Investigation of Quantum Liquid in Quasi One Dimensional Bose-Einstein Condensate

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Recent observations of droplets in dipolar and binary Bose-Einstein condensate (BEC) motivates us to study the theory of droplet formation in details. Precisely, we are interested in investigating the possibility of droplet formation in a quasi one dimensional geometry. The recent observation have concluded that the droplets are stabilized by the competition between effective mean-field and beyond mean-field interaction. Hence, it is possible to map the effective equation of motion to a cubic-quartic nonlinear Schrödinger equation (CQNLSE). We obtain two analytical solutions of the modified Gross-Pitaevskii equation or CQNLSE and based on their stability we investigate the parameter regime for which droplets can form. The effective potential allows us to conclude about the regions of soliton domination and self bound droplet formations.

I. INTRODUCTION

The collective behavior of particles at ultra-low temperature is a fascinating topic ever since the experimental observation of atomic Bose-Einstein condensate (BEC) [1-3]. The experimental success had raised the curtain for new domains of research [4-6]. Over the time, the experimentalists have achieved greater control over the atomic alkali gases by means of magneto-optic set up. More over they can tune the atom-atom interaction via Feshbach resonance which effectively means, changing of s-wave scattering length by tuning an external magnetic field. These unique features have enabled multi facet research in ultra-cold atomic gases [7].

Very recently, an unique liquid like state in a BEC mixture [8] has been reported. This quirky new state demands serious attention because the prevailing conception of liquid state is highly influenced by the theory of van der Waals. However, these newly emerged droplets in ultra-cold and extremely dilute atomic gases does not explicitly follows the common theoretical perception as predicted by van der Waals [9]. These are purely quantum mechanical in nature and manifestation of quantum fluctuations [10, 11]. These droplets are small clusters of atoms self-bound by the interplay of attractive and repulsive forces. The origin of the attractive force can be modeled in the purview of standard mean-field theory whereas the repulsive force originates from the beyond mean-field correction [12]. The underlying theory relies on the Lee-Huang-Yang’s (LHY) correction [13] to the mean-field Gross-Pitaevskii (GP) equation [14, 15]. In a binary BEC, the mean-field and LHY term depend on the balance of inter- and intra-species coupling constants. Even before the experiment, it was proposed theoretically that, if the square of the inter-species coupling is greater than the product of the intra-species coupling then the collapse of the binary mixture is suppressed and a dilute liquid like droplet state emerges [16]. Emergence of this phase has opened up several new avenues as these droplets describe truly many-body quantum effect.

The current framework was first proposed while discussing the possibility of collapse of due to attractive interaction in Bose-Bose mixture [17]. However, first experimental observation was on dipolar condensate of $^{164}$Dy [10, 11] and subsequently the theoretical description came to light [17]. Later the droplets were observed for a mixture of two hyperfine state in $^{39}$K [8, 18]. This was followed by a observation of transition from bright soliton to quantum droplets [19]. Quantum droplet is also observed in heteronuclear bosonic mixute of $^{41}$K and $^{87}$Rb [20]. Off late a self-consistent derivation of modified GP equation where LHY correction is incorporated through quantum fluctuation has been proposed [21]. We have also noted significant theoretical description on the collective modes across the soliton-droplet cross over [12], existence of vortex quantum droplets [22], dynamics of purely one dimensional droplet [23].

Here, we plan to analytically analyze the two component BEC in quasi 1 dimensional system as described in Fig. 1. At this juncture, it is worth mentioning that a numerical investigation of quantum liquid in for dipolar BEC in quasi one dimensional geometry has very recently been reported [24]. The formulation of the current problem follows the prescription of Ref. [8], where the mixture of two hyper-fine states of $^{39}$K was studied assuming that both the components occupy the same spatial mode. This ensures the two component nonlinear Schrodinger equation is reduced to an effective one component equation. Then we reduce the 3+1 dimensional problem to 1+1 dimensional problem following the prescription of Ref. [25]. The quasi 1-D system now consists to two non-linear term where the cubic term defines the effective mean-field (EMF) two body interaction and the quartic term is the signature of beyond mean-field or LHY contribution. Our primary goal is to find out an analytical solution for this quasi 1-D cubic-quartic nonlinear Schrödinger equation (CQNLSE). The next objective is to explore the droplet state. Here we note that, till date an analytical description of CQNLSE is almost nonexistent except our recent work [26]. However, in that work we have assumed an external driving force which is re-
required to stabilize the localized solutions. In this piece of work we concentrate on an effective one component quasi 1-D CQNLS system and derive couple of analytical solutions. Then we perform a linear stability analysis using Vakhitov-Kolokolov (VK) criterion [27] and realize that one solution is unstable. The other solution, which is stable in a certain parameter region, shows signature of binding together to form a droplet like state. To validate the droplet formation we calculate the corresponding equilibrium density where the pressure is zero and can be noted as the transition point between bright soliton like state to droplet state. Further, the critical density beyond which the droplets breaks down is calculated and corroborated with the theory. We also demonstrate the existence of a density plateau for higher number of particles signifying the creation of quantum liquid.

In this paper we report our results in the following sequence, in Sec. II we elaborate the theoretical model corresponding to the binary condensate, its mapping to one component extended Gross-Pitaevskii (GP) equation and dimensional reduction of the system from 3+1 to 1+1 dimension. We obtain the analytical solutions corresponding to the extended GP equation and we analyze the stability of the solutions in Sec. III. The possibility of droplet formation is explicated in Sec. IV. We draw our conclusion in Sec. V.

II. THEORETICAL MODEL

Here, we consider homonuclear bosonic mixture similar to that of Ref. [5], where two hyperfine states of $^{39}$K took part in the experiment, in a quasi one dimensional geometry. The situation can be visualized via Fig. 1. where the bluish and reddish spheres represents the different species of atoms (atoms in two hyperfine states) distributed in a effectively one dimensional cigar shaped trap. The intra-species interactions ($a_{11}$ and $a_{22}$) are repulsive in nature and the inter-species interaction ($a_{12}$ or $a_{21}$) is attractive. The experimentally observed droplets are small clusters of atoms, self-bound by the balance between the attractive and repulsive forces. In binary BEC based on the strength of intra-species and inter-species interactions, it possible to define three distinct ground states. If $a_{11}, a_{22}$ and $a_{12}$ all are repulsive then one expects a transition between the miscible and the immiscible phase. However the mixture can also collapse if the inter-species interaction is negative enough to counter the repulsive intra-species interactions. It can be shown that if $a_{12} > \sqrt{a_{11}a_{22}}$ then the mixture is in immiscible phase, if $-\sqrt{a_{11}a_{22}} < a_{12} < \sqrt{a_{11}a_{22}}$ then the condensate is in miscible phase and when $-\sqrt{a_{11}a_{22}} > a_{12}$ then the condensate collapses. Now, it is possible to model the EMF interaction strength $\delta a \propto (a_{12} + \sqrt{a_{11}a_{22}})$ which is close to the collapse regime. If $\delta a \ll 0$ then the beyond mean-field (BMF) contribution becomes significant. Let us defines the BMF contribution as $\delta a' \propto (\sqrt{a_{11}a_{22}})^{5/2}$ [19]. In the miscible phase and close to the collapse point, we can describe the system with an effective single component GP equation by neglecting the spin excitations. This criterion can be full filled by considering the two components occupy same spatial mode. The resulting one component equation of motion can be defined as [8],

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[\frac{-\hbar^2}{2m} \nabla^2 + V_{\text{trap}} \right] U|\Psi|^2 + U'||\Psi|^3 \right] \Psi,$$

where $U = \frac{4\pi\hbar}{m} \delta a$, $U' = \frac{256\sqrt{\pi} \delta a'}{15m}$ and $m$ being the mass of the atoms. The equation is quite unique as there exists two types of nonlinearity, the usual cubic nonlinearity as well as an additional quartic nonlinearity. Here, it can be noted that nonlinear Schrödinger equation with cubic and quintic nonlinearity, $(\propto |\Psi|^3 |\Psi|^5)$ is quite common in nonlinear optics [28] and BEC [29]. However, it is not the same for quartic nonlinearity $(\propto |\Psi|^3 |\Psi|^5)$. At this point we also like to note that, in the early days of 21st century, the possibility of droplet formation was explored via quintic nonlinearity as well [30]. Nevertheless, the repulsive term possessing an unusual quartic dependence manifests the beyond mean-field contribution, which is not well studied till date. Therefore, we are primarily motivated to obtain an analytical solution for a NLSE which has both cubic and quartic nonlinearity.

Further, in Eq. (1), $V_{\text{trap}}$ describes the external potential. It is possible to describe the external potential in terms of transverse component ($V_T(y,z) = \frac{1}{2} m \omega_T^2 (y^2 + z^2)$) and longitudinal component ($V_L(x)$). Here, $\omega_L$ is the transverse trap frequency. The potential along the longitudinal direction is defined as, $V_L(x) = \frac{1}{2} m \omega_L^2 x^2$ with $\omega_L$ being the longitudinal trap frequency. In cigar shaped BEC the transverse trapping frequency ($\omega_T$) is typically more than 10 times the longitudinal frequency ($\omega_L$). This implies that the interaction energy of the atoms is much less than the kinetic energy in the transverse direction. Consequently, it is possible to reduce Eq. (1) to an effective one dimensional equation. In order to perform the dimensional reduction, we have made use of the following ansatz,

$$\Psi(r,t) = \frac{1}{\sqrt{2\pi a_B a_\perp}} \psi \left( \frac{r}{a_\perp}, \omega_\perp t \right) e^{-i\omega_\perp t - \frac{x^2 + z^2}{2a_\perp^2}}$$

where $a_B$ is Bohr radius and $a_\perp = \sqrt{\frac{\hbar}{m \omega_\perp}}$.  

FIG. 1. (Color online) Schematic representation of two-component BEC in a quasi one dimensional confinement. The bluish and reddish spheres presents atoms in the two different hyper-fine states. The intra-species and inter-species interactions are repulsive and attractive respectively.
Applying the ansatz from Eq. (2) in Eq. (1) we obtain
\[
\frac{i}{\partial t} \frac{\partial \psi(x, t)}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi(x, t)}{\partial x^2} + \frac{1}{2} K x^2 + \tilde{g} |\psi(x, t)|^2 + \tilde{g}' |\psi(x, t)|^3 \psi(x, t),
\]
where \( \tilde{g} = 2 \delta a/a_B, \) \( \tilde{g}' = (64 \sqrt{2}/15 \pi) \delta a'/a_B^{3/2} a_\perp \) and \( K = \omega_0^2/\omega_\perp^2. \) Here, it is important to note that \( x \) and \( t \) are now actually dimensionless, i.e. \( x \equiv x/a_\perp \) and \( t \equiv \omega_\perp t. \) From here onward, we will follow this dimensionless notation of \( x \) and \( t. \) In this article, our main focus is to explicate the interplay between EMF and BMF interactions hence we exclude the effect of harmonic confinement and the system can become quasi homogeneous. Experimentally the system can be reduced to a quasi homogeneous setup by considering transverse confinement is much stronger compared to the longitudinal confinement \( (\omega_0 \ll \omega_\perp) \) resulting \( K \rightarrow 0. \) The next objective is to obtain analytical solution for Eq. (3) assuming \( K = 0. \)

III. SOLUTIONS

In this section, we elaborate on the mathematical scheme to derive the analytical solution for the extended GP equation and analyze the stability of the obtained solutions. To start with, we write the wave function such a way that, \( \psi(x, t) = \rho(x, t) \exp \{i (\gamma(x, t) + \mu_0 t) \} \) where \( \rho(x, t) \) leads to the amplitude contribution and \( \gamma(x, t) \) is the non trivial phase, \( \mu_0 \) being the chemical potential. Applying this ansatz in Eq. (3) we yield two equations, namely imaginary and real equation respectively such that,
\[
\rho_t = -\chi_x \rho_x - \frac{1}{2} \chi_{xx} \rho \quad (4)
\]
\[
-\chi_\rho = -\frac{1}{2} \left( \mu_{xx} - \chi_{x}^2 \rho \right) + \tilde{g} \rho^3 + \tilde{g}' \rho^4 + \mu_0 \rho. \quad (5)
\]
Eq. (4) leads to the continuity equation and if we transform the equation in center of mass frame, i.e., \( \zeta = x - ut, \) then we obtain,
\[
\chi_\zeta = u + \frac{C_0}{\rho^2}. \quad (6)
\]
Here, \( u \) defines velocity of the wave and \( C_0 \) is the integration constant. Eq. (5) in comoving frame can be rewritten as,
\[
\chi_\zeta u \rho = -\frac{1}{2} \left( \mu_{zz} - \chi_{z}^2 \rho \right) + \tilde{g} \rho^3 + \tilde{g}' \rho^4 + \mu_0 \rho. \quad (7)
\]
Applying Eq. (6) in Eq. (7) we obtain,
\[
\rho_{\zeta} + (u^2 - 2 \mu_0) \rho - 3 \tilde{g} \rho^3 - 2 \tilde{g}' \rho^4 = 0
\]
or, \( \frac{d^2 \rho}{d \zeta^2} + (3 \tilde{g} \rho^3 - 2 \tilde{g}' \rho^4 + 2 \gamma) \rho = 0. \quad (8)\]

the quasi one dimensional (cigar-shaped) extended GP equation as noted below,

\[
\psi(x, t) = \rho(x, t) \exp \left\{ i \left( \gamma(x, t) + \mu_0 t \right) \right\} \psi(x, t),
\]

where \( \psi(x, t) \) is the non trivial phase, \( \mu_0 \) being the chemical potential. Applying this ansatz in Eq. (3) we yield two equations, namely imaginary and real equation respectively such that,

To derive Eq. (8) it is important to consider that the phase and amplitude are uncorrelated which allows us to set \( C_0 = 0 \) [31]. We also note that \( \gamma = u^2/2 - \mu_0. \) Further, we assume \( g = -2 \tilde{g} \) and \( g' = 2 \tilde{g}' \) implying two-body effective mean-field interaction is attractive and LHY contribution is repulsive. The minimum criterion for droplet formation is that these two interactions must be competing. Otherwise we will not be able to see any qualitative change in the behavior of the system.

We consider an ansatz solution such that,
\[
\rho(\zeta) = \frac{A}{1 + \sqrt{1 - A \cosh(\sqrt{\xi} \zeta)}}, \quad (9)
\]
where \( \xi^2 \) is the inverse of coherence length. Applying the ansatz in Eq. (8) we obtain a set of condition for which Eq. (8) is a solution of Eq. (9). The constrained conditions read,
\[
A = \frac{-\xi \pm \sqrt{-6 \gamma g + 3 g \xi + \xi^2}}{g},
\]
\[
\xi = 6 \gamma + g,
\]
\[
\gamma = \frac{g' - g}{2},
\]
\[
|g| = 2 g', \text{ or } |g| = 3 g'. \quad (10)
\]
The last equation in Eq. (10) implies that it is possible to obtain an analytical solution iff the beyond mean-field interaction is half or one third (i.e., $g' = |g|/2$ or $|g|/3$) of the effective mean-field interaction and repulsive in nature. Hence, we can write the solutions as,

$$\rho(\zeta) = \frac{12\mu_g}{1 + \sqrt{1 - 12\mu_g \cosh (\sqrt{\frac{\gamma}{2}})}} \quad \text{for} \ |g| = 2g' \quad (11)$$

$$= \frac{1 + 12\mu_g}{1 + \sqrt{12\mu_g \cosh (\sqrt{\frac{\gamma}{2}})}} \quad \text{for} \ |g| = 3g'. \quad (12)$$

Here, $\mu_g = \mu_0/g$. Using the constrained conditions, we can also evaluate $\xi$ which is actually related to the two body interaction via $\xi = -|g|/2$ or $|g|$. This implies that the localized structures can only sustain iff $g < 0$ or the effective mean-field interaction is attractive.

In Fig. 2 we have described the typical density profile corresponding to Eq. (12). At this point we have used a set of arbitrary values for plotting such as $|g| = 1, \mu_g = 0.04$ and $\alpha = 0.5$. However, we will use $|g| = 1$ for uniformity. The effect of variation of $|g|$ can be a matter of future interest. We must note here that $\mu_g > 0$ implying $\mu < 0$ and correspondingly $\gamma > 0$. The plot depicts propagation of bright soliton like localized wave with respect to space and time coordinates. The robustness of the localized wave in the time domain against any kind of perturbation can be of serious interest however in this paper from here onward we plan to consider only the static solutions. Our major objective is to understand the interplay between EMF and BMF interaction for formation of droplets and the role of chemical potential. Hence, we define the relationship between normalization $N$ and chemical potential $\mu_0$ as,

$$N = \left\{ \begin{array}{ll}
\sqrt{\frac{2}{9}} \left[ \sqrt{\mu_1} \ln \left( \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} - 1} \right) - 2\mu_1 \right] \\
\left( \frac{1 + \mu_1}{2} \right) \left[ \sqrt{\mu_1} \ln \left( \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} - 1} \right) - 2\mu_1 \right] \end{array} \right. \left[ \sqrt{\frac{1}{1 - \mu_1}} \ln \left( \frac{1 - \mu_1}{2\mu_1} \left( \sqrt{1 - \mu_1} + 1 \right) \right) - 2 \right] \quad (13)$$

Here, the first equation derived from Eq. (11) (assuming $\mu_1 < 1$). Likewise $N$ is again calculated from Eq. (12) and noted in the second equation. We also recall that $\mu_g = \mu_0/g$ here for convenience of calculation we have denoted, $12\mu_g = \mu_1$. $N$ can also be noted as the number of particle associated with the formation of localized wave and scaled by $N_0$ where $N_0$ defines the particle number obtained from the constant background density solution such that $N_0 = 2\alpha_B \left( \frac{15\pi a_i}{64} \right)^2 \left( \frac{\delta a}{\delta a} \right)^2$.

### Stability Analysis

Before proceeding to any discussion related to droplet formation, it is important that we evaluate the stability of the obtained solutions. For this purpose we intend to employ the well-known Vakhitov-Kolokolov (VK) criterion [27]. The approach we use is based on a modification of the soliton perturbation theory [32] under the condition of slow (almost adiabatic) evolution of solitons near the instability threshold. The VK criterion has been widely used in determining the stability of the solutions of nonlinear Schrödinger equation (NLSE), which predicts the parameter regime in chemical potential where the solitons amplitude can grow or decay exponentially [33]. The VK criterion states that a necessary stability condition is a positive slope in the dependence of the number of atoms on the chemical potential. If, $N_\mu > 0$, the solution is found to be stable and for $N_\mu < 0$, the solution is unstable. One must note that the condition $N_\mu = 0$ provides the instability threshold (TH) where, $\mu = \mu_{TH}$ [33, 35]. Here $N_\mu = \frac{\partial N}{\partial \mu}$.

In order to obtain the stability criterion of the given solutions, we calculate $N_{\mu_0}$ from Eq. (11) as well as Eq. (12) as $N_{\mu_0} = \frac{\partial N}{\partial \mu_0}$, A primary inspection leads to the conclusion that the first case or Eq. (11) does not lead to any stable solution. However, it is possible to obtain a region where $N_{\mu_0}$ is positive thereby suggesting a stable solution regime from the second solution or Eq. (12). The behavior of $N_{\mu_0}$ is noted in Fig. 3. We observe that the threshold value is $-0.06$ over which $N_{\mu_0} > 0$. However, $N_{\mu_0}$ diverges as $\mu_0 \to 0$. We are unable to find any region of stability for positive $\mu_0$. Hence, from here on we concentrate on the second solution only and investigate the liquid phase.

### IV. QUANTUM DROPLET

The signature of droplet formation can be obtained from the spacial profile of the obtained solution which we provide in Fig. 3. The figure depicts the characteristic static density profile of Eq. (12). However, the chemical potential $\mu_0$ is obtained by numerically solving the second equation of Eq. (13) for different norm $(N)$ at a fixed EMF $(|g| = 1)$. We observe a nonuniform shape for small $N$ where kinetic energy actually relevant for determining
the shape as quantum pressure dominates over the potential energy. The situation is analogous to usual single component bright soliton solution with cubic nonlinearity. However, as we increase \(N\) we start observing a flattening of the top or accumulation of uniform density. This signature is observed for \(N \geq 10\) and it reminds of a classical liquid where density starts becoming spatially uniform with progressive accumulation of droplets. In the figure we have normalized all the profiles by peak density \(n_0 = n(x = 0) = \left(\frac{1 + 12n_g}{1 + \sqrt{12n_g}}\right)^2\), which also happens to be the bulk value. It is evident from the figure that for higher \(N\) the density plateau approaches the constant bulk value of \(n_0\).

It must be noted here that similar observation of density plateau is already reported in Ref.\(^{[23]}\). However, the density plateau was noted for an one dimensional system which implies the governing equation was a quadratic-cubic NLSE or QCNLSE, where as in this investigation we have concentrated on a quasi one dimensional system resulting a dynamical equation governed by cubic-quartic nonlinearities which we name CQNLS. Another important differentiator is the nature of the nonlinearities. In the mentioned reference the quadratic and cubic nonlinearities are attractive and repulsive respectively. In comparison, we consider obtain our solution for attractive cubic and repulsive quartic nonlinearity. It must be noted that the interaction strengths of similar nature was involved in the experimental observation of quantum liquid in binary condensate.\(^{[8]}\)

As noted above, when the droplets starts accumulating and create a puddle the potential energy plays the dominating role over the kinetic energy. Thus, we now concentrate on the effective potential energy which can be defined as, \(\mathcal{E}_I = \frac{1}{2}g n^2 + 2/5g n^{5/2}\). The first term is derived from the EMF interaction and the second term is BMF contribution. If both the interactions have the same sign, no qualitative change in behavior occurs. However, the system can exhibit novel behavior if the interactions are competing as discussed in this work. When the effective mean-field interaction is low then beyond-mean-field corrections are not necessarily negligible. Now at low density the quantum depletion remains weak, so the LHY level approximation remains valid. Since in our case \(g < 0\) but \(g' > 0\), thus \(\mathcal{E}_I = -1/2gn^2 + 2/5gn^{5/2}\) describes the actual effective potential. The situation is mimicked in Fig.\(^5\) where the green dashed-dotted line describes the attractive EMF interaction and red dashed line depicts the repulsive BMF interaction. The resultant interaction is represented by yellow short dashed line which is initially negative at low density but slowly grows and becomes positive at relatively higher density. The level crossing point is defined as the critical density \(n_c\) after which the droplets are expected to collapses. This point can be evaluated by inserting \(\mathcal{E}_I = 0\) and it turns out, \(n_c = \frac{25}{16}g^2 = 14.06\) (since, \(g' = |g|/3\) which matches exactly with Fig.\(^6\).

The minimum of the interaction resultant signifies the equilibrium density \(n_{eq}\). At this point the pressure is zero which implies \(P = \mathcal{E}_I - n \frac{\partial \mathcal{E}_I}{\partial n} = 0\), resulting \(n_{eq} = \frac{25}{36}g^2 = 6.25\). However, in Fig.\(^6\) the minimum is at about 8.94. As the equilibrium density signifies the point from where the solitons starts combining together to form the droplet as BMF effect takes over the EMF effect therefore to understand this anomaly is important. Hence, we analyze the chemical potential. The critical chemical potential \(\mu_{g_c}\) and equilibrium chemical potential \(\mu_{g_{eq}}\) can be expressed from effective potential energy as, \(\frac{25}{64g^2} = 3.51\) and \(\frac{25}{2187g^2} = -1.04\) respectively. Using \(n_c\) value from Fig.\(^6\) if we recalculate \(\mu_{g_c}\) as \(\mu_{g_c} = -n_c + \kappa N_c^{3/2}\), we obtain a good agreement. The equilibrium chemical potential or \(\mu_{g_{eq}}\) from Fig.\(^6\)
turns out as $-0.02$. Solving Eq.\[13\] numerically, we also observed that when $N$ is relatively large $\mu_g \rightarrow 0^-$ resulting the emergence of flat plateau as shown in Fig.\[4\]. Hence, the equilibrium density obtained from Fig.\[6\] corroborates well with the numerical result yet the departure from the theoretical value can be attributed to the constrain condition which defines existence of exact solution only for $g/g' = 3$. Direct numerical solution of the CQNLSE can be more insightful in this respect which we plan to accomplish next.

Nevertheless it is well accepted that the signature of plateau is one of the important evidence of formation of the liquid like state \[9\]. Hence, it is now possible to conclude that the droplet formation starts from the equilibrium point of density where the negative energy supports the bound state formation and further accumulation of particle happens as we increase the density till the point of $n_c$. The left hand side of $n_{eq}$, i.e., $n < n_{eq}$, describes the bright soliton like localized states as described in Fig.\[4\]. Albeit, the system will collapse for $\mu_g = -1.5$ as solving $\mu_g = -n + k n^{3/2}$ for density leads to the condition of $\mu_g = -\frac{1}{0.02}$ when the density collapses. One must also note here that the solution is stable in the vicinity of the liquid like state as stability criterion leads to $-0.06 \leq \mu_0 \leq 0.0$ for unit $|g|$.

Since $E_f$ has explicit dependence on density and interaction strength therefore variation of $\mu_g$ does not make any significant change in $E_f$ as shown in Fig.\[6\]. Another note worthy point in our calculation is that, though the liquid formation starts from negative chemical potential however, the chemical potential corresponding to critical density is positive. This is an notable departure from the existing understanding of quantum droplets, however from stability point of view the solution is not stable for positive chemical potential.

The observation of droplets are very recent and therefore there exists a considerable void in understanding this unique state, both theoretically and experimentally. Till now, the droplet formation in quasi one dimensional geometry is not yet observed. Only very recently a numerical study on droplets in quasi one dimension for dipolar BEC has been reported \[21\]. To the best of our knowledge our analytical attempt is the very first foray in this direction albeit for binary condensate. Thus we look forward for an experimental probing of binary condensate in quasi one dimension to validate our findings. We also plan to study the modulational instability of the obtained solution and full numerical analysis of CQNLSE in the coming days. We hope this will enrich us in understanding the phase diagram in the quasi one dimension.

**V. CONCLUSION**

In this article, we study the aspect of droplet formation in a quasi one dimensional binary BEC. We start from a three dimensional Gross Pitaevskii equation along with beyond mean-field contribution. The equation is effectively single component as we assume both the component occupy the same spatial mode \[19\]. Then we transfer our focus towards a cigar shaped condensate for which a systematic dimensional reduction was carried out. The resulting quasi one dimensional equation is unique as it contains an additional quartic nonlinearity along with the usual cubic nonlinearity. It must be noted that there exists very little understanding of CQNLSE. Here, we derive a pair of analytical solution corresponding to the CQNLSE. For existence of the solution it is required to satisfy a specific relation between the effective mean-field and beyond mean-field interaction strengths. Then we perform stability analysis of the obtained solutions using V-K criterion which results discarding of one solution. A close inspection of the second solutions allows us to comment on the self bound droplets as well as liquid like state. The static profile of the droplets reveal a density plateau for higher particle number. We further analyze the effective potential energy and realize that the equilibrium chemical potential corroborates well with the numerically obtained chemical potential for flat spatial density. More over the analytical solution is stable in the region where the droplets form. We also see that the critical density obtained via effective potential energy and from the analytical solution do match exactly.

In recent description of droplet in one dimensional geometry, the mean-field interaction is repulsive where as the LHY term is attractive \[23\] however in our case we follow the original experimental proposition where effective two-body interaction is attractive and beyond mean-field contribution is repulsive. Our result has close semblance with Ref.\[23\] however there lies couple of fundamental differences, (i) the reported analysis is in one dimension where as we study quasi one dimensional geometry. This necessitates dealing with CQNLSE instead of QCNLSE. (ii) While addressing the issue of droplet formation in a one dimensional system \[23\], the nature of the competing interaction in reverse in comparison to our model as it followed the experimental description \[8\].
It is undeniable fact that the topic of quantum liquid is one of the most discussed topic in last one year or two. Therefore, we believe, our analysis on quasi one dimensional system will be exciting to many and will lead to the experimental verification of the current findings.

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