A simple way to achieve black-and-white designs in topology optimization

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Abstract. In order to obtain crisp black-and-white designs, it is vital to use filtering techniques in density-based topology optimization. However, the applying of filters result in a large number of gray transition zones between the solid and void phases in the final designs. To solve this problem, a dynamic parameter tuning method is proposed in this paper. With the approach, the filter radius can be dynamically adjusted during the optimization process, thereby suppressing the gray elements in the final designs. The proposed method is tested by the classic MBB beam problems. The experimental results show that the proposed method can effectively inhibit the appearance of gray transition areas in the optimization results.

1. Introduction
In density-based topology optimization[1-2], the applying of filters effectively removes the checkerboard patterns[3] and ensures mesh independence[4], but the averaging nature of the filters causes a large number of gray transition zones between the solid and void phases in the final designs. In order to obtain a crisp black-and-white design, the averaging operation is often followed by a Heaviside projection[5], which is an element-wise operation that maps the intermediate densities to solid or void by defining projection threshold[6]. Apart from Heaviside projection, an alternative is to use the morphology-based filters[7], which are a class of density filters based on morphological operators used in image processing.

In order to allow for the use of a gradient-based optimization scheme, the discrete forms of the two approaches are replaced by a similar continuous form[5, 7]. A parameter $\beta$ is introduced to control the smooth of the approximation. To get a suitable value of $\beta$, a continuation approach is often used[8]. Although these two approaches have been proven to be effective in solving many of the known topology optimization problems, they are not as convenient as the original sensitivity and density filters due to the large amount of calculation[9] and the cumbersome parameter tuning process[10]. We reconsider the efficient sensitivity and density filters in this paper, a dynamic parameter tuning method is proposed to achieve the parameter tuning during the optimization process. The experimental results show that the original sensitivity and density filters can achieve almost clear black-and-white designs ($M_{\text{vol}} < 1\%$) without introducing additional constrains.

2. The proposed method
This section discusses the characteristics of sensitivity and density filters, and then introduces the dynamic parameter tuning method used for filter radius adjustment. The sensitivity filter modifies the sensitivities $\partial c/\partial x_e$ as follows:
\[
\frac{\Delta c}{\Delta x} = \frac{1}{\max(\gamma, x_i)} \sum_{i \in N_e} H_{ei} \sum_{i \in N_e} H_{ie} x_i, \frac{\Delta c}{\Delta x} 
\]  
(1)

where \(x_i\) are the design variable densities, \(N_e\) is the set of elements \(i\) for which the center-to-center distance \(\Delta(e, i)\) to element \(e\) is smaller than the filter radius \(r_{\text{min}}\), \(\gamma\) is a small positive number introduced in order to avoid division by zero\[9\].

The density filter transforms the original densities \(x_i\) as follows:

\[
\bar{x}_e = \frac{1}{\sum_{i \in N_e} H_{ei} \sum_{i \in N_e} H_{ie}} x_i 
\]  
(2)

In both filters, the convolution operator \(H_{ei}\) is defined as:

\[
H_{ei} = \max(0, r_{\text{min}} - \Delta(e, i)) 
\]  
(3)

It shows from equation (3) that all elements of \(H_{ei}\) take the same value when \(r_{\text{min}}\) is less than or equal to 1. In this case, the filtered sensitivity shown in equation (1) converges to the original sensitivity, and the same is true for density filter shown in equation (2). Taking the classic MBB beam topology optimization problem as an example, the density filter shown in equation (2) is used for the filtering process. In this experiment, two mesh sizes are tested, namely \(60 \times 20\) and \(150 \times 50\), the filter radii \(r_{\text{min}}\) are set to 1, the volume fractions are limited to 0.5, the penalization powers are set to 3. The optimization results of this experiment are displayed in Figure 1.

![Figure 1 Unfiltered MBB optimization results with (a) 60 × 20 mesh and (b) 150 × 50 mesh](image)

As seen in Figure 1, the optimization results are clear black-and-white designs, despite the presence of checkerboards and mesh dependence. The \(M_{nd}\) values of Figure 1(a) and Figure 1(b) are 0.75% and 0.15%, respectively. This indicates that if the optimization is carried out with filter radius \(r_{\text{min}} = 1\), the final design could be a clear black-and-white design, and the topological information can also be preserved. The evaluation criteria \(M_{nd}\) used in this experiment can be expressed as:

\[
M_{nd} = \frac{\sum_{i=1}^{N} 4\bar{x}_i (1 - \bar{x}_i)}{N} \times 100\%
\]  
(4)

where \(N\) represent the elements used to discrete the design domain.

In another experiment, the density filter shown in equation (2) is used for filtering, mesh size is set to \(150 \times 50\), volume fraction is limited to 0.5, and penalization power is set to 3. Firstly, the MBB beam is optimized with \(r_{\text{min}} = 3\), then the optimization result is re-optimized with \(r_{\text{min}} = 1\), the final designs are displayed in Figure 2.
The $M_{nd}$ value of optimization result shown in Figures 2(a) and 2(b) are 23.38% and 0.21%, respectively. As seen in Figure 2(b), the gray transition zones are greatly removed in the re-optimized design, and the topological features of the former design are also preserved.

In this experiment, the crisp black-and-white design is obtained by a two-step operation. The proposed dynamic parameter tuning method is based on this two-step operation. With the method, the following expression is used to gradually adjust the filter radius during the optimization process:

$$r_{\text{min}} = \lambda r_{\text{min}}^*, \quad r_{\text{min}} > 1, \quad 0 \leq \lambda \leq 1$$

where $\lambda$ is the decline power. The use of continuous dynamic parameter tuning method raises two issues: when to implement the dynamic parameter tuning method and how to choose the value of $\lambda$. These issues will be addressed in the next section.

### 3. Test examples

The test examples are implemented in the 88 line MATLAB code[9], the sensitivity filter shown in equation (1) is used in the experiment. The mesh size is set to 150 × 50, the volume fraction is limited to 0.5, the penalization power is set to 3, and the filter radius is set to 3.4. In test 1, we fix $\lambda$ to 0.95, and perform equation (5) after a certain number of iterations, namely 15, 95 and 200. The original topology phases are shown in Figure 3(a), and final designs with the use of the proposed method are displayed in Figure 3(b).

The $M_{nd}$ values of the final designs are 0.04%, 0.14% and 0 (0.00000015%), respectively, which indicates the final designs are almost clear black-and-white designs. It is also observed that the topological structures of final designs are closely related to the original topology phases. Therefore, the proposed method can be implemented when the objective function reaches the constant level. This provides a possible way to choose the starting point of the proposed method.

In test 2, the 15th iteration is chosen as starting point of the proposed method. The reason is that the topological structures are unformed at 15th iteration, and there are a lot of gray areas in the topology phase (see Figure 3(a) left). The decline power $\lambda$ are set to different values, which are 0.9, 0.8 and 0.7, respectively. The final designs are displayed in Figure 4.
Figure 4 Final designs of test 2 with (a) $\lambda = 0.9$, (b) $\lambda = 0.8$ and (c) $\lambda = 0.7$

The $M_{nd}$ values of the final designs are 0.26%, 0.13% and 0.12%, respectively. As seen in Figure 4, the slender features are retained when $\lambda$ is set to a lower value, and no checkerboards are generated.

The process data of the test examples are summarized in Table 1.

### Table 1 Optimization data of test examples

| Examples | Parameters | Object function | $M_{nd}$ (%) | Iterations |
|----------|------------|-----------------|--------------|------------|
| Original design | - | 205.7319 | 17.37 | 826 |
| Test 1 | starting point = 15 | 189.9684 | 0.04 | 65 |
| Test 1 | starting point = 95 | 188.0342 | 0.14 | 152 |
| Test 1 | starting point = 200 | 188.6567 | 0.00 | 260 |
| Test 2 | $\lambda = 0.9$ | 189.4106 | 0.26 | 54 |
| Test 2 | $\lambda = 0.8$ | 190.6516 | 0.13 | 49 |
| Test 2 | $\lambda = 0.7$ | 191.9395 | 0.12 | 44 |

Figure 5 displays the final design of $r_{\text{min}} = 6$, the density filter shown in equation (2) is used for filtering, other parameter settings are the same as the test example 1, $\lambda$ is set to 0.8, 95th iteration is selected as the starting point.

Figure 5 Final design of $r_{\text{min}} = 6$ with the use of proposed method

The object function value is 192.2168, the $M_{nd}$ value is 0.05%, and the iteration number is 124.

### 4. Conclusions

This paper proposes a dynamic parameter tuning method to obtain crisp black-and-white designs with the used of sensitivity and density filters in density-based topology optimization. The implementation of the method is addressed, and the performance of the method is investigated.

The proposed method is tested through the classic MBB beam problem. It is shown that the method is effective in eliminating the gray transition areas. It only requires a small revise of the filter radius, which makes it very easy to implement. By using this method, the sensitivity and density filters can easily achieve clear black-and-white designs in the topology optimization results ($M_{nd} < 1\%$).

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