Investigation of the temperature field in the areas of rock and coal concentration near the self-heating zone

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Abstract. The processes of coal self-heating, which, as a rule, have a local character, are the most acute problems for the coal industry. Experimental and theoretical studies show that self-heating is influenced by the processes of heat accumulation and temperature increase, which means that more heat is formed in the area of rock and coal concentration than released into the surrounding environment. Thus, self-heating can quickly and unexpectedly go into fire, and then into an endogenous fire. Any source of self-heating is a thermal “source”, which is the cause for spreading thermal field that inevitably leads to an increase in the temperature of the rocks surrounding the faces and then the air atmosphere of the faces. A mixed boundary-value problem for a one-dimensional parabolic equation describing the temperature field in the areas of rock and coal concentration in the presence of self-heating zone was formulated in order to reveal the regularities of temperature field propagation in the vicinity of the self-heating zone. A formula is obtained that determines the temperature of the rocky-coal cluster. Calculations were performed on the basis of which the temperature dependences of the temperature in the rocky clusters were plotted against its thermophysical parameters and certain regularities of the temperature field in the cluster were revealed.

1. Introduction
It is proved that self-heating zones significantly increase the probability of inflammation and burning of dust and gas mixtures in the mine workings [1]. During the movement of the flame front, fine coal particles rise from the walls, roof and soils of workings, mix with gas and create critical combustion conditions that can transform into explosion, which is catastrophic in the conditions of the mine. The largest catastrophes with human casualties occurred in the mines of the Kuznetsk Coal Basin. In the catastrophe that occurred on March 19, 2007 110 people died at Ulyanovskaya mine (Novokuznetsk), and at Raspadskaya mine (Mezhdurechensk) on May 8-9, 2010 the disaster claimed the lives of 91 people.

In connection with the facts mentioned above, it seems to us actual to study the temperature field in coal pillars and the areas of rock and coal concentration in the presence of self-heating zones.

The change in the temperature field in the conditions of coal mines is a complex non-stationary physical process [2]. Firstly, the area containing coal is three-dimensional and, secondly, the area of rock and coal concentration and rock massif is an inhomogeneous and anisotropic medium, and therefore, the thermophysical coefficients are functions of coordinates. Thirdly, because of technological measures the rock massif is “cut” by workings of various purposes that significantly
distort the temperature fields in the massif and cause difficulties in setting conditions on the boundaries of the considered areas.

2. Task setting
Based on the above, it is not possible to construct an exact solution of the problem of a nonstationary temperature field. Therefore, we will discuss this problem with the following simplifying assumptions:
1) within the considered areas the rock and coal concentrations will be assumed to be homogeneous and isotropic, so that the thermophysical coefficients will be assumed constant and identical in all directions; 2) at the initial temperature of the rock and coal concentration we take the temperature \( T_r \) of the rocks in the self-heating zone, but its location, which is a heat source, will be assumed to be outside the calculation area; 3) we will consider the rock and coal concentration as a formation of "beams-strips", the process of propagation of the temperature field in which occurs only along their length, and the neighboring "beams-strips" exchange heat along their lateral surfaces.

On the basis of accepted assumptions, we consider the problem of the non-stationary temperature field in the rock and coal concentrations of face 21-1-9 in the mine “Olzherasskaya-Novaya” (figure 1). The problem is formulated as follows.

![Figure 1](image_url)

**Figure 1.** The plan of mining works in face 21-1-9 of mine “O lzherasskaya-Novaya”.

There is a limited “beam-strip” (in figure 1 it is shown with a dotted line near axis \( x \)) with a length \( l \) and a square cross-section, the size of which \( b \) is equal to the thickness of the coal seam. A heat exchange takes place according to Newton’s law between the lateral surface of the “beam-strip” and the surrounding medium. The temperature of the medium surrounding the side surface of the “beam-strip” is assumed to be constant and equal to its initial temperature \( T_s \), and heat exchange with the surrounding medium occurs on the left and right ends of the “beam-strip”. Moreover, its left end contacts with the self-heating zone, the temperature in which \( T_r \), and the right one – with the face (figure 2), the wall temperature of which is \( T_w \). It is required to find the temperature distribution along the length of the “beam-strip” at any point time.
Since the size $b$ of the cross-section of the “beam-strip” is small in comparison with its length $l$, we can assume that the temperature difference in the cross-section of the “beam-strip” is insignificant [3, 4] and it can be neglected

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0 ,$$

thus, the problem under consideration degenerates into a one-dimensional problem.

**Figure 2.** Heat transfer scheme of “beam-strip” with the environment.

Heat transfer from the lateral surface of the “beam-strip” to the environment is assumed to be a negative source of heat $q$, the value of which, in accordance with Newton’s law, is defined as follows [3, 4]

$$q = -\alpha_1 \left[ T(x,t) - T_0 \right] P l = -\frac{\alpha_1}{h} \left[ T(x,t) - T_0 \right] , \quad (1)$$

where $\alpha_1$ is the coefficient of heat exchange between the side surface of the “beam-strip” and the surrounding rocky clusters, $h = S/P$ is the ratio of the cross-sectional area of the “beam-strip” to the perimeter of the cross section, which for the square cross section is $h = 1/4b$.

Thus, the heat equation in the one-dimensional problem under consideration is represented in the form

$$c_p \rho \frac{\partial T(x,t)}{\partial t} = \lambda \frac{\partial^2 T(x,t)}{\partial x^2} - \frac{\alpha_1}{h} \left[ T(x,t) - T_0 \right] . \quad (2)$$

where $c_p$ is the specific heat capacity of the rock coal concentration, determined at a constant pressure $p$; $\rho, \lambda$ are the density and thermal conductivity coefficient of the rock and coal concentration, respectively.

Let us divide equation (2) by $c_p \rho$ and, first, make in it a replacement

$$W = T(x,t) - T_0 , \quad \nu = \frac{\alpha_1}{c_p \rho h} , \quad (3)$$

which allows us to rewrite equation (2) as follows

$$\frac{\partial W}{\partial t} = a \frac{\partial^2 W}{\partial x^2} - \nu W , \quad (4)$$

and the second replacement
leads the inhomogeneous equation (4) to the classical heat equation

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2},$$

belonging to the parabolic type [5], in which the quantity $a = \frac{\lambda}{(c_p \rho)}$ is the coefficient of thermal diffusivity.

Adding to the equation (6) the initial condition

$$u \big|_{t=0} = \psi(x),$$

and two boundary conditions

$$\frac{\partial u}{\partial x} - \frac{\alpha_2}{\lambda} u \big|_{x=0} = 0, \quad \frac{\partial u}{\partial x} + \frac{\alpha_2}{\lambda} u \big|_{x=l} = 0,$$

we obtain the boundary value problem, where $\alpha_2$ is the coefficient of heat transfer between the environment and the end cross-sections of the “beam-strip”, which is different from $\alpha_1$.

The boundary value problem (6) – (8) describes the process of temperature propagation in the “ball-strip” provided that it is placed in the medium with an initial temperature $T_s$ and exchanges heat with the medium along its entire length, and at the ends of the “beam-stripes” the heat transfer of the self-heating zone with the “beam-strip” takes place at one of its end and the atmosphere of the face – at the other.

We need to note that the boundary conditions (8) contain not only the desired function $u$, but also its partial derivative, so that the boundary value problem (6) – (8) is a mixed-type problem.

3. Construction of the problem solution

To construct a mixed problem, we use the Fourier method and represent the particular solutions of (6) as the product of two functions [5-7]

$$u(x,t) = u_n(x)u_n(t),$$

and after placing (9) into (6) we obtain two ordinary differential equations

$$\frac{du_n(t)}{dt} = -a \Theta_n^2 u_n(t), \quad \frac{d^2 u_n(x)}{dx^2} + \Theta_n^2 u_n(x) = 0,$$

integrating which, we find their particular solutions [8, 9]

$$u_n(t) = a_n e^{-\Theta_n^2 at}, \quad u_n(x) = c_{1n} \cos(\Theta_n x) + c_{2n} \sin(\Theta_n x),$$

where the values of the index $n$ in formulas (9) – (11) belong to the interval $n \in 1, 2, \ldots, \infty$.

The eigenvalues $\Theta_n$ and the integration constants $a_n, c_{1n}, c_{2n}$, containing in equations (10) and (11), determine from the initial condition (7) and two boundary conditions (8). Placing the boundary conditions at the beginning into (11), we obtain a homogeneous system of equations
\[ Hc_{2n} - \Theta_n c_{2n} = 0, \]
\[ [H \cos(\Theta_n) - \Theta_n \sin(\Theta_n)]c_{2n} + [H \sin(\Theta_n) + \Theta_n \cos(\Theta_n)]c_{2n} = 0, \] (12)

which, according to Kronecker-Kapeli theorem [10], has at least one non-zero solution if the determinant of the system is zero, i.e.
\[ \begin{vmatrix} H & -\Theta_n \\ H \cos(\Theta_n) - \Theta_n \sin(\Theta_n) & H \sin(\Theta_n) + \Theta_n \cos(\Theta_n) \end{vmatrix} = 0, \]

where \( H = \alpha_2/\lambda. \)

Expanding the determinant and performing the transformations, we obtain the characteristic equation
\[ 2\text{ctg}(\mu_n) = \frac{\mu_n}{p} - \frac{\mu_n}{\mu_n}, \] (13)
in which
\[ \mu_n = \Theta_n l, \quad p = \frac{\alpha_2}{\lambda}. \] (14)

Equation (13) is transcendental and therefore has an innumerable set of roots, the first ten of which were found by with the help of MathCAD.

After that, from the first formula (14) we find the eigenvalues
\[ \Theta_n = \frac{\mu_n}{l}, \] (15)
then, taking \( c_{1n} = 1, \) from the first equation of system (12) we find \( c_{2n} = H/\Theta_n \) and, using relations (14), we obtain \( c_{2n} = p/\mu_n. \) Therefore, an eigenfunction corresponds to each eigenvalue
\[ u_n(x) = \cos \frac{\mu_n x}{l} + \frac{p}{\mu_n} \sin \frac{\mu_n x}{l}, \] (16)

which follows from (11).

Inserting formulas (16) and (10) taking into account (15) into equation (9), we find particular solutions of the problem under consideration
\[ u_n(x, t) = a_n e^{-\frac{\mu_n^2}{l^2} t} \left( \cos \frac{\mu_n x}{l} + \frac{p}{\mu_n} \sin \frac{\mu_n x}{l} \right), \]

and we build a general solution
\[ u(x, t) = \sum_{n=1}^{\infty} a_n e^{-\frac{\mu_n^2}{l^2} t} \left( \cos \frac{\mu_n x}{l} + \frac{p}{\mu_n} \sin \frac{\mu_n x}{l} \right). \] (17)

Then inserting (17) into the initial condition (7), we obtain the function
\[
\psi(x) = \sum_{n=1}^{\infty} a_n \left( \cos \frac{\mu_n x}{l} + \frac{p}{\mu_n} \sin \frac{\mu_n x}{l} \right),
\]

(18)
multiplying which by the function \( u_m(x) \), where \( m \in 1, 2, \ldots, \infty \), and then integrating and taking into account that the functions \( u_n(x), u_m(x) \) are orthogonal [5 - 9], we find \( a_n \)

\[
a_n = \frac{1}{B_n} \int_0^l \psi(x) u_n(x) dx,
\]

(19)
where

\[
B_n = \int_0^l u_n^2(x) dx = \int_0^l \left( \cos \frac{\mu_n x}{l} + \frac{p}{\mu_n} \sin \frac{\mu_n x}{l} \right)^2 dx = \frac{l}{2} \left( \mu_n^2 - p^2 \right) \cos \mu_n \sin \mu_n + p(p + 2) \mu_n + (\mu_n^2 - 2p^2 \cos^2 \mu_n) \mu_n.
\]

(20)
We find the integral in (19) by taking function \( \psi(x) \) as a constant value equal to \( T_s \)

\[
I_n = \int_0^l \psi(x) u_n(x) dx = \int_0^l T_0 \left( \cos \frac{\mu_n x}{l} + \frac{p}{\mu_n} \sin \frac{\mu_n x}{l} \right) dx = \frac{T_0}{\mu_n} \left( \mu_n \sin \mu_n - p \cos \mu_n + p \right).
\]

(21)
Taking into account the trigonometric relations

\[
\sin \mu_n = \frac{1}{\sqrt{1 + \tan^2 \mu_n}}, \quad \cos \mu_n = \frac{\tan \mu_n}{\sqrt{1 + \tan^2 \mu_n}}
\]

and formula (13), we find the relations

\[
\sin \mu_n = \frac{2p \mu_n}{\mu_n^2 + p^2}, \quad \cos \mu_n = \frac{\mu_n^2 - p^2}{\mu_n^2 + p^2},
\]

inserting which into formulas (20) and (21), we first find

\[
B_n = \frac{l}{2} \frac{p(p + 2) + \mu_n^2}{\mu_n^2}, \quad I_n = \frac{2p l T_0}{\mu_n^2},
\]

then by the formula (19) we find

\[
a_n = \frac{2T_0 p}{p(p + 2) + \mu_n^2}
\]

and inserting \( a_n \) into formula (17), we obtain the function
Finally, returning using formulas (3), (5) to the required variable \( T(x,t) \), we obtain the formula

\[
T(x,t) = T_0 \left[ 1 + 4e^{-\nu\mu} \sum_{n=1}^{\infty} \frac{P}{p(p+2)+\mu_n^2} \left( \cos \frac{\mu_n x}{l} + \frac{P}{\mu_n} \sin \frac{\mu_n x}{l} \right) e^{-\frac{\mu_n^2 t}{l}} \right],
\]

which is a solution of the mixed boundary-value problem under consideration.

4. Results and discussion
On the basis of the solution obtained, calculations and graphs are made (figures 3-6), in which value \( T \) represents the required temperature related to the initial temperature \( T_s \). The calculations were performed with the following initial data of the rock and coal concentration [11]: \( T_0 = 287 \) K; \( \rho = 1400 \) kg/m\(^3\); \( c_p = 0.835 \cdot 10^3 \) J/(kg K); \( \lambda = 46.5 \cdot 10^{-2} \) W/(m \( \cdot \) K); \( \alpha_1 = 1.5 \cdot 10^{-2} \) kW/(m \( \cdot \) K); \( \alpha_2 = 2 \cdot 10^{-2} \) kW/(m \( \cdot \) K); \( l = 130 \) m; \( b = 3 \) m. The remaining parameters were calculated from the previously obtained formulas \( h = 0.75 \) m; \( \nu = 0.559 \); \( \nu = 17.11 \cdot 10^{-9} \) sec\(^{-1}\); \( a = 0.39 \cdot 10^{-6} \) m\(^2\)/sec. Since ten eigenvalues \( \mu_n \) were found during calculation of the required temperature, the first ten eigenvalues were retained, then in formula (22) the difference in the temperature values found with the nine and ten retained members was only 0.17%.

Analyzing the graph in figure 3, we note that as the distance from the location of self-heating increases, the temperature of the rock and coal concentration decreases practically linearly.

![Figure 3. Dependence of the relative temperature of the rock and coal concentration on the distance to the zone of self-heating.](image)

Figures 4 and 5 show graphs of the dependences of the relative temperature of a rock and coal concentration on its thermophysical parameters.

As it follows from the presented graphs, when the coefficient of heat transfer of rock and coal concentration increases from 1.5 W/(m\(^2\) K) to 4 W/(m\(^2\) K) in the direction from the location of self-heating to the face, the relative temperature of rock and coal concentration decreases by 12%, and in the perpendicular direction – only by 1%.
Figure 4. Dependences of the relative temperature of the rock and coal concentration on the heat-transfer coefficients $\alpha_1$ (figure 4, a) and $\alpha_2$ (figure 4, b).

Figure 5. Dependences of the relative temperature of rock and coal concentration on the specific heat (figure 5a) and thermal conductivity coefficient (figure 5b).

With an increase in the thermal conductivity in the interval $\lambda \in [0.25; 0.75] \text{ W/(m} \cdot \text{K)}$ the relative temperature increases by 11%, and with the change in the specific heat capacity in the interval $c_p \in [250; 1200] \text{ J/(kg} \cdot \text{K)}$ – only by 0.4 %.

5. Conclusions
1. A mixed boundary-value problem is formulated for a one-dimensional parabolic equation describing the temperature field in the areas with rock and coal concentrations near self-heating zones.
2. Using the method of separation of variables, a solution of the mixed problem is constructed, on the basis of which a formula is obtained that makes it possible to calculate the temperature of the rock and coal concentration at any distance from the self-heating zone at any point of time.
3. During the analysis of the received formula it is established:
   a) when the coefficient of heat exchange of the rock and coal concentration increases from 1.5 $\text{ W/(m}^2\cdot\text{K)}$ to 4 $\text{ W/(m}^2\cdot\text{K)}$ in the direction from the zone of self-heating to the face, the relative concentration temperature decreases by 12 %, and in the perpendicular direction – only by 1%;
   b) with the increase in the thermal conductivity within the interval $[0.25; 0.75] \text{ W/(m} \cdot \text{K)}$, the relative temperature increases by 11%, and with a change in the specific heat capacity in the interval $[250; 1200] \text{ J/(kg} \cdot \text{K)}$ – only by 0.4 %.
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