Temperature and Magnetic Field Dependencies in a Model of Diamagnetic Hysteresis in Beryllium

Nathan Logoboy$^{1,2}$ and Walter Joss$^{1,3}$

$^1$Grenoble High Magnetic Field Laboratory, MPI-FKF and CNRS P.O. 166X, F-38042 Grenoble Cedex 9, France
$^2$The Racah Institute of Physics, The Hebrew University of Jerusalem, 91904 Jerusalem, Israel
$^3$Université Joseph Fourier, B.P. 53, F-38041 Grenoble Cedex 9, France

(Dated: March 23, 2022)

The model of diamagnetic hysteresis loop of strongly correlated electron gas at the conditions of dHvA effect is developed. It is shown, that in the framework of Rayleigh theory for magnetization loop, the coercive force and remnant magnetization in every period of dHvA oscillations are characterized by strong dependencies on temperature, magnetic field and Dingle temperature.

PACS numbers: 75.20.En, 75.60.Ch, 71.10.Ca, 71.70.Di, 71.25.-s; 71.25. Hc; 75.40-s; 75.40.Cx.

I. INTRODUCTION

At high magnetic field and low temperature the strongly correlated electron gas is characterized by instability due to magnetic interaction between electrons $^1$, which results in phase transition from homogeneous state into inhomogeneous one and stratiﬁcation of the sample into diamagnetic domains, e.g. Condon domains (CDs), with alternating from one domain to another diamagnetic moment. This instability of an electron gas is called diamagnetic phase transition and extensively studied, both theoretically and experimentally $^2$–$^6$. The diamagnetic domains were observed in Be by magnetization measurements $^2$; in Ag by nuclear magnetic resonance (NMR) $^5$ and recently by Hall probe spectroscopy $^6$. They have been observed in Be, Sn, Al and Pb by muon spin-rotation ($\mu$SR) spectroscopy $^8$. Usually, for plate-like samples, this non-uniform phase is realized as periodic domain structure $^1$–$^4$ with alternating from one domain to another diamagnetic moment. Magnetic interaction of strongly correlated electrons gas results in nonlinear relationship between local diamagnetic moments and external magnetic field. It was shown $^7$ that in CD phase the local diamagnetic moments, which contribute to the process of magnetization, are characterized by strong temperature and magnetic field dependencies.

The existence of metastable states for local diamagnetic moments in each period of dHvA oscillations can result in hysteresis, e.g. irreversible changes in magnetization due to change in external magnetic field, similarly to magnetic materials of spin origin. The microscopic mechanism of magnetization loop, relates to defects of crystal lattice, intrinsic stress and elastic deformations. The impurities, dislocations and other defects of crystal lattice cause an internal friction which opposes to the wall displacement. The domain walls are pinned by such defects. Unpinning is realized by small dissipative jumps through the increasing of the value of applied magnetic field.

Although, the possibilities of irreversible behavior of magnetization curve in diamagnetic materials have been discussed earlier $^1$, only recent experimental investigation of CDs in Be $^8$ reveals hysteresis loop in the magnetization process. The hysteresis magnetization curve of Be $^8$ was measured directly by Hall probes in dc field and standard ac method with different modulation levels, frequencies and magnetic field ramp rates, and the shape of magnetization curve was reconstructed by several higher harmonics of the ac pickup voltage. Detection of the hysteresis in Be $^8$ substantiates a far reaching analogy between an electron gas, which has unstable, metastable and stable states within one dHvA period, and the liquid-gas phase coexistence and transformation. Recent progress $^4$, $^8$ in experiment on observation of Condon domain structure in Ag and Be, in particular, the detection of diamagnetic hysteresis, provides a natural stimulus towards a more detailed understanding of the properties of strongly correlated electron gas in the conditions of dHvA effect. CDs in a $3d$ electron gas are the only type of magnetic structures, for which the magnetization process has not been considered so far.

In present paper we develop the model of diamagnetic hysteresis, which fits the recent data on observation of diamagnetic hysteresis in Be $^8$. The hysteresis loop appears periodically in every period of dHvA oscillations, when $\mu_0 \chi > 1$ ($\chi$ is differential magnetic susceptibility, $M$ is the oscillatory part of the magnetization, and $B$ is magnetic induction $^1$). There are three main contributions into the process of magnetization of normal metals in the condition of strong dHvA effect which have to be distinguished, the dependence of local diamagnetic moments on external magnetic field through the reduced amplitude of dHvA oscillations $^7$, the domain wall displacement and nucleation of the domains. The rotation processes, that are intrinsic property of magnetization loop in magnetic materials of spin origin, can be neglected in rearrangement of CD structure at least for metals with relatively simple Fermi surfaces $^1$ due to one-dimensional character of the diamagnetic moments. We show that the remnant magnetization and coerci-

*Electronic address: logoboy@phys.huji.ac.il
ity, as well as the hysteresis loop square, which are the fundamental properties of the magnetization curve, have strong dependence on temperature, magnetic field and Dingle temperature.

The paper is organized as follows. Section II is an introduction. In Section III we consider the model of diamagnetic hysteresis, based on Rayleigh model in the framework of Lifshitz-Kosevich-Shoenberg approximation. In Section IV we calculate the temperature and magnetic field dependence of remnant magnetization and coercivity. Section V is conclusions. The last section contains acknowledgements.

II. MODEL

The oscillator part of the thermodynamic potential density in the Lifshitz-Kosevich-Shoenberg formalism can be written in reduced form with taking into account the shape sample effects:

$$\Omega = \frac{1}{4\pi k^2} \left[ a \cos b + \frac{1}{2} a^2 (1 - n) \sin^2 b \right],$$

where $b = k(B - \mu_0 H) = k[\mu_0 h_{ex} + (1 - n) M] = x + (1 - n) y$, $\mu_0 H$ is the magnetic field inside the material at the center of dHvA period, $k = 2\pi F/(\mu_0 H)^2$, $F$ is the fundamental oscillation frequency, $H_{ex}$ is external magnetic field, $h_{ex} = H_{ex} - H$ is the small increment to the magnetic field $H$, $n$ is the demagnetization factor. All the components of vectors are taken along the direction of the magnetic induction. The quality $a = 4\pi k A = \mu_0 \partial M/\partial B$ is the reduced amplitude of the dHvA oscillations (the ratio between the amplitude of oscillations $A$ and their period $\Delta H$).

In the first harmonic approximation the magnetization is found from the implicit equation of state:

$$y = a \sin [x + (1 - n) y],$$

The validity of thermodynamic potential density $\Omega$, and the expression for the reduced magnetization $y$, derived from Eq. 1, is restricted by applicability to homogeneous phase only, where the existence of demagnetization coefficient $n$ is justified. In the conditions of strong magnetic interaction between electrons, when

$$a(\mu_0 H, T, T_D) \geq 1,$$

a state of lower thermodynamic potential is achieved over part of dHvA oscillation cycle by the formation of CD structures, for which the local value of magnetization alternates in sign from one domain to the next, resulting in essential reduce of magneto-static energy. The usually observed diamagnetic domain structure is of stripe-domain type, and the contribution of magneto-static energy into the free energy density is negligibly small in comparing to the magneto-static energy of homogeneous state. In the domain state, when the reduced amplitude of dHvA oscillations satisfies to the condition $\beta$, the demagnetization factor $n$, which is characteristic of the uniformly magnetized sample, is replaced by the coefficient $\alpha$. This coefficient depends on magnetic field, temperature, impurities and geometry of the sample and takes into account the magneto-static energy for given type of domain structure. Typically for plate-like sample, $\alpha \ll 1$, independently on the types of domain structures and important mainly in calculation of the steady-state period $2D$ of the domain structure by minimization of the free energy density, containing two terms, magneto-static energy and the surface energy of the domain walls. For observed CDs structures $\beta$, the inequality $D/L << 1$, where $L$ is the width of the sample, is fulfilled and $\alpha \sim D/L << 1$.

The equation

$$a(\mu_0 H, T, T_D) = 1,$$

defines the critical surface in three dimensions $\mu_0 H - T - T_D$. Above this surface the uniform diamagnetic phase exists, but below it, the CD phase can appear in the part of every period of the dHvA oscillations.

For proper calculation of the reduced amplitude of the dHvA oscillations a and constructing the hysteresis loop, the correct topology of Fermi surface has to be taken into account. In case of Be the standard procedure of calculation of the amplitude of dHvA oscillations, based on series expansions near the extreme cross-sections, requires considerable modification due to negligible small curvature of extreme cross-sections of the 'cigar'-like part of the Fermi surface, contributed to the dHvA oscillations. The Fermi surface of Be consists of the second zone monster ('coronet') and the third zone 'cigar'. It is well-established that the dHvA oscillations originates from the three maximum cross-sections of 'cigar' ('waist' and 'hips') which are characterized by a very small curvature. In particular, the small curvature of the cylinder like Fermi surface explains relatively high amplitude of dHvA oscillations in Be at magnetic field $H$ applied parallel to the hexagonal axis, e.g. $H || [0001]$, in comparing to Ag, where the small amplitude of dHvA oscillations is explained in the framework of spherical Fermi surface. In the framework of the model of slightly corrugated cylinder like Fermi surface of Be, the reduced amplitude of the dHvA oscillations takes a form

$$a = \frac{4\kappa(j_{0})^2}{\pi^3 m_c(\mu_0 H)^2}|Q(\mu_0 H)|R(\mu_0 H, T, T_D),$$

where $R(\mu_0 H, T, T_D)$ is a reduction factor, which takes into account the influence of the temperature $T$, impurities and on the amplitude of the dHvA oscillations.

The complex function

$$Q = Q_1 + jQ_2 = |Q| \exp (j\psi),$$

$$Q_1 = 3\beta \cos \lambda + 2(1 - 2\beta)j_0(\lambda), \quad Q_2 = \beta \sin \lambda$$

defines the critical surface in three dimensions $\mu_0 H - T - T_D$. Above this surface the uniform diamagnetic phase exists, but below it, the CD phase can appear in the part of every period of the dHvA oscillations.

For proper calculation of the reduced amplitude of the dHvA oscillations a and constructing the hysteresis loop, the correct topology of Fermi surface has to be taken into account. In case of Be the standard procedure of calculation of the amplitude of dHvA oscillations, based on series expansions near the extreme cross-sections, requires considerable modification due to negligible small curvature of extreme cross-sections of the 'cigar'-like part of the Fermi surface, contributed to the dHvA oscillations. The Fermi surface of Be consists of the second zone monster ('coronet') and the third zone 'cigar'. It is well-established that the dHvA oscillations originates from the three maximum cross-sections of 'cigar' ('waist' and 'hips') which are characterized by a very small curvature. In particular, the small curvature of the cylinder like Fermi surface explains relatively high amplitude of dHvA oscillations in Be at magnetic field $H$ applied parallel to the hexagonal axis, e.g. $H || [0001]$, in comparing to Ag, where the small amplitude of dHvA oscillations is explained in the framework of spherical Fermi surface. In the framework of the model of slightly corrugated cylinder like Fermi surface of Be, the reduced amplitude of the dHvA oscillations takes a form

$$a = \frac{4\kappa(j_{0})^2}{\pi^3 m_c(\mu_0 H)^2}|Q(\mu_0 H)|R(\mu_0 H, T, T_D),$$

where $R(\mu_0 H, T, T_D)$ is a reduction factor, which takes into account the influence of the temperature $T$, impurities and on the amplitude of the dHvA oscillations.

The complex function

$$Q = Q_1 + jQ_2 = |Q| \exp (j\psi),$$

$$Q_1 = 3\beta \cos \lambda + 2(1 - 2\beta)j_0(\lambda), \quad Q_2 = \beta \sin \lambda$$
where $\lambda = l^2 A_0 \xi$, depends on applied magnetic field through parameter $l^2 = \chi H/e \mu_H$ and describes the beatings in the amplitude of dHvA oscillations resulting from two different extreme cross-section of Fermi surface of Be. Here, $\kappa = 0.217 A^0 -1$ is the distance in reciprocal space between two extreme cross sections of 'cigar', $A_0$ is average cross section area, $J_0(\lambda)$ is Bessel function of the first order and $\beta$ is the half-width of the cylinder like extreme cross sections.

The Eqs. (4), (5) define in explicit form the critical temperature for diamagnetic phase transition as a function of magnetic field $\mu_0 H$ and Dingle temperature $T_D$. It allows us to construct the phase diagrams for Be and investigate the properties of the diamagnetic hysteresis.

### III. RESULTS AND DISCUSSIONS

At the condition $a(\mu_0 H, T, T_D) > 1$ the sample is divided into domains with up and down magnetization. In the center of the dHvA period $h = 0$ the domains are characterized by the same width. An external magnetic field removes the equivalence of the states, the energy balance is altered and rearrangements of the domain structure take place. For CD structure this process is realized not only through the motion of domain walls, which results in corresponding changes of domain volumes, and nucleation of the domains, but also through magnetic field dependence of the local magnetization in every adjacent domains. To understand the magnetization process in CD phase it is convenient to define the approximate anhysteretic, or pinning-free, magnetization

$$y^{(anhy)} = \frac{1}{2}(1 - \frac{x}{\Delta})y_+ + \frac{1}{2}(1 + \frac{x}{\Delta})y_-, \quad (7)$$

where the function $y_{\pm} = y_{\pm}(x) = -y_{\mp}(-x)$ is the solution of the equation $y_{\pm} = a \sin(x+y_{\pm})$ at the range $|x| \leq \Delta \leq k \cdot \Delta H/2$ in every period $\Delta H$ of the dHvA oscillations and describes magnetic field dependence of the local diamagnetic moments of the up (+) and down (−) domains (Fig. 1). In Eq. (7) we assumed that the volume fractions of the up and down domains are linear functions of the increment of the reduced magnetic field in every dHvA period. This assumption was justified by NMR experiments in Ag (2) and $\mu$SR spectroscopy in Be (5), (6). Both experiments reveal the domain structure by the appearance of a doublet corresponding to two domain magnetic inductions with line intensities, that show the linear dependence on the external magnetic field of the volume fractions occupied by the neighboring domains.

The range of existence of the CDs $2\Delta$ is defined by (7)

$$\Delta = \sqrt{a^2 - 1} \cos^{-1} \frac{1}{a}, \quad a \geq 0, \quad a \geq 1. \quad (8)$$

It follows from Eq. (7), that the anhysteretic magnetization in every period of the dHvA oscillations is a function of magnetic field $H$, temperature $T$ and Dingle temperature $T_D$. The expression for $y^{(anhy)}$ Eq. (7) can be simplified for relatively high reduced amplitude of the dHvA oscillations, e.g. $a \gg 1$. In this case $y_+ \approx y_0 - (x/\Delta)y_0$, $y_- \approx y_+ - 2y_0$, where $y_0$ is the half-splitting in the center of the period, defined explicitly form by the equation $y_0 = a(1,0)$, and the Eq. (7) is replaced by the following one

$$y^{(anhy)}_{\pm} \approx y_0(\frac{1}{\Delta} - \frac{1}{\pi})x, \quad |x| \leq \Delta \leq \pi. \quad (9)$$

Although, the local diamagnetic moments $y_{\pm} = y_{\pm}(x_{ex})$ are characterized by average negative slope as the functions of magnetic field increment $x_{ex}$ (2), it follows from Eq. (3), that the anhysteretic curve has a positive slope in every period of the dHvA oscillations (Fig. 1). We will see that the diamagnetic hysteresis is also characterized by the positive slope: in the process of the diamagnetic magnetization the volume fraction of the up-domains increases on expense of the volume fraction of the down-domains with increase of magnetic field increment $x_{ex}$ (1).

The real diamagnetic magnetization process in CD phase is characterized by hysteresis, or magnetization loop, which appears periodically in applied magnetic field in every period of dHvA oscillations, when the condition (3) is fulfilled. According to experimental data (4) the diamagnetic hysteresis is usually small, related to the domain wall displacement and domain nucleation. Thus, it can be described in the framework of the Lord Rayleigh model with two parameters: initial susceptibility $\chi$ and...
Rayleigh constant $\eta$ (see, e.g. [11]):

$$y_{a,d} = \pm y_s + \chi(x + \Delta) \pm \frac{1}{2} \eta(x + \Delta)^2,$$

(10)

where the upper (lower) sign corresponds to the ascending (descending) magnetization in increasing (decreasing) magnetic field (Fig. 1). The three fundamental parameters of the hysteresis loop: the remnant magnetization $y_r = y_r(a)$, the saturation magnetization $y_s$ and the coercivity $x_c = x_a$, which is the reverse field to bring the magnetization to zero from initial saturation, are characterized by strong dependence on the applied magnetic field, temperature and Dingle temperature through the reduced amplitude of the dHvA oscillations $a = a(\mu_0 H, T, T_D)$. The saturation magnetization $y_s = y_s$ can be calculated numerically from the explicit equation

$$y_s = a \sin [\Delta + (1 - n)y_s].$$

(11)

The Eq. (10) gives the following expressions for the coercivity factor $x_c$ and remnant magnetization $y_r$:

$$x_c = \frac{\delta \Delta^2}{1 - \delta \Delta + \sqrt{(1 - \delta \Delta)^2 + (\delta \Delta)^2}} \approx \delta \Delta^2,$$

(12)

$$y_r = \frac{1}{2} \eta \Delta^2,$$

(13)

where $\delta = \eta / \chi$. In calculation of Eqs. (12), (13) we used $\delta \Delta \ll 1$, which is justified by the observed narrow hysteresis loop in Be [8]. In this limit, coercivity factor $x_c$ and remnant magnetization $y_r$, being quadratic functions of the range of CD existence $\Delta$ [9], have the same temperature and magnetic field dependencies. The initial susceptibility $\chi$ can be calculated from measured remnant magnetization $y_r$ and coercivity factor $x_c$ according to the equation $\chi = 2y_r/x_c$. The hysteresis loop square which characterizes the losses in the process of diamagnetic magnetization, is defined as follows

$$S = \frac{4}{3} \eta \Delta^3.$$

(14)

and also temperature and magnetic-field dependent. The loop square $S$ Eq. (14) reaches its maximum $S_{\text{max}} = (4/3)\pi^2 \eta$ at $\alpha = 1$, inasmuch as the range of the existence of the CD 2$\Delta$ in every period of the dHvA oscillations is restricted by the period of the oscillations (2$\pi$ in relative units). Analysis of the Eqs. (12)-(14) with taking into consideration Eq. (3) shows that the parameters of the hysteresis loop $x_c, y_r, S \rightarrow 0$ at the point of diamagnetic phase transition, when $a \rightarrow 1$.

The magnetic field dependence of the coercivity $x_c = x_c(\mu_0 H)$ [12] at the Dingle temperature $T_D = 2K$ of the experiment [8] and three different temperatures $T$ from 0.5$K$ to 1.5$K$ is represented in Fig. 2. To construct the plots we used $\delta = 0.1$, which is justified by comparison with the data [13]. The function $x_c = x_c(\mu_0 H)$ is plotted at the whole range of applied magnetic field $H$ from 1$T$ till 6$T$ (Fig. 2(a)), where the non-uniform diamagnetic phase exists at given Dingle temperature $T_D = 2K$ [13]. The function $x_c = x_c(\mu_0 H)$ has asymmetric multi-bell like shape, typical for the phase diagrams of Be $\mu_0 H - T - T_D$ [12, 13]. The maximums of the function $x_c = x_c(\mu_0 H)$ appear periodically on the scale of inverse magnetic field with the period inversely proportional to the discrepancy $\Delta F = F_h - F_w$ of the two fundamental frequencies, $F_h$ and $F_w$, corresponding to two extreme cross sections of cigar-like Fermi surface of Be:

$$\Delta \left( \frac{1}{\mu_0 H} \right) = \frac{1}{\Delta F}.$$

(15)

For $F_h = 970.9T$ and $F_w = 942.2T$ [1, 8] we obtain $\Delta F = 28.7T$. The increase of the temperature $T$ results in decrease of the maximums of the function $x_c = x_c(\mu_0 H)$ till its disappearing, when the point of phase transition is reached ($a = 1$). The position of the maximums of the function $x_c = x_c(\mu_0 H)$ does not depend on temperature in accordance with the phase diagrams for Be [12]. The periodicity of the coercivity on reciprocal magnetic field is illustrated in Fig. 3(a), where the coercive force $X_c = x_c/k = x_c(\mu_0 H)$ measured in Gauss.
is plotted as a function of $(\mu_0H)^{-1}$ at $T = 1.3K$ and $T_D = 1.8K$. It follows from Fig. 3(a), that the preferable interval of applied magnetic field for measuring the coercive force is $2−4T$, where the function $X_c$ can reach maximum values from $1G$ to $2G$ at given values of $T = 1.3K$ and $T_D = 1.8K$.

The increase of the Dingle temperature $T_D = h/2\pi k_B \tau$ which is inversely proportional to the scattering lifetime $\tau$ of the conduction electrons, leads to the reduction of the amplitude of the dHvA oscillations, resulting in the phase transition into uniform diamagnetic phase. Thus, the range of the existence of the CD phase $\Delta$ Eq. (8) decreases ($\Delta \rightarrow 0$, when $a \rightarrow 1$). Assuming that this contribution to the coercive force $x_c$ Eq. (12) is dominant and neglecting an unessential change of the parameter $\delta = \chi/\eta$, one can come to the hells-like magnetic field dependencies of $x_c$, as illustrated in Fig. 3.

The temperature dependencies of the reduced coercivity factor $x_c = x_c(T)$ and measured coercive force $X_c = X_c(T)$ at $\mu_0H = 3.6T$ and different Dingle temperatures are shown in Fig. 4. The circles in Fig. 4(b) represent the results of measurement of the coercive force in Be. The theoretical curve $X_c = X_c(T)$ plotted at $\mu_0H = 3.6T$ and $T_D \approx 1.8K$ fits the experimental points. The reported Dingle temperature in [13] is $T_D = 2K$. There is a good agreement between predicted temperature dependence of the coercive force and data [13].

The observation of the hysteresis loop in Be provides the possibility of the verification of the developed theory of the diamagnetic hysteresis and evaluate parameters of the loop. The periodical irreversible behavior of the magnetization process in the rod-like sample $8 \times 2 \times 1mm^3$ of Be is demonstrated at $T = 1.3K$ in the whole range of applied magnetic field $\mu_0H \approx 2 − 6T$, where the CD phase exists. The Dingle temperature is $T_D = 2K$. The diamagnetic phase diagrams estimated from Eq. (11) with taking into account the Eq. (12) for the reduced amplitude of oscillations $a$, obtained in the framework of the slightly corrugated Fermi surface of Be [12], are consistent with the data [8]. At $\mu_0H = 3.6T$ the calculated period of the dHvA oscillations $\Delta H = 2\pi/k = (\mu_0H)^2/F_0 \approx 138G$ ($F_0 = 955T$ is average area of the extreme cross-sections of the Fermi surface sheet, relevant to dHvA oscillations $\delta$) is very close to the observed period of the dHvA oscillations $\Delta H \approx 134 − 138G$ [8]. According to the data [8], the coercive force $X_c \approx 1.2G$, the remnant magnetization $Y_r = y_r/k \approx 1.2G$, the saturation magnetization $Y_s$ and the range of existence of the non-uniform phase $2\Delta$ can be evaluated as $Y_s \approx 30G$ and $2\Delta \approx 34G$ consequently. Thus, the experimental value of the reduced parameter $\Delta^{(exp)} = X_\Delta k \approx 0.8$ can be compared with the corresponding theoretical value of $\Delta$ Eq. (8) in the following way. At $T = 1.3K, T_D = 2K$ and $\mu_0H = 3.6T$, the reduced amplitude of the dHvA oscillations according to the Eq. (11) is $a \approx 2$, giving the value of $\Delta \approx 0.7$ in accordance with the experimental value $\Delta^{(exp)} = X_\Delta k \approx 0.8$ [8]. According to this calculation, at the conditions of the experiment [8], CD phase occupies $0.25$ part of the period. The data [8] allow us to evaluate the parameters of the Rayleigh hysteresis loop, $\chi$ and $\eta$. First, we calculate $y_s = kY_s \approx 0.935$, that gives $\eta = 2y_s/\Delta^2 \approx 0.18$, $\chi = 2y_s/x_c \approx 2$ and $\delta = 0.09 \approx 0.1$. Then, we can compare the experimental and theoretical values for saturation magnetization. According to the data [8] $y_s^{(exp)} = kY_s \approx 1.4$. The theoretical value of the saturation magnetization $y_s$ can be calculated from Eq. (11). Using the value of the demagnetizing coefficient $n \approx 0.05$, calculated for prolate ellipsoid [14] $a \approx 2$ and $\Delta \approx 0.7$ one can obtain $y_s \approx 1.6$ in accordance with the experimental value of $y_s^{exp} = 1.4$.

IV. CONCLUSIONS

Magnetic interaction of strongly correlated electron gas results in non-linear relationship between magnetization and magnetic field, which can give rise an effect of hysteresis in magnetization curve due to existence of metastable states for local diamagnetic moments in each period of dHvA oscillations. There are three main contributions into the process of magnetization of normal
metals in the condition of strong dHvA effect: the domain wall displacement, the nucleation of domains and the dependence of local diamagnetic moments on external magnetic field.

We developed the model of diamagnetic hysteresis in CD phase in the framework of Rayleigh hysteresis loop. Rayleigh law for the magnetization process of spin origin is generally believed to be caused only by the wall displacement and in relatively small magnetic field. There are two circumstances that can justify the applicability of the Rayleigh model for magnetization process of strongly correlated electron gas. The first, although for observation of the dHvA effect the high magnetic field in the scale of $1-10T$ is need, the interplay between uniform and non-uniform phases realized at the fine magnetic field scale of the magnetic field increment $|h| \leq \Delta H = (\mu_0 H)^2 / F$, e.g. $\sim mT$, in every period of the dHvA oscillations. And, the second, the magnetization process for electron gas in the conditions of dHvA effect is dominated by domain wall motion which is similar to the magnetization process of soft thin magnetic films in the physics of magnetism of spin origin. There is a good agreement between the theoretical results obtained in the framework of the developed model of diamagnetic hysteresis and recent experimental results on observation of the hysteresis in Be. The diamagnetic hysteresis is usually small, appears periodically in every period of dHvA oscillations with changing the value of applied magnetic field, and is described in the framework of the Rayleigh model with two fundamental parameters: initial susceptibility $\chi$ and Rayleigh constant $\eta$. We calculate the parameters of the diamagnetic hysteresis from the data. The zero field saturation (remnant) magnetization $y_c$, coercivity $x_c$ and hysteresis loop square $S$ are characterized by strong dependence on temperature, magnetic field and impurity of the sample through the reduced amplitude of the dHvA oscillations $a(\mu_0 H, T, T_D)$. The irreversible changes in magnetization are the result of existence of metastable states due to defects of crystal lattice, intrinsic stress and deformation. Generally, the diamagnetic hysteresis occurs because the domain boundaries do not return to their original positions when the external magnetic field approaches to its value corresponding to the centre of period of dHvA oscillations. At the initial permeability range the domain walls are displaced reversibly from their stable positions which essentially determined by the homogeneity of materials. Beyond the initial permeability range, the intensity of magnetization increases and the process is irreversible. The irreversible magnetization range is attained by irreversible displacement of domain walls. Although the rotation processes, that are intrinsic property of magnetization loop in magnetic materials of spin origin, can be neglected in rearrangement of CD structure, the microscopic theory of diamagnetic loop has to include into consideration the possible bending of the domain wall due to magnetization current densities localized in the domain wall close to the sample surface. The irreversibility of the magnetization process is believed to be the intrinsic property of the CD structure for all normal metals. The present model of diamagnetic hysteresis can be applied for investigation of temperature and magnetic field dependencies of diamagnetic magnetization not only in Be, but in other systems, where the CDs were observed. We hope that our theoretical results will motivate the experimentalists to carry out further experiments in investigation such an exotic phenomenon as diamagnetic hysteresis.

Acknowledgments

We are indebted to V. Egorov and I. Sheikin for illuminating discussions. We are also grateful to R. Kramer for providing us with his experimental results before publication.
[1] D. Shoenberg, *Magnetic Oscillations in Metals* (Cambridge University Press, Cambridge, England, 1984).

[2] J. H. Condon, Phys. Rev. 45, 526 (1966).

[3] J. H. Condon and R. E. Walstedt, Phys. Rev. Lett. 21, 612 (1968).

[4] R. G. Kramer, V. S. Egorov, V. A. Gasparov, A. G. M. Jansen, and W. Joss, Phys. Rev. Lett. 95, 267209 (2005).

[5] G. Solt and V. S. Egorov, Physica B 318, 231 (2002).

[6] G. Solt, V. S. Egorov, C. Baines, D. Herlach, and U. Zimmermann, Physica B 326, 536 (2003).

[7] N. Logoboy, A. Gordon, I. D. Vagner, and W. Joss, Solid State Comm. 134, 497 (2005).

[8] R. B. G. Kramer, V. S. Egorov, A. G. M. Jansen, and W. Joss, Phys. Rev. Lett. 95, 187204 (2005).

[9] N. Logoboy, V. S. Egorov, and W. Joss, Solid State Comm. 137, 570 (2006).

[10] C. Kittel, Rev. Mod. Phys. 21, 41 (1949).

[11] S. Chikazumi, *Physics of Ferromagnetism* (Clarendon Press, Oxford, England, 1997).

[12] N. Logoboy and W. Joss, Solid State Comm. 139, 191 (2006).

[13] R. B. G. Kramer, (2006) (*private communication*).

[14] J. A. Osborn, Phys. Rev. 67, 351 (1945).