Entangled states of trapped ions allow measuring the magnetic field gradient produced by a single atomic spin

F. Schmidt-Kaler and R. Gerritsma

QUANTUM, Institut für Physik, Universität Mainz - D-55128 Mainz, Germany, EU

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Abstract - We propose detecting the magnetic field gradient produced by the magnetic dipole moment of a single atom by using ions in an entangled state trapped a few µm from the dipole. This requires measuring magnetic field gradients of order \(10^{-13}\) tesla/µm. We discuss applications in determining magnetic moments of a wide variety of ion species, for investigating the magnetic substructure of ions with level structures that are not suitable for laser cooling and detection, and for studying exotic or rare ions, and molecular ions. The scheme may also be used for measuring spin imbalances of neutral atoms or atomic ensembles trapped by optical dipole forces. As the proposed method relies on techniques that are well established in ion trap quantum information processing, it is within reach of current technology.

Introduction. - Recently developed quantum-enhanced precision measurement schemes in atomic physics have the potential to improve classical methods significantly [1]. The use of entangled states can for instance improve the sensitivity of Ramsey spectroscopy [2], used for frequency estimation and the detection of external fields. Furthermore, errors due to external field noise [3] can be circumvented by carefully designing quantum states [4] that are insensitive to the prevailing correlated noise in laboratory quantum systems, such as in ion traps and optical lattices [5], while still improving the sensitivity to the field of interest. Importantly, and in contrast to other emerging quantum technologies such as quantum computing and simulation, improvements already materialise for relatively small quantum systems. In trapped cold ion systems, entangled states of up to 14 ions [6-8] have been generated, making this an ideal system for performing quantum-enhanced precision spectroscopy.

A notable spectroscopic scheme making use of quantum effects is quantum logic spectroscopy [9] for realizing atomic clocks with relative frequency uncertainties of a few parts in \(10^{17}\). Here, a “logic” ion such as Be\(^{+}\) is used to read out the quantum state of a simultaneously trapped Al\(^{+}\) “clock” ion via a shared mode of motion. Modifications and new applications of this scheme have been subsequently proposed, such as for measuring the Lamb shift in He\(^{+}\) [10], in the context of optical frequency comb spectroscopy of metal ions [11,12], and for the molecular spectroscopy of N\(_2\)\(^{+}\) ions [13]. In all those cases the information on a successful laser excitation of the clock (or “spectroscopy”) ion is conveyed to the logic ion via a collective vibrational degree of freedom.

Here, we propose a fundamentally different approach, which does not rely on a collective mode of motion. Instead, direct coupling of the magnetic field, generated by the spins of the atoms or ions to be measured, to the spins of the measurement ions is used. In particular, we show that the field gradient of a source as small as a single electronic spin can be detected over a distance of a few µm using a specially designed quantum state of two ions that is insensitive to magnetic field noise.

We first describe the details of our method, discussing the most simple case of a spin detection in \(^{40}\)Ca\(^{+}\). Then, we exemplify the spectroscopy of magnetic properties of atomic or molecular ions that do not allow optical excitation or detection. Finally, we discuss the possibility of measuring the magnetic moments, or spin balance of ultra cold neutral atoms trapped in close proximity to the ions in a hybrid trap.

\(\text{(a)}\) E-mail: fsk@uni-mainz.de
Quantum-enhanced sensing of a magnetic field gradient. — The method is based on well-developed techniques in ion traps quantum processors [14]. For the implementation a micro-structured segmented ion trap would be preferred, where ions can be moved along the trap axis and positioned with high accuracy. Furthermore, the small trap dimensions allow for tight confinement with small inter-ion distances [15–18]. In the first step, a Bell state $\psi^+$ with two $^{40}$Ca$^+$ ions is generated

$$
\psi^+ = |\uparrow\rangle_1|\downarrow\rangle_2 + e^{i\phi}|\downarrow\rangle_1|\uparrow\rangle_2,
$$

where the phase $\phi$ is chosen equal to zero and where $|\downarrow\rangle$ denotes the $S_{1/2}$ m = $-1/2$ Zeeman level and $|\uparrow\rangle$ the m = $+1/2$ level, respectively, and we omitted the normalization factor for clarity. The index number corresponds to ion 1 and 2. This state may be generated in a Cirac-Zoller type [19], a Molmer-Sørenson type [20] or a geometric [21] type of gate operation, depending on the experimental preferences. Such gate operations currently reach state fidelities of about 99%. In the case of the Molmer-Sørenson operation, a dichromatic laser field can generate a Bell state such as $|S_{1/2}\rangle_1[D_{5/2}\rangle_2 + |D_{5/2}\rangle_1|S_{1/2}\rangle_2$ [22], where the Bell state is encoded in the $S$ ground and the metastable $D_{5/2}$ states. In a next step, the entanglement can be transferred from the optical qubit transition into the in $\text{Ca}^+$ ground state Zeeman superpositions $|\uparrow\rangle, |\downarrow\rangle$ using resonant laser pulses.

It has been shown experimentally that the Bell state $\psi^+$ exhibits a coherence time well above 10 seconds, even though no special care had been taken by, e.g., magnetic shielding [23]. The reason for this striking property comes from the advantages of decoherence-free subspaces: the linear Zeeman shift of the two ions leads to phase shifts acting in opposite directions, and since the ions are very close together (typically only a few $\mu$m apart) they experience similar magnetic field noise. This leads to an almost perfect cancellation of differential phase shifts. Another possible source of decoherence, spontaneous decay, is fully excluded since sublevels of the ground state are used.

Once the entangled state $\psi^+$ is generated, control voltages can be changed so that the third ion (X), whose magnetic field is to be determined, is brought close to the “logic” ions. Trap frequencies $\omega_z/(2\pi)$ of 5 to 10 MHz are typically reached and allow compressing the three-ion crystal to reach inter-ion distances $d$ of a few $\mu$m. The magnetic dipole field scales with $d^{-3}$, resulting in a gradient over the two-ion Bell state that leads to differential Zeeman shifts causing a rotation of the phase angle $\phi$, see fig. 1(a). After an interaction time $t$, one applies a $(\pi/2)$-pulse on the logic ions, transfers the entanglement from the $|\{\uparrow\rangle, |\downarrow\rangle\}$ states to the $|S_{1/2}\rangle, |D_{5/2}\rangle$ bases and reads out the laser induced fluorescence when applying resonant light on the dipole transitions from the $S_{1/2}$ to the $P_{1/2}$ state near 397 nm, and $D_{3/2}$ to the $P_{1/2}$ state near 866 nm, respectively. The fluorescence detection allows the determination of the quantum state of the two $\text{Ca}^+$ ions. Repeating the above sequence many times allows to determine the probabilities of finding the ions in each state, $p_1$ and $p_2$, from which the parity $P$ of the state $P = (p_1 + p_2) - (p_1 + p_2)$ is calculated. The shift of the phase $\phi$ of the Bell state leads to sinusoidal oscillations from $P(\phi = 0) = +1$ to $P(\phi = 2\pi) = -1$.

Next, we estimate the relevant parameters such as measurement time, noise sources and detection limits, for supporting the feasibility of the scheme under state-of-the-art experimental conditions. The three-ion crystal distance in a linear configuration reads as $d_{12} = 1.077(4\pi\epsilon_0\omega_c^2)^{1/3}$, where $m$ denotes the mass of one ion, $\epsilon_0$ their charge, $\epsilon_0$ the vacuum permittivity and $\omega_c$ [24] the axial trap frequency. A distance of $d_{12} = 1.03\mu$m is reached for $\omega_c/2\pi = 10$ MHz, which increases to 1.63$\mu$m for $\omega_c/2\pi = 5$ MHz. The magnetic dipole field of an atom or ion is given by

$$
B(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3\vec{p} \cdot (\vec{m} \times \vec{r}) - \vec{m} \cdot \vec{r}^2}{r^5},
$$

where $\mu_0$ denotes the permeability constant and $\vec{m}$ the magnetic dipole moment [25]. Let us assume that the quantization axis is determined by an external homogeneous field $B^{(0)}_z$ pointing in z-direction along the trap axis, the magnetic field $B_z(x,y,z)$ due to the dipole is then given by

$$
|B_z(x,y,z)| = \frac{3\mu_0}{4\pi} \frac{(3z^2 - 1)}{(x^2 + y^2 + z^2)^{3/2}},
$$

with $\mu_0 = 4\pi \cdot 10^{-7} H/m$ and the intrinsic magnetic moment $\mu = -9284.764 \cdot 10^{-26} J/T$ in SI units. From this equation one finds that the field generated by the third ion X drops from 7.8 $\cdot 10^{-13}$ T at the location of the second ion (at 1.03$\mu$m distance) by almost one order of magnitude to 9.7 $\cdot 10^{-14}$ T at the location of the first ion (at 2.06$\mu$m distance). For sensing the gradient field of ion X we apply an external gradient of opposite direction and the same

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Fig. 1: (Color online) Principle of the proposed scheme: a) The Bell state $1/\sqrt{2}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)$ made of two $^{40}$Ca$^+$ ions (dark grey circles) is trapped in a micro-trap (light grey) in close vicinity to a third ion X whose spin is to be measured (white circle). The magnetic dipole field affects mostly the neighboring ion and leads to a phase shift which can be detected in a parity signal (see text). b) The Greenberger-Horne-Zeilinger state $|1\rangle_1|2\rangle_2|3\rangle_4$ is used to measure the spin state of ion X in the middle (white circle).
Entangled states of trapped ions allow measuring the magnetic field gradient etc.

Fig. 2: Magnetic gradient between ion 1 and ion 2 as a function of their distance when ion 3 is spin polarized in the z-direction.

magnitude such that for the state $|\uparrow\rangle_X$ both contributions cancel and the entangled state parity remains constant with time, see fig. 2. For the state $|\downarrow\rangle_X$, however, both contributions add up to $\delta B = 2(B(z_{ion2}) - B(z_{ion1}))$ and the parity signal exhibits a phase evolution. For the above example $\delta B = 6.8 \cdot 10^{-13}$ T would lead to a sinusoidal rotation in $P(\phi)$ from +1 to −1 in 26 s.

We may detect smaller variations of the parity signal close to its zero-crossing. We reach ±30% for an evolution time of 5 s when the spin of the ion X is flipped from $|\uparrow\rangle_X$ to $|\downarrow\rangle_X$. Assuming quantum projection noise dominates, a signal-to-noise ratio of 2 could be achieved after about 10 repetitions of the experimental sequence (a total measurement time of less than 60 s). For first tests of the method, a three-ion crystal of $^{40}$Ca$^+$ ions could be used, where the spin of the third ion can be aligned with optical pumping. In such experiments, the long-term stability of the apparatus might have to be improved. Reducing the ambient magnetic gradient fluctuations below $10^{-13}$ T/µm for the relevant time scale may require passive magnetic shielding and active control. The technique using entangled, designed states of a two-ion crystal is an promising alternative way to overcome the ambient magnetic field noise, as compared to lock-in methods which are aiming for similar sensitivities [26].

Proposed applications. – The new method might be applied for yielding spectroscopic data of the magnetic sublevels in the case of ions without accessible dipole transitions for optical cooling and detection. The substructure of molecular ions could be investigated with the detection technique as well. In this scenario, a molecular ion trapped in a three-ion crystal together with the entangled Ca$^+$ pair is excited with a laser pulse of defined frequency and duration. Successful excitation of molecular transition could result in a state change and the corresponding modification of the magnetic moment could be detected via the parity signal of the entangled $^{40}$Ca$^+$ ions. The proposed method extends the idea of “designer ions” [4] —which have been used to accurately measure frequency shifts on an optical transition frequency— to ion crystals with multiple species ions. The method may be compared to destructive measurement approaches realized with large mixed ion crystals of $^{13}$N$^+_2$ ions sympathetically cooled in a $^{40}$Ca$^+$ crystal. A successful molecular transition event was either detected by the subsequent ion loss [13] or by charge transfer processes [27]. As compared with a recently proposed method, we do not require strong magnetic gradient fields or optical forces for the detection [28].

The method is not limited to detecting magnetic properties of atomic or molecular ions confined in Paul traps, as the interaction does not require a coupling to a common vibrational mode. Thus, the $^{40}$Ca$^+$ two-ion crystal might be positioned as a local probe in an ensemble of neutral atoms and measure the magnetic properties, e.g., for detecting quantum magnetic phases. The advantage of this system is that ions could be positioned much closer to the neutral atom spins then to other (positively) charged ions, which increases the sensitivity. In this way, the two-ion entangled state would directly serve to detect the difference of magnetic field at both of its sites, while rejecting any common magnetic field fluctuations. In recent experiments the operation of combined traps for ions and neutral atoms has been demonstrated [29,30] and it is a logical next step to use the advantages of entangled ion crystals also here. For instance, when the neutral atoms are trapped in a double-well potential, an entangled ion pair trapped in between could sense the imbalance of magnetic field generated by the atoms in each well, which in in typical realizations are a few µm apart. To exemplify the feasibility of this scheme we assume a double well separated by 4.4 µm, as in the experimental realization with optical dipole potentials and crossed beams in ref. [31]. The two-ion crystal with an inter-ion distance of 3.5 µm would perfectly fit in between the degenerate gas in the potential wells, and the ions are not immersed in the gas, such that collisions and heating effects are largely avoided. When we assume an interaction time of 2.5 s, and a magnetic gradient over both ions caused by an imbalance of only single atomic dipole we would have a differential $B$-field of $\delta B = 13 \cdot 10^{-12}$ T, which would appear as ±30% parity modulation signal. The signal directly reveals the difference of atom population $\Delta N$ in the two wells. The sensitivity reaches the single atom level $\Delta N = 1$ and could be used to detect number fluctuations at the quantum noise limit [32] and the Josephson dynamics of coupled potentials [33].

Let us compare our scheme with conventional magnetic field sensing techniques. Conventional magnetic sensing operates with SQUID, giant magnetoresistance or fluxgate sensors. The typical size of such devices ranges from a few mm to about 100 µm and pairs or even arrays of SQUID detectors are used for a measurement of magnetic field gradients. However, such devices do not reach the single spin level. Magnetic resonance force microscopy, on the other hand, has proven to reach single electron spin sensitivity at a measurement distance of up to 100 nm [34].
Alternatively, nitrogen color centers (NV) in diamond serve as magnetic field sensors [35]. The long coherence time of the single NV quantum state is critical for reaching the best sensitivity [35,36]. The width of the resonance between Zeeman shifted levels leads to a sensitivity limit of about $80 \times 10^{-6}$ T/nm for spin imaging [37]. A sensitivity, sufficiently high to detect a single spin, is reached in such devices at a distance of about 5 nm [37,38]. In contrast to these methods, the scheme proposed here uses i) entangled ions in a quantum state where only gradients of the magnetic field lead to a signal and where ii) the long coherence time of ions allows an improved sensitivity.

**Outlook.** – The method could be extended to linear ion crystals with $N$-particle entanglement and one or more embedded ions X. The phase evolution of the $N$-particle entangled state, as measured by the parity signal would be strongly influenced by the state of spin X and the corresponding local field gradients due to its magnetic moment. While there are certainly several experimental problems due to the complexity of this scheme, two advantages are clearly visible: the ion-distances become even smaller when the number of ions increases, which leads to stronger magnetic field gradients, and the multiparticle entanglement is leading to faster parity rotations [6] allowing a more accurate detection. For this case the $N$-ion Greenberger-Horne-Zeilinger (GHZ) state can be used, which can also be prepared in a decoherence-free, and thus long lived version, i.e.,

$$
\Psi^+ = \frac{1}{\sqrt{4}} |\uparrow_1, \downarrow_2, X, \downarrow_3, \uparrow_4\rangle + e^{i\phi} |\downarrow_1, \uparrow_2, X, \uparrow_3, \downarrow_4\rangle.
$$

Note that the generation of the GHZ state does not require individual addressing of $^{40}$Ca$^+$ ions with a laser beam. Finally, the magnetic field gradient due to X would be detected by a doubled phase shift induced by constructive addition of the opposite Zeeman effects to the ions 2 and 3, see fig. 1(b).

In conclusion, the proposed method extends the toolbox of quantum metrology. The entangled two-ion crystals serve as a very sensitive local magnetic field probes that directly reveal the difference of fields at the ion locations while any common magnetic noise contributions are suppressed. This quantum sensor may find applications in spectroscopy of ions of other species with transitions that are not easily accessible, for molecular ion spectroscopy and to detect the magnetic fields of ultra cold neutral atoms.

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