Tensor-Induced CMB Temperature-Polarization Correlation in Reionized Universes

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Abstract

We reexamine the temperature-polarization correlation function of the cosmic microwave background induced by tensor mode with a scale-invariant spectrum in reionized standard cold dark matter models. It is found that the sign of the correlation function is positive on all angular scales even in a model with substantial reionization.

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The detection of the large-angle anisotropy of the cosmic microwave background (CMB) by the COBE DMR experiment [1] has incited a lot of studies in this area [2]. It is now well established that CMB anisotropies are genuine imprint of the early universe, which could potentially be used to determine to a high precision virtually all cosmological parameters of interest [3]. Future missions such as NASA MAP and ESA Planck Surveyor would measure the anisotropy spectrum in high-precision; the polarization spectrum, expected with an order of magnitude below the anisotropy, would also be measured to provide complementary information to anisotropy.

It was first pointed out by Crittenden et al. that the correlation between the temperature and polarization anisotropies offers a test of physics on the last scattering surface, as well as a possibility of distinguishing the scalar and tensor perturbations [4,5]. They found that in a universe with standard recombination the sign of the tensor-induced temperature-polarization (TQ) correlation function is opposite to that for scalar mode on large scales. With substantial early ionization, polarization is greatly enhanced on large scales and a geometrical effect causes the tensor correlation function to reverse sign for \( \theta > 30^\circ \) [5].

TQ correlation will be measured by MAP and Planck at small scales. In addition, polarization experiments such as ground-based POLAR [6] and SPort/ISSA [7] will observe large-scale polarization as to test the thermal history of the early universe. In particular, the latter with full-sky coverage will be sensitive to TQ correlation [8]. Although TQ correlation has been studied extensively [9,10], it is worthwhile to reexamine and work out explicitly the TQ correlation function in reionized universes. We will follow closely the method of Ref. [4,5]. We find a correction term to their tensor correlation function that renders it a positive function irrespective of the reionization history. Our result should be useful to the large-scale polarization experiments.

Here we do not repeat detailed calculations. Only necessary steps for obtaining correlation functions are given. For tensor + -mode, the Stokes parameters induced by a Fourier mode with wavevector \( \mathbf{k} \) in the direction of an unit vector \( \hat{\mathbf{p}} \) are:
\[
\begin{pmatrix}
T'_{k} \\
Q'_{k} \\
U'_{k}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{2} \alpha(\mu)(1 - \mu^2) \cos 2\phi \\
\frac{1}{2} \beta(\mu)(1 + \mu^2) \cos 2\phi \\
\beta(\mu) \mu \sin 2\phi
\end{pmatrix},
\]  

(1)

where \(\mu \equiv \hat{p} \cdot \hat{k}\) and \(\phi\) is the azimuthal angle of \(\hat{p}\) about \(\hat{k}\). The \(\times\)-mode solution is given by the same expressions, except for replacing \(\cos 2\phi\) by \(\sin 2\phi\), and \(\sin 2\phi\) by \(-\cos 2\phi\). For scalar mode, \(T'_{k} = \alpha(\mu)\), \(Q'_{k} = \beta(\mu)\), and \(U'_{k} = 0\). Expanded in Legendre polynomials, \(\alpha(\mu) = \sum_i (2l + 1) \alpha_i P_l(\mu)\) and \(\beta(\mu) = \sum_i (2l + 1) \beta_i P_l(\mu)\). We evaluated all \(\alpha_i\)'s and \(\beta_i\)'s using the Boltzmann numerical code developed in Ref. [11].

To obtain the Stokes parameters defined with respect to a fixed orthonormal basis \((e_x, e_y, e_z)\) from those defined in the \(\hat{k}\)-basis [1], we need to perform the rotation [12]:

\[
\begin{pmatrix}
T_{k} \\
Q_{k} \\
U_{k}
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos 2\psi & \sin 2\psi \\
0 & -\sin 2\psi & \cos 2\psi
\end{pmatrix}
\begin{pmatrix}
T'_{k} \\
Q'_{k} \\
U'_{k}
\end{pmatrix},
\]  

(2)

where the rotation angle \(\psi\) is given by

\[
\cos \psi = \frac{\hat{k} \cdot e_z - (\hat{k} \cdot \hat{p})(e_z \cdot \hat{p})}{\sqrt{1 - (\hat{k} \cdot \hat{p})^2} \sqrt{1 - (e_z \cdot \hat{p})^2}},
\]

\[
\sin \psi = \frac{\hat{p} \cdot (e_z \times \hat{k})}{\sqrt{1 - (\hat{k} \cdot \hat{p})^2} \sqrt{1 - (e_z \cdot \hat{p})^2}}.
\]  

(3)

Summing up all \(k\)-mode contributions and two polarizations, the tensor TQ correlation function is obtained as

\[
\langle T(\hat{p}_1)Q(\hat{p}_2) \rangle = \frac{1}{4} \int d^3k \left[ (1 - \mu_1^2)(1 + \mu_2^2) \cos 2(\phi_1 - \phi_2) \cos 2\psi_2 \
- 2(1 - \mu_1^2)\mu_2 \sin 2(\phi_1 - \phi_2) \sin 2\psi_2 \right] \alpha(\mu_1)\beta(\mu_2),
\]  

(4)

where \(\mu_1 = \hat{p}_1 \cdot \hat{k}\) and \(\mu_2 = \hat{p}_2 \cdot \hat{k}\), \(\phi_1\) and \(\phi_2\) are respectively the azimuthal angles of \(\hat{p}_1\) and \(\hat{p}_2\) about \(\hat{k}\), and \(\psi_2\) is given by Eq. (3) with \(\hat{p}\) replaced by \(\hat{p}_2\). It is straightforward to prove the identities,
\begin{align*}
\cos(\phi_1 - \phi_2) &= \frac{\hat{p}_1 \cdot \hat{p}_2 - \mu_1 \mu_2}{\sqrt{1 - \mu_1^2} \sqrt{1 - \mu_2^2}}, \\
\sin(\phi_1 - \phi_2) &= \frac{(\hat{p}_2 \times \hat{p}_1) \cdot \hat{k}}{\sqrt{1 - \mu_1^2} \sqrt{1 - \mu_2^2}}.
\end{align*}

Without loss of generality, we choose \(\hat{p}_1 = \hat{q}\) and \(\hat{p}_2 = e_z\), and use the axes \(e_x\) and \(e_y\) to define the Stokes parameter \(Q(e_z)\). Hence, \(\mu_2 = \mu_k\) and \(\psi_2 = \phi_k\), where \(\mu_k, \phi_k\) are the spherical polar angles of \(\hat{k}\) in the coordinate \((e_x, e_y, e_z)\). Then, expanding

\[ A(\mu_1) \equiv \mu_1^2 \alpha(\mu_1) = \sum_l (2l + 1) A_l P_l(\mu_1), \]

\[ B(\mu_2) \equiv (1 + \mu_2^2) \beta(\mu_2) = \sum_l (2l + 1) B_l P_l(\mu_2), \]

and integrating over \(\mu_k\) and \(\phi_k\), we obtain

\[ \langle T(\hat{q})Q(e_z) \rangle = \frac{\pi}{2} \cos 2\varphi \int k^2 dk \sum_{l,l'} (2l + 1)(2l' + 1) \]

\[ \times \left[ \frac{(l' - 2)!}{(l' + 2)!} \left[ \alpha_{l'} B_l \cos^2 \theta - A_{l'} B_l \right] a_{ll'}^2 P_{l'}^2(\cos \theta) \right. \]

\[ + \frac{1}{2} \sin^2 \theta \alpha_{ll'} B_l \left( \delta_{ll'} \frac{2}{2l + 1} P_{l'}(\cos \theta) + \frac{(l' - 4)!}{(l' + 4)!} a_{ll'}^2 P_{l'}^4(\cos \theta) \right) \]

\[ - 2 \sin \theta \alpha_{ll'} \beta_l \left( \frac{(l' - 1)!}{(l' + 1)!} b_{ll'}^1 P_{l'}^1(\cos \theta) - \frac{(l' - 3)!}{(l' + 3)!} b_{ll'}^3 P_{l'}^3(\cos \theta) \right) \]

\[ \left. + \sin^2 \theta \alpha_{ll'} (B_l - \beta_l) \left( \delta_{ll'} \frac{2}{2l + 1} P_{l'}(\cos \theta) - \frac{(l' - 4)!}{(l' + 4)!} a_{ll'}^2 P_{l'}^4(\cos \theta) \right) \right], \tag{7} \]

where \((\theta, \varphi)\) are the spherical polar angles of \(\hat{q}\) about \(e_z\). The constants \(a_{ll'}^n\) and \(b_{ll'}^m\) are given respectively by \(a_{ll'}^n = \int_{-1}^1 dx P_l(x) P_l^n(x)\) and \(b_{ll'}^m = \int_{-1}^1 dx (1 - x^2) \hat{z} P_l(x) P_l^m(x)\). In Eq. (7), the first two terms are the expression for the tensor \(\langle TQ \rangle\) that was found in Ref. [5]. They come from integration of the first term containing \(\cos 2\psi_2\) in Eq. (4). The remaining terms are from integrating the \(\sin 2\psi_2\) term. Fig. 1 is the plot of \(\langle TQ \rangle\) from numerical calculations of Eq. (4) with \(\varphi = 0\) for reionized models with different optical depths. Apparently, the tensor \(\langle TQ \rangle\) functions are positive on all angular scales. The dashed line is the case of substantial reionization without the correction term, which is similar to the result obtained in Ref. [5]. We can see that the correction is sizable on large angular scales. This is expected because the effect of basis rotation operates on only large angular scales [13]. Thus, in
calculating small-scale correlation, one can simply use the small-angle approximation by making $\psi_2 = 0$, under which $\sin 2\psi_2 = 0$ and the correction term is then switched off. In the case of standard recombination, since the large-scale polarization is extremely small, the correction is not apparent at all.

To compare with the scalar mode, in Fig. 2 we plot the scalar $\langle TQ \rangle$ \cite{13,14},

$$\langle T(\hat{q})Q(e_\pm) \rangle = 2\pi \cos 2\varphi \sum_{l \geq 2} (2l + 1)C_l^{TQ} P_l^2(\cos \theta),$$

$$C_l^{TQ} = \frac{(l - 2)!}{(l + 2)!} \sum_{l'} (2l' + 1) a_{l'}^2 \int k^2 dk \alpha_l^{*} \beta_{l'}. \quad (8)$$

Note that the peak height of scalar $\langle TQ \rangle$ in the case of substantial reionization is significantly less than that in Ref. \cite{4}. This is due to the fact that they used a slightly different constant $a_{l'}^2$ (transposing $l'$ and $l$) for $C_l^{TQ}$ in the correlation function. Figs. 1 and 2 show that on large scales the tensor $\langle TQ \rangle$ has correlation whereas the scalar $\langle TQ \rangle$ has anticorrelation. In fact this can be easily explained by solving the collisional Boltzmann equation in the long-wavelength limit \cite{15}. It has been pointed out that in an open universe the sign of the scalar $\langle TQ \rangle$ on the largest scales is reversed, however, this effect will be destroyed by even minimal amounts of reionization \cite{10}. Thus the positivity of the large-scale TQ correlation is a direct indicator of a significant tensor component in metric fluctuations.

More recently, calculations of correlation functions were usually performed by using an elegant formalism developed by Zaldarriaga et al. \cite{16} and Kamionkowski et al. \cite{17}. Basically, the method is to expand $Q$ and $U$ in terms of spin-2 spherical harmonics. In Ref. \cite{17}, they gave

$$\langle T(\hat{q})Q(e_\pm) \rangle = -\cos 2\varphi \sum_l \frac{2l + 1}{4\pi} \sqrt{\frac{(l - 2)!}{(l + 2)!}} C_{Cl} P_l^2(\cos \theta), \quad (9)$$

where the angular spectrum $C_{Cl}$ can be expressed in terms of $\alpha_l$ and $\beta_l$. For scalar mode, their $\langle TQ \rangle$ is exactly the same as Eq. (8). But their tensor $\langle TQ \rangle$ has a different form from Eq. (8), where they can obtain a much simpler expression for $C_{Cl}$. Since their expansion scheme is different from ours, it is rather difficult to show the equivalence analytically.
However, we compare our numerical results of Eq. (7) with those using Eq. (9) with $C_{Ci}$'s evaluated by using the CMBFAST code [18], and good agreements are found.

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FIGURE CAPTIONS

Fig.1. Temperature-polarization correlation functions for scale-invariant tensor perturbation in reionized cold dark matter models ($\Omega_0 = 1, h = 0.5, \Omega_B = 0.05$) with different reionization optical depths $\tau$. Dashed line is for the case of $\tau = 20$ without the correction term. All curves are normalized to COBE data.

Fig.2. As Fig. 1, but for scale-invariant scalar perturbation. The height of the peak near $\theta \simeq 4^\circ$ is about a factor of 3 less than that in Ref. [4].
Fig. 1

Tensor

\( \langle TQ \rangle(\theta) \mu K^2 \)

\( \tau = 0 \)

\( \tau = 1 \)

\( \tau = 20 \)
Fig. 2

$\langle TQ(\theta) \rangle_{[\mu K^2]}$

$\theta$ [degrees]

$\tau=0$

$\tau=1$

$\tau=20$

Scalar