Nonlinear Interactions of Gravitational Wave with Matter in Magnetic-type Maxwell-Vlasov Description

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The interactions of gravitational waves with interstellar matter, dealing with resonant wave-particle and wave-wave interactions, are considered on the basis of magnetic-type Maxwell-Vlasov equations. It is found that the behavior of the fields, involving the “gravitoelectromagnetic” or “GEM” fields, the perturbed density field and self-generated gravitomagnetic field with low frequency, can be described by the nonlinear coupling equations Eqs. (6.10)-(6.12). Numerical results show that they may collapse. In other words, due to self-condensing, a stronger GME fields could be produced; and they could appear as the gravitational waves with high energy reaching on Earth. In this case, Weber results, perhaps, are acceptable.

PACS numbers: PACS 04.40.-b, 04.30.-w, 04.80.Nn

I. INTRODUCTION

It has often been noted that magnetism can be understood as the consequence of electrostatics plus Lorentz invariance. Similarly, Newtonian gravity together with Lorentz invariance in a consistent way must include a gravitomagnetic field. This is the case of gravitoelectromagnetic form of the Einstein equations in a medium. It is well known that in the slow-motion limit of general relativity, accurate to post-Newtonian (PN) order, where the field equations can be reduced as:

\[ \nabla^2 \phi = 4\pi G \rho, \tag{1.1} \]

\[ \nabla^2 A = 16\pi G \rho v/c; \tag{1.2} \]

moreover the harmonic gauge condition and the force on a unit mass reduced to

\[ \nabla \cdot A + \frac{4}{c} \frac{\partial \phi}{\partial t} = 0, \]

\[ \frac{dv}{dt} \approx -\nabla \phi - \frac{1}{c} \frac{\partial A}{\partial t} + \frac{v}{c} \times \nabla \times A, \]

where \( \phi \) is the Newtonian gravitational potential, \( A_i/c = g_{i0} \) is the mixed metric and \( G \) is in the Newton’s constant. One defines the GEM fields via

\[ E_g = -\nabla \phi - \frac{1}{c} \frac{\partial A}{\partial t}, \quad \nabla \times A = B_g, \tag{1.3} \]

in direct analogy with electromagnetism; it follows from these definitions the Maxwell-type field equations in the continuous medium:

\[ \nabla \times B_g = \frac{16\pi}{c} j_g + \frac{4}{c} \frac{\partial E_g}{\partial t}, \quad \nabla \cdot D_g = 4\pi \rho_g, \tag{1.4} \]

\[ \nabla \times E_g = -\frac{1}{c} \frac{\partial B_g}{\partial t}, \quad \nabla \cdot B_g = 0 \tag{1.5} \]
where \( \rho_g = -G\rho \) is “matter density” and \( \mathbf{j}_g = -G\rho \mathbf{v} = \rho_g \mathbf{v} \) is “matter current”. Then force on a unit mass has Lorentz-type form
\[
\frac{d\mathbf{v}}{dt} \approx \mathbf{E}_g + \frac{\mathbf{v}}{c} \times \mathbf{B}_g.
\] (1.6)

And all terms of \( O(v^4/c^4) \) are neglected in the above analysis. Now one decompose the total matter density and current in Eq. (1.4): \( \rho_g = \hat{\rho}_g + \rho_{g0} \) (or \( \rho = \hat{\rho} + \rho_0 \)), \( \mathbf{j}_g = \hat{\mathbf{j}}_g + \mathbf{j}_{g0} \), in which the \( \rho_{g0} \) (or \( \rho_0 \)) and \( \mathbf{j}_{g0} \) are the parts of a external field source in the medium, correspond to a local additional mass disturbance, for example, by nonlinear interactions between the fields and the medium, or some test particles. In this case it is convenient to define the vector quantity \( \mathbf{D}_g(t, \mathbf{r}) \) through ( in what follows one neglects the marks “~” in \( \hat{\rho}_g \) and \( \hat{\mathbf{j}}_g \) for simplicity)
\[
\mathbf{D}_g = \mathbf{E}_g + 4\pi \int_{-\infty}^{t} dt' \mathbf{j}_g(t', \mathbf{r});
\] (1.7)

by use of Eq. (1.7) and the mass continuity equation,
\[
\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot \rho_\alpha \mathbf{v} = 0, \quad (\alpha = g, g_0)
\]
Eqs. (1.4) and (1.5) are deduced to
\[
\nabla \times \mathbf{B}_g = \frac{16\pi}{c} \mathbf{j}_{g0} + \frac{4}{c} \frac{\partial \mathbf{D}_g}{\partial t}, \quad \nabla \cdot \mathbf{D}_g = 4\pi \rho_{g0},
\] (1.8)
\[
\nabla \times \mathbf{E}_g = -\frac{1}{c} \frac{\partial \mathbf{B}_g}{\partial t}, \quad \nabla \cdot \mathbf{B}_g = 0.
\] (1.9)

A dimensional analysis of equations (1.1), (1.3), (1.7) and (1.9) suggests that it is insightful to use the CGS units, since these field variables have the following dimensions:
\[
[D_g] \sim [E_g] \sim [B_g] \sim [\nabla \varphi] \sim v/s,
\]
thus, one has
\[
[E_g^2/G] \sim [B_g^2/G] \sim [\rho E_g^2/\rho G] \sim [\rho v^2];
\]
by substituting
\[
\mathbf{E} = -\mathbf{E}_g/4\sqrt{G}, \quad \mathbf{B} = -\mathbf{B}_g/4\sqrt{G}, \quad \mathbf{D} = -\mathbf{D}_g/4\sqrt{G},
\]
\[
\mathbf{j} = -\mathbf{j}_g/\sqrt{G} = \sqrt{G/\rho} \mathbf{v}, \quad \hat{\rho} = -\rho_g/\sqrt{G} = \sqrt{G/\rho},
\] (1.10)
the GEM field equations can be expressed in the standard form,
\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}_0 + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},
\] (1.11)
\[
\nabla \cdot \mathbf{D} = \pi \hat{\rho}_0, \quad \nabla \cdot \mathbf{B} = 0;
\] (1.12)
then Eqs. (1.7) and (1.6) become

$$D = E + \pi \int_{-\infty}^{t} dt' j(t', r),$$  \hspace{1cm} (1.13)

$$\frac{dv}{dt} \approx -4\sqrt{G} \left[ E + \frac{v}{c} \times B \right].$$  \hspace{1cm} (1.14)

To consider the responses of the medium on the GEM fields, one must introduce material relation, which describes the GEM properties of the medium. In view of Eq. (1.13), the states of the medium not only depend on a given time-space point \((t, r)\), but also depend on previous times and at any point of the medium. Hence, by general reasoning in physics (independent of a specific model for the medium) one can state that this is a no-local linear relation in the limit of linear response; whose Fourier representation is

$$j_i(\omega, k) = \sigma_{ij}(\omega, k) E_j(\omega, k).$$  \hspace{1cm} (1.15)

In special, for common continuous medium, where the “spatial dispersion” (dependency on \(k\)) is not important, the relation (1.15) is deduced to \((\sigma_{ij} \rightarrow \sigma_{ij})\)

$$j(\omega) = \sigma(\omega) E(\omega),$$

i.e. an “Ohm’s gravitational law” \(\square\), where \(\sigma\) is gravitational conductivity (see Refs \([10,11]\) for details). Then by use of Eqs. (1.13) and (1.15), one has

$$D_i(\omega, k) = \varepsilon_{ij}(\omega, k) E_j(\omega, k),$$  \hspace{1cm} (1.16)

here \(\varepsilon_{ij}\), called the dielectric tensor, is determined as

$$\varepsilon_{ij}(\omega, k) = \delta_{ij} + \frac{\pi i}{\omega} \sigma_{ij}(\omega, k) \quad (\omega \neq 0).$$  \hspace{1cm} (1.17)

In the next section we shall show that the material relations Eqs. (1.15) and (1.16) are relevant for GEM fields.

It has often been noted that the effects of GR can largely be understood and treated easily within the GEM framework. For example, the famous Lense-Thirring precession effect of GR is simpler and clearer to use the GEM equations \([12]\); and the gravitomagnetism is a useful insight for understanding the Schiff effect \([12]\) and the “Faraday” effect \([13]\). In special, just from our electromagnetic experience, we can infer that at a distance should present gravitational waves predicted by GR. Therefore, the existence of the GEM fields is equivalent of the existence of the gravitational waves predicted by GR \([13]\).

The many efforts that have been made to detect gravitational waves have so far given no convincing evidence that they have actually been seen. In the late 1960s and early 1970s, Weber announced that he had recorded simultaneous oscillations in detectors 1000 km apart, waves he believed originated from an astrophysical event. But many physicists were suspicious of the results that were several orders of magnitude higher than were theoretically predicted. In addition, observed high energies of gravitational waves by Weber have not been confirmed by these independent measurements. This is perhaps, due to the fact that gravitational waves with high energies are very rarer than many physicists had expected. On the other hand, it is highly possible that the gravitational waves can get largely increase rate as the waves interact with interstellar matter, so that Weber’s result, perhaps, are acceptable.

In order to study the nonlinear effects on a very larger scale, where the mean free path for collision between the particles of the matter is larger compared with the characteristic length appearing in the problem, a kinetic treatment is required: The physical systems should be described by the magnetic-type Maxwell-Vlasov equations. The small amplitude and high-frequency approximation are employed for the gravitational waves, such that perturbed techniques can be applied. The effects deal with resonant wave-particle and wave-wave interactions, inhomogeneities of the matter distribution and nonlinear self-collapsing. A previous work \([15]\) is just devoted to the study of the nonlinear interactions in the absence of gravitomagnetic field. The present paper considering gravitomagnetic component is a generalization of the work. And our another work \([16]\) is its analog of electromagnetism.
In Section II we establish the description of collisionless kinetics and give the linear effects. Then we present the nonlinear equations of the fields with low frequency and high frequency, starting from GME equations in Sections III and IV. In Section V we focus on the motion of matter disturbed by GME fields. As a result, the nonlinear controlling equations, involving the coupling of GEM fields with the perturbed density field and self-generated gravitomagnetic field, are presented in Section VI. A numerical integral of the controlling equations is given in Section VII. Finally we sketch the conclusions, stressing the possibility of detecting gravitational waves.

II. KINETIC DESCRIPTION AND LINEAR EFFECTS

Taking into account the presence of massive dark matter in the universe, one should treat a two-component self-gravitating system. The collisionless Boltzmann equations for the distribution function \( f_{\alpha} \) are

\[
\frac{\partial f_{\alpha}}{\partial t} + V \frac{\partial f_{\alpha}}{\partial r} + (a + \mathbf{F}) \cdot \frac{\partial f_{\alpha}}{\partial p} = 0 \quad (\alpha = 1, 2),
\]

where \( a \) is the non-gravitational term, \( \mathbf{F} \) is Lorentz-type force:

\[
\mathbf{F} = \frac{dp}{dt} \approx q_{\alpha} \left[ \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right],
\]

In which gravitoelectric field \( \mathbf{E} \) and gravitomagnetic field \( \mathbf{B} \) satisfy Eqs.\( 1.11 \) and\( 1.12 \), and the “charge” is

\[
q_{\alpha} = -4\sqrt{G}m_{\alpha}.
\]

The density \( n_{\alpha} \) and the current density \( j \) are connected with the particle distribution through

\[
n_{\alpha}(\mathbf{r}, t) = \int f_{\alpha}(\mathbf{r}, \mathbf{v}, t) \frac{dp}{(2\pi)^3},
\]

and

\[
j(\mathbf{r}, t) = -\frac{1}{4} \sum_{\alpha} q_{\alpha} \mathbf{v} f_{\alpha}(\mathbf{r}, \mathbf{v}, t) \frac{dp}{(2\pi)^3}.
\]

We can assume to the gravitation effects

\[
\rho_2 = n_0m_2 \ll \rho_2 = n_0m_2,
\]

where \( \rho_1 \) denotes the density of bright matter, and \( \rho_2 \) the density of dark matter. We divide \( f_{\alpha}, \mathbf{E} \) and \( \mathbf{B} \) into two parts: unperturbed and perturbed parts,

\[
f_{\alpha} = f_{\alpha}^R + f_{\alpha}^T, \quad \mathbf{E} = \mathbf{E}^R + \mathbf{E}^T, \quad \mathbf{B} = \mathbf{B}^R + \mathbf{B}^T.
\]

As the \( f_{\alpha} \) is closely coupling with GEM, the perturbed distribution can be expanded in power

\[
f_{\alpha}^T = \sum_i f_{\alpha}^{T(i)}
\]

provided that the perturbed field \( \mathbf{E}^T \) is weak, i.e.

\[
\bar{W} = \frac{|\mathbf{E}^T|^2}{8\pi n_0 T_0} << 1,
\]
where the index \( i \) indicates the \( i \)-th power of \( E^T \). In this case that equation for the unperturbed state

\[
\frac{\partial f_R^\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_R^\alpha}{\partial \mathbf{r}} + (\mathbf{a} + \mathbf{F}^R) \cdot \frac{\partial f_R^\alpha}{\partial \mathbf{p}} = 0 \tag{2.9}
\]

reduced to

\[
\frac{\partial f_R^\alpha}{\partial t} = 0,
\]

with a relevant solution

\[
f_R^\alpha,p_x \equiv \int f_R^\alpha dp_y dp_z = (2\pi)^{1/2} \frac{m^2 v^2 T_\alpha}{2m^2 v^4}. \tag{2.10}
\]

Substituting Eq.(2.6) into Eq.(2.1) and subtracting Eq.(2.9), putting \( a^g + g^R = 0 \), yield

\[
\frac{\partial f_T^\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_T^\alpha}{\partial \mathbf{r}} + \mathbf{F}^T \cdot \frac{\partial f_R^\alpha}{\partial \mathbf{p}} + \mathbf{F}^T \cdot \frac{\partial f_T^\alpha}{\partial \mathbf{p}} = 0. \tag{2.11}
\]

Substituting Eq.(2.7) into Eq.(2.11) and expanding \( A = (\mathbf{F}^T, f_\alpha) \) in a Fourier series

\[
A(r, v, t) = \int A_k e^{-i\omega t + ik \cdot \mathbf{r}} dk, \tag{2.12}
\]

we get from Eq.(2.11):

\[
i(\omega - k \cdot v) f_{\alpha,k}^{(1)} = \int \mathbf{F}^T_k \cdot \frac{\partial f_R^\alpha}{\partial \mathbf{p}} \delta(k - k_1 - k_2) dk_1 dk_2, \tag{2.13}
\]

\[
i(\omega - k \cdot v) f_{\alpha,k}^{(2)} = \int \mathbf{F}^T_{k_1} \cdot \frac{\partial f_T^{(1)}(1)}{\partial \mathbf{p}} \delta(k - k_1 - k_2) dk_1 dk_2, \tag{2.14}
\]

\[
i(\omega - k \cdot v) f_{\alpha,k}^{(3)} = \int \mathbf{F}^T_{k_1} \cdot \frac{\partial f_T^{(2)}}{\partial \mathbf{p}} \delta(k - k_1 - k_2) dk_1 dk_2. \tag{2.15}
\]

Dividing \( j \) into two parts,

\[
j = j^R + j^T, \tag{2.16}
\]

and expanding \( j^T \) in powers of \( E^T \)

\[
j^{(i)} = -\frac{1}{4} \sum_\alpha q_\alpha v_\alpha f^{(i)}(\alpha) \int \frac{d\mathbf{p}}{(2\pi)^3}, \tag{2.17}
\]

one can obtain the linear current from Eqs.(2.13), (2.10), and (2.17)

\[
j_{k,i}^T = \sigma_{ij}(\omega, k) E_{k,j}^T, \tag{2.18}
\]
This is the relation \(\text{Eq. (1.15)}\); where

\[
\sigma_{ij}(\omega, \mathbf{k}) = -\frac{1}{4} \sum_{\alpha} \int v_i q_{\alpha}^2 \left[ \delta_{js}(1 - \frac{k_j \mathbf{v}}{\omega}) + \frac{k_s v_s}{\omega} \right] \frac{\partial f_R^\alpha}{\partial p_s} \frac{dp}{(2\pi)^3} ; \tag{2.19}
\]

and here we have into account the slow change in \(f_R^\alpha\), i.e. \(f_R^\alpha \approx f_R^\alpha \delta(k_3)\); the term \(\delta\) arises from the Landau rule.

Similarly, we get the nonlinear currents (up to the third order) from Eqs. (2.14) and (2.15)

\[
j^{(2)}_k = \sum_{\alpha} \int S_{k,k_2}^{\alpha} E_{k_1} E_{k_2}^T dk_1 dk_2 \delta(k - k_1 - k_2), \tag{2.20}
\]

\[
j^{(3)}_k = \sum_{\alpha} \int G_{k,k_1,k_2,k_3}^{\alpha} E_{k_1} E_{k_2}^T E_{k_3}^T dk_1 dk_2 dk_3 \delta(k - k_1 - k_2 - k_3), \tag{2.21}
\]

with

\[
S_{k,k_2}^{\alpha} = \frac{1}{4} q_{\alpha}^2 \int v \left( \frac{\mathbf{e}_{k_1}^\alpha \cdot \mathbf{p}}{\mathbf{p}} \right) \left( \frac{\mathbf{e}_{k_2}^\alpha \cdot \mathbf{p}}{\mathbf{p}} \right) \frac{dp}{(2\pi)^3} \tag{2.22}
\]

\[
G_{k,k_1,k_2,k_3}^{\alpha} = -\frac{1}{4} q_{\alpha}^4 \int \frac{\mathbf{v} \cdot dp}{(2\pi)^3} \left( \frac{1}{\omega - \mathbf{k} \cdot \mathbf{v} + i\varepsilon} \right) \left( \frac{\mathbf{e}_{k_1}^\alpha \cdot \mathbf{p}}{\mathbf{p}} \right) \left( \frac{1}{\omega - \mathbf{k} \cdot \mathbf{v} + i\varepsilon} \right) \left( \frac{\mathbf{e}_{k_2}^\alpha \cdot \mathbf{p}}{\mathbf{p}} \right) \left( \frac{\mathbf{e}_{k_3}^\alpha \cdot \mathbf{p}}{\mathbf{p}} \right) f_R^\alpha. \tag{2.23}
\]

On the other hand, we can obtain the field equation from the Maxwell–type GEM equations

\[
\left( k^2 - \frac{\omega^2}{c^2} \varepsilon_k^\sigma \right) E_k^\sigma = \frac{4\pi i}{c^2} \omega(e_k^\sigma \cdot \sum_{n \geq 2} J_k^{(n)}), \tag{2.24}
\]

where

\[
\varepsilon_k^\sigma = \begin{cases} \varepsilon_{k,i}^\sigma(\omega, \mathbf{k}) \varepsilon_{k,i}^\sigma, & \text{ for } \sigma = l \text{ and } \text{transverse } \sigma = t \end{cases} \tag{2.25}
\]

is “dielectric constant” for \(\sigma\) mode (longitudinal mode, \(\sigma = l\) and transverse \(\sigma = t\)) and \(E_k^\sigma = E_k^\sigma \varepsilon_k^\sigma\) with \(e_k^\sigma \cdot e_k^\sigma = 1\).

Using Eqs. (2.17), (2.21) and (2.25), one has

\[
\varepsilon_k^l \equiv \varepsilon_{ij}(\omega, \mathbf{k}) \frac{k_i k_j}{k^2} = 1 - \sum_{\alpha} \frac{1}{k^2} \frac{\omega_{p\alpha}^2}{v_{T\alpha}} \left[ 1 - Z \left( \frac{\omega}{\sqrt{2kv_{T\alpha}}} \right) \right], \tag{2.26}
\]

\[
\varepsilon_k^t \equiv \begin{cases} \varepsilon_{ij}(\omega, \mathbf{k}) e_k^\sigma e_k^\sigma, & \text{ for } \sigma = l \end{cases} \equiv 1 + \sum_{\alpha} \frac{1}{k^2} \frac{\omega_{p\alpha}^2}{\omega} Z \left( \frac{\omega}{\sqrt{2kv_{T\alpha}}} \right) \tag{2.27}
\]

with

\[
\omega_{p\alpha}^2 = \frac{\pi q_{\alpha}^2 n_0}{4m_{\alpha}} = 4\pi G_{\rho\alpha}, \tag{2.28}
\]
where

\[ Z \left( \frac{\omega}{\sqrt{2}kv_T1} \right) \equiv \int_{-\infty}^{\infty} \frac{x}{\sqrt{\pi}} e^{-x^2} e^{-i\xi} d\xi \quad (2.29) \]

is dispersion function.\(^{[10]}\)

\[ Z(x) \approx 1 + \frac{1}{2x^2} + \frac{3}{4x^4} - i\sqrt{\pi}xe^{-x^2}, \quad x \gg 1, \quad (2.30) \]

\[ Z(x) \approx 2x^2 - i\sqrt{\pi}xe^{-x^2} \approx 2x^2 - i\sqrt{\pi}x, \quad x \ll 1. \quad (2.31) \]

For high-frequency field

\[ \omega \gg kv_T1 \gg kv_T2, \quad (2.32) \]

i.e., \( x \gg 1 \), therefore, neglecting the damping term, one obtain from Eqs.\(^{[2.26]}\) and \(^{[2.27]}\)

\[ \varepsilon'^l_k = 1 + \frac{\omega_p^2}{\omega^2} + \frac{\omega_{p1}^2}{\omega^2} + \frac{\omega_{p2}^2 k^2 v_T^2}{\omega^2} + \frac{\omega_{p1}^2 3k^2 v_T^2}{\omega^4} \quad (2.33) \]

and

\[ \varepsilon'^t_k \approx 1 + \frac{\omega_p^2}{\omega^2} + \frac{\omega_{p2}^2}{\omega^2}. \quad (2.34) \]

For the low-frequency fields, the following conditions are met

\[ v_T2 \gg \omega'/k' \ll v_T1, \quad \omega' \ll \omega_{p1}, \quad (2.35) \]

Then

\[ \varepsilon'^l_k = 1 + (\varepsilon'^{(l)}_{k'} - 1) + (\varepsilon'^{(2)}_{k'} - 1) \approx - \frac{\omega_{p1}^2}{k'^2 v_T^2} - \frac{\omega_{p2}^2}{k'^2 v_T^2} \quad (2.36) \]

\[ \varepsilon'^t_k \approx 1 + \frac{\omega_{p1}^2}{k'^2 v_T^2} + \frac{\omega_{p2}^2}{k'^2 v_T^2} - i\sqrt{\pi} \omega_{p2}^2 \omega_{p2} \omega_{p2} k' v_T2. \quad (2.37) \]

Taking Eq.\(^{[2.34]}\) into consideration, one gets the linear dispersion relationship from Eq.\(^{[2.24]}\) for the transverse oscillation with high-frequency:

\[ \omega^2 = k^2 c^2 - (\omega_{p1}^2 + \omega_{p2}^2). \quad (2.38) \]

Usually, there is not steady longitudinal high-frequency mode on basis of Eq.\(^{[2.33]}\), which consistent with the analyses for the electro-gravitation kinetics.\(^{[15]}\). Consider that only the oscillations with frequency is close to the proper frequency of the medium, which is similar to plasma case; this means

\[ \omega^2 \approx \omega_{pn}^2 + k^2 c^2 \approx \omega_{p2}^2, \quad (\omega_{pn}^2 \equiv \omega_{p1}^2 + \omega_{p2}^2 \gg \bar{k}^2 c^2) \quad (2.39) \]

Eq.\(^{[2.39]}\) is a branch of dispersion relationship [Eq.\(^{[2.38]}\)]. In what follows we omit the mark ‘\(~\)’ in Eq.\(^{[2.39]}\) for simplicity.
III. LOW-FREQUENCY TRANSVERSE FIELD EQUATION

First of all, let us study the case of the low-frequency transverse field $E_k^{T\sigma} = E_k^{Tt} = E_k^{TS}$ in Eq. (2.24). For the second order nonlinear current, because of delta function in Eq. (2.20), it must meet $k = k_1 + k_2, \omega = \omega_1 + \omega_2$; since $(k, \omega)$ belong to low-frequency wave, then $\omega_1, \omega_2$ must be high-frequency and are of opposite sign. In this case, the second order term in Eq. (2.20) becomes: $\left[ E_k^{T(+)} E_k^{T(-)} + E_k^{T(-)} E_k^{T(+)} \right]$, where upper indices “+” and “−” denote the positive and negative frequency parts of high-frequency perturbations, respectively. Hence

$$
\mathbf{j}_k^{(2)} = \sum_\alpha \int S_{k,k_1,k_2}^{\alpha(t)} \left( E_k^{T(+)} E_{k_1}^{T(-)} + E_k^{T(-)} E_{k_1}^{T(+)} \right) \delta (k - k_1 - k_2) \, dk_1 dk_2,
$$

where

$$
S_{k,k_1,k_2}^{\alpha(t)} = \frac{1}{4q_0^3} \int \mathbf{e}_{k_1} \cdot \frac{\partial}{\partial p} \mathbf{e}_{k_2} \cdot \frac{\partial}{\partial p} f_R \, dp.
$$

Using the substitution $k_1 \rightarrow k_2, k_2 \rightarrow k_1$ in the second integral term of Eq. (3.1), it yields

$$
\mathbf{j}_k^{(2)} = \sum_\alpha \int \left( S_{k,k_1,k_2}^{\alpha(t)} + S_{k,k_2,k_1}^{\alpha(t)} \right) E_k^{T(+)} E_{k_1}^{T(-)} \delta (k - k_1 - k_2) \, dk_1 dk_2;
$$

substituting Eq. (3.3) into Eq. (2.24) yields

$$
(k^2 c^2 - \omega^2 \varepsilon_k) E_k^{TS} = 4\pi i \omega \sum_\alpha \int \tilde{S}_{k,k_1,k_2}^{\alpha(t)} E_k^{T(+)} E_{k_1}^{T(-)} \delta (k - k_1 - k_2) \, dk_1 dk_2,
$$

where

$$
\tilde{S}_{k,k_1,k_2}^{\alpha(t)} \equiv \left( S_{k,k_1,k_2}^{\alpha(t)} + S_{k,k_2,k_1}^{\alpha(t)} \right) \cdot \mathbf{e}_k^* = \frac{1}{4q_0^3} \int \frac{\mathbf{e}_k^* \cdot \mathbf{v}}{\omega - k \cdot \mathbf{v} + i \varepsilon} \times
$$

$$
\left\{ \mathbf{e}_{k_1} \cdot \frac{\partial}{\partial p} \mathbf{e}_{k_2} \cdot \frac{\partial}{\partial p} + \mathbf{e}_{k_2} \cdot \frac{\partial}{\partial p} \mathbf{e}_{k_1} \cdot \frac{\partial}{\partial p} \right\} \int_{(2\pi)^3} f_R \, dp,
$$

with

$$
\mathbf{e}_k^* \equiv (1 - \frac{k \cdot \mathbf{v}}{\omega}) \mathbf{e}_k + \mathbf{v} \cdot \frac{\mathbf{e}_k}{\omega} \mathbf{k};
$$

and the second order distribution function is

$$
f_{\alpha,k}^{(2)} = \int \Sigma_{k,k_1,k_2}^{\alpha} E_k^{T(+)} E_{k_1}^{T(-)} \delta (k - k_1 - k_2) \, dk_1 dk_2,
$$

where

$$
\Sigma_{k,k_1,k_2}^{\alpha} = -q_0^2 \frac{1}{\omega - k \cdot \mathbf{v} + i \varepsilon} \left\{ \mathbf{e}_{k_1} \cdot \frac{\partial}{\partial p} \mathbf{e}_{k_2} \cdot \frac{\partial}{\partial p} + \mathbf{e}_{k_2} \cdot \frac{\partial}{\partial p} \mathbf{e}_{k_1} \cdot \frac{\partial}{\partial p} \right\} \int_{(2\pi)^3} f_R.
$$
IV. HIGH-FREQUENCY TRANSVERSE FIELD EQUATION

If $E^T_k$ in the left-hand side of Eq. (2.24) is high-frequency field, $E^T_k = E^{T(+)}_k$, the quadratic terms in the Eq. (2.20) should be the product of high-frequency and low-frequency fields: $E^T_k E^T_k = E^{T+} T^{th}_k E^{T+} T^{th}_k + E^{T-} T^{th}_k E^{T-} T^{th}_k$. The three-field product included in the current $j^{(3)}_k$ can be expressed in terms of high-frequency fields and cubic a mixed product of high-frequency and low-frequency fields. Using Eq. (3.3), then, in fact, this mixed term is the product of four high-frequency fields, which is higher order. Then $E^T_k E^T_k E^T_k \approx E^{T+} T^{th}_k E^{T+} T^{th}_k$. Due to the factor $[(\omega - \omega_1) - (k - k_1) \cdot \mathbf{v} + i\varepsilon]^{-1}$ in the Eq. (2.23), its contribution to the $j^{(3)}_k$ is important if $E^{k1}_{k1}$ is the positive high-frequency fields. As a result

$$E^T_k E^T_k E^T_k = E^{T(+)}_k E^{T(-)}_k E^{T(+)}_k E^{T(-)}_k \ .$$

Therefore, we get similarly the high-frequency field equation as follow

$$\begin{align*}
(k^2 c^2 - \omega^2 \varepsilon_k^T) E^{T(+)}_k &= 4\pi i\omega \left\{ \sum_\alpha \int S^{(k)}_k, k_2 E^{(k)} T^{(k)}_k \delta (k - k_1 - k_2) dk_1 dk_2 + \sum_\alpha \int S^{(k)}_k, k_1, k_3 E^{(k)} T^{(k)}_k E^{(k)} T^{(k)}_k \delta (k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3 \right\} \ ,
\end{align*}$$

where

$$\begin{align*}
\tilde{S}^{(k)}_k, k_1, k_2, k_3 &= \mathbf{e}^T_k \cdot \left( G^{(k)}_k, k_1, k_2, k_3 + G^{(k)}_k, k_2, k_1, k_3 \right) \\
&= -\frac{1}{4} \int d\alpha d^4 \mathbf{p} (\omega - k \cdot \mathbf{v} + i\varepsilon) \tilde{e}^T_{k1} \cdot \frac{\partial}{\partial \mathbf{p}} \left[ (\omega - \omega_1) - (k - k_1) \cdot \mathbf{v} + i\varepsilon \right] \times \tilde{S}^{(k)}_k, k_1, k_2, k_3, k_4 \ ,
\end{align*}$$

and the expression for $S^{(k)}_k, k_1, k_2$ is the same as $S^{(k)}_k, k_2, k_3$ (see Eq. (3.5)), except that $\omega_2$ is low-frequency now. And $E^{T+}_k$ is low-frequency fields, but it may different from the low-frequency fields $E^{T+}_k$ in the left-hand of Eq. (3.3). According to semiclassical theory, the fusion and decay interactions in Eqs. (4.1) and (4.1) can determine the field intensity with low-frequency, $N^T_k \sim |E^{T+}_k|^2$, in other words, they differ by a phase factor $e^{i\phi}$. The symbol $\sum_\alpha$ in those expressions above implies adding the contribution of dark matter($\alpha = 2$) and bright matter($\alpha = 1$). From Eqs. (3.5) and (4.2) one can see that $\int d\alpha d^4 \mathbf{p} \sim n_0, q_0 \sim m_\alpha, \tilde{S}^{(k)}_k \propto m_\alpha, G^{(k)}_k \propto m_\alpha$, namely that the matrix elements of interaction are proportional to the mass of particles. As the assumption of the dark matter mass far larger than the bright one[see Eq. (2.2)], we can neglect the contributions of the bright matter. Therefore we write $\tilde{S}^{(k)}_k, k_1, k_2, k_3, k_4$ and $G^{(k)}_k, k_1, k_2, k_3, k_4$ as $S^{(k)}_k, k_1, k_2, k_3, k_4$ and $G^{(k)}_k, k_1, k_2, k_3, k_4$.

In order to get the field equation in spectrum space for nonlinear interaction up to the third order, we must estimate in detail the integral value of matrix elements $\tilde{S}^{(k)}_k, k_1, k_2, k_3$ and $G^{(k)}_k, k_1, k_2, k_3$ in Eq. (4.1). Integrating Eq. (4.2) by parts and by using of Eq. (2.30) and (3.8), we get

$$4\pi i\omega \tilde{G}^{(k)}_k, k_1, k_2, k_3 \approx 4\omega^2 \varepsilon_k^T \tilde{e}^T_{k1} \cdot \frac{1}{m_0} \int d^3 \mathbf{p} \tilde{S}^{(k)}_k, k_1, k_2, k_3 \left( \frac{\mathbf{p}}{2\pi} \right)^3 \ ,$$

and for $\tilde{S}^{(k)}_k, k_1, k_2, k_3$, in which $\omega$ and $\omega_1$ are high-frequency and low-frequency, after integrating by parts, it reduces to

$$\tilde{S}^{(k)}_k, k_1, k_2, k_3 \approx \frac{1}{4\omega_1 \omega_2 m_2^2} \mathbf{e}^T_{k1} \times \left[ \mathbf{e}^T_{k2} \times \mathbf{e}^T_{k3} \right] \ .$$

Then first integral of Eq. (4.1) becomes
4\pi i \omega \int \frac{\hat{z}_{2(l)}^{(k,k_1,k_2)}}{S_{k,k_1,k_2}} E_{k_1}^{T(+)} S_{k_2}^{T} \delta (k - k_1 - k_2) \, dk_1 dk_2 \\
= 4i \frac{q_2 \omega p_2}{m_2 c} e_k^s \int \left[ E_{k_1}^{T(+)} \times B_{k_2}^{S} \right] \delta (k - k_1 - k_2) \, dk_1 dk_2,

where

\[ B_{k_2}^{S} = \frac{k_2 c}{\omega_2} \times \hat{E}_{k_2}^{T} \]

is low-frequency magnetic field, which are produced by the fields with positive and negative high-frequencies. And using Eq. (3.7), the second integral of Eq. (4.1) becomes

\[ 4\omega p_2^2 \int e_k^s \cdot (E_{k_1}^{T(+)} \frac{n_{k-k_1}^{(2)}}{n_0}) \, dk_1, \]

where \( n_{k'}^{(2)} \) is the second order of perturbed density,

\[ n_{k'}^{(2)} = \int f_{k,k'}^{(2)} \frac{dp}{(2\pi)^3}. \]

Therefore Eq. (4.1) is reduced to

\[ (k^2 c^2 - \omega^2 \varepsilon_k^s) E_{k}^{T(+)} = 4\omega p_2^2 \int (E_{k_1}^{T(+)} \frac{n_{k-k_1}^{(2)}}{n_0}) \, dk_1 + 4i q_2 \frac{\varepsilon}{m_2 c} \omega p_2 \int (E_{k_1}^{T(+)} \times B_{k-k_1}^{S}) \, dk_1. \]

As a result, we obtain from Eq. (4.9)

\[ \frac{2i}{\omega p_2^2} \frac{\partial E(r,t)}{\partial t} - \frac{e^2}{\omega p_2^2} \nabla \times \nabla \times E(r,t) = -2E(r,t) + 4 \frac{n_{k}^{(2)}(r,t)}{n_0} E(r,t) + 4i q_2 \frac{\varepsilon}{m_2 c \omega p_2} E(r,t) \times B_{S}(r,t), \]

here \( E(r,t) \) is the envelope for the high-frequency fields,

\[ E(r,t) e^{-i\omega t} = \int E_{k}^{T(+)} e^{-i\omega t + ik \cdot r} \, dk, \]

and because of slow change in \( E(r,t) \), we have neglected the term \((\frac{q_2^2}{\omega p_2^2})E(r,t)\).

**V. PERTURBED DENSITIES**

It is possible that two transverse fields with high frequencies can produce a longitudinal low-frequency field. In this case, the density perturbation of the first order can be exited. Then according to Eq. (3.4), one has

\[ \varepsilon_k^1 E_{k}^{T(l)} = -\frac{4\pi i}{\omega} \sum_{\alpha} \int \frac{\hat{z}_{2(l)}^{(k,k_1,k_2)}}{S_{k,k_1,k_2}} E_{k_1}^{T(+)} E_{k_2}^{T(-)} \delta (k - k_1 - k_2) \, dk_1 dk_2, \]

where the coupling matrix \( S_{k,k_1,k_2}^{2(l)} \) is the same as \( S_{k,k_1,k_2}^{2(l)} \) when \( e_k^s \to k/k \). The density perturbation produced by the longitudinal low-frequency wave is

\[ n_{k}^{(1)} = \int f_{k}^{T(l)} \frac{dp}{(2\pi)^3} = \frac{q_2}{i} \varepsilon_k^1 E_{k}^{T(l)} \int \frac{k \cdot \frac{\partial f_{k}^{T(l)}}{\partial p}}{k \cdot \varepsilon + i\varepsilon} \frac{dp}{(2\pi)^3}. \]
Taking account of the integral expression of $\varepsilon^{2(l)}_k$ [see Eq. (2.26)], we obtain

$$n^{(1)}_k = \frac{q_2}{\pi q_2} \left( \frac{2}{\varepsilon^{2(l)}_k} - 1 \right) E^{T_S(l)}_k.$$  \hspace{1cm} (5.3)

Using $\left( \varepsilon^{2(l)}_k - 1 \right) = \varepsilon^{1(l)}_k + \varepsilon^{1(l)}_k E^{T_S(l)}_k$, as the first-order approximation, is zero [see Eq. (5.1)], then

$$n^{(1)}_k = \frac{q_2}{\pi q_2} \varepsilon^{1(l)}_k E^{T_S(l)}_k.$$  \hspace{1cm} (5.4)

Similarly, one estimate the matrix as follow:

$$S^{2(l)}_{k_1,k_2} \approx \frac{k\omega}{\pi m^2 \omega^2} q_2 \left( \varepsilon^{2(l)}_k - 1 \right) (e^t_{k_2}, e^t_{k_1}).$$  \hspace{1cm} (5.5)

So the total perturbed density produced by two transverse fields with positive and negative high-frequencies, is

$$n'(r,t) = \frac{4}{\pi T^2} |E(r,t)|^2.$$  \hspace{1cm} (5.7)

In the case of taking account of longitudinal low-frequency fields, the first term to the right-hand side of Eq. (4.9) will be added to the coupling term of longitudinal fields,

$$4\pi i \omega \int \frac{S^{2(l)}_{k_1,k_2} E^{T_S(l)}_k}{E_{k_1}^{T_S(l)}} \delta (k - k_1 - k_2) dk_1 dk_2,$$  \hspace{1cm} (5.8)

where the coupling matrix $S^{2(l)}_{k_1,k_2}$ is similar to $S^{2(l)}_{k_1,k_2} (e^t_{k_1} \rightarrow k_2, k_2)$ in Eq. (4.9), its estimated value is

$$S^{2(l)}_{k_1,k_2} \approx -\frac{1}{4 q^2} \int \frac{e^t_{k_1} \cdot e^t_{k_2}}{\omega m_2} \frac{k_2 \cdot v}{\omega^2 - k_2 \cdot \omega + i\varepsilon} (2\pi)^3.$$  \hspace{1cm} (5.9)

And Eq. (5.8) is reduced to

$$4\omega^2 p^2 \frac{1}{n_0} e^t_{k_1} \cdot \int \frac{E^{T_S(l)}_k}{E_{k_1}^{T_S(l)}} \left[ \int f^{T(1)}_k \frac{dp}{(2\pi)^3} \right] dk_1 = 4\omega^2 p^2 e^t_{k_1} \cdot \int \frac{E^{T_S(l)}_k}{n_0} \frac{1}{k_1} dk_1.$$  \hspace{1cm} (5.10)

Taking this term into account, one gets from Eq. (4.9)

$$\frac{2i}{\omega p^2} \frac{\partial |E(r,t)|^2}{\partial t} - \frac{e^2}{\omega p^2} \nabla \times \nabla \times E(r,t) + 2E(r,t)$$  \hspace{1cm} (5.11)

$$-4 \frac{n'(r,t)}{n_0} E(r,t) - 4 \frac{i q_2}{m_2 \omega p^2} E(r,t) \times B^S(r,t) = 0.$$
VI. NON-LINEAR COUPLING FIELDS EQUATION

Now we study the low-frequency field equation (3.4). Differentiating with respect to section \( v \) in braces in Eq. (3.5) and taking account to delta function in Eq. (3.4), yields

\[
\tilde{S}_{k, k_1, k_2}^{2(t)} \approx -\frac{1}{4} \frac{q_0^3}{m_2^{\omega_1^2}} \left( \omega \right) e^{i\phi} \left[ e_{k_1}^\dagger (k \cdot e_{k_2}^\dagger) - e_{k_1}^\dagger (k \cdot e_{k_1}^\dagger) \right].
\]

(6.1)

Then Eq. (6.1) becomes

\[
(k^2 c^2 - \omega^2 \varepsilon_k^4) E_k^{TS} = -\frac{i q_2^2}{m_2} Z \left( \frac{\omega}{2 k v_{T\alpha}} \right) \int k \times \left( E_{k_1}^{T(+)} \times E_{k_2}^{T(-)} \right) \delta (k - k_1 - k_2) dk_1 dk_2.
\]

(6.2)

According to Eq. (4.6),

\[
e^{i\phi} B_k^S = \frac{c}{\omega} k \times E_k^{TS},
\]

(6.3)

we have

\[
(k^2 c^2 - \omega^2 \varepsilon_k^4) B_k^S e^{-i\phi} = -\frac{i q_2^2}{m_2} Z \left( \frac{\omega}{2 k v_{T\alpha}} \right) \times k \times \left[ k \times \int \left( E_{k_1}^{T(+)} \times E_{k_2}^{T(-)} \right) \delta (k - k_1 - k_2) dk_1 dk_2 \right].
\]

(6.4)

For very low frequency fields, \( v_{T1} \gg \omega/k \ll v_{T2} \), one has

\[
Z \left( \frac{\omega}{2 k v_{T\alpha}} \right) \approx \frac{\omega^2}{k^2 v_{T\alpha}^2} - i \frac{\pi}{\sqrt{k v_{T\alpha}}} \approx -i \frac{\pi}{\sqrt{k v_{T\alpha}}},
\]

\[
\varepsilon_k^4 \approx 1 + \frac{\omega^2_{p1}}{k^2 v_{T1}^2} + \frac{\omega^2_{p2}}{k^2 v_{T2}^2} - i \frac{\pi}{2} \frac{\omega_{p2}}{\omega} \approx -i \frac{\pi}{2} \frac{\omega_{p2}^2}{\omega v_{T2}},
\]

(6.5)

and the following condition obviously is satisfied

\[
\omega_{p2} \gg \left( \frac{k v_{T2}}{\omega} \right) k c.
\]

(6.6)

Then Eq. (6.4) become

\[
\omega B_k^S e^{-i\phi} = \frac{i q_2^2}{m_2} \frac{c}{\omega} k \times \left[ k \times \int \left( E_{k_1}^{T(+)} \times E_{k_2}^{T(-)} \right) \delta (k - k_1 - k_2) dk_1 dk_2 \right].
\]

(6.7)

Hence the coordinate representation of Eq. (6.7) is reduced to the following equation

\[
\frac{\partial}{\partial t} B_k^S (r, t) = i \frac{q_2^2}{m_2} \frac{c}{\omega} \nabla \times \nabla \times [E(r, t) \times E^* (r, t)],
\]

(6.8)

where we have chose \( \phi = \frac{\pi}{2} \) for getting real magnetic fields with low-frequency. Through the substitutions

\[
\xi = \frac{2}{3} \sqrt{\frac{\mu}{r v_{T2}/\omega_{p2}}} \quad \tau = \frac{2}{3} \frac{\mu \omega_{p2} t}{\omega}, \quad \mu = \frac{m_1}{m_2}, \quad \alpha = \frac{c^2}{3 v_{T2}^2},
\]

\[
E(\xi, \tau) = \frac{4\sqrt{3} E(r, t)}{\sqrt{\mu m_0 t_2}}, \quad B(\xi, \tau) = \frac{12 q_2}{4 \mu m_2 c^2 \omega_{p2}} B^*(r, t), \quad n = \frac{3}{4} n',
\]

(6.9)
we can now write Eqs. (5.7), (5.11) and (6.8) in the form

\[ n(\xi, \tau) = |E(\xi, \tau)|^2, \]  
\[ i\frac{\partial E(\xi, \tau)}{\partial \tau} - \alpha \nabla \times \nabla \times E(\xi, \tau) + \frac{3}{2\mu}E(\xi, \tau) - n(\xi, \tau)E(\xi, \tau) - iE(\xi, \tau) \times B(\xi, \tau) = 0, \]  
\[ \frac{\partial}{\partial \tau} B(\xi, \tau) = i\frac{1}{6} \nabla \times \nabla \times [E(\xi, \tau) \times E^*(\xi, \tau)]. \]

Therefore it may be seen that the gravitoelectric and gravitomagnetic fields, and self-generated gravito-magnetic fields with very low-frequency are completely determined by the closed Eqs. (6.10)-(6.12).

VII. NUMERICAL INTEGRAL

It is well known that the gravitational system can emit energy in quadrupole radiation. For example, consider a neutron star with mass \( M = M_\odot \), its radius, rotation inertia, rotational period and eccentricity are as follows:

\[ R \sim 10 km, \quad M \sim M_\odot, \quad I \sim 10^{45}, \quad P \sim 0.033, \quad e \sim 10^{-4}. \]

As a result, due to the quadrupole radiation, the energy loss for the system is\( 18 \)

\[ L \approx 10^{45} e^2 \sim 10^{37} \text{ (erg \cdot s}^{-1}). \]

Furthermore, one can estimate the radiation flux at \( r = D = r_i = 10 \times 10 km = 10^7 \text{ (cm)} \),

\[ F \sim \frac{L}{D^2} \sim \frac{L}{r_i^2} \sim 10^{23} \text{ (erg \cdot cm}^{-2} \cdot \text{s}^{-1}), \]

where \( r_i \) is the inner radius of accretion disk around the compact star, at which the corresponding parameters are

\[ T_0 = T_1 = T_2 = 10^7 K, \quad \rho_i = n_0 m_p = \rho_2/10 = 4(g \cdot cm}^{-3}). \]

Then we get immediately the estimated values of the field in the gravitational wave

\[ F \sim \frac{c |E(r, t)|^2}{8\pi}, \quad \frac{|E(r, t)|^2}{8\pi} \sim \frac{F}{c} \sim 3.3 \times 10^{12} \text{ (erg \cdot cm}^{-3}), \]

i.e.

\[ \frac{|E(r, t)|^2}{8\pi n_0 T_0} = \frac{3.3 \times 10^{14}}{1.38 \times 10^{-16} n_0 T_0} = \frac{3.3 \times 10^{14}}{1.38 \times 2.4 \times 10^{16}} = 10^{-3}; \]

and

\[ |E(r, t)|^2_{r=0} = \frac{384}{\mu} \bar{W}; \]

hence

\[ |E(\xi, \tau)|^2_{\tau=0} = 3.84, \quad \alpha = \frac{3.8 \times 10^{13}}{T_2} = 3.8 \times 10^6, \quad (\bar{W}_{\tau=0} = 10^{-3}). \]
We have solved numerically Eqs. (6.10), (6.11) and (6.12) in two dimensions with three field components using FFT. The initial condition with a periodic boundary condition is given as

\[ |E(\xi, \tau)|^2 \big|_{\tau=0} = E_0 \sin \frac{2\pi y}{y_0} \sec h \left( \frac{x}{L_0} \right) (e_x + e_z) - E_0 \frac{y_0}{2\pi L_0} \cos \frac{2\pi y}{y_0} \tanh \left( \frac{x}{L_0} \right) \sec h \left( \frac{x}{L_0} \right) e_y \]  \tag{7.1}

with

\[ L_0 = 2 \times 10^3, \quad y_0 = 5 \times 10^6. \]

The distribution of initial gravitoelectric field is shown in Fig.1, the dynamic evolution behavior for gravitoelectric fields and self-generated gravitomagnetic fields with the very low frequency is shown in Figs.2-7 and Figs.8-13. Quantities in Figs. 1-13 are dimensionless. The relations to dimensional ones are

\[ r = 3 \times 10^6 \sqrt{\frac{T_2}{\rho_2}} x_{F1g} = 1.5 \times 10^9 x_{F1g}, \] \tag{7.2}

\[ t = 1.6 \times 10^4 \frac{\tau}{\sqrt{\rho_2}} = 2.5 \times 10^3 \tau, \] \tag{7.3}

\[ \frac{|E(r, t)|^2}{8\pi} = 3.6 \times 10^{-20} (n_0 T_0) \quad |E|_{F1g}^2 = 8.6 \times 10^{11} |E|_{F1g}^2, \] \tag{7.4}

\[ B_s^2 = 5.6 \times 10^9 B_{F1g}. \] \tag{7.5}

Figs.2-7 give the collapse development of gravitoelectric field and Figs.8-13 the collapse development of self-generated gravitomagnetic field with very low frequency. While \( \tau = 1.65 \), the strengths of the self-generated gravitomagnetic fields are

\[ |E|_{\text{max}}^2 = 2748.75, \quad |B|_{\text{max}}^2 = 8.64 \times 10^{-9}. \]

Using Eqs. (7.2) –(7.5), while \( t = 3.3 \times 10^3 s \) we have

\[ |E(r, t)|_{\text{max}}^2 = 5.94 \times 10^{16} \text{erg/cm}^3 \]

and

\[ B_s^2 = 5.1 \times 10^8 (\text{erg/cm}^3)^\frac{1}{2}. \]

When \( \tau > 1.65 \), the collapses rapidly and leads to a very strong field, i.e. \( \tilde{W} = |E(r, t)|^2 / 8\pi n_0 T_0 > 1 \) and in this case the expansion Eq. (2.7) is no longer valid.

**VIII. CONCLUSION REMARKS**

From the above study, we arrive at the following conclusions: If there is very intense gravitational radiation with high frequency near by a source, the interactions of the wave decay and fusion and wave-particle develop, and nonlinear (matter) currents with low frequency are induced by the interactions, leading to increasing of local matter density and excitation of a very low-frequency gravitomagnetic field.

It is shown that the dynamic behavior and configuration for the GEM fields, the perturbed density and the gravitomagnetic field with very low-frequency are determined by Eqs. (6.10) - (6.12).
We investigate the numerical solution of Eqs. (6.10) - (6.12). It is shown that the fields, involving the GEM fields, the perturbed density field and self-generated gravitomagnetic field with very low frequency, may collapse, so that the gravitational waves are amplified by a factor of $10^3$. In other words, due to self-condensing, a stronger GME fields could be produced; and they could appear as the gravitational waves with high energy reaching on Earth. In this case, Weber’s results, perhaps, are acceptable.

Meanwhile, the increasing perturbed matter density and gravitomagnetic field with very low-frequency in the local region, where the gravitational waves get through, are in favor of the formation of a new rotating object.

Acknowledgments

This work was partly supported by the National Natural Science Foundation of China and the Natural Science Foundation of Jiangxi Province.

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FIG. 1: Initial gravitoelectric field distribution: (a) $\tau = 0.00$, $|E|^2_{\text{max}} = |E|_x^2 + |E|_y^2 + |E|_z^2 = 3.84$.

[1] S. Shapiro, Phys. Rev. Lett. 77, 4487 (1996).
[2] M. A. G. Bonilla and J. M. M. Senovilla, Phys. Rev. Lett. 11, 783 (1997).
[3] J. M. M. Senovilla, Mod. Phys. Lett. A 15,159 (2000).
[4] M. L. Ruggiero and A. Tartaglia, Nuovo Cimento B 117, 743 (2002).
[5] L. Iorio and D. M. Lucchesi, Class. Quantum Grav. 20, 2477 (2003).
[6] S. Weinberg, Gravitation and Cosmology, John Wiley & Sons, Inc, New York, (1972).
[7] V. B. Braginsky, C. M. Caves and Kip S. Thorne, Phys. Rev. D 15, 2045 (1977).
[8] V. B. Braginsky, A. G. Polnarev and Kip S. Thorne, Phys. Rev. Lett. 53, 863 (1984).
[9] C. Ciubotariu, Phys. Lett. A 158, 27 (1991).
[10] M. Agop, C. Gh. Buzea, P. Nica, Physica C 339, 120 (2000).
[11] I. Oprea, M. Agop, Studia Geoph. Et Geod. 42, 431 (1998).
[12] C.O. Hans and R. Ruffini, Gravitation and Spacetime, W.W.Norton and Company (1994).
[13] H. Peng, General Relati. Grav. 15, 725 (1983).
[14] S. Shapiro, Phys. Rev. Lett. 77, 4487 (1996).
[15] X. Q. Li, Astron. Astrophys. 227, 317 (1990).
[16] X. Q. Li, and Y. H. Ma, Astron. Astrophys. 270, 534(1993).
[17] V. E. Zakharov, in Basic Plasma Physics II, ed. A. A. Galeev and R. N. Sudan, North-Holland Phys. Pub., Amsterdam, p.84 (1984).
[18] J. P. Ostricker and J. E. Gunn, Astrophys. J. 157, 1395 (1969).
[19] S.Q. Liu and X. Q. Li, Astron. Astrophys. 364, 785 (2000).
FIG. 2: \( (a) \tau = 0.05, |E|_{\text{max}}^2 = 3.895 \)

FIG. 3: \( (b) \tau = 0.5, |E|_{\text{max}}^2 = 3.418 \)
FIG. 4: (c) $\tau = 1.4$, $|\mathbf{E}|^2_{\text{max}} = 5.213$

FIG. 5: (d) $\tau = 1.55$, $|\mathbf{E}|^2_{\text{max}} = 12.09$
FIG. 6: (e) $\tau = 1.6, |E|_{\text{max}}^2 = 30.55$

FIG. 7: (f) $\tau = 1.65, |E|_{\text{max}}^2 = 2749$
FIG. 8: (a) $\tau = 0.05$, $|\mathbf{B}|_{\text{max}}^2 = 1.171 \times 10^{-22}$

FIG. 9: (b) $\tau = 0.5$, $|\mathbf{B}|_{\text{max}}^2 = 6.938 \times 10^{-18}$
FIG. 10: (c) $\tau = 1.4$, $|B|^2_{\text{max}} = 5.705 \times 10^{-16}$

FIG. 11: (d) $\tau = 1.55$, $|B|^2_{\text{max}} = 1.154 \times 10^{-15}$
FIG. 12: (e) $\tau = 1.6, |B|_{\text{max}}^2 = 1.405 \times 10^{-15}$

FIG. 13: (f) $\tau = 1.65, |B|_{\text{max}}^2 = 8.641 \times 10^{-9}$