Analytical Method of Atomic Density Determination of Uranyl Nitrate Solution

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Abstract. A basic theory of an analytical calculation method to determine the atomic density of each individual nuclide contained in a homogeneous system of uranyl nitrate solution is presented. Atomic density or number density is a number of atoms or nuclides per cm³ or in a unit of atoms per barn-cm. This parameter is one of the important factors in defining the accuracy of neutronic calculations as such used in reactor core design. This work was carried out as part of neutronic design calculation of the SAMOP (subcritical assembly for ⁹⁹Mo isotope production) project, where uranyl nitrate solution was used as fuel with uranium enrichment of 20%. A homogeneous solution made by dissolving uranyl nitrate salt [UO₂(NO₃)₂] into H₂O solvent has been used as an approach to this model. Two kinds of solution were described i.e. one that was made based on the amount of UO₂(NO₃)₂ dissolved in the solution and another one based on the amount of uranium in the solution. Expectedly this method can be used as an alternative way in estimating the atom density, solution density (g/cc) and the corresponding atom and weight fractions which is important in criticality calculations.

Keywords: Analytical method, SAMOP, atomic density, solution, uranyl nitrate.

1. Introduction
Atomic density (N) or commonly termed as number density is number of atoms or nuclides per cm³ (atoms/cm³³), or in unit of atoms per 10²⁴ barn cm (atom / barn cm). This parameter is one of important factors that would contribute to the accuracy of the neutronic calculations result, as such used in reactor core design, core power distributions etc. There are several methods that can be accurately determine the material compositions, which can be performed by means of measurement using special equipment, such as mass spectrometry [1,2,3], or gamma spectrometry [4], and so forth.

In case when there is no such composition measuring equipment available, or lack of data available for neutronic calculations, which often requires many different variations of material composition, an analytical method might be helpful as an approach to perform the simulations. Recently study on a possible use of reactor using solution based fuel, commonly termed as aqueous homogeneous reactor (AHR) is being of great interest for some research institutions. This includes power generation reactor, such as molten salt reactors (MSR) [5,6,7] and those dedicated for producing medical radioisotope ⁹⁹Mo [8,9,10] which in many cases offers more benefit compared to that using solid fuel or ²³⁵U target as used in the present technique.

For the purpose of study related with the neutronic calculations, the atomic density of each nuclide contained in the solution is of important parameter, and for most cases of the data needed are not available. In this respect an analytical approach might help solve the problem, especially when needed simulation of solutions for many different variations of concentrations.
This paper presents a basic theory of the analytical method to determine the atomic density of each nuclide contained in uranyl nitrate solution, of different concentrations and $^{235}\text{U}$ enrichments. This method has been used as the basis of calculation in the previous study when dealing with the preliminary study of the proposed SAMOP (Subcritical Assembly for $^{99}$Mo Production), [11, 12]. The formula includes two kinds of uranyl nitrate solution i.e. the solution’s concentration which was prepared based on the amount (gram) of uranyl nitrate salt dissolved, and the one that based on the amount of uranium in the solution. Further, this method can be used as the basis approach in determining the atom density or the solution density (g/cc) and the associated weight or atom fractions of other homogeneous uranium solution as well.

2. Basic Theory

Atomic density of a nuclide (n) in general can be determined based on the following equation

$$n = \frac{M N_A}{V} = \frac{w N_A}{\rho N_A A}$$

(1)

where M is molarity of the material or nuclide (g/A), $N_A$ is Avogadro’s number = $6.0225 \times 10^{23}$ atom/mol, V is volume of the nuclide (cm$^3$), w is mass of nuclide (g), A is atomic mass or atom’s number of the nuclide, and $\rho$ is density of the nuclide (g/cm$^3$). For a material composed of several nuclides, the atomic density of each individual nuclide ($n_i$) can be determined based on the molarity of each nuclide contained in the material ($M_i$). For this case, data on the compositions i.e atomic fraction or weight fraction of each composing nuclide is needed.

Uranyl nitrate or sometimes referred as uranium nitrate is uranium salt, with chemical formula $\text{UO}_2(\text{NO}_3)_2$ and it is soluble in water[13,14]. Table 1 presents the physical properties and compositions of $\text{UO}_2(\text{NO}_3)_2$.

**Table 1.** Physical properties and compositions of uranyl nitrate [$\text{UO}_2(\text{NO}_3)_2$] [13, 14]

| Physical form | Density (g/cc) | Melting point (°C) | Boiling point (°C) | Molecular weight |
|---------------|----------------|--------------------|--------------------|-----------------|
| Solid         | 2.087          | 60                 | 118                | 394.04          |

| Nuclide       | Mass number | Number of atom | Molar mass (g/mol) |
|---------------|-------------|----------------|--------------------|
| U-total       | 238.028     | 1              | 238.028            |
| Nitrogen      | 14.007      | 2              | 28.014             |
| Oxygen        | 16          | 8              | 128.000            |
| Total         |             |                | 394.040            |

The molecular weight or mass number of uranyl nitrate as in the Table 1, is for natural uranium. This value will be slightly reduce with the increasing of $^{235}\text{U}$ enrichment.

Uranium, naturally consists of 3 main isotope i.e $^{234}\text{U}$, $^{235}\text{U}$ and $^{238}\text{U}$, with the compositions as presented in Table 2. The content of isotope $^{234}\text{U}$ is relatively very small compared to the other 2 isotopes, and in some cases such as used in criticality calculations, the presence can be omitted from the calculation. The compositions of the three uranium isotopes as a function of $^{235}\text{U}$ enrichment has been estimated based on a semi empirical approach [16]. Table 3 Presents the compositions of the 2 main isotope (without $^{234}\text{U}$) as a function of $^{235}\text{U}$ enrichment.

**Table 2.** Compositions of 3 uranium isotopes of natural uranium [15]

| Parameter | $^{234}\text{U}(A = 234.041)$ | $^{235}\text{U}(A = 235.0439)$ | $^{238}\text{U}(A = 238.0508)$ | Total |
|-----------|-------------------------------|-------------------------------|-------------------------------|-------|
| Atom (%)  | 0.0054%                       | 0.72%                         | 99.275%                       | 100%  |
To determine the number of hydrogen and oxygen atom from the component of water, the volume and a summation of two components i.e which is from the UO enrichment.

The uranyl nitrate solutions can be prepared through several ways, such as by dissolving the uranyl nitrate $[\text{UO}_2(\text{NO}_3)_2]$ into water solvent or by dissolving $\text{U}_3\text{O}_8$ with nitrate acid (HNO$_3$) solvent. In this work, the first method will be used as the base for evaluation, i.e by dissolving the $[\text{UO}_2(\text{NO}_3)_2]$ salt into water. The concentration of uranyl nitrate in the solution can be determined based on the amount (gram) of the uranyl nitrate salt dissolved, or based on the amount (gram) of uranium in the solution. In this respect, the determination of atomic density of each nuclide for the two cases will be evaluated here:

### 3. Methodology

Uranyl nitrate solutions can be prepared through several ways, such as by dissolving the uranyl nitrate $[\text{UO}_2(\text{NO}_3)_2]$ into water solvent or by dissolving $\text{U}_3\text{O}_8$ with nitrate acid (HNO$_3$) solvent. In this work, the first method will be used as the base for evaluation, i.e by dissolving the $[\text{UO}_2(\text{NO}_3)_2]$ salt into water. The concentration of uranyl nitrate in the solution can be determined based on the amount (gram) of the uranyl nitrate salt dissolved, or based on the amount (gram) of uranium in the solution. In this respect, the determination of atomic density of each nuclide for the two cases will be evaluated here:

### 3.1. Number density based on the mass of uranyl nitrate in the solution

For any weight of uranyl nitrate salt ($w_{\text{UN}}$) The number of $\text{UO}_2(\text{NO}_3)_2$ molecules can be determined based on the following equation:

$$n_{\text{UN}} = \frac{w_{\text{UN}}}{A_{\text{UN}}} N_A$$

where $A_{\text{UN}}$ is the molecular weight of uranyl nitrate which is about 394.040 for natural uranium. This value will be slightly reduce with the increasing of $^{235}\text{U}$ enrichment. Based on the isotopic compositions of uranyl nitrate, the total number of atom for each composing nuclide can then be determined using equation (3). The number of each uranium isotopes can then be found by multiplying with the nuclide fraction of the respective isotopes, which the values depending on the $^{235}\text{U}$ enrichment. When the uranyl nitrate salt is dissolved into water solvent, the number of oxygen atom is a summation of two components i.e which is from the $\text{UO}_2(\text{NO}_3)_2$ and from water (H$_2$O), and the amount would depend on the concentration of uranyl nitrate in the solution.

$$\#\text{Uranium} : n_u = n_{\text{UN}} = \frac{w_{\text{UN}}}{A_{\text{UN}}} N_A ;$$

$$\#\text{Nitrogen} : n_N = 2n_{\text{UN}} = 2 \frac{w_{\text{UN}}}{A_{\text{UN}}} N_A ;$$

$$\#\text{Oxygen} : n_O = 8n_{\text{UN}} = 8 \frac{w_{\text{UN}}}{A_{\text{UN}}} N_A ;$$

To determine the number of hydrogen and oxygen atom from the component of water, the volume and hence the weight fraction of water in the solution needs to be determined. For every one litre of solution, the volume fraction of uranyl nitrate ($V_{\text{UN}}$) and water($V_{\text{wat}}$) can be determined as follows:

| $^{235}\text{U}$ enrichment (w %) | $^{235}\text{U}$ weight fraction | $^{238}\text{U}$ weight fraction | Nuclide fraction | Atom Weight U (total) |
|----------------------------------|----------------------------------|----------------------------------|----------------|---------------------|
| 0.71                             | $7.110 \times 10^{-1}$          | $9.929 \times 10^{-1}$          | $7.200 \times 10^{-3}$ | 9.928 $\times 10^{-1}$ | 238.029 |
| 2.50                             | $2.500 \times 10^{-2}$          | $9.750 \times 10^{-1}$          | $2.531 \times 10^{-2}$ | 9.747 $\times 10^{-1}$ | 237.976 |
| 5.00                             | $5.000 \times 10^{-2}$          | $9.500 \times 10^{-1}$          | $5.061 \times 10^{-2}$ | 9.494 $\times 10^{-1}$ | 237.900 |
| 7.50                             | $7.500 \times 10^{-2}$          | $9.250 \times 10^{-1}$          | $7.589 \times 10^{-2}$ | 9.241 $\times 10^{-1}$ | 237.825 |
| 10.00                            | $1.000 \times 10^{-1}$          | $9.000 \times 10^{-1}$          | $1.011 \times 10^{-1}$ | 8.989 $\times 10^{-1}$ | 237.750 |
| 12.50                            | $1.250 \times 10^{-1}$          | $8.750 \times 10^{-1}$          | $1.264 \times 10^{-1}$ | 8.736 $\times 10^{-1}$ | 237.675 |
| 15.00                            | $1.500 \times 10^{-1}$          | $8.500 \times 10^{-1}$          | $1.516 \times 10^{-1}$ | 8.484 $\times 10^{-1}$ | 237.600 |
| 17.50                            | $1.750 \times 10^{-1}$          | $8.250 \times 10^{-1}$          | $1.768 \times 10^{-1}$ | 8.232 $\times 10^{-1}$ | 237.525 |
| 19.75                            | $1.975 \times 10^{-1}$          | $8.025 \times 10^{-1}$          | $1.995 \times 10^{-1}$ | 8.005 $\times 10^{-1}$ | 237.457 |
\[ V_{\text{UN}} = \frac{W_{\text{UN}}}{\rho_{\text{UN}}} = \frac{W_{\text{UN}}}{2,807} \, \text{cm}^3 \]
\[ V_{\text{wat}} = 1000 - V_{\text{UN}} \, \text{cm}^3 \]

where \( \rho_{\text{UN}} \) = density of uranyl nitrate salt \((2,807 \, \text{g/cm}^3)\). Hence the weight fraction of H\(_2\)O \((W_{\text{wat}})\) for each amount of uranyl nitrate to be solved can be determined as follows:

\[ w_{\text{wat}} = V_{\text{wat}} \rho_{\text{wat}} = \rho_{\text{wat}} \left(1000 - \frac{W_{\text{UN}}}{2,807}\right) \]

where \( \rho_{\text{wat}} \) = density of water \((\text{g/cm}^3)\).

The number of atom hydrogen and oxygen from component of water can then be determined as follows:

\[ n_H = \frac{2 \, W_{\text{wat}}}{A_{\text{wat}}} \, N_A = 2 \, \frac{W_{\text{wat}}}{18,016} \, N_A \]
\[ n_O = \frac{W_{\text{wat}}}{18,016} \, N_A \]

Hence the total number of oxygen atoms in the solution for any concentrations of uranyl nitrate solution becomes:

\[ n_O(\text{tot}) = \left(\frac{8 \, W_{\text{UN}}}{A_{\text{UN}}} + \frac{W_{\text{wat}}}{18,016}\right) \, N_A \]

Based on equation (3), (5), (6) and (7) the number density of each nuclide contained in one litre of solution can then be determined i.e by deviding the calculated total number of atom with 1000 cm\(^3\).

For uranium, the number density for the 3 isotopes, must be multiplied by their atom fractions, which are depending on the \(^{235}\text{U}\) enrichment, as presented in Table 3.

3.2. Number density based on the amount of uranium in the solution

In case the uranyl nitrate solution is prepared by dissolving an amount of uranium metal into nitride acid \((\text{HNO}_3)\) or other ways instead of dissolving the uranyl nitrate salt into water, the weight fraction or atom density of each nuclide can be estimated based on the following approach. First we need to calculate the weight of each nuclide of the associated \(\text{UO}_2(\text{NO}_3)_2\) salt. The weight of uranium isotopes for any amount \((\text{gram})\) of uranium \((w_U)\) can be determined based on eq. (8)

\[ w_{25} = f w_{25} \, \times w_u \]
\[ w_{28} = f w_{28} \, \times w_u \]

where \(f w_{25}\) and \(f w_{28}\) are the weight fractions of \(^{235}\text{U}\) and \(^{238}\text{U}\) respectively which correspond to the enrichment. The weight of nitrogen and oxygen can then be determined based on the following equation.

\[ w_N = \frac{2 \times 14}{A_U} \, w_u \]
\[ w_O = \frac{8 \times 16}{A_U} \, w_u \]

where \(A_U\) is the atom weight of uranium total which is a function of the \(^{235}\text{U}\) enrichment, as presented in Table 3. The weight of \(\text{UO}_2(\text{NO}_3)_2\) salt \((W_{\text{UN}})\) now can be found from eq. (8) and (9), from which the associated volume \((V_{\text{UN}})\) and the volume of water can be determined as in eq. (4). The weight of water \((w_{\text{wat}})\) in the solution can then be determined as in eq. (5). The total weight of the solution, the density \((\text{g/cc})\) and the weight fraction of each nuclide can be determined thereafter. Further, the number of atoms for each nuclide can then be determined using the general equations (3), (6) and (7). The corresponding number density of each nuclide in the solution can then be determined.
4. Result and discussion

Table 4 summarizes the calculated number densities of each nuclide contained in the solutions for several concentrations of $\text{UO}_2(\text{NO}_3)_2$ and $^{235}\text{U}$ enrichments, and Table 5 presents the corresponding weight fractions. The results show that the number density of uranium isotopes will change depending on both concentration and $^{235}\text{U}$ enrichment. The number density of hydrogen and oxygen are changing very slowly with the changes of concentration for both cases.

| Table 4. Atom density for several $\text{UO}_2(\text{NO}_3)_2$ concentrations and $^{235}\text{U}$ enrichments |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Concentration (g/l) | 150 | 200 | 300 | 400 | 500 |
| Density (g/cc) | 1.0781 | 1.1042 | 1.1563 | 1.2083 | 1.2604 |
| Nuclide | Atom density (atom / barn cm); Enrichment = 10 w% |
| $\text{U-235}$ | $3.2207 \times 10^{-3}$ | $3.0942 \times 10^{-3}$ | $4.6413 \times 10^{-3}$ | $6.1884 \times 10^{-3}$ | $7.7356 \times 10^{-3}$ |
| $\text{U-238}$ | $2.0622 \times 10^{-4}$ | $2.7496 \times 10^{-4}$ | $4.1244 \times 10^{-4}$ | $5.4992 \times 10^{-4}$ | $6.8741 \times 10^{-4}$ |
| $\text{N}$ | $4.5886 \times 10^{-4}$ | $6.1181 \times 10^{-4}$ | $9.1771 \times 10^{-4}$ | $1.2236 \times 10^{-3}$ | $1.5295 \times 10^{-3}$ |
| $\text{H}$ | $6.2107 \times 10^{-2}$ | $6.0504 \times 10^{-2}$ | $5.7298 \times 10^{-2}$ | $5.4091 \times 10^{-2}$ | $5.0885 \times 10^{-2}$ |
| $\text{O}$ | $3.2890 \times 10^{-2}$ | $3.2699 \times 10^{-2}$ | $3.2320 \times 10^{-2}$ | $3.1940 \times 10^{-2}$ | $3.1561 \times 10^{-2}$ |
| Nuclide | Atom density (atom / barn cm); Enrichment = 15 w% |
| $\text{U-235}$ | $3.4801 \times 10^{-5}$ | $4.6401 \times 10^{-5}$ | $6.9602 \times 10^{-5}$ | $9.2803 \times 10^{-5}$ | $1.1600 \times 10^{-4}$ |
| $\text{U-238}$ | $1.9472 \times 10^{-4}$ | $2.5962 \times 10^{-4}$ | $3.8943 \times 10^{-4}$ | $5.1924 \times 10^{-4}$ | $6.4905 \times 10^{-4}$ |
| $\text{N}$ | $4.5903 \times 10^{-4}$ | $6.1204 \times 10^{-4}$ | $9.1806 \times 10^{-4}$ | $1.2241 \times 10^{-3}$ | $1.5301 \times 10^{-3}$ |
| $\text{H}$ | $6.2107 \times 10^{-2}$ | $6.0504 \times 10^{-2}$ | $5.7298 \times 10^{-2}$ | $5.4091 \times 10^{-2}$ | $5.0885 \times 10^{-2}$ |
| $\text{O}$ | $3.2890 \times 10^{-2}$ | $3.2700 \times 10^{-2}$ | $3.2321 \times 10^{-2}$ | $3.1942 \times 10^{-2}$ | $3.1563 \times 10^{-2}$ |

| Table 5. Nuclide weight fraction for several $\text{UO}_2(\text{NO}_3)_2$ concentrations and $^{235}\text{U}$ enrichments |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Concentration (g/l) | 150 | 200 | 300 | 400 | 500 |
| Density (g/cc) | 1.0781 | 1.1042 | 1.1563 | 1.2083 | 1.2604 |
| Nuclide | Atom weight fraction; Enrichment = 10 w% |
| $\text{U-235}$ | $8.3502 \times 10^{-3}$ | $1.0812 \times 10^{-2}$ | $1.5488 \times 10^{-2}$ | $1.9761 \times 10^{-2}$ | $2.3680 \times 10^{-2}$ |
| $\text{U-238}$ | $7.5703 \times 10^{-2}$ | $9.8557 \times 10^{-2}$ | $1.4118 \times 10^{-1}$ | $1.8012 \times 10^{-1}$ | $2.1585 \times 10^{-1}$ |
| $\text{N}$ | $9.8937 \times 10^{-3}$ | $1.2880 \times 10^{-2}$ | $1.8450 \times 10^{-2}$ | $2.3540 \times 10^{-2}$ | $2.8209 \times 10^{-2}$ |
| $\text{H}$ | $9.5652 \times 10^{-2}$ | $9.0985 \times 10^{-2}$ | $8.2282 \times 10^{-2}$ | $7.4330 \times 10^{-2}$ | $6.7034 \times 10^{-2}$ |
| $\text{O}$ | $8.1045 \times 10^{-1}$ | $7.8677 \times 10^{-1}$ | $7.4260 \times 10^{-1}$ | $7.0225 \times 10^{-1}$ | $6.6523 \times 10^{-1}$ |
| Nuclide | Atom weight fraction; Enrichment = 15 w% |
| $\text{U-235}$ | $1.2463 \times 10^{-2}$ | $1.6225 \times 10^{-2}$ | $2.3241 \times 10^{-2}$ | $2.9652 \times 10^{-2}$ | $3.5534 \times 10^{-2}$ |
| $\text{U-238}$ | $7.1525 \times 10^{-2}$ | $9.3117 \times 10^{-2}$ | $1.3338 \times 10^{-1}$ | $1.7018 \times 10^{-1}$ | $2.0393 \times 10^{-1}$ |
| $\text{N}$ | $9.8975 \times 10^{-3}$ | $1.2885 \times 10^{-2}$ | $1.8457 \times 10^{-2}$ | $2.3549 \times 10^{-2}$ | $2.8220 \times 10^{-2}$ |
| $\text{H}$ | $9.5652 \times 10^{-2}$ | $9.0985 \times 10^{-2}$ | $8.2282 \times 10^{-2}$ | $7.4330 \times 10^{-2}$ | $6.7034 \times 10^{-2}$ |
| $\text{O}$ | $8.1046 \times 10^{-1}$ | $7.8679 \times 10^{-1}$ | $7.4264 \times 10^{-1}$ | $7.0229 \times 10^{-1}$ | $6.6528 \times 10^{-1}$ |
| Nuclide | Atom weight fraction; Enrichment = 19.75 w% |
| $\text{U-235}$ | $1.6415 \times 10^{-2}$ | $2.1370 \times 10^{-2}$ | $3.0612 \times 10^{-2}$ | $3.9056 \times 10^{-2}$ | $4.6803 \times 10^{-2}$ |
| $\text{U-238}$ | $6.7552 \times 10^{-2}$ | $8.7945 \times 10^{-2}$ | $1.2598 \times 10^{-1}$ | $1.6073 \times 10^{-1}$ | $1.9261 \times 10^{-1}$ |
| $\text{N}$ | $9.9011 \times 10^{-3}$ | $1.2890 \times 10^{-2}$ | $1.8464 \times 10^{-2}$ | $2.3558 \times 10^{-2}$ | $2.8230 \times 10^{-2}$ |
Table 6 and Table 7 present the results when using solution concentration based on the amount of uranium in the solution. The results show that both the number densities and the weight fractions of uranium isotopes will be higher than those calculated based on the uranyl nitrate [UO$_2$(NO$_3$)$_2$] concentration quite significantly. This phenomenon is caused by the change of water content in both cases, such as shown in Table 4 to Table 7. Figure 1 depicts comparison of the number density of total uranium atom in the solution for several concentration of both UO$_2$(NO$_3$)$_2$ and uranium in the solution for $^{235}$U enrichment of 19.75 w%. Figure 1 and 2 depict the comparison of both number density and the weight fraction of the uranium (total) as a function of concentration for the enrichment of 19.75 w%, for the two cases.

**Table 6.** Atom density for several uranium concentrations and $^{235}$U enrichments

| Concentration (g / l) | 150 | 200 | 300 | 400 | 500 |
|-----------------------|-----|-----|-----|-----|-----|
| Density (g / cc)      | 1.1295 | 1.1726 | 1.2589 | 1.3452 | 1.4315 |
| Nuclide               | Atom density (atom / barn cm) ; Enrichment =10 w% |
| U-235                 | 3.8441×10$^{-5}$ | 5.1255×10$^{-5}$ | 7.6883×10$^{-5}$ | 1.0251×10$^{-4}$ | 1.2814×10$^{-4}$ |
| U-238                 | 3.4161×10$^{-4}$ | 4.5548×10$^{-4}$ | 6.8322×10$^{-4}$ | 9.1097×10$^{-4}$ | 1.1387×10$^{-3}$ |
| N                     | 7.6011×10$^{-4}$ | 1.0135×10$^{-3}$ | 1.5202×10$^{-3}$ | 2.0270×10$^{-3}$ | 2.5337×10$^{-3}$ |
| H                     | 6.5032×10$^{-2}$ | 6.4403×10$^{-2}$ | 6.3146×10$^{-2}$ | 6.1890×10$^{-2}$ | 6.0633×10$^{-2}$ |
| O                     | 3.2516×10$^{-2}$ | 3.2202×10$^{-2}$ | 3.1573×10$^{-2}$ | 3.0945×10$^{-2}$ | 3.0316×10$^{-2}$ |
| Nuclide               | Atom density (atom / barn cm) ; Enrichment =15 w% |
| U-235                 | 5.7662×10$^{-3}$ | 7.6883×10$^{-3}$ | 1.1532×10$^{-4}$ | 1.5377×10$^{-4}$ | 1.9221×10$^{-4}$ |
| U-238                 | 3.2263×10$^{-4}$ | 4.3018×10$^{-4}$ | 6.4527×10$^{-4}$ | 8.6036×10$^{-4}$ | 1.0754×10$^{-3}$ |
| N                     | 7.6059×10$^{-4}$ | 1.0141×10$^{-3}$ | 1.5212×10$^{-3}$ | 2.0282×10$^{-3}$ | 2.5353×10$^{-3}$ |
| H                     | 6.5033×10$^{-2}$ | 6.4406×10$^{-2}$ | 6.3150×10$^{-2}$ | 6.1895×10$^{-2}$ | 6.0639×10$^{-2}$ |
| O                     | 3.2517×10$^{-2}$ | 3.2203×10$^{-2}$ | 3.1575×10$^{-2}$ | 3.0947×10$^{-2}$ | 3.0319×10$^{-2}$ |
| Nuclide               | Atom density (atom / barn cm) ; Enrichment =19.75 w% |
| U-235                 | 7.5922×10$^{-5}$ | 1.0123×10$^{-4}$ | 1.5184×10$^{-4}$ | 2.0246×10$^{-4}$ | 2.5307×10$^{-4}$ |
| U-238                 | 3.0460×10$^{-4}$ | 4.0614×10$^{-4}$ | 6.0921×10$^{-4}$ | 8.1228×10$^{-4}$ | 1.0153×10$^{-3}$ |
| N                     | 7.6105×10$^{-4}$ | 1.0147×10$^{-3}$ | 1.5221×10$^{-3}$ | 2.0295×10$^{-3}$ | 2.5368×10$^{-3}$ |
| H                     | 6.5035×10$^{-2}$ | 6.4408×10$^{-2}$ | 6.3154×10$^{-2}$ | 6.1899×10$^{-2}$ | 6.0645×10$^{-2}$ |
| O                     | 3.2518×10$^{-2}$ | 3.2204×10$^{-2}$ | 3.1577×10$^{-2}$ | 3.0950×10$^{-2}$ | 3.0322×10$^{-2}$ |

**Table 7.** Atom weight fraction for several uranium concentrations and $^{235}$U enrichments

| Concentration (g / l) | 150 | 200 | 300 | 400 | 500 |
|-----------------------|-----|-----|-----|-----|-----|
| Density (g / cc)      | 1.1295 | 1.1726 | 1.2589 | 1.3452 | 1.4315 |
| Nuclide               | Atom weight fraction ; Enrichment =10 w% |
| U-235                 | 1.3281×10$^{-2}$ | 1.7057×10$^{-2}$ | 2.3832×10$^{-2}$ | 2.9738×10$^{-2}$ | 3.4932×10$^{-2}$ |
| U-238                 | 1.1953×10$^{-1}$ | 1.5351×10$^{-1}$ | 2.1449×10$^{-1}$ | 2.6764×10$^{-1}$ | 3.1439×10$^{-1}$ |
| N                     | 1.5645×10$^{-2}$ | 2.0093×10$^{-2}$ | 2.8074×10$^{-2}$ | 3.5031×10$^{-2}$ | 4.1149×10$^{-2}$ |
| H                     | 8.6669×10$^{-2}$ | 7.9720×10$^{-2}$ | 6.7252×10$^{-2}$ | 5.6383×10$^{-2}$ | 4.6824×10$^{-2}$ |
| O                     | 7.6487×10$^{-1}$ | 7.2962×10$^{-1}$ | 6.6635×10$^{-1}$ | 6.1120×10$^{-1}$ | 5.6270×10$^{-1}$ |
| Nuclide               | Atom weight fraction ; Enrichment =15 w% |
| U-235                 | 1.9921×10$^{-2}$ | 2.5585×10$^{-2}$ | 3.5746×10$^{-2}$ | 4.4604×10$^{-2}$ | 5.2395×10$^{-2}$ |
| U-238                 | 1.1289×10$^{-1}$ | 1.4498×10$^{-1}$ | 2.0256×10$^{-1}$ | 2.5276×10$^{-1}$ | 2.9690×10$^{-1}$ |
In a nuclear reactor design calculation, some variations of $^{235}$U enrichment are commonly needed. This will affect the isotopic compositions of the main uranium isotopes i.e. $^{235}$U and $^{238}$U for the respective enrichment. From the neutronic calculations point of view, for a system containing homogeneous uranium solution the results prove that in general, there is a range of uranium concentration giving the smallest of critical dimension, which is as the result of two opposing factors, i.e., the increase of reaction rate on one side, and decreasing the H/$^{235}$U atomic ratio on the other side, as the uranium concentration increase [7,11]. Such phenomena is shown in Figure 3, where the density of hydrogen in the solution is slightly decreasing with the increasing of UO$_2$(NO$_3$)$_2$ and uranium concentration.

Figure 1. Comparison atom density of uranium in the solution as a function of UO$_2$(NO$_3$)$_2$ and uranium concentration.

Figure 2. Comparison atom weight fraction of uranium in the solution as a function of UO$_2$(NO$_3$)$_2$ and uranium concentration.

Figure 3. Comparison atom density of hydrogen in the solution as a function of UO$_2$(NO$_3$)$_2$ and uranium concentration.
5. Conclusion
A basic theory of an analytical method to determine the atomic or number density or solution density along with the associated atom and weight fraction of each nuclide contained in a homogeneous uranyl nitrate solution has been described. It includes some procedures either the solution concentration defined based on the amount of uranyl nitrate salt [UO$_2$(NO$_3$)$_3$] dissolved, or that was made based on the uranium content in the solution. The results conclude that the number density and the corresponding weight fraction of uranium isotopes of the solution defined based on the uranium content, tend to be higher by quite significantly compared with those defined based on the amount of uranyl nitrate salt [UO$_2$(NO$_3$)$_3$] dissolved. The difference is caused by the change of water content in both cases. This method can be used as the basic approach for neutronic calculations of a system containing homogeneous uranium solution.

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