Possible violation of the spin-statistics relation for neutrinos: cosmological and astrophysical consequences

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Abstract

We assume that the Pauli exclusion principle is violated for neutrinos, and consequently, neutrinos obey the Bose-Einstein statistics. Cosmological and astrophysical consequences of this assumption are considered. Neutrinos may form cosmological Bose condensate which accounts for all (or a part of) the dark matter in the universe. “Wrong” statistics of neutrinos could modify big bang nucleosynthesis, leading to the effective number of neutrino species smaller than three. Dynamics of the supernova collapse would be influenced and spectra of the supernova neutrinos may change. The presence of neutrino condensate would enhance contributions of the Z-bursts to the flux of the UHE cosmic rays and lead to substantial refraction effects for neutrinos from remote sources. The Pauli principle violation for neutrinos can be tested in the two-neutrino double beta decay.

1 Introduction

What is the next surprise neutrinos will bring us? Pauli has introduced neutrino to resolve paradoxes of the beta decay, in particular, an apparent violation of the spin-statistics relation. Could the neutrino itself violate this relation? Does the particle invented by Pauli respect the Pauli principle? Do we have any indication to that?

The puzzle of cosmological dark matter (DM) remains with us already for more than a half of century but we still do not know what are the constituents of this mysterious substance. The commonly accepted point of view is that dark matter is made of new elementary particles governed by the laws of the old established physics. Here we will explore a different possibility: old particles and new physics. Namely we assume that Fermi statistics for neutrinos is violated and, if so, neutrinos with a fraction of eV masses, as observed in the oscillation experiments, could condense and make all cosmological dark
We suggest that Pauli exclusion principle is violated for neutrinos and therefore neutrinos obey (at least partially) the Bose-Einstein statistics. Of course, a possibility to explain the dark matter is not the only consequence of the spin-statistics violation. One can expect some effects of the violation in any environment where large densities or fluxes of neutrinos exist. That includes the Early Universe in the epoch of the Big Bang nucleosynthesis, the cores of collapsing stars, etc.. Some consequences could be also seen in laboratory experiments (e.g., in the double beta decay).

Possible violation of the exclusion principle was discussed in a series of theoretical papers [1] though no satisfactory model has been proposed so far. (For a critical review see ref. [2].) Experimental searches of the Pauli principle violation for electrons [3] and nucleons [4] have given negative results. It may happen however that neutrinos due to their unique properties are much more sensitive to the violation and it is in the neutrino sector the effects can be seen first. Neutrinos may also possess kind of mixed or more general statistics than Bose or Fermi ones [5].

The assumption of violation of the Pauli exclusion principle reveals immediately a number of problems. The spin-statistics theorem follows from the canonical quantization to ensure a positive definiteness of energy. It is not clear how to overcome this problem and how serious it is for neutrinos. The CPT theorem follows, in particular, from the normal relation between spin and statistics, therefore the suggested scenario may also violate the CPT theorem. Actually, a possible violation of the latter in neutrino physics is under an active study now, see, e.g., refs. [6]. Last but not least, the spin-statistics violation in the neutrino sector is communicated due to the weak interactions to charged leptons and other fermions where the bounds are extremely strong. It is not clear if effects considered in this paper are consistent with these bounds, which depends on particular mechanism of the violation.

In what follows we put aside discussion of these problems. Instead, taking pure phenomenological approach, we concentrate on cosmological and astrophysical consequences of the neutrino “bosonization” in an attempt to find interesting observable effects or to restrict such a possibility.

## 2 Bosonic neutrinos: Context

The standard electro-weak theory puts the left-handed neutrinos and electrons into the same doublet and thus one would expect that neutrinos and electrons obey the same statistics. On the other hand, as we know, being the only neutral leptons, the neutrinos can have substantially different properties from those of the charged leptons. In particular, neutrinos can be the Majorana particles and induce the lepton number violation. The difference between the charged leptons and neutrinos is related to breaking of the electro-weak (EW) symmetry. The lepton number violation (in the context of seesaw mechanism) originates from very high scales.

Similarly, the neutrino sector might be a source of violation of the spin-statistics relation; this can also be connected to EW symmetry breaking and originate from some high
mass scale of Nature. One may consider scenario where violation of the Pauli principle occurs in a hidden sector of theory related to the Planck scale physics, or strings physics. It could be mediated by some singlets of the Standard model - (heavy) neutral fermions which mix with neutrinos when the EW symmetry is broken. Since only neutrinos can mix with the singlets, effects of the Pauli principle violation would be manifested first in neutrinos and then would communicate to other particles. Also one can consider a possibility that the messenger of the Pauli principle violation is the light sterile neutrino. It has a small mixing with the active components, and this small mixing quantifies the degree of violation in the observable sector. In this way a small or partial violation of relation between spin and statistics might occur.

As in the case of lepton number, a violation of the spin-statistic relation for other particles can be suppressed by an additional power of a small parameter relevant for the violation in the neutrino sector. In fact, the high accuracy of the validity of Fermi statistics for electrons [3] could put a strong bound on a possible "transfer" of wrong statistics from neutrinos to electrons. In this connection one can consider a scenario when significant effects of the spin-statistics violation develop on the cosmological times or in particular environments of the Early Universe.

A violation of the Pauli principle for neutrinos should show up in the elementary processes where identical neutrinos are involved. A realistic process for this test is the two-neutrino double beta decay, $A \rightarrow A' + 2\bar{\nu} + 2e^-$ (or similar with antineutrinos and positrons). The probability of the decay as well as the energy spectrum and angular distribution of electrons should be affected.

The difference between neutrino statistics would be strongly pronounced if the probabilities (see, e.g. [7]) are proportional to the bi-linear combinations of the type $K_{m}K_{n}$, $K_{m}L_{n}$, $L_{m}L_{n}$, where

$$
K_{m} \equiv \left[ E_{m} - E_{i} + E_{e1} + E_{\nu1} \right]^{-1} - \left[ E_{m} - E_{i} + E_{e2} + E_{\nu2} \right]^{-1},
$$

$$
L_{m} \equiv \left[ E_{m} - E_{i} + E_{e2} + E_{\nu1} \right]^{-1} - \left[ E_{m} - E_{i} + E_{e1} + E_{\nu2} \right]^{-1},
$$

where $E_{i}$ is the energy of the initial nuclei, $E_{m}$ is the energy of the intermediate nuclei state $m$, $E_{e_i}$, and $E_{\nu_i}$ are the energies of electrons and neutrinos respectively. The minus signs between the two terms in the above expressions are due to the bosonic character of neutrinos; in the case of fermionic neutrinos we would have plus signs [7]. For electrons we assume the normal Fermi statistics.

In the case of $0^+ \rightarrow 0^+$ transitions the combinations $K_{m}$ and $L_{m}$ can be approximated by

$$
K_{m} \approx \frac{E_{e2} - E_{e1} + E_{\nu2} - E_{\nu1}}{(E_{m} - E_{i} + E_{0}/2)^2}, \quad L_{m} \approx \frac{E_{e1} - E_{e2} + E_{\nu2} - E_{\nu1}}{(E_{m} - E_{i} + E_{0}/2)^2},
$$

whereas for the fermionic neutrinos

$$
K_{m} \approx L_{m} \approx \frac{2}{E_{m} - E_{i} + E_{0}/2}.
$$

Here $E_{0}/2 = E_{e} + E_{\nu}$ is the average energy of the leptonic pair. Appearance of the differences of the electron and neutrino energies in [2] leads to a suppression of the total
probability. It also modifies the energy distributions of electrons. The probabilities of the transitions $0^+ \rightarrow 2^+$ are proportional to the combinations $(K_m - L_m)(K_n - L_n)$, where

$$ (K_m - L_m) \approx \frac{2(E_{e2} - E_{e1})}{(E_m - E_i + E_0/2)^2}. $$

(4)

In the case of the fermionic neutrinos the combination has an additional factor $(E_{\nu2} - E_{\nu1})/(E_m - E_i + E_0/2)$ and the suppression is stronger.

A large number of already detected events \[8\] and especially future measurements \[9\] allow to make precision tests of the Pauli principle. The data seems to exclude the 100% violation of Fermi statistics for electronic neutrinos \[10\]. Notice however, that relation between the statistics of neutrinos and possible (small) violation of the Pauli principle is an open issue.

In what follows we will consider for simplicity an extreme case when neutrinos have purely bosonic statistics. In this case the equilibrium density of bosonic neutrinos in phase space would be equal to

$$ f_{\nu} = (2\pi)^3 n_0 \delta(p) + [\exp (E - \mu)/T - 1]^{-1}, $$

(5)

where $\mu$ is the chemical potential of neutrinos. The first term describes a possible neutrino condensate with $n_0$ being the number density of neutrinos in the condensate. The density \[5\] is somewhat larger than the fermionic one. The thermal part (second term) alone gives the energy density of bosonic neutrinos at $T \gg m_{\nu}$ larger than the fermionic one by the factor $8/7$.

Expression \[5\] is the equilibrium solution of the kinetic equations since the collision integral with such a distribution function vanishes for non-zero $n_0$ if $\mu = m_{\nu}$. For smaller values of $\mu$, $\mu < m_{\nu}$, the collision integral vanishes only if $n_0 = 0$. Therefore the condensate would be non-vanishing only if the charge asymmetry of neutrinos is so large that the maximum chemical potential $\mu = m_{\nu}$ is not sufficient to provide such an asymmetry. We will check this explicitly in the next section for the situation during the BBN epoch.

3  Big bang nucleosynthesis

The Big bang nucleosynthesis (BBN) is very sensitive to the neutrino statistics.

Let us first show that the distribution \[5\] is the equilibrium solution of the kinetic equations. For the processes

$$ e^- + p \leftrightarrow n + \nu_e, \quad e^+ + n \leftrightarrow p + \bar{\nu}_e $$

(6)

relevant for the BBN nucleosynthesis, the collision integral is given by the integral over the phase space of the function

$$ F[f] = f_{\nu} f_n (1 - f_e) (1 - f_p) - f_e f_p (1 - f_n) (1 + f_{\nu}), $$

(7)

if the invariance with respect to time reversal holds. Here $f_p$, $f_n$ and $f_e$ are the densities of protons, neutrons and electrons correspondingly. We assume that they are described
by the equilibrium Fermi-Dirac distributions

\[ f_F = \left[ \exp \left( \frac{E - \mu}{T} \right) + 1 \right]^{-1}, \quad F = e, p, n. \]  \hspace{1cm} (8)

Notice that the neutrino distribution enters the second them in (7) with plus sign.

Inserting (5) and (8) into (7) we find that \( F[f] \) is proportional to

\[ F \propto \left[ \exp \left( \frac{1}{T} (E_n + E_\nu - E_e - E_p + \mu_e + \mu_p - \mu_n - \mu_\nu) \right) - 1 \right] \left[ 1 - \exp \left( \frac{1}{T} (E_n - E_e - E_p - \mu_n + \mu_e + \mu_p) \right) \right] \]  \hspace{1cm} (9)

\[ + (2\pi)^3 n_0 \delta(p) \left[ 1 - \exp \left( \frac{1}{T} (E_n - E_e - E_p - \mu_n + \mu_e + \mu_p) \right) \right]. \]  \hspace{1cm} (10)

The first (thermal) term vanishes because of the conservations of energy, \( E_e + E_p = E_n + E_\nu \), and the chemical potential: \( \mu_e + \mu_p = \mu_n + \mu_\nu \). (The condition for \( \mu \) is true only in equilibrium.) The second term in (10) becomes zero if \( \mu_\nu = m_\nu \). Indeed, using the conservations of the energy and chemical potential we can rewrite the sum in the exponent as

\[ E_n - E_e - E_p - \mu_n + \mu_e + \mu_p = -E_\nu + \mu_\nu = -m_\nu + \mu_\nu, \]

where the last equality is valid since \( p = 0 \). So, for \( \mu_\nu = m_\nu \), \( F[f] \) vanishes and so does the collision integral, even for \( n_0 \neq 0 \). Thus, the usual equilibrium distributions remain true despite breakdown of the spin-statistics relation. If neutrinos obey mixed statistics their equilibrium distribution can be written as a sum of the Bose and Fermi ones at least in the case when a single neutrino participates in initial and/or final states of reactions.

If T-invariance is broken, then a more complicated expression appears in the collision integral which still vanishes for the equilibrium distribution because of the S-matrix unitarity \[11\].

Since chemical potential of bosons cannot exceed their mass and neutrino mass is at most about eV, the chemical potential of neutrinos would not be essential in the BBN range of temperatures, \( T = (1 - 0.07) \) MeV. A possible large value of lepton charge asymmetry would be hidden in the condensate and would not have a strong impact on chemical abundances of light elements. If this is the case, the restrictive BBN bounds \[12, 13\] on the magnitude of the cosmological lepton asymmetry would be inapplicable.

The equilibrium energy density of the bosonic neutrinos at \( T \gg m_\nu \) is 8/7 of the energy density of fermionic neutrinos and thus the change of statistics would lead to an increase of the effective number of neutrino species at BBN by \( \Delta N_\nu = 3/7 \) (for three neutrinos). On the other hand, a larger magnitude of the neutrino distribution function and the fact that it enters the kinetic equation (see (7)) as \( (1 + f_\nu) \) instead of \( (1 - f_\nu) \) makes the weak reactions of neutron-proton transformations \[6\] faster and the \( n/p \) freezing temperature becomes lower. The latter dominates and as a result the effective number of massless species becomes smaller than 3.

To estimate the effect we proceed in the following simplified way. The kinetic equation which governs the neutron-proton transformation has the form (see, e.g., ref. \[14\]):

\[ Hx \frac{dr}{dx} = \frac{(1 + 3g_A^2)G_E^2}{2\pi^3} \left[ A - (A + B) r \right], \]  \hspace{1cm} (11)

\(^1\)Since the violation of the Pauli principle is communicated to other sectors of theory, one may expect some deviation from the Fermi-Dirac distribution for other fermions as well, but the effects are probably small.
where \( r \) is the ratio of neutron number density to the total baryon number density, \( r \equiv n/(n+p) \), \( x \) is the cosmological scale factor, \( g_A = -1.267 \) is the axial coupling constant, the function \( A(T) = B(T) \exp(-\Delta m/T) \), \( \Delta m = 1.3 \) MeV is the neutron-proton mass difference. The function \( B(T) \) is the collision integral containing the equilibrium distribution functions of electrons and neutrinos.

If neutrinos have Bose-Einstein statistics, then the numerical value of \( B(T) \) would differ from the standard fermionic one. The ratio of fermionic to bosonic \( B \)'s at the moment of neutron-proton freezing \( (T \approx \Delta m/2) \) is \( B_f/B_b = 0.933 \). This rise in the reaction rate would result in a smaller \( (n/p) \)-freezing temperature. The same shift of the freezing temperature could be mimicked by a change in the number of the effective neutrino species which enters the expression for the Hubble parameter:

\[
H \sim \sqrt{g_\ast} = \sqrt{10.75 + 1.75\Delta N_\nu}. \tag{12}
\]

One can check that the increase in the reaction rate due to the larger \( B \) is equivalent to \( \Delta N_\nu = -0.8 \). Besides that one should take into account that a real change of the effective number of particle species changes not only the neutron-proton freezing temperature but also the moment when formation of the light elements begins. It takes place at about 0.07 MeV and with a smaller number of the particle species this temperature would be reached in a longer time and less neutrons would survive the decay. This effect brings about 20% into the total impact on \( \Delta N_\nu \) during BBN. Since the variation of \( B \) does not change the time of nucleus formation the total effect of this change on \( \Delta N_\nu \) should be \(-0.6\) instead of \(-0.8\) found above. Together with the positive contribution from three bosonic neutrinos into the total cosmological energy density, which is equivalent to \( \Delta N_\nu = 3/7 \), we find that the bosonic neutrinos would make the effective number of neutrino species smaller by \( \Delta N_\nu \approx -0.2 \). The detailed calculations using properly modified BBN code give \[15\] \( \Delta N_\nu = -0.94 \) for the reaction rate change effect, and

\[
\Delta N_\nu \approx -0.51 \tag{13}
\]

for the total effect.

The neutrino condensate would not noticeably change the equilibrium value of \( n/p \)-ratio:

\[
(n/p)_{eq} = \exp[-(\Delta m + \mu)/T] \tag{14}
\]

because the chemical potential of neutrinos is negligible, \( \mu = m_\nu \ll T \).

For a positive neutrino asymmetry, \( i.e. \), for \( n_\nu > n_\bar{\nu} \) the condensate can contribute to the transition \( \nu + n \to p + e \), while for a negative asymmetry the contribution of the condensate to the \( (n \leftrightarrow p) \)-transformation is kinematically forbidden. These effects are automatically included in the case of equilibrium, but when the \( n/p \)-ratio is out of equilibrium its evolution proceeds somewhat differently from the usual case, and additional processes with condensate should be taken into account. This would make \( \Delta N_\nu \) a few per cent more negative.

The negative value of \( \Delta N_\nu \) found above seems to be in concordance with the data on the light element abundances. According to the recent analysis \[16\] \( 2.67 < N_\nu < 3.85 \) at 68% CL, while ref. \[17\] presents the result \( \Delta N_\nu = -0.37^{+0.10}_{-0.11} \). Anyhow, taken at the
face value, the observed abundances of $^4\text{He}$ and $^2\text{H}$ seem to be in contradiction with the standard BBN calculations, especially if lower values of the observed abundances are taken. At the same time $\Delta N_\nu = -0.2$ opens some room for additional light degrees of freedom.

If neutrinos have mixed statistics, e.g., the Pauli principle is broken for sterile neutrinos which mix weakly with the standard ones, the effect of “wrong” statistics on $N_\nu$ would be further diminished.

4 Bosonic neutrinos and dark matter

If neutrinos obey the Bose-Einstein statistics, the Gunn-Tremaine [18] lower bound on their mass would be inapplicable. This bound is based on the Fermi statistics which does not permit to have too many neutrinos in a galaxy. Thus, to make all galactic dark matter, neutrinos (or any other fermions) must be sufficiently heavy, $m_\nu \sim 100 \text{ eV}$. However, this large mass contradicts the Gerstein-Zeldovich upper bound [19]. Bosonic neutrinos can be arbitrary light, even as light as axions, but still make all cosmological dark matter.

To play a noticeable role in the cosmological large scale structure (LSS) formation and, possibly, to make all the observed cold dark matter the neutrino condensate should be sufficiently large. For $m_\nu \approx 0.1 \text{ eV}$ the required energy density $\Omega_\nu = 0.25$ or $\rho_c = 1.25 \cdot 10^4 \text{ cm}^{-3}$ can be achieved if the number density of neutrinos in the condensate is $1.25 \cdot 10^4 \text{ cm}^{-3}$. It is approximately 100 times larger than the number density of neutrinos plus antineutrinos for any single neutrino species from the cosmological thermal bath. To create such a large number of cold neutrinos the cosmological lepton asymmetry must be much larger than 1.

The large lepton asymmetry can be created in a version of the Affleck-Dine scenario [20]. Suppose there is a scalar field, $\chi$, with non-zero lepton charge. This field can acquire a large vacuum expectation value during inflation. The potential of $\chi$ self-interaction, $U(\chi)$, is supposed to break the leptonic charge conservation, and therefore $U(\chi)$ is not invariant with respect to phase transformation, $\chi \to \chi \exp(i\beta)$. Usually it is assumed to have the form

$$U(\chi) = m_1^2|\chi|^2 + m_2^2\chi^2 + m_2^*\chi^2 + \lambda_1|\chi|^4 + \lambda_2\chi^4 + \lambda_2^*\chi^4,$$

where $m_1$ and $\lambda_1$ are real but $m_2$ and $\lambda_2$ may be complex so that the C-invariance is broken.

If the mass parameters $m_{1,2}$ are small in comparison with the Hubble constant during inflation, $H_I$, then $\chi^2$ could acquire a large vacuum expectation value

$$\langle \chi^2 \rangle \sim \frac{H_I^4}{m_1^2} \quad \text{or} \quad \frac{H_I^2}{\lambda_1}.$$

Moreover, along possible flat directions of the potential (valleys) $\chi^2$ would rise with time as $H_I^2 t$ (see the discussion and the list of references, e.g., in [21]).

The initial leptonic charge density of $\chi$ is equal to

$$L_\chi = \dot{\theta}|\chi|^2,$$
where $\theta$ is the phase of the field, $\chi = |\chi| \exp(i\theta)$. The energy density of $\chi$ is about the kinetic energy, $\dot{\theta}^2 |\chi|^2$, and the potential energy is normally of the same order of magnitude.

When inflation is over and the Hubble parameter becomes smaller than the mass of $\chi$, the field starts to relax down to zero and in the process of relaxation it transmits the accumulated leptonic charge into leptonic charge of the decay products. We assume that $\chi$ decays into two neutrinos, $\chi \rightarrow \nu\nu$, (and not antineutrinos), i.e., the leptonic charge of $\chi$ is two. In this case the lepton number density of the produced neutrinos would be

$$n_L = L_\chi.$$  \hspace{1cm} (18)

If the decay and the subsequent thermalization are fast, the energy density of the created plasma becomes

$$\rho_T = \dot{\theta}^2 |\chi|^2 = (\pi^2 g_*/30)T^4,$$  \hspace{1cm} (19)

where $g_*$ is the number of relativistic degrees of freedom in the plasma.

It follows from (17), (18) and (19) that the charge asymmetry of the plasma could be

$$\beta_L = n_L/T^3 \sim \sqrt{|\chi|/\dot{\theta}}.$$  \hspace{1cm} (20)

For the initial values $\dot{\theta} \sim H_I$ and $\chi \sim H_I^2/m$, the asymmetry equals

$$\beta_L \sim \frac{H}{m},$$  \hspace{1cm} (21)

and it does not look unreasonable, e.g., that $\beta_L > 100$.

After $\chi$ decayed into neutrino pairs, the plasma thermalized through the reactions $\nu\nu \rightarrow \nu\nu l\bar{l}$, etc.. Complete thermalization would produce the equilibrium distribution of neutrinos given by eq. (5). Because of a low rate of reactions with zero momentum neutrinos the complete equilibrium with condensation at $p_\nu = 0$ is never reached, still, according to the estimates presented below, neutrinos accumulate sufficiently close to $p = 0$ and the condensate practically forms.

The kinetic equilibrium could be achieved through elastic scattering reactions $\nu + l \rightarrow \nu + l$. If the initial temperature was above the electroweak scale and therefore the intermediate $W$ and $Z$ bosons could be considered massless, the low energy band of the spectrum of neutrinos would be filled in with the rate

$$\Gamma \sim g^4 E_\nu \sim \alpha^2 E_\nu,$$  \hspace{1cm} (22)

where $g$ is the electroweak coupling constant, $E_\nu$ is the neutrino energy and $\alpha = 1/137$. At temperatures below the masses of $W$ and $Z$ bosons, the rate becomes much lower:

$$\Gamma \sim G_F^2 T^4 E_\nu,$$  \hspace{1cm} (23)

where $G_F$ is the Fermi coupling constant.

Requiring $\Gamma \geq H$, we conclude that neutrinos “condense” down to energies $E_\nu/T \sim 10T/m_{Pl}$ if the process occurs before the EW phase transition, i.e. if $\chi$ decays at temperatures $T > m_{W,Z}$. When $T$ drops below $m_{W,Z}$ the cooling would become less efficient.
Effective energies to which neutrinos could cool down in the condensation process increases up to $m_W^4 / (T^2 m_{Pl})$. Though complete condensation with $p = 0$ could probably never be achieved, an accumulation of neutrinos at low energy part of the spectrum seems to be efficient enough to create cosmological cold dark matter of neutrinos.

In contrast to the case of large charge asymmetry carried by fermions (when the spectrum is populated by energetic particles), in the bosonic case the charge asymmetry is stored in the condensate, while chemical potential $\mu = m_\nu$ is very small and the energy density is dominated by relativistic particles with practically vanishing charge asymmetry. It remains true till temperature dropped down below $m_\nu$. After that the energy density would be dominated by non-relativistic condensate with large leptonic charge.

The energy density in the neutrino condensate equals

$$\rho_{\text{cond}} = m_\nu n_0.$$  

Requiring that at the BBN epoch $\rho_{\text{cond}} < 0.1 \rho_{\nu}^T$, where $\rho_{\nu}^T$ is the energy density in the thermal bath, we find the bound

$$n_0 < 0.3 \frac{n_{\nu}^T T_{\text{BBN}}}{m_\nu} \sim 10^6 n_{\nu}^T.$$  

Here $n_{\nu}^T$ is the neutrino number density in the thermal bath.

If the equilibrium concentration is not achieved and the condensate is not formed completely, so that $p \neq 0$, the BBN gives the following bound:

$$|p| < 0.3 \frac{T_{\text{BBN}} n_{\nu}^T}{n_0}.$$  

For $n_{\nu}^T / n_0 \sim 10^{-2}$ required to explain the dark matter we find $|p| < 3 \cdot 10^3$ eV.

The large scale structure formation with cold dark matter composed of the neutrino condensate would well fit the observed picture. The analysis of structure formation with the Bose condensate of normal bosons was done in ref. [22] and it can be extended to the case of neutrino without significant modifications.

One can be less ambitious and admit two coexisting forms of the cold dark matter: e.g., the usually accepted lightest supersymmetric particle (LSP) or axions, and the Bose-condensed neutrinos. It would make the scenario considered here less vulnerable but simultaneously less predictive.

Notice that because of smallness of the neutrino chemical potential a large lepton asymmetry in primordial plasma cannot be transformed into the baryon one by electroweak processes. Indeed, the chemical potential of neutrinos is tiny and in equilibrium the same must be true for chemical potentials of quarks. Hence the baryon asymmetry generated by sphalerons in equilibrium could be at most of the order $m_\nu / T \sim 10^{-12}$.

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2In the case that sterile neutrinos condense, the necessary for DM magnitude of the condensate should be scaled by the ratio of sterile/active masses.
5 Astrophysical consequences

The neutrino statistics plays the key role in the environments where neutrinos form dense degenerate gases.

Direct test of the “bosonic” nature of neutrinos can be probed by precise measurements of the neutrino energy spectrum from supernova. Instead of the Fermi-Dirac spectrum with pinching the distribution would be the Bose-Einstein one with some pinching effect too. So, generically, the spectrum of bosonic neutrinos should be narrower. To establish the difference one needs to measure the spectrum both in the low, $E < 3T$, and in the high, $E > 3T$ energy parts. Also pinching effect should be quantified rather precisely.

Violation of the Pauli principle can influence dynamics of the SN collapse. According to the usual scenario in the initial stages (formation of the hot proto-neutron star) the neutronization leads to production of high concentration of the electron neutrinos which are trapped in the core. The chemical potential of these neutrinos (due to the Pauli principle) can reach 70 - 100 MeV. These neutrinos heat the medium and diffuse from the core. Violation of the Pauli principle allows for the neutronization neutrinos to be produced with lower energies. These neutrinos escape easier the star leading to faster cooling and lower central temperatures. Also the evolution of the lepton number would change.

The presence of the neutrino condensate with large lepton number in the Universe may have a number of observable consequences.

High neutrino density in the condensate (especially if an additional clustering occurs) enhances rate of the $Z^0$-bursts produced by annihilation of the ultra high energy (UHE) cosmic neutrinos on the relic neutrinos [23, 24]. This in turn, enhances production of the UHE cosmic rays, and may help to explain the cosmic ray evens above the GZK cut-off.

The asymmetric neutrino condensate may produce strong refraction of the high energy neutrinos from remote sources (active galactic nuclei, gamma ray bursters). Apart from lensing, one may expect a substantial impact on neutrino oscillations [12].

Since the density of dark matter in galaxies is about 6 orders of magnitude larger than their average cosmological energy density, a condensation of cold neutrinos around the Earth might have an effect on the end point of the beta decay spectra, in particular, in the tritium decay experiments on search for neutrino mass.

6 Conclusions

If the Pauli principle is violated in Nature, neutrinos may be the first messengers of that. Indeed, neutrino properties may be related to physics at high energy scales and only neutrinos (due to their neutrality) can mix with particles of the hidden sector (singlets of the standard model symmetry group).

We have considered the cosmological and astrophysical consequences of the Pauli principle violation assuming that neutrinos obey the Bose-Einstein statistics. Such neutrinos would have the Bose-Einstein energy distributions and might form a condensate.
We have found that in the case of bosonic neutrinos, the effective number of the neutrino species during the BBN epoch is reduced by about $\Delta N_\nu = -0.5$, thus slightly facilitating an existence of new degrees of freedom, e.g., sterile neutrinos.

If in the Early Universe, a large lepton asymmetry was created, bosonic neutrinos should condense in the process of the cosmological cooling. Because of that the BBN bound on the neutrino density can be avoided, and so a large neutrino concentration and energy density at a later epoch become possible. In particular, it allows to explain all or a part of the cosmological dark matter. It opens a possibility of large lepton asymmetry of the Universe.

Bosonic neutrinos would influence dynamics of the stellar collapse and lead to modification of the energy spectra of SN neutrinos.

A large concentration of relic background neutrinos in the condensate at the present epoch would enhance the rate of the $Z^0$—bursts. It can produce strong refraction effect on neutrinos from remote sources, modifying the oscillation pattern.

A study of the double beta decay, in particular, of the energy spectrum and angular distribution of electrons from the decay can provide sensitive test of the Pauli principle violation and statistics of neutrinos.

Note added: After submission of the first version of our paper, we became aware of the paper [25] in which a possibility of bosonic statistics for neutrinos has been considered and its effects on the BBN have been studied. According to [25] the change of neutrino statistics is equivalent to the decrease of number of the effective neutrino species $\Delta N_\nu = -0.74$. This conclusion agrees qualitatively with our results of sec. 3, though quantitatively we find smaller decrease.

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