Green Modulations in Energy-Constrained Wireless Sensor Networks

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Abstract

Due to the unique characteristics of sensor devices, finding the energy-efficient modulation with a low-complexity implementation (referred to as green modulation) poses significant challenges in the physical layer design of Wireless Sensor Networks (WSNs). Toward this goal, we present an in-depth analysis on the energy efficiency of various modulation schemes using realistic models in the IEEE 802.15.4 standard to find the optimum distance-based scheme in a WSN over Rayleigh and Rician fading channels with path-loss. We describe a proactive system model according to a flexible duty-cycling mechanism utilized in practical sensor apparatus. The present analysis includes the effect of the channel bandwidth and the active mode duration on the energy consumption of popular modulation designs. Path-loss exponent and DC-DC converter efficiency are also taken into consideration. In considering the energy efficiency and complexity, it is demonstrated that among various sinusoidal carrier-based modulations, the optimized Non-Coherent M-ary Frequency Shift Keying (NC-MFSK) is the most energy-efficient scheme in sparse WSNs for each value of the path-loss exponent, where the optimization is performed over the modulation parameters. In addition, we show that the On-Off Keying (OOK) displays a significant energy saving as compared to the optimized NC-MFSK in dense WSNs with small values of path-loss exponent.

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Index Terms

Wireless sensor networks, energy efficiency, green modulation, M-ary FSK, Ultra-Wideband (UWB) modulation.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) have been recognized as a collection of distributed nodes to support a broad range of applications, including monitoring, health-care and detection of environmental pollution. In such configuration, sensors are typically powered by limited-lifetime batteries which are hard to be replaced or recharged. On the other hand, since a large number of sensors are deployed over a region, the circuit energy consumption is comparable to the transmission energy due to the short distance between nodes. Thus, minimizing the total energy consumption in both circuits and signal transmission is a crucial task in designing a WSN [1], [2]. Central to this study is to find energy-efficient modulations in the physical layer of a WSN to prolong the sensor lifetime. For this purpose, energy-efficient modulations should be simple enough to be implemented by state-of-the-art low-power technologies, but still robust enough to provide the desired service. In addition, since sensor devices frequently switch from sleep mode to active mode, modulator circuits should have fast start-up times. We refer to these simple and low-energy consumption schemes as green modulations.

In recent years, several energy-efficient modulations have been studied in the physical layer of WSNs (e.g., [3], [4]). In [3] the authors compare the battery power efficiency of PPM and OOK based on the Bit Error Rate (BER) and the cutoff rate of a WSN over path-loss Additive White Gaussian Noise (AWGN) channels. Reference [4] investigates the energy efficiency of a centralized WSN with an adaptive MQAM scheme. However, adaptive approaches impose some additional system complexity due to the multi-level modulation formats plus the channel state information fed back from the sink node to the sensor node. Most of the pioneering work on energy-efficient modulations, including research in [3], has focused only on minimizing the average energy consumption per information bit, ignoring the effect of the bandwidth and transmission time duration. In a practical WSN, however, it is shown that minimizing the total energy consumption depends strongly on the active mode duration and the channel bandwidth [5].
In this paper, we present an in-depth analysis (supported by numerical results) of the energy efficiency of various modulation schemes considering the effect of the “channel bandwidth” and the “active mode duration” to find the distance-based green modulations in a proactive WSN. For this purpose, we describe the system model according to a flexible duty-cycling process utilized in practical sensor devices. This model distinguishes our approach from existing alternatives [3]. New analysis results for comparative evaluation of popular modulation designs are introduced according to the realistic parameters in the IEEE 802.15.4 standard [6]. We start the analysis based on a Rayleigh flat-fading channel with path-loss which is a feasible model in static WSNs [3]. Then, we evaluate numerically the energy efficiency of sinusoidal carrier-based modulations operating over the more general Rician model which includes a strong direct Line-Of-Sight (LOS) path. Path-loss exponent and DC-DC converter efficiency (in a non-ideal battery model) are also taken into consideration. It is demonstrated that among various sinusoidal carrier-based modulations, the optimized Non-Coherent M-ary Frequency Shift Keying (NC-MFSK) is the most energy-efficient scheme in sparse WSNs for each value of the path-loss exponent, where the optimization is performed over the modulation parameters. In addition, we show that the On-Off Keying (OOK) has significant energy saving as compared to the optimized NC-MFSK in dense WSNs with small values of path-loss exponent. NC-MFSK and OOK have the advantage of less complexity and cost in implementation than MQAM and Offset-QPSK used in the IEEE 802.15.4 protocol, and can be considered as green modulations in WSN applications.

The rest of the paper is organized as follows. In Section II, the proactive system model and assumptions are described. A comprehensive analysis of the energy efficiency for popular sinusoidal carrier-based and UWB modulations is presented in Sections III and V. Section IV provides some numerical evaluations using realistic models to confirm our analysis. Finally in Section VI, an overview of the results and conclusions are presented.

For convenience, we provide a list of key mathematical symbols used in this paper in Table I.
**TABLE I**

**LIST OF NOTATIONS**

| Symbol | Description |
|--------|-------------|
| $B$    | Bandwidth   |
| $B_{eff}$ | Bandwidth efficiency |
| $d$    | Distance    |
| $M$    | Constellation size |
| $b$    | Number of bits per symbol |
| $E_t$  | Energy per symbol |
| $E_N$  | Total energy consumption for $N$-bit |
| $h_i$  | Fading channel coefficient |
| $L_d$  | Channel gain factor in distance $d$ |
| $N$    | Number of transmitted bits in active mode period |
| $M_N$  | Constellation size |
| $P_s$  | Symbol error rate |
| $P_c$  | Total circuit power consumption |
| $P_t$  | RF transmit power consumption per symbol |
| $T_s$  | Symbol duration |
| $T_{ac}$ | Active mode period |
| $T_{tr}$ | Transient mode period |
| $\eta$ | Path-loss exponent |
| $\Omega$ | $\Omega = \mathbb{E} [ |h_i|^2 ]$ |
| $\chi_e$ | Power transfer efficiency in DC-DC converter |
| $\gamma_i$ | Instantaneous SNR |
| $\mathbb{E} [ . ]$ | Expectation operator |
| $\Pr [ . ]$ | Probability of the given event |

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![Diagram](image.png)

**Fig. 1.** A practical duty-cycling process in a proactive WSN.

**II. SYSTEM MODEL AND ASSUMPTIONS**

In this work, we consider a proactive wireless sensor system, in which a sensor node transmits an equal amount of data per time unit to a designated sink node. The sensor and sink nodes synchronize with one another and operate in a duty-cycling manner as depicted in Fig 1. During *active mode* period $T_{ac}$, the sensed analog signal is first digitized by an Analog-to-Digital Converter (ADC), and an $N$-bit binary message sequence $M_N \triangleq \{ a_i \}_{i=1}^N$ is generated, where $N$ is assumed to be fixed. The bit stream is modulated using a pre-determined modulation scheme\(^1\) and then transmitted to the sink node. Finally, the

\(^1\)Because the main goal of this work is to find the distance-based green modulations, and noting that the source/channel coding increase the complexity and the power consumption, in particular codes with iterative decoding process, the source/channel coding are not considered.
sensor node returns to the \textit{sleep mode}, and all the circuits are powered off for the sleep mode duration $T_{sl}$.

We denote $T_{tr}$ as the \textit{transient mode} duration consisting of the switching time from sleep mode to active mode (i.e., $T_{sl \rightarrow ac}$) plus the switching time from active mode to sleep mode (i.e., $T_{ac \rightarrow sl}$), where $T_{ac \rightarrow sl}$ is short enough to be negligible. Under the above considerations, the sensor/sink nodes have to process one entire $N$-bit message $M_N$ during $0 \leq T_{ac} \leq T_N - T_{sl} - T_{tr}$, where $T_N \triangleq T_{tr} + T_{ac} + T_{sl}$ is assumed to be fixed for each modulation, and $T_{tr} \approx T_{sl \rightarrow ac}$. Note that $T_{ac}$ is an influential factor in choosing the energy-efficient modulation, since it directly affects the total energy consumption as we will show later.

We assume that both sensor and sink devices include a Direct Current to Direct Current (DC-DC) converter to generate a desired supply voltage from the embedded batteries. An DC-DC converter is specified by its \textit{power transfer efficiency} denoted by $\chi_e < 1$. In addition, we assume a linear model for batteries with a small discharge current, meaning that the stored energy will be completely used or released. This model is reasonable for the proposed duty-cycling process, as the batteries which are discharged during active mode durations can recover their wasted capacities during sleep mode periods.

Since sensor nodes in a typical WSN are densely deployed, the distance between nodes is normally short. Thus, the total circuit power consumption, defined by $P_c \triangleq P_{ct} + P_{cr}$, is comparable to the RF transmit power consumption denoted by $P_t$, where $P_{ct}$ and $P_{cr}$ represent the circuit power consumptions for the sensor and sink nodes, respectively. In considering the effect of the power transfer efficiency, the total energy consumption in the active mode period, denoted by $E_{ac}$, is given by

$$E_{ac} = \frac{P_c + P_t}{\chi_e} T_{ac}, \quad (1)$$

where $T_{ac}$ is a function of $N$ and the channel bandwidth as we will show in Section \text{III}. Also, it is shown in [7] that the power consumption during the sleep mode duration $T_{sl}$ is much smaller than the power consumption in the active mode (due to the low sleep mode leakage current) to be negligible. As a result, the \textit{energy efficiency}, referred to as the performance metric of the proposed WSN, can be measured by the total energy consumption in each period $T_N$ corresponding to $N$-bit message $M_N$ as follows:

$$E_N \approx \frac{1}{\chi_e} \left[ (P_c + P_t) T_{ac} + P_{tr} T_{tr} \right], \quad (2)$$
where \( \frac{P_{tr}}{\chi_c} T_{tr} \) is the circuit energy consumption during the transient mode period. We use (2) to investigate and compare the energy efficiency of various modulation schemes in the subsequent sections.

**Channel Model:** It is shown that for short-range transmissions including the wireless sensor networking, the root mean square (rms) delay spread is in the range of ns \([8]\) (and ps for UWB applications \([9]\)) which is small compared to the symbol duration \( T_s = 16 \) \( \mu \)s obtained from the bandwidth \( B = \frac{1}{T_s} = 62.5 \) KHz in the IEEE 802.15.4 standard \([6\text{ p. 49}]\). Thus, it is reasonable to expect a flat-fading channel model for WSNs. Under the above considerations, the channel model between the sensor and sink nodes is assumed to be Rayleigh flat-fading with path-loss. This assumption is used in many works in the literature (e.g., see \([3]\) for WSNs). We denote the fading channel coefficient corresponding to symbol \( i \) as \( h_i \), where the amplitude \( |h_i| \) is Rayleigh distributed with the probability density function (pdf) \( f_{|h_i|}(r) = \frac{2r}{\Omega^2} e^{-\frac{r^2}{\Omega^2}}, \ r \geq 0 \), where \( \Omega \triangleq \mathbb{E}[|h_i|^2] \) \([10]\). To model the path-loss of a link where the transmitter and receiver are separated by distance \( d \), let denote \( P_t \) and \( P_r \) as the transmitted and the received signal powers, respectively. For a \( \eta \)-th-power path-loss channel, the channel gain factor is given by

\[
\mathcal{L}_d \triangleq \frac{P_t}{P_r} = M_l d^\eta \mathcal{L}_1, \quad \text{with} \ \eta_{\min} \leq \eta \leq \eta_{\max},
\]

(3)

where \( M_l \) is the gain margin which accounts for the effects of hardware process variations and \( \mathcal{L}_1 \triangleq \frac{(4\pi)^2}{G_t G_r \lambda^2} \) is the gain factor at \( d = 1 \) meter which is specified by the transmitter and receiver antenna gains \( G_t \) and \( G_r \), and wavelength \( \lambda \) (e.g., \([3]\), \([5]\)). As a result, when both fading and path-loss are considered, the instantaneous channel coefficient becomes \( G_i \triangleq \frac{h_i}{\sqrt{\mathcal{L}_d}} \). Denoting \( x_i(t) \) as the transmitted signal with energy \( \mathcal{E}_i \), the received signal at the sink node is given by \( y_i(t) = G_i x_i(t) + z_i(t) \), where \( z_i(t) \) is AWGN with two-sided power spectral density given by \( \frac{N_0}{2} \). Thus, the instantaneous Signal-to-Noise Ratio (SNR) corresponding to an arbitrary symbol \( i \) can be computed as \( \gamma_i = \frac{|G_i|^2 \mathcal{E}_i}{N_0} \). Under the assumption of a Rayleigh fading channel, \( \gamma_i \) is chi-square distributed with 2 degrees of freedom, with pdf \( f_{\gamma_i}(\gamma_i) = \frac{1}{\gamma_i} \exp \left( -\frac{\gamma_i}{\Omega_L} \right) \), where \( \Omega_L \triangleq \mathbb{E}[|G_i|^2] \frac{\mathcal{E}_i}{N_0} = \frac{\Omega}{\mathcal{L}_d N_0} \) denotes the average received SNR.
III. ENERGY EFFICIENCY ANALYSIS OF SINUSOIDAL CARRIER-BASED MODULATIONS

In this section, we analyze the energy and bandwidth efficiency of three popular sinusoidal carrier-based modulations, namely MFSK, MQAM and OQPSK, over a Rayleigh flat-fading channel with path-loss. FSK is used in many low-complexity and energy-constrained wireless systems and some IEEE standards (e.g., [11]), whereas MQAM is used in modem and digital video applications. Also, OQPSK is used in the IEEE 802.15.4 standard which is the industry standard for WSNs. In the sequel and for simplicity of the notation, we use the superscripts ‘FS’, ‘QA’ and ‘OQ’ for MFSK, MQAM and OQPSK, respectively.

**M-ary FSK:** An M-ary FSK modulator with $M = 2^b$ orthogonal carriers benefits from the advantage of using the Direct Digital Modulation (DDM) approach, meaning that it does not need the mixer and the Digital to Analog Converter (DAC). This property makes MFSK has a faster start-up time than the other modulation schemes. Let denote $\Delta f = \frac{1}{\zeta T_{FS} s}$ as the minimum carrier separation with the symbol duration $T_{FS} s$, where $\zeta = 2$ for coherent and $\zeta = 1$ for non-coherent FSK [12, p. 114]. In this case, the channel bandwidth is obtained as $B \approx M \times \Delta f$, where $B$ is assumed to be fixed for all sinusoidal carrier-based modulations. Denoting $B_{eff}^{FS}$ as the bandwidth efficiency of MFSK defined as the ratio of data rate $R_{FS} = \frac{b T_{FS} s}{T_{FS} s}$ (b/s) to the channel bandwidth, we have

$$B_{eff}^{FS} \triangleq \frac{R_{FS}}{B} = \frac{\zeta \log_2 M}{M}, \text{ b/s/Hz.}$$  \hspace{1cm} (4)

It can be seen that using a small constellation size $M$ avoid losing more bandwidth efficiency in MFSK.

To address the effect of increasing $M$ on the energy efficiency, we first derive the relationship between $M$ and the active mode duration $T_{ac}^{FS}$. Since, we have $b$ bits during each symbol period $T_{FS} s$, we can write

$$T_{ac}^{FS} = \frac{N}{b} T_{FS} s = \frac{MN}{\zeta B \log_2 M}. \hspace{1cm} (5)$$

Recalling that $B$ and $N$ are fixed, an increase in $M$ results in an increase in $T_{ac}^{FS}$. However, the maximum value of $T_{ac}^{FS}$ is bounded by $T_N - T_{tr}^{FS}$ as illustrated in Fig.1. Thus, $M_{max} \triangleq 2^{b_{max}}$ in MFSK is calculated by the following non-linear equation:

$$\frac{M_{max}}{\log_2 M_{max}} = \frac{\zeta B}{N} (T_N - T_{tr}^{FS}). \hspace{1cm} (6)$$
At the receiver side, the received MFSK signal can be detected coherently to provide an optimum performance. However, the MFSK coherent detection requires the receiver to obtain a precise frequency and carrier phase reference for each of the transmitted orthogonal carriers. For large $M$, this would increase the complexity of the detector which makes a coherent MFSK receiver very difficult to implement. Thus, most practical MFSK receivers use non-coherent detectors. To analyze the energy efficiency of a NC-MFSK, we first derive $\mathcal{E}_t^{FS}$, the transmit energy per symbol, in terms of a given average Symbol Error Rate (SER) denoted by $P_s$. It is shown in [14, Lemma 2] that the average SER of a NC-MFSK is upper bounded by

$$P_s = 1 - \left(1 - \frac{1}{2 + \bar{\gamma}^{FS}}\right)^{M-1},$$  

(7)

where $\bar{\gamma}^{FS} = \frac{\Omega \varepsilon^{FS}}{Ld N_0}$. As a result, the transmit energy consumption per symbol is obtained from the above $P_s$ as

$$\mathcal{E}_t^{FS} \triangleq \mathcal{P}_t^{FS} T_s^{FS} = \left[\left(1 - (1 - P_s)^{\frac{1}{M-1}}\right)^{-1} - 2\right] \frac{Ld N_0}{\Omega}. $$

(8)

In considering the effect of the DC-DC converter and using (5), the output energy consumption of transmitting $N$-bit during $T_{ac}^{FS}$ is computed as

$$\frac{\mathcal{P}_t^{FS}}{\chi e} T_{ac}^{FS} = \frac{T_{ac}^{FS} \mathcal{E}_t^{FS}}{T_s^{FS} \chi e} = \left[\left(1 - (1 - P_s)^{\frac{1}{M-1}}\right)^{-1} - 2\right] \frac{Ld N_0 N}{\chi e \Omega \log_2 M}. $$

(9)

On the other hand, the total circuit energy consumption of the sensor/sink devices during $T_{ac}^{FS}$ is obtained from $\frac{\mathcal{P}_t^{FS} + \mathcal{P}_c^{FS}}{\chi e} T_{ac}^{FS}$. For the sensor node with the MFSK modulator, we denote the power consumption of frequency synthesizer, filters and power amplifier as $\mathcal{P}_{Sy}^{FS}$, $\mathcal{P}_{Filt}^{FS}$ and $\mathcal{P}_{Amp}^{FS}$, respectively. In this case,

$$\mathcal{P}_{ct}^{FS} = \mathcal{P}_{Sy}^{FS} + \mathcal{P}_{Filt}^{FS} + \mathcal{P}_{Amp}^{FS}. $$

(10)

It is shown that the relationship between $\mathcal{P}_{Amp}^{FS}$ and the transmission power of an MFSK signal is $\mathcal{P}_{Amp}^{FS} = \alpha^{FS}\mathcal{P}_t^{FS}$, where $\alpha^{FS}$ is determined based on the type of the power amplifier. For instance for a class B power amplifier, $\alpha^{FS} = 0.33$. For the circuit power consumption of the sink node, we use the fact that the optimum NC-MFSK demodulator consists of a bank of $M$ matched filters, each followed

\footnote{For the purpose of comparison, the energy efficiency of a coherent MFSK is fully analyzed in Appendix I in [13].}
by an envelope detector [15]. In addition, we assume that the sink node uses a Low-Noise Amplifier (LNA) which is generally placed at the front-end of a RF receiver circuit, an Intermediate-Frequency Amplifier (IFA), and an ADC, regardless of type of deployed modulation. Thus, denoting $\mathcal{P}_{LNA}^{FS}$, $\mathcal{P}_{Firr}^{FS}$, $\mathcal{P}_{IFA}^{FS}$ and $\mathcal{P}_{ADC}^{FS}$ as the power consumption of LNA, filters, envelope detector, IF amplifier and ADC, respectively, the circuit power consumption of the sink node is obtained as

$$\mathcal{P}_{cr}^{FS} = \mathcal{P}_{LNA}^{FS} + M \times (\mathcal{P}_{Firr}^{FS} + \mathcal{P}_{ED}^{FS}) + \mathcal{P}_{IFA}^{FS} + \mathcal{P}_{ADC}^{FS}. \quad (11)$$

Moreover, it is shown that the power consumption during the transient mode period $T_{tr}^{FS}$ is governed by the frequency synthesizer in both sensor/sink nodes [5]. Thus, the energy consumption during $T_{tr}^{FS}$ is obtained as $\frac{\mathcal{P}_{cr}^{FS}}{\chi_e T_{tr}^{FS}} = 2 \frac{\mathcal{P}_{Sy}^{FS}}{\chi_e} T_{tr}^{FS}$ [8]. Substituting (5) and (9) in (2), the total energy consumption of a NC-MFSK scheme for transmitting $N$-bit in each period $T_N$ and for a given $P_s$ is obtained as

$$E^{FS}_N = (1 + \alpha^{FS}) \left( \left[ 1 - (1 - P_s)^{\frac{1}{MN}} \right]^{-1} - 2 \right) \frac{L_d N_0}{\chi_e \Omega} N + \frac{1}{\chi_e} \left[ (\mathcal{P}_{c}^{FS} - \mathcal{P}_{Amp}^{FS}) \frac{MN}{B \log_2 M} + 2 \mathcal{P}_{Sy}^{FS} \frac{T_{tr}^{FS}}{T_{tr}} \right], \quad (12)$$

with the fact that $L_d = M_d T_d L_1$. Thus, the optimization goal is to determine the optimum constellation size $M$, such that the objective function $E^{FS}_N$ can be minimized, i.e.,

$$\hat{M} = \arg \min_M E^{FS}_N \quad \text{subject to} \quad \begin{cases} 2 \leq M \leq M_{\max} \\ \eta_{\min} \leq \eta \leq \eta_{\max} \\ d > 0, \end{cases} \quad (13)$$

where $M_{\max}$ derived from (6). To solve this optimization problem, we prove that (12) is a monotonically increasing function of $M$ for every value of $d$ and $\eta$. It is seen that the second term in (12) is a monotonically increasing function of $M$. Also, from the first term in (12), we have

$$\left( \left[ 1 - (1 - P_s)^{\frac{1}{MN}} \right]^{-1} - 2 \right) \frac{1}{\log_2 M} \approx \left( 1 - e^{-\frac{P_s}{MN}} \right)^{-1} - 2 \frac{1}{\log_2 M} \quad (14)$$

$$\approx \left( 1 - e^{-\frac{P_s}{M}} \right)^{-1} - 2 \frac{1}{\log_2 M} \quad (15)$$

$$\approx \left( \frac{M - 1}{P_s} - 2 \right) \frac{1}{\log_2 M}. \quad (16)$$
where (a) comes from the approximation $\ln(1 - z) \approx -z$, $|z| \ll 1$, and the fact that $P_s$ scales as $o(1)$. Also, (b) follows from the approximation $e^{-z} = \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{n!} \approx 1 - z$, $|z| \ll 1$. It is concluded from (16) that the first term in (12) is also a monotonically increasing function of $M$. As a result, the minimum total energy consumption $E_{FS}^N$ is achieved at $\hat{M} = 2$ for all values of $d$ and $\eta$.

**M-ary QAM:** For $M$-ary QAM with the square constellation, each $b = \log_2 M$ bits of the message is mapped to the symbol $S_i$, $i = 0, 1, ..., M - 1$, with the symbol duration $T_{QA}^S$. Assuming the raised-cosine filter is used for the pulse shaping, the channel bandwidth of MQAM is given by $B \approx 1 \frac{T_{QA}^S}{2T_s}$. Thus, using the data rate $R_{QA} = \frac{b}{T_{QA}^S}$, the bandwidth efficiency of MQAM is obtained as $B_{eff}^{QA} = \frac{R_{QA}}{B} = 2 \log_2 M$ which is a logarithmically increasing function of $M$. To address the impact of $M$ on the energy efficiency, we derive the active mode duration $T_{ac}^{QA}$ in terms of $M$ as follows:

$$T_{ac}^{QA} = \frac{N}{b} T_{QA}^S = \frac{N}{2B \log_2 M}.$$  \hfill (17)

It is seen that an increase in $M$ results in a decrease in $T_{ac}^{QA}$. Also compared to (5) for the NC-MFSK, it is concluded that $T_{ac}^{QA} = \frac{1}{2M} T_{ac}^{FS}$. Interestingly, it seems that the large constellation sizes $M$ would result in the lower energy consumption due to the smaller values of $T_{ac}^{QA}$. However, as we will show later, the total energy consumption of a MQAM is not necessarily a monotonically decreasing function of $M$. For this purpose, we obtain the transmit energy consumption $P_{t}^{QA} T_{ac}^{QA}$ with a similar argument as for MFSK.

It is shown in [16, pp. 226] and [13] that the average SER of a coherent MQAM is upper bounded by

$$P_s = \frac{4(M - 1)}{3 \tilde{\gamma}_{QA} + 2(M - 1)} \left(1 - \frac{1}{\sqrt{M}}\right),$$ \hfill (18)

where $\tilde{\gamma}_{QA} = \Omega \frac{\gamma_{QA}^t}{N_0}$ denotes the average received SNR with the energy per symbol $\gamma_{QA}^t$. As a result,

$$E_{t}^{QA} = P_{t}^{QA} T_{ac}^{QA} = \frac{2(M - 1)}{3} \left[2 \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{P_s} - 1\right] \frac{\mathcal{L}_d N_0}{\Omega}.$$ \hfill (19)

In considering the effect of the DC-DC converter, the output energy consumption of transmitting $N$-bit during the active mode period is computed as

$$P_{t}^{QA} T_{ac}^{QA} \chi_e \frac{E_{t}^{QA}}{T_{ac}^{QA} T_{s}^{QA} \chi_e} = \frac{2(M - 1)}{3} \left[2 \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{P_s} - 1\right] \frac{\mathcal{L}_d N_0}{\chi_e \Omega} \frac{N}{\log_2 M},$$ \hfill (20)
which is a monotonically increasing function of \( M \) for every value of \( P_s, d \) and \( \eta \). For the sensor node with the MQAM modulator,

\[
P_{ct}^{QA} = P_{DAC}^{QA} + P_{Sy}^{QA} + P_{Mix}^{QA} + P_{Amp}^{QA},
\]

(21)

where \( P_{DAC}^{QA} \) and \( P_{Mix}^{QA} \) denote the power consumption of DAC and mixer, respectively. It is shown that \( P_{Amp}^{QA} = \alpha^{QA} P_{ct}^{QA} \), where \( \alpha^{QA} = \frac{\xi}{\eta} - 1 \) with \( \xi = \frac{3\sqrt{M-1}}{\sqrt{M+1}} \) and \( \eta = 0.35 \) [5]. In addition, the circuit power consumption of the sink with the coherent MQAM is obtained as

\[
P_{cr}^{QA} = P_{LNA}^{QA} + P_{Mix}^{QA} + P_{Sy}^{QA} + P_{Filt}^{QA} + P_{IFA}^{QA} + P_{ADC}^{QA}.
\]

(22)

Also, with a similar argument as for MFSK, we assume that the circuit power consumption during transient mode period \( T_{tr}^{QA} \) is governed by the frequency synthesizer. As a result, the total energy consumption of a coherent MQAM for transmitting \( N \)-bit in each period \( T_N \) is obtained as

\[
E_N^{QA} = \left(1 + \alpha^{QA}\right)\frac{2(M-1)}{3} \left[2 \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{P_s} - 1\right] \frac{\mathcal{L}_d N_0}{\chi_e \Omega} \frac{N}{\log_2 M} + \frac{1}{\chi_e} \left[\left(P_{ct}^{QA} - P_{Amp}^{QA}\right) \frac{N}{2B \log_2 M} + 2P_{Sy}^{QA} T_{tr}^{QA}\right].
\]

(23)

Although, there is no constraint on the maximum size \( M \) for MQAM, to make a fair comparison to the MFSK scheme, we use the same \( M_{max} \) as MFSK. Taking this into account, the optimization problem is to determine the optimum \( M \in [4, M_{max}] \) subject to \( d > 0 \) and \( \eta_{min} \leq \eta \leq \eta_{max} \), such that \( E_N^{QA} \) can be minimized.

It is seen that the first term in (23) is a monotonically increasing function of \( M \) for every value of \( P_s, d \) and \( \eta \), while the second term is a monotonically decreasing function of \( M \) which is independent of \( d \) and \( \eta \). For the above optimization and for a given \( P_s \), we have two following scenarios based on the distance \( d \):

**Case 1:** For large values of \( d \) where the first term in (23) is dominant, the objective function \( E_N^{QA} \) is a monotonically increasing function of \( M \) and is minimized at \( M = 4 \), equivalent to the 4-QAM scheme.

**Case 2:** Let assume that \( d \) is small enough. One possible case may happen is when the total energy consumption \( E_N^{QA} \) for small sizes \( M \) is governed by the second term in (23). For this situation, either the
TABLE II

**SYSTEM EVALUATION PARAMETERS**

| Parameter | Value |
|-----------|-------|
| $\chi_e$  | 0.8   |
| $N_0$     | -180 dBm |
| $P_{DAC}$ | 7 mw   |
| $P_{ED}$  | 3 mw   |
| $P_{ADC}$ | 7 mw   |
| $P_{Sy}$  | 10 mw  |
| $P_{Mix}$ | 7 mw   |
| $P_{Tr}$  | 2.5 mw |
| $P_{Filt}$| 2.5 mw |
| $P_{LNA}$ | 9 mw   |
| $P_{Mix}$ | 7 mw   |
| $L_1$     | 30 dB  |
| $P_{Filt}$| 2.5 mw |
| $P_{IF A}$| 3 mw   |
| $\Omega$  | 1 P_{Filt} |
| $P_{LNA}$ | 9 mw   |

The objective function behaves as a monotonically decreasing function of $M$ for every value of $M$, or for a large constellation size $M$, the first term would be dominant, meaning that $\mathcal{E}_N^{QA}$ increases when $M$ grows.

In the former scenario, the optimum $M$ is achieved at $\hat{M} = M_{max}$; whereas in the latter scenario, there exists a minimum value for $\mathcal{E}_N^{QA}$, where the optimum $M$ for this point is obtained by the intersection between the first and second terms, i.e.,

$$(1 + \alpha_{QA}) \frac{2(M - 1)}{3} \left[ 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \frac{1}{P_s} - 1 \right] \frac{L_1 N_0}{\chi_e \Omega \log_2 M} = \frac{1}{\chi_e} \left[ N \left( P_{c}^{QA} - P_{Amp}^{QA} \right) / 2B \log_2 M + 2P_{Sy}^{QA} T_{tr}^{QA} \right].$$

Since, $P_s$ scales as $o(1)$ and ignoring the term $P_{Sy}^{QA} T_{tr}^{QA}$ to simplify our analysis, the optimum $M$ which minimizes (23) is obtained by the following equation:

$$M - 1 - \sqrt{M} + \frac{1}{\sqrt{M}} \approx \phi(d, \eta) \frac{1 + \alpha_{QA}}{1 + \alpha_{QA}}, \quad (24)$$

where $\phi(d, \eta) \triangleq \frac{P_{c}^{QA} - P_{Amp}^{QA}}{2B} \frac{3P_s \Omega}{4L_1 N_0}$ and the fact that $\alpha_{QA}$ is a function of $M$.

To gain more insight to the above optimization problem, we use a specific numerical example with the simulation parameters summarized in Table II. We assume $P_s = 10^{-3}$, $4 \leq M \leq 64$ and $2.5 \leq \eta \leq 6$. Fig. 2 illustrates the total energy consumption of MQAM versus $M$ for different values of $d$. It is seen that $\mathcal{E}_N^{QA}$ exhibits different trends depending on the distance $d$ and the path-loss exponent $\eta$. For instance, for large values of $d$ and $\eta$, $\mathcal{E}_N^{QA}$ is an increasing function of $M$, where the optimum value of $M$ is achieved at $\hat{M} = 4$ as expected. This is because, in this case, the first term in (23) corresponding to the RF signal energy consumption dominates $\mathcal{E}_N^{QA}$. Table III details the optimum values of $M$ which minimizes $\mathcal{E}_N^{QA}$ for

\[3\] For more details in the simulation parameters, we refer the reader to [13] and its references.
some values of $1 \leq d \leq 200 \text{ m}$ and $2.5 \leq \eta \leq 6$. We use these results to compare the energy efficiency of the optimized MQAM with the other schemes in the subsequent sections.

**Offset-QPSK:** For performance comparison, we choose the conventional OQPSK modulation which is used as a reference in the IEEE 802.15.4/ZigBee protocols. We also follow the same differential OQPSK structure mentioned in [6, p. 50] to eliminate the need for a coherent phase reference at the sink node. For this configuration, the channel bandwidth and the data rate are determined by $B \approx \frac{1}{T_{QPSK}}$ and $R_{QPSK} = \frac{2}{T_{QPSK}}$, respectively. As a result, the bandwidth efficiency of OQPSK is obtained as $B_{eff}^{OQ} = \frac{R_{QPSK}}{B} = 2 \text{ (b/s/Hz)}$.

Since we have 2 bits in each symbol period $T_{s}^{OQ}$, it is concluded that

$$T_{ac}^{OQ} = \frac{N}{2} T_{s}^{OQ} = \frac{N}{2B},$$  \hspace{1cm} (25)

Compared to (5) and (17), we have $T_{ac}^{OQ} = (\log_2 M) T_{ac}^{QA}$ with $M \geq 4$, while for the optimized MFSK, $T_{ac}^{OQ} = \frac{1}{4} T_{ac}^{FS}$. More precisely, it is revealed that $T_{ac}^{QA} < T_{ac}^{OQ} < T_{ac}^{FS}$. To determine the transmit energy consumption of the differential OQPSK scheme, we derive $E_{t}^{OQ}$ in terms of the average SER. It is shown in [17] and [13] that the average SER of the differential OQPSK is upper bounded by

$$P_{s} = \sqrt{\frac{1 + \sqrt{2}}{2} \frac{4}{(2 - \sqrt{2}) \tilde{\gamma}_{OQ} + 4}},$$  \hspace{1cm} (26)

where $\tilde{\gamma}_{OQ} = \frac{\eta E_{t}^{OQ}}{L_{d} N_{0}}$. With a similar argument as for MFSK and MQAM, the energy consumption of

| $d$ (m) | $\eta = 2.5$ | $\eta = 3$ | $\eta = 4$ | $\eta = 5$ | $\eta = 6$ |
|----------|-------------|-------------|-------------|-------------|-------------|
| 1        | 64          | 64          | 64          | 64          | 64          |
| 10       | 64          | 64          | 43          | 10          | 4           |
| 20       | 64          | 50          | 8           | 4           | 4           |
| 40       | 43          | 13          | 4           | 4           | 4           |
| 80       | 14          | 5           | 4           | 4           | 4           |
| 100      | 10          | 4           | 4           | 4           | 4           |
| 150      | 6           | 4           | 4           | 4           | 4           |
| 200      | 5           | 4           | 4           | 4           | 4           |

**TABLE III**

OPTIMUM $M$ IN THE MQAM OPTIMIZATION PROBLEM FOR $P_{s} = 10^{-3}$ AND DIFFERENT VALUES OF $d$ AND $2.5 \leq \eta \leq 6$. 

| $d$ (m) | $\eta = 2.5$ | $\eta = 3$ | $\eta = 4$ | $\eta = 5$ | $\eta = 6$ |
|----------|-------------|-------------|-------------|-------------|-------------|
| 1        | 64          | 64          | 64          | 64          | 64          |
| 10       | 64          | 64          | 43          | 10          | 4           |
| 20       | 64          | 50          | 8           | 4           | 4           |
| 40       | 43          | 13          | 4           | 4           | 4           |
| 80       | 14          | 5           | 4           | 4           | 4           |
| 100      | 10          | 4           | 4           | 4           | 4           |
| 150      | 6           | 4           | 4           | 4           | 4           |
| 200      | 5           | 4           | 4           | 4           | 4           |
Fig. 2. Total energy consumption $\mathcal{E}_N^{QA}$ vs. $M$ over a Rayleigh fading channel with path-loss for $P_s = 10^{-3}$, a) $\eta = 2.5$, and b) $\eta = 3.5$.

transmitting $N$-bit during $T_{ac}^{OOQ}$ is computed as

$$
\frac{P_{t}^{OOQ}}{\chi_e} T_{ac}^{OOQ} = \frac{T_{ac}^{OOQ} \mathcal{E}_t^{OOQ}}{T_s^{OOQ} \chi_e} = \left[ \frac{1}{2 - \sqrt{2}} \left( \frac{4}{P_s} \sqrt{\frac{1 + \sqrt{2}}{2}} - 4 \right) \right] \frac{L_d N_0 N}{\chi_e \Omega}.
$$

(27)

In addition, for the sensor node with the OQPSK modulator, $P_{ct}^{OOQ} \approx P_{DAC}^{OOQ} + P_{Sy}^{OOQ} + P_{Mix}^{OOQ} + P_{Filt}^{OOQ} + P_{Amp}^{OOQ},$
where we assume that the power consumption of the differential encoder is negligible, and $P_{Amp}^{OQ} = \alpha_{OQ} P_t^{OQ}$, with $\alpha_{OQ} = 0.33$. In addition, the circuit power consumption of the sink with the differential detection OQPSK is obtained as $P_{cr}^{OQ} = P_{LNA}^{OQ} + P_{Mix}^{OQ} + P_{Sy}^{OQ} + P_{IF}^{OQ} + P_{ADC}^{OQ}$. As a result, the total energy consumption of a differential OQPSK system for transmitting $N$-bit in each period $T_N$ is obtained as

$$E_N^{OQ} = (1 + \alpha_{OQ}) \left[ \frac{1}{2 - \sqrt{2}} \left( \frac{4}{P_s} \sqrt{1 + \frac{2}{2}} - 4 \right) \right] \frac{L_d N_0 N}{\chi_e \Omega} + \frac{1}{\chi_e} \left[ (P_c^{OQ} - P_{Amp}^{OQ}) \frac{N}{2B} + 2P_{Sy}^{OQ} T_{tr}^{OQ} \right].$$

(28)

**IV. Numerical Results**

In this section, we present some numerical evaluations using realistic parameters from the IEEE 802.15.4 standard and state-of-the-art technology to confirm the energy efficiency analysis discussed in Section III. We assume that all the modulations operate in the carrier frequency $f_0 = 2.4$ GHz Industrial Scientist and Medical (ISM) unlicensed band utilized in the IEEE 802.15.4 standard [6]. According to the FCC 15.247 RSS-210 standard for United States/Canada, the maximum allowed antenna gain is 6 dBi [18]. In this work, we assume that $G_t = G_r = 5$ dBi. Thus for the $f_0 = 2.4$ GHz, $L_1$ (dB) $\triangleq 10 \log_{10} \left( \frac{(4\pi)^2}{\mu \omega \lambda^2} \right) \approx 30$ dB, where $\lambda \triangleq \frac{3 \times 10^8}{f_0} = 0.125$ m. We assume that in each period $T_N$, the data frame $N = 1024$ bytes (or equivalently $N = 8192$ bits) is generated for transmission for all the modulations, where $T_N$ is assumed to be 1.4 s. The channel bandwidth is set to the $B = 62.5$ KHz, according to the IEEE 802.15.4 standard [6, p. 49]. In addition, we assume that the path-loss exponent is in the range of 2.5 to 6. We use the system parameters summarized in Table II for simulations. It is concluded from $M_{max} = \frac{\log_2 M_{max}}{\log_2 M_{max}} = \frac{\zeta B}{N} (T_N - T_{tr}^FS)$ that $M_{max} \approx 64$ (or equivalently $b_{max} \approx 6$) for NC-MFSK. Since, there is no constraint on the maximum $M$ in MQAM, we choose $4 \leq M \leq 64$ for MQAM to be consistent with MFSK.

Fig. 3 compares the energy efficiency of the modulation schemes investigated in Section III versus $M$ for $P_s = 10^{-3}$, $\eta = 3.5$ and different values of $d$. It is revealed from Fig. 3-a that for $M < 35$, NC-MFSK $^4 \eta = 2$ is regarded as a reference state for the propagation in free space and is unattainable in practice. Also, $\eta = 4$ is for relatively lossy environments, and for indoor environments, the path-loss exponent can reach values in the range of 4 to 6.
Fig. 3. Total energy consumption of transmitting $N$-bit vs. $M$ for MFSK, MQAM and differential OQPSK over a Rayleigh fading channel with path-loss and $P_s = 10^{-3}$, a) $d = 10$ m, and b) $d = 100$ m.
is more energy-efficient than MQAM, differential OQPSK and coherent MFSK for \( d = 10 \text{ m} \) and \( \eta = 3.5 \), while when \( M \) grows, 64-QAM outperforms the other schemes for \( d = 10 \text{ m} \). The latter result is well supported by the Case 2 in the MQAM optimization discussed in Section III. However, NC-MFSK for a small size \( M \) benefits from the advantage of less complexity and cost in implementation than 64-QAM. Furthermore, as shown in Fig. 3-b, the total energy consumption of both MFSK and MQAM for large \( d \) increase logarithmically with \( M \) which verify the optimization solutions for the NC-MFSK and Case 1 for the MQAM in Section III. Also, it is seen that NC-MFSK exhibits the energy efficiency better than the other schemes when \( d \) increases.

The optimized modulations for different transmission distance \( d \) and \( 2.5 \leq \eta \leq 6 \) are listed in Table IV. For these results, we use the optimized MQAM detailed in Table III and the fact that for NC-MFSK, \( \hat{M} = 2 \) is the optimum value which minimizes \( \mathcal{E}_N^{FS} \) for every \( d \) and \( \eta \). From Table IV, it is found that although 64-QAM outperforms NC-BFSK for very short range WSNs, it should be noted that using MQAM with a large constellation size \( M \) increases the complexity of the system. In particular, when we know that MQAM utilizes the coherent detection at the sink node. In other words, there exists a trade-off between the complexity and the energy efficiency in using MQAM for small values of \( d \).

Up to know, we have investigated the energy efficiency of the sinusoidal carrier-based modulations

### TABLE IV

| \( d \) (m) | \( \eta = 2.5 \) | \( \eta = 3 \) | \( \eta = 4 \) | \( \eta = 5 \) | \( \eta = 6 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 1         | 64QAM     | 64QAM     | 64QAM     | 64QAM     | 64QAM     |
| 10        | 64QAM     | 64QAM     | NC-BFSK   | NC-BFSK   | NC-BFSK   |
| 20        | 64QAM     | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   |
| 40        | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   |
| 80        | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   |
| 100       | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   |
| 150       | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   |
| 200       | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   | NC-BFSK   |
TABLE V
TOTAL ENERGY CONSUMPTION (IN JOULE) OF NC-MFSK, MQAM AND OQPSK OVER A RICIAN FADEING CHANNEL WITH PATH-LOSS FOR $P_s = 10^{-3}$ AND $\eta = 3.5$

| $K$ = 1 dB | $K$ = 10 dB | $K$ = 15 dB |
|----------|-------------|-------------|
| $M$       | OQPSK      | NC-MFSK    | MQAM      | OQPSK      | NC-MFSK    | MQAM      | OQPSK      | NC-MFSK    | MQAM      |
| d=10 m    |             |             |            |             |             |            |             |             |            |           |
| 4         | 1.1241     | 0.0173      | 0.5621     | 1.1241     | 0.0171      | 0.5620     | 1.1241     | 0.0171      | 0.5620     |
| 16        | 0.6558     | 0.1924      |            |             |             |            |             |             |            |           |
| 64        |             |             |            |             |             |            |             |             |            |           |
| d=100 m   |             |             |            |             |             |            |             |             |            |           |
| 4         | 1.2236     | 0.5835      | 0.8873     | 1.1445     | 0.0194      | 0.5652     | 1.1310     | 0.0175      | 0.5627     |
| 16        | 0.6570     | 0.2615      |            |             |             |            |             |             |            |           |
| 64        |             |             |            |             |             |            |             |             |            |           |

under the assumption of a Rayleigh fading channel with path-loss. It is also of interest to evaluate the energy efficiency of the aforementioned modulation schemes operating over the more general Rician model which includes the LOS path. For this purpose, let assume that the instantaneous channel coefficient correspond to symbol $i$ is $G_i = \frac{h_i}{\xi_d}$, where $h_i$ is assumed to be Rician distributed with pdf $f_{h_i}(r) = \frac{r}{\sigma^2}e^{-\frac{r^2 + A^2}{2\sigma^2}}I_0\left(\frac{rA}{\sigma^2}\right)$, $r \geq 0$, where $A$ denotes the peak amplitude of the dominant signal, $2\sigma^2 \triangleq \Omega$ is the average power of non-LOS multipath components [19, p. 78]. For this model, $\mathbb{E}[|h_i|^2] = A^2 + 2\sigma^2 = 2\sigma^2(1 + K)$, where $K(\text{dB}) \triangleq 10 \log \frac{A^2}{2\sigma^2}$ is the Rician factor. The value of $K$ is a measure of the severity of the fading. For instance, $K(\text{dB}) \to -\infty$ implies Rayleigh fading and $K(\text{dB}) \to \infty$ represents AWGN channel. Table V summarized the energy efficiency results of the previous modulations over a Rician fading channel with path-loss for $P_s = 10^{-3}$ and $\eta = 3.5$. It is seen from Table V that NC-MFSK with a small size $M$ has less total energy consumption than the other schemes in Rician fading channel with path-loss.

The above results make NC-MFSK with a small $M$ attractive for using in WSNs, since this modulation already has the advantage of less complexity and cost in implementation than MQAM, differential OQPSK and coherent MFSK, and has less total energy consumption. In addition, since for energy-constrained WSNs, data rates are usually low, using M-ary NC-FSK schemes with a small $M$ are desirable. The
sacrifice, however, is the bandwidth efficiency of NC-MFSK (when $M$ increases) which is a critical factor in band-limited WSNs. Since most of WSN applications operate in unlicensed bands where large bandwidth is available, NC-MFSK can surpass the spectrum constraint in WSNs. To have more insight into the above discussions for the NC-MFSK, we plot the total energy consumption of NC-MFSK as a function of $B_{FS_{eff}}$ for different values of $M$ and $d$ in Fig. 4. In all cases, we observe that the minimum $\mathcal{E}_{FS_{N}}$ is achieved at low values of distance $d$ and for $M = 2$, which corresponds to the maximum bandwidth efficiency $B_{FS_{eff}}^M = 0.5$.

The analysis and numerical evaluations so far implicitly focused on the sinusoidal carrier-based modulations with the bandwidth $B = 62.5$ KHz. To complete our analysis, it is of interest to compare the energy efficiency of the optimized NC-MFSK with the On-Off Keying (OOK), known as the simplest UWB modulation scheme. For this purpose, we first derive the total energy consumption of OOK with a similar manner as for sinusoidal carrier-based modulations. Then, we evaluate the energy efficiency of OOK in terms of distance $d$. For simplicity of the notation, we use the superscript ‘OK’ for OOK modulation scheme.

![Diagram](image-url)
V. Energy Consumption Analysis of OOK

For OOK, the number of bits per symbol is defined as \( b = \log_2 M = 1 \). An OOK transmitted signal corresponding to the symbol \( a_i \in \mathbb{M}_N \) is given by \( x_{i}^{\text{OK}}(t) = \sqrt{\mathcal{E}_t^{\text{OK}}} a_i p(t - iT_s^{\text{OK}}) \), where \( p(t) \) is an ultra-short pulse of width \( T_p \) with unit energy, \( \mathcal{E}_t^{\text{OK}} \) is the transmit energy consumption per symbol, and \( T_s^{\text{OK}} \) is the OOK symbol duration. The ratio \( \frac{T_p}{T_s^{\text{OK}}} \) is defined as the duty-cycle factor of an OOK signal, which is the fractional on-time of the OOK “1” pulse. The channel bandwidth and the data rate of an OOK are determined as \( B \approx \frac{1}{T_p} \) and \( R^{\text{OK}} = \frac{1}{T_s^{\text{OK}}} \), respectively. As a result, the bandwidth efficiency of an OOK is obtained as \( B_{\text{eff}}^{\text{OK}} \triangleq \frac{R^{\text{OK}}}{B} = \frac{T_p}{T_s^{\text{OK}}} \leq 1 \) (b/s/Hz) which controls by the duty-cycle factor.

Note that during the transmission of the OOK “0” pulse, the filter and the power amplifier of the OOK modulator are powered off. During this time, however, the receiver is turned on to detect zero pulses. For this reason, we still use the same definition for active mode period \( T_{ac}^{\text{OK}} \) as used for the sinusoidal carrier-based modulations as follows:

\[
T_{ac}^{\text{OK}} = \frac{N}{b} T_s^{\text{OK}} = NT_s^{\text{OK}}.
\]  

(29)

Depend upon the duty-cycle factor, \( T_{ac}^{\text{OK}} \) can be expressed in terms of bandwidth \( B \). For instance, for an OOK with the duty-cycle factor \( \frac{T_p}{T_s^{\text{OK}}} = \frac{1}{2} \), we have \( T_{ac}^{\text{OK}} = 2NT_p = \frac{2N}{B} \), and \( B_{\text{eff}}^{\text{OK}} = \frac{1}{2} \). Compared to (4) for the NC-MFSK, it is concluded that \( B_{\text{eff}}^{\text{OK}} \) with the duty-cycle factor \( \frac{T_p}{T_s^{\text{OK}}} = \frac{1}{2} \) is the same as that of the optimized MFSK (i.e., \( \hat{M} = 2 \)) in Section III. It is shown in [20, pp. 490-504] that the average SER of an OOK with non-coherent detection is upper bounded by

\[
P_s = \frac{1}{\bar{\gamma}_{\text{OK}} + 2},
\]

(30)

where \( \bar{\gamma}_{\text{OK}} = \frac{\Omega \ell_s^{\text{OK}}}{N_0} \) denotes the average received SNR. Thus, the transmit energy consumption per symbol is obtained as \( \mathcal{E}_t^{\text{OK}} \triangleq \mathcal{P}_t^{\text{OK}} T_p = \left( \frac{1}{T_s^{\text{OK}} - 2} \right) \frac{\ell_s N_0}{\Omega} \) which corresponds to transmitting “1” pulse. An interesting point is that \( \mathcal{E}_t^{\text{OK}} \) scales the same as \( \mathcal{E}_t^{\text{FS}} \) obtained in [8] for NC-BFSK. This can be easily proved by using the approximation method in (14)-(16). It should be noted that the energy consumption of transmitting \( N \)-bit during the active mode period, denoted by \( \frac{T_{ac}^{\text{OK}}}{\chi_e} T_{ac}^{\text{OK}} \), is equivalent to the energy consumption of transmitting \( L \)-bit “1” in \( \mathbb{M}_N \), where \( L \) is a binomial random variable with parameters
TABLE VI
OOK SYSTEM EVALUATION PARAMETERS

| Parameter | Value |
|-----------|-------|
| N         | 20000 |
| B         | 500 MHz |
| M         | 40 dB |
| L         | 30 dB |
| χ_e       | 0.8 |
| N_0       | -180 dB |
| P_{PG}    | 675 µw |
| P_{LNA}   | 3.1 mw |
| P_{ED}    | 3 mw |
| P_{ADC}   | 7 mw |
| P_{Int}   | 3 mw |
| P_{filt}  | 2.5 mw |
| P_{LNA}   | 3.1 mw |
| P_{ED}    | 3 mw |
| P_{ADC}   | 7 mw |
| P_{Int}   | 3 mw |
| P_{filt}  | 2.5 mw |
| P_{LNA}   | 3.1 mw |
| P_{ED}    | 3 mw |
| P_{ADC}   | 7 mw |
| P_{Int}   | 3 mw |

\( (N, q) \). Assuming uncorrelated and equally likely binary data \( a_i \), we have \( q = \frac{1}{2} \). Hence, \( \frac{P_{OK}^t}{\chi_e} T_{ac}^OK = \frac{L}{\chi_e} \mathcal{E}_t^OK = L \left( \frac{1}{P_s} - 2 \right) \frac{L_{dN_0}}{\chi_e \Omega} \) \( \) \( \) , where \( L \) has the probability mass function \( \Pr \{ L = \ell \} = \binom{N}{\ell} \left( \frac{1}{2} \right)^N \) with \( \mathbb{E}[L] = \frac{N}{2} \).

We denote the power consumption of pulse generator, power amplifier and filter as \( P_{PG}^OK, P_{Amp}^OK \) and \( P_{Filt}^OK \), respectively. Hence, the circuit energy consumption of the sensor node during \( T_{ac}^OK \) is represented as a function of the random variable \( L \) as \( P_{ct}^OK T_{ac}^OK = P_{PG}^OK T_{ac}^OK + LT_p (P_{filt}^OK + P_{Amp}^OK) \), where the factor \( LT_p \) comes from the fact that the filter and the power amplifier are active only during the transmission of \( L \)-bit “1”. We assume that \( P_{Amp}^OK = \alpha^OK P_t^OK \) with \( \alpha^OK = 0.33 \). In addition, the circuit energy consumption of the sink node with a non-coherent detection during \( T_{ac}^OK \) is obtained as \( P_{cr}^OK T_{ac}^OK = (P_{LNA}^OK + P_{ED}^OK + P_{Filt}^OK + P_{Int}^OK + P_{ADC}^OK) T_{ac}^OK \), where \( P_{Int}^OK \) is the power consumption of the integrator.

With a similar argument as for the sinusoidal carrier-based modulations, we assume that the circuit power consumption during \( T_{tr}^OK \) is governed by the pulse generator. As a result, the total energy consumption of a non-coherent OOK for transmitting \( N \)-bit is obtained as a function of the random variable \( L \) as follows:

\[
\mathcal{E}_N^OK(L) = (1 + \alpha^OK) L \left( \frac{1}{P_s} - 2 \right) \frac{L_{dN_0}}{\chi_e \Omega} + \frac{1}{\chi_e} \left[ (P_{cr}^OK + P_{PG}^OK) \frac{2N}{B} + \frac{L}{B} P_{filt}^OK + 2P_{PG}^OK T_{tr}^OK \right],
\]

where we use \( T_{ac}^OK = \frac{2N}{B} \). Since \( \mathbb{E}[L] = \frac{N}{2} \), the average \( \mathcal{E}_N^OK(L) \) is computed as

\[
\mathcal{E}_N^OK \triangleq \mathbb{E}[\mathcal{E}_N^OK(L)] = (1 + \alpha^OK) \left( \frac{1}{P_s} - 2 \right) \frac{L_{dN_0} N}{\chi_e \Omega} + \frac{1}{\chi_e} \left[ (P_{cr}^OK + P_{PG}^OK) \frac{2N}{B} + \frac{N}{2B} P_{filt}^OK + 2P_{PG}^OK T_{tr}^OK \right].
\]

It should be noted that the OOK scheme uses the channel bandwidth much wider than that of the
sinusoidal carrier-based modulations. Thus, to make a fair comparison with the optimized NC-MFSK, it is reasonable to use the total energy consumption per information bit defined as $E_b \triangleq \frac{E}{N}$, instead of using $E_N$. In our comparison, we use the simulation parameters shown in Table VI [13]. Fig. 5 compares the total energy consumption per information bit of the OOK with that of the optimized NC-MFSK versus communication range $d$ for $P_s = 10^{-3}$ and different values of $\eta$. We observe that a significant energy saving is achieved using OOK as compared to the optimized NC-MFSK, when $d$ and $\eta$ decrease. While, for the indoor environments where $\eta$ is large, the performance difference between OOK and the optimized NC-MFSK vanishes as $d$ increases. This is because, $i$) the transmission energy consumption in the active mode period is dominant when $d$ increases, $ii$) $E_{O^K}$ scales the same as $E_{F^K}$ obtained in (8) using (16). Since, UWB modulation schemes use the channel bandwidth much wider than that of the sinusoidal carrier-based modulations, the optimized NC-MFSK is desirable in use for the band-limited and sparse WSNs where the path-loss exponent is large.

VI. Conclusion

In this paper, we have analyzed the energy efficiency of some popular modulation schemes to find the distance-based green modulations in a WSN over Rayleigh and Rician flat-fading channels with path-loss. It was demonstrated that among various sinusoidal carrier-based modulations, the optimized NC-MFSK is the most energy-efficient scheme in sparse WSNs for each value of the path-loss exponent, where the optimization is performed over the modulation parameters. In addition, NC-MFSK with a small $M$ is attractive for using in WSNs, since this modulation already has the advantage of less complexity and cost in implementation than MQAM, differential OQPSK and coherent MFSK, and has less total energy consumption. Furthermore, MFSK has a faster start-up time than other schemes. Moreover, since for energy-constrained WSNs, data rates are usually low, using M-ary NC-FSK schemes with a small $M$ are desirable. The sacrifice, however, is the bandwidth efficiency of NC-MFSK when $M$ increases. Since most of WSN applications requires low to moderate bandwidth, a loss in the bandwidth efficiency can be tolerable, in particular for the unlicensed band applications where large bandwidth is available. It also
found that OOK has a significant energy saving as compared to the optimized NC-MFSK in dense WSNs with small values of path-loss exponent. While, for the indoor environments where the path-loss exponent is large, the performance difference between OOK and the optimized NC-MFSK vanishes as the distance between the sensor and sink nodes increases. In this case, the optimized NC-MFSK is attractive in use for the band-limited and sparse indoor WSNs.

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