Naturally Degenerate Neutrinos\footnote{Research partially supported by the Swiss National Foundation.}

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Abstract

The solar neutrino problem, atmospheric neutrino problem and the hot dark matter which seems to be required by galaxy formation can all be most economically described by three very degenerate neutrinos with a common mass of a few eV. Such neutrino masses could also be relevant for the recent data on double beta decay. We provide an existence proof for such a scenario by constructing an explicit model in which the desired hierarchy of neutrino mass splittings arises naturally.

1. Introduction

There have been a number of potential indications for new physics in the neutrino sector over the past five years \cite{1}, but of these only two have (so far) survived further scrutiny by later generations of experiments. These are:

1. \textit{The Solar-Neutrino (ν⊙) Deficit:} The ν⊙ flux, now observed with Cl, water and Ga detectors, remains in conflict with the predictions of the Standard Solar Model (SSM).
The observations appear to be consistent with (i) no suppression of the dominant \( p-p \) cycle neutrinos, (ii) partial suppression of the \( ^8\text{B} \) neutrinos, and (iii) virtually complete elimination of the \( ^7\text{Be} \) neutrinos.

2. The Atmospheric-Neutrino (\( \nu_{\text{atm}} \)) Problem: Both the Kamiokande and IMB detectors detect \( \nu_{\text{atm}} \)'s, but with a ratio of \( \nu_\mu/\nu_e \) which is roughly 60\% of what is theoretically expected — a result which is consistent with the null results of Fréjus and which more recently appears to be confirmed by SOUDAN-II measurements.

A third, more tentative, indication for new physics comes from cosmology and astrophysics [2]. There is now compelling evidence that the bulk of the matter of the universe is invisible, and reasonably persuasive arguments that much or most of this dark matter is not baryons. Furthermore, the observed distribution of matter on the largest scales, together with the recently measured pattern of fluctuations in the cosmic microwave background, appear to indicate that roughly 30\% of this particle dark matter should have been relativistic during the crucial epoch for galaxy formation. If this hot, dark matter should consist of the known neutrinos, certainly the most conservative assumption, then their masses should satisfy \( \sum_i m_{\nu_i} \simeq 7 \) eV.

As has recently been pointed out [3], all three of these hints for new physics in the neutrino sector can be very economically understood in terms of a specific pattern of masses involving only the three known neutrino species. Moreover, for only three neutrinos, the required pattern of masses and mixings is very tightly constrained by the requirement that all three phenomena be incorporated. The conditions are:

1. In order to account for the \( \nu_{\text{atm}} \) anomaly, one pair (either \( \nu_e - \nu_\mu \) or \( \nu_\mu - \nu_\tau \)) of neutrinos must have large mixings, \( \sin^2 2\theta_{ij} \simeq 0.5 \), with a squared-mass splitting: \( \Delta m_{ij}^2 \equiv m_{\nu_i}^2 - m_{\nu_j}^2 \simeq 10^{-2} \text{ eV}^2 \).

2. In order to provide an MSW explanation for the \( \nu_\odot \) deficit, another pair (either \( \nu_e - \nu_\mu \) or \( \nu_e - \nu_\tau \)) of neutrinos must have a squared-mass splitting: \( \Delta m_{ij}^2 \simeq 10^{-5} \text{ eV}^2 \) and either comparatively large mixings, \( \sin^2 2\theta_{ij} \simeq 0.5 \), or small mixings, \( \sin^2 2\theta_{ij} \simeq 10^{-2} \). Furthermore, if it is \( \nu_e - \nu_\mu \) oscillations that are the explanation for the \( \nu_{\text{atm}} \) anomaly, and it is \( \nu_e - \nu_\tau \) that resonantly oscillates inside the sun, then both of these oscillations deplete the solar neutrino flux, and the phenomenologically acceptable range of \( \nu_e - \nu_\tau \) mass splittings is enlarged to include \( \Delta m_{e\tau}^2 \simeq 10^{-4} \text{ eV}^2 \) [4].

3. Finally, the three species of light neutrinos can make up the required hot, dark matter.
provided their masses are degenerate, with their common mass being given by $m_{\nu_i} \simeq 2$ eV.

It is noteworthy that $m_{\nu_e}$ in this range would imply a $\beta\beta_{0\nu}$ signal that is just consistent with the present upper bound [5], $m_{\nu_e} \lesssim 1$ eV, keeping in mind a possible uncertainty of a factor of two in this bound due to difficulties in computing nuclear matrix elements. This would be of particular interest should the presently observed 2-σ bump at the electron endpoint persist in the $\beta\beta$ data.

The most striking feature about the required neutrino mass pattern is that it is very degenerate, with the pairs of oscillating states being respectively split by only $10^{-2}$ eV (and $10^{-4} - 10^{-5}$ eV) out of a total mass which is of order 1 eV. As was remarked in some of refs. [3], such a degenerate pattern is difficult to come by in a natural way in renormalizable gauge theories and, in particular, at first sight appears not to be what is expected of a ‘see-saw’ mechanism for generating neutrino masses. A corollary is that we stand to learn something interesting about the nature of the physics which underlies these neutrino masses if this should prove to be how neutrino masses are realized in nature.

It is the purpose of the present note to furnish an existence proof for models which can naturally generate the required type of light-neutrino masses and mixings. In fact, we do so within a see-saw framework in which the light-neutrino masses are intimately related to those of a family of heavy, sterile neutrinos. In particular, we imagine the existence of an approximate family symmetry which mixes the light neutrinos amongst themselves, but which is broken by the couplings of some of the heavy, singlet neutrinos. Then the required hierarchy of small mass splittings is obtained in terms of heavy mass ratios, and do not require the introduction of new dimensionless couplings that are much smaller than unity.

We organize our presentation in the following way. Section 2 describes some general features of a light-neutrino mass matrix which satisfies all of the phenomenological constraints. We explicitly exhibit its masses and mixing angles, and demonstrate that these include the parameter ranges that are required. An explicit model which produces this mass matrix is then constructed in Section 3. We finish with some concluding remarks in Section 4.
2. A Desirable Mass Matrix

The main observation for the purposes of model building is that the light-neutrino mass matrix should be a sum of successively smaller contributions,

\[ m = m_0 + m_1 + m_2, \]  

(1)

with \( m_0 = m_0 \mathbf{I} \) proportional to the 3 × 3 unit matrix, \( m_1 \) a rank-one matrix with elements which are \( O(10^{-2} m_0) \) in magnitude, and \( m_2 \) a generic matrix whose elements are \( O(10^{-4} m_0) \).

A representative example of a matrix of this type, and which is of the form that is actually generated by the models of the next section, is:

\[ m = m_0 \left[ \mathbf{I} + \epsilon \mathbf{e} \mathbf{e}^T - \delta (\mathbf{e} \mathbf{f}^T + \mathbf{f} \mathbf{e}^T) + \xi \mathbf{f} \mathbf{f}^T \right]. \]  

(2)

In this expression, \( m_0 \approx 2 \text{ eV} \) sets the scale of the common overall mass, \( \mathbf{e} \) and \( \mathbf{f} \) are arbitrary unit vectors in neutrino flavour-space, and \( \mathbf{e}^T \mathbf{e} \equiv \sum_{i=1}^{3} e_i^2 = 1 \) etc. \( \epsilon, \delta \) and \( \xi \) are three quantities which are all taken to be much smaller than unity.

In what follows it is worth keeping in mind the sizes for \( \epsilon, \delta \) and \( \xi \) that actually arise in the models considered in later section. There are two cases of interest. In Case I we typically find \( \epsilon \sim \epsilon^2 \gg \delta \sim \epsilon^3 \gg \xi \sim \epsilon^4 \), where \( \epsilon \lesssim 0.1 \) is a smallish parameter of the model. For case II, however, the sizes of \( \epsilon \) and \( \xi \) are reversed, so that: \( \epsilon \sim \epsilon^4 \ll \delta \sim \epsilon^3 \ll \xi \sim \epsilon^2 \).

It is straightforward to diagonalize the matrix of eq. (2). Denoting the cosine of the angle, \( \alpha \), between the vectors \( \mathbf{e} \) and \( \mathbf{f} \) by \( c_\alpha \), we have \( \mathbf{e} \cdot \mathbf{f} = c_\alpha \), and the eigenvalues of \( m \) are given by:

\[
m_{\pm} = m_0 \left\{ \left( 1 + \frac{\epsilon + \xi}{2} - \delta c_\alpha \right) \pm \frac{1}{2} \left[ \epsilon^2 + \xi^2 + 4\delta^2 - 4\delta(\epsilon + \xi)c_\alpha - 2\epsilon\xi(1 - 2c_\alpha^2) \right]^{1/2} \right\}
\]

and:

\[ m_3 = m_0. \]  

(3)

The corresponding eigenvectors are:

\[
\begin{pmatrix}
\mathbf{e}_+ \\
\mathbf{e}_-
\end{pmatrix} = \begin{pmatrix}
c_\theta & s_\theta \\ -s_\theta & c_\theta
\end{pmatrix} \begin{pmatrix}
\mathbf{e} \\
\mathbf{e}'
\end{pmatrix}, \quad \text{and} \quad \mathbf{e}_3 = \mathbf{e} \times \mathbf{e}',
\]  

(4)
where $e'$ is a unit vector which is defined to be orthogonal to $e$ and coplanar with $e$ and $f$. It is given explicitly by: $e' = \frac{1}{s_\alpha} (f - c_\alpha e)$. Finally, the angle $\theta$ which appears in the rotation matrix in eq. (4) is given by:

$$\tan 2\theta = \frac{2(\xi c_\alpha - \delta) s_\alpha}{\epsilon - 2\delta c_\alpha - \xi(1 - 2c_\alpha^2)}.$$  

(5)

If we take the original mass matrix to be in a basis for which the charged leptons are diagonal, then the relation between the weak-interaction and the propagation eigenstates is:

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
e_1 c_\theta + \frac{s_\theta}{s_\alpha} (f_1 - e_1 c_\alpha) & -e_1 s_\theta + \frac{c_\theta}{s_\alpha} (f_1 - e_1 c_\alpha) & \frac{1}{s_\alpha} (e_2 f_3 - e_3 f_2) \\
e_2 c_\theta + \frac{s_\theta}{s_\alpha} (f_2 - e_2 c_\alpha) & -e_2 s_\theta + \frac{c_\theta}{s_\alpha} (f_2 - e_2 c_\alpha) & \frac{1}{s_\alpha} (e_3 f_1 - e_1 f_3) \\
e_3 c_\theta + \frac{s_\theta}{s_\alpha} (f_3 - e_3 c_\alpha) & -e_3 s_\theta + \frac{c_\theta}{s_\alpha} (f_3 - e_3 c_\alpha) & \frac{1}{s_\alpha} (e_1 f_2 - e_2 f_1)
\end{pmatrix}
\begin{pmatrix}
\nu_+ \\
\nu_- \\
\nu_3
\end{pmatrix}$$

(6)

Notice that for one of the cases of practical interest, where $\epsilon \sim \epsilon^2 \gg \delta \sim \epsilon^3 \gg \xi \sim \epsilon^4$, we have the following approximate expressions:

$$m_+ = m_0 \left[ 1 + \epsilon - 2c_\alpha \delta + c_\alpha^2 \xi + \frac{\delta^2 s_\alpha^2}{\epsilon} + O(\epsilon^5) \right],$$

$$m_- = m_0 \left[ 1 + s_\alpha^2 \xi - \frac{\delta^2 s_\alpha^2}{\epsilon} + O(\epsilon^5) \right],$$

(7)

$$\tan 2\theta \approx \frac{-2\delta s_\alpha}{\epsilon} \sim O(\epsilon).$$  

(Case I)

These expressions imply the mass splittings: $\Delta m_{1+}^2 \sim \Delta m_{2+}^2 \sim O(\epsilon^2)$ and $\Delta m_{3+}^2 \sim O(\epsilon^4)$. Notice that the smaller splitting here is $O(\epsilon^4)$ even though the parameter $\delta$ is itself of order $\epsilon^3$.

In the alternative limit, Case II, we have approximate eigenvalue expressions which are simply obtained from eq. (7) by making the replacement $\epsilon \leftrightarrow \xi$, and so which give mass splittings which are the same order in $\epsilon$ as for Case I. By contrast, the mixing angle, $\theta$, is now given approximately by:

$$\tan 2\theta \approx \frac{-2c_\alpha s_\alpha}{1 - 2c_\alpha^2} = \tan 2\alpha \sim O(1)$$  

(Case II).  

(8)

Before turning to model building, we first pause to explicitly display two representative sets of parameters, corresponding to the two possible neutrino-mixing scenarios. Firstly,
for $\nu_e - \nu_\mu$ atmospheric-neutrino oscillations, and $\nu_e - \nu_\tau$ MSW mixing we take Case I, for which $\epsilon \simeq 10^{-2}, \delta \simeq 10^{-3}, \xi \simeq 10^{-4}$ and $\theta \simeq 0.1$. In this case if we choose $e_3 = f_1 = f_2 = 0, f_3 = 1, e_1 = s_\beta$ and $e_2 = c_\beta$, for some angle $\beta$, then $c_\alpha = 0$ and the mixing matrix of eq. (6) simplifies to:

$$
V = \begin{pmatrix}
 s_\beta c_\theta & -s_\beta s_\theta & c_\beta \\
 c_\beta c_\theta & -c_\beta s_\theta & -s_\beta \\
 s_\theta & c_\theta & 0
\end{pmatrix}.
$$

(9)

We see that the atmospheric oscillations are, in this scenario, controlled $\sin^2 2\beta \simeq 0.5$, and that MSW oscillations are governed by $\sin^2 2\theta \simeq 10^{-2}$. Notice that this last choice follows from the general result, $\theta \sim O(\epsilon)$, for Case I.

Alternatively, if we wish to describe atmospheric neutrinos with $\nu_\mu - \nu_\tau$ oscillations, and the MSW effect occurs with $\nu_e - \nu_\mu$ oscillations we work with Case II. Then $\epsilon \simeq 10^{-4}, \delta \simeq 10^{-3}, \xi \simeq 10^{-2}$ and $\theta = \alpha \sim O(1)$. We choose $e_3 = 0, e_2 = c_\beta, e_1 = s_\beta, f_3 = s_\alpha, f_2 = c_\alpha c_\beta$ and $f_1 = c_\alpha s_\beta$. In this case we again obtain the same mixing matrix as in eq. (9), but with $\theta = \alpha$ not required to be small. The large-angle $\nu_\mu - \nu_\tau$ oscillations are now achieved by choosing $\sin^2 2\alpha \simeq 0.5$, and the MSW oscillations are produced by taking $\sin^2 2\beta \simeq 10^{-2}$.

### 3. An Underlying Model

We now require a model which naturally produces a light-neutrino mass matrix as in eq. (2). In order to do so we must ensure, as a first approximation, a degenerate set of eV neutrinos. We can easily do so by demanding an approximate symmetry under which the light neutrinos rotate into one another in a real representation. We therefore require an approximate $O(3)$ symmetry under which the usual standard-model (SM) neutrinos, $\nu_i$, transform according to $\nu_i \to O_{ij} \nu_j$, with $O_{ij}$ a real, $3 \times 3$, orthogonal matrix. This symmetry can only be approximate since it is explicitly broken in the standard model by the charged-current weak interactions (or, equivalently, the charged lepton masses). An eV-size mass for these neutrinos is then obtained by introducing three electroweak-singlet neutrinos, $s_i$, which also transform as triplets under the approximate $O(3)$ symmetry.

With this particle content, the most general renormalizable and $O(3)$-invariant Yukawa couplings and mass terms (beyond the usual SM ones) are:

$$
L_{\text{inv}} = -\frac{M}{2} (s_i s_i) - \lambda (L_i s_i) H + \text{h.c.},
$$

(10)
where $H = (\phi^+ \phi^0)$ and $L_i = (\nu_i e_i)$ are respectively the SM Higgs and lepton doublets. If $v = \langle H \rangle \simeq 174 \text{ GeV}$ is the SM Higgs vev, and if $\lambda v \ll M$, then the light neutrinos acquire degenerate masses whose size is given by $m_0 = \lambda^2 v^2 / M$. Choosing $m_0 \simeq 2 \text{ eV}$ then requires $M/\lambda^2 \simeq 2 \times 10^{13} \text{ GeV}$.

For the theory as stated so far, the three light neutrinos are not exactly degenerate because electroweak radiative corrections can split them. At one loop this splitting is proportional to the corresponding charged-lepton masses, although this need not be so for higher loops [7]. Unfortunately, these radiatively-generated splittings are too small to account for the required mass pattern and so we must introduce a further source of $O(3)$ symmetry breaking. In order to produce the desired mass matrix of eq. (2), we require $O(3)$-breaking order parameters, $e_i$ and $f_i$, which transform as triplets. We therefore supplement the model with one more singlet neutrino, $N$, which is also neutral under $O(3)$. We permit the couplings of $N$ to the other fields to explicitly break the $O(3)$ symmetry. The most general such renormalizable couplings for the $N$ field are then

$$\mathcal{L}_N = -\frac{m}{2} (N N) - \mu_i (s_i N) - g_i (L_i N) H + \text{h.c.} \quad (11)$$

We return to the question of how natural it is to choose these particular symmetry-breaking couplings below.

For $\lambda v, g_i v \ll M, m, \mu_i$, the complete mass matrix for the light neutrinos that is generated at tree level by the interactions of eqs. (10) and (11) has the form of eq. (2), with

$$m_0 = \frac{\lambda^2 v^2}{M}, \quad \epsilon = \frac{\mu^2}{m M}, \quad \delta = \frac{\mu g}{m \lambda}, \quad \xi = \frac{g^2 M}{M^2 m}, \quad e_i = \frac{\mu_i}{\mu}, \quad f_i = \frac{g_i}{g} \quad (12)$$

where

$$\mu^2 \equiv \sum_i \mu_i^2 \quad \text{and} \quad g^2 \equiv \sum_i g_i^2. \quad (13)$$

Representative values for model parameters which reproduce Cases I and II of the previous sections are easily found. For example, Case I is reproduced by any of the following choices ($\varepsilon = 0.1$): (Ia) $m \sim M, \mu_i \sim \varepsilon M, g_i \sim \varepsilon^2 \lambda$; (Ib) $\mu_i \sim M \sim \varepsilon^2 m, g_i \sim \varepsilon \lambda$; (Ic) $M \sim \varepsilon^4 m, \mu_i \sim \varepsilon^3 m, g_i \sim \lambda$. Similarly, Case II is reproduced by any of the following:
\(\text{(IIa) } M \sim \varepsilon^2 m, \mu_i \sim \varepsilon^3 m, g_i \sim \lambda; \text{ (IIb) } \mu_i \sim \varepsilon^2 M, M \sim m, g_i \sim \varepsilon \lambda; \text{ (IIc) } \mu_i \sim M \sim \varepsilon^4 m, \lambda \sim \varepsilon g_i.\)

We finally consider the issue of naturalness. In this model, the required small splittings amongst the light-neutrino masses are ultimately due to the relative size of the symmetry breaking terms, \(\mu_i\) and \(g_i\), in comparison with the \(O(3)\)-invariant couplings, \(M, m\) and \(\lambda\). Notice however that the ratios of these couplings are actually not required to be extremely small, since they may all be chosen to be no smaller than \(\varepsilon^2 \approx 0.01\), say (c.f. examples (Ia), (Ib) and (IIb) above). This is more than adequate for producing the desired mass pattern.

The final question concerns loop corrections. Is it consistent to choose the \(O(3)\)-breaking interactions as large as we have and yet not to include any \(N\)-independent \(O(3)\)-breaking terms, such as \((s_i s_j), (L_i s_j)H\) or \((L_i L_j)HH\)? The answer to this question is a resounding ‘yes’, although this is subject to certain weak constraints on the various couplings of the model, which we now describe.

It is clear that loops involving the symmetry-breaking \(N\)-interactions must inevitably induce all possible \(O(3)\)-breaking couplings amongst the other neutrinos. But the point is that unless \(g_i\) or \(g_i/\lambda\) should be much larger than the range 0.1 to 0.001, these induced terms only perturb the above mass eigenvalues by amounts that are smaller than a part in \(10^4 - 10^5\), and so they are negligible in comparison to the effects which we have considered. This is particularly clear for the induced contributions to symmetry-breaking operators like \((s_i s_j)\), since the one-loop contributions to these are strongly suppressed by small mass insertions or loop factors. (These suppressions arise because any symmetry-breaking loop must necessarily involve a virtual \(N\) field, and this only becomes possible, given only two external \(s_i\) lines, at two loops and beyond, or at one loop with a number of mass insertions.)

Loop-induced symmetry-breaking operators of the form \((L_i s_j)H\) and \((L_i L_j)HH\ can arise at one loop, typically being generated through flavour-changing contributions to the various neutrino kinetic terms. Their generic size is respectively of order \(\lambda g_i g_j/(16\pi^2)\) and \(g_i g_j/(16\pi^2)\), which is innocuous provided that \(g_i\) and \(g_i/\lambda\) are smaller than \(O(0.1)\). (This therefore excludes examples Ic and IIa above.) The most dangerous of these contributions are those which are also enhanced by large logarithms, such as the one-loop contribution to the operator \((L_i L_j)HH\) which is induced by the Higgs self-coupling \(\zeta (H^\dagger H)^2\). This graph contributes an amount to \(\xi\) which is \(\zeta\)-dependent, but of order \(\zeta(g/4\pi\lambda)^2 \ln(m^2_H/M^2)\). For
$\zeta \sim 0.1$ and for small Higgs mass, $m_H \sim 80$ GeV, this can be numerically as large as $\sim 0.1(g/\lambda)^2$. Such an enhancement can strengthen the bound on $g/\lambda$ by factors of 3 or so.

4. Concluding Remarks

In summary, by constructing an explicit underlying model we provide an existence proof for the recently-proposed economical scenario in which the solar- and atmospheric-neutrino data, and the existence of hot dark matter are all explained in terms of the masses and mixings of the three known light neutrinos. This picture is attractive because it is tightly constrained, and so is quite predictive. In particular, it implies a $\beta\beta_0\nu$ signal at the current experimental sensitivity.

The underlying model realizes this mass pattern within a see-saw type framework in which the light neutrinos mix with a family of heavier, sterile neutrinos. The degeneracies of the light neutrino masses emerge in this picture as the low-energy residue of an approximate $O(3)$ neutrino-flavour symmetry, which is broken only by the interactions of one sector of the heavy neutrinos. All of the desired small mass splittings amongst the light neutrinos then naturally arise as ratios of the sterile-neutrino masses, without requiring the introduction of small dimensionless Yukawa couplings.

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Note Added: Shortly after completing this paper we received several others which approach the same problem from the point of view of $SO(10)$ grand-unified theories [8].
5. References

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