1. INTRODUCTION

The central regions of galaxies contain a great deal of information regarding their formation and evolution. The mass density profiles of these inner regions have been recognized for some time as important measures of cold dark matter models (Navarro et al. 1997; Keeton & Madau 2001), and more recently, the properties of central supermassive black holes (SMBHs) have emerged as important probes into the history of hierarchical growth models.

SMBHs are inferred compact objects with masses between \(10^6\) and \(10^9\) \(M_\odot\). Several empirical correlations have been identified between SMBH masses and the properties of their host galaxies, including correlations with the mass or luminosity of the galactic bulge (Magorrian et al. 1998; Kormendy et al. 2000; Laor 2001), with the bulge velocity dispersion (Gebhardt et al. 2000; Merritt & Ferrarese 2001), and also with the light concentration of the bulge (Graham et al. 2001). Recent analysis suggests that the SMBH mass may also be correlated with the circular velocity of the host galaxy, even beyond the optical radius (Ferrarese 2002; Baes et al. 2003), and thus, there may be a link between the SMBH mass and the total mass of the galaxy. Because of this apparently tight link between SMBHs and their host galaxies, including dark matter halos, it is likely that the formation of both were closely related. Most contemporary hierarchical growth models do in fact reflect this property (Silk & Rees 1998; Kauffmann & Haehnelt 2000; Burkert & Silk 2001; MacMillan & Henriksen 2002; Wyithe & Loeb 2002; Volonteri et al. 2003; Di Matteo et al. 2003).

The detection of SMBHs is difficult and traditionally has been achieved using stellar kinematics in the very central regions of galaxies. These techniques have been feasible only for relatively nearby galaxies \((D \leq 100\text{ Mpc}, z \leq 0.02;\) Tremaine et al. 2002) in which the cores can be resolved. SMBHs are presumed to be the inactive relics of active galactic nuclei, and thus have evolved over a long redshift history \((z \simeq 6\) to present). Motivated in part by the desire to observe this evolution, additional techniques for deducing SMBH mass in more distant galaxies have been developed. The most prominent of these is the use of gravitationally broadened emission lines in active galactic nuclei to infer SMBH mass in these objects (McLure & Dunlop 2001).

Gravitational lensing offers a compelling mechanism to explore the relationship between the properties of SMBHs and their host galaxies at intermediate redshifts, extending to \(z \simeq 1\) or greater. Examination of strong galaxy-galaxy (or galaxy-quasar) gravitational lenses under proper circumstances may simultaneously yield the mass of the SMBH, information on the inner mass density profile of the host galaxy, and the total mass of the galaxy, greatly simplifying existing observational hurdles.

Relatively minimal consideration has been given to the gravitational lensing properties of SMBHs in galaxies, however. Alexander & Loeb (2001) considered the SMBH in the Milky Way, discussing the influence of stellar neighbors on its lensing properties, as well as the effects of its gravitational shear on the microlensing of individual stars. For galaxy-galaxy lensing, only Mao et al. (2001) and Chen (2003) have made general analyses. The former demonstrate calculations for adding an SMBH to a standard galactic lensing potential, and the latter shows how SMBHs can affect image separations in lenses. Keeton (2003) considers a sample population of 73 mock lenses and calculates lensing statistics including SMBH effects. He is interested in observing core images, which contain information about the inner mass density profiles of galaxies, and finds that SMBHs should reduce the probability of observing the faintest of these images but otherwise do not generally affect core images. Using observational data, Rusin & Ma (2001) show that explicitly including SMBHs in models weakens constraints on...
mass profiles in the inner regions of galaxies, and Winn et al. (2004) have achieved an upper bound of $2 \times 10^8 M_\odot$ for any SMBH in the lens system PMN J1632–0033. Wambsganss & Paczynski (1992) discussed using VLBI observations to detect SMBHs in galaxy halos, and Wilkinson et al. (2001) and Augusto & Wilkinson (2001) placed limits on the abundance of supermassive compact objects in the universe.

In this paper, we expand on these initial efforts and provide a general context for interpreting SMBH effects in lensing studies. In particular, we elaborate on the discussions of Keeton (2003) and Mao et al. (2001) regarding the impact of SMBHs on observing core images and also perform calculations to determine the likelihood of directly observing SMBH lensing signatures. We begin in §2 by presenting two common galactic lens models and performing the calculations to describe the addition of an SMBH to each. From these modified models, we establish criteria that a lens-source system must meet for SMBH effects to be detectable. In §3 we use these criteria to compute cross sections for the production of observable lensing events. Using the observed local relation between SMBH mass and bulge velocity dispersion, these cross sections are convolved with the velocity dispersion distribution of galaxies to establish estimated optical depths to lensing. We conclude in §§4 and 5 by summarizing the lensing signatures of SMBHs and discussing their effects on other lensing studies, such as the search for core images, and also how SMBH signatures themselves might be observed with existing and proposed future facilities.

2. ANALYTICAL TREATMENT OF LENS MODELS

2.1. Lensing by a Single Galaxy with an SMBH

To determine the statistics of lensing by a SMBH in a galaxy core, we must first address the mechanics of the lensing. In this section, we review the properties of combined galaxy and black hole lensing, emphasizing those aspects that are important for the determination of cross sections for lensing statistics. The SMBH is modeled as a point mass, and we consider two models for the potential of a lensing galaxy, a cored isothermal sphere (CIS) and a broken power law. It has been recognized for some time that the observable surface brightnesses in the central regions of galaxies do not actually approach zero slope as implied by the canonical cored isothermal potential. This has been the motivation for various broken power-law parameterizations for surface brightness, such as the Nuker law (Lauer et al. 1995; Gusto & Wilkinson 2001) placed limits on the abundance of SMBHs in galaxy halos, and Wilkinson et al. (2001) and Mao et al. (2001) regarding the impact of SMBHs on observing core images and also perform calculations to describe the addition of an SMBH to each. From these modified models, we establish criteria that a lens-source system must meet for SMBH effects to be detectable. In §3 we use these criteria to compute cross sections for the production of observable lensing events. Using the observed local relation between SMBH mass and bulge velocity dispersion, these cross sections are convolved with the velocity dispersion distribution of galaxies to establish estimated optical depths to lensing. We conclude in §§4 and 5 by summarizing the lensing signatures of SMBHs and discussing their effects on other lensing studies, such as the search for core images, and also how SMBH signatures themselves might be observed with existing and proposed future facilities.

2.1.1. Point-Mass and Cored Isothermal Sphere (PM+CIS)

We begin by considering a point-mass (PM) potential added to a CIS potential. A similar treatment has already been performed by Mao et al. (2001), starting from a CIS mass-density. The distinction is subtle, and the resulting behavior is quite similar. The advantage of the approach followed here is that the characteristic equations are straightforward.

Assuming the lensing occurs in the weak-field limit, the standard one-dimensional lens equation is

$$\beta = \theta - \nabla \Psi,$$

where $\theta$ is the position of the image, $\beta$ is the position of the source as measured from the center of the lens, $\Psi$ is the two-dimensional projected potential of the lens, and $\nabla \Psi$ represents the deflection angle (Schneider et al. 1992, their § 5.1). For an isolated PM,

$$\Psi = \theta_{PM}^2 \ln \theta,$$

and for an isolated galaxy with a cored isothermal lensing potential,

$$\Psi = \theta_{SIS} \sqrt{\theta^2 + \theta_{c}^2}$$

(Narayan & Bartelmann 1996, their Table 2). The core radius of the galaxy potential is denoted by $\theta_c$, and the Einstein radii of the black hole and galaxy are denoted by $\theta_{PM}$ and $\theta_{SIS}$, respectively:

$$\theta_{PM} = \sqrt{\frac{4GM_{BH}}{c^2} \frac{D_{ls} D_{ol}}{D_{ls} D_{ol}}}$$

$$\theta_{SIS} = \frac{4\pi \sigma^2}{c} \frac{D_{ls}}{D_{os}}$$

The velocity dispersion of the galaxy is $\sigma$, the mass of the black hole is $M_{BH}$, and the distances from observer to lens, lens to source, and observer to source are $D_{ols}$, $D_{ls}$, and $D_{os}$, respectively.

For the weak-field limit, lensing potentials add linearly; thus, a reasonable projected potential for lensing by a galaxy with an SMBH is obtained by simply combining these two potentials to give

$$\Psi = \theta_{SIS} \sqrt{\theta^2 + \theta_{c}^2} + \theta_{PM}^2 \ln \theta$$

and results in the lens equation

$$\beta = \theta - \theta_{SIS} \frac{\theta}{\sqrt{\theta^2 + \theta_{c}^2}} - \frac{\theta_{PM}^2}{\theta} \ln \theta$$

For circularly symmetric lens potentials, the magnification at any image position is given by

$$\frac{1}{\mu} = \frac{\beta}{\theta} \frac{\partial \beta}{\partial \theta},$$

where $\mu$ is the magnification (Schneider et al. 1992, their § 2.3). The critical curves in the image plane can be found by setting this equation equal to zero ($\mu \rightarrow \infty$), which yields two conditions:

$$\frac{\beta}{\theta} = 1 - \frac{\theta_{SIS}}{\sqrt{\theta^2 + \theta_{c}^2}} - \frac{\theta_{PM}^2}{\theta^2} = 0$$

and

$$\frac{\partial \beta}{\partial \theta} = 1 - \frac{\theta_{SIS}}{\sqrt{\theta^2 + \theta_{c}^2}} + \frac{\theta_{SIS} \theta^2}{(\theta_{c}^2 + \theta^2)^{3/2}} + \frac{\theta_{PM}^2}{\theta^2} = 0.$$

The first condition produces a single caustic point at the origin of the source plane, while the second condition produces two concentric caustic circles. In the image plane, the first condition is responsible for the outer of two critical curves, and the second condition, for the inner curve plus a critical point at the origin. Figure 1 illustrates the transformation from the CIS
positions and magnifications of the images produced by the PM+CIS model (top) compared with the CIS (bottom). Image positions and the source position are given in units of the SIS Einstein ring radius, $\theta_{\text{SIS}}$. For this figure, we chose $\theta_{\text{PM}}/\theta_{\text{SIS}} = 0.01$ and $\theta_{\text{PM}}/\theta_{\text{SIS}} = 0.03$. The primary image is labeled “A,” and the secondary images are labeled “B,” “C,” and, in the case with the PM, also “D.” The dashed lines indicate the properties of the secondary image of an isolated PM.

2.1.2. Point-Mass and Broken Power-Law Isothermal Sphere (PM+POW)

We now derive a similar set of expressions for lensing by a PM potential added to a circularly symmetric broken power law (POW). In this case, the surface mass density is a more natural starting point and is given by

$$\Sigma_{\text{POW}}(\theta) = 2^{(\gamma - \eta)/\alpha} \Upsilon I_b \left( \frac{\theta}{\theta_b} \right)^{-\gamma} \left[ 1 + \left( \frac{\theta}{\theta_b} \right)^{\alpha} \right]^{(\gamma - \eta)/\alpha},$$

(11)

where $\gamma$ and $\eta$ are inner and outer power-law indices, respectively, $\theta_b$ is the break radius between the power laws, $I_b$ is the surface brightness at the break radius, $\Upsilon$ gives the sharpness of the break, and $\Upsilon$ is the mass-to-light ratio (Keeton 2003). This yields a lensing deflection angle,

$$\nabla \Psi = \frac{2^{1+(\gamma - \eta)/\alpha} \Upsilon I_b \theta_b}{2 - \gamma} \Sigma_{\text{cr}} \left( \frac{\theta}{\theta_b} \right)^{1-\gamma} \times \frac{\beta}{\alpha} \left( \frac{2 - \gamma}{\alpha} \right) \left( \frac{\theta}{\theta_b} \right)^{\alpha},$$

(12)

where $\Sigma_{\text{cr}}$ is the critical surface density for lensing and $\beta$ is the hypergeometric function (Keeton 2003). This model introduces several new parameters that are not present in the CIS. Later, we explore the effects of these new parameters and compare the outcome with that of the cored isothermal model. To facilitate that effort, we now restrict the outer power index to its isothermal equivalent, $\eta = 1$, and recognize that under this condition, in the limit $\theta_b \to 0$, the surface mass density must match that for the singular isothermal sphere (SIS). Doing so, we have

$$\lim_{\theta_b \to 0} \Sigma_{\text{POW}}(\theta : \eta = 1) = \Sigma_{\text{SIS}}(\theta).$$

(13)

Using

$$\Sigma_{\text{SIS}}(\theta) = \frac{\sigma^2}{2GD\theta} = \frac{\theta_{\text{SIS}}^2 D_{\text{os}}}{8\pi GD\theta},$$

(14)

we find the relation

$$2^{(1 - \gamma)/\alpha} \Upsilon I_b \theta_b = \frac{\theta_{\text{SIS}}^2 D_{\text{os}}}{8\pi GD\theta};$$

(15)

and are able to write the deflection angle for the combined PM and POW in terms of $\theta_{\text{SIS}}$:

$$\nabla \Psi = \frac{\theta^2}{\theta_{\text{PM}}} + \frac{\theta_{\text{SIS}}}{2 - \gamma} \left( \frac{\theta}{\theta_b} \right)^{1-\gamma} \times \frac{\beta}{\alpha} \left( \frac{2 - \gamma}{\alpha} \right) \left( \frac{\theta}{\theta_b} \right)^{\alpha}.$$  

(16)

2.1.3. Typical Values of $\theta_{\text{PM}}, \theta_c, \theta_b, \alpha$, and $\gamma$

The empirical correlation in the local universe between the masses of SMBHs and the velocity dispersions of their host galaxy bulges can be exploited to determine a typical value of $\theta_{\text{PM}}/\theta_{\text{SIS}}$. Observable lenses are dominated by elliptical lensing galaxies (Turner et al. 1984). For elliptical galaxies, we can...
and galaxy at velocity dispersion are shown. A standard cosmology of mass and the host galaxy velocity dispersion is used, and three values of velocity dispersion are shown. A standard cosmology of \( \Omega_m = 0.3, \Omega_\Lambda = 0.7 \), and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is used.

ignore the distinction between the velocity dispersion of the bulge and that of the galaxy as a whole. Therefore, we assign the SMBH its mass according to

\[
M_{\text{BH}} = (1.48 \pm 0.24) \times 10^8 M_\odot \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^{4.7},
\]

where \( \sigma \) is equal to the (bulge) velocity dispersion and \( M_\odot \) is the solar mass (Merritt & Ferrarese 2001). If we temporarily assume that this relationship holds for all galaxies, even beyond the local universe, we can generate characteristic curves for \( \theta_{\text{PM}}/\theta_{\text{SIS}} \) as a function of \( \sigma, D_{\text{SIS}}, D_{\text{SIS}}, \) and \( D_{\text{SIS}} \) by substituting the above result into equation (4). Figure 2 illustrates the behavior of \( \theta_{\text{PM}}/\theta_{\text{SIS}} \) for different lens and source galaxy distances. For lenses located roughly halfway to the source, the dominant case because it is most efficient for lensing (Turner et al. 1984, their Fig. 5), there is not much variation in \( \theta_{\text{PM}}/\theta_{\text{SIS}} \) for a fixed velocity dispersion. Galaxies with different velocity dispersions produce different \( \theta_{\text{PM}}/\theta_{\text{SIS}} \) ratios.

There is to date no empirical understanding of any relationship between galaxy cores and other observables similar to that for SMBHs. The parameterizations we have chosen use angular parameters. In Table 1 we compute the mean and median values for these parameters. In Table 1 we compute the mean and median values for the 26 cored galaxies listed in Faber et al. (1997, their Table 2).

\[
\theta_{\text{PM}}/\theta_{\text{SIS}} = \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^{4.7},
\]

On the basis of these findings, we take \( \gamma = 0.1 \) and \( \alpha = 2.0 \) for all subsequent calculations. From Table 1, it is also apparent that forcing \( \eta = 1 \), as we did above, was not a bad approximation to these observational results.

2.1.4. Shear and Ellipticity

We constrain our analysis to circularly symmetric lensing potentials for this paper to better illustrate the dependence on fundamental parameters. As is well known, however, gravitational lenses rarely seem to display such simple symmetry and are often best modeled by elliptical mass density potentials or by including a gravitational shear in the local environment. In the Appendix we present a justification for neglecting the contribution of these higher order factors in our exploration of the fundamental behavior of these systems.

2.2. Signatures of SMBH Lensing

The addition of an SMBH to an otherwise typical galactic lensing potential produces three significant effects: (1) two-image lensing extends infinitely beyond the Einstein radius of the galaxy; (2) the maximum number of images increases from three to four; and (3) the area of the region in the source plane expected to produce more than two images is reduced compared with a cored or broken power-law galactic potential with no SMBH. In general, for most source positions \( \beta \), the lensing characteristics of the combined SMBH and galaxy act as if they were merely superposed and conspire only in the central regions of the source plane to cancel out the secondary images that either would have produced alone.

Both the cored and broken power-law potentials demonstrate these properties. In both models, the source plane is divided into three regions by two concentric circular caustics and a degenerate caustic point at the origin. In fact, as shown in Figure 1 with the PM+CIS model, for different values of the source position, either one or three secondary images may form opposite the position of the primary image. For the cases in
which we are interested, the core radius and PM have little effect on the properties of the primary image (A) but can dramatically change the properties of the three possible secondary images (B, C, and D).

Image B, if it exists, is a saddle point in the light-travel time and has a negative magnification. It lies very close to the location of the secondary image that would form from an SIS potential alone and therefore provides little information about the properties of the core of the galaxy or the SMBH. Image C is a maximum of the light-travel time and has a positive magnification. It carries information about the degree of softening of the galaxy’s potential, generally becoming brighter for larger cores and fainter for smaller cores. Image D is another saddle point in the light-travel time and again has a negative magnification. For most source positions, it tracks the position and magnification of the image that would form from an isolated PM potential, thus providing information about the SMBH. Image D is present even without images B and C for source positions with sufficiently large $\beta$.

Keeton (2003) points out that including SMBHs in lensing calculations has the effect of swallowing faint core images. Figure 1 demonstrates that for sufficiently small impact parameters, for which the core images would be faintest, the SMBH does indeed eliminate the core image that would have formed in its absence. Furthermore, Figures 4 and 5 demonstrate that, for a given $\theta_{PM}/\theta_{SIS}$, the size of the region excluding core images depends on the other properties ($\theta_\gamma$, $\gamma$, and $\alpha$) of the inner region of the host galaxy. Referring again to Figure 5, it is easily seen that for both models, reducing the core radius pushes the caustics out and closer together until ultimately they merge (this actually occurs before the core radius goes to zero, although the exact configuration is not shown in the plotted sequence).

**TABLE 1**

| Parameter                | Mean  | Median |
|--------------------------|-------|--------|
| $\alpha$                 | 2.35  | 2.03   |
| $\gamma$                 | 0.09  | 0.08   |
| $\eta$ (arcsec)          | 1.28  | 1.33   |
| $\theta_\gamma$ (arcsec)| 1.71  | 1.29   |

**NOTES.—** These averages were calculated for the cored galaxies listed in Faber et al. (1997, their Table 2). Those authors use $\beta$ to indicate the outer power-law index, whereas we use $\eta$. 

**FIG. 4**—Influence of PM+POW parameters on lensing. Left two columns, top to bottom: Progression of the magnification curves for the PM+POW potential as the sharpness of the break, $\alpha$, is increased while holding $\eta = 1$, $\gamma = 0.1$, $\theta_\gamma/\theta_{SIS} = 0.01$, and $\theta_{PM}/\theta_{SIS} = 0.03$ constant. The profile of the normalized surface brightness, $\Sigma_{POW}/\Sigma_h$, is drawn alongside for reference. Right two columns, top to bottom: Progression as the inner power-law index, $\gamma$, increases while holding $\alpha = 2$, $\eta = 1$, $\theta_\gamma/\theta_{SIS} = 0.01$, and $\theta_{PM}/\theta_{SIS} = 0.03$ constant. The final frame, with $\gamma = 1$, is effectively an SIS plus an SMBH.
observed decrease in flux for lenses with a small core radius. As we have seen, the PM+CIS and PM+POW models produce lenses with a minimum of two images and a maximum of four images, depending on the source position. We would like to extend the analysis and gain insight into the likelihood of observing these images, particularly image C and image D. To do so, we must introduce a parameter that relates to the observational ability to detect an image. Let us assume for simplicity that an image can be detected if it produces a flux that is above a specified fraction of the flux produced by the primary image:

\[
\frac{F_{B,C,D}}{F_A} = \frac{\mu_{B,C,D}}{\mu_A} \geq \frac{1}{R},
\]

where \(R\) is the maximum detectable dynamic range between images and is the parameter we use in the following calculations. The cross section for detection is then defined as the area in the source plane that produces images that meet this requirement and in general is characterized by two extreme source positions, \(\beta_{\text{min}}\) and \(\beta_{\text{max}}\), such that the total angular area \(A\) is given by

\[
A = \pi(\beta_{\text{max}}^2 - \beta_{\text{min}}^2).
\]

In this expression, \(A\), \(\beta_{\text{min}}\), and \(\beta_{\text{max}}\) are implicitly dependent on the core radius, the SMBH Einstein radius, the galaxy Einstein radius, and the maximum dynamic range.

We are now in a position to calculate the characteristic cross sections for the PM+CIS and PM+POW models. We consider three cross sections: (1) one for detecting two images (A and B, or A and D), (2) one for detecting three images (A, B, and C), and (3) one for detecting four images (A, B, C, and D). The cross section for observing three images is, in fact, the sum of two of the annuli discussed above, one along the inner caustic and the other along the outer caustic. Because of the nature of the lensing potential equations, numerical methods were employed to determine \(\beta_{\text{min}}\) and \(\beta_{\text{max}}\) for a range of model parameters to a precision of 0.002\(\theta_{\text{SIS}}\). The results are discussed below.

### 3.1. Lensing Cross Sections

It is clear from Figure 5 that combining a PM and an isothermal sphere produces the possibility of a very large two-image cross section. In fact, without a maximum dynamic range constraint, the cross section would be theoretically unbounded, and even for finite dynamic range constraints, \(\beta_{\text{max}}\) may be well beyond the standard SIS limit of \(\beta_{\text{max}} = \theta_{\text{SIS}}\). To see more quantitatively how the dynamic range affects the cross section, Figure 7 plots \(A_2\) as a function of core radius for several values of \(R\), ranging from 1 to \(10^8\), with \(\theta_{\text{PM}}/\theta_{\text{SIS}}\) fixed at 0.03. There are two striking features in this plot. First, there is a degeneracy of cross section curves, around \(A_2/(\pi\theta_{\text{SIS}}^2) \approx 1\) (the same feature is visible in Fig. 8). This is caused by the disparity in magnification between image B and the image D tail, both of which can contribute to the two-image cross section. The disparity is reduced as the core radius is reduced and eventually, for cores less than about \(\theta_c/\theta_{\text{SIS}} \approx 10^{-2.8}\), disappears because the source plane is no longer divided into three regions but only one, which always produces two images (see Fig. 5, particularly the bottom row). This situation eliminates the distinction between our labels “B” and “D” for images, although for \(\beta \leq \theta_{\text{SIS}}\) the secondary image has properties similar to those of a typical image B, and for \(\beta \geq \theta_{\text{SIS}}\) the magnification drops toward that of a typical image D. This smooth decrease in magnification near \(\theta_{\text{SIS}}\) for lenses with small core radii is responsible for the second noticeable feature: cross sections increase rapidly with dynamic range for small core radii since there is no step function in magnification to overcome, as is the case for larger core radii.

---

**Fig. 5.—Progression of the magnification curves as the core radius is reduced. Left column, top to bottom, PM+CIS potential; right column, top to bottom, PM+POW potential with \(\alpha = 2, \eta = 1,\) and \(\gamma = 0.1\). For both cases, \(\theta_{\text{PM}}/\theta_{\text{SIS}} = 0.03\), and the behavior is qualitatively similar. As the core radius is reduced, the region of four images moves outward from the lens galaxy and its area is reduced until it disappears at a finite core radius (not shown).**

---

3. NUMERICAL ANALYSIS OF THE MODELS

3.1. Lensing Cross Sections

As we have seen, the PM+CIS and PM+POW models produce lenses with a minimum of two images and a maximum of four images, depending on the source position. We would like to extend the analysis and gain insight into the likelihood of observing these images, particularly image C and image D. To do so, we must introduce a parameter that relates to the area in the source plane that produces images that meet this requirement and in general is characterized by two extreme source positions, \(\beta_{\text{min}}\) and \(\beta_{\text{max}}\), such that the total angular area \(A\) is given by

\[
A = \pi(\beta_{\text{max}}^2 - \beta_{\text{min}}^2).
\]

In this expression, \(A\), \(\beta_{\text{min}}\), and \(\beta_{\text{max}}\) are implicitly dependent on the core radius, the SMBH Einstein radius, the galaxy Einstein radius, and the maximum dynamic range.

We are now in a position to calculate the characteristic cross sections for the PM+CIS and PM+POW models. We consider three cross sections: (1) one for detecting two images (A and B, or A and D), (2) one for detecting three images (A, B, and C), and (3) one for detecting four images (A, B, C, and D). The cross section for observing three images is, in fact, the sum of two of the annuli discussed above, one along the inner caustic and the other along the outer caustic. Because of the nature of the lensing potential equations, numerical methods were employed to determine \(\beta_{\text{min}}\) and \(\beta_{\text{max}}\) for a range of model parameters to a precision of 0.002\(\theta_{\text{SIS}}\). The results are discussed below.

### 3.1.1. Two Images

It is clear from Figure 5 that combining a PM and an isothermal sphere produces the possibility of a very large two-image cross section. In fact, without a maximum dynamic range constraint, the cross section would be theoretically unbounded, and even for finite dynamic range constraints, \(\beta_{\text{max}}\) may be well beyond the standard SIS limit of \(\beta_{\text{max}} = \theta_{\text{SIS}}\). To see more quantitatively how the dynamic range affects the cross section, Figure 7 plots \(A_2\) as a function of core radius for several values of \(R\), ranging from 1 to \(10^8\), with \(\theta_{\text{PM}}/\theta_{\text{SIS}}\) fixed at 0.03. There are two striking features in this plot. First, there is a degeneracy of cross section curves, around \(A_2/(\pi\theta_{\text{SIS}}^2) \approx 1\) (the same feature is visible in Fig. 8). This is caused by the disparity in magnification between image B and the image D tail, both of which can contribute to the two-image cross section. The disparity is reduced as the core radius is reduced and eventually, for cores less than about \(\theta_c/\theta_{\text{SIS}} \approx 10^{-2.8}\), disappears because the source plane is no longer divided into three regions but only one, which always produces two images (see Fig. 5, particularly the bottom row). This situation eliminates the distinction between our labels “B” and “D” for images, although for \(\beta \leq \theta_{\text{SIS}}\) the secondary image has properties similar to those of a typical image B, and for \(\beta \geq \theta_{\text{SIS}}\) the magnification drops toward that of a typical image D. This smooth decrease in magnification near \(\theta_{\text{SIS}}\) for lenses with small core radii is responsible for the second noticeable feature: cross sections increase rapidly with dynamic range for small core radii since there is no step function in magnification to overcome, as is the case for larger core radii.

---

**Fig. 5.—Progression of the magnification curves as the core radius is reduced. Left column, top to bottom, PM+CIS potential; right column, top to bottom, PM+POW potential with \(\alpha = 2, \eta = 1,\) and \(\gamma = 0.1\). For both cases, \(\theta_{\text{PM}}/\theta_{\text{SIS}} = 0.03\), and the behavior is qualitatively similar. As the core radius is reduced, the region of four images moves outward from the lens galaxy and its area is reduced until it disappears at a finite core radius (not shown).**
The SMBH has an effect on the cross section, as well. Figure 8 specifically addresses this dependence, and from it we see that the two-image cross section increases with $\theta_{PM}/\theta_{SIS}$. This is because the image D tail increases in magnification for larger SMBH mass scales. The effect is most dramatic for small core radii.

3.1.2. Three Images

For the three-image cross section, it is also useful to begin by examining the limit of no dynamic range constraints. In this case, the region in the source plane that produces three observable images is defined completely by the two concentric caustics as discussed at the end of § 2 and plotted in Figure 6. This represents an upper bound for the three-image cross section, which is plotted in Figures 8 and 7. Large dynamic ranges saturate to this limit. As the dynamic range is lowered, the cross section reduces, with the most rapid decline at small core radii. This can be understood from examination of Figure 5, once again noting that the magnification of image C generally decreases as the core radius decreases. It is no coincidence that the range of core radii that produce a significant three-image cross section corresponds to the range in which the two-image cross section is highly degenerate, since both trace the portion of parameter space that is dominated by the region in the source plane between the two concentric caustics.

Unlike for the two-image cross section, increasing the SMBH mass scale decreases the three-image cross section. This is another manifestation of the image swallowing previously described in § 2.

3.1.3. Four Images

The four-image cross section is qualitatively similar to the three-image cross section. However, since the overall magnification of image D is considerably less, the cross section curves are correspondingly reduced, as is clear in Figures 8 and 7.

3.2. Lensing Statistics

In this section we derive the optical depth to lensing for a given source using the calculated cross sections from above and following the standard method (Turner et al. 1984; Schneider et al. 1992, pp. 309 and 371). If we assume that the lensing objects have a local comoving number density that can be written as $n_1(\sigma, z)$, then the differential optical depth of a lensing event occurring at a redshift $z_l$ for a source at $z_s$ is

$$d\tau = n_1(\sigma, z_l)(1 + z_l)^3A(\sigma, z_l, z_s)D_{el}(z_l) d\sigma c dt.$$  

Here we constrain $\theta_{PM}/\theta_{SIS}$ according to equation (17) and have explicitly written the cross section, $A$, as a function of velocity dispersion, lens redshift, and source redshift to highlight its dependence on these parameters. It is, of course, still dependent on the choice of core radius and dynamic range, as well. $D_{el}$ is again the angular diameter distance between the observer and lens, and in a homogeneous, isotropic universe we have

$$c dt = \frac{R_0}{1 + z_l} \left[ \Omega_m (1 + z_l)^3 + \Omega_k (1 + z_l)^2 + \Omega_{\Lambda} \right]^{1/2} dz_l,$$  

where $c$ is the speed of light, $t$ is the look-back time, and $R_0$, $\Omega_m$, $\Omega_{\Lambda}$, and $\Omega_k$ are the standard cosmological parameters with $\Omega_k = 1 - \Omega_m - \Omega_{\Lambda}$ (Hogg 1999). Conceptually, all that remains is to select the form of the density distribution function. It is common in gravitational lens studies to utilize either the Press-Schechter formalism (Press & Schechter 1974) or the Schechter luminosity function (Schechter 1976) to generate this distribution. We have chosen to use the Schechter luminosity function combined with the Faber-Jackson relation and have also assumed no number density evolution with redshift to cast the density distribution in terms of only the velocity dispersion, giving

$$n_1(\sigma) = \gamma n_\ast \left( \frac{\sigma}{\sigma_\ast} \right)^{(\gamma - 1)} \exp \left[ - \left( \frac{\sigma}{\sigma_\ast} \right)^\gamma \right].$$  

We use the same choices of parameters $\alpha$, $\gamma$, and $\sigma_\ast$ and abundances, $n_\ast$, of spiral, S0, and elliptical galaxy types as Ofek et al. (2003) and reprint them in Table 2.

The redshift distribution function of extragalactic flat-spectrum radio sources is localized around $z_l \approx 1$ and has a mean value of $\langle z_l \rangle = 1.1$ (Muñoz et al. 2003). We assume that treating all sources as if they were at the mean redshift is sufficient and take $z_l = 1.1$. We are now prepared to calculate the optical depth to lensing for each cross section of interest by
integrating over velocity dispersion and redshift. Again, these calculations must be carried out numerically because the cross sections themselves are not analytical.

The results of these calculations are presented in Figure 9. For two-image lensing, we see that the optical depth to lensing, \( \tau \), parallels the cross section curves for this case. For a high maximum dynamic range, the probability is quite high, and as the dynamic range is lowered to more typical values of \( R \approx 1000 \), the probability approaches the expected value for lensing by a population of SISs of \( \tau \approx 0.001 \) (Browne et al. 2003), as it must, since this result has been observationally justified.

The optical depths to lensing for three- and four-image cases are also similar to their cross sections, with the four-image case suffering a more rapid decline in optical depth as \( R \) is decreased. Both probabilities exhibit a significant dependence on the assumed core radius of the lensing population, indicating that the addition of a PM to a softened isothermal lens model, if restricted to the relationship discussed in \( \S \) 2.1.3, would not hinder the use of lens statistics to learn about galaxy cores.

4. EMPIRICAL IMPLICATIONS OF THE MODELS

4.1. Relevant Facilities and Instruments

We have seen that if an intervening lens galaxy contains an SMBH, then because of the black hole potential an additional faint image is predicted that is not present in unmodified models. In fact, even in cases in which the source position and the lens are too far apart to create multiple images in the standard way, the SMBH still produces a faint image of the background object near the center of the foreground galaxy. In
this section we discuss the applicability of optical, infrared, and radio facilities to observing these images.

4.1.1. Optical and Infrared

Optical observations of these highly demagnified images would be difficult. To obtain a significant cross section for detecting image D, we saw in § 3 (and Fig. 7) that demagnification from the primary image by $10^6$, or 15 mag, must be observable. In the Hamburg-ESO Survey (Wisotzki et al. 2000) and the Sloan Digital Sky Survey (SDSS), the brightest quasars at redshift greater than 0.5 have magnitudes of 15. The demagnification due to the lensing alone would reduce this to 30th magnitude, and this would be compounded with the extinction that the image would suffer passing through the inner regions of the lensing galaxy. Such faint observations are far beyond the capabilities of current optical telescopes. To observe the lensing

---

**TABLE 2**

Luminosity Function Parameters for Spiral, S0, and Elliptical Galaxy Types

| Parameter | Spiral | S0 | Elliptical |
|-----------|--------|----|------------|
| $a$       | -1.16  | -0.54 | -0.54 |
| $\beta$   | $1.46 \times 10^{-2}$ | $0.61 \times 10^{-2}$ | $0.39 \times 10^{-2}$ |
| $\gamma$  | 2.6    | 4.0  | 4.0        |
| $\sigma$  | 144    | 206  | 225        |

REFERENCE.—Ofek et al. 2003.
of even the brightest quasars would require 30 m class or larger optical telescopes. Infrared observations would not suffer the severe extinction of the image passing through the foreground galaxy; however, it would still be problematic to separate the lens image from foreground galaxy emission. The use of spectral lines to distinguish two redshift components may be able to mitigate this problem.

4.1.2. **Radio**

Observations with radio telescopes, while still difficult, are more feasible. The brightest sources above a redshift of $z = 0.5$ in the NRAO-VLA Sky Survey (Condon et al. 1998) and the FIRST sky survey (Becker et al. 1995) have flux densities greater than 20 Jy. The flux density of image D from such a source would be on the order of $20 \mu$Jy, and at radio wavelengths any absorption through the lensing galaxy should be negligible. Long integrations with the Very Large Array (VLA) and VLBI arrays can already achieve sensitivities of $10 \mu$Jy beam$^{-1}$ (Garrett et al. 2003) or better, sufficient to observe faint images from the very brightest radio sources. Future improvements in radio telescope instrumentation, however, will dramatically increase the predicted number of observable D images. Four projects in particular are relevant to making sensitive observations of lenses and are summarized below.
VLA Expansion Project.—The VLA Expansion Project is already underway to supply the VLA with modern electronics and also plans to add up to eight additional antennas to the array at distances out to 350 km from the core facility. The resulting Expanded VLA (EVLA), scheduled for completion by 2012, is intended to improve performance in important areas by a factor of 10. This is expected to extend the detection threshold of faint sources to 1 \(\mu\)Jy or less. With this sensitivity, the EVLA should also be able to detect the C images for existing radio lenses.

Square Kilometer Array.—The Square Kilometer Array (SKA) is a proposed next-generation radio telescope that offers the most promise for observing lenses with faint images. The current design goals for the SKA are to achieve a dynamic range approaching \(10^8\) and a detection threshold that could reach the 10 nJy level.

Low Frequency Array.—A low-frequency radio telescope array has been proposed that would operate at low frequencies, between 30 and 240 MHz. The design is intended to achieve a baseline dynamic range of \(10^5\), extending to \(10^8\) in some cases, and a detection threshold of less than 1 \(\mu\)Jy. However, because of the low frequencies used, the expected best-case angular resolution of about 1" (at 240 MHz) of the instrument would prevent some lens configurations from being resolved.

Atacama Large Millimeter Array.—The Atacama Large Millimeter Array is a millimeter-wavelength telescope under design in Europe and North America. It will be located in Llano de Chagnetor, Chile and consist of at least 64 antennas, each 12 m. The intended resolution is 10 mas, with a sensitivity approximately 20 times greater than the VLA (Tofani 2003).

4.2. Lensing Expectations

Despite the limitations of current technology, it is not too early to consider candidate SMBH lensing targets. The first step is to estimate the predicted total number of lenses that should be observable. We can use the optical depth calculations from § 3.2, along with the known flux distribution of radio sources, to accomplish this. To estimate the number of lenses that an instrument with a limiting sensitivity, \(S\), should be able to detect, we integrate the product of the differential source flux distribution, \(dN/dF\), and the optical depth, \(\tau(R, \theta_c)\). The appropriate dynamic range, \(R\), is given by the ratio between the sensitivity of the instrument and the flux of the source \((R = F/S)\):

\[
N(S, \theta_c) = \int \tau \left(\frac{F}{S}, \theta_c\right) \frac{dN}{dF} dF. \tag{23}
\]

We are neglecting the dependence on source redshift of both \(\tau\) and \(dN/dF\). Again, we assume that treating all sources as if they were at the mean redshift, \(z_s = 1.1\), is sufficient. We still cannot perform this integral fully because we have evaluated \(\tau\) only for specific values of \(R\). However, we can achieve an approximate evaluation by converting the integral to a sum over flux bins centered on equivalent \(R\) values:

\[
N(S, \theta_c) \approx \sum \tau \left(\frac{F_i}{S}, \theta_c\right) \left(\frac{dN}{dF}\right)_{F_i} \Delta F_i. \tag{24}
\]

The differential number of radio sources as a function of flux in the range 14.5 \(\mu\)Jy \(\leq F \leq 1500\) \(\mu\)Jy is

\[
\frac{dN}{dF_{8.44}} = (-4.6 \pm 0.7) F^{-2.3\pm0.02} \text{ Jy}^{-1} \text{ sr}^{-1} \tag{25}
\]
There are over 800,000 sources in the FIRST radio catalog (White et al. 1997), which has a detection limit of 1 mJy and covers nearly 10,000 deg² of the sky, mostly in the northern hemisphere. For this analysis, we assume, as we did earlier, that all the sources in the catalog are at the redshift $z = 1.1$ and follow the same procedure as above. We divide the catalog into eight flux bins, shown in Table 5, and perform the sum for the two diagnostic regimes of the core radii.

The first regime, with $\theta_e/\theta_{\text{SIS}} = 10^{-4}$ and using the PM+CIS model, predicts the number of observable lenses for the area of the sky covered by the FIRST survey given in Table 6. Immediately, it is clear that, compared with Table 3, the total number of lenses detectable by sensitive instruments has been drastically reduced. This is because the FIRST survey contains no sources with flux below $F = 1$ mJy, and therefore, the extra sensitivity has no accessible faint sources to detect.

The second regime of core radii of interest is that for which $\theta_e/\theta_{\text{SIS}} = 10^{-1.3}$. The calculated values for these core radii are listed in Table 7. Although significantly fewer lenses are likely to be contained in the catalog, the relative number with an observable core image is much higher (between 50% and 80% for sensitivities comparable to or better than the EVLA sensitivities). In addition, most of the two-image lenses should already be detectable with the VLA. It is plausible, under these circumstances, that an appropriate search guided by the FIRST catalog today would be able to detect and characterize a population of two-image lenses that yield a significant number of third and fourth images when reobserved with future technology.

### Table 5
**Number of Sources in the FIRST Radio Catalog**

| Bin | Flux (mJy) | $N_{\text{tot}}$ |
|-----|------------|-----------------|
| 8   | $10^{3.5}$ | 27              |
| 7   | $10^{3.0}$ | 227             |
| 6   | $10^{2.5}$ | 1,444           |
| 5   | $10^{2.0}$ | 7,418           |
| 4   | $10^{1.5}$ | 26,914          |
| 3   | $10^{1.0}$ | 75,756          |
| 2   | $10^{0.5}$ | 181,766         |
| 1   | 1          | 517,565         |

Notes.—The number of sources in the FIRST radio catalog are divided into eight bins by flux. The flux values listed in the table represent the floor of each bin. The number of sources in each bin is used in combination with optical depth calculations for the PM+CIS lensing potential to calculate the predicted number of lenses contained in the catalog (Tables 6 and 7).

### Table 6
**Predicted Number of Gravitational Lenses in the FIRST Catalog Using $\theta_e/\theta_{\text{SIS}} = 10^{-4}$**

| $S$ (mJy) | Example               | $N_{\text{tot}}$ | $N_{\text{j}}$ | $N_{\text{4}}$ | $N_{\text{out}}$ |
|-----------|-----------------------|------------------|---------------|---------------|-----------------|
| 1000      | FIRST survey          | 90               | 2             | 0             | 0               |
| 100       | Typical VLA           | 410              | 20            | 0             | 0               |
| 10        | EVLA                  | 410              | 320           | 12            | 1               |
| 0.1       | EVLA                  | 410              | 320           | 70            | 7               |
| 0.01      | SKA                   | 580              | 320           | 230           | 86              |

Notes.—This table indicates the predicted number of gravitational lenses for a search with sensitivity $S$ in the FIRST catalog of radio sources. The choice of core radius is $\theta_e/\theta_{\text{SIS}} = 10^{-4}$.

The above calculations were performed for two specific core radii of particular interest, but they can be repeated for all the core radii that we considered in §3. This has been done and is plotted in Figure 10 for $S = 1$ mJy. The ratios of $N_3/N_{\text{tot}}$ and $N_4/N_{\text{tot}}$ are plotted, as well. For sensitivities better than $10 \mu$Jy, the $N_3/N_{\text{tot}}$ increases rapidly with core radius in the region $\theta_e/\theta_{\text{SIS}} \geq 10^{-2.6}$ and approaches unity for $\theta_e/\theta_{\text{SIS}} = 10^{-0.6}$ and $S = 10 \mu$Jy. Similarly, $N_4/N_{\text{tot}}$ increases in the same region, although it reaches a maximum of only $N_4/N_{\text{tot}} \approx 0.4$.

Searching for 500 lenses out of a catalog of over 800,000 sources is still a daunting task. We suggest that another object catalog may help reduce the list of candidates even more. SDSS covers 3324 deg² as of Data Release 2, overlapping the FIRST survey, and will eventually cover nearly 10,000 deg². It has identified over 88 million objects at optical wavelengths, many of which are extrasub-galactic sources with estimated redshifts in the range $0 \leq z \leq 0.5$ (Stoughton et al. 2002). This population of extragalactic objects makes an ideal foreground of potential lens galaxies for background radio sources in the FIRST catalog. Ivezic et al. (2002) have shown that the error in position between the two surveys is $\sim 0.4$ per coordinate, and the SDSS has included in its database a special flag to indicate objects that are spatially collocated with FIRST radio sources. If pairs of neighboring objects within approximately $1''$ of each other could be found in the SDSS database and if one were marked with the FIRST flag, then this would represent a sample of targets with an elevated likelihood of containing radio lenses. If it could be further reduced to pairs for which the radio source is known to be behind the neighboring object, the likelihood would be higher yet. Fortuitously, the SDSS aims to acquire 1 million spectra of galaxies, preferentially targeting

### Table 7
**Predicted Number of Gravitational Lenses Using $\theta_e/\theta_{\text{SIS}} = 10^{-1.3}$**

| $S$ (mJy) | Example               | $N_{\text{tot}}$ | $N_{\text{j}}$ | $N_{\text{4}}$ | $N_{\text{out}}$ |
|-----------|-----------------------|------------------|---------------|---------------|-----------------|
| 1000      | FIRST survey          | 90               | 2             | 0             | 0               |
| 100       | Typical VLA           | 410              | 20            | 0             | 0               |
| 10        | EVLA                  | 410              | 320           | 12            | 1               |
| 0.1       | EVLA                  | 410              | 320           | 70            | 7               |
| 0.01      | SKA                   | 580              | 320           | 230           | 86              |

Notes.—This table indicates the predicted number of gravitational lenses for a search with sensitivity $S$ in the FIRST catalog of radio sources. The choice of core radius is $\theta_e/\theta_{\text{SIS}} = 10^{-1.3}$. 

---

BOWMAN, HEWITT, & KIGER

Vol. 617
identified optical counterparts of FIRST radio sources. Thus, redshift information may be available for one or both of the objects in a pair directly from the database.

A complementary approach to finding new lens candidates has been described by Bolton et al. (2004). They have been able to identify multiple emission lines at two redshifts in a number of SDSS spectra, indicating a foreground and background galaxy within the $3\arcmin$ field of view, and are using these objects in an optical lens search.

4.2.2. Galaxy Cores

It is clear from the preceding estimates that the size of galaxy cores greatly affects the likelihood of observing core and SMBH images. It is beneficial, therefore, to consider what is known about galaxy cores to date. The structure of the central regions of galaxies has been studied extensively through surface brightness measurements, and a number of attempts have already been made to deduce the properties of galaxy cores from gravitational lensing studies.

Fits to the Nuker law surface brightness models, which we used earlier as the basis of our second lens model, indicate that galaxies appear to be of two types, those that are “cored” and those that are not. The cored galaxies are luminous, slowly rotating elliptical galaxies with boxy isophotes; the noncored, power-law galaxies are faint, rapidly rotating galaxies with disky isophotes. There is a fairly tight correlation between break radius and luminosity. Large elliptical galaxies dominate the lensing cross section, implying that the important galaxies

![Diagram of lensing predictions](image-url)

**Fig. 10.**—Number of lenses predicted to be observable in the FIRST radio source catalog. Top row: Search with limiting sensitivity $S = 1\ $µJy, plotted as a function of core radius. The curves are given for the number of lenses with at least two, three, or four detectable images ($N_{\text{tot}}$, $N_3$, and $N_4$, respectively). $N_{\text{out}}$ is the number of lenses with two detectable images if the second image would not be present without the SMBH. Middle row: Fraction of observable lenses with detectable third images. The curves, in this case, represent different limiting sensitivities, from $S = 1000\ $µJy down to $0.01\ $µJy. Bottom row: Similarly, the fraction of observable lenses with detectable fourth images. For all rows, the left column of graphs is for PM+CIS, and the right is for PM+POW. The vertical dashed lines are drawn for the values of $\theta_c$ used for the discussion in the text.
for SMBH lensing are also those that have cores. In fact, it has been hypothesized that in the hierarchical merger process involved in galaxy formation, SMBH binary systems form and spiral toward the center of the new system, cooling and possibly ejecting stars near the center. The SMBHs may therefore be responsible for the existence of the cores.

The presence of a core image (image C) in gravitational lenses, however, is exceedingly rare, with candidate images observed only in PMN J1632−0033 (Winn et al. 2004), MG 1131+0456 (Chen & Hewitt 1993), and APM 08279+5255 (Ibata et al. 1999). For the latter two systems, it remains uncertain whether the third component is an image or emission from the lens galaxy. If we do take the third components in MG 1131+0456 and APM 08279+5255 as lens images, the best-fit values of $\theta_c/\theta_{\text{SIS}} \approx 0.2$ (Chen et al. 1995) and $\theta_c/\theta_{\text{SIS}} \approx 0.7$ (Egami et al. 2000), respectively, are calculated.

The only system with convincing evidence of a core image detection is PMN J1632−0033 (Winn et al. 2004). Radio spectra of all three image components have been acquired, and the flux ratios $F_B/F_A$ and $F_C/F_A$ are nearly constant across the high-frequency measurements, indicating that the components are likely from the same source. The only discrepancy is with the lowest frequency measurement at 1.7 GHz, but, as Winn et al. (2004) argue, this is the range for which absorption from passing through the lens galaxy would be strongest. They have, furthermore, been able to place constraints on several quantities of interest (Winn et al. 2003). Using a POW model, the relation between the break radius and the inner power-law slope is constrained, and by using a cored isothermal mass density model, a core radius of $r_c \approx 31 h^{-1}$ pc, is derived. Compared with the estimated Einstein radius of the lens galaxy of $\theta_{\text{SIS}} \approx 0.074$ (Winn et al. 2002), the core radius corresponds to $\theta_c/\theta_{\text{SIS}} = 0.0075$, although this is not directly comparable to our core radius because of the differing model.

Keeton (2003) discusses in detail the implications of the behavior of the mass density profile at the core to the detectability of a third image and concludes that the paucity of third images in lens systems is consistent with their observed surface density profiles. Rusin & Ma (2001) have studied six double-lens systems in which the third image is not detected. Using the known detection limits for each of these sources, they employ a singular mass profile with a variable slope to determine how steep the inner density profile must be to explain the lack of this image. They find that for these six sources, the inner region must be quite steep. However, it is not clear that the constraints on the inner slope are relevant in constraining the possible flattening of the mass distribution that might occur right at the SMBH.

The above attempts have tended to be specific case studies of individual lenses, and therefore, it is difficult to get a sense of the overall probability of any particular core radius. The results of our modeling are then in a position to provide the backdrop for interpreting these results. For example, the results of a systematic lens survey with 1 mJy sensitivity could be compared with the ratio $N_C/N_{\text{tot}}$ in Figure 10 to learn about typical core radii values.

### 4.3. Structure in Image D

To this point we have considered only the overall magnification and position of image D. If these images are detected, however, any observed structure in them should be capable of providing additional important information about the lens system. The properties of observed structure in a lens image should be distinct from the properties of any radio emission from the foreground galaxy, and thus, resolved structure in faint emission at the center of a lens could be used to reduce confusion about whether an image or foreground emission has been observed. In addition, any structure in image D is capable of indicating the presence of binary SMBHs in the lensing galaxy. Both of these are discussed in detail below, but first we examine the ability to resolve structure in these images.

#### 4.3.1. Detecting Structure in Image D

The flux density of image D, as we have seen, is demagnified typically by a factor of at least $10^6$ from that of the original source. Since surface brightness is conserved by lensing, this corresponds to a reduction in the surface area of the image compared with the original source. Therefore, we expect each of the two spatial dimensions on the sky to be reduced by a factor of order $(10^6)^{1/2} = 10^3$.

VLBI measurements achieve angular resolutions approaching 0.1 mas. Thus, a radio source with 1 mas scale structure would produce an image D with $\sim 1$ mas scale structure, which should easily be distinguished by the VLBI.

#### 4.3.2. Foreground Galaxy Emission

An observation of a faint signal near the center of a lens galaxy does not provide sufficient evidence to prove that image D has been detected. It is possible that the signal may be explained by a lens model without an SMBH or that the emission may be from the lens galaxy itself.

In the former case, an analysis for plausible lens models of the probability of producing a faint image close to the center of the lens galaxy should typically be sufficient to clear up confusion. For example, a standard SIS model produces a faint secondary image in the rare cases in which the source impact parameter is just inside the Einstein radius. This image, in principle, could be confused with the image D tail of the models that we have discussed above. However, the probability of such a configuration in the SIS model is extremely low compared with that for the PM+CIS and PM+POW models.

In the latter case, structure in the signal could be used to rule out the possibility that the detected signal is emission from the foreground galaxy. Gravitational lenses produce a characteristic arcing in resolved images. In addition, structure in secondary images should be correlated with structure in the primary image.

As an example of the lensing that we expect because of an SMBH within a galaxy, we consider a radio jet QSO that is known to have nearby foreground galaxies. The radio and optical observations of 3C 9 are discussed in detail by Kronberg et al. (1991). This source lies at a redshift of $z_s = 2.012$ and has total flux of approximately 190 mJy, and its radio jets are approximately $10''$ across. A radio map of the source is shown in Figure 11, which was constructed from data provided by P. Kronberg. The especially interesting feature of 3C 9 is that there are two foreground galaxies near the jets. One (galaxy 1) lies approximately $10''$ from the jets and has an $R$-band magnitude of 19.5 and a measured redshift of $z_s = 0.254$. The other (galaxy 2) lies only about $3''$ from the jets and has an $R$-band magnitude of 23.0 and a less well determined redshift. We assume that the two galaxies lie at the same redshift ($z_s = 0.254$). Using a simple ray-tracing algorithm, we simulated the lensing that would occur from an SMBH in the center of each galaxy. Since the foreground galaxies are far from the radio source and in such cases the properties of image D are determined almost entirely by the SMBH without influence from the host galaxy.
potential, we are able to work under the simplifying approximation of treating the SMBH as an isolated PM. Figure 12 shows the image that would result from a $10^9 M_\odot$ SMBH in the center of galaxy 1. The total flux from this image is approximately 0.03 nJy. An SMBH with the same mass in galaxy 2 would produce a much brighter image, with total flux about 3 nJy. Figure 13 shows this case. These faint images of 3C 9 will not be detectable, even with the anticipated sensitivity of the SKA, and are presented merely as examples of how structure could be identified. The source 3C 9 therefore is not a realistic candidate for future SMBH lensing studies.

4.3.3. Mergers and Binary SMBHs

An interesting question arising from the hierarchical formation of SMBHs is that of how long the black holes of two merging galaxies take to coalesce. Indeed, if the timescale is long enough, we would expect some galaxies to contain binary SMBHs. Yu (2002) has carried out theoretical studies that determine how these merger timescales vary according to the type of host galaxy, its velocity dispersion, and the mass ratios of the SMBHs involved. The “surviving” binary systems (those whose merger time is larger than the Hubble time) are found to occur predominantly in galaxies that are not highly flattened and that have dispersion velocities greater than 90 km s$^{-1}$. In addition, SMBH pairs with equal masses are more likely to survive.

The separations of most of the remaining binary systems range from 0.01 to 10 pc. From equation (4), we calculate that for SMBHs with masses between $10^6$ and $10^9 M_\odot$, typical values of $\theta_{PM}$ correspond to physical distances between 10 and 400 pc. The binary separations therefore may be significant fractions of the SMBH Einstein radii, and binaries may be distinguishable in some lenses. Binary SMBHs with the widest anticipated separations may be sufficient to alter not only the properties of image D but also those of image C (and therefore the lensing statistics of these images), because for small source impact parameters image C is predicted to form typically within about 10 pc of the center of the lens (see Figs. 1 and 3). For large source impact parameters, however, image C does not exist and only image D is affected.

Figure 14 demonstrates the results of considering two $5 \times 10^8 M_\odot$ SMBHs in galaxy 2 of the 3C 9 example considered above. We calculate for these SMBHs that their Einstein radii correspond to 250 pc (using $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, and $H_0 = 70$ km s$^{-1}$). It is evident that, even for separations of a few pc, noticeable effects are visible in this scenario. A survey of a large
number of these faint lenses would provide insight not only into the distribution of the masses of SMBHs but also into whether the SMBHs occur as individual or binary systems and their correlation to the morphological type of the host galaxy.

It is also evident from theoretical simulations that, either through mergers or initial conditions, individual SMBHs may not be located precisely at the center of their host galaxies. If an SMBH has formed at a distance greater than approximately 100 pc (or more, depending on its mass) from the center of its host galaxy, Ullio et al. (2001, their Fig. 3) have predicted that dynamical friction is insufficient to produce spiral-in timescales shorter than the Hubble time. In addition, Brownian motion may cause an SMBH or binary pair to wander around the nucleus of a galaxy within a radius of order 0.01 pc (Merritt 2001). Observational evidence, on the other hand, suggests that in nearby galaxies SMBHs are within 0.1 pc of galaxy centers (Milosavljevic & Merritt 2001, their § 1). As was the case for binary systems, noncentrally located SMBHs may affect the magnification curves for small source impact parameters, and thus the lensing statistics for images C and D. However, they are not likely to significantly alter the properties of systems with large source impact parameters since in this limit the properties of image D approach those of an image from an isolated SMBH (see Appendix A.3 for a justification). Therefore, an off-center SMBH should not complicate detection or identification of an SMBH image in a two-image system with a large source impact parameter.

5. CONCLUSION

Observational measurements of the relationship between SMBHs and the properties of their host galaxies are an important method for probing theoretical hierarchical growth models. Gravitational lensing is a unique mechanism for acquiring this information in systems at cosmologically significant redshifts.

We have reviewed the calculations required to transform historically popular galactic lens models to account for the presence of SMBHs. The modified models, PM+CIS and PM+POW, exhibit an inner disk in the source plane where only two bright images are produced, surrounded by an annulus where two additional faint images are added near the center of the lens for a total of four images. For all source positions outside this region, the models predict one primary image and one highly demagnified image deep in the center of the lens, which traces very nearly the secondary image of an isolated SMBH. The relative cross sections of these three regions are very sensitive to galaxy core sizes and SMBH masses. We used these characteristic signatures to demonstrate the origin of the SMBH lensing effects reported in Rusin & Ma (2001), Keeton (2003), and Mao et al. (2001).

Determination of lensing cross sections over a range of parameter space and subsequent optical depth calculations revealed that, even for the most favorable lens configurations, observations of the highly demagnified fourth images will remain elusive until the next generation of radio telescopes are in use. Two scenarios, however, offer more immediate results. Since core images are detectable now, if galaxies tend to have large cores, deep VLBI observations (or EVLA) should make accessible a sufficiently large population of core images to provide meaningful statistics. On the other hand, if galaxies tend to have very small cores, then two-image lenses for which the secondary image is the highly demagnified SMBH image may already be observable. For any scenario, there is strong evidence suggesting that a targeted search today for new lenses, utilizing sky surveys such as FIRST to limit the number of candidate objects, should yield a precursory sample of two-image lenses that will, with improved technology, provide a significant number of third and fourth images. Finally, if these images are detected, they may even reveal binary SMBH systems.

We would like to thank Philipp Kronberg for providing the radio map of 3C 9. Support for this work was provided by the NSF through grant AST 00-71181.

APPENDIX A

MORE ON NONCIRCULARLY SYMMETRIC POTENTIALS

Many observed gravitational lenses do not produce simply two bright images, as a circularly symmetric potential would predict. Instead, four or more bright images are often observed. Two effective and common explanations are (1) that the lensing galaxies are...
Indeed not circularly symmetric but rather elliptical and (2) that the lensing galaxies lie in an external gravitational shear. In this appendix we provide a cursory analysis of how ellipticity and shear modify the results of the previous sections. In addition, we treat the case of an off-center SMBH in the large source impact parameter limit.

We have seen in the case of circularly symmetric potentials that, for large impact parameters ($\beta$), the magnification of image D is essentially independent of the lensing properties of the galaxy that contains the image. We show that neither ellipticity nor shear significantly alters the magnification of this image or the primary image, and therefore, the optical depths and statistics pertaining to observing these images calculated for the circularly symmetric potentials need not be altered. The properties of any other faint images, however, are complicated by the addition of the more complex potentials, and our previous calculations may not necessarily hold. Similarly, an off-center SMBH is shown not to alter the magnification of image D in the large source impact parameter limit, but other configurations and other images may be affected.

### A1. Ellipticity

It has been demonstrated (Evans & Hunter 2002) that an elliptical lensing potential derived from a CIS increases the maximum number of images produced from three to five. For our PM+CIS potential, considering ellipticity does not actually increase the maximum number of images but rather complicates the caustics in the source plane considerably (Mao et al. 2001). We expect the magnification of the SMBH image, if it exists, and of the primary image to be affected little by this change, but to demonstrate this quantitatively, we must return to the lens equation. Recalling from equation (1), but this time in two dimensions, the lens equation is

$$\beta = \theta - \nabla \Psi.$$  

(A1)

To describe a galaxy with an elliptical density distribution, we modify the circularly symmetric CIS lensing potential to

$$\Psi_{\text{elliptical}}(\theta_x, \theta_y) = \theta_{\text{SIS}} \sqrt{\theta^2_c + (1 - \epsilon) \theta^2_x + (1 + \epsilon) \theta^2_y}$$  

(A2)

(Narayan & Bartelmann 1996, their § 3.5.1). In this equation the variables $\theta_x$ and $\theta_y$ are the angular distances along the principal axes of the ellipse, and $\epsilon$ is the ellipticity of the potential. The cored elliptical lensing potential is not entirely realistic since, for large ellipticity, it results in dumbbell-shaped mass density contours. For small ellipticity, however, the contours are generally elliptical, and the potential is quite acceptable. Combining this with the PM potential yields the complete lensing potential for an elliptical galaxy with an SMBH:

$$\Psi(\theta_x, \theta_y) = \theta_{\text{SIS}} \sqrt{\theta^2_c + (1 - \epsilon) \theta^2_x + (1 + \epsilon) \theta^2_y} + \theta^2_{\text{PM}} \ln \sqrt{\theta^2_x + \theta^2_y}.$$  

(A3)

Using the lens equation, we find for the components of $\beta$

$$\beta_x = \theta_x \left[ 1 - \frac{\theta_{\text{SIS}}(1 - \epsilon) \theta^2_{\text{PM}} \theta^2_y}{\sqrt{\theta^2_c + (1 - \epsilon) \theta^2_x + (1 + \epsilon) \theta^2_y}} \right],$$  

(A4)

$$\beta_y = \theta_y \left[ 1 - \frac{\theta_{\text{SIS}}(1 + \epsilon) \theta^2_{\text{PM}} \theta^2_x}{\sqrt{\theta^2_c + (1 - \epsilon) \theta^2_x + (1 + \epsilon) \theta^2_y}} \right].$$  

(A5)

Since we are interested only in magnification of images for the present purposes, we can forgo completely solving the lens equation. For noncircularly symmetric potentials, the magnification of any image is given by

$$\mu^{-1} = \det A,$$  

(A6)

where $A$ is the Jacobian matrix that describes the lens properties (Schneider et al. 1992, their eq. [5.21]),

$$A = \frac{\partial \beta}{\partial \theta} = \begin{pmatrix} \partial_x \beta_x & \partial_x \beta_y \\ \partial_y \beta_x & \partial_y \beta_y \end{pmatrix}.$$  

(A7)

Unfortunately, writing out $A$ and calculating its determinant is not particularly illuminating, as it is difficult to see past the algebra. As a simplification, we consider the limiting cases of $\theta \ll \theta_{\text{PM}} \ll \theta_c$ and the opposite extreme $\theta \gg \theta_{\text{PM}}$ and $\theta_{\text{PM}} \ll \theta_c$. The former happens to correspond to the region of the image plane where SMBH images are expected to be observed because, from our earlier analysis, we might reasonably assume $\theta_c/\theta_{\text{SIS}} \simeq 10^{-4}$, $\theta_{\text{PM}}/\theta_{\text{SIS}} \simeq 10^{-2}$, and, for SMBH images, $\theta/\theta_{\text{SIS}} \simeq 10^{-3}$. The latter corresponds to the region where the primary image is observed, since $\theta/\theta_{\text{SIS}} \gtrsim 1$ for such images, and the other parameters, of course, do not change.
For the inner region of the image plane, where the SMBH image is expected to form, the components of $\beta$ reduce to

$$\lim_{\sigma \ll \sigma_{\text{PM}}} \beta_x = \theta_x \left( -\frac{\theta_{\text{PM}}^2}{\theta_x^2 + \theta_y^2} \right),$$

$$\lim_{\sigma \ll \sigma_{\text{PM}}} \beta_y = \theta_y \left( -\frac{\theta_{\text{PM}}^2}{\theta_x^2 + \theta_y^2} \right).$$

(A8)

(A9)

and thus, the magnification of an image in this region is given by

$$\mu^{-1}_{\text{SMBH}} = \det \mathbf{A} \approx -\frac{\theta_{\text{PM}}^4}{\left(\theta_x^2 + \theta_y^2\right)^2} = -\frac{\theta_{\text{PM}}^4}{\theta^4}. \quad (A10)$$

The magnification of these images would therefore not be affected by the ellipticity of the lens galaxy. It should be noted, however, that this approximation breaks down in this region if $\theta_v/\theta_{\text{SIS}}$ is sufficiently small ($\approx \theta^2/\theta_{\text{PM}}^2$).

For the outer region of the image plane, where a primary image forms, the components of $\beta$ reduce to another pair of relatively simple expressions,

$$\lim_{\sigma \gg \sigma_{\text{PM}}} \beta_x = \theta_x \left[ 1 - \frac{\theta_{\text{SIS}}(1 - \epsilon)}{\sqrt{(1 - \epsilon)\theta_x^2 + (1 + \epsilon)\theta_y^2}} \right],$$

$$\lim_{\sigma \gg \sigma_{\text{PM}}} \beta_y = \theta_y \left[ 1 - \frac{\theta_{\text{SIS}}(1 + \epsilon)}{\sqrt{(1 - \epsilon)\theta_x^2 + (1 + \epsilon)\theta_y^2}} \right].$$

(A11)

(A12)

Along the axes, $\hat{\Theta}_x$ and $\hat{\Theta}_y$, the magnification in this region is given by

$$\mu^{-1}_{\text{primary}} = \det \mathbf{A} \approx \begin{cases} 1 - \frac{\theta_{\text{SIS}}(1 + \epsilon)}{\theta_x \sqrt{1 - \epsilon}}, & \theta_y = 0, \\ 1 - \frac{\theta_{\text{SIS}}(1 - \epsilon)}{\theta_y \sqrt{1 + \epsilon}}, & \theta_x = 0. \end{cases} \quad (A13)$$

We can consider that, in effect, the addition of ellipticity simply modifies the value of $\theta_{\text{SIS}}$, depending on the position of the primary image. Since the value of $\epsilon$ is at the very greatest 1 and is usually much smaller, we can ignore this contribution as a minor correction.

A2. GRAVITATIONAL SHEAR

The lensing galaxy might be residing, on the other hand, in the presence of an external gravitational shear. It has been demonstrated (Evans & Hunter 2002) that the addition of an external shear to a CIS potential increases the maximum number of images produced from three to five, just as did the consideration of ellipticity. For our PM+CIS potential, the maximum number of images increases again to six; however, four images are a more probable outcome for most configurations, as shown in Figure 15. Again, we expect the SMBH image and the primary image to be affected little by this change. Returning to the two-dimensional lens equation, we begin by characterizing an external shear with

$$\Psi_{\text{shear}}(\theta_x, \theta_y) = \frac{\gamma}{2} \left( \theta_x^2 - \theta_y^2 \right).$$

(A14)

(Narayan & Bartelmann 1996, their § 3.5.2). Here $\gamma$ is the magnitude of the shear and, as in the case of the ellipticity, the angular distances $\theta_x$ and $\theta_y$ are measured along the axes of the shear. To consider how this affects the images, we begin by adding this contribution to the potential for a PM+CIS model, giving

$$\Psi(\theta_x, \theta_y) = \theta_{\text{SIS}} \sqrt{\theta_x^2 + \theta_y^2 + \theta_{\text{PM}}^2} \ln \left( \frac{\sqrt{\theta_x^2 + \theta_y^2}}{\theta_{\text{PM}}} \right) + \frac{\gamma}{2} \left( \theta_x^2 - \theta_y^2 \right).$$

(A15)
From the lens equation, we again can calculate the components of the vector $\beta$:

$$\beta_x = \theta_x \left( 1 - \frac{\theta_{\text{SIS}}}{\sqrt{\theta_x^2 + \theta_y^2 + \theta_c^2}} - \frac{\theta_{\text{PM}}^2}{\theta_x^2 + \theta_y^2 + \theta_c^2} - \gamma \right),$$  \hspace{1cm} (A16)

$$\beta_y = \theta_y \left( 1 - \frac{\theta_{\text{SIS}}}{\sqrt{\theta_x^2 + \theta_y^2 + \theta_c^2}} - \frac{\theta_{\text{PM}}^2}{\theta_x^2 + \theta_y^2 + \theta_c^2} + \gamma \right),$$  \hspace{1cm} (A17)

and for convenience we define an angle $\omega$ that measures the position of the image with respect to the positive $\hat{\Theta}_x$ axis such that

$$\theta_x = \theta \cos \omega,$$  \hspace{1cm} (A18)

$$\theta_y = \theta \sin \omega.$$  \hspace{1cm} (A19)

Calculating the determinant of the Jacobian matrix $A$ and grouping the terms by powers of $\gamma$ yields, after some simplification,

$$\det A = \left[ 1 - \frac{\theta_{\text{PM}}^4}{\theta_x^4} + \frac{\theta_{\text{SIS}}^2}{\theta_x^2 + \theta_y^2 + \theta_c^2} - \frac{\theta_{\text{SIS}}^2}{\theta_x^2 + \theta_y^2 + \theta_c^2} - \frac{\theta_{\text{PM}}^2}{\theta_x^2 + \theta_y^2 + \theta_c^2} \right] + \gamma \cos 2\omega \left[ 2\frac{\theta_{\text{PM}}^2}{\theta_x^2 + \theta_y^2 + \theta_c^2} + \frac{\theta_{\text{SIS}}}{\theta_x^2 + \theta_y^2 + \theta_c^2} \right] - \gamma^2.$$  \hspace{1cm} (A20)

Following the same approximations here as we did for the elliptical case, we consider the two regions of the image plane where $\theta_x \ll \theta_{\text{PM}} \ll \theta_c$ and $\theta_x \gg \theta_{\text{PM}} \ll \theta_c$. For the inner region of the image plane where an SMBH image is expected to be observed and $\theta_x \ll \theta_{\text{PM}} \ll \theta_c$, we substitute, as we did before, the values $\theta_c/\theta_{\text{SIS}} \simeq 10^{-1}$, $\theta_{\text{PM}}/\theta_{\text{SIS}} \simeq 10^{-2}$, and $\theta/\theta_{\text{SIS}} \simeq 10^{-3}$. In this limit, the above expression reduces to

$$\lim_{\theta_x \ll \theta_{\text{PM}} \ll \theta_c} \left( \det A \right) \rightarrow -\frac{\theta_{\text{PM}}^4}{\theta_x^4} + 2\gamma \cos 2\omega \frac{\theta_{\text{PM}}^2}{\theta_x^2} - \gamma^2.$$  \hspace{1cm} (A21)
Typical values of the shear parameter are generally of order $\gamma \lesssim 0.1$ (Casertano et al. 2003). Thus, it is sufficient to keep only the leading term of the expression for the magnification:

$$\mu_{\text{SMBH}}^{-1} \approx -\frac{\theta_{\text{PM}}^4}{\theta^4}. \quad (A22)$$

This is identical to the elliptical result for this region, and we conclude that shear does not substantially alter the magnification of an SMBH image for such a lens configuration.

For the outer region of the image plane, where a primary image forms and $\theta \gg \theta_{\text{PM}}$ and $\theta_{\text{PM}} \ll \theta$, we proceed by repeating the above simplification, but with typical values of $\theta/\theta_{\text{SIS}} \approx 1$. The determinant of $A$ is then reduced to

$$\lim_{\theta \gg \theta_{\text{SIS}}} \left( \det A \right) \rightarrow 1 - \frac{\theta_{\text{SIS}}}{\theta} \cos 2\omega \left( \frac{\theta_{\text{SIS}}}{\theta} \right)^2 - \gamma^2; \quad (A23)$$

and, when we use $\gamma \lesssim 0.1$,

$$\mu_{\text{primary}}^{-1} \approx 1 - \frac{\theta_{\text{SIS}}}{\theta} (1 - \cos 2\omega). \quad (A24)$$

The angular dependence in this equation makes sense, because symmetry dictates that two points separated by $180^\circ$ must have the same magnification. As was the case for the elliptical potential, this can be considered a small correction to $\theta_{\text{SIS}}$, and therefore, the external shear has not altered the ratio of the magnification of an SMBH image to a primary image in these limiting cases. Figure 15 plots the image magnifications for a representative system with $\gamma = 0.1$ as a function of source position for a path extending away from the center of the lens. Comparison with Figure 5 (top left), which has no shear but is otherwise the same configuration, demonstrates the minimal effect of shear on image magnifications for both core images in this case.

### A3. OFF-CENTER SMBHs

Another possible departure from circular symmetry, raised in §4.3.3, is that the SMBH may not be located in the center of the host galaxy. Returning to the lensing potential given in equation (6), we can explore the consequences of such an occurrence by inserting an offset, $\theta_0$, between the PM and the galaxy potential,

$$\Psi = \theta_{\text{SIS}} \sqrt{\left| \theta \right|^2 + \theta_{\text{PM}}^2 \ln |\theta - \theta_0|. \quad (A25)$$

This yields the lens equation

$$\beta = \theta - \theta_{\text{SIS}} \frac{\theta}{\sqrt{\left| \theta \right|^2 + \theta_{\text{PM}}^2 \ln |\theta - \theta_0|}} - \theta_{\text{PM}}^2 \frac{\theta - \theta_0}{|\theta - \theta_0|^2}. \quad (A26)$$

Following a similar approximation here as we did for the elliptical and shear cases, we consider the region of the image plane that corresponds to large source impact parameters. In this limit, we expect an image to form near the PM, and thus, $|\theta - \theta_0|$ is small. The last term in the lens equation therefore dominates, and we can simplify the lens equation to

$$\beta \approx -\theta_{\text{PM}}^2 \frac{\theta - \theta_0}{|\theta - \theta_0|^2} \quad (A27)$$

and rearrange to solve for the image position,

$$\theta_{\text{SMBH}} \approx \theta_0 - \theta_{\text{PM}}^2 \frac{\beta}{|\beta|^2}. \quad (A28)$$

The position of the SMBH image as a function of the source impact parameter is merely offset from the original location by the same amount as the SMBH itself. The magnification of the image,

$$\mu_{\text{SMBH}} \approx -\frac{|\theta - \theta_0|^4}{\theta_{\text{PM}}^4}, \quad (A29)$$

also has the familiar form, but with respect to the new SMBH location.

### A4. REMARKS

Neither ellipticity nor external shear (nor, under proper circumstances, an off-center SMBH) significantly changes the probability of observing lensing by an SMBH. The cross sections, optical depths, and statistics that we calculated for the circularly symmetric models therefore remain a reasonably good representation even under these more general circumstances. Other core images,
however, do not necessarily lie in the limiting-case regions of the image plane and may be affected by ellipticity and shear. We cannot assume that the results of the circularly symmetric calculations can be applied to deduce their statistics for these configurations, but we postpone further analysis for future work.

REFERENCES

Alexander, T., & Loeb, A. 2001, ApJ, 551, 223
Augusto, P., & Wilkinson, P. N. 2001, MNRAS, 320, L40
Baes, M., Buyle, P., Hau, G. K. T., & Dejonghe, H. 2003, MNRAS, 341, L44
Becker, R. H., White, R. L., & Helfand, D. J. 1995, ApJ, 450, 559
Bolton, A. S., Burles, S., Schlegel, D. J., Eisenstein, D. J., & Brinkmann, J. 2004, AJ, 127, 1860
Browne, I. W. A., et al. 2003, MNRAS, 341, 13
Burkert, A., & Silk, J. 2001, ApJ, 554, L151
Byun, Y.-I., et al. 1996, AJ, 111, 1889
Casertano, S., Ratmatunga, K. U., & Griffiths, R. E. 2003, ApJ, 598, L71
Chen, D. 2003, ApJ, 587, L55
Chen, G. H., & Hewitt, J. N. 1993, AJ, 106, 1719
Chen, G. H., Kochanek, C. S., & Hewitt, J. N. 1995, ApJ, 447, 62
Condon, J. J., Cotton, W. D., Greisen, E. W., Yin, Q. F., Perley, R. A., Taylor, G. B., & Broderick, J. J. 1998, AJ, 115, 1693
Di Matteo, T., Croft, R. A. C., Springel, V., & Hernquist, L. 2003, ApJ, 593, 56
Egami, E., Neugebauer, G., Soifer, B. T., Matthews, K., Ressler, M., Becklin, E. E., Murphy, T. W., & Dale, D. A. 2000, ApJ, 535, 561
Evans, N. W., & Hunter, C. 2002, ApJ, 575, 68
Faber, S. M., et al. 1997, AJ, 114, 1771
Ferrarese, L. 2001, ApJ, 551, 223
Garrett, M. A., Wrobel, J. M., & Morganti, R. 2003, NewA Rev., 47, 385
Gebhardt, K., et al. 2000, ApJ, 539, L13
Graham, A. W., Erwin, P., Caon, N., & Trujillo, I. 2001, ApJ, 563, L11
Hogg, D. W. 1999, preprint (astro-ph/9905116)
Ibata, R. A., Lewis, G. F., Irwin, M. J., Lehár, J., & Totten, E. J. 1999, AJ, 118, 1922
Ivezic, Ž., et al. 2002, AJ, 124, 2364
Kauffmann, G., & Haehnelt, M. 2000, MNRAS, 311, 576
Keeton, C. R. 2003, ApJ, 582, 17
Keeton, C. R., & Madau, P. 2001, ApJ, 549, L25
Kormendy, J., Gebhardt, K., & Richstone, D. 2000, BAAS, 196, 21.22
Kroonberg, P. P., Dyer, C. C., Burbidge, E. M., & Junkkarinen, V. T. 1991, ApJ, 367, L1
Laor, A. 2001, ApJ, 553, 677
Lauer, T. R., et al. 1995, AJ, 110, 2622
MacMillan, J. D., & Henriksen, R. N. 2002, ApJ, 569, 83
Magorrian, J., et al. 1998, AJ, 115, 2285
Mao, S., Witt, H. J., & Koopmans, L. V. E. 2001, MNRAS, 323, 301
McLure, R. J., & Dunlop, J. S. 2001, MNRAS, 327, 199
Merritt, D. 2001, ApJ, 556, 245
Merritt, D., & Ferrarese, L. 2001, in ASP Conf. Ser. 249, The Central Kiloparsec of Starbursts and AGN: The La Palma Connection, ed. J. H. Knappen, J. E. Beckman, I. Shlosman, & T. J. Mahoney (San Francisco: ASP), 335
Milosavljević, M., & Merritt, D. 2001, ApJ, 563, 34
Muñoz, J. A., Falco, E. E., Kochanek, C. S., Lehár, J., & Mediavilla, E. 2003, ApJ, 594, 684
Narayan, R., & Bartelmann, M. 1996, preprint (astro-ph/9606001)
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Ofek, E. O., Rix, H., & Maoz, D. 2003, MNRAS, 343, 639
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Rusin, D., & Ma, C. 2001, ApJ, 549, L33
Schechter, P. 1976, ApJ, 203, 297
Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses (Berlin: Springer)
Silk, J., & Rees, M. J. 1998, A&A, 331, L1
Stoughton, C., et al. 2002, AJ, 123, 485
Tofani, G. 2003, Mem. Soc. Astron. Italiana Suppl., 3, 292
Tremaine, S., et al. 2002, ApJ, 574, 740
Turner, E. L., Ostriker, J. P., & Gott, J. R. 1984, ApJ, 284, 1
Ullio, P., Zhao, H., & Kamionkowski, M. 2001, Phys. Rev. D, 64, 043504
Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
Wambagans, J., & Paczynski, B. 1992, ApJ, 397, L1
White, R. L., Becker, R. H., Helfand, D. J., & Gregg, M. D. 1997, ApJ, 475, 479
Wilkinson, P. N., et al. 2001, Phys. Rev. Lett., 86, 584
Windhorst, R. A., Fomalont, E. B., Partridge, R. B., & Lowenthal, J. D. 1993, ApJ, 405, 498
Winn, J. N., Rusin, D., & Kochanek, C. S. 2003, ApJ, 587, 80
———. 2004, Nature, 427, 613
Winn, J. N., et al. 2002, AJ, 123, 10
Wisotzki, L., Christlieb, N., Bade, N., Beckmann, V., Köhler, T., Vanelle, C., & Reimers, D. 2000, A&A, 358, 77
Wyithe, J. S. B., & Loeb, A. 2002, ApJ, 581, 886
Yu, Q. 2002, MNRAS, 333, 935
Zhao, H. 1997, MNRAS, 287, 525