Ordinary differential equations: students’ difficulty in solve the algorithm of the initial value problem with the integrating factor method

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Ordinary differential equations: students’ difficulty in solve the algorithm of the initial value problem with the integrating factor method

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Abstract. The algorithm of mathematical proof is very important possessed by students, especially in Mathematics Education Study Program in Differential Equation. One of the materials in Differential Equations is the integral factor. Many students have difficulties on the material factor of integral. This study aims to (1) analyze student errors in terms of mathematical proof capability on the material factor of integral, (2) analyze the difficulties faced by students in the algorithm. Subjects subjected to research are 33 sixth semester students of Mathematics Education Program UIN SunanGunungDjati Bandung totalling 33 students. This research is a qualitative research. Data collection by using the test is a test of mathematical proof of material factor of integral. The results showed that 65% of students were unable to complete the proof, 35% of the students did not use the proof algorithm, and all the students did not perform the verification check.

1. Introduction
The differential equation is an equations that have one or more variables [1]. Differential Equation that have one variable is called ordinary differential equations (ODE), while differentials have two variables or more partial differential equations (PDE) [2] [3] [4].

Differential Equation can used for daily activity such as population or bacterial growth rates, heating and cooling, radioactive decay, drug elimination, compound interest, torricelli law, science and automotive [2][1][5][6][7][8][9][10]. Because the differential equation are used in various fields of importance to be understood by students [11].

One of the most important in solving differential equations is the stages in finding the right solution to solve the problem of differential equations. Because proof is important in mathematics [12][13]. However, in reality there are still difficulties in solving differential equations. So it is necessary to analyze the algorithm in finding solutions of differential equations. To solve or find solutions in differential equations there are several methods [14] [15] and calculus theories as the basis [16] [17]. This research is different between the others, because this research is about students’ difficulty in solve the initial value problem with the integrating factor method.
2. Theory
There are several theories used in solving ordinary differential equations.

2.1. Theorem 1 existence
If \( f(x,y) \) and \( \frac{\partial f}{\partial y} \) is continuous function for all points \((x,y)\) in the field if \( f(x,y) \) and \( \frac{\partial f}{\partial y} \) is continuous function for all points \((x,y)\) in a rectangular plane \( R \) and limited, is \((a) |f| \leq K, (b) \frac{\partial f}{\partial y} \leq M\), for all \((x,y)\) in \( R \), then the initial value problem 
\[ y' = f(x,y), y(x_0) = y_0 \]
At least have one solution \( y(x) \).

2.2. Differential equations exact and non exact
Let \( u(x,y) \) is function of the variable \( x \) and \( y \) which is defined in \( D \), so that \( u \) have a first partial continuous in defined area.

The total differential or exact is:
\[
du(x,y) = \frac{\partial u(x,y)}{\partial x} dx + \frac{\partial u(x,y)}{\partial y} dy,
\]
\[ \text{for all } (x,y) \in D. \tag{1} \]
A first order differential equation in form:
\[
M(x,y)dx + N(x,y)dy = 0 \tag{2}
\]
Called exact, if the right side is total or exact differential from function \( u(r,s) \), is:
\[
M(x,y)dx + N(x,y)dy = \frac{\partial u(x,y)}{\partial x} dx + \frac{\partial u(x,y)}{\partial y} dy,
\]
\[ \text{so differential equation (2) can be written as } du = 0. \text{ By integrating, the settlement of the equation is obtained in the form} \]
\[ u(x,y) = c \tag{4} \]
By comparing the components of equation (3), it can be concluded that equation (2) is exact if there is a function \( u(x,y) = c \) such that:
\[ (a) \frac{\partial u}{\partial x} = M, (b) \frac{\partial u}{\partial y} = N \tag{5} \]
Let \( M \) and \( N \) defined and have a continuous first partial on the plane of \( rs \), then:
\[
\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial x} \right) = \frac{\partial^2 u}{\partial y \partial x}, \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right) = \frac{\partial^2 u}{\partial x \partial y} \tag{6}
\]
From the assumption of continuity, the two derivatives are the same, then:
\[
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{7}
\]
This condition is not only necessary but also enough \( M \ dx + N \ dy \) to be the exact differential equation. If (1) exact, function \( u(x,y) \) can be determined using a systematic way. From (a), by integrating to \( r \), is obtained
\[ u(x, y) = \int Mdx + k(y) \] 

(8)

By \( s \) looked as constant and \( k(s) \) act as constanta of integration. To determine \( k(y) \), equations (8) let off towards \( y \), it is \( \frac{\partial u}{\partial y} \) and use (5) (b) to get \( \frac{dk}{dy} \) then integrate \( \frac{dk}{dy} \) to get \( k(y) \).

Besides, being obtained from (a), \textit{we can use the form (5) (b) to obtain that form.}:

\[ u(x, y) = \int Mdx + k(y) \]

(9)

To get \( l(x) \), let off \( \frac{\partial u}{\partial y} \) from (9), use (5) (a) to get \( \frac{dl}{dx} \) and integrate it.

\textbf{Example}

Give the Differential Equations (ED):

\[(3x + 2y)dx + (2x + y)dy = 0\]

look for, is the differential equations exact? If that’s true, find the solution

\textbf{Answer:}

\textbf{First Step: Testing the accuration.}

For example \( M = 3x + 2y \) and \( N = 2x + y \). Then \( \frac{\partial M}{\partial y} = 2 \) and \( \frac{\partial N}{\partial x} = 2 \). Because \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \), so, the differential equations is exact.

\textbf{Second Step : Finding the differential equations.}

\[ u(x, y) = \int M dx + k(y) \]

\[ = \int (3x + 2y)dx + k(y) \]

\[ = \frac{3x^2}{2} + 2xy + k(y) \]

To find \( k(y) \), let off \( u \) towards \( y \) and use (5) (b), then obtained :

\[ \frac{\partial u}{\partial y} = 2x + \frac{dk}{dy} = N = 2x + y \]

That,

\[ \frac{dk}{dy} = y \rightarrow k(y) = \frac{1}{2} y^2 \]

So, the solution of the exact differential equations is

\[ u(x, y) = \frac{3x^2}{2} + 2xy + \frac{1}{2} y^2 = c \ or \]

\[ 3x^2 + 4xy + y^2 = c \]

\textbf{Third Step: Checking}

For checking, \( u(x, y) = c \) implicitly and will it show that \( \frac{dy}{dx} = -\frac{M}{N} \) or \( M dx + N dy = 0 \). In this matter, by let off \( u \) implicity towards \( x \), obtained :

\[ 6x + 4y + 4xy' + 2yy' = 0 \]

By \( M \) and \( N \) such as above, obtained \( M + Ny' = 0 \), so that,

\[ M(x, y)dx + N(x, y)dy = 0 \]
So, the solution is correct.

2.3. Factor integration
If a differential equations is not exact and shaped (2).

\[ P(x, y)dx + Q(x, y)dy = 0 \]  \hspace{1cm} (10)

If that equations multiply with a function \( F(x, y) \) which not zero, then the equations (10) be the exact of differential equations. It is

\[ FPdx + FQdy = 0 \]  \hspace{1cm} (11)

This Function \( F(x, y) \) called by factor integration from (10)
Determine integrating factor

\[ M(x, y)dx + N(x, y)dy = 0 \text{ and } FPdx + FQdy = 0 \]

then

\[ FPdx + FQdy = M(x, y)dx + N(x, y)dy \]

Theorem factor of intergration
If \( R(x) = \frac{1}{Q} \left( \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) \) then \( F(x) = exp \int R(x)dx \)

Or

If \( R(y) = \frac{1}{P} \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \) then \( F(y) = exp \int R(y)dy \)

3. Methods
This study using qualitative methods. Subjects subject to research are students of Mathematics Education UIN Sunan Gunung Djati Bandung 6th semester as many as 33 students. The researcher used purposive sampling technique in sampling. Data collection using test method. The test used is the mathematical test of integral factor matter in the course of differential equations. Students are given one problem about integral factor then result of student work analyzed about mathematical proof. There are two indicators analyzed that are algorithm and solution solution.

4. Result and discussion
Let’s solve the initial value problem of differential equations.

\[ (1 + 2x^2 + 4xy)dx + 2dy = 0, y(0) = 0 \]

Solving :

4.1. First step testing the accuration
From the differential equations above is known that \( P = 1 + 2x^2 + 4xy \) and \( Q = 2 \), then

\[ \frac{\partial P}{\partial y} = 4x, \text{and} \frac{\partial Q}{\partial x} = 0 \] so \( \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x} \) or not exact.

Because p.d. is not exact, then continue by finding the factor of integral.

\[ F(x) = exp \int 2x \, dx = exp(x^2) = e^{x^2} \]

Multiply \( F(x) \) to p.d. which exists till be p.d. new, it is
\[ e^x(1 + 2x^2 + 4xy)dx + 2e^x dy = 0 \]

Re-checking Step
\[
\frac{\partial F_P}{\partial y} = \frac{\partial(e^x(1 + 2x^2 + 4xy))}{\partial y} = 4xe^{x^2} \]
\[
\frac{\partial F_Q}{\partial x} = \frac{\partial(2e^{x^2})}{\partial x} = 4xe^{x^2} \]

\[ \frac{\partial F_P}{\partial y} = \frac{\partial F_Q}{\partial x} \rightarrow \text{ED Exact} \]

4.2. Second step finding the solution
\[
u = \int F_Qdy + l(x) = 2ye^{x^2} + l(x)\]
\[
\frac{\partial u}{\partial x} = 4xye^{x^2} + \frac{dl}{dx} = FP = e^{x^2}(1 + 2x^2 + 4xy) \]
\[
l(x) = xe^{x^2} \]

Then the new common solution of ED is
\[ 2ye^{x^2} + xe^{x^2} = c. \]
Because \( y(0) = 0 \), so, the initial requirement results \( c=0 \), till the solution of initial value problem ED is \( 2ye^{x^2} + xe^{x^2} = 0 \).

4.3. Third step checking
\[
\left(2ye^{x^2} + xe^{x^2}\right)' = (0)' \]
\[
(1 + 2x^2 + 4xy)e^{x^2} dy + 2e^x dx = 0 \]

The students which given by the test are the student took the subject twice. It was Differential Equations. It becomes the challenge because the student ever took this subject before. From the test which given, 65% the students can’t find the solution of initial value problem (IVP), 25% the students can’t finish the solution of ‘dari IVP, and just 1% the students who can find the solution of IVP. It’s mean the students can’t finish the solution which given. The result of student errors can show in figure 1a, and the result to find the solution from IVP can show in figure 1b.

**Figure 1a. Student error when use the differential.**

**Figure 1b. The result to find the solution from IVP.**
The result in use the algorithm the Students used algorithm but not finish it. 35% the students not use the algorithm to find the solution. And 1% the student use the algorithm and can finish it. So, it’s mean that the students ignore the algorithm to find the solution of ordinary differential equations (ODE) which given. And the most step that can’t finish it. It is the first step. The students don’t check the differential equations up which have given before. The students directly to the second step for finding the solution. This is same with the result of analyzing student errors in terms of differential equations. (18) the result can show in figure 2a and student error can show in figure 2b.

**Figure 2a.** The result in use the algorithm.  
**Figure 2b.** Student doesn’t finish the algorithm because error when they use differential.

5. **Conclusion**

Differential equations is very important, because it is used in many part of life. The Use of algorithm is very needed to find the solution of differential equations. The students’ difficulty and errors in solve the initial value problem with the integrating factor method is because they doesn’t use the differential as well. So, the differential and integral in ODE is very important.

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