Models for the self-heating evaluation of a gallium nitride-based high electron mobility transistor

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Abstract
We propose a novel model approach for temperature evaluation in the channel region of a InAlN/AIN/gallium nitride high electron mobility transistor (HEMT) due to self-heating effects. The heat transfer in a HEMT device has been investigated experimentally by the nearby temperature sensor and compared by theoretical models solved by both numerical and analytical methods. The average temperature of the channel area of almost 160 °C for dissipated power of 2 W was determined using the drain-source current variation analysis. The electrical and thermal behavioral numerical model under quasi-static conditions have been used to describe the HEMT device. In contrast, the one-dimensional thermal model for analytical evaluation has been proposed as an alternative approach. Surprisingly, the experimental results verified not only the validity of precise numerical simulation but also the simplified analytical model that makes it a reliable tool even for complex electronic devices.

Keywords: gallium nitride, field-effect transistor, high electron mobility transistor, average temperature, thermal resistance

(Some figures may appear in colour only in the online journal)

1. Introduction
The Internet of Things represents the coming technical revolution that will hit all areas of human society. The rising era of sensors, smart cities, and e-health results in an incredible load of wireless networks. Furthermore, the recent development of augmented reality, autonomous cars, and high-resolution video streaming creates a need for high-speed transmission, large bandwidth, or high capacity of the network. These requirements are beyond the capabilities of the present mobile network; hence, as a response, the fifth-generation (5G) mobile network has been envisioned as the next big thing that will change the world of data transmission. It should be mentioned here that the 5G network represents a broad interdisciplinary challenge that also requires advanced electronic devices and systems. The performance of a radio-frequency power amplifier dominates the overall transmitter performance. Imperfect amplifiers define the power and heat dissipation for the entire system. Even though silicon (Si) dominates in the area of consumer electronics, the III–V semiconductors are often proposed because of their superior frequency responses, high-temperature tolerance, or breakdown robustness. Among them, gallium nitride (GaN) is recognized as the most promising candidate for high-power and high-frequency devices since we are capable to fabricate a heterostructure that exhibits a presence of two-dimensional electron gas (2DEG). This outstanding nature gives us the potential to create high...
electron mobility transistors (HEMTs), which have emerged as excellent devices for applications in the power conversion field. However, the high mobility combined with a high density of the electrons in 2DEG leads to the possibility of the device self-heating due to Joule losses. The rise of temperature influences the device’s electrical properties as well as entire system performance and reliability [1–3]. Sup-pressing the negative impact of the elevated temperature substrates with high thermal conductance has been proposed. Hence, advanced power devices are grown on advanced substrates, e.g. silicon carbide (SiC) and Si exhibit better power dissipation in comparison to the conventional sapphire substrates [4]. However, even though high-cost substrates are applied, self-heating remains a crucial issue for these devices. Since the HEMT devices are microscopic in size, temperature study by conventional methods is not possible. Therefore, the temperature distribution in HEMT devices represents now the gap in the knowledge, and enormous effort has been put into the development of novel experimental techniques that can provide an insight into the heat dissipation. Optical methods such as Raman spectroscopy or interferometric mapping are widely applied since they offer a low-cost and contactless tool for accurate measurements [5–7]. However, the temperature evaluation often requires support from device simulation because it represents indirect measurement methods. The analytical approach that can compete with numerical methods remains a great challenge for such a complex system.

This work demonstrates the application of selected theoretical models for heat dissipation for temperature evaluation in the channel region of an InAlN/AlN/GaN HEMT device. The analytical approach is compared with the numerical simulation and experimental observation of the temperature distribution. An electrical and thermal behavioral model under quasi-static conditions is used to describe the HEMT device [8]. The average temperature of the active device area is evaluated and compared with the experimentally determined one.

2. Experimental

2.1. Structure design and experimental setup

The investigated In_{0.185}Al_{0.815}N/AlN/GaN HEMT structure including 6.3 nm In_{0.185}Al_{0.815}N/1.5 nm AlN/1830 nm GaN heterostructure was grown by MOVPE on 350 μm thick 4 H-SiC substrate. The backside Au contact is soldered to 1 mm thick CuMo leadframe by 60 μm thick AuSn solder. Top ohmic drain/source and gate contacts were created by standard Au-based metallization.

Gated transmission line model (GTLM) HEMT of width \( w \approx 100 \mu m \) with a gate of length \( d_G \approx 0.15 \mu m \) situated in the middle of the source to drain gap of length \( d_{SD} \approx 2.5 \mu m \) was investigated. As a temperature sensor, a parallel TLM structure of length \( d_{TLM} \approx 2.5 \mu m \) and width \( w \approx 100 \mu m \) located in the distance of \( d_{GT} \approx 120 \mu m \) from GTLM HEMT was utilized as shown in figure 1. The device is set in the open package placed on the Al thermal chuck preserved at a constant temperature.

Figure 1. Horizontal view and position of GTLM HEMT and TLM structure.

Figure 2. Output characteristics for gate-source voltage \( V_{GS} = 0 \) V at various ambient temperatures \( T_0 \).

Semiconductor parameter analyzer Agilent 4155C and controlled thermal chuck were utilized to acquire output characteristics at zero gate-source voltage \( V_{GS} \) and drain-source voltage \( V_{DS} \) varied from 0 V up to 20 V together with average TLM temperature \( T_S \) obtained as the current temperature dependence at constant voltage \( \sim 0.5 \) V in the low-power operating regime. The chuck temperature was set in the range 25 °C–85 °C to demonstrate and to compare methods based on ambient temperature and dissipated power variation. The device was recovered by white LED illumination for 1 min between measurements. The three-dimensional (3D) thermal FEM simulations of the device were performed by Synopsys TCAD Sentaurus [8, 9]. The 3D model was created according to device geometry, layout and thickness of individual layers. Material thermal conductivity values were obtained from the previous work and calibrated utilizing the measurements [10]. The boundary condition of the constant ambient temperature is set to the structure backside based on ideal heat
transfer to the heatsink. The structure self-heating is simulated by three thermal contacts placed along 2DEG between drain and source. The individual heat sources represent heat contribution from drain to source the access region, the region under the gate electrode and the pinch-off region located at the drain side gate edge [11].

2.2. Average channel temperature determination

The drain–source current $I_{DS}$ dependence on the drain–source voltage $V_{DS}$ for gate–source voltage $V_{GS} = 0$ V at varying ambient temperature $T_0$ in the range of 25 °C–85 °C and sensor temperature $T_S$ dependence on total dissipated power $P$ were acquired as shown in figures 2 and 3, respectively. Low temperature-dependent threshold voltage $V_{TH} \approx -3.8$ V acquired from semi-logarithmic transfer characteristics of the transistor, $V_{GS} = 0$ V and drain to gate voltage drop $V_{DG} \approx 1.5$ V allows us to estimate saturation voltage $V_{DS,SAT} = V_{GS} - V_{TH} + V_{DG}$ corresponding to the measured value $V_{DS,SAT} \approx 6$ V.

The electro-thermal model offers an approximate pinch-off area length increase under the gate $d_{PO}/V_{PO} \approx 0.4$ nm V$^{-1}$ and gate to drain area $d_{PD}/V_{PO} \approx 2.2$ nm V$^{-1}$ utilizing pinch-off voltage $V_{PO}$ in the saturation regime. 3D thermal FEM simulations are keeping up with HEMT dimensions such as $d_{PO}$ and $d_{PD}$. Assuming $dV_{PO} \approx dV_{DS}$ the channel length modulation caused by $d_{PO}$ results in the constant ratio $g_D/I_{DS} \approx (d_G - d_{PO})^{-1}d_{PO}/dV_{PO} \approx 2.7.10^{-3}$ V$^{-1}$ [8]. Small $d_G$ brings notable $g_D$ in comparison with measured output conductance $dI_{DS}/dV_{DS}$ including self-heating. Various methods are advisable to compare and obtain proper $g_D$ value having a significant impact on average channel temperature $T_A$ determination for short gated HEMT.

Differential thermal resistance $R_A(T_0)$ vs. $P$ and $R_A(T_0)$ vs. $T_0$ at different $T_0$ calculated by equation (11) (see appendix) is shown in figures 4 and 5, respectively. The increase of $R_A$ with $P$ at defined $T_0$ is partially caused by spatially distributed electrical parameters of the active device area such as $g_D$ variation which is caused mainly by non-linear pinch-off length.
Thermal resistance $R_A$ dependence on dissipated power $P$ at various ambient temperature $T_0$ utilizing equation (15) in the appendix.

Thermal resistance $R_A$ dependence on average temperature $T_A$ equal to the sensor temperature $T_S$.

Directly acquired $R_{S00} = \frac{dT_{SM0}}{dP_0}$ from sensor temperature $T_S$ dependence on power dissipation $P$ and $R_{S00}$ calculated by equations (11) and (22) (see appendix) are similar due to the negligible influence of dissipated power spatial distribution in linear and saturation regime on the far location of the sensing element despite TLM width. To include the device topology the distance $s/l$ ratio is defined where $s$ is the distance of the sensor device (TLM), and $l$ is the channel distance (GTLM) from the surrounding area with the ambient temperature. In other words, $s/l$ close to one represents sensor device nearby the channel region, whereas $s/l$ ratio approaching zero stands for sensor device far away from the channel region. Utilizing equations (14) and (22) and assuming $\Delta T_S \approx 1$ dissipated power variation method allows us to plot $s/l(T_S)$ vs. $P$, $R_A(T_S)$ vs. $P$ and $R_A(T_S)$ vs. $T_S$ dependence at different $T_0$ as shown in figures 6–8, respectively. The dissipated power variation method based on $T_S$ acquisition across the whole $T_A$ range requires much higher $V_{DS}$ as well as $g_D$ range than the ambient temperature variation method. Therefore, $R_A$ and $s/l$ deviation are more significant especially for $R_A$ being slightly dependent on $T_A$. Measured data approximation eliminates missing overlap of $R_A(T_S)$ vs. $T_S$ for different $T_0$ caused by $g_D$ variation and spatially distributed electrical parameters as shown in figure 8.

Approximate thermal resistance $R_A$ dependence on ambient temperature $T_0$ and sensor temperature $T_S$, respectively, depicted in figures 5 and 8 with consecutive linear fit gives us the opportunity to plot average temperature $T_A$ dependence on dissipated power $P$ depicted in figure 9 utilizing equations (13) and (16) (see appendix). For constant $V_{GS}$ applied to the InAlN/AlN/GaN HEMT self-heating results in $I_{DS}$ decrease caused mainly by thermally induced serial source area resistance increase [8]. Therefore, the average temperature of the source to the gate area is related to $T_A$. Simulated and calculated results are in good agreement as depicted in figure 9.
3D thermal FEM simulations were utilized to acquire HEMT channel temperature $T_C$ vs. distance $x_s$ from the drain side of the gate edge perpendicular to the gate in the middle of the HEMT width as shown in figure 10. Due to different dissipated power spatial distribution in the pinch-off area and source to drain region, the condition $x_s \gg d_{SD}$ needed for low $V_{DS}$ is gradually turned into $x_s \gg d_{PO} + d_{PD}$ for high $V_{OUT}$; these conditions are required for regular $s/l$ determination.

These conditions also allow acquisition of $x_s$ and $T_C$ vs. $s/l$ ratio dependencies for different dissipated power $P$ at $T_0 = 25 \degree C$ utilizing equations (23) and (24) (see appendix) and the quadratic fit of simulated average temperature $T_A$ vs. $P$ dependence, as shown in figure 11. Fulfilled condition $(d_{SP}/s) \ll (dP/P)$ results in $x_s$ vs. $s/l$ ratio plot independent of dissipated power.

The procedure mentioned above applied for $P = 2$ W at different $T_0$ makes it possible to obtain $x_s$ and $T_C$ vs. $s/l$ ratio dependencies as depicted in figure 12. Condition $(d_{SP}/s) \ll d_{TP}(PR_S)$ fulfilled for $P = 2$ W is assumed as well as for lower dissipated power resulting in $a_M \approx 1$ across the operating range.

3. Conclusions

The temperature sensor placed nearby the active device area was employed in order to minimize ambient temperature variation across the whole operating temperature range. The experimental part deals with an average InAlN/AIN/GaN HEMT channel temperature determination utilizing electrical parameters acquired by quasi-static measurements. Subsequently, the electrical and thermal models were analyzed to determine the average temperature of the active device area using ambient temperature and dissipated power variation methods under quasi-static conditions.

Average channel temperature $T_A \approx 160 \degree C$ indicating the major change of thermal resistance $R_T$ calculated for dissipated power $P = 2$ W in GTLM InAlN/AIN/GaN HEMT structure shows a good agreement with simulated results. Robust 3D thermal numerical simulations were a stimulus to develop a simplified analytical model.

Proper isothermal output conductance acquisition makes this method applicable for a field-effect transistor (FET), TLM structure or Schottky diode. Average channel temperature $T_A$ is related to the most thermally sensitive element of the device, $T_A$ evaluated by the proposed one-dimensional analytical model is close to the average temperature of the source to drain area of investigated HEMT determined by numerical simulations. The different influence of dissipated power and ambient temperature variation on heat flux distribution across the temperature sensor is negligible if the sensor is located far from the active area in comparison with a source to drain distance. The suggested analytical model represents a reliable and straightforward alternative for numerical simulations of complex HEMT devices.

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Appendix. Theory

Thermal model

In general under the quasi-static operating condition and dissipated power free space heat transfer equation describes heat flux vector $\vec{q}^s = -\lambda_i \nabla T$ dependent on temperature gradient $T$ and thermal conductivity $\lambda_i = (\lambda_{xx}, \lambda_{yy}, \lambda_{zz})$ in 3D orthogonal space using cartesian vectors $\hat{e}_x, \hat{e}_y, \hat{e}_z$ and coordinates $x, y, z$. The suggested analytical model represents a reliable and straightforward alternative for numerical simulations of complex HEMT devices.
\[ X_s = (x_s, y_s, z_s) \] \[ \vec{q}_l = -\lambda_s \frac{dT}{dx} x - \lambda_y \frac{dT}{dy} y - \lambda_z \frac{dT}{dz}. \] \[ (1) \]

Heat flux from device active area to defined ambient temperature \( T_0 \) boundary forms equithermal surface consisting of elementary surfaces \( dS = dS_x \vec{i}_d \) utilizing perpendicular \( \vec{i}_d \) vector. Equithermal elementary surfaces shifted by \( \vec{d}l = dS_x \vec{i}_d \) exhibit \( dT \) in \( \vec{q}_l = q_l \vec{i}_d \) direction and equation (1) result in \( \vec{q}_l = -\lambda_l (dT/dl) \vec{i}_d \) which is possible to be integrated across the equithermal surface utilizing spatially dependent \( dT \) and \( \lambda_l \):

\[ \iiint_{dS_x} q_l dS = -\iiint_{dS_x} \lambda_l [dT(X_s)/dl] \vec{i}_d dS_x. \] \[ (2) \]

The left side of equation (2) represents total dissipated power \( P \) of the active area for all power sources situated inside the space bounded by equithermal surface. The right side of equation (2) results in scalar form and makes transformation of 3D heat propagation into 1D possible by applying the following:

\[ \frac{dT}{dx} = \lambda_l^{-1}(T) q = r_l(T) q. \] \[ (3) \]

Constant heat flux \( q \) evenly produced at \( x = l > 0 \) results in temperature profile \( T(x) \) assuming zero \( \lambda_l \) for \( x > l \) and \( T(0) = T_0 \) set at zero \( x \). Thermal resistance \( r_l(T) = \lambda_l^{-1}(T) \) utilized in equation (3) for \( 0 < x < l \) is temperature-dependent.

If a block of fixed length and heat flux is prolonged by \( d_0 \) then \( dT(0) = dT_0 = r_l d_0 \) using \( r_l(0) = r_0 \) and subsequently \( dT_T(x) = r_l(x) d_0 \) as shown in figure 14(a). Length shift \( d_0 \) here is possible to be caused by ambient temperature shift \( dT_0 \) and the ratio between \( dT_T(x) \) and \( dT(0) \) \( dT_T(x) \) is determined:

\[ dT_T(x)/dT_0 = r_l(x)/r_0. \] \[ (4) \]

Two different heat flux intensities \( q_A, q_B \) with corresponding length differences \( d\lambda_A, d\lambda_B \) result in similar temperature difference \( dT(T) = r_l(T) q_A d\lambda_A = r_l(T) q_B d\lambda_B \) for the same temperature. Subsequent integration with boundaries \( 0, x_A \) and \( 0, x_B \) supposing \( T(x_A) = T(x_B) \) yields:

\[ q_{d\lambda A} = q_{d\lambda B}. \] \[ (5) \]

Heat flux increase \( dq = dP/S \) results in temperature increase \( dT_p(x) = r_l(x) dq_0 \) for \( x < l \) as depicted in figure 14(b). Substitution \( q_A = q, q_B = q + dq \) yields in equation (5) resulting in \( dq_0 = dq \) makes it possible to determine \( dT_p(x) \):

\[ dT_p(x) = r_l(x) dq_0. \] \[ (6) \]

Differential thermal resistance \( R(x) = dT_p(x)/dP \) brings \( R(x) = r_l(x) dP/S \). In similar way \( R_0(x) = r_0(x) dP/S \) is defined, therefore equation (4) yields:

\[ dT_T(x)/dT_0 = R(x)/R_0(x). \] \[ (7) \]

It is possible for \( dT_p(x) = R(x) dP \) to be invoked by dissipated power increase \( dP \) produced at \( l \). On the other hand at constant \( P \) ambient temperature increase, \( dT_0^* \) results in \( dT^*(x) \) consisting of \( dT_T(x) = [R(x)/R_0(x)] dT_0^* \) at constant dissipated power and \( dT_0^* = R(x) dP^* \) caused by dissipated power difference \( dP^* \) at constant \( T_0 \).

Subsequently substitution \( k(x) = dT(x)/dT^*(x) \) in \( dT^*(x) \approx dT_T(x) + dT_p(x) \) yields:

\[ R_0(x) = (dT_0^*/dP)/\left[k^{-1}(x) - \left(\frac{dP^*}{dP}\right)\right]. \] \[ (8) \]

As depicted in figure 1 figure average temperature \( T_A \) assumed as a constant value across the device active area defines the boundary conditions \( T(0) = T_0 \) at \( x = 0 \) and \( T(l) = T_A \) at \( x = l \) and \( dT^*(l) = dT_A^* \). dT_T(l) = dT_A^*, dT_p(l) = dT_A^*, R(l) = R_A, R_0(l) = R_0, k(l) = k_A. Thermal parameters \( T_s, dT_s^*, dT_s^*, dT_s^p, k_s, R_s, R_{s0}, k_s \) of spot temperature sensor placed at \( x = s \) utilizing transformation shown in figure 1 are assigned in the similar way.

Various spatial power density contribution for \( dP^* \) and \( dP^* \) has negligible influence on heat flux distribution relatively far from the active area. On the other hand, due to various temperature dependence of thermal conductivity a heat flux change across the structure caused by \( dP, dP_A, dP^* \) is unequal resulting in equithermal surface deformation causing \( x \) shift in 1D
thermal model. In general, the ratio between \( l \) and \( s \) varies with \( P \) and \( T_0 \).

**Average temperature active area electrical model**

Active device at operating point characterized by output, namely drain–source voltage and drain–source current \((V_{DS}, I_{DS})\), input, namely gate-source voltage \(V_{GS}\) and zero input, namely gate current at average active area temperature \(T_A\) is inspected under quasi-static operating conditions. Output current difference \(dI_{DS}\) is defined by device transconductance \(g_M = dI_{DS}/dV_{GS}\) and output conductance \(g_D = dI_{DS}/dV_{DS}\) at constant \(T_A\) and thermal coefficient \(k_{TH} = dI_{DS}/dT_A\) determined at constant \(V_{GS}\) and \(V_{DS}\) [13]:

\[
dI_{DS} = g_M dV_{GS} + g_D dV_{DS} + k_{TH} dT_A. \tag{9}
\]

Coefficients \(g_M\), \(g_D\), \(k_{TH}\) are determined from electrical parameters at operating point \((V_{GS}, V_{DS}, T_A)\) and are required to be independent of the entire device thermal parameters [14].

At constant \(V_{GS}\) and \(V_{DS}\) small ambient temperature increase \(dT_A^*\) results in \(dT_A^*\) inside the active area and current change \(dI_{DS}^*\) is supposed and \(dT_A^*\) is also influenced by dissipated power variation \(dP^* = V_{DS} dI_{DS}^*\).

Increasing \(V_{DS}\) at constant \(V_{GS}\) results in \(dT_A\), caused by \(dV_{DS}\) and \(dI_{DS}\) and in dissipated power change \(dP = V_{DS} dI_{DS} + I_{DS} dV_{DS}\). Whereas equation (9) results in \(dI_{DS} = g_D dV_{DS} + k_{TH} dT_A\) and \(dI_{DS} = k_{TH} dT_A\), consecutive substitution yields:

\[
k_A = dT_A/dT_A = (dI_{DS} - g_D dV_{DS})/dP^*. \tag{10}
\]

Rising \(V_{DS}\) at constant \(V_{GS}\) brings an increase of dissipated power mainly in the pinch-off area for field-effect transistor (FET in saturation regime, though dissipated power contribution in the entire active device area is evident especially for low \(V_{DS}\). The different mode of \(dP\) and \(dP^*\) influence on heat flux distribution across the spot temperature sensor is negligible for \(|dP/dP^*| \gg k_s\) or sensor location far from the active area in comparison with a source to drain distance. In the case of TLM structure the \(dP\) and \(dP^*\) spatial distribution plays a lesser role. In the case of Schottky diode or multi-fingered device, an appropriate location of the spot temperature sensor is coming out from the principles mentioned above.

Proper device simulations such as pulse measurements allow \(g_D\) acquisition taking leakage paths, trapping and other secondary effects into account [15]. Correct \(T_A\) determination is influenced by dissipated power produced by temperature sensor as well.

**Ambient temperature variation**

Although this method has been introduced [16], the theoretical background and requirements are mentioned here. Utilizing equation (8) in the active area yields:

\[
R_A = (dT_0^*/dP^*) / \left[ k_A^{-1} - \left( \frac{dP^*}{dP} \right) \right]. \tag{11}
\]

Coming out from equations (10) and (11) zero \(g_D\) results in:

\[
R_A = (dT_0^*/dI_{DS}) / (I_{DS} dV_{DS} dP^*). \tag{12}
\]

Variation of \(T_0\) across the assumed operating temperature range allows us to obtain \(R_A = r_I(l) l/s = r_T(0) l/s\) resulting in \(dT_A = R_A dP = R_A (T_A) dP\) for \(T_A \approx T_0\). Recurrent calculations of \(T_A\) are processed in this way:

\[
T_A = T_{00} + \sum R_A (T_0) dP. \tag{13}
\]

In equation (13) \(T_{00}\) represents the starting ambient temperature whereas \(R_A (T_0)\) is calculated using equation (11) at increased \(T_0\). This method requires accurate \(T_0\) determination across the whole \(T_A\) range independent on dissipated power. Otherwise \(R_A\) variation with changing \(V_{DS}\) at defined \(T_A \approx T_0\) points on improper \(g_D\) assumption or phenomena caused by spatially distributed electro-thermal parameters, temperature gradient and different allocation of dissipated power change \(dP\) and \(dP^*\) along the active device area.

**Dissipated power variation**

This method is advisable to acquire average temperature \(T_A\) if a small-sized temperature sensor is placed relatively far from an active device area because of the negligible influence of different dissipated power spatial distribution on sensor temperature \(T_S\). For FET and temperature sensor placed nearby the active device suppose \(|dP/dP^*| \gg k_s\) the \(T_A\) deviation is expected due to different dissipated power spatial distribution in the pinch-off area and source to drain area playing a lesser role with increasing \(V_{DS}\).

The ratio \(R_A / R_S = l/s\) coming out from constant \(r_I(x)\) at \(T_0\) ambient temperature and comparison of equation (11) applied for \(R_A\) and \(R_S\) yields:

\[
s/l = \left[ k_A^{-1} - \left( \frac{dP^*}{dP} \right) \right] / \left[ k_s^{-1} - \left( \frac{dP^*}{dP} \right) \right]. \tag{14}
\]

Similarly the same temperature-dependent \(r_T(x)\) at \(T_S\) utilizing \(r(x) = r_I(x) s/l\) applied for \(R_A\) and \(R_S = dT_S / dP)\) results in:

\[
R_A (T_S) = R_S (T_S) (s/l)^{-1} = (dT_S / dP) (s/l)^{-1}. \tag{15}
\]

Consecutive \(R_A\) utilization for \(T_S \approx T_A\) brings:

\[
T_A = T_{00} + \sum R_A (T_S) dP. \tag{16}
\]

Results from equations (14)–(16) are applicable in a straight way for the structure consisting of materials exhibiting a constant ratio of particular thermal conductivities within the operating temperature range. In this case, spatial heat flux distribution left intact for \(dT_0^*\) and evenly increased for \(dP\) and \(dP^*\) results in unchanged equithermal surface shape corresponding to particular spatial position.

However, various thermal conductivity dependence on temperature requires us to be aware of \(s\) position shift caused by
unequal heat flux change invoked by $dT_0^*$ or $dP$ and $dP^*$. For temperature sensor the difference between measured temperature shift $dT_{SM}$, $dT_{SM}^*$ and temperature shift $dT_S$, $dT_S^*$ mentioned above exhibit difference of $(R_S P_S) (dP/dT_{SM})$ where $dP_S$ is caused by $dT_0$ and $dP$ results in $dP_S$ and $dP^*$ proportionally in $dP_S/dP^*$. For $T_S ≈ T_0$ the conditions $dP ≈ P$ and $(dP_S/s_0) ≪ 1$ result in $R_{SM} = R_{SM}(T_0) = R_{SM0}$. Utilizing thermal resistance $R_{SM}/R_S = 1 = (dP_S/s_0)/(dP/P)$. Substitution $k_S = dT_S/dT_S^*$ and $R_S = dT_S/dP$ in $dT_S = dT_0^* (R_S/R_{SM0}) + R_S dP^*$ brings:

$$k_S = dT_S/dT_S^* = dP/[(dP_S/R_{SM0}) + dP^*]$$

$$s/l = R_{SM}/R_{SM0} = \left(k_A^{-1} - \left(\frac{dP^*}{dP}\right)\right) R_{SM0} dP/dT_0^*.$$  

(21)

Thermal model transformation

This part deals with $s/l$ ratio acquisition if thermal parameters of the active device area and temperature sensor $P$, $T_0$, $T_A$, $T_S$ are obtained from simulations or spatial calculations. The equi-thermal surface of temperature $T_S$ and $s/l$ ratio meets the requirements mentioned above. In a simple way equation (3) is able to be rewritten as:

$$dx = dT(x)/[r_T(T)P/S].$$

(23)

At defined $T_0$, the dissipated power $P_A$ needed for reaching $T_S$ in the active device area at $l$ position is applied. Subsequently the dissipated power $P_S$ at $l$ position needed for reaching $T_S$ nearby temperature sensor at $s$ position is set. Substitution $x_A = l$ and $x_B = s$ in equation (5) yields:

$$s/l = P_A/P_S.$$  

(24)

The procedure mentioned above allows us to acquire $T_S$ dependence on $s/l$ ratio for various $P$ and $T_0$ and subsequently calculate $(dP_S/s_0)$ and $(dP/S_0)$ for defined $s/l$. Utilizing thermal resistance $R_S(T_S) = dT_S/dP$, $R_{SM}(T_S) = dT_S/dT_0^*$ at defined $s/l$ and $R_{SM}^*(T_S) = dT_{SM}/dP$, $R_{SM0}(T_S) = dT_{SM}/dT_0^*$ at a spatial position corresponding to $s/l$ ratio allows us to verify them coming out from equations (17)–(19):

$$\frac{(dP_S/s_0)}{dP} = (R_{SM}/R_S - 1)/P$$

(25)

$$\frac{(dP_S/s_0)}{dP_0^*} = (R_{SM}^* - R_{SM0}^*)/(PR_S).$$

(26)

The shift ratio calculated as $a_M = R_S R_{SM}/R_{SM0} R_S$ brings correction of $s/l$ ratio calculations equation (14) from experimental data.

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References

[1] Ambacher O, Smart J, Shealy J R, Weimann N G, Chu K, Murphy M, Schaff W J and Eastman L F 1999 J. Appl. Phys. 85 3222
[2] Chattopadhyay M K and Tokekar S 2008 Microelectron. J. 39 1181
[3] Nigam A, Bhat T N, Rajamani S, Dolmanan S B, Tripathy S and Kumar M 2017 AIP Adv. 7 85015
[4] Chowdhury S 2015 Phys. Status Solidi a 212 1066
[5] Berthet F, Guhel Y, Gaulious H, Boudart B, Trolet J L, Piccione M and Gaquiére C 2011 Microelectron. Reliab. 51 1796
[6] Yuan Y L, Yong X X, Kai Z, Feng Z X, M X H and Yue H 2012 Chin. Phys. B 21 077304
[7] Kim J, Freitas J A, Mittereder I, Fitch R, Kang B S, Pearson S J and Ren F 2006 Solid-State Electron. 50 408
[8] Florovic M, Szobolovszky R, Kovac J, Kovac J Jr, Chvala A, Jacquet J C and Delage S 2019 Semicond. Sci. Technol. 34 065021

[9] Synopsys TCAD Sentaurus 2017 Sentaurus Device User Guide, v. L-2017.09 (San Jose, CA, USA)

[10] Chvala A, Szobolovszky R, Kovac J, Florovic M, Marek J, Cernaj L, Donoval D, Dua C, Delage S L and Jacquet J C J. Electron. Packag. 2019 141 031007

[11] Chvala A, Marek J, Pribytny P, Satka A, Stoffels S, Posthuma N, Decoutere S and Donoval D 2017 Microelectron. Reliab. 78 148

[12] Lienhard J H 2000 A Heat Transfer Textbook (Cambridge, MA: Phlogiston Press) pp 49–91

[13] Muthuswamy B and Banerjee S 2017 Introduction to Nonlinear Circuits and Networks (Berlin: Springer) pp 59–62

[14] Chvala A, Marek J, Cernaj L, Pribytny P, Kozarik J and Donoval D 2019 APCOM 2019 pp 47–50

[15] Chemming C H 2010 Modern Semiconductor Devices for Integrated Circuits (Upper Saddle River, NJ: Prentice Hall) pp 200–14

[16] Menozzi R, Membreno G A U, Nener B D, Parish G, Sozzi G, Faraone L and Mishra U K 2008 IEEE Trans. Device Mater. Reliab. 8 255