Hadron spectra in p+p collisions at RHIC and LHC energies

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Received Day Month Year
Revised Day Month Year

We present the systematic analysis of transverse momentum ($p_T$) spectra of identified hadrons in p+p collisions at RHIC ($\sqrt{s} = 62.4$ and 200 GeV) and at LHC energies ($\sqrt{s} = 0.9, 2.76$ and 7.0 TeV) using phenomenological fit functions. We review various forms of Hagedorn and Tsallis distributions and show their equivalence. We use Tsallis distribution which successfully describes the spectra in p+p collisions using two parameters, Tsallis temperature $T$ which governs the soft bulk spectra and power $n$ which determines the initial production in partonic collisions. We obtain these parameters for pions, kaons and protons as a function of center of mass energy ($\sqrt{s}$). It is found that the parameter $T$ has a weak but decreasing trend with increasing $\sqrt{s}$. The parameter $n$ decreases with increasing $\sqrt{s}$ which shows that production of hadrons at higher energies are increasingly dominated by point like qq scatterings. Another important observation is with increasing $\sqrt{s}$, the separation between the powers for protons and pions narrows down hinting that the baryons and mesons are governed by same production process as one moves to the highest LHC energy.

Keywords: Tsallis distribution; Hadron spectra.

PACS numbers: 13.85.Ni, 24.85.+p

1. Introduction

The heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) are performed to study strongly interacting matter at high energy density. The measurements in $p+p$ collisions are important to understand particle production mechanism and are also used as baseline for heavy ion (HI)
collisions. The hadron spectra provide insight into particle production as well as interaction in the hadronic and quark gluon plasma (QGP) phases. The hadron spectra at high transverse momentum ($p_T$) arise from fragmentation of high $p_T$ partons or jets from initial hard partonic collisions and are known to follow a power law distribution. The high $p_T$ hadrons are very important for QGP studies as they measure jet quenching$^3$ effect in QGP. The low $p_T$ hadrons form the bulk of the spectra arising from multiple scatterings and follow exponential distribution descriptive of particle distribution in a thermal system which may be more meaningful for heavy ion system. In addition, for heavy ion systems the hadron spectra at intermediate $p_T$ can arise from quark recombination.$^4, 5$

We review various forms of Hagedorn and Tsallis distributions which are used to describe p+p collisions at RHIC and LHC energies and show their equivalence. The Tsallis distribution$^6, 7$ can reproduce the full spectral shape of hadrons with just two parameters; temperature $T$ and a non-extensive parameter $q$ which is related to a measure of temperature fluctuations and the degree of non-equilibrium in the system.$^8$ The Tsallis functional form is mathematically same as Hagedorn distribution$^9$ with the parameter $q$ related to power $n$ of Hagedorn distribution which governs initial partonic collisions.$^{10}$ The Hagedorn function has been successfully used to describe the meson spectra in p+p collisions at RHIC energies with an additional parameter to describe heavy ion collisions as well.$^{11}$

In p+p collisions, the parameter $T$ governs soft collisions$^2$ which is related to freeze out temperature for larger systems produced in relativistic heavy ion collisions after collective effects are separated out. The parameter $n$ gives a good idea of initial production. A value of $n$ close to 4 indicates point-like qq scattering (leading twist) while a large value of $n$ is indicative of multiple scattering centers (higher twist effect).$^{12, 13}$ A successful description of particle spectra with the Tsallis distribution allow us to calculate the integrated yield which provide important information on the bulk properties of the soft particle production and also used to infer the degree of chemical equilibration when compared with the thermal model.$^{14}$

In the present work we use Tsallis distribution fit to draw systematics from the transverse momentum ($p_T$) spectra of identified hadrons measured in p+p collisions at RHIC ($\sqrt{s} = 62.4$ and 200 GeV) and at LHC energies ($\sqrt{s} = 0.9$, 2.76 and 7.0 TeV). We obtain the Tsallis parameters $T$ and $n$ for pions, kaons and protons and study them as a function of center of mass energy.

2. Particle spectra and the Tsallis distribution

The characteristics of transverse momentum spectra of particles produced in proton-proton or heavy ion collisions are keys to understand the particle production mechanisms. Hagedorn$^8$ successfully described the shape of transverse mass $m_T$ spectra of hadrons produced in heavy ion collisions in terms of two parameters by this form
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\[ E \frac{d^3N}{dp^3} = A \left( 1 + \frac{m_T}{p_0} \right)^{-n}. \]  

Here \( A, p_0 \) and \( n \) are fit parameters. The parameter \( A \) is related to \( dN/dy \) which we will show little later. At low transverse momenta it assumes an exponential form and at large transverse momenta it becomes a power law which mimics “QCD inspired” quark interchange model \(^{10}\) as follows:

\[ \left( 1 + \frac{m_T}{p_0} \right)^{-n} \approx \exp \left( -\frac{nm_T}{p_0} \right), \quad \text{for } p_T \to 0 \]  

\[ \approx \left( \frac{m_T}{p_0} \right)^{-n}, \quad \text{for } p_T \to \infty. \]  

The Hagedorn function has been successfully used to describe the meson spectra in p+p collisions at RHIC energies with an additional parameter to describe heavy ion collisions as well \(^{11}\).

The Tsallis distribution \(^{6,7}\) describes a thermal system in terms of two parameters \( T \) and \( q \) and is given by

\[ G_q(E) = C_q \left( 1 + (q-1) \frac{E}{T} \right)^{-1/(q-1)}. \]  

Here \( C_q \) is the normalization constant, \( E \) is the particle energy, \( T \) is the Tsallis temperature and \( q \) is the so-called nonextensivity parameter which measures the temperature fluctuations \(^3\) in the system as: \( q-1 = \text{Var}(T)/\langle T \rangle^2 \). The values of \( q \) lie between 1 < \( q < 4/3 \). For \( q \to 1 \), the distribution corresponds to an equilibrated system described by a pure exponential (Boltzmann-Gibbs) type distribution \(^{15,16}\)

\[ G_q(E) \to C_1 \exp \left( -\frac{E}{T} \right). \]  

Using the relations \( E = m_T = \sqrt{p_T^2 + m^2} \) at mid-rapidity and \( 1/(q-1) = n \), Eq. 4 takes the form

\[ E \frac{d^3N}{dp^3} = C_n \left( 1 + \frac{m_T}{nT} \right)^{-n}, \]  

which is same as Eq. 1 with \( p_0 = nT \). Larger values of \( q \) correspond to smaller values of \( n \) describe a system away from thermal equilibrium. In terms of QCD, smaller values of \( n \) imply dominant hard point-like scattering. Phenomenological studies suggest that, for quark-quark point scattering, \( n \sim 4 \) \(^{12,13}\) and when multiple scattering centers are involved \( n \) grows larger and can go up to 20 for protons.

In order to associate the Tsallis distribution with a probability distribution, which describes the invariant particle spectra defined over \( 0 < E < \infty \), Eq. 6 must satisfy a normalization and energy conservation condition. Using the unit
normalization condition, we can determine the co-efficient $C_n$, the resulting formula used for fitting the hadron spectra used by PHENIX\textsuperscript{17} and is given by

\[
E \frac{d^3N}{dp^3} = \frac{1}{2\pi} \frac{dN}{dy} \frac{(n-1)(n-2)}{(nT + m(n-1))(nT + m)} \left( \frac{nT + mT}{nT + m} \right)^{-n}
\] (7)

In some of the papers\textsuperscript{7,18} another variant of Tsallis distribution is written as

\[
G_q(E) = C_q \left(1 + (q-1)\frac{E}{T} \right)^{-q/(q-1)}
\] (8)

The corresponding normalized distribution in terms of $n$ can be obtained by replacing $n$ by $(n - 1)$ in Eq. (7).

The Tsallis distribution used by some publications of PHENIX\textsuperscript{19} STAR\textsuperscript{20,21} and CMS\textsuperscript{22} is of the form given by

\[
E \frac{d^3N}{dp^3} = \frac{1}{2\pi} \frac{dN}{dy} \frac{(n-1)(n-2)}{(nC + m(n-2))(nC)} \left(1 + \frac{mT - m}{nC} \right)^{-n}
\] (9)

This is same as Eq. (7) with the parameter $C$ of Eq. (9) related to $T$ by $nC \rightarrow nT + m$. In summary, all the forms of Hadron and Tsallis distribution described above are essentially same with parameter of one form is related to the parameter of the other form by a definite relation. We will use the form given in Eq. (7) in rest of the analysis.

3. Results and Discussions

In the present work, the measured $p_T$ distribution of hadrons are taken from PHENIX ($|y| < 0.35$)\textsuperscript{17,19,23} STAR ($|y| < 0.5, |y| < 0.75$)\textsuperscript{20,21} and CMS ($|y| < 1.0$)\textsuperscript{22} experiments. The average yields of charged pions, charged kaons and protons measured at all RHIC ($\sqrt{s} = 62.4$ and 200 GeV) and LHC energies ($\sqrt{s} = 0.9, 2.76$ and 7.0 TeV) are used in the analysis. The errors on the data are taken as quadratic sums of statistical and uncorrelated systematic errors wherever available.

First, we fit all the measured hadron (pion, kaon and proton) spectra with Tsallis distribution (Eq. (7) keeping both the parameters $T$ and $n$ as free. Figure 1 shows the variation of parameters $T$ and $n$ as a function of $\sqrt{s}$ along with their parameterizations by a function: $a - b(\sqrt{s})^{-\alpha}$. We notice that the parameter $T$ has a decreasing but weak dependence on $\sqrt{s}$ and the parameter $n$ has a definite decreasing trend with increasing $\sqrt{s}$. In case of pions, both the parameters vary smoothly as a function of collision energy. The solid line in Fig. 1(a) is obtained by fitting $n$ for all particles with a single parameterized form. The solid line in Fig. 1(b) is the same for $T$ for all particles. These curve mostly pass through the pion points.

There is correlation between parameters $T$ and $n$, if any of the two increases, the other also increases and thus we need to fix one of them by some method and study
the behaviour of the other as a function of particle type and $\sqrt{s}$. The parameter $T$ as defined in Eq. 7 can be assumed to be same for all particles at a particular energy. We can fix it to the value averaged over all particle types or simply use pion $T$ for all the particles. Thus first we fit the measured pion spectra with Tsallis distribution by keeping parameters $T$ and $n$ as free and then fix $T$ for kaon and proton spectra to get $n$.

![Graph](image.png)

Fig. 1. The variation of Tsallis parameters (a) $n$ and (b) $T$ as a function of center of mass energy $(\sqrt{s})$ for pion, kaon and proton. The solid curve in each figure are the function $a - b(\sqrt{s})^{-\alpha}$, fitted over all particles.
Figure 2 shows the invariant yields of pions, kaons and protons as a function of $p_T$ for p+p collisions at (a) $\sqrt{s} = 62.4$ GeV, and (b) $\sqrt{s} = 200$ GeV.

Figure 3 shows the invariant yields of pions, kaons and protons as a function of $p_T$ for p+p collisions at (a) $\sqrt{s} = 0.9$ TeV, (b) $\sqrt{s} = 2.76$ TeV.

Figure 4 shows the invariant yields of pions, kaons and protons as a function of $p_T$ for p+p collisions at 7.0 TeV.

The solid curves in all the figures are the fitted Tsallis Distribution of Eq. (7). In case of pions both $T$ and $n$ are kept free during fits. For kaons and protons the values of $T$ is fixed to the value obtained for pions. The values of $T$ and $n$ for pions and $n$ for kaons and protons so obtained are given in Table 1 at different collision energies. It can be noted that we get good $\chi^2$ values for fits at all energies with such procedure. In Fig 5, upper plot (a) shows the variation of Tsallis parameter $n$, and the lower plot (b) shows the variation of ratio $n_{\text{proton}}/n_{\text{pion}}$ and $n_{\text{kaon}}/n_{\text{pion}}$ as functions of $\sqrt{s}$. It is observed that the parameter $n$, for pions, kaons and protons, is monotonically decreasing with increasing $\sqrt{s}$, which could be understood in terms of changing production mechanism at different collision energies.

The QCD cross sections scale as $1/p_T^n$ in terms of the power $n = 2n_a - 4$ where $n_a$ is the number of participating quarks. If the dominant process in meson or baryon production is point like scattering $qq \rightarrow qq$ (referred as leading twist) the number of participating quarks is 4 and hence $n = 4$. The power can go up due to contribution of higher twists processes explained in the following. The mesons scattering on quark $q\pi \rightarrow q\pi$ gives $n = 2 \times 6 - 4 = 8$. For proton production if the subprocess $qq \rightarrow qqqq$ is dominant then $n = 8$. Proton scattering process on quark $(qp \rightarrow qp)$ gives $n = 12$. The value of $n$ for pions could go up to 16 for process $p\pi \rightarrow p\pi$ and that for protons could be 20 corresponding for process $pp \rightarrow pp$.

The decreasing value of parameter $n$, for pions, kaons and protons with collision energy in Table 1 indicates that the dominant production process is moving from higher twist to leading twist as one goes up in $\sqrt{s}$. We find that the separation of $n$ among pions, kaons and protons decreases as we go from lower center of mass energy (at RHIC) to higher center of mass energy (at LHC). This indicates that at higher collision energies the production process is more like $qq$ point scattering for all particles which gives lower and similar values of $n$ for different particles.

Lastly, we study all hadron spectra at all energies using fixed values of $T$. From the free fit results given in Fig 1(b) we choose two values $T = 110$ MeV and $T = 95$ MeV and fit all hadron spectra to obtain power $n$ for pions, kaons and protons as a function of $\sqrt{s}$. The results for $T = 110$ MeV and 95 MeV are given in Figs. 6(a) and (b) respectively. We find that for both the values of $T$, the parameter $n$ as a function of $\sqrt{s}$ decreases which is noticed in all our analysis. Although the values of $n$ at $T = 110$ MeV are greater than their respective values at $T = 95$ MeV the general conclusions about $n$ remain the same.
4. Conclusion

In the present work we use Tsallis fit to draw systematic trends from the transverse momentum spectra of identified hadrons measured in p+p collisions at RHIC and LHC energies. We also review various forms of Hagedorn and Tsallis distributions and show their equivalence. We obtain the Tsallis parameters $T$ and $n$ for pions,

$$T = 117.6 \text{ MeV} \pm 0.12$$

and kaons,

$$n_{\pi^+} = 11.87 \pm 0.48$$

$$n_{\pi^-} = 10.66 \pm 0.20$$

$$n_{K^+} = 15.51 \pm 0.35$$

and protons,

$$n_{p} = 9.30 \pm 0.12$$

$$n_{p^+} = 8.48 \pm 0.13$$

$$n_{p^-} = 11.84 \pm 0.14$$

Fig. 2. The invariant yields of pions, kaons and protons as a function of $p_T$ at (a) $\sqrt{s} = 62.4$ GeV and (b) $\sqrt{s} = 200$ GeV for p+p collisions. The solid curves are the fitted Tsallis distributions.
kaons and protons and study them as a function of center of mass energy. Since $T$ and $n$ are correlated we fix the value of $T$ for all particles at a particular energy to the pion temperature. In general, the Tsallis temperature $T$ has a decreasing but weak dependence on center of mass energy which means there are less soft collision processes as we move up in energy. The power $n$ determines if the particle is coming from point like qq scatterings (leading twist) or from multiple scattering centers.
Fig. 4. The invariant yields of pions, kaons and protons as a function of $p_T$ for $p+p$ collision at 7.0 TeV. The solid curves are the fitted Tsallis distribution.

involving many quarks (higher twists). The power $n$ decreases with decreasing $\sqrt{s}$ for all particles in all our analysis which indicates that the production process is moving from higher twist to leading twist as one goes up in $\sqrt{s}$. The maximum values of $n$ are found to be 16 for proton spectra 12 for pion spectra at the lowest RHIC energy. Another important observation is with increasing $\sqrt{s}$, separation of powers for protons and for pions narrows down hinting that the baryons and mesons are governed by same production process as one moves to the highest LHC energy. Separation of $n$ for protons and pions is more at lower energy. This would imply that higher twist process prefer pion production over proton at lower energies.

5. Acknowledgements

We acknowledge the financial support from Board of Research in Nuclear Sciences (BRNS) for this project. Prashant Shukla thanks his CMS colleagues for their comments.

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Fig. 5. (a) shows the variation of Tsallis parameter $n$ as a function of $\sqrt{s}$. Here the value of parameter $T$ obtained from fitting pion spectra has been used in kaon and proton spectra. and (b) shows the variation of $n_{\text{proton}}/n_{\text{pion}}$ and $n_{\text{kaon}}/n_{\text{pion}}$ as a function of $\sqrt{s}$.

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Fig. 6. The variation of Tsallis parameter $n$ as a function of $\sqrt{s}$, when $T$ is kept fixed at (a) 110 MeV and (b) at 95 MeV.

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Table 1. The values of Tsallis Parameters \( T \) and \( n \) for pions, kaons and protons. In case of pions both \( T \) and \( n \) are kept free during fits. For kaons and protons the values of \( T \) is fixed to the value obtained for pions.

| \( \sqrt{s} \) (TeV) | \( T \) (MeV) | \( n \) | \( \frac{dN}{dy} \) | \( \chi^2/ndf \) |
|----------------------|--------------|--------|-----------------|-----------------|
| **For pion**         |              |        |                 |                 |
| 0.062                | 117.6 ± 1.2  | 11.87 ± 0.48 | 0.812 ± 0.03   | 0.19            |
| 0.2                  | 112.6 ± 2.1  | 9.30 ± 0.12  | 9.204 ± 0.05   | 4.52            |
| 0.9                  | 104.5 ± 3.1  | 7.16 ± 0.22  | 1.917 ± 0.014  | 1.18            |
| 2.76                 | 97.5 ± 2.6   | 6.08 ± 0.17  | 2.44 ± 0.02    | 1.04            |
| 7.0                  | 94.4 ± 2.8   | 5.55 ± 0.15  | 3.083 ± 0.03   | 0.88            |
| **For Kaon**         |              |        |                 |                 |
| 0.062                | 117.6        | 10.66 ± 0.28 | 0.072 ± 0.002  | 0.17            |
| 0.2                  | 112.6        | 8.48 ± 0.13  | 0.493 ± 0.008  | 0.68            |
| 0.9                  | 104.5        | 6.53 ± 1.40  | 0.238 ± 0.002  | 0.07            |
| 2.76                 | 97.5         | 5.41 ± 0.08  | 0.322 ± 0.000  | 0.74            |
| 7.0                  | 94.4         | 4.70 ± 0.10  | 0.418 ± 0.006  | 0.33            |
| **For Proton**       |              |        |                 |                 |
| 0.062                | 117.6        | 15.51 ± 0.35 | 0.007 ± 0.00   | 0.35            |
| 0.2                  | 112.6        | 11.84 ± 0.14 | 0.083 ± 0.003  | 1.18            |
| 0.9                  | 104.5        | 8.95 ± 0.07  | 0.105 ± 0.00   | 1.06            |
| 2.76                 | 97.5         | 7.28 ± 0.06  | 0.137 ± 0.001  | 1.35            |
| 7.0                  | 94.4         | 6.07 ± 0.07  | 0.177 ± 0.001  | 1.05            |