Exchange-degenerate Regge trajectories: a fresh look from resonance and forward scattering regions

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Abstract

The exchange-degeneracy of the mesonic $f$, $\omega$, $\rho$ and $a_2$ Regge trajectories, dominant at moderate and high energies in hadron elastic scattering, is analyzed from two viewpoints. The first one concerns the masses of the resonances lying on these trajectories; the second one deals with the total cross-sections and the ratios of the real to the imaginary parts of the forward amplitudes of hadron and photon induced reactions. Neither set of data supports exact exchange-degeneracy.

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1 Introduction

A very convenient and useful method to group mesons and baryons in families with definite quantum numbers, makes use of the so called Chew-Frautschi plot (spin versus squared mass). It is a graphic representation of Regge trajectories for given quantum numbers. Early analyses of Regge trajectories hinted at remarkable properties: they appeared to be essentially linear and many of them coincide. The latter property came to be known as the principle of exchange-degeneracy (e-d) of Regge trajectories.

There are two kinds of exchange-degeneracy, qualified as strong and weak. In weak exchange-degeneracy, only the trajectories with different quantum numbers coincide. In strong exchange-degeneracy, in addition, the residues of the corresponding hadronic amplitudes coincide at the given pole in the \(j-\) plane. It was soon realized that strong exchange-degeneracy may be violated (for theoretical arguments, see [1]) and indeed experimental confirmations of this violation occurred.

Conclusive and definitive statements about weak exchange-degeneracy, however, are not possible without a sufficiently precise experimental information about the hadrons lying on each Regge trajectory. Therefore, lacking high precision data, general agreement with a weak exchange-degeneracy assumption, as well as with a linearity of meson Regge trajectories, was claimed in the past (see the references to old papers in [2]) and the hypothesis was applied repeatedly, for example, in models describing elastic scattering data (see references below). From this point of view, the most relevant trajectories are the \(f\), the \(\omega\), the \(\rho\), and the \(a_2\), which can variously be exchanged in the \(t\)-channel of many elastic reactions. These we are going to consider in what follows. The rôle of a unique Regge trajectory was repeatedly analyzed to describe hadron-hadron and photon-hadron total cross-sections in a most economical approach [3]. In spite, however, of an apparent agreement with the data, this model leads numerically to a quite large \(\chi^2\) when compared with more recent approaches [4, 5].

Today, the situation has changed somewhat. Three meson states are now known lying on each trajectory (except for the \(\omega\)-trajectory for which we know only two states) and, moreover, some of their masses are measured with very high precision [2] even though the data on highest spin resonances have not yet been confirmed. We believe, however, that a fairly conclusive analysis can be performed using, on the one hand data in the resonance region (Section 2) and, on the other hand, data on (near forward) elastic scattering (Section 3).

Our conclusion (Section 4), will suggest that the combined analysis of all data supports a breaking of the weak exchange-degeneracy principle.

2 Resonance region

To examine the agreement of weak exchange-degeneracy with the available data in the resonance region, we first assume that the four trajectories \(f\), \(\omega\), \(\rho\), \(a_2\) are linear and coincide. Writing the relevant exchange-degenerate linear trajectory as

\[
\alpha_{e-d}(m^2) = \alpha_{e-d}(0) + \alpha'_{e-d} m^2
\]

\((m\) is the mass of the bound state\), we determine the intercept \(\alpha_{e-d}(0)\) and the slope \(\alpha'_{e-d}\) by fitting 11 resonances lying on \(f\), \(\omega\), \(\rho\) and \(a_2\) trajectories. Using the MINUIT computer
code, we find (the precision is estimated as the usual one-standard deviation error)

\[ \alpha_{e-d}(0) = 0.4494 \pm 0.0007, \quad \alpha'_{e-d} = (0.9013 \pm 0.0011) \text{ GeV}^{-2}, \quad \text{with} \quad \chi^2/dof = 117.9. \quad (2) \]

The data are taken from Ref. [2]. The very high value of \( \chi^2/dof \) (dof stands for degree of freedom defined as the difference between the number of data points and the number of fitted parameters) is not surprising because (i) the data exhibit a known nonlinearity of the trajectories (see details below) and (ii) the masses of the low lying resonances are measured with very high precision. The corresponding degenerate trajectory one obtains is shown in Fig. 1 (solid line). For comparison, the trajectory with the parameters used in Ref. [6], \( \alpha(m^2) = 0.48 + 0.88m^2 \) (\( m \) in GeV), is also plotted (dashed line).

![Figure 1: Chew-Frautschi plot for the fully exchange-degenerate \( f, \omega, \rho \) and \( a_2 \) trajectories. The solid line denotes the trajectory with the parameters obtained in our fit; the dashed line is the trajectory from \[8\].](image)

Our conclusion is, thus, that in spite of a decent agreement with resonance data (plotted \( \text{à la Chew-Frautschi} \)), weak exchange degeneracy of the \( f, \omega, \rho, \) and \( a_2 \) trajectories is not supported by the resonance data when a more precise numerical analysis is performed.
In order to verify the possibility of a limited validity of exchange-degeneracy, we have considered a weaker (intermediate) version where the trajectories are grouped in pairs. Some of these combinations has been currently used to describe the behaviour of total cross-sections and of their differences. For convenience, we present the results in Table 1 where all the 6 possible groupings in pairs are considered.

|   |   | \( \omega \) | \( \rho \) | \( a_2 \) |
|---|---|---|---|---|
| f | \( \alpha(0) \) | 0.411 | 0.442 | 0.565 |
|   | \( \alpha'(\text{GeV}^{-2}) \) | 0.963 | 0.944 | 0.835 |
|   | \( \chi^2/\text{dof} \) | 66.84 | 57.34 | 194.26 |
| \( \omega \) | \( \alpha(0) \) | 0.445 | 0.456 |
|   | \( \alpha'(\text{GeV}^{-2}) \) | 0.908 | 0.890 |
|   | \( \chi^2/\text{dof} \) | 84.14 | 13.48 |
| \( \rho \) | \( \alpha(0) \) | 0.482 |
|   | \( \alpha'(\text{GeV}^{-2}) \) | 0.874 |
|   | \( \chi^2/\text{dof} \) | 1.30 |

Table 1. Intercepts \( \alpha(0) \), slopes \( \alpha' \) and \( \chi^2/\text{dof} \)’s obtained in the fits when exchange-degeneracy is assumed for each grouping in pairs of the trajectories. They are written at the intersections of the corresponding line and row.

For any grouping in pairs one can obtain the \( \chi^2 \) from the Table because each pair is considered independently of the other. What would appear as a natural grouping introducing just two pairs of degenerate trajectories (one crossing even \( f-a_2 \equiv R_+ \) and one crossing odd \( \omega-\rho \equiv R_- \), as in [7]), is clearly not supported by the resonance data under any reasonable common \( \chi^2 \).

An obvious general conclusion follows from this very simple analysis: under a careful numerical investigation, there are no experimental evidences from the resonance region that the \( f, \omega, \rho \) and \( a_2 \) trajectories can be assumed to be exchange-degenerate.

The available resonances are known with a good precision, allowing the determination of intercept and slope of each trajectory taken separately under the assumption of linearity

\[
\alpha_R(m^2) = \alpha_R(0) + \alpha'_R m^2, \quad R = f, \omega, \rho, a_2 .
\]  

(3)

The corresponding Chew-Frautschi plots obtained from the fit are shown in Fig.2. We obtained the following parameters

\[
\begin{align*}
\alpha_f(0) &= 0.6971 \pm 0.0029, & \alpha'_f &= (0.8014 \pm 0.0018) \text{ GeV}^{-2}, & \chi^2/\text{dof} &= 6.01, \\
\alpha_\omega(0) &= 0.4359, & \alpha'_\omega &= 0.9227 \text{ GeV}^{-2}, & \quad & \text{(not fitted)}, \\
\alpha_\rho(0) &= 0.4783 \pm 0.0011, & \alpha'_\rho &= (0.8800 \pm 0.0017) \text{ GeV}^{-2}, & \chi^2/\text{dof} &= 3.31, \\
\alpha_{a_2}(0) &= 0.5116 \pm 0.0009, & \alpha'_{a_2} &= (0.8567 \pm 0.0008) \text{ GeV}^{-2}, & \chi^2/\text{dof} &= 0.42 ,
\end{align*}
\]  

(4)

in qualitative agreement with the fits of [3]. The rather high values 3 and 6 obtained for the \( \chi^2 \) of two trajectories (which are anyhow much lower than in the assumption of e-d) can be attributed to the hypothesis of linearity. Actually, we should note that all trajectories, except the \( \omega \) (for which only two resonances are known), deviate from of a strict linear
Figure 2: Chew-Frautschi plots for $f$, $\omega$, $\rho$ and $a_2$ Regge trajectories taken separately assuming linearity (the figure below is an enlargement for low masses).
behaviour (see also [10]). Indeed, parametrizing in isolation the trajectories in a parabolic form instead of a linear one

\[ \alpha_R(m^2) = \alpha_R(0) + \alpha_R'(m^2) + \frac{\alpha_R''}{2}m^4, \quad (R = f, \omega, \rho, a_2), \]

one obtains from the experimental data on known resonances [2] (two for the \(\omega\) and three for the other Reggeons)

\(f\) trajectory:
\[ \alpha_f(0) = 0.9577 \pm 0.0023, \quad \alpha_f' = (0.5858 \pm 0.0014) \text{GeV}^{-2}, \quad \alpha_f'' = (0.0681 \pm 0.0015) \text{GeV}^{-4}, \]

\(\rho\) trajectory:
\[ \alpha_\rho(0) = 0.4404 \pm 0.0011, \quad \alpha_\rho' = (0.9566 \pm 0.0017) \text{GeV}^{-2}, \quad \alpha_\rho'' = (-0.0430 \pm 0.0028) \text{GeV}^{-4}, \]

\(a_2\) trajectory:
\[ \alpha_{a_2}(0) = 0.8759 \pm 0.0010, \quad \alpha_{a_2}' = (0.5987 \pm 0.0006) \text{GeV}^{-2}, \quad \alpha_{a_2}'' = (0.0876 \pm 0.0007) \text{GeV}^{-4}. \]

Of course such a parametrization cannot be satisfactory from a theoretical point of view (the negative sign of the second derivative of the \(\rho\) trajectory also is strange), it only suggests a nonlinearity of the given trajectories. A more detailed investigation of the phenomenon taking into account the actual widths of the resonances is desirable.

The deviation from linearity, dictated both by analyticity and unitarity, has often been discussed in the past. For a recent discussion on the nonlinearity of the \(f\) trajectory and its influence on the intercept, see, in particular [11, 12].

### 3 Forward scattering

#### 3.1 Generalities

The exchange-degeneracy hypothesis for the \(f, \omega, \rho, a_2\) trajectories can be checked also using elastic hadron scattering data. In particular, one can use forward scattering data, i.e. the total cross-sections for hadron hadron, \(\gamma\) hadron and \(\gamma\gamma\)-collisions. Following the arguments given in [5] we do not restrict our analysis to the data on total cross-sections, \(\sigma(t)(s)\), but include in the fits the ratios \(\rho\) of the real to the imaginary parts of the forward amplitudes.

Performing such an analysis requires an explicit parametrization for the amplitudes of the processes under investigation [6]. Like in the resonance-case, in order to check how well the exchange-degeneracy hypothesis works in the description of hadron and photon induced cross-sections, it is not sufficient to obtain agreement with the data which looks good. It is also necessary to compare this description with the one where the e-d assumption is removed. Clearly, removing the assumption of exchange degeneracy increases the number of parameters but the \(\chi^2\) referred to the number of degrees of freedom retains its comparative

\[5\] The best way to analyze exchange-degeneracy would be to consider some linear combinations of \(\sigma(t)(s)\) for several elastic processes. In principle, one can construct combinations that contain the contribution of one or two Reggeons and these could be compared with the experiment. The shortcoming of this procedure, however, lies in the fact that, usually, the required reactions are measured at different energies. As a consequence, while attractive, this procedure is heavily affected by the ambiguity of reconstructing data from interpolation. We shall not use this method.
validity. Thus, we analyze the data using the following explicit expressions for the forward amplitudes $A_{ab}(s, t = 0)$ of the 12 elastic reactions

$$
A_{p^+_p}(s, 0) = \mathcal{P}_{NN}(s) + f_{NN}(s) + a_{NN}(s) \mp \omega_{NN}(s) \mp \rho_{NN}(s), \\
A_{p^+_n}(s, 0) = \mathcal{P}_{NN}(s) + f_{NN}(s) + a_{NN}(s) \mp \omega_{NN}(s) \pm \rho_{NN}(s), \\
A_{p^+_n}(s, 0) = \mathcal{P}_{\pi N}(s) + f_{\pi N}(s) \mp \rho_{\pi N}(s), \\
A_{K^+_p}(s, 0) = \mathcal{P}_{KN}(s) + f_{KN}(s) + a_{KN}(s) \mp \omega_{KN}(s) \mp \rho_{KN}(s), \\
A_{K^+_n}(s, 0) = \mathcal{P}_{KN}(s) + f_{KN}(s) + a_{KN}(s) \mp \omega_{KN}(s) \pm \rho_{KN}(s), \\
A_{\gamma p}(s, 0) = \delta \mathcal{P}_{NN}(s) + f_{\gamma N}(s), \\
A_{\gamma N}(s, 0) = \delta^2 \mathcal{P}_{NN}(s) + f_{\gamma N}(s),
$$

\((p^+, p^-) \text{ stand for } p, \bar{p})\).

In addition to the main goal (to compare the e-d hypothesis with the data), we test two models of Pomeron, each one with two components: a constant background and an energy dependent term. One of them, explored in [6, 9], is \textit{universal}, in the sense that its asymptotic component, growing with energy, contributes equivalently to all processes

$$
\mathcal{P}_{ab}(s) = i\{Z_{ab} + X \mathcal{P}(s)\} \quad \text{for "universal" Pomeron}.
$$

The other one is \textit{non-universal}: its two components contribute differently to each process, but with a universal ratio of these two components.

$$
\mathcal{P}_{ab}(s) = i Z_{ab}\{X + \mathcal{P}(s)\} \quad \text{for "nonuniversal" Pomeron}.
$$

We remark that the suggestion to consider models with a "two-component" Pomeron is not new. Many times, this idea was successfully applied (see [4, 13] and references therein). Different constant terms $Z_{ab}$ in (8) or $Z_{ab}X$ in (9), often neglected, are intended to adjust the universal behavior of a unique Regge term and will reveal to improve the $t = 0$ fits of a simple one-component Pomeron especially at medium-high energies. From the phenomenological point of view, an energy-rising Pomeron component is only an asymptotic part of its contribution, unknown subasymptotic terms must also exist contributing to the amplitudes. We take into account effectively this part of Pomeron when adding a constant term to $A(s, 0)$.

It corresponds to a simple $j$-pole with a unit intercept. Various theoretical justifications of the existence of such an additive structure of the Pomeron (or of the total cross section) have been proposed: we note that some indication for such a background component has been found recently along with the ordinary BFKL Pomeron [14] (in contrast to the known hard component produced by two-gluon states, the new found one is constructed from the three-gluon states but with positive C-parity differing, from the three-gluonic Odderon with negative C-parity). At an equally fundamental level [15], such a constant term has been recognized as the nonperturbative contribution that one must add to the perturbative soft gluon radiation term responsible for the growth with energy of total hadronic cross sections.

We have considered two variants for the $s$–dependent Pomeron component, having in mind its properties in the complex angular momentum plane. The first one corresponds to a simple pole in the complex angular momentum plane with intercept $\alpha_{\mathcal{P}}(0) = 1 + \epsilon$, (the so-called \textit{Supercritical Pomeron} (SCP))

$$
\mathcal{P}(s) = (-is/s_0)^\epsilon, \quad s_0 = 1\text{GeV}^2.
$$
The second variant corresponds to the Dipole Pomeron (DP). In the $j$-plane it is described by a double pole with a unit intercept trajectory, $\alpha_P(0) = 1$

$$P(s) = \ln(-is/s_0).$$

(11)

For the secondary Reggeons we use the standard form

$$R_{a_2}(s) = \eta Y_{a_2}(-is/s_0)^{\alpha_{R}(0)} - 1 \quad R = f, a_2, \omega, \rho,$$

(12)

where $\eta = i$ for $f$ and $a_2$ while $\eta = 1$ for $\omega$ and $\rho$.

The above amplitudes are normalized according to

$$\sigma_{ab}^{(t)}(s) = 8\pi 3m A_{ab}(s, 0).$$

(13)

### 3.2 Results

We have taken into account the whole set of cross-section data for $(p^\pm p)$, $(p^\pm n)$, $(K^\pm p)$, $(K^\pm n)$, $(\pi^\pm p)$, $(\gamma p)$ and $(\gamma \gamma)$ interactions and of $\rho$ ratio data for all interactions excluding two last. Furthermore, in order to reasonably neglect the sub-leading meson trajectories, and to respect the stability if the $\chi^2$ and of the parameters (see the discussion below), we choose the energy range with $\sqrt{s} \geq 5$ GeV. No other wise selection of any kind is attempted (such as a filtering of the data suggested by some authors). In total there are 785 points available in the Data Base of Particle Data Group [7].

The values of the fitted parameters are given in Table 2 for the universal and the non universal Pomeron. If exchange-degeneracy is assumed, all intercepts of $f$, $a_2$, $\omega$ and $\rho$ Reggeons are equal (in Table 2, we have labeled the common intercept as $\alpha_f(0)$). We do not give the errors for the other parameters than intercepts, neither curves for the total cross-sections and $\rho$ ratios because they are only illustrative, not very important for the case in point.

One can see that in all considered cases, non degenerate trajectories lead to a better $\chi^2$, even though for the universal Supercritical Pomeron the difference is very small. The best agreement with the fitted data (“measured” by the $\chi^2$) is obtained either with the non universal non-degenerate DP or the SCP (compare columns 8 and 6 in Table 2). The reason lies in the similarity of these models when (as in the present case), the value of $\epsilon = \alpha_P(0) - 1$ is very small ($\epsilon \approx 0.0101$). Actually, as emphasized in [13, 4], when $\epsilon \ll 1$, the Supercritical Pomeron approximates very closely the Dipole Pomeron since, in the relevant energy range, and for $s_0$ given in (10)

$$Z_{ab} \left[ X + (-is/s_0)^\ell \right] \approx Z_{ab} \left[ 1 + X + \epsilon \ln(-is/s_0) \right] \equiv Z_{ab}^{(t)} \left[ X' + \ell \ln(-is/s_0) \right],$$

and this reflects in the numerical values found in Table 2.
The authors of the paper think that the model may be suitable for the data presented, but the authors of the paper have not found that the Superuniversal Pomeron model is suitable for the data presented. The authors of the paper state that the model is unsuitable for the data presented.

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and the model with $\sigma^{(t)} \propto \ell n^2 s$ are stable for $\sqrt{s} \geq \sqrt{s_{\text{min}}} = 9$ GeV, while the Dipole Pomeron model is stable for $\sqrt{s} \geq \sqrt{s_{\text{min}}} = 5$ GeV. We agree, in general, with this comment but, in our case, it is essential to keep $s_{\text{min}}$ as small as possible, because the contribution of the secondary Reggeons decreases with energy. Increasing $s_{\text{min}}$, we lose the data which allow to discriminate between the different Reggeons and to test the e-d assumption.

To check our conclusions, nevertheless, we have performed such a stability analysis. In Figs. 3-4, the results are reported for the meaningful cases of non degenerate trajectories. We show the dependence of the four Reggeon intercepts and of the $\chi^2$’s versus $\sqrt{s_{\text{min}}}$ for two representative models already discussed. The first is the Dipole Pomeron model with a non universal Pomeron term (Fig. 3) (as explained previously, due to the smallness of $\epsilon$, also the non universal SCP model is well approximated by this non universal DP model). The second one is the Supercritical Pomeron Model with a universal form of Pomeron (Fig. 4).

Figure 3: Stability of the $\chi^2$ (upper part) and of the Reggeons intercepts (lower part) versus the minimal c.m. energy limiting the fitted data, for the non degenerate non universal Dipole Pomeron model. The dashed lines join the central values for each Reggeon for visual indication. Some points are shifted slightly to the left (right) side to make the errors more easily distinguishable.
Figure 4: Same as in Fig. 3 for the non degenerate universal Supercritical Pomeron model.

One can see from these figures that the errors on the $\rho$ and, especially on the $a_2$ trajectories increase with $s_{\text{min}}$. This was expected since the $a_2$ contribution is determined mainly from rather poor $(p\pi)$ and $(K\pi)$ data. The situation with the $\rho$ contribution is better, due to the available $(\pi p)$ data. In general, we see that the scattering models used here are quite stable in the region $5 \text{ GeV} \leq \sqrt{s_{\text{min}}} \leq 7 \text{ GeV}$; this, in itself, is a justification of our 5 GeV minimal choice.

To complete this study, we performed also fits of the non universal Dipole Pomeron and universal Supercritical Pomeron with non degenerate Reggeons fixing their intercepts at the values determined in the Section 2 from the resonance data (6). For DP we obtained $\chi^2/\text{dof} = 1.034$ while for SCP $\chi^2/\text{dof} = 1.055$. Thus, no contradiction appears between the forward scattering and the spectroscopy data.

From this analysis of forward scattering data, we can argue that, taking into account the values of the intercepts ($\alpha(0)$), the errors in their determination ($\Delta$), and the $\chi^2$’s (Table 2) that the solution with nondegenerate Regge trajectories is definitely to be preferred.
4 Conclusions.

In the first part of this work, we have concluded that the assumption of exchange degeneracy of the $f$, $\omega$, $\rho$ and $a_2$ Regge trajectories (assumed to be linear), though qualitatively acceptable, is not compatible with a numerical best fit of the available data on the corresponding mesonic resonances.

Concerning the forward scattering data, considered in the second part, the situation is less clear because a reasonable description of the $t = 0$ data can be obtained under both hypotheses and is a priori model dependent. We have tried to eliminate or at least to weaken this model dependence in our conclusion by analyzing four models that provide a good description of forward scattering data. The fits with non-degenerate trajectories invariably improve the $\chi^2$'s, (all best fits occur for non-degenerate parametrizations), and this can be taken as an indication in favor of the non-degeneracy assumption.

For a more definitive conclusion we would need more precise data on meson-nucleon and proton-neutron or K-neutron cross-sections at higher energies. Even more conclusive, perhaps, would be to compare fits with and without exchange degeneracy involving all data both at $t = 0$ and at $t \neq 0$. This analysis puts much more stringent constraints on the free parameters as we have learned in previous experiences.

Thus, given that any model for scattering amplitudes should be in agreement with both types of data, from spectroscopy and from total cross-sections we conclude that the hypothesis of exact exchange degeneracy, even in its weak formulation, is not supported by the present data. In spite of this, due to its great economy in the number of parameters, exchange degeneracy associated with linear Regge trajectories retains its usefulness in practical calculations when only a rough approximation is sufficient.

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