Swampland Criteria in Slotheon Field Dark Energy

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Abstract

We explore in this work whether Slotheon model of Dark Energy obeys the Swampland criteria of string theory. Since de Sitter vacuum is very difficult to construct in string theory the cosmological constant as an explanation of Dark Energy is almost ruled out in string theory since it involves a scalar potential $V$ with positive local minimum that ends up to a stable (or meta stable) de Sitter (ds) vacuum. In quintessence model however if the derivative of the scalar potential $V$ is small and $|\nabla V|$ $\sim O(1)$ then in this situation the potential $V$ can be positive but the scalar field may not be at the minimum. For a consistent quantum theory of gravity the theory should not have any ds or meta stable ds vacua. In this regard the Swampland criterion is proposed which any low energy theory should obey to be consistent with quantum theory of gravity. This criterion is written as $|\nabla V|/V > c \sim O(1)$. In this work we consider a scalar field model for Dark Energy namely the Slotheon Dark Energy model inspired by the theories of extra dimensions and show that this Dark Energy model agrees better with the Swampland criteria than the quintessence Dark Energy model.

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1 Introduction

The scalar field theories such as quintessence as also the scalar fields inspired by the theories of extra dimensions are considered in the literature to account for the late time acceleration of the Universe and the Dark Energy of the Universe which is thought to have caused this acceleration. The basis of such theories for Dark Energy is generally the Einstein’s theory of general relativity which appears to work well, below the Planck scale. But beyond the Planck scale it is debatable whether such theories can connect to the more robust quantum theory of gravity in a string theory landscape. It appears that there exists an even bigger string Swampland [1] where some effective field theories coupled to gravity are inconsistent with the quantum theory of gravity. This has arisen from the difficulties in string theory in constructing the de Sitter vacuum and to find the possibility of the absence of de Sitter like vacuum in a consistent quantum theory of gravity. Therefore it is imperative to ensure that any such low energy theory of gravity obeys the string Swampland criteria for these theories to be a low energy theory of a consistent quantum theory of gravity and thus can be embedded in string theory. Therefore the string theory criteria can be used to constrain the Dark Energy models that originate from a scalar field theory.

In this work we explore whether the Slotheon model [2, 3, 4] for Dark Energy obeys the string Swampland criteria. The string swampland criteria for an effective field theory to be consistent with the string theory is given by

- The range $d$ traversed by the scalar fields should obey the bound $|\Delta \pi| < d \sim \mathcal{O}(1)$ (in reduced Planck units) [5].
- The quantity $|\nabla \pi V|/V > c \sim \mathcal{O}(1)$ in reduced Planck units [6], where $V$ is the potential of the scalar field $\pi$ and $V > 0$. This means that the derivative of the potential $V$ of the scalar field has a finite minima (a lower bound).

This second criterion is from the fact that it is difficult to construct the de Sitter vacuum. It is argued in [7] that the second criterion is relevant for Dark Energy. Therefore in the present context the second condition is important. In the present work we investigate the consistency of the Dark Energy from Slotheon scalar field theory with the string Swampland criteria. In this regard we explore the variations of Dark Energy equation of state parameters. We consider generalised thawing model [5] of Dark Energy [9] to construct the variations of Dark Energy equation of state where the experimental bounds are considered by choosing the parameter $\omega_0$ from Euclid [10] simulated data. We then compute the variations of $w(z)$ with $z$ for Slotheon Dark Energy scalar field model. This is done for different chosen values of $\lambda = \frac{M_{pl} V}{V}$, where $M_{pl}$ is the reduced Planck mass and $V$ is Slotheon scalar field potential. These are then compared with the thawing Dark Energy limits mentioned earlier to test the Swampland criteria for Slotheon Dark Energy model. We repeat this comparison for quintessence scalar field model and found that Slotheon Dark Energy model satisfies Swampland criteria better than the quintessence model.

In an earlier work L. Heisenberg et al [7] perform a similar test for Swampland criterion for scalar quintessence model. In that work they have considered a quintessence field and discussed about the Swampland criteria for the quintessence field Dark Energy. In doing so they have taken the experimental bounds by writing the variations of Dark Energy equation...
of state in the usual CPL \cite{11} parametrisation form and then translate it to obtain an upper bound of a reconstructed equation of state \( \omega(z) \sim \omega_0 + \frac{z}{1+z} \omega_a \). Here there are two parameters namely \( \omega_0 \) and \( \omega_a \). For this purpose they have taken the constraints on SNeIa, CMB, BAO and \( H_0 \) measurements data (Fig. 21 of Ref. \cite{12}). They have also repeated their analyses for Swampland criteria for comparing it by Euclid simulation of future data.

We have performed in this work a similar analysis of Swampland conditions in case of Slotheon field Dark Energy and standard quintessence field Dark Energy using Euclid simulated data but in our case we parametrise our Dark Energy equation of state with a generalised thawing Dark Energy model \cite{9}. We also repeat our analyses with the constraints used by L. Heisenberg e al \cite{7}.

We also mention here that in a recent work \cite{13} Brahma et al has done a similar study with the cubic Galileon term \( (\nabla \pi^2 \Box \pi) \) in their chosen Galileon action for Dark Energy. For the experimental bounds on Dark Energy equation of state they also consider (similar to that in Ref. \cite{7}) the CPL parametrisation. But in our work we explore the Swampland criterion for Slotheon field Dark Energy model. Also as mentioned earlier, for the experimental bounds we consider the Dark Energy equation of state in a generalised thawing model. Although both Galileon and Slotheon field arise from the DGP model (Dvali, Gabadadze, Porrati model \cite{14}) in its decoupling limit \( r_c \to \infty \) \cite{15, 16} \( r_c = \frac{M_{pl}^2}{2M_5^3} \), \( M_{pl} \) and \( M_5 \) are bulk and brane Plank masses respectively; \( r_c \) separates the 4-D and 5-D regimes), the Galileon field is described by a scalar field \( \pi \) from the DGP theory in Minkowski space time that obeys the shift symmetry \( \pi \to \pi + \alpha + b_\mu x^\mu \). The Slotheon field on the other hand arises when the Galileon transformation is generalised to curved space time and obeys the curved Galileon transformation \cite{4}

\[
\pi(x) \to \pi(x) + c + c_a \int_{x_0}^{x} \xi^a, \quad (1)
\]

where \( \xi^a \) is set of Killing vectors and \( x_0 \) is a reference point connected to \( x \) by a curve \( \gamma \) while \( c \) and \( c_a \) are a constant and a constant vector respectively.

This paper is organised as follows. In Section 2, we furnish the action for quintessence and Slotheon field and provide the necessary mathematical equations as also the dimensionless variables for both the cases that are required to calculate Dark Energy equation of state parameters of both the fields. Section 3 gives a brief account of the generalised thawing model as discussed in Ref. \cite{9} for choosing the Dark Energy equation of state parametrisation used in this work to obtain the variations of \( \omega(z) \) with \( z \) considering the experimental constraints. In Section 4 we furnish our calculations and results. We compute the variations of Dark Energy equation of state in the framework of Slotheon model for different values of \( \lambda = \frac{M_{pl}V'}{V} \) and compare them with those obtained using experimental constraints from Euclid simulated data \cite{10} and generalised thawing Dark Energy parametrisation. We also repeat the process for standard quintessence field of Dark Energy. Finally in Section 5 we give a summary and discussion.
2 Quintessence and Slotheon Fields

In this section we consider both the standard quintessence field $\phi$ and Slotheon field $\pi$ and calculate the equation of state of these fields.

**Quintessence Field**

The action of quintessence field is given as [17]

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{\text{pl}}^2 R - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right] + S_m .$$  \hspace{1cm} (2)

Where $M_{\text{pl}}$ is the reduced Planck mass, $g_{\mu\nu}$ is the metric while $g$ is the determinant of the metric and $R$ is Ricci scalar. In the above, $S_m$ is the action of standard matter field, $V(\phi)$ is the potential for the quintessence field $\phi$ and $\phi_{,\mu}$ denotes the covariant derivative of $\phi$.

By varying the action given in Eq. (2) with respect to the metric and $\phi$ respectively we obtain,

$$3M_{\text{pl}}^2 H^2 = \rho_m + \frac{\dot{\phi}^2}{2} + V(\phi) ,$$  \hspace{1cm} (3)

$$M_{\text{pl}}^2 (2\dot{H} + 3H^2) = -\frac{\dot{\phi}^2}{2} + V(\phi) ,$$  \hspace{1cm} (4)

$$\ddot{\phi} + 3H \dot{\phi} + V_{\phi} = 0 .$$  \hspace{1cm} (5)

In the above $\dot{A}, \ddot{A}$ denote derivative of $A$ w.r.t. time and double derivative of $A$ w.r.t. time respectively. Derivative of potential $V(\phi)$ w.r.t. $\phi$ is given as $V_{\phi}$ while $\rho_m$ denotes the matter energy density. In order to obtain the dynamics of the system it is convenient to introduce the following dimensionless variables,

$$x = \frac{\dot{\phi}}{\sqrt{6HM_{\text{pl}}}} ,$$  \hspace{1cm} (6)

$$y = \frac{\sqrt{V(\phi)}}{\sqrt{3HM_{\text{pl}}}} ,$$  \hspace{1cm} (7)

$$\lambda = -M_{\text{pl}} \frac{V_{\phi}}{V(\phi)} .$$  \hspace{1cm} (8)

With these, Eqs.(3)-(5) can be written as the following autonomous set of equations [18]

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2} \lambda y^2 + \frac{3}{2} x \left[ (1 - \omega_m)x^2 + (1 + \omega_m)(1 - y^2) \right] ,$$  \hspace{1cm} (9)

$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}} \lambda xy + \frac{3}{2} y \left[ (1 - \omega_m)x^2 + (1 + \omega_m)(1 - y^2) \right] ,$$  \hspace{1cm} (10)

$$\frac{d\lambda}{dN} = -\sqrt{6x} \lambda^2 \left( \frac{V_{\phi}}{V_{\phi}^2} - 1 \right) .$$  \hspace{1cm} (11)

Here $V_{\phi\phi}$ is the double derivative of $V(\phi)$ w.r.t. $\phi$, $\omega_m$ represents the equation of state for the matter field and $N = \ln a$ ($a$ is the scale factor of the Universe) is number of e-foldings.
Effective equation of state parameter $\omega_{\text{eff}}$ and equation of state parameter of Dark Energy $\omega_\phi$ for this system are obtained from Einstein’s equations (Eqs. 3 - 5) and given as

$$\omega_{\text{eff}} = \frac{p_{\text{total}}}{\rho_{\text{total}}} = \frac{p_m + p_\phi}{\rho_m + \rho_\phi} = -1 - \frac{2H}{3H^2}, \quad (12)$$

$$\omega_\phi = \frac{\omega_{\text{eff}}}{\Omega_\phi}. \quad (13)$$

In the above, density parameter $\Omega_\phi$ of the field $\phi$ is defined as $\Omega_\phi = \frac{\rho_\phi}{\rho_c}$ while $\rho_c$ is the critical density of the Universe and $\rho_\phi$ is the energy density of the quintessence field. Needless to mention that we obtain $\omega_{\text{eff}}$ and $\omega_\phi$ in terms of the dimensionless variables (Eqs. 6-8).

**Slotheon Field**

Slotheon field model is a scalar field model inspired by the theories of extra dimensions, which is a class of modified gravity models. Slotheon field model is followed from Dvali, Gabadadze and Porrati (DGP) model with one extra dimension.

Action of the Slotheon field $\pi$ is given as [3]

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( M_{\text{pl}}^2 R - \left( g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) \pi_{\mu} \pi_{\nu} \right) - V(\pi) \right] + S_m. \quad (14)$$

It can be noted from the above action that without the term $\frac{G^{\mu\nu}}{2M^2} \pi_{\mu} \pi_{\nu}$ in Eq. (14), both the actions of Eqs. (2) and (14) are identical. In the Slotheon action (Eq. (14)) $M$ represents an energy scale, $V(\pi)$ is the potential for Slotheon scalar field $\pi$, $G^{\mu\nu}$ denotes the Einstein’s tensor and all the other notations are same as, those in the standard quintessence scalar field case.

Einstein’s equations and equation of motion of Slotheon field $\pi$ are obtained by varying the action of Eq. (14) w.r.t. the metric $g^{\mu\nu}$ and $\pi$ respectively and are given as follows

$$3M_{\text{pl}}^2 H^2 = \rho_m + \frac{\dot{\pi}^2}{2} + \frac{9H^2 \dot{\pi}^2}{2M^2} + V(\pi), \quad (15)$$

$$M_{\text{pl}}^2 (2\dot{H} + 3H^2) = -\frac{\dot{\pi}^2}{2} + V(\pi) + (2\dot{H} + 3H^2) \frac{\dot{\pi}^2}{2M^2} + \frac{2H \dot{\pi} \ddot{\pi}}{M^2}, \quad (16)$$

$$0 = \ddot{\pi} + 3H \dot{\pi} + \frac{3H^2}{M^2} \left( \ddot{\pi} + 3H \dot{\pi} + \frac{2H \dot{\pi}}{H} \right) + V_\pi. \quad (17)$$

Here $V_\pi$ is the derivative of potential $V(\pi)$ w.r.t. $\pi$ and all the other symbols are same as in Eqs. (3 - 5).

Similar to the standard quintessence case, here too it is convenient to introduce some dimensionless variables to study the evolution of the system and the variables are defined as

$$x = \frac{\dot{\pi}}{\sqrt{6H} M_{\text{pl}}}, \quad (18)$$

$$y = \frac{\sqrt{V(\pi)}}{\sqrt{3H} M_{\text{pl}}}, \quad (19)$$

$$\lambda = -M_{\text{pl}} \frac{V_\pi}{V(\pi)}, \quad (20)$$

$$\epsilon = \frac{H^2}{2M^2}. \quad (21)$$
Using these dimensionless variables (Eqs. (18 - 21)) in the Eqs. (15 - 17), the following autonomous system of equations are constructed,

\[
\frac{dx}{dN} = \frac{P}{\sqrt{6}} - x \frac{\dot{H}}{H^2}, 
\]

\[
\frac{dy}{dN} = -y \left( \sqrt{\frac{3}{2}} \lambda x + \frac{\dot{H}}{H^2} \right), 
\]

\[
\frac{d\lambda}{dN} = -\sqrt{6}x \lambda^2 \left( \frac{VV_{\pi\pi'}}{V_{\pi}^2} - 1 \right), 
\]

\[
\frac{d\epsilon}{dN} = 2\epsilon \frac{\dot{H}}{H^2}. 
\]

In the above \( V_{\pi\pi} \) denotes the double derivative of potential \( V(\pi) \) w.r.t. \( \pi \) and

\[
P = \frac{3(12\sqrt{6}x^3\epsilon + y^2\lambda + \sqrt{6}x(-1 - 6\epsilon y^2))}{1 + 6\epsilon(1 + x^2(-1 + 18\epsilon))} 
\]

\[+ \frac{-18x^2y^2\epsilon\lambda}{1 + 6\epsilon(1 + x^2(-1 + 18\epsilon))}, \]

\[
\frac{\dot{H}}{H^2} = \frac{-3x^2(1 + 6\epsilon)(1 + 18\epsilon) + (1 + 6\epsilon)(-1 + 3y^2)}{2 + 12\epsilon(1 + x^2(-1 + 18\epsilon))} 
\]

\[+ \frac{12\sqrt{6}x\epsilon y^2\lambda}{2 + 12\epsilon(1 + x^2(-1 + 18\epsilon))}. \]

The effective equation of state parameter for Slotheon field is obtained from Eqs. (15) and (16) and is given as

\[
\omega_{\text{eff}} = -1 - \frac{2\dot{H}}{3H^2}, \]

which is similar to Eq. (12). Also the equation of state of Dark Energy \( \omega_{\pi} \) for Slotheon field \( \pi \) is defined as

\[
\omega_{\pi} = \frac{\omega_{\text{eff}}}{\Omega_{\pi}}, \]

where \( \Omega_{\pi} \) is the density parameter of the Slotheon field Dark Energy.

Therefore we obtain the equation of state parameters of standard quintessence model of Dark Energy (\( \omega_{\phi} \)) and Slotheon field Dark Energy model (\( \omega_{\pi} \)) in terms of the dimensionless variables defined in Eqs. (6 - 8) and Eqs. (18 - 21) respectively. The evolutions of \( \omega_{\phi} \) and \( \omega_{\pi} \) with red shift \( z \) can now be calculated by solving Eqs. (9 - 11) and Eqs. (22 - 25) respectively with proper initial conditions.

We consider exponential form of the potential \( V(X) \) as given below,

\[
V(X) = \exp \left( -\frac{CX}{M_{\text{pl}}} \right), \]

where \( X \equiv \phi, \pi \). From Eq. (8) or Eq. (20) it can be seen that for the exponential form of potential, \( \lambda = \frac{M_{\phi}V'}{V} = C \), while \( C \) is a constant. A discussion on its importance is given in Ref. [19].
3 Generalized Parametrisation of Thawing Dark Energy Model

We investigate in this paper the Swampland criteria for the case of Dark Energy from Slotheon scalar field model when confronted with the observational limits. We first obtain the observational limits for Dark Energy equation of state $\omega_{DE}$ in case of two parameter generalisation of different thawing Dark Energy models as given in Ref. [9]. The equation of state $\omega_{DE}$ for such a two parameter generalised thawing Dark Energy models take the form

$$\frac{d\omega_{DE}(a)}{da} = (1 + \omega_{DE}(a)) f(a), \quad (31)$$

where $f(a) = \frac{a}{a^n}$. In the above, $a$ represents the scale factor ($a = \frac{1}{1+z}$) and $c$, $n$ are the two parameters. It can be noted that for $c = 1$, $n = 1$ this generalised parametrisation can be reduced to CPL parametrisation form [11]. For $n \neq 1$, Eq. (31) can be reduced to the form

$$\omega_{DE}(a) = -1 + (1 + \omega_0) \exp \left[ \frac{c}{n-1} (1 - a^{(1-n)}) \right], \quad (32)$$

and it is straightforward to obtain from Eq. (31) that for $n = 1$

$$\omega_{DE}(a) = -1 + (1 + \omega_0) a^c. \quad (33)$$

In both the above two equations $\omega_0$ is equation of state in the present epoch.

We consider Euclid simulations for future data acquisition [10] and the values of $\omega_0$ in the above equations are adopted to be the different values of $\omega_0$ within 1-$\sigma$ range (Fig. 2.4 of Ref. [10]) as given in Euclid’s future simulations of data.

In the present work the region of variations of $\omega(z)$ with $z$ are obtained by using Eqs. (32, 33) within the range of values of the parameters $c$ and $n$ that obey the theoretical constraints as given in Fig. 2 of Ref. [9].

4 Calculations and Results

In this section we first calculate the variations of Dark Energy equation of state with the experimental constraints. As described in the last section, we adopt the generalised thawing Dark Energy parametrisation for obtaining the variations of Dark Energy equation of state with the parameter $\omega_0$ chosen from 1-$\sigma$ constraint as given in Euclid simulated results. We choose this as our $\omega_{DE}(z)$ vs $z$ with the 1-$\sigma$ upper bound for comparison with similar variations of Dark Energy equation of states with $z$ obtained from Slotheon model of Dark Energy with different values of $\lambda$. Similar computations are also performed for quintessence field Dark Energy equation of state and its agreement with Swampland criteria are tested to make a comparison between Slotheon Dark Energy model and quintessence model in terms of obeying the Swampland criteria. In this section we also compare our results of Slotheon field Dark Energy with the 1-$\sigma$, 2-$\sigma$, 3-$\sigma$ upper bounds of $\omega_{DE}$, constructed by CPL parametrisation with current cosmological results of Ref. [12] as given in Ref [7].
In Fig. 1 we furnish the variations of the upper bound of the equation of state $\omega_{DE}$ with $z$ when $\omega_0$ is within the 1-$\sigma$ range of the simulated future Euclid’s result and Eq. (32) and (33) are adopted for the equation of state with the values of the parameter $c$, $n$ within the theoretical constraints given in Ref. [9]. This 1-$\sigma$ range for $\omega_{DE}(z)$ vs $z$ is represented by the yellow band in Fig. 1. The band includes all allowed values of $c$ and $n$ as obtained from Fig. 2 of Ref. [9] with Eqs. (32) and (33) where $\omega_0$ is given by 1-$\sigma$ constraints of Euclid results. In Fig. 1 we also plot the calculated results of $\omega_{DE}(z)$ vs $z$ for standard Dark Energy quintessence model for four values of $\lambda$ namely $\lambda = 0.6, 0.8, 1, 1.2$. It may be mentioned here that $\omega_{DE}$ is nothing but $\omega_\phi$ for quintessence scalar field as mentioned in Section 2. From Fig. 1 it can be seen that for $\lambda = 0.6$ the quintessence model lies very much within the generalised thawing Dark Energy model region for 1-$\sigma$ range of $\omega_0$ as given by the Euclid simulation of future data. It is also seen from Fig. 1 that for $\lambda = 1$, the quintessence model does not quite satisfy the yellow region indicating that the standard quintessence model does not fully satisfy the Swampland criteria. The situation is even worse for $\lambda > 1$ for quintessence field Dark Energy. Therefore it appears from Fig. 1 that the general quintessence model barely satisfies the Swampland criteria when compared with the thawing Dark Energy model parametrisation with $\omega_0$ is adopted to be the values within 1-$\sigma$ region as obtained from the analysis of the simulated future Euclid data.

Figure 1: The variations of the upper bound of the equation of state $\omega_{DE}$ with $z$ when $\omega_0$ is within the 1-$\sigma$ range of the simulated future Euclid’s result and the variations of equation of state $\omega_{DE}$ with $z$ for standard Dark Energy quintessence model.

In Fig. 2 we make similar comparison for Slotheon field Dark Energy model in the context that Slotheon Dark Energy obeying the string Swampland criteria. In Fig. 2 the yellow region is the same as that in Fig. 1. We calculate the equation of state $\omega_{DE}(z)$ as it varies with $z$ for the case of Dark Energy from Slotheon scalar field model for four different values of $\lambda$ namely $\lambda = 0.6, 0.8, 1, 1.2$. Here for Slotheon scalar field model of Dark Energy, the equation of state $\omega_{DE}$ is the same as $\omega_\pi$ mentioned in Section 2. From Fig. 2 it is observed that for $\lambda = 0.8$ the Slotheon field model is well within the yellow region i.e., the generalised thawing Dark Energy model region with 1-$\sigma$ bound on $\omega_0$ (from Euclid). It
can also be noted from Fig. 2 that for $\lambda = 1$ the Slotheon model is marginally beyond the yellow region. From Fig. 1 and Fig. 2 it is clearly observed that the Slotheon field model better satisfies the Swampland criteria than the standard quintessence model and therefore the tensions with the Swampland criteria are less severe for Slotheon Dark Energy model than the general quintessence Dark Energy model.

We also compare our results for Slotheon field Dark Energy and standard quintessence field Dark Energy with the 1-$\sigma$, 2-$\sigma$ and 3-$\sigma$ upper bounds on equation of state $\omega_{DE}$, constructed by CPL parametrisation as given in Fig. 1 of Ref. [7] and plot them in Fig. 3(a). We show the variations of $\omega_{DE}$ with $z$ for Slotheon scalar field model in Fig. 3(a) for different values of $\lambda$, namely $\lambda = 0.8, 1, 1.2, 1.4, 1.6, 1.8$. It is observed from Fig 3(a) that for $\lambda = 1$ the equation of state for Slotheon field model is well below the 2-$\sigma$ and 3-$\sigma$ upper bound and therefore satisfies the Swampland criteria. It is noticed that even the equation of state for $\lambda = 1.8$ is below 3-$\sigma$ upper bound. Therefore the Slotheon Dark Energy model is fully in agreement with the string Swampland criteria when the current data (from Ref. [12]) as used in Ref. [7] is considered. It can also be noted from Fig. 3(a) (of this work) and Fig. 1 of Ref. [7] that Slotheon field Dark Energy better satisfies the string Swampland criteria than the general quintessence model of Dark Energy.
Figure 3: (a) Left panel: The Swampland criteria for the Slotheon field Dark Energy is explored for different values of $\lambda$ by computing the equation of state $\omega_{DE}(z)$ for different $z$ and comparing with the experimental 1-$\sigma$, 2-$\sigma$, 3-$\sigma$ upper bounds for the $\omega_{DE}(z)$ vs $z$ are adopted from Fig. 1 of Ref. [7] where the CPL parametrisation is used with experimental constraints obtained from SNeIa, CNB, BAO and $H_0$ measurements. (b) Right panel: Same as the left panel but using Euclid simulated constraints with CPL parametrisation as given in Fig. 2 of Ref. [7]

We now adopt the 1-$\sigma$ and 3-$\sigma$ upper bound of $\omega_{DE}(z)$ vs $z$ plots as obtained from CPL parametrisation from Euclid simulation data as given in Fig. 2 of Ref. [7] and compare them with the Slotheon Dark Energy equation of state for Swampland conditions. The results are furnished in Fig. 3(b). In Fig. 3(b) we show the variations of $\omega_{DE}$ with $z$ for Slotheon scalar field model for different values of $\lambda$, namely $\lambda = 0.3, 0.4, 0.5, 0.6, 0.8, 1$. It is observed from Fig. 3(b) that the plot corresponding to $\lambda = 0.5$ is well below the 3-$\sigma$ upper bound of $\omega_{DE}$ but the situations become worse when $\lambda > 0.5$. Therefore though the Slotheon field better satisfies the Swampland criteria than the general quintessence model but it appears to be in tension with string Swampland criteria if the experimental bound is adopted to be that obtained by CPL parametrisation of $\omega_{DE}(z)$ with Euclid bounds as given in Ref. [10]. On the other hand this can easily be observed from Fig. 2 and Fig. 3(b) that the agreement of Slotheon field with string Swampland criteria is better if generalised parametrisation of thawing Dark Energy model is adopted for the experimental bound instead of the CPL parametrisation of Dark Energy (in fact CPL parametrisation is already included within the yellow region of Fig. 1 and Fig. 2).

It can be mentioned here that to compute the plots of Fig. 2 and Fig. 3 we use the initial value of $\epsilon$ (Eq. (21)) is equal to $2.5\times10^7$. We have also observed that for higher initial values of $\epsilon$ (say $\epsilon = 4.5\times10^7, 6.5\times10^7$ etc.), Slotheon field model better satisfies string Swampland criteria and therefore it may be concluded that by increasing the initial values of $\epsilon$, the tension of string Swampland criteria for Slotheon Dark Energy field can further be reduced.
5 Summary and Discussions

The string Swampland conjectures lead us to investigate how the low energy effective field theories of general relativity of gravity which appear to work well below the Planck scale can be connected to the quantum theories of gravity in a string theory landscape which are theories beyond the Planck scale. The Swampland criteria give tight constraints to the Dark Energy models of the late time acceleration of the Universe as well as on inflationary models [20] of the early Universe. In this work we have studied the implication of string Swampland criteria on two scalar field models of Dark Energy, general quintessence and Slotheon model, on the basis of current and future cosmological observations. The scalar field models of Dark Energy have to satisfy specially the second Swampland criterion, which is in the context of the de Sitter constraint suggests that the slope of the potential of any effective scalar field theory should be related to the potential through a constant order one, to remain outside the Swampland. But it is observed that the general quintessence model leads to significant tension with this criterion in the view of current and future cosmological data. Therefore in the present work we study the Slotheon Dark Energy model which is inspired by extra dimensional theories in curved space time and explore whether it satisfies the string Swampland criteria.

In order to study the implications of string Swampland criteria on Slotheon Dark Energy model we calculate variations of Dark Energy equation of state $\omega_{DE}(z)$ with $z$ for Slotheon field with different values of $\lambda = \frac{M_0 V'}{V}$. We then compare it with experimental constraints by adopting generalised parametrisation of thawing Dark Energy models and by computing variations of $\omega_{DE}$ with parameter $\omega_0$ where the latter is chosen from 1-σ constraints as given in Euclid [10] simulated future cosmological results. Similar computations are also done for standard quintessence Dark Energy model. It is noted that Slotheon field better satisfies the string Swampland criterion than quintessence field.

In addition we also compare our results (of variations of $\omega_{DE}(z)$ with $z$ for Slotheon Dark Energy model) with 1-σ, 2-σ, 3-σ upper bounds on $\omega_{DE}$ constructed by CPL parametrisation as given in Fig. 1 and Fig. 2 of Ref. [7]. In Ref. [7] current data from Ref. [12] are used in Fig. 1 for the variations of $\omega_{DE}$ with $z$ and Euclid [10] simulated results are also used (in Fig. 2 of Ref. [7]) to construct experimental upper bound of $\omega_{DE}$. It is clearl from this work that Slotheon field Dark Energy is well in agreement with the Swampland criterion for current experimental constraints and it satisfies better the Swampland criterion than the quintessence field. It is also noted that for Euclid simulated future results, Slotheon Dark Energy model is more in agreement with the Swampland criteria for generalised parametrisation of thawing Dark Energy models than CPL parametrisation of Dark Energy. We also observe for Slotheon Dark Energy that by increasing the initial value of variable $\epsilon$ of Eq. [21] the tension with string Swampland criteria can further be relieved.

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