Doi, Yoshiaki; Konno, Norio; Nakamigawa, Tomoki; Sakuma, Tadashi; Segawa, Etsuo; Shinohara, Hidehiro; Tamura, Shunya; Tanaka, Yuuho; Toyota, Kosuke
On the average hitting times of the squares of cycles. (English) Discrete Appl. Math. 313, 18-28 (2022)

Summary: The exact formula for the average hitting time (HT, as an abbreviation) of simple random walks from one vertex to any other vertex on the square $C_{2N}^2$ of an $N$-vertex cycle graph $C_N$ was given by N. Chair [J. Stat. Phys. 154, No. 4, 1177–1190 (2014; Zbl 1291.82049)]. In that paper, the author gives the expression for the even $N$ case and the expression for the odd $N$ case separately. In this paper, by using an elementary method different from Chair (2014), we give a much simpler single formula for the HT’s of simple random walks on $C_{2N}^2$. Our proof is considerably short and fully combinatorial, in particular, has no-need of any spectral graph theoretical arguments. Not only the formula itself but also intermediate results through the process of our proof describe clear relations between the HT’s of simple random walks on $C_{2N}^2$ and the Fibonacci numbers.

MSC:
05C81 Random walks on graphs
11B39 Fibonacci and Lucas numbers and polynomials and generalizations
82B41 Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics
94C05 Analytic circuit theory

Keywords:
simple random walk; hitting time; square of a cycle; Fibonacci number; Kirchhoff index

Full Text: DOI

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