Study of $b \to u \ell \bar{\nu}$ Decays
with an Inclusive Generator

M. Battaglia
Dept. of Physics,
University of Helsinki (Finland)

Abstract

A generator for inclusive $b \to u \ell \bar{\nu}$ decays has been developed. Different prescriptions have been applied to describe the kinematics of the $b$ quark inside the hadron and the generation of the hadronic final states. Results are presented with particular attention to the invariant mass of the hadronic system recoiling against the lepton and its resonance decomposition. These studies are of special relevance for the extraction of $|V_{ub}|$ from semileptonic $B$ decays at LEP and at $B$ factories.
1 Introduction

The measurement of the branching ratio for the decay \( b \rightarrow u\ell\bar{\nu} \) provides the cleanest way to determine the \( |V_{ub}| \) element in the CKM mixing matrix. Evidence of the non-zero value of \( |V_{ub}| \) has been first obtained by both ARGUS and CLEO by observing leptons produced in \( B \) decays with momentum exceeding the kinematical limit for \( b \rightarrow c\ell\bar{\nu} \) transitions [1, 2].

The extraction of \( |V_{ub}| \) from the yield of leptons above the \( b \rightarrow c\ell\bar{\nu} \) endpoint is subject to large systematic uncertainties. More recently, exclusive \( B \rightarrow \pi\ell\bar{\nu} \) and \( B \rightarrow \rho\ell\bar{\nu} \) decays have been observed by CLEO and their rates measured [3, 4]. Still the derivation of \( |V_{ub}| \) from exclusive semileptonic decays, contrary to the case for \( |V_{cb}| \) in \( B \rightarrow D^{*}\ell\bar{\nu} \), has significant model dependence.

The extraction of \( |V_{ub}| \) from the shape of the invariant mass of the hadronic system recoiling against the lepton in \( b \rightarrow u\ell\bar{\nu} \) transitions was proposed several years ago [5] and it has been recently the subject of new studies [6, 7]. The proposed method starts from the observation that the hadronic system recoiling against the lepton in the decay has invariant mass lower than the charm mass for the majority of \( b \rightarrow u\ell\bar{\nu} \) decays. The model dependence in predicting the shape of this invariant mass distribution has been claimed to be under control within about 10 – 15% if \( b \rightarrow u\ell\bar{\nu} \) decays can be distinguished from the \( b \rightarrow c\ell\bar{\nu} \) ones for masses of the hadronic system up to cut values close enough to the \( D \) mass [7]. This corresponds to a model uncertainty of 5-7% on the extraction of \( |V_{ub}| \).

The predicted shape of the invariant mass distribution depends mainly on the kinematics of the heavy and spectator quarks inside the \( B \) hadron and on the quark masses. However from the experimental point of view, the hadronisation process, transforming the \( u\bar{q} \) system into the observable hadronic final state, represents a significant source of additional model uncertainties.

Several models have been proposed to describe both steps of the decay process. This note summarizes the results of a study performed by developing a dedicated \( b \rightarrow u\ell\bar{\nu} \) decay generator, BTOOL. This generator implements different prescriptions for the initial state kinematics and the resonance decomposition of the hadronic final states. Its results are used to define the model dependent systematics in the extraction of \( |V_{ub}| \) from the hadronic mass spectrum in semileptonic \( B \) decays.

2 The BTOOL Generator

The decay generator provides the four momenta of the stable decay products in \( b \rightarrow u\ell\bar{\nu} \) transitions. This requires a model for the kinematics of the \( b \) and spectator \( \bar{q} \) quarks inside the \( B \) hadron, the description of the \( Q^2 \) distribution of the virtual \( W \) and of the kinematics in the \( b \rightarrow uW \) and \( W \rightarrow \ell\nu \) decays, and finally a model for the hadronisation of the \( u\bar{q} \) system. In the following the implementation of the decay in the generator is discussed. In obtain to study model dependences and systematic effects, different prescriptions have been adopted.

2.1 ACCMM Model

In the ACCMM model [8] the \( B \) hadron consists of the \( b \) quark and the spectator \( \bar{q} \) quark moving back-to-back in the \( B \) rest frame. Their momenta \( p \) are distributed according to
the gaussian distribution:

$$\phi(p) = \frac{4}{\sqrt{\pi} p_F} e^{-\frac{p^2}{p_F^2}}$$

(1)

where the width $p_F$ is known as Fermi motion and represents a parameter in the model. The normalisation is chosen such that

$$\int_0^{+\infty} dp \ p^2 \phi(p) = 1.$$  

(2)

The choice of the $p_F$ parameter and of the mass of the spectator quark $m_q$ are discussed in details in the next section.

### 2.2 Parton Model

An alternative picture of the $b$ quark kinematics has been proposed as an application of the parton model to heavy quark decays [9]. In this model the decay is considered in a frame where the $B$ hadron moves with large momentum (infinite momentum frame or Breit frame). In this frame the $b$ quark behaves as a free particle carrying a fraction $z$ of the $B$ momentum, $p_b = z p_B$. The functional form of $f(z)$ can be extracted from the $b$ fragmentation function since the probability of finding the $b$ quark carrying a fraction $z$ of the hadron momentum corresponds to the probability for a $b$ quark to produce a $B$ hadron with a fraction $z$ of its energy. This is usually described by the Peterson fragmentation function [10] that has the form:

$$f(z) = \frac{N z (1-z)^2}{((1-z)^2 + \epsilon_b z)^2}$$

(3)

where $\epsilon_b$ is a free parameter. In this way, the parton model offers an advantage since the kinematics of the $b$ quark is described by a function that can be directly related with experimental data on $b$ fragmentation.

### 2.3 QCD Universal Structure Function

Recently there has been progress in defining the Fermi motion in the framework of QCD [11, 12, 13]. This has been achieved in terms of a universal structure function describing the distribution of the light-cone residual momentum of the heavy quark inside the hadron. At leading order and in the large $m_b$ limit, the light-cone residual momentum $k_+$ can be expressed as the difference between the $b$ quark pole mass and its effective mass $m_b^*$ inside the hadron: $m_b^* = m_b + k_+$. As a consequence $k_+ < \bar{\Lambda} = m_B - m_b$. An ansatz for the shape of the universal structure function has been suggested [13] in the form:

$$f(z) = z^a (1 - cz) e^{-cz}$$

(4)

where $z = 1 - \frac{k_+}{\bar{\Lambda}}$, and the coefficients $a$ and $c$ depend on the values of $\bar{\Lambda}$ and of the kinetic energy operator as discussed in the next section.
2.4 Decay kinematics

The $b$ quark decays as $b \rightarrow W u$, the $W$ and $u$ being emitted back to back in the $b$ rest frame with an isotropic distribution of their emission angle w.r.t the $b$ direction. The virtual $W$ is characterized by an effective mass $Q^2$. The kinematics of the $b$ and $W$ decays correspond to that for two body decays. Therefore the model dependence that propagates to the final state kinematics depends on the choice of the values of $p_F$ and $m_q$ and of the $Q^2$ distribution. By defining $x^2 = Q^2/m_b^2$ the differential decay rate can be expressed as:

$$\frac{d\Gamma}{dx} = \frac{G_F^2m_b^5|V_{ub}|^2}{192\pi^3}(F_0(x) - \frac{2\alpha_s}{3\pi}F_1(x)) \quad (5)$$

The functions $F_0(x)$ and $F_1(x)$ describe the tree-level contribution and the QCD corrections terms. Following Ref. [14], they can be written, in the limit $m_u \rightarrow 0$, as:

$$F_0(x) = 2(1 - x^2)^2(1 + 2x^2) \quad (6)$$

and

$$F_1(x) = (\pi^2 + 2S_{1,1}(x^2) - 2S_{1,1}(1 - x^2)) + 8x^2(1 - x^2 - 2x^4)ln(x)$$

$$+ 2(1 - x^2)(5 + 4x^2)ln(1 - x^2) - (1 - x^2)(5 + 9x^2 - 6x^4) \quad (7)$$

where $S_{1,1}(x)$ is the Nielsen polylogarithm. The resulting $Q^2$ distribution is shown in Figure 1.

![Figure 1: The $q^2$ distribution in the decay $b \rightarrow u\ell\bar{\nu}$ generated according to Eq. (5).](image)

The virtual $W$ is forced to decay to the lepton-neutrino pair. Due to the spin of the $W$, its decay is not isotropic and the angular distribution of the lepton varies as $1 + \cos^2\theta$. In computing the lepton and neutrino momenta in the $W$ rest frame the lepton mass is taken into account. After the decay the lepton and neutrino are boosted back to the $B$ rest frame. The $u$ quark is also boosted to the same system, and at this point the hadronic system is treated.
2.5 The hadronic system

The energy and invariant mass of the hadronic system correspond to those of the $u\bar{q}$ quark pair. The energy and invariant mass at the parton level can be compared with predictions obtained in QCD and heavy quark expansion. The observable final states are then generated by applying different prescriptions for describing the evolution of the quark fragmentation as discussed in the next section. These include a fully inclusive hadronisation scheme according to the JETSET parton shower model [13] and exclusive descriptions of the final states in the $b \to u\ell\bar{\nu}$ transition.

2.6 The JETSET 7.4 interface

The generator can be used to produce individual $B$ meson decays for dedicated studies. Timings for the generation of individual decays on different platforms are given in Table 1. BTOOL is also interfaced as an option of the LUDECY subroutine of JETSET to handle $b \to u\ell\bar{\nu}$ decays in the generation of $e^+e^- \to Z^0/\gamma \to \bar{b}b$ events. The BTOOL generator can be activated by setting the MDME(IDC,2) flag in the LUDAT3 common block of JETSET, where IDC refers to the $b \to u\ell\bar{\nu}$ decay channel.

| Platform      | OS          | secs./1000 decays |
|---------------|-------------|-------------------|
| HP 9000/778  | HP-UX 10.20 | 0.52              |
| Pentium-133  | Linux 2.0.27| 1.85              |

Table 1: Timing of the decay generator for different platforms

3 Results and Comparisons

The focal interest in the simulation study of $b \to u\ell\bar{\nu}$ decays is to determine the invariant mass spectrum of the hadronic system and its correlation with the lepton energy and then to define the uncertainties of these distributions due to the choice of the model and of its input parameters. Three different models have been applied for the definition of the kinematics of the $b$ and spectator $\bar{q}$ inside the hadrons and two models for the generation of the hadronic final states. The criteria chosen for the input parameters and their range of variation are discussed in the following subsection. The results for the hadronic system mass and multiplicities are presented in 3.2 and 3.3.

3.1 The Choice of Parameters

The ACCMM model introduces two free parameters: i) the Fermi momentum $p_F$ and ii) the spectator quark mass $m_{sp}$. There have been attempts to extract the values of these parameters from fits to experimental observables such as the momentum spectrum of leptons from $b \to c\ell\bar{\nu}$ decay [16,17], the photon spectrum in $b \to s\gamma$ decay [18] and the $J/\psi$ momentum distribution in $B \to J/\psi X$ [19]. Most of these determinations point to a value of $p_F \simeq 0.5$ GeV/c (Table 2).
Table 2: Experimental estimates of the $p_F$ value in the ACCMM model

| Channel     | $p_F$ (GeV/c) | $m_{sp}$ (GeV/c$^2$) | Ref. |
|-------------|--------------|---------------------|-----|
| $b \to c\ell\bar{\nu}$ | $0.27 \pm 0.04$ | $0.30$ | [16] |
| $b \to c\ell\bar{\nu}$ | $0.51^{+0.08}_{-0.07}$ | $0.0$ | [17] |
| $b \to s\gamma$ | $0.45$ | $0.0$ | [18] |
| $B \to J/\psi X$ | $0.57$ | $0.15$ | [19] |

However it has been pointed out that the value of $p_F$ obtained in a fit to $b \to c$ transitions may be not appropriate in the description of $b \to u$ decays [20]. At the same time it has also been shown that the ACCMM model is consistent with the QCD description of $b \to u\ell\bar{\nu}$ and $b \to s\gamma$ transitions and that the corresponding parameters can therefore be related. This is the direction followed in this study.

The effective $b$ quark mass $m_b$ depends on $p_F$ and $m_{sp}$ as:

$$m_b^2 = m_b^2(p_b) = m_B^2 + m_{sp}^2 - 2m_B\sqrt{p_b^2 + m_{sp}^2}$$

(9)

where $p_b$ is the momentum of the heavy quark in the hadron and $m_B$ is the $B$ hadron mass. The value of $m_B$ can be taken as a parameter, tunable such that the ACCMM model corresponding to an average $b$ mass $<m_b>$ can be compared with theory predictions obtained for a given value of $m_b$. Estimated values for the $b$ quark pole mass are in the range $4.72$ GeV/c$^2 < m_b < 4.92$ GeV/c$^2$ [14].

The value of $p_F$ is proportional to the average kinetic energy of the $b$ quark in the hadron since:

$$<p_b^2> = \int_0^{+\infty} dp_b \ p_b^2 \ (\phi(p_b) \ p_b^2) = \frac{3}{2}p_F^2$$

(10)

where $\phi(p)$ is given by Eq. (1). Through the two above equations the ACCMM model parameters are related to those of the QCD description of the heavy quark inside the hadron. In this framework $<p_b^2>$ corresponds to the value of the expectation value $\mu^2$ of the kinetic operator. The value $p_F = 0.5$ GeV/c corresponds to $<p_b^2> = 0.37$ GeV$^2$. Estimates of $<p_b^2>$ have been obtained both from theory and fits to measured spectra in $B$ decays as discussed below.

For the Parton Model, the fragmentation function for $b$ quarks has been measured at LEP. Averaging over the ALEPH, DELPHI and OPAL results, the fraction of the $b$ quark energy taken by the beauty hadron is $<x_B> = 0.702 \pm 0.008$ [21]. Also the observed shape in the preliminary DELPHI analysis [22] was compatible with that of the Peterson function. These results point to a value for the $\epsilon_b$ parameter of $\epsilon_b = 0.0040$. In the simulation of decays with the parton model, the spectator quark mass mass $m_q$ was set to zero and the $b$ quark mass was varied in the range $4.72$ GeV/c$^2 < m_b < 4.92$ GeV/c$^2$ as for the ACCMM model. It is interesting to point out that, by using the central value for $\epsilon_b$, the parton model gave $<p_b^2> = 0.35$ GeV$^2$ for $m_b = 4.72$ GeV/c$^2$, consistent with the value obtained in the ACCMM model for $p_F = 0.5$ GeV/c.

The use of the QCD universal structure function $f(k_+)$ allows a consistent comparison with the results obtained in the framework of QCD and Heavy Quark expansion. The
normalised moments $a_n = \frac{A_n}{\bar{\Lambda}^n}$ of $f(k_+)$, given by

$$a_n = \frac{1}{\bar{\Lambda}^n} \int dk_+ k_+^n f(k_+)$$

relate the function parameters with that of the theory. In particular the first two moments define the function normalisation and the third is proportional to the expectation value of the kinetic energy operator $[13]$:

$$a_0 = 1$$

$$a_1 = 0$$

$$a_2 = \frac{3\mu_\pi^2}{\bar{\Lambda}^2}$$

These relationships define the values of the parameters $a$ and $c$ in Eq. 4 as a function of the values of $\mu_\pi^2$ and $\bar{\Lambda}$. There have been several evaluations of $\mu_\pi^2$ and a selection of recent results is given in Table 3. Results are scheme dependent and, depending on the method used in their derivation, they point to the values of $\mu_\pi^2$ of 0.4 GeV$^2$ or 0.2 GeV$^2$.

| Method                  | $\mu_\pi^2$ (GeV$^2$) | Ref. |
|-------------------------|------------------------|------|
| QCD Sum Rules           | 0.50                   | 23   |
| Variational Method      | 0.44                   | 24   |
| Relativ. Potential      | 0.46                   | 25   |
| Virial Theorem          | 0.40 - 0.58            | 26   |
| Virial Theorem          | 0.15 ± 0.03            | 27   |
| $b \rightarrow c\ell\bar{\nu}$ | 0.19 ± 0.10         | 28   |
| $b \rightarrow c\ell\bar{\nu}$ | 0.14 ± 0.03         | 29   |
| $b \rightarrow s\gamma$ | 0.71$^{+1.16}_{-0.70}$ | 30   |

The two values of $\mu_\pi^2 = 0.2$ GeV$^2$ and 0.4 GeV$^2$ were chosen while $\bar{\Lambda}$ ranged between 0.36 GeV/c$^2$ and 0.56 GeV/c$^2$ for $4.72$ GeV/c$^2 < m_b < 4.92$ GeV/c$^2$ and $m_B = 5.28$ GeV/c$^2$. The masses of the light quarks, $u$ and spectator quark, were set to zero.

### 3.2 The Hadronic System Mass

The precise determination of the fraction of $b \rightarrow ul\bar{\nu}$ transitions yielding an hadronic system with mass $M_X$ below a given $M_{cut}$ value is crucial in the estimation of $|V_{ub}|$. As discussed above, at the quark level the hadronic mass corresponds to the mass of the $u\bar{q}$ system. This depends mainly on the masses and motion of the heavy and spectator quarks in the $B$ hadron. On the contrary the experimentally measureable hadronic mass is strongly affected by the resonant decomposition of the hadronic final states. In the simulation the hadronic system is analyzed in two steps.

Firstly, the $u\bar{q}$ pair is analyzed. This gives the hadronic mass distribution before resonant states are taken into account (see Figure 2) and can be compared with that computed using QCD and heavy quark expansion $[7]$. 

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Figure 2: The invariant mass distribution of the $u\bar{q}$ system obtained using the ACCMM model (left) and the QCD universal function (right) corresponding to $m_b = 4.72$ GeV$/c^2$ and $<p_b^2> = 0.4$ GeV$^2$.

Results are expressed in terms of the fraction $F_u(M_{cut})$ of the $b \rightarrow u\ell\bar{\nu}$ transitions resulting in a mass of the $u\bar{q}$ pair below a given cut value. The kinematics of the $b$ and spectator quark have been defined using the ACCMM model, the QCD universal structure function and the Parton Model. In order to compare the results, parameters have been chosen such that they correspond to $m_b = 4.80 \pm 0.10$ GeV$/c^2$ and $0.2$ GeV$^2 < <p_b^2> < 0.4$ GeV$^2$ as discussed above.

Table 4: Fraction $F_u(M_{cut})$ of $b \rightarrow u\ell\bar{\nu}$ decays with $M_X < M_{cut}$ for $<p_b^2> = 0.4$ GeV$/c^2$.

| $m_b$ (GeV$/c^2$) | $M_{cut}$ (GeV$/c^2$) | $F_u(M_{cut})$ | $F_u(M_{cut})$ | $F_u(M_{cut})$ | $F_u(M_{cut})$ | $F_u(M_{cut})$ |
|------------------|------------------|----------------|----------------|----------------|----------------|----------------|
|                  |                  | ACCMM          | QCD Funct.     | Parton         | Ref.           |
| 4.92             | 1.25             | 0.48           | 0.50           | 0.40           | 0.45           |
|                  | 1.50             | 0.65           | 0.60           | 0.62           | 0.60           |
|                  | 1.75             | 0.76           | 0.80           | 0.84           | 0.78           |
| 4.82             | 1.25             | 0.50           | 0.57           | 0.45           | 0.55           |
|                  | 1.50             | 0.65           | 0.72           | 0.74           | 0.72           |
|                  | 1.75             | 0.78           | 0.83           | 0.89           | 0.85           |
| 4.72             | 1.25             | 0.54           | 0.65           | 0.47           | 0.68           |
|                  | 1.50             | 0.68           | 0.75           | 0.71           | 0.81           |
|                  | 1.75             | 0.80           | 0.85           | 0.87           | 0.89           |

Table 4 summarizes the results for different choices of the input parameters. The first observation is that the universal function implemented in BTOOL reproduces the prediction from QCD and heavy quark expansion from [7]. Further it has been found that also the ACCMM model reproduces these results to better than 15%, for equivalent values of $m_b$ and $<p_b^2>$. This study confirms that the sensitivity to the value of the $b$ quark mass is significant in the region of low hadronic invariant masses. In this region also the model dependence is more pronounced. In order to estimate the overall uncertainty
in the estimate of the fraction of $b \to u \ell \bar{\nu}$ decays with hadronic mass below a given cut value $M_{cut}$, the different sources of systematics have been combined. The chosen range of variation of the parameters is $\pm \sigma(m_b) = \pm 0.10$ GeV/$c^2$ and $\pm \sigma(<p_b^2>) = \pm 0.1$ GeV$^2$. The corresponding relative systematic errors are summarized in Table 5.

| Source | $M_{cut} = 1.25$ | $M_{cut} = 1.50$ | $M_{cut} = 1.75$ |
|--------|-----------------|-----------------|-----------------|
| $m_b$  | 0.13            | 0.10            | 0.05            |
| $<p_b^2>$ | 0.08          | 0.04            | 0.02            |
| Model  | 0.08            | 0.04            | 0.04            |
| Total  | 0.17            | 0.12            | 0.07            |

Secondly, the hadronic final states corresponding to a given mass and energy of the $u\bar{q}$ pair can be predicted by a variety of methods ranging from the fully inclusive quark fragmentation approach to exclusive models.

Figure 3: The distribution of the $u$ quark energy versus the lepton energy (a) and the physical region in the $q^2$ versus $2m_B q_0$ plane (b) for inclusive $b \to u \ell \bar{\nu}$ decay.

At large enough recoil $u$ quark energies, the $u\bar{q}$ system moves away fast and this picture is similar to that of the evolution of a jet initiated by a light quark $q$ in $e^+e^- \to q\bar{q}$ annihilation. This is simulated by first arranging the $u\bar{q}$ system in a string configuration and then making it fragment according to the parton shower model. Exclusive models compute the decay amplitudes from the heavy-to-light form factors and the quark hadronic wave functions. The so-called ISGW2 model \cite{31} approximates the inclusive $b \to u \ell \bar{\nu}$ decay width by the sum over resonant final states, taking into account leading corrections to the heavy quark symmetry limit.

The range of applicability of the inclusive and exclusive models is restricted to particular regions of the accessible kinematic configurations. Figure 3 a) shows the physical
region in the $q^2$ versus $2m_B q_0$ plane where $q^2$ is the effective mass of the virtual $W$ and $q_0$ its energy in the $B$ rest frame. In this plot states of equal hadronic invariant mass $M_X$ correspond to lines $M_X^2 = m_B^2 - 2m_B q_0 + q^2$. Systems of large invariant mass correspond to low $q^2$ and $q_0$ values, i.e. large $u$ recoil as shown in Figure 3 b). In these cases the $u$ quark energy is typically large enough compared with that of the spectator quark that the analogy with jet fragmentation is justifiable. Conversely at low $u$ recoil energy, i.e. close to the upper kinematical limit in Figure 3 a), the relative momentum of the $u\bar{q}$ pair is small and they are therefore likely to form a bound state.

Figure 4: Comparison of the invariant mass of the hadronic final state from the ISGW2 model (upper left) and from parton shower fragmentation (upper right). In the lower plot the corresponding fractions $F_u(M_{cut})$ of the $b \to u\ell\bar{\nu}$ transitions resulting in a mass below a given cut value $M_{cut}$ are shown as a function of $M_{cut}$. The corresponding predictions for the invariant mass of the $u\bar{q}$ pair are shown with the continuous line ([7]) and the dash histogram (BTOOL). The CLEO results for the branching ratios for the $\pi\ell\bar{\nu}$ and $\rho\ell\bar{\nu}$ channels are also shown for comparison.

A satisfactory description of the inclusive $b \to u\ell\bar{\nu}$ decay must take these characteristics into account. Hybrid models have been proposed for this purpose [32]. The main feature of an hybrid model is to define a kinematical region in which the exclusive model is valid and its complement that can be treated by an inclusive fragmentation model. The
two regions must be chosen in order to have well behaved matching conditions for a set of relevant kinematical variables. Further constraints can be derived from the branching ratios for $B \to \pi \ell \bar{\nu}$ and $B \to \rho \ell \bar{\nu}$ decays measured by CLEO \cite{3,4}. Since the aim of this study is the definition of the systematic uncertainties in the description of the hadronic mass spectrum, results have been derived for the two extreme cases of fully inclusive and exclusive models. In the inclusive model, the probabilities for generating light vector and axial resonances have been tuned in JETSET in order agree with the measured rates in $Z^0$ decays. For the exclusive model the ISGW2 model has been used. The results are presented in Figure 4 in terms of the fraction of $b \to u \ell \bar{\nu}$ decays with the hadronic final state below a given mass value. The comparison of the predictions from the two models shows significant differences in the predicted mass spectrums due to the relative importance of resonant and non-resonant final states. For the region of $M_{cut} > 1.5 \text{ GeV}/c^2$, the relative difference of the two models correponds to 10 - 15% showing that the hadronic system fragmentation introduces an uncertainty of the same order as that from the $b$-quark mass and the heavy hadron kinematics.

3.3 The Hadronic System Multiplicity

An additional source of uncertainty in the modelling of the decay arises from the multiplicity of the hadronic system. This is of special relevance since the efficiency for reconstructing the decay depends on this multiplicity. In order to study this uncertainty, the different prescriptions for describing the final states discussed above have been analyzed in terms of the resulting decay multiplicity.

![Figure 5: Comparison of the generator predictions with experimental data on charged multiplicity in low-energy $e^+e^-$ collisions. The points represent the experimental data, the long thick line the best fit from Ref. \cite{33} and the shorter thinner lines the fits to the generator results for values of $p_F$ ranging from 0.0 GeV/c to 0.35 GeV/c.]
The charged multiplicity of the hadronic system from the $b \to u\ell\bar{\nu}$ decay can be compared with one half of that of a $q\bar{q}$ event at $\sqrt{s} = 2E_{\text{had}}$. The data on the event charged multiplicity from ADONE and MARK II at $2 \text{ GeV} < \sqrt{s} < 8 \text{ GeV}$ can be described with a function $<n_{\text{ch}}>=a+b \ln s$ with $a = 2.67\pm0.04$ and $b = 0.48\pm0.02$ [33] shown by the long thick line in Figure 5. These data have also been compared with the results of the simulation where a $u$ quark of energy $\sqrt{s}/2$ is paired with the spectator quark. The quark is either given Fermi motion according to the value of $p_F$ but no transverse momentum, or kept at rest (i.e. $p_F = 0$). The results are shown by the shorter thinner lines in Figure 5. The parton shower model reproduces reasonably well both the multiplicity and its scaling with the quark energy. The best agreement with the data is obtained by imposing $p_F = 0.2 \text{ GeV/c}$ which gives a fit with $a=2.71$ and $b=0.44$.

Multiplicities in semileptonic $B$ decays have also been predicted using the quark-gluon string model (QGSM) which also reproduces fairly precisely the same data on the charged event multiplicity in low energy $e^+e^-$ collisions [34]. The multiplicities obtained by the decay generator using the hybrid model are compared with the predictions from QGSM and ISGW2 in Table 6. The average charged multiplicity in the decay $<n_{\text{ch}}>$ agrees for the three models within $\pm 0.16$. This multiplicity is also quite close to that measured for $D$ meson decays [33], showing that the reconstruction of the hadronic system may be performed with comparable efficiency in semileptonic $b \to u$ and $b \to c$ decays.

Table 6: Decay multiplicities from the event generator compared with QGSM and ISGW2 models

| Final State     | Simulated | QGSM [34] | ISGW2 [31] |
|-----------------|-----------|-----------|------------|
| $0 \pi\pm$      | $1 \pi^0$ | .050      | .020       | .044       |
| $0 \pi\pm$      | $2 \pi^0$ | .016      | .002       | .010       |
| $0 \pi\pm$      | $3 \pi^0$ | .011      | .009       | .012       |
| $0 \pi\pm$      | $4 \pi^0$ | .005      | .004       | .004       |
| $1 \pi\pm$      | $0 \pi^0$ | .101      | .060       | .090       |
| $1 \pi\pm$      | $1 \pi^0$ | .124      | .140       | .135       |
| $1 \pi\pm$      | $2 \pi^0$ | .037      | .064       | .016       |
| $1 \pi\pm$      | $3 \pi^0$ | .008      | .025       | .008       |
| $1 \pi\pm$      | $4 \pi^0$ | .003      | .003       | -          |
| $2 \pi\pm$      | $0 \pi^0$ | .117      | .080       | .090       |
| $2 \pi\pm$      | $1 \pi^0$ | .176      | .175       | .167       |
| $2 \pi\pm$      | $2 \pi^0$ | .038      | .095       | .048       |
| $2 \pi\pm$      | $3 \pi^0$ | .004      | .015       | .005       |
| $3 \pi\pm$      | $0 \pi^0$ | .053      | .070       | .054       |
| $3 \pi\pm$      | $1 \pi^0$ | .062      | .098       | .061       |
| $3 \pi\pm$      | $2 \pi^0$ | .021      | .036       | -          |
| $4 \pi\pm$      | $0 \pi^0$ | .012      | .050       | .006       |
| $4 \pi\pm$      | $1 \pi^0$ | .017      | .020       | .003       |
| $<n_{\text{ch}}>$ |           | 1.91      | 2.06       | 1.70       |
4 The Lepton Spectrum

As already mentioned, the lepton spectrum is sensitive to the mass of the quark produced in the semileptonic $b$ decay. While the lepton yield in the region of lepton energies above the kinematical limit $\sim \frac{M_B^2-M_D^2}{2M_B}$ for $b \to c\ell\bar{\nu}$ transitions is subject to significant model dependences, a combined study of the mass of the hadronic system $M_X$ and the energy of the lepton in the $B$ rest frame $E_\ell^*$ may allow an extraction of the $|V_{ub}|$ with good sensitivity and improved control of the systematics. Figure 6 shows the correlation between the values of $E_\ell^*$ and $M_X$ before and after the hadronisation of the $u\bar{q}$ system. By selecting decays with low hadronic invariant mass, the lepton spectrum is depleted in its lower end without significantly affecting the region of lepton energies above 1.5 GeV/c that are relevant for separating $b \to u\ell\bar{\nu}$ from $b \to c\ell\bar{\nu}$ decays.

5 Conclusion

A generator for inclusive $b \to u\ell\bar{\nu}$ decays has been developed and used for studying the invariant mass and resonance decomposition of the hadronic system produced in the decay. These studies are of special relevance for the extraction of $|V_{ub}|$ from semileptonic $B$ decays at LEP and at the $B$ factories. The low invariant mass of the hadronic system emitted in these decays can be used to separate them from the CKM favoured $b \to c\ell\bar{\nu}$ transitions. Different models for defining the kinematics of the heavy quark inside the hadron and the $u\bar{q}$ system hadronisation have been compared. Systematic uncertainties in the fraction of decays giving an hadronic system with mass below a given cut arise from the value of the $b$ quark pole mass, the momentum distribution of the heavy and spectator quark inside the hadron and modelling of the $u\bar{q}$ hadronisation. This analysis confirmed the observation that model dependencies and these systematic uncertainties from the $b$-quark mass and the heavy quark kinematics can be kept at the 10% level, or below, if
the study of $b \rightarrow u\ell\bar{\nu}$ is performed including decays with hadronic final state masses up to $\simeq 1.6 \text{ GeV}/c^2$ or above. A comparable uncertainty arises from the hadronisation model when comparing an inclusive to a fully exclusive model. The combined analysis of the hadronic mass and lepton spectrum spectrum may provide an optimal separation of $b \rightarrow u\ell\bar{\nu}$ from $b \rightarrow c\ell\bar{\nu}$ decays.

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