Branching ratio and CP violation of $B_c \to D K$ decays in the perturbative QCD approach

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Abstract

In this paper, we calculate the branching ratio and direct $CP$ asymmetry parameter of $B_c^\pm \to D^0 K^\pm$ in the framework of perturbative QCD approach based on $k_T$ factorization. Besides the usual factorizable diagrams, both non-factorizable and annihilation type contributions are taken into account. We find that (a) the branching ratio is at the order of $10^{-5}$; (b) the tree annihilation diagrams and the penguin diagrams dominate the total contribution; and (c) the direct CP asymmetry is about 7%, which can be tested in the Large Hadron Collider beauty experiments (LHC-b) at CERN.

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I. INTRODUCTION

The rare B meson decays arouse more and more interest, since it is a good place for testing the Standard Model (SM), studying CP violation and looking for possible new physics beyond the SM. In recent years, the theoretical studies of \( B_{u,d} \) mesons have been done in the literature widely, which are strongly supported by the running \( B \) factories in KEK and Stanford Linear Accelerator Center (SLAC). \( B \) physics studies are further supported by the Large Hadron Collider beauty experiments (LHC-b), it is estimated that about \( 5 \times 10^{10} \) \( B_c \) mesons can be produced per year at LHC\[1, 2\]. So the studies of \( B_c \) meson rare decays are necessary in the next a few years.

The nonleptonic decays of the \( B_c \) mesons have been studied in previous literature\[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\] by employing Naive Factorization\[30\], QCD Factorization\[31\], perturbative QCD approach (PQCD)\[32\] and other approaches. The theoretical status of the \( B_c \) meson was reviewed in Ref.\[1\]. In this paper, we will study the \( B_c \to DK \) decays in the perturbative QCD approach. Our theoretical formulas for the decay \( B_c \to DK \) in PQCD framework are given in the next section. In section III, we give the numerical results of the branching ratio and CP asymmetries of \( B_c \to DK \) and the form factor of \( B_c \to D \) etc. At last, we give a short summary in section IV.

II. PERTURBATIVE CALCULATIONS

For decay \( B_c \to DK \), the related effective Hamiltonian is given by \[33\]

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q=u,c} V_{qs} V_{qb}^* \left[ C_1(\mu)O_1^q(\mu) + C_2(\mu)O_2^q(\mu) \right] + V_{tb} V_{ts} \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right\},
\]

where \( C_i(\mu)(i = 1, \cdots, 10) \) are Wilson coefficients at the renormalization scale \( \mu \) and \( O_i(i = 1, \cdots, 10) \) are the four quark operators

\[
\begin{align*}
O_1^q &= (\bar{b}_q \bar{q}_j)_{V-A}(\bar{q}_j s_i)_{V-A}, & O_2^q &= (\bar{b}_q q_i)_{V-A}(\bar{q}_j s_j)_{V-A}, \\
O_3 &= (\bar{b}_s i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, & O_4 &= (\bar{b}_s j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\
O_5 &= (\bar{b}_s i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, & O_6 &= (\bar{b}_s j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\
O_7 &= \frac{3}{2}(\bar{b}_s i)_{V-A} \sum_q e_q(\bar{q}_j q_j)_{V+A}, & O_8 &= \frac{3}{2}(\bar{b}_s j)_{V-A} \sum_q e_q(\bar{q}_j q_i)_{V+A}, \\
O_9 &= \frac{3}{2}(\bar{b}_s i)_{V-A} \sum_q e_q(\bar{q}_j q_j)_{V-A}, & O_{10} &= \frac{3}{2}(\bar{b}_s j)_{V-A} \sum_q e_q(\bar{q}_j q_i)_{V-A}.
\end{align*}
\]
Here $i$ and $j$ are $SU(3)$ color indices; the sum over $q$ runs over the quark fields that are active at the scale $\mu = O(m_b)$, i.e., $q \in \{u, d, s, c, b\}$. Operators $O_1, O_2$ come from tree level interaction, while $O_3, O_4, O_5, O_6$ are QCD-Penguins operators and $O_7, O_8, O_9, O_{10}$ come from electroweak-penguins.

In PQCD approach, the decay amplitude can be written as:

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr}[C(t)\Phi_{B_c}(k_1)\Phi_D(k_2)\Phi_K(k_3)H(k_1, k_2, k_3, t)],$$

(3)
where \( k_i \) are the momenta of light quarks included in each of the mesons, and \( \text{Tr} \) denotes the trace over Dirac and color indices. \( C(t) \) is the Wilson coefficient results from the radiative corrections at short distance. \( \Phi_M \) is the wave function which describes the hadronization of mesons. The wave functions should be universal and channel independent, but fortunately perturbative calculable.

Working at the rest frame of \( B_c \) meson, the momenta of the \( B_c, D \) and \( K \) can be written in the light-cone coordinates as:

\[
P_1 = \frac{M_{B_c}}{\sqrt{2}}(1, 1, 0_T), \quad P_2 = \frac{M_{B_c}}{\sqrt{2}}(r^2, 1, 0_T), \quad P_3 = \frac{M_{B_c}}{\sqrt{2}}(1 - r^2, 0, 0_T). \tag{4}
\]

where \( r = M_D/M_{B_c} \) and we neglect the kaon’s mass \( M_K \). Denoting the light (anti-)quark momenta in \( B_c, D \) and \( K \) as \( k_1, k_2 \) and \( k_3 \), respectively, we can choose:

\[
k_1 = (x_1p_1^+, 0, k_{1T}), \quad k_2 = (0, x_2p_2^-, k_{2T}), \quad k_3 = (x_3p_3^+, 0, k_{3T}). \tag{5}
\]

Then integration over \( k_1^-, k_2^+ \), and \( k_3^- \) in Eq.(3) leads to

\[
\text{Amplitude} \sim \int dx_1 dx_2 dx_3 db_1 db_2 db_3 \times \text{Tr} \left[ C(t) \Phi_{B_c}(x_1, b_1) \Phi_D(x_2, b_2) \Phi_K(x_3, b_3) H(x_i, b_i, t) \right] e^{-S(t)}, \tag{6}
\]

where \( b_i \) is the conjugate space coordinate of \( k_{iT} \), and \( t \) is the largest energy scale in \( H \). The exponential Sudakov factor \( e^{-S(t)} \) comes from the higher order radiative corrections to wave functions and hard amplitudes, it suppresses the soft dynamics effectively \cite{34} and thus makes a perturbative calculation of the hard part \( H \) reliable.

According to effective Hamiltonian \cite{11}, we draw the lowest order diagrams of \( B_c \to DK \) in Fig. 1. The usual factorizable diagrams (a) and (b) give the \( B_c \to D \) form factor if take away the Wilson coefficients. The operators \( O_1, O_2, O_3, O_4, O_9 \) and \( O_{10} \) are \((V-A)(V-A)\) currents, and the sum of their contributions is given by

\[
F_{c[C]} = \frac{8 f_{B_c}}{\sqrt{2} N_c} \pi C_F M_{B_c}^2 \int_0^1 dx_2 \int_0^\infty db_1 db_2 \phi_D(x_2, b_2) \times \left\{ \left[ (r^2 + 2r - 1)x_2 + (2 - r)r_b \right] \alpha_s(t_a^1) h_a^{(1)}(x_2, b_1, b_2) \exp[-S_B(t_a^1)] \\
- S_D(t_a^1) C(t_a^1) + [r(2 - r)] \alpha_s(t_a^2) h_a^{(2)}(x_2, b_1, b_2) \exp[-S_B(t_a^2)] \\
- S_D(t_a^2) C(t_a^2) \right\}, \tag{7}
\]

where \( r_b = M_b/M_{B_c} \), \( C_F = 4/3 \) is the group factor of the \( SU(3)_c \) gauge group. The expressions of the meson distribution amplitudes \( \phi_M \), the Sudakov factor \( S_X(t_i)(X = \)]
The operator of type \((V \pm A)(V \mp A)\), and the functions \(h_\alpha\) are given in the appendix. In above formula, the Wilson coefficients \(C(t)\) of the corresponding operators are process dependent.

The operator \(O_5, O_6, O_7, O_8\) have the structure of \((V-A)(V+A)\), their amplitude is

\[
F_{\alpha}^P[C] = \frac{16 f_B}{\sqrt{2N_c}} r_K \pi C_F M_{B_c}^2 \int_0^1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_D(x_2, b_2) \\
\times \{[(2r^2 - r)x_2 + (2 - r - r_b - 2r^2 + 4r_b r)]\alpha_s(t_a^1)h_a^{(1)}(x_2, b_1, b_2) \\
\times \exp[-S_B(t_a^1) - S_D(t_a^1)]C(t_a^1) + [3r - 4r^2] \alpha_s(t_a^2)h_a^{(2)}(x_2, b_1, b_2) \\
\times \exp[-S_B(t_a^2) - S_D(t_a^2)]C(t_a^2)\},
\]

where \(r_K = M_0K/M_{B_c} = M_K^2/[M_{B_c}(M_s + M_u)]\). For the non-factorizable diagrams (c) and (d), all three meson wave functions are involved. Using \(\delta\) function \(\delta(b_1 - b_3)\), the integration of \(b_1\) can be preformed easily. For the \((V-A)(V-A)\) operators the result is:

\[
M_e[C] = \frac{16}{N_c} \pi C_F f_B, M_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_D(x_2, b_2) \phi_K(x_3) \\
\times \{(1 - 2r + (r - r^2)x_2 - (1 - 2r^2)x_3)\alpha_s(t_c^1)h_c^{(1)}(x_2, x_3, b_2, b_3) \\
\times \exp[-S_B(t_c^1) - S_D(t_c^1)]C(t_c^1) + [2r - 1 + (1 - r - r^2)x_2 \\
- (1 - 2r^2)x_3] \alpha_s(t_c^2)h_c^{(2)}(x_2, x_3, b_2, b_3) \exp[-S_B(t_c^2) - S_D(t_c^2) \\
- S_K(t_c^2)]C(t_c^2)\},
\]

For the \((V-A)(V+A)\) operators, the formula is:

\[
M_{e}^P[C] = \frac{16}{N_c} \pi C_F f_B r_K M_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_D(x_2, b_2) \\
\times \{(1 + r - rx_2 - (1 + r)x_3)\phi_K(x_3) + (1 - r + rx_2 - (1 + r)x_3) \\
\times \phi_K^T(x_3)\alpha_s(t_c^1)h_c^{(1)}(x_2, x_3, b_2, b_3) \exp[-S_B(t_c^1) - S_D(t_c^1) - S_K(t_c^2)]C(t_c^1) \\
+ [(rx_2 - (1 + r)x_3)\phi_K^T(x_3) + (-2r + rx_2 + (1 + r)x_3)\phi_K^T(x_3)] \\
\times \alpha_s(t_c^2)h_c^{(2)}(x_2, x_3, b_2, b_3) \exp[-S_B(t_c^2) - S_D(t_c^2) - S_K(t_c^2)]C(t_c^2)\},
\]

Similar to (c),(d), the annihilation diagrams (e) and (f) also involve all three meson wave functions. Here we have two kinds of amplitudes, \(M_a\) is the contribution containing the operator of type \((V-A)(V-A)\), and \(M_{a}^P\) is the contribution containing the operator of
type \((V - A)(V + A)\).

\[
M_a[C] = \frac{16}{N_c} \pi C_F f_{B_c} M_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_D(x_2, b_2) \\
\times \{((1 - r - r_b - r^2 x_2 + (2r^2 - 1)x_3 + r_b r^2)\phi^A_K(x_3) + (2 - x_2 - x_3 - 4r_b) \\
x r r_K \phi^P_K(x_3) + (x_2 - x_3)r r_K \phi^T_K(x_3)\} \alpha_s(t_e^1) h_e^{(1)}(x_2, x_3, b_1, b_2) \exp[-S_B(t_e^1) \\
- S_D(t_e^1) - S_K(t_e^1)]C(t_e^1) + [(r + x_2)\phi^A_K(x_3) + (x_2 + x_3)r r_K \phi^P_K(x_3) \\
+ (x_2 - x_3)r r_K \phi^T_K(x_3)\} \alpha_s(t_e^2) h_e^{(2)}(x_2, x_3, b_1, b_2) \exp[-S_B(t_e^2) - S_D(t_e^2) \\
- S_K(t_e^1)]C(t_e^2),
\]

(11)

\[
M_a^P[C] = \frac{16}{N_c} \pi C_F f_{B_c} M_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_D(x_2, b_2) \\
\times \{((-r + r x_2 - r_b r^2)\phi^A_K(x_3) + (1 + r_b - r - x_3)r r_K \phi^P_K(x_3) + (1 + r_b \\
r - x_3)r r_K \phi^T_K(x_3)\} \alpha_s(t_e^1) h_e^{(1)}(x_2, x_3, b_1, b_2) \exp[-S_B(t_e^1) - S_D(t_e^1) \\
- S_K(t_e^1)]C(t_e^1) + [(r^2 - r x_2)\phi^A_K(x_3) + (x_3 - 2r)r r_K \phi^P_K(x_3) \\
+ (x_3 - 2r)r r_K \phi^T_K(x_3)\} \alpha_s(t_e^2) h_e^{(2)}(x_2, x_3, b_1, b_2) \exp[-S_B(t_e^2) - S_D(t_e^2) \\
- S_K(t_e^1)]C(t_e^2),
\]

(12)

The amplitude for the factorizable annihilation diagrams (g) and (h) result in \(F_a\) (for \((V - A)(V - A)\) type operators) and \(F_a^P\) (for \((V - A)(V + A)\) type operators):

\[
F_a[C] = 16\pi C_F M_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_D(x_2, b_2) \\
\times \{[(r^2 - 1)x_2 \phi^A_K(x_3) - 2r r_K (1 + x_2)\phi^P_K(x_3)\} \alpha_s(t_g^1) h_g(x_2, x_3, b_2, b_3) \\
\times \exp[-S_D(t_g^1) - S_K(t_g^1)]C(t_g^1) + [(-2r + x_3 - 2r^2 x_3)\phi^A_K(x_3) \\
+ (r r_K + 2r r_K r x_3)\phi^P_K(x_3) + (-r r_K + 2r r r x_3)\phi^T_K(x_3)] \\
\times \alpha_s(t_g^2) h_g(x_3, x_2, b_3, b_2) \exp[-S_D(t_g^2) - S_K(t_g^1)]C(t_g^2),
\]

(13)

\[
F_a^P[C] = 32\pi C_F M_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_D(x_2, b_2) \\
\times \{[r x_2 \phi^A_K(x_3) + 2r r_K \phi^P_K(x_3)\} \alpha_s(t_g^1) h_g(x_2, x_3, b_2, b_3) \exp[-S_D(t_g^1) \\
- S_K(t_g^1)]C(t_g^1) + [r \phi^A_K(x_3) + r r_K x_3 \phi^P_K(x_3) - r r_K x_3 \phi^T_K(x_3)] \\
\times \alpha_s(t_g^2) h_g(x_3, x_2, b_3, b_2) \exp[-S_D(t_g^2) - S_K(t_g^1)]C(t_g^2),
\]

(14)
From Equation (14), the total decay amplitude for \( B_c^+ \to D^0 K^+ \) can be written as

\[
A(B_c \to D^0 K^+) = f_R f_c [V_{us} V_{ub}^* (\frac{1}{3} C_1 + C_2) - V_{tb} V_{ts}^* (\frac{1}{3} C_3 + C_4 + \frac{1}{3} C_9 + C_{10})] \\
- f_K V_{tb} V_{ts} F_c^P [\frac{1}{3} C_5 + C_6 + \frac{1}{3} C_7 + C_8] + M_c [V_{us} V_{ub}^* C_1 - V_{tb} V_{ts}^* (C_3 + C_9)] \\
- V_{tb} V_{ts} M_c^P (C_5 + C_7) + M_a [V_{cs} V_{cb}^* C_1 - V_{tb} V_{ts}^* (C_3 + C_9)] \\
- V_{tb} V_{ts} M_a^P (C_5 + C_7) + f_B F_a [V_{cs} V_{cb}^* (\frac{1}{3} C_1 + C_2) - V_{tb} V_{ts}^* (\frac{1}{3} C_3 + C_4 \\
+ \frac{1}{3} C_9 + C_{10})] - f_B V_{tb} V_{ts} F_a^P [\frac{1}{3} C_5 + C_6 + \frac{1}{3} C_7 + C_8],
\]

(15)

and the decay width is expressed as

\[
\Gamma(B_c^+ \to D^0 K^+) = \frac{G_F^2 M_{B_c^+}^3}{128 \pi} (1 - r^2) |A(B_c^+ \to D^0 K^+)|^2.
\]

The decay amplitude of the charge conjugate channel \( B_c^- \to D^0 K^- \) can be obtained by replacing \( V_{qs} V_{qb}^* \) to \( V_{qs} V_{qb} \) and \( V_{ts} V_{ts}^* \) to \( V_{tb} V_{ts}^* \) in Eq. (15).

Using the unitary condition of the CKM matrix elements \( V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0 \), the decay amplitude of \( B_c^- \to D^0 K^+ \) in Eq. (15) can be parameterized as

\[
A = V_{ub}^* V_{us} T_u + V_{cb}^* V_{cs} T_c - V_{tb}^* V_{ts} P \\
= V_{ub}^* V_{us} (T_u + P)[1 + z e^{i(\delta - \gamma)}],
\]

(17)

where \( z = \frac{|V_{ub}^* V_{us}|}{|V_{ub}^* V_{us}||T_u + P|} \), \( V_{ub} \approx |V_{ub}| e^{-i \gamma} \) and \( \delta = \arg(T_u + P) \). \( z \) and \( \delta \) can be calculated from PQCD.

Similarly, the decay amplitude for \( B_c^- \to \bar{D}^0 K^- \) can be parameterized as

\[
\bar{A} = V_{ub} V_{us}^* T_u + V_{cb} V_{cs}^* T_c - V_{tb} V_{ts}^* P \\
= V_{ub} V_{us}^* (T_u + P)[1 + z e^{i(\delta + \gamma)}],
\]

(18)

and the averaged decay width for \( B_c^+ (B_c^-) \to D^0 K^\pm \) reads

\[
\Gamma(B_c^+ (B_c^-) \to D^0 K^\pm) = \frac{G_F^2 M_{B_c^\pm}}{128 \pi} (1 - r^2) (|A|^2/2 + |\bar{A}|^2/2) \\
= \frac{G_F^2 M_{B_c^\pm}}{128 \pi} (1 - r^2)|V_{ub}^* V_{us} (T_u + P)|^2 [1 + 2z \cos \gamma \cos \delta + z^2],
\]

(19)

which is a function of CKM angle \( \gamma \).
III. NUMERICAL EVALUATION

The following parameters have been used in our numerical calculation [24, 35, 36, 37, 38]:

\[ M_{B_c} = 6.286 \text{GeV}, \quad M_b = 4.8 \text{GeV}, \quad M_D = 1.685 \text{GeV}, \quad M_{0K} = 1.6 \text{GeV}, \]
\[ \omega_D = 0.2 \text{GeV}, \quad f_{B_c} = 489 \text{MeV}, \quad f_K = 160 \text{MeV}, \quad f_D = 240 \text{MeV}, \]
\[ a_D = 0.3, \quad \tau_{B_c} = 0.46 \times 10^{-12} \text{s}, \quad |V_{ub}^* V_{us}| = 0.0009, \quad |V_{cb}^* V_{cs}| = 0.0398. \]

We leave the CKM phase angle \( \gamma \) as a free parameter to explore the branching ratio and CP asymmetry parameter dependence on it. The averaged branching ratio of the decay \( B_c^+(B_c^-) \rightarrow D^0 K^\pm \) with respect to the parameter \( \gamma \) is shown in Fig. 2. Since the CKM angle \( \gamma \) is constrained as \( \gamma = (63^{+15}_{-12})^\circ \) [35], we can arrive from Fig. 2:

\[ 6.5 \times 10^{-5} < Br(B_c^+(B_c^-) \rightarrow D^0 K^\pm) < 6.7 \times 10^{-5}, \quad \text{for} 50^\circ < \gamma < 80^\circ. \]  

(21)

Our numerical analysis show that \( |V_{cb}^* V_{cs} T_c/V_{ub}^* V_{us} T_u| = 10 \) and \( |V_{tb}^* V_{ts} P/V_{ub}^* V_{us} T_u| = 7.4 \) which mean the tree level contributions from annihilation topology and the penguin contributions dominate in this decay and the branching ratio is not sensitive to \( \gamma \). In
our calculation, the only input parameters are wave functions, which represent the non-perturbative contributions. In all the three meson wave functions, the main uncertainty come from the value of $\omega_D$ in $D$ meson wave function (see appendix). We investigate the branching ratio dependence on the value of $\omega_D$ in Table I. By changing the value of $\omega_D$ from $0.2\, GeV$ to $0.4\, GeV$, we find the branching ratio of $B_c^+(B_c^-) \to D^0 K^\pm$ change little as shown in Table I.

| $\omega_D$ | $B_c^+ \to D^0 K^+$ | $B_c^- \to D^0 K^-$ |
|------------|------------------|------------------|
| $0.2\,GeV$ | 6.6              | 5.6              |
| $0.3\,GeV$ | 7.0              | 5.6              |
| $0.4\,GeV$ | 5.6              | 5.6              |

TABLE I: Branch ratios in the unit $10^{-5}$ using $\gamma = 60^\circ$ for different $\omega_D$

The diagrams (a) and (b) in Fig. I give the contribution for $B_c \to D$ transition form factor $F_{B_c \to D}(q^2 = M_K^2 \approx 0)$, where $q = P_1 - P_2$ is the momentum transfer. The sum of their amplitudes have been given by Eq. (7), so we can use PQCD approach to compute this form factor. Our result is $F_{B_c \to D}(0) = 0.24$, if $\omega_D = 0.2\, GeV$; and $F_{B_c \to D}(0) = 0.21$, if $\omega_D = 0.45\, GeV$. We can see that this form factor is not sensitive to the $D$ meson wave function. In the literature, there already exist a lot of studies on $B_c \to D$ transition form factor [8, 12, 27, 39, 40, 41, 42, 43, 44], we show their results in Table III. From which we find that there are large differences in these results (including ours) and eventually this form factor can be extracted from semi-leptonic experiments $B_c \to Dl \nu_l$ in the future LHC experiments.

| $F_{B_c \to D}(0)$ | DW [27] $^a$ | CNP [39] | NW [40] | IKL [41] | KKL [12] $^b$ | EFG [8] | ZH [42] | DSV [43] | WSL [44] |
|-------------------|------------|---------|--------|----------|-------------|--------|--------|----------|--------|
|                   | 0.255      | 0.13    | 0.1446 | 0.69     | 0.32[0.29] | 0.14   | 0.35   | 0.075    | 0.16   |

$^a$We quote the result with $\omega = 0.7\, GeV$.

$^b$The nonbracketed(bracketed) result is evaluated in sum rules(potential model).

TABLE II: $B_c \to D$ transition form factor at $q^2 = 0$ evaluated in the literature.

The direct CP violation $A_{CP}^{dir}$ is defined as

$$A_{CP}^{dir} = \frac{\Gamma(B_c^+ \to D^0 K^+) - \Gamma(B_c^- \to \bar{D}^0 K^-)}{\Gamma(B_c^+ \to D^0 K^+) + \Gamma(B_c^- \to \bar{D}^0 K^-)} = \frac{2z \sin \gamma \sin \delta}{1 + 2z \cos \gamma \cos \delta + z^2}$$

(22)
Using Eq. (17) and (18), we compute the parameter $A_{CP}^{dir}$. The direct CP asymmetry $A_{CP}^{dir}$ has a strong dependence on the CKM angle $\gamma$, as can be seen easily from Eq. (22) and Fig. 3. From this figure one can see that the direct CP asymmetry at $6\%-7.6\%$ for $50^\circ < \gamma < 80^\circ$. The small direct CP asymmetry is also a result of small tree level contribution from emission topology.

![Graph showing direct CP violation parameters of $B^\pm_c \to D^0 K^\pm$ decay as a function of CKM angle $\gamma$.]

**FIG. 3:** Direct CP violation parameters of $B^\pm_c \to D^0 K^\pm$ decay as a function of CKM angle $\gamma$.

**IV. SUMMARY**

In this work, we study the branching ratio and CP asymmetry of the decays $B^\pm_c \to D^0 K^\pm$ in PQCD approach. It is found that the branching ratio of $B^\pm_c \to D^0 K^\pm$ is at the order of $10^{-5}$. We also predict CP asymmetries in the process, which may be measured in the LHC-b experiments.

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APPENDIX A: FORMULAS FOR THE CALCULATIONS USED IN THE TEXT

In the appendix we present the explicit expressions of the formulas used in section II. First, we give the expressions of the meson distribution amplitudes $\phi_M$. For $B_c$ meson wave function, we use the function as \[29\]:

$$
\Phi_{B_c}(x) = \frac{f_{B_c}}{4N_c}(\phi_{B_c} + M_{B_c})\gamma_5\delta(x - M_c/M_{B_c}).
$$

(A1)

For $D$ meson wave function, we use the function as \[45\]:

$$
\phi_D(x, b) = \frac{3}{\sqrt{2N_c}}f_D x(1-x)\{1 + a_D(1-2x)\} \exp[-\frac{1}{2}(\omega_D b)^2],
$$

(A2)

We set $a_D = 0.3$ and $\omega_D = 0.2 GeV$ \[38\] in our numerical calculation.

We use the distribution amplitude $\phi_{A,P,T}^{\text{K}}$ of the K meson from Ref. \[46\]:

$$
\phi_{A}^{\text{K}}(x) = \frac{6f_{K}}{2\sqrt{2N_c}}x(1-x)[1 + 0.15t + 0.405(5t^2 - 1)],
$$

$$
\phi_{P}^{\text{K}}(x) = \frac{f_{K}}{2\sqrt{2N_c}}[1 + 0.106(3t^2 - 1) - 0.148(3 - 30t^2 + 35t^4)/8],
$$

$$
\phi_{T}^{\text{K}}(x) = \frac{f_{K}}{2\sqrt{2N_c}}t[1 + 0.1581(5t^2 - 3)],
$$

(A3)

where $t = 1 - 2x$, whose coefficients correspond to $m_0K = 1.6 GeV$.

$S_{B_c}$, $S_{D}$, $S_{K}$ used in the decay amplitudes are defined as

$$
S_{B_c}(t) = s(x_1P_1^+, b_1) + 2 \int_{1/b_1}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})),
$$

(A4)

$$
S_{D}(t) = s(x_2P_2^-, b_2) + 2 \int_{1/b_2}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})),
$$

(A5)

$$
S_{K}(t) = s(x_3P_3^+, b_3) + s((1 - x_3)P_3^+, b_3) + 2 \int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})),
$$

(A6)

where the so called Sudakov factor $s(Q, b)$ resulting from the resummation of double logarithms is given as \[47, 48\]

$$
s(Q, b) = \int_{1/b}^{Q} \frac{d\mu}{\mu} \left[ \ln \left( \frac{Q}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right]
$$

(A7)

with

$$
A = C_F \frac{\alpha_s}{\pi} + \left[ \frac{67}{9} - \frac{11}{27}r_f + \frac{2}{3\beta_0} \ln \left( \frac{e^{\gamma_E}}{2} \right) \right] \left( \frac{\alpha_s}{\pi} \right)^2,
$$

(A8)

$$
B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln \left( \frac{e^{2\gamma_E-1}}{2} \right).
$$

(A9)
Here $\gamma_E = 0.57722 \cdots$ is the Euler constant, $n_f$ is the active quark flavor number.

The functions $h_i (i = a, c.e.g)$ come from the Fourier transformation of propagators of virtual quark and gluon in the hard part calculations. They are given as

$$h^{(1)}_a (x_2, b_1, b_2) = S_t (x_2) K_0 (M_B \sqrt{(1 - x_2)(r-r^2)b_1})$$

$$\times [\theta(b_2 - b_1)\theta(r^2_b - (1-r^2)x_2) I_0 (M_B \sqrt{r^2_b - (1-r^2)x_2 b_1})$$

$$\times K_0 (M_B \sqrt{r^2_b - (1-r^2)x_2 b_2}) + (b_1 \leftrightarrow b_2)], \quad (A10)$$

$$h^{(2)}_a (x_2, b_1, b_2) = S_t (r) K_0 (M_B \sqrt{(1 - x_2)(r-r^2)b_2})$$

$$\times [\theta(b_2 - b_1)I_0 (M_B \sqrt{r-r^2 b_1})K_0 (M_B \sqrt{r-r^2 b_2}) + (b_1 \leftrightarrow b_2)], \quad (A11)$$

$$h^{(j)}_c (x_2, x_3, b_2, b_3) =$$

$$\left\{ \theta(b_2 - b_3) I_0 (M_B \sqrt{(1 - x_2)(r-r^2)b_3}) K_0 (M_B \sqrt{(1 - x_2)(r-r^2)b_2}) \right.$$  

$$\left. + (b_2 \leftrightarrow b_3) \right\} \times \left( \begin{array}{c} K_0 (M_B F_{e(j)} b_3), \quad \text{for } F_{e(j)}^2 > 0 \\ \frac{\pi i}{2} H_0^{(1)} (M_B \sqrt{|F_{e(j)}^2|} b_3), \quad \text{for } F_{e(j)}^2 < 0 \end{array} \right), \quad (A12)$$

where $H_0^{(1)} (z) = J_0 (z) + i Y_0 (z)$, and $F_{e(j)}$'s are defined by

$$F_{(1)}^2 = (1 - r - x_3 + r^2 x_3)(x_2 - 1), \quad F_{(2)}^2 = (1 - x_2)(r - r^2 - x_3 + r^2 x_3); \quad (A13)$$

$$h^{(j)}_c (x_2, x_3, b_1, b_2) =$$

$$\left\{ \theta(b_2 - b_1) \frac{\pi i}{2} H_0^{(1)} (M_B \sqrt{(1 - r^2) x_2 x_3 b_2}) I_0 (M_B \sqrt{(1 - r^2) x_2 x_3 b_1}) \right.$$  

$$\left. + (b_1 \leftrightarrow b_2) \right\} \times \left( \begin{array}{c} K_0 (M_B F_{e(j)} b_1), \quad \text{for } F_{e(j)}^2 > 0 \\ \frac{\pi i}{2} H_0^{(1)} (M_B \sqrt{|F_{e(j)}^2|} b_1), \quad \text{for } F_{e(j)}^2 < 0 \end{array} \right), \quad (A14)$$

where $F_{e(j)}$'s are defined by

$$F_{e(1)}^2 = r^2_b - (1-x_2)(1-r-x_3 + r^2 x_3), \quad F_{e(2)}^2 = r^2 - x_2(x_3 - r - r^2 x_3); \quad (A15)$$

$$h_y (x_2, x_3, b_2, b_3) = S_t (x_2) \frac{\pi i}{2} H_0^{(1)} (M_B \sqrt{(1 - r^2) x_2 x_3 b_3})$$

$$\times [\theta(b_3 - b_2)J_0 (M_B \sqrt{(1 - r^2) x_2 b_2}) \frac{\pi i}{2} H_0^{(1)} (M_B \sqrt{(1 - r^2) x_2 b_3}) + (b_2 \leftrightarrow b_3)]. \quad (A16)$$
We adopt the parametrization for $S_t(x)$ contributing to the factorizable diagrams [49],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)}[x(1 - x)]^c, \quad c = 0.3.$$  \hfill (A17)

The hard scale $t_s'$s in Eq.(7)-(14) are chosen as

$$t_1^a = \max(M_B\sqrt{(1 - x_2)(r - r^2)}, M_B\sqrt{r_b^2 - (1 - r^2)x_2}, 1/b_1, 1/b_2),$$

$$t_2^a = \max(M_B\sqrt{r - r^2}, 1/b_1, 1/b_2),$$

$$t_1^e = \max(M_B\sqrt{F_{e(1)}^2}, M_B\sqrt{(r - r^2)(1 - x_2)}, 1/b_1, 1/b_3),$$

$$t_2^e = \max(M_B\sqrt{F_{e(2)}^2}, M_B\sqrt{(1 - r^2)x_2x_3}, 1/b_1, 1/b_2),$$

$$t_1^g = \max(M_B\sqrt{(1 - r^2)x_2}, 1/b_1, 1/b_3),$$

$$t_2^g = \max(M_B\sqrt{(1 - r^2)x_3}, 1/b_2, 1/b_3).$$  \hfill (A18)

They are given as the maximum energy scale appearing in each diagram to kill the large logarithmic radiative corrections.

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