Research on Piano Batch Scheduling Method Based on Constraint Model

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Abstract: Research on the batch scheduling problem of piano production, with the goal of optimizing the production cycle, using a two-segment matrix coding method to solve the problems of batch division and sub-batch sorting, combining multicolor sets with genetic algorithms, and proposing a genetic algorithm based on model constraints (MCGA) in order to solve the problem of slow solution speed and low efficiency in batch scheduling problems. And by comparing the production cycle of overall scheduling, equal batch scheduling, and unequal batch scheduling, the experimental results show that non-equal batching has more advantages, and the feasibility and effectiveness of the algorithm are shown through example simulation.

1. Introduction
J Piano Manufacturing Company is a large-scale piano manufacturer in China, with an annual production capacity of 50,000 units. In the actual production process, there are many types and large quantities of piano parts, and the craftsmanship between different products is very different, and it is difficult to meet the actual production needs. In order to shorten the product production cycle, improve production efficiency, and reduce production costs, therefore, studying the batch scheduling problem of piano production has good theoretical and practical significance.

In addition to the allocation of processing resources and processing sequencing, the batch production scheduling problem also considers the optimization of production batches, which is a typical NP problem. Many scholars have conducted research on it. Paris [1] used particle swarm algorithm to solve the batch scheduling problem in assembly workshop, but did not consider the influence of adjustment time on batch scheduling in actual production in the workshop. Zhou Chao [2] used genetic algorithm to solve the equal batch scheduling problem for flexible job shops. Although it proved that the equal batch scheduling scheme is better than the overall scheduling scheme, it did not further consider the non-equal batch scheduling. Wang Haiyan [3] et al. proposed a pre-division sub-batch scheme according to the size of the machine load in the workshop, combined with an adaptive differential evolution algorithm to solve the multi-resource workshop scheduling problem. It provides a good idea of sub-batch division for batch scheduling problems. Zeng Chuifei [4] compared the equal batching scheme and the unequal batching scheme for the batch scheduling problem of the assembly workshop, and proved that the unequal batching scheduling scheme is better than the equal batching scheduling scheme, but the algorithm calculation efficiency And the convergence rate still has room for improvement. Ju Quanyong [5] and others combined particle swarm algorithm and genetic algorithm, and merged the sorted adjacent similar workpieces into a sub-batch in the evolution process. This method can effectively solve the small batch problem, but when the problem scale increases, Will affect the efficiency of the solution.
In this paper, the wooden shell production area in piano production is used as the research object, combined with the actual workshop scheduling production, a piano production batch scheduling model including batch optimization is established. Based on the unequal batch strategy, the description is established by using the multi-color collection enclosure matrix. Constraint model of the constraint relationship among workpiece batches, processes and machines. A genetic algorithm based on model constraints is proposed. Combining the constraint model, complete the steps of crossover, mutation and fitness value calculation. Use the constraint model to ensure the effectiveness of the solution space, so as to improve the search efficiency and reduce the complexity of the algorithm. Finally, the optimization results of the algorithm proposed in this article are compared and analyzed with the optimization results of other algorithms to verify the effectiveness and feasibility of the model and algorithm.

2. Piano production batch scheduling problem description

2.1. Description of the batch scheduling problem of piano production shell parts

Take the wooden case production area of J Piano Production Company as an example. The piano wooden case production area mainly produces piano shell parts, including door frames, pedals, piano feet, piano stools and other piano wooden shell parts. The batch scheduling problem of the wood shell workshop can be described as: there are N kinds of piano wood shell workpieces (hereinafter referred to as workpieces) that need to be processed on M machines, such as finishing, milling, drilling, and sanding. Each kind of workpiece The production batch is Q. The processing procedures of various workpieces are different, and the processing time of each procedure is different. According to the production needs, each workpiece can be divided into one or several sub-batch processing. The main problem of batch scheduling is to optimize the number of workpieces in each sub-batch, and arrange processing machines for each sub-batch workpiece, and determine the optimal processing order of each sub-batch workpiece on each machine, under the condition of satisfying various constraints.

Minimize the completion time of all workpieces.

2.2. Mathematical model

Based on the above assumptions, with the shortest workpiece completion time as the optimization goal, a batch scheduling model for the wooden shell production area is established:

\[
\text{min } Z = \max_{i = 1,2, \ldots, N} \{ C_{i[km]} \}
\]

s.t.

\[
B_{N_{\min}} \leq B_{N_i} \leq B_{N_{\max}} \quad \forall i
\]

\[
D_{M_i} = \sum_{k=1}^{B_{N_i}} B_{D_{ik}} \quad \forall i
\]

\[
\min V_m \leq B_{D_{ik}} \leq \max V_m
\]

\[
C_{M_{ijkm}} \geq S_{M_{ijkm}} + (r_m \times Q_{T_{ijm}} + B_{D_{ik}} \times t_{ijm}) \times \delta_{ikjm} \quad \forall i, j, k, m
\]

\[
r_m = \begin{cases} 1 \\ 0 \end{cases} \quad \forall i, j, k, m
\]

\[
C_{M_{ikjm}} \geq S_{M_{ikj}}(i-1)_{m'} \quad \forall i, j, k, m
\]

\[
S_{M_{ikjm}} \geq C_{M_{i[k]}}(i')_{m'} \quad \forall i, j, k, m
\]

Among them: i is the workpiece number, i=1,2,……N; j is the process number; k is the batch number of the workpiece, k=1,2,……,BN; m is the machine number; m=1,2,……,M. BN, BD, and r_m are decision variables. Equation (1) regards the minimum completion time of the workpiece as the optimization goal. Equation (2) represents the sub-batch constraint. Equation (3) represents the batch constraint. Equation (4) expresses the processing capacity constraint. Equation (5) represents the machine constraints. Equation (6) represents adjusting 0-1 variables. Equation (7) represents process constraints. Equation (8) indicates that any machine can only process the next sub-batch after the previous sub-batch is completed.
3. Batch scheduling constraint model based on multicolor set

3.1. Polychromatic set theory

Polychromatic set theory is a mathematical tool that combines system theory and information processing theory. It depicts the hierarchical structure and complex relationships between elements, and organizes and processes information through the set layer and the logic layer. Solve the low-level quantity problem through the quantity layer. Polychromatic set theory has obvious advantages in the formal research of problems, and is widely used in product conceptual design modeling, part manufacturing process modeling, product assembly process modeling, tolerance information modeling and other fields [6-7].

3.2 Confinement matrix constraint model

Taking a 4×6 job shop as an example, suppose the initial batch size of each workpiece is 40. Each kind of workpiece can be divided into 3 sub-batches at most, and the specific processing time is shown in Table 1.

| jobs | Processing batch | Process Optional processing machine | M1 | M2 | M3 | M4 | M5 | M6 |
|------|------------------|-----------------------------------|-----|----|----|----|----|----|
| J1   | 40               | O11 2 3 4 — — —                     |     |    |    |    |    |    |
|      |                  | O12 — 3 2 4 —                      |     |    |    |    |    |    |
|      |                  | O13 1 4 5 — —                      |     |    |    |    |    |    |
| J2   | 40               | O21 3 — 5 2 —                      |     |    |    |    |    |    |
|      |                  | O22 4 3 — 6 —                      |     |    |    |    |    |    |
|      |                  | O23 — 4 — 7 11                     |     |    |    |    |    |    |
| J3   | 40               | O31 5 6 — — —                      |     |    |    |    |    |    |
|      |                  | O32 4 — 3 — 5 — 6 —               |     |    |    |    |    |    |
|      |                  | O33 — 4 3 5 6 7 11                |     |    |    |    |    |    |
| J4   | 40               | O41 9 — 7 9 — —                   |     |    |    |    |    |    |
|      |                  | O42 — 6 — 4 — 5                   |     |    |    |    |    |    |
|      |                  | O43 1 — 3 — — —                   |     |    |    |    |    |    |

The constraint model based on the multi-color set confinement matrix well describes the logical and quantitative relationships among sub-batch batches, processes, and machines, and the constraint relationship between sub-batch processing orders is described at the logical level; sub-batch batches are described at the quantity level. The constraint relationship between sizes. As shown in Figure 1 and Figure 2.
In the figure, F1～F12 represent the process; M1～M6 represent the machines that can be selected during the processing of the workpiece; O1～O12 represent the 12 sub-batch workpieces processed on the machine. In order to distinguish the workpiece subbatch, they are represented by numbers 1～12 respectively. Sub-batch number.

In the constraint model, the logic layer [A×F(a)] describes the correspondence between different sub-batch and processing procedures, as well as processing procedures and machines. You can determine sub-batch processing operations and machines that can process operations. E.g:

F1=(111000000000000000111000)

It is described that the sub-batch with process 1 is the sub-batch numbered 1, 2, and 3, and the process 1 can be processed on any of the machines M1, M2, and M3.

The numerical constraint model [A×F(a)] further describes the quantitative constraint relationship between sub-batch and between processes and machines. Can determine the batch value of the workpiece and the processing time of a single piece of the process on the machine, for example:

F1=(101713000000000000000234000)

It describes that the batch values of sub-batch 1, 2, and 3 are 10, 17, and 13, respectively; the processing time of a single piece on machines M1, M2, and M3 in process 1 is 2, 3, and 4 unit processing times.

4. Genetic algorithm based on multicolor set model constraints

4.1. Coding based on constraint model

The chromosome should contain batch division and batch scheduling information. Here, two matrix coding methods are used to describe the batch and batch scheduling dyeing information respectively. The batch chromosomes and the chromosomes of the batch scheduling part adopt triple encoding, as shown in Figure 3 and Figure 4.

Figure 3 Batch-size chromosomes

Figure 4 Batch scheduling of partial fragments of chromosomes
4.2. Decoding based on constraint model
Decoding consists of two parts. The first part is: reading the batch size of each sub-batch in the batch matrix, and the second part is: the sequence of batch processes in the batch matrix and the selection of processing machines.

4.3. Cross operation
A two-point crossover method is used for batch chromosomes. In the process of offspring generation, invalid solutions will be produced. Therefore, it is necessary to repair the chromosomes and always ensure that the batch meets the constraints of the initial batch. The batch scheduling chromosome contains two levels of coding information: batch sorting and machine selection. Batch sorting adopts the sequential crossover method (OX). During the crossover process, the columns where the sub-batch and the process are located as a whole cross. The third column indicates the machine selection, just exchange the processing machines of the same process in the two matrices.

4.4. Mutation operation
For the batch matrix, follow the position-swapping mutation method, that is, swap the positions of the sub-batch genes in the chromosomes, and invert the entire chromosome to exchange positions to obtain a new variant chromosome. For the batch scheduling matrix, the interchange mutation method is used, and for the first layer of the batch sorting layer, the positions of two different genes in the chromosome are randomly exchanged. For the second-level coding, mutation is performed by searching the constraint matrix and using multi-position replacement.

4.5. Fitness value calculation
Due to the two-stage encoding method, the corresponding values in batch chromosomes and batch chromosomes are directly read, and the corresponding time in the numerical matrix in the constraint model is multiplied to obtain the size of the fitness value.

5. Example simulation and analysis
In order to verify the performance of the algorithm, take the processing data in Table 1 as an example for test instructions. Among them, the initial batch size of each workpiece is 8, and the data in the table are all unit processing time.

For this example, the genetic algorithm based on the constraint model proposed in this paper is used to solve the problem. The minimum processing batch is set to 1, the crossover rate $P_c=0.85$, the mutation rate $P_m=0.1$, and the population size is 50. The corresponding optimal production cycle is 141 and 103 and 101. Finally, the calculation results of the algorithm in this paper are compared with the calculation results of literature [2] and literature [3], as shown in Table 2.

For this calculation example, the calculation results of the algorithm in this paper and the calculation results of literature [2] and literature [3] are compared as shown in Table 2.

| algorithm | Batch strategy | Optimal production cycle | Machine idle time | Solve the optimal value CPU average running time (unit: second) |
|-----------|----------------|-------------------------|-------------------|-------------------------------------------------------------|
| literature[2]GA | overall | 136 | - | - |
| literature[2]GA | Equal | 108 | - | - |
| literature[3]DE | overall | 141 | - | 25.4 |
| MCGA | overall | 141 | 207 | 2.4 |
| MCGA | Equal | 103 | 116 | 23.6 |
| MCGA | Unequal | 101 | 96 | 25.4 |

The calculation results show that the production cycle of batch scheduling is significantly better
than the overall scheduling, and the production cycle of unequal batch scheduling is smaller than the result of equal batch scheduling. The adjustment time is considered in the model of this article, and the scheduling result is more reasonable. It can be seen from the display results in Figure 6-8 that batch scheduling has significantly reduced machine idle time and completion time. Due to the combination of constraint model and GA, although the unequal batch scheduling problem is more complicated, the solution efficiency has been significantly improved. Literature [3] obtained the optimization results of the overall scheduling after 500 iterations, while the CMGA in this paper only needs 50 generations to obtain the same optimization results. Even in the case of considering batch optimization, the complexity of the problem increases, CMGA can also obtain satisfactory optimization results in a relatively short time.

6. Conclusion
This paper analyzes the batch scheduling problem of the shell production workshop of J Piano Company and takes the completion time as the optimization goal. Considering the adjustment time in the actual production process, the batch scheduling problem model is established. A matrix encoding of quaternary gene values is proposed, which combines the multicolor set constraint model and genetic search to solve the piano batch scheduling problem, and compares the optimization of the overall scheduling strategy, equal batching strategy and unequal batching strategy. The results show that the unequal batching strategy is more conducive to reducing machine idleness and can effectively shorten the completion time. The model and algorithm are applied to the case scheduling problem, and good results are obtained, further verifying the feasibility and effectiveness of the model and algorithm proposed in this paper.

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