The 1980 discovery of the quantum Hall effect observed at ultra-low temperature and a strong magnetic field introduced topology to condensed matter physics. However, over the past decade or so, topological electronic phenomena have also emerged in seemingly ordinary bulk materials under non-extreme conditions, leading to a new understanding of the ubiquitous topological structure of quantum matter. These materials often host relativistic fermions, feature symmetry-protected bulk-boundary correspondence and exhibit quantized electronic excitations. These topological fermions usually take the form of Dirac, Weyl or Majorana solutions of the corresponding quantum field theory in high-energy physics. The topological bulk-boundary correspondence allows for making connections among various experimental techniques specifically sensitive to boundary states, bulk states or both. In addition, topologically protected quantized effects hold promise for new technology in areas such as energy efficiency and quantum information science. The study of weakly interacting topological insulators featuring an insulating bulk and conducting boundaries, for instance, has achieved success, further driving the field to new frontiers to search for more exotic phases of matter. With the advancement of the field, consideration of correlated materials is particularly fascinating, because they not only bring in the phenomena of magnetism and superconductivity at play but could also realize a many-body entangled version of topological order. More importantly, electron correlations present the possibility of emergent and hitherto unknown phenomena.

Topological materials with strong correlations can exhibit spin, orbital ordering or superconducting instabilities with intrinsic magnetic, orbital or electronic anisotropy, which call for advanced experimental tools to probe and elucidate the interplay between correlation and topology. Notably, external magnetic fields that break time-reversal symmetry can act as a strong perturbation to magnetism and superconductivity, as well as modify the band structure in the presence of strong spin–orbit interaction. The magnetic field applied on an electronic system leads to a nontrivial topology: the magnetic flux quantum \( \frac{\hbar}{e} \) and quantum Hall conductance \( \frac{N e^2}{h} \), related to Chern number \( N \), a topological invariant, are governed by the same set of fundamental constants, including the Planck’s constant \( h \) and elemental charge \( e \); the vector nature of the field can differentially interact with the chirality of topological matter to provide access to effects related with the topological invariant. Whereas transport techniques under a vector magnetic field can play a vital role, conventional transport for bulk materials (that are often not gate-tunable) is only sensitive to the electronic properties at the Fermi level, limiting its exploration necessary for probing the band interconnectivity and topology. Traditionally, in comparison with optical, X-ray and neutron techniques, photoemission spectroscopy is regarded as a powerful technique to map the interconnectivity and topology of the electronic band structure. However, an external magnetic field can be detrimental to detecting photoelectrons requiring complex orbital corrections, severely hindering its field application.
In this regard, scanning tunnelling microscopy (STM) is an indispensable, high-resolution spectroscopic technique that can work under a tunable vector magnetic field. In this Review, we outline the basics of STM and elaborate on its application in exploring the electronic and magnetic properties of topological materials. We elaborate on several proof-of-principle methods to link STM signals with quantum topology and broadly discuss how the technique can supplement, bridge and complement the traditional photoemission and transport, as well as other related methods in this research area. Finally, we point out several unique directions to extend the STM technique and its applications in exploring emergent topological matter.

**State-of-the-art scanning tunnelling microscopy**

Low-temperature STM is a cutting-edge microscopic and spectroscopic technique that can be employed under a strong magnetic field with energy \( E \), space \( r \) and scattering-vector \( q \) resolution. It utilizes the quantum tunnelling principle to probe the surface morphology and the local density of states (LDOS) of materials with atomic-scale precision and sub-meV energy resolution. When the scanning tip and sample are atomically close, their wavefunctions overlap and electrons can tunnel through their vacuum gap. The fast decay of the wavefunction leads to the exponential sensitivity of the tunnelling current \( I \) to the sample-tip distance \( d \). This exponential dependence of the electronic signal on physical distance allows for accurate feedback control of the scanning motion of the STM tip to probe the surface morphology, with atomic spatial resolution. Furthermore, fixing \( d \) by a rigid STM mechanical structure, through sweeping the bias voltage \( V \) added between the sample and the tip, the energy range of the tunnelling states associated with both the sample and the tip can be precisely tuned (Fig. 1a). By selecting the tip material so that its density of states is energy-independent and assuming that the tunnelling matrix element is space-independent, the differential tunnelling conductance is proportional to the LDOS \( \rho(E, r) \) of the sample at the location of the tip, convoluted with the energy derivative of the Fermi–Dirac distribution \( f(E, T) \) at the measuring temperature \( T \), such that \( \frac{dI}{dV}(E, r) \propto \int \rho(E - \omega, r) f'(\omega) d\omega \). The mathematical details for its derivation can be found in Ref. 22. Essentially, the \( dI/dV \) signal samples the LDOS \( \rho(E, r) \) with a thermal-induced finite energy resolution of \( 3.5k_B T \), where \( k_B \) is the Boltzmann constant. We also note that tunnelling spectroscopy becomes more challenging by progressively moving away from the Fermi level, where a non-constant tip density of states, as well as the larger background related to the increased tunneling probability, can complicate the interpretation of the spectroscopic data.

The combination of scanning and tunnelling provides comprehensive spectro-microscopy of materials, including morphology visualization and spectroscopic imaging. Figure 1b illustrates the microscopy of a material (for example, CoSn (Ref. 34)). The topographic image here reveals the surface morphology, which resolves both the Sn, honeycomb lattice layer and the CoSn kagome layer with atomic resolution. This atomic-lattice-resolving capability is crucial in probing the respective electronic structure properties of different lattice geometries, whose underlying real-space electronic structure can be simultaneously obtained by the \( dI/dV \) mapping. An energy slice of the \( dI/dV \) mapping can demonstrate the electronic difference between two atomic layers (lower-left panel of Fig. 1b). In particular, the defect-induced oscillation pattern is more evident on the kagome layer, and a Fourier transform of the \( dI/dV \) map taken on the kagome layer provides the essential quasi-particle interference (QPI) signal in this kagome paramagnet (lower-right panel of Fig. 1b). The topography, \( dI/dV \) spectrum and associated QPI constitute the primary data structure types for typical STM measurements.

To facilitate high-resolution microscopy, a rigid STM design is required, capable of an ultra-stable, picometre-level control of the tip position on the sample, despite all potential environmental sources of noise. As an example, the Pan-type STM (Fig. 1c), which utilizes the cooperative motion of multiple tightly clamped piezo legs against the moving scanner in a triangular configuration, is a useful rigid design widely adapted for applications at low temperature (down to 10 mK) and under a tunable (vector) magnetic field. In these
systems, a superconducting solenoid is used to generate a strong magnetic field along the z-axis, providing a primary vector field (commercially up to ±18 T). Pairs of split superconducting coils can be further included in the system to generate magnetic fields in the horizontal directions (commercially up to 5 T), providing a 3D vector magnetic field. Moreover, in contrast with magneto-transport measurements under a ramping magnetic field, STM measurements are usually performed under a stabilized magnet field to minimize the mechanical drift and electrical noise, which is associated with delicate STM tip retractions and approaches between ramping to different fields. Therefore, a full set of systematic STM data under a vector magnetic field requires a combination of reliable instrumentation, skilful control of the tip and suitable material platforms.

In the STM research of topological materials, besides the aforementioned technical challenges, a major question is how to link the tunnelling signal with the underlying topology. In this regard, we point out a general guideline. For a spectroscopic technique, new information often manifests as special spectroscopic patterns or peaks. Firstly, we search for an anomaly by comparing the tunnelling signal (unusual patterns and peaks) with those on well-known topologically trivial materials, including elemental metals and 2D electron systems. Secondly, we perturb the anomaly by tuning as many parameters as possible to provide substantial constraints for its interpretation. Lastly, we refer to other complementary experimental or theoretical techniques (Table 1, as an example) for a comprehensive understanding. More specifically, we highlight several proof-of-principle methodologies that have been developed in this field in the following discussion.

**Proof-of-principle methodologies**

**Quasi-particle interference method.** Prior to its application in the study of topological materials, the QPI signal was observed in noble metals and superconductors\(^{48-53}\), and has been developed as a general STM methodology. The QPI is reflected in the spectroscopic pattern through the Fourier transform of the measured real-space dI/dV image. The QPI signal stems from Friedel oscillations (or standing waves) around defects detected in real-space measurements (Fig. 2a). The periodicity of such oscillations gives rise to the \( q \) wavevector that is associated with the elastic scattering of quasi-particles and their interference between two momentum states \( (k_1, k_2) \).
Through measuring the QPI patterns as a function of energy, we can obtain the energy variation of $q$, which is the autocorrelation of the electronic band dispersion $E(k)$. For the same band dispersion, the intensity distribution of the QPI pattern can be different, depending on the quasi-particle scattering geometry. This geometry is related to the charge, spin and orbital texture of the electron distribution, which is associated with the spectral function $A(k, E)$ — the imaginary part of the Green’s function. Firstly, the spectral function intensity at different momenta can vary, and the QPI intensity is proportional to the intensities of the spectral function at $k_1$ and $k_2$. Secondly, the electronic structure can feature a momentum-dependent spin texture, and the QPI intensity is substantially reduced if the spins at $k_1$ and $k_2$ are in opposite directions, especially for QPI assisted by nonmagnetic defects. Thirdly, the electronic structure can feature a momentum-dependent orbital texture, and the QPI intensity is substantially reduced if the orbitals at $k_1$ and $k_2$ are orthogonal. In addition, there are also cases that the QPI intensity can be geometrically enhanced at $q$ that connects two parallel pieces of the band structure within a constant energy contour, known as the nesting effect. These factors form the basic selection rules for the general interpretation of the QPI data, and the mathematical details for such a physical picture including the joint density of states approximation can be found in Refs.28–32. For simple isotropic systems, the QPI signal near the Fermi level is a nearly isotropic ring, reflecting a plain circular Fermi surface. Thus, anisotropic, spot-like or disconnected-arc QPI data often present a hint of an unconventional band structure that could be further related to nontrivial band topology. As QPI data convolute the scattering effects from charge, spin and orbital textures, their final interpretation would require accurate knowledge of the bare band structure and scattering processes. In practice, QPI patterns can be simulated by considering the band structure obtained from angle-resolved photoemission data or density functional calculations under different scattering geometry considerations (note that photoemission cannot directly probe scattering geometry), and then it can be compared with the measured QPI pattern to gain insight of topology.

Here, we highlight several examples of QPI applications in topological materials. A rich QPI pattern is typically observed on the topological insulator Bi$_{1-x}$Sb$_x$ (Fig. 2b, top panel). The QPI signal is beyond simple circular pockets, indicative of nontrivial scattering geometry. The QPI data are compared with angle-resolved photoemission spectra, where there exist three sets of surface Fermi pockets with spin-momentum-locked chiral spin textures. The QPI data should be related with the quasi-particle scatterings among these pockets mediated by the random spatial potentials associated with the alloying. By applying the charge and spin selection rule, there can be three dominant scattering vectors along the Γ–M direction (Fig. 2b, white arrows in middle panel) while direct backscattering within each pocket is forbidden. A QPI simulation based on this selection rule explains the observed QPI pattern along the Γ–M direction (Fig. 2b, bottom panel), as well as other directions. The QPI demonstration of the lack of backscattering, therefore, supports the chiral spin texture, which

| Technique | Key parameter | Variables | Unique aspects | Connection with (in) scanning tunnelling microscopy | Limitations |
|-----------|---------------|-----------|----------------|-----------------------------------------------|-------------|
| Scanning tunnelling microscopy | Local density of states | Location; energy; magnetic field; magnetic or nonmagnetic tip; gating; temperature. | Probing the scattering geometry; atomic spatial resolution; sub-meV energy resolution; tunable magnetic field; Landau quantization; edge state; detecting zero modes. | Correspondence between single defects (vortices) and local density of states; correspondence between (bulk) energy gap and edge states. | Momentum resolution; requiring atomically flat surfaces; probing surface states and surface-projected bulk states; thermal smearing from tip. |
| Angle-resolved photoemission spectroscopy | Spectral function | Momentum; energy; photon wavelength; photon polarization; spin polarization; temperature. | Band crossing; spin/orbital-momentum locking and texture; probing both 2D surface and 3D bulk band structures. | Correspondence to momentum-integrated photoemission signal; energy gaps; charge/spin/orbital texture; interband/intra-band structure scattering. | Spatial resolution; requiring fresh and flat surface; occupied states; energy resolution; no magnetic field. |
| Magneto-transport | Conductivity tensor | Electrical field; magnetic field; Landau level sequence; pressure; temperature. | Fermi surface geometry; electron mobility; chiral anomaly; anomalous Hall and Hall quantization; phase transitions. | Effective quasi-particle dispersion and Fermi surface geometry; magnetic field response; effective mass, Fermi velocity and Fermi length. | No energy, spatial, momentum or spin resolution; extrinsic scattering mechanisms can contribute to the signal. |
| Density functional theory | Density functionals | Crystal structures; elements. | Band structures and wavefunction; topological index; bulk boundary correspondence. | Surface-dependent density of states and topography; quasi-particle interference; orbital magnetism. | Strongly correlated systems; phase transitions; surface reconstruction and disorder effects. |
| Tight-binding model | Wavefunction | Hopping strength; internal and external fields and interactions. | Analytical elaboration on the concept of topology and its measurable consequence. | Analytical evaluation of the symmetry-breaking response to topological band structure. | Model-dependent explanation; considering the full symmetry and ingredients in the experiments. |
is associated with the Berry phase of the topological matter. A simpler QPI pattern is observed on the topological insulator Bi$_2$Te$_3$ (Fig. 2c, where the deposited Ag impurities assist the QPI signal). This material also shows a simpler surface state with only one hexagonal Fermi pocket (Fig. 2c, bottom panel), and, thus, is close to a simple 3D topological insulator. Remarkably, the anisotropy of the QPI pattern (stronger along the Γ–M direction) is distinct from that of the Fermi surface intensity (stronger along the Γ–K direction), providing evidence that direct backscattering is forbidden.

The observed scattering vector along the Γ–M direction can be understood as the effect of hexagonal warping on surface states with a chiral spin texture, which will not be detectable without warping, as seen in Bi$_2$Se$_3$ and Bi$_2$Sb$_2$Te$_3$Se$_2$ (REFS[48,49]). The QPI signal in Bi$_2$Te$_3$ supports the unusual spin-momentum locking, which is a consequence of the underlying topology.

In topological insulators, the quantum topology is protected by the time-reversal symmetry; thus, magnetic impurities that break time-reversal symmetry can lead to strong perturbations. In particular, magnetism may modify the helical spin-momentum locking near the Dirac cone, and the magnetic impurities can mediate scattering between spin-reversed states, both of which would create new scattering channels in the QPI that are otherwise forbidden. Systematic experimental efforts along this direction are focused on the magnetic doped Bi$_2$Te$_3$ materials, revealing new results. While bulk magnetization has been seen in Bi$_2$MnTe$_3$ and Cr$_x$(Bi$_{0.1}$Sb$_{0.9}$)$_2$Te$_3$, no apparent new QPI signal is detected along the Γ–K direction, which seems to suggest the robustness of the forbidden scattering in these systems. However, the QPI of Bi$_2$FeTe$_3$ and Bi$_2$Te$_3$ with surface Mn impurities reveal a new scattering channel along the Γ–K direction emerging around 200 meV above the Dirac cone. This new scattering vector is identified as evidence for time-reversal symmetry breaking. These experiments suggest that details of the magnetic order pattern associated with the magnetic impurities are crucial to observe such an interplay between scattering and topology.
Besides the topological insulator, QPI has also been applied to probe other topological materials, including topological crystalline insulators and Weyl semimetals\textsuperscript{30–41}. In topological crystalline insulator (Pb,Sn)Se and Weyl semimetal NbP, the QPI anisotropy is linked to the orbital texture of the topological surface states\textsuperscript{30–35}. In Weyl semimetal TaAs and Mo\textsubscript{x}\textsubscript{1–x}Te\textsubscript{2}, signatures of the Fermi arcs, as well as the surface-bulk connectivity, have been studied\textsuperscript{32–41}. More recently, the QPI method is used to study the topological surface states in superconductors PbTaSe\textsubscript{2} and \( \beta\)-PdBi\textsubscript{2} (REF\textsuperscript{33,34}), uncover the tunable nematicity in topological kagome magnet Fe\textsubscript{2}Sn\textsubscript{1} (REF\textsuperscript{34}), resolve the heavy Dirac fermion in Kondo insulator SmB\textsubscript{6} (REF\textsuperscript{35}) and detect fermion–boson interplay in topological kagome paramagnet CoSn (REF\textsuperscript{36}).

We also note that, while QPI can visualize topological scattering geometry, QPI alone is often not sufficient to prove nontrivial topology. For instance, backscattering between states related by time-reversal symmetry is also forbidden in Rashba systems\textsuperscript{37}, which may not involve with nontrivial topology. Moreover, in systems with complex fermiology, QPI patterns may emerge to be more complex, making it challenging to pinpoint the topology.

**Landau quantization method.** Magnetic-field-induced Landau level quantization is reflected in the spectroscopic peaks through measuring the \( \text{d}I/\text{d}V \) signal. Landau levels are a sequence of discrete energy values formed due to the quantization of the cyclotron orbits of charged particles in magnetic fields. Prior to its application to topological materials, Landau levels in the LDOS have been observed in other materials, including 2D electron systems, graphite and graphene\textsuperscript{87–91}. It has been established that the Landau levels of the Dirac band are markedly different from those of a parabolic band, owing to the Berry phase effect. Figure 5a illustrates the Landau levels of a 2D Dirac band as a function of magnetic field.
of the magnetic field \( (B) \), which can be described by
\[ E_n = E_D + sgn(n)\sqrt{|2n|e\hbar B}, \]
where \( E_D \) is the energy of the Dirac cone relative to the Fermi level and \( v \) is the velocity of the Dirac band dispersion. In contrast, the energy of Landau levels of a parabolic band is described by
\[ E_n = \varepsilon_n \pm (n + \gamma)e\hbar B/m^* \]
where \( \varepsilon_n \) is the energy of the band edge, \( \gamma \) the Onsager phase and \( m^* \) the effective mass of the band.

The unique features of Dirac Landau levels include the following. The energy of the zeroth Landau level does not change with \( B \), the energy separations of Landau levels are not equal and decrease with increasing \( n \), and the non-zeroth Landau levels display a quadratic dispersion with \( B \). All of these features can be directly identified by the sequence of spectral peaks in the \( dI/dV \) data as a function of magnetic field strength. Moreover, the quantization nature of Landau levels enables a direct comparison with analytical models and magneto-transport data. The band dispersion can be extracted from Landau levels to compare with angle-resolved photoemission data and density functional theory. Therefore, Landau level imaging is a powerful method to diagnose the existence of Dirac or other topological fermions with linear band crossings in materials.

The detection of Landau level formation in real space often requires materials to be free from defects. A relevant length scale is the magnetic length, \( l = \sqrt{\hbar/eB} \), which is associated with the wavelength of the zeroth Landau level. Thus, the STM detection of Landau levels at a few Tesla would imply that the average defect inter-distance is larger or on the same order as \( l \), and the atomic defect rate is normally less than 0.1%, pointing to their quantum-limit nature. Here, we highlight several examples for Landau level imaging in materials. The \( dI/dV \) spectra taken at different magnetic fields in the topological insulator \( \text{Bi}_2\text{Se}_3 \), show evidence of the Landau quantization of surface Dirac fermions \((\text{Bi}_2\text{Se}_3)\). The Landau levels at different magnetic fields taken together provide a Landau fan diagram. The zeroth Landau level is identified as the peak at \(-200\text{meV}\) that barely shifts with \( B \), and the Landau levels for \( n > 0 \) are subsequently identified that exhibit energy differences proportional to \( \sqrt{nB} \) with respect to the zeroth Landau level. Fundamentally, the Berry phase of Dirac fermions would also imply that leaving the zeroth Landau level empty or completely filled gives rise to the half-integer quantum Hall effect. This expectation is visualized in the later transport experiment. The fixed zeroth Landau level relative to \( B \) can also be seen in the gate-tuned transport Landau fan measurement of \( \text{Bi}_2\text{Se}_3\), which is a close cousin of \( \text{Bi}_2\text{Se}_3 \), and exhibits surface-dominating conduction \((\text{Bi}_2\text{Se}_3)\). When the bottom surface is gated through the zeroth Landau level, an odd-integer quantum Hall plateau is observed. This observation is consistent with a half-integer quantum Hall effect of two degenerate Dirac bands arising from two surfaces \((\text{Bi}_2\text{Se}_3)\).

With real-space-imaging capability, researchers have also investigated the perturbations of local atomic impurities or charge potentials to the Landau levels \((\text{Bi}_2\text{Se}_3)\). Intriguingly, although the surface Dirac band exhibits spin–momentum locking in \( \text{Bi}_2\text{Se}_3 \), its Landau levels can be shifted and split by a charge potential in real space \((\text{Bi}_2\text{Se}_3)\), indicating their two-component nature with the possible formation of a real-space spin texture \((\text{Bi}_2\text{Se}_3)\).

Besides the topological insulators, Landau level imaging has also been applied to study other topological materials, including the topological crystalline insulator \( \text{(Pb,Sn)Se} \) (REFS\(^{93}\)), the Dirac semimetal \( \text{Cd}_3\text{As}_2 \) (REF.\(^{102}\)), as well as the Chern magnet \( \text{TbMn}_6\text{Sn}_6 \) (REF.\(^{109}\)). In these materials, the Landau levels exhibit additional complexity beyond features for a single Dirac cone. The complexity is associated with the interplay between symmetry-breaking order and band topology, which can often be described by the analytical Landau level model. The Landau fan of \( \text{(Pb,Sn)Se} \) features three sets of zeroth modes that do not change energy with varying \( B \) fields, indicating the coexistence of massless and massive Dirac fermions \((\text{Fig. 3c})\). The mass acquisition associated with the observed lattice distortion is a consequence of the crystalline quantum topology \((\text{Fig. 3d})\). Through further chemical engineering, the Landau level imaging method helps to identify a topological quantum phase transition associated with the Dirac mass generation \((\text{Fig. 3e})\). The Dirac-like Landau fan is obtained in the Dirac semimetal \( \text{Cd}_3\text{As}_2 \) \((\text{Fig. 3f})\), where the Landau levels can be attributed to bulk Dirac fermions projected on the surface \((\text{Fig. 3f})\). Two sets of Dirac-like Landau levels are observed, which provides an early indication of the splitting of Dirac fermions into Weyl fermions by the applied magnetic field.

**Emerging methods to discover topology**

QPI and Landau level quantization are both effective methods to characterize the nature of topological fermions. However, these STM studies in topological materials are often guided by other techniques and predictions that present the early evidence for topology. With the development of newer STM methodologies in probing topological matter, a question arises: can STM play a leading role in discovering new topological materials or phenomena? To achieve such innovation, we need to have a profound understanding of the fundamental principles and the measurable consequences of quantum topology, as well as materials science, at play. In this section, we outline case studies showing emerging methods where STM has played a leading role in unveiling the topological nature of a material.

**Topological correspondence**. The topological correspondence principle, which emphasizes the topology-governed connection between different physical properties, is a powerful way to establish quantum topology in STM studies. The bulk-boundary connectivity, for instance, is one topological correspondence that has been pointed out since the early research on the quantum Hall effect \((\text{QHE})\). In materials, the nontrivial topology of the bulk electronic structure can induce robust edge states around the boundary. As STM has the advantage of resolving step edges, the detection of robust edge states can often serve as an indication for nontrivial bulk topology, with examples including the topological step edge states observed in bismuth, \( \text{Bi}_n \text{Rh}_m \text{Te}_{n+m} \), \( \text{HfTe}_3 \), \( \text{FeSe/SrTiO}_3 \), \( \text{(Pb,Sn)Se} \), bismuthene, \( \text{WTe}_2 \) and \( \text{WSe}_2 \), and \( \text{TbMn}_6\text{Sn}_6 \) (REFS\(^{103}\)). This method is particularly powerful when the bulk energy gap can be independently...
identified away from the edge, as in the case of ZrTe5, HfTe5, WTe2, monolayer, bismuthene and TbMn6Sn6. Unusual fermions arising from lattices with unique geometry (such as honeycomb, kagome, Lieb and chiral lattices) are another relevant example of topological correspondence, as topologically nontrivial band degeneracies and band singularities can be protected by special lattice geometry. This correspondence is particularly relevant for STM, since STM has the advantage to resolve different lattice layers in a complex material. A well-known example is graphene, and a recent example includes the topological kagome magnet family (BOX 1), where STM can play a leading role in many novel observations34,84,101,116–125.

Here, we take the STM study34 on the kagome Chern magnet TbMn6Sn6 as a case in point to elaborate on the topological correspondence (FIG. 4a). The kagome lattice naturally hosts Dirac electrons at the Brillouin zone boundaries. The inclusion of spin–orbit coupling and out-of-plane ferromagnetic ordering in the kagome lattice effectively realizes the spinless Haldane model by generating Chern gapped topological fermions126–128. Essentially, the spin-polarized Dirac fermions in momentum space will open an energy gap at the Dirac cone with a non-zero Chern number (Chern gap), whereas the spin-orbit coupled kagome lattice carries a chiral edge state correspondingly (FIG. 4b). Among many other kagome magnets, TbMn6Sn6 uniquely features a pristine Mn kagome lattice with strong out-of-plane magnetization. STM study further finds that the Mn kagome lattice is almost free from defects and exhibits unique Landau quantization, while another hexagonal lattice in the same material does not feature Landau quantization. The Landau fan analysis demonstrates a spin-polarized Dirac band with a large Chern gap (FIG. 4b). The Landau levels can be described by $E_n = E_D \pm \sqrt{(\Delta/2)^2 + 2|n|\hbar v_F B B - \gamma g \mu_B}$, with $E_D = 130 \text{ meV}$, $\Delta = 34 \text{ eV}$, $v_F = 4.2 \times 10^5 \text{ m s}^{-1}$ and Landé $g$-factor, $g = 52$. Based on these parameters extracted from the Landau levels, the band dispersion can be obtained, which shows agreements with occupied bands observed by angle-resolved photoemission data and the density functional calculations. Precisely at the Chern gap energy position, a pronounced step edge state is observed (FIG. 4c) and the quasi-particle scattering is substantially reduced, as measured by the QPI on the sample edge, both of which are consistent with the existence of a chiral edge state within the Chern gap. Based on the obtained Dirac energy and Chern gap size, the Berry curvature contribution34 to the anomalous Hall conductivity can be estimated as $\sigma_{yx} = \frac{\Delta}{22} \times \frac{e^2}{h} = 0.13 \frac{e^2}{h}$ per kagome layer, which shows an agreement with the intrinsic anomalous Hall conductivity ($\sigma_{yx}^{\text{int}}$) obtained by magneto-transport as $\sigma_{yx}^{\text{int}} = 0.14 \frac{e^2}{h}$ per kagome layer within the error bar of the measurement (FIG. 4d). The agreements between Landau levels imaging, edge state (QPI) imaging and anomalous Hall conductivity collectively prove evidence for the topological bulk-boundary-Berry correspondence in identifying this Chern magnet. This case study also indicates that, with the guidance of topological correspondence principles, STM studies can present original key evidence for topological matter.

**Vector magnetic field control.** Besides the topological correspondence principles, the vector magnetic field capability of the STM technique provides a new channel to probe and manipulate topological matter. For

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**Box 1 | Topological kagome magnets**

Topological kagome magnets refer to a new class of magnetic quantum materials hosting kagome lattice and topological band structure. They include 3-1 materials (e.g. antiferromagnet Mn3Sn), 1-1 materials (e.g. paramagnet CoSn), 1-6-6 materials (e.g. ferrimagnet TbMn6Sn6), 3-2-2 materials (e.g. hard ferromagnet Co3Sn2S2) and 3-2 materials (e.g. soft ferromagnet Fe3Sn2), thus, demonstrating a variety of crystal and magnetic structures. They generally feature a 3d transition-metal-based magnetic kagome lattice with an in-plane lattice constant ± 5.5 Å. Their 3d electrons dominate the low-energy electronic structure in these quantum materials, and exhibit electronic correlation. Crucially, the kagome lattice electrons generally feature Dirac or Weyl band crossings and flat bands, which are the sources of nontrivial topology. Moreover, they all contain the heavy element Sn, which provides strong spin–orbit coupling to the system. Therefore, this is an ideal system to explore the rich interplay between geometry, correlation and topology. Scanning tunnelling microscopy (STM) has the advantage to uniquely resolve the kagome lattice layer of these materials and perturb the kagome electrons and magnetism with vector magnetic field, uncovering unprecedented topological phases and many-body effects34,84,101,117,322,324.

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instance, this technique has been used to study vortices in the anisotropic superconductor NbSe$_2$ (REF.129). Recently, it has also been applied to probe the vortices and QPI in the superconductors Cu$_2$Bi$_2$Se$_3$, LiFeAs and Bi$_2$Te$_3$–NbSe$_2$, all of which have topological surface states$^{130–134}$. The interplay between anisotropic Cooper pairing and topological surface states can be studied in detail. Another emerging direction is the application of topological magnets, where the vector magnetic field can control topological band structure, owing to the strong spin–orbit coupling and magnetic exchange interaction in these materials.

A notable example is the STM study of the topological soft magnet Fe$_3$Sn$_2$ (REF.135), in which the magnetization direction can be effectively controlled through a vector magnetic field. STM study of this material reveals that, upon increasing a magnetic field along the c-axis, a quantum state exhibits a progressive energy shift, which is in strong correlation with the bulk magnetization curve but is in stark contrast with the conventional Zeeman effect. Moreover, the rotation of the in-plane magnetization direction introduces an energy shift with strong two-fold anisotropy concerning the rotation angle. This nematicity is consistently observed through magneto-transport with rotating an in-plane field$^{135}$. The vector-magnetization-induced energy shift is summarized in the top panel of FIG. 5a. Particularly, it exhibits a node along the crystalline a-axis, indicating a spontaneous magnetization along the a-axis. Through QPI dispersion study, this energy shift is found to be associated with the magnetization tuning of the Dirac gap (FIG. 5a, bottom panel), with the quantum state corresponding to the band bottom of the upper Dirac branch. Intriguingly, the dI/dV mapping of this quantum state reveals spontaneous QPI nematicity along the a-axis as well (FIG. 5b, top panel). As these QPI data are taken in the absence of the applied magnetic field, the observed nematicity direction supports spontaneous magnetization along the a-axis and demonstrates an internal consistency with the anisotropic energy shift data in FIG. 5a. Magnetization along other directions by the applied magnetic field can alter, and, thus, control, the electron scattering symmetry (FIG. 5b). Such vector magnetic field control of the scattering geometry can be explained in the context of the interplay between vector magnetization and the underlying spin texture of the Dirac fermions$^{136}$. Another example is the STM study of the topological hard magnet Co$_3$Sn$_5$S$_3$ (REF.137), in which Berry-phase-induced orbital magnetism and flat band physics can be resolved$^{138–139}$. STM study of this material reveals that a flat band contributes to the tunneling signal and has a highly unusual vector magnetic field response. The flat band state (associated with the blue colour in FIG. 5c) exhibits a progressive energy shift with the magnetic field, and it shifts to the positive energy direction, irrespective of the vector magnetic field direction along the c-axis. The shift rate can be used to calculate an effective magnetic moment of $-3\mu_B$. This field-induced energy shift of the flat band state is highly unusual from two aspects. Firstly, the field-direction-independent energy shift is in strong contrast with the conventional Zeeman effect. As the magnetic field flips the bulk magnetization direction,
it introduces a magnetization-polarized Zeeman shift, thus the energy shift is insensitive to field directions. Secondly, the negative magnetic moment is beyond the spin Zeeman effects (~$+1\mu_B$ as the flat band features spin-up in density functional theory) and indicates the dominant negatively polarized orbital magnetization. Indeed, the Berry-phase-induced orbital magnetism for the flat band calculated from density functional theory (Fig. 5d) shows the same sign and order of magnitude as that obtained by STM. The STM experiment, thus, unveils the topological effect of the magnetic flat band state. More recently, STM studies under a vector magnetic field show that the impurity resonances also exhibit negative orbital magnetism$^{124,125}$, implying ubiquitous spin–orbit coupling and quantum phase effects in the topological magnet $\text{Co}_3\text{Sn}_2\text{S}_2$. Interestingly, the flat band orbital magnetism has also become a recent focus in the research of twisted bilayer graphene$^{139,140}$, which can be the underlying mechanism of the observed quantum anomalous Hall effect$^{141,142}$.

**Localized topological zero modes.** Similar to its vector magnetic field capability, STM is a unique tool to search for exotic quantum states at the extreme local scale in material, $k$ is no longer a suitable quantum number. These local quantum states include the single-atomic impurity-induced resonance$^{146}$ and the vortex core states$^{143}$ in superconductors. The combination of magnetism, topology and superconductivity enables the search for Majorana fermions$^{144–153}$. Majorana modes can manifest as a robust zero-energy peak (topological zero modes) inside a superconducting gap in the tunnelling spectra. Systematic efforts have been made to design artificially hybrid structures to visualize the topological zero modes based on various theoretical proposals; this is extensively discussed in other review articles$^{147–153}$. Here, we highlight an alternative approach for STM to search for originally unpredicted and naturally occurring topological zero modes in bulk $\text{Fe(Te,Se)}$ crystal and related materials, and this approach can be another useful guideline for future experimental discoveries.

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**Fig. 5 | Vector magnetic field control of quantum order in topological matter.** a | Vector-magnetization-induced energy shift of a quantum state in the topological magnet $\text{Fe}_3\text{Sn}_2$. The top panel shows the saturated energy shift $\Delta E$ as a function of the direction of the magnetization vector. The light-blue surface shows a 3D illustration of the energy shift $\Delta E$ as a function of the magnetization vector, which exhibits a nodal line along the $a$-axis. The lower panel shows the schematic of the magnetization-controlled Dirac gap, with the band bottom of the upper branch corresponding to the shifting state. The red and blue curves illustrate the band structures with in-plane and out-of-plane magnetizations, respectively. b | Quasi-particle interference (QPI) patterns of the shifting state as a function of the magnetization direction, which is indicated in the insets with respect to the lattice. The topmost QPI pattern shows the spontaneous nematicity along the $a$-axis. Magnetization along other directions can alter, and, thus, control, the electronic scattering symmetry. c | Vector magnetization control of a flat band state in the topological magnet $\text{Co}_3\text{Sn}_2\text{S}_2$. The inset illustrates that field is applied perpendicular to the kagome lattice plane. d | Orbital magnetism for the flat band from density functional theory. The inset illustrates the large negative orbital magnetism of the flat band. Panels a and b adapted from REF.$^{84}$, Springer Nature Limited. Panels c and d adapted from REF.$^{117}$, Springer Nature Limited.
Before the topological nature of Fe(\(\text{Te,Se}\)) gained appreciation, a robust zero-energy bound state at the native interstitial Fe impurity\(^4\) was observed by an early STM work (Fig. 6a). While measuring slightly away from the impurity, the bound state fades away but remains centred at zero energy. Remarkably, this state is robust against even an 8-T applied magnetic field, which would, otherwise, have induced a large splitting for any spin-degenerate state. This observation has no explanation in terms of classical impurity states in superconductors with \(s\)-wave symmetry. However, this state bears many characteristics of the Majorana mode proposed for topological superconductors, suggesting possible nontrivial topology in the system\(^4\). This observation stimulated further density functional theory calculations of Fe(\(\text{Te,Se}\)), which demonstrated nontrivial band topology\(^{155,156}\), also confirmed by angle-resolved photo-emission study\(^{157}\). The discovery of nontrivial quantum topology in these materials opened up new research directions in the study of iron-based superconductivity\(^{158}\). The observation of a robust zero-energy bound state at the Fe impurity stimulated further advanced theoretical proposals on topological excitations, including the concept of the quantum anomalous vortex, formally in analogy with the quantum anomalous Hall effect\(^{159}\). In \(s\)-wave superconductors with strong spin–orbit coupling, magnetic impurity ions can generate topological vortices in the absence of external magnetic fields. Such vortices, dubbed quantum anomalous vortices, support robust Majorana zero modes when superconductivity is induced in the topological surface states. The existence of quantum anomalous vortices at native interstitial Fe impurities is also consistent with the spontaneous Nernst signal in thermal transport measurement\(^{160}\). The quantum anomalous vortex appears to be a more robust way to generate Majorana zero modes than the field-induced vortex method\(^{159}\), as the topologically trivial vortex state is pushed to higher energies. This argument is consistent with the STM experiments\(^{133,161}\) in LiFeAs. In this system, the field-induced vortex shows no zero-energy.
bound state, even with systematic Co doping, which is attributed to the contamination of topologically trivial vortex states[186]. On the other hand, depositing Fe adatoms can generate a zero-energy bound state [Fig. 6b], which is robust against vector magnetic field perturbations[188]. These findings suggest that magnetic adatoms on superconductors with topological surface states can be a field-free platform for exploring topological zero modes. The robust zero-energy bound state is also observed on the Fe adatom deposited on Fe(Te,Se), which is insensitive to the magnetic field and can be further tuned through interaction with the STM tip[182].

Another unexpected observation is on the native line defect in Fe(Te,Se)/SrTiO3 (Fig. 6c), where the zero-energy peaks emerge at the line ends[183]. Benefiting from a higher superconducting transition temperature from interface coupling, this can serve as a high-temperature platform for topological zero modes. Another native defect, the crystalline domain wall in Fe(Te,Se), also exhibits nontrivial excitations[184]. The associated flat in-gap state is consistent with a linearly dispersing mode in 1D, suggesting the possibility of a propagating topological fermion.

With the identification of quantum topology in Fe(Te,Se), a natural direction is to search for the Majorana zero mode in magnetic-field-induced vortices, based on the original theoretical proposal[185,186]. This direction has been fruitful. STM measurements have found that there exist vortices with pronounced nonsplitting zero-energy bound state[187] as evidence for a topological zero mode (Fig. 6d). Ultra-high energy resolution STM further confirms their zero-energy nature for the core states of some vortices (Fig. 6d) and reveals ratios of zero-mode-carrying vortices as a function of magnetic field[188]. The topological nature of the vortices with and without zero-energy modes has been further explored[189,190] by an inspection of the side peaks of their core states and examination of the tunnelling conductance quantization[189]. In parallel, the nonsplitting zero-energy vortex bound state was observed in another iron-based superconductor (LiFe)OHFeSe (Fig. 6f) to exhibit nearly quantized conductance[190,191]. The nontrivial native defects and magnetic-field-induced vortices collectively present a zoo of potential Majorana platforms in iron-based superconductors. These experiments also stimulate discussions on the next frontier of Majorana research, including the possibility of braiding operations to demonstrate the non-abelian anyonic statistics[192], which will be the eventual smoking-gun evidence for Majorana interpretation of the existing results.

**More to discover**

STM is becoming an advanced technique in probing and uncovering emergent quantum effects and topological phases. Looking forward, there remains substantial room to improve the capabilities of STM. On the instrumentation side, for example, a new direction is the ability to rotate the STM head in a simple high field solenoid, which can push the effective vector magnetic field magnitude far beyond the current technical limit. This is possible by extending the STM design to incorporate rotational functionality. On the technique side, it has been possible to perform experiments under a ramping mode while compensating for the tip drift when the field ramping rate is sufficiently low[193]. This methodology allows for the faster Landau fan imaging, for example, in the topological magnet TbMn4S5, and will be valuable in the application to field-rotation-coupled STM measurements. Lastly, it has been recently pointed out that an electric-field-induced change in the electrons' charge density orientation can be an effective way to probe the Berry curvature field[194]. Therefore, the implementation of vector electric field tuning in STM systems will likely be another fruitful direction to extend the reach of the technique to access exotic topological phenomena.

The tunable vector magnetic field capability coupled with STM promises great potential in visualizing quantum engineering routines applied on topological materials. Besides the topological kagome magnet family with various magnetic structures, there are a number of other correlated topological materials featuring novel magnetic response that can be explored using STM. For instance, singular angular magnetoresistance has been observed in a magnetic nodal semimetal CeAlGe[REF. 177] and magnetic nodal lines have been experimentally observed in a soft magnet Co3MnGa [REF. 178], both of which are excellent candidates for the study of the interplay between Weyl-line fermions and vector magnetization, which is possible by STM. It is predicted that EuIn2As2 can either be an axion insulator or a topological crystalline insulator (exhibiting different hinge states), depending on the direction of the magnetic moment[195]. This promises to be another fertile platform for STM under a vector magnetic field to explore, manipulate and control the associated higher-order topology[196]. Moreover, nearly ferromagnetic spin-triplet superconductivity has been detected in UTe2 [REF. 179] and chiral edge states were observed by low-temperature STM [REF. 179]. Thus, low-temperature STM under a vector magnetic field can be used to elucidate the interplay between magnetism and chiral superconductivity. It has long been known that an in-plane magnetic field can induce chain vortex matter in Bi2Sr2CaCu2O8+δ while a strong spin-orbit effect was recently observed in the same material[196]. STM with an in-plane magnetic field can explore the spectral characteristics of the chain vortex matter and the possibility for high-temperature 1D topological superconductivity in cuprates. A novel charge order that breaks time-reversal symmetry[197] has been recently observed in the normal state of kagome superconductor KV3Sb5, serving as another example of STM driven discovery. The intertwined interplay between topological band structure, novel charge order and kagome superconductivity can be further explored by vector field based STM. There are also possibilities to explore more exotic systems, including a system of Majorana zero modes with random infinite-range interactions — the Sachdev–Ye–Kitaev model that theoretically exhibits an intriguing analogue to the horizons of extremal black holes[198,199]. Materials including Fe(FeO) with random excess Fe impurities carrying zero modes may be explored by STM to test these tantalizing ideas.

One important class of topological correspondence is the Wannier–Bloch duality[200,201], Wannier functions are...
the localized molecular orbitals and Bloch wavefunctions are related to the electron energy levels in a periodic crystal. The theoretical realization of the Wannier–Bloch duality links the local chemistry and extended wavefunction property approaches to describe electronic states, leading to a modern understanding of electric polarization and orbital magnetization, as well as allowing systematic first-principles predictions of topological materials. STM can be used to locally probe molecular orbitals, which can, additionally, be sensitive to Berry-phase-induced orbital magnetism with magnetic field perturbations. The ability to visualize Wannier–Bloch duality correspondence by STM can lead to new experimental directions in elucidating topology.

Quantum information science has emerged as one of the major research frontiers, which includes STM-based characterization and fabrication techniques to enable the bottom-up construction of qubits from the atomic components. Particularly notable is the progress in visualizing topological materials at the atomic scale that could yield inherently error-protected qubits expected to feature a high level of digital fidelity. However, a clear demonstration of topological protection of quantum information in materials remains an open question for further exploration. Collaboration between STM and device engineering would further enable the development of models for optimizing topological materials selection for desired functionality based upon controllable properties, such as density of states, tunneling energies, and the qubit-relevant characterization of materials-related decoherence channels.

Besides these exciting frontiers, there are potential research areas where the related questions are less defined and remain open-ended, including whether the STM technique can be utilized to explore the many-body entangled version of topological order. With the quantum-level interplay of geometry, correlation and topology resolved at the atomic scale, it is conceivable that there are more important, yet, hitherto unknown, emergent phenomena to be discovered through STM research on topological matter. In his last public talk in 2019, P.W. Anderson was asked: "What is your best suggestion to young researchers when they encounter a new phenomenon?" Anderson responded: "Patience, and it can take a lifetime long (to understand certain emergent phenomena substantially)." His remark highlighted the naturalness to be discovered 'unknown unknowns' in quantum materials. We believe that time is ripe in the field where tunnelling into topological matter will likely lead to genuinely new phenomena in the next few years, which will reap multiple lifetimes' worth of rewarding scientific vistas for our efforts now.

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