On subwavelength and open resonators involving metamaterials of negative refraction index

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Abstract. A one-dimensional (1D) subwavelength resonator utilizing a left-handed material (LHM) is first chosen as an example to show that for a reliable analysis, one should use the wave-field theory instead of the ray-trace theory. The resonant modes of a 2D subwavelength open resonator utilizing an LHM is calculated with wave-field simulation. An open resonator formed by a photonic crystal with negative effective index is also studied.

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1. Introduction

A left-handed material (LHM) having a negative refraction index due to simultaneously negative permeability and permittivity was first introduced by Veselago in 1967 [1]. LHMs have recently attracted much attention in the community of e.g., physics and electrical engineering due to their potential impacts and applications [2, 3]. Negative refraction can also be achieved in some special photonic crystals (PCs) [4]–[6]. Due to the negative refraction index, the optical path in a material of negative refraction index may cancel the optical path in a material of positive refraction index. Such a simple ray analysis can lead to some intuitions for the interesting ideas of a one-dimensional (1D) subwavelength resonator (introduced first by Engheta in [7]) and an open resonator (introduced first by Notomi in [4]). In the present paper, we want to emphasize that the ray conjecture may not provide the complete picture for the resonant conditions. We show that the wave-field theory should be used to analyse subwavelength and open resonators involving metamaterials with negative refraction index even though the ray-trace theory seems to be straightforward. For example, the ray conjecture cannot be fulfilled at resonance for a 1D subwavelength resonator. A subwavelength resonator is of great interest in various microwave or photonic applications, and LHM seems to provide a novel way for its realization. Open resonator is another amazing application of negative refraction index. An open resonator with a modal field of subwavelength size has potential applications in e.g., biosensors.

2. 1D subwavelength resonator

As shown in figure 1, the resonator consists of a conventional dielectric medium (RHM) layer of thickness \( d_1 \) and an LHM layer of thickness \( d_2 \) between two electrically perfectly conducting planes. The permittivity and permeability of the two layers are denoted by \( \varepsilon_i \) and \( \mu_i (i = 1, 2) \), where \( \varepsilon_1, \mu_1 > 0 \) and \( \varepsilon_2, \mu_2 < 0 \). The corresponding refractive indices are given by \( n_1 = \sqrt{\varepsilon_1 \mu_1} \) and \( n_2 = -\sqrt{\varepsilon_2 \mu_2} \).

Note that the sign of the refractive index for the LHM layer is negative. Since all the media are lossless, there is no phase change when the light passes across (normally) the RHM–LHM interface. Based on an intuitive thought (simple-ray analysis) of phase compensation between the RHM layer and the LHM layer, one may conjecture that the resonating condition would be \( n_1 d_1 + n_2 d_2 = 0 \) (called ray conjecture hereafter) if the total thickness \((d_1 + d_2)\) is required to be far less than the resonant wavelength. Later on we show that such a conjecture is not correct.

Firstly, we show that ray conjecture \( n_1 d_1 + n_2 d_2 = 0 \) cannot be fulfilled at resonance when the impedances of the two layers are not matched. From Maxwell’s equations together with the boundary conditions for the field components, one can easily obtain the following resonating condition (derived in [7] in the framework of wave-field theory)

\[
\frac{\text{tg}(n_1 k_0 d_1)}{\text{tg}(n_2 k_0 d_2)} = -\frac{n_1 \mu_2}{n_2 \mu_1},
\]

where \( k_0 = \frac{2\pi}{\lambda} \) is the wave number in vacuum. From equation (1), we obtain

\[
d_2 = -\frac{\text{tg}^{-1} \left[ \frac{\text{tg}(n_1 k_0 d_1) n_2 \mu_1}{n_1 \mu_2} \right]}{n_2 k_0}.
\]
As a numerical example, figure 2 shows the relation between \( k_0(n_1d_1 + n_2d_2) \) and \( \hat{\mu} \) (defined by \( \hat{\mu} = -\mu_2/\mu_1 \)) at the resonance when \( d_1 = \lambda/100, \varepsilon_1 = \mu_1 = 1 \) and \( n_2 = -1 \). From this figure, one sees that \( (n_1d_1 + n_2d_2) \) is zero only when \( \hat{\mu} = 1 \) (i.e., the impedances of the LHM and RHM layers are matched).

Furthermore, the stability of a subwavelength resonator cannot be predicted by the ray theory (one has to use the wave theory to analyse the stability). When the impedances of the two layers in the subwavelength resonator are matched, that is, \( n_1\mu_2/n_2\mu_1 = 1 \), the ray conjecture \( n_1d_1 + n_2d_2 = 0 \) satisfies resonating condition (1). However, such a resonance is not stable. Besides the non-stability due to a small loss of the LHM [8], the resonant wavelength is not stable under a small deviation (caused by any small change in the material parameters, e.g., \( \hat{\mu} \)) from the ray conjecture when the impedances of the two layers are matched. Below we show that the resonant wavelength is also unstable if the impedance mismatch is very small. Under condition \( d_1/(d_2\hat{\mu}) \approx 1 \) (i.e., we study the resonance around the ray conjecture \( n_1d_1 + n_2d_2 = 0 \) when the impedance mismatch is small), one can obtain the following approximate formula for the resonant wavelength (in vacuum) from equation (1)

\[
\lambda_r = n_1(d_1 + d_2) \left\{ \frac{1}{3} \left( \frac{2\pi\hat{\mu}}{1 + \hat{\mu}} \right)^2 \left[ \left( \frac{\hat{\varepsilon}}{\hat{\mu}} \right)^{2/3} + \left( \frac{\hat{\mu}}{\hat{\varepsilon}} \right)^{1/3} \right] + 1 \right\} \left( \frac{\hat{\varepsilon}\hat{\mu}}{d_1/d_2 - \hat{\mu}} \right)^{1/2},
\]

where \( \hat{\varepsilon} \equiv -\varepsilon_2/\varepsilon_1 \). Figure 3 shows how the resonant wavelength (scaled with \( n_1(d_1 + d_2) \)) varies when \( \hat{\mu} \) increases a little from 1.01 for the case of \( \hat{\varepsilon} = 1 \) and \( d_1/d_2 = 1.01 \) (to introduce a small
impedance mismatch around ray conjecture). From this figure, one sees that the resonating wavelength decreases rapidly if $\hat{\mu}$ increases a little from 1.01.

The above analyses indicate that when the size of the object is of the order of subwavelength (or wavelength), the ray theory can merely give us some rough guidance in some cases. For a reliable and accurate analysis, we should use the wave theory.

3. A subwavelength open resonator formed by LHM

The open resonator first suggested by Notomi [4] consists of four alternating rectangular blocks of two materials with opposite refractive indices. A simple ray-trace analysis can show that there exist many closed ray paths (with zero value of the optical path) running across the four interfaces and thus a kind of an open resonator with no surrounding reflective wall is formed. The idea of open resonator based on the cancellation of the optical path in the ray theory is straightforward. However, as we have discussed in the previous section, we should use the wave theory (instead of the simple ray theory) to give a reliable and accurate analysis for such an open resonator, particularly for a resonating mode whose modal field is distributed mainly in a region (around the centre) of subwavelength size. Therefore, it is important to give some numerical simulation (based on the wave theory) for the resonating modes, even though the idea of open...
resonator based on the ray theory seems already straightforward. Surprisingly, no one has done this (to the best of our knowledge) since Notomi introduced the idea of open resonator nearly 5 years ago.

We use the finite-difference time-domain method (FDTD) [9]–[11] to calculate the resonating modes in an open resonator consisting of two homogenous LHM squares in air (see figure 4). A lossless Drude model is used for the LHM: \( \varepsilon = \varepsilon_0 (1 - \omega_p^2/\omega^2) \) and \( \mu = \mu_0 (1 - \omega_p^2/\omega^2) \). As a numerical example, we choose \( \omega_p = 2\sqrt{2}\pi f_0 \) and \( f_0 = 3.0 \times 10^{10} \) Hz. At frequency \( \omega = 2\pi f_0 \), the refractive index of the LHM is \(-1\) and its impedance matches to that of the air. Each LHM square of the resonator has a size of \( \lambda_0 \times \lambda_0 \), where \( \lambda_0 = c/f_0 \) (\( c \) is the light speed in vacuum). The shortest distance between the resonator and Berenger’s perfectly matched layer (PML) [9] is \( 0.6\lambda_0 \), the thickness of the PML is \( 0.2\lambda_0 \), and the discretization stepsize of the mesh is \( \lambda_0/200 \). There exist even modes (with respect to a diagonal line; see figure 5(a)) and odd modes (see figure 5(b)) for such a resonator. Two point sources with equal amplitudes (both positive for the even modes; one positive and one negative for the odd modes) at appropriate positions are used to selectively excite one of the two kinds of modes. The time dependence of the point source excitation is

\[
s(t) = \exp \left[ -\left(\frac{t-t_0}{t_w}\right)^2 \right] \sin[2\pi f_0(t - t_0)],
\]

\( t, t_0, \) and \( t_w \) are time, temporal delay, and temporal width, respectively. The excitation of the odd mode is shown in figure 5(b). Figure 3 shows the relation between \( \lambda_r/[n_1(d_1 + d_2)] \) and \( \hat{\mu} \) for \( \hat{\varepsilon} = 1 \) and \( d_1/d_2 = 1.01 \).

**Figure 3.** The relation between \( \lambda_r/[n_1(d_1 + d_2)] \) and \( \hat{\mu} \) for \( \hat{\varepsilon} = 1 \) and \( d_1/d_2 = 1.01 \).
Figure 4. A subwavelength open resonator consisting of two homogenous LHM squares in air.

Figure 5. The symmetry of the field amplitude for (a) an even mode and (b) an odd mode.

where \( t_w = 20/f_0 \) and \( t_0 = 100/f_0 \). The two point sources are turned off at \( t = 220/f_0 \) and then the evolution of \( E_z \) (for \( E \)-polarization) at some point (not on the diagonal lines in figures 5(a) and (b)) in the resonator is recorded for a period \( (2500/f_0) \). The Padé approximation with Baker’s algorithm [12, 13] is used to compute accurately the resonant frequency and the quality factor \( Q = f_0/\Delta f \), where \( \Delta f \) is the full-width half maximum (FWHM) of the resonant frequency. Figures 6(a) and (b) show the calculated intensity distributions (normalized) of \( E_z \) for two well-confined modes (one is an even mode and the other an odd mode) whose resonant frequencies are closest to \( f_0 \) (but not identical to \( f_0 \); we have verified these two modal profiles with an independent simulation of finite element method). The corresponding resonant frequencies and quality factors for these two modes under the present configuration are \((f = 2.9953 \times 10^{10} \text{Hz}, Q = 4538)\) and \((f = 2.9947 \times 10^{10} \text{Hz}, Q = 21 239)\), respectively (one may be able to make a more effective analysis with a rigorous semi-analytical method).
Figure 6. The normalized intensity distributions of $E_z$ for (a) an even mode and (b) an odd mode. The boxes in (a) and (b) indicate the LHM squares of the open resonator.

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4. An open resonator formed by a PC with a negative effective refraction index

Negative refraction can also be achieved in some specially designed PCs [4]–[6]. In particular, for some PCs having negative effective refraction index $n_{\text{eff}}$ near the centre of the Brillouin zone [4, 6], the dot product of the Poynting vector and the wave vector is negative (like in an LHM), and consequently it may have a behaviour similar to that of an LHM (e.g., focusing slab-lens satisfying Snell’s law [2]).

Since a PC with negative effective index of refraction at an optical frequency is comparatively much easier to fabricate, one wishes to realize an open resonator with a PC of negative effective refraction index. We use the same 2D PC considered in [4], i.e., a triangular lattice of air holes (of radius $0.4a$; $a$ is the lattice constant) in GaAs background (with $n = 3.6$). For $E$-polarization, the equal-frequency surface (EFS) is almost circular (indicating that an isotropic $n_{\text{eff}}$ can be well defined at these frequencies) at the second band in a frequency window ranging from $0.29(c/a)$ to $0.34(c/a)$. Furthermore, EFS for a higher frequency has a smaller radius, and this indicates that the negative refraction has a left-handed behaviour. In particular, $n_{\text{eff}}$ is around $-1$ at frequency $f = 0.30(c/a)$.

It was shown in our previous work [14] that the reflection is large (i.e., small transmission) for any incident angle at an interface normal to the $\Gamma$–$K$ direction and the reflection is small for virtually all incident angles when the interface is normal to the $\Gamma$–$M$ direction. Therefore, the open resonator sketched by Notomi ([4]; cf figure 4) will not work due to the large reflection at

Figure 7. The proposed open resonator formed by three $60^\circ$ wedges of PC with negative effective index.
Figure 8. The $E_z$ distribution for the resonant mode of an open resonator with (a) $d/a = 0.12$ (field is not confined around apex $O$) and (b) $d/a = 0.49$ (field is confined around apex $O$).

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the air–PC interfaces (if one tries to reduce the reflection at one air–PC interface of a rectangular block by aligning the normal of this interface with the Γ–M direction, the reflection would be large at the other air–PC interface of the rectangular block).

Figure 7 shows the design of our open resonator, which consists of three 60° wedges of PC and three 60° wedges of air (in-between). The open resonator system is formed by letting the three 60° wedges of air ‘slice’ towards the centre (apex O) of the PC structure in a symmetrical way. The surface termination position of the PC wedges is determined by a parameter \( d \) (the distance between apex O and any of the tip points of the air wedges). The whole structure is designed in such a way that all the air–PC interfaces are normal to the Γ–M direction (to reduce the reflection at the air–PC interfaces). The inset of figure 7 shows the simple ray-trace sketch for a close optical path in such an open resonator.

A FDTD method (with a discretization stepsize of \( a/16 \)) similar to the one described in the previous section is used to calculate the resonant frequency and mode profile. The resonant field does not have to be confined around the apex. However, we found that the one around the apex has a larger \( Q \) and thus focus on it. For example, the quality factor for the case of \( d = 0.12a \) is only \( Q = 129 \) at the resonant frequency \( f = 0.309990(c/a) \). From the corresponding modal field in figure 8(a), one sees that it is not so well confined around the apex. Figure 8(b) shows the modal field for the case of \( d = 0.49a \) (its \( Q \) is over 1250). In both this non-confined case (figure 8(a)) and the confined case (figure 8(b)), the negatively refracted beams can be observed at the interface between the PC and air, even for such a small size structure.

The resonating frequency for figure 8(b) is \( f = 0.309675(c/a) \), which is a bit away from \( f = 0.30(c/a) \) at which \( n_{\text{eff}} \) is around −1. Such a deviation also indicates that one should use
the wave theory (instead of the ray theory) for an accurate analysis. Furthermore, the ray theory can give neither the field profile of the resonant mode nor the quality factor.

The present open resonator is appropriate for using as a biosensor. A small change of the refractive index of the measurand (filling only the three air wedges and not the holes in the PC) due to e.g., immobilized biomolecules will cause a shift in the resonant wavelength (see the solid line in figure 9), which can be measured by e.g., a wavemeter. Unlike a conventional PC resonator, the present open resonator has large air wedges into which a liquid measurand can flow easily (this makes the present open resonator more suitable for biosensing applications). The dashed line of figure 9 shows the quality factor of the PC open resonator. If we increase the structure size, the quality factor for an optimized open resonator can increase (over 2000; the modal profile will remain very similar if $d$ is fixed). However, for the application of biosensors, the sensitivity of the resonant wavelength and the small size of the sensing structure may be more important than the quality factor.

5. Conclusions

In the present paper, we have shown that the wave-field theory should be used to analyse subwavelength and open resonators involving a metamaterial with negative refraction index even though the ray-trace theory seems to be straightforward. The ray conjecture may not provide the complete picture for the resonant conditions. Some resonant modes (localized around the centre in a region of subwavelength size) of a 2D open resonator utilizing an LHM have been calculated with the FDTD method. An open resonator formed by a PC with negative effective index has also been studied and shown to be suitable for its use as e.g., a biosensor.

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