Zero temperature properties of mesons and baryons from an extended linear sigma-model

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Abstract. An extended linear sigma model with mesons ($q\bar{q}$ states) and baryons ($qqq$ states) is presented. The model contains a low energy multiplet for every hadronic particle type, namely a scalar, a pseudoscalar, a vector and an axialvector nonet, a baryon octet and a baryon decuplet. The model parameters are determined through a multiparametric minimalization with the help of well known physical quantities. It is found that the considered zero temperature quantities (masses and decay widths) can be described well at tree-level and are in good agreement with the experimental data.

1. Introduction
The vacuum properties of strong interaction are very hard to investigate within the framework of QCD – the fundamental theory of strong interaction – (see e.g.[1]), which is due to its subtlety at low energies. Consequently, instead of solving QCD one can set up an effective theory, which reflects some properties of the original theory. The underlying principle in the construction of such theories is that they share the same global symmetries as QCD.

For zero masses of the $u$, $d$ and $s$ quarks, the global symmetry of QCD is $U(3)_R \times U(3)_L$, the so-called chiral symmetry. In the vacuum, this chiral symmetry is spontaneously broken due to the existence of a quark-antiquark condensate. The chiral symmetry can be realized in different ways, nonlinearly [2] and linearly [3], which we choose here. Accordingly, in this paper we set up an extended linear sigma model, which contains mesonic and baryonic degrees of freedom. The previous versions of our model [4, 5] contained the scalar, pseudoscalar, vector- and axial-vector nonets. The vacuum phenomenology of mesons was described very well in that model. As an improvement, in this paper we include additionally the nucleon-octet and the Delta-decuplet to extend the vacuum phenomenology for baryons as well. Another approach to baryon phenomenology can be found in [6].

Our paper is organized as follows. In the Sec. 2 we briefly present the model, while in Sec. 3 we describe how to calculate various tree-level quantities. The Sec. 4 is dedicated to the results in the mesonic and baryonic sector and finally we conclude in Sec. 5.

2. The Model
The model can be defined through a Lagrangian consisting of a purely mesonic and a baryonic-mesonic part as $\mathcal{L} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{baryon}}$. The terms in the meson part are limited by chiral and dilaton symmetry (for details see [5]), while in case of the baryon part we included all the $SU(3)_V$
invariants which can produce baryon masses – with different masses for different particles in the given multiplet – and decuplet decays with the lowest possible dimension \((B - B - \Phi - \Phi, \Delta - \Delta - \Phi - \Phi \text{ and } \Delta - B - \Phi \text{ terms})\). The mesonic part has the following form

\[
\mathcal{L}_\text{meson} = \text{Tr}[(D_\mu \Phi)^\dagger(D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1[\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2
\]

\[
- \frac{1}{4} \text{Tr}(L_{\mu
u} + R_{\mu
u}) + \text{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L^2 + R^2)\right] + \text{Tr}(H(\Phi + \Phi^\dagger))
\]

\[
+ c_1(\det \Phi - \det \Phi^\dagger)^2 + i\frac{g_2}{2}\left[\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}\right]
\]

\[
+ \frac{g_3}{2}\text{Tr}(\Phi^\dagger \Phi)\left[\text{Tr}(L^2 + R^2) + h_2\text{Tr}(L \Phi)^2 + (\Phi R_{\mu})^2\right] + 2h_3\text{Tr}(L \Phi R^\mu \Phi^\dagger).
\]

\[
+ g_4\left[\text{Tr}(L_{\mu} L_{\nu} L^\mu L^\nu) + \text{Tr}(R_{\mu} R_{\nu} R^\mu R^\nu)\right] + g_3\left[\text{Tr}(L_{\mu} L^\mu L \Phi^\dagger) + \text{Tr}(R_{\mu} R^\mu R \Phi^\dagger)\right] + g_5\left[\text{Tr}(L_{\mu} L^\mu) \Phi^\dagger + \text{Tr}(L_\mu \Phi)\right]\]

\[
\text{Tr}(R_{\mu} R^\mu) + g_6\left[\text{Tr}(L_{\mu} L^\mu) \Phi^\dagger + \text{Tr}(R_{\mu} R^\mu)\right] + \text{Tr}(R_{\mu} R^\mu)\right]\]

\[
\right)\right),
\]

where

\[
D^\mu \Phi \equiv \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA^\nu[T_3, \Phi],
\]

\[
L^\mu = \partial^\mu \phi - ieA^\nu[T_3, \phi], \quad L^\mu = \{\partial^\mu \phi - ieA^\nu[T_3, \phi]\},
\]

\[
R^\mu = \partial^\mu R - ieA^\nu[T_3, R], \quad R^\mu = \{\partial^\mu R - ieA^\nu[T_3, R]\},
\]

The quantities \(\Phi = \sum_{i=1}^{8}(S_i + iP_i T_i), \quad L^\mu / R^\mu = \sum_{i=0}^{8}(V_{i}^\mu \pm A_{i}^\mu) T_i\) represent the scalar-pseudoscalar nonets and the left-/right-handed vector nonets. \(T_i \quad (i = 0, \ldots, 8)\) denote the generators of \(U(3)\), while \(S_i\) represents the scalar, \(P_i\) the pseudoscalar, \(V_i^\mu\) the vector, and \(A_i^\mu\) the axial-vector meson fields, and \(A_i^\mu\) is the electromagnetic field. \(H\) and \(\Delta\) are some constant external fields. It should be noted that in the \((0 - 8)\) sector\(^1\) of the scalars and pseudoscalars there is a mixing and it is more suitable to use the non strange – strange basis defined as \(\varphi_N = 1/\sqrt{3}(\varphi_0 + \varphi_8), \quad \varphi_S = 1/\sqrt{3}(\varphi_0 - \varphi_8)\) for \(\varphi_i \in (S_i, P_i, V_i^\mu, A_i^\mu)\).

Moving on to the baryonic-mesonic part, the Lagrangian is given by

\[
\mathcal{L}_\text{baryon} = \text{Tr}\left[\hat{B} i\partial^\mu - M_{(8)}\right]\hat{B},
\]

\[
- \text{Tr}\left[\hat{\Delta}_\mu, \left[(i\partial^\mu - M_{(10)}) \hat{B}\right]\right] - \xi_1 \text{Tr}\left[\hat{B}B\right] / \text{Tr}(\Phi^\dagger \Phi) - \xi_2 \text{Tr}\left[\hat{B}\{\Phi, \Phi^\dagger\}, B\right] + \xi_3 \text{Tr}\left[\hat{B}\{[\Phi, \Phi^\dagger], B\}\right]
\]

\[
- \xi_4 \text{Tr}\left[\hat{B}\Phi\right] / \text{Tr}(\Phi^\dagger \Phi) + \xi_5 \text{Tr}\left[\hat{B}\{\Phi, \Phi^\dagger\}, B\right] + \xi_6 \text{Tr}\left[\hat{B}\{[\Phi, \Phi^\dagger], B\}\right]
\]

\[
- \xi_7 \text{Tr}\left[\hat{B}\Phi\right] / \text{Tr}(\Phi^\dagger \Phi) + \xi_8 \text{Tr}\left[\hat{B}\{[\Phi, \Phi^\dagger], B\}\right] + \chi_1 \text{Tr}\left[\hat{\Delta} \cdot \Delta\right] \text{Tr}(\Phi^\dagger \Phi)
\]

\[
+ \chi_2 \text{Tr}\left[\{\hat{\Delta} \cdot \Delta\}, \Phi, \Phi^\dagger\right] + \chi_3 \text{Tr}\left[\{\hat{\Delta} \cdot \Phi\}(\Phi^\dagger \cdot \Delta) + (\hat{\Delta} \cdot \Phi^\dagger)(\Phi \cdot \Delta)\right]
\]

\[
+ \chi_4 \text{Tr}\left[\{\hat{\Delta} \cdot \Delta\}, [\Phi, \Phi^\dagger]\right],
\]

where \(B = \sqrt{2}\sum_{i=1}^{8}B_i T_a\) and \(\Delta_\mu\) stands for the baryon octet and decuplet. \(M_{(8)}\) and \(M_{(10)}\) are the bare masses of the baryon octet and decuplet. \(f\) is the pion decay constant, while \([\ , ]\)
and $\{,\}$ denote the commutator and the anticommutator. Here the meson-baryon interaction terms are all the possible $SU(3)_V$ invariants that can be written down with the given number of fields [7, 8]. The covariant derivatives are defined as

$$D_{\mu}B = \partial_{\mu}B + i[B, V_{\mu}] + \frac{1}{f} \{[A_{\mu}, \Phi], B\},$$

$$D_{\mu}\Delta_{ij}^{\mu} = \partial\Delta_{ij}^{\mu} + \left(\frac{1}{f}[A_{\mu}, \Phi]_i^j - iV_{\mu}^j\right)\Delta_{ij}^{\mu} + \left(\frac{1}{f}[A_{\mu}, \Phi]_i^j - iV_{\mu}^j\right)\Delta_{ij}^{\mu},$$

and the following dot notation is used:

$$(\vec{\Delta} \cdot \Delta)^{nm}_{\mu} \equiv \Delta_{ijk}\Delta_{ijm}^{\mu}, \quad (\vec{\Delta} \cdot \Phi)^{nm}_{\mu} \equiv \Delta_{ijk}\Phi_{ijkl}^{\mu}, \quad (\Phi \cdot \Delta)^{nm}_{\mu} \equiv \delta^{ijm}\Phi_{ijkl}^{\mu}. \quad (3)$$

From the given Lagrangian various tree-level quantities such as masses and decay widths can be calculated and used to determine the unknown parameters of the model. During this parametrization process we can check how well the physical spectrum is reproduced. In the next section calculation of tree-level quantities and the parametrization are discussed.

3. Tree-level quantities and parametrization

As a standard procedure in a spontaneously broken theory we assume non-zero vacuum expectation values (vev) to certain fields, in our case to the $\sigma_N \equiv \sqrt{3}(\sqrt{2}\sigma_0 + \sigma_3)$ and $\sigma_S \equiv 1/\sqrt{3}(\sigma_0 - \sqrt{2}\sigma_3)$ scalar fields, and denote their vev by $\phi_N$ and $\phi_S$. After that the $\sigma_N$ and $\sigma_S$ fields are shifted with their non zero vev’s $\phi_N$ and $\phi_S$. Consequently, the quadratic and three-coupling terms of the Lagrangian can be determined from which the masses and the decay widths originate. However, it should be noted that as a technical difficulty – due to the $\sigma_N$ and $\sigma_S$ field shifts – different particle mixings emerge. In detail, there will be mixings in the $N - S$ (or $0 - 8$) sector of the scalar and pseudoscalar octets and between the vector-scalar and axialvector-pseudoscalar nonets. The $N - S$ mixings can be resolved by some orthogonal transformation, while the other mixings by redefinition of certain (axial-)vector fields. The details can be found in [5] together with explicit expressions for the meson masses and various decay widths.

Regarding the baryon sector the tree-level octet and decuplet masses – from the terms of the Lagrangian quadratic in the fields $B$ and $\Delta_{\mu}$ – are found to be

$$m_p = m_n = M_{(8)} + \frac{1}{2} \xi_2(\Phi_N^2 + 2\Phi_S^2) + \frac{1}{2} \xi_3(\Phi_N^2 - 2\Phi_S^2),$$

$$m_{\Xi} = M_{(8)} + \frac{1}{2} \xi_2(\Phi_N^2 + 2\Phi_S^2) - \frac{1}{2} \xi_3(\Phi_N^2 - 2\Phi_S^2),$$

$$m_{\Sigma} = M_{(8)} + \xi_2\Phi_N^2,$$

$$m_{\Lambda} = M_{(8)} + \frac{1}{3} \xi_2(\Phi_N^2 + 4\Phi_S^2) + \frac{1}{3} \xi_4(\Phi_N - \sqrt{2}\Phi_S)^2, m_{\Delta} = M_{(10)} + \frac{1}{2} \chi_2\Phi_N^2,$$

$$m_{\Sigma^*} = M_{(10)} + \frac{1}{3} \chi_2(\Phi_N^2 + \Phi_S^2) + \frac{1}{6} \chi_3(\Phi_N - \sqrt{2}\Phi_S)^2,$$

$$m_{\Xi^*} = M_{(10)} + \frac{1}{6} \chi_2(\Phi_N^2 + 4\Phi_S^2) + \frac{1}{6} \chi_3(\Phi_N - \sqrt{2}\Phi_S)^2,$$

$$m_{\Omega} = M_{(10)} + \chi_2\Phi_N^2. \quad (4)$$

Beside the masses one can consider two-body decays of the decuplet baryons. According to PDG [9] there are four such physically allowed decays,

$$\Delta \to p\pi, \quad \Sigma^* \to \Lambda\pi, \quad \Xi^* \to \Xi\pi, \quad \Sigma^* \to \Sigma\pi. \quad (5)$$
And the decay widths are given by

\[ \Gamma_{\Delta \rightarrow \pi p} = \frac{k_\Delta^3}{24m_\Delta} (m_p + E_p)G^2, \quad \Gamma_{\Sigma^* \rightarrow \pi \Lambda} = \frac{k_{\Sigma^*}^3}{48m_{\Sigma^*}} (m_\Lambda + E_\Lambda)G^2, \]
\[ \Gamma_{\Xi^* \rightarrow \pi \Xi} = \frac{k_{\Xi^*}^3}{48m_{\Xi^*}} (m_\Xi + E_\Xi)G^2, \quad \Gamma_{\Sigma^* \rightarrow \pi \Sigma} = \frac{k_{\Sigma^*}^3}{72m_{\Sigma^*}} (m_\Sigma + E_\Sigma)G^2, \]  
(6)

with

\[ G^2 = C^2 z_\pi^2 \left( u_{a_1}^2 + \frac{1}{f^2} \right). \]

As it can be seen from the Lagrangian there are 30 unknown parameters of the model, 14 in the meson sector and 16 in the baryon sector. However some of them can be set to zero without the loss of generality, some of them not even appear in the formulas of the physical quantities considered here, while some of them appear only in certain combinations. All in all there is 19 parameters which should be determined. These parameters are determined through the comparison of the calculated tree-level expressions – from which we have 23 – of the spectrum and decay widths with their experimental value taken from [9] with artificially increased errors (5% for the masses and 10% for the decay widths, since we do not expect from a tree-level model to be more precise). Our strategy is that first we set the parameters of the meson sector [5] and then we fit the remaining parameters of the meson-baryon interaction terms. For this we used a \( \chi^2 \)-minimalization, which was realized with a multiparametric minimalization code (MINUIT [10]).

4. Results

Using the above mentioned \( \chi^2 \)-minimalization method in the meson sector we obtain results summarized in Table 1, which can also be found in [5].

In the baryon sector at first we only investigated the decuplet decays. The results are given in Table 2.

It can be seen that the meson observables are reproduced very well in general, while the decuplet decays show a fair correspondence. It is worth to note that in case of the the decuplet decays their tree-level expressions Eq. (6) only differ in their kinematic parts and we have only one parameter to fit for the four observables, which can cause the deviation from the experimental value.

5. Conclusion

We have presented an extended linear sigma model with meson and baryon degrees of freedom. This is a possible extension with baryon octet and decuplet of our previous meson model [5]. We included interaction terms, such as \( \Delta - B - P \) and \( B - B - \Phi - \Phi \) to describe \( \Delta \) decays and baryon masses. We calculated the tree-level masses and physically relevant decuplet decay widths and we found that in general they are in good agreement with the experimental data taken from the PDG [9].

As a continuation we plan to include the baryon masses to the fit and to add other (higher dimension) interaction terms containing derivatives which are important in case of scattering processes [6]. Our further aim is to go to finite temperature and/or densities with all these fields included in our model.

Acknowledgments

P. Kovács and Gy. Wolf were partially supported by the Hungarian OTKA funds NK101438 and K109462.

\[ \text{The isospin violation in some cases (e.g. for pion) has the order of 5\%} \]
Table 1. Calculated and experimental values of meson observables

| Observable | Fit [MeV] | PDG [MeV] | Error [MeV] |
|------------|-----------|-----------|-------------|
| $m_\pi$    | 141.0 ± 5.8 | 137.3     | ±6.9        |
| $m_K$      | 485.6 ± 3.0 | 495.6     | ±24.8       |
| $m_\eta$   | 509.4 ± 3.0 | 547.9     | ±27.4       |
| $m_{\eta'}$| 962.5 ± 5.6 | 957.8     | ±47.9       |
| $m_\rho$   | 783.1 ± 7.0 | 775.5     | ±38.8       |
| $m_{K^*}$  | 885.1 ± 6.3 | 893.8     | ±44.7       |
| $m_\phi$   | 975.1 ± 6.4 | 1019.5    | ±51.0       |
| $m_{a_1}$  | 1186 ± 6   | 1230      | ±62         |
| $m_{f_1(1420)}$ | 1372.5 ± 5.3 | 1426.4 | ±71.3       |
| $m_{a_0}$  | 1363 ± 1   | 1474      | ±74         |
| $m_{K^*_0}$| 1450 ± 1   | 1425      | ±71         |
| $\Gamma_\rho\to\pi\pi$ | 160.9 ± 4.4 | 149.1 | ±7.4         |
| $\Gamma_{K^*}\to K\pi$ | 44.6 ± 1.9 | 46.2 | ±2.3         |
| $\Gamma_\phi\to K\bar{K}$ | 3.34 ± 0.14 | 3.54 | ±0.18        |
| $\Gamma_{a_1}\to\rho\pi$ | 549 ± 43 | 425 | ±175         |
| $\Gamma_{a_1}\to\pi\gamma$ | 0.66 ± 0.01 | 0.64 | ±0.25        |
| $\Gamma_{f_1(1420)}\to K^*K$ | 44.6 ± 39.9 | 43.9 | ±2.2         |
| $\Gamma_{a_0}$ | 266 ± 12 | 265 | ±13         |
| $\Gamma_{K^*_0}\to K\pi$ | 285 ± 12 | 270 | ±80         |

Table 2. Calculated and experimental values of baryon observables

| Observable | Fit [MeV] | PDG [MeV] | Error [MeV] |
|------------|-----------|-----------|-------------|
| $\Gamma_{\Delta}\to p\pi$ | 67.3 | 110.0 | ±11.0 |
| $\Gamma_{\Sigma^*}\to \Lambda\pi$ | 27.0 | 32.0 | ±3.2 |
| $\Gamma_{\Sigma^*}\to \Sigma\pi$ | 4.9 | 4.3 | ±0.4 |
| $\Gamma_{\Xi^*}\to \Xi\pi$ | 11.2 | 9.5 | ±1.0 |

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