INFLATION FOR LARGE SCALE STRUCTURE

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Two extensions of ideas lying in the basis of the inflationary scenario of the early Universe and their effect on the large scale structure of the present-day Universe are discussed. The first of them is the possibility of fast phase transitions in physical fields other than an inflaton scalar field during inflation and not far from the end of it. This results in the appearance of specific features in the inflaton effective potential which, in turn, lead to the generation of localized spikes in the primordial perturbation spectrum. At present, there seems to exist one scale in the spectrum, \( k = 0.05h \, \text{Mpc}^{-1} \), around which we might see something of this type. The second one is the possibility that we are living at the beginning of a new inflation-like era now. Then observations of clustering of high-redshift objects can provide information sufficient for the unambiguous determination of the effective potential of a corresponding present inflaton scalar field.

1 Introduction

The main importance of the inflationary scenario of the early Universe for the theory of large scale structure in the present-day Universe is that the former scenario predicts (in its simplest realizations) an approximately flat, or scale-invariant, spectrum \( (n(k) \equiv d \ln P_0(k)/d \ln k \approx 1) \) of initial adiabatic perturbations. By the simplest realizations I mean, as usually, inflationary models with one effective slow-rolling scalar (inflaton) field. Of course, the physical nature of the inflaton may be completely different in these models, but it does not matter for observations, in particular, for the large scale structure. This prediction has been confirmed already, if by \( n(k) \) we understand the slope of the initial spectrum \( P_0(k) \) smoothed over the range \( \Delta \ln k \sim 1 \). To see this, it is even not necessary to use results for \( n \) following from the COBE experiment (though they also tell us the same), it is sufficient to compare the COBE normalization of perturbations for scales of the order of the present cosmological horizon \( R_h \) with the \( \sigma_8 \) normalization that follows, e.g., from the present cluster abundance. The difference in the amplitude of initial perturbations at these two scales which are divided by approximately 3 orders of magnitude is only \( 2 - 2.5 \) times for the pure CDM model and even less for other models, e.g., the \( \Lambda \)CDM model. In addition, these numbers give us an idea about the magnitude of expected deviations from the exact scale invariance: \( |n - 1| \leq 0.3 \) (once more, we are speaking about a smoothed \( n \)). Observational effects related to the part of \( P_0(k) \) between these two points (CMB temperature fluctuations at medium and small angles, galaxy-galaxy and cluster-cluster correlations, pe-
cular velocities of galaxies) also do not require larger smooth deviations from \( n = 1 \).

Still, it is well established now that the simplest cosmological model of the present Universe - the CDM model with the \( n \approx 1 \) initial spectrum of adiabatic perturbations (SCDM) - does not work. So, we have to go further and to introduce new elements (= new physics) into basic Lagrangians describing either inflation in the early Universe or the present dark matter content in the Universe. In the former case we change \( P_0(k) \). In the latter case we get a different dynamics of expansion of the Universe at recent time and change both a matter transfer function and a law of perturbation growth. Fortunately, the amount of required additional new physics can be parametrized by a few (1-2) new fundamental constants (in this respect, see the classification of cosmological models of the present Universe in [1]). Let me further discuss two interesting concrete possibilities. These new optional possibilities is what further development of inflationary ideas gives for the present-day cosmology and LSS - this explains the title of my talk.

2 How to produce steps and spikes in \( P_0(k) \)

The observational fact that the smoothed slope \( n \) cannot be significantly different from 1 does not exclude the possibility of local strong deviations from the flat spectrum, i.e., steps and/or spikes in \( P_0(k) \). Of course, one should not expect such a behaviour to be typical, we shall see below that if it happens at all, it occurs at some preferred scales which themselves become new fundamental parameters of a cosmological model. Do we have any observational evidence for an existence of such preferred scales at the Universe? At present, only one scale in the Fourier space, \( k = k_0 = 0.05 h \text{ Mpc}^{-1} \), remains a candidate for this role, and it seems that the spectrum is smooth for larger \( k \) (from galaxy-galaxy correlation data) and for smaller \( k \) (from CMB data). Here \( h \) is the present Hubble constant in terms of 100 km/s/Mpc. As for this scale itself, there exists an evidence for a peculiar behaviour (in the form of a sharp peak) in the Fourier power spectrum of rich Abell - ACO clusters (with richness class \( R \geq 0 \) and redshifts \( z \leq 0.12 \)) around it [2]. This anomalous behaviour persists if the distant border for the cluster sample is reduced to \( z = 0.07 - 0.08 \). If we assume that the cluster power spectrum is proportional to the power spectrum of the whole matter in the Universe (with some constant biasing factor), and calculate the corresponding rms multipole values \( C_l \) of angular fluctuations of the CMB temperature, they appear to be in a good agreement with existing results of medium-angle experiments [3]. Moreover, if \( \Omega_m = 1 \), the peak in the power spectrum inferred from the cluster data just explains an excess in \( C_l \) for
l = 200 – 300 observed in the Saskatoon experiment.

On the other hand, there is no peak at $k = k_0$ in the power spectrum of both APM clusters (which are generally less rich than Abell-ACO clusters) and APM galaxies (though some less prominent feature at this scale may still exist in the latter spectrum), and the maximum in these spectra seems to be shifted to $k \sim 0.03h$ Mpc$^{-1}$. Leaving a solution of this discrepancy to more complete future surveys, let us consider theoretical predictions.

It is possible to produce local features in the initial spectrum even remaining (at least, formally) inside the standard paradigm of one-field inflation. The only thing which should be relaxed is the requirement of the analyticity of an inflaton effective potential $V(\varphi)$ at all points. So, let me admit that $V(\varphi)$ has some kind of discontinuity at a point $\varphi = \varphi_0$. Of course, really this discontinuity is smoothed in a very small vicinity of $\varphi_0$. Three cases are the most interesting.

1. $[V] = [V'] = 0$, $[V''] \neq 0$ at $\varphi = \varphi_0$.

Here $[\ ]$ means the jump in the quantity considered, namely, $[A] \equiv A(\varphi_0 + 0) - A(\varphi_0 - 0)$, and the prime denotes the derivative with respect to $\varphi$. If we assume that the slow-roll conditions $V'' \ll 48\pi G V^2$, $|V'| \ll 24\pi G V$ are satisfied near the point $\varphi = \varphi_0$ (here and below $c = \hbar = 1$), then in the zero-order approximation the standard result for a perturbation spectrum is valid:

$$P_0(k) = \frac{k^4 R_h^2(t)}{400} h^2(k), \quad k^3h^2(k) = 18 \left( \frac{H^6}{V'^2} \right)_k, \quad H \equiv \frac{\dot{a}}{a} \approx \sqrt{\frac{8\pi G V}{3}}, \quad (1)$$

where the index $k$ means that the quantity is evaluated at the moment of the first Hubble radius crossing ($k = aH$) at the inflationary stage. The result for $P_0(k)$, in contrast to the metric perturbation $h^2(k)$ defined in the ultra-synchronous gauge ($h$ is equal to $1/3$ of the trace of a spatial metric perturbation in this gauge, see, e.g., the original paper), refers to the matter-dominated stage where $R_h(t) = 2/H = 3t$. Note also that there is no necessity in adding the multiplier $O(1)$ here.

So, in this case $P_0(k)$ is continuous at $k = k_0$ but its slope $n(k)$ has a step-like behaviour there (similar to the case considered in [3]). However, due to small corrections to Eq. (1), which are beyond the slow-roll approximation, it appears that $n$ cannot be obtained simply by differentiating (1), and I expect that the sharp behaviour in $n$ will be smoothed near $k_0$. This question is still under consideration.

2. $[V] = 0$, $[V'] \neq 0$ at $\varphi = \varphi_0$.

Now the second of the slow-roll conditions is violated, while we can choose parameters of the jump in such a way that the first condition is still valid.
Naive application of Eq.(1) would give a step in \( P_0(k) \). However, the slow-roll approximation is clearly not applicable. The exact solution for a local part of the spectrum near the point \( k_0 \) was obtained in \( 9 \). It reads:

\[
k^3 h^2(k) = \frac{18 H_0^6}{V_+''} D^2(y), \quad H_0 = \sqrt{\frac{8 \pi G V(\varphi_0)}{3}}, \quad V_+'' = V'(\varphi_0 \pm 0) > 0, \quad y = \frac{k}{k_0},
\]

\[
D^2(y) = 1 - 3 \left( \frac{V'}{V_+''} - 1 \right) \frac{1}{y} \left( \frac{1 - \frac{1}{y}}{1 + \frac{1}{y^2}} \right) \sin 2y + \frac{2}{y} \cos 2y \tag{2}
\]

\[
+ \frac{9}{2} \left( \frac{V'}{V_+''} - 1 \right)^2 \frac{1}{y^2} \left( 1 + \frac{1}{y^2} \right) \left( 1 + \frac{1}{y^2} + \left( 1 - \frac{1}{y^2} \right) \cos 2y - \frac{2}{y} \sin 2y \right).
\]

The function \( D^2(y) \) has a step-like behaviour with superimposed oscillations. Since \( D(0) = V'/V_+'' \), \( D(\infty) = 1 \), the spectrum approaches the flat spectrum if \( |\ln(k/k_0)| \gg 1 \). As compared with the flat spectrum, the spectrum (2) has more power at large scales (small \( k \)) if \( V'_- > V_+'' \), and more power on small scales in the opposite case. The shape of this function is universal (in the sense that it does not depend on a way of smoothing the jump in \( V' \), if it is made in a sufficiently small vicinity of \( \varphi_0 \)), it depends on the ratio \( V'/V_+'' \) only.

3. \( [V] \neq 0 \) at \( \varphi = \varphi_0 \).

In this case, there is no universal spectrum, and the answer depends on a concrete way of smoothing \( V(\varphi) \) at \( \varphi = \varphi_0 \). Some general results are presented in \( 9 \), typically \( P_0(k) \) acquires a large bump, however, a well may appear, too. Fortunately, there is no need in further consideration of this more complicated case, since observational data do not require such a strong non-analyticity. Looking at the results presented in \( 9 \), it is clear that they lie somewhere between the first two cases.

Therefore, a general lesson from these considerations is that peculiarities in \( V(\varphi) \) can produce local features in \( P_0(k) \) where the slope \( n \) is significantly different from unity. However, these features cannot be too sharp, in particular, both \( P_0(k) \) and \( n(k) \) are expected to be continuous functions of \( k \).

So far, the treatment of peculiar points was purely mathematical. However, if we are seeking for a physical explanation of such behaviour of \( V(\varphi) \), we have to go beyond the paradigm of one-field inflation to a more complicated case of two-field inflation. Inflation with two scalar field is a very rich physical model which includes double inflation, hybrid inflation, open inflation, etc. as specific cases. In our case it is sufficient to assume that the second scalar field \( \chi \), in contrast to the inflaton field \( \varphi \), is always in the regime \( |m_\chi^2| \gg H^2 \). So, it is not dynamically important during the whole inflation. However, it is coupled to \( \varphi \) (e.g., through the term \( g^2 \varphi^2 \chi^2 \)) and, as a result of change in \( \varphi \) during inflation,
the field $\chi$ experiences a fast phase transition approximately 60 e-folds before the end of inflation. If parameters of an interaction potential $V(\varphi, \chi)$ are such that the phase transition may be considered as an equilibrium one, then its net effect on inflation appears in the change of the equilibrium effective potential $V_{\text{eff}}(\varphi) \equiv V(\varphi, \chi_{\text{eq}}(\varphi))$ only. If the transition is a second-order one, with no jump in $\chi_{\text{eq}}$, the first case considered above takes place. If the transition is a first-order one, with a non-zero jump in $\chi_{\text{eq}}$, we arrive to the second case.

3 How to determine a variable cosmological term from observations

There exists a growing amount of evidence that the total energy density of matter in the Universe including baryonic and nonbaryonic dark matter is significantly less than unity ($\Omega_m \sim 0.3 - 0.4$). The most recent argument for this conclusion is based on the evolution of abundance of rich galaxy clusters with redshift $z$. Of course, as usually in modern cosmology, observational papers with the opposite conclusion have appeared almost immediately (once more, I leave the solution of this dilemma for future). A natural way to incorporate $\Omega_m < 1$ without abandoning the paradigm of one-field inflation is to assume the existence of a positive effective cosmological constant with the energy density (in terms of the critical one) $\Omega_\Lambda = 1 - \Omega_m$. This leads to the flat $\Lambda$CDM cosmological model with the $n \approx 1$ initial spectrum of adiabatic perturbations which is in a good agreement with all existing observational data for a rather large region in the space of parameters $H_0, \Omega_m$.

It is clear that the introduction of a cosmological constant requires new and completely unknown physics in the region of ultra-low energies. If we try to describe it phenomenologically by the same kind of physics which was so successively used in the paradigm of one-field inflation, namely, by a scalar field with some interaction potential $V(\varphi)$ minimally coupled to the Einstein gravity, then the conclusion is that generally this "constant" may be weakly time-dependent. Models with a time-dependent cosmological constant were introduced ten years ago, and different potentials $V(\varphi)$ (all inspired by inflationary models) were considered: exponential, inverse power-law, power-law, cosine.

However, it is clear that since we know essentially nothing about physics at such energies, there is no preferred theoretical candidate for $V(\varphi)$. On the other hand, using the cluster abundance $n(z)$ determined from observations and assuming the Gaussian statistics of initial perturbations (the latter follows from the paradigm of one-field inflation, too, and is in agreement with other observational data), it is possible to determine $\delta(z)$ - the time evolution of the linear density contrast in the CDM component for a fixed comoving scale.
\[ R \sim 8(1+z)^{-1}h^{-1}\text{Mpc up to } z \sim 1, \text{ using the Press-Schechter approximation.} \]

It can be shown that the knowledge of \( \delta(z) \) uniquely determines the form of the effective potential \( V(\varphi) \) required for this model (simultaneously, the present value of \( \Omega_m \) is determined from the same data, too). Therefore, I propose not to introduce \( V(\varphi) \) by hand, but to find it from data on clustering of high-redshift objects. Details will be published separately. Then a completely independent test of the model is provided by CMB temperature anisotropies.

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