Dyonic Black Holes in String Theory

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An exact solution of the low-energy string theory representing static, spherical symmetric dyonic black hole is found. The solution is labeled by their mass, electric charge, magnetic charge and asymptotic value of the scalar dilaton. Some interesting properties of the dyonic black holes are studied. In particular, the Hawking temperature of dyonic black holes depends on both the electric and magnetic charges, and the extremal ones, which have nonzero electric and magnetic charges, have zero temperature but nonzero entropy. These properties are quite different from those of electrically (or magnetically) charged dilaton black holes found by Gibbons et al. and Garfinkle et al., but are the same as those of the dyonic black holes found by Gibbons and Maeda.
1 Introduction

Superstring theories are the most promising candidates for a consistent quantum theory of gravity. It is of interest to investigate how the properties of black holes are modified when the low-energy effective string actions are considered. Recently, some new black hole solutions have been obtained in the low-energy string theories in which the Kalb-Ramond field, dilaton field and gauge field are incorporated \([1]-[14]\). Study of the minimal coupling theory of gravity and the Kalb-Ramond field indicate that there is no classical axionic hair in the model \([1][5][7]\). However, when dilaton and \(U(1)\) gauge field are included, the new classical axionic hair have been found \([1][14]\). There are some charged black holes, with electric or magnetic charge or dyon, in the effective string theories in which gravity coupled to axion, dilaton field and gauge field \([1][10]-[11]\). Reference \([15]-[17]\) are excellent reviews on the stringy black holes.

Here, we consider the four-dimensional effective string action in which gravity is coupled to dilaton and electromagnetic field only

\[
I = \int d^4x \sqrt{-g} \left[ -R + 2(\nabla \phi)^2 + e^{-2\phi}F^2 \right].
\]

(1)

Gibbons et al. \([8]\) and Garfinkle et al. \([9]\) showed that the properties of electrically or magnetically charged black holes can be drastically affected by incorporation of dilaton. Gibbons and Maeda \([8]\) also gave a dyonic black hole in the effective string theory. They reduced the field equations to a
‘Toda molecule’ form and obtained exact solutions by using a peculiar radial coordinate. Their solution is so complicated that can not be understood by a easy way. Therefore, it is interesting to find an exact dyonic black hole solution of the effective action, eq.(1), by using the usual radial coordinate, and present it in a clear form.

In this paper, we find a static, spherical symmetric solution in the low-energy effective string theory, eq.(1), in which gravity is coupled to electromagnetic field and dilaton only. It represents a dyonic black hole which is characterized by their mass $M$, electric charge $Q_e$, magnetic charge $Q_m$ and asymptotic value of the dilaton $\phi_0$. This solution is simpler and easier to interprete. In analogy to the properties of Reissner-Nordström black hole, the electrically (or magnetically) charged black holes found by Gibbons and Maeda, and by Grafinkle, Horowitz and Strominger, (GM-GHS solutions), are the limiting case of our results. Even the electrically (or magnetically) charged black holes are the special case of the dyonic black holes, however, unlike GM-GHS charged black holes, the Hawking temperature of the dyonic black hole depends on both the electric and magnetic charge, and vanishes as $Q_e$ and $Q_m$ tend to extremal values. These properties are the same as those of the dyonic black holes found by Gibbons and Maeda.

The plan of this paper is as follows. In Sec.2, we derive the static, spherically symmetric Einstein field equations of the theory eq.(1). In Sec.3, we
exhibit an exact dyonic black hole solution. We investigate the properties of the dyonic black holes in Sec.4. Finally, we present some concluding remarks.

2 The spherically symmetric field equations

The field equations of the effective string action, eq.(1), are

\[ \nabla_{\mu} (e^{-2\phi} F^{\mu\nu}) \ , \tag{2} \]

\[ \nabla^{2}\phi + \frac{1}{2} e^{-2\phi} F^2 = 0 \ , \tag{3} \]

\[ R_{\mu\nu} = 2\nabla_{\mu}\phi\nabla_{\nu}\phi + 2e^{-2\phi} F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{2} g_{\mu\nu} e^{-2\phi} F^2 \ . \tag{4} \]

The most general spherical symmetric metric can be written in the form

\[ ds^2 = -\Delta^2 dt^2 + \frac{\sigma^2}{\Delta^2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \ , \tag{5} \]

where \( t, r, \theta \) and \( \varphi \) are ordinary spherical coordinates and \( \Delta \) and \( \sigma \) are function of \( r \) only. Here, we introduce the tetrad frame with basis 1-forms

\[ \omega^0 = \Delta dt, \omega^1 = \frac{\sigma}{\Delta} dr, \omega^2 = rd\theta \quad \text{and} \quad \omega^3 = r \sin \theta d\phi. \]

The nonvanishing tetrad components of the \( U(1) \) gauge field strength \[ F^0 = -F^1 = f(r) \ , \tag{6} \]

\[ \text{[18]-[20]} \]

It is more convenient to generalize the electrically (or magnetically) charged black holes to dyonic black holes by using the spherically symmetric metric form, eq.(4), rather than that used by Garfinkle et al., \( ds^2 = -\lambda(r^*)^2 dt^2 + \frac{1}{\lambda(r^*)} dr^*^2 + R(r^*)^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \).
\[ F_{23} = -F_{32} = g(r) , \]

and the dilaton field \( \phi \) depend on \( r \) only.

Plugging eq. (5) and eq. (6) into eqs. (2)-(4), and the Bianchi identity of gauge field
\[
\nabla_\mu F_{\nu\lambda} + \nabla_\nu F_{\lambda\mu} + \nabla_\lambda F_{\mu\nu} = 0 ,
\]
we end up with the following six equations
\[
(r^2 e^{-2\phi} f)' = 0 , \tag{8}
\]
\[
(r^2 g)' = 0 , \tag{9}
\]
\[
(r^2 \Delta^2 \sigma)^{'} = r^2 \sigma e^{-2\phi}(f^2 + g^2) , \tag{10}
\]
\[
\left[ \frac{r\Delta}{\sigma} (r\Delta)' \right]' = \sigma , \tag{11}
\]
\[
\left( \frac{r\Delta^2}{\sigma} - \frac{r^2 \Delta \Delta'}{\sigma} \right)' - \sigma = -2r^2 \sigma e^{-2\phi}(f^2 + g^2) , \tag{12}
\]
\[
\left[ \frac{r\Delta}{\sigma} (r\Delta)' \right]' - 2\left( \frac{2r\Delta}{\sigma} \Delta' + \frac{\Delta^2}{\sigma} \right) + \sigma = -2r^2 \frac{\Delta^2}{\sigma} \phi^2 + 2r^2 \sigma e^{-2\phi}(f^2 + g^2) , \tag{13}
\]

where the prime denotes differentiation with respect to \( r \).
Eqs. (8) and (9) imply that
\[ f = \frac{Q_e}{r^2} e^{2\phi}, \]  
and
\[ g = \frac{Q_m}{r^2}, \]
where \( Q_e \) and \( Q_m \) are integration constants and are relate to electric and magnetic charge, respectively.

After replacing \( f, g \) by eqs. (14) and (15) and some suitable recombinations of eqs. (10)-(13), we can reduce eqs. (10)-(13) to four simpler equations
\[
\left( \frac{r^2 \Delta^2}{\sigma} \phi' \right)' = \frac{\sigma}{r^2} \left( Q_e^2 e^{2\phi} - Q_m^2 e^{-2\phi} \right),
\]  
\[
\left[ \frac{1}{2} \frac{(r^2 \Delta^2)'}{\sigma} \right]' = \sigma,
\]  
\[
\left( \frac{r^2 \Delta^2}{\sigma} \right)' = \sigma \left[ 1 - \frac{1}{r^2} (Q_e^2 e^{2\phi} + Q_m^2 e^{-2\phi}) \right],
\]  
\[
(\phi')^2 = \frac{1}{r^2} \frac{\sigma'}{\sigma},
\]
which we will solve directly in the following section.
3 Dyonic black hole solutions

In this section, we will find the solutions of eqs.(16)-(19). The key to solve these equations is eq.(19). Consider the asymptotic condition of $\phi$, $\phi(r \to \infty) = \phi_0$, it implies $\frac{\sigma'}{\sigma} \sim O(r^{-n})$, where $n > 1$, in the asymptotic regime. Therefore, we take a simply workable ansatz

$$\frac{\sigma'}{\sigma} = \frac{\rho^2}{r(r^2 + \rho^2)} ,$$

(20)

where $\rho$ is a real constant. Such that we get

$$\phi' = \pm \frac{\rho}{r\sqrt{r^2 + \rho^2}} ,$$

(21)

where the different signs of $\phi'$ will give the same result eventually, and we will study the “+” one only. Solving eq.(17), (20) and (21), we can determine the functions $\sigma(r)$, $\phi(r)$ and $\Delta(r)$ when the asymptotic conditions, $\phi(r) \to \phi_0, \sigma(r) \to 1$ and $\Delta(r) \to 1$ as $r \to \infty$, are imposed. eq.(16) gives us some constraints on the integration constants in $\phi(r)$, $\Delta(r)$ and $Q_e, Q_m$ and eq.(18) is automatically satisfied when constraints on integration constants are imposed.

An exact solution of eqs.(16)-(19) is

$$F_{01} = \frac{Q_e}{r^2} e^{2\phi} ,$$

(22)

$$F_{23} = \frac{Q_m}{r^2} ,$$

(23)
\[ \sigma^2 = \frac{r^2}{r^2 + \rho^2} , \]  

(24)

\[ \Delta^2 = 1 - \frac{2M}{r^2} \sqrt{r^2 + \rho^2} + \frac{\beta}{r^2} , \]  

(25)

\[ e^{2\phi} = e^{2\phi_0} \left( 1 - \frac{2\rho}{\sqrt{r^2 + \rho^2 + \rho}} \right) . \]  

(26)

The constraints on the parameters are

\[ \rho = \frac{1}{2M} \left( Q_e^2 e^{2\phi_0} - Q_m^2 e^{-2\phi_0} \right) , \]  

(27)

\[ \beta = \left( Q_e^2 e^{2\phi_0} + Q_m^2 e^{-2\phi_0} \right) . \]  

(28)

Here the integration constant \( M \) is the mass of black holes, and is determined by the asymptotical behavior of the metric. The solutions of eqs.(22)-(28), with different parameters, are related to one another by a simple dual transformation

\[ F \rightarrow \tilde{F} = \frac{1}{2} e^{-2\phi} \epsilon_{\mu\nu} \lambda^\rho F_{\lambda\rho} , \]  

(29)

\[ \phi \rightarrow \tilde{\phi} = -\phi , \]

which can be represented by \( (Q_e, Q_m, \phi) \rightarrow (Q_m, Q_e, -\phi) \) in our solution.

Moreover, we can find that the GM-GHS electrically (or magnetically) charged black hole is a special case of our solution, eqs.(22)-(28), in which \( Q_m = 0 \) (or \( Q_e = 0 \)) but \( Q_e \neq 0 \) (or \( Q_m \neq 0 \)). For example, when we set
\( Q_e = 0 \), the solution eqs.\((22)-(28)\) will reduce to magnetically charged black hole,

\[
F_{23} = \frac{Q_m}{r^2}, \tag{30}
\]

\[
\sigma^2 = \frac{r^2}{r^2 + \tilde{\rho}^2}, \tag{31}
\]

\[
\Delta^2 = 1 - \frac{2M\sqrt{r^2 + \tilde{\rho}^2}}{r^2} + \frac{2M\tilde{\rho}}{r^2}, \tag{32}
\]

\[
e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{2\tilde{\rho}}{\sqrt{r^2 + \tilde{\rho}^2}}\right), \tag{33}
\]

where

\[
\tilde{\rho} = \frac{Q_m^2}{2M}e^{-2\phi_0}. \tag{34}
\]

It is exactly the GM-GHS magnetic black hole solution

\[
F_{23} = \frac{Q_m}{r^2(1 - \frac{2\tilde{\rho}}{r^2})}, \tag{35}
\]

\[
e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{2\tilde{\rho}}{r^2}\right), \tag{36}
\]

\[
ds^2 = -(1 - \frac{2M}{r^*})dt^2 + \frac{1}{(1 - \frac{2M}{r^*})}dr^2 + r^2(1 - \frac{2\tilde{\rho}}{r^*})\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{37}
\]

by a simple coordinate transformation

\[
r^* = \sqrt{r^2 + \tilde{\rho}^2 + \tilde{\rho}}. \tag{38}
\]
4 Properties of dyonic black hole solutions

The structure of the dyonic black hole is similar to that of the Reissner-Nordström one. Three cases are considered as follows:

CASE I: DYONIC BLACK HOLE

For \( M > \frac{1}{\sqrt{2}}(|Q_e|e^{\phi_0} + |Q_m|e^{-\phi_0}) \), i.e., \( 2M^2 - \beta > 2|Q_e||Q_m| \), there are two zeros of \( \Delta^2(r) \) at \( r = r_\pm \), where

\[
r_\pm = \left(2M^2 - \beta \right) \pm \sqrt{\left(2M^2 - \beta \right)^2 - 4Q_e^2Q_m^2} \right]^{1/2},
\]

which correspond to two horizons. The Kretschmann scalar

\[
K = R_{\mu\nu\lambda\tau}R^{\mu\nu\lambda\tau}
= \frac{\left[\left(\Delta^2\right)'\right]^2}{\sigma^4} - \left(\Delta^2\right)'\left(\Delta^2\right)\frac{\sigma^2}{\sigma^6} + \frac{1}{4} \frac{\left[\left(\Delta^2\right)'\right]^2}{\sigma^8}
+ \frac{4}{r^2} \left\{ \frac{\left[\left(\Delta^2\right)'\right]^2}{\sigma^4} - \left(\Delta^2\right)'\left(\Delta^2\right)\frac{\sigma^2}{\sigma^6} \right\}
+ \frac{1}{2} \Delta^4 \left(\frac{\sigma^2}{\sigma^8}\right)^2 + \frac{4}{r^4} \left(1 - 2\frac{\Delta^2}{\sigma^2} + \frac{\Delta^4}{\sigma^4}\right),
\]

is finite at \( r_\pm \) and is divergent at \( r = 0 \), indicating that \( r_+ \) and \( r_- \) are regular horizons, and the singularity locates at \( r = 0 \). Since the Penrose diagram of these case is the same as that of the Reissner-Nordström for \( M > Q \), see Fig.1(a). Therefore, we may expect that \( r_+ \) corresponds to the regular event horizon and \( r_- \) corresponds to the unstable inner horizon [21].
The limiting case, \( Q_e = 0 \) or \( Q_m = 0 \), show that the inner horizon will shrink to the singularity \( r = 0 \). It means that the electrically or magnetically charged black holes have only one event horizon located at

\[
\tilde{r}_+ = \sqrt{4M^2 - 2Q_e^2 e^{2\phi_0}},
\]

(41)

or

\[
\tilde{r}_+ = \sqrt{4M^2 - 2Q_m^2 e^{-2\phi_0}},
\]

(42)

respectively, i.e., all of them are equivalent to \( r_{*+} = 2M \) in the coordinate used by Grafinkle et al.. The Penrose diagram of these limiting case is shown in Fig.1(b).

**CASE II: EXTREMAL DYONIC BLACK HOLE**

For \( M = \frac{1}{\sqrt{2}}(|Q_e| e^{\phi_0} + |Q_m| e^{-\phi_0}) \), i.e., \( 2M^2 - \beta = 2|Q_e||Q_m| \), there is only one root of \( \Delta^2(r) = 0 \) at \( r = r_0 \), where

\[
r_0 = \sqrt{2M^2 - (Q_e^2 e^{2\phi_0} + Q_m^2 e^{-2\phi_0})} = \sqrt{2|Q_e||Q_m|}.
\]

(43)

Two horizons \( r_+ \) and \( r_- \) match to form a regular event horizon. \( r = 0 \) is still a singularity in this case. The Penrose diagram of the extremal dyonic black hole is shown in Fig.1(c). According to reference [22], the extremal dyonic black hole solutions are expected to be the end points of Hawking evaporation and correspond to stable quantum states.

We also see that \( r_0 = 0 \) if \( Q_e = 0 \) or \( Q_m = 0 \). Since the Kretschmann scalar diverges at \( r_0 = 0 \), the extremal electrically or magnetically charged
solution does not describe a black hole at all but rather a naked singularity, see Fig.1(d).

**CASE III: NAKED SINGULARITY**

For \( M < \frac{1}{\sqrt{2}}(|Q_e|e^{\phi_0} + |Q_m|e^{-\phi_0}) \), the solution, eqs.(22)-(28), describes a naked singularity. According to the cosmic censorship, this case should be forbidden, and gives a extremal values for \( Q_e \) and \( Q_m \).

Besides investigate the structures of the black holes, we also study the Hawking temperature of the dyonic black hole. Based on Hawking’s remarkable discovery - the laws of black hole mechanics [23], we find that the entropy \( S \) and the Hawking temperature \( T_H \) of dyonic black holes are

\[
S = \frac{1}{4} \text{(area)} = \pi \left[ (2M^2 - \beta) + \sqrt{(2M^2 - \beta)^2 - 4Q_e^2Q_m^2} \right], \quad (44)
\]

and

\[
T_H = \frac{1}{4\pi M} \left[ \frac{\sqrt{(2M^2 - \beta)^2 - 4Q_e^2Q_m^2}}{\sqrt{(2M^2 - \beta)^2 - 4Q_e^2Q_m^2 + (2M^2 - \beta)}} \right]. \quad (45)
\]

They depend on electric and magnetic charge of the black hole, and show that the extremal dyonic black holes have non-zero entropy, \( S_0 = 2\pi|Q_e||Q_m| \), at zero temperature. We may consider it as a result of a dual symmetric which generates degenerate ground states of black hole. Moreover, the electrically or magnetically charged black holes have the charge independent Hawking
The Hawking temperature of dyonic black holes is demonstrated in Fig. 2. The temperature vanishes in the extreme limit on the boundary except at the points \((\pm \sqrt{2} Me^{-\phi_0}, 0)\) or \((0, \pm \sqrt{2} Me^{\phi_0})\). The discontinuity of temperature near the points \((\pm \sqrt{2} Me^{-\phi_0}, 0)\) or \((0, \pm \sqrt{2} Me^{\phi_0})\), which correspond to extremal electrically or magnetically charged solutions, reveals the limitations on the thermal description of black holes [24]. In Sec. 5 we will see that the thermal description of the dyonic black hole is inappropriate near the extreme limit.

## 5 Concluding remarks

In this paper, we give a dyonic black hole solution of the low-energy effective string theory in which gravity is coupled to dilaton and \(U(1)\) gauge field. These solutions are presented in an clear form by using usual radial coordinate. The structures and thermodynamic properties of dyonic black holes are similar to those of the conventional charged black holes, except for a purely electric or purely magnetic case. Those properties are the same as those of dyonic black holes found by Gibbons and Maeda.

As emphasized by Preskill et al. [24] and Holzhey et al. [25]. The description of a black hole as a thermal object must break down as the extreme limit is approached. They suggest that the extreme pure electrically or pure
magnetically charged solutions should be regraded as elementary particles rather than Reissner-Nordström black holes. Here, we will check the self consistent condition proposed by Preskill et al. for the near-extreme dyonic black holes.

Due to

\[
\left( \frac{\partial T}{\partial M} \right)_{Q_e,Q_m} \simeq \frac{1}{2\pi} \frac{1}{\sqrt{(2M^2 - \beta)^2 - 4Q_e^2Q_m^2}},
\]

and the heat capacity of black hole,

\[
C_{Q_e,Q_m} \simeq 0
\]

as the extremal limit \(2M^2 - \beta = 2|Q_e||Q_m|\) is approached. The temperature fluctuation compared to the temperature itself,

\[
\frac{\langle (\Delta T)^2 \rangle}{T^2} = \frac{1}{C_{Q_e,Q_m}}
\]

will diverge wildly near the extreme regime. It means that, under the assumption that the typical emitted quantum radiation carries energy of order \(T\) but no charge or angular momentum, the self-consistent condition for the thermal description

\[
\left| T\left( \frac{\partial T}{\partial M} \right)_{Q_e,Q_m} \right| \ll |T|
\]

is violated. Therefore, we conclude that the thermodynamic description of the near-extreme dyonic black hole is also inappropriate.
Note Added

After this paper was submitted for publication, D. Wiltshire told us that solutions, eqs. (22)-(28), are related to Gibbons-Maeda dyonic black hole solutions by a coordinate transformation and some parameters reparametization [26]. And, we were also informed that many of our results were previously obtained by Kallosh et al. [27]. The dyonic black hole solutions, eqs. (22)-(28), are also related to those of reference [27] by another coordinate transformation.

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Figure Captions

Fig. 1 The Penrose diagrams of dyonic black hole with a dilaton and its limiting cases, (a) dyonic black hole with both electric and magnetic charge, (b) electrically or magnetically charged black hole, (c) extremal dyonic black hole, (d) extremal electrically or magnetically charged solution does not describe a black hole at all but rather a naked singularity.

Fig. 2 The Hawking temperature \( T_H \) of dyonic black holes v.s. \( Q_e \) and \( Q_m \). Inside the square, we find a regular dyonic black hole. On the boundary except for points \((\pm \sqrt{2} Me^{-\phi_0}, 0)\) or \((0, \pm \sqrt{2} Me^{\phi_0})\) or, we find an extremal dyonic black hole. On the \( Q_e \) or \( Q_m \) axis, we find the electrically or magnetically charged black hole. The temperature is finite on the axes. Finally, points \((\pm \sqrt{2} Me^{-\phi_0}, 0)\) or \((0, \pm \sqrt{2} Me^{\phi_0})\) represent the extremal electrically or magnetically charged solutions.