Characteristic Relations of Type-III Intermittency in an Electronic Circuit

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Abstract

It is reported that the characteristic relations of type-II and type-III intermittencies, with respective local Poincaré maps of $x_{n+1} = (1 + \epsilon)x_n + ax_n^3$ and $x_{n+1} = -(1 + \epsilon)x_n - ax_n^3$, are both $-\ln(\epsilon)$ under the assumption of uniform reinjection probability. However, the intermittencies have various characteristic relations such as $\epsilon^{-\nu}$ ($1/2 \leq \nu \leq 1$) depending on the reinjection probability. In this Letters the various characteristic relations are discussed, and the $\epsilon^{-1/2}$ characteristic relation is obtained experimentally in an electronic circuit, with uniform reinjection probability.
Intermittency characterized by the appearance of intermittent short chaotic bursts between quite long quasiregular (laminar) periods is one of the critical phenomena that can be readily observed in nonlinear dynamic systems. The phenomenon was initially classified into three types according to the local Poincaré map (type-I, II, and III) by Pomeau and Manneville [1]. The local Poincaré maps of type-I, II, and III intermittencies are $y_{n+1} = y_n + ay_n^2 + \epsilon (a, \epsilon > 0)$, $y_{n+1} = (1 + \epsilon)y_n + ay_n^3(a, \epsilon, y_n > 0)$, and $y_{n+1} = -(1 + \epsilon)y_n - ay_n^3(\epsilon, a > 0)$, respectively [2]. The characteristic relation of type-I intermittency is $\langle l \rangle \propto \epsilon^{-1/2}$, where $\langle l \rangle$ is the average laminar length and $\epsilon$ is the channel width between the diagonal and the local Poincaré map. Those of type-II and III intermittencies are $\langle l \rangle \propto \ln(1/\epsilon)$ where $1 + \epsilon$ is the slope of the local Poincaré map around the tangent point under the assumption of uniform reinjection probability distribution (RPD). On the other hand, some monographs [3] suggested that the standard scaling should be $\langle l \rangle \propto 1/\epsilon$.

Recently, however, it was found that the reinjection mechanism is another important factor of the scaling property of the intermittency. In the case of type-I intermittency, various characteristic relations appear dependent on the RPD for the given local Poincaré map, such as $-\ln \epsilon$ and $\epsilon^{-\nu}$ ($0 \leq \nu \leq 1/2$). When the lower bounds of the reinjection (LBR) are below and above the tangent point the critical exponent is always $-1/2$ and 0, respectively, irrespective of the RPD. However when the LBR is at the tangent point the characteristic relations have various critical exponents dependent on the RPD, such that when the RPDs are uniform, fixed and of the form $x^{-1/2}$, the characteristic relations are $-\ln \epsilon$, $\epsilon^{-1/2}$, and $\epsilon^{-1/4}$ respectively [4,5]. In the case of type-II and III intermittencies, the characteristic relations also have various critical exponents for a given local Poincaré map such as $\epsilon^{-\nu}$ ($1/2 \leq \nu \leq 1$) dependent on the RPD. When RPDs are uniform, of the form $x^{-1/2}$ around the tangent point, and fixed very close to the tangent point, the characteristic relations are $\epsilon^{-1/2}$, $\epsilon^{-3/4}$, and $\epsilon^{-1}$, respectively [5]. In this report we discuss the characteristic relations of type-III intermittency analytically, and obtain $\epsilon^{-1/2}$ characteristic relation experimentally in an electronic circuit that consists of inductor, resistor, and diode with uniform RPD.
Since the local Poincaré map of type-III intermittency can be described to be \( y_{n+2} = (1 + 2\epsilon)y_n + by_n^3 \), which is the same as that of type-II intermittency [3], it is enough to discuss the characteristic relations of type-II intermittency according to the RPD without loss of generality. For the given local Poincaré map of type-II intermittency, if we set a gate such that \(|y_{in}| \leq c\) on deviations in the laminar region, the laminar length \( l(y_{in}, c) \) for the reinjection at \( y_{in} \) is obtained in the long laminar length approximation

\[
l(y_{in}, c) = \frac{2 \ln \left[ \frac{c}{y_{in}} \right] - \ln \left[ \frac{ac^2 + \epsilon}{a(y_{in})^2 + \epsilon} \right]}{2\epsilon}. \tag{1}
\]

This is the result obtained in Ref. [3] without consideration of RPD. Here if we consider a normalized RPD \( P(y_{in}) \) the average laminar length \( \langle l \rangle \) is given by

\[
\langle l \rangle = \int_{\Delta}^{c} l(y_{in}, c)P(y_{in})dy_{in} \tag{2}
\]

where \( \Delta \) is the value of \( y_{in} \) representing the LBR.

We first consider the case of uniform RPD of the form \( 1/(c - \Delta) \). In this case the average laminar length is given by

\[
\langle l \rangle = \frac{\tan^{-1} \left( c\sqrt{\Delta} \right) - \tan^{-1} \left( \Delta\sqrt{\frac{c}{a}} \right)}{\sqrt{\epsilon a(c - \Delta)}} - \frac{2\Delta \ln \left( \frac{c}{\Delta} \right) - \Delta \ln \left( \frac{ac^2 + \epsilon}{a\Delta^2 + \epsilon} \right)}{2\epsilon(c - \Delta)}. \tag{3}
\]

In this equation if \( \Delta \) is very close to the tangent point, that is \( \Delta^2 \ll \epsilon \), the second term is negligible in the limit \( \epsilon \to 0 \) because of the factor \( \Delta \) in the numerator and then the characteristic relation is \( \langle l \rangle \propto \epsilon^{-1/2} \). However, when \( \Delta \) is within the gate and not close to the tangent point, the second term is still negligible in the limit \( \epsilon \to 0 \). Then the characteristic relation is determined by the first term alone and is a power type with critical exponent zero as discussed in Ref. [3]. What is interesting here is that these results are different from those of Pomeau and Manneville’s although the RPD is uniform.

We next consider the fixed RPD of which the form is \( \delta(y_{in} - \Delta) \). In this case the average laminar length is
\[
\langle l \rangle = \frac{2 \ln \left( \frac{\Delta}{\epsilon} \right) - \ln \left( \frac{a^2 + \epsilon}{a^3 + \epsilon} \right)}{2 \epsilon}.
\]

When \( \Delta^2 \ll \epsilon \), the average laminar length can be reduced to \( \langle l \rangle \approx [\ln(\epsilon) - \ln(a\Delta^2)]/2\epsilon \) in the limit \( \epsilon \to 0 \). Then the characteristic relation is \( \langle l \rangle \propto \epsilon^{-1} \) since the constant \( \ln(a\Delta^2) \) is dominant over \( \ln(\epsilon) \). Similarly when \( \Delta \) is far from the tangent point, the characteristic relation is a power type whose critical exponent is zero.

In the above results since uniform and fixed RPDs are two extreme cases of power type RPD, we are able to give a general argument. Type-II intermittency has power type characteristic relations such that \( \epsilon^{-\nu}(1/2 \leq \nu \leq 1) \) according to the RPD when \( \Delta^2 \ll \epsilon \) and has a power form whose critical exponent is zero when \( \Delta \) is not close to the tangent point.

To show this argument we further consider the case of nonuniform RPD of the form \( 1/(2\sqrt{\Delta^2 + c}\sqrt{y_{in} + \Delta}) \), for which analytic calculation of the characteristic relation is possible. The characteristic relation of the average laminar length is \( \langle l \rangle \propto \epsilon^{-3/4} \) in the limit \( \epsilon \to 0 \) when \( \Delta^2 \ll \epsilon \), and has a power form whose critical exponent is zero when \( \Delta \) is far from the tangent point. These results show that type-II and III intermittencies have various characteristic relations according to the RPD.

These characteristic relations of type-III intermittency were studied experimentally in the electronic circuit consisting of inductors, resistors, and diodes shown in Fig. 1. A 1N4007 silicon junction diode and a 100 mH inductor (165 \( \Omega \) dc resistance) connected in series were forced by a function generator and a series connected 100 mH inductor and a 1N4007 diode are connected in parallel with the first inductor. In this kind of electronic circuit the characteristic relations of type-I intermittency were observed by varying the amplitude or the offset of the external force \[\text{[S]}.\] In this circuit the amplitude of the external force can be varied by multiplying a sinusoidal signal from a function generator to a dc voltage from a digital-analog converter using a multiplier (MPY100), and the dc voltage is controlled by a personal computer. This apparatus can tune the amplitude very precisely in the limit of noise from electronic elements. The frequency and the bias voltage were fixed at 30 kHz and 0.4 volt, respectively. All the external forces were added by using an operational amplifier,
and the noise from the power sources were reduced using by-pass capacitors.

The rectified voltages across the second diode were measured and each rectified pulse was integrated using an integrating circuit, to obtain the experimental data. Before integration 0.6 volt dc voltage was added to the rectified pulses because the voltage drop across the silicon diode was $-0.6$ volt. And after the adding, the rectified voltages were reduced using a variable resistor, to prevent distortion due to the peak of the integrated voltage being higher than 15 volts. Throughout the experiment we checked that the peaks of the rectified pulses corresponded to those of the integrated pulses. The peaks of integrated pulses were stored in the 40 M byte memory of the pentium computer by using expanded memory manager via a 12-bit analog-digital converter. All the systems were synchronized one another. The digitized time of the analog-digital converter was $12 \mu$sec. The digitized value of $\pm 2048$ corresponds to $\pm 5$ V, respectively. The chaotic outputs of the rectified and integrated pulses were also monitored, by using a digital storage oscilloscope (LeCroy 9310).

In the circuit various transitions from chaotic bands to stable fixed points (or vice versa) were observed when the dc voltage from the computer was varied, because of the nonlinear capacitance of the junction diode [9]. To show the temporal behavior of type-III intermittency in this system, we have obtained them around the tangent bifurcation point near period-2 window as shown in Fig. 2. Figure 2 (a) and (b) show long and short laminar phases from when the amplitude of the external force was about $V = 8.0V$ and $8.01V$, respectively. The shapes are the typical temporal behaviors of type-III intermittency. The continuously increasing and decreasing amplitudes alternate, and are interrupted by the chaotic bursts.

To show more clearly that the temporal behaviors are type-III intermittency, the $x_n$ vs $x_{n+2}$ return maps of the temporal behaviors are obtained as given in Fig. 3 (a). In the figure, lines I and II are the return maps of Fig. 2 (a) and (b), respectively. Figure 3 (b) is $x_n$ vs $x_{n+4}$ return map that implies the local Poincaré map of type-III intermittency can be expressed as that of type-II. In the figures, the return maps of type-III are continuous near the tangent point. This means that the LBR is very close to the tangent point. Also the figures show the differences in slope around the tangent point, which are related to
the average laminar length. To confirm that Fig. 3(b) is the typical local Poincaré map of type-II intermittency, the return maps near the tangent point are fitted with the cubic function, $y_{n+1} = (1+\epsilon)y_n + ay_n^3$. The parameters of lines I and II are $a = 1.2 \times 10^{-5}$ together, $\epsilon = 0.050$ and $\epsilon = 0.083$, respectively. This means that the change in the external amplitude causes the changes in the slope of the local Poincaré map.

To obtain the RPD from the return map, the reinjecting region, between $V_t = 1.31V$ and $1.67V$, is divided into 150 sections and the number of reinjections at each section is counted. Figure 4 is a log-log plot of the total number of the reinjection at each section, where $V_r$ is the voltage of reinjections and $V_t$ is the voltage of the tangent point. As given in the figure, the slope of the RPD turns out close to zero if the gate size is small, which means that the RPD is uniform.

In the intermittency region, the average laminar lengths were obtained by varying the amplitude of the external force. In this measurement, we reduced the voltage from digital-analog converter by a factor of 50 using resistors connected in series. This was done to enable a fine tuning of $\epsilon$ after the addition of a further dc voltage to bring the amplitude of the forcing signal near to the bifurcation point. The step size of the voltage from the digital-analog converter was about 0.05 mV. As the voltage reduced, the length of the laminar was counted and when the longest laminar was less than $10^4$, the computer began storing the data. We assume that the last point at which the length of the longest laminar is larger than $10^4$ is the bifurcation point. In the experiment, if the length of the longest laminar was larger than $10^4$, we could not observe chaotic bursts between regular periods. Also, at each voltage, $10^4$ laminar phases were obtained and 300 steps of total step size were varied, which corresponds to about 15 mV of total variation. Figure 5 is a log-log plot of the average laminar lengths vs $\epsilon$ where $\epsilon = |V - V_t|$. In the figure the dots are experimental results and the solid lines are the fitting of the experimental data. The slope of the solid lines are the critical exponents of the characteristic relations. Line A clearly shows that the characteristic relation is $\langle l \rangle \propto |V - V_t|^{-\nu}$ with approximately $\nu \approx 0.5$. The critical exponent is remarkably similar to the result obtained theoretically with the LBR very close to the tangent point.
We also obtained the critical exponent of zero with LBR above the tangent point, labelled by line B in Fig. 5. The line was obtained when the external force was around 12.4V and bias voltage is 0.8V. In this region hysteresis crisis appears and the stable fixed point is divided into two chaotic bands (one is above and the other below the tangent point) after the tangent bifurcation. This means the LBR is far from the tangent point. To recapitulate, if LBR is far from the tangent point, the critical exponent is zero irrespective of RPD. Line B clearly shows constant laminar lengths when $\varepsilon$ is small. The result again agrees well with the theoretical predictions for the LBR far from the tangent point. The two lines clearly show the characteristic relations due to the RPD in experiment.

In summary, we have obtained various characteristic relations of type-II and III intermittencies such as $\langle l \rangle \propto \varepsilon^{-\nu} \ (1/2 \leq \nu \leq 1)$ depending on the RPDs when the LBR is very close to the tangent point. Also zero critical exponent is obtained irrespective of the RPD when the LBR is far from the tangent point. In an inductor-resistor-diode circuit, these characteristic relations were studied in experiment. The RPD of type-III intermittency appearing in this circuit is uniform and the local Poincaré map can be replaced with type-II intermittency whose local Poincaré map is of the form $y_{n+1} = (1 + \varepsilon)y_n + ay_n^3$. Thus, $-1/2$ and 0 critical exponents are obtained when the LBR is very close to and far from the tangent point, respectively. These experimental results well agree with those of the theoretical analysis.

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FIGURES

FIG. 1. Schematic diagram of experimental setup.

FIG. 2. Temporal behaviors of intermittency near period-2 window for (a) long and (b) short laminar phases.

FIG. 3. (a) $x_n$ vs $x_{n+2}$ and (b) $x_n$ vs $x_{n+4}$ return maps near period-2 window. Line I and II are the maps of Fig. 2 (a) and (b).
FIG. 4. RPD near period 2-window. The dots are experimental data and the solid line is the fitting of the data when the gate size is small. The slope of the solid line is 0, so the RPD is approximately uniform.

FIG. 5. The average laminar lengths vs $|V - V_t|$. The figure shows well that the critical exponents are $-1/2$, and 0, when the LBRs are very close to (line A) and above (line B) the tangent point, respectively. The dots are experimental data and the solid lines are the fitting of the data with the slopes of $-1/2$ and 0, respectively.