Testing weak form informational efficiency on the Tunisian stock market using long memory models

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ABSTRACT

The purpose of this paper is to test the weak-form market efficiency of the Tunisian stock market using recent developments in time series econometrics. The efficiency hypothesis was tested by using the class of long memory models namely ARFIMA-FIGARCH. For this, we will attempt to examine the long memory behavior in the returns and the volatility series of the Tunisian stock market index namely Tunindex. Our empirical study covers a sample covering the Tunindex during the period: 02/01/1998 to 16/03/2018. Our results show the presence of the long memory property in the return and volatility specified respectively by an ARFIMA and FIGARCH process. This result implies that it is possible to predict future stock prices and an extraordinary gain could be obtained when trading in this market, which displays that the Tunisian stock market is not efficient in its weak-form.

Keywords: long memory, ARFIMA, FIGARCH, heteroscedasticity, efficiency
1. Introduction
The efficiency of financial markets poses the problem of the integration of information into prices which are considered as non-biased signals reflecting that information. (FAMA, 1970), defines an efficient market as a market in which prices reflect simultaneously and completely all available and relevant information: the best predictor of future price is the price of today. This definition assumes the nullity of information and transaction costs. The process specifying a testable hypothesis of this version of efficiency is the random walk on log prices. Therefore, the concept of market efficiency is related to the absence of memory: the efficiency hypothesis is then associated with the random walk model. Stock prices follow a random walk, and returns are white noise: the observed price fluctuates randomly around its fundamental value. In fact, Fama has shown in 1970 then in 1991 that short memory does not affect the efficiency assumption that since some significant short-term autocorrelations cannot be used to speculate. However, the presence of long memory poses more problems. In 1986, (SUMMER 1986) studied in detail the property of the mean reversion in price: after a shock, the price deviates from its fundamental value but always ends up returning. This property implies the possible existence of a gap between the market price and fundamental value. However, if the gap is sustainable, this can be reflected by the presence of a long memory. This idea will go in contrast with the property of efficiency.

We propose first to continue the work of (MEESE et al, 1983) by studying the dynamics of returns series through an ARFIMA modeling (Auto Regressive Integrated Moving Average fractionally). Second, we extend the study to the volatility by taking a fractional GARCH modeling. Such modeling is intended to display the long memory phenomena in time series. The purpose of this work is thus to determine whether the explicit modeling of the long term dynamics in the return price series provides better forecasts than a simple random walk model.

The study of long-term dependence structures in the financial time series has been the subject of many contributions. (Granger and Joyeux 1980) introduced the ARFIMA processes which are known to be capable of modeling long-run persistence in financial time series; they generalize Box-Jenkins models, when the order d of differencing is allowed to be fractional. Bollerslev and Mikkelsen (1996) apply FIEGARCH model for long memory of volatility of S&P500 returns in U.S. stock market. Huang and Yang (1999) confirm the presence of long memory in the NYSE and NASDAQ indexes by using a modified R/S technique. Resende and Teixeira (2002) investigate the existing of long memory property in Brazilian Stock Market by using ARFIMA model. They show that Brazilian Stock Market is efficient. Henry (2002) finds that Germany, Japan, South Korea and Taiwan Stock Markets have long memory and these markets are inefficient. Kumar (2004) displays that India Stock Market has long memory due to the presence of conditional heteroscedasticity in returns by using ARFIMA-GARCH models. Vougas (2004) finds that Athens Stock Market is an efficient market by using ARFIMA model. Kilç (2004) reveals that daily returns of Turkish Stock Market are not characterized by long memory by using FIGARCH model. Assaf and Cavalcante (2005) cannot find any evidence of long memory when they investigate long memory in return and volatility of Brazilian Stock Market. Thupayagale (2010) displays mix evidence of long memory property for 11 African stock markets. Macheshchandra (2012) examines the existence of long memory in the Indian stock market by using ARFIMA-FIGARCH models. He finds that ARFIMA model implies the absence of long memory in return series, however, FIGARCH model shows strong evidence of long memory in volatility of stock returns. Lopez-Herrera et al. (2012) investigate long memory behavior in the returns of the Mexican Stock Market by using ARFIMA mod-
The authors find that the Mexican Stock Market has long memory and it is weak-form inefficient.

2. Fractionally integrated processes
Granger (1980), Granger and Joyeux (1980) and Hosking (1981) suggested an ARFIMA approach to model financial time series characterized by long memory processes.

**The ARFIMA process**
An ARFIMA \((p, d, q)\) model is specified as follows:

\[
\Phi(L)(1-L)^d (y_t - \mu) = \Theta(L)\epsilon_t
\]

where:

\[
\Phi(L) = 1-\varphi_1 L - \ldots - \varphi_p L^p
\]

\[
\Theta(L) = 1-\theta_1 L - \ldots - \theta_q L^q
\]

\(d\) is the fractional integration parameter \((0 < d < 1)\) defined by:

\[
(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(1-k)} = 1-dL - \frac{d(d+1)L^2}{2!} - \frac{d(d+1)(d+2)L^3}{3!} + \ldots
\]

\(\Gamma(.)\) is the Gamma function:

\[
\Gamma(g) = \int_0^\infty x^{g-1}e^{-x}dx.
\]

If \(d = 0\), then, \(y_t\) is stationary, however, if \(d = 1\), then \(y_t\) is stationary in the first difference \((I(1))\).

The ARFIMA model generalizes ARIMA process by allowing the integration parameter \(d\) to take real values. Thus, there are:

- for \(|d| < 0.5\), the process \(y_t\) exhibits a stationary and invertible ARMA process with geometrically bounded autocorrelations (Mills 2006).
- for \(|d| > 0.5\), \(y_t\) exhibits non-stationary process and has an infinite variance.

3. Volatility and long memory
Long memory in financial time series is still the subject of intensive theoretical and empirical research. In recent years, one may notice the special interest of researchers in the long memory of assets return volatility expressed by the absolute values of returns or alternatively by squared returns. To capture long memory property in volatility, Baillie and al. (1996) and Ding and al. (1996) propose a FIGARCH model.

3.1 Fractional integrated GARCH (or FIGARCH)
By analogy to the ARFIMA process modeling long-term dependencies in the conditional mean of the time series in economics, similar issues will be raised in the modeling of the conditional variance. This observation has led (Engle, 1982) and (Bollerslev, 1986) to propose a class of the integrated GARCH model or IGARCH, which process is characterized by the presence of a unit root. In fact, the effect of a shock on the optimal forecast of future conditional variance will be evaluated by the corresponding cumulative response impulses which tend to a nonzero constant, and where predictions can increase linearly with the adopted horizon.

3.2 Definition and properties of FIGARCH process
For a series \(y_t\), the standard GARCH process is given by:

\[
\epsilon_t = y_t - E_{t-1}(y_t)
\]

\[
\text{var}_{t-1}(y_t) = E_{t-1}(\epsilon_t^2) = h_t = w + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j} = w + \alpha(L)\epsilon_t^2 + \beta(L)h_t
\]

Where, \(h_t\) is the conditional variance, \(\epsilon_t\) is the error term, \(\alpha(L)\) and \(\beta(L)\) are lag polynomials. \(\alpha_i\) and \(\beta_i \geq 0\), \(\sum_{i=1}^q \alpha_i + \sum_{i=1}^p \beta_i < 1\) to ensure non-negativity of the conditional variance.
By analogy to the ARFIMA \((k, d, l)\) process, FIGARCH \((p, d, q)\) of \(\{\varepsilon_t\}\) is defined by:

\[
\varphi(L)(1-L)^d \varepsilon_t^2 = w + [1 - \beta(L)] \varepsilon_t,
\]

where \(0 < d < 1\) and all the roots of \(\varphi(L)\) and \([1 - \beta(L)]\) are outside the unit circle.

An alternative representation to the FIGARCH model \((p, d, q)\) is given by:

\[
[1 - \beta(L)] h_t = w + \left[1 - \beta(L) - \varphi(L)(1-L)^d\right] \varepsilon_t^2
\]

In this case, the conditional variance of \(\varepsilon_t\) is expressed as:

\[
h_t = w[1 - \beta(L)]^{-1} \left[1 - [1 - \beta(L)]^{-1} \varphi(L)(1-L)^d\right] \varepsilon_t^2
= w[1 - \beta(L)]^{-1} + \lambda(L) \varepsilon_t^2
\]

The FIGARCH model provides greater flexibility for modeling the volatility as it nests GARCH. It is identical to the GARCH(p,q) model for \(d=0\) and to the IGARCH(p,q) model for \(d=1\), we can see that the FIGARCH process includes the GARCH and IGARCH processes as special cases.

### 3.3 Extension model of FIGARCH

A further extension was provided by the Fractionally Integrated Exponential GARCH model (FIEGARCH) (Bollerslev and Mikkelsen, 1996), which incorporated the asymmetric dynamics of the EGARCH model of Nelson, 1991.

#### 3.3.1 EGARCH model

\[
\ln(h_t) = \alpha_0 + \alpha_1 \ln(h_{t-1}) + \gamma_0 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma_1 \left[\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - E\left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}}\right)\right]
\]

When the distribution of standardized residuals is normal, \(E\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right) = \sqrt{2/\pi}\) and when it follows a GED:\footnote{GAD: generalized error distribution.}

\[
E\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right) = \frac{\Gamma(\gamma + 1)}{\Gamma(\gamma + \frac{1}{2})} \left[\frac{\gamma}{\pi}\right]^\gamma \frac{1}{\Gamma(\gamma /2)} \frac{1}{\gamma！}
\]

If the coefficient \(\gamma_0\) is negative, the volatility increases in case of negative shocks and decay otherwise. The "amplitude" effect is determined by the absolute value of last period’s volatility shock.

#### 3.3.3 FIEGARCH model

The idea of fractional integration was extended also to the EGARCH model. Therefore, Fractionally Integrated EGARCH or FIEGARCH models have not only the capability of modeling clusters of volatility (as in ARCH and GARCH) and capturing its asymmetry (as in
the EGARCH) but they also take into account the characteristic of long memory in the volatility (as in the FIGARCH).

\[
\begin{align*}
\ln(h_t) &= w_t + (1-\psi)L(1-\varphi L)^{-1}(1-L)^{-d} g(\varepsilon_t) \\
g(\varepsilon_t) &= \theta \varepsilon_{t-1} + \theta_2 \left[ \varepsilon_{t-1} - E[\varepsilon_{t-1}] \right] \\
w_t &= w + \ln(1 + \delta N_t)
\end{align*}
\] (11)

4. Empirical Results

In order to test the weak-form efficient hypothesis of the Tunisian stock market, it is proposed to study the dynamic of the Tunindex returns series over a period ranging between 02/01/1998 to 03/16/2018, a series of 5000 daily observations. The daily stock returns are defined as the logarithmic first-difference of the daily closing index values. Figure 1 shows the Tunindex and its returns series. From this figure, it is clear that there is a change in the trend and variance. Indeed, we note a rapid increase in the index that seems characteristic of new markets such as Tunisia.

According to the figure 1, we note that tunindex is not stationary at level, since its mean and its variance both depend on time. We can check this by a unit root test, for this purpose, we carry out the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and KPSS tests that have the primary purpose of checking the stationarity of the series. The results are presented in Table 1. Referring to the calculated values of the ADF and PP tests, the results indicate the reject of unit root null hypothesis for the returns series (with constant, with constant and trend, and without constant and without trend) at the significance level of 5%. The same result is confirmed by the KPSS test where the stationarity hypothesis is accepted at the significance level of 5%. However, the result is reversed with the tunindex series at level.

FIGURE 1: RETURNS AND TUNINDEX SERIES GRAPHS
The critical values of the tests used at a significance level of 5% are respectively: ADF:-2.862, PP:-2.862 and KPSS: 0.463

Table 1 - Stationarity tests

|       | Tunindex | Returns |
|-------|----------|---------|
| ADF   | -2.862   | -2.862  |
| PP    | -58.266  | 0.151   |
| KPSS  | 8.612    | 0.463   |

Descriptive statistics for the returns series \( r_t \) are given in Table 2. Given this, we note that the kurtosis for \( r_t \) of 28.41 is higher than that of a normal distribution which is 3 (an indication of fat tails). The returns have a negative skewness of -0.56 (meaning that negative returns are more likely to happen than positive). The Jarque-Bera normality test statistic is far beyond the critical value which suggests that \( r_t \) is far from a normal distribution.

Furthermore, from the absolute returns series presented in Figure 2, we can clearly see the observation of Mandelbrot (1963) and Fama (1965) that large absolute returns are more likely than small absolute returns to be followed by a large absolute return.

4.1 Modeling the conditional mean

According to the Box and Jenkins methodology, the identification of the ARMA process takes place in comparing the same empirical characteristics of a chronic and a theoretical ARMA process.

The examination of the autocorrelations and the partial autocorrelations functions presented in Figure 3, let us choose an AR(2) process to specify the returns dynamic. Indeed, the partial autocorrelations become insignificant from the order 3. For reasons of parsimony, one can consider that the returns process can be specified by an R(2), ARMA(1,1), ARMA(1,2) A and
The estimation results of the selected models using daily returns allowed us to choose an ARMA(1,1) structure to specify the returns series dynamic. This can be also justified by the use of an appropriate modeling criteria as AIC and BIC presented in table 3. Indeed, ARMA(1,1) is characterized by the lowest information criterias.

| Model          | AIC      | BIC      | Log-likelihood |
|----------------|----------|----------|----------------|
| AR (2)         | 1.642    | 1.647    | -4098.552      |
| ARMA (1,1)     | 1.641    | 1.646    | -4097.665      |
| ARMA (1,2)     | 1.642    | 1.648    | -4097.721      |
| ARMA (2,1)     | 1.643    | 1.648    | -4097.851      |

**Table 3 - selection criterias model of ARMA process**

4.2 Dual long memory in mean and volatility: ARFIMA-FIGARCH model

In order to verify the possibility of the existence of a long memory property in the returns series of Tunisian stock market and whether it is an efficient market, we take to estimate a different models ARFIMA(p,d,q). Estimation results by maximum likelihood method of the appropriate specification, ARFIMA(1,d,1) is displayed in table 4. According to table 4, it is clear that the returns series has a long memory property. Indeed, the d-ARFIMA parameter is significantly different from zero. From this, we see a predictable behavior of Tunisian stock market returns which display that this market is the weak-form market inefficiency.

**Dependent variable: Rtunindex**

| Mean equation: ARFIMA(1,d,1) model |
|-----------------------------------|
| Coefficient | Std Error | t-value | t-prob |
| Cst(M)       | 0.024     | 0.014   | 1.672  | 0.094  |
| d-Arfima     | 0.097     | 0.021   | 4.465  | 0.000  |
| AR(1)        | 0.151     | 0.089   | 1.694  | 0.090  |
| MA(1)        | -0.029    | 0.081   | -0.368 | 0.712  |

Q-Statistics on Standardized Residuals
---> P-values adjusted by 2 degree(s) of freedom
Q(10) = 21.983 [0.004]**
Q(15) = 38.393 [0.000]**
Q(20) = 43.034 [0.000]**

Q-Statistics on Squared Standardized Residuals
---> P-values adjusted by 2 degree(s) of freedom
Q(10) = 116.879 [0.000]**
Q(15) = 118.275 [0.000]**
Q(20) = 121.598 [0.000]**

**Table 4 - Estimated ARFIMA model (1,d,1)**
Moreover, according to the ARCH test result displayed in table 4, conditional variance of returns series exhibit conditional heteroscedasticity properties and therefore, the existence of significant ARCH effect in volatility. Now, to model this effect a GARCH(p,q) model is entertained for the ARFIMA residuals. The different ARFIMA(1,d,1)-GARCH(p,q) models were estimated and compared in terms of AIC and SC information criteria. Estimation results let us choose an ARFIMA(1,d,1)-GARCH(1,1) process as a most appropriate model to specify the behavior of Tunisian stock market returns.

As presented in table 5, all parameters of GARCH specification are significant showing the variability in conditional variance. The long memory d-ARFIMA parameter remains statistically significant which indicate the presence of long memory property in mean.

| Dependent variable: Rtunindex |
|-------------------------------|
| Mean equation: ARFIMA(1,d,1) model |
| Variance equation: GARCH(1,1) model |
| Coefficient | Std.Error | t-value | t-prob |
| Cst(M)       | 0.008     | 0.012   | 0.684  | 0.493 |
| d-Arfima     | 0.097     | 0.020   | 4.817  | 0.000 |
| AR(1)        | 0.134     | 0.086   | 1.550  | 0.121 |
| MA(1)        | -0.012    | 0.079   | -0.155 | 0.876 |
| Cst(V)       | 0.031     | 0.010   | 2.876  | 0.004 |
| ARCH(Alpha1) | 0.302     | 0.054   | 5.508  | 0.000 |
| GARCH(Beta1) | 0.584     | 0.096   | 6.312  | 0.000 |
| Student(DF)  | 5.159     | 0.432   | 11.930 | 0.000 |

Table 5 - estimated ARFIMA model (1, d, 1) -GARCH (1,1)

4.2.1 Long memory in volatility process:
To investigate the persistence phenomena and to test the presence of long memory property in volatility process, we use at the first time, the autocorrelation function of the squared residuals issued from the ARFIMA-GARCH model presented in figure 4. From this figure it clear that the volatility is characterized by a long-term dependency.

![Figure 4: Squared residuals autocorrelation function](image)

In this situation, the long memory property in the volatility process of the Tunisian stock market returns can be evaluated by using a new class of GARCH model, named FIGARCH model. Estimation results by maximum likelihood method of the most appropriate FIGARCH(p,d,q) model selected from different processes with different lags of (p and q) are displayed in table 6. From the estimation results of the ARFIMA(1,d,1)-FIGARCH(1,d,1) model, we note that the long-term dynamics in the volatility, which are modeled by the fractional differentiation d-FIGARCH parameter, appear statistically significant, in fact, the estimated value is between zero and one and significant indicating the presence of long memory behavior in the volatility process.
Dependent variable: Rtunindex
Mean equation: ARFIMA(1,d,1) model
Variance equation: FIGARCH(1,d,1) model

| Coefficient | Std.Error | t-value | t-prob |
|-------------|-----------|---------|--------|
| Cst(M)      | 0.008     | 0.011   | 0.715  | 0.474  |
| d-Arfima    | 0.096     | 0.020   | 4.767  | 0.000  |
| AR(1)       | 0.128     | 0.086   | 1.478  | 0.139  |
| MA(1)       | -0.004    | 0.079   | -0.059 | 0.952  |
| Cst(V)      | 0.160     | 0.030   | 5.334  | 0.000  |
| d-Figarch   | 0.136     | 0.030   | 4.477  | 0.000  |
| ARCH(Phi1)  | 0.694     | 0.073   | 9.480  | 0.000  |
| GARCH(Beta1)| 0.513     | 0.087   | 5.839  | 0.000  |
| Student(DF)| 5.662     | 0.474   | 11.940 | 0.000  |

Table 6 - estimation results of the ARFIMA(1, d, 1)–FIGARCH(1, d, 1) model

4.2.2 Leverage effect and FIEGARCH model

FIEGARCH processes also explain volatility clusters and asymmetry. Thus, these models offer better modeling capability than FIGARCH ones as they don’t suffer from FIGARCH drawbacks since the variance under FIEGARCH is defined in terms of the logarithm function. Indeed, they increase the flexibility of the conditional variance specification by allowing an asymmetric response of conditional variance to positive and negative shocks. To explain the dual long memory of the mean and volatility of Tunisian stock market returns, we fitted different combinations of ARFIMA(1,d,1)–FIEGARCH(p,d,q) model models. The most appropriate model selected by referring to model selection criteria is an ARFIMA(1,d,1)–FIEGARCH(1,d,1) which their results are given in table 7.

Dependent variable: Rtunindex
Mean equation: ARFIMA(1,d,1) model
Variance equation: FIEGARCH(1,d,1) model

| Coefficient | Std.Error | t-value | t-prob |
|-------------|-----------|---------|--------|
| Cst(M)      | 0.005     | 0.009   | 0.560  | 0.576  |
| d-Arfima    | 0.098     | 0.015   | 6.610  | 0.000  |
| AR(1)       | 0.130     | 0.025   | 5.177  | 0.000  |
| MA(1)       | 0.001     | 0.030   | -0.013 | 0.990  |
| Cst(V)      | -1.857    | 0.158   | -11.750| 0.000  |
| d-Figarch   | 0.387     | 0.068   | 5.693  | 0.000  |
| ARCH(Phi1)  | -0.440    | 0.158   | -2.788 | 0.005  |
| GARCH(Beta1)| 0.651     | 0.136   | 4.792  | 0.000  |
| EGARCH(Theta1)| 0.012   | 0.018   | 0.654  | 0.514  |
| EGARCH(Theta2)| 0.533   | 0.032   | 16.430 | 0.000  |
| Student(DF)| 5.457     | 0.470   | 11.610 | 0.000  |

Table 7 – estimation results of ARFIMA (1, d, 1)–FIEGARCH (1, d, 1) model

According to Table 7, In the FIEGARCH(1,d,1) model, the long memory parameter is 0<d<0.5. So, the returns series is stationary and has long memory. However, the coefficient (EGARCH (Theta1)) is not significant, implying that there is not asymmetry in news impact on volatility (no leverage effect).

To evaluate the robustness of these models fitted above, we try to compare their forecasting performance in the following subsection.
4.3 Forecasts evaluation for the selected models

The main objective of the financial series fitting is the prediction of the future values of interest variable. Thus, Evaluation of forecasts for different specifications is important as it helps us

to understand the forecasting accuracy of the selected models. For this, we used two empirical measures commonly used in literature, namely the mean square error (MSE) and the root mean squared error (RMSE). These measures are defined as:

\[ MAE = \frac{1}{N_k} \sum_{k} |\hat{e}_k| \quad \text{et} \quad RMSE = \sqrt{\frac{1}{N_k} \sum_{k} e_k^2} \]  

(14)

Where, \( N_k \), is the total number of forecasts and \( \hat{e}_k \), the difference between the predicted value \( t \) for \( (t+k) \) and the value observed in \( (t+k) \).

In the following and from a sample of 4500 observations, we will examine the prediction (by out of sample method) of different processes: ARMA(1,1)-GARCH(1,1), ARFIMA(1,1)-FIGARCH(1,d,1), ARFIMA(1,1)-FIEGARCH(1,d,1) and a random walk model on the returns stock market series. Then, for each of these models, we will compare the MAE and the RMSE defined above for: 1 day, 2 days, 15 days, 30 days, 60 days, 90 days and 500 days. Forecast evaluation results are displayed in table 8.

|                  | Random walk | ARFIMA-GARCH | ARMA-GARCH | ARFIMA-FIGARCH | ARFIMA-FIEGARCH |
|------------------|-------------|--------------|------------|----------------|----------------|
| 1 day            | RMSE        | 0.3101       | 0.2155     | 0.2128         | 0.2179         |
|                  | MAE         | 0.2907       | 0.2155     | 0.2128         | 0.2179         |
| 2 days           | RMSE        | 0.3619       | 0.2246     | 0.3227         | 0.3262         |
|                  | MAE         | 0.3325       | 0.2104     | 0.3083         | 0.3122         |
| 15 days          | RMSE        | 0.7326       | 0.7634     | 0.7634         | 0.7635         |
|                  | MAE         | 0.5692       | 0.6059     | 0.6057         | 0.6061         |
| 30 days          | RMSE        | 0.5925       | 0.6082     | 0.6082         | 0.5783         |
|                  | MAE         | 0.4425       | 0.4606     | 0.4605         | 0.4103         |
| 60 days          | RMSE        | 0.4709       | 0.4798     | 0.4797         | 0.4198         |
|                  | MAE         | 0.3351       | 0.3436     | 0.3436         | 0.3137         |
| 90 days          | RMSE        | 0.4113       | 0.4174     | 0.4173         | 0.3874         |
|                  | MAE         | 0.2878       | 0.2939     | 0.2929         | 0.173         |
| 500 days         | RMSE        | 0.3750       | 0.3784     | 0.3784         | 0.1783         |
|                  | MAE         | 0.2758       | 0.2784     | 0.2780         | 0.1662         |

Table 8 - Forecasts evaluation of different selected models

In the short-term, we note that the random walk is beaten by all other selected models. Indeed, the ARFIMA forecasts only supplant the forecasts resulting from the random walk in the short term. In the long-term, forecasts of the random walk are better than those of ARFIMA.

In the short term, the ARMA-GARCH models show an improvement of forecasts as compared to ARFIMA-FIGARCH and ARFIMA-FIEGARCH. While, in a long term ARFIMA-FIEGARCH gives better forecasts than ARMA-GARCH, and further ARFIMA-FIEGARCH models proved to explain Tunisian stock returns series better than other selected models.

5. Conclusion

In this paper, long memory property in Tunisian stock market returns has been examined by using an ARFIMA-FIGARCH model and weak-form efficient market hypothesis has been tested. This hypothesis assumes that current stock prices reflect all available information,
and that past price performance has no relationship with the future. In other words, this form of the hypothesis says that using technical analysis to achieve exceptional returns is impossible. Thus, asset prices fluctuate as random walk which is satisfied by unpredictable behavior of asset returns. However, in the presence of long memory, returns are not independent over time and their future values can be predicted by using past prices.

Using ARFIMA-FIGARCH model types, our study finds that there is strong evidence of dual long memory in its conditional mean and volatility. Moreover, ARFIMA–FIEGARCH model is the best to analyze long memory and asymmetric volatility among others. Indeed, it outperforms other ARMA-GARCH-type models in terms of prediction. Our study also shows that the leverage effect is not significant in Tunisian index returns. Thus, the presence of dual long memory in Tunisian stock market returns contradicts the weak form market efficiency hypothesis.

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