Two-photon-exchange effects in the electro-excitation of the ∆ resonance

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We evaluate the two-photon exchange contribution to the eN → e∆(1232) → eπN process at large momentum transfer with the aim of a precision study of the ratios of electric quadrupole (E2) and Coulomb quadrupole (C2) to the magnetic dipole (M1) γ∗NΔ transitions. We relate the two-photon exchange amplitude to the N → ∆ generalized parton distributions and obtain a quantitative estimate of the two-photon effects. The two-photon exchange corrections to the C2/M1 ratio is a constant corrected by double-logarithms of Q2, where Q2 is the photon virtuality. Of special interest are the ratios of the electric (G_E^*) and Coulomb (G_C^*) quadrupole transitions to the dominant magnetic dipole (G_M^*) transition:

\[ R_{EM} \equiv -\frac{G_E^*}{G_M^*}, \quad R_{SM} \equiv -\frac{Q_+ Q_-}{(2M_N)^2} \frac{G_C^*}{G_M^*}, \quad (1) \]

where \( Q_\pm = \sqrt{Q^2 + (M_\Delta \pm M_N)^2} \) with nucleon mass \( M_N = 0.938 \) GeV, and Δ mass \( M_\Delta = 1.232 \) GeV.

The γ∗NΔ transition form factors are usually studied in the pion electroproduction \((eN \rightarrow e\pi N)\) process in the Δ-resonance region. Denoting the invariant mass of the final πN system by \( W_{\pi N} \), we consider \( W_{\pi N} = M_\Delta \). The 5-fold differential cross-section of this process is commonly written as:

\[ \frac{d\sigma}{dE_e' d\Omega_e'^{lab} d\Omega_\pi} \equiv \Gamma_v \frac{d\sigma}{d\Omega_\pi}, \quad (2) \]

where, in 1γ approximation, \( d\sigma/d\Omega_\pi \) has the interpretation of a γ∗N → πN virtual photon absorption cross section, and the virtual photon flux factor \( \Gamma_v \) is given by:

\[ \Gamma_v = \frac{e^2}{(2\pi)^3} \left( \frac{E_{e'}^{\gamma}}{E_e} \right)^2 \frac{(W_{\pi N}^2 - M_N^2)/(2M_N)}{Q^2 (1 - \varepsilon)}, \quad (3) \]

where \( E_{e'}^{\gamma} (E_e^{\gamma}) \) are the initial (final) electron lab energies, \( e \) is the electric charge, and \( \varepsilon \) denotes the photon polarization parameter. The pion angles \( \theta_\pi, \Phi \) are defined in the πN c.m. frame, with \( \theta_\pi \) the pion polar angle and \( \Phi \) the angle between the hadron and lepton planes (\( \Phi = 0 \) corresponds with the pion emitted in the same half-plane as the leptons). For unpolarized nucleons, the cross section at \( W_{\pi N} = M_\Delta \) can, in general, be parametrized as:

\[ \frac{d\sigma}{d\Omega_\pi} = \sigma_0 + \varepsilon \cos(2\Phi) \sigma_{TT} + (2h) \varepsilon \sin(2\Phi) \sigma_{TT}, \quad \varepsilon = \sqrt{2\varepsilon_+ \cos \Phi \sigma_{LT} + (2h) \sqrt{2\varepsilon_+} \sin \Phi \sigma_{LT}}, \quad (4) \]
with \( h = \pm 1/2 \) the lepton helicity and \( \epsilon_\pm \equiv \sqrt{1 \pm \varepsilon} \). The cross sections \( \sigma_{TT}, \sigma_{LT} \) are new responses which appear beyond the \( 1\gamma \)-exchange approximation. For the cross-sections \( \sigma_0, \sigma_{TT}, \sigma_{LT} \) the \( 2\gamma \) exchange induces corrections of order \( \varepsilon^2 \) relative to \( 1\gamma \).

In order to evaluate the \( 2\gamma \) contribution to the \( N \to \Delta \) electroproduction amplitudes at large momentum transfers, we will consider a partonic model in the handbag approximation, illustrated in Fig. 1, as studied before for the \( eN \to eN \) process. This calculation involves a hard scattering subprocess on a quark, which is then embedded in the proton by means of the \( N \to \Delta \) GPDs \([12, 13, 14]\). We consider only the vector GPD in terms of the following two characteristic integrals:

\[
A^* = \int_1^{-1} \frac{dx}{x} \left[ \frac{\hat{s} - \hat{u}}{Q^2} g_M^{hard} + g_A^{(2\gamma)} \right] \sqrt{\frac{2}{3}} \frac{1}{6} H_M^{(3)} \quad \text{and} \quad C^* = \int_1^{-1} \frac{dx}{x} \left[ \frac{\hat{s} - \hat{u}}{Q^2} g_A^{(2\gamma)} + g_M^{hard} \right] sgn(x) \frac{1}{6} C_1^{(3)},
\]

where the factor \( 1/6 \) results from the quadratic quark charge combination \((e_q^2 - e_d^2)/2\), and all hard scattering quantities in the square brackets are given in Ref. [1].

The GPDs \( H_M^{(3)} \) \((C_1^{(3)})\) are linked with the \( N \to \Delta \) vector (axial-vector) transition form factors \( G_M \) \((C_5^A\) as introduced by Adler \([17]\) respectively through the sum rules:

\[
\int_1^{-1} dx H_M^{(3)}(x, 0, Q^2) = 2 G_M^* (Q^2), \quad \int_1^{-1} dx C_1^{(3)}(x, 0, Q^2) = 2 C_5^A (Q^2).
\]

At \( Q^2 = 0 \), \( G_M^* \) is extracted from pion photoproduction experiments as: \( G_M^* (0) \simeq 3.02 \). For small \( Q^2 \), PCAC leads to a dominance of the form factors \( C_5^A \), for which a Goldberger-Treiman relation for the \( N \to \Delta \) transition yields: \( \sqrt{3/2} C_5^A (0) = g_A f_{\pi N \Delta} (2 f_{\pi NN}) \). Using the phenomenological values \( f_{\pi N \Delta} \simeq 1.95, f_{\pi NN} \simeq 1.00 \), and \( g_A \simeq 1.267 \) one obtains \( C_5^A (0) \simeq 1.01 \).

We can now compute the \( 2\gamma \) effects in observables. We start by multipole expanding Eq. (1) for the \( ep \to e\Delta^- \to e\pi N \) process as:

\[
\sigma_0 = A_0 + \frac{1}{2} (3 \cos^2 \theta_\pi - 1) A_2,
\]

\[
\sigma_{TT} = \sin^2 \theta_\pi C_0,
\]

\[
\sigma_{LT} = \frac{1}{2} \sin (2 \theta_\pi) D_1,
\]

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\]

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\]

where \( A_0 \) can be written as:

\[
A_0 = T \frac{e^2}{4\pi} \frac{Q^2 \theta^0 (M_\Delta + M_N)}{4 M_N^2 (M_\Delta - M_N) M_\Delta g_{\Delta}} (G_M^*)^2 \sigma_R, \quad (10)
\]

where \( g_{\Delta} \simeq 0.120 \) GeV is the \( \Delta \) width, and \( T \) denotes an isospin factor which depends on the final state in the \( \Delta^+ \to \pi^+ N^- \) decay as: \( T (p^0 p) = 2/3 \) and \( T (\pi^+ n) = 1/3 \). Furthermore in Eq. (10), the reduced cross section \( \sigma_R \), including \( 2\gamma \) corrections evaluated in the handbag model, is given by:

\[
\sigma_R = 1 + 3 R_{EM}^2 + \frac{16 M_N^2 Q^2}{Q^2 - M_\Delta^2} R_{SM}^2
\]

\[
+ \frac{1}{G_M^*} \left[ \frac{A^* Q^2 \varepsilon_+ - e_\varepsilon}{2 Q^2 - M_\Delta^2} + 2 C_5^A Q^2 \varepsilon^2 - M_N^2 (M_N + M_\Delta) \right].
\]

We next discuss the \( 2\gamma \) corrections to \( R_{EM} \) and \( R_{SM} \) as extracted from \( \sigma_{TT} \) and \( \sigma_{LT} \). Experimentally, these ratios have been extracted at \( W_{\pi N} = M_\Delta \) using:

\[
R_{EM}^{exp.} = \frac{3 A_2 - 2 C_0}{12 A_0} \quad \text{and} \quad R_{SM}^{exp.} = \frac{Q_+ Q_- D_1}{2 M_\Delta 6 A_0} \equiv R_{SM} - R_{SM} R_{EM} + \ldots
\]

where the omitted terms involve cubic products of \( R_{EM} \) and \( R_{SM} \). These formulas are usually applied by neglecting the smaller quantities \( R_{SM}^2 \) and \( R_{EM} \cdot R_{SM} \).

We will keep the quadratic terms here and show that the \( R_{SM}^2 \) term leads to a non-negligible contribution in the extraction of \( R_{EM} \). A second way of extracting \( R_{EM} \), which avoids corrections at the one-photon level, is:

\[
R_{EM}^{exp. II} = - \frac{(A_0 - A_2) - 2 C_0}{3 (A_0 - A_2) - 2 C_0} \equiv R_{EM}.
\]

We can usefully denote the corrections to \( R_{EM} \) and \( R_{SM} \) generically by:

\[
R \simeq R^{exp} + \delta R^{1\gamma} + \delta R^{2\gamma}.
\]

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\]
The term $\delta R^{1\gamma}$ denotes the corrections due to the quadratic terms in Eqs. (12) [13], which are:

$$\delta R^{1\gamma}_{EM} = -\frac{4 M^2 \gamma^2}{Q^2 Q^2_{\gamma}} R^2_{SM},$$

$$\delta R^{1\gamma}_{EM} = 0,$$

$$\delta R^{1\gamma}_{EM} = R_{EM} \cdot R_{SM}. \quad (15)$$

The two-photon exchange corrections to $R_{EM}$ and $R_{SM}$ are obtained in the handbag model and are:

$$\delta R^{2\gamma}_{EM} = -\frac{1}{8} \sqrt{\frac{3}{2}} \frac{Q^2}{Q^2_{\gamma}} \frac{\varepsilon^2}{\varepsilon^2 + 1} \frac{M^2}{G^2_{EM}} A^\ast,$$

$$+ \frac{1}{16} \frac{3}{Q^2_{\gamma}} \frac{\varepsilon^2}{\varepsilon^2 + 1} \frac{M}{(M + M_{\Delta})} \frac{1}{G^2_{EM}} C^\ast,$$

$$\delta R^{2\gamma}_{EM} = 2 \delta R^{2\gamma}_{EM},$$

$$\delta R^{2\gamma}_{SM} = \frac{2}{Q^2_{\gamma}} \left( \frac{Q^2 - M^2_{\Delta} + M_{N}^2}{4 M_{\Delta}^2} \right) \frac{Q^2}{Q^2_{\gamma}} \frac{1}{\varepsilon^2} \frac{M}{\varepsilon^2 + 1} \frac{M}{(M + M_{\Delta})} \frac{1}{G^2_{EM}} C^\ast,$$

$$\times \frac{M_N}{(M + M_{\Delta})} \frac{1}{G^2_{EM}} C^\ast. \quad (16)$$

To provide numerical estimates for the $2\gamma$ corrections, we need a model for the two ‘large’ GPDs which appear in the integrals $A^\ast$ and $C^\ast$. Here we will be guided by the large $N_c$ relations discussed in [12] [13]. These relations connect the $N \rightarrow \Delta$ GPDs $H^{(3)}_{EM}$ and $C^{(3)}$ to the $N \rightarrow N$ isovector GPDs $E^u - E^d$, and $H^u - H^d$ respectively as:

$$H^{(3)}_{EM}(x, 0, Q^2) = 2 \frac{G^s_{EM}(0)}{\kappa_V} \left[ E^u - E^d \right] (x, 0, Q^2),$$

$$C^{(3)}(x, 0, Q^2) = \sqrt{3} \left[ H^u - H^d \right] (x, 0, Q^2), \quad (17)$$

with $\kappa_V = \kappa_p - \kappa_n = 3.706$. For the nucleon GPDs $E^u$ and $H^u$ appearing in Eq. (17), we use the modified Regge model [18], which was applied before [19] to estimate $2\gamma$ corrections to the $eN \rightarrow eN$ process. As an example, we show in Fig. 2 the form factor $G^i_{EM}$, obtained by evaluating the sum rule of Eq. (7) using the modified Regge GPD model for the GPD $H^{(3)}_{EM}$, adjusting the Regge slope parameter as $\alpha'_2 = 1.3$ GeV$^{-2}$. One sees that a good description is obtained over the whole range of $Q^2$.

In Fig. 3 we show the effect of the $1\gamma$ and $2\gamma$ corrections on $R_{EM}$ and $R_{SM}$. For the $1\gamma$ correction, we see that the effect on $R_{SM}$ is negligible, whereas it yields a systematic downward shift of the $R_{EM}$ result. This shift becomes more pronounced at larger $\varepsilon$, and it is found that for $Q^2$ around 5 GeV$^2$ such as for the upcoming data of Ref. [20], it decreases $R_{EM}$ by around 1%. To avoid such a correction, it calls for extracting $R_{EM}$ according to the procedure II as we outlined above. Furthermore, we show the $2\gamma$ corrections in Fig. 3 (right panels), estimated using the modified Regge GPD model. We see that the $2\gamma$ effects are mainly pronounced at small $\varepsilon$ and larger $Q^2$. For $R_{EM}$ they are well below 1%, whereas they yield a negative correction to $R_{SM}$ by around 1%, when $R_{SM}$ is extracted from $\sigma_{LT}$ according to Eq. (12).
FIG. 4: (Color online) Rosenbluth plot of the reduced cross section $\sigma_R$ of Eq. (11) for the $eN \rightarrow e\Delta$ reaction at $Q^2 = 3 \text{ GeV}^2$. The dashed curve corresponding with $R_{SM} = -10\%$ is the 1σ result, whereas the solid curve represents the result including $2\gamma$ corrections for the same values of $R_{SM}$. The dotted curve corresponding with $R_{SM} = -13\%$ corresponds with a linear fit to the total result in an intermediate $\varepsilon$ range.

change in the slope of the Rosenbluth plot. When fitting the total result by a straight line in an intermediate $\varepsilon$ range, one extracts a value of $R_{SM}$ around 3 percentage units lower than its value as extracted from $\sigma_{LT}$. This is sizable, as it corresponds with a 30% correction on the absolute value of $R_{SM}$. The situation is similar to extracting the elastic proton form factor ratio $G_E/G_M$ using the Rosenbluth method [3]. It will be interesting to confront this to new Rosenbluth separation data in the $\Delta$ region up to $Q^2 \approx 5 \text{ GeV}^2$ which are presently under analysis [21].

Summarizing, in this work we estimated the $2\gamma$ contribution to the $eN \rightarrow e\Delta(1232) \rightarrow e\pi N$ process at large momentum transfer in a partonic model. We related the $2\gamma$ amplitude to the $N \rightarrow \Delta$ generalized parton distributions. For $R_{EM}$, the $2\gamma$ corrections were found to be small, below the 1% level. We showed however that the neglect of a quadratic term $R_{SM}^2$ in the usual method to extract $R_{EM}$ yields corrections at the 1% level. The $2\gamma$ corrections to $R_{SM}$ were found to be substantially different when extracting this quantity from an interference cross section or from Rosenbluth type cross sections, as has been observed before for the elastic $eN \rightarrow eN$ process. It will be interesting to confront these results with upcoming new Rosenbluth separation data at intermediate $Q^2$ values in order to arrive at a precision extraction of the large $Q^2$ behavior of the $R_{EM}$ and $R_{SM}$ ratios.

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