Investigating the relationship between freeway rear-end crash rates and macroscopically modeled reaction time

Ishtiak Ahmed a, Billy M. Williams b, M. Shoaib Samandar b and Gyoung-hoon Chun a

aDepartment of Civil, Construction, and Environmental Engineering, North Carolina State University, Raleigh, NC, USA; bInstitute for Transportation Research and Education (ITRE), North Carolina State University, Raleigh, NC, USA

ABSTRACT
This study tests the hypothesis that an analytically estimated driver reaction time required for asymptotic stability, based on the macroscopic Gazis-Herman-Rothery (GHR) model, serves as an indicator of the impact of traffic oscillations on rear-end crashes. If separate GHR models are fit discontinuously for different traffic regimes, the local drop in required reaction time between these regimes can also be estimated. This study evaluates the relationship between rear-end crash rates and that drop in required reaction time. Traffic data from 28 sensors were used to fit the GHR model. Rear-end crash rates, estimated from four years of crash data, exhibited a positive correlation with the drop in required reaction time at the congested regime’s density-breakpoint. A linear relationship provided the best fit. These results motivate follow-on research to incorporate macroscopically derived reaction time in road-safety planning. More generally, the study demonstrates a useful application of a discontinuous macroscopic traffic model.

ARTICLE HISTORY
Received 23 April 2020
Accepted 2 April 2021

KEYWORDS
Required reaction time; car-following; discontinuous traffic flow model; rear-end crashes; traffic stream stability

Introduction

Understanding traffic characteristics that may lead to rear-end crashes is important not only for improving safety, but also to alleviate non-recurrent congestion on the roads. Past studies showed that rear-end crashes are strongly associated with traffic instability and oscillations (Tanaka, Ranjitkar, and Nakatsuji 2008; Touran, Brackstone, and McDonald 1999). Due to traffic oscillations, drivers often fail to react in time and collide with the vehicle in front. To identify rear-end crash-prone locations, the state of the practice is documented in the Highway Safety Manual (American Association of State Highway and Transportation Officials 2010). However, highway safety engineering is founded on statistical analysis of crash occurrences. Therefore, the identification of rear-end crash-prone locations would be improved if the crash risk could be predicted using non-crash data, for instance, the macroscopic traffic characteristics of a site.

May (1990) demonstrated that the car-following model proposed by Gazis, Herman, and Rothery (1961) (also known as the GHR model) has potential applications to investigate
traffic instability – the primary reason for most rear-end crashes on freeways. Traffic instability can be caused by the unstable behavior of a driver. The car-following behavior of a driver is considered unstable when the driver is slow to react (i.e. high reaction time), but when reacts, does so abruptly (i.e. high sensitivity). May explained that to maintain the car-following stability for a series of vehicles, the product of driver reaction time and sensitivity must be less than a certain threshold. This analytical formula enables to estimate the upper limit of driver reaction time required for stability for a given state using the GHR car-following model coefficients.

Because of the challenges associated with observing microscopic car-following behavior, it is difficult to fit the GHR car-following model to observed traffic data. However, for steady-state conditions, the macroscopic traffic flow model equivalent to the GHR car-following model enables one to estimate the model coefficients using macroscopic traffic data. Consequently, the steady-state macroscopic flow model provides a rational means to estimate the upper limit of reaction time required for traffic stream stability. Here, it is hypothesized that this analytically estimated maximum driver reaction time for asymptotic stability can serve as an effective indicator of the impact of traffic oscillations on rear-end crashes.

This study investigates the relationship between freeway rear-end crash rates and macroscopically derived maximum reaction time required for asymptotic stability. Since rear-end crashes tend to occur more during traffic state transitions, this study specifically focuses on the change in the required maximum reaction time when the traffic state shifts from the uncongested to the congested regime. To investigate this change, a discontinuous form of macroscopic GHR model is calibrated to traffic sensor data. The relationship between long-term rear-end crash rates of a freeway segment and the change in the required maximum reaction time during traffic state transition at that location is investigated using regression analysis. An important assumption of this experiment, which was verified by a past study (Liu, Williams, and Rouphail 2012), is that the fitted parameter values of the macroscopic traffic model for a site remain reasonably stable over the study period if the corresponding traffic data do not contain significant outliers.

This paper is organized as follows. A review of past studies on the topic is presented in the next section. The following section presents the mechanism of developing and fitting to macroscopic traffic data a discontinuous form of GHR macroscopic model, estimating driver required reaction time from the fitted model, and estimating the rear-end crash rate from police reports of crashes from the selected freeway sites. Results from the fitted models, rear-end crash analysis, and regression analysis are discussed in the following section. Finally, the summary of the results and limitations of the study are presented in the last section.

**Literature review**

May (1990) defined an unstable driver behavior as when a following driver responds slowly to the change in the speed of the leading vehicle, but when responds, exerts large acceleration or deceleration rates. Therefore, the stability of car-following behavior of two vehicles, or in a broader term, of a traffic stream is a function of the driver reaction time and sensitivity. The General Motor researchers developed an analytical formulation of the sensitivity term and showed that the product of this sensitivity and
reaction time dictates the stability of car-following behavior both on a local and asymptotic scale. Details on the mathematical condition of asymptotic stability are discussed later in this paper. In recent decades, several studies investigated the relationships among these critical car-following parameters, namely, reaction time, sensitivity, headway (Kim and Michael Zhang 2011; Zielke, Bertini, and Treiber 2008; Ahn, Laval, and Cassidy 2010; Xu and Laval 2019; Rahman and Kang 2020). Kim and Zhang investigated the relationship among these parameters based on the GHR and other simpler car-following models using car-following behavior data extracted from the NGSIM (USDOT 2006) dataset. Another study (Zielke, Bertini, and Treiber 2008) conducted a comparative investigation of traffic oscillation in terms of the amplitude, propagation velocity, and frequency of shockwave across three countries. Ahn et al. (2010) analytically proved with a triangular-shaped fundamental diagram that the amplitude of an oscillation through a queued traffic stream dampens as it passes an on-ramp and magnifies as it passes an off-ramp.

In addition to car-following behavior and traffic stability analysis, a few studies extended their focus on these parameters’ effects on crash risk (Tanaka, Ranjitkar, and Nakatsuji 2008; Misener et al. 2000; Chatterjee and Davis 2016; Davis and Swenson 2006). Using Wu’s (2002) 4-phase traffic state model, Xu et al. (2015) showed that traffic states drastically impact crash likelihood, type, and severity. They furthermore revealed an association between traffic states, collision types and injury severity. Tanaka et al. (2008) used a microscopic dataset of ten trajectories to investigate the effect of required reaction time and sensitivity on asymptotic stability using different safety indicators. Although several safety indicators showed that traffic oscillation propagates downstream when the product of sensitivity and required reaction time exceeds the threshold proposed by Chandler et al. (1958), other safety indicators yielded an inconclusive result. Nonetheless, it successfully demonstrated the effect of reaction time on traffic stability. To explore the causal relationship between crash occurrence and following headway along with reaction time, Davis and Swenson (2006) used video-recorded microscopic traffic data and the kinematic theory developed by Brill (1972) and simulated three real-world rear-end crashes. It revealed that had the colliding vehicles or a few vehicles further downstream in the sequence maintained a higher following headway than their reaction time, the collisions would probably have been avoided. Chatterjee and Davis (2016) extended the analytical formulation of crash occurrence using car-following theory by Brill for a series of vehicles in a platoon. It perceived the stopping distance for the braking of the first car in a platoon as a shared resource and this resource is either consumed or contributed by the following vehicles. If the sum of the consumption of this resource by the following vehicles exceeds a threshold, rear-end crash occurs. This theory was verified by observing 41 shockwaves in which 15 rear-end crashes or swerving events to avoid crashes were occurred. Li et al. (2020) used time-to-collision index and NGSIM trajectory data to study the transition conditions of rear-end crashes with a focus on the transition mechanism that vehicles portray when moving from a safe to a dangerous situation. They employed a new index (derivative of TTC or TTCD) and identified 13 TTC-based transition types. They found that 3 out of 13 transition types are critical and that the value of TTCD at transition start point is much smaller compared to the end and the average points. To develop a probabilistic model for rear-end crash occurrence using trajectory data, Oh and Kim (2010) used two probability measures, namely probability of changing lanes and probability of the following vehicle hitting the leading one for a given ‘Time to
Collision’. However, the accuracy of the proposed approach was not tested against field crash data. While the availability of microscopic information on driver reaction time level is increasing with the increasing research using instrumented vehicles (Tanvir, Chase, and Roupahil 2019), microscopic traffic data like acceleration, following headway, and instantaneous reaction time is still difficult to obtain by transport agencies. Therefore, the practical application of the studies discussed above is limited since these demand several micro-level inputs like acceleration, following headway, and reaction time.

Several studies attempted to predict the rear-end crash risk of a roadway using macroscopic traffic data (Lord, Manar, and Vizioli 2005; Abdel-Aty et al. 2004). Among these, Lord et al. used traffic density and volume-to-capacity ratio as explanatory variables to predict freeway crashes. Chou and Nichols (2015) used surrogate safety measures related to end-of-queue such as; queue duration, queue impact area, and number of vehicles exposed to end-of-queue to predict freeway rear-end collisions. Abdel-Aty et al. showed that a combination of high coefficient of variation of speed and high occupancy at a downstream segment is a potential crash pre-cursor in the upstream segment of a roadway. Pande and Abdel-Aty (2006) divided rear-end crashes into two groups based on whether they occur before or during congestion. Average and coefficient of variation of speed, average occupancy, and presence of a downstream ramp were the statistically significant variables to predict rear-end crashes. However, the discrepancies in the findings of these studies indicate that the findings are mostly site-specific.

The above survey of literature shows that while many studies analyzed the car-following model parameters related to driver reaction time, only a few focused on its relation to rear-end crash rates. These few studies used microscopic trajectory level data that are difficult to obtain on a network-level. Although several studies investigated the crash precursor potential of various macroscopic traffic characteristics, those mostly focused on a single site and the findings are mostly site-specific.

Methodology

In this section, first, the development of the two-regime macroscopic model equivalent to the GHR car-following model is explained. Next, the derivation of driver maximum reaction time required for asymptotic stability is described. Finally, the description of the study site and crash data collection and analysis method are presented.

The macroscopic model equivalent to the GHR car-following model

The basic form of the macroscopic model equivalent to the fifth GHR car-following model (May 1990) is

\[
q_{M,i} = k_i \cdot u_i \cdot \left(1 - \left(\frac{k_i}{k_j}\right)^{-1}\right)^{-m}.
\]  

Here \(i = 1, 2, 3 \ldots \) observation index; \(q_{M,i} \) = model flow for observation \(i\) (pc/hr/ln); \(u_i \) = free flow speed (mph); \(k_i \) = average density for observation \(i\) (pc/mi/ln); \(k_j \) = jam density (pc/mi/ln); \(l \) = distance headway exponent; and \(m \) = speed difference exponent.
Variety of versions of this model form has been proposed by past studies by incorporating multiple regimes and discontinuity in the fundamental diagram. Since our target was to derive the drop in maximum required reaction time during the transition of two regimes, we adopted the multi-regime discontinuous form that is also known as the inverse lambda-shaped flow-density model form.

A few versions of the inverse lambda-shaped flow-density form are found in the literature (Edie 1961; Ahmed, Williams, and Shoaib Samandar 2018). Among these, a relatively recent study by the authors (Ahmed, Williams, and Shoaib Samandar 2018) proposed a two-regime fundamental diagram with an overlap of the regimes near the capacity. Moreover, this study also proposed an algorithm to filter the steady-state observations from side-fire radar sensors. A slightly modified version of that approach is adopted in this study.

The proposed algorithm for fitting a two-regime GHR model to macroscopic traffic data from a freeway site can be divided into three steps: (i) data pre-processing to filter steady-state observations (ii) initial fitting of a two-regime GHR Model (iii) iteration of the second step with robust regression to remove remaining outliers. For the details of the first step, readers are suggested to review the previous research (Ahmed, Williams, and Shoaib Samandar 2018).

In order to introduce a transition regime in the GHR model, it was assumed that the transition regime includes a mixture of observations from both uncongested and congested regimes. Empirical observations of traffic stream data also depict that the data points in the transition range follow either the uncongested or the congested regime’s characteristics. Within the overlap of the uncongested and congested regime, each data point was proposed to be modeled either as uncongested or congested regime, whichever results in the smallest absolute error. Equations (2)–(4) demonstrates the mechanism of the proposed model.

**Observed Density Formula for model flow**

\[
\begin{align*}
k_i \leq k_{b,r=2} & \quad q_{M,i,r=1} = k_i \times u_{r=1} \times \left( \frac{k_i}{k_{j,r=1}} \right)^{l_{r=1}-1} \frac{1}{1-l_{r=1}}, \quad (2) \\
k_i \geq k_{b,r=1} & \quad q_{M,i,r=2} = k_i \times u_{r=2} \times \left( \frac{k_i}{k_{j,r=2}} \right)^{l_{r=2}-1} \frac{1}{1-l_{r=2}}, \quad (3) \\
k_{b,r=2} < k_i < k_{b,r=1} & \quad q_{M,i,r} = \begin{cases} q_{M,i,r=1} & \text{if } |q_{M,i,r=1} - q_i| < |q_{M,i,r=2} - q_i| \\ q_{M,i,r=2} & \text{Otherwise} \end{cases}. \quad (4)
\end{align*}
\]

Here \( r \) = Regime index. 1 represents uncongested and 2 represents congested regime. \( k_b = \) Density breakpoint. Note that \( k_{b,r=2} < k_{b,r=1} \).

In Equation (2) through 4, the two density breakpoints \( (k_{b,r=1} \text{ and } k_{b,r=2}) \) define the upper and lower limit of the transition regime, respectively. This proposed discontinuous GHR model is fitted to observed traffic flow data by minimizing the sum of squared error – where the error is the difference between the modeled flow and observed flow – using a nonlinear optimizing tool available in MATLAB.

The above algorithm fits an inverse lambda-shaped flow-density model to the observed data. However, it could not capture some high-flow observations near the capacity of a
segment. Moreover, the queue discharge flow rate according to the model was significantly lower than the expected range. To tackle these issues, two thresholds based on the Highway Capacity Manual’s (HCM) basic freeway segment analysis method were incorporated into the algorithm (Transportation Research Board 2016). An example of the resulting model after applying these thresholds is shown in Figure 3.

According to the first threshold, the slope at the capacity of the flow-density model must be less than or equal to the HCM derived slope at capacity value. The second threshold implies that the queue discharge flow rate should be within 2% to 20% of the capacity as specified in the HCM. The application of these thresholds resulted in a well-fitted model with reasonable estimates of capacity and queue discharge flow rate. Note that a third threshold was applied in the previous study (Ahmed, Williams, and Shoab Samandar 2018) to keep the jam density within a reasonable range. However, that threshold is omitted here as empirical observation showed that artificially capping the jam density value may result in a poor fit of the congested regime curve to the observed data.

To further remove outliers from field observations, the so-called Robust Regression technique was used to fit the model and remove outliers based on the fitted model iteratively. From the initial model fit, the standard error for each data point is estimated. Data points with a standard error higher than a certain threshold are removed from the original dataset. Then, the model is fitted again with the updated dataset. The process is continued until the maximum standard error becomes lower than the threshold. The formula for estimating standard error (SE) in terms of flow is shown in Equation (5).

$$\text{SE}_{i,r} = \frac{q_{M,i,r} - q_i}{\text{Std}_r}.$$  

(5)

where $\text{Std}_r = \text{Standard deviation of flow error for regime } r$, and $q_i$ is the observed error.

The major issue with the selection of the threshold was that while measurement errors are present in both regimes, mixed-state observations are more prevalent in the congested regime than in the uncongested regime. Two different thresholds are applied to the two regimes to remove both measurement errors and mixed-state data. To remove the measurement errors from the uncongested regime which are symmetric in nature, a symmetric threshold of $\pm 3.5$ is applied to the uncongested regime standard error. Mixed state observations, on the other hand, are likely to be distributed asymmetrically as these tend to be on the left side of the flow-density curve. Hence, an asymmetric threshold of $+2$ is applied for all but the final step of the robust regression. In the final step of robust regression, a threshold of $-3.5$ is applied to exclude any remaining outliers from the congested regime on the right side of the flow-density curve. The $\pm 3.5$ standard error threshold represents a confidence interval of approximately 99.95%, assuming that the residuals follow Gaussian distribution. The number of data points removed after applying the robust regression process varied for different sites from 60 to 2421. The range in percentage was 0.07%–2.4% of the total number of observations from each site.

**Estimating required driver reaction time for different traffic states**

The formula for estimating the required driver reaction time for asymptotic stability needs to be derived from the microscopic form of the fifth and final GHR car following model, which is expressed in Equation (6). Here, the acceleration of the $(n+1)$th vehicle in a traffic
stream at time \((t + \Delta t)\) [termed as \(x''_{n+1}(t + \Delta t)\)] in response to the relative speed between the \(n\)th and \((n+1)\)th vehicle at time \(t\) is expressed as the product of the sensitivity term and the relative speed between the two vehicles.

\[
x''_{n+1}(t + \Delta t) = \alpha \left[ \left( \frac{[x'_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]} \right) \ast [x'_n(t) - x'_{n+1}(t)] \right]. \tag{6}
\]

Here \(n\) = Position of a driver in a traffic stream. \((n = 1\) is for the most downstream driver). \(x_n\) = Location of the \(n\)th driver with respect to a reference point. \(x'_n(t)\) = Speed of the \(n\)th driver at time \(t\).

According to May (1990), the parameter \(\alpha\) can be expressed as shown in Equation (7).

\[
\alpha = \frac{(l - 1)u_f^{1-m}}{(1 - m)k_j^{l-1}}. \tag{7}
\]

Thus, the sensitivity factor is equivalent to what is shown in Equation (8). It is a mathematical expression for how much a driver reacts (accelerate or decelerate) in response to the change in the relative speed of the vehicle it is following.

\[
\text{Sensitivity factor} = \frac{(l - 1)u_f^{1-m}}{(1 - m)k_j^{l-1}} \ast \left( \frac{[x'_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]} \right) \ast [x'_n(t) - x'_{n+1}(t)]. \tag{8}
\]

For steady-state observations, individual vehicle speed represents the traffic stream’s speed and the spacing between two successive vehicles represents the inverse of the traffic density. Thus, Equation (9) can be written as

\[
\text{Sensitivity factor} = \frac{(l - 1)u_f^{1-m}}{(1 - m)k_j^{l-1}} \ast \frac{u_m}{u_m^*} \left( \frac{1}{k} \right)^l. \tag{9}
\]

As mentioned earlier, the car-following behavior of a driver is considered unstable when the driver is slow to react (i.e. high reaction time), but when reacts, does so abruptly (i.e. high sensitivity). According to May (1990), for a traffic stream to be asymptotically stable, the product of the reaction time and sensitivity must be less than or equal to 0.5. Hence, the expression for the maximum reaction time required for asymptotic stability can be derived as:

\[
t_i = \frac{(1 - m)k_j^{l-1}}{2(l - 1)u_f^{1-m}} \ast \frac{u_m^*}{u_m}. \tag{10}
\]

Equation (10) gives the formula to estimate the maximum reaction time required for stability for each observation of flow, speed, and density.

**Drop in required reaction time between two regimes**

As shown earlier, fitting the proposed discontinuous flow-density model enabled the research team to investigate the drop in required reaction time when the traffic state moves from uncongested to the congested regime. This estimated drop in the required time for a freeway segment is illustrated in Figure 1(a). This figure shows the
change in the required reaction time with average traffic density, but it requires further explanation.

Assume that we have a sample of observations of flow rate and average density from a site. Suppose the sample size is sufficiently large for fitting the proposed macroscopic discontinuous GHR model. In that case, we can do so for that site based on the algorithm described earlier and using a nonlinear optimization tool. Thus, we will obtain an estimate of the model coefficients \((m, l, k_j, k_b, \text{ and } u_f)\) for each regime \(r\) for that site based on the observed traffic flow data. By plugging these coefficient estimates in Equation (10), we can estimate the upper limit of reaction time required for traffic stream stability \((t_i)\) for any density value \((k_i)\). Note that the density breakpoint for each regime \((k_b)\) is necessary for defining the regimes. Also, the traffic stream speed \((u_i)\) in Equation (10) is estimated for a density value \(k_i\) by combining the fundamental equation of traffic flow (i.e. \(u_i = q_i/k_i\)) and the fitted GHR model as described by Equations (3)–(5).

Figure 1(a) shows an example plot of the required reaction time \((t_i)\) vs. average density, where \(t_i\) is estimated per Equation (10) for a series of density values \((k_i = 1, 2, 3 \ldots, k_j)\). The resulting curves are distinguished based on their respective regimes by their line colors.

Upon scrutinizing the same plot of required reaction time vs. density for different locations, it was found that \(t_i\) for the congested regime does not change significantly. Moreover, \(t_i\) for a very low density value (such as less than 20 pc/mi/ln) may not have any significance as traffic stream barely follows the car-following model. On the other hand, if the transition regime is focused here [bounded by the two vertical arrows in Figure 1(a)], there are two \(t_i\) values for each density point within this regime. As the traffic state transfers from the uncongested to the congested regime, the required upper limit of the driver reaction time also drops to get adapted to the change in the traffic state. The research team hypothesizes that the higher the drop of \(t_i\), higher the risk of a rear-end crash to occur during this transition regime.

The two drops in reaction time shown in Figure 1(a) are of particular interest here. These two drops termed as \(\Delta t_1\) and \(\Delta t_2\) are the drops in required reaction time at the beginning and the end of the transition regime, respectively (assuming that the transition is from uncongested to congested regime). \(\Delta t_1\) is the largest drop (shown by the black arrow) in

![Figure 1.](image-url)
within the transition regime. It is the value by which the upper limit of driver required reaction time changes when traffic state transfers from the uncongested to the congested regime at a density of $k_{b,r=2}$. On the other hand, $\Delta t_2$ is the change in maximum driver reaction time when traffic state transfers the regime upon reaching the capacity and at a density of $k_{b,r=1}$ (shown by the green arrow).

Figure 1(b) illustrates the flow-density plot corresponding to Figure 1(a), fitted according to the proposed discontinuous GHR macroscopic model. An example of such a plot including the underlying field data is shown later in this paper [Figure 3(a)]. Figure 1(b) helps to investigate the flow rates associated with $\Delta t_1$ and $\Delta t_2$, shown by a black and a green arrow, respectively. We focus on the uncongested regime flow rates associated with $\Delta t_1$ and $\Delta t_1$, i.e. $q_{r=1}(k_{b,r=2})$ and $q_{r=1}(k_{b,r=1})$, respectively. It is apparent that $\Delta t_2$ is associated with a transition from the maximum sustainable flow rate, i.e. $q_{r=1}(k_{b,r=1})$. However, field data often do not contain observations close to this maximum flow rate due to the presence of downstream bottlenecks. Hence, for such cases, $q_{r=1}(k_{b,r=1})$ represents the capacity of a downstream bottleneck rather than representing the characteristics of the study site. On the contrary, field observations with flow rate equal to $q_{r=1}(k_{b,r=2})$, which associates with $\Delta t_1$, is more common than with flow rate equal to $q_{r=1}(k_{b,r=1})$. This reasoning hints that $\Delta t_1$ represents the characteristics of the study-site whereas $\Delta t_2$ might represent the characteristics of a bottleneck-site located further downstream of the study-site. Nonetheless, we explore in this paper the correlation of both $\Delta t_1$ and $\Delta t_2$ against the rear-end crash rate for each study site.

**Data description**

*Traffic data from sensors*

In this study, the discontinuous macroscopic model described above is proposed to be fitted with field data collected from side-fire radar sensors located on different locations of the freeway system of the Triangle Region of North Carolina. The radar sensors, each of which has a range of 200 ft., recorded the vehicle count, class, and instantaneous speed across each directional freeway segment. From these records, flow rate ($q$), space mean speed ($u$), and – plugging these two variables into the fundamental traffic flow equation – average density ($k$) was estimated. Data from 28 directional sensors are collected for the calendar year of 2013 in a time resolution of 5 min.

These sensors are located on three interstates namely I-40, I-440, and I-540. Since this study primarily hinges on car-following model, only basic freeway segments (see HCM for definition) are selected to ensure that the lane changing activities are minimum near the sensors. Figure 2 shows the location of these sensors in the study area. Past studies showed that several recurring bottlenecks exist in the proximity of some of these locations (Ahmed, Rouphail, and Tanvir 2018). The numbers show the tag of each station and the alphabets following the number indicate the travel direction.

Prior to fitting the model with these data, a filtering algorithm was applied to remove mixed-state and inconsistent observations as much as possible. Details of the filtering algorithm are presented in a past study (Ahmed, Williams, and Shoaib Samandar 2018). After removing those mixed-state and inconsistent observations, the number of 5-minute observations for each sensor ranged from 75,844 to 103,423 (total number of observations for all 28 sensors was 2,679,995).
Crash data

To estimate the rear-end crash rate associated with each sensor, a critical task was to select an appropriate freeway segment surrounding the sensor location. As explained in the Highway Safety Manual, the crash rate at a segment can be attributed to several geometric features including the vertical and horizontal curvature, lane width, number of lanes, and presence of ramps. Here, each segment was selected in such a way that it is away from any ramp, does not have any tight curve, and the number of lanes and lane width is consistent throughout the segment.

Four years of police-reported crash data from the selected segments are collected for the period from 2011 to 2014. This duration is appropriate for avoiding any regression-to-the-mean bias as the Highway Safety Manual recommends using at least three years of crash data for crash rate analysis. Further, it was confirmed through the historical imagery from Google Earth that the selected freeway segments did not exhibit any significant geometric change during this period. A tool called ‘Traffic Engineering Accident Analysis System (TEAAS)’ (North Carolina Department of Transportation 2020) was used to extract the police reports. Each crash report was carefully investigated to decide if that crash occurred within the corresponding segment. The number of rear-end crashes for the selected years varied for those 28 locations over a wide range of 3–65.

The average AADT for each segment across the selected years is obtained from the archive of the North Carolina Department of Transportation. Finally, the following equation was used to estimate the rear-end crash rate (crashes per 100 million VMT) for each
Crash rate = $100,000,000 \times \frac{C}{(365 \times N \times \overline{AADT} \times 0.5 \times L)}$ \hspace{1cm} (11)

Here $N$ = number of years over which crash data were collected ($N = 4$), $C$ = total frequency of crashes in $N$ years, $\overline{AADT}$ = average AADT over $N$ years, $L$ = segment length in miles.

After estimating the crash rate for each segment, its correlation with the two drops in required reaction time ($\Delta t_1$ and $\Delta t_2$) are investigated using statistical modeling through regression analysis.

**Analysis and results**

This section is divided into three major parts. In the first part, results from fitting the proposed two-regime traffic flow model is presented. The fitted parameter values and estimated drops in required reaction time are also discussed here. In the second part, results from the rear-end crash rate analysis are described. Finally, the relationship between driver reaction time drop and rear-end crash rates are assessed.
Fitted two-regime traffic flow models

To fit the traffic stream model by imposing the constraints described earlier, a nonlinear optimization tool available in MATLAB was used. The fundamental diagrams for the sensor 18W are shown Figure 3. The parameter values used to plot these diagrams are the fitted parameters obtained from the convergence of the proposed robust regression algorithm. The fundamental diagrams (flow-density plots) for all sensors are provided in Appendix 1. Table 1 presents the estimated parameter values, their standard deviations, and the resulting required reaction time drops ($\Delta t_1$, $\Delta t_2$). Standard deviation for such nonlinear optimization models is calculated using a method described in a past study (Smith 2013).

The fundamental diagrams shown in Figure 3 illustrate that the fitted models reasonably follow through the steady-state observations. Here, the value of the parameters that have physical interpretation needs to be discussed. The distance headway ($l$) and speed exponent ($m$) for both regimes, free-flow speed for the congested regime, and jam density for the uncongested regime do not have any physical interpretations. The free flow speed of the uncongested regime varies from about 56 to 70 mph across these sites. The jam density of the congested regime varies over a very wide and somewhat unrealistic range of 229 to 4207 pc/mi/ln. This is due to the fact that observations near jam density condition is very scarce. Hence, the congested regime curve extrapolates to a very high density value when flow = 0 in that regime. Unlike the previous research (Ahmed, Williams, and Shoaib Saman- dar 2018), we are not artificially capping the jam density value which caused the congested regime to have a very poor fit to the observed data for some sensors.

The two density breakpoint values ($k_{br1}$ and $k_{br2}$) are of particular interests in this study since these two constitute the overlap and consequently, the required reaction time drops. Here, $k_{br1}$ ranges from 37 to 50 pc/mi/ln. However, in most sensors, it is less than the density at capacity (45 pc/mi/ln) for basic freeway segment specified by HCM. This wide-range variation of density breakpoint underscores that the national average value provided by HCM needs to be calibrated with field data if high fidelity analysis is desired.

The difference between the two density breakpoints represents the overlap range, which appears to be a unique characteristic for these sensors. The fitted overlap in density varies from 0 to 8 pc/mi/ln. The values for the first and second drop in driver reaction time vary from 0.74 to 2.95 and from 0.55 to 1.68, respectively, indicating that the sites have significant variations in the estimated drops in required reaction time.

The standard deviations of the estimates for the parameters to which the model is sensitive are of small magnitude, as shown within parenthesis in Table 1. This reveals that most of these estimates are significant and the error in the estimates of $\Delta t_1$ and $\Delta t_2$ should be within an acceptable range. It should be noted that the standard deviations for the two breakpoints ($k_b$) cannot be estimated because these are only classifiers of the regimes. The standard deviation for the congested regime free-flow speed is very high because the fitted models are insensitive to this parameter. For the same reason, the standard deviation of the uncongested regime jam density for a few sensors are high (e.g. for 4W and 25E).

A fundamental assumption of this study is that the traffic model coefficients remain stable during the period over which the crash data were analyzed (2011–2014). As mentioned earlier, this assumption was validated by a past study (Liu, Williams, and Rouphail 2012). To further check the validity of this assumption, we applied the proposed discontinuous
| ID | Uncongested Regime | Congested Regime | \( \Delta t_1 \) | \( \Delta t_2 \) |
|----|-------------------|------------------|----------------|----------------|
| 1W | 65 (5.9E-3)       | 40               | 4.418 (2.8E-3) | 0.609 (1.4E-3) |
| 2E | 62 (8.0E-3)       | 39               | 10.37 (5.7E-2) | 0.985 (8.0E-4) |
| 3E | 56.4 (6.9E-3)     | 42               | 9.333 (2.5E-1) | 0.99 (3.4E-3)  |
| 4W | 59.2 (6.0E-3)     | 39               | 8.215 (4.5E-1) | 0.99 (9.5E-2)  |
| 5W | 61.7 (5.0E-3)     | 44               | 5.47 (2.5E-3)  | 0.993 (3.3E-5) |
| 6E | 57.7 (1.1E-2)     | 50               | 3.964 (1.6E-3) | 0.977 (1.4E-5) |
| 7E | 63.3 (1.0E-2)     | 39               | 11.016 (9.0E-2)| 0.981 (1.5E-3) |
| 8W | 61.7 (9.5E-3)     | 42               | 6.716 (8.5E-3) | 0.993 (8.7E-5) |
| 9W | 65.3 (6.5E-3)     | 48               | 4.862 (4.2E-2) | 0.99 (1.2E-3)  |
| 10E| 62.9 (1.2E-2)     | 42               | 2.973 (1.0E-3) | 0.955 (1.3E-4) |
| 11E| 63.6 (1.3E-2)     | 37               | 4.149 (1.6E-1) | 0.996 (1.7E-5) |
| 12W| 65.1 (4.8E-3)     | 40               | 4.352 (1.3E-3) | 0.998 (7.6E-6) |
| 13W| 61.9 (6.6E-3)     | 44.6             | 5.609 (3.6E-2) | 0.99 (8.1E-4)  |
| 14W| 61.4 (8.1E-3)     | 45               | 3.395 (7.1E-4) | 0.994 (1.3E-5) |
| 15E| 62.4 (9.1E-3)     | 40               | 1.973 (1.4E-3) | 0.205 (4.3E-3) |
| 16W| 67.1 (1.6E-2)     | 42               | 3.056 (1.7E-3) | 0.792 (6.7E-4) |
| 17E| 67.8 (6.1E-1)     | 40               | 6.09 (1.4E-2)  | 0.99 (2.6E-4)  |
| 18W| 62.8 (4.8E-3)     | 37               | 11.207 (5.6E-2)| 0.971 (1.3E-3) |
| 19E| 58.8 (4.9E-3)     | 44               | 5.546 (9.7E-3) | 0.99 (2.0E-4)  |
| 20W| 63.3 (6.8E-3)     | 40               | 3.846 (5.0E-3) | 0.99 (1.5E-4)  |
| 21E| 63.3 (3.9E-3)     | 38               | 3.526 (2.7E-3) | 0.752 (1.2E-3) |
| 22E| 66 (6.3E-3)       | 39               | 5.83 (5.2E-3)  | 0.996 (2.7E-6) |
| 23W| 69.9 (6.3E-3)     | 42               | 4.559 (1.0E-3) | 0.999 (2.7E-6) |
| 24E| 63.8 (8.4E-3)     | 44               | 4.512 (1.0E-2) | 0.99 (2.7E-4)  |
| 25E| 61.8 (9.8E-3)     | 42               | 6.95 (5.2E-1)  | 0.99 (1.1E-2)  |
| 26W| 62.7 (8.7E-3)     | 46               | 3.718 (2.0E-3) | 0.989 (5.8E-5) |
| 27E| 67.2 (1.2E-2)     | 41               | 4.403 (1.9E-2) | 0.99 (5.8E-4)  |
| 28W| 68.1 (8.2E-3)     | 41               | 4.188 (8.6E-3) | 0.99 (2.4E-4)  |
macroscopic GHR model to two sites for the remaining years (i.e. 2011, 2012, and 2014). The percentage differences in the parameter estimates and $\Delta t_1$ between 2013 and the remaining years are shown in Table B1 in Appendix 2 for sensor IDs 17E and 20W. It was found that most parameter estimates for the remaining years, particularly the ones with a physical meaning (i.e. the two density breakpoints, free flow speed for the uncongested regime, and jam density for the congested regime) exhibited only up to 8% difference when compared to those for 2013. Most importantly, $\Delta t_1$ for these years differed by less than 13% for all but one case. The highest difference in $\Delta t_1$ was $-17.97\%$, which is for sensor ID 17E in 2012. Later in this paper, it is shown that even this highest difference in $\Delta t_1$ does not have a significant impact on the final outcomes of this research.

**Analysis of crash data**

Figure 4 shows the estimated rear-end crash rates (per 100 million VMT) for all the 28 sensors. The segment length, which ranges from about 0.15 to 2 miles, is showed in this figure by color-coding the bars. The figure shows that the selected sites have significant variability in terms of crash frequency since the crash rate and segment length appear to be uncorrelated. The crash rate spans over a wide range of about 10–65 rear-end crashes per 100 million VMT.

**Crash rates vs. drop in required reaction time**

Figure 5(a,b) show the scatterplots of rear-end crash rates and the two drops in required reaction time – $\Delta t_2$ and $\Delta t_1$, respectively, as demonstrated in Figure 1(a). We investigated three forms of relationship: exponential, linear, and logarithmic. In addition to estimating the correlation coefficient, regression models were fitted to gain further insights into each type of relationship. Table 2 shows the results.

It is apparent from Figure 5(a) that no correlation exists between the rear-end crash rates and $\Delta t_2$. It could be because $\Delta t_2$ is very sensitive to the selection of break point density.

![Figure 4. Rear-end crash rates for different sensor stations and the length of the freeway segments.](image-url)
**Figure 5.** Rear-end crash rates vs. (a) \( \Delta t_2 \) (b) \( \Delta t_1 \).

| Relationship type | Model form | Residual SE | Adj. R-sq. | Param. Estimate | Std. Error | p-value |
|-------------------|------------|-------------|------------|----------------|------------|---------|
| Exponential       | \( y = e^{mx+c} \) | 11.63        | 0.37       | \( m \) 0.48   | 0.12       | \( 3 \times 10^{-4} \) |
|                   |            |              |            | \( c \) 2.43    | 0.22       | \( 3 \times 10^{-11} \) |
| Linear            | \( y = mx + c \) | 11.53        | 0.38       | \( m \) 14.79   | 3.55       | \( 3 \times 10^{-4} \) |
|                   |            |              |            | \( c \) 3.74    | 6.77       | 0.59    |
| Logarithmic       | \( y = \ln(x) + c \) | 11.86        | 0.34       | \( m \) 24.20   | 6.25       | \( 6 \times 10^{-4} \) |
|                   |            |              |            | \( c \) 17.63   | 3.99       | \( 2 \times 10^{-4} \) |

for the uncongested regime, which makes this predictor less robust to any potential outliers. Furthermore, the effect of the presence of downstream bottlenecks as explained in Figure 1(b) can play a critical role here. The \( R^2 \) values for the regression models were between 0.004 and 0.01.

On the other hand, a strong positive correlation is apparent in the rear-end crash rates vs. \( \Delta t_1 \) plot, as shown in Figure 5(b). The correlation coefficient between these two variables are +0.63. Since this evidence is mostly empirical, it is difficult to pre-determine about what should best explain the relationship. Nonetheless, all three forms of relationship showed a reasonable fit.

Note that the objective of this study is to investigate the correlation between rear-end crash rates and \( \Delta t \), which is an important first step for building a fully-evolved model in the form of a safety performance function or crash modification factor (Hauer 2015). Developing such a fully-evolved safety model is a multi-step procedure. The steps are: selecting the model form (for parametric models), choosing the appropriate measure to optimize, testing relevant safety variables as predictors (e.g. geometric properties of the road), and evaluating the goodness of fit. The state-of-the-art practice for these steps is described in Hauer (2015) and implemented by recent studies (Srinivasan et al. 2016). The purpose of showing the performance of the three model forms that we tested here was to gain insights into the correlation type between rear-end crash rates and \( \Delta t \). Therefore, the findings reported in this study may serve as a stepping stone for future research focusing on developing a safety model for rear-end crash rates with the change in required reaction time as one of the potential predictors.
At this point, it is crucial to describe the fitted models and their goodness of fit for the three forms. Table 2 shows the form of the fitted models, the fitted parameter values along with several statistical measures to evaluate the relationship.

Here, the response variable \( y \) is the rear-end crash rate (in crashes per 100 million VMT), the explanatory variable \( x \) is \( \Delta t_1 \) (in seconds), and \( m \) and \( c \) are the model coefficients. The exponential and linear form exhibited similar performance measures in terms of the residual standard error (SE) and adjusted \( R^2 \)-squared. On the other hand, the logarithmic model, while showing a satisfactory fit, generated a slightly higher residual error and a lower \( R^2 \)-squared. The \( p \)-value column shows that except for the Y-intercept of the linear model, other coefficients of all three models are statistically significant at least at a level of 0.001.

Here, it is essential to explain the interpretation of the correlation from the perspective of this study. For instance, the \( R^2 \)-squared of 0.38 generated by the linear model interprets that the model explains 38% of the variation in the rear-end crash rates. Note that while \( \Delta t_1 \) is associated only with the transition from uncongested to the congested regime, not all rear-end crash rates happened during the transition regime. Moreover, a lot of such crashes are not directly related to the driver reaction time required to maintain asymptotic stability. Rather, several other factors including distracted driving, driving under the influence, animal crossing, and abrupt lane changing maneuver may lead to rear-end crashes, which are not reflected in the long-term traffic stream characteristics of a segment. Nonetheless, a lot of rear-end crashes are associated with the onset of congestion and the transition between the two regimes. Hence, only a significant portion of all rear-end crashes are expected to be explained by the fitted models. Having said that, a correlation coefficient of \(+0.63\) or an \( R^2 \)-squared value of 0.38 appears to be satisfactory for the purpose of this study.

As suggested by Hauer (2015), it is important to assess the goodness of fit of a model both in terms of a single measure (like \( R^2 \)-squared) and the distribution of the residuals. Despite having a satisfactory \( R^2 \)-squared, a model might be heteroscedastic if the residual shows a trend with the fitted response variable. To check for heteroscedasticity, the square root of standardized residuals vs. the fitted response variable plot, also known as the scale-location plot, is commonly used (University of Virginia Library, Charlottesville). Figure 6 shows such plots for all three form of relationships along with a smoothed trend line and the confidence interval.

Figure 6 shows that the linear and the logarithmic form have a very mild increasing trend with the increase of the fitted values. However, the residuals are more uniformly distributed across the fitted values in the case of the linear than the logarithmic form. The scale-location plot for the exponential form has a stronger trend than the other two – implying that the exponential form has more heteroscedasticity.

It is apparent that overall, the linear form performed best among the three forms of relationship that we investigated here. The slope of the model interprets that for each second increment in \( \Delta t_1 \), the estimated rear-end crash rate increases by 14.79 crashes per 100 million VMT. Note that the high \( p \)-value of the intercept term of the linear relationship infers that the null hypothesis that the intercept is zero cannot be rejected. However, the intercept term is a trait obtained by extrapolating the linear model; the drop in driver reaction time cannot be zero in reality.
In Table B1 in Appendix 2, we showed for two sites that $\Delta t_1$ does not vary significantly across different years within the study period. Consider the highest difference in $\Delta t_1$ in this table ($-17.97\%$), which is for sensor ID 17E between year 2013 and 2012. According to the linear model shown in Table 2, the difference in the estimated rear-end crash rate between 2013 and 2012 for 17E would be only 5.35 per 100 million VMT. This difference is less than half of the model residual standard error, which is 11.53 per 100 million VMT. It implies that at least for the two example sites shown in Table B1, the most extreme deviance of $\Delta t_1$ does not have a significant impact on the modeling results.

**Conclusions**

The study investigates the relationship between rear-end crash rates and macroscopically derived driver reaction time required for asymptotic stability. Separate GHR models are fitted discontinuously for the uncongested and congested flow regimes using traffic sensor data collected for one year. The drops in reaction time at the beginning and end of the transition regime are mainly focused here. Data from 28 sensors located in basic freeway segments of three interstates are used in this study. A freeway segment surrounding each sensor is selected for crash data analysis such that the road geometric characteristics remain constant throughout the segment. Rear-end crash rate for each segment is then estimated by examining police-reported crash data.

Results from the fitted models and crash data analysis showed that the selected sites have significant variability in terms of both traffic characteristics and rear-end crash risks.
Standard deviations of the estimates were of small magnitude for the parameters that have a physical interpretation and to which the model is sensitive. One of the assumptions of this research, the traffic model coefficients remain stable over the period for which crash data were analyzed, was verified by a past study cited above. Furthermore, the validity of this assumption was tested for two sites in this study. Although this limited multi-year investigation found that the assumption of model coefficient stability to be reasonable, follow-up research to provide more extensive testing of the robustness of the model coefficients across different periods, data types, and sample sizes is recommended.

Three model forms, namely linear, exponential, and logarithmic were used to test the correlation between crash rates and the drops in required reaction time. The second drop in the required reaction time exhibited no significant correlation with rear-end crash rates. The first drop in the required reaction time exhibited a strong positive correlation with rear-end crash rates. The correlation coefficient was found +0.63. The three models exhibited $R^2$-squared values ranging from 0.34 to 0.38, with the linear model performing best both in terms of $R^2$-squared and standard error. Analysis of the residuals for each model revealed that the linear and logarithmic models have reasonable homoscedasticity.

While an $R^2$-squared within this range might seem low, it should be noted that not all rear-end crashes are associated with the transition of traffic regimes and car-following behavior. Hence, all rear-end crash occurrences at a location cannot be explained by the drop in the required reaction time. Having said that, the statistical correlation revealed here can be considered strong enough to motivate follow-on research to incorporate macroscopically derived reaction time in safety and planning level studies.

More generally, this study demonstrates a useful application of a discontinuous macroscopic traffic model. However, a few limitations need to be highlighted here to be addressed in future research. A sample size of 28 falls slightly short of the generally accepted sample size of 30 for the assumption of normality. As much as it is difficult to extract accurate crash data, it is important to increase the number of study sites. Further, if detailed data are available, it is recommended to filter the crashes that are directly associated with the transition of traffic regimes and car-following behavior.

Acknowledgements
The authors would like to thank the National Transportation Center at Maryland for their financial support in this research.

Disclosure statement
No potential conflict of interest was reported by the author(s).

Funding
This work was supported by National Transportation Center [grant number NTC2015-MU-R-07].

Author contributions
The authors confirm contribution to the paper as follows: study conception and design: B. Williams, I. Ahmed; data collection: I. Ahmed, S. Samandar, G. Chun; analysis and interpretation of results: I. Ahmed, B. Williams, S. Samandar; draft manuscript preparation: I. Ahmed,
B. Williams, S. Samandar, G. Chun. All authors reviewed the results and approved the final version of the manuscript.

ORCID

Ishtiak Ahmed http://orcid.org/0000-0002-2946-5392

References

Abdel-Aty, Mohamed, Nizam Uddin, Anurag Pande, M. Fathy Abdalla, and Liang Hsia. 2004. “Predicting Freeway Crashes from Loop Detector Data by Matched Case-Control Logistic Regression.” *Transportation Research Record* 1897 (1): 88–95. doi:10.3141/1897-12.

Ahmed, Ishtiak, Nagui M. Rouphail, and Shams Tanvir. 2018. “Characteristics and Temporal Stability of Recurring Bottlenecks.” *Transportation Research Record* 2672 (42): 235–246. doi:10.1177/0361198118799991.

Ahmed, Ishtiak, Billy M. Williams, and M. Shoaib Samandar. 2018. “Application of a Discontinuous Form of Macroscopic Gazis-Herman-Rothery Model to Steady-State Freeway Traffic Stream Observations.” *Transportation Research Record* 2672 (42): 51–62. doi:10.1177/0361198118799166.

Ahn, Soyoung, Jorge Laval, and Michael J. Cassidy. 2010. “Effects of Merging and Diverging on Freeway Traffic Oscillations.” *Transportation Research Record* 2188 (1): 1–8. doi:10.3141/2188-01.

American Association of State Highway and Transportation Officials. 2010. *Highway Safety Manual*. Vol. 1. Washington, DC: AASHTO.

Brill, Edward A. 1972. “Car-Following Model Relating Reaction Times and Temporal Headways to Accident Frequency.” *Transportation Science* 6 (4): 343–353. doi:10.1287/trsc.6.4.343.

Chandler, Robert E., Robert Herman, and Elliott W. Montroll. 1958. “Traffic Dynamics: Studies in Car Following.” *Operations Research* 6 (2): 165–184. doi:10.1287/opre.6.2.165.

Chatterjee, Indrajit, and Gary A. Davis. 2016. “Analysis of Rear-End Events on Congested Freeways by Using Video-Recorded Shock Waves.” *Transportation Research Record* 2583 (1): 110–118. doi:10.3141/2583-14.

Chou, Chih Sheng, and Andrew P. Nichols. 2015. “Deriving a Surrogate Safety Measure for Freeway Incidents Based on Predicted End-of-Queue Properties.” *IET Intelligent Transport Systems* 9 (1): 22–29. doi:10.1049/iet-its.2013.0199.

Davis, Gary A., and Tait Swenson. 2006. “Collective Responsibility for Freeway Rear-Ending Accidents? An Application of Probabilistic Causal Models.” *Accident Analysis and Prevention* 38 (4): 728–736. doi:10.1016/j.aap.2006.01.003.

Edie, Leslie C. 1961. “Car-Following and Steady-State Theory for Noncongested Traffic.” *Operations Research* 9 (1): 66–76. doi:10.1287/opre.9.1.66.

Gazis, Denos C., Robert Herman, and Richard W. Rothery. 1961. “Nonlinear Follow-the-Leader Models of Traffic Flow.” *Operations Research* 9 (4): 545–567. doi:10.1287/opre.9.4.545.

Hauer, Ezra. 2015. *The Art of Regression Modeling in Road Safety*. New York: Springer.

Kim, Taewan, and H. Michael Zhang. 2011. “Interrelations of Reaction Time, Driver Sensitivity, and Time Headway in Congested Traffic.” *Transportation Research Record* 2249 (1): 52–61. doi:10.3141/2249-08.

Li, Ye, Dan Wu, Jaeyoung Lee, Min Yang, and Yuntao Shi. 2020. “Analysis of the Transition Condition of Rear-End Collisions Using Time-to-Collision Index and Vehicle Trajectory Data.” *Accident Analysis and Prevention* 144. doi:10.1016/j.aap.2020.105676.

Liu, C., B. M. Williams, and N. M. Rouphail. 2012. “Temporal Stability of Freeway Macroscopic Traffic Stream Models.” *Transportation Research Record* 2315 (1): 131–140.

Lord, Dominique, Abdelaziz Manar, and Anna Vizioli. 2005. “Modeling Crash-Flow-Density and Crash-Flow-V/C Ratio Relationships for Rural and Urban Freeway Segments.” *Accident Analysis and Prevention* 37 (1): 185–199. doi:10.1016/j.aap.2004.07.003.

May, A. D. 1990. *Traffic Flow Fundamentals*. 
Misener, J. A., H. S. J. Tsao, B. Song, and A. Steinfeld. 2000. “Emergence of a Cognitive Car-Following Driver Model: Application to Rear-End Crashes with a Stopped Lead Vehicle.” Transportation Research Record 1724 (1): 29–38. doi:10.3141/1724-05.

North Carolina Department of Transportation. 2020. https://connect.ncdot.gov/resources/safety/Pages/TEAAS-Crash-Data.

Oh, Cheol, and Taejin Kim. 2010. “Estimation of Rear-End Crash Potential Using Vehicle Trajectory Data.” Accident Analysis and Prevention 42 (6): 1888–1893. doi:10.1016/j.aap.2010.05.009.

Pande, Anurag, and Mohamed Abdel-Aty. 2006. “Comprehensive Analysis of the Relationship Between Real-Time Traffic Surveillance Data and Rear-End Crashes on Freeways.” Transportation Research Record 1953 (1): 31–40. doi:10.3141/1953-04.

Rahman, Moynur, and Min Wook Kang. 2020. “Safety Evaluation of Drowsy Driving Advisory System: Alabama Case Study.” Journal of Safety Research 74: 45–53. doi:10.1016/j.jsr.2020.04.005.

Smith, Ralph C. 2013. Uncertainty Quantification: Theory, Implementation, and Applications. Philadelphia: Siam.

Srinivasan, Raghavan, Michael Colety, Geni Bahar, Brent Crowther, and Matt Farmen. 2016. “Estimation of Calibration Functions for Predicting Crashes on Rural Two-Lane Roads in Arizona.” Transportation Research Record 2583 (1): 17–24. doi:10.3141/2583-03.

Tanaka, Mitsuru, Prakash Ranjitkar, and Takashi Nakatsuji. 2008. “Asymptotic Stability and Vehicle Safety in Dynamic Car-Following Platoon.” Transportation Research Record 2088 (1): 198–207. doi:10.3141/2088-21.

Tanvir, Shams, R. T. Chase, and N. M. Roupahil. 2019. “Development and Analysis of Eco-Driving Metrics for Naturalistic Instrumented Vehicles.” Journal of Intelligent Transportation Systems, 1–14. doi:10.1080/15472450.2019.1615486.

Touran, Ali, Mark A. Brackstone, and Mike McDonald. 1999. “A Collision Model for Safety Evaluation of Autonomous Intelligent Cruise Control.” Accident Analysis and Prevention 31 (5): 567–578. doi:10.1016/S0965-8564(99)00013-5.

Transportation Research Board. 2016. Highway Capacity Manual. 6th ed. Washington, DC: The National Academies of Sciences, Engineering, and Medicine.

University of Virginia Library, Charlottesville. “Understanding Diagnostic Plots for Linear Regression Analysis”. https://data.library.virginia.edu/diagnostic-plots/.

USDOT. 2006. Next Generation Simulation (NGSIM). https://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm.

Wu, Ning. 2002. “A New Approach for Modeling of Fundamental Diagrams.” Transportation Research Part A: Policy and Practice 36 (10): 867–884. doi:10.1016/S0965-8564(01)00043-X.

Xu, Tu, and Jorge A. Laval. 2019. “Analysis of a Two-Regime Stochastic Car-Following Model: Explaining Capacity Drop and Oscillation Instabilities.” Transportation Research Record 2673 (10): 610–619. doi:10.1177/0361198119850464.

Xu, Chengcheng, Wei Wang, Pan Liu, and Fangwei Zhang. 2015. “Development of a Real-Time Crash Risk Prediction Model Incorporating the Various Crash Mechanisms Across Different Traffic States.” Traffic Injury Prevention 16 (1): 28–35. doi:10.1080/15389588.2014.909036.

Zielke, Benjamin A., Robert L. Bertini, and Martin Treiber. 2008. “Empirical Measurement of Freeway Oscillation Characteristics: An International Comparison.” Transportation Research Record 2088 (1): 57–67. doi:10.3141/2088-07.
Appendices

Appendix 1: Flow-density plots for all sensors – fitted curve and observed data
Appendix 2: Difference in parameter estimates and $\Delta t_1$ between 2013 and rest of the years for two sites

| ID | Year | $u_f$ | $k_b$ | $k_j$ | $l$  | $m$  | $u_f$ | $k_b$ | $k_j$ | $l$  | $m$  | $\Delta t_1$ |
|----|------|-------|-------|-------|------|------|-------|-------|-------|------|------|-------------|
| 17E | 2011 | −1.19%| 7.62% | 32.24%| −33.29%| −0.89%| 0.00% | 3.20% | −0.10%| 1.61%| 1.15%| 8.29% |
|    | 2012 | −0.92%| 6.86% | 0.61% | −20.24%| −0.98%| 0.00% | 4.22% | 0.01% | 1.17%| 0.78%| −17.97%|
|    | 2014 | 1.51% | 3.58% | 0.55% | −8.93%| −0.90%| 0.00% | −1.37%| 0.01% | 0.97%| 0.72%| 12.90%|
| 20W | 2011 | −3.91%| −7.99%| 0.01% | 24.12% | 2.28% | 0.00% | −5.36%| −6.17%| −1.85%| −5.58%| 1.63% |
|    | 2012 | 0.04% | −3.17%| 0.00% | −0.83%| −0.70%| 0.00% | −3.20%| 0.00% | −1.07%| −3.15%| 3.66% |
|    | 2014 | −0.03%| −0.89%| 0.03% | 6.71% | −0.49%| 0.00% | 1.17% | 1.02% | −0.29%| −0.96%| 10.98% |