Uncertainty propagation of storage reliability evaluation using two-level MCS/NIPC

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Abstract. In order to solve the problem of limited sample size and life data in the storage reliability evaluation of high-value weapons, the storage reliability evaluation method of electronic product based on storage performance modeling and simulation was studied. The problem of epistemic-aleatory mixed uncertainty propagation in product storage performance model with time degradation factor was discussed emphatically. The calculation method of two-level Monte Carlo simulation and non-intrusive polynomial chaos is proposed, which solved the problem that the computational capacity of storage performance simulation would increase explosively when the number of uncertainty parameters is large. Finally, the effectiveness of the method is verified by an example of circuit storage reliability evaluation, and the calculation efficiency is about 100 times higher than that of the two-lever MCS method.

1. Introduction

For some high-reliability, long-life electronic products, such as missile control systems, aerospace electronic equipment, etc., there will be aging and corrosion of materials during long-term storage, resulting in changes in material properties, strength decreasing, and drift of electrical performance parameters, eventually causing the product’s key performance parameters do not meet the requirements of use. In order to assess the impact of performance degradation on the storage life of electronic products, a large number of accelerated or natural storage life tests are often required to obtain the degradation laws of the parameters. However, due to the complexity of failure mechanism and the limitation of funds, it is difficult to carry out large sample tests at the whole machine or system level. In order to solve the problem of storage life assessment under a small sample of test ban weapons, the Sandia National Laboratory of the United States proposed the QMU(Quantification of Margin and Uncertainty) evaluation method [1], which first establishes a high-fidelity physical model of the product, and then by quantifying the uncertainty of the model parameters and the propagation of uncertainty in the model, the performance degradation law, margins and uncertainties, confidence factor, storage life and other information of key parameters of the product are obtained.

The QMU method has attracted the attention of many scholars at home and abroad, especially the quantification and transmission of uncertainty, and has carried out a great deal of research. Earlier studies on uncertainty quantification have raised two challenging issues [2] for Oberkampf of Sandia National Laboratory in the United States. Many scholars have studied these challenge problems using possibility theory [3], Info-gap theory [4], evidence theory [5-7], and stochastic set theory [8]. Subsequently, domestic and foreign scholars studied the uncertainty quantification under parameter
correlation[9], the uncertainty quantification based on fuzzy set theory under stochastic-cognitive mixed uncertainty[10], and the aleatory-epistemic mixed uncertainty separation [11], fast algorithm of uncertainty quantification[12,13], uncertainty quantification in multidisciplinary optimization[14,15], stochastic field uncertainty quantification [16,17], and so on.

For the storage reliability assessment problem, on the one hand, due to the complexity of the storage failure mechanism and the limitations of cognition, the model parameters often contain both aleatory uncertainty and epistemic uncertainty. On the other hand, the response of the product storage performance model is varying over time. At present, although many achievements have been made in the field of uncertainty quantification, the research on its application in storage reliability assessment is relatively fewer.

In this paper, the interval number is used to describe the epistemic uncertainty in the storage performance model parameters, and the probability distribution is used to describe the aleatory uncertainty. The mixed uncertainty propagation problem in the system storage performance model when the parameters contain time degradation factors is studied. When there are more uncertain parameters and at the same time contains time factors, the amount of calculation will show explosive growth. Aiming at this problem, the two-level Monte Carlo sampling algorithm is improved, and the calculation method of two-level Monte Carlo Simulation(MCS) and non-intrusive polynomial chaos(NIPC) is proposed. The validity of the method is verified by a case of circuit storage reliability evaluation.

2. Basic theory

2.1. P-boxes basic theory

Definition 1: \( F \) and \( F \) are the increasing functions defined on the interval \([0, 1]\) in the real field \( \mathbb{R} \), and \( F \leq F \), \( x \in \mathbb{R} \). By defining a series of inexact known cumulative distribution functions, \( F(x) = P(X < x) \) (among them, \( F(x) \leq F(x) \leq \bar{F}(x) \)) is the probability boundary, denoted as \([F, \bar{F}]\).

According to the definition, it is known that the probability boundary contains the upper and lower boundaries of the distribution function of the stochastic variable, and even if the distribution of the random variable is unknown, it is possible to obtain a robust distribution of the stochastic variable. That is to say, for the stochastic variable \( X \), its distribution function \( F \) is unknown, but the known distribution function is the probability boundary \([F, \bar{F}]\), then \( F(x) \) is the lower boundary of \( F(x) \) (representing the probability that the stochastic variable \( X \) is less than or equal to \( x \)). Similarly, \( \bar{F}(x) \) is the upper boundary of the corresponding same probability. If the lowest probability \( P \) of the stochastic variable \( X \) is known, then the upper and lower boundaries of the distribution function can be calculated.

\[
\begin{align*}
\bar{F}_x(x) &= 1 - P(X > x) \\
F_x(x) &= P(X \leq x)
\end{align*}
\]

(1)

The probability boundary describes the general framework for reconstructing inexact special distributions. It can describe not only the distribution of unknown parameters, but also the distribution function of unknown types. For example, if the concern is the probability that the system response occurs when the random variable is at \( x_i \), then the probability interval of the system response is \([F(x_i), \bar{F}(x_i)]\); if the concern is the value of the stochastic variable when the cumulative probability of the response is \( F(x_i) \), then the interval of the stochastic variable is \([\hat{x}_i, \bar{x}_i]\), as shown in Figure 1. Figure 1 also shows the concept of a 95% confidence interval for probability boundaries (confidence from 2.5% to 97.5%).
2.2. Quantification and propagation of aleatory uncertainty based on NIPC

The polynomial chaos method is based on some deterministic solutions to estimate the coefficients in the polynomial chaotic expansion, and then obtain the statistical properties of the response such as mean and standard deviation. The polynomial chaos method originated from the random variable spectrum expansion in Wiener’s isotropic chaos theory. Then Xiu[18] extended the early Hermite polynomial chaos based on Gaussian stochastic variables to Askey polynomial chaos, which is suitable for more general stochastic variable probability distribution (even distribution, normal distribution, etc.).

Let the system’s solution have the following form:

\[ U(x,t,\xi(\theta)) = \sum_{i=0}^{\infty} \tilde{U}_i(x,t)\psi_i(\xi(\theta)) \]  \hspace{1cm} (2)

Where \( \tilde{U}_i(x,t) = (\hat{v}_i, \hat{a}_i, \hat{E}_i) \) is a deterministic vector function, \( \psi_i(\xi) \) is a polynomial function of the stochastic variable, and \( \{\psi_i(\xi)\}_{i=0}^{\infty} \) is an orthogonal polynomial family. \( \xi \) is the stochastic variable on probability space \( (\Omega, F, P) \), \( U(x,t,\xi) \) is the stochastic variable on probability space \( (\Omega, F, P) \), \( x \) is the sample space, \( F \) is the algebra on which \( P:F \rightarrow [0,1] \) is the probability measure, \( \theta \in \Omega \). Different stochastic variable types correspond to different polynomials \( \psi_i \) [18], as shown in Table 1.

| distribution type of random variables | Wiener–Askey chaos | range of random variables |
|--------------------------------------|--------------------|--------------------------|
| **Continuous**                       |                    |                          |
| Gaussian                             | Hermite            | \((-\infty, +\infty)\)   |
| Gamma                                | Laguerre           | \([0, +\infty)\)        |
| Beta                                 | Jacobi             | \([a,b]\)                |
| Uniform                              | Legendre           | \([a,b]\)                |
| **Discrete**                         |                    |                          |
| Possion                              | Charlier           | \([0,1,2,\ldots]\)      |
| Binomial                             | Krawtchouk         | \([0,1,2,\ldots,n]\)    |

Table 1. The type of Wiener-Askey polynomial chaos with random variables.
Negative binomial  Meixner  \{0,1,2,\ldots\}
Hypergeometric  Hahn  \{0,1,2,\ldots,n\}

According to the different methods of solving \( U_i(x,t) \), polynomial chaos methods can be divided into intrusive polynomial chaos (IPC) and non-intrusive polynomial chaos (NIPC). NIPC uses the existing numerical solution program as a black box. In a stochastic space, a number of sample points are obtained through a certain sampling method. The sample points are input into a deterministic program solution, and then the deterministic output results are statistically analyzed. The statistical characteristics of the relevant numerical solution results are obtained to evaluate the influence of the uncertainty of the input parameters or calculation conditions during the calculation process. NIPC uses existing numerical simulation programs that do not require modifications to the governing equations and do not require reprogramming. This paper applies a non-intrusive chaotic polynomial expansion based on the stochastic collocation method [19], which is a method for solving polynomial coefficients without modifying the original model parameters. Figure 2 is a flow chart for establishing the polynomial chaos expansion (PCE) model to estimate the system response.

![Flow chart of PCE model construction and output responses estimation](image)

**Figure 2.** Flow chart of PCE model construction and output responses estimation.

In Figure 2, the orthogonal polynomial basis is first selected according to the probability distribution type of the stochastic input variable. Assuming that each independent stochastic variable is a normal distribution, the Hermite polynomial is selected as the base and the finite term expansion is used.

Assume:

\[
U(x,t,\xi(\theta)) = \sum_{i=0}^{N} \hat{U}_i(x,t)\psi_i(\xi(\theta))
\]  

(3)
where \( N = \frac{(d + n)!}{d!n!} - 1 \) is the number of stochastic variables, and \( d \) is the polynomial order.

Then, the coefficient of the chaotic polynomial is solved by the stochastic collocation method. Finally, the output response is estimated from the established PCE model.

3. Uncertainty quantification and transmission based on two-level MCS/NIPC method

Uncertainty quantification and propagation are the process of determining the influence of input uncertainty on the output response uncertainty, while input uncertainty is generally divided into two types: aleatory uncertainty and epistemic uncertainty. For these two kinds of uncertainties alone, the current uncertainty quantification methods, such as the classical probabilistic methods (including traditional probabilistic methods and Bayesian methods), can effectively solve the quantification and transmission problems of stochastic uncertainties. Otherwise, using non-probability methods, such as fuzzy set theory, interval analysis, evidence theory, possibility theory, etc., can effectively deal with cognitive uncertainty. When the two uncertainties coexist, the two uncertainties need to be effectively separated to analyze the contribution degree of various uncertainties to the overall uncertainty in the response.

At present, the double-sampling technique is mainly used to deal with the mixed uncertainty of the coexistence of these two uncertainties. This type of method is mainly divided into three methods: probability boundary method, second order probability theory(SOP) and Dempster-Shafer theory(DST). The three methods differ in the processing of epistemic uncertainty in the outer loop: the probabilistic boundary method treats epistemic uncertainty as interval values, the SOP method treats epistemic uncertainty as subjective probability, and the DST method treats epistemic uncertainty as a trust structure. In this paper, epistemic uncertainty is treated by interval analysis. The quantification and transmission of aleatory and epistemic mixed uncertainties will be achieved based on Monte Carlo Simulation and non-intrusive polynomial chaos.

The P-box is used to characterize the mixing uncertainty, which is usually quantified by mass sampling using a two-layer nested MCS method. The method contains two parts: the cycle and the inner cycle. The outer loop samples the epistemic uncertainty, and the inner loop samples the aleatory uncertainty. However, under the premise of ensuring the accuracy of the calculation, the two-leveled MCS method for uncertainty analysis requires a large number of samples, especially for considering the influence of various uncertainties and ensuring the accuracy of uncertainty. The flutter boundary problem, such a method is obviously difficult to apply in practical engineering.

This paper uses a more efficient two-layer nested MCS/NIPC method. The analysis steps are shown in Figure 3. The outer loop quantifies the amount of epistemic uncertainty by the MCS method, extracts a sample from each epistemic uncertainty variable, and the epistemic uncertainty variable is regarded as a fixed value in the inner loop. On this basis, The loop quantifies the aleatory uncertainty by the NIPC method. First, the PCE model of the actual model is established, and then \( N \) random sampling parameters of the aleatory uncertainty are sampled. The sampled value is input into the PCE model to calculate the output response value. When the number of samples \( N \) is sufficient. When large, each time an outer loop is performed, a CDF with an output response can be drawn, and \( M \) pieces of CDFs can be drawn through \( M \) times outer loops to form a P-box whose output response value is under the current uncertainty condition.
4. Example: power circuit storage reliability assessment
A power supply circuit is mainly composed of DC/DC module SWH03-12S05, voltage regulator 78D33S and multiple diodes and capacitors. Among them, SWH03-12S05 is a complex analog device, and others are discrete devices. The functional performance requirements of the power supply circuit are: SWH03-12S05 input 12±3V, output 5±0.5V, 3.3±0.5V. The schematic diagram of the circuit diagram is shown in Figure 4.
Figure 4. Power supply circuit schematic diagram.

First, establish the normal performance model of the power circuit and test it. The results are shown in 5 and 6.

Figure 5. Simulation test model for power supply circuit.

Figure 6. Simulation waveform of power supply circuit standard function.

It can be seen from Figure 6 that the nominal function verification result of the power supply circuit is: ZK_5V port output voltage value is 5.0335V, the relative error is 0.67%; ZK_3.3V port output voltage value is 3.3242V, the relative error is 0.73%, and the simulation result satisfies its functional performance requirements.

During long-term storage, the power circuit will degrade the key performance parameters, which will lead to circuit performance degradation. In order to evaluate the storage reliability of the power supply circuit, it is necessary to carry out the storage life test to obtain the degradation law of the key parameters of the components with time, and then bring into the power supply circuit simulation model to obtain the degradation rules of two key parameters ZK_5V and ZK_3.3V. Finally, the reliability evaluation conclusion of power supply circuit storage is obtained.

Through FA and FMECA, it is determined that the DC/DC module SWH03-12S05 and the three-terminal regulator 78D33S are storage sensitive components, and the key parameters are voltage. The
degradation laws of SWH03-12S05 and 78D33S were obtained by accelerated storage test. Since SWH03-12S05 has a small sample size, only the interval value can be obtained; 78D33S sample size is large, and the distribution type of the degradation process is normal distribution. According to the degenerate function, the mean value of the output voltage of swh03_12s05 at 0h, 10k h, 20k h, 30k h, 40k h, 50k h, 60k h, 70k h, 80k h, 90k h, 100k h (11 moments in total) is obtained. And the standard deviation, and the interval value of the 78D33S output voltage, as shown in Table 2 and Table 3.

**Table 2. Aleatory uncertain input parameters of supply circuit at 11 discrete times.**

| Degradation time (h) | Voltage of 78D33S (V) |Aleatory uncertainties |
|---------------------|------------------------|------------------------|
|                     | Mean                   | Standard deviation     | Distribution type |
| 0                   | 680.0                  | 2.2667                 | normal            |
| 10k                 | 693.6                  | 2.4417                 | normal            |
| 20k                 | 707.2                  | 2.9042                 | normal            |
| 30k                 | 720.8                  | 3.5433                 | normal            |
| 40k                 | 734.4                  | 4.2806                 | normal            |
| 50k                 | 748.0                  | 5.0735                 | normal            |
| 60k                 | 761.6                  | 5.8996                 | normal            |
| 70k                 | 775.2                  | 6.7468                 | normal            |
| 80k                 | 788.8                  | 7.6079                 | normal            |
| 90k                 | 802.4                  | 8.4788                 | normal            |
| 100k                | 816.0                  | 9.3567                 | normal            |

**Table 3. Interval uncertain input parameters of supply circuit at 11 discrete times.**

| Degradation time (h) | Interval uncertainties |
|---------------------|------------------------|
|                     | Voltage of swh03_12s05 (V) |
|                     | Upper bound            | Lower bound            | Distribution type |
| 0                   | 4.9000                 | 5.1000                 | interval          |
| 10k                 | 4.9980                 | 5.2020                 | interval          |
| 20k                 | 5.0922                 | 5.3078                 | interval          |
| 30k                 | 5.1832                 | 5.4168                 | interval          |
| 40k                 | 5.2717                 | 5.5283                 | interval          |
| 50k                 | 5.3582                 | 5.6418                 | interval          |
| 60k                 | 5.4433                 | 5.7567                 | interval          |
| 70k                 | 5.5274                 | 5.8726                 | interval          |
| 80k                 | 5.6106                 | 5.9894                 | interval          |
| 90k                 | 5.6933                 | 6.1067                 | interval          |
| 100k                | 5.7755                 | 6.2245                 | interval          |

The output voltage of the power supply circuit is calculated by the two-layer Monte Carlo sampling method in the literature [20] and the two-layer MSC/NIPC method proposed in this paper. Here, only the calculation result of the ZK_5V parameter is given, which is similar for the ZK_3.3V calculation process.

4.1. Two-level Monte Carlo sampling method
The external loop performs 50 Monte Carlo sampling on the output voltage of the cognitive uncertainty parameter swh03_12s05 according to the uniform distribution; the internal loop performs
500 Monte Carlo sampling on the random uncertainty parameter 78D33S, and the corresponding decommissioning conditions are generated at 11 times. 11 CDF curves. 50 sets of 11 CDF curves were generated in 50 external cycles to form 11 sets of p-boxes, as shown in Figure 7. The calculation time is 56364.4 seconds.

![Figure 7. Uncertainty propagation results of supply circuit using 2-order MCS method.](image)

4.2. Two-level MSC/NIPC method

The output voltage of the power supply circuit is then calculated using a two-layer MSC/NIPC method. The outer loop performs 50 Monte Carlo sampling on the output voltage of the cognitive uncertainty parameter swh03_12s05. The inner loop extracts 500 samples for the random uncertainty parameters, and quantizes each random uncertainty combination sample by the second-order NIPC method. The 11 times of the decommissioning conditions correspondingly generate 11 CDF curves. 50 sets of 11 CDF curves were generated in 50 external cycles to form 11 sets of p-boxes, as shown in Figure 8. The calculation time is 642.54 seconds.

![Figure 8. Uncertainty propagation results of supply circuit using 2-order MCS/NIPC method.](image)

At \( t=100\text{k} \), comparing the results of the two-level MSC method and the two-level MSC/NIPC method (Figure 9), the average value of \( ZK\_5V \) obtained by the two-level MSC/NIPC method is 5.2765, and the standard deviation is 0.0674, which is obtained by the two-level MCS. The mean \( ZK\_5V \) is 5.2742 and the standard deviation is 0.0681. It can be seen that the MSC/NIPC method has a good prediction effect.
It can be seen from the calculation result of Figure 8 that the output voltage of the power supply circuit increases with the storage time course, and the failure threshold of the power supply circuit ZK_5V parameter is 5.5V, which can obtain the storage reliability function of the power supply circuit, as shown in Figure 10.

**Figure 9.** The result contrast of 2-order MCS and MCS/NIPC method at t=100kh.

**Figure 10.** Storage reliability function of supply circuit.

5. Conclusion
In this paper, the aleatory-epistemic mixed uncertainty transfer problem in storage reliability evaluation is studied. Based on the circuit storage performance transfer model, based on the MCS/NIPC two-level nested loop solution, the response p-box is obtained and the product storage reliability is evaluated. The studies have shown that for time-varying circuit simulations, the p-box of the system response at each degraded time can be obtained. The simulation results show that compared with the traditional two-layer Monte Carlo sampling method, the mean and standard deviation of the obtained system response are basically consistent, and the calculation accuracy is reliable. However, MCS/NIPC shows obvious in calculating the time cost. The computational efficiency advantage is about 100 times better than the two-level Monte Carlo sampling method.
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