Time variation of proton-electron mass ratio and fine structure constant with runaway dilaton

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Abstract

Recent astrophysical observations indicate that the proton-electron mass ratio and the fine structure constant have gone through nontrivial time evolution. We discuss their time variation in the context of a dilaton runaway scenario with gauge coupling unification at the string scale $M_s$. We show that the choice of adjustable parameters allows them to fit the same order magnitude of both variations and their (opposite) signs in such a scenario.

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I. INTRODUCTION

In unified theories of fundamental interactions, a variety of fundamental constants are not necessarily “constant” but can vary as a function of spacetime. Therefore, many experiments and observations have been done to test the constancy of various fundamental constants [1]. Among them, several groups report nonvanishing time variation of some of the fundamental constants. For example, Murphy et al. report time variability of the fine structure constant \( \alpha \) by use of absorption systems in the spectra of distant quasars [2]. They found that the fine structure constant \( \alpha \) was smaller in the past,

\[
\frac{\Delta \alpha}{\alpha} = (-0.543 \pm 0.116) \times 10^{-5},
\]

for the redshift range \( 0.2 < z < 3.7 \),\(^1\) though similar observations of other groups do not necessarily reproduce this result [4, 5]. The linear interpolation of such a change yields the rate of the change,

\[
\frac{\dot{\alpha}}{\alpha} = O(10^{-15}) \text{ yr}^{-1} = O(10^{-65}) M_G,
\]

where the dot represents the time derivative and \( M_G = 1/\sqrt{8\pi G} \) is the reduced Planck scale.

The observations of \( \text{H}_2 \) spectral lines in the Q 0347-383 and Q 0405-443 quasars also suggest a fractional change in the proton-electron mass ratio \( \mu = m_p/m_e \),

\[
\frac{\Delta \mu}{\mu} = (2.4 \pm 0.6) \times 10^{-5},
\]

for a weighted fit [6], which implies that it has decreased over the last 12 Gyr. The linear interpolation of such a change yields the rate of the change,

\[
\frac{\dot{\mu}}{\mu} \simeq -2.0 \times 10^{-15} \text{ yr}^{-1} = -1.7 \times 10^{-65} M_G.
\]

Thus, though there are still large uncertainties, the hints of the time variation of fundamental constants are found.

On the other hand, from the theoretical point of view, it is natural to allow time and space dependence of fundamental constants. In fact, superstring theory, which is expected to unify all fundamental interactions, predicts the existence of a scalar partner \( \phi \) (called dilaton) of the tensor graviton, whose expectation value determines the string coupling constant \( g_s = e^{\phi/2} \) [7]. The couplings of the dilaton to matter induces the violation of the equivalence principle and hence generates deviations from general relativity. Therefore, though the dilaton is predicted to be massless at tree level, it is usually assumed that it acquires a sufficiently large mass, associated with supersymmetry breaking, to satisfy the present experimental constraints on the equivalence principle.

However, Damour and Polyakov proposed another possibility which can naturally reconcile a massless dilaton with experimental constraints [8]. They pointed out that full string-loop effects modify the four dimensional effective low-energy action as

\[
S = \int d^4x \sqrt{\bar{g}} \left( \frac{B_3(\phi)}{\alpha'} \bar{R} + \frac{B_3(\phi)}{\alpha'} \left[ 2 \Box \phi - (\nabla \phi)^2 \right] - \frac{1}{4} B_F(\phi) \bar{F}^2 - V + \cdots \right),
\]

where \( \bar{g} \) is the Einstein frame metric.

\(^1\) In [3], the data is updated but the time variation has almost the same value \( \Delta \alpha/\alpha = (-0.57 \pm 0.11) \times 10^{-5} \).

\(^2\) In this paper, we use the units of \( c = \hbar = 1 \).
Here $B_i(\phi)$ ($i = g, \phi, F, \ldots$) are $\phi$ dependent coupling functions. String scale $M_s$ is given by $M_s = \alpha'^{-1/2}$. In the weak coupling limit $g_s \to 0$ ($\phi \to -\infty$), they are expanded into

$$B_i(\phi) = e^{-\phi} + c_0^{(i)} + c_1^{(i)} e^\phi + c_2^{(i)} e^{2\phi} + \cdots,$$

which comes from genus expansion of string theory with $B_i = \sum_n g_s^{2(n-1)} c_n^{(i)} (n = 0, 1, 2, \ldots)$. Assuming a universality of the dilaton coupling functions, that is, the existence of a value $\phi_m$ of $\phi$ which extremizes all the coupling functions $B_i^{-1}(\phi)$, it has been shown that, during a primordial inflationary stage, the dilaton evolves towards the special value $\phi_m$, at which it decouples from matter (so-called "Least Coupling Principle")\(^8\). Subsequent (slight) change of the dilaton induces the time variation of fundamental constants. Note that the dilaton becomes almost homogeneous in space during inflation so that spatial variations of fundamental constants are expected to be much smaller than their time variations.

On the other hand, in the infinite bare coupling limit $g_s \to \infty$ ($\phi \to +\infty$), it is suggested that all the coupling functions have smooth finite limits\(^8\),

$$B_i(\phi) = C_i + A_i e^{-\phi} = C_i (1 + d_i e^{-\phi}),$$

with $d_i \equiv A_i/C_i$. Since $A_i$ ($d_i$) is expected to be positive in the “large N”-type toy model of\(^8\), we assume that all $A_i$’s are positive in this paper. All the coupling functions are extremized (minimized) at $\phi_m = +\infty$. In this case, also, the dilaton evolves towards its fixed point at infinity during inflation so that it decouples from matter\(^8\). In Ref.\(^8\), assuming the dilaton coupling to dark matter and/or dark energy, the magnitude of the time variation of the fine structure constant is estimated.

In this paper, we discuss time variations of the proton-electron mass ratio and the fine structure constant in the context of a dilaton runaway scenario with gauge coupling unification at the string scale $M_s$. We show that our model can account for the putative time variation of these constants of the same magnitude with the opposite signs.

The rest of the paper is organized as follows. In the next section, we calculate time variation of the proton-electron mass ratio and that of the fine structure constant by taking into account effects associated with thresholds in renormalization group running and variations in the vacuum expectation value (VEV) of the Higgs field.\(^3\) We then apply it to the specific case of a runaway dilaton scenario in §III. In §IV we show that our model can not only fit the observations well but also is testable by the experiments to verify the equivalence principle. Finally §V is devoted to the conclusion.

II. RENORMALIZATION GROUP ANALYSIS OF THE TIME VARIATION OF THE FUNDAMENTAL CONSTANTS

In this section, we calculate time variation rates of fundamental constants such as the proton-electron mass ratio and the fine structure constant from a fundamental point of view using renormalization group analysis. As for the particle contents, we concentrate on the standard model and its minimal supersymmetric extension, although our discussions could proceed in the same way for more extended models as well.

\(^{3}\) For related works in the context of GUT, see\(^\text{[12]}\).
A. $\dot{\mu}$

First of all, we focus on the time variation of the proton-electron mass ratio $\mu = m_p/m_e$. The time variation of $\mu$ is given by

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{m}_p}{m_p} - \frac{\dot{m}_e}{m_e}. \quad (8)$$

Though the proton mass $m_p$ depends not only on the QCD scale $\Lambda_{QCD}$ but also on the masses of the up quark and the down quark, we set $m_p$ to be proportional to $\Lambda_{QCD}$ because these quark masses are much smaller than $\Lambda_{QCD}$. Then assuming $|\dot{m}_u|, |\dot{m}_d| \ll |\dot{\Lambda}_{QCD}|$, the time variation of $m_p$ is given by

$$\frac{\dot{m}_p}{m_p} = \frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}}. \quad (9)$$

The QCD scale $\Lambda_{QCD}$ can be extracted from the Landau pole of the renormalization group equations as

$$0 = \frac{1}{\alpha_3(\Lambda_{QCD})} = \frac{1}{\alpha_X(M_s)} + \frac{b_3^s}{2\pi} \ln \left( \frac{M_s}{M_{SUSY}} \right) + \frac{b_3^b}{2\pi} \ln \left( \frac{M_{SUSY}}{m_t} \right) + \frac{b_3^{b-c}}{2\pi} \ln \left( \frac{m_t}{m_b} \right) + \frac{b_3^c}{2\pi} \ln \left( \frac{m_c}{\Lambda_{QCD}} \right). \quad (10)$$

Here the parameters $b_i$ are given by $b_i = (b_1, b_2, b_3) = (41/10, -19/6, -7), b_i^s = (b_1^s, b_2^s, b_3^s) = (33/5, 1, -3), b_i^{b-c} = -23/3, b_3^{b-c} = -25/3, b_3^c = -9$. $\alpha_X = \alpha_X(M_s)$ is the gauge coupling unified at the string scale $M_s$, $M_{SUSY}$ is the supersymmetry (SUSY) breaking scale, and $m_t, m_b, m_c$ are the masses of top, bottom, and charm quarks, respectively. Reduction to the case of nonsupersymmetric theory would be obvious, that is, we take $M_{SUSY} = M_s$.

Then, the time variation of the QCD scale is given by

$$\frac{b_3^s}{2\pi} \frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} = -\frac{\dot{\alpha}_X}{\alpha_X^2} + \frac{b_3^s}{2\pi} \frac{\dot{M}_s}{M_s} + \frac{b_3 - b_3^{b-c}}{2\pi} \frac{\dot{M}_{SUSY}}{M_{SUSY}} - \frac{b_3}{2\pi} \frac{\dot{m}_t}{m_t} + \frac{b_3^c}{2\pi} \frac{\dot{m}_c}{m_c} + \frac{b_3^{b-c}}{2\pi} \left( \frac{\dot{m}_b}{m_b} - \frac{\dot{m}_t}{m_t} \right) + \frac{b_3^c}{2\pi} \left( \frac{\dot{m}_c}{m_c} - \frac{\dot{m}_b}{m_b} \right). \quad (11)$$

Here and hereafter, for simplicity, we assume the universality of the time dependence of fermion masses, that is, the time variation of all the fundamental fermion masses except three light quarks, $u, d,$ and $s$ is identical, which is denoted by $\dot{m}_f/m_t$. Later we present a set of sufficient conditions to realize such a universality in the context of a dilaton runaway scenario. Under such a universality, the last two terms of the right hand side of the above equation are dropped. Then, the time variation of the proton mass is given by

$$\frac{\dot{m}_p}{m_p} = \frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} = \frac{2\pi}{9} \frac{\dot{\alpha}_X}{\alpha_X^2} + \frac{1}{3} \frac{\dot{M}_s}{M_s} + \frac{4}{9} \frac{\dot{M}_{SUSY}}{M_{SUSY}} + \frac{2}{9} \frac{\dot{m}_t}{m_t}. \quad (12)$$

Note that the light quark masses may contribute to the proton mass, which may lead to the slight change of the coefficient in Eq. (12).

The current masses of these light quarks are irrelevant to our analysis.
After all, the time variation of the proton-electron mass ratio \( \mu = m_p/m_e \) becomes
\[
\frac{\dot{\mu}}{\mu} = \frac{2\pi}{9} \frac{\dot{\alpha}_X}{\alpha_X^2} + \frac{1}{3} \frac{\dot{M}_s}{M_s} + \frac{4}{9} \frac{\dot{M}_{\text{SUSY}}}{M_{\text{SUSY}}} - \frac{7}{9} \frac{\dot{m}_f}{m_f},
\]
where we have used the universality of the time variation of fermion masses \( \dot{m}_e/m_e = \dot{m}_f/m_f \).

B. \( \dot{\alpha} \)

Next, we discuss the time variation of the fine structure constant \( \alpha \). The renormalization group equations yield the scale dependence of the gauge couplings \( \alpha_i \) (\( i = 1,2 \))
\[
\frac{1}{\alpha_i(M_{\text{EW}})} = \frac{1}{\alpha_X} + \frac{b_i}{2\pi} \ln \left( \frac{M_{\text{SUSY}}}{M_{\text{EW}}} \right) + \frac{b_i}{2\pi} \ln \left( \frac{M_{\text{SUSY}}}{M_{\text{EW}}} \right),
\]
where \( M_{\text{EW}} \simeq M_Z \) is the electroweak symmetry breaking scale. Assuming \( SU(5) \) GUT, for example, the fine structure constant \( \alpha \) can be related to the gauge couplings \( \alpha_i \) (\( i = 1,2 \)) at any scale as
\[
\frac{1}{\alpha(M_{\text{EW}})} = \frac{5}{3 \alpha_1(M_{\text{EW}})} + \frac{1}{\alpha_2(M_{\text{EW}})} = \frac{8}{3} \frac{1}{\alpha_X} + \frac{6}{\pi} \ln \left( \frac{M_s}{M_{\text{SUSY}}} \right) + \frac{11}{6\pi} \ln \left( \frac{M_{\text{SUSY}}}{M_{\text{EW}}} \right),
\]
for \( M_{\text{EW}} \leq M_{\text{SUSY}} \). After the electroweak symmetry breaking, charged fields acquire their masses. Therefore, taking their mass thresholds into account, the fine structure constant \( \alpha \) is given by
\[
\frac{1}{\alpha} = \frac{1}{\alpha(M_{\text{EW}})} + \frac{b_\alpha}{2\pi} \ln \left( \frac{M_{\text{EW}}}{m_{f_i}} \right) + \sum_{f_i} \frac{b_{f_i-f_{i+1}}}{2\pi} \ln \left( \frac{m_{f_i}}{m_{f_{i+1}}} \right).
\]
Here, the third term in the right hand side corresponds to fermion mass thresholds, where \( f_i = t, b, \cdots, e \), \( b_\alpha = 32/3 \) and \( b_{f_i-f_{i+1}} \) denotes beta-function coefficients for fermion mass thresholds between \( f_i \) and \( f_{i+1} \).

Then, the time variation of the fine structure constant is given by
\[
\frac{\dot{\alpha}}{\alpha^2} = \frac{80}{27} \frac{\dot{\alpha}_X}{\alpha_X^2} - \frac{50}{9\pi} \frac{\dot{M}_s}{M_s} + \frac{257}{54\pi} \frac{\dot{M}_{\text{SUSY}}}{M_{\text{SUSY}}} - \frac{7}{2\pi} \frac{\dot{M}_{\text{EW}}}{M_{\text{EW}}} + \frac{116}{27\pi} \frac{\dot{m}_f}{m_f},
\]
where we have used the universality of the time variation of fermion masses except three light quarks u, d, s, and \( m_u \simeq m_d \simeq m_s \simeq \Lambda_{\text{QCD}} \) because light quarks are confined and effectively have a mass of the order of \( \Lambda_{\text{QCD}} \).

We have assumed that the gauge couplings are unified to \( \alpha_X \). The gauge coupling unification is consistent with experimental values on the gauge couplings within the framework of the minimal supersymmetric standard model, but not in the (non-SUSY) standard model. For the latter case, we need some corrections to gauge couplings at \( M_s \). Such corrections can appear from gauge kinetic functions, which depend on moduli fields other than the dilaton. We assume that moduli-dependent corrections to the gauge couplings do not vary while only the dilaton varies in the time range relevant to our analysis.
C. Universality of Time Variation of Fermion Masses

The four dimensional effective low-energy action related to the generation of fermion masses is given by

\[ S \supset \int d^4x \sqrt{-g} \left[ B_{H_u} (\phi) (D_\mu \tilde{H}_u)^\dagger D^\mu \tilde{H}_u + B_{H_d} (\phi) (D_\mu \tilde{H}_d)^\dagger D^\mu \tilde{H}_d ight. \\
+ i B_{Q_j \bar{Q}_j} \bar{D}_j \bar{Q}_j + i B_{u_i \bar{u}_i} \bar{D}_i \bar{u}_i + i B_{d_j \bar{d}_j} \bar{D}_j \bar{d}_j + i B_{L_j \bar{L}_j} \bar{D}_j \bar{L}_j + i B_{e_j \bar{e}_j} \bar{D}_j \bar{e}_j \\
+ i B_{y_u} \tilde{y}_u \bar{Q}_j \tilde{u}_j \tilde{H}_u + i B_{y_d} \tilde{y}_d \bar{Q}_j \tilde{d}_j \tilde{H}_d + i B_{y_e} \tilde{y}_e \bar{L}_j \tilde{e}_j \tilde{H}_d \right], \tag{18} \]

where \( \bar{D} = \gamma^\mu D_\mu \), \( D_\mu \) represents the covariant derivative, and \( j \) is the generation index. Canonically normalizing all fields yield effective Yukawa couplings and fermion masses,

\[ y_{u_j} = \frac{B_{y_{u_j}}}{\sqrt{B_{Q_j} B_{u_j} B_{H_u}}}, \quad m_{u_j} = \frac{y_{u_j} v_u}{\sqrt{2}}, \]
\[ y_{d_j} = \frac{B_{y_{d_j}}}{\sqrt{B_{Q_j} B_{d_j} B_{H_d}}}, \quad m_{d_j} = \frac{y_{d_j} v_d}{\sqrt{2}}, \]
\[ y_{e_j} = \frac{B_{y_{e_j}}}{\sqrt{B_{L_j} B_{e_j} B_{H_d}}}, \quad m_{e_j} = \frac{y_{e_j} v_d}{\sqrt{2}}. \tag{19} \]

Uplike and downlike fermions acquire masses \( m_{u_j} = y_{u_j} v_u / \sqrt{2} \) and \( m_{d_j} = y_{d_j} v_d / \sqrt{2} \) through the Higgs mechanism, respectively. Here \( y_{ij} \) is a Yukawa coupling constant and \( v_u \) and \( v_d \) are the VEV of the up-type and down-type Higgs fields, respectively. Note that \( v_u(v_d) \) is replaced by the VEVs of the standard Higgs field \( v \) in the case of the nonsupersymmetric standard model with a single Higgs doublet.

Then, the time variation of fermion masses is given by

\[ \frac{\dot{m}_{u_j}}{m_{u_j}} = \frac{\dot{y}_{u_j}}{y_{u_j}} + \frac{\dot{v}_u}{v_u}, \]
\[ \frac{\dot{m}_{d_j}}{m_{d_j}} = \frac{\dot{y}_{d_j}}{y_{d_j}} + \frac{\dot{v}_d}{v_d}, \]
\[ \frac{\dot{m}_{e_j}}{m_{e_j}} = \frac{\dot{y}_{e_j}}{y_{e_j}} + \frac{\dot{v}_d}{v_d}. \tag{20} \]

Thus, the universality of the time variation of fermion masses is realized, for example, if the following conditions are satisfied,

\[ \frac{\dot{y}_{u_j}}{y_{u_j}} = \frac{\dot{y}_{d_k}}{y_{d_k}} = \frac{\dot{y}_{e_l}}{y_{e_l}} = \frac{\dot{y}_t}{y_t}, \quad \frac{\dot{v}_u}{v_u} = \frac{\dot{v}_d}{v_d} = \frac{\dot{v}}{v}, \tag{21} \]

where \( j, k, l \) are generation indices and the last equality comes from \( v^2 = v_u^2 + v_d^2 \). Note that the second condition is unnecessary in the case of the nonsupersymmetric minimal standard model. Here, we have implicitly assumed that the scale dependence of the Yukawa couplings is negligible, that is, the time dependence of the Yukawa couplings is dominated by their dilaton dependence.
Here, we comment on radiative corrections on Yukawa couplings. Renormalization group effects on Yukawa couplings are the same among quarks except the top quark. Thus, radiative corrections do not violate the universal time variation of Yukawa couplings among quarks except the top quark. Furthermore, the mass ratios of other quarks to the top quark do not change drastically between \( M_s \) and \( M_Z \), i.e. \( m_t(M_s)/m_t(M_Z) \sim m_t(M_Z)/m_t(M_Z) \). Hence, even including radiative corrections, the universal time variation of quark Yukawa couplings would be a reasonable assumption, and such corrections on the time variation on \( \Lambda_{QCD} \) would be sufficiently small. The same discussion holds true for the universal time variation of Yukawa couplings only among leptons. However, radiative corrections on quark Yukawa couplings are different from those on lepton Yukawa couplings, because of corrections from \( \alpha_3 \). Such difference would be estimated as \( \dot{Y}_q/Y_q - \dot{Y}_l/Y_l = a \dot{\alpha} X \), where \( |a| \leq O(1) \), and it has some effect on \( \dot{\alpha}/\alpha \) Eq. (17), but it can be neglected compared with the first term in Eq. (17).

D. Electroweak scale, Higgs VEV, and SUSY scale

Before investigating the time dependence of the VEV of the Higgs fields \( \dot{v}/v \), we discuss the time dependence of the electroweak symmetry breaking scale \( M_{EW} \) characterized by the gauge boson mass \( M_Z \),

\[
M_{EW} \simeq M_Z = \frac{v}{2} \sqrt{g^2 + g'^2} = v \sqrt{\frac{3}{5}} \alpha_1 + \alpha_2 ,
\]

(22)

where \( \alpha_i = \alpha_i(M_{EW}) \), and \( g \) and \( g' \) are \( SU(2)_L \) and \( U(1)_Y \) gauge coupling constants respectively. Then, the time variation of the electroweak symmetry breaking scale \( \dot{M}_{EW}/M_{EW} \) is given by

\[
\frac{\dot{M}_{EW}}{M_{EW}} = \frac{3\dot{\alpha}_1 + 5\dot{\alpha}_2}{2(3\alpha_1 + 5\alpha_2)} \frac{\dot{v}}{v} .
\]

(23)

Inserting Eq. (14) into this equation yields

\[
\frac{\dot{M}_{EW}}{M_{EW}} \simeq \frac{3\dot{\alpha}_1 + 5\dot{\alpha}_2}{2(3\alpha_1 + 5\alpha_2)} \frac{\dot{\alpha} X}{\alpha X} - \frac{99\alpha_1^2 + 25\alpha_2^2}{20\pi(3\alpha_1 + 5\alpha_2)} \frac{\dot{M}_s}{M_s} + \frac{45\alpha_1^2 + 125\alpha_2^2}{24\pi(3\alpha_1 + 5\alpha_2)} \frac{\dot{M}_{SUSY}}{M_{SUSY}} + \frac{\dot{\alpha}}{v} \simeq \frac{\dot{v}}{v} ,
\]

(24)

where we have used \( \alpha_1 \sim 1/60 \ll 1 \) and \( \alpha_2 \sim 1/30 \ll 1 \).

We now investigate the time variation of the VEV of the Higgs fields, \( \dot{v}/v \). First, we consider the nonsupersymmetric minimal standard model. The Lagrangian density related to the standard Higgs field is expected to read

\[
\mathcal{L} \supset B_\lambda(\phi) (D_\mu \tilde{H})^\dagger D^\mu \tilde{H} - B_\lambda(\phi) \tilde{\lambda} \left( \tilde{H}^\dagger \tilde{H} - \frac{v^2}{2} \right)^2 = (D_\mu H)^\dagger D^\mu H - \lambda \left( H^\dagger H - \frac{v^2}{2} \right)^2 ,
\]

(25)

with \( H \equiv \sqrt{B_H} \tilde{H}, \lambda \equiv B_\lambda \tilde{\lambda} / B_H^2 \), and \( v^2 \equiv B_H \tilde{v}^2 \). Assuming that \( \tilde{v} \) is intrinsic and has no time dependence, the time variation of the VEV of the standard Higgs field is given by

\[
\frac{\dot{v}}{v} = \frac{1}{2} \frac{\dot{B}_H}{B_H} .
\]

(26)
Next, we consider the SUSY model. The neutral components of up and down sector Higgs fields, \( h_u \) and \( h_d \), have the following potential,

\[
V = m_1^2 |h_d|^2 + m_2^2 |h_u|^2 + b(h_d h_u + h.c.) + \frac{1}{8} (g^2 + g'^2) \left(|h_d|^2 - |h_u|^2\right)^2 ,
\]

(27)

with

\[
m_1^2 = m_{H_d}^2 + \mu_H^2, \quad m_2^2 = m_{H_u}^2 + \mu_H^2,
\]

(28)

where \( m_{H_u,d}^2 \) are SUSY breaking scalar masses squared of Higgs fields, \( h_u,d \), and \( \mu_H \) is the supersymmetric mass parameter. In addition, the parameter \( b \) is also SUSY breaking parameter with mass dimension two, that is, the so-called \( b \)-term.

By using the stationary conditions, \( \partial V / \partial h_u,d = 0 \), we obtain

\[
\frac{1}{4} (g^2 + g'^2) v^2 = -m_1^2 - m_2^2 + \frac{\tan^2 \beta + 1}{\tan^2 \beta - 1} (m_1^2 - m_2^2),
\]

(29)

\[
b (\tan \beta + \cot \beta) = m_1^2 + m_2^2,
\]

(30)

where \( \tan \beta = v_u/v_d \). Furthermore, for a moderate and/or large value of \( \tan \beta \), i.e. \( \tan^2 \beta > O(1) \), these equations reduce to

\[
\frac{1}{4} (g^2 + g'^2) v^2 = -m_2^2,
\]

(31)

\[
b \tan \beta = m_1^2 + m_2^2.
\]

(32)

That implies that if the time variation of mass parameters is the same, i.e.

\[
\frac{d}{dt} \ln \mu_H^2 = \frac{d}{dt} \ln m_{H_u}^2 = \frac{d}{dt} \ln m_{H_d}^2 = \frac{d}{dt} \ln b,
\]

(33)

\( \tan \beta \) does not vary, that is, \( \dot{v}_u/v_u = \dot{v}_d/v_d \). The above assumption might be plausible for mass parameters at tree level, but \( m_{H_u}^2 \) has a significant radiative correction due to stop mass,

\[
\delta m_{H_u}^2 \sim - \frac{3 y_t^2 m_t^2}{4\pi^2} \ln(M_s/m_t),
\]

(34)

where \( m_t \) is SUSY breaking stop mass. Thus, in general, the value of \( \tan \beta \) varies in time, and the time variations of \( \dot{v}_u/v_u \) and \( \dot{v}_d/v_d \) are different. To take into account this aspect, we have to consider the situation that the time dependence of up-type quark masses are different from those of down-type quark masses and lepton masses, and we have to introduce another parameter to represent such difference. Such extension of our analysis is straightforward and would enlarge a favorable parameter space. (Note that because of \( M_{\text{SUSY}}/M_{\text{SUSY}} \) the SUSY model has more degrees of freedom than the non-SUSY standard model.) Similarly, the time variation of \( v \) also depends on those of several values, \( \mu_H, m_{H_u}^2, y_t, m_t^2 \) as well as the gauge couplings. To simplify our analysis, we use the same parameterization as the non-SUSY model Eq.(26).

Now let us discuss the time dependence of the SUSY breaking scale \( M_{\text{SUSY}} \). Although it strongly depends on the SUSY breaking model, we give one example based on the gaugino condensation and gravity mediation model. We consider a hidden sector, in which a hidden gauge group with a coupling \( \alpha_h \) blows up and hence gauginos \( \lambda^a \) condensate at some scale.
\( M_c \), which breaks the SUSY. Then, repeating the same argument as the case of the QCD scale, the condensation scale \( M_c \) is given through the RG flow by

\[
M_c = M_s e^{-\frac{2\pi}{\alpha_h(M_s) b_h}}, \quad \langle \lambda^a \lambda^a \rangle = M_c^3 \neq 0,
\]

where \( b_h \) is the beta-function coefficient which, for example, is given by \( b_h = 3 N_c \) for the gauge group \( SU(N_c) \). If this breaking is transmitted to the visible sector through the gravitational interaction, the SUSY breaking scale \( M_{\text{SUSY}} \) is given by

\[
M_{\text{SUSY}} = \frac{8\pi GM_c^3}{M_G^2}.
\]

In this case, the time variation of the SUSY breaking scale is given by

\[
\frac{\dot{M}_{\text{SUSY}}}{M_{\text{SUSY}}} = 3 \frac{\dot{M}_c}{M_c} - 2 \frac{\dot{M}_G}{M_G} = -\frac{6\pi}{b_h} \frac{\dot{\alpha}_h(M_s)}{\alpha_h^2(M_s)} - 2 \frac{\dot{M}_G}{M_G}
\]

### III. TIME VARIATION IN A RUNAWAY DILATON SCENARIO

#### A. Runaway Dilaton

Now, we estimate the time variation of the proton-electron mass ratio and the fine structure constant in the context of a runaway dilaton scenario. From the four dimensional effective low-energy action (5), we have the following relations,

\[
B_g(\phi) M_s^2 = \frac{1}{16\pi G},
\]

\[
B_F(\phi) = \frac{1}{8\pi \alpha_X}.
\]

Since the coupling functions are given in Eq. (7), the dilaton dependence of the gravitational coupling \( G \) and the unified gauge coupling \( \alpha_X \) is given by

\[
M_G^2 = (8\pi G)^{-1} = 2 M_s^2 B_g(\phi) = 2 M_s^2 C_g(1 + d_g e^{-\phi}),
\]

\[
\alpha_X^{-1} = 8\pi B_F(\phi) = 8\pi C_F(1 + d_F e^{-\phi}),
\]

which leads to

\[
\frac{\dot{M}_G}{M_G} = -\frac{d_g}{2(1 + d_g e^{-\phi})} \dot{\phi} + \frac{\dot{M}_s}{M_s} \approx -\frac{1}{2} d_g e^{-\phi} \dot{\phi} + \frac{\dot{M}_s}{M_s},
\]

\[
\frac{\dot{\alpha}_X}{\alpha_X^2} = 8\pi A_F e^{-\phi} \dot{\phi} > 0,
\]

where we set \( \dot{\phi} > 0 \) without generality and we have assumed \( e^{-\phi} \ll 1 \). We regard the string scale \( M_s \) as fundamental and hence it has no time dependence \( \dot{M}_s = 0 \).

In the context of the dilaton runaway scenario, the sufficient conditions for the universality of the time variation of Yukawa couplings, (21), are satisfied, for example, in the case that
the dilaton dependent functions have the following properties,

\[
B_{Q_i} = B_{L_k} \equiv B_D = C_D + A_D e^{-\phi} = C_D (1 + d_D e^{-\phi}), \\
B_{u_j} = B_{d_k} = B_{e_l} \equiv B_S = C_S + A_S e^{-\phi} = C_S (1 + d_S e^{-\phi}), \\
B_{y_{u_j}} = B_{y_{d_k}} = B_{y_{e_l}} \equiv B_Y = C_Y + A_Y e^{-\phi} = C_Y (1 + d_Y e^{-\phi}), \\
B_{H_u} = B_{H_d} \equiv B_H = C_H + A_H e^{-\phi} = C_H (1 + d_H e^{-\phi}).
\]

Hereafter we assume that the dilaton dependent functions satisfy the above conditions (41).

In this case, the universal time variation of fermion masses \( \dot{m}_f/m_f \) is given by

\[
\frac{\dot{m}_f}{m_f} = \frac{\dot{y}_f}{y_f} + \frac{\dot{\nu}}{\nu},
\]

where the universal time variation of Yukawa couplings \( y_f \) is estimated as

\[
\frac{\dot{y}_f}{y_f} \approx -d e^{-\phi} \dot{\phi},
\]

with

\[
d \equiv d_y - (d_D + d_S + d_H)/2,
\]

and (26) reads

\[
\frac{\dot{\nu}}{\nu} = -d_H e^{-\phi} \dot{\phi} \approx -\frac{1}{2} d_H e^{-\phi} \dot{\phi}.
\]

Note that \( d \) can take either a positive value or a negative one, which is an important point to account for the decline of \( \mu \).

After all, the universal time variation of fermion masses is given by

\[
\frac{\dot{m}_f}{m_f} = \frac{\dot{y}_f}{y_f} + \frac{\dot{\nu}}{\nu} \approx -\left( d + \frac{d_H}{2} \right) e^{-\phi} \dot{\phi}.
\]

B. \( \dot{\mu} \) and \( \dot{\alpha} \) in the Runaway Dilaton Scenario

Finally, we show that the choice of adjustable parameters allows them to fit the observed time variation of the proton-electron mass ratio and the fine structure constant in the dilaton runaway scenario, and give constraints on the parameters.

In the case of the non-SUSY model (\( M_{\text{SUSY}} = 0 \)), the time variation of the proton-electron mass ratio \( \mu = m_p/m_e \) reads

\[
\frac{\dot{\mu}}{\mu} = \frac{2\pi \dot{\alpha}_X}{9 \alpha_X^2} - \frac{7 \dot{m}_f}{9 m_f} \\
\approx \frac{2\pi}{9} e^{-\phi} \dot{\phi} \left[ 8\pi A_F + \frac{7}{2\pi} \left( d + \frac{d_H}{2} \right) \right],
\]

where we have used Eqs. (10) and (46) in the second equality. As is seen in Eq. (14), \( d \) can be negative. On the other hand, the time variation of the fine structure constant is given by

\[
\frac{\dot{\alpha}}{\alpha^2} = \frac{80 \dot{\alpha}_X}{27 \alpha_X^2} - \frac{7}{2\pi} \frac{\dot{M}_{\text{EW}}}{\alpha^2 M_{\text{EW}}} + \frac{116 \dot{m}_f}{27\pi m_f},
\]
\[
\frac{29}{160\pi^2} \left( d + \frac{43}{464} d_H \right) \lesssim A_F \lesssim -\frac{7}{16\pi^2} \left( d + \frac{d_H}{2} \right) .
\]

These constraints can be easily satisfied if \( d < 0 \) and \( |d| \gg d_H \) and \( A_F \lesssim 29|d|/(160\pi^2) \). Though the time variations of the proton-electron mass ratio and the fine structure constant depend on the evolution of the dilaton field \( \phi \), from Eqs. (1), (3), (47), and (48), we can expect that observed variation can be fitted by taking our model parameters appropriately around order of unity.

In the case of the SUSY model, the time variation of the proton-electron mass ratio \( \mu = m_p/m_e \) becomes

\[
\frac{\dot{\mu}}{\mu} = \frac{2\pi}{9} \frac{\dot{\alpha}_X}{\alpha_X^2} + \frac{4}{9} \frac{\dot{M}_{\text{SUSY}}}{M_{\text{SUSY}}} - \frac{7}{9} \frac{\dot{m}_t}{m_t} = \frac{2\pi}{9} e^{-\phi} \phi \left[ 8\pi A_F \left( 1 - \frac{12}{A_{m_b}} \frac{A_{m_b}}{A_{m_b}^2} \right) + 7 \frac{2\pi}{d + \frac{d_H}{2} + \frac{4d_g}{7}} \right] ,
\]

where we have used Eqs. (37), (40), and (46). On the other hand, time variation of the fine structure constant is given by

\[
\frac{\dot{\alpha}}{\alpha^2} = \frac{80}{27} \frac{\dot{\alpha}_X}{\alpha_X^2} + \frac{257}{54\pi} \frac{\dot{M}_{\text{SUSY}}}{M_{\text{SUSY}}} - \frac{7}{2\pi} \frac{\dot{M}_{\text{EW}}}{M_{\text{EW}}} + \frac{116}{27\pi} \frac{\dot{m}_t}{m_t} = \frac{80}{27} e^{-\phi} \phi \left[ 8\pi A_F \left( 1 - \frac{771}{80} \frac{A_{m_b}}{A_{m_b}^2} \right) - \frac{29}{20\pi} \left( d + \frac{43}{464} d_H - \frac{257}{232} d_g \right) \right] .
\]

In this case, too, from Eqs. (1), (3), (50), and (51), we can expect that our parametrization can fit the observed variation naturally. In fact, the SUSY model has more degrees of freedom than the non-SUSY standard model because of \( \dot{M}_{\text{SUSY}}/M_{\text{SUSY}} \). Furthermore, if we introduce the time variation of the ratio of \( \dot{v}_u/v_u \) to \( \dot{v}_d/v_d \) as well, we have a wider parameter space to fit the observed variation.

\section{Comparison with Observations}

Having formulated the time variation of the fundamental constants in the particle-physics context and given their explicit form in the dilaton runaway scenario, we now solve cosmological evolution of the dilaton field to show our model can account for the observed variation. Before giving an explicit result, however, we must consider other experimental consequences of the dilaton coupling which impose a stringent constraints on the parameter space.
A. Experimental Constraints on Dilaton Coupling

The dilaton coupling to hadronic matter induces deviations from general relativity: post-Newtonian deviations from general relativity and the violations of the equivalence principle \[8, 11\]. After integration by parts, the action of the dilaton is rewritten as

\[
S = \int d^4x \sqrt{g} \left( \frac{B_g(\phi)}{\alpha'} \tilde{R} - \frac{Z(\phi)}{\alpha'} (\tilde{\nabla})^2 \phi - V(\phi) + \cdots \right),
\]

where \(Z(\phi) = C_\phi - A_\phi e^{-\phi}\) and \(B_g(\phi) = C_g(1 + d_g e^{-\phi})\).

From the effective mass \(m(\phi)\) of a test particle (composed of hadronic matter) in the Einstein frame metric \(g_{\mu\nu} = B_g(\phi)g_{\mu\nu}\), using Eq. (12), the strength of the coupling of the dilaton to hadronic matter, \(\alpha_{had}\), is given by \[8, 11\]

\[
\alpha_{had} \approx \sqrt{\frac{2B_g(\phi)}{Z(\phi)} d \ln m(\phi)} \approx \sqrt{\frac{2B_g(\phi)}{Z(\phi)} \left[ \frac{d \ln m_p(\phi)}{d\phi} - \frac{1}{2} \frac{d m(\phi)}{d\phi} \right]},
\]

\[
\approx e^{-\phi} \sqrt{\frac{2C_\phi}{C_g}} \left[ \frac{16\pi^2}{9} A_F - \frac{2}{9} \left( d + \frac{d_H}{2} \right) + \frac{1}{2} d_g \right],
\]

where the non-SUSY standard model is assumed in the second line. Since the mass of a test particle depends on the dilaton, the test particle experiences an acceleration, \(-\nabla \ln m\), in addition to the usual free-fall acceleration \(g\), which results in the violation of the universality of free-fall. \(\alpha_{had}\) is related to the Eddington parameter \(\gamma\) measuring a post-Newtonian deviation from general relativity and to the Eötvös ratio \(\eta\) measuring the difference in accelerations, \(a_I\), between the two test masses \((I = A, B) [8, 11]\):

\[
\gamma - 1 = -\frac{2\alpha_{had}^2}{1 + \alpha_{had}^2} \approx -2\alpha_{had}^2,
\]

\[
\eta \equiv \frac{a_A - a_B}{a_A + a_B} \approx 5.2 \times 10^{-5} \alpha_{had}^2 \approx -2.6 \times 10^{-5}(\gamma - 1).
\]

The present experimental constraints are \(\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} [14]\) and, \(\eta = (-1.9 \pm 2.5) \times 10^{-12}\) for \(A = \text{Be}\) and \(B = \text{Cu} [15]\), and \(\eta = (0.1 \pm 4.4) \times 10^{-13}\) from the measurement of the composite-dependent differential acceleration of Earth and Moon toward the Sun, which are made of several materials such as Fe, Ni, Si, Mg, O [16].

B. Dilaton Evolution

We numerically solve the cosmological evolution of the dilaton to give a concrete example which can account for the time variations of \(\mu\) and \(\alpha\). Its dynamics is determined by specifying the dilaton potential \(V(\phi)\). We assume the following form of \(V(\phi)\) in accord with Eq. (7): \(V(\phi) = V_0(1 + d_n e^{-\phi})\). We further assume that the magnitude of \(V_0\) is similar to that of the cosmological constant, \(V_0 \approx 10^{-120}M_G^4\), which is miniscule in the unit of the string scale in order for the dilaton to play the role of dark energy.\(^6\)

\(^6\) We expect that the value of \(V_0\) may be determined by the expectation value of other fields \(\chi\) which are orthogonal to the dilaton and the smallness of \(V_0\) may be related to the largeness of such fields as,
FIG. 1: The time evolution of $\mu$ and $\alpha$ (solid curve) in runaway dilaton scenario. The upper panel is $\mu$ determined from the spectral lines of hydrogen molecules. The data are taken from [6]. The lower panel is $\alpha$ determined from quasar absorption lines at several redshifts. Open circles are the binned data from [2] (in each bin the value of $\Delta\alpha/\alpha$ is the weighted mean), crosses are from [4] and filled circles are from [5]. The datum at $z \simeq 0.16, 0.44$ is the Oklo bound [19] and the $^{187}$Re bound [20], respectively.

Before proceeding to numerical calculations, let us make analytic estimate. From (53) and (55), we find

$$\eta \sim 10^{-5} \alpha_{had}^2 \sim 10^{-5} e^{-2\phi} \frac{2Cg}{C_0} \left[ \frac{16\pi^2}{9} A_F - \frac{2}{9} \left( d + \frac{d_H}{2} \right) + \frac{1}{2} d_g \right]^2 \lesssim 10^{-13}. \quad (56)$$

Assuming the last two factors are of order of unity, we find $e^{-2\phi} \lesssim 10^{-7}$ which implies $\phi \gtrsim 8$. Therefore the dilaton must have run away much beyond the Planck scale by now. In order

$V_0 \simeq M_5^4 e^{-\chi}$. Alternatively, one may argue the smallness of $V_0$ is due to a nontrivial vacuum structure which may lead to $V_0 = M_5^4 e^{-S_E}$ where $S_E$ is a Euclidean action connecting two degenerate vacua [17]. Since the purpose of this paper is to demonstrate that it is possible to fit the observational results, we will not pursue this issue (the problem of the cosmological constant) any further here.
to reproduce the observed time variation, \( \phi \) must change appropriately in the cosmological time scale. Specifically, from (2) and (18) or (51), we require \( \dot{\phi}/\alpha \sim a e^{-\phi} \phi \sim 10^{-65} \), in the Planck unit with \( M_G = 1 \). Using the slow-roll equation of motion we estimate

\[
\dot{\phi} \sim \frac{V'[\phi]}{3H} \sim \frac{V_0 d_v e^{-\phi}}{\sqrt{V_0}} \sim \sqrt{V_0} d_v e^{-\phi} \sim 10^{-60} d_v e^{-\phi},
\]

in the same unit. From these results we find that \( d_v \) must take a fairly large value, \( d_v \gtrsim 10^4 \). We also find a similar value of \( d_v \) from (1).

The presence of nonzero \( A_\phi \) hardly affects the evolution of the dilaton since it is moving slowly: it only slightly facilitates the rolling of \( \phi \) because the equation of motion of \( \phi \) is divided by \( C_\phi - A_\phi e^{-\phi} \). However, the presence of nonzero \( d_g \) greatly affects the evolution since it induces a negative effective potential for \( \phi \): \( V_R(\phi) \equiv -C_g d_g e^{-\phi} R/\alpha' \). \( V_R \) practically vanishes during the radiation dominated epoch when \( R \sim -(a^4/2B_g)T \sim 0 \) with \( T \) being the trace of the energy momentum tensor, and hence the dilaton does not move. It becomes non-negligible during matter dominated epoch. The magnitude of \( V_R \) can be larger than \( V(\phi) \) and hence \( \phi \) can decrease rather than increase [18], quite the opposite to what we want. Therefore, \( d_g \) must be small enough in order to be consistent with the observations.

Provided these conditions are satisfied, there are a wide allowed region in the parameter space that can account for the observed variation, so that we only give one specific example in Fig. 1 where the time variations of \( \mu \) (upper graph) and \( \alpha \) (lower graph) are given. The data of \( \mu \) are taken from [6]. The open circles in \( \alpha \) data which are binned and averaged by redshift are taken from [2], crosses from [4] and filled circles from [5]. We also plot the bound of \( \Delta \alpha/\alpha \) from the analysis of isotope abundances in the Oklo natural reactor operated 2Gyr ago (corresponding to \( z \approx 0.16 \)). We adopt a conservative bound by Damour and Dyson [19]. The bound on \( \Delta \alpha/\alpha \) over the age of the solar system \( \sim 4.6 \)Gyr (\( z \approx 0.44 \)) from the comparison of the \( ^{187}\text{Re} \) meteoritic measurements of the time averaged \( ^{187}\text{Re} \) decay rate with the laboratory measurements is also included. We adopt a conservative bound by Fujii and Iwamoto [20].

In this example, we have considered a non-SUSY theory and taken \( d_g = 0.01, C_\phi = A_\phi = 1, d_v = 10^4 \) for dilaton couplings to gravitational part and \( A_F = 0.001, d = -0.4, d_H = 0.01 \) for those to gauge/matter part. \( C_g \) is used to normalize the gravitational constant and set to be 0.5, which corresponds to \( M_s \sim M_G/\sqrt{2C_g} \sim 2.4 \times 10^{18} \) GeV in this example. However, one should notice that the action (52) is invariant under the following scale transformation: \( M_s \to a M_s, \phi \to \phi, C_g \to C_g/a^2, C_\phi \to C_\phi/a^2, A_\phi \to A_\phi/a^2 \). Therefore, the dynamics of the dilaton is unchanged under the transformation. If we take \( a \approx 1/10 \), the string scale can be lowered to the GUT scale. The initial condition of the dilaton is \( \phi = 8, \dot{\phi} = 0 \) at \( z = 10^{10} \). As is seen in the figure, this example can fit the observational data. Damour and Dyson obtained \( \Delta \alpha/\alpha = (-0.9 \sim 1.2) \times 10^{-7} \) at \( z \approx 0.16 \) from the Oklo mine samples [19]. In our example, \( \Delta \alpha/\alpha = -7.8 \times 10^{-8} \) at \( z = 0.16 \), which satisfies the Oklo bound. The \( ^{187}\text{Re} \) bound is \( |\Delta \alpha/\alpha| < 1.5 \times 10^{-6} \) at \( z \approx 0.44 \) [20], but it is to be noted that the

7 In these nuclear bounds on varying \( \alpha \), if quark masses are allowed to vary, there may be correlations between the effects of varying \( \alpha \) and those of varying quark masses, since nuclear effects depend both on electromagnetic forces and on nuclear forces. These effects would lead to different bounds on \( \alpha \). However, we will not consider such a possibility here for simplicity.
bound depends on the way of time dependence of the $^{187}$Re decay rate $^{20}$. The bound is also satisfied in our example: $\Delta \alpha/\alpha = -1.8 \times 10^{-7}$ at $z = 0.44$. The comparison of different atomic clocks over several years yields a bound on the present-day time variability of $\alpha$ ($\dot{\alpha}/\alpha = (-0.3 \pm 2.0) \times 10^{-15}$ yr$^{-1}$ $^{23}$), which is also satisfied in our example. We also note that this example reproduces the cosmic expansion law of the present universe correctly since the dilaton potential plays the role of the dark energy with $\Omega_d = 0.74$ and the equation-of-state parameter $w \simeq -0.91$.

Although the allowed region in the parameter space is large we mention that there is a tension to account for both the smallness of $\eta$ and the observed values of $\dot{\alpha}/\alpha$ and $\dot{\mu}/\mu$ simultaneously, for the latter requires a relatively large $\phi$ sourced by a large $d_v$. In the present example, we find $\gamma - 1 \simeq -2.9 \times 10^{-9}$ and $\eta \simeq 7.6 \times 10^{-14}$.

Finally we comment on the time variation of the gravitational constant $G$. It is given by $\dot{G}/G = 2\alpha_{\text{had}}\dot{\alpha}_{\text{had}}/(1 + \alpha^2_{\text{had}}) < 0$. However, its magnitude is very small due to the smallness of $\alpha_{\text{had}}$: $|\dot{G}/G| \sim -2\alpha_{\text{had}}|\dot{\alpha}|/24\alpha^2 \sim 10^{-18}$ yr$^{-1}$ and safely satisfies the present experimental constraint $^{24}$: $\dot{G}/G = (4 \pm 9) \times 10^{-13}$ yr$^{-1}$. The tests of the weak equivalence principle put severe limits on dilaton models. More precise experiments of the weak equivalence principle could discover its violation by a runaway dilaton.

V. CONCLUSION

Motivated by the observational evidence that indicates nontrivial time evolution of the fundamental constants such as the proton-electron mass ratio and the fine structure constant, we have theoretically calculated their time variations based on the standard particle-physics model and its supersymmetric extension with the help of the renormalization group approach. We have employed several simple assumptions such as the universality of the time variation of the masses of the fundamental fermions as a first step. We have applied our formalism to a specific scenario of the runaway dilaton and found that we can account for the observed time variation with some natural choice of model parameters.

We have also found that there is some tension between the observed magnitude of the time variation and the experimental constraint imposed by the verification of the equivalence principle. Indeed we typically find $\eta$ to be larger than $10^{-13}$ unless we adopt a sufficiently small value of $A_F$. One can regard this feature of our model as a prediction, that is, if one performs a more precise experiment on the equivalence principle to measure $\eta$ with a higher accuracy, one would be able to discover its violation by a runaway dilaton.

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$^{8}$ The difference between $^{20}$ and $^{21}$ comes from the difference in the adopted laboratory measurements of the decay rate. We use a more recent measurement $^{22}$. 
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