Mathematical Representation Ability by Using Project Based Learning on the Topic of Statistics

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Abstract. Seeing the importance of the role of mathematics in everyday life, mastery of the subject areas of mathematics is a must. Representation ability is one of the fundamental ability that used in mathematics to make connection between abstract idea with logical thinking to understanding mathematics. Researcher see the lack of mathematical representation and try to find alternative solution to solve it by using project based learning. This research use literature study from some books and articles in journals to see the importance of mathematical representation ability in mathematics learning and how project based learning able to increase this mathematical representation ability on the topic of Statistics. The indicators for mathematical representation ability in this research classifies namely visual representation (picture, diagram, graph, or table); symbolize representation (mathematical statement, Mathematical notation, numerical/algebra symbol) and verbal representation (written text). This article explain about why project based learning able to influence student’s mathematical representation by using some theories in cognitive psychology, also showing the example of project based learning that able to use in teaching statistics, one of mathematics topic that very useful to analyze data.

1. Introduction
Mastery mathematics is a must to face globalization today. Mathematics is a foundation and framework to develop technologies and another subjects that will be useful in human vital skills to the learners. National Council of Teachers of Mathematics has developed the characteristics of mathematical abilities as principle and process standards in mathematics, they are problem solving, reasoning, communication, connection, and representation. The five of characteristics are the goal to be achieved in mathematics learning. So mathematics is a learning that has final result more than a score in the final report, mathematics can improve the ability of logical thinking, accuracy, and spatial awareness, also gives effort the ability to solve challenging problems [1]. In this article, researcher will focus on representation ability. Representation is the way as fundamental in people can understand mathematical ideas. Students use representations as tools to support their mathematical understanding by constructing abstract idea into concrete idea by using logical thinking because representation is a sign or a configuration of signs, characters, or objects which mark and configuration to stand to represent, describe, or represent something other than itself [2]. So it will support students for learning and communicating, connecting mathematics concept to solve the problem in the given project. From this explanation, students which has low ability of representation will show a lack of skilled students in generating ideas, ask questions and respond to questions or opinions of others.

This argument also shown from some researches result such as representations could be effectively used to teach word problem solving and fractions [3], there is a bidirectional relation between
representation and cognitive abilities [4], and the use of visual representations is a highly recommended instructional practice in mathematics [5], so strong representation ability is needed to demonstrated for developing mathematical understanding [6-8]. In fact our student still has low representation ability. Based on the observation did in a school located in Medan by using Bruner’s theory about how to represent world through action, visual images, words and language [2]. From 37 students who answer the questions given, 66.67% of them have not been able yet to build their visual representations in making table, while 70.27% of students also have not been able yet to build their mathematical representations ability in equation or mathematical expression aspects especially in making the equation and 65.49% of students have not been able yet to represent their ideas or knowledge in writing the text form [9]. Furthermore, to see the factors that influence this, researcher also give questionnaire to students and found the result that only 39% of students learn mathematics more than equal to 3 hours every week outside the school activity, for the example at house or course place. It seems that they don’t have motivation to learn mathematics. Actually, 86% of students know how important mathematics in their daily life. But it still doesn’t enough for them. 55% students said that mathematics is difficult, 45% said it is not so difficult if they learn it seriously and none said mathematics is easy. 38% of students give explanation that they have difficulty in understanding mathematics, 33% explain it because of teacher doesn’t explain clearly, 19% of students the difficulty of mathematics is depend on the matter and 10% of students said it can be happen because they hate mathematics [9]. From the result above, we can see that student’s difficulty in understanding mathematics not just because they have problem in their self (internal factor), but environment and teacher (external factor) also influence it. So teacher can reduce this problem by changing student’s perception about mathematics. One of the solution is giving fun and innovative learning strategies as “learning by doing” in teaching and learning process. Because students will remember the learning material until 90% by doing something [10] Such as project based learning.

In contrast to conventional learning models, project based learning emphasizes mathematics application into interdisciplinary, holistic, student-centred, and integrated with practice and real-world issues by giving complex problems in which students must be able to conduct an investigation to understand and oriented to the product that made by students. That’s why project based learning has a great potential to make meaningful and interesting learning experience. In this model, students need to have a good team management to build team building skills, decision making cooperative and make a problem solving for their project given. This model has a syntax which the phase of each activity, they are getting ideas, designing project, setting the project, making project and project showcase. From this phases, project based learning has shown its effectivity in increasing student’s understanding. Such as found that students who were educated by project-based learning was more successful and have higher attitude levels towards the lesson than the students who were educated by the instruction based on student’s text books [11]. It also showed that socio-affective skills are developed during project-based learning [12]. Adults from the project based school has also moved into significantly more professional jobs because they have greater understanding, and more appreciation of mathematics than a traditional approach [13-15]. Seeing this successfull of project based learning and how important representation ability in mathematics understanding. However researcher will try to analyze that project based learning able to increase student’s representation ability. Also on the topics of statistics which use to represent data in another way, so it can be analyze to conclude about something.

1.1. Statistics
Statistics is the branch of mathematics in which facts and information are collected, sorted, displayed, and analyzed. Statistics are used to make decisions and predict what may happen in the future. Data that have been collected but not organized in any way are called raw data. Raw data are difficult to interpret, so it can be arranged in a frequency distribution table. A frequency distribution table shows the number of times (frequency) each value occurs. Tallying is a system of recording and counting results using diagonal lines grouped in fives. Each time five is reached, a horizontal line is drawn through the tally marks to make a group of five. The next line starts a new group. Statistical data is of two types – Ungrouped and Grouped data. The grouped data is the type of data which is subdivided into classes. Grouped data is not purely raw data. In fact, in grouped data, some analysis has been already done and it is classified.
Ungrouped data has not been classified or has not been subdivided in the form of groups. This type of data is totally the raw data. Ungrouped data is just in the form of number list. It is the data collected in original form. We can say that ungrouped data is an array of numbers.

Grouped data has been subdivided in the form of groups. To put the data in a more condensed form, we make groups of suitable size, and mentions the frequency of each group. Such a table is called a grouped frequency distribution table. Class-Interval: Each class is bounded by two figures, which are called class limits. The figure on the left side of a class is called its lower limit and that on its right is called its upper limit. Grouping a set of data into class intervals requires the following steps:

1. Range = \( R = \) the highest value – the lowest value
2. The number of class intervals = \( k \)
   \( k = 1 + 3.3 \log n \), where \( n = \text{number of data} \)
3. The class length or class width = \( C \)
   \( C = \frac{R}{k} \)
4. All of data must have a class, include the lowest value and the highest value.
5. Each class of a data is only one.

We can represent data in some types of graphs to understand about data easily. For example, bar graphs, pie charts (circle graph), and line charts. In bar charts or bar graphs, data are represented in a series of bars that are equally wide. The width itself is not significant, but all the bars should be the same width.

Figure 1. Bar Graphs

A pie chart is a circle graph in which the angles of the sectors represent the frequency.

Figure 2. Pie Charts

Use lines between data points to depict the magnitude of data for two items or for one item over time. The heights of the line allow the user to compare magnitudes easily.
Figure 1 shows the example of bar charts, figure 2 shows the example of pie charts, so do figure 3 that shows the example of line charts.

2. Experimental Method
This article use literature review to analyze that project based learning able to increase student’s representation ability to understand mathematics also giving explanation for the result of my research before which give conclusion that project based learning and problem based learning give positive impact for student’s mathematical representation ability on the topic of statistics in grade X SMA Negeri 1 Percut Sei Tuan [9]. To complete analyze of this article, researcher also read some books and another articles in some journals which talking about student’s representation ability and project based learning during 1982 – 2017.

3. Result and Discussion
Let’s talk about the research did by Boaler and Selling about how far project based learning able to contribute student’s life from Phoenix Park, a project based school [15]. An amazing result that after 3 years of working on open-ended projects and worked in heterogeneous groups and they could take tasks in any direction that they wanted, and the mathematics that they encountered was not predetermined by the mathematics question. It makes students at Phoenix Park scored at significantly higher achievement levels than the Amber Hill (a traditional school) students on a range of assessments, including the UK’s national examination (General Certificate of Secondary Education, or GCSE). In their adult life (after they’ve leaved school for 8 years) they has moved into significantly more professional jobs because they have greater understanding, and more appreciation of mathematics than a traditional approach. All of the Phoenix Park adults reported enjoyment of maths now and spoke in positive ways when describing their mathematics classes. They reflected mathematics lessons, using very positive words such as “brilliant,” “ideal,” and “brave.”

It can be happen because students are trained well to use their own ideas and their mathematical thinking is valued in the classroom, it makes they develop form a mathematical identity that is active, engaged, and positive as learners and doers of mathematics and using their human agency. The students were “required to provide critiques of each other’s idea, ask questions, propose ‘theories’, suggest the direction of mathematical problem solving, and ‘author’ some of the mathematical methods and directions in the classroom”. The Phoenix Park participants appeared to have moved seamlessly from their mathematics classrooms into the mathematical demands of the workplace, whereas the Amber Hill participants noted a distinct disconnect. From this explanation, we can see how far they use their representation ability.
Figure 4. The Relationship between Internal and External Representation in Developing Child’s Understanding of the Concept of Numeracy.

Figure 4 shows about representation ability processes. For example, while we said “five”, students will represent “five” in their mind as another thing that can make they able to understand as mental image of counting and numeracy. It can be represent as a form of number, written, or images like amount of girls, squares, etc. Then they able to make conclusion what “five” is. Representation ability is the basic ability to concept their understanding about mathematics. Representation ability in mathematics arise from internal and external representation processes while both of them have a reciprocal relationship to understand mathematics in class. Since internal representation is an ability that has owned by students as a mental activity in mind, so it can’t be seen with the naked eye. External representation is represent internal representation to a thing as result that can be seen may be as words, symbol, diagrams that can be expressed verbally and in writing.

So we know now that representation is a fundamental of students to use their connection and reasoning ability, then use their problem solving ability and communicate the result use their communication ability. Without representation, there is no image that will be transfer to their mind and process it to get mathematical idea so they can solve problem and finish their project given. It is also explained by Habok that Implementation of a project-based concept mapping developmental programme to facilitate children’s experiential reasoning and comprehension of relations [16]. Can not be denied that mathematic objects are abstract so that to learn and understanding of abstract ideas require representation. Especially for statistics matter that has many concepts to determine something, such as mean, median, modus. They have their own formula. But in fact it can be easy understand if students able to apply the concepts to read the reality arround them. Such as in the traditional market, how many people go there to do buying and selling transaction.

Project based learning in Statistics in class X may be can be do by asking students to make data about their environmental reality, then represent data on to project board, then make mini exhibition near by canteen and they must explain about data on their project board, then another students beside their classmates can vote the best project board based on their opinion. Then group which get the most vote is the winner and teacher will give them gift as apresiation.
Another activity that can be hold by project based learning is asking students to do selling and buying by making their own product and making school as their market for one week. Then students must make financial report and analyse how much they get profit in one week. Then they represent it in front of class to announce everything they get also give and take advice from another group. Group which get most profit is a winner and each profit can be owned by each group. So more they get the profit, they will get more mony for every person in the groupe. For this way, beside to understanding the topic, students also hope to learn about financial literacy by doing effort to get money.

From this two examples, students will use their representation to construct the idea of problem given and process it to produce something to finish their project, then they need representation again to make another people understand about what they’ve did in their project and announce every data they get.

There are five kinds of representations that are useful for mathematical understanding: (a) real life experiences, (b) manipulative models, (c) pictures or diagrams, (d) spoken words, and (e) written symbols [2]. All of this exactly happen in the situation that explained by students from Phoenix Park before. Real life experience is very essential make a figure in mind. It will be use to make connection about abstract thing to be something easy to understant. Usually we also know this as prior knowledge. Matlin explained that when we recognize an object, we manage to figure out the identity of the distal stimulus, even when the information available in the proximal stimulus is far from perfect. Then your visual system has some assistance from one of its other components. Your sensory memory is a large capacity storage system that records information from each of the senses with reasonable accuracy. How long information records depend on how important the information and it involves attention. To be specific, iconic memory or visual sensory memory, preserves an image of a visual stimulus for a brief period after the stimulus has disappeared. It can be applied in mathematics learning by making stimulus that easy to catch by visual sensory memory [17]. Prior knowledge in term identification was the only skill that had a consistent, positive, and significant influence on learner-interface [18]. So if students able to manage good prior knowledge, you will easy to understand next experience because you able to make good connection in your mind. The whole processes is expected to be able to change student’s perception about how mathematics become a killer for them become mathematics is fun, very usefull and aplicable do solve every problem in their daily life by using their logical thinking by mastery their representation ability. Using project based learning means using student’s representation ability. It will imply that project based learning able to increase representation ability for students.

4. Conclusion
The activities to solve problem in project based learning able to influence student’s representation ability especially on the topic of Statistics and it can explain clearly why it can be happen. By doing active and fun learning, also applying mathematics directly to solve problem in the project given, students are expected have good prior knowledge to make them easier in connected every topics in mathematics and change student’s perception about mathematics is difficult become mathematics is very useful and aplicable to solve problem in their daily life.

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