Sub-wavelength imaging at optical frequencies using canalization regime

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Imaging with sub-wavelength resolution using a lens formed by periodic metal-dielectric layered structure is demonstrated. The lens operates in canalization regime as a transmission device and it does not involve negative refraction and amplification of evanescent modes. The thickness of the lens have to be an integer number of half-wavelengths and can be made as large as required for ceratin applications, in contrast to the other sub-wavelength lenses formed by metallic slabs which have to be much smaller than the wavelength. Resolution of \( \lambda/20 \) at 600 nm wavelength is confirmed by experimental results for a 300 nm thick structure formed by a periodic stack of 10 nm layers of silicon with \( \varepsilon = 2 \) and 5 nm layers of metal-dielectric composite with \( \varepsilon = -1 \). Resolution of \( \lambda/60 \) is predicted for a structure with same thickness, period and operating frequency, but formed by 7.76 nm layers of silicon with \( \varepsilon = 15 \) and 7.24 nm layers of silver with \( \varepsilon = -14 \).

Competitive alternatives of left-handed media at optical frequencies are photonic crystals \([11, 12]\). The negative refraction effect in photonic crystals at the frequencies close to the band-gap edges was reported by Notomi in \([13, 14]\) and the sub-wavelength imaging using flat lenses formed by photonic crystals was demonstrated both theoretically \([15, 16, 17, 18, 19]\) and experimentally \([20, 21]\). Unfortunately, the resolution of such lenses is strictly limited by period of the crystal. This fundamental restriction was formulated and proven in \([22]\). It means that it is impossible to get very good sub-wavelength resolution using lenses formed by photonic crystals since they operate in the regime when the wavelength in the crystal is comparable with lattice period, but this wavelength can not be shortened too much due to the lack of naturally available high-contrast materials.

During studies of negative refraction and imaging in photonic crystals it was noted that in the certain cases sub-wavelength imaging happens due to the other principle than that in left-handed materials. Actually, the negative refraction in photonic crystals is observed either in forward wave regime, that usually corresponds to the frequencies from the first propagation band \([17, 18, 22]\), or in backward wave regime, that corresponds to the second propagation band \([16, 17, 18]\). The evidence of non-negative refraction was reported by numerous authors \([23, 24, 25, 26, 27]\) for the crystals operating in the first frequency band. The lenses formed by such crystals indeed operate in the regime which does not involve negative refraction and amplification of evanescent waves. This regime was called in \([27]\) as canalization. The slab of photonic crystal in the canalization regime operates not like usual lens which focus radiation into the focal point, it effectively works as a transmission device which delivers sub-wavelength images from front interface of the lens to the back one. The implementation of such a regime becomes possible if the crystal has a flat iso-frequency contour and the thickness of the slab fulfils Fabry-Perot con-
dition (an integer number of half-wavelengths) \[27\]. The flat iso-frequency contour allows to transform all spatial harmonics produced by the source, including evanescent modes, into propagating eigenmodes of the crystal. This preserves sub-wavelength details of the source which usually disappear with distance due to rapid spatial decay of evanescent harmonics. These propagating eigenmodes transmit image across the slab from the front interface to the back one. The possible reflections from the interfaces are eliminated with the help of Fabry-Perot resonance for transmission which in this case holds for all incidence angles due to flatness of iso-frequency contour.

The lenses operating in the canalization regime have same restrictions on the resolution provided by periodicity as those working in the left-handed regime: in order to get sub-wavelength resolution it is required to have period of the structure to be much smaller than the wavelength. In microwave region the canalization regime with \(\lambda/6\) resolution for \(s\)-polarization \[27\] was implemented using an electromagnetic crystal formed by a lattice of wires periodically loaded by capacitances \[28\]. Such a crystal has a resonant band-gap at very low frequencies (with wavelength/period ratio \(\lambda/a = 14\)) and does not contain high-contrast materials. The theoretical and numerical estimations \[27\] were confirmed by experimental verification \[29\] and \(\lambda/10\) resolution was demonstrated. The higher resolution can be achieved using loading by larger capacitances, but the real implementations of these ideas meet with such problems as strong losses and very narrow band-width of operation.

An excellent possibility to realize the canalization regime for \(p\)-polarization is provided by a wire medium, a material formed by a lattice of parallel conducting wires \[30, 31, 32, 33\]. This material supports very special type of eigenmodes, so-called transmission line modes \[33\], which transfer energy strictly along wires with the speed of light and can have arbitrary transverse wave vector components. It means that such modes correspond to completely flat iso-frequency contour which is the main requirement for implementing the canalization regime. The detailed analytical, numerical and experimental studies \[34\] show that flat lenses formed by the wire medium are capable to transmit sub-wavelength images with a resolution equal to double period of the structure which can be made as small as required. Effectively, such lenses work as multi-conductor transmission lines (telegraph) or bundle of sub-wavelength waveguides, which perform pixel-to-pixel imaging. It is important that such lenses are matched to free space and do not experience parasitic reflections from the interfaces. The sub-wavelength imaging with \(\lambda/15\) resolution at 1 GHz was demonstrated in the work \[34\]. The measured bandwidth of operation is 18\%. Moreover, the structure is nearly not sensitive to the losses. Thus, the lens can be made as thick as required. The only restriction is that the thickness should be an integer number of half-wavelengths (in order to fulfil Fabry-Perot condition).

The lens formed by wire medium is an unique sub-wavelength imaging device for microwave frequencies where metals are ideally conducting. At the higher frequencies including visible range such a lens will not operate properly since the metals at these frequencies have plasma-like behavior. In the present paper we propose a different structure which can operate in the canalization regime at optical frequency range. This is a sub-wavelength optical telegraph which operates completely in the same principle as the slab of wire medium at microwaves. It is known that the wire medium \[32\] can be described by spatially dispersive permittivity tensor of the form

\[
\bar{\varepsilon} = xx + yy + \varepsilon_{zz}, \quad \varepsilon(\omega, q_z) = 1 - \frac{k_0^2}{k^2 - q_z^2}, \tag{1}
\]

where \(z\)-axis is oriented along wires, \(k = \omega/c\) is wave number of the host medium, \(k_0 = \omega_0/c\) is wave number corresponding to the plasma frequency \(\omega_0\) which depends on the lattice period and radius of wires, \(q_z\) is \(z\)-component of wave vector \(\mathbf{q}\), \(c\) is the speed of light. For the transmission line mode \(q_z = k\) and effective permittivity \(\varepsilon\) becomes infinite. Thus, transmission line modes effectively propagate in the medium with permittivity tensor of the form

\[
\bar{\varepsilon} = xx + yy + \infty zz. \tag{2}
\]

In order to achieve in optical range the same properties as the wire medium has at microwave frequencies it is required to find some uniaxial optical material which has permittivity of the form \[2\]. Usually, it is assumed that in optical range it is impossible to get very high values of permittivity. It is true for natural materials, but for metamaterials, especially uniaxial, it is not so. The high permittivity can be achieved in layered metal-dielectric structures \[33\]. Let us consider a layered structure presented in Fig. 1. Such a metamaterial can be described using permittivity tensor of the form:

\[
\bar{\varepsilon} = \varepsilon_\parallel (xx + yy) + \varepsilon_\perp zz, \tag{3}
\]

where

\[
\varepsilon_\parallel = \frac{\varepsilon_1 d_1 + \varepsilon_2 d_2}{d_1 + d_2}, \quad \varepsilon_\perp = \left[\frac{\varepsilon_1^{-1} d_1 + \varepsilon_2^{-1} d_2}{d_1 + d_2}\right]^{-1}.
\]

FIG. 1: Geometry of layered metal-dielectric metamaterial.
In order to get $\varepsilon_\parallel = 1$ and $\varepsilon_\perp = \infty$, and obtain a material with permittivity tensor of the form \( \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \) required for implementation of the canalization regime, it is necessary to choose parameters of the layered material so that $\varepsilon_1/\varepsilon_2 = -d_1/d_2$ and $\varepsilon_1 + \varepsilon_2 = 1$. From the first equation it is clear that one of the layers should have negative permittivity and thus, the structure has to be formed by one dielectric layer and one metallic layer. For example, one can choose $\varepsilon_1 = 2$, $\varepsilon_2 = -1$ and $d_1/d_2 = 2$, or $\varepsilon_1 = 15$, $\varepsilon_2 = -14$ and $d_1/d_2 = 15/14$.

Note, that no layered structure required for canalization regime can be formed using equally thick layers $d_1 = d_2$. The layered metal-dielectric structures considered in [8, 9, 10] have completely different properties as compared to the structures considered in the present paper. As it is noted in [10], the structures with $d_1 = d_2$ and $\varepsilon_1 = -\varepsilon_2$ (as in [8, 9, 10]) correspond to $\varepsilon_\perp = 0$ and $\varepsilon_\parallel = \infty$, and operate as an array of wires embedded into the medium with zero permittivity. Such a structure can be considered as unmatched uniaxial analogue of so-called material with zero-index of refraction [36]. The absence of matching ($\mu = 1$, but not 0, as it is required) causes strong reflections and restricts slab thickness to be thin. In contrast to this case, in the canalization regime the reflections from the slab are absent due to the Fabry-Perot condition which holds for all angles of incidence.

In order to demonstrate how canalization regime can be implemented using the suggested metal-dielectric layered structure, we present results of numerical simulation using CST Microwave Studio package. A sub-wavelength source (a loop of the current in the form of P-letter) is placed at 20 nm distance from a 300 nm thick multilayer slab composed of 10 nm and 5 nm thick layers with $\varepsilon_1 = 2$ and $\varepsilon_2 = -1$, respectively. The detailed geometry of the structure is presented in Fig. 2. The wavelength of operation $\lambda$ is 600 nm. The field distributions in the planes parallel to the interface of the lens plotted in Fig. 3 clearly demonstrate imaging with 30 nm resolution ($\lambda/20$). Figs. 3.a,b show the field produced by the source in free space at 20 nm distance. It is practically identical to the field observed at the front interface of the lens, Figs. 3.c,d. It confirms that the reflections from the front interface are negligibly small. Actually, the main contribution into reflected field comes from diffraction at corners and wedges of the lens. Figs. 3.e,f show the field at the back interface of the lens. The image is clearly visible, but it is a little bit distorted by plasmon-polariton modes excited at the back interface. Being diffracted into the free space this distribution forms an image without distortions produced by plasmon-polariton modes.
see Figs. 3.g.h presenting field distribution at 20 nm distance from the back interface.

The resolution of the proposed layered lens is restricted by its period \( d = d_1 + d_2 \). The model of uniaxial dielectric \( \epsilon_1 \) is valid only for restricted range of wave-vector components. In order to illustrate this we calculated isofrequency contour for the layered structure under consideration treating it as 1D photonic crystal and using an analytical dispersion equation available in [37]. The result is presented in Fig. 4. While \(|q_x d/\pi| < 0.5\) the isofrequency contour is flat, the homogenized model \( \epsilon_3 \) is valid and the lens works in the canalization regime. The spatial harmonics which have \(|q_x d/\pi| > 0.5\) will be lost by the lens and this defines \( d/0.5 \approx \lambda/20 \) resolution. The calculation of isofrequency contour for the case of \( \epsilon_1 = 15, \epsilon_2 = -14, d_1 = 7.76 \text{ nm and } d_2 = 7.24 \text{ nm for the same wavelength of 600 nm revealed its flatness for } |q_x d/\pi| > 1.5 \). This allows us to predict \( d/1.5 \approx \lambda/60 \) resolution for this case. Note, that the similar restrictions on resolution by the periodicity are applicable for the layered structures considered in [38]. In spite of the authors’ claims that resolution of such structures is mainly limited by losses. The general study of limitations on homogenization of periodic layered structures will be published elsewhere.

In conclusion, we have demonstrated possibility of the imaging with sub-wavelength resolution using the lens formed by a layered metal-dielectric structure. The lens works in canalization regime as a transmission device and does not involve negative refraction and amplification of evanescent modes. The simulation was done for 300 nm thick structure comprising of 10 nm dielectric layers with \( \epsilon_2 = 2 \) and 5 nm metal layers with \( \epsilon = -1 \) at wavelength of 600 nm and resolution of \( \lambda/20 = 30 \text{ nm} \) was shown. The metal with \( \epsilon = -1 \) at 600 nm wavelength can be created by doping some lossless dielectric by small concentration of silver which has \( \epsilon = -15 \) at such frequencies, in the similar manner to the ideas of works [35] or [38]. Even more promising resolution of \( \lambda/60 = 10 \text{ nm} \) was predicted for the layered structure comprising of 7.76 nm layers of dielectric with \( \epsilon = 15 \) and 7.24 nm layers of metal with \( \epsilon = -14 \). The last structure can be constructed using silicon as dielectric and silver as metal, but very accurate fabrication with error no more than 0.05 nm will be required in order to get proper result. The losses in silver in both cases are already reduced by operating at rather long wavelength of 600 nm, but in accordance with our estimations they are still high enough to destroy quality of the sub-wavelength resolution. This problem can be solved by using of active materials, for example doped silicon [10, 38].

\[ \text{FIG. 4: Isofrequency contour of layered metal-dielectric structure with } \epsilon_1 = 2, \epsilon_2 = -1, d_1 = 10 \text{ nm, } d_2 = 5 \text{ nm, } d = d_1 + d_2 = 15 \text{ nm for } \lambda = 600 \text{ nm. The region where the dispersion curve is flat } (|q_x d/\pi| < 0.5) \text{ is shadowed.} \]

\[ \begin{array}{c}
|q_x d/\pi| \\
-0.2 \quad -0.1 \quad -0.0 \quad 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4
\end{array} \]

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