A SIMPLE NON-PARAMETRIC TEST AGAINST RENEWAL INCREASING MEAN RESIDUAL LIFE CLASS

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ABSTRACT. When a device is experiencing random number of shocks governed by a homogeneous Poisson process, the concept of renewal increasing mean residual life is very much useful to study the properties of age replacement model. In this paper, we propose a simple non-parametric test for testing exponentiality against renewal increasing mean residual life class. We derive the exact null distribution of the test statistic and then find the critical values for different sample sizes. The test statistic is shown to be asymptotically normal and consistent against the alternatives. The Pitman’s asymptotic efficacy value shows that our test perform well. Some numerical results are presented to demonstrate the performance of the testing method and then we illustrate the test procedure using a real data.

KEYWORDS: Exponential distribution; Mean residual life; renewal increasing mean residual life; Pitman’s asymptotic efficacy; Replacement model; U-statistics.

1. Introduction

Quite often, in life testing analysis we assume that the lifetime of a device follows exponential distribution. This assumption implies that a used item

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is stochastically as good as a new one. That is, the unit in question does not age with time. Hence, there is no reason to replace a unit which is working. However, this is not always a realistic assumption and it is important to know which life distribution deserves membership benefits of an ageing class. In this scenario, test designed to detect the departure from exponentiality towards the relevant alternative hypothesis have important in reliability analysis. The problem of testing exponentiality against a particular ageing class has been well studied in the literature; see Lai and Xie (2006) for an overview of such procedures.

Due to its importance in the analysis of age replacement model, testing the null hypothesis that a lifetime is exponential (ageless) against the alternatives that it has decreasing (increasing) mean residual life (MRL) class has received considerable attention during the last two decades. Test for exponentiality against decreasing mean residual life class alternatives was first considered by Hollander and Proschan (1975). Subsequently, various authors used different types of approaches in deriving the test statistics; see Chen et al. (1983), Ahmad (1992), Lim and Park (1993), Belzunce et al. (2000), Abu- Youssef (2002) and Anis (2010).

If a device is experiencing a random number of shocks governed by a homogeneous Poisson process, then renewal increasing mean residual life is useful concept to study age replacement model. In this context testing exponentiality against renewal increasing mean residual life class can be used to determine whether to adopt a planned replacement model over unscheduled one. After giving some characterization result about the renewal increasing mean residual life class, Sepehrifar et al. (2015) consider the same testing problem and obtained a critical region based on asymptotic theory of U-statistics and their calculation of asymptotic variance seems to be incorrect (see Remark 2.1). Motivated by Sepehrifar et al. (2015), we develop a new
non-parametric test for testing exponentiality against renewal increasing mean residual life class.

The rest of the paper is organized as follows. In Section 2, based on U-statistics we propose a simple non-parametric test for testing exponentiality against renewal increasing mean residual life class. In Section 3, we derive the exact null distribution of the test statistic and then calculated the critical values for different sample sizes. The asymptotic normality and the consistency of the proposed test statistic are proved in Section 4. The Pitman’s asymptotic efficacy value is also given in this section. In Section 5, we report the result of the simulation study carried out to assess the performance of the proposed test. We illustrate our test procedure using a real data. Finally, in Section 6 we depict the conclusions of our study.

2. Test statistic

Let $X$ be the lifetime of a device which has absolutely continuous distribution function $F(.)$. Suppose $\bar{F}(x) = P(X > x)$ denotes the survival function of $X$ at $x$. Also let $\mu = E(X) = \int_0^{\infty} \bar{F}(t)dt < \infty$. Assume that the device under consideration is experiencing a random shock. Suppose $N(t)$ denotes the total number of shocks up to time $t$ with probability mass function $P(N(t) = j) = F^j(t) - F^{j+1}(t), j = 0, 1, 2, \ldots$. Suppose that the random variable $W_j, j = 0, 1, 2$, quantify the amount of hidden lifetime absorbed by the $j$th shock with $W_0 = 0$ and having common distribution function $G(x) = P(W_j \leq x)$. The total cumulative life damage up to time $t$ is defined as $Z(t) = \sum_{j=0}^{N(t)} W_j$ with the cumulative distribution function $Q(x) = P(Z(t) \leq x) = \sum_{j=0}^{\infty} G^{(j)}[F^{(j)}(t) - F^{(j+1)}(t)]$. We refer to Glynn
and Whitt (1993), Roginsky (1994) and Sepehrifar et al. (2015) for discussion related this framework. It is assumed that the unit fails when the total life-damage exceeds a pre-specified level \( x > 0 \).

Let \( X^* = X - Z(t) \) be the residual lifetime of an operating device with cumulative damage \( Z(t) \). Note that the realizations of \( X^* \) is available to us for further analysis. Consider a device subjected to \( N(t) \) number of shocks up to time \( t \). Given that such a device is in an operating situation at time instant \( t \) after installation, the MRL function of \( X^* \) denoted by \( m^*(t) \) is defined by \( m^*(t) = E(X^* - t | X^* \geq t) \). Note that the total life-damage will not exceed the threshold level \( x \). From the definitions it is evident that the random variables \( X^* \) and \( Z(t) \) are independent.

Next we give the definition of RIMRL\textsubscript{shock} class (Sepehrifar et al., 2015).

**Definition 2.1.** The mean residual life of a device under shock model (MRL\textsubscript{shock}) at time \( t \) is defined as

\[
m^*(t) = \frac{1}{\bar{r}(t)} \int_t^\infty \bar{r}(z)dz,
\]

where \( \bar{r}(z) = \int_0^z \bar{F}(z + w)dQ(w) \).

**Definition 2.2.** The random variable \( X \) belongs to the RIMRL\textsubscript{shock} class if the function \( m^*(t) \) is a non-decreasing function for all \( t > 0 \).

Next, we develop a simple non-parametric method for testing exponentiality against RIMRL\textsubscript{shock} class. We wish to test the null hypothesis

\[ H_0 : F^* \text{ is exponential} \]

against

\[ H_1 : F^* \text{ is RIMRL\textsubscript{shock} (and not exponential)}, \]
A non-parametric test against IRMRL class 5 on the basis of a random sample $X_1^*, X_2^*, ..., X_n^*$ from an absolutely continuous distribution $F^*$.

For the above testing problem Sepehrifar et al. (2015) considered the parameter defined by

$$\Delta^*(F^*) = \frac{1}{\mu} E_{f^*}(\min(X_1^*, X_2^*) - \frac{1}{2}X_1^*).$$

It can be easily verify that $\Delta^*(F)$ is zero under $H_0$ and positive under the alternative $H_1$. Denote $\Delta(F^*) = E_{f^*}(\min(X_1^*, X_2^*) - \frac{1}{2}X_1^*)$. Then the $\Delta^*(F^*)$ can be rewritten as

$$\Delta^*(F^*) = \frac{\Delta(F^*)}{\mu}.$$

A U-statistic based on symmetric kernel $h(X_1^*, X_2^*) = \frac{1}{2}(2\min(X_1^*, X_2^*) - \frac{1}{2}X_1^* - \frac{1}{2}X_2^*)$ given by

$$\hat{\Delta} = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j<i}^{n} h(X_i^*, X_j^*),$$

is an unbiased estimator of $\Delta(F^*)$. Hence the test statistic is given by

$$\hat{\Delta}^* = \frac{\hat{\Delta}}{\bar{X}^*},$$

where $\bar{X}^* = \frac{1}{n} \sum_{i=1}^{n} X_i^*$. After simplification, we can rewrite the above expression as

$$\hat{\Delta}^* = \frac{1}{2n(n-1)\bar{X}^*} \sum_{i=1}^{n} (3n - 4i + 1)X_{(i)}^*, \tag{1}$$

where $X_{(i)}^*, i = 1, 2, ..., n$, is the i-th order statistics based on the random sample $X_1^*, X_2^*, ..., X_n^*$, from $F^*$.

Note that, under the null hypothesis $H_0$, the test $\hat{\Delta}^*$ is asymptotically distribution free which we will prove in Section 4. Hence the test procedure
is to reject the null hypothesis $H_0$ in favour of the alternative hypothesis $H_1$ for large values of $\hat{\Delta}^*$.

**Remark 2.1.** Sepehrifar et al. (2015) developed a test procedure for testing the same problem discussed above and obtained a critical region based on the asymptotic variance $\frac{7}{48}$. However as we shown in our calculation the asymptotic variance is $\frac{1}{12}$. We could not find enough details in their paper to explain the discrepancy.

3. Exact null distribution

In this section, we derive the exact null distribution of the test statistic. Then we calculate the critical values for different sample sizes. Note that the exponential random variable with rate $\frac{1}{2}$ is distributed same as $\chi^2$ random variable with 2 degrees of freedom. Hence we use the Theorem 3.1 of Box (1954) to find the exact null distribution of the test statistic.

**Theorem 3.1.** Let $X^*$ be continuous non-negative random variable with $F^*(x) = e^{-\frac{x}{2}}$. Let $X^*_1, X^*_2, ..., X^*_n$ be independent and identical samples from $F^*$. Then for fixed $n$

$$P(\hat{\Delta}^* > x) = \sum_{i=1}^{n} \prod_{j=1, j\neq i}^{n} \left(\frac{d_{i,n} - x}{d_{i,n} - d_{j,n}}\right)I(x, d_{i,n}),$$

provided $d_{i,n} \neq d_{j,n}$ for $i \neq j$, where

$$I(x, y) = \begin{cases} 
1 & \text{if } x \leq y \\
0 & \text{if } x > y
\end{cases} \text{ and } d_{i,n} = \frac{(n - 2i + 1)}{2(n - 1)}.$$

**Proof:** Rewrite the test statistic given in equation (1) as

$$\widehat{\Delta} = \sum_{i=1}^{n} X^*_{(i)} \left[\frac{(n - i + 1)^{2}}{n(n - 1)} - \frac{(n - i)^{2}}{n(n - 1)} - \frac{(n + 1)}{2n(n - 1)}\right].$$
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Or
\[
\hat{\Delta} = \frac{n}{(n-1)} \sum_{i=1}^{n} X^*_i \left[ \frac{(n-i+1)^2}{n^2} - \frac{(n-i)^2}{n^2} - \frac{(n+1)^2}{2n^2} \right].
\] (2)

Hence, in terms of the normalized spacings, \( D_i = (n-i+1)(X^*_i - X^*_{i-1}) \), with \( X^*_0 = 0 \), we can express \( \hat{\Delta}^* \) as
\[
\hat{\Delta}^* = \frac{\sum_{i=1}^{n} d_{i,n} D_i}{\sum_{i=1}^{n} D_i},
\]
where \( d_{i,n} \)’s are given by
\[
d_{i,n} = \frac{1}{(n-1)} \left[ (n-i+1) - \frac{(n+1)}{2} \right] = \frac{(n-2i+1)}{2(n-1)}.
\]

Hence the result follows from Theorem 3.1 of Box (1954).

The critical values of the test for different \( n \) under the null distribution is tabulated in Table 1.

**Table 1. Critical Values**

| \( n \) | 90% level | 95% level | 97.5% level | 99% level |
|-------|----------|----------|-------------|----------|
| 2     | 0.4000   | 0.4500   | 0.4750      | 0.4900   |
| 3     | 0.2764   | 0.3419   | 0.3883      | 0.4292   |
| 4     | 0.2189   | 0.2678   | 0.323       | 0.3693   |
| 5     | 0.1883   | 0.2383   | 0.28        | 0.325    |
| 6     | 0.1679   | 0.2131   | 0.2508      | 0.2927   |
| 7     | 0.1529   | 0.1944   | 0.2293      | 0.2682   |
| 8     | 0.1413   | 0.1799   | 0.2125      | 0.2492   |
| 9     | 0.1319   | 0.1682   | 0.1989      | 0.2336   |
| 10    | 0.1243   | 0.1586   | 0.1877      | 0.2208   |
| 15    | 0.0993   | 0.1271   | 0.1508      | 0.178    |
| 20    | 0.0852   | 0.109    | 0.1295      | 0.1531   |
| 25    | 0.0758   | 0.097    | 0.1153      | 0.1363   |
| 30    | 0.0689   | 0.0882   | 0.1049      | 0.1241   |
| 40    | 0.0594   | 0.0761   | 0.0905      | 0.1072   |
| 50    | 0.0529   | 0.0679   | 0.0808      | 0.0957   |
| 75    | 0.0431   | 0.0552   | 0.0658      | 0.078    |
| 100   | 0.0373   | 0.0477   | 0.0569      | 0.0675   |
4. Asymptotic properties

In this section, we investigate the asymptotic properties of the proposed test statistic. The test statistic is shown to be asymptotically normal and consistent against the alternative under consideration. Making use of the asymptotic distribution we also calculate the Pitman’s asymptotic efficacy.

4.1. Consistency and asymptotic normality. As the proposed test is based on U-statistics, we use the asymptotic theory of U-statistics to discuss the limiting behaviour of $\hat{\Delta}^*$.

**Theorem 4.1.** The $\hat{\Delta}^*$ is a consistent estimator of $\Delta^*(F^*)$.

**Proof:** Since $\hat{\Delta}$ is a U-statistic it is a consistent estimator of $\Delta(F^*)$ (Lehmann, 1951). By weak law of large number $\bar{X}^*$ converges to $\mu^*$ in probability. As we can write

$$\hat{\Delta}^* = \frac{\hat{\Delta}}{\Delta(F^*)} \frac{\Delta(F^*)}{\mu^*} \frac{\mu^*}{\bar{X}^*},$$

the proof of the theorem is immediate.

Next we find the asymptotic distribution of the test statistic.

**Theorem 4.2.** The distribution of $\sqrt{n}(\hat{\Delta} - \Delta(F^*))$, as $n \to \infty$, is Gaussian with mean zero and variance $4\sigma_1^2$, where $\sigma_1^2$ is the asymptotic variance of $\hat{\Delta}$ and is given by

$$\sigma_1^2 = \frac{1}{4} \text{Var} \left( 2X^* \bar{F}^*(X^*) + 2 \int_0^{X^*} ydF^*(y) - \frac{1}{2} X^* \right).$$

**Proof:** Since the kernel has degree 2, using the central limit theorem for U-statistics (Hoeffding, 1948), $\sqrt{n}(\hat{\Delta} - \Delta(F^*))$ has limiting distribution

$$N(0, 4\sigma_1^2), \quad \text{as} \quad n \to \infty,$$
where the value of $\sigma_1^2$ is specified in the theorem. For finding $\sigma_1^2$, consider

$$E(h(x, X^*_2)) = \frac{1}{2} E\left(2x I(x < X^*_2) + 2X^*_2 I(X^*_2 < x) - \frac{1}{2} x - \frac{1}{2} X^*_2\right)$$

$$= \frac{1}{2} \left(2xF^*(x) + 2 \int_0^x ydF^*(y) - \frac{1}{2} x - \frac{1}{2} \mu^*\right).$$

Hence

$$\sigma_1^2 = \frac{1}{4} V\left(2X^*F^*(X^*) - 2 \int_0^{X^*} ydF^*(y) - \frac{1}{2} X^*\right),$$

which completes the proof.

**Corollary 4.1.** Let $X^*$ be continuous non-negative random variable with $F^*(x) = e^{-\frac{x}{\lambda}}$, then under $H_0$, as $n \to \infty$, $\sqrt{n}(\hat{\Delta} - \Delta(F^*))$ is Gaussian random variable with mean zero and variance $\sigma_0^2 = \frac{\lambda^2}{12}$.

**Proof:** Under $H_0$, we have

$$E(X^* - x|X^* > x) = \lambda. \quad (4)$$

That is

$$\int_x^{\infty} ydF^*(y) = (x + \lambda)F^*(x).$$

Since

$$\int_0^x ydF^*(y) + \int_x^{\infty} ydF^*(y) = \lambda,$$

we have

$$\int_0^x ydF^*(y) = \lambda - (x + \lambda)F^*(x).$$
Hence using (3) we obtain

\[ \sigma_0^2 = 4\sigma_1^2 \]
\[ = V\left( 2X^*\tilde{F}^*(X^*) - 2\lambda - 2(X^* + \lambda)\tilde{F}^*(X^*) - \frac{1}{2}X^* \right) \]
\[ = V\left( 2\lambda F^*(X^*) - \frac{1}{2}X^* \right) = \frac{\lambda^2}{12}. \]

Using Slutsky’s theorem, the following result can be easily obtained from Corollary 4.1.

**Corollary 4.2.** Let \( X^* \) be continuous non-negative random variable with \( \tilde{F}^*(x) = e^{-\frac{x}{\lambda}} \), then under \( H_0 \), as \( n \to \infty \), \( \sqrt{n}(\tilde{\Delta}^* - \Delta^*(F^*)) \) is Gaussian random variable with mean zero and variance \( \sigma_0^2 = \frac{1}{12} \).

Hence in case of the asymptotic test, for large values of \( n \), we reject the null hypothesis \( H_0 \) in favour of the alternative hypothesis \( H_1 \), if

\[ \sqrt{12n}(\tilde{\Delta}^*) > Z_\alpha, \]

where \( Z_\alpha \) is the upper \( \alpha \)-percentile of \( N(0,1) \).

**Remark 4.1.** One can also look at the problem of testing exponentiality against the dual concept renewal decreasing mean time to failure (RDMRL\textsubscript{shock}) class. We reject the null hypothesis \( H_0 \) in favour of RDMRL\textsubscript{shock} class, if

\[ \sqrt{12n}(\tilde{\Delta}^*) < -Z_\alpha. \]

**4.2. Pitman’s asymptotic efficacy.** The Pitman efficiency is the most frequently used index to make a quantitative comparison of two distinct asymptotic tests for a certain statistical hypothesis. The efficacy value of a test statistic can be interpreted as a power measure of the corresponding test. A test with maximum efficacy is the asymptotically locally most powerful
A NON-PARAMETRIC TEST AGAINST IRMRL CLASS (ALMP) test. The Pitman’s asymptotic efficacy (PAE) is defined as

\[ \text{PAE}(\Delta^*(F^*)) = \frac{|\frac{d}{d\lambda} \Delta^*(F^*)|_{\lambda \rightarrow \lambda_0}}{\sigma_0}, \]  

(5)

where \( \lambda_0 \) is the value of \( \lambda \) under \( H_0 \) and \( \sigma_0^2 \) is the asymptotic variance of \( \hat{\Delta}^* \) under \( H_0 \). In our case, the PAE is given by

\[ \text{PAE}(\Delta^*(F^*)) = \frac{|\frac{d}{d\lambda} \Delta^*(F^*)|_{\lambda \rightarrow \lambda_0}}{\sigma_0} = \sqrt{\frac{12}{\lambda_0}} (W'(\lambda_0) - W(\lambda_0) \mu^*_a(\lambda_0)), \]

where \( W = E(\min(X^*_1, X^*_2)) \) and \( \mu^*_a \) is the mean of \( X^* \) under the alternative hypothesis and the prime denotes the differentiation with respect to \( \lambda \). We calculate the PAE value for three commonly used alternatives which are the members of IRMRL\(_{\text{shock}}\) class

(i) the Weibull family: \( \bar{F}^*(x) = e^{-x^\lambda} \) for \( \lambda > 1, x \geq 0 \)

(ii) the linear failure rate family: \( \bar{F}^*(x) = e^{(-x-\frac{\lambda}{2}x^2)} \) for \( \lambda > 0, x \geq 0 \)

(iii) the Makeham family: \( \bar{F}^*(x) = e^{-x-\lambda(e^{-x}+x-1)} \) for \( \lambda > 0, x \geq 0 \).

By direct calculations, we observe that the PAE for Weibull distribution is equal to 1.2005; while for linear failure rate distribution and the Makeham distribution these values are, 0.8660 and 0.2828, respectively. This shows that the proposed test has very good efficacy value for Weibull and linear failure rate alternatives.

5. Simulation and data analysis

Next, we report a simulation study for evaluating the performance of our test against various alternatives. The simulation was done using R program. Finally, we illustrate our test procedure using a real data.

5.1. Monte carlo study. First we find the empirical type 1 error of the proposed test. we simulate random sample from the exponential distribution
Table 2. Empirical type 1 error of the test

| $n$ | Type 1 Error (5% level) | Type 1 Error (1% level) |
|-----|-------------------------|-------------------------|
| 10  | 0.0635                  | 0.0123                  |
| 20  | 0.0540                  | 0.0115                  |
| 30  | 0.0518                  | 0.0107                  |
| 40  | 0.0520                  | 0.0110                  |
| 50  | 0.0517                  | 0.0107                  |
| 60  | 0.0516                  | 0.0102                  |
| 70  | 0.0515                  | 0.0102                  |
| 80  | 0.0511                  | 0.0100                  |
| 90  | 0.0504                  | 0.0103                  |
| 100 | 0.0504                  | 0.0101                  |

with cumulative distribution function $F(x) = 1 - \exp(-x), x \geq 0$. Since the test is scale invariant, we can take the scale parameter to be unity, while performing the simulations. A random sample of different sample size is drawn from the exponential distribution specified above and the value of the test statistic is calculated. We check whether this particular realization of the test statistic accepts or rejects the null hypothesis of exponentiality. Then we repeat the whole procedure ten thousand times and observe the proportion of times the proposed test statistic takes the correct decision of rejecting the null hypothesis of exponentiality and this gives the empirical type I error. The procedure has been repeated for different values of $n$ and is reported in Table 2. From the Table 2 it evident that the empirical type 1 error is a very good estimator of the size of the test even for small sample size. For finding empirical power against different alternatives, we simulate observations from the Weibull, linear failure rate and Makeham distributions with various values of $\lambda$ where the distribution functions were given in the Section 4. As pointed out earlier these are typical members of the IRMRL$_{shock}$ class. The empirical powers for the above mentioned alternatives are given in Tables 3, 4 and 5. From these tables we can see that empirical powers of the test approaches to one when the $\theta$ values are
going away from the null hypothesis value as well as when $n$ takes large values.

**Table 3. Empirical Power: Weibull distribution**

| $n$ | $\lambda = 1.2$ | $\lambda = 1.4$ | $\lambda = 1.6$ | $\lambda = 1.8$ |
|-----|-----------------|-----------------|-----------------|-----------------|
|     | 5% level        | 1% level        | 5% level        | 1% level        | 5% level        | 1% level        | 5% level        | 1% level        |
| 60  | 0.5023          | 0.2307          | 0.9361          | 0.7689          | 0.9978          | 0.9797          | 1.0000          | 0.9995          |
| 70  | 0.5563          | 0.2738          | 0.9651          | 0.8432          | 0.9997          | 0.9919          | 1.0000          | 1.0000          |
| 80  | 0.6081          | 0.3189          | 0.9805          | 0.8911          | 0.9999          | 0.9979          | 1.0000          | 1.0000          |
| 90  | 0.6499          | 0.3642          | 0.9900          | 0.9348          | 0.9999          | 0.9988          | 1.0000          | 1.0000          |
| 100 | 0.6952          | 0.4125          | 0.9942          | 0.9568          | 1.0000          | 0.9999          | 1.0000          | 1.0000          |

Next we illustrate our test procedure using a real data set.

5.2. **Data analysis.** To demonstrate our testing method, we apply it to the data set consists of $n = 27$ observations of the intervals between successive failures (in hours) of the air-conditioning systems of 7913 jet air planes of a fleet of Boeing 720 jet air planes as reported in Proschan (1963). The data is given in Table 6.

**Table 4. Empirical Power: Linear failure rate distribution**

| $n$ | $\lambda = 0.2$ | $\lambda = 0.4$ | $\lambda = 0.6$ | $\lambda = 0.8$ |
|-----|-----------------|-----------------|-----------------|-----------------|
|     | 5% level        | 1% level        | 5% level        | 1% level        | 5% level        | 1% level        | 5% level        | 1% level        |
| 60  | 0.4993          | 0.2284          | 0.6847          | 0.3854          | 0.7900          | 0.5178          | 0.8763          | 0.6570          |
| 70  | 0.5599          | 0.2771          | 0.7452          | 0.4684          | 0.8481          | 0.6125          | 0.9245          | 0.7465          |
| 80  | 0.6083          | 0.3211          | 0.8016          | 0.5383          | 0.8928          | 0.6845          | 0.9481          | 0.8100          |
| 90  | 0.6500          | 0.3631          | 0.8348          | 0.5967          | 0.9198          | 0.7434          | 0.9700          | 0.8667          |
| 100 | 0.6917          | 0.4108          | 0.8771          | 0.6566          | 0.9471          | 0.8043          | 0.9827          | 0.9072          |

**Table 5. Empirical Power: Makeham distribution**

| $n$ | $\lambda = 0.2$ | $\lambda = 0.4$ | $\lambda = 0.6$ | $\lambda = 0.8$ |
|-----|-----------------|-----------------|-----------------|-----------------|
|     | 5% level        | 1% level        | 5% level        | 1% level        | 5% level        | 1% level        | 5% level        | 1% level        |
| 60  | 0.3708          | 0.1469          | 0.4968          | 0.2241          | 0.6569          | 0.3605          | 0.875           | 0.6375          |
| 70  | 0.4219          | 0.1714          | 0.5505          | 0.2653          | 0.7290          | 0.4382          | 0.9205          | 0.7273          |
| 80  | 0.4660          | 0.2018          | 0.6051          | 0.3119          | 0.7850          | 0.4985          | 0.9457          | 0.7995          |
| 90  | 0.5120          | 0.2334          | 0.6518          | 0.3508          | 0.8240          | 0.5632          | 0.9646          | 0.8465          |
| 100 | 0.5566          | 0.2736          | 0.7024          | 0.4089          | 0.8614          | 0.6255          | 0.9826          | 0.9047          |
The value of the test statistic corresponds to this particular data set is 0.1674. When $\alpha = 0.05$, the critical values of the exact test statistic correspond to $n = 27$ is 0.0932, hence we reject the null hypothesis of exponentially in favour of IRMRL\textsubscript{shock} class at 0.05 level.

6. Conclusions

In order that a device or system is able to perform its intended functions without disruption due to failure, several types of maintenance strategies that spell out schemes of replacement before failure occurs, have been devised in reliability engineering (Kayid et al., 2013). If the device is experiencing a random number of shocks governed by a homogeneous Poisson process, then renewal increasing mean residual life is used to study age replacement model. Testing exponentiality against IRMRL\textsubscript{shock} class enables reliability engineers to decide whether to adopt a planned replacement policy over unscheduled one. To address this issue, a new testing procedure for exponentiality against IRMRL\textsubscript{shock} class was introduced and studied. It is simple to devise, calculate and have exceptionally high efficiency for some of the well-known alternatives. We obtained the exact null distribution of the test statistic and then obtained critical values for different sample sizes. Using the asymptotic theory of U-statistics, we showed that the test statistic was consistent and has limiting normal distribution. Finally, we illustrated our test procedure using a real data.
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