Inverse problems of restoring the geometric dimensions of a construction defects by thermal fields analysis

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Abstract. The proposed work deals with the problem of finding the geometric dimensions of a cavity-type defect by measuring the thermal fields. As a structural element, a plate with a rectangular cavity is considered.

1. Introduction
The proposed work deals with the problem of finding the geometric dimensions of a cavity-type defect by measuring the thermal fields [1-22]. The method of solving this problem is based on the optimization approach. Optimization is carried out using methods of finding extremums without calculating the derivative, in this paper this is the Hook-Jeeves method. The results of solving this problem make it possible to improve the accuracy of determining the location of the defect and show the advisability of generalizing the problem under consideration to more complex structural elements. The results obtained in this paper can be used in problems of thermal non-destructive testing. Thermal control is based on monitoring, measuring and analyzing changes in the temperature background on the surface and inside the monitored objects. Thermal control is applicable to the object under one main condition, namely, the presence in the controlled object of convection of heat flows. The process of recycling thermal energy to the environment, the allocation or absorption of heat within the object, movement through the entire volume of the object, invariably leads to a slow change in temperature relative to the surrounding space. The nature of the distribution of the temperature background along the surface of the monitored object is an important technical parameter in the thermal control method, since it contains data on the process of heat transfer, the constant and temporary modes of operation of the object, its external and internal arrangement, and the presence of hidden internal defects. The occurrence of heat fluxes in a controlled object is due to external influences and various internal causes.

The aim of the work is to increase the effectiveness of the method of nondestructive testing of aerospace products. One of the methods of nondestructive testing is thermal diagnostics, which makes it possible to draw conclusions about the internal structure of the objects under study from the measured thermal fields. Non-destructive testing methods are the main means of testing the reliability of assemblies and aggregates of aerospace and construction equipment. Among the methods of non-destructive testing, the method of thermal diagnostics is one of the main methods for detecting a cavity-type defect, since it allows for diagnostics in a non-contact way.

2. Mathematical model
The model problem of thermal diagnostics in the following formulation is considered. An element of a product of aerospace engineering in the form of a plate with a cavity of rectangle type is considered.
The plate on one side is heated by a stationary flow, the two sides of the plate are thermally insulated, and the thermal imager measures the thermal flux of radiation from the fourth side. The boundary conditions [2-6] on each side are recorded.

The problem under consideration reduces to a parabolic equation with nonlinear boundary conditions and taking into account reradiation. This problem was solved by the finite difference method together with the Fredholm integral equation of the second kind [1].

\[
q(x) - k(x) \int G(x, \xi) q(\xi) d\xi = q_0(x)
\]

at \( \Gamma_1 \): \( K \frac{\partial u}{\partial n} = 0 \)  

at \( \Gamma_2 \): \( K \frac{\partial T}{\partial n} + \alpha(T - T_c) + \sigma(T^4 - T_c^4) = 0 \)  

at \( \Gamma_3 \): \( K \frac{\partial u}{\partial n} = 0 \)  

at \( \Gamma_4 \): \( K \frac{\partial u}{\partial n} + q_0(x) = 0 \)  

at \( \gamma_1, \gamma_2, \gamma_3, \gamma_4 \): \( K \frac{\partial u}{\partial n} + \alpha q(x) = 0 \)

The results of solving this problem make it possible to improve the accuracy of determining the location of the defect and show the advisability of generalizing the problem under consideration to more complex structural elements.

The obtained results showed the importance of accounting for reradiation in solving the problem of restoring geometric dimensions and location of defects in the plate.

3. Formulation of the inverse problem

The inverse problem is to restore the geometric dimensions of a plate with a cavity-type defect. This problem reduces to an optimization problem with four parameters (Figure 2).

\[
T(\Gamma) = A(x_{01}, x_{02}, a, b)
\]

Figure 1. Type of plate boundary conditions

Figure 2. Parameters for restoring the geometric dimensions of the cavity
4. The coordinated problem of heat exchange by radiation and thermal conductivity

Of greatest interest is the problem of taking into account reradiation for non-convex bodies, when it is necessary to take heat transfers into the body. Here, such a coordinated problem of taking into account heat transfer by radiation and thermal conductivity is considered under the conditions of the two-dimensional problem for the next model area (Figure 1):

For non-convex bodies or a system of bodies, it is necessary to take into account the heat fluxes that fall on the surface elements of the body from other parts of the surface. This leads to additional heat fluxes, an increase in surface temperature and, thus, an increase in the thermal radiation flux itself. In such conditions it is natural to talk about additional radiation, re-emission.

The task of calculating the processes of heat exchange must be considered in an agreed formulation. The intensity of thermal radiation depends on the surface temperature, which, in turn, depends on the heat fluxes on the surface, on the thermal radiation.

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Let us consider the formulation of a stationary problem. The environment is assumed to be isotropic, so the thermal state is described by the equation:

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = 0$$

We assume that the boundaries $\Gamma_1, \Gamma_3$ are thermally insulated. We write the boundary conditions:

$$K \frac{\partial u}{\partial n} = 0, \quad x \in \Gamma_1, \Gamma_3$$

(10)

The boundary $\Gamma_4$ is supplied with heat flow, the boundary condition:

$$K \frac{\partial u}{\partial n} + q^*(x) = 0, \quad x \in \Gamma_4$$

(11)

On the remaining part of the boundary $\Gamma_2$ there is a heat exchange with the environment, the boundary condition:

$$K \frac{\partial u}{\partial n} + \alpha (u - u_c) + \sigma (u^4 - u_c^4) = 0, \quad x \in \Gamma_2$$

(12)

We rewrite the boundary condition in the following form:

$$K \frac{\partial u}{\partial n} + \alpha u + \sigma u^4 = \alpha u_c + \sigma u_c^4, \quad x \in \Gamma_2$$

(13)

$$K \frac{\partial u}{\partial n} + \sigma_1 u + \sigma_2 u^4 = \tilde{q}, \quad x \in \Gamma_2$$

(14)

Where

$$\tilde{q} = \alpha u_c + \sigma u_c^4$$

(15)

Where $u_l$ - temperature of a solid at a point $\left(x_1, x_2\right) \in \Omega$,
ambient temperature, $u_c$, coefficient $\sigma_1 > 0$ characterizes convective heat exchange with the environment, $\sigma_2 > 0$ - blackness. The $\gamma$ carries heat transfer by radiation, the boundary condition is written as:

$$K \frac{\partial u}{\partial n} + q_{per}(x) = 0, \quad x \in \gamma_1, \gamma_2, \gamma_3, \gamma_4,$$

(16)

Where $K$ - coefficient of thermal conductivity, $q_{per}(x)$ - resultant radiation flux.

Let $q(x)$ be the radiation flux at the point $x$ of the surface $\gamma$. It is defined as the sum of its own radiation $q_0(x)$ and part of the incident radiation, which is reflected. For $q(x)$ and part of the incident radiation, which is reflected:

$$q(x) - k(x) \int G(x, \xi)q(\xi)d\xi = q_0(x)$$

(17)

- Fredholm integral equation of the second kind,

Where $0 < k(x) < 1$ - reflection coefficient.

The intrinsic radiation is due only to the body temperature, and in accordance with the Stefan-Boltzmann law, the right-hand side is determined by the expression:

$$q_0(x) = \sigma_2(x)g^4(x),$$

(18)

where $\sigma_2(x)$ - emissivity of the body.

$$q_{per}(x) = q(x) - k(x) \int G(x, \xi)q(\xi)d\xi$$

(19)

The kernel of the integral equation $G(x, \xi)$ is defined by the formula:

$$G(x, \xi) = \frac{1}{\pi r(x, \xi)} \cos\left(r(x, \xi), n(x)\right) \cos\left(r(\xi, x), n(\xi)\right)$$

(20)

Where $r(x, \xi)$ - distance between points $x$ and $\xi$, $\cos(n(\xi), r)$. The cosine of the angle between the normal $k$ and the segment joining $x$ and $\xi$.

We note some properties of the integral equation (4), which is an integral Fredholm equation of the second kind. When $k(x) = const$ the kernel of the integral equation, as follows from (14), is symmetric. In addition, the kernel is nonnegative, i.e.

$$G(x, \xi) = G(\xi, x) \geq 0$$

(21)

For positive $k(x) \neq const$ the integral equation (11) can easily be reduced to an integral equation with a symmetric kernel.

The asymmetry is caused by a factor $k(x)$. We divide equation (11) by $\left(k(x)\right)^{\frac{1}{2}}$ and introduce a new unknown: 

$$\tilde{u}(x) = u(x)\left(k(x)\right)^{-\frac{1}{2}},$$
\[ \hat{q}(x) = \frac{q(x)}{(k(x))^{\frac{1}{2}}} \]  
Equation (11) takes the form:
\[ \hat{q}(x) - k(x) \int_{\gamma} \hat{G}(x, \xi) \hat{q}(\xi) d\xi = \hat{q}_0(x), \ x \in \gamma \]  
(23)

Here the right-hand side:
\[ \hat{q}_0(x) = \frac{q_0(x)}{(k(x))^{\frac{1}{2}}} \]  
(24)
And the kernel looks like this:
\[ \hat{G}(x, \xi) = (k(x)k(\xi))^{\frac{1}{2}} G(x, \xi) \]  
(25)

Because of the symmetry of the kernel \( G(x, \xi) \) the kernel \( \hat{G}(x, \xi) \) of the integral equation is also symmetric.

5. **Block diagram of the solution of the problem**

We represent the algorithm of the Hook-Jeeves method in the form of a block diagram (Figure 3).

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**Figure 3.** Block diagram of the Hook-Jeeves method for the inverse problem

\( x_1, x_2 \) - coordinates of the lower left corner of the cavity; \( x_3 \) - length of the upper and lower walls of the cavity; \( x_4 \) - length of the left and right walls of the cavity

The choice of the initial condition is as follows: the defect is located in the middle of the plate and its geometric dimensions are twice less than the specified dimensions of the plate.
6. The results of solving the inverse problem

The Hook-Jeeves method was programmed in C ++ in Visual Studio. The geometric dimensions of the cavity were restored. Figure 3 shows the results. Blue contour - the present location of the cavity.

![Image](image_url)

Figure 4. Results of solving the inverse problem

7. Conclusion

The results of solving this problem show the importance of taking into account reradiation and allow increasing the accuracy of defect location and show the expediency of generalizing the problem under consideration to more complex structural elements. The results obtained in this paper can be used in problems of thermal non-destructive testing.

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