Mathematical methods in geometrization of coal field

D N Shurygin, V M Kalinchenko, V A Tkachev, A Ya Tretyak

Platov South Russia State Polytechnic University, 132, Prosveshchenia St., Novocherkassk, Rostov Region, 346428, Russia

E-mail: Shurygind@mail.ru

Abstract. In the work, the approach to increase overall performance of collieries on the basis of an increase in accuracy of geometrization of coal thicknesses is considered. The sequence of stages of mathematical modelling of spatial placing of indicators of a deposit taking into account allocation of homogeneous sites of thickness and an establishment of quantitative interrelations between mountain-geological indicators of coal layers is offered. As a uniform mathematical method for modelling of various interrelations, it is offered to use a method of the group accounting of arguments (MGUA), one of versions of the regressive analysis. This approach can find application during delimitation between geological homogeneous sites of coal thicknesses in the form of a linear discriminant function. By an example of division into districts of a mine field in the conditions of mine "Sadkinsky" (East Donbass), the use of the complex approach for forecasting of zones of the small amplitude of disturbance of a coal layer on the basis of the discriminant analysis and MGUA is shown.

1. Introduction

In the course of realisation of actions for re-structuring of the coal industry of the Russian Federation, technical and economic indicators of work of mines, such as loading on a clearing face and labour productivity have considerably grown. In this connection it rises a problem of updating the requirements for reliability of prospecting works at all stages and phases, especially at a stage of more sharp operation of deposits.

The analysis of long-term practice of mountain works on collieries shows that the major factors, negatively influencing the productivity of extracting complexes, are zones of an unstable roof and intensive fractures, sharp fluctuations of capacity of a coal layer, presence of explosive infringements, etc. Therefore the detailed and authentic forecast of mountain-geological conditions of a deposit is one of the major problems of modern mountain manufacture and exerts a serious impact on improvement of quality of planning of mountain works, labour productivity and the extraction level of a mineral [1-10].

To increase the efficiency of planning of mountain works development on the basis of geometrization of coal thicknesses, the following scientific results have been received. The analysis of existing methods of geometrization of mountain-geological conditions of working out the coal layers, influencing an optimum choice of rational ways of conducting of mountain works, is carried out. Spatial statistical regularities between mountain-geological conditions of working out coal layers and parameters of coal thicknesses (physical, mechanical and lithological signs of rocks) are established.

The method of allocation and geometrization of homogeneous sites of coal thicknesses has been developed, each of which is characterised by the same effective technology of conducting mountain
works. The method of forecasting and geometrization of mountain-geological indicators in interwell space, taking into account their spatial variability revealed according to prospecting chinks, is developed. The geometrization method of the degree of rocks stability of the immediate roof and soil of a coal layer in underground mining developments on the basis of the facial-phase analysis of carboniferous adjournment, taking into account variability of parameters of coal thicknesses in the interwell space, has been developed.

The basic stages of geometrization of coal thicknesses are presented in Figure 1, which are considered in the subsequent sections in detail.

| Stages of mathematical modelling and geometrization of coal thicknesses | Application of MGUA for a choice of the optimum equation |
|---|---|
| Formation of vectors of parameters values of coal thicknesses in prospecting developments | Representation of paragenic interrelation in the form of the regress equation |
| Establishment of quantitative interrelations between parameters of coal thicknesses | Representation of the discriminant function in the form of the regress equation |
| Allocation and geometrization of sites of coal thicknesses with homogeneous mountain-geological conditions on a basis of the cluster analysis | Representation of variogram in the form of the regress equation |
| Geometrization of spatial position of borders of homogeneous sites of coal thicknesses on the basis of the discriminant analysis | Representation of a degree of stability of rocks in the form of the regress equation |
| Geometrization of mountain-geological conditions of working out coal layers in interwell space by the kriging method taking into account variograms, which parameters, reflecting spatial variability of coal thicknesses parameters | |
| Geometrization of the degree of rocks stability of an immediate roof and soil of a coal layer on the basis of the facial-phase analysis of carboniferous adjournment, taking into account variability of parameters of coal thicknesses in interwell space | |

**Figure 1.** Application of the regressive analysis (MGUA) during geometrization of coal thicknesses

2. **Mathematical modelling of coal thicknesses and its geometrization**

The choice of the parameters, characterising coal thickness, should consider characteristic spatial features of a structure and the lithologic composition, as well as physical and mechanical properties of a file of rocks. For mathematical modelling of such system, it is necessary to use multidimensional geometrization of indicators of a deposit, which implies a set of methods of construction of mountain-geometrical (mathematical) models. The essence of the technique consists in revealing and the
quantitative description of the multidimensional dependences, caused by genesis, of the geometrized indicator with other elements (indicators) of the system "deposit".

During detailed investigation, the features of a coal deposit are studied on the basis of a network of prospecting wells, for each of which it is possible to generate a set of geological signs. The basic rock, forming the geological section, is a coal and coaly differences, mudstone (clay slate), aleurolite (sandy-argillaceous and sandy slate), sandstone and limestone. For each rock, containing a coal layer, it is possible to define its lithological, physical and mechanical properties from the file of a prospecting well by the description of a core and logging diagrammes. Capacity and the graduation factor (the average size of composing particles) of the rock, the structure and texture, the presence of impurities, intensity of fractures, durability, porosity, humidity, etc. are referred to these properties.

The first stage of mathematical modelling of coal thicknesses is formation of values vectors of geological signs (parameters) of coal thicknesses in prospecting developments. Already at this step, it is possible to carry out geometrization of various indicators of a coal deposit on the basis of griding of undercuttings of rocks by prospecting wells. For further analysis, the establishment of quantitative interrelations between parameters of coal thicknesses in wells is necessary. Such interrelations, revealed on the basis of methods of the regress analysis, will be further used for the characteristic of geological homogeneous sites of thickness.

3. The method of the group accounting of arguments (MGAA)

Split-hair accuracy of the forecast of mountain-geological conditions of coal layers is provided with application of methods of the regressive analysis, in particular, a method of the group accounting of arguments (MGAA). This method allows one to make a choice of an optimum mathematical model (by the minimum set of the most informative mountain-geological signs) which is also characterised by the minimal RMS error when selecting a regression equation for describing experimental data. In such approach, the principle of external addition, when the information on parameters of coal thicknesses in prospecting developments is divided in two samples - training and control, is used. On the basis of the data from the first sample, a set of regression equations from different sets of parameters is constructed, and the best model is selected, proceeding from the adequacy of the regression equations to the data of control sample.

MGAA's algorithm consists in the following. For prospecting developments, from the training sample \(x_i\) and \(x_j\), the dependence of predicted mountain-geological indicator \(P\) on a pair of features is constructed.

\[
P = a_{i1}x_i + a_{2j}x_j.
\]  

For an estimation of values of factors \(a_{i1}, a_{2j}\) the method of the least squares is applied. The first index (accepting values 1 or 2) designates the feature number in a chosen pair. The second index – the number of a feature among all \(t\) features.

If a training sample contains \(n\) wells for a method of least squares, the following matrixes are formed:

\[
A = \begin{bmatrix} a_{i1} \\ a_{2j} \end{bmatrix}, \quad X = \begin{bmatrix} x_i, x_j \\ x_{i1}, x_{ij} \\ x_{i2}, x_{i2j} \\ \vdots \\ x_{it}, x_{ijt} \end{bmatrix}, \quad P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}.
\]

The solution of the method of least squares in a matrix form is obtained by the formula:

\[
A = \left(X^T X\right)^{-1} X^T P.
\]

The choice of the equations, describing experimental data with the minimal RMS error, is called selection. Let us designate the regression equation as \(P_{ij}\) corresponding to the pair of features \(x_i\) and
In the first row of selection, for all pairs of features, the coefficients of the equations of type (equation (1)) are calculated:

\[ P_1 = a_{11}x_1 + a_{22}x_2, \quad P_2 = a_{11}x_1 + a_{23}x_3, \ldots, \quad P_S = a_{t_{m-1}}x_{i_{m-1}} + a_{t_m}x_{i_m}. \]  

(2)

Models (equation (2)) comply with various interrelations between features, from which it is necessary to choose an optimum model. In \( m \) wells, taken from the control sample, the quality of coefficients factors, not participating in calculation, is checked by the following criterion:

\[ \delta_{ij}^{(1)} = \frac{1}{m} \sum_{k=1}^{m} (P_i - a_{i_{1}}x_{i_{1}} - a_{i_{2}}x_{i_{2}})^2. \]  

(3)

The figure in brackets in the top index corresponds to number of a row of the selection. Further, all equations are ranged by criterion (equation (3)) and \( t \), the best of them (by the minimum value) are accepted as arguments in the equations in the second row of selection. After that, at points of the training sample, the factors of new dependences are calculated:

\[ Q_1 = a_{11}P_1 + a_{22}P_2, \quad Q_2 = a_{11}P_1 + a_{23}P_3, \ldots, \quad Q_S = a_{t_{m-1}}P_{m-1} + a_{t_m}P_m. \]  

(4)

At points of control sample, criterion \( \delta^{(2)} \) is again calculated by formula (3) for each equation; selection \( t \) of the best equations (4) are ranged and transmitted to the third row. If \( \delta_{\min}^{(1)} > \delta_{\min}^{(2)} \), in the third row of selection, the described procedure repeats. The bottom index «min» in the previous inequality corresponds to the least value of criterion (equation (3)).

Dependence of the selection criterion (modelling error) at control points on a number of selection of the model is presented in Figure 2.

Model construction is conducted until the inequality \( \delta_{\min} = \delta_{\min}^{(v-1)} < \delta_{\min}^{(v)} \) will be executed. The model, obtained in a \( v-1 \) row of selection and having a minimum of the criterion (equation (3)) on external addition (control sample), is accepted as an optimum model (the regression equation).

After definition of number of an optimum row of selection, it is necessary to return in reverse order to the first row. At that, arguments of models of the previous rows for definitive expression of the optimum equation through features \( \{x_i\} \) of the first row of selection are consecutively found.

**Figure 2.** The graph of changing the criterion of selection by rows

4. Application of MGAA during geometrization of predictive zones of display of small-amplitude disorder of coal layer
For definition of conditional wells, in which occurrence of tectonic infringements is predicted on borders of types of thickness or in the area close to them, it is necessary to find the equation of their interrelation with properties and the structure of coal thicknesses for each allocated homogeneous site. The training sample for finding the equation of the discriminant function within the allocated types of thickness consists of two groups of conditional wells: the first one includes the conditional broken wells, the second – conditional unbroken wells.

The part of these wells in the discriminant analysis with a step-by-step choice of variables on the basis of MGAA is allocated into control sample for realization of accuracy of estimations (for pairs of signs). Conditional wells of thickness type, in which infringements are predicted, are referred to the training sample. In Figure 3, the fragment of the plan of mountain works with 45 numbered conditional broken wells is given, each of which is the nearest to the known infringements revealed during realization of mountain works. By triangles of one colour, geological homogeneous sites of coal thicknesses are allocated. Let us consider two sites (wells of the first site are designated by the dark blue colour, the of the second one – by red).

The border equation between them, calculated by MGAA, looks as follows (accuracy of recognition on training sample – 97.2 %)

\[ D = 1.387x_1 - 0.061x_2 - 0.236x_3 - 0.184x_4 + 0.470x_5 - 0.593x_6. \]

In the tectonic relation, the first site of rock thickness essentially surpasses the second one, which is a prerequisite of localization of small infringements of a coal layer on sites with low-power sandstone \( x_1 \) in its roof breeds. Localization of ruptures in transitive zones between types of carboniferous cuts

![Figure 3. The division into districts of coal thicknesses into geological homogeneous sites](image)

conditional broken well  conditional unbroken chink

---

For more information, please refer to the attached document.
that is caused, on the one hand, by accumulation of the tectonic pressures, which do not have an opportunity to be discharged in strong thickness of rocks, and on the other – by relatively smaller durability of the thickness assuming a possibility of its rupture is thus most probable. On the second site, it is possible to note replacement of sandstone in a roof of a coal layer by clay aleurolite $x_2$ and mudstone $x_3$. Clay aleurolites four times, and mudstone three times by the capacity surpass corresponding lithologic types on the first site. It is interesting that total capacity of rocks of the immediate and basic roof on the first site makes almost 19 m, and on the second — only 11 m. Thus, the capacity of rocks of the immediate roof $x_5$ differs twice, which influences propensity of a false and immediate roof to plastic deformations under tectonic influence.

Parameter $x_6$ (type of lamination of rocks of a coal rhythm), characterizing sedimentation dynamics, also varies on different sites of thickness of rocks. For the first site (the central part of a mine field) rocks are basically either slantwise or curved the layered ones. The rocks of the second site are either not layered (monolithic), or horizontally layered that allows to assume about accumulation of a sedimentary material of the first type mainly in a zone of development of the river delta; of the second type - in coastal-sea and lagoon conditions.

Presence of splitting of a coal layer testifies about the transgressive and regressive cycle of sedimentation in a cut of thickness with displacement of delta and the coastal-lagoon zone in the plan. Northern and southern parts of the mine field, most likely, were coasts of the river; in the eastern part, the coal layer is split into two packs; along splitting, a coastal line of the sea probably passed.

5. Conclusion
For a number of stages of geometrization of coal thicknesses, it is possible to use MGAA for a choice of the optimum equation of regression of construction of borders of homogeneous sites of thickness. Use of this approach when forecasting the zones of small amplitude disorder of coal layers has been presented above.

MGAA also finds wide application during the analysis of paragenetic interrelations between mountain-geological parameters, selection of theoretical model of the variogram of a mountain-geological indicator, definition of a degree of stability of rocks in underground developments. The coal geometrization thicknesses technique is recommended to be realized as a component of the automated system of forecasting mountain-geological conditions of working off coal layers with the help of which effective planning the development of mountain works is carried out.

References
[1] Carpentier S, Gamache M, Dimitrakopoulos R 2016 Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology 125(2) 93-102
[2] Marinho A, Tipe L M 2015 Proceedings of the 37th International Symposium, APCOM 2015 316-325
[3] Mello P R G, De Lemos Peroni R, Costa J F, Wolff R 2015 Proceedings of the 37th International Symposium, APCOM 2015 1098-1106.
[4] Montiel L, Dimitrakopoulos R, Kawahata K 2016 Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology 125(1) 2-14
[5] Navarra A, Montiel L, Dimitrakopoulos R 2016 International Journal of Mining, Reclamation and Environment 1-17
[6] Rahmanpour M, Osanloo M 2016 Journal of the Southern African Institute of Mining and Metallurgy 116(3) 229-236
[7] Shaykhutdinov D, Shurygin D, Aleksanyan G, Grushko I, Leukhin R, Stetsenko I, Oganyan R, Bondarenko V, Leukhin V 2016 Asian Journal of Information Technology 9(15) 1443-1446
[8] Shaykhutdinov D, Shurygin D, Gorbatenko N, Grechikhin V, Leukhin R, Stetsenko I, Oganyan R, Bondarenko V, Leukhin V 2016 Research Journal of Applied Sciences 11(8) 628-631
[9] Shurygin D N, Gorbatenko N I, Grechikhin V V, Shaykhutdinov D V 2016 Journal of
Engineering and Applied Sciences 11 2764-2768
[10] Silva V M, Coimbra Leite Costa J F 2016 Transactions of the Institutions of Mining and Metallurgy, Section B: Applied Earth Science 1-7