CP-VIOLATION AND MIXING IN CHARMED MESONS

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The Standard Model contribution to $D_0 - \bar{D}_0$ mixing is dominated by the contributions of light $s$ and $d$ quarks. Neglecting the tiny effects due to $b$ quark, both mass and lifetime differences vanish in the limit of $SU(3)_F$ symmetry. Thus, the main challenge in the Standard Model calculation of the mass and width difference in the $D_0 - \bar{D}_0$ system is to estimate the size of $SU(3)$ breaking effects. We prove that $D$ meson mixing occurs in the Standard Model only at second order in $SU(3)$ violation. We consider the possibility that phase space effects may be the dominant source of $SU(3)$ breaking. We find that $y = (\Delta \Gamma)/(2\Gamma)$ of the order of one percent is natural in the Standard Model, potentially reducing the sensitivity to new physics of measurements of $D$ meson mixing. We also discuss the possibility of observing lifetime differences and CP violation in charmed mesons both at the currently operating and proposed facilities.

1. Introduction

One of the most important motivations for studies of weak decays of charmed mesons is the possibility of observing a signal from new physics which can be separated from the one generated by the Standard Model (SM) interactions. The low energy effect of new physics particles can be naturally written in terms of a series of local operators of increasing dimension generating $\Delta C = 1$ (decays) or $\Delta C = 2$ (mixing) transitions. For $D^0 - \bar{D}^0$ mixing these operators, as well as the one loop Standard Model effects, generate contributions to the effective operators that change $D^0$ state into $\bar{D}^0$ state leading to the mass eigenstates

$$|D_2\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle,$$

where the complex parameters $p$ and $q$ are obtained from diagonalizing the $D^0 - \bar{D}^0$ mass matrix. The mass and width splittings between these
eigenstates are parameterized by

\[ x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \tag{2} \]

where \( m_{1,2} \) and \( \Gamma_{1,2} \) are the masses and widths of \( D_{1,2} \) and the mean width and mass are \( \Gamma = (\Gamma_1 + \Gamma_2)/2 \) and \( m = (m_1 + m_2)/2 \). Since \( y \) is constructed from the decays of \( D \) into physical states, it should be dominated by the Standard Model contributions, unless new physics significantly modifies \( \Delta C = 1 \) interactions. On the contrary, \( x \) can receive contributions from all energy scales, so it is usually conjectured that new physics can significantly modify \( x \) leading to the inequality \( x \gg y \). As we discuss later, this signal for new physics is lost if a relatively large \( y \), of the order of a percent, is observed. It is known experimentally that \( D^0 - \bar{D}^0 \) mixing proceeds extremely slow, which in the Standard Model is usually attributed to the absence of superheavy quarks destroying GIM cancellations\(^1\).

Another possible manifestation of new physics interactions in the charm system is associated with the observation of (large) CP-violation. This is due to the fact that all quarks that build up the hadronic states in weak decays of charm mesons belong to the first two generations. Since \( 2 \times 2 \) Cabbibo quark mixing matrix is real, no CP-violation is possible in the dominant tree-level diagrams that describe the decay amplitudes. In the Standard Model CP-violating amplitudes can be introduced by including penguin or box operators induced by virtual \( b \)-quarks. However, their contributions are strongly suppressed by the small combination of CKM matrix elements \( V_{cb}V_{ub}^* \). It is thus widely believed that the observation of (large) CP violation in charm decays or mixing would be an unambiguous sign for new physics. This fact makes charm decays a valuable tool in searching for new physics, since the statistics available in charm physics experiment is usually quite large.

As in B-physics, CP-violating contributions in charm can be generally classified by three different categories: (I) CP violation in the decay amplitudes. This type of CP violation occurs when the absolute value of the decay amplitude for \( D \) to decay to a final state \( f \) (\( A_f \)) is different from the one of corresponding CP-conjugated amplitude (“direct CP-violation”); (II) CP violation in \( D^0 - \bar{D}^0 \) mixing matrix. This type of CP violation is manifest when \( R^2_m = |p/q|^2 = (2M_{12} - i\Gamma_{12})/(2M_{12}^* - i\Gamma_{12}^*) \neq 1 \); and (III) CP violation in the interference of decays with and without mixing. This type of CP violation is possible for a subset of final states to which both \( D^0 \) and \( \bar{D}^0 \) can decay.
For a given final state \( f \), CP violating contributions can be summarized in the parameter
\[
\lambda_f = \frac{q \overline{A}_f}{p A_f} = R_m e^{i(\phi + \delta)} \left| \frac{A_f}{\overline{A}_f} \right|
\]  
(3)
where \( A_f \) and \( \overline{A}_f \) are the amplitudes for \( D^0 \rightarrow f \) and \( \overline{D}^0 \rightarrow f \) transitions respectively and \( \delta \) is the strong phase difference between \( A_f \) and \( \overline{A}_f \). Here \( \phi \) represents the convention-independent weak phase difference between the ratio of decay amplitudes and the mixing matrix.

2. Present and perspective experimental constraints

Presently, experimental information about the \( D^0 - \overline{D}^0 \) mixing parameters \( x \) and \( y \) comes from the time-dependent analyses that can roughly be divided into two categories. First, more traditional studies look at the time dependence of \( D \rightarrow f \) decays, where \( f \) is the final state that can be used to tag the flavor of the decayed meson. The most popular is the non-leptonic doubly Cabibbo suppressed decay (DCSD) \( D^0 \rightarrow K^+\pi^- \). Time-dependent studies allow one to separate the DCSD from the mixing contribution \( D^0 \rightarrow \overline{D}^0 \rightarrow K^+\pi^- \),
\[
\Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |A_{K^+\pi^-}|^2
\times \left[ R + \sqrt{R} R_m (y'\cos \phi - x'\sin \phi) \Gamma t + \frac{R_m^2}{4} (y^2 + x^2)(\Gamma t)^2 \right]
\]  
(4)
where \( R \) is the ratio of DCS and Cabibbo favored (CF) decay rates. Since \( x \) and \( y \) are small, the best constraint comes from the linear terms in \( t \) that are also linear in \( x \) and \( y \). A direct extraction of \( x \) and \( y \) from Eq. (4) is not possible due to unknown relative strong phase \( \delta \) of DCS and CF amplitudes\(^2\), as \( x' = x \cos \delta + y \sin \delta \), \( y' = y \cos \delta - x \sin \delta \). This phase can be measured independently\(^3\). The corresponding formula can also be written\(^4\) for \( \overline{D}^0 \) decay with \( x' \rightarrow -x' \) and \( R_m \rightarrow R_m^{-1} \).

Second, \( D^0 \) mixing can be measured by comparing the lifetimes extracted from the analysis of \( D \) decays into the CP-even and CP-odd final states. This study is also sensitive to a linear function of \( y \) via
\[
\frac{\tau(D \rightarrow K^-\pi^+)}{\tau(D \rightarrow K^+K^-)} - 1 = y \cos \phi - x \sin \phi \left[ \frac{R_m^2 - 1}{2} \right]
\]  
(5)
Time-integrated studies of the semileptonic transitions are sensitive to the quadratic form \( x^2 + y^2 \) and at the moment are not competitive with the analyses discussed above.
The construction of a new tau-charm factory at Cornell (CLEO-c) will introduce new \textit{time-independent} methods that are sensitive to a linear function of \( y \). In particular, one can use the fact that heavy meson pairs produced in the decays of heavy quarkonium resonances have the useful property that the two mesons are in the CP-correlated states\(^5\). By tagging one of the mesons as a CP eigenstate, a lifetime difference may be determined by measuring the leptonic branching ratio of the other meson. The initial \( D^0\bar{D}^0 \) state is prepared as
\[
|D^0\bar{D}^0\rangle_L = \frac{1}{\sqrt{2}} \left\{ |D^0(k_1)\bar{D}^0(k_2)\rangle + (-1)^L |D^0(k_2)\bar{D}^0(k_1)\rangle \right\} ,
\]
where \( L \) is the relative angular momentum of two \( D \) mesons. There are several possible resonances at which CLEO-c will be running, for example \( \psi(3770) \) where \( L = 1 \) and the initial state is antisymmetric, or \( \psi(4114) \) where the initial \( D^0\bar{D}^0 \) state can be symmetric due to emission of additional pion or photon in the decay. In this scenario, the CP quantum numbers of the \( D(k_2) \) can be determined. The semileptonic width of this meson should be independent of the CP quantum number since it is flavor specific. It follows that the semileptonic \textit{branching ratio} of \( D(k_2) \) will be inversely proportional to the total width of that meson. Since we know whether \( D(k_2) \) is tagged as a (CP-eigenstate) \( D^+ \) or \( D^- \) from the decay of \( D(k_1) \) to \( S^\sigma \), we can easily determine \( y \) in terms of the semileptonic branching ratios of \( D^\pm \). This can be expressed simply by introducing the ratio
\[
R^L_\sigma = \frac{\Gamma[\psi_L \to (H \to S^\sigma)(H \to X\nu\bar{\nu})]}{\Gamma[\psi_L \to (H \to S^\sigma)(H \to X)]} \frac{\text{Br}(H^0 \to X\nu\bar{\nu})}{\text{Br}(H^0 \to X\nu)} ,
\]
where \( X \) in \( H \to X \) stands for an inclusive set of all final states. A deviation from \( R^L_\sigma = 1 \) implies a lifetime difference. Keeping only the leading (linear) contributions due to mixing, \( y \) can be extracted from this experimentally obtained quantity,
\[
y \cos \phi = (-1)^L \sigma \frac{R^L_\sigma}{R^L_\sigma} - 1 .
\]

The current experimental upper bounds on \( x \) and \( y \) are on the order of a few times \( 10^{-2} \), and are expected to improve significantly in the coming years. To regard a future discovery of nonzero \( x \) or \( y \) as a signal for new physics, we would need high confidence that the Standard Model predictions lie well below the present limits. As was recently shown\(^6\), in the Standard Model \( x \) and \( y \) are generated only at second order in \( SU(3) \) breaking,
\[
x, \ y \sim \sin^2 \theta_C \times [SU(3) \text{ breaking}]^2 ,
\]
where $\theta_C$ is the Cabibbo angle. Therefore, predicting the Standard Model values of $x$ and $y$ depends crucially on estimating the size of $SU(3)$ breaking. Although $y$ is expected to be determined by the Standard Model processes, its value nevertheless affects significantly the sensitivity to new physics of experimental analyses of $D$ mixing.

Theoretical calculations of $x$ and $y$, as will be discussed later, are quite uncertain, and the values near the current experimental bounds cannot be ruled out. Therefore, it will be difficult to find a clear indication of physics beyond the Standard Model in $D^0 - \bar{D}^0$ mixing measurements alone. The only robust potential signal of new physics in charm system at this stage is CP violation.

CP violation in $D$ decays and mixing can be searched for by a variety of methods. For instance, time-dependent decay widths for $D \to K \pi$ are sensitive to CP violation in mixing (see Eq.(4)). Provided that the $x$ and $y$ are comparable to experimental sensitivities, a combined analysis of $D \to K \pi$ and $D \to K K$ can yield interesting constraints on CP-violating parameters.

Most of the techniques that are sensitive to CP violation make use of the decay asymmetry,

$$A_{CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})} = \frac{1 - |A_f/A_{\bar{f}}|^2}{1 + |A_f/A_{\bar{f}}|^2}. \quad (10)$$

Most of the properties of Eq.(10), such as dependence on the strong final state phases, are similar to the ones in B-physics. Current experimental bounds from various experiments, all consistent with zero within experimental uncertainties, can be found in Ref. 8.

Other interesting signals of CP-violation that are being discussed in connection with tau-charm factory measurements are the ones that are using quantum coherence of the initial state. An example of this type of signal is a decay $(D^0 \bar{D}^0) \to f_1 f_2$ at $\psi(3770)$ with $f_1$ and $f_2$ being the different final CP-eigenstates with $CP|f_1\rangle = CP|f_2\rangle$. This type of signals are very easy to detect experimentally. It is easy to compute this CP-violating decay rate for the final states $f_1$ and $f_2$

$$\Gamma_{f_1 f_2} = \frac{2 + x^2 - y^2}{2R_m^2(1 + x^2)(1 - y^2)} |\lambda_{f_1} - \lambda_{f_2}|^2 \Gamma_f \Gamma_{\bar{f}}. \quad (11)$$

The result of Eq. (11) represents a generalization of the formula given in Ref. 9. It is clear that both terms in the numerator of Eq. (11) receive
contributions from CP-violation of the type I and III, while the second term is also sensitive to CP-violation of the type II. Moreover, for a large set of the final states the first term would be additionally suppressed by SU(3) symmetry. For instance, \( \lambda_{\pi\pi} = \lambda_{KK} \) in the SU(3) symmetry limit.

It is easy to see that only the second term survives if only CP violation in the mixing matrix is retained,

\[
\Gamma_{f_1f_2} \propto |1 - R_m^2|^2 \propto A_m^2.
\]

This expression is of the second order in CP-violating parameters. As it follows from the existing experimental constraints on rate asymmetries, CP-violating phases are quite small in charm system, regardless of whether they are produced by the Standard Model mechanisms or by some new physics contributions. In that respect, it looks unlikely that the SM signals of CP violation would be observed at CLEO-c with this observable.

While the searches for direct CP violation via the asymmetry of Eq. (10) can be done with the charged D-mesons (which are self-tagging), investigations of the other two types of CP-violation require flavor tagging of the initial state. This severely cuts the available dataset. It is therefore interesting to look for signals of CP violation that do not require identification of the initial state. One possible CP-violating signal involves the observable obtained by summing over the initial states, \( \sum \Gamma_i = \Gamma_i + \Gamma_i \) for \( i = f, \bar{f} \). A CP-odd observable that can be formed out of \( \sum \Gamma_i \) is an asymmetry

\[
A_{CP}^U = \frac{\sum \Gamma_f - \sum \Gamma_\bar{f}}{\sum \Gamma_f + \sum \Gamma_\bar{f}}.
\]  

(12)

Note that this asymmetry does not require quantum coherence of the initial state and therefore is accessible in any D-physics experiment. The final states must be chosen such that \( A_{CP}^U \) is not trivially zero. As we shall see below, decays of \( D \) into the final states that are CP-eigenstates would result in zero asymmetry, while the final states like \( K^+K^- \) or \( K_S\pi^+\pi^- \) would not. A non-zero value of \( A_{CP}^U \) in Eq. (12) can be generated by both direct and indirect CP-violating contributions. These can be separated by appropriately choosing the final states. For example, indirect CP violating amplitudes are tightly constrained in the decays dominated by the Cabibbo-favored tree level amplitudes, while singly Cabibbo suppressed amplitudes also receive contributions from direct CP violating amplitudes. Neglecting small CP-violation in the mixing matrix (\( R_m \to 1 \)) one obtains,

\[
A_{CP}^U = \frac{\Gamma_f - \Gamma_\bar{f} - \Gamma_f + \Gamma_\bar{f}}{\Gamma_f + \Gamma_\bar{f} + \Gamma_f + \Gamma_\bar{f}} + \frac{2y}{\Gamma_f + \Gamma_\bar{f} + \Gamma_f + \Gamma_\bar{f}}
\]

\[
\times \left[ \cos \phi \left( Re A_f \bar{A}_f - Re A_\bar{f} A_f \right) + \sin \phi \left( Im A_f \bar{A}_f + Im A_\bar{f} A_f \right) \right]
\]

(13)
It is easy to see that, as promised, this asymmetry vanishes for the final states that are CP-eigenstates, as $\Gamma_f = \Gamma_f^\tau$ and $\Gamma_f - \Gamma_f^\tau = \Gamma_f^\tau - \Gamma_f$.

3. Theoretical expectations for mixing parameters

Theoretical predictions of $x$ and $y$ within and beyond the Standard Model span several orders of magnitude. Roughly, there are two approaches, neither of which give very reliable results because $m_c$ is in some sense intermediate between heavy and light. The “inclusive” approach is based on the operator product expansion (OPE). In the $m_c > \Lambda$ limit, where $\Lambda$ is a scale characteristic of the strong interactions, $\Delta M$ and $\Delta \Gamma$ can be expanded in terms of matrix elements of local operators. Such calculations yield $x, y < 10^{-3}$. The use of the OPE relies on local quark-hadron duality, and on $\Lambda/m_c$ being small enough to allow a truncation of the series after the first few terms. The charm mass may not be large enough for these to be good approximations, especially for nonleptonic $D$ decays. An observation of $y$ of order $10^{-2}$ could be ascribed to a breakdown of the OPE or of duality, but such a large value of $y$ is certainly not a generic prediction of OPE analyses. The “exclusive” approach sums over intermediate hadronic states, which may be modeled or fit to experimental data. Since there are cancellations between states within a given $SU(3)$ multiplet, one needs to know the contribution of each state with high precision. However, the $D$ is not light enough that its decays are dominated by a few final states. In the absence of sufficiently precise data on many decay rates and on strong phases, one is forced to use some assumptions. While most studies find $x, y < 10^{-3}$, Refs. obtain $x$ and $y$ at the $10^{-2}$ level by arguing that $SU(3)$ violation is of order unity, but the source of the large $SU(3)$ breaking is not made explicit.

In what follows we first prove that $D^0 - \bar{D}^0$ mixing arises only at second order in $SU(3)$ breaking effects. The proof is valid when $SU(3)$ violation enters perturbatively. This would not be so, for example, if $D$ transitions were dominated by a single narrow resonance close to threshold. Then we argue that reorganization of “exclusive” calculation by explicitly building $SU(3)$ cancellations into the analysis naturally leads to values of $y \sim 1\%$ if only one source of $SU(3)$ breaking (phase space) is taken into account.

The quantities $M_{12}$ and $\Gamma_{12}$ which determine $x$ and $y$ depend on matrix elements $\langle \bar{D}^0 | H_w H_w | D^0 \rangle$, where $H_w$ denote the $\Delta C = -1$ part of the weak Hamiltonian. Let $D$ be the field operator that creates a $D^0$ meson and annihilates a $\bar{D}^0$. Then the matrix element, whose $SU(3)$ flavor group
theory properties we will study, may be written as

$$\langle 0 \rangle D H_w H_w D \langle 0 \rangle.$$ (14)

Since the operator $D$ is of the form $\bar{c}u$, it transforms in the fundamental representation of $SU(3)$, which we will represent with a lower index, $D_i$. We use a convention in which the correspondence between matrix indexes and quark flavors is $(1, 2, 3) = (u, d, s)$. The only nonzero element of $D_i$ is $D_1 = 1$. The $\Delta C = -1$ part of the weak Hamiltonian has the flavor structure $(\bar{q}_i c)(\bar{q}_j q_k)$, so its matrix representation is written with a fundamental index and two antifundamentals, $H^{ij}_k$. This operator is a sum of irreps contained in the product $3 \times 3 \times 3 = \overline{15} + 6 + \overline{3}$. In the limit in which the third generation is neglected, $H^{ij}_k$ is traceless, so only the $\overline{15}$ and $6$ representations appear. That is, the $\Delta C = -1$ part of $H_w$ may be decomposed as $\frac{1}{2}(O_{\overline{15}} + O_6)$, where

$$O_{\overline{15}} = (\bar{s}c)(\bar{u}d) + (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) + s_1(\bar{u}c)(\bar{d}d) - s_1(\bar{s}c)(\bar{u}s) - s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) - s_1^2(\bar{u}c)(\bar{d}s),$$

$$O_6 = (\bar{s}c)(\bar{u}d) - (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) - s_1(\bar{u}c)(\bar{d}d) - s_1(\bar{s}c)(\bar{u}s) + s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) + s_1^2(\bar{u}c)(\bar{d}s),$$ (15)

and $s_1 = \sin \theta_C$. The matrix representations $H(\overline{15})^{ij}_k$ and $H(6)^{ij}_k$ have nonzero elements

$$H(\overline{15})^{ij}_k: \quad H^{13}_2 = H^{31}_2 = 1, \quad H^{12}_2 = H^{21}_2 = s_1, \quad H^{13}_3 = H^{31}_3 = -s_1, \quad H^{12}_3 = H^{21}_3 = -s_1^2,$$

$$H(6)^{ij}_k: \quad H^{13}_2 = -H^{31}_2 = 1, \quad H^{12}_2 = -H^{21}_2 = s_1, \quad H^{13}_3 = -H^{31}_3 = -s_1, \quad H^{12}_3 = -H^{21}_3 = -s_1^2.$$ (16)

We introduce $SU(3)$ breaking through the quark mass operator $M$, whose matrix representation is $M^i = \text{diag}(m_u, m_d, m_s)$ as being in the adjoint representation to induce $SU(3)$ violating effects. We set $m_u = m_d = 0$ and let $m_s \neq 0$ be the only $SU(3)$ violating parameter. All nonzero matrix elements built out of $D_i$, $H^{ij}_k$ and $M^i$ must be $SU(3)$ singlets.

We now prove that $D^0 - \overline{D^0}$ mixing arises only at second order in $SU(3)$ violation, by which we mean second order in $m_s$. First, we note that the pair of $D$ operators is symmetric, and so the product $D_i D_j$ transforms as a 6 under $SU(3)$. Second, the pair of $H_w$’s is also symmetric, and the product $H^{ij}_k H^{jm}_n$ is in one of the reps which appears in the product

$$[\{\overline{15} + 6\} \times \{\overline{15} + 6\}]_S = (\overline{15} \times \overline{15})_S + (\overline{15} \times 6) + (6 \times 6)_S$$

$$= (66 + 2\overline{15} + 15 + 15' + \overline{3}) + (42 + 24 + 15 + \overline{3} + 3) + (15' + \overline{3}).$$ (17)
A direct computation shows that only three of these representations actually appear in the decomposition of $H_w H_w$. They are the $\overline{60}$, the $42$, and the $15'$ (actually twice, but with the same nonzero elements both times). So we have product operators of the form (the subscript denotes the representation of $SU(3)$)

$$D D = D_6, \quad H_w H_w = O_{\overline{60}} + O_{42} + O_{15'}.$$  \hspace{1cm} (18)

Since there is no $\overline{6}$ in the decomposition of $H_w H_w$, there is no $SU(3)$ singlet which can be made with $D_6$, and no $SU(3)$ invariant matrix element of the form (14) can be formed. This is the well known result that $D^0 - \overline{D^0}$ mixing is prohibited by $SU(3)$ symmetry. Now consider a single insertion of the $SU(3)$ violating spurion $M$. The combination $D_6 M$ transforms as $6 \times 8 = 24 + \overline{15} + 6 + \overline{3}$. There is still no invariant to be made with $H_w H_w$, thus $D^0 - \overline{D^0}$ mixing is not induced at first order in $SU(3)$ breaking. With two insertions of $M$, it becomes possible to make an $SU(3)$ invariant. The decomposition of $D M M$ is

$$6 \times (8 \times 8)_S = 6 \times (27 + 8 + 1)$$

$$= (60 + 1\overline{2} + 24 + \overline{15} + \overline{15'} + 6) + (24 + \overline{15} + 6 + \overline{3}) + 6.$$  \hspace{1cm} (19)

There are three elements of the $6 \times 27$ part which can give invariants with $H_w H_w$. Each invariant yields a contribution to $D^0 - \overline{D^0}$ mixing proportional to $s_1^2 m_s^2$. Thus, $D^0 - \overline{D^0}$ mixing arises only at second order in the $SU(3)$ violating parameter $m_s$.

We now turn to the contributions to $y$ from on-shell final states, which result from every common decay product of $D^0$ and $\overline{D^0}$. In the $SU(3)$ limit, these contributions cancel when one sums over complete $SU(3)$ multiplets in the final state. The cancellations depend on $SU(3)$ symmetry both in the decay matrix elements and in the final state phase space. While there are $SU(3)$ violating corrections to both of these, it is difficult to compute the $SU(3)$ violation in the matrix elements in a model independent manner. Yet, with some mild assumptions about the momentum dependence of the matrix elements, the $SU(3)$ violation in the phase space depends only on the final particle masses and can be computed. We estimate the contributions to $y$ solely from $SU(3)$ violation in the phase space. We find that this source of $SU(3)$ violation can generate $y$ of the order of a few percent.

The mixing parameter $y$ may be written in terms of the matrix elements for common final states for $D^0$ and $\overline{D^0}$ decays,

$$y = \frac{1}{\Gamma} \sum_n \int [P.S.]_n \langle D^0 | H_w | n \rangle \langle n | H_w | D^0 \rangle ,$$  \hspace{1cm} (20)
where the sum is over distinct final states \( n \) and the integral is over the phase space for state \( n \). Let us now perform the phase space integrals and restrict the sum to final states \( F \) which transform within a single \( SU(3) \) multiplet \( R \). The result is a contribution to \( y \) of the form

\[
\frac{1}{\Gamma(D^0)} H_w \left\{ \eta_{CP}(F_R) \sum_{n \in F_R} |n\rangle \rho_n(n) \right\} H_w |D^0\rangle ,
\]

(21)

where \( \rho_n \) is the phase space available to the state \( n \), \( \eta_{CP} = \pm \frac{1}{6} \). In the \( SU(3) \) limit, all the \( \rho_n \) are the same for \( n \in F_R \), and the quantity in braces above is an \( SU(3) \) singlet. Since the \( \rho_n \) depend only on the known masses of the particles in the state \( n \), incorporating the true values of \( \rho_n \) in the sum is a calculable source of \( SU(3) \) breaking.

This method does not lead directly to a calculable contribution to \( y \), because the matrix elements \( \langle n|H_w|D^0\rangle \) and \( \langle D^0|H_w|n\rangle \) are not known. However, \( CP \) symmetry, which in the Standard Model and almost all scenarios of new physics is to an excellent approximation conserved in \( D \) decays, relates \( \langle D^0|H_w|n\rangle \) to \( \langle D^0|H_w|\bar{n}\rangle \). Since \( |n\rangle \) and \( |\bar{n}\rangle \) are in a common \( SU(3) \) multiplet, they are determined by a single effective Hamiltonian. Hence the ratio

\[
y_{F,R} = \frac{\sum_{n \in F_R} \langle D^0|H_w|n\rangle \rho_n(n) \langle n|H_w|D^0\rangle}{\sum_{n \in F_R} \langle D^0|H_w|n\rangle \rho_n(n) \langle n|H_w|D^0\rangle} = \frac{\sum_{n \in F_R} \langle D^0|H_w|n\rangle \rho_n(n) \langle n|H_w|D^0\rangle}{\sum_{n \in F_R} \Gamma(D^0 \rightarrow n)}
\]

(22)

is calculable, and represents the value which \( y \) would take if elements of \( F_R \) were the only channel open for \( D^0 \) decay. To get a true contribution to \( y \), one must scale \( y_{F,R} \) to the total branching ratio to all the states in \( F_R \). This is not trivial, since a given physical final state typically decomposes into a sum over more than one multiplet \( F_R \). The numerator of \( y_{F,R} \) is of order \( s_i^2 \) while the denominator is of order 1, so with large \( SU(3) \) breaking in the phase space the natural size of \( y_{F,R} \) is 5%. Indeed, there are other \( SU(3) \) violating effects, such as in matrix elements and final state interaction phases. Here we assume that there is no cancellation with other sources of \( SU(3) \) breaking, or between the various multiplets which occur in \( D \) decay, that would reduce our result for \( y \) by an order of magnitude. This is equivalent to assuming that the \( D \) meson is not heavy enough for duality to enforce such cancellations. Performing the computations of \( y_{F,R} \), we see\(^6\) that effects at the level of a few percent are quite generic. Our results are summarized in Table 1. Then, \( y \) can be formally constructed from
the individual $y_{F,R}$ by weighting them by their $D^0$ branching ratios,

$$y = \frac{1}{\Gamma} \sum_{F,R} y_{F,R} \left[ \sum_{n \in F_R} \Gamma(D^0 \to n) \right].$$

(23)

However, the data on $D$ decays are neither abundant nor precise enough to disentangle the decays to the various $SU(3)$ multiplets, especially for the three- and four-body final states. Nor have we computed $y_{F,R}$ for all or even most of the available representations. Instead, we can only estimate individual contributions to $y$ by assuming that the representations for which we know $y_{F,R}$ to be typical for final states with a given multiplicity, and then to scale to the total branching ratio to those final states. The total branching ratios of $D^0$ to two-, three- and four-body final states can be extracted from the Review of Particle Physics\textsuperscript{14}. Rounding to the nearest 5\% to emphasize the uncertainties in these numbers, we conclude that the branching fractions for $PP$, $(VV)_s\text{-wave}$, $(VV)_d\text{-wave}$ and $3P$ approximately amount to 5\%, while the branching ratios for $PV$ and $4P$ are of the order of 10\%\textsuperscript{6}.

We observe that there are terms in Eq. (23), like nonresonant $4P$, which could make contributions to $y$ at the level of a percent or larger. There, the rest masses of the final state particles take up most of the available energy, so phase space differences are very important. One can see that $y$ on the order of a few percent is completely natural, and that anything an order of magnitude smaller would require significant cancellations which do not appear naturally in this framework. Cancellations would be expected only if they were enforced by the OPE, or if the charm quark were heavy enough that the “inclusive” approach were applicable. The hypothesis underlying the present analysis is that this is not the case.

4. Conclusions

We proved that if $SU(3)$ violation may be treated perturbatively, then $D^0 - \overline{D}^0$ mixing in the Standard Model is generated only at second order in $SU(3)$ breaking effects. Within the exclusive approach, we identified an $SU(3)$ breaking effect, $SU(3)$ violation in final state phase space, which can be calculated with minimal model dependence. We found that phase space effects alone provide enough $SU(3)$ violation to induce $y \sim 10^{-2}$. Large effects in $y$ appear for decays close to $D$ threshold, where an analytic expansion in $SU(3)$ violation is no longer possible.

Indeed, some degree of cancellation is possible between different multiplets, as would be expected in the $m_c \to \infty$ limit, or between $SU(3)$
breaking in phase space and in matrix elements. It is not known how effective these cancellations are, and the most reasonable assumption in light of our analysis is that they are not significant enough to result in an order of magnitude suppression of $y$, as they are not enforced by any symmetry arguments. Therefore, any future discovery of a $D$ meson width difference should not by itself be interpreted as an indication of the breakdown of the Standard Model.

At this stage the only robust potential signal of new physics in charm system is CP violation. We discussed several possible experimental observables that are sensitive to CP violation.
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