Recent Advances in Adversarial Training for Adversarial Robustness

Tao Bai∗, Jinqi Luo1, Jun Zhao1, Bihan Wen1, Qian Wang2
1Nanyang Technological University, Singapore
2Wuhan University, China
{bait0002, luoj0021, junzhao, bihan.wen}@ntu.edu.sg, qianwang@whu.edu.cn

Abstract
Adversarial training is one of the most effective approaches defending against adversarial examples for deep learning models. Unlike other defense strategies, adversarial training aims to promote the robustness of models intrinsically. During the last few years, adversarial training has been studied and discussed from various aspects. A variety of improvements and developments of adversarial training are proposed, which were, however, neglected in existing surveys. For the first time in this survey, we systematically review the recent progress on adversarial training for adversarial robustness with a novel taxonomy. Then we discuss the generalization problems in adversarial training from three perspectives. Finally, we highlight the challenges which are not fully tackled and present potential future directions.

1 Introduction
The adversarial vulnerability of deep neural networks has attracted significant attention in recent years. With slight but carefully-crafted perturbations, the perturbed natural images, namely adversarial examples [Szegedy et al., 2014], can mislead state-of-the-art (SOTA) classifiers to make erroneous predictions. Besides classification, adversarial examples appear in various tasks like semantic segmentation, object detection, super-resolution, etc (see the summary in [Yuan et al., 2019]). The existence of adversarial examples raises concerns from the public and motivates the proposals of defenses [Goodfellow et al., 2015; Huang et al., 2015; Madry et al., 2018]. Naturally, the defenses also stimulate the development of stronger attacks, seeming like an arms race.

Among various defense strategies, Adversarial Training (AT) [Goodfellow et al., 2015; Madry et al., 2018] currently proves to be the most effective against adversarial attacks [Pang et al., 2020a; Maini et al., 2020; Schott et al., 2019], receiving considerable attention from the research community. The idea of adversarial training is straightforward: it augments training data with adversarial examples in each training loop. Thus adversarially trained models can behave more normally facing adversarial examples than standardly trained models. Mathematically, adversarial training is formulated as a min-max problem, searching for the best solution to the worst-case optimum. The main challenge of adversarial training is to solve the inner maximization problem, which researchers are actively working on. The last few years have witnessed tremendous efforts made by the research community. The recent advances have resulted in a variety of new techniques in the literature, which deserves a comprehensive review.

To our best knowledge, surveys focusing on adversarial training do not exist so far, and recently proposed surveys [Yuan et al., 2019; Silva and Najafirad, 2020] mainly summarize existing adversarial attacks and defense methods. Our goal in this paper is to give a brief overview of adversarial training. We believe that this survey can provide up-to-date findings and developments happening on adversarial training. Notably, we carefully review and analyze adversarial training from different perspectives, uniquely discuss the known issues, and present future research directions.

2 Preliminaries
Adversarial Attacks. Adversarial attacks refer to finding adversarial examples for well-trained models. In this paper, we consider only the situation where the training/test data are initially from the same distribution. Taking classification as an example, we use \( f(x; \theta) : \mathbb{R}^{h \times w \times c} \rightarrow \{ 1 \ldots k \} \) to denote an image classifier that maps an input image \( x \) to a discrete label set \( C \) with \( k \) classes, in which \( \theta \) indicates the parameters of \( f \), and \( h, w, c \) represent image height, width and channel, respectively. Given the perturbation budget \( \epsilon \), the adversary tries to find a perturbation \( \delta \in \mathbb{R}^{h \times w \times c} \) to maximize the loss function, e.g., cross-entropy loss \( L_{ce} \), so that \( f(x + \delta) \neq f(x) \). Therefore, \( \delta \) is estimated as

\[
\delta^* := \arg \max_{|\delta|_p \leq \epsilon} L_{ce}(\theta, x + \delta, y),
\]

where \( y \) is the label of \( x \), and \( p \) can be 0, 1, 2 and \( \infty \). In most cases, \( \epsilon \) is small so that the perturbations are imperceptible to human eyes. Note that we only consider the \( l_p \)-based attacks for classification in this paper.

The adversarial counterpart \( x' \) of \( x \) is expressed as

\[
x' := x + \delta^*.
\]
3.1 The Origin of Adversarial Training

The initial idea of adversarial training is first brought to light by [Szegedy et al., 2014], where neural networks are trained on a mixture of adversarial examples and clean examples. Goodfellow et al. (2015) went further and proposed FGSM to produce adversarial examples during training. Yet, their trained models remain vulnerable to iterative attacks [Tramèr et al., 2018] as these approaches utilized a linear function to approximate the loss function, leading to sharp curvature near data points on the decision surface of the corresponding deep models, which is also known as gradient masking [Papernot et al., 2017].

Unlike the prior works in that models are trained on a mixture of clean data and adversarial data, a line of research trains models with adversarial data only. For the first time, Huang et al. (2015) defined a min-max problem that the training procedure is forced to minimize classification error against an adversary who perturbs the input and maximizes the classification error. Shaham et al. (2018) considered this min-max problem from a robust optimization perspective and proposed the framework of adversarial training, which is formulated as follows:

$$\min_{\delta} \mathbb{E}_{(x,y) \sim D} \left[ \max_{\delta \in B(x, \varepsilon)} \mathcal{L}_{ce}(\theta, x + \delta, y) \right],$$

(3)

where \( (x, y) \sim D \) represents training data sampled from distribution \( D \) and \( B(x, \varepsilon) \) is the allowed perturbation set, expressed as \( B(x, \varepsilon) := \{ x + \delta \in \mathbb{R}^m | \| \delta \|_p \leq \varepsilon \} \).

Madry et al. (2018) gave a reasonable interpretation of this formulation: the inner maximization problem is finding the worst-case samples for the given model, and the outer minimization problem is to train a model robust to adversarial examples. The key to solving this min-max problem is finding strong adversarial examples [Huang et al., 2015]. With such connection, Madry et al. (2018) employed a multi-step gradient based attack known as PGD attack for solving the inner problem as follows:

$$x^{t+1} = \text{Proj}_{x + B(x, \varepsilon)} \left( x^t + \alpha \text{sign} \left( \nabla_{x} \mathcal{L}_{ce}(\theta, x^t, y) \right) \right),$$

(4)

where \( t \) is the current step and \( \alpha \) is the step size.

Further, they investigated the inner maximization problem from the landscape of adversarial examples and gave both theoretical and empirical proofs of local maxima’s tractability with PGD. Through extensive experiments, their approach (PGD-AT) significantly increased the adversarial robustness of deep learning models against a wide range of attacks, which is a milestone of adversarial training methods. As most derivative works followed their designs and settings, PGD-AT became a critical benchmark and is regarded as the standard way to do adversarial training in practice.

3.2 Taxonomy of Adversarial Training

In this subsection, we review the recent advances of adversarial training in last few years, categorized by different understandings on adversarial training. A summary of selected adversarial training methods is provided in Table 1.
Adversarial Regularization

The idea of adversarial regularization first appears in [Goodfellow et al., 2015]. Besides cross-entropy loss, they added a regularization term in the objective function, which is based on FGSM and expressed as \( \mathcal{L} (\theta, x + \epsilon \text{sign}(\nabla_x \mathcal{L}(\theta, x, y))) \). Kurakin et al. (2017) extended this FGSM-based regularization term by controlling the ratio of adversarial examples in batches so that it can scale up to ImageNet. Their methods’ effectiveness is validated on single-step attacks as they believe the linearity of neural networks is attributed to the existence of adversarial examples [Goodfellow et al., 2015]. However, Qin et al. (2019) calculated the absolute difference between the adversarial loss and its first-order Taylor expansion, concluding that more robust models usually have smaller values of local linearity. Correspondingly, they replaced the FGSM-based regularization with a Local Linearity Regularization for adversarial robustness.

Distinct from previous methods, [Zhang et al., 2019b] decomposed the robust error \( R_{rob} \) as the sum of natural error \( R_{nat} \) and boundary error \( R_{db} \). Boundary error occurs when the distance between data and the decision boundary is sufficiently small (less than \( \epsilon \)), which is also the reason for adversarial examples’ existence. So they proposed TRADES to minimize the \( R_{db} \) by solving the following problem:

\[
\min_{f} \mathbb{E}_{\epsilon} \left\{ \mathcal{L}(f(x), y) + \max_{x' \in \mathcal{B}(x, \epsilon)} \mathcal{L}\left( f(x), f(x') \right) / \lambda \right\},
\]

where \( \lambda \) is a coefficient determining the strength of regularization. Such decomposition is proved to be effective, and TRADES outperforms PGD-AT on CIFAR-10 with error rates reduced by 10%. One problem of TRADES is that the regularization term is designed to push natural examples and their adversarial counterparts together, no matter natural data are classified correctly or not. [Wang et al., 2020] investigated the influence of misclassified examples and proposed Misclassification Aware adversarial training (MART), which emphasizes on misclassified examples using the output probability \( 1 - p_y(x, \theta) \).

Due to the amplification of deep models, imperceptible noises could lead to substantial changes in feature space [Goodfellow et al., 2015]. Some works analyze adversarial training from the perspective of representation. Kannan et al. (2018) proposed Adversarial Logit Pairing (ALP), encouraging logits for pairs of examples to be similar. But ALP initially is not useful due to the wrong formulation of adversarial training objectives [Engstrom et al., 2018]. Further to enhance the alignments of representations of natural data and their adversarial counterparts, Mao et al. (2019) adopted the prevalent triplet loss for regularization, which uses adversarial examples as anchors.

Adversarial regularization is an essential variant of adversarial training [Shaham et al., 2018]. Compared to the original formulation of adversarial training, adversarial regularization is more flexible and requires a deep understanding of adversarial robustness. Also, the decomposition of robust error indeed paves the way for unlabeled data to enhance adversarial robustness.

Curriculum-based Adversarial Training

According to the formulation of adversarial training, the inner problem is always trying to find the worst-case samples. One natural question is: are those worst-case samples always suitable for adversarial training? Zhang et al. (2020) found that adversarial examples generated by strong attacks significantly cross over the decision boundary and are close to natural data. As PGD-AT only utilizes adversarial examples for training, this leads to overfitting to the adversarial examples [Cai et al., 2018].

For alleviating the overfitting, researchers adapt the idea of curriculum training to adversarial training. Cai et al. (2018) proposed Curriculum Adversarial Training (CAT), with an assumption that PGD with more steps generates stronger adversarial examples. Starting from a small number of steps, CAT gradually increases the iteration steps of PGD until the model achieves a high accuracy against the current attack. Different from CAT, Friendly Adversarial Training (FAT) [Zhang et al., 2020] adapts early stopping when performing PGD attacks and returns adversarial data near the decision boundary for training. Both CAT and FAT adjust the attacks’ strength in a practical way, where a quantitative criterion is missing. From the convergence point of view, Wang et al. (2019) designed First-Order Stationary Condition (FOC) to estimate the convergence quality of the inner maximization problem. The closer the FOSC to 0, the stronger the attack.

Such curriculum-based methods help improve the generalization of clean data while preserving adversarial robustness. One possible reason for their success is weak attacks in early training stages are associated with generalization [Wang et al., 2019]. In addition to relieving overfitting, curriculum-based methods reduce training time due to the varying iteration numbers of PGD for solving the inner maximization problem.

Ensemble Adversarial Training

Tramèr et al. (2018) firstly introduces ensemble learning into adversarial training, called Ensemble Adversarial Training (EAT), where training data is augmented with adversarial examples generated from different target models instead of a single model. The advantage of EAT is that it helps alleviate the sharp curvatures caused by the single-step attacks e.g., FGSM. However, the interaction among different target models is neglected in [Tramèr et al., 2018]. Specifically, standardly trained models may have similar predictions or feature presentations [Dauphin et al., 2014] and share the adversarial subspace [Tramèr et al., 2017], which potentially hurts the performance of EAT.

For promoting the diversity among target models, several improvements are proposed, such as the adaptive diversity promoting regularizer [Pung et al., 2019], forcing different models to be diverse in non-maximal predictions; maximizing the cosine distances among each target models’ input gradients [Kariyappa and Qureshi, 2019] (input gradients refer to the gradients of the loss function w.r.t. the input); and maximizing the vulnerability diversity [Yang et al., 2020a], which is defined as the sum of losses for two models with crafted images containing non-robust
features [Ilyas et al., 2019].

Intrinsically, such ensemble methods are useful for approximating the optimal value of the inner maximization problem in adversarial training. As proved in [Tramèr et al., 2018], models trained with EAT have better generalization abilities regardless of the perturbation types. To conclude, adding the number and diversity of target models in training is a practical and useful way to approximate the space of adversarial examples, which is challenging to be described explicitly.

**Adversarial Training with Adaptive $\epsilon$**

As shown in Equation (3), the parameters of threats or attacks are predefined and fixed during training, e.g., $\epsilon$. Some papers [Balaji et al., 2019; Ding et al., 2020] argued individual data points might have different intrinsic robustness, i.e., different distances to the classifier’s decision boundary; however, adversarial training with fixed $\epsilon$ treats all data equally.

Considering the individual characteristic of adversarial robustness, researchers propose to do adversarial training at the instance level. Balaji et al. (2019) firstly presented Instance Adaptive Adversarial Training (IAAT) where $\epsilon$ is selected to be as large as possible, ensuring images within $\epsilon$-ball of $x$ are from the same class. This strategy helps IAAT relieve the trade-off between robustness and accuracy, though there is a slight drop in robustness. Unlike IAAT, another work called Margin Maximization Adversarial Training (MMA) [Ding et al., 2020] directly maximizes the margin-distances between data points and the model’s decision boundary, which is estimated by the adversarial perturbations with the least magnitudes. The manner of choosing $\epsilon$ in MMA is more reasonable as $\epsilon$ is sufficiently small. Such small $\epsilon$ in spatial domain hardly changes the classes of images substantially, especially for high-resolution images. The following work, Customized Adversarial Training (CAT) [Cheng et al., 2020] further applies adaptive label uncertainty to prevent over-confident predictions based on adaptive $\epsilon$.

Adversarial training with adaptive $\epsilon$ is a good exploration. However, empirical evidence shows many standard datasets are distributionally separated, i.e., the distances between classes are larger than $\epsilon$ used for attacks [Yang et al., 2020b]. This reflects the limitation of current training methods on finding proper decision boundaries.

**Adversarial Training with Semi/Unsupervised Learning**

One key observation in supervised adversarial training methods [Madry et al., 2018; Zhang et al., 2019b] is adversarial accuracy in testing is much lower than in training. There is a large generalization gap in adversarial training (see Figure 1 in [Schmidt et al., 2018]). The recent work [Schmidt et al., 2018] studied this problem from the perspective of sample complexity. It is theoretically proved that adversarially robust training requires substantially larger datasets than standard training. However, quality datasets with labels are expensive to collect, which is of particular interest in practice. Alternatively, several works appeared concurrently, exploring the possibility of training with additional unlabeled data.

Following the analysis of Gaussian models in [Schmidt et al., 2018], a couple of works [Alayrac et al., 2019; Carmon et al., 2019; Zhai et al., 2019] theoretically show that training with unlabeled data significantly reduces the sample complexity gap between standard training and robust training. They share the same idea of decomposing the adversarial robustness like TRADES and utilize unlabeled data for stability while labeled data for classification. Empirically, they investigated the impact of different factors on adversarial training like label noise, distribution shift, and the amount of additional data. On the other hand, Najafi et al. (2019) introduced some new complexity measures like Adversarial Rademacher Complexity and Minimum Supervision Ratio for theoretical analysis on generalization. It is also observed that adversarial robustness is benefited by self-supervised training [Hendrycks et al., 2019].

It is inspiring to see the improvement of adversarial robustness brought by extra unlabeled data. However, theoretical or empirical guarantees that how much additional data are needed precisely still lack. Besides, the cost of such methods should not be neglected, including collecting and training adversarially on data multiple times larger than original datasets.

**Efficient Adversarial Training**

One well-known limitation of conventional adversarial training methods like PGD-AT is that they take 3-30 times longer than standard training before the model converges [Shafahi et al., 2019]. The main reason is the min-max problem described in Equation (3) is solved iteratively. This line of research of adversarial training aims to reduce the time cost while keeping the performances of adversarial training.

As the first attempt, the core idea of free adversarial training (Free-AT) [Shafahi et al., 2019] is to reuse the gradients computed in the backward pass when doing forward pass. In Free-AT, both the model parameters and image perturbations are updated simultaneously. Concretely, for the same mini-batch data, the same operation is done for $m$ times in a row, equivalent to utilizing strong adversarial examples in PGD-AT. Further upon Free-AT, Wong et al. (2020) proposed fast adversarial training (FAST-AT), which combines FGSM-AT with random initialization and is as effective as the PGD-AT. They also attributed the failure of FGSM-AT to the catastrophic overfitting and zero-initialized perturbation. However, it is found [Andriushchenko and Flammarion, 2020] that the above fast training methods [Shafahi et al., 2019; Wong et al., 2020] suffer from catastrophic overfitting like FGSM-AT. Andriushchenko and Flammarion (2020) also pointed out randomization in [Wong et al., 2020] helps because it slightly reduces the magnitude of perturbations. Kim et al. (2021) supported the above finding and demonstrated catastrophic overfitting is because single-step adversarial training uses only adversarial examples with maximum perturbations. For the purpose of preventing catastrophic overfitting, many improvements are proposed like GradAlign [Andriushchenko and Flammarion, 2020], dynamic schedule [Vivek and Babu, 2020b], inner interval verification [Kim et al., 2021], domain adaptation [Song et al., 2019] and regularization methods [Vivek and Babu, 2020a; Huang et al., 2020].
Intrinsic from the above works, Zhang et al. (2019a) proposed YOPO from the perspective of Pontryagin’s Maximum Principle. According to their analysis on adversarial training, they observed that adversarial gradients update is only related to the first layer of the neural networks. This property enables YOPO to focus on the first layer of the proposed network architecture for adversary computation while other layers are frozen, significantly reducing the numbers of forward and backward propagation. The authors claimed that Free-AT is a particular case of YOPO.

Other Variants
In addition to the above branches of adversarial training methods, several other variants of adversarial training are summarized as follows. Some works modify the learning objectives of vanilla adversarial training, like adversarial distributional training [Dong et al., 2020] where a distribution-based min-max problem is derived from a general view; bilateral adversarial training [Wang and Zhang, 2019] where the model is training on both perturbed images and labels; and adversarial training based on feature scatter [Zhang and Wang, 2019], which utilizes a distance metric for sets of natural data and their counterparts and produces adversarial examples in feature space. Some replace the fundamental components of models for better performances, like hypersphere embedding [Pang et al., 2020b], and smoothed ReLU function [Xie et al., 2020]. Last, some propose to augment adversarial examples by interpolation, such as AVmixup [Lee et al., 2020] and adversarial interpolation training [Zhang and Xu, 2020].

3.3 Generalization Problem in Adversarial Training

For deep learning algorithms, generalization is a significant characteristic. Though most efforts in this research community are paid for improving adversarial training under given adversarial attacks, the voice of discussions on generalization is getting louder. This subsection mainly reviews the studies on the generalization of adversarial training from three aspects: standard generalization, adversarially robust generalization, and generalization on unseen attacks.

Standard Generalization
Despite the success in improving the robustness of neural networks to adversarial attacks, adversarial training is observed that hurts standard accuracy badly [Madry et al., 2018], leading to the discussion of relationships between adversarial robustness and standard accuracy. We refer to it as standard generalization.

One popular point is the trade-off between adversarial robustness and standard accuracy. Tsipras et al. (2018) claimed that standard accuracy and adversarial robustness might at odds and demonstrated the existence of the trade-off via a binary classification task. Su et al. (2018) evaluated the recent SOTA ImageNet-based DNN models on multiple robustness metrics. They concluded a linearly negative correlation between the logarithm of model classification accuracy and model robustness. Zhang et al. (2019b) decomposed the robust error as the sum of natural error and the boundary error and provided a tight upper bound for them, theoretically characterizing the trade-off.

However, some works have different opinions that adversarial robustness and standard accuracy are not opposing. Stutz et al. (2019) studied the manifold of adversarial examples and natural data. They confirmed the existence of adversarial examples on the manifold of natural data, adversarial robustness on which is equivalent to generalization. Yang et al. (2020b) investigated various datasets, from MNIST to Restricted ImageNet, showing these datasets are distributionally separated, and the separation is usually larger than 2ε (the values of ε differ in different datasets, e.g., 0.1/1 for MNIST, 8/255 for CIFAR). It indicates the existence of a robust and accurate classifier. They also claim existing training methods fail to impose local Lipschitzness or insufficiently generalized. Experiments in [Raghunathan et al., 2019] support this statement, where additional unlabeled data is proved to help mitigate the trade-off.

As suggested in [Yang et al., 2020b], the trade-off might not be inherent but a consequence of current adversarial training methods. Though researchers haven’t reached a consensus on the reason of the trade-off, existing evidence does reveal some limitations on adversarial training. Adversarial robustness should not be at the cost of standard accuracy. Some variants of adversarial training show better standard generalization empirically, such as adaptive ε for adversarial training reviewed in Section 3.2, robust local features [Song et al., 2020] and L1 penalty [Xing et al., 2020].

Adversarially Robust Generalization

The phenomenon that adversarially trained models do not perform well on adversarially perturbed test data is first observed in [Madry et al., 2018]. In other words, there is a large gap between the training accuracy and test accuracy on adversarial data. Other than CIFAR-10, similar experimental results are observed on multiple datasets, such as SVHN, CIFAR-100, and ImageNet [Rice et al., 2020]. These gaps indicate a severe overfitting happens in current adversarial training methods. Such overfitting is firstly studied by [Schmidt et al., 2018], who refers to it as adversarially robust generalization.

Schmidt et al. (2018) reveals the difficulty of obtaining a robust model with the fact that more training data are required for adversarially robust generalization. Later many efforts are made to improve generalization empirically, such as adversarial training with semi/unsupervised Learning, AVmixup, and robust local feature [Song et al., 2020]. In contrast, Rice et al. (2020) systematically investigated various techniques like ξ1 and ξ2 regularization, cutout, mixup and early stopping, where early stopping is found to be the most effective. Pang et al. (2020a) confirms the effectiveness of early stopping.

On the other hand, though researchers attempt to analyze this generalization problem with different tools like Rademacher complexity [Yin et al., 2019] and VC dimension [Cullina et al., 2018], theoretical progress is, in fact, limited, and the generalization problem is far from being solved.
Generalization on Unseen Attacks

The last significant property of adversarial training is to generalize on unseen attacks. It is proved that the specific type of attacks are not sufficient to represent the space of possible perturbations [Tramèr et al., 2018; Goodfellow et al., 2015]. However, in adversarial training, the inner maximization problem’s constraints: $l_p$ norm and $\epsilon$ are pre-fixed. Thus, adversarially trained models, which are robust to a specific attack, e.g., $l_\infty$ adversarial examples, can be circumvented easily by different types of attacks, e.g., other $l_p$ norms, or larger $\epsilon$, or different target models [Kang et al., 2019]. Simply combining perturbations with different $l_p$ norms in adversarial training proves to be useless as well [Tramer and Boneh, 2019]. This kind of poor generalization to other attacks significantly degrades the reliability of adversarial training.

Such limitation is intrinsically caused by adversarial training itself, and the key is how to solve the inner problem properly. The research line in EATier can be seen as the first attempt to approximate the optimal solutions to the inner problem by increasing the number and diversity of targeted models during training. Similarly, Maini et al. (2020) adopt adversaries under different $l_p$ norm and use the steepest descent to approximate the optimal value for the inner problem. Dong et al. (2020) proposed to explicitly model the distribution of adversarial examples around each sample, replacing the the perturbation set $B(x, \epsilon)$. From a different view, Stutz et al. (2020) suggested calibrating the confidence scores during adversarial training.

Though significant, the generalization problem of adversarial training on unseen attacks is only occasionally studied at this time. One possible reason is that our understanding of adversarial examples is limited and incomplete. The truth of adversarial examples is still underground, which also needs much effort.

4 Conclusion and Future Directions

In this paper, we reviewed the current adversarial training methods for adversarial robustness. To our best knowledge, for the first time, we give a novel summary of the research been conducted and discussed the generalization problem of adversarial training. We also summarize the benchmarks and provide performance comparisons of different methods. Despite extensive efforts, the performance of adversarial training is far away from being satisfied. Several open problems remain yet to solve, summarized as follows.

Min-Max Optimization in Adversarial Training. Adversarial training is formulated as a min-max problem. However, due to the non-convexity of deep neural networks, it is very challenging to obtain the global optimum for adversarial training. In existing methods, PGD is a prevalent technique for approximating the optimum, as Madry et al. (2018) empirically proved the tractability of adversarial training with PGD. But it can hardly provide a “robustness certificate” after solving the problem [Razaviyayn et al., 2020]. In other words, the robustness of adversarially trained models is not guaranteed. For this purpose, the development of new techniques for solving non-convex min-max problems is necessary and crucial.

Overfitting in Adversarial Training. Overfitting is an old topic in deep learning, and there are effective countermeasures to alleviate the overfitting. But, in adversarial training, overfitting seems to be more severe. Those common countermeasures used in deep learning help little [Rice et al., 2020]. The generalization gap between adversarial training accuracy and testing accuracy is very large. From the perspective of generalization, the theory of sample complexity [Schmidt et al., 2018] explains such a phenomenon partially, and is supported by experimental results in derivative works [Alayrac et al., 2019; Carlini et al., 2019]. As suggested by [Schmidt et al., 2018], it is essential to explore the intersections between robustness, classifier model and data distribution. Some open problems can be found in [Schmidt et al., 2018].

Beyond Adversarial Training. Though many theories have been proposed for improving adversarial training, it is undeniable that these improvements are less effective than claimed [Pang et al., 2020]. Some basic settings, e.g., training schedule, early stopping, seem to owe much on adversarial training’s performance. The shreds of evidence in [Yang et al., 2020; Stutz et al., 2019] show that adversarial training might not be the optimal solution for obtaining models with desirable robustness and accuracy. The trade-off between robustness and generalization can also be seen as an intrinsic limitation of adversarial training. Thus it is critical and necessary to investigate new methods beyond adversarial training in the future.

References

[Alayrac et al., 2019] Jean-Baptiste Alayrac, Jonathan Uesato, Po-Sen Huang, Alhussein Fawzi, Robert Stanforth, and Pushmeet Kohli. Are Labels Required for Improving Adversarial Robustness? In NeurIPS, 2019.

[Andriushchenko and Flammarion, 2020] Maksym Andriushchenko and Nicolas Flammarion. Understanding and improving fast adversarial training. ArXiv, 2020.

[Athalye and Carlini, 2018] Anish Athalye and Nicholas Carlini. On the robustness of the cvpr 2018 white-box adversarial example defenses. arXiv preprint arXiv:1804.03286, 2018.

[Balaji et al., 2019] Yogesh Balaji, Tom Goldstein, and Judy Hoffman. Instance adaptive adversarial training: Improved accuracy tradeoffs in neural nets. arXiv preprint arXiv:1910.08051, 2019.

[Buckman et al., 2018] Jacob Buckman, Aurko Roy, Colin Raffel, and Ian Goodfellow. Thermometer Encoding: One Hot Way To Resist Adversarial Examples. In ICLR, 2018.

[Cai et al., 2018] Qi-Zhi Cai, Chang Liu, and Dawn Song. Curriculum Adversarial Training. In {IJCAI-18}, 2018.

[Carmon et al., 2019] Yair Carmon, Aditi Raghunathan, Ludwig Schmidt, John C Duchi, and Percy S Liang. Unlabeled Data Improves Adversarial Robustness. In NeurIPS, 2019.
[Cheng et al., 2020] Minhai Cheng, Qi Lei, Pin-Yu Chen, Inderjit Dhillon, and Cho-Jui Hsieh. Cat: Customized adversarial training for improved robustness. arXiv preprint arXiv:2002.07689, 2020.

[Cullina et al., 2018] Daniel Cullina, Arjun Nitin Bhagoji, and Prateek Mittal. Pac-learning in the presence of evasion adversaries. arXiv preprint arXiv:1806.01471, 2018.

[Dauphin et al., 2014] Yann Dauphin, Razvan Pascanu, Caglar Gulcehre, Kyunghyun Cho, Surya Ganguli, and Yoshua Bengio. Identifying and attacking the saddle point problem in high-dimensional non-convex optimization. arXiv preprint arXiv:1406.2572, 2014.

[Ding et al., 2020] Gavin Weiguang Ding, Yash Sharma, Kry Yik Chau Lui, and Ruitong Huang. MMA Training: Direct Input Space Margin Maximization through Adversarial Training. In ICLR, 2020.

[Dong et al., 2020] Yinpeng Dong, Zhijie Deng, Tianyu Pang, Jun Zhu, and Hang Su. Adversarial Distributional Training for Robust Deep Learning. In NeurIPS, 2020.

[Engstrom et al., 2018] Logan Engstrom, Andrew Ilyas, and Anish Athalye. Evaluating and understanding the robustness of adversarial logit pairing. arXiv preprint, 2018.

[Goodfellow et al., 2015] Ian J Goodfellow, Jonathon Shlens, and Christian Szegedy. Explaining and Harnessing Adversarial Examples. In ICLR, 2015.

[Hendrycks et al., 2019] Dan Hendrycks, Mantas Mazeika, Saurav Kadavath, and Dawn Song. Using self-supervised learning can improve model robustness and uncertainty. CoRR, 2019.

[Huang et al., 2015] Ruitong Huang, Bing Xu, Dale Schuurmans, and Csaba Szepesvári. Learning with a strong adversary. arXiv preprint arXiv:1511.03034, 2015.

[Huang et al., 2020] Tianjin Huang, Vlado Menkovski, Yulong Pei, and Mykola Pechenizkiy. Bridging the performance gap between fgsm and pgd adversarial training, 2020.

[Ilyas et al., 2019] Andrew Ilyas, Shibani Santurkar, Dimitris Tsipras, Logan Engstrom, Brandon Tran, and Aleksander Madry. Adversarial examples are not bugs, they are features. arXiv preprint arXiv:1905.02175, 2019.

[Kang et al., 2019] Daniel Kang, Yi Sun, Tom Brown, Dan Hendrycks, and Jacob Steinhardt. Transfer of adversarial robustness between perturbation types. arXiv preprint, 2019.

[Kannan et al., 2018] Harini Kannan, Alexey Kurakin, and Ian J Goodfellow. Adversarial Logit Pairing. 2018.

[Kariyappa and Qureshi, 2019] Sanjay Kariyappa and Moinuddin K Qureshi. Improving adversarial robustness of ensembles with diversity training. arXiv preprint, 2019.

[Kim et al., 2021] Hoki Kim, Wooyin Lee, and Jaewook Lee. Understanding catastrophic overfitting in single-step adversarial training. In Proceedings of AAAI 2021, 2021.

[Kurakin et al., 2016] Alexey Kurakin, Ian J Goodfellow, and Samy Bengio. Adversarial examples in the physical world. CoRR, abs/1607.0, 2016.

[Kurakin et al., 2017] Alexey Kurakin, Ian J Goodfellow, and Samy Bengio. Adversarial Machine Learning at Scale. In ICLR. OpenReview.net, 2017.

[Lee et al., 2020] Saehyung Lee, Hyungyu Lee, and Sungroh Yoon. Adversarial Machine Learning at Scale. In CVPR, 2020.

[Madry et al., 2018] Aleksander Madry, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. Towards Deep Learning Models Resistant to Adversarial Attacks. In ICLR. OpenReview.net, 2018.

[Maini et al., 2020] Pratyush Maini, Eric Wong, and Zico Kolter. Adversarial Robustness Against the Union of Multiple Perturbation Models. In ICML, 2020.

[Mao et al., 2019] Chengzhi Mao, Ziyuan Zhong, Junfeng Yang, Carl Vondrick, and Baishakhi Ray. Metric Learning for Adversarial Robustness. In NeurIPS, 2019.

[Najafi et al., 2019] Amir Najafi, Shin-ichi Maeda, Masanori Koyama, and Takeru Miyato. Robustness to Adversarial Perturbations in Learning from Incomplete Data. In NeurIPS, pages 5542–5552. 2019.

[Pang et al., 2019] Tianyu Pang, Kun Xu, Chao Du, Ning Chen, and Jun Zhu. Improving Adversarial Robustness via Promoting Ensemble Diversity. In ICML, 2019.

[Pang et al., 2020a] Tianyu Pang, Xiao Yang, Yinpeng Dong, Hang Su, and Jun Zhu. Bag of tricks for adversarial training. arXiv preprint arXiv:2010.00467, 2020.

[Pang et al., 2020b] Tianyu Pang, Xiao Yang, Yinpeng Dong, Kun Xu, Hang Su, and Jun Zhu. Boosting Adversarial Training with Hypersphere Embedding, 2020.

[Paperot et al., 2017] Nicolas Papernot, Patrick McDaniel, Ian Goodfellow, Somesh Jha, Z. Berkay Celik, and Ananthram Swami. Practical black-box attacks against machine learning. ASIA CCS 2017, pages 506–519, 2017.

[Qin et al., 2019] Chongli Qin, James Martens, Sven Gowal, Dilip Krishnan, Krishnamurthy Dvijotham, Allhussein Fawzi, Soham De, Robert Stanforth, and Pushmeet Kohli. Adversarial Robustness through Local Linearization. In NeurIPS, 2019.

[Raghunathan et al., 2019] Aditi Raghunathan, Sang Michael Xie, Fanny Yang, John C Duchi, and Percy Liang. Adversarial training can hurt generalization. arXiv preprint, 2019.

[Razaviyayn et al., 2020] Meisam Razaviyayn, Tianjian Huang, Songtao Lu, Maher Nouiehed, Maziar Sanjabi, and Mingyi Hong. Nonconvex min-max optimization: Applications, challenges, and recent theoretical advances. IEEE Signal Processing Magazine, 37(5):55–66, 2020.

[Rice et al., 2020] Leslie Rice, Eric Wong, and J. Zico Kolter. Overfitting in adversarially robust deep learning, 2020.
NeurIPS. 2020.

Dimitris Tsipras, Shibani Santurkar, Logan Engstrom, Alexander Turner, and Aleksander Madry. Robustness may be at odds with accuracy, 2018.

Vivek and Babu, 2020a B. S. Vivek and R. Venkatesh Babu. Regularizers for single-step adversarial training, 2020.

Vivek and Babu, 2020b B. S. Vivek and R. Venkatesh Babu. Single-step adversarial training with dropout scheduling. In CVPR, pages 947–956, 2020.

Wang and Zhang, 2019 Jianyu Wang and Haichao Zhang. Bilateral Adversarial Training: Towards Fast Training of More Robust Models Against Adversarial Attacks. In ICCV, 2019.

Wang et al., 2019 Yisen Wang, Xingjun Ma, James Bailey, Jinfeng Yi, Bowen Zhou, and Quanquan Gu. On the Convergence and Robustness of Adversarial Training. In ICLR, pages 6586–6595, 2019.

Wang et al., 2020 Yisen Wang, Diful Zou, Jinfeng Yi, James Bailey, Xingjun Ma, and Quanquan Gu. Improving Adversarial Robustness Requires Revisiting Misclassified Examples. In ICLR, 2020.

Wong et al., 2020 Eric Wong, Leslie Rice, and J. Zico Kolter. Fast is better than free: Revisiting adversarial training. ArXiv, abs/2001.03994, 2020.

Xie et al., 2020 Cihang Xie, Mingxing Tan, Boqing Gong, Alan Yuille, and Quoc V Le. Smooth adversarial training. arXiv preprint arXiv:2006.14536, 2020.

Xing et al., 2020 Yue Xing, Qifan Song, and Guang Cheng. On the Generalization Properties of Adversarial Training. arXiv preprint arXiv:2008.06631, 2020.

Yang et al., 2020a Huanrui Yang, Jingyang Zhang, Hongliang Dong, Nathan Inkawhich, Andrew Gardner, Andrew Touchet, Wesley Wilkes, Heather Berry, and Hai Li. DIVERGE: Diversifying Vulnerabilities for Enhanced Robust Generation of Ensembles. In NeurIPS, 2020.

Yang et al., 2020b Yao-Yuan Yang, Cyrus Rashtchian, Hongyang Zhang, Ruslan Salakhutdinov, and Kamalika Chaudhuri. A closer look at accuracy vs. robustness, 2020.

Yin et al., 2019 Dong Yin, Kannan Ramchandran, and Peter Bartlett. Rademacher Complexity for Adversarially Robust Generalization. In ICMl, 2019.

Yuan et al., 2019 Xiaoyong Yuan, Pan He, Qile Zhu, and Xiaolin Li. Adversarial Examples: Attacks and Defenses for Deep Learning. IEEE TNNLS, 2019.

Zhai et al., 2019 Runtian Zhai, Tianle Cai, Di He, Chen Dan, Kun He, John E Hopcroft, and Liwei Wang. Adversarially Robust Generalization Just Requires More Unlabeled Data. CoRR, abs/1906.0, 2019.

Zhang and Wang, 2019 Haichao Zhang and Jianyu Wang. Defense Against Adversarial Attacks Using Feature Scattering-based Adversarial Training. In NeurIPS, 2019.

Zhang and Xu, 2020 Haichao Zhang and Wei Xu. Adversarial interpolation training: A simple approach for improving model robustness, 2020.
[Zhang et al., 2019a] Dinghuai Zhang, Tianyuan Zhang, Yipeng Lu, Zhanxing Zhu, and Bin Dong. You only propagate once: Accelerating adversarial training via maximal principle. *ArXiv*, abs/1905.00877, 2019.

[Zhang et al., 2019b] Hongyang Zhang, Yaodong Yu, Jiantao Jiao, Eric Xing, Laurent El Ghaoui, and Michael Jordan. Theoretically Principled Trade-off between Robustness and Accuracy. In *ICML*, 2019.

[Zhang et al., 2020] Jingfeng Zhang, Xilie Xu, Bo Han, Gang Niu, Lizhen Cui, Masashi Sugiyama, and Mohan Kankanhalli. Attacks Which Do Not Kill Training Make Adversarial Learning Stronger. In *ICML*, volume 119, 2020.