Observational Critical QCD

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A detailed study of correlated scalars, produced in collisions of nuclei and associated with the \( \sigma \)-field fluctuations, \( \langle \delta \sigma \rangle^2 \approx \langle \sigma^2 \rangle \), at the QCD critical point (critical fluctuations), is performed on the basis of a critical event generator (Critical Monte-Carlo) developed in our previous work. The aim of this analysis is to reveal suitable observables of critical QCD in the multiparticle environment of simulated events and select appropriate signatures of the critical point, associated with new and strong effects in nuclear collisions.

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The existence of a critical point in the phase diagram of QCD, for nonzero baryonic density, is of fundamental significance for our understanding of strong interactions and so its experimental verification has become an issue of high priority \( ^1 \). For this purpose an extensive programme of event-by-event searches for critical fluctuations in the pion sector has already been started in experiments with heavy ions from SPS to RHIC energies \( ^2 \). In \( ^2 \) we have emphasized, however, that in order to reveal critical density fluctuations in multiparticle environment, one has to reconstruct the \( \sigma \)-sector from pion pairs \( (\sigma \rightarrow \pi^+\pi^-) \) and study correlations of sigmas as a function of the invariant mass near the two-pion threshold. In fact, the QCD critical point, if it exists, communicates with a zero mass scalar field \( \sigma \)-field which at lower temperatures \( (T < T_c) \) may reach the two-pion threshold and decay in very short time scales owing to the fact that its coupling to the two-pion system is very strong. Obviously, the fundamental, underlying pattern of \( \sigma \)-field fluctuations, built-up near the critical point by the universal critical exponents of QCD \( ^3 \), is phenomenologically within reach if and only if the study of correlated sigmas, reconstructed near the two-pion threshold, becomes feasible. In this Letter we perform a detailed feasibility study of the observables related to the detection of the QCD critical point in nuclear collisions. In order to proceed we summarize, first, the principles on which the behaviour of a critical system of sigmas is based \( ^1 \) \( ^2 \) \( ^3 \).

(a) The geometrical structure of the critical system in transverse space (after integrating in rapidity) consists of \( \sigma \)-clusters with a fractal dimension \( d_F = \frac{2(\delta - 1)}{\delta + 1} \) leading to a power law, \( \langle \sigma^2 \rangle \sim |\vec{x}|^{-\frac{2(\delta - 1)}{\delta + 1}} \), for the \( \sigma \)-field fluctuations, within each cluster (\( \delta \) : isothermal critical exponent).

(b) In transverse momentum space the \( \sigma \)-fluctuations obey a power law \( \langle \sigma^2 \rangle \sim \overline{|p_T|^2(\delta - 1)} \) leading to observable intermittent behaviour of factorial moments: \( F_2(M) \sim (M^2)^{\frac{2(\delta - 1)}{\delta + 1}} \) where \( M^2 \) is the number of 2D cells.

(c) The density fluctuations of pion pairs \( (\pi^+\pi^-) \) with invariant mass close to two-pion threshold \( (2m_\pi) \) simulate to a good approximation the sigma-field fluctuations, \( \langle \delta \sigma \rangle^2 \approx \langle \sigma^2 \rangle \), at the critical point, under the assumption that the sigma mass reaches the two-pion threshold \( (m_\sigma \approx 2m_\pi) \) at a freeze-out temperature close to the critical value.

(d) The QCD critical point belongs to the universality class of the 3D Ising system in which \( \delta \approx 5 \).

On the basis of these principles and the fact that critical clusters in the above universality class interact weakly \( ^3 \), one may construct a generator of critical events (Critical Monte-Carlo: CMC) containing only sigmas, correlated according to the above prescription \( ^3 \). The numerical experiment is completed by a self-consistent treatment of \( \sigma \)-reconstruction in the simulated events (after the process of decay of sigmas to pions) and a comparative study between the associated critical correlations-fluctuations and the corresponding behaviour of a conventional Monte-Carlo (HIJING).

The input parameters of the simulation are the size of the system in rapidity \( (\Delta) \) and transverse space \( (R_L) \) as well as the proper time scale \( (\tau) \) characteristic for the formation of the critical system. The CMC generator can be used to obtain a large set of critical events. The analysis of these events could serve as a guide for the search of the critical point in the data of real \( A + A \) processes in SPS and RHIC experiments. This will be shown in a more transparent way in what follows.

We will investigate two sets of critical events. One set corresponding to a rather small system with mean charged pion multiplicity per event \( < n_{ch} > \approx 40 \) and another set describing a larger system \( ( < n_{ch} > \approx 120) \). Each data set consists of \( 10^5 \) CMC generated events. First we will consider the properties of the small system. For the values of the input parameters in this case we take \( \Delta = 6 \) adapted to the SPS energies and \( R_L = 15 \) fm. In order to reveal underlying critical fluctuations, at the level of observation, one has first to perform factorial moment analysis in small cells of the phase space \( ^4 \). We have chosen transverse momenta for this analysis, the reason being that density fluctuations in this plane are practically independent of the geometry of the critical

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events. The significance of this self-consistent reconstruction of the sigmas, based on this sector, is simply a search for the sigma-decay to charged pions and the reconstruction of the produced charged pions.

FIG. 1: The second factorial moment in transverse momentum space of the negative pion sector produced through the decay of the critical sigmas in the 10^5 CMC events. For comparison we show in the same plot the second moment for the sigmas before their decay.

system θ. Applying factorial moment analysis to the transverse momenta (p_x, p_y) of the negative pions in the sample of the 10^5 critical events we obtain for the second moment the behaviour shown in Fig. 1. We observe a weak intermittency effect \( F_2 \sim M^{2\epsilon} \) with \( s_2 \approx 0.07 \) much smaller than the expected to occur in the critical system \( (s_2 = 2/3) \). For comparison we show in the same plot the corresponding factorial moment for the sigmas, before their decay, which follows closely the theoretical prediction: \( s_2 \approx 0.63 \). The pions analysed in this figure originate from the decay of the CMC sigmas using pure kinematics and taking into account the isospin properties of the decay amplitude. The mass of the decaying sigmas is treated as an almost uniformly distributed random variable θ.

The reason for the suppression of fluctuations in the pionic sector is the kinematical distortion of the self-similar pattern formed in the sigma sector due to the sigma-decay. The strength of this distortion increases with momentum transfer: \( Q = \sqrt{m_\sigma^2 - 4m_\pi^2} \) and becomes negligible near the two-pion threshold \( (m_\sigma \approx 2m_\pi) \). Here \( m_\pi^2 = (p_{\pi^+} + p_{\pi^-})^2 \) where \( p_{\pi^\pm} \) are the four momenta of the produced charged pions.

In this treatment we consider only the visible mode of the sigma-decay to charged pions and the reconstruction of the sigmas, based on this sector, is simply a search for \((\pi^+, \pi^-)\) pairs with invariant mass close to \(2m_\pi\). The aim of this procedure is to establish a self-consistent mechanism for producing critical sigmas either directly from the event generator (CMC) or through reconstruction from the final states \((\pi^+, \pi^-)\) in the same data set of simulated events. The significance of this self-consistent reconstruction approach in a search for critical \(\sigma\)-fluctuations in real events becomes self-evident. In practice this can be done by looking, in each recorded event, for \((\pi^+, \pi^-)\) pairs fulfilling the criterion A:

\[
A = \{(\pi^+, \pi^-)|4m_\pi^2 \leq (p_{\pi^+} + p_{\pi^-})^2 \leq (2m_\pi + \epsilon)^2\} \quad (1)
\]

with \(\frac{\epsilon}{2m_\pi} \ll 1\). The momentum of the sigma-particle is then obtained as: \(p_\sigma = p_{\pi^+} + p_{\pi^-}\). In general the pairing will not be one-to-one, allowing for different possibilities to pair the charged pions in order to get sigmas. It turns out that the full pairing forming all possible pairs \((\pi^+, \pi^-)\) fulfilling (1) for a given \(\pi^+\), produces more accurately the geometric properties of the critical sigma sector in the limit \(\epsilon \to 0\). Due to finite statistics this limit can only be taken using data sets consisting of events with reconstructed sigmas, obtained for a sequence of \(\epsilon\)-values approaching to zero, and then extrapolate smoothly to \(\epsilon = 0\). Thus we have performed factorial moment analysis in transverse momentum space using values of \(\epsilon\) in the region \(8 - 15 \text{ MeV}\). We have calculated the second moment \(F_2\) for different sets of events with reconstructed sigmas, each obtained using a fixed \(\epsilon\). It exists a wide window in the resolution scale \(M\) for which \(F_2 \sim (M^2)^{s_2(\epsilon)}\). Using linear extrapolation the full pairing leads to the value \(s_2(0) \approx 0.70 \pm 0.03\) very close to the theoretically predicted \(s_2(0) = \frac{2}{3}\). In an alternative rule for pairing, choosing for each \(\pi^+\) randomly the corresponding \(\pi^-\) partner, from all possible ones fulfilling (1), and repeating the same moment analysis we find \(s_2(0) \approx 0.44 \pm 0.01\) deviating by 35% from the theoretically expected value. Thus the full pairing reproduces successfully the geometrical properties of the critical system due to the fact that in this way one reconstructs all critical sigmas even if it is impossible to identify them.

The above algorithm besides the reconstruction of the real critical sigmas also creates a number of fake sigmas. Some of these are generated by combining pions belonging to different critical sigmas and contribute significantly to the strong intermittency effect occurring in the reconstructed isoscalar sector. Therefore the intermittency effect in the factorial moment analysis is a necessary but not sufficient condition to identify critical, strongly correlated isoscalars. There is a need to subtract the effect of the fake sigmas in order to reveal the correlated critical isoscalar sector. To achieve this goal one has to calculate the pair-correlation density of the sigmas defined by:

\[
C(1, 2) = \langle \rho_2(1, 2) \rangle = \langle \rho_2^{(m)}(1, 2) \rangle \quad (2)
\]

where \(\langle \rho_2 \rangle\) is the two-particle density averaged over the events. The arguments 1, 2 refer to the momentum coordinates of two sigma-particles. The second term is the two-particle density estimated using mixed events θ. In fact for a given set of events one can construct a set of mixed events by shuffling the momentum coordinates of the pions in the initial data set. As a consequence the (possible) correlations in the original data, including the
formation of sigmas, are destroyed in the mixed set. In general the two-particle density is given by the number of particle pairs per event and per phase space cell of volume \(d\Omega\). Therefore eq. 1 can be written as:

\[
C(1,2) = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \frac{dn_{1,2,i}}{d\Omega} - W \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \frac{dn_{1,2,i}}{d\Omega} (3)
\]

where \(i\) labels the events, \(N_{ev}\) is the total number of CMC events fulfilling the condition \(\phi\) while \(N_{ev}(m)\) is the total number of mixed events, obtained from the shuffling of the tracks in the CMC events, fulfilling \(\phi\). The coefficient \(W\) is the ratio of numbers of events:

\[
W = \frac{N_{ev}(m)}{N_{ev}}.
\]

The correlator \(G(M)\) is defined as the integral of \(C(1,2)\) over a finite phase space cell with volume \(\delta\Omega = \frac{1}{M^D}\), where \(\Omega\) is the total available phase space in \(D\)-dimensions (here \(D = 2\)). Performing the integration and normalizing properly we can express the correlator in terms of the second order factorial moments \(W\). Taking into account the fact that the resulting distributions of the sigmas depend on the parameter \(\epsilon\) through \(\phi\) we finally get:

\[
G(M, \epsilon) = F_2(M, \epsilon) - W(\epsilon) \left( \frac{m}{n_\sigma} \right)^2 F_2(m, \epsilon)
\]

where \(\langle n_\sigma \rangle\) is the mean multiplicity of sigmas within a phase space cell for the CMC events while \(\langle n_\sigma (m) \rangle\) is the corresponding quantity for the mixed events. These two multiplicities are in fact almost equal and therefore their ratio appearing in eq. 1 is very close to one for all \(\epsilon\). The weight factor \(W\) has a transparent physical interpretation: it describes the relative weight of the fake sigmas in the reconstructed set. If \(W(\epsilon) \approx 1\) the fake sigmas dominate while for \(W(\epsilon) \rightarrow 0\) they are suppressed relative to the real sigmas. It turns out that the dependence of \(W\) on \(\epsilon\) is crucial for the determination of the critical isoscalar sector. Accordingly, if \(W \approx 0\) the correlator follows the properties of the second moment \(F_2\) and the effect of the mixed events is marginal, while for \(W \approx 1\) the correlator becomes negligible and no critical effect can be identified. At threshold \((\epsilon = 0)\) we expect to fully recover the critical isoscalar sector therefore \(W(0)\) must vanish if real critical sigmas exist in the data set of the reconstructed sigmas. This should be the case when analyzing the CMC events. In fact, it turns out that in order to avoid fake sigmas at threshold, in the reconstruction procedure, events with sufficient multiplicity of pion pairs \((n_\pi)\) must be considered in the analysis \((n_\pi > 2)\). In addition the correlator \(G(M, \epsilon)\) should behave like \(\sim M^{2\phi_2(\epsilon)}\) if self-similar patterns of correlated sigmas are present in the corresponding phase space. The exponent \(\phi_2(\epsilon)\) is a new intermittency exponent having the property \(\phi_2(0) = \phi_2(0)\) as \(G(M, 0) \approx F_2(M, 0)\). Finally, the strength of the correlations in the sigma sector is given by the magnitude of \(G(M, \epsilon)\). Therefore in terms of the correlator we have three very strict requirements in order to reveal the critical isoscalars in a data set of charged pions:

- Dominance of the real sigmas with respect to the fake sigmas when approaching the two-pion threshold \((\lim_{\epsilon \to 0} W(\epsilon) = 0)\)
- Large positive values of the correlator at threshold \((G(M, 0) \gg 1)\) at high resolution scales.
- Correct power-law dependence of the correlator on the resolution scale \(M\), at threshold: \(G(M, 0) \sim M^{1.34}\)

In a conventional hadronic system with no critical isoscalar sector at least one of the above properties will not be met.

To test the above ideas we applied the sigma-reconstruction algorithm to \(10^5\) CMC events describing a small system as well as to 33000 HIJING events for the \(C + C\) system at the SPS. The mean charged pion multiplicity per event in these two systems is practically the same. The CMC data set is expected to possess a recoverable critical isoscalar sector. On the contrary the HIJING simulator describes, in the low \(p_t\) region, conventional hadronic dynamics and therefore no critical isoscalars are expected to occur in the corresponding data set.

We have first calculated the correlator for these two systems choosing \(n_\pi \geq 5\) for the multiplicity of pion pairs. The results are given in Fig. 2 for \(\epsilon\)-values in the region of \(8 - 15\) \(MeV\), suggested by finite statistics.

As we can see the CMC-correlator behaves very differently from the HIJING-correlator for all values of \(\epsilon\). In Fig. 3 we also show the weight factor \(W(\epsilon)\) for both systems. It is obvious that in the CMC case \(W\) approaches zero for \(\epsilon \to 0\) while in HIJING \(W(\epsilon)\) is of order of one independently of the \(\epsilon\)-value. The interpretation of this very important result is that in the case of HIJING, i.e. in a conventional hadronic system, the reconstructed sigmas are fake for every value of \(\epsilon\).

Finally in Fig. 4 we plot \(\phi_2(\epsilon)\) for the CMC case. The linear extrapolation leads to \(\phi_2(0) = 0.69 \pm 0.04\) in very good agreement with the theoretically expected value of \(2/3\). According to the analysis above we have recovered in a self-consistent manner the critical isoscalar sector in the CMC data set as a result of \(\sigma\)-reconstruction. On the contrary the HIJING events do not contain real critical sigmas, as expected.

We also examined the dependence of our results on the charged pion multiplicity. We have produced CMC events for a larger system with size in rapidity \(\Delta = 11\) and \(R_1 = 25 \text{ fm}\). In this case the total charged pion multiplicity is \(\approx 120\) per event coming from the decay of sigmas with mean multiplicity \(\approx 96\). We found that \(W(0)\) is very close to zero (linear extrapolation: \(W(0) = -0.03\)) and \(\phi_2(0) = 0.664 \pm 0.02\) in very good agreement with the estimations in the smaller critical system (Fig. 4). Thus the results concerning the description of the critical isoscalar sector are not sensitive to the size of the system,
as expected. What is changing is the correlation length which for a second order phase transition is proportional to the size of the system. In fact for the two systems considered here the critical correlation length is $\approx 6 \, fm$ (small system) and $\approx 10 \, fm$ (large system) respectively.

In order to support further our claim that in the CMC description the sector of $\pi^+\pi^-$ pairs near two-pion threshold, has a distinct dynamical behaviour, associated with $\sigma$-field fluctuations at the critical point, we have examined the behaviour of $\pi^+\pi^+$ pairs in the same region ($\epsilon \lesssim 4 \, MeV$). As expected no effect is found, the pairing $\pi^+\pi^+$ has no dynamical origin ($W \approx 1$) and the corre-
FIG. 5: The correlator \( G(M, \epsilon) \) for \((\pi^+, \pi^+)^\ast\) pairs (stars) as well as for sigmas (open triangles) using \( \epsilon = 4 \text{ MeV} \).

The correlator \( G(M, \epsilon) \) of these pairs is strongly suppressed. We illustrate this in Fig. 5 where we compare, for \( \epsilon = 4 \text{ MeV} \), the correlators of sigmas \((\pi^+\pi^-)^\ast\) pairs and \(\pi^+\pi^+\) pairs for the small system \( < n_{ch} > \approx 40 \). It must be noted that for this value of \( \epsilon \) the Coulomb correlations are negligible \((Q^2 = -\left(p_{\pi^+} - p_{\pi^-}\right)^2 \approx 50 \text{ MeV}^2\)\). Similar results are found if the system is larger \((< n_{ch} > \approx 120)\).

In conclusion, we have shown that a set of well prescribed observables (factorial moments, correlators, statistical weight of fake sigmas, intermittency exponents) associated with the existence of a critical point in quark matter, can be established in nuclear collisions. These observables belong to the reconstructed \(\sigma\)-sector describing massive scalars \((\pi^+\pi^-)^\ast\) near the two-pion threshold and their behaviour reveals strong critical effects suggested by \(\sigma\)-field fluctuations near the critical point. We claim that a search for such a critical behaviour in heavy ion experiments is feasible within the framework of a reconstruction procedure of massive scalars, discussed in this work. Since the critical effects in this sector are strong and their pattern remains robust for systems of different size, our proposal is to study, using the above observables, different processes at the SPS and RHIC with the aim to scan a substantial area of the phase diagram, in a systematic search for the QCD critical point in collisions of nuclei.

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