Comment on nonperturbative effects in $\bar{B} \rightarrow X_s \gamma$

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Abstract

Uncertainties in the theoretical prediction for the inclusive $\bar{B} \rightarrow X_s \gamma$ decay rate are discussed. We emphasize that there is no operator product expansion for this process. Nonetheless, some nonperturbative effects involving a virtual $c \bar{c}$ loop are calculable using the operator product expansion. They give a contribution to the decay rate that involves the $B$ meson matrix element of an infinite tower of operators. The higher dimension operators give effects that are only suppressed by powers of $m_b \Lambda_{\text{QCD}}/m_c^2 \sim 0.6$, but come with small coefficients.
The inclusive $B \to X_s \gamma$ decay has received considerable attention in recent years [1–15], since it is sensitive to physics beyond the standard model [3] ($X_s$ denotes a final state with strangeness $-1$ and charm $0$). The photon spectrum also carries information on nonperturbative physics that can help us better understand other $B$ decays [13,14]. The recent CLEO measurement [1] excludes large deviations from the standard model. Therefore, it is important to know the standard model predictions as precisely as possible.

Since the $b$ quark is heavy compared to the QCD scale, one would hope that the inclusive $\bar{B} \to X_s \gamma$ decay rate can be calculated in a systematic QCD-based expansion [16]. The dominant contribution to the decay rate comes from the matrix element of the electromagnetic penguin operator (usually denoted by $O_7$). In the $m_b \to \infty$ limit, it is given by the free quark decay result. The leading nonperturbative corrections to this contribution are suppressed by $(\Lambda_{\text{QCD}}/m_b)^2$. Provided the photon energy is not restricted to be too close to its maximal (i.e., end-point) value, they are quite small, around $-3\%$ [12]. With the recent completion of the full next-to-leading order perturbative calculation [11], it is usually argued that theoretical uncertainties in the prediction for the inclusive $\bar{B} \to X_s \gamma$ decay rate are not larger than 10%.

The effective weak interaction Hamiltonian at a scale $\mu$ (of order $m_b$) is given by

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i(\mu).$$

(1)

In the conventional notation, $O_2 = (\bar{s}_L\gamma_\mu b_L)(\bar{c}_L\gamma^\mu c_L)$, $O_1$ only differs from $O_2$ in the way color indices are contracted, $O_3$–$O_6$ are four-quark operators involving all flavors below the scale $\mu$, $O_7 = (e/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} b_R$, and $O_8$ is obtained from $O_7$ by replacing $eF_{\mu\nu}$ by $g_s G_{\mu\nu}$. Using perturbative QCD to evaluate the $\bar{B} \to X_s \gamma$ decay rate, in the leading logarithmic approximation the matrix element of $C_7(\mu) O_7(\mu)$ dominates for large enough photon energies.

A systematic computation of the $O_7$ contribution to the inclusive $\bar{B} \to X_s \gamma$ decay rate involves performing an operator product expansion (OPE) for the time ordered product

$$T_{77} = \frac{i}{2m_B} \int d^4x \, e^{-iq \cdot x} \langle \bar{B}(v) | T\{O_7^\mu(x) O_7^\nu(0)\} | \bar{B}(v) \rangle g_{\mu\nu},$$

(2)
to all orders in the strong interaction. Here $O_7^\mu = (i e/8\pi^2) m_b \bar{s}_L \sigma^{\mu\lambda} q_\lambda b_R$. At fixed $q^2 = 0$, this time ordered product has a cut in the complex $v\cdot q$ plane along $v\cdot q < m_b/2$ corresponding to final hadronic states $X_s$, and another cut along $v \cdot q > 3m_b/2$ corresponding to final hadronic states $X_{bb\bar{s}}$. The contribution of the magnetic moment operator $O_7$ to the $\bar{B} \to X_s \gamma$ decay rate is given by the discontinuity across the cut in the region $0 < v \cdot q < m_b/2$,

$$\frac{d\Gamma}{dE_\gamma} = \frac{4G_F^2 |V_{ts}^* V_{tb}|^2 C_7^2}{\pi^2} E_\gamma \text{Im} T_{77}.$$

(3)

Since the cuts are well-separated, one can compute this contribution to the $\bar{B} \to X_s \gamma$ decay rate assuming local duality at the scale $m_b$. (The integration of $T_{77}$ over the contour $C$ in Fig. 1 pinches the physical cut at $v \cdot q = 0$, but at that point the hadronic final states have invariant mass $m_{X_s} = m_B$.)

At leading order in the OPE, the dimension-three operator $\bar{b} \gamma_\mu b$ occurs. Its matrix element gives a calculable contribution to the photon energy spectrum proportional to $\delta(E_\gamma - m_b/2)$. Higher dimension operators give terms proportional to derivatives of this delta function, and the matrix elements of the operators with dimension greater than five are not known. In order to justify retaining only the lowest dimension operators, the photon energy must be averaged over a region $\Delta E_\gamma \gg \Lambda_{\text{QCD}}$. At the present time these higher dimension
operators introduce a significant uncertainty, since the photon spectrum is only measured experimentally over a region about 500 MeV from the end-point \([1]\).

When operators in \(H_{\text{eff}}\) other than \(O_7\) are included, the \(\bar{B} \to X_s \gamma\) decay receives contributions from diagrams in which the photon couples to light quarks. It is well-known that for such processes, there are uncalculable contributions suppressed by \(\alpha_s\), but not by powers of the scale associated with the process. Typically, the leading logarithms are calculable \([17]\), but terms suppressed by a logarithm (or equivalently by \(\alpha_s\), but not by a power) can only be estimated using information on the fragmentation functions \(D_{q \to \gamma X}\) and \(D_{g \to \gamma X}\) deduced from other experiments or from models. While this may be worrisome, experience shows that usually the leading order perturbative QCD calculation provides an order of magnitude estimate of these effects \([18]\). Perturbative computations indicate that for weak radiative \(B\) decays into hard photons both the contribution of light quark loops \([9]\), and the effects related to decay functions of light partons into a photon \([8]\), are very small. Therefore, these nonperturbative effects which are not power suppressed constitute less than five percent uncertainty in the theoretical prediction for the \(\bar{B} \to X_s \gamma\) decay rate.

There is no OPE that allows one to parametrize nonperturbative effects from the photon coupling to light quarks in terms of \(B\) meson matrix elements of local operators. Given this, it is perhaps not surprising that nonperturbative effects that come from the photon coupling to the charm quark contain \(B\) meson matrix elements of local operators that are suppressed by \((\Lambda_{\text{QCD}}/m_c)^2\) rather than \((\Lambda_{\text{QCD}}/m_b)^2\). Recently, Voloshin identified such a nonperturbative correction to the \(\bar{B} \to X_s \gamma\) decay rate \([15]\). This contribution arises from the interference of \(O_2\) with \(O_7\) corresponding to the diagram shown in Fig. 2, and can be studied using the operator product expansion.

\[\text{For soft photons these effects are important. There are also interference effects where the photon couples to a light quark and to the charm quark, or to a light quark and through} \ O_7. \text{These are also small for hard photons.}\]
FIG. 2. Feynman diagram that gives rise to $T_{27}$ in Eq. (4). Interchange of the photon and gluon couplings to the charm loop is understood.

For a sufficiently heavy charm quark, nonperturbative corrections to the contribution of the interference of $O_2$ and $O_7$ to the decay rate can be computed from the discontinuity of the diagram in Fig. 2. Analogous diagrams with more gluons give effects suppressed by additional powers of $\Lambda_{\text{QCD}}/m_c$. Denoting the gluon momentum by $k$, we work to all orders in $k \cdot q/m_c^2$ since the photon momentum $q$ is of order $m_b$, but neglect terms of order $k \cdot q/m_b^2$, $k^2/m_c^2$, and $m_s/m_b$. The result of the loop integration is

$$T_{27} = -\frac{1}{2m_B} \langle \bar{B}(v) | \bar{b} m_b \sigma^\mu q_\mu \frac{m_b \gamma^\mu - \gamma_5}{(m_b v - q)^2 + i\epsilon} \gamma^\nu (1 - \gamma_5) I_{\mu\nu} b | \bar{B}(v) \rangle. \quad (4)$$

$I_{\mu\nu}$ is a complicated operator involving all powers of $(q \cdot iD)/m_c^2$. It is given by

$$I_{\mu\nu} = \left( \frac{e}{16\pi^2} \right)^2 \frac{2}{9m_c^2} \left[ \sum_{n=0}^\infty \frac{3 \cdot 2^{n+3} [(n + 1)!]^2}{(2n + 4)!} \left( \frac{-q \cdot iD}{m_c^2} \right)^n \right] \varepsilon_{\mu\nu\lambda\beta} q^\lambda q_\eta g_s G^{\lambda\eta}. \quad (5)$$

Here $G^{\lambda\eta}$ is the gluon field strength tensor and $D$ denotes the covariant derivative. The contribution of $T_{27}$ to the $\bar{B} \to X_s \gamma$ decay rate is given by Eq. (4) with $C_1^2 T_{77}$ replaced by $2C_2 C_7 T_{27}$.

For the leading $n = 0$ term in Eq. (5), the matrix element in Eq. (4) can be computed using the identity [19]

$$\frac{1}{2m_B} \langle \bar{B}(v) | \bar{b} \Gamma g_s G_{\alpha\beta} b | \bar{B}(v) \rangle = \frac{\lambda_2}{8} \text{Tr} \{ \Gamma (1 + \gamma^5) \sigma_{\alpha\beta} (1 + \gamma^5) \}, \quad (6)$$

valid for any Dirac structure $\Gamma$. The ratio of the decay rate from the $n = 0$ term in Eq. (5) to that from $O_7$ is

$$\frac{\delta\Gamma(\bar{B} \to X_s \gamma)}{\Gamma(\bar{B} \to X_s \gamma)} = -\frac{C_2}{9C_7} \frac{\lambda_2}{m_c^2}. \quad (7)$$
The measured $B^*-B$ mass splitting gives $\lambda_2 = 0.12$ GeV$^2$. Using this value for $\lambda_2$ and the values of $C_2 = 1.11$ and $C_7 = -0.32$ in Ref. [3], Eq. (7) implies that this $O_2-O_7$ interference is about a three percent effect. This is an order of magnitude larger than the perturbative estimate of the contribution from the interference of $O_2$ and $O_7$ to the $\bar{B} \to X_s \gamma$ decay rate (which contains a gluon in the final state).

The contribution of all terms in $I_{\mu\nu}$ to $\text{Im} T_{27}$ is

$$\text{Im} T_{27} = \frac{1}{2m_B} \sum_{n=0}^{\infty} \frac{a_n (-1)^n m_b^{n+3}}{m_c^{2n+2}} \hat{q}^{\mu_1} \cdots \hat{q}^{\mu_n} \times \langle \bar{B}(v) | \bar{b} \Gamma^{\alpha\beta}(\hat{q}, v) (iD_{\mu_1} \cdots iD_{\mu_n}) g_\alpha G_{\alpha\beta} | \bar{B}(v) \rangle \delta(\hat{q} \cdot v - 1/2),$$

where $\hat{q} = q/m_b$. $\Gamma^{\alpha\beta}$ and $a_n$ are dimensionless and can be deduced from Eqs. (4) and (5). The indices $\mu_1 \cdots \mu_n$ are symmetrized, since they are dotted into $\hat{q}^{\mu_1} \cdots \hat{q}^{\mu_n}$. Note that the derivatives $iD_{\mu_1} \cdots iD_{\mu_n}$ act on the gluon field $G_{\alpha\beta}$, and are determined by the spacetime dependence of the chromomagnetic field in the $B$ meson. The $n = 0$ term in Eq. (8) is a special case in that the $\langle \bar{B}(v) | \bar{b} \Gamma^{\alpha\beta} g_\alpha G_{\alpha\beta} | \bar{B}(v) \rangle$ matrix element is known from the $B^*-B$ mass splitting. The $n = 1$ matrix element vanishes by the equations of motion [20]. The $n > 1$ terms in Eq. (8) depend on an infinite series of unknown matrix elements. Estimating $\langle \bar{B}(v) | \bar{b} \Gamma^{\alpha\beta}(\hat{q}, v) (iD_{\mu_1} \cdots iD_{\mu_n}) g_\alpha G_{\alpha\beta} | \bar{B}(v) \rangle/(2m_B) \sim (\Lambda_{\text{QCD}})^{n+2}$, we see that the $n > 1$ terms are “suppressed” compared to the $n = 0$ term considered by Voloshin only by powers of $m_b \Lambda_{\text{QCD}}/m_c^2$.

In the limit where $m_c$ is fixed and $m_b \to \infty$, the higher order terms in Eq. (8) become successively more important and the expansion we have made is clearly inappropriate. (The whole sum in Eq. (8) is, up to logarithms, of order $\Lambda_{\text{QCD}}/m_b$.) In the limit where $m_b/m_c$ is held fixed and both masses become very large, the $n \geq 1$ terms in Eq. (8) are suppressed by powers of $\Lambda_{\text{QCD}}/m_c$. Then the $n = 0$ result, which is of order $\Lambda_{\text{QCD}}^2/m_c^2$, dominates the sum. In the physical world, $m_b \Lambda_{\text{QCD}}/m_c^2 \sim 0.6$ (an equally reasonable estimate would be $E_{\gamma}^{\text{max}} \Lambda/m_c^2$, which is also about 0.6). From Eq. (3) we see that $a_1/a_0 = 4/15$, $a_2/a_0 = 3/35$, $a_3/a_0 = 16/525$, etc., and asymptotically $a_n/a_0 \to 3\sqrt{\pi}/(2^{n+1} n^{3/2})$ as $n \to \infty$. The values of $a_n/a_0, n = 1, 2, \ldots$, are small. This together with the asymptotic formula for large $n$ suggests
that the \( n \geq 1 \) terms in Eq. \((8)\) do not introduce a nonperturbative uncertainty greater than the value of the leading \( n = 0 \) term. Nonperturbative effects from the interference of \( O_1 \) with \( O_7 \) are expected to be smaller.

Near the photon end-point region, another set of corrections become large. Expanding factors of \( iD \) that occur in the denominator of the strange quark propagator (these were neglected in Eq. \((4)\)) yields corrections suppressed by powers of \( m_b \). However, these corrections are proportional to derivatives of the delta function \( \delta(E_\gamma - m_b/2) \), and they become as important as those in Eq. \((8)\) in the end-point region.

Consider next the contribution to the \( \bar{B} \rightarrow X_s \gamma \) decay rate coming from the square of \( C_1 O_1 + C_2 O_2 \). Diagrams like that in Fig. 2 should give a smaller nonperturbative contribution to the decay rate than the interference of \( O_2 \) with \( O_7 \) (i.e., these are order \( \Lambda_{QCD}^3/m_c^4 \) instead of order \( \Lambda_{QCD}^2/m_c^2 \)). But we know in this case that there is a contribution to the \( \bar{B} \rightarrow X_s \gamma \) decay rate from \( B \rightarrow X_s J/\psi \) followed by \( J/\psi \rightarrow \gamma X \), which is much larger than the perturbative calculation of the effect of \( (C_1 O_1 + C_2 O_2)^2 \). The combined branching ratio for this process is about \( 10^{-4} \), while the perturbative estimate of the contribution of \( (C_1 O_1 + C_2 O_2)^2 \) is less than \( 10^{-5} \). This might not present a serious difficulty for the comparison of experiment with theory, since the process \( \bar{B} \rightarrow X_s J/\psi \) followed by \( J/\psi \rightarrow \gamma X \) does not favor hard photons, and in any case it can be treated as a background and subtracted away. Further work on this issue is warranted.

In this letter we examined uncertainties in the theoretical prediction for the weak radiative decay rate of \( B \) mesons into hard photons that come from nonperturbative strong interaction physics. We focused on effects that arise from photon couplings to light quarks and to charm quarks. For hard photons the first of these sources of theoretical uncertainty is less than five percent. This is smaller than the uncertainty in the Wilson coefficient \( C_7(m_b) \) from uncalculated order \( \alpha_s^2 \) terms in its perturbative expansion. For the photon coupling to the charm quark, more work is needed to decide the size of the theoretical uncertainty associated with nonperturbative effects.

The present experimental data on \( \bar{B} \rightarrow X_s \gamma \) focuses on photon energies in the region
$E_\gamma \gtrsim 2.2 \text{ GeV}$ [1]. For comparison with this data, the largest theoretical uncertainty is from the contribution of higher dimension operators to the time ordered product $T_{77}$ which become more important in the end-point region. This uncertainty would be substantially smaller if the photon energy cut were reduced.

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