EXISTENCE AND WELL-POSEDNESS FOR EXCESS DEMAND EQUILIBRIUM PROBLEMS

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Abstract. In this paper, we study excess demand equilibrium problems in Euclidean spaces. Applying the Glicksberg’s fixed point theorem, sufficient conditions for the existence of solutions for the reference problems are established. We introduce a concept of well-posedness, say Levitin–Polyak well-posedness in the sense of Painlevé–Kuratowski, and investigate sufficient conditions for such kind of well-posedness.

1. Introduction. The general equilibrium theory has been found as one of the great achievements in economic theory during the last couple of decades. The main goal of such problem is to investigate properties of supply, demand and prices in a certain economy, and to explain how and why most of the markets get closer to equilibrium in the long run. In the 1970s, Sonnenschein [29], Debreu [12], and Mantel [26] were the ones who laid the foundation of researching general equilibrium theory via the excess demand function. They had been successful in showing that the excess demand functions in an economy can be characterized by Walras’ law, homogeneity, and continuity. Since then, the characterizations of excess demand functions and their applications have attracted immense attention from researchers. Bottazzi and Hens [8] studied characterizations of such functions around noncritical spot price systems in two-period exchange economies with incomplete markets and real assets. Chiappori and Ekeland [11] considered the general equilibrium incomplete markets and proved that any non-vanishing analytic function, which satisfies the natural extension of the Walras’ law, is the excess demand function of such markets provided that the number of consumers is not less than the dimension of

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the admissible trades’ space. Momi [27] investigated the characterizations of aggregate excess demand functions in an economy with incomplete real asset markets, and showed that such functions can be still characterized by homogeneity, Walras’ law, and continuity around critical prices. Also, by using the mathematical fixed point theorems to economic models, sufficient conditions for the existence of a price vector have been established. Wong [34] gave characterizations for excess demand functions and the equilibrium price set for a class of economics, and also proved that Brouwer’s fixed-point theorem is implied by the Arrow-Debreu equilibrium existence theorem for economics. Quah [28] provided an equilibrium existence proof in which the excess demand function of an exchange economy obeys the weak axiom of revealed preference. Recently, Tian [30] has established the existence of price equilibrium in economies where commodities can be indivisible but excess demand functions can be discontinuous or do not have any other structural property beyond Walras’ law.

The issue of well-posedness plays a crucial role in the study of optimization theory. The classical concept of well-posedness dealing with unconstrained optimization problem was due to Tykhonov [31] in 1966. This concept is based on two aspects inspired by the numerical methods, one being the unique existence of minimizer and the other being the convergence of every minimizing sequence to the unique minimizer. Since then, Tykhonov well-posedness and its extensions have been among the very interesting and important topics which have attracted the attention of many mathematicians; see, for instance, [1, 2, 4, 9, 10, 15, 17]. It should be noted that the fundamental requirement in Tykhonov well-posedness is that every minimizing sequence must lie in the feasible region. In many practical situations, however, the minimizing sequences produced by numerical methods, such as augmented Lagrangian methods and exterior penalty methods, are not necessarily feasible to the problem. Taking this observation into account, Levitin and Polyak [25] proposed another notion of well-posedness which is known as Levitin–Polyak well-posedness. It is an extension of the Tykhonov well-posedness because it allows a minimizing sequence to be outside but gets closer and closer to the feasible set. There have been recently many works dealing with Levitin–Polyak well-posedness for problems related to optimization; see, for instance, [3, 5, 7, 13, 16, 19, 21, 22, 24, 32, 33] and the references therein.

From the above observations, in this paper, our aim is to investigate qualitative properties of solutions of excess demand equilibrium problems which are standard general equilibrium models with flexible prices and many traders. We first use the Glicksberg’s fixed point theorem to provide results for the existence of solutions of the reference problems. Then, we propose a concept of Levitin–Polyak well-posedness in the sense of Painlevé–Kuratowski and study its sufficient conditions. The layout of the paper is as follows. In Section 2, we recall preliminaries and describe the model of excess demand equilibrium problems. Sufficient conditions for the existence of such problems are studied in Section 3. In Section 4, based on Painlevé–Kuratowski convergence of solutions for perturbed excess demand equilibrium problems to the solutions of the original problem, we introduce the concept of Levitin–Polyak well-posedness for such problems. Also, sufficient conditions of this well-posedness for the reference problems are studied in this section.
2. Excess demand equilibrium problems. Let $\mathbb{R}$ be a real number set, $\mathbb{R}_+$ be the set of all nonnegative real numbers and $Q$ be a set-valued map from $\mathbb{R}^l$ into $\mathbb{R}^l$. Firstly, we recall notions and their properties used in what follows.

**Definition 2.1.** (See [18, Definition 2.3, page 38]) The map $Q$ is said to be upper semicontinuous (usc) at $x_0$ if for any neighborhood $U$ of $Q(x_0)$, there is a neighborhood $N$ of $x_0$ such that $Q(N) \subset U$.

**Lemma 2.2.** (See [18, Proposition 2.19, page 41]) If $Q(x_0)$ is compact, then $Q$ is usc at $x_0$ if and only if, for any sequence $\{x_n\}$ converging to $x_0$ and for any $y_n \in Q(x_n)$, there is a subsequence $\{y_{n_k}\}$ converging to some $y_0 \in Q(x_0)$.

We consider an economic equilibrium problem with $l$ commodities and $m$ economic agents dealing with such commodities. For $\mathcal{M} = \{1, \ldots, m\}$, we divide $\mathcal{M}$ into two subsets $\mathcal{M}_s$ and $\mathcal{M}_c$ which correspond to producers and consumers, respectively. Let $A := \{p \in \mathbb{R}^l_+ \mid 0 \leq p_i \leq \hat{p}_i\}$, where $\hat{p}_i$ is set by the government to prevent the market price from rising above a certain level, that is, it is a maximum price of the $i$-th product for each $i = 1, \ldots, l$. We denote the total supply of the $j$-th producer and the total demand of the $k$-th consumer with respect to the price vector $p \in A$ by $S_j(p)$ and $D_k(p)$, respectively.

We assume that each $j$-th producer maximizes profit subject to a production processes $K \subset \mathbb{R}^l_+$ with $K$ being a convex, closed and bounded set. Thus, for a price vector $p \in A$, the producer finds activities $y^* \in K$ such that, for all $y \in K$,

$$\langle p, y^* \rangle \geq \langle p, y \rangle.$$

We define $S_j(p)$ in the following

$$S_j(p) := \{y^* \in K \mid \langle p, y^* \rangle \geq \langle p, y \rangle \forall y \in K\}.$$

Then, $S_j(p)$ is the set of all production processes, which brings maximum profit for $j$-th producer. This shows that $S_j(p)$ is nonempty, convex, bounded and closed, and further that the map $S_j$ is usc. The market supply function $S$ is defined as the sum of the individual supply functions

$$S(p) := \sum_{j \in \mathcal{M}_s} S_j(p),$$

and hence it retains all of the properties of the individual supply functions.

Let $b_k$ be the individual budget of the $k$-th consumer, we can define his/her total demand correspondence as

$$D_k(p) := \{x \in K \mid \langle p, x \rangle \leq b_k\}.$$

It can be shown that for any $p \in A$, $D_k(p)$ is a nonempty, closed, convex and bounded set, and the map $D_k$ is upper semicontinuous. The market demand function $D$, defined as the sum of the individual demand functions, is given by

$$D(p) := \sum_{k \in \mathcal{M}_c} D_k(p).$$

Therefore, we can define an excess demand map $Z : \mathcal{A} \rightrightarrows \mathbb{R}^l$ as follows

$$Z(p) := D(p) - S(p).$$

The following result is derived from properties of the demand function $D$ and the supply function $S$. 
Lemma 2.3. The map $Z$ is upper semicontinuous and nonempty, compact, convex valued on $A$.

Now we consider the following excess demand equilibrium problem.

(EDEP) Find $p^* \in A$ such that

$$Z(p^*) \cap \mathbb{R}_+^l \neq \emptyset,$$

where $\mathbb{R}_+^l = -\mathbb{R}_+^l$. We denote the solution set of this problem by

$$\text{Sol}(Z, A) := \{ p \in A \mid Z(p) \cap \mathbb{R}_+^l \neq \emptyset \}.$$

Next, we will specify properties of the excess demand map $Z(A)$ (Walras's law) for all $p \in A$.

(A1) $Z$ is homogeneous of degree zero map on $A$, i.e., for every scalar $\lambda > 0$ and $p \in A$,

$$Z(\lambda p) = Z(p).$$

In the other word, the value of the excess demand function $Z(p)$ does not change when one multiplies each nonzero, nonnegative price vector by some positive number.

(A2) (Walras's law) For all $p \in A$ and $z \in Z(p)$, $\langle z, p \rangle = 0$. Noting that this condition is always assumed for general equilibrium models. The idea behind this assumption is that for every nonzero, nonnegative price vector, the value of all demands must be equal to that of all supplies.

3. Existence conditions. In this section, using a fixed point theorem introduced by Glicksberg [14], we study sufficient conditions for the existence of solutions to excess demand equilibrium problems.

Lemma 3.1. (Glicksberg’s fixed point theorem) Let $\mathcal{X}$ be a nonempty, compact, convex set in a Euclidean space and $\Phi : \mathcal{X} \rightrightarrows \mathcal{X}$ be upper semicontinuous and nonempty-closed-convex-valued. Then, $\Phi$ has a fixed point, that is, there exists an element $x \in \mathcal{X}$ such that $x \in \Phi(x)$.

Theorem 3.2. If Assumption (A2) is satisfied, then $\text{Sol}(Z, A)$ is nonempty and compact.

Proof. Let $Z$ be a compact and convex subset of $\mathbb{R}^l$ such that it contains all of the sets $Z(p)$ for all $p \in A$. Define a map $T : Z \rightrightarrows A$ as follows

$$T(z) := \{ \tilde{p} \in A \mid \langle z, \tilde{p} \rangle \geq \langle z, p \rangle \forall p \in A \}.$$

Because of the compactness of $A$, $T$ is nonempty valued. Now, we prove that $T$ is closed and convex valued on $Z$. Indeed, for any $\{p_n\} \subset T(z)$ converging to $p_0$, one has $p_0 \in A$ as $A$ is a closed set. Combining $p_n \in T(z)$ and the continuity of the linear function $q \mapsto \langle z, q \rangle$, we get $\langle z, p_0 \rangle \geq \langle z, p \rangle$ for all $p \in A$. So, $p_0 \in T(z)$, that is, the set $T(z)$ is closed. For each $z \in Z$, $p_1, p_2 \in T(z)$ and $\lambda \in [0, 1]$, we have

$$\langle z, \lambda p_1 + (1 - \lambda)p_2 \rangle = \lambda \langle z, p_1 \rangle + (1 - \lambda) \langle z, p_2 \rangle \geq \lambda \langle z, p \rangle + (1 - \lambda) \langle z, p \rangle = \langle z, p \rangle,$$

for all $p \in A$, and hence $T(z)$ is a convex set.

Next, let $\{z_n\} \subset Z$ be a sequence converging to $z_0$ and $u_n \in T(z_n)$. By the compactness of $A$, there is a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ such that $u_{n_k} \to u_0 \in A$. Because $u_{n_k} \in T(z_{n_k})$, we have $\langle z_{n_k}, u_{n_k} - u \rangle \geq 0$ for all $u \in A$. Using the continuity property of the related functions, we conclude that $\langle z_0, u_0 - u \rangle \geq 0$ for all $u \in A$. 


Consequently, \( u_0 \in T(z_0) \). As \( T \) is compact-valued, Lemma 2.2 implies that \( T \) is use on \( Z \).

In order to use Lemma 3.1 to prove the existence of an excess demand equilibrium problem, we note that \( A \times Z \) is closed, convex and bounded, and so the required conditions for Lemma 3.1 are satisfied. We consider a set-valued map \( F \) from \( A \times Z \) into itself defined by \( F(p, z) = T(z) \times Z(p) \). Clearly, \( F \) is upper semicontinuous and nonempty, closed, convex valued. By Lemma 3.1, there exists a vector \((p^*, z^*)\) satisfying \( p^* \in T(z^*) \) and \( z^* \in Z(p^*) \). Combining (A2) and \( z^* \in Z(p^*) \), we get \( \langle z^*, p^* \rangle = 0 \). This together with \( p^* \in T(z^*) \) implies that for all \( p \in A \),

\[
0 = \langle z^*, p^* \rangle \geq \langle z^*, p \rangle.
\]

For each \( k \in \{1, \cdots, l\} \), choosing a vector price \( \bar{p} \in A \) such that all its coordinates equal to zero except the \( k \)-th component, in view of the setting of \( A \), (1) implies \( z_k^* \leq 0 \). Since \( k \) is arbitrary, we conclude that \( p^* \) is a solution of the excess demand equilibrium problem.

Finally, we prove that \( \text{Sol}(Z, A) \) is compact. For any sequence \( \{p_n\} \subset \text{Sol}(Z, A) \) converging to \( p_0 \), then \( p_0 \in A \) and \( Z(p_n) \cap \mathbb{R}^+_{-} \neq \emptyset \). For each \( n \), clearly, there exists \( z_n \in Z(p_n) \cap \mathbb{R}^+_{-} \), and hence we can pick up a sequence \( \{z_n\} \) such that \( z_n \in Z(p_n) \cap \mathbb{R}^+_{-} \) for all \( n \). Combining this with the upper semicontinuity of \( Z \) and the closedness of \( \mathbb{R}^+_{-} \), by [18, Proposition 2.46, p. 53], we can find a subsequence \( \{z_{n_k}\} \) of \( \{z_n\} \) such that \( z_{n_k} \rightharpoonup z_0 \in Z(p_0) \cap \mathbb{R}^+_{-} \). So, \( p_0 \in \text{Sol}(Z, A) \), and hence \( \text{Sol}(Z, A) \) is compact.

**Remark 1.** Usually, to establish the existence of solutions to (EDEP), most of authors assume that the excess demand function holds the homogeneity property. Then, employing the auxiliary excess demand equilibrium problem with its constraint set defined by \( \{p \in A \mid \sum_{i=1}^{l} p_i = 1\} \), sufficient conditions of the existence of (EDEP) are considered, for more details we refer the readers to [6, 20, 30, 34] and the references therein. Herein, we apply Glicksberg’s fixed point theorem to investigate the solvability of (EDEP) without assuming the homogeneity property of excess demand functions. Therefore, the approach and obtained result of this paper are different from the existing ones.

4. **Levitin–Polyak well-posedness in the sense of Painlevé–Kuratowski.** In this section, we study the Levitin–Polyak well-posedness in the sense of Painlevé–Kuratowski for the excess demand equilibrium problems perturbed by convergent sequence of objective maps in the sense of Gamma convergence.

**Definition 4.1.** (See [23, page 359]) Let \( A \) be a nonempty subset of \( \mathbb{R}^l \), a sequence of nonempty subsets \( \{A_n\} \) of \( \mathbb{R}^l \) is called the upper Hausdorff convergence to \( A \) (denoted by \( A_n \xrightarrow{H} A \)) if \( e(A_n, A) \to 0 \), where \( e(A, B) := \sup_{a \in A} d(a, B) \) and \( d(a, B) := \inf_{b \in B} d(a, b) \) for all nonempty subsets \( A, B \) of \( \mathbb{R}^l \).

**Definition 4.2.** Let \( Z_n, Z : \mathbb{R}^l \rightrightarrows \mathbb{R}^l \) be set-valued maps. Then, the sequence \( \{Z_n\} \) is said to be converging to \( Z \) in the sense of the upper \( \Gamma \)-convergence (denoted by \( Z_n \xrightarrow{\Gamma} Z \)), if for every sequence \( \{p_n\} \) converging to \( p \), there exists a subsequence \( \{p_{n_k}\} \) of \( \{p_n\} \) such that \( Z_{n_k}(p_{n_k}) \xrightarrow{H} Z(p) \).

Let \( u \) be an element of \( \text{int} \mathbb{R}^+_{+} \), we propose the following notions.
Proof. By Theorem 3.2, we have \( \text{Sol}(\mathcal{A}) \) is generalized Levitin–Polyak well-posed in the sense of Painlevé–Kuratowski. 

Assume that Assumption (A2) is satisfied. Then, the problem 

Theorem 4.6. 

(a) every Levitin–Polyak approximating sequence in the sense of Painlevé–Kuratowski 

(b) every Levitin–Polyak approximating sequence in the sense of Painlevé–Kuratowski

The problem (EDEP) is said to be 

Definition 4.5. The problem (EDEP) is said to be Levitin–Polyak well-posed in the sense of Painlevé–Kuratowski for (EDEP) wrt \( v \) if and only if it is the corresponding one wrt \( v' \).

Proof. Let \( \{p_n\} \) be a LP approximating sequence in the sense of PK for (EDEP) wrt \( v \), then there exist a sequence \( \{\varepsilon_n\} \subset \mathbb{R}_+ \), \( \varepsilon_n \rightarrow 0 \), and a sequence of maps \( \{Z_n\} \), \( Z_n \overset{\Gamma}{\rightarrow} Z \), such that 

\[
\delta_n = \varepsilon_n \max_{i \in \{1, \ldots, l\}} \left\{ \frac{v_i}{v_i'} \right\},
\]

we get

\[
z_n - \delta_n v' = z_n - \varepsilon_n v - (\delta_n v' - \varepsilon_n v) = z_n - \varepsilon_n v - \varepsilon_n \left( \max_{i \in \{1, \ldots, l\}} \left\{ \frac{v_i}{v_i'} \right\} v' - v \right) \in \mathbb{R}_-.
\]

This together with \( d(p_n, A) \rightarrow 0 \) implies that \( \{p_n\} \) is a LP approximating sequence in the sense of PK for (EDEP) corresponding to \( v' \). The proof is complete. \( \square \)

Employing Lemma 4.4, we introduce a concept of Levitin–Polyak well-posedness in the sense of Painlevé–Kuratowski for (EDEP) as follows.

Definition 4.5. The problem (EDEP) is said to be Levitin–Polyak well-posed in the sense of Painlevé–Kuratowski iff

(a) it has solutions;

(b) every Levitin–Polyak approximating sequence in the sense of Painlevé–Kuratowski

has a subsequence converging to a solution.

Theorem 4.6. Assume that Assumption (A2) is satisfied. Then, the problem (EDEP) is generalized Levitin–Polyak well-posed in the sense of Painlevé–Kuratowski.

Proof. By Theorem 3.2, we have \( \text{Sol}(Z, \mathcal{A}) \neq \emptyset \). For a given \( v \in \text{int} \mathbb{R}_+ \), let \( \{p_n\} \) be a LP approximating sequence in the sense of PK for (EDEP) wrt \( v \). We show that there is a subsequence of \( \{p_n\} \) converging to some solution of (EDEP). For such sequence \( \{p_n\} \), there exist a sequence \( \{\varepsilon_n\} \subset \mathbb{R}_+ \), \( \varepsilon_n \rightarrow 0 \), and a sequence of maps \( \{Z_n\} \) with \( Z_n \overset{\Gamma}{\rightarrow} Z \) such that 

\[
d(p_n, A) \leq \varepsilon_n, \quad (2)
\]

and

\[
(Z_n(p_n) - \varepsilon_n v) \cap \mathbb{R}_+ \neq \emptyset. \quad (3)
\]

The compactness of \( \mathcal{A} \) implies that, for each \( n \in \mathbb{N} \), there is an element \( \hat{p}_n \in \mathcal{A} \), such that \( d(p_n, \hat{p}_n) = d(p_n, A) \). Using the compactness of \( \mathcal{A} \) again, we can assume,
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without loss of generality, that \( \{ \hat{p}_n \} \) converges to some vector \( p \) in \( A \). Hence, from (2), we have

\[
d(p_n, p) \leq d(p_n, \hat{p}_n) + d(\hat{p}_n, p) \leq d(p_n, A) + d(\hat{p}_n, p) \leq \varepsilon_n + d(\hat{p}_n, p).
\]

Since \( \hat{p}_n \to p \) as \( n \to \infty \), we conclude that \( \{ p_n \} \) also converges to \( p \). It follows from the upper \( \Gamma \)-convergence of \( Z \) that there exists a subsequence \( \{ p_{n_k} \} \) of \( \{ p_n \} \) satisfying \( Z_{n_k}(p_{n_k}) \xrightarrow{H} Z(p) \). Taking into account (3), there exist the corresponding subsequence \( \{ \varepsilon_{n_k} \} \) of \( \{ \varepsilon_n \} \), and \( z_{n_k} \in Z_{n_k}(p_{n_k}) \) such that \( z_{n_k} - \varepsilon_{n_k} v \in R_{-} \). By the compactness of \( Z(p) \), we can choose a sequence \( \{ \hat{z}_{n_k} \} \subset Z(p) \) such that \( d(z_{n_k}, Z(p)) = d(z_{n_k}, \hat{z}_{n_k}) \) and \( \hat{z}_{n_k} \to z \in Z(p) \). Then,

\[
d(z_{n_k}, z) \leq d(z_{n_k}, \hat{z}_{n_k}) + d(\hat{z}_{n_k}, z) \leq \varepsilon(\varepsilon_{n_k}(p_{n_k}), Z(p)) + d(\hat{z}_{n_k}, z).
\]

Combining this with \( Z_{n_k} \xrightarrow{r} Z \) and \( \hat{z}_{n_k} \to z \), we get \( z_{n_k} \to z \). Consequently, the sequence \( \{ z_{n_k} - \varepsilon_{n_k} v \} \) converges to the vector \( z \) belonging to \( Z(p) \cap R_{-} \) as \( R_{-} \) is closed. Hence, \( Z(p) \cap R_{-} \neq \emptyset \), and so \( p \in \text{Sol}(Z, A) \). Therefore, (EDEP) is generalized LP well-posed in the sense of PK. The proof is complete.

\( \square \)

Remark 2. Up to now, most of existing works in the literature, which deal with excess demand equilibrium problems, are mainly focused on establishing the existence of solutions and studying properties of the excess demand maps as mentioned in Introduction. To the best of our knowledge, there is no work with contributions to investigating well-posedness properties for excess demand equilibrium problems, and so the obtained results in this section are new.

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