Tensionless Strings:  
Physical Fock Space and Higher Spin Fields

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Abstract

I study the physical Fock space of the tensionless string theory with perimeter action, exploring its new gauge symmetry algebra. The cancellation of conformal anomaly requires the space-time to be 13-dimensional. All particles are massless and there are no tachyon states in the spectrum. The zero mode conformal operator defines the levels of the physical Fock space. All levels can be classified by the highest Casimir operator $W$ of the little group $E(11)$ for massless particles in 11-dimensions. The ground state is infinitely degenerated and contains massless gauge fields of arbitrary large integer spin, realizing the irreducible representations of $E(11)$ of fixed helicity. The excitation levels realize CSR representations of little group $E(11)$ with an infinite number of helicities. After inspection of the first excitation level, which, as I prove, is a physical null state, I conjecture that all excitation levels are physical null states. In this theory the tensor field of the second rank does not play any distinctive role and therefore one can suggest that in this model there is no gravity.
1 Introduction

A string model which is based on the concept of surface perimeter or length was suggested in [1]. It describes random surfaces embedded in D-dimensional space-time with the following action

\[ S = m \cdot L = \frac{m}{\pi} \int d^2 \zeta \sqrt{h} \sqrt{K_{ab} K^{ab}}, \]  

where \( m \) has dimension of mass, \( h_{ab} \) is the induced metric and \( K_{ab} \) is the second fundamental form (extrinsic curvature). There is no Nambu-Goto area term in this action. Because the action has dimension of length \( L \), alternative to the area, we have introduced a new dimensional parameter \( m \) which is very similar to the mass parameter in the action for relativistic point particle. This observation can be justified better if one takes a limit in which a surface shrinks into a single world-line, in that case the action (1) reduces to its length

\[ S \rightarrow m \int ds, \]  

and allows to identify a new parameter \( m \) with the quantity which is in clear analogy with the point particle mass. Geometrically, the action is a natural extension of the geometrical concept of length, which is well defined for space-time curves, into two-dimensional surfaces.

The best way to understand the concept of surface perimeter is to use simple analogy with general relativity. In gravity the action measures the area of the four-dimensional universe alternative to its four-volume. The action includes scalar curvature term \( [R] = cm^{-2} \) and together with the volume element \( [\sqrt{g} d^4 X] = cm^4 \) has dimension \( cm^2 \). The coupling constant in front of the action should be therefore of dimension mass square: \( m^2 = 1 / G_N \). In our case dimension of extrinsic curvature is \( [K] = cm^{-1} \) and volume element has dimension \( [d^2 \zeta \sqrt{h}] = cm^2 \), therefore our coupling constant has dimension of mass.

At the classical level this model is tensionless because for the flat Wilson loop the action is equal to its perimeter \( S = m(R + T) \) [1]. One can guess therefore that the spectrum of the theory is massless or even continuous, but because of string rigidity there are nontrivial vibrational modes and the spectrum is massless. The solution of this nonlinear, high-derivative, two-dimensional world-sheet CFT is [22]:

\[ X^\mu_\mathcal{L} = x^\mu + \frac{1}{m} \pi^\mu \zeta^+ + i \sum_{n \neq 0}^\frac{1}{n} \beta^\mu_n e^{-in\zeta^+}, \]

\[ \Pi^\mu_\mathcal{L} = me^\mu + k^\mu \zeta^+ + i \sum_{n \neq 0}^\frac{1}{n} \alpha^\mu_n e^{-in\zeta^+}, \]

where \( k^\mu \) is momentum operator and \( \alpha_n, \beta_n \) are oscillators with the following commutator relations \( [x^\mu, k^\nu] = i \eta^{\mu\nu}, \ [\alpha^\mu_n, \beta^\nu_k] = n \eta^{\mu\nu} \delta_{n+k,0} \). The similar expansion exists also for the right moving modes \( \tilde{\alpha}_n, \tilde{\beta}_n \). The essential difference between tensionless and standard string theories is the appearance of additional zero mode \( [e^\mu, \pi^\nu] = i \eta^{\mu\nu}, \) meaning that the wave function is a function of two continuous variables \( k^\mu \) and \( e^\mu \):

\[ \Psi_{\text{phys}} = \Psi(k, e). \]

\[ ^1 \text{In [4] the action is proportional to the spherical angle and has dimensionless coupling constant.} \]

\[ ^2 \text{This model also differs from the tensionless string models based on Schild's work on "null" string [5, 6, 7, 8] with its most likely continuous spectrum [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].} \]
It was suggested in [22] that $e^\mu$ should be interpreted as a polarization vector. This fact has very important physical consequences. In particular it leads to high degeneracy of all physical states.

Our aim is to study spectral properties of the model and the structure of its physical Fock space in analogy with the standard string theory [23, 24, 25, 26, 27, 28]. This is quite possible using the above mode expansion of the fields and exploring corresponding new gauge symmetries. In particular one should be able to prove that the physical Fock space is positive definite, i.e. ghost-free.

This tensionless theory is invariant under new extended symmetries [22]. It is invariant with respect to the standard conformal transformation with generators

$$L_n = \sum_l : \alpha_{n-l} \cdot \beta_l :$$

and with respect to the infinite-dimensional Abelian transformation (11),(18) generated by the operator $\Theta = \Pi^2 - m^2$ with the following components (10),(32),(37):

$$k^2, \ k \cdot e, \ k \cdot \alpha_n, \ k \cdot \bar{\alpha}_n, \ \Theta_{nl}.$$ 

Therefore in covariant quantization scheme the space-time equations which define physical Fock space have the following form [22]:

$$\begin{pmatrix}
  k^2 \\
  k \cdot e \\
  k \cdot \alpha_n \\
  k \cdot \bar{\alpha}_n \\
  \Theta_{nl} \\
  L_n \\
  \bar{L}_n
\end{pmatrix} \Psi_{\text{phys}} = 0, \quad n, l = 0, 1, 2, .... \quad (4)$$

As one can clearly see from the first equation in (4) all particles, with arbitrary large spin, are massless. This pure massless spectrum is consistent with the tensionless character of the theory and, what is very important, also tells us that there are no tachyon states.

Our aim is to describe physical meaning of different operators in this system and analyze solutions of these space-time equations in details. They are similar to the Virasoro equations in Gupta-Bleuler quantization of standard string theory:

$$(L_0 - 1)\Psi_{\text{phys}} = 0, \quad L_n \Psi_{\text{phys}} = 0 \quad n = 1, 2, ....$$

where the first equation is a generalization of Klein-Gordon equation because $L_0 = \alpha' k^2 + \hat{N}$ and $\hat{N} = \frac{1}{2} \sum_{n \neq 0} \alpha_{-n} \alpha_n$ is a mass level operator. The rest of Virasoro equations allow to prove the absence of negative norm states on every mass level of physical Fock space and require the space-time to be 26-dimensional: $D_c = 26$.

In our previous consideration of the tensionless strings we found that the critical dimension is $D_c = 13$ [22]. This follows from the cancellation of the conformal anomaly. The contribution to the central charge from bosonic coordinates is $2 \times \frac{D_{12}}{12} = \frac{D}{6}$, which is twice bigger than in the standard bosonic string theory. The contribution of Faddeev-Popov ghosts to the central charge remains the same, therefore the absence of conformal anomaly requires the space-time to be 13-dimensional.
It is also important that \( L_0 \) in our case has different meaning, because it has the form

\[ L_0 = k \cdot \pi/m + \hat{\Xi}, \]

where \( \hat{\Xi} = \sum_{n \neq 0} \alpha_n \beta_n \). We shall define \( \hat{\Xi} \) as our new level operator with eigenvalues \( \Xi = 0, 1, 2... \) and shall identify the operator \( k \cdot \pi/m \) with the length of the highest Casimir operator of the Poincaré group.

A natural question arose, whether the physical Hilbert space on every level \( \Xi \) is positive-definite, i.e. ghost-free in analogy with the well known No-ghost theorem [23, 24, 25, 26, 27, 28] in the standard string theory. We shall demonstrate that first two levels \( \Xi = 0 \) and \( \Xi = 1 \), ground and excitation states, are well defined and that there are no negative norm waves. The vacuum state \( \Xi = 0 \) is infinitely degenerated and contains massless particles of increasing tensor structure \( A^{\mu_1...\mu_s}(k) \ s = 1, 2, ... \). After inspection of the first level \( \Xi = 1 \), which happens to be a physical null state, we conjecture that all excitation levels \( \Xi = 1, 2, 3, ... \) are physical null states.

Let us first describe the ground state, \( \Xi = 0 \). The wave function \( \Psi_0(k,e) \equiv |k,e,0> \) is defined as \( \alpha_n |0,k,e> = \beta_n |0,k,e> = 0, \ n = 1, 2, .. \) and the system of equations (4) reduces to the following four equations [22]:

\[ k^2 \Psi_0 = 0, \quad e \cdot k \Psi_0 = 0, \quad (e^2 - 1) \Psi_0 = 0, \quad \frac{k \cdot \pi}{m} \Psi_0 = -\Xi \Psi_0 = 0. \] (5)

It follows from the third equation that \( \Psi_0 \) is a function defined on a unit sphere \( e^2 = 1, \eta^{\mu\nu} = (- + ... +) \) and can be expanded in the corresponding bases:

\[ \Psi_0 = |0,k,e> = A(k) + A^{\mu_1}(k) e_{\mu_1} + A^{\mu_1,\mu_2}(k) e_{\mu_1} e_{\mu_2} + ... |0,k> . \]

We clearly see that because we have additional variable \( e_\mu \) the vacuum state in infinitely degenerated. The fields \( A^{\mu_1...\mu_s}(k) \) describe massless particles of increasing tensor structure. They are symmetric traceless tensor fields because they are harmonic functions on a sphere \( e^2 = 1 \), and it follows from the last equation in (5), that \( k_{\mu_1} A^{\mu_1...\mu_s}(k) = 0 \). Thus the fields \( A^{\mu_1...\mu_s} \) are: 1) symmetric traceless tensors of increasing rank \( s = 0, 1, 2, ... \), 2) divergent free \( k_{\mu_1} A^{\mu_1...\mu_s} = 0 \) and 3) satisfying massless wave equations \( k^2 A^{\mu_1...\mu_s} = 0 \). All the above conditions are sufficient to describe integer spin fields [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]. Thus the ground state is infinitely degenerated and contains gauge particles of arbitrary large fixed helicity. Every tensor gauge field \( A^{\mu_1...\mu_s}, s = 0, 1, 2, .. \) appears only once in the spectrum. The infinite degeneracy of the vacuum state reflects enhanced symmetry of the model. In some sense the leading Regge trajectory degenerates into a single ground state and one can only speculate that it may describe unbroken phase of standard string theory when \( \alpha' \rightarrow \infty \) [2, 3].

Two remarks are in order. The first remark is connected with the infinite degeneracy of the vacuum state. It has been found that statistical system of random surfaces with the same perimeter energy functional (1) can be formulated on a cubit lattice, and is equivalent to a spin system with competing ferromagnetic and anti-ferromagnetic interaction [50]. It is a remarkable fact that this spin system also has exponential degeneracy of the vacuum and excitation states of order \( 2^N \), where \( N \) is the size of the lattice. This degeneracy should be compared with only double degeneracy of the vacuum state of the Ising model with area energy functional.
The second remark is connected with the four vacuum equations (5), they actually coincide with the four equations introduced by Wigner in 1963 [43, 44, 45, 46, 47, 48, 49] in order to describe the irreducible representations of the little group of the Poincaré group. Therefore these equations naturally follow from the tensionless theory, which is formulated in terms of Lagrangian mechanics and action principle. They describe representations of massless particles of fixed helicity. For these representations the highest Casimir operator \( W = (k \cdot \pi / m)^2 = \Xi^2 \) of the little group \( E(11) \), which is Euclidean group of \( SO(11) \) rotations and displacements \( T(11) \) for massless particles in 11-dimensions, is equal to zero: \( \Xi = 0 \). Alternatively, the excitation levels realize the second class of infinite dimensional representations \( 0(\Xi) \) for which \( \Xi \neq 0 \) and describe states with an infinite number of helicities. As we shall see in tensionless theory they are physical null states and therefore do not produce explicit contradiction with the obvious experimental fact that there are no particles with infinite number of helicities.

Indeed, the first excitation state \( \Xi = 1 \) of the closed string depends on four polarization tensors \( \xi_{\mu\nu}(e), \omega_{\mu\nu}(e), \zeta_{\mu\nu}(e) \) and \( \chi_{\mu\nu}(e) \). The restriction on these tensors follows from our space-time equations (4). Solving them we shall see that \( \chi_{\mu\nu}(e) = 0, \omega^{\mu\nu} = \omega(e)k_\mu k_\nu = -\zeta^{\mu\nu} \) and \( \xi_{\mu\nu}(e) \) remain arbitrary, thus

\[
\Psi_1 = [ \xi_{\mu\nu}(e)\alpha_{-1}^\mu \alpha_{-1}^\nu + \omega_{\mu\nu}(e)(\alpha_{-1}^\mu \beta_{-1}^\nu - \beta_{-1}^\mu \alpha_{-1}^\nu) ]|0, k, e>. 
\]

The norm of this state is \( <\Psi_1|\Psi_1> = -\omega^* \omega^{\mu\nu} = -|\omega|^2 (k^2) \) and therefore is equal to zero! It is also normal to the ground state \( <\Psi_0|\Psi_1> = 0 \). Thus \textit{the first excitation is a physical null state} and realizes the infinite-dimensional representation \( 0(\Xi) \) with \( \Xi = 1 \). This consideration allows to make a conjecture that \textit{all excitations are physical null states} and therefore define gauge parameters of huge gauge group. In this respect new extended algebra of our space-time equations (4) should play a crucial role defining positive definite metric in physical Fock space on every level.

In the next sections we shall review solution of our basic world-sheet equations in conformal gauge together with the mode expansion of these fields (3) and calculation of the critical dimension in path integral formulation of the theory and shall formulate our space-time equations (4) in Gupta-Bleuler quantization scheme [22]. I shall also describe extended symmetries of the model in terms of new \( \Theta \) algebra (34), (35) and (36). It is an Abelian extension of conformal algebra and equations (33) suggest its oscillator representation. To this author it is not known, whether this algebra has appeared also somewhere else.

Our main goal is to solve these space-time equations for the first few levels and to find out the appropriate interpretation of the corresponding operators. We shall see that the vacuum is infinitely degenerated and every massless fixed helicity state appears only once in the physical spectrum and there are no tachyon states. In this model the tensor field of the second rank does not play any distinctive role and therefore one can suggest that in this model there is no gravity. The inspection of the first level shows that it is a physical null state defining gauge transformation of the ground state wave function.

In the last section I shall also present a short review of the finite- and infinite-dimensional representations of the little group of the Poincaré group defined by Wigner [43, 44]. These representations naturally appear in this tensionless string theory on different levels of the physical Fock space.
2 Equations and symmetries of the Model B

The gonihedric action (1) can be represented in the form [22]
\[
S = \frac{m}{\pi} \int d^2 \zeta \sqrt{h} \sqrt{\left(\Delta(h)X_\mu\right)^2} = \frac{1}{\pi} \int d^2 \zeta \, \Pi^\mu \sqrt{h} \Delta(h)X_\mu,
\]
(7)
here \( h_{ab} \) is the world-sheet metric, \( \Delta(h) = 1/\sqrt{h} \partial_a \sqrt{h} h^{ab} \partial_b \) is Laplace operator and I shall consider the model \( B \), in which two field variables \( X^\mu \) and \( h_{ab} \) are independent. This is nonlinear, high-derivative, two-dimensional world-sheet conformal field theory which can be solved exactly.

To get classical equations and world sheet energy-momentum tensor one should compute variation of the action with respect to the coordinates \( X^\mu \)
\[
(I) \quad \Delta(h) \Pi^\mu = 0 \tag{8}
\]
and the metric \( h^{ab} \)
\[
(II) \quad T_{ab} = \partial_{(a} \Pi^{b)} \partial^\mu X^\mu - h_{ab} h^{cd} \partial_c \Pi^d \partial_d X^\mu = 0, \tag{9}
\]
where we have introduced the operator \( \Pi^\mu \):
\[
\Pi^\mu = m \frac{\Delta(h)X^\mu}{\sqrt{\left(\Delta(h)X^\mu\right)^2}}.
\]
It has a property very similar to the constraint equation for a point-like relativistic particle but with the essential difference that it is a space-like vector,
\[
(III) \quad \Theta \equiv \Pi^\mu \Pi^\mu - m^2 = 0. \tag{10}
\]
The energy momentum tensor is conserved \( \nabla^a T_{ab} = 0 \) and is traceless \( h^{ab}T_{ab} = 0 \), thus we have high-derivative nonlinear two-dimensional world-sheet conformal field theory. Equations (8), (9) and (10) completely define the system. We have equation of motion (8) together with the constraint equations (9) and (10). We should stress that in addition to the conformal generators (9) we have an enhanced symmetry generators (10) responsible for the symmetry transformation[22]:
\[
\delta \left(\sqrt{h} \Delta(h)X^\mu\right) = \Omega \Pi^\mu, \quad \delta \Pi^\mu = 0, \quad \delta h = 0. \tag{11}
\]
Indeed the variation of the action is
\[
\delta S = \frac{1}{\pi} \int d^2 \zeta \, \Pi^\mu \delta \left(\sqrt{h} \Delta(h)X^\mu\right) = \frac{m^2}{\pi} \int d^2 \zeta \, \Omega,
\]
(12)
and is a total derivative because \( \Omega = \nabla^a \sqrt{h} \partial_a \). This transformation can be partially integrated out if we add the term \( \chi_a \sqrt{h} \partial^a \Pi^\mu \) to the transformation (11). The action is still invariant, because \( \Pi^2 - m^2 = 0 \) and \( \Pi^\mu \partial_a \Pi^\mu = 0 \). Then \( \delta(\sqrt{h} \Delta(h)X^\mu) = \Pi^\mu \nabla^a \sqrt{h} \partial_a + \sqrt{h} \partial_a \partial^a \Pi^\mu = \nabla^a (\Pi^\mu \sqrt{h} \partial_a \Pi^\mu \partial_a \) and we have our new symmetry transformation in the form
\[
\delta (\partial_a X^\mu) = \Pi^\mu \partial_a, \quad \delta \Pi^\mu = 0, \quad \delta h = 0. \tag{13}
\]
One can also check that \( \delta T_{ab} = 0 \). As we shall see, the theory avoids many obstacles of high derivative field theory, thanks to this extended symmetry.
3 Solution in Conformal Gauge

We can fix the conformal gauge $h_{ab} = \rho_{ab}$ using reparametrization invariance of the action and represent it in the form (see (7))

$$\dot{S} = \frac{m}{\pi} \int d^2 \zeta \sqrt{(\partial^2 X)^2} = \frac{1}{\pi} \int d^2 \zeta \; \Pi^\mu \partial^2 X^\mu, \quad \Pi^\mu = m \frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X)^2}}. \quad (14)$$

In this gauge the equations of motion are more simple

$$(I) \quad \partial^2 \Pi^\mu = 0 \quad (15)$$

and they should be accompanied by the constraint equations (9) and (10)

$$(II) \quad T_{ab} = \partial_a \Pi^\mu \partial_b X^\mu - \eta_{ab} \partial^\epsilon X^\mu = 0, \quad (16)$$

$$(III) \quad \Theta = \Pi^a \Pi^b - m^2 = 0.$$ (17)

In the light cone coordinates $\zeta^\pm = \zeta^0 \pm \zeta^1$ $T_{ab}$ takes the form

$$T_{++} = 2 \partial_+ \Pi^\mu \partial_+ X^\mu, \quad T_{--} = 2 \partial_- \Pi^\mu \partial_- X^\mu, \quad (17)$$

with trace equal to zero $T_{+-} = 0$. The conservation of the energy momentum tensor takes the form $\partial_+ T_{++} = \partial_- T_{--} = 0$ and requires that its components are analytic $T_{++} = T_{++}(\zeta^+)$ and anti-analytic $T_{--} = T_{--}(\zeta^-)$ functions. Thus our system has infinite number of conserved charges. This residual symmetry can be easily seen in gauge fixed action (14) written in light cone coordinates $\int \sqrt{(\partial_+ \partial_- X^\mu)^2} \; d\zeta^+ d\zeta^-$, it is invariant under the transformations $\zeta^+ = f(\tilde{\zeta}^+), \quad \zeta^- = g(\tilde{\zeta}^-)$ where $f$ and $g$ are arbitrary functions.

Our new symmetry transformation is

$$\delta(\partial_\mu X^\mu) = \Pi^\mu \omega^\mu, \quad \Pi^\mu \rightarrow \Pi^\mu. \quad (18)$$

The classical equation is $\partial^2 \Pi^\mu = 0$, therefore $\Pi^\mu$ is a function of the form

$$\Pi^\mu = m \frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X)^2}} = \frac{1}{2} (\Pi^\mu_L(\zeta^+) + \Pi^\mu_R(\zeta^-)). \quad (19)$$

One can find now that $\partial_+ \partial_- X^\mu = \frac{1}{2} [\Pi^\mu_L(\zeta^+) + \Pi^\mu_R(\zeta^-)] \Omega(\zeta^+, \zeta^-)$, where $\Omega(\zeta^+, \zeta^-)$ is arbitrary function of $\zeta^+$ and $\zeta^-$. Thus $X^\mu$ is a sum of inhomogeneous and homogeneous solutions

$$X^\mu = \frac{1}{2} [X^\mu_L(\zeta^+) + X^\mu_R(\zeta^-)] + \frac{1}{2} \int_0^{\zeta^+} \int_0^{\zeta^-} [\Pi^\mu_L(\tilde{\zeta}^+) + \Pi^\mu_R(\tilde{\zeta}^-)] \Omega(\tilde{\zeta}^+, \tilde{\zeta}^-) d\tilde{\zeta}^+ d\tilde{\zeta}^- \quad (20)$$

where $X^\mu_L(\zeta^+), \; X^\mu_R(\zeta^-)$ are arbitrary functions of $\zeta^+$ and $\zeta^-$. We have therefore two left $X^\mu_L(\zeta^+), \; \Pi^\mu_L(\zeta^+)$ and two right movers $X^\mu_R(\zeta^-), \; \Pi^\mu_R(\zeta^-)$. These degrees of freedom are twice bigger than in the standard string theory, this is simply because world-sheet equations here are of the forth order. The constraints (17) take the form

$$T_{++} = \frac{1}{2} \Pi^\mu_L(\zeta^+) \dot{X}^\mu_L(\zeta^+), \quad T_{--} = \frac{1}{2} \Pi^\mu_R(\zeta^-) \dot{X}^\mu_R(\zeta^-), \quad (21)$$

verifying the fact that they are indeed functions of only one light cone variable and are $\Omega$ independent.
4 Critical Dimension $D_c = 13$

The action (14) is invariant under the global symmetries $\delta X^\mu = \Lambda^{\mu\nu} X_\nu + a^\mu$, where $\Lambda^{\mu\nu}$ is a constant antisymmetric matrix, while $a^\mu$ is a constant. The translation invariance of the action (14) $\delta a X^\mu = a^\mu$ results into the conserved momentum current

$$P_a^\mu = \partial_a \Pi^\mu, \quad \partial^a P_a^\mu = 0, \quad P^\mu = \int P_0^\mu d\sigma \quad (22)$$

$(\zeta^1 \equiv \sigma)$ and Lorentz transformation $\delta\Lambda X^\mu = \Lambda^{\mu\nu} X_\nu$ into angular momentum current

$$M^{a\mu} = X^\mu \partial_a \Pi^\nu - X^\nu \partial_a \Pi^\mu + \Pi^a \partial_a X^\nu - \Pi^\nu \partial_a X^\mu, \quad \partial^a M^{a\mu} = 0, \quad M^{a\mu} = \int M_0^{a\mu} d\sigma. \quad (23)$$

Last two terms in $M^{a\mu}$ define internal angular momentum current and we shall find an appropriate physical interpretation for them considering representations of the Wigner’s little group of the Poincaré group in the next sections. They actually describe helicities of the massless particles. One can also check that $\delta M^{a\mu} = 0$ under transformation (13),(18).

From its definition the momentum density $P_0^\mu(\zeta^0, \zeta^1) = \partial_0 \Pi^\mu$ is conjugate to $X^\mu(\zeta^0, \zeta^1)$ and therefore $[X^\mu(\zeta^0, \zeta^1), P_0^\nu(\zeta^0, \zeta^1)] = i\eta^{\mu\nu}\delta(\zeta^1 - \zeta^1)$. From (14) we can deduce the propagator $<\Pi^\mu(\zeta) X^\nu(\zeta')> = \frac{i}{2} \eta^{\mu\nu} \ln(\zeta - \zeta')$. Using explicit solution (20) one can get

$$<\Pi^\mu_R(\zeta^-) X^\nu_R(\zeta^-) > = \eta^{\mu\nu} \ln[(\zeta^- - \zeta^-)\mu], <\Pi^\mu_L(\zeta^+) X^\nu_L(\zeta^+) > = \eta^{\mu\nu} \ln[(\zeta^+ - \zeta^+)\mu]$$. 

Now we are in a position to compute the two-point correlation function of the energy momentum operator:

$$<T \ T_{++}(\zeta^+) \ T_{++}(\tilde{\zeta}^+) > = \frac{1}{4} \frac{D}{(\zeta^+ - \tilde{\zeta}^+)^4}. \quad (24)$$

The ghost contribution to the central charge remains the same as for the standard bosonic string:

$$<T \ T_{++}^{gh}(\zeta^+) \ T_{++}^{gh}(\tilde{\zeta}^+) > = -\frac{13}{4} \frac{1}{(\zeta^+ - \tilde{\zeta}^+)^4},$$

therefore the absence of conformal anomaly requires the space-time to be 13-dimensional, close to the 11-dimensional space-time of M-theory [22]

$$D_c = 13. \quad (25)$$

This result can be qualitatively understood if one takes into account the fact that the field equations here are of the fourth order and therefore we have two left movers and two right movers of $X$ and $\Pi$ fields, thus two times more degrees of freedom than in the standard bosonic string theory $2 \times \frac{D}{8} = \frac{D}{4}$. We shall confirm this calculation also using mode expansion of conformal generators $L_n$ in the next section.

5 Mode Expansion and Quantization

Let us now find mode expansion of different operators in the general solution (19),(20). The appropriate boundary condition for closed strings is simply periodicity of the coordinates $X^\mu(\zeta^0, \zeta^1) = X^\mu(\zeta^0, \zeta^1 + 2\pi)$. The arbitrary periodic functions $X^\mu_L$ and $X^\mu_R$ can
The unusual term in this expression is $E$ where $X$ can be written as normal mode expansions in the form [22]:

$$X_L^\mu = x^\mu + \frac{1}{m} \pi^\mu \zeta^+ + i \sum_{n \neq 0} \frac{1}{n} \beta_n^\mu e^{-in\zeta^+},$$

$$X_R^\mu = x^\mu + \frac{1}{m} \pi^\mu \zeta^- + i \sum_{n \neq 0} \frac{1}{n} \bar{\beta}_n^\mu e^{-in\zeta^-},$$

where $X^\mu = \frac{1}{2}(X_L^\mu(\zeta^+) + X_R^\mu(\zeta^-))$, and in similar manner $\Pi^\mu = \frac{1}{2}(\Pi_L^\mu(\zeta^+) + \Pi_R^\mu(\zeta^-))$

$$\Pi_L^\mu = me^\mu + k^\mu \zeta^+ + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\zeta^+},$$

$$\Pi_R^\mu = me^\mu + k^\mu \zeta^- + i \sum_{n \neq 0} \frac{1}{n} \bar{\alpha}_n^\mu e^{-in\zeta^-}.$$  \hspace{1cm} (26)

Substituting the above mode expansion into the basic commutator $[X_{L,R}^\mu(\zeta^\pm), P_{L,R}^\nu(\zeta^\pm)] = 2\pi \eta^\mu\nu \delta(\zeta^\pm - \zeta')$ gives

$$[e^\mu, \pi^\nu] = [x^\mu, k^\nu] = i\eta^\mu\nu, \hspace{0.5cm} [\alpha_n^\mu, \beta_k^\nu] = n \eta^\mu\nu \delta_{n+k,0},$$

where $\alpha_0^\mu = k^\mu$, $\beta_0^\nu = \pi^\mu/m$ are zero modes momentum. We can also deduce a less trivial commutator [22]

$$[\partial_\pm X_{R,L}^\mu(\zeta^\pm), \Pi_{R,L}^\nu(\zeta') = -2\pi \eta^\mu\nu \delta(\zeta^\pm - \zeta'),$$

which tells us that $\Pi$ and $E \equiv -\dot{X}$ form a second pear of canonically conjugate variables of our higher derivative theory. All other commutators are equal to zero. It is important that commutators and momentum operator $\Pi$ are invariant under transformation (13),(18).

Using above mode expansion we can find the generators of the Lorentz group $M^\mu\nu$

$$M^\mu\nu = J^\mu\nu + E^\mu\nu + O^\mu\nu,$$ \hspace{1cm} (29)

where

$$J^\mu\nu = x^\mu k^\nu - x^\nu k^\mu = i(k^\mu \partial^\nu k^\mu - k^\mu \partial^\mu k^\nu),$$

$$E^\mu\nu = e^\mu \pi^\nu - e^\nu \pi^\mu = i(e^\mu \partial^\nu e^\nu - e^\nu \partial^\mu e^\mu),$$

$$O^\mu\nu = 2i \sum_{n \neq 0} \frac{1}{n} (\beta_n^\mu \alpha_n^\nu - \beta_n^\nu \alpha_n^\mu + \bar{\beta}_n^\mu \bar{\alpha}_n^\nu - \bar{\beta}_n^\nu \bar{\alpha}_n^\mu).$$ \hspace{1cm} (30)

The unusual term in this expression is $E^\mu\nu$, which acts on the new variable $e^\mu$. Indeed, as we explained in the introduction, the difference between tensionless and standard string theories is the appearance of additional zero mode $[e^\mu, \pi^\nu] = i\eta^\mu\nu$, meaning that the wave function is a function of two continuous variables $k^\mu$ and $e^\mu$, the momentum and polarization, $\Psi_{phys} = \Psi(k,e)$. This means that one should define not only initial and final momenta of the tensionless strings, but also the initial and final directions of the polarization vector. This fact has very important consequences. In particular it leads to high degeneracy of the physical states. This can be seen if one expands the wave function in terms of new vector variable $e^\mu$: $\Psi_{phys} = \sum_s \Psi_{\mu_1,...,\mu_s} e^{\mu_1}...e^{\mu_s}$, these fields $\Psi_{\mu_1,...,\mu_s}$ belong to the same level of the Fock space $\Xi$. 

9
These fields are nontrivial helicity eigenstates of the highest Casimir operator $W = w^2_{D-3}$ of the Poincaré group, which is the square of the D-3 Pauli-Lubanski form $w_{D-3} = k \wedge M = k \wedge E$, and defines $\Theta_0$ or $\Theta(\Xi)$ representations of the little group E(11) depending on whether $W = \Xi^2$ is zero or not. We shall analyze this expansion in the next sections.

To guarantee that string states are Lorentz multiplets one can check the commutators

$$[L_n, M^{\mu\nu}] = 0, \quad [\Theta_{n,m}, M^{\mu\nu}] = 0. \tag{31}$$

6 Enhanced Gauge Algebra, $\Theta$-Algebra

Let us now define the Fourier expansion of our constraint operators, they are the conformal algebra has here its classical form but with twice larger central charge

**The conformal algebra has here its classical form but with twice larger central charge**

Thus we have [22]

$$L_n = <e^{in\zeta^+} : P_L^\mu \partial_L X_\mu : >, \quad \Theta_{n,l} = <e^{in\zeta^+ + il\zeta^-} : \Pi^\mu \Pi^\nu - m^2 : > \tag{32}$$

We thus have [22]

$$L_n = \sum_l : \alpha_{n-l} \cdot \beta_l : \quad \tilde{L}_n = \sum_l : \tilde{\alpha}_{n-l} \cdot \tilde{\beta}_l :$$

$$\Theta_{0,0} = m^2(e^2 - 1) + \frac{1}{4n} \sum_{n \neq 0} : (\alpha_{-n} \alpha_n + \tilde{\alpha}_{-n} \tilde{\alpha}_n) :$$

$$\Theta_{n,0} = \frac{im}{n} e \cdot \alpha_n - \frac{1}{4} \sum_{l \neq 0, n} : (n-l) : \alpha_{n-l} \cdot \alpha_l : \quad n = \pm 1, \pm 2, ..$$

$$\Theta_{0,n} = \frac{im}{n} e \cdot \tilde{\alpha}_n - \frac{1}{4} \sum_{l \neq 0, n} : (n-l) : \tilde{\alpha}_{n-l} \cdot \tilde{\alpha}_l : \quad n = \pm 1, \pm 2, ..$$

$$\Theta_{n,k} = -\frac{1}{2nk} : \alpha_n \cdot \tilde{\alpha}_k : \quad n, k = \pm 1, \pm 2, ... \tag{33}$$

The conformal algebra has here its classical form but with twice larger central charge

$$[L_n, L_k] = (n-k)L_{n+k} + \frac{D}{6}(n^3 - n)\delta_{n+k,0} \tag{34}$$

and with the similar expression for right movers $\tilde{L}_n$. The reason that the central charge is twice bigger than in the standard bosonic string theory $2 \times \frac{D}{12} = \frac{D}{6}$ is simply because we have two left and two right movers of $X_\mu$ field, twice bigger degrees of freedom.\(^3\)

The full extended gauge symmetry algebra of constraints (32) takes the form

$$[L_n, \Theta_{0,0}] = -2n\Theta_{n,0}, \quad \tilde{L}_n, \Theta_{0,0} = -2n\Theta_{0,n}$$

$$[L_n, \Theta_{0,k}] = -(n+k)\Theta_{n+k,0}, \quad \tilde{L}_n, \Theta_{0,k} = -(n+k)\Theta_{0,n+k}$$

$$[L_n, \Theta_{k,0}] = -2n\Theta_{n,k}, \quad \tilde{L}_n, \Theta_{0,k} = -(n+k)\Theta_{0,n+k}$$

$$[L_n, \Theta_{k,l}] = -(n+k)\Theta_{n+k,l}, \quad \tilde{L}_n, \Theta_{k,l} = -(n+l)\Theta_{k,n+l}$$

$$[L_n, k^2] = 0, \quad \tilde{L}_n, k^2 = 0$$

$$[L_n, k \cdot \alpha] = -\frac{i}{m} k \cdot \alpha_n, \quad \tilde{L}_n, k \cdot \alpha = -\frac{i}{m} k \cdot \tilde{\alpha}_n$$

$$[L_n, k \cdot \tilde{\alpha}] = -l k \cdot \alpha_{n+l}, \quad \tilde{L}_n, k \cdot \tilde{\alpha}_l = -l k \cdot \tilde{\alpha}_{n+l} \tag{35}$$

\(^3\)Such doubling of modes is reminiscent of the bosonic part of the $\mathcal{N} = 2$ superstring [51]. In the last model there was an essential problem in identifying the $Y^\mu$ coordinates which are introduced in addition to the normal coordinates $X^\mu$ [51]. In our model the coordinate field $X$ has simply two sets of commuting oscillators and the conjugate fields are described by a separate field $\Pi^\mu$.\(^3\)
where we have included commutators with the operators $k^2$, $k \cdot e$, $k \cdot \alpha_l$, $k \cdot \tilde{\alpha}_l$ because they are constituents of the $\tau$ dependent part of the operator $\Theta$ as one can see from the formula (37). Therefore we added them to the full algebra of gauge operators. One should stress that it is an essentially Abelian extension

$$\left[ \Theta_{n,k}, \Theta_{l,p} \right] = 0, \quad n, k, l, p = 0, \pm 1, \pm 2, \ldots$$

(36)

One can easily check that Jacobi identities between all these operators are satisfied, therefore the relations (34), (35) and (36) define Abelian extension of conformal algebra and hint that there exist symmetries higher than the conformal algebra. The equations (33) suggest its oscillator representation. To this author it is not known, whether this algebra has appeared also somewhere else. In the next section we shall analyze the physical Fock space, new generators play fundamental role in defining the particle spectrum of this theory.

7 Physical Fock Space

To define the physical Hilbert-Fock space we should first impose our new constraint $\Theta = \Pi^2 - m^2$. The reason is that as we shall see it defines the spectrum of the theory [22]. Indeed the last operator has a linear and quadratic $\zeta^0 \equiv \tau$ dependence which in fact uniquely defines the spectrum of this string theory:

$$(\Pi^2 - m^2) = k^2 \tau^2 + 2\left\{ m \cdot k + k \cdot \Pi_{oscil} \right\} \tau + \Pi_{oscil}^2 + 2m \cdot \Pi_{oscil} + m^2(e^2 - 1).$$

(37)

The first operator diverges quadratically with $\tau$ and the second one linearly. Therefore in order to have normalizable states in physical Hilbert-Fock space one should impose corresponding constraints. We are enforced to define the physical space as

$$k^2 \Psi_{phys} = 0, \quad e \cdot k \Psi_{phys} = 0, \quad k \cdot \alpha_n \Psi_{phys} = 0, \quad k \cdot \tilde{\alpha}_n \Psi_{phys} = 0, \quad n = 1, 2, \ldots$$

(38)

The first equation states that all physical states with different spins are massless. This is consistent with the tensionless character of the theory. The rest of the constraints which come from the oscillatory part of the operator $\Theta = \Pi^2 - m^2$ take the form

$$\Theta_{n,k} \Psi_{phys} = 0, \quad n, k = 0, 1, 2, \ldots$$

(39)

We have to impose the conformal constraints as well

$$(L_0 - a)\Psi_{phys} = 0 \quad (\bar{L}_0 - a)\Psi_{phys} = 0$$

$$L_n \Psi_{phys} = 0 \quad \bar{L}_n \Psi_{phys} = 0 \quad n = 1, 2, \ldots$$

(40)

together with "level"\footnote{We shall define this concept below.} matching condition

$$(L_0 - \bar{L}_0)\Psi_{phys} = 0.$$  \quad (41)

Thus the physical Fock space is defined by the equations (38),(39),(40) and (41). We shall make considerable efforts to understand and solve these space-time equations.
We are now interested to study "level by level" physical space solving the full system of space-time equations described above. As we just explained the essential difference with the standard string theory is the appearance of additional zero modes
\[ [e^\mu, \pi^\nu] = [x^\mu, k^\nu] = i\eta^\mu\nu \]
meaning that the wave function is a function of the momentum and polarization vectors \( k^\mu \) and \( e^\mu \)
\[ \Psi_{phys} = \Psi(k, e). \quad (42) \]
The other difference is that despite the fact that we have the same conformal algebra as in the standard string theory nevertheless there is an essential difference consisting in the oscillator representation of conformal operators. In particular, as we have seen above, the operators \( L_0 \) and \( \tilde{L}_0 \) do not define a mass spectrum any more, instead, as we shall see, they define the helicities of the massless representations of the little group of Poincaré group. To make these things more clear it is helpful to separate zero modes \( \alpha_0 \) and \( \beta_0 \) from other high modes. The \( L_0 \) and \( \tilde{L}_0 \) operators will take the form
\[
\{ \alpha_0\beta_0 + \sum_{n \neq 0} \alpha_{-n}\beta_n - a \} \Psi_{phys} = \left\{ \frac{k \cdot \pi}{m} + \sum_{n=1}^{\infty} \alpha_{-n}\beta_n + \beta_{-n}\alpha_n \right\} \Psi_{phys} = 0,
\]
\[
\{ \alpha_0\beta_0 + \sum_{n \neq 0} \tilde{\alpha}_{-n}\tilde{\beta}_n - a \} \Psi_{phys} = \left\{ \frac{k \cdot \pi}{m} + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}\tilde{\beta}_n + \tilde{\beta}_{-n}\tilde{\alpha}_n \right\} \Psi_{phys} = 0, \quad (43)
\]
where the constant \( a \) should be tuned to eliminate vacuum oscillations. After introducing the "level" operators \( \hat{\Xi}_L \) and \( \hat{\Xi}_R \) as
\[
\hat{\Xi}_L = \sum_{n=1}^{\infty} (\alpha_{-n}\beta_n + \beta_{-n}\alpha_n), \quad \hat{\Xi}_R = \sum_{n=1}^{\infty} (\tilde{\alpha}_{-n}\tilde{\beta}_n + \tilde{\beta}_{-n}\tilde{\alpha}_n), \quad (44)
\]
the above space-time equations, which involve \( L_0 = k \cdot \pi/m + \hat{\Xi}_L \) and \( \tilde{L}_0 = k \cdot \pi/m + \hat{\Xi}_R \), will take the form
\[
\frac{k \cdot \pi}{m} \Psi_{phys} = -\hat{\Xi}_L \Psi_{phys} = -\hat{\Xi}_R \Psi_{phys}. \quad (45)
\]
In the vacuum state of all oscillators the eigenvalues of these operators are equal to zero \( \Xi_L = \Xi_R = 0 \), thus we can define the vacuum as a zero level state, and the equations reduce to the following one
\[
\frac{k \cdot \pi}{m} \Psi_0 = 0. \quad (46)
\]
In general, when oscillators are in high excitation states, the eigenvalues of these operators are integer numbers \( \Xi_L = \Xi_R = \Xi = 0, 1, 2... \) and we can use them to define levels of our Fock space. In this theory, with all its massless states, these numbers allow to identify subsets \( \mathbb{R}^\Xi \) of states in the full Hilbert-Fock space. At the same time these numbers actually define irreducible representations \( \mathbf{0}_\Xi \) or \( \mathbf{0}(\Xi) \) of the Euclidean little group \( \text{E}(11) \). Therefore string states represent an explicit realization of different irreducible representations of the \( (D-2) \)-dimensional Euclidean group \( E(D-2) \) consisting of \( SO(D-2) \) rotations in the transverse hyperplane and \( D-2 \) displacements \( T(D-2) \).
Now let us consider the constraint $\Theta_{0,0}$, it has the form

$$\{ m^2(e^2 - 1) + \sum_{n=1}^{\infty} \frac{1}{2n^2}(\alpha_n + \tilde{\alpha}_{-n}) \} \Psi_{\text{phys}} = 0$$

(47)

and again for the vacuum state it reduces to the equation

$$(e^2 - 1) \Psi_0 = 0.$$ 

(48)

One should study in great details this Hilbert-Fock space in order to learn more about the spectrum of the theory and to prove the absence of the negative norm states. Let us start from the ground state.

8 \hspace{1cm} \textit{Ground state, } \Xi = 0

First of all let us build the table of equations as follows, on the left hand side we shall show symbolically all our space-time equations (38),(39), (40),(41) and on the right hand side the equations to which they reduce for the vacuum state

$$\begin{pmatrix} k^2 \\ k \cdot e \\ k \cdot \alpha_n \\ k \cdot \tilde{\alpha}_n \\ \Theta_{nm} \\ L_n \\ \tilde{L}_n \end{pmatrix} \Psi_{\text{phys}} = 0, \quad \Rightarrow \quad \begin{pmatrix} k^2 \\ k \cdot e \\ 0 \\ 0 \\ (e^2 - 1) \\ k \cdot \pi \\ k \cdot \pi \end{pmatrix} \Psi_0 = 0.$$ 

(49)

Thus for the ground state $\Psi_0 \equiv |k, e, 0>$ which is defined as

$$\alpha_n |0, k, e > = \beta_n |0, k, e > = 0, \quad n = 1, 2, ..$$

from the above consideration we have four equations

$$k^2 \Psi_0 = 0, \quad e \cdot k \Psi_0 = 0, \quad (e^2 - 1) \Psi_0 = 0, \quad k \cdot \pi \Psi_0 = 0.$$ 

(50)

If one changes by hand the first equation in this system introducing a mass term $(k^2 + M^2)\Psi_0 = 0$ then it is easy to see that consistency condition, which comprises in computing all commutators between four operators in (50) will require $M = 0$. Indeed $[L_0, e \cdot k] = [k \cdot \pi, e \cdot k] = -ik^2 = iM^2 = 0$.

The metric signature is $\eta_{\mu\nu} = (-1, +1, ..., +1)$ and the $e^\mu$ is a space-like unit vector. This fact is not accidental because the initial vector $\Pi_\mu$ was a space-like vector (10) and therefore its zero frequency part $e^\mu$ (see mode expansion (27)) inherited this property. Therefore the first three equations allow to interpret the vector $e_\mu$ as a polarization vector normal to the momentum vector $k_\mu$ [22]. The space-time wave function $\Psi_0 = \Psi(k, e)$ depends therefore on two independent variables: space-time momentum $k^\mu$ and polarization vector $e^\mu$ (42). The wave function is a function on a unit sphere $e^2 = 1$ and can be expanded in the corresponding bases

$$\Psi_0 = |0, k, e >= \{ A(k) + A^{\mu_1}(k) e_{\mu_1} + A^{\mu_1, \mu_2}(k) e_{\mu_1} e_{\mu_2} + ...\}|0, k >.$$ 

(51)
We clearly see that because in this model we have additional variable \( e_\mu \) the vacuum state in infinitely degenerated. Every field \( A^{\mu_1 \ldots \mu_s}(k) \) describes massless particle of increasing tensor structure. It is a symmetric traceless tensor field because it is a harmonic function on a sphere \( e^2 = 1 \). The constrain (40), (45),(46) reduces to the last equation in (50)

\[
k \cdot \pi |k,e,0 > = i k_\mu \frac{\partial}{\partial e_\mu} |k,e,0 > = i \left\{ k_\mu A^{\mu_1}(k) + k_{\mu_1} A^{\mu_1 \ldots \mu_s}(k) e_{\mu_2} + \ldots \right\} |0,k > = 0 \tag{52}\]

and it follows from this equation that

\[
k_\mu A^{\mu_1 \ldots \mu_s}(k) = 0. \tag{53}\]

Thus the fields \( A^{\mu_1 \ldots \mu_s} \) are: 1) symmetric tensors of increasing rank \( s = 0, 1, 2, \ldots \), 2) traceless in the sense that \( \eta_{\mu_1 \mu_2} A^{\mu_1 \ldots \mu_s} = 0 \), 3) divergent free \( k_\mu A^{\mu_1 \ldots \mu_s} = 0 \) and 4) satisfying massless wave equations \( k^2 A^{\mu_1 \ldots \mu_s} = 0 \). All the above conditions are sufficient to describe integer spin fields [29]. In the vacuum we have therefore massless fields \( A^{\mu_1 \ldots \mu_s}(k) \) of integer spin and momentum vector \( k^\mu \), they are fixed helicity states of the little group \( E(11) \). Thus the vacuum is infinitely degenerated and every fixed helicity state appears only once in the physical spectrum and there are no tachyon states. What is also important, our tensor fields expansion is invariant under Abelian gauge transformation

\[
e_\mu \rightarrow e_\mu + \chi(k) k_\mu, \tag{54}\]

where \( \chi(k) \) is an arbitrary function of momentum \( k \).

9 First Excited Level \( \Xi = 1 \)

Let us now consider the first excitation of the most general form

\[
|\Psi_1 > = \left[ \xi_{\mu \nu}(e) \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} + \omega_{\mu \nu}(e) \alpha^{\mu}_{1} \tilde{\beta}^{\nu}_{-1} + \zeta_{\mu \nu}(e) \beta^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} + \chi_{\mu \nu}(e) \beta^{\mu}_{1} \tilde{\beta}^{\nu}_{-1} \right] |k,e,0 >, \tag{55}\]

where polarization tensors \( \xi_{\mu \nu}(e), \omega_{\mu \nu}(e), \zeta_{\mu \nu}(e) \) and \( \chi_{\mu \nu}(e) \) are in general a function of the vector \( e_\mu \). Using operator algebra of \( (\alpha, \beta) \) oscillators one can see that

\[
|\hat{\Xi}_L|\Psi_1 > = \Xi_R |\Psi_1 > = |\Psi_1 >,
\]

thus justifying the fact that it is a level one state \( \Xi_L = \Xi_R = 1 \) (see equation (45). The equation (45) now takes the form

\[
\frac{1}{m} k \cdot \pi |\Psi_1 > = \frac{i}{m} k_\mu \frac{\partial}{\partial e_\mu} |\Psi_1 > = -|\Psi_1 >. \tag{56}\]

Let us now consider the rest of the equations for the first level wave function:

\[
\begin{pmatrix}
k^2 \\
k \cdot e \\
k \cdot \alpha_1 \\
k \cdot \tilde{\alpha}_1 \\
\Theta_{00} \\
\Theta_{10} \\
\Theta_{01} \\
\Theta_{11} \\
L_1 \\
\tilde{L}_1 \\
\end{pmatrix}
\begin{pmatrix}
|\Psi_1 > \\
\end{pmatrix}
= \begin{pmatrix}
k^2 \\
k \cdot e \\
k \cdot \alpha_1 \\
k \cdot \tilde{\alpha}_1 \\
m^2(e^2 - 1) + \frac{1}{2}(\alpha_{-1} \alpha_1 + \tilde{\alpha}_{-1} \tilde{\alpha}_1) \\
im \ e \cdot \alpha_1 \\
im \ e \cdot \tilde{\alpha}_1 \\
-\frac{1}{2} \alpha_1 \tilde{\alpha}_1 \\
\alpha_0 \beta_1 + \beta_0 \alpha_1 \\
\alpha_0 \tilde{\beta}_1 + \tilde{\beta}_0 \alpha_1 \\
\end{pmatrix}
\begin{pmatrix}
|\Psi_1 > \\
\end{pmatrix}
= 0. \tag{57}\]
The equation $\Theta_{00}|\Psi_1> = 0$ takes the form
\[
(e^2 - 1)\xi^{\mu\nu} + \frac{1}{2} \zeta^{\mu\nu} + \frac{1}{2} \omega^{\mu\nu} = (e^2 - 1)\chi^{\mu\nu} + \frac{1}{2} \zeta^{\mu\nu} + \frac{1}{2} \omega^{\mu\nu} = 0
\]
with the solution $\chi^{\mu\nu} = 0$, $\zeta^{\mu\nu} + \omega^{\mu\nu} = 0$, $e^2 - 1 = 0$ and we are left with the state
\[
|\Psi_1> = \left[ \xi_{\mu\nu}(e^\mu_{\nu} - 1) + \alpha_{\mu\nu}(e)(\beta_{\nu\mu} - \beta_{\mu\nu}) \right]|k, e, 0>
\]
(58)
The equations $k\alpha_1|\Psi_1> = k\tilde{\alpha}_1|\Psi_1> = 0$ tell us that
\[
\omega_{\mu\nu} = 0.
\]
(59)
The equations $\Theta_{10}|\Psi_1> = \Theta_{01}|\Psi_1> = \Theta_{11}|\Psi_1> = 0$ take the form
\[
e^{\mu}\omega_{\mu\nu} = \omega_{\mu\nu} e^{\nu} = 0
\]
(60)
and finally our constraints $L_1|\Psi_1> = \tilde{L}_1|\Psi_1> = 0$ are
\[
\omega_{\mu\nu} = 0
\]
(61)
and
\[
k^\mu\xi_{\mu\nu} = \xi_{\mu\nu} k^\nu = 0.
\]
(62)
Having in mind the equations (51), (52) for the vacuum wave function the first equation reduces to the form
\[
\omega_{\mu\nu} A_{\mu\nu} = 0.
\]
(63)
We can also compute the norm of this state:
\[
<\Psi_1|\Psi_1> = -2\omega^*_{\mu\nu}\omega^{\mu\nu},
\]
(64)
it is remarkable that it is independent on the tensor $\xi_{\mu\nu}$. Thus we have to analyze solutions of the equations (56), (59), (60),(62) and (63). From (60) it follows that nonzero components of $\omega_{\mu\nu}$ are only $\omega_{00}$, $\omega_{0,D-1}$, $\omega_{D-1,0}$, $\omega_{D-1,D-1}$ simply because this tensor should be orthogonal to $D - 2$ space-like mutually orthogonal polarization vectors
\[
e_1^\mu, e_2^\mu, ..., e_{D-2}^\mu, \quad e_i e_j = \delta_{ij}
\]
(65)
and the equations (59) tell us that $\omega_{00} = \omega_{0,D-1} = \omega_{D-1,0} = \omega_{D-1,D-1}$, thus
\[
\omega_{\mu\nu} = \omega(e)k_{\mu}k_{\nu}.
\]
It follows now that the norm (64) of our first level physical state (58) is zero! The only remaining restriction on polarizations $\xi_{\mu\nu}$ and $\omega_{\mu\nu}$ are equations (62) and (56)
\[
k^\mu\xi_{\mu\nu} = \xi_{\mu\nu} k^\nu = 0, \quad k_{\mu}\frac{\partial}{\partial e_\mu} \xi_{\lambda\rho}(e) = im \xi_{\lambda\rho}(e), \quad k_{\mu}\frac{\partial}{\partial e_\mu} \omega(e) = im \omega(e).
\]
(66)
Because the first level physical wave function (58) has zero norm, it defines gauge transformation of the ground state wave function $\Psi_0$. We have to learn how this gauge transformation acts on the high spin fields $A_{\mu_1...\mu_s}$. 

15
10 Enhanced Gauge Invariance

Let us summarize our findings. The first two levels of our physical Fock space are: the ground state wave function $\Psi_0$ which uniquely describes massless states of increasing helicities $A^{\mu_1...\mu_j}$ and the first excited state wave function $\Psi_1$ of the form:

$$\Psi_1 = [ \xi_{\mu\nu} \alpha^{-1}_\mu \tilde{\alpha}^\nu_{-1} + \omega_{\mu\nu} (\alpha^{-1}_\mu \beta^\nu_{-1} - \beta^{-1}_\mu \tilde{\alpha}^\nu_{-1}) ] |k, e, 0 >,$$

with nonzero tensors $\xi_{\mu\nu}(e)$, and $\omega_{\mu\nu} = k_{\mu} k_{\nu} \omega(e)$, and these tensors are subject to the constraint (66). The norm of the state $\Psi_1$ is equal to zero and it is orthogonal to the ground state wave function

$$< \Psi_1 | \Psi_1 >= 0, \quad < \Psi_0 | \Psi_1 >= 0. \quad (68)$$

Thus $\Psi_1$ is physical null state, and so we can add it to any physical state with no physical consequences. We could therefore impose an equivalence relation

$$\Psi_0 \sim \Psi_0 + \Psi_1. \quad (69)$$

Let us first consider the case when $\omega(e) = 0$. Expanding the tensor $\xi_{\mu\nu}(e)$ in $e$ series

$$\sum_J \xi_{\mu\nu}^{\mu_1...\mu_s} e_{\mu_1} ... e_{\mu_s}$$

from equations (66) we can get its properties

$$k^\mu \xi_{\mu\nu}^{\mu_1...\mu_s} = k^\nu \xi_{\mu\nu}^{\mu_1...\mu_s} = 0, \quad k_{\mu_s} \xi_{\mu\nu}^{\mu_1...\mu_{s-1}} \mu_s = i m \xi_{\mu\nu}^{\mu_1...\mu_{s-1}}, \quad \Xi = 1. \quad (70)$$

Then using explicit form of the $\Psi_1$ function

$$\Psi_1 = (\sum_s \xi_{\mu\nu}^{\mu_1...\mu_s} e_{\mu_1} ... e_{\mu_s} ) (\sum_s A_{\mu_1...\mu_s} e^{\mu_1} ... e^{\mu_s} ) \alpha^{-1}_\mu \tilde{\alpha}^\nu_{-1} |0, k >$$

the equivalence relation $\delta \Psi_0 = \Psi_1$ will imply the gauge transformation of the tensor fields $A^{\mu_1...\mu_s}$ of the form

$$\delta A = A \delta \varphi$$

$$\delta A^{\mu_1} = A^{\mu_1} \delta \varphi + A \delta \varphi^{\mu_1}$$

$$\delta A^{\mu_1 \mu_2} = A^{\mu_1 \mu_2} \delta \varphi + A^{\{\mu_1} \{ \delta \varphi^{\mu_2} \} + A \delta \varphi^{\mu_1 \mu_2} \quad (71)$$

$$\vdots$$

where

$$\delta \varphi = \xi_{\mu\nu} \alpha^{-1}_\mu \tilde{\alpha}^\nu_{-1}$$

$$\delta \varphi^{\mu_1} = \xi_{\mu\nu}^{\mu_1} \alpha^{-1}_\mu \tilde{\alpha}^\nu_{-1}$$

$$\delta \varphi^{\mu_1 \mu_2} = \xi_{\mu\nu}^{\mu_1 \mu_2} \alpha^{-1}_\mu \tilde{\alpha}^\nu_{-1} \quad (72)$$

These gauge parameters have following properties:

$$k_{\mu_s} \delta \varphi^{\mu_1...\mu_{s-1}} \mu_s = i m \delta \varphi^{\mu_1...\mu_{s-1}}. \quad (73)$$

Analogous transformation is generated by the tensor $\omega_{\mu\nu}(e) = \omega(e) k_{\mu} k_{\nu}$: $\omega(e) = \sum_s \omega^{\mu_1...\mu_s} e_{\mu_1} ... e_{\mu_s}$, with the level one condition, $\Xi = 1$

$$k_{\mu_s} \omega^{\mu_1...\mu_{s-1}} \mu_s = i m \omega^{\mu_1...\mu_{s-1}},$$

thus in this case

$$\delta \varphi = \omega k_{\mu} k_{\nu} (\alpha^{-1}_\mu \beta^\nu_{-1} - \beta^{-1}_\mu \tilde{\alpha}^\nu_{-1})$$

$$\delta \varphi^{\mu_1} = \omega^{\mu_1} k_{\mu} k_{\nu} (\alpha^{-1}_\mu \beta^\nu_{-1} - \beta^{-1}_\mu \tilde{\alpha}^\nu_{-1})$$

$$\vdots \quad \vdots$$

(74)
All Lorentz transformations $L$ which leave a fixed null vector $k_\mu = k(1,0,0,1)$ invariant $Lk = k$ form a subgroup called "little group" [43]. The group of Lorentz transformations which leave a null vector $k_\mu$ invariant is a two-dimensional Euclidean group of rotations $R(\theta) = \exp (-iM_{xy} \theta)$ in the plane transverse to the vector $\vec{k}$ and displacements $T'(\alpha) = \exp (-i\alpha \pi')$, $T''(\beta) = \exp (-i\beta \pi'')$, which are induced by Lorentz generators

$$M_{xy}, \quad \pi' = M_{xz} - M_{tx}, \quad \pi'' = M_{zy} - M_{ty}. \quad (75)$$

They form the Euclidean algebra $E(2)$

$$[\pi', \pi''] = 0, \quad [M_{xy}, \pi'] = i\pi'', \quad [M_{xy}, \pi''] = -i\pi'. \quad (76)$$

Two Casimir operators of the Poincaré group are given by the operators $P = k^2$ and $W = w^2$ - square of the Pauli-Lubanski vector $w_\mu = \epsilon^{\mu\lambda\rho} k_\nu M_{\lambda\rho}$

$$W = \pi'^2 + \pi''^2. \quad (77)$$

where we have used

$$w_0 = M_{xy}, \quad w_z = -M_{xy}, \quad w_y = M_{zx} - M_{tx} = \pi', \quad w_x = M_{zy} - M_{ty} = \pi''. \quad (78)$$

To describe representations of the little group one can take $M_{xy}$ in a diagonal form with integer eigenvalues $s = 0, \pm 1, \pm 2, \ldots$ and then from Lorentz subalgebra (76) it follows that both $\pi$ generators have Jacobi: form [43]

$$M_{xy} = \begin{pmatrix} -2 & -1 & 0 & +1 \\ -1 & 0 & 0 & +2 \\ 0 & 0 & \Xi/2 & \Xi/2 \\ +1 & +2 & \Xi/2 & \Xi/2 \end{pmatrix}, \quad \pi' = \begin{pmatrix} 0 & \Xi/2 & 0 & \Xi/2 \\ \Xi/2 & 0 & \Xi/2 & \Xi/2 \\ 0 & \Xi/2 & 0 & \Xi/2 \\ \Xi/2 & \Xi/2 & 0 & 0 \end{pmatrix}, \quad (78)$$

$$\pi'' = \begin{pmatrix} 0 & i\Xi/2 & i\Xi/2 & 0 \\ -i\Xi/2 & 0 & 0 & i\Xi/2 \\ i\Xi/2 & 0 & 0 & i\Xi/2 \\ -i\Xi/2, & 0 & i\Xi/2 & 0 \end{pmatrix}. \quad (79)$$

These infinite-dimensional representations can be characterized by the parameter $\Xi$ which can be assumed to be any positive real number. One can now compute the Casimir operator $W$ for these representations:

$$W = \pi'^2 + \pi''^2 = \Xi^2. \quad (80)$$

---

5For simplicity we shall consider in details only four-dimensional space-time. The extension to high dimension can be found in [44], see also [45, 46, 47, 48, 49].
Let us define the state with fixed helicity \( s \) \((0, \pm 1, \pm 2, \ldots)\) as \( M_{xy}|s> = s|s> \). It can be represented as an infinite vector with entry one in the row \( s \):

\[
|s> = \begin{pmatrix}
0 \\
1 \\
0 \\
0 \\
0
\end{pmatrix}
\]  

(81)

The action of the \( \pi' \) generators on this vector can be found by using (79)

\[
\pi'|s> = \frac{\Xi}{2}(|s-1> + |s+1>) , \quad \pi''|s> = \frac{i\Xi}{2}(|s-1> - |s+1>).
\]

These relations tell us that under the Lorentz boosts the state with helicity \( |s> \) transforms into the states with helicities \( |s \pm 1> \) with the amplitude proportional to \( \Xi/2 \).

Only in the case when \( \Xi = 0 \) we shall have invariant pure helicity states \( |s> \) which decouple from each other. These are well known physical representations \( O_s \) appearing in standard field theory of massless particles, where \( W = \Xi^2 = 0 \).

When \( \Xi \neq 0 \) we shall have representations, in which helicity of the ”particle” can take any integer value \( s = 0, \pm 1, \pm 2, \ldots \). If one prepares a ”particle” in a pure helicity state \( s = N \) then after Lorentz boosts one can find the same ”particle” in different helicity states with nonzero amplitude. These are so called ”continuous spin representations - \( O(\Xi) \)” CSR [43]. Let us introduce the coherent state of different helicities

\[
|\varphi> = \sum_s e^{is\varphi}|s>.
\]

Under rotation \( R(\theta) \) in xy-plane it will transform as

\[
R(\theta)|\varphi> = \sum_s e^{is\varphi}R(\theta)|s> = \sum_s e^{is\varphi}e^{in\theta}|s> = |\varphi + \theta>
\]

and under Lorentz boosts as

\[
\pi'|\varphi> = \sum_s e^{is\varphi}\pi'|s> = \sum_s e^{is\varphi}\frac{\Xi}{2}(|s-1> + |s+1>) = \Xi \cos \varphi|\varphi>
\]

and \( \pi''|\varphi> = \Xi \sin \varphi|\varphi> \).

In high dimensions the displacement generators are \( \pi_i, \ i = 1, \ldots, D-2 \) and the little group for the massless particles in \( D \)-dimensions is a \( (D-2) \)-dimensional Euclidean group \( E(D-2) \) induced by the following Lorentz generators:

\[
M_{ij}, \quad \pi_i = M_{D-1i} - M_{0i}, \quad i = 1, \ldots, D-2
\]

(83)

with commutators

\[
[\pi_i, \pi_j] = 0, \quad [M_{ij}, \pi_k] = i(\delta_{ik}\pi_j - \delta_{jk}\pi_i), \quad [M_{ij}, M_{kl}] = i(\delta_{ik}M_{jl} - \delta_{il}M_{jk} + \delta_{jl}M_{ik} - \delta_{jk}M_{il}).
\]

(84)
The highest Casimir operator of the Poincaré algebra is given by \( W = w^2_{D-3} \) - the square of the \( D - 3 \) Pauli-Lubanski form \( w_{D-3}^{\mu_1...\mu_{D-3}} = \frac{1}{\sqrt{2(D-3)!}} \epsilon^{\mu_1...\mu_{D-3}\nu\lambda\rho} k_\nu M_{\lambda\rho} \) and has the form
\[
W = \sum_i \pi_i^2
\] (85)

If \( W = \Xi^2 = 0 \) we have again finite-dimensional irreducible representations of the transverse group \( SO(D-2) \) which we shall identify as pure "generalized helicity" states \( 0_s \). By generalized helicity one should understand an irreducible representation with \( SO(D-2) \) quantum numbers. If on the other hand \( W = \Xi^2 \neq 0 \) the irreducible representations of the little group \( E(D-2) \) are infinite-dimensional and contain infinite number of irreducible representations of \( SO(D-2) \), and I suggest to denote them again as \( 0(\Xi) \).

Discussing the representations of the Poincaré group Wigner was arguing that the relativistic wave function should depend on two variables \( x \) and \( \xi \) in order to describe irreducible representations of the little group. For that he suggested three equations (equations (6.5),(6.6) and (6.7) in [43]):
\[
\begin{align*}
(p \cdot p)\psi &= M^2\psi \\
(\xi \cdot \xi)\psi &= \psi \\
(p \cdot \xi)\psi &= 0
\end{align*}
\] (86)

(in massless case \( M^2 = 0 \)) and postulated that the Lorentz generators should have the form
\[ M^{\mu\nu} = i(p^\mu \partial_\nu - p^\nu \partial_\mu + \xi^\mu \partial_\nu - \xi^\nu \partial_\mu) = J^{\mu\nu} + E^{\mu\nu}. \]

Computing the Casimir operator \( W = w^2 = (p \wedge M)^2 \) of the Poincaré group for this representation of \( M^{\mu\nu} \), that is the square of the Pauli-Lubanski vector \( w^\mu \), he has found that \( W = V^2 \), where \( V = ip\partial_\xi \). The eigenvalue \( \Xi \) of the last operator \( V\psi = \Xi\psi \) defines the irreducible representation of the little group with \( W = \Xi^2 \). Thus the forth Wigner equation is (equation (6.17) in [43])
\[
ip \frac{\partial}{\partial \xi} \psi = \Xi\psi.
\] (87)

When \( \Xi = 0 \) the representations describe usual fixed helicity states of massless particles \( 0_s \). When \( \Xi \neq 0 \) the representations are infinite-dimensional with infinite helicities \( 0(\Xi) \), so called "continuous spin representations" CSR.

This analysis tells us that our equations (43) and definition of the "level" operator (44), (45) naturally correspond to the representation of the little group \( E(11) \) with \( \Xi_L = \Xi_R = \Xi \). In our case \( \Xi \) is a level number and takes only integer values \( \Xi = 0, 1, 2,... \). The ground state corresponds to the representations \( 0_s \) with \( \Xi = 0 \) and all high level states of \((\alpha_n, \beta_n)\) oscillators correspond to the CSR representations \( 0(\Xi) \) with \( \Xi = 1, 2,... \).

12 Conclusion

In conclusion let us consider in brief the high levels. The \( \Xi = 2 \) level contains twenty five tensors because we have five relevant operators in the left sector
\[
\alpha^\nu_{-1}\alpha^\nu_{-1}, \quad \alpha^\nu_{-1}\beta^\nu_{-1}, \quad \beta^\nu_{-1}\beta^\nu_{-1}, \quad \alpha^\mu_{-2}, \quad \beta^\mu_{-2}
\]
and the same amount in the right sector. Corresponding tensors are subject to the constraints

\[
\begin{pmatrix}
  k^2 \\
k \cdot e \\
k \cdot \alpha_1 \\
k \cdot \tilde{\alpha}_1 \\
k \cdot \alpha_2 \\
k \cdot \tilde{\alpha}_2 \\
\Theta_{00} \\
\Theta_{10} \\
\Theta_{20} \\
\Theta_{01} \\
\Theta_{02} \\
\Theta_{12} \\
\Theta_{21} \\
\Theta_{11} \\
\Theta_{22} \\
L_1 \\
\tilde{L}_1 \\
L_2 \\
\tilde{L}_2
\end{pmatrix}
\begin{pmatrix}
k^2 \\
k \cdot e \\
k \cdot \alpha_1 \\
k \cdot \tilde{\alpha}_1 \\
k \cdot \alpha_2 \\
k \cdot \tilde{\alpha}_2 \\
m^2(e^2 - 1) + \frac{1}{2}(\alpha_1 + \tilde{\alpha}_1) + \frac{1}{8}(\alpha_2 + \tilde{\alpha}_2) \\
im e \cdot \alpha_1 + \frac{1}{2} \tilde{\alpha}_1 \\
im e \cdot \alpha_1 + \frac{1}{2} \tilde{\alpha}_1 \\
im e \cdot \alpha_1 - \frac{1}{2} \tilde{\alpha}_1 \\
im e \cdot \alpha_2 - \frac{1}{2} \tilde{\alpha}_1 \\
-\frac{1}{2} \alpha_1 \tilde{\alpha}_2 \\
-\frac{1}{2} \alpha_2 \tilde{\alpha}_1 \\
-\frac{1}{2} \alpha_1 \tilde{\alpha}_1 \\
-\frac{1}{2} \alpha_2 \tilde{\alpha}_2 \\
\alpha_0 \beta_1 + \beta_0 \alpha_1 + \alpha_1 \beta + \beta_1 \alpha_2 \\
\alpha_0 \beta_1 + \beta_0 \tilde{\alpha}_1 + \tilde{\alpha}_1 \beta + \beta_1 \tilde{\alpha}_2 \\
\alpha_2 \beta_0 + \beta_2 \alpha_0 + \alpha_1 \beta_1 \\
\alpha_0 \beta_1 + \beta_0 \tilde{\alpha}_1 + \tilde{\alpha}_1 \beta_1
\end{pmatrix} \begin{pmatrix}
|\Psi_2 > = 0
\end{pmatrix}
\]

It is rather complicated system of space-time equations and we shall describe solution of these equations in a separate place.

I should also mention that in our early attempts [53, 54] to write down space-time equations for the tensionless strings we were using the general finite and infinite-dimensional representations of the Lorentz group defined as \((j, \lambda)\), where \(j\) is integer or half-integer spin and \(\lambda\) is arbitrary complex number, very similar to the parameter \(\Xi\) introduced in the previous section. If \(\lambda\) is real and is equal to \(j\) plus natural number \(N = 1, 2, 3, \ldots\) then representation \((j, \lambda)\) is finite-dimensional, otherwise it is infinite-dimensional and contains all representations of the little group with spins \(j + 1, j + 2, \ldots\). The number of representations used in our approach was growing polynomially with spin in order to produce physically acceptable spectrum. This grows should be compared with the exponential grows of spin states in Ramond generalization of the Dirac equation [52].

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