Electromagnetic interaction of a magnetized rotating star with a conducting disk

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Abstract A conducting disk significantly changes the generation of the electromagnetic radiation excited by the rotation of the magnetic field frozen to a star. Due to the reflection of waves from a disk there appear waves propagating toward a star, not only outward a star as it takes place for the magneto-dipole radiation. Because the angular momentum can be transformed from a disk to a star when the inner edge of a disk approaches the light surface of a rotating star. This is purely electromagnetic effect. At some distance of a disk from a star, \( r_d \approx r^* \approx c/\omega_s \), the stellar angular momentum losses due to the electromagnetic radiation become zero. It results the stable stellar rotation.

Keywords stars: neutron

1 Introduction

The interaction of an accretion disk with a magnetized star is very complicated and is not understandable yet. The observations of X–ray binaries demonstrate many physical phenomenon [1]. It seems useful to solve a set of simple physical problems throwing light on the nature of interaction of an accretion flow with a star. One of these problems is the interaction of a rotating magnetic field with a conducting matter of a disk. The phenomena depends on the conductivity of the disk matter \( \sigma \). The physics is different for the high and low conductivities. Aly [2] consider the disk conductivity is high and the magnetic field does not penetrate to a disk. Opposite, Bardou and Heyvaerts [3] consider the conductivity is low due to abnormal resistivity of a turbulent disk plasma, and the field penetrate to a disk freely.

The interesting combination of penetration and repulsion of magnetic field lines was suggested by Lovelace at. al [4]. We consider here a disk to be stationary, without accretion and with the classical properties.

The ionized plasma of an accretion disk has the conductivity \( \sigma = 10^{13}(T_e/1eV)^{3/2}(\Lambda/10)^{-1}s^{-1} \), which is high enough to consider a disk as an ideal conductor. Here \( T_e \) is the temperature of electrons in a disk which is larger than 10eV, \( \Lambda \) is the Coulomb logarithm. At such conductivity \( \sigma \) the width of the skin layer \( \lambda_{sk} = (\tau c^2/\sigma)^{1/2} \) is less than the disk width \( H \). The value of \( \tau \) is the characteristic time of turbulent motion in the \( \alpha \)- disk, \( \tau = H/v_k \), \( v_k \) is the Keplerian velocity of the disk rotation. The condition \( \sigma \gg c^2/Hv_k \) is well fulfilled in the inner parts of a disk.

Here we consider the oblique magnetic field rotating with the frequency of the star rotation \( \omega_s \), and a disk having the infinite conductivity and finite dimensions. The environment of this system is the vacuum. We will see that these conditions though the boundary condition on a disk surface results to the repulsion of the magnetic field from a disk.

2 Statement of problem

We are solving the following problem: a rotating neutron star with the angular velocity \( \omega_s \) and the magnetic momentum \( \mu \) is surrounded by a disk (see figure 1). The angle between \( \omega_s \) and \( \mu \) is \( \chi \). The matter of a disk is ideal, i.e. the conductivity \( \sigma = \infty \). It means \( E' = 0 \), where \( E' \) is the electric field in the frame of a disk. The field in the laboratory frame is

\[
E = -\frac{1}{c}[v_d B].
\]

Here \( v_d \) is the rotational velocity of the disk matter around the a star \( v_d = v_ke_\phi \). We consider a disk is
rotating with the Keplerian velocity $v_k \propto r^{-1/2}$ in the \( \phi \) direction
\[
v_d = \left( \frac{GM_s}{r} \right)^{1/2} \sin \vartheta \phi.
\] (2)

Here \( M_s \) is the stellar mass, \( G \) is the gravitational constant and \( r, \vartheta, \phi \) are the spherical coordinates. To create the electric field (1) the matter of a disk must polarize and produces the charge density

\[
\varrho = \frac{1}{4\pi} \nabla \mathbf{E} = -\frac{v_k}{4\pi cr} \left[ \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} (\sin^2 \vartheta B_r) - \frac{1}{r^{1/2}} \frac{\partial}{\partial r} (r^{3/2} B_\vartheta) \sin \vartheta \right].
\] (3)

The rotating charge density produces the toroidal electric current \( j_\phi \), \( j_\phi = v_k \varrho \sin \vartheta \),
\[
j_\phi = -\frac{v_k^2}{4\pi cr} \left[ \frac{\partial}{\partial \vartheta} (\sin^2 \vartheta B_r) - \frac{1}{r^{1/2}} \frac{\partial}{\partial r} (r^{3/2} B_\vartheta) \sin^2 \vartheta \right].
\]

We consider that a disk occupies the region \( r_d < r < \infty, \pi/2 - \delta < \vartheta < \pi/2 + \delta \), where \( r_d \) is inner edge of a disk and \( 2\delta \) is its width in the \( \theta \) direction. Integrating the Maxwell equation \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \) producing the magnetic current \( \mathbf{J} = \frac{\mu_0}{c} \nabla \times \mathbf{E} \), we obtain the condition connecting two values of \( B_r \) on both sides of a disk
\[
B_r(\frac{\pi}{2} - \delta) - B_r(\frac{\pi}{2} + \delta) = \left( 1 - \frac{v_k^2 \cos^2 \delta}{c^2} \right)^{-1} \int_{\delta - \frac{\pi}{2}}^{\frac{\pi}{2} + \delta} \frac{B_\vartheta d\vartheta}{\sin \vartheta}.
\] (4)

Here the current \( j_\phi \) is the toroidal electric current exciting in a disk by the infinitely small electric field \( E_\phi \). The boundary condition (4) is the condition for the current \( j_\phi \) arising inside a disk. Another two boundary conditions follow from (1)
\[
E_\phi(\frac{\pi}{2} - \delta) = E_\phi(\frac{\pi}{2} + \delta) = 0, r \geq r_d;
\] (5)

\[
(E_r - \frac{v_k}{c} \sin \vartheta B_\vartheta)(\vartheta = \frac{\pi}{2} - \delta) = 0, r \geq r_d;
\]

\[
(E_r - \frac{v_k}{c} \sin \vartheta B_\vartheta)(\vartheta = \frac{\pi}{2} + \delta) = 0, r \geq r_d.
\] (6)

3 Solution

The general solution of the wave equation in vacuum in the spherical coordinates consists of the sum of the multipole components, magnetic and electric. The magnetic multipole of the \( n \)-th order is

\[
B_n = \left\{ \begin{array}{l}
P_n^1 (\cos \vartheta) z^{-\frac{\delta}{2}} [a_n^B J_{n+\frac{1}{2}} + b_n^B J_{n-\frac{1}{2}}], \\
\quad \frac{1}{n(n+1)} \frac{\partial^2}{\partial \vartheta^2} - \frac{\partial}{\partial \vartheta} \left[ \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \right] \left[ z^{-\frac{\delta}{2}} [a_n^B J_{n+\frac{1}{2}} + b_n^B J_{n-\frac{1}{2}}] \right], \\
\quad \frac{1}{n(n+1)} \frac{\partial^2}{\partial \vartheta^2} + \frac{\partial}{\partial \vartheta} \left[ \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \right] \left[ z^{-\frac{\delta}{2}} [a_n^B J_{n+\frac{1}{2}} + b_n^B J_{n-\frac{1}{2}}] \right], \\
\quad \varrho, \varphi.
\end{array} \right.
\]

Here and below \( P_n^l (\cos \vartheta) \) are the associated Legendre functions, \( J_{n+1/2}(z) \) is the Bessel functions of the \( (n+1/2) \) order, \( z = r \omega_s/c \). The electric multipole of the \( n \)-th order is

\[
E_n = \left\{ \begin{array}{l}
P_n^0 (\cos \vartheta) z^{-\frac{\delta}{2}} [a_n^E J_{n+\frac{1}{2}} + b_n^E J_{n-\frac{1}{2}}], \\
\quad \frac{1}{n(n+1)} \frac{\partial^2}{\partial \vartheta^2} - \frac{\partial}{\partial \vartheta} \left[ \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \right] \left[ z^{-\frac{\delta}{2}} [a_n^E J_{n+\frac{1}{2}} + b_n^E J_{n-\frac{1}{2}}] \right], \\
\quad \frac{1}{n(n+1)} \frac{\partial^2}{\partial \vartheta^2} + \frac{\partial}{\partial \vartheta} \left[ \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \right] \left[ z^{-\frac{\delta}{2}} [a_n^E J_{n+\frac{1}{2}} + b_n^E J_{n-\frac{1}{2}}] \right], \\
\quad \varrho, \varphi.
\end{array} \right.
\]

All quantities \( B_n \) and \( E_n \) are oscillating functions with the frequency of the stellar rotation \( \omega_s \), \( (B, E)_n \propto e^{i(\omega - \omega_s)t} \). Multipoles (7,8) contain waves travelling in the positive direction from a neutron star to the infinity, in this case \( b_n = i(-1)^n a_n \), and the combination \( a_n J_{n+1/2} + b_n J_{n-1/2} = a_n H_{n+1/2}^{(1)} \) is the Hankel function of the first kind

\[
H_{n+1/2}^{(1)} = e^{iz} \left( \frac{1}{2\pi n} \right)^{1/2} Z_n \left( \frac{1}{z} \right).
\]

Here \( Z_n(1/z) \) is the polynomial of the \( n \)-th order of the argument \( z^{-1} \). Also for the wave propagating in the negative direction from the infinity \( b_n = i(-1)^n a_n \), and we have the Hankel function of the second kind, \( H_{n+1/2}^{(2)} = e^{-iz} (2/\pi n)^{1/2} Z_n(1/z) \). When the disk is absent the condition at the infinity demands the relation \( b_n = i(-1)^{n+1} a_n \). In the presence of a disk the part of the wave energy can be reflected from a disk and we

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Fig. 1 A rotating neutron star with a dipole magnetic field is surrounded by a disk.
have the mixture of both types of waves. In general case the coefficients \(a_n^{B,E}, b_n^{B,E}\) are the arbitrary complex numbers and can be determined from the boundary conditions (5,6). The boundary condition on the stellar surface \(r = R_s (z = z_0 = R_s \omega_s / c)\) are

\[B_r(z = z_0) = B_r(inside); E_\theta(z = z_0) = E_\theta(inside)\]

\[E_\phi(z = z_0) = E_\phi(inside).\]

Fields inside are

\[\mathbf{B} = \frac{\mu_0}{r^3} \sin \chi e^{i\phi - i\omega_0 t} \begin{cases} 2 P_1^1 (\cos \theta), & \mathbf{e}_r \\ - \frac{\partial P_1^1}{\partial \theta} (1 - \frac{z}{a}), & \mathbf{e}_\theta \\ - i \frac{\partial P_1^1}{i \sin \theta} (1 - \frac{z}{a}), & \mathbf{e}_\phi \end{cases} \tag{9}\]

\[\mathbf{E} = \frac{\mu_0}{r^3} \sin \chi e^{i\phi - i\omega_0 t} \begin{cases} - \frac{1}{2} P_1^1 (1 - \frac{z}{a}), & \mathbf{e}_r \\ - \frac{\partial P_1^1}{\sin \theta} (1 - \frac{z}{a}) + \frac{1}{2} \frac{\partial P_1^1}{\partial \theta}, & \mathbf{e}_\theta \\ - i \frac{\partial P_1^1}{i \sin \theta} + i \frac{1}{2} \frac{\partial P_1^1}{\partial \phi}, & \mathbf{e}_\phi \end{cases} = 0, \quad \mathbf{e}_\phi.\]

Here \(\mu\) is the magnetic moment of a star. We consider here only the varying component of the magnetic dipole, proportional to \(\sin \chi\), which produces an electromagnetic radiation. From (5,6) and (9) it follows

\[(a_n^B J_{3/2}(z) + b_n^B J_{-3/2}(z))\bigg|_{z = z_0} = \frac{2 \mu_0}{R_s^3} z_0^{3/2} \sin \chi; \tag{10}\]

\[(a_n^B J_{n+1/2}(z) + b_n^B J_{n-1/2}(z))\bigg|_{z = z_0} = 0, \quad n \neq 1; \tag{11}\]

\[
\left( \frac{\partial}{\partial z} z^{1/2} (a_n^E J_{5/2} + b_n^E J_{-5/2}) \right)\bigg|_{z = z_0} = \frac{2 \mu_0}{R_s^3} z_0^{3/2} \sin \chi; \tag{12}\]

\[
\left( \frac{\partial}{\partial z} z^{1/2} (a_n^E J_{n+1/2} + b_n^E J_{n-1/2}) \right)\bigg|_{z = z_0} = 0, \quad n \neq 2.\tag{13}\]

Introducing the notations

\[\varrho_n^B = \left( J_{n+1/2}(z) \right)\bigg|_{z = z_0}, \quad \varrho_n^E = \left( \frac{\partial}{\partial z} (z^{1/2} J_{n+1/2}) \right)\bigg|_{z = z_0},\]

and the dimensionless quantities \(a_n, b_n\), dividing them on the value \(2\mu_0 \sin \chi / R_s^3\), we rewrite (10) in the form

\[b_1^B = -a_1^B \varrho_1^B + s_1, \quad b_2^B = -a_2^B \varrho_2^B, \quad n \neq 1, \tag{14}\]

\[b_2^E = -a_2^E \varrho_2^E + s_2, \quad b_2^E = -a_2^E \varrho_2^E, \quad n \neq 2.\]

Here the values \(s_1\) and \(s_2\) are

\[s_1 = \frac{z^{3/2}}{J_{-3/2}(z)}\bigg|_{z = z_0}, \quad s_2 = \frac{z^2}{J_{5/2}(z)}\bigg|_{z = z_0}. \tag{15}\]

Let us note that because \(z_0 \ll 1 (R_s \ll c/\omega_s)\) the coefficients \(\varrho_n^B, \varrho_n^E, s_1, s_2\) are small values. In the lowest order of \(z_0\) they are

\[\varrho_n^B \approx \left. \frac{\Gamma(1/2 - n)}{\Gamma(3/2 + n)} \right|_{z_0} \left( \frac{z_0}{2} \right)^{2n+1} ; \tag{16}\]

\[\varrho_n^E \approx \left. \frac{-n(1/2 - n)}{n\Gamma(3/2 + n)} \right|_{z_0} \left( \frac{z_0}{2} \right)^{2n+1} ; \tag{17}\]

\[s_1 \approx 2^{-3/2} \Gamma(-1/2) z_0^3 = -2^{-1/2} \pi^{1/2} z_0^3 ; \tag{18}\]

\[s_2 \approx 2^{-7/2} \Gamma(-3/2) z_0^5 = \frac{2^{-3/2}}{\pi^{1/2}} z_0^5. \tag{19}\]

Here \(\Gamma(x)\) is the Gamma function. In the case of a disk Eq. (12) are the connections between the coefficients \(b_n\) and \(a_n\). When a disk is absent connections are conditions \(b_n = i(-1)^{n+1} a_n\), that is the condition for the radiation toward the infinity when there are no reflected waves. Then Eq. (12) shows that nonzero coefficients are \(a_1^B\) and \(a_2^E\) only

\[a_1^B = \left( \frac{-i + \varrho_1^E}{1 + \varrho_1^B} \right) s_1 \approx -i s_1 = i \left( \frac{\pi}{2} \right)^{1/2} z_0^3. \tag{20}\]

\[a_2^E = \left( \frac{i + \varrho_2^B}{1 + \varrho_2^E} \right) s_2 \approx i s_2 = -i \left( \frac{\pi}{8} \right)^{1/2} z_0^5. \tag{21}\]

It means that a rotating magnetized star without a disk radiates the magnetodipole radiation with the amplitude \(a_1^B \approx z_0^3\) and also the electroquadrupole radiation with the amplitude \(a_2^E \ll a_1^B\). The disk changes this picture, and a star radiates also many multipoles \(n \neq 1, 2\).

To find the coefficients \(a_n^{B,E}\) we need use the boundary conditions on a disk (5,6). The boundary condition (5) means

\[\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \left[ \frac{P_n^1(\phi_0)}{\sin \phi_0} \right] z^{\frac{3}{2}} \frac{\partial}{\partial z} \left( z^{1/2} J_{n+1/2} \right) - \frac{\partial P_n^1(\phi_0)}{\partial \phi_0} \left( a_n^B J_{n+1/2} + b_n^B J_{n-1/2} \right) = 0; \quad z \geq z_d, \phi_0 = \frac{\pi}{2} \pm \delta. \tag{22}\]

We use the recurrent relations for the Bessel functions

\[\frac{\partial}{\partial z} z^{3/2} J_{n+1/2} = z^{1/2} \left[ \frac{n+1}{2n+1} J_{n+1/2} - \frac{n}{2n+1} J_n \right]; \tag{23}\]

\[\frac{\partial}{\partial z} z^{3/2} J_{n-1/2} = z^{1/2} \left[ \frac{n}{2n+1} J_{n-1/2} - \frac{n+1}{2n+1} J_{n+1/2} \right]. \tag{24}\]

Then multiplying Eq. (22) on the Gegenbauer polynomial \(A_{k,1/2}(z)\) and integrating it over the contour passing through the infinity as shown on the figure 2a, we obtain the matrix equation connected the coefficients \(a_n^B\) and \(a_n^E\). The Gegenbauer polynomial \(A_{k,1/2}\) is the
function conjugated to the Bessel functions \( J_{n+1/2}(z) \),

\[
A_k(z) = 2^{-1/2} \left( k + \frac{1}{2} \right) \sum_{m=0}^{\leq k/2} \frac{\Gamma \left( \frac{1}{2} + k - m \right)}{m!} \left( \frac{z}{2} \right)^{2m-k-1},
\]

and possesses the property [5]

\[
\frac{1}{2\pi} \int_C J_{n+1/2}(z) A_{k,1/2}(z) z^{-1/2} dz = \delta_{k,n}.
\]

The result is

\[
A_k(z) \approx 2^{-1/2} \left( k + \frac{1}{2} \right) \sum_{m=0}^{\leq k/2} \frac{\Gamma \left( \frac{1}{2} + k - m \right)}{m!} \left( \frac{z}{2} \right)^{2m-k-1},
\]

and possesses the property [5]

\[
\frac{1}{2\pi} \int_C J_{n+1/2}(z) A_{k,1/2}(z) z^{-1/2} dz = \delta_{k,n}.
\]

The resulting contour \( C \) has the shape as shown on the figure 2b.

\[
\begin{aligned}
&\int \left( \frac{\partial P^i}{\partial \sigma_0} \frac{\sin \vartheta_0}{\eta_0} \left( d^+_n, \delta_{k,n} \right) + \frac{1}{2} \eta_0 \right) \\
&\quad \left[ \left( \frac{\delta_{k,n}}{\rho_0} - \frac{\delta_{k,n+1}}{\rho_0} \right) + \eta_0 \left( C_n - C_{n+1} \right) \right] a_n^E = \\
&\quad \frac{1}{2} \frac{\partial P^i}{\partial \sigma_0} s_1 C_{k,1} + \frac{1}{10} \frac{\partial P^i}{\partial \sigma_0} \sin \vartheta_0 s_2 (C_{k,1} - \frac{2}{3} C_{k,3}).
\end{aligned}
\]

(15)

Here coefficients \( C_{k,n} \) are

\[
C_{k,n} = \frac{1}{2\pi} \int_C A_{k,1/2} J_{n-1/2} z^{-1/2} dz =
\]

\[
(-1)^l \left( k + \frac{1}{2} \right) \sum_{n=0}^{\ell/2} \frac{(1 - m) \Gamma \left( \frac{1}{2} + k + m \right)}{m!(l+1-m) ! \Gamma \left( \frac{1}{2} + k + m - l \right)}.
\]

The coefficients \( C_{k,n} \) are not equal to zero if \( n + k = 2l + 1, l \) is the integer number.

If we introduce two matrices \( \Lambda^B_{k,n}, \Lambda^E_{k,n} \) and the vector \( \sigma^k_1 \),

\[
\Lambda^B_{k,n} = \frac{\partial P^i}{\partial \sigma_0} \left( \frac{1}{n(n+1)} + i \delta_{k,n} \right) \left( \rho_0^B C_{k,n} - \delta_{k,n} \right);
\]

\[
\Lambda^E_{k,n} = \frac{\partial P^i}{\partial \sigma_0} \left( \frac{1}{n(n+1)} + i \delta_{k,n} \right) \left( \rho_0^E C_{k,n} - \delta_{k,n} \right);
\]

\[
\sigma^k_1 = \frac{1}{2} \frac{\partial P^i}{\partial \sigma_0} s_1 C_{k,1} + \frac{1}{10} \frac{\partial P^i}{\partial \sigma_0} \sin \vartheta_0 s_2 (C_{k,1} - \frac{2}{3} C_{k,3}),
\]

then we obtain

\[
\Lambda^B_{k,n} a^B_n + \Lambda^E_{k,n} a^E_n = \sigma^k_1.
\]

(17)

The second matrix equation connecting \( a^B_n \) and \( a^E_n \) is followed from the condition (6). Doing the same procedure (multiplying on the Gegenbauer polynomial and integrating in the complex plane \( z \)), but taking into account that the Keplerian velocity, \( \psi_0 \propto z^{-1/2} \), is the two fold function, we have to introduce the cut in the complex plane from the point \( z = z_d \) to the infinity. The resulting contour \( C \) has the shape as shown on the
4 Torque acting on a star

Our purpose is to determine the change of the torque acting on a star under the influence of a rotating disk,

\[ K = \frac{1}{c} \int r \times J_s \times B \, ds, \]

(22)

where \( J_s \) is the surface current on the star surface

\[ J_s = \frac{c}{4\pi r} \times \{ B \}, \quad \{ B \} \equiv B|_{r=R_s+0} - B|_{r=R_s-0}. \]

(23)

\( \{ B \} \) is the discontinuous of the tangential magnetic field. The torque is

\[ K = \frac{R_s}{c} \int J_s B_{sr} \, ds, \]

(24)

where \( B_{sr} \) is the radial magnetic field on the stellar surface

\[ B_{sr} = \frac{2\mu}{R_s^3} \sin \chi P_s^1 e^{i\phi-i\omega t}. \]

Integrating in Eq. (24) over the surface we find the component of the torque \( K_z \) along the axis of the rotation. It turns out to be proportional to two coefficients of the expansion of the vacuum field over multipoles (7,8)

\[ K_z = -\frac{2}{3} \frac{\mu^2 \sin^2 \chi}{R_s^3} \int m(a^B_n) \left[ \frac{1}{2} \frac{\partial}{\partial z} (z^{1/2} J_{5/2}) - \right. \]

\[ \left. \frac{1}{2} \frac{\partial}{\partial z} (z^{1/2} J_{3/2}) \right]_{z=z_0} + \frac{2}{3} \frac{\mu^2 \sin^2 \chi}{R_s^3} \int m(a^E_n) \left[ z^{-3/2} J_{5/2} - z^{-1/2} J_{3/2} \right] \right. \]

(25)

That means that only two types of radiation, the magnetodipole and the electroquadrupole, retard the star rotation. The contribution of the magnetodipole component is much larger than the electroquadrupole one because \( z_0 << 1 \). In the absence of a disk the torque is

\[ K_z = K_z^{MD} = -\frac{2}{3} \frac{\mu^2 \sin^2 \chi}{R_s^3} z_0^{-3} = -\frac{2}{3} \frac{\mu^2 \omega^3 \sin^2 \chi}{c^3}, \]

(26)

\[ |K_z^{MD}| = 1.5 \cdot 10^{30} B_{12}^2 \left( \frac{R}{10^{12} \text{km}} \right)^6 \left( \frac{P_s}{1 \text{s}} \right)^{-3} \sin^2 \chi \text{ erg}. \]

An conducting disk distorts the radiation. There appears another multipole components of the radiation. The amplitudes of them are connected by the relation (21). The coefficients \( \Lambda_{k,n} \) contain also the imaginary parts which are proportional to the Keplerian parameter \( \kappa \). The disk also changes the conditions at the infinity - a disk reflects the wave energy. As a result a star can as lost the angular momentum as obtain it from a disk. The effect depends on the disk parameter \( z_d = r_d \omega_s/c \). The qualitative estimation demands the solution of the matrix equation (21) numerically.

The result of calculations for the usual neutron star parameters \( R = 10 \text{km}, M_s = 1M_\odot, \omega_s = 1 \text{s}^{-1} \) and \( z_d = 1, \delta = 10^\circ \) is presented on the table 1.

Table 1. Amplitudes of n-th magnetic multipoles \( a^B_n \) and n-th electric multipoles \( a^E_n \) for the first sixth waves (7,8). They are normalized by their values in the vacuum: \( a^B_n \) by \( Ima^B_1 = (\pi/2)^{1/2} z_0^3 \) and \( a^E_n \) by \( Ima^E_1 = -(\pi/72)^{1/2} z_0^5 \).

We see that though the main component is \( a^B_1 \), there appear compatible amplitudes \( a^E_2, a^B_3, a^E_3 \). Among electric multipoles the electrodipole amplitude \( a^E_1 \) becomes general. For the larger \( n > 4 \) the amplitudes of multipoles fall down. It permits us to restrict ourself by the finite dimension of vectors \( \vec{a}, \vec{\sigma} \) in equation (21), \( k = 12 \). The calculation of the torque \( K_z \), acting on a star, is presented on the figure 3. For comparison we draw also the result of calculation for \( k = 8 \). It is seen that the difference between \( k = 8 \) and \( k = 12 \) is not significant and our approximation to use finite numbers of equations in infinite matrix equation (21) is valid. The main result is the change of sign of \( K_z \) at \( z_d = z^* \approx 1 \). For the chosen parameters \( z^* = 0.8 \). The behavior of the torque \( K_z \) over \( z_d \) can be well approximated the by the simple formula

\[ K_z = K_z^{MD} \left( 1 - \frac{z^*}{z_d} \right). \]

(27)

This expression does not take into account the small maximum of \( K_z \) at \( z_d \approx 2 - 3 \), when a star transmits its angular momentum not only to the electromagnetic radiation, but also to a disk.

At \( z_d < z^* \) a disk transmits the angular momentum to a star, it spins up. Let us note that \( z^* \approx 1 \) is just the region where the electromagnetic radiation forms. A disk changes radiation conditions in the zone where it originates. Such electromagnetic stellar spin up exists independently of the mechanical angular momentum transition, which is proportional to the value of the accretion matter rate \( \dot{M} \).

The structure of the magnetic field is shown on the figure 4. Here we present only the magnetic field produced by the varying component of the magnetic dipole, proportional to \( \mu \sin \chi \), which generates the electromagnetic radiation.

We see that the magnetic field lines repeal from an ideal disk. It is the result of calculations under the
Fig. 2  The contour of integration in the complex plane $z$. The contour a) corresponds to the boundary condition (5), and the contour b) corresponds to the boundary condition (6) on the disk surface.

Fig. 3  The torque $K_z$, acting on a star, in the units of $K_z^{MD}$ versus the inner edge of a disk $z_d$. $\delta = 10^\circ$, a) $k=12$, b) $k=8$.

Fig. 4  Projection of magnetic field lines of the oblique dipole onto the plane orthogonal to the direction of the magnetic moment $\mu$. Owing to the projective view and the large scale of the order of light cylinder radius $c/\omega_s$, some details on this figure look like peculiarities (cusps, touches), but they are not in reality.
boundary conditions (5,6) on the disk surface. It is not postulated ad hoc as did in [2].

5 Discussion

We showed that presence of a conducting disk in the magnetosphere of a rotating magnetized star changes significantly the electromagnetic radiation of a star if the inner edge of a disk $r_d$ is closer than the radius of the stellar light cylinder $c/\omega_s$. The torque acting on a star $K_z$ changes its sign when $r_d < r^* \approx c/\omega_s$. Instead of the spinning down of the star rotation, a star begins to spin up, getting the angular momentum from a disk. At the position of the inner disk radius near the light cylinder radius, $r_d = r^*$, the torque $K_z$ becomes zero. It means that a star will not change its angular momentum and the period of its rotation will be constant. It can happen after the period of the gas accretion onto a star when it gains the angular momentum, value of which is proportional to the accretion rate $\dot{M}$. As was shown [6], under the mass accretion onto a magnetized star, the torque $K_z$ has always definite sign, and there is no situation of the stable star rotation. When the accretion stops, $\dot{M} = 0$, a star pushes out a disk from its vicinity, that is, so called, the propeller regime. The inner edge of a disk moves toward the light cylinder distance where a disk prevents the stellar spin down due to the radiation of electromagnetic waves. The period of the stellar rotation will not change when a disk is presented at the distance from a star $r_d \approx c/\omega_s$. Such period of the star life is observed among X-rays binaries [1].

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