Long-range proximity effects in superconductor-ferromagnet structures

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We analyze the proximity effect in a superconductor/ferromagnet (S/F) structure with a local inhomogeneity of the magnetization in the ferromagnet near the S/F interface. We demonstrate that not only the singlet but also the triplet component of the superconducting condensate is induced in the ferromagnet due to the proximity effect. Although the singlet component of the condensate penetrates into the ferromagnet over a short length \( \xi_h \approx \sqrt{D/\hbar} \) (\( \hbar \) is the exchange field in the ferromagnet and \( D \) the diffusion coefficient), the triplet component, being of the order of the singlet one at the S/F interface, penetrates over a long length \( \sqrt{D/\epsilon} \) (\( \epsilon \) is the energy). This long-range penetration leads to a significant increase of the ferromagnet conductance below the superconducting critical temperature \( T_c \).

In recent experiments on S/F structures a considerable increase of the conductance below the superconducting critical temperature \( T_c \) was observed [1,3]. Although in a recent work [4] it was suggested that such an increase may be due to scattering at the S/F interface, a careful measurement of the conductance demonstrated that the entire change of the conductance was due to an increase of the conductivity of the ferromagnet [1,2].

Such an increase would not be a great surprise if instead of the ferromagnet one had a normal metal N. It is well known (see for review [5,6]) that in S/N structures proximity effects can lead to a considerable increase of the conductance of the N wire provided its length does not exceed the phase breaking length \( L_\phi \). However in a S/F structure, if the superconducting pairing is singlet, the proximity effect is negligible at distances exceeding a much shorter length \( \sim \xi_h \). This reduction of the proximity effect due to the exchange field \( h \) of the ferromagnet is clear from the picture of Cooper pairs consisting of electrons with opposite spins. The proximity effect is not considerably affected by the exchange energy only if the latter is small \( h < T_c \). As concerns such strong ferromagnets as Fe or Co used in the experiments [1,3], whose exchange energy \( h \) is by several orders of magnitude larger than \( T_c \), a singlet pairing is impossible due to the strong difference in the energy dispersions for the two spin bands. At the same time, an arbitrary exchange field cannot destroy a triplet superconducting pairing because the spins of the electrons forming Cooper pairs are already parallel. A possible role of the triplet component in transport properties of S/F structures has been noticed in Refs. [13], where the triplet component arose only as a result of mesoscopic fluctuations. However, in both cases the corrections to the conductance are much smaller than the observed ones.

In this paper, we suggest a much more robust mechanism of formation of the triplet pairing in S/F structures, which is due to a local inhomogeneity of the magnetization \( M \) in the vicinity of the S/F interface. We show that the inhomogeneity generates a triplet component of the superconducting order parameter with an amplitude comparable with that of the singlet pairing. The penetration length of the triplet component into the ferromagnet is equal to \( \xi_T = \sqrt{D/\epsilon} \), where the energy \( \epsilon \) is of the order of temperature \( T \) or the Thouless energy \( E_T = D/L^2 \), \( L \) is the sample size. The length \( \xi_T \) is of the same order as that for the penetration of the superconducting pairs into a normal metal and therefore the increase of the conductance due to the proximity effect can be comparable with that in an S/N structure.

We consider a structure shown in Fig.1 and assume that the magnetization orientation varies linearly from \( \alpha = 0 \) at \( x = 0 \) to \( \alpha_w = Qw \) at \( x = w \). Here \( \alpha \) is the angle between \( M \) and the \( z \)-axis. The case \( Qw = \pi \) corresponds to a domain wall with thickness \( w \) located at the S/F interface, while the model with the homogeneous magnetization is recovered by putting \( Q = 0 \). This variation of \( M \) may also be brought about by an external magnetic field (see [3] and references therein). Of course, the variation of the magnetization considered is the simplest model of what may happen at the inter-
depends on the coordinate expression into Eq. (1) and assuming that the solution may assume a weak proximity effect, which presumably interface if the transparency is high. In both cases one Fermi surfaces. Moreover, the order parameter in the S/F interface is small due to a strong mismatch of the siclassical Green’s functions. These functions are $4 \times 4$ Green’s function $\hat{\epsilon}$ where
can be sought in the form $\hat{f}$. The equation containing spin variables [12]. The equation is equation between the exchange field and spins of the conduc-
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can be written for temperature anomalous Green functions $\hat{f}$ at Matsubara frequencies $\omega$, by replacing $\epsilon \rightarrow i|\omega|$ and multiplying the last term by $\text{sgn}(\omega)$ [13]. Eq. (1) is supplemented by the boundary conditions at the interface that can also be linearized [13]. Assuming that there are no spin-flip processes at the S/F interface, we have
\[
\partial_x \hat{f} \bigg|_{x=0} = (\rho/R_b) \hat{f}_S ,
\]
where $\rho$ is the resistivity of the ferromagnet, $R_b$ is the S/F interface resistance per unit area in the normal state, and $\hat{f}_S = \hat{\sigma}_3 \Delta / \sqrt{\epsilon^2 - \Delta^2}$. The solution of Eq. (1) is trivial in the superconductor but needs some care in the ferromagnet. In the region $0 < x < w$ the solution $\hat{f}$ can be sought in the form $\hat{f} = \hat{U}(x)f_n U(x)$, where $\hat{U}(x) = \hat{\sigma}_0 \cos(\alpha x) + i \hat{\sigma}_1 \sin(\alpha x)$. Substituting this expression into Eq. (1) and assuming that the solution depends on the coordinate $x$ only we obtain the following equation for $\hat{f}_n$
\[
-iD\partial_{xx}^2 \hat{f}_n + i \left( DQ^2 / 2 \right) \left( \hat{f}_n + \hat{\sigma}_i \hat{f}_n \hat{\sigma}_i \right) + DQ \left\{ \partial_x \hat{f}_n, \hat{\sigma}_i \right\} + 2\epsilon \hat{f}_n + h \left\{ \hat{\sigma}_3, \hat{f}_n \right\} = 0 \tag{3}
\]
Here $\left\{ \ldots \right\}$ is the anticommutator. In the region $x > w$, $\hat{f}_n$ satisfies Eq. (3) with $Q = 0$.

We see from Eq. (3) that the singlet component, commuting with $\hat{\sigma}_3$, and triplet component, anticommuting with $\hat{\sigma}_3$, are mixed by the rotating exchange field $h$. In the region $x > w$ the triplet and the singlet components decouple and their amplitudes should be found by matching the solutions at $x = w$. One should also use the boundary condition, Eq. (2), and match the solutions in the ferromagnet and superconductor. It is clear that the singlet and triplet components of the anomalous function $\hat{f}_n$ inevitably coexist in the ferromagnet. This fact is also known for the case of magnetic superconductors with $Q \neq 0$ [4]. In the region $x > w$ the singlet part decays sharply but the triplet one survives over long distances. We are able to confirm these statements solving Eq. (3) with the boundary condition Eq. (2). In the case of a homogeneous magnetization ($Q = 0$) the triplet pairing cannot be induced, which follows immediately from Eq. (2) connecting separately the singlet and triplet components at the opposite sides of the interface and Eq. (3).

Eq. (3) can be solved exactly. The solution $\hat{f}_n$ can be written in the form
\[
\hat{f}_n = \hat{\sigma}_0 A(x) + \hat{\sigma}_3 B(x) + i \hat{\sigma}_1 C(x) \tag{4}
\]
The function $C(x)$ in Eq. (4) is the amplitude of the triplet pairing, whereas the first and the second term describe the singlet one. Substituting Eq. (4) into Eq. (3) we obtain a system of three equations for the functions $A$, $B$ and $C$, which can be sought in the form
\[
A(x) = \sum_{i=1}^{3} \left( A_i \exp(-\kappa_i x) + \bar{A}_i \exp(\kappa_i x) \right) \tag{5}
\]
where $\kappa_i = \sqrt{\kappa^2 - \epsilon^2 - Q^2}$ and $\bar{\kappa}_i = \sqrt{\kappa^2 + \epsilon^2 - Q^2}$. The functions $B(x)$ and $C(x)$ can be written in a similar way. The eigenvalues $\kappa_i$ obey the algebraic equations
\[
(\kappa^2 - \kappa^2_0 - Q^2) C - 2(Q\kappa) A = 0
\]
\[
(\kappa^2 - \kappa^2_0) B - \kappa^2_h A = 0 \tag{6}
\]
where $\kappa^2_h = -2\epsilon / D$ and $\kappa^2_h = -2i\hbar / D$ (indices $i$ were dropped). The eigenvalues $\kappa$ are the values at which the determinant of Eqs. (3) turns to zero. From the first equation of Eqs. (3) we see that in the homogeneous case ($Q = 0$) the triplet component has a characteristic penetration length $\sim \kappa^{-1}_0$, but we see from Eq. (2) that its amplitude is zero. If $Q \neq 0$, the triplet component $C$ is coupled to the singlet component ($A, B$) induced in the ferromagnet according to the boundary condition Eq. (3) (proximity effect). If the width $w$ is small, the triplet component changes only a little in the region $(0, w)$ and spreads over a large distance of the order $|\kappa^{-1}_h|$ in the region $(0, L)$. In the case of a strong exchange field $h$, $\xi_h$ is very short ($\xi_h \ll w, \xi_T$), the singlet component
decays very fast over the length $\xi_3$, and its slowly varying part turns out to be small. In this case the first two eigenvalues $\kappa_{1,2} \approx (1 \pm i)/\xi_3$ can be used everywhere in the ferromagnet ($0 < x < L$), where $L$ is the length of the ferromagnet. As concerns the third eigenvalues, we obtain $\kappa_3 = \sqrt{\kappa_2^2 + Q^2}$ in the interval $(0, w)$, and $\kappa_3 = \kappa_e$ in the interval $(w, L)$. The amplitude $B_3$ of the slowly varying part of the singlet component is equal to $B_3 = (Q \kappa_3/\kappa_e^3) C_3 \ll C_3$.

All the amplitudes should be chosen to satisfy the boundary conditions at $x = 0$ (Eq. (3)) and zero boundary condition at $x = L$. For the triplet component we obtain (we restore the indices $R(A)$)

$$C_{R(A)}(x) = \mp i \{ Q B(0) \sinh (\kappa_e (L - x)) \}$$

$$[\kappa_e \cosh \Theta_e \cosh \Theta_3 + \kappa_3 \cosh \Theta_3 \sinh \Theta_3 \sinh \Theta_3]^{-1} R(A)$$

(7)

where $\kappa_3 = \sqrt{\kappa_2^2 + Q^2}$ in the interval $(0, w)$, and $\kappa_3 = \kappa_e$ in the interval $(w, L)$. One can see that the difference $C^R - C^A$ is an even function of $\epsilon$. This is a direct consequence of the fact that $C^R - C^A$ is proportional to the Fourier transform of the correlator $K(t) = \langle \psi_\uparrow(t) \psi_\uparrow(0) \psi_\downarrow(0) \psi_\downarrow(t) \rangle$, which is even in time. In the Matsubara representation, $C_{R(A)}(\Omega)$ should be replaced by $C_{C_{\uparrow}}$ with $\Theta_{\Omega} \alpha_{\Omega}$ instead of $\Theta_3$ and $\kappa_3 \cosh \Theta_3 \sinh \Theta_3$ instead of $\kappa_3 \cosh \Theta_3 \sinh \Theta_3$, respectively. Thus, $C_{\uparrow}$ corresponding to the temperature correlator $K(\tau) = - < T \psi_\uparrow(0) \psi_\downarrow(\tau) >$ is an odd function of $\omega$ and the sum over all $\omega$ is zero in accordance with $K(0) = K(\Omega) = 0$.

It is clear from Eq. (7), that the triplet component is of the same order of magnitude as the singlet one at the interface. Indeed, for the case $w < L$ we obtain from Eq. (7) $|C(0)| \sim B(0)/\sinh \alpha_w$, where $\alpha_w = Qw$ is the angle characterizing the rotation of the magnetization. Therefore if the angle $\alpha_w \ll 1$ and the S/F interface transparency is not too small, the singlet and triplet components are not small. They are of the same order in the vicinity of the S/F interface, but while the singlet component decays abruptly over a short distance ($\sim \xi_3$), the triplet one varies smoothly along the ferromagnet, turning to zero at the F reservoir. In Fig.2 we plot the spatial dependence of the singlet $|B(x)|$ and the triplet $|C(x)|$ components for two different $Q$. One can see that the singlet component decays abruptly undergoing the well known oscillations [13] while the triplet one decays to zero slowly. This decay in the region $(0, w)$ increases with increasing $Q$.

Thus, we come to a remarkable conclusion: the penetration of the superconducting condensate into a ferromagnet may be similar to the penetration into a normal metal. The only difference is that, instead of the singlet component in the case of the normal metal, the triplet one penetrates into the ferromagnet. Of course, in order to induce the triplet component one needs an inhomogeneity of the exchange field at the interface.

The presence of the condensate function (triplet component) in the ferromagnet can lead to interesting long-range effects. One of them is a change of the conductance of a ferromagnetic wire in a S/F structure (see inset in Fig.1) when the temperature is lowered below $T_c$. This effect was observed first in S/N structures and later was successfully explained (see, e.g. reviews [14]). Now we consider the S/F structure shown in the inset of Fig.1. The normalized conductance variation $\delta G = (G - G_n)/G_n$ is given by the expression [15]:

$$\delta G = -\frac{1}{32 T} \text{Tr} \int d\epsilon F(V) \left\langle \left[ \hat{f}^R(x) - \hat{f}^A(x) \right] \right\rangle^2$$

(8)

for $G_n$ is the conductance in the normal state, $F(V) = 1/2 \left[ \text{cosh}^{-2}((\epsilon + eV)/2T) + \text{cosh}^{-2}((\epsilon - eV)/2T) \right]$, and $< .. >$ denotes the average over the length of the ferromagnetic wire between the F reservoirs. Substituting Eqs. (4) into Eq. (8) one can determine the temperature dependence $\delta G(T)$. Fig.3 shows this dependence.

We see that $\delta G$ increases with decreasing temperature and saturates at $T = 0$. This monotonic behaviour of $\delta G$ contrasts with the so called reentrant behaviour of $\delta G$ in S/N structures [15,19] and is a result of broken time-reversal symmetry of the system under consideration.

Available experimental data are still controversial. It has been established in a recent experiment [16] that the conductance of the ferromagnet does not change below $T_c$ and all changes in $\delta G$ are due to changes of the S/F interface resistance $R_0$. However, in other experiments $R_0$ was negligibly small [17]. The mechanism suggested in our work may explain the long-range effects observed in the experiments [18]. At the same time, the result of the experiment [16] is not necessarily at odds with our findings. The inhomogeneity of the magnetic moment at the interface, which is the crucial ingredient of our theory, is not a phenomenon under control in these experiments. One can easily imagine that such inhomogeneity existed in the structures studied in Refs. [16,18] but was absent in those of Ref. [16]. The magnetic inhomogeneity near the interface may have different origins. Anyway, a more careful study of the possibility of a rotating magnetic moment should be performed to clarify this question.

In order to explain the reentrant behaviour of $\delta G(T)$ observed in Refs. [16,18] one should take into account other mechanisms, as those analyzed in Refs. [17,19]. However, this question is beyond the scope of the present paper.

We note that at the energies $\epsilon$ of the order of Thouless energy $\epsilon \sim E_T$ the triplet component spreads over the full length $L$ of the ferromagnetic wire (see Fig.2). This long-range effect differs completely from the proximity effect in a ferromagnet with a uniform magnetization consid-
erated recently in Ref. [20]. In the latter case the characteristic wave vector is equal to \( \kappa_{1,2} = \sqrt{-2i(\epsilon \pm h)/D} \) (cf. Eqs. [3]). It was noted in Ref. [20] that if \( \epsilon \to \pm h \), then \( \kappa_{1,2} \to 0 \) and the singlet component penetrates in the ferromagnet. If the characteristic energies \( \epsilon_{ch} \sim E_T, T \) are much less than \( h \), the penetration length \( |\kappa_{1,2}|^{-1} \) is of the order \( \xi_h \) and is much shorter then \( \xi_T \) or \( L \).

\[ \xi_{w} = \pi/5 \]

FIG. 2. Spatial dependence of the singlet (dashed line) and triplet (solid line) components of \( \hat{F} \) in the F wire for different values of \( \alpha_w \). Here \( w = L/5, \epsilon = E_T \) and \( h/E_T = 400 \). \( E_T = D/L^2 \) is the Thouless energy.

\[ \alpha_w = \pi/5 \]

\[ \alpha_w = \pi/3 \]

\[ \alpha_w = \pi/10 \]

FIG. 3. The \( \delta G(T) \) dependence. Here \( \gamma = \rho \xi_h/R_0 \). \( \Delta/E_T \gg 1 \) and \( w/L = 0.05 \)

It is also interesting to note that a triplet component of the condensate function with the same symmetry (odd in frequency \( \omega \) and even in momentum \( p \)) was suggested by Berenziskii [21] as a possible phase in superfluid \(^3\)He (this, so called “odd” superconductivity, was discussed in a subsequent paper [22]). Being symmetric in space, this component is not affected by potential impurities, in contrast to the case analyzed in Ref. [23] where the triplet component of the condensate was odd in space. While in \(^3\)He this hypothetical condensate function is not realized (in \(^3\)He it is odd in \( p \) but not in frequency), in our system this odd (in \( \omega \)) triplet component does exist, although under special conditions described above.

In conclusion, we have shown that in the presence of a local inhomogeneity of magnetization near the S/F interface, both the singlet and triplet components of the condensate are created in the ferromagnet due to the proximity effect. The singlet component penetrates into the ferromagnet over a short length \( \xi_h \), whereas the triplet component can spread over the full mesoscopic length of the ferromagnet. This long-range penetration of the triplet component should lead to a significant variation of the ferromagnet conductance below \( T_c \).

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