The analysis of the preliminary RHIC data on $\pi^+ - \Xi^-$ correlation function is carried out. The $\Xi^*(1530)$ resonance is reasonably described. The value of the fireball radius has been estimated and the sensitivity to the $\pi^+ - \Xi^-$ S-wave scattering lengths has been tested.

Measurement of momentum correlations of the two low relative momentum particles produced in heavy ion collisions is an important method to study the spatio-temporal picture of the emission source at the level of $fm = 10^{-15}$ m. This type of analysis acquired the name of femtoscopy and has been reviewed in e.g. [1-4].

At the early stages the studies were focused on the production of identical pions, since then, measurements have been performed for different systems of both identical and non-identical hadrons, high-statistics data sets were accumulated in heavy ion experiments at AGS, SPS, RHIC accelerators [5-8]. Correlations are significantly affected by the Coulomb and/or strong final state interactions (FSI) between outgoing particles. The non-identical particles correlations due to FSI provide information not only about space-time characteristics of the emitting source, but also about the average relative space-time separation asymmetry between the emission points of the two particle species in the pair rest frame [9]. Maybe the most exotic system studied recently by STAR collaboration is $\pi - \Xi$ [10,11], the particles composing the pair have one order of magnitude difference in mass plus $\Delta B=1/\Delta S=2$ gap in baryon/strangeness quantum numbers. It is challenging to study FSI of such exotic meson-baryon system and to extract information about the $\pi - \Xi$ S-wave scattering lengths. The other important reason to study $\pi - \Xi$ correlations is that multistrange baryons are expected to decouple earlier, than other particle species because of their small hadronic cross-sections [12], allowing one to extract the space-time interval between the different stages of the fireball evolution.

Preliminary results for the $\pi \Xi$ system are available from STAR Collaboration [10,11]. The following important observations were made:

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• Decomposition of the correlation function $C(\mathbf{k}) \equiv C(k, \cos \theta, \varphi)$ from 10% of the most central Au+Au collisions into spherical harmonics, provided the first preliminary values of $R = (6.7 \pm 1.0)$ fm and $\Delta_{\text{out}} = (-5.6 \pm 1.0)$ fm. The negative value of the shift parameter $\Delta_{\text{out}}$ indicates that the average emission point of $\Xi$ is positioned more to the outside of the fireball than the average emission point of the pion.

• In addition to the Coulomb interaction seen in previous non-identical particle analysis the $\pi^+\Xi^-$ correlations at small relative momenta provide sufficiently clear signal of the strong FSI that reveals itself in a peak corresponding to the $\Xi^*(1530)$ resonance. The peak’s centrality dependence shows a high sensitivity to the source size.

• Comparison with the FSI model[13] confirms that theoretical calculations in the Coulomb region are in a qualitative agreement with the data. They however over-predict the peak in the $\Xi^*(1530)$-region.

Below we present the first results of our calculations of the $\pi^+\Xi^-$ FSI and make the comparison with the experimental data. The problem of FSI in the $\pi^+\Xi^-$ system is highly intricate since one has to take into account the following factors[13,16]:

• The superposition of strong and Coulomb interactions
• The presence of $\Xi^*(1530)$ resonance
• The spin structure of the w.f. including spin-flip.
• The fact that the $\pi^+\Xi^-$ state is a superposition of $I = 1/2$ and $I = 3/2$ isospin states and that $\pi^+\Xi^-$ state is coupled to the $\pi^0\Xi^0$ and that the thresholds of the two channels are non-degenerate.
• The contribution from inner potential region where the structure of the strong interaction is unknown.

The outgoing multichannel wave functions (w.f.’s) of $\pi^+\Xi^-$ system $\Psi_i^{(-)}(\mathbf{k}, \mathbf{r})$ enter as building blocks into the correlation function (CF)[13,16] $C(\mathbf{k})$:

$$C(\mathbf{k}) = \sum_i \int d\mathbf{r} S_i(\mathbf{r}) |\Psi_i^{(-)}(\mathbf{k}, \mathbf{r})|^2 = \sum_i \int d\mathbf{r} S(\mathbf{r}) |\Psi_i^{(-)}(\mathbf{k}, \mathbf{r})|^2,$$

(1)

where $\mathbf{k}$ is a relative momentum of the pair, $S(\mathbf{r})$ is a universal source function. The out-state w.f.’s $\Psi_i^{(-)}(\mathbf{k}, \mathbf{r})$ have the asymptotic form

$$\Psi_i^{(-)}(\mathbf{k}, \mathbf{r}) \simeq e^{ik_1 r} \delta_{i1} + f_{ii}^{+}(-\cos \theta) e^{-ik_1 r} \frac{\mu_i}{R} \left( \frac{\mu_i}{ \mu_1} \right)^{1/2},$$

(2)

where $\mu_i$ is the reduced mass (or, for relativistic particles, reduced energy) of the particles in channel $i = 1-4$, ($i = \pi^+\Xi^-, \pi^0\Xi^0$ without and with the spin flip).

We considered the Gaussian (in the pair rest frame) model for the source function:

$$S(\mathbf{r}) = (8\pi^{3/2} R^3)^{-1} \exp(-r^2/4R^2).$$

(3)

The low energy region of $\pi\Xi$ interaction up to the $\Xi^*(1530)$ resonance is dominated by $S$- and $P$-waves. Therefore the w.f. contains two phase shifts with $I = 1/2, 3/2$ for $S$-wave and four phase shifts with $I = 1/2, 3/2$ and $J = 1/2, 3/2$ for $P$-wave ($J = l \pm 1/2$ is the total momentum).

To reduce the number of parameters we have assumed that the dominant interaction in $P$-wave occurs in a state with $J = 3/2, I = 1/2$ containing the $\Xi^*(1530)$ resonance. Since the parameters of $\Xi^*(1530)$ are known from the experiment we are left with two $S$-wave phase shifts which are expressed in terms of the two scattering lengths $a_{1/2}$ and $a_{3/2}$ with isospin $I = 1/2$ and $I = 3/2$ correspondingly.

Leaving the technical details for the future full-size publication we present the resulting expression for the sum of the squares of w.f.’s in Eq. (1).
\[
\sum_{i=1}^{4} |\Psi_i^{(-)}(E, r)|^2 = |\Psi_{Coul}^{*}(-E, r)|^2 + \frac{2}{\sqrt{3}} T_0 \varphi_0 Y_0^0 + \frac{2}{\sqrt{3}} T_1 \varphi_1 Y_1^0 + \sqrt{\frac{2}{3}} T_1 \varphi_1 Y_1^{1*} + \frac{k_2}{k_1} \left( |R_0 \chi_0 Y_0^0 + \frac{2}{\sqrt{3}} R_1 \chi_1 Y_1^0|^2 + \sqrt{\frac{2}{3}} R_1 \chi_1 Y_1^{1*}|^2 \right),
\]

(4)

here \( \Psi_{Coul} \) is the pure Coulomb w.f., \( k_1 \) and \( k_2 \) are the c.m. momenta in \( \pi^+ \Xi^- \) and \( \pi^0 \Xi^0 \) channels; the spherical harmonics \( Y_1^m = Y_1^m(\pi - \theta, \phi + \pi) \) correspond to the reversed direction of the vector \( k \), the functions

\[
\varphi_l(\eta_1, \rho_1) = \sqrt{4\pi} (-i)^l e^{-i\sigma_1(m)} H_l^{(-)}(\eta_1, \rho_1) / \rho_1
\]

\[
\chi_l(\eta_1, \rho_2) = (\mu_i / \mu_1)^{1/2} \sqrt{4\pi} (-i)^l e^{-i\sigma_1(m)} H_l^{(-)}(0, \rho_2) / \rho_2,
\]

(5)

where \( H_l^{(-)}(\eta, \rho) \) is a combination of the regular and singular Coulomb functions \( F_l \) \( G_l \) at a given orbital angular momentum \( l \) with the asymptotics \( H_l^{(-)}(\eta, \rho) \rightarrow e^{-i(\rho + \sigma_1(m))ln2 \rho - 1\pi/2} \), \( \rho_1 = k_1 r, \rho_2 = k_2 r, \eta_1 = (a_1 k_1)^{-1}, a_1 = -214 \) fm is a Bohr radius of the \( \pi^+ \Xi^- \) system taking into account the negative sign of the Coulomb repulsion. The quantities \( T_l = k_1 f_{21}^{J,l1s} \) and \( R_l = - (k_1 k_2)^{1/2} f_{l}^{J,21s} \) contain the elastic \( (1 \rightarrow 1) \) and inelastic \( (1 \rightarrow 2) \) scattering amplitudes \( f_{l}^{J,11} \) and \( f_{l}^{J,21} \) at a given total and orbital angular momentum \( J \) and \( l \). For the \( S \)-waves \( (l = 0, J = 1/2) \), they are expressed through the scattering lengths \( a_{1/2} \) and \( a_{3/2} \) is a similar way as in pion-nucleon scattering (see, e.g. \[15\,16\]). For the resonance \( \Xi^*(1530) \) \( P \)-wave,

\[
T_1 = - \frac{\Gamma_1/2}{E - E_0 - i\Gamma/2}, \quad R_1 = - \frac{(\Gamma_1\Gamma_2)^{1/2}/2}{E - E_0 - i\Gamma/2},
\]

(6)

where \( \Gamma = \Gamma_1 + \Gamma_2, \Gamma_1 \div 2\Gamma/3, \Gamma_2 \div \Gamma/3 \).

Expression (4) describes the region \( r > \epsilon \sim 1 \) fm where the strong potential is assumed to vanish. In the inner region \( r < \epsilon \), we substitute Eq. (4) by \( |\Psi_{Coul}|^2 \) and take into account the effect of strong interaction in a form of a correction which depends on the strong interaction time (expressed through the phase shift derivatives) and can be calculated without any new parameters unless the \( S \)-wave effective radii are extremely large. It is important that the complete CF does not depend on \( \epsilon \) provided the source function is nearly constant in the region \( r < \epsilon \[14\,15\].

Fig. 1 presents the results of calculations and the experimental data from \[10\,11\]. The solid curve corresponds to the source size \( R = 7.0 \) fm and zero \( S \)-wave scattering lengths. The results are however practically the same even for the \( S \)-wave scattering lengths of the order of one fm. We may conclude that at present experimental errors, the CF at \( R > 7 \) fm is practically independent of the \( S \)-wave scattering parameters.

Similar to the FSI model \[13\], our calculations are in agreement with the data in the low-\( k \) Coulomb region. Contrary to this model, they are however much closer to the experimental peak in the \( \Xi^*(1530) \) region though, they still somewhat overestimate this peak. The predicted peak is however expected to decrease due to a strong angular asymmetry of a more realistic source function obtained from Blast-wave like simulations.

In summary, using a simple Gaussian model for the source function, we have reasonably described the experimental data on the \( \pi^+ \Xi^- \) CF, estimated the emission source radius and tested the sensitivity to the low energy parameters of the strong interaction.
Figure 1: The CF of $\pi^+\Xi^-$ system for $R = 7.0$ fm and zero scattering lengths (solid line), the experimental data points are from the STAR collaboration.

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