Abstract

Hadronic $B$ decays containing an even-parity charmed meson in the final state are studied. Specifically we focus on the Cabibbo-allowed decays $\bar{B} \to D^{**}\pi(\rho)$, $D^{**}\bar{D}_s^*(s)$, $\bar{D}_s^{**}D(s)$ and $\bar{B}_s \to D_s^{**}\pi(\rho)$, where $D^{**}$ denotes generically a $p$-wave charmed meson. The $B \to D^{**}$ transition form factors are studied in the improved version of the Isgur-Scora-Grinstein-Wise quark model. We apply heavy quark effective theory and chiral symmetry to study the strong decays of $p$-wave charmed mesons and determine the magnitude of the $D_1^{1/2}/D_1^{3/2}$ mixing angle (the superscript standing for the total angular momentum of the light quark). Except the decay to $D_1^{1/2}(2427)\pi^-$ the predictions of $B^-(B^+ \to D^{**0}\pi^-)$ agree with experiment. The sign of the $D_1^{1/2}/D_1^{3/2}$ mixing angle is found to be positive in order to avoid a severe suppression on the production of $D_1^{1/2}(2427)\pi^-$. The interference between color-allowed and color-suppressed tree amplitudes is expected to be destructive in the decay $B^- \to D_1^{1/2}(2427)\rho^-$. Hence, an observation of the ratio $D_1^{1/2}(2427)\rho^-/D_1^{1/2}(2427)\pi^-$ exceeds the present upper limit, it leads to the ratio $D_1^{1/2}(2427)\rho^-/D_1^{1/2}(2427)\pi^- \approx 2.6$ as expected from the factorization approach and from the ratio $f_\rho/f_\pi \approx 1.6$. Therefore, it is crucial to have a measurement of this mode to test the factorization hypothesis. For $\bar{B} \to D_s^{**}D$ decays, it is expected that $\overline{D}_{s0}^* D \geq \overline{D}_{s1}^* D$ as the decay constants of the multiplet $(D_{s0}^*, D_{s1}^*)$ become identical in the heavy quark limit. The experimental observation of less abundant production of $\overline{D}_{s0}^* D$ than $\overline{D}_{s1}^* D$ is thus a surprise. What is the cause for the discrepancy between theory and experiment remains unclear. Meanwhile, it is also important to measure the $B$ decay into $\overline{D}_{s1}(2536)D^{(*)}$ to see if it is suppressed relative to $\overline{D}_{s1}(2460)D^{(*)}$ to test the heavy quark symmetry relation $f_{\overline{D}_{s1}(2536)} \ll f_{\overline{D}_{s1}(2460)}$. Under the factorization hypothesis, the production of $\overline{D}_{s2}^* D$ is prohibited as the tensor meson cannot be produced from the $V-A$ current. Nevertheless, it can be induced via final-state interactions or nonfactorizable contributions and hence an observation of $\overline{B} \to \overline{D}_{s2}^* D^{(*)}$ could imply the importance of final-state rescattering effects.
I. INTRODUCTION

The interest in even-parity charmed mesons is revived by a recent discovery of a new narrow resonance by BaBar [1]. This state which can be identified with \( J^P = 0^+ \) is lighter than most theoretical predictions for a \( 0^+ c\bar{s} \) state [2]. Moreover, a renewed lattice calculation [3] yields a larger mass than what is observed. This unexpected and surprising disparity between theory and experiment has sparked a flurry of many theory papers. For example, it has been advocated that this new state is a four-quark bound state [4,5],\(^*\) or a \( D_K \) molecular [9] or even a \( D\pi \) atom [10]. On the contrary, it has been put forward that, based on heavy quark effective theory and chiral perturbation theory, the newly observed \( D_s(2317) \) is the \( 0^+ c\bar{s} \) state and that there is a \( 1^+ \) chiral partner with the same mass splitting with respect to the \( 1^- \) state as that between the \( 0^+ \) and \( 0^- \) states [11,12]. The existence of a new narrow resonance with a mass near 2.46 GeV which can be identified with \( 1^+ \) state was first hinted by BaBar and has been observed and established by CLEO [13] and Belle [14].

Although the \( D^*_s(2317) \) and \( D_{s1}(2460) \) states were discovered in charm fragmentation of \( e^+e^- \rightarrow c\bar{c} \), it will be much more difficult to measure the counterpart of \( D^*_s(2317) \) and \( D_{s1}(2460) \) in the non-strange charm sector, namely \( D_2^0 \) and \( D_1 \), owing to their large widths. Indeed, the broad \( D_2^0 \) and \( D_1 \) resonances were explored by Belle [14] in charged \( B \) to \( D^+\pi^-\pi^- \) and \( D^{*+}\pi^-\pi^- \) decays (see Table I). The study of even-parity charmed meson production in \( B \) decays, which is the main object of this paper, also provides an opportunity to test heavy quark effective theory.

This work is organized as follows. The masses and widths of \( p^- \)wave charmed mesons are summarized in Sec. II. In order to determine the mixing angle of the axial-vector charmed mesons, we apply heavy quark effective theory and chiral symmetry to study their strong decays. The decay constants of \( p^- \)wave charmed mesons and \( B \rightarrow D^{**} \) form factors are

\(^*\)The low-lying non-charm scalar mesons in the conventional \( qq' \) states are predicted by the quark potential model to lie in between 1 and 2 GeV, corresponding to the nonet states \( f_0(1370), a_0(1450), K_0^*(1430) \) and \( f_0(1500)/f_0(1710) \). The light scalar nonet formed by \( \sigma(600), \kappa(800), f_0(980) \) and \( a_0(980) \) can be identified primarily as four-quark states [6]. It has been argued [7] that a strong attraction between \( (qq)_3^* \) and \( (q\bar{q})_3 \) [6,8], where \( 3^* \) and \( 3 \) here refer to color, and the absence of the orbital angular momentum barrier in the \( s^- \)wave 4-quark state, may explain why the scalar nonet formed by four-quark bound states is lighter than the conventional \( q\bar{q} \) nonet. By the same token, it is likely that a scalar \( cn\bar{n}c \) 4-quark state, where \( n = u, \bar{d} \), will be lighter than the \( 0^+ \) \( p^- \)wave \( c\bar{s} \) state, where a typical potential model prediction gives 2487 MeV [2]. It has been suggested in [4] to search for exotic 4-quark \( cqq\bar{q} \) charmed meson production in \( B \) decays. Particularly noteworthy are resonances in the doubly charged \( D^+_s(1^+) \) and \( D^+K^+ \), and wrong pairing \( D^+K^- \) channels. However, contrary to the case of scalar resonances, the \( 1^+ D^s(2460) \) state is unlikely a four-quark state as it is heavier than the axial-vector meson formed by \( c\bar{s} \). A non-observation of a heavier and broad \( 0^+ c\bar{s} \) state will not support the four-quark interpretation of \( D^s(2317) \).
studied in Sec. III within the Isgur-Scora-Grinstein-Wise quark model. The production of 
$p$-wave charmed mesons in $B$ decays is studied in detail in Sec. IV. Conclusions are presented
in Sec. V.

TABLE I. The masses and decay widths of even-parity charmed mesons. We follow the naming
scheme of Particle Data Group [15] to add a superscript $^{*+}$ to the states if the spin-parity is in
the “normal” sense, $J^P = 0^+, 1^-, 2^+, \cdots$. The four $p$-wave charmed meson states are thus denoted
by $D_0^*$, $D_1$, $D_1'$ and $D_2^*$. In the heavy quark limit, $D_1$ has $j = 1/2$ and $D_1'$ has
$j = 3/2$ with $j$ being the total angular momentum of the light degrees of freedom.

| State     | Mass (MeV)   | Width (MeV) | Reference |
|-----------|--------------|-------------|-----------|
| $D_0^*(2308)^0$ | 2308 ± 17 ± 15 ± 28 | 276 ± 21 ± 18 ± 60 | [14] |
| $D_1(2427)^0$ | 2427 ± 26 ± 20 ± 15 | 384^{+107}_{-75} ± 24 ± 70 | [14] |
| $D_1'(2420)^0$ | 2422.2 ± 1.8 | 18.9^{+4.6}_{-3.5} | [15] |
| $D_1'(2420)^{\pm}$ | 2421.4 ± 1.5 ± 0.4 ± 0.8 | 23.7 ± 2.7 ± 0.2 ± 4.0 | [14] |
| $D_2^*(2460)^0$ | 2427 ± 5 | 28 ± 8 | [15] |
| $D_2^*(2460)^{\pm}$ | 2458.9 ± 2.0 | 23 ± 5 | [15] |
| $D_2^*(2473)^{0}$ | 2461.6 ± 2.1 ± 0.5 ± 3.3 | 45.6 ± 4.4 ± 6.5 ± 1.6 | [14] |
| $D_2^*(2573)^{0}$ | 2459 ± 4 | 25^{+8}_{-1} | [15] |
| $D_0^*(2317)$ | 2317.3 ± 0.4 | < 7 | [1,13,14] |
| $D_{s1}(2460)$ | 2458.9 ± 1.2 | < 7 | [13,14] |
| $D_{s1}'(2536)$ | 2535.35 ± 0.34 ± 0.5 | < 2.3 | [15] |
| $D_{s2}'(2573)$ | 2572.4 ± 1.5 | 15^{+5}_{-4} | [15] |

II. MASS SPECTRUM AND DECAY WIDTH

In the quark model, the even-parity mesons are conventionally classified according to the
quantum numbers $J, L, S$: the scalar and tensor mesons correspond to $^{2S+1}L_J = ^3P_0$ and
$^3P_2$, respectively, and there exist two different axial-vector meson states, namely, $^1P_1$ and
$^3P_1$ which can undergo mixing if the two constituent quarks do not have the same masses.
For heavy mesons, the heavy quark spin $S_Q$ decouples from the other degrees of freedom in
the heavy quark limit, so that $S_Q$ and the total angular momentum of the light quark $j$ are
separately good quantum numbers. The total angular momentum $J$ of the meson is given
by $\vec{J} = \vec{j} + \vec{S}_Q$ with $\vec{S} = \vec{s} + \vec{S}_Q$ being the total spin angular momentum. Consequently, it is
more natural to use $L^j_J = P_0^{3/2}, P_1^{3/2}, P_1^{1/2}$ and $P_0^{1/2}$ to classify the first excited heavy meson
states where $L$ here is the orbital angular momentum of the light quark. It is obvious that
the first and last of these states are $^3P_2$ and $^3P_0$, while [17]

$$|P_0^{3/2}\rangle = \sqrt{\frac{2}{3}}|^{1}P_1\rangle + \sqrt{\frac{1}{3}}|^{3}P_1\rangle, \quad |P_1^{3/2}\rangle = -\sqrt{\frac{1}{3}}|^{1}P_1\rangle + \sqrt{\frac{2}{3}}|^{3}P_1\rangle. \quad (2.1)$$
In the heavy quark limit, the physical eigenstates with \( J^P = 1^+ \) are \( P_1^{3/2} \) and \( P_1^{1/2} \) rather than \( 3P_1 \) and \( 1P_1 \).

The masses and decay widths of even-parity (or p-wave)\(^\dagger\) charmed mesons \( D_J^* \) and \( D_{sJ}^* \) are summarized in Table I. We shall use \( 1^+ \) and \( 1'^+ \) or \( D_1 \) and \( D_1' \) to distinguish between two different physical axial-vector charmed meson states. As we shall see below, the physical \( 1^+ \) state is primarily \( P_1^{1/2} \), while \( 1'^+ \) is predominately \( P_1^{3/2} \). A similar broad \( D_1 \) state not listed in Table I was reported by CLEO [18] with \( M = 2461^{+42}_{-51} \) MeV and \( \Gamma = 290^{+102}_{-83} \) MeV. For the known narrow resonances, the Belle’s measurement of the \( D_2^* \) width (see Table I) is substantially higher than the current world average of \( 23 \pm 5 \) MeV [15].

In the heavy quark limit, the states within the chiral doublets \( (0^+, 1^+) \) with \( j = 1/2 \) and \( (1'^+, 2^+) \) with \( j = 3/2 \) are degenerate. After spontaneous chiral symmetry breaking, \( 0^+ \) states acquire masses while \( 0^- \) states become massless Goldstone bosons. As shown in [11], the fine splitting between \( 0^+ \) and \( 0^- \) is proportional to the constituent quark mass. The hyperfine mass splittings of the four \( p^- \) wave charmed meson states arise from spin-orbit interactions and tensor-force interaction [see Eq. (2.21) below], while the spin-spin interaction is solely responsible for the hyperfine splitting within the multiplet \( (0^-, 1^-) \).

From Table I and the given masses of pseudoscalar and vector charmed mesons in the PDG [15], it is found empirically that the hyperfine splitting within the chiral multiplets \( (0^+, 1^+) \), \( (1'^+, 2^+) \) and \( (0^-, 1^-) \) are independent of the flavor of the light quark:

\[
m(D_s^*) - m(D_s^{'*}) \approx \frac{3}{2} m(D_s) = 430 \text{ MeV},
\]

\[
m(D_{s1}) - m(D_{s0}) \approx m(D_{s1}' - m(D_s) = 144 \text{ MeV},
\]

\[
m(D_{s1}) - m(D_{s0}) \approx m(D_{s1}' - m(D_s) = 143 \text{ MeV}.
\]

However, the fine splittings

\[
m(D_{s0}^*) - m(D_s) \approx m(D_{s1}) - m(D_s^*) \approx 350 \text{ MeV},
\]

\[
m(D_{s0}^*) - m(D_s) \approx m(D_{s1}) - m(D_s^*) \approx 430 \text{ MeV},
\]

depend on the light quark flavor.\(^\ddagger\) Since the fine splitting between \( 0^+ \) and \( 0^- \) or \( 1^+ \) and \( 1^- \) should be heavy-flavor independent in heavy quark limit, the experimental result (2.3)

\(^\dagger\)If the even-parity mesons are the bound states of four quarks, they are in an orbital \( s^- \) wave. In this case, one uses \( J^P = 0^+ \) rather \( 3P_0 \) to denote scalar mesons, for example.

\(^\ddagger\)Likewise, considering the spin-averaged masses of the doublets \( (0^+, 1^+) \) and \( (1'^+, 2^+) \)

\[
\overline{m}_{01}(D) \equiv \frac{1}{4} m(D_0^*) + \frac{3}{4} m(D_1), \quad \overline{m}_{12}(D) \equiv \frac{3}{8} m(D_1') + \frac{5}{8} m(D_s^*),
\]

the hyperfine mass splittings

\[
\overline{m}_{12}(D) - \overline{m}_{01}(D) \approx 48 \text{ MeV}, \quad \overline{m}_{12}(D_s) - \overline{m}_{01}(D_s) \approx 132 \text{ MeV},
\]
implies that the fine splitting is light quark mass dependent. Indeed, if the first line rather than the second line of Eq. (2.3) is employed as an input for the fine splittings of non-strange charmed mesons one will predict \[\begin{align*} M(D_0^{±}) &= 2217 \text{ MeV}, \quad M(D_0^0) = 2212 \text{ MeV}, \\
M(D_1^+) &= 2358 \text{ MeV}, \quad M(D_1^0) = 2355 \text{ MeV}, \end{align*}\] (2.6) which are evidently smaller than what are measured by Belle \[\cite{14}\].

It is interesting to note that the mass difference between strange and non-strange charmed mesons is of order \(100 \sim 110 \text{ MeV}\) for \(0^-, 1^-, 1'^+ \) and \(2^+\) as expected from the quark model. As a consequence, the experimental fact that \(m(D_{s0}) \approx m(D_0^0)\) and \(m(D_{s1}) \sim m(D_1)\) is very surprising.

In the heavy quark limit, the physical mass eigenstates \(D_1\) and \(D_1'\) can be identified with \(P_{1/2}^1\) and \(P_{3/2}^1\), respectively. However, beyond the heavy quark limit, there is a mixing between \(P_{1/2}^1\) and \(P_{3/2}^1\), denoted by \(D_{1/2}^1\) and \(D_{3/2}^1\) respectively,

\[
D_1(2427) = D_{1/2}^1 \cos \theta + D_{3/2}^1 \sin \theta, \\
D_1'(2420) = -D_{1/2}^1 \sin \theta + D_{3/2}^1 \cos \theta.
\] (2.7)

Likewise for strange axial-vector charmed mesons

\[
D_{s1}(2460) = D_{s1}^{1/2} \cos \theta_s + D_{s1}^{3/2} \sin \theta_s, \\
D_{s1}'(2536) = -D_{s1}^{1/2} \sin \theta_s + D_{s1}^{3/2} \cos \theta_s.
\] (2.8)

Since \(D_{1/2}^1\) is much broader than \(D_{3/2}^1\) as we shall see shortly, the decay width of \(D_1'(2420)\) is sensitive to the mixing angle \(\theta\). Our task is to determine the \(D_{1/2}^1 - D_{3/2}^1\) mixing angle from the measured widths. In contrast, the present upper limits on the widths of \(D_{s1}(2460)\) and \(D_{s1}'(2536)\) do not allow us to get any constraints on the mixing angle \(\theta_s\). Hence, we will turn to the quark potential model to extract \(\theta_s\) as will be shown below.

It is suitable and convenient to study the strong decays of heavy mesons within the framework of heavy quark effective theory in which heavy quark symmetry and chiral symmetry are combined \[\cite{21}\]. It is straightforward to generalize the formalism to heavy mesons in \(p\)-wave excited states \[\cite{22}\]. The decay \(D_0^*\) undergoes a \(s\)-wave hadronic decay to \(D\pi\), while \(D_{1/2}^1\) can decay into \(D^*\) by \(s\)-wave and \(d\)-wave pion emissions but only the former is allowed in the heavy quark limit \(m_c \to \infty\):

\[
\Gamma(D_0^* \to D\pi) = g_{D_0^*D\pi}^2 \frac{P_c}{8\pi m_{D_0}^2}, \\
\Gamma(D_{1/2}^1 \to D^*\pi) = g_{D_{1/2}^1D^*\pi}^2 \frac{P_c}{8\pi m_{D_{1/2}^1}^2},
\] (2.9)

also depend on the light quark flavor. Based on a quark-meson model, the spin-weighted masses \(\overline{m}_{01}(D) = 2165 \pm 50 \text{ MeV} \ [19]\) and \(\overline{m}_{01}(D_s) = 2411 \pm 25 \text{ MeV} \ [20]\) were predicted, while experimentally they are very similar (see Table I).
where \( p_c \) is the c.m. momentum of the final-state particles in the \( B \) rest frame. The tensor meson \( D_2^* \) decays into \( D^* \) or \( D \) via \( d \)-wave pion emission. In the heavy quark limit where the total angular momentum \( j \) of the light quark is conserved, \( D_{1/2}^{3/2} \to D \pi \) is prohibited by heavy quark spin symmetry. The explicit expressions for the decay rates are [22],

\[
\Gamma(D_{1/2}^{3/2} \to D^*\pi) = \frac{1}{6\pi} \frac{m_{D^*}}{m_{D_{1/2}^{3/2}}} \frac{h}{f_\pi} \left( \frac{p_c}{m_{D_{1/2}^{3/2}}} \right)^5, \\
\Gamma(D_2^* \to D^*\pi) = \frac{1}{10\pi} \frac{m_{D^*}}{m_{D_2}} \frac{h}{f_\pi} \left( \frac{p_c}{m_{D_2}} \right)^5, \\
\Gamma(D_2^* \to D\pi) = \frac{1}{15\pi} \frac{m_D}{m_{D_2}} \frac{h}{f_\pi} \left( \frac{p_c}{m_{D_2}} \right)^5, 
\]

(2.10)

where \( \Lambda_\chi \) is a chiral symmetry breaking scale, \( f_\pi = 132 \) MeV and \( h' \) is a heavy flavor independent coupling constant. The \( p_c^5 \) dependence of the decay rate indicates the \( d \)-wave nature of pion emission. From Eq. (2.10) we obtain

\[
\frac{\Gamma(D_2^0 \to D^{+}\pi^-)}{\Gamma(D_2^0 \to D^{+}\pi^0)} = \frac{2}{3} \frac{m_D}{m_{D^*}} \left( \frac{p_c(D_2^* \to D\pi)}{p_c(D_2^* \to D^*\pi)} \right)^5 = 2.3, 
\]

(2.11)

in excellent agreement with the measured value of \( 2.3 \pm 0.6 \) [15].

Since the \( d \)-wave decay is severely phase-space suppressed, it is evident that \( D_0^* \) and \( D_1 \) are very broad, of order 250 MeV in their widths, whereas \( D_1' \) and \( D_2^* \) are narrow with widths of order 20 MeV.

The strong couplings appearing in Eq. (2.9) are given by

\[
g_{D_0^*D^*\pi} = \sqrt{\frac{m_{D_0}^2 - m_{D}^2}{m_{D_0}^2}} \frac{h}{f_\pi}, \\
g_{D_{1/2}^{1/2}D^*\pi} = \sqrt{\frac{m_{D_{1/2}^{1/2}}^2 - m_{D^*}^2}{m_{D_{1/2}^{1/2}}^2}} \frac{h}{f_\pi}, 
\]

(2.12)

with \( h \) being another heavy-flavor independent coupling constant in the effective Lagrangian [22]. It can be extracted from the measured width of \( D_0^*(2308) \) (see Table I) to be

\[
h = 0.65 \pm 0.12. 
\]

(2.13)

From the averaged width \( 29.4 \pm 4.2 \) MeV measured for \( D_2^{*0} \) we obtain

\[
\frac{h'}{\Lambda_\chi} = 0.67 \pm 0.05 \text{ GeV}^{-1}. 
\]

(2.14)

Substituting the couplings \( h \) and \( h' \) into Eqs. (2.9) and (2.10) leads to

\[
\Gamma(D_{1/2}^{3/2} \to D^*\pi) = 10.5 \pm 1.5 \text{ MeV}, \quad \Gamma(D_{1/2}^{1/2} \to D^*\pi) = 248 \pm 92 \text{ MeV}, 
\]

(2.15)
where we have assumed that $D_{1}^{3/2}$ has a mass close to $D_{1}^{0}(2420)$ and $m(D_{1}^{3/2}) \approx m(D_{1}(2427))$. Therefore, $D_{1}^{3/2}$ is much narrower than $D_{1}^{1/2}$ owing to the phase-space suppression for $d$-wave. However, the physical state $D_{1}^{0}(2420)$ can receive a $s$-wave contribution as there is a mixing between $D_{1}^{1/2}$ and $D_{1}^{3/2}$ beyond the heavy quark limit. The observed narrowness of $D_{1}^{0}(2420)$ indicates that the mixing angle should be small; that is, $D_{1}^{0}(2420)$ should be dominated by $D_{1}^{3/2}$ while $D_{1}(2427)$ is primarily the $D_{1}^{1/2}$ state. Using

$$\Gamma[D_{1}^{0}(2420)] = \Gamma(D_{1}^{3/2} \to D^{*}\pi) \cos^{2}\theta + \Gamma(D_{1}^{1/2} \to D^{*}\pi) \sin^{2}\theta$$

(2.16)

and the averaged width $20.9 \pm 2.1$ MeV for $D_{1}^{0}(2420)$, it is found \(^5\)

$$\theta = \pm(12.1^{+6.6}_{-4.4})^\circ.$$  

(2.17)

We shall see in Sec. IV that a positive mixing angle is preferred by a study of $D_{1}(2427)^{0}\pi^{-}$ production in $B$ decays.

The scalar resonance $D_{s0}^{*}(2317)$ is below the threshold of $DK$ and its only allowed strong decay $D_{s0}^{*}(2317) \to D_{s}\pi^{0}$ is isospin violating. Therefore, it is extremely narrow with a width of order 10 keV [4,11,23]. As for $D_{s1}(2460)$, it is below $D^{*}K$ threshold and its decay to $DK$ is forbidden by parity and angular momentum conservation. Hence, the allowed strong decays are $D_{s}^{*}\pi^{0}$, $D_{s0}^{*}\pi^{0}$, $D_{s}\pi\pi$ and $D_{s}\pi\pi\pi$. The isospin-violating decay $D_{s1}(2460) \to D_{s}^{*}\pi^{0}$ has a rate similar to $D_{s0}^{*}(2317) \to D_{s}\pi^{0}$. At a first sight, it is tempted to argue that $D_{s}\pi\pi$ could dominate over $D_{s}^{*}\pi^{0}$ as the former can proceed without violating isospin symmetry. However, a detailed analysis shows this is not the case. The decay $D_{s1} \to D_{s}\pi\pi$ arises from the weak transitions $D_{s1}(2460) \to D_{s}\{\sigma(600), f_{0}(980), \cdots\}$ followed by the strong decays $\{\sigma(600), f_{0}(980), \cdots\} \to \pi\pi$. Consider the dominant contributions from the intermediate states $\sigma(600)$, $f_{0}(980)$ and their mixing

$$f_{0} = s\bar{s} \cos \phi + n\bar{n} \sin \phi, \quad \sigma = -s\bar{s} \sin \phi + n\bar{n} \cos \phi,$$

(2.18)

with $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$. The $\sigma - f_{0}$ mixing angle can be inferred from various processes, see [24] for a summary. In general, the mixing angle is small so that $f_{0}(980)$ has a large $s\bar{s}$ component while $\sigma$ is primarily $n\bar{n}$. The $f_{0}$ production is favored by the weak decay of $D_{s1}$ into $D_{s}$, but its contribution to $\pi\pi$ is suppressed by the large off-shellness of $f_{0}(980)$, recalling that the mass difference between $D_{s1}(2460)$ and $D_{s}$ is only 494 MeV. In contrast, $\sigma(600)$ is favored by phase space consideration and yet its contribution is suppressed by the small $\sigma - f_{0}$ mixing angle. As a net result, although the strong decay into $D_{s}\pi\pi$ is isospin conserving, its OZI suppression is more severe than the isospin one for $D_{s}^{*}\pi^{0}$. This is confirmed by the recent measurement of CLEO [13]

\(^5\)The $D_{1}^{1/2} - D_{1}^{3/2}$ mixing angle was just reported to be $\theta = 0.10 \pm 0.03 \pm 0.02 \pm 0.02$ radians = $(5.7 \pm 2.4)^\circ$ by Belle through a detailed $B \to D^{*}\pi\pi$ analysis [14]. This is consistent with the result of Eq. (2.17).
\[
\frac{\mathcal{B}(D_{s1}(2460) \to D_s\pi^+\pi^-)}{\mathcal{B}(D_{s1}(2460) \to D_s^0\pi^0)} < 0.08. \tag{2.19}
\]

As for the electromagnetic decays of \(D_{s1}(2460)\), CLEO and Belle found

\[
\frac{\mathcal{B}(D_{s1}(2460) \to D_s\gamma)}{\mathcal{B}(D_{s1}(2460) \to D_s^0\pi^0)} = \begin{cases} 
< 0.49 & \text{CLEO [13],} \\
0.38 \pm 0.11 \pm 0.04 & \text{Belle [14],} \\
0.63 \pm 0.15 \pm 0.15 & \text{Belle [16].}
\end{cases} \tag{2.20}
\]

There are two different measurements of the radiative mode by Belle: the measured value 0.38 \(\pm 0.11 \pm 0.04\) is determined from \(B \to DD_{s1}\) decays, while the other value comes from charm fragmentation of \(e^+e^- \to c\bar{c}\). These two measurements are consistent with each other, though the central values differ significantly. Hence, just as its \(0^+\) partner \(D_{s0}^*(2317)\), \(D_{s1}(2460)\) is also extremely narrow. A theoretical estimation yields 38.2 keV for its width [11].

Eq. (2.14) leads to \(\Gamma(D_s^2) = 12.6\) MeV, in agreement with experiment (see Table I). Since \(\Gamma(D_{s1}^{3/2}) = 280\) keV followed from Eqs. (2.10) and (2.14) and \(D_{s1}^{1/2}\) is very narrow as its mass is close to \(D_{s1}(2460)\) which is below \(D^*K\) threshold, the decay width of \(D'_{s1}(2536)\) is thus at most of order 0.3 MeV and is consistent with the experimental limit 2.3 MeV [15]. In short, while \(D_{s0}^*\) and \(D_{s1}\) are quite broad as they are allowed to have \(s\)-wave hadronic decays. In sharp contrast, \(D_{s0}^*\) and \(D_{s1}\) are even much narrower than \(D'_{s1}\) as their allowed strong decays are isospin violating.

Since the width of \(D'_{s1}(2536)\) has not been measured, we will appeal to the quark potential model to estimate the \(D_{s1}^{1/2} - D_{s1}^{3/2}\) mixing angle [see Eq. (2.8)]. It is known that spin-orbital and tensor force interactions are responsible for the mass splitting of the four \(p\)-wave charmed mesons. In the quark potential model the relevant mass operator has the form [25]

\[
M = \lambda \ell \cdot s_1 + 4\tau \ell \cdot s_2 + \tau S_{12}, \tag{2.21}
\]

with \(s_1\) and \(s_2\) refer to the spin of the light and heavy quarks, respectively, and

\[
\lambda = \frac{1}{2m_q^2} \left[ \frac{V'}{r} \left( 1 + \frac{2m_q}{m_c} \right) - \frac{S'}{r} \right],
\]

\[
\tau = \frac{1}{4m_q m_c} \frac{V'}{r}, \tag{2.22}
\]

where \(V(r)\) is the zero-component of a vector potential, \(S(r)\) is a scalar potential responsible for confinement, and \(S_{12}\) is the tensor force operator

\[
S_{12} = 3\sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2. \tag{2.23}
\]

Note that the assumption of a Coulomb-like potential for \(V(r)\) has been made in deriving Eq. (2.21) [25]. Under this hypothesis, the mass splitting is governed by the two parameters \(\lambda\) and \(\tau\).
Following Cahn and Jackson [25], the masses of the \( J = 2 \) and \( J = 0 \) states read

\[
M_2 = \frac{\lambda}{2} + \frac{8}{5}\tau + c, \quad M_0 = -\lambda - 8\tau + c, \quad (2.24)
\]

while the masses of the two \( J = 1 \) states obtained by diagonalizing the matrix in the \(|J, j, m\rangle = |1, 3/2, m\rangle \) and \(|1, 1/2, m\rangle \) bases are (up to a common mass \( c \))

\[
\left( \begin{array}{cc}
\frac{\lambda}{2} - \frac{8}{3}\tau & -\frac{2\sqrt{2}}{3}\tau \\
-\frac{2\sqrt{2}}{3}\tau & -\frac{\lambda}{3} + \frac{8}{3}\tau \\
\end{array} \right). \quad (2.25)
\]

It is clear that the mixing vanishes in the heavy quark limit \( m_c \to \infty \). However, \( 1/m_c \) corrections will allow charm quark spin to flip and mix \( D^{1/2}_1 \) and \( D^{3/2}_1 \). The two eigenmasses for \( J = 1 \) are then

\[
M_{1\pm} = -\frac{\lambda}{4} + c \pm \sqrt{\frac{\lambda^2}{16} + \frac{1}{2}(\lambda - 4\tau)^2}. \quad (2.26)
\]

From Eqs. (2.8) and (2.25) we arrive at

\[
\theta = \sin^{-1}\left( \frac{-R_+}{\sqrt{1 + R^2_+}} \right) \quad \text{with} \quad R_+ = \frac{\frac{\lambda}{2} - \frac{8}{3}\tau - M_{1+}}{\frac{2\sqrt{2}}{3}\tau}. \quad (2.27)
\]

The parameters \( \lambda \) and \( \tau \) are obtained by a global fit to the charm spectroscopy. For \( D^{**}_s \) mesons, it is found

\[
\tau \approx 12 \text{ MeV}, \quad \lambda \approx 104 \text{ MeV}, \quad \theta \approx 7^\circ. \quad (2.28)
\]

As pointed out in [25], a positive spin-orbit energy \( \lambda \) implies a less important scalar potential \( S \). On the contrary, the existing potential model calculation such as the one by Di Pierro and Eichten [2] yields \( \lambda < 0 \) [25] or a very strong confining potential \( S \). This will cause a reversed splitting, namely the \( j = 1/2 \) states lying above the \( j = 3/2 \) states. However, we will not address this issue here. For \( D^{**} \) mesons, the parameters \( \lambda \) and \( \tau \) fall into some large regions because of large uncertainties associated with the measured masses of \( D^*_0 \) and \( D_1 \). Hence, the magnitude and even the sign of the \( D^{1/2}_1 - D^{3/2}_1 \) mixing angle \( \theta \) at present cannot be fixed within this approach. Instead, we have used heavy quark effective theory together with the measured widths to extract \( |\theta| \). As will be seen below, the sign of \( \theta \) can be inferred from a study of the \( p \)-wave charmed meson production in \( B \) decays.

### III. DECAY CONSTANTS AND FORM FACTORS

#### A. Decay constants

The decay constants of scalar and pseudoscalar mesons are defined by
\begin{align*}
\langle 0 | A_\mu | P(q) \rangle &= i f_P q_\mu, \\
\langle 0 | V_\mu | S(q) \rangle &= f_S q_\mu.
\end{align*}

(3.1)

It is known that the decay constants of non-charm light scalar mesons are smaller than that of pseudoscalar mesons as they vanish in the SU(3) limit. For the neutral scalars $\sigma(600)$, $f_0(980)$ and $a_0^0(980)$, the decay constant must be zero owing to charge conjugation invariance or conservation of vector current:

\begin{equation}
\begin{aligned}
f_\sigma &= f_{f_0} = f_{a_0^0} = 0.
\end{aligned}
\end{equation}

(3.2)

Applying the equation of motion, it is easily seen that the decay constant of $K^{*+}_0$ ($a_0^+$) is proportional to the mass difference between the constituent $s$ ($d$) and $u$ quarks. Consequently, the decay constant of the charged $a_0(980)$ is very small, while the one for $K^{*+}_0(1430)$ is less suppressed. A calculation based on the finite-energy sum rules \cite{26} yields

\begin{equation}
\begin{aligned}
f_{a_0^+} &= 1.1 \text{ MeV}, \\
f_{K^{*+}_0} &= 42 \text{ MeV}.
\end{aligned}
\end{equation}

(3.3)

Contrary to the non-charm scalar resonances, the decay constant of the scalar charmed meson is not expected to be suppressed because of charm and light quark mass imbalance. Applying the equation of motion again leads to

\begin{equation}
\begin{aligned}
m_{K^{*+}_0}^2 f_{K^{*+}_0} &= i (m_s - m_u) \langle K^{*+}_0 | \bar{s}u | 0 \rangle, \\
m_{D^{*+}_0}^2 f_{D^{*+}_0} &= i (m_c - m_u) \langle D^{*+}_0 | \bar{c}u | 0 \rangle.
\end{aligned}
\end{equation}

(3.4)

For a crude estimate, we assume $\langle D^{*+}_0 | \bar{c}u | 0 \rangle \approx \langle K^{*+}_0 | \bar{s}u | 0 \rangle$ and obtain

\begin{equation}
f_{D^{*+}_0} \approx 160 \text{ MeV}.
\end{equation}

(3.5)

This is comparable to $f_D \approx 200$ MeV, the decay constant of the pseudoscalar $D$ meson.

The decay constants of the axial-vector charmed mesons are defined by

\begin{equation}
\begin{aligned}
\langle 0 | A_\mu | D^{1/2}_1(q, \varepsilon) \rangle &= f_{D^{1/2}_1} m_{D^{1/2}_1} \varepsilon_\mu, \\
\langle 0 | A_\mu | D^{3/2}_1(q, \varepsilon) \rangle &= f_{D^{3/2}_1} m_{D^{3/2}_1} \varepsilon_\mu.
\end{aligned}
\end{equation}

(3.6)

It has been shown that in the heavy quark limit \cite{27–29}

\begin{equation}
f_{D^{1/2}_1} = f_{D^{3/2}_0}, \\
f_{D^{3/2}_1} = 0.
\end{equation}

(3.7)

Since the decay constant of $D^{*+}_2$ vanishes irrespective of heavy quark symmetry (see below), the charmed mesons within the multiplet $(0^+, 1^+)$ or $(1^+, 2^+)$ thus have the same decay constant. This is opposite to the case of light $p$-wave mesons where the decay constant of $^1P_1$ meson vanishes in the SU(3) limit \cite{30} based on the argument that for non-charm axial vector mesons, the $^3P_1$ and $^1P_1$ states transfer under charge conjugation as

\begin{equation}
M_a^b(3P_1) \rightarrow M_b^a(3P_1), \\
M_a^b(1P_1) \rightarrow -M_b^a(1P_1), \quad (a = 1, 2, 3),
\end{equation}

(3.8)

where the axial-vector mesons are represented by a $3 \times 3$ matrix. Since the weak axial-vector current transfers as $(A_\mu)^a_b \rightarrow (A_\mu)^b_a$ under charge conjugation, it is clear that the decay constant of the $^1P_1$ meson vanishes in the SU(3) limit \cite{30}.
The polarization tensor $\varepsilon_{\mu\nu}$ of a tensor meson satisfies the relations

$$
\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}, \quad \varepsilon_{\mu}^{\mu} = 0, \quad p_\mu \varepsilon^{\mu\nu} = p_\nu \varepsilon^{\mu\nu} = 0.
$$

Therefore,

$$
\langle 0 | (V - A)_\mu | D_2^* (\varepsilon, p) \rangle = a \varepsilon_{\mu\nu} p_\nu + b \varepsilon_{\nu}^\nu p_\mu = 0.
$$

The above relation in general follows from Lorentz covariance and parity consideration. Hence the decay constant of the tensor meson vanishes; that is, the tensor meson $D_2^*$ cannot be produced from the $V - A$ current.

Beyond the heavy quark limit, the relations (3.7) receive large $1/m_c$ corrections which have been estimated in [29] using the relativistic quark model. In the present paper we shall use $f_\rho = 216$ MeV and (in units of MeV)

$$
f_D = 200, \quad f_{D_s} = 230, \quad f_{D_s^*} = 230,
$$

$$
f_{D_{0}} = 160, \quad f_{D_{1/2}} = 120, \quad f_{D_{3/2}} = 40,
$$

$$
f_{D_{s0}} = 60, \quad f_{D_{s1}^{1/2}} = 145, \quad f_{D_{s1}^{3/2}} = 50.
$$

The decay constants for $D_s^{**}$ are obtained essentially by fitting to experiment to be discussed in Sec. IV.B. Note that the measurements of $B \to D_s^{(*)} D^{(*)}$ [15,31] indicate that the decay constants of $D_s^*$ and $D_s$ are similar.

**B. Form factors**

Form factors for $B \to M$ transitions with $M$ being a parity-odd meson are given by [32]

$$
\langle P(p) | V_\mu | B(p_B) \rangle = \left( (p_B + p)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu \right) F_1^{BP}(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0^{BP}(q^2),
$$

$$
\langle V(p, \varepsilon) | V_\mu | B(p_B) \rangle = \frac{2}{m_P + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon^\nu p_B^\alpha p_V^\beta V(q^2),
$$

$$
\langle V(p, \varepsilon) | A_\mu | B(p_B) \rangle = i \{ (m_P + m_V) \varepsilon^\mu A_1(q^2) - \frac{\varepsilon^\mu \cdot p_B}{m_P + m_V} (p_B + p)_\mu A_2(q^2)
$$

$$- 2m_V \frac{\varepsilon^\mu \cdot p_B}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] \},
$$

where $q = p_B - p$, $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$, and

$$
A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2).
$$

(3.12)

For $B \to P$ and $B \to V$ form factors, we will use the Melikhov-Stech (MS) model [33] based on the constituent quark picture. Other form factor models give similar results.

The general expressions for $B \to D^{**}$ transitions ($D^{**}$ being a $p$-wave charmed meson) are given by [34]
\[
\langle D_0^*(p)|A_\mu|B(p_B)\rangle = i\left[u_+(q^2)(p_B + p)_\mu + u_-(q^2)(p_B - p)_\mu\right],
\]
\[
\langle D_{1/2}^1(p, \epsilon)|V_\mu|B(p_B)\rangle = i\left[\ell_{1/2}(q^2)\epsilon_\mu^* + c_{1/2}^\ast(q^2)(\epsilon^* \cdot p_B)(p_B + p)_\mu + c_{1/2}^\ast(q^2)(\epsilon^* \cdot p_B)(p_B - p)_\mu\right],
\]
\[
\langle D_{1/2}^1(p, \epsilon)|A_\mu|B(p_B)\rangle = -q_{1/2}(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\sigma}(p_B + p)^\rho(p_B - p)^\sigma,
\]
\[
\langle D_{3/2}^1(p, \epsilon)|V_\mu|B(p_B)\rangle = i\left[\ell_{3/2}(q^2)\epsilon_\mu^* + c_{3/2}^\ast(q^2)(\epsilon^* \cdot p_B)(p_B + p)_\mu + c_{3/2}^\ast(q^2)(\epsilon^* \cdot p_B)(p_B - p)_\mu\right],
\]
\[
\langle D_{3/2}^1(p, \epsilon)|A_\mu|B(p_B)\rangle = -q_{3/2}(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\sigma}(p_B + p)^\rho(p_B - p)^\sigma,
\]
\[
\langle D_2^*|\nu\mu|B(p_B)\rangle = \mu(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\sigma}(p_B + p)^\rho(p_B - p)^\sigma,
\]
\[
\langle D_2^*|\nu\mu|A_\mu|B(p_B)\rangle = -i\left[k(q^2)\epsilon_\nu^* P_B^\nu + b_+(q^2)\epsilon_\mu^* P_B(\sigma) + P_B(\sigma)\right] + b_-(q^2)\epsilon_\alpha^* P_B^\alpha(\sigma) \mu.
\]
(3.14)

In order to know the sign of various form factors appearing in Eq. (3.14), it is instructive to check the heavy quark limit behavior of \(B \to D^{**}\) transitions which has the form [17]

\[
\langle D_0^*(v')|A_\mu|B(v)\rangle = \sqrt{m_B m_{D_0}} 2\tau_{1/2}(\omega) i(v' - v)_\mu,
\]
\[
\langle D_{1/2}^1(v', \epsilon)|V_\mu|B(v)\rangle = \sqrt{m_B m_{D_{1/2}}} 2\tau_{1/2}(\omega) i\left[(\omega - 1)\epsilon^*_\mu - (\epsilon^* \cdot v)v'_\mu\right],
\]
\[
\langle D_{1/2}^1(v', \epsilon)|A_\mu|B(v)\rangle = \sqrt{m_B m_{D_{1/2}}} 2\tau_{1/2}(\omega)(-\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\sigma} v^\rho v^\sigma),
\]
\[
\langle D_{3/2}^1(v', \epsilon)|V_\mu|B(v)\rangle = \sqrt{m_B m_{D_{3/2}}} 2\tau_{3/2}(\omega) i\left[(1 - \omega^2)\epsilon^*_\mu - (\epsilon^* \cdot v)[3v_\mu - (\omega - 2)v'_\mu]\right],
\]
\[
\langle D_{3/2}^1(v', \epsilon)|A_\mu|B(v)\rangle = \sqrt{m_B m_{D_{3/2}}} 2\tau_{3/2}(\omega)(\omega + 1)\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\sigma} v^\rho v^\sigma,
\]
\[
\langle D_2^*(v', \epsilon)|V_\mu|B(v)\rangle = \sqrt{m_B m_{D_2}} 2\tau_{3/2}(\omega) i\left[(\omega + 1)\epsilon^*_\mu v^\nu + \epsilon^* v^\nu + \epsilon^* v^\nu v'_\mu\right],
\]
\[
\langle D_2^*(v', \epsilon)|A_\mu|B(v)\rangle = \sqrt{m_B m_{D_2}} 2\tau_{3/2}(\omega)(-i)\left[(\omega + 1)\epsilon^*_\mu v^\nu - \epsilon^* v^\nu v^\nu v'_\mu\right],
\]
(3.15)

where \(\omega \equiv v \cdot v'\) and there are two independent functions \(\tau_{1/2}(\omega)\) and \(\tau_{3/2}(\omega)\) first introduced in [17]. It is easily seen that the matrix elements of weak currents vanish at the zero recoil point \(\omega = 1\) owing to the orthogonality of the wave functions of \(B\) and \(D^{**}\). The universal functions \(\tau_{1/2}(\omega)\) and \(\tau_{3/2}(\omega)\) are conventionally parametrized as

\[
\tau_i(\omega) = \tau_i(1)\left[1 - \rho_i^2(\omega - 1)\right]
\]
(3.16)

for \(i = 1/2\) and 3/2. The slope parameter \(\rho^2\) can be related to \(\tau_{1/2}(1)\) and \(\tau_{3/2}(1)\) via the Bjorken sum rule. Comparing Eq. (3.14) with Eq. (3.15) we see that heavy quark symmetry requires that the form factors \(u_+, \ell_{1/2}, q_{1/2}, c_{1/2}^\ast, h, k\) and \(b_+\) be positive, while \(u_-, \ell_{3/2}, q_{3/2}, c_{3/2}^\ast, c_{3/2}^\ast, c_{3/2}^\ast\) and \(b_-\) be negative. Heavy quark symmetry also demands the relations \(c_{1/2}^\ast + c_{3/2}^\ast = 0\) and \(b_+ + b_- = 0\). It is easily seen that these heavy quark symmetry requirements are satisfied in realistic model calculations shown below.

In the present paper, we shall use the improved version, the so-called ISGW2 model [35], of the non-relativistic quark model by Isgur-Scora-Grinstein-Wise (ISGW) [34] to compute
the $B \to D^{**}$ transition form factors.** In general, the form factors evaluated in the original version of the ISGW model are reliable only at $q^2 = q_m^2$, the maximum momentum transfer. The reason is that the form-factor $q^2$ dependence in the ISGW model is proportional to $\exp[-(q_m^2 - q^2)]$ and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. This has been improved in the ISGW2 model in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$ which is expressed in terms of a certain polynomial term. In addition to the form-factor momentum dependence, the ISGW2 model incorporates a number of improvements, such as the constraints imposed by heavy quark symmetry, hyperfine distortions of wave functions, etc., \cdots [35]. The results of the ISGW2 model predictions for various form factors are shown in Tables II-IV. Evidently, the signs of various calculated form factors are consistent with what are expected from heavy quark symmetry.

**TABLE II.** The form factors at various $q^2$ for $B \to D_0^*$ and $B_s \to D_{s0}^*$ transitions calculated in the ISGW2 model.

| Transition | $u_+(m_π^2)$ | $u_-(m_π^2)$ | $u_+(m_μ^2)$ | $u_-(m_μ^2)$ | $u_+(m_π^2)$ | $u_-(m_π^2)$ |
|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $B \to D_0^*$ | 0.175 | -0.462 | 0.178 | -0.471 | 0.198 | -0.524 |
| $B_s \to D_{s0}^*$ | 0.196 | -0.515 | 0.200 | -0.527 | 0.230 | -0.605 |

**TABLE III.** The form factors at $q^2 = m_π^2$ for $B \to D_1^{1/2}$ and $B \to D_1^{3/2}$ transitions calculated in the ISGW2 model, where $\ell_{1/2}$ and $\ell_{3/2}$ are in units of GeV and all others are in units of GeV$^{-1}$.

| Transition | $q_{1/2}$ | $\ell_{1/2}$ | $c_{1/2}^{1/2}$ | $c_{1/2}$ | $q_{3/2}$ | $\ell_{3/2}$ | $c_{3/2}^{1/2}$ | $c_{3/2}$ |
|------------|-----------|---------------|----------------|----------|-----------|---------------|----------------|----------|
| $B \to D_1^{1/2}$ | 0.057 | 0.54 | -0.064 | 0.068 |           |               |               |           |
| $B \to D_1^{3/2}$ |         | -0.057 | -1.15 | -0.043 | -0.018 |               |               |           |
| $B_s \to D_{s1}^{1/2}$ | 0.063 | 0.66 | -0.072 | 0.078 |           |               |               |           |
| $B_s \to D_{s1}^{3/2}$ |         | -0.063 | -1.31 | -0.048 | -0.023 |               |               |           |

In realistic calculations of decay amplitudes it is convenient to employ the dimensionless form factors defined by [32]

$$
\langle D_0^*(p_{D_0})|A_μ|B(p_B)\rangle = i \left[ \left( p_B + p_{D_0} \right)_μ - \frac{m_B^2 - m_{D_0}^2}{q^2} q_μ \right] F_1^{BD_0}(q^2) + \frac{m_B^2 - m_{D_0}^2}{q^2} q_μ F_0^{BD_0}(q^2),
$$

**Note that in the original version of the ISGW model [34], the form factors for $B$ to axial-vector charmed meson transition are evaluated for $D_1^{(1P_1)}$ and $D_1^{(3P_1)}$. As a result, one has to apply Eq. (2.1) to obtain $B \to D_1^{1/2}$ and $B \to D_1^{3/2}$ form factors.**
TABLE IV. The $B \to D_2^*$ and $B_s \to D_{s2}^*$ form factors at $q^2 = m^2_\pi$ calculated in the ISGW2 model, where $k$ is dimensionless and $h, b_+, b_-$ are in units of GeV$^{-2}$.

| Transition  | $h$  | $k$  | $b_+$ | $b_-$ |
|-------------|------|------|-------|-------|
| $B \to D_2^*$ | 0.011 | 0.60 | -0.010 | 0.010 |
| $B_s \to D_{s2}^*$ | 0.013 | 0.70 | -0.011 | 0.012 |

TABLE V. The form factors $F_0$ and $F_1$ at various $q^2$ for $B \to D_0^*$ and $B_s \to D_{s0}^*$ transitions calculated in the ISGW2 model.

| Transition  | $F_0(m^2_\pi)$ | $F_0(m^2_\pi)$ | $F_1(m^2_\pi)$ | $F_0(m^2_{P1})$ | $F_1(m^2_{D1})$ | $F_0(m^2_{D_{s1}})$ |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $B \to D_0^*$ | 0.175 | 0.175 | 0.178 | 0.166 | 0.198 | 0.108 |
| $B_s \to D_{s0}^*$ | 0.196 | 0.196 | 0.200 | 0.187 | 0.230 | 0.130 |

\[
\langle D_1(p_{D_1}, \varepsilon)|V_\mu|B(p_B)\rangle = i \left\{ (m_B + m_{D_1})\varepsilon_\mu V_1^{BD_1}(q^2) - \frac{\varepsilon^* \cdot p_B}{m_B + m_{D_1}}(p_B + p_{D_1})_\mu V_2^{BD_1}(q^2) - 2m_{D_1} \frac{\varepsilon^* \cdot p_B}{q^2}(p_B - p_{D_1})_\mu [V_3^{BD_1}(q^2) - V_0^{BD_1}(q^2)] \right\},
\]

\[
\langle D_1(p_{D_1}, \varepsilon)|A_\mu|B(p_B)\rangle = \frac{2}{m_B + m_{D_1}} \epsilon_{\mu\rho\sigma} \varepsilon^* \rho \sigma V_3^{BD_1}(q^2),
\]

with

\[
V_3^{BD_1}(q^2) = \frac{m_B + m_{D_1}}{2m_{D_1}} V_1^{BD_1}(q^2) - \frac{m_B - m_{D_1}}{2m_{D_1}} V_2^{BD_1}(q^2),
\]

and $V_3^{BD_1}(0) = V_0^{BD_1}(0)$. The form factors relevant for $B \to D_0^*P$ decays are $F_0^{BD_0}$ and $F_1^{BP}$. Note that only the form factor $V_0^{BP}$ or $F_1^{BD_1}$ will contribute to the factorizable amplitude of $B \to D_1P$ as one can check the matrix elements $q^\mu \langle D_1(p_{D_1}, \varepsilon)|V_\mu|B(p_B)\rangle$ and $\varepsilon^\mu(P|V_\mu|B)$. The ISGW2 model predictions for the form factors $F_{0,1}$, $V_{0,1,2}$ and $A$ are summarized in Tables V-VI. It is evident that the form factor $F_{0,1}(0) \approx 0.18$ for $B \to D_0^*$ transition is much smaller than the typical value of 0.65 $\sim$ 0.70 for the $B \to D$ transition form factor at $q^2 = 0$.

IV. ANALYSIS OF $\bar{B} \to D^{**}M$, $\bar{D}_{s}^{**}M$ DECAYS

A. Factorization

In the present work we focus on the Cabibbo-allowed decays $\bar{B} \to D^{**}\pi(\rho)$, $D^{**}\bar{D}_s^{(*)}$, $\bar{D}_s^{**}D^{(*)}$ and $\bar{B}_s \to D_s^{**}\pi(\rho)$, where $D^{**}$ denotes generically a p-wave charmed meson. We will study these decays within the framework of generalized factorization in which the hadronic
TABLE VI. The dimensionless form factors $A$ and $V_{0,1,2}$ at $q^2 = m_{\pi}^2$ for $B \to D_{1/2}^1$ and $B \to D_{3/2}^1$ transitions calculated in the ISGW2 model.

| Transition  | $A$  | $V_0$ | $V_1$ | $V_2$ |
|-------------|------|-------|-------|-------|
| $B \to D_{1/2}^1$ | $-0.43$ | $-0.18$ | $0.070$ | $0.49$ |
| $B \to D_{3/2}^1$ | $0.44$ | $-0.43$ | $-0.15$ | $0.33$ |
| $B_s \to D_{s1/2}$ | $-0.49$ | $-0.20$ | $0.085$ | $0.57$ |
| $B_s \to D_{s3/2}$ | $0.50$ | $-0.47$ | $-0.17$ | $0.38$ |

The decay amplitude is expressed in terms of factorizable contributions multiplied by the universal (i.e. process independent) effective parameters $a_i$ that are renormalization scale and scheme independent. Since the aforementioned $B$ decays either proceed through only via tree diagrams or are tree dominated, we will thus neglect the small penguin contributions and write the weak Hamiltonian in the form

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* \left[ a_1 (\bar{c} b)(\bar{s} c) + a_2 (\bar{s} b)(\bar{c} c) \right] + V_{cb} V_{ud}^* \left[ a_1 (\bar{c} b)(\bar{d} u) + a_2 (\bar{s} b)(\bar{c} u) \right] + h.c. \right\} \quad (4.1)$$

with $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_{\mu} (1 - \gamma_5) q_2$. For hadronic $B$ decays, we shall use $a_1 = 1.15$ and $a_2 = 0.26$.

Under the factorization hypothesis, the decays $B^- \to D^{*+} \bar{D}_s^-$, $B^{0} \to D^{*+} \bar{D}_s^-$ and $\bar{B}^{0} \to D^{*+} \pi^-$ receive contributions only from the external $W$-emission diagram. As stated before, the penguin contributions to the first two decay modes are negligible.

Apart from a common factor of $G_F V_{cb} V_{ud}/\sqrt{2}$, the factorizable amplitudes for $B^- \to D^{*+} \pi^-$ read

$$A(B^- \to D_0^* (2308)^0 \pi^-) = -a_1 f_\pi (m_B^2 - m_{D_0}^2) F_{0}^{BD_0}(m_\pi^2) - a_2 f_{D_0} (m_B^2 - m_{D_0}^2) F_{0}^{B\pi}(m_{D_0}^2),$$

$$A(B^- \to D_1(2427)^0 \pi^-) = -2(\varepsilon^* \cdot p_B) \left\{ a_1 f_\pi \left[ V_0^{BD_1^{3/2}} (m_\pi^2) m_{D_1^{3/2}}^2 \sin \theta + V_0^{BD_{1/2}} (m_\pi^2) m_{D_1^{1/2}}^2 \cos \theta \right] + a_2 \left[ F_{1}^{BD_1^{3/2}} (m_{D_1^{3/2}}^2) m_{D_1^{3/2}}^2 f_{D_1^{1/2}} \sin \theta + F_{1}^{B\pi} (m_{D_1^{1/2}}^2) m_{D_1^{1/2}}^2 f_{D_1^{1/2}} \cos \theta \right] \right\},$$

$$A(B^- \to D_1'(2420)^0 \pi^-) = -2(\varepsilon^* \cdot p_B) \left\{ a_1 f_\pi \left[ V_0^{BD_1'^{3/2}} (m_\pi^2) m_{D_1'^{3/2}}^2 \cos \theta - V_0^{BD_{1/2}} (m_\pi^2) m_{D_1'^{1/2}}^2 \sin \theta \right] + a_2 \left[ F_{1}^{BD_1'^{3/2}} (m_{D_1'^{3/2}}^2) m_{D_1'^{3/2}}^2 f_{D_1'^{1/2}} \cos \theta - F_{1}^{B\pi} (m_{D_1'^{1/2}}^2) m_{D_1'^{1/2}}^2 f_{D_1'^{1/2}} \sin \theta \right] \right\},$$

$$A(B^- \to D_2^* (2460)^0 \pi^-) = i a_1 f_\pi \varepsilon_{\mu \nu} \rho_{\mu \nu}^{BD_B} \left[ k (m_\pi^2) + b_+ (m_B^2 - m_{D_2}^2) + b_- (m_\pi^2) m_{D_2}^2 \right]. \quad (4.2)$$

Note that except $B^- \to D_2^{*0} \pi^-$ all other modes receive contributions from color-suppressed internal $W$-emission. The decay rates are given by

$$\Gamma(B \to D_0^* \pi) = \frac{p_c}{8 \pi m_B^2} |A(B \to D_0^* \pi)|^2,$$

15
\[ \Gamma(B \to D_1\pi) = \frac{p_c^3}{8\pi m_{D_1}^2} |A(B \to D_1\pi)/(\varepsilon^* \cdot p_B)|^2, \]
\[ \Gamma(B \to D_2^*\pi) = \frac{p_c^5}{12\pi m_{D_2}^2} \left( \frac{m_B}{m_{D_2}} \right)^2 |M(B \to D_2^*\pi)|^2, \] (4.3)

where \( A(B \to D_2^*\pi) = \varepsilon^*_\rho p_B^\mu \rho_B^\nu M(B \to D_2^*\pi) \) and \( p_c \) is the c.m. momentum of the pion. The \( p_c^{2L+1} \) dependence in the decay rate indicates that only \( s, p \) and \( d \)-waves are allowed in \( D_1\pi \), \( D_1\pi \) and \( D_2^*\pi \) systems, respectively. The factorizable decay amplitudes for \( B^- \to D_0^0\rho^- \) and \( B^- \to D_2^0\rho^- \) are (up to a common factor of \( G_F V_{ud} V_{us}/\sqrt{2} \))
\[
A(B^- \to D_0^0(2308)^0\rho^-) = 2(\varepsilon^* \cdot p_B) \left[ a_1 f_{\rho} m_\rho F_1^{BD_0}(m_\rho^2) + a_2 f_{D_0} m_{D_0} A_1^{\rho}(m_{D_0}^2) \right],
\]
\[
A(B^- \to D_2^0(2460)^0\rho^-) = a_1 f_{\rho} m_\rho^2 \varepsilon^* \rho(p_B - p_{D_2}) \lambda \left[ i h(m_\rho^2) \epsilon^{\mu\nu\lambda\sigma} g_{\alpha\nu}(p_\rho)_\beta(g_{\rho\sigma}(p_\rho)_\alpha) \right]
+ k(m_\rho^2) \delta_\alpha \delta_\lambda + b_+(m_\rho^2)(p_\rho)_\alpha(p_\rho)_\beta g^{\alpha\lambda}. \quad (4.4)
\]

The expression for \( \overline{B} \to D_1\rho \) is more complicated. In the absence of the \( D_{11/2}^- - D_{33/2}^- \) mixing, one has
\[
A(B^- \to D_1^-(2420)^0\rho^-) = -ia_1 f_{\rho} m_\rho \left[ (\varepsilon^*_\rho \cdot \varepsilon_{D_1}^*)(m_B + m_{D_1}) V_{1}^{BD_1}(m_\rho^2) \right.
- \left( \varepsilon^*_\rho \cdot p_B \right)(\varepsilon_{D_1}^* \cdot p_B) \frac{2V_{1}^{BD_1}(m_\rho^2)}{m_B + m_{D_1}} + i\epsilon_{\mu\nu\alpha\beta} \varepsilon_{D_1}^* \varepsilon_{\rho} \frac{2A_1^{BD_1}(m_\rho^2)}{m_B + m_{D_1}} \]
- \left[ a_2 f_{D_1} m_{D_1} \left( (\varepsilon^*_\rho \cdot \varepsilon_{D_1}^*)(m_B + m_{D_1}) A_1^{BD_1}(m_{D_1}^2) \right) \right.
- \left( \varepsilon^*_\rho \cdot p_B \right)(\varepsilon_{D_1}^* \cdot p_B) \frac{2A_1^{BD_1}(m_{D_1}^2)}{m_B + m_{D_1}} + i\epsilon_{\mu\nu\alpha\beta} \varepsilon_{D_1}^* \varepsilon_{\rho} \frac{2V_{1}^{BD_1}(m_{D_1}^2)}{m_B + m_{D_1}} \right]. \quad (4.5)
\]

In the presence of the \( D_{11/2}^- - D_{33/2}^- \) mixing, it is more convenient to express the decay amplitude as
\[
A(B^- \to D_1^0(\rho, p_{D_1}) \rho^- (\varepsilon^*_\rho, p_{\rho}^\nu) \propto \varepsilon_{D_1}^* \varepsilon_{\rho}^{\nu}[S_1 g_{\mu\nu} + S_2(p_B)_\mu(p_B)_\nu + iS_3 \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{\rho}^\beta], \quad (4.6)
\]
where \( \epsilon^{0123} = +1 \) in our convention, the coefficient \( S_3 \) corresponds to the \( p \)-wave amplitude, and \( S_1, S_2 \) to the mixture of \( s \) and \( d \)-wave amplitudes:
\[
S_1 = a_1 f_{\rho} m_\rho \left[ (m_B + m_{D_1}^{1/2}) V_{1}^{BD_1^{3/2}}(m_\rho^2) \cos \theta - (m_B + m_{D_1}^{1/2}) V_{1}^{BD_1^{1/2}}(m_\rho^2) \sin \theta \right]
+ a_2 m_B m_{D_1}^{1/2} \left[ m_{D_1}^{1/2} V_{1}^{BD_1^{1/2}}(m_\rho^2) \cos \theta - m_{D_1}^{1/2} V_{1}^{BD_1^{3/2}}(m_\rho^2) \sin \theta \right],
\]
\[
S_2 = a_1 f_{\rho} m_\rho \left[ \frac{1}{m_B + m_{D_1}^{3/2}} V_{1}^{BD_1^{1/2}}(m_\rho^2) \cos \theta - \frac{1}{m_B + m_{D_1}^{1/2}} V_{1}^{BD_1^{3/2}}(m_\rho^2) \sin \theta \right]
+ a_2 \left[ \frac{1}{m_B + m_{D_1}^{1/2}} m_{D_1}^{1/2} f_{D_1}^{3/2} A_{2}^{BD_1^{3/2}}(m_\rho^2) \cos \theta - m_{D_1}^{1/2} f_{D_1}^{1/2} A_{2}^{BD_1^{1/2}}(m_\rho^2) \sin \theta \right],
\]
\[
S_3 = a_1 f_{\rho} m_\rho \left[ \frac{1}{m_B + m_{D_1}^{3/2}} A_{1}^{BD_1^{1/2}}(m_\rho^2) \cos \theta - \frac{1}{m_B + m_{D_1}^{1/2}} A_{1}^{BD_1^{3/2}}(m_\rho^2) \sin \theta \right]
+ a_2 \left[ \frac{1}{m_B + m_{D_1}^{1/2}} m_{D_1}^{1/2} f_{D_1}^{3/2} V_{1}^{BD_1^{1/2}}(m_\rho^2) \cos \theta - m_{D_1}^{1/2} f_{D_1}^{1/2} V_{1}^{BD_1^{3/2}}(m_\rho^2) \sin \theta \right]. \quad (4.7)
\]
Then the helicity amplitudes $H_0$, $H_\pm$ and $H_-$ can be constructed as

$$H_0 = \frac{1}{2m_D,m_\rho} \left[ (m_B^2 - m_{D_1}^2 - m_\rho^2)S_1 + 2m_B^2 p_\rho^2 S_2 \right],$$

$$H_\pm = S_1 \pm m_B p_\rho S_3. \quad (4.8)$$

For $B^- \to D_1(2427)^0 \rho^-$, the amplitudes $S_{1,2,3}$ are the same as Eq. (4.7) except for the replacement of $\cos \theta \to \sin \theta$ and $\sin \theta \to -\cos \theta$. The decay rates read (up to the common factor of $G_F^2 |V_{cb}V_{ud}^*|^2 / 2$)

$$\Gamma(B \to D_0^* \rho) = \frac{p_\rho^3}{8\pi m_D^2} |A(B \to D_0^* \rho)/(\varepsilon^* \cdot p_B)|^2,$$

$$\Gamma(B \to D_1 \rho) = \frac{p_\rho}{8\pi m_B^2} (|H_0|^2 + |H_+|^2 + |H_-|^2),$$

$$\Gamma(B \to D_2^* \rho) = \frac{f_\rho^2}{24\pi m_D^2} (a p_\rho^7 + b p_\rho^5 + c p_\rho^3), \quad (4.9)$$

with

$$a = 8m_D^4 b_+^2, \quad c = 5m_D^2 m_\rho^2 k^2,$$

$$b = 2m_B^2 [6m_D^2 m_\rho^2 h^2 + 2(m_B^2 - m_D^2 - m_\rho^2)kb_+ + k^2]. \quad (4.10)$$

### B. Results and discussions

Given the decay constants and form factors discussed in Sec. III, we are ready to study the $B$ decays into $p$-wave charmed mesons. The predicted branching ratios are shown in Tables VII and VIII. The experimental results are taken from PDG [15] and Belle [14]. For $B^- \to D_2^0 \pi^-$ we combine the Belle measurements [14]

$$B(B^- \to D_2^0 \pi^-)B(D_2^0 \to D^+ \pi^-) = (3.4 \pm 0.3 \pm 0.6 \pm 0.4) \times 10^{-4},$$

$$B(B^- \to D_2^0 \pi^-)B(D_2^0 \to D^{*+} \pi^-) = (1.8 \pm 0.3 \pm 0.3 \pm 0.2) \times 10^{-4}, \quad (4.11)$$

to arrive at

$$B(B^- \to D_2^* \pi^-)B(D_2^* \to D^+ \pi^-, D^{*+} \pi^-) = (5.5 \pm 0.8) \times 10^{-4}. \quad (4.12)$$

Using $B(D_2^0 \to D^+ \pi^-, D^{*+} \pi^-) = 2/3$ followed from the assumption that the $D_2^0$ width is saturated by $D\pi$ and $D^*\pi$, we are led to $B(B^- \to D_2^0 \pi^-) = (7.8 \pm 1.4) \times 10^{-4}$ as shown in Table VII.

From Table VII we see that except $D_1(2427)^0 \pi^-$ the predictions of $B(B^- \to D^{*\pi^0})$ agree with experiment. It is worth mentioning that the ratio

$$R = \frac{B(B^- \to D_2^0(2460)^0 \pi^-)}{B(B^- \to D_1^0(2420)^0 \pi^-)} \quad (4.13)$$
TABLE VII. The predicted branching ratios for $B^- \rightarrow D^{*+0}(\pi^-, \rho^-, \bar{D}_s)$ and $B^- \rightarrow \bar{D}_s^{*+}D^{(*)}$ decays, where $D^{**}$ denotes a generic $p$-wave charmed meson. Experimental results are taken from PDG [15] and Belle [14]. The numbers in parentheses for $\bar{D}_{s0} D$ and $\bar{D}_{s1} D$ productions are fits to the central values of the experimental data. The axial-vector meson mixing angles are taken to be $\theta = 17^\circ$ for $D_1 - D'_1$ systems and $\theta_s = 7^\circ$ for $D_{s1} - D'_{s1}$ systems. The parameters $a_{1,2}$ are taken to be $a_1 = 1.15$ and $a_2 = 0.26$.

| Decay | This Work | KV [36] | CM [37] | KLO [38] | Expt |
|-------|-----------|---------|---------|----------|------|
| $B^- \rightarrow D_0^0(2308)^0\pi^-$ | $7.7 \times 10^{-4}$ | $4.2 \times 10^{-4}$ | $(9.2 \pm 2.9) \times 10^{-4}$ [14] |
| $B^- \rightarrow D_1(2427)^0\pi^-$ | $3.6 \times 10^{-4}$ | $2.4 \times 10^{-4}$ | $(7.5 \pm 1.7) \times 10^{-4}$ [14] |
| $B^- \rightarrow D'_1(2420)^0\pi^-$ | $1.1 \times 10^{-3}$ | $2.1 \times 10^{-3}$ | $(1.0 \pm 0.2) \times 10^{-3}$ [14] |
| $B^- \rightarrow D_2^0(2460)^0\pi^-$ | $6.7 \times 10^{-4}$ | $7.2 \times 10^{-5}$ | $4.1 \times 10^{-4}$ | $3.5 \times 10^{-4}$ | $(7.8 \pm 1.4) \times 10^{-4}$ [14] |
| $B^- \rightarrow D_0^0(2308)^0\rho^-$ | $1.3 \times 10^{-3}$ | & & & & |
| $B^- \rightarrow D_1(2427)^0\rho^-$ | $1.1 \times 10^{-3}$ | & & & & |
| $B^- \rightarrow D'_1(2420)^0\rho^-$ | $2.8 \times 10^{-3}$ | & & $< 1.4 \times 10^{-3}$ [15] | & & |
| $B^- \rightarrow D_2^0(2460)^0\rho^-$ | $1.8 \times 10^{-3}$ | $1.1 \times 10^{-3}$ | $9.8 \times 10^{-4}$ | $< 4.7 \times 10^{-3}$ [15] | & |
| $B^- \rightarrow D_0^0(2308)^0\bar{D}_s^-$ | $8.0 \times 10^{-4}$ | $2.7 \times 10^{-3}$ | & & & |
| $B^- \rightarrow D_1(2427)^0\bar{D}_s^-$ | $9.6 \times 10^{-4}$ | $1.4 \times 10^{-3}$ | & & & |
| $B^- \rightarrow D'_1(2420)^0\bar{D}_s^-$ | $1.3 \times 10^{-3}$ | $5.0 \times 10^{-3}$ | & & & |
| $B^- \rightarrow D_2^0(2460)^0\bar{D}_s^-$ | $4.2 \times 10^{-4}$ | $1.0 \times 10^{-4}$ | $2.7 \times 10^{-4}$ | $4.9 \times 10^{-4}$ | & |
| $B^- \rightarrow D_0^0(2308)^0\bar{D}_s^{*-}$ | $3.5 \times 10^{-4}$ | & & & & |
| $B^- \rightarrow D_1(2427)^0\bar{D}_s^{*-}$ | $6.0 \times 10^{-4}$ | & & & & |
| $B^- \rightarrow D'_1(2420)^0\bar{D}_s^{*-}$ | $1.6 \times 10^{-3}$ | & & & & |
| $B^- \rightarrow D_2^0(2460)^0\bar{D}_s^{*-}$ | $1.1 \times 10^{-3}$ | $1.0 \times 10^{-4}$ | $1.1 \times 10^{-3}$ | $1.2 \times 10^{-3}$ | & |
| $B^- \rightarrow \bar{D}_{s0}(2317)^-D^0$ | $(9.3 \times 10^{-4})$ | 0 | & & & |
| $B^- \rightarrow \bar{D}_{s1}(2460)^-D^0$ | $(3.1 \times 10^{-3})$ | $3.5 \times 10^{-3}$ | & | see text | |
| $B^- \rightarrow \bar{D}_{s0}(2536)^-D^0$ | $1.3 \times 10^{-4}$ | $3.4 \times 10^{-3}$ | & | see text | |
| $B^- \rightarrow \bar{D}_{s2}(2572)^-D^0$ | & | & | & 0 | |
| $B^- \rightarrow \bar{D}_{s0}(2317)^-D^{*-}$ | $5.0 \times 10^{-4}$ | & & & & |
| $B^- \rightarrow \bar{D}_{s1}(2460)^-D^{*-}$ | $1.2 \times 10^{-2}$ | & & & & |
| $B^- \rightarrow \bar{D}_{s0}(2536)^-D^{*-}$ | $5.3 \times 10^{-4}$ | & & & & |
| $B^- \rightarrow \bar{D}_{s2}(2572)^-D^{*-}$ | & | & | & | |
TABLE VIII. Same as Table VII except for neutral B and Bs mesons.

| Decay                  | This Work | KV [36] | CM [37] | KLO [38] | Expt |
|------------------------|-----------|---------|---------|----------|------|
| $B^0 \rightarrow D_0^*(2308)^+\pi^-$ | 2.6 × 10^{-4} | 4.1 × 10^{-4} | | | |
| $B^0 \rightarrow D_1(2427)^+\pi^-$ | 6.8 × 10^{-4} | 1.2 × 10^{-4} | | | |
| $B^0 \rightarrow D'_1(2420)^+\pi^-$ | 1.0 × 10^{-3} | 2.4 × 10^{-3} | | | |
| $B^0 \rightarrow D_2^*(2460)^+\pi^-$ | 6.1 × 10^{-4} | 7.1 × 10^{-5} | 4.1 × 10^{-4} | 3.3 × 10^{-4} | < 2.2 × 10^{-3} [15] |
| $B^0 \rightarrow D_0^*(2308)^+\rho^-$ | 6.4 × 10^{-4} | | | | |
| $B^0 \rightarrow D_1(2427)^+\rho^-$ | 1.6 × 10^{-3} | | | | |
| $B^0 \rightarrow D'_1(2420)^+\rho^-$ | 2.6 × 10^{-3} | | | | |
| $B^0 \rightarrow D_2^*(2460)^+\rho^-$ | 1.7 × 10^{-3} | 1.1 × 10^{-3} | 9.2 × 10^{-4} | < 4.9 × 10^{-3} [15] |
| $B^0 \rightarrow D_0^*(2308)^+\bar{D}_s^-$ | 7.3 × 10^{-4} | 2.6 × 10^{-3} | | | |
| $B^0 \rightarrow D_1(2427)^+\bar{D}_s^-$ | 8.8 × 10^{-4} | 1.3 × 10^{-3} | | | |
| $B^0 \rightarrow D'_1(2420)^+\bar{D}_s^-$ | 1.2 × 10^{-3} | 4.9 × 10^{-3} | | | |
| $B^0 \rightarrow D_2^*(2460)^+\bar{D}_s^-$ | 3.8 × 10^{-4} | 1.0 × 10^{-4} | 2.7 × 10^{-4} | 4.6 × 10^{-4} |
| $B^0 \rightarrow D'_0(2308)^+\bar{D}_s^-$ | 5.5 × 10^{-4} | | | | |
| $B^0 \rightarrow D'_1(2420)^+\bar{D}_s^-$ | 1.5 × 10^{-3} | | | | |
| $B^0 \rightarrow D_2^*(2460)^+\bar{D}_s^-$ | 1.0 × 10^{-3} | 1.0 × 10^{-4} | 1.1 × 10^{-3} | 1.1 × 10^{-3} | |
| $B^0 \rightarrow B_{s0}(2317)^-D^+$ | (8.6 × 10^{-4}) | 0 | | | see text |
| $B^0 \rightarrow B_{s1}(2460)^-D^+$ | (2.8 × 10^{-3}) | 3.4 × 10^{-3} | | | see text |
| $B^0 \rightarrow B_{s1}(2536)^-D^+$ | 1.2 × 10^{-4} | 3.3 × 10^{-3} | | | |
| $B^0 \rightarrow B_{s2}(2572)^-D^+$ | – | 0 | | | |
| $B^0 \rightarrow B_{s0}^*(2317)^-D^{**}$ | 4.6 × 10^{-4} | | | | |
| $B^0 \rightarrow B_{s1}^*(2460)^-D^{**}$ | 1.1 × 10^{-2} | | | | |
| $B^0 \rightarrow B_{s1}^*(2536)^-D^{**}$ | 4.9 × 10^{-4} | | | | |
| $B^0 \rightarrow B_{s2}^*(2572)^-D^{**}$ | – | | | | |
| $B^0 \rightarrow D_{s0}(2317)^+\pi^-$ | 3.3 × 10^{-4} | | | | |
| $B^0 \rightarrow D_{s1}(2460)^+\pi^-$ | 5.2 × 10^{-4} | | | | |
| $B^0 \rightarrow D_{s1}'(2536)^+\pi^-$ | 1.5 × 10^{-3} | | | | |
| $B^0 \rightarrow D_{s2}'(2572)^+\pi^-$ | 7.1 × 10^{-4} | | | | |
| $B^0 \rightarrow D_{s0}(2317)^+\rho^-$ | 8.3 × 10^{-4} | | | | |
| $B^0 \rightarrow D_{s1}(2460)^+\rho^-$ | 1.3 × 10^{-3} | | | | |
| $B^0 \rightarrow D_{s1}'(2536)^+\rho^-$ | 3.8 × 10^{-3} | | | | |
| $B^0 \rightarrow D_{s2}'(2572)^+\rho^-$ | 1.9 × 10^{-3} | | | | |
The present limit on $D_1(2427)^0\pi^-$ is too small by a factor of 2. This is ascribed to a destructive interference between color-allowed and color-suppressed tree amplitudes because the form factors $V^{BD_1^1/2}_0$ and $V^{BD_3^3/2}_0$ have signs opposite to that of $F^{B\pi}_1$ as required by heavy quark symmetry (see Eq. (4.2) and Table VI). In contrast, the production of $D_1(2427)^+\pi^-$ is larger than $D_1(2427)^0\pi^-$ by a factor of about 2 because the former does not receive the destructive contribution from internal $W$-emission. Hence, a measurement of the ratio $D_1(2427)^0\pi^-/D_1(2427)^+\pi^-$ can be used to test the relative signs of various form factors as implied by heavy quark symmetry. Note that for the aforementioned discrepancy between theory and experiment. Moreover, the ratio $D_0^*+\pi^-/D_0^0\pi^-$ is predicted to be 0.34 instead of unity because of the absence of the color-suppressed tree contribution to the former.

At a first glance, it appears that the prediction $B(B^- \rightarrow D_1(2420)^0\rho^-) = 2.8 \times 10^{-3}$ already exceeds the experimental limit $1.4 \times 10^{-3}$ [15]. However, it should be noticed that the $D_1^0\rho^-$ rate is about three times larger than that of $D_1^0\pi^-$ as expected from the factorization approach and from the ratio $f_\rho/f_\pi \approx 1.6$ (see Table IX below). Hence, it appears that the present limit on $D_1^0\rho^-$ is not consistent with the observed rate of $D_1^0\pi^-$. Of course, it is crucial to measure $B \rightarrow D_1'(2420)\rho$ in order to clarify the issue.

Apart from the external $W$-emission diagram, the $D^{**}\pi(\rho)$ productions in neutral $B$ decays also receive $W$-exchange contributions which are neglected in the present work. This will constitute a main theoretical uncertainty for $B \rightarrow D^{**}\pi^- (\rho^-)$.

For $B \rightarrow \overline{D}D^*_s$ decays, their factorizable amplitudes are rather simple

$$A(B \rightarrow \overline{D}D^*_s) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \langle D^*_s|\bar{s}c|0\rangle \langle D|\bar{c}b|\overline{B}\rangle. \quad (4.14)$$

The decay rates are thus governed by the decay constants. Since the tensor meson cannot be produced from the $V-A$ current, the $B$ decay into $\overline{D}D^*_2D^{(*)}$ is prohibited under the factorization hypothesis. For $D_{s0}^*$ and $D_{s1}$ mesons, it is expected that $f_{D_{s0}^*} \sim f_{D_{s1}(2460)} \gg f_{D_{s1}(2536)}$ as $f_{D_{s1}^1/2} = f_{D_{s0}^*}$ and $f_{D_{s1}^{1/2}} = 0$ in the heavy quark limit [see Eq. (3.7)]. When the charmed quark mass is finite, the $D_{s1}^{1/2} - D_{s2}^{3/2}$ mixing effect has to be taken into account via

$$f_{D_{s1}} = f_{D_{s1}^{1/2}} \cos \theta_s + f_{D_{s1}^{3/2}} \sin \theta_s,$$

$$f_{D_{s1}^*} = -f_{D_{s1}^{1/2}} \sin \theta_s + f_{D_{s1}^{3/2}} \cos \theta_s,$$

$$\sin \theta_s = \frac{f_{D_{s1}}^2 - f_{D_{s1}^*}^2}{f_{D_{s1}^{1/2}}^2}.$$
where use of Eqs. (2.8) and (3.11) has been made. Consequently, it is anticipated that \( \overline{D}D_{s0}^* \gtrsim \overline{D}D_{s1}(2460) \gg \overline{D}D_{s1}(2536) \) after taking into account phase space corrections. However, the recent data on \( \overline{D}D_{s0}^* \) production indicate a surprise.

The results of Belle measurements read [40]

\[
\begin{align*}
B[B \to \overline{D}D_{s0}^*(2317)]B[D_{s0}^*(2317) \to D_s \pi^0] & = (8.5^{+2.1}_{-1.9} \pm 2.6) \times 10^{-4}, \\
B[B \to \overline{D}D_{s1}(2460)]B[D_{s1}(2460) \to D_s^* \pi^0] & = (17.8^{+4.5}_{-3.9} \pm 5.3) \times 10^{-4}, \\
B[B \to \overline{D}D_{s1}(2460)]B[D_{s1}(2460) \to D_s \gamma] & = (6.7^{+1.3}_{-1.2} \pm 2.0) \times 10^{-4}.
\end{align*}
\]

Since \( D_{s0}^*(2317) \) is dominated by its hadronic decay to \( D_s \pi^0 \), the branching ratio of \( B \to D D_{s0}^*(2317) \) is of order \( 0.9 \times 10^{-3} \). This means that the production rate of \( \overline{D}D_{s0} \) in \( B \) decays is smaller than \( B(B^+ \to \overline{D}D_s) = (1.3 \pm 0.4)\% \) [15] by one order of magnitude. Since this decay proceeds only via external \( W \)-emission, it can be used to determine the decay constant of \( D_{s0}^* \). It is found that \( f_{D_{s0}^*} \sim 60 \text{ MeV} \) \( ^{\dagger\dagger} \) which seems too small, recalling that \( f_{D_{s0}^*} \sim 160 \text{ MeV} \) is needed to account for the production of \( D_{s0}^* \pi^- \). To estimate the branching ratios of \( D_s^* \pi^0 \) and \( D_s \gamma \) in \( D_{s1}(2460) \) decay, we need some experimental and theoretical inputs. Besides the values \( 0.38 \pm 0.12 \) and \( 0.63 \pm 0.21 \) measured by Belle for the ratio \( D_s \gamma / D_s \pi^0 \) [see Eq. (2.20)], the ratios \( D_{s0}^* \gamma / D_{s0}^* \pi^0 \) and \( D_{s0}^* \pi^+ \pi^- / D_{s0}^* \pi^0 \) are found to be less than 0.16 and 0.08, respectively, by CLEO [13]. Theoretically, the \( M1 \) transition \( D_{s1} \to D_{s0} \gamma \) turns out to be quite small [41]. Assuming that the \( D_{s1}(2460) \) decay is saturated by \( D_s^* \pi^0 \), \( D_s \gamma \), \( D_{s0}^* \gamma \) and \( D_s \pi \pi \), we are led to

\[
0.56 \lesssim \mathcal{B}(D_{s1}(2460) \to D_{s0}^* \pi^0) \lesssim 0.75,
\]

where the averaged value \( 0.44 \pm 0.10 \) for the ratio \( D_{s0}^* \gamma / D_{s0}^* \pi^0 \) has been employed. This in turn implies \( \mathcal{B}(B \to \overline{D}D_{s1}(2460)) = (1.5 \sim 4.4) \times 10^{-3} \). As a result, the decay constant of \( D_{s1}(2460) \) is found to be

\[
f_{D_{s1}(2460)} \approx 100 - 175 \text{ MeV}.
\]

This is in sharp contrast to the heavy quark symmetry expectation that the decay constants of \( D_{s0}^* \) and \( D_{s1} \) become identical in the heavy quark limit. Therefore, the large disparity between \( f_{D_{s0}^*} \) and \( f_{D_{s1}} \) is a surprise. It is not clear to us what is the underlying reason for the discrepancy between theory and experiment.

Although the \( B \) decay into \( \overline{D}_{s2} D^{(*)} \) is prohibited under the factorization hypothesis, nevertheless it can be induced via final-state interactions (FSIs) and/or nonfactorizable contributions. For example, it can be generated via the color-allowed decay \( B^- \to \overline{D}_s D_{s0}^0 \)

\( ^{\dagger\dagger} \)The upper limit on the ratio \( \Gamma(D_{s0}^* \to D_s \gamma) / \Gamma(D_{s0}^* \to D_s \pi^0) < 0.059 \) was set recently by CLEO [13].

\( ^{\dagger\dagger} \)Interestingly, this happens to be the original estimate of \( f_{D_{s0}^*} \) made in [4] for \( D_{s0}^* \) in the four-quark state.
followed by the rescattering process $\overline{D}_s D_s^{*0} \to \overline{D}_s^s D^0$. Since the nonfactorizable term is of order $c_2/N_c$ with the Wilson coefficient $c_2(m_b) \approx -0.20$, it is likely to be suppressed relative to the FSIs. Hence, an observation of $B \to \overline{D}_s^s D^{(*)}$ could imply the importance of final-state rescattering effects.

Since heavy quark symmetry implies $f_{D_{s1}(2536)} \ll f_{D_{s1}(2460)}$, it is important to measure the $B$ decay into $\overline{D}_{s1}(2536)D^{(*)}$ to see if it is suppressed relative to $\overline{D}_{s1}(2460)D^{(*)}$ to test heavy quark symmetry. In order to compute $\mathcal{B} \to \overline{D}_{s1}^s D^*$ we have fixed the decay constants of $D_{s0}^*, D_{s1}^{1/2}$ and $D_{s1}^{3/2}$ as shown in Eq. (3.11) so that the resulting production rates of $\overline{D}_{s0}^s D^0$ and $\overline{D}_{s1}^s D^0$ are consistent with experiment (see Tables VII and VIII). We see that the decay $\mathcal{B} \to \overline{D}_{s1}(2460)D^{*0}$ has the largest branching ratio of order $10^{-2}$ in two-body hadronic $B$ decays involving a $p$-wave charmed meson in the final state. It is essential to test all these anticipations in the near future. A recent CLEO measurement yields [42]

$$\mathcal{B}[B^- \to (\bar{D}_s + \bar{D}_s^*)(D_1^0 + D_1^0 + D_2^0)] = (2.73 \pm 0.78 \pm 0.48 \pm 0.68)\%.$$ (4.19)

Our prediction of $6 \times 10^{-3}$ is slightly small.

It is interesting to consider the ratios $\Gamma(\mathcal{B} \to D^{**}V)/\Gamma(\mathcal{B} \to D^{**}P)$ and $\Gamma(\mathcal{B} \to D^{**}V)/\Gamma(\mathcal{B} \to D^{**}P)$. The calculated results are shown in Table IX. Several remarks are in order: (i) The ratios $D^{*0} \rho^-/D^{*0} \pi^-$ and $D^{***} \rho^-/D^{***} \pi^-$ for $D^{**} = D_0^*$ and $D_1$ are not the same as the former receives an additional color-suppressed internal $W$-emission contribution. (ii) Whether the ratio $D^{**}V/D^{**}P$ is greater than unity or not depends essentially on the ratio of the decay constants. For example, $D_0^* \bar{D}_s^*/D_0^* \bar{D}_s = 0.43$ if the decay constants of $D_0^*$ and $D_s$ are similar, while $D_0^{*+} \rho^-/D_0^{*+} \pi^- = 2.5$ for $f_\rho/f_\pi = 1.6$. It should be stressed that the proximity of the ratio $D^{***} \rho^-/D^{***} \pi^-$ to 2.5 has less to do with the three degrees of freedom of $\rho$, rather it is mainly related to the decay constant ratio of $f_\rho/f_\pi$. (iii) The ratio of $D_s^{*} \rho^-/D_s^{*} \pi^-$ in $B_s$ decay is the same as $D^{***} \rho^-/D^{***} \pi^-$ in $B$ decay as they proceed via external $W$-emission.

**TABLE IX.** The ratios $\Gamma(\mathcal{B} \to D^{**}V)/\Gamma(\mathcal{B} \to D^{**}P)$ and $\Gamma(\mathcal{B} \to D^{**}V)/\Gamma(\mathcal{B} \to D^{**}P)$. The last column is for the ratio $\Gamma(\mathcal{B}_{s} \to D^{**}_{s} \rho^-)/\Gamma(\mathcal{B}_{s} \to D^{**}_{s} \pi^-)$.

| $D^{**}(D_{s0}^*)$ | $D^{*0}\rho^-_{D^{**}\pi}$ | $D^{***}\rho^-_{D^{***}\pi}$ | $D^{***}\rho^+_{D^{***}\pi}$ | $D^{**}\rho^+_{D^{**}\pi}$ | $D^{***}\rho^+_{D^{***}\pi}$ |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $D_0^*(D_{s0}^*)$ | 1.7             | 2.4             | 2.5             | 0.43            | 0.54            |
| $D_1(D_{s1}^*)$   | 3.1             | 2.4             | 3.8             | 0.63            | 2.5             |
| $D_1'(D_{s1}^*)$  | 2.6             | 2.6             | 4.0             | 1.2             | 2.6             |
| $D_2(D_{s2}^*)$   | 2.7             | 2.7             | 2.7             | 2.7             | 2.7             |

Because the scalar resonances $D_0^*$ and $D_1$ have widths of order 300 MeV, we have checked the finite width effects on their production in $B$ decays and found that the conventional narrow width approximation is accurate enough to describe the production of broad resonances owing to the large energy released in hadronic two-body decays of $B$ mesons.
C. Comparison with other works

The decays $B \to D^{**}(\pi, D_s)$ and $B \to D_s^{**}D$ have been studied previously by Katoch and Verma (KV) [36]. López Castro and Muñoz (CM) [37], Kim, Lee and Oh (KLO) [38], and Lee [43] also have a similar study with focus on the tensor charmed meson production. We shall comment on their works separately.

In the paper of KLO, the $B \to D_s^*$ form factors are evaluated using the ISGW2 model. However, the predicted rates for $D_s^*\pi$ and $D_s^*\rho$ by KLO are smaller than ours by a factor 2 (see Tables VII and VIII); that is, their $B \to D_s^*$ form factor differs from ours roughly by a factor of $\sqrt{2}$. In contrast, the results for $D_s^*D_1^{(*)}$ are similar owing to the fact that KLO employ $f_{D_s} = 280$ MeV and $f_{D_s^*} = 270$ MeV which are larger than ours [see Eq. (3.11)]. Instead of employing the QCD sum rule to study the universal function $\tau_{3/2}(\omega)$ to the leading order [43]. He found $\mathcal{B}(B^- \to D_s^{*0}\pi^-) = 12.9 \times 10^{-4}$ for $a_1 = 1.15$, which is two times larger than ours.

The work of KV and CM is based on the original version of the ISGW model. However, as stressed by KLO [38], the exponentially decreasing behavior of the form factors in the ISGW model is not realistic and justified. This has been improved in the ISGW2 model which provides a more realistic description of the form-factor behavior at large ($q_m^2 - q^2$).

The values of form factors at small $q^2$ in the ISGW2 model can be a few times larger than that obtained in the ISGW model as the maximum momentum transfer $q_m^2$ in $B$ decays is large. However, the expected form-factor suppression does not appear in the calculations of KV as they calculated the form factors at $q_m^2$ and then employed them even at low $q^2$. In contrast, CM did compute the form factors at proper $q^2$. The fact that CM and KLO have similar results for $B \to D_s^*(\rho)$ and $B \to D_s^*D_1^{(*)}$ (see Tables VII and VIII) is surprising as the $B \to D_s^*$ transition is evaluated in two different versions of the ISGW model. To check this, we find that $\eta(q^2) \equiv k(q^2) + b_+(q^2) + b_-(q^2) = 0.286$ and $0.386$ for $q^2 = m_\pi^2$ in the ISGW and ISGW2 models, respectively, which in turn imply the respective branching ratios, $6.7 \times 10^{-4}$ and $3.8 \times 10^{-4}$ for $B^- \to D_s^{*0}\pi^-$. Therefore, our result obtained in the ISGW model is consistent with that of CM and the estimate of $\eta(q^2)$ within the ISGW2 model by KLO is likely too small, as noted in passing.

The axial-vector charmed meson production considered by KV is for $D_1(1P_1)$ and $D_1(3P_1)$ rather than $D_1^{1/2}$ and $D_1^{3/2}$. Therefore, the expressions of the decay amplitudes involving $D_1$ or $D_{a1}$ by KV should be modified by taking into account a proper wave function combination, Eq. (2.1). For the decay constants, KV assumed that $f_{D_1^0} = 0$ and $f_{D_1(1P_1)} = 0$. As a consequence, $B^- \to D_1^{*0}\pi^-$ is too small compared to experiment and the decay into $D_{a0}^*D$ is not allowed. This is not consistent with the heavy quark symmetry relation $f_{D_1^{3/2}} = f_{D_1^{1/2}}$. Finally, the predicted rate of $B^- \to D_2^{*0}\pi^-$ by KV is too small by one order of magnitude compared to experiment. This is ascribed to a missing factor of $\left(m_B/m_T\right)^2$ in their calculation of decay rates.
V. CONCLUSIONS

The hadronic decays of $B$ mesons to a $p$-wave charmed meson in the final state are studied. Specifically we focus on the Cabibbo-allowed decays $\overline{B} \to D^{**}(\rho)$, $D^{**}D_s^{(*)}$, $\overline{D}_s^{(*)}D^{(*)}$ and $\overline{B}_s \to D_s^{**}\pi(\rho)$. The main conclusions are as follows:

- We apply heavy quark effective theory in which heavy quark symmetry and chiral symmetry are unified to study the strong decays of $p$-wave charmed mesons and determine the magnitude of the $D_1^{1/2} - D_3^{1/2}$ mixing angle. In contrast, the present upper limits on the widths of $D_{s1}(2460)$ and $D_{s1}'(2536)$ do not provide any constraints on the $D_1^{1/2} - D_3^{1/2}$ mixing angle $\theta_s$. Therefore, we appeal to the quark potential model to extract $\theta_s$.

- Various form factors for $B \to D^{**}$ transitions and their $q^2$ dependence are studied using the improved version of the Isgur-Scora-Grinstein-Wise quark model. Heavy quark symmetry constraints are respected in this model calculation.

- The predicted branching ratios for $B^- \to D^{**}\pi^-$ agree with experiment except $D_1(2427)0^\pi^-$. The $D_1^{1/2} - D_3^{1/2}$ mixing angle is preferred to be positive in order to avoid a severe suppression on the production of $D_1(2427)0^\pi^-$. The decay $B^- \to D'_1(2420)0^\rho^-$ is predicted at the level of $3 \times 10^{-3}$. Although it exceeds the present experimental limit of $1.4 \times 10^{-3}$, it leads to the ratio $D'_1(2420)\rho^-/D'_1(2420)\pi^- \approx 2.6$ as expected from the factorization approach and from the ratio $f_\rho/f_\pi \approx 1.6$. Therefore, it is crucial to have a measurement of this mode to test the factorization hypothesis.

- The predicted rate for $B^- \to D_1(2427)0^\pi^-$ is too small by a factor of 2 owing to a destructive interference between color-allowed and color-suppressed tree amplitudes as the relevant form factors for $B \to D_1^{1/2}$ and $B \to D_3^{1/2}$ transitions are negative. It is crucial to measure the production of $D_1(2427)^+\pi^-$ to see if it is larger than $D_1(2427)0^\pi^-$ by a factor of about 2 because the former does not receive the internal $W$-emission contribution.

- Under the factorization hypothesis, the production of $\overline{D}_{s2}^*D^{(*)}$ in $B$ decays is prohibited as the tensor meson cannot be produced from the $V – A$ current. Nevertheless, the decays $B \to \overline{D}_{s2}^*D^{(*)}$ can be induced via final-state interactions and/or the nonfactorizable contributions. Since the latter is suppressed by the order of $c_2/N_c$, an observation of $B \to \overline{D}_{s2}^*D^{(*)}$ could imply the importance of final-state rescattering effects.

- For $\overline{B} \to \overline{D}_s^{**}D$ decays, it is expected that $\overline{D}_{s0}^*D \gtrsim \overline{D}_{s1}D$ as the decay constants of the multiplet $(D_{s0}^*,D_{s1})$ become identical in the heavy quark limit. The experimental observation indicates that $\overline{D}_{s0}D/\overline{D}_{s1}D \approx (0.2 – 0.6)$ and that $f_{D_{s1}} \approx (100 – 175)$ MeV and $f_{D_{s0}} \sim 60$ MeV. The less abundant production of $\overline{D}_{s0}D$ than $\overline{D}_{s1}D$ and the large disparity between $f_{D_{s1}}$ and $f_{D_{s0}}$ are surprising. The reason for the discrepancy between
theory and experiment remains a puzzle. In the meantime, it is also important to measure the decay to $D_{s1}(2536)D^{(*)}$ to see if it is suppressed relative to $D_{s1}(2460)D^{(*)}$ to test the heavy quark symmetry relation $f_{D_{s1}(2536)} \ll f_{D_{s1}(2460)}$.

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Note Added:

After this work was completed, we noticed the appearance of the related works on $B \to D\bar{D}_{s}^{**}$ decays by C.H. Chen and H.n. Li, hep-ph/0307075; A. Datta and P.J. O’Donnell, hep-ph/0307106; M. Suzuki, hep-ph/0307118.
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