Research Article

Reliability Modeling and Evaluation for Complex Systems Subject to New Dependent Competing Failure Process

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The objective of this study is to construct some system-level reliability models subject to new dependent competing failure process of the soft failure process and hard failure process. In those complex system-level models, the soft failure caused by continuous degradation and sudden degradation increases by random shocks and hard failure due to the same shock process; in addition, the bidirectional effects between natural degradation and random shock are considered in this study; in other words, the random shock can bring additional degradation, and the natural degradation may also impact the shock process. Therefore, the soft failure process and hard failure process are dependent. By using the cumulative shock model and Gaussian stochastic degradation process with line mean value and line standard deviation, five types of reliability models were established: (1) single component model; (2) series system model; (3) parallel system model; (4) series-parallel system model; and (5) parallel-series system model. Finally, an example of fatigue crack growth dataset is taken to verify the effectiveness of the established model and method, and some sensitivity analyses are given. The results show that the bidirectional effects can significantly accelerate the system failure, and the impact of natural degradation process to random shocks should not be ignored. In addition, the results also show that random shock and failure threshold have significant effects on the different complex systems.

1. Introduction

Reliability analysis and maintenance policies for complex degradation systems have been widely used in different fields, such as in aerospace [1, 2], electronics industry [3], civil engineering [4–6], and other fields with high reliability and long-life requirements. In the above different research fields, the competitive failure model based on degradation and random shock has been deeply studied. Huang and Askin [7] studied a competitive failure reliability model in which sudden failure and degradation process were independent on each other. Lehman [8] proposed a reliability model under degradation and random shock. Klutke and Yang [9] and Li and Pham [10] studied reliability analysis and maintenance model subject to degradation and shock. Zhu et al. [11] studied maintenance policies based on an independent competitive failure reliability model. In addition, some other shock models, such as cumulative shock model [12], extreme shock model [13], mixed shock model [14], and $\delta$-shock model [15] were also considered in competitive failure model. Most of the above competitive failure models assumed that the degradation process and the random shock process are independent on each other. However, in practice, each shock may have an impact on both failure processes, and independent assumptions are inconsistent with reality. When these failure processes depend on each other, the reliability assessment of system is a very worth studying problem.

Recently, some research on reliability analysis of dependent competitive failure process (DCFP) has been carried out. Rafiee et al. [16] established a DCFP system with the degradation rate changing. Qi et al. [17] studied a DCFP system with self-recovery capability. Liu et al. [18] proposed a competitive failure process with damage self-healing ability. Gao et al. [19] studied a DCFP system with multiple species of shocks. Qiu and Cui [20] proposed a novel
dependent two-stage failure process model with competing failures. Wang et al. [21] established a new DCFP model based on age and current state. Li et al. [22] proposed a DCFP system subject to different failures shifting threshold. In addition, some reliability assessment and maintenance strategies method for the complex degradation system with DCFP model have also established, such as Peng et al. [23], Wang et al. [24], Rassoul and Kamyar [25], and Jiang et al. [26]. Most of the previous studies only considered the reliability analysis of a single-component system with dependent competing failure process. There are few studies on reliability analysis of multiple components complex systems with DCFP [27, 28], and these papers only used simple linear model and extreme shock model. In addition, the above studies only assume that each shock loads will cause additional abrupt degradation, in other words, only considering the impact from random shocks to natural degradation process. In fact, the impact of natural degradation process to random shocks should also be considered, because the random shock may come more frequently with the system performance reduction.

Therefore, the bidirectional effects between natural degradation and random shock are considered in this paper, and we extend the previous model presented in References [13, 27–29] to a multi-component complex system by developing some system-level reliability model. Firstly, the reliability model of single-component system with DCFP is established via the cumulative shock model and Gaussian stochastic degradation process with linear mean and linear standard deviation. Secondly, the reliability models of series and parallel systems with DCFP are conducted, respectively. Then, some complex series-parallel reliability models subjected to DCFP are also conducted. At last, an example of fatigue crack growth dataset is taken to verify the effectiveness of the established model and method.

2. Degradation Model of the Element

Suppose that the complex system is made up of multiple components in a defined structure, and each component contains a hard failure process and a soft failure process. Due to the same shock loads, these two failure processes are dependent. When any failure process exceeds its threshold, the element will fail.

Figure 1 shows that X_i(t) is the continuous degradation value of the i-th component at time t. For the i-th component, suppose that the j-th shock load will cause additional abrupt degradation as Y_ij (i = 1, 2, ..., n, j = 1, 2, 3, ...), which represents the acceleration degradation value. Let the total degradation X_i(t) be made up of X_i(t) and Y_ij, and when X_i(t) > H_i, the soft failure occurs.

Figure 2 shows that W_ij represents the magnitude of the i-th component caused by the j-th shock load. When the cumulative magnitudes W_ij of those loads surpass the hard threshold value D_i, the failure occurs.

2.1. Hard Failure Modelling. During the normal operation, assume that the component is continuously affected by a Poisson shock process with rate \( \lambda \). Let \( N(t) \) represent the total number of shocks in \((0, t]\), then we can get

\[
P[N(t) = j] = \frac{(\lambda t)^j}{j!} e^{-\lambda t}, \quad j = 0, 1, 2, \ldots.
\]

Meanwhile, as shown in Figure 2, when the cumulative magnitude is less than the threshold value \( D_i \), the hard failure will not occur. Then, we can get

\[
F_{W_i}(D_i) = P \left( \sum_{j=1}^{N(t)} W_{ij} < D_i \right), \quad i = 1, 2, \ldots, n,
\]

\[
j = 0, 1, 2, \ldots.
\]

If \( W_{ij} \) follows the normal distribution \( W_{ij} \sim \mathcal{N}(\mu_w, \sigma_w^2) \) and i.i.d., then each component survival probability of shock loads becomes

\[
F_{W_i}(D_i) = \Phi \left( \frac{D_i - j \mu_w}{\sqrt{j} \sigma_w} \right), \quad i = 1, 2, \ldots, n, \quad j = 0, 1, 2, \ldots.
\]

2.2. Soft Failure Modelling. In this section, suppose that each shock loads will cause an additional abrupt degradation. Let
these abrupt degradations be measured sizes as \{Y_{i1}, Y_{i2}, \ldots \} and \( S_i(t) \) be the total damage size of the \( i \)th component until time \( t \), then we can get
\[
S_i(t) = \begin{cases} 
\sum_{j=1}^{N(t)} Y_{ij}, & \text{if } N(t) = 0, \\
0, & \text{if } N(t) = 0.
\end{cases}
\] (4)

Figure 1 shows that the overall value \( X_{i\theta}(t) \) of the \( i \)th component is made up of \( X_i(t) \) and \( S_i(t) \). For the \( i \)th component, the degradation process \( X_i(t) \) of the \( i \)th component can be described by Gaussian stochastic degradation process as
\[ X_i(t) \sim N(\mu_i(t)). \] (5)

Assuming that the shock damage sizes \( Y_{ij} \) follows the normal distribution and i.i.d as
\[
Y_{ij}(t) \sim N(\mu_{ij}, \sigma_{ij}^2), \quad i = 1, 2, \ldots, n; \quad j = 0, 1, 2, \ldots
\] (6)

Based on equations (4)-(7), considering the independence of \( X_i(t) \) and \( S_i(t) \), we know that \( X_{i\theta}(t) \) follows the normal distribution as follows:
\[
X_{i\theta}(t) \sim N(\mu_{i\theta}(t) + N(\mu(t)) \mu_{ij}, \sigma_{ij}^2 N(t) \sigma_{ij}^2).
\] (7)

Suppose that \( T \) is the soft failure time of the \( i \)th component, \( H_i \) is the threshold value of the \( i \)th component. Considering that in most engineering degradation processes, the mean and standard deviation generally increases in terms of time, then, we can get the soft failures function of the \( i \)th component as
\[
R_{i\theta}(t) = P(T > t) = P(X_{i\theta}(t) < H_i) \\
= P(X_i(t) + S_i(t) < H_i) \\
= P(X_i(t) < H_i | N(t) = 0) P(N(t) = 0) \\
+ \sum_{k=1}^{\infty} \left( P(X_i(t) + \sum_{j=1}^{N(t)} Y_{ij} < H_i | N(t) = k) \right) \cdot P(N(t) = k) \\
= \Phi \left( \frac{H_i - \mu_i(t)}{\sigma_i(t)} \right) \cdot e^{-\lambda t} \\
+ \sum_{k=1}^{\infty} \Phi \left( \frac{H_i - (k \mu_{ij} + \mu_i(t))}{\sqrt{k \sigma_i^2 + k \sigma_{ij}^2}} \right) \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k.
\] (8)

2.3. Reliability Modelling of DCFP. As shown in Figures 1 and 2, we know that the \( i \)th component can work normally only when \( X_{i\theta}(t) < H_i \) and \( \sum_{j=1}^{N(t)} Y_{ij} < D_i \). Then, the reliability function of the \( i \)th component under the model \( M_1 \) can be obtained as
\[
R_{i\theta}(t) = P(X_{i\theta}(t) < H_i, \sum_{j=1}^{N(t)} Y_{ij} < D_i) \\
= P(X_i(t) + \sum_{j=1}^{N(t)} Y_{ij} < H_i, \sum_{j=1}^{N(t)} Y_{ij} < D_i) \\
= \sum_{k=0}^{\infty} P(X_i(t) + \sum_{j=1}^{N(t)} Y_{ij} < H_i, \sum_{j=1}^{N(t)} Y_{ij} < D_i | N(t) = k) \cdot P(N(t) = k) \\
= \Phi \left( \frac{H_i - \mu_i(t)}{\sigma_i(t)} \right) \cdot e^{-\lambda t} \\
+ \sum_{k=1}^{\infty} \Phi \left( \frac{H_i - (k \mu_{ij} + \mu_i(t))}{\sqrt{k \sigma_i^2 + k \sigma_{ij}^2}} \right) \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k.
\] (9)

2.4. New Reliability Modelling of DCFP. In the above DCFP model from equations (5) to (9), only suppose that each shock loads will cause additional abrupt degradation, in other words, only considering the impact from random shocks to natural degradation process. In fact, the impact of natural degradation process to random shocks should also be considered, because the random shock may come more frequently with the system performance reduction. Therefore, the bidirectional effects between natural degradation and random shock are considered in this paper, and the following model \( M_2 \) is used to describe the degradation path of the \( i \)th component:
\[
Y_{i\theta}(t) = X_i(t) + f(X_i(t)S_i(t)) \tag{10}
\]
where \( f(X_i(t)) \) is an increasing function of natural degradation process \( X_i(t) \), it can describe the impact of natural degradation process to random shocks. In this paper, we assume \( f(X_i(t)) = a X_i(t) + b \), and we know that if \( a = 0, b = 1 \), the equation (11) turns into equation (5).
Then, the reliability function of the $i$th component for new DCFP is as
\[
R_i(t) = P\left( Y_{ix}(t) < H_i, \sum_{j=1}^{N(t)} W_{ij} < D_i \right) \\
= P\left( X_i(t) + (a X_i(t) + b) \sum_{j=1}^{N(t)} Y_{ij} < H_i, \sum_{j=1}^{N(t)} W_{ij} < D_i \right) \\
= \sum_{k=0}^{\infty} P\left( X_i(t) \left( 1 + a \sum_{j=1}^{N(t)} Y_{ij} \right) \\
+ b \sum_{j=1}^{N(t)} Y_{ij} < H_i, \sum_{j=1}^{N(t)} W_{ij} < D_i \right) | N(t) = k \)
\cdot P(N(t) = k).
\] (11)

Based on equations (6), (7), and (12), the reliability function of the $i$th component under the model $M_2$ can be obtained as
\[
R_i(t) = P(X_i(t) < H_i | N(t) = 0)P(N(t) = 0) \\
+ \sum_{k=1}^{\infty} \Phi\left( \frac{H_i - \mu_{W_i}}{\sigma_{W_i}} \right) \\
\cdot P\left( X_i(t) < H_i - b \sum_{j=1}^{N(t)} Y_{ij} + \sum_{j=1}^{N(t)} W_{ij} \right) | N(t) = k \\
\cdot P(N(t) = k), \\
= \Phi\left( \frac{H_i - \mu_{W_i}}{\sigma_{W_i}} \right) e^{-\lambda t} \\
+ \sum_{k=1}^{\infty} \Phi\left( \frac{D_i - k \mu_{W_i}}{\sqrt{k} \sigma_{W_i}} \right) \\
\cdot \left[ \int_{0}^{H_i} \Phi\left( \frac{H_i - \sum_{j=1}^{N(t)} Y_{ij}}{\sqrt{1 + az} \sigma_{i}(t)} \right) dF_{Z}^{(n)}(z) \right] \\
\cdot e^{-\lambda t} \frac{1}{k!} (\lambda t)^k,
\] (12)
where $F_i(z)$ is the distribution of the damage $Y_{ij}$ for shocks in the damage zone and $F_{Z}^{(n)}(z)$ is the $n$-fold Stieltjes convolution with $F_i(z)$ itself.

### 3. DCFP Model of the Series System

#### 3.1. Series System Modeling via the Element Model $M_1$

Figure 3 shows that a series system is made up of $n$ components. Only when each component for DCFP works properly, the system can run normally. That is, each component satisfied the total degradation $X_{ix}(t) < H_i$ and the cumulative magnitude $\sum_{j=1}^{N(t)} W_{ij} < D_i$, the series system can run normally.

![Figure 3: Series system example.](image)

\[
R(t) = P\left( \bigcap_{j=1}^{N(t)} W_{ij} < D_i \right) \\
\cdot P\left( \bigcap_{j=1}^{N(t)} Y_{ij} < H_i \right) \\
\cdot P\left( \bigcap_{j=1}^{N(t)} X_{ij} < N(t) \right)
\] (13)

By using the conditional probability, (13) can also be expressed as
\[
R(t) = \sum_{k=0}^{\infty} P\left( \bigcap_{j=1}^{N(t)} W_{ij} < D_i \right) \\
\cdot P\left( \bigcap_{j=1}^{N(t)} Y_{ij} < H_i \right) \bigcap \cdots \bigcap \left( \bigcap_{j=1}^{N(t)} X_{ij} < N(t) \right) | N(t) = k \)
\cdot P(N(t) = k).
\] (14)

Then, we can get the reliability function of the series system as
\[
R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left\{ P\left( \sum_{j=1}^{N(t)} W_{ij} < D_i \right) \\
\cdot P\left( \sum_{j=1}^{N(t)} Y_{ij} < H_i \right) \right\} | N(t) = k \}
\cdot \exp(-\lambda t)(\lambda t)^k.
\] (15)

According to (3) and (7), equation (15) can be shown as
\[
R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left\{ \Phi\left( \frac{D_i - k \mu_{W_i}}{\sqrt{k} \sigma_{W_i}} \right) \Phi\left( \frac{H_i - (k \mu_{Y_i} + \mu_{i}(t))}{\sqrt{k} \sigma_{Y_i}(t) + k \sigma_{Y_i}^2} \right) \right\} \\
\cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k.
\] (16)

For equation (16), another expression can also be obtained according to literature [27, 30]. Firstly, we introduce a conditional system reliability given the total number of shocks $N(t)$ as follows:
\[ R(t|N(t) = k) = \prod_{i=1}^{n} \Phi \left( \frac{D_i - k\mu_W}{\sqrt{k} \sigma_W} \right) \Phi \left( \frac{H_i - (k\mu_Y + \mu_i(t))}{\sqrt{\sigma_i^2(t) + k\sigma_Y^2}} \right). \]

\text{(17)}

By using the law of total probability, we can get

\[ R(t) = \sum_{k=0}^{\infty} R(t|N(t) = k)P(N(t) = k) \]

\[ = \sum_{k=0}^{\infty} \sum_{i=1}^{n} \Phi \left( \frac{D_i - k\mu_W}{\sqrt{k} \sigma_W} \right) \Phi \left( \frac{H_i - (k\mu_Y + \mu_i(t))}{\sqrt{\sigma_i^2(t) + k\sigma_Y^2}} \right) e^{-\lambda t} \left( \frac{\lambda t}{k!} \right)^k. \]

\text{(19)}

Next, we consider a special case for the series system with two components \((n=2)\). Based on equation (19), we can get

\[ R(t) = \sum_{k=0}^{\infty} \left\{ \Phi \left( \frac{D_1 - k\mu_W}{\sqrt{k} \sigma_W} \right) \Phi \left( \frac{H_1 - (k\mu_Y + \mu_1(t))}{\sqrt{\sigma_1^2(t) + k\sigma_Y^2}} \right) \right\} e^{-\lambda t} \left( \frac{\lambda t}{k!} \right)^k. \]

\text{(20)}

3.2. Series System Modeling via the Element Model \( M_2 \).

Similarly, suppose that each component satisfied the total degradation \( Y_{is} < H_i \) and the cumulative magnitude \( \sum_{j=1}^{N(t)} W_{ij} < D_i \), the series system can run normally. Then, we can get

\[ R(t) = P \left\{ \sum_{j=1}^{N(t)} W_{1j} < D_1, Y_{1j}(t) < H_1 \right\} \cap \cdots \cap \sum_{j=1}^{N(t)} W_{nj} < D_n, Y_{nj}(t) < H_n \right\}. \]

\text{(21)}

By using the conditional probability, (21) can also be expressed as

\[ R(t) = \sum_{k=0}^{\infty} \left\{ \sum_{j=1}^{N(t)} W_{1j} < D_1, X_1(t) + (aX_1(t) + b) \sum_{j=1}^{N(t)} Y_{1j} < H_1 \right\} \cap \cdots \cap \sum_{j=1}^{N(t)} W_{nj} < D_n, X_n(t) + (aX_n(t) + b) \sum_{j=1}^{N(t)} Y_{nj} < H_n \} N(t) = k \} \cdot \lim_{N(t) \to \infty} P(N(t) = k). \]

\text{(22)}

Then, we can get the reliability function of the series system as

\[ R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left\{ P \left( \sum_{j=1}^{N(t)} W_{ij} < D_i \right) P \left( X_i(t) + (aX_i(t) + b) \sum_{j=1}^{k} Y_{ij} < H_i \right) \right\} N(t) = k \} \cdot \lim_{N(t) \to \infty} P(N(t) = k). \]

\text{(23)}
According to (3) and (12), the equation (23) can be shown as

\[
R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left\{ \Phi \left( \frac{D_i - k\mu_{W_i}}{\sqrt{k}\sigma_{W_i}} \right) \int_{0}^{H} \Phi \left( \frac{H_i - bz - (1 + az)\mu_i(t)}{1 + az}\sigma_i(t) \right) dF_Z^{(i)}(z) \right\} \cdot e^{-\lambda t} \frac{\lambda^k}{k!}.
\]

(24)

Without loss of generality, we also consider a special case for the series system with two components \(n = 2\). Based on equation (24), we can get

\[
R(t) = \sum_{k=0}^{\infty} \left\{ \Phi \left( \frac{D_1 - k\mu_{W_1}}{\sqrt{k}\sigma_{W_1}} \right) \int_{0}^{H} \Phi \left( \frac{H_1 - bz - (1 + az)\mu_1(t)}{1 + az}\sigma_1(t) \right) dF_Z^{(1)}(z) \right\} \cdot e^{-\lambda t} \frac{\lambda^k}{k!}.
\]

\[
\Phi \left( \frac{D_2 - k\mu_{W_2}}{\sqrt{k}\sigma_{W_2}} \right) \int_{0}^{H} \Phi \left( \frac{H_2 - bz - (1 + az)\mu_2(t)}{1 + az}\sigma_2(t) \right) dF_Z^{(2)}(z) \}
\]

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(25)

4. DCFP Model of the Parallel System

4.1. Parallel System Modeling via the Element Model \(M_i\).

Figure 4 shows that a parallel system is made up of \(n\) components. Similarly, the system can run normally when at least one component satisfied the total degradation \(X_{ts}(t) < H_i\) and the cumulative magnitude \(\sum_{j=1}^{N(t)} W_{ij} < D_i\).

\[
R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left\{ 1 - R_i(t|N(t) = k) \right\} \cdot \frac{\lambda^k}{k!}.
\]

(27)

Similarly, to a series system, we can get conditional parallel system reliability given the number shocks \(N(t)\) as

\[
R(t|N(t) = k) = 1 - \prod_{i=1}^{n} \left\{ 1 - R_i(t|N(t) = k) \right\}.
\]

(26)

Then, the parallel system reliability is obtained as

\[
R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left\{ 1 - R_i(t|N(t) = k) \right\} \cdot \frac{\lambda^k}{k!}.
\]

(27)

Without loss of generality, we also consider a special case for the parallel system with two components \(n = 2\). Based on equation (27), we can get
5.1. Series-Parallel System Modeling via the Element Model $M_1$

Figure 5 shows a series-parallel system made up of $n$ subsystems, and the $i$th subsystem has $m_i$ components. Similarly, suppose that the $k^{th}$ load of the $j^{th}$ component of the $i$th subsystem $W_{ijk}$, assumed to be i.i.d. random variables distributed as a normal distribution, $W_{ijk} \sim N(\mu_{W_{ijk}}, \sigma_{W_{ijk}}^2)$, and assume that the corresponding shock damage sizes $Y_{ijk}$ are independently distributed random variables taking the

\[ R(t) = 1 - \sum_{k=0}^{\infty} \left\{ 1 - \Phi \left( \frac{D_1 - k \mu_{W}}{\sqrt{k} \sigma_{W}} \right) \int_0^t \Phi \left( \frac{H_1 - (k \mu_{W} + \mu_i(t))}{\sqrt{\sigma_i^2(t) + k \sigma_{W}^2}} \right) e^{-\lambda t} \frac{dt}{k!} \right\} \]
form as $Y_{ijk} \sim N(\mu_{Y_{ij}}, \sigma^2_{Y_{ij}})$. Suppose the degradation model of the $j^{th}$ component of the $i^{th}$ subsystem $X_{ij}(t)$ describes by the Gaussian process, $X_{ij}(t) \sim N(\mu_{ij}(t), \sigma^2_{ij}(t))$. Then, similarly, the reliability of this system at time $t$ is the probability that the system survives each of the $N(t)$ shock loads ($\sum_{k=1}^{N(t)} W_{ijk} < D_{ij}$, for $i = 1, 2, \ldots, n; j = 1, 2, \ldots, m_i; k = 1, 2, \ldots, N(t)$) and the total degradation is less than the threshold level ($X_{Sij}(t) < H_{ij}$) for each component.

By using the condition on the number of shocks, we can get conditional series-parallel system reliability as

$$R(t|N(t) = k) = \prod_{i=1}^{n} \left(1 - \prod_{j=1}^{m_i} \left[1 - R_{ij}(t|N(t) = k)\right]\right). \quad (31)$$

Then, we can get the system reliability as

$$R(t) = \sum_{k=0}^{\infty} \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left[1 - \prod_{j=1}^{m_i} \left[1 - R_{ij}(t|N(t) = k)\right]\right] \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k$$

$$= \sum_{k=0}^{\infty} \prod_{i=1}^{n} \sum_{j=1}^{m_i} \left[1 - \prod_{j=1}^{m_i} \left[1 - \Phi\left(\frac{D_{ij} - k\mu_{W_{ij}}}{\sqrt{k \sigma_{W_{ij}}}}\right)\right]\right] \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k. \quad (32)$$

Without loss of generality, we also consider a special case for the series-parallel system with four components ($n = 2$, $m_i = 2$). Based on equation (32), we can get

$$R(t) = \sum_{k=0}^{\infty} \prod_{i=1}^{2} \sum_{j=1}^{2} \left[1 - \prod_{j=1}^{2} \left[1 - \Phi\left(\frac{D_{ij} - k\mu_{W_{ij}}}{\sqrt{k \sigma_{W_{ij}}}}\right)\right]\right] \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k. \quad (33)$$

5.2. Series-Parallel System Modeling via the Element Model $M_2$. Similarly, suppose that each component satisfied the total degradation $Y_{ij}(t) < H_{ij}$ and the cumulative magnitude $\sum_{k=1}^{N(t)} W_{ijk} < D_{ij}$, the series-parallel system can run normally. Then, we can get

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**Figure 5: Series-parallel system example.**
Without loss of generality, we also consider a special case for the series-parallel system with four components \( n = 2, m_i = 2 \). Based on equation (21), we can get

\[
R(t) = \sum_{k=0}^{\infty} R(t|N(t) = k) \cdot P(N(t) = k)
\]

\[
= \sum_{k=0}^{\infty} \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m_i} \left[ 1 - R_{ij}(t|N(t) = k) \right] \right) \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k
\]

\[
R(t) = \sum_{k=0}^{\infty} \left( \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m_i} \left[ 1 - \Phi \left( \frac{D_{ij} - k\mu_{ij}}{\sqrt{k} \sigma_{ij}} \right) \right] \right) \int_0^H \Phi \left( \frac{H_i - bz - (1 + az)\mu_i(t)}{(1 + az)\sigma_i(t)} \right) dF_{Z_{ij}}(z) \right) \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k.
\]  

(35)
Without loss of generality, we also consider a special case for the parallel-series system with four components \((n = 2, m = 2)\). Based on equation (37), we can get

\[
R(t) = 1 - \sum_{k=0}^{\infty} \left[ \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m} R_{ij}(t) N(t) = k \right) \right] \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k
\]

\[
= 1 - \sum_{k=0}^{\infty} \left[ \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m} \Phi \left( \frac{D_{ij} - k \mu W_{ij}}{\sqrt{k} \sigma W_{ij}} \right) \right) \Phi \left( \frac{H_{ij} - (k \mu + \mu_j(t))}{\sqrt{\sigma^2_{ij}(t) + k \sigma^2_{ij}}} \right) \right] \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k.
\]

6.2. Parallel-Series System Modeling via the Element Model

\(M_2\). Similarly, suppose that each component satisfied the total degradation \(Y_{sij}(t) < H_{ij}\) and the cumulative magnitude \(\sum_{k=1}^{N(i)} W_{ijk} < D_{ij}\), the parallel-series system can run normally. Then, we can get

\[
R(t) = 1 - \sum_{k=0}^{\infty} \left[ \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m} \Phi \left( \frac{D_{ij} - k \mu W_{ij}}{\sqrt{k} \sigma W_{ij}} \right) \right) \Phi \left( \frac{H_{ij} - (k \mu + \mu_j(t))}{\sqrt{\sigma^2_{ij}(t) + k \sigma^2_{ij}}} \right) \right] \cdot \frac{e^{-\lambda t}}{k!} (\lambda t)^k.
\]
\[ R(t) = \sum_{k=0}^{\infty} R(t|N(t) = k) \cdot P(N(t) = k) \]

\[ = \sum_{k=0}^{\infty} \left( \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m_i} \Phi \left( \frac{D_{ij} - k\mu_{W_{ij}}}{\sqrt{k} \sigma_{W_{ij}}} \right) \right) \right) \int_0^{H_i} \Phi \left( \frac{H_i - bz - (1 + az)\mu_i}{(1 + az)\sigma_i(t)} \right) \]

\[ = 1 - \sum_{k=0}^{\infty} \left( \prod_{i=1}^{n} \left( 1 - \prod_{j=1}^{m_i} \Phi \left( \frac{D_{ij} - k\mu_{W_{ij}}}{\sqrt{k} \sigma_{W_{ij}}} \right) \right) \right) \int_0^{H_i} \Phi \left( \frac{H_i - bz - (1 + az)\mu_i}{(1 + az)\sigma_i(t)} \right) \]

\[ \left(39\right) \]
Without loss of generality, we also consider a special case for the parallel-series system with four components \((n = 2, m_i = 2)\). Based on equation (39), we can get

\[
R(t) = 1 - \sum_{k=0}^{\infty} \prod_{i=1}^{2} \left(1 - \Phi\left(\frac{D_{ij} - k\mu_{W_{ij}}}{\sqrt{k}\sigma_{W_{ij}}}\right)\right) \left[\int_0^{H} \Phi\left(\frac{H_j - bz - (1 + az)\mu_i(t)}{1 + az}\right) dF^{(2)}_Z(z)\right] e^{-\lambda t} \frac{(\lambda t)^k}{k!}.
\]

(40)
7. Numerical Example

In this section, a case as the fatigue crack growth data of an alloy [31] is taken to verify the effectiveness of the established model and method. The dataset consists of 20 experimental samples, and Table 1 lists all the data set information. In order to verify the proposed model, relevant information about the shock process needs to be given. In this paper, reasonable assumptions are made for the information of the shock process based on real data characteristics and literature.

Similar to the References [13, 31], for the all component, we suppose that \( W_i \sim N(2, 0.5) \), and \( Y_i \sim N(0.02, 0.01) \), the soft failure threshold \( H = 2.0 \) inches, the hard failure threshold \( D = 35 \) units, and the shock arrival rate \( \lambda = 0.5 \times 10^{-4} \). In addition, we suppose that \( a = 0.8, b = 1 \).

Figures 7 and 8 show fatigue crack growth paths of component 1 and component 2 in the data-set in Table 1, respectively. Figures 9 and 10 plot the mean and standard deviation of component 1 and component 2, respectively. Figures 9 and 10 imply that evolution of the two parameters (mean and standard deviation) can be appropriately fitted by a linear model. Similar to Reference [30], for the degradation process, we use the formulation \( \mu_i(t) = a_i + b_i t \) as the mean function of the \( i \)th component, and use the formulation \( \sigma_i(t) = c_i + d_i t \) as the standard deviation function of the \( i \)th component. Based on the dataset in Table 1, by using the
least square method, the unknown parameters $a_i$, $b_i$, $c_i$, and $d_i$ can be obtained, the results are as follows:

$$
\begin{align*}
\mu_1(t) &= 0.8643 + 5.7539 \times 10^{-6} t, \\
\sigma_1(t) &= -0.0055 + 8.1343 \times 10^{-7} t, \\
\mu_2(t) &= 0.8823 + 3.6158 \times 10^{-6} t, \\
\sigma_2(t) &= -0.0017 + 5.5458 \times 10^{-7} t,
\end{align*}
$$

(41)

where $t$ is the number of cycles.

7.1. System Reliability about the Two Components Series System. According to (19) and (24), the reliability curves of series system with random shock case and without shock case are plotted in Figure 11. From Figure 11, we can find that, when the random shocks are not considered, the reliability of series system almost keeps 1 when $t < 1.5 \times 10^5$ cycles, and when $t > 1.5 \times 10^5$ cycles, the reliability declines rapidly. If those random shocks are considered under the DCFP model $M_1$, we can find that the system reliability almost keeps 1 when $t < 0.75 \times 10^5$ cycles, and when $t > 0.75 \times 10^5$ cycles, the reliability declines rapidly. But, if the
bidirectional effects between natural degradation and random shock are considered under the new DCFP model $M_2$, we can find that the system reliability almost keeps 1 when $t < 0.25 \times 10^5$ cycles, and when $t > 0.25 \times 10^5$ cycles, the reliability declines more quickly. The results show that the random shocks can significantly accelerate the series system failure. In addition, the results also show that the bidirectional effects can significantly accelerate the series system failure. In other words, the impact of natural degradation process to random shocks should not be ignored.

In addition, some sensitivity analyses are also given under the new DCFP model $M_2$. Firstly, the system reliability curves under different soft failure thresholds are plotted in Figure 12. From Figure 12, it can be found that the soft failure threshold $H$ has an important effect on system reliability. Secondly, the system reliability curves under different hard failure thresholds are plotted in Figure 13. From Figure 13, it can be found that the hard failure threshold $D$ has an effect on system reliability. Compared with the hard failure threshold, the system reliability is more sensitive to the soft failure threshold. Thirdly, the sensitivity analysis of the shock arrival rate $\lambda$ is also given. The system reliability curves under different $\lambda$ are plotted in Figure 14. From Figure 14, we can find that the shock arrival rate $\lambda$ has a great impact on the series system reliability. We can also find that the reliability curves shift to the left when the shock arrival
rate $\lambda$ increases from 0.5 to 1. That is, the reliability of system will be lower when there is a higher arrival rate $\lambda$.

At last, the effect analysis about the component number is given. The different component number of series system is plotted in Figure 15. Without loss of generality, we assume that components 1 and 3 have the same degradation parameter with $\mu_1(t) = \mu_3(t) = 0.8643 + 5.7539 \times 10^{-6} t$, $\sigma_1(t) = \sigma_3(t) = -0.0055 + 8.1343 \times 10^{-7} t$, and they are different from component 2 with degradation parameter $\mu_2(t) = 0.8823 + 3.6158 \times 10^{-6} t$, $\sigma_2(t) = -0.0017 + 5.5458 \times 10^{-7} t$.

From Figure 15, in the series system, we can find that the reliability of component number $n = 1$ (1-component) is higher than component number $n = 2$ (2-component), and the reliability of component number $n = 2$ (2-component) is higher than component number $n = 3$ (3-component). That is to say, when the series system has the fewer components, the system has a higher reliability level.

7.2. System Reliability about the Two Components Parallel System. Similarly, according to equations (28) and (30), reliability curves of the parallel system with random shock case and without shock case are plotted in Figure 16. We can find that, when random shocks are not considered, the parallel system reliability almost keeps 1 when $t < 2.2 \times 10^5$ cycles, and when $t > 2.2 \times 10^5$ cycles, the reliability declines slowly. If those random shocks are considered under the DCFP model $M_1$, we can find that the system reliability almost keeps 1 when $t < 1.2 \times 10^5$ cycles, and when $t > 1.2 \times 10^5$ cycles, the reliability declines rapidly. But, if the bidirectional effects between natural degradation and random shock are considered under the new DCFP model $M_2$, we can find that the system reliability almost keeps 1 when $t < 0.6 \times 10^5$ cycles, and when $t > 0.6 \times 10^5$ cycles, the reliability declines more quickly. The results show that the random shocks can significantly accelerate the parallel-series system failure. In addition, the results also show that the bidirectional effects can significantly accelerate the parallel-series system failure.

In addition, the sensitivity analyses about the soft failure thresholds, the hard failure thresholds, and the shock arrival rate $\lambda$ are plotted in Figures 21–23, respectively. From Figures 21–23, we can find the soft failure threshold, the hard failure threshold, and the shock arrival rate all have an important impact on system reliability.

7.3. System Reliability about the Four Components Parallel-Series System. Without loss of generality, we assume that components 11 and 21 have the same degradation parameter with $\mu_{11}(t) = \mu_{21}(t) = 0.8643 + 5.7539 \times 10^{-6} t$, $\sigma_{11}(t) = \sigma_{21}(t) = -0.0055 + 8.1343 \times 10^{-7} t$, and they are different from component 12 and component 22 with degradation parameter $\mu_{12}(t) = \mu_{22}(t) = 0.8823 + 3.6158 \times 10^{-6} t$, $\sigma_{12}(t) = \sigma_{22}(t) = -0.0017 + 5.5458 \times 10^{-7} t$.

Similarly, according to equations (32) and (35), reliability curves of the parallel-series system with random shock case and without shock case are plotted in Figure 20. We can find that, when random shocks are not considered, the parallel-series system reliability almost keeps 1 when $t < 2.2 \times 10^5$ cycles, and when $t > 2.2 \times 10^5$ cycles, the reliability declines slowly. If those random shocks are considered under the DCFP model $M_1$, we can find that the system reliability almost keeps 1 when $t < 1.2 \times 10^5$ cycles, and when $t > 1.2 \times 10^5$ cycles, the reliability declines rapidly. But, if the bidirectional effects between natural degradation and random shock are considered under the new DCFP model $M_2$, we can find that the system reliability almost keeps 1 when $t < 0.6 \times 10^5$ cycles, and when $t > 0.6 \times 10^5$ cycles, the reliability declines more quickly. The results show that the random shocks can significantly accelerate the parallel-series system failure. In addition, the results also show that the bidirectional effects can significantly accelerate the parallel-series system failure.

In addition, the sensitivity analyses about the soft failure thresholds, the hard failure thresholds, and the shock arrival rate $\lambda$ are plotted in Figures 25–27, respectively. From Figures 25–27, we can find the soft failure threshold, the hard failure threshold, and the shock arrival rate all have an important impact on system reliability.
8. Conclusions
This paper considers the reliability of complex systems for the new DCFP model. Under the bidirectional effects between natural degradation and random shock, five types of reliability model based on Gauss stochastic degradation process and cumulative shock process were obtained, and some sensitivity analyses are given. A numerical example of fatigue crack growth dataset is taken to verify the effectiveness of the established model and method, and some sensitivity analyses are given.

This paper only considers the Gauss degradation process. Some other stochastic process can be used to model the degradation path in future research, such as Wiener process, Gamma process.

Data Availability
The data used to support the findings of this study are available from the corresponding authors upon request.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors’ Contributions
The authors contributed equally to the presented mathematical framework and the writing of the paper. All authors have read and approved the final manuscript.

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