ANALYTICAL INVESTIGATION IN BENDING CHARACTERISTIC OF TWISTED STACKED-TAPE CABLE CONDUCTOR

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Abstract. An analytical model to evaluate bending strains of a twisted stack-tape cable (TSTC) conductor has been developed. Through a comparison with experimental results obtained for a soldered 32-tape YBCO TSTC conductor, it has been found that a Perfect-Slip Model (PSM) taking into account the slipping between tapes in a stacked-tape cable during bending gives much better estimation of the bending performance compared to a No-Slip Model (NSM). In the PSM case the tapes can slip so that the internal longitudinal axial strain can be released. The longitudinal strains of compression and tension regions along the tape are balanced in one twist-pitch and cancel out evenly in a long cable. Therefore, in a cable the strains due to bending can be minimized. This is an important advantage of a TSTC conductor. The effect of the cable diameter size on the bending strain is also expected to be minor, and all tapes composing a TSTC conductor have the same strain response under bending, therefore the cable critical current can be characterized from a single tape behaviour.

1. Introduction

The second generation High Temperature Superconductor (HTS) Rare-Earth Barium Copper Oxide (REBCO) tapes are very attractive to various applications of high field magnets and transmission power cables. Cabling methods for the HTS flat tapes cabling such as Roebel Assembled Coated Conductor (RACC) [1], Conductor-On-Round Core (CORC) [2], Twisted Stacked-Tape Cable (TSTC) [3] and a few other alternatives [4], [5] have been proposed and are being investigated for high current, high field applications.

We have been developing TSTC cabling method, which consists of stacking flat tapes and twisting them along the stack axis. This compact cabling technique using REBCO tapes is very useful for both high field magnet conductors and power transmission. TSTC conductors have been proposed and fabricated by several methods, including sheathing the stack of tapes with a copper tube and embedding the stack in single and multiple helical grooves formed in a circular rod [3], [6]. In real applications, the bendability of a REBCO tape cable is very important to effectively fabricate and transport a long cable and ultimately wind a magnet [5], [6]. We have experimentally examined bendability of a TSTC conductor [7]. Bending characteristic of various TSTC conductors is investigated by an analytical calculation method. Two models, No-Slip Model (NSM) and Perfect-Slip Model (PSM), are presented, and critical current degradation due to bending is discussed.

2. Analysis Modelling
When a TSTC conductor is circularly bent as shown in figure 1, the bending neutral axis of the cable is at the center of its axis, which is on the bending plane. Therefore the portions of the tape located on the outside of the cable neutral plane experiences tension, while the inner portions experience compression. Note that the cable neutral plane means a circular plane determined by the cable neutral axis, which is perpendicular to the cable bending plane.

Since the thickness of the coated conductor tape and the superconductor layer thickness are much smaller than the other dimensions (tape width, the cable diameter and the cable bending radius), the tape thickness effect on the bending strain is neglected in this calculation.

Figure 1. Illustration of a 40 tape Twisted Stacked-Tape Cable (TSTC) conductor bent with the bending radius $r_o=500$ mm.

Figure 2. Schematic illustrations showing a bending model of a TSTC conductor: (a) a cross-section view of a tape in a TSTC conductor bent with the radius $r_o$ and (b) an enlarged view of a part of a twisted tape.

The distance between the tape center axis and the cable center axis is defined by $h$ (named the tape off-center distance in this manuscript), as shown in figure 2(a). Figure 2(b) illustrates an enlarged view of a segment of a twisted tape, which is one of the tapes composing the stacked-tape cable. A segment on the tape is denoted by $x$ in the tape width direction and $z$ in the tape center axis, as shown in the figure. The axial bending strain $\varepsilon_b$ of an element $(dx\cdot dz)$ on a tape due to a twisted-tape cable bending is given for the bending radius $r_o$ as

$$\varepsilon_b = \left(\frac{x}{cos\alpha}\right) \sin \theta + h \cos \theta \cos \alpha$$  \hspace{1cm} (1)

$$\varepsilon_b = \frac{x \sin \theta + h \cos \theta \cos \alpha}{r_o}$$  \hspace{1cm} (2)

where $\Theta$ is the twist angle, and $\alpha$ is the cable twist angle of a TSTC conductor which are given by the tape off-center distance $h$ and the twist-pitch $L_p$ as follows,
$$\alpha = \tan^{-1}(2\pi h/L_p)$$

$$\Theta \equiv \frac{2\pi \cos \alpha}{L_p} \approx \frac{2\pi}{L_p}$$

It is noted that in the present analysis of the bending strain of a TSTC conductor composed of twisted tapes, the strains induced in the tapes during the twisting of the stack are neglected, as they are negligible compared with the bending strain. Therefore we calculate only the additional strain due to bending. It was shown that the twisting strain effect on the critical current for a TSTC conductor of 4 mm REBCO tapes for twist-pitches longer than 200 mm is much smaller than the bending strains discussed here [7].

Figure 3. (a) The bending strain distribution at $x = +w$ for NSM along a twisted tape of 4 mm width and 200 mm twist-pitch bent with the radius of $r_o=300$ mm with various tape off-center distance $h$ values up to 5.5 mm. (b) The insert figures show a single stack cable with $0 \leq h \leq 2$ mm and three-stack cable with $1.5 \leq h \leq 5.5$ mm.

Figure 3 shows the bending strains at $x = +w$ for one of the 4 mm tapes of a twisted stacked-tape cable with a twist-pitch of 200 mm. The bending strain is evaluated using equation (2) as a function of the twist angle $\Theta$ with various values of the tape off-center distance $h$. Figure 3(b) shows two TSTC conductors where the tapes have various $h$-values. Distributions of the bending strain $\epsilon_b$ significantly change with the tape off-center distance $h$. The second term, $h \cos \alpha \cos \Theta$, in the numerator of equation (2) contributes the internal longitudinal strain along a tape. As seen in figure 3, the larger the tape off-center distance $h$ is, the larger are the bending strains.

We consider two limiting cases to understand the bending behaviour of a TSTC conductor: No-Slip Model (NSM) and Perfect-Slip Model (PSM). In the case of the No-Slip Model, the tapes in the stacked-tape cable cannot slip on one-another and behave like a solid block during bending due to high friction between the tapes. On the other hand, in the case of the Perfect-Slip Model the tapes can slip freely as friction is assumed to be zero between the tapes.

In the case of PSM, equation (2) can be simplified. In the right hand of equation (2) the first term of $x$ reflects a strain due to the tape width, and the second term of $h$ component induces a longitudinal strain along a tape. If the tapes can freely slip in a stack during bending, the longitudinal strain can be released by slipping. In a TSTC conductor tapes are twisted, therefore compressive and tensional stresses in a full twist-pitch balance and cancel out over the short length of one twist-pitch even for a long cable. Therefore the longitudinal strain due to the second term of equation (2) can be disregarded. In this case the bending strain $\epsilon_b$ from equation (2) reduces to

$$\epsilon_b = \frac{x \sin \theta}{r_o}$$
This equation is independent of the tape off-center distance $h$.

The strain distributions along both tape edges ($x = \pm w$) for both NSM and PSM were calculated from equations (2) and (5), respectively, and are shown in figure 4. As expected, in the case of PSM described by equation (5), all stacked tapes composing a cable experience the same strain distribution, as the strain does not depend on the tape off-center distance $h$. The neutral axis of each tape is the center axis of the tape. The distribution of the bending strains $\varepsilon_b$ along the width is symmetrical with equal amounts of compression and tension, as seen in figure 4(b).

Bending strains for various bending radiiues for both NSM with the tape off-center distance $h=2$ mm and PSM obtained from equations (2) and (5), respectively, are shown in figure 5. These figures show the bending strains at the tape edge $x = \pm w$. The peak strains exceed 2%, and they become very large in the NSM case. The large strain resulted from those bending values will be used for the critical current evaluation in the following calculations. We should notice that the large strains will cause the yielding of the material and permanent degradations of the critical current. Including those phenomena in the models discussed in this paper is beyond the scope of this work.

![Figure 4](image1.png)

**Figure 4.** Bending strain distributions for a tape of 4 mm width and the twisted pitch 200 mm along a tape as a function of the twist angle $\Theta$. The curves show the strains at the edges of $x = \pm w$ for the bending radius $r_o=100$ mm and 300 mm. (a) No-Slip Model (NSM) with $h=2$ mm, and (b) Perfect-Slip Model (PSM).

![Figure 5](image2.png)

**Figure 5.** Bending strains at the tape edge $x=\pm w$ for (a) No-Slip Model (NSM) with $h=2$ mm and (b) Perfect-Slip Model (PSM) calculated for various bending radiiues $r_o$. The tape width, $2w$, and twist-pitch, $L_p$, are 4 mm and 200 mm, respectively.
3. Critical Current

The critical current locally varies with the bending strains at locations given with $x$ in the width and $z$ or the twist angle $\Theta$ along the length. To simplify the calculation of the critical current of the twisted bent tape, first the critical current is averaged over the width between $x = +w$ and $x = -w$ in order to evaluate the strain distribution effect over the width. The average critical current $I_{c,\Theta}$ over the width is given by integration,

$$I_{c,\Theta} = \frac{1}{2w} \int_{-w}^{w} I_c(\varepsilon_{b\theta}, x) \, dx$$  \hspace{1cm} (6)

The critical current as a function of the strain was approximated using reported data, which was given by the following polynomial equation [3], [8],

$$I_c/I_{c,0} = 0.057713 \times 10^{12} \varepsilon_{b\theta}^6 - 0.03979215 \times 10^{10} \varepsilon_{b\theta}^5 - 0.2090279 \times 10^8 \varepsilon_{b\theta}^4 + 0.02385557 \times 10^6 \varepsilon_{b\theta}^3 - 0.1668065 \times 10^4 \varepsilon_{b\theta}^2 - 0.003662115 \times 10^2 \varepsilon_{b\theta} + 1.0$$  \hspace{1cm} (7)

This equation was obtained by curve fitting based on the experimental data of the strain ranges between about -1.0% and +0.52%, however in this analysis the critical current was evaluated using the equation for strain ranges beyond those values.

Equation (6) was solved by a numerical method using the Gaussian integration method of order 40 with Microsoft Excel®. The strain $\varepsilon_{b\theta}$ is given by equation (2) for NSM and equation (5) for PSM.

The critical current evaluated with equation (6) is given as a function of the twist angle $\Theta$ along the tape length. That is, the critical current varies along the length. When the tape current is given, the voltage distribution during a transition from superconducting to normal state varies as shown in figure 6. Here, the electric field distribution calculated for a given strain using equation (5) (PSM case) and $E/E_c=(I_c/I_{c,0})^n$ with $n=25$ is shown as a function of the twist angle $\Theta$ along the length. The critical current of a tape in the stacked tape cable is obtained from the criterion of $E_c$ (100 $\mu$V/m) for the integrated total voltage of one full twist-pitch length $L_p$. Similar curves can be obtained for the NSM case and various $h$ values.

**Figure 6.** Electric-field distribution calculated from bending strain distribution along the tape of one twist pitch when the total voltage of one twist pitch reached the critical current criterion of $E_c$. Tape width $2w=4$ mm, twist-pitch $L_p=200$ mm, $h=0$, $n=25$, $r_o=200$ mm.

The total voltage of one twist-pitch at the tape current $I_o$ is given by

$$V = \int_0^{L_p} \Delta V \, dz = \int_0^{L_p} E_c I_o \left(\frac{I}{I_{c,0}}\right)^n \, dz = E_c I_o \int_0^{L_p} I_{c,\Theta}^{-n} \, dz$$  \hspace{1cm} (8)

On the other hand the voltage $V$ of one twist-pitch $L_p$ is given for a criterion $E_c$ by $V= E_c L_p$. Using equations (8) and (3), the tape current critical $I_{c, \text{Tape}}$ for a twisted bent tape is given by
The tape critical current was evaluated with \( \Theta = 5 \) degree step analysis using Microsoft Excel®. In this case equation (9) is reduced to

\[
I_{c,\text{Tape}} = n \int_0^{L_p} \frac{L_p}{\sqrt{j_0 I_{c0}^{-n}}} \, dt = n \frac{2\pi}{\sqrt{j_0 I_{c0}^{-n}}} \tag{10}
\]

The critical current of a tape was analysed using equation (10). Cable critical current of a cable composing of \( k \) tapes can be obtained from \( k I_{c,\text{Tape}} \) for PSM as all the tapes experience the same strain during bending.

4. Critical Current Results Analysed and Comparison with Experimental Results

Figure 7 shows the critical current characteristics with various values of the tape off-center distances \( h \) as a function of the bending diameter. They are obtained for the tape width \( 2w = 4 \) mm, the twist pitch \( L_p = 200 \) mm and \( n = 25 \) using equation (10). The critical currents degrade significantly as the tape off-center distance \( h \) increases. The critical current for the PSM case corresponds to the case of \( h = 0 \) in figure 7.

![Figure 7](image)

**Figure 7.** Normalized critical current in NSM for various values of \( h \) as a function of the bending diameter. Tape width \( 2w = 4 \) mm, twist-pitch \( L_p = 200 \) mm, and \( n = 25 \). The solid circles of \( h = 0 \) also apply to the critical current in PSM (The same solid circle curve is shown in figure 8.).

![Figure 8](image)

**Figure 8.** Normalized critical current for PSM with various tape width \( 2w \) as a function of the bending diameter, comparing with an experimental result of 4 mm width, 32-tape soldered YBCO TSTC conductor. Twist-pitch \( L_p = 200 \) mm, and \( n = 25 \).

Figure 8 shows the critical currents in PSM (\( h = 0 \)) for 2 mm, 3 mm and 4 mm width tapes with experimental data obtained for a 32-tape, 4 mm width cable made of SuperPower YBCO tapes. The cable was soldered before bending in a straight condition [7]. A narrower width tape has smaller degradation due to bending. The calculated critical currents for \( 2w = 4 \) mm degrade much more with the
bending diameter than those obtained experimentally. The experimental result for 4 mm tape was similar to the calculation value for 2 mm width. The experimental work, as mentioned in [7], was performed to wind a soldered 32-tape TSTC conductor on the side surface of a disk of a given diameter without a groove. The cable was not forced to make a very tight winding with the disk, so that the overall cable bending diameter was correctly achieved. This caused some local waving of the cable due to the nature of a twisted cable made of stacked flat-tapes. Such short range waving of the cable was observed in a few places. This might be a reason why the experimental result showed less critical current degradation than the calculated one (locally the tapes experienced less bending). Overall this result together with what observed in figure 7 (NSM vs. PSM) suggests that the model analysis of Perfect-Slip Model (PSM) can give a good evaluation for bending characteristics of a TSTC conductor and it is more appropriate in describing the bending behaviour of a TSTC compared to the No-Slip Model (NSM).

5. Discussion

The maximum value of the bending strain, \( e_{b\text{Max}} \) for NSM is given from equation (2) as

\[
e_{b\text{Max}} = \frac{\sqrt{w^2 + (h \cos \alpha)^2}}{r_0}
\]

This maximum value is established at the hard bending (in-plane bending) [9], [10] where a flat tape bends in the tape plane. For a TSTC conductor the maximum value of \( h \) will be approximately the size of the cable radius. Bending strain increases with \( h \). However, our experiments of the critical current tests for TSTC conductors did not show the significant degradations expected from the strain evaluated with equation (11).

In the right end side of equation (11) the first term of \( w \) reflects a strain due to the tape width, and the second term of \( h \) component induces a longitudinal strain. Given the experimental observations (lower degradations than what evaluated with equation (11)) we can conclude that the tapes in a TSTC conductor are not rigidly held and that the PSM case could be acceptable and more appropriate in describing the behaviour of a TSTC conductor under bending. Therefore it is reasonable to say that the slipping of 2G stacked tapes in a TSTC conductor can occur, and the tapes can freely slip in a stack during bending. The stacked tapes are twisted, therefore in each half cycle of the twisting the strains change from tension to compression. It follows that the internal longitudinal strains given by the \( h \) term in the bending strain equation can be released in a full twist-pitch even for a long cable. This is an important and beneficial consequence of the twisting of a TSTC conductor.

In the slipping condition (PSM) the bending strain of equation (2) can be reduced to equation (5), which becomes independent of the tape off-center distance \( h \). All tapes of a TSTC conductor experience the same bending strain. That is, a cable critical current can be obtained from multiplying the number of tapes with the single tape critical current.

Based on the Perfect-Slip Model (PSM), the critical current of a twisted tape was evaluated for one full twist-pitch length and compared with experimental results. The agreement was not good as seen in figure 8, however it was much better than what estimated with the No-Slip Model (NSM) as seen in figure 7. The calculated result for PSM showed more degradations than the experimental results. This could be due to the soft bending applied in the experiment to respect the natural irregularity during cable bending. Taking into account this observation the analytical results seem to show a fairly good agreement with the experimental results.

In the present work the tape width and the twist pitch were 4 mm and 200 mm, respectively. As seen in equation (5) for the PSM case, the bending strain is proportional to the width. A narrower tape gives less degradation for bending. The twist-pitch \( L_p \) in equation (3) affects \( \cos \alpha \) in the strain of equation (2), however its effect on the bending strain is minor.

6. Conclusions

In order to analyse bending characteristics of a HTS Twisted Stacked-Tape Cable (TSTC) conductor No-Slip Model (NSM) and Perfect-Slip Model (PSM) have been developed. Comparing with
experimental results, it was found that the PSM case fits the experimental results much better than the NSM case.

In the NSM case, 2G tapes are rigidly stacked and they do not slip between tapes. On the other hand in the PSM case the tapes can slip so that the internal axial strain can be released. The longitudinal strains of compression and tension regions along the tape are balanced in one twist-pitch and cancel out even for a long cable. Therefore the strains due to a cable bending can be minimized. Additional strains over the width are induced during the bending of the cable. This strain causes the critical current degradation due to bending. The bending strain effect on the critical current of a TSTC conductor was evaluated for various bending diameters.

The Perfect-Slip Model analysis has shown that the tape distance (the tape off-center distance $h$) from the cable center does not affect the magnitude of the strain due to bending, since a tape can slip and release the longitudinal strain. The bending strains are induced equally into all stacked tapes in a TSTC conductor. Therefore the cable current can be characterized from the behavior of one single tape, and the total cable current can be obtained from a multiplication of the current of one tape by the number of tapes. Further investigation to explain the discrepancy between experimental results and the model analysis will be necessary using both experiments of various TSTC conductors and Finite Element Analysis.

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