A Closer Look at $R_D$ and $R_{D^*}$

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The measurement of $R_D$ ($R_{D^*}$), the ratio of the branching fraction of $\overline{B} \rightarrow D\tau\bar{\nu}_\tau (\overline{B} \rightarrow D^*\tau\bar{\nu}_\tau)$ to that of $\overline{B} \rightarrow Dl\bar{\nu}_l (\overline{B} \rightarrow D^*l\bar{\nu}_l)$, shows 1.9σ (3.3σ) deviation from its Standard Model (SM) prediction. The combined deviation is at the level of 4σ according to the Heavy Flavour Averaging Group (HFAG). We perform an effective field theory analysis (at the dimension 6 level) of these potential New Physics (NP) signals assuming $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance. We first show that, in general, $R_D$ and $R_{D^*}$ are theoretically independent observables and hence, their theoretical predictions are not correlated. We identify the operators that can explain the experimental measurements of $R_D$ and $R_{D^*}$ individually and also together. Motivated by the recent measurement of the $\tau$ polarisation in $\overline{B} \rightarrow D^*\tau\bar{\nu}_\tau$ decay, $P_{\tau}^{D^*}$ by the BELLE collaboration, we study the impact of a more precise measurement of $P_{\tau}^{D^*}$ (and a measurement of $P_{\tau}^{D}$) on the various possible NP explanations. Furthermore, we show that the measurement of $R_{D^*}$ in bins of $q^2$, the square of the invariant mass of the lepton neutrino system, along with the information on $\tau$ polarisation, can completely distinguish the various operator structures.

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1. Introduction

The quantity $R_{D^{(*)}}$ is defined as the following ratio between two branching ratios:

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\bar{\nu})}$$  \hspace{1cm} (1.1)

where $l = e, \mu$. This quantity, being a ratio, is a ‘clean’ observable devoid of the systematic uncertainties that plague individual measurements of branching ratios. Experimental measurements of these two quantities - 0.397 ± 0.028 for $R_D$ and 0.316 ± 0.019 for $R_{D^{*}}$ - don’t match with the theoretical predictions from the Standard Model (SM) - 0.300 ± 0.011 for $R_D$ and 0.254 ± 0.004 for $R_{D^{*}}$. This corresponds to deviations 1.9σ and 3.3σ significance for $R_D$ and $R_{D^{*}}$ respectively, while the discrepancy for the two taken together is quite large ~ 4σ.\footnote{It is worth noting that even though the quoted results suggest a large deviation, a recent measurement of $R_{D^{*}}$ by the BELLE collaboration \cite{4} is consistent with the SM value, although the measurement is quite imprecise.}

This might well be a signal for new physics and we perform a model-independent analysis of the process using six-dimensional operators; in this analysis, we assume that any NP only affects the third leptonic generation.

Besides $R_D$ and $R_{D^{*}}$, we also consider the binned value of $R_D$ and $R_{D^{*}}$, the polarisation of the final state $\tau$ lepton, $P^{D}_{\tau}$ and $P^{D^{*}}_{\tau}$ and the forward-backward asymmetry in the two processes, $\mathcal{A}^{D}_{FB}$ and $\mathcal{A}^{D^{*}}_{FB}$. While a recent measurement of $P^{D^{*}}_{\tau}$ has been reported by BELLE for the first time (although with large errors) \cite{4}, none of the other quantities have been experimentally measured as yet. The definitions of the observables is given below:

\begin{align*}
\text{Binned } R_{D^{(*)}} : R_{D^{(*)}}[q^2 \text{ bin}] &= \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)[q^2 \text{ bin}]}{\mathcal{B}(B \to D^{(*)}\bar{\nu})[q^2 \text{ bin}]} \hspace{1cm} (1.2) \\
\text{Tau Polarisation : } P^{D^{(*)}}_{\tau} &= \frac{\Gamma^{D^{(*)}}_{\tau}(+)}{\Gamma^{D^{(*)}}_{\tau}(+)} - \frac{\Gamma^{D^{(*)}}_{\tau}(-)}{\Gamma^{D^{(*)}}_{\tau}(-)} \hspace{1cm} (1.3) \\
\text{FB Asymmetry : } \mathcal{A}^{D^{(*)}}_{FB} &= \frac{\int_{-\pi/2}^{\pi/2} \frac{d\Gamma^{D^{(*)}}_{\tau}(\cos \theta)}{d\theta} d\theta - \int_{-\pi/2}^{\pi/2} \frac{d\Gamma^{D^{(*)}}_{\tau}(\cos \theta)}{d\theta} d\theta}{\int_{-\pi/2}^{\pi/2} \frac{d\Gamma^{D^{(*)}}_{\tau}(\cos \theta)}{d\theta} d\theta - \int_{-\pi/2}^{\pi/2} \frac{d\Gamma^{D^{(*)}}_{\tau}(\cos \theta)}{d\theta} d\theta} \hspace{1cm} (1.4)
\end{align*}

The branching ratio can be written as

$$\frac{d^2\mathcal{B}^{D^{(*)}}_{\tau}}{dq^2 d(cos \theta)} = \mathcal{N} \left| p_{D^{(*)}} \right| \left( a^{D^{(*)}}_{l} + b^{D^{(*)}}_{l} \cos \theta + c^{D^{(*)}}_{l} \cos^2 \theta \right)$$  \hspace{1cm} (1.5)

where

$$\mathcal{N} = \frac{\tau_B G_F^2 |V_{cb}|^2 q^2}{256\pi^3 M_B^2} \left( 1 - \frac{m^2_{\ell}}{q^2} \right)^2 \quad \text{and} \quad \left| p_{D^{(*)}} \right| = \frac{\sqrt{\lambda(M_B^2, M_{D^{(*)}}^2, q^2)}}{2M_B}$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$ and $\theta$ is the angle between the lepton and $D^{(*)}$ meson in the lepton-neutrino centre-of-mass frame.

The decay amplitude for the process can be factorised into two parts - the hadronic part an the leptonic part. The hadronic part of the decay amplitude cannot be calculated exactly and is parameterised using form factors. These form factors are calculated in some theoretical and numerical framework and, in this work, we choose to simply borrow those results.\footnote{In principle, various differential distributions are also sensitive to the different NP Lorentz structures, see e.g., \cite{3}.}
2. Lagrangian and Operator Basis

The effective six-dimensional Lagrangian for $b \to c \ell \bar{\nu}_\ell$ we use for the analysis is given by:

\begin{align*}
\mathcal{O}^{\text{chf}}_{\text{VL}} &= [\bar{c} \gamma^\mu b] [\bar{\ell} \gamma_\mu P_L \nu] \\
\mathcal{O}^{\text{chf}}_{\text{AL}} &= [\bar{c} \gamma^\mu \gamma_5 b] [\bar{\ell} \gamma_\mu P_L \nu] \\
\mathcal{O}^{\text{chf}}_{\text{SL}} &= [\bar{c} b] [\bar{\ell} P_L \nu] \\
\mathcal{O}^{\text{chf}}_{\text{PL}} &= [\bar{c} \gamma_5 b] [\bar{\ell} P_L \nu] \\
\mathcal{O}^{\text{chf}}_{\text{TL}} &= [\bar{c} \sigma^{\mu\nu} b] [\bar{\ell} \sigma_{\mu\nu} P_L \nu]
\end{align*}

and the set of Wilson Coefficients (WCs) corresponding to these operators are defined at the renormalization scale $\mu = m_b$.

In the SM, we would have $C^{\text{chf}}_{\text{VL}} = -C^{\text{chf}}_{\text{AL}} = 1$. We wish to go beyond the SM, but we shall respect the full gauge invariance of the SM and consequently only consider the operators listed on the left in (2.1). Further, since it is difficult to build a microscopic model with tensor interaction, we neglect its contribution in this note. (For the study of tensor operators, refer to the Appendix of [1]).

3. Form Factors

3.1 For $B \to D$ decay

The non-zero hadronic matrix elements for $\bar{B} \to D$ transition (ignoring the tensor) are parameterized by

\begin{align*}
\langle D(p_D,M_D)|\bar{c}\gamma^\mu b|\bar{B}(p_B,M_B)\rangle &= F_+(q^2) \left[ (p_B + p_D)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] \\
&+ F_0(q^2) \frac{M_B^2 - M_D^2}{q^2} q^\mu \\
\langle D^*(p_D',M_D')|\bar{c}\gamma_\mu b|\bar{B}(p_B,M_B)\rangle &= F_0(q^2) \frac{M_B^2 - M_D^2}{m_b - m_c} A_1(q^2)
\end{align*}

Calculations for the form factors $F_0(q^2)$ and $F_+(q^2)$ are known in a lattice framework [5]. The axial vector and the pseudoscalar matrix elements are zero from symmetry considerations and thus only the WCs $C^{\text{chf}}_{\text{VL}}$ and $C^{\text{chf}}_{\text{SL}}$ contribute to this decay.

3.2 For $B \to D^*$ decay

The non-zero hadronic matrix elements for $\bar{B} \to D^*$ transition are parameterised by

\begin{align*}
\langle D^*(p_D',M_D')|\bar{c}\gamma_\mu b|\bar{B}(p_B,M_B)\rangle &= i \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu*} p^\rho_B p_D^{\sigma*} \frac{2\mathbf{V}(q^2)}{M_B + M_D'} \\
\langle D^*(p_D',M_D')|\bar{c}\gamma_\mu \gamma_5 b|\bar{B}(p_B,M_B)\rangle &= 2M_D' \frac{\varepsilon^* \cdot q}{q^2} q_\mu A_0(q^2) + (M_B + M_D') \left[ \varepsilon_5^* - \frac{\varepsilon^* \cdot q}{q^2} q_\mu \right] A_1(q^2) \\
&- \frac{\varepsilon^* \cdot q}{M_B + M_D'} \left[ (p_B + p_D')_\mu - \frac{M_B^2 - M_D'^2}{q^2} q_\mu \right] A_2(q^2) \\
\langle D^*(p_D',M_D')|\bar{c}\gamma_\mu \gamma_5 b|\bar{B}(p_B,M_B)\rangle &= -\varepsilon^* \cdot q \frac{2M_D'}{m_b + m_c} A_0(q^2)
\end{align*}

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While no lattice calculations exist for the form factors in this case, they have been calculated in a Heavy Quark Effective Theory (HQET) framework [6] and we borrow those results. In this case, symmetry dictates that the scalar current is zero and thus there is no contribution to the decay width from $C_{SL}^\tau$.

### 3.3 Independence of $R_D$ and $R_{D^*}$

We see that while $C_{VL}^\tau$ and $C_{SL}^\tau$ contribute to the $B \to D$ decay process, $C_{V_L}^\tau$, $C_{A_L}^\tau$, and $C_{P_L}^\tau$ contribute to the other one. Thus, given the independence of the WCs, the two processes are independent of each other since they depend of different sets of WCs. In other words, $R_D$ and $R_{D^*}$ are theoretically independent measurements and allow for separate explanations.

### 4. Explaining $R_D$ Alone

The quantities $a_D^{\tau}$, $b_D^{\tau}$, and $c_D^{\tau}$ (in 1.5) can be calculated for a particular helicity of the final state lepton using a helicity amplitude approach. The complete expressions are given in [1] and it is not repeated here. Since only $C_{VL}^\tau$ and $C_{SL}^\tau$ are relevant, we can plot $R_D$ as a variation of the two WCs and note the range of values for which it satisfies the experimental bounds. This is done in Fig. 1.

![Figure 1](image1.png)

**Figure 1:** The dependence of $R_D$ with respect to the variation of the WCs $C_{VL}^\tau$ (left) and $C_{SL}^\tau$ (right), where the red (brown) band corresponds to the $1\sigma$ ($2\sigma$) value on the experimental measurement.

We can use this range of the WCs to make a prediction of the value of the binned $R_D$, and for the values of $P_D^{\tau}$ and $\alpha_{FB}^D$. These are shown in Fig. 2 and Fig. 3 respectively.

![Figure 2](image2.png)

**Figure 2:** The binwise $R_D$ for four $q^2$ bins. On the left, $C_{VL}^\tau$ is varied, while on the right, $C_{SL}^\tau$ is varied within their $1\sigma$ allowed ranges.

where the red (brown) band corresponds to the $1\sigma$ ($2\sigma$) value on the experimental measurement.
5. Explaining $R_{D^*}$ Alone

We can carry out a similar treatment for the case of $B \to D^*$ decay. In this case, three WCs - $C^\tau_{VL}$, $C^\tau_{AL}$ and $C^\tau_{PL}$ - contribute. The plots of $R_{D^*}$ as a function of the different WCs are given in Fig. 4 As before, the 1σ (2σ) bands are indicated by the red (brown) bands. The prediction for

![Figure 3: Predictions for the polarisation fraction $P_\tau(D)$ (left) and $\omega_{FB}^D$ (right)](image)

![Figure 4: The dependence of $R_{D^*}$ with respect to the variation of the WCs $C^\tau_{VL}$ (left), $C^\tau_{AL}$ (middle) and $C^\tau_{PL}$ (right). A thin vertical line shows the SM values of the WCs.](image)

![Figure 5: The binwise $R_{D^*}$ for four $q^2$ bins. On the left, $C^\tau_{VL}$ is varied, in the middle $C^\tau_{AL}$ is varied, and on the right, $C^\tau_{PL}$ is varied within their 1σ allowed ranges. The SM predictions are shown in red.](image)

the binned $R_{D^*}$ is given in Fig. 5. In this case, we do have a measurement of $P_\tau^{D^*}$, but it is quite imprecise. In the left plot of Fig. 6, the size of the errors indicated for the BELLE measurement is a projection with 20 $ab^{-1}$ data, which is expected to be collected by the year 2021; the central value
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**Figure 6:** Predictions for the polarisation fraction $P_\tau(D^*)$ (left), $\mathcal{R}_{FB}^{D^*}$ (right). In the left plot, the Belle II 20 ab$^{-1}$ projection is shown.

indicated is the current central value. As a matter of completion, we also plot the prediction for $\mathcal{R}_{FB}^{D^*}$ on the right of Fig. 6, although no measurement of this quantity exists as yet. We can combine

**Figure 7:** The predictions for $P_\tau^{D^*}$, $R_{D^*}$ in the last bin and $\mathcal{R}_{FB}^{D^*}$ are shown in three different planes for the ranges of the three WCs $C_{\tau V L}^\tau$, $C_{\tau A L}^\tau$ and $C_{\tau P L}^\tau$.

the predictions for the binned $R_{D^*}$ restricted to the highest $q^2$ bin, $P_\tau^{D^*}$ and $\mathcal{R}_{FB}^{D^*}$ to construct three planes. When plotted in these three planes, the regions of the allowed values of the WCs all separate out nicely as shown in Fig. 7. A future measurement of any two of these three observables would help in restricting us to a particular region, thus limiting the scope of any NP model.

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