The Laws of Physics and Cryptographic Security

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Abstract

This paper consists of musings that originate mainly from conversations with other physicists, as together we’ve tried to learn some cryptography, but also from conversations with a couple of classical cryptographers. The main thrust of the paper is an attempt to explore the ramifications for cryptographic security of incorporating physics into our thinking at every level. I begin by discussing two fundamental cryptographic principles, namely that security must not rely on secrecy of the protocol and that our local environment must be secure, from a physical perspective. I go on to explain why by definition a particular cryptographic task, oblivious transfer, is inconsistent with a belief in the validity of quantum mechanics. More precisely, oblivious transfer defines states and operations that do not exist in any (complex) Hilbert space. I go on to argue the fallaciousness of a “black box” approach to quantum cryptography, in which classical cryptographers just trust physicists to provide them with secure quantum cryptographic sub-protocols, which they then attempt to incorporate into larger cryptographic systems. Lest quantum cryptographers begin to feel too smug, I discuss the fact that current implementations of quantum key distribution are only technologically secure, and not “unconditionally” secure as is sometimes claimed. I next examine the concept of a secure lab from a physical perspective, and end by making some observations about the cryptographic significance of the (often overlooked) necessity for parties who wish to communicate having established physical reference frames.

Nihil est dictum, quod non est dictum prius

Joe Kilian began his well known STOC paper [1] on oblivious transfer with the words Cryptographers seldom sleep well. Today some physicists would, rather smugly, qualify this by inserting the word “Classical” in front of “Cryptographers”. As I begin writing this article at 4am on a freezing cold Viennese morning, I can assure you that quantum cryptographers also seldom sleep well. By the end of this article I hope you realize that this is not due solely to jetlag.

Two basic cryptographic principles

The field of Quantum Information has brought attention to the fact that information theory needs to be considered in the light of physics. In this paper I attempt to consider the ramifications for cryptographers of thinking physically about security.

The two cryptographic principles I will begin by focusing on are the following:

Principle 1: (Kerckhoff) The security of a cryptographic protocol must not depend on features/details of the protocol itself being kept secret.

Principle 2: All parties involved in a cryptographic protocol must operate in a secure local environment.

Since we are concerned in this paper with the role of physics in cryptography, it is useful to rewrite these principles in a manner that places particular emphasis on the physics:

[1] By inserting this caveat at the start I am essentially trying to avoid a whole bunch of people being upset with me. I am writing this article off the top of my head, with only reference [1] actually in front of me, and reference [2] ingrained in my thick skull. Thus it should be understood that any work which I fail to cite must be of such quality that it has passed into the realms of “common knowledge”; accordingly I hereby completely disavow any claims as to the originality of any thought expressed in this paper. Thankfully, since I can’t imagine this paper appearing anywhere other than the quant-ph archive, I have no anxiety about anyone’s citation rates suffering.
Physicist’s Version of Principle 1: The security of a cryptographic system must not rely on keeping secret either the specific physical devices or the sequence of physical/experimental operations being used to implement the protocol.

Physicist’s Version of Principle 2: All parties involved in a cryptographic system operate in secure laboratories, and each party trusts only in the integrity of their laboratory and not in any way on physical events taking place in the universe external to it.

A cryptographic task is defined by an abstract set of requirements. A cryptographic protocol is designed to achieve these requirements, and is normally presented as an abstract series of procedures to be undertaken by the parties involved. The abstract protocol makes reference to such notions as “bits”, or “qubits” and actions such as ‘choose at random’ and so on. Whether the ‘bits’ used are black and white pigeons or flashes of blue and green lights is irrelevant to this abstract formulation of the protocol. When it comes to implementing a cryptographic protocol however, the parties involved need to decide upon a sequence of physical operations to perform using actual physical devices. What philosophers call a bridge principle - between the formal mathematics and idealized statements/instructions of the abstract protocol and the “dirty experimental implementations” - is necessarily invoked. This is not news to a physicist: the study of physics is founded upon the quest to uncover and elucidate the bridge principles between mathematics and experimental procedures and observations.

Of course there are usually many different possible physical realizations of any given abstract cryptographic protocol, and it is important to realize that once one particular implementation is chosen, we are adding a link to the ‘chain of belief’, upon which hangs our feelings of security. No matter how much we may believe the formal laws of mathematics, information theory or physics, which are invoked in proving security of the abstractly defined protocol, we need also to believe that the particular sequences of experimental operations being performed are, in fact, isomorphic to the abstract protocol under the bridge principle we have chosen to believe. This observation is important beyond merely metaphysical musings. In general, the practical realization of the abstract protocol involves many more physical elements and degrees of freedom than those theoretically required to implement the protocol in its purest and simplest form. These are generally ignored while proving formal notions of security for the abstract protocol. However they must be considered if we wish to argue that a particular implementation is truly as secure as the abstract protocol might suggest. For example, the two polarization states of a photon might make a particularly simple physical realization of a bit; but one must understand that a photon is described by more degrees of freedom: its frequency, orbital angular momentum or spatial mode for example. These degrees of freedom are potential carriers of information too. Such degrees of freedom are not normally innocent bystanders - for example we might need to couple to them for transmission or measurement purposes.

The Physicist’s Version of Principle 1 is simply the statement that, in addition to not relying on the details of the abstract protocol being kept secret, we must not rely on keeping secret the sets of physical devices and operations being used to implement the protocol. Thus we assume that our adversaries know what type of photodetectors and computers and lasers and so on we have in our laboratories, how we intend to use them, and what physical systems we are exchanging to affect communication.

The Physicist’s Version of Principle 2 is meant to encapsulate the fact that our feelings of security should not rely on presumptions about systems which are not under our direct control and observation. Ideally, we should feel secure despite the possibility that the whole ‘big bad outside world’ is controlled by our adversaries, and that they have arbitrarily large resources. That is, they are not limited technologically in the attacks they can mount against us, they are limited only by the laws of physics.

The necessity of incorporating physics in cryptography

From the conversations I have had with a few classical cryptographers, I will wildly extrapolate and speculate that the majority of classical cryptographers would prefer to ignore the much-hyped excitement about quantum information theory and its implications for cryptography. Who, after all, wants to get involved in learning dirty physics when we can study cryptography through beautiful and pure mathematics? It is a sentiment I am not altogether unsympathetic towards. A slightly more enlightened group seem to want “black box” quantum cryptography. They would like physicists to give them black box modules of quantum cryptographic sub-protocols and routines, the absolute security of which we will affirm (and they will, rather
amazingly, believe). They will then go off and try and incorporate these into a larger (classical) cryptographic framework. In this section I am going to argue that both of these perspectives are fundamentally flawed. I’ll do so by arguing that the abstract definition of a standard cryptographic task is incompatible with a belief in the validity of the laws of quantum mechanics. Thus attempts to find protocols for this task are inevitably doomed to failure. I will then illustrate that building larger protocols out of ‘promised secure’ sub-protocols is a very tricky business, which necessitates incorporating physical considerations at every step.

These days, of course, a quantum information theorist might try to make the case for cryptographers learning some physics based around the well-known story of quantum computation, Shor’s factoring algorithm and its obvious implications for RSA (and many other cryptographic protocols). However this argument would imply that cryptographers somehow thought that factorisation was more than just technologically difficult. Protocols based on one-way functions have always had (and will always have) security that is fundamentally founded in a belief about the technological capabilities of one’s adversary.

A popular conception of cryptography is that it investigates how well two co-operating parties can send secret messages. This is, of course, only the beginning. One of my favourite areas of cryptography is surely one of the simplest, the study of two-party protocols. These are protocols designed to complete cryptographic tasks, such as coin-flipping, bit commitment or oblivious transfer, generally between two antagonistic parties. Two-party protocols are, in fact, some of the most problematic in classical cryptography; there are no known secure such protocols. In [1], which has the wonderfully ambitious title *Founding Cryptography on Oblivious Transfer*, Kilian explains:

> [In a two-party protocol] both parties possess the entire transcript of the conversation that has taken place between them. Thus, in a protocol between A and B, A can determine exactly what B knows about A’s data. Because of this knowledge symmetry condition, there are impossibility proofs for seemingly trivial problems. [Classical] Cryptographic protocols “cheat” by setting up situations in which A may determine exactly what B can infer about her data, from an information theoretic point of view, but does not know what he can easily (i.e. in probabilistic polynomial time) infer about her data. From an information theoretic point of view, of course, nothing has been accomplished. A message encrypted using a one-way permutation, for example, might as well be sent in the clear in terms of information theory. (emphasis added)

Faced with this obstacle, cryptographers have attempted to understand the structure of two party protocols by examining the relationship between different tasks under assumptions that perfectly secure protocols for certain fundamental tasks exist. A particularly interesting such fundamental task is oblivious transfer (OT), upon (a perfectly secure version of) which arbitrarily secure versions of all other two-party protocols could presumably be built. OT is designed specifically to break the ‘knowledge symmetry’ referred to above, and can be defined as follows [1]:

**Oblivious Transfer**: Alice has a secret bit $b$. At the end of the protocol, one of the following two events occurs, each with probability 1/2. (i) Bob learns the value of $b$. (ii) Bob gains no further information about the value of $b$ (other than what Bob knew before the protocol).

At the end of the protocol, Bob knows which of these two events actually occurred, and Alice learns nothing.

It is useful to describe heuristically how an OT protocol would be viewed from a physical perspective. Alice and Bob engage in rounds of communication, which is implemented by physical systems exchanged between them. Alice follows one of two different sequences of actions, according to whether her secret bit is 0 or 1. These actions amount to different physical interactions, both with apparatuses in her laboratory and with the systems that are being exchanged. At the end of the protocol Bob reads the output of a device in his possession. The output of the device is, with equal likelihood, either $b$ or $\#$, and Alice cannot tell which of the two outcomes occurs.

It is also useful to make the following observation: cryptographic tasks do not require subjective decisions by human observers. Thus they can be completely automated. We can imagine that Alice and Bob have each programmed the devices in their lab to follow some sequence of actions; the decisions that we more normally imagine Alice and Bob themselves making, are instead made by a Turing machine. In this way we can assume that they do not actually look at the pieces of paper on which are written the outcomes of measurements until the termination of the protocol. This (possibly pedantic) assertion is an attempt to bypass the pitfalls (and nasty emails) which arise from failing to address the pet philosophies of everyone

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2Both the voices in my head assure me that you need at least two parties for meaningful communication.
who has an opinion on what is commonly known as the ‘quantum measurement problem’. If we believe the
laws of quantum mechanics, then the automated physical devices that Alice and Bob must use to implement
OT (or for that matter any cryptographic protocol) will always have a description in some (generally very
large) Hilbert space. All the actions which take place during the protocol are described by unitary evolution
within subspaces of this Hilbert space. When Alice and Bob finally interact with their apparatuses at the
culmination of the protocol, they perform a measurement which tells them what the outcome of the protocol
in fact was.

The definition of any cryptographic task can then be viewed as a set of constraints on the large state of
all the automated physical systems of Alice and Bob, as well as things in the universe external to their labs
(such as communication channels etc). In particular, we generally have specific requirements on the statistics
of outcomes which Alice and Bob can “collapse” this large state to, when they finally observe the output
of their devices. Thus it is natural to ask whether or not the constraints imposed by a given cryptographic
task can be fulfilled in principle; more precisely, whether there exists states in an arbitrarily large complex
Hilbert space, along with positive operators corresponding to the possible outcomes of the task, which satisfy
the requirements by which the task under consideration is defined. If there does not exist such states and
operators, I will refer to the task as being inconsistent with a belief in quantum mechanics, or, for conciseness,
simply as an inconsistent task.

To see that OT is an inconsistent task, we need to precisely formulate the requirements OT in quantum
shmantum language. The first requirement is two states |ψ₁^₀^⟩⟨ψ₁^₀^⟩ and |ψ₂^₀^⟩⟨ψ₂^₀^⟩, which describe completely,
just prior to Bob finding out either b or #, the systems of all Alice and Bobs’ apparatuses and all the
systems they exchanged during the protocol, as well as systems which lie outside their labs. The labels A, B
and U are used to identify systems lying in Alice’s lab, Bob’s lab and the rest of the universe respectively.
The states are labeled by a 0 and 1, since Alice follows a different sequence of actions if she’s transferring
a 0 than if she’s transferring a 1. Its important to note that these states must be pure if Principle 2 is to
be followed: If my adversary has an arbitrary powerful technology (within the laws of physics), and
they can control everything external to my lab, then I must assume they hold a purification of any mixed
state that I hold. Put another way: A method of ensuring that both parties describe the complete system
by a mixed state, maybe through a guaranteed noisy channel or more generally a guaranteed decohering
equipment, is equivalent to having some form of a trusted third party - and this moves us beyond two-party
protocols. The next requirement is that there exist a three outcome POVM measurement {E^B, E^₁, E^#} which Bob
implements at the culmination of the protocol, on the systems of B, that has the properties:

\[ \text{Tr}(\rho^B E^B) = \frac{1}{2}, \text{Tr}(\rho^B E^#) = 0, \text{Tr}(\rho^B E^#) = 0. \]

where \( \rho^B = \text{Tr}_A |\psi^0^ABU⟩⟨\psi^0^ABU|\). We also require that Bob cannot estimate whether Alice was sending
a 0 or a 1, better than by implementing the three outcome measurement and randomly guessing in the
event of the E^# outcome. This supplies an extra constraint which be written as D(\( \rho^B_{AU} \), \( \rho^B_{BU} \)) = 1/2, where

\[ \rho^B_{BU} = \text{Tr}_A |\psi^0^ABU⟩⟨\psi^0^ABU| \] and D(\( ., . \)) denotes the trace norm distance[^4]. Note that this requirement has been
explicitly formulated to allow for the fact that Bob might have access to the rest of the universe, an
assumption that Alice must make. Finally, although Alice knows whether the state is |\( \psi^0^ABU \rangle \) or |\( \psi^0^ABU \rangle \),
we require that there is no measurement she can perform which allows her to estimate whether Bob got b or #
with probability better than 1/2. Lets imagine Alice was transferring a 0. This constraint implies that for
all POVM’s \{F^A, I - F^A\} we require

\[ \langle \psi^0^ABU | F^A \otimes E^B | \psi^0^ABU \rangle = \langle \psi^0^ABU | F^A \otimes E^# | \psi^0^ABU \rangle. \]

Once again we have incorporated an assumption that Bob must make, namely that a cheating Alice has access to the
rest of the universe.

Having formulated the requirements mathematically, we can now ask whether they are consistent. Unfor-
unately we find that, states and operators satisfying the above requirements do not exist in any dimension
complex Hilbert space. I leave the proof of this as an exercise for the reader. It is important to realize that
even if OT were a consistent task, the question would still remain as to whether there actually exists a
protocol to implement it. That is, we would have to decide whether there existed a way of Alice and Bob
engaging in rounds of communication such that they both feel secure that the states |\( \psi^0^ABU \rangle \) are attained.
The task of secure bit commitment (discussed below) is also inconsistent with quantum mechanics, hence
we should not be surprised (with the benefit of hindsight!) that it has been proven that there exists no
protocol to implement it. By contrast, coin flipping (which I will not discuss further here) is a consistent
task, however the question remains open as to whether there exists a protocol to implement it[^3].

[^3]: D(x,y) = 1/2 Tr|x-y| is a measure of the distinguishability of two density operators x, y.
[^4]: For the sake of clarity, in this discussion I am avoiding the issues that arise if we generalize our definitions of tasks to allow
The second point I want to raise in this section is directed at those cryptographers who would like physicists to supply them with secure black-box protocols.

As mentioned above, OT forms a powerful building block for implementing secure versions of other cryptographic tasks. In particular let us examine how an arbitrarily secure bit commitment (BC) could presumably be founded on a secure OT protocol. First we define bit commitment:

**Bit Commitment:** Alice has a secret bit $b$. A bit commitment protocol consists of two phases.

Commitment phase: Alice and Bob supplies Bob with some token/information that depends on the value of $b$.

Unveiling phase: Alice provides Bob with a further token/information from which Bob can determine $b$.

Ideally, Alice is unable to change the value of $b$ after the commitment phase, and Bob is unable to determine the value of $b$ before the unveiling phase.

Now lets look at a proposal (based on the one in [1], where it is attributed to Crépeau) for building arbitrarily secure BC on perfectly secure OT:

**BC built on OT:**

Commitment Phase: Alice randomly chooses $N$ strings, each of $M$ bits. She chooses each of the $N$ strings such that their sum modulo 2 (their parity), is $b$. She then obliviously transfers each bit of every string to Bob, using the perfectly secure OT protocol.

Unveiling Phase: Alice announces the value of every bit.

The intuition behind the security of this BC built on perfectly secure OT is as follows. If Bob is cheating, he needs to know the parity of one of the $N$ strings. For any given string, the probability he does not obtain the outcome $\#$ is $(1/2)^M$. Thus the probability of him being able to determine Alice’s bit $b$, is $N/2^M$. If Alice is cheating and she wants to change the parity of a given string, she needs to announce one bit value to be different from the one she actually transferred. She will get away with this only if Bob obtained the outcome $\#$ for this bit, that is, she succeeds with probability $1/2$. To change her commitment she will need to change the parity of $N$ strings, her probability of cheating successfully is therefore $1/2^N$. Thus we conclude that the probability of either party cheating successfully can be made arbitrarily small by choosing $M$ and $N$ large enough.

Now let us imagine that we only have a partially secure oblivious transfer protocol. By this I mean a protocol which, regardless of Alice’s actions, she cannot increase the probability of Bob obtaining the $\#$ outcome to greater than $p$ for some $p < 1$ and, regardless of Bob’s actions, he cannot estimate $b$ with probability greater than $q$ for some $q < 1$. Could such a protocol be used to implement BC? Generalizing the above argument, Bob’s probability of successfully estimating the bit is $Nq^M$, while Alice’s probability of successfully changing her commitment is $p^N$. These can be made arbitrarily small by choosing $N, M$ large enough for any $p, q < 1$.

It turns out that there are, in fact, partially secure OT protocols in quantum cryptography. Here is a simple example: If Alice is honest then she obliviously transfers $b$ by sending Bob $|\psi_b\rangle$, where $|\psi_b\rangle = \cos \theta |0\rangle + \sin \theta |1\rangle$ for some $\theta < \pi/4$. If Bob is honest then he implements the POVM for unambiguously discriminating these two states: $E_0 = \frac{1}{1+\cos 2\theta} (\sin^2 \theta \ |\sin \theta \cos \theta \rangle \langle \sin \theta \cos \theta |$, $E_1 = \frac{1}{1+\cos 2\theta} (\cos^2 \theta - \sin \theta \cos \theta \ |\sin \theta \cos \theta \rangle \langle \sin \theta \cos \theta |$, $E_\# = I - E_0 - E_1$. With probability $1 - \cos 2\theta$ he obtains the correct outcome $b = 0, 1$; otherwise he obtains the $\#$ outcome (indicating that we should choose $\cos 2\theta = 1/2$). If Alice is cheating, however, she can maximize the probability of Bob obtaining the $\#$ outcome by sending him the state $|0\rangle$; then he obtains $\#$ with probability $p = 2 \cos 2\theta/(1 + \cos 2\theta)$. If Bob is cheating he can maximize his information gain about which bit Alice is sending by implementing a Helstrom measurement; with probability $q = (1 + \sin 2\theta)/2$ he would correctly identify $b$.

In accordance with the arguments presented above, we might conclude that we could take this partially secure quantum OT protocol and use it as a black box module to build arbitrarily secure BC. However this is not the case, as is hopefully clear from what follows:

Let us imagine imagine Alice is equally likely to want to unveil $b = 0, 1$, and we are building BC on the partially secure quantum OT protocol described above. Here is possible way for Alice to cheat: She follows for their implementation to only be “arbitrarily close to secure” (security is achieved as some parameter in the protocol is taken to infinity). Thus, I more precisely should have said that ideal coin flipping is a consistent task, but Lo and Chau have proven that it is impossible to find a protocol to implement it. Arbitrarily secure coin flipping is also a consistent task, and it is actually the question of whether a protocol exists to implement it that remains open.
honestly the protocol for committing \( b = 0 \), with one small twist. The protocol specifies she must randomly choose a whole set of even parity bit strings. She could do this by using some random number generator, recording the even parity strings and following the OT protocols accordingly. However she can also take some ancillary systems that are in a quantum superposition of every possible even parity string of \( M \) bits. She then follows the sequence of actions for obliviously transferring 0’s or 1’s controlled on the value of a corresponding ancilla bit\(^5\). This is really the toughest part to explain to someone without any knowledge of quantum mechanics. Suffice to say that this leaves Alice with a large entangled state between the ancillary systems used for choosing the parity strings, and the systems being sent to Bob to implement the OT. She holds what is sometimes known as a “purification” of the systems being held by Bob. Its important to realize that Alice is playing absolutely honestly at the level of each individual OT, it is just that the random choices which the protocol for building BC on OT specified she should make are being performed using quantum systems instead of classical coin tosses. Bob has no way of checking how Alice chose to generate the random strings\(^6\). In actual fact, at this stage Alice has no way of knowing for each bit whether she has actually obliviously transferred a 0 or a 1 (she could easily find out by measuring her ancillary systems - the equivalent of looking at the outcome of a coin toss, but that would destroy her ability to cheat\(^7\)).

If Alice decides she wants to unveil a 0 she makes a certain measurement on all the ancilla systems she holds, and announces the outcomes to Bob as the even parity strings she sent. If, however, she decides she wants to unveil a 1, then she performs a different measurement on the ancilla systems and announces those outcomes as the odd parity strings she sent. She is certainly not guaranteed of passing Bob’s checks, but she succeeds in unveiling the bit of her choice with considerably better probability of success than one might expect from the naive intuition discussed above. In fact her probability of successfully unveiling the bit she wants (and not being caught cheating!) is \( (1 + f^2)/2 \), and I’ll talk a little more about the value of \( f \) below.

Bob meanwhile can cheat in a different way. Bob’s goal is not to actually determine the value of every bit that Alice obliviously transferred. What he needs to know is whether the set of systems as a whole are in a state corresponding to even or odd parity. It turns out that he can make a joint measurement on all the systems, which is designed to distinguish only these two possibilities. His probability of successfully identifying the bit Alice is committing to is \( (1 + d)/2 \).

A bit of tedious algebra shows that as we vary \( M \) and \( N \), we certainly do vary the parameters \( f \) and \( d \) between 0 and 1. However, for this particular pair of cheating strategies one finds that we always have \( f + d \geq 1 \). That is, no choice of \( M, N \) gives \( f \to 0, d \to 0 \), as we would wish for an arbitrarily secure protocol\(^8\). In fact the cheating strategy described here is not the optimal one for Alice (the one for Bob is). If we solve for their optimal cheating strategies we do however find that \( f_{\text{max}} < 1, d_{\text{max}} < 1 \), and thus on top of our OT protocol with partial security we have built a BC protocol with partial security.

I should emphasize that the above argument only proves that building BC on top of partially secure OT using this one particular suggestion does not result in an arbitrarily secure protocol. The question of exactly how secure BC can actually be made is still open, but it is known that it can never be made arbitrarily secure from the work of Mayers, Lo and Chau. Reasonably tight bounds on the best possible security for BC are discussed in [2]. The crucial point I want to make with this example however, is that we must be very careful when constructing larger cryptographic systems based on (even well understood) sub-systems.

Now I know that there are a whole lot of physicists who (if they actually read this paper) will be ready to jump down my throat with arguments against an implicit assumption I’ve been making, that quantum mechanics is somehow applicable to the physical systems of any protocol regardless of their size, shape, color or ethnicity. And this raises exactly the third essential point I’m trying to make in this section. If someone truly believes this and designs a protocol based around it, then their faith in the security of a cryptographic scheme is still founded in their (supposed) understanding of physics; moreover it is founded in an belief which is not, by my reckoning, the accepted norm amongst physicists. A less dogmatic argument would be to assert that perhaps some cheating strategies are technologically infeasible; for example they might require Alice to maintain large amounts of entanglement. This may well be true, but then we are still back to admitting we

\(^5\)It was Dominic Mayers’ crucial insight about the ability to keep random choices at the quantum level which allowed him to extend the MLC theorem to proving the impossibility of any arbitrarily secure quantum bit commitment.

\(^6\)Since it is generally believed there is no other way of generating true randomness than quantum mechanically, it could be argued she is playing “more honestly”!

\(^7\)In quantum shmantum language, Alice holds a purification of \( W_0 \), where we define \( W_0 = \rho_0 \otimes \rho_0 \otimes (N\text{copies}), W_1 = \rho_0 \otimes \rho_0 \otimes (N\text{copies}), \rho_0 \) is an equal mixture of all even (odd) parity tensor products of \(|\psi_0\rangle, |\psi_1\rangle\).\(|\psi_0\rangle, |\psi_1\rangle\).

\(^8\)In quantum shmantum language, Alice’s probability of unveiling the bit of her choosing for this cheating strategy is given by \( (1 + F(W_0, W_1)^2)/2 \), where \( F(\omega, \tau) = Tr[\sqrt{\omega} \sqrt{\tau}] \) is the fidelity between two mixed states. Bob’s probability of cheating is \( (1 + D(W_0, W_1))/2 \)
have a sense of security which is founded on a belief about the technological capabilities of our adversaries.

The desirability of incorporating physics in cryptography

In this section I will briefly discuss reasons I think two-party quantum cryptography should be of interest to all cryptographers.

Firstly there is an analogy with one-way functions that non-orthogonal quantum states provide. As Kilian pointed out, because the difficulty of inverting one-way functions is only computational, founding cryptographic protocols on 1-way functions does not provide ‘information-theoretic’ security. One-way functions are generically used in scenarios where Alice and Bob have agreed on the particular function, Alice sends Bob one (or some) outputs \{y_i\} of the function and she feels confident that Bob has not the computational resources to find the corresponding \{x_i\}. We can find a quantum analogy as follows: Alice and Bob agree on a particular set of states, Alice sends Bob one (or some) states \{\psi_i\}. If the states are non-orthogonal then Alice knows that the laws of physics prevent Bob determining exactly which states have been sent. Unfortunately they also prevent Bob verifying with absolute certainty what state Alice has sent. But I suspect that the fundamental tradeoff between these two degrees of certainty lies at the heart of determining the absolute limits to the security of all two-party protocols.

Secondly, quantum cryptography provides an avenue for the breaking of “knowledge symmetry”. As discussed above, OT protocols with bounded degrees of security are certainly possible in quantum cryptography, and these break the knowledge symmetry referred to by Kilian. This also has implications for game theory, the analysis of two player games shares a lot in common with the analysis of two-party cryptography. Unfortunately no arbitrarily secure two-party quantum cryptographic protocols are known, in fact the only such protocol we really know a lot about in the context of quantum cryptography is BC. Interestingly, if the two parties share a trusted resource of entangled states, arbitrarily secure BC is still impossible, and in fact the security bounds remain unaffected. However, coin-flipping with trusted shared prior entanglement is trivial. There is clearly an intricate hierarchy of these protocols.

Finally, quantum protocols naturally display cheat sensitivity. When we use a classical one-way function, we implicitly assume that our adversary is trying their best to invert it. They have nothing to lose by doing so. However when communication is being implemented by transfer of quantum states, the actions of cheating parties generically lead to disturbances of the states and this in turn leads to situations wherein one can detect whether one’s adversary is trying to cheat or not. Since we must presume that the two-parties engaged in any two-party protocol are not so adversarial that they have no motivation to communicate at all, it is not unreasonable to start analyzing situations in which the parties assign certain costs to being caught cheating. Furthermore, we generally find that when one party cheats in a quantum protocol it results in a lowering of their ability to detect whether the other party is also cheating. There are all sorts of interesting scenarios and tradeoffs along these lines which are almost completely unexplored; they can surely fail to interest only the most apathetic and unimaginative persons in the cryptographic community.

Different conceptions of security

Up until now I have been throwing the word security around a little carelessly. In this section I am going to try to differentiate more precisely the three concepts of security that I carry around in my mind on a day to day basis.

The first concept is Technological Security, or T-security. This is most common type of security employed in our day to day cryptography. T-security is of course the reason that there is a seemingly endless game of back and forth between cryptographers and cryptanalysts, and this keeps a lot of people employed. There is no need to worry about their job security yet - later I will argue that ‘almost all’ security is technological, despite what quantum cryptographers might try and assert to the contrary.

Within the category of T-security one can identify two themes. Consider first RSA public key cryptography, the security of which is based on the difficulty of factorising large numbers. If I use the RSA algorithm I do so in the belief that my adversaries do not have the technological capability to factorise the large numbers required to compromise my messages - factorisation is not in principle impossible. Moreover, I must accept that if my adversary wishes to, they can keep trying for the next 20 years to decode my message, and perhaps a quantum computer will be built before a decoding of the message has lost its value. Now consider quantum

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9One way functions in classical cryptography are functions for which \(f(x) = y\) is easy to compute, but finding \(x = f^{-1}(y)\) is computationally difficult.
key distribution (QKD). In the next section I will argue that most, if not all, current implementations of quantum key distribution are also only T-secure. However there seems to be an intrinsic difference with the T-security of RSA. If the eavesdropper does not have the technological capability to render the QKD insecure during the implementation of the protocol, then it would seem she has lost forever the chance to do so.

The second concept of security is Information Theoretic Security (IT-security). This is the level of security most desired by classical cryptographers - it is security that does not rest on complexity-theoretic assumptions such as the difficulty of factoring, and is therefore viewed by classical cryptographers as being superior to T-security. However, as I have discussed above, IT-security might not actually be as attainable as one might hope. An important question is whether in fact any protocols are IT-secure? From one perspective we could look at Quantum Information Theory (QIT) as just being a larger theory of information which incorporates Classical Information Theory. We can then argue that as long as a protocol is QIT-secure, it really and truly is secure, and the problem with the standard IT-security is that it was actually only “Classical IT”-secure. While a part of me wants to believe this (if only because I work in quantum information theory!), I’m not sure I can make an unassailable argument for the ultimate ubiquity and universal applicability of the principles of QIT.

As such, the third concept of security I like to carry around in my head is Physical Security (P-security), which is security promised upon the truth of the laws of physics as we currently understand them. This type of security is clearly still a “degree of belief” - it is not unknown for laws of physics to be modified as we deepen our understanding of the universe. Moreover; we do not have one universal theory of physics applicable to all energy scales. Thus when confusion could arise, we should in fact qualify which particular physical theory we are basing our cryptographic protocol around. The most obvious candidates are non-relativistic quantum mechanics, quantum field theory, special relativity and general relativity.

The multiplicity of physical theories upon which P-secure protocols could be based might make a non-physicist distinctly uncomfortable. For this reason, cryptographers attempting to construct P-secure protocols should do so by trying to use those features of the physical theory which are expected to survive in the “über-theory” that most of us believe will one day be discovered. Ideally, P-secure protocols would manifestly rely only on some general physical principles - such as conservation laws, no-signaling or perhaps, for quantum mechanical theories, linearity. It should be emphasized that even if we discover an über-theory of physics which withstands repeated experimental test across all accessible energies, security based on this theory is still founded on a degree of belief in it. Nature will never hand us out a certificate congratulating us on having, in fact, discovered everything that is to be discovered. It is interesting to speculate that experimental physicists would primarily place their faith in an über-theory based upon it passing multiple experimental tests, whereas the foundation of a theoretical physicist’s faith would presumably be the mathematical beauty and elegance of the theory. Faith, trust or belief in something is ultimately a personal and often very subjective decision. The majority of people who use cryptography are in fact placing their faith in cryptographers; they have neither attempted to design an algorithm for factorization nor to test the laws of quantum mechanics.

Why quantum cryptographers should not be too smug

As promised near the beginning of the previous section, in this section I am going to try and argue that most, if not all, current experiments implementing QKD are actually only T-secure. This particular attack relies heavily on the ideas received from multiple communications with Richard Gill and Gregor Weihs. Let me say emphatically that I am not asserting the insecurity of QKD in an idealized, abstract protocol, but rather I’m using one particular (and presumably not optimal) technological attack on current implementations of QKD. Essentially I want to raise the question about where, in our metaphorical chain of belief, we are really placing our faith when we use QKD?

I will use the example of entanglement based QKD, due originally to Ekert, although as Cˇ aslav Brukner pointed out to me the attack described works equally well against the current implementations of BB84 or B92 QKD. Entanglement based QKD gives quantum cryptographers a nice, warm fuzzy feeling inside, because it exploits what is (unfortunately) commonly termed “quantum nonlocality”. More precisely, it exploits the inability of any set of local hidden variables to reproduce the correlations in data recorded at two spacelike separated locations. Ideally the QKD works because even an eavesdropper with an arbitrarily

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10In fact Kent has devised a P-secure two-party protocol akin to bit commitment based on special relativity. I say “akin” because the protocol requires an extra ‘holding phase’ during which communication still takes place, as well as guarantees about the spatial proximity or otherwise of certain communicating parties. These constraints are difficult to fit into a standard cryptographic framework, but the protocol is certainly nonetheless interesting.
powerful technology (i.e. constrained only by the laws of quantum mechanics) cannot obtain a significant amount of information about the key without degrading these correlations.

Let me give now a slightly idealized description of most current implementations of QKD. Much of my understanding of such things is due to Thomas Jennewein who patiently tolerates my questions - however he should not be held responsible for inaccuracies! Alice uses a parametric downconverter to produce entangled pairs of photons\(^1\). One of the output beam of photons is sent to Bob (and it is these photons which Eve may intercept and interact with). The other beam of photons is retained by Alice. Alice sends her beam through a polarizer, which is rapidly and randomly set to one of a choice of several rotation angles, and a measurement of the photon's polarization is then made. We'll assume here that Alice's photon detectors are 100% efficient. Alice records the time of measurement of any photon using an atomic clock. The rate of switching polarizer settings is fast compared to the photon production rate of the source - most of the time no photons go through any given setting. Meanwhile Bob is doing the same thing with the beam of photons which has been sent to him.

After a suitable number of photons have been detected, Alice and Bob communicate publicly. In the idealized protocol they would announce which setting they used for each pair of photons, the measured polarization outcomes of the subset of photons for which they chose different polarizer settings would then also be announced, and sufficient violation of the Bell inequality for this subset would indicate that Eve had not gained enough information to compromise the security of the QKD.

In reality however, Alice and Bob must also announce their photon arrival times, and only consider photons which arrived in coincidence. This is because photons get lost: The pairs are not emitted perfectly in opposite directions, the quantum channel (optical fiber/free space) is lossy and so on. Photons which do not arrive in coincidence are not part of an entangled pair and therefore not useful for the QKD. This allows Eve the following attack: Eve replaces the lossy channel with one of much lower loss rate. She then acts just like Bob - she makes random choices of polarizer settings and records the outcomes/detection times of measurements on the photons that were meant for Bob. She then constructs a "demonic" photon. This is a photon (or wavepacket of photons) which is programmed to go along the channel into Bob's lab, but to only arrive in coincidence with Alice's photon if the setting on Bob's polarizer is the same as the setting Eve randomly chose. If it does arrive in coincidence, then it is programmed to give the measurement outcome that Eve obtained. If Eve and Bobs' settings are different, the photon aborts its mission (e.g. slows down/speeds up) in order to avoid arriving in coincidence. Thus for all photons which Alice and Bob accept as having arrived in coincidence, Eve actually knows the outcome Bob obtained.

Now I know that the same people who didn't like my implicit use of macroscopic entanglement earlier on are really going to hate the usage of demonic photons. How, after all, do I propose to construct such a photon. In fact the question is irrelevant - what is important is whether the construction of such a photon is forbidden by the laws of quantum mechanics. We want our security to be founded only on our belief in those laws, and perhaps more specifically on the aforementioned warm fuzzy feeling we get about Bell's inequality. Note that here the Bell inequality would still be violated, even though the demonic photon is programmed with a local hidden variable\(^2\). It seems to me we have reverted back to T-security: can proponents of current experimental implementations categorically assure their users that no demonic photons are present? Since no-one to date has formulated the minimal amount of complexity required for construction of such demonic photons, I doubt it. In fact the demonic photon does not need to be extremely sophisticated: If the two polarizer settings in Bobs lab have slightly different optical properties (birefringence, or dispersion characteristics say) then Eve may be able to utilize this to create quite unintelligent but effective demons (recall that by the Physicist's Version of Principle 1 she knows all the equipment that Bob is using).

However; the astute reader will be able to argue against the relevance of above attack on QKD with a much better argument than "well it seems unlikely". Essentially Eve’s attack can be viewed as violating the Physicist's Version of Principle 2. After all, who can claim to feel safe and secure in a laboratory with photonic demons floating around it? More seriously, it is important to realize that the information we receive down a quantum channel is actually physical stuff. Proofs of security of abstract protocols generally implicitly assume that what comes into a lab through such a channel contains states with support only in a certain specified subspace of the full Hilbert space. We might say that the channel is presumed to be exorcised. However such exorcism is not in general trivially accomplished\(^3\), since we generally rely on the

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\(^1\) The photon source could in fact be located in-between Alice and Bob and therefore controlled by Eve, however this unnecessary for Eve to be able to mount the attack I’m describing here.

\(^2\) It should be pointed out that this attack is really just an exotic exploitation of the well known “detector loophole”, and therefore not of fundamental philosophical significance to Bell inequality type experiments.

\(^3\) I think even the bible of QIT (Nielsen and Chuang’s textbook) does not provide us with an operational definition of the
that we feel secure only about the actions and events taking place in our lab, and that all the outside world is potentially controlled by our adversary. To quote Descartes:

I will suppose that...some malicious demon of the utmost power and cunning has employed all his energies in order to deceive me. I shall think that the sky, the air, the earth, colours, shapes, sounds and all external things are merely the delusions of dreams which he has devised to ensnare my judgement. (emphasis in original)

Is it possible to have a lab, the security of which is guaranteed by the laws of physics? Such a lab must be secure against both active external attacks (impenetrable) and passive outside monitoring (emanation free). Impenetrability must be guaranteed for two types of penetration attacks: (i) Attacks which involve penetration and subsequent gathering of information by the adversary from the penetrating agents (e.g. X-rays) and (ii) Attacks in which the penetrating agents attempt to modify the physical conditions/devices inside the lab. Such penetration attacks can of course be countered by the person in the lab trying to detect the penetration, but then we are definitely back to a technological level of security again; we would be trusting that we are as smart and advanced as the persons attempting the penetration. The second type of penetration attack particularly troubles me. As an extreme example, how can I be sure that my adversary is not able to wave a large mass outside my lab in such a way as to distort the spacetime inside my lab to their advantage? In the limit of an infinitesimally small lab this is not a problem, but I’m not infinitesimally small!

An emanation free lab would not be very comfortable, since information processing at absolute zero is unlikely to be much fun - although it is not impossible. Completely emanation free labs are presumably unnecessary - for example, we would not in general care if our adversaries could only measure emanations that inform them as to our temperature. However; if the whole lab is at thermal equilibrium, not much information processing will be going on inside it anyway. (I could try and elucidate on a tenous link here to Maxwell’s demon, however two demons in any one paper should be enough for anybody.) Thus a natural physical question, to which I do not have an answer, is whether we can construct (hypothetical idealized) labs for which all emanations are guaranteed to leak only an exponentially small amount of useful information to the outside world.

The impenetrability/no emanation conditions are presumably satisfied by a 'black hole lab', and perhaps this is the only way that they can be. Unfortunately being inside a black hole lab will present significant hurdles to communication. In fact there seems to be an intrinsically physical aspect to communication that is often overlooked in quantum cryptography. If two parties are communicating classically, it is generally presumed that they both can distinguish a 1 from a 0, and in the majority of circumstances this is trivial. However when we progress to exchanging very small systems for communication, things are considerably more subtle. Consider first using different two-level quantum systems for classical communication. Exchanging two level atoms would present no problems for classical communication - the ground and excited states require no specialized reference frame to identify. For degenerate systems however things are more problematic. The “spin up” or “spin down” states of spin-1/2 systems can only be used as classical bits if the two parties have agreed upon a reference frame, in this case which direction is up. The setting up of such a spatial reference frame is a physical process, requiring either the exchange of physical systems or agreement about external reference points (such as a particular fixed star to define a spatial direction say, which then presumes proximity of the parties). Using the horizontal and vertical polarizations of a photon is in fact a little more subtle. The polarization lies in the plane defined with respect to the photon’s direction of propagation. Thus a single ‘external fixed star’ method of establishing a reference frame will not work unless the two communicating parties also know the orientation of their labs with respect to each other - in which case the reference frame is presumably trivially fixed. Of course, in these latter two cases, appropriate reference frames

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measurement operator corresponding to projection onto a non-demonic set of states - and if you can’t find mention of demons in a bible, where else are you going to find them?

I admit it - I do in fact have more than just [1] in front of me - for when I get bored I also have The Great Philosophers edited by Ray Monk and Frederic Raphael. Note that Descartes’ demon is very powerful and so presumably much higher up in the underworld hierarchy than the photonic demon of the previous section.
can be set up by exchanging a large number of photons or spin-1/2 particles, communicating classically by other means, (using the two-level atoms say) and then playing around with the relevant polarizers/Stern-Gerlach apparatuses until a reference frame is fixed. The need to communicate classically by other means could possibly be avoided if some standard protocol was already in place; however, it is clear that even at the simple level of exchanging classical bits with these objects something extra is required. And remember, we need to feel sure that the process of establishing this reference frame does not allow our adversary some sophisticated cheating strategies...

Once we want to exchange quantum systems in coherent superpositions then even the two-level atoms are no longer a help - the relative phase between the ground and excited states must be fixed using some form of time standard. Now it is unreasonable to expect (in fact its quantum mechanically impossible) any reference frame to remain fixed for an arbitrary amount of time. This leads to the following problem. If two parties are engaged in quantum communication they will eventually be faced with one of these two choices: (i) They must agree to use some external reference points to re-calibrate their reference frame, or (ii) they must exchange some physical systems to perform the re-calibration. Either possibility is not ideal from a cryptographic standpoint. For example, under (i) I might be forced to accept that my adversaries do not have the ability to move around the fixed stars (or at least to install faux stars between my lab and the real stars). As such I am trusting in the security of something external to my lab. Under (ii) I might use a reference frame which is completely determined by my adversaries, this too leads to obvious potential problems. We might take the view that shared physical systems that establish reference frames are in themselves an information theoretic resource.

InConclusions

I have tried to explore just a tiny subset of the issues that arise when we approach cryptography by incorporating physics into our thinking at every level. Of course I do not anticipate the day the NSA employs more physicists than mathematicians and computer scientists arriving any time soon. The issues discussed here are, for the moment, completely irrelevant - there are much weaker links in present cryptographic systems than those raised here. It seems inevitable to me however, that discussion about the ‘ultimate’ limits to security will always lie in the realm of physics. We are physical creatures, ‘security’ has no existence in Plato’s universe and therefore cannot be ‘proven’; security is a subjective feeling based on a particular individual’s beliefs.

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