STATUS OF FRACTIONAL SUPERSTRINGS

S.-H. Henry Tye
Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853-5001

Abstract

We argue why it is important to search beyond the superstring for consistent string theories that have space-time supersymmetry with critical space-time dimensions less than 10. We give a lengthy introduction on the motivation and approach. We discuss briefly some promising possibilities, namely, fractional superstrings. More specifically, we consider the $K = 4$, or spin-$4/3$ fractional superstring, which has a 3-dimensional flat Minkowski space-time representation with bosons and fermions in its spectrum. Its central charge of 5 is less than the critical value of 10. The no-ghost theorem for the bosonic sector is proved, using the recently constructed Kac determinants. We summarize the present status and point out some open questions.

Talk at the conference “Strings 93”, Berkeley, May 23-29, 1993.

1 Introduction

String theory is the only known theory with the potential for describing all matter and forces in nature in a unified way. In particular, the superstring theory and the closely related heterotic string theory entail many structures, including gravity, gauge fields and chiral fermions, that are central to the present understanding of our universe. However, their critical space-time dimension is ten, and there are numerous mechanisms to reduce the number
of observable space-time dimensions. Although the number of models in ten dimensions is very limited, namely the $SO(32)$ and the $E_8 \otimes E_8$ models[1], the number of models with four flat space-time dimensions is huge. Furthermore, there is no known compelling reason why the superstring theory should have only four large space-time dimensions. While it is important to search for dynamical and/or symmetry reasons explaining how and why our universe can be realized in the heterotic/superstring framework, it is also important to search for a new string theory that gives four large space-time dimensions naturally. Whether such a string theory exists or not is a question that we should be able to answer definitively.

For self-consistency, it is reasonable to request that the new string theory have space-time supersymmetry(SUSY). Let us recall the superstring case. Ensional superstring model with spacetime SUSY automatically has a zero cosmological constant to all orders in perturbation theory. On the other hand, a non-supersymmetric string model has, generically, a non-zero cosmological constant through string loop corrections. Such a cosmological constant is typically over a hundred orders of magnitude too big. One may argue that this does not happen in superstring theory, where space-time SUSY is broken spontaneously. The argument is based on two observations: (1) in string theory, a non-zero cosmological constant will destroy the factorization property in multi-loop string diagrams, thus invalidating the consistency of string perturbation expansions; (2) starting with a consistent quantum theory, spontaneous/dynamical symmetry breaking will occur only in a self-consistent way. These observations imply that, if a superstring model with spacetime SUSY is a consistent quantum theory, then, after SUSY breaking, it must remain a consistent theory, i.e., the resulting cosmological constant in flat Minkowski spacetime must remain precisely zero, since a consistent string perturbative expansion must exist in the new non-supersymmetric vacuum. Of course, this argument sheds no light on whether or not (or how) SUSY breaking takes place in string theory.

So our goal is to search for the existence of new consistent string theories that have space-time SUSY and lower critical space-time dimensions. Our approach starts from the string world-sheet. String theories are characterized by the local symmetries of two-dimensional conformal field theories on the string world-sheet. The bosonic string is invariant under diffeomorphisms and local Weyl rescalings on the world-sheet; whereas the superstring is characterized by a locally supersymmetric version of these symmetries.
This involves a spin-3/2 supercurrent, in addition to the energy-momentum tensor, on the string world-sheet. It is natural to ask whether other symmetries on the world-sheet can give rise to consistent string theories, which have lower space-time dimensions. To obtain a string theory from the world-sheet symmetry, the symmetry algebra is treated as a constraint algebra on the string Fock space. This approach have been successfully applied to the bosonic string and the superstring. It is natural to conjecture that, given any world-sheet symmetry algebra, there is a corresponding string theory with that algebra as the constraint algebra. Recently, a lot of progress have been made on the W-string\cite{2}, supporting this conjecture.

The first step of this approach is to understand the properties of world-sheet symmetries. I give an overall view of the possible world-sheet symmetry algebras. Specific examples\cite{3} are given to illustrate the richness of possibilities. In Section 2, I discuss some specific possibilities of new string theories, namely, fractional superstrings. Since the properties of fractional superstrings were described in detail in the literature\cite{4-13}, I shall restrict myself to a brief summary of the present status and some remarks. Then I shall discuss a specific example, namely, the $K = 4$, or spin-4/3 fractional superstring\cite{10}. This theory is the simplest non-trivial example, and is characterized by a chiral algebra involving a pair of spin-4/3 currents on the world-sheet in addition to the energy-momentum tensor\cite{14}. Scattering amplitudes of this theory are briefly described. They satisfy both spurious state decoupling and cyclic symmetry (duality). Examples of such amplitudes are calculated using an explicit $c = 5$ realization of the spin-4/3 current algebra\cite{10}. This representation has a 3-dimensional flat Minkowski space-time interpretation, with fermions and bosons only in its spectrum\cite{11}. The no-ghost theorem for the bosonic sector is proved using the recently constructed Kac determinants\cite{12}. Again, I shall restrict myself to a brief introduction and a summary of the present status. Section 3 contains some remarks.

The structure of two-dimensional conformal field theories(CFT) is in large part determined by their underlying conformal symmetry algebra, which organizes all the fields in a CFT into sets of primary fields of definite conformal dimensions, and their associated infinite towers of descendant fields. The fundamental conformal symmetry is the Virasoro algebra. The most general extended conformal symmetry algebra can be written as follows. Consider a set of currents $J_i(z)$, primary with respect to $T(z)$, with conformal dimen-
sions $h_i$, where $i \in \{1, 2, \ldots, N\}$:

$$T(z)T(w) = \frac{c/2}{(z - w)^4} + \frac{2T(w)}{(z - w)^2} + \frac{\partial T(w)}{(z - w)} + \ldots,$$

$$T(z)J_i(w) = \frac{h_i J_i(w)}{(z - w)^2} + \frac{\partial J_i(w)}{(z - w)} + \ldots.$$  \hspace{1cm} (1)

The operator product expansions (OPEs) among the currents have the following generic form:

$$J_i(z)J_j(w) = q_{ij}(z - w)^{-h_i - h_j}(1 + \ldots) + \sum_k f_{ijk}(z - w)^{-h_i - h_j + h_k} (J_k(w) + \ldots).$$  \hspace{1cm} (2)

where $q_{ij}$ and $f_{ijk}$ are structure constants. The ellipses stand for current algebra descendants, whose dimensions differ from those of the identity and the $J_k$ by positive integers. Since the algebra is chiral, i.e., independent of $\bar{z}$, the conformal dimensions are the spins of the fields. The parameters $c$, $h_i$, $q_{ij}$ and $f_{ijk}$ are not free and must be chosen such that the algebra is associative. This condition places strong constraints on the set of consistent conformal dimensions $h_i$ and restricts the structure constants $q_{ij}$ and $f_{ijk}$ as functions of the central charge $c$.

Extended conformal algebras naturally fall into three classes:

(i) Local algebras: the simplest type, where all powers of $(z - w)$ appearing in (2) are integers. This class includes the most familiar extended algebras, such as the superconformal, Kač-Moody and $W_n$ algebras. These examples are unitary and therefore consist of currents with only integer and half-integer spins.

When fractional powers of $(z - w)$ appear in some of the OPEs in (2), some of the currents will necessarily have fractional spins. In this case, the algebra is non-local, due to the presence of Riemann cuts in the complex plane. Such algebras are more complicated to construct and analyze than the local ones. Among non-local algebras there is a further division, again along lines of complication.

(ii) Abelian non-local algebras: also known as parafermion (PF) or generalized parafermion current algebras, were first constructed by Zamolodchikov and Fateev[16]. They are the simplest type of non-local algebras, involving at most one fractional power of $(z - w)$ in each OPE in (2). Any two currents in a PF algebra obey abelian braiding relations, i.e., upon braiding the
two currents (analytically continuing one current along a path encircling the other), any correlation function involving these two currents only changes by a phase. The analysis of the associativity conditions for PF theories can be carried out using algebraic methods.

(iii) Non-abelian non-local algebras: or non-abelian algebras for short, since they are necessarily non-local. This is the most general class of extended algebras. Their characteristic feature is that their OPEs involve multiple cuts, i.e., there are terms in at least one of the OPEs in (2) with different fractional powers of \((z - w)\). Any two fractional spin currents appearing in one of these OPEs will in general obey non-abelian braiding properties. The analysis of the associativity conditions for non-abelian algebras requires more powerful methods.

In general, the holomorphic \(n\)-point correlation functions of the currents, \(\langle J_i(z_1)J_j(z_2)\ldots J_k(z_n) \rangle\), can be expressed as a linear combination of some set of conformal blocks. The relative coefficients of the various conformal blocks are fixed by the closure condition and the associativity condition. The closure condition is simply the requirement that no new currents beyond the currents of the algebra should appear. Associativity is the condition that the particular linear combination of conformal blocks that appears in the \(n\)-point function is invariant under fusion transformations (i.e., duality). For the local algebras, each conformal block involves only integer powers of \((z_i - z_j)\); for the abelian non-local algebras, each correlation function of the parafermion currents has exactly one conformal block, even though it involves fractional powers of \((z_i - z_j)\). Of course, for the most general case we expect each correlation function to have multiple conformal blocks, and the conformal blocks to involve different fractional powers of \((z_i - z_j)\). This general case corresponds to the non-abelian non-local algebras. From this point of view we see that upon braiding the currents, the correlation function is, in general, transformed into an independent linear combination of conformal blocks. This reflects the different phases that are picked up upon analytically continuing the different fractional powers of \((z_i - z_j)\).

The different braiding properties of the different types of extended algebras described above are reflected in the moding of their currents. On a given state in any representation of the algebra, we can obtain new states by acting with current modes

\[
J_{-n_1}^{i_1}J_{-n_2}^{i_2}\ldots J_{-n_m}^{i_m}\Phi|\Phi\rangle, \tag{3}
\]
where the $n_j$ are integers and the $r_j$ are fractional in general. For local unitary algebras, the half-integer spin currents can have only integer or half-integer modings, and $r_1 = r_2 = \cdots = r_m = r$ where either $r = 0$ or $r = \frac{1}{2}$, depending on the state $|\Phi\rangle$. For PF theories, the situation is slightly more complicated. Generically the $r_i$ are different, determined by the state $|\Phi\rangle$ and the currents that preceded it. For non-abelian algebras, the moding of a particular current in (3) is not unique; it depends both on the state it operates on as well as on the state we want it to create.

Another useful way to classify symmetry algebras is their linearity property. An algebra is said to be linear if tensoring any two of its representations automatically gives a representation of the same algebra. The Virasoro and the superconformal algebras are linear in this sense. Most other algebras are non-linear, i.e., they do not have this property. The construction of representations for these algebras are rather non-trivial.

The first set of non-abelian algebras was conjectured in Ref.[13] and explicitly constructed in Ref.[3]. Consider a fractional supercurrent $G$ with conformal dimension (i.e., spin) $\Delta = (K + 4)/(K + 2)$. The following OPEs define the level $K$ FSCA with arbitrary central charge $c$:

\[
T(z)T(w) = \frac{(c/2)}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} + \ldots ,
\]

\[
T(z)G(w) = \frac{\Delta}{(z-w)^2}G(w) + \frac{1}{(z-w)}\partial G(w) + \ldots ,
\]

\[
G(z)G(w) = (z-w)^{-2\Delta} \left \{ \frac{c}{\Delta} + 2(z-w)^2T(w) + \ldots \right \} + \\
\lambda(c)(z-w)^{-\Delta} \left \{ G(w) + \frac{1}{2}(z-w)\partial G(w) + \ldots \right \} .
\]

The associativity condition for this algebra fixes the structure constant $\lambda^2(c)$:

\[
\lambda^2(c) = \frac{2K^2c_{111}^2}{3(K+4)^2}(c_1 - c) .
\]

Here $c_{111}$ is the $SU(2)_K$ structure constant for the OPE of two chiral spin-1 primary fields to give a chiral spin-1 field. It depends only on $K$ and is explicitly given in Ref.[3]; $c_1 = 24/K + c_0$, and $c_0=3K/(K + 2)$.

For $K = 1$, $G(z)$ is absent and we recover the Virasoro algebra. For $K = 2$, $c_{111}$ is zero and we recover the usual superconformal algebra, which
has no cuts in the complex plane in its OPEs. The absence of cuts is an enormous simplification, and is the reason why the superconformal algebra is substantially easier to study than the $K > 2$ cases. We call these algebras the fractional superconformal algebras (FSCAs), since they are the natural generalization of the superconformal algebra. The closed algebra generated by the currents $T(z)$ and $G(z)$ is the level $K$ FSCA. It is closely associated with the $SU(2)_K$ WZW model, in the sense that a representation of level $K$ FSCA can be constructed from the currents and the $j = 1$ fields of $SU(2)_K$. We can also construct representations using $\mathbb{Z}_K$ parafermions. We note that there exist other level $K$ FSCAs with additional fractional supercurrents [3].

It is not hard to convince oneself that, for each affine Lie algebra, there is at least one such corresponding extended conformal symmetry algebra; some examples are $W_N$ algebra associated with $SU(N)_1$ [17] and the PF current algebras associated with $so(N)_2$ [18]. The usefulness of these symmetry algebras in organizing CFTs is illustrated by the application of FSCA to the $SU(2)$ WZW coset models [15, 19].

2 Fractional superstrings

It is natural to ask whether symmetries other than the superconformal symmetry on the world-sheet can give rise to consistent string theories, which have lower space-time dimensions. Recently, we find strong evidence for new string theories based on FSCAs, called fractional superstrings (FSS), which have interesting phenomenologies in space-times with critical dimensions less than 10. The evidence come from the following:

1. Critical central charge and null states; any consistent string theory is expected to have extra sets of physical null states at a special value of the central charge, known as the critical central charge. This property is presumably a reflection of the underlying gauge symmetry of the string theory. Recently the Kac determinants for the level $K$ FSCA have been constructed using the BRST cohomology techniques developed in the conformal field theory [12]. In particular, we reproduce the Kac determinants for the Virasoro ($K = 1$) and the superconformal ($K = 2$) algebras. The Kac determinant formulae allow us to examine the null state structure of plausible critical FSS with an arbitrary level $K$ world-sheet fractional supersymmetry. The critical central
charge is found to be\cite{4,12}

\[ c_{\text{critical}} = c_1 + c_0 = \frac{6K}{K + 2} + \frac{24}{K} \tag{6} \]

Note that this recovers the well-known results for the bosonic ($K = 1$, with $c_{\text{critical}} = 26$) and the superstring ($K = 2$, with $c_{\text{critical}} = 15$) theories. This result also agrees with the more explicit determination of the spin-4/3 ($K = 4$, with $c_{\text{critical}} = 10$) FSS given in Appendix E of Ref.\cite{3}.

(2) One-loop modular invariant partition function; since the fractional superstring involves world-sheet fractional spin fields, i.e., parafermions, we search for closed string modular invariant partition functions that: (i) involve parafermion characters, (ii) contain the massless graviton, and (iii) are tachyon-free. Rather unique solutions are obtained, and they are consistent with space-time supersymmetry\cite{5,6}. The FSS with spin-3/2 ($K = 2$), 4/3 ($K = 4$), 6/5 ($K = 8$) and 10/9 ($K = 16$) fractional supercurrent on the world-sheet seem to have maximum flat dimension 10, 6, 4 and 3 respectively. Of course, the spin-3/2 case is simply the usual superstring. It is natural to view the other cases as non-trivial generalizations of the superstring case.

(3) Tree-level scattering amplitudes; we construct tree-level scattering amplitudes for the spin-4/3 fractional superstring\cite{10}. Despite the non-local and non-linear structure of the world-sheet symmetry, we construct the $N$-point scattering amplitude for bosons and demonstrate its cyclic symmetry (duality) and the decoupling of spurious states to the physical states. In a particular representation which has a three dimensional Minkowski space-time, we also argue that only space-time bosons and fermions are present in this model\cite{10,11}. The closed string version contains the massless graviton.

(4) No-ghost theorem; we apply the Kac determinant formula for the $K=4$ FSC algebra to the above three-dimensional Minkowski space-time representation of the spin-4/3 fractional superstring\cite{12}. We prove the no-ghost theorem for the space-time bosonic sector of this model; that is, at the tree-level scattering, its physical spectrum is free of negative-norm states.

Here, a few remarks on the above properties are in order:

(a) In the construction of the modular invariant partition functions, we found the critical dimension to be, for $K > 1$, $D_{\text{crit}} = 2 + 16/K$. There, we tensor $(D_{\text{crit}} - 2)$ copies of $Z_K$ parafermions plus boson $X^\mu$ in the light-cone gauge, each copy with central charge $c_0$. Naively, this would imply
that the critical central charge is $c_{\text{crit}}=D_{\text{crit}}c_0=6(K+8)/(K+2)$, which disagrees with (6). However, a remarkable cancellation happens in the modular invariant partition functions, within the characters of the sectors containing the massless fields. This delicate cancellation, referred to as “internal projection” [6], reduces the central charge in the light-cone gauge from $c_{lc} = (D_{\text{crit}} - 2)c_0 = 48/(K + 2)$ to an effective value $c_{lc} = 24/K$. This can agree with (6) if we add back the central charge $2c_0$ for the time and longitudinal dimensions[9]. This “internal projection” feature is not surprising, for the following reason. The level $K$ FSCA are non-linear for $K > 2$, so its representations cannot be obtained from tensoring copies of $Z_K$ parafermion plus boson $X^\mu$. Inside the space of tensored copies, one can imagine a projection to a subspace of states which does form a representation of the algebra. The explicit construction discussed later in this section is motivated by this observation. Since the $K = 2$ FSCA (the superconformal algebra) is linear, there is no internal projection for the superstring case.

(b) It was argued that the critical dimensions for the $K = 4$ and $K = 8$ FSS are 6 and 4 respectively. However, a closer examination[7] of the modular invariant partition functions for these two cases indicates that they do not have the necessary Lorentz symmetry: the $K = 4$ modular invariant partition function allows at most a four-dimensional Poincare symmetry while the $K = 8$ case allows at most a three-dimensional Poincare symmetry. This seems to imply that the two of the flat directions in the $K = 4$ FSS cannot be normal space-time dimensions. To avoid a continuous spectrum, they must be compactified. This maximum possible space-time dimension was referred to as the “natural dimension”, as opposed to the critical dimension. In this sense, the $K = 4$ FSS is the phenomenologically interesting theory to pursue, since its natural space-time dimension is 4. As we shall see in the next section, the $K = 4$ FSS is also by far the easiest case (among all FSS with non-linear FSCA) to study.

(c) In uncompactified superstrings, the spectrum does not contain chiral fermions in fundamental representations of gauge groups. They appear only after appropriate compactification. On the other hand, it is the compactification that removes the uniqueness of the superstring theory, and raises the non-trivial dynamical questions of why and how does it happen. So it is intriguing to see that the $K = 4$ FSS has natural dimension of 4, which is less than its critical dimension of 6; in this situation, one can hope the compactification of the extra dimensions to appear more naturally[7].
course, an explicit realization of this feature remains to be found.

To be concrete, let us consider the $K = 4$ FSCA. This is the simplest non-trivial case because one can split the $G$ current into two pieces, $G(z) = G^+(z) + G^-(z)$, which then satisfy abelian braid relations:

\[
G^\pm(z)G^\mp(w) = \frac{1}{(z-w)^{8/3}} \left\{ \frac{3c}{8} + (z-w)^2 T(w) + \ldots \right\},
\]

\[
G^\pm(z)G^\pm(w) = \frac{\lambda}{(z-w)^{4/3}} \left\{ G^\mp(w) + \frac{1}{2}(z-w)\partial G^\mp(w) + \ldots \right\}.
\]

Here, $\lambda^2 = (8 - c)/6$. In this split form, this algebra is the spin-4/3 parafermion current algebra constructed by Zamolodchikov and Fateev [14]. Appendix C of Ref.[8] gives a detailed construction of this algebra. To construct a string theory based on this algebra, we use an approach that mimics the original construction of the superstring[20]. By analogy with the superconformal gauge of the superstring, the stress-energy tensor and fractional supercurrents are assumed to generate the physical state conditions. In particular, physical states, taken to be annihilated by all the positive modes $L_n$ and $G^\pm_r$ of $T$ and $G^\pm$ respectively, are the highest-weight states of the FSC algebra. This FSC algebra has a $\mathbb{Z}_3$ symmetry. In particular, the currents $G^+$ and $G^-$ can be assigned $\mathbb{Z}_3$ charges $q = 1$ and $-1$, respectively, while the energy-momentum tensor $T$ (as well as the identity) has charge $q = 0$. So the highest-weight modules of this algebra are organized by the $\mathbb{Z}_3$ symmetry of the algebra. Highest-weight states with $\mathbb{Z}_3$ charge $\pm 1$ are said to belong to D-modules, while those with $\mathbb{Z}_3$ charge 0 are in S-modules.

The consistency of the FSS can be demonstrated by defining tree scattering amplitudes and showing that they are consistent with the assumed physical state conditions following from the spin-4/3 FSCA. In other words, in tree scattering, physical states never scatter to unphysical states, and null states can also be consistently decoupled from scattering of other physical states. This property is commonly referred to as spurious state decoupling, and can be shown in a representation-independent way. The argument for spurious state decoupling follows closely that used in the “old covariant formalism” [21] for ordinary superstring amplitudes.

Scattering of D-module states can be written in three physically equivalent “pictures,” reflecting the $\mathbb{Z}_3$ symmetry of the fractional superconformal algebra, in which the vertex operators for scattering can be one of $W^\pm$ of
conformal dimension $1/3$ and $\mathbb{Z}_3$ charge $\pm 1$ or $V$ of conformal dimension $1$ and $\mathbb{Z}_3$ charge $0$. In the old covariant formalism, the N-point scattering amplitude can be written in three pictures

$$\mathcal{A}_N = 2\langle W_N^+ | V_{N-1}(1) \Delta \ldots \Delta V_2(1) | W_1^- \rangle$$

$$= 2\langle W_N^- | V_{N-1}(1) \Delta \ldots \Delta V_2(1) | W_1^+ \rangle$$

$$= \langle V_N | V_{N-1}(1) \tilde{\Delta} \ldots \tilde{\Delta} V_2(1) | V_1 \rangle. \quad (8)$$

Here the propagators are $\Delta = (L_0 - \frac{1}{3})^{-1}$ and $\tilde{\Delta} = (L_0 - 1)^{-1}$; and the vertex operators are related by

$$[G^r_{\pm}, V(1)] = \left( L_0 + r - \frac{1}{3} \right) W^\pm(1) - W^\pm(1) \left( L_0 - \frac{1}{3} \right) \quad (9)$$

for all $r \in \mathbb{Z}/3$. This is closely analogous to the two different pictures for scattering of Neveu-Schwarz sector states in the old covariant formalism for the ordinary superstring, in which vertex operators can be either G-parity even dimension-$1/2$ operators or G-parity odd dimension-1 operators (this property follows from the $\mathbb{Z}_2$ symmetry of the superconformal algebra). Scattering of S-module states is problematic due to the absence of an appropriate dimension-1 vertex operator in that sector. Since the S-module typically contains the tachyonic state, we are fortunate that they can be decoupled from tree scattering amplitudes of D-module states by a $\mathbb{Z}_3$ analog of the GSO projection \cite{22}. A separate issue that can be addressed at tree level is the unitarity of scattering amplitudes. In particular, spurious state decoupling implies unitarity only if one can prove that the space of physical states has non-negative norm. This latter property is called the no-ghost theorem. Using the recently constructed Kac determinants for FSCAs, such a theorem is proven in Ref.\cite{12} for the three-dimensional model that we shall now discuss.

This model is a particular conformal field theory representation of the spin-4/3 FSC algebra with central charge $c = 5$. It is made up of three free coordinate boson fields $X^\mu$ on the world-sheet and a two-boson representation of the $so(2,1)_2$ WZW model, which is closely related to the $SU(3)_1$ WZW model; (note that, conformally, $SU(3)_1 = SU(2)_4 = so(3)_2$). This model has a global three-dimensional Poincare invariance. The non-linear nature of the spin-4/3 FSC algebra makes the existence of such a representation non-trivial. Also, the states in the model are found to be space-time bosons or fermions, showing that the existence of fractional-spin constraints on the
world-sheet need not imply fractional spins in space-time. The untwisted sectors of this FSCA describe space-time bosonic physical states in this representation. In particular, the lowest-mass D-module states describe massless gauge fields for the open string and a graviton for closed fractional superstrings.

Since three-dimensional Minkowski space-time is too small to describe nature, it is encouraging that the central charge of the three-dimensional model is less than the critical value of 10, allowing the possibility of a critical spin-$4/3$ fractional superstring containing four-dimensional Minkowski space-time.

### 3 Discussion and outlook

Of course, the existence of FSS remains to be demonstrated. There are many open questions remain to be answered. More specifically, we are still steps away from showing that

1. consistent fractional superstring theory exists and
2. it is phenomenologically interesting.

However, we are encouraged by some non-trivial “coincidences” that are worth emphasizing. Let me mention two:

1. In the construction of modular invariant partition functions, typically one finds there are more constraints than the number of parameters available in finding a solution. A priori, solutions should not exist, but they do.

2. In the construction of scattering amplitudes for D-module states in the $K = 4$ FSS, which includes the massless vector particle as the ground state, the constraints necessary for spurious state decoupling and for picture-changing to work seems to be tighter than the freedom allowed by the parameters present. Naively, the picture-changing formula (9) should not exist a priori, but it does.

We believe that these “coincidences” are really a reflection of an underlying consistent theory being looked at from a funny angle. They clearly suggest that FSS is tighter than the usual superstring theory. This can be seen in other ways as well:

1. the number of possible modular invariant partition functions of both the FSS type and the heterotic type are a lot fewer than in the corresponding cases in usual superstring, at any fixed space-time dimensions. This comes about because of (i) the initial critical dimension of FSS starts out
to be smaller than that of the superstring; so there are less dimensions to compactify; and (ii) the FSCAs underlying the FSS are non-linear, hence there are fewer representations for fixed central charge.

(2) the tachyonic state (or any state in the same sector) in the Neveu-Schwarz sector is completely consistent in the superstring theory, even though it is undesirable phenomenologically; the same tachyonic state (or any state in the same sector) in the $K = 4$ FSS is not consistent: the four-point scattering amplitude of such states does not exist in the old covariant formalism. So, the GSO-like projection is necessary in the FSS case even at the tree-level.

At the moment, we are trying to construct the BRST ghosts and the BRST invariance of the $K = 4$ FSS. Here, the ghosts are expected to couple to the matter fields, much like the Yang-Mills theory. In this case, one should seek a $c = 0$ representation of the FSC algebra that contains both the matter and the ghost fields and permits the construction of a nilpotent BRST charge. Within this $c = 0$ representation, the goal is to find a $c = 10$ sub-representation that has a four-dimensional Poincare invariance. Unfortunately, there is no clear guideline on how to search for such a representation. It is interesting to note that, since $so(4) = so(3) \otimes so(3)$, the world-sheet symmetry algebra corresponding to $so(4)_2$ is simply two copies of the spin-4/3 algebra; however, the coordinate bosons coupled to the $so(3)_2$ model in our $c = 5$ representation do not transform in the vector representation of $so(4)_2$, and so cannot give a flat space-time interpretation.

In general, other fractional superstrings will be technically more difficult to work with than the spin-4/3 FSS. The main reason is that the general FSC algebra are non-abelian. Since conformally, $su(2)_L = so(3)_L$, we expect all of them (at least the ones with even $K$) to have a representation (similar to the $c = 5$ one in the $K = 4$ FSS) with a three-dimensional Poincare invariance. However, like the $K = 4$ case, none of them will be at the critical central charge except when $K$ is infinite. In this limit, the fractional supercurrent has conformal dimension 1, with $c_{\text{critical}} = 6$ for the corresponding FSS. The representation is made up of three free coordinate boson fields $X^\mu$ on the world-sheet and a three-boson representation of the $so(2,1)_\infty$ WZW model, so the central charge equals $c_{\text{critical}}$. This string theory has critical space-time dimension 3, and it may be used to describe the three-dimensional Ising model. Since the OPEs in this $K = \infty$ FSCA has only poles, the analysis may be quite tractable.
Acknowledgements

This talk reports on joint work with Stephen Chung, Keith Dienes, Jim Grochocinski, Zurab Kakushadze, Andre LeClair, Ed Lyman, and especially Philip Argyres. It is a pleasure to thank them all for fruitful collaborations and many useful discussions. This work was supported in part by the National Science Foundation.

References

[1] D.J. Gross, J.A. Harvey, E. Martinec, and R. Rohm, Nucl. Phys. B256 (1985) 253.

[2] See e.g., Talk by C. Pope in this conference.

[3] P.C. Argyres, J. Grochocinski and S.-H.H. Tye, Nucl. Phys. B367 (1991) 217, hep-th/9110052. Nucl. Phys. B391 (1993) 409, hep-th/9202007.

[4] P.C. Argyres, A. LeClair and S.-H.H. Tye, Phys. Lett. 253B (1991) 306.

[5] P.C. Argyres and S.-H.H. Tye, Phys. Rev. Lett. 67 (1991) 3339, hep-th/9109001.

[6] P.C. Argyres, K.R. Dienes and S.-H.H. Tye, Commun. Math. Phys. 154 (1993) 471, hep-th/9201078.

[7] K.R. Dienes and S.-H.H. Tye, Nucl. Phys. B376 (1992) 297, hep-th/9112015.

[8] P.C. Argyres, E. Lyman and S.-H.H. Tye, Phys. Rev. D46 (1992) 4533, hep-th/9205113.

[9] P.C. Argyres and K.R. Dienes, Phys. Rev. Lett. 71 (1993) 819, hep-th/9305093; K.R. Dienes, The Worldsheet Conformal Field Theory of the Fractional Superstring, McGill preprint McGill-93-01, to appear in Nucl. Phys. B, hep-th/9305094.

[10] P.C. Argyres and S.-H.H. Tye, Tree scattering amplitudes of the spin-4/3 fractional superstring I: The untwisted sectors, IASSNS-HEP-93/57, CLNS 92/1176, hep-th/9310131.
[11] P.C. Argyres and S.-H.H. Tye, *Tree scattering amplitudes of the spin-4/3 fractional superstring II: The twisted sectors*, IASSNS-HEP-93/58, CLNS 93/1251, to appear.

[12] Z. Kakushadze and S.-H.H. Tye, *Kac and new determinants for fractional superconformal algebras*, CLNS 93/1243, hep-th/9310160.

[13] P. Frampton and J. Liu, *Phys. Rev. Lett.* 70 (1993) 130, hep-th/9210049; G.B. Cleaver and P.J. Rosenthal, *Aspects of Fractional Superstrings*, Caltech preprint CALT-68-1756, hep-th/9302071.

[14] A.B. Zamolodchikov and V.A. Fateev, *Theo. Math. Phys.* 71 (1987) 451.

[15] D. Kastor, E. Martinec and Z. Qiu, *Phys. Lett.* 200B (1988) 434; J. Bagger, D. Nemeschansky and S. Yankielowicz, *Phys. Rev. Lett.* 60 (1988) 389; F. Ravanini, *Mod. Phys. Lett.* A3 (1988) 397.

[16] A.B. Zamolodchikov and V.A. Fateev, *Sov. Phys. J.E.T.P.* 62 (1985) 215; *Sov. Phys. J.E.T.P.* 63 (1986) 913.

[17] A.B. Zamolodchikov, *Theo. Math. Phys.* 63 (1985) 1205; V.A. Fateev and S.L. Lykyanov, *Int. J. of Mod. Phys.* A3 (1988) 507.

[18] P. Goddard and A. Schwimmer, *Phys. Lett.* 206B (1988) 62.

[19] S.-w. Chung, E. Lyman and S.-H.H. Tye, *Int. J. Mod. Phys.* 7A (1992) 3339.

[20] P. Ramond, *Phys. Rev.* D3 (1971) 2415; A. Neveu and J. H. Schwarz, *Nucl. Phys.* B31 (1971) 86.

[21] A. Neveu, J.H. Schwarz and C.B. Thorn, *Phys. Lett.* B35 (1971) 529; for a detailed explanation of this formalism, see, e.g., Chapter 7 of M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press (1987).

[22] F. Gliozzi, J. Scherk and D. Olive, *Nucl. Phys.* B122 (1977) 253.