ACCELERATOR, REACTOR AND ATMOSPHERIC 
NEUTRINO DATA: A THREE FLAVOUR OSCILLATION 
ANALYSIS

Srubarati Goswami $^a$, Kamales Kar $^b$, Amitava Raychaudhuri $^a$

$^a$ Department of Pure Physics, 
University of Calcutta, 
92 Acharya Prafulla Chandra Road, 
Calcutta 700 009, INDIA.

$^b$ Saha Institute of Nuclear Physics, 
1/AF, Bidhannagar 
Calcutta 700 064, INDIA.

P.A.C.S. Nos.: 14.60.Pq, 14.60.Lm, 96.40.Tv

ABSTRACT

We perform a three flavour analysis of the atmospheric, accelerator and reactor neutrino data from the Kamiokande, LSND and Bugey experiments respectively. Choosing the values of $\Delta m^2$ obtained from two flavour fits of the first two experiments, the allowed ranges of the three generation mixing angles are determined. The accelerator experiments CHORUS and NOMAD are found to be most sensitive to regions of the allowed parameter space which correspond to genuine three generation solutions for the atmospheric neutrino anomaly.

May 24, 1995
In the standard model of electroweak theory the neutrinos are assumed to be massless. But there is no compelling theoretical reason behind this assumption. If the neutrinos are massive then, as in the quark sector, the weak interaction basis of neutrinos may be different from the mass eigenstate basis – leading to mixing between different flavours. A way for probing such mixing and small neutrino masses is provided by neutrino oscillations. Two well known neutrino puzzles that can be explained by flavour oscillation of neutrinos are the solar neutrino problem and the atmospheric neutrino anomaly. The recent declaration by the Liquid Scintillator Neutrino Detector (LSND) collaboration [1] that they are observing an excess of $\nu_e$s (over the expected background) which can be attributed to $\nu_\mu - \nu_e$ oscillations has added a new dimension to the issue of neutrino mass and mixing. LSND is most sensitive to $\Delta m^2 \sim 6 eV^2$ [2] and the significance of this result for particle physics, astrophysics and cosmology has been investigated [3, 4]. One notes that the three phenomena mentioned above – namely, the solar neutrino problem, the atmospheric neutrino anomaly and the $\nu_\mu - \nu_e$ oscillations observed by the LSND group – require vastly different mass ranges. The solar neutrino problem can be explained either by Mikheyev-Smirnov-Wolfenstein oscillation [5] for $\Delta m^2 \sim 6 \times 10^{-6} eV^2$ and $\sin^2 2\theta \sim 7 \times 10^{-3}$ (non-adiabatic solution) and $\Delta m^2 \sim 9 \times 10^{-6} eV^2$ and $\sin^2 2\theta \sim 0.6$ (large mixing angle solution) [6] or by oscillation in vacuum for $\Delta m^2 \sim (0.45 - 1.2) \times 10^{-10} eV^2$ and $\sin^2 2\theta \sim (0.6 - 1.0)$ [7] in a two generation scenario. The atmospheric anomaly can be explained by either $\nu_\mu - \nu_e$ or $\nu_\mu - \nu_\tau$ oscillations in a two generation picture. The analysis of the new multi-GeV data as well as the previous sub-GeV data of the Kamiokande collaboration predicts the following best-fit parameters ($\Delta m^2, \sin^2 2\theta$) = ($1.8 \times 10^{-2} eV^2, 1.0$) for $\nu_\mu - \nu_e$ oscillation and ($1.6 \times 10^{-2} eV^2, 1.0$) for $\nu_\mu - \nu_\tau$ oscillation [8].

Although each experiment can be explained by two flavour neutrino oscillations, there are several motivations to go beyond this approximation. The LEP result that there are three light neutrinos is also supported by the requirements of nucleosynthesis in the early universe. In the quark sector, mixing between three generations is well
established. A natural question then is how do experiments constrain three neutrino mixing? We stress that in a realistic three flavour framework it is important to do a combined analysis to find out the allowed range of parameters rather than using separate two flavour schemes. In particular, this might uncover regions in the parameter space sensitive to the presence of the third generation which cannot be probed in the two flavour limit.

In this paper we perform a three flavour analysis of the atmospheric and LSND data assuming that the presently reported values will not change significantly as more results accumulate. The constraints obtained from the reactor experiments are also incorporated. We take the $\Delta m^2$'s as: $\Delta_{12} \simeq \Delta_{13} = 6eV^2$ in the LSND range and $\Delta_{23} = 10^{-2}eV^2$ as preferred by the atmospheric neutrino data. The $\simeq$ sign means we neglect $10^{-2}$ as compared to 6. It will become clear as we proceed that most of our analysis does not depend on this specific choice as long as the order of magnitude remains the same. The three mixing angles are allowed to cover the whole range from 0 to $\pi/2$. For atmospheric neutrinos, we find, in addition to the two flavour results, genuine three generation solutions where both $\nu_\mu - \nu_e$ and $\nu_\mu - \nu_\tau$ oscillation channels simultaneously contribute. The implications of the allowed areas thus obtained for the accelerator experiments CHORUS and NOMAD searching for $\nu_\mu - \nu_\tau$ oscillations are also discussed. Such an analysis for constraining the mixing angles has been performed in [9] under the approximation of an effective two flavour interpretation of the atmospheric neutrino problem either in the $\nu_\mu - \nu_e$ or $\nu_\mu - \nu_\tau$ oscillation mode, instead of a full three flavour investigation. A detailed analysis combining the accelerator, reactor, solar and atmospheric neutrino data had been carried out earlier (pre-LSND) [10] taking a different spectrum for $\Delta m^2$ and assuming the mixing angles to be less than $\pi/4$.

The measurement of atmospheric neutrino fluxes is being carried out by the following groups – Kamiokande, IMB, Fréjus, Nusex and Soudan2. So far, data of most statistical significance have been collected by the Kamiokande and the IMB collaborations. To reduce the uncertainty in the absolute flux values the usual practice is to
present the ratio of ratios \( R \) which is defined as,

\[
R = \frac{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)_{\text{obsvd}}}{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)_{\text{MC}}}
\]

where MC denotes the Monte-Carlo simulated ratio. For neutrinos of energy less than \( \sim 1 \) GeV, IMB finds \( R = 0.54 \pm 0.05 \pm 0.12 \) \cite{11} in agreement with the Kamiokande data \( R = 0.60^{+0.06}_{-0.05} \pm 0.05 \) in this energy range \cite{8, 12}. Recently the Kamiokande collaboration has published the results of the measurement of the flux ratio in the multi-GeV energy range \cite{8}. They found \( R = 0.57^{+0.08}_{-0.07} \pm 0.07 \) in good agreement with the sub-GeV value. All these data show that \( R \) is smaller than the expected value of unity, a result that might be explained by neutrino flavour oscillation \cite{13}. Another aspect of this measurement that can independently point towards neutrino oscillation is the dependence of \( R \) on the zenith-angle. The multi-GeV Kamiokande data reveals a dependence on the zenith-angle unlike the sub-GeV data, though the statistical significance of this result has been questioned \cite{14}. For the purpose of this paper we use the sub-GeV Kamiokande results.

The LSND group searches for \( \overline{\nu}_\mu \to \overline{\nu}_e \) oscillations using \( \overline{\nu}_e \) appearance. The \( \overline{\nu}_e \)s produce neutrons \emph{via} the reaction \( \overline{\nu}_e p \to e^+ n \) which in turn are captured by protons producing a 2.2 MeV \( \gamma \). An excess of beam-on events with a \( \gamma \) of the above energy in time and space coincidence with an electron in the energy range \( 36 \text{ MeV} < E_e < 60 \) MeV is considered as a signal for \( \overline{\nu}_e \). The mean source-detector distance is 30 metres. The initial LSND data reports an excess of \( 16.4^{+9.7}_{-8.9} \pm 3.3 \) events over the estimated background which, if interpreted in terms of neutrino oscillations, corresponds to a probability \( P_{\overline{\nu}_\mu \overline{\nu}_e} \) of \( (0.34^{+0.20}_{-0.18} \pm 0.07)\% \).

Other appearance experiments searching for \( \overline{\nu}_\mu \to \overline{\nu}_e \) oscillations are KARMEN at the ISIS spallation neutron facility \cite{15} and the BNL-E776 \cite{16}. These experiments are consistent with no neutrino oscillation. KARMEN has so far quoted an upper limit on the oscillation probability as \( P_{\overline{\nu}_\mu \overline{\nu}_e} \leq 3.1 \times 10^{-3} \) (90\% C.L.) whereas from the two flavour exclusion areas presented by BNL one gets \( P_{\overline{\nu}_\mu \overline{\nu}_e} \leq 1.5 \times 10^{-3} \) (90\% C.L.).
ref. [1] the LSND group has shown that some of the areas allowed by them in a two
flavour analysis are disfavoured by KARMEN and BNL-E776. In this paper we confine
ourselves to the LSND data for constraining the parameters.

Reactor experiments searching for neutrino oscillation are GÖSGEN, KRASNO-
YARSK and Bugey. These experiments provide exclusion regions in the \( \Delta m^2 - \sin^2 2\theta \) parameter space by non-observance of neutrino oscillation. The maximum exclusion
is by Bugey which measures the spectrum of \( \nu_e \), coming from the Pressurized Water
Reactors running at the Bugey nuclear power plant, at 15, 40, and 95 metres using
neutron detection techniques. The 90\% C.L. exclusion contour implies \( 1 - P_{\nu_e\nu_e} \leq 0.05 \).

The general expression for the probability that an initial \( \nu_\alpha \) of energy \( E \) gets
converted to a \( \nu_\beta \) after travelling a distance \( L \) in vacuum is

\[
P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta j} U_{\alpha j} \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right)
\]

where \( \lambda_{ij} = 2.47 m(E_\nu/MeV)(eV^2/\Delta_{ij}) \), \( \Delta_{ij} = m_j^2 - m_i^2 \). The actual forms of the
various survival and transition probabilities depend on the spectrum of \( \Delta m^2 \) assumed
and the choice of the mixing matrix \( U \) relating the flavour eigenstates to the mass
eigenstates. The most suitable parametrisation of \( U \) for the mass spectrum chosen
by us is \( U = R_{13}R_{12}R_{23} \) where \( R_{ij} \) denotes the rotation matrix in the \( ij \)-plane. This
yields:

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13}c_{23} - s_{13}s_{23} & c_{13}s_{12}s_{23} + s_{13}c_{23} \\
-s_{12} & c_{12}c_{23} & c_{12}s_{23} \\
-s_{13}c_{12} & -s_{13}s_{12}c_{23} - c_{13}s_{23} & -s_{12}s_{13}s_{23} + c_{13}c_{23}
\end{pmatrix}
\]

where \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \). We have assumed CP-invariance so that \( U \) is real.

The above choice of \( U \) has the advantage that \( \theta_{23} \) does not appear in the expressions
for the probability for LSND and Bugey. We now turn to the implications of the above
mixing matrix and the chosen mass ranges on the various probabilities.
(i) LSND
In order to see the impact of three neutrino generations, we first note that for the energy and length scales relevant for LSND $\lambda_{23} >> L$ and the term involving $\sin^2(\pi L/\lambda_{23}) \to 0$. Further, $\lambda_{13} \simeq \lambda_{12}$ and (2) simplifies to

$$P_{\nu_\mu \nu_e} = 4c_{12}^2 s_{12}^2 c_{13}^2 \sin^2(\pi L/\lambda_{12})$$

(4)

(ii) Bugey
For Bugey, the neutrino energy ranges from 2.8 - 7.8 MeV whereas $L$ is typically $\sim 40$ metres. Then $\lambda_{23} >> L$, so that $\sin^2(\pi L/\lambda_{23}) \to 0$. On the other hand $\lambda_{12} = \lambda_{13} << L$ so that $\sin^2(\pi L/\lambda_{12})$ and $\sin^2(\pi L/\lambda_{13})$ average out to 1/2. Then the relevant probability is

$$P_{\nu_\mu \nu_e} = 1 - 2c_{13}^2 c_{12}^2 + 2c_{13}^4 c_{12}^4$$

(5)

(iii) Atmospheric neutrinos
In a three flavour mixing scheme (3) is given in terms of the neutrino transition and survival probabilities as

$$R = \frac{P_{\nu_\mu \nu_\mu} + r_{MC} P_{\nu_\mu \nu_e}}{P_{\nu_\mu \nu_e} + \frac{1}{r_{MC}} P_{\nu_\mu \nu_e}}$$

(6)

where $r_{MC} = (\nu_e + \overline{\nu}_e)/(\nu_\mu + \overline{\nu}_\mu)$ as obtained from a Monte-Carlo simulation. Notice that for neutrinos in the energy range $\sim (0.1 - 1)$ GeV travelling through a distance ranging from $\sim (10 - 10^4)$ km, $\lambda_{12} = \lambda_{13} << L$ and $\sin^2(\pi L/\lambda_{12})$ and $\sin^2(\pi L/\lambda_{13})$ can be replaced by their average value 1/2. Taking this into account, the probabilities appearing in (3) can be expressed as

$$P_{\nu_\mu \nu_e} = 1 - 2c_{13}^2 c_{12}^2 + 2c_{13}^4 c_{12}^4 - 4(c_{13} s_{12} c_{23} - s_{13} s_{23})^2 (c_{13} s_{12} s_{23} + s_{13} c_{23})^2 \sin^2(\pi L/\lambda_{23})$$

(7)

$$P_{\nu_\mu \nu_\mu} = 2c_{13}^2 c_{12}^2 s_{12}^2 - 4c_{12}^2 c_{23} s_{23} (c_{13} s_{12} c_{23} - s_{13} s_{23}) (c_{13} s_{12} s_{23} + s_{13} c_{23}) \sin^2(\pi L/\lambda_{23})$$

(8)

$$P_{\nu_\mu \nu_e} = 1 - 2c_{12}^2 s_{12}^2 - 4c_{12}^2 c_{23}^2 s_{23}^2 \sin^2(\pi L/\lambda_{23})$$

(9)

The results of the combined analysis of the above three experiments are presented in figs. 1 and 2 in the large $s_{13}^2$ and small $s_{13}^2$ limits respectively. It is sufficient to
consider these limits as the allowed values of $s_{13}^2$ are confined in these ranges. As seen in (4), the parametrisation chosen for the mixing matrix $U$ ensures that the probability relevant for LSND is independent of the mixing angle $s_{23}$. From the value of $P_{\nu_{\mu}\nu_e}$ quoted by the LSND group [1] one can find the allowed area in the $s_{12}^2 - s_{13}^2$ parameter space for fixed values of the ratio $\Delta m^2 L/E$. The following constraints are found:

for $s_{12}$ very small ($\sim 0$) or very large ($\sim 1$), $s_{13}$ ranges from $0 \leq s_{13} < 1$ while for intermediate values of $s_{12}$, only very large $s_{13}$ values are allowed. This is between the solid lines in fig. 1 (2) for large (small) values of $s_{13}$, in the limit of $\sin^2(\pi L/\lambda_{12}) \sim 1$. From eq. (5) the probability for Bugey is also a function of the same mixing angles $s_{12}$ and $s_{13}$ only, so that, using their result one can further rule out a portion of the parameter space – namely, intermediate $s_{13}$ values at small $s_{12}$ – which were allowed by LSND. This is shown by the dashed lines in figs. 1 and 2 implying the following limits for small $s_{12}^2 \,(< 0.0018)$: either $s_{13}^2 > \sim 0.97$ (fig. 1) or $s_{13}^2 < 0.026$ (fig. 2). In the other regions of the parameter space the LSND data puts more severe constraints than Bugey.

Our approach next is to determine how much of the combined allowed area from LSND and Bugey is consistent with the atmospheric data for fixed values of $s_{23}$. The sub-GeV Kamiokande data implies

$$0.48 \leq R \leq 0.73 \,(90\%\text{C.L.}) \quad (10)$$

Imposing this constraint, one finds that the allowed parameter space (shown shaded in figs. 1 and 2) depends on the chosen $s_{23}^2$. In general there are three regions:

(i) The large $s_{13}^2 \,(> \sim 0.97)$ - small $s_{12}^2 \,(< \sim 0.1)$ region shown in fig. 1. In this limit it is the $\nu_{\mu} - \nu_e$ oscillation that dominates. Considering the limiting case of $s_{12} \to 0$ and $s_{13} \to 1$, the relevant probabilities assume the forms:

$P_{\nu_e\nu_e} \simeq 1 - 2c_{23}^2 s_{23}^2$, $P_{\nu_{\mu}\nu_e} \simeq 2c_{23}^2 s_{23}^2$, $P_{\nu_{\mu}\nu_{\mu}} \simeq 1 - 2c_{23}^2 s_{23}^2$

From these expressions it is clear that in this limit $P_{\nu_{\mu}\nu_e} \simeq 0$.

(ii) The large $s_{13}^2$ and intermediate $s_{12}^2$ zone also shown in fig. 1. To understand the
transitions in this range we examine the various probabilities in the limit $s_{13}^2 \to 1$. In this limit eqs. (7) - (9) become

$$P_{\nu_e \nu_e} \simeq 1 - 2c_{23}^2 s_{23}^2, \quad P_{\nu_\mu \nu_e} \simeq 2c_{12}^2 c_{23} s_{23}^2, \quad P_{\nu_\mu \nu_\mu} \simeq 1 - 2c_{12}^2 s_{12}^2 - 2c_{23}^4 s_{23}^4$$

This is the region where the depletion can be due to both the channels simultaneously excepting in the special case of $s_{23} \to 0$ when this reduces to solely $\nu_\mu - \nu_\tau$ oscillation. From fig. 1 one also notices that irrespective of the choice of $s_{23}$, large values of $s_{12}^2$ around $\sim (0.85-1)$ are disfavoured by the atmospheric data. In this region $\nu_\tau - \nu_\tau$ conversion is effective.

(iii) The small $s_{12}^2 - s_{13}^2$ zone – $0 < s_{12}^2 < 1.8 \times 10^{-3}$, $0 \leq s_{13}^2 \leq 0.01$ – a look at the various survival and transition probabilities reveals that this is a region where the depletion is mainly due to $\nu_\mu - \nu_\tau$ oscillation. This can be easily seen by considering the limiting cases $s_{12}, s_{13} \to 0$, when eqs. (7) - (9) give $P_{\nu_e \nu_e} \simeq 1, P_{\nu_\mu \nu_e} \simeq 0, P_{\nu_\mu \nu_\mu} \simeq 1 - 2c_{23}^2 s_{23}^2$. Substituting these in (10) one finds $0.162 < s_{23}^2 < 0.838$. There is a sharp cut-off as $s_{23}^2$ crosses 0.162 and for practically all intermediate values upto 0.838, the whole of the parameter space allowed by LSND and Bugey is consistent with the atmospheric neutrino data. Thus in this regime we show the allowed region for only one representative $s_{23}^2$. We have numerically checked that the allowed region is the same as the one presented in fig. 2 for all other $s_{23}^2$ in the above range.

In our analysis we have fixed $\Delta_{12} \simeq \Delta_{13}$ at $6eV^2$, where LSND is most sensitive and $\sin^2(\pi L/\lambda_{12}) \to 1$, maximising the oscillatory term. As discussed in [4] it remains to be seen what best-fit value, consistent with KARMEN and BNL-E776, emerges when more data is accumulated. Our results remain unchanged as long as it is permissible to use the above limit.

For the atmospheric neutrino case we approximate the $\sin^2(\pi L/\lambda_{23})$ factor by its averaged value 0.5 as is often done in the context of the sub-GeV data [18, 19, 20]. This approximation can be improved by an averaging over the incident neutrino energy spectrum, the zenith-angle of the beam as well as the final lepton energy [18, 11]. $r_{MC}$ is taken to be 0.45 from a detailed Monte-Carlo simulation including the effects of
muon polarisation [21].

Finally let us discuss the implications of the parameter space allowed by the Kamiokande atmospheric neutrino, LSND and Bugey data for the $\nu_\mu - \nu_\tau$ oscillation search at CHORUS [22] and NOMAD [23]. These experiments use the $\nu_\mu$ beam from the CERN SPS with the mean energy $\sim 30$ GeV and the approximate source-detector distance is 0.8 km so that $\lambda_{23} \gg L$ and

$$P_{\nu_\mu \nu_\tau} = 4c_{12}^2s_{12}^2s_{13}^2\sin^2(\pi L/\lambda_{13})$$

(11)

With the CERN SPS designed to deliver $2.4 \times 10^{19}$ protons, CHORUS and NOMAD are sensitive to a minimum oscillation probability of $10^{-4}$. For $\Delta_{12} = \Delta_{13}$ in the LSND range of $\sim 6eV^2$, $\sin^2(\pi L/\lambda_{13}) \sim 0.04$, whence (11) is $P_{\nu_\mu \nu_\tau} \simeq 0.16c_{12}^2s_{12}^2s_{13}^2$. Then for the three allowed regions in the $s_{12}^2 - s_{13}^2$ plane one gets:

(i) In the large $s_{13}^2$, small $s_{12}^2$ zone $P_{\nu_\mu \nu_\tau}$ can be slightly greater than $10^{-4}$ being marginally within the reach of these experiments. This is the $\nu_\mu - \nu_e$ oscillation zone for atmospheric neutrinos.

(ii) For large $s_{13}^2$ and intermediate values of $s_{12}^2$, $P_{\nu_\mu \nu_\tau}$ is $\sim 10^{-2}$, which is well within the reach of CHORUS and NOMAD. Recall that this is the genuine three generation oscillation regime for atmospheric neutrinos where both $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_e$ modes are operative, excepting for the special case of $s_{23} \simeq 0$ for which it is just $\nu_\mu - \nu_\tau$.

(iii) In the limit of both $s_{12}^2, s_{13}^2$ small, $P_{\nu_\mu \nu_\tau}$ is very small and this regime, where the atmospheric anomaly is due to $\nu_\mu - \nu_\tau$ oscillation, cannot be probed by CHORUS and NOMAD.

For the chosen values of the mass-differences a simultaneous solution to the solar neutrino problem is unobtainable unless one invokes a sterile neutrino. Work is in progress in this direction [24].

In conclusion, we have obtained the mixing angles compatible with atmospheric, LSND and reactor experiments (in particular Bugey) in the context of three flavour mixing. Keeping $\Delta m^2$ fixed at the best fit values obtained from two generation analyses
of the LSND and atmospheric data, we find three regions of parameter space that can account for all three experiments simultaneously. Our results differ from an analysis presented in [9] in that we find the mixing angles to be less restricted. Our method, which takes into account the possibility that the depletion of the atmospheric neutrinos can be simultaneously due to $\nu_\mu - \nu_e$ and $\nu_\mu - \nu_\tau$ oscillations, is more general and includes the constraints obtained in [9] as a special case. A direct comparison of the values obtained for the mixing angles is, however, not proper because the definitions of the mixing matrices are different. The sensitivity of the accelerator based neutrino oscillation experiments at CERN, CHORUS and NOMAD, is different in the three allowed zones and thus they can distinguish between these sectors of the parameter space. We find that CHORUS and NOMAD are most sensitive to that part of the combined allowed area where the atmospheric neutrino anomaly is due to $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_e$ oscillation modes simultaneously.

S.G. is supported by the Council of Scientific and Industrial Research, India while A.R. has been supported in part by the Department of Science and Technology and Council of Scientific and Industrial Research, India. We thank S. Mohanty for sending us some of the useful literature.
FIGURE CAPTIONS

Figure 1: The 90 % C.L. allowed region in the $s_{12}^2 - s_{13}^2$ plane from LSND is between the solid lines, that from Bugey is above the dashed line while the combined allowed area including the Kamiokande sub-GeV data is shown shaded.

Figure 2: Same as in fig. 1 excepting the region allowed from Bugey is below the dashed line.
References

[1] W.C. Louis, Nucl. Phys. (Proc. Suppl.) B38, 229 (1995); C. Athanassopoulos et al., nucl-ex/9504002 (1995).

[2] D.O. Caldwell, Nucl. Phys. (Proc. Suppl.) B38, 375 (1995).

[3] J.R. Primack et al., Phys. Rev. Lett. 74, 2160 (1995); G.M. Fuller, J.R. Primack and Y.Z. Qian, astro-ph/9502081; D.O. Caldwell and R.N. Mohapatra, Preprint No. UCSB-HEP-95-1, hep-ph/9503316.

[4] G. Raffelt and J. Silk, hep-th/9502306.

[5] L. Wolfenstein Phys. Rev. D34 969 (1986); S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl Phys. 42(6) 913 (1985); Nuovo Cimento 9c 17 (1986).

[6] N. Hata and P. Langacker, Phys. Rev. D50, 632 (1994).

[7] N. Hata, Univ. of Pennsylvania Preprint No. UPR-0605T, 1994.

[8] Y. Fukuda et al., Phys. Lett. B335, 237 (1994).

[9] H. Minakata, Preprint No. TMUP-HEL-9502, March 15, 1995.

[10] G.L. Fogli, E. Lisi and D. Montanio, Phys. Rev. D49, 3626 (1994).

[11] D. Casper et al., Phys. Rev. Lett. 66, 2561 (1991); R. Becker-Szendy et al., Phys. Rev. D46, 3720 (1992).

[12] K. S. Hirata et al., Phys. Lett. B280, 146 (1992).

[13] The usage of the ‘ratio of ratios’ R as a valid indicator of the neutrino anomaly has been recently critically examined. see G.L. Fogli and E. Lisi, Institute for Advanced Study report IASSNS-AST 95/21 (unpublished).
[14] D. Saltzberg, Report no. hep-ph-9504343 (unpublished).

[15] B. Seligmann in Proc. of the XXVII Int. Conf. on High Energy Physics (Glasgow), Eds. P.J. Bussey and I.G Knowles, Inst. of Phys. Publising, Bristol, (1995) p683; G. Drexlin, Prog. in Part. and Nucl. Phys., 32, 375 (1994); B. Armbruster et al. (KARMEN Collaboration), Nucl. Phys. (Proc. Suppl.) B38, 235 (1995).

[16] L. Borodovsky et al., Phys. Rev. Lett. 68, 274 (1992).

[17] B. Achkar et al., Nucl. Phys. B434, 503 (1995).

[18] V. Barger and K. Whisnant, Phys. Lett. B209, 365 (1988).

[19] A. Acker, J.G. Learned, S. Pakvasa and T.J. Weiler, Phys. Lett. B298, 149 (1993).

[20] A. Acker, A.B. Balantekin and F. Loreti, Preprint No. Mad/NT/93-07, MAD/PH/# 774, July (1993).

[21] G. Barr, T.K. Gaisser and T. Stanev, Phys. Rev D39 3532 (1989); T.K. Gaisser, T. Stanev and G. Barr, ibid D38, 85 (1988).

[22] M. de Jong et al., CERN-PPE/93-131 (1993); N. Armenise et al., CERN-SPSC/90-42 (1990).

[23] P. Astier et al., CERN-SPSLC/91-21, CERN-SPSLC/91-48, CERN-SPSLC/P261 Add.1 (1991).

[24] S. Goswami, in preparation.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9505395v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9505395v1