Flood Risk Assessment in Small Watershed Based on Catastrophe Theory

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Abstract. Flood system is a complex system affected by many control factors and has typical system characteristics, such as irreversibility, fuzzy characteristics, gray characteristics, etc. Catastrophe theory is a well developed singularity theory, which has been initially applied in some leaping disaster systems. However, in the field of flood risk management, the application of catastrophe theory is relatively few. In this paper, a small watershed in the mountainous area of southwest China is taken for an example. The catastrophe assessment theory is adopted to conduct flood risk assessment. By analyzing the characteristics of the flood system and selecting flood risk factors, the flood risk assessment index system is established and the catastrophe theory flood risk assessment model is conducted, thus conducting flood risk zoning and management according to the assessment result, which provides a new reliable method for flood risk assessment and offers foundation for disaster reduction department.

1. Introduction
In recent years, flood disasters occur frequently all over the world. The severity of flood disasters lies in the destruction of the ecological environment on which human beings depend for survival, the direct loss of people’s lives and property, and the serious impact on economic development and social progress. In order to prevent and mitigate flood disasters, a series of flood control and disaster mitigation measures need to be taken. Among them, risk assessment is a necessary step to develop appropriate and effective disaster reduction policies and measures. Flood risk analysis refers to the identification, estimation and evaluation of various risk factors in the process of flood disaster and reduction, and hence managing disasters and making decisions. Due to the limitation of cognition and evaluation methods, the current situation of flood risk analysis mainly focuses on single risk, but the formation of flood is the result of combined activities of astronomy, geography, meteorology, climate, hydrology and vegetation and human beings, thus requiring comprehensive analysis of flood analysis and risk management, otherwise it will lack reliability. Huang D et al. obtained the index weight through expert scoring, and established the vulnerability model of disaster-bearing body\cite{1}. Luo P et al. used fuzzy comprehensive assessment method and other techniques to study storm disaster risk\cite{2}. Li L et al. used analytic hierarchy process to construct storm disaster risk assessment model\cite{3}. In this paper, the catastrophe theory evaluation method is adopted to evaluate the flood risk, a small watershed in the mountainous area of southwest China is taken for an example.
2. Catastrophe theory

Catastrophe evaluation method is a comprehensive evaluation method based on catastrophe theory. Catastrophe theory is a kind of topological theory founded by French mathematician Rene Thom in 1960 based on the system stability theory[4], singularity theory and other mathematical theories, and then studied catastrophe phenomenon on the basis, which is known as a revolution in mathematics after calculus. Catastrophe theory has been initially applied in some multi-objective evaluation problems like leaping disaster systems. However, in the field of flood risk management, the application of catastrophe theory is relatively few. Multi-objective evaluation refers to the selection and sorting of multiple objects (indicators), and these objects (indicators) show different quality states, so the catastrophe mathematical model can be used to select and sort the multiple objects (indicators) . On one hand, the quantification of the importance of each index by catastrophe theory is based on the internal contradiction status and mechanism of the index in the formula itself, which reduces the subjectivity of weight setting and improves the practicality of operation. On the other hand, the catastrophe theory is suitable for systems with unknown internal functions, while the flood disaster system is affected by many factors, and the interaction mechanism of all factors is not clear, so the catastrophe theory is relatively appropriate. In conclusion, compared with other evaluation method, the catastrophe evaluation method avoids the arbitrariness of determining the subjective weight in ahp, and over objectivity of principal component analysis method, which is more comprehensive and can reflect the evaluated problem more comprehensively and truly.

2.1 Catastrophe theory basic model

The characteristics of the catastrophe theory is to classify the critical points of the system according to the potential function of the system, study the characteristics of the discontinuous changing states near the classification critical points, and then conclude several elementary catastrophe models. One catastrophe corresponds to only one potential function. There are state variables and control variables in a mutation system: state variables $x, y$ describe the behavior of the system, while control variables $a, b, c, d$ represent the control factors that affect the system state change. Rene. Thom studies the effects of different parameters and variables and proves that, when the number of state variables is no more than two and the number of control variables is no more than four, there are seven mutation forms of the elementary mutation theory, namely seven mutation models. Their potential functions are shown as table 1. Among them, folded, cusp, swallowtail, butterfly catastrophe models are used more common. From the potential function $V$, the equilibrium surface $M$, the singularity set $S$, and the bifurcation set $B$ can be obtained[5].

| Table 1. Basic types of catastrophe functions |
|-----------------------------------------------|
| Catastrophe model               | Control variable dimensions | State variable dimensions | Potential function         |
|-----------------------------------------------|
| Folded catastrophe                | 1                           | 1                         | $V_a(x) = x^3 + ax$         |
| Cusp catastrophe                  | 2                           | 1                         | $V_{ab}(x) = x^4 + ax^2 + bx$ |
| Swallowtail catastrophe           | 3                           | 1                         | $V_{abc}(x) = x^5 + ax^3 + bx^2 + cx$ |
| Butterfly catastrophe            | 4                           | 1                         | $V_{abcd}(x) = x^6 + ax^4 + bx^3 + cx^2 + dx$ |

2.2 Normalized operation

Normalization operation is a recursive operation using catastrophe theory to conduct comprehensive quantization and basic operation formula for comprehensive analysis and evaluation to transform the different states of control variable in the system into the same kind of state that can be compared. By normalizing the bifurcation set of the catastrophe model, the normalization formula of the catastrophe model can be obtained, as shown in table 2[6].
Table 2. Normalization formula of catastrophe model

| Catastrophe model       | Normalization formula |
|------------------------|-----------------------|
| Cusp catastrophe        | $x_a = a^2, x_b = b^3$ |
| Swallowtail catastrophe | $x_a = a^2, x_b = b^3, x_c = c^3$ |
| Butterfly catastrophe  | $x_a = a^2, x_b = b^3, x_c = c^3, x_d = d^3$ |

In the normalization formula, the importance of state variable $x, y$, control variable $a, b, c, d$ gradually decreases. After normalization, the catastrophe fuzzy membership function with same unit and order of magnitude is obtained, and the value range of control variable and state variable is limited to $0~1$[7]. This is the core of catastrophe multi-criteria evaluation method. The two transformation formulas are as follows:

- The bigger the factor, the better: $\frac{(x_i - x_{\text{min}})}{(x_{\text{max}} - x_{\text{min}})}$ (1)
- The smaller the factor, the better: $\frac{(x_{\text{max}} - x_i)}{(x_{\text{max}} - x_{\text{min}})}$ (2)

2.3 Comprehensive assessment

In using catastrophe theory for evaluation, firstly dividing the total evaluation index, then decomposing the total index into tree-like indexes, processing each tree-like index dimensionless and carrying out normalization operation. Calculating recursively from the bottom layer to the top layer to obtain the evaluation result. In the evaluation process, three different catastrophe criterias can be selected according to different actual problems. First, the complementary criterion: each control variable can make up for the deficiency of each other, usually using "mean value". Second, non-complementary criteria: the role of control variables in the system can be replaced by each other and can not make up for each other, following the principle of "minimax criterion". Third, the criterion of over-threshold complementary criterion: the control variables can only be make up for each other only after reaching a certain threshold, following the principle of "taking the mean value after reaching the threshold".

3. Flood risk assessment index system

Flood system is a complex system affected by many control factors, including external and internal factors, natural and unnatural factors and it has typical system characteristics, such as irreversibility, fuzzy characteristics, gray characteristics, etc. Based on the analysis of the characteristics of flood disaster in the watershed, this paper starts with the basic concept of flood risk, selecting risk factors according to system theory and disaster science theory, and establishes the risk assessment index system. This paper uses causation analysis method to establish the flood risk assessment index system, including three levels: target level $A$, criterion level $B$ and index level $C$. Among them, the criterion layer analyzes the disaster drivers $B_1$, disaster environment $B_2$, disaster bearers $B_3$ and disaster bearing capacity $B_4$. From these four aspects, this paper expounds their impact on flood disaster, selecting representative and quantifiable impact factors as index factors of risk division to establish the evaluation system. Index level $C$ is the specific evaluation index selected based on the criterion layer. The specific index of the criterion layer is analyzed as follows:

- Disaster drivers $B_1$: Flood disaster driver factors are mainly the variation characteristics of regional flood itself. The higher the frequency and the greater the intensity is, the greater the damage is. The average annual precipitation $C_{11}$ and rainfall patterns $C_{12}$ (the relationship between the maximum 1h
rainfall in 20 years and the average annual 1h rainfall)[8] are selected to describe the disaster driver factors.

Disaster environment $B_2$: The environment factors mainly refer to the sensitivity of the external environment threatened by flood, which mainly includes the environmental conditions such as topography, water system and vegetation. The slope of river $C_{21}$ and forest area $C_{22}$ are used to describe the topographic conditions of the underlying surface. The lower the slope of river and the larger the forest area is, the stronger the flood drainage capacity is, and the less the impact of rainstorm disaster is.

Disaster bearers $B_3$: The vulnerability of disaster-bearing mainly considers from the aspects of population and economy. Population density $C_{31}$ is selected to describe the population situation, GDP $C_{32}$ and regional property $C_{33}$ are selected to describe the economic situation.

Disaster bearing capacity $B_4$: The flood control and disaster bearing capacity mainly considers the flood control and monitoring capacity of individual society and the support capacity of social foundation. Index of water holding capacity $C_{41}$ is selected (control area of small watershed to total area)[9].

In conclusion, the assessment system is established as shown in Figure 1.

![Flood risk assessment index system](image)

**Figure 1.** Flood risk assessment index system

### 4. Flood risk assessment results

In this paper, a small watershed in the mountainous area of southwest China is taken for an example. Assessing flood risk in four natural villages across the small watershed: Weigu, Hongyan, Mawo, Luoduo. The flood disaster risk assessment scheme of this region is constructed by using flood risk assessment index. According to the collected data, the actual values of flood risk assessment factors are shown as table 3.

| Table 3. Flood risk assessment factors actual values |
|---------------------------------|-----------------|--------------|-------------|-------------|
| Village | Average annual precipitation/mm | Weigu | Hongyan | Mawo | Luoduo |
| Disaster driver | Rainfall patterns | 620 | 1210 | 976 | 1148 |
| Disaster environment | Slope of river | 343 | 278 | 376 | 789 |
| | Forest area/ $km^2$ | 0.065 | 0.072 | 0.058 | 0.060 |
| | Water holding capacity | 7.28 | 5.32 | 6.72 | 6.85 |
According to the catastrophe theory evaluation method, the specific application of the criterion layer is as follows: For disaster driver. Firstly, the bigger the value of \( C_{11} \) and \( C_{12} \) is, the greater the flood risk is. So formula (1) is adopted to standardize the data. The standard values of flood risk assessment factors are shown as table 4.

**Table 4. Flood risk assessment factors standard values**

| Village | Disaster driver | Disaster environment | Disaster bearers | Disaster bearing capacity |
|---------|-----------------|----------------------|-----------------|--------------------------|
|         | Average annual precipitation | Rainfall patterns | Slope of river | Forest area | Population dentist | GDP | Property | Water holding capacity |
| Weigu   | 0               | 0.1441               | 0.5             | 0           | 0.3077           | 0.1235 | 0.0977 | 0.5913 |
| Hongyan | 1               | 0.2173               | 1               | 0.2857      | 0.0077           | 1      | 1       | 0.5486 |
| Mawo    | 0.6034          | 0.1429               | 0.1429          | 0.2194      | 0.4615           | 0.3887 | 0.2395 | 1       |
| Luoduo  | 0.8949          | 1                    | 0.2194          | 0.2395      | 1               |        |         |         |

Secondly, taking Weigu for example, \( C_{11} \) and \( C_{12} \) constitute cusp catastrophe model, so the cusp catastrophe formula is used for normalization. \( X_{C_{11}} = C_{11}^{\frac{1}{3}} = 0, X_{C_{12}} = C_{12}^{\frac{1}{3}} = 0.5243 \). Lastly, \( C_{11} \) and \( C_{12} \) are complementary indicators, and \( B_1 \) is obtained based on the principle of average. \( B_1 = 0.2622 \).

Similarly, according to catastrophe model basic principle, the criterion layer assessment results of each village is obtained. The comprehensive assessment result is controlled by four variables \( B_1, B_2, B_3, B_4 \), constituting butterfly catastrophe model, so the butterfly catastrophe formula is used for normalization. Finally, according to the complementary principle, the flood risk in each village is comprehensively evaluate. The larger the value \( A \) is, the greater the risk is, as shown in table 5.

**Table 5. Flood risk assessment results**

| Village | Criterion layer assessment results | A | Ranking |
|---------|-----------------------------------|---|---------|
|         | \( B_1 \) | \( B_2 \) | \( B_3 \) | \( B_4 \) |
| Weigu   | 0.2622          | 0.3536          | 0.4980          | 0.7689          | 0.7520          | 2 |
| Hongyan | 0.5              | 0.5             | 1               | 0               | 0.6252          | 3 |
| Mawo    | 0.689            | 0.3293          | 0               | 0.7407          | 0.6156          | 4 |
| Luoduo  | 0.9730           | 0.4906          | 0.6793          | 1               | 0.9207          | 1 |
According to the flood risk assessment results, the risk level of each village from high to low is: Luoduo, Weigu, Hongyan, Mawo. The risk level of each village obtained from the analysis results is basically consistent with the actual flood risk level of villages involved in this area, which verifies that the application of catastrophe theory to flood risk assessment is feasible and reasonable, and can comprehensively reflect the characteristics of flood risk.

5. Conclusions
This paper takes a small watershed of mountainous area in southwest China as an example, considering the risk source factors of flood disaster, and using catastrophe theory method to evaluate flood risk. Firstly, the total index of flood risk is decomposed, risk assessment factors are selected and arranged according to the importance of the index, so as to reduce the subjectivity of the evaluation results and make the evaluation results closer to the actual situation, verifying the rationality of applying catastrophe theory to flood risk assessment. However, It is still necessary to further study and improve the indicator division of each layer, the order of importance of indicators and the judgment of relationship between indicators.

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