Fluctuation-induced Nambu–Goldstone bosons in a Higgs–Josephson model

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Abstract
We present a new mechanism of fluctuation-induced Nambu–Goldstone bosons in a scalar field theory of Higgs–Josephson systems. We consider a simple scalar field model with $U(1)^n$ rotational symmetry. When there is an interaction which violates the rotational symmetry, the Nambu–Goldstone bosons become massive and massless bosons are concealed. We present a model where the massive boson becomes a massless boson as a result of the perturbative fluctuation. In our model the $Z_2$-symmetry associated with the chirality is also broken. The transition occurs as a first-order transition at the critical point. The ground state at absolute zero will flow into the state with more massless bosons due to fluctuation effects at finite temperature.

Keywords: Higgs–Josephson model, Nambu–Goldstone boson, fluctuation-induced massless boson, superconductors

Introduction

When global and continuous symmetries are spontaneously broken, gapless excitation modes, called Nambu–Goldstone bosons, exist and govern the long-distance behaviors of the system [1–3]. When the $U(1)$ rotational symmetry is spontaneously broken, there is a massless Nambu–Goldstone boson. When there is an interaction that violates the $U(1)$ symmetry, we have no massless boson. An interesting question is whether such an interaction will
continuously conceal the Nambu–Goldstone bosons when the perturbative corrections are taken into account. We present a model that exhibits a fluctuation induced Nambu–Goldstone boson in this paper. This means that a massless boson appears in spite of an interaction that hides the Nambu–Goldstone bosons. We propose the mechanism of a fluctuation induced Nambu–Goldstone boson.

We consider a model of an n-component scalar field with Josephson interactions, the so-called Higgs–Josephson model [4–7]. Let us consider the action given as

$$ S = \frac{1}{k_B T} \int d^4x \sum_j \left( \alpha_j |\phi_j|^2 + \frac{\beta_j}{2} |\phi_j|^4 \right) $$

$$ + \frac{1}{k_B T} \int d^4x \left[ \sum_j K_j |\nabla \phi_j|^2 + \sum_{i \neq j} \gamma_{ij} \phi_i^\dagger \phi_j \right], $$

(1)

where $\phi \equiv (\phi_1, \ldots, \phi_n)$ is a complex n-component scalar field. We write $\phi_j$ as

$$ \phi_j = e^{i\theta_j} |\phi_j| = e^{i\theta_j} \rho_j, $$

(2)

where $\rho_j$ is a real scalar field. The last term in the action is the Josephson term. We assume that $\gamma_{ij}$ are real and $\gamma_{ij} = \gamma_{ji}$. Without this interaction, the phase modes $\theta_j$ represent massless modes. Because of this term, we have $n - 1$ phase massive modes and one massless mode as shown by expanding $\cos(\theta_i - \theta_j)$ in terms of $\theta_i - \theta_j$. We adopt that $\beta_j$ is positive so that the action has a minimum. When $\alpha_j$ is negative, $\rho_j$ takes a finite value at the minimum of the potential. We set this value as $\Delta_j$ and write $\rho_j = \Delta_j H_j + \lambda \Lambda \theta_j$. $H_j$ is the Higgs field and represents fluctuation of the scalar field around the minimum $\Delta_j$. We simply assume that $K = K_j$, $\Lambda = |\Lambda|$ and $\gamma_{ij} = \gamma_{ji} \equiv \gamma$. Then the action for the phase variables $\theta_j$ is

$$ S[\theta] = \frac{\Lambda^{d-2}}{t} \int d^4x \left( \sum_j (\nabla \theta_j)^2 + \lambda \Lambda^2 \sum_{i < j} \cos(\theta_i - \theta_j) \right), $$

(3)

where $t/\Lambda^{d-2} = k_B T / (K \Delta^2)$ and $\lambda \Lambda^2 = \gamma / K$. We have introduced the cutoff $\Lambda$ so that $t$ and $\lambda$ are dimensionless parameters. We assume that $\lambda > 0$ in this paper. We now focus on $\theta_j$ and consider the case $n = 3$. Since the potential term is written as

$$ V \equiv \left( \lambda \Lambda^{d-2} / t \right) \left( \cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1) \right), $$

(4)

the mode of the total phase $\theta_1 + \theta_2 + \theta_3$ remains massless. We do not consider this mode because the coupling to the gauge field turns this mode into a gapped mode (Higgs mechanism). The other $n - 1$ modes do not become massive by the coupling to the gauge field. Let us consider the case $\lambda > 0$. As is easily shown, the ground state of $V$ has a $2\pi/3$ structure, namely, $\theta_2 - \theta_1 = 2\pi/3$ and $\theta_3 - \theta_2 = 2\pi/3$ as shown in figure 1(a). The state in figure 1(b) has also the same energy. Two states are indexed by the chirality for (a) and $\kappa = -1$ for (b), where $\kappa$ is defined by $\kappa = (2/3\sqrt{3})(\sin(\theta_2 - \theta_1) + \sin(\theta_3 - \theta_2) + \sin(\theta_1 - \theta_3))$ [8–15]. We set $\varphi_1 = \theta_3 - \theta_1$ and $\varphi_2 = \theta_1 - 2\theta_2 + \theta_3$ to write the potential density as
\( V = \frac{\lambda \Lambda^d}{t} \left( \cos (\varphi_1) + 2 \cos \left( \frac{\varphi_1}{2} \right) \cos \left( \frac{\varphi_2}{2} \right) \right). \)  

(5)

\( V \) has a minimum at \( \varphi_1 = 4\pi/3 \) and \( \varphi_2 = 0 \). We mention here that an \( S_3 \) symmetry of the Josephson potential is not lost when we express the potential in terms of \( \varphi_1 \) and \( \varphi_2 \). When \( V \) has a minimum at some value of \( \varphi_{\theta \theta} = -\frac{1}{3} \), \( V \) has also a minimum when \( \theta_3 - \theta_2 \) takes the same value (modulo \( 2\pi \)). When the former has the chirality \( \kappa = 1 \), the latter has \( \kappa = -1 \). We consider the fluctuation around this minimum. For this purpose, we perform a unitary transformation by defining \( \varphi_1 = 4\pi/3 + \sqrt{2} \eta_1 \) and \( \varphi_2 = \sqrt{6} \eta_2 \):

\[
\begin{align*}
\theta_1 &= \frac{2\pi}{3} - \frac{1}{\sqrt{2}} \eta_1 + \frac{1}{\sqrt{6}} \eta_2 + \frac{1}{\sqrt{3}} \eta_3, \\
\theta_2 &= -\frac{2}{\sqrt{6}} \eta_2 + \frac{1}{\sqrt{3}} \eta_3, \\
\theta_3 &= \frac{2\pi}{3} + \frac{1}{\sqrt{2}} \eta_1 + \frac{1}{\sqrt{6}} \eta_2 + \frac{2}{\sqrt{3}} \eta_3.
\end{align*}
\]

(6)

(7)

(8)

where \( \eta_i \ (i = 1,2,3) \) indicate fluctuation fields. \( \eta_3 \) describes the total phase mode, \( \eta_3 = (\theta_1 + \theta_2 + \theta_3)/\sqrt{3} \), and is not important in this paper because this mode turns out to be a plasma mode by coupling with the long-range Coulomb potential. We obtain \( \Sigma_i (V\vartheta_i)^2 = \Sigma_i (V\eta_i)^2 \), and then the action \( S[\eta] = S[\theta] \) is

\[
S[\eta] = \frac{\Lambda^{d-2}}{t} \int d^d x \left[ \sum_j (V\eta_j)^2 + \lambda \Lambda^2 \left( \cos \left( \frac{\sqrt{2}}{\sqrt{3}} \eta_1 + \frac{\sqrt{3}}{2} \eta_2 \right) \right) \right].
\]

(9)

The potential term has a minimum at \( \eta_1 = \eta_2 = 0 \). Both of \( \eta_1 \) and \( \eta_2 \) represent massive modes with mass \( 3\lambda/(2t) \).

**Fluctuation induced Nambu–Goldstone boson**

The potential \( V \) corresponds to the potential of a two-dimensional XY model on the triangular lattice with a frustrated interaction. The ground state has a well-known \( 2\pi/3 \)-structure. We
consider the role of fluctuation and show the existence of a fluctuation-induced massless mode. We examine the following free energy density by neglecting the kinetic term:

\[ f = k_B T \frac{\lambda A^d}{t} \left[ \cos \left( \sqrt{2} \eta_1 + \frac{4\pi}{3} \right) + 2 \cos \left( \frac{1}{\sqrt{2}} \eta_1 + \frac{2\pi}{3} \right) \cos \left( \frac{3}{\sqrt{2}} \eta_2 \right) \right]. \]  (10)

The partition function is given by

\[ Z = \int \! d\eta_1 \! d\eta_2 \exp \left( -\frac{F}{k_B T} \right), \]  (11)

for the free energy functional \( F \). Using the formula for the modified Bessel function,

\[ I_0(z) = \frac{1}{\pi} \int_0^\pi e^{z \cos \varphi} \, d\varphi, \]  (12)

we have, by using \( q_2/2 = \sqrt{3/2} \eta_2 \),

\[ \int_0^{2\pi} d\varphi_2 \exp \left[ -\frac{2\lambda A^d}{t} \cos \left( \frac{q_2}{2} \right) \cos \left( \frac{q_1}{2} \right) \right] = 2\pi I_0 \left( \frac{2\lambda A^d}{t} \cos \left( \frac{q_1}{2} \right) \right). \]  (13)

We use \( I_0(-x) = I_0(x) \) and the asymptotic formula \( I_0(z) \sim e^z/\sqrt{2\pi z} \) \((z > 0)\) at low temperature. Then the effective free-energy density for \( \eta_1 \) is

\[ \frac{f[\eta_1]}{A^d} = \epsilon_0 \cos \left( \sqrt{2} \eta_1 + \frac{4\pi}{3} \right) - 2\epsilon_0 \left| \cos \left( \frac{1}{\sqrt{2}} \eta_1 + \frac{2\pi}{3} \right) \right| + \frac{1}{2} \frac{k_B I}{\lambda A^d} \ln \left( \frac{\lambda A^d}{\pi t} \left| \cos \left( \frac{1}{\sqrt{2}} \eta_1 + \frac{2\pi}{3} \right) \right| \right), \]  (14)

where \( \epsilon_0 \equiv k_B T \lambda /t \). We have an effective entropy term being proportional to the temperature \( T \). \( F[\eta_1] \) has a minimum at \( \eta_1 = 0 \) \((q_1 = 4\pi/3)\) at absolute zero \( T = 0 \). In contrast, at finite temperature \( T > 0 \), the minimum is at \( \eta_1 = -\sqrt{2} \pi/6 \) and \( q_1 = \pi \). This is shown in figure 2 where we present the potential \( f[\eta_1]/\epsilon_0 \) as a function of \( \varphi \equiv q_1 = 4\pi/3 + \sqrt{2} \eta_1 \) for \( t = \lambda \) with setting \( A = 1 \). At \( \varphi = \pi \), \( \eta_2 \) becomes a massless boson because the free-energy density in equation (10) becomes independent of \( \eta_2 \) with the vanishing of the mass term. This is due to the fluctuation of the \( \eta_2 \) field at finite temperature. The qualitatively same result is obtained by Gaussian integration with respect to \( \eta_2 \) after expanding the cosine function as

\[ \cos \left( \sqrt{3/2} \eta_2 \right) = 1 - (3/4) \eta_2^2 + \cdots \] and assuming that \( \cos (\eta_2/\sqrt{2} + 2\pi/3) < 0 \). This state is shown in figure 1(c) using a spin analogue where we have two antiferromagnetic spins and one vanishing spin. This means that the \( \eta_2 \)-mode is massless and \( q_2 = \sqrt{6} \eta_2 = \theta_1 - 2\theta_2 + \theta_3 \) can take any value. At the absolute zero, we have the index of chirality \( \kappa = \pm 1 \) as shown in figures 1(a) and (b). The chirality disappears at finite temperature leading to the emergency of a Nambu–Goldstone boson. This represents a phenomenon wherein the Nambu–Goldstone boson appears due to a fluctuation effect.
We next consider the kinetic terms of $\eta_j$. For this purpose, we use the expansion of cosine term and write the action in the form

$$S = \frac{A^{d-2}}{t} \int d^dx \left[ \sum_j \left( \nabla \eta_j \right)^2 + \lambda \Lambda^2 \left( \cos \left( \sqrt{2} \eta_1 + \frac{4\pi}{3} \right) \right) \right. \\
- \left. 2 \left[ \cos \left( \frac{1}{\sqrt{2}} \eta_1 + \frac{2\pi}{3} \right) \right] + \frac{3\lambda \Lambda^2}{2} \left[ \cos \left( \frac{1}{\sqrt{2}} \eta_1 + \frac{2\pi}{3} \right) \right] \right] \eta_2^2 \right].$$

When $\cos (\varphi_1/2) < 0$, we use $\cos (\sqrt{3/2} \eta_2) = 1 - (4/3) \eta_2^2 + \cdots$. Around the minimum at $\varphi_1 = 2\pi/3$ and $\varphi_2 = 2\pi$ (with chirality $\kappa = -1$), we use instead the expansion by defining $\varphi_2 = 2\pi + \sqrt{6} \eta_2$. We integrate out the field $\eta_2$ to obtain the effective action, using $\varphi \equiv \varphi_1 = 4\pi/3 + \sqrt{2} \eta_1$,

$$S = \frac{A^{d-2}}{t} \int d^dx \left[ \frac{1}{2} (\nabla \varphi)^2 + \lambda \Lambda^2 \left( \cos \varphi - 2 \left| \cos \left( \frac{\varphi}{2} \right) \right| \right) \right] \\
+ \frac{1}{2} \text{Tr} \ln \left( -\frac{A^{d-2}}{t} \nabla^2 + \frac{3\lambda \Lambda^2}{2t} \left| \cos \left( \frac{\varphi}{2} \right) \right| \right).$$

When we neglect the kinetic term $-\nabla^2$, this action is reduced to the previous effective free energy. We ensure that the spatial variation of $\varphi$ field is very slow so that we can perform the

**Phase transition at finite temperature**

Figure 2. Potential terms $V_1$, $V_2$ and $V_{\text{total}} = V_1 + V_2$ as a function of $\varphi \equiv 4\pi/3 + \sqrt{2} \eta_2$ for $t/\lambda = 1$ and $A = 1$. $V_1 = \cos (\varphi) - 2t \cos (\varphi/2)t$ and $V_2 = 2t \cos (\varphi/2)t - (t - \lambda) \ln (2\pi t)(2\lambda/\pi) \cos (\varphi/2)t)$. $V_1$ and $V_2$ are symmetric with respect to the axis of $\varphi = \pi$. The total potential $V_{\text{total}} = V_1 + V_2$ has a minimum at $\varphi = \pi$ due to the logarithmic term. Minima of $V_1$ correspond to the state of chirality $\kappa = 1$ and $\kappa = -1$, respectively.
$k$-summation for $-V^2 = k^2$. In the two-dimensional case ($d = 2$), the effective free-energy density is obtained as

$$ \frac{f[\varphi]}{\Lambda^2} = \frac{1}{2} K \Lambda^2 A^2 (V\varphi)^2 + \epsilon_0 \left( \cos \varphi - 2 \left| \cos \left( \frac{\varphi}{2} \right) \right| \right) + \frac{1}{2} k_B T \frac{c}{4\pi} \ln \left( \frac{c \Lambda^d t}{2t} + \frac{3\lambda \Lambda^d}{2t} \left| \cos \left( \frac{\varphi}{2} \right) \right| \right) + k_B T \frac{3\lambda}{16\pi} \left| \cos \left( \frac{\varphi}{2} \right) \right| \ln \left( 1 + \frac{2c}{3\lambda} \left| \cos \left( \frac{\varphi}{2} \right) \right| \right) \right|^{-1}, $$

where we have chosen the cutoff $k_0$ in the momentum space as $k_0^2 = c \Lambda^2$ for a constant $c$.

The spatial fluctuation softens the thermal fluctuation effect and there is a finite critical temperature where the minimum at $\varphi = 4\pi/3$ disappears and simultaneously the chirality vanishes. We show the potential term as a function of $\varphi$ for $t = 2c$ and $\lambda = c$ with $c = 4\pi$ in figure 3 where we subtracted the term $k_B T \ln \Lambda^d$ which is independent of $\varphi$ (or equivalently we set $\Lambda = 1$). We have a minimum at $\varphi = \pi$ when $t$ is large as shown in figure 3. The critical temperature $t_c$ is scaled by $\lambda/c$:

$$ t_c = t_c(\lambda/c). $$

$t_c$ is estimated by the equation $V(\varphi = 4\pi/3) = V(\varphi = \pi)$, which gives

$$ \frac{k_B T_c}{K \Lambda^2} = t_c = \frac{\lambda}{\frac{c}{4\pi} \ln \left( 1 + \frac{3\lambda}{4c} \right) + \frac{3\lambda}{16\pi} \ln \left( 1 + \frac{4c}{3\lambda} \right)}. $$

For small $\lambda \to 0$, $t_c$ is small: $t_c \approx 16\pi/(3 \ln (1/3\lambda))$. When $\lambda$ is large, $\lambda \gg 1$, $t_c$ is also large $t_c \approx 4\pi\lambda/c \ln (3\lambda/4c)$. In figure 4 we show the potential for $t/c = 0.5, 0.935, 2$, and $\lambda/c = 1$.
with $c = 4\pi$. When $t$ is small, the potential has a minimum at $\varphi = 4\pi/3$ or at $\varphi = 2\pi/3$ indicating that the ground state has the $2\pi/3$ structure with the chirality $\pm 1$. In contrast, when $t$ is large, we have a minimum $\varphi = \pi$. There is a transition at finite temperature $t = t_c$. This is a first-order transition since we have the double-minimum potential in the range $\pi \leq \varphi \leq 2\pi$.

This should be called a weak first-order transition because the change of $V_{\text{total}}(\varphi = \pi)$ is slow as $t$ is varied near the critical temperature. The minimum point of $\varphi$ changes gradually from $\pi$ and changes suddenly to $\pi$ at the critical temperature. For $t > t_c$, the $\eta_2$-mode represents a massless boson. We show $t_c$ as a function of $\lambda/c$ in figure 5.

We discuss a relation to the classical XY model on a two-dimensional triangular lattice. The ground state of the 2D XY model has the $2\pi/3$-structure to minimize energy. There is a

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4}
\caption{Potential as a function of $\varphi$ for $t/c = 0.5$, 0.935 and 2, respectively, where we set $\lambda/c = 1$ and $c = 4\pi$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5}
\caption{$t_c$ as a function of $\lambda/c$. where we set $\lambda/c = 1$ and $c = 4\pi$. $t_c$ is a increasing function of $\lambda/c$.}
\end{figure}
transition of the chirality at finite temperature. The critical temperature $T_c$ is of the order of the exchange coupling $J$ because $\lambda/t = J/k_BT$ in this case. The Kosterlitz–Thouless (KT) transition also occurs in the XY model on the 2D triangular lattice. The critical temperature of the KT transition $T_{KT}$ is determined by the renormalization group equation. In general, $T_{KT}$ is different from the critical temperature of the chiral transition $T_{chiral} \equiv K\Delta^2 t_c$.

A similar phenomenon occurs for an $n = 4$ theory with the potential

$$V = \frac{\lambda A^d}{t} \left[ \cos (\theta_1 - \theta_2) + a \cos (\theta_1 - \theta_3) + \cos (\theta_1 - \theta_4) + \cos (\theta_2 - \theta_3) + a \cos (\theta_2 - \theta_4) + \cos (\theta_3 - \theta_4) \right],$$

(20)

where $a \geq 1$ is a constant. This model has a close relation with the 2D antiferromagnetic XY model on a square lattice [16, 17]. One of the ground states is given by $(\theta_1, \theta_2, \theta_3, \theta_4) = (0, \theta, \pi, \theta + \pi)$ where real $\theta$ is arbitrary and the ground state is degenerate with respect to $\theta$. We define $\varphi_1 = \theta_1 - \theta_2 - \theta_3 + \theta_4 = \eta_1$, $\varphi_2 = \theta_1 + \theta_2 - \theta_3 - \theta_4 = \eta_2 - 2\pi$, $\varphi_3 = \theta_1 - \theta_2 + \theta_3 - \theta_4 = \eta_3 - 2\theta$, and the total phase $\Phi = \theta_1 + \theta_2 + \theta_3 + \theta_4$. Then the potential becomes

$$V = \frac{\lambda A^d}{t} \left[ -2a + \frac{1}{4}(a - \cos \theta)\eta_1^2 + \frac{1}{4}(a + \cos \theta)\eta_2^2 + \cdots \right],$$

(21)

where $\cdots$ indicates higher order terms. The $\eta_1$-mode becomes massless and the ground state energy $-2a$ is independent of $\theta$. This is the $n - 3$ series state [18] which we call the type I. When $a = 1$, the $\eta_1$- or $\eta_2$-mode is massless in the case $\theta = 0$ or $\pi$. This is the $n - 2$ series state. The effective potential $V_{\text{eff}}$ is obtained by integrating out the $\eta_1$ and $\eta_2$ variables in a similar way to the case $n = 3$ in two dimensions:

$$\frac{V_{\text{eff}}}{k_B T \Lambda^2} = \frac{1}{2} \ln \left( (4\pi + \lambda a)^2 - \lambda^2 \cos^2 \theta \right)$$

$$+ \frac{1}{8\pi} \ln \left( \frac{(4\pi + \lambda a)^2 - \lambda^2 \cos^2 \theta}{\lambda^2 (a^2 - \cos^2 \theta)} \right)$$

$$+ \frac{1}{8\pi} \cos \theta \ln \left( \frac{4\pi + \lambda (a + \cos \theta) \lambda (a - \cos \theta)}{4\pi + \lambda (a + \cos \theta) \lambda (a - \cos \theta)} \right).$$

(22)

where we used the cutoff $k_0$ in the momentum integral satisfying $k_0^2 / (4\pi) = \Lambda^2$. The potential is shown in figure 6 for several parameters where the ground state is at $\theta = m\pi$ for an integer $m$. This indicates that a Nambu–Goldstone boson emerges for $a = 1$ as a result of fluctuation of the $U(1)$ phase variables. We can regard the sign of $\sin \theta$ as a kind of chirality. The emergence of a new massless boson is accompanied by the vanishing of chirality.

We can generalize our argument to an $n$-component scalar field with Josephson couplings. The potential

$$V = \frac{\lambda A^d}{t} \sum_{i < j} \cos (\theta_i - \theta_j),$$

(23)

has a series of massless bosons; there are two types of ground states called type I and II [18]. In ground state I one has $n - 3$ massless bosons and in ground state II one has $n - 2$ massless
bosons. (The $n - 2$ series exists only for even $n$.) Two ground states I and II are degenerate for the potential $V$. However, ground state II becomes more stable than state I due to fluctuation effect. Thus, when we are in ground state I first, the fluctuation effect leads us to state II with an increase in the number of Nambu–Goldstone bosons.

**Order to order transition by disorder**

The chiral transition considered in this paper is a transition from the $2\pi/3$-structure in figure 1(a) (or (b)) to the antiferromagnetic state in figure 1(c). We can say that the ordered state with a massless boson in figure 1(c) is induced by disorder, namely, thermal fluctuation. We call this an order to order transition by disorder. We discuss here the fluctuation effect on the induced Nambu–Goldstone boson. For this purpose, we write $\phi_2 = \phi_1 + \phi_1$ so that $\phi_1$ indicates the fluctuation mode in the neighborhood of $\pi$. The action is written as

$$S = \frac{\Lambda^{d-2}}{t} \int d^dx \left[ \frac{1}{2} (\nabla \phi_1)^2 + \frac{1}{2} \lambda \Lambda^2 \phi_1^2 + \frac{1}{6} (\nabla \phi_2)^2 - \lambda \Lambda^2 \phi_1 \cos \left( \frac{\phi_2}{2} \right) \right].$$

(24)

The $\phi_2$-mode is obviously a massless mode, but there is an interaction with $\phi_1$. This interaction will generate an effective potential of $\phi_2$ that is proportional to $\cos^2(\phi_2/2) = (\cos \phi_2 + 1)/2$. Then, the effective action for $\phi_2$ is given by the sine-Gordon model:

$$S_{\phi_2} = \frac{\Lambda^{d-2}}{t} \int d^dx \left[ \frac{1}{6} (\nabla \phi_2)^2 - \frac{\lambda}{4} \Lambda^2 \cos \phi_2 \right].$$

(25)

The low-energy property is determined by the values of $\lambda$ and $t$ as indicated by the renormalization group equations [26, 27] near two dimensions. The critical value of $t$, denoted by $t_{2c}$, is $t_{2c} = 8\pi/3$. We assume that $t > t_c > t_{2c}$. When $\lambda$ is small, $\lambda$ is renormalized to 0 following the renormalization flow. This indicates that the $\phi_2$-mode remains massless for small $\lambda$. When $\lambda$ is large, $\lambda$ is renormalized to be a large value, showing that the potential term dominates the behavior of $\phi_2$-mode and then that $\phi_2$ takes the value near 0. In this case the

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**Figure 6.** Effective potential as a function of $\theta$ for $n = 4$. From the top, we set $\lambda = 1$ and $a = 1.2$, $\lambda = 1$ and $a = 1.001$ and $\lambda = 0.5$ and $a = 1.2$, respectively. The potential has minima at $\theta = m\pi$ for integer $m$. 
massless $\phi_2$-mode becomes massive, that is, a gapped mode again. Basically $\phi_2$-mode may remain massless because the Josephson coupling $\lambda$ is small in real superconductors.

Summary

We have proposed a mechanism of fluctuation induced Nambu–Goldstone bosons. In an $n$-component scalar field theory with frustrated Josephson interactions, massless bosons appear due to fluctuations at finite temperatures. In the 3-component theory discussed in the paper, a massless boson appears and the chirality vanishes as the temperature is increased, that is, the $Z_2$-symmetry breaking is driven by the chirality. This shows that nature prefers massless bosons. In fact, in an $n$-component model, the ground state at absolute zero will flow into the state with more massless bosons as the temperature is increased from $(n - 3)$-state to $(n - 2)$-state.

The excitation modes in our model are analogous to the vibration modes of a molecule CH$_2$. Two modes, the scissoring mode and the rocking mode, are important in determining the excitation spectra of CH$_2$ [28, 29]. The modes shown by $\phi_1 = \theta_3 - \theta_1$ and $\phi_2 = \theta_1 - 2\theta_2 + \theta_3$ represent the scissoring and rocking modes, respectively. In our model, the rocking mode plays a significant role. The fluctuation effect of the rocking mode becomes large as the temperature is increased and gives rise to the phase transition. The model presented in the paper appears as an effective free energy in multi-band superconductors [13–15, 19–23]. Low energy excitation states are important in superconductors, and the existence of massless modes have been pointed out [18, 21, 24, 25]. In this paper we have presented a new mechanism of the emergence of Nambu–Goldstone bosons.

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