The idea of a collective intelligence behind the complex natural structures built by organisms suggests that the organization of social networks is selected so as to optimize problem-solving competence at the group-level. Here we study the influence of the social network topology on the performance of a group of agents whose task is to locate the global maxima of NK fitness landscapes. Agents cooperate by broadcasting messages informing on their fitness and use this information to imitate the fittest agent in their influence networks. In the case those messages convey accurate information on the proximity of the solution (i.e., for smooth fitness landscapes) we find that high connectivity as well as centralization boost the group performance. For rugged landscapes, however, these characteristics are beneficial for small groups only. For large groups, it is advantageous to slow down the information transmission through the network to avoid local maximum traps. Long-range links and modularity have marginal effects on the performance of the group, except for a very narrow region of the model parameters.

I. INTRODUCTION

The benefits of cooperative work have long been explored by nature as revealed by the collective structures built by termites and slime molds. These structures may be thought of as the organisms’ solutions to the problems that endanger their existence [1, 2]. Within this (admittedly controversial) perspective, competence in problem solving should be viewed as a selection pressure on the size and organization of groups of gregarious animals, in addition to the more traditional pressures such as defense against predation and foraging success [3]. In fact, given the ubiquity of network optimization in biology [4], one would expect that the size and the topology of social networks are also selected for optimization of species adaptation and surveillance [5, 6]. Social networks are considered optimal if they improve communication and decision making at the group level, while keeping a minimum number of (costly) connections between the individuals [6].

The information in social networks flows between individuals via social contacts and, in the problem-solving context, the relevant process is imitative learning or, more generally, social learning [7], as expressed in this quote by Bloom “Imitative learning acts like a synapse, allowing information to leap the gap from one creature to another” [1]. In fact, social learning has inspired the planning of a variety of optimization heuristics, such as the particle swarm optimization algorithm [8] and the adaptive culture heuristic [9, 10]. Although there has been some progress on the understanding of the factors that make cooperative group work effective [11–13], a workable minimal model to study group problem-solving via imitative learning was proposed only very recently [14, 15]. Our goal here is to investigate how the social network properties are related to efficiency at the group level within that minimal model framework. By efficiency we mean that the time to solve a problem must scale superlinearly with the number of individuals or resources employed in the task.

The agent-based model we consider consists of a group of $L$ agents that can perform individual trial-and-test searches to explore a fitness landscape and, most importantly, can imitate a model agent – the best performing agent in their influence network at the trial – with a certain probability $p$. Thus the model exhibits the two critical ingredients of a collective brain: imitative learning and a dynamic hierarchy among the agents [1]. It is the exchange of messages between agents informing each other on their partial success (i.e., their fitness at the current trial) towards the completion of the task that characterizes the model as a distributed cooperative problem-solving system [11].

The task of the agents is to find the global maxima of smooth and rugged fitness landscapes created by the NK model and the performance or efficiency of the group is measured by the number of trials required to find those maxima. We assume that the links between agents are costless. Differently from a previous study that considered only fully connected networks (i.e., each agent influences and is influenced by all the other agents in the group) [10], here we focus on the structure of the social network that connects the agents within the group. In particular, we consider the effect of varying the connectivity (or coordination number) of one-dimensional lattices as well as of varying the structure of the network while keeping the average connectivity of the network fixed.

We find that in both smooth and rugged landscapes the best group organization comprises an optimal number $L^*$ of fully connected agents. For smooth landscapes, decrease of the connectivity and hence of the speed of
The solution space of the NK landscape consists of the distinct binary strings of length $N$. This parameter determines the size of the solution space, $2^N$. The other parameter $K = 0, \ldots, N-1$ influences the number of local maxima on the landscape. In particular, for $K = 0$ the corresponding (smooth) landscape has one single maximum. For $K = N-1$, the (uncorrelated) landscape has on the average $2^N / (N+1)$ maxima with respect to single bit flips [19].

The rest of the paper is organized as follows. In Section II we present a brief description of the NK model of rugged fitness landscapes. In Section III we describe in some detail the imitative learning search strategy and introduce a convenient measure of the performance of the search for the global maximum on NK landscapes. In Section IV we study the performance of the imitative search in the case the imitation network is modeled by a regular one-dimensional lattice with coordination number $M$ and in Section V in the case it is modeled by more complex network topologies, namely, scale-free, small-world and community networks. Finally, Section VI is reserved to our concluding remarks.

II. NK MODEL OF RUGGED FITNESS LANDSCAPES

The NK model was introduced by Kauffman [18] to model the adaptive evolution process as walks on rugged fitness landscapes. The main advantage of the NK model of rugged fitness landscapes is the possibility of tuning the ruggedness of the landscape by changing the two integer parameters of the model, namely, $N$ and $K$. The NK landscape is defined in the space of binary strings of length $N$. This parameter determines the size of the solution space, $2^N$. The other parameter $K = 0, \ldots, N-1$ influences the number of local maxima on the landscape. In particular, for $K = 0$ the corresponding (smooth) landscape has one single maximum. For $K = N-1$, the (uncorrelated) landscape has on the average $2^N / (N+1)$ maxima with respect to single bit flips [19].

The solution space of the NK landscape consists of the $2^N$ distinct binary strings of length $N$, which we denote by $x = (x_1, x_2, \ldots, x_N)$ with $x_i = 0, 1$. To each string $x$ we associate a fitness value $\Phi(x)$ which is an average of the contributions from each component $i$ in the string, i.e.,

$$\Phi(x) = \frac{1}{N} \sum_{i=1}^{N} \phi_i(x),$$

where $\phi_i$ is the contribution of component $i$ to the fitness of string $x$. It is assumed that $\phi_i$ depends on the state $x_i$ as well as on the states of the $K$ right neighbors of $i$, i.e.,

$$\phi_i = \phi_i(x_i, x_{i+1}, \ldots, x_{i+K})$$

with the arithmetic in the subscripts done modulo $N$. We notice that $\Phi$ measures the degree of interaction (epistasis) among the components of the strings. It is assumed, in addition, that the functions $\phi_i$ are $N$ distinct real-valued functions on $\{0,1\}^{K+1}$. As usual, we assign to each $\phi_i$ a uniformly distributed random number in the unit interval $[0,1]$. Hence $\Phi \in (0,1)$ has a unique global maximum. Figure 1 illustrates the calculation of $\Phi(x)$.

For $K = 0$ there are no local maxima and the sole maximum of $\Phi$ is easily located by picking for each component $i$ the state $x_i = 0$ if $\phi_i(0) > \phi_i(1)$ or the state $x_i = 1$, otherwise. However, for $K > 0$ finding the global maximum of the NK model is a NP-complete problem [20], which means that the time required to solve the problem using any currently known deterministic algorithm increases exponentially fast with the length $N$ of the strings [21]. The increase of the parameter $K$ from 0 to $N-1$ decreases the correlation between the fitness of neighboring configurations (i.e., configurations that differ at a single component) in the solution space. For $K = N-1$, those fitness values are uncorrelated so the NK model reduces to the Random Energy model [22, 23]. Hence $\Phi \in (0,1)$ has a unique global maximum. Figure 1 illustrates the calculation of $\Phi(x)$.

Since the functions $\phi_i$ are random, the ruggedness measures (e.g., the number of local maxima) of a particular realization of a NK landscape are not fixed by the parameters $N$ and $K$. In fact, those measures can vary considerably between landscapes characterized by the same values of $N$ and $K > 0$ [18], which implies that the per-
formance of any search heuristic based on the local correlations of the fitness landscape depends on the particular realization of the landscape. Hence in order to highlight the role of the parameters that are relevant to imitative learning – the group size $L$, the network topology and average connectivity $M$, and the imitation probability $p$ – here we compare the performance of groups characterized by different sets of those parameters for the same realization of the NK fitness landscape. In particular, we consider two types of landscape: a smooth landscape with $N = 12$ and $K = 0$ and a rugged landscape with $N = 12$ and $K = 4$. The effects of averaging over different landscape realizations is addressed briefly in Section V.

III. THE MODEL

We consider a group composed of $L$ agents. Each agent operates in an initial binary string drawn at random with equal probability for the bits 0 and 1. Henceforth we use here the terms agent and string interchangeably. At any trial $t$, a target agent can choose between two distinct processes to operate on the strings. The first process, which happens with probability $1 - p$, is the elementary move in the solution space that consists of picking a bit $i = 1, \ldots, N$ at random with equal probability and then flipping it. Through the repeated application of this operation, the agent can produce all the $2^N$ binary strings starting from any arbitrary string. The second process, which happens with probability $p_i$, is the imitation of a model string. We choose the model string as the highest fitness string at trial $t$ among the (fixed) subgroup of agents that can influence the target agent. The string to be updated (target string) is compared with the model string and the different bits are singled out. Then the agent selects at random one of the distinct bits and flips it so that this bit is now the same in both the target and the model strings. Hence, as a result of the imitation process the target string becomes more similar to the model string. Figure 2 illustrates the steps involved in the model.

The parameter $p \in [0, 1]$ is the imitation probability. The case $p = 0$ corresponds to the baseline situation in which the $L$ agents explore the solution space independently. The imitation procedure described above is motivated by the mechanism used to simulate the influence of an external media in the celebrated agent-based model proposed by Axelrod to study the process of culture dissemination. We notice that in the case of the independent search it can be shown that the mean rescaled computational cost is given by

$$C = \frac{L}{2^N \left[ 1 - \left( \frac{\lambda_N}{L} \right)^L \right]},$$

(2)

where $\lambda_N$ is the second largest eigenvalue of a tridiagonal stochastic matrix $T$ whose elements are $T_{i,j} = (1 - j/N) \delta_{i,j+1} + j/N \delta_{i,j-1}$ for $j = 1, \ldots, N - 1$, $T_{i,0} = \delta_{i,1}$, and $T_{i,N} = \delta_{i,N}$, where $\delta_{i,j}$ is the Kronecker delta. The notation $\langle \ldots \rangle$ stands for the average over independent searches on the same landscape. The results of the simulations exhibited in the next sections were obtained by averaging over $10^5$ searches. Since the expected number of trials for a group of $L$ independent agents to find the global maximum is $\langle t^* \rangle L$, where $\langle t^* \rangle \approx 4545$. Since $\langle \lambda_{12} \rangle^L \approx e^{-L(1-\lambda_{12})}$ we have $\langle C \rangle \approx \langle t^* \rangle / 2^{12} \approx 1.110$ for $L \ll \langle t^* \rangle$ and $\langle C \rangle \approx L / 2^{12}$ for $L \gg \langle t^* \rangle$.

FIG. 2. Illustration of the update rules of the model. At any step, a target agent is selected and its binary string is changed either by flipping one randomly chosen bit or by copying a bit from the highest fitness (model) string. The former process occurs with probability $1 - p_i$, whereas the latter with probability $p_i$. Notice that when the imitation operation is selected, the updated string becomes closer to the model string.
the risk of producing disconnected clusters. one can vary the connectivity of the lattice. This configuration is important because \( L \) at the represented by several copies when \( K \) landscape (\( \text{ring} \)) with coordination number \( M = 2(\bigcirc), M = 10(\blacktriangledown) \), and \( M = 50(\bigcirc) \). The results for the fully connected lattice \( M = L - 1(\bullet) \) as well as for the independent search (+) are also shown. The solid curve is Eq. (2), the dashed line is the linear function \( \langle C \rangle = L/2^{12} \) and the dotted line is the fitting \( \langle C \rangle = 0.76 \left( L/2^{12} \right)^{1/2} \) in the range \( L \in [10^2, 10^4] \). The parameters of the NK landscape are \( N = 12 \) and \( K = 0 \). The landscape exhibits a single maximum.

### IV. REGULAR LATTICES

Here we consider the case that the agents are fixed at the \( L \) sites of a one-dimensional lattice with periodic boundary conditions (i.e., a ring) and can interact with their \( M/2 \) left neighbors as well as with their \( M/2 \) right neighbors. Hence \( M \leq L - 1 \) is the coordination number of the lattice. This configuration is important because one can vary the connectivity \( M \) of the lattice without the risk of producing disconnected clusters.

Figure 3 shows the performance of a group of \( L \) agents at the task of finding the global optimum of a smooth landscape (\( K = 0 \)) in the case the copying and the elementary move operations are equally probable, i.e., \( p = 0.5 \). We find that, regardless of the number of agents \( L \), the best performance is achieved by the fully connected lattice \( M = L - 1 \) and the performance degrades smoothly as the connectivity decreases. This can be explained by the absence of local maxima in the landscape for \( K = 0 \). Since all sequences display faithful information about the location of the global maximum, the higher the information flow between them, the better the performance of the group. In addition, we find that regardless of the connectivity the computational cost decreases as the imitation probability increases. We recall that for \( p \approx 1 \) only the model string, which is represented by several copies when \( L \) is large, is likely to perform the elementary move; all other strings imitate the model. This reduces greatly the effective group size since the strings are concentrated in the vicinity of the model string, which cannot accommodate more than \( L = N = 12 \) strings without duplication of work. This is the reason we observe the degradation of the performance when the group size increases beyond the optimal value \( L \approx N \). Notice that for \( K = 0 \) the imitative learning search always performs better than the independent search.

At this stage it is prudent to point out that a constant computational cost implies that the time \( t^* \) necessary to find the global maximum decreases linearly with the number of agents. On the other hand, a computational cost that grows linearly with the group size means that adding more agents to the group does not affect \( t^* \). For group sizes in the range \( L \in [10^2, 10^3] \), Fig. 3 reveals a sublinear decrease of \( t^* \), since \( \langle C \rangle \) increases with \( L^{1/2} \) and, therefore, \( t^* \) decreases with \( L^{-1/2} \). According to our criterion for efficiency, as a collaborative strategy the imitative search is efficient only in the range \( L < 10 \) where we find the superlinear scaling \( t^* \propto L^{-2} \).

Figure 4 shows the performance for a rugged landscape (\( K = 4 \)) having 52 local maxima and a single global maximum. Because of the presence of local maxima that may trap the sequences in their neighborhoods, the computational cost exhibits a much more complex dependence on the model parameters than in the previous case of a single-maximum landscape. This figure reveals many instructive facts about the imitative search strategy. In particular, this strategy may be disastrous, in the sense that it performs much worse than the independent search, for large groups with high connectivity.
The parameters of the NK landscape are and L

increasing the connectivity of the lattice are startling as increasing are fully connected. In particular, L as well as on details of the landscape, where the agents information quickly through the group. For groups of intermediate size (say, L = 100) the detrimental effects of increasing the connectivity of the lattice are startling as illustrated in Fig. 5. The same conclusion holds true in the case the number of agents is greater than the size of the solution space (L > 2^12) since it is very likely that some string will be close to the global maximum. It is thus advantageous to disseminate this information quickly through the group. For groups of intermediate size (say, L = 100) the detrimental effects of increasing the connectivity of the lattice are startling as illustrated in Fig. 5.

The main conclusion of this analysis is that the optimal setting for the imitative search is a small group of maximum and it is thus advantageous to disseminate this information quickly through the group. For groups of intermediate size (say, L = 100) the detrimental effects of increasing the connectivity of the lattice are startling as illustrated in Fig. 5.

The main conclusion of this analysis is that the optimal setting for the imitative search is a small group of size L*, which depends on the imitation probability p as well as on details of the landscape, where the agents are fully connected. In particular, L* decreases with increasing p as illustrated in Fig. 6 for p > 0.98 we find L* = 2 whereas for p → 0 we find L* ≈ 2^N. Although the decrease of the connectivity between agents can boost significantly the performance of the group in the regime L* < L ≪ 2^N, the leading factor for optimality is the group size, with the lattice connectivity playing a coadjuvant role (see Fig. 5).

V. COMPLEX NETWORKS

The previous section focused on the influence of varying the connectivity of the regular network on the problem-solving performance of the group. Here we consider networks with the same average connectivity but very different organizations, as reflected by the degree distribution – the degree k_i of a node i is given by its number of connections. In particular, we consider scale-free networks generated by the Barabási-Albert algorithm [28], a star topology and a regular chain with nearest-neighbors links. In the three topologies there are L nodes and L − 1 links. In addition, to examine the influence of long-range interactions we consider small-world networks [30] as well as a network exhibiting community structure [31].

A. Scale-free networks

A scale-free network is a network whose degree distribution follows a power law when the number of nodes L is very large. For our purposes, the interesting feature of this topology, besides being a good approximation to social and biological networks [28], is that it introduces a distinction between agents, since the nodes of the network can exhibit very different degrees. Here we use the Barabási-Albert algorithm for generating random scale-free networks which exhibit the degree distribution \( P(k) \sim k^{-\gamma} \) in the asymptotic limit \( L \rightarrow \infty \) [28]. This algorithm is based on a preferential attachment mechanism and network growth. Thus, at each time step, a new node i with m_0 connections is added to the network. The probability that a node j, which is present in the network, will receive a connection from i is proportional to \( k_j / \sum k_l \), i.e., \( P(i \rightarrow j) = k_j / \sum k_l \). In the case we start with a single node and m_0 = 1, the resulting network will exhibit L − 1 connections at the Lth step. In addition, if m_0 = 1 then the network will
imitate the central agent only. Actually, that piece of information can even be lost if the fittest agent is chosen as the target agent before it is imitated by the central one.

In the presence of local maxima \((K = 4)\), we find a similar pattern, as shown in Fig. 8 but the failure of the star topology already for \(L > 30\) is probably due to the spreading of inaccurate information about the global optimum by the central agent. In fact, once the central agent reaches a local maxima it quickly attracts the rest of the group towards it.

We may interpret the central agent of the star topology as a type of blackboard, which the other agents must access to get hints about the location of the global maximum \([22]\). In fact, when the central agent is selected as the target and the imitation operation is chosen, it copies the best performing agent of the previous trial and this information become available to the other agents, which can then access the blackboard with probability \(p\). Given the severe bottleneck to the information flow in the star topology, it comes to a surprise that it works so well for small groups.

For topologies where the degree of the nodes (or agents) is not constant, we can estimate the probability \(P_h\) that the agent with the highest degree is the one that finds the global maximum. Since in the case all agents have the same probability of finding the global maximum \((e.g., for the regular lattices discussed in Section IV) this probability is \(1/L\), it is convenient to consider the ratio \(r_h = P_h/(1/L) = LP_h \geq 1\) which gives a measure of the influence of the degree of an agent on its chances of hitting that maximum. In Fig. 9 we show \(r_h\) as function of the group size for the star and the scale-free topologies. In the limit of very large \(L\), chances are that the global maximum already shows up in the setting of the initial population, so we have \(r_h \to 1\) in this limit. The influence of the degree on the ratio \(r_h\) depends on the ruggedness of the landscape: it is significant for smooth landscapes but marginal for rugged landscapes. It is in-
interesting that for the star topology the regime where \( r_{L} \) decreases with increasing \( L \) roughly coincides with the regime where the star topology performs less well than the other topologies considered in Figs. 7 and 8.

B. Community structure and small-world networks

To explore the effects of the structure of the population on the performance of the imitative search, we consider a network with \( L = 200 \) agents divided in 4 clusters of 50 agents each as illustrated in Fig. 10. The probability that two nodes of the same cluster are connected is 0.3, whereas nodes in different clusters are connected according to a probability of 0.001. The total number of links is 1519 so the average connectivity of the network is approximately 15.2. We consider, in addition, small-world networks which allow the interpolation between structureless (random) graphs and regular lattices [30].

In particular, the small-world networks we consider here are generated using the Watts and Strogatz algorithm. Explicitly, we begin with a one-dimensional lattice of \( L \) nodes, each node connected to \( M \) neighbors (\( M/2 \) on each side), and periodic boundary conditions. Then for every node \( i = 1, \ldots, L \) we rewire the links between \( i \) and \( j = i + 1 \) (the sums are done modulo \( L \)) with probability \( \beta \). Rewiring of a link is done by replacing the original neighbor of node \( i \) by a random node chosen uniformly among all possible nodes that avoid self-loops and link duplication. This procedure is then repeated for the links between \( i \) and \( j = i + 2 \), \( i \) and \( j = i + 3 \), and so on until the links between \( i \) and \( j = i + M/2 \). The case \( \beta = 1 \) corresponds to random graphs with average connectivity \( M \) and the case \( \beta = 0 \) to a regular ring with number of coordination \( M \).

Figures 11 and 12 show the mean computational cost for small-world networks with rewiring probability \( \beta = 0, 0.2 \) and 1, and for the network with community structure shown in Fig. 10. We find that \( \langle C \rangle \) is completely insensitive to variations on the structure of the network for smooth landscapes (see Fig. 11). This conclusion applies to rugged landscapes as well, except for \( p \) close to the boundary of the region where the imitative search fails to find the global maximum (see Fig. 12). In this case the sensitivity is extreme. For instance, for \( p = 0.58 \) searches using random networks \( (\beta = 1) \) fail to find the global maximum, whereas searches using regular \( (\beta = 0) \) or community networks succeed. In fact, for large values of the imitation probability, the risk of the search being stuck in local maxima is very high and so long-range links which speed up the flow of information through the network may become notably detrimental to the imitative search performance. However, in the absence of local maxima (Fig. 11) the best performance is attained by imitating the model string and allowing only their clones to explore the landscape through the elementary move.

To check whether the specific realization of the NK rugged fitness landscape we used in our study has unduly influenced our conclusions, we have considered four random realizations of the landscape with \( N = 12 \) and \( K = 4 \) in addition to that realization. We note that for smooth landscapes \( (K = 0) \) all realizations are equivalent. The comparison between the mean computational costs for those five realizations is shown in Fig. 13. The
FIG. 11. (Color online) Mean rescaled computational cost $\langle C \rangle$ as function of the imitation probability $p$ for the network with community structure illustrated in Fig. 10 ( ), and for small-world networks with average connectivity 14 and rewiring probability $\beta = 1$ ( ), $\beta = 0.2$ ( ), and $\beta = 0$ ( ). The number of nodes is $L = 200$ for the three networks and the parameters of the smooth NK landscape are $N = 12$ and $K = 0$.

FIG. 12. (Color online) Same as Fig. 11 but for the rugged NK landscape of parameters $N = 12$ and $K = 4$.

FIG. 13. (Color online) Mean rescaled computational cost $\langle C \rangle$ as function of the probability of imitation $p$ for the network with community structure illustrated in Fig. 10 and five realizations (different symbols) of the NK fitness landscape with $N = 12$ and $K = 4$.

VI. DISCUSSION

The tenet of our approach to study cooperative problem-solving systems is that group size and organization are selected so as to maximize group-level performance as measured by the number of trials necessary to find the global maxima of fitness landscapes. Differently from the definition of network optimality in biology, where there is a direct selection pressure to reduce the number of connections between entities because of their building and maintenance costs [33], here we assume that the connections between agents are costless. As a result, we find that the optimal group organization is obtained by a relatively small group of fully connected agents. The optimal group size $L^*$ is a decreasing function of the imitation probability $p$ (see Fig. 6) and is roughly proportional to the logarithm of the size of the solution space [16]. In our model, the problem-solving efficiency decreases with the group size for $L > L^*$ because of the duplication of work resulting from the decrease of the diversity of the group members due to imitation. More traditional selection pressures that contribute to limit the size of groups are resource competition, parasite transmission and the managing of the social relationships [34, 35].

The manner the information on the location of the global maximum spreads among the agents, which is determined by the network structure, as well as the accuracy of that information can greatly affect the problem-solving performance of the group. For instance, we find that for smooth landscapes the decrease of the agents’ connectivity is always harmful to the performance, regardless of the group size $L$. For rugged landscapes, however, this decrease can be highly beneficial for large groups (i.e., $L > L^*$). In fact, because the model agent may broadcast misleading information in this case, it is
advantageous to slow down the information transmission so as to allow the agents more time to explore the solution space away from the neighborhoods of the local maxima. The same conclusion holds in the case of centralization: the presence of super-spreaders, which enhance the diffusion of information, is beneficial provided the information is accurate. Decentralized networks are less susceptible to the diffusion of inaccurate information. It is interesting that for small groups (i.e., $L < L^*$) both centralization and full connectivity are beneficial, regardless of the accuracy of the information. In addition, the presence of long-range links has no effect on the performance of the group, except for a very narrow region of the model parameters. Notice that the evolution of other dynamical processes, such as synchronization or cascade failures, are strongly affected by the network structure [36]. Therefore, the weak influence of long-range links on the performance of the imitative learning search is not an expected result.

An interesting question that we can address within our minimal model framework is whether the fixed topology of a social network can enhance the fitness of some group members [37]. In this case the accuracy of the transmitted information seems to be the crucial ingredient. In fact, for smooth landscapes we find that highly-connected agents are very likely to be the ones that find the global maximum, but for rugged landscapes the fitness advantage of the central agents is marginal only (see Fig. 9). Another interesting issue, which we plan to address in a future work, is the study of adaptive networks, in which the network topology itself changes during the search [38, 39]. The difficulty here is to find a suitable procedure to update the topology, since simply adding more connection to the model (fittest) agent can be catastrophic if that agent is near a local maximum.

Finally, we note that our study departs from the vast literature on the game theoretical approach to the evolution of cooperation [10] where it is usually taken for granted that mutual cooperation is beneficial to the players. Here we consider a problem solving scenario and a specific cooperation mechanism (imitation) aiming at determining in which conditions cooperation is beneficial. The findings that in some cases cooperation results in maladaptive behavior [11, 12] and that for very large groups efficiency is achieved by minimizing communication [13] support our conjecture that the efficacy of imitative learning could influence the size and organization of the groups of social animals.

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