A Lattice Study of the Glue in the Nucleon

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Abstract

By introducing an additional operator into the action and using the Feynman–Hellmann theorem we describe a method to determine both the quark line connected and disconnected terms of matrix elements. As an illustration of the method we calculate the gluon contribution (chromo-electric and chromo-magnetic components) to the nucleon mass.
1 Introduction

One of the earliest experimental indications that the nucleon consists not only of three quarks, but also has a gluonic contribution came from the measurement of the fraction of the nucleon momentum carried by the quarks. That this did not sum up to 1 as is required from the energy–momentum sum rule gave evidence for the existence of the gluon. Denoting \( \langle x \rangle_f \) as the fraction of the nucleon momentum carried by parton \( f \) we have

\[
\sum_q \langle x \rangle_q + \langle x \rangle_g = 1,
\]

where for the quarks \( f \equiv q = u, d, \ldots \) and for the gluon \( f \equiv g \). Experimentally \( \langle x \rangle_{u+d} \sim 0.4 \) so the missing component is large ~ 50% of the total nucleon momentum. Both \( \langle x \rangle_q \) and \( \langle x \rangle_g \) have similar definitions and so analogously to the definition of \( \langle x \rangle_q \) we have, with \( M \) denoting Minkowski space

\[
\langle N(p)| [\hat{O}^{M(g)\mu_1\mu_2} - \frac{1}{4} \eta^{\mu_1\mu_2} \hat{O}^{M(g)\alpha_1\alpha_2}]|N(p')\rangle = 2 \langle x \rangle_g [p^{\mu_1} p^{\mu_2} - \frac{1}{4} \eta^{\mu_1\mu_2} m_N^2] ,
\]

where

\[
O^{M(g)\mu_1\mu_2} = -\text{tr}_c F^{M\mu_1\alpha} F^{M\mu_2\alpha},
\]

(where \( O(t) = \int d^3x O(t, \vec{x}) \) and with normalisation \( \langle N(p)|N(p')\rangle = 2E_N \delta(\vec{p} - \vec{p}') \)). Note that we can generalise from a nucleon to an arbitrary hadron (averaging over polarisations if necessary). Higher moments can also be considered, by inserting covariant derivatives between the \( F_s \). These occur when using the Wilson operator product expansion which relates them to moments of structure functions in a twist expansion.

There have been many lattice estimates of the quark momentum fraction \( \langle x \rangle_q \) both for the nucleon (see e.g. [1, 2] for a review) and the pion e.g. [3, 4], but few attempts for the gluon part, \( \langle x \rangle_g [5, 6, 7] \). This is due to the fact that a lattice simulation must compute a quark line disconnected term, which is extremely noisy and gives a poor signal. These are direct calculations; in this letter we propose a new method using the Feynman–Hellmann theorem, to determine the gradient of \( E_N \) as a function of a parameter of an operator which has been introduced into the action \( S \rightarrow S(\lambda) = S + \lambda S_O \). An obvious disadvantage of this method is that it requires dedicated simulations for each operator of interest, but the gain, as we shall see, is a much cleaner signal.

While the method is general, we shall demonstrate its practicability here by determining \( \langle x \rangle_g \) in the quenched case.

2 The Feynman–Hellmann theorem

We first briefly describe the Feynman–Hellmann theorem, in a Euclidean form that will be useful for the case to be considered here. Let \( S \) depend on some
parameter $\lambda$, so $S \rightarrow S(\lambda)$. Now as by definition the (Euclidean) correlation function is given by

$$\langle N(t)\overline{N}(0) \rangle_\lambda \equiv \frac{\int [dU] N(t)\overline{N}(0)e^{-S(\lambda)}}{\int [dU] e^{-S(\lambda)}}$$ \hspace{1cm} (4)

(the unpolarised case for the nucleon and where we make the obvious replacements $N$ by $H$ and $\overline{N}$ by $H^\dagger$ for other hadrons), then we have

$$\frac{\partial}{\partial \lambda} \langle N(t)\overline{N}(0) \rangle_\lambda = -\left\langle N(t) \left( \frac{\partial S(\lambda)}{\partial \lambda} - \langle \frac{\partial S(\lambda)}{\partial \lambda} \rangle_\lambda \right) \overline{N}(0) \right\rangle_\lambda \hspace{1cm} (5)$$

We now use the transfer matrix formalism on both sides of this equation. Ignoring finite size effects this gives

$$\langle N(t)\overline{N}(0) \rangle_\lambda = A_N(\lambda)e^{-E_N(\lambda)t} + \text{exp. smaller terms} \hspace{1cm} (6)$$

so on the LHS of eq. (5),

$$\frac{\partial}{\partial \lambda} \langle N(t)\overline{N}(0) \rangle_\lambda = -\frac{\partial E_N(\lambda)}{\partial \lambda} \langle N(t)\overline{N}(0) \rangle_\lambda t + \text{exp. smaller terms} \hspace{1cm} (7)$$

Furthermore, if $\Omega(\tau)$ is any operator (local in time), then using the transfer matrix formalism again the associated 3-point function gives

$$\langle N(t)\Omega(\tau)\overline{N}(0) \rangle_\lambda = \left\{ \begin{array}{ll} \frac{1}{2E_N(\lambda)} \langle N|\hat{\Omega}|N \rangle_\lambda + \text{exp. small terms} & 0 \ll \tau \ll t \\ \text{exp. small terms} & \text{otherwise} \end{array} \right\} \hspace{1cm} (8)$$

Note that we have inserted a $2E_N$ in the denominator of the RHS to account for the mis-match of normalisations, i.e. to agree with those of eq. (2). Hence summing over $\tau$ also gives a linear term in $t$. Thus from this equation, replacing $\sum_\tau \Omega(\tau)$ by the operator in the RHS of eq. (5), and together with eq. (7) we have the Feynman–Hellmann theorem

$$\frac{\partial E_N(\lambda)}{\partial \lambda} = \frac{1}{2E_N(\lambda)} \left\langle N \left| \frac{\partial S(\lambda)}{\partial \lambda} \right| N \right\rangle_\lambda \hspace{1cm} (9)$$

(where : : : : : means that the vacuum term has been subtracted). Thus by suitably choosing $S_O$ and by identifying numerically the gradient of $E_N(\lambda)$ at $\lambda = 0$ we can determine the desired matrix element.
3 The lattice method

3.1 Gluon operators

Before considering the lattice, let us first Euclideanise the gluon operators\(^1\) to give us an indication of what we might add to the action. Defining

\[
O_{\mu\nu} = -\text{tr}_c F_{\mu\alpha} F_{\nu\alpha} ,
\]

\((\text{tr}_c F^2 = \frac{1}{2} F^a F^a)\) this then gives the two obvious operator choices \((a)\) and \((b)\),

\[
O_{ai} = O_{i4} = \text{tr}_c (\vec{E} \times \vec{B})_i \\
O_b = O_{44} - \frac{1}{4} O_{jj} = \frac{2}{3} \text{tr}_c (-\vec{E}^2 + \vec{B}^2) 
\]

\((O_{a\text{M}(g)}^{M(g)} \rightarrow iO_a) \text{ and } O_{b\text{M}(g)}^{M(g)} \rightarrow O_b)\). The relation to \(\langle x \rangle_g\) is given by

\[
\langle N(\vec{p}) | \hat{O}_{ai} | N(\vec{p}) \rangle = -2i E_N p_i \langle x \rangle_g \\
\langle N(\vec{p}) | \hat{O}_b | N(\vec{p}) \rangle = 2( m_N^2 + \frac{4}{3} \vec{p}^2 ) \langle x \rangle_g ,
\]

with

\[
\hat{O}_{ai} = \text{tr}_c (\vec{\hat{E}} \times \vec{\hat{B}})_i , \quad \hat{O}_b = \frac{2}{3} \text{tr}_c (-\vec{\hat{E}}^2 + \vec{\hat{B}}^2) .
\]

Both choices have their difficulties: operator \((a)\) always needs a non-zero momentum \(\vec{p}\), while operator \((b)\) requires a delicate subtraction between two terms similar in magnitude.

Note that, because of Euclideanisation (footnote\(^1\) the energy has a negative \(E^2\) term, while the action (see section 3.2) has a positive \(E^2\) term.

3.2 The action

We now turn to the lattice. We shall use the Wilson gluonic action

\[
S = \frac{1}{3} \beta \sum_{x, \mu < \nu} \text{Re} \text{tr}_c \left[ 1 - U_{\mu\nu}^\square (x) \right] ,
\]

(i.e. sum over plaquettes), with \(\beta = 6/g^2\). As

\[
\text{Re} \text{tr}_c \left[ 1 - U_{\mu\nu}^\square (x) \right] = \frac{1}{2} a^4 g^2 F_{\mu\nu}^a(x)^2 + \ldots ,
\]

\(^1\)Our conventions follow \[3\]. So \(E^{M_i} = F^{M_i0} \rightarrow iF_{i4} = iE_i\) and \(B^{M_i} = -\frac{1}{2} \epsilon_{ijk} F^{M_jk} \rightarrow \frac{1}{2} \epsilon_{ijk} F_{jk} = B_i\).
this motivates the simplest definition of electric and magnetic field on each time slice as
\[ \frac{1}{2} \mathcal{E}^a(\tau) = 3 \beta a \sum_{\vec{x}_i} \text{Re} \text{tr} \left[ 1 - U_{ia}^\square(\vec{x}, \tau) \right] \]
\[ \frac{1}{2} \mathcal{B}^a(\tau) = 3 \beta a \sum_{\vec{x}_i<j} \text{Re} \text{tr} \left[ 1 - U_{ij}^\square(\vec{x}, \tau) \right] , \]
respectively. For the action we thus take
\[ S(\lambda) = a \sum_\tau \left( \frac{1}{2} \mathcal{E}^a(\tau) + \mathcal{B}^a(\tau) \right) - \lambda a \sum_\tau \left( -\frac{1}{2} \mathcal{E}^a(\tau) + \mathcal{B}^a(\tau) \right) , \]
or in terms of the gauge plaquettes
\[ S(\lambda) = 3 \beta (1 + \lambda) \sum_i \text{Re} \text{tr} \left[ 1 - U_{ia}^\square(\vec{x}, \tau) \right] + 3 \beta (1 - \lambda) \sum_{i<j} \text{Re} \text{tr} \left[ 1 - U_{ij}^\square(\vec{x}, \tau) \right] . \]

Of course for \( \lambda = 0 \), then this reduces to the standard action, eq. (14).

### 3.3 Gluon moment

Comparing the results of sections 3.1 and 3.2 we see that they can be applied to operator (b) only; operator (a) would require the clover definition of the field strength tensor. Using eq. (11) together with eq. (12) and eq. (9) gives from the Feynman–Hellmann theorem
\[ \frac{\partial E_N(\lambda)}{\partial \lambda} = -\frac{1}{2 E_N(\lambda)} \langle N(\vec{p}) \mid \frac{1}{2} ( -\hat{\mathcal{E}}^a + \hat{\mathcal{B}}^a ) \mid N(\vec{p}) \rangle_\lambda , \]
which leads to
\[ \left. \frac{\partial E_N(\lambda)}{\partial \lambda} \right|_{\lambda=0} = -\frac{3}{2 E_N} \left( m_N^2 + \frac{4}{3} \vec{p}^2 \right) \langle \vec{x} \rangle_{\text{lat}} , \]
where the \( \text{lat} \) superscript on \( \langle \vec{x} \rangle_{\text{lat}} \) signifies that it is now the lattice operator.

The vacuum term which appears in section 2 has been dropped, because
\[ \langle 0 \mid \frac{1}{2} ( -\hat{\mathcal{E}}^a + \hat{\mathcal{B}}^a ) \mid 0 \rangle = 0 . \]
This follows from rotation symmetry. In the Euclidean vacuum the time and space directions are equivalent, so the average trace of the chromo-electric plaquettes, \( U_{ia}^\square \), is the same as that of the chromo-magnetic plaquettes, \( U_{ij}^\square \), in eq. (16), leading to perfect cancellation in eq. (21).
4 Lattice results

We work with quenched Wilson clover fermions at $\beta = 6.0$, $c_{sw} = 1.769$ and $\kappa = 0.1320, 0.1324, 0.1333, 0.1338, 0.1342$ on a $24^3 \times 48$ lattice with antiperiodic time boundary conditions for the fermion. We have generated $O(500)$ configurations for each ensemble. We use standard nucleon interpolating operators together with Jacobi smeared source/sink as in e.g. [3]. The results were generated using the Chroma program suite, [8]. We have only considered the case $\vec{p} = 0$ so eq. (20) reduces to

$$\langle x \rangle_{lat} = -\frac{2}{3am_N} \frac{\partial am_N(\lambda)}{\partial \lambda} \bigg|_{\lambda = 0}. \quad \text{(22)}$$

To estimate the gradient at $\lambda = 0$, we have generated data at $\lambda = -0.03333, 0.0, 0.03333$ which enables us to straddle the $\lambda = 0$ point. The raw data results are given in Table 1.

$$\begin{array}{l|llll}
\kappa & \lambda = -0.03333 & \lambda = 0 & \lambda = 0.03333 \\
0.1320 & 1.0033(29) & 0.9772(33) & 0.9564(34) \\
0.1324 & 0.9537(30) & 0.9283(34) & 0.9077(36) \\
0.1333 & 0.8357(33) & 0.8117(40) & 0.7923(41) \\
0.1338 & 0.7649(38) & 0.7413(47) & 0.7236(47) \\
0.1342 & 0.7044(47) & 0.6799(62) & 0.6647(55) \\
\end{array}$$

Table 1: Nucleon masses, $am_N$, as a function of $\lambda$ for five quark masses, $\kappa$, calculated on ensembles with fixed $\beta = 6.0$ and $c_{sw} = 1.769$.

In Fig. 1 we plot the nucleon mass, $am_N$, against $\lambda$ for the five quark masses. The data show no $O(\lambda^2)$ effects for the $\lambda$ values chosen. These gradients (at $\lambda = 0$) together with the nucleon masses (again at $\lambda = 0$) determine $\langle x \rangle_{lat}$ from eq. (22) which are given in Table 2.

$$\begin{array}{l|lll}
\kappa & am_x & \langle x \rangle_{lat} \\
0.1320 & 0.55499(48) & 0.4826(456) \\
0.1324 & 0.51745(49) & 0.4985(502) \\
0.1333 & 0.42531(52) & 0.5383(644) \\
0.1338 & 0.36711(55) & 0.5620(811) \\
0.1342 & 0.31433(62) & 0.5893(1062) \\
\end{array}$$

Table 2: The pion mass and $\langle x \rangle_{lat}$ for the five different quark masses.
5 Renormalisation

As gluon operators are singlets, they can mix with the quark singlet. However there exists a combination of singlet operators with vanishing anomalous dimension. (This is due to the conservation of the energy-momentum tensor, eq. (1).) We follow [6] and first write

\[ \langle x \rangle_{g}^{\text{bare}} + \sum_{q} \langle x \rangle_{q}^{\text{bare}} = 1 + O(a^2), \] (23)

where

\[ \langle x \rangle_{g}^{\text{bare}} = Z_{g} \langle x \rangle_{g}^{\text{lat}}, \quad \langle x \rangle_{q}^{\text{bare}} = Z_{q} \langle x \rangle_{q}^{\text{lat}}. \] (24)

Together with the change to a scheme (here taken as \( \overline{MS} \))

\[ \left( \begin{array}{c} \langle x \rangle_{g}^{\overline{MS}}(\mu) \\ \sum_{q} \langle x \rangle_{q}^{\overline{MS}}(\mu) \end{array} \right) = \left( \begin{array}{cc} \frac{Z_{g}^{\overline{MS}}}{\sum_{q} Z_{g}^{\overline{MS}}(\mu)} & 1 - \frac{Z_{g}^{\overline{MS}}}{Z_{g}^{\overline{MS}}(\mu)} \\ 1 - \frac{Z_{q}^{\overline{MS}}}{Z_{q}^{\overline{MS}}(\mu)} & \frac{Z_{q}^{\overline{MS}}}{\sum_{q} Z_{q}^{\overline{MS}}(\mu)} \end{array} \right) \left( \begin{array}{c} \langle x \rangle_{g}^{\text{bare}} \\ \sum_{q} \langle x \rangle_{q}^{\text{bare}} \end{array} \right), \] (25)

this completes the renormalisation procedure. As we are considering quenched QCD only there is a simplification as \( Z_{g}^{\overline{MS}} = 1 \),

\[ \langle x \rangle_{g}^{\overline{MS}}(\mu) = \langle x \rangle_{g}^{\text{bare}} + [1 - Z_{g}^{\overline{MS}}] \sum_{q} \langle x \rangle_{q}^{\text{bare}} \]

\[ \langle x \rangle_{q}^{\overline{MS}}(\mu) = \frac{Z_{q}^{\overline{MS}}}{Z_{g}^{\overline{MS}}(\mu)} \langle x \rangle_{q}^{\text{bare}}, \] (26)
(\(Z_{\text{bare qq}}(\mu)\) is common for all the quarks). We thus need to determine \(Z_g\), \(Z_q\) and \(Z_{\text{bare qq}}(\mu)\). We can find \(Z_g\) by following [10] in considering an alternative interpretation of the action [18]. We motivated this action by adding a multiple of the gluon \(x\) operator to the standard action, but we could also write the action as

\[
S = \frac{1}{3} \beta_t \sum_i \text{Re tr}_c \left[ 1 - U_{ij}(\vec{x}, \tau) \right] + \frac{1}{3} \beta_s \sum_{i<j} \text{Re tr}_c \left[ 1 - U_{ij}(\vec{x}, \tau) \right],
\]

which is the standard way of writing a gluon action on an anisotropic symmetric lattice, with differing spatial and temporal lattice spacings, \(a_s \neq a_t\). This action has been studied in detail, in particular the way in which the anisotropy \(\xi = a_s/a_t\) depends on \(\beta_s\) and \(\beta_t\) is known both perturbatively and non-perturbatively [11]. At tree-level the anisotropy is given by

\[
\xi_{\text{tree}}^2 = \frac{\beta_t}{\beta_s}.
\]

\(Z_g\) can be found by comparing the anisotropy actually produced by splitting \(\beta_s\) and \(\beta_t\) with this tree-level value. The result is

\[
Z_g = 1 - \frac{g^2}{2} (c_\sigma - c_\tau)\]

where the anisotropy coefficients \(c_\sigma\) and \(c_\tau\) are defined in [11]. Using the perturbative values for \(c_\sigma, \tau\) [12] yields

\[
Z_g = 1 - 0.16677 g^2 + \cdots
\]

as the 1-loop perturbative \(Z_g\). In [9] this result was combined with non-perturbative determinations of \(c_\sigma, \tau\), [11], to give a Padé expression

\[
Z_g = 1 - \frac{1.0225 g^2 + 0.1305 g^4}{1 - 0.8557 g^2}, \quad \beta \geq 5.7,
\]

(with an error of \(\sim 1\%\)). So for \(\beta = 6.0\) this gives \(Z_g = 0.748\).

To estimate \(Z_q\) we use the results for \(\langle x \rangle^\text{lat}_g\) from Table 2 together with those for \(\langle x \rangle^\text{lat}_u, \langle x \rangle^\text{lat}_d\), from [13] (i.e. \(v_2\)) together with eqs. (23) and (24). In Fig. 2 we plot \(\langle x \rangle^\text{lat}_u + \langle x \rangle^\text{lat}_d\) against \(\langle x \rangle^\text{lat}_g\). From eq. (23) we would expect that the y-intercept is given by \(1/Z_g\) and the x-intercept is given by \(1/Z_q\). At present we do not have enough results for a determination, so we shall just check for consistency by fixing the y-intercept as \(1/0.748\) and the x-intercept as 1, [6]. This gives consistency so we shall adopt here \(Z_q = 1\) together with a 10% error.

Also from [13], we have for \(\mu = 2\) GeV,

\[
Z_{\text{bare qq}}^\text{MS}(\mu = 2\text{ GeV})Z_q = \frac{Z_{v_2}^{\text{RG}}}{Z_{v_2}^{\text{RI-MOM}}} \times [\Delta Z_{v_2}^\text{MS}(\mu = 2\text{ GeV})]^{-1} = 1.45 \times 0.732(9) = 1.06(1),
\]

where the second equation uses the notation of that article (the non-perturbative \(RI - MOM\) scheme is converted to an \(RGI\) form and then back to the \(MS\) scheme). Further values of \(\Delta Z_{v_2}^\text{MS}(\mu)\) are also given in [13]. With \(Z_q\) this then gives \(Z_{\text{bare qq}}^\text{MS}\).
6 Results and conclusion

We are now in a position to determine $\langle x\rangle_{\text{lat}}^{\text{MS}}(\mu = 2 \text{ GeV})$. Using the first equation in eq. (26) together with eq. (28) (evaluated at $\beta = 6.0$) and eq. (29) gives $\langle x\rangle_{g}^{\text{MS}}(\mu = 2 \text{ GeV})$. In Fig. 3 we plot using eq. (26), $\langle x\rangle_{g}^{\text{MS}}(\mu = 2 \text{ GeV})$ versus $(am_{\pi})^2$. This gives a value for $\langle x\rangle_{g}^{\text{MS}}(\mu = 2 \text{ GeV})$ of

$$\langle x\rangle_{g}^{\text{MS}}(\mu = 2 \text{ GeV}) = 0.43(7)(5),$$

as our final result, where the first error is in the determination of $\langle x\rangle_{g}^{\text{lat}}$ and the second is due to the renomalisation procedure. This is a significant improvement of our previous estimate 0.53(23) based on generating $O(5000)$ configurations, [5] (with error given just for $\langle x\rangle_{g}^{\text{lat}}$).

Direct measurements of gluonic expectation values are notoriously plagued by noise problems, because the gluons are bosonic fields. We have seen here that a cheaper alternative, modifying the gluon action and using the Feynman-Hellmann theorem to find expectation values from mass measurements, works well. Here we have performed a test calculation in the quenched case. The method is a generalisation of that used to determine the sigma term (see e.g. [14] and references therein), $\beta$-function, e.g. [15], or singlet terms, e.g. [16]. It is clearly interesting to repeat this with dynamical fermions.
Figure 3: $\langle \chi \rangle_{\overline{MS}}(\mu = 2 \text{ GeV})$ versus $(am_\pi)^2$ for the five $\kappa$ values, together with a linear chiral extrapolation.

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