Observer-Based Coordinated Tracking Control for Nonlinear Multi-Agent Systems with Intermittent Communication under Heterogeneous Coupling Framework

Yuhang Zhang, Yulian Jiang and Shenquan Wang

Abstract—In this article, the observer-based coordinated tracking control problem for a class of nonlinear multi-agent systems (MASs) with intermittent communication and information constraints is studied under dynamic switching topology. First, a state observer is designed to estimate the unmeasurable actual state information in the system. Second, adjustable heterogeneous coupling weighting parameters are introduced in the dynamic switching topology, and the distributed coordinated tracking control protocol under heterogeneous coupling framework is proposed. Then, a new Lemma is constructed to realize the cooperative design of observer gain, state feedback gain and heterogeneous coupling gain matrices. Furthermore, the stability of the system is further proved, and the range of communication rate is obtained. On this basis, the intermittent communication mode is extended to three time interval cases, namely normal communication, leader-follower communication interruption and all agents communication interruption. Moreover, the distributed coordinated tracking control method is improved to solve this problem. Finally, simulation experiments are conducted with nonlinear MASs to verify the correctness of methods.

Index Terms—nonlinear multi-agent systems; distributed cooperative tracking control; heterogeneous coupling framework; observer; intermittent communication.

I. INTRODUCTION

The problem of cooperative control of multi-agent systems (MASs) has attracted increasing attention in various scientific communities recently including synchronization of complex networks, cooperative control of multiple vehicles, coordination of multiple robotic fish, teaming of robots, and so on [1–6]. According to the control objectives of distributed control of MASs, the existing coordination control problems are divided into regulation cooperative control [7] and coordinated tracking control [8]. However, it is important to design a suitable control strategy for whichever control goal you want to achieve. In the meantime, an appropriate framework should be chosen to address the phenomenon of information interaction within and outside the system. Distributed control protocols are designed for all agents via their available local state information to achieve a specific number of interests into agreement, which can categorized into leaderless control protocol [9, 10] and leader-follower control protocol [11, 12] for MASs. In the latter case, which is the concern of this paper, all agents are expected to follow the trajectory of expectations generated by the leader.

In recent years, many meaningful results have been generated on studies related to behavioral consensus. State feedback based on linear combinations of relative state measurements between neighboring agents was used to implement the design of the proposed protocol in [13–15]. However, in practical or engineering applications, many state quantities in the system are not directly accessible because of economic costs, environmental constraints, etc., and thus the measurable output information is used to design a state observer, thus providing an elegant alternative that can be used for observation, diagnosis, and estimation purposes.

In order to estimate the leader’s velocity, a new neighbor-based protocol was proposed for first-order followers in [16], and it was improved to second-order followers [17, 18] and high-order systems with input saturation [19], time delays [20], and external disturbance [21]. Li et al. proposed a distributed observer-based consensus protocol [22] for general linear MASs and further investigated corresponding reduced-order case [23]. A fixed-time distributed observer was developed in [24] with high-order integrator dynamics and matched external disturbances. In addition, the cooperative output regulation problem of linear discrete-time time-delay MASs was developed in [25] utilizing adaptive distributed observers. Nevertheless, most of the current research on observers was focused on low-order integral systems or general linear systems. The study of observer problems in nonlinear MASs is more complicated and requires dealing with nonlinear dynamics through Lipschitz conditions.

It is noteworthy that the above distributed control problems are solved based on the fact that the communication between neighboring agents is continuous all the time. However, communication bandwidth, physical device failures and economic costs lead to communication constraints in real systems, which means that communication between neighboring agents may be intermittent or interrupted [26, 29]. How to address observer-based coordinated tracking control for nonlinear MASs with intermittent communication and dynamic switching topology via heterogeneous coupling framework is
This article mainly studies observer-based consensus control problems for nonlinear MASs with intermittent communication under heterogeneous coupling network. The main contributions are summarized as follows: (1) A class of nonlinear observers is investigated based on each agent only temporally sharing its relative output with neighbors to estimate the actual and unavailable states for nonlinear MASs. (2) The distributed tracking controllers based heterogeneous coupling framework with nonlinear observers are designed for considering intermittent constrained and intermittent coordinated constrained, in which some additional adjustable weighting parameters are constructed via dynamic interaction topology and heterogeneous coupling network. (3) Utilizing the switching system theory, Lyapunov function and LMI technology, the stability criterion. Lyapunov function and LMI technology, the stability of the system is proved and communication conditions are given, meanwhile, the co-design of heterogeneous coupling weighting, feedback gain and observer gain matrices are realized. (4) The result presented in this work can resolve the coordinated tracking consensus for higher order nonlinear MASs with unavailable states under intermittent communication and dynamic switching topology.

II. GRAPH THEORY AND PROBLEM FORMULATION
A. Graph Theory And Problem Formulation

The communication topology between agents is described by the directed graph $G(\tilde{v}, \zeta, A)$, in which $\tilde{v} = \{\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_N\}$ denotes the set of nodes, $\zeta \subseteq \tilde{v} \times \tilde{v}$ denotes the set of edges, and $A = [a_{ij}]_{N \times N}$ is the weighted adjacency matrix with elements $a_{ij}$. If $(\tilde{v}_i, \tilde{v}_j) \in \zeta$, the information in the $j$th node can be passed to the $i$th one. Then $a_{ij} = 1$. If $(\tilde{v}_i, \tilde{v}_j) \notin \zeta$, $a_{ij} = 0$. The directed dynamic switching topology $G$ consists of $N + 1$ nodes, i.e., generated by the directed graph $G$ of the follower and the leader. Let $L^{(t)}$ be the Laplace matrix of the directed graph $G$ and $D^{(t)} = \text{diag} \{d_1^{(t)}, d_2^{(t)}, \cdots, d_N^{(t)}\}$ denote the diagonal matrix of the different communication paths between the followers and the leader, where $\Theta(t) : [0, +\infty) \to \{1, 2, \cdots, p\}$ denotes the switching signal. If the followers can obtain the leader information at time $t \geq 0$, then $d_i^{(t)} = 1$, otherwise $d_i^{(t)} = 0$. In addition, $\tilde{G}^{(t)} \in \tilde{G}$, where $\tilde{G} = \{\tilde{G}^1, \tilde{G}^2, \cdots, \tilde{G}^p\}$ denotes the set of topologies composed of different interaction methods under the switching signal.

B. Problem Formulation

The nonlinear leader-following MASs considered in this research consists of $N$ agents named followers and one more agent as a leader, the dynamic model is

$$
\begin{align*}
\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + f(x_i(t)) \\
\dot{x}_0(t) &= Ax_0(t) + f(x_0(t)) \\
y_i(t) &= Cx_i(t) \\
y_0(t) &= Cx_0(t)
\end{align*}
$$

where $i = 1, 2, \ldots, N$, $x_i(t) \in \mathbb{R}^n$, $y_i(t) \in \mathbb{R}^2$ and $u_i(t) \in \mathbb{R}^m$ represent the $i$th follower’s state, output and control input, respectively. In addition, $x_0(t) \in \mathbb{R}^n$ and $y_0(t) \in \mathbb{R}^2$ denote the leader’s state and output. $A$, $B$ and $C$ are matrices of appropriate dimensions. $f(x_i(t))$ and $f(x_0(t))$ are continuously differentiable vector-valued functions, represent the intrinsic nonlinear dynamics of the $i$th agent and the leader.

**Assumption 1**: The digraph $\tilde{G}$ contains a directed spanning tree, and the leader as one agent is the root.

**Assumption 2**: For a nonlinear function $f(x_i(t))$, there exists a nonnegative constant $\varepsilon$ such that:

$$
\|f(x_i(t)) - f(x_0(t))\| \leq \varepsilon \|x_i(t) - x_0(t)\|
$$

then it means that $f(x_i(t))$ satisfies the Lipschitz condition.

Some lemmas will be provided here to derive the proposed criterion.

**Lemma 1** [30]: Define $L^{(t)}$ be a structure matrix of derivative directed graph $G$. Then, $G$ has a spanning tree if and only if $L^{(t)}$ has a nonzero eigenvalues with multiplicity $1$ and all the other nonzero eigenvalues have positive real parts.

**Definition 1**: Let $Z_n \in \mathbb{R}^{n \times n}$ be the set of all square matrices of dimension $n$ with nonpositive off-diagonal entries. A matrix $A \in \mathbb{R}^{n \times n}$ is said to be a nonsingular M-matrix if all its eigenvalues have positive real parts.

**Lemma 2** [31]: Suppose that a matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ satisfies $a_{ij} < 0$ for any $i \neq j$. Then, the following statements are equivalent: 1) $A$ is a nonsingular M-matrix; 2) there exists a positive definite $n \times n$ diagonal matrix $\Theta$ such that $A^T \Theta + \Theta A > 0$; and 3) all eigenvalues of $A$ have positive real parts.

**Lemma 3** [32]: Suppose that $M \in \mathbb{R}^{n \times n}$ is the symmetric positive-define matrix and $\bar{R} \in \mathbb{R}^{n \times n}$ is the symmetric matrix, then, for any vector $x \in \mathbb{R}^n$, the following formula holds

$$
\lambda_{\min}(M^{-1}\bar{R}) x^T M x \leq x^T \bar{R} x \leq \lambda_{\max}(M^{-1}\bar{R}) x^T M x
$$

**Lemma 4** [33]: For $\forall a, b$, one has $2a^T b \leq l a^T a + l^T b^T b$, where $\forall a, b \in \mathbb{R}^n$ and $l > 0$.

**Definition 2**: Consider the nonlinear MASs (1), if

$$
\lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0 \quad (\forall i = 1, 2, \cdots, N)
$$

is satisfied for any initial states, the nonlinear MASs (1) achieve consensus tracking.

III. STABILITY ANALYSIS

This session introduces the problem about observer-based tracking consensus control with intermittent communication and dynamic switching topology for nonlinear MASs, which can ensure the system (1) stable and the heterogeneous coupling gain and feedback gain matrices derived.

In order to estimate the unmeasurable states of nonlinear MASs, by using the measurable output information, a type of nonlinear observers can be designed as follows:

$$
\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B u_i(t) + f(\hat{x}_i(t)) + \tilde{G}(\hat{y}_i(t) - y_i(t))
$$

where $\hat{x}_i(t)$ is the estimated state of $x_i(t)$ and $\tilde{G}$ is the observer gain matrix, $f(\hat{x}_i(t))$ is the nonlinear function satisfying Assumption 2, $\hat{y}_i(t)$ is the output of followers.
Here, the communication mode is considered as periodic intermittent communication. Suppose that \( t \in [kw, kw + \delta) = T^m \) represents normal communication time period, while \( t \in [kw + \delta, (k + 1) w) = T^m \) represents communication interruption time period, where \( w > \delta > 0, k \in \mathbb{Z}, t \in [kw, (k + 1) w) \) indicates that within each periodic intermittent communication time period \( w \), the normal communication time period \( \delta \) and the communication interruption time period \( w - \delta \) are included, where \( k \) refers to the first few periodic intermittent communication periods. In order to achieve \( w > \delta > 0 \), the following protocol under heterogeneous coupling framework is presented:

\[
\Psi (t) = \Psi (0) + \int_{0}^{t} \mathbf{A} \Psi (\tau) d \tau,
\]

where \( \mathbf{A} = \mathbf{A}^{\delta} + \mathbf{A}^{\omega} \), and \( \mathbf{A}^{\delta} = \mathbf{A}^{\delta}_1 + \mathbf{A}^{\delta}_2 + \mathbf{A}^{\delta}_3 \), where \( \mathbf{A}^{\delta}_1, \mathbf{A}^{\delta}_2, \mathbf{A}^{\delta}_3 \) represent the heterogeneous network topology.

We give the assumption for constructing dynamic interactive network topology.

**Assumption 3**: Assume that there exists an infinite sequence of consistent bounded non-overlapping time intervals \( t \in [kw, (k + 1) w), k \in \mathbb{N} \), where \( (k - 1) w = t_k, kw = t_{k+1}, t_1, t_2, \ldots, t_n \) denotes the switching sequence of communication network topology in MASs. When \( t_1 = 0 \), \( \inf_{k \in \mathbb{N}} (t_{k+1} - t_k) \geq 0, \sup_{k \in \mathbb{N}} (t_{k+1} - t_k) < \varepsilon_1 \).

Suppose that the switching signal is \( \vartheta(t) : [0, +\infty) \rightarrow \{1, \ldots, p\} \). Let \( \hat{G}^{\vartheta(t)}(\hat{t}) \in \hat{G} \) be the digraph for nonlinear MASs (1), and the set of all directed graphs can be represented as \( \hat{G} = \{\hat{G}^1, \ldots, \hat{G}^p\} \), where \( p \geq 1 \). After the above analysis, we are obtain, for \( t \geq 0 \), communication topology \( \hat{G}(t) = \hat{G}^{\vartheta(t)}(\hat{t}) \in \hat{G}, \hat{G} = \{\hat{G}^1, \ldots, \hat{G}^p\} \).

Further, the following Lemma is proposed to realize the design of heterogeneous coupling gain matrix, observer gain matrix and state feedback gain matrix.

**Lemma 5**: Define \( \hat{L}^{\vartheta(t)}, \hat{L}^{\vartheta(t)} \) and \( \Gamma^{\vartheta(t)} \) as the switched directed graph, its corresponding switching structure matrix and heterogeneous coupling gain matrix, respectively. Since the leader trajectory is not affected by the followers, we can obtain:

\[
\hat{L}^{\vartheta(t)} = \begin{pmatrix} 0 & 0 \end{pmatrix}^{T} \begin{pmatrix} \hat{L}_1^{\vartheta(t)} & \hat{L}_2^{\vartheta(t)} \end{pmatrix}
\]

where \( \hat{L}_1^{\vartheta(t)} \in R^{n \times n}, \hat{L}_2^{\vartheta(t)} = L^{\vartheta(t)} + D^{\vartheta(t)}(\hat{t}) \in R^{N \times N} \), and there exists a bounded scalar \( 0 < \beta \leq \lambda_{\min}(\hat{A}^{\vartheta(t)}) \) can make the following LMI holds

\[
Q^1 = \begin{pmatrix} AP_1 + P_1 A^T + \rho^2 I_N + +BP_1 - \beta(1 - \frac{1}{2}) BB^T & P_1 \end{pmatrix}^* < 0
\]

\[
Q^2 = \begin{pmatrix} AP_2 + P_2 A^T + \rho^2 I_N + +\hat{Q} + \hat{Q}^T + \beta P_2 & M^T \end{pmatrix}^* < 0
\]

\[
Q^3 = AP_1 + P_1 A^T + \rho^2 I_N + +P_1 T^P P_1
\]

\[
Q^4 = AP_2 + P_2 A^T + \rho^2 I_N + +\hat{Q} + \hat{Q}^T + P_2^T P_2
\]

where \( \lambda^{\vartheta(t)} = \lambda(\hat{A}^{\vartheta(t)})^{-1} \Phi^{\vartheta(t)}(\hat{A}^{\vartheta(t)})^{-1}, \hat{\Xi}^{\vartheta(t)} = \sqrt{\hat{P}^{\vartheta(t)}(\hat{L}^{\vartheta(t)})}, \hat{L}^{\vartheta(t)} = (\hat{L}^{\vartheta(t)} + D^{\vartheta(t)}) \Gamma^{\vartheta(t)}, \Phi^{\vartheta(t)} = \Pi^{\vartheta(t)} \Xi^{\vartheta(t)} \hat{L}^{\vartheta(t)} + \left( \hat{L}^{\vartheta(t)} \right)^T \Pi^{\vartheta(t)} \Xi^{\vartheta(t)} \),

\[
\Xi^{\vartheta(t)} = diag\left\{ \xi_1^{\vartheta(t)}, \ldots, \xi_p^{\vartheta(t)} \right\}, \xi_i^{\vartheta(t)} = 1/\theta_i^{\vartheta(t)} \text{ and } \theta_i^{\vartheta(t)} = \left( \theta_i^{\vartheta(t)}, \ldots, \theta_p^{\vartheta(t)} \right), \text{ and } (\hat{L}^{\vartheta(t)} + D^{\vartheta(t)}) \Gamma^{\vartheta(t)} = I_p, \Pi^{\vartheta(t)} = diag\left\{ \pi_1^{\vartheta(t)} I_{n_1}, \ldots, \pi_p^{\vartheta(t)} I_{n_p} \right\}, \pi_i^{\vartheta(t)} \text{ is the appropriate positive number to be selected. Matrices } \hat{M} = \hat{G} P_2 \text{ and } M = KP_2 \text{, and feedback gain matrix } K = -B^T P_1^{-1}.
\]

**Proof**: It follows from Lemmas 1 and 2 that there exists a matrix \( \Xi_i^{\vartheta(t)} \) such that:

\[
\Xi_i^{\vartheta(t)} \left( L_i^{\vartheta(t)} + D_i^{\vartheta(t)} \right) + \left( L_i^{\vartheta(t)} + D_i^{\vartheta(t)} \right)^T \Xi_i^{\vartheta(t)} > 0
\]
Define \( \Xi^{\theta(t)} = \text{diag}\{\Xi_1^{\theta(t)}, \ldots, \Xi_p^{\theta(t)}\} \), \( \Xi_i^{\theta(t)} = 1/\hat{q}_i^{\theta(t)} \) and \( \theta^{\theta(t)} = (\theta_1^{\theta(t)}, \ldots, \theta_p^{\theta(t)})^T \), and it satisfies \((L_i^{\theta(t)} + D^{\theta(t)})\Gamma^{\theta(t)} = I_p \) and \( \Pi^{\theta(t)} = \text{diag}\{\pi_1^{\theta(t)} I_n, \ldots, \pi_p^{\theta(t)} I_n\} \), next we can obtain

\[
\Pi^{\theta(t)}\Xi^{\theta(t)}L^{\theta(t)} + (\hat{L}^{\theta(t)}\Pi^{\theta(t)}\Xi^{\theta(t)} > 0 \tag{17}
\]

Let \( \Phi_1^{\theta(t)} = \pi_1^{\theta(t)} \left( \Xi_1^{\theta(t)} I_{n+1} + (\hat{L}^{\theta(t)}\Pi^{\theta(t)}\Xi^{\theta(t)} \right)^T \) and

\[
\Pi^{\theta(t)}\Xi^{\theta(t)}\hat{L}^{\theta(t)} + (\hat{L}^{\theta(t)}\Pi^{\theta(t)}\Xi^{\theta(t)} = \Phi_p^{\theta(t)} \tag{18}
\]

It thus follows (16) that \( \Phi_1^{\theta(t)} > 0 \) for any positive number \( \pi_1^{\theta(t)} \). Choosing a suitable \( \pi_{i+1}^{\theta(t)} \) guarantees \( \Phi_i^{\theta(t)} > 0 \), in addition by Schur complement lemma and (16), we can get

\[
\Phi_i^{\theta(t)} = \pi_i^{\theta(t)} \left( \Xi_i^{\theta(t)} I_{n+1} + (\hat{L}_i^{\theta(t)}\Pi^{\theta(t)}\Xi^{\theta(t)} \right)^T \Pi_i^{\theta(t)}\Xi^{\theta(t)} = \Phi_p^{\theta(t)} \tag{19}
\]

where \( \Pi_i^{\theta(t)}\Xi^{\theta(t)}\hat{L}_i^{\theta(t)} + (\hat{L}_i^{\theta(t)}\Pi_i^{\theta(t)}\Xi^{\theta(t)} > 0 \).

It can be seen that if \( \Xi_{j+1}^{\theta(t)} \) is sufficiently less than \( \Xi_i^{\theta(t)} \) for \( j < i \), equation (18) holds. And for \( i = 1, \ldots, p \), select a group of \( \pi_i^{\theta(t)} \), the following equation holds:

\[
\Lambda^{\theta(t)} = (\Xi^{\theta(t)})^{-1} \Phi^{\theta(t)} (\Xi^{\theta(t)})^{-1} \tag{20}
\]

where \( \Lambda^{\theta(t)} > 0 \) and \( \Xi^{\theta(t)} = \sqrt{\Pi^{\theta(t)}\Xi^{\theta(t)}} \).

Assuming that \((A, B)\) is stable, there exists \( 0 < \beta < \lambda_{\min}(\Lambda^{\theta(t)}) \), matrix \( P_1 > 0, P_2 > 0, P_3 > 0 \) and \( P_4 > 0 \) such that the following inequality is constructed:

\[
AP_1 + P_1 A^T + \beta^2 I_n + \beta P_1
\]

\[
- \beta \left( 1 - \frac{1}{2} \right) B B^T + P_1 P_1^T < 0
\]

\[
AP_2 + P_2 A^T + \beta^2 I_n + Q + \bar{Q}^T
\]

\[
+ \beta P_2 + P_2 P_2^T + 2\beta^{-1} M M^T < 0
\]

This completes the proof.

Stability analysis and proofs are performed for the designed distributed coordinated tracking controllers (4) based state observers (3), and the main results are given by the following theorem.

**Theorem 1**: Suppose Assumptions 1-3 hold, the nonlinear MAsSs (1) under switching topology with intermittent communication can achieve cooperative consensus tracking by the proposed Lemma 5, if the communication condition (20) holds:

\[
\delta_{\max} > \frac{\bar{\hat{w}}}{\gamma_{\min} + \bar{l}}, \delta_{\min} > \frac{\bar{\hat{w}}}{\gamma_{\max} + \bar{l}} \tag{21}
\]

where \( \gamma_{\min} = \beta_{\min} \min(\lambda_{\min}(P_1^{-1}P_1^{-1}), \eta_{\min}(\max)) \), \( \gamma_{\max} = \beta_{\max} \min(\lambda_{\max}(P_1^{-1}P_1^{-1}), \eta_{\min}(\max)) \), \( \bar{l} = \lambda_{\max}(Q^TQ) \lambda_{\max}(P_1^{-1}P_1^{-1}) \), \( \lambda_{\min}(\max) \) indicates the minimum (maximum) value among eigenvalues of the matrix.

**Proof**: Set Lyapunov function candidate as:

\[
V(t) = e^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_1^{-1}) \right) e(t)
\]

\[
+ \Psi^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_2^{-1}) \Psi(t) \right) \tag{22}
\]

where \( \Pi^{\theta(t)} \) and \( \Xi^{\theta(t)} \) are the adjustable parameters which can be defined by Lemma 5, \( P_1 \) and \( P_2 \) are the solutions of (12) and (13). Next, the stability is analyzed in two time intervals: normal communication \( t \in [kw, kw + \delta] \) and communication interruption \( t \in [kw + \delta, (k + 1)w] \).

For \( t \in [kw, kw + \delta] \)

\[
V(t) = 2e^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_1^{-1} A) \right) e(t)
\]

\[
+ 2\Psi^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_2^{-1}) \Psi(t) \right) \tag{23}
\]

Based on Lemma 4, we can obtain

\[
2\hat{e}^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_1^{-1}) \right) \hat{e}(t)
\]

\[
\leq e^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_1^{-1} P_1^{-1} + I)) \right) e(t) \tag{24}
\]

\[
2\Psi^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_2^{-1}) \right) \hat{e}(t)
\]

\[
\leq \Psi^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_2^{-1} P_2^{-1} + I)) \right) \Psi(t) \tag{25}
\]

Substituting (23) and (24) into (22) proceeds

\[
\hat{V}(t) \leq 2e^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes P_1^{-1} A) \right) e(t)
\]

\[
+ e^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_1^{-1} P_1^{-1} + I)) \right) e(t)
\]

\[
+ 2\Psi^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (P_1^{-1} A + P_2^{-1} G C)) \right) \Psi(t)
\]

\[
- 2\hat{e}^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_1^{-1} P_1^{-1} + I)) \right) \hat{e}(t)
\]

\[
+ D^{\theta(t)} \Gamma^{\theta(t)} \otimes P_1^{-1} B K(e(t) + \Psi(t))
\]

\[
+ \Psi^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_2^{-1} P_2^{-1} + I)) \right) \Psi(t) \tag{26}
\]

According to Lemma 3, we have

\[
2\hat{e}^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_1^{-1} P_1^{-1} + I)) \right) e(t)
\]

\[
+ 2\hat{e}^T(t) \left( \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (\beta^2 P_1^{-1} P_1^{-1} + I)) \right) e(t)
\]

\[
\leq e^T(t) \left[ \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (I P_1^{-1} B) P_1^{-1} B F) \right] e(t)
\]

\[
+ \Psi^T(t) \left[ \Pi^{\theta(t)}(\Xi^{\theta(t)} \otimes (I_{\max} F T F) \right] \Psi(t) \tag{27}
\]
Then, substituting $K = -B^T P_1^{-1}$ into (25) yields

\[
\dot{V}(t) \leq \varepsilon^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes (P_1^{-1} A + A^T P_1^{-1}) \right) e(t)
+ e^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes (\rho^2 P_1^{-1} + I) \right) e(t)
+ \Psi^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes (P_1^{-1} A + A^T P_1^{-1}) \right) \Psi(t)
+ \Psi^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes (2P_2^{-1} GC) \right) \Psi(t)
+ 2e^T(t) \left( \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \mathcal{L}(\theta(t)) \otimes \hat{P} \right) e(t)
+ 2e^T(t) \left( \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \mathcal{L}(\theta(t)) \otimes \hat{P} \right) e(t)
+ \Psi^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes (\rho^2 P_1^{-1} + I) \right) \Psi(t)
+ e^T(t) \left( \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \mathcal{L}(\theta(t)) \otimes I \right) \hat{P} \right) \Psi(t)
- e^T(t) \left( \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \mathcal{L}(\theta(t)) \otimes I \right) \hat{P} \right) \Psi(t)
- \Psi^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \mathcal{L}(\theta(t)) \otimes (I^{-1} \hat{P}) \right) \Psi(t)
\]

(27)

where

\[
\hat{P} = P_1^{-1} B B^T P_1^{-1}
\]

and define

\[
\bar{\varepsilon}(t) = \left( \varepsilon^T_1(t), \ldots, \varepsilon^T_N(t) \right)^T, \quad \bar{\Psi}(t) = \left( \Psi^T_1(t), \ldots, \Psi^T_N(t) \right)^T, \quad \bar{\varepsilon}_i(t) = \sqrt{\Pi^{\theta(t)} \Xi^{\theta(t)} P_1^{-1} e_i(t)},
\]

\[
\bar{\Psi}_i(t) = \sqrt{\Pi^{\theta(t)} \Xi^{\theta(t)} P_1^{-1} \Psi_i(t)}.
\]

Then, we have

\[
\dot{V}(t) \leq \varepsilon^T(t) \left( I_N \otimes (A P_1 + P_1 A^T + \rho^2 + P_1 T P_1) \right) \bar{\varepsilon}(t)
- \bar{\varepsilon}^T(t) \left( \Lambda^{\theta(t)} \otimes \left( B B^T + \frac{1}{2} B B^T \right) \right) \bar{\varepsilon}(t)
+ \Psi^T(t) \left( I_N \otimes (A P_2 + P_2 A^T) \right) \bar{\Psi}(t)
+ \bar{\Psi}^T(t) \left( I_N \otimes (2G C P_2 + \rho^2 + P_2 T P_2) \right) \bar{\Psi}(t)
- \frac{1}{2} \bar{\Psi}^T(t) \left[ \Lambda^{\theta(t)} \otimes (I^{-1} F P_2 (F P_2)^T) \right] \bar{\Psi}(t)
\]

(28)

According to Lemmas 2, 3 and 5, we can obtain $0 < \beta \leq \lambda_{\min}(\Lambda^{\theta(t)})$, and the time derivative of $V(t)$ for the communication network topology $G^{\theta(t)}$ under any switching signal $\theta(t)$ is taken as

\[
\dot{V}(t) \leq \varepsilon^T(t) \left( I_N \otimes (A P_1 + P_1 A^T + \rho^2 + P_1 T P_1) \right) \bar{\varepsilon}(t)
- \bar{\varepsilon}^T(t) \left( I_N \otimes \left( \beta (B B^T + \frac{1}{2} B B^T) \right) \right) \bar{\varepsilon}(t)
+ \Psi^T(t) \left( I_N \otimes (A P_2 + P_2 A^T + 2G C P_2) \right) \bar{\Psi}(t)
+ \bar{\Psi}^T(t) \left( I_N \otimes (\rho^2 + P_2 T P_2) \right) \bar{\Psi}(t)
- \bar{\Psi}^T(t) \left( I_N \otimes \left( \frac{1}{2} \beta I^{-1} F P_2 (F P_2)^T \right) \right) \bar{\Psi}(t)
\]

(29)

Combining (12) with (13) in Lemma 5, (29) can be written as

\[
\dot{V}(t) \leq -\beta e^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes P_1^{-1} \right) e(t)
- \beta \Psi^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes P_1^{-1} \right) \Psi(t)
\]

(30)

To obtain the maximum value of $\delta_{\text{max}}$, define

\[
\eta_{\text{max}} = \lambda_{\max}(\Pi^{\theta(t)} \Xi^{\theta(t)}), \quad \eta_{\text{min}} = \lambda_{\min}(\Pi^{\theta(t)} \Xi^{\theta(t)}).
\]

According to (19) and Lemma 3, substituting $V(t)$ into (19) gives

\[
\dot{V}(t) \leq \lambda_{\max}(P_1^{-1}, P_2^{-1}) \eta_{\text{max}} e^T(t) e(t) + + \lambda_{\max}(P_1^{-1}, P_2^{-1}) \eta_{\text{max}} \Psi^T(t) \Psi(t)
\]

(31)

where

\[
\gamma_{\text{min}} = \frac{\eta_{\text{min}} \lambda_{\min}(P_1^{-1}, P_2^{-1})}{\eta_{\text{max}} \lambda_{\max}(P_1^{-1}, P_2^{-1})}, \quad P_1 \quad \text{and} \quad P_2 \quad \text{are the positive-definition matrix, so we can get} \quad \gamma > 0.
\]

Similarly, based on Lemmas 3 and 4, and some transformations of the necessary form, equation (32) is rewritten as

\[
\dot{V}(t) = 2e^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes P_1^{-1} A \right) e^T(t)
+ 2\Psi^T(t) \left( \Pi^{\theta(t)} \Xi^{\theta(t)} \otimes P_1^{-1} \right) \tilde{f}(x(t))
\]

(32)

Similarly, based on Lemmas 5 and 6, we have

\[
\dot{V}(t) \leq \lambda_{\max}(Q^4, Q^4)^T \tilde{e}^T(t) \tilde{e}(t)
+ \lambda_{\max}(Q^4, Q^4)^T \Psi^T(t) \Psi(t)
\]

(33)

Utilizing Lemma 5 and 6, we can obtain

\[
\dot{V}(t) \leq \lambda_{\max}(Q^4, Q^4)^T \tilde{e}^T(t) \tilde{e}(t)
+ \lambda_{\max}(Q^4, Q^4)^T \Psi^T(t) \Psi(t)
\]

(34)

where $\tilde{I} = \lambda_{\max}(Q^4, Q^4)$, $\lambda_{\max}(P_1^{-1}, P_2^{-1})$, $P_1$ and $P_2$ are the positive-definition matrix, so we can get $\tilde{I} > 0$. According to the minimum value $\gamma_{\text{min}}$ and the maximum value $\gamma_{\text{max}}$, we can obtain
where \( \tilde{\Upsilon}_1 = -\gamma \delta_0 + \tilde{u}(w-\delta_0) \). The range of \( \tilde{\Upsilon}_1 \) can be obtained by (20), that is \( \tilde{\Upsilon}_1 > 0 \). For any positive integer \( k \),

\[
V ((k+1)w) \leq V (0) e - \frac{\tilde{\Upsilon}}{\bar{\omega}_{k+1}} \sum_{j=1}^{k} \tilde{\Upsilon}_j
\]  

where \( \tilde{\Upsilon}_j = \gamma \delta_j - \bar{\omega} (w-\delta_j) > 0, j = 1, \ldots, k \). Set \( sw \leq t \leq (s+1)w \) for any \( s \in \mathbb{R} \). Define \( [kw, (k+1)w] \), \( k \in \mathbb{N} \), is a uniformly bounded time sequence, \( \bar{\omega}_{\text{max}} = \max_{1 \leq k \leq k} \{(k+1)w, kw\} \) and \( \tilde{\Upsilon} = \min_{j \in \mathbb{N}} \{ \tilde{\Upsilon}_j \} \). Finally, according to the recursion principle, we can obtain

\[
V (t) \leq V (kw) e - \frac{\tilde{\Upsilon}}{\bar{\omega} \text{max}} t
\]

\[
\leq e^{-\tilde{\Upsilon} t} V (0) e - \Sigma_{j=1}^{k} \tilde{\Upsilon}_j
\]

\[
\leq e^{-\tilde{\Upsilon} t} V (0) e - \frac{\tilde{\Upsilon}}{\bar{\omega}_{\text{max}}} t
\]

\[
= \Omega_0 e^{-\Omega_1 t}
\]

where \( \Omega_0 = e^{-\tilde{\Upsilon} t} V (0) \) and \( \Omega_1 = \tilde{\Upsilon} / \bar{\omega}_{\text{max}} \). Through the above process, it is demonstrated that the nonlinear MASs (1) under a dynamically switched communication network topology subject to the combined constraints of intermittent communication and information unpredictability is able to achieve the consensus tracking goal (2) under the combined action of a state observer (3) and a distributed coordinated tracking controller (4).

This completes the proof.

Lemma 6: Similar to Lemma 5, there exists a bounded positive scalar \( \beta \), and the switching consensus protocol (38) under heterogeneous coupling framework with intermittent communication and dynamic switching topology for nonlinear MASs (1) can be constructed by following matrices:

\[
M^1 = \begin{pmatrix}
AP_1' + PP_1'AT + \rho^2 I_N & P_1' \\
+ \beta P_1' - \beta \left(1 - \frac{1}{2} l \right) BB^T & -I
\end{pmatrix} < 0
\]  

(39)

\[
M^2 = \begin{pmatrix}
AP_2' + PP_2'AT + \rho^2 I_N & P_2' \\
+ \bar{Q}' + \bar{Q}'B & \beta P_2' - I \\
- I & 0
\end{pmatrix} < 0
\]  

(40)

\[
M^3 = \begin{pmatrix}
AP_1' + PP_1'AT + \rho^2 I_N + P_1'TP_1' - \beta \left(1 - \frac{1}{2} l \right) BB^T \\
+ \bar{Q}' + \bar{Q}'B & \beta P_2' - I \\
- I & 0
\end{pmatrix} < 0
\]  

(41)

\[
M^4 = \begin{pmatrix}
AP_2' + PP_2'AT + \rho^2 I_N + \bar{Q}' + \bar{Q}'B + PP_2'TP_2' \\
+ \bar{Q}' + \bar{Q}'B & \beta P_2' - I \\
- I & 0
\end{pmatrix} < 0
\]  

(42)

\[
M^5 = \begin{pmatrix}
AP_1' + PP_1'AT + \rho^2 I_N + P_1'TP_1' \\
+ \bar{Q}' + \bar{Q}'B & \beta P_2' - I \\
- I & 0
\end{pmatrix} < 0
\]  

(43)

\[
M^6 = \begin{pmatrix}
AP_2' + PP_2'AT + \rho^2 I_N + \bar{Q}' + \bar{Q}'B + PP_2'TP_2' \\
+ \bar{Q}' + \bar{Q}'B & \beta P_2' - I \\
- I & 0
\end{pmatrix} < 0
\]  

(44)

Corollary 1: Suppose Assumptions 1-3 hold, the nonlinear HMASs (1) under switching communication with intermittent communications can achieve cooperative consensus tracking by the proposed Lemma 6, if the following condition (45) holds:

\[
\hat{\gamma}_{\text{max}}' \delta_j + \bar{\omega} (h_j + \delta_j) - \hat{\chi} (h_j - \delta_j) > \bar{\omega}'
\]

(45)

where \( \hat{\gamma}_{\text{max}}' = \frac{\beta \bar{\eta}_{\text{min}} \bar{\eta}_{\text{max}} (P_{2}^{-1}, P_{2}^{-1})}{\bar{\eta}_{\text{max}} (P_{2}^{-1}, P_{2}^{-1})} \), \( \hat{\chi} = \lambda_{\text{min}} \left(M_{3}, M_{4}\right) \lambda_{\text{max}} \left(P_{1}^{-1}, P_{1}^{-1}\right) \), \( \bar{\omega}_{\text{max}} = \lambda_{\text{min}} \left(M_{3}, M_{4}\right) \lambda_{\text{max}} \left(P_{1}^{-1}, P_{1}^{-1}\right) \), \( \bar{\omega}_{\text{max}} \) indicates the minimum (maximum) value among eigenvalues of the matrix.

Proof: Set Lyapunov function candidate as:

\[
\tilde{V}' (t) = e^T (t) \left( \Pi^0(t) \Xi^0(t) \otimes P_1^{-1} \right) e (t)
\]

+ \Psi^T (t) \left( \Pi^0(t) \Xi^0(t) \otimes P_2^{-1} \right) \Psi (t)
\]

(46)

where \( \Pi^0(t) \) and \( \Xi^0(t) \) are the adjustable parameters which can be defined by Lemma 6, \( P_1' \) and \( P_2' \) are the solutions of (39) and (40).

When \( t \in [kw, kw + \delta] \), by the similar process in Theorem...
1, we can obtain

\[
\dot{V}' (t) = 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{1}^{t-1} A \right) e^T (t) \\
+ 2 \Psi^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{2}^{t-1} \right) \dot{f} (x (t)) \\
- 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes \left( \Pi^{(t)} T^{(t)} \right) \right) e^T (t) \\
+ D^{(t)} (T^{(t)}) \otimes P_{1}^{t-1} BK) (e (t) + \Psi (t)) \\
+ 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{1}^{t-1} \right) \dot{f} (x (t)) \\
+ 2 \Psi^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes \left( P_{2}^{t-1} A + P_{2}^{t-1} GC \right) \right) \Psi (t) \\
\leq \varepsilon^T (t) \left( I_N \otimes \left( A P_{2}^t + P_{1}^t A^T + {\rho}^2 + P_{1}^t P_{1}^t \right) \right) \varepsilon (t) \\
- \varepsilon^T (t) \left( I_N \otimes \left( \beta \left(BB^T + {\frac{1}{2}} {BB}^T \right) \right) \right) \varepsilon (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( A P_{2}^t + P_{1}^t A^T + 2GC P_{2}^t \right) \right) \Psi (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( \rho^2 + P_{1}^t P_{2}^t \right) \right) \Psi (t) \\
- \Psi^T (t) \left( I_N \otimes \left( \frac{1}{2} \beta \left(1 - FP_{2}^t (FP_{2}^t) \right) \right) \right) \Psi (t) \\
\leq - \beta \lambda_{\min} \left( P_{1}^{t-1}, P_{2}^{t-1} \right) e (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes I_N \right) e (t) \\
- \beta \lambda_{\min} \left( P_{2}^{t-1}, P_{2}^{t-1} \right) \Psi (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes I_N \right) \Psi (t) \\
\leq - \hat{\gamma}' V' (t) \\
(47)
\]

When \([kw + \delta, kw + h)\), its time derivative as

\[
\dot{V}' (t) = 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{1}^{t-1} A \right) e^T (t) \\
- 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{1}^{t-1} BK \right) e (t) \\
- 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{2}^{t-1} BK \right) e (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( A P_{2}^t + P_{1}^t A^T + {\rho}^2 + P_{1}^t P_{1}^t \right) \right) \Psi (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( \frac{1}{2} \beta \left(1 - FP_{2}^t (FP_{2}^t) \right) \right) \right) \Psi (t) \\
\leq \varepsilon^T (t) \left( I_N \otimes \left( A P_{2}^t + P_{1}^t A^T + {\rho}^2 + P_{1}^t P_{1}^t \right) \right) \varepsilon (t) \\
- \varepsilon^T (t) \left( I_N \otimes \left( \beta \left(BB^T + {\frac{1}{2}} {BB}^T \right) \right) \right) \varepsilon (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( A P_{2}^t + P_{1}^t A^T + 2GC P_{2}^t \right) \right) \Psi (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( \rho^2 + P_{1}^t P_{2}^t \right) \right) \Psi (t) \\
- \Psi^T (t) \left( I_N \otimes \left( \frac{1}{2} \beta \left(1 - FP_{2}^t (FP_{2}^t) \right) \right) \right) \Psi (t) \\
\leq \lambda_{\max} \left( M^5, M^6 \right) \left( \varepsilon^T (t) \varepsilon (t) + \bar{\Psi}^T (t) \bar{\Psi} (t) \right) \\
\leq \hat{\bar{\gamma}} V' (t) \\
(48)
\]

For \(t \in [kw + h, (k + 1) w)\), we have

\[
\dot{V}' (t) = 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{1}^{t-1} A \right) e^T (t) \\
+ 2 \Psi^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{2}^{t-1} \right) \Psi (t) \\
+ 2e^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes P_{1}^{t-1} \right) \Psi (t) \\
+ 2 \Psi^T (t) \left( \Pi^{(t)} \Xi^{(t)} \otimes \left( P_{2}^{t-1} A + P_{2}^{t-1} GC \right) \right) \Psi (t) \\
\leq \varepsilon^T (t) \left( I_N \otimes \left( (A P_{2}^t + P_{1}^t A^T + {\rho}^2) \right) \right) \varepsilon (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( (A P_{2}^t + P_{1}^t A^T + {\rho}^2) \right) \right) \Psi (t) \\
+ \bar{\Psi}^T (t) \left( I_N \otimes \left( 2GC P_{2}^t + P_{2}^t P_{2}^t \right) \right) \bar{\Psi} (t) \\
+ \Psi^T (t) \left( I_N \otimes \left( P_{1}^t P_{1}^t \right) \right) \Psi (t) \\
\leq \lambda_{\max} \left( M^5, M^6 \right) \left( \varepsilon^T (t) \varepsilon (t) + \bar{\Psi}^T (t) \bar{\Psi} (t) \right) \\
\leq \hat{\bar{\gamma}} V' (t) \\
(49)
\]

where \(\hat{\bar{\gamma}} = \lambda_{\max} \left( M^5, M^6 \right) \lambda_{\max} \left( P_{1}^{t-1}, P_{2}^{t-1} \right)\). Note that the cooperative constrained weighting \(\hat{\bar{\gamma}}\) is vary in \([-\beta, \hat{\bar{\gamma}}]\) and that depends on the solution of (39) and (40). Thus, we can get

\[
V' (w) \leq V' (h) e^{\hat{\bar{\gamma}} \left( w - h \right)} \\
= V' (0) e^{-\sum_{j=1}^{k} \hat{\bar{\gamma}}_j} \\
(50)
\]

where \(\hat{\bar{\gamma}}_j = \gamma_j - \hat{\bar{\gamma}}_j (w - h - \hat{\bar{\gamma}}_j - \hat{\bar{\gamma}} (h - \hat{\bar{\gamma}}_j), j = 1, \ldots, k. Set 2sw \leq t \leq (2s + 1) w\) for any \(s \in R\). Define \([kw, (k + 1) w)\), \(k \in N_+\) is a uniformly bounded time sequence, \(\omega_{\max} = \max_{k \in e, \{ (k + 1) w, kw \} \{ \hat{\bar{\gamma}}_j \}}, \bar{T} = \min_{j \in N_+} \{ \hat{\bar{\gamma}}_j \}\). Finally, according to the recursion principle, we can obtain

\[
V' (t) \leq V' (2sw) e^{\omega_{\max} \left( \gamma_j + \hat{\bar{\gamma}} + \hat{\bar{\gamma}} \right)} \\
\leq e^{\omega_{\max} \left( \gamma_j + \hat{\bar{\gamma}} + \hat{\bar{\gamma}} \right)} V' (0) e^{-\sum_{j=1}^{k} \hat{\bar{\gamma}}_j} \\
\leq \Omega_0 e^{-\Omega_1 t} \\
(53)
\]

where \(\Omega_0 = e^{\omega_{\max} \left( \gamma_j + \hat{\bar{\gamma}} + \hat{\bar{\gamma}} \right)} V' (0)\) and \(\Omega_1 = \bar{T} / \omega_{\max}\).

This completes the proof.

IV. NUMERICAL EXAMPLES

In this section, two simulation examples are provided to verify the effectiveness of the obtained results. Both nonlinear MASs consist of four followers and a leader, with the directed dynamic interaction topology as shown in Fig. 1, where 0 is the leader and 1-4 are the followers.
The dynamic of the $i$th agent for nonlinear MASs (1) can be written with
\[ x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \\ x_{i4}(t) \end{bmatrix}, \]
\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, \]
\[ B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \]

The nonlinear dynamic functions are:
\[ f(x_i(t)) = \begin{bmatrix} 0 & 0 & 0 & -3.33 \sin(x_{i3}) \end{bmatrix}^T, \]
\[ f(x_0(t)) = \begin{bmatrix} 0 & 0 & 0 & -3.33 \sin(x_{03}) \end{bmatrix}^T, \forall i = 1, 2, \ldots, n. \]

Consider the nonlinear MASs (1) compose of four followers and a leader, whose dynamic switching communication network is shown in Fig. 1, where node 0 represents the leader and the remaining nodes 1-4 are the followers. For the convenience of analysis, suppose that the underlying derivative directed network topologies can be constructed as shown in Fig. 1(a), (b) and (c).

**Example 1**: First, we consider the intermittent constrained consensus for cooperative tracking control. The intermittent communication mode is shown in Fig. 2, in which, if $y=1$, it indicates that the communication between adjacent agents in the system is working, and $y=0$ shows the neighboring agents can not communicate with each other.

According to Theorem 1, there exists an infinite time sequence of uniformly bounded and nonoverlapping time intervals $t \in [kw, (k+1)w)$, where $w = 5$. To further analyze the stability, the switching sequence within the time interval $t \in [kw, kw + \delta)$ is defined as $G_1 \rightarrow G_2 \rightarrow G_3 \rightarrow$...
The trajectories of the actual states and estimates states of four followers are provided in Figs. 3-6, respectively for first state, second state, third state and fourth state. It can be proved that the constructed observers (3) can estimate the actual states asymptotically. The state trajectories of the closed-loop MASs (1) under the heterogeneous coupling framework protocol (4) are shown in Figs. 7-10 show that the coordinated tracking control with intermittent communication and dynamic switching topology is indeed achieved.

**Example 2**: Second, we consider the observer-based consensus control under second intermittent coordinated constrained with three types of intervals. The intermittent communication mode is shown in Fig. 11, in which, if \( y=2 \), it indicates that the communication between adjacent agents and the leader in the system is working, if \( y=1 \), it indicates that each follower evolves only based on the information of itself and
Fig. 10: The fourth state errors for four followers and the leader under the designed controller (4).

its neighbors, and y=0 shows the neighboring agents can not communicate with each other.

Three types of time intervals \( t = [5k, 5k+4) = T_m \), \( t = [5k, 5k+4.5) = T_q \) and \( t = [5k+4.5, 5k+5) = T_n \) respectively are considered for simulation.

Fig. 11: Intermittent Coordinated Constrained

Fig. 12: Tracking errors with \( x_{i1} \) and \( x_{01} \) under the designed controller (38)

Figs. 12-15 demonstrate respectively the four states \( x_{i1}, x_{i2}, x_{i3} \) and \( x_{i4} \) of four agents could achieve the goal

Fig. 13: Tracking errors with \( x_{i2} \) and \( x_{02} \) under the designed controller (38)

Fig. 14: Tracking errors with \( x_{i3} \) and \( x_{03} \) under the designed controller (38)

Fig. 15: Tracking errors with \( x_{i4} \) and \( x_{04} \) under the designed controller (38)
of consensus control based on improved distributed coordinated tracking control protocol (38) with second intermittent coordinated constrained. We can summarize that the proposed distributed control protocol based on heterogeneous coupling framework and observers are effective and asymptotically stable for nonlinear MASs with intermittent communication and dynamic switching topology.

V. CONCLUSIONS

In our work, the coordinated tracking control problem for nonlinear MASs under heterogeneous coupling network with intermittent communication has been studied. Especially, considering the unavailable states, nonlinear observers have been constructed for each agents. Then, a lemma has been correspondingly constructed to calculate the heterogeneous coupling gain, feedback gain and observer gain matrices. The system stability has been analyzed utilizing Lyapunov stability theory, switching system theory and LMI technology, and the allowed maximum communication threshold have been obtained. The results have been extended to the case of the intermittent communication with three types of intervals. Finally, two simulation examples have been provided to prove the effectiveness and correctness of the proposed methods.

REFERENCES

[1] Z. Li, G. Wen, Z. Duan and W. Ren, “Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs,” IEEE Transactions on Automatic Control, vol. 60, no. 4, pp. 1152–1157, 2015.
[2] J. Yu, C. Wang and G. Xie, “Coordination of multiple robotic fish with applications to underwater robot competition,” IEEE Transactions on Industrial Electronics, vol. 62, no. 2, pp. 1280–1288, 2016.
[3] H. Li, Y. Shi, W. Yan and F. Liu, “Receding horizon consensus of general linear multi-agent systems with input constraints: An inverse optimality approach,” Automatica, vol. 91, pp. 10–16, 2018.
[4] H. Su, H. Wu, X. Chen and M. Chen, “Positive edge consensus of complex networks,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 48, no. 12, pp. 2242–2250, 2018.
[5] Y. Zhao, W. Zhang, H. Su and J. Yang, “Observer-based synchronization of chaotic systems satisfying incremental quadratic constraints and its application in secure communication,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 50, no. 12, pp. 5221-5232, 2020.
[6] J. Wang, L. Han, X. Dong and Q. Li, “Distributed sliding mode control for time-varying formation tracking of multi-UAV system with a dynamic leader,” Aerospace Science and Technology, vol. 111, no. 1, pp. 106549, 2021.
[7] B. Wang, W. Chen, B. Zhang and Y. Zhao, “Regulation cooperative control for heterogeneous uncertain chaotic systems with time delay: A synchronization errors estimation framework,” Automatica, vol. 108, no. 1, pp. 108486, 2019.
[8] Y. Zhang, G. Lu, X. He and Y. Zhang, “Distributed active fault-tolerant cooperative control for multiagent systems with communication delays and external disturbances,” IEEE Transactions on Cybernetics, no. 99, pp. 1–11, 2021.
[9] Y. Jiang, H. Wang and S. Wang, “Distributed $H_{\infty}$ consensus control for nonlinear multi-agent systems under switching topologies via relative output feedback,” Neural Computing and Applications, vol. 31, no. 1, pp. 1–9, 2019.
[10] H. Wang, W. Ren, W. Yu and D. Zhang, “Fully distributed consensus control for a class of disturbed second-order multi-agent systems with directed networks,” Automatica, vol. 132, no. 1, pp. 109816, 2021.
[11] Y. Yang, H. Modares, D. Wunsch and Y. Yin, “Leader-follower output synchronization of linear heterogeneous systems with active leader using reinforcement learning,” IEEE Transactions on Neural Networks and Learning Systems, vol. 29, no. 6, pp. 2139–2153, 2018.
[12] Z. Jia, L. Wang, J. Yu and X. Ai, “Distributed adaptive neural networks leader-following formation control for quadrotors with directed switching topologies,” ISA Transactions, vol. 93, pp. 93–107, 2019.
[13] Z. Li, W. Ren, X. Liu and M. Fu, “Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols,” IEEE Transactions on Automatic Control, vol. 58, no. 7, pp. 1786–17912, 2013.
[14] C. Ong and B. Hou, “Consensus of heterogeneous multi-agent system with input constraints,” Automatica, vol. 134, no. 12, pp. 109895, 2021.
[15] J. Zhang, H. Zhang, Y. Cai and W. Wang, “Consensus control for nonlinear multi-agent systems with event-triggered communications,” Applied Mathematics and Computation, vol. 408, no. 6, pp. 126341, 2021.
[16] Y. Hong, J. Hu and L. Gao, “Tracking control for multi-agent consensus with an active leader and variable topology,” Automatica, vol. 42, no. 7, pp. 1177–1182, 2006.
[17] W. Yu, L. Yang, G. Wen, X. Yu and J. Cao, “Observer design for tracking consensus in second-order multi-agent systems: Fractional order less than two,” IEEE Transactions on Automatic Control, vol. 62, no. 2, pp. 894–900, 2017.
[18] Y. Bai and J. Wang, “Observer-based distributed fault detection and isolation for second-order multi-agent systems using relative information,” Journal of the Franklin Institute, vol. 358, no. 7, pp. 3779–3802, 2021.
[19] H. Chu, J. Yuan and W. Zhang, “Observer-based adaptive consensus tracking for linear multi-agent systems with input saturation,” IET Control Theory and Applications, vol. 9, no. 14, pp. 2124–2131, 2015.
[20] Y. Han, Y. Kao and J. Park, “Robust $H_{\infty}$ nonfragile observer-based control of switched discrete singular systems with time-varying delays: A sliding mode control design,” International Journal of Robust and Nonlinear Control, vol. 29, no. 5, pp. 1462–1483, 2019.
[21] B. Methallari and J. Sahabahit, “Disturbance observer
based distributed consensus control strategy of multi-agent system with external disturbance in a standalone DC microgrid,” *Asian Journal of Control*, vol. 23, no. 2, pp. 920–936, 2020.

[22] Z. Li, Z. Duan, G. Chen and L. Huang, “Consensus of multiagent systems and synchronization of complex networks: a unified viewpoint,” *IEEE Transactions on Circuits and Systems I Regular Papers*, vol. 57, no. 1, pp. 213–224, 2010.

[23] Z. Li, X. Liu, P. Lin and W. Ren, “Consensus of Linear Multi-agent Systems with Reduced-order Observer-based Protocols,” *Systems and Control Letters*, vol. 60, no. 7, pp. 510–516, 2011.

[24] Z. Zuo, B. Tian and M. Defoort, “Fixed-time consensus tracking for multi-agent systems with high-order integrator dynamics,” *IEEE Transactions on Automatic Control*, vol. 63, no. 2, pp. 563–570, 2018.

[25] Y. Yan and Z. Chen, “Cooperative output regulation of linear discrete-time time-delay multi-agent systems by adaptive distributed observers,” *Neurocomputing*, vol. 331, no. 28, pp. 33–39, 2019.

[26] G. Wen, Z. Duan and W. Ren, “Distributed consensus of multi-agent systems with general linear node dynamics and intermittent communications,” *International Journal of Robust and Nonlinear Control*, vol. 24, no. 16, pp. 2438–2457, 2015.

[27] Q. Song, F. Liu, G. Wen, J. Cao and X. Yang, “Distributed position-based consensus of second-order multi-agent systems with continuous/intermittent communication,” *IEEE Transactions on Cybernetics*, vol. 47, no. 8, pp. 1860–1871, 2017.

[28] Y. Liu, D. Xie and L. Shi, “Consensus of general linear multi-agent systems with intermittent communications,” *International Journal of Systems Science*, vol. 51, no. 12, pp. 2293–2305, 2020.

[29] B. Wang, W. Chen, B. Zhang, Y. Zhao and P. Shi, “Cooperative control-based task assignments for multi-agent systems with intermittent communication,” *IEEE Transactions on Industrial Informatics*, vol. 17, no. 10, pp. 6697–6708, 2020.

[30] W. Ren and R. Beard, “Consensus seeking in multiagent systems under dynamically changing interaction topologies,” *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[31] B. Wang, L. Wang, L. Zhang and B. Zhang, “Robust adaptive consensus tracking for higher-order multi-agent uncertain systems with nonlinear dynamics via distributed intermittent communication protocol,” *International Journal of Adaptive Control and Signal Processing*, vol. 30, no. 3, pp. 511–533, 2016.

[32] N. Huang, Z. Duan and Y. Zhao, “Leader-following consensus of second-order non-linear multi-agent systems with directed intermittent communication,” *IET Control Theory and Applications*, vol. 8, no. 10, pp. 782–795, 2014.

[33] R. Horn and C. Johnson, “Matrix analysis,” *Cambridge University Press, Cambridge*, 2012.