Theoretical analysis and experiment performance of slow-light based on stimulated Brillouin scattering (SBS)

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Abstract. Slow light technology will play a key role in future all-optical communication. The slow-light technology based on stimulated Brillouin scattering has become a research highlight because of its additional advantages, such as compatibility of the devices with existing telecommunication systems, room-temperature operation, and tunable at arbitrary wavelengths. According to the propagation of a cw pulse through a Brillouin fiber amplifier, whose frequency is near the Stokes resonance, via three-wave coupling equations, both pump depletion and fiber losses taken into consideration, the principle of how slow-light effect based on stimulated Brillouin scattering produced and the mathematical expression of time delay are strictly deduced. A delay of 8 ns is obtained when the input Stokes pulse is 200ns and the SBS (stimulated Brillouin scattering) gain G is ~18 in our designed experiment of SBS slow-light system. Then the extent of transformation from pump waves to Stokes waves is measured using MATLAB numerical simulation according to the experiment dates, based on the relation between output pump light power and input pump light power and also the relation between output Stokes light power and input pump light power. And the relation between the input light power and propagation distance is discussed as well. Finally the relation between slow light pulse delay and SBS gain is also obtained.

1. INTRODUCTION
Slow light is widely used in many important sections in all-optical communication, such as optical buffering, optical storage, optical equilibrator and optical information processing, and it can also carry out all-optical route and all-optical swap. Slow light based on stimulated Brillouin scattering has become a hot topic. It is an important breakthrough in optical fiber communication that pulse time...
delay is achieved based on stimulated Brillouin scattering—nonlinear effect in fiber. Kwang Yong Song \(^{11}\) got 30 ns time delay for a 100 ns pulse for the first time based on stimulated Brillouin scattering in fiber in 2005, and Zhu ZH M \(^{12,13}\) got 47 ns in 2007. Meanwhile, Kwang Yong Song research group improved wavelength to 25 GHz, and got 10.9 ps for a pulse of 37 ps \(^{14}\).

Zhu ZH M \(^6\) made theoretical analysis on how slow light produced based on stimulated Brillouin scattering, according to three-wave equations in weak signal regime. Here we deduce the relations between emerged pump power and emerged Stokes power and incident pump power when fiber losses and pump depletion are both considered in the general case, and the relation between time delay and gain. A delay of 8 ns is obtained when the input Stokes pulse is 200 ns and the SBS gain G is ~18 in the designed experiment of SBS.

2. THE PRINCIPLE OF SLOW-LIGHT BASED ON STIMULATED BRILLOUIN SCATTERING

The pump wave generates acoustic waves through the process of the electrostriction, which in turn causes a periodic modulation of the refractive index. Finally, the pump-induced refractive index grating scatters the light through Bragg diffraction \(^7\). Brillouin scattering is then stimulated along the fiber by the electrostriction. Then the produce of slow light based stimulated Brillouin scattering can be realized as interaction of three waves: a backward strong pump wave (-z direction), a forward weak Stokes wave (+z direction), and a backward acoustic wave, when special frequency condition achieved, that is \( \omega_p = \omega_s + \Omega_b \). Because the coupling action among the three waves, then slow light emerges \(^6\).

\[
\begin{align*}
-\frac{\partial E_p}{\partial z} + \frac{n_{f_0}}{c} \frac{\partial E_p}{\partial t} &= -\frac{\alpha}{2} E_p + \frac{g_b}{2 A_{eff}} |E_s|^2 E_p + ig_z E_s \rho \\
\frac{\partial E_s}{\partial z} + \frac{n_{f_0}}{c} \frac{\partial E_s}{\partial t} &= -\frac{\alpha}{2} E_s - \frac{g_b}{2 A_{eff}} |E_p|^2 E_s + ig_z E_p \rho \\
\frac{\partial \rho}{\partial t} + \left( \frac{\Gamma_b + i \Delta \omega}{2} \right) \rho &= i \frac{g}{\eta} E_p^* E_s
\end{align*}
\]

Here \( E_p \), \( E_s \), \( \rho \) are the field amplitudes of pump, Stokes, and the acoustic, \( n_{f_0} \) is group index of the fiber mode, \( \alpha \) is fiber loss coefficient, \( \Gamma_b / 2\pi \) is the bandwidth (FWTH), \( \Delta \omega = (\omega_{p_0} - \Omega_b) - (\omega_b - \omega_{s_0}) \) is the detuning from SBS frequency shift, \( \Omega_b \) is the SBS frequency. \( \omega_{p_0} (\omega_{s_0}) \) is the center angular frequency of the pump (Stokes) wave, \( g = \gamma_c \rho \eta \epsilon_0 \). \( g_z = \gamma_c \rho / (4 \pi \eta n_{f_0} ) \) \( \eta = \frac{1}{2} c \varepsilon_0 n_{f_0} \); \( \gamma_c \) is electrostriction coefficient; \( v_a \) is the speed of the acoustic, \( n_{f_0} \) is mode index of the fiber mode, \( \rho_0 \) is the material density. \( |E_p|^2 |E_s|^2 \) is optical intensity of the pump (Stokes) wave.
\[ \frac{\partial \rho^*}{\partial t} + \left( \frac{\Gamma_s}{2} + i \Delta \omega \right) \rho^* = -i \frac{g_s}{\eta} E^* E_s \]  

(1c*)

We use a Fourier-transform (FT) technique, \( FT[E_s(z,t)] = \tilde{E}_s(z,\omega) \), \( FT[E_p(z,t)] = \tilde{E}_p(z,\omega) \), \( FT[\rho(z,t)] = \tilde{\rho}(z,\omega) \), transforming the equations into the frequency domain, then (1b)(1c) can be rewritten as follows:

\[ \frac{\Gamma_s}{2} i (\omega - \omega_{so}) \tilde{E}_s - \frac{n_{fE}}{c} \tilde{E}_s = -\alpha \tilde{E}_s - \frac{g_s}{2 A_{eff}} |\tilde{E}_p|^2 \tilde{E}_s + ig_s \tilde{E}_p \tilde{E}_s^* + \frac{i g_s}{\eta 2\pi} \tilde{E}_p^* \tilde{E}_s \]  

(2b)

\[ \frac{\Gamma_s}{2} i (\omega - \omega_{so} + \Omega_g) \tilde{\rho} = -i \left( \frac{g_s}{\eta} \right) \frac{1}{2\pi} \tilde{E}_p^* \tilde{E}_s \]  

(2c)

Then \( \tilde{\rho} = \frac{\Gamma_s}{2} \left( \frac{g_s}{\eta} \right) \frac{1}{2\pi} \tilde{E}_p^* \tilde{E}_s \)  

(3)

Then (2b) is

\[ \frac{\partial \tilde{E}_s}{\partial z} - i (\omega - \omega_{so}) \tilde{E}_s = -\alpha \tilde{E}_s - \frac{g_s}{2 A_{eff}} |\tilde{E}_p|^2 \tilde{E}_s + \frac{1}{4\pi} \frac{g_s}{\eta} I_p(z) \tilde{E}_s + \frac{1}{4\pi} \frac{g_s}{\eta} \frac{1}{2\pi} \tilde{E}_p^* \tilde{E}_s \]  

(4)

\[ g_o = \frac{4 g_s g_2}{\eta / \Gamma_g}, \delta \omega = \omega - \omega_{so} + \Omega_g \]

So

\[ \frac{\partial \tilde{E}_s}{\partial z} = i (\omega - \omega_{so}) \tilde{E}_s - \frac{n_{fE}}{c} \tilde{E}_s - \frac{\alpha}{2} \tilde{E}_s - \frac{g_s}{2 A_{eff}} I_p(z) \tilde{E}_s + \frac{1}{4\pi} \frac{g_s}{\eta} I_p(z) \tilde{E}_s \]  

(5)

Comparing (5) with the SVEA wave equation in the frequency domain for Stokes pulse, we can get:

\[ \left( \frac{\omega}{c} - \frac{\omega_{so}}{c} \right) n_{fE} - i \frac{4\pi g_o I_p(z)}{1 - i 2\delta \omega / \Gamma_g} + i \frac{\alpha}{2} - i \frac{g_s}{2 A_{eff}} = k_s(\omega) - k_s(\omega_{so}) \]

(6)

\[ n_s = n_f - i \frac{4\pi g_o I_p(z)}{\omega 1 - i 2\delta \omega / \Gamma_g} + i \frac{\alpha c}{\omega} + i \frac{g_s c}{\omega A_{eff}} \]

(7)

The new index of the medium is

\[ n_s = n_f - i \frac{4\pi g_o I_p(z)}{\omega 1 - i 2\delta \omega / \Gamma_g} + i \frac{\alpha c}{\omega} + i \frac{g_s c}{\omega A_{eff}} \]  

(8)
Group index is 
\[ n_g(\omega) = n_s + \omega (dn_s/d\omega) = n_s + \frac{1}{\Gamma_B} \frac{1 - 4\delta\omega^2/\Gamma_B^2}{(1 + 4\delta\omega^2/\Gamma_B^2)^2} I_p \] (9)

Figure 1: the relation between the change of group index and the detuning of incident Stokes from resonance frequency

The propagation group velocity 
\[ v_g = \frac{\omega}{n_g} = \frac{c}{n_s + \omega \frac{dn_s}{d\omega}} \] (10) when the change of index \( dn_s/d\omega \) is great, group velocity will change greatly accordingly. \( dn_s/d\omega \gg 1 \), then group velocity is small, and slow light emerges.

3. OPTICAL POWER ANALYSIS IN A LOSSY SYSTEM

3.1 INTENSITY COUPLING EQUATION

When pump depletion is considered, the interaction is governed by the following set of two coupled equations:
\[ \frac{dP_p}{dz} = -\gamma P_p P_s - \alpha P_p \]  
\[ \frac{dP_s}{dz} = -\gamma P_p P_s + \alpha P_s \] (11)

Here, we have normalized length \( \xi = \frac{z}{L}, 0 \leq \xi \leq 1 \) \( ^{109} \), \( w = \frac{P_s(z)}{P_s(0)} \), measurement of pump depletion, \( u = \frac{P_s(z)}{P_p(0)} \), measurement of how much pump energy transformed to Stokes, \( a = \alpha L \), measurement of the total linear absorption, \( k = \gamma P_p(0)L \), \( \gamma = \frac{g_u}{\omega_0} \), \( \epsilon = \frac{P_s(L)}{P_p(0)} \).
\[
\frac{dw}{d\xi} = -kw - aw \quad \frac{du}{d\xi} = -kw + au
\]  
(12)

Boundary conditions: \( w(0) = 1 \quad u(1) = \varepsilon \)

We recognize the unknowns through perturbations to the lossless systems,

\[
w(\xi) = A(\xi)[1 - \rho(\xi)] \quad u(\xi) = B(\xi)[1 - \mu(\xi)]
\]  
(13)

In the lossless systems:

\[
\frac{dA}{d\xi} = -kAB \quad \frac{dB}{d\xi} = -kAB
\]  
(14)

Boundary conditions: \( A(0) = 1 \quad B(0) = \varepsilon \)

Then \( A(\xi) = C_0[1 - (1 - C_0)\exp(-C_0k\xi)]^{-1} \quad B(\xi) = C_0(1 - C_0)[\exp(C_0k\xi) - 1 + C_0]^{-1} \)

\[
C_0 = A(\xi) - B(\xi)
\]  
(15)

According to (13) (14) and (12) we obtain

\[
\frac{d\rho}{d\xi} = -kB\mu + a(1 - \rho) \quad \frac{d\mu}{d\xi} = -(kA\rho + a)(1 - \mu)
\]  
(16) (17)

Boundary conditions: \( \rho(0) = 0 \quad u(1) = 0 \)

For small perturbation, we discard perturbation to \( \rho \), then \[ \mu(\xi) \approx 1 - e^{\alpha(\xi - 1)} \]

(18)

We can get the follows from (18) and (16):

\[
\int_{\xi}^{\xi_0} B(\xi)(1 - e^{\alpha(\xi - 1)})d\xi - a\xi \approx k(1 - e^{-a}) \int_{0}^{\xi} B(\xi)d\xi - a\xi
\]  
(19)

Because \( \frac{dA}{d\xi} = -kAB \Rightarrow k\int_{0}^{\xi} B(\xi)d\xi = -\ln A(\xi) \)

Then \( 1 - \rho(\xi) = A(\xi)e^{(a-1)}e^{-a\xi} \)

(20)

We can obtain the followings from (13) and (20)

\[
w(\xi) \approx A(\xi)^{\exp(-a)}\exp(-a\xi) \quad u(\xi) \approx B(\xi)\exp[a(\xi - 1)]
\]  
(21) (22)

Boundary conditions: \( B(0) = \varepsilon = C_0(1 - C_0)[\exp(C_0k) - (1 - C_0)]^{-1} \)

\[ C_0 \text{ can be written as } C_0 \approx \frac{1}{k}[\Lambda + \ln(\Lambda(1 - \frac{\Lambda}{k}))] \quad (23), \quad \Lambda = -\ln(\varepsilon k) = -\ln[\gamma P_s(L)L], \varepsilon k < < 1 \]

According to (15), (21) - (22), (23), we can obtain the simulated result of how pump light intensity \( w(\xi) \) and Stokes light intensity \( u(\xi) \) vary along the propagation distance.
Figure 2: Normalized power of pump wave $w(\xi)$ versus propagation distance $\xi$.

FIGURE 2 shows pump wave power attenuation constantly along the propagation distance $\xi$;

FIGURE 3 shows Stokes wave power become stronger constantly along the propagation distance.

We can come to the conclusion that pump wave translates energy largely to Stokes.

3.2 SBS experiment

3.2.1 composition of SBS experiment.
Altering different incident pump wave intensity, we get different curve according to the dates of spectrometer 1 and 2, showing the relation between output pump light power $I_{p}(0)$ and input pump light power $I_{p}(0)$ and also the relation between output Stokes light power $I_{s}(0)$ and input pump light power $I_{p}(0)$.

The following figures are the results according to the experiment dates.
Figure 6: the relation between output pump light power $I_{p}(0)$ and input pump light power $I_{p}(0)$.

Figure 7: the relation between output Stokes light power $I_{s}(0)$ and input pump light power $I_{p}(0)$.

We can come to the conclusion: ① when pump light intensity is weak, output pump light intensity and output Stokes light intensity become stronger because of input pump light intensity’s becoming stronger, and nearly linear enlarged; ② only when input pump light intensity comes to a value, two curves change steeply, meaning both of output pump light intensity and output Stokes light intensity amplify rapidly. output pump light intensity is still large compared to its depletion; while Stokes light intensity largely is amplified because of SBS gain; ③ then input pump light intensity become stronger and accompanied is energy transferring largely to Stokes, output pump light intensity trends to be a gentle value due great depletion: output Stokes light intensity obtains gain, and turn into
saturation regimes. So if the input pump light intensity is strong, its depletion is a problem that can’t be ignored.

In our model we use the common single-mode fiber SMF-28 at 1550nm, and the value are as follows: \( A_g = 50 \text{m} \), \( \Gamma_g/2\pi = 4 \text{MHz} \), \( g_0 = 5 \times 10^{-13} \text{m}^2/\text{W} \), \( L = 50 \text{m} \), \( \lambda = 1550 \text{nm} \), \( n_g = 1.45 \), \( \alpha = 0.2 \text{dB/km} \).

Then the pulse time delay can be written \(^{[6]}\):

\[
\Delta T_d = \frac{L}{c} \left( \frac{1}{n_g} - \frac{1}{n_{fg}} \right) = \frac{G}{\Gamma_g} \left( 1 - \frac{4 \delta \omega^2 / \Gamma_g^2}{1 + 4 \delta \omega^2 / \Gamma_g^2} \right) \left( 1 - 2 \delta \omega^2 / \Gamma_g^2 \right) \tag{24}
\]

Gain parameter \( G = g_0 L I_p(0)^{1/6} \). If detuning is ignored (\( \delta \omega = 0 \)), then \( \Delta T_d \propto G \propto I_p(0) \), and the time delay trend of the Stokes light is shown in fig 6.

![Figure 8: the relation between slow-light time delay and gain parameter G](image)

3.2.2 the experiment performance of the SBS-slow light. In our experiment system, we get a delay of 8ns when the FWHM of input Stokes is 200ns, and G is 18.
FIGURE 9 shows power spectrum of Stokes pulse time delay. The difference between the output and input light is 104 ns when there is no pump as shown in figure (a); and 112 ns when pump light exists. So we can obtain a time delay of 8 ns. But severe distortion emerged.

4. CONCLUSION

In this paper we consider both fiber absorption and pump depletion for light pulse propagating along the propagation distance. According to three-wave coupling equations, we deduce the principle of slow-light based on SBS, that is signal frequency causes resonance near Stokes frequency, and medium index become greater, then group slow down. In our experiment system, we get a delay of 8 ns when the FWHM of input Stokes is 200 ns, and G is 18. And according to the experiment dates, we also get the qualitative relationship between pulse delay and gain parameter. Time delay changes in line with pump power, that is pump depletion makes a big difference to time delay. So in order to get satisfactory results of time and gain, appropriate fiber length and input pump power must be attentively considered.

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REFERENCES

[1] SONG K Y, HERRAEZ M G, THEVENAZ L 2005 Opt. Express. 13 82-88
[2] ZHU ZH M, DAWES A M C, GAUTHIER D J 2007 Journal of Lightwave Technology 25 201-206
[3] ZHU ZH M, DAWES A M C, GAUTHIER D J 2006 Optical Fiber Communication Conference (OFC) PDPI
[4] SONG K Y, HOTATE K 2007 Optical Fiber Communication Conference (OFC)
[5] Yin J C, Xiao X S, Yang C X 2010 ACTA PHYSICA SINICA 59 3986-91
[6] Zhu ZH M, Daniel J. Gauthier 2005 J. Opt. Soc. Am. B 22 2378-84
[7] Aydin Yeniay, Jean-Marc Delavaux, Jean Toulouse 2002 Journal of Lightwave Technology 20 1425-32
[8] G. P. Agrawal 2001 Nonlinear Fiber Optics, 3rd ed chap9 ACADEMIC PRESS 335-384
[9] Andrey Kobyakov, Sergey Darmanyan, Michael Sauer 2006 OPTICS LETTERS 31 1960-62
[10] K. Y. Song, M. G. Herrera and L. Thevenaz 2005 Opt.Lett.30 1782
[11] Yoshitomo Okawachi, Jay E. Sharping, and Alexander L. Gaeta 2005 Phys. Rev. Lett. 94 153902
[12] Miguel González Herráez, Kwang Yong Song, Luc Thévenaz 2006 Optics Express.14 1395-1400
[13] David Dahan and Gadi Eisenstein, 2005 Optics Express 13 6234-49
[14] M. G. Herrraez ,K. Y. Song and L. Thevenaz 2005 Appl.Phys.Lett.87 081113