Performance of tuned tandem mass dampers based on shape memory alloy

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Abstract. Tuned tandem mass dampers based on shape memory alloy (SMA-TTMD) are proposed in this paper. The SMA-TTMD is formed by replacing the connection damping between two mass blocks in the TTMD with a shape memory alloy spring. Firstly, the stochastic equivalent linearization of non-linear SMA is presented. It is attached to a single-degree-of-freedom structure excited by stationary filtered white noise to study the performance of SMA-TTMD. Based on the minimum displacement variance of the main structure, the structure random vibration method is used to study the optimal control performance. The effectiveness and stroke of the SMA-TTMD under the changes of mass ratios are studied, and the optimal parameters of the system are proposed. The results are compared with those of TTMD. The numerical analysis shows that the control performance of SMA-TTMD is better than that of TTMD. The results demonstrate that the stroke of SMA-TTMD is obviously superior to that of TTMD.

1. Introduction

Since Frahm first proposed passively tuned mass damper (TMD) in 1909, vibration control using vibration absorber is a common means of vibration reduction.[1] There are many innovations in terms of models for passive control. Li and Yang[2] proposed tuned tandem mass dampers (TTMD), which has better vibration reduction effect than TMD. Recently, SMA materials have been introduced into the field of civil engineering, mainly for the production of passive energy dissipation devices. The Graessler-Cozzarelli model[3] is a good description of the superelastic properties of SMA. Mishra[4] proposed an improved tuned mass damper (SMA-TMD) assisted by a shape memory alloy spring, which reduces the mass block stroke and improves the effectiveness of vibration reduction. The shape memory alloy modified liquid column damper (SMA-LCD)[5] was proposed by Gur et al. The study of vibration control under random excitation has more practical significance for the engineering application of SMA materials. Sun[6] analyzed three kinds of typical stochastic seismic motion models.

Based on the research of TTMD, tuned tandem mass dampers based on shape memory alloy (SMA-TTMD) is proposed to further improve the vibration control effect of TTMD.

2. Equivalent linearization of shape memory alloy

The equivalent linearization method[7] is used to predict the response of SMA superelastic system under random excitation. A simple polygonal superelastic model is proposed for this purpose. Figure 1[4]
Figure 1. Yan-Nie model and Graesser model.

Figure 2. Mechanical model of SMA-TTMD.

presents the Yan-Nie model and Graesser model. The restoring force of the shape memory alloy spring can be expressed as

\[ F(x, \dot{x}) = \alpha kx + (1-\alpha)kZ \]

(1)

In the above formula, the first item on the right is the elastic force, and the second item is the hysteretic force. where Z is the hysteretic displacement. \( k \) is the stiffness of SMA in the austenite state, \( \alpha \) is the stiffness ratio of martensite and austenite state. Z is a non-linear function of \( x \) and \( \dot{x} \), expressed as

\[ Z = c_x \dot{x} + k_x x \]

(2)

Where \( x \) and \( \dot{x} \) are the displacement and velocity of SMA. The expressions of stochastic linearization stiffness and damping are given as follows[7]

\[ c_x = \frac{b-a}{\sqrt{2\pi}\sigma_x} \left[ 1 - \text{erf} \left( \frac{a}{\sqrt{2}\sigma_x} \right) \right]; \quad k_x = \frac{a+b}{\sqrt{2\pi}\sigma_x} \exp \left( -\frac{a^2}{2\sigma_x^2} \right) \]

(3)

Where \( a \) and \( b \) respectively represent the displacement of the elastic limit and the displacement corresponding to the point of starting to induce martensite phase transition, which can be determined by experiments. \( \sigma_x \) and \( \sigma_{\dot{x}} \) are the standard deviation of displacement and velocity of SMA.

3. Formulation of SMA-TTMD

The mechanical model of the structure-SMA-TTMD system is shown in Figure 2. The dynamic equation of the structure-SMA-TTMD system can be expressed as:

\[ m_1(\ddot{y}_1 + \ddot{x}) + c_1 \dot{y}_1 + k_1(y_1 - y_s) - c_1(y_1 - \ddot{y}_1) - k_2(y_2 - y_s) - c_2(y_2 - \ddot{y}_2) = 0 \]

(4)

\[ m_2(\ddot{y}_2 + \ddot{x}) + k_1(y_1 - y_s) + c_1(y_1 - \ddot{y}_1) + F = 0 \]

(5)

\[ m_s(\ddot{y}_s + \ddot{x}) + k_2(y_2 - y_s) + c_2(y_2 - \ddot{y}_2) - F = 0 \]

(6)

\[ F = \alpha k(y_1 - y_2) + (1-\alpha)k \left[ \ddot{y}_1 - \ddot{y}_2 + \dddot{y}_2 - \dddot{y}_1 \right] \]

(7)

Where \( m_1, m_2, \) and \( m_s \) are the mass of the TMD1, TMD2 and structure, respectively; \( c_1, c_2, \) and \( c_s \) refer to the damping of the TMD1, TMD2 and structure, respectively; \( k_1, k_2, \) and \( k_s \) represent the stiffness of the TMD1, TMD2 and structure, respectively; \( y_1, y_2, \) and \( y_s \) denote the displacements of the TMD1, TMD2 and structure with respect to the ground, respectively. The earthquake ground motion acceleration is given by \( \dddot{x}_g \), which is modeled as stationary random processes in this paper. \( F \) stands for restoring force produced by SMA spring between the TMD1 and TMD2. By equivalent linearization of SMA, it can be simplified to (8). \( \alpha, k, \ddot{y}, \dddot{y} \) are as mentioned previously. In order to obtain a compact formulation of the SMA-TTMD, we beforehand introduce the following new variables:

\[ \mu = \frac{m_1 + m_2}{m_s}, \eta = \frac{m_1}{m_2}, \xi_s = \frac{c_s}{2m_s\omega_s}, \omega_s^2 = \frac{k_s}{m_s} \]
The transformation force \( F_{ys} \) of the SMA spring is normalized as

\[
F_0 = \frac{F_{ys}}{(m_1 + m_2)g}
\]

Then the stiffness \( k \) of SMA in the austenite state is becoming

\[
k = \frac{F_{ys}}{u_s} = \frac{F_0(m_1 + m_2)g}{u_s}
\]

Where \( u_s \) is the displacement corresponding to the forward phase transformation in the SMA spring. The equation (5-8) are organized as follow

\[
\dot{\ddot{\mathbf{\Gamma}}} + \dot{\mathbf{C}} \dot{\mathbf{\Gamma}} + \mathbf{K} \mathbf{\Gamma} = -\mathbf{M} \ddot{\mathbf{x}}_g \quad \text{(8)}
\]

Where \( \mathbf{M}, \mathbf{C}, \mathbf{K} \) is mass, damping and stiffness matrix of the structure-SMA-TTMD system; \( \mathbf{u} \) is the displacement component, which includes the displacement of the main structure and TMDs, i.e. \( \mathbf{u} = (y_1, y_1, y_2)^T \); and \( \mathbf{r} = (1, 1, 1)^T \).

The Kanai-Tajimi stochastic model[6] assumes that the absolute acceleration of bedrock is an ideal white noise with an average value of zero, and the spectral density is \( S_0 \). The soil layer on the bedrock is equivalent to a filter, which is simplified as a linear single-degree-of-freedom system. The ground acceleration process \( \ddot{x}_g \) can be expressed as the stationary response of the following filters:

\[
\begin{cases}
\dddot{\mathbf{y}}_1 + 2\zeta_g \omega_g \mathbf{\ddot{y}}_1 + \omega_g^2 \mathbf{\mathbf{y}}_1 = -\ddot{\mathbf{U}}_g \\
\dddot{\mathbf{x}}_g = \mathbf{\ddot{y}}_1 + \ddot{\mathbf{U}}_g
\end{cases}
\]

Where \( \dddot{\mathbf{y}}_1 \) and \( \dddot{\mathbf{U}}_g \) is the acceleration of the surface relative to the bedrock and acceleration of bedrock motion; \( \omega_g \) and \( \zeta_g \) are the characteristic frequency and characteristic damping ratio of the overburden soil, respectively. Then the state vector of the system is expressed as

\[
\mathbf{y} = (u \quad \dot{u} \quad Y \quad \dot{Y})^T
\]

So the equation of motion in the state space is

\[
\dot{\mathbf{P}} = A\mathbf{P} + \mathbf{W}
\]

Where \( A \) is the augmented matrix. According to the random vibration theory, the mean of the excitations is zero, so the mean of the \( \mathbf{P} \) is also zero. And the covariance matrix \( \Gamma \) for \( \mathbf{P} \) has the following equation[8]

\[
\dot{\Gamma} + \Gamma \dot{\mathbf{A}}^T + \mathbf{D}_S = 0
\]

Where \( \mathbf{D}_{\mathbf{S}} \) is a matrix of zero except for the last term \( 2\pi S_0 \). Then the element \( \Gamma_{11}, \Gamma_{22} \) and \( \Gamma_{33} \) are the stationary displacement variances of the structure, TMD1 and TMD2, expressed as

\[
\sigma_{\gamma_1}^2 = \Gamma_{11}; \quad \sigma_{\gamma_2}^2 = \Gamma_{22}; \quad \sigma_{\gamma_3}^2 = \Gamma_{33}
\]

Since the matrix \( A \) contains displacement variances, formula (12) needs to be solved iteratively.

4. Performance evaluation of SMA-TTMD

The optimal objective function of SMA-TTMD is defined as

\[
R = \min (\sigma_{\gamma_1})
\]

For SMA-TTMD systems, the parameters that need to be optimized are \( \gamma_1, \gamma_2, \xi_1, \xi_2 \) and \( F_0 \). The range of \( \gamma_1 \) and \( \gamma_2 \) is 0.6-1.5; The range of \( \xi_1 \) and \( \xi_2 \) is 0-0.3; The range of \( F_0 \) is 0-1.5. Considering the complexity of objective function (14), genetic algorithm[9] is used to optimize the system. GA is an iterative adaptive probabilistic search algorithm based on natural selection and natural genetics. It is a robust intelligent optimization algorithm. The specified parameter values of SMA-TTMD, structure and earthquake are listed in table 1.

Figure 3 shows the variation trends of the minimum displacement variances of the structure with
respect to structure time periods for three $\eta$: (a) 0.25, (b) 0.5, (c) 1.0 when $\mu=0.01$. With the increase of time period of structure, the variance of structure displacement becomes larger and larger. The size of variance represents the effectiveness of vibration control. The smaller the variance is, the better the effectiveness of vibration reduction is. The variation trends and magnitude of the minimum displacement variances of the structure of SMA-TTMD are almost the same as that of TTMD. Therefore, the control effectiveness of SMA-TTMD and TTMD is almost the same when $\mu$ is 0.01.

| Parameters                          | Range or value                  |
|------------------------------------|---------------------------------|
| Total mass ratios $\mu$            | 0.01, 0.0025                    |
| Relative mass ratios $\eta$        | 0.25, 0.5, 1                    |
| Damping ratio of structure $\xi_s$ | 0.02                            |
| Properties of SMA                  | $\alpha = 0.1$, $a = 0.005m$, $b = 0.05m$, $us = 0.05m$ |
| Ground motion $\omega_p = 9\pi$ rad$s^{-1}$, $\xi_p = 0.6$, $S_0 = 0.05m^2$s$^{-3}$ |

Table 1. The specified parameter values.

Figure 4 presents the histograms of variation trends of displacement variances of the TMD1 with respect to structure time periods for three $\eta$: (a) 0.25, (b) 0.5, (c) 1.0 when $\mu=0.01$. Compared with TTMD, the stroke of TMD1 of SMA-TTMD decreases greatly. Compared with TTMD, the stroke of TMD1 of SMA-TTMD decreased by 59% to 69% at $\eta = 0.25$; the stroke of TMD1 of SMA-TTMD decreased by 43% to 60% at $\eta = 0.5$; the stroke of TMD1 of SMA-TTMD decreased by 34% to 50% at $\eta = 1.0$.

Figure 3. Variation trends of the minimum displacement variances of the structure with respect to structure time periods for three $\eta$: (a) 0.25, (b) 0.5, (c) 1.0 at $\mu=0.01$.

Figure 4. Variation trends of displacement variances of the TMD1 with respect to structure time periods for three $\eta$: (a) 0.25, (b) 0.5, (c) 1.0 at $\mu=0.01$. 

IOP Conf. Series: Earth and Environmental Science 304 (2019) 052014 doi:10.1088/1755-1315/304/5/052014
Figure 5. Variation trends of displacement variances of the TMD2 with respect to structure time
periods for three \( \eta \): (a) 0.25, (b) 0.5, (c) 1.0 at \( \mu = 0.01 \).

The histograms of variation trends of displacement variances of the TMD2 with respect to structure
periods for three \( \eta \): (a) 0.25, (b) 0.5, (c) 1.0 when \( \mu = 0.01 \) are plotted in Figures 5. Compared
with TTMD, the stroke of TMD1 of SMA-TTMD decreases greatly. Compared with TTMD, the stroke
of TMD2 of SMA-TTMD decreased by 15% to 23% at \( \eta = 0.25 \); the stroke of TMD2 of
SMA-TTMD decreased by 24% to 35% at \( \eta = 0.5 \); the stroke of TMD2 of SMA-TTMD decreased
by 35% to 50% at \( \eta = 1.0 \); Combining the stroke reduction of TMD1 and TMD2, the average stroke
reduction rate of each TMD is about 40%.

Figure 6. Variation trends of the minimum displacement variances of the structure with respect to
structure time periods for three \( \eta \): (a) 0.25, (b) 0.5, (c) 1.0 at \( \mu = 0.0025 \).

Figure 7. Variation trends of displacement variances of the TMD1 with respect to structure time
periods for three \( \eta \): (a) 0.25, (b) 0.5, (c) 1.0 at \( \mu = 0.0025 \).
Figure 8. Variation trends of displacement variances of the TMD2 with respect to structure time periods for three $\eta$: (a) 0.25, (b) 0.5, (c) 1.0 at $\mu=0.0025$.

In Figure 6, the plot of variation trends of the minimum displacement variances of the structure is displayed with the change of flexibilities of the structure at $\mu=0.0025$. Because of the increasing flexibility of the structure, the displacement variance becomes larger. And control effectiveness of SMA-TTMD and TTMD is roughly the same.

From Figure 7, it can be clearly concluded that the stroke of SMA-TTMD is obviously superior to that of TTMD when $\mu$ is 0.0025. Relative to TTMD, the stroke of TMD1 of SMA-TTMD is reduced by 50% to 64% at $\eta = 0.25$; the stroke of TMD1 of SMA-TTMD is reduced by 44% to 54% at $\eta = 0.5$; the stroke of TMD1 of SMA-TTMD is reduced by 42% to 49% at $\eta = 1.0$.

Figure 8 shows the histograms of variation trends of displacement variances of the TMD2 with the change of structure time periods for three $\eta$: (a) 0.25, (b) 0.5, (c) 1.0 when $\mu$ is 0.0025. Relative to TTMD, the stroke of TMD2 of SMA-TTMD is reduced by 18% to 25% at $\eta = 0.25$; the stroke of TMD2 of SMA-TTMD is reduced by 28% to 36% at $\eta = 0.75$; the stroke of TMD2 of SMA-TTMD is reduced by 38% to 45% at $\eta = 1.0$. And the average stroke of each TMD is approximately reduced by 40%. Hence, SMA-TTMD has absolute advantages in stroke.

Table 2. The $min.\sigma^2_{y_1}, \sigma^2_{y_1}, \sigma^2_{y_2}$ and the optimum parameters of TTMD and TMD for different structure time periods at $\mu = 0.01$ and $\eta = 0.25$.

| Structure time period(s) | $min.\sigma^2_{y_1}$ | $\sigma^2_{y_1}$ | $\sigma^2_{y_2}$ | $\gamma_1$ | $\xi_1$ | $\gamma_2$ | $\xi_2$ | $F_o/F_T$ |
|--------------------------|----------------------|------------------|------------------|-------------|--------|-------------|--------|-----------|
| TTMD                     | 1                    | 0.0083           | 0.911            | 0.497       | 1.057  | 0.000       | 0.956  | 0.035     | 0.006     |
| SMA-TTMD                 | 1.5                  | 0.0086           | 0.328            | 0.412       | 1.408  | 0.000       | 0.860  | 0.059     | 0.689     |
| TTMD                     | 1                    | 0.0265           | 2.924            | 1.599       | 1.057  | 0.000       | 0.956  | 0.035     | 0.006     |
| SMA-TTMD                 | 1.5                  | 0.0276           | 1.090            | 1.312       | 1.429  | 0.001       | 0.848  | 0.062     | 0.537     |
| TTMD                     | 2.5                  | 0.1203           | 16.372           | 6.807       | 1.057  | 0.000       | 0.956  | 0.035     | 0.006     |
| SMA-TTMD                 | 2.5                  | 0.1246           | 5.179            | 5.845       | 1.264  | 0.000       | 0.903  | 0.063     | 0.221     |

Tables 2 show a comparison of TTMD and TMD in terms of both the effectiveness and stroke and the optimal parameters are given at $\mu = 0.01$ and $\eta = 0.25$. These can be used for reference in engineering application.

5. Conclusion

Tuned tandem mass dampers based on shape memory alloy (SMA-TTMD) are proposed in this paper. Through theoretical derivation and definition of the optimal parameter evaluation criteria, the optimal control performance of SMA-TTMD is numerically studied under the stationary filtering model of white noise by using genetic algorithm optimization. And it is compared with TTMD. The numerical analysis shows that the control performance of SMA-TTMD is better than that of TTMD. Specifically, SMA-TTMD has absolute advantages in stroke. Compared with TTMD, the stroke of SMA-TTMD can be reduced by about 15% to 69%, with an average reduction of about 40%. And SMA-TTMD and TTMD have almost the same effectiveness. In addition, the paper provides some parameter design
tables of SMA-TTMD for reference in engineering application.

6. Appendices
Details of the augmented matrix in equation (12) are given as

\[
A = \begin{pmatrix}
\mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} & 0 & 0 \\
-M_{3\times 3}^{-1} K_{3\times 3} & -M_{3\times 3}^{-1} C & 2\xi \omega_r \mathbf{r}_{3\times 1} & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -\omega_r^2 & -2\xi \omega_r
\end{pmatrix}
\]

(A.1)

The system matrix in equation (8) are given as follows

\[
M = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{\mu}{\eta + 1} & 0 & 0 \\
0 & 0 & \frac{\mu}{\eta + 1} & 0
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
2\omega_r (\xi^2 + \gamma^2 \xi + \frac{\mu}{\eta + 1} + \frac{\mu}{\eta + 1}) & -2\gamma \omega_r \frac{\mu}{\eta + 1} & -2\gamma \omega_r \frac{\mu}{\eta + 1} \\
-2\gamma \omega_r \frac{\mu}{\eta + 1} & 2\gamma \omega_r \xi + \frac{\mu}{\eta + 1} + (1 - \alpha) \frac{F g}{\mu \tau} & -2\gamma \omega_r \xi + \frac{\mu}{\eta + 1} + (1 - \alpha) \frac{F g}{\mu \tau} \\
-2\gamma \omega_r \xi + \frac{\mu}{\eta + 1} & -2\gamma \omega_r \xi + \frac{\mu}{\eta + 1} & -2\gamma \omega_r \xi + \frac{\mu}{\eta + 1} + (1 - \alpha) \frac{F g}{\mu \tau}
\end{pmatrix}
\]

(A.2)

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