Collective excitations of an imbalanced fermion gas in a 1D optical lattice

R. Mendoza  
Posgrado en Ciencias Físicas, UNAM; Instituto de Física, UNAM

Mauricio Fortes and M. A. Solís  
Instituto de Física, UNAM, Apdo. Postal 20-364, 01000 México D.F., México

The collective excitations that minimize the Helmholtz free energy of a population-imbalanced mixture of a $^6$Li gas loaded in a quasi one-dimensional optical lattice are obtained. These excitations reveal a rotonic branch after solving the Bethe-Salpeter equation under a generalized random phase approximation based on a single-band Hubbard Hamiltonian. The phase diagram describing stability regions of Fulde-Farrell-Larkin-Ovchinnikov and Sarma phases is also analyzed.

PACS numbers: 74.70.Tx, 74.25.Ha, 75.20.Hr  
Keywords: Superfluidity, Bethe-Salpeter, Collective excitations, Phase diagram

I. INTRODUCTION

Optical lattices are tailored made to study strongly-correlated Fermi systems, the stability of different phases and the effects of dimensionality in population-imbalanced mixtures of different species of ultra cold gases under attractive interactions. The FFLO phase competes with a number of other phases, such as the Sarma phases, who suggested that the order parameter is a superposition of two plane waves. In the latter case, the Fermi surfaces of each species are no longer aligned and Cooper pairs have non-zero total momenta $2q$. Such phases were first studied by Fulde and Ferrell (FF), who used an order parameter that varies as a single plane wave, and by Larkin and Ovchinnikov (LO), who suggested that the order parameter is a superposition of two plane waves.

The mean-field treatment of the FFLO phase in a variety of systems, such as atomic Fermi gases with population imbalance loaded in optical lattices, shows that the FFLO phase competes with a number of other phases, such as the Sarma ($q=0$) states, but in some regions of momentum space the FFLO phase is more stable as it provides the minimum of the mean-field expression of the Helmholtz free energy. In addition, recent calculations on the FFLO phase of the same system in two- and three-dimensional optical lattices suggest that the region of stability of this phase as a function of polarization increases when the dimensionality of the periodic lattice is lowered.

In this work, we use a Bethe-Salpeter approach to obtain the collective excitations of the two-particle propagator of a polarized mixture of two hyperfine states $|\uparrow\rangle$ and $|\downarrow\rangle$ of a $^6$Li atomic Fermi gas with attractive interactions loaded in a quasi one-dimensional optical lattice described by a single-band Hubbard Hamiltonian.

II. HUBBARD MODEL

The Hamiltonian of a two-component Fermi gas under an attractive contact interaction in a periodic lattice with constant $a$ is

$$H = -J_x \sum_{\langle i,j \rangle \sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} - J_y \sum_{\langle i,j \rangle \sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} - J_z \sum_{\langle i,j \rangle \sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} - \sum_i \left( \mu_\uparrow \hat{c}_{i,\uparrow}^\dagger \hat{c}_{i,\uparrow} + \mu_\downarrow \hat{c}_{i,\downarrow}^\dagger \hat{c}_{i,\downarrow} \right) + U \sum_i \hat{c}_{i,\uparrow}^\dagger \hat{c}_{i,\uparrow} \hat{c}_{i,\downarrow}^\dagger \hat{c}_{i,\downarrow},$$

where $J_\nu$ is the tunneling strength of the atoms between nearest-neighbor sites in the $\nu$-direction; $U$ is the on-site attractive interaction strength; $\mu_{\uparrow,\downarrow}$ is the chemical potential of species $|\uparrow\rangle$, $|\downarrow\rangle$, and the Fermi operator $\hat{c}_{i,\sigma}^\dagger$ ($\hat{c}_{i,\sigma}$) creates (destroys) an atom on site $i$.

We assume a system with a total number of atoms $M = M_\uparrow + M_\downarrow$ distributed along $N$ sites of the optical-lattice potential. For a quasi one-dimensional (1D) system the tunneling strengths satisfy $J_x \gg J_y = J_z$ and the usual tight-binding lattice dispersion energies are $\xi_{\uparrow,\downarrow}(k) = 2 \sum_\nu J_\nu \left( 1 - \cos k_\nu a \right) - \mu_{\uparrow,\downarrow}$.

The order parameter $\Delta_i = U \langle \hat{c}_{i,\downarrow} \hat{c}_{i,\uparrow} \rangle$ of the FFLO states is assumed to vary as a single plane wave, $\Delta_i = \Delta \exp (2q \cdot r_i)$, where $2q$ is the pair center-of-mass momentum, $r_i$ the coordinate of site $i$, and $\Delta$ is the usual BCS gap.

III. PHASE DIAGRAMS

Within the mean field approximation and using a Bogoliubov transformation to diagonalize the Hamiltonian, the grand canonical partition function $Z$ can be obtained in terms of both, electronlike and holelike dispersion $\omega_{\pm} = E_q(k) \pm \eta_q(k)$, where $\eta_q(k) = \frac{1}{2} \left[ \xi_\uparrow(k+q) - \xi_\downarrow(q-k) \right]$ and $E_q(k) = \sqrt{\chi_q^2(k) + \Delta^2}$.
The thermodynamic potential \( \Omega = -\frac{1}{\beta} \ln Z \) is

\[
\Omega = \frac{1}{N} \sum_{k} \left[ \chi_{\uparrow}(k) + \omega_{\uparrow}(k, q) + \frac{\Delta^2}{U} \right] - \frac{1}{\beta} \sum_{k} \left[ \ln \left( 1 + e^{-\beta \omega_{\uparrow}(k, q)} \right) + \ln (1 + e^{\beta \omega_{\downarrow}(k, q)}) \right],
\]

where \( \chi_{\uparrow}(k) = \frac{1}{2} \left[ \xi_{\uparrow}(k + q) + \xi_{\uparrow}(q - k) \right] \) and \( \beta = 1/k_{B}T \). The Helmholtz free energy \( F(\Delta, f_{\uparrow}, f_{\downarrow}, T) = \Omega + \mu_{\uparrow}f_{\uparrow} + \mu_{\downarrow}f_{\downarrow} \) determines the stable phases of this system as a function of temperature and polarization \( P \equiv f_{\downarrow} - f_{\uparrow} \).

\[
f_{\uparrow} = \frac{1}{N} \sum_{k} \left[ u_{\uparrow}^{2}(k)f(\omega_{\uparrow}(k, q)) + v_{\uparrow}^{2}(k)f(-\omega_{\uparrow}(k, q)) \right],
\]

\[
f_{\downarrow} = \frac{1}{N} \sum_{k} \left[ u_{\downarrow}^{2}(k)f(\omega_{\downarrow}(k, q)) + v_{\downarrow}^{2}(k)f(-\omega_{\downarrow}(k, q)) \right],
\]

\[
1 = \frac{U}{N} \sum_{k} \left[ 1 - f(\omega_{\uparrow}(k, q)) - f(\omega_{\downarrow}(k, q)) \right] \frac{\partial \chi_{\uparrow}(k)}{\partial q_{x}} \frac{\partial \chi_{\downarrow}(k)}{\partial q_{x}} \left[ 1 - f(\omega_{\downarrow}(k, q)) - f(\omega_{\downarrow}(k, q)) \right]
\]

where \( u_{\uparrow}(k) = \sqrt{\frac{1}{2} \left[ 1 + \frac{\chi_{\uparrow}(k)}{E_{q}(k)} \right]} \), \( v_{\downarrow}(k) = \sqrt{\frac{1}{2} \left[ 1 - \frac{\chi_{\downarrow}(k)}{E_{q}(k)} \right]} \)

and \( f(x) \) is the Fermi distribution function.

The phase diagram of the quasi 1D system is shown in Fig. 3. It is interesting to note that the FFLO phase is dominant over quite a large region of the phase-diagram and is stable at higher values for the polarization (up to \( P \approx 0.64 \)) compared to our previous work in 2D and 3D systems with the same composition and dynamical parameters. In addition, the mixed phase or phase separation regime in which an unpolarized BCS core and a polarized normal fluid (in momentum space) coexist at very low temperatures and moderate polarizations is no longer present in this system in contrast to the 3D system (and to a lesser extent in the 2D optical lattice).

### IV. COLLECTIVE STATES

The spectrum of the collective modes can be obtained from the poles of the two-particle Green’s function \( K(1, 2; 3, 4) \), where we use the compact notation \( 1 = \{ \sigma_{1}, r_{1}, t_{1} \} \), \( 2 = \{ \sigma_{2}, r_{2}, t_{2} \} \), ... with \( \sigma_{i} \) denoting the spin variables, \( r_{i} \) the vector for lattice site \( i \), and \( t_{i} \), the time variable. \( K \) satisfies the following Dyson equation:

\[
K = K_{0} + K_{0}IK,
\]
where \( K_0(1, 2; 3, 4) \) is the two-particle free propagator which is defined by a pair of fully dressed single-particle Green’s function,

\[
K_0(1, 2; 3, 4) = G(1; 3)G(4; 2),
\]

and the interaction kernel \( I \) is given by functional derivatives of the mass operator.

Using the generalized random phase approximation, we replace the single-particle excitations with those obtained by diagonalizing the Hartree-Fock (HF) Hamiltonian while the collective modes are obtained by solving the Bethe-Salpeter (BS) equation in which the single-particle Green’s functions are calculated in the HF approximation, and the BS kernel is obtained by summing ladder and bubble diagrams. The resulting equation for the BS amplitudes \( \hat{\Psi}_q(k, Q) \) is

\[
\hat{\Psi}_q(k, Q) = -U \hat{D} \sum_p \hat{\Psi}_q(p, Q) + U \hat{M} \sum_p \hat{\Psi}_q(p, Q),
\]

where \( \hat{\Psi}_q(k, Q) \) is a vector with four components and the \( 4 \times 4 \) matrices \( U \hat{D} \) and \( U \hat{M} \) represent the contribution resulting from the direct and exchange interactions, respectively. The dispersion \( \omega(Q) \) for the collective excitations is obtained from the solutions of the \( 4 \times 4 \) secular determinant defined by (5). In Fig. 3 we show the collective excitations of the 1D system. For small \( Q \) the Goldstone mode is clearly present with a sound velocity of \( v_s = 4.92 \text{ mm/s} \). For larger \( Q \), a rotonlike minimum appears with a gap \( \Delta_r = 0.0157E_R \) and a critical flow velocity \( v_f = 1.05 \text{ mm/s} \).

V. CONCLUSIONS

We have shown that superfluid phases of the FFLO and Sarma types are present in ultra cold Fermi gases loaded in quasi one-dimensional optical lattices. The region of stability of the FFLO states in the phase diagram is larger and supports higher population imbalances than identical systems in 2D and 3D. The energy dispersion of collective excitations have the usual Goldstone-mode behavior with a sound velocity of 4.92 mm/s. In addition, for higher momenta a rotonic branch is also present.

Acknowledgments

We acknowledge the partial support from UNAM-DGAPA grants IN105011, IN-111613 and CONACyT grant 104917.

1 T. Esslinger, Ann. Rev. Condensed Matter Phys. 1, 129 (2010).
2 Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Nature 451, 689 (2008).
3 W. Ketterle, Y. Shin, A. Schirotzek and C. H. Schunk, J. Phys, Condensed Matter 21, 164206 (2009).
4 Yean-an Liao, A.S.C. Rittner, T. Paprotta, W. Li, G.B. Partridge, R.G. Hulet, S.K. Baur and E.J. Mueller, Nature 467, 567 (2010).
5 P. Fulde, and R. A. Ferrell, Phys. Rev. 135, A550 (1964).
6 A. I. Larkin, and Y. N. Ovchinnikov, Zh. Eksp. Teor. Fiz., 47, 1136 (1964) [Sov. Phys. JETP 20, 762 (1965)].
7 T. Koponen et al., New Journal of Physics 8, 179 (2006)
8 T. Koponen et al., Phys. Rev. Lett. 99, 120403 (2007); T. Paananen, T. K. Koponen, P. Törma, and J.P. Martikainen, Phys. Rev. A 77, 053602, (2008).
9 Tung-Lam Dao, A. Georges, and M. Capone, Phys. Rev. B 76, 104517 (2007); Q. Chen et al., Phys. Rev. B 75,
014521 (2007); Xia-Ji Liu, H. Hu, and P. D. Drummond, Phys. Rev. A 76, 043605 (2007); M. Rizzi, et al., Phys. Rev. B 77, 245105 (2008); Xia-Ji Liu, Hui Hu, and P. D. Drummond, Phys. Rev. A 78, 023601 (2008); M. Reza Bakhhtiari, M. J. Leskinen, and P. Törma, Phys. Rev. Lett. 101, 120404 (2008); A. Lazarides and B. Van Schaeybroeck, Phys. Rev. A 77, 041602 (2008); T Paananen, J. Phys. B: At. Mol. Opt. Phys. 42, 165304 (2009); X. Cui and Y. Wang, Phys. Rev. B 79, 180509(R) (2009); A. Mishra and H. Mishra, Eur. Phys. J. D 53, 75 (2009); B. Wang, Han-Dong Chen, and S. Das Sarma, Phys. Rev. A 79, 051604(R) (2009); Y. Yanase, Phys. Rev. B 80, 220510(R) (2009); A. Ptok, M. Mäska, and M. Mierzejewski, J. Phys.: Condens. Matter 21, 295601 (2009); Yan Chen et al., Phys. Rev. B 79, 054512 (2009); Yen Lee Loh and N. Trivedi, Phys. Rev. Lett. 104, 165302 (2010); A. Korolyuk, F. Massel, and P. Törma, Phys. Rev. Lett. 104, 236402 (2010); F. Heidrich-Meisner et al., Phys. Rev. A 81, 023629 (2010); S. K. Baur, J. Shumway, and E. J. Mueller, Phys. Rev. A 81, 033628 (2010); A. Korolyuk, F. Massel, and P. Törma, Phys. Rev. Lett. 104, 236402 (2010); M. J. Wolak et al., Phys. Rev. A 82, 013614 (2010); L. Radzihovsky and D. Sheehy, Rep. Prog. Phys. 73, 076501 (2010).

K. Machida, T. Mizushima and M. Ichioka, Phys. Rev. Lett. 97 120407 (2006).

11. T.L. Dao, M. Ferrero, A. Georges, M. Capone and O. Parcollet, Phys. Rev. Lett. 101 236405 (2008).

12. Z. G. Koinov, R. Mendoza and M. Fortes, Phys. Rev. Lett. 106, 100402 (2011).

13. R. Mendoza, M. Fortes, M.A. Solís and Z.G. Koinov, arXiv:1306.4706 (2013).

14. G. Sarma, J. Phys. Chem. 24, 1029 (1963).

15. W.V. Liu and F. Wilczek, Phys. Rev. Lett., 90, 047002 (2003).

16. P. F. Bedaque, H. Caldas, and G. Kupak, Phys. Rev. Lett. 91, 247002 (2003); H. Caldas, Phys Rev. A 69, 063602 (2004); H. Caldas, C. W. Morais and A. L. Mota, Phys. Rev. D 72, 045008 (2005); S. Sachdev and K. Yang, Phys. Rev. B 73, 174504 (2006).

17. Z. G. Koinov, Physica C 407, 470 (2010); Physica Status Solidi (B) 247, 140 (2010); Ann. Phys. (Berlin) 522, 693 (2010); cond-mat/1010.1200.

18. K. V. Samokhin, Phys. Rev. B 81, 224507 (2010).

19. Y.-P. Shim, R. A. Duine, and A. H. MacDonald, Phys. Rev. A 74, 053602 (2006).