Connection between dark matter abundance and primordial tensor perturbations

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Primordial inflation and Dark Matter (DM) could both belong to the hidden sector. It is therefore plausible that the inflaton, which drives inflation, could couple to the DM either directly or indirectly, thus providing a common origin for both luminous and non-luminous matter. We explore this novel possibility and show that in certain scenarios the DM mass can be correlated with the tensor-to-scalar ratio. This unique correlation can provide us with a new window of opportunity for unraveling the properties of DM.

Primordial inflation is one of the simplest paradigms to explain the formation of large scale structures in our Universe, and in particular, to understand the observed features of the Cosmic Microwave Background spectrum \cite{1}. Inflation is driven by the vacuum energy density of a scalar field, known as the inflaton, whose origin usually requires some new physics beyond the Standard Model (SM). For a review on inflationary models, see e.g. \cite{2}.

On the other hand, various astrophysical and cosmological observations strongly suggest the existence of a non-luminous, non-baryonic form of matter, known as the Dark Matter (DM), which follows a universal density profile as an essential ingredient for the hierarchical structure formation \cite{3}, as suggested by detailed N-body simulations \cite{4}, and semi-analytics \cite{5}.

Although we still do not know the masses and interactions of either inflaton or DM, we do know that both must couple to the SM in some way, if not directly. Technically speaking, they can be considered to be SM gauge singlets, and therefore, could both belong to the dark or the hidden sector. In this case, one can imagine that the net DM, which we assume to be cosmologically stable, can be created via two processes:

(a) Decays: The inflaton $\phi$ could directly couple to the DM $\chi$ via renormalizable interactions, as shown in Fig. 1(a). For concreteness, we assume the DM to be fermionic, so that the interaction is of the Yukawa type, i.e. $\phi \chi \bar{\chi}$. We could also have imagined a scalar DM with a trilinear coupling to inflaton. Apart from the inflaton itself, any heavy hidden sector scalar field $X$ could also directly decay to DM via renormalizable interactions.

(b) Scatterings: Inflaton must couple to the SM degrees of freedom (d.o.f) for the success of Big Bang Nucleosynthesis. Since the SM fermions are chiral, a SM singlet inflaton can in principle couple to the right-handed (RH) fermions $\bar{\psi}_R$ via renormalizable interactions of the form $\phi \bar{\psi}_R \psi_R$. Therefore, by virtue of the previous point, we would have a scenario, where we can create DM via inflaton mediation, as shown in Fig. 1(b). One can generalize this scenario to envisage that any heavy scalar mediator $X$ could connect the SM d.o.f with the dark sector.

A rather natural outcome of this simple scenario is that the scale of inflation, determined by the inflaton potential $V(\phi)$, is correlated with the DM properties in a rather intriguing way. The DM can in principle keep the memory of how it was excited at the first instance, either (a) directly via the inflaton decay, or (b) indirectly via scatterings mediated by the inflaton or a heavy scalar field. In case (a), the DM is essentially decoupled from the thermal bath since its creation. This leads to the usual non-thermal DM scenario, where the DM relic abundance is directly determined by the initial inflaton energy density \cite{6}. In case (b), if the effective coupling of the DM to the SM d.o.f is too small to fully thermalize the DM with the bath, but sufficient enough to produce the observed abundance of DM, this leads to the Feebly Interacting Massive Particle (FIMP) or freeze-in DM scenario \cite{7}.

The aim of this paper will be to show that, in either of the two scenarios discussed above, it is possible to establish a deep connection between the DM and inflaton sectors via the scale of inflation, which can be determined by measuring the primordial tensor-to-scalar ratio. Note that for the usual thermal DM scenario with a relatively large DM coupling to the SM d.o.f, there are plethora of observational constraints from direct/indirect searches \cite{8}, but there exist very little constraints on the non-standard DM scenarios, i.e. (a) and (b) discussed...
As for the thermal contribution, one can easily show that the relevant contribution comes from the X-mediated DM production in Fig. 1(b) [12]. In particular, \( \psi_R \psi_R \rightarrow \phi \rightarrow \bar{\chi} \chi \) would yield a sub-dominant DM contribution compared to \( \psi_R \bar{\psi}_R \rightarrow X \rightarrow \bar{\chi} \chi \). This is due to the fact that both thermal and non-thermal contribution to DM abundance in the inflaton-mediation case are proportional to \( y_{\chi} \), see Eq. (4), which has to be small in order not to non-thermally overproduce DM [13, 15].

The thermal contribution due to inflaton mediation dominates only when \( T_{\text{reh}} \gg m_\phi \), in which case the thermal corrections to inflaton decay rate also become important [17]. On the other hand, such complications do not arise in case of the X-mediation as long as \( m_X \gtrsim T_{\text{max}} \).

By assuming \( B_\chi \ll 1 \), we can trace the time evolution of the inflaton decay products by solving the following set of coupled Boltzmann equations:

\[
\frac{dp_\phi}{dt} + 3H \rho_\phi = -\Gamma_\phi \rho_\phi \tag{5}
\]

\[
\frac{d\rho_{\text{rad}}}{dt} + 4H \rho_{\text{rad}} = (1 - B_\chi) \Gamma_\phi \rho_\phi \tag{6}
\]

\[
\frac{dn_\chi}{dt} + 3H n_\chi = \frac{B_\chi \Gamma_{\phi \rho_\phi}}{m_\phi} + \sum_\psi \gamma(\bar{\psi}_R \psi_R \rightarrow \bar{\chi} \chi), \tag{7}
\]
where \( \rho_\phi \) (\( \rho_{\text{rad}} \)) denotes the inflaton (radiation) energy density, \( n_\chi \) is the DM number density, and \( \gamma \) is the DM thermal production rate. For the sake of simplicity, we focus on the case of heavy mediator i.e. \( m_X \gg T_{\text{max}} \). Then on dimensional grounds, the cross section \( \sigma(\psi_R \psi_R \rightarrow \bar{\chi} \chi) \sim \frac{g_\psi^2}{\pi} \frac{g_\chi T^2}{m_X^4} \), and since the DM thermal production rate \( \gamma \propto \sigma \), we have:

\[
\gamma(\psi_R \psi_R \rightarrow \bar{\chi} \chi) = I(\psi_R \psi_R \rightarrow \bar{\chi} \chi) \frac{T^8}{m_X^4}, \tag{8}
\]

where \( I \sim \frac{g_\psi^2}{\pi} \frac{g_\chi T^2}{m_X^4} \) is a constant for \( T \gg m_\chi, m_\psi \) and the factor \( T^8 \) arises from the two \( \psi_R \)'s being in a thermal plasma. In the numerical calculations we use the exact integral expression for \( \gamma \) and also take into account the plasma induced thermal masses for RH fermions \([19]\).

The total DM relic abundance for temperatures \( T < T_{\text{rh}} \) is given by the sum of the thermal (th) and non-thermal (non-th) components:

\[
\Omega_{\chi h^2} = \Omega_{\chi \text{non-th}} h^2 + \Omega_{\chi \text{th}} h^2 \nonumber \\
\simeq 2.74 \times 10^8 m_\chi \left( n_{\chi \text{non-th}} + n_{\chi \text{th}} \right)/s, \tag{9}
\]

where \( h \) is the scaled Hubble rate, \( s = (2\pi^2/45)g_\ast T^3 \) is the entropy density with \( g_\ast \) being the corresponding number of relativistic d.o.f. We take \( g_\ast = g \), which is valid for most of the thermal history of the Universe.

Eqs. (3)-(7) can be simplified by defining dimensionless comoving energy and number densities: \( \Phi \equiv \rho_\phi x^3/m_\phi^2 \), \( R \equiv \rho_{\text{rad}} x^3/m_\phi^2 \) and \( X \equiv n_\chi x^3/m_\phi^2 \) with \( x(t) \equiv a(t)/\epsilon \) where \( a(t) \) is the scale factor.

During reheating, the energy density of the Universe is dominated by the coherent oscillations of the inflaton, which slowly decays until \( t = \Gamma_{\phi}^{-1} \). Deep inside inflaton domination (id) epoch, i.e when \( H \gg \Gamma_\phi \), \( \Phi \simeq \Phi_I \) where \( \Phi_I = \rho_{\phi,I} x^3/I^2 \) and \( R_I \simeq X_I \simeq 0 \). With this approximation Eq. (6) can be easily solved to yield \( \rho_{\text{rad}}(x) \simeq (2\sqrt{3}/5) \alpha_\phi M_p m_\phi^{2/3}(x_I/x)^{4/3} \). This allows us to estimate the temperature of the ambient relativistic d.o.f. during the inflaton domination epoch: \( T_{\text{id}}(x) = \left[ \frac{30}{\pi^2 g_\ast} \rho_{\text{id}} \right]^{1/4} \)

\[
\approx \left( \frac{332}{\pi^2 g_\ast} \right)^{1/8} \alpha_\phi^{1/4} M_p^{1/4} m_\phi^{1/4} \rho_{\phi,I} (x_I/x)^{3/8}. \tag{10}
\]

Now Eq. (7) can be easily solved for the non-thermal DM component, which is sourced by the first term on the RHS of Eq. (7), to yield \( n_{\chi \text{non-th}}(x) \simeq (2/\sqrt{3}) B_\chi \alpha_\phi M_p \rho_{\phi,I} (x_I/x)^{4/3} \). Evaluating this expression for the non-thermal number density at \( T_{\text{rh}} \), given by Eq. (10), and accounting for the inflaton population decaying at \( T < T_{\text{rh}} \) and the accompanying entropy release \([20]\), we obtain \( [18] \)

\[
\frac{\Omega_{\chi \text{non-th}} h^2}{0.12} \simeq 0.228 \times 10^5 B_\chi \left( \frac{m_X}{1 \text{ GeV}} \right) \left( \frac{T_{\text{rh}}}{10^9 \text{ GeV}} \right). \tag{11}
\]

Note that in order to avoid the overclosure of the Universe due to the non-thermal DM production from the inflaton, one typically requires a small branching fraction, which is a challenge for many models of inflation, in particular string compactification models \([21, 22]\), due to the proliferation of hidden sectors. We assumed that the \( X \) field is not excited after inflation, since it is kinematically forbidden if \( m_X > m_\phi/2 \), and resonant production is suppressed due to small couplings between \( \phi \) and \( X \).

For thermal production, since \( \gamma \propto T^8 \) [see Eq. (8)], \( \chi \) decouples from the thermal bath during the inflaton domination, shortly after the temperature reaches its maximum at \( a \sim 1.48 a_I \), where \( a_I \) denotes the initial scale factor at the end of inflation. The evolution of the thermal yield \( Y_{\chi \text{th}} = n_{\chi \text{th}}/s \) with respect to the scale factor \( a_I \), is shown in Fig. 2. The abundance \( Y_{\chi \text{th}} \) builds up very quickly as the thermal scattering rate \( \gamma \) increases with temperature [see Eq. (8)], until \( \chi \) completely decouples from the thermal bath at \( a_{\text{dec}} \) shortly after \( \gamma \) reaches its maximum at \( a_{\text{max}} \). However due to the ongoing entropy production from the inflaton decay, and the resulting dilution effect, the abundance decreases as \( Y_{\chi \text{th}} \propto T \propto a^{-3/8} \), roughly until the end of reheating at \( a_{\text{rh}} \). Fig. 2 also shows the evolution of the scaled energy densities for inflaton (\( \Phi \)) and radiation (\( R \)) for comparison. Therefore, for the thermal DM abundance, it is sufficient to evaluate the yield function \( n_{\chi \text{th}}/s \) at \( T_{\text{rh}} \), and account for the entropy released after reheating. Substituting Eqs. (9) and (10) into Eq. (7), we obtain \( n_{\chi \text{th}}(x) \simeq 288 \sqrt{3}/(\pi^4 g_\ast^2) I_0 \alpha_\phi^4 M_p^2 m_\phi^4 (x_I/x)^{4/3} \) with \( I \simeq (126/\pi^4 g_\ast^2 \sqrt{2} g_\chi x_{\chi}^2) \), from which we get the following DM thermal abundance:

\[
\frac{\Omega_{\chi \text{th}} h^2}{0.12} \simeq 0.72 \times 10^5 \left( \frac{g_{106.75}}{1 \text{ GeV}} \right)^{-3/2} \left( \frac{m_X}{1 \text{ GeV}} \right)^4 \left( \frac{T_{\text{rh}}}{10^9 \text{ GeV}} \right)^3 \left( \frac{m_X}{10^{13} \text{ GeV}} \right)^{-4}. \tag{12}
\]
FIG. 3: Map of the DM thermal abundance as a function of the masses $m_\chi$ and $m_X$ for $m_\phi = 10^{13}$ GeV and $T_{\text{rh}} = 10^9$ GeV. The unshaded region to the right is ruled out by overclosure constraints, while the region on the left side is still allowed, though for practical purposes, the corresponding thermal DM abundance becomes negligible.

Here we have used Eq. (3) for $\alpha_\phi$, and Eq. (10) to change the dependence from $\alpha_{\text{rh}}$ to $T_{\text{rh}}$. Fig. 3 shows the DM thermal abundance map as a function of $m_\chi$ and $m_X$ for $m_\phi = 10^{13}$ GeV and $T_{\text{rh}} = 10^9$ GeV, which corresponds to $T_{\text{max}} \approx 10^{12}$ GeV. For very small values of the branching fraction $B_X \ll 1$, the thermal contribution given by Eq. (12) can be dominant over the non-thermal contribution given by Eq. (11), and can account for the observed DM abundance for $m_\chi$ as low as roughly $1 \text{ GeV}/(y_{X, \chi})^2$. The unshaded region to the right is ruled out by overclosure constraints, while the region on the left side is still allowed, though for practical purposes, the corresponding thermal DM abundance becomes negligible, and one has to invoke some other DM candidates to explain the observed abundance (see e.g. [22]). Note that for $m_X > T_{\text{rh}}$, the thermal production rate is reduced due to a smaller phase space. This can be seen from the top-right part of the allowed ($m_\chi, m_X$) parameter space in Fig. 3. For higher values of $T_{\text{rh}}$, the allowed range of $(m_\chi, m_X)$ shifts to lower values of $m_\chi$ and vice-versa.

Since the DM couples to the inflaton sector via Eq. (2), it is possible to correlate the scale of inflation to the DM relic density allowed by the current Planck data, using Eq. (1). By taking into account the energy density during inflation and at the time of inflaton oscillations, we are able to place an interesting constraint on DM abundance with respect to the tensor-to-scalar ratio, $r = P_T/P_s$. Since $P_T = 16H_T^2/(\pi M_P^2) = 16\rho_{\phi, T}/(3\pi M_P^2)$, we can relate the non-thermal DM abundance with $r$, as follows:

$$\Omega_{\chi, \text{non-th}}^2 \simeq 1.70 \times 10^5 \frac{B_X}{(y_{X, \chi})^2} \left( \frac{g}{106.75} \right)^{-1/4} \left( \frac{m_\chi}{1 \text{ GeV}} \right)^{3/2} \left( \frac{\alpha_\phi}{10^{-13}} \right)^{1/2} \left( \frac{r}{0.1} \right)^{1/8},$$  (13)

while for the thermal case, we have

$$\frac{\Omega_{\chi, \text{th}}^2}{0.12} \simeq 0.30 \frac{y_{X, \chi}^2}{y_{\alpha, \psi}} \left( \frac{g}{106.75} \right)^{-9/4} \left( \frac{m_\chi}{1 \text{ GeV}} \right)^4 \left( \frac{\alpha_\phi}{10^{-13}} \right)^{3/2} \left( \frac{m_X}{10^{13} \text{ GeV}} \right)^{-4} \left( \frac{r}{0.1} \right)^{3/8}. \quad (14)$$

Thus, in future, if the recent claim by BICEP [23] of a positive detection of the primordial tensor modes is verified, it will provide an important indirect handle on the dark sector physics along with a clear evidence of a quantum nature of gravity [24]. The results given by Eqs. (13)-(14) are intriguing, especially the latter one, since it relates thermal production of DM at the time of reheating and thermalization of the Universe with the value of $r$, which is highly non-trivial. As a matter of fact, this connection provides a promising probe for the FIMP-like scenario in near future.

Before we conclude, let us briefly mention that we could have also imagined a similar mechanism for producing baryon/lepton (B/L) asymmetry, either from the direct decay of the inflaton, if the inflaton were carrying any B/L number [24], or from the intermediate condensate $X$ carrying B/L number [25], as e.g. in GUT-baryogenesis [8]. As a concrete example, in non-supersymmetric models, we can realize high-scale thermal leptogenesis via the production and decay of RH heavy Majorana neutrinos [26]. In either case, we would be able to relate the scale of inflation, and therefore the tensor-to-scalar ratio $r$, with the magnitude of the L-asymmetry. In the latter case, one would require a weak-wash-out regime in order to retain the sensitivity towards the initial conditions. A detailed discussion of these issues will be given elsewhere.

To conclude, if the future observations could pin down the exact value of the tensor-to-scalar ratio, it would definitely serve as an interesting way to constrain the hidden sector, including the properties of the DM feebly interacting with the SM d.o.f, which are otherwise very hard to probe. Although for the sake of illustration we have used chaotic inflation to derive our central results given in Eqs. (13)-(14), the idea of connecting the DM abundance to the primordial tensor perturbations should hold true for a generic model of inflation, including multi field driven inflation [27].

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[1] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. (2014) [arXiv:1303.5076 [astro-ph.CO]].
[2] A. Mazumdar and J. Rocher, Phys. Rept. 497, 85 (2011) [arXiv:1001.0993 [hep-ph]].
[3] G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005) [hep-ph/0404175].
[4] J. F. Navarro, C. S. Frenk and S. D. M. White, Astrophys. J. 462, 563 (1996) [astro-ph/9508025].
[5] P. Dayal, A. Ferrara, J. Dunlop and F. Pacucci, Mon. Not. Roy. Astron. Soc. 445, 2545 (2014) [arXiv:1405.4862 [astro-ph.GA]].
[6] R. Allahverdi and M. Drees, Phys. Rev. Lett. 89, 091302 (2002) [hep-ph/0203118]; Phys. Rev. D 66, 063513 (2002) [hep-ph/0205246].
[7] L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, JHEP 1003, 080 (2010) [arXiv:0911.1120 [hep-ph]].
[8] E. W. Kolb and M. S. Turner, “The Early Universe,” Front. Phys. 69 (1990) 1.
[9] A. D. Linde, Phys. Lett. B 129, 177 (1983).
[10] M. Blennow, E. Fernandez-Martinez and B. Zaldívar, JCAP 1401, 003 (2014) [arXiv:1309.7348 [hep-ph]]; H. Baer, K. Y. Choi, J. E. Kim and L. Roszkowski, arXiv:1407.0017 [hep-ph]; F. Elahi, C. Kolda and J. Unwin, arXiv:1410.6157 [hep-ph].
[11] Y. Shtanov, J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 51, 5438 (1995) [hep-ph/9407247]; L. Kohan, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994) [hep-th/9405187].
[12] R. Allahverdi, et.al, Ann. Rev. Nucl. Part. Sci. 60, 27 (2010) [arXiv:1001.2600 [hep-th]].
[13] G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, JHEP 9908, 014 (1999) [hep-ph/9905242].
[14] K. Enqvist, S. Kasuya and A. Mazumdar, Phys. Rev. Lett. 89, 091301 (2002) [hep-ph/0204270]; Phys. Rev. D 66, 043505 (2002) [hep-ph/0206272].
[15] D. J. H. Chung, E. W. Kolb and A. Riotto, Phys. Rev. Lett. 81, 4048 (1998) [hep-ph/9805473]; Phys. Rev. D 60 (1999) 063504 [hep-ph/9809453].
[16] A. Mazumdar and B. Zaldívar, Nucl. Phys. B 886, 312 (2014) [arXiv:1310.5143 [hep-ph]].
[17] M. Drewes, JCAP 1411, 020 (2014) [arXiv:1406.6243 [hep-ph]].
[18] P. S. B. Dev, A. Mazumdar and S. Qutub, Front. Phys. 2, 26 (2014) [arXiv:1311.5297 [hep-ph]].
[19] H. A. Weldon, Phys. Rev. D 26, 2789 (1982).
[20] G. F. Giudice, E. W. Kolb and A. Riotto, Phys. Rev. D 64 (2001) 023508 [hep-ph/0005123].
[21] M. Cicoli and A. Mazumdar, JCAP 1009, 025 (2010) [arXiv:1005.5076 [hep-th]].
[22] D. Chialva, P. S. B. Dev and A. Mazumdar, Phys. Rev. D 87, 063522 (2013) [arXiv:1211.0250 [hep-ph]].
[23] P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]].
[24] A. Ashoorioon, P. S. B. Dev and A. Mazumdar, Mod. Phys. Lett. A 29 (2014) 30, 1450163 [arXiv:1211.4678 [hep-th]].
[25] H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Phys. Rev. Lett. 70, 1912 (1993).
[26] K. Enqvist and A. Mazumdar, Phys. Rept. 380, 99 (2003) [hep-ph/0209244].
[27] S. Davidson, E. Nardi and Y. Nir, Phys. Rept. 466, 105 (2008) [arXiv:0802.2962 [hep-ph]].
[28] A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D 58, 061301 (1998) [astro-ph/9804177].