Quark Number Susceptibility and Thermodynamics in HTL approximation

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Abstract

In HTL perturbation theory we obtain leading order quark number susceptibility as a response to an external disturbance, viz., chemical potential ($\mu$) that generates density fluctuation, which is related to the correlation function through the thermodynamic sum rule associated with the symmetry of the system. We also obtain various thermodynamic quantities in leading order.

Key words: Quark-Gluon Plasma, Hard Thermal Loop Approximation, Quark Number Susceptibility
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1. Introduction

The fluctuations of conserved charges such as baryon number related to quark number susceptibility (QNS) is generally considered as a useful probe for quark gluon plasma (QGP). Various hard thermal loop (HTL) approaches and methods [1–3] have led to different results for QNS. We reconsider [4] various thermodynamic quantities and QNS calculations by reformulating the HTL perturbation theory (HTLpt) to an external probe ($\mu$) and show that the leading order (LO) results agree quite well despite the use of different approaches. The QNS is defined through the thermodynamic sum rule as [4,5]

$$\chi(T) = \left. \frac{\partial \rho}{\partial \mu} \right|_{\mu=0} = \left. \frac{\partial^2 P}{\partial \mu^2} \right|_{\mu=0} = \int d^4 x \, \langle J_0(0,x)J_0(0,0) \rangle = -\lim_{p \rightarrow 0} \text{Re} \Pi_{00}(0,p),$$

(1)

where $\Pi_{\sigma\nu}(t,x) = \langle J_\sigma(t,x)J_\nu(0,0) \rangle$ with external current $J_\sigma(t,x)$, $P = (T/V) \ln Z$ is the pressure, $\rho$ is the quark number density and $Z$ is the partition function. For a given

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order in coupling $\alpha_s$, the thermodynamic consistency is automatic in (1) for conventional perturbation theory whereas for resummed case one needs to take proper measure [4].

2. HTL Perturbation Theory and Thermodynamic Sum Rule

The HTL Lagrangian density [6] for quark can be written as

$$\mathcal{L}_{HTL} = \mathcal{L}_{QCD} + \delta \mathcal{L}_{HTL} = \bar{\psi} i \gamma_{\mu} D^\mu \psi + m_q^2 \bar{\psi} \gamma_{\mu} \Gamma_0 \psi \left( \frac{R^\mu}{i R \cdot D} \right) \psi ,$$  \hspace{1cm} (2)

where $R$ is a light like four-vector and $\langle \rangle$ is the angular average. A HTLpt has been developed [7] around $m_q^2$ by treating it as a parameter ($\sim (gT)^0$ and $(g\mu)^0$) much like a rest mass of a quark. In presence of an external source $\mu$, $D^\mu$ is defined as $\tilde{D}^\mu = (D^\mu - i R^\mu \mu)$. Expanding the second term around $\delta \mu$, we can write (2) as [4]

$$\mathcal{L}_{HTL}(\mu + \delta \mu) = \bar{\psi} \left( i \tilde{D} + \Sigma \right) \psi + \delta \mu \bar{\psi} \Gamma_0 \psi + \delta \mu^2 \bar{\psi} \frac{\Gamma_{00}}{2} \psi + O(\delta \mu^3),$$  \hspace{1cm} (3)

where $\Sigma, \Gamma_0, \Gamma_{00}$ are the various HTL $N$-point functions in coordinate space. The net quark number density (with $N_f$ flavour and $N_c$ color) becomes [4]

$$\rho(\beta, \mu) = \frac{\partial P}{\partial \mu} \bigg|_{\mu=0} = N_c N_f T \int \frac{d^3 k}{(2\pi)^3} \sum_{k_0=(2n+1)\pi T+\mu} \text{Tr} \left[ S^*(K) \Gamma_0(K, -K) \right].$$  \hspace{1cm} (4)

Similarly, the QNS becomes [4]

$$\chi(\beta) = \frac{\partial \rho}{\partial \mu} \bigg|_{\mu=0} = \frac{\partial^2 P}{\partial \mu^2} \bigg|_{\mu=0} = -N_c N_f T \int \frac{d^3 k}{(2\pi)^3} \sum_{k_0=(2n+1)\pi T} \times \text{Tr} \left[ S^*(K) \Gamma_0(K, -K; 0) S^*(-K) \Gamma_0(K, -K; 0) - S^*(K) \Gamma_{00}(K, -K; 0, 0) \right].$$  \hspace{1cm} (5)

The first term in (5) is a one-loop self-energy whereas the second term is a tadpole in HTLpt with effective $N$-point functions at the external momentum $P = (\omega_p, \mu) = 0$. This is thermodynamic sum rule for LO HTLpt associated with quark number conservation (gauge symmetry). As we will see below it does not matter which of the equivalent expression in (5) is used to obtain QNS in contrast to Ref. [3].

3. Thermodynamics and Quark Number Susceptibility

We intend first to obtain the LO number density $\rho(T, \mu)$ from (4) and then various thermodynamic quantities and QNS. Now considering only first term of (4), the quasiparticle (QP) part of the LO quark number density (with subscript $I$) becomes [4]

$$\rho_{I}^{QP}(T, \mu) = 2 N_c N_f \int \frac{d^3 k}{(2\pi)^3} \left[ n(\omega_+ - \mu) + n(\omega_- - \mu) - n(k - \mu) - \{ \mu \rightarrow -\mu \} \right],$$  \hspace{1cm} (6)

which agrees with that of Ref. [1]. Now, the Landau damping (LD) part [4] reads as

$$\rho_{I}^{LD}(T, \mu) = N_c N_f \int \frac{d^3 k}{(2\pi)^3} \int d\omega \left( \frac{2m_q^2}{\omega^2 - k^2} \right) \beta_+(\omega, k) \left[ n(\omega - \mu) - n(\omega + \mu) \right],$$  \hspace{1cm} (7)

where $n(y)$ is the FD function and $\beta_{\pm}$ is the cut-spectral function of HTL propagator [4]. From this number density (QP+LD) expression, pressure and entropy density can be obtained [4] using thermodynamics relations and they agree with those of Refs. [1,7].
The LO QNS is obtained [4] by taking the derivative of \( \rho \) w.r.t \( \mu \) as

\[
\chi_{II}^{QP}(T) = 4N_cN_f\beta \int \frac{d^3k}{(2\pi)^3} \left[ n(\omega_+) (1 - n(\omega_+)) + n(\omega_-) (1 - n(\omega_-)) - n(k) (1 - n(k)) \right],
\]

\[
\chi_{II}^{LD}(T) = 2N_cN_f\beta \int \frac{d^3k}{(2\pi)^3} \int d\omega \left( \frac{2m_q^2}{\omega^2 - k^2} \right) \beta_+(\omega, k) \ n(\omega) (1 - n(\omega)),
\]

where the \( \mu \) derivative is performed only to the explicit \( \mu \) dependence and agrees with Ref. [1]. The LO HTL quark density and entropy density for 2 flavour as a function of \( T/T_c \) are shown in Fig. 1 whereas HTL QNS is shown in the left panel of Fig.2.

\[
\begin{align*}
\text{Fig. 1. (Color online)} & \quad \text{Left panel: Scaled LO quark number density with } T^3 \text{ vs } T/T_c \text{ for various } \mu/T_c. \\
& \quad \text{Right panel: Scaled entropy density with free one vs } T/T_c \text{ for } \mu = 0.2T_c. \text{ } T \\
& \quad \text{dependent } \alpha_s \text{ is used with scalar } Q \text{.}
\end{align*}
\]

4. Correlation Function, Thermodynamic Sum Rule and QNS

Now, we obtain QNS (denoted by subscript II) from the correlation function of (5) that relates the thermodynamic sum rule associated with the conserved baryon number. The QP and LD part of QNS are obtained [4], respectively, as

\[
\begin{align*}
\chi_{II}^{QP}(T) &= 4N_cN_f\beta \int \frac{d^3k}{(2\pi)^3} \left[ n(\omega_+) (1 - n(\omega_+)) + n(\omega_-) (1 - n(\omega_-)) \\
& \quad - n(k) (1 - n(k)) - \frac{1}{\beta_k} (1 - 2n(k)) \right],
\end{align*}
\]

\[
\begin{align*}
\chi_{II}^{LD}(T) &= 2N_cN_f\beta \int \frac{d^3k}{(2\pi)^3} \int d\omega \left( \frac{2m_q^2}{\omega^2 - k^2} \right) \beta_+(\omega, k) \ n(\omega) (1 - n(\omega)) \\
& \quad + N_cN_f \int \frac{d^3k}{(2\pi)^3} \int d\omega \left( \frac{2m_q^2}{\omega^2 - k^2} \right) \frac{\beta_+(\omega, k)}{\omega + k} \ (1 - 2n(\omega)).
\end{align*}
\]

Both expressions in (9) agree to those, respectively, in (8) but with an additional term. For LO QNS one can neglect [4] these additional terms in (9). Thus, the LO thermodynamic sum rule associated with quark number conservation in HTLpt is guaranteed. Below we show the correct inclusion of LO to QNS in HTLpt in a strict perturbative sense.

5. QNS in Perturbative Leading Order (\( g^2 \))

In conventional perturbation theory, the QNS has been calculated for massless QCD [1] upto order \( g^4 \log(1/g) \) at \( \mu = 0 \) as
\[
\frac{x_p}{x_f} = 1 - \frac{1}{2} \left( \frac{g^2}{\pi} \right)^2 + \sqrt{1 + \frac{N_f}{6} \left( \frac{g^2}{\pi} \right) - 3 \left( \frac{g^2}{\pi} \right)^4 \log \left( \frac{1}{g^2} \right)} + O(g^4) .
\]

(10)

In the middle panel of Fig. 2 we display the LO HTL and perturbative QNS scaled with free one vs \(m_q/T\). In the weak coupling limit both approach unity whereas HTL has a little slower deviation from ideal gas value. Now, in the right panel we plot a ratio \[ R = \left( \frac{\chi_{\text{htl}}}{\chi_{\text{pQCD}}} \right) \left( \frac{x_p(g^2)}{x_f} \right) \), which measures the deviation of interaction of \(\chi_{\text{htl}}\) from that of pQCD in order \(g^2\). It approaches unity in the weak coupling limit implying the correct inclusion \[1\] of order \(g^2\) in our approach in a truly perturbative sense.

6. Conclusion

We have formulated a thermodynamically consistent HTLpt at the first derivative level of the thermal potential by incorporating an external source, viz., the quark chemical potential to the system. We show that the various thermodynamic quantities and the QNS agree with the other HTL approaches \[1\] existing in the literature. In addition we also obtained the QNS from the correlation functions associated with the conserved number density fluctuation of the system. The equivalence of the results obtained for the QNS in LO HTLpt in two ways (Sec. 3 and 4) guarantees the thermodynamic sum rule, which was not shown earlier within HTL approximation earlier. We also reproduce the LO perturbative results in the weak coupling limit, which indicates the correct inclusion of the LO in a strict perturbative sense in our approach.

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