AN INTEGRATED POWER FLOW SOLUTION OF FLEXIBLE AC TRANSMISSION SYSTEMS CONTAINING WIND ENERGY CONVERSION SYSTEMS

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Keywords: Wind energy conversion systems (WECS), wind farm, FACTS devices, Newton-Raphson algorithm, power flow.

Abstract

This paper presents a general steady-state modelling approach of power networks having wind farms and FACTS devices. A mathematical formulation is then proposed where the state variables associated with wind farms and FACTS devices are combined with the nodal voltage magnitudes and angles of the transmission network for a unified iterative solution of the power flow problem. Additionally, a suitable wind generator model based on a permanent magnet synchronous generator for power flow studies is introduced, which allows for a direct voltage magnitude’s control at the high-voltage side of the wind generator’s transformer. Lastly, two numerical examples are presented which show how the suggested approach performs.

1 Introduction

Even though the use of wind generators for converting wind energy into electricity is beneficial from the environmental standpoint, their consideration in the active power dispatch makes the already complex task of achieving power system controllability even more demanding. Consequently, the quantification of the effects that large-scale integration of wind generation will cause on the network is a very important matter that requires special attention when planning and operating an electrical power system. Arguably, power flow analysis is the most popular computational calculation performed in a power system’s planning and operation, and this study has been selected to quantify the electrical response of wind generators when they are integrated in Flexible AC Transmission Systems.

Mathematical models of several types of wind generators have been developed in which their active and reactive power outputs are obtained based on the steady-state equivalent representation of the induction machine. The power injection method is then used to include these models into the power flow formulation, which is solved by using a sequential approach to obtain an operating point of the power system. In this approach, only the network’s state variables are calculated through a conventional power flow algorithm, while a subproblem is formulated for updating the state variables of wind generators as well as their power injections at the end of each power flow’s iteration.

Instead of using the power injection concept, another way of representing a wind generator is by means of an equivalent variable impedance expressed in terms of the slip of the generator and its rotor and stator winding parameters. This impedance is included in the system’s admittance matrix, and the network nodal voltages are computed through the power flow analysis. Based on these voltages, the air-gap power of the wind generator is calculated and used to iteratively compute the value of the slip of the induction generator that produces the match between the air-gap power and the mechanical power extracted from the wind.

In general terms, all the methods discussed above share the characteristic of using a sequential approach to calculate the state variables of the wind generators, and none of them considers the integration of FACTS controllers in the network where the WECSs are embedded.

A fundamentally different approach for the modelling of WECS, within the context of the power flow problem, is a method that simultaneously combines the state variables associated with the wind generators, the FACTS controllers and the transmission network in a single frame-of-reference for a unified iterative solution through a Newton-Raphson (NR) technique. From the convergence standpoint, the unified method is superior to the sequential one because the interaction between the network, FACTS controllers, and wind generators is better represented during the iterative solution. Furthermore, it arrives at the solution with a quadratic convergence regardless of the network size. Hence, the key contribution of this work is to provide a comprehensive and general approach for the analysis of power flows in Flexible AC Transmission systems containing wind generators in a unified single-frame of reference.

2 Power flow including FACTS controllers and wind generators

In this paper, the unified approach suggested in [1] is extended to compute the power flow solution of a power

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system containing FACTS controllers and WECS and represented by a single set of nonlinear power flow mismatch equations \( f(X_{SC}, X_{S}, X_{WG}) = 0 \), where \( X_{SC} \) is a vector of all nodal voltage magnitudes and angles, \( X_{S} \) stands for the state variables of the FACTS controllers, and \( X_{WG} \) is a vector of all state variables associated with the wind generators. The linearized power flow mismatch equations corresponding to the wind farms are then combined with those associated with the FACTS controllers and the rest of the network, as given by (1), which are solved iteratively by the NR method:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\Delta P_f \\
\Delta R_{WG}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P}{\partial \theta} V & \frac{\partial P}{\partial \theta} X_f & \frac{\partial P}{\partial X_{SG}} & \frac{\partial P}{\partial X_{SG}} \\
\frac{\partial Q}{\partial \theta} V & \frac{\partial Q}{\partial \theta} X_f & \frac{\partial Q}{\partial X_{SG}} & \frac{\partial Q}{\partial X_{SG}} \\
\frac{\partial P}{\partial \theta} V & \frac{\partial P}{\partial \theta} X_f & \frac{\partial P}{\partial X_{SG}} & \frac{\partial P}{\partial X_{SG}} \\
\frac{\partial R}{\partial \theta} V & \frac{\partial R}{\partial \theta} X_f & \frac{\partial R}{\partial X_{SG}} & \frac{\partial R}{\partial X_{SG}} \\
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V \\
\Delta X_f \\
\Delta X_{SG}
\end{bmatrix}
\]  

(1)

where \( \Delta P_f \) and \( \Delta R_{WG} \) represent the mismatch equations of the FACTS controllers and wind generators, respectively. The NR method starts from an initial guess for all the state variables and updates the solution at each iteration \( i \) until a pre-defined tolerance is fulfilled. In this unified solution, all the state variables are adjusted simultaneously in order to compute the steady-state operating condition of the power system. Hence, this method has strong convergence characteristics.

3 Modelling of FACTS devices

Among all FACTS devices used to improve the steady-state performance of power systems [1], the Static Var Compensator (SVC) and Thyristor-Controlled Series Capacitor (TCSC) are the controllers considered in this paper.

3.1 Static VAR compensator

An integrated SVC and step-down transformer model is obtained by combining the admittances of both components \( Y_{T SVC} = Y_T + Y_{SVC} \) as proposed in [1]. The linearised power flow equations are given by (2) considering the firing angle \( \alpha_{SVC} \) of the SVC as the state variable within the NR method:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\Delta \phi
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{\partial G_{T SVC}}{\partial \alpha_{SVC}} V^2 & 0 \\
0 & \frac{\partial B_{T SVC}}{\partial \alpha_{SVC}} V^2 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\Delta \phi \alpha_{SVC}
\end{bmatrix}
\]  

(2)

where \( G_{T SVC} \) and \( B_{T SVC} \) are functions dependent on \( \alpha_{SVC} \):

\[
G_{T SVC} = \frac{R_T}{R_T^2 + X_{L}^2}, \quad B_{T SVC} = -\frac{X_L}{R_T^2 + X_{L}^2}, \quad X_{L} = X_T + X_{SVC},
\]

\[
X_{SVC} = \frac{X_L X_{T SVC}}{X_C - X_{T SVC}}, \quad X_{T SVC} = \frac{\pi X_L}{2(\pi - \alpha_{SVC}) + \sin(2\alpha_{SVC})}.
\]

3.2 Thyristor-controlled series compensator

The TCSC firing angle power flow model is represented as an equivalent series reactance which is associated with the firing angle \( \alpha_{TCSC} \). This reactance can be expressed as [1] (see Appendix):

\[
X_{TCSC} = -X_C + C_i \left\{ (\pi - \alpha) \sin [2(\pi - \alpha)] \right\} - C_i \cos^2 (\pi - \alpha) \left\{ \tan [\alpha (\pi - \alpha)] - \tan (\pi - \alpha) \right\}
\]

(3)

Assuming the TCSC controls the active power flowing from bus \( k \) to bus \( m \) to a specified value of \( P_{km}^* \), the set of linearised power flow equations is

\[
\begin{bmatrix}
\Delta P_k \\
\Delta Q_k \\
\Delta P_m \\
\Delta Q_m
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P_k}{\partial \theta} V_k & \frac{\partial P_k}{\partial \phi} V_k & \frac{\partial P_k}{\partial X_{SG}} & \frac{\partial P_k}{\partial X_{SG}} \\
\frac{\partial Q_k}{\partial \theta} V_k & \frac{\partial Q_k}{\partial \phi} V_k & \frac{\partial Q_k}{\partial X_{SG}} & \frac{\partial Q_k}{\partial X_{SG}} \\
\frac{\partial P_m}{\partial \theta} V_m & \frac{\partial P_m}{\partial \phi} V_m & \frac{\partial P_m}{\partial X_{SG}} & \frac{\partial P_m}{\partial X_{SG}} \\
\frac{\partial Q_m}{\partial \theta} V_m & \frac{\partial Q_m}{\partial \phi} V_m & \frac{\partial Q_m}{\partial X_{SG}} & \frac{\partial Q_m}{\partial X_{SG}} \\
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_k \\
\Delta V_k \\
\Delta X_{SG}
\end{bmatrix}
\]  

(4)

where the power flow mismatch for the TCSC module is defined as \( \Delta P_{km} = P_{km}^* - P_{km} \), \( P_{km} = V_k V_m B_{km} \sin (\theta_k - \theta_m) \) and \( B_{km} = 1/X_{TCSC} \).

4 Modelling of wind generators

Wind generators are categorized according to how they operate when they are connected to the grid. The Fixed-Speed Wind Generators (FSWG) are called so because their speed is mainly set according to the system’s frequency [2]. In this category are the Stall-Regulated Fixed-Speed Wind Generators (SR-FSWG) and the Pitch-Regulated Fixed-Speed Wind Generators (PR-FSWG). A variant of the previous models is the semi-variable speed wind generator (SSWG), which uses a wound-rotor induction generator with an external resistor added to the rotor circuit in order to achieve a power regulation when wind speeds are above the rated one [3]. Also, variable-speed wind generators are being employed worldwide with the doubly-fed induction generator being the most used. However, another emergent topology that is being widely accepted is the wind generator based on a Permanent Magnet Synchronous Generator (PMSG) with a full-scale converter in which the gearbox can be omitted [4].

In this paper, the mathematical modelling of FSWG, SSWG and PMSG-based wind generators for power flow studies is addressed. In reference [5], the models of fixed- and semi-variable speed wind generators are suitably derived for power flow analysis and can be readily integrated in the formulation presented herein. For this reason, only a brief description of these models is given next.
4.1 Fixed-speed wind generators

This generator is directly connected to the network through a step-up transformer, and its final operating point depends upon the electrical frequency as well as the nodal voltage at the generator’s terminals. The generated reactive and active powers are given by Equations (5) and (6), respectively, and the stator and rotor currents of the induction generator can be expressed according to Equations (7) and (8) [5]:

\[ Q_e(V,s) = -V^2 \left( A + Bs^2 \right) \frac{A + Bs^2}{\left(C - D s + F s^2\right)} \]

\[ P_e(V,s) = -V^2 \left( K + H s + L s^2 \right) \frac{K + H s + L s^2}{\left(C - D s + F s^2\right)} \]

\[ I_1^2(V,s) = V^2 \left( M s + N s^2 \right) \frac{M s + N s^2}{\left(C - D s + F s^2\right)} \]

\[ I_2^2(V,s) = V^2 \left( T s - W s^2 \right) \frac{T s - W s^2}{\left(C - D s + F s^2\right)} \]

where \( s \) is the machine’s slip, \( V \) is the terminal voltage, and the constants from \( A \) to \( W \) are as given in the Appendix. Furthermore, the power converted from mechanical to electrical form \( P_{\text{conv}} \) can be computed by using Equation (9), where \( R_2 \) represents the rotor resistance.

\[ P_{\text{conv}} = -I_2^2 R_2 \left( \frac{1-s}{s} \right). \]

4.1.1 Stall-regulated fixed-speed wind generator

The mechanical power \( P_m \) [W] extracted from the wind by this generator is given by [6]

\[ P_m = 0.5 \cdot \rho \cdot c_e \left( \frac{c_s}{\lambda} - c_l - c_{sl} - c_{cl} \right) \left( 1-s \right) \eta_{\text{gen}} \cdot A V_w^2 \]

where \[ \lambda = \left( \frac{1}{\lambda + c_l} \right) \left( \frac{c_e}{B^2 + 1} \right) \]

\[ \eta_{\text{gen}} = \frac{R \cdot n_{\text{sl}} - \omega_T \cdot (1-s)}{V_w} \]

and \( \rho \) is the air density [kg/m\(^3\)], \( A \) is the swept area of the blades [m\(^2\)], \( V_w \) is the wind speed [m/s], \( R \) is the radius of the rotor [m], \( n_{\text{sl}} \) is the gearbox ratio, \( \omega_T \) is the angular synchronous speed [rad/s], \( \beta \) is the pitch angle [degrees], \( \omega_T \) is the angular speed of the turbine [rad/s], and the constants \( c_i \) to \( c_{sl} \) are the parameters of the wind turbine’s design. Thus, assuming that the SR-FSWG is connected at bus \( k \), the power mismatches equations are (11)-(13), and the set of linearised equations that has to be assembled and combined with the Jacobian matrix and the power mismatches vector of the entire network is shown in Equation (14) [5]:

\[ \Delta P_1 = P_1(V,s) - P_{1,k} - P_{1,k}^{\text{cal}} = 0 \]  

\[ \Delta Q_1 = Q_1(V,s) - Q_{1,k} - Q_{1,k}^{\text{cal}} = 0 \]  

\[ \Delta P_{\text{conv}} = -\left( P_m - P_{\text{conv}} \right) = -\left( P_m + I_2^2 R_2 \left( \frac{1-s}{s} \right) \right) = 0 \]  

where \( P_1(V,s) \) and \( Q_1(V,s) \) are given by (6) and (5), respectively, \( P_{1,k} \) and \( Q_{1,k} \) represent the active and reactive powers drawn by the load at bus \( k \), respectively, and \( P_{1,k}^{\text{cal}} \) and \( Q_{1,k}^{\text{cal}} \) are active and reactive power injections given by

\[ P_{1,k}^{\text{cal}} = V_k^2 G_{1,k} + V_k \sum_{n=1}^{\text{N}} G_{1,n} \left( V_n \cos(\theta_n - \theta_k) + B_{1,n} \sin(\theta_n - \theta_k) \right) \]

\[ Q_{1,k}^{\text{cal}} = -V_k^2 B_{1,k} + V_k \sum_{n=1}^{\text{N}} Q_{1,n} \left( V_n \sin(\theta_n - \theta_k) - B_{1,n} \cos(\theta_n - \theta_k) \right) \]

4.1.2 Pitch-regulated fixed-speed wind generator

Since this wind generator has a blade pitch angle mechanism which acts to limit the power extracted from the wind [7], the generated active power \( P_{g,pr}^{\text{cal}} \) can be obtained from its power curve and is considered constant at a value \( P_{g,pr}^{\text{cal}} \) through the iterative process; however, the reactive power \( Q_{g,pr}^{\text{cal}} \) needs to be calculated [5]. Therefore, the internal power equilibrium point in the wind generator has to be computed by Equation (17):

\[ P_{n,pr} = P_{g,pr}^{\text{cal}} + P_{\text{losses,pr}} + P_{\text{losses,r}} = P_{g,pr}^{\text{cal}} + 3I_1^2 R_1 + 3I_2^2 R_2 \]

where \( P_{\text{losses,pr}} \) and \( P_{\text{losses,r}} \) are the three-phase stator and rotor power losses, respectively, and the core losses in the induction machine are neglected. As mentioned previously, the set of linearised power flow mismatch equations regarding the PR-FSWG is given by Equations (18)-(21) when the generator is connected at bus \( k \):

\[ \Delta P_1 = P_{g,pr}^{\text{cal}} - P_{1,k} - P_{1,k}^{\text{cal}} = 0 \]

\[ \Delta Q_1 = Q_{g,pr}^{\text{cal}}(V,s) - Q_{1,k} - Q_{1,k}^{\text{cal}} = 0 \]

\[ \Delta P_{\text{conv}} = -\left( P_n - P_{\text{conv}} \right) = -\left( P_n + I_2^2 R_2 \left( \frac{1-s}{s} \right) \right) = 0 \]

\[ \Delta P_{\text{conv}} = 0 \]

\[ \Delta Q_{\text{conv}} = \frac{\partial Q_{\text{conv}}}{\partial \theta_1} \left( V_1 \right) \frac{\partial P_{\text{conv}}}{\partial \theta_1} \left( V_1 \right) \Delta \theta_1 \]

\[ \Delta Q_{\text{conv}} = \frac{\partial Q_{\text{conv}}}{\partial \theta_1} \left( V_1 \right) \frac{\partial P_{\text{conv}}}{\partial \theta_1} \left( V_1 \right) \Delta \theta_1 \]

4.2 Semi-variable speed wind generator

In this type of generator, the slip of the induction machine cannot be regarded as the state variable because of the external resistance \( R_{ext} \) added in the rotor circuit [5]. Hence, a total rotor circuit resistance, \( R_k = (R_k + R_{ext}) / s \), is considered as a single-state variable associated to the rotor circuit, which is
adjusted to satisfy the power mismatch equations during the NR power flow calculation [5,8]. Hence, the reactive and active powers, the stator and rotor currents as well as the power converted from mechanical to electrical form will be dependent functions on $R_s$ and can be expressed as

$$Q_{g,s}(V, R_s) = -V^2 \left[ \frac{A' R_s^2 + B}{[C' R_s - D']^2 + [E' R_s + F']^2} \right]$$  \hspace{1cm} (22)

$$P_{g,s}(V, R_s) = -V^2 \left[ \frac{K' R_s^2 + H' R_s + L}{[C' R_s - D']^2 + [E' R_s + F']^2} \right]$$  \hspace{1cm} (23)

$$I_{1,s}(V, R_s) = V^2 \left[ \frac{K' R_s^2 + H' R_s + L}{[C' R_s - D']^2 + [E' R_s + F']^2} \right]$$  \hspace{1cm} (24)

$$I_{2,s}(V, R_s) = V^2 \left[ \frac{[M' R_s + N]^2 + [E' R_s + F']^2}{[C' R_s - D']^2 + [E' R_s + F']^2} \right]$$  \hspace{1cm} (25)

Therefore, when the SSWG is connected at bus $k$, its set of mismatch power flow equations is

$$P_{w,k,s} = P_{g,s} + P_{conv} + P_{loss,s} = P_{g,s} + 3 I_{1,s}^2 R_s + 3 I_{2,s}^2 R_s$$  \hspace{1cm} (27)

where the constants $A'$, $C'$, $E'$, $H'$, $K'$, $M'$ and $T$ are given in the Appendix. Since the generated active power $P_{g,s}$ is set to a fixed value $P_{g,s}^*$ obtained from the wind generator power curve, and assuming no core losses, the mechanical power of the induction generator $P_{m,s}$ can be estimated as follows:

$$P_{w,s} = P_{g,s}^* + P_{conv} + P_{loss,s} = P_{g,s}^* + 3 I_{1,s}^2 R_s + 3 I_{2,s}^2 R_s$$

The proposed model for power flow studies is shown in the Fig. 1(b) in which $P_{g,pw}$ represents the output power set by the wind generator power curve for a given wind speed, $V_{soc}$ and $V_{g,s}$ are the voltage at the machine-side converter and grid-side converter terminal, respectively, and $Z_{st}$ is the step-up transformer impedance.

![PMSG-based wind generator](image)

The power flow equations for the PMSG-based wind generator are derived assuming the following voltage at the grid-side converter terminal: $V_{soc} = V_{g,s}\sin(\delta_{g,s})$. Based on Fig. 1(b), the active and reactive powers flowing from the grid-side converter terminal to the $k$-th bus are:

$$P_{g,s} = V_{g,s}^* G_{g,s} + V_{g,s}^* V_{g,s}^* \left[ C_{g,s} \cos(\delta_{g,s} - \theta_s) + B_{g,s} \sin(\delta_{g,s} - \theta_s) \right]$$  \hspace{1cm} (32)

$$Q_{g,s} = V_{g,s}^* G_{g,s} + V_{g,s}^* V_{g,s}^* \left[ Q_{g,s} \sin(\delta_{g,s} - \theta_s) - B_{g,s} \cos(\delta_{g,s} - \theta_s) \right]$$  \hspace{1cm} (33)

For the active and reactive powers at bus $k$, the subscripts $g,s$ and $k$ are exchanged in Equations (32) and (33). Therefore, the NR-based power flow formulation is given by

$$P_{w,k} = -P_{g,s} - P_{k,k} - P_{k,k}^* = 0$$  \hspace{1cm} (34)

$$Q_{w,k} = -Q_{g,s} - Q_{k,k} - Q_{k,k}^* = 0$$  \hspace{1cm} (35)

$$P_{w,k} = P_{g,pw} - P_{k,k} = 0$$  \hspace{1cm} (36)

Note that Equation (36) represents the power constraint in the AC/DC/AC converter in which active power losses are neglected.

4.3 PMSG-based wind generator

This type of wind generator possesses a PMSG and a full-rated converter to connect the generator to the network, resulting in complete speed and reactive power control [9]. Hence, all the generated power is supplied to the power system through a machine-side converter and grid-side converter. The schematic diagram of this topology is shown in Fig. 1(a). Reactive power support is one of the characteristics that make this machine attractive for wind power production. In this case, the inclusion of the explicit representation of the wind generator step-up transformer is considered, which allows for direct voltage magnitude control at the high-voltage side of the transformer.
5 Case studies with wind farms and FACTS devices

This section shows how the proposed approach performs when considering a power system having FACTS devices and wind generators.

5.1 5-bus test system with FSWGs, SSWGs and a SVC

The typical 5-bus test system is used to provide an example with the inclusion of a wind farm consisting of ten SR-FSGWs, ten PR-FSGWs and ten SSGWs operating at a wind speed of 16 m/s with which all wind generators are injecting their maximum power. Additionally, a SVC is placed at bus 5 in order maintain its terminal voltage magnitude at 1 pu. The conventional generators are set to control voltage magnitudes at 1 pu. Parameters of the wind farm are given in the Appendix.

In order to show the effect of wind farms and SVC in the operation of the power system, the following three scenarios are presented: (a) the base case where the wind farm and SVC are not considered, (b) the case where only the wind farm is running and (c) the case where the SVC is connected at the bus at which the wind farm is connected in order to provide voltage support. The results regarding each case are reported in Table 1.

The simulated wind farm is a reactive power consumer since it lacks a reactive power control as seen from Table 1. When no SVC is considered, its reactive power absorption exceeds 10 MVAr, resulting in a low voltage magnitude at node 6. On the other hand, when the SVC is set in operation, not only the low-voltage side of the wind farm transformer is boosted, but also the system voltage profile.

Since all wind generators are operating at the same wind speed, clearly the state variables calculated for each group of wind generators corresponding to each model will result in the same value. Furthermore, if the system voltage profile changes, as occurs with the inclusion of the SVC, another operating point is found at each wind generator as shown in Table 2.

| Scenario | SR-FSWG | PR-FSWG | SSWG | SVC |
|----------|---------|---------|------|-----|
| (b)      | -0.00506 | -0.00679 | -67.69437 | --- |
| (c)      | -0.00467 | -0.00613 | -75.16952 | 136.35024 |

Table 2: Computed wind generators and SVC state variables.

5.2 5-bus test system with PMSG-based wind generators and a TCSC

In this case, a PMSG-based wind farm is located at bus 4 with 30 wind generators operating at rated wind speed, i.e. 15 m/s. Be aware that each PMSG-based wind generator provides reactive power support by controlling its terminal voltage magnitude. Also, a TCSC is placed for controlling the active power flowing through the transmission line connected between nodes 4 and 5 at $P_{tr}=20$ MW, as shown in Fig 2. The next scenarios are analysed: (a) the base case where the wind farm and TCSC are not considered, (b) the power system including only the PMSG-based wind farm and (c) the network having simultaneously the TCSC and the wind farm. The results are reported in Table 3.

The system voltage profile is improved when the PMSG-based wind farm is integrated to the grid as seen from Table 3. This is mainly due to two reasons; one is the fact that there is a redistribution of power flows in the network, e.g. the load connected at node 4 is being supplied locally by the wind farm. On the other hand the PMSG-based wind farm is providing voltage support, resulting in a voltage magnitude of 1 pu at the low-voltage side of the wind farm transformer even when the TCSC is set in operation to increase the power transfer to 20 MW in the line connecting nodes 4 and 5. The wind generators and TCSC state variables estimated by the NR algorithm for each scenario are the following: scenario (b) $V_{gsc} = 1.017$ pu, $\delta_{gsc} = 9.071^\circ$, and scenario (c) $V_{gsc} = 1.016$ pu, $\delta_{gsc} = 8.862^\circ$, $\alpha_{TCSC} = 143.499^\circ$.

| Results | Scenario |
|---------|----------|
| $V_1$   | (a) 1.000 (b) 1.000 (c) 1.000 |
| $V_2$   | 1.000     |
| $V_3$   | 0.971     |
| $V_4$   | 0.971     |
| $V_5$   | 0.967     |
| $V_6$   | --- 0.945 |
| $P_{PMSG}$ | --- 25.349 |
| $Q_{PMSG}$ | --- -10.802 |
| $Q_{SVC}$ | --- 36.897 |

Table 1: Power flow simulation results with wind farm and SVC.

Figure 1: Modified 5-bus test system used to incorporate an SVC and a wind farm composed of several wind generator models.

Figure 2: Modified 5-bus test system used to incorporate a TCSC and a PMSG-based wind farm.
6 Conclusions

This paper has put forward the NR-based power flow algorithm which is capable of computing the steady-state operating point of electric networks containing WECS and FACTS devices. The solution problem is formulated in a single-frame of reference, resulting in an efficient iterative solution. Additionally, a PMSG-based wind generator model for power flow studies is presented, which allows for direct voltage magnitude control at the high-voltage side of the wind generator transformer. Numerical examples have shown that FACTS controllers are a practical alternative to integrate WECS into power systems without degrading their operational performance.

Appendix A

- TCSC parameters:

\[ C_i = \frac{X_c + X_{LC}}{\pi}, \quad C_2 = \frac{4X_{LC}^2}{\pi X_L}, \quad X_{LC} = \frac{X_c X_l}{X_c - X_l}, \quad \sigma = \left( \frac{X_c}{X_L} \right)^{1/2} \]

- FSWG parameters:

\[ A = R_f^2 (X_l + X_m), \quad B = (X_l + X_m)(X_l X_m + X_l (X_l + X_m)), \quad C = R_l R_f, \quad D = X_l X_m + X_l (X_l + X_m), \quad E = R_l (X_l + X_m), \quad F = R_l (X_l + X_m), \quad H = R_l X_m, \quad K = K_l R_f, \quad L = R_l (X_l + X_m), \quad M = X_m R_l (X_l + X_m), \quad N = X_m R_l (X_l + X_m), \quad T = R_l R_f X_m, \quad W = X_m [X_l X_m + X_l (X_l + X_m)] \]

- SSWG parameters:

\[ A' = (X_l + X_m), \quad C' = R_f, \quad E' = (X_l + X_m), \quad H' = X_m, \quad K' = R_l, \quad M' = X_m (X_l + X_m), \quad T' = R_l X_m \]

The data for each wind farm is (on a base power of 100 MVA): wind farm step-up transformer impedance is 0.2 pu and the impedance of each wind generator transformer is 4.1667 pu. Also, the data of each wind generator model are given in Table A.1

| Results | Scenario |
|---------|----------|
| (a)     | (b)      | (c)     |
| $V_{r1}$ | 1.000    | 1.000   | 1.000   |
| $V_{r2}$ | 1.000    | 1.000   | 1.000   |
| $V_{r3}$ | 0.971    | 0.984   | 0.984   |
| $V_{r4}$ | 0.971    | 0.987   | 0.987   |
| $V_{r5}$ | 0.967    | 0.973   | 0.971   |

The table for wind generator data is as follows:

| Scenario | $Z_r$ | $Z_g$ | $V_{nom}$ |
|----------|-------|-------|-----------|
| (a)      | 0.0027 + j0.025 | 0.00073 + j0.10906 | 690 |
| (b)      | 0.0022 + j0.046 | 0.000219 + j0.04599 | 690 |
| (c)      | 0.00269 + j0.072605 | 0.00269 + j0.072605 | 690 |

The results are from the simulation in Power Networks, shown in Table A.1.

Acknowledgements

The authors gratefully acknowledge the financial support granted to MSc. Luis M. Castro by the Consejo Nacional de Ciencia y Tecnología (CONACYT) México, and the University of Michoacán (U.M.S.N.H) for allowing him to undertake PhD studies.

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