INTRODUCTION

Non-diffracting Bessel beams are a set of solutions of the free space Helmholtz equation and have transverse intensity profiles that can be described by the Bessel functions of the first kind. Since their discovery in 1987, they have exhibited many interesting properties such as non-diffraction, self-reconstruction and even providing optical pulling forces. The scalar form of Bessel beams propagating along the z axis can be described in cylindrical coordinates (r, θ, z) by:

$$E(r, \theta, z) = A \cdot \exp(ik_z z) \cdot J_n(k_r r) \cdot \exp(\pm \imath n \phi)$$  \hspace{1cm} (1)

where A is the amplitude, k_r and k_z are the corresponding longitudinal and transverse wavevectors that satisfy the equation \(\sqrt{k^2_r + k^2_z} = k = \frac{\lambda}{\sin \theta}\) (where \(\lambda\) is the wavelength). Equation (1) shows that the transverse intensity profiles of Bessel beams are independent of the z coordinate, which gives rise to their non-diffracting characteristic. It also indicates that any higher-ordered Bessel beam (\(n \neq 0\)) must carry orbital momentum and have zero intensity along the z axis at \(r = 0\) because of the phase singularity resulting from the \(\exp(\pm \imath n \phi)\) term.

Ideal Bessel beams are not spatially limited and carry infinite energy; therefore, they can only be approximated within a finite region by the superposition of multiple plane waves. This can be achieved by symmetrically refracting incident plane waves toward the optical axis of a conical prism, an axicon, to generate a \(J_0\) Bessel beam. Figure 1a shows the schematic diagram of a conventional axicon. The numerical aperture (NA) of an axicon is related to the angle \(\alpha\) (Figure 1a) by:

$$\text{NA} \equiv \sin(\theta) = \sin^{-1}\left(n \cdot \sin(\alpha) \right)$$  \hspace{1cm} (2)

where \(n\) is the refractive index of the constituent material, often glass. This equation and Figure 1a show that for a given refractive index, achieving high NA axicons requires an increase in \(n\). Considering a refractive index of 1.5, typical of most silica glasses, total internal reflection occurs when \(\alpha > 42^\circ\). Thus, the NA of a conventional axicon cannot exceed 0.75 (Supplementary Fig. S1). This, in turn, also limits the minimum achievable full width at half maximum (FWHM) of the Bessel beam. In addition, the tip of a refractive axicon is rounded instead of being perfectly sharp because of limitations in glass polishing, which again affects the FWHM of the Bessel beam. Herein, the FWHM of the zeroth-order Bessel beam \(J_0\) is defined as the distance between two points at half of the maxima intensity of the center bright spot, and can be derived from Equation (1) as:

$$\text{FWHM}_{J_0} = \frac{2.25}{k_r} = \frac{0.3582}{\text{NA}}$$  \hspace{1cm} (3)

where \(k_r = \frac{2\pi}{\lambda} \cdot \text{NA}\). Similarly, the FWHM of a \(J_1\) Bessel beam is defined as twice the distance from the dark spot center to the point at its closest ring with the half maxima intensity, and given by:

$$\text{FWHM}_{J_1} = \frac{1.832}{k_r} = \frac{0.2926}{\text{NA}}$$  \hspace{1cm} (4)

Generation of wavelength-independent subwavelength Bessel beams using metasurfaces

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Bessel beams are of great interest due to their unique non-diffractive properties. Using a conical prism or an objective paired with an annular aperture are two typical approaches for generating zeroth-order Bessel beams. However, the former approach has a limited numerical aperture (NA), and the latter suffers from low efficiency, as most of the incident light is blocked by the aperture. Furthermore, an additional phase-modulating element is needed to generate higher-order Bessel beams, which in turn adds complexity and bulkiness to the system. We overcome these problems using dielectric metasurfaces to realize meta-axicons with additional functionalities not achievable with conventional means. We demonstrate meta-axicons with high NA up to 0.9 capable of generating Bessel beams with full width at half maximum about as small as \(\sim \lambda/3\) (\(\lambda = 405\) nm). Importantly, these Bessel beams have transverse intensity profiles independent of wavelength across the visible spectrum. These meta-axicons can enable advanced research and applications related to Bessel beams, such as laser fabrication, imaging and optical manipulation.

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In the case of conventional axicons, the NA is almost constant within the visible region due to the weak dispersion of glass. Thus, the FWHM of the $J_0$ beam is proportional to wavelength and varies accordingly. For example, changing the wavelength from 400 to 700 nm can result in a difference of 175% in the FWHM. Alternatively, a high NA objective lens paired with an annular aperture is usually used to generate Bessel beams with subwavelength FWHMs, as shown in Figure 1b. However, this configuration is not efficient as most of the incident light is blocked by the aperture. Both methods require the addition of phase-modulating elements, such as spatial light modulators or spiral phase plates, to generate higher-ordered Bessel beams.

In recent years, metasurfaces, consisting of subwavelength-spaced phase shifters, have been demonstrated to fully control the optical wavefront\textsuperscript{7–12}. Various compact optical components have been reported including lenses\textsuperscript{13–17}, holograms\textsuperscript{18–21}, modulators\textsuperscript{22–24} and polarization-selective devices\textsuperscript{25–28}. Metallic and dielectric metasurface axicons were reported in Ref. 29 and Ref. 27, respectively; both had low NAs. Unlike conventional phase-modulating devices (for example, spatial light modulators), metasurface-based devices can provide subwavelength spatial resolution, which is essential in order to deflect light by very large angles. This is mandatory to realize high NA optical components, including axicons and lenses capable of generating beams with even smaller FWHM. Various applications, including (but not limited to) scanning microscopy\textsuperscript{29,30}, optical manipulation\textsuperscript{32–34} and lithography\textsuperscript{35,36}, all require subwavelength FWHM to achieve high spatial resolution, strong trapping force and subwavelength feature sizes, respectively. Here we report meta-axicons with high NA up to 0.9 in the visible region that are capable of generating, not only the zeroth-order, but also higher-ordered Bessel beams with FWHM about one-third of the wavelength without the use of additional phase elements. In addition, by appropriately designing the metasurfaces’ phase shifters, the transverse field intensity profiles are maintained independent of the wavelength.

**MATERIALS AND METHODS**

Figure 1c shows a schematic diagram of a meta-axicon. The basic elements of the meta-axicon are identical rotated titanium dioxide (TiO$_2$) nano-fins with height $h$, length $L$ and width $w$, arranged in a square lattice. To maximize the performance of the meta-axicon, each nano-fin should act as a half-waveplate at the design wavelength, converting incident circularly polarized light to its orthogonal polarization state. To tailor the required phase profiles, we use geometric phase associated with the rotation angle of the nano-fin, known as the Pancharatnam–Berry phase\textsuperscript{37,38}. The $h$, $L$ and $w$ parameters are determined using the three-dimensional finite difference time domain (FDTD from Lumerical Inc.) method to maximize the circular polarization conversion efficiency\textsuperscript{39,40}. At the design wavelength $\lambda_d = 405$ nm, simulated polarization conversion efficiencies $> 90\%$ were obtained. The efficiencies decrease as NA increases due to the sampling criterion (Supplementary Fig. S2). To determine the polarization conversion efficiency, we arranged an array of TiO$_2$ nano-fins in such a way to deflect light to a particular angle and then the efficiency is calculated by dividing the total deflected optical power by
the input optical power. Perfectly matched layer boundary conditions were used normal to the propagation of the incident circularly polarized light and periodic conditions were used for the remaining boundaries.

For the generation of a zeroth-order Bessel beam, a meta-axicon requires a radial phase profile \( \varphi(r) \) with a phase gradient \( \frac{d\varphi}{dr} = -\frac{2\pi}{\lambda_d} \sin(\theta) \) (5)

This can be understood from the generalized Snell’s law\(^*\) as the condition for all light rays to be refracted by the same angle \( \theta \) at the design wavelength \( \lambda_d \), where \( \sin(\theta) \) is the NA. Integrating Equation (5) gives:

\[
\varphi(x, y) = 2\pi \frac{2\pi}{\lambda_d} \sqrt{x^2 + y^2} \cdot \text{NA} \quad (6)
\]

where \( \sqrt{x^2 + y^2} = r \). The generation of the high-order Bessel beams requires the addition of a term \( n\phi \), where \( \phi = \arctan(\frac{y}{x}) \) is the azimuthal angle, which represents the phase of an optical vortex imparted to the deflected light. Equation (6) then becomes

\[
\varphi(x, y) = 2\pi \frac{2\pi}{\lambda_d} \sqrt{x^2 + y^2} \cdot \text{NA} + n\phi \quad (7)
\]

This phase profile is imparted by the rotation of each nano-fin at a position \((x, y)\) by an angle \( \theta_{rot}(x, y) = \frac{\varphi(x, y)}{2\pi} \) for the case of left-handed polarized incidence. TiO\(_2\)-based meta-axicons are fabricated using the approach described in Ref.41, which can prevent tapered sidewall\(^*\). Figure 1d shows a scanning electron microscope image of the center part of a fabricated meta-axicon. We used a custom-built microscope to characterize the meta-axicons. A schematic diagram of the set-up and the optical components used can be found in Supplementary Fig. S3.

### RESULTS AND DISCUSSION

Figure 2a and 2d shows the measured transverse intensity profile of the \( J_0 \) and \( J_1 \) Bessel beams at \( \lambda = 405 \text{ nm} \), whereas Figure 2b and 2e shows the intensity along a horizontal cut across the centers of Figure 2a and 2d, respectively. The measured FWHM of the \( J_0 \) Bessel beam is observed to be \( \sim 163 \text{ nm} \) with \( 3.5 \text{ nm} \) standard deviation, which is very close to its theoretical limit of \( 160 \text{ nm} \) (Equation (3)). The measured FWHM of the \( J_1 \) Bessel beam is \( 130 \text{ nm} \) with \( 1.75 \text{ nm} \) standard deviation, which agrees well with its theoretical value of \( 131 \text{ nm} \) (Equation (4)). Figure 2c and 2f shows the intensity profile along the beam propagation direction of the \( J_0 \) and \( J_1 \) Bessel beams. Their FWHMs at different planes normal to the propagation \( z \)-axis are provided in Supplementary Fig. S4. Both the \( J_0 \) and \( J_1 \) Bessel beam have a depth of focus of \( 75 \mu\text{m} \) (150\( \lambda \)). This value is close to the theoretical value using geometric optics, that is, \( \frac{D}{\text{NA}} = 72 \mu\text{m} \), where \( D = 300 \mu\text{m} \) is the diameter of the meta-axicons.

Polarization properties of a Bessel beam generated by a high NA meta-axicon can be very different from that of a corresponding axicon with low NA\(^43,44\). In order to understand the polarization properties of the \( J_0 \) and \( J_1 \) Bessel beams generated by the meta-axicons with \( \text{NA} = 0.9 \), we show in Figure 3 their theoretical (first row) and simulated (second row) normalized electric field intensities \( |E_x|^2 \), \( |E_y|^2 \) and \( |E_z|^2 \). Only a portion of the meta-axicon (30 \( \mu\text{m} \) in diameter), but with the same NA, was simulated due to limited computational resources.

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**Figure 2** Optical characterization of meta-axicons with \( \text{NA} = 0.9 \) designed at wavelength \( \lambda_d = 405 \text{ nm} \). (a, b) Measured intensity profile (a) for the meta-axicon designed to generate \( J_0 \) (scale bar = 250 nm) and its corresponding horizontal cut (b). (c) Normalized intensity profile of the Bessel beam (\( J_0 \)) along the propagation direction. (d, e) Measured intensity profile (d) for the meta-axicon designed to generate \( J_1 \) (scale bar = 250 nm) and its corresponding horizontal cut (e). (f) Normalized intensity profile of the Bessel beam (\( J_1 \)) along the propagation direction.

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resources. A slight deviation of the simulations from theory results from the uncoupled light of nano-fins and the effects of the boundary in the simulations. We note that for either $|E_{z}|^{2}$ (Figure 3a and 3d) or $|E_{y}|^{2}$ (Figure 3h and 3e), the shape of the $J_{0}$ and $J_{1}$ Bessel beams at their respective centers is elliptical rather than circular. For example, as shown in Figure 3d and 3e, for $J_{1}$, the center regions are accompanied by two brighter spots at the end of the long axis of the ellipse. Moreover, the intensity of $|E_{z}|^{2}$ and $|E_{y}|^{2}$ show variation on their side lobes. These are also observed in their corresponding simulation results (Figure 3j and 3k). In addition, $|E_{z}|^{2}$ is described by a Bessel function one order larger than its transverse electric field. These properties can be explained by considering the vector form of Bessel beams. The theoretical vector solutions of the electric field of a Bessel beam propagating along the z direction in cylindrical coordinates are:

$$E(r, \phi, z) = E_0 \cdot e^{ikz} e^{i(n+1)\phi} \left( \begin{array}{l}
(C_{TM} |f(kr)+J_{n+1}(kr)|/2 + i \cdot C_{TM} \eta \sqrt{1 - \pi \eta^2} |f(kr)-J_{n+1}(kr)|/2 \right)

(C_{TM} \eta \sqrt{1 - \eta^2} |f(kr)+J_{n+1}(kr)|/2 + i \cdot C_{TM} \eta \sqrt{1 - \eta^2} |f(kr)-J_{n+1}(kr)|/2)

NA \cdot C_{TM} \eta \cdot J_{n+1}(k_{r} r) z

\right)

(8)

where $C_{TM}$ and $C_{TE}$ are complex numbers associated with the constituent transverse magnetic (TM) and transverse electric (TE) waves of Bessel beams, and $\eta$ is the phase difference between $C_{TM}$ and $C_{TE}$.

For our case (circularly polarized Bessel beams), $C_{TE}$ is equal to $C_{TM}$, $\eta = \pm \pi/2$, $NA = 0.9$, and $n = 0$ and $n = 1$ for $J_{0}$ and $J_{1}$ Bessel beam, respectively. For high NA Bessel beams, the electric field of the z component results from TM waves: the higher the NA, the higher the contribution the TM waves make to $E_{z}$. The $E_{r}$ and $E_{\phi}$ components mainly result from TE wave contribution, as the term $\sqrt{1 - \eta^2}$ is relatively small for the high NA case. The term $J_{n+1}(k_{r} r)$ contributes to localized intensity near the center spot or the most inner ring, as $J_{n}(k_{r} r) \approx \sqrt{\pi/2} \cos(r - \pi n/2)$ for large $r$, such that $J_{n}(k_{r} r)$ and $J_{n+1}(k_{r} r)$ cancel each other due to a $\pi$ phase difference.

The intensity distribution of side lobes away from the center of a Bessel beam is due to another term $J_{n+1}(k_{r} r)$. When we transform cylindrical $(r,\phi)$ to Cartesian coordinates $(x,y)$ using $E_{x} = E_{r} \cdot \cos(\phi) - E_{\phi} \cdot \sin(\phi)$, we will have $\cos(\phi)$ modulation resulting in the elliptical center, and a corresponding modulation of $\sin(\phi)$ for the side lobe, which is shown in Figure 3a and 3d, respectively. This is a signature feature of high NA Bessel beams (see a comparison with a lower NA in Supplementary Fig. S5). Due to the spatially varying intensity of $E_{x}$ and $E_{\phi}$, the Bessel beams for high NA are not homogeneously polarized, but rather show space-variant polarization states (see Supplementary Fig. S6 for plots of ellipticity and polarization orientation angle). Therefore, only the center part of the $J_{0}$ Bessel beam can be circularly polarized in the case of high NA case.

By the judicious design of our metasurface, we compensate the wavelength dependency of the FWHM of Bessel beams (Equations (3) and (4)). As mentioned previously, the transverse intensity profile is determined by the factor $k_{r} = \pi/2 \cdot NA$. In our case, the NA = $\pi/2\sqrt{\sqrt{\eta^2} - \eta^2}$, where $\eta$ follows Equation (7). Therefore, $k_{r}$ only depends on the phase gradient $\nabla \phi(x,y,z)$, which can be designed to be wavelength-independent using the Pancharatnam–Berry phase concept. In this case, the phase gradient is a constant, and the NA is only proportional to the wavelength $\lambda$. This manifests experimentally, in the form of the increasing ring diameters in the Fourier plane of the meta-axicons for increasing wavelength (Supplementary Fig. S7). To demonstrate this unique characteristic across a broad wavelength region, we use the same method to design two meta-axicons for $J_{0}$ and $J_{1}$ with the NA = 0.7 at the wavelength $\lambda = 532 \text{ nm}$. Each nano-fiber ($L = 210 \text{ nm}$, $W = 65 \text{ nm}$ and $h = 600 \text{ nm}$) for this case is arranged in a square lattice, with a lattice constant of 250 nm. We show in Figure 4a–4d and 4f–4i the corresponding $J_{0}$ and $J_{1}$ Bessel beams in false color for different wavelengths ($\lambda = 480, 530, 590$ and $660 \text{ nm}$) with a bandwidth of $5 \text{ nm}$ at the z plane about 60 $\mu$m away from the surface of meta-axicons. Note that the efficiency of the meta-axicon is dependent on wavelength. The efficiency was measured and is shown in Supplementary Fig. S8. The FWHM$_{0}$ and FWHM$_{1}$ for each wavelength spanning 470–680 nm are shown in Supplementary Fig. S9. Figure 4 explicitly indicates that the intensity profile for different wavelengths vary weakly, confirming wavelength-independent behavior. It is notable that for these measurements the distance between the meta-axicon and objective lens was kept unchanged. We also repeated the measurements using a super-continuum laser of bandwidth 200 nm centered at 575 nm (see Supplementary Fig. S10 for its spectrum). The intensity profiles (Figure 4e and 4j) for both $J_{0}$ and $J_{1}$ remarkably changed weakly. It is important to note that in order to generate high NA and wavelength-independent Bessel beams, the Nyquist sampling theorem

**Figure 3** Polarization properties of meta-axicons with $NA = 0.9$ designed at wavelength $\lambda_{d} = 405 \text{ nm}$. A comparison of the square of the electric fields from theoretical calculation using Equation (8) (first row) and FDTD simulations (second row). The first (a, g), second (b, h) and third columns (c, i) show $|E_{z}|^{2}$, $|E_{x}|^{2}$ and $|E_{y}|^{2}$ for the $J_{0}$ Bessel beam, respectively. The corresponding plots for the $J_{1}$ Bessel beam are shown on the right side of the black line. Scale bar = 250 nm.
and wavelength-independent phase gradient conditions both need to be satisfied. According to the Nyquist sampling theorem, one needs to sample the phase profile given by Equation (7) in the spatial domain with a rate that is at least twice the highest transverse spatial frequency. This requires the size of the unit cell to be equal to or smaller than \( \frac{\lambda}{2\pi} \), which cannot be satisfied by conventional diffractive elements. For example, spatial light modulators usually have \( \sim 6 \mu m \) pixel sizes\(^{46}\) and photo-aligned liquid crystal devices are usually limited to a phase gradient of \( \sim \pi/\mu m^{37,47} \), corresponding to a maximum achievable NA of about 0.03 and 0.26 in the visible region, respectively. It is also important to note that the phase profile of metasurfaces can also be designed by varying the geometrical sizes (length, width or diameter and so on) of the nanostructures, pixel by pixel\(^{7,9}\). However, these metasurfaces, not designed by the Pancharatnam–Berry phase, are accompanied by strong amplitude differences between each pixel at wavelengths away from the design wavelength. This becomes more significant within the absorption region of the constituent materials used. In addition, the unwanted amplitude difference between each pixel can result in the deflection of light to multiple angles\(^{48}\), changing the profile of the Bessel beam. Utilizing the Pancharatnam–Berry phase approach minimizes the relative amplitude difference between each nano-fin for all wavelengths in the case of circularly polarized illumination, as each nano-fin is identical, and consequently, has the same size. We experimentally demonstrate this concept by measuring meta-axicons consisting of silicon nano-fins from the near-infrared to the visible spectrum, where silicon becomes intrinsically lossy (Supplementary Fig. S11). It is clearly observed that the sizes of the Bessel beam remain constant over the wavelength range of 532–800 nm.

**CONCLUSIONS**

In summary, as a superior alternative to using conventional prism axicons or an objective paired with an annular aperture, we demonstrate high NA meta-axicons capable of generating Bessel beams of different orders in a single device in a much more efficient and compact way. The FWHM of \( J_0 \) and \( J_1 \) Bessel beams are shown to be as small as \( \sim 160 \) and 130 nm at the design wavelength \( \lambda = 405 \) nm. This size is maintained for an exceptionally large distance of 150\( \mu m \) (depth of focus). Their polarization is space-variant due to high NA. By tailoring the phase profile of the meta-axicons, the FWHMs of generated Bessel beams are made independent of the wavelength of incident light. These meta-axicons can be mass-produced with large diameter using today’s industrial manufacturing (deep ultraviolet steppers). These properties show great promise in potential applications ranging from laser lithography and manipulation to imaging.

**CONFLICT OF INTEREST**

The authors declare no conflict of interest.

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