Binary Source Parallactic Effect in Gravitational Micro-lensing

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ABSTRACT

The first micro-lensing event discovered towards the Small Magellanic Cloud by the MACHO collaboration (Alcock et al. 1997b) had a very long time scale, \( t_0 = 123 \) days. The EROS collaboration (Palanque-Delabrouille et al. 1997) discovered a \( \sim 2.5\% \) brightness variation with a period \( P = 5.1 \) days. The OGLE collaboration (Udalski et al. 1997) established that the variation persists while the micro-lensing event is over, and the variable star is the one which has been micro-lensed, not its blend.

The simplest explanation of the periodic variability is in terms of a binary star with the orbital period \( P_{\text{orb}} = 10.2 \) days, with its component(s) tidally distorted. Such objects are known as ellipsoidal variables. The binary nature should be verified spectroscopically.

Binary motion of the source introduces a parallactic effect into micro-lensing light curve, and a few examples are shown. The effect is relatively strong if the light center and the mass center of a binary are well separated, i.e. if the binary has a large photometric dipole moment. The diversity of binary parameters is large, and the corresponding diversity of photometric effects is also large. The presence or absence of the effect may constrain the lens mass and its distance from the source.

Subject headings: galaxy: halo – gravitational lensing

1. Introduction

The searches for micro-lensing events towards the Magellanic Clouds have as their hopeful goal the determination of what the dark matter is made of (Paczyński 1996, and references therein). While just over a handful of robust micro-lensing events have been detected towards the LMC by the MACHO collaboration (Alcock et al. 1997a), the interpretation of the result has not been agreed upon. The review of the vigorous discussion of the subject is beyond the scope of this short paper.
A very intriguing event has been recently discovered towards the Small Magellanic Cloud by the MACHO collaboration (Alcock et al. 1997b), and confirmed by the EROS collaboration (Palanque-Delabrouille et al. 1997). It had a very long time scale, $t_0 = 123$ days (247 days on the MACHO scale). It has been found to be a periodic variable with the amplitude $A_{el} = 0.025$ and the period of 5.1 days (Palanque-Delabrouille et al. 1997). However, it was not clear what varied, the lensed star or its unresolved blend. This has been clarified by the OGLE collaboration (Udalski et al. 1997), which resolved the two stars thanks to a smaller pixel size and a better seeing: the variable is the lensed star. The variability continues with the same amplitude as it was during the micro-lensing event.

The persistence of periodic variability implies that the lensed star is likely to be an ellipsoidal variable (Udalski et al. 1997). Such stars are common. Their variability is a result of a tidal distortion in a binary with the orbital period twice the photometric period, in this case 10.2 days.

An alternative explanation for the periodic variability might be a slowly pulsating B stars (SPB) as discovered by the Hipparcos mission (Waelkens et al. 1997). However, such oscillations, just as the $\beta$ Cephei pulsations (Dziembowski & Pamiętny 1993), are likely to be driven by iron opacity, and therefore they are not likely to be present in the SMC. In any case, the nature of the periodic oscillations, ellipsoidal variability or SPB, should be settled with spectroscopic observations. This paper is based on the assumption that the lensed star is a binary ellipsoidal variable.

The purpose of this paper is to describe parallactic effect due to binary motion of the source. This is analogous to the parallactic effect of Earth’s orbital motion (Gould 1992, Alcock et al. 1995).

### 2. Binary parallactic effect

Consider a source which is a binary, with the orbital period $P_{\text{orb}}$, and a circular orbit with the angular diameter $\varphi_a$. The orbit has an inclination $i$ to the celestial plane, with $i = 90^\circ$ corresponding to the observer located in the orbital plane. The orbit appears as an ellipse with the minor axis $\varphi_b = \varphi_a \cos i$. The angle between the major axis and the direction of the source’s proper motion with respect to the lens is $\alpha$. An example of such lensing geometry is shown in Fig.1, where the two binary components have equal mass, and the two orbital angles are: $i = 45^\circ$, $\alpha = 45^\circ$.

For the purpose of this analysis I shall adopt a hypothesis that the lens is in the SMC, and that the relative proper motion of the source with respect to the lens is
Fig. 1.— Geometry of micro-lensing of a binary source by a single point mass, shown as a black point located at the center of a dashed circle, its Einstein ring. The binary source moves along the straight horizontal line with the impact parameter $u_{\text{min}} = 0.4$. The microlensing time scale is $t_0 = 123$ days. The orbital period is $P_{\text{orb}} = 10.2$ days. The binary orbit is shown for $i = 45^\circ$ and $\alpha = 45^\circ$.

$\phi \approx 30 \text{ km s}^{-1} / 60 \text{ kpc}$. The angular Einstein ring radius is

$$\varphi_E = \phi t_0 = 36 \mu s \times \left( \frac{V}{30 \text{ km s}^{-1}} \right), \quad t_0 = 123 \text{ days}. \tag{1}$$

Let the two stellar masses be $M_1 = M_2 = 2 \, M_\odot$, and the orbital period $P_{\text{orb}} = 10.2$ days. The diameter of the circular orbit is

$$A = [G (M_1 + M_2)]^{1/3} \left( \frac{P_{\text{orb}}}{2\pi} \right)^{2/3} = 2.2 \times 10^{12} \text{ cm} \left( \frac{M_1 + M_2}{4 \, M_\odot} \right)^{1/3}. \tag{2}$$

The corresponding angular orbital diameter at the distance of 60 kpc is

$$\varphi_a \equiv \frac{A}{60 \text{ kpc}} = 2.4 \mu s \left( \frac{M_1 + M_2}{4 \, M_\odot} \right)^{1/3}. \tag{3}$$
A model light curve was calculated for the recent SMC event assuming that the blend contributed 26% to the total light at minimum (Udalski et al. 1997), the impact parameter was \( u_{\min} = 0.4 \), the lensed source had its brightness modulated with the period \( P_{\text{orb}}/2 = 5.1 \) days, and the amplitude \( A_{\text{ell}} = 0.025 \). First, no lensing effects due to binary motion of the source were taken into account. The difference between the modulated light curve and a micro-lensing light curve with identical parameters but without any modulation is shown at the top of Fig.2 and Fig.3.

If the periodic light modulation is due to tidal distortion of the binary components, then the periodic displacement of the two stars with respect to the binary center of mass modifies the lensing magnification. Let the fractional contribution of the two components and the blend to the minimum light be: \( f_1, f_2, f_3 \), respectively, with \( f_1 + f_2 + f_3 = 1 \), and the blend contribution \( f_3 = 0.26 \) (Udalski et al. 1997). Let the two components have the mass fractions \( m_1, m_2 \), respectively, with \( m_1 + m_2 = 1 \), and the total mass \( M_{\text{tot}} = 4 \, M_\odot \).
The binary center of mass moves along a straight line along the ‘x’ direction. The time dependence of the two angular coordinates is given as
\[
\varphi_{x,\text{cm}} = \dot{\varphi} (t - t_{\text{max}}), \quad \varphi_{y,\text{cm}} = \varphi_0 = u_{\text{min}} \varphi_E, \quad (4)
\]
where \(t_{\text{max}}\) is the time of maximum magnification. The primary component moves around the binary center of mass according to
\[
\Delta \varphi_{x1} = \Delta \varphi_{a1} \cos \alpha - \Delta \varphi_{b1} \sin \alpha, \quad \Delta \varphi_{y1} = \Delta \varphi_{a1} \sin \alpha + \Delta \varphi_{b1} \cos \alpha, \quad (5)
\]
where
\[
\Delta \varphi_{a1} = \frac{m_2}{m_2} \varphi_a \cos \left(\frac{2\pi t}{P_{\text{orb}}}\right), \quad \Delta \varphi_{b1} = \frac{m_2}{m_2} \varphi_a \sin \left(\frac{2\pi t}{P_{\text{orb}}}\right) \cos i, \quad (6)
\]
and the secondary’s orbit is given as
\[
\Delta \varphi_{x2} = -\frac{m_1}{m_2} \Delta \varphi_{x1}, \quad \Delta \varphi_{y2} = -\frac{m_1}{m_2} \Delta \varphi_{y1}, \quad (7)
\]
Finally, the trajectories of both components are given as
\[
\varphi_{x,i} = \varphi_{x,\text{cm}} + \Delta \varphi_{xi}, \quad \varphi_{y,i} = \varphi_{y,\text{cm}} + \Delta \varphi_{yi}, \quad i = 1, 2. \quad (8)
\]

The magnification with respect to the combined non-lensed brightness of the binary and the blend can be calculated as
\[
A = (A_1 f_1 + A_2 f_2) \left[1 + A_{\text{ell}} \cos \left(\frac{\pi t}{P_{\text{orb}}}\right)\right] + f_3, \quad (9)
\]
where
\[
A_i = \frac{u_i^2 + 2}{u_i \sqrt{u_i^2 + 4}}, \quad u_i^2 = \frac{\varphi_{x,i}^2 + \varphi_{y,i}^2}{\varphi_E^2}, \quad i = 1, 2. \quad (10)
\]
Note, that the ellipsoidal light variations with the amplitude \(A_{\text{ell}}\) reach the local light maximum at the time of maximum angular separation between the two components, when the two stars are seen ‘sideways’ and the tidal distortion makes them appear somewhat larger, and brighter, than a quarter of the orbital period later (or earlier).

A few examples of binary modulation of a micro-lensing light curve are are shown in Fig.2 and Fig.3. In both figures the uppermost curve corresponds to the photometric modulation due to binary components’ tidal distortion. The following three light curves present the effects of binary source modulation of the lensing. In Fig.2 these are the differences between the four models and the standard micro-lensing light curve. In Fig.3 the three lower light curves are the differences between the corresponding curves in Fig.2, and the topmost curve. The first of these three corresponds to a binary with both components having identical masses and luminosities, and the orbit diameter set to be 0.067 of the Einstein ring radius, following equations (1) and (3). The last two curve corresponds to a binary with only one bright component separated from the center of mass by 0.013 Einstein ring radii. All curves are labeled with the values of orbital angles \(i\) and \(\alpha\).
Fig. 3.— The uppermost light curve is identical to the uppermost light curve in Fig. 3. The following 3 light curves are the differences between the corresponding light curves in Fig. 2 and the uppermost light curve. The dashed lines represent the amplitudes of oscillations calculated with equations (16). D and Q stand for dipole and quadrupole variations.

3. Discussion

The main practical problem with the binary parallactic effect is the very large number of parameters which make model fitting impractical unless the binary nature of the source is constrained with spectroscopic observations. In the case of the recent SMC event the spectroscopy is needed to verify the hypothesis that the source is a binary.

Fig. 2 and Fig. 3 demonstrate that the effect of binary modulation is likely to be small in the recent SMC event. Therefore, the amplitude of the binary effect can be estimated expanding the binary modulation in a power series by writing equation (9) as

\[ A \approx A_0 (f_1 + f_2) + A' h (f_1 m_2 - f_2 m_1) + 0.5 A'' h^2 (f_1 m_2^2 + f_2 m_1^2) + f_3, \]  

\[ A_0 = \frac{u^2 + 2}{u \sqrt{u^2 + 4}}, \quad A' \equiv \frac{dA}{du} = -\frac{8}{u^2 (u^2 + 4)^{3/2}}, \quad A'' \equiv \frac{d^2 A}{du^2} = \frac{8(5u^2 + 8)}{u^3 (u^2 + 4)^{5/2}}, \]
where
\[ h = \frac{\varphi_a}{\varphi_E} \left[ 1.0 - \cos^2(\alpha - \beta) \sin^2 i \right]^{1/2}, \quad \tan \beta = \frac{\varphi_{y,cm}}{\varphi_{x,cm}}. \]  

\[ (13) \]

The angle \( \beta \) is between the line joining the lens and the binary center of mass and the trajectory of that center. The small range of distances from the lens covered by the orbital ellipse is calculated as \( h \). The lens magnification is expanded in a power series, with the first two small terms corresponding to the dipole and quadrupole moments of the binary light distribution:
\[ D = h (f_1 m_2 - f_2 m_1), \quad Q = h^2 \left( f_1 m_2^2 + f_2 m_1^2 \right), \]

\[ (14) \]

The corresponding amplitudes of dipole and quadrupole variations are calculated as
\[ A_D = A'D, \quad A_Q = 0.5 A''Q. \]

\[ (15) \]

The amplitudes calculated with these formulae are shown with dashed lines in Fig.3.

If the lensed binary has a photometric dipole moment then the lensing variations are much larger than in the case when there is no dipole, as quadrupole variations are very small (cf. Fig.3). It follows that binary lens modulation is much stronger if the two components are not identical. If future spectroscopic observations of the SMC star confirm its binary nature, then it will be very important to determine not only the amplitude of the orbital radial velocity variations, but also to estimate the luminosity ratio of the two components.

Only MACHO collaboration has the full event well covered photometrically. It will be very interesting to find out if that data will provide useful information about the presence or absence of the binary parallactic effect. The presence or absence of the effect will provide additional constraint on the location of the lens.

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