Bubble wall velocity: heavy physics effects

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Abstract

We analyse the dynamics of the relativistic bubble expansion during the first order phase transition focusing on the ultra relativistic velocities $\gamma \gg 1$. We show that fields much heavier than the scale of the phase transition can significantly contribute to the friction and modify the motion of the bubble wall leading to interesting phenomenological consequences. NLO effects on the friction due to the soft vector field emission are reviewed as well.
1 Introduction

First order phase transitions (FOPT) in the early universe are very interesting phenomena which can lead to a plethora of cosmological observations, i.e. production of stochastic gravitational wave signals [1], matter-antimatter asymmetry [2, 3] or primordial magnetic fields [4]. During the FOPT the change of phase of the system occurs due to the bubble nucleation and it becomes crucial to understand the dynamics of this process. In this paper, we will focus on the dynamics of the bubble wall expansion and on the friction effects which are induced due to the interaction with the hot plasma (for the previous studies see also [5, 6, 7, 8, 9, 10]).

Ideally, in order to answer this question one has to perform the out-of-equilibrium quantum field theory calculation. However in the case of very relativistic bubbles with a very large Lorentz factor, $\gamma \gg 1$, a quasi-classical calculation can provide reliable results [11, 12, 13, 14, 15]. To avoid dealing with complicated quantum out-of-equilibrium effects, in this study, we thus consider only the bubble expansions with $\gamma \gg 1$. We will review the results by [11, 12] and show that, in the presence of new heavy particles, there is an additional unsuppressed contribution to the friction which can prevent the runaway behaviour of the bubble, which is the main result of this paper. We demonstrate the importance of this effect using a two-scalars toy model with FOPT. Next we move on to the discussion of the Next-To-Leading Order (NLO) friction effects along the lines of [12] and present an alternative derivation using Equivalent Photon Approximation (EPA)[16, 17, 18, 19], which we believe offers more intuitive understanding of the friction.

The manuscript is organized as follows: in the section 2 we review the LO friction following [11], then in the section 3, we derive the friction from the heavy particles and provide an example where this can lead to observational effects. In the section 4 we discuss NLO effects and in the section 5 we finally conclude and summarize our main results.

2 Transition pressure

Let us start by reviewing the origin of the friction effects focusing on the bubbles which are expanding relativistically $\gamma \gg 1$. Our discussion will follow closely the presentation in [11, 12, 13]. Suppose we are looking at the effects coming from a particle $A$ hitting the wall and producing an $X$ final state (which can perfectly be a multiparticle state) (see Fig. 1), then the pressure will be given by

$$P_{A \rightarrow X} = \int \frac{p_z d^3p}{p_0(2\pi)^3} f_A(p) \times \sum_X \int dP_{A \rightarrow X}(p^Z_A - \sum_X p^Z_X), \quad (1)$$

where the first factor is just a flux of incoming particles and the second includes the differential probability of the transition from $A$ to $X$, $dP_{A \rightarrow X}$, as well as momentum transfer to the wall $(p^Z_A - \sum_X p^Z_X)$. Note that the equation above is valid if only the mean free path of the particles is much larger than the width of the wall, so that we can ignore the thermalization effects inside the wall and consider individual particle collision with the wall [14]. The probability of transition can be calculated as follows

$$dP_{A \rightarrow X} = \prod_{i \in X} \frac{d^3k_i}{(2\pi)^3 2k_0} \langle \phi | T | X \rangle \langle X | T | \phi \rangle, \quad (2)$$

where $\phi$ is the wave-packet building the one-particle normalized state

$$| \phi \rangle = \int \frac{d^3k}{(2\pi)^3 2k_0} | \phi(k) \rangle | k \rangle, \quad \langle p | k \rangle = 2p_0(2\pi)^3 \delta^3(p - k)$$

$$\int \frac{d^3p}{(2\pi)^3 2p_0} | \phi(p) \rangle^2 = 1. \quad (3)$$
Combining all of this and using the energy and transverse momentum conservation we arrive at the following expression for the pressure from transition [12]

\[ P_{A \rightarrow X} = \int \frac{d^3p}{(2\pi)^3 2p_0} f_p \prod_{i \in X} \int \frac{d^3k_i}{(2\pi)^3 2k_0} (2\pi)^3 \delta(p_\perp - \sum_{i \in X} k_\perp) \delta(p_0 - \sum_{i \in X} k_0) |M|^2 (p_A^2 - \sum_{i \in X} k_i^2) \]

where we have ignored the high density effects for the final particles and \( M \) is defined as follows

\[ \langle p | H_{\text{int}} | k_1 ... \rangle = (2\pi)^3 \delta^3(p_\perp - \sum_{i \in X} k_\perp) \delta(p_0 - \sum_{i \in X} k_0) M, \]

\[ M = \int dz \chi_p(z) \prod_{i \in X} \chi_i(z) V. \]

Armed with this expression we can proceed to the calculation of the friction effects.

### 2.1 Leading order (LO) friction

In this first section, we review the Leading-Order (LO) effects i.e. when the initial and the final state contain one particle (1 → 1 transition). We will be focusing on the very relativistic bubble expansions and, in particular, on regimes where the \( WKB \) approximation is valid, which is when

\[ p_z L \gg 1, \]

where \( L \) is a typical width of the wall and \( p_z \) is the momentum of the incident particle. Let us suppose that the mass \( m_1 \), in the symmetric phase, changes when passing through the wall to \( m_2 \) in the broken phase, \( m_1 \rightarrow m_2 \). The matrix element for this transition is equal according to Eq.3 to

\[ \langle p | k \rangle = 2p_0 (2\pi)^3 \delta^3(p - k) \Rightarrow M_{1 \rightarrow 1} = 2p_0. \]

Then the pressure for the relativistic particles is equal to:

\[ P_{1 \rightarrow 1} = \int \frac{d^3p}{(2\pi)^3} f_p (p^*_z - p_z^0) \approx \int \frac{d^3p}{(2\pi)^3} f_p \times \frac{m_2^2 - m_1^2}{2p_0} = \int \frac{d^3p}{(2\pi)^3} f_p \times \frac{\Delta m^2}{2p_0} \]

where we have expanded the momenta in \( m_{1,2}^2/p_0^2 \) and defined \( \Delta m^2 \equiv m_2^2 - m_1^2 \). It is well known that the quantity \( \frac{d^3p}{p_0^2} \) is invariant under boost, which allowed the authors of [11] to conclude that the Leading-Order friction is independent of \( \gamma \) and scales as

\[ P_{1 \rightarrow 1} \simeq \frac{\Delta m^2 T^2}{24}. \]
Let us make a few comments about this result. For simplicity, let us assume first that \( m_1 = 0 \) then it is obvious that the value in Eq. 9 will be reached only for the \( \gamma \) factors satisfying
\[
\gamma \gg \frac{m_2}{T},
\]
otherwise initial particles simply will not have enough energy to pass through the wall (see more details in the Appendix A as well as [15] for analytical results). Now let us look at the scenario when the initial mass as well is non-zero, \( m_1 \neq 0 \). In this case the particle will contribute only if its mass is smaller than the temperature
\[
m_1 \lesssim T
\]
otherwise the contribution of this particle to the pressure will be exponentially suppressed by a Boltzmann factor (see Appendix A).

3 Friction from mixing

In the context of very relativistic bubble, in the rest frame of the wall, the particles colliding it can reach very high energies \( \sim \gamma T_{\text{nucl}} \), much larger than the temperature of the transition \( \sim T_{\text{nucl}} \) and symmetry breaking parameter \( \sim \langle s \rangle \). Then it becomes interesting whether new degrees of freedom absent in the low energy lagrangian describing the phase transition can play a role in the dynamics of the bubble acceleration. The simplest example where this phenomena can occur is the following: let us consider the lagrangian of a massless fermion mixed with another heavy vectorlike fermion
\[
\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi + i\bar{N}\gamma^\mu N + M\bar{N}N + Ys\bar{\psi}N
\]
where \( s \) is a field developing a VEV via the phase transition, \( \psi \) is the light fermion and \( N \) is the heavy fermion. In the regime \( M \gg \langle s \rangle \sim T_{\text{nucl}} \), then at the temperature of the transition, the species \( N \) can be ignored (they are Boltzmann suppressed and are not part of the plasma), so its contribution to pressure naively should be zero. However let us consider a process of \( \psi \) hitting the wall. We suppose that the energy of the incident \( \psi \) particles is much larger than the mass of the heavy species \( N \); \( E \gg M \Rightarrow \gamma T \gg M \). Note that the mass eigenstates inside and outside of the bubble are different due to the VEV of the \( \langle s \rangle \) and in particular there will be mixing between the \( \psi \) field and the heaviest mass eigenstate in the broken phase. The mixing angle \( \theta_{\psi N} \) is approximately given
\[
\sin \theta_{\psi N} \sim \frac{Y\langle s \rangle}{M}.
\]
In the limit when the energy of the incident particle is much larger than the inverse of the wall width, \( p_z L \gg 1 \), the mass eigenstates will change adiabatically and there will be a probability of transition \( \psi \to N \) of the form
\[
P(\psi \to N) \sim \sin^2 \theta_{\psi N} \sim \frac{Y^2\langle s \rangle^2}{M^2}.
\]
We can estimate the pressure due to this mixing using the results of the previous section. We obtain
\[
P_{\text{mixing}} \sim \int \frac{d^3p}{(2\pi)^3} f_p \frac{P(\psi \to N)}{M^2} \times \frac{Y^2\langle s \rangle^2}{2E} \theta(E - M)
\]
\[
\sim Y^2\langle s \rangle^2 T^2 \theta(\gamma T - M),
\]
\[
(15)
\]
where we always have to assume that $\gamma T \gg M$ so that the heaviest state can be produced and contribute to the pressure. Note that this new contribution to the pressure is not suppressed by the large $M$ mass and can be present even if $M \gg \langle s \rangle$.

### 3.1 Friction from mixing: more details

One can derive the expression for the friction force in Eq. 15 using the master equation (Eq.5) for the pressure from transition $\psi \rightarrow N$. Indeed, in the the WKB approximation, the solutions for the wave functions are equal to

$$\chi(z) \simeq \sqrt{\frac{k_{z,s}}{k_z(z)}} \exp \left( i \int_0^z k_z(z') dz' \right),$$

where $k_{z,s}$ is the $z$ component of the momenta on the symmetric side of the wall. Then assuming that the change of the wave functions inside the wall is small, the product of wave functions will be:

$$\chi \psi(z) \chi_N(z) \sim \exp \left[ iz \left( p^\psi_z - p^N_z \right) \right]$$

where $p^\psi_z, p_N^z$ are different on the two sides of the wall and are calculated using energy and transverse momentum conservation. Then for the matrix element we obtain:

$$\mathcal{M} = V_s \int_{-\infty}^0 dz \chi\psi(z)\chi_N(z) + V_h \int_0^\infty dz \chi\psi(z)\chi_N(z) = i \left( \frac{V_h}{(p^\psi_z - p^N_z)}|_h - \frac{V_s}{(p^\psi_z - p^N_z)}|_s \right),$$

where the $h, s$ subscripts denote the interactions and momenta inside and outside of the bubble. The process under study is only possible on the broken side of the wall and thus $V_s = 0, \quad V_h \neq 0$. We have

$$|V_h|^2 = 2Y^2\langle s \rangle^2 p^\psi_z (p^\psi_z - p^N_z).$$

On top of this there will be an effect due to the mass modification of $\psi$ and $N$, however this effect will be subleading and suppressed by additional powers of $Y^2\langle s \rangle^2 / M^2$. Combining the Eq. 18 and 19 the matrix element becomes

$$|\mathcal{M}|^2 = \frac{2Y^2\langle s \rangle^2 p^\psi_z}{p^\psi_z - \sqrt{(p^\psi_z)^2 - M_N^2}}.$$

Plugging it in the master Eq.5, we obtain the following estimate for the mixing pressure\footnote{The integral below is assumed to be taken for the values of $p_z > M_N$, otherwise the process is forbidden.}

$$P_{\text{mixing}} = \int \frac{d^3p}{(2\pi)^3} f_p \times \frac{Y^2\langle s \rangle^2}{2\sqrt{p_z^2 - M_N^2}}.$$

When going back to the plasma frame we recover our estimate in Eq.15. We can see that the pressure from the mixing is not suppressed by the mass of the heavy particles and in general can be present if we treat our theory as an effective field theory (EFT) with heavy degrees of freedom integrated out. One can ask what could be the maximal pressure from the mixing in this case. We can estimate it by using unitarity arguments on the maximal value of the mixing coupling $Y^{\max} \sim 4\pi$. So that, the maximal pressure from mixing is

$$P_{\text{mixing}}^{\max} \simeq \frac{T^2}{24} (16\pi^2)\langle s \rangle^2 \theta(\gamma T - M).$$

In the appendix B, we give other examples of friction induced by the otherwise decoupled particles in the theories with only scalars.
3.2 Importance of friction from mixing

One can wonder whether this friction from mixing can be phenomenologically important, since in any case we are looking at the very relativistic bubble expansion velocities \( v \rightarrow 1 \). However the relativistic bubble which have reached terminal velocity have vanishingly small fraction of the energy stored in the wall and most of the energy released in the phase transition is transferred to the motion of the plasma \([20, 21, 22]\). This distribution of energy has important phenomenological consequences on the spectrum of stochastic gravitational wave background since the bubble wall collisions signal \( \Omega_\phi \) and plasma motion signal \( \Omega_{sw} \) lead to different shape of the spectrum (see for example \([23]\)). Namely, the most obvious difference is the fall of the signal at high frequencies;

\[
\Omega_{sw,f} \rightarrow \infty \propto f^{-4} \quad \text{(No Runaway)}, \quad \Omega_{\phi,f} \rightarrow \infty \propto f^{-3/2} \quad \text{(Runaway)}.
\] (23)

In order to understand whether the friction from mixing can indeed prevent the runaway bubble case let us consider the following toy model \([24, 25]\)

\[
L = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \eta)^2 - \lambda_\phi \phi^4 - \frac{\lambda_\eta}{4} \eta^4 - \frac{\lambda_{\phi\eta}}{2} \phi^2 \eta^2.
\] (24)

On top of the tree-level potential, at one loop, the usual Coleman-Weinberg potential is generated \([26]\) for the fields \( \phi, \eta \) (in \( \overline{MS} \) scheme)

\[
V_{CW} = \sum_{i=\eta,\phi} \frac{m_i^4}{64\pi^2} \left[ \log \frac{m_i^2}{\mu_R^2} - \frac{3}{2} \right].
\] (25)

The thermal corrections can be taken into account by adding the thermal potential \( V_T \) defined as follows

\[
V_T = \sum_{i=\eta,\phi} \frac{T^4}{2\pi^2} J \left( \frac{m_i^2}{T} \right), \quad J(y^2) \equiv \int_0^\infty dx x^2 \log(1 - \exp(-\sqrt{x^2 + y^2})).
\] (26)

Higher loop corrections due to the daisy diagrams can be taken into account using the truncated full dressing procedure \([27]\)

\[
V(\phi, \eta, T) = V_{tree}(\phi, \eta) + \sum_{i=\eta,\phi} V_{CW}(m_i^2 + \Pi_i^2) + V_T(m_i^2 + \Pi_i^2).
\] (27)

In the case of the model (24), the thermal mass corrections are given by

\[
m_\phi^2 + \Pi_\phi^2 = 3\lambda_\phi \phi^2 + \lambda_{\phi\eta} \eta^2 + T^2 \left( \frac{\lambda_\phi}{4} + \frac{\lambda_{\phi\eta}}{12} \right)
\] (28)

\[
m_\eta^2 + \Pi_\eta^2 = \lambda_{\phi\eta} \phi^2 + 3\lambda_\eta \eta^2 + T^2 \left( \frac{\lambda_\eta}{4} + \frac{\lambda_{\phi\eta}}{12} \right).
\] (29)

Generically we have to analyze the phase transition in the \((\phi, \eta)\) field space, however the discussion simplifies if we put the coupling \( \lambda_\phi = 0 \). Indeed in this case, along the line \( \eta = 0 \), the tree-level potential is vanishing and only the one loop potential will be controlling the phase transition. The tree-level \( \eta^4\)-potential is stabilizing the \( \eta \)-direction, thus it is obvious that the tunnelling must happen along \( \eta = 0 \) direction. The only mass parameter in this construction is the renormalization scale \( \mu_R \equiv \lambda_{\phi\eta} w \) which fixes the value of the VEV of the field \( \langle \phi \rangle \sim w \).

The transition from the false to the true vacuum, separated by the potential barrier, can be calculated using the usual bounce action (see \([28, 29, 30]\))
\[ \Gamma(T) \sim \max \left[ T^4 \left( \frac{S_3}{2 \pi T} \right)^{3/2} \exp(-S_3/T), R_0^{-4} \left( \frac{S_4}{2 \pi} \right)^2 \exp(-S_4) \right]. \]  

(30)

However the phase transition in the early universe will occur when the rate of transition becomes comparable to the expansion rate of the universe. This condition is approximately given by

\[ \Gamma(T_{\text{nuc}}) = H^4(T_{\text{nuc}}), \]

\[ H^2 \equiv \frac{\rho_{\text{rad}} + \rho_{\text{vac}}}{3M_{\text{pl}}^2} = \frac{1}{3M_{\text{pl}}^2} \left( \frac{\pi^2 g_* T^4}{30} + \Delta V \right), \]  

(31)

where \( M_{\text{pl}} \equiv 2.435 \times 10^{18} \text{ GeV} \) is the reduced Planck mass and \( T_{\text{nuc}} \) is the nucleation temperature.

We are prepared now to discuss the friction effects. The bubble will have runaway behaviour if the LO friction, which in our model is equal to

\[ P_{\text{LO}} \approx T_{\text{nuc}}^2 \lambda \langle \phi \rangle^2 \theta(\gamma T - \langle \phi \rangle \sqrt{\lambda \langle \phi \rangle}), \]  

(32)

cannot overcome the potential difference, providing the driving force for the expansion of the bubble. This amounts to the condition

\[ \Delta V > P_{\text{LO}} \quad \text{(runaway condition)}. \]  

(33)

At the same time, as we have seen, there can be an additional friction induced by the mixing effect

\[ P_{\text{mixing}} \approx T_{\text{nuc}}^2 Y_{\text{mixing}}^2 \langle \phi \rangle^2 \theta(\gamma T - M). \]  

(34)

which can prevent the runaway behaviour. Now if the condition

\[ P_{\text{LO}} + P_{\text{mixing}} > \Delta V > P_{\text{LO}} \]  

(35)

is satisfied we are in the situation when the mixing pressure is preventing the bubbles from the otherwise runaway motion. To analyse the \( P_{\text{mixing}} \) effects in our model we have deferred from performing the full parameter scan and instead have fixed the symmetry breaking scale to be \( 10^5 \text{ GeV} \) and the mixing coupling \( Y_{\text{mixing}} = 1 \). Then the region of the parameter space where the mixing effect is important is displayed on the Fig.2.

In order to find the upper bound on the masses on the states which can be produced in mixing we need to estimate the maximum value of the Lorentz \( \gamma_{\text{max}} \) factor that would have been reached if the bubbles keep accelerating till the moment of the collision. It can be estimated from the ratio of initial and final radii of the bubble and is approximately equal to [21, 25]

\[ \gamma_{\text{max}} \approx 2R_*/3R_0 \left( 1 - \frac{P_{\text{LO}}}{\Delta V} \right). \]  

(36)

The initial bubble radius and the bounce solution can be found numerically, while the final radius can be estimated according to [31] by the derivative of the bounce action:

\[ R_* \left( \frac{8\pi}{\beta(T_n)} \right)^{1/3}, \quad \beta(T) = HT \frac{d}{dT} \left( \frac{S_3}{T} \right), \]  

(37)
Driving Force

| \( \lambda_{\phi} \) | \( \omega^{-4} \Delta V \) | \( \omega^{-4} P_{\nu,0}(T_{\text{nucl}}) \) | \( \omega^{-4} P_{\text{mixing}}(T_{\text{nucl}}) \) | \( \omega^{-4} P_{\text{hot}}(T_{\text{nucl}}) \) |
|----------------|----------------|----------------|----------------|----------------|
| 0.5            | Runaway        | Runaway        | No Runaway     | No Runaway     |
| 1.0            |                |                |                |                |
| 1.5            |                |                |                |                |
| 2.0            |                |                |                |                |

Pressure

No Runaway

Runaway if no mixing

Runaway

Figure 2: Right- the potential difference and various contributions to the pressure as a function of the coupling \( \lambda_{\phi} \). The scale of the symmetry breaking was fixed to be \( w = 10^5 \) GeV so that \( \langle \phi \rangle \sim 10^5 \) GeV. Left- The maximal mass of the heavy particle defined by the Eq.38 as a function of \( \lambda_{\phi} \).

where \( H \) is the Hubble constant. Then the friction from mixing can be generated only by the states satisfying

\[
M < M_{\text{max}} = \gamma_{\text{max}} T_{\text{nucl}}. \tag{38}
\]

We report the value \( M_{\text{max}} \) on the Fig.2. In our model, we can see that states as heavy as \( 10^{15} \) GeV can lead to non-vanishing friction effects. Generically one can estimate \( M_{\text{max}} \) as follows

\[
M_{\text{max}} = \gamma_{\text{max}} T_{\text{nucl}} \sim \frac{R_*}{R_0} T_{\text{nucl}}. \tag{39}
\]

The values of the initial and the final radii are very roughly equal to:

\[
R_0 \sim \frac{1}{T_{\text{nucl}}}, \quad R_* \sim H^{-1} \sim \frac{M_{\text{pl}}}{\text{scale}^2}, \tag{40}
\]

where \( M_{\text{pl}} \) is the Planck mass and the “scale” refers to the energy scale of the potential. Combining all of this together we can find the estimate for the maximal mass to be

\[
M_{\text{max}} \sim M_{\text{pl}} \left( \frac{T_{\text{nucl}}}{\text{scale}} \right)^2. \tag{41}
\]

Of course this estimate is valid only for the theories where the bubbles are runaway without the friction from mixing. In the next section however we will review the NLO effects from the gauge field which generically prevent bubbles from infinite acceleration.

4 NLO effects (review of [12])

So far we have been looking at the effects appearing in \( 1 \rightarrow 1 \) transition, now let us move to the \( 1 \rightarrow 2 \) transitions (we closely follow the discussion in [12]). Again we will assume that we are in the regime where the WKB approximation is valid i.e. \( p_z L \gg 1 \), where \( L \) is the width of the wall. The calculation of the \( 1 \rightarrow 2 \) splitting simplifies in the limit when \( k_z \gg m, k_\perp \) and, in this case, it becomes easy to find the solution for the free wave functions

\[
\chi(z) \simeq \sqrt{\frac{k_{z,s}}{k_z(z)}} \exp \left( i \int_0^z k_z(z')dz' \right). \tag{42}
\]
Using the following notation for the initial and final momenta

\[ p = (p_0, 0, 0, \sqrt{p_0^2 - m_A^2(z)}) \]
\[ k^{(1)} = (p_0(1 - x), 0, k_z, \sqrt{p_0^2(1 - x)^2 - k_z^2 - m_C^2(z)}) \]
\[ k^{(2)} = (p_0x, 0, -k_z, \sqrt{p_0^2x^2 - k_z^2 - m_B^2(z)}) \]

(43)

the product of three wave functions in \( 1 \rightarrow 2 \) splitting is equal to

\[ \chi_A(z)\chi_B^*(z)\chi_C^*(z) \sim \exp \left[ \int_0^z \left( \frac{m_A^2(z) - m_B^2(z) + k_z^2}{2p_0} - \frac{1}{2k_0^{(1)}} - \frac{m_C^2(z) + k_z^2}{2k_0^{(2)}} \right) \right]. \]

(44)

Then the matrix element is equal to

\[ \mathcal{M} = V_s \int_{-\infty}^0 \exp \left[ iz \frac{A_s}{p_0} \right] + V_h \int_0^\infty \exp \left[ iz \frac{A_h}{p_0} \right] = 2ip_0 \left( \frac{V_h}{A_h} - \frac{V_s}{A_s} \right) \]
\[ A = -\frac{k_z^2}{x(1 - x)} + m_A^2 - \frac{m_C^2}{1 - x} - \frac{m_B^2}{x}, \]

(45)

so we end up with

\[ |\mathcal{M}|^2 = 4p_0^2 \left| \frac{V_h}{A_h} - \frac{V_s}{A_s} \right|^2. \]

(46)

The reference [12] has studied various splitting effects and it was shown that only the production of the vector particles gaining the mass during the phase transition can lead to a friction effect growing with the Lorentz factor \( \gamma \). Let us apply this formalism for the case of the QED-like theory. In other words let us consider the process \( \psi \rightarrow A\psi \), where the fermion splits into a vector boson and the fermion. This process, which is obviously forbidden by momentum conservation in the absence of the wall, can happen when the wall is present and the matrix element becomes equal to

\[ V_h = V_s = \frac{\sqrt{2k_z}}{x}, \]
\[ |\mathcal{M}_V|^2 = \frac{8p_0^2k_z^2}{x^2} \left| \frac{A_h - A_s}{A_h - A_s} \right|^2 = \frac{8p_0^2m_V^2}{(k_z^2 + m_V^2)^2k_z^2}, \]

(47)

(we are looking at \( \psi \rightarrow \psi A^T \) at the production of the transversely polarized vector bosons). Focusing on the limit \( k_z \sim m \ll k_0 \sim k_z \), we recover the following expression for the pressure

\[ \mathcal{P}_{\psi \rightarrow A\psi} = \int \frac{d^3p}{8p_0^2(2\pi)^6}f_p \int \frac{dk_0^{(2)}}{k_0^{(2)}} \int d^2k_\perp |\mathcal{M}|^2 \frac{k_\perp^2 + m_V^2}{2p_0x}. \]

(48)

Plugging in our expression for the matrix element (47) we get:

\[ \mathcal{P}_{\psi \rightarrow A\psi} = \int \frac{d^3p}{8p_0^2(2\pi)^6}f_p \int \frac{dk_0^{(2)}}{k_0^{(2)}} \int d^2k_\perp \frac{8p_0^2m_V^4}{(k_z^2 + m_V^2)^2k_z^2} \frac{k_\perp^2 + m_V^2}{p_0x} \]
\[ = \int \frac{d^3p}{p_0(2\pi)^6}f_p \int dx \frac{x^2}{k_z^2} \frac{m_V^4}{k_z^2 + m_V^2} \]
\[ = \int \frac{d^3p}{p_0(2\pi)^6}f_p \pi m_V^2 \log(m_V^2/(eT)^2) \times \left[ \int \frac{dx}{x^2} = \frac{p_0}{m_V} \right] \]
\[ = \int \frac{d^3p}{(2\pi)^3p_0}f_p \left[ \frac{m_Vp_0}{8\pi^2} \log(m_V^2/(eT)^2) \right]. \]

(49)
Let us make a few comments regarding this expression. We can see that in the wall frame the pressure is proportional to $\Delta P \propto \int d^3 p f_p$, however $d^3 p$ is not invariant under the boost and in the plasma frame it will lead to the additional $\gamma$ factor

$$\Delta P \propto \gamma T^3 m_V.$$ (50)

Another important point we would like to stress is that the minimal value of the transverse momenta is cut in the IR at the scale $k_{\perp} \sim eT$, due to the screening of the long wavelength modes by the temperature effects ($e$ is the gauge coupling). We can see that the pressure is dominated by the emission of the soft photons, which provides the $\gamma$ enhancement. In the next subsection we will rederive the same result using semi-classical equivalent photon approximation.

### 4.1 Equivalent photon approximation

It is well-known that the effect of the soft and collinear photons can be taken into account using the equivalent photon approximation (EPA)\(^{16, 17, 18, 19}\) (see for review \cite{32, 33, 34}). In other words, an initial fermion state can be thought as a state made of photons and fermions with the photons distributed according to the Weizsacker-Williams parton distribution function

$$f_\gamma(x) = \frac{e^2}{8\pi^2} \log \frac{m^2_V}{(eT)^2} \left[ 1 + \frac{(1 - x)^2}{x} \right],$$ (51)

where we are using the information from Eq. 49 that the pressure is dominated by the $k_{\perp} \lesssim m_V$ and the minimal value of transverse momenta scales as $\sim eT^2$. We also know that a photon with a phase-dependent mass going through the wall will lose (deposited in the wall) $z$-momenta, of the order $\Delta p_z \sim m^2 V^2 p x$. Then the pressure can be trivially estimated to be

$$P_{\text{reflection}}^{eq, \gamma} = \int \frac{d^3 p}{(2\pi)^3} f_p \int_{m_V/p}^{1} dx f_\gamma(x) \times \frac{m^2_V}{2px}$$

$$= \int \frac{d^3 p}{(2\pi)^3} f_p \times \left[ \frac{e^2}{8\pi^2} m^2_V \right] \log \frac{m^2_V}{e^2 T^2}$$ (52)

which leads to the exactly same result as the expression in Eq. 49. Intuitively the $\gamma$ factor in the pressure comes from the two following effects: both the photon distribution function as well as momentum transfer to the wall are enhanced by the factor $1/x$, which together allows to enhance the pressure by the additional factor $p_0/m_V \sim \gamma$. One may wonder what will be the effect of the particles which do not have enough energy to pass through the wall, since for them the photon distribution function will be even larger. However, in that case the momentum transfer to the wall will scale as $p_0 x$, so that the pressure will scale as

$$P_{\text{reflection}}^{reflection} = \int \frac{d^3 p}{(2\pi)^3} f_p \int dx f_\gamma(x) \times 2p_0 x$$

$$= \int \frac{d^3 p}{(2\pi)^3} f_p \times \left[ \frac{e^2}{2\pi^2} (x_{\text{max}} - x_{\text{min}}) p_0 \right] \log \frac{m^2_V}{e^2 T^2}$$ (53)

$$x_{\text{min}} \sim k_{\perp}/p_0 \sim T/p_0, \quad x_{\text{max}} \sim m_V/p_0$$ (54)

One can also argue that $k_{\perp} \lesssim m_V$ by noting that in the limit $px, k_{\perp} \gg m_V$ four momentum is approximately conserved, which should strongly suppress the splitting.
leading to the pressure from reflection

\[ \mathcal{P}_{1 \to 2}^{\text{reflection}} \simeq \int \frac{d^3p}{(2\pi)^3} f_p \left[ \frac{e^2 m_V}{2\pi^2} \log \frac{m_V^2}{e^2 T^2} \right]. \] (55)

We have again the friction effect growing with the Lorentz factor \( \gamma \). However, note that our calculation becomes questionable in this regime, since we need \( p_0 x L \gg 1 \) in order to remain in the WKB validity range.

We can generalize the Eq.52 for arbitrary splitting and the resulting pressure will be

\[ \mathcal{P}_{A \to BC} = \int \frac{d^3p}{(2\pi)^3} f_p \int_{m_{B/p}}^1 dx \frac{m_B^2}{2px} \frac{\alpha}{2\pi} \log \frac{m_B^2}{e^2 T^2} P_{B \to A}(x) \] (56)

where \( B \) is the soft particle and \( P_{B \to A}(x) \) are Altarelli-Parisi [35] splitting functions. Then it is obvious that a friction proportional to \( \propto \gamma \) can appear only from the splitting when the splitting functions scale as \( 1/x \) for small values of \( x \). This is the case only when the soft final state is a vector boson, which confirms the results of [12].

The expression of the pressure in Eq. 56 was derived assuming single soft vector boson emission, and it corresponds to the solution of the DGLAP equations [35, 36, 37]

\[ \frac{df_B(x, Q)}{d\log Q} = \frac{\alpha}{\pi} \int_x^1 dz \frac{P_{B \to A}(z)}{z} f_A \left( \frac{x}{z}, Q \right), \] (57)

up to the order \( \mathcal{O}(\alpha^2) \) starting with initial conditions at the scale \( Q = eT \)

\[ f_B(x, eT) = 0, \quad f_A(x, eT) = \delta(x - 1). \] (58)

The multiple emissions can be taken into account by solving the system of the DGLAP equations, however these will lead to only higher order in \( \mathcal{O}(\alpha \log \frac{m_V e}{eT}) \) corrections. We observe that in absence of log-enhancement \( \log \frac{m_V e}{eT} \sim \mathcal{O}(1) \), any multiple emission will be suppressed by powers of the coupling \( \alpha \) with respect to the single-emission result.

### 4.2 Another calculation of the NLO friction effects

Recently there was another calculation [38\textsuperscript{3}] of the friction which tried to take into account effects of the soft emission. The resulting friction pressure for the fermion emitting soft vector bosons was found to scale as

\[ \mathcal{P}^{[38]} \sim \alpha \gamma^2 T^4. \] (59)

However note that Eq. 59 does not have the correct \( m_\psi, m_V \to 0 \) limit (vanishing masses of the fermion and the vector boson). Indeed in the case when both \( m_\psi, m_V = 0 \) the particles do not interact with the wall and it becomes completely transparent. However the particles which do not interact with the bubble wall cannot induce any friction so that \( \mathcal{P}_{\text{friction}} \mid_{m_\psi, m_V \to 0} \to 0 \). This signals the inconsistency of the Eq. 59. On general grounds the inconsistency of Eq. 59 can be seen directly by noting that there is no dependence on the order parameter differentiating two phases separated by the bubble wall.

\textsuperscript{3}We acknowledge J. Turner and A. Long for discussions on the results of Ref. [38].
5 Summary

In summary, we recapitulate the main results of this paper. We have studied the friction forces acting on the relativistically expanding bubble at leading and next-to-leading order in the coupling $\alpha$. We have shown that generically new heavy particles, even if they are completely decoupled at the scale of the phase transition, can provide a significant contribution to the friction force. This effect can significantly modify the dynamics of the bubble wall expansion and in particular it can prevent the runaway behaviour of the bubble expansion, which results in different stochastic gravitational backgrounds. We have illustrated the effect using a toy model example where we show that new states, as heavy as $10^{15}$ GeV, can be active source of friction and prevent the infinite acceleration of the bubbles.

Beside this new result we have reviewed the NLO friction results of the Ref.[12], where it was shown that the soft vector boson emission leads to a new component of the friction pressure which scales proportionally to $\propto \gamma$. We have presented an alternative derivation of this effect using the equivalent photon approximation, which provides an intuitive picture of the origin of the friction $\propto \gamma$ as well as commented on the importance of the higher order effects and the ways to include them in the calculation.

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A Transition pressure

In this appendix we will review, for the sake of completeness, the pressure from transition of the particles through the wall. We will focus on the limit $\gamma \gg 1$ and will always assume that the mean free path of the particles is larger than the wall width. In this case we can treat particles quasi-classically and consider only individual interactions with the wall. This discussion is not new and was already presented in the papers [13, 14] and recently reviewed in [15](where the analytical results for the pressure have been reported). The particle will follow the usual thermal distribution, which in the frame of the wall becomes

$$f(E, p, T) = f\left(\frac{p_\mu u^\mu}{T}\right) = f\left(\frac{\gamma(E + vp_z)}{T}\right),$$

(60)

where we have assumed like in Fig.1 that the wall moves along the positive $z$ direction with velocity $v$. We will assume that the particle is incident on the wall with mass $m_1$ and on the other side it has mass $m_2$. The pressure on the wall is originating from the following three processes (we follow closely the notations of [13]).

- Reflection from the wall, when the incident particle does not have enough momentum or energy to pass through the wall:

$$\Delta P^r = \frac{2}{4\pi^2} \int_{m_1}^{m_2} dE \int_{0}^{\sqrt{E^2 - m_1^2}} dp_z \left[p_z^2 f\left(\frac{\gamma(E + vp_z)}{T}\right)\right]$$

$$+ \frac{2}{4\pi^2} \int_{m_2}^{\infty} dE \int_{\sqrt{m_2^2 - m_1^2}}^{0} dp_z \left[p_z^2 f\left(\frac{\gamma(E + vp_z)}{T}\right)\right]$$

(61)

in this case the momentum transfer to the wall is $\Delta p_z = 2p_z$. 

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• Transition through the wall, the pressure is generated due to the change of momenta of the particle with \( \Delta p_z = p_z + \sqrt{p_z^2 - (m_2^2 - m_1^2)} \):

\[
\Delta P^{t+} = \frac{1}{4\pi^2} \int_{m_2}^\infty dE \int_{-\sqrt{E^2 - m_2^2}}^{\sqrt{E^2 - m_2^2}} dp_z \left[ p_z(p_z + \sqrt{p_z^2 - (m_2^2 - m_1^2)}) f \left( \frac{\gamma(E + vp_z)}{T} \right) \right]
\]

(62)

• Transition in the opposite direction with \( \Delta p_z = \sqrt{p_z^2 + m_2^2 - m_1^2} - p_z \):

\[
\Delta P^{t-} = \frac{1}{4\pi^2} \int_{m_2}^\infty dE \int_{0}^{\sqrt{E^2 - m_2^2}} dp_z \left[ p_z(\sqrt{p_z^2 + m_2^2 - m_1^2} - p_z) f \left( \frac{\gamma(E + vp_z)}{T} \right) \right]
\]

(63)

Let us start by considering the transition pressure Eq.62. Introducing the new variables

\[
Y \equiv \frac{\gamma(E + vp_z)}{T}, \quad k \equiv -p_z T,
\]

the expression for the pressure becomes:

\[
\Delta P^{t+} = \frac{T^4}{4\pi^2\gamma} \int_{\sqrt{m_2^2 - m_1^2}/T}^\infty dk (k - \sqrt{k^2 - (m_2^2 - m_1^2)/T^2}) \int_{\gamma(k^2+m_2^2/T^2-vk)}^\infty f(Y) dY
\]

\[
= -\frac{T^4(m_2^2 - m_1^2)}{8\pi^2\gamma} \int_{\sqrt{m_2^2 - m_1^2}/T}^\infty dk \int_{\gamma(k^2+m_2^2/T^2-vk)}^\infty f(Y) dY,
\]

(64)

where we have expanded the momentum difference \( (k - \sqrt{k^2 - (m_2^2 - m_1^2)/T^2}) \) in the large \( k \) limit. We can see that the integral is non-vanishing if the lower limit of the second integral is small

\[
\left\{ \gamma(\sqrt{k^2 + m_2^2/T^2} - vk) \right\} \sim \frac{k}{2\gamma} + \frac{m_1^2 \gamma}{2T^2 k} \lesssim O(1) \Rightarrow m_1 \lesssim T.
\]

(65)

Otherwise the pressure effects will be strongly suppressed by the Boltzmann factor \( \exp[-m_1 T] \), which is obvious, since the energy of the massive particle is always larger than its mass. On top of this, looking at the lower limit of the \( k \) integration is we can conclude that

\[
Y \lesssim 1 \Rightarrow \sqrt{m_2^2 - m_1^2} \lesssim \gamma T,
\]

(66)

which is just the necessary condition for the particle to pass through the wall. Combining these two conditions we observe that the friction is efficient if only

\[
m_2 < \gamma T, \quad m_1 < T.
\]

(67)

Performing the integration we will obtain for the friction

\[
\Delta P^{t+}\big|_{\gamma T/m_2 \to \infty} = \frac{m_2^2 - m_1^2}{24} T^2.
\]

(68)

Using a similar analysis we can argue that the reflection pressure and the transmission from the opposite side are vanishingly small in \( \gamma \to \infty \) limit. Indeed setting \( m_1 \to 0 \) for simplicity
and using the same variable redefinition as in Eq. 65 we will get

\[
\Delta \mathcal{P}_r = I_1 + I_2 \\
I_1 = \frac{2}{4\pi^2} \int_0^{m_2} dE \int_{-\sqrt{E^2-m_1^2}}^0 d\rho \left[ p_z^2 f \left( \frac{\gamma(E+\rho p_z)}{T} \right) \right] \\
= \frac{T^4}{2\pi^2\gamma} \int_0^{m_2/T} dk k^2 \int_{\frac{k}{\gamma}}^{\gamma(m_2/T-k)} dY f(Y) \propto \gamma^{-1} \rightarrow 0 \\
I_2 = \frac{T^4}{2\pi^2\gamma} \int_0^{m_2/T} dk k^2 \int_{\gamma(m_2/T-vk)}^{\infty} f(Y) dY \propto \gamma^{-2} \rightarrow 0.
\]

(70)

At last the pressure from the transition in the opposite direction \( \Delta \mathcal{P}_{t-} \) is always suppressed since the argument of the distribution function is always larger than one \( \sim \frac{\gamma m}{T} \gg 1 \). We confirm these estimates using our numerical calculation illustrated on the Figure 3, where we plotted the various contributions to the total pressure. For the various contributions to the pressure the following approximate relations are true in the mass range \( \frac{m}{T} \sim 1 - 10 \):

\[
\mathcal{P}_r \approx \mathcal{P}_z \approx 0.4 \times \mathcal{P}_{\gamma \rightarrow \infty} \text{ for } \gamma T = m_0 \\
\mathcal{P}_{t-} \approx \mathcal{P}_z \approx 0, \quad \mathcal{P}_{t+} \approx 0.9 \times \mathcal{P}_{\gamma \rightarrow \infty} \text{ for } \gamma T = 10m_0.
\]

(71)  \( \quad \text{(72)} \)

B Examples of the friction induced by the heavy particles

In Section 3, we have shown that the mixing of a light and a heavy fermion can lead to the friction, which we called mixing pressure. We can find a similar effect in the theories with scalars fields only. In this appendix we will present two such examples of the non-vanishing pressure from the heavy fields. Let us start by considering the following model:

\[
\mathcal{L} = \frac{1}{2} (\partial s)^2 + \frac{1}{2} (\partial \phi)^2 - B s^2 \phi - \frac{M^2 \phi^2}{2},
\]

(73)

where the phase transition occurs along the \( s \) field direction and there is a hierarchy between the VEV of the \( s \) field and the mass of the \( \phi \), \( M \gg \langle s \rangle \). In this case, following the lines of Section 3 the mixing between the \( s \) field and the heavy mass eigenstate inside the wall will scales as

\[
\theta_{s-\phi} \sim \frac{2B\langle s \rangle}{M^2}
\]

(74)
which leads to the friction effect

\[ P_{\text{mixing}} \sim T^2 \frac{B^2(s)^2}{M^2} \theta(\gamma T - M) \] (75)

Note that the friction is suppressed by a factor \( \frac{B^2}{M^2} \) with respect to the pressure induced by fermionic mixing. This suppression disappears in the limit \( B \rightarrow M \), which is the maximal value of \( B \) allowed by the technical naturalness arguments.

Another example of friction from heavy particles effect can be observed in the following model:

\[ \mathcal{L} = \left( \partial s \right)^2 + \frac{\partial \phi}{2} - V(s) - \frac{M^2 \phi^2}{2} - \lambda \phi^2 s^2, \] (76)

where again we will be interested in the limit \( M \gg \langle s \rangle \). We will consider the process \( s \rightarrow \phi \phi \), where again \( s \) is a field getting the VEV, and \( \phi \) is a heavy field. Following the procedure outlined in the section 4 we will get for the matrix element

\[ A = -k^2 \theta(x(1 - x)) + m_s^2 - \frac{M^2}{1 - x} - \frac{M}{x} \]

then the pressure will be:

\[ P \sim \int \frac{d^3 p}{p^0} f_p \int \frac{dx}{x(1 - x)} \int \frac{dk^2}{(k^2 + M^2)^2} \left[ \frac{k^2 + M^2}{2p_0 x(1 - x)} \right] \theta(p_0 - 2M) \]

\[ \sim \int \frac{d^3 p}{p_0} f_p \lambda^2(s)^2 \int dx \int \frac{dk^2}{k^2 + M^2} \theta(p_0 - 2M) \] (78)

where the \( \theta \) function appears from the trivial requirement that we need enough energy to produce the two heavy states. Thus the pressure becomes

\[ P \sim \int \frac{d^3 p}{p_0} f_p \lambda^2(s)^2 \theta(p_0 - 2M) \propto \lambda^2(s)^2 T^2 \theta(\gamma T - 2M), \] (79)

so that again the friction is not suppressed by the large mass of the field \( \phi \).

\section*{C Friction from higher dimensional operators}

In this appendix, we briefly sketch the pressure that could be generated by higher dimensional operators. In general if the integral for the pressure in Eq.5 is UV dominated then it becomes trivial to estimate the \( \gamma \) of such contribution to the pressure. Indeed, all one has to do is to estimate the dependence of the \( |\mathcal{M}|^2 \) on the coupling constants and VEV’s. Using dimensional analysis for the n-point interaction, we obtain

\[ [\mathcal{M}] = 3 - n \] (80)

where brackets \([\]\) denote the dimensionality of the operator inside the brackets. On general grounds, we know that

\[ \mathcal{M} = \Lambda^p E^{3-n-p} \] (81)

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where Λ is the combination of the couplings and masses which are different on two sides of the wall and $E$ is energy of the process. Plugging this expression in the master formula for the pressure we get

$$\mathcal{P} \sim \int \frac{d^3p}{p_0} f_p [\Lambda]^{2p} \frac{2}{p_0^{2-2p}} \sim \gamma^{2-2p}.$$  (82)

From this expression, we can distinguish two extremal cases; along the wall the interaction is constant (or phrased differently, the interaction is identical on both sides of the wall) and only the WKB phases change or we consider that the interaction is only possible in the broken phase, and thus the interaction switches on along the wall.

- **Constant vertex** Let us consider the case when the interactions do not change during the passage through the wall. Then following Eq. 46 the matrix element in UV will scale as

  $$|M| \propto \Delta m^2 \times \text{coupling}. \quad (83)$$

  Consequently, we can see that UV contributions can not give contributions rising with $\gamma$ unless we are dealing with the couplings with $-2$ dimension (dimension six operators).

  $$\mathcal{P}^\text{mass \ dim 6} \sim \frac{\Delta m^4}{\Lambda^4} T^4 \gamma^2.$$  (84)

- **Varying vertex** The other possibility is when the interaction is modified during the passage through the wall (this is for example the case of the scalar splitting in two). In this case the matrix element scales as

  $$|M| \propto \Delta V. \quad (85)$$

  Then we can get the following effect in the friction

  $$\mathcal{P}^\text{vertex} \sim \Delta V^2 T^4 - 2[\Delta V] \gamma^{2-2[\Delta V]}.$$  (86)

To illustrate this effect in the second case, let us consider the Weinberg-like operator,

$$\Delta \mathcal{L} = \frac{g \psi^2 s^2}{\Lambda}, \quad (87)$$

where $\psi$ is a fermion and $s$ is scalar degree of freedom. Then only inside the bubble there will be a Yukawa interaction

$$\Delta \mathcal{L}^\text{broken phase} = 2g \psi^2 s \times \frac{\langle s \rangle}{\Lambda}. \quad (88)$$

Thus we expect the contribution to the pressure to scale as

$$\mathcal{P} \sim \left( \frac{\langle s \rangle}{\Lambda} \right)^2 \gamma^2 T^4.$$  (89)

Of course this effect estimate is valid only up to the velocities satisfying $\gamma T \lesssim \Lambda$, as long as the EFT description remains valid. As a consequence, we expect the pressure to saturate around

$$\mathcal{P}^\text{max} \sim \langle s \rangle^2 T^2,$$  (90)

which of the same order as the friction from mixing in Eq.15.
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