1. INTRODUCTION

The first indication of charge-exchange (CE) reactions was found in $\pi^- p$ and in $pp$ scattering from cosmic rays (see [1]) detected with proportional chambers. CE in $pp$ interactions is defined as

$$ p + p \rightarrow n + \pi^- + p + N_\pi, $$

where $n$ is a neutron and $N_\pi$ is the number of created pions. The ongoing “Thermalization” project [2] detects high multiplicity events in $pp$ interactions with $50 \text{ GeV}/c$ beam at U-70 (IHEP, Protvino). CE reactions as well can be investigated at this facility.

From the theoretical point of view, CE has been described in different approaches. Cumulative production of pions in high energy $pp$ interactions has been described, for example, in frame of statistical models [3] or, more recently of the gluon dominance model (GDM). In GDM, the multiplicity distributions in hadron interactions are constructed by the convolution of two steps. At first, in the parton stage, the kinetic energy of the colliding particles is dissipated into thermal energy and some of the gluons become free. This step can be described by a truncated Poisson distribution. In a second step, these gluons are transformed in hadrons. The hadronization stage is described phenomenologically through a binomial distribution. In this model, part of the gluons hadronizes and the ones which stay in the interaction region can be source of thermal photons [4].

In the fragmentation region, models based on effective Regge theory exist, but they are far from being predictive, see, for example [5], where effective reggeized gluon–particle vertices have been calculated. In leading logarithmic approximation BFKL violates unitarity. Different solutions have been proposed: within the eikonal approximation, or by introducing composite reggeized gluons. Effective Lagrangians for multi-Regge kinematics of gluons in intermediate state of $s$ and $u$ channels have also been written, where the Feynman diagrams are factorized in the product of integrals over longitudinal and transverse subspaces [6].

The one pion exchange model has been successfully applied at high energies for the processes which involve resonance production at low momentum transfer ($t \sim m_\pi^2$). In this condition, the one pion exchange graph gives the dominant (infinite) contribution. The angular dependence and the cross section can be reasonably reproduced, provided corrections to avoid violation of unitarity are taken into account. For a discussion on this class of models see [7].

In this work we estimate the CE contribution, using the formalism [8, 9]. The emission by the initial proton of a charged light meson-$\pi$ or $\rho$-meson in proton–proton(anti-proton) collisions transforms high energy protons (for example in a proton beam) into neutrons. Possible applications at existing and planned facilities are considered.
CE reactions may occur in antiproton beams, too. High energy, high intensity antiproton beams will be available in next future at PANDA [12], FAIR [13]. Hard π and ρ meson can be detected with high efficiency. Charged mesons will be deviated by the 2T magnetic field of the central spectrometer, and (anti)neutrons, which are produced at high rate, might be used as a secondary high energy beam. We are interested here in “initial state emission”, as a mechanism to produce a (anti)neutron beam with comparable energy of the (anti)proton beam.

In this work, we propose a description of CE reactions referring to the known QED process of emission of a hard real photons by electron (positron) beams at e+e− colliders. Such process enhances the cross section when the energy loss from one of the incident particles lowers the total energy up to the mass of a resonance. This is known as “return to resonance” mechanism. In the case of creation of a narrow resonance this mechanism appears through a radiative tail: it is the characteristic behavior of the cross section which gradually decreases for energies exceeding the resonance mass. This mechanism provides, indeed, an effective method for studying narrow resonances like J/ψ.

For the emission in a narrow cone along the directions of the initial (final) particles, the emission probability has a logarithmic enhancement, which increases with the energy of the “parent” charged particle. In frame of QED this mechanism is called as “quasi-real electron” mechanism (QRE) [8].

In this work we apply the QRE mechanism to the case of hadrons and, in particular, to the collinear emission of a light meson from a (anti)proton beam. We evaluate the cross section for this process for single as well as multi pion production, where pions can be neutral or charged. Our derivation concerns a kinematical region outside the resonance formation region.

The plan of the paper is as follows. In Section 1.1. we recall the formalism of QRE hard photon emission, and then (Section 1.2.) we extend it to hadronic reactions and give numerical estimations of the cross section for the simplest CE processes. Discussion and conclusions follow.

1.1. Quasi-Real Electron Kinematics with Hard Photon Emission

Let us consider the radiative process e(p1) + T(p2) → e(p1 – k) + γ(k) + X (the four momenta of the particles are written in parenthesis), T stay for any nuclear target (proton p, or nucleus A), and the final state X is undetected, Fig. 1.

The virtual electron after the hard (collinear) photon emission is almost on mass shell [8]. This property allows to express the matrix element of the radiative process e(p) + T → e(p – k) + γ(k) + X in terms of the matrix element of the non-radiative process e + T → e + X:

\[ M(p_1, p_2) = e \bar{\psi} (p_2) \gamma^\mu \frac{\hat{p}_1 - \hat{k} + m - 2p_1 k}{-2p_1 k} \epsilon(\mu) u(p_1). \]  

(1)

In the case when the denominator of the intermediate electron’s Green function is small \(|(p_1 - k)^2 - m^2| \ll 2p_1 p_2\) one can write \(\hat{p}_1 - k + m = \sum_i u_i(p_1 - k) \bar{u}_i(p_1 - k)\) and the matrix element has a factorized form.

The square of the matrix element, summed over the spin states of the photon is:

\[ \sum |M_i|^2 = e^2 \left[ \frac{E_p + E_{\gamma}}{E_p - \omega} \int (kp) \right] ^2 \sum |\bar{\psi} (p_2) u(p_1 - k)|^2, \]  

(2)

where \(\sum |\bar{\psi} (p_2) u(p_1 - k)|^2\) is the Born matrix element squared with shifted argument.

In the case of unpolarized particles, the cross section of the process e(p1) + T(p2) → e + γ + T may be written in factorized form:

\[ d\sigma_{\gamma}(s, x) = d\sigma_{\gamma}(\epsilon_\gamma) dW_{\gamma}(x), \quad x = 1 - x, \]
\[ dW_{\gamma}(x) = \frac{\alpha}{\pi x} \left[ 1 - x + \frac{1}{2} x^2 \right] \ln \frac{E_0^2}{m_e^2} (1 - x), \]  

(3)

where \(E_0 = E \cos \theta, \quad 0 < \theta_0 \ll 1, \quad \frac{E_0}{m_e} \gg 1, \) where \(E_0\) is the energy of the initial electron (center of mass frame implied \(\hat{p}_1 + \hat{p}_2 = 0\), and \(s = (p_1 + p_2)^2\).

It is assumed here that the initial electron transforms into an electron with energy fraction \(1 - x\) and a hard photon with energy fraction \(x\) which is emitted within the cone \(\theta < \theta_0\) along the direction of initial electron. Moreover it is implied that \(\exists s > s_{thr}\) where \(s_{thr}\) is the threshold energy of the process without photon emission. The logarithmic enhancement originates from the small values of the mass of the intermediate
electron, which is almost on mass shell. This justifies the name of Quasi Real Electron (QRE) method.

Below we consider a possible extension of the QRE method to the processes with “quasi real” (anti)nucleon intermediate state.

1.2. Application to Hadron Physics

Let us apply this formalism to the case of initial high energy proton (anti–proton) beams and the emission of a hard pion or vector meson in forward direction, collinear to the beam. We do not consider the emission of the pion from the final proton (final state emission), which corresponds to a different kinematical region for the intermediate particle.

For the case of emission of a positively charged $\rho^+$ or $\pi^+$ meson by the high energy (anti)proton, the final state consists in a high energy (anti)neutron, accompanied by a positively charged meson. The charged meson can be deflected by an external magnetic field, providing the possibility to select a high energy neutron beam. In case of emission of the neutral meson, it can be identified by measuring its decay channels.

Let us consider the reactions (Fig. 2):

$$p + T \rightarrow n + T + h^+,$$ (4)

$$\bar{p} + T \rightarrow \bar{n} + T + h^-,$$ (5)

where $h = \rho$ or $\pi$ and $T$ may be any target ($p$, $n$, nucleus...). In QED the interaction is assumed to occur through the exchange of a virtual photon. In the present case the particle can be a vector meson. However, the nature of the exchanged particle is irrelevant for the present considerations, where we focus on small angle charged pion emission and on the factorization properties of the cross section.

The matrix element for collinear $\pi(\rho)$ emission can be written as:

$$\mathcal{M}_{p+T \rightarrow n+h^+} = \mathcal{M}_{n+T \rightarrow h^+}(p_1, p_2)\mathcal{T}_{h^+}^p(p_1, p_1 - k),$$

$$\mathcal{M}_{\bar{p}+T \rightarrow \bar{n}+h^-} = \mathcal{M}_{\bar{n}+T \rightarrow h^-}(p_1, p_2)\mathcal{T}_{h^-}^\bar{p}(p_1, p_1 - k),$$ (6)

where

$$\mathcal{T}_{h^+}^p(p_1, p_1 - k) = \frac{g}{m^2_{h^+} - 2p_1k} \bar{u}_n(p_1 - k)\gamma_5u_\rho(p_1),$$

$$\mathcal{T}_{h^-}^\bar{p}(p_1, p_1 - k) = \frac{g}{m^2_{h^-} - 2p_1k} \bar{u}_n(p_1 - k)\gamma_5u_\rho(p_1),$$ (7)

are the matrix elements of the subprocesses: $p \rightarrow n + \pi^+$ and $p \rightarrow n + \rho^+$ (or $\bar{p} \rightarrow \bar{n} + \pi^-$ and $\bar{p} \rightarrow \bar{n} + \rho^-$). The matrix element of emission of a charged pion is expressed in factorized form of (6) of a term connected to the target and a ‘universal’ factor for the hard meson emission, which generalizes the QED result. Below, we will focus on this last term.

The relevant cross sections are:

$$d\sigma_{\rho^+T \rightarrow n+h^+}^p(s, x) = \sigma_{\rho^+T \rightarrow n+h^+}^{s-\bar{p}_n}(\bar{x}s)dw_{\rho^+}(x),$$

$$d\sigma_{\rho^-T \rightarrow \bar{n}+h^-}^{\bar{p}}(s, x) = \sigma_{\rho^-T \rightarrow \bar{n}+h^-}^p(\bar{x}s)dw_{\rho^-}(x),$$

$$d\sigma_{\rho_T \rightarrow \bar{n}+h^-}^{\rho}(s, x) = \sigma_{\rho_T \rightarrow \bar{n}+h^-}^\rho(\bar{x}s)dw_{\rho}(x).$$ (8)

The quantity $dw_{\rho}(x)$ can be inferred using the QED result:

$$\frac{dW_{\rho}(x)}{dx} = \frac{g^2}{4\pi^2\hbar} \left[1 - \frac{m^2_{\rho}}{x^2E^2}\left[(1 - x + \frac{1}{2}x^2)L - (1 - x)^2\right]\right],$$ (9)

$$1 > x = \frac{E_\rho}{\bar{E}} > \frac{m^2_{\rho}}{E^2}, \quad L = \ln\left(1 + \frac{E^2\theta^2}{M^2}\right),$$

$$\rho^+ = \rho^+, \rho^-, \rho^0,$$

where $M$, $m_{\rho}$, $E$, and $E_\rho$ are the masses and the energies of the initial proton and the emitted $\rho$-meson (laboratory reference frame implied).

For the probability of hard pion emission we have

$$\frac{dW_{\pi}}{dx} = \sum |\mathcal{T}_{\pi}(p_1, p_1 - k)|^2 \frac{g^2k}{16\alpha\pi^2},$$ (10)

with

$$\sum |\mathcal{T}_{\pi}(p_1, p_1 - k)|^2 = \frac{g^2}{[m^2_{\pi} - 2(p_1k)]^2} \times \mathrm{Tr}(\hat{p}_1\tilde{k} + M)\gamma_5\gamma_5^\rho = \frac{4(p_1k)^2}{[m^2_{\pi} - 2(p_1k)]^2},$$ (11)

$$(p_1k) = E\omega(1 - bc), \quad 1 - b^2 = \frac{m^2_{\pi} + M^2}{\omega^2E^2},$$

Fig. 2. Feynman diagrams for collinear hard charged pion (a) and (anti)nucleon meson emission in $\rho(p) + T$ collisions.
with \( c = \cos \theta_0 \). The angular integration in the region \( 1 - (\theta_0/2) < \theta < 1 \) leads to

\[
\frac{dW_h(x)}{dx} = \frac{g^2}{8\pi^2} \left[ 1 - \frac{m_n^2}{x^2E^2} \right] \left[ L + \ln \frac{1}{d(x)} + \frac{m_n^2}{xd(x)M^2} \right],
\]

\[ 1 > x = \frac{E_0}{E}, \quad d(x) = 1 + \frac{m_\pi^2}{M^2x^2}, \]

\[ \bar{x} = 1 - x, \quad \pi^0 = \pi^+ - \pi^-, \]

where \( g = g_{\text{ppp}} = g_{\text{ppp}} \approx 6 \) is the strong coupling constant. The quantities \( dW_h(x)/dx \) as functions of the energy fraction \( x = E_h/E \) for \( E = 15 \text{ GeV} \) and two different values of \( \theta_0 \) are drawn in Fig. 3. For \( E = 15 \text{ GeV} \) and two different values of \( \theta_0 \) are presented in Appendix.

The expressions of the integrated probabilities are:

\[
W_h = \int_{x^1}^{1} \frac{dW_h(x)}{dx} dx, \quad A^\rho = I_0(\rho^0) - I_1(\rho^0) + \frac{1}{2} I_2(\rho^0),
\]

\[
B^\rho = -I_0(\rho^0) + I_1(\rho^0), \quad A^\pi = \frac{1}{2} I_1(\pi^0), \quad B^\pi = I_1(\pi^0),
\]

where \( x^h = E_h/E \) with \( E_h = m^h \) — threshold value of the energy of the detected particle, \( h = \rho, \pi \). The analytic expressions of the functions \( I_i(\theta), i = 0, 1, 2 \) are presented in Appendix.

The renormalized probabilities \( W_{\pi, \rho}^*, P_{\pi, \rho} \) in Eq. (13) are illustrated in Fig. 4 as a function of the incident energy for two values of the hadron emission angle. Notations as in Fig. 3.

Note that the probability of emission of ‘soft’ neutral pions follows a Poisson distribution, which is not the case for the emission of charged pions. Fortunately, in our case, it is sufficient to consider the emission of charged pion at lowest order (the process of one charged pion emission) plus any number of real and virtual pions with total charge zero. In such configuration, this vertex has the form of the product of the Born probability of emission of a single pion times the Poisson-like factor:

\[
P_{\pi, \rho} = e^{-W_{\pi, \rho}},
\]

which takes into account virtual corrections.

The renormalized probabilities \( W_{\pi, \rho}^*, P_{\pi, \rho} \) are drawn in Fig. 4 as a function of the incident energy for two different \( \theta_0 \) angles.
Keeping in mind the possible processes of emission of \( n \) real soft neutral pions escaping the detection, the final result can be obtained using the replacement
\[
\sigma(s) \rightarrow \sigma(s) \times \mathcal{R}_\pi, \quad \mathcal{R}_\pi = P_\pi \sum_{k=0}^{n} \frac{W_{\pi k}}{k!}.
\]

The renormalization factor \( \mathcal{R}_\pi \) is illustrated in Fig. 5, for the probability of emission of 2 (solid line), 3 (dashed line), 4 (dotted line) pions.

The quantity \( P_\pi \) can be compared to the experimentally measurable phenomena [10]: the fraction of protons in the final state of proton–proton collisions is approximately one half, \( P_\pi \approx 0.5 \). The commonly accepted explanation is that charge exchange reactions are responsible for changing protons into neutrons.

Let us consider the antiproton–proton annihilation into two and three pions \( \bar{p} + p \rightarrow \pi^+\pi^- \) or \( \bar{p} + p \rightarrow \pi^+\pi^-\pi^0 \).

Concerning the production of two charged pions, accompanied by a final state \( X \), we can write:
\[
d\sigma^{\bar{p}p \rightarrow \pi^+\pi^-} = 2 \frac{dW_{\pi}(x)}{dx} \sigma^{\bar{p}p \rightarrow \pi^+\pi^-}(\bar{p}s) \times P_\rho, \tag{16}
\]
where the factor of two takes into account two kinematical situations, corresponding to the emission along each of the initial particles and \( P_\rho \) is the survival factor (14) which takes into account virtual radiative corrections. The characteristic peak at \( x = x_{\text{max}} \) has the same nature as for the QED process \( e^+ + e^- \rightarrow \mu^+ + \mu^- + \gamma \). As explained in [15], it is a threshold effect, correspond-

![Fig. 5. Renormalization factor \( R_{\pi} \), Eq. (15), for the probability of emission of 2 (solid line), 3 (dashed line), 4 (dotted line) neutral pions, as a function of the incident energy for \( \theta_0 = 10^\circ \).](image)

\[
\sigma^{\bar{p}p \rightarrow \rho^0 X}, \text{mb}
\]

![Fig. 6. The cross section \( d\sigma(\rho, \bar{p} \rightarrow \rho^0 X) \) is plotted as function of the \( \rho \) energy fraction for two values of the incident energy and of the \( \rho \) emission angle: \( E = 10 \text{ GeV} \) and \( \theta_0 = 10^\circ \) (solid line), \( E = 10 \text{ GeV} \) and \( \theta_0 = 20^\circ \) (dashed line), \( E = 20 \text{ GeV} \) and \( \theta_0 = 10^\circ \) (dotted line), \( E = 20 \text{ GeV} \) and \( \theta_0 = 20^\circ \) (dash-dotted line).](image)

ing to the creation of a muon pair, where \( x_{\text{max}} = 1 - 4M_{\mu}^2/s \), \( M_{\mu} \) is the muon mass.

The cross section (16) is illustrated in Fig. 6 for two different values of the laboratory energy and of the emitted angle as a function of the \( \rho \) meson energy fraction.

In case of three pion production, assuming that the process occurs through a \( \pi^0\pi^0 \) initial state emission, we find:
\[
d\sigma(\rho, \bar{p} \rightarrow \rho^0 X) = dW_{\rho}(x_\rho) dW_{\pi}(x_\pi) \times \left[ \frac{d\sigma(p - p_\rho, \bar{p} - p_\pi \rightarrow \rho^0 X)}{dx} + \frac{d\sigma(p - p_\rho, \bar{p} - p_\pi \rightarrow \pi^+\pi^-)}{dx} \right] P_\pi P_\rho,
\]

implying the subsequent decay \( \rho^0 \rightarrow \pi^+\pi^- \).

It is interesting to note that the cross sections for the interaction of high energy neutron (anti-neutron) beams with a hadronic target can be calculated using the cross section of a proton beam interacting with the same target with the emission of the charged meson. We obtain (see Eqs. (9), (12)):
\[
\sigma^{\pi^+\pi^-\pi^0 \rightarrow X} = \frac{d\sigma^{\pi^+\pi^-\pi^0 \rightarrow X}}{dx}, \tag{18}
\]
and similarly for the anti-proton beams.

Experimental data from coincidence experiments with the selection of a hard meson at small angles are not available, and we can not test the factorisation pre-
diction. However the values of the cross section for \( \bar{p}p \rho^0 X \) illustrated in Fig. 6 are not in contradiction with the corresponding total cross sections (for a compilation, see [16]).

Inversely, if we use the experimental data for the total cross section of process \( \bar{p}p \to \bar{n}h^0 p \approx 1 \text{ mb} \), we can predict the value of the total cross section of the process \( \bar{n}p \to X \):

\[
P_\pi W_\pi(E_1, \theta_0)\sigma^{\bar{n}p \to X}(E - E_1) = \sigma^{\bar{n}p \to x}(E), \quad (19)
\]

with \( W_\pi(E, \theta_0) \) given in Eqs. (9), (15).

2. CONCLUSIONS

We have extended the QRE method to light meson emission from an (anti)proton beam and calculated the probabilities and the relative cross section for multi-pion emission. The considered processes can be measured at present and planned hadron facilities. We have also suggested a possible application. The collinear light meson emission could be used to produce secondary (anti)neutron beams, at a high energy (anti)proton accelerator. This would constitute an alternative to the usual way, when high-energy neutron beams are produced as secondary beams by break-up of deuterons on a hadronic target.

Our result for the matrix element squared, Eq. (11), agrees with Adler principle: the cross section vanishes when the pion momentum vanishes. This result can be inferred as well from the reduction formula of current algebra [17]. Note that the probabilities to create a \( \pi \) or \( \rho \)-meson by a proton can also be obtained using the infinite momentum reference frame ([18], Eq. (52)).

This allows one to obtain the relation between the cross section \( d\sigma_\pi(s(1 - x)) \) and \( d\sigma_\rho(1) \). This relation has the form of conversion on the energy fraction of the emitted particle \( x = k_1^0/p_1^0 \) of the probability of emission by the initial projectile \( dW_\pi(x) \) with cross section \( d\sigma_\pi(s(1 - x)) \). We underline that the details of interaction of projectile with the target are irrelevant, assuming that they are the same for shifted and unshifted kinematics. The contribution of the final state hard hadron emission as well as the interference of the relevant amplitude with the one describing the initial state emission vanish in the limit \( \theta_0 \to 0 \).

Let us discuss the limits of our approach and the accuracy of the present calculation. Concerning the kinematical region where this mechanism is important, we consider charged pion emission at angles smaller than an angle \( \theta_0 \), i.e. \( 0 < \theta_0 < 1 \). Moreover \( E\theta \gg M \), which insures the presence of large logarithm. In this case, the main contribution is due to the large quantity \( L = \ln(E\theta^2/M^2) \gg 1 \). In the calculations, we keep the terms containing \( L \) and omit the terms proportional to \( \theta^2 \). The resulting accuracy is given by \( 1 + O(1/L) \). Taking \( L = 10 \), the accuracy of the calculation is of the order of 10%.

We considered the case when the initial and the virtual projectiles are fermions. For instance we used the probability density to find a fermion in the initial fermion (see (2, 3)) \( P_\xi(z) = [1 + (1 - z^2)]/z \). Instead, one can consider other relations of the same kind, which hold when the initial projectile is a boson (photon, \( \rho \)-meson, gluon). We will not discuss this topic here.

Another interesting possibility is the conversion of the initial lepton to a photon or a neutral \( Z \)-boson. In this case we must use \( P_\xi(z) = z^2 + (1 - z^2) \) [15].

The arguments given above have a phenomenological character and are formulated in terms of hadrons. A similar idea, at quark level, was introduced in [19], where the emission of \( \rho \)-meson by quark and the \( \rho \)-meson production in quark-antiquark annihilation was studied. Special attention was paid to polarization phenomena of the created \( \rho \)-meson.

We have also suggested a possible application. The collinear light meson emission could be used to produce secondary (anti)neutron beams, at a high energy (anti)proton accelerator. This would constitute an alternative to the usual way where high-energy neutron beams are produced as secondary beams by break-up of deuterons on a hadronic target.

In frame of the “Gluon Dominance Model”, developed by one of us [20], the ratio of the inelastic CE cross section to the total inelastic cross section in \( pp \) scattering is estimated as 40%, in reasonable agreement with the experimental data [1].

The collinear light meson emission mechanism in (anti)proto-proton collisions provides a possible source of events with rather high multiplicities of (charged and neutral) pion production. For such events, the emission of hadrons in initial as well as in final states must be taken into account.

We considered here only initial proton emission. In this case, the resonance formation is kinematically forbidden since the four momentum of the virtual nucleon is Space-Like. The simplest CE processes \( pp \to n\bar{n}, \bar{p}n \to \bar{\Lambda}\Sigma^- \) can be in principle measured at PANDA. Other reaction mechanisms can contribute to these processes. In the frame of a description in terms of a single pseudoscalar meson \( \pi^\mp, K^\mp \) exchange, information on the strange meson-baryon constant can be extracted. Note that neutrons created by \( \rho \) and \( \pi \) emission form a (anti)proton beam with the mechanism considered here (initial state radiation) can not be formed by subsequent decay of a nucleon resonance, as \( \Delta \). Nucleon resonances can not be produced in the initial state, because the neutron has the time like momentum squared (the propagator is \( (p - k)^2 - M^2 < 0 \)).
The simplest CE processes $p\bar{p} \rightarrow n\Lambda, \bar{p}n \rightarrow \Lambda\Sigma^-$ can be in principle measured at PANDA. Other reaction mechanisms can contribute to these processes. In the frame of a description in terms of a single pseudoscalar meson $\pi^+, K^+$ exchange, information on the strange meson-baryon constant can be extracted.

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4. APPENDIX

The analytic expressions for calculating the integrals

$$I_a(z) = \int_0^1 \frac{dx}{x^n} \sqrt{1 - (\frac{z}{x})^2}$$

are:

$$I_0(z) = \frac{1}{2} \ln \frac{1 + r - r}{1 - r}$$

$$I_1(z) = r + z \arcsin(z)$$

$$I_2(z) = \frac{1}{2} \frac{r - z^2}{4} \ln \frac{1 + r}{1 - r}$$

with $r = \sqrt{1 - z^2}$.

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