A clarification on a common misconception about interferometric detectors of gravitational waves

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Abstract

The aims of this letter are two. First, to show the angular gauge-invariance on the response of interferometers to gravitational waves (GWs). In this process, after resuming for completeness results on the Transverse-Traceless (TT) gauge, where, in general, the theoretical computations on GWs are performed, we analyse the gauge of the local observer, which represents the gauge of a laboratory environment on Earth. The gauge-invariance between the two gauges is shown in its full angular and frequency dependences. In previous works in the literature this gauge-invariance was shown only in the low frequencies approximation or in the simplest geometry of the interferometer with respect to the propagating GW (i.e. both of the arms of the interferometer are perpendicular to the propagating GW).

Second, as far as the computation of the response functions in the gauge of the local observer is concerned, a common misconception about interferometers is also clarified. Such a misconception purports that, as the wavelength of laser light and the length of an interferometer’s arm are both stretched by a GW, no effect should be visible, invoking an analogy with cosmological redshift in an expanding universe.

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The scientific community aims in a first direct detection of GWs in next years (for the current status of GWs interferometers see [1]) confirming the indirect, Nobel Prize Winner, proof of Hulse and Taylor [2].

Detectors for GWs will be important for a better knowledge of the Universe and either to confirm or to rule out, in an ultimate way, the physical consistency of General Relativity, eventually becoming an observable endorsing of Extended Theories of Gravity, see [3] for details.

In the framework of General Relativity, computations on GWs are usually performed in the so-called TT gauge [3, 4]. This kind of gauge is historically called Transverse-Traceless, because in these particular coordinates GWs have a transverse effect, are traceless and the computations are, in general, simpler [4]. As interferometers work in a laboratory environment on Earth, the gauge in which the space-time is locally flat and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics has to be used [4]. In this gauge, called the gauge of the local observer [4, 5, 6], GWs manifest themselves by exerting tidal forces on the masses (the mirrors and the beam-splitter in the case of an interferometer, see Figure 1). At this point, when approaching the first direct detection of GWs, it is very important, whatever the frequency and the direction of propagation of the GW will be, to demonstrate that the signal in the TT gauge, which has been computed only in theoretical approaches in the literature, is equal to the one computed in the gauge of the local observer (which is the gauge where the detection will be observed on Earth).

In this letter, such a gauge-invariance on the response of interferometers to GWs between the two mentioned gauges is shown. In this process, after a resume of results on the TT gauge which have been obtained in [3], which is due for completeness, the response functions of interferometers are computed directly in the gauge of the local observer, obtaining the same result of the computation in the TT gauge. In this way, the gauge-invariance is shown in its full angular and frequency dependences. In previous works in the literature, this gauge-invariance was shown only in the low frequencies approximation (i.e. wavelength of the GW much large than the linear distance between test masses, see [7, 8] for example) or in the simplest geometry of the interferometer in respect to the propagating GW (i.e. both of the arms of the interferometer are perpendicular to the propagating GW) [9].

Of course, we do not have any idea concerning the direction of the propagating GW that will arrive to detectors, exactly like we do not have any idea concerning its frequency. Thus, a generalization which will take into account both of the full angular and frequency dependences is due. The presented results are consistent with previous approximations. As far as the computation of the response functions in the gauge of the local observer is concerned, a common misconception about interferometers is also clarified. Such a misconception purports that, as the wavelength of laser light and the length of an interferometer’s arm are both stretched by a GW, no effect should be visible, invoking an analogy with cosmological redshift in an expanding universe [17].

This issue has been raised in some papers in the literature [17, 20].
In [18] a qualitative intuitive explanation of the issue has been discussed. In [19] the issue has been addressed by recalling that this is the most common question asked about interferometric detectors of GWs. The author provided a qualitative answer [19]: “Does the wavelength of the light in the gravitational wave get stretched and squeezed the same manner as these mirrors move back and forth? ... The answer is no, the spacetime curvature influences the light in a different manner that it influences the mirror separations ... the influence on the light is negligible and it is only the mirrors that get moved back and forth and the light’s wavelength does not get changed at all ...”. In [20] the issue has been raised again, by invoking the analogy with the cosmological situation. An analogy between the gauge freedom of General Relativity and the Aharonov-Bohm effect in quantum mechanics has been discussed too [20]. In both situations gauge-dependent quantities appear in the equations describing the physics but the final physical results calculated are gauge-independent [20]. Again, the answer is that at the end the only physical quantity measured (the phase shift between two laser beams in an interferometric detector) is gauge independent. Finally, in [17], a direct calculation was performed, but with the assumption of wavelength of the GW much large than the linear distance between the beam splitter and the mirror of the interferometer (i.e. within the low-frequency approximation). Here the issue is ultimately clarified with a new direct calculation and without any approximation.

Following [3], we work with \( G = 1, c = 1 \) and \( \hbar = 1 \) and we call \( h_+(t + z) \) and \( h_{\times}(t + z) \) the weak perturbations due to the + and the \( \times \) polarizations which are expressed in terms of synchronous coordinates in the TT gauge. In this way, the most general GW propagating in the \( z \) direction can be written in terms of plane monochromatic waves [3, 4]

\[
h_{\mu\nu}(t + z) = h_+(t + z)\epsilon^{(+)}_{\mu\nu}(t + z) + h_{\times}(t + z)\epsilon^{(\times)}_{\mu\nu}(t + z) = h_{+0} \exp i\omega(t + z)\epsilon^{(+)}_{\mu\nu}(t + z) + h_{\times0} \exp i\omega(t + z)\epsilon^{(\times)}_{\mu\nu}(t + z),
\]

and the correspondent line element will be

\[
ds^2 = dt^2 - dz^2 - (1 + h_+)dx^2 - (1 - h_+)dy^2 - 2h_{\times}dxdx.
\]

The wordlines \( x, y, z = \text{const.} \) are timelike geodesics, representing the histories of free test masses [3, 4], that, in our case, are the beam-splitter and the mirrors of an interferometer, see Figure 1 and ref. [6].

In order to obtain the response functions in the TT gauge, a generalization of the analysis in [9] has been used in [3], the so called “bouncing photon method”: a photon can be launched from the beam-splitter to be bounced back by the mirror (Figure 1). This method has been generalized to scalar waves, angular dependences and massive modes of GWs in [3, 6, 10]. This includes the more general problem of finding the null geodesics of light in the presence of a weak GW [11, 12, 13, 14].

Defining [3]
Figure 1: photons can be launched from the beam-splitter to be bounced back by the mirror, adapted from ref. [6]

\[ \tilde{H}_u(\omega, \theta, \phi) \equiv \frac{-1+\exp(2i\omega L)}{2i\omega(1+\sin^2 \theta \cos^2 \phi)} + \frac{-\sin \theta \cos \phi ((1+\exp(2i\omega L)-2 \exp i\omega(1-\sin \theta \cos \phi)))}{2i\omega(1+\sin^2 \cos^2 \phi)} \] (3)

and

\[ \tilde{H}_v(\omega, \theta, \phi) \equiv \frac{-1+\exp(2i\omega L)}{2i\omega(1+\sin^2 \theta \sin^2 \phi)} + \frac{-\sin \theta \sin \phi ((1+\exp(2i\omega L)-2 \exp i\omega(1-\sin \theta \sin \phi)))}{2i\omega(1+\sin^2 \theta \sin^2 \phi)} \] (4)

the total frequency and angular dependent response function (i.e. the detector pattern) of an interferometer to the + polarization of the GW has been obtained in [3] in the TT gauge:

\[ \tilde{H}^+(\omega) \equiv \Upsilon_u^+(\omega) - \Upsilon_v^+(\omega) = \frac{(\cos^2 \theta \cos^2 \phi - \sin^2 \phi)(\frac{-1+\exp(2i\omega L)}{2i\omega(1+\sin^2 \theta \cos^2 \phi)} \tilde{H}_u(\omega, \theta, \phi) + \frac{(\cos^2 \theta \sin^2 \phi - \cos^2 \phi)(\frac{-1+\exp(2i\omega L)}{2i\omega(1+\sin^2 \theta \sin^2 \phi)} \tilde{H}_v(\omega, \theta, \phi)}{2i\omega(1+\sin^2 \theta \sin^2 \phi)} \right. \] (5)

that, in the low frequencies limit (\(\omega \to 0\)) gives the well known low frequency response function of [15] for the + polarization:

\[ \tilde{H}^+(\omega \to 0) = \frac{1}{2}(1 + \cos^2 \theta \cos 2\phi) . \] (6)

The same analysis works for the × polarization, see [3] for details. In that case, one obtains that the total frequency and angular dependent response function of an interferometer to the × polarization is:
that, in the low frequencies limit \((\omega \to 0)\), gives the low frequency response function of \([15, 16]\) for the \(\times\) polarization:

\[
\tilde{H}^\times(\omega \to 0) = -\cos \theta \sin 2\phi.
\]

The importance of these results is due to the fact that in this case the limit where the wavelength is shorter than the length between the splitter mirror and test masses is calculated. The signal drops off the regime, while the calculation agrees with previous calculations for longer wavelengths \([15, 16]\). The contribution is important especially in the high-frequency portion of the sensitivity band.

Now, let us see the computation in the gauge of the local observer.

A detailed analysis of the gauge of the local observer is given in Ref. \([4]\), Sect. 13.6. Here, we only recall that the effect of GWs on test masses is described by the equation for geodesic deviation in this gauge

\[
\ddot{x}^i = -\tilde{R}^i_{0k0} x^k, \tag{9}
\]

where \(\tilde{R}^i_{0k0}\) are the components of the linearized Riemann tensor \([4]\).

In the computation of the response functions in this gauge, a common misconception about interferometers will be also clarified. Once again, let us recall this issue. The famous misconception purports that, as the wavelength of the laser light and the length of an interferometer’s arm are both stretched by a GW, no effect should be present, invoking an analogy with the cosmological redshift of the expanding Universe. This misconception has been recently clarified in a good way in \([17]\), but only in the low frequency approximation. Here the misconception will be clarified in the full angular and frequency dependences of a GW, showing that the variation of proper time due to the photon’s redshift is different from the variation of proper time due to the motion of the arms.

We start with the \(+\) polarization. In the gauge of the local observer the equation of motion for the test masses are \([4, 9]\)

\[
\ddot{x} = \frac{1}{2} \dot{h}_+ x, \quad \ddot{y} = -\frac{1}{2} \dot{h}_+ y, \quad \ddot{z} = 0, \tag{10-12}
\]

which can be solved using the perturbation method \([4]\), obtaining

\[
x(t) = l_1 + \frac{1}{2}[l_1 h_+(t) - l_2 h_\times(t)] \\
y(t) = l_2 - \frac{1}{2}[l_2 h_+(t) + l_1 h_\times(t)] \\
z(t) = l_3, \tag{13}
\]
where $l_1$, $l_2$ and $l_3$ are the coordinates of the mirror of the interferometer in absence of GWs, with the beam-splitter located in the origin of the coordinate system. But the arms of the interferometer are in the $\vec{u}$ and $\vec{v}$ directions, while the $x,y,z$ frame is the proper frame of the propagating GW. Then, a spatial rotation of the coordinate system has to be performed:

\[
\begin{align*}
u &= -x \cos \theta \cos \phi + y \sin \phi + z \sin \theta \cos \phi \\
v &= -x \cos \theta \sin \phi - y \cos \phi + z \sin \theta \sin \phi \\
w &= x \sin \theta + z \cos \theta,
\end{align*}
\]

Figure 2: a GW propagating from an arbitrary direction, adapted from ref. [10]

In this way, the GW is propagating from an arbitrary direction $\vec{r}$ to the interferometer (see Figure 2 and ref. [10]).

As a result, the $u$ coordinate of the mirror is

\[
u = L + \frac{1}{2}L A h_+ (t + u \sin \theta \cos \phi),
\]

where

\[
A \equiv \cos^2 \theta \cos^2 \phi - \sin^2 \phi,
\]

and $L = \sqrt{l_1^2 + l_2^2 + l_3^2}$ is the length of the interferometer arms.

We consider a photon which propagates in the $u$ axis. The unperturbed (i.e. in absence of GWs) propagation time between the two masses is

\[
T = L.
\]

From eq. (15), the displacements of the two masses under the influence of the GW are

\[
ds u_\phi(t) = 0
\]
and

\[ \delta u_m(t) = \frac{1}{2} L A h_+ (t + L \sin \theta \cos \phi). \]  \hspace{1cm} (19)

In this way, the relative displacement, which is defined by

\[ \delta L(t) = \delta u_m(t) - \delta u_b(t), \]  \hspace{1cm} (20)

gives

\[ \frac{\delta T(t)}{T} = \frac{\delta L(t)}{L} = \frac{1}{2} L A h_+ (t + L \sin \theta \cos \phi). \]  \hspace{1cm} (21)

But, for a large separation between the test masses (in the case of Virgo the distance between the beam-splitter and the mirror is three kms, four in the case of LIGO), the definition (20) for relative displacements becomes unphysical because the two test masses are taken at the same time and therefore cannot be in a casual connection \[6, 9, 10\]. The correct definitions for the bouncing photon are

\[ \delta L_1(t) = \delta u_m(t) - \delta u_b(t - T_1) \]  \hspace{1cm} (22)

and

\[ \delta L_2(t) = \delta u_m(t - T_2) - \delta u_b(t), \]  \hspace{1cm} (23)

where \( T_1 \) and \( T_2 \) are the photon propagation times for the forward and return trip correspondingly. According to the new definitions, the displacement of one test mass is compared to the displacement of the other at a later time, in order to allow a finite delay for the light propagation. The propagation times \( T_1 \) and \( T_2 \) in eqs. (22) and (23) can be replaced with the nominal value \( T \) because the test mass displacements are already first order in the GW's field \[6, 9, 10\]. Thus, the total change in the distance between the beam splitter and the mirror, in one round-trip of the photon, is

\[ \delta L_{r.t.}(t) = \delta L_1(t - T) + \delta L_2(t) = 2 \delta u_m(t - T) - \delta u_b(t) - \delta u_b(t - 2T), \]  \hspace{1cm} (24)

and, in terms of the amplitude of the + polarization of the GW:

\[ \delta L_{r.t.}(t) = L A h_+ (t + L \sin \theta \cos \phi - L). \]  \hspace{1cm} (25)

The change in distance (24) leads to changes in the round-trip time for photons propagating between the beam-splitter and the mirror:

\[ \frac{\delta T(t)}{T} = \frac{\delta L}{L} = \frac{1}{2} L A h_+ (t + L \sin \theta \cos \phi + L). \]  \hspace{1cm} (26)

In the last calculation (variations in the photon round-trip time which come from the motion of the test masses induced by the GW), it has been implicitly
assumed that the propagation of the photon between the beam-splitter and the mirror of the interferometer is uniform as if it were moving in a flat space-time. But the presence of the tidal forces indicates that the space-time is curved \([4, 6, 9, 10]\). As a result, one more effect after the first discussed has to be considered, and it requires spacial separation (note: in \([9]\) the effects considered were three, but the third effect vanishes putting the beam splitter in the origin of the coordinate system \([6]\)). This is exactly the contribution of the photons redshift. If it results different from the contribution of the test masses motion in previous analysis (i.e. the sum of the two contributions is different from zero), it also clarifies the misconception purporting that, because the wavelength of the laser light and the length of an interferometer’s arm are both stretched by a GW, no effect should be present.

From equations (10), (11) and (14) the tidal acceleration of a test mass caused by the GW in the \(u\) direction is

\[
\ddot{u}(t + u \sin \theta \cos \phi) = \frac{1}{2} LA \ddot{h}_+(t + u \sin \theta \cos \phi),
\]

(27)

Equivalently, one can say that there is a gravitational potential \([4, 6, 9]\):

\[
V(u, t) = -\frac{1}{2}LA \int_0^u \ddot{h}_+(t + l \sin \theta \cos \phi)dl,
\]

(28)

which generates the tidal forces, and that the motion of the test mass is governed by the Newtonian equation

\[
\ddot{\vec{r}} = -\nabla V.
\]

(29)

In the framework of weak-field gravity, the interval in the gauge of the local observer is given by \([4, 5]\)

\[
ds^2 = g_{00}dt^2 + du^2 + dv^2 + dw^2.
\]

(30)

Equations like eq. (27) work for the \(v\) and the \(w\) directions too. Thus, photon momentum in these directions is not conserved and photons launched in the \(u\) axis will deflect out of this axis. But here this effect can be neglected, because the photon deflections into the \(v\) and \(w\) directions will be at most of order \(h_+\) \([4, 6, 9]\). Then, to first order in \(h_+\), the \(dv^2\) and \(dw^2\) terms can be neglected. Thus, the line element (30), for photons propagating along the \(u\)-axis, can be rewritten as

\[
ds^2 = g_{00}dt^2 + du^2.
\]

(31)

The condition for a null trajectory \((ds = 0)\) and the well known relation between Newtonian theory and linearized gravity \((g_{00} = 1 + 2V\) \([4, 5]\)) give the coordinate velocity of the photons

\[
v_c^2 \equiv \left(\frac{du}{dt}\right)^2 = 1 + 2V(t, u),
\]

(32)
which, to first order in $h_+$, is approximated by

$$v_c \approx \pm [1 + V(t, u)], \quad (33)$$

with + and − for the forward and return trip respectively. Knowing the coordinate velocity of the photon, the propagation time for its travelling between the beam-splitter and the mirror can be defined:

$$T_1(t) = \int_{u_b(t-T_1)}^{u_m(t)} \frac{du}{v_c} \quad (34)$$

and

$$T_2(t) = \int_{u_m(t-T_2)}^{u_b(t)} \frac{(-du)}{v_c}. \quad (35)$$

The calculations of these integrals would be complicated because the $u_m$ boundaries of them are changing with time:

$$u_b(t) = 0 \quad (36)$$

and

$$u_m(t) = L + \delta u_m(t). \quad (37)$$

But, to first order in $h_+$, these contributions can be approximated by $\delta L_1(t)$ and $\delta L_2(t)$ (see eqs. 22 and 23). Thus, the combined effect of the varying boundaries is given by $\delta \tau T(t)$ in eq. 26. Then, only the times for photon propagation between the fixed boundaries 0 and $L$ have to be computed. Such a propagation times will be indicated with $\Delta T_{1,2}$ to distinguish from $T_{1,2}$. In the forward trip, the propagation time between the fixed limits is

$$\Delta T_1(t) = \int_0^L \frac{du}{v_c(t', u)} \approx L - \int_0^L V(t', u) du, \quad (38)$$

where $t'$ represents the delay time which corresponds to the unperturbed photon trajectory (i.e. $t$ is the time at which the photon arrives in the position $L$, so $L - u = t - t'$):

$$t' = t - (L - u).$$

Similarly, the propagation time in the return trip is

$$\Delta T_2(t) = L - \int_L^0 V(t', u) du, \quad (39)$$

where now the delay time is given by

$$t' = t - u.$$
The sum of $\Delta T_1(t-T)$ and $\Delta T_2(t)$ gives the round-trip time for photons travelling between the fixed boundaries. Then, the deviation of this round-trip time (distance) from its unperturbed value $2T$ is

$$\delta_2 T(t) = - \int_0^L [V(t - 2L + u, u) du +$$

$$- \int_0^L V(t - u, u) du],$$

and, using eq. (28),

$$\delta_2 T(t) = \frac{1}{2} LA \int_0^L \left[ \int_0^u \dot{\tilde{h}}_+(t - 2T + l(1 + \sin \theta \cos \phi)) dl +$$

$$- \int_0^u \dot{\tilde{h}}_+(t - l(1 - \sin \theta \cos \phi)) dl \right] du.$$ (40)

Thus, the total round-trip proper time in presence of the GW is:

$$T_t = 2T + \delta_1 T + \delta_2 T,$$ (42)

and

$$\delta T_u = T_t - 2T = \delta_1 T + \delta_2 T$$ (43)

is the total variation of the proper time for the round-trip of the photon in presence of the GW in the $u$ direction.

Using eqs. (26), (41) and the Fourier transform of $h_+$, defined by

$$\tilde{h}_+(\omega) = \int_{-\infty}^{\infty} dt \ h(t) \exp(i\omega t),$$ (44)

the quantity (43) can be computed in the frequency domain as

$$\tilde{\delta} T_u(\omega) = \tilde{\delta}_1 T(\omega) + \tilde{\delta}_2 T(\omega)$$ (45)

where

$$\tilde{\delta}_1 T(\omega) = \exp[i\omega L(1 - \sin \theta \cos \phi)] LA \tilde{h}_+(\omega)$$ (46)

$$\tilde{\delta}_2 T(\omega) = - \frac{LA}{2} \left[ -1 + \exp[i\omega L(1 - \sin \theta \cos \phi)] - iL\omega(1 - \sin \theta \cos \phi) + \right.$$

$$\left. \frac{1}{(1 - \sin \theta \cos \phi)^2} \right]$$

$$+ \frac{\exp(2i\omega L)(1 - \exp[i\omega L(1 - \sin \theta \cos \phi)] + iL\omega(1 - \sin \theta \cos \phi))}{(1 - \sin \theta \cos \phi)^2} \tilde{h}_+(\omega).$$ (47)

In the above computation, derivative and translation Fourier transform theorems have been used.

Then, using eqs. (26), (46), (47) and the definition $\tilde{\theta}$ a signal can be defined:

$$\frac{\tilde{\delta} T_u(\omega)}{T} = \tilde{\Theta}_u(\omega) \tilde{h}_+(\omega),$$ (48)

where
\[ \Upsilon_u^+(\omega) = \frac{(\cos^2 \theta \cos^2 \phi - \sin^2 \phi)}{2L} \tilde{H}_u(\omega, \theta, \phi) \]  

is the response function of the u arm of the interferometer to the GW.

Note: the fact that this response function is, in general, different from zero implies that the contribution to the total signal, due to the motion of the test masses, will be, in general, different from the contribution due to the gravitational redshift of the GW. In this way the misconception on interferometers is clarified in the full angular and frequency dependence of GWs.

The same analysis works the v arm. One gets the total response function in the v direction for the GWs, which is

\[ \Upsilon_v^+(\omega) = \frac{(\cos^2 \theta \sin^2 \phi - \cos^2 \phi)}{2L} \tilde{H}_v(\omega, \theta, \phi). \]

Thus, in the gauge of the local observer, the total frequency-dependent response function (i.e. the detector pattern) of an interferometer to the + polarization of the GW is given by:

\[ \tilde{H}^+(\omega) \equiv \Upsilon_u^+(\omega) - \Upsilon_v^+(\omega) = \]

\[ = \frac{(\cos^2 \theta \cos^2 \phi - \sin^2 \phi)}{2L} \tilde{H}_u(\omega, \theta, \phi) + \]

\[ - \frac{(\cos^2 \theta \sin^2 \phi - \cos^2 \phi)}{2L} \tilde{H}_v(\omega, \theta, \phi), \]

which is the same result of the TT gauge (eq. (5)). This gauge-invariance agrees with lots of results which are well known in the literature, where the analysis has been made in the low frequency approximation, i.e. in the case in which the wavelength of the GW is much larger than the length of the interferometer’s arms (see [16] for example). Note that the gauge-invariance obtained with the equality between equation and equation is more general than the one in [9], where the computation was performed in the simplest geometry of the interferometer in respect to the propagating gravitational wave, i.e. in the case in which the arms of the interferometer are perpendicular to the propagating GW, an only for one arm. Putting \( \theta = \phi = 0 \) and \( v = 0 \) in equations and , the result of [9] for one arm of an interferometer is recovered. Once again, we recall that we do not have any idea concerning the direction of the propagating GW that will arrive to detectors, exactly like we do not have any idea concerning its frequency. Thus, our generalization which will take into account both of the full angular and frequency in the gauge of the local observer dependences is important.

A similar analysis works for the × polarization. One obtains the same result of eq. in the TT gauge.

Then, the total response functions of interferometers for the + and × polarization of GWs, in their full angular and frequency dependences, are equal in the TT gauge and in the gauge of a local observer. In this way, the gauge-invariance has been totally generalized.
Conclusions

In this letter, the gauge-invariance on the response of interferometers to GWs has been shown. In this process, after resuming, for completeness, results on the Transverse-Traceless (TT) gauge, where, in general, the theoretical computations on GWs are performed, the gauge of the local observer has been analysed. The gauge-invariance between the two gauges has been shown in its full angular and frequency dependences while in previous works in the literature this gauge-invariance was shown only in the low frequencies approximation or in the simplest geometry of the interferometer in respect to the propagating gravitational wave. We remark that one has not any idea concerning the direction of the propagating GW that will arrive to detectors, exactly like we do not know its frequency. Thus, the present results, which analyse both of the full angular and frequency dependences in the gauge of the local observer, are important for a sake of completeness.

As far as the computation of the response functions in the gauge of the local observer has been concerned, a common misconception about interferometers has been also clarified. Such a misconception purports that, as the wavelength of laser light and the length of an interferometer’s arm are both stretched by a GW, no effect should be visible, invoking an analogy with cosmological redshift in an expanding universe. This issue has been raised in some papers in the literature [17]-[20] and has been ultimately clarified with a new direct calculation and without any approximation in this letter.

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