Pnictogen height as a possible switch between high-$T_c$ nodeless and low-$T_c$ nodal pairings in the iron based superconductors

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We study the effect of the lattice structure on the spin-fluctuation mediated superconductivity in the iron pnictides adopting the five-band models of several virtual lattice structures of LaFeAsO as well as actual materials such as NdFeAsO and LaFePO obtained from the maximally-localized Wannier orbitals. Random phase approximation is applied to the models to solve the Eliashberg equation. This reveals that the gap function and the strength of the superconducting instability are determined by the cooperation or competition among multiple spin fluctuation modes arising from several nestings among disconnected pieces of the Fermi surface, which is affected by the lattice structure. Specifically, the appearance of the Fermi surface $\gamma$ around $(\pi, \pi)$ in the unfolded Brillouin zone is sensitive to the pnictogen height $h_{\text{Pn}}$ measured from the Fe plane, where $h_{\text{Pn}}$ is shown to act as a switch between high-$T_c$ nodeless and low-$T_c$ nodal pairings. We also find that reduction in the lattice constants generally suppresses superconductivity. We can then combine these to obtain a generic superconducting phase diagram against the pnictogen height and lattice constant. This suggests that NdFeAsO is expected to exhibit a fully-gapped, sign-reversing d-wave superconductivity with a higher $T_c$ than in LaFeAsO, while a nodal pairing with a lower $T_c$ is expected for LaFePO, which is consistent with experiments.

I. INTRODUCTION

The discovery of superconductivity in the iron-based compounds by Hosono’s group and subsequent increase in the transition temperature ($T_c$) exceeding 50 K$^2$ in the same family of compounds are seminal not only because of high values of $T_c$, but also because this poses a fundamental question on electronic mechanisms of high $T_c$ superconductivity in a wider class of compounds other than cuprates.

Theoretically, a phonon mechanism was shown to be unlikely for this system$^3$ and a spin-fluctuation mediated pairing has been proposed from the very early stage of the study$^3$. In these studies the nesting between disconnected pieces (pockets) of the Fermi surface is shown to induce spin fluctuations associated with the nesting vector. This can give rise to a superconducting gap, which is basically $s$-wave but changes sign between different pockets, hence termed as $s$±wave or sign-reversing $s$-wave first proposed by Mazin et al.$^4$ (see Fig.1). Although recent experimental as well as theoretical studies suggest that the magnetism in the undoped material is not driven solely by Fermi surface nesting, the spin fluctuation originating from the nesting has been considered to be a possible origin of the pairing interaction by a number of authors.$^5,7,8,9,10,11,12,13,14,15,16,17,18$

In particular, the present authors with Tanaka and Kontani have constructed a minimal model, which has turned out to be five-band, for LaFeAsO based on first principles calculation, and investigated spin-fluctuation mediated superconductivity with random-phase approximation (RPA).$5,19,20,21$ In that study it was pointed out that, along with the sign-reversing d-wave, a d-wave pairing can also be a candidate depending on the band filling. Our five-band model has subsequently been adopted in various studies, among which are a perturbation study by Nomura$^{22}$, a fluctuation exchange study by Ikeda$^{13}$, and a functional renormalization group study by Wang et al.$^{14}$ An analysis on the normal state spin dynamics has also been performed using our five-band model$^{22}$, where good agreement with inelastic neutron scattering experiments$^{23,24}$ has been obtained. On the other hand, Graser et al.$^{25,26}$ recently applied RPA to a five-band model that is based on a band structure obtained by Cao et al.$^{27}$ The study finds that a sign-reversing s-wave that has nodes intersecting the Fermi surface closely competes with d-wave pairing (see Fig.1). It has further been proposed in ref.$^{28}$ that this nodal s-wave pairing is intrinsic to the iron pnictide superconductors, while a full gap can occur as a consequence of the presence of impurities. It is also worth noting that a competition or mixture of sign-reversing s and d pairings have also been discussed on the basis of a 16 band $d$-$p$ model$^{12}$ and a two-orbital
exchange coupling model \((J_1-J_2 \text{ model})\). It is the purpose of the present paper to explore systematically the material- and structure-dependences on the strength and gap symmetry of superconductivity in terms of the five-band model.

Experimentally, the fully-gapped, sign-reversing \(s\)-wave scenario is consistent with a number of measurements on arsenides, such as angle-resolved photoemission spectroscopy (ARPES),\(^{30,31}\) penetration depth measurements,\(^{32}\) and muon spin relaxation \((\mu SR)\),\(^{33,34,35,36}\) which suggest that the gap is open on the whole Fermi surface, although the magnitude of the gap may vary along the surface. The fully-gapped, sign-reversing \(s\) is also consistent with some neutron scattering results\(^{37,38}\) that observe a resonance peak predicted theoretically.\(^{39,40,41}\) On the other hand, the weak effect of nonmagnetic impurities such as Co on \(T_c\) or even the appearance of superconductivity upon Co doping\(^{42}\) has cast doubt on the sign-reversing gap, but some theoretical studies\(^{43,44,45}\) have shown that these experiments in fact do not necessarily contradict with the sign-reversing gap. In particular, Senga and Kontani showed that the effect of the inter-pocket scattering due to nonmagnetic impurities becomes irrelevant in the unitarity limit.\(^{46,47}\)

One interesting and important feature in the iron pnictides is the unusually strong dependence of \(T_c\) on materials, which ranges from \(\approx 5\) K in LaFePO\(^{48}\) to 55 K in SmFeAsO\(^{2}\) even within the same group of elements. More systematically, Lee \textit{et al.}\(^{50}\) pointed out that we can parametrize the value of \(T_c\) in terms of the Fe-Pn-Fe (Pn: pnictogen) bond angle \(\alpha\), where \(T_c\) seems to have a peak around the bond angle \((\approx 109)\) at which the pnictogens form a regular tetrahedron, while \(T_c\) is low for materials with large \(\alpha\) such as LaFePO. The importance of the bond angle has also been pointed out by Zhao \textit{et al.}\(^{51}\). On the other hand, it has also been shown by Miyazawa \textit{et al.} that the chemical trend for LnFeAsO has the maximum \(T_c\) increasing with the decreasing lattice constant \(a\) for Ln=La \(\rightarrow\) Nd, but nearly constant for Ln=Nd \(\rightarrow\) Dy.\(^{52}\) Pressure effects have also been experimentally elaborated. For LaFeAsO, \(T_c\) first increases with pressure, but then decreases when the pressure becomes too large,\(^{53,54,55,56}\) which is contrasted with materials having \(T_c \geq 50\) K at ambient pressure such as NdFeAsO for which \(T_c\) monotonically and rapidly decreases with pressure.\(^{57}\) All these experimental results indicate that \(T_c\) is unusually sensitive to the lattice structure in the iron-based compounds.

If we move on to the symmetry of the superconducting gap, we have various pieces of experimental evidence for strong material dependence as well: while a number of experiments on arsenides suggests that the gap is fully open on the Fermi surface as mentioned, a recent penetration depth measurement on LaFePO shows that there are nodes in the superconducting gap.\(^{58,59}\) Arsenides and LaFePO also exhibit sharp contrast in nuclear magnetic resonance (NMR) experiments. In LaFeAsO, some experiments show that the NMR relaxation rate \(1/T_1\) has no coherence peaks, and decays as \(T^3\) below \(T_c\).\(^{60,61,62,63,64,65}\) while a recent experiment by Kobayashi \textit{et al.}\(^{66}\) indicates a more rapid decay. In LaFePO, \(1/T_1\) below \(T_c\) is shown to decay even more slowly than above \(T_c\).\(^{67}\) These results strongly suggest that even the symmetry of the superconducting gap can be unusually sensitive to the lattice structure.

So the crucial theoretical question is how we can understand these sensitivities. In analyzing the structure dependence, there is one factor to which we have to pay attention. Previous theoretical studies have shown that the position of the pnictogen with respect to the Fe plane affects the band structure, in particular the character of the bands that lie close to the Fermi level near the \(\Gamma\) point (in the folded Brillouin zone) as well as the band width (see Fig.\(^7\)). Local spin-density approximation studies have shown that the tendency towards magnetism becomes stronger when the pnictogen lies farther from the Fe plane, which is expected to enhance superconductivity if the pairing is mediated by spin fluctuations.\(^{31}\)

Given this background, in the present study we investigate the lattice structure dependence of the spin-fluctuation mediated superconductivity, where we construct five-band models for several virtual lattice structures of LaFeAsO as well as actual materials such as NdFeAsO and LaFePO, and apply RPA to solve the Eliashberg equation. We shall show that the position of the pnictogen is indeed the key factor that determines both \(T_c\) and the form of the superconducting gap, namely, the “pnictogen height” above the Fe plane (Fig.\(^7\)) can act as a \textit{switch between a high-}\(T_c\), fully-gapped, sign-reversing \(s\)-wave and a low-\(T_c\), nodal \(s\)- or \(d\)-wave pairings. We also show that the reduction in the lattice constant is generally unfavorable for superconductivity. Combining these results for the lattice structure dependence, we then obtain a generic “phase diagram” against the pnictogen height and the lattice constants. Based on the phase diagram, we argue that the systematic dependence of \(T_c\) against the bond angle found in ref.\(^{60}\) can be accounted for as a combined effect of the pnictogen height and the lattice constants. In order to get higher \(T_c\), we propose to seek for materials that have high position of the pnictogen and large lattice constants simultaneously.

| FIG. 1: (Color online) Lattice structure of one Fe-Pn layer, with the pnictogen height indicated. | FIG. 1: (Color online) Lattice structure of one Fe-Pn layer, with the pnictogen height indicated. | FIG. 1: (Color online) Lattice structure of one Fe-Pn layer, with the pnictogen height indicated. | FIG. 1: (Color online) Lattice structure of one Fe-Pn layer, with the pnictogen height indicated. | FIG. 1: (Color online) Lattice structure of one Fe-Pn layer, with the pnictogen height indicated. |

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II. THE BAND STRUCTURE AND THE FERMI SURFACE

LaFeAsO has a tetragonal layered structure, where Fe atoms form a square lattice in each layer, which is sandwiched by As atoms (Figs. 12(a)). Due to the tetrahedral coordination of As, there are two Fe atoms per unit cell. The experimentally determined lattice constants are \(a = 4.036\, \text{\AA}\) and \(c = 8.739\, \text{\AA}\), with two internal coordinates \(z_{\text{Fe}} = 0.142\) and \(z_{\text{As}} = 0.6512\). We have obtained the band structure (Fig. 2(b)) with the local density approximation with a plane-wave basis.

We then construct the maximally-localized Wannier functions (MLWFs). These MLWFs, centered at the two Fe sites in the unit cell, have five orbital symmetries \((d_{\Delta Z}, d_{XYZ}, d_{XZ}, d_{XZ-YZ}, d_{XY})\), where \(X, Y, Z\) refer to those for the unit cell with two Fe sites as shown in Fig. 2(a)). The two Wannier orbitals in each unit cell are equivalent in that each Fe atom has the same local arrangement of surrounding atoms. We can then take a unit cell that contains only one orbital (for each orbital symmetry) by unfolding the Brillouin zone, and we end up with an effective five-band model on a square lattice, where \(x\) and \(y\) axes are rotated by 45 degrees from \(X-Y\), to which we refer for all the wave vectors hereafter. We define the band filling \(n\) as the number of electrons/number of sites (e.g., \(n = 10\) for a full filling). The doping level \(x\) in LaFeAsO_{1-x}Fe_x is related to the band filling as \(n = 6 + x\).

The five bands are heavily entangled as shown in Fig. 2(b) reflecting the strong hybridization of the five 3d orbitals, which is physically due to the tetrahedral coordination of As atoms around Fe. Hence we conclude that the minimal electronic model requires all the five bands. In Fig. 2(b), the Fermi surface at \(k_z = 0\) for \(n = 6.1\) (corresponding to \(x = 0.1\)) is shown in the unfolded Brillouin zone. The Fermi surface consists of four pieces (pockets in 2D): two concentric hole pockets (denoted as \(\alpha_1, \alpha_2\)) around \((k_x, k_y) = (0, 0)\), two electron pockets around \((\pi, 0)\) \((\beta_1)\) or \((0, \pi)\) \((\beta_2)\), respectively. Besides these pieces of the Fermi surface, there is a portion of the band around \((\pi, \pi)\) that touches the \(E_F\) at \(n = 6.1\), so that this portion acts as a “quasi Fermi surface” (which we call \(\gamma\)). As for the orbital character, \(\alpha\) and some portions of \(\beta\) near the Brillouin zone edge have mainly \(d_{XZ}\) and \(d_{YZ}\) character, while the portions of \(\beta\) away from the Brillouin zone edge and \(\gamma\) have mainly \(d_{XZ-YZ}\) orbital character. An interesting feature in the band structure is the presence of Dirac cones, i.e., places where the upper and the lower bands make a conical contact. The ones closest to the Fermi level correspond to the crossing points of the \(d_{XZ-YZ}\) and the \(d_{XZ}/d_{YZ}\) bands below the \(\beta\) Fermi surface.

III. MANY-BODY HAMILTONIAN AND RANDOM-PHASE APPROXIMATION

For the many body part of the Hamiltonian, we consider the standard interaction terms that comprise the intra-orbital Coulomb \(U\), the inter-orbital Coulomb \(U'\), the Hund’s coupling \(J\), and the pair-hopping \(J'\). The many body Hamiltonian then reads

\[
H = \sum_{\mu} \left[ \epsilon_{\mu} n_{\mu \sigma} + \sum_{\mu' \sigma'} \sum_{\sigma} t_{\mu \mu'} c_{\mu \sigma} \right] c_{\mu' \sigma'} + U \sum_{\sigma} n_{\mu \sigma} n_{\mu \sigma} + U' \sum_{\mu > \nu} \sum_{\sigma, \sigma'} n_{\mu \sigma} n_{\nu \sigma'}
\]


\(-J \sum_{\mu \neq \nu} S_{\mu \nu} \cdot S_{\mu \nu} + J' \sum_{\mu \neq \nu} e_{\mu \mu}^{\dagger} e_{\nu \nu}^{\dagger} c_{\mu \mu}^{\dagger} c_{\nu \nu}^{\dagger} \) \hspace{1cm} (1)

where \( i, j \) denote the sites and \( \mu, \nu \) the (five \( d \)) orbitals, and \( t_{ij}^{\mu \nu} \) is the obtained in the previous section. The orbitals \( d_{x^2-r^2}, d_{xz}, d_{yz}, d_{x^2-y^2}, d_{xy} \) are labeled as \( \nu = 1, 2, 3, 4, \) and \( 5 \), respectively. As for the electron-electron interactions, there have been theoretical studies that estimate the parameter values. Some studies give \( U = 2.2 - 3.3 \) and \( J = 0.3 - 0.6^{78,79} \) in units of eV, while others have \( U \sim J \sum \). Here we assume that \( U > J \) and take the values \( U = 1.2, U' = 0.9 \), and \( J = J' = 0.15 \). We also examine orbital-dependent interactions as introduced in section VC. We have taken the values somewhat smaller than those obtained in ref \( \ref{78,79} \) because the self-energy correction is not taken into account in the present RPA calculation, so that small interaction parameters are needed to avoid magnetic ordering at high temperatures.

Having constructed the model, we move on to the five-band RPA calculation, where the modification of the band structure due to the self-energy correction is not taken into account. Multiorbital RPA is described in e.g. ref.\( \ref{78,79} \). In the present case, Green’s function \( G_{\text{im}}(k) \) (\( k \equiv (k, i \omega_n) \)) is a \( 5 \times 5 \) matrix. The irreducible susceptibility matrix

\[ \chi_{i_1i_2i_3i_4}(q) = \sum_{k} G_{i_1i_2}(k + q) G_{i_3i_4}(k) \] \hspace{1cm} (2)

\((i_1 = 1, ..., 5) \) has \( 5^4 \) components, and the spin and the charge (orbital) susceptibility matrices are obtained from matrix equations,

\[ \hat{\chi}_s(q) = \frac{\hat{\chi}_0(q)}{1 - S\hat{\chi}_0(q)} \] \hspace{1cm} (3)

\[ \hat{\chi}_c(q) = \frac{\hat{\chi}_0(q)}{1 + C\hat{\chi}_0(q)} \] \hspace{1cm} (4)

where

\[ S_{i_1i_2i_3i_4} = \begin{cases} U, & l_1 = l_2 = l_3 = l_4 \\ U', & l_1 = l_3 \neq l_2 = l_4 \\ J, & l_1 = l_2 \neq l_3 = l_4 \\ J', & l_1 = l_4 \neq l_2 = l_3, \end{cases} \] \hspace{1cm} (5)

\[ C_{i_1i_2i_3i_4} = \begin{cases} U, & l_1 = l_2 = l_3 = l_4 \\ -U' + J, & l_1 = l_3 \neq l_2 = l_4 \\ 2U' - J, & l_1 = l_2 \neq l_3 = l_4 \\ J', & l_1 = l_4 \neq l_2 = l_3. \end{cases} \] \hspace{1cm} (6)

We denote the largest eigenvalue of the spin (charge) susceptibility matrix for \( i \omega_n = 0 \) as \( \chi_s(k) \) \( (\chi_c(k)) \).

The Green’s function and the effective singlet pairing interaction,

\[ \hat{V}^s(q) = \frac{3}{2} \hat{S}\hat{\chi}_s(q)\hat{S} - \frac{1}{2} \hat{C}\hat{\chi}_c(q)\hat{C} + \frac{1}{2} (\hat{S} + \hat{C}), \] \hspace{1cm} (7)

are plugged into the linearized Eliashberg equation,

\[ \lambda \phi_{i_1i_4}(k) = -\frac{T}{N} \sum_q \sum_{l_2l_3l_6} V_{i_1i_2i_3i_4}(q) \times G_{i_2i_3}(k - q) \phi_{i_3i_4}(k - q) G_{i_3i_6}(q - k). \] \hspace{1cm} (8)

The \( 5 \times 5 \) matrix gap function \( \phi_{\text{im}} \) in the orbital representation along with the associated eigenvalue \( \lambda \) is obtained by solving this equation. The gap function can be transformed into the band representation with a unitary transformation. The calculation is performed at \( T = 0.02 \) eV taking a three dimensional \( k \)-point mesh of \( 32 \times 32 \times 4 \) and 512 Matsubara frequencies. All the results for the spin susceptibility and the superconducting gap will be presented for the lowest Matsubara frequency and at \( k_\epsilon = 0 \) or \( q_\epsilon = 0 \). The eigenvalue of the Eliashberg equation \( \lambda \) at the fixed temperature of 0.02eV will be adopted as a measure of the strength of the superconducting instability, since directly obtaining \( T_c \), especially for low \( T_c \) systems, requires more \( k \)-point meshes and Matsubara frequencies.

IV. ORBITAL-DEPENDENT NESTING AND THE PAIRING SYMMETRY COMPETITION

Let us first look in Fig\[3\] at the result for the (orbital-diagonal components of) spin susceptibility, \( \chi_{ss\alpha\beta} \) and \( \chi_{ss\gamma\gamma} \), which are the two largest components. \( \chi_{ss\gamma\gamma} \) has peaks solely around \((\pi, 0)\) and \((0, \pi)\), which reflects the nesting between \( d_{xz}, d_{yz} \) portions of \( \alpha \) and \( \beta \) pockets as shown in the lower panel of Fig\[3\] where the thickness of the Fermi surface represents the strength of the \( d_{xz} \) - \( d_{yz} \) character. On the other hand, \( \chi_{ss\alpha\beta} \) has peaks around \((\pi, 0), (0, \pi)\) and \((\pi, \pi/2), (\pi/2, \pi)\). The former is due to the nesting between the \( \gamma \) pocket and the \( d_{xz} - d_{yz} \) portion of the \( \beta \) pocket, while the latter originates from the nesting between the \( d_{xz} - d_{yz} \) portion of the \( \beta \) and \( \beta' \) \( \ref{5,21,25,26,80} \).

The superconducting gap should be determined by the cooperation or competition between the multiple nestings mentioned above. Specifically, the \( \alpha \)-\( \beta \) and \( \gamma \)-\( \beta \) nestings tend to favor the fully-gapped, sign-reversing \( s \)-wave, in which the gap changes sign between \( \alpha \) and \( \beta \) but has a constant sign on each pocket as shown in Fig\[4\]. On the other hand, \( \beta_1 \)-\( \beta_2 \) nesting tends to change the sign of the gap between these pockets, which can result in either \( d \)-wave or an \( s \)-wave pairing with nodes on the \( \beta \) Fermi surface, as shown schematically in Fig\[2,7,20\]. For the band structure of LaFeAsO (obtained by using the experimentally determined lattice structure), the sign-reversing \( s \)-wave with no nodes intersecting the Fermi surface dominates for the present set of parameter values with \( n = 6.1 \) as shown in Fig\[5\]. The eigenvalue of the Eliashberg equation at \( T = 0.02 \) eV is \( \lambda = 0.90 \) for \( s \)-wave, against \( \lambda = 0.54 \) for \( d \)-wave.

As for the band filling dependence, we plot the eigenvalue of the Eliashberg equation of \( s \)- and \( d \)-wave pairings in Fig\[4,a\]. We can see for the band structure
of LaFeAsO with the present set of interaction values that the sign-reversing \( s \)-wave pairing with a full gap for each pocket dominates for the band filling \( n \leq 6.2 \). For \( n \geq 6.3 \), the \( \gamma \) pocket becomes less effective, and the \((\pi,0)\) peak in \( \chi_{4444} \) disappears as seen in the right panel of Fig. 6(b). The \( \alpha \) pocket becomes less effective as well, and the \((\pi,0)\) peak in \( \chi_{3333} \) becomes small. Thus in this region, \( d \)-wave pairing begins to dominate, and the subdominant \( s \)-wave gap has nodes almost touching the \( \beta \) as seen in the right panel of Fig. 6(c). For small doping levels when the \( \gamma \) Fermi surface is effective and \( s \)-wave dominates, the magnitude of the \( s \)-wave gap has maxima at the positions along the \( \beta \) pocket facing the \( \Gamma \) point (Fig. 4 lower left), but when the doping increases to \( n = 6.3 \), the \( s \)-wave gap has minima at these points. The gap turns out to be nearly constant on the \( \beta \) Fermi surface (Fig. 4(c), left) for the band filling \( n \approx 6.2 \). In this case, the gap on \( \alpha \) (not shown) and \( \beta \) have nearly the same magnitude.

Note however that the present analysis on the band filling dependence does not take account of the doping dependence of the band structure itself, which should occur mainly due to the change of the As position caused by doping. We will come back to this point in section VI B taking NdFeAsO as an example.

V. THE EFFECT OF THE LATTICE STRUCTURE

A. Pnictogen height dependence

We now investigate the effect of the “pnictogen height \( h_{Pn} \)”, namely, the distance between a pnictogen atom and the Fe layer, \((z_{Pn} - 0.5) \times c\), where \( z_{Pn} \) is the internal coordinate of the pnictogen atom and \( c \) the c-axis lattice constant. As shown in previous studies, \( z_{Pn} \) controls the relative position of \( d_{X^2-Y^2} (= d_{xy}) \) and \( d_{Z^2} \) bands near the \( \Gamma \) point of the folded (original) Brillouin zone. In the unfolded Brillouin zone, these bands appear near \((\pi,\pi)\). In Fig. 7 we show the band structure in
the unfolded Brillouin zone for virtual lattice structure with $z_{As} = 0.658$ and 0.6304 with the lattice constants and $z_{La}$ fixed at the original values for LaFeAsO. For the original $z_{As} = 0.651$, the As height is $h_{As} = 1.32\text{Å}$. For $z_{As} = 0.658$, $h_{As}$ increases to 1.38Å, which is the same as in the optimally doped NdFeAsO, while $z_{As} = 0.6304$ ($h_{As} = 1.14\text{Å}$ corresponds to the height of P in LaFePO).

We see that the $d_{X^2-Y^2}$ band that forms the $\gamma$ Fermi surface around $(\pi, \pi)$ rises as $h_{As}$ is increased, while the $d_{Z^2}$ band sinks below the Fermi level. The reason for these shifts in the band positions can be understood in terms of the hopping integrals. As $h_{As}$ increases, the nearest-neighbor hopping for the $d_{X^2-Y^2}$ orbital decreases as shown in Table I. If we approximate the $d_{X^2-Y^2}$ portion of the bands by

$$
\varepsilon_{X^2-Y^2}(k) = -2t_{X^2-Y^2}[\cos(k_x) + \cos(k_y)] - 4't_{X^2-Y^2} \cos(k_x) \cos(k_y),
$$

where $t'_{X^2-Y^2}$ stands for the second-nearest-neighbor hopping, the energy difference between $(0, 0)$ and $(\pi, \pi)$ is proportional to $t_{X^2-Y^2}$ (Fig 2c), so that the reduction in $t_{X^2-Y^2}$ acts to push up the $d_{X^2-Y^2}$ band at $(\pi, \pi)$. Besides the variation in the $d_{X^2-Y^2}$ hoppings, the increase in $h_{As}$ results in an overall reduction in the hopping integrals of other orbitals because the effective hopping path Fe→As→Fe becomes less effective.

The effect of varying $h_{As}$ on the spin susceptibility is shown in Fig 3. The $d_{Y^2}$ orbital component always has peaks around $(\pi, 0), (0, \pi)$ reflecting the $\alpha$-$\beta$ nesting. On the other hand, the $d_{X^2-Y^2}$ orbital component of the spin susceptibility $\chi_{s\pi444}$ exhibits a strong variation with $h_{As}$: when $h_{As}$ is large and the $\gamma$ pocket is present, $\chi_{s\pi444}$ (Fig 3a, right) is strongly peaked at $(\pi, 0)$, reflecting the $\gamma$-$\beta$ nesting and also the strong electron correlation due to the overall reduction in the band width. However, as $h_{As}$ is reduced, the structure around $(\pi, \pi/2)$, which arises from the $\beta_1$-$\beta_2$ nesting, dominates (Fig 3b, right). The effect of reducing $h_{As}$ resembles the effect of electron doping, but in the case of electron doping, not only the effect of the $\gamma$ pocket, but also that of the $\alpha$ becomes weak, so that $\chi_{s\pi333}$ is suppressed.

The effect on the spin susceptibility in turn affects superconductivity. When $h_{As}$ is large, $\chi_{s\pi444}$ and $\chi_{s\pi333}$ spin fluctuations near $(\pi, 0)$ cooperate to mediate the fully-gapped, sign-reversing $s$-wave superconductivity. When $h_{As}$ is small, by contrast, the $(\pi, \pi/2)$ spin fluctuations begin to favor $d$-wave and nodal $s$-wave pair-
FIG. 6: (Color online) (a) Eigenvalues of the Eliashberg equation for s-wave and d-wave, respectively, for the five-band model of LaFeAsO plotted against the band filling \( n \). The light red (or gray) symbol for the s-wave at \( n = 6.3 \) indicates that the gap is nearly nodal. (b) \( \chi_{s333} \) and \( \chi_{s444} \) for \( n = 6.3 \). (c) The s-wave gap function for band 4 with \( n = 6.2 \) (left) and \( n = 6.3 \) (right).

In Fig. 7, we plot against \( z_{As} \) (lower scale) or against \( h_{As} \) (upper scale) the eigenvalue of the Eliashberg equation for the s-wave and d-wave pairings, respectively. For large \( h_{As} \), where the fully-gapped, sign-reversing s-wave (Fig. 10, upper right) dominates, \( \lambda \) is large because the strong spin fluctuations arising from \( \alpha-\beta \) and \( \gamma-\beta \) nestings cooperate. This is contrasted with the case of small \( h_{As} \), where d-wave or nodal s-wave begin to dominate (Fig. 10, left). In this region \( \lambda \) is small because the \( \gamma-\beta \) nesting is no longer effective, or to be more precise, its remaining effect competes with the effect of the \( \beta_1-\beta_2 \) nesting. It is worth noting that if we adopt \( z_{As} = 0.638 \), which is the value determined by theoretical structure optimization, we have the closely competing d-wave and nodal s-wave pairings. This is consistent with a recent RPA calculation by Graser et al., who adopted a band structure determined by a theoretical structure optimization. The message of the present analysis, then, is that the pnictogen height can act as a “switch” between the fully-gapped, sign-reversing high-\( T_c \) s-wave and the low-\( T_c \) gapless (either d-wave or nodal s-wave) superconductivity.

B. Lattice constant dependence

We now turn to the effect of the lattice constants. We consider virtual lattice structures where one of the lattice constants, \( a \) or \( c \), is varied, while the pnictogen height is fixed at the original value for LaFeAsO. When we reduce the lattice constant \( a \), we find that the nearest-neighbor hopping \( t_{X^2-Y^2} \) decreases, probably because the Fe-As-Fe angle is reduced, which may cause a suppression of the effective hopping via the path Fe→As→Fe. Nonetheless, most of the other in-plane hopping integrals (including the ones not listed in the table) are enhanced as intuitively expected. On the other hand, a reduction in the lattice constant \( c \) is found to mainly enhance the in-plane \( d_{X^2-Y^2} \) hopping (apart from the obvious enhancement of the hopping in the \( c \) direction). This may be because the As wave function is pushed toward the Fe plane for a reduced layer-layer distance, and the hopping between \( d_{X^2-Y^2} \) orbitals, which is elongated in the direction of the As atom positions, is enhanced by this deformation.

The eigenvalue of the Eliashberg equation is plotted...
as functions of the lattice constants in Fig.11. We find that the reduction in the lattice constants tends to suppress superconductivity, which can be attributed to the increased hopping integrals and associated suppression of the electron correlation. In fact, the reduction in \( z_\text{As} \) (c) enhances the \( XZ, YZ \) (\( X^2 - Y^2 \)) hopping integrals, which leads to suppressed \( \chi_{3333} \) (\( \chi_{4444} \)) as seen from the comparison between Fig.8 and the lower panels of Fig.11. The effect of reduced lattice constants is small for the competition between \( s \) and \( d \) waves (i.e., two curves move roughly in parallel). We note here that, although an increased \( h_{\text{As}} \) and a decreased lattice constant \( a \) both lead to a reduction in the Fe-As-Fe bond angle \( \alpha \), they have opposite effects on the eigenvalue of the Eliashberg equation (compare Figs.9 and 11).

**C. Effect of the orbital-dependent interactions**

In refs.75,76 it is pointed out that the interaction parameters have significant orbital dependence. In ref.75, the intraorbital repulsions are \( U = 3.27, 2.77, 2.20, \) and 3.31 (eV) for \( d_{3z^2-R^2} \), \( d_{XZ/YZ} \), \( d_{X^2-Y^2} \), and \( d_{XY} \) orbitals, respectively. This variation comes from the fact
that each Fe 3$d$ orbital hybridizes with As 4$p$ quite differently. Namely, while the local basis of the five-band model, \{d$^i$\}, can be represented as a linear combination of atomic \(d^i\) and \(\tilde{p}^i\) orbitals (as \(\alpha_i d^i + \sum_j \beta_{ij} \tilde{p}^j\)), the coefficients \(\alpha_i, \beta_{ij}\) have a strong orbital dependence. For example, the ratio \(\beta_{ij}/\alpha_i\) is large for \(i = X^2 - Y^2\) but small for e.g. \(i = 3Z^2 - R^2\). Therefore, if we adopt a common value for the interaction parameters for all five orbitals in the five-band model, electron correlations are relatively overestimated for \(X^2 - Y^2\), because \(\tilde{p}\) orbitals are more weakly correlated than \(d\) orbitals. In order to avoid this problem, we should use orbital dependent interactions, where the interaction for \(i = X^2 - Y^2\) is small compared to others.

So we study in this section the effect of the orbital dependence of the interactions, taking into account the orbital dependence of \(U', J, J'\) as well. Since the self-energy correction is not taken into account in RPA, it is again necessary to reduce the interactions to avoid magnetic ordering at high temperatures. Here we multiply all the interaction parameters in ref. \[33\] by a factor of \(f = 0.42\), so the intraorbital interaction is taken to be 1.37, 1.16, 0.92, and 1.39 (eV) for \(d_{3Z^2-R^2}, d_{XZ/YZ}, d_{X^2-Y^2}, \) and \(d_{XV}\) orbitals, respectively. Since the \(d_{X^2-Y^2}\) orbital has the smallest intraorbital interaction (0.92eV in the present calculation as compared with 1.2eV in the calculation for orbital-independent interactions), the effect of the \(d_{X^2-Y^2}\) orbital is expected to be reduced compared with the results obtained by using the orbital independent interactions.

First, we take the model for LaFeAsO to study the band-filling dependence. As shown in Fig. 12(a), we find that the s-wave pairing becomes nodal for \(n > 6.2\), i.e., the s-wave becomes nodal for smaller electron doping compared to the case with orbital independent interactions (see Fig. 6(a)). Also, the nodal s-wave pairing still slightly dominates over d-wave even at \(n = 6.3\), at which, for the case of orbital-independent interactions, the s gives way to d. If we turn to the pnictogen-height dependence of the eigenvalue of the Eliashberg equation in Fig. 12(b), the s-wave is again enhanced with the increased height, but the enhancement is smaller than in the case of Fig. 9 which can be attributed to the reduction in the \(\gamma\) (namely, the \(d_{X^2-Y^2}\) Fermi surface effect). As for the lattice constant dependence depicted in Fig. 12(c)/(d), we find that the reduction in \(c\) suppresses \(\lambda\), while that of \(c\) has small effect. This is because the reduction in \(c\) mainly enhances the \(d_{X^2-Y^2}\) hopping, and the suppression of the electron correlation within this orbital has small effect when the intraorbital interaction is small.

Another effect of adopting orbital-dependent interac-
VI. CALCULATION FOR ACTUAL MATERIALS

In this section, we calculate the band structure of actual materials other than LaFeAsO, i.e., the phosphate and Nd compound, using the experimentally determined lattice structure to construct the five-band model. The band filling will be fixed mainly at $n = 6.1$ to make a direct comparison with the results for LaFeAsO. The results are interpreted in view of the general trend obtained in the study of the virtual lattice structures.

A. LaFePO

The band structure of the five-band model for LaFePO is shown in Fig.14. In the case of LaFePO, the lattice constants are small compared to LaFeAsO (while closer to NdFeAsO below). However, the hopping integrals are similar to or larger than those for the virtual structure for LaFeAsO with $z_{As} = 0.6304$. Thus, the main difference from LaFeAsO is caused by the height of P. The top of the $d_{X^2-Y^2}$ band at $(\pi, \pi)$ is indeed pushed below the Fermi level, and this makes the $(\pi, \pi/2)$ spin fluctuations arising from the $\beta$-$\beta$ nesting dominate in $\chi_{s_{4444}}$ as shown in Fig.15.

This behavior in the spin fluctuation for LaFePO acts to make the $d$-wave pairing (Fig.15 right) dominate for the orbital-independent interactions, while the sign-reversing $s$-wave with nodes intersecting the $\beta$ Fermi

FIG. 14: (Color online) The band structure of the five-band model of the optimally doped NdFeAsO (upper panels) and LaFePO (lower). The Fermi surface at $k_z = 0$ for $n = 6.1$ is shown on the right.
FIG. 15: (Color online) Upper panels: $\chi_{s3333}$ and $\chi_{s4444}$ for the five-band model for LaFePO with $n = 6.1$, $U = 1.7$, $U' = 1.4$, and $J = J' = 0.15$ (orbital-independent interactions). Lower panels: $s$-wave (left) and $d$-wave (right) gap functions for bands 3 and 4 for the same parameter values.

The band structure of NdFeAsO is shown in Fig.14. The low-temperature lattice structure of the optimally doped sample (sample 4) in ref.50 is adopted here. Here, we have performed an LDA calculation with the plane wave basis set using the pseudopotential of Nd obtained with the open-core treatment for the $f$ electrons. The $d_{X^2-Y^2}$ band at $(\pi, \pi)$ is seen to cross the Fermi level even at $n = 6.1$ as expected, and the $s$-$d$ spin fluctuation strongly dominates in $\chi_{s4444}$ as shown in the right panel of Fig.17(a), and the fully-gapped, sign-reversing $s$-wave shown in Fig.18(a) strongly dominates over $d$-wave. For the orbital-independent interactions, the eigenvalue perconducting gap.58,59

B. NdFeAsO

The band structure of NdFeAsO is shown in Fig.14. The low-temperature lattice structure of the optimally doped sample (sample 4) in ref.50 is adopted here. Here, we have performed an LDA calculation with the plane wave basis set using the pseudopotential of Nd obtained with the open-core treatment for the $f$ electrons. The $d_{X^2-Y^2}$ band at $(\pi, \pi)$ is seen to cross the Fermi level even at $n = 6.1$ as expected, and the $s$-$d$ spin fluctuation strongly dominates in $\chi_{s4444}$ as shown in the right panel of Fig.17(a), and the fully-gapped, sign-reversing $s$-wave shown in Fig.18(a) strongly dominates over $d$-wave. For the orbital-independent interactions, the eigenvalue perconducting gap.58,59
of the Eliashberg equation is \( \lambda = 1.20 \) for \( s \)-wave, which is indeed greater than that for LaFeAsO (\( \lambda = 0.90 \)), but not as large as the virtual lattice structure of LaFeAsO where the As height is increased to the value of NdFeAsO (\( \lambda = 1.91 \)). The latter property can mainly be attributed to the reduction in the lattice constants \( a \) and \( c \) in NdFeAsO as compared to those of LaFeAsO.

When we adopt the orbital-dependent interactions introduced in section V C, the enhancement of \( \lambda \) from the LaFeAsO value is weaker than in the case of orbital-independent interactions, as expected from the previous discussion. Namely, \( \lambda_{\text{La}} = 0.70 \) and \( \lambda_{\text{Nd}} = 0.72 \) for the multiplication factor \( f = 0.42 \) while \( \lambda_{\text{La}} = 1.20 \) and \( \lambda_{\text{Nd}} = 1.32 \) for \( f = 0.45 \). The magnitude of the gap for the \( s \)-wave pairing is nearly constant on each of the all pockets as shown in Fig. 18(b), which is also expected from the argument in section V C.

We have also performed a calculation for NdFeAsO with the lattice structure in the underdoped regime (sample 1 in ref.50). Here we take the band filling of \( n = 6.03 \) and adopt orbital-independent interactions. As shown in Fig.17 the maximum value of \( \chi_{\text{4444}} \) (and also \( \chi_{\text{s}} \), not shown) is larger than in the case of the optimally doped lattice structure because the nesting is better for the underdoped case. Nonetheless, the eigenvalue of the Eliashberg equation is found to be smaller, \( \lambda = 1.03 \) for the \( s \)-wave as compared to \( \lambda = 1.2 \) for the optimally doped sample. This shows that removing the electrons to make the \( \gamma \) Fermi surface more effective does not necessarily favor superconductivity. This may be because lowering the Fermi level results in the decrease in the \( d_{X2−Y2} \) density of states on the \( \beta \) Fermi surface, where the \( d_{X2−Y2} \) band forms a Dirac cone. Doping the electrons raises the Fermi level, thereby increasing the \( d_{X2−Y2} \) density of states on the \( \beta \) Fermi surface, and at the same time increasing the pnictogen height, which pushes up the \( d_{X2−Y2} \) band at \( (\pi, \pi) \) so as to catch up with the raised Fermi level. The increased \( d_{X2−Y2} \) density of states upon doping is seen in the broad \( (\pi, 0) \) peak structure in \( \chi_{\text{4444}} \) in the optimally doped case as compared to that in the underdoped regime (Fig. 17).

The tendency that electron doping tends to increase the pnictogen height should be general because the increase in the negative charge in the Fe layers suppresses the attractive interaction between the positively charged iron and the negatively charged pnictogen. Therefore, the doping dependence of \( \lambda \) shown in Fig. 18(a) with a fixed band structure may be too naive in that the effect of the \( \gamma \) Fermi surface monotonically decreases with the higher electron doping. As for the doping dependence of superconductivity, other than the effect of the change in the band structure, there have also been theoretical studies that suggest the importance of the “unscreening effect” of the Coulomb interaction\(^{22} \), or the importance of the electron correlation\(^{24} \).

We have finally examined the effect of pressure on NdFeAsO. The lattice structure data for NdFeAsO\(_{1−y}\) under a pressure of 3.8GPa is taken from ref.81. Applying pressure on NdFeAsO tends to reduce \( h_{\text{As}} \) as well as the lattice constants. At 3.8GPa, the height is reduced to \( h_{\text{As}} = 1.33 \AA \). This results in a suppression of the eigen-value of the Eliashberg equation, and the \( s \)-wave eigen-
value is reduced regardless of the choice of the electron-electron interactions, e.g., for the orbital-independent interactions, $\lambda$ at $T = 0.02$ reduces (from 1.2) to 0.67. The suppression of $\lambda$ is at least qualitatively consistent with the experiment.\textsuperscript{57,83}

VII. DISCUSSIONS

A. Validity of the five-band model

One of the most important next steps in the microscopic study on superconductivity in the iron-based superconductors should be an examination of the present scenario based on the Fermi surface nesting by means of self-consistent calculations. Indeed, if we are interested in the behavior of the present five-orbital (d-only) model with moderate (realistic) size of the Hund coupling\textsuperscript{77} or the Coulomb interactions\textsuperscript{77,76} estimated by various \textit{ab initio} methods, we have obviously to take account of the self-energy (otherwise we have magnetic ordering at rather high temperatures), where the self-energy correction generally affects the effect of Fermi surface nesting, which is usually overestimated in RPA.

One possible way to go beyond RPA is employing the fluctuation exchange (FLEX) approximation.\textsuperscript{84} However, it has recently been recognized that FLEX does not work so successfully for the five-orbital model with moderate correlations. Namely, while we naively expect that the model should have a strong instability for the stripe-type antiferromagnetic ordering for undoped LaFeAsO, the spin susceptibility in FLEX has a peak at $(\pi, \pi)$, which corresponds to the checkerboard-type antiferromagnetic instability. Even in the weakly correlated regime, Ikeda\textsuperscript{52} had to introduce artificial level shifts for $d_{z^2}$ and $d_{x^2-y^2}$ to the original LDA band in order to avoid a large $d_{z^2}$ Fermi surface.

These problems seem to come from the fact that the self-energy correction generally has a strong orbital dependence in the five-orbital model rather than from some problems in FLEX. Even in the simple Hartree approximation for the paramagnetic case, the band structure and the Fermi surface dramatically change from those in LDA due to the Hartree field ($\sim U\langle n_i \rangle$), since the filling of each of the five orbitals varies so differently (e.g., $n \sim 0.8(0.5)$ for $d_{z^2}$ and $d_{x^2-y^2}$).

On the other hand, in the \textit{dpp} model that takes account of Fe $3d$ and As $4p$ and O $2p$\textsuperscript{72} for which five Fe $3d$ orbitals are similarly filled, we do not have such problems as far as we introduce a gap (the so-called double-counting term $\Delta$) that depends on the difference between the correlations in Fe $3d$ and As $4p$.\textsuperscript{75} In fact, this double-counting term in the \textit{dpp} model makes the situation in the five-orbital model subtle. For the \textit{dpp} model, we can safely assume that $\Delta$ does not have a serious orbital dependence. On the other hand, if we translate $\Delta$ in terms of the five-orbital model, we have to assume that $\Delta$ has a non-trivial orbital dependence, since each Fe $3d$ hybridizes with As $4p$ differently (e.g., while the hybridization between $d_{x^2-y^2}$ and As $4p$ is strong, those between $d_{z^2}$ and As $4p$ are weak). This should be one reason why Ikeda had to introduce an orbital-dependent level shift in his FLEX calculation.\textsuperscript{52}

Therefore, we believe that it is impossible to obtain any meaningful results in self-consistent calculation for the five-orbital model without considering the orbital-dependent double-counting term, while we have still no guarantee that the double-counting term can really make the five-orbital model mimic the original \textit{dpp} model. An interesting observation is that the situation is in sharp contrast with the case of high $T_c$ cuprates. For cuprates, aside from the issue of the validity of the single-band Hubbard model or the $t$-$J$ model, we can naively expect that the self-consistent solutions of these models can at least describe some model or the the $t$-$J$ model. This is why we consider this an important future problem.

B. Phase diagram

In the preceding sections, it has been shown that the pnictogen height can act as a switch between high $T_c$ nodeless and low $T_c$ nodal superconductivities. We have also shown that the increase in the lattice constants is unfavorable for superconductivity. These tendencies can be incorporated in a schematic phase diagram shown in Fig.19. In the upper panel, we take the pnictogen height as the horizontal axis, and lattice constants as the vertical axis. In LnFeAsO, the lattice constants decrease monotonically in the chemical trend La $\rightarrow$ Nd $\rightarrow$ Dy.\textsuperscript{52} On the other hand, the As height monotonically increases, and these effects may cancel with each other to result in a nearly constant $T_c$ between Ln=Nd and Dy.\textsuperscript{22} In the lower panel of Fig.19 we adopt the As-Fe-As bond angle $\alpha$ as the horizontal axis to make clear comparison with ref.50, in which it was shown that the maximum $T_c$ seems to be reached when the pnictogens form a regular tetrahedron. As schematically indicated by a curved arrow, the appearance of the maximum $T_c$ may be a consequence of the combined effect of the bond angle and the lattice constants. Thus, as far as the present theoretical study is concerned, the pnictogen height is a better parameter than the angle $\alpha$ to draw a phase diagram, since $\alpha$ is affected by both the height and the lattice constant $a$, which have opposite effects on $T_c$.

To attain higher $T_c$ on the basis of this phase diagram, it is desirable to have higher position of the pnictogen while keeping lattice constants not reduced. On the other hand, we have to keep in mind that such a variation in the lattice parameters also enhances the tendency toward magnetism. Experimentally, magnetic ordering...
The dashed lines are schematic contours of $T_c$agram on the h FIG. 19: (Color online) Schematic superconducting phase di-
tion among the so-called “spin density wave” state ob-
tivity . A better understanding of the competition be-
to remove the phase transition to result in superconduc-
tivity in that the magnetism/structural phase tran-
seem to be linked. In this sense, the shorthands for materials are the same as in Table I.

d is known to occur after the structural phase transition, and the tendency toward magnetism and the structural phase transition seem to be linked. In this sense, the superconductivity will have to compete with the magnetism/structural phase transition in a more severe manner as the height and/or the lattice constants are increased. Namely, too much increase in the height and/or the lattice constants can be unfavorable for superconductivity in that the magnetism/structural phase transition, which may take place at high temperatures, can dominate over superconductivity. In such a case, applying pressure to reduce the lattice constants and/or doping carriers can be effective to suppress the magnetism or the structural phase transition. This seems to be the case for the undoped LaFeAsO$_4$ and CaFe$_2$As$_2$ where structural phase transition and magnetism take place at ambient pressure, while applying pressure seems to remove the phase transition to result in superconductivity. A better understanding of the competition between superconductivity and magnetism should require further understanding of the magnetic state sitting next to the superconducting state. Namely, the mutual relation among the so-called “spin density wave” state ob-
served experimentally, the magnetic state obtained in first principles calculations, and the spin fluctuation obtained in the downfolded five-band models has to be made clearer.

Another point that should be kept in mind is the material dependence of the electron-electron interactions, which is not considered in the present phase diagram. In particular, the difference in the lattice structure in the 11 systems such as FeSe$^{93}$ and in the hole doped 122 systems such as BaFe$_2$As$_2$, can affect the effective electron-electron interaction within the FeAs planes. Also, the screening effect of the $f$ orbitals may be different between LaFeAsO and NdFeAsO. On the $f$ electrons, the present analysis adopts open-core treatment for the $f$ electrons of Nd, whose validity may have to be examined. We cannot completely rule out the possibility that the hy-
ridization between Nd$^f$ and Fed electrons can affect the electronic states and thus the superconductivity. In these senses, the actual phase diagram for the entire family of the iron-based superconductors should have more axes than presented here.

VIII. CONCLUSION

In the present study, we have investigated how the lattice structure affects the spin-fluctuation mediated superconductivity in iron pnictides. The obtained picture is that the gap function and $T_c$ are determined by the competition or cooperation of the multiple spin fluctuation modes arising from $\alpha$-$\beta$, $\beta_1$-$\beta_2$, and $\gamma$-$\beta$ Fermi surface nestings, which depends on the materials, band filling, and/or pressure. In particular, the competition between $\beta_1$-$\beta_2$ and $\gamma$-$\beta$ nestings within portions of the Fermi surface having strong $d_{x^2-y^2}$ character governs the form of the superconducting gap as well as the strength of the superconducting instability. The relevance of the $\gamma$ (quasi) Fermi surface is determined by the pnictogen height, and consequently, the pnictogen height plays the role of a switch between high-$T_c$, nodeless and low-$T_c$, nodal pairings, which may give the answer to the question of why the form of the superconducting gap as well as $T_c$ are vastly different between LaFeAsO and LaFePO. An intriguing observation is, since $d$-wave and the nodal $s$-wave tend to be closely competing for low pnictogen heights, there is a possibility of exotic pairing such as $s+id_{xy}$ 键 The lattice constant also affects superconductivity in the manner that the reduction in $a$ (c) mainly suppresses the electron correlation within $d_{x^2-y^2}/d_{XZ}$ ($d_{XZ}$-$d_{YZ}$) orbitals and thus degrades superconductivity. A schematic phase diagram has been obtained by combining the effect of the pnictogen height and the lattice constants.

The low $T_c$ in cases where the $\beta_1$-$\beta_2$ nesting dominates over $\gamma$-$\beta$ nesting can be naturally understood in the sense that there is a kind of frustration between $\beta_1$-$\beta_2$, $\alpha$-$\beta_1$, and $\alpha$-$\beta_2$ nestings in determining the form of the gap because the sign of the gap has to be changed across each.

FIG. 19: (Color online) Schematic superconducting phase diagram on the $h_{pn}$-$[a,c]$ (upper) and $\alpha$-$[a,c]$ (lower) planes. The dashed arrow in the lower panel schematically indicates how the lattice parameters vary as the bond angle $\alpha$ is decreased in ref.50. The shorthands for materials are the same as in Table I.
of the multiple nesting vectors (see Fig.1). Conversely, the high $T_c$ in the case where the $\gamma$-$\beta$ nesting, along with $\alpha$-$\beta$, dominates over $\beta_1$-$\beta_2$ is natural in that the unfrustrated gap fully opens on all five disconnected pieces of the Fermi surface, $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$, and $\gamma$. In this sense, high $T_c$ in iron pnictides can be understood as a realization of the theoretical proposal that one can look for high $T_c$ superconductors in systems with disconnected Fermi surfaces.\cite{94,95}

The form of the $s$-wave gap is shown to be nonuniversal, even when the gap is fully open, and its variation along the Fermi surface strongly depends on the band structure (i.e. the lattice structure), the band filling, and the electron-electron interactions. When the $s$-wave gap varies significantly along the Fermi surface, it is also expected to be affected by the presence of the impurities, as pointed out by Mishra et al.\cite{101} In this sense, the form of the gap should experimentally be determined by a combination of multiple experiments on the same material, desirably on the same sample. From this viewpoint, the discrepancy between the NMR experiments for LaFeAsO and other experiments suggesting nearly isotropic gap may be a consequence of the nonuniversality of the superconducting gap, especially because LaFeAsO lies close to the nodeless/nodal boundary. In fact, anisotropic $s\pm$-wave pairing where the gap varies strongly on the $\beta$ Fermi surface has been proposed to explain the $T^3$ decay in the NMR experiment.\cite{102} This view may also give some clue as to why some of the tunneling spectroscopy measurements exhibit zero bias conductivity peak,\cite{103} which is an indication of unconventional sign reversing pairing,\cite{104} while others do not.\cite{105} It is worth noting that recent theoretical studies show that it is unlikely to observe the zero bias conductivity peak for the fully gapped sign reversing $s$-wave pairing.\cite{106,107}

In the present study, we have focused on the LnFeAsO (1111) systems. In the 11 systems such as FeSe, experiments under pressure also suggest strong structure dependence of superconductivity.\cite{108,109,110,111} However, there may be some discrepancies from the 1111 systems regarding the lattice structure dependence since the LnO layer is not present. Also, the extremely high position of the chalcogen atom in 11 systems (1.47Å in FeSe\cite{112} and 1.76Å in FeTe\cite{113}) may affect the lattice structure dependence. Our study focusing on the 11 system is now underway. Similarly, the hole-doped 122 systems are also expected to have some discrepancies with the electron-doped 1111 systems, which also deserves future study.

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