PILOT: Physics-Informed Learned Optimal Trajectories for Accelerated MRI

Tomer Weiss, Ortal Senouf, Sanketh Vedula, Oleg Michailovich, Michael Zibulevsky, Alex Bronstein

Abstract—Magnetic Resonance Imaging (MRI) has long been considered to be among “the gold standards” of diagnostic medical imaging. The long acquisition times, however, render MRI prone to motion artifacts, let alone their adverse contribution to the relative high costs of MRI examination. Over the last few decades, multiple studies have focused on the development of both physical and post-processing methods for accelerated acquisition of MRI scans. These two approaches, however, have so far been addressed separately. On the one hand, recent works in optical computational imaging have demonstrated growing success of concurrent learning-based design of data acquisition and image reconstruction schemes. Such schemes have already demonstrated substantial effectiveness, leading to considerably shorter acquisition times and improved quality of image reconstruction. Inspired by this initial success, in this work, we propose a novel approach to the learning of optimal schemes for joint acquisition and reconstruction of MRI scans, with the optimization carried out simultaneously with respect to the time-efficiency of data acquisition and the quality of resulting reconstructions. To be of a practical value, the schemes are encoded in the form of general $k$-space trajectories, whose associated magnetic gradients are constrained to obey a set of predefined hardware requirements (as defined in terms of, e.g., peak currents and maximum slew rates of magnetic gradients). With this proviso in mind, we propose a novel algorithm for the end-to-end training of a combined acquisition-reconstruction pipeline using a deep neural network with differentiable forward- and back-propagation operators. We also demonstrate the effectiveness of the proposed solution in application to both image reconstruction and image segmentation, reporting substantial improvements in terms of acceleration factors as well as the quality of these end tasks.

Index Terms—Magnetic Resonance Imaging (MRI), fast image acquisition, image reconstruction, segmentation, neural networks, deep learning.

I. INTRODUCTION

Magnetic Resonance Imaging (MRI) is a leading modality of modern medical imaging, particularly favoured due to its noninvasive nature, the lack of harmful radiation, and superb imaging contrast and resolution. However, its relatively long image acquisition times adversely affect the use of MRI in multiple clinical applications, including emergency radiology and dynamic imaging. During the past few years, compressed sensing [1] and, later, deep learning [2], [3], [4] have been in the forefront of accelerated MRI acquisition, leading to substantial improvements in terms of the time-efficiency of image acquisition, with a hardly discernible reduction in image quality. Most of these studies have focused on the stage of image restoration, trying to recover high-quality MRI scans from reduced (a.k.a. incomplete) sets of their measurements available through partial sampling of the $k$-space (i.e., the Fourier domain that is dual to the physical space of spatial coordinates). At the same time, more recent studies have attempted to optimize directly over the geometry of the $k$-space trajectories, showing further sizable improvements. Inspired by these initial findings, the problem of data-driven optimization of image acquisition-reconstruction pipelines has recently been addressed using the tools of machine and deep learning [5], [6], [7]. Thus, for example, in our previous work [8], we suggested to learn the optimal $k$-space trajectories (or, equivalently, sub-sampling schemes) concurrently with maximizing the quality of resulting reconstructions. However, the main drawback of this method (and of some other related works as well [9], [10], [11]) remains in its reliance on Cartesian trajectories which are defined in the absence of any practical considerations. At the same time, it was shown in [1] that variable-density sampling with a gradual reduction in the sampling density towards the periphery of the $k$-space is likely to be ideal from the perspective of compressed sensing. Unfortunately, the ability of Cartesian trajectories to approximate such non-uniform densities is rather limited.

Addressing the above problem has yielded a number of interesting solutions, which have effectively extended the space of admissible trajectories to include a larger class of parametric curves, with spiral, radial and rosette being among the most well-known examples. Unfortunately, the use of less regular trajectories raised additional challenges and concerns related to the lack of an explicit compliance between the algorithmic solutions and hardware requirements[1]. Some of these challenges have been recently addressed by the SPARKLING trajectories proposed in [12]. However, this solution does not exploit the strengths of data-driven learning methods, merely advocating the importance of constrained optimization in application to finding the trajectories that best fit predefined sampling distributions, on the one hand, while obeying additional hardware-related constraints, on the other. On the contrary, in our present work, we suggest a new method of cooperative learning which is driven by the data acquisition, reconstruction, and end task-related criteria. In this case, the resulting $k$-space trajectories are optimized for a particular end application, with image reconstruction and

$^1$These requirements are usually specified in the form of constraints on the maximum amplitude of magnetic gradients and/or their maximum slew rates, which are in turn directly related to the velocity and the acceleration of the trajectories.
Fig. 1: **PILOT for reconstruction and segmentation tasks.** The top row depicts the performance of PILOT on the reconstruction task. The fixed and reconstruction-optimized trajectories are depicted in the second row. The third row depicts the performance of PILOT on the segmentation task. The last row depicts the fixed and the segmentation-optimized trajectories. Reconstruction results are presented along with the PSNR metric with respect to the fully-sampled ground truth image (shown in the top row, left). The segmentation results are presented along with the DICE and IOU scores with respect to the groundtruth segmentation mask (shown in the 3rd row, left). The middle and right columns present the best reconstruction and segmentation results obtained through a network trained with fixed physically-feasible spiral trajectories and PILOT-TSP, respectively. The optimized trajectories obey the following physical constraints: maximum gradient $G_{\text{max}} = 40 \text{ mT/m}$, maximum slew-rate $S_{\text{max}} = 200 \text{ T/m/s}$ and sampling interval $dt = 10 \mu\text{s}$. The presented results correspond to decimation rate 20.

Another important innovation proposed in this work lies in its dependence on single-shot echo planar imaging (SS-EPI), whereas previous works (including SPARKLING) took advantage of multi-shot EPI acquisition. At the same time, with the development of more powerful MRI scanners with less stringent hardware limitations, comes the possibility of designing single-shot trajectories, which can, in turn, lead to substantially shorter data acquisition, thereby improving the time-efficiency of MRI examination.

**A. Contribution**

This paper makes three main contributions: Firstly, we introduce PILOT (Physics Informed Learned Optimal Trajectory), a deep-learning based method for joint optimization of physically-viable $k$-space trajectories. To the best of our knowledge, this is the first time when the hardware constraints are introduced into the learning pipeline to model the $k$-space trajectories conjointly with the optimization of the image segmentation being two important standard examples that are further exemplified in our numerical experiments.
image reconstruction network. Furthermore, we demonstrate that PILOT is capable of producing significant improvements in terms of time-efficiency and reconstruction quality, while guaranteeing the resulting trajectories to be physically feasible.

Secondly, when initialized with a spiral SS-EPI trajectory, PILOT has been observed to produce trajectories that are (globally) relatively close to the initialization. A similar phenomenon has been previously observed in [12], [5]. As a step towards finding globally optimal trajectories, we propose PILOT-TSP, a training strategy that uses an approximated solution to the traveling salesman problem to globally update the \(k\)-space trajectories during learning. We then demonstrate that PILOT-TSP provides additional improvements when compared to both PILOT and parametric trajectories (see Fig. 1).

Lastly, we further demonstrate the effectiveness of PILOT and PILOT-TSP through their application to a different end-task of image segmentation (resulting in task-optimal trajectories). Again, to the best of our knowledge, this is the first attempt to design \(k\)-space trajectories in a physics-aware task-driven framework.

The remainder of this paper is organized as follows. In Section II, the proposed methods PILOT and PILOT-TSP are presented. Section III summarises the experiments conducted and discusses the results. Finally, the main conclusions are presented in Section IV.

II. Method

It is convenient to view our approach as a single neural network combining the forward (acquisition) and the inverse (reconstruction) models (see Fig. 2 for a schematic depiction). The input to the model is the fully sampled \(k\)-space. The input is faced by a sub-sampling layer modeling the data acquisition along a \(k\)-space trajectory, a regridding layer producing an image on a Cartesian grid in the space domain, and an end-task model operating in the space domain and producing a reconstructed image (reconstruction task) or a segmentation mask (segmentation task).

In what follows, we detail each of the three ingredients of the PILOT pipeline. It should be mentioned that all of its components are differentiable with respect to the trajectory coordinates, denoted as \(k\), in order to allow training the latter with respect to the performance of some end-task.

A. Sub-sampling layer

The goal of the sub-sampling layer, denoted as \(S\), is to create the list of measurements to be acquired by the MRI machine, forming a trajectory in the \(k\)-space. The trajectory is parametrized as a list \(k\) of a fixed number \(m\) points in the plane. The measurements themselves constitute an \(m\)-dimensional complex vector emulated by means of bilinear interpolation \(\tilde{x} = S_k(X)\) on the full Cartesian sampling \(X \in \mathbb{C}^{n \times n}\) of the \(k\)-space. We refer to the ratio \(n^2/m\) as the decimation rate.

B. Regridding Layer

For transforming regularly sampled MRI \(k\)-space measurements to the image domain, the inverse fast Fourier transform (IFFT) is applied. However, in our case of non-Cartesian sampling trajectories, we chose to use the non-uniform inverse FFT [13], henceforth denoted as \(\hat{F}^{-1}_k\). The non-uniform Fourier transform first performs regridding (resampling and interpolation) of the irregularly sampled points onto a regular grid followed by the FFT. The result is a (distorted) MR image, \(Z_{\text{dis}} = \hat{F}^{-1}_k(\tilde{x})\).

C. Task Model

The goal of the task model is to extract the representation of the input image \(Z_{\text{dis}}\) that will contribute the most to the performance of the end-task such as reconstruction or segmentation. At training, the task-specific performance is quantified by a loss function, which is described in sub-section II-E. The model is henceforth denoted as \(Z = R_\theta(Z_{\text{dis}})\), with \(\theta\) representing its learnable parameters. The input to the network is the distorted MR image, \(Z_{\text{dis}}\), and the output varies
Stage 1

full $k$-space time-budget initial mask → PILOT without constraints

Stage 2

TSP solver

Stage 3

PILOT physical constraints

Fig. 3: PILOT-TSP training pipeline. At the first stage, an unordered set of $k$-space points is learned without enforcing any hardware constraints. At the second stage, a greedy solver to the traveling salesman problem is employed to construct a trajectory passing through the sampled points. At the final stage, the obtained trajectory is used as the initialization the PILOT training pipeline with physical constraints enforced during training.

according to the specific task. For example, in reconstruction, the output is an MR image (typically, on the same grid as the input), while in segmentation it is a mask representing the segments.

In our work, to implement the task models we used a multi-resolution encoder-decoder network with symmetric skip connections, also known as the U-Net architecture [14]. U-Net is widely-used in medical imaging tasks in general and in MRI reconstruction [4] and segmentation [15] in particular. It is important to emphasize that the scope of this work is not for any hardware constraints. At the second stage, a greedy solver to the traveling salesman problem is employed to construct a trajectory passing through the sampled points. These penalties remain zero as long as the solution is feasible and grow linearly with the violation of each of the constraints. The relative importance of the velocity (peak current) and acceleration (slew rate) penalties is governed by the parameters $\lambda_v$ and $\lambda_a$, respectively. Their values for the different experiments presented in this paper are given in the supplementary material.

The training is performed by solving the optimization problem

$$\min_{k, \theta} \sum_{(X, Z)} L_{\text{task}}(R_{\theta}(\hat{F}^{-1}_{k}(S_{k}(X))), Z) + L_{\text{const}}(k),$$

where the loss is summed over a training set comprising the pairs of fully sampled data $X$ and the corresponding ground-truth output $Z$ (in the case of reconstruction tasks, $Z = F^{-1}(X)$). Since optimization problem (1) is highly non-convex, iterative solvers are guaranteed to converge only locally.

D. Physical Constraints

A feasible sampling trajectory must follow the physical hardware constraint of the MRI machine, specifically the peak-current (translated into the maximum gradient value $G_{\text{max}}$), and maximum slew-rate $S_{\text{max}}$ of the gradient coils. These requirements can be translated into geometric constraints on the first- and second-order derivatives of each of the spatial coordinates of the trajectory,

$$|\dot{k}| \approx \frac{|k_{i+1} - k_i|}{dt} \leq v_{\text{max}} = \gamma G_{\text{max}},$$

and

$$|\ddot{k}| \approx \frac{|k_{i+1} - 2k_i + k_{i-1}|}{dt^2} \leq a_{\text{max}} = \gamma S_{\text{max}},$$

where $\gamma$ is the appropriate gyromagnetic ratio.

E. Loss Function and Training

The training of the proposed pipeline is performed by simultaneously learning the trajectory $k$ and the parameters of the task model $\theta$. A loss function is used to quantify how well a certain choice of the latter parameters performs, and is composed of two terms: a task fitting term and a constraints fitting term, $L = L_{\text{task}} + L_{\text{const}}$.

The aim of the first term is to measure how well the specific end task is performed. In the supervised training scenario, this term penalizes the discrepancy between the task-dependent output of the model, $\hat{Z}$, and the desired ground truth outcome $Z$. For the reconstruction task, we chose the $L_1$ norm to measure the discrepancy between model output image $\hat{Z}$ and the ground-truth image $Z = F^{-1}(X)$, derived from the fully sampled $k$-space,

$$L_{\text{task}} = \|Z - F^{-1}(X)\|_1.$$ Similarly, for the segmentation task, we chose to use the cross-entropy operator to measure the discrepancy between the model output and the ground truth. This time, the model output $\hat{Z}$ was the estimated segmentation map while $Z$ is the ground truth segmentation map. The Task loss is defined to be

$$L_{\text{task}} = H(\hat{Z}, Z).$$

The second term $L_{\text{const}}$ in the loss function applies to the trajectory $k$ only and penalizes for the violation of the physical constraints. We chose the hinge functions of the form $\max(0, |k| - v_{\text{max}})$ and $\max(0, |k| - a_{\text{max}})$ summed over the trajectory spatial coordinates and over all sample points. These penalties remain zero as long as the solution is feasible and grow linearly with the violation of each of the constraints. The relative importance of the velocity (peak current) and acceleration (slew rate) penalties is governed by the parameters $\lambda_v$ and $\lambda_a$, respectively. Their values for the different experiments presented in this paper are given in the supplementary material.

As we discuss in details in the following Section (III), the trajectory $k$ learned by solving optimization problem (1) significantly depends on the initialization, for which we use some of the standard single-shot trajectories, EPI, spiral etc. A naive way to mitigate the effect of such local convergence is to randomly initialize the trajectory coordinates. However, our experiments show that random initialization invariably
result in sequences of points that are far from each other and, thus, induce huge velocities and accelerations violating the constraints by a large margin. The minimization of the previously described loss function with such an initialization appears to be unstable and does not produce useful solutions. To overcome this difficulty, we introduce PILOT-TSP, an extension of PILOT for optimizing MRI trajectories from random initialization. TSP stands for the Traveling Salesmen Problem [16] – a classical problem in computer science which tries to answer the following question: “Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits all cities exactly once?” Translated to our terminology, TSP aims at finding an ordering of  \( m \) given \( k \)-space locations such that the path connecting them has minimal length. Since TSP is NP-hard, we used a greedy approximation algorithm [17] for solving it, as implemented in TSP-Solver.\(^2\) With this approach, we first optimize for the best unconstrained solution (i.e., the \( L_{\text{const}} \) term in the loss is set to zero), and then find the closest solution satisfying the constraints.

The flow of PILOT-TSP, described schematically in Fig. 3, proceeds as follows:

1) Trajectory coordinates \( k \) are initialized by sampling \( m \) i.i.d. vectors from a multivariate normal distribution;
2) The parameters \( k \) of the trajectory and \( \theta \) of the task model are optimized by solving (1) with the second loss term set to zero. Note that this makes the solution independent of the ordering of \( k \).
3) Run the TSP approximate solver to order the elements of the vector \( k \) and form the trajectory. This significantly reduces albeit still does not completely eliminate constraint violation, as can be seen in the input to stage 3 in Fig. 6.
4) The trajectory is fine-tuned to obey the constraints by further performing iteration on the loss in (1) this time with the constraints term.

III. EXPERIMENTS AND DISCUSSION

A. Datasets

We used two datasets in the preparation of this article: the NYU fastMRI initiative database \(^3\), and the medical segmentation decathlon [18]. The fastMRI dataset contains 1372 knee MRI volumes. Since our work focuses on designing \( k \)-space trajectories and the provided test set in fastMRI is already sub-sampled, we split the training set into two sets: one containing 973 volumes (34700 slices) for training and validation (80/20 split), and 199 volumes (7100 slices) for testing. The fastMRI dataset consists of data obtained from multiple machines through two pulse sequences: proton-density and proton-density fat suppression. We included both modalities in our datasets. Further, the fastMRI dataset is acquired from multiple 3T scanners (Siemens Magnetom Skyra, Prisma and Biograph mMR) as well as 1.5T machines (Siemens Magnetom Aera). All data were acquired through the fully-sampled Cartesian trajectories using the 2D TSE protocol.

For the segmentation task, we used data obtained from the medical image segmentation decathlon challenge [18]. Within the decathlon challenge, we used the brain tumors dataset that contained 750 4D volumes of multi-modal (FLAIR, T1w, T1gd & T2w) brain MRI scans from patients diagnosed with either glioblastoma or lower-grade glioma. Gold standard annotations for all tumor regions in all scans were approved by expert board-certified neuroradiologists as detailed in [18]. Within this dataset, we used only the T2-weighted images. Since the goal of our experiment is to design trajectories that are optimal for segmenting tumors, we only considered images containing tumors, and used only the training set for all the experiments since they are the only ones that contain ground-truth segmentations. These data were split into two sets: one containing 400 volumes (26000 slices) for training and validation, and the other one containing 84 volumes (5300 slices) for testing.

B. Training settings

We trained both the sub-sampling layer and the task model network with the Adam [19] solver. The learning rate was set to 0.001 for the task model, while the sub-sampling layer was trained with learning rates varying between 0.001 to 0.1 in different experiments. More information about the hyper-parameters is provided in the supplementary material, Table I. We implemented the differentiable regridding layer using PyTorch autograd tools, adopting the code available in the sigpy package\(^4\).

C. Physical constraints.

The following physical constraints were used in all our experiments: \( G_{\text{max}} = 40 \text{mT/m} \) for the peak gradient, \( S_{\text{max}} = 200 \text{mT/m} \) for the maximum slew-rate, and \( dt = 10 \mu \text{sec} \) for the sampling time. We convert these parameters into the \( k \)-space dimensions by assuming an image field-of-view (FOV) of 0.2m or, equivalently, the \( k \)-space pixel resolution of \( \Delta k = 5 \text{m}^{-1} \).

D. Reference single-shot MR trajectories

Standard trajectories used in single-shot MR imaging are spiral and EPI. In this experiment, we use these trajectories as the initialization to the PILOT scheme (Fig. 2), i.e., \( k \) is initialized with these trajectory coordinates. We then update these coordinates jointly with the reconstruction network while including the penalties on the gradient and the slew rate as explained in Section II-E.

Cartesian Echo Planar Imaging (EPI). Cartesian EPI is the most commonly used single-shot MR imaging modality. EPI involves full sampling along the frequency-encoding direction and uniform skips along the phase-encoding direction. This is realized in single-shot by performing a zig-zag scan, keeping the gradient along the \( x \) direction constant, while performing a gradient blip along the \( y \) direction at the end of each line in order to shift the phase. Usually EPI is limited by the peak current and slew rate along one of the gradient directions (as

\(^2\)https://github.com/dmishin/tsp-solver
\(^3\)https://github.com/mikgroup/sigpy
\(^4\)https://github.com/mikgroup/sigpy
can be observed in the Fig. 5), leading to geometric aberrations in the images.

**Spiral.** Spiral trajectories were also proposed in the context of single-shot MR imaging [20], although not as prevalent as EPI in the single-shot scenario. In order to initialize feasible spirals that obey the hardware constraints we used [21] and produced spiral which cover the $k$-space with the length of $m$ samples.

### E. Image Reconstruction with PILOT

In order to quantitatively evaluate our method, we use the peak signal-to-noise ratio (PSNR) and the structural-similarity (SSIM) [22] measures, portraying both the pixel-to-pixel and perceptual similarity. In all our experiments, we compare our algorithms to the baseline training only the reconstruction model for measurements obtained with fixed handcrafted trajectories, henceforth referred to “fixed trajectory”. Table I presents the quantitative results comparing fixed spiral initialization to PILOT. The results demonstrate that PILOT outperforms the fixed trajectory across all decimation rates. The observed improvement is in the range of $0.23 - 1.23$ dB PSNR and $0.009 - 0.019$ SSIM points. Figure 4 depicts the initial and the optimized trajectories as well as the corresponding velocity (peak current) and acceleration (slew rate) encountered along the trajectory.

Table II also summarizes the quantitative results obtained for the EPI initialization. The quantitative results again suggest that the learned trajectories perform better compared to the fixed counterparts, with the improvement in the range of $2.21 - 2.88$ dB PSNR and $0.063 - 0.090$ SSIM points for the decimation rates 10 and 20. The fixed and learned trajectories with their corresponding gradient and slew rate are presented in Figure 5. We can notice that the fixed trajectories always violate the physical constraints on the slew rate and peak current by an order of magnitude. In practice, such trajectories are realized by a rather gradual increase in the $y$-axis gradient in order to fit the constraints. However, using PILOT, we obtain a feasible trajectory that satisfies the constraints in all regions, making it feasible on a real machine. For decimation rate of 80, no quantitative improvement is observed, yet but our method succeed in bringing the highly infeasible initial EPI trajectory inside the feasible area while retaining its performance (Fig 5, column 3).

Comparing the quantitative metrics between the spiral and EPI trajectories, we can notice that the spiral trajectory performs consistently better than the EPI (both when fixed and after optimization). This makes us doubt whether EPI, albeit simple to implement, is a good choice for performing single-shot imaging when used in conjunction with a reconstruction network.

Another interesting observation from the PILOT experiment is that the overall structure and shape of the trajectory remains similar to the initialized version, yet with significant improvement in quantitative metrics. This phenomenon is expressed more prominently in the case of spiral, due to the fact that the velocity of the spiral, by construction, is always at the highest allowed value. Therefore, we cannot significantly change the coordinates without violating the hardware constraints. This suggests that PILOT can be employed as a “fine-tuning” scheme to the trajectories that are used in practice today; since the overall shape of the trajectory is unchanged, it can be used in concert with the existing pulse timings, to achieve desired SNR at the desired places along the path.

### F. Image Reconstruction with PILOT-TSP

An observation from the PILOT experiment is that the learned trajectories depend on the initialization and do not deviate from it largely. This behavior could be attributed to the optimization algorithms that converge to a local minimum around the initial point. Another possibility for such local convergence is that the penalty applied to the trajectory vanishes once the latter is feasible, leaving less space for exploration. In order to allow for further exploration of the trajectory space, we employ a global optimization scheme as previously explained in the description of the PILOT-TSP algorithm. We follow the four-stage training procedure described in Section II-E.

**Optimal infeasible scatter map.** As a baseline, we report the performance of the optimal scatter of frequencies obtained without any constraints enforced on the velocity (peak current) or acceleration (slew rate) of the trajectory. This baseline is essentially the output of the second stage of the PILOT-TSP scheme. However, as can be seen in Figure 6, the obtained trajectory through this scatter plot would be highly infeasible, exceeding gradient and slew rate at almost all regions in the $k$-space. The subsequent two stages of the PILOT-TSP receive this highly infeasible trajectory and update it such that it falls into the feasible region.

The quantitative results of PILOT-TSP are presented in Table III. Observe that PILOT-TSP provides improvements in the range of $1.05 - 2.73$ dB PSNR when compared to the learned trajectory with the spiral initialization and a total of $1.94 - 3.96$ dB overall improvement over fixed trajectories, in the case of 20 and 80 decimation rates. For smaller decimation rates (e.g. 10), PILOT-TSP does not outperform PILOT initialized with spiral trajectory. The reason could be that the optimal infeasible scatter map obtained after the first stage of PILOT-TSP has high density of points at the central region of the $k$-space. Such a configuration is challenging to be efficiently explored while keeping the solution feasible in the single-shot regime (which is limited by the slew-rate constraint), therefore dropping the performance of the obtained reconstruction. The fixed and learned trajectories with their corresponding gradient and slew rate are presented in Figure 6.
Fig. 4: PILOT with Spiral initialization. Comparison of fixed (blue) and learned (red) trajectories for the Spiral initialization. The three columns depict different decimation rates (left-to-right: 10, 20, and 80). Top row depicts the trajectories in the $k$-space with the corresponding image reconstruction PSNR. Two magnified regions are depicted in the inserts. The following two rows depict the $x$- and the $y$-axis components of the gradient. The fixed spiral and learned trajectories obey the following constraints: maximum gradient $G_{\text{max}} = 40$ mT/m, maximum slew-rate $S_{\text{max}} = 200$ T/m/s and sampling interval $dt = 10 \mu$s.

![Decimation Rate - 10](image1)
![Decimation Rate - 20](image2)
![Decimation Rate - 80](image3)

### TABLE II: Comparison of image reconstruction PSNR and SSIM of the fixed spiral trajectory, its optimized version using PILOT, and the trajectory learned with PILOT-TSP. The last row reports the results obtained using the physically infeasible trajectory obtained at the end of Stage 2 of PILOT-TSP (training without hardware constraints).

| Trajectory          | 10       | 20       | 80       | 10       | 20       | 80       |
|---------------------|----------|----------|----------|----------|----------|----------|
| Fixed (Spiral)      | 29.86dB  | 27.81dB  | 24.71dB  | 0.706    | 0.640    | 0.541    |
| PILOT (Spiral)      | 30.09dB  | 28.70dB  | 25.94dB  | 0.715    | 0.658    | 0.560    |
| PILOT-TSP           | 30.05dB  | 29.75dB  | 28.67dB  | 0.692    | 0.653    | 0.609    |
| Best infeasible     | 32.80dB  | 31.80dB  | 30.20dB  | 0.748    | 0.695    | 0.633    |

### TABLE III: Comparison of DICE and IOU scores for the segmentation task using the fixed spiral trajectory, its optimized version using PILOT, and the trajectory learned with PILOT-TSP. Three different decimation rates are reported.

| Trajectory          | 10       | 20       | 80       | 10       | 20       | 80       | 10       | 20       | 80       |
|---------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Fixed (Spiral)      | 0.810    | 0.719    | 0.720    | 0.669    | 0.725    | 0.609    | 0.816    | 0.725    | 0.639    |
| PILOT (Spiral)      | 0.825    | 0.737    | 0.735    | 0.716    | 0.725    | 0.639    | 0.818    | 0.725    | 0.639    |
| PILOT-TSP           | 0.816    | 0.726    | 0.754    | 0.669    | 0.725    | 0.639    | 0.818    | 0.725    | 0.639    |

A visual comparison of results is available in Fig. 1 for decimation rate 20 and in Fig. 5 for the rest.

### G. Optimal trajectory design for segmentation

We demonstrate the ability of the proposed methods to learn a trajectory optimized for a specific end-task on the tumor segmentation task. The details of the dataset used are described in Section III-A. We use the standard Dice similarity coefficient (DICE) and the intersection-over-union (IOU) metrics to quantify segmentation accuracy.

The results for segmentation are reported in Table III. We observe that the learned trajectories achieve an improvement of $0.015 - 0.044$ DICE points and $0.018 - 0.056$ IOU points when compared to the fixed trajectories they were initialized with. We notice that the PILOT-TSP performs better than PILOT with spiral initialization at decimation rates 20 and 80, while PILOT performs better than PILOT-TSP at decimation rate 10, consistent with what was observed in the reconstruction.
H. Cross-task and multi-task learning

In order to verify end-task optimality of the found solution, we evaluated the learned models on tasks they were not learned on. Specifically, we trained PILOT-TSP with the same dataset once on the segmentation task and another time on the reconstruction task. We then swapped the obtained trajectories between the tasks, fixed them, and re-trained only the task model (U-Net) for each task (while keeping the trajectory as it was learned for the other task). The results for this experiment are reported in Table IV. We observe that the trajectories obtained from segmentation are optimal for that task alone and perform worse on reconstruction, and vice-versa. From the output trajectories learned for segmentation and reconstruction, visualized in Fig [10] we can see that the reconstruction trajectory is more centered around the DC frequency, whereas the segmentation trajectory tries to cover higher frequencies. This could be explained by the fact that most of the image information is contained within the...
Fig. 6: **Trajectory evolution in PILOT-TSP.** (a) The initial variable density sampling used as the input to the PILOT-TSP pipeline (Stage 1). (b) The learned sampling pattern obtained after unconstrained optimization (Stage 2). (c) The trajectory obtained by ordering the samples with the approximate TSP solver (Stage 3). (d) The fine-tuned trajectory obtained after the optimization (Stage 4) in PILOT-TSP pipeline. The trajectories are colored by the slew-rate. Notice that the trajectory obtained in (c) is not feasible and exceeds the slew rate constraints in most regions; this is resolved after fine tuning in (d).

Fig. 7: **PILOT-TSP.** Trajectories learned with PILOT-TSP. The three columns depict different decimation rates (left-to-right: 10, 20, and 80). Top row depicts the trajectories in the \( k \)-space with the corresponding image reconstruction PSNR. Two magnified regions are depicted in the inserts. The following two rows depict the \( x \)- and \( y \)-axis components of the gradient. The learned trajectories obey the following constraints: maximum gradient \( G_{\text{max}} = 40 \text{ mT/m} \), maximum slew-rate \( S_{\text{max}} = 200 \text{ T/m/s} \) and sampling interval \( dt = 10 \mu s \).
lower frequencies in the \( k \)-space, therefore covering them, contributes more to the pixel-wise similarity measure of the reconstruction task. For the segmentation task, contrarily, the edges and the structural information present in the higher frequencies, is more critical for the success of the task. This is an interesting observation because it paves the way to design certain accelerated MRI protocols that are optimal for a given end-task that is not necessarily reconstruction.

Another important use can be to jointly learn several tasks such as segmentation and reconstruction in order to find the best segmentation while preserving the ability to reconstruct a meaningful human-intelligible image. For this purpose, we build a model with one encoder and two separate decoders (separated at the bottleneck layer of the U-Net), each for a different task. The loss function is defined to be the sum of both the reconstruction loss (from the first decoder) and the segmentation loss (pixel-wise cross-entropy, from the second decoder). The results for this experiment are reported in the last row of Table IV. Interestingly, we can observe that the presence of the reconstruction tasks aids segmentation as well. This is also consistent with the results observed in a recent study [23].

IV. CONCLUSION AND FUTURE DIRECTIONS

We have demonstrated, as a proof-of-concept, that learning the \( k \)-space sampling trajectory simultaneously with an image reconstruction neural model improves the end image quality of an MR imaging system. To the best of our knowledge, the proposed PILOT algorithm is the first to do end-to-end learning over the space of all physically feasible MRI \( k \)-space trajectories. We also showed that end-to-end learning is possible with other tasks such as segmentation. On both tasks and across decimation rates, we observed that starting from an infeasible trajectory (EPI), PILOT learns a feasible trajectory that, in most cases, outperforms the baseline infeas-
Fig. 9: **Tumor segmentation with PILOT and PILOT-TSP.** Compared are the fixed spiral trajectory (second column), PILOT with spiral initialization (third column), and PILOT-TSP (rightmost column). The first row depicts the MR images as obtained after the inverse Fourier transform with no further processing. The groundtruth image obtained from the fully sampled k-space is depicted in the leftmost column. The second row depicts the predicted segmentation map (red) and the groundtruth map (blue) with the corresponding DICE and IOU scores.

Fig. 10: **Task-specific PILOT-TSP.** Comparison of the learned trajectories (decimation rate 20) optimized for different tasks: segmentation (left) and reconstruction (right). Quantitative performance of these trajectories is presented in Table IV.

We also suggested the PILOT-TSP procedure capable of learning a feasible task-optimal trajectory starting with a random initialization. Experimental evaluation shows superior performance of PILOT-TSP across tasks and decimation rates. Neither PILOT or PILOT-TSP take into account the decay of the measurement SNR during the scan and the effect of the time each measurement has been taken at. We intend to address this important issue in future research.

**ACKNOWLEDGMENT**

This work was supported by the ERC StG RAPID.

**REFERENCES**

[1] M. Lustig, D. Donoho, and J. M. Pauly, “Sparse mri: The application of compressed sensing for rapid mr imaging,” Magnetic Resonance in Medicine: An Official Journal of the International Society for Magnetic Resonance in Medicine, vol. 58, no. 6, pp. 1182–1195, 2007.

[2] K. Hammernik, T. Klatzer, E. Kobler, M. P. Recht, D. K. Sodickson, T. Pock, and F. Knoll, “Learning a variational network for reconstruction of accelerated mri data,” Magnetic resonance in medicine, vol. 79, no. 6, pp. 3055–3071, 2018.

[3] J. Sun, H. Li, Z. Xu et al., “Deep admm-net for compressive sensing mri,” in Advances in neural information processing systems, 2016.

[4] J. Zbontar, F. Knoll, A. Sriram, M. J. Muckley, M. Bruno, A. Delazio, M. Parente, K. J. Geros, J. Katznelson, H. Chandarana et al., “fastmri: An open dataset and benchmarks for accelerated mri,” arXiv preprint arXiv:1811.08839, 2018.

[5] S. Vedula, O. Senouf, G. Zurakhov, A. Bronstein, O. Michailovich, and M. Zibulevsky, “Learning beamforming in ultrasound imaging,” in Proceedings of The 2nd International Conference on Medical Imaging with Deep Learning, ser. Proceedings of Machine Learning Research, vol. 102. PMLR, July 2019, pp. 493–511.

[6] O. Menashe and A. Bronstein, “Real-time compressed imaging of scattering volumes,” in 2014 IEEE International Conference on Image Processing (ICIP). IEEE, 2014, pp. 1322–1326.
[7] H. Haim, S. Elmalem, R. Giryes, A. M. Bronstein, and E. Marom, “Depth estimation from a single image using deep learned phase coded mask,” IEEE Transactions on Computational Imaging, vol. 4, no. 3, pp. 298–310, 2018.
[8] T. Weiss, S. Vedula, O. Senouf, A. Bronstein, O. Michailovich, and M. Zibulevsky, “Learning Fast Magnetic Resonance Imaging,” arXiv e-prints, p. arXiv:1905.09324, May 2019.
[9] B. Gözcü, R. K. Mahabadi, Y.-H. Li, E. Ilıcak, T. Çukur, J. Scarlett, and V. Cevher, “Learning-based compressive mri,” IEEE transactions on medical imaging, vol. 37, no. 6, pp. 1394–1406, 2018.
[10] Z. Zhang, A. Romero, M. J. Muckley, P. Vincent, L. Yang, and M. Drozdzal, “Reducing uncertainty in undersampled mri reconstruction with active acquisition,” arXiv preprint arXiv:1902.03051, 2019.
[11] K. H. Jin, M. Unser, and K. M. Yi, “Self-supervised deep active accelerated mri,” arXiv preprint arXiv:1901.04547, 2019.
[12] C. Lazarus, P. Weiss, N. Chauffert, F. Maucouduit, L. El Gueddari, C. Destrieux, I. Zemmoura, A. Vignaud, and P. Ciuciu, “Sparkling: variable-density k-space filling curves for accelerated t2*-weighted mri,” Magnetic Resonance in Medicine, vol. 81, no. 6, pp. 3643–3661, 2019.
[13] A. Dutt and V. Rokhlin, “Fast fourier transforms for nonequispaced data,” SIAM J. Comput., vol. 14, no. 6, pp. 1368–1393, Nov. 1993. [Online]. Available: http://dx.doi.org/10.1137/0914081
[14] O. Ronneberger, P. Fischer, and T. Brox, “U-net: Convolutional networks for biomedical image segmentation,” in International Conference on Medical image computing and computer-assisted intervention. Springer, 2015, pp. 234–241.
[15] F. Isensee, P. F. Jaeger, P. M. Full, I. Wolf, S. Engelhardt, and K. H. Maier-Hein, “Automatic cardiac disease assessment on cine-mri via time-series segmentation and domain specific features,” in International workshop on statistical atlases and computational models of the heart. Springer, 2017, pp. 120–129.
[16] H. Wang, X. Wang, Y. Zhou, Y. Chang, and Y. Wang, “Smoothed random-like trajectory for compressed sensing mri,” vol. 2012, 08 2012, pp. 404–7.
[17] J. J. Grefenstette, R. Gopal, B. J. Rossmaita, and D. V. Gucht, “Genetic algorithms for the traveling salesman problem,” in ICGA, 1985.
[18] A. L. Simpson, M. Antonelli, S. Bakas, M. Bilello, K. Farahani, B. van Ginneken, A. Kopp-Schneider, B. A. Landman, G. Litjens, and B. Menze, “A large annotated medical image dataset for the development and evaluation of segmentation algorithms,” arXiv e-prints, p. arXiv:1902.09063, Feb 2019.
[19] D. P. Kingma and J. Ba, “Adam: A Method for Stochastic Optimization,” arXiv e-prints, p. arXiv:1412.6980, Dec 2014.
[20] K. T. Block and J. Frahm, “Spiral imaging: A critical appraisal,” Journal of Magnetic Resonance Imaging, vol. 21, no. 6, pp. 657–668, 2005.
[21] B. Hargreaves. Variable-density spiral design functions. [Online]. Available: https://mrsrl.stanford.edu/~brian/vdspiral
[22] Zhou Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, “Image quality assessment: from error visibility to structural similarity,” IEEE Transactions on Image Processing, vol. 13, no. 4, pp. 600–612, 2004.
[23] L. Sun, Z. Fan, Y. Huang, X. Ding, and J. Paisley, “Joint CS-MRI Reconstruction and Segmentation with a Unified Deep Network,” arXiv e-prints, p. arXiv:1805.02165, May 2018.