MCGDM based on VIKOR and TOPSIS methods using spherical interval valued fuzzy soft with aggregation operators

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Abstract: Spherical interval valued fuzzy soft set (SIVFS set) has much stronger ability than Pythagorean interval valued fuzzy soft set and interval valued intuitionistic fuzzy soft set. Now, we talk about aggregated operation for aggregating SIVFS decision matrix. TOPSIS and VIKOR methods are strong point of view for multi criteria group decision making (MCGDM), which is a various extensions of interval valued fuzzy soft sets. We talk through a score function based on aggregating TOPSIS and VIKOR method to the SIVFS-positive ideal solution and the SIVFS-negative ideal solution. Also TOPSIS and VIKOR methods are provides the weights of decision makings. To find out the optimal alternative under closeness is introduced.

Keywords: spherical interval valued fuzzy soft set, MCGDM, VIKOR, aggregation operator.

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1 Introduction

Decision making (DM) problem indicates the finding of best optional alternatives. Hwang and Yoon [6] was discussed by multiple criteria decision making (MCDM) methods. The matrix form of MCDM problem as:

\[
\begin{array}{c}
\mathcal{A}_1 \quad \mathcal{A}_2 \quad \ldots \quad \mathcal{A}_n \\
\mathcal{B}_1 \\
\mathcal{B}_2 \\
\vdots \\
\mathcal{B}_m \\
\mathcal{P}_{n \times m} = \\
\begin{pmatrix}
x_{11} & x_{12} & \ldots & x_{1m} \\
x_{21} & x_{22} & \ldots & x_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \ldots & x_{nm}
\end{pmatrix}
\end{array}
\]

Here \( \mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \) are called possible alternatives means which decision makers have to choose, \( \mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_m \) are called criteria means which alternative effecting are calculated and \( x_{ij} \) means estimate of \( \mathcal{A}_i \) with respect to \( \mathcal{B}_j \).

These two methods (TOPSIS and VIKOR) for DM problems have been studied by Adeel et al. [1], Akram et al. [2], Boran et al. [4], Eraslan et al. [5], Peng et al. [17], Xu et al. [22] and Zhang et al. [27]. In 2021, Zulqarnain et al. discussed the TOPSIS extends to interval valued intuitionistic fuzzy soft sets (shortly IVIFSS). He also discussed a new type of correlation coefficient under IVIFSS’s [28]. In TOPSIS method consists of distances to positive ideal solution (PIS) and negative ideal solution (NIS), and calculate a preference order is ranked under relative closeness, and finding a combination of these two distance measures. In VIKOR method focal point on ranking and selecting from a set of alternatives, and compute compromise solutions for a problem with inconsistent criteria, which can help the decision makers to get a final decision [14, 15]. Opricovic et al. [16] discussed VIKOR method using fuzzy logic. Tzeng et al. [19] discussion about comparison of VIKOR with TOPSIS methods using public transportation problem.
2 Preliminaries

Definition 2.1 [9] Let \( \mathbb{U} \) be a non-empty set of the universe, spherical interval valued fuzzy set \( X \) in \( \mathbb{U} \) is of the following form: \( \tilde{X} = \left\{ u, (\tilde{\delta}_X(u), \tilde{\eta}_X(u), \tilde{\gamma}_X(u)) | u \in \mathbb{U} \right\} \), where \( \tilde{\delta}_X(u) = [\delta^l_X(u), \delta^u_X(u)] \) and \( \tilde{\eta}_X(u) = [\eta^l_X(u), \eta^u_X(u)] \) and \( \tilde{\gamma}_X(u) = [\gamma^l_X(u), \gamma^u_X(u)] \) represent the degree of positive, neutral and negative-membership of \( X \) respectively. Consider the mapping \( \tilde{\delta}_X : \mathbb{U} \to D[0,1], \tilde{\eta}_X : \mathbb{U} \to D[0,1], \tilde{\gamma}_X : \mathbb{U} \to D[0,1] \) and \( 0 \leq (\tilde{\delta}_X(u))^2 + (\tilde{\eta}_X(u))^2 + (\tilde{\gamma}_X(u))^2 \leq 1 \) means \( 0 \leq (\tilde{\delta}^U_X(u))^2 + (\tilde{\eta}^U_X(u))^2 + (\tilde{\gamma}^U_X(u))^2 \leq 1 \). The degree of refusal is determined as \( \tilde{\pi}_X(u) = [\tilde{\pi}^l_X(u), \tilde{\pi}^U_X(u)] = \left[ \sqrt{1 - (\tilde{\delta}^U_X(u))^2} - (\tilde{\eta}^U_X(u))^2, \sqrt{1 - (\tilde{\delta}^U_X(u))^2} - (\tilde{\eta}^U_X(u))^2 \right] \). Here \( \tilde{X} = ([\delta^l_X, \delta^U_X], [\eta^l_X, \eta^U_X], [\gamma^l_X, \gamma^U_X]) \) is called an interval valued spherical fuzzy number (SIVFN).

Definition 2.2 Let \( \mathbb{U} \) and \( \mathbb{E} \) be the universe and set of parameter respectively. The pair \((\mathbb{T}, X)\) or \( \tilde{\mathbb{T}}_X \) is called a SIVFS set on \( \mathbb{U} \) if \( X \subseteq \mathbb{E} \) and \( \mathbb{T} : X \to SIVF^\mathbb{U} \), where \( SIVF^\mathbb{U} \) is denote the set of all spherical interval valued fuzzy subsets of \( \mathbb{U} \). (ie)

\[ \tilde{\mathbb{T}}_X = \left\{ \left( e, \left\{ \left[ \tilde{\delta}^l_X(u), \tilde{\delta}^U_X(u), \tilde{\eta}^l_X(u), \tilde{\eta}^U_X(u), \tilde{\gamma}^l_X(u), \tilde{\gamma}^U_X(u) \right] \right\} \right) : e \in X, u \in \mathbb{U} \right\} . \]

Remark 2.3 Let \( \tilde{\varphi}_{ij} = \tilde{\delta}^l_X(e_j)(u_i) \), \( \tilde{\varphi}_{ij} = \tilde{\eta}^l_X(e_j)(u_i) \) and \( \tilde{\varphi}_{ij} = \tilde{\gamma}^l_X(e_j)(u_i) \), where \( i \) varies from 1 to \( m \) and \( j \) varies from 1 to \( n \). Then the SIVFS set \( \tilde{\mathbb{T}}_X \) defined in matrix form:

\[ \tilde{\mathbb{T}}_X = \begin{bmatrix} (\tilde{\varphi}_{11}, \tilde{\varphi}_{12}, \tilde{\varphi}_{1n}) & (\tilde{\varphi}_{21}, \tilde{\varphi}_{22}, \tilde{\varphi}_{2n}) & \cdots & (\tilde{\varphi}_{m1}, \tilde{\varphi}_{m2}, \tilde{\varphi}_{mn}) \\ (\tilde{\varphi}_{11}, \tilde{\varphi}_{21}, \tilde{\varphi}_{n1}) & (\tilde{\varphi}_{12}, \tilde{\varphi}_{22}, \tilde{\varphi}_{n2}) & \cdots & (\tilde{\varphi}_{m1}, \tilde{\varphi}_{m2}, \tilde{\varphi}_{mn}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{\varphi}_{11}, \tilde{\varphi}_{21}, \tilde{\varphi}_{n1}) & (\tilde{\varphi}_{12}, \tilde{\varphi}_{22}, \tilde{\varphi}_{n2}) & \cdots & (\tilde{\varphi}_{m1}, \tilde{\varphi}_{m2}, \tilde{\varphi}_{mn}) \end{bmatrix} \]

This matrix is called spherical interval valued fuzzy soft matrix (SIVFSM).

Remark 2.4 Using fundamental operations of arithmetic leads to the following.

(i) \( [u, v] + [w, x] = [u + w, v + x] \)

(ii) \( [u, v] - [w, x] = [u - w, v - x] \)

(iii) \( [u, v] \cdot [w, x] = [uw, vx] \) whenever \( u \geq 0 \) and \( v \geq 0 \)

(iv) \( \frac{1}{[u, v]} = \left[ \frac{1}{v}, \frac{1}{u} \right] \) whenever \( 0 \notin [u, v] \), \( u, v, w, x \in \mathbb{R} \).

3 MCGDM based on SIVFS sets

Definition 3.1 The cardinal set of the SIVFS set \( \tilde{\mathbb{T}}_X \) over \( \mathbb{U} \) is a SIVFS set over \( \mathbb{E} \) and is defined as \( c\tilde{\mathbb{T}}_X = \left\{ \left[ \left[ \delta^l_{\varphi_X}(e), \delta^U_{\varphi_X}(e), \eta^l_{\varphi_X}(e), \eta^U_{\varphi_X}(e), \gamma^l_{\varphi_X}(e), \gamma^U_{\varphi_X}(e) \right] : e \in \mathbb{E} \right\} \right\} = \left\{ \left[ \left[ \delta^l_{\varphi_X}(e), \delta^U_{\varphi_X}(e), \eta^l_{\varphi_X}(e), \eta^U_{\varphi_X}(e), \gamma^l_{\varphi_X}(e), \gamma^U_{\varphi_X}(e) \right] : e \in \mathbb{E} \right\} \right\} \), where \( \delta_{\varphi_X}, \eta_{\varphi_X} \) and \( \gamma_{\varphi_X} : \mathbb{E} \to D[0,1] \) are mapping respectively, where \( \delta_{\varphi_X}(e) = \left[ \delta_X(e) \right] \), \( \eta_{\varphi_X}(e) = \left[ \eta_X(e) \right] \) and \( \gamma_{\varphi_X}(e) = \left[ \gamma_X(e) \right] \) denote the scalar cardinalities of the SIVFS sets \( \varphi_X(e), \eta_X(e) \) and \( \gamma_X(e) \) respectively, and \( \left[ \cdot \right] \) represents cardinality of the universe \( \mathbb{U} \). The collection of all cardinal sets of SIVFS sets of \( \mathbb{U} \) is represented as cSIVF^\mathbb{U}. If \( X \subseteq \mathbb{E} = \left\{ e_i : i = 1, 2, \ldots, n \right\} \), then \( c\tilde{\mathbb{T}}_X \in c\text{SIVF}^\mathbb{U} \) may be represented in matrix form as

\[ \begin{bmatrix} \left[ \left[ p^l_{1i}, p^u_{1i}, q^l_{1i}, q^u_{1i}, r^l_{1i}, r^u_{1i} \right] \right] \\ \vdots \\ \left[ \left[ p^l_{ni}, p^u_{ni}, q^l_{ni}, q^u_{ni}, r^l_{ni}, r^u_{ni} \right] \right] \end{bmatrix} \]
Let \( \mathbf{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) \) be an SIVFS set. Suppose that \( \mathbf{y} = (y_1, y_2, \ldots, y_n) \) such that \( y_i \leq x_i \) for all \( i \). Then \( \mathbf{y} \) is an SIVFS set.

We can make a MCGDM based on SIVFS set by the following algorithms:

**Algorithm-I**

**Step 1:** Form SIVFS set \( \mathbf{y}_X \) over the universal \( \mathbf{u} \).

**Step 2:** Calculate the cardinalities and cardinal set \( \tilde{\mathbf{y}}_X \) of \( \mathbf{y}_X \).

**Step 3:** Compute aggregate SIVFS set \( \mathbf{y}_X \) of \( \mathbf{y}_X \).

**Step 4:** Find the score function \( S_c(u) = \frac{\left( \sum_{i=1}^{n} w_i \tilde{x}_{ij}^2 \right)}{\sum_{i=1}^{n} w_i} \) and \( -1 \leq S_c(u) \leq 1 \), \( u \in \mathbf{u} \).

**Step 5:** Find the best alternative by \( \max S_c(u) \).
Example 3.5 An automobile company produces ten different types of scooter $\mathbb{U} = \{S_1, S_2, \ldots, S_{10}\}$ and five parameters namely $E = \{e_1, e_2, \ldots, e_5\}$ consists of fuel tank capacity, better style, better price, more mileage, more durable respectively. Suppose that a customer has to establish which scooter to be purchased? Each scooter is evaluated and which is a subset of parameters. That is $X = \{e_1, e_2, e_3, e_4\} \subseteq E$. We appeal to algorithm-I as follows.

Step-1: Form SIVFS set $\tilde{\Gamma}_X$ of $\mathbb{U}$ is defined below:

$$\tilde{\Gamma}_X = \left\{ e_1, \begin{array}{ccc} 0.35,0.4 & 0.2,0.25 & 0.35,0.4 \\ 0.3,0.5 & 0.3,0.45 & 0.2,0.25 \\ 0.35,0.35 & 0.25,0.3 & 0.2,0.25 \\ 0.35,0.45 & 0.2,0.25 & 0.35,0.35 \\ 0.35,0.45 & 0.2,0.25 & 0.35,0.35 \\ 0.35,0.4 & 0.2,0.25 & 0.35,0.35 \\ 0.25,0.3 & 0.2,0.25 & 0.35,0.35 \\ 0.25,0.3 & 0.2,0.25 & 0.35,0.35 \\ 0.25,0.3 & 0.2,0.25 & 0.35,0.35 \\ 0.25,0.3 & 0.2,0.25 & 0.35,0.35 
\end{array} \right\}$$

Step-2: The cardinal set of $\tilde{\Gamma}_X$ as $c\tilde{\Gamma}_X = \left\{ (0.185,0.27), (0.19,0.25), (0.205,0.25), (0.155,0.21), (0.185,0.245), (0.155,0.235), (0.205,0.24), (0.195,0.255), (0.18,0.255) \right\}$

Step-3: The aggregate SIVFS set $\bar{\tilde{\Gamma}}_X$ of $\tilde{\Gamma}_X$ is $\bar{\tilde{\Gamma}}_X = \frac{M\tilde{\Gamma}_X \times M\tilde{\Gamma}_X}{|E|}$
Step-3: Compute score value of SIVFS matrix (weights are unequal).

Step-4: The score function \( S_c(S_i) \) as follows.

| Scooter | \( S_c(S_i) \) |
|---------|----------------|
| \( S_1 \) | -0.001973 |
| \( S_2 \) | -0.02583 |
| \( S_3 \) | -0.00994 |
| \( S_4 \) | -0.001815 |
| \( S_5 \) | -0.002391 |
| \( S_6 \) | -0.001575 |
| \( S_7 \) | -0.002583 |
| \( S_8 \) | -0.002468 |
| \( S_9 \) | -0.001724 |
| \( S_{10} \) | -0.002609 |

Step 5: Since \( \max_i S_c(S_i) = -0.00094 \). Hence the customer to be purchased by the scooter \( S_3 \).

Algorithm-II

Step-1: Form spherical interval valued fuzzy soft matrix (SIVFS matrix) on the basis of the parameters.

Step-2: Case-I Obtain the choice matrix for the positive, neutral and negative-membership of SIVFS matrix (weights are equal).

Case-II Find the choice matrix for the positive, neutral and negative-membership of SIVFS matrix (weights are unequal).

Step-3: Compute score value \( S_c(u) = \frac{\sum_{i=1}^{2} \delta^{2L}(\hat{\eta}^{2L}_{U} - \hat{\eta}^{2L}_{L}) + \sum_{i=1}^{2} \delta^{2U}(\hat{\eta}^{2U}_{U} - \hat{\eta}^{2U}_{L})}{2} \), \( -1 \leq S_c(u) \leq 1, u \in \mathbb{U} \).

Step-4: Find the best alternative by \( \max_i S_c(u_i) \).

Case-I: By Example 3.5,

\[
\hat{C}(X) = \left\{ \begin{array}{ccc}
[0.0745, 0.0925] & [0.016, 0.025] & [0.0425, 0.062] \\
[0.044, 0.103] & [0.071, 0.1085] & [0.0485, 0.115] \\
[0.005, 0.0105] & [0.017, 0.026] & [0.0425, 0.064] \\
[0.0045, 0.099] & [0.019, 0.045] & [0.058, 0.099] \\
[0.0205, 0.0615] & [0.0625, 0.099] & [0.019, 0.0665] \\
[0.005, 0.0065] & [0.0445, 0.0965] & [0.068, 0.068] \\
[0.0305, 0.03] & [0.026, 0.0565] & [0.0445, 0.068] \\
[0.0067, 0.1045] & [0.0665, 0.1345] & [0.0405, 0.0645] \\
[0.0305, 0.056] & [0.0565, 0.0905] & [0.056, 0.0845] \\
[0.037, 0.071] & [0.044, 0.1425] & [0.0305, 0.045]
\end{array} \right\}
\]

Score value

\[
\begin{array}{ccc}
\text{Scooter} & \text{\( S_c(S_i) \)} & \\
\hline
S_1 & 0.002348 & \\
S_2 & -0.005923 & \\
S_3 & 0.0053 & \\
S_4 & -0.003956 & \\
S_5 & 0.007144 & \\
S_6 & -0.008206 & \\
S_7 & -0.003521 & \\
e_8 & -0.006452 & \\
S_9 & -0.008882 & \\
S_{10} & -0.009394 & \\
\end{array}
\]

Case-II: Weights \( \tilde{w}_j = \{ [0.16, 0.165], [0.14, 0.145], [0.18, 0.19], [0.17, 0.175], [0.15, 0.155] \} \).
By Example 3.5,
\[
\mathcal{C}_w(X) = \left\{ \begin{array}{c}
[0.0748, 0.0992], \\
[0.0407, 0.1019], \\
[0.0709, 0.1172], \\
[0.0675, 0.111], \\
[0.0193, 0.0611], \\
[0.0495, 0.0705], \\
[0.0385, 0.0536], \\
[0.0708, 0.1195], \\
[0.0316, 0.0621], \\
[0.0355, 0.0728], \\
\end{array} \right\}
\]

Therefore the customer to be purchased by the scooter is $S_3$.

### 3.1 Comparison Analysis for SIVFS-Methods:

Comparison analysis of final ranking as follows:

| Methods | Ranking of alternatives | Optimal alternatives |
|---------|-------------------------|----------------------|
| Algorithm – I | $S_1 \leq S_2 \leq S_3 \leq S_4 \leq S_5 \leq S_6 \leq S_7 \leq S_8 \leq S_9 \leq S_{10}$ | $S_3$ |
| Algorithm – II Case – (i) | $S_1 \leq S_2 \leq S_3 \leq S_4 \leq S_5 \leq S_6 \leq S_7 \leq S_8 \leq S_9 \leq S_{10}$ | $S_3$ |
| Algorithm – II Case – (ii) | $S_1 \leq S_2 \leq S_3 \leq S_4 \leq S_5 \leq S_6 \leq S_7 \leq S_8 \leq S_9 \leq S_{10}$ | $S_3$ |
| Algorithm – III | $S_1 \leq S_2 \leq S_3 \leq S_4 \leq S_5 \leq S_6 \leq S_7 \leq S_8 \leq S_9 \leq S_{10}$ | $S_3$ |

Therefore the customer to be purchased by the scooter is $S_3$.

### 4 MCGDM based on SIVFS-TOPSIS aggregating operator

#### Algorithm-IV (SIVFS-TOPSIS)

**Step-1:** Suppose that the finite decision makers namely $\mathcal{D} = \{\mathcal{D}_i : i \in \mathbb{N}\}$ and the finite collection of alternatives namely $\mathcal{C} = \{\mathcal{C}_i : i \in \mathbb{N}\}$ and finite family of parameters namely $\mathcal{D} = \{\mathcal{D}_i : i \in \mathbb{N}\}$. 

**Step-2:** Compute the score function $\mathcal{S}_c(u) = (\mathcal{S}_c^2 - \mathcal{S}_c^1 - \mathcal{S}_c^2)/2$ and $-1 \leq \mathcal{S}_c(u) \leq 1$, $u \in \mathcal{U}$.

**Step-3:** Find the best alternative by max $\mathcal{S}_c(u_i)$. 

Weights $(\mathcal{w}_j) = \{0.16, 0.165, 0.14, 0.145, 0.18, 0.19, 0.17, 0.175, 0.15, 0.155\}$.
Step-2: Form a linguistic variable with weighted parameter matrix

\[ \mathcal{P} = [\omega_{ij}^{L}, \omega_{ij}^{U}]_{n \times m} = \begin{bmatrix}
[\omega_{11}^{L}, \omega_{11}^{U}] & [\omega_{12}^{L}, \omega_{12}^{U}] & \cdots & [\omega_{1m}^{L}, \omega_{1m}^{U}]
[\omega_{21}^{L}, \omega_{21}^{U}] & [\omega_{22}^{L}, \omega_{22}^{U}] & \cdots & [\omega_{2m}^{L}, \omega_{2m}^{U}]
[\omega_{11}^{L}, \omega_{11}^{U}] & [\omega_{12}^{L}, \omega_{12}^{U}] & \cdots & [\omega_{1m}^{L}, \omega_{1m}^{U}]
\vdots & \vdots & \ddots & \vdots
[\omega_{11}^{L}, \omega_{11}^{U}] & [\omega_{12}^{L}, \omega_{12}^{U}] & \cdots & [\omega_{1m}^{L}, \omega_{1m}^{U}]
\end{bmatrix} \]

Here the weight \(\omega_{ij}\) means \(D_i\) to \(P_j\) by considering linguistic variables.

Step-3: Obtain weighted normalized decision matrix

\[ \tilde{\mathcal{N}} = [\tilde{n}_{ij}^{L}, \tilde{n}_{ij}^{U}]_{n \times m} = \begin{bmatrix}
[\omega_{11}^{L}, \omega_{11}^{U}] & [\omega_{12}^{L}, \omega_{12}^{U}] & \cdots & [\omega_{1m}^{L}, \omega_{1m}^{U}]
[\omega_{21}^{L}, \omega_{21}^{U}] & [\omega_{22}^{L}, \omega_{22}^{U}] & \cdots & [\omega_{2m}^{L}, \omega_{2m}^{U}]
[\omega_{11}^{L}, \omega_{11}^{U}] & [\omega_{12}^{L}, \omega_{12}^{U}] & \cdots & [\omega_{1m}^{L}, \omega_{1m}^{U}]
\vdots & \vdots & \ddots & \vdots
[\omega_{11}^{L}, \omega_{11}^{U}] & [\omega_{12}^{L}, \omega_{12}^{U}] & \cdots & [\omega_{1m}^{L}, \omega_{1m}^{U}]
\end{bmatrix} \]

where \([\tilde{n}_{ij}^{L}, \tilde{n}_{ij}^{U}] = \left[\frac{\omega_{ij}^{L}}{\sqrt{\sum_{i=1}^{n} \omega_{ij}^{L}}}, \frac{\omega_{ij}^{U}}{\sqrt{\sum_{i=1}^{n} \omega_{ij}^{U}}} \right]\) is the normalized parameter and weighted vector

\[ W = (m_{11}^{L}, m_{11}^{U}], m_{12}^{L}, m_{12}^{U}], \ldots, m_{m1}^{L}, m_{m1}^{U} \)

where \(m_{ij}^{L}, m_{ij}^{U}] = \left[\frac{\omega_{ij}^{L}}{\sqrt{\sum_{i=1}^{n} \omega_{ij}^{L}}}, \frac{\omega_{ij}^{U}}{\sqrt{\sum_{i=1}^{n} \omega_{ij}^{U}}} \right]\) is the weight of the \(j^{t}\) parameter and \(\omega_{ij}^{L}, \omega_{ij}^{U}] = \left[\sum_{i=1}^{n} \tilde{n}_{ij}^{L}, \sum_{i=1}^{n} \tilde{n}_{ij}^{U} \right] \]

Step-4: Form SIVFS decision matrix

\[ \mathcal{D}_i = [c_{ij}^{L}, c_{ij}^{U}]_{1 \times m} = \begin{bmatrix}
[c_{11}^{L}, c_{11}^{U}] & [c_{12}^{L}, c_{12}^{U}] & \cdots & [c_{1m}^{L}, c_{1m}^{U}]
[c_{21}^{L}, c_{21}^{U}] & [c_{22}^{L}, c_{22}^{U}] & \cdots & [c_{2m}^{L}, c_{2m}^{U}]
[c_{31}^{L}, c_{31}^{U}] & [c_{32}^{L}, c_{32}^{U}] & \cdots & [c_{3m}^{L}, c_{3m}^{U}]
\vdots & \vdots & \ddots & \vdots
[c_{n1}^{L}, c_{n1}^{U}] & [c_{n2}^{L}, c_{n2}^{U}] & \cdots & [c_{nm}^{L}, c_{nm}^{U}]
\end{bmatrix} \]

Here \([c_{ij}^{L}, c_{ij}^{U}]\) is a SIVFS element for \(i^{t}\) decision maker \([\mathcal{D}_i, \mathcal{D}_j] \) for each \(i\). Find the aggregating matrix by \([\mathcal{X}_i^{L}, \mathcal{X}_i^{U}] = \left[\frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{n} + \frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{n} + \cdots + \frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{n} \right] = [c_{ij}^{L}, c_{ij}^{U}]_{1 \times m} \)

Step-5: Find the decision weighted SIVFS matrix

\[ [\mathcal{Y}_i^{L}, \mathcal{Y}_i^{U}] = \left[\frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{n} \right]_{1 \times m} = \begin{bmatrix}
[c_{11}^{L}, c_{11}^{U}] & [c_{12}^{L}, c_{12}^{U}] & \cdots & [c_{1m}^{L}, c_{1m}^{U}]
[c_{21}^{L}, c_{21}^{U}] & [c_{22}^{L}, c_{22}^{U}] & \cdots & [c_{2m}^{L}, c_{2m}^{U}]
[c_{31}^{L}, c_{31}^{U}] & [c_{32}^{L}, c_{32}^{U}] & \cdots & [c_{3m}^{L}, c_{3m}^{U}]
\vdots & \vdots & \ddots & \vdots
[c_{n1}^{L}, c_{n1}^{U}] & [c_{n2}^{L}, c_{n2}^{U}] & \cdots & [c_{nm}^{L}, c_{nm}^{U}]
\end{bmatrix} \]

Where \([c_{ij}^{L}, c_{ij}^{U}] = [m_{ij}^{L} \times c_{ij}^{L}, m_{ij}^{U} \times c_{ij}^{U}] \).

Step-6: Calculate SIVFSV-PIS and SIVFSV-NIS. Now, SIVFSV-PIS = \( \left[\frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{1}, \frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{2}, \ldots, \frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{1} \right] \)

\[ = \left\{ \bigvee_k \left[ c_{ij}^{L}, c_{ij}^{U} \right], \bigwedge_k \left[ c_{ij}^{L}, c_{ij}^{U} \right], \bigwedge \left[ c_{ij}^{L}, c_{ij}^{U} \right] : k = 1, 2, \ldots, m \right\} \]

and SIVFSV-PIS = \( \left[\frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{1}, \frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{2}, \ldots, \frac{\mathcal{D}_i^{L}, \mathcal{D}_i^{U}}{1} \right] \)
Here \( \mathcal{D} \) the collection of companies/alternatives namely \( \mathcal{L} \) and \( \mathcal{U} \).\n
### Step-7:
Find the SIVFS-Euclidean distances from SIVFSV-PIS and SIVFSV-NIS. Now
\[
\begin{align*}
\left[ d_j^L+, d_j^U+ \right] &= \sqrt{\sum_{k=1}^{m} \left( \left( \delta^L_{jk} - \delta^L_j \right)^2 + \left( \eta^L_{jk} - \eta^L_j \right)^2 + \left( \gamma^L_{jk} - \gamma^L_j \right)^2 \right)}, \\
\left[ d_j^L-, d_j^U- \right] &= \sqrt{\sum_{k=1}^{m} \left( \left( \delta^L_{jk} - \delta^L_j \right)^2 + \left( \eta^L_{jk} - \eta^L_j \right)^2 + \left( \gamma^L_{jk} - \gamma^L_j \right)^2 \right)}
\end{align*}
\]
where \( j = 1, 2, ..., n \).

### Step-8:
Find the closeness of ideal solution by \( C^{L*}(\bar{c}_j), C^{U*}(\bar{c}_j) = \left[ \frac{d_j^L-}{d_j^L+ + d_j^U-}, \frac{d_j^U-}{d_j^L+ + d_j^U-} \right] \) hence \( C^*(\bar{c}_j) = \frac{C^{L*}(\bar{c}_j) + C^{U*}(\bar{c}_j)}{2} \in [0, 1] \).

### Step-9:
Find the rank of alternatives using closeness coefficients under the order of decreasing (or) increasing.

### Step-10:
The conclusion of the best alternative.

**Example 4.1** A company plans to invest some cash in stock exchange by purchasing some shares of best five companies. In order to minimize the factor, they establish to invest their cash percentage of 30, 25, 20, 15 and 10. Find the top five ranked companies.

### Step-1:
A finite set of decision makers namely \( \mathcal{D}^L, \mathcal{D}^U \) = \( \{ \mathcal{D}^L, \mathcal{D}^U \} : i = 1, 2, 3, 4, 5 \} \), the collection of companies/alternatives namely \( \mathcal{C} = \{ \bar{c}_i : i = 1, 2, ..., 10 \} \) and finite family of parameters namely \( \mathcal{E} = \{ e_i : i = 1, 2, ..., 5 \} \), put \( e_1 = \text{Momentum}, e_2 = \text{Value}, e_3 = \text{Growth}, e_4 = \text{Volatility}, e_5 = \text{Quality}. \)

### Step-2:
Obtain weighted parameter matrix under the linguistic variables

| Linguistic variables | Interval valued fuzzy weights |
|----------------------|-----------------------------|
| Very Good Crucial(VGC) | [0.9, 0.95] |
| Good Crucial(GC) | [0.8, 0.9] |
| Average Crucial(AC) | [0.65, 0.8] |
| Poor Crucial(PC) | [0.5, 0.65] |
| Very Poor Crucial(VPC) | [0.35, 0.5] |

Form weighted parameter matrix
\[
\mathcal{P} = [\omega^L_{ij}, \omega^U_{ij}]_{5 \times 5}
\]
\[
\begin{bmatrix}
\omega^L_{ij} & \omega^U_{ij} \\
PC & VGC & VPC & GC & AC & VPC \\
VGC & AC & VGC & PC & VPC & VGC \\
VPC & VGC & AC & GC & PC & VPC \\
VGC & PC & GC & AC & VPC & VGC
\end{bmatrix}
\]
\[
\begin{bmatrix}
[0.5, 0.65] & [0.35, 0.5] & [0.4, 0.95] & [0.35, 0.5] & [0.8, 0.9] \\
[0.65, 0.8] & [0.35, 0.5] & [0.5, 0.65] & [0.9, 0.95] & [0.65, 0.8] \\
[0.9, 0.95] & [0.65, 0.8] & [0.9, 0.95] & [0.5, 0.65] & [0.35, 0.5] \\
[0.35, 0.5] & [0.9, 0.95] & [0.65, 0.8] & [0.8, 0.9] & [0.5, 0.65] \\
[0.9, 0.95] & [0.5, 0.65] & [0.8, 0.9] & [0.65, 0.8] & [0.35, 0.5]
\end{bmatrix}
\]

Here \([\omega^L_{ij}, \omega^U_{ij}]\) means weight of the \( \mathcal{D}_i \) to \( \mathcal{P}_j \).
Step-3: The weighted normalized decision matrix

\[
\mathbf{K} = \left[ \frac{\tilde{v}_{ij} - \tilde{v}_{ij}^{\min}}{\tilde{v}_{ij}^{\max} - \tilde{v}_{ij}^{\min}} \right]_{n \times p}
\]

Weighted vector \( \mathbf{W} \) as

\[
\mathbf{W} = (0.0971, 0.1322, 0.103, 0.1575, 0.092, 0.1182, 0.0968, 0.1366, 0.1027, 0.1641)
\]

Step-4: Obtain the aggregated decision matrix \( \mathbf{X} = [\mathbf{x}^L, \mathbf{x}^U] = \frac{\mathbf{g}_1^L + \mathbf{g}_1^U + \mathbf{g}_2^L + \mathbf{g}_2^U + \cdots + \mathbf{g}_n^L + \mathbf{g}_n^U}{n} \)

Step-5: The weighted decision SIVFS matrix \( \mathbf{Y} = [\mathbf{y}^L, \mathbf{y}^U] = \mathbf{m}_k^L \times \mathbf{c}_j^L, \mathbf{m}_k^U \times \mathbf{c}_j^U \)

Step-6: We find SIVFSV-PI and SIVFSV-NIS can be written as
Step-7: We found SIVFS euclidean distances from SIVFS-PI and SIVFS-NIS.

\[
\begin{array}{c|c|c}
\text{Alternative} & \mathcal{e}_i^+ & \mathcal{e}_i^- \\
\hline
(\mathcal{e}_1^+, \mathcal{e}_1^-) & (0.0867, 0.1183) & (0.0741, 0.1202) \\
(\mathcal{e}_2^+, \mathcal{e}_2^-) & (0.0655, 0.0867) & (0.0632, 0.0772) \\
(\mathcal{e}_3^+, \mathcal{e}_3^-) & (0.0543, 0.0804) & (0.0619, 0.0651) \\
(\mathcal{e}_4^+, \mathcal{e}_4^-) & (0.0859, 0.1054) & (0.1003, 0.1018) \\
(\mathcal{e}_5^+, \mathcal{e}_5^-) & (0.0535, 0.1126) & (0.0658, 0.1144) \\
(\mathcal{e}_6^+, \mathcal{e}_6^-) & (0.0798, 0.1267) & (0.0732, 0.1309) \\
(\mathcal{e}_7^+, \mathcal{e}_7^-) & (0.0457, 0.1117) & (0.0604, 0.1048) \\
(\mathcal{e}_8^+, \mathcal{e}_8^-) & (0.0671, 0.0854) & (0.0448, 0.0811) \\
(\mathcal{e}_9^+, \mathcal{e}_9^-) & (0.0504, 0.0722) & (0.0556, 0.0481) \\
(\mathcal{e}_{10}^+, \mathcal{e}_{10}^-) & (0.0544, 0.0772) & (0.0596, 0.1037) \\
\end{array}
\]

Step-8: We calculate closeness coefficients from SIVFS-PI and SIVFS-NIS.

\[
\begin{array}{c|c|c}
\text{Alternative} (\mathcal{e}_i) & \mathcal{e}_i^{L^+}, \mathcal{e}_i^{U^+} & \mathcal{e}_i^+ \\
\hline
\mathcal{e}_1 & (0.3106, 0.7475) & 0.5291 \\
\mathcal{e}_2 & (0.3979, 0.5613) & 0.4796 \\
\mathcal{e}_3 & (0.4254, 0.5603) & 0.4929 \\
\mathcal{e}_4 & (0.4841, 0.5469) & 0.5155 \\
\mathcal{e}_5 & (0.2807, 0.9589) & 0.6243 \\
\mathcal{e}_6 & (0.2841, 0.8553) & 0.5907 \\
\mathcal{e}_7 & (0.2849, 0.9465) & 0.6157 \\
\mathcal{e}_8 & (0.2906, 0.7023) & 0.4964 \\
\mathcal{e}_9 & (0.3650, 0.7558) & 0.5604 \\
\mathcal{e}_{10} & (0.3295, 0.9095) & 0.6195 \\
\end{array}
\]

Step-9: The order of the alternatives for \( \mathcal{e}_i^+ \) is \( \mathcal{e}_5 \geq \mathcal{e}_{10} \geq \mathcal{e}_7 \geq \mathcal{e}_6 \geq \mathcal{e}_9 \geq \mathcal{e}_1 \geq \mathcal{e}_4 \geq \mathcal{e}_8 \geq \mathcal{e}_3 \geq \mathcal{e}_2 \).

![Graphical representation using MCGDM based on TOPSIS.](image)

Step-10: The above ranking, it conclude that the company \( \mathcal{e}_5 \) invest 30\%, \( \mathcal{e}_{10} \) invest 25\%, \( \mathcal{e}_7 \) invest 20\%, \( \mathcal{e}_6 \) invest 15\% and \( \mathcal{e}_9 \) invest 10\%.

5 MCGDM based on SIVFS-VIKOR aggregating operator
Algorithm-V (SIVFS-VIKOR)

**Step-1:** Suppose that the finite decision makers namely \( D = \{ D_i : i \in \mathbb{N} \} \) and the finite collection of alternatives namely \( C = \{ c_i : i \in \mathbb{N} \} \) and finite family of parameters namely \( D = \{ e_i : i \in \mathbb{N} \} \).

**Step-2:** Form a linguistic variables with obtain weighted parameter matrix

\[
\mathcal{P} = \left[ \omega^L_{ij}, \omega^U_{ij} \right]_{n \times m} = \begin{bmatrix}
[\omega^L_{11}, \omega^U_{11}] & [\omega^L_{12}, \omega^U_{12}] & \cdots & [\omega^L_{1m}, \omega^U_{1m}] \\
[\omega^L_{21}, \omega^U_{21}] & [\omega^L_{22}, \omega^U_{22}] & \cdots & [\omega^L_{2m}, \omega^U_{2m}] \\
\vdots & \vdots & \ddots & \vdots \\
[\omega^L_{n1}, \omega^U_{n1}] & [\omega^L_{n2}, \omega^U_{n2}] & \cdots & [\omega^L_{nm}, \omega^U_{nm}] 
\end{bmatrix}
\]

Here the weight \( \left[ \omega^L_{ij}, \omega^U_{ij} \right] \) means \( D_i \) to \( P_j \).

**Step-3:** Form weighted normalized decision matrix

\[
\mathcal{N} = \left[ \tilde{\tilde{n}}^L_{ij}, \tilde{\tilde{n}}^U_{ij} \right]_{n \times m} = \begin{bmatrix}
[\tilde{\tilde{n}}^L_{11}, \tilde{\tilde{n}}^U_{11}] & [\tilde{\tilde{n}}^L_{12}, \tilde{\tilde{n}}^U_{12}] & \cdots & [\tilde{\tilde{n}}^L_{1m}, \tilde{\tilde{n}}^U_{1m}] \\
[\tilde{\tilde{n}}^L_{21}, \tilde{\tilde{n}}^U_{21}] & [\tilde{\tilde{n}}^L_{22}, \tilde{\tilde{n}}^U_{22}] & \cdots & [\tilde{\tilde{n}}^L_{2m}, \tilde{\tilde{n}}^U_{2m}] \\
\vdots & \vdots & \ddots & \vdots \\
[\tilde{\tilde{n}}^L_{n1}, \tilde{\tilde{n}}^U_{n1}] & [\tilde{\tilde{n}}^L_{n2}, \tilde{\tilde{n}}^U_{n2}] & \cdots & [\tilde{\tilde{n}}^L_{nm}, \tilde{\tilde{n}}^U_{nm}] 
\end{bmatrix}
\]

Here \( \left[ \tilde{\tilde{n}}^L_{ij}, \tilde{\tilde{n}}^U_{ij} \right] \) is the normalized parameter and weighted vector

\[
\mathcal{W} = \left[ \sum_{i=1}^{n} \tilde{\tilde{n}}^L_{i1}, \sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i1} \right], \left[ \sum_{i=1}^{n} \tilde{\tilde{n}}^L_{i2}, \sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i2} \right], \ldots, \left[ \sum_{i=1}^{n} \tilde{\tilde{n}}^L_{im}, \sum_{i=1}^{n} \tilde{\tilde{n}}^U_{im} \right], \left[ \frac{\omega^L_{11} \sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^L_{i1}}}{\sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i1}}} \right], \left[ \frac{\omega^U_{11} \sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i1}}}{\sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i1}}} \right]
\]

is the weight of the \( j^{th} \) parameter and \( \left[ \omega^L_{ij}, \omega^U_{ij} \right] = \left[ \frac{\omega^L_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^L_{i1}}}, \frac{\omega^U_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i1}}} \right] \) is the weight of the \( j^{th} \) parameter and \( \left[ \omega^L_{ij}, \omega^U_{ij} \right] = \left[ \frac{\omega^L_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^L_{i1}}}, \frac{\omega^U_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{\tilde{n}}^U_{i1}}} \right] \) is the weight of the \( j^{th} \) parameter.

**Step-4:** Form decision SIVFS matrix

\[
\mathcal{D}_i = \left[ c^L_{ik}, c^U_{ik} \right]_{1 \times m} = \begin{bmatrix}
[\dot{c}^L_{11}, \dot{c}^U_{11}] & [\dot{c}^L_{12}, \dot{c}^U_{12}] & \cdots & [\dot{c}^L_{1m}, \dot{c}^U_{1m}] \\
[\dot{c}^L_{21}, \dot{c}^U_{21}] & [\dot{c}^L_{22}, \dot{c}^U_{22}] & \cdots & [\dot{c}^L_{2m}, \dot{c}^U_{2m}] \\
\vdots & \vdots & \ddots & \vdots \\
[\dot{c}^L_{n1}, \dot{c}^U_{n1}] & [\dot{c}^L_{n2}, \dot{c}^U_{n2}] & \cdots & [\dot{c}^L_{nm}, \dot{c}^U_{nm}] 
\end{bmatrix}
\]

Here \( \left[ c^L_{ik}, c^U_{ik} \right] \) means \( i^{th} \) decision maker \( \left[ \mathcal{D}_i, \mathcal{D}_i^U \right] \) for each \( i \). Then obtain the aggregating matrix

\[
\mathcal{D}^L = \mathcal{D}^L_1 + \mathcal{D}^L_2 + \ldots + \mathcal{D}^L_n = \left[ \frac{\dot{c}^L_{jk}, \dot{c}^U_{jk}}{m} \right]_{1 \times m}.
\]

**Step-5:** Find the weighted decision SIVFS matrix

\[
\mathcal{Y} = \left[ \dot{c}^L_{jk}, \dot{c}^U_{jk} \right]_{1 \times m} = \begin{bmatrix}
[c^L_{11}, c^U_{11}] & [c^L_{12}, c^U_{12}] & \cdots & [c^L_{1m}, c^U_{1m}] \\
[c^L_{21}, c^U_{21}] & [c^L_{22}, c^U_{22}] & \cdots & [c^L_{2m}, c^U_{2m}] \\
\vdots & \vdots & \ddots & \vdots \\
[c^L_{n1}, c^U_{n1}] & [c^L_{n2}, c^U_{n2}] & \cdots & [c^L_{nm}, c^U_{nm}]
\end{bmatrix}
\]

Where \( \left[ c^L_{jk}, c^U_{jk} \right] = \left[ m^L_{jk} \times c^L_{jk}, m^U_{jk} \times c^U_{jk} \right] \).

**Step-6:** Calculate SIVFS-PIS and SIVFS-NIS. Now,
\[
SIVFSV-PIS = \left\{ \left[ \frac{p_1^{+}}{c_1^{-}}, \frac{p_1^{-}}{c_1^{+}} \right], \left[ \frac{p_2^{+}}{c_2^{-}}, \frac{p_2^{-}}{c_2^{+}} \right], ..., \left[ \frac{p_l^{+}}{c_l^{-}}, \frac{p_l^{-}}{c_l^{+}} \right] \right\}
\]

and \(SIVFSV-PIS = \left\{ \left[ \frac{p_1^{+}}{c_1^{-}}, \frac{p_1^{-}}{c_1^{+}} \right], \left[ \frac{p_2^{+}}{c_2^{-}}, \frac{p_2^{-}}{c_2^{+}} \right], ..., \left[ \frac{p_l^{+}}{c_l^{-}}, \frac{p_l^{-}}{c_l^{+}} \right] \right\}\).

**Step-7:** Find the values of utility \( S_L^i, S_U^i \), individual regret \( R_L^i, R_U^i \) and compromise \( Q_L^i, Q_U^i \),

where \( Q = S_L^i + S_U^i \), where \( S_L^i, S_U^i \) and \( R_L^i, R_U^i \) are calculated as follows.

**Step-8:** Obtain the ranks of choices and derive the compromise solution. Arrange \( \bar{Q} \) in increasing order to make ranking list. The alternative \( \bar{c}_a \) will be declared compromise solution if it ranks the best (having least value) in \( \bar{Q} \), and the following two conditions satisfies simultaneously:

**C1** admissible: If \( \bar{c}_a \) and \( \bar{c}_\beta \) represent top alternatives in \( \bar{Q} \), then \( \bar{Q}(\bar{c}_\beta) - \bar{Q}(\bar{c}_a) \geq \frac{1}{n-1} \), where \( n \) is the counting of parameters.

**C2** admissible: The alternative \( \bar{c}_a \) should be best ranked by \( S_L^i, S_U^i \) and compromise \( R_L^i, R_U^i \).

If **C1** and **C2** are not satisfied simultaneously, then there exist multiple compromise solutions:

(i) If \( C1 \) is satisfied, then the alternatives \( \bar{c}_a \) and \( \bar{c}_{\beta} \) are called compromise solutions:

(ii) If \( C1 \) is not satisfied, then the alternatives \( \bar{c}_a, \bar{c}_{\beta}, ..., \bar{c}_\zeta \) are called the compromise solutions, where \( \bar{c}_\zeta \) is found by \( \bar{Q}(\bar{c}_\zeta) - \bar{Q}(\bar{c}_a) \geq \frac{1}{n-1} \).

**Example 5.1** Now, we can appeal Example 4.1 to Algorithm-V. We conclude that the first five steps are the same process. Hence we solve the problem using VIKOR method entering Step 6.

**Step-6:** Find SIVFSV-PIS and SIVFSV-NIS are listed as follows.

| \( S_L^i, S_U^i \) | SIVFSV-PIS | \( S_L^i, S_U^i \) | SIVFSV-NIS |
|---------------------|-----------|---------------------|-----------|
| \( [c_+^{+}, c_+^{-}] \) | \( [0.0534, 0.0994], [0.0194, 0.0542], [0.0262, 0.0377] \) | \( [c_+^{+}, c_+^{-}] \) | \( [0.0252, 0.0568], [0.0544, 0.0805], [0.0505, 0.0793] \) |
| \( [c_+^{+}, c_+^{-}] \) | \( [0.0587, 0.1024], [0.0299, 0.0614], [0.0185, 0.0409] \) | \( [c_+^{+}, c_+^{-}] \) | \( [0.0237, 0.0552], [0.0567, 0.1024], [0.0567, 0.1024] \) |
| \( [c_+^{+}, c_+^{-}] \) | \( [0.0586, 0.0804], [0.0239, 0.0426], [0.0285, 0.0447] \) | \( [c_+^{+}, c_+^{-}] \) | \( [0.0258, 0.0449], [0.0497, 0.065], [0.0561, 0.0839] \) |
| \( [c_+^{+}, c_+^{-}] \) | \( [0.0474, 0.0847], [0.0136, 0.0464], [0.0145, 0.0451] \) | \( [c_+^{+}, c_+^{-}] \) | \( [0.0252, 0.0382], [0.0474, 0.0778], [0.0649, 0.0929] \) |
| \( [c_+^{+}, c_+^{-}] \) | \( [0.0534, 0.0968], [0.0277, 0.0673], [0.0277, 0.0644] \) | \( [c_+^{+}, c_+^{-}] \) | \( [0.0195, 0.0476], [0.0627, 0.1051], [0.0647, 0.1083] \) |

**Step-7:** Taking \( \kappa = 0.5 \), we found that the values of utility \( S_L^i, S_U^i \), individual regret \( R_L^i, R_U^i \) and compromise \( Q_i \) for each alternative \( [c_1^{+}, c_1^{-}] \).
The authors declare no conflict of interest.

Conflicts of Interest

Some technique. Also we have inserted various sorts of statistical charts to image the rankings operator. Again we interact SIVFS aggregation operator and score function values based on last two algorithms follow by SIVFS linguistic TOPSIS and VIKOR approaches under aggregation function. We can finding ranking of values using an aggregating function. The best VIKOR utilize to linear normalization approach. The major difference between two methods looks in 7 Conclusion:

Comparison and discussion

These two methods are assume a scalar component for each criterion and these two methods are different from normalization approach. In TOPSIS utilize to vector normalization approach and VIKOR utilize to linear normalization approach. The major difference between two methods looks in the aggregation function. We can finding ranking of values using an aggregating function. The best ranked alternative by VIKOR is closest to the ideal solution. However, the best ranked alternative by TOPSIS is the best using ranking index, but doesn’t closest to the ideal solution. Hence advantage of VIKOR gives to be compromise solution.

6 Comparison and discussion

Step-8: The rank of alternatives for $Q_i: \bar{c}_9 \leq \bar{c}_1 \leq \bar{c}_4 \leq \bar{c}_7 \leq \bar{c}_{10} \leq \bar{c}_8 \leq \bar{c}_6 \leq \bar{c}_2$.

Now, $Q(\bar{c}_1) - Q(\bar{c}_9) = 0.0794 \geq \frac{1}{4}$. Thus C1 is false, further more $Q(\bar{c}_5) - Q(\bar{c}_9) = 0.4178 \geq \frac{1}{4}$.

Therefore, we establish $\bar{c}_9, \bar{c}_1, \bar{c}_4, \bar{c}_7, \bar{c}_5$ are multiple compromise solutions. Hence the company should invest 30% on $\bar{c}_9$, 25% on $\bar{c}_1$, 20% on $\bar{c}_4$, 15% on $\bar{c}_7$ and 10% on $\bar{c}_5$.

6 Comparison and discussion

These two methods are assume a scalar component for each criterion and these two methods are different from normalization approach. In TOPSIS utilize to vector normalization approach and VIKOR utilize to linear normalization approach. The major difference between two methods looks in the aggregation function. We can finding ranking of values using an aggregating function. The best ranked alternative by VIKOR is closest to the ideal solution. However, the best ranked alternative by TOPSIS is the best using ranking index, but doesn’t closest to the ideal solution. Hence advantage of VIKOR gives to be compromise solution.

7 Conclusion:

In this present communication, the first three algorithms follow by MCGDM under SIVFS and last two algorithms follow by SIVFS linguistic TOPSIS and VIKOR approaches under aggregation operator. Again we interact SIVFS aggregation operator and score function values based on some technique. Also we have inserted various sorts of statistical charts to image the rankings of alternatives under consideration.

Conflicts of Interest

The authors declare no conflict of interest.
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