Reliability of building structures in case of an air blast wave

Oleg Mkrtchyan and Anton Savenkov

Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, Russia

E-mail: mkrtchyan@yandex.ru; savenkov.asp@mail.ru

Abstract. The analysis shows that the parameters of the air shock wave vary significantly and have a pronounced random nature. In general, in structural analysis, design parameters of the shock wave are random functions or random variables; therefore, the reaction parameters of the system under consideration will also be random. Then, fulfillment of certain conditions (for example, strength) is only possible with a certain probability, and the reliability of the system under consideration can be estimated using methods of building structures reliability, methods of mathematical statistics and probability theory. As a result, a probabilistic model of an air shock wave has been developed and the reliability of a free-standing wall has been estimated for random exposure parameters, as well as for random strength characteristics of materials. Based on the calculation results, it was concluded that the calculation of reliability using methods of mathematical and statistical probability theory makes it possible to bring the strength analysis of building structures closer to actual operating conditions.

1. Introduction

When solving the problem of the theory of reliability of building structures using random variables, the probability of failure as the main quantitative characteristic of reliability is equal to a multidimensional integral:

\[ P_{rob}\{g(x_1, x_2, \ldots, x_n) < 0\} = \int_{\Omega_n} \cdots \int f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \]  \hspace{1cm} (1)

where \( \Omega_n \) is the area of failure statues in the \( n \)-dimensional space of all random variables \((x_1, x_2, \ldots, x_n)\), the boundary of which is determined by the condition \( g = 0 \);

\( f(x_1, x_2, \ldots, x_n) \) — joint probability density of all random variables.

2. Study objective

The purpose of the calculation is:

- determination of the random parameters that affect the main parameters in the front of the front of air blast.
- development of a probabilistic model of the air blast effect.
- evaluation of the reliability of a building structure in case of the air blast effect at random parameters, as well as at random parameters of the strength of reinforced concrete on the example of a reinforced concrete free-standing wall.
3. Materials and methods

3.1. Solving the problem of interaction of a shock wave with a structure

To solve the problem of interaction of the wave front with the structure, a numerical modeling was used, for example, this problem is most efficiently solved by a non-linear dynamic method. The study was carried out in the LS-DYNA software package which implements a non-linear dynamic method that allows solving problems in the time domain using explicit schemes for direct integration of the equations of motion.

To describe highly non-linear current explosion processes in the software package, the Eulerian approach [1] is used, which is based on the principle of studying the behavior of media moving through a fixed computational grid. At that, all parameters of the medium are considered as functions of coordinates and time, which ensures the best result when studying the behavior of liquids or gases.

To perform structural analysis of the air blast effect, a method was proposed in [2], which consists in setting a pressure graph at the boundary of the computational domain with all necessary parameters in the wave front ($\Delta P$, $\tau$) in accordance with current regulatory documents.

To describe the profile of overpressure in the air blast front, the Friedlander equation [3, 4] is most often used in design practice:

$$P(t)=P_0 + P_{pos} e^{\left(\frac{t}{t_0}\right)} \left(1 - \frac{t}{t_0}\right)$$

(2)

where $t$ is the time that has elapsed since the explosion occurred, $s$;
$t_0$ – compression phase time, $s$;
$P_0$ – atmospheric pressure, kPa;
$P_{pos}$ – the maximum pressure in the front of air blast in the compression phase, kPa.

3.2. Reliability assessment

The reliability assessment of building structures should be performed by the statistical modeling method (hereinafter referred to as SSM) [5]. The SSM is based on the laws of large numbers, namely, on two limit Bernoulli’s and Chebyshev’s theorems.

According to Chebyshev’s theorem:

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} p[|\bar{y} - m_y| < \varepsilon] = 1$$

(3)

where $\varepsilon > 0$.

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} p[|\bar{y} - m_y| < \varepsilon] = 1$$

(4)

where $\varepsilon > 0$.

Bernoulli’s theorem allows to determine the probability of an event.

Tests are carried out as follows. The deterministic solution of certain system is known. It has several input parameters ($x_1, ..., x_n$), for example, the load value, strength parameters of the load-bearing structure, as well as output parameters ($y_1, ..., y_n$), for example, stresses, deflections, relative plastic strains, failures data, etc. Input data are accepted as random variables. The following should be known for each random variable:

- distribution function.
- mathematical expectation and variance.

Output values are unknown. To determine them, $n$ implementations are performed. Since it is often impossible to investigate the entire general set of implementations of random variables because of their large number, the $n$ number of tests is set based on computer time resources, but in this case a confidence interval with a given security shall be established. For each implementation, input parameters are generated in the form of random variables or processes. As a result of each implementation, the output variable also takes on a certain random value:

$$y_n = f(x_1^n, x_2^n, ..., x_n^n)$$

(5)
According to this method, numerical characteristics of random values of the input parameters of the system are entered in a computer. In the program (for example Excel, Mathcad, etc.), a cycle is established to determine random implementations of the indicated variables with the given distribution laws. Calculations at each stage are performed by substituting the implementation data in the calculation scheme. As a result, implementations of the system output parameters are obtained. Having collected the required number of such implementations (according to the required accuracy), the numerical characteristics (moments) of the output parameters are determined by considering them as a random variable.

The failure frequency is determined by the formula:

\[ \nu = \frac{m}{n} \]  

(6)

where \( \nu \) is the failure rate; \( m \) is the number of failures, \( n \) is the total number of tests.

At \( n \to \infty \), \( \nu \to P_f \), where \( P_f \) is the probability of failure.

\[ \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \]  

(7)

At that, with \( n \to \infty \), the average value \( \bar{y} \) tends in probability to its mathematical expectation \( m \), according to the Chebyshev's limit theorem.

The condition for non-exceedance of the boundary of the permissible states area of structures can be defined as:

\[ R \cdot F > 0 \]  

(8)

The probability of non-destruction of the structure or reliability \( N \) is the probability of non-exceedance of a random variable characterizing the limiting state. If the distribution curve of this variable is somehow determined, then based on the integral probability distribution curve \( P_s \), it will be possible to determine the quantile of probability \( N \) of the implementation of the random variable \( S \) that will be less than this quantile, cutting off the ordinate \( P_s = N \).

The probability of structural failure:

\[ Q = \int_{-\infty}^{0} P_s(S) dS = P_s(0) \]  

(9)

Figure 1. Probability distribution curve \( P_s \).

where \( p_s(S) \) is the distribution density of the strength reserve; \( p_s(0) \) is the value of the distribution function of the strength reserve at \( S=0 \) (the probability that \( S \leq 0 \), i.e., fracture).

When \( R \) and \( F \) do not correlate:

\[ p_f = P(R \leq 0, F \leq 0) = P(S \leq 0) = \Phi \left( \frac{0 - \mu_G}{\sigma_G} \right) \]  

(10)

\( p_f \) – the probability of failure; \( \mu_G \) and \( \sigma_G \) are the first moments of the distribution (mathematical expectation and standard); \( \Phi \) – normal distribution function.

4. Statement of a deterministic problem

Let us consider the deterministic problem of the air blast effect on a free-standing reinforced concrete wall with a front pressure of \( \Delta P_f = 30 \) kPa and a compression phase duration of \( \tau_c \approx 0.25 \) s. This load is the starting point for structural analysis in accordance with IAEA standards [6].
The calculation area is the volume of air with initial parameters corresponding to normal atmospheric conditions: density $\rho_0 = 1.225 \text{ kg/m}^3$, temperature $T = 298.15 \text{ K}$, static pressure $E_0 = 101325 \text{ Pa}$, heat capacity $C_p = 1004 \text{ Дж/кг} \cdot \text{К}$, coefficients $C_0 = C_1 = C_2 = C_3 = C_4 = 0$, $C_5 = C_6 = 0.4$ [7,8]. The lower boundary of the circuit is rigidly fixed. At the external boundaries of the computational domain, nonreflecting boundary conditions [9] were applied, which ensure that the pressure exits the boundaries of the domain.

The schematic diagram of the calculation model and pressure graph $\Delta P_f$ is presented in figure 2.

**Figure 2.** (a) Calculation model (b) Pressure graph $\Delta P_f$, Pa.

4.1. The results of calculating the deterministic problem

**Figure 3.** Pressure isofields at the front of air blast, Pa (a), (b) view from above (c), (d) side view.

**Figure 4.** (a) X-displacement, m, (b) Moment $M_y$, H·m.
As a result of wall analysis, the maximum horizontal displacement was 45 \text{ mm}; this displacement reaches 0.1 \text{ s} at a time after the beginning of the air blast on the wall. Also, at this point in time, the maximum value of the bending moment \( M_y = 86982 \text{ H‧m} \) is accepted. The calculation also showed that plastic strains do not occur in the wall structures, and it works in the elastic stage. According to the calculation results, it was assumed that a wall section with a thickness of 300 \text{ mm} is sufficient to accommodate the existing loads.

5. Solving the probabilistic problem

5.1. Random characteristics of air blast effect

The simulation method of setting the load on the computational domain in the form of a graph with the specified overpressure and operating time parameters allows to reproduce the load in accordance with regulatory documents where the main parameters of the air blast are determined as follows: front pressure \( \Delta P_f \) and compression phase time \( \tau_c \).

Let us consider the results of numerical or field tests performed in order to determine which values affect the main parameters of the air blast. In Soviet times, a large number of tests were carried out and their results were most fully summarized and processed in \[10\], where it was noted that the charge form, type of shell, air temperature, explosive density, chemical losses at an explosion, explosive weight and distance from the explosion center affect the parameters in the air blast front. With this in mind, formulas were obtained for determining the main parameters of the air blast front. One of which subsequently was named the Sadovsky's formula:

\[
\Delta P_f = \left( \frac{0.92}{\tilde{R}} + \frac{3.5}{\tilde{R}^2} + \frac{10.6}{\tilde{R}^3} \right) \times 10^5
\]

(11)

where the reduced distance \( \tilde{R} \) is determined from the relation:

\[
\tilde{R} = \frac{R}{\sqrt{Q_{ef}}}
\]

(12)

where \( R \) is the distance from the explosion center, m; \( Q_{ef} \) is the effective explosive weight, determined by the formula:

\[
Q_{ef} = (1-\varepsilon)\alpha M
\]

(13)

where \( \varepsilon \) is the fraction of the explosion energy spent for funneling (for solid rocks \( \varepsilon = 0.05 \); for soft soils \( \varepsilon = 0.2 \); with conservative estimates, \( \varepsilon = 0 \) \[11\] should be taken).

\( \alpha \) is the ratio of the specific explosion energy of the considered explosive to the specific energy of TNT, determined by reference data (for TNT \( \alpha = 1 \)).

The duration of the compression phase at \( 1.2 \leq \tilde{R} < 10 \text{m} / \sqrt{\text{kg}} ^{1/3} \) is determined by the formula:

\[
\tau_c = 1.7 \times 10^{-3} \frac{Q_{ef} \sqrt{\tilde{R}}}{\sqrt{\tilde{R}}}
\]

(14)
The specific impulse of the compression phase, numerically equal to the area under the pressure curve in the compression phase at $\frac{\bar{R}}{R} < 10 \text{m/kg}^{1/3}$, is determined by the formula:

$$i_s = 350 \sqrt{\frac{Q_{ef}}{R}}$$  \hspace{1cm} (15)

In foreign sources, very few studies describe the observed variability of explosive loads. Some of them get statistics based on numerical or field tests [12, 13], where the observed statistics also show the greatest influence of $Q_{ef}$ and $R$ on the air blast parameters. Also, for example, foreign regulatory documents [14] contain the instructions for increasing the weight of explosives by 20% when designing explosion-proof building structures. But [15] shows that the level of reliability or the probability of non-exceedance of design explosive loads varies from 0.72–0.95 and 0.86–0.99 for pressures and pulses, respectively, but $Q_{ef}$ and $R$ in that model were accepted as deterministic, and only air conditions were variable. These studies were developed in [16], showing the significant effect that $Q_{ef}$ and $R$ also have on the explosion variability and design load parameters. The test was carried out for military, commercial and terrorist explosives, where it was shown that the explosive weight $Q_{ef}$ and the distance $R$, being random variables, can vary from 0.0 to 0.3. The lognormal law was adopted as the law of distribution.

To describe the explosion process, let us take $Q_{ef}$ and $R$ as random variables. Thus, many implementations of random pressure functions $P(t)$, which together constitute a random process are obtained.

Let us develop a probabilistic impact model for the initial data of the above deterministic problem. To accomplish this, considering the domestic and foreign experience, let us take $Q_{ef}$ and $R$ as random variables with the following parameters $\sigma = 0.3$, $\mu_{Q_{ef}} = 140000 \text{ kg}$, $\mu_{R} = 316 \text{ m}$ distributed according to the lognormal distribution law.

Let us perform 100 implementations of random functions $P_i(x_i)$ (see figure 6) the combination of which form a random process $P(x)$ (see figure 6). Figure 7 shows 6 of the 100 completed implementations $P_i(x_i)$ of the random process $P(t)$. For a complete description of the sections $P_i(x_i)$, it is necessary to investigate the laws of distribution of values of a random process in its sections. To do this, let us perform cross sections at times 0 s, 0.06 s, and 0.5 s (see figure 6).
Figure 6. Implementations $P_i(x_i)$ of the random process.

Figure 6 shows that the histograms of empirical frequencies in various sections of the random process are well described by the theoretical lognormal distribution curve.

5.2. Random concrete strength

The probabilistic nature of the strength of concrete as a material is based on the normative value of the cubic strength of concrete, taken with the reliability of 0.95 and determined by the formula [17]:

$$R_n = m_r (1 - 1.64\nu)$$  \hspace{1cm} (16)

where $R_n$ is the standard cubic strength; $\nu$ is the coefficient of variation characterizing the uniformity of strength. Having accepted the normative coefficient of variation for heavy concrete $\nu_n = 0.135$ for heavy concrete, $R_n = 0.78R$ is obtained. For stretched concrete, $\nu_n = 0.135$.

From the formula above, the average value of $m_R$ is determined by the formula:

$$m_R = \frac{R_n}{1 - 1.64\nu}$$  \hspace{1cm} (17)

$m_{R_h} = 23.76\text{MPa}; S_{R_h} = m_{R_h} \cdot \nu_{R_h} = 3.208\text{MPa}$.

The mathematical expectation and standard deviation of tensile strength of concrete B25:

$m_{R_t} = 2.13\text{MPa}; S_{R_t} = m_{R_t} \cdot \nu_{R_t} = 0.35\text{MPa}$.

The distribution of concrete strength is best described by Weibull's law. Distribution of density in shown in figure 7.
5.3. Reliability calculation results
In the course of calculations, it was found that relative plastic strains in some implementations exceeded the limit values for concrete. For example, figure 8 shows the distribution of bending moment in time at the wall base. Here it can be seen that at 0.05 s the concrete reaches its limiting values of relative strains equal to 0.0035 and the bending moment has a constant value. So, it can be concluded that the section after the concrete destruction ceases to perceive the current moment exceeding $1.16 \times 10^5$ N$m$. At this moment, the wall receives significant displacements of 250 mm in the direction of impact at its top (see figure 10), after which it loses its equilibrium state, collapsing under the action of the rarefaction phase in the opposite direction. In this case, the reinforcement bars in the wall are not destroyed, but are in the phase of plastic straining (see figure 9). Based on the analysis of the stress-strain state of the reinforced concrete wall, let us take the maximum bending moment at the base (foundation) of the wall as the criterion for failure or fault, after which the wall loses its equilibrium state and collapses (see figure 11).
As a result of the calculation, histogram of bending moments $M_y$ at the base of a reinforced concrete wall has been obtained (see figure 12), that is described by the distribution law as close to normal. The probability of failure of the reinforced concrete wall was $P_f = 0.06$. The probability of failure-free operation is $P_f = 1-0.06=0.94$. This estimate is within the confidence interval of 0.923–0.961 with the reliability of 0.95.

6. Conclusion
As a result of the calculation:

- the deterministic problem of determining the bearing capacity of a free-standing, monolithic, reinforced concrete wall under the air blast effect from detonative explosion has been resolved.
the effect of the explosive weight $Q_{ef}$ and the distance from the center of the explosion to the building structure $R$ on the parameters in the front air blast wave has been estimated.

- probabilistic modeling of the air blast effect in the form of a random process has been performed;
- random strength characteristics of reinforced concrete have been determined.
- criteria for the destruction of a reinforced concrete wall have been specified.
- an assessment of the reliability of a free-standing, monolithic, reinforced concrete wall under the impact of the air blast effect has been obtained.

Thus, the calculation of the reliability of the building structure has been performed with random parameters of the explosive impact and strength characteristics of the materials.

References
[1] Livermore Software Technology Corporation (LSTC) 2017 LS-DYNA Keyword user's manual. Vol. I, Version 971
[2] Mkrtchyan O V Savenkov A Y 2020 Methods of simulating the front of the air shock wave for calculating the industrial structure (Vestnik MGSU) 15 (2) (Moscow: Moscow State University Press) pp 223-234
[3] Nevskaya E E 2017 Main Methods of the Blast Waves Parameters Assessment at Emergency Explosions. Principles of Designing Blast Resistant Buildings and Structures (Occupational Safety in Industry) 9 (Moscow: ZAO NTTS PB) pp 20-29.
[4] Novozhilov Y V 2017 14th International CAFDEM / ANSYS User Conference (Metodiki modelirovaniya vzryvov v ANSYS LS-DYNA) (St. Petersburg)
[5] Mkrtchyan O V Rayzer V D 2016 Teoriya nadezhnosti v proyektirovani stroitelnikh konstruktsii (Moscow: Izdatelstvo ASV)
[6] IAEA Safety Standards No. SSR-2/1 2016 Safety of nuclear power plants: design. (Vienna: International atomic energy agency).
[7] Andreev S G Babkin A V Baum F A Imhovik N A Kobylkin I F Kolpakov V I et al 2004 Physics of a Blast (Moscow: Fizmatlit Publ)
[8] Bazhenova T V Gvozdeva L G 1977 Non-stationary shock wave interactions (Moscow: Science Publ)
[9] Grote M J Sim I 2009 On local non-reflecting boundary conditions for time dependent wave propagation (Chinese Annals of Mathematics, Series B) 30 (5) (Germany: Fudan University and Springer-Verlag GmbH) pp 589-606
[10] Sadovsky M A 1952 Mekhanicheskiye deystviye vozduzhnykh udarnykh voln vzryva po dannym eksperimental'nykh issledovaniy (Fizika vzryva) p 20
[11] Birbraer A N Roleder A J 2009 Extreme actions on structures (St. Petersburg: Publishing House of the Politechnical University)
[12] Low H Y Hao H 2002 Reliability analysis of direct shear and flexural failure modes of RC slabs under explosive loading (ENG STRUCT) 24 (2) (American Elsevier) pp 89–98
[13] Bogosian D Ferritto J Shi Y 2002 30nd the explosives safety seminar (Atlanta, Georgia)
[14] UFC3-340-02 2008 Structures to Resist the Effects of Accidental Explosions. Unified Facilities Criteria (Washington)
[15] Stewart M G Netheerton M D 2015 Reliability-based design load factors for explosive blast loading (J PERFORM CONSTR FAC) 29 (5) (American Society of Civil Engineers) B4014010
[16] Stewart M G 2018 Reliability-based load factor design model for explosive blast loading. (Structural Safety) 71 (Amsterdam: Elsevier) pp 13-23
[17] Mkrtchyan O V Dorozhinskiy V B Sidorov D S 2016 Nadezhnost' stroitel'nikh konstruktsiy pri vzryvakh i pozharakh (Moscow: Izdatelstvo ASV)