Basic principles for describing the process of vibratory with an external source of dynamic impact

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Abstract. The modern development of mechanical engineering requires high quality manufacturing of machine parts. The production experience testifies that high requirements to the quality of surface treatment of the parts need to be improved, in particular, in the finishing and clearing operations (FCO). Progressive method of processing parts is vibratory (V) on machines with U-shaped containers. Despite the positive practical experience of vibratory using in mechanical engineering, this method faces a number of problems related to the development of its theoretical foundations: equipment and tools are still imperfect; in the initial stage are works on managing of the V process; it is insufficiently studied the mechanism of surface layer formation at V. These problems hamper further development and improvement of the vibratory, which is very effective by its technological capabilities, and limit its wider introduction into industry. Efficiency of processing is provided by the right choice of equipment, schemes, and modes of processing, types of working environments. The mass of loading in the container of the machine does not absorb all the energy transmitted by the vibratory machine, but only part of it, the other part is non-productively dissipated. Therefore, the choice of the constructive scheme of the machine, on which the redistribution of these energies depends, is a crucial step, both in its design and in the management of the machining process.

1. Introduction
In modern mechanical engineering when creating new types of products, there are new operational requirements that ensure reliability and durability of these products. An important condition for solving these problems is the development of progressive forms of finishing technology, in particular, methods of finishing. One of the effective methods of processing is vibratory processing of parts in different technological environments using vibrations of different types. Vibrating processing provides change of physical and mechanical properties of a surface layer of a detail as a result of influence by low-frequency vibrations of a working environment with certain frequency and amplitude. It possesses wide technological possibilities and universality. For increase of productivity of processing creation of the tool - single abrasive granules from positions of revealing of the most rational geometrical form of a granule which will allow to provide reception of the maximum removal of metal from surfaces of processed details and preservation of working capacity of a granule for a long time is also important. Vibration machining is widely used for various parts, and primarily for complex shapes, but there are limitations due primarily to the size of parts, these are small flat and long parts. The first one - due to the fact that during the treatment with water or solutions of reactive substances, parts stick to the walls of the container and are collected in bags (this phenomenon disrupts the processing process, reduces the quality of processing of small flat parts), the second - due to uneven processing, associated with the
difficulty of ensuring stable circulation in long vibrating containers. Therefore, it is necessary to search for new technological solutions for the V using of such parts.

2. Main text

Modern mathematical constructions on the theory of vibration processing of parts [1–3] are based on the same unified approach: the initial position is to consider a container without fixed geometry, filled with a working medium together with the workpieces, and under conditions of arbitrarily long periodical vibration excitation. The description of the movement of the working medium is a contact problem that takes into account the boundary conditions. It is easy to see that the exact solution to this problem assumes its deterministic nature, which is the result of a complete specification of the behavior of the contents of the container at its border. However it is not possible even for the simplest geometric configuration of the border. Thus, the creation of the theory of vibration processing in a single container did not, does not and will not lead to a description of the general laws of the interaction of the processing medium with the machined parts, which would bring the research process to specific technological solutions and recommendations, including the possibility of controlling the processing productivity based on a combination of the values of the design parameters of the task [4–6]. Without a new consistent systematic approach to creating a theory of vibration processing, it can not be kept in view clear applied research perspective based on effective technology. This situation underlines a certain weakness in the previously mentioned concept of the vibratory process (and in some cases the use of completely unacceptable research schemes), when the design of containers is improved as a result of the analysis of numerous experimental data by the trial and error method without taking into account strict performance criteria and in the absence of theoretical general results character.

Therefore, it is necessary to consider the task related to the description of the process of medium motion that fills at standstill in the initial moment of time \( t = 0 \) the whole Euclidean plane \( \mathbb{R}^2 \) with a Cartesian rectangular coordinate system \( X^OY \). Here and elsewhere, two states of the working medium will be distinguished: \( a \) - state (a priori rest state) and \( e \) - state (state of excitation). It should be pointed out that the working medium approximately passes from a homogeneous and isotropic \( a \) - state to \( e \) - state ("a – e" transition) using an external alternating plane-parallel field \( \mathcal{F}(N, t) \), \( N(x, y) \in \mathbb{R}^2 \), which turns on at a moment of time \( t = 0 \) and which is general model of vibrational excitation. The term “general model” should be understood in the sense that, with a special task of the vibration field \( \mathcal{F}(N, t) \), the plane problem in a bounded simply connected area \( G \subset \mathbb{R}^2 \) can be studied. \( a \) - state and \( e \) - state of the working medium will be characterized by their sets of parameters, which will be fixed with indices respectively “\( a \)” or “\( e \)”.

For example, the mass surface density of the working medium is indicated by the symbol \( \rho = m_n^e \), where \( m_n^e \) - is the mass of an individual granule, \( n_n^e \) - is the surface density of granules, etc [7].

The formulation of the problem of “\( a – e \)” transition of the working medium is as follows: to study the response of the considered many-particle system to an external variable field \( \mathcal{F}(N, t) \) and the process of interaction of the resulting mass flows with the workpieces as the main theoretical and technological working mechanism. It should be noted that a complete description of the response of the working medium to the action of a power vibrofield \( \mathcal{F}(N, t) \) should be carried out in terms of quantities \( \{ \rho_s(N, t), \tilde{x}_s(N, t), \varepsilon_s(N, t) \} \), where \( \rho_s(N, t) \) - is the surface density of the mass, \( \tilde{x}_s(N, t) \) - is the surface density of the pulse, \( \varepsilon_s(N, t) \) - is the surface energy density. In this case, the main parameter is the mass density \( \rho_s(N, t) \), through which the remaining parameters are easily expressed. This circumstance is due to the fact that the movement of the working medium in the presence of an external field \( \mathcal{F}(N, t) \) is forced and, therefore, is not of a relaxation nature. Moreover, dissipative processes in a moving working medium are caused only by mechanical scattering of the mass during two-, three- and, in the general case, \( n \)-partial collisions of individual granules of the working medium with their wear. This so-called "internal friction" is not an effect of intermolecular interaction and is not related to the classical transport...
phenomena in liquids and gases, which can be interpreted in the language of the laws of conservation of mass, momentum and energy, but is induced by the vortex properties of the force field \( \mathbf{F}(N,t) \), which leads to the tensor form of the velocity distribution in a working environment. We now formulate the basic conditions and principles (axiom system) for constructing a model of the process under study:

A1. Working medium in \( a \)-state fills the entire Euclidean plane \( \mathbb{R}^2 \) and has a surface mass density \( \rho_a = \mu_a n_a \).

A2. At the moment of time an external variable plane-parallel force field providing “a-\( e \)” transition and having a surface density of forces is switched on [8].

\[
\mathbf{F}(N,t) = \mathbf{F}_{x,p}(N,t) + \mathbf{F}_{r,d}(N,t),
\]

(1)

where \( \mathbf{F}_{x,p}(N,t) \) - solenoidal potential field for which, except for a maximum number of countable points \( \text{div} \mathbf{F}_{x,p}(N,t) = 0 \) and \( \text{rot} \mathbf{F}_{x,p}(N,t) = 0 \), \( \mathbf{F}_{r,d}(N,t) \), represents a turbulent component with \( \text{rot} \mathbf{F}_{r,d}(N,t) \neq 0 \) and \( \text{div} \mathbf{F}_{r,d}(N,t) \neq 0 \). It is easy to see that the surface density of forces \( \mathbf{F}_{r,d}(N,t) \) is a vectorial random value given on some initial probabilistic space \( \{ \Omega, U, P \} \), where \( \Omega = \{ \omega \} \) - space of elementary events, \( U \) - algebra (or \( \sigma \)-algebra) of random events \( A \) and \( P = P(A) \) - probability function. The choice of the corresponding probability space is determined by the peculiarities of the vibration-excitation mechanism and will not be investigated in detail in this work.

A3. In \( e \)-state working medium is considered as a two-component medium with qualitatively different components: the first component - the actual granules of the working medium, consisting of particles in mass \( m_{1e}(t) \) and having a density of mass flow

\[
\mathbf{v}_{1e}(N,t) = n_{1e}(N,t)\mathbf{v}_{1e}(N,t),
\]

(2)

where \( n_{1e}(N,t) \) - the density of the number of particles in a point \( N = (x,y) \), and \( \mathbf{v}_{1e}(N,t) \) - the velocity of the working medium in a point \( N = (x,y) \), and the second component - the set of point vortexes with mass \( m_{2e}(t) \) and mass flow density

\[
\mathbf{v}_{2e}(N,t) = n_{2e}(N,t)\mathbf{v}_{2e}(N,t),
\]

(3)

where \( n_{2e}(N,t) \) is the density of the number of vortexes in a point \( N = (x,y) \), and \( \mathbf{v}_{2e}(N,t) \) the speed of vortexes flow in a point \( N = (x,y) \).

The dependence between the speeds \( \mathbf{v}_{1e}(N,t), \mathbf{v}_{2e}(N,t) \) and the external power field \( \mathbf{F}(N,t) \) will be set lower.

A4. The presence in the structure of the external force field (1) of the turbulent component \( \mathbf{F}_{r,d}(N,t) \) contributes to the appearance of vortexes density \( n_{2e}(N,t) \) eddies in the working environment, which is a random function. Let us determine the average density value \( n_{2e}(N,t) \) by means of dynamic limit transition

\[
n_{2e}(t) = \left\{ n_{2e}(N,t) \right\} = \lim_{S \to \infty} \frac{1}{\mu(S)} \int_S n_{2e}(N,t) d\sigma_S,
\]

(4)

where \( d\sigma = dx dy \), and \( \mu(S) \) the area of the single-linked area \( S \subset \mathbb{R}^2 \).

Thus, the dynamic vortex density \( n_{2e}(t) \) can be considered as a random process with a discrete or continuous parameter \( t, t \geq 0 \). It should be noted that the meaning of the limit transition (4) is to exclude the influence of boundaries and thus to match the spirit of the main task.
A5. The vortex is a combination at one point $M_0(x_0,y_0) \in R^2$ of the vortex with the intensity $E(t)$ and the virtual flow with the intensity $H(t)$, so that

$$H(t) = \lambda E(t), \quad t \geq 0, \quad 0 < \lambda < 1,$$

where $\lambda$ is the scattering coefficient of the aggregate mass of the granules in the vortex.

Correlation (5) indicates the specificity of the vortex: as $\tau \to 0$ it leads to the action of runoff at the point $M_0(x_0,y_0)$. The point $M_0(x_0,y_0)$ moves in the working medium along the fault line of the function $\tilde{F}_{v,d}(N,t)$ along the trajectory

$$x_0 = x_0(t), \quad y_0 = y_0(t), \quad t \geq 0.$$  

A6. Mass transfer of the two-component working medium in the presence of an external power field $F(N,t)$ (1) with a turbulent component $\tilde{F}_{v,d}(N,t)$ occurs along the current lines of the power solenoidal-potential field $\tilde{F}_{sp,d}(N,t)$ at a rate:

- for the first component

$$v_{1ei}(N,t) = \int_0^T \int_{R^2} \varepsilon_{ik}^{(1)}(N-P,t-\tau) F_{s,p,k}(P,\tau) d\tau d\sigma_p,$$

and for the second component moving along the field current line $\tilde{F}_{sp,d}(N,t)$ along the trajectory of the turbulent component break line $\tilde{F}_{v,d}(N,t)$

$$w_{2ei}(N,t) = \int_0^T \int_{R^2} \varepsilon_{ik}^{(2)}(N-P,t-\tau) F_{s,p,k}(P,\tau) d\tau d\sigma_p.$$  

The bivalent tensors in formulas (7) and (8) should be considered as coefficients of the first and second "viscosity" in a two-component working medium, $\varepsilon_{ik}^{(1)}(N,t), \varepsilon_{ik}^{(2)}(N,t)$ respectively.

A7. Let us assume that the vortexes radius $r$ is distributed according to the normal law with mathematical expectation $\mu = 0$ and standard deviation $\sigma = \sigma(t)$ which is a function of the parameter $t$, i.e. the probability density of a random value $r$ is equal to

$$P_{0\sigma}(r) = \frac{1}{\sigma^2(t)} e^{-\frac{r^2}{2\sigma^2(t)}}.$$  

Action of a vibrofield $\tilde{F}(N,t)$ (1) occurs on a time interval $[0,T]$ where $T$ - time of technological functionality of a working environment when the mass scattering caused by wear of granules, has not reached critical level. If the critical mass of a granule is equal to $m_{c\beta} = \beta m_a, \quad 0 < \beta < 1$, the value of the parameter $T$ it can be deduced from

$$m_{c\beta}(T) = \beta m_a.$$  

If $\tau_{eq}$ – the processing time of the workpiece in the presence of the vibrating field (1) $\tilde{F}(N,t)$, the efficiency criterion of this process is as follows.

$$\tau_{eq} \leq T.$$  

where \( T \) is the solution to equation (10). Returning to the correlation (9), it is easy to notice that
\[
\lim_{t \to T} \sigma(t) = 0.
\]
(12)

The study of the scattering processes shown in, which are "fine structure" in the formation of the macro parameter \( n(N,t) \), is of independent scientific interest and will not be conducted in this research.

It should be noted that in the formula (12) the values \( m_e(t) \), \( n_e(N,t) \), \( m_v(t) \) have a deterministic character, and the value \( n_v(N,t) \) is a random function. First of all, let us find the mass of the vortex \( m_v(t) \). As the geometrical characteristics of the vortex do not depend on its position on the plane \( R^2 \), we will place the vortex at the origin of coordinates \( O(0,0) \), assuming its radial symmetry, for simplicity of the following constructions. Let's find with the help of formula (9) the middle radius of the vortex [9-10].

\[
\tau = \tau(t) = \int_0^\infty \rho_{0,\tau}(\tau) \sigma d\tau = \int_0^\infty r^2 e^{-\frac{\tau^2}{2\sigma^2(t)}} \sigma d\tau = \sqrt{2\pi} \sigma(t).
\]
(13)

Let's calculate the point vortex circulation (Fig. 1).

\[
E(t) = \oint_{C} \left( \mathbf{F}_{r.d}(N,t), dS_r \right),
\]
(14)

where the vortex mass density \( \rho_{r.d}(N,t) \) is equal
\[
\rho_{r.d}(N,t) = m_{v}(t)n_{v}(N,t)\vec{v}(N,t),
\]
(15)

\( \vec{v}(N,t) = K(N,t)\mathbf{F}_{r.d}(N,t) \).

In formula (15) \( K(N,t) \) means the coefficient of turbulence of flows in the field \( \mathbf{F}_{r.d}(N,t) \).

By substituting (1) in (15), it will be obtained
\[
E(t) = m_{v}(t)\oint_{C} n_{v}(N,t)K(N,t)\left( \mathbf{F}_{r.d}(N,t), dS_{r} \right).
\]
(16)

3. Conclusions

The mathematical constructions carried out in the research have a general character and largely determine the direction in which the theory of vibroprocessing should develop. More detailed research of geometry of a vector field will lead to the concrete technological recommendations connected with parameters of used in practice forms of containers of \( V \) machines during their work.

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