Properties of Elastic Waves in a non-Newtonian (Maxwell) Fluid-Saturated Porous Medium

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Abstract. The present study investigates novelties brought about into the classic Biot’s theory of propagation of elastic waves in a fluid-saturated porous solid by inclusion of non-Newtonian effects that are important, for example, for hydrocarbons. Based on our previous results (Tsiklauri & Beresnev: 2001, Phys. Rev. E, 63, 046304), we have investigated the propagation of rotational and dilatational elastic waves, through calculating their phase velocities and attenuation coefficients as a function of frequency. We found that the replacement of an ordinary Newtonian fluid by a Maxwell fluid in the fluid-saturated porous solid results in: (a) an overall increase of the phase velocities of both the rotational and dilatational waves. With the increase of frequency these quantities tend to a fixed, higher, as compared to the Newtonian limiting case, level which is not changing with the decrease of the Deborah number \( \alpha \). (b) the overall decrease of the attenuation coefficients of both the rotational and dilatational waves. With the increase of frequency these quantities tend to a progressively lower, as compared to the Newtonian limiting case, levels as \( \alpha \) decreases. (c) Appearance of oscillations in all physical quantities in the deeply non-Newtonian regime.

Keywords: Fluid-saturated porous medium, Biot’s theory, Non-Newtonian Fluids, Maxwell Fluid, Elastic waves, Phase Velocities, Attenuation Coefficients.

1. Introduction

Apart from fundamental interest, there are at least three major reasons to study the dynamics of fluid in porous media under oscillatory pressure gradient and oscillating pore walls, as well as to investigate propagation of elastic waves in porous media.

First, in petroleum geophysics, regional exploration seismology needs direct methods of discovering oil-filled bodies of rock, and these should be based on models of propagation of elastic waves in porous media with realistic fluid rheologies (Carcione & Quiroga-Goode, 1996).

Second, the investigation of the dynamics of fluid in porous media under oscillatory pressure gradient is of prime importance for the recently emerged technology of acoustic stimulation of oil reservoirs.
It is known that the natural pressure in an oil reservoir generally yields no more than approximately 10 percent oil recovery. The residual oil is difficult to produce due to its naturally low mobility, and the enhanced oil recovery operations are used to increase production. It has been experimentally proven that there is a substantial increase in the net fluid flow through porous space if the latter is treated with elastic waves (Beresnev & Johnson, 1994), (Drake & Beresnev, 1999).

Third, in the environment conservation, treatment of ground water aquifers contaminated by organic liquids, such as hydrocarbons, by elastic waves proved to be successful for quick and efficient clean up (Beresnev & Johnson, 1994), (Drake & Beresnev, 1999).

A quantitative theory of propagation of elastic waves in a fluid-saturated porous solid was formulated in the classic papers by Biot (Biot, 1956a,b). One of the major findings of Biot’s work was that there is a breakdown in Poiseuille flow above a certain characteristic frequency specific to the fluid-saturated porous material. Biot theoretically studied this phenomenon by considering the flow of a viscous fluid in a tube with longitudinally oscillating walls under an oscillatory pressure gradient. In Biot’s theory, the two-phase material is considered as a continuum and the microscopic, pore-level effects are ignored. As a reminder, the theory assumes that: (a) the wavelength is large with respect to the dimensions of pores in order to make continuum mechanics applicable; this also implies that scattering dissipation is negligible; (b) the displacements are small, therefore the macroscopic strain tensor is related to them by the lowest second-order approximation; (c) the liquid phase is continuous, such that the pores are connected and isolated pores are treated as part of solid matrix; and (d) the permeability is isotropic and the medium is fully saturated.

Biot demonstrated the existence of the two kinds of compressional waves in a fluid-saturated porous medium: the fast wave for which the solid and fluid displacements are in phase, and the slow wave for which the displacements are out of phase. At low frequencies, the medium does not support the slow wave as it becomes diffusive. On the other hand, at high frequencies, tangential slip takes place, inertial effects dominate, and the slow wave becomes activated.

Biot’s theory can be used to describe interaction of fluid-saturated solid with the sound for a classic Newtonian fluid; however, oil and other hydrocarbons often exhibit significant non-Newtonian behavior. In paper I (Tsiklauri & Beresnev, 2001), we have incorporated non-Newtonian effects into the classical theory of Biot (Biot, 1956a,b). Using the recent results of del Rio et al. (1998), who presented a study of enhancement in the dynamic response of a viscoelastic (Maxwell) fluid.
flowing through a stationary (non-oscillating) tube under the effect of an oscillatory pressure gradient, we have combined their theory with the effect of the acoustic oscillations of the walls of the tube introduced by Biot (Biot, 1956a,b), thus providing a complete description of the interaction of Maxwell fluid, filling the pores, with acoustic waves. We have generalized the expression for the function $F(\kappa)$, which measures the deviation from Poisseuille flow friction as a function of frequency parameter $\kappa$ (Tsiklauri & Beresnev, 2001). As a next step, in the present work we investigate the propagation of rotational and dilatational elastic waves through the porous medium filled with Maxwell fluid, by calculating their phase velocities and attenuation coefficients as a function of frequency.

This paper is organized as follows: we formulate theoretical basis for our numerical calculations in section 2. In sections 3 and 4 we study numerically properties of the rotational and dilatational elastic waves, respectively, and, finally, in Section 5 we close with a discussion of our main results.

2. Theory

The theory of propagation of elastic waves in a fluid-saturated porous solid was formulated by Biot (Biot, 1956a,b). He demonstrated that the general equations which govern propagation of rotational and dilatational high-frequency waves in a fluid-saturated porous medium are the same as in the low-frequency range provided the viscosity is replaced by its effective value as a function of frequency. In practice, it means replacing the resistance coefficient $b$ by $bF(\kappa)$.

The equations describing dynamics of the rotational waves are (Biot, 1956a,b)

\[
\frac{\partial^2}{\partial t^2}(\rho_{11}\vec{\omega} + \rho_{12}\vec{\Omega}) + bF(\kappa) \frac{\partial}{\partial t}(\vec{\omega} - \vec{\Omega}) = N\nabla^2 \vec{\omega}, \tag{1}
\]

\[
\frac{\partial^2}{\partial t^2}(\rho_{12}\vec{\omega} + \rho_{22}\vec{\Omega}) - bF(\kappa) \frac{\partial}{\partial t}(\vec{\omega} - \vec{\Omega}) = 0, \tag{2}
\]

where, $\rho_{11}, \rho_{12}$ and $\rho_{22}$ are mass density parameters for the solid and fluid and their inertia coupling; $\vec{\omega} = \text{curl} \, \vec{u}$ and $\vec{\Omega} = \text{curl} \, \vec{U}$ describe rotations of solid and fluid with $\vec{u}$ and $\vec{U}$ being their displacement vectors, while the rigidity of the solid is represented by the modulus $N$. Substitution of a plane rotational wave of the form

\[
\omega = C_1e^{i(\lambda x + \chi t)}, \quad \Omega = C_2e^{i(\lambda x + \chi t)}, \tag{3}
\]
into Eqs. (1) and (2) allows us to obtain a characteristic equation

$$
\frac{Nl^2}{\rho \chi^2} = E_r - iE_i,
$$

(4)

where \( l \) is wavenumber, \( \chi = 2\pi f \) is wave cyclic frequency, \( \rho = \rho_{11} + 2\rho_{12} + \rho_{22} \) is the mass density of the bulk material.

The real and imaginary parts of Eq.(4) can be written as

$$
E_r = \frac{(\gamma_{11}\gamma_{22} - \gamma_{12}^2)(\gamma_{22} + \epsilon_2) + \gamma_{22}\epsilon_2 + \epsilon_1^2 + \epsilon_2^2}{(\gamma_{22} + \epsilon_2)^2 + \epsilon_1^2},
$$

(5)

and

$$
E_i = \frac{\epsilon_1(\gamma_{12} + \gamma_{22})^2}{(\gamma_{22} + \epsilon_2)^2 + \epsilon_1^2},
$$

(6)

where \( \gamma_{ij} = \rho_{ij}/\rho \), \( \epsilon_1 = (\gamma_{12} + \gamma_{22})/(f_c/f) \ Re[F(\kappa)] = (\gamma_{12} + \gamma_{22})(f_c/f) \ Re[F(\delta \sqrt{f/f_c})] \), \( \epsilon_2 = (\gamma_{12} + \gamma_{22})/(f_c/f) \ Im[F(\kappa)] = (\gamma_{12} + \gamma_{22})(f_c/f) \ Im[F(\delta \sqrt{f/f_c})] \). The function \( F(\kappa) \) was written here more conveniently as a function of frequency \( f \), i.e. \( F(\kappa) = F(\delta \sqrt{f/f_c}) \) (Biot, 1956a,b), where \( \delta \) is a factor dependent on pore geometry. For the hollow cylinder-like pores, \( \delta = \sqrt{8} \) (Biot, 1956a,b) and we use this value throughout the paper. \( f_c \) is the critical frequency above which the Poiseuille flow breaks down, and it equals \( b/(2\pi \rho_2) = b/(2\pi \rho(\gamma_{12} + \gamma_{22})) \).

In order to obtain phase velocity and attenuation coefficient of the rotational waves, we put \( l = Re[l] + iIm[l] \). Thus, the phase velocity is then \( v_r = \chi/|Re[l]| \). Introducing a reference velocity as \( V_r = \sqrt{N/\rho} \), we obtain the dimensionless phase velocity as

$$
\frac{v_r}{V_r} = \frac{\sqrt{2}}{\left[\sqrt{E_i^2 + E_i^2 + E_r}\right]^{1/2}}.
$$

(7)

To obtain the attenuation coefficient of the rotational waves, we introduce a reference length, \( L_r \), defined as \( L_r = V_r/(2\pi f_c) \). The length \( x_a \) represents the distance over which the rotational wave amplitude is attenuated by a factor of \( 1/e \). Therefore we can construct the dimensionless attenuation coefficient as \( L_r/x_a \),

$$
\frac{L_r}{x_a} = \frac{f}{f_c} \left[\sqrt{E_i^2 + E_i^2 - E_r}\right]^{1/2}.
$$

(8)

The equations describing dynamics of the dilatational waves are (Biot, 1956a,b)

$$
\nabla^2(Pe + Q\epsilon) = \frac{\partial^2}{\partial t^2} ((\rho_{11}e + \rho_{12}\epsilon) + bF(\kappa)) \frac{\partial}{\partial t}(e - \epsilon),
$$

(9)
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\[ \nabla^2 (Q\varepsilon + R\varepsilon) = \frac{\partial^2}{\partial t^2} (\rho_{12}\varepsilon + \rho_{22}\varepsilon) - b F(\kappa) \frac{\partial}{\partial t} (e - \varepsilon), \]  

(10)

where, \( P, Q \) and \( R \) are the elastic coefficients, \( e = \text{div } \vec{u} \) and \( \varepsilon = \text{div } \vec{U} \) are the divergence of solid and fluid displacements. Again, substitution of a plane dilatational wave of the form

\[ e = C_1 e^{i(lx + \chi t)}, \quad \varepsilon = C_2 e^{i(lx + \chi t)}, \]  

(11)

into Eqs.(9) and (10) allows us to obtain a characteristic equation

\[ (z - z_1)(z - z_2) + iM(z - 1) = 0, \]  

(12)

where \( z = l^2 V_c^2 / \chi^2, \) \( V_c^2 = (P + R + 2Q)/\rho \) represents the velocity of a dilatational wave when the relative motion between fluid and solid is absent, \( z_{1,2} = V_{c,1}^2 / V_{1,2}^2 \) with \( V_{1,2} \) being the velocities of the purely elastic waves with subscripts 1,2 referring to the two roots of Eq.(12), and finally \( M = (\epsilon_1 + i\epsilon_2)/ (\sigma_{11}\sigma_{22} - \sigma_{21}^2) \) with \( \sigma_{11} = P/(P + R + 2Q), \) \( \sigma_{22} = R/(P + R + 2Q) \) and \( \sigma_{12} = Q/(P + R + 2Q). \)

Eq.(12) has two complex roots \( z_I \) and \( z_{II}. \) Phase velocities of the two kinds of dilatational waves can be defined as

\[ \frac{v_I}{V_c} = \frac{1}{\text{Re}[\sqrt{z_I}]}, \quad \frac{v_{II}}{V_c} = \frac{1}{\text{Re}[\sqrt{z_{II}}]}, \]  

(13)

while the corresponding attenuation coefficients can be also introduced as

\[ \frac{L_c}{x_I} = \text{Im}[\sqrt{z_I}] f_{cI}, \quad \frac{L_c}{x_{II}} = \text{Im}[\sqrt{z_{II}}] f_{cII}. \]  

(14)

In paper I, we generalized Biot’s expression for \( F(\kappa) \) to the case of a non-Newtonian (Maxwell) fluid, which reads

\[ F(\kappa) = -\frac{1}{4} \frac{\kappa \sqrt{i + \kappa^2/\alpha}}{1 - i\kappa^2/\alpha} \left[ J_1(\kappa \sqrt{i + \kappa^2/\alpha})/J_0(\kappa \sqrt{i + \kappa^2/\alpha}) \right]^{1} \]  

(15)

Here, \( \kappa = a\sqrt{\omega/\nu} \) is the frequency parameter, \( a \) is the radius of the pore, \( \nu = \eta/\rho \) is the ratio of the viscosity coefficient to the fluid mass density, \( J_0 \) and \( J_1 \) are the Bessel functions, and, finally, \( \alpha \) denotes the Deborah number del Rio et al. (1998), which is defined as the ratio of the characteristic time of viscous effects \( t_v = a^2/\nu \) to the relaxation time \( t_m, \) i.e., \( \alpha = t_v/t_m = a^2/(\nu t_m). \)

Eq.(15) was derived by solving the equations of incompressible hydrodynamics, namely, the continuity equation, linearized momentum
equation, and rheological equation of a Maxwell fluid, in the frequency domain for a cylindrical tube whose walls oscillate harmonically in time. By calculating the ratio of the total friction force exerted on the tube wall to the average velocity of a Maxwell fluid, and noting that \( F(\kappa) \) is proportional to this ratio, we generalized the classical result obtained by Biot (see details in Paper I).

As noted in del Rio et al. (1998), the value of the parameter \( \alpha \) determines in which regime the system resides. Beyond a certain critical value (\( \alpha_c = 11.64 \)), the system is dissipative, and viscous effects dominate. On the other hand, for small \( \alpha (\alpha < \alpha_c) \), the system exhibits viscoelastic behavior which we call the non-Newtonian regime. Note, that the Newtonian flow regime can be easily recovered from Eq. (15) by putting \( \alpha \rightarrow \infty \).

In order to investigate the novelties brought about into classical Biot’s theory of propagation of elastic waves in porous medium (Biot, 1956a,b) by the inclusion of non-Newtonian effects, we have studied the full parameter space of the problem. We have calculated the normalized phase velocities and attenuation coefficients for both rotational and dilatational waves using our general expression for \( F(\kappa) \) given by Eq. (15).

3. Numerical Results for Propagation of Rotational Waves

In all our numerical calculations, we have used polynomial expansions of \( J_0 \) and \( J_1 \) with absolute error not exceeding \( 10^{-6} \% \). Thus, our calculation results are accurate to this order. Also, in order to catch a true oscillatory structure of our solutions (see below), number of data points in all our plots is 10000, as opposed to paper I where only 100 points per curve were taken.

In all forthcoming results, we calculate phase velocities and attenuation coefficients for the case 1 from Table I taken from (Biot, 1956b), which is \( \sigma_{11} = 0.610, \sigma_{22} = 0.305, \sigma_{12} = 0.043, \gamma_{11} = 0.500, \gamma_{22} = 0.500, \gamma_{12} = 0, z_1 = 0.812 \), and \( z_2 = 1.674 \).

We calculated normalized phase velocity of the plane rotational waves, \( v_r/V_r \), and the attenuation coefficient \( L_r/x_a \) using our more general expression for \( F(\kappa) \) (Maxwell fluid filling the pores) given by Eq. (15).

In Fig. 1 we plot phase velocity \( v_r/V_r \) as a function of frequency for the three cases: the thick curve corresponds to \( \alpha \rightarrow \infty \) (Newtonian limit), the dashed curve corresponds to a slightly sub-critical value of \( \alpha = 10 \) (recall that \( \alpha_c = 11.64 \)) and thin solid curve corresponds to the case of \( \alpha = 1 \). Note that the \( \alpha \rightarrow \infty \) case perfectly reproduces the
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For $\alpha = 10$ we notice a deviation from the classic Newtonian behavior in the form of overall increase of phase velocity and appearance of small oscillations on the curve, which means that we have entered the non-Newtonian regime. Note that when $\alpha = 1$ the phase velocity settles at somewhat higher value and this onset happens already for smaller frequencies than in the case of Newtonian fluid. Also, much more pronounced oscillatory structure of the solution can be observed.

Fig. 2 shows the attenuation coefficient $L/r$ as a function of frequency for the three values of $\alpha$: the thick curve corresponds to $\alpha \to \infty$ (Newtonian limit), the dashed curve corresponds to a slightly sub-critical value of $\alpha = 10$ and thin solid curve corresponds to the case of $\alpha = 1$. Note that $\alpha \to \infty$ case coincides with curve 1 in Fig. 6 from (Biot, 1956b). For $\alpha = 10$, there is a noticeable deviation from the classic Newtonian behavior in the form of overall decrease of the attenuation coefficient and appearance of small oscillations on the curve indicating that the wave has entered the non-Newtonian regime. For $\alpha = 1$, the attenuation coefficient settles at a somewhat lower value, and this happens already for smaller frequencies than in the case of Newtonian fluid. Also, much more pronounced oscillatory structure of the solution can be noticed.

4. Numerical Results for Propagation of Dilatational Waves

We calculated normalized phase velocities of the plane dilatational waves, $v_I/V_c$ and $v_{II}/V_c$, and the attenuation coefficients $L_c/x_I$ and $L_c/x_{II}$ using our more general expression for $F(\kappa)$ (Maxwell fluid filling the pores) given by Eq. (15).

In Fig. 3 we plot phase velocity $v_I/V_c$ as a function of frequency for the case of $\alpha \to \infty$, in order to recover the Newtonian limit obtained by Biot. Note that this case reproduces curve 1 in Fig. 11 from (Biot, 1956b).

Fig. 4 shows phase velocity $v_I/V_c$ as a function of frequency for the case of $\alpha = 1$, corresponding to the deeply non-Newtonian regime. We notice again appearance of an oscillatory structure of the solution. Also, phase velocity $v_I/V_c$ settles at a somewhat higher value than in the Newtonian case, and this happens already for smaller frequencies.

In Fig. 5 we plot phase velocity $v_{II}/V_c$ as a function of frequency for the cases of $\alpha \to \infty$ and $\alpha = 1$. Note that the case of $\alpha \to \infty$ (thick solid line) perfectly reproduces the Newtonian limit obtained by Biot curve 1 in Fig. 12 from (Biot, 1956b). For $\alpha = 1$, again we notice an oscillatory structure of the solution. Besides, we observe that the phase
velocity $v_I/V_c$ settles again at a somewhat higher value than in the Newtonian case.

In Fig. 6 we plot the attenuation coefficient $L_c/x_I$ as a function of frequency for the three cases: the thick curve corresponds to $\alpha \to \infty$ (Newtonian limit), the dashed curve corresponds to a slightly sub-critical value of $\alpha = 10$ and thin solid curve corresponds to the case of $\alpha = 1$. The $\alpha \to \infty$ case reproduces curve 1 in Fig. 13 from (Biot, 1956b). For $\alpha = 10$, we notice deviation from the classic Newtonian behavior in the form of the overall increase of the attenuation coefficient and appearance of small oscillations on the curve, which indicates that the wave has entered the non-Newtonian regime. The large spike at low frequencies is also due to non-Newtonian effects. For the case of $\alpha = 1$ the attenuation coefficient settles at somewhat lower values, and this happens already for smaller frequencies than in the case of a Newtonian fluid. Also, a much more pronounced oscillatory structure of the solution can be noticed.

In Fig. 7 we plot the attenuation coefficient $L_c/x_{II}$ as a function of frequency for the three cases: the thick curve corresponds to $\alpha \to \infty$ (Newtonian limit), the dashed curve corresponds to $\alpha = 10$ and thin solid curve corresponds to the case of $\alpha = 1$. Note that the $\alpha \to \infty$ case perfectly matches curve 1 in Fig. 14 from (Biot, 1956b). For $\alpha = 10$, we notice a deviation from the classic Newtonian behavior in the form of the overall decrease in the attenuation coefficient and appearance of small oscillations on the curve. The jump at $f/f_c = 1$ (dashed curve) should be attributed to the non-Newtonian effects. For the case of $\alpha = 1$ the attenuation coefficient settles again at somewhat lower value and this happens already for smaller frequencies than in the case of the Newtonian fluid. Also, much more pronounced oscillatory structure of the solution can be noticed again.

5. discussion

In this paper, we have studied the non-Newtonian effects in the propagation of elastic waves in porous media by calculating phase velocities and attenuation coefficients of the rotational and dilatational waves as a function of frequency. Originally, Biot (Biot, 1956a,b) performed similar analysis for a Newtonian fluid-saturated porous medium. Using our recent results [Paper I], and motivated by a current need in models of propagation of elastic waves in porous media with realistic fluid rheologies, we have generalized the work of Biot to the case of a non-Newtonian (Maxwell) fluid-saturated porous medium.
In summary, we found that replacement of an ordinary Newtonian fluid by a Maxwell fluid in the fluid-saturated porous medium results in

- an overall increase of the phase velocities of both the rotational and dilatational waves. With the increase of frequency these quantities tend to a fixed, higher, as compared to the Newtonian limiting case, level which is not changing with the decrease of the Deborah number $\alpha$.

- the overall decrease of the attenuation coefficients of both the rotational and dilatational waves. With the increase of frequency these quantities tend to a progressively lower, as compared to the Newtonian limiting case, levels as $\alpha$ decreases.

- Appearance of oscillations in all physical quantities in the deeply non-Newtonian regime when $\alpha \ll \alpha_c = 11.64$.

The investigation of properties of elastic waves is important for a number of applications. The knowledge of phase velocities and attenuation coefficients of elastic waves in a realistic [such as saturated with Maxwell fluid] porous medium is necessary, for example, to guide the oil-field exploration applications, acoustic stimulation of oil producing fields (in order to increase the amount of recovered residual oil), and the acoustic clean up of contaminated aquifers (Beresnev & Johnson, 1994), (Drake & Beresnev, 1999).

The idea of the paper was to use the function, $F(\kappa)$, that measures the deviation from Poisseuille flow friction, extended to Maxwell fluids, and to substitute it into Biot’s equations of poroelasticity without changing the latter. However, Biot’s equations have been derived under a number of assumptions. One of these assumptions is that deviatoric (shear) components of the macroscopic stress in the fluid are negligible Pride et al. (1992). Pride et al. (1992) have shown that this assumption is justified when $\eta \omega \ll G$, where $\eta$ is the dynamic viscosity of the fluid, $\omega$ is frequency, and $G$ is frame shear modulus. Simple analysis shows that for typical Newtonian fluids such as water, this condition is only violated at frequencies $\omega > 10^9$ 1/s, or $f = \omega/(2\pi) > 10^8$ Hz. Thus, for all frequencies below 1 MHz Biot’s assumption is justified. However, when we introduce the Maxwell fluid, the situation changes in that we introduce the real (in addition to imaginary) shear stresses. In summary, for any rheology (including Maxwellian) Biot’s theory is only valid if macroscopic shear stresses are negligible. In order to prove that, we note from the rheological equation for a Maxwell fluid

$$t_m \frac{\partial \tilde{\tau}}{\partial t} = -\eta \nabla \tilde{v} - \tilde{\tau},$$
where $\tilde{\tau}$ represents the viscous stress tensor, that in the frequency domain we can effectively obtain

$$\tilde{\tau} = -\eta \nabla \vec{v} / (1 + it_m \omega).$$

This means that we can roughly replace $\eta$ in all our estimates with $\eta' / (1 + it_m \omega)$. There are two limiting cases. When $\omega \ll 1/t_m$, then the fluid is effectively Newtonian and Biot’s theory is valid. When $\omega \gg 1/t_m$, i.e. when the fluid is essentially non-Newtonian, we effectively have $\eta' = \eta / (it_m \omega)$, which in this case is smaller than $\eta$ in absolute value. Thus, when substituted into the shear stress, $S$, it produces $S = i\eta' \omega = \eta / t_m$, which is smaller than $\eta \omega$. Therefore, we conclude that inequality $\eta' \omega \ll G$ still holds for the Maxwellian fluid, i.e. Biot’s equations are valid for Maxwell rheology.

This study, similarly to the results of Paper I, has clearly shown the transition from dissipative to non-Newtonian regime in which sharp oscillations of all physical quantities are found. We would like to comment on these unexpected strong oscillations that were demonstrated by our numerical analysis. The results are based on equation (15), which has been derived for a circular cylindrical geometry. This is the same geometry that was used in classical works of Biot and others. For Newtonian fluids the use of such an idealized geometry for porous materials was backed up by an analysis that showed that the results are not very sensitive to the particular geometry (see, e.g., Johnson et al. (1987)). Of course, the magnitude of these oscillations depends on fluid parameters and permeability, and may not be as high for many fluids. However, even if parameters are such that oscillations are large, it is unclear at this stage whether this oscillatory behavior will hold for more realistic geometry, i.e. when curved pore walls (tortuosity) are considered. There is a possibility that with tortuosity effects included the obtained oscillations will be smeared. However, our goal was to constrain ourselves with simple cylindrical geometry, and a separate study is needed to analyze the tortuosity effects on our results.

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Figure captions

Fig. 1 Behavior of dimensionless, normalized phase velocity of the rotational wave, $v_r/V_r$, as a function of frequency. The thick solid curve corresponds to the Newtonian limit when $\alpha \to \infty$, while dashed and thin solid curves represent the non-Newtonian cases $\alpha = 10$ and $\alpha = 1$ respectively.

Fig. 2 Behavior of dimensionless, normalized attenuation coefficient of the rotational wave, $L_r/x_{\alpha}$, as a function of frequency. The thick solid curve corresponds to the Newtonian limit when $\alpha \to \infty$, while dashed and thin solid curves represent the non-Newtonian cases $\alpha = 10$ and $\alpha = 1$ respectively.

Fig. 3 Behavior of dimensionless, normalized phase velocity of the dilatational wave, $v_I/V_c$, as a function of frequency. Here the Newtonian limiting case, when $\alpha \to \infty$, is plotted.

Fig. 4 Same as in Fig. 3 but for $\alpha = 1$.

Fig. 5 Behavior of dimensionless, normalized phase velocity of the dilatational wave, $v_{II}/V_c$, as a function of frequency. Here the Newtonian limiting case, when $\alpha \to \infty$, is plotted with thick curve, while the thin one corresponds to the non-Newtonian case of $\alpha = 1$.

Fig. 6 Behavior of dimensionless, normalized attenuation coefficient of the dilatational wave, $L_c/x_I$, as a function of frequency. The thick solid curve corresponds to the Newtonian limit when $\alpha \to \infty$, while dashed and thin solid curves represent the non-Newtonian cases $\alpha = 10$ and $\alpha = 1$ respectively.

Fig. 7 Behavior of dimensionless, normalized attenuation coefficient of the dilatational wave, $L_{II}/x_{II}$, as a function of frequency. The thick solid curve corresponds to the Newtonian limit when $\alpha \to \infty$, while dashed and thin solid curves represent the non-Newtonian cases $\alpha = 10$ and $\alpha = 1$ respectively.
