Quarks, Leptons as Fermion-Boson Composite Objects and Flavor-Mixings by Substructure Dynamics

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Abstract

A fermion-boson-type composite model for quarks and leptons is proposed. Elementary fields are only one kind of spin-1/2 and spin-0 preon. Both are in the global supersymmetric pair with the common electric charge of “e/6” and belong to the fundamental representations of $(3,2,2)$ under the spontaneously unbroken $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R$ gauge symmetry induced necessarily by the concept of “Cartan connection” equipped with “Soldering Mechanism”. Preons are composed into subquarks which are “intermediate clusters” towards quarks and leptons. The mechanism of making higher generations is obtained by adding neutral scalar subquark composed of a preon-antipreon pair in the $3$-state of $SU(2)_{L,R}$. This model predicts the CKM matrix elements : $|V_{ts}| = 2.6 \times 10^{-2}, |V_{td}| = 1.4 \times 10^{-3}$; the neutral pseudoscalar meson mass differences : $\Delta M_D \approx 10^{-14} \text{ GeV}, \Delta M_{B_s} \approx 10^{-11} \text{ GeV}$, $\Delta M_{T_u} \approx 10^{(-10\sim-9)} \text{ GeV}$ and $\Delta M_{T_c} \approx 10^{(-8\sim-7)} \text{ GeV}$; the phases of CP violation : $\theta_K = \theta_D = \theta_{B_s} = \theta_{T_c} \approx (1/2)\theta_{B_d} \approx (1/2)\theta_{T_u}$. This model also suggests that $\Delta M^2 \approx 10^{(-8\sim-7)} \text{ eV}^2$ with large mixing angle in $(\nu_e,\nu_\mu)$-oscillation.
1 Introduction

The discovery of the top-quark[1] has finally confirmed the existence of three quark-lepton symmetric generations. So far the standard $SU(2)_L \otimes U(1)$ model (denoted by SM) has successfully explained various experimental evidences. Nevertheless, as is well known, the SM is not regarded as the final theory because it has many arbitrary parameters, e.g., quark and lepton masses, quark-mixing parameters and the Weinberg angle, etc. Therefore it is meaningful to investigate the origins of these parameters and the relationship among them. In order to overcome such problems some attempts have done, e.g., Grand Unification Theory (GUT), Supersymmetry, Composite model, etc. In the GUT scenario quarks and leptons are elementary fields in general. On the contrary in the composite scenario they are literally the composite objects constructed from the elementary fields (so called “preon”). The lists of various related works are in ref.[2]. If quarks and leptons are elementary, in order to solve the above problems it is necessary to introduce some external relationship or symmetries among them. On the other hand the composite models have ability to explain the origin of these parameters in terms of the substructure dynamics of quarks and leptons. Further, the composite scenario naturally leads us to the thought that the intermediate vector bosons of weak interactions ($W, Z$) are not elementary gauge fields but composite objects constructed from preons (same as $\rho$-meson from quarks). Many studies based on such conception have done after Bjorken’s[3] and Hung and Sakurai’s[4] suggestions of the alternative way to unified weak-electromagnetic gauge theory[5-11]. In this scheme the weak interactions are regarded as the effective residual interactions among preons. The fundamental fields for intermediate forces are massless gauge fields belonging to some gauge groups and they confine preons into singlet states to build quarks and leptons and $W, Z$.

Recently CDF Collaboration at the Fermilab Tevatron Collider has released the data that the excess of the inclusive jet differential cross section in the jet transverse energy region of $200 \sim 400$ GeV in the $\bar{p}p$ collision experiments at $\sqrt{s} = 1.8$ TeV[12]. Although several arguments are going on concerning next-to-leading order QCD calculations[13], they suggest the possibility of the presence of quark compositeness scale limits the range 1.6 to 1.8 TeV.[12]. In this article we consider a composite model for quarks and leptons and also quark-flavor-mixing phenomena in terms of the substructure dynamics. The conception of our model is that the fundamental interacting forces are all originated from massless gauge fields belonging to the adjoint representa-
tions of some gauge groups which have nothing to do with the spontaneous breakdown and that the elementary matter fields are only one kind of spin-1/2 preon and spin-0 preon carrying common “e/6” electric charge (e > 0). Quarks, leptons and W, Z are all composites of them and usual weak interactions are regarded as effective residual interactions. Based on this model we suggest that there exists the relations between the quark mass spectrum and the quark-flavor-mixings at the level of the substructure dynamics and also discuss the mass difference and CP violation in neutral pseudoscalar meson systems.

2 Gauge theory inspiring quark-lepton composite scenario

In our model the existence of fundamental matter fields (preon) are inspired by the gauge theory with Cartan connections[14]. Let us briefly summarize the basic features of that. Generally gauge fields, including gravity, are considered as geometrical objects, that is, connection coefficients of principal fiber bundles. It is said that there exist some different points between Yang-Mills gauge theories and gravity, though both theories commonly possess fiber bundle structures. The latter has the fiber bundle related essentially to 4-dimensional space-time freedoms but the former is given, in an ad hoc way, the one with the internal space which has nothing to do with the space-time coordinates. In case of gravity it is usually considered that there exist ten gauge fields, that is, six spin connection fields in $SO(1, 3)$ gauge group and four vierbein fields in $GL(4, R)$ gauge group from which the metric tensor $g^{\mu\nu}$ is constructed in a bilinear function of them. Both altogether belong to Poincaré group $ISO(1, 3) = SO(1, 3) \otimes R^4$ which is semi-direct product. In this scheme spin connection fields and vierbein fields are independent but only if there is no torsion, both come to have some relationship. Seeing this, $ISO(1, 3)$ gauge group theory has the logical weak point not to answer how two kinds of gravity fields are related to each other intrinsically.

In the theory of Differential Geometry, S.Kobayashi has investigated the theory of “Cartan connection”[15]. This theory, in fact, has ability to reinforce the above weak point. The brief recapitulation is as follows. Let $E(B_n, F, G, P)$ be a fiber bundle (which we call Cartan-type bundle) associated with a principal fiber bundle $P(B_n, G)$ where $B_n$ is a base manifold with dimension “n”, $G$ is a structure group, $F$ is a fiber space which is homogeneous and diffeomorphic with $G/G'$ where $G'$ is a subgroup of $G$. Let $P' = P'(B_n, G')$ be a principal fiber bundle, then $P'$ is a subbundle of $P$. Here
let it be possible to decompose the Lie algebra $g$ of $G$ into the subalgebra $g'$ of $G'$ and a vector space $f$ such as:

$$g = g' + f, \quad g' \cap f = 0, \quad (1)$$

$$[g', g'] \subset g', \quad (2)$$

$$[g', f] \subset f, \quad (3)$$

$$[f, f] \subset g', \quad (4)$$

where $\dim f = \dim F = \dim G - \dim G' = \dim B_n = n$. The homogeneous space $F = G/G'$ is said to be “weakly reductive” if there exists a vector space $f$ satisfying Eq.(1) and (3). Further $F$ satisfying Eq.(4) is called “symmetric space”. Let $\omega$ denote the connection form of $P$ and $\overline{\omega}$ be the restriction of $\omega$ to $P'$. Then $\overline{\omega}$ is a $g$-valued linear differential 1-form and we have:

$$\omega = g^{-1}\overline{\omega}g + g^{-1}dg, \quad (5)$$

where $g \in G$, $dg \in T_g(G)$. $\omega$ is called the form of “Cartan connection” in $P$.

Let the homogeneous space $F = G/G'$ be weakly reductive. The tangent space $T_{O}(F)$ at $o \in F$ is isomorphic with $f$ and then $T_{O}(F)$ can be identified with $f$ and also there exists a linear $f$-valued differential 1-form (denoted by $\theta$) which we call the “form of soldering”. Let $\omega'$ denote a $g'$-valued 1-form in $P'$, we have:

$$\overline{\omega} = \omega' + \theta. \quad (6)$$

The dimension of vector space $f$ and the dimension of base manifold $B_n$ is the same “$n$”, and then $f$ can be identified with the tangent space of $B_n$ at the same point in $B_n$ and $\theta$s work as $n$-bein fields. In this case $\omega'$ and $\theta$ unifyingly belong to group $G$. Here let us call such a mechanism “Soldering Mechanism”.

Drechsler has found out the useful aspects of this theory and investigated a gravitational gauge theory based on the concept of the Cartan-type bundle equipped with the Soldering Mechanism[16]. He considered $F = SO(1,4)/SO(1,3)$ model. Homogeneous space $F$ with $dim = 4$ solders 4-dimensional real space-time. The Lie algebra of $SO(1,4)$ corresponds to $g$ in Eq.(1), that of $SO(1,3)$ corresponds to $g'$ and $f$ is 4-dimensional vector space. The 6-dimensional spin connection fields are $g'$-valued objects and vierbein fields are $f$-valued, both of which are unified into the members of $SO(1,4)$ gauge group. We can make the metric tensor $g^{\mu\nu}$ as a bilinear function of
f-valued vierbein fields. Inheriting Drechsler’s study the author has investigated the quantum theory of gravity[14]. The key point for this purpose is that $F$ is a symmetric space because $f$s are satisfied with Eq.(4). Using this symmetric nature we can pursue making a quantum gauge theory, that is, constructing $g'$-valued Faddeev-Popov ghost, anti-ghost, gauge fixing, gaugeon and its pair field as composite fusion fields of $f$-valued gauge fields by use of Eq.(4) and also naturally inducing BRS-invariance.

Comparing such a scheme of gravity, let us consider Yang-Mills gauge theories. Usually when we make the Lagrangian density $\mathcal{L} = tr(F \wedge F^*)$ ($F$ is a field strength), we must borrow a metric tensor $g_{\mu\nu}$ from gravity to get $F^*$ and also for Yang-Mills gauge fields to propagate in the 4-dimensional real space-time. This seems to mean that “there is a hierarchy between gravity and other three gauge fields (electromagnetic, strong, and weak)”. But is it really the case? As an alternative thought we can think that all kinds of gauge fields are “equal”. Then it would be natural for the question “What kind of equality is that?” to arise. In other words, it is the question that “What is the minimum structure of the gauge mechanism which four kinds of forces are commonly equipped with?” For answering this question, let us make a assumption: “Gauge fields are Cartan connections equipped with Soldering Mechanism.” In this meaning all gauge fields are equal. If it is the case three gauge fields except gravity are also able to have their own metric tensors and to propagate in the real space-time without the help of gravity. Such a model has already investigated in ref.[14].

Let us discuss them briefly. It is found that there are four types of sets of classical groups with small dimensions which admit Eq.(1,2,3,4), that is, $F = SO(1, 4)/SO(1, 3)$, $SU(3)/U(2)$, $SL(2, C)/GL(1, C)$ and $SO(5)/SO(4)$ with $dimF = 4[17]$. Note that the quality of “dim 4” is very important because it guarantees $F$ to solder to 4-dimensional real space-time and all gauge fields to work in it. The model of $F = SO(1, 4)/SO(1, 3)$ for gravity is already mentioned. Concerning other gauge fields, it seems to be appropriate to assign $F = SU(3)/U(2)$ to QCD gauge fields, $F = SL(2, C)/GL(1, C)$ to QED gauge fields and $F = SO(5)/SO(4)$ to weak interacting gauge fields. Some discussions concerned are following. In general, matter fields couple to $g'$-valued gauge fields. As for QCD, matter fields couple to the gauge fields of $U(2)$ subgroup but $SU(3)$ contains, as is well known, three types of $SU(2)$ subgroups and then after all they couple to all members of $SU(3)$ gauge fields. In case of QED, $GL(1, C)$ is locally isomorphic with $C^1 \cong U(1) \otimes R$. Then usual Abelian gauge fields are assigned to $U(1)$ subgroup of $GL(1, C)$. Georgi and Glashow suggested that the reason why the electric charge is quantized comes from the fact that $U(1)$ electromagnetic gauge group is a unfactorized
subgroup of $SU(5)$[18]. Our model is in the same situation because $GL(1, C)$ a unfactorized subgroup of $SL(2, C)$. For usual electromagnetic $U(1)$ gauge group, the electric charge unit “$e$”($e > 0$) is for one generator of $U(1)$ but in case of $SL(2, C)$ which has six generators, the minimal unit of electric charge shared per one generator must be “$e/6$”. This suggests that quarks and leptons might have the substructure simply because $e, 2e/3, e/3 > e/6$. Finally as for weak interactions we adopt $F = SO(5)/SO(4)$. It is well known that $SO(4)$ is locally isomorphic with $SU(2) \otimes SU(2)$. Therefore it is reasonable to think it the left-right symmetric gauge group : $SU(2)_L \otimes SU(2)_R$. As two $SU(2)s$ are direct product, it is able to have coupling constants $(g_L, g_R)$ independently. This is convenient to explain the fact of the disappearance of right-handed weak interactions in the low-energy region. Possibility of composite structure of quarks and leptons suggested by above $SL(2, C)$-QED would introduce the thought that the usual left-handed weak interactions are intermediated by massive composite vector bosons as $\rho$-meson in QCD and that they are residual interactions due to substructure dynamics of quarks and leptons. The elementary massless gauge fields , “as connection fields”, relate intrinsically to the structure of the real space-time manifold but on the other hand the composite vector bosons have nothing to do with it. Considering these discussions, we set the assumption : “All kinds of gauge fields are elementary massless fields, belonging to spontaneously unbroken $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{e.m}$ gauge group and quarks and leptons and $W, Z$ are all composite objects of the elementary matter fields.”

3 Composite model

Our direct motivation towards compositeness of quarks and leptons is one of the results of the arguments in Sect.2, that is, $e, 2e/3, e/3 > e/6$. However, other several phenomenological facts tempt us to consider a composite model, e.g., repetition of generations, quark-lepton parallelism of weak isospin doublet structure, quark-flavor-mixings, etc.. Especially Bjorken[3]’s and Hung and Sakurai[4]’s suggestion of an alternative to unified weak-electromagnetic gauge theories have invoked many studies of composite models including composite weak bosons[5-11]. Our model is in the line of those studies. There are two ways to make composite models, that is, “Preons are all fermions.” or “Preons are both fermions and bosons (denoted by FB-model).” The merit of the former is that it can avoid the problem of a quadratically divergent self-mass of elementary scalar fields. However, even in the latter case such a disease is
overcome if both fermions and bosons are the supersymmetric pairs, both of which carry the same quantum numbers except the nature of Lorentz transformation (spin-1/2 or spin-0)[19]. Pati and Salam have suggested that the construction of a neutral composite object (neutrino in practice) needs both kinds of preons, fermionic as well as bosonic, if they carry the same charge for the Abelian gauge or belong to the same (fundamental) representation for the non-Abelian gauge[20]. This is a very attractive idea for constructing the minimal model. Further, according to the representation theory of Poincaré group both integer and half-integer spin angular momentum occur equally for massless particles[21], and then if nature chooses “fermionic monism”, there must exist the additional special reason to select it. Therefore in this point also, the thought of the FB-model is minimal. Based on such considerations we propose a FB-model of “only one kind of spin-1/2 elementary field (denoted by Λ) and of spin-0 elementary field (denoted by Θ)” (preliminary version of this model has appeared in Ref.[14]). Both have the same electric charge of “e/6” (Maki has first proposed the FB-model with the minimal electric charge e/6. [22]) and the same transformation properties of the fundamental representation (3, 2, 2) under the spontaneously unbroken gauge symmetry of $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R$ (let us call $SU(2)_L \otimes SU(2)_R$ “hypercolor gauge symmetry”). Then Λ and Θ come into the supersymmetric pair which guarantees 'tHooft’s naturalness condition[23]. The $SU(3)_C$, $SU(2)_L$ and $SU(2)_R$ gauge fields cause the confining forces with confining energy scales of $\Lambda_c << \Lambda_L < (or \cong) \Lambda_R$ (Schrempp and Schrempp discussed them elaborately in Ref.[11]). Here we call positive-charged primons (Λ, Θ) “matter” and negative-charged primons (Λ, Θ) “antimatter”. Our final goal is to build quarks, leptons and W, Z from Λ (Λ) and Θ (Θ). Let us discuss that scenario next.

At the very early stage of the development of the universe, the matter fields (Λ, Θ) and their antimatter fields (Λ, Θ) must have broken out from the vacuum. After that they would have combined with each other as the universe was expanding. That would be the first step of the existence of composite matters. There are ten types of them:

| spin1/2 | spin0 | e.m.charge | Y.M.representation |
|--------|-------|------------|-------------------|
| ΛΘ     | ΛΛ, ΘΘ | e/3        | (3, 1, 1) (3, 3, 1) (3, 1, 3), (7a) |
| ΛΘ, ΛΘ | ΛΛ, ΘΘ | 0          | (1, 1, 1) (1, 3, 1) (1, 1, 3), (7b) |
| ΛΘ     | ΛΛ, ΘΘ | − e/3      | (3, 1, 1) (3, 3, 1) (3, 1, 3), (7c) |

1 The notations of Λ and Θ are inherited from those in Ref.[22]. After this we call Λ and Θ “Prion” named by Maki which means “primordial particle”[22].
In this step the confining forces are, in kind, in $SU(3) \otimes SU(2)_L \otimes SU(2)_R$ gauge symmetry but the $SU(2)_L \otimes SU(2)_R$ confining forces must be main because of the energy scale of $\Lambda_L, \Lambda_R \gg \Lambda_c$ and then the color gauge coupling $\alpha_s$ and e.m. coupling constant $\alpha$ are negligible. As is well known, the coupling constant of $SU(2)$ confining force are characterized by $\varepsilon_i = \sum_a \sigma^a_p \sigma^a_q$, where $\sigma$s are $2 \times 2$ matrices of $SU(2)$, $a = 1, 2, 3$, $p, q = \Lambda, \overline{\Lambda}, \Theta, \overline{\Theta}$, $i = 0$ for singlet and $i = 3$ for triplet. They are calculated as $\varepsilon_0 = -3/4$ which causes the attractive force and and $\varepsilon_3 = 1/4$ causing the repulsive force. As concerns, $SU(3)_C$ octet and sextet states are repulsive but singlet, triplet and antitriplet states are attractive and then the formers are disregarded. Like this, two primons are confined into composite objects in more than one singlet state of any $SU(3)_C, SU(2)_L, SU(2)_R$. Note that three primon systems cannot make the singlet states of $SU(2)$. Then we omit them. In Eq.(7b), the $(1, 1, 1)$-state is the “most attractive channel”. Therefore $(\Lambda\overline{\Theta}), (\overline{\Lambda}\Theta), (\Lambda\Lambda)$ and $(\Theta\overline{\Theta})$ of $(1, 1, 1)$-states with neutral e.m. charge must have been most abundant in the universe. Further $(\overline{3}, 1, 1)$- and $(3, 1, 1)$-states in Eq.(7a,c) are next attractive. They presumably go into $\{(\Lambda\Theta)(\overline{\Lambda}\overline{\Theta})\}, \{(\Lambda\Lambda)(\overline{\Lambda}\overline{\Lambda})\}$, etc. of $(1, 1, 1)$-states with neutral e.m. charge. These objects may be the candidates for the “cold dark matters” if they have even tiny masses. It is presumable that the ratio of the quantities between the ordinary matters and the dark matters firstly depends on the color and hypercolor charges (maybe the ratio is more than $1/(3 \times 3)$). Finally the $(\ast, 3, 1)$- and $(\ast, 1, 3)$-states are remained $($ is $1, 3, \overline{3})$. They are also stable because $|\varepsilon_0| > |\varepsilon_3|$. They are, so to say, the “intermediate clusters” towards constructing ordinary matters (quarks, leptons and $W, Z$). 

\[ \begin{array}{cccc}
\text{Y.M. representation} & \text{spin} & \text{e.m.charge} \\
\alpha & (\Lambda\Theta), & \alpha_L : (\overline{3}, 3, 1), & \alpha_R : (\overline{3}, 1, 3) \\
\beta & (\Lambda\overline{\Theta}), & \beta_L : (1, 3, 1), & \beta_R : (1, 1, 3) \\
x & (\Lambda\Lambda, \Theta\Theta), & x_L : (\overline{3}, 3, 1), & x_R : (\overline{3}, 1, 3) \\
y & (\Lambda\overline{\Lambda}, \Theta\overline{\Theta}), & y_L : (1, 3, 1), & y_R : (1, 1, 3) \\
\end{array} \]

and there are also their antisubquarks[9].

Now we come to the step to build quarks and leptons. The gauge symmetry of

\footnote{Such thoughts have been proposed by Maki in Ref.[22]}

\footnote{The notations of $\alpha, \beta, x$ and $y$ are inherited from those in Ref.[9] written by Fritzsch and Mandelbaum, because ours is, in the subquark level, similar to theirs with two fermions and two bosons. R. Barbieri, R. Mohapatra and A. Masiero proposed the similar model[9].}
the confining forces in this step is also $SU(2)_L \otimes SU(2)_R$ because the subquarks are in the triplet states of $SU(2)_{L,R}$ and then they are combined into singlet states by the decomposition of $3 \times 3 = 1 + 3 + 5$ in $SU(2)$. We make the first generation of quarks and leptons as follows:

\begin{align*}
< u_h | &= < \alpha_h x_h | & 2e/3 & (3, 1, 1), \\
< d_h | &= < \overline{\alpha_h} \overline{x_h} x_h | & -e/3 & (3, 1, 1), \\
< \nu_h | &= < \alpha_h \overline{x_h} | & 0 & (1, 1, 1), \\
< e_h | &= < \overline{\alpha_h} \overline{x_h} \overline{x_h} | & -e & (1, 1, 1),
\end{align*}

where \( h \) stands for \( L \) (left handed) or \( R \) (right handed)[5]. Here we note that $\beta$ and $y$ do not appear. In practice $(\beta y) : (1, 1, 1)$-particle is a candidate for neutrino. But as Bjorken has pointed out[3], non-vanishing charge radius of neutrino is necessary for obtaining the correct low-energy effective weak interaction Lagrangian[11]. Therefore $\beta$ is assumed not to contribute to forming ordinary quarks and leptons. However $(\beta y)$-particle may be a candidate for “sterile neutrino”. Presumably composite $(\beta \beta)$-; $(\beta \beta)$-; $(\beta \beta)$-states may go into the dark matters. It is also noticeable that in this model the leptons have finite color charge radius and then $SU(3)$ gluons interact directly with the leptons at energies of the order of, or larger than $\Lambda_L$ or $\Lambda_R$[19].

Concerning the confinements of primons and subquarks, the confining forces of two steps are in the same spontaneously unbroken $SU(2)_L \otimes SU(2)_R$ gauge symmetry. It is known that the $\alpha_W(Q^2)$ (the running coupling constant of the $SU(2)$ gauge theory) satisfies the following equation:

\begin{align*}
1/\alpha_W(Q^2_1) &= 1/\alpha_W(Q^2_2) + b_2 \ln(Q^2_1/Q^2_2), \\
b_2 &= 1/(4\pi) \{ 22/3 - (2/3) \cdot N_f - (1/12) \cdot N_s \},
\end{align*}

where $N_f$ and $N_s$ are the numbers of fermions and scalars contributing to the vacuum polarizations. Here let us assume that subquarks in quarks are confined at the energy of 1.6 TeV (if admitting CDF’s data[12]). By use of Eq.(10b) we calculate $b_2 = 0.35$ which comes from that the number of confined fermionic subquarks are 4 ($\alpha_i, i = 1, 2, 3$ for color freedom, $\beta$) and 4 for bosons ($x_i, y$) contributing to the vacuum polarization.

\footnote{Subquark configurations in Eq.(9) are essentially the same as those in Ref.[5] written by Królikowski, who proposed the model of one fermion and one boson with the same e.m. charge $e/3$.}
Using $b_2 = 0.35$ we get $\alpha_W = 0.040$ at $Q = 10^{10}$ GeV and extrapolating from this value we get the confining energy of primons $(\Lambda, \Theta)$ is $1.6 \times 10^2$ TeV, where we use $b_2 = 0.41$ (by Eq. (23b)) which is calculated with three kinds of $\Lambda$ and $\Theta$ owing to three color freedoms. In sum, the radii of $\alpha$, $\beta$, $x$ and $y$ are the inverse of $1.6 \times 10^2$ TeV and the radii of quarks are the inverse of $1.6$ TeV.

Next let us see the higher generations. Harari and Seiberg have stated that the orbital and radial excitations seem to have the wrong energy scale (order of $\Lambda_{L,R}$) and then the most likely type of excitations is the addition of preon-antipreon pairs [6, 25]. Then using $y_{L,R}$ in Eq. (8, d) we construct them as follows:

\[
\begin{align*}
&\{<c| = <\alpha xy|, \quad \{<\nu_\mu| = <\alpha xy|, \\
&<s| = <\alpha xy|, \quad \{<\mu| = <\alpha xy|, \\
&<t| = <\alpha xy|, \quad \{<\nu_\tau| = <\alpha xy|, \\
&<b| = <\alpha xy|, \quad \{<\tau| = <\alpha xy|,
\end{align*}
\]

where the suffix $L, Rs$ are omitted for brevity. We can also make vector and scalar particles with $(1, 1, 1)$:

\[
\begin{align*}
&\{<W^+| = <\alpha^\dagger\alpha^\dagger x|, \quad \{<Z^0_1| = <\alpha^\dagger\alpha^\dagger |, \\
&<W^-| = <\alpha^\dagger\alpha^\dagger \overline{x}|, \quad \{<Z^0_2| = <\alpha^\dagger\alpha^\dagger x\overline{x}|, \\
&<S^+| = <\alpha^\dagger\alpha^\dagger x|, \quad \{<S^0_1| = <\alpha^\dagger\alpha^\dagger |, \\
&<S^-| = <\alpha^\dagger\alpha^\dagger \overline{x}|, \quad \{<S^0_2| = <\alpha^\dagger\alpha^\dagger x\overline{x}|,
\end{align*}
\]

where the suffix $L, Rs$ are omitted for brevity and $\uparrow, \downarrow$ indicate spin up, spin down states. They play the role of intermediate bosons same as $\pi, \rho$ in the strong interactions. As Eq. (9) and Eq. (12) contain only $\alpha$ and $x$ subquarks, we can draw the “line diagram” of weak interactions as seen in Fig (1). Eq. (9d) shows that the electron is constructed from antimatters only. We know, phenomenologically, that this universe is mainly made of protons, electrons, neutrinos, antineutrinos and unknown dark matters. It is said that protons and electrons in the universe are almost same in quantity. Our model show that one proton has the configuration of $(\textbf{uud}) = (2\alpha, \overline{\alpha}, 3x, \overline{x})$; electron :$(\pi, 2\overline{x})$; neutrino :$(\alpha, \overline{x})$; antineutrino :$(\overline{\alpha}, x)$ and the dark matters are presumably constructed from the same amount of matters and antimatters because of their neutral charges. Note that proton is a mixture of matters and anti-matters and electrons is composed of anti-matters only. This may lead the thought that “the universe is the matter-antimatter-even object.” And then there exists a conception-leap between “proton-electron abundance” and “matter abundance” if our composite scenario is admitted (as for the possible way to realize the proton-electron excess universe, see
This idea is different from the current thought that the universe is made of matters only. Then the question about CP violation in the early universe does not occur.

Our composite model contains two steps, namely the first is “subquarks made of primons” and the second is “quarks and leptons made of subquarks”. Here let us discuss about the mass generation mechanism of quarks and leptons as composite objects. Our model has only one kind of fermion : Λ and boson : Θ. The first step of “subquarks made of primons” seems to have nothing to do with ’tHooft’s anomaly matching condition[23] because there is no global symmetry with Λ and Θ. Therefore from this line of thought it is impossible to say anything about that α, β, x and y are massless or massive. However, if it is the case that the neutral (1,1,1)-states of primon-antiprinmon composites (as is stated above) become the dark matters, the masses of them are presumably less than the order of MeV from the phenomenological aspects of astrophysics. In this connection it is interesting that Królikowski has showed one possibility of constructing massless composite particles(fermion-fermion or fermion-boson pair) controled by relativistic two-body equations[34]. Then we may assume that these subquarks are massless or almost massless compared with Λ_{L,R} in practice, that is, utmost a few MeV. In the second step, the arguments of ’tHooft’s anomaly matching condition are meaningful. The confining of subquarks must occur at the energy scale of Λ_{L,R} >> Λ_c and then it is natural that α_s, α \rightarrow 0 and that the gauge symmetry group is the spontaneously unbroken SU(2)_L \otimes SU(2)_R gauge group. Seeing Eq.(9), we find quarks and leptons are composed of the mixtures of subquarks and antisubquarks. Therefore it is proper to regard subquarks and antisubquarks as different kinds of particles. From Eq.(8,a,b) we find eight kinds of fermionic subquarks ( 3 for α, \bar{α} and 1 for β, \bar{β}). So the global symmetry concerned is SU(8)_L \otimes SU(8)_R. Then we arrange :
\[(β, \bar{β}, α_i, \bar{α}_i \ i = 1, 2, 3 )_{L,R} \quad \text{in} \quad (SU(8)_L \otimes SU(8)_R)_{\text{global}}, \quad (13)\]
where is are color freedoms. Next, the fermions in Eq.(13) are confined into the singlet states of the local SU(2)_L \otimes SU(2)_R gauge symmetry and make up quarks and leptons as seen in Eq.(9) (eight fermions). Then we arrange :
\[(ν_e, e_i, u_i, d_i \ i = 1, 2, 3 )_{L,R} \quad \text{in} \quad (SU(8)_L \otimes SU(8)_R)_{\text{global}}, \quad (14)\]
where is are color freedoms. From Eq.(13) and Eq.(14) the anomalies of the subquark level and the quark-lepton level are matched and then all composite quarks and leptons
(in the 1st generation) are remained massless. Note again that presumably, $\beta$ and $\bar{\beta}$ in Eq.(13) are composed into “bosonic” $(\beta\beta)$, $(\beta\bar{\beta})$ and $((\beta\bar{\beta}))$, which vapour out to the dark matters. Schrempf and Schrempf have discussed about a confining $SU(2)_L \otimes SU(2)_R$ gauge model with three fermionic preons and stated that it is possible that not only the left-handed quarks and leptons are composite but also the right-handed are so on the condition that $\Lambda_R/\Lambda_L$ is at least of the order of 3\[11\]. If CDF’s data\[12\] truly indicates the compositeness of quarks, $\Lambda_L$ is presumably around 1.6 TeV. As seen in Eq.(12a) the existence of composite $W_R$, $Z_R$ is predicted. As concerning, the fact that they are not observed yet means that the masses of $W_R$, $Z_R$ are larger than those of $W_L$, $Z_L$ because of $\Lambda_R > \Lambda_L$. Owing to ‘tHooft’s anomaly matching condition the small mass nature of the 1st generation comparing to $\Lambda_L$ is guaranteed but the evidence that the quark masses of the 2nd and the 3rd generations become larger as the generation numbers increase seems to have nothing to do with the anomaly matching mechanism in our model, because, as seen in Eq.(11a,b), these generations are obtained by just neutral adding scalar $y$-particles. This is different from Abott and Farhi’s model in which all fermions of three generations are equally embedded in $SU(12)$ global symmetry group and all members take part in the anomaly matching mechanism\[8,26\]. Concerning this, let us discuss a little about subquark dynamics inside quarks. According to “Uncertainty Principle” the radius of the composite particle is, in general, roughly inverse proportional to the kinetic energy of the constituent particles moving inside it. The radii of quarks may be around $1/\Lambda_{L,R}$. So the kinetic energies of subquarks may be more than hundreds GeV and then it is considered that the masses of quarks essentially depend on the kinetic energies of subquarks and such a large binding energy as counterbalances them. As seen in Eq.(11a,b) our model shows that the more the generation number increases the more the number of the constituent particles increases. So assuming that the radii of all quarks do not vary so much (because we have no experimental evidences yet), the interaction length among subquarks inside quarks becomes shorter as generation numbers increase and accordingly the average kinetic energy per one subquark may increase. Therefore integrating out the details of subquark dynamics it could be said that the feature of increasing masses of the 2nd and the 3rd generations is essentially described as a increasing function of the sum of the kinetic energies of constituent subquarks. From Review of Particle Physics\[29\] we can phenomenologically parameterized the mass spectrum of quarks and leptons as
follows:

\begin{align}
M_{UQ} &= 1.2 \times 10^{-4} \times (10^{2.05})^n \text{ GeV for } u, c, t, \quad (15a) \\
M_{DQ} &= 3.0 \times 10^{-4} \times (10^{1.39})^n \text{ GeV for } d, s, b, \quad (15b) \\
M_{DL} &= 3.6 \times 10^{-4} \times (10^{1.23})^n \text{ GeV for } e, \mu, \tau, \quad (15c)
\end{align}

where \( n = 1, 2, 3 \) are the generation numbers. They seem to be geometric ratio-like. The slope parameters of the up-quark sector and down-quark sector are different, so it seems that each has different aspects in subquark dynamics. It is interesting that the slope parameters of both down sectors of quark and lepton are almost equal, which suggests that there exist similar properties in substructure dynamics and if it is the case, the slope parameter of up-leptonic (neutrino) sector may be the same as that of up-quark sector, that is, \( M_{UL} \sim 10^{2n} \). From Eq.(15) we obtain \( M_u = 13.6 \text{ MeV, } M_d = 7.36 \text{ MeV and } M_e = 6.15 \text{ MeV.} \) These are a little unrealistic compared with the experiments[29]. But considering the above discussions about the anomaly matching conditions (Eq.(13,14)), it is natural that the masses of the members of the 1st generation are roughly equal to those of the subquarks, that is, a few MeV. The details of their mass-values may depend on the subquark dynamics owing to the effects of electromagnetic and color gauge interactions. These mechanism has studied by Weinberg[32] and Fritzsch[33].

One of the experimental evidences inspiring the SM is the “universality” of the coupling strength among the weak interactions. Of course if the intermediate bosons are gauge fields, they couple to the matter fields universally. But the inverse of this statement is not always true, namely the quantitative equality of the coupling strength of the interactions does not necessarily imply that the intermediate bosons are elementary gauge bosons. In practice the interactions of \( \rho \) and \( \omega \) are regarded as indirect manifestations of QCD. In case of chiral \( SU(2) \otimes SU(2) \) the pole dominance works very well and the predictions of current algebra and PCAC seem to be fulfilled within about 5%[19]. Fritzsch and Mandelbaum[9,19] and Gounaris, Kögerler and Schildknecht[10,27] have elaborately discussed about universality of weak interactions appearing as a consequence of current algebra and \( \mathbf{W} \)-pole dominance of the weak spectral functions from the stand point of the composite model. Extracting the essential points from their arguments we mention our case as follows. In the first generation let the weak charged currents be written in terms of the subquark fields as:

\[ J^+_{\mu} = \overline{U}h_{\mu}D, \quad J^-_{\mu} = \overline{D}h_{\mu}U, \quad (16) \]
where \( U = (\alpha x), \ D = (\alpha xx) \) and \( h_\mu = \gamma_\mu (1 - \gamma_5) \). Reasonableness of Eq.(16) may given by the fact that \( M_W << \Lambda_{L,R} \) (where \( M_W \) is \( W \)-boson mass). Further, let \( U \) and \( D \) belong to the doublet of the global weak isospin \( SU(2) \) group and \( W^+, W^-, (1/\sqrt{2})(Z^0_0 - Z^0_2) \) be in the triplet and \( (1/\sqrt{2})(Z^0_1 + Z^0_2) \) be in the singlet of \( SU(2) \). These descriptions seem to be natural if we refer the diagrams in Fig.(1). The universality of the weak interactions are inherited from the universal coupling strength of the algebra of the global weak isospin \( SU(2) \) group with the assumption of \( W^-, Z \)-pole dominance. The universality including the 2nd and the 3rd generations are investigated in the next section based on the above assumptions and in terms of the flavor-mixings.

4 \( \Delta F = 1 \) flavor-mixing by subquark dynamics

The quark-flavor-mixings in the weak interactions are usually expressed by Cabbibo-Kobayashi-Maskawa (CKM) matrix based on the SM. Its nine matrix elements (in case of three generations) are "free" parameters (in practice four parameters with the unitarity) and this point is said to be one of the drawback of the SM along with non-understanding of the origins of the quark-lepton mass spectrum and generations. In the SM, the quark fields (lepton fields also) are elementary and then we are able to investigate, at the utmost, the external relationship among them. On the other hand if quarks are the composites of substructure constituents, the quark-flavor-mixing phenomena must be understood by the substructure dynamics and the values of CKM matrix elements become materials for studying these. Terazawa and Akama have investigated quark-flavor-mixings in a three spinor subquark model with higher generations of radially excited state of the up (down) quark and stated that a quark-flavor-mixing matrix element is given by an overlapping integral of two radial wave functions of the subquarks which depends on the momentum transfer between quarks[28,31].

In our model “the quark-flavor-mixings occur by creations or annihilations of \( y \)-particles inside quarks”. The \( y \)-particle is a neutral scalar subquark in the 3-state of \( SU(2)_L \) group and then couples to two hypercolor gluons (denoted by \( g_h \)) (see Fig.(2)). Here we propose the important assumption : “The \( (y \rightarrow 2g_h) \)-process is factorized from the net \( W^\pm \) exchange interactions.” This assumption is plausible because the effective energy of this process may be in a few TeV energy region comparing to a hundred GeV energy region of \( W \)-exchange processes. Let us write the contribution of \( (y \rightarrow 2g_h) \)-process to charged weak interactions as :

\[
A_i = \alpha_W (Q^2_i)^2 \cdot B \quad i = s,c,b,t,
\]

(17)
where $\alpha_W$ is a running coupling constant of the hypercolor gauge theory appearing in Eq.(10), $Q_i$ is the effective four momentum of $g_h$-exchange among subquarks inside the $i$-quark and $B$ is a dimensionless complex free parameter which is originated from the unknown primon dynamics and may depend on $|<0|\bar{\Lambda}\gamma_\mu\Lambda, (\bar{\Theta}\partial_\mu\Theta)|y>|$ / $|<0|\bar{\Lambda}\gamma_\mu\Lambda, (\bar{\Theta}\partial_\mu\Theta)|0>|$.

The weak charged currents of quarks are taken as the matrix elements of subquark currents between quarks which are not the eigenstates of the weak isospin[28]. Using Eq.(11), (16) and (17) with the above assumption we have:

\[
\begin{align*}
V_{ud}\bar{u}\mu d &= <\bar{u}|\bar{U}h_\mu D|d>, \\
V_{us}\bar{u}\mu s &= <\bar{u}|\bar{U}h_\mu(Dy)|s> \equiv <\bar{u}|\bar{U}h_\mu D|s> \cdot A_s, \\
V_{ub}\bar{u}\mu b &= <\bar{u}|\bar{U}h_\mu(Dyy)|b> \equiv <\bar{u}|\bar{U}h_\mu D|b> \cdot 2A_b^2, \\
V_{cu}\bar{c}\mu d &= <\bar{c}|(\bar{U}y)h_\mu D|d> \equiv <\bar{c}|\bar{U}h_\mu D|d> \cdot A_c, \\
V_{cs}\bar{c}\mu s &= <\bar{c}|(\bar{U}y)h_\mu(Dy)|s>, \\
V_{cb}\bar{c}\mu b &= <\bar{c}|(\bar{U}y)h_\mu(Dyy)|b> \equiv <\bar{c}|\bar{U}h_\mu(Dy)|b> \cdot A_b, \\
V_{td}\bar{t}\mu d &= <\bar{t}|(\bar{U}yy)h_\mu D|d> \equiv <\bar{t}|\bar{U}h_\mu D|d> \cdot 2A_t^2, \\
V_{ts}\bar{t}\mu s &= <\bar{t}|(\bar{U}yy)h_\mu(Dy)|s> \equiv <\bar{t}|\bar{U}h_\mu(Dy)|s> \cdot A_t, \\
V_{tb}\bar{t}\mu b &= <\bar{t}|(\bar{U}yy)h_\mu(Dyy)|b>,
\end{align*}
\]

where $V_{ij}$s are CKM-matrices and $\{u, d, s, \text{etc.}\}$ in the left sides of the equations are quark-mass eigenstates. Here we need some explanations. In transitions from the 3rd to the 1st generation in Eq.(18c,g) there are two types: One is that two $(y \rightarrow 2g_h)$-processes occur at the same time and the other is that $y$ annihilates into $2g_h$ in a cascade way. Then we can describe the case of Eq.(18c) as:

\[
<\bar{u}|\bar{U}h_\mu(Dyy)|b> \equiv <\bar{u}|\bar{U}h_\mu D|b> \cdot A_b^2 + <\bar{u}|\bar{U}h_\mu(Dy)|b> \cdot A_b \\
\equiv <\bar{u}|\bar{U}h_\mu D|b> \cdot A_b^2 + <\bar{u}|\bar{U}h_\mu D|b> \cdot A_b^2 \\
= <\bar{u}|\bar{U}h_\mu D|b> \cdot 2A_b^2.
\]

The case of Eq.(18g) is also same as this (here the phase-difference between the 1st and the 2nd term is disregarded for simplicity). If we admit the assumption of factorizability of $(y \rightarrow 2g_h)$-process, it is natural that the universality of the net weak interactions among three generations are realized. The net weak interactions are essentially same as $(u \rightarrow d)$-transitions(Fig.(1)).
Then we may think that:

\[ \left< \mathbf{u}^\mu \mathbf{d} \right> \cong \left< \mathbf{u}^\mu \mathbf{s} \right> \cong \left< \mathbf{u}^\mu \mathbf{b} \right> \]
\[ \cong \left< \mathbf{c}^\mu \mathbf{d} \right> \cong \left< \mathbf{t}^\mu \mathbf{d} \right>, \]
(20a)

\[ \left< \mathbf{c}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{s} \right> \cong \left< \mathbf{c}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{b} \right> \cong \left< \mathbf{t}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{s} \right> \cong \left< \mathbf{t}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{b} \right>. \]
(20b)

and additionally we may assume:

\[ \left< \mathbf{u}^\mu \mathbf{d} \right> \cong \left< \mathbf{c}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{s} \right> \cong \left< \mathbf{t}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{s} \right> \cong \left< \mathbf{t}^\mu \left( \mathbf{U} \mathbf{y} \right)^\mu \mathbf{b} \right>. \]
(21)

In Eq.(20b) and (21) y-particles are the spectators for the weak interactions. Concerning the left sides of Eq.(18a-i), The \{\mathbf{u}^\mu, \mathbf{h}^\mu \mathbf{d}, \mathbf{u}^\mu \mathbf{s}, \mathbf{u}^\mu \mathbf{b}, \mathbf{c}^\mu \mathbf{d}, \mathbf{c}^\mu \mathbf{s}, \mathbf{c}^\mu \mathbf{b}, \mathbf{c}^\mu \mathbf{t}, \mathbf{t}^\mu \mathbf{d}, \mathbf{t}^\mu \mathbf{b}, \mathbf{t}^\mu \mathbf{s}, \mathbf{t}^\mu \mathbf{t} \} operate coordinately as the function of the current operator (that is, just as the function of coupling to the “common” W-boson current) when only weak interactions switch on. In practice weak interactions occur as the residual ones commonly among subquarks inside any kinds of quarks. Therefore in this scenario (quark-subquark correspondence) it seems natural to assume that such equations work in the weak interactions as:

\[ \mathbf{u}^\mu \mathbf{d} = \mathbf{u}^\mu \mathbf{s} = \mathbf{u}^\mu \mathbf{b} = \mathbf{c}^\mu \mathbf{d} = \ldots. \]
(22)

Using Eq.(17),(18),(20),(21) and (22) we find:

\[ |V_{us}| / |V_{ud}| = |A_s| = \alpha_W (Q_s^2)^2 \cdot |B|, \]
(23a)

\[ |V_{cd}| / |V_{ud}| = |A_c| = \alpha_W (Q_c^2)^2 \cdot |B|, \]
(23b)

\[ |V_{cb}| / |V_{us}| = |A_b| = \alpha_W (Q_b^2)^2 \cdot |B|, \]
(23c)

\[ |V_{tb}| / |V_{cs}| = |A_t| = \alpha_W (Q_t^2)^2 \cdot |B|, \]
(23d)

\[ |V_{ub}| / |V_{ud}| = 2 |A_b|^2 = 2 \{ \alpha_W (Q_b^2)^2 \cdot |B| \}^2, \]
(23e)

\[ |V_{td}| / |V_{ad}| = 2 |A_t|^2 = 2 \{ \alpha_W (Q_t^2)^2 \cdot |B| \}^2. \]
(23f)

Here let us investigate the substructure dynamics inside quarks referring to the above equations. In our composite model quarks are composed of \( \alpha, \mathbf{x}, \mathbf{y} \). Concretely from Eq.(11) c-quark is composed of three subquarks; t-quark : four subquarks; s-quarks : four subquarks; b-quark : five subquarks. From the discussions in Sect.3, let the quark mass be proportional to the sum of the average kinetic energies of the subquarks (denoted by \( < T_i >, i = s, c, b, t \)). The proportional constants are assumed
common in the up (down)-quark sector and different between the up- and the down-quark sector according to the discussions in Sect.3. Then we denote them by $K_s$ ($s = \text{up, down}$). The $<T_i>$ may considered inverse proportional to the average interaction length among subquarks (denoted by $<r_i>$). Further, it is presumable that $\sqrt{Q_i^2}$ ($Q_i$ is the effective four momentum of $g_h$-exchange among subquarks inside the $i$-quark in Eq.(17)) is inverse proportional to $<r_i>$.

Then we have:

$$M_b/M_s = 5K_{down} <T_b> / (4K_{down} <T_s>) = (5/4) \cdot (<r_s> / <r_b>)$$

$$= (5/4) \cdot (\sqrt{Q_b^2/Q_s^2}), \quad (24a)$$

$$M_t/M_c = 4K_{up} <T_t> / (3K_{up} <T_c>) = (4/3) \cdot (<r_c> / <r_i>)$$

$$= (4/3) \cdot (\sqrt{Q_t^2/Q_c^2}), \quad (24b)$$

where $M_i$s are the masses of $i$-quarks. In the Review of Particle Physics[29] we find:

$M_b/M_s = 30 \pm 15$ and $M_t/M_c = 135 \pm 35$, using which we get by Eq.(24):

$$Q_b^2/Q_s^2 \cong (24)^2, \quad (25a)$$

$$Q_t^2/Q_c^2 \cong (100)^2. \quad (25b)$$

Note again that it seems to be meaningless to estimate $Q_s^2/Q_t^2$ or $Q_c^2/Q_b^2$ because the up-quark sector and the down-quark sector possibly have the different aspects of substructure dynamics.

The absolute values of CKM-matrix elements: $|V_{ij}|$s are reported as the “experimental” results (without unitarity assumption)[29] that:

$$|V_{ud}| = 0.9740 \pm 0.0010, \quad |V_{us}| = 0.2196 \pm 0.0023,$$

$$|V_{cd}| = 0.224 \pm 0.016, \quad |V_{cb}| = 0.0395 \pm 0.0017, \quad (26)$$

$$|V_{cs}| = 1.04 \pm 0.16, \quad |V_{ub}|/|V_{cb}| = 0.08 \pm 0.02.$$

Relating these data to the scheme of our composite model, we investigate the quark-flavor-mixing phenomena in terms of the substructure dynamics. Using Eq.(23a), (23c) and $|V_{us}|, |V_{cb}|$ in Eq.(26) we get:

$$\alpha_W(Q_s^2)/\alpha_W(Q_b^2) = 2.32, \quad (27)$$

where we assume $|V_{ud}| = |V_{cs}|$. Applying $N_f = N_s = 4$ (as is stated in Sect.3) to Eq.(10b) we have:

$$b_2 = 0.345. \quad (28)$$
Here we rewrite Eq.(10a) in Sect.3 as:

$$\alpha_W(Q_1^2) = \{1 - \alpha_W(Q_1^2)/\alpha_W(Q_2^2)\}/\{b_2 \ln(Q_1^2/Q_2^2)\}. \quad (29)$$

Inserting the values of Eq.(25,a), (27) and (28) into Eq.(29) we have:

$$\alpha_W(Q_s^2) = 0.602, \quad (30)$$

where $$Q_s, Q_b$$ corresponds to $$Q_1, Q_2$$ in Eq.(29). Combining $$|V_{ud}|, |V_{us}|$$ in Eq.(26) and Eq.(30) with Eq.(23a) we obtain:

$$|B| = 0.629, \quad (31)$$

and using Eq.(30) to Eq.(27) we get:

$$\alpha_W(Q_b^2) = 0.259. \quad (32)$$

By use of $$|V_{ud}|, |V_{cd}|$$ in Eq.(26) and Eq.(31) to Eq.(23b) we have:

$$\alpha_W(Q_c^2) = 0.605. \quad (33)$$

Using Eq.(10a) with Eq.(25b), (28) and (33) (setting $$t(c)$$ to 1(2)) we obtain:

$$\alpha_W(Q_t^2) = 0.207. \quad (34)$$

Inserting Eq.(31), (32) to the right side of Eq.(23e) we have:

$$|V_{ub}| = 0.00345. \quad (35)$$

Comparing this with the experimental value of $$|V_{ub}| = 0.003 \pm 0.001$$ (obtained from the values of $$|V_{cb}|$$ and $$|V_{ub}|/|V_{cb}|$$ in Eq.(26)), the consistency between the prediction and the experiment seems good. This result is also consistent with the first exclusive determinations of $$|V_{ub}|$$ from the decay $$B \to \pi l \nu_l$$ and $$B \to \rho l \nu_l$$ by the CLEO experiment to obtain $$|V_{ub}| = (3.3 \pm 0.4 \pm 0.7) \times 10^{-3}$$[59].

Finally using Eq.(31), (34) to Eq.(23d,f) we predict:

$$|V_{ts}| = 2.62 \times 10^{-2}, \quad |V_{td}| = 1.40 \times 10^{-3}, \quad (36)$$

where we use $$|V_{ud}| = 0.974, |V_{cs}| = 0.974$$[29]. Comparing the values of Eq.(36) with $$|V_{ts}| = 0.039 \pm 0.004$$ and $$|V_{td}| = 0.0085 \pm 0.0045$$[29] obtained by assuming the three generations with unitarity, we find that our results are smaller by a factor than them. The origin of these results presumably is in that the top-quark mass is heavy. We wish the direct measurements of ($$t \to d, s$$) transitions in leptonic and/or semileptonic decays of top-quark mesons.
Recently CP violation in heavy neutral mesons ($D^0$, $B^0$) has been discussed as the experiments concerned were carried out (though have not yet been observed) and in the near future the real features will become clear. So the totally understandable scenario for CP violation phenomena is now required. The mass difference ($\Delta M_P$) between heavier neutral pseudo scalar meson ($P$) and lighter one is considered to be essentially connected with CP violation because the origin of both phenomena comes from the off diagonal matrix elements of the mass matrix ($M_{ij}$, $i, j = 1, 2$) and the decay matrix ($\Gamma_{ij}$, $i, j = 1, 2$). Comparing with CP violation, the experiments of $\Delta M_P$ are a little abundant, e.g., there are $\Delta M_K$, $\Delta M_{B_d}$, the upper bound of $\Delta M_D$, and the lower bound of $\Delta M_{B_s}$[29]. Theoretical analyses about them are roughly in two ways, e.g., the estimation of $M_{12}$ by the superweak (SW) theory or the box diagram calculation in the SM with (or without) long distance contributions. Therefore the subject of the present stage is to clarify what kind of dynamics controls $M_{ij}$ and $\Gamma_{ij}$.

In the following discussions we investigate these issues in the context of our Fermion-Boson-type subquark model (FB-model)[61].

a. Mass difference $\Delta M_P$ by $P^0 - \bar{P}^0$ mixing

The typical $\Delta F = 2$ phenomenon is the mixing between a neutral pseudo scalar meson ($P^0$) and its antimeson ($\bar{P}^0$). There are six types, e.g., $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B^0_d - \bar{B}^0_d$, $B^0_s - \bar{B}^0_s$, $T^0_u - \bar{T}^0_u$ and $T^0_c - \bar{T}^0_c$ mixings. Usually they have been considered to be the most sensitive probes of higher-order effects of the weak interactions in the SM. The basic tool to investigate them is the “box diagram”. By using this diagram to the $K_L - K_S$ mass difference, Gaillard and Lee predicted the mass of the charm quark[38]. Later, Wolfenstein suggested that the contribution of the box diagram which is called the short-distance (SD) contribution cannot supply the whole of the mass difference $\Delta M_K$ and there are significant contributions arising from the long-distance (LD) contributions associated with low-energy intermediate hadronic states[38]. As concerns, the LD-phenomena occur in the energy range of few hundred MeV and the SD-phenomena around 100 GeV region. Historically there are various investigations for $P^0 - \bar{P}^0$ mixing problems[36][39-48] and many authors have examined them by use
of LD- and SD-contributions. In summary, the comparison between the theoretical results and the experiments about $\Delta M_P$ ($P = K, D$ and $B_d$) are as follows:

$$\Delta M^L_{K} \approx \Delta M^S_{K} \approx \Delta M^\text{exp}_{K}, \quad (37a)$$

$$\Delta M^L_{D} \ll \Delta M^S_{D} (\ll \Delta M^\text{exp}_{D}, \text{upper bound}), \quad (37b)$$

$$\Delta M^L_{B_d} \ll \Delta M^S_{B_d} \approx \Delta M^\text{exp}_{B_d}. \quad (37c)$$

Concerning Eq.(37a) it is explain that $\Delta M_K = \Delta M^S_{K} + D \Delta M^L_{K}$ where “$D$” is a numerical value of order $O(1)$. As for Eq.(37c), they found that $\Delta M^L_{B_d} \approx 10^{-15}$ GeV and $\Delta M^S_{B_d} \approx 10^{-13}$ GeV, then the box diagram is the most important for $B_d^0 \bar{B}_d^0$ mixing. Computations of $\Delta M^S_{B_d}$ and $\Delta M^S_{B_s}$ from the box diagrams in the SM give

$$\Delta M^S_{B_s} / \Delta M^S_{B_d} \simeq (M_{B_s} / M_{B_d}) |V_{ts} / V_{td}|^2 (B_{B_s} f^2_{B_s} / B_{B_d} f^2_{B_d}) \zeta, \quad (38)$$

where $V_{ij}$s stand for CKM matrix elements; $M_P$ : P-meson mass; $\zeta$ : a QCD correction of order $O(1)$; $B_B$ : Bag factor of B-meson and $f_B$ : decay constant of B-meson. Measurements of $\Delta M^\text{exp}_{D}$ and $\Delta M^\text{exp}_{B_d}$ are, therefore, said to be useful to determine $|V_{ts} / V_{td}|$[49][50]. Concerning Eq.(37b), they found that $\Delta M^L_{D} \approx 10^{-15}$ GeV and $\Delta M^S_{D} \approx 10^{-17}$ GeV[36][44] but the experimental measurement is $\Delta M^\text{exp}_{D} < 1.6 \times 10^{-13}$ GeV[29]. Further there is also a study that $\Delta M^L_{D}$ is smaller than $10^{-15}$ GeV by using the heavy quark effective theory[45]. Then many people state that it would be a signal of new physics beyond the SM if the future experiments confirm that $\Delta M^\text{exp}_{D} \approx 10^{-14} \sim 10^{-13}$ GeV[39-45][60]. Above investigations are based on the calculations of SD-contributions with (or without) LD-contributions in the SM.

On the other hand some authors have studied these phenomena in the context of the theory explained by the single dynamical origin. Cheng and Sher[51], Liu and Wolfenstein[47], and Gérard and Nakada[48] have thought that all $P^0-\bar{P}^0$ mixings occur only by the dynamics of the TeV energy region which is essentially the same as the SW-idea originated by Wolfenstein[35]. They extended the original SW-theory (which explains CP violation in the K-meson system) to other flavors by setting the assumption that $\Delta F = 2$ changing neutral spin 0 particle with a few TeV mass (denoted by $H$) contributes to the “real part” of $M_{ij}$ which determines $\Delta M_P$ and also the “imaginary part” of $M_{ij}$ which causes the indirect CP violation. The ways of extensions are that $H$-particles couple to quarks by the coupling proportional to $\sqrt{m_i m_j}[47][51]$, $(m_i / m_j)^n \quad n = 0, 1, 2 [47]$ and $(m_i + m_j)[48]$ where $i, j$ are flavors of quarks coupling to $H$. It is suggestive that the SW-couplings depend on quark masses (this idea is adopted in
our model discussed below). Cheng and Sher[51] and Liu and Wolfenstein[47] obtained that 
\[ \Delta M_D = (m_c/m_s) \Delta M_D^{exp} \approx 10^{-14} \text{ GeV} \] with the assumption that \( H \)-exchange mechanism saturates the \( \Delta M_D^{exp} \) bound, which is comparable to \( \Delta M_K^{exp} < 1.6 \times 10^{-13} \text{ GeV}[29] \). Concerning \( B \)-meson systems they found that 
\[ \Delta M_{B_s}/\Delta M_{B_d} \approx 10^{-14} \text{ GeV} \] with the assumption that \( H \)-exchange mechanism saturates the \( \Delta M_{B_s}/\Delta M_{B_d}^{exp} \) bound, which is comparable to \( \Delta M_{B_s}/\Delta M_{B_d}^{exp} > 20 \) [29]. However using their scheme it is calculated that 
\[ \Delta M_{B_d}/\Delta M_K = \left( B_{B_d} f_{B_d}^2 / B_K f_K^2 \right) (M_{B_d}/M_K) (m_b/m_s) \approx 300, \] where we use \( m_b = 4.3 \text{ GeV}, m_s = 0.2 \text{ GeV}, M_{B_d} = 5.279 \text{ GeV}, M_K = 0.498 \text{ GeV}, B_{B_d} f_{B_d}^2 = (0.22 \text{GeV})^2, B_K f_K^2 = (0.17 \text{GeV})^2 \). This is larger than \( \Delta M_{B_d}/\Delta M_K^{exp} = 87 \) [29] and is caused by large b-quark mass value.

Now let us discuss \( P^0-\bar{P^0} \) mixings by using our FB-model. The discussions start from the assumption that the mass mixing matrix \( M_{ij}(P) (i,j) = 1(2) \) denotes \( P^0(\bar{P^0}) \) is saturated by the SW-type interactions causing a direct \( \Delta F = 2 \) transitions. We usually calculate \( \Delta M_P \) as 
\[ M_{12}(P) = \langle P^0 | \mathcal{H}_{SW}^{\Delta F=2} | P^0 \rangle, \] 
\[ \Delta M_P = M_H - M_L \approx 2 |M_{12}(P)|, \] where we assume \( \text{Im}M_{12} \ll \text{Re}M_{12} \) which is experimentally acceptable[36][52], and \( M_H(L) \) stands for heavier (lighter) \( P^0(\bar{P^0}) \)-meson mass. Applying the vacuum-insertion calculation to the hadronic matrix element as 
\[ \langle P^0 | [q_i \gamma_\mu (1-\gamma_5) q_j] | P^0 \rangle \sim B_P f_P^2 M_P^2 \] we get 
\[ M_{12}(P) = (1/12\pi^2) B_P f_P^2 M_P \] 
(41)

The details of \( \mathcal{M}_P \) are model-dependent, e.g., the box diagram in the SM; the neutral \( spin \text{ } 0 \) particle exchange in the SW-theory. In case of our FB-model, the diagrams contributing to \( \mathcal{M}_P \) are seen in Fig.(3), and \( P^0(\bar{P^0}) \) mixings occur due to \( \text{“}y\text{-exchange} \) between two quarks inside the present \( P^0(\bar{P^0}) \)-meson. This is a kind of the realizations of Wolfenstein’s SW-idea[35]. The schematic illustration is as follows : two particles (quarks) with radius order of \( 1/\Lambda_q \) (a few TeV\(^{-1}\)) are moving to and fro inside a sphere (meson) with radius order of GeV\(^{-1}\). The \( y\)-exchange interactions would occur when two quarks inside \( P^0(\bar{P^0}) \)-meson interact in contact with each other because \( y\)-particles are confined inside quarks. As seen in Fig.(3), the contributions of \( y\)-exchanges seem common among various \( P^0(\bar{P^0}) \)-mesons. Upon this, setting the assumption : \text{“universality of the \( y\)-exchange interactions”,} we rewrite \( \mathcal{M}_P \) as 
\[ \mathcal{M}_P = n_P \eta(P) \tilde{M}_l(P), \] 
(42)
where \( n_P = 1 \) for \( P = K, D, B_d, T_u \); \( n_P = 2 \) for \( P = B_s, T_c \); \( l = 1 \) for \( K, D, B_s, T_u \); \( l = 2 \) for \( B_d, T_c \). Then the universality means explicitly that

\[
\tilde{M}_1(K) = \tilde{M}_1(D) = \tilde{M}_1(B_s) = \tilde{M}_1(T_c),
\]

(43a)

\[
\tilde{M}_2(B_d) = \tilde{M}_2(T_u).
\]

(43b)

The explanation of \( n_P \) is such that \( K \) and \( D \) have one \( y \)-particle and one \( y \)-particle exchanges; \( B_d \) and \( T_u \) have two \( y \)-particles and both of them exchange simultaneously, so for them we set \( n_P = 1 \). On the other hand \( B_s \) and \( T_c \) have two \( y \)-particles but one of them exchanges, so they have \( n_P = 2 \) because the probability becomes double. The " \( l \) " means the number of exchanging \( y \)-particles in the present diagram. Concerning \( \eta(P) \), we explain as follows: In our FB-model \( P^0_{\overline{P}^0} \)-mixing occurs by the "contact interaction" of two quarks colliding inside \( P^0_{\overline{P}^0} \)-meson. Therefore the probability of this interaction may be considered inverse proportional to the volume of the present \( P^0_{\overline{P}^0} \)-meson, e.g., the larger radius \( K \)-meson gains the less-valued probability of the colliding than the smaller radius \( D \)- (or \( B_s \)-) meson. The various aspects of hadron dynamics seem to be successfully illustrated by the semi-relativistic picture with "Breit-Fermi Hamiltonian"[53]. Assuming the power-law potential \( V(r) \sim r^{\nu} (\nu \text{ is a real number}) \), the radius of \( P^0_{\overline{P}^0} \)-meson (denoted by \( r_P \)) is proportional to \( \mu_P^{-1/(2+\nu)} \), where \( \mu_P \) is the reduced mass of two quark-masses inside \( P^0_{\overline{P}^0} \)-meson[53]. Then the volume of \( P^0_{\overline{P}^0} \)-meson is proportional to \( r_P^3 \sim \mu_P^{3/(2+\nu)} \). After all we could assume for \( \eta(P) \) in Eq.(42) as

\[
\eta(P) = \xi(\mu_P/\mu_K)^{1} \quad \text{for linear – potential},
\]

(44a)

\[
\eta(P) = \xi(\mu_P/\mu_K)^{1.5} \quad \text{for log – potential},
\]

(44b)

where \( \xi \) is a dimensionless numerical factor depending on the details of the dynamics. The \( \eta(P) \) is normalized by \( \mu_K \) (reduced mass of \( s \)- and \( d \)-quark in \( K \) meson) for convenience. We may think that the \( y \)-exchange is described by the overlapping of the wave functions of two quarks inside \( P^0_{\overline{P}^0} \)-meson. Then we write as

\[
|\tilde{M}_l(P)| = (1/\Lambda_q^2)|\kappa_l| \int \Psi_q(r)\Psi_{q'}(r)d^3r|,
\]

(45a)

\[
\simeq |\kappa_l| \int \Psi_q(r)\Psi_{q'}(r)d^3r| \times 10^{-7} \quad \text{GeV}^{-2},
\]

(45b)

where \( \Psi_q(r) \) is a radial wave function of \( q \)-quark, \( \kappa_l \) is a dimensionless complex numerical factor caused by unknown subquark dynamics and may depend on \( |<q'|\Psi(\partial_\mu y)|q >| \). In Eq.(45b) we estimate a few TeV as \( \Lambda_q \).
From the experimental informations the complex $M_{12}(K)$ is evaluated\[36\] as

$$M_{12}^{\exp}(K) = -(0.176 + i0.114 \times 10^{-2}) \times 10^{-14} \text{ Gev.} \quad (46)$$

Then, setting $P=K$ in Eq.(41) we obtain

$$|M_K| = |M_{12}(K)|/\{(1/12\pi^2)B_Kf_K^2M_K\} \simeq 0.15 \times 10^{-10} \text{ GeV}^{-2}, \quad (47)$$

where we use $|M_{12}(K)| = |M_{12}^{\exp}(K)|$ from Eq.(46) by SW-saturating assumption, $B_Kf_K^2 = (0.17\text{GeV})^2$ and $M_K = 0.498 \text{ GeV}$. Further setting $P = K$ in Eq.(42), (44), and (45) we have

$$|M_K| = \xi |\tilde{M}_1(K)| = \xi |\kappa_1 \int \Psi_s(r)\Psi_d(r)d^3r| \times 10^{-7} \text{ GeV}^{-2}, \quad (48)$$

From Eq.(47) and (48) we obtain

$$\xi |\kappa_1 \int \Psi_s(r)\Psi_d(r)d^3r| \simeq 10^{-4} \text{ GeV}^{-2}. \quad (49)$$

If we expect that

$$\int \Psi_s(r)\Psi_d(r)d^3r \simeq O(10^{-2}) \sim O(10^{-1}), \quad (50)$$

we have

$$\xi |\kappa_1| \simeq O(10^{-3}) \sim O(10^{-2}). \quad (51)$$

Eq.(50) and Eq.(51) have to be ascertained in future by using some dynamical model describing the subquark physics. The above investigations are compared to the scheme of the Higgs (of a few TeV mass value) exchange in the SW-theory\[47\][51][54][56][57]\. Note that the mass value of $y$-particle itself is less than a few MeV as seen in Sect.(3).

The present experimental results of $\Delta M_P$ are as follows\[29\] :

$$\Delta M_K = (3.510 \pm 0.018) \times 10^{-15} \text{ Gev}, \quad (52a)$$
$$\Delta M_D < 1.6 \times 10^{-13} \text{ Gev}, \quad (52b)$$
$$\Delta M_{B_d} = (3.05 \pm 0.12) \times 10^{-13} \text{ Gev}, \quad (52c)$$
$$\Delta M_{B_s} > 6.0 \times 10^{-12} \text{ Gev.} \quad (52d)$$

Using Eq.(40), (41) and (52), we have

$$|\mathcal{M}_D| < 8.1|\mathcal{M}_K|, \quad (53a)$$
$$|\mathcal{M}_{B_d}| = 4.92|\mathcal{M}_K|, \quad (53b)$$
$$|\mathcal{M}_{B_s}| > 74.0|\mathcal{M}_K|. \quad (53c)$$
At the level of $\mathcal{M}_P$, it seems that

$$|\mathcal{M}_P|/|\mathcal{M}_K| \simeq O(1) \sim O(100),$$  \hspace{0.5cm} (54)$$

where $P = D, B_d, B_s$.

Here let us go on to more precise investigations. In Eq.(43) assuming that $|\tilde{\mathcal{M}}_1(K)| \simeq |\tilde{\mathcal{M}}_2(B_d)|$ and using Eq.(42), (44) and (53b) we obtain

$$\mu_{B_d}/\mu_K = 4.91 \quad \text{for linear–potential,} \quad (55a)$$
$$= 2.88 \quad \text{for log–potential,} \quad (55b)$$

where $B_{B_d}f_{B_d}^2 = (0.22\text{GeV})^2$, $B_Kf_K^2 = (0.17\text{GeV})^2$ are used. Note that, comparing with the case of Eq.(39), we can evade the large enhancement by $b$-quark mass effect. This is because the quark mass dependence is introduced through the reduced mass (in which the effect of heavier mass decreases). Some discussions are as follows: If we adopt the pure non-relativistic picture it may be that $\mu_K \simeq \mu_{B_d} \simeq m_d \simeq (\mu_D \simeq \mu_{T_u})$ but from the semi-relativistic standpoint it seems preferable that $\mu_K(< \mu_D) < \mu_{B_d}(< \mu_{T_u})$ because the effective mass value of “$d$-quark” in $B_d$-meson is considered larger than that in $K$-meson. It may be caused by that the kinetic energy of “$d$-quark” in $B_d$-meson is larger than that in $K$-meson owing to the presumption: $r_{B_d} < r_K$ where $r_p$ means the radius of $p$-meson (Refer to discussions in Sect.3). Then we can expect the plausibility of Eq.(55). Of course it may be also a question whether $|\tilde{\mathcal{M}}_1(K)| \simeq |\tilde{\mathcal{M}}_2(B_d)|$ is good or not (this point influences Eq.(55)), which will become clear when the experimental result about $\Delta M_{T_u}$ is confirmed in future and compared with $\Delta M_{B_d}$.

Next, let us study $\Delta M_D$. In order to estimate the lower limit of $\Delta M_D^{SW}$ (denoted by $(\Delta M_D^{SW})_{LL}$) we set $\mu_D = \mu_K$ tentatively in Eq.(44) and obtain

$$(\Delta M_D^{SW})_{LL} = 4.67 \times \Delta M_K = 1.6 \times 10^{-14} \quad \text{GeV}, \quad (56)$$

where we use $B_Df_D^2 = (0.19\text{GeV})^2$ and Eq.(40), (41), (42), (43a) and (52a). In the same way, assuming $\mu_D = 1.5 \times \mu_K$ for example and using Eq.(44) we have

$$\Delta M_D^{SW} = (2.9 \sim 5.4) \times 10^{-14} \quad \text{GeV}, \quad (57)$$

the parenthesis means that (linear-potential $\sim$ log-potential). This result is consistent and comparable with Eq.(52b). These values are similar to the results by Cheng and Sher[51] and Liu and Wolfenstein[47].
The study of $\Delta M_{B_s}$ is as follows. Both $s$- and $b$-quark in $B_s$-meson are rather massive and then supposing availability of the non-relativistic scheme we have

$$\mu_{B_s} = m_s m_b / (m_s + m_b) = 0.19 \text{ GeV}, \quad (58)$$

where $m_s = 0.2 \text{ GeV}$ and $m_b = 4.3 \text{ GeV}$ are used. If we adopt $\mu_K = 0.01 \text{ GeV} (\simeq m_d)$ for example we obtain

$$\eta(B_s) = 19.0 \xi \quad \text{ for linear potential,} \quad (59a)$$

$$= 82.8 \xi \quad \text{ for log potential,} \quad (59b)$$

By using Eq.(40b), (41), (42) and (43a) we have

$$\Delta M_{B_s}^{SW} = 2(B_{B_s} f_{B_s}^2 M_{B_s} \eta(B_s) / B_{K} f_{K}^2 M_{K} \eta(K)) \Delta M_{K}^{SW}, \quad (60)$$

where factor 2 comes from $n_{B_s} = 2$ in Eq.(42). Assuming that $\Delta M_{K}^{SW} = \Delta M_{K}^{exp}$ (that is, the SW exchange saturates the $\Delta M_{K}^{exp}$ bound) and using Eq.(59) we obtain

$$\Delta M_{B_s}^{SW} = (0.31 \sim 1.4) \times 10^{-11} \text{ GeV}, \quad (61)$$

where we use $B_{B_s} f_{B_s}^2 = (0.25 \text{GeV})^2[49]$ (the parenthesis means the same as Eq.(57)). From Eq.(52c) and(61) we get

$$\Delta M_{B_s}^{SW} / \Delta M_{B_d}^{SW} = (10 \sim 50), \quad (62a)$$

$$x_s = \tau_{B_s} \Delta M_{B_s} = (8 \sim 30), \quad (62b)$$

where we set $\Delta M_{B_d}^{SW} = \Delta M_{B_d}^{exp}$ and use $\tau_{B_s} = 2.4 \times 10^{12} \text{ GeV}^{-1}[29]$, and the parenthesis means the same as Eq.(57). Note that the present experimental result is $\Delta M_{B_s}^{exp} / \Delta M_{B_d}^{exp} > 20[29]$. If we adopt the box diagram calculation in the SM and use Eq.(38) with the unitary assumption of CKM-matrix elements, it is found that[50][51]

$$\Delta M_{B_s}^{SD} / \Delta M_{B_d}^{SD} = 10 \sim 100. \quad (63)$$

Therefore, from the above studies of $\Delta M_{B_d}$ and $\Delta M_{B_s}$ it is difficult to clarify which scheme (SW or SD in the SM) is true, at least until the future experiments confirm the values of $|V_{ts}/V_{td}|$ and $\Delta M_{B_s}$.

Finally let us estimate $\Delta M_{T_u}^{SW}$ and $\Delta M_{T_c}^{SW}$. Setting $\mu_{T_u} = \mu_{B_d}$ (though $\mu_{T_u} > \mu_{B_d}$ in practice) and using Eq.(40b), (41), (42), (43b) and (44) we estimate the lower limit of $\Delta M_{T_u}^{SW}$ (denoted by $\Delta M_{T_u}^{SW}_{LL}$) as

$$(\Delta M_{T_u}^{SW})_{LL} = (B_{T_u} f_{T_u}^2 M_{T_u} / B_{B_d} f_{B_d}^2 M_{B_d}) \Delta M_{B_d}^{SW} = 7.3 \times 10^{-10} \text{ GeV}, \quad (64)$$
where we use $B_{Tu} f_{Tu}^2 = (1.9\text{GeV})^2[36]$, $M_{Bu} = 5.279 \text{ GeV}$, $M_{Tu} = 171 \text{ GeV}$ and set $\Delta M_{Bu}^{SW} = \Delta M_{Bu}^{exp}$ in Eq.(52c). Note that $|\tilde{M}_2(T_u)| = |\tilde{M}_2(B_d)|$ is used in Eq.(64). Cheng and Sher’s scheme[51] predicts $\Delta M_{Tu} \simeq 10^{-7} \text{ GeV}$ which is order of $10^3$ larger than Eq.(64). (In Ref.[51] they estimated $\Delta M_{Tu} \simeq 10^{-10} \text{ GeV}$ using smaller $t$-quark mass value than 170 GeV). For evaluating $\Delta M_{Tc}$, we calculate

$$\mu_{Tc} = m_c m_t / (m_c + m_t) = 1.34 \text{ GeV}, \quad (65)$$

where $m_c = 1.35 \text{ GeV}$ and $m_t = 170 \text{ GeV}$ are used. Then we get from Eq.(44)

$$\eta(T_c) = \begin{cases} 134 \xi & \text{for linear potential}, \\ 1551 \xi & \text{for log potential}, \end{cases} \quad (66a)$$

where we set $\mu_K = 0.01 \text{ GeV}$ for example. After all with Eq.(66) we obtain

$$\Delta M_{Tc}^{SW} = 2(B_{Tc} f_{Tc}^2 M_{Tc} / B_K f_K^2 M_K) (\eta(T_c) / \eta(K)) \Delta M_{K}^{SW} = (4 \sim 47) \times 10^{-8} \text{ GeV}, \quad (67)$$

where we adopt $n_{Tc} = 2$, $B_{Tc} f_{Tc}^2 = (1.9\text{GeV})^2[36]$, $M_{Tu} = 171 \text{ GeV}$ and $\Delta M_{K}^{SW} = \Delta M_{K}^{exp}$ and the parenthesis means the same as Eq.(57). Note that $|\tilde{M}_1(T_c)| = |\tilde{M}_1(K)|$ is used in Eq.(67).

**b. CP violation in $P^0$-$\bar{P}^0$ mixing**

Presently observed CP violation is only in the Kaon system and are still not inconsistent with the SW-model though there exists the discrepancy between E731 and NA31 experiments concerning $Re(\epsilon'/\epsilon)_K$. If $Re(\epsilon'/\epsilon)_K = 0$ is confirmed by experiments, CKM-mixing matrix has no phase factor[56] and CP violation in $K$-systems can be explained only by the SW-theory. On the other hand there exist several models explaining $Re(\epsilon'/\epsilon)_K \neq 0$(which implies the existence of direct CP violation by decay modes), e.g., only the standard CKM-theory with (or without) the LD-contributions; the SW-interactions coexisting with the SM, etc.. Concerning the heavy mesons no evidence of CP violation has found yet. As widely discussed, CP asymmetries in $B_s \to \psi K_s$ and $B \to 2\pi$ may give us the crucial clues[48][58].

Here we discuss CP violation by mass-mixings which is assumed to be saturated by the SW-interactions. In the CP-conserving limit in the $P^0$-$\bar{P}^0$-meson systems, $M_{12}(P)$s are supposed to be real positive. Note that $CP|P_H >= -|P_H > and
$CP|P_L \geq |P_L >$ where $H \ (L)$ means heavy (light). If the CP-violating SW-interactions are switched on, $M_{12}(P)$ becomes complex. Following Gérard and Nakada’s notation[48][52], we write as

\[ M_{12} = |M_{12}| \exp(i\theta_P), \]

with

\[ \tan \theta_P = \text{Im}M_{12}(P)/\text{Re}M_{12}(P). \]

As we assume that the SW-interaction saturates CP violation, we can write

\[ \text{Im}\langle \overline{P^0}|H_{SW}^{\Delta F=2}|P > = \text{Im}M_{12}(P). \] (70)

From Eq.(40), (41) and (42) we obtain

\[ \text{Im}M_{12}(P) = A \cdot \text{Im}\tilde{M}_i(P), \] (71)

where $A = (1/12\pi^2)B_Pf_P^2M_P\eta(P)$. Therefore the origin of CP violation of $P^0(\overline{P^0})$-meson system is only in $\tilde{M}_i(P)$. The Factor “$A$” in Eq.(71) is common also in $\text{Re}M_{12}(P)$ and then we have

\[ \text{Im}M_{12}(P)/\text{Re}M_{12}(P) = \text{Im}\tilde{M}_i(P)/\text{Re}\tilde{M}_i(P). \] (72)

If the universality of Eq.(43) is admitted, we obtain

\[ \theta_K = \theta_D = \theta_{B_s} = \theta_{T_c}, \]

\[ \theta_{B_d} = \theta_{T_u} \simeq 2\theta_K. \] (73b)

These are the predictions about CP violation from the standpoint of our FB-model.

Concerning Eq.(73b) if two $y$-particles in $B_d$ and $T_u$ (See Fig.(3).) exchange without any correlation, it is possible that CP phases become double of $\theta_K$ and if there exists some correlation they become less than $2\theta_K$.

6 Summary and Discussion

The motivation of our composite model is inspired by the studies about the gauge mechanisms by which four interacting forces are commonly controlled. Namely, all gauge fields are Cartan connections equipped with “Soldering Mechanism”. In case of the electromagnetic gauge field, its gauge symmetry group $G$ (including the habitual $U(1)$ gauge symmetry) is $SL(2,C)$ with six generators, which leads that the
minimal electric charge is \(|e/6|\). The fact that the charges of \(u\)-quark, \(d\)-quark and electron \((|2e/3|, |−e/3|, |−e|)\) are larger than \(|e/6|\) naturally induces the concept of compositeness of quarks and leptons. Following Pati and Salam’s investigation we choose the FB-model (preons are both fermionic and bosonic). Further, learning Hung and Sakurai’s and Bjorken’s thought of the alternative to spontaneously broken unified gauge theories we adopt the idea that the weak interactions at low energies are remnants of the spontaneously unbroken confining forces governing the substructure dynamics of quarks and leptons. \(W\)- and \(Z\)-bosons are also composites of the preons.

As the fundamental confining gauge symmetry we choose \(SU(2)_L \otimes SU(2)_R\) gauge symmetry, which is not the ad hoc assumption but induced from the concept of Cartan connection, that is, \(SU(2) \otimes SU(2)\) is locally isomorphic to \(SO(4)\) which takes part in constructing the homogeneous space: \(F = SO(5)/SO(4)\) in the Cartan-type fiberbundle. The elementary matter fields are only one kind of fermion (\(\Lambda\)) and scalar (\(\Theta\)) (both named “Primon”), belonging to the same fundamental representation of \((3, 2, 2)\) in the \(SU(3)_C \otimes SU(2)_L \otimes SU(2)_R\) gauge group and having the same electric charge “\(e/6\)”. Following Harari and Seiberg’s idea the higher generations are constructed by adding scalar \(y\)-particles without introducing any more freedoms and just this mechanism explains the flavor-mixing phenomena. Namely, the annihilations of \(y\)-particles into two hypercolor gluons occur coincidentally with the composite \(W\)-boson exchange.

Here let us discuss some points. In the stage of this article, the unification of gauge fields are not considered. In fact the insouciant extrapolations of the running coupling constants to the energy of \(10^{19}\) GeV show that \(\alpha_W = 0.040\) and \(\alpha_s = 0.017\) (normalized with \(\alpha_s = 0.12\) at \(10^2\) GeV) and then they have no crossing point. But it seems to be dangerous to require the matching of them as the GUT scenario in which quarks and leptons are the elementary fields, because if we take a stand point of the composite model we have too few informations in the energy range of \(10^2\) GeV to \(10^{19}\) GeV to understand the dynamics of that energy range. If we pursue the unification of the gauge symmetries, such gauge group must contain not \(U(1), SU(2)\) and \(SU(3)\) but \(SO(1, 4), SU(3), SL(2, C)\), and \(SO(5)\).

Concerning the flavor changing neutral current(FCNC) the SM usually explains it by the GIM mechanism. In our model the vanishing FCNC may occur by the interference between two amplitudes to which \(Z_1^0\) and \(Z_2^0\) contribute with simultaneously occurring \((y \rightarrow 2g_h)\)-process. Namely the phases of two amplitudes are almost inverse. The dynamical origin of such mechanisms has to be studied in future.

The \(y\)-subquarks which are responsible for constructing the higher generations carry
the hypercolor charge and then \((b \rightarrow s\gamma)\)-process cannot occur in the subquark level. The \((\mu \rightarrow e\gamma)\)-process also cannot.

As for the leptonic flavor-mixings, if they exist, they are originated from \((y \rightarrow 2g_h)\)-process as the cases of quarks in our model. Maki, Nakagawa and Sakata first proposed the idea of “flavor-changing neutrino oscillation” on the assumption that neutrinos have small masses[62]. Recently the observations of the atmospheric neutrino at Super Kamiokande suggested the possibility of \((\nu_\mu, \nu_\tau)\)-oscillation with large mixing angle[63] and it is reported that

\[
\Delta M^2 (= |M^2_{\nu_\tau} - M^2_{\nu_\mu}|) \simeq 10^{(-4\sim-3)} \text{ eV}^2, \tag{74a}
\]

\[
\sin^2 2\theta_{\mu\tau} \geq 0.82. \tag{74b}
\]

From Eq(74a) we can evaluate \(M_{\nu_\mu} \simeq \Delta M \simeq 10^{(-2\sim-1.5)} \text{ eV}\). Combining this results with our discussions of Eq.(15) in Sect.3 about the slope parameter of up-leptonic sector, that is, \(M_{UL} \sim 10^{2n}\) it is suggested that \(M_{\nu_\mu} \simeq 10^{(-4\sim-3.5)} \text{ eV}\) and \(\Delta M^2 (= |M^2_{\nu_\mu} - M^2_{\nu_e}|) \simeq 10^{(-8\sim-7)} \text{ eV}^2\). Concerning mixing angles, \((y \rightarrow 2g_h)\)-processes may be thought to play the same role in both \((\nu_e, \nu_\mu)\)- and \((\nu_\mu, \nu_\tau)\)-oscillation, namely they occur by creation or annihilation of one \(y\)-particle and then it is expected that \((\nu_e, \nu_\mu)\)-oscillation occurs with large mixing angle. About \((\nu_e, \nu_\tau)\)-oscillation the probability may become less because it occurs by creation or annihilation of two \(y\)-particles. Above suppositions must be compared with the solar neutrino experiments.

In our model the existence of the 4th generation is, in kind, not inhibited because the generation-making mechanism is just to add \(y\)-subquarks. In fact, if the experimental evidence of \(1-|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2|=0.0017 \pm 0.0015\) at the 1\(\sigma\) level[31] is taken seriously[30], the possibility of the 4th generation is not to be said nothing. But whether the 4th generation really exists or not may depend on the details of the substructure dynamics, that is, the possibility of the existence of the dynamical stable states with the addition of three \(y\)-subquarks : namely, whether the sum of the kinetic energies of the constituent subquarks may balance to the binding energy to form the stable states, or not. If the non-existence of the 4th generation is finally confirmed, that fact will offer one of the clue to solve the substructure dynamics. Referring Eq.(14), it predicts \(M_\nu \simeq 110\text{ GeV}\) and \(M_{\tau'} \simeq 30\text{ GeV}\) for \(n = 4\).

Concerning CP violation we know the experimental result[47] as

\[
\theta_K = (6.5 \pm 0.2) \times 10^{-3}. \tag{75}
\]
Therefore if this FB-model is admissible and Eq.(73) is the case, the indirect CP violation of other mesons are also very small and difficult to observe. But as Gérard and Nakada[48] and Soares and Wolfenstein[58] have pointed out, the measurements of asymmetries of $B \to \psi K$ and $2\pi$ decays will distinguish the standard CKM-model from the SW-model. For the same purpose it is hoped to carry out the precise measurements of $\Delta M_D$; the dilepton charge asymmetry and also the total charge asymmetry of $D^0-\bar{D}^0$ system, which surely discriminate which model is true one. If the future experiments confirm that $\Delta M_D \simeq 10^{-14} \sim 10^{-13}$ GeV and $\theta_K \simeq \theta_D \simeq \theta_{B_s} \simeq (1/2)\theta_{B_d}$, it could be said that the subquark-level physics in TeV energy region totally controls $\Delta M_P$ and the indirect CP violation.

To conclude, we have discussed the possibility that the subquark dynamics play the essential role in all $\Delta F = 1, 2$ flavor-changing phenomena.

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**Figure Caption**

Fig.(1) Subquark-line diagrams of the weak interactions.

Fig.(2) *The*($y \to 2g_h$)-process by primon-level diagram.

Fig.(3) Schematic pictures of $P^0 - \overline{P^0}$ mixings by $y$-exchange interactions.
Fig.(1)

Subquark-line diagrams of the weak interactions
Fig. (2)

The \((y \rightarrow 2g_h)\)-process by prion-level diagram

\[ \Lambda (\Theta) \quad g_h \]

\[ y \]

\[ \Lambda (\Theta) \quad g_h \]
Fig. (3)
Schematic illustrations of $P^0$-$\overline{P^0}$ mixings by $y$-exchange interactions

$K^0$ ($D^0$)

$d$ ($\overline{c}$)  \hspace{2cm} $\bar{b}$ ($t$)

$B_s^0$ ($T_c^0$)

$s$ ($\overline{c}$)  \hspace{2cm} $d$ ($\overline{u}$)

$B_d^0$ ($T_u^0$)

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