THE PULSATING PULSAR MAGNETOSPHERE

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ABSTRACT

Following the basic principles of a charge-separated pulsar magnetosphere, we consider the magnetosphere to be stationary in space, instead of corotating, and the electric field to be uploaded from the potential distribution on the pulsar surface, set up by the unipolar induction. Consequently, the plasma of the magnetosphere undergoes guiding center drifts of the gyromotion due to the forces transverse to the magnetic field. These forces are the electric force, magnetic gradient force, and field line curvature force. Since these plasma velocities are of drift nature, there is no need to introduce an emf along the field lines, which would contradict the \( E \cdot B = 0 \) plasma condition. Furthermore, there is also no need to introduce the critical field line separating the electron and ion open field lines. We present a self-consistent description where the magnetosphere is described in terms of electric and magnetic fields and also in terms of plasma velocities. The fields and velocities are then connected through the space-charge densities self-consistently. We solve the pulsar equation analytically for the fields and construct the standard steady-state pulsar magnetosphere. By considering the unipolar induction inside the pulsar and the magnetosphere outside the pulsar as one coupled system, and under the condition that the unipolar pumping rate exceeds the Poynting flux in the open field lines, plasma pressure can build up in the magnetosphere, in particular, in the closed region. This could cause a periodic opening up of the closed region, leading to a pulsating magnetosphere, which could be an alternative to pulsar beacons. The closed region can also be opened periodically by the build up of toroidal magnetic field through a positive feedback cycle.

Key words: pulsars: general

1. ROTATING MAGNETOSPHERE

The basic model of a pulsar has long been recognized as a perfect conductor (neutron star) rotating with an angular velocity \( \Omega_0 \) in a hyper magnetic field \( B \) (Gold 1968). Considering the rotational axis and the magnetic axis to both be aligned the same way, charged particles in the neutron star are driven by the magnetic force to the surface according to their signs, with electrons toward the polar regions and ions toward the equatorial region. This is the Faraday unipolar induction, which operates the homopolar generator and extracts the energy of a conducting rigid rotator. Due to its rotation in the presence of a magnetic field, the conducting pulsar surface is not an equipotential surface, contrary to the static case. If the pulsar were surrounded by a vacuum, the separated charges would accumulate on the pulsar surface according to their signs, and a counter-electric field would build up inside the neutron star to counteract the magnetic force. A steady-state surface charge distribution or potential distribution would be reached when the two forces inside the pulsar cancel out, and there would be a quadrupole electric field in space surrounding the neutron star. The unipolar induction would cease to operate then. However, the seminal work of Goldreich & Julian (1969) noted that such a vacuum configuration was unstable, and there had to be a pulsar magnetosphere to channel the surface charges. As a result, the counter-electric field inside the pulsar would be small, and the unipolar induction would continue to operate. The steady-state potential distribution on the pulsar surface would be uploaded to the equipotential magnetic field lines of the magnetosphere. Quite often, field line breaking near the pulsar is invoked to provide an electron–positron background pair plasma to fill the magnetosphere.

The equation of motion of a charge particle in the magnetosphere is given by

\[
\rho \frac{\partial \vec{v}}{\partial t} = \rho_0 \vec{E} + \vec{J} \times \vec{B} - \vec{v} \times \vec{E} - \nu \rho \vec{v} + \vec{g}.
\]

With the plasma inertia on the left-hand side and plasma pressure, collision, and gravitational force on the right-hand side neglected and comparing to the hyper magnetic and electric fields, the macroscopic structure of the magnetosphere is determined only by the balance between electric and magnetic forces, which is the force-free field model (Scharlemann & Wagoner 1973; Okamoto 1974) with

\[
\rho_0 \vec{E} + \vec{J} \times \vec{B} = 0. \tag{1}
\]

Although plasma inertia and plasma pressure are neglected in the macroscopic description, these terms in the equation of motion give rise to charged particle gyromotion about magnetic field lines and transverse drift motions across field lines in the microscopic single particle description (Thompson 1962; Schmidt 1966; Jackson 1975). As a result of these drifts, macroscopic drift currents and plasma flows, corresponding to the guiding center motions, are generated in Equation (1), as will be addressed in Section 3.

With the surface charges having ready access to the field lines, the plasma density will be high enough in the magnetosphere such that the plasma conductivity \( \sigma \) tends to be infinite; Ohm’s law reads

\[
\vec{E} + \nu \times \vec{B} = \frac{1}{\sigma} \vec{J} = 0. \tag{2}
\]
For a quasi-neutral normal MHD plasma, the plasma velocity \( v \) in Equation (2) amounts to the center-of-mass velocity. For a charge-separated plasma, this equation coincides with Equation (1) other than an overall factor of the space-charge density \( \rho_c \). Nevertheless, this does not mean that Equation (2) is redundant, since it describes the magnetosphere in terms of velocities while Equation (1) does it in terms of fields. These two descriptions are then coupled through space-charge densities.

Probably because of the analogy of pulsar beacons to lighthouses, the pulsar magnetosphere is often interpreted as having magnetic field lines rigidly anchored on the pulsar surface. The entire magnetosphere thus corotates with the pulsar at its angular velocity, and the charge-separated plasma is being dragged along with it. For an aligned rotator and with axisymmetry, however, whether or not the magnetosphere is rotating is indistinguishable. Nevertheless, in an oblique rotator, it could generate the lighthouse beacons and magnetic dipole radiation. In this scenario, the rotating magnetic field \( B \) drags the plasma, generating a rigid rotor velocity \( v \). Applying this rotating plasma velocity \( v \) to a stationary frame in space, and considering the magnetic field \( B \) to be stationary, which is indistinguishable from a rotating one, Equation (2) then gives the magnetospheric electric field \( E \) in the stationary frame. We remark that only under the axisymmetric aligned rotator case can the magnetic field be considered either stationary and/or rotating. So, the plasma velocity \( v \) is an input parameter in Equation (2), the driving energy is the rotating magnetic field, and the electric field \( E \) is the response called the rotation–induced electric field. Some authors introduce a non-rotation–induced longitudinal field \( \sim V V \) by taking into consideration the interstellar plasma potential or by considering the centrifugal outflow at the light cylinder (LC; Mestel et al. 1985; Fitzpatrick & Mestel 1988; Goodwin et al. 2004; Timokhin 2006). Other authors have considered a very stringent supply of positive charges from the pulsar surface, probably due to the surface physics of neutron stars, and proposed that vacuum gaps could be formed, separating the charged regions from the pulsar surface and/or separating the charged regions from each other (Ruderman & Sutherland 1975; Jackson 1976; Michel 1979).

In this corotating magnetic field model, the unipolar induction in the neutron star is clearly decoupled from the magnetosphere. The magnetic dipole radiation, the angular momentum outflow of stellar winds, and the current loop in the presence of \( \sim V V \) are attributed to the braking torque on the pulsar indirectly through the general energy conservation principle. However, the use of the energy conservation principle implies a direct coupling with the neutron star, which is not reflected in this scenario. As a matter of fact, if the magnetosphere field lines are anchored on the pulsar surface, the (dipole) magnetic field inside the pulsar will be rotating as well. Consequently, there is no relative rotation between the pulsar and the magnetic field inside. For this reason, there will be no unipolar induction as well.

As a matter of fact, a steady-state magnetosphere decoupled from the pulsar implies that the emission problem of the beacons (Melrose 1978) can be constructed from the equilibrium magnetosphere. Although the validity of separating equilibrium and emission into two different issues cannot be disproved, this approach could have oversimplified the pulsar dynamics. Recently, a time-dependent description of the magnetosphere where the plasma equations are coupled to the Maxwell equations to account for induction effects has been presented. This broader approach has been adopted to study the formation of magnetosphere current sheets (Komissarov 2006; Spitkovsky 2006; Tchekhovskoy et al. 2013) and the linear instabilities in magnetosphere that could be responsible for emissions (Urpin 2012, 2014).

2. NEUTRON STAR–STATIONARY MAGNETOSPHERE COUPLED SYSTEM

To overcome the rotational ambiguities of plasma velocity and magnetic field, we consider the magnetic field \( B \) to be stationary in space, and the electric field \( E \) uploaded to the equipotential field lines of the magnetosphere according to the potential distribution on the pulsar surface established by the unipolar induction. This means that the charge-separated magnetospheric plasma has a negligible contribution to the electric field there. In the case of a stationary magnetic field \( B \), the driving energy is the uploaded electric field \( E \) which is the input parameter in Equation (2), and the plasma velocity \( v \) is the response. The magnetosphere is directly coupled to the unipolar induction of the pulsar. The charge-separation flow inside the pulsar generates a current. The interaction of this current with the magnetic field produces a direct braking torque in addition to the contributions of dipole radiation and stellar winds, which slow down the rotation rate. Finally, we remark that a magnetic field is a real physical quantity, while a magnetic field line is only a physical concept derived from the field line equation to help visualize the action of the magnetic field. It is rather unphysical to consider field lines being rigidly attached to the pulsar surface corotating with it. The notion of a rotating magnetosphere is probably cultivated in the oblique rotator case, where the magnetosphere is seen to rotate obliquely about the pulsar rotational axis. As obliqueness goes to zero, we then have an aligned rotator. For an oblique rotator, we choose to describe the magnetosphere as wobbling (not rotating) about the rotational axis. Consequently, as the obliqueness goes to zero, the magnetosphere stops wobbling and becomes stationary in space.

In this paper, we reconsider the Goldreich & Julian (1969) analysis of charge-separated plasmas. In particular, there are two inconsistencies in their work. The first inconsistency is the space-charge density expression which is incompatible with the curvature of the closed ion magnetic field lines near the pulsar. To resolve this inconsistency, they resorted to a corotating electron cloud above the closed magnetic field lines. The second one is the introduction of the far zone quasi-neutral interstellar plasma potential to the near zone pulsar magnetosphere to draw plasma outflows in the open field lines. The field line that has the potential of the interstellar plasma is the critical field line with no charge flow. This interstellar emf is inconsistent with the \( E_\parallel = 0 \) plasma condition in the magnetosphere. Actually, many publications in recent decades have concentrated on the pulsar equation itself without mentioning either the question of space-charge density or the condition of the critical field line. These simplifications are equivalent to considering the pulsar magnetosphere as filled with a quasi-neutral normal plasma instead of a charge-separated plasma.

The charge-separated plasma in this stationary magnetosphere undergoes an \( E \times B \) drift which happens to have the angular velocity of the pulsar, instead of being dragged along by the corotating magnetosphere. As for the plasma outflow,
we consider it to be driven by plasma drifts, not by interstellar emf. Finally, with a stationary magnetosphere in space, coupled to the unipolar induction in the pulsar interior, we offer an alternative pulsar beacon mechanism by suggesting a pulsating magnetosphere. To understand this dynamic picture, we assume that the unipolar induction is delivering more power to the magnetosphere than the open field lines can drain. This leads to an energy build-up in the closed field lines represented by plasma pressure. Although plasma pressure is neglected in Equation (1), it appears indirectly through plasma drifts, which could open up the closed field lines near LC.

Observationally, radio pulsars are known for their stable repetitive pulse profiles. However, the pulse profile that characterizes each radio pulsar is obtained as the average over many shots. From shot to shot, the profiles are far from identical. Therefore, the period of the oblique rotator is not well defined from shot to shot, which is not supposed to be in the lighthouse scenario. On the other hand, such shot to shot variations are expected in a pulsating magnetosphere.

We review the basic concepts of a charge-separated pulsar magnetosphere and the plasma velocity \( \mathbf{v} \) of Equation (2) in Section 3. The space-charge density \( \rho_p \) is derived in the presence of a poloidal and toroidal field, and in the presence of drift velocities in Section 4. Using the concept of guiding center drifts, the idea of a critical field line is reconsidered in Section 5. We then follow the earlier works of Sturrock (1971), Scharlemann & Wagoner (1973), Okamoto (1974), Beskin & Malyskhin (1998), and Goodwin et al. (2004) to solve the magnetosphere of an isolated pulsar analytically via the force-free pulsar equation. Nevertheless, we take a different approach to split the pulsar equation into two coupled equations through a separating function. Each of the coupled equations is solved by separation of variables for the global structure of the pulsar magnetosphere in Section 6. The standard monopole-like steady-state magnetosphere is presented in Section 7. A self-consistent description of the magnetosphere in terms of fields, velocities, and space-charge densities is outlined in Section 8. Under continuous unipolar pumping, the standard magnetosphere is generalized to the pulsating magnetosphere in Section 9, with conclusions given in Section 10.

### 3. PLASMA VELOCITY

It is important to remark that Equation (2) is written in an inertial frame where the plasma moves with velocity \( \mathbf{v} \) with respect to the stationary magnetic and electric fields. In the plasma moving frame, the Lorentz-transformed electric field is \( \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} \) and Ohm’s law reads \( \mathbf{E}' = 0 \) there. This Ohm’s law in the limit of infinite \( \sigma \) is known as the frozen field condition in MHD plasmas, and happens to be identical to Equation (1), and we have \( \mathbf{E} \cdot \mathbf{v} = 0 \) and \( E_\parallel = 0 \). As a matter of fact, for a charge-separated plasma, Equation (1) and Equation (2) always amount to the same equation. However, Equation (1) describes the magnetosphere in terms of fields, while Equation (2) considers the magnetosphere in terms of plasma velocities. The fields and the velocities are then connected through the space-charge density.

Here we call attention to the fact that the condition of \( E_\parallel = 0 \) allows an arbitrary longitudinal velocity \( v_\parallel \) along the magnetic field line. Consequently, the plasma velocity can be projected in components parallel and perpendicular to the magnetic field as

\[
\mathbf{v} = v_\parallel \mathbf{\hat{v}} + v_\perp \\
v_\perp = v_E = \frac{1}{B^2}(E_\perp \times \mathbf{B}).
\]

The frozen field plasma condition shows that \( v_E \) is given by the \( \mathbf{E} \times \mathbf{B} \) guiding center drift of the gyromotion due to the transverse part of the electric field \( \mathbf{E} \) (Thompson 1962; Schmidt 1966; Jackson 1975), with the constraint \( v_E < c \), which implies that the electric field strength is \( E_\perp < cB \) for a magnetized plasma. This \( v_E \) drift is charge independent, and for a quasi-neutral normal plasma, this drift generates no current and represents only the fluid drift velocity. But for charge-separated plasmas, the \( \mathbf{E} \times \mathbf{B} \) drift corresponds a current flow. Consequently, we have the picture of a stationary pulsar magnetosphere with plasma drifting with respect to the magnetic field.

In cylindrical coordinates \((r, \phi, z)\), the axisymmetric magnetic field can be represented as

\[
\mathbf{B} = A_0 (\nabla \Psi \times \nabla \phi + \alpha A \nabla \phi)
\]

\[
= A_0 \left( -\frac{1}{r} \frac{\partial \Psi}{\partial z}, 1, -\frac{1}{r} \frac{\partial \Psi}{\partial r}, \frac{1}{r} \frac{\partial \Psi}{\partial \phi} \right) \\
\mu_0 J = A_0 \frac{1}{r} \left( -\alpha \frac{\partial A}{\partial \phi}, -\nabla^2 \Psi + \frac{2}{r} \frac{\partial \Psi}{\partial r} + \frac{\partial A}{\partial r} \right). \tag{3}
\]

where \( A_0 \) carries the dimension of the poloidal magnetic flux such that \( \Psi \) is a dimensionless flux function, and \( \alpha = A_\phi / A_0 \) represents the relative ratio of the toroidal flux to the poloidal flux. The function \( A \) is related to the axial current through

\[
2\pi \alpha A_0 A = \mu_0 I_c. \tag{4}
\]

From the magnetic field line equation, it is easy to show that the poloidal field lines are given by the contours of \( \Psi \)

\[
\Psi(r, z) = C. \tag{5}
\]

The fact that the longitudinal electric field vanishes means that any potential difference along the magnetic field line will be shorted out by plasma mobility. With the longitudinal electric field \( E_\parallel = 0 \) and the magnetic field \( \mathbf{B} \) represented by Equation (3), the transverse electric field in the magnetosphere can be expressed through the equipotential field lines of Equation (5) as

\[
E_\perp = -V_0 \nabla \Psi,
\]

where \( V_0 \) is the equator–pole voltage drop on the pulsar surface, and \( \Psi \) is assigned by the corresponding surface potential label.

Alternatively, we can also project the plasma velocity in poloidal and toroidal components by writing \( \mathbf{v} = v_\parallel \mathbf{\hat{v}} + v_\perp \mathbf{B} \) and \( \mathbf{B} = B_\parallel \mathbf{B}_\parallel + B_\perp \mathbf{B}_\perp \). From Equation (2), we get \( 0 = -E_\phi = v_\parallel \times B_\perp \) by axisymmetry, and \( -E_\perp = v_\parallel \times B_\phi + v_\perp \times B_\perp \). We therefore have respectively

\[
v_\parallel = \kappa_\parallel B_\parallel, \tag{6}
\]

\[
-E_\perp = \left( -\kappa_\parallel B_\phi + v_\parallel \right) \times B_\perp. \tag{7}
\]
Loading the potential distribution of the pulsar surface onto the equipotential magnetosphere poloidal field lines, we have

\[ -E_p = r \Omega_0 \mathbf{\hat{B}} \times \mathbf{B}_p, \]

where \( r \) measures the radial position of a point \( r \) in the magnetosphere. Substituting this result to Equation (7) then gives \( v_\phi = \kappa_r \mathbf{B}_p + r \Omega_0 \mathbf{\hat{B}}, \) and therefore

\[ \mathbf{v} = v_p + v_\phi = \kappa_r \mathbf{B} + r \Omega_0 \mathbf{\hat{B}}. \]  

(8)

We note that \( v_\phi \) drifts with the angular velocity of the pulsar. This equation is often interpreted as the “beads on a wire” model, where the plasma (beads) slides with a velocity \( \kappa_r \mathbf{B}_p \) along a rotating \( r \Omega_0 \mathbf{\hat{B}} \) field line (wire). We have rederived this equation with a stationary magnetosphere where the rotation comes from the guiding center drift.

We have expressed the plasma velocity in two orthogonal projections. The first one is with respect to the magnetic field lines, and the second one is with respect to the cylindrical coordinate system. These two projections are independent of each other. Only when the magnetic field is purely poloidal with \( \mathbf{B}_p = 0 \) does the two expressions correspond, and we can identify

\[ v_\parallel = v_p = \kappa_r \mathbf{B}_p, \]
\[ v_\perp = v_\phi = v_E = r \Omega_0 \mathbf{\hat{B}}. \]

4. SPACE-CHARGE DENSITY

In the magnetosphere, the electric field \( \mathbf{E} \) is not the only transverse force that causes plasma drift. Magnetic field line curvature and field gradient are two major transverse forces. Therefore, besides the \( v_E \) drift, we also have the curvature and gradient \( \mathbf{B} \) drifts (Thompson 1962; Schmidt 1966; Jackson 1975) given by

\[ v_{\text{RC}} = + \frac{2 \epsilon_\parallel}{q} \frac{R_c}{R_s^2} \left( \frac{R_c}{R_s} \times \mathbf{B} \right), \]  

(9a)
\[ v_{\text{VB}} = - \frac{\epsilon_\perp}{q} \frac{1}{B^2} \left( \frac{\nabla B}{B} \times \mathbf{B} \right). \]  

(9b)

where \( \epsilon_\parallel = m v_\parallel^2 / 2 \) and \( \epsilon_\perp = m v_\perp^2 / 2 \) are the charge particle energies parallel and perpendicular to the magnetic field. Therefore, these drifts are mass dependent as well as field dependent. In a thermal plasma, these energies are measured by the plasma pressure. Although plasma inertia and pressure are neglected in Equation (1), they appear indirectly through these two drift velocities. The direction of these two drifts depends on the sign of the charge \( q \). Even in a quasi-neutral normal plasma, these are current-generating drifts, while the electric drift \( v_E \) represents the plasma fluid velocity only. In a charge-separated plasma, all three drifts generate currents. These plasma drifts are driven by transverse forces, and they are not emf-driven velocities. To understand this, we recall that for a uniform magnetic field, charge particles follow a circular gyromotion along the magnetic field lines according to the equation of motion due to plasma inertia. In the presence of a force transverse to the magnetic field, e.g., an electric field, a centrifugal force due to curvature, and an inhomogeneous magnetic field, etc., the circular gyromotion will be transformed into a displacing cycloid across the field lines (Thompson 1962; Schmidt 1966; Jackson 1975), such that the gyro guiding center presents a transverse drift. Although Equation (2) gives the guiding center electric drift directly, the gyromotion itself is due to the plasma inertia. Therefore, Equation (2) actually includes the plasma inertia implicitly. We remark that these drift velocities can be very large and become relativistic. Should the electric field \( E \) be time varying, there would be a polarization drift as well, which we will ignore here.

We should point out that in Equation (2) only the \( v_E \) drift is represented on the left side. The other two current-generating drifts, which are contained in \( \mathbf{J} \) on the right side of Equation (2), do not appear in Equation (2) because the plasma conductivity \( \sigma \) is infinite. For this reason, the plasma drift velocity is given by

\[ v_{\text{drift}} = (v_{\text{RC}} + v_{\text{VB}}) + v_E. \]

Since the plasma drift velocities are obtained from the equation of motion of each charged plasma, the plasma drift velocity \( v_{\text{drift}} \) is beyond the scope of Equation (2), as discussed in Section 1, in the sense that the MHD expression of \( \mathbf{v} = v_\parallel + v_E \) differs from \( v_{\text{drift}} \) because \( v_\parallel \neq (v_{\text{RC}} + v_{\text{VB}}) \).

The space-charge density, which connects the fields of Equation (1) and the velocities of Equation (2), in the magnetosphere is, by Equation (2),

\[ \rho_q = -\epsilon_0 \nabla \cdot (v_E \times \mathbf{B}) = \epsilon_0 (v_E \cdot \mu_0 \mathbf{J} - B \cdot \nabla \times v_E). \]

Writing \( \mathbf{J} = \rho_q v_{\text{drift}} = \rho_q (v_E + v_{\text{RC}} + v_{\text{VB}}), \) considering a general poloidal and toroidal fields, and representing \( v_E \) by the projection of Equation (8) to evaluate the \( -B \cdot \nabla \times v_E \) term on the right side, we get

\[ \left[ \rho_q - \epsilon_0 \mu_0 (v_E - \kappa_r \mathbf{B}) \cdot \mathbf{J} \right] = \left[ 1 - \epsilon_0 \mu_0 (v_E - \kappa_r \mathbf{B}) \cdot v_{\text{drift}} \right] \rho_q \]
\[ = -2 \epsilon_0 B \cdot \Omega_0. \]

(10)

In the particular case of a purely poloidal field, we can identify \( v_E = r \Omega_0 \mathbf{\hat{B}} \), and Equation (10) becomes

\[ \left( \rho_q - \epsilon_0 \mu_0 v_E \cdot J \right) = (1 - \epsilon_0 \mu_0 v_E \cdot v_{\text{drift}}) \rho_q \]
\[ = -2 \epsilon_0 B \cdot \Omega_0. \]

Goldreich & Julian (1969) has derived a space-charge density (their Equation (8)) with \( \mathbf{J} = \rho_q v_E \) alone which reads

\[ (1 - \epsilon_0 \mu_0 v_E^2) \rho_q = -2 \epsilon_0 B \cdot \Omega_0. \]

In the absence of \( v_{\text{RC}} \) and \( v_{\text{VB}}, \) the left side of the above equation looks like the relativistic Lorentz factor, which is just coincidental. This equation predicts \( B_c < 0 \) for closed ion field lines, which is consistent only with the outer portion of the dipole-like closed ion field lines. For the inner portion with \( B_c > 0, \) they postulated a negatively charged cloud corotating with the field lines.

We anticipate the result in Section 8 that \( \kappa_r < 0, \) such that the \( (v_E - \kappa_r \mathbf{B}) \) term is positive. Near the pulsar, where the plasma pressure is very high, \( v_{\text{RC}} \) and \( v_{\text{VB}} \) are large on the left
side, and we could have \( B_i > 0 \). Farther out from the pulsar, with plasma pressure decreasing, we have \( B_i < 0 \). Since the \( \nu_{\text{rc}} \) and \( \nu_{\text{vb}} \) drifts are mass dependent, they would be dominant again as the closed field lines approach LC, due to their relativistic inertia. We would have \( B_i > 0 \) on approaching LC. For this reason, the closed region of a steady-state magnetosphere can only extend to some intermediate radius, staying away from LC. Although plasma pressure and plasma inertia are not considered in Equation (1) explicitly, plasma inertia appears implicitly through the guiding center drifts of the gyromotion and plasma pressure through the coefficients of the gradient \( B \) and curvature drifts. We have given a qualitative understanding of these effects in the closed region through the drift velocities, and we will return to this issue in Section 8.

### 5. CRITICAL FIELD LINE

In terms of cylindrical coordinates, the toroidal components of Equation (1), which describes the magnetosphere in terms of fields, give

\[
(\alpha \nabla A \times \nabla \phi) \times (\nabla \psi \times \nabla \phi) = 0, \quad A = A(\psi),
\]

whereas the poloidal components yield the pulsar equation

\[
\left(1 - \left(\frac{r}{r_c}\right)^2\right) \nabla^2 \psi - \frac{2}{r} \frac{\partial \psi}{\partial r} + \alpha^2 A(\psi) \frac{\partial A(\psi)}{\partial \psi} = 0, \quad (12)
\]

where \( r_c = c/\Omega_0 \) is the LC radius. Because of the singular coefficient of the highest derivative which vanishes at \( r = r_c \), Equation (12) is a singular equation. As indicated by Scharlemann & Wagoner (1973) in Section III, the steady-state magnetosphere, which is stationary in space, is solved within LC, and is then analytically continued beyond LC where the pulsar equation is not valid with the tangential velocity \( v_\phi = r \Omega_0 > c \). Since plasma parameters like plasma pressure and velocities are not considered in the pulsar equation, analytic continuation of the field structure beyond LC amounts to solving the pulsar equation in that region with solutions on both sides matched across LC, as done numerically by Contopoulos et al. (1999), Timokhin (2006), and others.

Analytic solutions of this equation have been extensively studied, yet global solutions were not obtained explicitly (Scharlemann & Wagoner1973; Okamoto1974). So far, solutions have been developed by numerical iterations using the boundary condition of a dipole field at the center, notably by Contopoulos et al. (1999), Ogura & Kojima (2003), Gruzinov (2005), and Contopoulos (2005) where the open and closed regions rotate differentially, and Timokhin (2006), where the braking index is evaluated in terms of the magnetosphere dynamics.

In the open field lines, electrons and ions are assumed to stream out along their respective open field lines with equal flux. These currents will be connected to form a return loop at a distant load (interstellar plasma). The electron and ion outflows are thought to be drawn by the action of the far zone quasi-neutral normal interstellar plasma floating potential \( \mathbf{E}_{\text{interp}} = \pm V_{\text{interp}} \mathbf{\nabla} \Phi \), where \( V_{\text{interp}} \) is the potential difference between the far interstellar plasma and the near zone electron or ion open field lines, and \( \Phi \) is a dimensionless scalar potential function. In particular, there will be a field line having the same potential as the interstellar plasma. This is the critical field line \( \Phi_c \), where there is no charge and no poloidal current flow, which separates the electron and ion field lines. This means that \( \partial A(\psi)/\partial \psi = 0 \), and \( A(\psi) \) should be stationary at \( \Phi_c \). This critical field line should pierce LC with \( B_i = 0 \).

Nevertheless, using the interstellar emf to draw current flows in the open field lines is clearly in contradiction to the \( E_i = 0 \) condition. This inconsistency can be removed if we note that the presence of a toroidal and a poloidal magnetic field in the open region can drive a poloidal and a toroidal \( v_\psi \) drift, respectively

\[
v_\psi = v_p + v_\phi = \frac{1}{B^2} E_\perp \times (B_\phi + B_\psi).
\]

The same can be applied to the \( \nu_{\text{rc}} \) and \( \nu_{\text{vb}} \) drifts. All these drift velocities will, therefore, produce a component along the magnetic field lines plus a toroidal rotation, as in Equation (8), and there is no need to introduce either the interstellar potential to the near zone or the concept of a critical field line.

### 6. ANALYTIC PULSAR SOLUTION

By considering the magnetic field \( \mathbf{B} \) to be stationary in space, and the electric field \( \mathbf{E} \) to be uploaded to the field lines from the pulsar surface, our model has the plasma velocity \( \mathbf{v} \) as the response in Equation (2). With \( \mathbf{E} = r \Omega_0 \mathbf{\phi} \), our model simply implies that the electric potential on the pulsar surface cannot be fully uploaded to the magnetosphere to all distances, even beyond LC. This means that the plasma conductivity \( \sigma \) is finite in Equation (2) on approaching LC, and collisional effect, or equivalently, inertia effect, has to be taken into account. The same is true for Equation (1). This should be the subject of further investigations. For the moment, we leave Equation (12) as it is.

To construct the standard magnetosphere, we denote \( \xi = r/r_c \) as the normalized radial coordinate and take a different approach by recasting the pulsar equation as

\[
\nabla^2 \psi = \frac{1}{(1 - \xi^2)} \left[ \frac{1}{r_c^2} \frac{2}{\xi} \frac{\partial \psi}{\partial \xi} - \alpha^2 A(\psi) \frac{\partial A(\psi)}{\partial \psi} \right] = f(\psi),
\]

where we have required both sides of the equality be equal to a separating function \( f(\psi) \). We attempt to solve this equation analytically by choosing

\[
f(\psi) = k^2 \psi,
\]

which renders

\[
\frac{\partial^2 \psi}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \psi}{\partial \xi} + \frac{\partial^2 \psi}{\partial \xi^2} = (k r_c)^2 \psi,
\]

\[
2 \frac{\partial \psi}{\partial \xi} - (\alpha r_c)^2 A(\psi) \frac{\partial A(\psi)}{\partial \psi} = (1 - \xi^2)(k r_c)^2 \psi,
\]

where \( \xi = z/r_c \) is the normalized axial coordinate. We write \( \Psi(\xi, z) = R(\xi) Z(\xi) \) by separation of variables, and Equation (15a) gives

\[
\frac{\partial^2 Z}{\partial \xi^2} - \left( (k r_c)^2 - m^2 \right) Z = 0,
\]
\[
\frac{\partial^2 R}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial R}{\partial \xi} = m^2 R, \tag{16b}
\]

where \(m^2\) is the separation constant. We have denoted the two constants in Equations (14) and (16b) by \(k^2\) and \(m^2\) respectively in quadratic form, by choice. However, \(k^2\) and \(m^2\) are not necessarily positive. As for Equation (15b), we take a linear \(A(\Psi)\) and get

\[
A(\Psi) = -k_z \Psi, \tag{17a}
\]

\[
2 \frac{\partial R}{\xi} = \left[ \left( ak_z r_L \right)^2 + (kr_L)^2 \right] R. \tag{17b}
\]

Here, \(k_z\) has the dimension of an inverse scale length. We have chosen a minus sign explicitly in Equation (17a) to represent an overall downward axial current in the northern polar region. With the understanding that electrons and ions flow along their field lines by plasma drifts, \(A(\Psi)\) does not have to have a stationary point on LC, which allows the linear choice in Equation (17a). Such choice of \(A(\Psi) = -k_z \Psi\) was also used by Scharlemann & Wagneron (1973) and Beskin & Malyshekin (1998). Our analytic approach to the pulsar equation has introduced a separating function \(f(\Psi) = k_z^2 \Psi\), a separating constant \(m^2\), and an axial current function \(A(\Psi) = -k_z \Psi\).

The \(Z(\xi)\) function can be solved readily to render

\[
Z_a(\xi) = C_a e^{-\kappa \xi}, \tag{18a}
\]

\[
Z_b(\xi) = C_b e^{\kappa \xi}, \tag{18b}
\]

with \(\kappa^2 = ((kr_L)^2 - m^2) > 0\), and

\[
Z_a(\xi) = C_a \cos \kappa \xi, \tag{19a}
\]

\[
Z_b(\xi) = C_b \sin \kappa \xi, \tag{19b}
\]

with \(\kappa^2 = -\kappa^2 = -(m^2 - a_{2L}^2) < 0\). To solve for \(R(\xi)\), we make use of Equation (17b) to cast Equation (16b) as

\[
2 \frac{\partial^2 R}{\partial \xi^2} + \left[ \left( (ak_z r_L)^2 + (kr_L)^2 - 2m^2 \right) - (kr_L)^2 \xi^2 \right] R
\]

\[
= 2 \frac{\partial R}{\partial \xi} + \left[ (a_{2L}^2 + a_{2L}^2 - 2m^2) - a_{2L}^2 \xi^2 \right] R
\]

\[
= 2 \frac{\partial R}{\partial \xi} + \left[ a_{2L}^2 - a_{2L}^2 \xi^2 \right] R = 0. \tag{20}
\]

This equation can be solved through a power series. Because of the boundary condition of \(B_{\phi}(r, z) = 0\) on the polar axis \(r = 0\), we should have \(R(\xi) = \xi^2\) as \(\xi\) goes to zero. We therefore have

\[
R(\xi) = \xi^2 \sum a_n \xi^n,
\]

\[
a_2 = -\frac{1}{24} a_{2L}^2 a_0,
\]

\[
a_3 = -\frac{1}{40} a_{2L}^2 a_1, \tag{21}
\]

\[
a_n = -\frac{1}{2(n+2)(n+1)} (a_{2L}^2 a_{n-2} - a_{2L}^2 a_{n-4}). \tag{22}
\]

We note that \(a_0\) and \(a_1\) are the two independent constants of the second order differential equation, Equation (20). Other coefficients are generated by the recursion formula, Equation (22). The coefficients \(a_0\) and \(a_1\) generate even and odd power terms of the series, respectively. Thus \(a_0\) and \(a_1\) are the multipliers of the even and odd series so we can write

\[
R(\xi, a_0, a_1) = a_0 R_{\text{even}}(\xi, 1, 0) + a_1 R_{\text{odd}}(\xi, 0, 1). \tag{23}
\]

7. STANDARD MAGNETOSPHERE

We note that the radial function \(R(\xi)\) is governed by the parameters \((a_{2L}^2, a_{2L}^2)\) and the choice of the two independent coefficients \((a_0, a_1)\). With positive \(a_{2L}^2\) and \(a_{2L}^2\), the coefficients \(a_n\) are alternating in sign, and \(R(\xi)\) could have oscillating solutions for small \(\xi\). This functional form is suitable to construct the open ion field lines and the closed region of the pulsar magnetosphere. As \(\xi\) increases, \(R(\xi)\) becomes monotonically increasing, which is suitable for the split monopole structure far from the pulsar. As for the axial function \(Z_a(\xi)\), it is a decreasing function of \(\xi\). When this is combined with the radial function, together they could generate contours adequate for open field lines. For a given \(a_{2L}^2\), we could scan \(a_{2L}^2\) over a given range to generate a set of field line morphologies. As a result, we represent the electron open field lines by taking the poloidal flux function as

\[
\Psi(\xi, \phi) = R(\xi) Z_a(\xi) = C. \tag{24}
\]

For simplicity, we consider the even terms of the power series by taking \(a_0 = 1\) and \(a_1 = 0\). We choose to set \(a_{2L}^2 = 20\), say, for the radial solution. Taking \(a_{2L}^2 = 8\), the function \(R(\xi)\) is a monotonically increasing function, as is shown in Figure 1, which is appropriate for electron open field lines. We should emphasize that the monotonic functional form of \(R(\xi)\) can be obtained under a wide range of parameters, not restricted to the chosen ones. Furthermore, we set \(\kappa^2 = 5\), for example, and we have the self-consistent parameters \(a_{2L}^2 = 18\), \(m^2 = 3\). With \(\kappa^2 = 5\), the axial function \(Z_a(\xi)\) is shown in Figure 2, and a set of poloidal field lines is shown in Figure 3.

According to Equation (3), the poloidal and toroidal magnetic fields are given by

\[
B_p = A_0 \frac{1}{r_L \xi} \nabla \Psi \times \hat{\phi}, \tag{25a}
\]
\[ \alpha \xi \xi = - \Psi = - \Psi \]

where the last equation gives the ratio of the two fields in the magnetosphere. However, this ratio is expressed in terms of their respective fluxes, \( A_0 \) and \( A_\phi \), without which a specific ratio cannot be determined.

For the ion open field lines, we use the same representation as Equation (24), and keep \( a_\phi^2 = 20 \) and \( \kappa^2 = 5 \). However, we change \( a_\xi^2 = 6 \) and other parameters become \( a_\xi^2 = 16, m^2 = 1 \). By changing \( a_\xi^2 = 8 \) to \( a_\xi^2 = 6 \), the radial function begins to present oscillations for small \( \xi \), which is appropriate for the ion field lines. The functions \( R(\xi) \) and \( Z_a(\xi) \) are given in Figures 4 and 5, respectively. We note that \( R(\xi) \) rises from zero and passes through a maximum and a minimum locally before it takes off monotonically. The local maximum and minimum can become more pronounced by taking a smaller \( a_L^2 \). A set of poloidal field lines is shown in Figure 6 which reflects the monopole-like solution of a charge-separated magnetosphere in the near zone. In the boundary zone, the plasma tends to be quasi-neutral as it proceeds to meet the interstellar plasma in the far zone. The \( \xi < 1 \) part of the open field lines is the solution of the pulsar equation, and the \( \xi > 1 \) part is the analytic continuation across LC.
To construct the closed field lines of the ion plasma, known as the dead zone, we note that, with ion current outflow and electron return current outflow (closed at a distant load) along the open field lines equal in magnitude, \( B_\phi = 0 \) in the closed region which requires \( a_\xi^2 = 0 \). To represent the closed field lines, we need a radial function bounded by a root \( R(\xi) = 0 \) within LC. We also need a bounded axial function. We therefore write

\[
\Psi(\xi, \zeta) = R(\xi)Z'_\zeta(\zeta) = C. \tag{26}
\]

In order to have \( R(\xi) \) bounded by a root within LC, we choose \( a_0 = +1 \) and \( a_1 = -1.5 \) with different signs. Under the condition \( a_\xi^2 = 0 \), we take \( a_\phi^2 = -17 \), \( a_\zeta^2 = 7 \), and \( \kappa^2 = 5 \) \((\kappa^2 = -5)\), we have \( m^2 = 12 \), and the functions \( R(\xi) \) and \( Z'_\zeta(\zeta) \) are shown in Figures 7 and 8, respectively. Once more, recall that these parameters are neither unique nor restrictive ones. There is a good range in parameter space that reproduces the closed region. The boundary condition \( B_\phi = 0 \) on the equatorial plane at \( \zeta = 0 \) is satisfied by \( Z'_\zeta(\zeta) \). A corresponding set of poloidal field lines is illustrated with contour values \( C = 0.015, 0.020, 0.025 \) with the same ordering. These field lines become monopole-like for large \( \xi \).}

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Equation (2). We should note that the force-free approximation is only valid for the field description. As for the velocity-description, plasma inertia and plasma pressure are the prime factors. These two descriptions are then connected by Equation (10) which describes the magnetosphere in terms of space-charge densities. All these three aspects have to be self-consistent with each other. With general poloidal and toroidal magnetic fields, the plasma velocities are given by Equation (8), derived from plasma drifts. As for the space-charge density, it is reflected by the $B \cdot \Omega_0$ on the right side of Equation (10).

Before addressing the consistency of the space-charge density, let us first establish the magnitudes of the $v_{RB}$ and $v_{rB}$ drifts by noting their ratio

$$\frac{v_{rB}}{v_{RB}} = \frac{L}{R_c},$$

where $L$ is the scale length defined by $1/L = \nabla B/B$. Next, to compare with $v_E$, we rewrite $v_E$ in terms of $\Psi$ which reads

$$v_E = \frac{1}{B^2}(E_\perp \times B) = -V_0 \frac{1}{B^2}(\nabla \Psi \times B).$$

For a poloidal magnetic field with $\Psi = C$ representing the field line, $\nabla \Psi$ is directed inward toward the equatorial plane, and $v_E$ corresponds to a toroidal rotation of $v_E = r \dot{\Omega}_0 \hat{\phi}$. For the $v_{rB}$ drift of Equation (9b), we can evaluate $\nabla B$ by using the $\Psi = C$ representation to get

$$\nabla B = A_0 a^2 \nabla \Psi,$$

where $a$ has the dimension of the inverse scale length. Equivalently, $A_0 a^2$ has the dimension of the reference poloidal magnetic field. As a result, Equation (9b) reads

$$v_{rB} = -\frac{c_l}{q} A_0 a^2 \frac{1}{B^2}(\nabla \Psi \times B).$$

We note that this equation is identical in form to Equation (28) for a positive charge $q$. As for the $v_{rC}$ drift of Equation (9a), we note that the radius of curvature $R_c$ is directed from the center of curvature outward to the field lines. Therefore, $R_c$ and $-\nabla \Psi$ are pointing outward, although they are not parallel. Consequently, both $v_{rB}$ and $v_{rC}$ drifts are in the toroidal direction, like $v_E$.

As for their amplitudes, based on Equations (28), (30), and (9a), all three velocities have the same $1/B^2$ scaling, so their strengths can be compared by other factors. For $v_{rC}$, we note that $R_c$ is small when it is near the pulsar and when it is near the apex on returning back to the conjugate point of the southern equatorial region. In between, the field lines are relatively flat, as shown in Figure 9, and $R_c$ is large. As for $v_{rB}$, the ratio of this velocity to $v_E$ is

$$\frac{v_{rC}}{v_{rB}} = \frac{q V_0}{c_l A_0 a^2},$$

Assuming the reference poloidal field $A_0 a^2$ is comparable to $B$, these two velocities become comparable when the plasma energy (pressure) $c_l = q V_0$. The same can also be said for $c_l$ for $v_{rB}$. This high plasma pressure can be found close to the pulsar surface, and it decreases as $r$ increases.

For a toroidal magnetic field in $+\hat{\phi}$ direction, the $v_E$ drift will be poloidal but in the direction opposite the closed field lines, likewise for $v_{rB}$ and $v_{rC}$. From Equation (6), we therefore have $\kappa_\phi < 0$, and the factor $(v_E - \kappa_\phi B)$ in Equation (10) is positive. Should we consider the toroidal magnetic field to be in the $-\hat{\phi}$ direction, we would get $\kappa_\phi > 0$, and the factor $(v_E - \kappa_\phi B)$ would be positive again. We remark that the space-charge density of Equation (10) is valid for the entire magnetosphere, not just for the closed region. In the presence of poloidal and toroidal fields, the scalar product in the square bracket on the right side of Equation (10) has a poloidal and a toroidal contribution. This space-charge density has to be calculated together with the fields and plasma drifts in numerical solutions.
9. PULSATING MAGNETOSPHERE AND MAGNETOSPHERE INSTABILITY

We have constructed the standard magnetosphere in Section 7 where the closed region with $B_\phi = 0$ is described by Equation (26). On the other hand, with the self-consistent description of the magnetosphere based on fields, velocities, and space-charge densities in Section 8, there should be a $B_\phi$ in the closed region. On the question of a toroidal field, we note that, in a charge-separated magnetosphere, a toroidal magnetic field can arise spontaneously. If we begin with a purely poloidal magnetic field in the polar region with open electron field lines, equilibrium condition requires the electron plasma to be at rest. This is an unstable configuration, since any longitudinal movement of the electron plasma corresponds to a poloidal current that would generate a toroidal magnetic field. This toroidal magnetic field would enhance the poloidal drift velocity (current) itself. Since the field lines are open, an equilibrium configuration of fields and currents could be reached. With $\rho_0 < 0$ for electron field lines, and with small $v_\phi$ due to small $\xi$, it is reasonable that the square bracket be positive and thus $B_z > 0$. As for the ion open field lines, the toroidal field here begins to decline due to the ion outflows. With $\rho_0 > 0$ for ion field lines, it is likely that we could have $B_z > 0$ in some parts of the field line and $B_z < 0$ in other parts.

As for the ion field lines in the closed region, there still is a toroidal field due to the imperfect shielding of the electron axial current by the ion outflow in the open region. With $\rho_0 > 0$, and with the presence of a toroidal field and a high plasma pressure near the pulsar, Equation (10) could have $B_z > 0$ near the pulsar. Farther out with large $\xi$, the toroidal field becomes small; likewise, with the plasma pressure, we could expect $B_z < 0$. With the poloidal field lines described by Equation (26), the corresponding toroidal field in the closed region is represented by Equation (25b). Because of the mirror symmetry between the upper (north) and lower (south) hemispheres, the toroidal fields have opposite sign in the two parts, which drives an outward radial current sheet on the equatorial plane. On reaching the apex of the closed region, this current sheet divides into two parts, returning from the northern and southern closed poloidal field lines. We therefore have two current loops, one north and one south, to enhance the corresponding toroidal fields. This in turn enhances the radial current sheet and the current loops. In a closed finite region, this positive feedback cycle can only be stabilized in the presence of dissipations. Naturally, there is the usual toroidal current sheet on the equatorial plane beyond LC for the split monopole poloidal field lines.

To find an alternative beacon mechanism, we first recall that the current model pictures a corotating magnetic field that drags the magnetospheric plasma. In response to this, a rotation-induced electric field is generated in the stationary frame of space. In this picture, the rotating magnetic field is the prime driver, and the magnetosphere is decoupled from the unipolar induction of the pulsar. Here, we view the unipolar induction inside the pulsar and the magnetosphere outside the pulsar as one coupled system. The unipolar induction delivers charges separated by their signs to the polar and equatorial regions at a given rate. The current flow inside the neutron star interacts with the magnetic field, generating a braking torque. The charges are then channeled to the magnetosphere along the field lines, and are ejected into interstellar space as stellar winds through open field lines; this allows the unipolar induction to operate continuously. Considering that the unipolar energy pumping rate exceeds the Poynting outflow rate along the open field lines, plasma density and pressure will build up in the magnetosphere, particularly in the closed region. In the outer part of the closed region extending to intermediate $\xi$ with $B_z < 0$, $v_{Fe}$ and $v_{Re}$ are small. However, as the plasma pressure build-up reaches there, these velocities begin to increase. Eventually, they get to the point where $B_z > 0$, flipping the field lines and releasing the trapped plasma and magnetic energies. Furthermore, the field lines can also be flipped open in the outer part of the closed region by the positive feedback on the toroidal field, which appears in the $(v_{Fe} - \kappa_1B)$ factor of Equation (10).

Once depleted of plasmas, in the sense that the plasma density is so low that Equation (2) is not warranted, the magnetosphere will then be recharged all over to the standard configuration, and the whole cycle starts anew. Instead of a stationary magnetosphere, we therefore have the scenario of a pulsating magnetosphere where the opening of the closed field lines occurs either by unipolar pumping on drift velocities or by positive feedback on the toroidal field working as a magnetic switch, which offers an alternative mechanism for pulsar beacons. In this alternative picture, an observer aligned with the magnetic axis sees the beacons separated by the unipolar induction charging period, instead of the pulsar rotation period.

Here, we have reached the pulsating picture of the magnetosphere through a steady-state approach by viewing the unipolar induction inside the pulsar and the magnetosphere outside as a coupled system. As a matter of fact, the steady-state approach may not be adequate to describe a charge-separated magnetosphere. We call attention to the fact that a charge-separated plasma is a very unstable plasma since any plasma flow (drift) is a current flow generating a magnetic field that drives drifts. For this reason, we might have to let go of the lighthouse paradigm and consider the time-dependent model with Maxwell Equations coupled to the plasma equations (Komissarov 2006; Spitkovsky 2006; Tchekhovskoy et al. 2013; Urpin 2012, 2014). Furthermore, the inherently unstable nature of a charge-separated magnetosphere could be the cause of periodic beaming of electromagnetic waves.

10. CONCLUSIONS

We have interpreted the magnetic field $B$ of the magnetosphere as being stationary in space, and the electric field $E$ as established by uploading the potential distribution of the pulsar surface to the magnetic field lines. The plasma velocity $v$ is, therefore, the drift response in Equation (2), which rotates with the angular velocity of the pulsar. The presence of a magnetosphere allows the pulsar surface charges be channeled to the field lines and keeps the counter-electric field in the pulsar interior low. This warrants the continuous operation of the unipolar induction that constantly pumps energy to the magnetosphere. In this scenario, the energetics of the magnetosphere is coupled to the unipolar induction of the neutron star.

By introducing the gradient $B$ and curvature plasma drifts and the presence of a toroidal field, we have removed the inconsistency between the space-charge density and the curvature of the dipole-like closed ion field lines. Also, the inconsistency between introducing the Far zone interstellar plasma floating potential to the near zone open field lines to generate an emf-driven plasma flow along open field lines and
the $E_{||} = 0$ plasma condition can be removed as well by drift velocities. Although plasma inertia and pressure are neglected in the force-free pulsar equation, they enter the magnetospheric dynamics implicitly through $E \times B$, gradient $B$, and curvature plasma drifts, which are the respective guiding center drifts of the basic gyromotion. Since the gradient $B$ and curvature drifts are mass dependent, the closed region should be bound at a distance within $LC$.

We have devised a method to solve the pulsar equation analytically for the standard split monopole magnetosphere. The solutions are given in terms of three parameters, the separation function parameter $f(\Psi) = k^2 \Psi$, the axial current parameter $A(\Psi) = -k, \Psi$, and the separation constant $m^2$.

Recognizing that the magnetosphere is described by the fields of Equation (1) under force-free approximation and by the plasma drift velocities of Equation (2), where plasma inertia and plasma pressure are prime factors, and these two descriptions are connected by the space-charge densities of Equation (10), we have presented a self-consistent description of the pulsar magnetosphere. Through the coupled system between the unipolar induction inside the neutron star and the magnetosphere outside, and considering the unipolar induction rate exceeds the open field line Poynting flux, we have constructed a dynamic pulsating magnetosphere. In this pulsating model, the accumulated magnetic and plasma energies of the closed field lines can be released periodically by switching open the field lines either as the plasma pressure gets high enough or as the toroidal magnetic field gets large enough. As an alternative to the lighthouse paradigm, this pulsating magnetosphere offers a different mechanism for generating pulsar beacons.

REFERENCES

Beskin, V. S., & Malyshkin, L. M. 1998, MNRAS, 298, 847
Contopoulos, I. 2005, A&A, 442, 579
Contopoulos, I., Kazanas, D., & Fenidzi, C. 1999, ApJ, 511, 351
Fitzpatrick, R., & Mestel, L. 1988, MNRAS, 232, 277
Gold, T. 1968, Natur, 218, 731
Goldreich, P., & Julian, W. H. 1969, ApJ, 157, 869
Goodwin, S. P., Mestel, J., Mestel, L., & Wright, G. A. E. 2004, MNRAS, 349, 213
Gruzinov, A. 2005, PhRvL, 94, 021101
Jackson, J. D. 1975, Classical Electrodynamics (New York: Wiley) Chapter 12
Jackson, E. A. 1976, ApJ, 206, 831
Komissarov, S. S. 2006, MNRAS, 367, 19
Melrose, D. B. 1978, ApJ, 225, 557
Mestel, L., Robertson, J. A., Wang, Y. M., & Westfold, K. C. 1985, MNRAS, 217, 443
Michel, F. C. 1979, ApJ, 227, 579
Ogura, J., & Kojima, Y. 2003, PThPh, 109, 619
Okamoto, I. 1974, MNRAS, 167, 457
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51
Scharlemann, E. T., & Wagener, R. V. 1973, ApJ, 182, 951
Schmidt, G. 1966, Physics of High Temperature Plasmas (New York: Academic) Chapter 2
Spitkovsky, A. 2006, ApJ, 648, 51
Sturrock, P. A. 1971, ApJ, 164, 529
Tchekhovskoy, A., Spitkovsky, A., & Jason, G. L. 2013, MNRAS, 435, 1
Thompson, W. B. 1962, An Introduction to Plasma Physics (London: Addison-Wesley) Chapter 7
Timokhin, A. N. 2006, MNRAS, 368, 1055
Urpin, V. 2012, A&A, 541, A117
Urpin, V. 2014, A&A, 563, A29