KINEMATICAL LORENTZ-SYMMETRY TESTS AT PARTICLE COLLIDERS

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Violations of Lorentz symmetry are typically associated with modifications of one-particle dispersion relations. The physical effects of such modifications in particle collisions often grow with energy, so that ultrahigh-energy cosmic rays provide an excellent laboratory for measuring such effects. In this talk we argue that collisions at particle colliders, which involve much smaller energies, can nevertheless yield competitive constraints on Lorentz breaking.

1. Introduction

One of the earliest ideas for testing Lorentz symmetry in the context of quantum gravity involves kinematical searches for modifications in one-particle dispersion relations. Such modifications can indeed be accommodated in various theoretical approaches to more fundamental physics.¹

For phenomenological purposes in this context, corrections to energy–momentum relations need to be modeled.² To this end, it is appropriate to consider the long-distance limit of general Lorentz violation in underlying physics. This limit can be described by the Standard-Model Extension (SME).³ To date, the SME has provided the basis for numerous experimental⁴ and theoretical⁵ studies of Lorentz violation. As a consistent dynamical framework, the SME permits the extraction of acceptable Lorentz-violating dispersion relations. For example, the modified dispersion relation for spin-½ fermions in the flat-spacetime minimal SME (mSME) is given in Ref. 6; the one for photons in the full flat-spacetime SME can be found in Ref. 7.

These Lorentz-breaking dispersion relations typically affect particle collisions, which can cause observable effects. For example, particle reactions that are kinematically forbidden in conventional physics can now occur above certain thresholds; and vice versa, processes that normally occur may
be forbidden above certain energy scales when Lorentz violation is present.

Since the Lorentz violations in such dispersion relations often grow with the particle’s momentum, the novel observable signals tend to be enhanced at higher energies. Common belief therefore holds that kinematical Lorentz-symmetry studies are best performed with ultrahigh-energy cosmic rays (UHECRs). However, an UHECR collision process involves the (modified) dispersion relations of all participating particles including the primary, but the exact type of the primary particle is often unknown. For this reason, it can be interesting to consider also Lorentz tests with particle collisions in a controlled laboratory environment at much lower energies. In what follows, it is argued that such dispersion-relation tests of special relativity at colliders can yield competitive constraints on Lorentz breakdown.

The Lorentz tests at accelerators discussed below are particularly sensitive to the electron–photon sector of the flat-spacetime mSME. Currently, the $c^{\mu\nu}$ and $\tilde{k}^{\mu\nu} \equiv (k_F)_\alpha^{\mu\nu} \alpha$ coefficients in this sector obey the weakest experimental constraints. We can therefore focus exclusively on $c^{\mu\nu}$ and $\tilde{k}^{\mu\nu}$.

From a conceptual viewpoint, it is vital to note that $c^{\mu\nu}$ and $\tilde{k}^{\mu\nu}$ are physically equivalent in an electron–photon system. This equivalence stems from the fact that judiciously chosen coordinate rescalings freely transform the $\tilde{k}^{\mu\nu}$ and $c^{\mu\nu}$ parameters into one another.\textsuperscript{9,10} Intuitively, this reflects the fact that one may choose to measure distances with a ‘ruler’ composed of electrons ($c^{\mu\nu} = 0$), or with a ‘ruler’ composed of photons ($\tilde{k}^{\mu\nu} = 0$), or any other ‘ruler’ ($c^{\mu\nu}$, $\tilde{k}^{\mu\nu} \neq 0$). We utilize this freedom by making the specific choice $c^{\mu\nu} = 0$ (corresponding to an ‘electron ruler’) in intermediate calculations. However, we present all final results in a scaling-independent (i.e., ruler-independent) way by reinstating the $c^{\mu\nu}$ coefficient for generality.

The $\tilde{k}^{\mu\nu}$ coefficient possesses nine independent components: it is traceless and symmetric. We will discuss its isotropic component denoted by $\tilde{\kappa}_{tr}$ and the three parity-violating anisotropic components usually grouped into the antisymmetric $3 \times 3$ matrix $\tilde{\kappa}_{o+}$. The remaining five components describe parity-even anisotropies and are not discussed here.

2. The isotropic component $\tilde{\kappa}_{tr}$

An mSME analysis establishes that the isotropic component of $\tilde{k}^{\mu\nu}$ parametrized by $\tilde{\kappa}_{tr}$ leads to the following modified dispersion relation:\textsuperscript{9}

$$E_\gamma^2 - (1 - \tilde{\kappa}_{tr})p^2 = 0.$$ (1)

Equation (1) holds at leading order in $\tilde{\kappa}_{tr}$, and $p^\mu \equiv (E_\gamma, \vec{p})$ denotes the photon’s 4-momentum. Notice that the physical speed of light is $(1 - \tilde{\kappa}_{tr})$, \textsuperscript{9}
which is different from the usual $c = 1$. In what follows, we treat the two cases $\tilde{\kappa}_{tr} > 0$ and $\tilde{\kappa}_{tr} < 0$ separately because they are associated with different phenomenological signatures.

The case $\tilde{\kappa}_{tr} > 0$.—For positive $\tilde{\kappa}_{tr}$, the speed of light $(1 - \tilde{\kappa}_{tr})$ is slower than the conventional value $c = 1$. This implies in particular that the maximal attainable speed (MAS) of the electrons is greater than the speed of the photons. In analogy to ordinary electrodynamics inside a macroscopic medium, we expect a Cherenkov-type effect: \textsuperscript{11} charges moving faster than the modified speed of light $(1 - \tilde{\kappa}_{tr})$ would be unstable against the emission of light. With the modified photon dispersion relation (1), one can indeed show that electrons at energies above the threshold

$$E_{VCR} = \frac{1 - \tilde{\kappa}_{tr}}{\sqrt{2 - \tilde{\kappa}_{tr}}} m_e = \frac{m_e}{\sqrt{2\tilde{\kappa}_{tr}}} + \mathcal{O} \left( \sqrt{\tilde{\kappa}_{tr}} \right)$$

emit Cherenkov photons. We remark that the threshold (2) can also be obtained from the ordinary Cherenkov condition requiring that the electrons be faster than the speed of light $(1 - \tilde{\kappa}_{tr})$.

At LEP, where electrons attained the energy $E_{LEP} = 104.5$ GeV, this Cherenkov effect was not observed. This essentially means that the LEP electrons must have been below the Cherenkov threshold $E_{LEP} < E_{VCR}$. Equation (2) then yields

$$\tilde{\kappa}_{tr} - \frac{4}{3} c^{00} \leq 1.2 \times 10^{-11},$$

where $c^{00}$ has been reinstated for generality. Note that we have implicitly used the dynamical result\textsuperscript{10,11} that Cherenkov radiation must be highly efficient to deduce $E_{LEP} < E_{VCR}$ from the non-observation of $e \rightarrow e \gamma$.

The case $\tilde{\kappa}_{tr} < 0$.—The speed of light $(1 - \tilde{\kappa}_{tr})$ is now greater than the conventional value $c = 1$. In particular, all photons move faster than the MAS of the electrons. Paralleling the above Cherenkov case, one would then expect that the photon can now become unstable. An mSME calculation with the dispersion relation (1) indeed confirms that for photon energies above the threshold

$$E_{pair} = \frac{2m_e}{\sqrt{\tilde{\kappa}_{tr} (\tilde{\kappa}_{tr} - 2)}} = \sqrt{\frac{2}{-\tilde{\kappa}_{tr}}} m_e + \mathcal{O} \left( \sqrt{\tilde{\kappa}_{tr}} \right),$$

photon decay into an electron–positron pair is kinematically allowed.\textsuperscript{10,13} The D0 experiment at the Tevatron has observed photons with energy in excess of $E_{D0} = 300$ GeV, so $E_{pair}$ must be greater than this value. We then
arrive at the constraint

$$-5.8 \times 10^{-12} \lesssim \tilde{\kappa}_{tr} - \frac{4}{3} c^{00},$$

(5)

where we have reinstated the electron’s $c^{00}$ coefficient. Again, we have implicitly used the dynamical result \(^{10,13}\) that photon decay must be highly efficient to deduce $E_{D0} < E_{\text{pair}}$ from the non-observation of $\gamma \to e^+ e^-$.  

We finally remark that the one-sided limits (3) and (5) have recently been improved by roughly three orders of magnitude with an alternative method involving colliders. The idea is that the synchrotron losses of charges moving on a circular path are highly sensitive to $\tilde{\kappa}_{tr}$. Since such losses were accurately determined at LEP, a bound at the level of a few parts in $10^{15}$ has been obtained.\(^{14}\)

3. The anisotropic parity-violating components $\tilde{\kappa}_{o+}$

It can be shown that the $\tilde{\kappa}_{o+}$ components of $\tilde{k}^{\mu\nu}$ modify the photon dispersion relation as follows:

$$E_\gamma = (1 - \tilde{\kappa} \cdot \hat{p}) |\vec{p}| + O(\kappa^2).$$

(6)

Here, the three components of $\tilde{\kappa}_{o+}$ have been assembled into a 3-vector: $\tilde{\kappa} \equiv (\tilde{\kappa}_{o+}^{23}, (\tilde{\kappa}_{o+}^{31}), (\tilde{\kappa}_{o+}^{12}))$ and $\hat{p} \equiv \vec{p}/|\vec{p}|$. For a given photon momentum $|\vec{p}|$, the photon energy $E_\gamma$ depends on the direction of propagation $\hat{p}$ exposing anisotropies; reversing the direction of propagation reveals parity violation.

Consider now Compton scattering with the dispersion relation (6), where the incoming photon and electron are counter-propagating. A leading-order mSME calculation then establishes that the Compton edge (CE), which is the maximal energy of the backscattered photon, is\(^{15}\)

$$\lambda' \simeq \lambda_{CE} \left[ 1 + \frac{2 \gamma^2}{(1 + 4 \gamma \lambda / m)^2} \tilde{\kappa} \cdot \hat{p} \right].$$

(7)

Here, $\lambda_{CE}$ denotes the conventional CE energy, $\gamma$ is the relativistic boost factor of the incoming electron, and $\lambda$ the magnitude of the incoming photon 3-momentum. It follows that in present context, the CE depends on the direction $\hat{p}$ of in the incoming electron (i.e., $-\hat{p}$ for the incoming photon). In a terrestrial particle collider, the direction $\hat{p}$ changes constantly due to the rotation of the Earth. According to Eq. (7), this should lead to sidereal variations in the CE. Such variations have not been observed at ESRF’s GRAAL facility. This can be used to extract the competitive constraint\(^{15}\)

$$\sqrt{[2c_{TX} - (\tilde{\kappa}_{o+})^{YZ}]^2 + [2c_{TY} - (\tilde{\kappa}_{o+})^{ZX}]^2} < 1.6 \times 10^{-14}, \quad 95\% \text{ CL},$$

(8)
where we have again included the electron coefficients for generality. Similar limits with different methods were recently obtained by B. Altschul.\textsuperscript{16}

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