Assessment of the state of the technical system in the presence of parametric perturbations and the absence of information about the input actions

D N Demyanov

Kazan Federal University, Naberezhnye Chelny Institute, 423812, Russia, Naberezhnye Chelny, Prospekt Syuyumbike 10A
demyanovdn@mail.ru

Abstract. The paper deals with the problem of assessment of the state of a technical system described by first-order differential equations whose parameters change over time according to a certain law. The conditions of solvability of the synthesis problem in terms of matrix canonization technology are formed. An algorithm is proposed for calculating the coefficients of the state observer of the luenberger type, providing an asymptotic tendency to zero of the assessment error.

1. Introduction

It is known that one of the most important conditions for the effective functioning of complex technical systems is a regular monitoring of their state[1]. Most often, it is possible to assess the state with a sufficient degree of accuracy by processing the readings of sensors or statistical analysis of performance characteristics. However, in a number of cases, these methods are not effective enough for a number of technical and economic reasons, therefore so-called state observers are used to evaluate the state of technical systems [2]. At present, the theory of state observers for systems described by first-order linear differential equations is well developed and is set out in the relevant literature. In this case, the problems of designing state observers for nonlinear or non-stationary dynamical systems are much worse and contain a large number of unresolved problems [3, 4], the solution of which is a very urgent problem.

2. Statement of the problem

Let the mathematical model of the considered technical system be given by equations in the state space:

\[
\begin{align*}
    x_1 &= A_{11}x_1 + A_{12}x_2 + vF_{11}x_1 + vF_{12}x_2 + Bu; \\
    x_2 &= A_{21}x_1 + A_{22}x_2 + vF_{21}x_1 + vF_{22}x_2 + Bu; \\
    y &= x_1.
\end{align*}
\]

Here, \(x_1, x_2\) are the elements of the state vector of the system under consideration; \(u\) is the input effect; \(y\) is output signal; \(v\) is a time function that determines the law of changing the parameters of the system; \(A_{11}, A_{12}, A_{21}, A_{22}, F_{11}, F_{12}, F_{21}, F_{22}\) are numerical matrices of the coefficients.

It is assumed that the output signal is accessible to measurement with a high degree of accuracy,
the function \( v \) is known, the vectors \( x_2 \) and \( u \) can not be measured.

From the substantial point of view, the formulated conditions represent the description of a linear dynamic system with parametric disturbances (matrix coefficients of the equation of dynamics change over time by a certain law). The most obvious example is the wear and tear of elements of the technical system during operation.

It is necessary to synthesize the observer of the state of the dynamic system (1), which allows for known \( y \) and \( v \) to form a vector of evaluation \( z \) such that:

\[
\lim_{t \to \infty} e(t) = \lim_{t \to \infty} \left[ z(t) - x_2(t) \right] = 0.
\]  (2)

### 3. Synthesis of the equation of the observer

Let's compose the equation of the observer on the basis of the second equation of the system (1), having supplemented it with the component proportional to a difference of an assessment and true value of a vector \( x_2 \):

\[
\dot{z} = A_{22} y + A_{22} z + vF_{22} y + vF_{22} z + B_2 u + L \left( A_{12} z + vF_{12} z - A_{12} x_2 - vF_{12} x_2 \right).
\]  (3)

Substituting in equation (3) the true values of the terms \( A_{12} x_2 \) and \( vF_{12} x_2 \) from the first equation of the system (1), we obtain:

\[
\dot{z} = \left( A_{22} + LA_{12} \right) z + v \left( F_{22} + LF_{12} \right) y + \left( F_{21} + LF_{11} \right) y + \left( B_2 + LB_1 \right) u - Ly.
\]  (4)

The equation of dynamics of the error of estimation of the vector \( x_2 \) can be obtained by subtracting from the equation (3) the second equation of the system (1):

\[
\dot{\varepsilon} = \left( A_{22} + LA_{12} \right) \varepsilon + v \left( F_{22} + LF_{12} \right) \varepsilon.
\]  (5)

In general, the solution of equation (5) depends on function \( v \), and the law of change of the estimation error can be quite complicated. However, the problem can be significantly simplified by choosing a matrix of observations from the condition:

\[
F_{22} + LF_{12} = 0.
\]  (6)

In this case, the equation of the dynamics of the estimation error will look like the following:

\[
\dot{\varepsilon} = \left( A_{22} + LA_{12} \right) \varepsilon.
\]  (7)

If the pair \((A_{22}, A_{12})\) is completely observable by Kalman, then the corresponding choice of the matrix \( L \) can ensure the asymptotic tending to zero of the estimation error, that is, the condition (2) is satisfied.

According to the accepted restrictions, the input signal \( u \) is inaccessible to measurement, therefore it should also be excluded from the dynamics equation of the observer. For this it is sufficient to ensure the following condition:

\[
B_2 + LB_1 = 0.
\]  (8)

When constraints (6) and (8) are satisfied, equation (4) is essentially simplified and takes the following form:

\[
\dot{z} = \left( A_{22} + LA_{12} \right) z + \left( A_{21} + LA_{11} \right) y + \left( F_{21} + LF_{11} \right) y - Ly.
\]  (9)

It should be noted that equation (9) includes a component proportional to the derivative of the output signal. At the same time, it is known that in practice, the operation of differentiation, as a rule, leads to the appearance of additional errors and it is recommended to avoid it. To do this, we modify the calculation scheme by introducing an auxiliary variable \( w(t) \):

\[
w(t) = z(t) + Ly(t).
\]  (10)
We rewrite equation (9) taking into account relation (10):
\[
\dot{w} = (A_{22} + LA_{21})\dot{w} + (A_{21} + LA_{12} - A_{22}L - LA_{21}L)y + v(F_{21} + LF_{11})y.
\]
(11)
The equation to estimate the vector \( x_2 \) will look like the following:

\[
z(t) = w(t) - Ly(t). \quad (12)
\]

Thus, in the case of the observability of the pair \((A_{22}, A_{12})\) under the conditions (6) and (8), it is possible to make the observer equations ensuring the asymptotic tending to zero of the estimation error of the unmeasured component of the state vector of the technical system (1).

4. Analysis of solvability conditions

We define the conditions under which the observer equations have the form (9) or (11) – (12). To do this, we combine the conditions (6) and (8), presenting them as one linear matrix equation:

\[
K_1 + LK_2 = 0.
\]
(13)

Here is denoted: \( K_1 = [F_{22} \quad B_2] \); \( K_2 = [F_{12} \quad B_1] \).

The solution of equation (13) exists if condition [5] is satisfied:

\[
K_1^R K_2^L = 0. \quad (14)
\]

Here and in what follows, for an arbitrary matrix \( M \), the symbols \( \tilde{M}^R \) and \( \tilde{M}^L \) will denote non-zero matrices of maximal rank such that \( \tilde{M}^R \tilde{M}^L = 0 \) and \( \tilde{M}^L M = 0 \) respectively. In addition, we define a matrix construction \( \tilde{M} = \tilde{M}^R \tilde{M}^L \) in which \( \tilde{M}^R \) and \( \tilde{M}^L \) are matrices of maximal rank such that \( \tilde{M}^L \tilde{M}^R = I \). Detailed information on the properties of these matrices and a description of the algorithm for their calculation are given in the paper [5].

In the general case, the solution set of equation (13) is given by the formula:

\[
\{L\}_q = -K_1 \tilde{K}_2 + \eta \tilde{K}_2^L.
\]
(15)

Here \( \eta \) is an arbitrary numerical matrix of the corresponding dimension.

We rewrite the matrix of coefficients from the right-hand side of equation (7), taking into account condition (15):

\[
A_{22} + LA_{12} = A_{22} - K_1 \tilde{K}_2 A_{12} + \eta \tilde{K}_2^L A_{12}.
\]

Let's introduce new symbols:

\[
A^* = \left( A_{22} - K_1 \tilde{K}_2 A_{12} \right)^T; \quad B^* = \left( \tilde{K}_2^L A_{12} \right)^T.
\]
(16)

Then the condition (2) is satisfied by the required arrangement of eigenvalues of the matrix \( A^* + B^* \eta \), the setting of which by the matrix selection \( \eta \) is a classical problem of modal control.

Thus, the solution of the problem is possible if the condition (14) is satisfied, and the pair \((A^*, B^*)\) defined by formulas (16) is fully controlled by Kalman.

5. Synthesis algorithm

Summarizing the results obtained, we formulate an algorithm for calculating the observer’s coefficients for the state of the dynamic system in the presence of parametric perturbations and the lack of information on input effects.

1. To set the numerical values of the matrices of coefficients of model (1), the desired distribution
of the poles of the observer, information about the permissibility or non-permissibility of differentiation of the output signal.

2. To verify the fulfillment of condition (14). If this is done, you can proceed to the next step. Otherwise, the proposed method is not applicable.

3. To calculate the matrix of coefficients by formulas (16) and estimate the controllability of the pair \((A^*, B^*)\). If it is fully controllable, then you can proceed to the next step. Otherwise, it is recommended to change the requirements for the dynamics of the observer or use a different calculation algorithm.

4. Solve the problem of modal control by providing the choice of a matrix \(\eta\) with the required arrangement of the eigenvalues of the matrix \(A^* + B^*\eta^T\).

5. To calculate a matrix of observations according to the formula (15).

6. To make the equations of the observer. If differentiation of the output signal is allowed, then equation (9) is used. Otherwise, the equation of the dynamics of the observer is given by (11), and the expression for estimating the vector \(x_2\) is given by formula (12).

The end of algorithm.

6. Conclusions

A new method of assessing the state of the technical system in the presence of parametric perturbations and the absence of information about the input actions is proposed. The solvability conditions of the problem and a step-by-step algorithm for calculating the coefficients of the observer providing an asymptotic tendency to zero evaluation errors are formulated. The obtained results can be used in practice to build diagnostic systems of technical objects whose characteristics change over time according to the known law.

7. Acknowledgment

The work is executed at financial support of RFBR (grant No. 17-08-00516).

References

[1] Mironovskij L A 1998 *Functional diagnostics of dynamic systems* (SPb.: Nauchnoe izdanie) p 256
[2] Kuzovkov N T 1976 *Modal control and monitoring devices* (M.: Mashinostroenie) p 184
[3] Zheng G, Boutat D, Wang H 2017 *A nonlinear Luenberger-like observer for nonlinear singular systems* Automatica Vol 86 pp 11 – 17
[4] Asanov A Z, Dem'yanov D N 2017 *Analytical synthesis of the functional observer of the state of bilinear dynamical system* Avtometriya No 4 pp 26–34
[5] Bukov V N 2006 *Systems investment. Analytical approach to the analysis and synthesis of matrix systems* (Kaluga: Publ. N.F. Bochkareva) p 720