MULTIPLE SUB-JET MODEL OF GAMMA-RAY BURSTS AND POSSIBLE ORIGIN
OF X-RAY PRECURSORS AND POSTCURSORS

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ABSTRACT

We assume that internal shocks of gamma-ray bursts (GRBs) consist of multiple sub-jets with a collimation half-angle of about several times $\gamma_i^{-1}$, where $\gamma_i$ is the Lorentz factor of each sub-jet. If by chance a sub-jet is first emitted off-axis from the line of sight, the observed peak energy can be in the X-ray region. Next, if by chance a subsequent sub-jet is emitted along the line of sight, then the peak energy will be in the gamma-ray region and the gamma ray may arrive after the X-ray precursor from the former sub-jet depending on parameters. This model predicts a new class of GRBs with extremely weak and short gamma-ray emission but X-ray precursors and/or postcurors as well as an afterglow.

Subject heading: gamma rays: bursts

1. INTRODUCTION

Recently, evidence for collimation of GRBs in the afterglow came out for GRB 990123 (Kulkarni et al. 1999; Fruchter et al. 1999; Galama et al. 1999; Castro-Tirado et al. 1999; see also Dai & Lu 1999 for nonjet interpretation) and GRB 990510 (Harrison et al. 1999; Stanek et al. 1999). The rapid decline rate of GRB 980519 (Halpern et al. 1999) also suggests the collimation of GRBs. From the fact that the power-law index of the decline rate of the afterglow changed at 1–2 days for GRB 990123 and GRB 990510, the collimation half-angle $\Delta \theta_i$ of the afterglow is estimated as $\sim 0.1$ (Rhoads 1997; Kulkarni et al. 1999; Harrison et al. 1999), where the subscript $i$ stands for afterglow. The unusually slowly declining X-ray afterglow of GRB 980425 ($\propto T^{-0.2}$) can also be explained\(^1\) if the afterglow is beamed and the angle between the jet and the line of sight is $\sim 30^\circ$ (Nakamura 1999).

In an internal shock model (Piran 1998), which is one of the promising models of GRBs, the relativistic beaming half-angle ($= \gamma_i^{-1} < 0.01$), with $\gamma_i$ being the initial gamma factor of the internal shock, is smaller than the suggested collimation half-angle of the afterglow ($\Delta \theta_i \sim 0.1$). This means that it is possible to assume that the collimation half-angle of the internal shock is much smaller than the collimation half-angle of the afterglow. We here assume that the beam of internal shocks consists of such sub-jets with the collimation half-angle $\Delta \theta_i < \Delta \theta_i$. This kind of model can interpret no correlation between the isotropic luminosity of gamma rays and that of the afterglow as well as the diversity of the isotropic luminosity for GRBs with known redshifts (Kumar & Piran 1999).

Here I would like to point out that such a sub-jet model has the potential ability to interpret X-ray precursors, which were first observed by the Ginga satellite (Murakami et al. 1991). The evidence for X-ray precursors has been confirmed by the Granat/WATCH (Sazonov et al. 1998) catalog with six X-ray precursor events in 95 GRBs. This suggests that about $\sim 6\%$ of GRBs have precursors. Recently, BeppoSAX found the X-ray precursor and prompt X-ray emission from GRB 980519 (in ‘t Zand et al. 1999). For GRB 980519, an analysis of the evolution of the X-ray spectrum in terms of a single power-law spectrum shows that the photon index evolved from $-2.0$ to $-1.1$ to $-2.4$. The onset of burst has such a soft spectrum that the 2–27 keV emission appears to precede the greater than 107 keV emission by about 70 s. One more important fact for GRB 980519 is that the X-ray postcursor is seen after $\sim 100$ s from the gamma-ray emission.

Paczyński (1998) has once suggested that precursors of GRBs might be the evidence for the association of GRBs with supernovae, that is, precursors might suggest a baryon-contaminated environment of GRBs, while the gamma ray might come from the clean fireball. Here I would like to interpret precursors in the framework of the standard fireball model which has been very successful so far (Piran 1998).

In this Letter, I assume that internal shocks of gamma-ray bursts (GRBs) consist of multiple sub-jets with the collimation half-angle $\sim$ several times $\gamma_i^{-1}$, where $\gamma_i \approx 100–1000$ is the Lorentz factor of each sub-jet. If by chance a sub-jet is first emitted off-axis from the line of sight, the observed peak energy can be in the X-ray region. Next, if by chance a subsequent sub-jet is emitted along the line of sight, then the peak energy will be in the gamma-ray region and, depending on parameters, the gamma ray can arrive after the X-ray precursor from the former sub-jet. This emission pattern agrees with that of GRBs with precursors. The event rate of precursors ($\sim 6\%$) is compatible with this model since precursors occur by chance. If the time order of the former and the latter sub-jets is reversed by chance, the gamma ray comes first and the X-ray comes later. This emission pattern is compatible with the postcursor. In the next section, we will argue a simple model in this picture.

2. SIMPLE MODEL

Let us consider a simple model of GRBs in the framework of an internal shock picture. First, at $t = 0$ in the laboratory frame, a slow proton jet with Lorentz factor $\gamma_s > 100$ with the collimation half-angle $\Delta \theta_s$ starts from the central engine toward the direction $\theta_s$, where $\theta_s$ is the angle between the direction to the detector and the axis of the sub-jet. This slow jet continues for $\Delta t_s$. At $t = t_s + \Delta t$, a rapid proton jet with Lorentz factor $\gamma_r > \gamma_s$ with the collimation half-angle $\Delta \theta_r$ starts from the central engine toward the same direction $\theta_s$ as the slow jet. For simplicity, we consider only the case of $\Delta \theta_s = \Delta \theta_r < \theta_s$. Then the rapid jet catches up the slow jet at $t_r = 2t_s \gamma_r^2 S/(S^2 - 1) + \Delta t_r$, where $S \equiv \gamma_r/\gamma_s > 1$. The internal shock breaks out at $t_s = t_r + 2\Delta t \gamma_r^2 S^2/(S^2 - 1)$, where $S' \equiv$...
\( \gamma_{sh}/\gamma_s > 1 \), with \( \gamma_s \) being the gamma factor of the internal shock in the laboratory frame. Let us consider the case of \( \theta_s > \gamma_s^{-1} \). Then the relation of the observed time \( T \) to \( t \) is given by

\[
T = (1 - \beta \cos \theta_s)t \sim t\theta_s^2/2,
\]

where we set \( T = 0 \) as the time when the photon starts from the central engine at \( t = 0 \) arrives at the detector. The observed frequency \( \nu \) at the detector is related to \( \nu' \) in the shock frame as

\[
\nu = \frac{\nu'}{\gamma(1 - \beta \cos \theta_s)} \sim 11.1 \nu' \left( \frac{\gamma_{sh}}{200} \right)^{1} \left( \frac{\theta_s}{0.03} \right)^{-2}.
\]

Since in the internal shock model the peak energy in the comoving frame should be in the soft X-ray band (say \( \sim \mathrm{keV} \)), the observed peak energy from the off-axis emission can be in X-ray band (say \( \sim 10 \mathrm{keV} \)) for appropriate values of \( \gamma_{sh} \) and \( \theta_s \). For \( t_s \sim \Delta t_s \), the arrival time of this off-axis emission \([T_s(\theta_s)\]) is given by

\[
T_s(\theta_s) = 9t_s \left( \frac{\gamma_{sh}}{100} \right)^{1} \left( \frac{\theta_s}{0.03} \right)^{2} \frac{S^2}{S^2 - 1},
\]

while the duration from the catch-up time to the shock breakout time \( \Delta T(\theta_s) \) is given by

\[
\Delta T(\theta_s) = 9\Delta t_s \left( \frac{\gamma_{sh}}{100} \right)^{1} \left( \frac{\theta_s}{0.03} \right)^{2} \frac{S^2}{S^2 - 1}.
\]

Now let us assume that at \( t = t_0 \), the central engine starts to emit another pair of the same slow and rapid sub-jets toward the line of sight (\( \theta = 0 \)). Then the catch-up time \( (t'_c) \) and the breakout time \( (t'_b) \) become \( t'_c = t_0 + t_s \) and \( t'_b = t_0 + t_r \), respectively, while the relation of the observed time \( T \) to \( t \) becomes

\[
T(\theta_s = 0) = t_0 + \frac{t - t_0}{2\gamma_s^2}.
\]

Then

\[
T_s(\theta_s = 0) = t_0 + t_s \frac{1}{S^2 - 1},
\]

while the duration from the catch-up time to the shock breakout time \( \Delta T(\theta_s = 0) \) is given by

\[
\Delta T(\theta_s = 0) = \Delta t_s \frac{1}{S^2 - 1}.
\]

In this case, the observed frequency \( \nu \) at the detector is related to \( \nu' \) in the shock frame as

\[
\nu = 400\nu' \left( \frac{\gamma_{sh}}{200} \right).
\]

This means that the observed emission will be in the gamma-ray band if \( \nu' \) is in the soft X-ray band (\( \sim \mathrm{keV} \)).

Now the condition for the occurrence of the precursor \([T_s(\theta_s) > T(\theta_s)]\) is rewritten as

\[
\frac{t_0}{\tau_s} > \left[ 9 \left( \frac{\gamma_{sh}}{100} \right)^{1} \left( \frac{\theta_s}{0.03} \right)^{2} - \frac{S^2}{S^2 - 1} \right].
\]

Let us consider a specific numerical example, case 1:

\[
\gamma_s = 100, \quad \gamma_{sh} = 200, \quad \gamma_s \sim 200, \quad t_s = 6 \text{ s}, \quad \Delta t_s = 6 \text{ s}, \quad \theta_s = 0.03.
\]

Then

\[
T_s(\theta_s = 0) = t_0 + 2 \text{ s}, \quad \Delta T(\theta_s = 0) = 2 \text{ s}.
\]

If the peak energy in the shock frame is \( \sim 0.5 \) keV, then the peak energy from 72 to 144 is \( \sim 5 \) keV, while the peak energy from \( t_0 + 2 + t_s + 4 \) is \( \sim 200 \) keV. Therefore, if \( t_0 > 70 \) s, the X-ray emission starts \( (t_0 - 70 \text{ s}) \) before the gamma-ray emission. The duration of the X-ray precursor is \( \sim 70 \) s, while the duration of the gamma-ray emission is \( \sim 2 \) s. The duration of the X-ray precursor in this specific example happens to be comparable to those observed by Ginga for GRB 900126 (Murakami et al. 1991) and by BeppoSAX for GRB 980519 (in ’t Zand et al. 1999), while the duration of the gamma ray is somewhat smaller.

The above model is extremely simple but clearly demonstrates a possible origin of precursors. In case 1, by choosing \( t_s \) much smaller than 70 s, we may explain the postcursor. For example, if we choose \( t_s = 20 \) s, then the gamma-ray emission starts at 22 and ends at 24 s, while the X-ray postcursor emission starts at 72 s.

3. X-RAY FLUX FROM THE PRECURSOR AND THE POSTCURSOR IN A SUB-JET MODEL

In the previous section, we discussed only the peak energy and the temporal structure of precursors and postcursors. In this section, we estimate the X-ray flux from the precursor in the sub-jet model. The observed X-ray flux of the precursor and postcursor of GRB 980519 (in ’t Zand et al. 1999) is \( \sim 5 \times 10^{-9} \text{ergs s}^{-1} \text{ cm}^{-2} \) in the 2–10 keV band. The average X-ray flux of the precursor of GRB 900126 (Murakami et al. 1991) is \( \sim 2.5 \times 10^{-9} \text{ergs s}^{-1} \text{ cm}^{-2} \). We argue that these values are compatible with our model of precursors and postcursors.

For \( \theta_s = 0 \) and \( \gamma \Delta \theta \gg 1 \), we assume that the observed spectrum of the gamma ray from the sub-jet is given by

\[
F \propto \frac{1}{[1 + (\nu / \nu_0)^2]^{\beta/2}},
\]

where \( \beta \sim 1.5 \) and \( \nu_0 \sim 150 \text{ keV} \) are constants. The above spectrum is essentially the same as the Band spectrum (Band et al. 1993) with \( \alpha = -1 \) and \( \beta = -\beta_s - 1 \), where \( \alpha \) and \( \beta \) are the spectrum parameters in the Band spectrum. The peak frequency \( \nu_{\text{peak}} \), that is, the maximum of \( \nu F \), is \( [1/(\beta_s - 1)]^{1/2} \nu_0 \).

Granot, Piran, & Sari (1998) as well as Woods & Loeb (1999) derived a general formula to compute the off-axis emission from beamed GRBs. Here we adopt their formulations and notations. Let us use a spherical coordinate system \( r = \)
\( (r, \theta, \phi) \), where the coordinates are measured in the lab frame; let the \( \theta = 0 \) axis (\( z \)-axis) point to the detector and \( r = 0 \) be the central engine. Let also \( D \) be the distance to the source and \( \alpha = r \sin \theta / D \) be the angle that a given ray makes with the normal to the detector. Then the observed flux is given by

\[
F_\gamma (T) = \frac{\nu D}{\gamma \beta} \int_0^{2\pi} \int_0^{\alpha_{\text{m}}} \alpha^2 d\alpha \times \int_{\gamma (1-\beta)}^{\gamma (1+\beta)} \frac{d\nu}{\nu^2} \frac{\nu^2}{(1-\mu^2)^{3/2}},
\]

\( \alpha_{\text{m}}, T, \text{and } j'_\gamma \) are the maximum value of \( \alpha \), the arrival time of a photon at the detector, and the rest-frame emissivity measured in units of ergs \( s^{-1} \ cm^{-1} \ sr^{-1} \), respectively. Note here that a prime means the physical quantity in the rest frame.

In the case of \( \theta = 0 \), we assume that \( j'_\gamma \) is expressed as

\[
j'_\gamma \propto H(\Delta \theta_\gamma - \theta) f(\nu'),
\]

where \( f(\nu') \) and \( H(\cdot) \) are a certain function and the Heaviside step function, respectively. Then \( F_\gamma (T) \) is given by (Nakamura 1999)

\[
F_\gamma (T) = 2\pi A \nu \int_{\nu_0}^{\gamma_\Delta \theta_\gamma^2/2} f(\nu') \frac{d\nu'}{\nu^2},
\]

where \( A \) is a factor depending on the density, the gamma factor, the strength of the magnetic fields, and the location of the shock as well as the distance to the source. Since we assume that the spectrum has the form of equation (12) for \( \gamma \Delta \theta_\gamma \gg 1, f(\nu') \) is given by

\[
f(\nu') = \frac{1 + (\beta_\gamma + 1)(2\gamma \nu'/\nu_0)^2}{2\gamma [1 + (2\gamma \nu'/\nu_0)^2]^{3/2}}.
\]

Neglecting the term proportional to \( 1/\gamma^2 \Delta \theta_\gamma^2 \), we have

\[
F_\gamma (T) = 2\pi A \nu_0 \left[ \frac{1}{[1 + (\nu/\nu_0)^2]^{3/2}} \right]^{\nu_0/\nu_0}. \quad (19)
\]

Then the peak frequency is in the gamma-ray region as \( \nu_{\text{peak}} \sim \nu_0 \sim 150 \text{ keV} \) and the peak flux is given by

\[
(v F_\gamma)' \sim 2\pi A \nu_0 / 2 \nu_0^2. \quad (20)
\]

Now let us consider the off-axis case (\( \theta > \Delta \theta_\gamma \)). In this case, \( j'_\gamma \) is expressed as

\[
j'_\gamma \propto H(\cos \phi - \cos \Delta \theta_\gamma - \cos \theta_\gamma \cos \phi) \sin \theta_\gamma \sin \theta \times H(\Delta \theta_\gamma - |\theta - \theta_\gamma|) f(\nu').
\]

The flux is given by (Nakamura 1999)

\[
F_\gamma (T) = 2\pi A \nu \int_{\gamma (1-\beta \cos(\theta_\gamma + \Delta \theta_\gamma))}^{\gamma (1+\beta \cos(\theta_\gamma + \Delta \theta_\gamma))} \phi_\gamma f(\nu') \frac{d\nu'}{\nu^2},
\]

where \( \cos \phi_\gamma = (\cos \Delta \theta_\gamma - \cos \theta_\gamma \cos \phi_\gamma) / \sin \phi_\gamma \sin \theta_\gamma \).

Equation (22) is evaluated as

\[
F_\gamma (T) = 8A \left( \frac{\Delta \theta_\gamma}{\theta_\gamma} \right)^2 \left( \frac{1}{\gamma \theta_\gamma} \right)^2 2\gamma f(\nu_\gamma \theta_\gamma^2 / 2). \quad (23)
\]

Then the peak frequency is in the X-ray region as \( \nu_{\text{peak}} \sim \nu_\gamma (\gamma \theta_\gamma^2) \sim 4 (\gamma / 200)^2 (\theta_\gamma / 0.03)^2 \text{ keV} \), and the peak flux is given by

\[
(v F_\gamma)' \sim 8A \left( \frac{\Delta \theta_\gamma}{\theta_\gamma} \right)^2 \left( \frac{1}{\gamma \theta_\gamma} \right)^4 2+2 (\frac{\beta_\gamma}{\gamma \theta_\gamma^2}) 2. \quad (24)
\]

The ratio of the X-ray flux observed from the off-axis of the sub-jet to the gamma ray flux observed along the axis is given by

\[
\frac{v F_\gamma (T)}{v F_\gamma (T)} = 10^{-3} (1 + (\beta_\gamma / 2)) \left( \frac{\gamma \theta_\gamma^2}{200} \right)^4 \left( \frac{\theta_\gamma}{0.03} \right)^{-4}. \quad (25)
\]

For the gamma-ray flux of \( \sim 5 \times 10^{44} \text{ ergs} \text{ s}^{-1} \text{ cm}^{-2} \), we have the X-ray flux of \( \sim 5 \times 10^{44} \text{ ergs} \text{ s}^{-1} \text{ cm}^{-2} \). This X-ray flux is similar to the observed ones in the precursors of GRB 980519 (in 't Zand et al. 1999) and GRB 900126 (Murakami et al. 1991). Therefore, our model is compatible with the X-ray flux of the precursor and the postcursor.

4. DISCUSSION

Let us try to interpret the X-ray photon index evolution of GRB 980519 (in 't Zand et al. 1999) stated in § 1. In our model, X-ray precursors and postursors come from the off-axis emission from sub-jets so that the spectrum is expressed by equation (23), which leads to the photon index \( \beta = -\beta_\gamma - 1 \sim -2.5 \). On the other hand, the main gamma-ray emission comes from the on-axis emission from the sub-jet so that the spectrum is expressed by equation (19). In equation (19), the photon index in the X-ray band is \(-1\). Therefore, our model predicts that the photon index evolves from \(-2.5\) to \(-1\) to \(-2.5\), which agrees with the observations.

As we discussed in § 2, the duration of the emission from the sub-jet depends on the viewing angle. From equations (4) and (7), we have

\[
\frac{\Delta T(\theta_\gamma)}{\Delta T(\theta_\gamma = 0)} = \frac{\gamma_\theta^2}{100} \left( \frac{\theta_\gamma}{0.03} \right)^2 \left( \frac{\gamma_\theta}{\gamma_\theta} \right)^2. \quad (26)
\]

This means that the duration of the lower energy emission (\( \theta_\gamma = 0 \)) is longer than that of the higher energy emission (\( \theta_\gamma = 0 \)). If the X-ray precursors or postursors occur by chance, our model in this Letter predicts that their duration is longer than the duration of the main gamma-ray emission. This agrees with observations of precursors and postursors (Murakami et al. 1991; in 't Zand et al. 1999).

However, for most cases precursors and postursors do not occur. Then what does our model predict for usual GRBs? In our model, in general, there exists a sub-jet whose axis makes an angle \( \theta_\gamma \neq 0 \) with the normal to the detector. This sub-jet mainly emits delayed X-rays after the main gamma-ray emission from the sub-jet with \( \theta_\gamma \sim 0 \). The duration of X-ray emission is longer than the main gamma-ray emission, so that our model predicts that prompt and delayed X-ray emission with longer duration should associate with GRBs. The emission patterns of GRB 960720, GRB 970111, GRB 970228, GRB 970508, GRB 971224, GRB 980329, and GRB 980425 qual-
itatively agree with this prediction (Frontera et al. 1999), while other possible explanations may exist.

What is a new prediction from our model? Suppose that by chance none of the sub-jets point to the line of sight. In this case, gamma-ray emission is extremely weak and short or absent while the X-ray precursors and/or postcursors as well as X-ray, optical, and radio afterglows exist. If such an event is found, that will be compatible with our sub-jet model discussed in this Letter.

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Note added in proof.—After I submitted this Letter, F. Frontera et al. (preprint [astro-ph/0002527] [2000]) reported another example of a GRB with an isolated precursor (GRB 981226). The gamma-ray fluence of GRB 981226 is smaller than the X-ray fluence, so that GRB 981226 might belong to a new class of GRBs suggested in this Letter.