Observer-Based Controllers for Incrementally Quadratic Nonlinear Systems With Disturbances

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Abstract—Robust global stabilization of nonlinear systems by observer-based feedback controllers is a challenging task. This article investigates the problem of designing observer-based stabilizing controllers for incrementally quadratic nonlinear systems with external disturbances. The nonlinearities considered in the system model satisfy the incremental quadratic constraints, which are characterized by incremental multiplier matrices and encompass many common nonlinearities. The simultaneous search for the observer and the controller gain matrices is formulated as a feasibility problem of linear matrix inequalities, for two parameterizations (i.e., the block diagonal parameterization and the block antitriangular parameterization) of the incremental multiplier matrices, respectively. The closed-loop system implementing the observer-based feedback controller is proven to be input-to-state stable with respect to external disturbances. Using the proposed continuous-time observer-based controllers, event-triggered controllers with time regularization are constructed for globally Lipschitz systems, such that the closed-loop system is Zeno-free and input-to-state practically stable.

Index Terms—Event-triggered control (ETC), incrementally quadratic nonlinearity, observer-based control, stabilization of nonlinear systems.

I. INTRODUCTION

A

THE state variables of a system are difficult or expensive to measure in practice, output feedback control design has received a lot of attention (see, e.g., [1–4]) and found applications in biological systems [5–7], mechanical systems [8–10], power systems [11], [12], and networked control systems [13], [14], among others. For linear systems, the output feedback stabilizing control design problem can be solved by designing the state-feedback controller and the state observer independently, which is known as the controller-observer separation principle. For nonlinear systems, a certainty-equivalence implementation of a globally stabilizing state-feedback controller with an asymptotic observer can lead to finite escape time (e.g., see the counter-examples in [15] and [16]), which makes observer-based stabilizing controller design a challenging problem [17]. By using a high-gain observer [18], [19], separation principles for input–output linearizable systems were studied in [20]–[23], and semiglobal asymptotic stability of the resulting closed-loop systems was proved in these papers. Separation principles for some other special class of nonlinear systems were also investigated, such as bilinear systems [24], [25], nonaffine nonlinear systems [26], systems with nondecreasing or slope-restricted nonlinearities [27]–[29], and cascaded systems [30], [31]. Apart from the certainty-equivalence approach, interdependent design of the controller and the observer was investigated in [32]–[34].

Linear matrix inequalities (LMIs) provide a computationally efficient approach for the synthesis of observer-based output feedback controllers [35], where the main difficulty lies in the coupling between the unknown matrices of the observer and the controller and the Lyapunov matrices. For linear systems, LMI-based conditions were proposed for the robust observer-based stabilization of linear systems with state perturbations [36] or with parametric uncertainties [37], [38]. For nonlinear systems, the synthesis problem is often formulated as the feasibility of bilinear matrix inequalities (BMIs), which is known to be an NP-hard problem [39]. Different approaches that aim to transform the nonconvex BMI conditions to convex LMI conditions have been proposed: Zemouche et al. [38] studied observer-based controller design for Lipschitz nonlinear systems with uncertain parameters, and developed an LMI-based design technique that relies on the linearization of the corresponding BMIs; Wang et al. [40] investigated observer-based control design for the interconnection of a linear system and an uncertain nonlinear operator satisfying the integral quadratic constraint, and proposed a sequential LMI algorithm to solve BMIs; Kim and Braatz [41] studied output feedback control of discrete-time parametric uncertain Lure systems, and developed an LMI-based iterative algorithm to solve BMIs. Moreover, Grandvallet et al. [42] investigated $H_{\infty}$ stabilization of discrete-time globally Lipschitz nonlinear systems, and provided LMI-based conditions that compute simultaneously the observer and controller gains; Ekramian [43] considered asymptotic stabilization

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of continuous-time Lipschitz nonlinear systems and developed LMI-based conditions that synthesize the gain matrices of the observer-based controller.

This article considers observer-based output feedback global stabilization of a class of nonlinear systems whose nonlinearities satisfy incremental quadratic constraints. The incremental quadratic constraint is characterized by an incremental quadratic inequality with incremental multiplier matrices [44]–[49]. This characterization with incremental multiplier matrices provides a general framework to represent many common classes of nonlinearities (e.g., globally Lipschitz nonlinearities, incrementally sector bounded nonlinearities, nondecreasing nonlinearities, and the polytopic Jacobian nonlinearities), implying a wide range of applicability for the proposed theoretical results. Observer design for systems with nonlinearities satisfying incremental quadratic constraints was studied in [48], which was later generalized to the systems with bounded exogenous disturbances in [50]. Observer-based control design for some special classes of incrementally quadratic nonlinear systems have been investigated in [28], [43], [51]–[53].

Motivated by the development of networked control systems, event-triggered control (ETC) has recently received a lot of attention as it provides a new control paradigm to reduce the resource consumption of networked control systems whose communication bandwidth and computational power are usually limited [54]–[56]. Most of the ETC results assume that full-state information is available, but this assumption is restrictive since many systems only have information on their measured outputs. Extending results on observer-based, ETC design from linear systems (e.g., see [57]–[59]) to nonlinear systems is difficult [60]. Existing results on ETC design for nonlinear systems mostly assume that the continuous-time observer-based controllers are already given, but the observers and controllers themselves can be hard to construct. When external disturbances or measurement noise are present, the triggering rules also need to be carefully designed to rule out the Zeno phenomenon [60]–[62], e.g., using time regularization to enforce a built-in lower bound for interexecution times.

The main contributions of the article are summarized as follows. For incrementally quadratic nonlinear systems affected by external disturbances and measurement noise, LMI-based sufficient conditions are developed for the design of robust stabilizing observer-based controllers. The simultaneous search for the observer and the controller gain matrices is formulated as a feasibility problem of LMIs when the incremental multiplier matrices are parameterized as the block diagonal matrices or the block antitriangular matrices. The resulting closed-loop system is proven to be input-to-state stable (ISS) with respect to disturbances. Using the proposed continuous-time observer-based controller, event-triggered controllers are constructed for globally Lipschitz systems affected by external disturbances and measurement noise where the triggering rule is designed with an enforced positive lower-bound on interexecution times. The resulting closed-loop system is Zeno-free and input-to-state practically stable (ISpS) with respect to external disturbances.

A preliminary version of this article appeared in [63]. The present article is different from [63] in the following important ways: the system model considered is subject to external disturbances and measurement noise; the event-triggered mechanism is considered; complete proofs are included, and more discussion is added. The remainder of the article is organized as follows: Section II introduces preliminaries on incremental quadratic constraints and input-to-state practical stability, Section III develops LMI-based conditions for the design of robust stabilizing observer-based controllers for two parameterizations of the incremental multiplier matrices, Section IV presents the event-triggered controller design, Section V provides a simulation example, and Section VI provides the conclusions.

Notation: $\mathbb{R}_0^+$ denotes the set of non-negative real numbers; $\|x\|$ denotes the 2-norm of a vector $x$; $\|P\|$ denotes the maximum singular value of a matrix $P$; $\lambda_{\min}(P)$ and $\lambda_M(P)$ denote the minimum and maximum eigenvalues of a symmetric matrix $P$, respectively; $I_n$ identifies an identity matrix of size $n$; $O_{n_1 \times n_2}$ and $O_o$ denote the zero matrix of size $n_1 \times n_2$ and the zero vector of size $n$, respectively, where the subscript will be omitted when clear from context. For symmetric matrices, * denotes entries whose values follow from symmetry. For a matrix $M$, $M > 0$, $M \geq 0$, $M < 0$, $M \leq 0$ mean $M$ is positive definite, positive semidefinite, negative definite, and negative semidefinite, respectively. A continuous function $f : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ belongs to class $K$ (denoted as $f \in K$) if it is strictly increasing and $f(0) = 0$; $f$ belongs to class $K_{\infty}$ (denoted as $f \in K_{\infty}$) if $f \in K$ and $f(r) \rightarrow \infty$ as $r \rightarrow \infty$. A continuous function $f : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ belongs to class $K_{\mathcal{L}}$ (denoted as $f \in K_{\mathcal{L}}$) if for each fixed $s$, the function $f(\cdot, s) \in K_{\infty}$ and for each fixed $r$, the function $f(r, \cdot)$ is decreasing and $f(r, s) \rightarrow 0$ as $s \rightarrow 0$.

## II. Preliminaries

Consider the following nonlinear system:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Ep(q) + E_w w \\
y &= Cx + Du + F_w w \\
q &= C_q x
\end{align*}
\]

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the control input, $y \in \mathbb{R}^{n_y}$ is the measured output, $p : \mathbb{R}_0^{n_q} \rightarrow \mathbb{R}_0^{n_p}$ is the known nonlinearity of the system, $w \in \mathbb{R}^{n_w}$ is the unknown external disturbance or measurement noise, and $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_x}$, $D \in \mathbb{R}^{n_y \times n_u}$, $C_q \in \mathbb{R}^{n_q \times n_x}$, $E \in \mathbb{R}^{n_x \times n_w}$, $E_w \in \mathbb{R}^{n_x \times n_w}$, $F_w \in \mathbb{R}^{n_y \times n_w}$ are constant matrices of appropriate dimensions.

The characterization of the nonlinearity $p$ is based on incremental multiplier matrices [48], [49].

**Definition 1:** Given a function $p : \mathbb{R}_0^{n_q} \rightarrow \mathbb{R}_0^{n_p}$, a symmetric matrix $M \in \mathbb{R}^{(n_q+n_p) \times (n_q+n_p)}$ is called an incremental multiplier matrix ($\delta$-MM) for $p$ if it satisfies the following incremental quadratic constraint for all $q_1, q_2 \in \mathbb{R}^{n_q}$:

\[
\begin{pmatrix} \delta q \\ \delta p \end{pmatrix}^\top M \begin{pmatrix} \delta q \\ \delta p \end{pmatrix} \geq 0
\]

where $\delta q = q_2 - q_1$, $\delta p = p(q_2) - p(q_1)$. 
For a given nonlinearity \( p \), its \( \delta \)-MM is not unique. Denote \( M \) as the set of incremental multiplier matrices for \( p \). If \( M \in \mathcal{M} \), then \( \lambda M \in \mathcal{M} \) for any \( \lambda \geq 0 \).

**Remark 1:** The global Lipschitz condition \( \|p(q_2) - p(q_1)\| \leq \gamma \|q_2 - q_1\| \) where \( \gamma > 0 \) can be expressed in the form of (2) with

\[
M = \begin{pmatrix} \gamma^2 I & 0 \\ 0 & -I \end{pmatrix}.
\]  

(3)

The incrementally sector bounded nonlinearity \((\delta p - K_1 \delta q) S(\delta p - K_2 \delta q) \leq 0 \) where \( S = S^\top \) can be expressed in the form of (2) with

\[
M = \begin{pmatrix} -K_1^\top S K_2 - K_2^\top S K_1 & * \\ S(K_1 + K_2) & -2S \end{pmatrix}.
\]  

(4)

The nondecreasing nonlinearity, which satisfies \( \delta p \top \delta q \geq 0 \), can be expressed in the form of (2) with

\[
M = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.
\]  

(5)

Refer to [48] and [49] for some other nonlinearities that can be expressed using the incremental quadratic constraint.

Next, we introduce input-to-state practical stability and its characterization using Lyapunov functions. Consider the system

\[
\dot{x} = f(x, u)
\]  

(6)

where \( f : \mathbb{R}^{n_\delta} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_\delta} \) is a locally Lipschitz function and \( u : \mathbb{R} \to \mathbb{R}^{n_u} \) is a measurable essentially bounded input. Define \( x(t, x_0, u) \) as the solution of (6) with initial state \( x_0 \) and input \( u \), which satisfies \( x(0, x_0, u) = x_0 \).

**Definition 2 (see [64, Def. 2.1]):** The system (6) is called ISpS w.r.t. \( u \), if there exist functions \( \beta_1 \in \mathcal{K} \mathcal{L} \), \( \beta_2 \in \mathcal{K} \), and a non-negative constant \( d \) such that for every initial state \( x_0 \) and every measurable essentially bounded \( u \) defined on \([0, \infty)\), the solution \( x(t, x_0, u) \) exists on \([0, \infty)\) and satisfies

\[
\|x(t, x_0, u)\| \leq \beta_1(\|x_0\|, t) + \beta_2(\|u\|_\infty) + d \quad \forall t \geq 0
\]  

(7)

where \( \|u\|_\infty := \text{esssup}_{t \geq 0} \|u(t)\| \).

When (7) is satisfied with \( d = 0 \), the system is said to be ISS w.r.t. \( u \) [65].

**Definition 3 (see [64, Remark 2.2]):** A smooth function \( V : \mathbb{R}^n \to \mathbb{R} \) is said to be an ISpS-Lyapunov function for the system (6) if \( V \) is radially unbounded, positive definite, and there exist functions \( \gamma \in \mathcal{K}_\infty \), \( \chi \in \mathcal{K} \) and a non-negative constant \( d \) such that the following condition holds:

\[
\nabla V(x)^T f(x, u) \leq -\gamma(\|x\|) + \chi(\|u\|) + d.
\]  

(8)

Instead of requiring inequality (8), the ISpS-Lyapunov function can be also defined equivalently as follows: a smooth, positive definite, radially unbounded function \( V \) is an ISpS-Lyapunov function for the system (6) if there exist a positive-definite function \( \gamma \), a class \( \mathcal{K} \) function \( \chi \) and a non-negative constant \( d \) such that the following condition holds ( [64, Def. 2.2]):

\[
\|x\| \geq \chi(\|u\|) + d \Rightarrow \nabla V(x)^T f(x, u) \leq -\gamma(\|x\|).
\]  

(9)

The existence of an ISpS-Lyapunov function is a necessary and sufficient condition for the ISpS property.

**Proposition 1 (see [66]):** The system (6) is ISpS (resp. ISS) if and only if it has an ISpS (resp. ISS) Lyapunov function.

In particular, if there exist a symmetric and positive definite matrix \( P = P^\top \geq 0 \), two constants \( \alpha > 0 \), \( d \geq 0 \) and a function \( \chi \in \mathcal{K}_\infty \) such that the positive definite function \( V(x) = x^\top P x \) satisfies

\[
\nabla V(x)^T f(x, u) \leq -\alpha V(x) + \chi(\|u\|) + d
\]  

(10)

then \( V \) is an ISpS-Lyapunov function satisfying (8) with \( \gamma(\|x\|) = \alpha \lambda_m(P) \|x\|^2 \), implying that (6) is ISpS w.r.t. \( u \).

### III. LMI-BASED CONDITIONS FOR ROBUST GLOBAL STABILIZATION OF INCREMENTALLY QUADRATIC NONLINEAR SYSTEMS

Consider a system described by (1) where the nonlinear term \( p \) satisfies the incremental quadratic constraint (2) for some \( M \in \mathcal{M} \). In this section, a continuous-time observer and a feedback controller will be designed for (1), such that the closed-loop system is ISS w.r.t. \( u \). LMI-based sufficient conditions will be given for the simultaneous design of the observer and controller gain matrices.

The following observer is proposed:

\[
\begin{align*}
\dot{\hat{x}} &= A \hat{x} + Bu + E p(\hat{q} + L_1(\hat{y} - y)) + L_2(\hat{y} - y) \\
y &= C \hat{x} + Du &
\end{align*}
\]  

(11)

where \( L_1, L_2 \) are gain matrices to be designed. This observer contains a copy of the plant and two correction terms, the nonlinear injection term \( L_1(\hat{y} - y) \) and the Luenberger-type correction term \( L_2(\hat{y} - y) \). Based on observer (11), we design the feedback controller \( u \) as

\[
u = k(\hat{x})
\]  

(12)

where \( k : \mathbb{R}^{n_\delta} \to \mathbb{R}^{n_u} \) is a function that has the form of

\[
k(x) = K_1 x + K_2 p(C_q x)
\]  

(13)

with gain matrices \( K_1 \in \mathbb{R}^{n_\delta \times n_\delta}, K_2 \in \mathbb{R}^{n_\delta \times n_p} \) to be designed. Defining the estimation error by

\[
e(t) = x(t) - \hat{x}(t)
\]  

the input (12) can be rewritten as

\[
u = k(x) - \Delta k(x, \hat{x})
\]  

where \( \Delta k(x, \hat{x}) = k(x) - k(\hat{x}) \). Recalling (13), \( \Delta k \) can be expressed as \( \Delta k = K_1 e - K_2 \Delta p \) where

\[
\Delta p = p(\hat{q}) - p(q).
\]  

(14)

The closed-loop system resulting from the observer-based controller (12) can now be expressed as

\[
\begin{align*}
\dot{x} &= (A + BK_1) x + (E + BK_2) p - B \Delta k + E w \\
\dot{\hat{e}} &= (A + L_2 C) e - E \delta p + (E_w + L_2 F_w) w
\end{align*}
\]  

(15)
where
\[
\begin{align*}
\delta p &= p(q + \delta q) - p(q), \\
\delta q &= -(C_q + L_1 C) e - L_1 F w.
\end{align*}
\] (16)

Defining \( z = (x e) \), dynamics (15) are expressed compactly as
\[
\dot{z} = A_c z + H_1 p + H_2 \delta p + H_3 \Delta p + H_4 w.
\] (17)

where \( \Delta p \) is given in (14), \( \delta p \) is given in (16), and
\[
A_c = \begin{pmatrix}
A + BK_1 & -BK_1 \\
0 & A + L_2 C
\end{pmatrix}
\] (18)

\[
H_1 = \begin{pmatrix}
E + BK_2 \\
0
\end{pmatrix}, \quad H_2 = \begin{pmatrix}
0 \\
-E
\end{pmatrix}
\] (19)

The following proposition provides a sufficient condition for the closed-loop system (17) to be ISS w.r.t. \( w \).

**Proposition 2:** Consider the system described by (1)-(2) with \( p(0) = 0 \). Suppose that there exist matrices \( L_1 \in \mathbb{R}^{n_q \times n_e}, L_2 \in \mathbb{R}^{n_q \times n_p}, K_1 \in \mathbb{R}^{n_u \times n_q}, K_2 \in \mathbb{R}^{n_u \times n_r}, P \in \mathbb{R}^{2n_q \times 2n_e} \) with \( P > 0 \), and real numbers \( \alpha_0 > 0, \mu > 0, \sigma_1 > 0, \sigma_2 > 0, \sigma_3 > 0 \) such that
\[
\begin{pmatrix}
S_0 & S_1 \\
* & 0
\end{pmatrix} + \sigma_1 S_2^T M S_2 + \sigma_2 S_3^T M S_3 + \sigma_3 S_4^T M S_4
\]
\[
- \mu S_5^T S_5 \leq 0
\] (20)

where
\[
S_0 = PA_c + A_c^T P + \alpha_0 P
\]
\[
S_1 = (PH_1 P H_2 P H_3 P H_4)
\]
\[
S_2 = \begin{pmatrix}
C_q & 0_{n_q \times (n_q+3n_p+w)} \\
0_{n_q \times 2n_e} & 0_{n_p \times (2n_q+n_p)}
\end{pmatrix}
\]
\[
S_3 = \begin{pmatrix}
0_{n_q \times n_r} & -(C_q + L_1 C) & 0_{n_q \times 3n_p} & -L_1 F_w \\
0_{n_r \times 2(n_q+n_p)} & I_{n_p} & 0_{n_p \times (n_p+n_u)}
\end{pmatrix}
\]
\[
S_4 = \begin{pmatrix}
0_{n_q \times n_r} & -C_q & 0_{n_q \times (3n_p+n_u)} \\
0_{n_r \times (2n_q+2n_e)} & I_{n_p} & 0_{n_p \times n_u}
\end{pmatrix}
\]
\[
S_5 = \begin{pmatrix}
0_{n_w \times (2n_q+3n_r)} & I_{n_w}
\end{pmatrix}
\]

Then, the closed-loop system (17) is ISS w.r.t. \( w \) and satisfies \( \dot{V} \leq -\alpha_0 V + \mu \|w\|^2 \) where \( V(z) = z^T P z \).

**Proof:** Since \( \dot{M} \) is a \( \delta \)-MM for \( p \) and \( p(0) = 0 \), it holds that
\[
M^{-T} M \geq 0, \quad \begin{pmatrix}
\delta q \\
\delta p
\end{pmatrix} \geq 0, \quad \begin{pmatrix}
\Delta q \\
\Delta p
\end{pmatrix} \geq 0
\]

where \( \delta p, \delta q \) are given in (16), \( \Delta p \) is given in (14), and
\[
\Delta q = C_q \dot{x} - C_q x = -C_q e.
\] (21)

With \( \xi = (x^T e^T \ p^T \ \delta p^T \ \Delta p^T \ w^T)^T \),
\[
\begin{pmatrix}
q \\
p
\end{pmatrix} = S_2 \xi, \quad \begin{pmatrix}
\delta q \\
\delta p
\end{pmatrix} = S_3 \xi, \quad \begin{pmatrix}
\Delta q \\
\Delta p
\end{pmatrix} = S_4 \xi.
\]

Hence, \( \xi^T S_2^T M S_2 \xi \geq 0, \xi^T S_3^T M S_3 \xi \geq 0, \xi^T S_4^T M S_4 \xi \geq 0 \).

Pre and postmultiply (20) by \( \xi^T \) and \( \xi \), respectively. Since \( \sigma_1, \sigma_2, \sigma_3 \) are non-negative, we obtain that
\[
\xi^T \begin{pmatrix}
S_0 & S_1 \\
* & 0
\end{pmatrix} \xi - \mu \xi^T S_5^T S_5 \xi \leq 0.
\] (22)

Consider the positive definite function defined by \( V(z) = z^T P z \). Then, it is easy to check that \( \dot{V} + \alpha_0 V - \mu \|w\|^2 \) is equal to the left-hand side of (22) where \( V \) is the derivative of \( V \) along the trajectories of (17). Therefore, \( V \) is an ISS-Lyapunov function since \( V \leq -\alpha_0 V + \mu \|w\|^2 \). The conclusion follows from Proposition 1.

Clearly, matrix inequality (20) is not an LMI. In the next two sections, we will consider two parameterizations of the \( \delta \)-MM and provide LMI conditions, which can be used to solve for \( M \) and gain matrices \( L_1, L_2, K_1, K_2 \) simultaneously.

### A. Block Diagonal Parameterization

This section considers a block diagonal parameterization of the \( \delta \)-MM for \( p \). We first make the following two assumptions on the parameterizations of \( M \).

**Assumption 1:** There exist a set \( N_1 \) of matrix pairs \( (X_1, Y_1) \) with \( X_1 \in \mathbb{R}^{n_q \times n_q}, Y_1 \in \mathbb{R}^{n_p \times n_p} \) symmetric, and an invertible matrix \( T_1 \) with
\[
T_1 = \begin{pmatrix}
T_{11} & T_{12} \\
T_{13} & T_{14}
\end{pmatrix}
\] (23)

and \( T_{14} \in \mathbb{R}^{n_p \times n_p} \) invertible, such that \( M_1 \) given below is a \( \delta \)-MM of \( p \) for all \((X_1, Y_1) \in N_1\):
\[
M_1 = T_1^T \tilde{M}_1 T_1 \text{ where } \tilde{M}_1 = \begin{pmatrix} X_1 & 0 \\ 0 & -Y_1 \end{pmatrix}.
\] (24)

**Assumption 2:** There exist a set \( N_2 \) of matrix pairs \( (X_2, Y_2) \) with \( X_2 \in \mathbb{R}^{n_q \times n_q}, Y_2 \in \mathbb{R}^{n_p \times n_p} \) symmetric and invertible, and an invertible matrix \( T_2 \) with
\[
T_2 = \begin{pmatrix}
T_{21} & T_{22} \\
T_{23} & T_{24}
\end{pmatrix}
\] (25)

and \( T_{24} \in \mathbb{R}^{n_p \times n_p} \) invertible, such that \( M_2 \) given below is a \( \delta \)-MM of \( p \) for all \((X_2, Y_2) \in N_2\):
\[
M_2 = T_2^T \tilde{M}_2 T_2 \text{ where } \tilde{M}_2 = \begin{pmatrix} X_2^{-1} & 0 \\ 0 & -Y_2^{-1} \end{pmatrix}.
\] (26)

**Remark 2:** For the globally Lipschitz nonlinearity \( \|p(q_2) - p(q_1)\| \leq 1 \|q_2 - q_1\| \), the matrix \( M \) in (3) satisfies Assumptions 1 and 2 if we choose
\[
T_1 = T_2 = \begin{pmatrix}
I & 0 \\
0 & I
\end{pmatrix}, \quad N_1 = N_2 = \{(\lambda I, \lambda I) | \lambda > 0 \}.
\]
For the incrementally sector bounded nonlinearity \((\delta p - K_1 \delta q)^T S (\delta p - K_2 \delta q) \leq 0\) where \(S\) is symmetric and invertible, the matrix \(M\) in (4) satisfies Assumptions 1 and 2 if we choose
\[
T_1 = T_2 = \begin{pmatrix} K_2 - K_1 & 0 \\ K_2 + K_1 & -2I \end{pmatrix}, \quad N_1 = N_2 = \{(\lambda, S, \lambda S) | \lambda > 0\}.
\]
For the nondecreasing nonlinearity \(\delta p^T \delta q \geq 0\), the matrix \(M\) in (5) satisfies Assumptions 1 and 2 if we choose
\[
T_1 = T_2 = \begin{pmatrix} I & I \\ I & -I \end{pmatrix}, \quad N_1 = N_2 = \{((\lambda, I, \lambda I)) | \lambda > 0\}.
\]


\(N_1\) and \(N_2\) do not have to be the set of scalings of a matrix pair as in the examples above. For instance, for the nonlinearity whose Jacobian is confined within a polytope or a cone, \(N_1\) that satisfies Assumption 1 (or \(N_2\) that satisfies Assumption 2) is characterized via matrix inequalities (see [48, Sec. V] for more details). Furthermore, \(T_1\) does not necessarily has to be chosen to be equal to \(T_2\).

Because \(T_1\) in Assumption 1 and \(T_2\) in Assumption 2 are invertible, the matrix \(\Gamma_{i1}(i = 1, 2)\) defined as
\[
\Gamma_{i1} = T_{i1} - T_{i2} T_{i4}^{-1} T_{i3}
\]
is also invertible by the matrix inversion lemma. Furthermore, we define the matrix \(\Gamma_{i2}(i = 1, 2)\) as
\[
\Gamma_{i2} = T_{i2} T_{i4}^{-1}.
\]

The following theorem provides sufficient conditions for the design of matrices \(L_1, L_2\) in the observer (11) and matrices \(K_1, K_2\) in the controller (12), when the \(\delta\)-MM can be parameterized in a block diagonal manner.

**Theorem 1:** Consider the system described by (1)–(2) with \(p(0) = 0\). Suppose that
1) Assumption 1 holds;
2) Assumption 2 holds with \(M_2 = \begin{pmatrix} M_{21} & M_{22} \\ M_{31} & M_{24} \end{pmatrix}\) where \(M_{24} \in \mathbb{R}^{n_p \times n_p}\) and \(M_{24} < 0\);
3) there exist positive numbers \(\alpha_1, \alpha_2, \mu_1, \mu_2, \mu_4\), matrices \(R_1, R_2, R_3, R_4\), symmetric and positive definite matrices \(P_1, P_2, X_1, X_2, Y_2\) and a symmetric matrix \(X_1\), such that \((X_1, Y_1) \in N_1, (X_2, Y_2) \in N_2\) and
\[
\begin{pmatrix} \Phi - \varphi^T Y_1 \varphi & \phi^T \\ \phi & -X_1 \end{pmatrix} \leq 0
\]
(observer ineq.)
\[
\begin{pmatrix} \Psi - \varphi^T Y_2 \varphi & \psi^T \\ \psi & -X_2 \end{pmatrix} \leq 0
\]
(controller ineq.)
where
\[
\Phi = \begin{pmatrix} \Phi_0 & -P_1 \hat{E}_1 & P_1 E_w + R_1 \\ * & 0 & 0 \\ * & * & -\mu_1 I \end{pmatrix}
\]
\[
\Phi_0 = \tilde{A}_1 P_1 + P_1 \tilde{A}_1 + C^T R_1^T + R_1 C + \alpha_1 P_1
\]
\[
\Psi = \begin{pmatrix} \Psi_0 & \tilde{E}_2 Y_2 + BR_4 & E_w \\ * & 0 & 0 \\ * & * & -\mu_2 I \end{pmatrix}
\]
\[
\Psi_0 = \tilde{A}_2 P_2 + P_2 \tilde{A}_2 + BR_3 + R_2^T B^T + \alpha_2 P_2
\]
\[
\phi = -(X_1 \Gamma_{11} C_q + R_2 C), \quad X_1 \Gamma_{12}, -R_2 F_w
\]
\[
\varphi = (0_{n_p \times n_z}, I_{n_p}, 0_{n_q \times n_w})
\]
\[
\psi = (\Gamma_{21} C_2 P_2, \Gamma_{22} Y_2, 0_{n_q \times n_w})
\]
\[
\tilde{A}_i = A - ET_{i4}^{-1} T_{i3} C_q, \quad i = 1, 2
\]
\[
\tilde{E}_i = ET_{i4}^{-1}, \quad i = 1, 2
\]
with \(\Gamma_{11}(i = 1, 2)\) given in (27) and \(\Gamma_{12}(i = 1, 2)\) given in (28).

Then, the closed-loop system (15) is ISS w.r.t. \(w\) with
\[
\begin{cases}
L_1 = \Gamma_{11}^{-1} X_{11}^{-1} R_2 \\
L_2 = P_{13}^{-1} R_1 + ET_{14}^{-1} T_{13} L_1 \\
K_1 = R_1 P_2^{-1} + K_2 T_{24}^{-1} T_{23} C_q \\
K_2 = R_2 Y_2^{-1} T_{24}.
\end{cases}
\]

**Proof:** The proof proceeds in five steps.

1) First, we derive dynamics of the system under transformations of variables \(q\) and \(p\) via \(T_1\) and \(T_2\). Since \(M_1\) (resp. \(M_2\)) satisfies Assumption 1 (resp. Assumption 2) with an invertible matrix \(T_1\) (resp. \(T_2\)), we introduce variable transformations from \((q, p)\) to \((\hat{q}_i, \hat{p}_i)\) as follows:
\[
\begin{pmatrix} \hat{q}_i \\ \hat{p}_i \end{pmatrix} = T_i \begin{pmatrix} q \\ p \end{pmatrix}
\]
Since \(\hat{p}_i = T_{i3} \hat{q} + T_{i4} p\) and \(T_{i4}\) is invertible, we have \(p = T_{i4}^{-1} \hat{p}_i - T_{i4}^{-1} T_{i3} \hat{q}\) and \(\hat{q}_i = T_{i1} \hat{q} + T_{i2} \hat{p}_i\) for \(i = 1, 2\), where \(\Gamma_{11}, \Gamma_{12}\) are given in (27) and (28). Recall that \(\Gamma_{11}\) is invertible since \(T_1\) is invertible.

Substituting \(p = T_{i4}^{-1} \hat{p}_2 - T_{i4}^{-1} T_{i3} \hat{q}\) into (15), we have
\[
\dot{\hat{x}} = (\hat{A}_2 + B \hat{K}_2) \hat{x} + (\hat{E}_2 + B \hat{K}_2) \hat{p}_2 - B \Delta k + E_w w
\]
where \(\hat{p}_2 = \hat{p}_2(C_q x)\), \(\hat{A}_2\) is given in (38), \(\hat{E}_2\) is given in (39)
\[
\hat{K}_1 = K_1 - K_2 T_{24}^{-1} T_{23} C_q, \quad \hat{K}_2 = K_2 T_{24}^{-1}
\]
and
\[
\Delta k = \hat{K}_1 \epsilon - \hat{K}_2 \Delta \hat{p}
\]
where
\[
\Delta \hat{p} = \hat{p}_2(C_q \hat{x}) - \hat{p}_2(C_q x).
\]
Define \((\Delta \hat{q}, \Delta \hat{p}) = T_i (\Delta q, \Delta p)\). Then, \(\Delta \hat{p}_i = \Gamma_{i1} \Delta \hat{q} + T_{i4} \Delta \hat{p}_i\), which implies that \(\delta \hat{p}_i = T_{i4}^{-1} \Delta \hat{q} - T_{i4}^{-1} T_{i3} \Delta \hat{q}\). Substituting this form of \(\delta \hat{p}\) and (16) into (15), we have
\[
\dot{\epsilon} = (\hat{A}_1 + \hat{L}_2 C) \epsilon - \hat{E}_1 \hat{p}_1 + (E_w + \hat{L}_2 F_w) w
\]
where \(\hat{A}_1\) is given in (38), \(\hat{E}_1\) is given in (39), and \(\hat{L}_2\) is defined as
\[
\hat{L}_2 = L_2 - ET_{i4}^{-1} T_{i3} L_1.
\]
Equations (42) and (46) are the dynamics of the closed-loop system after transformations of variables via $T_1$ and $T_2$.

2) We now consider the performance of the observer.

From (40), we have $R_1 = P_1L_2$ where $L_2$ is given in (47), and $R_2 = X_1\Gamma_1L_1$. Plugging $R_1$ into $\Phi$ in (31), we have $\Phi_0 = P_1(A_1 + L_2C) + (A_1 + L_2C)^T P_1 + \alpha_1 P_1$, and the (1,3) entry of $\Phi$ to be $P_1E_w + R_1F_w = P_1(E_w + L_2F_w)$; plugging $R_2$ into $\Phi$ in (35), we have $\phi = X_1 \phi_0$ where $\phi_0 := (-\Gamma_1(C_q + L_1C), \Gamma_1L_1F_0)$. Recalling $\varsigma$ given in (36) and applying Schur’s complement to (29), we have

$$\Phi + \begin{pmatrix} \phi_0 & \varphi \end{pmatrix}^T \tilde{M}_1 \begin{pmatrix} \phi_0 & \varphi \end{pmatrix} \leq 0.\quad (48)$$

Define $\xi_1 = \{e^T, \varphi^T, w^T\}^T$. Pre and postmultiplying the inequality (48) by $\xi_1^T$ and $\xi_1$, respectively, we have

$$\xi_1^T \Phi \xi_1 + \xi_1^T \begin{pmatrix} \phi_0 & \varphi \end{pmatrix}^T \tilde{M}_1 \begin{pmatrix} \phi_0 & \varphi \end{pmatrix} \xi_1 \leq 0.\quad (49)$$

Note that $\delta \bar{\eta} = T_{11} \delta \bar{q} + T_{12} \delta \bar{p} = T_{11} \delta \bar{q} + T_{12} T_{24}^T \delta \bar{p} - T_{12} T_{14}^T \delta \bar{p} = \Gamma_1 \delta \bar{q} + \Gamma_2 \delta \bar{p} = -\Gamma_1(C_q + L_1C)e + \Gamma_2 \delta \bar{p} = \Gamma_1L_1F_0.\quad (50)$

Define $V_1(e) = e^T P_1 e$. Then, the derivative of $V_1$ along the trajectory of (46) satisfies

$$\dot{V}_1 = 2e^T P_1 [A + L_2C]e - E_1 \delta \bar{p} + (E_w + L_2F_w)w \leq -\alpha_1^e e^T P_1 e + \mu_1 \|w\|^2.\quad (50)$$

3) We now prove that $\|\Delta k\|/\|e\|$ is bounded where $\Delta k$ is given in (44). Since $M_{24} = T_{22}X_2^{-1}T_{22} - T_{24}X_2^{-1}T_{24} \leq 0$ and $T_{21}$ is invertible, we have

$$M_{21}^{-1}X_2^{-1}T_{22} = T_{24}^{-1}M_{24}T_{24}^{-1} < 0.\quad (51)$$

Recall that $\Delta q = -C_q \varphi$ in (21) and define $\Delta \bar{q} := \tilde{q}_2(C_q \tilde{x}) - \tilde{q}_2(C_q x)$. Then, $\Delta \bar{q} = -\Delta \bar{q} + \Gamma_2 \Delta \bar{p}$ where $\Delta \bar{p}$ is given in (45). Define $\zeta = (e^T, \Delta \bar{p})^T$. Therefore

$$\zeta^T \begin{pmatrix} \Gamma_2 C_q & \Gamma_2 \end{pmatrix} \begin{pmatrix} \Gamma_2 & 0 \\ 0 & I \end{pmatrix} \tilde{M}_2 \begin{pmatrix} \Gamma_2 C_q & \Gamma_2 \end{pmatrix} \zeta \leq 0.\quad (52)$$

where $\kappa_1 = \lambda_{\max}(C_q \Gamma_2^{-1}X_1^{-1}\Gamma_2 C_q)$, $\kappa_2 = 2\|C_q \Gamma_2^{-1}X_1^{-1}\Gamma_2 C_q\|_2$, and $\kappa_3 = \lambda_{\min}(Y_2^{-1} - \Gamma_2 X_2^{-1}\Gamma_2 C_q)$. Clearly, $\kappa_1, \kappa_2, \kappa_3 \geq 0$. Therefore, we have $\|\Delta \bar{p}\| \leq \kappa \|e\|$, where $\kappa := (\kappa_2^{1/2} + \sqrt{\kappa_3^{1/2} - 4\kappa_2 \kappa_3})/2\kappa_3 > 0$.

Since $\Delta k = \tilde{K}_1 \varphi + \tilde{K}_2 \Delta \bar{p}$ by (44), we have

$$\|\Delta k\| \leq \kappa \|e\|\quad (53)$$

for all $x, e$, where $\tilde{K}_1 = \|\tilde{K}_1\| + \|\tilde{K}_2\| \|\kappa \| > 0$, which bounds $\|\Delta k\|/\|e\|$.

4) Next, we analyze controller performance. From (40), we have $R_3 = \tilde{K}_1 P_2$ and $R_4 = \tilde{K}_2 P_2$ where $\tilde{K}_1, \tilde{K}_2$ are given in (43). Plugging $R_3, R_4$ into (33), we have $\psi_0 = (\tilde{A}_2 + B\tilde{K}_1)P_2 + P_2(\tilde{A}_2 + B\tilde{K}_1)^T + \alpha_2 P_2$, and the (1,2) entry of $\psi$ to be $(\tilde{E}_2 + B\tilde{K}_2)V_2$. Pre and postmultiplying the inequality (30) by the matrix diag($I_n, Y_2^{-1}, I_{n_w}, I_{n_q}$), and then applying Schur’s complement, we have

$$\tilde{\Psi} + \psi_1^T \tilde{M}_2 \psi_1 \leq 0.\quad (54)$$

where

$$\tilde{\Psi} = \begin{pmatrix} \psi_0 & \tilde{E}_2 + B\tilde{K}_2 & E_w \\ \ast & 0 & 0 \\ \ast & \ast & -\mu I \end{pmatrix} \quad (55)$$

and $\psi_0 = (\Gamma_2 C_q, \Gamma_2 \psi_0, 0_{n_q \times n_w})$ and

$$\tilde{\Psi} = \begin{pmatrix} \psi_0 & P_3(\tilde{E}_2 + B\tilde{K}_2) & P_3E_w \\ \ast & 0 & 0 \\ \ast & \ast & -\mu I \end{pmatrix} \quad (56)$$

Define $\xi_2 = (x^T, \tilde{p}_2, w^T)^T$. Pre and postmultiplying the inequality (54) by $\xi_2^T$ and $\xi_2$, respectively, we have

$$\xi_2^T \tilde{\Psi} \xi_2 + \xi_2^T \psi_0 \psi_0 \xi_2 \leq 0.\quad (57)$$

By (26) and (41), we have $(\psi_0 \psi_0^T) \tilde{M}_2 (\psi_0 \psi_0^T) \geq 0$. Since $\tilde{q}_2 = \Gamma_1 q + \Gamma_2 \tilde{p}_2 = \Gamma_1 C_q x + \Gamma_2 P_2 + (\tilde{E}_2 + B\tilde{K}_2)\tilde{p}_2 + E_w w + \alpha_2 \tilde{p}_2 x - \mu_2 \|w\|^2 \leq 0.$

Let $V_2(x) = x^T P_3 x$. Then, the derivative of $V_2$ along the trajectory of (42) satisfies

$$\dot{V}_2 = 2x^T P_3 [(\tilde{A}_2 + B\tilde{K}_1)x + (\tilde{E}_2 + B\tilde{K}_2)\tilde{p}_2 - B\Delta k + E_w w] \leq -\alpha_2 x^T P_3 x + \mu_2 \|w\|^2 + 2\|P_3 B\| \|x\| \|\Delta k\|.\quad (58)$$
Recalling (52), we have
\[ \dot{V}_2 \leq -\alpha_2 x^\top P_3 x + \mu_2 \|w\|^2 + \theta \|x\| \|e\| \]  
(58)
where \( \theta = 2 \|P_2 B\| \). 

5) Finally, we prove that the closed-loop system expressed by (42) and (46) is ISS with respect to \( w \). Choose two constants \( c_1, c_2 \) as \( c_1 = \alpha_1 \lambda_M(P_3)/\lambda_M(P_1) \), \( c_2 = \alpha_1 \lambda_M(P_3)/\lambda_M(P_3) \). Since \( c_1 > 0, c_2 > 0 \), we can choose two constants \( \alpha_0 > 0, \beta_0 > 0 \) such that \( \alpha_0 \leq \min\{c_1, c_2\}, \beta_0 \geq \frac{\alpha_0}{\theta} \).

Then, it is easy to check that the matrix \( \tilde{P}_0 := \begin{pmatrix} P_0 & \theta/2 \\ \theta/2 & P_0 \end{pmatrix} \) is negative semidefinite where \( \tilde{P}_0 = -\alpha_2 \lambda_M(P_3) + \alpha_0 \lambda_M(P_3) \) and \( \tilde{P}_0 = \beta_0(-\alpha_1 \lambda_M(P_1) + \alpha_0 \lambda_M(P_1)) \). Define a matrix \( P \) as \( P = \begin{pmatrix} \tilde{P}_0 & 0 \\ 0 & \tilde{P}_0 \end{pmatrix} \). Clearly, \( P \) is positive definite.

We can verify that the candidate Lyapunov function \( V(x,e) := z^\top P_2 z \) satisfies \( V(x,e) = \beta_0 V_1(e) + V_2(x) \), and its derivative along the trajectory of (42) and (46) satisfies
\[ \dot{V} + \alpha_0 V \leq -\alpha_1 \beta_0 x^\top P_1 e - \alpha_2 x^\top P_3 x + \theta x \|e\| + \alpha_0 V \leq (\|x\|, \|e\|) \tilde{P}_0 (\|x\|, \|e\|)^\top + (\mu_1 \beta_0 + \mu_2) \|w\|^2 \leq (\mu_1 \beta_0 + \mu_2) \|w\|^2. \]  
(59)

Theorem 2: Consider the system described by (1)–(2) with \( p(0) = 0 \). Suppose that
1) Assumption 3 holds for some \( T_1, T_3 \) and \( T_2, T_4 \) respectively, where \( T_1 \) and \( T_2 \) are partitioned as in (23) and in (25), respectively, with \( T_1, T_2 \) invertible;
2) there exist positive constants \( \alpha_1, \alpha_2, \mu_1, \mu_2, \) matrices \( R_1, R_2, R_3, R_4, X_1, Y_1, X_2, Y_2 \) and symmetric and positive definite matrices \( P_1, P_2 \), such that \( (\tilde{X}_1, \tilde{Y}_1) \in N_1, (\tilde{X}_2, \tilde{Y}_2) \in N_2 \), and
\[ \bar{M} = T^\top \bar{M}T \] where
\[ \bar{M} = \begin{pmatrix} 0 & X \\ X^\top & 0 \end{pmatrix}. \]  
(60)

In this section, we consider a block antitriangular parameterization of the \( \delta \)-MM for \( p \). The following assumption on the parameterization of \( M \) is given first.

Assumption 3: There exist a set \( \mathcal{N} \) of matrix pairs \( (X, Y) \) with \( X \in \mathbb{R}^{n_x \times n_p}, Y \in \mathbb{R}^{n_y \times n_p}, \) and an invertible matrix \( T \in \mathbb{R}^{(n_x+n_p) \times (n_q+n_q)} \), such that \( M \) given below is a \( \delta \)-MM of \( p \) for all \( (X, Y) \in \mathcal{N} \):
\[ M = T^\top \bar{M}T \] where
\[ \bar{M} = \begin{pmatrix} 0 & X \\ X^\top & 0 \end{pmatrix}. \]  
(60)

where
\[ \tilde{M} = \begin{pmatrix} 0 & X_1 \\ X_1^\top & 0 \end{pmatrix}, \tilde{M} = \begin{pmatrix} 0 & X_2 \\ X_2^\top & 0 \end{pmatrix} \]  
(64)
and
\[ \Psi = \begin{pmatrix} \Psi_0 & \tilde{E}_2 + BR_4 & E_w \\ * & 0 & 0 \\ * & * & -\mu_2 \end{pmatrix}. \]  
(65)

The following theorem provides sufficient conditions for the design of matrices \( L_1, L_2, K_1, K_2 \) when the \( \delta \)-MM \( M \) can be parameterized in a block antitriangular manner.

Remark 3: In the proof of Theorem 1, we first prove that the observer error \( e \) is ISS w.r.t. \( w \) (see (50) in step 2), then the state \( x \) is ISS w.r.t. \( w \) and \( e \) (see (58) in step 4), and finally \( (x,e) \) is ISS w.r.t. \( w \) (see (59) in step 5). This procedure of proving global stabilization is similar to the certainty equivalence proof used in [25].

Remark 4: If \( \alpha_1 \) is fixed, then (29) is an LMI in decision variables \( \mu_1, P_1, R_1, R_2, X_1, Y_1 \) that are used to determine observer gains \( L_1, L_2 \); if \( \alpha_2 \) is fixed, then (30) is an LMI in decision variables \( \mu_2, P_2, R_3, R_4, X_2, Y_2 \) that are used to determine controller gains \( K_1, K_2 \). There is no coupling in decision variables in LMIs (29) and (30), implying a separation of the controller and observer designs.

Remark 5: The proof of Theorem 1 indicates that larger \( \alpha_1, \alpha_2 \) result in a larger function \( \gamma(\cdot) \in \mathcal{K}_\infty \) in (8), which, in turn, indicates a faster convergence rate for the system (17). The convergence rate guarantee given in the proof of Theorem 1, \( \alpha_0 \), can be improved by finding a new ISS-Lyapunov function \( V = z^\top P_2 z \) via Proposition 2.

Remark 6: The condition \( M_{24} < 0 \) in Theorem 1 is used to prove that \( \|\Delta k\|/\|e\| \) is bounded, which holds automatically when \( p \) is globally Lipschitz. The condition \( M_{24} < 0 \) can be replaced by a growth condition on \( p \) similar to [28, Theorem 2]. Specifically, for a system described by (1)–(2) where \( E_w = F_w = 0 \), if all the conditions of Theorem 1 but \( M_{24} < 0 \) hold, and there exist a function \( g_1 \) and a nondecreasing function \( g_2 : [0, \infty) \to [0, \infty) \) such that \( \|p(C_q(x + \Delta x)) - p(C_qx)\| \leq g_1(\|\Delta x\|)\|C_qx\| \) for all \( x, \Delta x \) that satisfy \( \|C_qx\| \geq g_2(\|\Delta x\|) \), then the closed-loop system (15) is globally exponentially stable.
with $\Gamma_{11}(i = 1, 2)$ given in (27), $\Gamma_{12}(i = 1, 2)$ given in (28), $\Phi$ given in (31), $\Phi_0$ given in (32), $\Psi_0$ given in (34), $\varphi$ given in (36), $\tilde{A}_i(i = 1, 2)$ given in (38), and $\tilde{E}_i(i = 1, 2)$ given in (39). If $X_1$ has full row rank, then the closed-loop system (15) is ISS w.r.t. $w$ with $L_2$, $K_1$ given by (40), and $L_1, K_2$ given by

$$L_1 = \Gamma_{11}^{-1} X_1 R_2, \quad K_2 = R_4 T_{24}$$

(67)

where $X_1^\dagger$ is the right inverse of $X_1$.

Proof: As shown in (42) and (46), dynamics of the closed-loop system under transformations can be described as

$$\dot{x} = (\tilde{A}_2 + B\tilde{K}_1)x + (\tilde{E}_2 + B\tilde{K}_2)\tilde{p}_2 - B\Delta k + E_w w$$

$$\dot{\tilde{p}} = (\tilde{A}_1 + \tilde{L}_2 C)e - \tilde{E}_1 \delta \tilde{p}_1 + (E_w + \tilde{L}_2 F_w)w$$

where $\tilde{K}_1, \tilde{K}_2$ are given in (43), $\Delta k$ is given in (44), $\tilde{p}_2$ is given in (41), and $L_2$ is given in (47). From (40) and (67), we have $R_1 = P_1 L_2$ and $R_2 = X_1 \Gamma_{11} L_1$. We claim that (61) is equivalent to

$$\Phi + Q_1^\top M_1 Q_1 \leq 0$$

(68)

and (62) is equivalent to

$$\Psi + Q_2^\top M_2 Q_2 \leq 0$$

(69)

where

$$Q_1 = \begin{pmatrix} -\Gamma_{11}(C_q + L_1 C) & \Gamma_{12} & -\Gamma_{11} L_1 F_w \\ 0_{n_p \times n_x} & I_n & 0_{n_p \times n_w} \end{pmatrix}$$

and

$$Q_2 = \begin{pmatrix} \Gamma_{21} C_q P_2 & \Gamma_{22} & 0_{n_p \times n_x} \\ 0_{n_p \times n_x} & I_n & 0_{n_p \times n_w} \end{pmatrix}.$$
**Remark 8:** The LMIs (29) and (30) both have dimensions \((n_x + n_p + n_q + n_w) \times (n_x + n_p + n_q + n_w)\), the LMIs (61) and (62) both have dimensions \((n_x + n_p + n_w) \times (n_x + n_p + n_w)\), and the LMI (63) has dimension \(n_p \times n_p\). These LMIs can be solved reliably and efficiently by the interior point method algorithms of convex optimization with a polynomial-time complexity. Exploring for what class of systems these LMIs are guaranteed to be feasible (i.e., analytical verification of feasibility) is still under our investigation. Furthermore, these LMI conditions might be conservative compared with specific results that focus on certain special nonlinearities such as the globally Lipschitz nonlinearity.

### IV. ETC Design

In this section, we discuss event-triggering mechanisms (ETMs) within the observer-based controller designed in the preceding section for the system described by (1) and (2) where the nonlinearity \(p\) is assumed to be globally Lipschitz. For certain incrementally quadratic nonlinearities that imply the global Lipschitzness (such as the incremental sector bounded nonlinearity and the nonlinearities with Jacobians in polytopes [48]), using their corresponding incremental matrix characterizations, instead of the matrix characterizations for global Lipschitzness, makes the associated LMIs in the design procedure less conservative, while benefiting from having the Lipschitz property needed for the upcoming ETM-related results to hold.

#### A. Configuration I: The Controller Channel Is Implemented by ETM

In this section, we discuss the configuration shown in Fig. 1 where the plant is described by (1) and (2), the observer is given in (11), the continuous-time feedback controller is given in (12), and the ETM only has the information of \(\hat{x}\), the state of the observer. We will assume that \(\|w\|_\infty \leq \omega_0\) where \(\omega_0\) is a positive constant indicating the bound of the disturbance in this section and the next section.

The feedback controller \(u(t)\) is implemented by an ETM such that it is only updated at certain triggering times \(t_1, t_2, \ldots\) where \(t_k < t_{k+1}\) for any \(k \geq 0\) and kept constant during consecutive time instances. Define \(t_0 = 0\) and the piecewise constant signal \(\hat{x}_s\) as

\[
\hat{x}_s(t) = \hat{x}(t_k) \quad \forall t \in [t_k, t_{k+1}).
\]

Then, the control input \(u(t)\) is given by

\[
u(t) = K_1\hat{x}_s(t) + K_2p(C_q\hat{x}_s(t))
\]

where \(K_1, K_2\) are matrices to be designed. The input \(u(t)\) has the same form as that in (12), but it is updated at triggering time instances \(t_1, t_2, \ldots\) which are determined by the following type of triggering rule:

\[
t_{k+1} = \inf \{t \mid t \geq t_k + \tau, \|\hat{x}_e(t)\| > \sigma\|\hat{x}(t)\| + \epsilon\}
\]

where \(\hat{x}_e\) is defined as \(\hat{x}_e(t) = \hat{x}_s(t) - \hat{x}(t)\) and \(\tau, \sigma, \epsilon\) are all positive numbers to be specified. The time-updating rule (72) guarantees that the interexecution times \(\{t_{k+1} - t_k\}\) lower bounded by the built-in positive constant \(\tau\), which means that Zeno phenomenon (i.e., infinite executions happen in a finite amount of time) will not occur [55].

**Remark 9:** The triggering rule (72) only depends on the information of \(\hat{x}\) and \(\hat{x}_s\), which are available from the proposed observer. This triggering rule is a combination of a mixed ETM and a time regularization technique. There are several motivations for choosing this type of rule. It was known that even in the absence of disturbances, interexecution times of many ETMs converge to zero for output-based control configurations [59]. To exclude the Zeno phenomenon, time regularization or periodic ETC, which enforces a built-in lower bound for interexecution times, has been utilized in recent works on observer-based ETMs [57, 61, 62]. Furthermore, mixed ETM is known to be robust to external disturbances or measurement noise, while relative ETM and absolute ETM have zero robustness to disturbance/noise [60]. Additionally, an event-triggering rule with time regularization can benefit from using mixed ETMs in terms of the number of events that are generated (e.g., see [60, Example 3]).

The closed-loop system that combines systems (1), (2), observer (11), and event-triggered controller (71) is expressed compactly as

\[
z = A_c z + H_1 p + H_2 \delta p + H_3 \delta \hat{p} + H_4 w + H_5 \hat{x}_e
\]

where \(\delta p, \delta q\) are given in (16), \(A_c\) is given in (18), \(H_1, H_2, H_3, H_4\) are given in (19), and

\[
\delta \hat{p} = p(C_q\hat{x}_s) - p(C_q x)
\]

\[
H_5 = \begin{bmatrix} BK_1 \\ 0 \end{bmatrix}.
\]

**Theorem 3:** Consider the configuration shown in Fig. 1 where the plant is described by (1) and (2) with \(p(0) = 0\) and \(\|w\|_\infty \leq \omega_0\) with \(\omega_0\) a positive number. Suppose that there exists \(\ell > 0\) such that \(\|p(r) - p(s)\| \leq \ell \|r - s\|\) for any \(r, s\). Suppose that there exist positive numbers \(\alpha_0 > 0, \mu > 0, \) and matrices \(P > 0, K_1, K_2, L_1, L_2\) such that the closed-loop system (17) with controller (12) and observer (11) satisfies

\[
\dot{V} \leq -\alpha_0 V + \mu \|w\|_2^2
\]

where \(V = z^\top P z\). Choose any \(\epsilon > 0\) and

\[
\sigma = \frac{\alpha_0 \lambda_{\min}(P)}{2\sqrt{2\epsilon}} > 0
\]

where \(0 < \varrho < 1\) and \(s = \|PH_5\| + \ell\|PH_2\|\|C_q\|\). Choose \(\tau > 0\) as the solution to the equation \(\varphi(\tau) = 1\) where \(\varphi\) is the
solution of the following ODE:

\[ \dot{\phi} = \sqrt{2}(\eta_4 + \eta_2 \phi)(1 + \sigma \phi), \quad \phi(0) = 0 \]

with

\[ \begin{cases} 
\eta_1 = ||A_c|| + \ell \sqrt{b_1^2 + b_2^2} \\
\eta_2 = ||H_5|| + \ell ||H_6|| ||C_1|| \\
\eta_3 = \ell ||H_2|| ||L_1 F_w|| + ||H_4|| \\
\eta_4 = \frac{\rho_0}{\epsilon} + \frac{\sigma_0}{\epsilon} \\
b_1 = ||H_1|| ||C_1|| \\
b_2 = ||H_2|| ||C_1 + L_1 C|| + ||H_3|| ||C_1||.
\end{cases} \tag{77} \]

Then, the closed-loop system (73) that implements the triggering rule (72) is ISS w.r.t. \( w \).

Proof: Since the derivative of \( V \) along the trajectory of the closed-loop system (17) satisfies \( \dot{V} \leq -\alpha_0 V + \mu ||w||^2 \), the derivative of \( V \) along the trajectory of the closed-loop system (73) satisfies \( \dot{V} \leq -\alpha_0 V + \mu ||w||^2 + 2z^T P[H_5 \dot{x} + H_3 (\sigma \dot{p} - \Delta p)] \leq -\alpha_0 \lambda_m(P) ||z||^2 + \mu ||w||^2 + \frac{q}{2} ||z||^2 + ||P H_5|| ||x|| + ||P H_4|| ||(\sigma \dot{p} - \Delta p)|| \). Clearly, \( ||(\sigma \dot{p} - \Delta p)|| \leq \ell ||C_1 (\dot{x} - \dot{x})|| \leq \ell ||C_1|| ||x||. \) Then, we have

\[ \dot{V} \leq -\alpha_0 \lambda_m(P) ||z||^2 + \mu ||w||^2 + 2 \|z\| ||x|| \leq -(1 - \rho) \alpha_0 \lambda_m(P) ||z||^2 + \mu ||w||^2 + \frac{q}{2} ||z||^2 \geq \|z\| / \sqrt{2}, \]

meaning that \( \|x\| \leq \sqrt{2} ||z|| \).

Therefore, the condition

\[ \|\dot{x}\| \leq \sigma \|x\| + \epsilon \tag{79} \]

implies

\[ \|\dot{x}\| \leq \sqrt{2} \sigma ||z|| + \epsilon \tag{80} \]

which is equivalent to the inequality \( 2s \|\dot{x}\| - \rho_0 \alpha_0 \lambda_m(P) \|z\| \leq 2s \epsilon \).

Choose a constant \( c \) such that \( 0 < c < (1 - \rho) \alpha_0 \lambda_m(P) \).

Then, as long as (80) holds, from (78), we have

\[ \dot{V} \leq -[(1 - \rho) \alpha_0 \lambda_m(P) - c] \|z\|^2 + \mu ||w||^2 + \frac{s^2 \epsilon^2}{c}. \tag{81} \]

Recalling that \( p(0) = 0 \) and \( \ell \) is the Lipschitz constant of \( p \), we have \( ||p|| \leq \ell ||C_1|| ||x||, \quad ||\delta p|| \leq \ell ||\delta q|| \leq \ell ||C_1 + L_1 C|| ||c|| + ||L_1 F_w|| ||w|| \), and \( ||\delta p|| \leq \ell ||C_1|| (||\dot{x}|| + ||e||) \).

Therefore, from (73), we have

\[ \|\dot{z}\| \leq ||A_c \| ||z|| + ||H_5|| ||\dot{x}\| + \ell ||H_1|| ||C_1|| ||x|| + \ell ||H_2|| (||C_1 + L_1 C|| ||c|| + ||L_1 F_w|| ||w||) + \ell ||H_3|| ||C_1|| (||\dot{x}|| + ||e||) + ||H_4|| ||w|| \leq \eta_1 ||z|| + \eta_2 ||\dot{x}|| + \eta_3 ||w|| \]

where the second inequality follows from Cauchy’s inequality \( b_1 ||x|| + b_2 ||e|| \leq \sqrt{b_1^2 + b_2^2} ||z|| \).

Because \( \|\dot{z}\| = \sqrt{\|\dot{z}\|^2 + \|e\|^2} = \sqrt{\|\dot{x}\|^2 + \|\dot{x} - \dot{x}\|} = \sqrt{\|\dot{x}\|^2 + 2\|\dot{x}\|^2 - 2\dot{x}^T \dot{x} \geq \|\dot{x}\|/\sqrt{2} } \) and \( \|\dot{x}\| = \|\dot{x}\| \), we have \( ||\dot{x}\| \leq \sqrt{2} ||\dot{z}||. \)

Let \( v(t) = \frac{\|\dot{x}\|(t)}{2 \sigma ||z(t)|| + \epsilon} \). Then, for any \( h > 0 \)

\[ v(t + h) - v(t) = \frac{\|\dot{x}\|(t + h)}{2 \sigma ||z(t + h)|| + \epsilon} - \frac{\|\dot{x}\|(t)}{2 \sigma ||z(t)|| + \epsilon} \]

\[ = \frac{\|\dot{x}\|(t + h)||2 \sigma ||z(t + h)|| + \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon}{(2 \sigma ||z(t + h)|| + \epsilon)(2 \sigma ||z(t)|| + \epsilon)} \]

\[ = \frac{\|\dot{x}\|(t + h)||2 \sigma ||z(t + h)|| - ||\dot{x}\|(t)||2 \sigma ||z(t)||}{(2 \sigma ||z(t + h)|| + \epsilon)(2 \sigma ||z(t)|| + \epsilon)} \]

and hence

\[ D^+ v(t) = \lim \sup_{h \to 0^+} \frac{v(t + h) - v(t)}{h} = \frac{D^+ \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon}{(2 \sigma ||z(t)|| + \epsilon)^2}. \tag{82} \]

When \( z(t) \neq 0 \), \( D^+ \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \) and, therefore,

\[ |D^+ \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \leq |\|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \].

When \( z(t) = 0 \), \( D^+ \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \leq \lim \sup_{h \to 0^+} \|\dot{x}(t + h)||2 \sigma ||z(t)|| + \epsilon \| = \|\dot{x}(t)||2 \sigma ||z(t)|| + \epsilon \|.

Thus, in all cases \( |D^+ \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \leq |\|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \| \).

Similarly, \( D^+ \|\dot{x}\|(t)||2 \sigma ||z(t)|| + \epsilon \| \). Dropping the argument \( t \), it now follows from (82) that

\[ D^+ v \leq \frac{\|\dot{x}\|(t)}{2 \sigma ||z(t)|| + \epsilon} + \frac{2 \sigma ||\dot{x}\||(t)||\dot{x}\|}{(2 \sigma ||z(t)|| + \epsilon)^2} \]

\[ \leq \frac{\sqrt{2} \|\dot{x}\|}{2 \sigma ||z|| + \epsilon} + \frac{\sqrt{2} \|\dot{x}\| ||\dot{x}\|}{(2 \sigma ||z|| + \epsilon)^2} \]

\[ \leq \frac{\sqrt{2} \|\dot{x}\|}{2 \sigma ||z|| + \epsilon} + \frac{1 + \sigma ||\dot{x}\||}{2 \sigma ||z|| + \epsilon} \]

\[ \leq \frac{\sqrt{2}}{2} \left( \eta_4 + \eta_2 \frac{\|\dot{x}\|}{\sqrt{2} \sigma ||z|| + \epsilon} \right) \left( 1 + \sigma \frac{\|\dot{x}\|}{\sqrt{2} \sigma ||z|| + \epsilon} \right) \]

\[ = \frac{\sqrt{2}}{2} \left( \eta_4 + \eta_2 \sigma \right) \left( 1 + \sigma v \right) \]

where the following facts are used to derive the last inequality:

\[ \frac{\eta_1 ||z||}{\sqrt{2} \sigma ||z|| + \epsilon} \leq \frac{\eta_1 ||w||}{\sqrt{2} \sigma ||z|| + \epsilon} + \frac{\eta_3 ||w||}{\sqrt{2} \sigma ||z|| + \epsilon} \]

Because \( v(t_k) = 0 \), it now follows from the comparison lemma that \( v(t) \leq \phi(t - t_k) \). Since the time it takes for \( v \) to evolve from \( 0 \) to \( 1 \) is lower bounded by \( \tau \), (80) holds during the time interval \([t_k, t_k + \tau] \). For any \( k \geq 0 \), if \( t_{k+1} = t_k + \tau \), then (80) holds during the interval \([t_k, t_{k+1}] \) as shown above; if \( t_{k+1} > t_k + \tau \), then, during the interval \([t_k + \tau, t_{k+1}] \), condition (79) holds, which implies that (80) holds. Therefore, (80) holds during any interval \([t_k, t_{k+1}] \) for any \( k \geq 0 \), i.e., it holds for any \( t \geq 0 \). Since satisfaction of (80) implies the inequality (81), we conclude that
the function $V$ is an ISpS-Lyapunov function since it satisfies (8) for any $t \geq 0$ with $\gamma(\|z\|) = \|z\|^2 \in K$, and $\dot{H} = \inf \{\|w\|^2 \in K\}$ and $d = s^2/\epsilon > 0$. The conclusion follows by Proposition 1.

Remark 10: In the proof of Theorem 3, the equation for $\tau$ is given explicitly, and any $\tau^0 \in (0, \tau]$ also makes the proof valid. The parameter $\epsilon$ can be chosen arbitrarily, but there are tradeoffs in choosing $\epsilon$: on one hand, the value of $d$ in the inequality (8) or (10) increases as $\epsilon$ increases, meaning that the ultimate bound for $x$ increases as $\epsilon$ increases; on the other hand, the explicit equation of $\tau$ depends on $\epsilon$, with $\tau$ decreasing to 0 when $\epsilon$ approaches 0. Hence, parameters in the triggering rule should be chosen appropriately to balance the execution times and the performance. Finding the maximal lower-bound of the interexecution times is an interesting and challenging problem that will be investigated in our future work.

B. Configuration II: The Controller and Observer Channels Are Both Implemented By ETMs

In this section, we discuss the configuration shown in Fig. 2 where the ETM for the output is triggered by the information of $\hat{x}$, in an asynchronous manner.

Consider a system described by (1) and (2). The observer in the configuration of Fig. 2 only has sampled information $y_s(t)$ of the output $y(t)$ where $y_s(t)$ is updated at time instances $t_k^0, t_k^1, \ldots$ by

$$y_s(t) = y(t_k^0) \quad \forall t \in [t_k^0, t_{k+1}^0].$$

Here, $t_k^0 = 0$ and the triggering times $t_1^0, t_2^0, \ldots$ are determined by the following triggering rule:

$$t_{k+1}^0 = \inf\{t \mid t \geq t_k^0 + \tau_y, \|y_s(t)\| > \sigma_y\|y(t)\| + \epsilon_y\} \quad (83)$$

where $y_s(t) = y(t) - y(t)$ and $\tau_y, \sigma_y, \epsilon_y$ are all positive numbers to be specified.

With the sampled information $y_s(t)$, the observer now becomes

$$\begin{aligned}
\dot{x} &= A\dot{x} + Bu + E_p p(\dot{y} + L_1(y - y_s)) + L_2(\dot{y} - y_s) \\
\dot{y} &= C\dot{x} + Du \\
\dot{\hat{x}} &= C_q\dot{x}
\end{aligned}$$

(84)

where $L_1, L_2$ are matrices to be designed.

The observer-based controller $u(t)$ has the form shown in (71) where $\hat{x}_s(t)$ is updated at time instances $t_k^1, t_k^2, \ldots$ by

$$\dot{\hat{x}}_s(t) = \hat{x}_s(t_k^0) \quad \forall t \in [t_k^0, t_k^1, \ldots]$$

Here, $t_k^0 = 0$ and the triggering times $t_1^1, t_2^1, \ldots$ are determined by the following triggering rule:

$$t_{k+1}^1 = \inf\{t \mid t \geq t_k^1 + \tau_u, \|\dot{\hat{x}}_s(t)\| > \sigma_u\|\dot{\hat{x}}(t)\| + \epsilon_u\} \quad (85)$$

where $\dot{\hat{x}}_s(t) = \hat{x}(t_k^1) - \hat{x}(t)$ and $\tau_u, \sigma_u, \epsilon_u$ are all positive numbers to be specified. Note that the information of $\hat{x}$ and $\hat{x}_s$ are available from the proposed observer.

The time-updating rule (83) (or (85)) provides a built-in positive lower bound $\tau_u$ (or $\tau_u$) for interexecution times $\{t_{k+1}^1 - t_k^1\}$ (or $\{t_{k+1}^0 - t_k^0\}$), implying that the Zeno phenomenon will not occur. Although there is no bound guarantee on the interexecution times between $t_k^0$ and $t_{k+1}^0$, this will not cause a problem since these two ETMs are implemented separately.

Since $y_s(t) = y_s(t) - y(t)$, the closed-loop system that com- bines system (1) and (2), observer (84) and event-triggered controller (71) is expressed compactly as

$$\dot{z} = A_z z + H_1p + H_2\delta \hat{p} + H_3\delta \hat{q} + H_4w + H_5\hat{x} + H_6y_e$$

(86)

where $A_z, H_1, H_2, H_3, H_4, H_5$ are given in (18), (19), (75), respectively, $\delta \hat{p}$ is given in (74), and

$$\begin{aligned}
\delta \hat{p} &= p(q + \delta \hat{q}) - p(q) \\
\delta \hat{q} &= -(C_q + L_1C)e - L_1F_u w - L_1y_e \\
H_6 &= \begin{pmatrix} 0 \\ L_2 \end{pmatrix}
\end{aligned}$$

Theorem 4: Consider the configuration shown in Fig. 2 where the plant is described by (1) and (2) with $D = 0, p(0) = 0$, and $\|w\|_\infty \leq \omega_0$ with a positive number. Suppose that there exists $\ell > 0$ such that $|p(r) - p(s)| \leq \ell|r - s|$ for any $r, s$. Suppose that there exist constants $\sigma_0 > 0, \mu > 0$, and matrices $P > 0, K_1, K_2, L_1, L_2$ such that the closed-loop system (17) with controller (12) and observer (11) satisfies $V \leq -\sigma_0 V + \mu\|w\|^2$ where $V = z^T Pz$. Choose any $\epsilon_y, \epsilon_u > 0$, and

$$\sigma_y = \frac{\alpha_2 \alpha_0 \lambda_m(P)}{2(C_s^2)\|s_2\|^2}, \quad \sigma_u = \frac{\alpha_1 \alpha_0 \lambda_m(P)}{2(C_s^2)\|s_1\|^2}$$

(87)

where $0 < \varrho < 1, s_1 = \|PH_5\| + \ell\|PH_3\|^2\|C_q\|, s_2 = \|PH_6\| + \ell\|PH_2\|^2\|L_1\|^2, \alpha_1, \alpha_2$ are two constants satisfying $0 < \alpha_1, \alpha_2 < 1$ and $\alpha_1 + \alpha_2 = 1$. Choose $\tau_u > 0$ as the solution to the equation $\phi_1(\tau_u) = 1$ where $\phi_1$ is the solution of the following ODE

$$\dot{\phi}_1 = \sqrt{2}(1 + \sigma_0 \phi_1)(\eta_5 + \eta_2 \phi_1 + d_1 \eta_7), \quad \phi_1(0) = 0$$

and choose $\tau_y > 0$ as the solution to the equation $\phi_2(\tau_y) = 1$ where $\phi_2$ is the solution of the following ODE:

$$\dot{\phi}_2 = ||C||(1 + \sigma_y \phi_2)(\eta_6 + \eta_7 \phi_2 + d_2 \eta_7), \quad \phi_2(0) = 0$$
where

\[
\begin{align*}
\eta_5 &= \frac{\eta_z}{\sqrt{\sigma_y C}} + \frac{2\eta_0}{\epsilon_y}, \\
\eta_6 &= \frac{\eta_z}{\sqrt{\sigma_y C}} + \frac{2\eta_0}{\epsilon_y}, \\
\eta_7 &= \ell |H_2| L_1 + |H_0|, \\
d_1 &= \max\left\{ \frac{\eta_5}{\sqrt{\sigma_y C}}, \frac{\eta_6}{\sqrt{\sigma_y C}} \right\}, \\
d_2 &= \max\left\{ \frac{\eta_5}{\sqrt{\sigma_y C}}, \frac{\eta_6}{\sqrt{\sigma_y C}} \right\},
\end{align*}
\]

and \(\eta_1, \eta_2, \eta_3\) are given in (77). Then, the closed-loop system (86) that implements triggering rules (83) and (85) is ISpS w.r.t. \(w\).

**Proof:** If the derivative of \(V\) along the trajectory of (17) satisfies \(\dot{V} \leq -\alpha_0 V + \mu \|w\|^2\), then the derivative of \(V\) along the trajectory of the closed-loop system (86) satisfies

\[
\dot{V} \leq -\alpha_0 V + \mu \|w\|^2 + 2z^TP[H_2(\delta\hat{p} - \delta p) + H_3(\delta\hat{p} - \delta p) + H_5\hat{x}_e + H_6 y_e] \leq -(1 - \theta)\alpha_0 \lambda_m(P) \|z\|^2 + \mu \|w\|^2 + \|z\| \left(2\epsilon_1 \|\hat{x}_e\| + 2\epsilon_2 \|y_e\| - \epsilon_0 \alpha_0 \lambda_m(P) \|z\|\right)
\]

(89)

From (89) and (90), we have

\[
\dot{V} \leq -(1 - \theta)\alpha_0 \lambda_m(P) \|z\|^2 + \mu \|w\|^2 + \|z\| \left(2\epsilon_1 \|\hat{x}_e\| + 2\epsilon_2 \|y_e\| - \epsilon_0 \alpha_0 \lambda_m(P) \|z\|\right)
\]

(91)

The condition \(\|\hat{x}_e\| \leq \sigma_u \|\hat{x}\| + \epsilon_u\) implies

\[
\|\hat{x}_e\| \leq \sqrt{2\sigma_u} \|z\| + \epsilon_u
\]

(92)

and the condition \(\|y_e\| \leq \sigma_y \|y\| + \epsilon_y\) implies

\[
\|y_e\| \leq \sigma_y \|C\| \|z\| + \epsilon_y
\]

(93)

As long as (91) and (92) hold, we have

\[
\dot{V} \leq -(1 - \theta)\alpha_0 \lambda_m(P) - \epsilon_0 \|z\|^2 + \mu \|w\|^2 + \epsilon_0 \frac{\epsilon_0}{4c}
\]

(94)

It is easy to verify that \(\frac{\eta_7 \|y_e\|}{\sqrt{2\sigma_u} \|z\| + \epsilon_u} \leq d_1 \frac{\eta_7 \|y_e\|}{\sigma_y \|C\| \|z\| + \epsilon_y}\). Hence, from (94), we have

\[
\frac{d}{dt} \left(\frac{\|\hat{x}_e\|}{\sqrt{2\sigma_u} \|z\| + \epsilon_u}\right) \leq \sqrt{2} \left(1 + \frac{\sigma_u \|\hat{x}_e\|}{\sqrt{2\sigma_u} \|z\| + \epsilon_u}\right) \times \left(\eta_5 + \frac{\eta_7 \|\hat{x}_e\|}{\sqrt{2\sigma_u} \|z\| + \epsilon_u} + d_1 \frac{\eta_7 \|y_e\|}{\sigma_y \|C\| \|z\| + \epsilon_y}\right).
\]

When \(\|\hat{x}_e\| = 0\) or \(\|z\| = 0\), the upper right-hand derivative of \(\frac{\|\hat{x}_e\|}{\sqrt{2\sigma_u} \|z\| + \epsilon_u}\) can be calculated similar to the proof of Theorem 3, which can still be captured by the inequality above.

Since \(|\|y_e\| - \|y\|\| \leq \|C\| \|\hat{x}\| \leq \|C\| \|z\|\) can be shown similar to the proof of Theorem 3, we can show that the following inequality holds using arguments similar to those used above:

\[
\frac{d}{dt} \left(\frac{\|y_e\|}{\sigma_y \|C\| \|z\| + \epsilon_y}\right) \leq \|C\| \left(1 + \frac{\sigma_y \|y_e\|}{\sigma_y \|C\| \|z\| + \epsilon_y}\right)
\]

Therefore, the discussion on using the upper right-hand derivative is omitted since it is similar to that used in the proof of Theorem 3.

It is not hard to show that the time it takes for \(\|\hat{x}_e\|\) to evolve from 0 to \(\sqrt{2\sigma_u} \|z\| + \epsilon_u\) is lower bounded by \(\tau_u\) (resp. \(\tau_y\), which implies that (91) holds during \([t_{k-1}, t_k + \tau_u]\), and (92) holds during \([t_{k-1}, t_k + \tau_y]\), for any \(k \geq 0\). Recalling that \(\|\hat{x}_e\| \leq \sigma_u \|\hat{x}\| + \epsilon_u\), we can show that \(\|y_e\| \leq \sigma_y \|y\| + \epsilon_y\) implies (91) and \(\|y_e\| \leq \sigma_y \|y\| + \epsilon_y\) implies (92), the triggering rules (83) and (85) guarantee that (91) holds during the interval \([t_{k-1}, t_k + \tau_y]\), for any \(k \geq 0\). Hence, (93) holds for any \(t \geq 0\), implying that the function \(V\) is an ISpS-Lyapunov function since it satisfies (88) with \(\tau((\|z\|) = (1 - \theta)\alpha_0 \lambda_m(P) - \epsilon_0 \|z\|^2 \in K_{\infty}, \chi((\|w\|) = \mu \|w\|^2 \in K\) and \(d = \epsilon_0^2/4c > 0\). The conclusion follows by Proposition 1.

**Remark 11:** The assumption \(\dot{V} \leq -\alpha_0 V + \mu \|w\|^2\) in Theorems 3 and 4 can be verified by using Theorem 1. Therefore, Theorems 1, 3, and 4 altogether provide a systematic and constructive approach to design observer-based event-triggered controllers. One limitation of this ETC design, however, is that it relies on the global Lipschitz constant, which is normally very conservatively computed.

**Remark 12:** Similar to Remark 10, there are tradeoffs in choosing parameters in triggering rules (83) and (85); for example, smaller \(\epsilon_u, \epsilon_y\) reduces the ultimate bounds but decreases the int exe execution times.

**V. SIMULATION EXAMPLE**

In this section, we use a single-link robot arm example given in [62] and the configuration of Fig. 2 to illustrate Theorem 4. Dynamics of the single-link robot arm are expressed as

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\sin(x_1) + u + w \\
y = x_1
\]
where $x = (x_1, x_2)^T$ is the state representing the angle and the rotational velocity, $u$ is the input representing the torque, and $w$ is the external disturbance. The system can be written in the form of $(1)$ with $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $C = (1, 0)$, $D = 0$, $E = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $E_w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $F_w = 0$, $C_q = (1, 0)$ and $p(q) = \sin(q)$. The non-linearity $p$ satisfies $(2)$ with $M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Recalling Remark 2, $p$ satisfies Assumptions 1 and 2 with $T_1 = T_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $N_1 = \{(\lambda_1, I, \lambda_1 I) | \lambda_1 > 0\}, N_2 = \{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} I, \frac{1}{\sqrt{2}} I) | \lambda_2 > 0\}$, which means that $X_1 = Y_1 = \lambda_1 I$, $X_2 = Y_2 = \lambda_2 I$. Additionally, the corresponding $M_{24} = -1 < 0$. By letting $\alpha_1 = \alpha_2 = 1$, $\mu_1 = \mu_2 = 0.1$, the LMIs (29) and (30) with variables $\lambda_1, \lambda_2$, $P_1, P_2$ are feasible, from which we can obtain matrix gains $L_1 = -1$, $L_2 = \begin{pmatrix} -0.1294 \\ -18.0052 \end{pmatrix}$, $K_1 = (-7.3936, -3.9937)$, $K_2 = 1$. The observer is given in (84) with $L_1, L_2$ above, and the controller is given in (71) with $K_1, K_2$ above. We then let $\alpha_0 = 0.25$, $w_0 = 0.02$ and recompute $P$ via (20) with the objective to be minimizing the condition number of $P$. With $q = 0.8$, $\alpha_1 = \alpha_2 = 0.5$, $\epsilon_u = \epsilon_y = 0.005$, we can calculate that $\sigma_y = 0.0017$, $\sigma_u = 0.0023$, and $\tau_u \geq 0.07 \times 10^{-4}$ s, $\tau_u \geq 0.78 \times 10^{-5}$ s. In the simulations, we suppose that the random disturbance $w$ is uniformly generated from $[-w_0, w_0]$, and the initial conditions of the plant and the observer are $(0.1, -0.15)$ and $(-0.1, 0.05)$, respectively. The simulation results are shown in Fig. 3 through Fig. 7.

Figs. 3 and 4 show trajectories of the state $x$ and the estimation error $e$, respectively. Both $x$ and $e$ eventually enter a small neighborhood of the origin as expected. Fig. 5 shows interexecution times $\{t_k^{e+1} - t_k^{e}\}$ in the observer ETM (83), and Fig. 6 shows interexecution times $\{t_k^{u+1} - t_k^{u}\}$ in the controller ETM (85). Fig. 7 shows the trajectory of the piecewise constant input $u(t)$ that is fed into the plant. It is readily seen that the control input $u(t)$ updates its values at each sampling time $t = t_k^u$, which is determined by the triggering rule (85).

Denote $\tau_{\text{min}}^{[T_1, T_2]}$ and $\tau_{\text{avg}}^{[T_1, T_2]}$ as the minimal and average interexecution times during the time interval $[T_1, T_2]$, respectively. The values of $\tau_{\text{min}}^{[0, 20]}$, $\tau_{\text{avg}}^{[0, 20]}$, $\tau_{\text{min}}^{[3, 20]}$, $\tau_{\text{avg}}^{[3, 20]}$ for the observer ETM and the controller ETM are summarized in Table I. We notice that after 3 s, the controller input is updated about every 0.36 s...
on average, and the plant output is updated about every 1.09 s on average, which shows the effectiveness of our control design.

VI. CONCLUSION

In this article, we studied observer-based, global stabilizing control design for incrementally quadratic nonlinear systems affected by external disturbances and measurement noise. We proposed LMI-based sufficient conditions for the simultaneous design of the observer and the controller in the continuous-time domain for two parameterizations of the incremental multiplier matrices. Based on that, we investigated ETM design within the observer-based controller setting for globally Lipschitz systems. The simulation example showed the effectiveness of the controller design and the proposed triggering rule.

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