On the resistance of arbitrarily ring-stiffened welded bins subject to axial compression

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1 Introduction

Ring-stiffened shells are widely used as tanks or silos to store a variety of goods, such as oil, fertilizers, water, sewage and sludge. Material efficiency is achieved by combining thin plates with ring-stiffeners. When the bin is covered by a dome or cone roof, high meridional forces due to live or snow loads may trigger axial buckling of the shell. Ring stiffeners are beneficial, not just against buckling due to wind, but to reduce the risk of stability failure of tanks and silos caused by meridional compression as well.

This paper presents the outcome of a parametrical study concerned with axial buckling and the most effective placement of the stiffening rings depending on the stiffener distance, cross-sectional area and slenderness of the cylinder. Buckling curve parameters are deduced using the modified capacity curve. These allow the direct application of the analysis results in a hand calculation procedure that enables practitioners to take into account the benefits, which come with ring stiffeners against buckling due to axial compression.

Keywords
Cylindrical Shell, Buckling, Axial compression, Weld Imperfection

While for unstiffened shells a weld type A shape deviation according to [5] leads to similar results as the axis-symmetric eigenform-affine imperfection, for ring-stiffened shells no specific studies are available on ring-stiffened cylinders that allow a comparison. Besides [6] there is no study available specifically dealing with the placement of a weld in relation to the stiffeners and the influence of this particular configuration on the resistance against axial compression.

For this paper the results of [6] have been extended for two stiffening ratios $k_s$ (eq. 1) to gain an overview for practically relevant parameter combinations. Effects of certain parameters are discussed briefly and a calculation procedure is proposed. For that, equations for the buckling curve parameters were extracted from the modified capacity curve [7] for all combinations of parameters. It is shown that interpolation between few reference parameter combinations allow a safe-sided design procedure.

$$k_{st} = \frac{A_{st}}{a_{st} \cdot t} \quad (1)$$

where $A_{st}$ = cross-sectional area of the stiffener, $a_{st}$ = stiffener spacing and $t$ = thickness of the connected shell.
2 Analysis procedure and parameters

The analysis was carried out using the commercial FEM software Sofistik®, which is widely used in structural engineering practice and has been approved for many finite element calculations [8]. The “QUAD” element, a four node shell element using linear shape functions to calculate the deformations, was used. The reference steel grade S 355 was used without hardening (f_y = f_u = 355 MPa) for all calculations. The mesh convergence study carried out for [6] showed that a meridional length of 0.24√t yields satisfactory results when the circumferential length does not exceed 0.96√t.

The length l of the shell was kept approximately at a length to radius (l/r) ratio of two if the stiffener configuration allowed it. Depending on the spacing of the stiffeners α, the length had to be adjusted so that in any case three rings were present. Only a quarter of the cylinder was modelled due to geometrical and load symmetry. Appropriate boundary conditions were applied on the vertical edges. The horizontal edges were chosen to represent continuity with very weak radial springs of 1 kN/m stiffness to ensure numerical stability. The edge was clamped around its tangential direction. The bottom had restrained vertical movement while the top was free to move vertically. The load was applied as a uniform line load at the top edge.

The geometrically and materially nonlinear analysis (GMNA; GMNIA with weld type A imperfection [5]) was carried out employing the Newton-Raphson iteration scheme. Sofistik uses the modified approach in combination with a line-search. However, in some cases the full Newton-Raphson algorithm allowed for better convergence. For comparison, a dynamically stabilized quasi-static analysis was used. The increments were chosen as one percent of the critical load. The resistance was determined by carrying out an eigenvalue check for every four increments along the load deflection path. When the critical load factor switched from above one to below one between two load increments, the limit load was determined by linear interpolation between the adjacent load increments.

The radius over thickness (r/t) ratio is not directly varied but instead the yield strength of the material is changed to eliminate the effect of geometrical nonlinearity. The reference radius r is kept constant at 1000 mm with a thickness t of 1 mm.

3 Unstiffened shells

It was shown in [6] that the numerical results derived during the study align well with findings published in the literature. It is important to note that the weld type imperfection creates a length dependence of the capacity [10], which cannot be observed when an eigenform-affine imperfection is chosen.

The proposed modification of the formulation of the elastic buckling reduction factor α (eq. 2) was derived from [9] with an optimization in the range of imperfection depths Δw/t up to 2, which is approximately equal r/t = 2500 for fabrication tolerance class B (FQC B), and r/t = 1000 for FQC C. Very slender cylinders in any case achieve a buckling capacity of about 12% of the critical load [11]. With the current codified formulation for α this effect may not be accounted for. Hence, for this paper different equations for α, β and η were used to more economically describe the behavior.

\[ a_x = \frac{a}{b + c \Delta w/t} \]  
\[ a_x = \frac{a}{b + (\Delta w/t)^2} \]  
\[ a_x = \frac{a}{b + (\Delta w/t)^3} \]

4 Parameter study of ring-stiffened shells

4.1 Chosen parameters

The stiffener spacing was varied by multiples of the bending half-wave length (BeHW, l_beww = 2.44√t). It was anticipated that less heavy rings need to be placed closer to achieve a remarkable effect while heavy rings stiffen the shell even if they are more distant to each other. Hence, for the ring parameter k_r = 0.40 the values for α_out / l_beww were chosen to 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0 and for k_r = 1.00 the parameter α_out / l_beww was set to 2, 3, 4, 5, 6, 8, 10. The imperfection was considered to be in the line of the stiffener (Δwimp = 0) and away from the ring at Δwimp = a_out / 8, a_out / 4, 3 a_out / 8, a_out / 2.

4.2 Influence on elastic imperfection reduction factor

The variation of capacity with increasing imperfection depth Δw/t is depicted in fig. 3 for several positions of the weld imperfection. It is observed that close and heavy stiffeners reduce the imperfection sensitivity efficiently, especially when the shape deviation is in the vicinity of the stiffening ring. When an imperfection is in the line of the stiffener, circumferential compression due to the inward deformation, which results form the imperfection and meridional compression, is beared mostly by the strong ring. Since the shell suffers only minor circumferential compression, a chequer-board pattern is
The best placement for a weld is close to the stiffener. This is demonstrated in fig. 4. Closely spaced rings allow for high capacity even at deep imperfections. However, the reduction of resistance does not drop below around $\alpha_{x,rst} = 0.5$ even for stiffening rings, which are so far away from each other that no stiffening effect was anticipated. The capacity is in each case higher than for the unstiffened shell.

A criterion should be specified in the design code, to define at which spacing a stiffened cylinder becomes an unstiffened one and for which parameters further weld depressions or similar imperfections need to be taken into account to make sure that the determined capacity is sufficiently safe. In practice, standard format plates or coils may already set up this criterion for large tanks. For smaller bins however, the linear bending half-wave may be quite small so that special care needs to be taken to make sure that the assumptions of a structural analysis and the erection on site align.

As expected, the stiffened shells' resistance is close to an unstiffened cylinder as can be observed in fig. 5. As the distance between two rings gets larger, the tendency for increased strength with higher imperfection amplitude gets more pronounced. Again, several modifications in the model and the analysis procedure have been attempted to alter the results. However, the derived capacities remained approximately the same in each case. While it is well known that imperfection may enhance the capacity rather than lowering it, in the case of ring-stiffened shells, this effect may have a mechanical background rather than numerical reasons. As the imperfection gets deeper, the meridional bending moment increases, which leads to increased circumferentially alternating tension and compression membrane forces. As the stiffener keeps the shell circular, buckling can only be triggered away from the stiffeners. Each circumferential compression area is elastically supported by adjacent circumferential tension. The load at which circumferential buckling triggers the collapse may therefore increase because the tension force's support outweighs the negative influence of the deeper imperfection.

$$
(r/t)_{eq} = \frac{1000 f_y}{355}
$$

Comparing figs. 6 and 7, the dependence of the elastic imperfection reduction factor $\alpha_{x,rst}$ from the effective $r/t$ ratio is illustrated. Like unstiffened shells, weakly stiffened shells show only few to no strength gain when the $r/t$ ratio is increased (fig. 7). For the present case this is true for $f_y \geq 89$ MPA (eq. $r/t \approx 240$) up to $\Delta w/t = 2$. A deeper imperfection causes a huger drop of capacity due to yielding prior to buckling when the shell is thick. Material strength lesser effects the
resistance with decreasing shell thickness. Buckling then becomes elastic as it is usually expected for unstiffened shells for a large range of $r/t$ ratios. It can be observed in fig. 7 that exceeding the yield strength of 533 MPa (eq. $r/t = 15000$) does not result in any further gain in capacity, which means that all thinner shells buckle elastically even at very deep imperfections of 5 times the shell thickness.

When heavy rings at a close distance are employed, even at very deep imperfections, high capacities may be achieved when thinner shells are employed (fig. 6). In the case of $f_y = 3550$ MPa (eq. $r/t = 10000$), even the classical bifurcation stress can be achieved while for thicker cylinders very high elastic imperfection reduction factors were calculated. Exceeding $\alpha = 1$ is only possible due to numerical effects, especially at deeper imperfections.

Ring stiffeners efficiently reduce the imperfection sensitivity, which is lowest for stocky, thick-walled shells and very high for very thin-walled cylinders. Ring stiffeners may reduce the imperfection sensitivity so much that even for $r/t = 10000$ the yield strength may alter the resistance of the shell [4].

Hence, it is clear that accounting for the elastic imperfection reduction factor may alone is not sufficient to determine the capacity of the shell and considering the interaction of elastic and plastic buckling is much more important for heavily stiffened shells as in the case of unstiffened cylinders. The range in which plasticity is involved in the buckling process may be evaluated using the plastic limit relative slenderness $x_{rst}$ determined by eq. 6. The plastic range parameter $\beta_{rst}$ needs to be explored more in detail. This is best achieved by plotting the numerical results in the modified capacity curve.

$$x_{rst} = \sqrt{\frac{\alpha_{rst}}{1 - \beta_{rst}}}$$  \hspace{1cm} (6)

4.3 Modified capacity curve

The plastic range parameter cannot be directly calculated from the numerical results but can be determined by evaluating numerical values plotted in the modified capacity curve.

In many cases, clear trends of $\beta$ can be determined, as can be seen in fig. 8. With increasing imperfection depth smaller values of $1 - \beta$ are found. At a constant value of $\alpha$, decreasing $1 - \beta$ results in an extended elastic plastic interaction range. Since $\alpha$ and $\beta$ usually decline as the imperfection gets deeper, only small variation of $x_{rst}$ is determined.

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No clear trend can be deduced from the modified capacity curve for the heavily stiffened shell, which is depicted in fig. 10. An interpolation of the buckling curve parameters is anyway possible (fig. 11). While the curve for $\alpha_{rst}$ shows almost no reduction with imperfection depth, a clear descending path of the curve of $1 - \beta_{rst}$ can be observed. This means that with increasing imperfection depth the range of elastic-plastic interaction during the buckling process extends. Since by code more slender shells are deemed to have deeper imperfections consequently these shells profit more from heavy stiffening as the plasticity is involved in the buckling process even for very thin cylinders.

The plastic limit relative slenderness remains almost constant for the parameters chosen in figs. 8 and 9. The trends of the curves of $\alpha_{rst}$ and $\beta_{rst}$ quotient converge to be parallel (fig. 9).
4.4 The effect of the weld position

The influence of the weld position is depicted in fig. 12 as a relative value to $\alpha_{x,\text{ref}}$, which was chosen as for $k_{st} = 1$, $a_{st} = 2$ BeHW, S 355, $\alpha_{imp} = 0$. As a further reference, the unstiffened shell has been included as $\alpha_{imp} = \infty$. It can be expected that the loss of resistance is most detrimental for the heaviest stiffened shell with closest stiffener spacing. For comparison the curves are plotted for a stiffener spacing of $a_{st} = 5$ BeHW in fig 13. Clearly, the loss of capacity is less detrimental for this case.

$$\Delta w/t = \frac{1}{\sqrt{a}} \left( 1 - 0.8 k_{st} \right)$$

(8)

Furthermore, weaker stiffeners do not yield the same effect as heavy stiffeners. However, even small rings may enhance the resistance considerably. This effect may be approximately accounted for by reducing the part of the elastic imperfection reduction factor, which exceeds $\alpha_s$ of the unstiffened shell according to eq. 9.

$$\alpha_{x,\text{eff}} = \alpha_s + \left( \alpha_{x,\text{eff}} - \alpha_s \right) (1 - k_{st})$$

(9)

4.6 Influence of the stiffener spacing

It can be taken from figs. 3 and 4 that at $a_{st} = 3$ BeHW a minimum of $\alpha_{x,\text{ref}}$ is determined for the group of distances of rings from 3 to 8 BeHW. It is hence assumed for the simplified design procedure that a straight line interpolation between 2 and 3 BeHW as well as between 8 and the unstiffened shell (10 BeHW) yields satisfactory results. The elastic imperfection reduction factor is constant between 3 and 8 BeHW.
5 Proposal for a simplified design procedure

The procedure presented in this section is valid for ring parameters up to $k_{st} = 1$ and a $r/t$ ratio $\leq 10000$. The minimum considered ring spacing is 2 BeHW.

To determine the characteristic capacity of a ring-stiffened shell with arbitrarily placed welds between the rings, the following steps have to be undertaken.

1. Determination of the equivalent characteristic imperfection amplitude $\Delta w/t_{eq}$ with eq. 8

$$\alpha_{x,rst,2} = -8.466 - 9.095 + 0.415 \Delta w/t$$  (10)

2. Calculation of the elastic imperfection reduction factor of heavily stiffened shell, $k_{st} = 1.0$, $a_{st} = 2$ BeHW

$$\alpha_{x,rst,3} = -0.992 - 2.101 + 0.616 \Delta w/t$$  (11)

3. Determination of the elastic imperfection reduction factor of a heavily stiffened shell, $k_{st} = 1.0$, $a_{st} = 3$ BeHW

$$\alpha_{x,min} = \alpha_x = -0.084 - 1.084 + 0.761 \Delta w/t$$  (12)

4. Calculation of the elastic imperfection reduction factor of unstiffened cylinder, $\lambda_{x0} = 0.0$, $a_{st} = \infty$

5. Linear interpolation of $a_x$ between $a_{st} = 2$ BeHW, 3 BeHW and the unstiffened cylinder (10 BeHW)

Alternatively, the appropriate wider spacing may be used. For 3 BeHW to 8 BeHW, the value of $a_x$ remains constant.

6. Reduction of $a$ according to eq. 7 with parameters from tab. 1

7. Definition of the plastic limit relative slenderness according to $\lambda_{x,rst}$

8. Definition of the plastic range factor and the buckling curve exponent taken from the unstiffened shell if $\lambda_{x,0} < \lambda_{x,rst} < \lambda_{x,p,rst}$

$$\beta_x = \frac{0.650}{0.773 + (\Delta w/t)^{0.640}}$$  (13)

$$\eta_x = \frac{0.848}{0.130 + (\Delta w/t)^{0.640}}$$  (14)

9. Calculation of the buckling reduction factor $\chi_{x,rst}$

$$\lambda_{x,rst} < \lambda_{x,0} < \lambda_{x,p,rst}$$

$$\lambda_{x,0} \leq \lambda_{x,rst} \quad 1$$  (15)

$$\lambda_{x,0} < \lambda_{x,rst} < \lambda_{x,p,rst} \quad 1 - \beta_{x,rst} \left( \lambda_{x,rst} - \lambda_{x,0} \right) \eta_{x,rst}$$  (16)

$$\lambda_{x,p,rst} < \lambda_{x,rst} \quad \alpha_{x,rst} \lambda_{x,rst}^2$$  (17)

6 Check of the design procedure

The design procedure has been checked against the outcome of the FEM study for the ring parameters $k_{st} = 0.40$ and $1.00$ (figs. 13, 14) using FQT C ($Q = 16$).

Less scatter is achieved when linear interpolation between all the parameter combinations is used. The required coefficients determined from the modified capacity curve are available upon request.

6.1 Weld in line of stiffener

In the stocky slenderness range a major part of the results is unconservative. This is due to the consideration of all imperfection depths. Typically, thick-walled cylinders have only little characteristic imperfections so that in fact the appropriate calculated design results do not overestimate the FEM calculations.

As the slenderness grows, results align in a band between the values $\chi_{x,rst,FEM} / \chi_{x,rst,cal} = 0.90$ and 1.3 (fig. 13), respectively 0.90 and 1.60 for the medium stiffened shell. The design procedure hence is safe and, taking into account the complexity, allows quite accurate predictions of the numerically derived resistance.

![Figure 13](image)

**Figure 13** Comparison of FEM results against calculated resistances for heavily stiffened shell ($k_{st} = 1.00$). $a_{imp} = 0$

![Figure 14](image)

**Figure 14** Comparison of FEM results against calculated resistances for medium stiffened shell ($k_{st} = 0.40$), $a_{imp} = 0$

6.2 All parameter combinations considered

When all parameter combinations are considered the scatter is larger than for the situation of welds in the line of stiffener. The scatter ranges from about 0.8 up to 5 in the case of the heavily stiffened shell (fig. 15) while for the medium stiffened cylinder the scatter is slightly reduced (fig. 16).
Figure 1 Comparison of FEM results against calculated resistances for heavily stiffened shell ($k_{st} = 1.00$), $a_{im}$ = 0 ... $a_{st}$ / 2

Figure 16 Comparison of FEM results against calculated resistances for medium stiffened shell ($k_{st} = 0.40$), $a_{im}$ = 0 ... $a_{st}$ / 2

This outcome is partially due to increasing resistance with increasing imperfection depth in the FEM analysis, which cannot be captured with a hand calculation procedure that assumes descending capacity with increasing imperfection depth. Furthermore, if only the appropriate characteristic imperfection depths are considered, the scatter is reduced.

Less scatter may be achieved with variations in the proposed formulæ. However, to capture the behavior of the about 3250 data points acquired by numerical calculation with simple algebraic formulæ is a quite ambitious undertaking, especially as all the considered parameters depend non-linearly from each other.

The more promising and more economical approach would be to use the equations determined using the modified capacity curve and programming the formulæ in a spread sheet. The resistances may then easily be determined using linear interpolation of the buckling reduction factors between adjacent parameter sets.

7 Conclusion

A finite element study concerned with the axial buckling resistance of arbitrarily ring-stiffened welded shells has been conducted with a special focus on the effect of the distance of the imperfection to the ring stiffener. The influence of the weld position, the ring parameter and the stiffener spacing on the capacity has been briefly described. A simplified design procedure has been worked out, which uses equations for the buckling curve parameters of two parameter sets of the stiffened shell and the unstiffened cylinder as reference resistances. Linear interpolation between the three cases allows to safely determine the enhanced buckling capacity of arbitrarily ring-stiffened shells with randomly placed welds using a hand calculation procedure.

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