Quantum mechanics without quantum logic

D.A. Slavnov

Department of Physics, Moscow State University,
Moscow 119899, Russia. E-mail: slavnov@goa.bog.msu.ru

We describe a scheme of quantum mechanics in which the Hilbert space and linear operators are only secondary structures of the theory. As primary structures we consider observables, elements of noncommutative algebra, and the physical states, the nonlinear functionals on this algebra, which associate with results of single measurement. We show that in such scheme the mathematical apparatus of the standard quantum mechanics does not contradict a hypothesis on existence of an objective local reality, a principle of a causality and Kolmogorovian probability theory.

1 Introduction

Quantum field theory has achieved significant successes in the last decades. These successes are related to creation of non-Abelian (noncommutative) gauge models. The newest quark physics has arisen on their basis. Abelian gauge model, quantum electrodynamics, was known for a long time. Transition to non-Abelian models became a qualitative leap in development of quantum field theory.

At the same time, this transition has not caused any essential revision of the basic concepts of quantum field theory. Especially, it has not demanded any changes in logic and mathematics.

In present paper the idea is carried out that transition from classical to quantum physics is similar to transition from Abelian to non-Abelian gauge models. Of course, the quantum physics is qualitatively new theory. However, for successful development of the quantum theory it is completely not necessary to refuse the main concepts of the classical theory: the formal logic, the classical probability theory, the principle of causality, idea on the objective physical reality.

The base notions of the modern standard quantum mechanics are the Hilbert space and linear operators in this space. Von Neumann mathematically precisely has formulated quantum mechanics on the basis these concepts. The matrix mechanics of Heisenberg and the wave mechanics of Schrödinger are concrete realization of the von Neumann’s abstract method.

The formalism of the Hilbert space became the mathematical basis of those tremendous successes which were achieved by quantum mechanics. However, these successes have also a reverse side. There is some worship of the Hilbert space. Physicists have ceased to pay attention that the Hilbert space rather specific mathematical object. It has appeared a
excellent basis for calculation of expectation values of observable quantities and their probabilistic distributions. At the same time, it is completely not self-evident that observables are operators in the Hilbert space.

Attempts to use the formalism of the Hilbert space for the description of the single physical phenomena not so are successful. Reasonings which are used in this case, frequently appear not indisputable.

So von Neumann [1] resorts to rather doubtful idea on "internal I" to coordinate the concept of the Hilbert space to the results of single measurements. Abandonment of the causality principle also is hard to perceive.

The same concerns the idea on the determinative influence of the observer onto quantum-mechanical processes. In this respect quantum mechanics is a unique subdiscipline of physics and of the science in general. The idea (see for example, [2]) is put forward on the active role of conscious in the quantum phenomena.

In attempt to give slightly more objective form to similar notion Everett [3] has put forward rather exotic idea on existence in the nature of set of the parallel worlds. Happy-go-lucky the observer appears at each measurement in any one of these worlds. This idea has got very many supporters despite all extravagance.

Probably, this specifies that though the Hilbert space rather useful mathematical object, its base role is completely not indisputable. It is not a new idea that the Hilbert space and linear operators are not primary elements of the quantum theory. Namely this idea became a basis of the algebraic approach to quantum field theory (see for example, [4, 5, 6]).

2 Observables and states

Base notion of classical physics is "observable". This notion seems self-evident and does not demand definition. It is possible to multiply them by real numbers, to sum up and multiply together. In other words, they form a real algebra $A_{cl}$.

The elements $\hat{A}$ of this algebra are the latent form of observable variables. The explicit form of an observable should be some number. It means that the explicit form of an observable corresponds to the value of some functional $\varphi(\hat{A}) = A$ ($A$ is a real number), defined on the algebra $A_{cl}$.

Physically, the latent form of the observable $\hat{A}$ becomes explicit as a result of measurement. This means that the functional $\varphi(\hat{A})$ describes a measurement result of the observable $\hat{A}$.

Experiment shows that the sum and the product of measurement results correspond to the sum and the product of observables:

$$\hat{A}_1 + \hat{A}_2 \rightarrow A_1 + A_2, \quad \hat{A}_1 \hat{A}_2 \rightarrow A_1 A_2.$$  \hspace{1cm} (1)

In this connection further there will be useful a following definition [7].

Let $B$ be a real commutative algebra and $\check{\varphi}$ be a linear functional on $B$. If

$$\check{\varphi}(\hat{B}_1 \hat{B}_2) = \check{\varphi}(\hat{B}_1) \check{\varphi}(\hat{B}_2) \text{ for all } \hat{B}_1 \in B \text{ and } \hat{B}_2 \in B,$$

then the functional $\check{\varphi}$ is called a real homomorphism on algebra $B$.

Now we can introduce the second base notion of classical physics — a state of the object under consideration.
The state is a real homomorphism on the algebra of observables. The result of any measurement of the classical object is determined by its state.

In principle, in classical physics it is possible to measure observables in any combinations. Always it is possible to pick up such system of measuring devices that measurement results of several observables will not depend on sequence in what observables are measured. For example, if we carry out measurement of an observable \( \hat{A} \), then an observable \( \hat{B} \), then again an observable \( \hat{A} \) and an observable \( \hat{B} \) results of repeated measurements of observables coincide with results of primary measurements. We name the corresponding measuring instruments compatible devices.

Let us pass to discussion of the situation in quantum physics. In quantum physics it seems also natural to accept “observable” as base notion. Quantum observables also possess algebraic properties.

However quantum measurements significantly differ from classical one. There are systems of compatible measuring devices not for any observables. Accordingly, in quantum physics observables are subdivided into compatible (simultaneously measurable) and incompatible (additional). There are systems of compatible measuring devices for compatible observables. Such systems of devices do not exist for incompatible observables. As it is told in paper by Zeilinger [8]: "Quantum complimentarity then is simply expression of the fact that in order to measure quantities, we would have to use apparatuses which mutually exclude each other".

For incompatible quantum observables the measurement results depend on sequence of measurement of these observables. This fact leaves traces on rules of multiplication of observables. Two ways are applied. The first way is used in so-called Jordan algebra [3, 4]. In this case, observables form real commutative algebra, but the operation of multiplication is not associative. Use of Jordan algebra has not led to to appreciable successes in the quantum theory.

The standard quantum mechanics is based on the other method of multiplication of observables. In this case, operation of multiplication is associative, but noncommutative. Besides product of two observables not necessarily is an observable, i.e. observables do not form algebra.

In order to use the advanced mathematical apparatus of algebras full-scale, it is convenient to leave for framework of the directly observable variables and to consider its complex combinations. Hereinafter, we call these combinations the dynamic variables.

Having in view of told above, we accept

**Postulate 1:**

Dynamic variables correspond to elements of an involutive, associative, and (in general) noncommutative algebra \( \mathfrak{A} \), satisfying the following conditions: for each element \( \hat{R} \in \mathfrak{A} \), there exists a Hermitian element \( \hat{A} \) \((\hat{A}^* = \hat{A})\) such that \( \hat{R}^* \hat{R} = \hat{A}^2 \), and if \( \hat{R}^* \hat{R} = 0 \), then \( \hat{R} = 0 \).

We assume that the algebra has a unit element \( \hat{I} \) and that Hermitian elements of algebra \( \mathfrak{A} \) correspond to observable variables. We let \( \mathfrak{A}_+ \) denote the set of these elements.

In the standard quantum mechanics this postulate is accepted in considerably stronger form. It is supposed that dynamic variables \( \hat{R} \) are linear operators in the Hilbert space.

Postulate 2 directly follows from quantum measurements.
Postulate 2:
Mutually commuting elements of the set $\mathbb{A}_+$ correspond to compatible (simultaneously measurable) observables.

Because of Postulate 2, commutative subalgebras of the algebra $\mathbb{A}$ have an important role in the further analysis. (see [10]).

For the further it is useful to recollect definition of the spectrum $\sigma(\hat{A}; \mathbb{A})$ of an element $\hat{A}$ in the algebras $\mathbb{A}$. The number $\lambda$ is a point of the spectrum of the element $\hat{A}$ if and only if the element $\hat{A} - \lambda \hat{I}$ does not have an inverse element in the algebra $\mathbb{A}$. Generally, an element may have different spectra in an algebra and its subalgebra. However, if the subalgebra $\mathbb{Q}$ is maximal, $\sigma(\hat{Q}; \mathbb{Q}) = \sigma(\hat{Q}; \mathbb{A})$ for any $\hat{Q} \in \mathbb{Q}$ (see for example, [7]).

The Hermitian elements of the algebra $\mathbb{A}$ are the latent form of the observable variables. The same as in a classical case the latent form of an observable $\hat{A}$ becomes explicit as a result measurements. Only mutually commuting observables can be are measured in individual experiment. Experiment shows that the same relations (1) are carried out for such observables like for classical observables.

Generalizing definition of a state in classical physics, we accept the central postulate of the proposed approach (see [11]).

Postulate 3:
The result of the observation which we carry out on the quantum system is determined by a physical state of this system. The physical state is described by a functional $\varphi(\hat{A})$ (generally, multivalued), with $\hat{A} \in \mathbb{A}_+$, whose restriction $\varphi_\xi(\hat{A})$ to each subalgebra $\mathbb{Q}_\xi$ is single-valued and is a real homomorphism ($\varphi_\xi(\hat{A}) = A$ is a real number).

The functionals $\varphi_\xi(\hat{A})$ can be shown to have the following properties [7]:

1) $\varphi_\xi(0) = 0$;
2) $\varphi_\xi(\hat{I}) = 1$;
3) $\varphi_\xi(\hat{A}^2) \geq 0$;
4) if $\lambda = \varphi_\xi(\hat{A})$, then $\lambda \in \sigma(\hat{A}; \mathbb{Q}_\xi)$;
5) if $\lambda \in \sigma(\hat{A}; \mathbb{Q}_\xi)$, then $\lambda = \varphi_\xi(\hat{A})$ for some $\varphi_\xi(\hat{A})$.

The corresponding properties of individual measurements are postulated in the standard quantum mechanics but are a consequence of the third postulate here.

On the other hand, properties (2.4) and (2.5) allow to construct all functionals $\varphi$, appearing in the Postulate 3. Clearly that for construction of the functional $\varphi$ sufficiently to construct all its restriction $\varphi_\xi$ on subalgebras $\mathbb{Q}_\xi$. In its turn, each functional $\varphi_\xi$ can be constructed as follows. In each subalgebra $\mathbb{Q}_\xi$ it is necessary to choose arbitrarily system $G(\mathbb{Q}_\xi)$ independent generators. Further we require $\varphi_\xi$ to be a certain mapping $G(\mathbb{Q}_\xi)$ to a real number set (allowable points of the spectra for the corresponding elements of the set $G(\mathbb{Q}_\xi)$). On the other elements of $\mathbb{Q}_\xi$, the functional $\varphi_\xi$ is constructed by linearity and multiplicativity. Sorting out all possible mappings of the set $G(\mathbb{Q}_\xi)$ into points of the spectrum, we construct all functionals $\varphi_\xi$.

On other subalgebra $\mathbb{Q}_\xi'$ the functional $\varphi_{\xi'}$ is constructed similarly. It is clearly that this procedure is always possible if functionals $\varphi_\xi$ and $\varphi_{\xi'}$ are constructed independently.
Thus, the set of physical states (functionals $\varphi$) is completely defined by the algebra $\mathfrak{A}$ (set of its maximal real commutative subalgebras and spectra of these subalgebras). It is clearly that these functionals, generally, are multivalued. Moreover, it is possible to show [12] that there are algebras having physical sense for which it is impossible to construct the single-valued functional

However, it is always possible to construct a functional $\varphi$ that is single-valued on all observables belonging any preset subalgebra $\mathfrak{Q}_\xi$. For this, suffice it to assign number one (set $\xi = 1$) to the subalgebra $\mathfrak{Q}_\xi$ and define the restriction $\varphi_1$ of $\varphi$ to $\mathfrak{Q}_1$ as follows. Let $G(\mathfrak{Q}_1)$ be a set of generators of $\mathfrak{Q}_1$. We define the restriction $\varphi_1$ to be some mapping of $G(\mathfrak{Q}_1)$ to a real number set $S_1$. We next choose another subalgebra $\mathfrak{Q}_2$. With $\mathfrak{Q}_1 \cap \mathfrak{Q}_2 = \mathfrak{Q}_{12} \neq \emptyset$, we first construct a set of generators $G_{12}$ of $\mathfrak{Q}_{12}$, and then supplement it with the set $G_{21}$ to the complete set of generators of $\mathfrak{Q}_2$. If $\hat{A} \in G_{12}$, then $\varphi_2(\hat{A}) = \varphi_1(\hat{A})$. If $\hat{A} \in G_{21}$, then the functional $\varphi_2$ is defined such that it is a mapping of $G_{21}$ to some allowable set of points in the spectra of the corresponding elements of the algebra $\mathfrak{Q}_2$. We must next exhaust all subalgebras $\mathfrak{Q}_i$ (of type $\mathfrak{Q}_\xi$) that have nonempty intersections with $\mathfrak{Q}_1$. To construct the restriction $\varphi_i$ of $\varphi$ to each $\mathfrak{Q}_i$, suffice it to use the recipe used for $\varphi_2$. By construction, such a functional $\varphi$ is single-valued on all elements belonging to $\mathfrak{Q}_1$. Different subalgebras $\mathfrak{Q}_i$ can have common elements that do not belong to $\mathfrak{Q}_1$. On these elements, the functional $\varphi$ can be multivalued.

Physically the multivaluedness of the functional $\varphi$ can be justified as follows. The result of observation may depend not only on an observable quantum object, but also on properties of the measuring device used for observation. A typical measuring device consists of an analyzer and a detector. The analyzer is a device with one input and several output channels. As an example, we consider the device measuring an observable $\hat{A}$. For simplicity, we assume that the spectrum of this observable is discrete. Each output channel of the analyzer must then correspond to a certain point of the spectrum. The detector registers the output channel through which the quantum object leaves the analyzer. The corresponding point of the spectrum is taken to be the value of the observable $\hat{A}$ registered by the measuring device.

In general, the value not of one observable $\hat{A}_i$ but of an entire set of compatible observables can be registered in one experiment. All these observables must belong to a single subalgebra $\mathfrak{Q}_\xi$. Naturally the analyzer must be constructed appropriately. The main technical requirement to this construction consists in the following. Sorting of researched quantum objects on values of one of the observables belonging $\mathfrak{Q}_\xi$, should not deform sorting on values of other observable belonging $\mathfrak{Q}_\xi$.

Therefore, measuring devices can be are marked by an index $\xi$. We say that the corresponding device belongs to the type $\mathfrak{Q}_\xi$.

Let now we want to measure value of the observable $\hat{A} \in \mathfrak{Q}_\xi \cap \mathfrak{Q}_{\xi'}$. For this purpose we can use the device of the type $\mathfrak{Q}_\xi$ or the device of the type $\mathfrak{Q}_{\xi'}$. These devices have a different construction. Therefore, it is absolutely not necessary that we obtain the same result if we use a different devices for investigation of the same quantum object. It just corresponds to what the functional describing the physical state of the quantum object, is multivalued on the observable $\hat{A}$.

We cannot use both devices in one experiment as subalgebras $\mathfrak{Q}_\xi$ and $\mathfrak{Q}_{\xi'}$ contain incompatible observables. Therefore, in each concrete experiment the measurement result is single-valued. However, this result can depend not only on internal characteristics of the observable object (its physical state), but also on the characteristic (type) of the measuring device. Accordingly, the functional $\varphi$ describes not the value of each observable in a given
physical state but the response of the measuring device of defined type, to this physical state.

Here there is an essential difference between the proposed approach and so-called PIV model described in the review [13]. In this model it is supposed that the value of each observable is uniquely predetermined for the quantum object.

If the functional \( \varphi \) is single-valued at a point \( \hat{A} \), we say that the corresponding physical state \( \varphi \) is stable on the observable \( \hat{A} \). In this case, we can say that the observable \( \hat{A} \) has a definite value in the physical state \( \varphi \) and this value is the physical reality.

The commutative algebra \( \mathfrak{A} \) has only one maximal real commutative subalgebra. Therefore, in this case all measuring devices belong to one type and all physical states are stable on all observables.

It is natural to impose a condition of "minimality" on algebra \( \mathfrak{A} \) with the following physical sense. If we cannot distinguish two observables by any experiment it is one element of algebra. Therefore, we accept

\[
\text{Postulate 4:} \\
\text{If } \sup_{\varphi_\xi} |\varphi_\xi(\hat{A}_1 - \hat{A}_2)| = 0, \text{ then } \hat{A}_1 = \hat{A}_2.
\]

We note that the standard quantum mechanics implies the stronger assumption that the observables coincide if all their average values coincide.

It is obvious that in case of commutative algebra, the set of the physical states passes into usual classical phase space, and a separate functional \( \varphi \) comes to a point in this space. However, for noncommutative algebras the physical state differs that is understood as a state in quantum physics. Further we use the term "a quantum state" for latter.

Let us note that the physical state can not be fixed uniquely in quantum measurement. Really, the devices for measurement of incompatible observables are incompatible. Therefore, we can measure the observables belonging to one maximal commutative subalgebra \( \mathfrak{Q}_\xi \) in one experiment. As a result we establish only values of the functional \( \varphi_\xi \) which is a restriction of the functional \( \varphi \) (the physical state). In other respects the functional \( \varphi \) is uncertain. Repeated measurement with use of different device does not help the situation. Because the quantum object passes into a new physical state for which the values of the functional found in the first experiment \( \varphi_\xi \) is useless.

We say that the functionals \( \varphi \) are \( \varphi_\xi \)-equivalent if they have the same restriction \( \varphi_\xi \) on the subalgebra \( \mathfrak{Q}_\xi \). Thus, in quantum measurement we can ascertain only the class of equivalence \( \{ \varphi \}_{\varphi_\xi} \) to which the physical state belongs. Because the functional \( \varphi \) has continual set of restrictions on various maximal commutative subalgebras the set \( \{ \varphi \}_{\varphi_\xi} \) has a potency of the continuum. Only this class of equivalence can claim a name "quantum state".

Actually there is one more restriction. Experiment shows that if the quantum object is in the quantum state corresponding \( \{ \varphi \}_{\varphi_\xi} \) the measurement result of the observable \( \hat{A} \in \mathfrak{Q}_\xi \) does not depend on the type of used measuring device. It means that the physical state should be stable on the subalgebra \( \mathfrak{Q}_\xi \). We call such class of equivalence the quantum state and denote by \( \Psi(\cdot | \varphi_\xi) \).

Strictly speaking, the above definition of the quantum state is valid only for a physical system that does not contain identical particles. Describing identical particles requires some generalization of the definition of the quantum state.

The measuring instrument cannot distinguish which of the identical particles entered it. Therefore, we slightly generalize the definition of a quantum state. Let the physical system contain identical particles. Let \( \{ \varphi \}_{\varphi_\xi} \) be the set of \( \varphi_\xi \)-equivalent functionals. We say
that a functional $\varphi'$ is weakly $\varphi_\xi$-equivalent to the functionals $\varphi$ if its restriction $\varphi'_\xi$ on the subalgebra $\mathcal{Q}_\xi$ coincides with the restriction $\varphi_\xi$ of the functional $\varphi$ on the subalgebra $\mathcal{Q}_\xi$. The set $\{\hat{Q}'\}$ must be an image of the set $\{\hat{Q}\}$ under a mapping such that the observables corresponding to one of the identical particles are changed to the respective observables corresponding to the other particle.

Hence, the definition of the quantum state $\Psi(\cdot | \varphi_\xi)$ should refer to the set of all weakly $\varphi_\xi$-equivalent functionals. Hereinafter, we let the symbol $\{\varphi\}^{\varphi_\xi}$ denote the set of a weakly $\varphi_\xi$-equivalent functionals.

### 3 Probability theory and quantum ensemble

Now the Kolmogorovian probability theory [14] is the most consistent and mathematically rigorous. It is usually considered that it does not approach for the description of quantum systems. In present paper the opposite opinion is protected: the Kolmogorovian probability theory very well approaches for the description of the quantum phenomena, it is necessary only to take into account peculiarity of quantum measurements [15].

We recall the foundations of Kolmogorov’s theory probability (see, for example [14, 16]). The probability theory scheme is based on the so-called probability space $(\Omega, \mathcal{F}, P)$. The first component $\Omega$ is set (space) of the elementary events $\omega$. The physical sense of the elementary events specially is not stipulated, but it is considered that they are mutually exclusive. One and only one elementary event is realized in each test.

Besides the elementary event the concept of ”event” (or ”random event”) is introduced. Each event $F$ is identified with some subset of set $\Omega$. It is supposed that we can ascertain, the event is carried out or failed in a experiment under consideration. Such assumption is not done about the elementary event.

Collections of subsets of the set $\Omega$ (including the set $\Omega$ and the empty set $\emptyset$) are supplied with the structure of Boolean algebras. Algebraic operations are: intersection of subsets, joining of them, and complement with respect to $\Omega$. A Boolean algebra, closed in respect of denumerable number of operations of joining and intersection, is called a $\sigma$-algebra.

The second component of the probability space is some $\sigma$-algebra $\mathcal{F}$. The set $\Omega$ in which the particular $\sigma$-algebra $\mathcal{F}$ is chosen, refers to as measurable space. Further on the measurability will play a key role.

Finally, the third component of the probability space is the probability measure $P$. It represents a mapping of the algebra $\mathcal{F}$ onto a set of real numbers satisfying the following conditions for any countable set of nonintersecting subsets $F_j \in \mathcal{F}$: a) $0 \leq P(F) \leq 1$ for all $F \in \mathcal{F}$, $P(\Omega) = 1$; b) $P(\bigcup_j F_j) = \sum_j P(F_j)$ Let us pay attention that the probabilistic measure is defined only for the events which are included in the algebra $\mathcal{F}$. For the elementary events the probability, generally speaking, is not defined.

A real random quantity on $\Omega$ is defined as a mapping $X$ of the set $\Omega$ onto the extended real straight line $\bar{R} = [\infty, +\infty]$, $X(\omega) = X \in \bar{R}$.

The set $\bar{R}$ is assumed to have the measurability property. The Boolean algebra $\mathcal{F}_R$ generated by the semi-open intervals $(x_i, x_j]$, i.e., the $\sigma$-algebra that results from applying the algebraic operations to all such intervals, can be chosen as the $\sigma$-algebra in the set $\bar{R}$. Let $\{\omega : X(\omega) \in F_R\}$, where $F_R \in \mathcal{F}_R$ be the subset of elementary events $\omega$ such that $X(\omega) \in F_R$. The subsets $F = \{\omega : X(\omega) \in F_R\}$ form the $\sigma$-algebra $\mathcal{F}$ in the space $\Omega$. 

7
We consider now the application of formulated main principles of probability theory to a problem of quantum measurements. We associate an elementary event with a physical state. Accordingly, we associate the set of physical states of a quantum object with the space $\Omega$. Further, we need to make this space measurable, i.e. to choose certain $\sigma$-algebra $\mathcal{F}$. Here, a peculiarity of quantum measurement, which has the name “principle of complementarity” in standard quantum mechanics, has crucial importance. We can organize each individual experiment only in such a way that compatible observables are measured in it. The results of measurement can be random. That is, observables correspond to the real random quantities in probability theory.

The main goal of a typical quantum experiment is to obtain the probability distributions for one or another observable quantities. We can obtain such distribution for certain collection of compatible observables if an appropriate measuring device is used. From the point of view of probability theory we choose certain $\sigma$-algebra $\mathcal{F}$, choosing the certain measuring device. For example, let us use the device intended for measurement of momentum of a particle. Let us suppose that we can ascertain by means of this device that the momentum of particle hits an interval $[p_i, p_j]$. For definiteness we have taken a semi-open interval though it is not necessary. Hit of momentum of the particle in this or that interval is the event for the measuring device, which we use. These events are elements of certain $\sigma$-algebra. Thus, the probability space $(\Omega, \mathcal{F}, P)$ is determined not only by the explored quantum object (by collection of its physical states) but also by the measuring device which we use.

Let us assume that we carry out some typical quantum experiment. We have an ensemble of the quantum systems, belonging to a certain quantum state. For example, the particles have spin 1/2 and the spin projection on the x axis equals 1/2. Let us investigate the distribution of two incompatible observables (for example, the spin projections on the directions forming angles $\theta_1$ and $\theta_2$ with regard to the x axis). We cannot measure both observables in one experiment. Therefore, we should carry out two groups of experiments which use different measuring devices. "Different" is classically distinct. In our concrete case the devices should be oriented by various manners in the space.

We can describe one group of experiments with the help of a probability space $(\Omega, \mathcal{F}_1, P_1)$, another group with the help of $(\Omega, \mathcal{F}_2, P_2)$. Although in both cases the space of elementary events $\Omega$ is the same, the probability spaces are different. Certain $\sigma$-algebras $\mathcal{F}_1$ and $\mathcal{F}_2$ are introduced in these spaces to give them the property of measurability.

Mathematically, a $\sigma$-algebra $\mathcal{F}_{12}$ that include the algebra $\mathcal{F}_1$ as well as algebra $\mathcal{F}_2$ can be formally constructed.

It is said that such an algebra is generated by the algebras $\mathcal{F}_1$ and $\mathcal{F}_2$. In addition to the subsets $F_i^{(1)} \in \mathcal{F}_1$ and $F_j^{(2)} \in \mathcal{F}_2$ of the set $\Omega$, it also contains all possible intersections and unions of the subsets $F_i^{(1)} \in \mathcal{F}_1$ and $F_j^{(2)} \in \mathcal{F}_2$.

But this $\sigma$-algebra is unacceptable physically. Indeed, the event $F_{ij} = F_i^{(1)} \cap F_j^{(2)}$ is an event in which the values of two incompatible observables of one quantum object belong to a strictly determined domain. For a quantum system, it is impossible in principle to set up an experiment that could distinguish such an event. Therefore, the probability concept does not exist for such event. In other words, there is no probability measure corresponding to the subset $F_{ij}$, and the $\sigma$-algebra $\mathcal{F}_{12}$ cannot be used to construct the probability space. This illustrates the following fundamental point that should be kept in mind when applying the theory of probability to quantum systems: not all mathematically possible $\sigma$-algebras are physically acceptable.
The probability definition implies numerous tests. These tests must be performed under same conditions. This applies to the object being tested as well as to the measuring instrument. It is obvious that the microstates of either the object or the instrument cannot be fully controlled. Therefore, the term "the same conditions" should refer to some equivalence classes for the states of the quantum object and the measuring instrument.

For a quantum object under study, such fixation is normally realized by choosing a certain quantum state. For example, in the case of spin particles, the particles with a certain spin orientation are selected.

For the measuring instrument, we also must choose a definite classical characteristic to be used to fix a certain equivalence class. For example, the initial single beam of particles in the instrument should split into a few well-separated beams corresponding to different values of the spin projection on some distinguished direction.

Thus, what corresponds to an element of the measurable space \((\Omega, F)\) in an experiment is the ensemble of quantum objects (which can be in a definite quantum state) and a measuring instrument of a certain type that allows registering an event of a definite form. Each such instrument can distinguish events that correspond to some set of compatible observables. As it was already mentioned, the result of individual an measurement may depend not only on intrinsical properties of the measured object (the physical state), but also on the type of the measuring device. In terms of the probability theory, this can be expressed as follows for a quantum system, a random variable \(X\) can be a multivalued function of the elementary event \(\omega\).

In the classical case, all observables are compatible. Accordingly, all measuring instruments belong to one type; therefore, the classical random quantity \(X\) is a single-valued function of \(\omega\). We note that in the quantum case, if the quantity \(X\) is interpreted as a function on the measurable space \((\Omega, F)\) rather than the space \(\Omega\), then this function is single-valued.

All this motivates us to reconsider the interpretation of the result obtained in [12], where a no-go theorem was proved. Essentially, the theorem states that there is no intrinsic characteristic of a particle with spin 1 that unambiguously predetermines the squares of the spin projections on three mutually orthogonal directions.

The conditions of the Kochen-Specker theorem are not carried out in the approach described in present paper. Really, used in paper [12] the observables \(\hat{S}_x^2, \hat{S}_y^2, \hat{S}_z^2\) are compatible. The observables \(\hat{S}_x^2, \hat{S}_y^2, \hat{S}_z^2\) are also compatible. Here, the \(x, y, z'\) directions are orthogonal among themselves, but the \(y, z\) directions are not parallel to the \(y', z'\) directions. The observables \(\hat{S}_y^2, \hat{S}_z^2\) are not compatible with the observables \(\hat{S}_y^2, \hat{S}_z^2\). The devices coordinated with the observables \(\hat{S}_x^2, \hat{S}_y^2, \hat{S}_z^2\) and the \(\hat{S}_x^2, \hat{S}_y^2, \hat{S}_z^2\) belong to different types. Therefore, these devices not necessarily should give the same result for square of spin projection on the \(x\) direction. It is impossible to carry out the experiment for check of this statement, as we cannot use simultaneously two types of measuring devices in one experiment.

Let us consider an ensemble of physical systems which are in the quantum state \(\Psi(\cdot \mid \varphi_\xi)\). We consider the physical states \(\varphi\) of these systems as an elementary events \(\omega\) and the quantum state \(\Psi(\cdot \mid \varphi_\xi)\) (the class of equivalence \(\{ \varphi \}_{\varphi_\xi}\)) as a space \(\Omega(\varphi_\xi)\) of the elementary events. The observable \(\hat{A}\) is a random variable

\[
\varphi \xrightarrow{\hat{A}} A \equiv \varphi(\hat{A}).
\]
Let value of the observable be measured in experiment \( \hat{A} \in \mathfrak{A}_{\xi'} \) and the device of the type \( \mathfrak{A}_{\xi'} \) be used. We denote the measurable space of the elementary events by \((\Omega(\varphi_{\xi}), \mathcal{F}_{\xi'})\). It corresponds to the quantum state \( \Psi(\cdot | \varphi_{\xi}) \) and to the \( \sigma \)-algebra \( \mathcal{F}_{\xi'} \) (to the measuring device of the type \( \mathfrak{A}_{\xi'} \)). Let \( P_{\xi'} \) be a probabilistic measure on this space, i.e. \( P_{\xi'}(F) \) is probability of the event \( F \in \mathcal{F}_{\xi'} \).

Let us consider that the event \( \tilde{A} \) is realized in experiment if the registered value of the observable \( \hat{A} \) is no more \( \tilde{A} \). We denote probability of this event by \( P_{\xi'}(\tilde{A}) = P(\varphi : \varphi_{\xi}(\hat{A}) \leq \tilde{A}) \).

Knowing probabilities \( P_{\xi'}(\tilde{A}) \), we can find probability \( P_{\xi'}(\tilde{A}) \) with the help of corresponding summations and integrations. Distribution \( P_{\xi'}(\tilde{A}) \) is marginal for the probabilities \( P_{\xi'}(F) \).

The observable \( \hat{A} \) can belong not only the subalgebra \( \mathfrak{A}_{\xi'} \) but also other maximal subalgebra \( \mathfrak{A}_{\xi'} \). Therefore, for definition of probability of event \( \tilde{A} \) we can use the device of the type \( \mathfrak{A}_{\xi'} \). In this case for probability we could obtain other value \( P_{\xi'}(\tilde{A}) \). However, experiment shows that the probabilities do not depend on a used measuring device. Therefore, we should accept one more postulate.

**Postulate 5:**

Let the observable be \( \hat{A} \in \mathfrak{A}_{\xi} \cap \mathfrak{A}_{\xi'} \), then the probability to find out the event \( \tilde{A} \) for the system which are in the quantum state \( \Psi(\cdot | \varphi_{\xi}) \), does not depend on of the type used device, i.e. \( P(\varphi : \varphi_{\xi}(\hat{A}) \leq \tilde{A}) = P(\varphi : \varphi_{\xi}(\hat{A}) \leq \tilde{A}) \).

Therefore, although the functional \( \varphi \) can be multivalued, we have the right to use notations \( P(\varphi : \varphi(\hat{A}) \leq \tilde{A}) \) for probability of the event \( \tilde{A} \).

Let we have ensemble of the quantum systems which are in the quantum state \( \Psi(\cdot | \varphi_{\xi}) \). For this ensemble we carry out a series of experiments in which the observable \( \hat{A} \) is measured. We deal the finite set of the physical states in any real series. In the ideal series this set can be denumerable. We let \( \{ \varphi \}^A_{\varphi_{\xi}} \) denote a random denumerable sample in the space \( \Omega(\varphi_{\xi}) \) which contains all the physical states of the real series. By the law of the large numbers (see for example [10]) the probabilistic measure \( P_{\hat{A}} \) in this sample is uniquely determined by the probabilities \( P(\varphi : \varphi(\hat{A}) \leq \tilde{A}) \).

The probabilistic measure \( P_{\hat{A}} \) determines average value of the observable \( \hat{A} \) in the sample \( \{ \varphi \}^A_{\varphi_{\xi}} \):

\[
\langle \hat{A} \rangle = \int_{\{ \varphi \}^A_{\varphi_{\xi}}} P_{\hat{A}}(d\varphi) \varphi(\hat{A}) \equiv \Psi(\hat{A}|\varphi_{\xi}). \quad (3)
\]

This average value does not depend on concrete sample, and is completely determined by the quantum state \( \Psi(\cdot | \varphi_{\xi}) \).

Formula (3) defines a functional (quantum average) on set \( \mathfrak{A}_+ \). We denote this functional also by \( \Psi(\cdot | \varphi_{\xi}) \). The totality of all quantum experiments specifies that we must accept the following postulate.

**Postulate 6:**

The functional \( \Psi(\cdot | \varphi_{\xi}) \) is linear on the set \( \mathfrak{A}_+ \).

It implies that

\[
\Psi(\hat{A} + \hat{B}|\varphi_{\xi}) = \Psi(\hat{A}|\varphi_{\xi}) + \Psi(\hat{B}|\varphi_{\xi}) \text{ also in the case where } [\hat{A}, \hat{B}] \neq 0.
\]
Any element \( \hat{R} \) of the algebra \( \mathfrak{A} \) is uniquely represented as \( \hat{R} = \hat{A} + i\hat{B} \), where \( \hat{A}, \hat{B} \in \mathfrak{A}_+ \). Therefore, the functional \( \Psi(\cdot | \varphi_\xi) \) can be uniquely extended to a linear functional on \( \mathfrak{A} \):
\[
\Psi(\hat{R} | \varphi_\xi) = \Psi(\hat{A} | \varphi_\xi) + i\Psi(\hat{B} | \varphi_\xi).
\]
Let us define norm of the element \( \hat{R} \) by equality
\[
\|\hat{R}\|^2 = \sup_{\varphi_\xi} \varphi_\xi(\hat{R}^* \hat{R}).
\]
Such definition is allowable. Due to the property (2.3) we have \( \|\hat{R}\|^2 \geq 0 \). It follows from property (2.1) and Postulate 4 that if \( \|\hat{R}\|^2 = 0 \), then \( \hat{R} = 0 \). Further, by virtue of definition of the probabilistic measure
\[
\Psi(\hat{R}^* \hat{R} | \varphi_\xi) = \int_{\{\varphi\}} P_{\hat{R}^* \hat{R}}(d\varphi) \varphi(\hat{R}^* \hat{R}) \leq \sup_{\varphi_\xi'} \varphi_\xi'(\hat{R}^* \hat{R}).
\]
If \( \hat{R}^* \hat{R} \in \mathfrak{Q}_\xi \), then \( \Psi(\hat{R}^* \hat{R} | \varphi_\xi) = \varphi_\xi(\hat{R}^* \hat{R}) \). Therefore,
\[
\|\hat{R}\|^2 = \sup_{\varphi_\xi} \varphi_\xi(\hat{R}^* \hat{R}) = \sup_{\varphi_\xi} \Psi(\hat{R}^* \hat{R} | \varphi_\xi).
\]
Because \( \Psi(\cdot | \varphi_\xi) \) is a linear positive functional, the Cauchy-Bunyakovskii-Schwars inequality
\[
|\Psi(\hat{R}^* \hat{S} | \varphi_\xi)\Psi(\hat{S}^* \hat{R} | \varphi_\xi)| \leq \Psi(\hat{R}^* \hat{R} | \varphi_\xi)\Psi(\hat{S}^* \hat{S} | \varphi_\xi).
\]
is satisfied.
Because \( \varphi_\xi([\hat{R}^* \hat{R}]^2) = [\varphi_\xi(\hat{R}^* \hat{R})]^2 \), we have \( \|\hat{R}^* \hat{R}\| = \|\hat{R}\|^2 \). Therefore, if we complete the algebra \( \mathfrak{A} \) with respect to the norm \( \|\cdot\| \), then \( \mathfrak{A} \) turns out \( C^* \)-algebra [18].
Thus, a necessary condition of consistency of the postulate of the linearity is the following strengthening of Postulate 1.

Postulate 1:

The set of the dynamical variables is algebra which can be is equipped with structure \( C^* \)-algebra.

The reason that we did not accepted this formulation of the first postulate initially because it follows from the experiment that the observables have algebraic properties and the quantum mean values have the linearity property. But mathematical relations included in the definition of a \( C^* \)-algebra are not directly related to the experiment.

4 Time evolution and the ergodicity condition

In the standard quantum mechanics the time evolution is determined by the unitary automorphism
\[
\hat{A}(t) = \hat{U}^{-1}(t)\hat{A}(0)\hat{U}(t) \quad \hat{A}(0) = \hat{A},
\]
(4)
where \( \hat{A}(t) \) and \( \hat{U}(t) \) are operators in some Hilbert space. The operators of evolution \( \hat{U}(t) \) realize unitary representation of one-parameter group. But for (4) to preserve its physical meaning, it suffices to consider \( \hat{A}(t) \) and \( \hat{U}(t) \) as elements of some algebra (in particular, of \( \mathfrak{A} \)).
In our case, the evolution equation can be rewritten in terms of physical states. We therefore accept

**Postulate 7.**

A physical state of a quantum system evolves in time as

$$\varphi(\hat{A}) \rightarrow \varphi_t(\hat{A}) \equiv \varphi(\hat{A}(t)),$$

where $\hat{A}(t)$ is defined by Eq. (4).

Equation (5) describes time evolution of a physical state entirely unambiguously. It is a different story, though, that an observation allows determining the initial value $\varphi(\hat{A})$ of a functional only up to its belonging to a certain quantum state $\{\varphi\}_{\varphi_i}$. Most of our predictions regarding the time evolution of a quantum object are therefore probabilistic. In addition, Eqs. (4) and (5) are valid only for systems that are not exposed to first-class actions (in von Neumann’s terminology [1]), i.e., do not interact with a classical measuring device.

We now return to the fifth and the sixth postulates. From the experimental standpoint, these postulates are well justified. But it is not quite clear whether they can be realized within the mathematical scheme considered here. It turns out that these postulates can be related to the time evolution of the quantum system. For this, we must impose restrictions on the elements $\hat{U}(t)$.

We now accept

**Postulate 8.**

The elements $\hat{U}(t)$ are unitary elements, which have an integral representation of the form

$$\hat{U}(t) = \int \hat{\rho}(dE) \exp[iEt],$$

where $\hat{\rho}(dE)$ are orthogonal projectors. The spectrum of $\hat{U}(t)$ contains at least one discrete nondegenerate value $E_0$.

Hereinafter integrations (and also limits) on algebra $\mathfrak{A}$ are understood in sense of the weak topology of $C^*$-algebra [18].

Somewhat conventionally, we can represent $\hat{\rho}(dE)$ as

$$\hat{\rho}(dE) = \hat{\rho}_p(dE) + \hat{\rho}_c(dE) = \sum_n \hat{\rho}_n \delta(E - E_n) dE + \hat{\rho}_c(dE).$$

Here $\hat{\rho}_p(dE)$ and $\hat{\rho}_c(dE)$ concern to point and continuous spectrums, accordingly. Besides, $\hat{\rho}_n\hat{\rho}_m = \hat{\rho}_m\hat{\rho}_n = 0$ for $m \neq n$, $\hat{\rho}_n\hat{\rho}_c(dE) = \hat{\rho}_c(dE)\hat{\rho}_n = 0$. The sum over $n$ in (10) must necessarily involve at least one term ($n = 0$) with a nondegenerate value $E_0$.

In addition to this last restriction, other requirements are always assumed in considering any quantum mechanics model. Requiring a discrete point in the spectrum does not seem too restrictive either. For example, a one-particle quantum system can have a purely continuous energy spectrum. But it can be considered as a one-particle state of an extended system that can also be in the vacuum state in addition to the one-particle state. The energy spectrum of the extended system already has a discrete nondegenerate point in the spectrum.
By the nondegeneracy of $E_0$, we assume that the projector $\hat{p}_0$ in decomposition (10) is one-dimensional. A projector $\hat{p}$ is said to be onedimensional if it cannot be represented as

$$\hat{p} = \sum \alpha \hat{p}_\alpha, \quad \hat{p}_\alpha \neq \hat{p}, \quad \hat{p} \hat{p}_\alpha = \hat{p}_\alpha \hat{p} = \hat{p}_\alpha.$$

Let us remark that, if two elements $\hat{A}_1$ and $\hat{A}_2$ of algebra $\mathfrak{A}$ have identical spectral representation, then they obey the fourth postulate. Therefore, such elements coincide.

We call physical state $\varphi^0_\alpha$ a ground state if $\varphi^0_\alpha(\hat{p}_0) = 1$.

**Statement.** If $\hat{A} \in \mathfrak{A}_+$, then $\hat{A}_0 \equiv \hat{p}_0 \hat{A} \hat{p}_0$ has the form $\hat{A}_0 = \hat{p}_0 \Psi_0(\hat{A})$, where $\Psi_0(\hat{A})$ is the linear, positive functional. It satisfies the normalization condition $\Psi_0(\hat{I}) = 1$.

**Proof.** Because $[\hat{A}_0, \hat{p}_0] = 0$ it follows that $\hat{A}_0$ and $\hat{p}_0$ have the common spectral decomposition of unity. Since the projector $\hat{p}_0$ is one-dimensional, the spectral decomposition $\hat{A}_0$ has the form $\hat{A}_0 = \hat{p}_0 \Psi_0(\hat{A}) + \hat{A}'_0$ where $\hat{A}'_0$ is orthogonal to $\hat{p}_0$. Therefore, $\hat{A}_0 = \hat{p}_0 \hat{A}_0 = \hat{p}_0 \hat{p}_0 \Psi_0(\hat{A}) + \hat{p}_0 \hat{A}'_0 = \hat{p}_0 \Psi_0(\hat{A})$.

Linearity:

$$\hat{p}_0 \Psi_0(\hat{A} + \hat{B}) = \hat{p}_0 \hat{A} \hat{B} \hat{p}_0 = \hat{p}_0 \Psi_0(\hat{A}) + \hat{p}_0 \Psi_0(\hat{B}).$$

From here follows $\Psi_0(\hat{A} + \hat{B}) = \Psi_0(\hat{A}) + \Psi_0(\hat{B})$.

By linearity, the functional $\Psi_0(\hat{A})$ can be expanded to the algebra $\mathfrak{A}$, $\Psi_0(\hat{A} + i \hat{B}) = \Psi_0(\hat{A}) + i \Psi_0(\hat{B})$, where $\hat{A}, \hat{B} \in \mathfrak{A}_+$.

Positivity:

$$\Psi_0(\hat{R}^* \hat{R}) = \varphi^0_\alpha(\hat{p}_0 \Psi_0(\hat{R}^* \hat{R})) = \varphi^0_\alpha(\hat{p}_0 \hat{R}^* \hat{R} \hat{p}_0) \geq 0,$$

by virtue of the property (2.3).

Normalization:

$$\Psi_0(\hat{I}) = \varphi^0_\alpha(\hat{p}_0 \Psi_0(\hat{I})) = \varphi^0_\alpha(\hat{p}_0 \hat{I} \hat{p}_0) = 1.$$

To find the physical meaning of the functional $\Psi_0$, it is necessary to consider an element $\hat{A}$ in the algebra $\mathfrak{A}$ that corresponds to an observable $\hat{A}$ averaged in time.

$$\bar{A} = \lim_{L \to \infty} \frac{1}{2L} \int_{-L}^{L} dt \hat{A}(t) = \lim_{L \to \infty} \frac{1}{2L} \int_{-L}^{L} dt \hat{U}^{-1}(t) \hat{A} \hat{U}(t).$$

It is possible to show (see [19]) that

$$\Psi_0(\hat{A}) = \varphi^0_\alpha(\bar{A}).$$

That is, the value of the observable $\hat{A}$ in the quantum ground state $\Psi_0$ is equal to the value of the observable $\bar{A}$ in the physical ground state $\varphi^0_\alpha$. This value is the same in all physical ground states.

The functional $\Psi_0$ has all the properties that must be possessed by a functional determining quantum mean values. It is linear, is positive, and is equal to unit on the unit element. In addition it is continuous, as a linear functional on the $C^*$-algebra. Therefore, we can accept the ergodicity axiom.

**Postulate 9:**
The mean value of an observable $\hat{A}$ in the quantum ground state is equal to the value of the observable $\bar{A}$ (the observable $\hat{A}$ average in time) in any physical ground state.

Thus, averaging in quantum ensemble can be reduced to averaging in time. We note that the Postulate 9 does Postulates 5 and 6 superfluous.

To construct the standard mathematical formalism of quantum mechanics, we can now use the canonical construction of Gelfand-Naimark-Segal (GNS) (see, e.g., [11]).

We consider two elements $\hat{R}, \hat{S} \in \mathfrak{A}$ equivalent if the condition $\Psi_0(\hat{K}^*(\hat{R} - \hat{S})) = 0$ is valid for any $\hat{K} \in \mathfrak{A}$. We let $\Phi(\hat{R})$ denote the equivalence class of the element $\hat{R}$ and consider the set $\mathfrak{A}(\Psi_0)$ of all equivalence classes in $\mathfrak{A}$. We make $\mathfrak{A}(\Psi_0)$ a linear space setting $a\Phi(\hat{R}) + b\Phi(\hat{S}) = \Phi(a\hat{R} + b\hat{S})$. The scalar product in $\mathfrak{A}(\Psi_0)$ is defined as $(\Phi(\hat{R}), \Phi(\hat{S})) = \Psi_0(\hat{R}^*\hat{S})$. This scalar product generates the norm $\|\Phi(\hat{R})\|^2 = \Psi_0(\hat{R}^*\hat{R})$ in $\mathfrak{A}(\Psi_0)$.

Completion with respect to this norm makes $\mathfrak{A}(\Psi_0)$ a Hilbert space. Each element $\hat{S}$ of the algebra $\mathfrak{A}$ is uniquely assigned a linear operator $\Pi_\Psi(\hat{S})$ acting in this space as $\Pi_\Psi(\hat{S})\Phi(\hat{R}) = \Phi(\hat{S}\hat{R})$.

5 Examples

To illustrate the above, we consider two simple examples.

First we consider a quantum system whose observable quantities are described by Hermitian $2 \times 2$ matrices. The Hamiltonian $\hat{H}$ and the elements $\hat{p}_0, \hat{A}$ are given by

$$\hat{H} = \begin{bmatrix} E_0 & 0 \\ 0 & -E_0 \end{bmatrix}, \quad \hat{p}_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Obviously, $\hat{p}_0 \Psi_0(\hat{A}) = \hat{p}_0 \hat{A} \hat{p}_0 = \hat{p}_0 d$, i.e.,

$$\Psi_0(\hat{A}) = d. \quad (9)$$

In addition,

$$\bar{A} = \lim_{L \to \infty} \frac{1}{2L} \int_{-L}^{L} dt e^{-iE\tau_3} \hat{A} e^{iE\tau_3} = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix},$$

where $\tau_i$ are Pauli matrices.

All physical states can easily be constructed. We consider a Hermitian matrix $\hat{A}$, i.e., with $a^* = a$, $d^* = d$, and $c = b^*$. Any such matrix can be represented as

$$\hat{A} = r_0 \hat{I} + r \hat{\tau}(\hat{n}),$$

where $\hat{n}$ is the unit three-dimensional vector, $\hat{\tau}(\hat{n}) = (\hat{\tau}_1 \hat{n})$. For Eq.(11) to be valid, we must

$$r = \left( \frac{(ad)^2}{4} + bb^* \right)^{1/2}, \quad r_0 = \frac{a + d}{2},$$

$$n_1 = \frac{b + b^*}{2r}, \quad n_2 = \frac{b - b^*}{2ir}, \quad n_3 = \frac{a - d}{2r}.$$  

The commutator of the matrices $\hat{\tau}(\hat{n})$, $\hat{\tau}(\hat{n}')$ is nonvanishing for $\hat{n}' \neq \pm \hat{n}$. Therefore, each matrix $\hat{\tau}(\hat{n})$ (up to a sign) is a generator of a real maximal commutative subalgebra. Because $\hat{\tau}(\hat{n})\hat{\tau}(\hat{n}) = \hat{I}$, the spectrum of $\hat{\tau}(\hat{n})$ consists of two points $\pm 1$. 

14
Let \( \{ f(\bar{n}) \} \) be the set of all functions taking the values \( \pm 1 \) and such that \( f(-\bar{n}) = -f(\bar{n}) \). A physical state is described by a functional whose value coincides with one of the points in the spectrum of the corresponding algebra element. For each point of the spectrum, there exists an appropriate functional. Therefore, to the set of physical states, there corresponds a set of functionals defined by

\[
\varphi(\hat{r}(\bar{n})) = f(\bar{n}).
\]

Taking properties (2) into account (which must be possessed by each physical state), we obtain

\[
\varphi(A) = r_0 + r f(\bar{n}).
\]

The ground state is any functional \( \varphi_{0\alpha} \) such that

\[
f(n_1 = 0, n_2 = 0, n_3 = 1) = -1.
\]

Substituting the element \( \bar{A} \) (10) in (12), we obtain

\[
\varphi_{0\alpha}(\bar{A}) = \frac{a + d}{2} - \frac{a - d}{2} = d.
\]

This agrees with (9).

Because all maximal commutative subalgebras have one independent generator in this model, the physical states are described by the single-valued functionals. If there were several generators, then multivalued functionals would inevitably arise. It corresponds to the result obtained (in other terms) by Kochen and Specker [12].

As the second example we consider a harmonic oscillator.

In this case the algebra of dynamical variables is algebra with two noncommuting Hermitian generators \( \hat{Q} \) and \( \hat{P} \) satisfying the commutative relation

\[
[\hat{Q}, \hat{P}] = i.
\]

Time evolution in the algebra controls the Hamiltonian

\[
\hat{H} = \frac{1}{2}(\hat{P}^2 + \nu^2 \hat{Q}^2).
\]

The elements \( \hat{Q}, \hat{P} \) and \( \hat{H} \) are unbounded. Therefore, they do not belong to the \( C^* \)-algebra. However, their spectral projectors are elements of the \( C^* \)-algebra, i.e. \( \hat{Q}, \hat{P} \) and \( \hat{H} \) is the elements joined to the \( C^* \)-algebra. Thus, in this case the \( \mathfrak{A} \)-algebra is a \( C^* \)-algebra with the joined elements.

It is convenient to turn from the Hermitian elements \( \hat{Q} \) and \( \hat{P} \) to elements

\[
\hat{a}^- = \frac{1}{\sqrt{2\nu}}(\nu \hat{Q} + i\hat{P}), \quad \hat{a}^+ = \frac{1}{\sqrt{2\nu}}(\nu \hat{Q} - i\hat{P})
\]

with the commutative relation

\[
[\hat{a}^-, \hat{a}^+] = 1 \quad (13)
\]

and simple time dependence

\[
\hat{a}^-(t) = \hat{a}^- \exp(-i\nu t), \quad \hat{a}^+(t) = \hat{a}^+ \exp(+i\nu t).
\]
Let us calculate a generating functional for Green functions. In standard quantum mechanics the $n$-time Green function is defined by the equation
\[
G(t_1, \ldots, t_n) = \langle 0|T(\hat{Q}(t_1) \ldots \hat{Q}(t_n))|0\rangle,
\]
where $T$ is an operator of chronological ordering and $|0\rangle$ is a quantum ground state.

According to Postulate 9 in the proposed approach the Green function is defined by the equation
\[
\hat{p}_0 T(\hat{Q}(t_1) \ldots \hat{Q}(t_n))\hat{p}_0 = G(t_1, \ldots, t_n)\hat{p}_0,
\]
where $\hat{p}_0$ is a spectral projector $\hat{H}$ corresponding to the minimal value of energy.

It is easy to make sure that $\hat{p}_0$ can be represented in form
\[
\hat{p}_0 = \lim_{r \to \infty} \exp(-r\hat{a}^+\hat{a}^-).
\]
As we have earlier, here the limit is understood in sense of weak topology of the $C^*$-algebra.

First we prove the auxiliary statement:
\[
\hat{J} = \lim_{r_1, r_2 \to \infty} \exp(-r_1 k - r_2 l)\Psi((\hat{a}^+)^k(\hat{a}^-)^l) = 0.
\]
Let $\Psi$ be an any positive linear functional. Then
\[
\Psi(\hat{J}) = \lim_{r_1, r_2 \to \infty} \exp(-r_1 k - r_2 l)\Psi((\hat{a}^+)^k(\hat{a}^-)^l).
\]
Here, we have used a continuity of the functional $\Psi$ and the commutative relation. Further,
\[
|\Psi(\hat{J})| \leq \lim_{r_1, r_2 \to \infty} \exp(-r_1 k - r_2 l)|\Psi((\hat{a}^+)^k(\hat{a}^-)^l)|^{1/2} \times |\Psi((\hat{a}^+)^l(\hat{a}^-)^k)|^{1/2}
\]
\[
\leq \lim_{r_1, r_2 \to \infty} \exp(-r_1 k - r_2 l)|\Psi((\hat{a}^+)^k(\hat{a}^-)^l)|^{1/2} |\Psi((\hat{a}^+)^k(\hat{a}^-)^l)|^{1/2}
\]
Here we considered that $\|\exp(-r\hat{a}^+\hat{a}^-)\| \leq 1$. It follows from (17) that $|\Psi(\hat{J})| = 0$, i.e. it is valid.

We now prove (16). In terms of the elements $\hat{a}^+, \hat{a}^-$ the Hamiltonian $\hat{H}$ has form
\[
\hat{H} = \nu(\hat{a}^+\hat{a}^- + 1/2).
\]
According (17),
\[
\lim_{r_1, r_2 \to \infty} \exp(-r_1\hat{a}^+\hat{a}^-)\hat{H}\exp(-r_2\hat{a}^+\hat{a}^-) = \frac{\nu}{2} \lim_{r_1, r_2 \to \infty} \exp(-(r_1+r_2)\hat{a}^+\hat{a}^-) = \frac{\nu}{2} \lim_{r \to \infty} \exp(-(r)\hat{a}^+\hat{a}^-).
\]
It proves the equality (16).

It follows from the equation (15) that
\[
G(t_1, \ldots, t_n)\hat{p}_0 = \left(\frac{1}{t}\right)^n \delta^n \hat{p}_0 T \exp \left(i \int_{-\infty}^{\infty} dt \hat{J}(t)\hat{T}(t)\right) \hat{p}_0 |_{j=0}.
\]
By the Wick theorem (see (20))
\[
T \exp \left(i \int_{-\infty}^{\infty} dt \hat{J}(t)\hat{T}(t)\right) =
\]
\[
= \exp \left(\frac{1}{2t} \int_{-\infty}^{\infty} dt_1 dt_2 \frac{\delta}{\delta\hat{T}(t_1)} D^c(t_1 - t_2) \frac{\delta}{\delta\hat{T}(t_2)}\right) \exp \left(i \int_{-\infty}^{\infty} dt \hat{J}(t)\hat{T}(t)\right).
\]
Here \( \mathcal{D}^c(t_1-t_2) = \frac{1}{2\pi} \int dE \exp\left(-i(t_1-t_2)E\right) \frac{1}{\nu^2 - E^2 - i0} \)

Carrying out a variation over \( \hat{Q} \) in the right-hand side (21) and taking into account (17), we have

\[
\hat{p}_0 T \exp \left( i \int_{-\infty}^{\infty} dt \, j(t) \hat{Q}(t) \right) \hat{p}_0 = \exp \left( -\frac{1}{2i} \int_{-\infty}^{\infty} dt_1 dt_2 \, j(t_1) \mathcal{D}^c(t_1-t_2) j(t_2) \right) \times \hat{p}_0 \exp \left( i \int_{-\infty}^{\infty} dt \, j(t) \hat{Q}(t) \right) \hat{p}_0 = \hat{p}_0 \exp \left( -\frac{1}{2i} \int_{-\infty}^{\infty} dt_1 dt_2 \, j(t_1) \mathcal{D}^c(t_1-t_2) j(t_2) \right).
\]

Comparing with (20), we obtain

\[
G(t_1 \ldots t_n) = \left( \frac{1}{i} \right)^n \delta^n Z(j) \bigg|_{j=0},
\]

where

\[
Z(j) = \exp \left( \frac{i}{2} \int_{-\infty}^{\infty} dt_1 dt_2 \, j(t_1) \mathcal{D}^c(t_1-t_2) j(t_2) \right)
\]
is the generating functional.

This method of calculation of a generating functional for Green functions is easily to generalize for more substantial quantum models, in particular, quantum-field models.

6 Bell inequality

We now investigate how the measurability condition on the probability space is manifested in the important case of the derivation of the Bell inequality [21]. There are many forms of this inequality. Hereinafter, we refer to the version proposed in [22]. This variant is usually designated by the abbreviation CHSH.

Let particle with the spin 0 decay into two particles \( A \) and \( B \) with spin 1/2. These particles move apart, and the distance between them becomes large. The projections of their spins are measured by two independent devices \( D_a \) and \( D_b \). Let the device \( D_a \) measures the spin projection of the particle \( A \) on the \( a \) direction, and the device \( D_b \) measures the spin projection of the particle \( B \) on the \( b \) direction. We let \( \hat{A} \) and \( \hat{B} \) denote the corresponding observables and let \( A_a \) and \( B_b \) denote the measurement results.

Let us assume that the initial particle has some physical reality that can be marked by the parameter \( \lambda \). We use the same parameter to describe the physical realities for the decay products. Accordingly, it is possible to consider measurement results of the observables \( \hat{A} \), \( \hat{B} \) as the function \( A_a(\lambda), B_b(\lambda) \) of the parameter \( \lambda \). Let the distribution of the events with respect to the parameter \( \lambda \) be characterized by the probabilistic measure \( P(\lambda) \):

\[
\int dP(\lambda) = 1, \quad 0 \leq P(\lambda) \leq 1.
\]

We introduce the correlation function \( E(a, b) \):

\[
E(a, b) = \int dP(\lambda) \, A_a(\lambda) \, B_b(\lambda) \quad (22)
\]
and consider the combination
\[ I = |E(a, b) - E(a, b')| + |E(a', b) + E(a', b')| = \] (23)
\[ = \left| \int P(d\lambda) A_a(\lambda) [B_b(\lambda) - B_{b'}(\lambda)] \right| + \left| \int P(d\lambda) A_{a'}(\lambda) [B_b(\lambda) + B_{b'}(\lambda)] \right|. \]

The equalities
\[ A_a(\lambda) = \pm 1/2, \quad B_b(\lambda) = \pm 1/2 \] (24)
are satisfied for any directions \( a \) and \( b \). Therefore,
\[ I \leq \int P(d\lambda) \left| [A_a(\lambda)] [B_b(\lambda) - B_{b'}(\lambda)] + [A_{a'}(\lambda)] [B_b(\lambda) + B_{b'}(\lambda)] \right| = \]
\[ = 1/2 \int P(d\lambda) \left[ |B_b(\lambda) - B_{b'}(\lambda)| + |B_b(\lambda) + B_{b'}(\lambda)| \right]. \]

Due to the equality (24) for each \( \lambda \) one of the expressions
\[ |B_b(\lambda) - B_{b'}(\lambda)|, \quad |B_b(\lambda) + B_{b'}(\lambda)| \] (26)
is equal to zero and the other is equal to unity. Here it is crucial that the same value of the parameter \( \lambda \) appears in both expressions. Hence, the Bell inequality (CHSH) then follows:
\[ I \leq 1/2 \int dP(\lambda) = 1/2. \] (27)

The correlation function can be easily calculated within standard quantum mechanics. We obtain
\[ E(a, b) = -1/4 \cos \theta_{ab}, \]
where \( \theta_{ab} \) is the angle between the directions \( a \) and \( b \). For the directions \( a = 0, b = \pi/8, a' = \pi/4, b' = 3\pi/8 \) we have
\[ I = 1/\sqrt{2}. \]

It contradicts the inequality (27).

Experiments that have been performed corresponded to quantum-mechanical calculations and did not confirm the Bell inequality. These results have been interpreted as decisive argument against the hypothesis of the existence of local objective reality in quantum physics. It is easy to see that if the probability theory is properly applied to quantum system, then the above derivation of the Bell inequality is invalid.

Because the \( \sigma \)-algebra and accordingly probability measure depend on the measuring device used in a quantum case, it is necessary to make replacement \( dP(\lambda) \rightarrow dP_{\hat{A}\hat{B}}(\varphi) \) in the equation (22). If we are interested in correlation function \( E(a', b') \), it is necessary to make replacement \( P(d\lambda) \rightarrow P_{\hat{A}\hat{B}}(d\varphi) \) in the equation (22). Although we used the same symbols \( d\varphi \) in both cases for notation of the elementary volume in the space of the physical states, it is necessary to remember that sets of the physical states corresponding \( d\varphi \), are different. The matter is that these sets should be elements of the \( \sigma \)-algebras. If observables \( \hat{A}, \hat{B} \) are incompatible with observables \( \hat{A}', \hat{B}' \), then \( \sigma \)-algebras are different. Moreover, there are no physically allowable \( \sigma \)-algebra which has these algebras as subalgebras.

Besides in experiment we deal not with the complete probability space \( \Omega(\varphi_\xi) \) but with random denumerable sample \( \{\varphi\}_{\varphi_\xi}^{AB} \). Finally, the equation (22) should be replaced on
\[ E(a, b) = \int_{\{\varphi\}_{\varphi_\xi}^{AB}} P_{\hat{A}\hat{B}}(d\varphi) \varphi(\hat{A}\hat{B}). \]
Accordingly, the equation (23) now has the form

\[
I = \left| \int_{\{\varphi\}}^{AB} P_{AB}(d\varphi) \varphi(\hat{A}\hat{B}) - \int_{\{\varphi\}}^{AB'} P_{AB'}(d\varphi) \varphi(\hat{A}\hat{B}') \right| + \\
+ \left| \int_{\{\varphi\}}^{A'B} P_{A'B}(d\varphi) \varphi(\hat{A}'\hat{B}') + \int_{\{\varphi\}}^{A'B'} P_{A'B'}(d\varphi) \varphi(\hat{A}'\hat{B}') \right|. 
\]

If the directions \(a\) and \(a'\) (\(b\) and \(b'\)) are not parallel to each other, then the observables \(\hat{A}\hat{B}, \hat{A}'\hat{B}', \hat{A}'\hat{B}, \hat{A}'\hat{B}'\) are mutually incompatible. Therefore, there is no physically acceptable universal \(\sigma\)-algebra that corresponds to the measurement all these observables. It follows that there is no probability measure common for these observables. Besides, the sets \(\{\varphi\}^{AB}, \{\varphi\}^{A'B}, \{\varphi\}^{A'B}, \{\varphi\}^{A'B'}\) are different random denumerable samples from continual space \(\Omega(\varphi_\xi)\). The probability of their crossings is equal to zero. Therefore, the probability of occurrence of combinations of the type (26) is equal to zero. As a result, the reasoning which have led to to an inequality (27) appears unfair for the physical states.

Thus, the hypothesis that local objective reality does exist in the quantum case does not lead to the Bell inequalities. Therefore, the numerous experimental verifications of the Bell inequalities that have been undertaken in the past and at present largely lose theoretical grounds.

7 The possible carrier of "the objective local reality"

In the previous sections of the paper we tried to show that, contrary to a popular belief, the mathematical formalism of the standard quantum mechanics does not contradict the hypothesis on existence of an objective local reality. In the developed approach this reality is identified with concept "the physical state". The mathematical essence of this concept is defined quite uniquely (the functional \(\varphi\)). It would be desirable to have some physical filling of this concept.

In a present section we heuristically consider one of the variants (23) of such filling. This variant is not unique. At the same time, it gives simple and obvious interpretation to the problem phenomenon such as a collapse of the quantum state.

Within the framework of the algebraic approach the quantum object is a finite region in the space-time, which the certain noncommutative local algebra (algebra of quantum local observables) is associated with.

On the other hand, the quantum object is a source of some field. Obviously, any quantum object is a source of the gravitational field. Till now all attempts consistently to quantize the gravitational field were not crowned with success. Probably, it is related to that the gravitational field is classical, i.e. the corresponding algebra of observables is commutative.

Probably, quantum objects are also sources of other classical fields, in particular, the classical electromagnetic field. This field should be very weak. In this case for microobjects it is unobservable on background of the quantum electromagnetic field.

It is natural to assume that the classical field radiated by a microobject, is coherent to this object. Thus, it is a carrier of the information about the object. If the microobject is multipartial then the radiated field is coherent to both the separate parts of the object, and to all collective. In this case the field is a carrier of the information about correlations.
Macroscopic bodies can act on the radiated classical field essentially. This action can be two types. The first type is action which destroys the coherence of the field. Such action is irreversible. The second type is action which preserves the coherence. So the mirror acts on radiation. Usually, both type of interactions are present at measuring devices. The coherence is preserved in the analyzer and destroyed in the detector.

On account of weaknesses the classical field exercises negligibly small effect on both micro- and macroobjects. Unique exception is the microobject, coherent to this field. In this case the action is resonant. Therefore, even very small action can lead to appreciable result if the quantum object is in the state of the bifurcation. That is, in the state where without taking into account this action several variants of the further evolution are possible. In this case such action can play a role of random force which forces the quantum object to choose one of the variants. In this sense the classical field can be considered as a pilot field. It is possible to assume that the configuration of the classical field, coherent to the quantum object, is the objective reality which determines the physical state of this object.

For an illustration of this phenomenon we discuss experiment whose scheme is represented on Figure 1.

![Diagram]

Figure 1.

The device consists of four mirrors (1,2,3,4) and three detectors (D₁, D₂, D₃). The mirrors 1 and 4 are semipermeable. The detectors D₁ and D₃ are necessary only for registration of photons. The detector D₂ plays central role in the phenomenon of collapse. At the device the photon and the coherent classical field either are reflected from mirrors, or pass through them. After reflection from the mirrors the phase changes by $\pi/2$, at passage through a semipermeable mirror the phase does not change.

Second, the mirror 1 is a point of the bifurcation for the photon. Without taking into account interaction with the oscillations excited by the classical field in the mirror, both channels for the photon are equally allowable. Oscillations are very weak but they are coherent to the photon. Therefore, interaction is resonant. Due to this interaction the information which is stored in the classical field (the physical state) dictates to the photon a choice of the channel.

From the point of view of quantum mechanics this choice is random. The fact is that quantum mechanics deals not with physical state, but only with its generalized characteristic — quantum state. The various configurations of the coherent classical field correspond to
the same quantum state. On the other hand, with the help of preliminary measurements we can receive an information only about the quantum state. Therefore, the choice of the routes by the photon is random for us.

The phases of the photon and separate parts of the classical field can vary when they pass channels, but their coherency is kept. According to rules of the classical optics in the mirror 4 the separate parts of the classical field interfere so that after the mirrors 4 the field does not propagate to the detector $D_3$. Physically the classical field raises the small collective oscillations in the mirror 4 coherent to the field. The scattering occurs on these oscillations. The photon also is coherent to these oscillations and scatters the same as the classical field. Therefore, it also does not hit the detector $D_3$.

We now consider the second variant of the experiment when the detector $D_2$ is switched on. In the mirror 1 everything happen the same way as in the first variant. Two scenarios are further possible, in which the photon goes by the route 1-3-4 or does by the route 1-2-4.

In the scenarios with the route 1-3-4 the photon hits the detector $D_2$. There the photon participates in interaction with the classical device. The device goes out of unstable equilibrium due to interaction with the photon. The catastrophic process develops in the device. This process has macroscopically observable result and the quantum object is registered.

The detector exerts strong action on the photon. Its state changes, and it loses a coherency with earlier radiated classical field. Again radiated classical field is coherent to the photon in the new state. At the same time, the field in the channel 1-2-4 is not coherent to the photon. Because only the coherent classical field determines the physical state of the quantum object, the field in the channel 1-2-4 is effectively lost for the photon.

This process results in a sharp modification of the coherent classical field of the quantum object. The quantum state of the object also changes sharply thereof. This phenomenon has all features of the collapse. However, any inconsistency with a relativity theory does not arise, as in the channel 1-2-4 the classical field does not change. The modifications happen in the channel 1-3-4. Thus, the classical fields in the channels 1-2-4 and 1-3-4 do not disappear in the collapse, but these fields lose the coherence with each other. Therefore, in the mirror 4 interference is absent. Latter corollary agrees with the corollary adduced in the review [24].

Let us consider now the second scenario in which the photon goes by the route 1-2-4, and the photon-free classical field goes through the detector $D_2$. This field exercises negligibly small effect on the detector. In this case the cause generating catastrophic process in the detector is absent. Any macroscopically observable of the reaction of the device is not present.

On the contrary, the action of the detector affects strongly the classical field in the channel 1-3-4. This field loses coherence with the field and the photon in the channel 1-2-4. The situation is the same as in the first scenario.

Quite similarly it is possible to interpret so-called delayed-choice experiment [25]. It is usually considered that this experiment testifies to absence of the local physical reality in the quantum phenomena.

We now can look at the experiment double-slits [26] in a new fashion. The distinct interference pattern is observed in this experiment. If to reject any verbal ornaments standard interpretation of this experiment is reduced to the following. Up to the slits the indivisible quantum object passes simultaneously through the slits divided by the macroscopic distance then it again becomes indivisible.

The proposed approach allows to interpret this experiment much more evidently. The indivisible quantum object hits one of the slits and scatters on it. It can scatter in region behind the slit or in the opposite side. From the point of view of the standard quantum
mechanics this process is random. In terms of the present paper it means that the slit is region of the bifurcation for the quantum object. In this region the behaviour of the concrete quantum object is determined by the random force, i.e. by classical, coherent to the quantum object, the field in the area of the slit.

According to rules of classical physics the part of the classical field, scattered on both slits, interfere among themselves. Therefore, presence of the second slit influences structure of the field in the area of the first slit. Accordingly, presence of the second slit influences on chance of passage of the quantum object through the first slit. Thus, the ensemble of the quantum objects, which have passed through simultaneously open slits, is not a mix of two ensembles of the quantum objects, which have passed through any one of the slits when another is closed. As a result there is the interference pattern. If we place the detector in one of slits, it disturbs the coherency of the parts of the classical field, scattered on the different slits, and the interference becomes impossible.

It is possible to look at this experiment from slightly other position. It is possible to use these two slits as the device determining localization of the quantum object. In this case we can consider that if the quantum object is found out behind the plane of the slits at the moment of passage of these slits, then it is located inside one or two slits depending on what slits are open. Permeability of each slit depends on that, other slit is open or closed. Therefore, corollary about localization of the concrete quantum object depends not only on properties of the object but also on properties of the measuring device. Whether both slits or only one of them are open in a present concrete case. It is possible such, paradoxical from the point of view of classical physics situation when two open slits appear impenetrable for the quantum object which could pass through one open slit when another closed.

8 Conclusions

In principle, a physical state can be considered as a special hidden parameter [27]. Hidden parameters have had a "bad reputation" in quantum mechanics since the works of von Neumann. In his illustrious monograph [1], von Neumann argued that models with hidden parameters are incompatible with the fundamentals of quantum mechanics. As basic fundamentals of quantum mechanics, he accepted, in particular, axioms: Linear operators in the Hilbert space correspond to quantum observables; the state of quantum system is described by a linear functional (density matrix).

In the approach proposed here these statements are considerably weakened. It is supposed only that observables are elements of some algebra. They become linear operators only in representation which is generated by linear functional of a special form (quantum average). The physical state of the quantum system is described by the nonlinear functional. The von Neumann’s reasonings are not valid for this functional.

Essentially, it was shown in [1] that the linearity of the state is incompatible with causality and the hypothesis on hidden parameters. Therefore, von Neumann concluded that there is no causality at the microscopic level while it appears at the macroscopic level because of averaging over a large number of noncausal events.

The approach formulated in this work allows avoiding this conflict in just the opposite way. It can be said that causality does exist at the level of individual microscopic phenomena, but linearity does not. The linearity of the (quantum) state appears because of averaging over the quantum ensemble. The passage from individual phenomena to quantum ensembles
implies the passage from the initial determinism to the probabilistic interpretation.

Because of the multivaluedness of the functional $\varphi$, the conditions of the Kochen and Specker no-go theorem are not satisfied for this hidden parameter. Finally, the conditions of Bell’s theorem are not satisfied for the functional $\varphi$. Therefore, the arguments that are usually adduced by opponents of the use of hidden parameters in quantum mechanics become inapplicable for the physical state.

Regarding the multivaluedness of the functional $\varphi$, the physical process that is usually called a measurement should rather be called an observation. The term "observation" better expresses the fact that the readout of the device has two causes: the physical state of the observed object, and the type of the device.

It then follows that the concept of a local objective reality and, traced back to Bohr, ”the situational (contextual) approach” are not in such antagonistic contradiction as it is considered to be. It is quite allowable that there is a physical reality which is inherent to the quantum object under consideration and which predetermines the result of any experiment. However, this result can depend on conditions in which this experiment is carried out. One of these conditions is the classical characteristic (type) of the measuring device, which is used in a concrete case.

The quantum logic is not used in the present paper. Actually, the quantum logic is a formulary for subspaces of the Hilbert space and for projectors on these subspaces. By the way, these rules are obtained with the help of the formal classical logic. They are useful to the description of the Hilbert space. However it cannot be the basis for attributing to these rules of general character in the question "what is a true”. Namely this question is in the competence of logic.

The Hilbert space is not a primary element in the proposed approach to quantum mechanics. It appears as some by-product alongside with others. Therefore, ”quantum logic” is not the tool which is useful in the present approach.

References

[1] Von Neummann J. Mathematical Foundation of Quantum, New York: Mechanic Prentice-Hall (1952).
[2] M.B. Mensky. Physic-Uspekhi, 43 (2000) 585.
[3] H. Everett. Rev.Mod.Phys., 29 (1957) 454.
[4] G.G. Emch. Algebraic Methods in Statistical Mechanics and Quantum Field Theory, Wiley-Interscience, a Division of John Wiley and Sons, INC, New York (1972).
[5] S.S. Horuzhy. Introduction to Algebraic Quantum Field Theory, Klwer, Dordrecht (1990).
[6] N.N. Bogoliubov, A.A. Logunov, A.I. Oksak, and I.T. Todorov. General Principles of Quantum Field Theory, Klwer, Dordrecht (1990).
[7] W. Rudin. Functional Analysis, New York: McGraw-Hill Company, (1973)
[8] A. Zeilinger. Rev. Modern. Phys., 71 (1999) S289.
[9] P. Jorrdan. Comm. Math. Phys., 80 (1933) 285.
[10] D.A. Slavnov. Theor. Math. Phys., 129 (2001) 87; D.A. Slavnov ArXiv: quant-ph/0101139
[11] D.A. Slavnov. Theor. Math. Phys., 132 (2002) 1264;
[12] S. Kochen, E.P. Specker. Journ. of Mathematics and Mechanics, 17 (1967) 59.
[13] D. Home, M.A.B. Whitaker. Phys. Rep., 210 (1992) 223.
[14] A.N. Kolmogorov, Foundation of the theory of probability, Chelsea, New York (1956).
[15] D.A. Slavnov. Theor. Math. Phys., 136 (2003) 1273
[16] J. Neveu. Bases mathématiques du calcul des probilités, Paris: Masson (1968)
[17] A. Einstein, B. Podolsky, and N. Rosen. Phys. Rev., 47 (1935) 777.
[18] J. Dixmier. Les $C^*$ algèbres et leurs représentations, Paris: Gauthier – Villars Éditeur 1969.
[19] D.A. Slavnov. ArXiv:quant-ph/0101139
[20] N.N. Bogoliubov and D.V. Shirkov. Introduction to Quantum Fields, Wiley, New York 1980.
[21] J.S. Bell. Physics (Long Island City, N.Y. 1 (1965) 195
[22] J.F. Clauser, M.A. Horn, A. Shimony, R.A. Holt. Phys. Rev. Lett., 23 (1969) 880
[23] D.A. Slavnov. Theor. Math. Phys., 106 (1996) 220.
[24] M. Namiki, S. Pascazio. Phys. Rep. 232 (1993) 301.
[25] J.A. Wheeler. Mathematical Founation of Quantum Theory, New York: Academic Press 1978
[26] A. Tonomura. Phys. Today, 41 (1990) 22.
[27] D. Bohm. Phys. Rev., 85 (1952) 166