Cavity-QED based on collective magnetic dipole coupling: spin ensembles as hybrid two-level systems

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We analyze the magnetic dipole coupling of an ensemble of spins to a superconducting microwave stripline structure, incorporating a Josephson junction based transmon qubit. We show that this system is described by an embedded Jaynes-Cummings model: in the strong coupling regime, collective spin-wave excitations of the ensemble of electrons pick up the nonlinearity of the cavity mode, such that the two lowest eigenstates of the coupled spin-wave + microwave-cavity + Josephson-junction system define a hybrid two-level system. The proposal described here enables the use of spin ensembles as qubits which can be coherently manipulated and coupled using the same nonlinear-cavity. Possibility of strong-coupling cavity-QED with magnetic-dipole transitions opens up the possibility of extending previously proposed quantum information processing protocols to spins in silicon or graphene, without the need for single-electron confinement.

As compared to the coupling of a single emitter, the strength of optical excitations out of an ensemble of two-level emitters is enhanced by the square-root of the number of emitters ($\sqrt{N}$). This collective enhancement of light-matter coupling in free-space has played a central role in quantum memory and repeater protocols [1]. In the context of cavity quantum electrodynamics (QED), the $\sqrt{N}$ enhancement comes at the expense of the desirable nonlinearity of the coupled cavity-emitter system [2]. Nevertheless, strong electric-dipole coupling of a number of diverse systems including inter-band excitons [2], intersubband plasmons [3], and cold-atomic ensembles [5] to high quality ($Q$) factor optical cavities have been demonstrated: the signature of strong coupling for these systems is the appearance of Vacuum Rabi-splitting of two dressed modes, each with a perfectly harmonic spectrum. Direct magnetic-dipole coupling of spins to cavity modes on the other hand, have been totally ignored, even though collective excitations out of an ensemble of $\sim 10^6$ spins could easily reach a corresponding linear strong coupling regime using the superconducting microstrip (SCM) cavities recently realized in the context of circuit-QED experiments [6, 7].

In this Letter, we describe how two-level hybrid-spin qubits can be defined using magnetic-dipole coupling of an ensemble of spins to SCM cavities with a built-in nonlinear element such as a transmon qubit [8]. We consider a geometry where the transmon qubit, with ground and excited states denoted by $|a\rangle$ and $|b\rangle$, is introduced at an electric-field maximum of the SCM cavity. In contrast, an ensemble of spins are placed at a location where the magnetic field is maximum (Fig. 1). The Hamiltonian of the combined system in the interaction picture is given by

$$\hat{H} = \hbar g_e (\hat{\sigma}_{ba} \hat{a}_c e^{-i\delta t} + h.c.) + \hbar g_m \sum_{i=1}^{N_{s}} (\hat{\sigma}_{-i}^{\dagger} \hat{a}_c e^{-i\Delta t} + h.c.)$$

(1)

where $\hat{\sigma}_{ba} = |b\rangle \langle a|$ and $\hat{\sigma}_{-i}^{\dagger}$ denotes the spin lowering operator of the $i^{th}$ spin. $\hat{a}_c$ is the annihilation operator of the SCM cavity-mode. $g_e$ ($g_m$) denotes the electric (magnetic) dipole coupling strength of the transmon-qubit (single-spin) to the SCM cavity mode. We assume here that the transmon-qubit (spins) is (are) red detuned from the cavity-mode by $\delta$ ($\Delta$).

Before proceeding, we highlight the recent work discussing a very similar scenario where an ensemble of polar molecules are coupled to a SCM cavity containing a josephson-junction qubit [9]. The underlying idea...
for this pioneering work was the use of strong electric-dipole coupling strength of polar molecules to achieve an interface between a solid-state and an ensemble molecular qubit, with the goal of using the molecular qubit as quantum memory. The present proposal in contrast is based on strong coupling regime of magnetic dipole interaction in systems where electric-dipole coupling is vanishingly small: such systems have the advantage of being immune to charge noise and could constitute qubits with much longer coherence times.

A key element of the proposal we describe here is the unprecedented values of cavity-qubit electric-dipole coupling strength \( g_c \) achieved using a transmon qubits in SCM cavities, saturating the fundamental limit \( g_c = \sqrt{\alpha} \omega_c \). As \( \sqrt{\alpha} \) and \( \omega_c \) denote the fine-structure constant and the bare-cavity resonance frequency, respectively. In this regime, the condition \( g_c \gg g_m \sqrt{N_s} \) is satisfied and we can start our analysis by neglecting the second term in Eq. 1: we then obtain the celebrated Jaynes-Cummings (JC) spectrum for the coupled cavity-transmon system, with an anharmonic spectrum. In the first part of our analysis, we will focus on the resonant case with \( \delta = 0 \).

We now turn our attention to the coupling of the cavity-transmon molecule to the ensemble of spins. If we choose \( \Delta \) such that \( |g_c - \Delta| \ll g_c \), and the cavity is high enough to satisfy \( g_c \gg \kappa_c \approx \omega_c/Q \), we would only need to consider the coupling of collective excitations of the spins to the two lowest-energy eigenstates of the JC ladder; namely the ground state \( |0\rangle = |a,0_c\rangle \) and the symmetric state of the single-excitation \( |1\rangle \) manifold \( |1\rangle = (|a,1_c\rangle + |b,0_c\rangle) / \sqrt{2} \). This is a consequence of the fact that the energy separation between the lowest energy states of the one \( |n=1\rangle \) and two \( |n=2\rangle \) photon excitation manifolds in the JC-ladder is given by \( g_c(\sqrt{2} - 1) \), which, is, by our initial assumption, much larger than the energy scale \( g_m \sqrt{N_s} \) associated with coupling to spins. Since the eigenstates of the spin ensemble form a harmonic ladder, the coupling of the two-level system spanned by the eigenstates \( |0\rangle \) and \( |1\rangle \) to collective spin excitations maps on to an embedded JC ladder. The scenario we envision is depicted in Figure 2 where the encircled states form the embedded JC model. Provided that the corresponding coupling strength, given by \( g_m \sqrt{N_s} \), is larger than the decoherence rate of the spin excitation \( \kappa_{spin} \), transmon qubit \( \gamma_{JJ} \) and the SCM cavity-mode \( \kappa_c \), the two lowest energy states of the embedded JC ladder define a two-level hybrid spin qubit. For \( \delta = 0 \) and \( \Delta = g_c \), the Hilbert space of this qubit is spanned by the states

\[
\begin{align*}
|\tilde{0}\rangle &= |G, a, 0_c\rangle \\
|\tilde{1}\rangle &= \frac{1}{\sqrt{2}}(|E, a, 0_c\rangle + |G, b, 0_c\rangle + \frac{1}{2}|G, a, 1_c\rangle)
\end{align*}
\]

where \( |G\rangle \) and \( |E\rangle = (\sum_{i=1}^{N_s} \hat{\phi}^+_i) |G\rangle / \sqrt{N_s} \) denote the fully polarized ground and first excited (single collective spin-flip) states of the spin ensemble. To estimate the validity of the two-level approximation, we compare the degree of the anharmonicity given by \( g_m \sqrt{N_s} (\sqrt{2} - 1) \) with the decoherence rates typical for the system: \( \kappa_c \approx 1 \times 10^8 \approx \gamma_{JJ} \geq \gamma_{spin} \). The single-spin magnetic-dipole coupling strength is given by \( g_m = \mu_B \sqrt{\mu_0 (\omega_c - g_c) / \sqrt{2} V_c} \), where \( \mu_B \) is the Bohr magneton, \( \mu_0 \) is the vacuum permeability and \( V_c \) is the SCM cavity-mode volume. For a cavity-volume \( V_c = 10^{-12} m^3 \) (corresponding to a separation of \( \sim 10 \mu m \) between the center conductor and the ground planes), \( \omega_c = 2 \pi \times 10^7 \) rad/sec. This estimation implies that we would need \( N_s \geq 10^8 \) for the two-level approximation to be valid. A 10 \( \mu m \) thick bulk-doped silicon with a dopant density \( n = 10^{16} cm^{-3} \) and an area of 10 \( \mu m \) by 100 \( \mu m \) would yield \( N_s \sim 10^8 \) spins.

The hybrid qubit defined by the computational states \( |G\rangle \) and \( |E\rangle \) brings up a number of benefits, such as collective enhancement of coupling to external coherent fields, the possibility of working with spin systems with weak spin-orbit interaction and avoiding the requirement to isolate a single spin. However, using the anharmonic spin-wave excitations as a qubit would require that the ensemble of spins are either completely polarized or are in a mixture of dark-state that could not be further polarized due to the symmetry \( |1\rangle \rightarrow |1\rangle \). Fortunately, the same strong cavity coupling utilized in defining the hybrid qubit could also be used to cool down the spin ensemble into a mixture of dark states. This would be achieved ideally in the limit where the cavity-Q is reduced and the spin excitation is transferred to phonons via non-radiative cavity dissipation. Provided that the cav-
ity/substrate temperature satisfies $T_s \leq 70 \text{mK}$, it should be possible to ensure that the spins are in a mixture of dark states with mean number of collective excitations much smaller than unity. However, the inhomogeneous broadening due to different collective coupling strengths associated with different dark states may become a limitation if the degree of spin polarization is not high \[13\]. A temporary reduction in cavity-Q could be achieved by laser irradiation that induces ohmic losses. We emphasize that the initial temperature of the spins need not be low.

Coupling of spins to the cavity mode would require that the Zeeman splitting $\omega_z$ satisfies $\omega_z = \omega_c - g_c$, which in turn implies the presence of an effective magnetic field of $B \sim 0.5 \text{Tesla}$. Such magnetic fields will substantially reduce the cavity-Q. Ideally, this problem can be remedied by inducing the electron spin-splitting via polarized nuclear spins of the host material: it is well known that dynamical polarization techniques lead to effective Overhauser fields that can exceed 2 T esla and that survive for more than a minute after active polarization is turned off. Since these effective fields are predominantly due to Fermi-contact hyperfine interaction, cavity-Q would remain unaffected.

Alternatively, we could envision a local external field of $B \sim 1 \text{Tesla}$ that vanishes at the site of the transmon qubit. Such an external field will reduce the cavity-Q and the condition $g_c \gg g_m \sqrt{N_s} \gg \kappa_c$ may no longer be satisfied. However, provided that $Q > 100$, $g_c \gg \kappa_c$ is still satisfied and it is possible to overcome the limitation of a lossy cavity: to this end, we assume that the transmon qubit is detuned from the cavity mode in a way to ensure that it is resonant with the electron spins ($\Delta = \delta + \tilde{g}_c^2 / \delta$). In the limit $\hbar |\delta| \gg$ all other energy scales, we use a Schrieffer-Wolff transformation to eliminate the first term in Eq. 1 and find the transformed Hamiltonian to lowest order

$$\hat{H} = \hbar \frac{g_m g_c}{\Delta} \sum_{i=1}^{N_s} (\hat{a}_i^\dagger \hat{a}_{i+1} + h.c.) \quad (3)$$

where we assumed that $\delta$ is large enough to ensure that $|\delta - \Delta| \ll |\Delta|$ and that $g_m \sqrt{N_s} \ll g_c$. The transformed Hamiltonian is also of the JC-form since the collective spin lowering operator $(\sum_{i=1}^{N_s} \hat{a}_i^\dagger) / \sqrt{N_s}$ approximates a bosonic creation operator in the limit $N_s \gg 1$ for low excitation manifolds. The condition for strong coupling in this case is given by $g_c g_m \sqrt{N_s} / \Delta > \kappa_c g_c^2 / \delta^2$, $\gamma_{spins} \gamma_{JJ}$; in this limit, the cavity-mode acts as a quantum-bus and its virtual excitations mediate the long distance JC interaction between the transmon qubit and the collective spin excitations.

Having demonstrated that the two lowest energy collective excitations of a spin ensemble could define an anharmonic two-level system, we address the possibilities for manipulation of quantum information. Arbitrary unitary rotations of the two-level system defined by the states $|0\rangle$ and $|1\rangle$ could be effected by an external resonant ESR field or by Raman transitions induced by two phase-locked laser fields. In order to couple spin-ensemble qubits, one could consider a cavity containing several transmon qubits, each with a different detuning. By swapping the spin qubits to and from the same transmon qubit, it is possible to effect square-root of swap operation \[13\]. Each transmon qubit will then constitute a different channel for parallel implementation of two-qubit gates \[14\].

An advantage of ensemble spin qubits over that of transmon qubits is their potentially much longer decoherence times. This would particularly be true for ensemble spins in pure silicon-28 with no hyperfine interaction and ultra-small spin-orbit coupling. In GaAs, the ensemble spin approach will enable conversion of quantum information carried by the spin/transmon system to that of a propagating photon. As compared to GaAs single-electron spin qubits, ensemble spins have the advantage of reduced hyperfine decoherence \[16\]. It should be emphasized however, that spin-orbit induced decoherence eventually becomes prominent in bulk GaAs structures and limits the spin coherence time to $\sim 1 \mu\text{sec}$ \[17\].

Another interesting system for which the formalism discussed here applies is an ensemble of cold atoms trapped 10$\mu$m above the SCM cavity structure. At such distances, the adverse effects of the solid-interface on ground-state atoms can be avoided \[18\]. Without the need for strong external magnetic fields, the magnetic dipole coupling of the hyperfine transition of the atomic ensemble to the nonlinear cavity mode will then lead to an anharmonic energy level diagram for the collective atomic (hyperfine) spin excitations. In addition to providing a long-lived memory, a cigar shaped atomic ensemble coupled to a cavity-transmon system would enable near-unity efficiency conversion of a microwave photon to an optical photon that can be collimated using a low numerical aperture lens \[10\].

In conclusion, we demonstrate that collective enhancement of magnetic dipole coupling of an ensemble of spins would lead to the strong coupling regime of cavity-QED. The presence of a transmon qubit in the SCM cavity would lead to the realization of an embedded JC model where the harmonic emitter (spin) system picks up the nonlinearity of the cavity. In the context of quantum information processing, our findings generalize the conclusions previously drawn for polar molecules to a larger class of systems, ranging from trapped ground-state atomic gases to spins in bulk silicon or graphene.

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