Assessment of anisotropy and revisiting Kolmogorov constant in particle-laden turbulent channel flows

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A large number of models which address the dynamics of particle-laden turbulent flows have been developed based on the assumption of local isotropy and use the Kolmogorov constant that correlates the spectral distribution of turbulent kinetic energy with the turbulent dissipation rate. Many research works have been performed to estimate the Kolmogorov constant for a wide range of single-phase turbulent flows like homogeneous isotropic turbulence, turbulent boundary layers, etc. In earlier work, Sreenivasan [1] has consolidated and analyzed a large number of experimental data and concluded that the Kolmogorov constant is independent of the Reynolds number and may be considered a universal constant. In the present work, we assess the extent of local anisotropy in the inertial and dissipation range of particle-laden channel flows. We estimate the Kolmogorov constant using a second-order velocity structure function and compensated spectra. Our study reveals that the Kolmogorov constant decreases with an increase in the particle volume fraction. The present analysis of variation of the Kolmogorov constant with particle volume loading will provide insight to develop advanced models for two-phase turbulent flows.

I. INTRODUCTION

Particle-laden turbulent flows are encountered in a large number of geophysical and industrial processes. One of the major focuses in this area is understanding the dynamics of fluid and solid phases. With the advent of high-speed computational systems, although performing direct numerical simulation for small system size has become possible, modeling still plays a vital role in studying large-scale systems with practical applications. Most of the modeling techniques are based on the approximation of isotropy in the inertial and dissipation range. Kolmogorov’s similarity hypothesis for the inertial subrange states that for every turbulent flow at a sufficiently high Reynolds number, the statistics of the motions of scale \( r \) in the range, \( \eta \ll r \ll L \), have a universal form that is uniquely determined by \( \epsilon \) and independent of \( \nu \) [2]. Here, \( L \) is the integral length scale, \( \eta \) is the Kolmogorov length scale, \( \nu \) is the kinematic viscosity, and \( \epsilon \) is the mean viscous dissipation rate of turbulent kinetic energy. The spectral energy in the inertial subrange is expressed as \( E(k) = C\epsilon^{2/3}k^{-5/3} \), where \( k \) is the wavenumber and \( C \) is the proportionality prefactor known as the Kolmogorov constant. The Kolmogorov constant is obtained based upon the Kolmogorov hypothesis for different flows such as boundary layers [1, 3], channel flows [4, 5], homogeneous isotropic turbulence [6–8], etc. The Kolmogorov constant for different experiments and simulations are summarized by Sreenivasan [1], Yeung and Zhou [8], and Lien and D’Asaro [9]. The Kolmogorov constant has been used in...
stochastic turbulence models [10–17] and other turbulence models which have been developed based on large scale turbulent structures, like Smagorinsky model [18–20], and other eddy viscosity based models [20], which are used in large eddy simulations (LES). Another implication of the Kolmogorov constant ($C_0$) is that in the Lagrangian velocity structure function, it controls the integral time scale through relation $T_L = 2\sigma_u^2 C_0 \epsilon$; which in turn controls the magnitude of turbulent diffusivity $K = \sigma_u^2 T_L$ for large times [6]. Here, $\sigma_u^2$ is the variance of velocity fluctuation.

In his seminal work, Sreenivasan [1] has summarized that the Kolmogorov constant is universal and independent of the flow configuration and Reynolds number. At sufficiently high Reynolds number where local isotropy is satisfied at dissipation scale and inertial range, will lead to a universal value of Kolmogorov constant [8]. However, at moderate and low Reynolds numbers, the Kolmogorov constant may differ from the universal value [1, 8]. Antonia et al. [4] performed experiments for channel flow and observed a lower value of the Kolmogorov constant. They analyzed the second and third-order velocity structure functions and mentioned that small-scale isotropy is to be satisfied for the existence of a universal inertial range. Yeung and Zhou [8] mentioned that for the existence of inertial range, isotropy should also be present along with $-5/3$ scaling. The Kolmogorov constant may attain a different value (other than two) if isotropy is not satisfied in the inertial range. Heinz [10] discussed the variations of the Kolmogorov constant for equilibrium turbulent boundary layer and homogeneous isotropic stationary turbulence and mentioned that for Stochastic modeling, the value is near two if anisotropic velocity fluctuation and acceleration fluctuations dominate in the energy budget. Furthermore, the value is near six if those contributions disappear. The above studies point out the deviation of the Kolmogorov constant from universality due to anisotropy at low and moderate Reynolds numbers for unladen cases.

In turbulent flows, the isotropy may also be affected due to the presence of particles in a particle-laden turbulent flow. Depending on the solid phase loading, the particles can affect the turbulence intensity of the gas phase. Gualtieri et al. [21] have commented that care should be taken while applying Kolmogorov theory as anisotropy is increased for particle-laden flows. In a very recent study, Rohilla et al. [22] have demonstrated that the LES models, like Smagorinsky and dynamic Smagorinsky models, perform poorly to predict the turbulence modulation and the critical volume loading at which turbulence collapse completely. The scale-similarity and mixed models are found to perform better than Smagorinsky and dynamic models in predicting local energy flux [23]. The authors mentioned that the former models could capture the backscatter and thus perform well. However, the dynamic model allows the backscattering effect. In earlier work, the deviation between DNS and LES in predicting statistical properties was attributed to the modeling error by Dritselis and Vlachos [24]. All these observations demand a rigorous analysis to check the local isotropy of small scales, which is the basis of LES formulation. In the present study, we want to explore whether particles can induce or modify the extent of anisotropy in dissipation or inertial range scales of the gas phase, and if so, what is the effect of increased anisotropy on the Kolmogorov constant? Direct numerical simulations (DNS) are performed for turbulent channel flow at Reynolds numbers 3300 and 5600 based on average gas velocity and channel width to answer the above questions. Kolmogorov constant has been computed following different methods using the simulations results.

II. SIMULATION PARAMETERS

The fluid phase has been considered incompressible and described by the Navier-Stokes equation. The discrete smooth point particles are simulated using Newton’s second law of motion. The detailed simulation procedure for calculation of feedback force, near-wall corrections in lift and drag, corrections for the undisturbed velocity field at the particle locations are discussed in our earlier works.
Particle-particle and particle-wall interactions have been accounted through the tracking of the particle center of mass. The particle to fluid density ratios considered in the present study is \(\approx 1000\) or higher. Therefore, the buoyancy and Basset history effects are neglected in the particle’s equation of motion.

The simulations have been performed in a vertical channel with \(8\pi\delta \cdot 2\delta \cdot (4/3)\pi\delta\) in streamwise \((x)\), wall-normal \((y)\), and spanwise \((z)\) directions, respectively. Where \(\delta\) is half channel width, no-slip boundary conditions are applied on the walls in the \(y\)-direction. The bulk Reynolds numbers \((Re_b = \rho_f \cdot \bar{u} \cdot 2\delta / \mu_f)\) are fixed at 3300 and 5600 based on the channel width \((2\delta)\) and average fluid velocity \((\bar{u})\), which corresponds to \(Re_\tau\) of 115 and 180 respectively based on the unladen frictional velocity and half-channel width. The pressure gradient is adjusted to maintain a constant bulk flow rate for all the solid volume loadings. The range of Stokes numbers are mentioned in Table I.

TABLE I: The different values of Stokes numbers \((St = \tau_p / \tau_f)\) where \(\tau_p = \rho_p d_p^2 / 18 \mu_f\), \(\tau_f = 2\delta / \mu\), \(\rho_p\) is the particle density, \(d_p\) is the particle diameter, \(\mu_f\) is the fluid dynamic viscosity, \(\delta\) is the half channel width and \(\bar{u}\) is the average fluid velocity.

| \(Re_b\) | \(\rho_p\) | St  |
|--------|---------|-----|
| 3300   | 1000    | 52.73 |
| 2000   | 105.47  |
| 4000   | 210.93  |
| 5600   | 1200    | 105.47 |
| 2400   | 210.93  |

III. RESULTS

Simulations are performed for two bulk Reynolds numbers of 3300 and 5600 for a range of volume fractions and different Stokes numbers with the same code as in Ref. [25]. It is observed that the turbulence attenuation increases with an increase in volume loading steadily up to a certain volume fraction, and then there is a sudden collapse in the turbulence intensities [22]. A detailed analysis to find out the effect of volume fraction and Stokes number on the turbulence attenuation is presented in earlier studies. In this work, we present the effect of turbulence attenuation on the extent of anisotropicity in turbulence intensities. The local isotropy of the small scales across the channel width can be accounted from the ratio of Kolmogorov time scale to mean shear time scale [3] [29] [30]. The necessary condition for the small scale to be isotropic was provided by Corrsin [29] as

\[
\left(\frac{\nu}{\epsilon}\right)^{1/2} \ll \frac{1}{S} \tag{1}
\]

or

\[
S_c^* \ll 1 \tag{2}
\]

where \(S = dU / dy\) is the mean shear rate, \(\epsilon\) is the mean energy dissipation rate and \(S_c^* = S (\nu / \epsilon)^{1/2}\). However, Antonia and Kim [31] mentioned that this condition is too restrictive and can be relaxed with \(S_c^* \leq 0.2\) for the small scales to be isotropic. Antonia and Kim [30] performed the DNS study for channel flow and found a value of \(S_c^* = 2.5\) at the wall and it reduces to a low value for \(y^+ > 60\). The \(S_c^*\) is plotted along the wall-normal direction for \(St = 105.47\) and 210.93 for Reynolds numbers of 3300 and 5600 respectively at different volume fractions \((\phi)\) as shown in Fig. I. It is observed that the \(S_c^*\) is 2.6 and 3.2 at the wall for \(Re_b = 3300\) and 5600 respectively for unladen cases. A
decrease in the $S^*$ is observed away from the wall. The $S^*$ increases across the channel width as the particle volume loading is increased, suggesting an increase in anisotropy of the small scales for particle-laden cases, shown in Fig. 1(a and b). The complete turbulence collapse is observed at $\phi = 0.001$ and $\phi = 0.0028$ for Reynolds number of 3300 and 5600 respectively [22, 25, 31]. The $S^*$ at the wall for $\phi = 0.0027$ (for $Re_b = 5600$) becomes almost 50% higher than of the unladen flow.

FIG. 1: The ratio of Kolmogorov time scale to mean shear time scale, $(S^* = S(\nu/\epsilon)^{1/2})$, for different volume fractions

![Graph](a) $Re_b = 3300, St = 105.47$

![Graph](b) $Re_b = 5600, St = 210.93$

FIG. 2: Second-order velocity structure functions multiplied with $r^2$ plotted for (a) unladen cases (b) for a range of volume fraction ($\phi$) at $y^+ = 180$ for $Re_b = 5600$ and $St = 210.93$. The dashed black line is $(1/15)$ line.

The local isotropy at the small scales can also be checked using the expressions of the second-order velocity structure-function and the mean energy dissipation rate for the homogeneous isotropic turbulence. In the limit of small $r$, the moment of longitudinal velocity fluctuation is defined with
the below expression \[2\]

\[
\langle (\delta u)^2 \rangle r^{-2} = \left( \frac{\partial u}{\partial x} \right)^2
\]  \hspace{1cm} (3)

where ‘r’ being the distance between the two points and, \( \delta u = u(x + r) - u(x) \), with \( u \) being the longitudinal fluctuations. The mean energy dissipation(\( \epsilon \)) for homogeneous isotropic turbulence \[19\] is defined as

\[
\epsilon = 15 \nu \left( \frac{\partial u}{\partial x} \right)^2
\]  \hspace{1cm} (4)

Using the above two equations, local isotropy of dissipation range can be explained as

\[
\langle (\delta u^*)^2 \rangle (r^*)^{-2} = \frac{1}{15}
\]  \hspace{1cm} (5)

where ‘(*)’ denote the non-dimensionalized quantities. \( \delta u \) is normalized using Kolmogorov velocity scale, \( u_k = (\nu \epsilon)^{1/4} \), and \( r \) is normalized with Kolmogorov length scale, \( \eta = \nu^{3/4}/\epsilon^{1/4} \). Using the DNS, we have computed \( \langle (\delta u^*)^2 \rangle (r^*)^{-2} \) and compared the unladen cases with the Eqn. \[5\] and experimental data of Antonia et al. \[4\] for both the Reynolds numbers in Fig. \[2\] (a). The profiles are presented for two locations, one in the near-wall region \( (y^+ \sim 15) \) and the other at the channel center \( (at \ y^+ \sim 180 \ for \ Re_b = 5600 \ and \ at \ y^+ \sim 115 \ for \ Re_b = 3300) \). A good agreement is observed for all the cases, Fig. \[2\] (a). It is observed in Fig. \[2\] (a) that for both the Reynolds numbers, the channel center location is nearly isotropic for unladen cases. However, the near-wall region follows the experimental trend \[4\] and is slightly away from the isotropic condition. In Fig. \[2\] (b), we have presented the effect of particle volume loading on local isotropy at channel center for \( St = 210.93 \) and \( Re_b = 5600 \). We find that deviation from local isotropy increases with an increase in particle volume loading.

A similar observation is found for other cases of Stokes number, channel locations, and Reynolds numbers, which are not shown here for the sake of brevity. It is mentioned by Antonia et al. \[4\] that the isotropy of the inertial range is unlikely to be achieved if there is no isotropy at the small
scales. Thus, it is expected that the decrease in the local isotropy of small scales with an increase in particle volume loading will affect the local isotropy of inertial range and the Kolmogorov constant which has been examined via second-order velocity structure-function and compensated spectra. The reduction in isotropy for particle-laden flows is related to the difference in the extent of attenuation of the components of fluid velocity fluctuations. It is observed that the decrease in the transverse velocity component is more than the streamwise component. Richter and Sullivan who mentioned that inertial particles reduce the ability of the carrier phase to transfer the momentum flux in a wall-normal direction. This increased anisotropy can affect the Kolmogorov constant for particle-laden cases which is analyzed hereafter.

The second-order velocity structure-function for \( r \) in the inertial range is defined as

\[
\langle (\delta u)^2 \rangle = C_2 (\epsilon r)^{2/3},
\]

where \( \epsilon \) is the mean viscous dissipation rate and \( \delta u = u(x+r) - u(x) \), with \( u \) being the longitudinal fluctuations. The \( r \) is described as, \( \eta \ll r \ll L \), with \( L \) as the integral length scale. In the above expression, \( C_2 \) is the Kolmogorov constant and angular brackets denote the time averaging. The second-order velocity structure-function for unladen case is plotted in Fig. 3 using the Eqn. 6. The (*) denotes the normalized variables. The profiles are plotted for both the Reynolds number at two locations, one in the near-wall region (\( y^+ \sim 15 \)) and the other at the channel center (at \( y^+ \sim 180 \) for \( Re_b = 5600 \) and at \( y^+ \sim 115 \) for \( Re_b = 3300 \)), and unladen cases are validated against the experimental data of Antonia et al. [4], shown in Fig. 3 (a). There is a good agreement between the experimental data of Antonia et al. [4] and the present DNS results for both the channel locations and Reynolds numbers. In Fig. 3 the peak value of \( C_2 \) (from plateau in the inertial range where statistical properties are only dependent on mean energy dissipation rate) is considered as Kolmogorov constant. The majority values of \( C_2 \) (peak value of the second-order velocity structure-function) in the literature reported are two or more. However, a lower value of \( C_2 \) happens due to lower Reynolds number or if there is a deviation of isotropy in dissipation and inertial-range.

In Fig. 3 (b), the second-order velocity structure function associated with the inertial range is shown for different particle volume loadings at the channel center for Reynolds number of 5600 with particles of \( St = 210.93 \). There is a decrease in the value of the second-order velocity structure-function at all the \( r^* \) locations for an increase in volume fraction. The peak value decreases nearly to 0.4 for \( \phi = 0.0027 \) from 1.2 for unladen flow which essentially indicates that the two-point correlation of fluid velocity fluctuation becomes weak with an increase in solid volume loading. The decrease in the value of the second-order velocity structure-function (Fig. 3(b)) is an indicator of the increase in anisotropy across the channel width.

In Fig. 4 the peak value \( C_2 \) of the second-order velocity structure-function is plotted for two channel locations, one in near-wall (\( y^+ \sim 15 \)) and the other at the channel center (at \( y^+ \sim 180 \) for \( Re_b = 5600 \) and at \( y^+ \sim 115 \) for \( Re_b = 3300 \)), for a range of volume fraction \( (\phi_{av}) \). In Fig. 4 (a and b), the \( C_2 \) is plotted for Reynolds number of 3300 and \( St = 52.73, 105.47 \). It is seen that the Kolmogorov constant \( (C_2) \) decreases significantly with increase in volume loading at both the channel locations. Also, with an increase in volume loading, the decrease in \( C_2 \) is more in the channel center location than in the near-wall region. Interestingly, a crossover occurs as the volume loading is increased, and the \( C_2 \) value becomes less in the channel center than the near-wall region for all the Stokes numbers. It is also observed that this crossover occurs at low volume loading for low Stokes number, Fig. 4 (a, b and c). In Fig. 4(d), the peak value of \( C_2 \) is presented for all the Stokes and Reynolds numbers over a range of volume fractions. An almost linear decrease in peak values of \( C_2 \) with volume fractions are observed. The observation suggests that the Kolmogorov constant...
is a function of particle volume fraction ($\phi$), bulk Reynolds number ($Re_b$) and wall-normal direction ($y^+$) for particle-laden flows. The effect of Stokes number on the variation of Kolmogorov constant seems insignificant for the range of bulk Reynolds numbers.

The second-order velocity structure-function (Eqn. 6) can be expressed as $E(k) = C k^{5/3} \epsilon^{-2/3}$ using the Fourier transformation [35] where $C$ is called as Kolmogorov constant [1]. Then, the Kolmogorov constant ($C$) is also plotted using the compensated spectra, $C = E(k) k^{5/3} \epsilon^{-2/3}$ in near channel center locations (for $y^+ = 150$ for $Re_b = 5600$ and $y^+ = 100$ for $Re_b = 3300$) for both the Reynolds numbers and different Stokes numbers, shown in Fig. 5. The value of $C$ decreases from 1.3 to 0.4 for a change in $\phi$ from $2 \times 10^{-4}$ to $2.7 \times 10^{-3}$ for $Re_b = 5600$ and approximately to 0.8 at $\phi = 0.0011$ for $Re_b = 3300$. It also observed in compensated spectra that the $C$ value decreases at a faster rate near the channel center location than the near-wall region (near-wall plots are not shown for the brevity).

The larger decrease in the Kolmogorov constant at the channel center compared to the near-wall
FIG. 5: Kolmogorov constant \( (C) \) plotted from compensated spectra at near channel center location for \( Re_b = 3300 \) at \( y^+ = 100 \) and for \( Re_b = 5600 \) at \( y^+ = 150 \). The solid lines without the symbols are the fitting curve.

FIG. 6: The ratio of fluctuating turbulent kinetic energy \( (k) \) and square of Kolmogorov velocity scale plotted across the channel width for different volume fraction.

region with an increase in particle loading can be explained by the ratio of velocity scales. The ratio of fluctuating turbulent kinetic energy to the square of the Kolmogorov velocity scale is plotted as a function of wall-normal direction in Fig. \( \text{(a and b)} \) for different particle volume loading. It is observed that the ratio decreases faster in the channel center than in the near-wall region with an increase in volume loading. A decrease in the ratio signifies the decrease in scale separation of the velocity scales, and consequently, it will increase the small scale anisotropy.

**IV. CONCLUSIONS**

We report that the Kolmogorov constant decreases with an increase in volume loading at low and moderate Reynolds numbers due to an increase in anisotropy in the inertial and the dissipation range. The Kolmogorov constant increases from the wall to the channel center for unladen wall-bounded flows. However, it is observed in the present study that in the case of particle-laden flows,
the Kolmogorov constant at the channel center is less than the near-wall region for high volume fraction. The small scales are assumed to be isotropic which is the basis of LES formulation such as Smagorinsky, dynamic Smagorinsky, spectral eddy viscosity, or models based on the energy spectra \[20\]. These small scales are no more isotropic for the particle-laden cases. Thus, care should be taken while implementing these models. The demonstrated variations of the Kolmogorov constant in the present study will be helpful to develop better turbulence models in LES and the stochastic modeling approach for particle-laden turbulent flows. It is worth mentioning the limitations of the present study. The simulations are carried out at low and moderate Reynolds numbers. Thus, simulations must be performed at a much higher Reynolds number to quantify the effect of particle volume loading on the assumption of Kolmogorov theory.
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