Identification and influence factors analysis of blade crack mistuning in hard-coated blisk based on modified component mode mistuning reduced-order model

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Abstract
Blade crack will cause severe mistuning of hard-coated blisks, which will lead to vibration localization. To identify crack mistuning and analyze influence factors, in this study, a mistuning identification method of blade cracks in hard-coated blisks is presented based on modified component mode mistuning reduced-order model, in which the hard-coated blisk with blade crack is decomposed into a substructure of tuned hard-coated blisk and a substructure of coated blade with cracks. Crack mistuning of each coated blade can be obtained by a single identification calculation. After verifying the rationality of this identification method, the influence factors of blade crack mistuning are analyzed. The influence factors include the crack location on the coated blade (cracks occurring only in coating or only in blade substrate or both in blade substrate and coating), crack length, crack position in the radial direction of the blisk, and modal data type of coated blisk used for mistuning identification calculation. The research results show that, with the increase of crack length, the mistuning of crack occurring only in the coating does not increase continuously but decreases firstly and then increases. For the first bending modes, the closer the blade crack is to the blade root, the larger the mistuning is. For the second bending modes, the blade crack located at the position of maximum modal displacement will produce large mistuning. For hard-coated blisk with blade crack, these crack mistuning variation rules are of great significance to the dynamic analysis and the determination of the crack location.

Keywords
Blade crack, mistuning identification, reduced-order model, hard-coated blisk, influence factors analysis

Introduction
As one of the core components of modern aeroengines, the blisk (integral bladed disk) is exposed to a harsh working environment of high temperature, high pressure, and high rotation speed for a long time.¹,² Under the combined action of shocks and loads of centrifugal, aerodynamic, and thermal, the blisk will produce high-temperature creep, corrosion, high-frequency fatigue damage, and other failure forms.³,⁴ In order to improve service life, the hard coatings with vibration reduction, heat insulation, wear resistance, corrosion resistance, and other functions have been widely used on the blisk.⁵–⁸ As an important form of high-frequency fatigue failure, blade cracks will lead to vibration localization of the hard-coated blisk.⁹–¹² The intensification of vibration will accelerate crack propagation, which is likely to cause catastrophic failure.¹³–¹⁵ Therefore, for the cracked blisk or
bladed disk, the researches on the dynamics and the crack location method have been widely concerned. The crack on the coated blade will cause severe mistuning of the hard-coated blisk, which is the important cause of vibration localization. However, because the hard coating material and the blisk substrate material are usually different, the hard-coated blisk is a kind of composite structure. For the hard-coated blade, the cracks may occur only in the coating, or only in the blade substrate, or both in coating and blade substrate. Due to different mechanical properties of coating material and blade substrate material, the mistuning caused by the cracks occurring at different parts of the coated blade will also be different. Moreover, with the change of the crack length and the crack position in the radial direction of the blisk, these blade crack mistuning will also change accordingly. Therefore, compared with the blisk of a single material, the variation rules of blade crack mistuning on hard-coated blisk are more complicated. In order to more accurately study the vibration location and other dynamic behaviors of hard-coated blisks with blade cracks, for the hard-coated blisk, it is necessary to investigate blade crack mistuning identification method and crack mistuning influence factors. At the same time, the obtained variation rules of blade crack mistuning can also provide reliable reference data for the determination of crack location and crack length.

In recent years, researches on the blade crack mistuning of the bladed disk or blisk have been carried out. In some researches, the blade crack mistuning is indirectly investigated by analyzing the vibration localization of the bladed disk or blisk with blade crack. For example, Kuang and Huang\textsuperscript{16–18} approximated blades on the disk as Euler Bernoulli beams and the Galerkin method to derive the equation of motion of the mistuned system with the blade crack for investigating the effects of blade crack on mode localization in rotating bladed disks. Fang et al.\textsuperscript{19} developed analytical solutions for the free and forced vibrations of the bladed disk with a single crack and applied the $U$-transformation approach to study the effect of a crack on the vibratory response of a simplified aeroengine bladed disk model. Wang et al.\textsuperscript{20} investigated the effects of multiple cracks on the forced response of centrifugal impellers using a finite element-based component mode synthesis (CMS) method. Zeng et al.\textsuperscript{21} introduced typical fatigue cracks into the ANSYS finite element model (FEM) of a rotating compressor blade to discuss the effects of angular acceleration, amplitude of aerodynamic force, and crack parameters on the dynamic characteristics of a cracked compressor blade during run-up process. Tien et al.\textsuperscript{22} modified and combined the X-Xr method and the generalized bilinear amplitude approximation technique to present a technique for analyzing the dynamics of mistuned bladed disks with cracks, and the influence of mistuning patterns and cracks on the vibrational response of the bladed disk was discussed. Saito et al.\textsuperscript{23} employed a hybrid-interface method of CMS to generate a reduced-order model (ROM), in which the crack surfaces were retained as physical degrees of freedom. Using this reduced-order modeling and analysis framework, the effects of the cracked blade on the system response of an example rotor were investigated for various mistuning levels and rotation speeds. Wang et al.\textsuperscript{24} employed a hybrid-interface method of CMS to generate an ROM for the cracked impeller and investigated the effects of mistuning and cracks on the vibration features of centrifugal impellers. Shukla and Harsha\textsuperscript{25} used the finite element method and experimental modal analysis technique to understand the vibration behavior of the blade for the varying sizes of cracks. In addition, other researchers directly studied the crack mistuning of the bladed disk or blisk. For example, Hou\textsuperscript{26} formulated an analytical model for a bladed disk with a through-crack at the root using lumped-mass beams and studied the mechanisms of cracking-induced mistuning in bladed disks. Jung et al.\textsuperscript{27} employed a hybrid-interface method based on CMS to develop ROMs for the presence of mistuning in the tuned system with a cracked blade, and the effects of the cracked blade on the mistuned system were investigated. Zhang et al.\textsuperscript{28} performed an experimental and numerical investigation of the crack blade and another noncrack blade, and results of natural frequencies and mode shapes of both blades have been compared to identify the main differences in modal behavior when a crack appears. Sun et al.\textsuperscript{29} further investigated the cracking elements method (CEM) in simulating complex crack growth, regarding propagations of existed cracks as well as initiations of new cracks. Salmi et al.\textsuperscript{30} used the extended finite element method to investigate the stress intensity factors of 3D cracks. Through the studies of the above literature, it is found that most of the researches reflect the mistuning effect generated by the blade crack based on the analysis of the vibration location of the blisk or bladed disk. Although a few studies have been carried out focused directly on the blade crack mistuning, there is no method to identify or evaluate blade crack mistuning in each sector of the blisk or bladed disk. Moreover, the research objects of all the above researches are the blisk or bladed disk of a single material, and the researches on blade crack mistuning identification and analysis of mistuning influence factors for hard-coated blisk have not been carried out yet.

Compared with the structure of a single material, the dynamic characteristics of composite structure are more complex. In recent years, studies on the dynamics of composite structures are also carried out. For example, Zhang et al.\textsuperscript{31,32} utilized the third-order shear deformation plate theory, von Karman geometry nonlinearity, and
Hamilton’s principle to study the nonlinear dynamics of the rotating laminated composite cantilever rectangular plate and the rotating tapered cantilever cylindrical panel with the graphene coating layers. Bai et al. 33 proposed a new methodology called extremum response surface method-based improved substructural component modal synthesis to improve the computational efficiency of vibration characteristics and reliability analysis for a detailed numerical model of the mistuned turbine bladed disk. Niu et al. 34 investigated the free vibrations of the rotating pretwisted functionally graded (FG) composite cylindrical panels reinforced with the graphene platelets (GPLs) based on the first-order shear deformation theory (FSDT), Chebyshev-Ritz method. Yao et al. 35–37 investigated nonlinear vibrations of the blade with varying rotating speed, nonlinear dynamics of the high-speed rotating plate, nonlinear oscillations, and resonant responses of a compressor blade based on the FSDT, von-Karman nonlinear geometric relationship and Hamilton’s principle. Wang and Zhang 38 employed Bolotin method and multiple timescale method to discuss the stability of a spinning blade having periodically time-varying coefficients for both linear model and geometric nonlinear model. He 39 adopted a semi-inverse method to search for the variational formulation from the governing equations and obtained a few new variational principles for the 3D unsteady flow, which can avoid the Lagrange crisis. Then, He and Sun 40 used the semi-inverse method to the establishment of a variational formulation for the thin film equation and given a detailed derivation process. Li et al. 41 established a damping model of fiber-reinforced composite thin plate with consideration of amplitude-dependent property using the Jones-Nelson nonlinear theory in conjunction with the classical laminated plate theory, polynomial fitting method, and strain energy method. Qin et al. 42 proposed a unified method to analyze free vibrations of laminated FG shallow shells reinforced by GPLs under arbitrary boundary conditions based on the FSDT, artificial spring technique, and Rayleigh Ritz method. The above researches are of great reference significance to this paper.

In order to study the blade crack mistuning of hard-coated blisk, a mistuning identification method for coated blade crack mistuning is urgently needed. Among the previous mistuning identification methods of the blisk or bladed disk, the mistuning identification methods based on finite element ROMs have been widely studied. One kind of method is the mistuning identification method of integral structure-based subset of nominal modes ROM. 43 This kind of method also includes the mistuning identification methods based on the fundamental model of mistuning ROM 44,45 and the modified modal domain analysis ROM. 46 The other kind of method is the mistuning identification method of component separate based CMS ROM. 47,48 The mistuning identification method based on component mode mistuning (CMM) ROM 49 also belongs to this category. Among the above mistuning identification methods, the mistuning identification method based on CMM ROM can identify various mistuning of the blade, such as small mistuning, large mistuning, and geometric mistuning. Therefore, this identification method has strong applicability and has been widely used. 50–53 Considering the advantages of the CMM mistuning identification method and it focuses on blade mistuning, the identification concept of this method is also used for reference in this study. However, since this method has not been applied to the identification of blade crack mistuning in hard-coated blisk before, the classical identification method based on CMM ROM needs to be improved accordingly in this study.

In this paper, the simplified hard-coated blisk with blade cracks is taken as the research object. A mistuning identification method of blade cracks for hard-coated blisks is presented. Meanwhile, the influence factors of blade crack mistuning are analyzed based on this identification method. This study is organized as follows. In the “Theory” section, the improvement of the classical CMM ROM is introduced, in which the hard-coated blisk with blade crack is decomposed into a substructure of tuned hard-coated blisk and a substructure of the coated blade with cracks. And then, the theoretical derivation of the identification algorithm is completed. In the “Numerical verification for effectiveness of identification method” section, a numerical case is used to verify the rationality of the identification method. In the “Influence factors analysis of crack mistuning” section, the influence factors of crack mistuning are studied based on the identification method proposed in the “Theory” section. The mistuning variation rules of three kinds of cracks (cracks occurring only in coating or cracks occurring only in blade substrate or cracks occurring both in hard coating and blade substrate) are compared and analyzed. For each kind of crack, the mistuning variation rules with the variation of crack lengths and the mistuning variation rules with the variation of location in the radial direction of blisk are all studied. Moreover, the variation rules of crack mistuning calculated by the first and second bending mode family data are also analyzed. Finally, some conclusions are listed in the “Conclusions” section.

Theory

In this study, the modal data of the hard-coated blisk with blade cracks are expected to be used to identify the blade crack mistuning. Therefore, the undamped free vibration state of the hard-coated blisk with blade cracks is
taken as the research object. Firstly, an undamped free vibration model of the hard-coated blisk with blade cracks is established based on the finite element method. Then, the FEM is reduced according to the reduced-order principle of the CMM ROM. Finally, the calculation formula of blade crack mistuning is obtained according to the reduced-order FEM.

Based on the finite element method, the free vibration frequency domain equation of the hard-coated blisk with blade cracks can be expressed as

$$(-\omega^2 M + K)\Phi = 0$$

where $M$ and $K$ are, respectively, the mass and stiffness matrices of the coated blisk with blade cracks, $\omega$ is the free vibration frequency of the coated blisk with blade cracks, and $\Phi$ is the displacement vector of free vibration response of the coated blisk with blade cracks.

Since the response of a bladed disk is much more sensitive to mistuning in blades than that in the disk, only blade mistuning is considered in CMM ROM, and the mistuned system is represented by the full tuned system and virtual mistuning components. For the hard-coated blisk with blade crack, CMM ROM is improved, that is, the hard-coated blisk with blade crack is decomposed into a substructure of tuned hard-coated blisk and a substructure of the coated blade with cracks. The substructure decomposition method is shown in Figure 1. Herein, blade cracks may occur only in the hard coating or only in the blade substrate or both in the hard coating and the blade substrate (as shown in Figure 2).

In this paper, the mistuning caused by cracks of the coated blade is studied. Because the mass of the coated blade is not significantly changed by the crack, the mass mistuning is not considered in this study. According to

![Figure 1. Substructuring of a hard-coated blisk with blade crack.](image1)

![Figure 2. Cracks occur in different parts of coated blade.](image2)
the substructure decomposition in Figure 1, $K$ can be written as

$$K = K^{ca} + K^{cr}$$  \hspace{1cm} (2)

where $K^{ca}$ is the stiffness matrix of tuned coated blisk, and $K^{cr}$ is the mistuned stiffness matrix caused by cracks. Then, equation (1) becomes

$$(-\omega^2 M + K^{ca} + K^{cr})\Phi = 0$$  \hspace{1cm} (3)

The response of coated blisk with blade cracks in the tuned modal space can be expressed as

$$\Phi = \Theta \Psi$$  \hspace{1cm} (4)

where $\Theta$ is the mass normalized mode-shape matrix of the tuned coated blisk, and $\Psi$ is the modal coordinates vector. Substituting equation (4) into equation (3), it is converted into

$$(-\omega^2 \Theta^T M \Theta + \Theta^T K^{ca} \Theta + \Theta^T K^{cr} \Theta)\Psi = 0$$  \hspace{1cm} (5)

where $\Theta^T M \Theta = I$, $\Theta^T K^{ca} \Theta = \Omega$, and $I$ is the identity matrix; $\Omega$ is the generalized stiffness matrix of tuned coated blisk. So equation (5) can be further expressed as

$$(-\omega^2 I + \Omega + \Theta^T K^{cr} \Theta)\Psi = 0$$  \hspace{1cm} (6)

Since only the mistuning caused by coated blade cracks is considered in this study, the mistuning is considered to be only located on the coated blades according to the CMM ROM. Then, $\Theta^T K^{cr} \Theta$ can be expressed as

$$\Theta^T K^{cr} \Theta \approx \sum_{j=1}^{J} (\Theta_j)^T K_j^{cr} \Theta_j$$  \hspace{1cm} (7)

where $j = 1, 2, \cdots, J$, $J$ is the total number of coated blades, $K_j^{cr}$ is the crack mistuned stiffness matrix of $j$th coated blade, and $\Theta_j$ is the mode-shape matrix of the $j$th blade of tuned coated blisk and can be defined as

$$\Theta_j = \Theta^{cb} \Gamma_j$$  \hspace{1cm} (8)

where $\Theta^{cb}$ is the mass normalized mode-shape matrix of the tuned cantilevered coated blade, and $\Gamma_j$ is the modal participation factor of the $j$th blade of tuned coated blisk.

The following formula can be obtained from equation (8)

$$\Theta^{cb} K^{cb} \Theta = \Theta^{cb} K^{cr} \Theta^{cb} \Gamma_j$$  \hspace{1cm} (9)

where $K^{cb}$ is the stiffness matrix of the tuned cantilevered coated blade.

In equation (9), $\Theta^{cb} K^{cb} \Theta = \Omega^{cb}$, where $\Omega^{cb}$ is the generalized stiffness matrix of the tuned cantilevered coated blade.

Then, $\Gamma_j$ can be computed by the following formula

$$\Gamma_j = (\Omega^{cb})^{-1} \Theta^{cb} K^{cb} \Theta_j$$  \hspace{1cm} (10)

Therefore, $\Theta^T K^{cr} \Theta$ can be written as

$$\Theta^T K^{cr} \Theta \approx \sum_{j=1}^{J} (\Gamma_j)^T (\Theta^{cb})^T K_j^{cr} \Theta^{cb} \Gamma_j$$  \hspace{1cm} (11)
According to the further reduction mode of CMM ROM, the off-diagonal terms representing the coupling between cantilevered-blade modes can be neglected. In equation (11), \( (\Theta^{cb})^T K^{cr} \Theta^{cb} \) can be written as

\[
(\Theta^{cb})^T K^{cr} \Theta^{cb} \approx \text{Diag}_{n \in N}(k_{jn})
\]

(12)

where \( k_{jn} \) is the frequency eigenvalue deviations relative to tuned cantilevered coated blade caused by stiffness mistuning of \( j \)-th coated blade with crack, \( N \) is the number of the retained cantilevered-blade normal mode orders, and \( n = 1, 2, \ldots, N \).

In this study, \( k_{jn} \) is used as the mistuning identification parameter to quantify the stiffness mistuning produced by the crack.

Then, \( \Theta^T K^{cr} \Theta \) can be further expressed as

\[
\Theta^T K^{cr} \Theta = \sum_{j=1}^{J} (\Gamma_j)^T [\text{Diag}_{n \in N}(k_{jn})] \Gamma_j
\]

Therefore, equation (6) can be written as

\[
\left\{ \sum_{j=1}^{J} (\Gamma_j)^T [\text{Diag}_{n \in N}(k_{jn})] \Gamma_j \right\} \Psi = (-\omega^2 I + \Omega) \Psi
\]

(14)

Further, the parameter \( k_{jn} \) can be computed by

\[
\begin{bmatrix}
(\Gamma_{1,1})^T \Psi \\
(\Gamma_{1,2})^T \Gamma_{1,2} \Psi \\
\vdots \\
(\Gamma_{j,n})^T \Gamma_{j,n} \Psi \\
\vdots \\
(\Gamma_{J,N})^T \Gamma_{J,N} \Psi
\end{bmatrix} = (-\omega^2 I - \Omega) \Psi
\]

(15)

where \( \Gamma_{j,n} \) is the \( n \)-th dominating modal participation factor for the \( j \)-th blade of tuned coated blisk, that is, it is the \( n \)-th row of \( \Gamma_j \).

Since equation (15) is derived from the free vibration equation, the modal data of the coated blisk with blade cracks can be used to calculate the identification parameter \( k_{jn} \).

**Numerical verification for effectiveness of identification method**

In this section, the blade crack mistuning of two numerical cases is identified by the identification algorithm proposed in “Theory” section. The two numerical cases are two FEMs of hard-coated blisk with blade cracks established by ANSYS software. Modal data are obtained by modal analysis of two FEMs. The blade crack mistuning is calculated through the modal data of the two numerical hard-coated blisks with blade cracks. In the two numerical cases, except for the difference in crack length, the geometry structures of coatings, blade, and disk on each sector are the same, and the distances between the cracks on each blade and the center of the coated blisk are the same. Therefore, the change of crack mistuning is only related to the crack length. Furthermore, the effectiveness of the identification method is verified by comparing the variation trend of the mistuning identification results and that of the crack lengths, that is, the identification method is effective if the two change trends have a good consistency.

**Crack mistuning identification**

In order to verify the effectiveness of the identification method, two numerical hard-coated blisks are used. The numerical hard-coated blisks all have eight blades. The upper and lower surfaces of the blade are all coated with hard coating. The numerical hard-coated blisks are shown in Figure 3, where the coatings and
blade substrate are penetrated by blade cracks in the Z-direction. The distances between the cracks on each blade and the center of the hard-coated blisk ($L_1$) are all 107.5 mm. The widths of the cracks in the radial direction of the hard-coated blisk ($L_2$) are all 0.01 mm. The lengths of the cracks on each blade in the circumferential direction of the hard-coated blisk ($L_3$) are different and listed in Table 1. The crack lengths distribution of the numerical coated blisk 1 is random, and the crack lengths of the numerical coated blisk 2 increase linearly with the change of the blade number. The purpose of formulating these two crack distribution schemes is to verify the rationality of identification methods more fully. The material property parameters of the blisk substrate and hard coating are listed in Table 2. The geometry dimensions of the corresponding tuned hard-coated blisk are shown in Table 3. Herein, the numerical hard-coated blisks are created by ANSYS software. The upper and lower surfaces of disk lug boss are completely constrained, and modal analyses are carried out. Then, the natural frequencies and modal displacements of the coated blades are extracted. The constraints of numerical hard-coated blisks and the locations of the vibration pickup points are shown in Figure 4.

In order to complete the mistuning identification, the modal data of the tuned structures are needed. The tuned structures include a tuned cantilever coated blade and a tuned hard-coated blisk. According to the geometries of Table 3, FEMs of the tuned structures are created and shown in Figure 5. The end face of the tuned cantilever coated blade is completely restrained, and upper and lower surfaces of the lug boss of tuned hard-coated blisk are also completely restrained. Then, the modal analyses of the two tuned structures are performed. The natural frequencies and modal shapes of the tuned cantilever coated blade and the tuned hard-coated blisk are extracted. Finally, according to the identification algorithm proposed in “Theory” section, the crack mistuning of the coated blades of the numerical hard-coated blisks is calculated. Herein, the modal data of the 1–4 modal families

![Figure 3. Numerical hard-coated blisks with cracks in coated blades.](image)

| Blade number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------|---|---|---|---|---|---|---|---|
| Numerical hard-coated blisk 1 (mm) | 8 | 6 | 6 | 8 | 10 | 12 | 10 | 12 |
| Numerical hard-coated blisk 2 (mm) | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |

Table 2. Material property parameters of hard-coated blisk.

| Structure      | Material                | Density (kg/m³) | Modulus of elasticity (GPa) | Poisson’s ratio |
|----------------|-------------------------|-----------------|-----------------------------|----------------|
| Blisk substrate| Steel 45                | 7890            | 209                         | 0.265          |
| Hard coating   | NiCoCrAlY + YSZ         | 5600            | 54.49                       | 0.3            |

YSZ: yttria-stabilized zirconia.
of the cracked hard-coated blisk are selected for identification calculation. The 1–4 modes include the first bending mode family, the first swing mode family, the second bending mode family, and the first torsion mode family. Their natural frequencies are shown in Figure 6, and the mode shapes in each mode family are shown in Figure 7. The identification results of crack mistuning of numerical hard-coated blisk 1 and numerical hard-coated blisk 2 ($k_{j,n}$) are, respectively, listed in Tables 4 and 5. It is worth noting that the crack mistuning results are different according to the modal data of different mode families of the cracked hard-coated blisk. In order to more clearly
Figure 6. Natural frequencies of 1–4 modal families.

Figure 7. Mode shapes of 1–4 modal families: (a) first bending modal family, (b) first swing modal family, (c) second bending modal family, and (d) first torsion modal family.
Table 4. Identification results of crack mistuning of numerical hard-coated blisk 1 calculated by modal data of 1–4 modal families $k_{jn}$ (rad$^2$/s$^2$).

| Blade number | First bending | First swing | Second bending | First torsion |
|--------------|---------------|-------------|----------------|---------------|
| 1            | -24929.97     | -28666.666 | -6079668.30    | -8358390.84   |
| 2            | -13106.82     | -5830273.2 | -321762.67     | -23931945.28  |
| 3            | -13127.48     | -14859594.7 | -3305319.11   | -27309926.82  |
| 4            | -24799.73     | -7048427.5 | -6074924.67    | -3176795.46   |
| 5            | -430035.74    | -2993878.7 | -10374236.89   | -338270550.49 |
| 6            | -675800.99    | -5816120.8 | -1673149.14    | -315432395.52 |
| 7            | -420032.27    | -14858069.6 | -103380378.51  | -273628903.13 |
| 8            | -670403.37    | -6093238.5 | -166358675.41  | -85837042.21  |

Table 5. Identification results of crack mistuning of numerical hard-coated blisk 2 calculated by modal data of 1–4 modal families $k_{jn}$ (rad$^2$/s$^2$).

| Blade number | First bending | First swing | Second bending | First torsion |
|--------------|---------------|-------------|----------------|---------------|
| 1            | -10797.20     | -3389797.9 | -5155157.27    | -108902120.19 |
| 2            | -38709.14     | -8199681.6 | -13528441.3    | -323220383.79 |
| 3            | -135476.93    | -17421399.4 | -32292352.11  | -359601697.57 |
| 4            | -240420.34    | -9532560.7 | -60488070.57   | -429549349.74 |
| 5            | -41051.16     | -3538339.0 | -105445156.46  | -441554429.55 |
| 6            | -697399.82    | -8197730.5 | -166965065.51  | -426416660.26 |
| 7            | -1071887.23   | -17363199.1 | -241575945.22  | -356102600.95 |
| 8            | -1775087.39   | -8340090.8 | -358754254.64  | -116034603.13 |

Table 6. Frequency eigenvalue deviation rates of numerical hard-coated blisk 1 $\lambda_n$ (%).

| Blade number | First bending | First swing | Second bending | First torsion |
|--------------|---------------|-------------|----------------|---------------|
| 1            | -2.9          | -18.93      | -18.21         | -12.7         |
| 2            | -1.53         | -38.5       | -9.77          | -36.37        |
| 3            | -1.53         | -9812.96    | -9.9           | -41.98        |
| 4            | -2.89         | -46.55      | -18.19         | -48.28        |
| 5            | -5.01         | -19.8       | -31.07         | -51.56        |
| 6            | -7.87         | -38.41      | -49.93         | -47.94        |
| 7            | -4.89         | -9811.95    | -30.96         | -41.59        |
| 8            | -7.81         | -40.24      | -49.82         | -13.05        |

reflect the crack mistuning state of each coated blade, the frequency eigenvalue deviation rates are calculated. The calculation formula is as follows

$$\lambda_n = \frac{k_{jn}}{(\omega_{ch})^2}$$

(16)

where $\lambda_n$ is the frequency eigenvalue deviation rate of $n$th coated blade with crack, and $\omega_{ch}$ is the natural frequency of the tuned cantilevered coated blade. When calculating the frequency deviation rate, corresponding to the frequency eigenvalue deviation $k_{jn}$ calculated by the modal data of the 1–4 modal families, the 1–4 order natural frequencies of the cantilevered coated blade are used correspondingly. The frequency eigenvalue deviation rates are listed in Tables 6 and 7.

**Discussion on identification results**

In order to verify the correctness of mistuning identification results, the frequency eigenvalue deviation rates (listed in Tables 6 and 7) are compared with the lengths of the cracks in the circumferential direction of the hard-
Comparison results of numerical hard-coated blisk 1 and numerical hard-coated blisk 2 are shown in Figures 8 and 9. For No.1–No.8 blades of the numerically hard-coated blisk 1, the variation trend of frequency eigenvalue deviation rates calculated by the modal data of first and second bending mode families has a good consistency with that of crack lengths (as shown in Figure 8(a) and (c)). Moreover, the frequency eigenvalue deviation rates of coated blades with the same crack length are approximately equal. However, the variation trend of frequency eigenvalue deviation rates calculated by the modal data of first swing mode family and first torsion mode family is in poor agreement with that of crack lengths (as shown in Figure 8(b) and (d)). For the numerical hard-coated blisk 2, the same phenomenon is shown. Figure 9(a) and (c) shows that the variation trend of the frequency eigenvalue deviation rates calculated by the modal data of the first bending mode family and the second bending mode family is in good agreement with that of the crack lengths. Figure 9(b) and (d) shows that the variation trend of frequency eigenvalue deviation rates calculated by the modal data of first swing mode family and first torsion mode family is not consistent with that of crack lengths. Through the above comparison results, it is shown that the identification method is not sensitive to the modal data of first swing mode family and first torsion model.

Table 7. Frequency eigenvalue deviation rates of numerical hard-coated blisk 2 $\lambda_n$ (%).

| Blade number | First bending | First swing | Second bending | First torsion |
|--------------|--------------|-------------|----------------|--------------|
| 1            | -0.13        | -22.39      | -1.54          | -16.55       |
| 2            | -0.45        | -54.15      | -4.05          | -49.12       |
| 3            | -1.58        | -11504.72   | -9.67          | -54.65       |
| 4            | -2.8         | -62.95      | -18.11         | -65.27       |
| 5            | -4.79        | -23.37      | -31.58         | -67.11       |
| 6            | -8.13        | -54.14      | -50.00         | -64.81       |
| 7            | -12.49       | -11466.29   | -72.34         | -54.12       |
| 8            | -20.68       | -55.08      | -107.43        | -17.63       |

Figure 8. Comparison results of frequency eigenvalue deviation rate and crack length of numerical hard-coated blisk 1: (a) deviation rates calculated by first bending mode family, (b) deviation rates calculated by first swing mode family, (c) deviation rates calculated by second bending mode family, and (d) deviation rates calculated by first torsion modal family.
mode family of the cracked hard-coated blisk, that is, the crack mistuning of the coated blade can be effectively identified by the modal data of the first bending mode family and the second bending mode family.

In addition, compared with the irregular distribution of the crack lengths of numerical hard-coated blisk 1, the crack lengths of the numerical hard-coated blisk 2 are increased linearly from No.1 blade to No.8 blade. However, through Figure 9(a) and (c), it can be seen that the crack mistuning is not linearly increased from No.1 blade to No.8 blade. Therefore, the relationship between crack mistuning and crack length is not linear, which also shows the complexity of the mistuning caused by the crack.

Through the above discussion, it is shown that although this identification method can simultaneously obtain the crack mistuning of each blade for all four modes, the identification method only has a good identification effect for first and second bending modes.

**Influence factors analysis of crack mistuning**

**Mistuning identification of cracks with different lengths at different locations**

In this section, the relationship between crack length and mistuning is further studied. At the same time, the influence of crack location on the mistuning is also studied. There are two meanings of “crack location.” One is the location of the crack in the radial direction of the hard-coated blisk, that is, the location of the crack reflected by \( L_1 \) in Figure 3. The other is the place where the crack exists (as shown in Figure 2), that is, the cracks occurring only in the hard coating, only in the blade substrate, and both in the hard coating and the blade substrate. In order to obtain the relationship between the length and location of the cracks and the mistuning, the mistuning of coated blisks with three types of cracks is identified. For each type of crack shown in Figure 2, the lengths of the cracks in the circumferential direction of the coated blisk are taken as 16 values, i.e. \( L_3 = 1, 2, 3, \ldots, 16 \) mm. The location dimensions of the cracks in the radial direction of the coated blisk are also taken as 16 values, i.e. \( L_1 = 70, 75, 80, \ldots, 145 \) mm. In order to reduce the number of FEMs of crack coated blisk, two FEMs are established for each type of crack coated blisk, which are shown in Figure 10. In the two FEMs, the crack lengths \( L_3 \) on each blade are equal. In Figure 10(a), the crack location dimensions are \( L_1 = 70, 80, 90, \ldots, 140 \) mm. In Figure 10(b), the crack location dimensions are \( L_1 = 75, 85, 95, \ldots, 145 \) mm. Through the modal data of the two FEMs, the mistuning of the cracks with uniform length at 16 locations on the coated blades can be calculated. Then, according to \( L_3 = 1, 2, 3, \ldots, 16 \) mm, the crack lengths of the two FEMs are changed respectively, and 32 FEMs of crack coated blisk are established. Through the modal data of these 32 FEMs, the mistuning of cracks of

**Figure 9.** Comparison results of frequency eigenvalue deviation rate and crack length of numerical hard-coated blisk 2: (a) deviation rates calculated by first bending mode family, (b) deviation rates calculated by first swing mode family, (c) deviation rates calculated by second bending mode family, and (d) deviation rates calculated by first torsion modal family.
16 lengths at 16 locations on the coated blade can be obtained. In the “Numerical verification for effectiveness of identification method” section, it is found that the crack mistuning can be effectively identified by the mode data of the first and second bending mode families of the cracked coated blisk. Therefore, the mistuning of the three types of cracks is all identified by the modal data of the first and second bending modal families. Then, the frequency eigenvalue deviation rates are calculated by equation (16) and shown in Figures 11 to 13. Note that since most of the deviation rates are negative, in order to show the variation trend of the deviation rates more clearly, the z-axis in the coordinate system is displayed in the opposite direction in Figures 11 to 13.

Discussion on mistuning variation rules of three types of cracks

Through the comparative analysis of Figures 11 to 13, it is found that there is a great difference in the stiffness mistuning caused by the three types of cracks. For cracks of equal length at the same location, in the three types of crack mistuning, the least mistuning is generated by the crack occurring only in the coating, the mistuning is slightly larger by the crack occurring only in the blade substrate, and the largest mistuning is generated by the crack occurring both in the coating and blade substrate. Moreover, it is important to note that the mistuning of the cracks occurring both in hard coating and blade substrate is not equal to the sum of the other two kinds of crack mistuning. Compared with the mistuning of the other two kinds of cracks, the mistuning of the cracks occurring both in hard coating and blade substrate increases greatly. This shows that the cracks in the coating and the substrate at the same location will seriously reduce the stiffness of the coated blade. Moreover, the phenomenon that the mistuning of cracks occurring only in the blade substrate is much smaller than that of the cracks occurring both in hard coating and blade substrate is also clearly shown.
In these figures, the curve fittings of the discrete points are carried out by using the sixth-degree polynomial to obtain the trend lines of the crack mistuning variation. In addition, the mistuning variation trends of the three kinds of cracks with the change of crack locations and lengths are different. For the stiffness mistuning calculated by the first bending mode family, the mistuning variation trend of the crack occurring only in the coating is different from that of the other two kinds of cracks. For the stiffness mistuning calculated by the second bending mode family, the mistuning variation trend of the cracks occurring both in the coating and blade substrate is different from that of the other two kinds of cracks. Specific variation rules will be discussed in “Discussion on mistuning variation rules of cracks at different locations in the radial direction of the hard-coated blisk” section and “Discussion on mistuning variation rules of cracks with different lengths” section.

**Discussion on mistuning variation rules of cracks with different lengths**

In order to further analyze the variation rule of mistuning with crack length, variation trends of crack mistuning varying with crack lengths at 16 locations are compared. The mistuning calculated by the first bending mode data is shown in Figure 14, and the mistuning calculated by the second bending mode data is shown in Figure 15. In these figures, the curve fittings of the discrete points are carried out by using the sixth-degree polynomial to obtain the trend lines of the crack mistuning variation.

**Figure 12.** Frequency eigenvalue deviation rates caused by cracks occurring only in blade substrate: (a) deviation rates calculated by first bending mode family and (b) deviation rates calculated by second bending mode family.

**Figure 13.** Frequency eigenvalue deviation rates caused by cracks occurring both in hard coating and blade substrate: (a) deviation rates calculated by first bending mode family and (b) deviation rates calculated by second bending mode family.

occurring both in hard coating and blade substrate indicates that the hard coating can effectively control the mistuning of the coated blade even if the cracks only exist in blade substrate.
Figure 14. Variation trend of frequency eigenvalue deviation rates with crack length calculated by first bending mode family data: (a) mistuning of cracks occurring only in hard coating, (b) mistuning of cracks occurring only in blade substrate, and (c) mistuning of cracks occurring both in hard coating and blade substrate.
Figure 15. Variation trend of frequency eigenvalue deviation rates with crack length calculated by second bending mode family data: (a) mistuning of cracks occurring only in hard coating, (b) mistuning of cracks occurring only in blade substrate, and (c) mistuning of cracks occurring both in hard coating and blade substrate.
Figure 14(a) shows mistuning variation trends of cracks occurring only in hard coating calculated by the first bending mode data. It can be easily observed that mistuning variation trends of cracks at 16 locations are consistent. Except for a small amount of positive mistuning in the location of 145 mm, there are all negative mistuning in other locations. With the increase of crack length, the magnitudes of these negative mistuning all decrease first and then increase, and the minimum value appears when the crack length is 9 mm. It is worth noting that the mistuning at 145 mm becomes positive mistuning (0.0042%) when the crack length is 9 mm, while the minimum negative mistuning (0.00095%) occurs when the crack length is 7 mm. The mistuning variation trends of cracks occurring only in the hard coating are not consistent with that of our preliminary assumption.

Figure 14(b) shows mistuning variation trends of cracks occurring only in blade substrate calculated by the first bending mode data. Except for a small amount of positive crack mistuning at the location of 145 mm, all the mistuning at other locations is negative. Two kinds of variation trends are presented here. One is the mistuning variation trend of the crack location ranging from 110 mm to 145 mm. The negative mistuning first decreases and then increases with the increase of the crack length, and the minimum values occur when the crack length is 9 mm. These variation trends are very similar to that of cracks occurring only in the hard coating. The other is the mistuning variation trend of the crack locations ranging from 70 mm to 105 mm. The magnitude of negative mistuning keeps increasing with the increase of the crack length. Meanwhile, it is worth noting that the increase of crack length is linear, but the increase of crack mistuning is not linear.

Figure 14(c) shows mistuning variation trends of cracks occurring both in blade substrate and coating calculated by the first bending mode data. The positive crack mistuning occurred at six locations ranging from 120 mm to 145 mm. There are also two mistuning variation trends here. One is the crack mistuning variation trend at six locations ranging from 120 mm to 145 mm. That is, with the increase of the crack length, the magnitude of negative crack mistuning decreases and turned into negative positive mistuning, and then the magnitude of positive crack mistuning increases. The other is the crack mistuning variation trend at 10 locations ranging from 70 mm to 115 mm. With the linear increase of the crack length, the crack mistuning also shows a nonlinear increasing trend.

Figure 15(a) shows mistuning variation trends of cracks occurring only in hard coating calculated by the second bending mode data. It is shown that the crack mistuning at locations of 145 mm, 140 mm, 135 mm, and 70 mm is all positive mistuning, while the crack mistuning at other locations is all negative mistuning. Although the variation trend of the crack mistuning trend line at 16 locations is basically the same, the magnitude of mistuning shows two kinds of variation trends, that is, with the increase of the crack length, the magnitude of positive mistuning first increases and then decreases, while the magnitude of negative mistuning first decreases and then increases.

Figure 15(b) shows mistuning variation trends of cracks occurring only in blade substrate calculated by the second bending mode data. The crack mistuning at locations of 145 mm, 140 mm, and 135 mm is all positive mistuning, and their variation rules are not obvious. There is both positive and negative mistuning at the location of 70 mm. With the increase of the crack length, the magnitude of positive mistuning decreases firstly and turns into negative mistuning, and then the magnitude of negative mistuning increases. In addition, the crack mistuning at the other 12 locations is all negative mistuning, and the same variation trend is shown, that is, the magnitude of negative mistuning keeps increasing with the increase of the crack length.

Figure 15(c) shows mistuning variation trends of cracks occurring both in blade substrate and coating calculated by the second bending mode data. The crack mistuning at locations of 145 mm and 140 mm is all positive mistuning, and the magnitude of positive mistuning keeps increasing with the increase of crack length. In addition, the crack mistuning at the other 14 locations is negative. With the linear increase of the crack length, the magnitude of negative mistuning presents a nonlinear increasing trend.

Through the above analysis, the following variation rules of crack mistuning with crack length are found:

1. For the first bending and second bend mode families, the mistuning magnitude of the crack occurring only in hard coating does not increase with the increase of the crack length. Therefore, the crack mistuning on the hard-coated blade does not always increase with the increase of crack length. In spite of this, the mistuning of the cracks occurring only in hard coating at all locations varies according to the same regular trend.

2. For the first bending and second bend mode families, the mistuning of cracks occurring only in blade substrate and the mistuning of cracks occurring both in hard coating and blade substrate shows a nonlinear increase trend with the linear increase of crack length at most locations. However, at locations of 135 mm, 140 mm, and 145 mm which are near the blade tip, there are no obvious rules to follow for the variation of the magnitude of mistuning.
With the variation of crack length, the variation trends of three kinds of crack mistuning are all regular, and these variation rules can provide a reference for the determination of the crack length of the coated blade.

Discussion on mistuning variation rules of cracks at different locations in the radial direction of the hard-coated blisk

In order to further analyze the variation rule of mistuning with the crack location in the radial direction of the hard-coated blisk, mistuning trends of cracks of 16 kinds of lengths varying with crack locations are compared. The mistuning calculated by the first bending mode data is shown in Figure 16, and the mistuning calculated by the second bending mode data is shown in Figure 17. In the figures, the curve fittings of the discrete points are also carried out by using the sixth-degree polynomial to obtain the trend lines of the crack mistuning variation.

Figure 16(a) shows mistuning variation trends of cracks occurring only in hard coating calculated by the first bending mode data. As the crack moves from the blade root to the blade tip, the crack mistuning of 16 kinds of lengths showed the same variation trend, that is, the magnitude of the negative mistuning of the cracks increased slightly and then decreased continuously.

Figure 16(b) shows mistuning variation trends of cracks occurring only in blade substrate calculated by the first bending mode data. As the cracks moved from the blade root to the blade tip, the crack mistuning of 16 kinds of lengths also showed the same variation trend, that is, the magnitude of the negative mistuning of the cracks decreased continuously.

Figure 16(c) shows mistuning variation trends of cracks occurring both in blade substrate and coating calculated by the first bending mode data. Herein, the variation trends of mistuning are relatively complex. As cracks move from blade root to blade tip, the negative mistuning shows a decreasing trend firstly. And then, at the location of 120 mm, except for crack mistuning of the location of 1 mm and 2 mm is still negative and continues to decrease, the mistuning of other crack lengths begins to turn into positive mistuning and presents an increasing trend.

Figure 17(a) shows mistuning variation trends of cracks occurring only in hard coating calculated by the second bending mode data. It can be observed from this figure that the mistuning of cracks of 16 kinds of lengths has the same variation trend. The crack at the root of the blade produces positive mistuning. With the change of crack location, the positive mistuning decreases and turns into negative mistuning. Then, the negative mistuning increases with the change of crack location. After the negative mistuning reaches its maximum value at the location of 105 mm, it starts to decrease and turns into positive mistuning. Subsequently, positive mistuning continues to increase.

Figure 17(b) shows mistuning variation trends of cracks occurring only in blade substrate calculated by the second bending mode data. As shown in Figure 17(b), the mistuning variation trends of the cracks of 16 kinds of lengths are roughly the same, except that the mistuning variation trends of the cracks near the blade root are slightly different. Three kinds of mistuning variation trends are shown here. One is the crack mistuning variation trend of six lengths with a crack length ranging from 1 mm to 6 mm. The mistuning of these cracks is transformed from positive mistuning to negative mistuning, and then the negative mistuning begins to increase. The second is the mistuning variation trend of 5 crack lengths ranging from 7 mm to 11 mm, the crack mistuning is negative mistuning initially and shows the increasing trend. The third is the crack mistuning variation trend of five crack lengths ranging from 12 mm to 16 mm. Initially, the mistuning of these cracks is also negative mistuning, and it shows a trend of decreasing first and then increasing. After the location of 80 mm, the mistuning of cracks of 16 kinds of length has the same change trend, that is, as the crack moves towards the blade tip, the magnitude of negative mistuning continues to increase, and the maximum value appears at the location of 105 mm, then the negative mistuning starts to decrease and turns into positive mistuning. After that, the positive mistuning continues to increase.

Figure 17(c) shows mistuning variation trends of cracks occurring both in blade substrate and coating calculated by the second bending mode data. The mistuning of cracks 16 kinds of lengths varies with the same variation rule. The mistuning generated by the crack at the blade root is negative mistuning. As the crack moves towards the blade tip, the magnitude of negative mistuning decreases firstly and then increases. After that, it decreases again and turns into positive mistuning. However, it is important to note that the largest negative mistuning occurs in different locations. For cracks of 12 lengths ranging from 1 mm to 12 mm, the largest negative mistuning appears at the location of 110 mm, and for cracks of 4 lengths ranging from 13 mm to 16 mm, the largest negative mistuning appears at the location of 70 mm (blade root).
Figure 16. Variation trend of frequency eigenvalue deviation rates with crack location calculated by first bending mode family data: (a) mistuning of cracks occurring only in hard coating, (b) mistuning of cracks occurring only in blade substrate, and (c) mistuning of cracks occurring both in hard coating and blade substrate.
Figure 17. Variation trend of frequency eigenvalue deviation rates with crack location calculated by second bending mode family data: (a) mistuning of cracks occurring only in hard coating, (b) mistuning of cracks occurring only in blade substrate, and (c) mistuning of cracks occurring both in hard coating and blade substrate.
Through the above analysis, it is found that crack mistuning is very sensitive to the change of crack location on the blade, and the following variation rules of crack mistuning with crack location are obtained:

1. For the first bending mode family, the closer the three types of cracks are to the blade root, the greater the mistuning of the cracks will be (as shown in Figure 18(a)). Therefore, for the first bending mode of the hard-coated blisk, enough attention should be paid to the crack at the blade root.

2. For the second bending mode family, the mistuning of cracks occurring only in blade substrate or only in the hard coating is maximized when the crack is located in the middle of the blade (as shown in Figure 18(b)). For cracks occurring both in blade substrate and coating, the mistuning of cracks of 12 lengths ranging from 1 mm to 12 mm is also maximized when the crack is located in the middle of the blade (as shown in Figure 18(b)). However, for cracks of four lengths ranging from 13 mm to 16 mm, although the maximum mistuning occurs when the crack is located at the blade root, a relatively large mistuning also occurs when the crack is located in the middle of the blade. Therefore, for the second bending mode family, the cracks in the middle of the blade should be paid more attention. At the same time, enough attention should be paid for the relatively long cracks at the blade root.

3. According to the comparison of Figure 18(b), for the second bending mode family, the maximum modal displacement of the coated blade occurs in the middle of the blade, which is also the location where the maximum or relatively large crack mistuning occurs. Therefore, under the second bending mode, it is surmised that the crack located at the maximum modal displacement of the blade will produce a relatively large crack mistuning.

4. With the variation of crack location, the variation trend of three kinds of crack mistuning is all regular, which can provide a reference for the determination of the crack location of coated blade.

**Conclusions**

As a composite structure, the blade crack mistuning state of the hared-coated blisk is very complex. In this paper, the classical mistuning identification method based on CMM ROM is improved to study the blade crack mistuning of hard-coated blisk, and the following conclusions are obtained.
1. The effectiveness of the mistuning identification method is verified by numerical cases. It is found that the method proposed in this paper can simultaneously obtain the crack mistuning of each blade. However, it should be noted that this identification method can effectively identify the crack mistuning of the first and second bending modes, but it is not applicable for the first swing mode and first torsion mode.

2. By comparing the mistuning of three kinds of cracks (cracks occurring only in the hard coating, cracks occurring only in blade substrate, and cracks occurring both in hard coating and blade substrate), it is found that the mistuning of the cracks occurring both in hard coating and blade substrate is much greater than that of the cracks occurring only in blade substrate, which indicates that the hard coating can effectively control the mistuning degree of the whole hard-coated blade when the cracks occur only in blade substrate.

3. For the first bending and second bend mode families, only the mistuning of the crack occurring only in coating decreased first and then increased with the linear increase of crack length. For the other two kinds of cracks, the mistuning of the crack at most locations tends to increase nonlinearly with the linear increase of crack length.

4. For the first bending mode family, the closer the three types of cracks are to the blade root, the greater the mistuning of the cracks will be. Therefore, for the first bending mode of the hard-coated blisk, enough attention should be paid to the crack at the blade root.

5. For the second bending mode family, the mistuning of cracks occurring only in the blade substrate or only in the coating is maximized when the crack is located in the middle of the blade. For cracks occurring both in blade substrate and coating, the mistuning of cracks of most kinds of lengths is also maximized when the crack is located in the middle of the blade. However, for cracks of other few kinds of lengths, although the maximum mistuning occurs when the crack is located at the blade root, a relatively large mistuning also occurs when the crack is located in the middle of the blade. Therefore, for the second bending mode family, the cracks in the middle of the blade should be paid more attention. At the same time, enough attention should be paid for the relatively long cracks at the blade root.

6. For the three kinds of cracks, with the change of crack length and location in the radial direction of hard-coated blisk, the variation of crack mistuning can be followed by the rule, which can provide effective reference data for determination of the blade crack length and location of the hard-coated blisk.

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