Photon in a cavity—a Gedankenexperiment

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(Dated: May 7, 2014)

Abstract

The inertial and gravitational mass of electromagnetic radiation (i.e., a photon distribution) in a cavity with reflecting walls has been treated by many authors for over a century. After many contending discussions, a consensus has emerged that the mass of such a photon distribution is equal to its total energy divided by the square of the speed of light. Nevertheless, questions remain unsettled on the interaction of the photons with the walls of the box. In order to understand some of the details of this interaction, a simple case of a single photon with an energy $E_\nu = h \nu$ bouncing up and down in a static cavity with perfectly reflecting walls in a constant gravitational field $g$, constant in space and time, is studied and its contribution to the weight of the box is determined as a temporal average.
I. INTRODUCTION

Massive particles and electromagnetic radiation (photons) in a box have been considered by many authors. Most of them have discussed the inertia of an empty box in comparison with a box filled with a gas or radiation.

In the presence of a constant gravitational field \( g \) (pointing downwards in Fig. 1), the effect on the weight of the box is another topic that has been studied. In very general terms, this problem has been treated in Refs. 6 and 7 with the conclusion that, in a closed system in equilibrium, all types of energy \( E_n \) contribute to the mass according to

\[
\Delta M = \sum E_n c_0^2 ,
\]

where \( M \) refers to the passive gravitational mass.

For a gas and even for a single massive particle, this can easily be verified with the help of the energy and momentum conservation laws as a temporal average. For radiation the situation has been debated over the years\(^{10,14}\). Kolbenstvedt in Ref. 11 studied photons in a uniformly accelerated cavity and found a mass contribution

\[
\text{[...]} \text{ in agreement with Einstein’s mass-energy formula.}
\]

It might, therefore, be instructive to describe the problem with the help of a Gedankenexperiment in the simple case of a photon bouncing up and down in a cavity of a box with perfectly reflecting inner walls at rest in a constant gravitational field. The height \( h \) is measured as fall height in the field direction, which will be indicated by the unit vector \( \hat{n} \) in equations and figures. The mass of the box, \( M \), includes the mass of the walls and the equivalent mass of any unavoidable energy content, such as thermal radiation, except the test photon.

II. DEFINITION OF GRAVITATIONAL POTENTIALS

The gravitational potentials will be defined as \( U_t \) at the top of the cavity and as \( U_b \) at the bottom with

\[
U_b - U_t = \Delta U = -g \cdot h < 0 \quad (2)
\]

and the relations

\[
- c_0^2 < U_b < U < U_t < 0 , \quad (3)
\]
FIG. 1. Box of height $h = ||\mathbf{h}||$ with mass $M$ and reflecting inner walls on a weighing scale (indicated by the black bar) to determine its weight in a constant gravitational field $\mathbf{g} = g \hat{\mathbf{n}}$. The gravitational potentials at the bottom and top are $U_b$ and $U_t$, respectively. One cycle with a period of $T = t_1 - t_0$ and the continuation into the next cycle (dashed arrow) are schematically shown for a photon with an energy $E_\nu = h \nu = h_b (1 + U_b / c_0^2)$ bouncing (nearly) vertically up and down. The values of the speed of light $c_b$ and $c_t$ in the cavity corresponding to the potentials $U_b$ and $U_t$ are given at the bottom and the top as well as for the center as a mean speed of $c_0 (1 + 2 \overline{U}/c_0^2)$. 

\[ U_t = U_b - \Delta U = U_b + \langle g, h \rangle < 0 \]

\[ c_t = c_0 (1 + 2 \overline{U}/c_0^2) \]

\[ c_b = c_0 [1 + (U_b + U_t)/c_0^2] \]
where \( c_0 \) is the speed of light in vacuum without a gravitational field and \( \overline{U} = (U_b + U_t)/2 \). Under the weak-field condition, so defined, the speed of light measured on the coordinate or world time scale is\(^{15-19}\)

\[
c(U) = c_0 \left(1 + \frac{2U}{c_0^2}\right).
\] (4)

Einstein originally derived from the equivalence principle

\[
c = c_0 \left(1 + \frac{\phi}{c^2}\right)
\] (5)

with \( \phi \) as symbol for the gravitational potential \( U \) used here.\(^{20}\) Application of Huygens’ principle then led him to expect a deflection of 0.83" for a Sun-grazing light beam. A value of 0.84" had been obtained by Soldner in 1801.\(^{21}\) However, in 1916 Einstein predicted a deflection of 1.7" based on his general theory of relativity.\(^{22}\) This was first verified during a solar eclipse in 1919, and since then values between 1.75" and 2.0" have been found in many observational studies.\(^{23-25}\) Consequently, it can be concluded that observations are not consistent with Eq. (5), but are—within the uncertainty margins—in agreement with Eq. (4).

The relativistic energy \((E)\) and momentum \((p_0)\) equation for a free body with mass \( m \) in vacuum is

\[
E^2 = m^2 c_0^4 + ||p_0||^2 c_0^2.
\] (6)

It reduces for a massless particle to

\[
E = p_0 c_0
\] (7)

with \( p_0 = ||p_0|| \).\(^{26-28}\) In a static gravitational field the energy of a photon

\[
E_\nu = h \nu = p(U) c(U)
\] (8)

measured in the coordinate time system is constant.\(^{18}\) Its speed, however, varies according to Eq. (4), whilst the momentum changes inversely to this speed.

**III. PHOTON REFLECTION SCENARIOS**

At least three different scenarios can be conceived to describe the situation inside the box:
1. A naive application of Eqs. (4) and (8), i.e., direct reflections at the walls without any further interaction, leads to the result that the photon contributes

$$\Delta m \approx 2 \frac{E_{\nu}}{c_0^2} \left(1 + \frac{2U}{c_0^2}\right)$$

(9)

to the mass of the box, which is nearly a factor of two higher than expected from Eq. (1). To show this, let us consider the oppositely directed momentum transfers during photon reflections at the bottom and the ceiling of the cavity. The values are twice

$$\frac{E_{\nu}}{c_b} \hat{n} \approx p_{\nu} \left(1 - 2 \frac{U_b}{c_0^2}\right)$$

(10)

and

$$-\frac{E_{\nu}}{c_t} \hat{n} \approx -p_{\nu} \left(1 - 2 \frac{U_t}{c_0^2}\right),$$

(11)

where $c_b$ and $c_t$ are determined from Eq. (4) taking into account the conditions in (3) to justify the approximations, i.e., neglecting orders equal or higher than $(U/c_0^2)^2$ against unity. The momentum $p_{\nu} = (E_{\nu}/c_0) \hat{n}$ of a photon with energy $E_{\nu} = h \nu$ at $U_0 = 0$ in vacuum has been introduced. A complete cycle lasts for

$$T = t_1 - t_0 = \frac{2h}{c_0 \left(1 + 2 \frac{U}{c_0^2}\right)},$$

(12)

i.e., $2h$ divided by the mean speed. Within this time interval, the total momentum transfer of

$$\Delta P^C = 2 \left(\frac{E_{\nu}}{c_b} - \frac{E_{\nu}}{c_t}\right) \hat{n} \approx 4 p_{\nu} \frac{g \cdot h}{c_0^2},$$

(13)

is obtained from Eqs. (2), (10) and (11). A momentum vector with upper index C refers in this and later equations to a momentum inside the cavity. Division by $T$ of Eq. (12) gives a mean force of

$$\overline{F_1} \approx 2 \frac{E_{\nu}}{c_0^2} \left(1 + \frac{2U}{c_0^2}\right) g,$$

(14)

confirming Eq. (9), which has been found to be inconsistent with Eq. (1).

2. The assumption that the reflections occur at the walls of the box without further effects might not be correct. Consequently, the next scenario is based on an intermediate storage of the energy $E_{\nu}$, i.e., as elastic energy, and a transfer of a momentum calculated under the assumption that the speeds $c_b$ and $c_t$ valid in the cavity at $U_b$
and \( U_t \), respectively, are also applicable for the complete reflection process. This gives a momentum transfer of

\[
2p_\nu \left( 1 - \frac{2U_b}{c^2} \right)
\]

during the absorption and emission at the bottom and

\[
-2p_\nu \left( 1 - \frac{2U_t}{c^2} \right)
\]

at the top in opposite directions. Comparison with Eqs. (10) and (11) shows that the resulting force \( F_2 \) after application of Eqs. (12) to (14) equals \( F_1 \) and leads to the same surprising result.

3. The previous concept was based on the assumption that the reflections actually occur in regions of the wall, where the effective speed of light has been modified by the local gravitational potential. However, the process of a photon reflection has to be accomplished by interactions with electrons in the walls. The huge ratio of the electrostatic to the gravitational forces between elementary particles makes it unlikely that such an interaction is directly influenced significantly by weak fields of gravity. However, the photon absorption and emission will be affected as is evident from the gravitational redshift. This redshift—since its prediction in 1908 by Einstein in Ref. 6—has been theoretically and experimentally studied in many investigations resulting in a quantitative validation of Einstein’s statement in Ref. 20:

\[
\nu_0 - \nu = -\frac{\phi}{c^2} = 2 \cdot 10^{-6}.
\]

The (negative) difference of the gravitational potentials between the Sun and the Earth is denoted by \( \phi \) in Eq. (17).

Einstein’s early suggestion was that the transition of an atom is an intra-atomic process, i.e. it is not dependent on the gravitational potential:
FIG. 2. The box is shown on a weighing scale as support system. The bottom and the top of the cavity are complemented by “interaction regions”. Energy release and photon emission processes occur in these regions as required by energy and momentum conservation. The initial energy release of \( E_b = \Delta m_b c_0^2 \) happens in the bottom interaction region accompanied by a momentum transfer to the box and the support system of \( +p_b \). The conversion of \( \Delta m_b \) into energy at \( U_b \) is subject to an increase of the potential energy of the bottom interaction region by \( -U_b \Delta m_b \) taken from \( E_b \). The related differential momentum contribution \( +\delta p_b \) acts on the support system. It will be compensated by the opposite effect of the following energy-to-mass conversion at \( U_b \) after one period \( T \). The corresponding steps at \( U_t \) are described in the text.
Da der einer Spektrallinie entsprechende Schwingungsvorgang wohl als ein intraatomischer Vorgang zu betrachten ist, dessen Frequenz durch das Ion allein bestimmt ist, so können wir ein solches Ion als eine Uhr von bestimmter Frequenzzahl $\nu_0$ ansehen.\(^6\)

(Since the oscillation process corresponding to a spectral line probably can be envisioned as an intra-atomic process, the frequency of which is determined by the ion alone, we can consider such an ion as a clock with a distinct frequency $\nu_0$.)

This means that the energy $E_b$ initially released at the gravitational potential $U_b$ by an elementary process equals the energy $E_0$ released by the same process at the potential $U_0 = 0$. In both cases the process will be accompanied by a momentum pair

$$\pm \mathbf{p}_b = \pm \frac{E_b}{c_0} \hat{n} = \pm \frac{E_0}{c_0} \hat{n}.$$  

(18)

It has, however, to be noted in this context that Einstein later concluded:

Die Uhr läuft also langsamer, wenn sie in der Nähe ponderabler Massen aufgestellt ist. Es folgt daraus, daß die Spektralinien von der Oberfläche großer Sterne zu uns gelangenden Lichtes nach dem roten Spektralende verschoben erscheinen müssen.\(^22\)

(The clock is thus delayed, if it is placed near ponderable masses. Consequently, it follows that the spectral lines of light reaching us from the surface of large stars must be shifted towards the red end of the spectrum.)

Both statements can be reconciled by postulating that the redshift occurs during the actual emission process and realizing that energy trapped in a closed system has to be treated differently from propagating radiation energy.

In addition, the importance of the momentum transfer during the absorption or emission of radiation was emphasized by Einstein:

Bewirkt ein Strahlenbündel, daß ein von ihm getroffenes Molekül die Energiemenge $h\nu$ in Form von Strahlung durch einen Elementarprozeß aufnimmt oder abgibt (Einstrahlung), so wird stets der Impuls $\frac{h\nu}{c}$ auf das
A beam of light that induces a molecule to absorb or deliver the energy $h \nu$ as radiation by an elementary process (irradiation) will always transfer the momentum $\frac{h \nu}{c}$ to the molecule, directed in the propagation direction of the beam for energy absorption, and in the opposite direction for energy emission. [...]  

However, in general one is satisfied with the consideration of the energy exchange, without taking the momentum exchange into account.)

The energy and momentum conservation principles lead to a relationship between the energy $E_b$ and the emitted photon energy $E_\nu$ at $U_b$ of

\[
E_\nu = E_b - ||\delta p|| c_0 = ||p_b - \delta p|| c_0 = \\
||p_b + \delta p_b|| c_b = ||p(U_b)|| c_b
\]  

(19)

by introducing a differential momentum $\delta p_b$ parallel to $p_b$ (neglecting the very small recoil energy). The evaluation gives, together with Eq. (4),

\[
\delta p_b = -p_b \frac{U_b}{c_0^2}
\]  

(20)

and

\[
E_\nu = E_b \left( 1 + \frac{U_b}{c_0^2} \right).  
\]  

(21)

Such a scenario has recently been discussed in the context of the gravitational redshift.42

With the basic assumption that the photon is reflected from both walls with the same energy $E_\nu$, the relation for the ceiling is:

\[
E_\nu = E_t \left( 1 + \frac{U_t}{c_0^2} \right),
\]  

(22)
The corresponding momentum transfers during absorption and emission expected according to Eqs. (18), (21) and (22) are

\[ 2 \mathbf{p}_b \approx 2 \frac{E_\nu}{c_0} \left( 1 - \frac{U_b}{c_0^2} \right) \hat{n} = 2 \mathbf{p}_\nu \left( 1 - \frac{U_b}{c_0^2} \right) \]  
\[ \text{(23)} \]

at the bottom and

\[ -2 \mathbf{p}_t \approx -2 \frac{E_\nu}{c_0} \left( 1 - \frac{U_t}{c_0^2} \right) \hat{n} = -2 \mathbf{p}_\nu \left( 1 - \frac{U_t}{c_0^2} \right) \]  
\[ \text{(24)} \]

at the top. Note in this context that the elementary processes at the bottom and the top must have slightly different energy levels for constant \( E_\nu \). Comparison of the momentum values of Eqs. (23) and (24) with those of Eqs. (10) and (11) shows that the force \( \mathbf{F}_3 \) obtained in analogy to Eq. (13) equals \( \mathbf{F}_1/2 \) and thus gives a value expected from Eq. (1).

This encouraging result will now be analysed in detail. The initial energy release is assumed in the “Interaction region (Bottom)” at the potential \( U_b \) in Fig. 2 on the left according to

\[ E_b + \Delta m_b c_0^2 = 0 \]  
\[ \text{(25)} \]

accompanied by a momentum pair of \( \pm \mathbf{p}_b \) as mentioned above. The exact release process is of no importance for the present discussion, but the initial release must be controlled by the elementary process alone. A speed of \( c_b \) at the bottom of the cavity, together with energy and momentum conservation laws, then requires that a photon can only be emitted with an energy \( E_\nu \) given by Eqs. (19) and (21) (the rest energy of the mass \( \Delta m_b \) at the gravitational potential \( U_b \)) and a momentum in the upward direction of

\[ -\mathbf{p}_b^C = -\mathbf{p}_b - \delta \mathbf{p}_b \approx -\mathbf{p}_\nu \left( 1 - \frac{2 U_b}{c_0^2} \right) , \]  
\[ \text{(26)} \]

where we have used Eqs. (20) and (23). The energy difference, corresponding to the potential energy of a mass \( \Delta m_b \) at \( U_0 \) relative to \( U_b \),

\[ E_b - E_\nu = -U_b \Delta m_b \]  
\[ \text{(27)} \]

will be transferred to the box. This process is accompanied by a corresponding differential momentum transfer of \( +\delta \mathbf{p}_b \) and by a mass increase of

\[ \delta m_b = -\frac{U_b \Delta m_b}{c_0^2} . \]  
\[ \text{(28)} \]
The momentum $+\mathbf{p}_b$ from the initial release and $+\delta \mathbf{p}_b$ will thus be acting on the support system.

The reflections at the top and bottom have to be considered in several steps—illustrated in detail in Fig. 2:

- The photon $E_\nu$ arrives at the top with $c_t$, obtained from Eq. (4), and a momentum of
  \[
  -p_t^C = -p_b \left( 1 - U_b/c_0^2 \right) \left( 1 + 2 U_b/c_0^2 \right) \approx -p_t - \delta p_t \approx -p_\nu \left( 1 - \frac{2 U_t}{c_0^2} \right)
  \]
  (29)
  derived from Eqs. (8) and (24) with
  \[
  \delta p_t = -p_t \frac{U_t}{c_0^2}.
  \]
  (30)

- The change of the speed from $c_t$ to $c_0$ in the interaction region will entail a change of the momentum in Eq. (29) to a new value, which can be written in our approximation as $-p_t + \delta p_t$.

- The energy-to-mass conversion at the potential $U_t$ according to
  \[
  \Delta m_t + \frac{E_t}{c_0^2} = 0
  \]
  (31)
  in the upper interaction region can only be accomplished by adding the potential energy term
  \[
  E_t - E_\nu = -U_t \Delta m_t
  \]
  (32)
  and a momentum of $-\delta \mathbf{p}_t$, which will be provided by the conversion of $\delta m_t$ into energy. The momentum $+\delta \mathbf{p}_t$ of the momentum pair will act on the box. In total a momentum of $-p_t + \delta \mathbf{p}_t$ has thus to be taken up by the box.

These processes can be seen as the reversed actions performed by Eqs. (25) to (28), but now at $U_t$.

- The return trip essentially occurs in the reverse order as shown on the right side of Fig. 2. The energy release of $E_t$ will be accompanied by a momentum pair $\pm \mathbf{p}_t$. The photon will be emitted in the downward direction with a momentum of
  \[
  p_t^C = +\mathbf{p}_t + \delta \mathbf{p}_t \approx +\mathbf{p}_\nu \left( 1 - \frac{2 U_t}{c_0^2} \right),
  \]
  (33)
cf., Eqs. (24) and (29). The energy difference $E_t - E_\nu$ restores the potential energy of Eq. (32). A momentum of $-p_t - \delta p_t$ will be transferred to the box in analogy to the processes at $U_b$.

- The photon momentum at the bottom of the cavity will be
  \[
P_b^C = p_t \frac{(1 - U_t/c_0^2)(1 + 2 U_t/c_0^2)}{1 + 2 U_b/c_0^2} \approx + p_b + \delta p_b \approx + p_\nu \left(1 - \frac{2 U_b}{c_0^2}\right) \quad (34)
  \]
as expected from Eq. (26).

- In analogy to the situation at $U_t$, we find a momentum of $+p_p - \delta p_b$ that has to be transferred to the support system.

- The total external momentum thus is with Eqs. (2), (3), (18), (21) and (22):
  \[
  \Delta P = (p_b + \delta p_b) - (p_t + \delta p_t) + (p_b - \delta p_b) + (p_t - \delta p_t) = 2(p_b - p_t) = 2 \left(\frac{E_b}{c_0} - \frac{E_t}{c_0}\right) \approx -2 \frac{E_\nu}{c_0} \frac{\Delta U}{1 + 2 U/c_0^2} = 2 \frac{E_\nu}{c_0} \frac{g \cdot h}{1 + 2 U/c_0^2}. \quad (35)
  \]

Averaged over the time interval $T$ from Eq. (12) for a full cycle gives a mean force of
  \[
  \overline{F}_3 = \frac{\Delta P}{T} \approx \frac{E_\nu}{c_0^2} g \quad (36)
  \]
in agreement with Eq. (1).

A short comment is required on the intermediate mass storage processes at the bottom and the top. Compared to $T$ the storage times are so small that the contributions to the mean mass and thus the weight can be neglected. Since a factor of two was in question, there was also no need to complicate the calculations even further by retaining terms which are very small under the weak-field conditions assumed.

A multi-step process including Einstein’s assumption of an intra-atomic energy liberation thus leads to the correct result within our approximations. It involves an external box which cannot be completely rigid as it must be able to enact the required gravitational energy and momentum transfers. Another objection against a rigid box is the limited speed of any signal transmission in the side walls.\textsuperscript{10,14}
IV. DISCUSSION

The assumption of a constant field $g$ in the environment of the box—made in the interest of simplifying the calculations—calls for some explanations. Einstein discussed in Ref. 43 a static gravitational field without mass ("... ein massenfreies statisches Gravitationsfeld [...]"") and clarified in a footnote that such a field can be thought of as generated by masses at infinity. Bondi in Ref. 44 went even further in his scepticism against constant gravitational fields. Only the non-uniformity of the field would be observable. A field with constant magnitude and direction should not be regarded as a field.

Indeed, there can be no perfectly homogeneous gravitational field. Nevertheless very good approximations remote from the generating masses exist between neighbouring potential surfaces, and the effects are observable. These remarks are of relevance for the considerations in the previous section, as the calculations could have significantly been shortened by setting $U_t = 0$, but no realistic configuration would correspond to such a definition. It can thus be concluded that Eq. (2) should be amended by $\lvert \Delta U \rvert \ll \lvert U_0 \rvert \approx \lvert U_t \rvert$ in order to justify the assumption of a (nearly) constant field.

The present authors argued in Ref. 42 that the interaction of the liberated energy during an atomic transition with the kinetic energy of the emitter and its momentum discussed by Fermi in relation to the Doppler effect45 has some resemblance with the gravitational redshift, if the kinetic energy in the multi-step process leading to the Doppler shift is replaced by the potential energy. The same argument appears to be relevant for the photon reflection process discussed here. The influence of gravity could indeed be cancelled by the Doppler effect in the experiment of Pound and Rebka (Ref. 46) in such a way that the emission and absorption energies of X-ray photons generated by the 14.4 keV transition of iron (Fe$^{57}$) were the same in the source and the receiver positioned at different heights in the gravitational field of the Earth47.

Although we have studied a single photon bouncing in a vacuum cavity, our result might have some implications for the Abraham–Minkowski controversy48,49 about the momentum of light in a medium with a refractive index $n = c_0/c > 1$. Modern expositions of this problem have been presented, for instance, in Refs. 50 and 51. The Abraham momentum is smaller in the medium than in vacuum, whereas the Minkowski form gives a greater momentum inside the medium. In a recent paper52 the dilemma could be resolved by identifying
the Abraham form with the kinetic momentum and the Minkowski form with the canonical momentum. Both expressions are identical in vacuum but this is only true in the absence of a gravitational field. With such a field present, the Minkowski momentum is clearly compatible with Eqs. (4) and (8) in the cavity. In the interaction regions, the effective speed during the momentum conversion processes appears to be the speed of light in vacuum \(c_0\) not affected by the weak gravitational field, see Eq. (18). Consequently, both Minkowski’s and Abraham’s momentum calculations will lead to the same result.

V. CONCLUSION

The temporally averaged increase of the passive gravitational mass and thus the weight of a box containing a single photon bouncing up and down in a weak gravitational field could be obtained in agreement with Eq. (11) by assuming interactions between the photon and the walls of the cavity as well as momentum and energy conservation.

Preston formulated and wished some 130 years ago:

[...] let us work towards the great generalization of the Unity of Matter and Energy.

Most of the work has been done by now, but it has also become quite clear that the interactions between photons and matter in a gravitational field are complicated as outlined by Bondi in his article entitled “Why gravitation is not simple”.

ACKNOWLEDGMENTS

We thank the editor and an anonymous referee for constructive comments. This research has made extensive use of the Astrophysics Data System (ADS).

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