Static spherically symmetric solutions for conformal gravity in three dimensions

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Static spherically symmetric solutions for conformal gravity in three dimensions are found. Black holes and wormholes are included within this class. Asymptotically the black holes are spacetimes of arbitrary constant curvature, and they are conformally related to the matching of different solutions of constant curvature by means of an improper conformal transformation. The wormholes can be constructed from suitable identifications of a static universe of negative spatial curvature, and it is shown that they correspond to the conformal matching of two black hole solutions with the same mass.

Keywords: Conformal gravity, wormholes, black holes.

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1. Introduction

The lack of propagating degrees of freedom for General Relativity (GR) in three dimensions stems from the fact that the solutions must be spacetimes of constant curvature (see e.g. Ref.[1]). Nonetheless, for negative cosmological constant, non-trivial solutions including black holes can be found from suitable identifications of anti-de Sitter (AdS) spacetime [2]. Besides, few is known about conformal gravity in three dimensions, whose field equations correspond to the vanishing of the Cotton tensor,

\[ C = \frac{1}{4} g^{\mu \nu} \nabla_\mu \nabla_\nu R = 0; \]  

which are fulfilled if and only if the spacetime metric is locally conformally flat. Exact solutions for this theory have been recently explored in Refs.[3,4]. The purpose
of this paper is showing that, since the field equations (1) are conformally invariant, interesting nontrivial solutions can be found not only from suitable identifications, but also from proper conformal transformations of maximally symmetric space-times. It is worth pointing out that worm holes as well as asymptotically locally at or de Sitter (dS) black hole solutions arise within this new set. This can be seen as follows: It is possible to choose the gauge such that the static spherically symmetric solution of (1) reads

$$\text{ds}^2 = -a^2 + b r + c \text{ dt}^2 + \frac{\text{dr}^2}{a^2 + b r + c} + r^2 \text{ d}^2; \quad (2)$$

where $a$, $b$, and $c$ are integration constants. These solutions are asymptotically of constant curvature, which by means of a trivial (proper) global conformal transformation can be rescaled to 1 or zero. For vanishing $b$ the metric (2) has constant curvature, and it reduces to the usual solution of standard GR. Thus, switching on the constant $b$ relaxes the asymptotic behavior of the metric as compared with the one of GR, enlarging the space of allowed solutions. Indeed, for $b = 0$ the Ricci scalar is given by $R = 6a - 2b^2$, which is singular at the origin. Depending on the value of the integration constants, this singularity could be surrounded by one or two horizons.

In the case of vanishing $a$, for $b > 0$ and $c < 0$, the metric (2) describes an asymptotically locally at black hole with a spacelike singularity at the origin surrounded by an event horizon located at $r = r_+ = c b^{-1}$. Its causal structure coincides with the one of the Schwarzschild black hole (see Fig. 1 C.1).

The case $a = 1$, corresponds to an asymptotically dS black hole with a spacelike singularity at the origin enclosed by event and cosmological horizons located at $r$, and $r_+$, respectively, provided $b = r_+ - r$, and $c = r_+ r$. As shown in Fig. 1 B.1, this black hole shares the same causal structure with the Schwarzschild-dS metric.

It is worth to remark that static black holes cannot be obtained from three-dimensional GR with non-negative cosmological constant in vacuum.

A spherically At dS black holes are obtained for the case $a = 1$. For $c > 0$ and $b < 0$ the curvature singularity is timelike and it is surrounded by a Cauchy and an event horizon located at $r$ and $r_+$, respectively, provided $b = (r-r_+)$ and $c = r_+ r$. In this case the causal structure corresponds to the one of the Reissner-Nordstrom-AdS black hole (see Fig. 1 A.1). The extremal case is obtained for $r_+ = r$. For negative $c$ ($c < 0$) the black hole possesses a spacelike curvature singularity at the origin, surrounded by a single event horizon located at $r_+$, and its causal structure reduces to the one of the Schwarzschild-AdS black hole, as it is depicted Fig. 1 D.1.

The Schwarzschild gauge" is not the only option. Actually, a de event gauge xing leads to the following static spherically symmetric solution:

$$\text{ds}^2 = \text{ dt}^2 + \text{ d}z^2 + x_0^2 \cosh^2(z) \text{ d}^2; \quad (3)$$
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Fig. 1. The causal structure of the conformal gravity black holes $\mathbb{B}$ and the worm hole $\mathbb{C}$ is shown in (A-D)1 and E1, respectively. These spacetimes can be constructed from the conformal matching of different patches of constant curvature spacetimes at the corresponding spatial infinities, as depicted in (A-E)2. Each square diagram appearing in Figs (B-E)2 describes the causal structure of static BTZ black holes. In Fig. A2 the square diagrams correspond to the causal structure of static solutions with negative mass and positive cosmological constant. In the figures A2 and B2, the region $< r_c$ must be excised and for D.2 the excised region correspond to $> r_c$. Here $r_c$ corresponds to the matching surface where the constant curvature spacetimes are smoothly glued; for the worm hole the matching surface is located at the neck ($z = 0$).