Abstract—In this letter, we study the reference signal-aided channel estimation concept which is a crucial requirement to address the realistic performance of spatial media-based modulation (SMBM) systems where the radio frequency mirrors are deployed along with the multiple transmit antennas. Accordingly, least squares and linear minimum mean square error-based channel estimation schemes are proposed for MBM-based systems for the first time in the literature where former studies mainly assume either perfect channel state information or an error model on channel coefficients. In addition, corresponding symbol detection performance is studied. To measure the efficiency of the proposed channel estimation approaches, the theoretical upper bounds on average bit error rate are derived and shown to be well overlapped with the computer simulations for the medium and high signal-to-noise ratio regions. This study is important due to the implementation of channel estimation as well as the theoretical derivation of detection bounds for MBM-based communication systems.

Index Terms—Average bit error rate, channel estimation, media-based modulation, spatial modulation, symbol detection.

I. INTRODUCTION

Spatial modulation (SM) inherently incorporates one extra transmit unit through allocating information bits to the corresponding transmit antenna selection for communication in addition to the transmission of conventionally modulated data symbols [1], [2]. Thus, the overall spectral efficiency, which is one of the crucial design issue of next generation communication systems, can be increased.

Media-based modulation (MBM), proposed recently [3], is one of the novel channel modulation techniques where the wireless channel environment can be manipulated through deployed electronic radio frequency (RF) mirrors on the transmit antennas [4]. Based on the digital on/off condition of controllable RF mirrors, which is specified by the RF source bits, different channel environments are realized. This helps to diversify the channel fading realizations that leads to improved system performance due to converting Rayleigh fading channel to additive white Gaussian noise (AWGN) channel [5].

On the other hand, spatial MBM (SMBM), which is the combination of SM and MBM principles belonging to index modulation (IM) family [6], [7], is able to provide both increased spectral efficiency especially when applied to orthogonal frequency division multiplexing (OFDM)-based systems and channel robustness. Accordingly, there are many studies concentrating on IM family schemes from various perspectives on the system performance [5]-[15]. In [8], the performance of SM is investigated in the presence of channel estimation errors. In [9], block pilot-based channel estimation is proposed for generalized SM (GSM)-OFDM systems where the IM idea is extended to frequency domain. Similarly, only detection performance of generalized multiple-mode OFDM with IM is demonstrated in [10]. In [11], quadrature channel modulation is proposed as a combination of quadrature SM and MBM, and, its corresponding detection performance analysis is evaluated. In [12], GSM-MBM system detection performance is examined in terms of bit-error-rate (BER). In [13], the effect of imperfect channel estimation is studied on the BER performance of MBM by simply introducing an error model on the perfect channel state information (P-CSI). In [14], least squares (LS)-based channel estimation is applied to IM based OFDM system by employing pilot symbols into the OFDM data. On the other hand, a compressed sensing-based joint user and symbol detection approach is proposed for MBM-enable massive machine-type communications [15].

In the studies addressed in [1]-[15], the main concentration is to obtain detection performance without performing channel estimation except [14] and [15] where authors implement channel estimation for OFDM-IM and massive machine-type systems separately, respectively. The common view is to use channel estimation errors to model the performance under imperfect CSI (I-CSI) to get rid of challenging channel estimation problem.

In this letter, LS and linear minimum mean square error (LMMSE)-based channel estimation schemes are considered for SMBM systems operated under block-wise time-varying Rayleigh fading channels where the effects of channel estimation to the performance of SMBM systems has not been investigated yet. The effect of number of RF mirrors deployed in SMBM on the system performance is investigated. In addition, symbol detection performance is evaluated in terms of BER for different modulation types. Moreover, an upper bound on average BER (ABER) is derived theoretically and shown to be attainable for the proposed channel estimation scheme even in I-CSI conditions. Computer simulation results confirm that channel estimation performance improves with the increasing number of RF mirrors due to the realization of additional channel environments. In the medium-to-high signal-to-noise ratio (SNR) regime, LS-based estimation performance is capable of achieving the LMMSE performance.
II. SYSTEM MODEL

In this section, SMBM system model is introduced by providing transmitter and receiver characteristics. The transceiver model of SMBM system is illustrated in Fig. 1 where SMBM system is assumed to have \( \mathcal{N}_r \) transmit and \( \mathcal{N}_r \) receive antennas. In addition, each transmit antenna has a number of deployed \( \mathcal{N}_{rf} \) RF mirrors. Each RF mirror either reflects the incident wave or passes the incident wave which is called on/off status of RF mirrors. On/off status of an RF mirror can be controlled by one information bit and each on/off combination of RF mirrors creates different radiation patterns.

A. Transmitter Model

In one communication interval of SMBM systems

\[
\eta = \log_2 M + \log_2 \mathcal{N}_r + \mathcal{N}_{rf}
\]

bits per channel use (bpcu) are generated by the information source and these bits can be classified as

\[
q = \begin{bmatrix} \log_2 M \text{ bits} & \log_2 \mathcal{N}_r \text{ bits} & \mathcal{N}_{rf} \text{ bits} \end{bmatrix} = \begin{bmatrix} \cdots & 010 & \cdots & 110 & \cdots & 01 \cdots \end{bmatrix},
\]

where the first \( \log_2 M \) bits are used to obtain modulated data signals depending on its modulation type (\( M \)-quadrature amplitude modulation, \( M \)-QAM or \( M \)-phase shift keying, \( M \)-PSK signal), while \( \log_2 \mathcal{N}_r \) and \( \mathcal{N}_{rf} \) bits are used to determine the active transmit antenna for SM and the on/off status of RF mirrors (i.e., channel state) for MBM, respectively.

Accordingly, in the mapper block of SMBM transmitter, one transmit antenna and one channel state is selected for transmission by the source bits allocated to determine the active transmit antenna and channel realization, respectively. Since there are possible \( \mathcal{N}_r \) transmit antennas and \( 2^{\mathcal{N}_{rf}/2} \) channel states, the transmitted signal vector is represented as

\[
x = \begin{bmatrix} \mathcal{N}_r \text{ zeros} & \mathcal{N}_r \text{ zeros} & \mathcal{N}_r - 1 \text{ zeros} & \mathcal{N}_r \text{ zeros} \end{bmatrix} \begin{bmatrix} 0 \cdots 0 & 0 \cdots d \cdots 0 & 0 \cdots 0 \end{bmatrix}^T \in \mathbb{C}^{\mathcal{N}_r 2^{\mathcal{N}_{rf}/2}},
\]

where the modulated data symbol, \( d \) is conveyed at the \( m \)th coordinate that can be formulated as \( m = (k-1)\mathcal{N}_r + j \) with \( k \in \{1, 2, \ldots, 2^{\mathcal{N}_{rf}/2}\} \) and \( j \in \{1, 2, \ldots, \mathcal{N}_r\} \) denoting the indices of channel state and active transmit antenna, respectively. Here, \( [\cdot]^T \) stands for the transpose operation. Note that transmit signal vector \( x \) is naturally sparse vector since it consists of one nonzero modulated signal value and \( \mathcal{N}_r 2^{\mathcal{N}_{rf}/2} - 1 \) zeros thanks to the SMBM technique.

B. Receiver Model

The received signal model of SMBM communication system can be represented as

\[
y = Gx + w,
\]

where \( G \) is the channel matrix comprising of Rayleigh fading coefficients and \( w \) represents the zero-mean AWGN with variance \( \sigma_w^2 \). The channel matrix can be decomposed as

\[
G = \begin{bmatrix} G^1, G^2, \ldots, G^{2^{\mathcal{N}_{rf}/2}} \end{bmatrix},
\]

where

\[
G^k = \begin{bmatrix} h_{k,1}^1, h_{k,1}^2, \ldots, h_{k,1}^{\mathcal{N}_r} \\
\vdots & \ddots & \vdots \\
0 & \cdots & h_{k,\mathcal{N}_r,1}^{\mathcal{N}_r} \\
\end{bmatrix},
\]

is the sub-channel matrix belonging to the \( k \)th channel state with \( h_{k,j}^{i} \) denoting the channel coefficient between \( i \)th receiver and \( j \)th transmitter antennas at the \( k \)th state for \( i \in \{1, 2, \ldots, \mathcal{N}_r\} \).

III. CHANNEL ESTIMATION AND SYMBOL DETECTION

In this section, the channel estimation and symbol detection are explained in SMBM systems.

We consider block-wise time-varying Rayleigh fading channel where coefficients remain unchanged for a certain communication interval. In order to estimate the channel, a reference signal (i.e., pilot symbol) \( p \) with unit energy is used first regarding all the possible channel coefficients. Then, communication still continues for the remaining signaling interval until channel changes to detect the data symbols by using the estimated channel coefficients.

A. Channel Estimation

The received signal expression defined in (4) can be rearranged alternatively in accordance with the reference signal-aided channel estimation process as

\[
r = Ph + n,
\]

where \( r, P, h \) and \( n \) denote the received reference signal vector, transmission matrix, channel vector and zero-mean AWGN with variance \( \sigma_n^2 \), respectively. Since same reference signal is used throughout the estimation process and only one symbol is transmitted actively in a signaling duration, reference signal transmission matrix \( P \) appears as a diagonal matrix. Therefore, transmission matrix can be written as

\[
P = \begin{bmatrix} p & 0 & \cdots & \cdots & 0 \\
0 & p & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & p \\
\end{bmatrix},
\]

where \( n \) is the corresponding zero-mean AWGN with variance \( \sigma_n^2 \). However, channel vector \( h \) contains all possible channel coefficients and can be expressed as

\[
h = \begin{bmatrix} h_{1,1}^1, h_{1,1}^{2^{\mathcal{N}_{rf}/2}}, h_{2,1}^1, \ldots, h_{2,1}^{2^{\mathcal{N}_{rf}/2}}, \ldots, h_{\mathcal{N}_r,1}^1, h_{\mathcal{N}_r,1}^{2^{\mathcal{N}_{rf}/2}} \end{bmatrix}^T.
\]

After reference signal transmission process, channel coefficients can be estimated by applying LS and LMMSE considering the linear model defined in (7), respectively, as

\[
\hat{h} = (P^TP)^{-1}P^Tr,
\]

\[
\hat{h} = (P^TP + \sigma_n^2 R_h^{-1})^{-1}P^Tr,
\]
where $R_h = E[hh^H]$ represents the autocorrelation matrix of Rayleigh distributed channel coefficients which is a diagonal matrix whose diagonal components are all equal to channel power, $\sigma_h^2$ for this case while $\hat{h}$ contains estimated channel coefficients and $[\cdot]^\dagger$ stands for the Hermitian conjugate operation. Note that the diagonal autocorrelation matrix becomes unitary matrix since the channel power is normalized to unity.

When the channel estimation phase is completed, the channel estimation performance can be evaluated in terms of mean square error (MSE) that can be expressed as

$$\text{MSE} = \frac{1}{2N_r N_t} E[(\hat{h} - h)^\dagger(\hat{h} - h)],$$

(12)

where $E$ denotes the expectation operation.

### B. Symbol Detection

Symbol detection phase is operated following the channel estimation phase. To make the detection process of transmitted symbols convenient, the channel coefficients between $j$th transmitter and all receivers at $k$th channel state can be expressed as

$$h^s_j = [h^s_{1,j}, h^s_{2,j}, \ldots, h^s_{N_r,j}]^T.$$ 

(13)

Accordingly, received signal demodulation is applied using maximum-likelihood (ML) detection. In order to clarify detection phase, let us first define $h^d_j$ as the estimation of the channel vector, $h^d_j$ defined in (13). Through ML detection, data symbol, transmitter antenna index and RF mirror state are estimated by finding minimum Euclidean distance as represented below

$$[\hat{d}, \hat{j}, \hat{k}] = \arg\min_{d,j,k} \left\| y - \hat{h}^d_j \right\|^2.$$ 

(14)

### IV. AVERAGE BIT ERROR PROBABILITY ANALYSIS

In this section, average bit error probability (ABEP), i.e., an upper bound on ABER, for SMBM system is calculated by the derivation of average pair-wise error probability (PEP) for a single receiver case and generalization of this derivation to the systems with multiple receivers.

**A. ABEP for Single Receive Antenna**

In a single receive antenna (SRA) system, probability of error can be defined and analytically calculated as [16]

$$P_e = P_r((h_l,s_l) \rightarrow (h_l,s_l)|\hat{h}) = Q \left( \sqrt{ \frac{||h_ls_l - h_ls_l||^2}{2(\sigma^2_s|s|^2 + \sigma^2_h)} } \right) = Q(\sqrt{\gamma}),$$

(15)

where $h_l$ represents the $l$th element of channel vector corresponding to the related transmitter antenna and RF mirror combination while $s_l$ and $\sigma^2_h$ denote the $l$th modulated symbol and the variance of disruptive effects in channel estimation, respectively. Probability density function of $\gamma$ can be found as $p_\gamma(\gamma) = \frac{1}{2} \exp(-\frac{\gamma}{2})$. Using partial derivation method, average PEP for an SRA can be calculated as

$$\text{PEP}_{\text{SRA}} = \int_0^\infty P_e p_\gamma(\rho) \, d\rho$$

$$= \frac{1}{2} \left( 1 - \frac{\sqrt{\gamma/2}}{1 + \sqrt{\gamma/2}} \right),$$

(16)

where

$$\gamma = \frac{E_s \sigma^2_s (1 + \sigma^2_h)}{2(\sigma^2_h + \sigma^2_s|s|^2)} \times \begin{cases} |s_l - s_l|^2, & \text{if } l = l' \\ |s_l|^2 + |s_l|^2, & \text{if } l \neq l' \end{cases}.$$ 

(17)

Thus, ABER for SRA case in SMBM systems is upper bounded by the ABEP as

$$\text{ABER} \leq \frac{1}{\eta 2^n} \sum_{l,f} \sum_{l,\ell} \text{PEP}_{\text{SRA}} e(l,\ell \rightarrow l,\ell),$$

(18)

where $e(l,\ell \rightarrow l,\ell)$ is number of erroneously detected bits related with $(l,\ell \rightarrow l,\ell)$ event and the right-hand side of the inequality in (18) corresponds to the closed-form expression of theoretical ABEP for SRA case.

**B. ABEP for Multiple Receive Antennas**

In multiple receive antenna (MRA) case, the square of Euclidean distance between two transmitted signals can be
written as a sum of squared Euclidean distances between these two transmitted signals in each antenna. Therefore, error probability for a given channel matrix, considering an MRA case can be analytically calculated as

\[
P(e|\rho) = P_r((h_i,s_i) \rightarrow (h_{i'},s_{i'})) \hat{G}
\]

\[
= Q \left( \sqrt{\frac{\sum_{i=1}^{N_r} \left| h_{i,s} - h_{i',s'} \right|^2}{2(N_r - 1 + i) \sigma_n^2 + \sigma_n^2}} \right)
\]

\[
\triangleq Q(\sqrt{\gamma_{MRA}}), \tag{19}
\]

where \( \hat{G} \) stands for the estimated channel matrix.

The series sum of \( \gamma \) variable for an SRA provides a random variable with chi-square distribution for the MRA case. When the chi-square distributed random variable is integrated for all possible values in a similar manner, average PEP for multiple receive antenna can be derived as

\[
\text{PEP}_{MRA} = (\text{PEP}_{SRA})^N \sum_{i=0}^{N_r - 1} \binom{N_r - 1 + i}{i} (1 - \text{PEP}_{SRA})^i. \tag{20}
\]

Finally, ABER for MRA case in SMBM systems is upper bounded by the ABEP as

\[
\text{ABER} \leq \frac{1}{\eta 2^N} \sum_{i=0}^{N_r - 1} \sum_{i'=0}^{N_r - 1} \text{PEP}_{MRA} e(i, i' \rightarrow \hat{i}, \hat{i}') \tag{21}
\]

where the right-hand side of the inequality in (21) corresponds to the closed-form expression of theoretical ABEP for MRA case.

V. SIMULATION RESULTS

In the following, Monte-Carlo channel estimation and symbol detection simulation results of SMBM system will be given along with the theoretical ABER for P-CSI and I-CSI, utilizing block-wise time varying uncorrelated Rayleigh fading channel with respect to signal-to-noise ratio (SNR) defined as \( \text{SNR} = E_b/\sigma_n^2 \), where \( E_b \) denotes average energy per bit. Accordingly, channel estimation and symbol detection performances of SMBM systems are provided by taking into account different channel and modulation circumstances for different SNR values.

Starting with Fig. 2, channel estimation error performances of LS and LMMSE estimation techniques are shown for SMBM system with four transmit antennas, four receive antennas (i.e., \( N_t = N_r = 4 \)) and various numbers of RF mirrors (i.e., \( N_{rf} = \{1,2,4\} \)). Thanks to the utilization of channel statistics such as correlation and noise variance, LMMSE is superior to LS for the low SNR region (i.e., between -4dB and 4dB) where the noise variance is significant. In addition, it can be pointed out that LS and LMMSE based estimation schemes perform better for the systems with higher numbers of RF mirrors due to increase in diversity against fading channel on the transmitter side.

Secondly, symbol detection performance results and analytically calculated ABEP curves using (21) belonging to LMMSE estimation are given for P-CSI conditions (i.e., ideal case) in addition to the I-CSI conditions (i.e., realistic case) in Fig. 2 considering an SMBM system for a quadrature PSK (QPSK) and 8-PSK modulated communication scenario specialized as \( N_t = N_r = 4, N_{rf} = 2 \). In this context, the spectral efficiencies of data transmission utilizing QPSK and 8-PSK modulations are \( \eta = \log_2(M) + N_{rf} + \log_2(N_t) = 6 \) bpcu and \( \eta = 7 \) bpcu, respectively. Increasing modulation level leads to the detection performance degradation, as expected. In the realistic case (i.e., I-CSI, where channel estimation is applied), detection performance is worse almost 3dB SNR compared to ideal case (i.e., P-CSI) for the same BER value. On the other hand, derived theoretical upper bounds on ABER perfectly match with the simulations for CSI conditions as well as modulation types for a certain region of SNR values.

Third, BER performances for 16-level modulations (16-QAM and 16-PSK) resulting in 8 bpcu utilization are shown in Fig. 3. Similarly, theoretical ABEP curves are tight bounds for ABER curves. In addition, BER results of 16-QAM are better...
than 16-PSK due to the higher distance among constellation points. Furthermore, compared to 8-PSK, it is also worth noting that, there is an around 2.5dB performance gap on BER values of 16-PSK among P-CSI and I-CSI cases.

VI. CONCLUSION

In this letter, LS and LMMSE based channel estimation schemes for SMBM systems under a Rayleigh fading channel are first proposed to demonstrate SMBM communication system performance in a more realistic manner. In addition, a theoretical upper bound for ABER is derived and shown to be the tight bound for Monte-Carlo simulation results. The effects of number of RF mirrors deployed on the transmit antennas and modulation types are investigated in detail. An increase in RF mirror deployment provides better channel estimation results thanks to the diversity in MBM. Moreover, detection performance of I-CSI case is found to be worse than P-CSI case with a gap of 2-3dB. These outcomes are important for providing a practical channel estimation perspective to SMBM communication system performance.

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