Big Rip: heating by Hawking radiation and a possible connection to conformal cyclic cosmology

Rafael Ruggiero
São Paulo, Brazil
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In the Big Rip cosmological scenario, a FRW universe containing dark energy with \( w < -1 \) in its equation of state (phantom energy) expands in such a way that, in a finite timespan, the scale factor diverges to infinity and the size of the cosmic horizon goes to zero. Here we revisit this scenario in light of the fact that Hawking radiation is expected to be generated at the apparent horizon of a FRW universe, and show that the energy density and temperature of that radiation both diverge at the Big Rip. We then use this fact to propose a new variant of Penrose’s conformal cyclic cosmology model in which the future spacetime metric becomes conformally invariant at the Big Rip instead of in the remote future of a de Sitter universe; this removes the need for mass decay from the model and makes it consistent with current physical laws.

I. INTRODUCTION

The ultimate fate of the Universe is a topic of great philosophical significance. In order to speculate about what will take place, it is crucial to understand what factors are involved in its expansion. In particular, the future evolution strongly depends on the exact nature of dark energy.

In a generalized formulation, dark energy is a fluid with equation of state \( P = w \rho \). Current observations suggest that \( w \) is close to \(-1\) (e.g. \([1]\)), and an often adopted base scenario is that its value is exactly \(-1\), making dark energy a negative pressure fluid that can be more simply interpreted as a cosmological constant in the Einstein equations, since the behavior is exactly the same. Dark energy with \( w < -1 \) in its equation of state receives the name of phantom energy \([2]\), and it results in an extreme future expansion that diverges in a finite timespan and causes the size of the cosmic horizon to go to zero. That would disrupt all structures, from galaxies to molecules and atoms, hence why this scenario is called Big Rip \([3]\).

The metric for an homogeneous and isotropic universe is the Friedmann-Robertson-Walker (FRW) metric. One important property of a FRW universe is that Hawking radiation is expected to be generated at its apparent horizon. This has been demonstrated in \([4]\) using the so called tunneling approach \([5]\), in which the process of particle creation at the horizon is conceptualized as the tunneling of particles from beyond it. This result generalizes the previous finding by Gibbons and Hawking that the apparent horizon of a de Sitter universe is expected to radiate \([6]\), obtained just a few years after Hawking first considered the problem of particle creation at the event horizon of black holes \([7]\).

The temperature of the Hawking radiation generated at the apparent horizon of a FRW universe is inversely proportional to the radius \( r_\Lambda \) of the horizon, and is given by \([4, 8]\):

\[
T = \frac{\hbar c}{k_B 2\pi r_\Lambda}. \quad (1)
\]

This result holds for any spatial curvature. In the case of a flat universe \((k = 0)\), the radius of the horizon is simply \([8]\)

\[
r_\Lambda = \frac{c}{H}. \quad (2)
\]

where \( H \equiv \dot{a}/a \) is the Hubble parameter.

As intriguing as the fate of the universe is its origin. The question remains as to whether the Big Bang was an ultimate beginning or if it was preceded by something. Cyclic cosmological models hold to the latter view; a prominent such model has been proposed by Roger Penrose and is called Conformal Cyclic Cosmology (CCC, \([3,10]\)). The basic idea of this model is that, as long all mass in the universe asymptotically decays in an unspecified way in the remote future, the spacetime metric will eventually become conformally invariant, just like it was at the Big Bang when radiation was the only relevant energy component, making it possible to reinterpret the final state of the universe as something equivalent to its initial state. In this framework, each iteration of the cyclic universe is referred to as an aeon, and infinitely many of these aeons are assumed to take place in sequence, without a beginning or an end. Tests of this model based on attempting to find gravitational signatures of a previous aeon on the CMB have been carried out with positive results \([11,12]\), but their statistical relevance has been disputed \([13]\).

In this paper, we revisit the Big Rip scenario considering the Hawking radiation that is generated at the cosmic horizon of a FRW universe containing phantom energy. In Sec. \([II]\) we show that the temperature and energy density of that radiation both diverge at the Big Rip. In Sec. \([III]\) we propose a variant of the CCC model in which the final, conformally invariant limit of the universe is taken to be the Big Rip instead of the remote future of a de Sitter universe. We then discuss and contextualize our results in Sec. \([IV]\) and present our conclusions in Sec. \([V]\)
II. HEATING BY HAWKING RADIATION

To calculate the heating of a phantom universe by Hawking radiation, we start from the Friedmann equation. For an universe similar to ours at the present moment, assumed to be flat and with matter and dark energy being the only relevant energy components, it can be written as [3]:

\[
\frac{1}{H_0^2} \frac{\dot{a}^2}{a^2} = \Omega_{m,0} a^{-3} + (1 - \Omega_{m,0}) a^{-3(1+w)},
\]

where \( a \) is the scale factor, \( H_0 \) is the current Hubble parameter and \( \Omega_{m,0} \) is the current matter density parameter.

If \( w < -1 \), then the term \( \Omega_{m,0} a^{-3} \) will eventually become much smaller than the second one, so that the equation will become simply

\[
\frac{1}{H_0^2} \frac{\dot{a}^2}{a^2} = (1 - \Omega_{m,0}) a^{-3(1+w)}.
\]

The solution for this equation is of the form \( a(t) \propto (A - Bt)^{-C} \), where \( A \) and \( C \) are positive numbers determined by \( w \), and \( B \) is a positive number determined by \( H_0 \) and \( \Omega_{m,0} \). This solution diverges when \( A - Bt = 0 \); an explicit calculation shows that this happens for

\[
t_{\text{rip}} = \frac{2}{3(1+w) H_0 \sqrt{1 - \Omega_{m,0}}},
\]

and that \( H \) also diverges at this moment. Figure 1 illustrates some values of \( t_{\text{rip}} \) as a function of \( w \).

The Hawking radiation temperature given by Eq. 1 has been shown in [4] to apply to any universe obeying the FRW metric, a special case of which is the universe containing phantom energy that we are considering. Since this temperature is proportional to \( H \) (as a consequence of Eq. 2), we conclude that it will diverge at the Big Rip. The same is true for the energy density associated to that temperature, which assuming thermal equilibrium is given by the Stefan-Boltzmann law for black-body radiation:

\[
U = \frac{4\sigma T^4}{c} = \frac{4\sigma}{c} \left( \frac{hc}{k_B 2\pi FA} \right)^4.
\]

We started the discussion assuming that dark energy would be the only relevant energy component in the universe, and ended up concluding that a new energy component (Hawking radiation) appears as a result. The effect of this additional energy component is to further accelerate the expansion, but without any qualitative change to its outcome. This exact mechanism of cosmological back-reaction of Hawking radiation has been proposed to be a viable mechanism of inflation [14, 15], this will be relevant for us in Sec. III.

III. CONFORMAL CYCLIC COSMOLOGY

Now that we have established that the energy density of Hawking radiation diverges at the Big Rip, we turn to considering how this might establish a connection between the Big Rip scenario and conformal cyclic cosmology. The main observation to be made is that the energy density of matter relative to radiation goes to zero at the Big Rip, causing the spacetime metric to become conformally invariant at that moment. With this, the procedure of “squashing down” a divergent metric that is employed in the CCC model can also be applied in this case. It consists of reinterpreting the metric using a smooth positive scalar field \( \Omega \):

\[
g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}.
\]

The conformal factor \( \Omega \) is taken to be infinitesimal, and the result of the transformation is to bring a future divergent scale factor to a finite size. This may sound like an unreasonable procedure at first glance, but the removal of isotropic spacetime singularities through conformal rescalings is mathematically well defined, as outlined in [16]. A side effect of this transformation is to make the energy density of phantom energy insignificant, given that it is proportional to \( a^{-3(1+w)} \) while \( a \) is being made small, thus bringing the universe to a state completely dominated by radiation that would correspond to a new Big Bang, exactly as in CCC.

This way, it can be seen that the Big Rip scenario leads to a variant of the CCC model, but with some differences. The first is that the need for all particles in the universe to lose their masses in the future is removed, since the spacetime metric becomes conformally invariant due to the radiation energy density increasing instead of the matter energy density decreasing. The second is that the age of the universe at the end of each aeon is fi-
nate in this variation, whereas it is infinite in the original formulation.

Regarding inflation, in the CCC model it is loosely assumed that the exponential expansion of a previous aeon will play the role of inflation without the need for any ad hoc inflaton field. In our variant of the model, the expansion prior to the Big Rip could perhaps be taken to play the same role, but we consider it more interesting to adhere to the view of inflation proposed in [14] and [15], in which it is driven by the Hawking radiation generated at the apparent horizon. The basic idea is that a very dense universe with a fast expansion rate and a small cosmic horizon sees a high energy density of Hawking radiation, which causes the expansion to self-reinforce and become exponential, but in a way that does not go on forever: once the original density of the universe gets diluted enough by the expansion, Hawking radiation becomes insignificant and the exponential expansion ends. This has been quantitatively shown to be a viable mechanism for inflation, and it is one that is compatible with our framework, given that a high energy density in the beginning of a new aeon is an outcome of it.

IV. DISCUSSION

Relative to the original CCC formulation, our version has the advantage of not requiring all the massive fermions and massive charged particles in the universe to disappear in the future. In fact, not even the evaporation of the black holes in the universe is a requirement. But it comes with the need for a new ingredient, namely the presence of phantom energy. Current observations are in agreement with this requirement, and will remain this way as long as \( w = -1 \) remains a valid option, since error bars will always exist and any value \( w < -1 \), no matter how close to \(-1\), suffices to cause a future Big Rip, as pointed out in [8]. All that is changed is how far into the future this moment will be. Perhaps the biggest disadvantage of phantom energy relative to the \( w = -1 \) base scenario is that it cannot be interpreted as a cosmological constant. So in order for its effects to be seen without the introduction of an ad hoc energy component in the universe, a more sophisticated modification of General Relativity would have to be developed. It is worth noting that an equation of state parameter \( w \) that remains constant in time is a simplifying assumption; the Big Rip is also bound to happen e.g. for a time varying dark energy model which satisfies \( w < -1 \) at all times.

Regarding observational tests of our model, we consider that the same ones that apply to the original CCC model also apply to it, namely looking for signatures of a previous aeon on the CMB [11] and possibly on the gravitational wave background of the universe, which has been shown to be sensitive to the primordial density fluctuations [17]. Both of these lie on the fact that gravitational radiation is able to cross the divergent spacetime boundary in the CCC framework [10]. The predictions could turn out to have discerning characteristics in our case because, since the previous aeon would have ended in a finite timespan and in a more abrupt way, the irregularities in its final energy distribution would have been presumably different than in the more gradual scale factor divergence of standard CCC.

V. CONCLUSION

In this paper, we have demonstrated that the Hawking radiation generated at the apparent horizon of a FRW universe containing phantom energy (dark energy with \( w = -1 \) in its equation of state) diverges both in temperature and energy density at the Big Rip. Then we used this fact to propose a variant of the conformal cyclic cosmology model based on the observation that the energy density of matter relative to radiation will become insignificant at the Big Rip, making the spacetime metric conformally invariant at that moment. This variant makes the model more compatible with known physical laws, since the need for mass decay is removed from it. The dark energy requirements are also consistent with the best cosmological parameter estimations available today, given that values of \( w \) smaller than \(-1\) are within their confidence intervals. Tying this framework to a model of inflation based on Hawking radiation, we believe that a fully self-consistent cyclic cosmological model can be obtained.

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