Medium effects in $K^+$ nucleus interaction from consistent analysis of integral and differential cross sections

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Abstract

Self consistency in the analysis of transmission measurements for $K^+$ on several nuclei in the momentum range of 500-700 MeV/c is achieved with a $t_{eff}(\rho)\rho$ potential and new results are derived for total cross sections. The imaginary part of the $t_{eff}$ amplitude is found to increase linearly with the average nuclear density in excess of a threshold value of $0.088 \pm 0.004$ fm$^{-3}$. This phenomenological density dependence of the $K^+$ nucleus optical potential also gives rise to good agreement with recent measurements of differential cross sections for elastic scattering of 715 MeV/c $K^+$ by $^6$Li and C.

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Total cross sections for the interaction of 500-700 MeV/c K\(^+\) with several nuclei were derived from transmission experiments performed in recent years \([1-4]\). Since in this energy range the KN interaction is relatively weak and does not vary strongly with energy, one expects that optical potentials close to the ‘\(t\rho\)’ approximation will be adequate. Consequently such potentials indeed were used for extracting total cross sections from the data \([1-4]\). However, attempts to reproduce these total cross sections with optical potentials, including in addition various degrees of conventional nuclear physics sophistication beyond the \(t\rho\) starting point, invariably gave rise \([3]\) to calculated values smaller than the measured cross sections. This as well as earlier indications led to speculations about nonconventional medium modifications of the KN interaction \([6-10]\). Very recently the same transmission experiments \([4]\) were re-analyzed \([11]\) to yield reaction cross sections, for the first time, and also revised values for the total cross sections.

The derivation of reaction (\(\sigma_R\)) and total (\(\sigma_T\)) cross sections from transmission measurements involves the use of an optical potential \(V_{opt}\). An obvious question of self consistency is whether these derived \(\sigma_R\) and \(\sigma_T\) values are consistent with values predicted by \(V_{opt}\), or more generally, do optical potentials which are constructed to fit \(\sigma_R\) and \(\sigma_T\) values lead to the same values (within errors) when used in a re-analysis of transmission experiments. The answer, unfortunately, has been negative so far. As an example we quote results for Si at 714 MeV/c; the derived reaction and total cross sections are \([11]\) 317.5±3.6 and 396.5±2.3 mb respectively whereas the corresponding calculated values, based on a \(t\rho\) potential, are 253 and 328 mb respectively. Attempting to fit the data for \(^6\)Li, C, Si and Ca with \(t_{eff}\rho\), or with density-dependent potentials that include higher powers of \(\rho\), generally reduces the values of \(\chi^2\) per point from a few hundreds to a few tens. However, if any of these modified potentials is used in reanalyses of the transmission measurements then the process diverges. It is therefore obvious that the question of self consistency is essential to the derivation of integral cross sections from transmission measurements and to the proper handling of systematic errors. In the present work we report on an empirical specific density dependent potential that achieves excellent fits to the measured reaction and total cross sections within a fully
consistent picture. In addition it fits well recent measurements [12] of angular distributions
for the elastic scattering of 715 MeV/c K\(^+\) by \(^6\)Li and C.

The interaction of K\(^+\) with nuclei is described in the present work by the Klein Gordon
equation

\[
\left[ \nabla^2 + k^2 - \left( 2\varepsilon_{\text{red}}(A) V - V_c^2 \right) \right] \psi = 0 \quad (\hbar = c = 1) \tag{1}
\]

where \(k\) and \(\varepsilon_{\text{red}}(A)\) are the wave number and reduced energy respectively in the c.m. system,
\((\varepsilon_{\text{red}}(A))^{-1} = E_p^{-1} + E_t^{-1}\) in terms of the c.m. energies for the projectile and target particles,
respectively. \(V_c\) is the Coulomb potential due to the charge distribution of the nucleus, and
\(V = V_c + V_{\text{opt}}\). The simplest possible \(t\rho\) form was chosen for the optical potential, namely

\[
2\varepsilon_{\text{red}}(A) V_{\text{opt}}(r) = -4\pi F_k b_0 \rho(r), \quad F_k = \frac{M_A \sqrt{s}}{M(E_t + E_p)}, \tag{2}
\]

where \(F_k\) is a kinematical factor resulting from the transformation of amplitudes between the
KN and the K\(^+\) nucleus c.m. systems and \(b_0\) is the value of the KN scattering amplitude in
the forward direction. \(M\) is the free nucleon mass, \(M_A\) is the mass of the target nucleus, \(\sqrt{s}\) is
the total projectile-nucleon energy in their c.m. system and the nuclear density distribution
\(\rho(r)\) is normalized to \(A\), the number of nucleons in the target nucleus. Another choice of
kinematics and wave equation will be mentioned below.

It was shown in [11] that by comparing the ratios of measured to calculated \(\sigma_R\) and \(\sigma_T\)
values (based on \(t\rho\) potentials) for C, Si and Ca to the corresponding ratios for \(^6\)Li, a clear
dependence on the nucleus emerges that appears to be independent of beam momentum. An
alternative way of displaying the dependence of \(\sigma_R\) and \(\sigma_T\) on the nuclear medium, beyond
the obvious dependence on \(A\) and \(\rho\) implied by eqs.(1) and (2), is to fit the parameter \(b_0\) of
\(V_{\text{opt}}\) to the \(^6\)Li data and then use this potential to calculate reaction and total cross sections
for the other, much denser nuclei. Figure 1 shows ratios of experimental to calculated
integral cross sections for C, Si and Ca. These ratios deviate considerably from the value of
one in a way which is largely independent of the beam momentum.

The choice of as light a nucleus as \(^6\)Li to serve as a basis for studying K meson nuclear
medium effects in terms of optical potentials might appear questionable. Nevertheless, the
use of a potential can be bypassed at the relatively high energies encountered here by using
Glauber’s eikonal multiple scattering expansion. For example, in very light nuclei the bulk
contribution to the total cross section comes from single scattering, i.e. \( \sigma_T \sim A\sigma \), where
\( \sigma \) is the KN total cross section. For the double scattering contribution at 714 MeV/c we
calculate for \(^6\)Li a value of \(-5 \pm 2\) mb compared to the single scattering contribution of
84 mb. The uncertainty of \(\pm 2\) mb reflects uncertainty in the value of \(b_0\) (particularly its
real part) and modifications due to nuclear center of mass motion and Pauli and shell model
correlations which provide the longest range two-body nuclear correlations. The neglect of
1/A corrections involved in using an eikonalized optical potential to approximate the multiple
scattering expansion does not apply to the dominant single scattering term. Consequently
it amounts to only about 1 mb in the present case. Thus we estimate that the use of an
optical potential at these energies for forward K\(^+\) \(^6\)Li scattering does not introduce errors
larger than about 2\%. This is considerably smaller than the medium effects in the heavier
nuclei discussed in the present work.

Since integral cross sections are sensitive particularly to the imaginary part of the optical
potential, we have tried in a first step to see whether a modification of the imaginary part of
a \(t_{\text{eff}}\rho\) optical potential, via multiplication by a rescaling factor, separately for each target
nucleus, is capable of providing good fits to the data. Excellent agreement with the C, Si and
Ca data was achieved when the imaginary potential was increased by 17-25\%, depending
on the nucleus, on top of the increase of 5-15\% in \(ImV_{\text{opt}}\), with respect to the free KN
interaction, needed to fit the \(^6\)Li data as described above. Moreover, this target-dependent
rescaling factor turned out to be independent of the beam momentum. Next we proceeded
to replace this \textit{ad hoc} approach by a more general model that depends on parameters of the
target nuclei. We failed to reach good agreement with the data by using the local density
in modifying the potential. However, it was possible to achieve good fits to the data by
correlating \(ImV_{\text{opt}}\) with the \textit{average} nuclear density, defined as follows:

\[
\bar{\rho} = \frac{1}{A} \int \rho^2 d\mathbf{r}. \tag{3}
\]
Values of $\rho$ can be obtained quite reliably from Hartree-Fock calculations or from simpler single-particle calculations \cite{13,14} that are constrained by r.m.s. radii of nuclear charge distributions. The results for the nuclei considered here are 0.049, 0.103, 0.110 and 0.107 fm$^{-3}$ for $^6$Li, C, Si and Ca respectively, with an uncertainty of about $\pm 1\%$ for the last three cases. The enhancement of $ImV_{opt}$ was then assumed to take place above a threshold average density $\rho_c$, by multiplying the imaginary part of the conventional potential eq.(2) by the rescaling factor $1 + \beta(\rho - \rho_c)\Theta(\rho - \rho_c)$, and a fit was made at each momentum varying the parameters $b_0$, $\beta$ and $\rho_c$. The last two parameters were found to have the same values at all four beam momenta. Table I shows that excellent fits are obtained within this linear rescaling model. In contrast, attempting to replace the threshold function $\Theta(\rho - \rho_c)$ by a power of the average density failed to produce fits to the data. The average values of $\rho_c=0.088\pm0.004$ fm$^{-3}$ and $\beta=13.0\pm3.4$ fm$^3$ were used in these calculations. This value of $\rho_c$ is considerably larger than $\rho(\!^6\!\text{Li})$ so that the $^6$Li data are fitted, as above, merely by adjusting the parameter $b_0$. Note that the inequality $\rho(\!^{\text{Ca}}\!)>\rho(\!^\text{C}\!)$ (see above), which implies stronger rescaling of $ImV_{opt}$ for Ca than for C, does not translate into larger $\sigma(\text{exp.})/\sigma(\text{calc.})$ values (see fig.1) for Ca than for C since multiple scattering is more effective in the former. The departure of the fitted (medium) KN forward scattering amplitude $b_0$ from its free-space value (given in parentheses in table I) is worth a comment. $Imb_0$ is found to gradually increase with energy up to 16% above its free-space value. This increase is quantitatively consistent with the estimates of García-Recio et al. \cite{10} of the contribution due to meson exchange currents. However, we have no clues for the theoretical significance of $\rho_c$ in providing a threshold density above which new reaction channels in the K-nucleus interaction open up, beyond those accounted for by the reactive content of the impulse approximation. As for $Reb_0$, we observe from table I a clear tendency for less repulsive KN interaction with increasing energy than that implied by the free space values. Such additional attraction could be expected due to the proximity of the $K^*N$ and $K\Delta$ channels.

The next question is that of self consistency. With the above enhancement of $ImV_{opt}$ we re-analyzed all the transmission measurements and obtained new values for $\sigma_R$ that almost
agree, within errors, with the previous values based on a $t\rho$ potential \[11\]. Values of $\sigma_T$ are higher a little more (3-5\%) than the previous ones \[11\], as expected on grounds of the extent of theoretical input involved in the analysis of transmission measurements \[11,15\].

Fits to these new $\sigma_R$ and $\sigma_T$ values left the parameters of table \[1\] essentially unchanged and another round of re-analysis of the transmission measurements showed that full convergence had been achieved, at all four beam momenta. The parameters in table \[1\] correspond to the converged, self consistent values of $\sigma_R$ and $\sigma_T$.

A further test of the whole procedure is now possible at the highest momentum thanks to the very recent publication of experimental differential cross sections for K$^+$ elastic scattering at 715 MeV/c on $^6$Li and C \[12\]. These angular distributions were therefore used in optical model fits, with a variety of potentials including the one of table \[1\]. The normalization of the data was allowed to vary too, within the quoted range of $\pm$15\%. The best fits to the elastic scattering data without rescaling $ImV_{opt}$ achieved reasonable fits to the angular distributions but with $\chi^2$ per point of about 20 for the reaction and total cross sections (for $^6$Li, C, Si and Ca) at this momentum. In contrast, when $ImV_{opt}$ was rescaled as in table \[1\], values of $\chi^2$ per point for the reaction and total cross sections were in the range of 1.6 to 2.0. The normalization factor for the angular distribution data was well determined at the acceptable value of 0.92$^{+0.05}_{-0.05}$. Figure \[2\] shows the good agreement between calculated and measured angular distributions. The potential of fig.\[2\] was then used in a further re-analysis of the transmission measurements at 714 MeV/c to yield yet another set of reaction and total cross sections, based this time on potentials that fit also the available angular distributions.

Table \[1\] summarizes $\sigma_R$ and $\sigma_T$ values as obtained from transmission measurements using (a) the $t\rho$ potential (eqs.\[1\] and \[2\]) and (b), (c) two potentials incorporating the $\Theta(\bar{\rho} - \rho_c)$ rescaling, where (c) is also constrained by fits to differential cross sections for elastic scattering. For (b) and (c) the process converges, which is not the case for (a). Values of $\sigma_T$ are larger by 3-5\% than those found in non self-consistent analyses. Values of $\sigma_R$ remain practically unchanged.

One final comment on the kinematics and wave equation is in order. Since there is no
agreed relativistic wave equation to be used with a finite mass target once a potential is introduced, we have checked the dependence of our results on the wave equation and kinematics by repeating all the calculations with the Goldberger-Watson equation [16]. Details do change but the overall picture remains unchanged. In particular, there is a threshold average nuclear density of about 0.09 fm$^{-3}$ above which the imaginary potential must be rescaled in order to achieve agreement between calculated and measured integral cross section within a consistent analysis. A major advantage of using the wave equation (1) with the potential (2) is that it yields in the eikonal approximation an attenuation factor which agrees with the semiclassical expression $(\sigma \rho)^{-1}$ for the mean free path. This ensures the correct reactive content of the procedure adopted here.

To summarize, empirical rescaling of the imaginary part of the optical potential with the average nuclear density in excess of a threshold value solves the long standing problem of inconsistencies in the extraction of reaction and total cross sections from transmission measurements for the interaction of 500-700 MeV/c K$^+$ with nuclei. The self consistent values of total cross sections are 3-5% larger than previously published values. This potential is also very successful in reproducing the available angular distributions of elastically scattered K$^+$ by $^6$Li and C. The empirical finding that the absorptive part of $V_{opt}$ increases with the average nuclear density above a threshold value $\rho_c$, may signal that novel degrees of freedom are excited in K$^+$ nucleus interactions.

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FIGURES

FIG. 1. Ratios between experimental and calculated cross sections for calculations based on fits to $^6$Li. Full squares represent $\sigma_R$ and open circles represent $\sigma_T$.

FIG. 2. Fits to angular distributions for elastic scattering of 715 MeV/c K$^+$ by $^6$Li and C (see text).
TABLES

TABLE I. Fits to $K^+$ nucleus $\sigma_R$ and $\sigma_T$ values obtained by rescaling $ImV_{opt}$ by the factor $1 + \beta(\overline{\rho} - \rho_c)\Theta(\overline{\rho} - \rho_c)$ with $\rho_c=0.088$ fm$^{-3}$, $\beta=13.0$ fm$^3$. Values in parentheses are for the free KN interaction.

| $p$(MeV/c) | $Reb_0$(fm)     | $Imb_0$(fm)     | $\chi^2/N$ |
|------------|-----------------|-----------------|------------|
| 488        | -0.154±0.012    | 0.160±0.002     | 0.6        |
|            | (-0.178)        | (0.153)         |            |
| 531        | -0.119±0.012    | 0.186±0.002     | 1.2        |
|            | (-0.172)        | (0.170)         |            |
| 656        | -0.035±0.062    | 0.241±0.002     | 0.2        |
|            | (-0.165)        | (0.213)         |            |
| 714        | -0.044±0.064    | 0.265±0.001     | 1.0        |
|            | (-0.161)        | (0.228)         |            |
TABLE II. K$^+$ nucleus reaction and total cross sections derived from transmission measurements at 714MeV/c, based on potentials (a), (b) and (c) (see text).

| potl. | $\sigma_R$ (mb) | $\sigma_T$ (mb) |
|-------|-----------------|-----------------|
|       | Li      | C      | Si      | Ca      | Li       | C      | Si      | Ca      |
| (a)   | 80.7±1.2 | 151.8±1.5 | 318.7±3.6 | 413.7±5.5 | 86.8±0.6 | 177.4±0.9 | 392.0±2.3 | 523.2±2.8 |
| (b)   | 82.2±1.2 | 152.8±1.5 | 320.2±3.6 | 417.1±5.5 | 88.5±0.6 | 183.8±0.9 | 411.3±2.3 | 550.4±2.8 |
| (c)   | 82.1±1.2 | 150.8±1.5 | 317.3±3.6 | 416.8±5.5 | 89.1±0.6 | 183.1±0.9 | 411.9±2.3 | 554.7±2.8 |
$\log_{10} d\sigma/d\Omega$ (mb/sr) vs c.m. angle (deg.)

$K^+ 715$ MeV/c

$^6$Li (x1/10)

$^6$Li (x1/10)