Invariant-region-preserving DG methods for multi-dimensional hyperbolic conservation law systems, with an application to compressible Euler equations

Yi Jiang, Hailiang Liu

PII: S0021-9991(18)30156-6
DOI: https://doi.org/10.1016/j.jcp.2018.03.004
Reference: YJCPH 7896

To appear in: Journal of Computational Physics

Received date: 10 November 2017
Revised date: 27 February 2018
Accepted date: 2 March 2018

Please cite this article in press as: Y. Jiang, H. Liu, Invariant-region-preserving DG methods for multi-dimensional hyperbolic conservation law systems, with an application to compressible Euler equations, J. Comput. Phys. (2018), https://doi.org/10.1016/j.jcp.2018.03.004

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.
INVARIANT-REGION-PRESERVING DG METHODS FOR
MULTI-DIMENSIONAL HYPERBOLIC CONSERVATION LAW
SYSTEMS, WITH AN APPLICATION TO COMPRESSIBLE EULER
EQUATIONS

YI JIANG AND HAILIANG LIU

Abstract. An invariant-region-preserving (IRP) limiter for multi-dimensional hyper-
bolic conservation law systems is introduced, as long as the system admits a global
invariant region which is a convex set in the phase space. It is shown that the order of
approximation accuracy is not destroyed by the IRP limiter, provided the cell average
is away from the boundary of the convex set. Moreover, this limiter is explicit, and
easy for computer implementation. A generic algorithm incorporating the IRP limiter
is presented for high order finite volume type schemes. For arbitrarily high order dis-
continuous Galerkin (DG) schemes to hyperbolic conservation law systems, sufficient
conditions are obtained for cell averages to remain in the invariant region provided the
projected one-dimensional system shares the same invariant region as the full multi-
dimensional hyperbolic system does. The general results are then applied to both one
and two dimensional compressible Euler equations so to obtain high order IRP DG
schemes. Numerical experiments are provided to validate the proven properties of the
IRP limiter and the performance of IRP DG schemes for compressible Euler equations.

1. Introduction

The multi-dimensional hyperbolic conservation law systems are given by
\[
\partial_t w + \sum_{j=1}^{d} \partial_{x_j} F_j(w) = 0, \quad x \in \mathbb{R}^d, \quad t > 0
\]  
(1.1)
with the unknown vector \( w \in \mathbb{R}^l \) and the flux function \( F_j(w) \in \mathbb{R}^l \) for \( j = 1, \ldots, d \). We consider the initial value problem for system (1.1) with the initial data \( w(x, 0) = w_0(x) \).

For simplicity, we take periodic or compactly supported boundary conditions.

It is well known that entropy inequalities should be considered for general hyperbolic
conservation laws so to single out the physically relevant solution among many weak
solutions (see, e.g., [18]). In application problems, the pointwise range of solutions may
be known from physical considerations, instead of total entropy. For scalar conserva-
tion laws, the entropy solution satisfies a strict maximum principle. For hyperbolic
conservation law systems, the notion of maximum principle does not apply and must

\begin{flushleft}
Date: November 1, 2017; revised Feb 28, 2018.
2000 Mathematics Subject Classification. 65M60, 35L65, 35L45.

Key words and phrases. Invariant region, hyperbolic conservation laws, compressible Euler equation, discontinuous Galerkin methods.

This work was supported by the National Science Foundation under Grant DMS1312636 and by NSF Grant RNMS (Ki-Net) 1107291.
\end{flushleft}
