Optical vortex discrimination with a transmission volume hologram

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**Abstract.** Transmissive volume holograms are considered as mode-selective optical elements for the de-multiplexing and detecting of optical vortex modes according to the topological charge or mode number. Diffraction of vortex modes by a fundamental mode hologram is modeled using a physical optics model that treats the volume hologram as an angle-dependent transfer function. Diffracted irradiance profiles and diffraction efficiencies are calculated numerically as a function of the incident mode number. The results of the model are compared with experimental results obtained with volume holograms of fundamental and higher-order vortex modes. When considered as a function of detuning between the incident and recorded mode numbers, the measured diffraction efficiencies are found to be invariant with respect to the recorded mode number, provided that the order difference remains unchanged, and in close agreement with the predictions of the model. Measurements are made with a 1.3 mm thick permanent photo-thermo-refractive glass hologram and a 9 mm thick re-writable photorefractive lithium niobate hologram. A liquid-crystal spatial light modulator generates the vortex modes used to record and read the holograms. The results indicate that a simple volume hologram can discriminate
between vortex modes; however, adjacent mode discrimination with low crosstalk would require a very thick hologram. Furthermore, broadening of the vortex angular spectrum, due to diffraction at a finite aperture, can adversely affect diffraction efficiencies.

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1. Introduction

Optical vortices, characterized by the azimuthal nature of their transverse optical phase, are central to a variety of classical and quantum mechanical phenomena. Classically, optical vortices occur as paraxial modes of lasers with cylindrical symmetry [1] and branch points in atmospheric optics [2]. Quantum mechanically, the azimuthal phase in the photon probability amplitude gives rise to quantized orbital angular momentum (OAM) [3]. Optical vortices show promise in new applications, including optical tweezers [4] and optical vortex coronagraphs [5]. Mathematically, the complex optical fields associated with optical vortices are orthogonal over circular apertures of arbitrary radius. This suggests that, in analogy to orthogonal polarization states, it may be possible to spatially multiplex and de-multiplex vortex modes for multi-spatial-channel communications [6] and utilize vortex modes as bases for projective quantum measurements for multi-level quantum key distribution (QKD) [7, 8].

Generation, multiplexing and de-multiplexing of OAM states, or vortex modes, are therefore of interest. Spatial light modulators (SLMs) have been utilized to generate OAM states and superpositions of OAM states using modulo $2\pi$ phase control [9, 10] and computer-generated holography [12] to achieve both amplitude and phase modulation. Thin holograms in conjunction with single-mode optical fibers have been used to detect OAM states, but do not provide a means of separating OAM states with high efficiency [7, 8, 10, 11]. Several approaches to efficiently de-multiplexing OAM states have been described in the literature. Leach et al [13] presented a technique for sorting single photons according to the parity of the OAM states. In this approach, a cascade of interferometers would be required to de-multiplex more than two OAM states. Anguita et al proposed the use of cascaded transmission volume holograms for multiplexing and de-multiplexing optical vortices for multi-channel free-space optical communications [6]. In a related concept, multiplexed volume holograms have been analyzed by Lou et al for segregating optical fields according to the wavefront radii of curvature [14]. Recently, Berkhout et al [15] presented a novel optics design that maps azimuthal phases onto planar phases, reducing the problem to one of separating optical fields whose wavefronts differ by integer waves of tilt over the aperture.
Figure 1 shows a conceptual schematic diagram of a four-state de-multiplexer based on cascaded transmission volume holograms as proposed in [6]. Four optical vortex modes, denoted by $|m\rangle$, where $m = 1, 2, 3$ and 4, define four free-space channels. Photons prepared in each of these modes co-propagate to a receiver where each photon propagates through the holograms until encountering the mode-matched hologram. The matched hologram diffracts the photon to a detector to reveal the mode of the photon.

De-multiplexing optical vortices as suggested in figure 1 places specific requirements on the performance of the holograms. In order to minimize channel crosstalk, each hologram should diffract only the matched mode and transmit all non-matched modes. Furthermore, achieving high diffraction efficiency can be important for applications such as QKD where losses can adversely affect bit rates. The mode selectivity for an ideal holographic mode selector is illustrated in figure 2 for the case of a hologram recorded with mode number $n$ and illuminated with modes of arbitrary number $m$. The diffraction efficiency is unity for the matched mode and zero elsewhere.

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In this paper, the performance of a Leith–Upatnieks (i.e. planar reference wave) transmission volume hologram is evaluated for differentiating optical vortices according to their topological charge. For the purpose of modeling, optical vortices are defined within a circular aperture and the angular spectrum is calculated numerically via fast Fourier transform. Interaction of the plane-wave components with the fundamental mode, \( m = 0 \), hologram is considered and the angle-dependent transfer function of the hologram is calculated numerically using the well-known results of coupled wave theory. The results of this model are compared with the experimental results obtained with holograms recorded with various mode numbers. When considered as a function of detuning between the incident and recorded mode numbers, the measured diffraction efficiencies are found to be invariant with respect to the recorded mode number, provided the order difference remains unchanged and in close agreement with the predictions of the model. The results show that increased mode selectivity can be obtained with increased hologram thickness, but adjacent mode discrimination with low crosstalk would require a very thick hologram and could result in relatively low diffraction efficiency. Experimental results are obtained using photo-thermo-refractive (PTR) glass [16] and photorefractive lithium niobate (LiNbO\(_3\)) transmission volume holograms. A liquid-crystal SLM generates the vortex modes used to record and read the holograms.

2. Optical vortices

The transverse optical phase \( \phi \) of an optical vortex is given in cylindrical coordinates by \( \phi(\rho, \theta) = m\theta \), where \( \rho \) and \( \theta \) are the radial and azimuthal coordinates, respectively. The integer \( m \) spans the range \( -\infty < m < \infty \) and denotes both the topological charge for classical vortices and the quantized OAM, in units of Planck’s constant, for single photons. Figure 3 illustrates the transverse phase in gray scale for mode numbers given by \( m = -2, -1, 0, 1 \) and 2. Note that the \( m = 0 \) phase profile is constant or planar and all adjacent modes differ by the same azimuthal phase \( \Delta \phi(\theta) = \theta \). The phases are constant along the radial direction. Since the phase increases from zero to \( m2\pi \) around a contour of any radius, the phase slope is smallest at the edge of the aperture and becomes arbitrarily large near the center. For a given radius, the phase slope increases with increasing \( m \).
3. Physical optics model of a fundamental mode transmission volume hologram as a mode selective element

Consider a transmission volume hologram recorded with an optical vortex of mode number \( n \) and illuminated with a vortex of mode number \( m \). Since the difference in phase between the illuminating and recorded mode, \( \Delta \phi(\theta) = (m - n)\theta \), depends on the recorded mode number only through its contribution to the mode detuning, we postulate that an analysis of the \( m = 0 \) hologram should be sufficient to predict the performance of a hologram recorded with arbitrary vortex mode number, provided that the order difference remains unchanged upon readout. Note that this assumption ignores the increased resolution required to record holograms of vortices with increasing mode number. This postulation, which will be validated by experiment in section 4, simplifies the problem to one of treating the interaction of optical vortices with the fundamental mode hologram. The fundamental mode hologram is then approximated as a plane-wave-interference hologram whose characteristics are given by the coupled wave theory of Kogelnik \[17\]. This assumes the hologram to be infinite in transverse extent and ignores the effects of diffraction introduced by a finite aperture when recording the hologram. For the cases where the hologram is sufficiently larger than the read beam, this approximation can be considered appropriate. The interaction of non-planar wavefronts with crystalline structures \[18, 19\] and volume holograms \[20, 21\] has been treated by Fourier decomposition and by considering the interaction of each plane-wave component with the diffracting medium. Following this approach, this section treats diffraction of optical vortex fields by a plane-wave transmission volume hologram through decomposition of the vortex fields into their constituent plane-wave components and multiplying the plane-wave spectrum by the angle-dependent transfer function of the hologram as described by coupled wave theory.

While optical vortices are appropriately represented in cylindrical coordinates, the angular selectivity of a plane-wave transmission volume hologram is most easily understood in Cartesian coordinates. Figure 4 illustrates a plane-wave interference volume hologram with grating period \( \Lambda \) and thickness \( d \) along the \( z \)-axis as described in \[17\]. The grating vector \( \mathbf{K} \) is oriented in the \( x-z \)-plane at angle \( \phi \) relative to the \( z \)-axis. Angles \( \alpha \) and \( \beta \) denote optical wave-vector angles in air in the \( x-z \)- and \( y-z \)-planes, respectively. Angles \( \alpha' \) and \( \beta' \) denote the corresponding wave vectors inside the hologram. It is assumed that the plane waves and

![Figure 4](image-url)
hologram are infinite in the transverse dimension and effects associated with diffraction due to finite apertures are excluded in the recording process.

For a lossless dielectric medium, upon readout with a unit amplitude plane wave $R$ at angle $\alpha'$ in the medium, the transmitted complex field $S$ is given by

$$S(\alpha') = -i \left( \frac{c_R}{c_S} \right)^{1/2} \exp\left(-i\xi\right) \frac{\sin\left(v^2 + \xi^2\right)^{1/2}}{\left(1 + \frac{\xi^2}{v^2}\right)^{1/2}},$$

(1)

where

$$v = \frac{\pi n_1 d}{\lambda (c_R c_S)^{1/2}},$$

(2)

$$\xi = \frac{\Delta \alpha' K d \sin(\phi - \alpha'_0)}{2c_S}.$$  

(3)

Obliquity factors $c_R$ and $c_S$ are given by $c_R = \cos \alpha'$ and $c_S = \cos \alpha' - (\lambda/n_0 \Delta \alpha) \cos \phi$. The Bragg angle inside the medium is given by $\alpha'_0$, $\lambda$ is the optical wavelength in free space and $\Delta \alpha' = \alpha' - \alpha'_0$ is the deviation of the readout angle from the Bragg angle inside the medium. The refractive index modulation is given by $n(x) = n_0 + n_1 \cos(2\pi K \cdot r / \Lambda)$, where $n_0$ describes the average refractive index, $n_1$ describes the magnitude of the index modulation and $r$ is a vector to any point in the medium. The diffraction efficiency $\eta$ is given by

$$\eta = \frac{\sin^2\left(\frac{v^2 + \xi^2}{\sqrt{v^2}}\right)^{1/2}}{1 + \frac{\xi^2}{v^2}}.$$  

(4)

Equation (4) captures several characteristics of transmission volume holograms that are important to the analysis. A plane-wave optical field, incident at the Bragg angle, will diffract with propagation in the medium to reconstruct the reference field. With continued propagation, the conversion efficiency will increase, peak at unity and then decrease with propagation as energy transfers back into the input field. The cycle of conversion and backconversion repeats as the fields propagate through an arbitrarily thick hologram. For a hologram of a given thickness, there exist multiple values for the index modulation $n_1$ that will result in unity conversion efficiency. The smallest of these values corresponds to the case where the first conversion peak coincides with the exit face of the hologram and also results in the highest degree of angular selectivity while minimizing side-lobe peaks.

Consistent with the lithium niobate hologram described in section 4, consider an average index $n_0$ of 2.32, symmetric recording angles of $\alpha_0 = \pm 18^\circ$ in air and a 532 nm optical wavelength. Figure 5 shows the plane-wave diffraction efficiency as a function of incidence angle in air, calculated from equation (4). Four hologram thicknesses are considered, and for each thickness $d$, the index modulation $n_1$ is chosen to maximize the diffraction efficiency and angular selectivity. This is accomplished by solving numerically for the smallest value of $n_1$ that results in unity conversion efficiency at the output of the hologram. Angular selectivity curves are shown for $(d \text{[mm]}, n_1)$ given by $(1.0, 2.63 \times 10^{-4})$, $(3.0, 8.78 \times 10^{-5})$, $(9.0, 2.92 \times 10^{-5})$ and $(27.0, 9.76 \times 10^{-6})$. As the thickness increases, the angular selectivity of the holograms increases correspondingly. For small angular deviations in the orthogonal direction, the diffraction efficiencies are constant. Consequently, in the diffracted order, non-matched input modes will be attenuated due to angular mismatch along the direction defined by the reference wave only.
Figure 5. Calculated angular response of a plane-wave transmission volume hologram for 1, 3, 9 and 27 mm thickness.

Figure 6. Schematic diagram of a numerical model used to quantify optical vortex discrimination with a fundamental mode volume hologram. The angular spectrum is calculated via Fourier transform and multiplied by the angle-dependent transfer function to give the angular spectrum of the diffracted optical field. The squared modulus of the inverse Fourier transform then yields the irradiance profile of the diffracted beam.

Figure 6 illustrates the physical optics model describing the interaction of the optical vortices with the hologram. In reading the hologram, the phase of the incident vortex mode is defined within a circular aperture. One effect of the finite aperture is to place a lower bound on the magnitude of the azimuthal phase tilt. In addition, diffraction associated with truncating the vortex mode leads to broadening of the angular spectrum. Consistent with the experiments described in section 4, the diameter of the aperture is taken to be 7.68 mm. The complex angular spectrum is calculated numerically by taking the fast Fourier transform of the complex field associated with the azimuthal phase function and circular aperture. For graphical purposes, the real-valued power spectrum is shown in the figure. Equation (1) for the input-angle-dependent output field defines the complex transfer function for the hologram. Again for graphical purposes, the real-valued diffraction efficiency is shown in the figure. The product of the input angular spectrum and the hologram transfer function gives the angular spectrum of the diffracted field. The irradiance profile of the diffracted field is then given by the squared modulus of the inverse Fourier transform.

Calculated irradiance profiles are shown in figure 7 for the case of the 1 mm thick hologram described above and for input vortex mode numbers \( m = -40, -5, 0, +5 \) and +40. For the mode-matched case, \( m = 0 \), the diffraction efficiency is nearly unity throughout the aperture. For the
Figure 7. Calculated diffracted irradiance profiles for a 1 mm thick plane-wave hologram illuminated by optical vortex modes $m = -40, -5, 0, +5$ and $+40$.

Figure 8. Calculated diffraction efficiency versus incident mode number for various hologram thicknesses.

In non-mode-matched cases, the cylindrical symmetry of the angular spectra and the Cartesian symmetry of the angle-dependent transmittance lead to the spatially varying irradiance profiles shown. More specifically, along the $x$-axis of the beam profiles, the wave-vector angles of the incident vortices are entirely along $y$ where the transfer function yields diffraction efficiencies near unity, independent of the magnitude of the angle. Along the $y$-axis, the wave-vector tilts are entirely along $x$ where the transfer function varies with the magnitude of the angle. In this case, attenuation is greatest near the center of the vortex where the wave-vector angles are greatest.

The net diffraction efficiency is found by integrating the diffracted irradiance over the aperture and normalizing to the integrated input power. The calculated diffraction efficiencies are shown as a function of incident mode number in figure 8 for 1, 3, 9, 27 and 81 mm thick holograms. Calculated values are represented by circles and the lines are added as an aid to the viewer. In each case, the diffraction efficiency reaches a peak for the mode-matched case and decreases as the mode detuning increases. For the 1 mm thick case, the matched mode is diffracted with 98% efficiency; however, neighboring modes are diffracted with nearly the
Figure 9. Conceptual diagram showing key portions of the experiments.

same efficiency and the width of the curve spans nearly 80 modes over the full-width at half-maximum (FWHM). In a de-multiplexing application, the diffracted matched mode would be highly contaminated by non-matched modes. Increasing the hologram thickness from 1 to 81 mm reduces the width of the curve to only two modes at FWHM, thereby considerably improving mode discrimination. For the case of the 81 mm thick hologram, the matched mode is diffracted with 27% efficiency. Adjacent modes, given by \( m = \pm 1, 2 \) and 3, are diffracted with 13, 9 and 7% efficiency, respectively. The decline in peak efficiency seen in the progression of curves is understood as follows. As the hologram thickness increases from 1 to 81 mm, the width of the hologram transfer function decreases relative to the width of the Airy function associated with the angular spectrum of the fundamental mode. This, in turn, leads to losses in the diffracted power, even for the mode-matched case. Note these results are obtained for an input aperture of a specific size.

4. Experimental results with transmission volume holograms

Experimental measurements of hologram mode selectivity are carried out using holograms prepared in PTR glass and crystalline photorefractive iron-doped lithium niobate. The PTR glass hologram is a permanent plane-wave interference hologram and the lithium niobate holograms are prepared in situ by interfering modes \( m = 0, 3, 5 \) and 10 with a planar reference wave. Key portions of the experimental setup are shown in figure 9. Figure 9(a) illustrates the configuration used to read the PTR glass hologram and figure 9(b) illustrates the configuration used to record and read holograms in lithium niobate. The light source (not shown) is a 5 W Coherent Verdi continuous wave frequency-doubled Nd: YAG laser. The output is expanded, and collimated to produce a near-uniform intensity beam at the pupil. A variable beam splitter divides the light to create the signal and reference fields. The signal field is directed onto a Boulder Nonlinear Systems 512 × 512 element liquid-crystal SLM, which is shown in the figure. The SLM introduces up to \( 2\pi \) phase modulation at the 532 nm wavelength, operates in reflection.
and is illuminated at an angle of 7.5° relative to the surface normal. The SLM is programmed with modulo $2\pi$ optical phase functions defined over a 7.68 mm diameter circular aperture, which defines the pupil. The topological charges range from $m = -40$ to $+40$. An iris, placed in close proximity to the SLM, prevents light from illuminating pixels beyond the pupil region of the SLM. A 1 : 1 afocal pupil relay (shown) transfers the optical vortex field from the SLM onto the hologram. The diffracted beam is directed to a detector (not shown) to measure the diffraction efficiency. Alternatively, a second pupil relay is used to image the hologram onto a camera to record the diffracted beam profiles. In the case of the lithium niobate hologram, shown in figure 9(b), a reference beam with 10 mm diameter is introduced for the purpose of recording. The reference beam is sufficiently large to interact with the signal beam throughout the volume of the hologram. After exposure, the reference wave is blocked and the hologram is illuminated with optical vortices generated by the SLM.

The results obtained with the PTR glass hologram are summarized in figures 10 and 11. The hologram is a 13.5 mm × 13.5 mm aperture by 1.27 mm thick permanent hologram fabricated by OptiGrate and recorded by interfering two planar wavefronts [16]. The hologram is described by a grating period of 3 µm with grating planes oriented at 3.25° relative to the surface normal, an average index $n_0$ of 1.49 and a combination of thickness and index modulation ($d$ [mm], $n_1$) given by (1.27, 2.09 × 10$^{-4}$). The index modulation value is inferred from the coupled-wave model of the hologram and yields very good agreement with measured angular selectivity data. Figure 10 shows phase and intensity profiles for illuminating mode numbers given by $m = -40$, −5, 0, 5 and 40. The first row shows the theoretical phase modulo $2\pi$. In order to generate optical vortex fields with the SLM, the theoretical phase functions are first scaled to compensate for phase nonlinearities in the SLM and then added to a second phase function that compensates for wavefront errors introduced by irregularities in the reflective substrate of the SLM [22]. The resulting phase function is then displayed modulo $2\pi$ within a circular region on the SLM.

Figure 10. Calculated and measured phase and intensity profiles for incident and diffracted vortices with a plane-wave interference hologram as a function of incident mode number.
Figure 11. Hologram diffraction efficiency as a function of input mode number for a plane-wave hologram. Squares represent measured values and the circles represent values calculated with the numerical model. The solid line is added to help distinguish modeled from measured values.

On-axis interferograms of the optical field generated by the SLM and recorded with the iris open are shown in the second row. The interferograms show the circular region of the pupil in which the vortex phase is defined. Outside the pupil, the interferograms show the phases associated with the non-actuated SLM elements. At the center of the vortex functions, where the phase slopes become arbitrarily large, pixilation of the SLM will lead to artifacts in the vortex phase. These artifacts include reduced irradiance due to a loss of diffraction efficiency and erroneous phase slopes due to aliasing effects [23]. These artifacts can be seen at the center of the $m = \pm 40$ interferograms. The third row shows the intensity profiles of the first-order diffracted beam calculated with the model described in section 3 using parameters relevant to the PTR glass hologram. The fourth row shows the measured irradiance profiles. Aside from artifacts associated with the modulo $2\pi$ phase resets on the SLM, the model and experiment show qualitative agreement including the relative intensity maxima located within the central minima. We associate this feature with the secondary maxima of the angular selectivity curves.

Figure 11 shows the calculated and measured diffraction efficiencies for illuminating mode numbers in the range $-40 \leq m \leq 40$. The experimental values, shown as squares, represent the internal diffraction efficiency, which is determined by measuring the first-order diffracted power and dividing this value by the sum of the zero- and first-order diffracted power. In this way, any losses due to Fresnel reflections, absorption or scattering from surface imperfections or bulk inhomogeneities are excluded from the measured efficiency. The circles represent the calculated values obtained from the model. The model and measurement are in close agreement and display similar characteristics. The model predicts a peak efficiency of 0.99, while the measured peak value is 0.94.

Holograms of the $m = 0, 3, 5$ and 10 vortex modes are recorded with a 10 mm $\times$ 14 mm aperture by 9 mm thick sample of lithium niobate. The LiNbO$_3$ sample has a persistence of many hours and may be erased and re-recorded repeatedly. Signal and reference beams are introduced at symmetric angles of $\pm 18^\circ$ relative to the surface normal with the crystalline $c$-axis.
Figure 12. Measured diffraction efficiency as a function of vortex mode number $m$ for a 9 mm thick lithium niobate hologram and recorded mode numbers given by $n = 0, 3, 5$ and 10 and measured values represented by squares, diamonds, triangles and circles, respectively. The solid lines are added to help identify individual data sets.

Figure 13. Hologram diffraction efficiency as a function of mode detuning for a 9 mm thick lithium niobate hologram. Holograms recorded with mode numbers of $n = 0, 3, 5$ and 10 are represented by ‘×’ symbols, triangles, diamonds and circles, respectively. The calculated values taken from figure 8 are represented by a dashed line.

oriented as shown in figure 9(b). The irradiance of each beam is approximately 100 mW cm$^{-2}$. An exposure time of 21 s was empirically determined to be the shortest exposure to yield maximum diffraction efficiency for the fundamental mode hologram. Note that this corresponds to the criteria adhered to in the model and, for the 9 mm thick sample, implies a refractive index modulation of $n_1 = 2.92 \times 10^{-5}$. It should also be noted that this number is comparable to inhomogeneity values cited for lithium niobate. For each recording, the hologram is first erased with light from a tungsten-halogen lamp. Figure 12 shows the measured diffraction efficiencies for illuminating mode numbers in the range $-40 \leq m \leq 40$. The experimentally measured values, shown as discrete symbols, represent the internal diffraction efficiency as defined above. Results obtained from holograms recorded with mode numbers $m = 0, 3, 5$ and 10
are represented by squares, diamonds, triangles and circles, respectively. Each data set shows a peaked efficiency for the case of the matched mode. Peak internal diffraction efficiencies for the \( m = 0, 3, 5 \) and 10 holograms are 0.85, 0.86, 0.85 and 0.83, respectively. Aside from the shift in the data, the measured mode selectivity plots show similar widths and functional dependences. The overlap of the plots quantifies the degree of channel crosstalk wherein a given incident mode can experience appreciable diffraction by multiple holograms. For example, the \( m = 0 \) hologram diffracts the \( m = 0, 3, 5 \) and 10 vortex modes with efficiencies of 0.85, 0.58, 0.34 and 0.23, respectively.

Figure 13 shows the experimental results of figure 12 plotted as a function of detuning of the incident mode from the recorded mode. The dashed line represents the calculated values shown previously in figure 8 for the 9 mm thick hologram. When plotted as a function of mode detuning, the measured values overlap and are in close agreement with the theoretical values calculated for the fundamental mode hologram. Figure 13 illustrates that when considered as a function of detuning between the incident and recorded mode numbers, the measured diffraction efficiencies are invariant with respect to the recorded mode number, provided that the order difference remains unchanged. The peak efficiency predicted by the model is 0.86, in close agreement with the measured values.

5. Conclusions

The performance of a transmission volume hologram for discriminating optical vortex modes is quantified through a physical optics model that predicts diffracted irradiance profiles and mode selectivity characteristics. The model considers the case of the fundamental mode hologram and uses results from coupled-wave theory to define an angle-dependent transfer function acting on the plane-wave spectra of the incident vortex modes. The results of the model are in close agreement with empirically determined values. Mode selectivities measured with higher-order mode holograms, when considered as a function of detuning between the incident and recorded mode numbers, are also found to be in close agreement with the results of the model. Modeling results suggest that a sufficiently thick hologram can achieve a significant degree of adjacent mode discrimination but, under the assumptions of the model, such a hologram could also experience reduced diffraction efficiency. This occurs in the model when diffraction associated with the input aperture causes the diffraction-limited divergence of the fundamental mode to exceed the angular acceptance of the infinite plane-wave hologram. Extensions of this analysis directed towards optimizing efficiency and mode discrimination for a given application might include increasing the transverse dimension of the interaction between the vortex and hologram, including the effects of diffraction due to a finite aperture in the hologram recording process, amplitude modulation of the incident beam profiles to mitigate diffraction effects at the aperture and analysis of alternative hologram recording geometries.

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