Consistent massive graviton on arbitrary background

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We obtain the fully covariant linearized field equations for the metric perturbation in the de Rham-Gabadadze-Tolley (dRGT) ghost free massive gravities. For a subset of these theories, we show that the non dynamical metric that appears in the dRGT setup can be completely eliminated leading to the theory of a massive graviton moving in a single metric. This has a mass term which contains non trivial contributions of the space-time curvature. We show further how 5 covariant constraints can be obtained including one which leads to the tracelessness of the graviton on flat space-time and removes the Boulware-Deser ghost. The 5 constraints are obtained for a background metric which is arbitrary, i.e. which does not have to obey the background field equations.

Lately there has been a renewal of interest in massive gravity with interesting applications to cosmology (see e.g. Ref. 1 for reviews). The only consistent linear theory for a massive graviton on flat space-time has been known for a long time since the work of Fierz and Pauli 2. It propagates 5 degrees of freedom of positive energy, those of a transverse, traceless, symmetric, two times covariant tensor $h_{\mu\nu}$. It can easily be extended to an Einstein space-time background keeping the same number of propagating polarizations 3. However, a similar theory for an arbitrary background metric has not been written so far. A starting point to do so is the set of fully non linear theories formulated by de Rham, Gabadadze and Tolley (dRGT in the following) 4. Such a theory was shown to contain only 5 dynamical degrees of freedom and hence is devoid from a pathology long thought unavoidable: the presence of an extra ghost-like sixth degree of freedom in dRGT family a consistent linear theory for a massive graviton moving in a space-time endowed with two background metrics which has various drawbacks (see e.g. Ref. 6). Moreover, this linearization is not easy, in part because the dRGT theories involve a matrix square root of a transverse, traceless, symmetric, two times covariant $\mathbf{g}_{\mu\nu}$. It can easily be extended to an Einstein space-time and removes the Boulware-Deser ghost 15. This last constraint involves combinations of the curvature of the background metric which become trivial when this metric describes a flat space-time or a more general Einstein space-time. Our starting point is the set of massive gravity theories defined by the following action in 4 dimensions 4

$$S_{g,m} = M_p^2 \int d^4 x \sqrt{|g|} \left[ R(g) - 2m^2 \sum_{n=0}^{3} \beta_n e_n(S) \right] , \quad (2)$$

the $\beta_n$ being dimensionless parameters, and $e_n(S)$ the n’th order elementary symmetric polynomial of the eigenvalues of its matrix argument $S$. One has in particular $e_0 = 1$ and $e_1 = \text{Tr}[S]$, where here and henceforth $\text{Tr}[X] = X_{\rho}^{\rho}$ indicates a matrix trace operation and we do not write out any more the functional dependence of the $e_n$ when they depend only on $S$ (i.e. it is to be understood that $e_n \equiv e_n(S)$). The $e_n$ can be constructed iteratively (with $e_0 = 1$) from the relation

$$e_n = -\frac{1}{n} \sum_{k=1}^{n} (-1)^k \text{Tr}[S^k] e_{n-k} , \quad n \geq 1 , \quad (3)$$

where $S^k$ is the k-th power of the tensor $S_{\mu\nu}$ (considered as a matrix), and $S^0$ is just the identity. The field equations deriving from the action 2 for the dynamical metric $g_{\mu\nu}$ are

$$E_{\mu\nu} \equiv G_{\mu\nu} + m^2 V_{\mu\nu} = 0 , \quad (4)$$

where $G_{\mu\nu}$ is the Einstein tensor built from the metric $g_{\mu\nu}$, and $V_{\mu\nu}$ is given by

$$V_{\mu\nu} = g_{\mu\rho} \sum_{n=0}^{3} \sum_{k=0}^{n} (-1)^{n+k} \beta_n [S^{n-k}]_{\rho}^{\rho} \epsilon_k . \quad (5)$$

The next step in our derivation consists in linearizing these field equations around a background solution for
the dynamical metric $g_{\mu\nu}$, calling $h_{\mu\nu}$ the small perturbation of this metric. This may seem at first sight an easy task, however, it is not because it involves in general computing the variation at first order in $h_{\mu\nu}$ of the matrix square root $S$. This variation $\delta S$ obeys, (with obvious notations) as seen from \[1,\]
\[S^\mu_{\nu}(\delta S)^\nu_{\sigma} + (\delta S)^\mu_{\nu}S^\nu_{\sigma} = \delta S^\mu_{\sigma}, \quad (6)\]
which is a special kind of Sylvester matrix equation, 

\[
\frac{\delta S^\lambda_{\mu}}{\delta g_{\sigma\tau}} = \frac{1}{2}g^{\rho\lambda}\left[ e_4 c_1 \left( \delta^\rho_{\sigma}\delta^\mu_{\tau} + \delta^\sigma_{\tau}\delta^\mu_{\rho} - g_{\mu\rho}g_{\sigma\tau} \right) + e_4 c_2 \left( S^\rho_{\mu}\delta^\sigma_{\tau} + S^\sigma_{\tau}\delta^\rho_{\mu} - S_{\mu\rho}g^{\sigma\tau} - g_{\mu\sigma}S^{\rho\tau} \right) - e_3 c_1 \left( \delta^\rho_{\sigma}\delta^\mu_{\tau} + \delta^\sigma_{\tau}\delta^\mu_{\rho} \right) + e_4 c_3 \left[ \delta^\rho_{\mu}[S^2]_{\sigma\tau} + \delta^\sigma_{\tau}[S^2]_{\rho\mu} - g^{\rho\sigma}[S^2]_{\mu\tau} + \delta^\rho_{\sigma}[S^2]_{\mu\tau} + \delta^\sigma_{\tau}[S^2]_{\rho\mu} - g_{\mu\sigma}[S^2]_{\rho\tau} \right] + (e_2 c_1 - e_4 c_3 - e_3 c_2) S_{\mu\rho} \delta S^{\sigma\tau} \\
- e_3 c_2 \left( S^\rho_{\mu}\delta^\sigma_{\tau} + S^\sigma_{\tau}\delta^\rho_{\mu} - S_{\mu\rho}g^{\sigma\tau} - g_{\mu\sigma}S^{\rho\tau} \right) + (e_3 c_3 - e_1 c_1) \left( S^{\rho\sigma}[S^2]_{\mu\tau} + S_{\mu\sigma}[S^2]_{\rho\tau} \right) - (e_3 c_2 - e_3 c_3) \left( [S^2]_{\mu\rho}^\sigma[S^2]_{\tau\mu}^\rho + [S^2]_{\mu\sigma}^\rho[S^2]_{\tau\mu}^\rho \right) + e_4 [S^2]_{\mu\rho}^\sigma[S^2]_{\tau\mu}^\rho + 2 \left( [S^3]_{\mu\rho}^\sigma[S^3]_{\rho\sigma} + S_{\mu\sigma}[S^3]_{\rho\sigma} \right) + 2 \left( [S^3]_{\mu\rho}^\sigma[S^3]_{\rho\sigma} + [S^2]_{\mu\sigma}[S^3]_{\rho\sigma} \right) + 3 \left( [S^3]_{\mu\rho}^\sigma[S^3]_{\rho\sigma} \right), \quad (7)\]

where the coefficients $c_i$ are given by,
\[c_1 = \frac{e_3 - e_1 e_2}{-e_1 e_2 e_3 + e_2^2 + e_1^2 e_4}, \quad c_2 = \frac{-e_1^2 e_3 + e_3^2 + e_1^2 e_4}{e_1 e_2 e_3 + e_2^2 + e_1^2 e_4}, \quad c_3 = \frac{-e_1}{-e_1 e_2 e_3 + e_2^2 + e_1^2 e_4}, \quad c_4 = \frac{e_3 - e_1^3}{-e_1 e_2 e_3 + e_2^2 + e_1^2 e_4}, \quad (8)\]
and here and henceforth all indices are moved with the metric $g_{\mu\nu}$. Obviously, expression \[7\] makes sense only if $-e_1 e_2 e_3 + e_2^2 + e_1^2 e_4$ does not vanish, which in turn can be shown to be equivalent to the non intersection of the spectra of $S^\mu_{\nu}$ and $-S^\nu_{\mu}$ mentioned above.

This result was checked to agree with the one we know from the mathematical literature \[12\] using in particular non trivial identities -syzygies- which also play a fundamental role for the derivation of the covariant constraints below. These identities, as a consequence of the "second fundamental theorem" of invariant theory \[14\], can be derived using the Cayley-Hamilton theorem stating that for an arbitrary $4 \times 4$ matrix $M$, one has
\[M^4 = e_1 (M) M^3 - e_2 (M) M^2 + e_3 (M) M - e_4 (M) 1. \quad (9)\]
One can then apply this to a matrix $M$ built out of 4 arbitrary matrices $A, B, C, D$ and four arbitrary real numbers $\{x_i\}$ in the form, $M = x_0 A + x_1 B + x_2 C + x_3 D$. Now, because the $\{x_i\}$ as well as the matrices $A, B, C, D$ are arbitrary, it means that in equation \[9\] the terms which have the same degree of homogeneity in the $\{x_i\}$ must each yield separate identities between the matrices $A, B, C, D$. Once these identities are obtained, one can replace in them $A$ by $h$, $B$ by $S$, $C$ by $S^2$ and $D$ by $S^3$ to get non trivial matrix syzygies, denoted here as $[L_x]^{\mu\nu} = 0$, between the tensors of interest here. Notice that the above Cayley-Hamilton equation \[12\] can also be used iteratively, when applied to the matrix $S$, to replace any power of $S$, $S^k$, with $k \geq 4$ by a linear combination of powers of $S, S^3$ with $i \leq 3$. This was done systematically in order to reach the expression \[7\].

Using \[7\], one can get the linearization of field equations \[14\] around an arbitrary metric $g_{\mu\nu}$ reading \[19\].
\[\delta E_{\mu\nu} = \delta G_{\mu\nu} + m^2 \mathcal{M}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} = 0. \quad (10)\]
where $\delta G_{\mu\nu}$ is the linearization of the Einstein tensor
\[\delta G_{\mu\nu} = -\frac{1}{2} \left[ \delta^\rho_{\mu}\delta^\sigma_{\nu}\nabla^2 + g^{\rho\sigma}\nabla_{\mu}\nabla_{\nu} - \delta^\rho_{\mu}\nabla_{\sigma}\nabla_{\nu} \right] h_{\rho\sigma} \]
\[-\delta^\rho_{\mu}\nabla_{\rho}\nabla_{\mu} - g_{\mu\nu}g^{\rho\sigma}\nabla^2 + g_{\mu\nu}\nabla_{\rho}\nabla_{\sigma} \right] h_{\rho\sigma} + \frac{1}{2} \left[ g_{\mu\nu} R^{\rho\sigma} - \delta^\rho_{\mu}\delta^\sigma_{\nu} R \right] h_{\rho\sigma} \quad (11)\]
and the "mass matrix" $\mathcal{M}_{\mu\nu}^{\rho\sigma}$ is defined through,
\[
\mathcal{M}_{\mu\nu}^{\rho\sigma} = \frac{\partial V_{\mu\nu}}{\partial g_{\rho\sigma}}, \quad (12)\]
and is given by the following expression,
\[
\mathcal{M}_{\mu\nu} = \frac{1}{2} \mu^\sigma \delta_\nu^\rho - \frac{1}{2} ((\beta_2 \delta_\mu^\lambda + \beta_3 (e_1 \delta_\mu^\lambda - S_\mu^\lambda)) [S^{2}]^\lambda_\rho \delta_\nu^\sigma + \frac{1}{4} \sum_{n=1}^{3} \sum_{k=1}^{n} \sum_{m=1}^{k} (-1)^{n+k+m} \beta_n \epsilon_{k-m} [S^{n-k}]^\lambda_\mu g_{\nu\lambda} G^{\rho} [S^m]^\rho_\tau
\]

\[
- \frac{1}{2} (\beta_1 \delta^\tau_\lambda + \beta_2 e_1 \delta^\tau_\lambda + \beta_3 (e_2 \delta^\tau_\lambda + [S^2]^\tau_\lambda)) \frac{\delta S^\lambda}{\delta g_{\mu\nu}} g_{\tau\tau} + (\mu \leftrightarrow \nu)
\]  

(13)

In the most general case, equations (18) still contains two different metrics, the background dynamical metric \( g_{\mu\nu} \) and the non dynamical metric \( f_{\mu\nu} \) which can be traded for the tensor \( S^2 \). However, there is a subclass of dRGT models where one can explicitly get rid of this second metric and obtain field equations for a massive graviton on the background of just one metric \( g_{\mu\nu} \). This subclass of models (called here \( \beta_1 \) models) is defined by setting to zero the parameters \( \beta_2 \) and \( \beta_3 \). In this case, indeed, \( V_{\mu\nu} \) is linear in \( S \) and the background field equations can just be reexpressed as [16]

\[
S^\rho_\nu = \frac{1}{\beta_1 m^2} \left[R^\rho_\nu - \frac{1}{6} \delta^\rho_\nu R - \frac{m^2 \beta_0}{3} \delta^\rho_\nu \right],
\]  

(14)

where \( R_{\mu\nu} \) is the Ricci tensor of the metric \( g_{\mu\nu} \), and \( R \) the corresponding Ricci curvature. Using (16), this can equivalently be seen (by squaring the above equation) as expressing \( f_{\mu\nu} \) in terms of \( g_{\mu\nu} \) and its Ricci curvatures. This remarkable feature means that in the linearized equations of motion, we can eliminate any and all occurrences of the auxiliary metric \( f_{\mu\nu} \) in favor of \( g_{\mu\nu} \) and its curvature. This feature only requires a non-vanishing \( \beta_1 \) [20].

Having carried out this elimination, we take the obtained linearized field equation (18) as a new starting point, and ask if one can show from these equations that the graviton \( h_{\mu\nu} \) propagates 5 polarizations (or less) for a completely generic metric \( g_{\mu\nu} \) (i.e. without assuming it obeys the background equations). The idea here is to try to parallel what can be done for a massive graviton on flat space-time with metric \( \eta_{\mu\nu} \) (and easily extended to a massive graviton on an Einstein space-time [3]). In this case, the linearized Bianchi identities lead, by taking one derivative of the field equations, to 4 constraints reading

\[
\partial^\mu h_{\mu\nu} - \partial_\nu h = 0.
\]  

(15)

Taking another derivative of this equation and substracting this from the trace (tracing with \( \eta_{\mu\nu} \)) of the field equations, one then concludes that \( h \), defined as \( h = \eta^{\mu\nu} h_{\mu\nu} \), vanishes in vacuum. Together with (15) this gives 5 Lagrangian constraints, which eliminate as many degrees of freedom out of the a priori 10 dynamical degrees of freedom of \( h_{\mu\nu} \).

Let us then try to follow a similar path from the field equations (18). First, it is easy to find four vector constants similar to (18). Indeed, as a consequence of the Bianchi identities, one has

\[
\nabla^\mu \delta G_{\mu\nu} \sim 0
\]  

(16)

where here and henceforth \( \nabla \) denotes the covariant derivative taken with respect to the background metric and two expressions separated by the symbol “\( \sim \)” are by definition equal off-shell (i.e. without using the field equations) up to terms containing no second or higher order derivatives acting on \( h_{\mu\nu} \). Hence, the field equations yield the four vector constraints (being first order in derivatives)

\[
\nabla^\mu \delta E_{\mu\nu} = 0.
\]  

(17)

In analogy with the flat space case we are interested in finding a fifth scalar constraint which generalizes the constraint \( h = 0 \). This fifth constraint should be the one which eliminates the Boulware Deser ghost and reduces the number of degrees of freedom from 6 to 5. Accordingly, we look for a linear combination of scalars made by tracing over the field equations (18), and its second derivatives, which would not contain any derivatives of \( h_{\mu\nu} \) of order strictly higher than one. However, we have now at hand two (symmetric) tensors which can be used to take traces, namely the metric \( g_{\mu\nu} \) and its Ricci curvature \( R_{\mu\nu} \). Equivalently we can also use the metric and the tensor \( S_{\mu\nu} \) trading \( R_{\mu\nu} \) for \( S_{\mu\nu} \) via equation (17). Choosing the second solution turns out to be more convenient for technical reasons. We stress however that the two possibilities are strictly equivalent and do not impose any restriction on \( g_{\mu\nu} \), since (17) can just be considered as a definition of \( S_{\mu\nu} \) in terms of \( R_{\mu\nu} \) and \( g_{\mu\nu} \) (as opposed to a background field equation). Hence we define the scalars \( \Phi_i \) obtained by tracing the equations of motion with powers of \( S \)

\[
\Phi_i \equiv [S^i]^{\mu\nu} \delta E_{\mu\nu},
\]  

(18)

together with the scalars \( \Psi_i \) obtained by tracing the derivative of the divergence of the equations of motion in various ways,

\[
\Psi_i \equiv \frac{1}{2} [S^i]^{\mu\nu} \nabla_\nu \nabla^\lambda \delta E_{\lambda\mu}.
\]  

(19)

An exhaustive set of linearly independents scalar is obtained by restricting \( i, 0 \leq i \leq 3 \), due the the Cayley-Hamilton identity. To summarize we look for a specific
linear combination of the scalars $\Phi_i$ and $\Psi_i$, $i = 0 \ldots, 3$ with scalar coefficients $\{u_i, v_i\}$ to be determined, such that

$$\sum_{i=0}^{3} (u_i \Phi_i + v_i \Psi_i) \sim 0,$$

(20)

i.e. which contains no second (or higher) derivatives of $h_{\mu\nu}$. Computing explicitly the scalars $\Phi_i$ and $\Psi_i$ [13], one obtains that, in these scalars, the second derivatives of $h_{\mu\nu}$ appear in the form of linear combinations (with $S$-dependent coefficients) of 26 different scalars $\bar{\xi}_i$ made by contracting $\nabla_\rho \nabla_\sigma h_{\rho\sigma}$ with powers of $S$ (including zeroth power which is simply the metric) in various ways. Two of these scalars are e.g. $\nabla_\rho \nabla_\sigma h^{\rho\sigma}$ and $[S^3]^{\mu\nu} [S^3]^{\rho\sigma} \nabla_\rho \nabla_\sigma h_{\mu\nu}$.

We get a priori 26 equations for the seven unknowns $\{u_i, v_i\}$ [21] by setting to zero each coefficient of the $\bar{\xi}_i$ which appears in (20). However, one can show that not all the scalars $\bar{\xi}_i$ are independent, thanks to the syzygies $\tilde{\xi}_k$. Indeed, the equation $\nabla_\rho \nabla_\nu [\tilde{\xi}_k]^{\mu \nu} = 0$, together with the use of the Cayley-Hamilton theorem for $S$ yields four independent identities between the scalars $\bar{\xi}_i$ which vanish up to terms $\sim 0$. These identities are just enough to reduce to seven the number of equations to be solved to fulfill (20). This yields a unique solution for the coefficients $\{u_i, v_i\}$ which translates into the identity

$$\frac{m^2 \beta_1 e_4}{4} \Phi_0 - e_3 \Psi_0 + e_2 \Psi_1 - e_1 \Phi_2 + \Psi_3.$$

(21)

Hence, using the field equations, we get the scalar constraint

$$\frac{m^2 \beta_1 e_4}{4} \Phi_0 - e_3 \Psi_0 + e_2 \Psi_1 - e_1 \Phi_2 + \Psi_3 = 0,$$

(22)

valid now for an arbitrary metric $g_{\mu\nu}$. Notice that the curvature of this metric enters this constraint in a non trivial way via the tensor $S$. It can also be verified that, when one sets $g_{\mu\nu}$ to be equal to the flat metric $\eta_{\mu\nu}$, this constraint reduces to $h = 0$ which also shows that (22) is independent of the vector constraints (17), as it should be.

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[18] Note that Guarato & Durrer [15] obtained the quadratic action of dRGT theories around an arbitrary metric. However, these authors worked in a framework where $S$ was taken to be in upper triangular form and hence did not provide covariant expressions, neither did they provide in fact field equations.

[19] We keep otherwise $\beta_0$ and $\beta_1$ arbitrary which means that these models, if one sets there $f_{\mu\nu}$ equals to the flat spacetime metric $\eta_{\mu\nu}$, will not always have $g_{\mu\nu} = \eta_{\mu\nu}$ as a background solution, and hence will not always be non linear extensions of Fierz-Pauli theories stricto sensu. It is however always possible to get a flat space-time solution for $g_{\mu\nu}$ by adding a constant conformal factor in $f_{\mu\nu}$.

[20] There are eight scalars $\{u_i, v_i\}$, but they only need to be determined up to an overall factor.