\)

\[
Br( B_s \to \mu \bar{\mu}) = 4.1 \cdot 10^{-9} \left[ \frac{F_B}{230 \text{ MeV}} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{3.12} \left[ \frac{|V_{ts}|}{0.040} \right]^2 \left[ \tau_{B_s} / 1.6 \text{ ps} \right]
\]

The impact of NLO calculations is best illustrated by giving the scale uncertainties in the leading order and after the inclusion of the next-to-leading corrections:

\[
Br( K^+ \to \pi^+ \nu \bar{\nu}) = (1.00 \pm 0.20) \cdot 10^{-10} \Rightarrow (1.00 \pm 0.05) \cdot 10^{-10}
\]

\[
Br( K_L \to \pi^0 \nu \bar{\nu}) = (3.00 \pm 0.30) \cdot 10^{-11} \Rightarrow (3.00 \pm 0.04) \cdot 10^{-11}
\]

\[
Br( B_s \to \mu \bar{\mu}) = (4.10 \pm 0.50) \cdot 10^{-9} \Rightarrow (4.10 \pm 0.05) \cdot 10^{-9}
\]

The reduction of the scale uncertainties is truly impressive.

The present experimental bound on \( Br(K^+ \to \pi^+ \nu \bar{\nu}) \) is \( 5.2 \cdot 10^{-9} \) \([106]\) (a preliminary result from this group is \( 3.0 \cdot 10^{-9} \)). An improvement by one order of magnitude is expected at AGS in Brookhaven for the coming years. The present upper bound on \( Br(K_L \to \pi^0 \nu \bar{\nu}) \) from Fermilab experiment E799I is \( 5.8 \cdot 10^{-5} \) \([107]\). FNAL-E799II expects to reach the accuracy \( O(10^{-8}) \) and the future experiments at FNAL and KEK will hopefully be able to reach the standard model expectations. The latter are given for both decays at present as follows:

\[
Br( K^+ \to \pi^+ \nu \bar{\nu}) = (1.1 \pm 0.4) \cdot 10^{-10}, \quad Br( K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 2.0) \cdot 10^{-11}
\]

5 Finalists

5.1 General Remarks

From tree level K decays sensitive to \( V_{us} \) and tree level B decays sensitive to \( V_{cb} \) and \( V_{ub} \) we have:

\[
\lambda = 0.2205 \pm 0.0018 \quad | V_{cb} | = 0.041 \pm 0.003 \quad \Rightarrow \quad A = 0.85 \pm 0.06
\]
\[
\frac{|V_{ub}|}{V_{cb}} = 0.08 \pm 0.03 \Rightarrow \sqrt{\vartheta^2 + \eta^2} = 0.36 \pm 0.14
\]  

The main recent progress here, is the improved determination of \(|V_{ub}|\) due to experimental \cite{108} and theoretical efforts \cite{113, 112}. Although some further reduction of the errors could be expected in the future, it is difficult to imagine at present that in tree level B-decays a better accuracy than \(\Delta |V_{ub}| = \pm 2 \cdot 10^{-3}\) and \(\Delta |V_{ub}/V_{cb}| = \pm 0.01\) \((\Delta R_b = \pm 0.04)\) could be achieved unless some dramatic improvements in the theory and experiment will take place. It is therefore of interest to look simultaneously at other decays in order to improve the determination of these parameters. For instance as stressed in \cite{109, 110}, it is in principle possible to determine all CKM parameters without any hadronic uncertainties although this will require heroic experimental efforts. Indeed using the loop induced decays or transitions which are fully governed by short distance physics simultaneously with CP asymmetries in B-decays clean and precise determinations of \(|V_{cb}|\), \(|V_{ub}/V_{cb}|\), \(|V_{td}|\), \(\vartheta\) and \(\eta\) can be achieved.

In this respect the most promising from the theoretical point of view are the following four: i) CP-Asymmetries in \(B^o\)-Decays, ii) \(K_L \to \pi^o \nu \bar{\nu}\), iii) \(K^+ \to \pi^+ \nu \bar{\nu}\) and iv) \((B_d^o - \bar{B}_d^o)/(B_s^o - \bar{B}_s^o)\).

Let us summarize their main virtues one-by-one.

\subsection*{5.2 CP-Asymmetries in \(B^o\)-Decays}

The CP-asymmetry in the decay \(B_d^o \to \psi K_S\) allows in the standard model a direct measurement of the angle \(\beta\) in the unitarity triangle without any theoretical uncertainties. This has been first pointed out by Bigi and Sanda \cite{111}, analyzed in detail already in \cite{9} and during the past years discussed by many authors \cite{112}. Similarly the decay \(B_d^o \to \pi^+ \pi^-\) gives the angle \(\alpha\), although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions \cite{113}. The determination of the angle \(\gamma\) from CP asymmetries in neutral B-decays is more difficult but not impossible \cite{114}. Also charged B decays could be useful in this respect \cite{115}.

Since in the usual unitarity triangle one side is known, it suffices to measure two angles to determine the triangle completely. This means for instance that the measurements of \(\sin(2\beta)\) and \(\sin(2\alpha)\) through the asymmetries \(A_{CP}(\psi K_S)\) and \(A_{CP}(\pi^+ \pi^-)\) can determine the parameters \(\vartheta\) and \(\eta\). The main virtues of this determination are as follows:

- No hadronic or \(\Lambda_{MS}\) uncertainties.
- No dependence on \(m_t\) and \(V_{cb}\) (or \(A\)).

As various analyses \cite{4, 116, 63} of the unitarity triangle show, \(\sin(2\beta)\) is expected to be large: \(\sin(2\beta) \approx 0.6 \pm 0.2\). The predictions for \(\sin(2\gamma)\) and \(\sin(2\alpha)\) are very uncertain on the other hand.

\subsection*{5.3 \(K_L \to \pi^o \nu \bar{\nu}\)}

As we have discussed above \(K_L \to \pi^o \nu \bar{\nu}\) is the theoretically cleanest decay in the field of rare K-decays. Moreover it proceeds almost entirely through direct CP violation \cite{118}. The main features of this decay are:

- No hadronic uncertainties
- \(\Lambda_{MS}\) and renormalization scale uncertainties at most \(\pm 1\%)\.
- Strong dependence on \(m_t\) and \(V_{cb}\) (or \(A\)).
5.4 $K^+ \to \pi^+ \nu \bar{\nu}$

$K^+ \to \pi^+ \nu \bar{\nu}$ is CP conserving and receives contributions from both internal top and charm exchanges. $K^+ \to \pi^+ \nu \bar{\nu}$ is the second best decay in the field of rare decays. The main features of this decay are:

- Hadronic uncertainties below 1% [103].
- $\Lambda_{\text{MS}}$, $m_c$ and renormalization scales uncertainties at most $\pm(5-10)$% [34].
- Strong dependence on $m_t$ and $V_{cb}$ (or $A$).

5.5 $(B_d^0 - \bar{B}_d^0)/(B_s^0 - \bar{B}_s^0)$

Measurement of $B_d^0 - \bar{B}_d^0$ mixing parametrized by $x_d$ together with $B_s^0 - \bar{B}_s^0$ mixing parametrized by $x_s$ allows to determine $R_t$:

$$R_t = \frac{1}{\sqrt{R_{ds}}} \sqrt{\frac{x_d}{x_s}} \quad R_{ds} = \frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{m_{B_d}}{m_{B_s}} \left[ \frac{F_{B_d} \sqrt{B_{B_d}}}{F_{B_s} \sqrt{B_{B_s}}} \right]^2$$

(48)

where $R_{ds}$ summarizes SU(3)–flavour breaking effects. Note that $m_t$ and $V_{cb}$ dependences have been eliminated this way and $R_{ds}$ contains much smaller theoretical uncertainties than the hadronic matrix elements in $x_d$ and $x_s$ separately. Provided $x_d/x_s$ has been accurately measured a determination of $R_t$ within $\pm 10\%$ should be possible. Indeed the most recent lattice result [5] gives $F_{B_d}/F_{B_s} = 1.22 \pm 0.04$. It would be useful to know $B_{B_s}/B_{B_d}$ with a similar precision. For $B_{B_s} = B_{B_d}$ I find $R_{ds} = 0.62 \pm 0.07$. Consequently rescaling the results of [3], obtained for $R_{ds} = 1$, the range $12 < x_s < 39$ follows. Such a large mixing will not be easy to measure. The main features of $x_d/x_s$ are:

- No $\Lambda_{\text{MS}}$, $m_t$ and $V_{cb}$ dependence.
- Hadronic uncertainty in SU(3)–flavour breaking effects of roughly $\pm 10\%$.

5.6 $\sin(2\beta)$ from $K \to \pi \nu \bar{\nu}$

It has been pointed out in [119] that measurements of $Br(K^+ \to \pi^+ \nu \bar{\nu})$ and $Br(K_L \to \pi^0 \nu \bar{\nu})$ could determine the unitarity triangle completely provided $m_t$ and $V_{cb}$ are known. In view of the strong dependence of these branching ratios on $m_t$ and $V_{cb}$ this determination is not precise however [117]. On the other hand it has been noticed [117] that the $m_t$ and $V_{cb}$ dependences drop out in the evaluation of $\sin(2\beta)$. Consequently $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ offer a clean determination of $\sin(2\beta)$ which can be confronted with the one possible in $B^0 \to \psi K_S$ discussed above. Any difference in these two determinations would signal new physics. Choosing $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ and $Br(K_L \to \pi^0 \nu \bar{\nu}) = (2.5 \pm 0.25) \cdot 10^{-11}$, one finds [117]

$$\sin(2\beta) = 0.60 \pm 0.06 \pm 0.03 \pm 0.02$$

(49)

where the first error is "experimental", the second represents the uncertainty in $m_c$ and $\Lambda_{\text{MS}}$ and the last is due to the residual renormalization scale uncertainties. This determination of $\sin(2\beta)$ is competitive with the one expected at the B-factories at the beginning of the next decade.
Using the first two finalists and $\lambda = 0.2205 \pm 0.0018$ [20] it is possible to determine all the parameters of the CKM matrix without any hadronic uncertainties [10]. As illustrative examples we consider in table 2 three scenarios. The first four rows give the assumed input parameters and their experimental errors which are expected in the next decade. The remaining rows give the results for selected parameters. The experimental errors on $Br(K_L \to \pi^0 \nu \bar{\nu})$ to be achieved in the next 15 years are most probably unrealistic, but I show this exercise anyway in order to motivate this very challenging enterprise. Table 2 shows very clearly the potential of CP asymmetries in B-decays and of $K_L \to \pi^0 \nu \bar{\nu}$ in the determination of CKM parameters. It should be stressed that this high accuracy is not only achieved because of our assumptions about future experimental errors in the scenarios considered, but also because $\sin(2\alpha)$ is a very sensitive function of $\rho$ and $\eta$ [4], $Br(K_L \to \pi^0 \nu \bar{\nu})$ depends strongly on $|V_{cb}|$ and most importantly because of the clean character of the quantities considered.

This results should be compared with the expectations from a "standard" analysis of the unitarity triangle which is based on $\varepsilon_K$, $x_d$, $|V_{cb}|$, and $|V_{ub}/V_{cb}|$ with the last two extracted from tree level decays. As a typical analysis [4] shows, even with optimistic assumptions about the theoretical and experimental errors it will be difficult to achieve the accuracy better than $\Delta \rho = \pm 0.15$ and $\Delta \eta = \pm 0.05$ this way.

In the last two rows of table 2 we show the results for $|V_{cb}|$ and $|V_{td}|$ obtained using $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ for the scenario I and $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.05) \cdot 10^{-10}$ for scenarios II and III in place of $Br(K_L \to \pi^0 \nu \bar{\nu})$ with all other input parameters unchanged. We observe that due to the uncertainties present in the charm contribution to $K^+ \to \pi^+ \nu \bar{\nu}$, which was absent in $K_L \to \pi^0 \nu \bar{\nu}$, the determinations of $|V_{cb}|$ and $|V_{td}|$ are less accurate, but still very interesting. In particular the error on $|V_{td}|$ is much smaller than the one given in [27].

An alternative strategy is to use the measured value of $R_t$ instead of $\sin(2\alpha)$. The result of this exercise is shown in table 3. Again the last two rows give the results when $K_L \to \pi^0 \nu \bar{\nu}$ is replaced by $K^+ \to \pi^+ \nu \bar{\nu}$.

The consistency of the determinations presented in tables 2 and 3 will offer an important test of the standard model. Of particular interest will also be the comparison of $|V_{cb}|$ determined as suggested here with the value of this CKM element extracted from tree level semi-leptonic B-decays. Since in contrast to $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$, the tree-level decays are to an excellent approximation insensitive to any new physics contributions from

| Parameter | Central | I | II | III |
|-----------|---------|---|----|----|
| $\sin(2\alpha)$ | 0.40 | ±0.08 | ±0.04 | ±0.02 |
| $\sin(2\beta)$ | 0.70 | ±0.06 | ±0.02 | ±0.01 |
| $m_t$ | 170 | ±5 | ±3 | ±3 |
| $10^{13} Br(K_L)$ | 3 | ±0.30 | ±0.15 | ±0.15 |
| $\rho$ | 0.072 | ±0.040 | ±0.016 | ±0.008 |
| $\eta$ | 0.389 | ±0.044 | ±0.016 | ±0.008 |
| $|V_{ub}/V_{cb}|$ | 0.087 | ±0.010 | ±0.003 | ±0.002 |
| $|V_{cb}|/10^{-3}$ | 39.2 | ±3.9 | ±1.7 | ±1.3 |
| $|V_{td}|/10^{-3}$ | 8.7 | ±0.9 | ±0.4 | ±0.3 |
| $|V_{cb}|/10^{-3}$ | 41.2 | ±4.3 | ±3.0 | ±2.8 |
| $|V_{td}|/10^{-3}$ | 9.1 | ±0.9 | ±0.6 | ±0.6 |

Tab. 2: Determinations of various parameters in scenarios I-III

5.7 Precise Determinations of the CKM Matrix
very high energy scales, the comparison of these two determinations of $|V_{cb}|$ would be a good test of the standard model and of a possible physics beyond it.

6 Final Remarks

In this compact review we have concentrated on rare decays and CP violation in the standard model. The structure of rare decays and of CP violation in extensions of the standard model may deviate from this picture. Consequently the situation in this field could turn out to be very different from the one presented here. However in order to distinguish the standard model predictions from the predictions of its extensions it is essential that the theoretical calculations reach acceptable precision. In this context we have emphasized the importance of the QCD calculations in rare and CP violating decays. During the recent years a considerable progress has been made in this field through the computation of NLO contributions to a large class of decays. This effort reduced considerably the theoretical uncertainties in the relevant formulae and thereby improved the determination of the CKM parameters to be achieved in future experiments. At the same time it should be stressed that whereas the theoretical status of QCD calculations for rare semileptonic decays like $K \to \pi \nu \bar{\nu}$, $B \to \mu \bar{\mu}$, $B \to X_s e^+ e^-$ is fully satisfactory and the status of $B \to X_s \gamma$ should improve in the coming years, a lot remains to be done in a large class of non-leptonic decays or transitions where non-perturbative uncertainties remain sizable.

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\begin{tabular}{|c|c|c|c|c|}
\hline
 & Central & I & II & III \\
\hline
$R_t$ & 1.00 & ±0.10 & ±0.05 & ±0.03 \\
$\sin(2\beta)$ & 0.70 & ±0.06 & ±0.02 & ±0.01 \\
$m_t$ & 170 & ±5 & ±3 & ±3 \\
$10^{41}Br(K_L)$ & 3 & ±0.30 & ±0.15 & ±0.15 \\
\hline
$\rho$ & 0.076 & ±0.111 & ±0.053 & ±0.031 \\
$\eta$ & 0.388 & ±0.079 & ±0.033 & ±0.019 \\
$|V_{ub}/V_{cb}|$ & 0.087 & ±0.014 & ±0.005 & ±0.003 \\
$V_{cb}/10^{-3}$ & 39.3 & ±5.7 & ±2.6 & ±1.8 \\
$V_{td}/10^{-3}$ & 8.7 & ±1.2 & ±0.6 & ±0.4 \\
$|V_{cb}|/10^{-3}$ & 41.3 & ±5.8 & ±3.7 & ±3.3 \\
$|V_{td}|/10^{-3}$ & 9.1 & ±1.3 & ±0.8 & ±0.7 \\
\hline
\end{tabular}

Tab. 3: As in table 2 but with $\sin(2\alpha)$ replaced by $R_t$.
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