Topological defects and the spin glass phase of cuprates

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Abstract. – We propose that the spin glass phase of cuprates is due to the proliferation of topological defects of a spiral distortion of the antiferromagnet order. Our theory explains straightforwardly the simultaneous existence of short-range incommensurate magnetic correlations and complete a-b symmetry breaking in this phase. We show via a renormalization group calculation that the collinear $O(3)/O(2)$ symmetry is unstable towards the formation of local non-collinear correlations. A critical disorder strength is identified beyond which topological defects proliferate already at zero temperature.

Introduction. – Until recently, the spin glass (SG) phase of La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), with $0.02 < x < 0.05$, was believed to have short-ranged commensurate correlations, with incommensurate (IC) correlations appearing only at the onset of superconductivity $x \sim 0.05$ [1]. New data on LSCO have however revealed IC spin correlations also in the SG regime [2–4]. The short-range nature of the correlations may point to the existence of a strongly disordered stripe glass, in accord with our previous analysis [5] which predicted an instability of the stripe array towards a disorder-dominated phase. Within the picture of disordered stripes, one is however immediately confronted with a problem of two fundamentally different scales: the inter-stripe distance, $\ell_s$, which scales as $\ell_s \sim 1/x$ [1] and the average separation, $\ell_d$, between disorder centers (Sr ions) which scales as $\ell_d \sim 1/\sqrt{x}$. A conflict arises because, for pinned stripes, both scales are associated with the charge sector. As $\ell_d \ll \ell_s$ in the SG regime, the stripes must break up into short segments or drastically reduce the on-stripe charge density to take advantage of the disorder potential [6].

Such a conflict does not arise in models which assume that holes localize individually near their dopants, as indicated by transport measurements [7], and act as frustration centers to the AF order [8–11]. A polarization of the frustration centers can then lead to the appearance of IC correlations [12]. In such a scenario, the scale $\ell_s$ exists only in the spin sector, whereas $\ell_d$ is characteristic of the charge distribution. In the following we revisit this model, assuming that holes produce dipolar frustration, and show that it can well explain the experiments. Our analysis sheds light on the origin of the cluster spin glass phase which in our analysis arises from topological defects of a spiral spin texture.
Because dipolar frustration produces a long-range distortion of the AF order, this problem can be treated within a continuum field theory approach. A non-linear σ-model (NLσM) for this problem reads [9,11]

$$\tilde{H}_{col} = \frac{\rho_s}{2T} \int d^2x (\partial_\mu n)^2 + \frac{\rho_s}{T} \int d^2x f_\mu \cdot \partial_\mu n,$$

where $\rho_s$ is the spin stiffness, $T$ the temperature and $n$ a three-component unit vector representing the local staggered moment (we set $k_B = 1$). The quenched field $f_\mu$ represents the dipoles, with zero mean and a Gaussian distribution. An analysis of (1) leads to a finite correlation length at $T = 0$ for any finite disorder strength and to fair agreement with the temperature and doping dependence of the correlation length of LSCO [9,11]. As it stands, the model however predicts commensurate AF correlations, in contradiction with experiments [2–4].

It is well known that the dipole model can lead to IC correlations [12]. The Hamiltonian (1) favors the formation of a spiral phase, with a non-zero average twist $\partial_\mu n$ of the AF order and a simultaneous alignment of the dipoles, $\langle f_\mu \rangle \neq 0$, as long as the lattice and spin degrees of freedom of dipoles are annealed. While the spiral phase has been found to be unstable in the case of an annealed hole distribution towards a local enhancement of the spiral pitch, this instability arises from the fermionic susceptibility [13] and hence is absent in the model considered here, where all fermionic degrees of freedom are quenched. Further, as the lattice part of the $f_\mu$ vector should reflect the symmetries of the underlying lattice, a discrete set of favored lattice vectors for the formation of the spiral exists. Thus, the $a$-$b$ (or square lattice) symmetry breaking associated with the formation of spiral correlations can have truly long-range order. The continuous symmetry of spin space, on the other hand, inhibits long-range magnetic order in the 2D system for either finite temperature or finite disorder. The experimental observation of a macroscopic $a$-$b$ asymmetry [4] but very short spin correlation lengths thus clearly motivate the study of the dipole model.

We here investigate the dipole model assuming a random spatial distribution of the dipoles but allowing for non-zero–ordered moments. The non-collinearity of the ground state which arises from ordered moments requires a serious modification of previous collinear approaches for two reasons: First, the symmetry breaking scheme of non-collinear magnets leads to the appearance of three rather than two Goldstone modes which couple to disorder. Second, a spiral phase allows for pointlike topological defects (TDs) with $Z_2$ charges [14], which originate from a chiral degeneracy of the spiral ground state and which are absent in collinear models. Below, we investigate the influence of disorder on both the Goldstone modes, using a renormalization group (RG) approach, and on the creation of TDs, using a free-energy argument. A strongly disordered regime, as associated with a SG, is only found once topological defects of the spin texture are accounted for. The situation encountered here thus resembles that of disordered planar XY models, where the coupling of the disorder to spin waves was also found to lead to a simple renormalization of the spin stiffness, whereas a phase transition to a disordered phase is driven by a proliferation of TDs [15]. There are however profound differences between these simpler XY models and the present model. Specifically, the renormalization of the spin stiffness in the latter case is primarily driven by the curvature of the order parameter space while in the former case the renormalization arises from the presence of TDs. Further, the influence of TDs on the renormalization of non-collinear Heisenberg models is at present not understood and a unified renormalization group approach which accounts for both TDs and curvature terms is not available [14]. We therefore derive the RG equations without incorporating TDs, while we address the importance of TDs in a separate analysis below, where we determine the critical disorder threshold of the unbinding transition. Below this threshold, but not above, the RG describes the system correctly.
**Derivation of the model.** – As in the presence of spiral correlations the $O(3)$ symmetry of the spins is broken completely, the $O(3)/O(2)$ AF model (1) is not adequate. We need a formulation of the problem which incorporates the order parameter for a spiral, which is an element of $O(3) \times O(2)/O(2)$ [16]. Thus, there are three instead of the two Goldstone modes of collinear magnets. A possible representation of the local spiral order is in terms of orthonormal $n_k, k = 1, 2, 3$, with $n_k^a n_k^a = \delta_{kq}$. A derivation of a continuum field theory for a spiral state from a lattice Heisenberg model can be found in [17]. Using $S_{ij} = n_1 \cos(k_S \cdot r_{ij}) - n_2 \sin(k_S \cdot r_{ij})$ and $n_3 = n_1 \times n_2$, where $S_{ij}$ is the spin at the lattice site $(i, j)$, $k_S = (\pi, \pi) + q_S$ and $q_S$ is the IC ordering wave vector of the spiral, the effective classical Hamiltonian can be written in the form

$$\tilde{H} = \frac{1}{2} \int d^2 x p_k (\partial_\mu n_k)^2 + s_\mu \int d^2 x \ n_1 \cdot \partial_\mu n_2,$$

(2)

where we ignore small anisotropies of order $q_S^2$ in the stiffnesses $p_k$. Roughly, $p_{1,2} \simeq J/(2T)$ and $p_3 \simeq 0$. The vector $s$ is to lowest order given by $s = Jq_S/T$. The second term makes this Hamiltonian unstable, which simply shows that the pure Heisenberg model does not support a spiral ground state. We show below that this term is cancelled by a term originating from the coupling of the spins to the dipoles. Thus, the ordered dipoles stabilize the spiral phase.

We use a phenomenological form for the coupling of the spiral order parameter to the dipoles. The dipoles locally cant the spin order and couple to the first spatial derivatives of the coupling of the spins to the dipoles. Thus, the ordered dipoles stabilize the spiral phase.

**Renormalization.** – The RG is obtained in the $SU(2)$ representation [19], where $n_k^a = \text{tr} [\sigma^a g \sigma^b g^{-1}]/2$ ($\sigma^a$ are Pauli matrices and $g \in SU(2)$). Introducing $A_\mu^a = -i \text{tr} [\sigma^a g^{-1} \partial_\mu g]/2$ [20], eq. (3) can be written in the form

$$\tilde{H} = \frac{1}{t} \int d^2 x \left[ A_\mu^2 + a A_\mu^2 \right] + 2 \int d^2 x \ p_k \epsilon_{ijk} \epsilon_{abc} \ A_\mu^i n_j^a n_k^b Q_\mu^b,$$

(5)
where \( t^{-1} = 2(p_1 + p_3) \) and \( a = (p_1 - p_3)/(p_1 + p_3) \). At the point \( a = 0 \) the symmetry is enhanced to \( O(3) \times O(3)/O(3) \simeq O(4)/O(3) \) while at \( a = -1 \) the model is collinear. The one-loop RG equations are obtained by splitting the field \( g \) into slow and fast modes, \( g = g' \exp[i\phi^a\sigma^a] \) and tracing out the fast modes \( \phi^a \) with fluctuations in the range \([\Lambda, 1]\) (where the original UV cutoff is 1). For \( \lambda \ll t \) the RG equations are given by

\[
\begin{align*}
4\pi \partial_t t &= 2(1-a)t^2 + (2-a + a^2)\lambda t, \\
4\pi \partial_t a &= -4a(1+a)t - a(1+a)(3-a)\lambda,
\end{align*}
\]

where \( \ell = \ln(\Lambda) \). For \( \lambda = 0 \), these equations describe the RG of a clean spiral [19], while for the collinear point \( a = -1 \), the equations reproduce the RG of the stiffness for disordered collinear models [11]. From (6) it is seen that there are two fixed points for \( a \), see also fig. 1. The collinear point \( a = -1 \) is unstable whereas \( a = 0 \) is stable, irrespective of the disorder. The coupling to weak disorder thus only renormalizes the spin stiffness but does not not lead to any new fixed points.

The validity of (6) is however limited to high temperatures, for low temperatures the RG of \( \lambda \) must be accounted for. We have calculated the RG of \( \lambda \) using the method described in [11]. However, for \( a \neq 0 \) new disorder coupling terms appear. Specifically, the RG generates transverse correlated fields which couple to the \( n_{1,2} \) vectors even at \( a = -1 \) \( (i.e. \ p_{1,2} = 0) \), whereas in the original Hamiltonian with \( a = -1 \) they only couple to \( n_3 \), see eq. (4). Such transverse correlated fields destroy the collinear fixed point. Thus, even if the original AF order is collinear \( (i.e. \ \text{in the absence of dipole ordering}) \), the disorder drives the system to a non-collinear state.

We recover the RG equations for the collinear model obtained in [11] only if we ignore the correlated transverse field coupling. A purely collinear analysis is thus not valid in the presence of dipoles and cannot describe the low-temperature regime correctly. This is in fact expected, because a random canting of the spins destroys the remaining \( O(2) \) spin symmetry of the collinear model. We conjecture that the system will always flow to the point \( a = 0 \), although this remains to be proven.

At the highest symmetry point \( a = 0 \), however, no new coupling terms are generated. It is then straightforward to arrive at the following RG equations, valid for \( a = 0 \) but any initial ratio of \( \lambda/t \):

\[
\begin{align*}
2\pi \partial_t t &= t^2 + \lambda t \; ; \\
4\pi \partial_t \lambda &= \lambda^2,
\end{align*}
\]

which can be simplified through \( z = t + \lambda/2 \) to get \( 2\pi \partial_t z = z^2 \). So, for \( a = 0 \), the presence of disorder leads to an additive renormalization of the stiffness, \( t \rightarrow t + \lambda/2 \). In the presence of any amount of disorder, the IC correlation length \( \xi \) at \( T = 0 \) is finite, as can be inferred from an integration of the RG equation, yielding \( \xi \propto \exp[CL\lambda^{-1}t] \) at \( T = 0 \) with some constant \( C \).

While the disorder scales to strong coupling, the relative disorder strength with respect to the stiffness, \( \lambda/t \), always scales to zero so that at long wavelengths the disorder becomes less relevant. This is surprisingly different to the situation with \( a = -1 \) fixed [11], where the ratio \( \lambda/t \) was found to diverge below a certain initial value of \( \lambda_0/t_0 \) which was interpreted as the scaling towards a new disorder-dominated regime. Thus, if one correctly takes into account non-collinearity, this crossover to a strongly disordered phase disappears.

To summarize, within the perturbative RG analysis disorder leads only to a simple renormalization of the spin stiffness which in turn leads to a finite correlation length already at \( T = 0 \). As mentioned already in the introduction, disorder may have a more dramatic effect via the creation of TDs in the spin texture. Below, we analyse the stability of the spin system against the creation of such defects.
Topological defects. – TDs in spirals are related to the chiral degeneracy of the spiral, i.e. a spiral can turn clock- or anti-clockwise \([14,21]\). These defects are pointlike and at a TD the spiral changes its chirality. Saddle point solutions of TDs for spirals are of the form \([21]\)

\[
g(x) = \exp[im^a \sigma^a \Psi(x)/2],
\]

where \(m\) is a space-independent unit vector and \(\Psi(x)\) satisfies \(\nabla^2 \Psi(x) = 0\) with singular solutions \(\Psi(x,y) = \arctan(y/x)\). In fig. 2 the spin distribution around an isolated defect in a homogeneous spiral is shown for \(a \geq 0\). The energy of a TD diverges logarithmically with the linear system size \(R\), \(\beta E = [1 + (m^z)^2 a^2] \pi \ln R/2t\), so for \(a < 0\) the lowest energy defects have \(m^z = \pm 1\) while for \(a > 0\), \(m^z = 0\) is preferred. Because of the logarithmic divergence of the energy, isolated TDs are absent in clean systems at sufficiently low enough temperatures and in this regime the predictions of the NL\(\sigma\)M analysis should hold. Indeed, NL\(\sigma\)M predictions for related frustrated Heisenberg models were found to be in excellent agreement with numerical simulations at low temperatures \([22]\). Only at higher temperatures an abrupt decrease of the correlation length was observed which has been attributed to the appearance of unpaired TDs \([22,23]\). It has been argued that the high-temperature regime is similar to the equivalent phase of XY models, as both are governed by the plasma phase of the Coulomb gas model \([23]\). The spiral defects differ however from their XY counterparts in that the logarithmic divergence of the energy appears only for length scales smaller than the correlation length obtained from the NL\(\sigma\)M analysis. In an expansion around the saddle points the defects do not decouple from the spin waves even at the lowest (Gaussian) level. Also, TDs of spirals carry a \(Z_2\) charge, whereas XY vortices have \(Z\) charges. Despite these differences, the critical temperature, at which free TDs first appear, can in both cases be well estimated using a free-energy argument \([24]\).

A comparison to XY models is further useful as the disorder coupling we employed is very similar to the one appearing in XY models with randomly fluctuating gauge fields \([15]\). In these models, if one ignores vortices, the influence of the disorder was shown to amount to a
simple renormalization of the spin stiffness [15], and no disordering transition as a function of the disorder strength is found. Once TDs are included in the analysis, randomness leads to a disordered phase even at $T = 0$ through the creation of unpaired defects if the disorder fluctuations are stronger than some critical value [15,25]. Similar to the estimate of the critical temperature at which thermal fluctuations lead to the appearance of TDs, the critical disorder strength is well estimated from the free energy of an isolated defect in the presence of disorder [15,26]. Here, we use this approach to calculate the critical disorder strength for the spiral strength is well estimated from the free energy of an isolated defect in the presence of disorder.

To judge whether or not TDs play a role in the LSCO SG phase, we must estimate $\lambda$ roughly linearly with doping which points to a doping-independent ordered fraction of dipoles.

Experiments found the incommensurability to scale $x \rightarrow x(1-\gamma z)$ (experiments indicate $\gamma \sim 2$ [28]) as Zn atoms placed close to a localized hole strongly distort the hole wave function which leads to a reduction or complete destruction of the AF frustration induced by the hole. Thus, co-doping with Zn has two effects: First, it lowers the amount of frustration in the sample and thus enhances the correlation length [28,29]. Secondly, Zn doping also lowers the total amount of ordered dipoles which leads to a decrease of the incommensurability by a factor $1 - \gamma z$ which should be observable experimentally.

Conclusions. – We propose a novel description of the SG phase as a strongly disordered spiral state, which can account for the IC correlations and the a-b asymmetry without invoking...
any kind of charge order. We show that a collinear analysis is inadequate to describe the strongly disordered regime. No sharp transition towards a disorder-dominated phase is found within the NLσM model analysis and TDs of the spiral must be accounted for to explain the extremely short correlations observed in experiments [30]. The picture of the cluster SG phase as proposed in [10] has some similarities to our work in that it invokes the presence of defects. However, in this picture, the defects are those of an $O(3)/O(2)$ spin system (skyrmions). Also, while the skyrmion model predicts IC correlations only for $x > 0.05$ [10], our analysis includes IC correlations from the outset.

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