Influence of Warping on Bearing Capacity of Steel I-Beam with Non-Fork Support

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Abstract. The European Standard 3 does not provide a direct answer about how to verify the bearing capacity of a beam with non-fork support. For a pinned beam it assumes, that there is no constraint on lateral bending and warping or full restraint. But in the most of design cases the constrainment is somewhere between them. There are some guidelines in scientific literature and computer programs that enhance the calculation of the value of a Critical Moment (which governs the Lateral-Torsional Buckling and as a result – the Bearing Capacity of Bending) for a flexible support. But there is no official information about which of method is the best for a certain case. To confine the variables only an influence of warping on a Lateral-Torsional Buckling was taken into the consideration. It was created by adding an endplate on one or both ends of a beam. Few approaches to calculate the Critical Moment were compared. A beam based on cross section of IS-300/150/10.7/7.1 with the length of 5.0m were taken into the consideration. The first method is the General Formula from pre-code of the European Standard 3 (1992) with the additional equations for calculating the stiffness of endplates according to Lindner (1994). The second approach bases on the General Formula, but with modification of C1 factor derived by Lopez, Yong and Serna (2006). The third method uses very similar equation to the G. F., but with other value of C1 factor. It was derived by Lindner (1994). Those approximated formulas were compared with Finite Element Method calculation. It was conducted using three different computer programs. In the first one, ABAQUS CAE, the spatial shell model of a beam was created. An influence of a type of a discretisation on results was validated. Secondly, LTBeamN, the dedicated program for calculating the value of Critical Moment written by CTICM, was used. Thirdly, computing was made using the RFEM – an application for civil engineers which allows to create models of beams with seven degrees of freedom in each node. The results show that an endplate’s influence on a value of a Critical Moment is significant. Even the thinnest plates can noticeably increase the bearing capacity of a cross-section. But over the certain value of plate’s thickness there is no further growth of the Critical Moment value. The results from equations based on the General Formula and calculations with the FEM are approximate for endplates below 4.0cm of thickness. It is very important to take few approaches to discretise the model of beam with endplates using ABAQUS, because there could be a major difference between them in outcomes.

1. Introduction

In a bending of a steel, pinned I-beam the lateral-torsional buckling is correlated with the out-of-plane buckle of a compressed flange. This phenomenon can occur where the stiffness about the minor axis is relatively small in comparison to the stiffness about the major axis. It is determined by a parameter $M_{cr}$
(elastic critical moment). The lower the value of the $M_{cr}$, the higher the relative slenderness, therefore lower plastic moment capacity of the cross-section. In Eurocode 3 [1] the influence of lateral-torsional buckling is denoted as a reduction factor $\chi_{LT}$.

$$M_{b,Rd} = \chi_{LT} \frac{W_y f_y}{\gamma_{M1}}$$  \hspace{1cm} (1)

where:
- $M_{b,Rd}$ – design buckling resistance moment,
- $\chi_{LT}$ – reduction factor for lateral-torsional buckling,
- $W_y$ – section modulus about y-y axis,
- $f_y$ – yield strength of steel,
- $\gamma_{M1}$ – partial safety factor.

The calculation of $\chi_{LT}$ is necessary to determine the value of $M_{cr}$. However, the European Standard 3 does not provide applicable equation. In the pre-code [2] and decommissioned Polish code [3] the critical moment is determined by general formula:

$$M_{cr} = C_1 \pi^2 E I_z \left( \frac{k_z}{k_w} \right)^2 \left( \frac{L}{I_w} \right)^2 \left( \frac{G I_T}{E I_z} \right) + \left( C_2 \cdot z_g - C_3 \cdot z_j \right)^2 - C_2 \cdot z_g - C_3 \cdot z_j$$  \hspace{1cm} (2)

where:
- $M_{cr}$ – the critical moment of bending,
- $C_1$, $C_2$, $C_3$ – factors depending on the loading and end restraint conditions,
- $k_z$ – the effective length factor of lateral bending,
- $k_w$ – the effective length factor of warping,
- $E$ – the Young’s modulus,
- $G$ – the shear modulus,
- $L$ – the length of the beam,
- $I_w$ – the warping constant,
- $I_T$ – the torsion constant,
- $I_z$ – the second moment of area about the weak axis,
- $z_g$ – the distance between the point of load application and the shear center about the strong axis,
- $z_j$ – the distance between the point of load application and the shear center about the weak axis.

Simultaneously as in buckling, the equation contains additional effective length factor parameters that refer to end rotation on plan ($k_z$) and end warping ($k_w$). Values of $k_z$ and $k_w$ vary from 0.5 for full fixity to 1.0 for no fixity, with 0.7 for one end fixed and one end free [4]. The code however does not determine the terms “fixity” and “no fixity” and their classification. This paper analyses the influence of end-plate addition on warping, thus value of critical moment and bearing capacity of pinned I-beam.

The simplified equation (2) is derived from a differential equation from [5]. If there is a beam with ends prevented from freely warping or lateral bending the equation is transcendental and it can be solved only by trial and error method. That is why there are still computational factors (for instance $k_z$ and $k_w$) in the equation for calculation the critical moment.

2. Material and Methods

2.1. Beam

A steel, IS-300/150/10.7/7.1 pinned I-beam with the length of 5.0 m was used for analytical considerations. The cross section corresponds to the IPE300 excluding the radius in the connection between the flanges and a web (figure 1). The seventeen different types of the end-plates fixed at the
end or ends of the beam have been taken into account. The study was conducted on models without, with single or end-plate at both ends with thickness of 5 mm, 8 mm, 10 mm, 12 mm, 15 mm and increasing up to 100 mm. Initial assumptions were adopted that the beam and plates are made with S355JR steel and work in elastic state with stresses under 355 MPa. The element is under influence of a evenly distributed load of 10 kN/m applied to the centre of the top flange’s face (figure 2).

Figure 1. Geometry of the beam

2.2. Modified General Formula
According to Eq. 1) and the research of Lindner [6] the parameters $k_c$ and $k_w$ can be calculated using Saint-Venant stiffness of the endplate:

$$s_{st,ini} = GL_f (h - t_f) = G \frac{bt_f^3}{3} (h - t_f)$$

where:
- $s_{st,ini}$ – the stiffness of an endplate,
- $t_f$ – the thickness of flanges,
- $h$ – the height of cross section of a beam,
- $b$ – the width of cross section of a beam,
- $t_p$ – the thickness of an endplate.

Comparison of Lindner’s equation with other taken from [7]:

$$k_w = \mu_{st} = 0.5 + 0.28 \kappa_{st} + 0.21 \kappa_{st}^2$$

and [8]:

$$k_w = \mu_{st} = 0.5 + 0.14 (\kappa_{st,1} + \kappa_{st,2}) + 0.055 (\kappa_{st,1} + \kappa_{st,2})^2$$

$$\kappa_{st} = \frac{1}{1 + \frac{s_{st,ini} l}{2EI_{st}}}$$

where:
- $\kappa_{st}$ – the coefficient of the stiffness of fixations on both ends,
- $\kappa_{st,1}, \kappa_{st,2}$ – the coefficients of the stiffness of fixations - respectively in the left and right support,

The equation (4) assumes that there are end-plates with the same thickness at both sides of the beam. In (5) assumes the possibility of different end-plates or a single end-plate.

In steel, pinned I-beam with end-plates Lindner [6] advice to assume the value of $k_c$ as 1.0. According to [2] and based on the bending moment diagram the values of $C_1$ and $C_2$ should be taken respectively as 1.132 and 0.459. The IS-beam cross-section is double symmetric, meaning that $z_f=0$.

2.3. $C_i$ factor by Lopez, Yong and Serna
According to [1] the $C_i$ factor depends only on the bending moment diagram and effective length factor in end rotation on plan. Lopez et. al. in [8] derived a formula to determine $C_1$ factor which take into account the coefficients $k$ and $k_w$. 


where:

- $k$ – the coefficient of stiffness of fixation at both supports for warping and lateral bending,
- $k_1$ – the coefficient of stiffness of fixation at the left support for warping and lateral bending,
- $k_2$ – the coefficient of stiffness of fixation at the right support for warping and lateral bending;
- $M_1, M_2, M_3, M_4, M_5, M_{\text{max}}$ – the values from the bending moment diagram.

There is however an inconvenience, as the $k_1$ and $k_2$ factors depends on both lateral bending and warping condition respectively on the left and right side. For this research an assumption was made, that $k_1$ and $k_2$ are the arithmetic mean of both $k_z$ and $k_w$ from (2). It is a great simplification and should be taken into account in further considerations. More accurate relation between $k$ and $k_w$ should be determined.

### 2.4. Lindner’s formula

For a pinned I-beam with end-plates Lindner in [4] proposed an formula:

$$\alpha_i = 1 - k_2 \quad \alpha_2 = \frac{k_1^3}{k_2} \quad \alpha_3 = 5 \left( \frac{1}{k_1} + \frac{1}{k_2} \right) \quad \alpha_4 = \frac{5}{k_1^3} \quad \alpha_1 = 1 - k_1$$

Equation (12) works for I-beams and two identical end-plates on both sides. There are no details how to proceed with I-beams with only one end-plate or end-plates with different thickness. What is important – there is an inversion of the vertical axis compared previous formulas (the values of the $z_g$ coefficient are inverted for downward forces applied above the shear center).

### 2.5. LTBeam and RFEM

In 2002 the CTICM (Le Centre Technique de la Construction Métallique) from France released a software founded by European Community for Steel and Coal to help calculate the critical moment of steel I-beams. This computer program is based on Finite Element Method. It applies to straight single- or multi-span beams, under simple bending about their strong axis with doubly- or mono-symmetrical cross-sections about the plane of bending. Lateral restraints against Lateral-Torsional-Buckling (LTB) may be discrete or continuous and may concern the lateral displacement, the torsional rotation, the
lateral flexural rotation and the warping. Similarly to loading the lateral restraints can be applied at a distance above or beneath the shear centre of the cross-sections. The LTBeam v. 1.0.11 however has a problem with defining the $\theta'$ value, which corresponds to a warping of an end of a beam. This matter came out during the calculations for this paper and are currently being fixed by the CTICM.

The CTICM released the other version of LTBeam called LTBeamN, where it is possible to put a normal load and determine the buckling of an element. The new version of software correctly computes the critical moment from given value of $\theta'$ The results acquired from LTBeamN v. 1.0.3 were taken for comparison. The RFEM is a complex software for structural analysis in 2D and 3D with implemented Finite Element Method. There are 7 degrees of freedom (DOF) in each node (3 diagonals, 3 rotational and warping).

2.6. ABAQUS

In this environment the I-beam was modelled as a shell structure whereas the end-plates as a solid. The 3D I-beam has to be modelled concerning the boundary conditions of a fork support (as shown of figure 2).

![Figure 2. Scheme of pinned I-beam supports assumed in ABAQUS software – taken from [10]](image)

When two types of structure are connected (the long beam and proportionally small object such as end-plate) it is important to assume the proper type of mesh. The higher the number of Finite Elements the longer the computation time. Otherwise, the bigger FE the less accurate the results. Different types of mesh were taken into consideration in this study.

3. Results and Discussion

3.1. General Formula

According to the modified general formula from [2] and (equation 3) the difference in cross-section’s bending capacity of a fork and full-fixed support is about 30 kNm. For a steel end-plate of 5 cm and more, the $k_w$ factor has similar values as in full-fixation. For plates between 1-3 cm there is a considerable increase of a critical moment (figure 4).
3.2. Influence of a type of discretisation on results in ABAQUS

The shell model assumed for the IS-beam in ABAQUS was created by extrusion of a flat I-shape in two ways. The first method (mesh #1) determines the I-shape by its outer flanges. The second (mesh #2) assumed the location of shells in the geometric centre of flanges. Different types and dimensions of Finite Elements of the end-plates were taken into account. Two sizes of S4R shell mesh (square shaped with size of 1.0 and 2.5cm) and two types of solid mesh (C3D8R and C3D20R, cube size of 2.5cm) were taken into consideration. The model with C3D20R mesh on end-plates and S4D mesh on the beam has obtained the best convergence with the General Formula. The results were also approximate to results obtained with smaller, more precise mesh. There is a characteristic pitch in the development of diagram for model with C3D8R mesh. It occurs when the thickness increases from 4.0 to 5.0 cm, doubling the number of FE.
3.3. Beam with one end-plate

In case of a beam with one end-plate with a thickness ranging between 0.0 and 3.0 cm the calculations similar outcomes. With increase of plate’s thickness the differences in critical moment values acquired with different methods increases. The biggest value of a critical moment was obtained from LTBeamN and RFEM. Almost identical results were acquired from ABAQUS, General Formula and it’s modification by Lopez. These three methods provide the most secure approach to the issue. There is no result from the Lindner formula because it only regards beams with end-plates on both sides.
3.4. Beam with two end-plates

The very similar curve of a diagram was obtained for a beam with two end-plates of the same thicknesses. The Formula presented Lindner approximate outcomes to General Formula and results from ABAQUS. The RFEM and LTBeamN gave the biggest values of $M_{cr}$ for plates above 4.0cm thick.

![Graph showing influence of end-plates at the left ($t_{p,1}$) and right side ($t_{p,2}$) of the beam on the $k_w$ factor.](image)

**Figure 7.** An influence of end-plates at the left ($t_{p,1}$) and right side ($t_{p,2}$) of the beam on the $k_w$ factor.

4. Conclusion

The end-plate’s influence on a value of a critical moment is significant. Even the thinnest plates widely applied in civil engineering (about 1.0-3.0 cm thick) can noticeably increase the bearing capacity of a cross-section. The similar results were obtained in [11] and [12]. There was an 5-20% increase of load bearing capacity of bending for endplates 10-20mm thick connected to beams with IPE and HEB cross-section.

Past 20 years brought several new approaches and computer application for critical moment’s calculation. However, those equations are still included in Codes, therefore majority of engineers assume the values corresponding to fork-support or full-fixation. It is possible, that these formulas are not yet checked thoroughly and can give under the line of safety results. Classifying all of no-fixity elements under bending as fork-supported is a great overestimation. Be that as it may it puts the designed structure over the probability of failure line to safe zone and causes unnecessary use of building material. There is a growing number of computer programs with implemented Finite Element Method that allows to calculated the critical moment with an adequate accuracy for engineering purposes. Unfortunately, not all of civil engineers have access to the newest FEM software. Calculating the critical moment in a computer program is a risk, if it not followed by a thoughtful double-check by the engineer.

This paper concerns only the most typical scheme with the most typical load. To verify included formulas it is necessary to conduct further studies. Laboratory tests on a real-time model are necessary to prove the analytical formulas. It is important to incorporate a uniform solution for calculating the value of critical moment in the Code for general access.
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