Control of the index of refraction in optically dense medium

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Abstract. In this work we investigate conditions of emergence and destruction of the electromagnetically induced transparency effect in the optically dense medium of atoms which are excited along closed contour of excitation by the three frequency radiation. We found the conditions under which index of refraction of the medium acquires spatial quasi-periodic dependence. It is possible to control this dependence by variation of the initial phases and intensities of the electromagnetic fields.

1. Introduction
The mediums with spatial periodical dependence of index of refraction, which is capable of the Bragg diffraction of light have the properties of photonic crystals [1-2]. However, variation of the optical properties of such crystals, specifically variation of the index of refraction in different layers, is possible only at growing of a crystal. At the same time increased interest is attracted to the mediums in which it is possible to induce the quasi-periodic structure of the resonance index of refraction by electromagnetic fields under the electromagnetically induced transparency (EIT) effect [3].

This work is devoted to investigation of emergence of stationary quasi-periodic structure of the index of refraction in the medium of cold atoms which interact with three electromagnetic fields, propagating collinear. In this calculation we suppose that the microwave field inside the medium is known only. It interacts with atoms weakly because the magnetic dipole moment of microwave transition is much smaller than electric dipole moments of the optical transitions. We suppose that two electromagnetic fields of optical range are unknown in the thickness of the medium and calculate them by the self-consistent solution of the Maxwell-Bloch equations. It allows us to take into consideration the energy exchange between frequency components of radiation.

2. Mathematical model and results
We consider the medium of three-level atoms which has two lower levels and one excited level. In this system the quantum transitions are excited by the electromagnetic fields which have closed contour of the excitation. In this system there are two optical range transitions and one microwave range transition (Δ - scheme). Each transition is bound by the resonant electromagnetic field (figure 1 (a)). We suppose that the medium is optically thick (figure 1 (b)). It means that \( n_0 \sigma L > 1 \), where \( n_0 \) is the concentration of atoms, \( \sigma \) is the cross section of scattering of photons by an atom and \( L \) is the length of the medium (figure 1 (b)). Simultaneously the average distance between atoms is sufficiently large: \( n_0 \lambda^3 < 1 \), where \( \lambda \) is the wavelength of the scattered radiation. The latter assumption allows us to ignore collective effects. The electric dipole transitions \(|1\rangle \leftrightarrow |3\rangle\) and \(|2\rangle \leftrightarrow |3\rangle\) interact with the fields
of optical range with the Rabi frequencies $\Omega_1$ and $\Omega_2$. The magnetic dipole transition $|1\rangle \leftrightarrow |2\rangle$ interacts with the field of microwave range with the Rabi frequency $U$. Thus the three fields form a closed contour of excitation. It leads to dependence of optical properties of the medium on the relative phase $\Phi_0 = \varphi_1 - \varphi_2 - \varphi_3$ of fields at the entrance, where $\varphi_j$ are the initial phases of each field $(j=1,2,3)$.

![Image](image-url)

**Figure 1.** (a) – Level system and excitation scheme of atoms ($\Delta$ - scheme). Here $\nu_1$ and $\nu_2$ are the frequencies of radiation of optical range, $\nu_3$ is the frequency of microwave field; $\Omega_1$ and $\Omega_2$ are the Rabi frequencies of radiation of optical range; $U$ is the Rabi frequency of microwave field; $\gamma$ is the rate of decay of excited level; $\Gamma$ is the rate of decay of microwave coherence; $\Delta_1$ and $\Delta_2$ are the one-photon detunings. (b) – Propagation of the three-frequency radiation in the medium, which consists of atoms with concentration $n_a$ non-interacting with each other.

The problem is described by the Maxwell-Bloch equations (1) for the slowly changing amplitudes of electric field $E_j^0(x,t)$ of waves and the atomic density matrix $\hat{\rho}_{ik} = \rho_{ik}e^{i\omega_j t}$:

$$\begin{bmatrix}
\frac{\partial E_j^0(x,t)}{\partial x} + \frac{1}{c} \frac{\partial E_j^0(x,t)}{\partial t} \\
\frac{\partial \hat{\rho}_{ik}}{\partial t} = -\frac{i}{\hbar} \sum_l [H_{li}\hat{\rho}_{lk} - \hat{\rho}_{lk}H_{il}] + \sum_{l,m} I_{lk,lm} \hat{P}_{lm}
\end{bmatrix} = 4\pi P_j^0(x,t)k_j,$$

where the index $j=1,2$ corresponds to each optical field. The wave vectors $\vec{k}_j = \vec{\epsilon}_j k_j$ are parallel to the axis $x$. $H$ is the Hamiltonian and $\Gamma$ is the relaxation matrix. $P_j^0(x,t)$ is the slowly changing amplitude of polarization of the medium. $c$ is the speed of light, $\hbar$ is the Plank constant. We suppose that the microwave field $U$ does not decay in the medium because the magnetic dipole moment of microwave transition $|1\rangle \leftrightarrow |2\rangle$ is $10^4$ times less than electric dipole moments of optical transitions. Therefore, $U(x,t) = U(t)$. Below we consider the stationary case: the derivatives are equal to zero. Solution of the Maxwell-Bloch equations gives us spatial evolution of the optical fields $\Omega_j(x)$ and the slowly changing amplitudes of atomic density matrix $\rho_{ij}(x)$ in the medium. The values $\text{Re}(\rho_{13})$ and $\text{Re}(\rho_{23})$ are proportional to the resonant indexes of refractions of the corresponding transitions.
In figure 2 one can see the dependencies of index of refraction on $x$-coordinate for two sets of boundary conditions: 

1. $\Phi_0 = 0$ and $\Omega_1(0) \neq \Omega_2(0)$ (figure 2 (a,c));
2. $\Omega_1(0) = \Omega_2(0)$ and $\Phi_0 = \pi / 4$ (figure 2 (b, d)).

In the case of different Rabi frequencies of the optical fields and zero relative phase the indexes of refraction of two optical transitions are not equal to zero and to each other. This leads to phase shifts of the fields along the medium and prevents transition to the state of electromagnetically induced transparency.

In the case of equal Rabi frequencies and nonzero relative phase the phase shifts arise due to nonzero relative phase which destroys the state of electromagnetically induced transparency. On the figure 2 (c,d) one can see spatial evolution of the relative phase. Also spatial oscillations of intensity take place due to energy exchange between the frequency components of radiation. Such oscillations induce a quasi-periodic spatial distribution of the index of refraction in the medium which is typical for photonic crystals (figure 2 (a,b)).

Figure 2. Dependencies of the real part of amplitude of atomic coherence of the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ and the relative phase $\Phi$ on the coordinate $x$ for the boundary conditions, corresponding to the first way (a,c) and the second way (b,d) of excitation of the spatial oscillations. Parameters for (a,c): $\Phi_0 = 0$, $\Omega_1(0) = \gamma$, $\Omega_2(0) = 2\gamma$, $U = 3.5\gamma$, $n_u = 2 \times 10^{11} \text{cm}^{-3}$; Parameters for (b,d): $\Phi_0 = \pi / 4$, $\Omega_1(0) = \Omega_2(0) = \gamma$, $U = 3.5\gamma$, $n_u = 2 \times 10^{11} \text{cm}^{-3}$. 
In figure 2 the envelope of oscillations of $\text{Re}(\rho_{13})$ and $\text{Re}(\rho_{23})$ decreases exponentially. If we enhance intensity of the optical fields the probability of stimulated emission will grow up and probability of spontaneous emission will fall down. It leads to reduction of radiation losses sideways therefore envelope of intensity of the optical fields and the real part of amplitude of atomic coherences falls down slower. For increasing the value of resonant index of refraction it is necessary to gain the high atomic concentration. For the value of atomic concentration $n_a \sim 10^{16} \text{cm}^{-3}$ the amplitude of refraction oscillations will be tens percent of non-resonant value [4,5]. Such concentrations can be achieved in rare earth crystals [3].

3. Conclusions
In this work we investigate interaction of the medium which consists of the three-level atoms with two electromagnetic fields of optical range and one microwave field. The fields form a closed contour of excitation (Δ - scheme). The calculation was made for the optically dense medium. It is found that two types of boundary conditions lead to pumping of energy between the optical fields, and index of refraction has the spatial quasi-periodic dependence. We analyzed such dependencies for different conditions of excitation and identified the boundary conditions which provide maximal amplitude of oscillations of the index of refraction. For enhancing of this amplitude it is necessary to increase the atomic concentration and intensity of the fields. The spatial quasi-periodic dependence of the index of refraction can be used for creation of the medium with controllable photonic band. Such schemes may be implemented in atomic gases as well as in solid state matrix.

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