Effective vector-field theory and long-wavelength universality of the fractional quantum Hall effect

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Abstract
We report on an effective vector-field theory of the fractional quantum Hall effect that takes into account projection to Landau levels. The effective theory refers to neither the composite-boson nor composite-fermion picture, but properly reproduces the results consistent with them, thus revealing the universality of the long-wavelength characteristics of the quantum Hall states. In particular, the dual-field Lagrangian of Lee and Zhang is obtained, and an argument is given to verify the identification by Goldhaber and Jain of a composite fermion as a dressed electron. The generalization to double-layer systems is also remarked on.

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1. Introduction
The early studies of the fractional quantum Hall effect (FQHE) based on Laughlin’s variational wave functions [1-4] gave impetus to descriptions of the FQHE in terms of electron–flux composites. The Chern-Simons (CS) theories [5-7] realize the composite-boson and composite-fermion pictures of the FQHE and have been successful in describing the long-wavelength characteristics of the fractional quantum Hall states. The CS approaches, however, make no explicit use of projection to the lowest Landau level, a procedure which is crucial in the wave-function approach. One might naturally wonder if and how the former are compatible with the Landau-level structure and quenching of the electronic kinetic energy. The present paper is motivated to answer this question affirmatively.

In this paper we wish to report on an effective vector-field theory of the FQHE that takes into account projection to the Landau levels [8]. The effective theory, constructed via the electromagnetic responses and bosonization, does not refer to either the composite-boson or composite-fermion picture, but properly reproduces the results consistent with them, thus revealing the universality of the long-wavelength characteristics of the quantum Hall states. In particular, the dual-field Lagrangian of Lee and Zhang [6] is obtained. An argument is given to substantiate the identification by Goldhaber and Jain [9] of the composite fermion as a dressed electron.

2. Electromagnetic response of Hall electrons
Consider electrons in a plane with a perpendicular magnetic field $B > 0$, and study how they respond to weak external potentials $A_\mu(x) = (A_0, A)$. The eigenstates of a freely orbiting electron are Landau levels of energy $\omega_c(n + \frac{1}{2})$ with $\omega_c \equiv eB/m^*$ and $n = 0, 1, \cdots$. This level structure is modified in the presence of $A_\mu(x)$, and the effect of level mixing it causes is calculated by diagonalizing the Hamiltonian $H$ with respect to the true levels $\{n\}$. 

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Suppose now that an incompressible many-body state of uniform density $\bar{\rho}$ is formed via the Coulomb interaction (for $A_\mu = 0$). The projected Hamiltonian then serves to determine its response to weak electromagnetic potentials $A_\mu(x)$, with the result

$$L[A; \bar{\rho}] = \bar{\rho}[-A_0 - \ell^2 \frac{1}{2} A_\mu e^{\mu\nu\lambda} \partial_\nu A_\lambda + \frac{\ell^2}{2\omega_c} (A_{k0})^2 - \frac{\sigma(\nu)}{2m^*}(A_{12})^2] + O(\partial^3),$$

where $\ell \equiv 1/\sqrt{eB}$, $A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and the electric charge $e > 0$ has been suppressed by rescaling $eA_\mu \rightarrow A_\mu$; $e^{\mu\nu\lambda}$ is a totally-antisymmetric tensor with $e^{012} = 1$ and $\sigma(\nu)$ is some factor of $O(1)$, whose value is not needed in what follows.

3. Bosonization and effective response

The electromagnetic response we have calculated is essentially the partition function $W[A] = \exp(i \int d^3 x L[A; \bar{\rho}])$, which is written as a functional integral over the electron fields. Once such $W[A]$ is known it is possible to reconstruct it through the quantum fluctuations of a boson field. The rules of functional bosonization [10] tell us that the response $W[A]$ is reconstructed from a bosonic theory of a 3-vector field $b_\mu = (b_0, b_1, b_2)$, with the action $Z[b]$ given by Fourier transforming (given) $W[v]$,

$$e^{iZ[b]} \equiv \int [dv_\lambda] e^{i\int d^3 x b_\mu e^{\mu\nu\lambda} \partial_\nu v_\lambda} W[v].$$

The $b_\mu$ field is coupled to $A_\mu$ through the coupling $-A_\mu e^{\mu\nu\lambda} \partial_\nu b_\lambda$.

Let us now substitute $L[A; \bar{\rho}]$ into Eq. (2) and construct an equivalent bosonic theory that recovers it. The relevant functional integration is best carried out in the following way: First make a shift $v_\mu = v'_\mu + f_\mu[b]$ and choose $f_\mu[b]$ so as to decouple $v'_\mu$ and $b_\mu$ in the exponent. All the $b_\mu$ dependence is then isolated in the background piece $f_\mu = (1/\bar{\rho}\ell^2) b_\mu + O(\partial b)$, leading to the action $Z[b] = \int d^3 x L^{(0)}[b]$ with

$$L^{(0)}[b] = -\frac{1}{\ell^2} b_0 + \frac{\pi}{\nu} b_\mu e^{\mu\nu\lambda} \partial_\nu b_\lambda + \frac{\pi}{\nu \omega} (b_{k0})^2 - \frac{\sigma(\nu)}{\nu m^*} (b_{12})^2 + O(\partial^3),$$

where the filling factor $\nu = 2\pi/\ell^2 \bar{\rho}$ and $b_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$.

An advantage of the bosonic theory is that the Coulomb interaction is handled exactly. It further admits inclusion of new degrees of freedom, vortices, which describe Laughlin’s quasiparticle excitations over the FQH states. Eventually the bosonic theory that takes into account both the Coulomb interaction and vortices is described by the Lagrangian

$$L_{\text{eff}}[b] = -A_\mu e^{\mu\nu\lambda} \partial_\nu b_\lambda - 2\pi \tilde{J}_\mu b_\mu + L^{(0)}[b] - \frac{1}{2} \delta b_{12} V \delta b_{12},$$

where $\tilde{J}_\mu$ stands for the vortex 3-current; $\delta b_{12} V \delta b_{12} \equiv \int d^2 y \delta b_{12}(x) V(x - y) \delta b_{12}(y)$ denotes the (bosonized) Coulomb interaction and $\delta b_{12} = b_{12} - \bar{\rho}$. Upon quantization the CS term $bc\partial b$ in $L^{(0)}[b]$ combines with the kinetic term $(b_{10})^2$ to yield a “mass” gap $\omega$ while the $(b_{12})^2$ term yields only a tiny $O(\partial^2)$ correction to it.

This $L_{\text{eff}}[b]$ almost precisely agrees with the dual-field Lagrangian of Lee and Zhang (LZ) [6], derived within the Chern-Simons-Landau-Ginzburg (CSLG) theory. The only difference to $O(\partial^2)$ lies in the $(b_{12})^2$ term, which, however, is unimportant at long wavelengths.
The fact that we have practically arrived at the LZ dual Lagrangian without invoking the composite-boson picture would thus reveal the following: (1) The long-wavelength characteristics of the FQH states, as governed by the LZ dual Lagrangian, are determined universally by the filling factor and some single-electron characteristics, independent of the details of the FQH states. (2) In the CSLG theory the random-phase approximation properly recovers the effect of the Landau-level structure, crucial for determining the electromagnetic response.

It is instructive to read off the vortex charge from the $-2\pi\tilde{j}_\mu b_\mu$ coupling in $L_{\text{eff}}[b]$. The $b_\mu$ field acquires, in the presence of the electromagnetic coupling $-A_\mu\epsilon^{\mu\nu\rho}\partial_\nu b_\rho$, a background component, i.e., $b_\mu = b'_\mu + f_\mu$ with $f_\mu = (\nu/2\pi)A_\mu + O(\partial A)$, so that

$$-2\pi\tilde{j}_\mu b_\mu = -\nu\tilde{j}_\mu A_\mu + \cdots,$$

which reveals the vortex charge of $-\nu e$.

4. Composite fermions

The fermionic CS theory [7] realizes the composite-fermion description of the FQHE. There the composite fermions are visualized as electrons carrying an even number $2p$ of flux quanta of the CS field $c_\mu(x)$. The fractional quantum Hall states at the principal filling fractions $\nu \equiv 2\pi\ell^2\bar{\rho} = \nu_{\text{eff}}/(2p\nu_{\text{eff}} \pm 1)$ are thereby mapped into an integer quantum Hall state of composite fermions in the reduced field $B_{\text{eff}} = (1 - 2p\nu)B$ at $\nu_{\text{eff}} = 1, 2, \cdots$.

It is possible to cast, via the electromagnetic response of the composite fermion, this fermionic CS theory into an equivalent bosonic version, with the Lagrangian

$$L_{\text{eff}}^{\text{CF}}[b, c] = -e(A_\mu + c_\mu)\epsilon^{\mu\nu\rho}\partial_\nu b_\rho - 2\pi\tilde{j}_\mu b_\mu + L^{(0)}[b; B_{\text{eff}}] + L_{\text{CS}}[c],$$

where $L^{(0)}[b; B_{\text{eff}}]$ stands for $L^{(0)}[b]$ of Eq. (3) with $B \rightarrow B_{\text{eff}}$ and $\nu \rightarrow \nu_{\text{eff}}$.

Let us here try to eliminate $c_\mu$ from $L_{\text{eff}}^{\text{CF}}[b, c]$ and derive an equivalent theory of the $b_\mu$ field alone. We omit the detail here. The result precisely coincides with $L_{\text{eff}}[b]$ in Eq. (4), apart from (unimportant) $(b_1^2)^2$ terms. This verifies, in particular, that the composite-boson and composite-fermion descriptions of the FQHE as well as our (projection+bosonization) approach are perfectly consistent in the long-wavelength regime.

As for the nature of the composite fermion Goldhaber and Jain [9] identified it as a dressed electron with bare charge $-e$, and argued on the basis of Laughlin’s wave functions that the bare charge is screened in the composite-fermion medium to yield local charge $-\nu e$.

It is possible to substantiate such characteristics of composite fermions within the CS approach. First, applying Laughlin’s reasoning reveals that the vortex excitations over the composite-fermion ground state are nothing but the composite fermions (or holes) themselves. Counting of the vortex charges [or simply Eq. (6)] shows that each composite fermion has “bare” charge $-e$ in response to the “bare” field $A_\mu + c_\mu$. Note next that, upon integration over $b_\mu$ with $L_{\text{CF}}^{\text{CF}}[b, c]$, the CS field $c_\mu$ acquires a background piece proportional to $A_\mu$, leading to substantial dressing (or renormalization) of the bare field

$$A_\mu + c_\mu = \pm (\nu/\nu_{\text{eff}})A_\mu + \cdots,$$

which shows that a composite fermion has fractional charge

$$Q_{\text{CF}} = -(\nu/\nu_{\text{eff}})e = -e/(2p\nu_{\text{eff}} \pm 1)$$
in response to $A_{\mu}$. (In the above the ± sign refers to the sign of $B_{\text{eff}}/B$.) Hence the bare charge $-e$ coupled to $A_{\mu} + c_{\mu}$ is the same as the renormalized charge $Q_{\text{CF}}$ probed with $A_{\mu}$. This special dressing of the bare field has a natural consequence: The composite fermion with bare charge $-e$ feels an effective electric field $E_{y}^{A+c} \equiv E_{y} + c_{20} = \pm(\nu/\nu_{\text{eff}})E_{y} + \cdots$ in the effective magnetic field $B_{\text{eff}} = \pm(\nu/\nu_{\text{eff}})B$ so that it drifts with the same velocity as the electron, as it should,

$$v_{x}^{\text{drift}} = E_{y}^{A+c}/B_{\text{eff}} = E_{y}/B.$$  

5. Concluding remarks

In this paper we have presented an effective vector-field theory of the FQHE. It does not refer to either the composite-boson or composite-fermion picture, and simply supposes an incompressible FQH ground state of uniform density. Our approach by itself does not tell at which filling fraction $\nu$ such a state is realized. Instead, it tells us that once such an incompressible state is formed its long-wavelength characteristics are fixed universally, independent of the composite-boson and composite-fermion pictures. In this sense, our approach is complementary to the picture-specific CS approaches.

The CS approaches without Landau-level projection encounter some subtle difficulties [11], when generalized to double-layer systems where intra-Landau-level collective excitations play an important role in the low-energy regime. In our approach it turns out possible to construct an effective theory that properly incorporates the effect of the dipole-active inter-layer out-of-phase collective mode (via its electromagnetic response in the single-mode approximation) [12]. Details will be reported elsewhere.

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