Vibration pile driving system based on Sliding Mode Control

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Abstract. The sliding mode variable structure synchronous control of vibration pile driving system driven by double DC motors is studied. Firstly, the model of double machine driven vibratory pile driving system is established, and the theoretical derivation of system synchronization and stability is carried out. Secondly, the sliding mode variable structure control method is used to realize the phase speed synchronous control of two motors in the vibration system. Finally, Simulink is used to verify the correctness of the theoretical derivation, and the vibration displacement of the pile driving system is simulated and analyzed. The research shows that the sliding mode variable structure control can realize the synchronous control of the double machine driven vibratory pile driving system, which provides a theoretical and practical engineering basis for the design of this type of vibratory pile driving system.

1. Introduction

Pile sinking system is widely used in engineering. It uses the upper excitation load to be transmitted to the deep hard and less compressible soil layer or rock stratum through the pile. This pile sinking mechanism has the advantages of high bearing capacity, good stability and uniform settlement [1]. In addition, there are many construction methods of pile sinking mechanism: hammer driving pile, static pile pressing and vibration pile sinking [2]. Vibratory pile sinking is to install a vibratory pile hammer on the top of the pile, and use the exciting force generated by the vibratory pile hammer to vibrate the pile and sink it into the soil. This construction method has the outstanding advantages of strong penetration force, high pile sinking efficiency, good pile sinking quality, low noise and simple operation [3-4], which is also the main object of this paper.

The research on vibration pile driving abroad started in the 1930s and has made many important achievements. Barkan determined a series of parameters affecting the construction of vibration pile sinking, including the peak value of vibration acceleration, pile amplitude, vibration frequency, etc. [5]. Kondner and Edwards proposed that there is an optimal vibration frequency in the process of vibration pile sinking. When the pile is driven at this frequency, the soil resistance to the pile can be reduced to the greatest extent and the pile sinking speed is the fastest, but they did not find the solution [6]. Nogami et al. Studied the impact of vibration pile driving construction on surrounding buildings [7]. Holeyman believes that the most important problem in the analysis of vibration pile sinking is the problem of pile-soil interaction during pile sinking [8]. Masoumi et al. Used a dynamic formula for the interaction between soil and superstructure to predict the free vibration of surrounding buildings caused by vibration pile sinking [9].

The domestic research on vibratory pile driving mainly focuses on two aspects: one is the research on the modeling and dynamic characteristics of vibratory pile driving, and the other is the research on
the phenomenon of soil "vibratory liquefaction" in the process of vibratory pile driving. Jia Wuxue analyzed the vibration pile driving process, established the single degree of freedom vibration system and dynamic differential equation, and studied the amplitude frequency characteristics of the system [10]. Wang Jian, Jiang Yunfei and others considered the vibration damping device between the cross beam of the pile machine and the vibrating pile hammer, established a two degree of freedom system mechanical model based on the single degree of freedom linear damping vibration model, numerically calculated the model with the help of C language programming and MATLAB software, and predicted the dynamic characteristics of the system [11-12]. By studying the mechanism of vibration pile driving, Cui Yuhang established the mechanical models of two degree of freedom and three degree of freedom vibration pile driving system respectively, simulated the model with the help of MATLAB, Adams and other software, studied the influence of the change of construction parameters on the dynamic characteristics of the system, and obtained good results [13].

The significance of this paper is that the introduction of sliding film control can make the two motors on the pile sinking hammer reach the synchronous state faster and work more stably, which is of far-reaching significance to the development of pile sinking engineering machinery, provide theoretical basis for future related research and help for practical production and application.

2. Dynamics model

2.1. Mechanical model of vibratory pile driving system

The double motor vibration pile driving system is composed of rigid body $M$, two excitation motors, support spring $k$, and damping $c$. Its mechanical model is shown in Figure 1. The excitation motor 1 and the excitation motor 2 are arranged symmetrically. Under normal working conditions, excitation motor 1 and excitation motor 2 rotate in reverse at the same speed. Fig. 1 mass of eccentric rotor in excitation motor is $m_1 \sim m_2$, rotation center is $O_1 \sim O_2$, rotation radius is $r_1 \sim r_2$ and phase is $\phi_1 \sim \phi_2$. Because the excitation motor in the vibration pile driving system rotates symmetrically and at the same speed, it mainly produces the excitation force in the vertical direction, so the degree of freedom of the system is mainly the movement in the vertical direction $y$ direction.

From the Lagrange equation, the $y$ direction motion differential equation of the double motor vibration pile driving system and the rotation equation of two eccentric rotors can be obtained:

$$
M \ddot{y} + cy + ky = \sum_{i=1}^{2} m_i r_i \left( \dot{\phi}_i \sin \phi_i - \dot{\phi}_i \cos \phi_i \right)
$$

$$
J_i \ddot{\phi}_i + f_i \dot{\phi}_i = T_{mi} - T_p - m_i r_i \left( y \cos \phi_i + g \cos \phi_i \right) (i = 1, 2)
$$

Where: $\ddot{y}$ is the displacement in the vertical direction of the pile sinking system driven by the double excitation motor; $\dot{y}$ is the vertical speed of the pile sinking system driven by the double excitation motor; $\ddot{y}$ is the acceleration in the vertical direction of the pile sinking system driven by the double excitation motor; In addition, $\dot{\phi}_i$ is the angular velocity of eccentric block $i (i = 1, 2)$ on the double excitation motor on the pile sinking system; $\ddot{\phi}_i$ is the angular acceleration of the eccentric block; $f_i$ is the rotational
damping of the eccentric block; \( J_{ei} \) is the moment of inertia of the eccentric block around point O; \( T_{Me} \) is the electromagnetic torque of the motor; \( T_{fi} \) is the load torque of the motor.

### 2.2. Mathematical model of DC motor

DC motor has many advantages, such as simple structure, easy manufacture, convenient maintenance, reliable operation and so on. Compared with AC motor, DC motor also has good starting performance and speed regulation performance. Therefore, for the pile sinking system with high speed regulation requirements, frequent braking and multi unit coordinated operation, the use of DC motor drive is reasonable. Since it mainly studies the synchronization when the vibration pile driving system is stable, the armature resistance of the DC motor is set as a constant value, and the mechanical characteristic equation of the DC motor is as follows:

The relationship between electromagnetic torque and armature current is as follows:

\[
T_m = J \frac{d\omega}{dt} \tag{2}
\]

Where, \( T_m \) is the motor electromagnetic torque, \( T_f \) is the motor load torque, \( J \) is the armature moment of inertia, and \( \omega \) is the motor speed.

The relationship between electromagnetic torque and armature current is as follows:

\[
T_m = K_i \phi_i \tag{3}
\]

Where, \( \phi \) is the magnetic flux of the motor pole, \( K_i \) is the motor structure constant, and \( i_a \) is the armature current.

### 3. Design of force variable structure sliding mode controller for vibratory pile driving system

Based on the master-slave control structure, the combination of master-slave motor speed difference sliding mode control and master-slave motor phase difference sliding mode control is adopted.

#### 3.1. Exponential approach rate sliding mode

Sliding mode variable structure control is a variable structure control strategy designed for complex control systems. It can realize the effect that the controlled system slides along the set trajectory on the switching surface until it tends to be stable. Sliding mode motion includes approach motion and sliding mode. The movement of the system from any initial state to the switching surface until it reaches the switching surface is called approaching motion. Namely. According to the sliding mode variable structure principle, the sliding mode reachability condition only ensures that the moving point at any position in the state space reaches the switching surface in a limited time, but does not limit the specific trajectory of the approaching motion. The approach rate method can improve the dynamic quality of approach motion.

The exponential approach rate can gradually reduce the approach speed from a large value to 0, the approach time is short, and the speed of the moving point reaching the switching surface is small, so the exponential approach rate in the following form is adopted:

\[
s = -E_s \operatorname{sgn}(s) - k_s s \tag{4}
\]

Where, \( E_s \) and \( k_s \) are positive numbers.

#### 3.2. Design of main motor speed controller

Take the main motor state variable as:

\[
\begin{align*}
x_1 &= \omega^* - \omega_i \\
x_2 &= \dot{x}_i = -\dot{\omega}_i
\end{align*} \tag{5}
\]

Where \( \omega^* \) is the given speed; \( \omega_i \) is main motor speed. Substituting equations 2 and 3 into equation 5 and deriving, we can get:
\[
\begin{align*}
\dot{x}_1 &= -\frac{K_i \phi_i - T_f}{J} \\
\dot{x}_2 &= -\frac{K_i \phi_i}{J}
\end{align*}
\] (6)

Let: \( A = -\frac{K_i \phi}{J}, u_i = \dot{i}_i \), and obtain the system state equation as:
\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} + \begin{pmatrix}
0 \\
A
\end{pmatrix} u
\] (7)

The sliding surface of the system is:
\[ s = cx_1 + x_2 \] (8)

c is a positive number and satisfies the Hurwitz condition.

The expression of control quantity \( i_i \) according to equations 4 and 8:
\[
i_i = \frac{1}{A} \left( cx_2 + \varepsilon \text{sgn}(s) \right) dt
\] (9)

3.3. Design of slave motor phase controller

Take the slave motor state variable as:
\[
\begin{align*}
x_3 &= \varphi_1 - \varphi_s \\
x_4 &= \dot{x}_3 = \dot{\phi}_1 - \dot{\phi}_s
\end{align*}
\] (10)

The sliding surface of the system is:
\[ s = c_s x_1 + x_4 \] (11)

The exponential approach rate is also adopted, get
\[
\dot{s} = -E_s \text{sgn}(s) - k_s s
\] (13)

Which, \( E_s, k_s \) are positive numbers.

The control quantity \( i_s \) expression from equation 11-13:
\[
i_s = -\frac{1}{A} \left( \rho_i + \varepsilon_s s \text{sgn}(s) + k_s s + \frac{T_{f2}}{J_2} \right)
\] (14)

because \( \varepsilon, K \) is a constant greater than zero, so for both controllers have \( \lim_{s \to 0} \varepsilon \dot{s} < 0 \) it is known from Lyapunov stability theory that the master-slave motor sliding mode control system is stable.

4. Sliding mode control simulation of vibration pile driving system

In order to verify the rationality of the system control synchronization design, Simulink is used to build the simulation model of motor vibration system, as shown in Figure 2. Runge Kutta algorithm and variable step size control strategy are used to substitute the system parameters in Table 1 for simulation analysis. When the angular velocity and phase difference of motor 1 and 2 tend to be stable, the vibration system enters the synchronous state.
Figure 2. Mechanical model and dynamic model of vibratory pile driving system

Table 1. The parameters of the vibrating system

| $M$/kg | $(m_1-m_2)$/kg | $I/(kg \cdot m^2)$ | $(J_1-J_2)/(kg \cdot m^2)$ | $(r_1-r_4)/m$ | $k_1 \cdot \phi$ | $Jm$ |
|--------|----------------|-------------------|---------------------------|--------------|----------------|------|
| 148    | 3.5            | 17                | 0.01                      | 0.05         | 0.23           | 1.26 |

(a). Speed of main motor

(b). Speed of slave motor

(c). Speed difference between master and slave motors

(d). Displacement in Y direction of pile sinking system

Figure 3. Simulation result
Figure 3(a) shows the change of main motor speed $\omega_1$, it can be seen that $\omega_1$ has stabilized near the preset value of 20 rad / s in about 20s; Figure 3(b) shows the change of slave motor speed $\omega_2$, and $\omega_2$ also follows the main motor speed $\omega_1$ to reach the preset value in about 20s. Figure 3(c) shows the change of the phase difference $\varphi_{12}$ of the master-slave motor with time. The phase difference reaches the peak value in 0 ~ 10s. This is because the slave motor speed control has the inherent characteristic of hysteresis. The phase difference gradually decreases to 0 in 10 ~ 40s and is better maintained near 0. Figure 3(d) shows the displacement in y direction of pile driving system, which changes sharply in 0 ~ 20s. After 15s, the y value gradually transits to regular sinusoidal motion and finally reaches steady state. It can be seen from the above figure that under the action of the master-slave controller, the master-slave motor basically realizes synchronous movement and the vibration pile sinking system reaches steady state in about 40s.

5. Conclusion
(1) The dynamic model of vibration pile driving system is established by using Lagrange equation
(2) The synchronous control of the system is realized by using the master-slave control structure based on the speed control of the master motor - the phase control of the slave motor.

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