Electromagnetic field with induced massive term: Case with scalar field

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We consider an interacting system of massless scalar and electromagnetic field, with the Lagrangian explicitly depending on the electromagnetic potentials, i.e., interaction with broken gauge invariance. The Lagrangian for interaction is chosen in such a way that the electromagnetic field equation acquires an additional term, which in some cases is proportional to the vector potential of the electromagnetic field. This equation can be interpreted as the equation of motion of photon with induced nonzero rest-mass. This system of interacting fields is considered within the scope of Bianchi type-I (BI) cosmological model. It is shown that, as a result of interaction the electromagnetic field vanishes at \( t \to \infty \) and the isotropization process of the expansion takes place.

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I. INTRODUCTION

The hypothesis of possible nonzero photon mass has long been discussed in the literature [1–4]. The modern experimental data do not contradict this hypothesis [5–13]. So it is interesting to consider some additional arguments for or against this hypothesis. As one of such arguments can serve experimental data of modern observational cosmology, which witnesses the isotropy of the Universe. It is interesting to combine this fact with the description of matter by means of system of interacting fields including the electromagnetic one. It comes out that in a number of cases consideration of such system in cosmology happens to be equivalent to the photon mass. In this paper we consider one of the simplest systems comprising with mass-less scalar and electromagnetic fields and study the influence of such interaction on the expansion of the Universe in the asymptotic region.

II. BASIC EQUATIONS AND THEIR GENERAL SOLUTIONS

We choose the Lagrangian of the interaction electromagnetic and massless scalar fields within the framework of a BI cosmological gravitational field in the form

$$\mathcal{L} = \frac{R}{2\kappa} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \varphi^i \varphi^j \mathcal{G}_{ij} \left(1 + \Phi(I)\right), \quad \mathcal{G} = \left(1 + \Phi(I)\right),$$

with $I = A_{\mu} A^{\mu}$.

We consider the BI metric in the form

$$ds^2 = e^{2\alpha} dt^2 - e^{2\beta_1} dx^2 - e^{2\beta_2} dy^2 - e^{2\beta_3} dz^2.$$  (2.2)

The metric functions $\alpha, \beta_1, \beta_2, \beta_3$ depend on $t$ only and obey the coordinate condition

$$\alpha = \beta_1 + \beta_2 + \beta_3.$$  (2.3)

Written in the form

$$R^v_{\mu} = -\kappa \left(T^v_{\mu} - \frac{1}{2} \delta^v_{\mu} T\right),$$

the Einstein equations corresponding to the metric (2.2) in account of (2.3) read

$$e^{-2\alpha} \left(\ddot{\alpha} - \dot{\alpha}^2 + \dot{\beta}_1^2 + \dot{\beta}_2^2 + \dot{\beta}_3^2\right) = -\kappa \left(T^0_0 - \frac{1}{2} T\right),$$  (2.5a)

$$e^{-2\alpha} \ddot{\beta}_1 = -\kappa \left(T^1_1 - \frac{1}{2} T\right),$$  (2.5b)

$$e^{-2\alpha} \ddot{\beta}_2 = -\kappa \left(T^2_2 - \frac{1}{2} T\right),$$  (2.5c)

$$e^{-2\alpha} \ddot{\beta}_3 = -\kappa \left(T^3_3 - \frac{1}{2} T\right),$$  (2.5d)

where over dot means differentiation with respect to $t$ and $T^\mu_\nu$ is the energy-momentum tensor of the matter fields.

Variation of (2.1) with respect to electromagnetic field gives

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left(\sqrt{-g} F^{\mu\nu}\right) - \left(\varphi, \varphi^i\right) \mathcal{G}_i A^\mu = 0, \quad \mathcal{G}_i = \frac{d\mathcal{G}}{dI}. \quad (2.6)$$
The scalar field equation corresponding to the Lagrangian (2.1) has the form

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \phi \phi' \right) = 0. \tag{2.7}
\]

The energy-momentum tensor of the interacting matters fields has the form

\[
T^\nu_\mu = \left[ \phi, \phi \phi' - F_{\mu\eta} F^{\nu\eta} + (\phi, \phi) \phi A_\mu A^\nu \right],
\]

\[
- \partial^\nu \left[ \frac{1}{2} (\phi, \phi') \phi' - \frac{1}{4} F_{\eta\rho} F^{\eta\rho} \right]. \tag{2.8}
\]

We consider the case when the electromagnetic and scalar fields are the functions of \( t \) only. Taking this in mind we choose the vector potential in the following way:

\[
A_\mu = (0, A_1(t), A_2(t), A_3(t)). \tag{2.9}
\]

In this case the electromagnetic field tensor \( F^{\mu\nu} \) has three non-vanishing components, namely

\[
F_{01} = \dot{A}_1, \quad F_{02} = \dot{A}_2, \quad F_{03} = \dot{A}_3. \tag{2.10}
\]

On account of (2.9) and (2.10) we now have

\[
I = -A_1^2 e^{-2\beta_1} - A_2^2 e^{-2\beta_2} - A_3^2 e^{-2\beta_3}, \tag{2.11}
\]

\[
F_{\mu\nu} F^{\mu\nu} = -2 e^{-2\alpha} (A_1^2 e^{-2\beta_1} + A_2^2 e^{-2\beta_2} + A_3^2 e^{-2\beta_3}). \tag{2.12}
\]

Let us now solve the scalar field equation. Taking into account that \( \phi = \phi(t) \), from the scalar field equation one finds

\[
\dot{\phi} = \frac{\phi_0}{\phi'}, \quad \Rightarrow \phi, \phi' = \frac{\phi_0^2}{\phi'^2} e^{-2\alpha}, \quad \phi_0 = \text{const.} \tag{2.13}
\]

On account of (2.10) and (2.13) for electromagnetic field we find

\[
\frac{d}{dt} \left( \dot{A}_1 e^{-2\beta_1} \right) - \frac{\phi_0^2}{\phi'^2} P A_1 e^{-2\beta_1} = 0, \tag{2.14a}
\]

\[
\frac{d}{dt} \left( \dot{A}_2 e^{-2\beta_2} \right) - \frac{\phi_0^2}{\phi'^2} P A_2 e^{-2\beta_2} = 0, \tag{2.14b}
\]

\[
\frac{d}{dt} \left( \dot{A}_3 e^{-2\beta_3} \right) - \frac{\phi_0^2}{\phi'^2} P A_3 e^{-2\beta_3} = 0, \tag{2.14c}
\]

where we set \( P(I) = 1/\phi'(I) \).

Finally, let us solve the Einstein equations. In doing so, let us first write the nonzero components of the energy momentum tensor of material fields. In view of (2.13) from (2.8) we find
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From (2.18) one finds

\[ T_0^0 = \left[ \frac{\phi_0^2 P}{2} + 1 \left( A_1^2 e^{-2\beta_1} + A_2^2 e^{-2\beta_2} + A_3^2 e^{-2\beta_3} \right) \right] e^{-2\alpha}, \]  

(2.15a)

\[ T_1^1 = \left[ -\frac{\phi_0^2 P}{2} + 1 \left( A_1^2 e^{-2\beta_1} - A_2^2 e^{-2\beta_2} - A_3^2 e^{-2\beta_3} \right) + \phi_0^2 P_1 A_1^2 e^{-2\beta_1} \right] e^{-2\alpha}, \]  

(2.15b)

\[ T_2^2 = \left[ -\frac{\phi_0^2 P}{2} + 1 \left( A_2^2 e^{-2\beta_2} - A_3^2 e^{-2\beta_3} - A_1^2 e^{-2\beta_1} \right) + \phi_0^2 P_2 A_2^2 e^{-2\beta_2} \right] e^{-2\alpha}, \]  

(2.15c)

\[ T_3^3 = \left[ -\frac{\phi_0^2 P}{2} + 1 \left( A_3^2 e^{-2\beta_3} - A_1^2 e^{-2\beta_1} - A_2^2 e^{-2\beta_2} \right) + \phi_0^2 P_3 A_3^2 e^{-2\beta_3} \right] e^{-2\alpha}, \]  

(2.15d)

\[ T_2^1 = \left( A_1 A_2 + \phi_0^2 P_1 A_1 A_2 \right) e^{-2\alpha - 2\beta_1}, \]  

(2.15e)

\[ T_3^2 = \left( A_2 A_3 + \phi_0^2 P_1 A_2 A_3 \right) e^{-2\alpha - 2\beta_2}, \]  

(2.15f)

\[ T_1^3 = \left( A_3 A_1 + \phi_0^2 P_1 A_3 A_1 \right) e^{-2\alpha - 2\beta_3}. \]  

(2.15g)

From (2.15) one also finds

\[ T = \left[ -\phi_0^2 P + \phi_0^2 P_1 \left( A_1^2 e^{-2\beta_1} + A_2^2 e^{-2\beta_2} + A_3^2 e^{-2\beta_3} \right) \right] e^{-2\alpha} = -\phi_0^2 [P + IP] e^{-2\alpha}. \]  

(2.16)

The triviality of off-diagonal components of the Einstein tensor for BI metric leads to

\[ T_2^1 = T_3^2 = T_1^3 = 0, \]  

(2.17)

that gives

\[ \frac{\dot{A}_1 A_2}{A_1 A_2} = \frac{\dot{A}_2 A_3}{A_2 A_3} = \frac{\dot{A}_3 A_1}{A_3 A_1} = -\phi_0^2 P_1. \]  

(2.18)

From (2.18) one finds

\[ \frac{\dot{A}_1}{A_1} = \frac{\dot{A}_2}{A_2} = \frac{\dot{A}_3}{A_3}, \]  

(2.19)

that leads to the following relations between the three components of vector potential:

\[ A_1 = A, \quad A_2 = C_{21} A, \quad A_3 = C_{31} A, \]  

(2.20)

with \( C_{21} \) and \( C_{31} \) being constants of integration.

In view of (2.19) and (2.18) the diagonal components of the energy momentum tensor take the form:

\[ T_0^0 = -T_1^1 = -T_2^2 = -T_3^3 = \frac{\phi_0^2}{2} \left[ P + IP \right] e^{-2\alpha}. \]  

(2.21)

Inserting (2.21) into (2.5) for the metric functions one finds:

\[ \alpha = \alpha^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 = -\alpha \phi_0^2 [P + IP], \]  

(2.22a)

\[ \dot{\beta}_1 = 0, \]  

(2.22b)

\[ \dot{\beta}_2 = 0, \]  

(2.22c)

\[ \dot{\beta}_3 = 0. \]  

(2.22d)

From (2.22b), (2.22c) and (2.22d) we find

\[ \beta_1 = b_1 t + \beta_{10}, \quad \beta_2 = b_2 t + \beta_{20}, \quad \beta_3 = b_3 t + \beta_{30}. \]  

(2.23)
Here $b_i$ and $\beta_{i0}$ are integration constants. It should be noted that in order to maintain the same scaling along all the axes, the constants $\beta_{i0}$ should be the same, hence, without losing generality one can set $\beta_{i0} = 0$.

Now let us go back to the electromagnetic field equations (2.14), which can be arranged as

\begin{align}
\left(\frac{\dot{A}_1}{A_1}\right)^2 + 2\left(\frac{\dot{A}_1}{A_1}\right) \beta_1 - \phi_0^2 P_1 &= 0, \\
\left(\frac{\dot{A}_2}{A_2}\right)^2 + 2\left(\frac{\dot{A}_2}{A_2}\right) \beta_2 - \phi_0^2 P_1 &= 0, \\
\left(\frac{\dot{A}_3}{A_3}\right)^2 + 2\left(\frac{\dot{A}_3}{A_3}\right) \beta_3 - \phi_0^2 P_1 &= 0.
\end{align}

In view of (2.19) from (2.24) we conclude that

\[
\dot{\beta}_1 = \dot{\beta}_2 = \dot{\beta}_3,
\]
which is equivalent to $b_1 = b_2 = b_3 = b$ in (2.23), i.e.

\[
b_1 = b_2 = b_3 = bt.
\]

As one sees from (2.26) we have isotropy at any given time.

Now taking into account that both $A_2$ and $A_3$ can be expressed in term of $A_1$ one could solve only one of the three equations of (2.14). In view of (2.20) and (2.26) let us first rewrite (2.11) as follows

\begin{equation}
I = -QA e^{-2bt}, \quad Q = [1 + C_{21}^2 + C_{31}^2].
\end{equation}

In view of (2.18) and (2.26) the equation for $A$ now reads

\begin{equation}
A \dddot{A} + \dot{A}^2 = 2bA\ddot{A} = 0.
\end{equation}

One of the solutions to the equation (2.28) takes the form

\begin{equation}
A = De^{bt},
\end{equation}
with $D$ being the constant of integration.

Let us now find the interaction corresponding to the solutions obtained. Here we consider a few cases.

**Case I** Let us assume $P$ be the power law of $I$: $P(I) = \lambda I^n$, where $\lambda$ is the coupling constant. For $\lambda = 0$, we have $Q = 1$. This corresponds to the self-consistent system of scalar and electromagnetic fields with minimal coupling. The off-diagonal component of the energy-momentum tensor we write in the form

\begin{equation}
\dot{A}^2 + \phi_0^2 P_1 A^2 = 0.
\end{equation}

Inserting $A$ from (2.29) in this case we find

\begin{equation}
b^2 = -\lambda n \phi_0^2 I^{n-1}.
\end{equation}
From (2.31) it becomes obvious that $n = 1$, that gives

\begin{equation}
b^2 = -\lambda \phi_0^2.
\end{equation}
From (2.32) one concludes that $\lambda$ is negative. Setting $\lambda = -\zeta$ one finds
\[ b^2 = \zeta \phi_0^2. \] (2.33)
On the other hand from (2.22a) we find
\[ 3b^2 = -\zeta \kappa \phi_0^2 I. \] (2.34)
From (2.33) and (2.34) we find
\[ I = -\frac{3}{\kappa} = -D^2(1 + C_{21}^2 + C_{31}^2), \] (2.35)
where the second equality follows from (2.27).

Let us now once again go back to the electromagnetic field equations, which in this case reads
\[ \ddot{A} + 2b\dot{A} + b^2 A = 0, \] (2.36)
which is a Fock-Proca type equation. The equation (2.36) shows that the photon mass is directly related to the gravitational field. In case of $b = 0$, i.e., in case of flat space-time we have usual Maxwell equation
\[ \ddot{A} = 0. \] (2.37)

\textbf{Case II} Let us assume
\[ G = \sum_{n=0}^{\infty} \lambda_n I^n. \] (2.38)
Recalling that $G = 1 + \Phi(I)$ one immediately finds $\lambda_0 = 1$. Taking into account that $P = 1/G$ from off-diagonal components of energy-momentum tensor we find
\[ \phi_0^2 G I = b^2 G^2. \] (2.39)
In view of (2.39) equation (2.22a) now reads
\[ \left( \frac{6}{\kappa} + I \right) G I = G. \] (2.40)

The system (2.39) and (2.40) possesses a large number of solutions. One of the simplest solution is
\[ \lambda_1 = \frac{\kappa}{6}, \quad \lambda_2 \neq 0, \quad \lambda_{n>2} = 0. \] (2.41)
In this case we get
\[ I = -\frac{12}{\kappa} = -D^2(1 + C_{21}^2 + C_{31}^2). \] (2.42)

\textbf{Case with minimal coupling}
Let us now consider the case with minimal coupling. In this case we have $G = 1/P = 1$. The purpose of this study is to clarify the role of interaction in isotropization process. From the off-diagonal components of energy-momentum tensor in this case we find
\[ \dot{A}_1 \dot{A}_2 = \dot{A}_2 \dot{A}_3 = \dot{A}_3 \dot{A}_1 = 0. \] (2.43)
From (2.43) follows that at least two of the three components \( A_i \) are constant, which means only one of the components of \( F_{\mu \nu} \) is nonzero. Let us assume that \( \dot{A}_1 = \dot{A}_3 \neq 0 \). In this case we have

\[
T_0^0 = \left( \frac{\phi_0^2}{2} + \frac{1}{2} \dot{A}^2 e^{-2\beta_1} \right) e^{-2\alpha}, \tag{2.44a}
\]

\[
T_1^1 = \left( -\frac{\phi_0^2}{2} + \frac{1}{2} \dot{A}^2 e^{-2\beta_1} \right) e^{-2\alpha}, \tag{2.44b}
\]

\[
T_2^2 = \left( -\frac{\phi_0^2}{2} - \frac{1}{2} \dot{A}^2 e^{-2\beta_1} \right) e^{-2\alpha}, \tag{2.44c}
\]

\[
T_3^3 = \left( -\frac{\phi_0^2}{2} - \frac{1}{2} \dot{A}^2 e^{-2\beta_1} \right) e^{-2\alpha}. \tag{2.44d}
\]

In view of \( \dot{A}_2 = \dot{A}_3 = 0 \) from the electromagnetic field equations in this case we have

\[
A = C \int e^{2\beta_1} dt + C_1, \quad A_2 = \text{const.}, \quad A_3 = \text{const.}, \tag{2.45}
\]

with \( C \) and \( C_1 \) being some arbitrary constants. Einstein field equations in this case take the form

\[
\ddot{\alpha} - \dot{\alpha}^2 + \dot{\beta}_1^2 + \dot{\beta}_2^2 + \dot{\beta}_3^2 = -\frac{\kappa}{2} C^2 e^{2\beta_1} + \kappa \phi_0^2, \tag{2.46a}
\]

\[
\dot{\beta}_1 = -\frac{\kappa}{2} C^2 e^{2\beta_1}, \tag{2.46b}
\]

\[
\dot{\beta}_2 = \frac{\kappa}{2} C^2 e^{2\beta_1}, \tag{2.46c}
\]

\[
\dot{\beta}_3 = \frac{\kappa}{2} C^2 e^{2\beta_1}. \tag{2.46d}
\]

In view of coordinate condition for \( \alpha \) we find

\[
\dot{\alpha} = \frac{\kappa}{2} C^2 e^{2\beta_1}. \tag{2.47}
\]

From (2.46b), (2.46c) and (2.46d) immediately follows that in the case in question, no isotropization process takes place. From (2.46b) one finds the following expression for \( \beta_1 \):

\[
e^{2\beta_1} = \frac{2 \eta^2}{\kappa C^2 \cosh^2(\psi_0 - \eta t)}, \quad \eta^2 = \text{const.}, \quad \psi_0 = \text{const.} \tag{2.48}
\]

Inserting \( e^{2\beta_1} \) into (2.46c), (2.46d) and (2.47) one finds

\[
e^{2\alpha} = e^{2\beta_2} = e^{2\beta_3} = \cosh^2(\psi_0 - \eta t). \tag{2.49}
\]

From (2.48) and (2.49) we get

\[
\dot{\alpha} = -\dot{\beta}_1 = \dot{\beta}_2 = \dot{\beta}_3 = -\eta \tanh(\psi_0 - \eta t). \tag{2.50}
\]

Inserting (2.47), (2.48) and (2.50) into (2.46a) we find

\[
\eta^2 = \frac{\kappa \phi_0^2}{2}. \tag{2.51}
\]
Thus the system with minimal coupling is completely solved. It is shown that in the case concerned, no isotropization process takes place.

**Case in absence of gravitational field**

Let us consider the case when the influence of the gravitational field is not taken into account. In this case we have \( \alpha = \beta_1 = \beta_2 = \beta_3 = 0 \), i.e., the space-time is flat. For the scalar field in this case we have

\[
\dot{\phi}^2 = \frac{\phi_0^2}{\phi^2},
\]

whereas for the electromagnetic field we have

\[
\ddot{A} + \phi_0^2 P_1 A = 0.
\]

Unlike the case with gravitational field, now there is no restrictions imposed on \( P(I) \). If set, \( P = \lambda_1 I^n \), where \( I = A^2 \) we now have

\[
\ddot{A} + \phi_0^2 \lambda A^{2n-1} = 0.
\]

In this case \( n \) may take any value, with \( n = 1 \) giving the Fock-Proca type equation.

### III. CONCLUSION

Within the framework of Bianchi type-I cosmological model we studied the evolution of the initially anisotropic space-time in presence of an interacting system of electromagnetic and massless scalar fields. We consider the case when the Lagrangian density of electromagnetic field was given as a sum of Maxwellian part and the one explicitly depending on scalar invariant \( (\phi, \varphi^\mu ; \varphi^\nu) \) and the invariants \( I = A_\mu A^\mu \), that was expressed as power law function. It was shown that the model allows a set of partial solutions, a few of which is described explicitly in this paper.

In case of interacting electromagnetic and scalar fields on account of gravitational one, the Fock-Proca type equation with induced massive-term was obtained. It was shown that only in case of interacting material fields the isotropization process takes place.

It is shown that introduction of gravitational field, depending on the concrete form of metric, imposes additional restriction to the components of the vector potential.

In case of minimal coupling the isotropization process remains absent.

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