Creffield and Sols Reply: Benenti et al. [1] assert that in our Letter [2] we claimed that the ratchet current we observed for time-symmetric driving would persist indefinitely. Their assertion is false. In our Letter we clearly indicated that, in general, the ratchet currents would be transient, and indeed wrote that we estimated them only to be “stable over time scales ... of the order of 50 driving periods”. Unfortunately Benenti et al. appear not to have read our paper with sufficient care to have noted our discussion of this point, since we did not claim, or even imply, that the ratchet currents would be of infinite duration.

To arrive at our estimate for the stability of the current, we used a technique developed in Ref. [3] to estimate the Ehrenfest time of the system. In our study we considered a completely coherent time evolution, and accordingly the current is given by a coherent sum

$$I(t) = \sum_{m,n} c_n^* c_m e^{i(\epsilon_n - \epsilon_m)} \int_0^{2\pi} dx \langle \phi_n(t) | p_x | \phi_m(t) \rangle$$

(1)

where $c_n$ are expansion coefficients in the Floquet basis, $\epsilon_n$ are the quasienergies, $| \phi_m(t) \rangle$ are the Floquet states, and $p_x$ is the standard momentum operator. It is important to note the off-diagonal interference terms $\exp [i(\epsilon_n - \epsilon_m)]$. If the system were strongly chaotic, level repulsion would imply that the quasienergy separations are generally large, and so these interferences would rapidly average to zero. This yields the approximate formula given in Eq. 1 of Ref. [1], in which solely the diagonal terms of the current are retained, collapsing the coherent sum to an incoherent one. This strong chaoticity would correspond to a short Ehrenfest time, and so our analysis would similarly predict a short time scale for the stability of the ratchet current.

When the quasienergy spectrum contains degeneracies the corresponding interference terms in Eq. [1] will not decay (for exact degeneracies), or will only decay extremely slowly (when the degeneracy is approximate). Although the analysis of Benenti et al. cannot describe this situation, our approach would simply yield a longer Ehrenfest time, indicating the enhanced stability of the current. Such a quasidegeneracy is actually present (see Fig. 1) in the numerical results presented in the Comment. For a value of the asymmetry parameter $\alpha = 0.32$, a very narrow avoided crossing appears, producing the long-lived current plotted in the inset on Fig. 1 of Ref. [1]. The conclusion of Benenti et al. that “no asymptotic directed transport occurs for any value of K” is thus not generally correct – it depends on the detailed form of the quasienergy spectrum.

Benenti et al. correctly note that “the stroboscopically averaged current... remains finite forever”. We do not dispute this point, but it is irrelevant. This would be an issue only if we had attempted to deduce the time scale for the decay of the current by making a fit of the time-dependence of the stroboscopically averaged current. As we emphasise above, this was not our procedure. Even making use of the continuous time-average proposed by Benenti et al., in place of the more experimentally-relevant stroboscopic average plotted in Fig. 3 of Ref. [2], the conclusions of our Letter would be unaffected. In Fig. [2] we show the decay rates of the continuously-averaged current, which clearly show that even for time-symmetric driving, significant ratchet currents are produced over timescales that are very long in comparison to typical experimental observation times [1]. Although the interacting case ($g \neq 0$) is not amenable to Floquet analysis, very similar results are numerically obtained for the values of nonlinearity considered in Fig. 3 of Ref. [2].

Benenti et al. further attempt to support their case by considering the behavior of the harmonic oscillator. This example is trivial; it is not even periodically-driven. A more telling comparison would be with the phenomenon of dynamical localization [3]. Here a particle on a lattice, subjected to a driving potential, periodically expands and collapses when the parameters of the driving are adjusted to certain specific ratios. Viewed stroboscopically the particle appears to be frozen. The purely stroboscopic character of this phenomenon does not prevent it from being a genuine physical effect, as reflected in the name “dynamical localization”.

In summary, in our Letter we never claimed that for time-symmetric driving a ratchet current would last forever (although in the present Reply we point out that it could be possible if exact quasienergy degeneracies existed). A stroboscopic simulation may indeed overestimate the decay time of the ratchet current. However, our decay estimate was based on general quantum chaos theory arguments. Moreover, even a continuously time-averaged current may exhibit ratchet behavior for times longer than present experimental times. The conclusions of our Letter thus remain unaffected.

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[1] G. Benenti, et al., Phys. Rev. Lett. 104, 228901 (2010).
[2] C.E. Creffield and F. Sols, Phys. Rev. Lett. 103, 200601 (2009).
[3] J. Martin, B. Georgeot, and D. L. Shepelyansky, Phys. Rev. Lett. 101, 074102 (2008).
FIG. 1: Quasienergy spectrum of the ratchet system for timesymmetric driving ($K = 2.4$, $\omega = 1$, $\beta = 0$). The majority of the quasienergies show little dependence on the spatial asymmetry $\alpha$, but the narrow avoided crossings (with a gap of $\Delta \epsilon \approx 0.0014$) at $\alpha = \pm 0.32$ (highlighted by red circles) give rise to long-lived transient currents, with a duration much longer than typical experimental observation times, even though the temporal symmetry of the driving is not broken.

FIG. 2: Decay of the continuously averaged current $\langle I(t) \rangle$ for the strongly driven ratchet system, $K = 2.4$, $\omega = 1$, for different asymmetry parameters $\alpha$ and $\beta$. A typical experimental observation time, indicated by the vertical green line, is taken from the recent work of the Bonn group [4]. For $\beta = 0.2$ (black solid line) time-symmetry of the driving is explicitly broken and $\langle I(t) \rangle$ approaches a non-zero asymptotic value. When $\beta = 0$, however, the time-symmetry of the driving is not broken and asymptotically the ratchet current must decay to zero. For $\alpha = 0.32$ (red dashed line) the average current decays extremely slowly, due to a narrow avoided crossing in the quasienergy spectrum (see Fig. 1). We also plot the stroboscopically averaged value of the current (green dot-dashed line) for these driving parameters, which asymptotically approaches a constant value. For other values of $\alpha$ (blue dash-dot-dot line) the current decays more rapidly, but nonetheless remains significant over timescales much longer than those used in experiment.

[4] T. Salger, et al., Science 326, 1241 (2009).
[5] D H. Dunlap and V.M. Kenkre, Phys. Rev. B 34, 3625 (1986).