Analysis of students’ mathematical problem solving ability as the effect of constant ill-structured problem’s employment

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Abstract. This study aimed to give more detailed comprehension on how students’ mathematical problem solving ability improved when ill-structured problem constantly employed for a particular period of time. The study was descriptive qualitative research and conducted in MTs Surya Buana, Malang, East Java, Indonesia. The subject consisted of 27 students from eighth-grade, where they were distributed into 7 groups of 3 or 4. The most significant improvements were students gained new skill in (1) formulating problems as well as new insight on seeing all possibilities in solving problems; (2) constructing resolution plan; (3) doing investigation as well as judging the validity of the information retrieved; and (4) evaluating solutions according to mathematical concepts. Based on these results, the use of ill-structured problem and active scaffolding from the teacher are highly recommended in mathematics instruction.

1. Introduction

The importance of problem solving skill frequently mentioned and studied in the mathematics field [1,2]. Problem solving requires a variety of skills including interpreting information, planning and working methodically, checking results and trying alternative strategies [3]. Problem solving skill is needed not only in education but also in daily life [4]. Possessing problem solving skill is considered the foundation to enhance other abilities, such as critical thinking, creative thinking, reasoning, and communication [1,5,6,7,8]. Stakeholders have been putting huge efforts to improve this 21st-century skill, such as revising the curriculum, developing instructional models, media, and books [9]. However, those importance and efforts don’t translate into significant positive outcome for Indonesian students. The result of TIMSS and PISA in mathematics showed that Indonesia is consistently placed at a lower rank and below average [10]. The results indicated Indonesian students met big challenges when they had to apply mathematical concepts in solving a problem, especially real life problem. This shows it is notable for the students to be consistently faced with authentic real world problem, i.e., ill-structured problem.

Ill-structured problems are contextualized, require learners to define the problems as well as determine the information and skills needed to solve them [11]. Their characteristics are as follows: first, aspects of the situation are not concrete; second, the problems are not well-defined; third, they are based on real-life situations and have openness; and finally, complex situations are presented [2]. This kind of problem demands the students to identify the given and unknown necessary information in order to solve the problem [12]. This demand will lead the students to engage in an investigation. Ill-structured
problem allows students to monitor and evaluate the whole process as well as the solution of the problem [13]. Studies suggest that students engaged with ill-structured problem gain significant improvement in problem solving skill, mathematical thinking capacity, mathematical abstraction, reasoning, and creativity [11,14].

Solving ill-structured problem is different with solving well-defined problem. Solvers must develop arguments by gathering evidences, expert opinions, and reasons, and by integrating or synthesizing diverse opinions and information for supporting their decision [15]. Ill-structured problem solving can be summarized as three processes: a) representation problems, b) solution process, and c) monitor and evaluation [16]. A representation problem is established by constructing a problem space which includes defining problems, searching and selecting information, and developing justification of the selection. This information enables the solvers to enter the next problem state. The solution process involves generating and selecting solutions. Finally, the monitoring and evaluating process requires assessing the solution by developing justification.

Most of ill-structured problem studies resulted on the final grade of problem solving ability. In this study, the researchers focused on analyzing the development of five abilities separately over time, which are understanding the problem, constructing plan, applying the plan, concluding, and evaluating. The purpose of the study is to give more detailed comprehension and whole picture on how students improved significantly when ill-structured problem constantly employed for particular period of time.

2. Methods
This study categorized as descriptive qualitative research. Based on preliminary research, the study took place in MTs Surya Buana, Malang, East Java, Indonesia, because this school possessed the problem conjectured beforehand. The subject consisted of 27 students from eighth-grade, where they were distributed into 7 groups of 3 or 4. Each week, students were faced with ill-structured problem, with missing information, in order for the students to be involved in investigation. The problem designed in real life context based on three dimensional material, from cube, cuboid, prism, and pyramid. Before field test, all instrument validated by 2 validators with qualifications as follows: hold doctoral degree and expert in instructional model. Validators stated that the instruments were valid with minor revision.

For research purpose, this study assessed students’ problem solving ability, which synthesized Polya’s problem solving phase and Hong’s ill-structured problem solving process [16,17]. Therefore, the assessment focused on 5 abilities, namely understanding the problem, constructing plan, applying, concluding, and evaluating. The main data for this research were students’ performance and students’ statements in semi structured interview on what they think about the instruction and the problem. Students’ performance from the worksheet would be analyzed to identify students’ difficulties and their development over time as the instruction and guidance kept on going.

3. Result and Discussion
At the first meeting, all groups showed less than satisfying works. The problem given as follow:

“Wulan wants to throw a birthday party with her friends. To celebrate her birthday, she bought three packages of snacks. Package 1 contains 12 Tango Wafer of 171 grams. Package 2 contains 27 Serena Broniz of 80 grams. Package 3 contains Yupi Berri Bonz of 50 grams. Each package put inside a box. If there is a box with a shape of cube that contains exactly one package, then which packet is in that box? What is the surface area of the box?”

The results showed that students didn’t earn good score in all indicators. Figure 1 until Figure 4 show students work at the first meeting.

Figure 1 shows students’ performance in identifying information. They just rewrote given information, without identifying additional necessary information in order to solve the problem. This because they didn’t see the problem as ill-structured problem [18].
Same thing happened in formulating questions. One group rewrote the problem in another equivalent sentence (Figure 2(a)). However, this kind of answer indicated that the group’s way of thinking wasn’t much developed [19]. They only rewrote the question without formulating additional questions that would helped them to construct plans to solve the problem. Other group were not even able to rewrote the question as showed in figure 2(b). Those groups wrote a question that showed the group inability to figure out the way to solve the problem.

The inability to understand the problem affected students’ performances for the rest of the works. Figure 3 is the evidence that lack of understanding towards the problem resulted in an absurd solving plan. In figure 3(a), a group of students only wrote one plan that was deemed unable to solve the problem. Figure 3(b) shows a group began writing plans in the right direction, but the plan was incomplete so with the plan, the desired final solution might not achieved [17].
Despite of the bad solving plan, with intense guidance by the teacher [20], especially in searching the measurements of these boxes, some groups were able to write down the solution quite well as shown in figure 4. According to researchers, one of the factors of this phenomenon is that students might not be accustomed in providing extended explanations of given information, formulating questions, making a problem-solving plan, or investigate necessary information, but the students understand the intent of the problem, since they used to well-structured problem [21,22]. In addition to the factor of students' understanding of the purpose of the problem, the role of the teacher model was also considered to have a major effect on this outcome.

At the second meeting, there were some enhancements in students’ performance in identifying information and formulating question. In explaining information, students began to explain the given information and identified additional information they needed to be able to solve the problem.
Figure 5 shows that students were able to identify their needs for the size of container to find surface area. However, there were still shortcomings in the answers. The student didn’t identify the fact that the purchase of paint must be a whole number. This later affected students’ final answer [23].

In formulating questions, it appeared that the groups of students were not only able to write down the questions listed in the problem, but also write down other questions that held important role in the solving process. However, in the students’ answers, the sequence of the questions was still not hierarchically arranged. Writing the questions in the right order will help students in constructing plan in a clearer and more detailed manner [17].

In constructing plan, the students’ answers appeared to be more elaborated. This was the effect of students' ability to identified necessary information and formulate questions increased. The shortage that still occurred in this activity was the way students expressed their plan, as seen in Figure 7.

From an interview with a member of the group, they meant the order as follows:

a) Finding container’s surface area (\( L_p \))

b) Divide the area by 12, because 1 liter is enough to cover a surface of 12 m\(^2\).

c) The results of the division (x) multiplied by 95,000 because 1 liter of paint costs Rp 95,000.00

The plan made by the students was not perfect because the containers to be painted were 10 containers.

In problem solving activities, majority of students had been able to carry out these activities well. However, the detail and reason were still underdeveloped. Figure 8 shows that students made mistakes in calculations. They also made mistakes in writing rule of multiplication over addition (circle). In addition, the reasoning of when multiplying the number of litters needed to paint and the price of paint per litter, the student also did not round from 11.302 to 12. This is due to the students’ incomplete explanation in clarifying the facts [24].
Figure 8. Student performance on problem solving activities at the second meeting

In the next meeting, students’ performance was very satisfying, especially their ability to evaluate. Figure 9 shows a solution by one group.

Figure 9. Performance of students in problem solving activities at the fourth meeting

Based on this solution, other group gave their evaluation (figure 10). They claimed that the solution offered was not correct. Because when they applied Pythagorean Theorem, the solution will be inconsistent.

Figure 10. Students' evaluation towards a solution
From those performances, it could be concluded that students’ ability for every indicators in solving problem increased significantly, even when minority of students still struggle and made some errors.

According to the students, the challenges in solving ill-structured problem were divided into 3 major problems that consistently resulted lower than other 2 indicators as follows:

a. Formulating questions. At first, they didn’t realize they had to pose questions that weren’t mentioned in the problem because they only been exposed with well-structured problem [12,25]. They just rewrote or rephrased the questions. Consequently, they stuck at this step and couldn’t go any further [12]. Actually, they did have the sense that something wasn’t right. They kept telling each other “it’s blurry”. They noticed some important information went missing, but couldn’t expressed it. After the teacher guide them to write down all the confusion, they slowly realized that those questions were parts of foundation in order to solve the problem. By this step, the students gained new skill in formulating problems as well as new insight on how to see every possibilities in solving problems.

b. Constructing plan. Interestingly, the root of obstacle in constructing plan according to the students was “they didn’t know what they didn’t know”. In general, they didn’t have problem on mathematical procedure but because they couldn’t determine necessary information needed. Ill-structured problem requires students to find what’s missing in order to solve the problem [11,12]. Once the teacher guided them in formulating questions and use them as basis of investigation, they started to understand that they need to write the plan was to find all the missing elements through investigation [24]. They learned how to seek some information, and judged them to determine which information is valid and trustworthy. By this step, the students gained new skill in doing investigation. They learned that a problem could have more than one solution, since the information retrieved were various depends on the source. When they constructed the plan successfully, applying mathematical formula in the computation wasn’t much of a problem. Indeed, there were some students who did some errors, but this procedural difficulties can be minimize with constant guidance from the teacher and determination of the students to learn.

c. Evaluating the solution. The students found a lot of obstacles in evaluating, because they were never asked to evaluate any problem handed by the teacher since the problems were well-defined. They need to realize that the smallest factor in real life would have such big impact in concluding the solution [26,27]. Like the fact that we can’t buy paint with the amount of fraction or the fact that the surface of container is not flat, which affecting it’s surface area. Furthermore, they never thought to link one mathematical concept to another before this instruction implemented. They knew that one material could be prerequisite for another material, but they didn’t realize if one material could be used as evaluation tool to others, like in Pythagorean case provided in figure 10. Once the teacher guided them in this term, they gained new insight on how to evaluate things accordingly and enhanced their critical thinking ability [28].

4. Conclusion

There were 5 abilities as the focus of this research. The result showed that 3 of them were the hardest to achieve. Applying mathematical procedure wasn’t a problem to the students since they used to face mathematical problem. Same thing goes to conclusion. However, since the students had been exposed with well-structured problems rather than ill-structured problem, their ability of formulating questions, constructing plan, investigating, and evaluating weren’t much developed. Constant employment of ill-structured problem gave positive effect for these abilities. The students improved in formulating questions as well as how to see all possibilities in solving problems. This led to improvement in constructing resolution plan, seek some information and judged them to determine which information is valid and trustworthy. They gained new insight on how to evaluate things according to mathematical concepts. This result also stated the importance of the role of teacher in guiding the students. Therefore, the use of ill-structured problem and active scaffolding from the teacher are highly recommended in mathematics instruction.
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