Double-Regge Exchange Limit for the $\gamma p \rightarrow K^+K^-p$ Reaction

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We apply the generalized Veneziano model ($B_3$ model) in the double-Regge exchange limit to the $\gamma p \rightarrow K^+K^-p$ reaction. Four different cases defined by the possible combinations of the signature factors of leading Regge exchanges ($(K^+, a_2/f_2)$, $(K^+, \rho/\omega)$, $(K^2_2, a_2/f_2)$, and $(K^2_2, \rho/\omega)$) have been simulated through the Monte Carlo method. Suitable event candidates for the double-Regge exchange high-energy limit were selected employing Van Hove plots as a better alternative to kinematical cuts in the $K^+K^-p$ Dalitz plot. In this way we predict and analyze the double-Regge contribution to the $K^+K^-p$ Dalitz plot, which constitutes one of the major backgrounds in the search for strangeonia, hybrids and exotics using $\gamma p \rightarrow K^+K^-p$ reaction. We expect that data currently under analysis, and that to come in the future, will allow verification of the double-Regge behavior and a better assessment of this component of the amplitude.

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I. INTRODUCTION

A host of new experiments dedicated to precision studies of the hadron spectrum will begin operations in the near future. These will complement and extend the reach of recently completed and other ongoing experiments that, among other discoveries, found intriguing structures in the hadron spectrum [1–7]. Existence of these structures demonstrates that the hadron spectrum is far more complex than predicted by the valence quark model [8]. Nevertheless, it remains to be seen if these structures are to be associated with new resonances. This is because identification of new states requires detailed understanding of reaction dynamics. The tools that enable us to constrain and interpret reaction amplitudes are based on principles of the S-matrix theory, which include analyticity, crossing relations and unitarity. In practice, rigorous implementation of these principles is impossible. It would require knowledge of an infinite number of amplitudes describing all coupled channels for all reactions related by crossing. Nevertheless, for a given reaction it is possible to kinematically isolate regions where specific processes dominate and use analyticity to constrain amplitudes in other regions of interest, e.g. correlate amplitude parametrization in the low- and the high-energy regions using finite energy sum rules [9].

It follows from S-matrix principles that in relativistic scattering resonance formation in the direct channel is dual to Regge exchanges, aka Reggeons, in the cross-channels. Leading Reggeons in the cross-channel determine the high energy behavior of the direct channel. Thus, because of analyticity, contributions from resonances at low-energies are smoothly connected with Reggeon contributions at the higher energies. Therefore identification of resonances has to be made simultaneously with studies of the high-energy behavior and cross-channel Regge exchanges [10].

A class of models that incorporates resonance-Regge duality has been extensively studied in the past [11–14]. These dual models are based on an extension of the Veneziano [15] approach for amplitudes connecting four external particles to reactions with an arbitrary number, $N$, of external particles [16–18]. The simplest, so-called $B_N$ dual model satisfies crossing and resonance-Regge duality for linear trajectories. Even though the $B_N$ model lacks proper unitarity, which would require non-linear trajectories, it is expected to provide a reasonable description of reaction amplitudes when averaged over resonance widths. Various extensions that enable unitarity, and as a consequence implement complex trajectories [19, 20], have been proposed but they lack the simplicity of the original formulation [21–24].

In this paper we apply the $B_5$ model [25–27] in the double-Regge exchange limit (DRL) to the reaction $\gamma p \rightarrow K^+K^-p$. The analysis of this reaction is currently underway based on the data collected by the CLAS collaboration at JLab using the highest photon energy beam, $E_\gamma \leq 5.5$ GeV, delivered to date at CEBAF to CLAS. The $KK$ spectrum produced in photon dissociation is expected to be dominated by vector resonances, but higher-spin states are also possible.
At present, however, there is little evidence for $K\bar{K}$ decay modes of higher mass meson resonances [28], suggesting resonance signals in the $K\bar{K}$ channel of $\gamma p \to K^+K^-p$ may be weak. This makes studies of non-resonant processes even more relevant. The Regge/Pomeron exchange is the dominant process in the kinematical domain where all sub-channel invariants are large. According to the hypothesis of two-component duality [29], cross-channel Regge exchanges are dual to direct channel resonances. Thus analysis of the $K\bar{K}$ spectrum in photon-production will benefit from understanding of the DRL of this reaction.

The rest of the paper is organized as follows. In Sec. II, we discuss properties of the $B_N$ dual models focusing on $B_4$ (the Veneziano model) and $B_5$, which are of relevance to the process of interest here. In Sec. III A, we describe the double-Regge limit of the $B_5$ amplitude. Once the structure of the dual amplitude in the double-Regge limit is obtained, in Sec. III B, we introduce appropriate modifications related to the presence of external particles with non-zero spin that make the $B_5$ model more suitable for analysis of kaon-pair photoproduction. There we also present results of our numerical analysis. The summary and outlook are given in Sec. V.

II. DUAL AMPLITUDE MODEL

The Veneziano model [15] describes an amplitude of four external scalar particles. As such, it is a function of the three Mandelstam variables $s,t,u$, which are related by

$$s + t + u = \Sigma,$$

where $\Sigma$ is the sum of squares of masses of the external particles. In the following, all dimensional quantities are measured in units of GeV. The building block of the Veneziano model is the $B_4$ amplitude. It is a function of two variables, e.g. $s$ and $t$. For four particles, any pair of Mandelstam variables corresponds to invariant mass squared in two overlapping channels. The $B_4$ amplitude has Regge behavior in each channel and for linear trajectories, $\alpha(x) = \alpha_0 + \alpha'x$, exhibits duality between resonances in one channel and Reggeons in the overlapping channel. Assuming, for simplicity that the reaction is in $s \leftrightarrow t$ symmetric, i.e. resonance/Regge trajectories in the $s$ and $t$ channels are identical, a $B_4$ amplitude can be written as,

$$B_4(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} = \sum_{n=0}^{\infty} \frac{\beta_n(t)}{n - \alpha(s)} = \sum_{n=0}^{\infty} \frac{\beta_n(s)}{n - \alpha(t)}.$$

For linear trajectories the residue function,

$$\beta_n(x) = \frac{\Gamma(-\alpha(x))}{\Gamma(-n - \alpha(x))},$$

is a polynomial in $x$ of order $n$. The two alternative forms in Eq. (2) represent the amplitudes for spinless particle scattering in terms of an infinite series of narrow resonances in either the $s$ or the $t$ channel. The solution of the equation $\alpha(m_B^2) = n$ gives the mass $m_B$ of the resonances. In the model, at given $n$ there are $n+1$ degenerate resonances with spins ranging from 0 to $n$. The couplings of these resonances to the external particles are computed by expanding the residue function $\beta_n(x)$ in terms of Legendre polynomials. In the Veneziano model couplings are fixed and determined by the ratio of $\Gamma$ functions in Eq. (3). A model with adjustable couplings may be obtained by taking combinations of the $B_4$’s with different parameters, i.e. trajectory intercept $\alpha_0$, slope $\alpha'$ and the overall normalization [30, 31].

The asymptotic behavior of the amplitude in Eq. (2) in the limit when one of the channel variables, e.g. $s$ is taken to infinity $s \to \infty$, follows from the Stirling’s formula (shown in Eq. (A4)) and is given by

$$B_4(s,t) \to (-\alpha(s))^{\alpha(t)}\Gamma(-\alpha(t)),$$

which, except for the signature factor, is the behavior expected for $t$-channel exchange of a Regge trajectory. Regge signature factors are recovered by taking appropriate linear combinations of the $B_4$ amplitudes with different channel variables as arguments. For example, at fixed-$t$, the amplitude is symmetric under $s \leftrightarrow u$ crossing, contains only signature-even $t$-channel Reggeons and corresponds to a combination $B_4(s,t) + B_4(u,t)$. The leading behavior in the $s \to \infty$, limit is then given by

$$B_4(s,t)+B_4(u,t) \to \left[(-\alpha(s))^{\alpha(t)} + (-\alpha(u))^{\alpha(t)}\right]\Gamma(-\alpha(t)).$$

So far we have assumed that all trajectory functions are linear. If one also assumes a common slope then it

![Diagramatic representation of the three independent $B_4$ amplitudes. The Mandelstam invariants, $s_{ij} = s_{ji} = (p_i + p_j)^2$ of the neighboring overlapping pairs are the arguments of the amplitude. The amplitudes, from left to right are given by, $B_4(s_{12}, s_{13}), B_4(s_{12}, s_{14})$ and $B_4(s_{13}, s_{14})$.](image)
the leading Regge pole. At fixed $t$ where $\xi$ represents an antiparticle, the reaction
follows from Eq. (1) that,
$$\alpha(s) + \alpha(t) + \alpha(u) = \text{const.} \quad (6)$$
and the leading behavior at large-$s$ of the combination in
Eq. (5) reduces to
$$B_4(s, t) + B_4(u, t) \to (-\alpha(s))^n(t) \xi(t) \Gamma(-\alpha(t)) \quad (7)$$
where $\xi(t) = 1 + \tau e^{i\pi\alpha(t)}$ with $\tau = +1$ is the
proper signature factor for the spin-even $t$-channel
Regge exchange. Thus Eq. (7) is consistent with the
expectations from Regge theory for the contribution of
the leading Regge pole. At fixed $t$ and large and positive
$s, s \to \infty, u \to -\infty$ the amplitude without $t$-channel
poles, i.e. $B_4(u, s)$ behaves as
$$B_4(u, s) \to e^{i\pi\alpha(s)} \sim e^{-i\pi\alpha(u)} \quad (8)$$
When the simple linear trajectory is replaced with a
realistic one, which has a positive imaginary part that
grows as $s \to \infty$, the $B_4(u, s)$ amplitude becomes
exponentially suppressed. Thus, as expected from Regge
theory, in the $B_4$ dual model, out of the three possible
diagrams shown in Fig. 1, only two contribute in the
Regge limit, $s \to \infty$ and $t$-fixed.

The leading meson trajectory is approximately equal
to $\alpha(s) = 0.5 + s$. Therefore, the first pole in the
amplitude of Eq. (2), which corresponds to $\alpha = 0$, would
behave as a resonance of spin-0 and negative mass
squared $m_R^2 = -0.5$. A spurious pole like this is easily
removed by replacing $-\alpha$ by $1 - \alpha$ in the arguments
of the $\Gamma$ functions. With such a shift, the first pole in the
amplitude corresponds to a spin-1 or spin-0 resonance
with mass $m_R^2 = 0.5$ GeV$^2$.

The amplitude $B_3(x, y)$ in Eq. (2) is identical to the
Euler’s Beta-function. Thus $B_4$ can also be written
using an integral representation, which is defined for
$\alpha(s), \alpha(t) < 0$, to be
$$B_4(s, t) = \int_0^1 du u^{-\alpha(s)-1}(1 - u)^{-\alpha(t)-1}. \quad (9)$$
For other values of $\alpha(s)$ and $\alpha(t)$, the amplitude is
obtained from Eq. (2), i.e. by analytical continuation.

The integral representation provides the basis for
generalization of the Veneziano amplitude to an arbitrary
number of external particles. In particular the $B_5$
amplitude can be written as,

$$B_5(s_{AB}, s_{A1}, s_{12}, s_{23}, s_{B3}) = \int_0^1 dt \int_0^1 du t^{-\alpha_{12}-1}u^{-\alpha_{23}-1}(1 - t)^{-\alpha_{A1}-1}(1 - u)^{-\alpha_{B3}-1}(1 - tu)^{-\alpha_{AB}+\alpha_{12}+\alpha_{23}} \quad (10)$$
where $\alpha_{ij} = \alpha_{ij} + \alpha'_{ij}$ and $s_{ij} = (p_i + p_j)^2$ are
the channel variables. We adopt the labeling convention from
[27] with all particle momenta, $p_i$ taken as incoming,
cf. Fig. 2 and $i, j = A, B, 1, 2, 3$. Using the bar to
represent an antiparticle, the reaction $A + B \to 1 + 2 + 3$ corresponds the physical channel of the reaction
of interest, i.e. $\gamma = A, p($target$) = B, K^+ = 1, K^- = 2, p($recoil$) = 3$. The reaction amplitude involving
five particles depends on five independent kinematical
variables, which we choose as the consecutive two-body
channel invariants, $s_{ij}$ as shown in Fig. 2. It follows
from the integral representation in Eq. (10) that $B_5$ is
symmetric under cyclic permutation and the reflection of
the arguments. The integral representation in Eq. (10) is
valid when all trajectories are negative, $\alpha_{ij} < 0$, which is
outside the physical region of the reaction of interest.

The amplitude in the physical region is obtained by
analytic continuation. As in the case of the $B_4$ model,
the analytical continuation of Eq. (10) is performed
once the integral is represented in terms of analytical
functions. In particular it can be expressed in terms of
Euler Beta-functions, ($B_4$ amplitudes) and a generalized
hypergeometric function of unit argument,
The relevant properties of generalized hypergeometric functions are given in the Appendix. The series expression for the hypergeometric function of unit argument, \( _3F_2(a, b, c; d, e) \) is convergent provided \( \text{Re}(a + b + c - d - e) < 0 \), which implies Eq. (11) is well defined for \( \alpha_{AB} < 0, (s_{AB} < 0) \) but ill-defined for \( \alpha_{AB} > 0, (s_{AB} > 0) \). It is the latter that corresponds to the physical region of \( \gamma p \to K^+K^-p \). The symmetry properties of the \( B_5 \), e.g.

\[
B_5(s_{AB}, s_{A1}, s_{12}, s_{23}, s_{B3}) = B_4(-\alpha_{12} - \alpha_{A1}) B_4(-\alpha_{23} - \alpha_{B3})
\times _3F_2(\alpha_{AB} - \alpha_{12} - \alpha_{23}, -\alpha_{A1}, -\alpha_{B3}; -\alpha_{12} - \alpha_{A1}, -\alpha_{23} - \alpha_{B3}).
\] (11)

enables the analytical continuation of Eq. (11) to the physical region of \( A + B \to 1 + 2 + 3 \). Alternatively, one can use the relations between hypergeometric functions given in the Appendix (cf. Eq. (A3)) [27] to continue to the region \( (s_{AB} > 0) \). Both continuations result in the same amplitude [32].

III. DOUBLE-REGGE LIMIT OF \( B_5 \) AMPLITUDE

A. Scalar Amplitudes

Taking into account symmetries implied by Eq. (10) out of \( 5! \) permutations of the external particles there are only 12 independent amplitudes. These are the equivalent to the 3 independent \( B_4(x, y) \), \( x, y = s, t, u \) amplitudes of the Veneziano model shown in Fig. 1. The twelve amplitudes for the reaction \( \gamma p \to K^+K^-p \), denoted by \( B_5(i), i = 1 \ldots 12 \), are depicted in Fig. 3. The first six diagrams have the photon and incident proton next to each other and the other six diagrams have one particle between the incident photon and proton. Before imposing additional symmetry constraints, e.g. Bose symmetry, the most general 5-point amplitude is given by a linear combination of the 12 independent \( B_5 \) amplitudes.

For a 2–to–3 reaction, the double-Regge limit corresponds to large values of the channel energies and small momentum transfers,

\[
s_{AB}, s_{12}, s_{23} \to \infty, \quad \frac{s_{12}s_{23}}{s_{AB}} = \text{fixed}
\]

\[
t_{A1}, t_{B3} \to 0
\] (13)

where \( t_{A1} = (p_A - p_1)^2 \) and \( t_{B3} = (p_B - p_3)^2 \). To compute the double-Regge limit of the \( B_5(1) \) amplitude we use the relations (A2) and (A3) in the Appendix to obtain

\[
B_5 = B_4(-\alpha_{AB} - \alpha_{A1}) B_4(-\alpha_{23} - \alpha_{B3} + \alpha_{A1})
\times _3F_2(-\alpha_{A1}, -\alpha_{23}, -\alpha_{12} - \alpha_{A1} + \alpha_{B3};
1 - \alpha_{A1} + \alpha_{B3} - \alpha_{AB} - \alpha_{A1})
\]

\[
+ B_4(-\alpha_{AB} - \alpha_{B3}) B_4(-\alpha_{12} - \alpha_{A1} + \alpha_{B3})
\times _3F_2(-\alpha_{B3}, -\alpha_{12} - \alpha_{23} - \alpha_{A1}, -\alpha_{B3} + \alpha_{A1};
1 - \alpha_{B3} + \alpha_{A1} - \alpha_{AB} - \alpha_{B3}).
\] (14)

In the above expression three out of the five arguments in the two hypergeometric functions are large. Using the
relation
\[ s F_2(a, \mu, \nu; b, \lambda; z) \to 1 F_1(a, b; \frac{\mu \nu z}{\lambda}), \] (15)
valid in the limit \( \lambda, \mu, \nu \to \infty \), Eq. (14) reduces to
\[ B_5(1) \to B_4(-\alpha_{AB}, -\alpha_{A1}) B_4(-\alpha_{23}, -\alpha_{B3} + \alpha_{A1}) \]
\[ \times F_1(-\alpha_{A1}; 1 - \alpha_{A1} + \alpha_{B3}; -\eta) \]
\[ + B_4(-\alpha_{AB}, -\alpha_{B3}) B_4(-\alpha_{12}, -\alpha_{A1} + \alpha_{B3}) \]
\[ \times F_1(-\alpha_{B3}; 1 - \alpha_{B3} + \alpha_{A1}; -\eta), \] (16)
where \( \eta = \alpha_{12} \alpha_{23} / \alpha_{AB} \). Further simplification is obtained when in the two limits, \( \eta \to 0 \) or \( \eta \to \infty \). In the former (case a) \( 1 F_1(a; b; -\eta) \to 1 \), and Eq. (16) reduces to,
\[ B_5(1, a) = B_4(-\alpha_{AB}, -\alpha_{A1}) B_4(-\alpha_{23}, -\alpha_{B3} + \alpha_{A1}) \]
\[ + B_4(-\alpha_{AB}, -\alpha_{B3}) B_4(-\alpha_{12}, -\alpha_{A1} + \alpha_{B3}). \] (17)
For \( \eta \to \infty \) one can use \( 1 F_1(a; b; -\eta) \to (\eta)^{-a} \Gamma(b) / \Gamma(b - a) \) and obtain,
\[ B_5(1, b) = B_4(-\alpha_{12}, -\alpha_{A1}) B_4(-\alpha_{23}, -\alpha_{B3}). \] (18)
In the following we will also take the limit \( s_{12}, s_{23} \to \infty \) in the \( B_4 \) functions. In the double-Regge limit specified by Eq. (13), four of the twelve diagrams shown in Fig. 2 dominate. These are the diagrams that have the same trajectory in \( A1(\gamma K^+) \) and \( B3(\eta \bar{p}) \) channels. The four diagrams are related by exchanging \( A \leftrightarrow 1 \) or \( B \leftrightarrow 3 \) and are the diagrams \( B_5(1), B_5(7), B_5(12), B_5(9) \) in Fig. 3. The remaining eight diagrams are exponentially suppressed [33].

We demonstrate this suppression using the diagram \( B_5(2) \) as an example. Using the integral representation for the hypergeometric function, the amplitude \( B_5(1) \) can be written as,
\[ B_5(1) = \frac{\Gamma(-\alpha_{12}) \Gamma(-\alpha_{23})}{\Gamma(\alpha_{AB} - \alpha_{12} - \alpha_{23})} \]
\[ \times \frac{1}{2\pi i} \int_{i\infty}^{i\infty} \frac{\Gamma(-s) \Gamma(s - \alpha_{A1}) \Gamma(s - \alpha_{B3})}{\Gamma(s - \alpha_{12} - \alpha_{23}) (-1)^s} ds, \] (19)
The \( B_5(2) \) amplitude is obtained from \( B_5(1) \) by exchanging lines \( 1 \leftrightarrow 3 \),
\[ B_5(2) = \frac{\Gamma(-\alpha_{12}) \Gamma(-\alpha_{23})}{\Gamma(\alpha_{AB} - \alpha_{12} - \alpha_{23})} \]
\[ \times \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{\Gamma(-s) \Gamma(s - \alpha_{A3}) \Gamma(s - \alpha_{B1})}{\Gamma(s - \alpha_{23} - \alpha_{A3})} \]
\[ \times \frac{\Gamma(s + \alpha_{AB} - \alpha_{12} - \alpha_{23}) (-1)^s}{\Gamma(s - \alpha_{12} - \alpha_{B1})} ds, \] (20)
and one finds in the double-Regge limit that it behaves as
\[ B_5(2) \to \Gamma(-\alpha_{12}) \Gamma(-\alpha_{23}) \int_{-i\infty}^{i\infty} \Gamma(-s) \]
\[ \times (s - \alpha_{A3})^{\alpha_{23}} (s - \alpha_{B1})^{\alpha_{12}} (-\alpha_{AB})^{-s} ds. \] (21)
The amplitude thus contains the factor \( e^{i\pi \alpha_{12}} e^{i\pi \alpha_{23}} \).
In the physical region of the reaction considered here, \( \alpha_{12} \) and \( \alpha_{23} \)'s have positive and increasing imaginary parts which makes the amplitude \( B_5(2) \) exponentially suppressed. This mechanism is analogous to the suppression of the \( B_4(u, s) \) amplitude in the \( s \)-channel physical region at large \( s \) and fixed-\( t \) (cf. discussion following Eq. (8)).

To obtain the DRL of the amplitudes corresponding to the diagrams \( B_5(7) \) and \( B_5(12) \) one only needs to exchange \( A \leftrightarrow 1 \) (upper vertex) and \( B \leftrightarrow 3 \) (lower vertex), respectively. The last diagram, \( B_5(9) \), is obtained from \( B_5(1) \) by exchanging particles in both the upper and the lower vertex. The combination of the four diagrams generates the signature factors of the two Reggeons in channels \( A1 \) and \( B3 \). The corresponding amplitudes are given by,
\[ B_5(7) = B_4(-\alpha_{B1}, -\alpha_{A1}) B_4(-\alpha_{23}, -\alpha_{B3} + \alpha_{A1}) \]
\[ \times F_1(-\alpha_{A1}; 1 - \alpha_{A1} + \alpha_{B3}; -\eta_1) \]
\[ + B_4(-\alpha_{B1}, -\alpha_{B3}) B_4(-\alpha_{A2}, -\alpha_{A1} + \alpha_{B3}) \]
\[ \times F_1(-\alpha_{B3}; 1 - \alpha_{B3} + \alpha_{A1}; -\eta_1), \]
\[ B_5(12) = B_4(-\alpha_{A3}, -\alpha_{A1}) B_4(-\alpha_{B2}, -\alpha_{B3} + \alpha_{A1}) \]
\[ \times F_1(-\alpha_{A1}; 1 - \alpha_{A1} + \alpha_{B3}; -\eta_2) \]
\[ + B_4(-\alpha_{A3}, -\alpha_{B3}) B_4(-\alpha_{B2}, -\alpha_{A1} + \alpha_{B3}) \]
\[ \times F_1(-\alpha_{B3}; 1 - \alpha_{B3} + \alpha_{A1}; -\eta_2), \]
\[ B_5(9) = B_4(-\alpha_{A3}, -\alpha_{A1}) B_4(-\alpha_{B2}, -\alpha_{B3} + \alpha_{A1}) \]
\[ \times F_1(-\alpha_{A1}; 1 - \alpha_{A1} + \alpha_{B3}; -\eta_3) \]
\[ + B_4(-\alpha_{A3}, -\alpha_{B3}) B_4(-\alpha_{B2}, -\alpha_{A1} + \alpha_{B3}) \]
\[ \times F_1(-\alpha_{B3}; 1 - \alpha_{B3} + \alpha_{A1}; -\eta_3), \] (22)
where \( \eta_1 = \alpha_{A2} \alpha_{23} / \alpha_{B1}, \eta_2 = \alpha_{12} \alpha_{B2} / \alpha_{A3} \) and \( \eta_3 = \alpha_{A2} \alpha_{B2} / \alpha_{A3} \). There are new trajectories that appear in these amplitudes. For example \( \alpha_{B1} \) in \( B_5(7) \) originates from \( \alpha_{AB} \) in \( B_5(1) \) after replacing \( A \) with 1. In principle these two trajectories have different functional dependence on the channel invariants since they represent resonances coupled to a different pair of particles. In \( B_5(7) \), \( \alpha_{A1} \) contains resonances in the \( K^- p \) channel while \( \alpha_{AB} \) in \( B_5(1) \) describes resonances in the \( \gamma p \) channel. As discussed in the case of the Veneziano model, to achieve resonance-Regge duality it is necessary to use a common slope for all trajectories (see discussion in Sec. II). In this case, trajectories can be related to each other using kinematical relations between channel invariants, analogous to that in Eq. (1), e.g.
\[ s_{AB} = s_{13} + s_{12} + s_{23} + \text{const.}, \] (23)
where the constant is given by sum of masses squared. In particular one finds
\[
\begin{align*}
\alpha_{13} &= \alpha_{AB} - \alpha_{12} - \alpha_{23} + \text{const.}, \\
\alpha_{A3} &= \alpha_{12} - \alpha_{B3} - \alpha_{AB} + \text{const.}, \\
\alpha_{B1} &= \alpha_{23} - \alpha_{A1} - \alpha_{AB} + \text{const.}, \\
\alpha_{A2} &= \alpha_{B3} - \alpha_{A1} - \alpha_{12} + \text{const.}, \\
\alpha_{B2} &= \alpha_{A1} - \alpha_{B3} - \alpha_{23} + \text{const.,}
\end{align*}
\] (24)
which in the double-Regge limit lead to
\[
\begin{align*}
\alpha_{13} &\sim \alpha_{AB} - \alpha_{12} - \alpha_{23} \sim \alpha_{AB}, \\
\alpha_{A3} &\sim -\alpha_{AB} \ , \ \alpha_{B1} \sim -\alpha_{AB}, \\
\alpha_{A2} &\sim -\alpha_{12} \ , \ \alpha_{B2} \sim -\alpha_{23},
\end{align*}
\] (25)
and \(\eta_1 = \eta_{12} = \eta_{23} \rightarrow \eta\). Combining the four surviving amplitudes in the double-Regge limit,
\[
A_5 = B_5(1) + \tau_{A1} B_5(7) + \tau_{B3} B_5(12) + \tau_{A1} \tau_{B3} B_5(9)
\] (26)
with \(\tau_i = \pm 1\), one finds
\[
\begin{align*}
A_5 &= (-\alpha_{AB})^{\alpha_{A1}} (-\alpha_{23})^{\alpha_{B3}} \frac{\Gamma(-\alpha_{A1}) \Gamma(-\alpha_{A1} - \alpha_{B3})}{\Gamma(-\alpha_{A1} - \alpha_{23})} \\
&\times (1 + \tau_{A1} e^{i \pi \alpha_{A1}} \tau_{B3} e^{i \pi \alpha_{B3}} + \tau_{A1} \tau_{B3} e^{i \pi \alpha_{B3}}) \\
&\times V_1(t_{A1}, t_{B3}, \eta) \\
&+ (-\alpha_{AB})^{\alpha_{A1}} (-\alpha_{12})^{\alpha_{A1}} \Gamma(-\alpha_{B3} - \alpha_{A1}) \\
&\times (1 + \tau_{A1} e^{i \pi \alpha_{A1}} + \tau_{B3} e^{i \pi \alpha_{B3}} + \tau_{A1} \tau_{B3} e^{i \pi \alpha_{B3}}) \\
&\times V_2(t_{A1}, t_{B3}, \eta),
\end{align*}
\] (27)
where the functions \(V_i\) represent the Reggeon-Reggeon-particle coupling at the middle-vertex as shown in Fig. 4.

The equation above has the general structure for the leading Regge pole contributions to the double-Regge limit [34], in which \(V_1\) and \(V_2\) are analytical functions of their variables in the kinematical domain of the double-Regge limit. In particular, the \(B_5\) model used gives the following prediction for the middle-vertex functions,
\[
\begin{align*}
V_1(t_{A1}, t_{B3}, \eta) &= 1 F_1(-\alpha_{A1} - \eta; -\alpha_{A1} + \alpha_{B3} + \eta), \\
V_2(t_{A1}, t_{B3}, \eta) &= 1 F_1(-\alpha_{B3} + \eta; -\alpha_{B3} + \alpha_{A1} - \eta) .
\end{align*}
\] (28)

In the double-Regge limit, the resonances-trajectories in the production channels, \(\alpha_{AB}, \alpha_{12}\) and \(\alpha_{23}\), are proportional the channel variables, \(\alpha_{ij} \rightarrow s_{ij}\), and
\[
\begin{align*}
A_5 &= (s_{AB})^{\alpha_{A1}} (s_{23})^{\alpha_{B3}} \frac{\Gamma(-\alpha_{A1}) \Gamma(-\alpha_{A1} - \alpha_{B3})}{\Gamma(-\alpha_{A1} - \alpha_{23})} \\
&\times (1 + \tau_{A1} e^{i \pi \alpha_{A1}} + \tau_{B3} e^{i \pi \alpha_{B3}} + \tau_{A1} \tau_{B3} e^{i \pi \alpha_{B3}}) \\
&\times V_1(t_{A1}, t_{B3}, \eta') \\
&+ (s_{AB})^{\alpha_{B3}} (s_{12})^{\alpha_{A1}} \frac{\Gamma(-\alpha_{B3} - \alpha_{A1})}{\Gamma(-\alpha_{B3} + \alpha_{A1})} \\
&\times (1 + \tau_{B3} e^{i \pi \alpha_{B3}} + \tau_{A1} e^{i \pi \alpha_{A1}} + \tau_{A1} \tau_{B3} e^{i \pi \alpha_{B3}}) \\
&\times V_2(t_{A1}, t_{B3}, \eta'),
\end{align*}
\] (29)
where \(\eta' = s_{12}s_{23}/s_{AB}\). We have neglected slowly varying exponential form factors proportional to \(\exp(\alpha_{i} \log(\alpha'))\), where \(i = A1, B3\), since for leading trajectories the slope parameter \(\alpha'\) is very close to one.

We observe, that just as in Eq. (2), the above amplitude also contains ghost poles in the \(A1\) and \(B3\) channels for positive signatures \(\tau_i = 1\) when \(\alpha_{A1}\) and \(\alpha_{B3}\) are the leading trajectories \(\alpha(s) \sim 0.5 + s\). In the present work, we focus only on the latter case and the ghost poles have to be removed. As discussed in the previous section, these can be eliminated by shifting trajectories, \(\alpha_{A1} \rightarrow \alpha_{A1} - 1\) and \(\alpha_{B3} \rightarrow \alpha_{B3} - 1\). The replacement guarantees that the double-Regge amplitude has no resonances in the \(A1\) and \(B3\) channels in the physical region of the \(A + B \rightarrow 1 + 2 + 3\) reaction, where these are the exchange channels and \(\alpha_{A1}, \alpha_{B3} \leq 0\).

Shifting the trajectories and redefining the signature accordingly we obtain
\[
\begin{align*}
A_5 &= (s_{AB})^{\alpha_{A1}} (s_{23})^{\alpha_{B3}} \frac{\Gamma(1 - \alpha_{A1}) \Gamma(-\alpha_{A1} - \alpha_{B3})}{\Gamma(-\alpha_{A1} - \alpha_{23})} \\
&\times (1 + \tau_{A1} e^{i \pi \alpha_{A1}} + \tau_{B3} e^{i \pi \alpha_{B3}} + \tau_{A1} \tau_{B3} e^{i \pi \alpha_{B3}}) \\
&\times V_1(t_{A1}, t_{B3}, \eta') \\
&+ (s_{AB})^{\alpha_{B3}} (s_{12})^{\alpha_{A1}} \frac{\Gamma(1 - \alpha_{B3}) \Gamma(-\alpha_{B3} - \alpha_{A1})}{\Gamma(1 - \alpha_{B3} + \alpha_{A1})} \\
&\times (1 + \tau_{B3} e^{i \pi \alpha_{B3}} + \tau_{A1} e^{i \pi \alpha_{A1}} + \tau_{A1} \tau_{B3} e^{i \pi \alpha_{B3}}) \\
&\times V_2(t_{A1}, t_{B3}, \eta').
\end{align*}
\] (30)
When \(\alpha_{A1}\) and \(\alpha_{B3}\) are equal to even/odd positive integers, the amplitude has poles when \(\tau_{A1}, \tau_{B3} = +1, -1\), respectively. The amplitude in the \(B3\) channel

FIG. 4. The four diagrams for double-Regge limit \(\gamma p \rightarrow K^+ K^- p\) reaction. The diagrams correspond to, from top left to bottom right, \(B_5(1), B_5(7), B_5(12), B_5(9)\) in Fig. 3, respectively. \(K^+\) and \(K^-\) are the particles exchanged in the \(A1\) channel while \(p/\omega\) and \(f_2/\omega_2\) are exchanged in the \(B3\) channel.
(lower Reggeon) has the first pole for $\alpha_{B3} = 1$. The pole appears in the amplitude with $\tau_{B3} = -1$ and, depending on the isospin, corresponds to the $\rho$ or $\omega$ meson exchange.

At $\alpha_{B3} = 2$ there is a pole in the right-signature, $\tau_{B3} = +1$ amplitude. It corresponds to the lightest spin-2 tensor mesons, the $a_2$ and the $f_2$, depending on the isospin.

In the $A1$ channel, the leading pole at $\alpha_{A1} = 1$ in the right-signature amplitude ($\tau_{A1} = -1$) corresponds to the exchange of the lightest strange, spin-1 meson, the $K^*(890)$. The strange, tensor meson pole in the right-signature amplitude at $\alpha_{A1} = 2$ corresponds to the $K^*_2(1430)$. The amplitude in the double-Regge limit is therefore associated with the exchange of the following meson combinations ($K^*, \rho/\omega$), ($K^*, a_2/f_2$), ($K^*_2, \rho/\omega$), ($K^*_2, a_2/f_2$), corresponding to the amplitude with $\tau_{A1, B3} = (-,-), (++, -)$, respectively.

We note that the unnatural parity trajectory in the $A1$ channel of the $K$-meson is located below the leading trajectory. In the double-Regge limit of the $B_5$ model it is suppressed compared to the exchange of vector and tensor mesons. Even though the $K$-meson exchange in the upper vertex is suppressed, the Pomeron exchange in the lower vertex, to which it can couple, may dominate over the leading $B3$ meson Regge exchange. Since the $B_5$ model does not include the Pomeron exchange, $K$ exchange is not considered here.

Finally we also give the expressions for the DRL amplitudes in the two limiting cases discussed earlier, $\eta \to 0$ or $\eta \to \infty$. One finds,

$$A_5 \rightarrow (-s_{AB})^{\alpha_{A1}-1}(-s_{B3})^{\alpha_{B3}-\alpha_{A1}} \Gamma(1-\alpha_{A1})^{\alpha_{A1}-\alpha_{B3}} \times (1+\tau_{A1}e^{i\pi\alpha_{A1}}+\tau_{B3}e^{i\pi\alpha_{B3}}+\tau_{A1}\tau_{B3}e^{i\pi(\alpha_{B3}-\alpha_{A1})})$$

$$+ (-s_{AB})^{\alpha_{B3}-1}(-s_{A1})^{\alpha_{A1}-\alpha_{B3}} \Gamma(1-\alpha_{B3})^{\alpha_{B3}-\alpha_{A1}} \times (1+\tau_{A1}e^{i\pi\alpha_{A1}}+\tau_{B3}e^{i\pi\alpha_{B3}}+\tau_{A1}\tau_{B3}e^{i\pi(\alpha_{A1}-\alpha_{B3})}),$$

for $\eta \to 0$ and

$$A_5 \rightarrow (-s_{A1})^{\alpha_{A1}-1}(-s_{B3})^{\alpha_{B3}-1} \Gamma(1-\alpha_{A1})^{\alpha_{A1}-\alpha_{B1}} \times (1+\tau_{A1}e^{i\pi\alpha_{A1}}+\tau_{B3}e^{i\pi\alpha_{B3}}+\tau_{A1}\tau_{B3}e^{i\pi(\alpha_{B3}+\alpha_{A1})}),$$

for $\eta \to \infty$, respectively.

### B. Spin Structure

As discussed above, the dual model contains only the information about the resonance content and not about the external particles. In particular, it is agnostic about external particle spin. The amplitudes of the model should therefore be used in general to describe the kinematically-free scalar amplitudes appearing in a Lorentz decomposition of helicity amplitudes [35]. For $\gamma p \rightarrow K^+ K^- p$ in the expression for the helicity amplitude $M$,

$$M = \sum_{\alpha} \bar{u}(p_3, \lambda_3)J^\mu u(p_B, \lambda_B)\epsilon_\mu(p_A, \lambda_A),$$

the current $J^\mu$ is given in terms of Dirac matrices combined with the four independent particle momenta and multiplied by scalar functions of the invariant, Mandelstam variables. It is these scalar functions which can be represented by the $B_5$ amplitudes of the dual model.

In the numerical study that follows, we test a particular model for the current operator. The model is based on the analysis of perturbation theory diagrams with the Reggeons replaced by lightest mass particle on the leading trajectory, i.e. the vector mesons. This is most accurate for the $(\tau_{A1}, \tau_{B3}) = (-,-)$ amplitude, c.f. discussion above, while the other three combinations should include at least one tensor structure associated with the exchange of a tensor meson. We note that exchange of higher-spin states does not require further modification of the spin-tensors, since as far as the spin-structure is concerned, the only difference between i.e. spin-3 and spin-1 meson exchange is an analytical function of the channel variables. The dependence on these variables is already fixed by the dual Regge limit. In the following, we make a simplifying approximation and use the same spin structure for all four signature combinations.

The (upper) vertex representing a coupling of an external (vector) photon to a (pseudoscalar) kaon, via exchange of a vector meson in the $A1$ channel is given by

$$V^u(\lambda_{A1}) = \epsilon_{\mu\nu\alpha\beta}\epsilon_\mu(p_A, \lambda_A)\epsilon^\nu(p_A, \lambda_A)p^\alpha p^\beta_\lambda.$$

Here $p_A = p_4 + p_1$ and $\lambda_A$ and $\lambda_{A1}$ are the helicities of the photon and the exchanged vector meson, respectively. Similarly, the bottom vertex represents the coupling of a vector meson in the $B3$ channel to the two nucleons is given either by

$$V^l\lambda_{B3} = \epsilon^\mu(p_B, \lambda_B)\bar{u}(p_3, \lambda_3)\gamma_\mu u(p_B, \lambda_B)$$

or by

$$V^l\lambda_{B3} = \epsilon^\mu(p_B, \lambda_B)\bar{u}(p_3, \lambda_3)i\sigma_{\mu\nu}p^\nu_B u(p_B, \lambda_B),$$

with $p_B = p_4 + p_1$ and $\lambda_{B3}$ representing momentum and helicity of the exchanged vector meson in the $B3$ channel. The $V^l_{II}$ vertex represents dominantly the helicity-flip/non-flip amplitude in the $s$-channel, respectively. The $\rho$ meson exchange, for example, is expected to be dominantly helicity flip and corresponds to the vertex $V^l_{II}$. In the following, we will use $V^l_{II}$ for the bottom vertex for the four Reggeon combinations discussed in the preceding section.

In Eq. (28), the vertex functions $V_i$ describes the middle-vertex where the $A1$ and $B3$ exchanges couple to
Van Hove plots. in [36, 37] based on the analysis of the longitudinal distributions guided by the selection criteria proposed following we analyze the limit by studying the Dalitz plot from the resonance region is quite smooth. In the exchanges, and summing over helicities of the intermediate vector by the product of the three vertices, Eqs. (34, 36, 37), Regge limit resides in the predicted dependence of the Mandelstam variables. The key feature of the double- an intensity function that is a regular function of the When the spin is averaged, the spin-amplitudes produce the pseudoscalar kaon. This coupling is given by our case, the coupling of the two exchanged vectors to the pseudoscalar kaon. This coupling is given by

\[ V^m(\lambda_{A1}, \lambda_{B3}) = \varepsilon_{\mu\nu\alpha\beta} \epsilon^\mu(p_{A1}, \lambda_{A1}) \epsilon^\nu(p_{B3}, \lambda_{B3}) p_{A1}^\alpha p_{B3}^\beta. \]  

(37)

The final amplitude \( M \) is obtained by multiplying \( A_5 \) by the product of the three vertices, Eqs. (34, 36, 37), and summing over helicities of the intermediate vector exchanges,

\[ M = A_5 \sum_{\lambda_{A1}, \lambda_{B3}} V^m(\lambda_{A1}) V^m(\lambda_{A1}, \lambda_{B3}) V^m_{IJ}(\lambda_{B3}). \]  

(38)

When the spin is averaged, the spin-amplitudes produce an intensity function that is a regular function of the Mandelstam variables. The key feature of the double-Regge limit resides in the predicted dependence of the sub-channel energies, \( s_{12}, s_{23} \) and \( s_{AB} \), which, far away from the resonance region is quite smooth. In the following we analyze the limit by studying the Dalitz plot distributions guided by the selection criteria proposed in [36, 37] based on the analysis of the longitudinal component of the momentum, also called longitudinal or Van Hove plots.

IV. DATA SIMULATION

In this section, we use the amplitude obtained above, Eq. (38), to simulate the \( \gamma p \to K^+ K^- p \) reaction. We set the photon energy to \( E_\gamma = 5.5 \text{ GeV} (\sqrt{s} = 3.5 \text{ GeV}) \) in the lab-frame (target rest frame) which corresponds to the highest photon energy of the \( K^+ K^- \) data collected by the CLAS collaboration at JLab [38, 39]. Once the data are analyzed they can be compared with the simulations presented below. We use a standard Monte Carlo (MC) method to generate the events according the the amplitudes discussed above.

For the leading trajectories, as discussed in Sec. III A, we use meson trajectories from [40]

\[ \alpha_{A1}(t) = \alpha_{K^*} = 0.318 + 0.839t, \]
\[ \alpha_{B3}(t) = \alpha_\rho = 0.456 + 0.887t. \]  

(39)

The numerical computation of the \( A_5 \) amplitude must be done carefully when evaluating the Regge-Regge-particle vertices given by Eq. (38). Each term separately is singular when \( \alpha_{A1} \) and \( \alpha_{B3} \) are not integers but \( \alpha_{A1} - \alpha_{B3} \) is an even integer, although the full amplitude is finite at those points. To avoid singularities in the \( V_i \) we add a small imaginary part to the real axis where the amplitude is evaluated.

We study four amplitudes defined by the four possible combinations of the signature factors, which as discussed in Sec. III A, correspond to the following cases:

I: \( \tau_{A1} = -1, \tau_{B3} = +1, \text{ for } (K^*, a_2/f_2); \)

II: \( \tau_{A1} = -1, \tau_{B3} = -1, \text{ for } (K^*, \rho/\omega); \)

III: \( \tau_{A1} = +1, \tau_{B3} = +1, \text{ for } (K^*_2, a_2/f_2); \)

IV: \( \tau_{A1} = +1, \tau_{B3} = -1, \text{ for } (K^*_2, \rho/\omega). \)

In this analysis we do not distinguish between different isospins, but we do study the spin structure described in Sec. III B.

A. Data Selection and the Van Hove Plot

At fixed \( s_{AB} \), we integrate over \( t_{A1} \) and \( t_{B3} \), the cross-section then becomes a function of \( s_{12} \) and \( s_{23} \) only and can be represented in a Dalitz plot. The double-Regge limit corresponds to low values of the momentum transfer variables, \( t_{A1} \) and \( t_{B3} \). To isolate the corresponding DRL in the Dalitz variables it is best to employ the procedure developed by Van Hove [36, 37].

For a 2–to–3 process, the Van Hove plot is a two-dimensional plot of the longitudinal momenta of the three produced particles. In the center of mass (c.o.m.) frame, the incident photon defines the positive-\( z \) axis and the longitudinal components of the outgoing particles are defined by the projection of the momenta onto the \( z \) axis. Longitudinal momentum conservation
mandates that only two out of the three particle’s longitudinal momenta are independent. Furthermore, energy and transverse-momentum conservation require that events are distributed inside a bounded region of the two dimensional space defined by the independent longitudinal momenta. The longitudinal momenta of the outgoing particles are parameterized using polar coordinates with radius \( q = (p_{K+L}^2 + p_{K-L}^2 + p_{PL}^2)^{1/2} \) and a polar angle \( \omega \) defined as:

\[
\begin{align*}
 p_{K+L} &= \sqrt{\frac{2}{3}} q \sin \omega, \\
 p_{K-L} &= \sqrt{\frac{2}{3}} q \sin \left( \frac{2}{3} \pi + \omega \right), \\
 p_{PL} &= \sqrt{\frac{2}{3}} q \sin \left( \frac{4}{3} \pi + \omega \right).
\end{align*}
\]

With the lines corresponding to \( p_{K+L} = 0, p_{K-L} = 0, \) and \( p_{PL} = 0 \) drawn at 60° angle on a 2-dimensional plot, aka longitudinal plot, cf. Fig. 5. Each point satisfies longitudinal momentum conservation. In the limiting case where particle masses and transverse momenta are ignored, the boundary of the longitudinal plot for \( \gamma p \rightarrow K^+ K^- p \) corresponds to the hexagon in Fig. 5. Otherwise it is given by the smooth curve shown by the inner elliptical-line defined by vanishing transverse momenta.

With the parametrization given in Eq. \((40)\), the Dalitz variables, \( s_{12} \) and \( s_{23} \), and the two momentum transfer, \( t_{A1} \) and \( t_{B3} \), become functions of \( \omega \) and the transverse momenta. For example, given a photon beam energy of 5.5 GeV, which corresponds to \( s_{AB} = 11.2 \text{ GeV}^2 \) and fixing \( |p_{K+T}| = |p_{K-T}| = |p_{PT}| = 0.6 \text{ GeV} \), the \( \omega \) dependence of the Dalitz and momentum transfer variables are shown in Fig. 6.

The longitudinal momentum \( p_{K+L} \) is positive for \( \omega \in [0^\circ, 180^\circ] \) and \( p_{PL} \) is negative for \( \omega \in [60^\circ, 240^\circ] \), hence, the events concentrate in the upper left and center sectors of the plot in Fig. 5. The overlap of these two regions \( \omega \in [60^\circ, 240^\circ] \) corresponds to low momentum transfers in \( A1 \) and \( B3 \) channels.

The Dalitz variables \( s_{12} \) and \( s_{23} \) are periodic with 180° period and their minima in the region that overlaps with the region of small momentum transfer are \( \omega \in [110^\circ, 170^\circ] \) for \( s_{12} \) and \( \omega \in [70^\circ, 110^\circ] \) for \( s_{23} \). These define the single-Regge limits that correspond to large \( s_{AB} \) and small, either, \( s_{12} \) or \( s_{23} \). These are labeled as the R12 and R23 wedges in Fig. 5.

The region were both \( s_{12} \) and \( s_{23} \) are largest that overlaps with the region of small momentum transfers corresponds to \( \omega \in [110^\circ, 130^\circ] \). This is the region which is closest to the kinematic domain of the double-Regge limit and it is marked by the wedge labeled D123 in Fig. 5. In this region, in the center of mass frame,
the $K^+$ and the recoiling proton have large momentum components in the $+z$ and $-z$ direction, respectively while the 3-momentum components of the $K^-$ are small.

To generate events in the double-Regge region we use the following procedure. First, we generate a large sample $O(10^3)$ events uniformly distributed in the three particle phase space. In Fig. 7(a) we show the generated Dalitz plot and the distribution of the momentum transfer $t_{A1}$ and $t_{B3}$. Next, we limit the momentum transfers by constraining the transverse momenta $|p_{K^+}|$, $|p_{K^-}|$, and $|p_{PT}|$ to the $[0, 0.6]$ GeV range. The center of the longitudinal plot in Fig. 5 corresponds to vanishing longitudinal momenta $q = (p_{K^+L}^2 + p_{K^-L}^2 + p_{T_L}^2)^{1/2} = 0$ and maximal value of the transverse momenta. Because of the cut off on the value of transverse momenta, a hole appears in the longitudinal plot which also shows up in the Dalitz plot in Fig. 7(b). The middle and right panel in Fig. 7(b) shows that, despite of the cuts in transverse momenta, there are still contributions from large $|t_{A1}|$ and $|t_{B3}|$ that need to be removed. As discussed above, the contribution from the large momentum transfers is eliminated by restricting $\omega$ to the D123 region of $\omega \in [110^0, 130^0]$. After the cut on $\omega$ events with momentum transfers, $|t_{A1}|$ and $|t_{B3}|$, larger than 1.5 GeV$^2$ are removed as shown in in Fig. 7(c). The final sample is reduced to approximately $5 \times 10^6$ events. The Dalitz plot distribution is shown in the left panel in Fig. 7(c) and it agrees with that of [41].

The final amplitude depends on the spin structure of the external particles as explained in Sec. III B. In order to study the spin structure, in Fig. 8 we plot the amplitude $M$ in Eq. (38) with $A_3 = 1$ for the selected events. Comparing the Dalitz plots in Fig. 7(c) and Fig. 8 we notice that event concentration shifts from the bottom-left of the Dalitz plot to the center. Fig. 8 also shows that the kinematical factor suppressed the events in the forward direction due to the spin-flip nature of the bottom vertex.

Once we have chosen the kinematical region carefully and we understand the impact of the spin structure in the amplitude, we can simulate the full amplitude $M$ in Eq. (38) for the four cases described in Sec. IV. The four cases have similar, but distinguishable, characteristics that are apparent in Dalitz plots shown in Fig. 9. For instance, in all the cases, the events are concentrated in the middle of the selected kinematical region. All four amplitudes share the same spin structure and the same dependence on $s_{12}$ and $s_{23}$ but have different combinations of exchanged Reggeons. Because of the small $|t_{A1}|$ and $|t_{B3}|$, the signature factors become approximately constant and therefore do not introduce significant differences among the four amplitudes, as shown in Fig. 9.

Finally we discuss the results for the limits, $\eta \to 0$ and $\eta \to \infty$, corresponding to the amplitudes in Eqs. (31) and (32).

Specifically we take $\tau_{A1} = -1$ and $\tau_{B3} = +1$, corresponding to $K^+$ and $a_2/f_2$, resulting in Fig. 10. As expected the limit $\eta \to \infty$ favors events which maximize the channel invariants in the middle of Dalitz plot.
V. CONCLUSIONS AND OUTLOOK

Experiments have been performed and are planned for the future in which multiparticle final states are produced with a view to identifying new resonances, as we as confirming the properties of previously found states. Having only nucleon targets, these final states inevitably involve both baryon and meson resonances. A common analysis procedure adopted is to regard the mesons as an unwanted background to the study of baryon resonances, and the $N^*$'s (or here the $A$'s and $\Sigma$'s) as an annoying background to the study of meson resonances. Kinematic cuts are made to eliminate one or the other, and so enhance the baryon or meson signal as required. Of course, this has the deficiency of throwing away not just a 'background', but part of the signal too, as well as the essence of the production process. Dalitz plot analyses together with analysis of momentum transfer distribution highlight how the overlap of kinematic regimes contains valuable information that can help to elucidate the signals under study.

In this article we have studied the double-Regge exchange limit for the $\gamma p \rightarrow K^+ K^- p$ reaction employing a generalized Veneziano model ($B_5$ model). The equations necessary to describe the double-Regge limit are obtained by taking the high-energy limit, $s_{AB}, s_{12}, s_{23} \rightarrow \infty$, as well as restricting the momentum transfers. When the 2-to-3 amplitude is saturated by the two Regge poles, the dependence on the sub-channel energies is fixed by the Regge pole trajectories and can be tested by studying the Dalitz plot distributions as a function of the (small) momentum transfers. We have shown that suitable event candidates for the double-Regge exchange in the high-energy limit can be selected by means of longitudinal momentum distribution which provide the adequate cuts in the sub-channel invariants and momentum transfers. The importance of the spin structure in the amplitude has been investigated and momentum transfers. The importance of the production process. Dalitz plot analyses together with analysis of momentum transfer distribution highlight how the overlap of kinematic regimes contains valuable information that can help to elucidate the signals under study.

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Because of the small range of momentum transfer the signature factors become approximately constant and the leading trajectories are approximately exchange degenerate. Consequently, we find little sensitivity to exchange dynamics.

The double-Regge limit operates in the kinematic regime where the two sub-channel invariants are large and outside the resonance region in each channel. Analyticity implies that amplitude in the resonance region, in both meson and baryon channels should connect smoothly to the amplitude in double-Regge kinematics. This constrain can be formalized using finite energy sum rules [42]. Having a common framework to analyze both baryonic and mesonic signals is key to making combined multiparticle analyses tractable. The isolation of the double-Regge limit is a step in that direction by providing a realistic and accurate modeling that feeds into both the dynamics of mesons and baryons. The single Regge limit and the above mentioned analytical continuation for $\gamma p \rightarrow K^+ K^- p$ will be presented in future works.

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Appendix A: Useful Relations

Throughout this manuscript we make extensive use of the hypergeometric function defined by

$$
_\nu F_\mu(x_1, \cdots, x_p; y_1, \cdots, y_q; z) = \sum_k \frac{\Gamma(x_1 + k) \cdots \Gamma(x_p + k) \Gamma(y_1) \cdots \Gamma(y_q) z^k}{\Gamma(y_1) \cdots \Gamma(x_p) \Gamma(y_1 + k) \cdots \Gamma(y_q + k) k!},
$$

(A1)

and its properties. If we write $\nu F_\mu(x_1, \cdots, x_p; y_1, \cdots, y_q) = \nu F_\mu(x_1, \cdots, x_p; y_1, \cdots, y_q; 1)$, for $z = 1$, the series converges if $\text{Re}(\sum y_q - \sum x_p) > 0$. Outside of its domain of convergence, the hypergeometric function is defined by analytical continuation, which can be performed aided by the following relations

$$
_3 F_2(a, b, c; d, e) = \frac{\Gamma(1-a)\Gamma(d)\Gamma(e)\Gamma(c-b)}{\Gamma(1-b)\Gamma(d-b)\Gamma(1+b-a)\Gamma(e)} \times _3 F_2(b, 1+b-d, 1+b-e; 1+b-c, 1+b-a)
$$

$$
+ \frac{\Gamma(1-a)\Gamma(d)\Gamma(e)\Gamma(b-c)}{\Gamma(1-b)\Gamma(d-c)\Gamma(1+c-a)\Gamma(b)} \times _3 F_2(c, 1+c-e, 1+c-d; 1+c-b, 1+c-a),
$$

(A2)
\[
\frac{3F_2(a, b, c; d, e)}{\Gamma(s)\Gamma(d)\Gamma(e)} = \frac{3F_2(d - c, d - b, a; s + a, d)}{\Gamma(e - a)\Gamma(s + a)\Gamma(d)} = \frac{3F_2(s, d - a, e - a; s + b, s + c)}{\Gamma(a)\Gamma(s + b)\Gamma(s + c)},
\]

where \( s = d + e - a - b - c \). We employ these relations to obtain Eq. (14) from Eq. (11) by substituting Eq. (A2) and then using Eq. (A3). Another relation that we use is the Stirling’s formula for \( \Gamma(z) \) function in the \(|z| \to \infty\) and \(|\arg z| < \pi\) limit:

\[
\Gamma(z) \to \sqrt{2\pi} e^{-\frac{z^2}{2}},
\]

for pole isolation in the \( B_4 \) and \( B_5 \) models.

[1] S. K. Choi et al., Physical Review Letters 91, 262001 (2003).
[2] S. K. Choi et al., Physical Review Letters 100, 142001 (2008).
[3] K. Chilikin et al., Physical Review D 88, 074026 (2013).
[4] R. Aaij et al., Physical Review Letters 112, 222002 (2014).
[5] A. Bondar et al., Physical Review Letters 108, 122001 (2012).
[6] A. Ablikim et al., Physical Review Letters 110, 252001 (2013).
[7] D. Powell, Nature 498, 280 (2013).
[8] M. Gell-Mann, Physics Letters 8, 214 (1964).
[9] R. Dolen, D. Horn, and C. Schmid, Physical Review Letters 19, 402 (1967).
[10] P. D. B. Collins, An Introduction to Regge Theory and High Energy Physics (Cambridge University Press, 1977).
[11] G. F. Chew and A. Pignotti, Physical Review Letters 20, 1078 (1968).
[12] V. Barger and L. Durand, Physics Letters B 26, 588 (1968).
[13] V. Alessandrini, D. Amati, and E. Squires, Physics Letters B 27, 463 (1968).
[14] P. Collins, G. Ross, and E. J. Squires, Nuclear Physics B 10, 475 (1969).
[15] G. Veneziano, Nuovo Cimento 57, 190 (1968).
[16] H.-M. Chan, Physics Letters B 28, 425 (1969).
[17] C.-J. Geobel and B. Sakita, Physical Review Letters 22, 257 (1969).
[18] Z. Koba and H. B. Nielsen, Nuclear Physics B 10, 633 (1969).
[19] A. I. Bugril et al., Fortsch.Phys. 21, 427 (1973).
[20] L. L. Jenkovszky, Sov.J.Nucl.Phys 21 (1973).
[21] J. Bartsch et al., Nuclear Physics B 23, 1 (1970).
[22] B. Petersson and N. A. Tönqvist, Nuclear Physics B 13, 629 (1969).
[23] K. W. J. Barnham et al., Nuclear Physics B 28, 291 (1971).
[24] R. Baier, H. Kühnelt, and F. Widder, Nuclear Physics B 27, 372 (1971).
[25] K. Bardakci and H. Ruegg, Physics Letters B 28, 342 (1969).
[26] K. Bardakci and H. Ruegg, Physics Letters B 28, 671 (1969).
[27] A. Bialas and S. Pokorski, Nuclear Physics B 10, 399 (1969).
[28] K. A. Olive et al., Chin. Phys. C 38, 0900001 (2014).
[29] H. Harari, Physical Review Letters 20, 1395 (1968).
[30] D. Sivers, Rev. Mod. Phys 43, 125 (1971).
[31] A. P. Szczepaniak and M. R. Pennington, Physics Letters B 737, 283 (2014).
[32] W. F. Perger, A. Bhalla, and M. Nardin, Comput. Phys. Comm 77, 249 (1993).
[33] W. J. Zakrzewski, Nuclear Physics B 14, 458 (1969).
[34] R. C. Brower, C. E. DeTar, and J. H. Weis, Physics Reports 14, 257 (1974).
[35] M. D. Scandron and H. F. Jones, Physical Review 173, 1734 (1968).
[36] L. Van Hove, Physics Letters B 28, 429 (1969).
[37] L. Van Hove, Nuclear Physics B 9, 331 (1969).
[38] B. Dey et al., Physical Review C 89, 055208 (2014).
[39] P. Eugenio, “Search for new forms of hadronic matter in photoproduction,” (2003), jLAB-E04-005.
[40] D. Ebert, R. N. Faustov, and V. O. Galkin, Physical Review D 79, 114029 (2009).
[41] H.-M. Chan et al., Nuovo Cimento LI, 696 (1967).
[42] P. Hoyer and J. Kwieciński, Nuclear Physics B 60, 26 (1973).