On “Sub-Threshold” Reactions Involving Nuclear Fission

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We analyze reactions of several types that are naively below threshold but can proceed because of the release of binding energy from nuclear fission and occasionally the formation of Coulombic bound states. These reactions include (i) photofission with pion production and (ii) charged current neutrino-nucleus reactions that lead to fission and/or formation of a Coulomb bound state of a $\mu^-$ with the nucleus of a fission fragment. We comment on the possible experimental observation of these reactions.

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I. INTRODUCTION

There are several types of reactions that can proceed even with incident particles whose energies are apparently below threshold, because of certain mechanisms that make the requisite energy available. This is true, in particular, for reactions in which the target is a nucleus, denoted here by \((Z,A)\), where \(Z\) denotes the number of protons and \(A\) denotes the mass number. Here we explore reactions involving heavy nuclei which can proceed, although the energy of the incident particle is “below threshold”, because the reaction leads to the fission of the nucleus, thereby releasing a substantial amount of energy, \(E_{\text{fiss.}} \sim 150\) MeV, which may be partially available to exceed a threshold. We also consider the small energy release due to the formation of a Coulomb bound state of a \(\mu^-\) with the nucleus or with a fission fragment.

II. PHOTOFISSION WITH PION PRODUCTION

Fission in heavy nuclei can be induced in several ways, in addition to sometimes occurring spontaneously; these ways include bombardment by slow (thermal and resonance) neutrons with energies from about 0.02 to \(O(10)\) eV, by medium and fast neutrons with energies from keV to MeV, by other hadronic projectiles such as protons, deuterons, and \(\pi\) mesons, by photons, and by the capture of \(\mu^-\)'s (some reviews are [1,2], a recent conference is [3], and a recent paper on photofission is [4]). Typical fission energy barriers (measured, e.g., in fast neutron or photofission reactions) are around 5 MeV. In general, from the roughly 0.1 eV widths of resonances observed in fission induced by neutrons with energies between 0.2 and \(10^2\) eV, it has been inferred that the time taken for fission to occur is \(t_{\text{fiss.}} \sim 10^{-14}\) sec [1].

Fission induced by incident photons, photofission, has been extensively studied. The fissioning nucleus breaks into two fragments:

\[
\gamma + (Z,A) \rightarrow (Z_1,A_1) + (Z_2,A_2)
\]

with \(Z_1 + Z_2 = Z\), \(A_1 + A_2 = A\). The fission daughter nuclei are typically produced in excited states and de-excite with prompt neutron and \(\gamma\)-ray emission. The cross section for photofission in a heavy element such as \(^{238}\text{U}\) rises from threshold for photon energies of about \(E_\gamma \approx 5\) MeV to a pronounced peak of about 100 mb at \(E_\gamma \approx 14\) MeV due to the excitation of the giant dipole resonance [4]. For higher photon energies, the cross section decreases, and then increases again for \(E_\gamma \gtrsim 100\) MeV. This second increase is interpreted as being due to the production of virtual excited states of a nucleon in the nucleus, in particular, \(\Delta(1232)\), which transfer energy and thereby catalyze the fission process [4,5].

One may also consider photoproduction reactions yielding a pion in the final state, including

\[
\gamma + (Z,A) \rightarrow (Z,A)_{\text{g.s.}} + \pi^0
\]

\[
\gamma + (Z,A) \rightarrow (Z \pm 1,A)_{\text{g.s.}} + \pi^\pm
\]
where g.s. means ground state, and the fission processes

\[ \gamma + (Z, A) \rightarrow (Z_1, A_1) + (Z_2, A_2) + \pi^0 \] (4)

and

\[ \gamma + (Z, A) \rightarrow (Z_1, A_1) + (Z_2, A_2) + \pi^\pm \] (5)

The \( \pi^0 \) decays to \( \gamma \gamma \) with a mean lifetime of \( \sim 0.8 \times 10^{-16} \) sec, so that the signature for reaction (4) would be the detection of the two fission fragments together with the two photons whose invariant mass reconstructs to that of the \( \pi^0 \). The signature for reaction (5) would be the detection of the two fission fragments together with the \( \pi^\pm \), which, however, may undergo charge-exchange reactions in the nucleus, so that one would actually detect the diphoton signal from the decay of the \( \pi^0 \). Before one takes account of the fission energy the naive threshold (thr.) energy for the reaction (4) is

\[ E_{\gamma, 0, \text{thr.}} = m_{\pi^0} + \frac{m_{\pi^0}^2}{2M(Z, A)}. \] (6)

With the definition of the difference in nuclear ground state (g.s.) energies

\[ \Delta E_{Z\pm 1, Z; A} = E_{(Z\pm 1, A), \text{g.s.}} - E_{(Z, A), \text{g.s.}}. \] (7)

the analogous naive threshold photon energy for the reaction (5) is

\[ E_{\gamma, \pm, \text{thr.}} = m_{\pi^\pm} + \frac{m_{\pi^\pm}^2}{2M(Z, A)} + \Delta E_{Z\pm 1, Z; A}. \] (8)

Of course, one must also take account of Fermi momentum and Pauli blocking in the reaction kinematics. Now consider a case where the available energy, including the effect of the Fermi momentum of the struck nucleon, is below threshold for pion production without fission. However, with fission, pion production might still occur because of the energy \( E_{\text{fiss.}} \) may be partially transferred by strong, as well as Coulombic, interactions, from the final state fission fragments to the pion.

In the relevant range of photon energy, slightly below the naive threshold of about 140 MeV for pion production, the total photofission cross sections on \( ^{238}\text{U} \) and \( ^{232}\text{Th} \) are approximately 150 mb and 45 mb, respectively [34]. Since the transfer of the fission energy to the pion is via the strong interaction, this transfer should not significantly reduce the cross section. However, for total energies only slightly beyond the true threshold, there would be substantial phase space suppression of the three-body reactions (4) and (5). Although the full reaction involves the integral over phase space of the squared amplitude \(|\mathcal{M}|^2\) for the reaction, one can obtain a rough measure of the kinematic threshold suppression by considering the threshold dependence of the phase space factor by itself. Recall that the usual Lorentz-invariant \( n \)-body final state phase space integration is
$$R_n = (2\pi)^{4-3n} \int \prod_{j=1}^n \frac{d^3p_j}{2E_j} \delta(P - \sum_{j=1}^n p_j)$$  \hspace{1cm} (9)$$

where $p_j^\lambda = (E_j, p_j)$ denotes the 4-momenta of the $j$'th final-state particle, and $P^\lambda = (\sqrt{s}, 0, 0, 0)$ in the center-of-mass-frame. Let us denote the overall energy release as

$$Q = \sqrt{s} - \sum_{j=1}^n m_j. \hspace{1cm} (10)$$

Near threshold, i.e. for $Q \to 0$, $R_n$ has the expansion (e.g., \cite{7})

$$R_n \simeq \frac{(2\pi^3)^{\frac{n-1}{2}}}{2\Gamma\left(\frac{3}{2}(n-1)\right)} \left(\prod_{j=1}^n m_j\right)^{1/2} Q^{\frac{n-2}{2}}. \hspace{1cm} (11)$$

Hence, near threshold, while the two-body phase space factor relevant for the photofission reaction (1) (before photon or prompt neutron emission by the excited fission fragments) only involves a square root suppression $R_2 \propto Q^{1/2}$, the three-body phase space factor relevant for the pion production reactions (4) and (5) involves a more severe quadratic suppression, $R_3 \propto Q^2$. A more detailed estimate would require calculation of the Coulomb distortion of the outgoing plane wave representing the pion, which would be represented by a factor analogous to the Fermi function in nuclear beta decay. However, the phase space considerations discussed above suggest that it would be quite difficult to observe the pion production reactions near to their thresholds. Slightly above threshold, $\pi^0$ production by photofission on a heavy nucleus such as $^{238}$U might show low energy $\pi^0$'s and higher energy $\pi^0$'s which have gained energy from the fission. In addition, one should mention the possibility that in the case where a $\pi^-$ is produced, it may form a Coulomb bound state with the larger-$Z$ fission fragment. This channel would suffer less kinematic suppression for two reasons: first, because it is a two-body final state, and second because of the small additional release of energy due to the Coulomb binding of the $\pi^-$. However, the $\pi^-$ would rapidly be absorbed by the fission fragment to which it binds.

### III. “SUB-THRESHOLD” CHARGED-CURRENT NEUTRINO REACTIONS

We next proceed to neutrino-induced fission reactions, namely the charged-current processes

$$\nu_\mu + (Z, A) \to \mu^- + (Z + 1, A) \to \text{fission} \hspace{1cm} (12)$$

and

$$\bar{\nu}_\mu + (Z, A) \to \mu^+ + (Z - 1, A) \to \text{fission}. \hspace{1cm} (13)$$
Suitable targets include $^{232}\text{Th}$, $^{238}\text{U}$, and the stable heavy nuclei of $\text{Pb}$ and $\text{Bi}$. The naive threshold energy for a charged-current (CC) reaction on a nucleus $(Z,A)$ with incident $\nu_\mu$ (or $\bar{\nu}_\mu$) is

$$E_{\nu,\text{thr.}} \simeq \Delta E_{Z\pm 1,Z,A} + m_\mu + \Delta E_{F,\text{PB}}$$

where $\Delta E_{F,\text{PB}}$ represents the effect of the Fermi momentum, $p_F \sim 270$ MeV, of the struck nucleon and the effect of Pauli blocking. The Fermi momentum smears out the threshold. We envision a situation in which, without fission, the incident energy $E_\nu$ would be below threshold, but the reaction (12) or (13) is rendered possible by energy transferred from the fission. (Clearly, some of the energy transfer $q^0$ from the incident neutrino would be taken up to push the nucleus over the fission energy barrier of about 5 MeV.)

In the case of an incident $\nu_\mu$, there would be a significant probability for the resultant $\mu^{-}$ to form a Coulomb bound state with one of the two fission fragments, preferentially the one with higher $Z$. The $\mu^{-}$ could be captured in an excited state and de-excite to the ground state with photon emission. The Coulombic binding energy, say in the ground state, is $E_B \sim 5$ MeV for a fission fragment with $Z \sim 60$ [5]-[11], so this would provide an additional source of energy for the reaction. If we assume that the mechanism of fission via the deformation of the nucleus [12] also applies to neutrino-induced fission reactions, then the resultant time scale is expected to be similar to that characterizing neutron-induced fission, of order $10^{-14}$ sec. In the fraction of the reactions where the muon does form a Coulomb bound state with the nucleus of one of the fission fragments, the time $\tau_\mu = \Gamma_\mu^{-1}$ during which the muon stays bound, before it either decays or undergoes a CC reaction with the nucleus of the fragment to which it is bound, is thus considerably longer than the time characterizing the initial fission.

Near threshold, the usual charged-current neutrino cross sections on nuclei vary strongly with energy. For example, for the well-studied $A = 12$ system, the cross section for the reaction $^{12}\text{C}(\nu_\mu, \mu^{-})^{12}\text{N}$ has been calculated to vary from less than $10^{-42}$ cm$^2$ slightly above the threshold of about 120 MeV to $\sim 0.7 \times 10^{-40}$ cm$^2$ for $E_\nu \gtrsim 150$ MeV [13]. Similar energy dependence is found in calculations of neutrino reactions on heavier nuclei such as $^{56}\text{Fe}$ and $^{208}\text{Pb}$ [14] (see also the related works [15]-[18]) and would be expected to hold for the analogous reactions on $^{232}\text{Th}$ or $^{238}\text{U}$. Since these reactions involve a two-body final state, the phase space factor has the $Q^{1/2}$ dependence in this threshold region as $Q \to 0$. In the present case, for the CC reactions yielding an outgoing $\mu^{\pm}$, the cross section would be further suppressed near threshold, since the three-body phase space factor by itself goes like $Q^2$ for $Q \to 0$. However, for the fraction of the reactions with an incident $\nu_\mu$ in which the $\mu^{-}$ forms a Coulomb bound state with the nucleus of the higher-$Z$ fission fragment, one would still have the less severe $Q^{1/2}$ factor.

To obtain a rough estimate of the cross section for the reaction (12) leading to a Coulomb bound state of the $\mu^{-}$ with the nucleus of a fission fragment, with $E_\nu$ below threshold for the production of an asymptotic $\mu^{-}$ state, but above the true threshold, taking into account the release of fission energy and the smaller energy release from the Coulomb binding, we proceed as follows. The first step consists of the reaction $\nu_\mu + n \to (p + \mu^{-})_{\text{virtual}}$ on one of the
neutrons in the nucleus. This elementary process is characterized by a usual weak amplitude \( \propto G_F \) and a time \( t_W \sim 1/m_W \sim 10^{-26} \text{ sec} \). Assuming that the energy transfer to the nucleus is sufficient to push it over the fission barrier of about 5 MeV, the nucleus fissions at a later time, around \( t_{fiss} \sim 10^{-14} \text{ sec} \). The fission energy can be transferred by the exchange of a virtual photon. To incorporate this in the amplitude one would multiply by a factor \( \sim (Z\alpha)^2 \) in the cross section. For the formation of the Coulomb bound state, one would multiply the rate by a factor \( |\psi(0)|^2 \) where \( \psi \) is the quantum mechanical wavefunction describing the \( \mu^- \) and the nuclear fission fragment with charge \( Z_f \) to which it binds \[19\]. If this fission fragment were a point charge then, since the Bohr radius \( a \) of the ground state of the Coulomb bound state is much smaller (by the factor \( m_e/m_\mu = 1/207 \)) than those of the electrons, there would not be strong screening of the nuclear charge, and one would have simply \( |\psi(0)|^2 = (\pi a^3)^{-1} = (Z_f\alpha)^3 m_\mu^3/\pi \). However, since \( a \) is comparable to the nuclear radius, one must take account of the non-pointlike nature of the nuclear charge. Combining the factor of \( (Z_f\alpha)^3 \) with the factor of \( (Z\alpha)^3 \) for the energy transfer from the fissioning nucleus, one has an overall factor of \( f_{NIFM} \approx Z^2 Z_f^3 \alpha^5 \), where \( NIFM \) stands for neutrino-induced fission with formation of a muonic Coulomb bound state. Substituting \( Z = 93 \) for the virtual \( ^{238}\text{Np} \) nucleus resulting from the elementary reaction \( \nu_\mu n \rightarrow \mu^- p \), and a typical \( Z_f = 60 \) for the larger-\( Z \) fission fragment, this factor is \( f_{NIFM} \approx 0.04 \). Therefore, a lowest-order estimate of the cross section for the neutrino-induced fission with \( \mu^- \) binding to a fission fragment could be obtained by starting with the cross section for the corresponding nonfission reaction, making the substitution \( E_\nu \rightarrow E_\nu + aE_{fiss.} + E_B \), and then multiplying by the factor \( f_{NIFM} \), where for a given target nucleus \( (Z,A) \) the value of \( E_{fiss.} \) depends on which fission products are produced, and the factor \( a \) represents the fraction of the fission energy available for transfer to the \( \mu^- \). Thus if one uses \( ^{238}\text{U} \) as the target nucleus, and (i) \( E_\nu \) is below the naive threshold by, say, 30 MeV, (ii) the fission energy release is 150 MeV, of which about 80 MeV is transferred to the muon, and (iii) the muon Coulomb binding energy is 5 MeV, it follows that the true energy is about 55 MeV above the true threshold, then, taking into account the factor \( f_{NIFM} \) with \( Z_f \approx 60 \), our rough estimate of the cross section for the neutrino-induced fission reaction leading to the formation of a Coulomb bound state of the \( \mu^- \) with the nucleus of the \( Z \approx 60 \) fission fragment suggests that this cross section could be as large as a few percent of the corresponding weak charged-current reaction a similar interval of 55 MeV above threshold, and hence of order \( 10^{-43} - 10^{-42} \text{ cm}^2 \).

One could also carry out a similar estimate for the reaction channels leading to the production of an outgoing \( \mu^\pm \). For these channels, one would not have to multiply by \( |\psi(0)|^2 \), but the cross section would be suppressed by the more severe \( Q^2 \) dependence of the phase space near threshold for the three-body final state. Furthermore, if the effective energy is only slightly above the true threshold, the \( \mu^+ \) from an incident \( \bar{\nu}_\mu \) and the \( \mu^- \) from an incident \( \nu_\mu \) (if the \( \mu^- \) is not bound in a muonic atom with a fission fragment) will have low energy, so that the Coulomb correction to the rate will be substantial. In the absence of fission, this would be approximately given by the Fermi function

6
\[ F(E, Z) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \]  

(15)

where

\[ \eta = \mp \frac{Z_f \alpha}{\beta} \]  

(16)

where the \(-\) (+) sign applies for an outgoing \(\mu^+\) (\(\mu^-\)), \(Z_f\) is again the charge of the final state nucleus, and \(\beta = v/c\) is the dimensionless velocity of the outgoing \(\mu^\pm\) \[^{20}[21]\]. This factor suppresses (enhances) the emission of \(\mu^+\) (\(\mu^-\)). In the present case, the Coulomb effect is more complicated to compute because the outgoing charged lepton interacts not just with the Coulomb field of a single final-state nucleus, but with the Coulomb fields of the two fission fragments, with two relative velocities, \(\beta_1\) and \(\beta_2\). However, the qualitative effect would be similar to that for a charged lepton recoiling against a single nucleus.

In estimating the cross sections for these “below-threshold” charged-current neutrino reactions leading to an outgoing \(\mu^\pm\), it is useful to observe that in one respect they are analogous to a rare type of fission in which the final state consists not just of the two daughter fission fragments (usually with rather asymmetric mass distributions), but also a long-range \(\alpha\) particle or higher-\(A\) cluster \[^{21}[22]\]. In these processes the energy of the fission products is partially transferred to other particles, e.g. long-range \(\alpha\)’s and ternary fission clusters. The emission of energetic \(\alpha\)’s is peaked near to 90\(^\circ\) relative to the axis of motion of the two outgoing fission fragments and hence is denoted as “equatorial emission”. A plausible explanation of this process, within the general liquid-drop model of fission, is that as the two lobes of the droplet are pulling apart, the \(\alpha\) particle is emitted from a region of the drawn-out neck between these two lobes, and the \(\alpha\) particle is then accelerated roughly away from the axis of the receding droplets by the Coulomb repulsion with the nuclei of these fission fragments. A typical energy for the \(\alpha\) particle is 20 - 30 MeV. For a heavy nucleus such as \(^{235}\text{U}\), this ternary fission with emission of an energetic \(\alpha\) occurs with a frequency, relative to the usual binary fission, of a few parts in \(10^3\). This thus gives some measure of the suppression due to phase space and Coulombic energy transfer. A rarer type of ternary fission occurs when an \(\alpha\) particle or heavier cluster is emitted roughly along the polar axis defined by the two receding fission fragments, termed “polar emission” \[^{22}\]. This emission has been interpreted as being due in part to the excitation of a giant dipole resonance due to the fission process. Estimates of the energy transfered in the decay of this giant dipole resonance to the \(\alpha\) particle or heavier cluster are consistent with the observation that these latter particles tend to have higher energies than the \(\alpha\) particles emitted in an equatorial manner.

The phenomenon of “below-threshold” charged-current neutrino reactions on heavy nuclei may also be relevant to supernova neutrinos. We recall that neutrinos from supernovas would, in general, be comprised of a mix of \(\nu_\ell\) and \(\bar{\nu}_\ell\), \(\nu_\ell = \nu_e, \nu_\mu, \nu_\tau\). The \(\nu_\mu\) and \(\bar{\nu}_\mu\) have average energies of about 25 MeV and flux distributions, as a function of energy, that extend up to about 50 MeV \[^{23}\]. A method for observing these (anti)neutrinos via neutral current reactions on \(^{16}\text{O}\) involving proton or neutron emission, populating excited
states of $^{15}$N and $^{15}$O which then undergo photon emission, has been discussed in [24]. We note that although the above range of energies of supernova (anti)neutrinos is below threshold for usual charged-current $\mu^\pm$ production, this could be rendered possible by the neutrino-induced fission that we have discussed here. Thus, these charged-current reactions could, in principle, help as a means for the detection of supernova-generated $\nu_\mu$’s and $\bar{\nu}_\mu$’s, explicitly distinguishing these from other types (flavors) of (anti)neutrinos, which neutral current reactions do not do. Although it is not straightforward to calculate accurate cross sections for our neutrino-induced fission reactions going beyond the rough estimates given here, and these cross sections may well be somewhat smaller than those for the neutral current reactions of [24], our reactions have the useful property of yielding information on the type of incident (anti)neutrino, $\nu_\mu$ or $\bar{\nu}_\mu$, and are subject to different systematics. The detection of our reactions would use an appropriate $^{238}$U target, from which the neutrons due to the fission (adding to those from neutral current reactions) could be detected. Since the the time of the initial arrival of neutrinos from the supernova would have been obtained from a parallel neutrino detector containing hydrogen (e.g., in water) via the $\bar{\nu}_e p \rightarrow e^+ n$ reaction, any background from spontaneous fission during the $O(10)$ sec. duration of the supernova neutrino signal would be very small. For incident $\bar{\nu}_\mu$ one might also be able to detect the outgoing $\mu^+$ via the high-energy positron from its decay.

IV. CONCLUDING REMARKS

In conclusion, we have discussed several types of reactions that are naively below threshold but can proceed because of the release of binding energy from fission and the formation of Coulombic bound states. These include photofission reactions with pion production and charged-current neutrino reactions on heavy nuclei. It would be of interest to search for these reactions experimentally.

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