THE ANISOTROPIC TWO-POINT CORRELATION FUNCTIONS OF THE NONLINEAR TRACELESS TIDAL FIELD IN THE PRINCIPAL-AXIS FRAME

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ABSTRACT

Galaxies on the largest scales of the universe are observed to be embedded in the filamentary cosmic web, which is shaped by the nonlinear tidal field. As an efficient tool to quantitatively describe the statistics of this cosmic web, we present the anisotropic two-point correlation functions of the nonlinear traceless tidal field in the principal-axis frame, which are measured using numerical data from an N-body simulation. We show that both the nonlinear density and traceless tidal fields are more strongly correlated along the directions perpendicular to the eigenvectors associated with the largest eigenvalues of the local tidal field. The correlation length scale of the traceless tidal field is found to be $\sim 20 h^{-1} \text{Mpc}$, which is much larger than that of the density field $\sim 5 h^{-1} \text{Mpc}$. We also provide analytic fitting formulae for the anisotropic correlation functions of the traceless tidal field, which turn out to be in excellent agreement with the numerical results. We expect that our numerical results and analytical formulae are useful to disentangle cosmological information from the filamentary network of the large-scale structures.

Key words: cosmology: theory – large-scale structure of universe

1. INTRODUCTION

On the largest scales, the spatial distribution of galaxies exhibits the pattern of a filamentary network, which is commonly referred to as the “cosmic web”. N-body simulations have long predicted the web-like pattern in the clustering of matter and galaxies (e.g., Shandarin & Zel’Dovich 1986), and recent large galaxy surveys have confirmed its presence (e.g., Colless et al. 2001), and theoretical endeavors have been able to explain the naturalness of its occurrence in the context of the cold dark matter (CDM) paradigm (Bond & Myers 1996; Bond et al. 1996). It is now well accepted that the cosmic web is a real, natural, and common phenomenon.

Nonetheless, the most crucial part has been missing for the comprehension of the cosmic web—an optimal statistical tool to quantify it. The standard N-point statistics, of course, can be used in principle to quantify it, but they cannot capture the essence of the cosmic web that it is a nonlinear manifestation of the primordial tidal interactions. It is because in N-point statistics what is emphasized is the role of gravity in establishing the high-order correlations of the density field. The cosmic web is a nonlinear manifestation of the spatial coherence of the tidal field (Bond et al. 1996). Thus, to quantify the cosmic web and capture its essence, it is necessary to have a statistical tool by which the role and effect of the tidal field and its spatial correlation are highlighted.

Patchy efforts have so far been made to account for the effect of the tidal field on structure formation. For instance, the failure of the Press–Schechter mass function (Press & Schechter 1974) has been attributed to the deviation of the gravitational process from spherical dynamics caused by the effect of the tidal forces (Monaco 1995; Audit et al. 1997; Lee & Shandarin 1998; Sheth et al. 2001). The galaxy angular momentum has been ascribed to the intertidal interactions with the surrounding matter distribution (Doroshkevich 1970; White 1984; Catelan & Theuns 1996; Lee & Pen 2000; Porciani et al. 2002a, 2002b; Navarro et al. 2004; Trujillo et al. 2006). The intrinsic galaxy alignments are also found to be explained due to the effect of nonlinear tidal interactions between the galaxies (Catelan et al. 2001; Crittenden et al. 2001; Jing 2002; Lee et al. 2005; Lee & Pen 2007, 2008; Lee & Park 2006; Aragón-Calvo et al. 2007). The ellipticity correlation functions have been developed to quantify the preferential orientations of galaxies relative to the surrounding large-scale structure (Lee et al. 2008; Lee 2006; Faltenbacher et al. 2009). The cosmic web, however, requires not a patchy but a unified way in which the multiple aspects of the nonlinear tidal field can be quantified.

The difficulty in quantifying the effect of the tidal field lies in the fact that the correlations of the tidal field are highly anisotropic even in the linear regime. Lee & Pen (2001) and Catelan & Porciani (2001) have for the first time derived analytically the anisotropic two-point correlations of the tidal field. But, their results were all expressed in the fixed Cartesian coordinate frame. To fully appreciate the anisotropic nature of the correlations of the tidal field, it is desirable to find the correlations of the tidal field in the principal-axis frame, since the filamentary cosmic web is generated by the anisotropic collapse of matter along the principal axes of the tidal field (Bond et al. 1996). Unfortunately, it is extremely difficult to derive analytically the correlations of even the linear tidal field in the principal-axis frame, which rotates from point to point.

The goal of this paper is to measure numerically the anisotropic two-point correlations of the nonlinear tidal field in the principal-axis frame and to provide an analytic fitting formula as a sufficient statistical tool for the description of the cosmic web. This paper is organized as follows. In Section 2, we provide a brief description of the numerical data. In Section 3, we present the numerical results and analytic fitting formula as well. In Section 4, we summarize the results and discuss the implications.

2. DATA

We use the N-body results derived by Hahn et al. (2007) using the tree-PM code GADGET-2 (Springel et al. 2005; Springel 2005). The simulations followed the evolution of $512^3$ particles
starting from redshift \( z = 52.4 \) to \( z = 0 \) in a periodic box of linear size \( 180 \, h^{-1} \, \text{Mpc} \). A flat \( \Lambda \)CDM cosmology is assumed with key parameters given as \( \Omega_m = 0.25 \) (matter density), \( \Omega_\Lambda = 0.75 \) (vacuum energy density), \( \Omega_b = 0.045 \) (baryon density), \( \sigma_8 = 0.9 \) (power spectrum amplitude), \( H_0 = 0.73 \) (Hubble constant), and \( n_s = 1 \) (spectral index).

Hahn et al. (2007) constructed the overdensity field \( \delta \) (dimensionless density contrast) on \( 1024^3 \) grids with the help of the Cloud-In-Cell interpolation method and smoothed it by a Gaussian kernel with a filtering radius \( R_s \). The peculiar gravitational potential field \( \phi \) was also derived from the overdensity field by solving the Poisson’s equation \( \nabla^2 \phi \propto \delta \). In simulations, the overdensity field is converted to the Fourier space with the help of a fast Fourier transform (FFT). The potential field is obtained as \( \phi \propto k^{-2} \delta \) where \( k \) represents the magnitude of the wave vector in the Fourier space. For a detailed description of the N-body simulations and the derivation of the density and potential fields, see Hahn et al. (2007). For the smoothing of the density (and potential) fields, we use the small smoothing scale \( R_s = 0.5 \, h^{-1} \, \text{Mpc} \) to minimize the effect of the smoothing on the determination of the correlations of the tidal fields.

The tidal tensor \( (T_{ij}) \) is defined as the Hessian of the peculiar gravitational potential \( T_{ij} \equiv \partial_i \partial_j \phi \). In Fourier space, it is written as \( T_{ij} = k_i k_j \phi \). Using the FFT again, we calculate the tidal tensor at each of the randomly selected 250,000 grid points. The sum of the diagonal components of \( T_{ij} \) (i.e., the trace of the tidal field) equals the dimensionless density contrast, \( \delta \). To sort out the true tidal effect from the gravitational effect, we calculate the traceless tidal field \( (\tilde{T}_{ij}) \) as \( \tilde{T}_{ij} = T_{ij} - (\delta/3)I_{ij} \) where \( I_{ij} \) denotes the identity tensor. Diagonalizing \( \tilde{T}_{ij} \) at each of the 250,000 sampling points, we find its three eigenvalues, \( \{\lambda_1, \lambda_2, \lambda_3\} \) (in a decreasing order, i.e., \( \lambda_1 > \lambda_2 > \lambda_3 \)), and the corresponding normalized eigenvectors with unit magnitude, \( \{\hat{e}_1, \hat{e}_2, \hat{e}_3\} \). Regardless of whether a given volume element is expanding or contracting (negative or positive \( \delta \)), the traceless tidal tensor \( \tilde{T}_{ij} \) gives the deformation of that volume element (relative to spherical expansion or contraction). By construction, we have \( \lambda_1 > 0 \) and \( \lambda_3 < 0 \). The second largest eigenvalue \( \lambda_2 \) can take on either sign. The maximal (relative) compression of a given volume element by gravity occurs along the direction parallel to \( \hat{e}_1 \) associated with \( \lambda_1 \). Figure 1 plots the probability density distributions of \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) as solid, dotted, and dashed line, respectively.

Figure 2 shows the isotropic two-point correlations \( \xi_{ij} (r) \) of the three eigenvalues of the traceless tidal field as a function of \( r \) (solid, dashed, and dotted line for \( \lambda_1, \lambda_2, \) and \( \lambda_3 \), respectively) at \( z = 0 \). This isotropic correlation function \( \xi_{ij} (r) \) is obtained by taking the average \( \langle \lambda_i (x) \lambda_j (x+r) \rangle \) over the points whose separation distances belong to the differential distance bin, \([r, r + dr]\). The tendency of \( \xi_{ij} (r) \) being flat for \( r < 2 \, h^{-1} \, \text{Mpc} \) is due to the effect of the smoothing of the tidal field. The isotropic two-point correlation of \( \delta \) is also plotted for comparison (thin solid line). As it can be seen, the correlations of \( \lambda_1 \) and \( \lambda_3 \) decrease with \( r \) much more slowly than that of \( \delta \). In other words, the largest and smallest eigenvalues of the traceless tidal field are coherent over larger scales than the density field \( \delta \). Let the correlation length scale \( l_c \) correspond to the distance at which the correlation strength becomes an order of magnitude lower than its value at \( r = 0 \). We find that \( l_c \approx 20 \, h^{-1} \, \text{Mpc} \) for...
Figure 3. Rescaled isotropic correlation functions of $\lambda_1$ (right panel), $\lambda_2$ (middle panel), and $\lambda_3$ (left panel) as a function of $r$ and their variations with polar angle $\theta$ in the principal-axis frame of the local tidal field. In each panel, the dots represent the numerical results, while the solid line is the analytic fitting formula. The errors represent one standard deviation in the measurement of the mean values.

Figure 4. Anisotropic correlation functions of $\lambda_1$ (right panel), $\lambda_2$ (middle panel), and $\lambda_3$ (left panel) as a function of $r$ and their variations with polar angle $\theta$ in the principal-axis frame of the local tidal field. The polar angle $\theta$ represents the angle between the position vector and the first eigenvector of the local tidal field. In each panel, the solid, dotted, dashed, long-dashed, and dot-long dashed lines correspond to the range $0^\circ \leq \theta < 15^\circ$, $15^\circ \leq \theta < 30^\circ$, $30^\circ \leq \theta < 45^\circ$, $45^\circ \leq \theta < 60^\circ$, $60^\circ \leq \theta < 75^\circ$, and $75^\circ \leq \theta < 90^\circ$, respectively. The anisotropic correlation functions of $\delta$ are also shown for comparison (thin solid lines).

Figure 5. Anisotropic correlation function of $\lambda_1$ (right panel), $\lambda_2$ (middle panel), and $\lambda_3$ (left panel) as a function of $\theta$ at $z = 0$. In each panel, the dots represent the numerical results while the solid line is the analytic fitting formula.

Table 1

| Parameters | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
|------------|-------------|-------------|-------------|
| $n$        | 1.28        | 2.76        | 1.65        |
| $r_0$      | 3.03        | 12.58       | 21.51       |
| $A$        | 0.37        | 0.41        | 0.38        |

$\lambda_1$ and $\lambda_3$, while $\delta$ and $\lambda_2$ has $l_c \approx 3h^{-1}$Mpc. The smallest eigenvalue $\lambda_3$ has the highest amplitude at all distances, while the second largest eigenvalue $\lambda_2$ has the lowest amplitude at $r \leq 10h^{-1}$Mpc. The correlation amplitude of $\delta$ is comparable to that of $\lambda_1$ at small distance $r \leq 2h^{-1}$Mpc but decreases much more rapidly with $r$ than $\lambda_1$, reaching the lowest value at $r \geq 10h^{-1}$Mpc.

For the rescaled isotropic correlation $\xi_{i}(r)/\xi_{i}(0)$, we find the following analytic formula:

$$\xi_{i}(r)/\xi_{i}(0) = \left(\frac{r}{r_0} + 1\right)^n r^{n-3}, \quad \text{for } \lambda_1, \lambda_3, \quad (2)$$

$$\xi_{i}(r)/\xi_{i}(0) = \left(\frac{r}{r_0} + 1\right)^{n} r^{1/2-n}, \quad \text{for } \lambda_2, \quad (3)$$

where $r_0$ and $n$ are two fitting parameters. By comparing the numerical results with the above analytic formula, we empirically determine the best-fit values of $r_0$ and $n$, which are listed in Table 1. Figure 3 shows the rescaled isotropic correlations of $\lambda_1$ (left panel), $\lambda_2$ (middle panel), and $\lambda_3$ (right panel) at $z = 0$. In each panel, the dots and the solid line represent the numerical results and the analytic model with best-fit parameters, respectively. The error at each bin is calculated as one standard deviation in the measurement of the mean value. As it can be seen, the analytic models (Equations (2)–(3)) fit the numerical results very well for $r \geq 2h^{-1}$ Mpc. On smaller distances $r < 2h^{-1}$ Mpc, the numerical results become flatter than the analytic curve due to the smoothing effect.

Figure 4 shows the anisotropic two-point correlations of $\lambda_1$ (left panel), $\lambda_2$ (middle panel), and $\lambda_3$ (right panel) as a function of $r$ and their variation with $\theta$ at $z = 0$. These anisotropic correlations $\xi_{i}(r, \theta)$ are obtained by taking the average of $\lambda_i(x + r)$ over the points whose separation distances belong to the differential distance bin $[r, r + dr]$ and at which the angles between the separation vectors and the first principal axes are in the differential range $[\theta, \theta + d\theta]$. In each panel, the thick solid, dotted, dashed, long-dashed, and dot-long dashed lines correspond to the range $0^\circ \leq \theta < 15^\circ$, $15^\circ \leq \theta < 30^\circ$, $30^\circ \leq \theta < 45^\circ$, $45^\circ \leq \theta < 60^\circ$, $60^\circ \leq \theta < 75^\circ$, and $75^\circ \leq \theta \leq 90^\circ$, respectively. The fluctuations of $\xi_{i}(r, \theta)$ in the small-$r$ section ($r \leq 2h^{-1}$Mpc) are due to the numerical noise. The anisotropic two-point correlation of $\delta$ is also plotted for comparison as thin lines in each panel. As it can be seen, both correlations of $\lambda$ and $\delta$ increase with $\theta$ in the principal-axis frame at all distances. That is, the correlations become stronger toward the directions normal to the first principal axes of the tidal field. This behavior is consistent with the scenario that matter maximally compress along the first principal axes of the tidal field, and thus is more likely to lie in the plane normal to the first principal axis of the tidal field. The variation of $\lambda_2$ with $\theta$ is stronger than that of $\lambda_1$ and $\lambda_3$. Note also that the degree of variation of $\xi_{i}(r, \theta)$ with $\theta$ decreases with $r$. In other words, the correlations tend to become less anisotropic at large distances ($r \geq 10h^{-1}$Mpc).

We also obtain $\xi_{i}(\theta)$ by taking the average of $\lambda_i(x)\lambda_i(x + r)$ over the points at which the angles between the first principal axis and the separation vectors belong to the differential angle bin, $[\theta, \theta + d\theta]$. The following analytic formulae are found to
fit the numerical results well:

\[
\frac{\xi(\theta)}{\xi(0)} = \frac{1}{1 - A \sin^2 \theta}, \quad \text{for} \quad \lambda_1, \lambda_3,
\]

\[
\frac{\xi(\theta)}{\xi(0)} = \frac{1}{1 - A \sin^4 \theta}, \quad \text{for} \quad \lambda_2,
\]

where \( A \) is an adjustable parameter whose best-fit value is determined via the \( \chi^2 \) statistics (see Table 1). Figure 5 shows the anisotropic correlation function of \( \lambda_1 \) (right panel), \( \lambda_2 \) (middle panel), and \( \lambda_3 \) (left panel) as a function of \( \theta \) in the principal-axis frame of the local tidal field. In each panel, the dots represent the numerical results while the solid line is the analytic fitting formula. As it can be seen, the analytic models fit the numerical results very well for each case. To improve readability, we do not show errorbars which are negligibly small anyway. These results show clearly how anisotropic the correlations of the eigenvalues of the traceless tidal field are with respect to the first principal axis. The difference between the correlations at \( \theta = 0 \) and \( \theta = 90^\circ \) reaches approximately 60% for \( \lambda_1 \) and \( \lambda_3 \), while it increases up to 70% for \( \lambda_2 \).

4. SUMMARY AND DISCUSSION

The web-like pattern in the large-scale matter distribution is an imprint produced by the effect of the nonlinear tidal field in the late universe. While the trace part of the tidal field is responsible for the collapse of matter, its traceless part deforms the cosmic volume elements, resisting the overall spherical expansion or contraction. In consequence, the traceless tidal field induces anisotropy in the large-scale correlations of matter distribution. An efficient statistical tool has long been looked for to quantitatively describe this cosmic web and to retrieve cosmological information encoded in the cosmic web.

We have numerically determined the two-point correlations of the three eigenvalues of the nonlinear traceless tidal field defined as the Hessian of the gravitational potential in the frame of the principal axes of the tidal field. The numerical findings indicate that the correlation functions of the traceless tidal field and the density field are all anisotropic relative to the principal axes. Their correlations have much larger correlation length scales than that of the density field and increase along the directions normal to the first principal axes of the tidal field. The analytic fitting formula for the correlation functions of the three eigenvalues of the tidal field are determined and found to fit the numerical results well. Our numerical and analytical results are thus able to provide a physical description of the large-scale filamentary structure.

It will be interesting to know how the anisotropic two-point correlations of the traceless tidal field depend on the background cosmology. Our future work is in this direction.

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