INTERACTING THREE FLUID SYSTEM AND THERMODYNAMICS OF THE UNIVERSE BOUNDED BY THE EVENT HORIZON

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The work deals with the thermodynamics of the universe bounded by the event horizon. The matter in the universe has three constituents namely dark energy, dark matter and radiation in nature and interaction between them is assumed. The variation of entropy of the surface of the horizon is obtained from unified first law while matter entropy variation is calculated from the Gibbs’ law. Finally, validity of the generalized second law of thermodynamics is examined and conclusions are written point wise.

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I. INTRODUCTION

At present it is strongly believed that our universe is experiencing an accelerated expansion. The various cosmological observations (for examples data collected from SNeIa [1], WMAP [2], SDSS [3] and X-ray [4]) suggest that the acceleration is driven by a missing energy density with negative pressure, known as dark energy (DE). Although cosmological constant is a possible solution for accelerated expansion but it is normally discarded due to fine-tuning problem [5]. There are various candidates for DE model namely variable Cosmological Constant [6], scalar field or quintessence field [7], phantom field [8] (a scalar field with negative kinetic term) or quintom field [9] (a unified model of quintessence and phantom). From the effective quantum field theory and gravitational effect some speculation about the nature of DE is made and is known as holographic dark energy model (HDE) [10]. (One may note that from the recently proposed Horava gravity [11] the DE model may have strong quantum gravitational back ground [12]). Although, these DE models are in satisfactorily agreement with observational evidences but a new type of ‘coincidence’ problem [5] has come into picture- the density of vacuum energy and that of dark matter (DM) are of the same order although they have quite distinct energy scale during expansion era. A possible way out of this problem is to consider interaction between DE and DM. Further, from the analysis of the cosmic microwave background radiation, our universe still have some remnant of this back ground radiation. So it is quite natural to consider DE interacting with both DM and radiation.

In the present work, we perform a thermodynamical analysis of the universe bounded by the event horizon and the matter is chosen as above i.e. DE interacting with DM and radiation. Thermodynamical study of the universe bounded by the apparent horizon is common in the literature as they form a bekenstein system [13] while any definite character is still unknown for event horizon. Basically we examine the validity of the generalized second law of thermodynamics on the event horizon and the required constraints are analyzed. Also a comparison with earlier results are attempted.

II. FORMULATION OF THE MODEL:

Let us consider our universe to be homogeneous and isotropic FRW model bounded by the event horizon. The universe is assumed to be filled up with DE interacting with DM and radiation. The space time geometry is described by the line element

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\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \]

\[ = h_{ab} dx^a dx^b + R^2 d\Omega^2 \quad (1a) \]

where

\[ h_{ab} = \text{diag} \left( -1, \frac{a^2}{1 - kr^2} \right) , \quad (a, b = 0, 1 \text{ with } x^0 = t, x^1 = r) \]

is the metric on the plane normal to the spherical surface of symmetry and

\[ d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \] is the metric on unit two sphere.

\[ R = ar \] is the radius of the sphere(area-radius), \( a(t) \) is the scale factor and \( k = 0, \pm 1 \) stands for flat, closed and open model of our universe respectively. Let us denote by \((\rho_c, p_c), (\rho_m, p_m), (\rho_r, p_r)\) the matter density and thermodynamic pressure of dark energy, dark matter and radiation respectively. Assuming barotropic equation of state for the individual matter components we write

\[ p_l = \omega_l \rho_l , \quad (l \equiv (e, m, r)) \quad (2) \]

The two Friedmann equations for the present model are

\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \]

\[ \dot{H} - \frac{k}{a^2} = -4\pi G (\rho + p) \]

where \( \rho = \rho_c + \rho_m + \rho_r \) is the total matter density and the resulting thermodynamic pressure \( p = p_c + p_m + p_r \). Due to interaction among the matter constituents the energy conservation relations are

\[ \dot{\rho}_c + 3H(1 + \omega_c)\rho_c = -\Gamma \]

\[ \dot{\rho}_m + 3H(1 + \omega_m)\rho_m = \Gamma' \quad (5a) \]

and

\[ \dot{\rho}_r + 3H(1 + \omega_r)\rho_r = \Gamma - \Gamma' \]

(5b)

Here the two interaction terms \( \Gamma \) and \( \Gamma' \) are in general not constants (may have arbitrary forms) and distinct. The sign of \( \Gamma \) and \( \Gamma' \) will indicate the direction of matter flow. For example, if \( \Gamma > 0 \) then there is an energy flow from DE to the other two matter distribution while \( \Gamma' < 0 \) indicates an energy loss from the DM sector to the other two constituents and so on. According to ref.[14] the ansatzs for the interaction terms may be chosen as

\[ \Gamma = \mu_c \rho_c \text{ and } \Gamma' = \mu_c \rho_c \quad (6) \]

and we have three non-interacting fluids with conservation equations

\[ \dot{\rho}_c + 3H(1 + \omega^{eff}_c)\rho_c = 0 \]

\[ \dot{\rho}_m + 3H(1 + \omega^{eff}_m)\rho_m = 0 \]

(7)
and
\[ \dot{\rho}_r + 3H(1 + \omega_{r}^{eff})\rho_r = 0 \]

Here the form of the modified state parameters are
\[ \omega_{D}^{eff} = \omega_e + \frac{\mu_e}{3H}, \quad \omega_{m}^{eff} = \omega_m - \frac{\mu_m}{3H} \quad \text{and} \quad \omega_{r}^{eff} = \omega_r - \frac{\mu_r u}{3H} + \frac{\mu_m v}{3H} \quad (8) \]

where \( u = \frac{\rho_e}{\rho_r}, \quad v = \frac{\rho_m}{\rho_r} \) is the ratio of two energy densities. Hence from (7) we have
\[ \dot{\rho} + 3H(1 + \omega)\rho = 0 \quad (7a) \]

where
\[ \omega = \alpha \omega_D + \beta \omega_m + \delta \omega_r \quad (8a) \]

with
\[ \alpha = \frac{\rho_d}{\rho}, \quad \beta = \frac{\rho_m}{\rho} \quad \text{and} \quad \delta = \frac{\rho_r}{\rho}. \]

Thus the interacting 3-components fluid distribution can be considered as a single fluid with equation of state parameter given by (8a).

III. THERMODYNAMICS OF THE UNIVERSE BOUNDED BY THE EVENT HORIZON:

We start this section with the idea of horizons for the present model. For the space-time metric given by equation (1a) the apparent horizon \( (R_A) \) is defined as
\[ h^{ab}\partial_a R \partial_b R = 0 \]

or explicitly it gives
\[ R_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} \quad (9) \]

One may note that this apparent horizon coincides with the trapping horizon [15] and for flat case apparent horizon and Hubble horizon \( (R_H = \frac{1}{H}) \) coincides. The radius of the event horizon \( (R_E) \) is mathematically given by
\[ R_E = a \int_{t}^{\infty} \frac{dt}{a} = a \int_{a}^{\infty} \frac{da}{Ha^2} \quad (10) \]

It is to be noted that event horizon exists only for accelerating universe. In terms of the conformal time \( \tau \) [16]
\[ \tau = -\int_{t}^{\infty} \frac{dt}{a(t)} \quad |\tau| < \infty \quad (11) \]

the event horizon can be written as [16]
\[ R_E = -a \sinh(\tau) \quad k = -1 \]
\[ R_E = -a\tau \quad k = 0 \] (12)

\[ R_E = -a \sin(\tau) \quad k = +1 \]

Note that if \(|\tau| = \infty\), event horizon does not exist.

The change of different horizon radii with the evolution of the universe are given by

\[ \dot{R}_H = -\frac{\dot{H}}{H^2} \] (13)

\[ \dot{R}_A = -H \left( \frac{\dot{H}}{a^2} \right) R_A^3 \] (14)

\[ \dot{R}_E = H R_E - \sqrt{1 - \frac{k}{a^2} R_E^2} \] (15)

We see that radius of the event horizon increases throughout the evolution of the universe so long as \( R_E > R_A \) and it does not depend on the nature of the matter in the universe. On the other hand both \( R_A \) and \( R_H \) increases with the evolution of the universe in the quintessence era but decreases in the phantom era. (For detailed discussion see ref[17]).

To find the entropy variation of the surface of the event horizon we start with unified first law

\[ dE = A\Psi + WdV \] (16)

where,

\[ E = \frac{R}{2G} (1 - h^{ab}\partial_a R\partial_b R) \] (17)

is the Misner-sharp energy.

\[ \Psi = \psi_a dx^a, \quad \text{is the energy supply} \]

\[ \psi_a = T^a_b \partial_b R + \partial_a R, \quad \text{is the energy flux} \] (18)

and

\[ W = -\frac{1}{2} Trace(T), \quad \text{is the work function} \]

For the present model we have

\[ A\Psi = 2\pi R^2 (\rho + p) [-2HRdt + dR] \]

\[ WdV = 2\pi R^2 (\rho - p)dR \] (19)

Hence from the Clausius relation on the event horizon

\[ T_E dS_E = dQ = -dE = 4\pi R^3_E H (\rho + p) dt \]
\[ T_E \frac{dS_E}{dt} = 4\pi R_E^2 H (\rho + p) \] (20)

Where \( S_E \) and \( T_E \) are respectively the entropy and the temperature on the event horizon. For equilibrium thermodynamics we assume \( T_E \) as the temperature of the matter inside the event horizon. For variation of the entropy \( (S_I) \) of the matter distribution we take help of the Gibb’s equation

\[ T_E dS_I = dE_I + p dV_I \] (21)

where \( V_I = \frac{4}{3}\pi R_E^3 \), the volume bounded by the event horizon and \( E_I = \rho V_I \). Thus using the combined energy conservation relation (7a) and the variation of the radius of the event horizon, i.e., eq. (15) we obtain

\[ T_E \frac{dS_I}{dt} = -4\pi R_E^2 (\rho + p) \sqrt{1 - \frac{k}{a^2 R_E^2}} \] (22)

Hence combining (20) and (22) we obtain

\[ T_E \frac{d}{dt} (S_E + S_I) = 4\pi R_E^2 (\rho + p) \left[ HR_E - \sqrt{1 - \frac{k}{a^2 R_E^2}} \right] \] (23)

From the above thermodynamical analysis the conclusions are the following:

1. The time variation of the entropy of the horizon and that of the matter distribution inside the horizon are of opposite character. In the quintessence era (i.e., when the resulting matter satisfies weak energy condition) \( S_E \) is an increasing function while \( S_I \) decreases with the evolution. However in the phantom era the entropy functions exchange their role, i.e., entropy of the horizon decreases while entropy of the matter distribution increases with the evolution.

2. The validity of the generalized second law of thermodynamics (GSLT) depends both on the nature of the matter and on the evolution of the horizons, i.e., GSLT will hold if
   
   (a) \( \rho + p > 0 \) and \( \dot{R}_E > 0 \), i.e., \( R_E > R_A \)
   
   (b) \( \rho + p < 0 \) and \( \dot{R}_E < 0 \), i.e., \( R_E < R_A \)

   The first possibility indicates that the resulting matter should be of quintessence nature, i.e.,

   \[ 1 + \omega > 0 \]

   \[ 1 + \omega_d + \beta \omega_m + \delta \omega_r > 0 \]

   \[ 1 + \frac{\Omega_d \omega_d + \Omega_m \omega_m + \Omega_r \omega_r}{1 - \Omega_k} > 0 \] (24)

   For the other possibility the restrictions are

   \[ 1 - \Omega_k + \Omega_d \omega_d + \Omega_m \omega_m + \Omega_r \omega_r < 0 \] (25)

   and \( R_E < R_A \).

3. The validity of GSLT does not depend on the interaction terms, it only depends on the equation of state parameter for each component.

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