Strong field effects on binary systems in Einstein–aether theory

Brendan Z. Foster

Institute for Theoretical Physics, Utrecht University,
Leuvenlaan 4, NL-3584 CE Utrecht, The Netherlands

(Dated: September 23, 2008)

Abstract

“Einstein–aether” theory is a generally covariant theory of gravity containing a dynamical preferred frame. This article continues an examination of effects on the motion of binary pulsar systems in this theory, by incorporating effects due to strong fields in the vicinity of neutron star pulsars. These effects are included through an effective approach, by treating the compact bodies as point particles with nonstandard, velocity dependent interactions parametrized by dimensionless “sensitivities”. Effective post-Newtonian equations of motion for the bodies and the radiation damping rate are determined. More work is needed to calculate values of the sensitivities for a given fluid source; therefore, precise constraints on the theory’s coupling constants cannot yet be stated. It is shown, however, that strong field effects will be negligible given current observational uncertainties if the dimensionless couplings are less than roughly 0.01 and two conditions that match the PPN parameters to those of pure general relativity are imposed. In this case, weak field results suffice. There then exists a one-parameter family of Einstein–aether theories with “small-enough” couplings that passes all current observational tests. No conclusion can be reached for larger couplings until the sensitivities for a given source can be calculated.

PACS numbers: 04.50.+h, 04.30.Db, 04.25.Nx, 04.80.Cc

B.Z.Foster@phys.uu.nl
I. INTRODUCTION

This article examines the motion of stellar systems in “Einstein–aether” theory—an alternative theory of gravity that permits breaking of Lorentz symmetry through a dynamical preferred frame. The general theory contains four dimensionless couplings whose values can be constrained by comparing the predictions of the theory with observations—in particular, observations of binary pulsar systems. It will be demonstrated that all current tests will be passed by a one-parameter family whose couplings are “small-enough”—that is, on the order of 0.01 or less. Identifying whether there is a viable extension of this family to large coupling values requires additional work beyond this article.

The question of whether the physical world is exactly Lorentz invariant has received increasing attention in recent years. This interest is sourced largely by hints of Lorentz violation in popular candidates for theories of quantum gravity—for instance, string theory [1], loop quantum gravity [2], and noncommutative field theory [3]. More broadly, challenging the rule of Lorentz symmetry means challenging the fundamentals of all of modern physics, and doing that is just plain exciting.

The review [4] discusses a wide variety of theoretical models that feature Lorentz-symmetry violating effects, and observational searches for violations. So far no conclusive sign of Lorentz variance has been identified, and very strong bounds exist on the size of couplings for Lorentz-violating effects in standard model extensions [4, 5]. The effects of Lorentz violation in a gravitational context, however, are not covered by these bounds.

Einstein–aether theory—or “ae-theory” for short—is a classical metric theory of gravity that contains an additional dynamical vector field. The vector field “aether” is constrained to be timelike everywhere and of fixed norm. The aether can be thought of as a remnant of unknown, Planck-scale, Lorentz-violating physics. It defines a preferred frame, while its status as a dynamical field preserves diffeomorphism invariance. The fixed norm, which can always be scaled to unity, ensures that the aether picks out just a spacetime direction and removes instabilities in the unconstrained theory [6].

Much of past work on ae-theory has focussed on placing observational bounds on the values of the four free parameters $c_n$ appearing in the ae-theory action, Eqn. (7). Constraints have been derived from the rate of primordial nucleosynthesis [7], the rate of Čerenkov radiation [6], the requirements of stability and energy positivity of linearized wave modes [8],
and the parameterized post-Newtonian (PN) form of the theory [8, 9, 10]. A summary of these constraints was presented in [8], where it was shown that they are met by a large two-parameter subset of the original four-parameter class of theories.

Additional constraints on the $c_n$ come from observations of binary pulsar systems. Study of the predictions of ae-theory for binary pulsars was begun in [11]. There, an expression for the rate of radiation damping in $N$-body systems was derived to lowest non-trivial PN order and neglecting effects due to strong fields in the vicinity of the bodies. It was shown that a one-parameter subset of the two-parameter family allowed by the collected constraints discussed in [8] would pass tests from binary pulsar systems if there were justification for ignoring strong field effects. That neglect is dangerous, though, since the fields inside neutron star pulsars should be very strong. Justification requires an unclear assumption on the values of the $c_n$.

In this article, I will incorporate strong field effects on binary pulsar systems, calculating the PN equations of motion and the rate of radiation damping of a system of strongly self-gravitating bodies. The effects will be handled via an effective approach in which the compact bodies are treated as point particles whose action contains nonstandard couplings that depend on the velocity of the particles in the preferred frame. The effective approach to $N$-body dynamics in relativistic gravity theories has previously been employed in pure general relativity (GR) and other alternative theories; see for example [12, 13, 14, 15, 16, 17]. The new interactions are parametrized by dimensionless coefficients, or “sensitivities”, whose values can be calculated for a given stellar source by matching the effective theory onto the exact, perfect fluid theory. Prior work [11] reveals just the form of the “first” sensitivity at lowest order in the self-potential of a body.

The expressions obtained can be used to constrain the allowed class of ae-theories. Observations of binary pulsar systems allow for measurement of “post-Keplerian” (PK) parameters that describe perturbations of the binary’s Keplerian orbit due to relativistic effects. These parameters are mostly “quasi-static” ones, whose expressions can be derived from the non-radiative parts of the PN forms of the gravitational fields and the effective equations of motion for the bodies. In addition, there is the radiation damping rate, whose expression depends on the radiative parts of the fields. The ae-theory expressions for the PK parameters differ from those of pure GR in that they depend on the $c_n$, the sensitivities, and the center-of-mass velocity of the system of bodies. Stating precise constraints for general
$c_n$ values will require work beyond the scope of this article—specifically, what is needed is a method for dealing with dependence on the unmeasurable center-of-mass velocity and a calculation of the values of the sensitivities of a given source.

For the time being, a few comments can be made, which will be defended below. A crucial piece of information learned by comparing the weak field limit of the effective theory with the weak field limit of the perfect fluid theory [11] is that the sensitivities will be “small”. That is, they will be at least as small as $(G N m/d)^2$, where $m$ is the body’s mass and $d$ its size, times a $c_n$ dependent coefficient that must scale at least linearly with $c_n$ in the small $c_n$ limit. For neutron stars in pure GR, $(G N m/d) \sim (0.1 \sim 0.3)$; it is reasonable to expect something similar in ae-theory based on studies of stellar solutions [18] and the fact that ae-theory is generally “close” in the small-$c_n$ limit, to GR plus a non-dynamical vector field.

It then follows that bounds on the magnitude of violations of the strong equivalence principle [19] constrain the $c_n$ dependent factor to be less than $(0.01)(G N m/d)^{-2}$. It further follows that the strong field corrections fall below the level of current observational uncertainties when $|c_n| \lesssim 0.01$ and the two conditions that match the ae-theory PPN parameters to those of GR are imposed. Thus, weak field analysis [11] suffices for small enough $c_n$, and implies the existence of a one-parameter family of theories that passes all current tests from binary pulsar systems.

I will now present the strong field formulas. First, the effective particle action is constructed, and the exact field equations are defined in Sec. II. The PN expansions of the metric and aether fields are then given in Sec. III A and used to express the PN equations of motion for a binary system in Sec. III B. The rate of radiation damping is then determined in Sec. IV. Comments on dealing with center-of-mass velocity and sensitivity dependence are given in Sec. V along with the argument for the viability of the weakly coupled family of theories.

I follow the conventions of Wald [20]. In particular, I use units in which the flat space speed-of-light $c = 1$, and I use metric signature $(-,+,+,+)$. This signature is opposite to that employed in [11], but it is much more convenient for calculations involving a time-space decomposition. The ae-theory action is defined here in such a way as to permit easy comparison between [11] and this article. The following shorthand conventions for
combinations of the $c_n$ will be used:

\begin{align}
    c_{14} &= c_1 + c_4, \\
    c_{123} &= c_1 + c_2 + c_3, \\
    c_\pm &= c_1 \pm c_3.
\end{align}

When covariant equations are expanded in Minkowskian coordinates, the following conventions are observed. Spatial indices will be indicated by lowercase Latin letters from the middle of the alphabet: $i, j, k, \ldots$. One exception is when the coefficients $c_{1,2,3,4}$ are referred to collectively as $c_n$. Indices will be raised and lowered with the flat metric $\eta_{ab}$. Repeated spatial indices will be summed over, regardless of vertical position: $T_{ii} = \sum_{i=1,\ldots,3} T_{ii}$. Time indices will be indicated by a 0; time derivatives will be denoted by an overdot: $\dot{f} \equiv \partial_0 f$.

II. EFFECTIVE ACTION AND FIELD EQUATIONS

A. Particle action

The aim of this work is to treat within ae-theory a system of compact bodies that potentially possess strong internal gravitational fields. The complicated internal workings of the bodies will be dealt with via an effective approach that reproduces the bulk motion of the bodies and the fields far from them. Each body will be treated as a point particle with the composition dependent effects encapsulated in nonstandard couplings in the particle action.

The form of the effective action can be deduced from the following considerations. The one-particle action $S_A$ will have the rough form $S_A = -\tilde{m} \int dt \mathcal{O}$, where the integral is along the particle worldline parametrized by $t$, $\tilde{m}$ has dimensions of mass, and $\mathcal{O}$ is a sum of dimensionless local scalar quantities. The fundamental theory has only one dimensionful parameter $G$. For a first approximation, the spin of the body can be neglected. Derivative couplings in the particle theory are then suppressed by powers of $(d/R)$, where $d$ is the size of the underlying finite-sized body and $R$ is the radius of curvature of the background spacetime. In addition, $S_A$ presumably reduces to the standard free particle action if the particle is comoving with the local aether and must be invariant under reparametrization of the particle worldline.
These considerations imply the following one-particle action:

$$S_A = -\tilde{m}_A \int d\tau_A (1 + \sigma_A (u^a v_a + 1) + \frac{\sigma^\prime_A}{2} (u^a v_a + 1)^2 + \cdots), \quad (4)$$

where $A$ labels the body, $\tau_A$ is the proper time along the body’s curve, $v^a$ is the body’s unit four-velocity, and $u^a$ is the aether. The quantity $u^a v_a$ expressed in a PN expansion with the aether purely timelike at lowest order, is of order $v^2$, the square of the velocity of the body in the aether frame. By assumption, $v^2$ is first PN order (1PN). The 1PN corrections to Newtonian equations of motion will follow from the part of the action that is $m_A \times (2\text{PN})$, so only the terms in $S_A$ written above are needed for current purposes. For a system of $N$ particles, the action is given by the sum of $N$ copies of $S_A$.

This action can be thought of as a Taylor expansion of the standard worldline action, but with a mass that is a function of $\gamma \equiv -u^a v_a$:

$$S_A = -\int d\tau \tilde{m}_A[\gamma]. \quad (5)$$

The expansion is made about $\gamma = 1$. The parameters $\sigma, \sigma^\prime$ are then defined as

$$\sigma_A = -\frac{d \ln \tilde{m}_A}{d \ln \gamma} \big|_{\gamma=1}, \quad \sigma^\prime_A = \sigma_A + \sigma_A^2 + \tilde{\sigma}_A, \quad \tilde{\sigma}_A = \frac{d^2 \ln \tilde{m}_A}{d (\ln \gamma)^2} \big|_{\gamma=1}. \quad (6)$$

This form of $S_A$ suggests that that $\sigma_A, \tilde{\sigma}_A$ can be determined by considering asymptotic properties of perturbations of static stellar solutions.

### B. Field equations

The full action is the four-parameter ae-theory action $S$

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{|g|} \left( R - K^{ab}_{\, \, cd} \nabla_a u^c \nabla_b u^d + \lambda (u^a u^b g_{ab} + 1) \right), \quad (7)$$

plus the sum of $N$ copies of $S_A \quad (4)$, retaining only the terms explicitly written above. Here,

$$K^{ab}_{\, \, cd} = (c_1 g^{ab} g_{cd} + c_2 \delta^a_c \delta^b_d + c_3 \delta^a_d \delta^b_c - c_4 u^a u^b g_{cd}). \quad (8)$$

While the sign of the $c_4$ term looks awkward, it permits easier comparison with the results of $\text{[11]}$.

The field equations are then as follows. There are the Einstein equations

$$G_{ab} - S_{ab} = 8\pi G T_{ab}, \quad (9)$$
where

\[ G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}, \]  

\[ S_{ab} = \nabla c \left( K^c_{(a} u_{b)} - K^c_{(a} u_{b)} - K^c_{ab} u^c \right) \]

\[ + c_1 \left( \nabla_a u_c \nabla_b u^c - \nabla_a u_b \nabla^c u_c \right) + c_4 (u^c \nabla_c u_a) (u^d \nabla_d u_b) \]

\[ + \lambda u_a u_b + \frac{1}{2} g_{ab} (K^c_d \nabla_c u^d), \]  

with

\[ K^a_c = K^a_{cd} \nabla_b u^d, \]  

and \( T_{ab} \) is the particle stress tensor

\[ T_{ab} = \sum_A \tilde{m}_A \delta A \left[ A^1_A v^a_A v^b_A + 2 A^2_A u^a(v^b_A) \right], \]  

with a covariant delta-function

\[ \tilde{\delta}_A = \frac{\delta^3(\vec{x} - \vec{x}_A)}{v_A^0 \sqrt{|g|}}, \]  

and

\[ A^1_A = 1 + \sigma_A - \frac{\sigma'_A}{2} \left((u^c_A v^c_A)^2 - 1 \right), \]

\[ A^2_A = -\sigma_A - \sigma'_A (u^c_A v^c_A + 1). \]  

The aether field equation is

\[ \nabla_b K^{ba} = c_4 (u^c \nabla_c u_b) \nabla^a u^b + \lambda u^a + 8 \pi G \sigma^a, \]  

where

\[ \sigma^a = \sum_A \tilde{m}_A \delta A A^2_A v^a_A. \]  

Varying \( \lambda \) gives the constraint \( g_{ab} u^a u^b = -1 \). Eqn. (17) can be used to eliminate \( \lambda \), giving

\[ \lambda = -u^a \left( \nabla_b K^{ba} - c_4 (\nabla_a u^b)(u^c \nabla_c u_b) - 8 \pi G \sigma^a \right). \]  

The covariant equation of motion for a single particle has the form

\[ \nabla_b T^{ab}_A - \nabla_b \left((\sigma\xi)^a u^b\right) - (\sigma\xi)_b \nabla^a u^b = 0, \]  

where \( T^{ab}_A \) and \( (\sigma\xi)^a \) are the one-particle summands in (13) and (18). This can be written more explicitly as

\[ v^b_A \nabla_b (A^1_A v^a_A + A^2_A u^a) - A^2_A u^b \nabla^a u^b = 0. \]
III. POST-NEWTONIAN EXPANSION

A. Fields

The PN expansion of the fields can be determined by iteratively solving the field equations in a weak field, slow motion approximation [8, 21]. A background of a flat metric and constant aether is assumed, and a Lorentzian coordinate system with the time direction defined by the background aether is chosen. Following the procedures of [8] gives

\[
g_{00} = -1 + 2 \sum_A \frac{G_N m_A}{r_A} - 2 \sum_{A,B} \frac{G_N^2 m_A m_B}{r_A r_B} - 2 \sum_{A,B \neq A} \frac{G_N^2 m_A m_B}{r_A r_B AB} + 3 \sum_A \frac{G_N m_A}{r_A} v_A^2 (1 + \sigma_A),
\]

\[
g_{ij} = \left(1 + 2 \sum_A \frac{G_N m_A}{r_A}\right) \delta_{ij},
\]

\[
g_{0i} = \sum_A B_A^- \frac{G_N m_A}{r_A} v_A^i + \sum_A B_A^+ \frac{G_N m_A}{r_A^3} (v_A^j r_A^j) r_A^i,
\]

where \( r_A^i = x^i - x_A^i \), \( r_{AB}^i = x_A^i - x_B^i \),

\[
B_A^\pm = \frac{3}{2} + \frac{1}{4} \left( \alpha_1 - 2 \alpha_2 \right) \left( 1 + \frac{(2 - c_{14})}{(2c_+ - c_{14})} \sigma_A \right) - \frac{1}{4} \left( 8 + \alpha_1 \right) \left( 1 + \frac{c_-}{2c_1} \sigma_A \right),
\]

and

\[
G_N = \frac{2}{2 - c_{14}} G,
\]

\[
\alpha_1 = -\frac{8(c_3^2 + c_1 c_4)}{2c_1 - c_+ c_-},
\]

\[
\alpha_2 = \frac{\alpha_1}{2} - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_2 + c_3 + c_4)}{(2 - c_{14})c_{123}}.
\]

The numerical values of the PPN parameters \( \alpha_1 \) and \( \alpha_2 \) are constrained to be very small by weak field experiments, via analysis that allows for a possible lack of Lorentz symmetry in the underlying theory [21]. There are two independent pairs of conditions on the \( c_n \) that will set \( \alpha_1 \) and \( \alpha_2 \) to zero. One pair is

\[
c_2 = -\frac{2c_1^2 + c_1 c_3 - c_3^2}{3c_1}, \quad c_4 = -\frac{c_3^2}{c_1}.
\]

The other is \( c_+ = c_{14} = 0 \). With this second pair, the spin-1 and spin-0 wave speeds diverge (Sec. [IV]); also, the spin-0 linearized energy density vanishes while that of spin-1 remains
finite\textsuperscript{22}. Observational signatures of this behavior have not been worked out, and I will not consider these conditions further here. Hence, the first pair of conditions is assumed below whenever attention is restricted to the case of vanishing \(\alpha_1\) and \(\alpha_2\).

The aether to order of interest is

\[
\begin{align*}
    u^0 &= 1 + \sum_A \frac{G_N \tilde{m}_A}{r_A}, \\
    u^i &= \sum_A C_A^{-} \frac{G_N \tilde{m}_A}{r_A} (v_A)^i + \sum_A C_A^{+} \frac{G_N \tilde{m}_A}{r_A^3} (v_A^j r_A^j) r_A^i,
\end{align*}
\]

where

\[
C_A^{\pm} = \left(\frac{8 + \alpha_1}{8c_1}\right) \left(c_+ - (1 - c_+) \sigma_A\right) \pm \frac{(2 - c_{14})}{2} \left(\frac{(\alpha_2 - \frac{\alpha_1}{2})}{(c_1 + 2c_3 - c_4) + 1/c_{123}}\right) \sigma_A.
\]

The results of this section are equivalent to the weak field expressions obtained in \textsuperscript{8} when \(\sigma_A\) is set to zero.

**B. Post-Newtonian equations of motion**

The equations of motion for the system of compact bodies follow by expressing the exact result (20) in a PN expansion using the forms of the fields given above. The Newtonian order result can be used to define the effective two-body coupling \(G\) and the “active” gravitational mass \(m\):

\[
\dot{v}_A^i = \sum_{B \neq A} \frac{-G_N \tilde{m}_B}{(1 + \sigma_A) r_{AB}^3} r_{AB}^i \equiv \sum_{B \neq A} \frac{-G_{AB} m_B}{r_{AB}^3} r_{AB}^i,
\]

with the two-body coupling

\[
G_{AB} = \frac{G_N}{(1 + \sigma_A)(1 + \sigma_B)},
\]

and the active gravitational mass

\[
m_B = (1 + \sigma_B) \tilde{m}_B.
\]

These definitions arise by requiring that \(G_{AB} = G_{BA}\) and that \(m_B/\tilde{m}_B\) depend on just \(\sigma_B\).

Using the Newtonian result and continuing with the expansion leads to the 1PN equations of motion, expressed here just for the case of a binary system:
\[ \dot{v}_i = \frac{G m_2}{r^2} \hat{r}^i \left[ -1 + 4 \frac{m_2}{r} + \left( 1 - \frac{2}{1 + \sigma_2} D \right) \frac{m_1}{r} \right. \\
\left. \quad - \frac{1}{2} \left( 2 + 3 \sigma_1 + \frac{\sigma'_1}{1 + \sigma_1} \right) v_1^2 - \left( \frac{3}{2} (1 + \sigma_2) + (E - D) \right) v_2^2 \right. \\
\left. \quad - 2Dv_1^i v_2^j + 3(E - D)(v_2^j \hat{r}^j) \right]^2 \\
+ \frac{G m_2}{r^2} \left[ v_1^i \left( 4 + 3 \sigma_1 - \frac{\sigma'_1}{1 + \sigma_1} \right) - 3(1 + \sigma_1) v_2^i \hat{r}^j \right] \\
\left. + v_2^i \left( 2Dv_1^i \hat{r}^j - 2E v_2^i \hat{r}^j \right) \right], \quad (33)
\]

where \( G = G_{12} \), \( r^i = r_1^i - r_2^i \), and

\[ D = -\frac{1}{4} (8 + \alpha_1) \left( 1 + \frac{c_-}{2c_1} (\sigma_1 + \sigma_2) + \frac{(1 - c_-)}{2c_1} \sigma_1 \sigma_2 \right), \quad (34) \]

\[ E = -\frac{3}{2} - \frac{1}{4} (\alpha_1 - 2\alpha_2) \left( 1 + \frac{(2 - c_{14})}{(c_1 + 2c_3 - c_4)} (\sigma_1 + \sigma_2) + \frac{(2 - c_{14})}{2c_{123}} \sigma_1 \sigma_2 \right). \quad (35) \]

The expression for \( \dot{v}_2^i \) is obtained by exchanging all body-1 quantities and body-2 quantities, including the switch \( r^i \rightarrow -r^i \).

The “Einstein–Infeld–Hoffman” Lagrangian \[12\]—that is, the effective Lagrangian expressed purely in terms of particle quantities—can be determined by working backwards from the equations of motion. It is

\[ L = - (m_1 + m_2) + \frac{1}{2} (m_1 v_1^2 + m_2 v_2^2) \\
+ \frac{1}{8} \left( \left( 1 - \frac{\sigma'_1}{1 + \sigma_1} \right) v_1^4 + \left( 1 - \frac{\sigma'_2}{1 + \sigma_2} \right) v_2^4 \right) \\
+ \frac{G m_1 m_2}{r} \left[ 1 + \frac{3}{2} (1 + \sigma_1) v_1^2 + (1 + \sigma_2) v_2^2 \right] \\
- \frac{1}{2} \left( \frac{G m_1}{r} (1 + \sigma_2) + \frac{G m_2}{r} (1 + \sigma_1) \right) \\
+ D(v_1^i v_2^j) + E(v_2^i \hat{r}^j v_2^k \hat{r}^k) \right]. \quad (36) \]

This Lagrangian is not Lorentz invariant unless \( \sigma_A = \sigma'_A = 0 \). This follows from the analysis of Will \[23\] and the list of criteria therein. In particular, the action and the equations of motion depend on the velocity of the system’s center of mass in the aether frame.

**IV. RADIATION DAMPING RATE**

The radiation damping rate is the rate at which the particle system loses energy via gravity-aether radiation. This energy loss manifests as a change in the orbital period of a
binary system, equating the energy radiated to minus the change in mechanical energy. The expression for the rate in the effective particle theory can be determined by adapting the methods of [11], which were used to find the rate for a system of weakly self-gravitating perfect fluid bodies in ae-theory. It will be convenient to introduce the parameter $s_A$

$$s_A = \frac{\sigma_A}{1 + \sigma_A},$$

and to work with the active gravitational mass

$$m_A = (1 + \sigma_A)\bar{m}_A = \bar{m}_A/(1 - s_A).$$

### A. Wave forms

The method of [11] begins by assuming a background of a flat metric and constant aether, with a coordinate system with respect to which the background metric is the Minkowski metric $\eta_{ab}$ and the background aether is aligned with the time direction. The metric and aether perturbations are then decomposed into irreducible transverse and longitudinal pieces. The spatial vectors $u^i$ and $h_{0i}$ are written as:

$$h_{0i} = \gamma_i + \gamma_i, \quad u^i = \nu^i + \nu, i,$$

with $\gamma_{i,i} = \nu_{i,i} = 0$. The spatial metric $h_{ij}$ is decomposed into a transverse, trace-free tensor, a transverse vector, and two scalar quantities giving the transverse and longitudinal traces:

$$h_{ij} = \phi_{ij} + \frac{1}{2} P_{ij}[f] + 2\phi_{(i,j)} + \phi_{ij},$$

where

$$0 = \phi_{ij,j} = \phi_{jj} = \phi_{i,i},$$

and

$$P_{ij}[f] = \delta_{ij} f_{kk} - f_{ij};$$

hence, $P_{ij}[f,j] = 0$, and $h_{ii} = (f + \phi)_{ii}$. Further, define

$$F = f_{,jj}.$$
obvious extension of it, does not usefully simplify the ae-theory field equations. Instead, the following convenient conditions will be imposed:

\[ 0 = u^i_{,i} = h_{0i,i} = h_{i[j,k]i}, \]  

(44)

or equivalently,

\[ 0 = \nu = \gamma = \phi_i. \]  

(45)

Because \( \phi_i \) is transverse, these constitute just four conditions.

Following [11], the field equations can then be linearized and expressed in terms of the above variables, and sorted to obtain a set of wave equations with matter terms and nonlinear terms as sources. Having done this, the linear contributions can be seen by inspection to satisfy a conservation law. This fact implies the existence of a conserved source \( \tau^{ab} \)

\[ \tau^{ab} = T^{ab} - \sigma^a \delta^b_0 + \tilde{\tau}^{ab}, \]  

(46)

where \( T^{ab} \) and \( \sigma^a \) are as defined in Eqns. (13) and (18), and \( \tilde{\tau}^{ab} \) is constructed from nonlinear terms—its precise form will not be needed. The non-symmetric \( \tau^{ab} \) satisfies the conservation law with respect to the right-index only: \( \tau^{ab}_{,b} = 0 \). The corresponding conserved total energy \( E \) and momentum \( P^i \) to lowest PN order are

\[ E = \int d^3 x \tau^{00} = \sum_A \bar{m}_A = \sum_A (1 - s_A) m_A, \]  

(47)

\[ P^i = \int d^3 x \tau^{i0} = \sum_A m_A v^i_A. \]  

(48)

Conservation of \( P^i \) means that the system center-of-mass \( X^i \) defined via \( m_A \)

\[ X^i = \frac{\sum_A m_A x^i_A}{\sum_A m_A}, \]  

(49)

is unaccelerated to lowest order.

The field equations reduce to the following. For spin-2,

\[ \frac{1}{w^2_2} \ddot{\phi}_{ij} - \phi_{ij,kk} = 16\pi G T^{TT}_{ij}, \]  

(50)

where TT signifies the transverse, trace-free components, and

\[ w^2_2 = \frac{1}{1 - c^+_+}. \]  

(51)
For spin-1,
\[
\frac{1}{w_1^2} (\ddot{\nu} + \dot{\gamma}_i) = \frac{16\pi G}{2c_1 - c_+ c_-} (c_+ \tau_{i0} + (1 - c_+) \sigma^i)^T, \tag{52}
\]
\[
(c_+ \nu^j + \gamma_i)_{,kk} = -16\pi G \tau_{i0}^T, \tag{53}
\]
where T signifies the transverse components, and
\[
w_1^2 = \frac{2c_1 - c_+ c_-}{2(1 - c_+) c_14}. \tag{54}
\]
For the spin-0 variables, the constraint gives to linear order
\[
u \dot{u}^0 = 1 + \frac{1}{2} h_{00}. \tag{55}
\]
Non-linear corrections to this are of uninteresting order, as explained in more detail in [11].

The other equations are
\[
\frac{1}{w_0^2} \ddot{F} - F_{,kk} = \frac{16\pi G c_{14}}{2 - c_14} \left( \tau_{kk} - \frac{2 + 3c_2 + c_+}{c_{123}} \tau^L_{kk} + \frac{2}{c_{14}} \tau_{00} \right), \tag{56}
\]
\[
(F - c_{14} h_{00})_{,kk} = -16\pi G \tau_{00}, \tag{57}
\]
\[
(1 + c_2) \dot{\Phi}_{,i} + c_{123} \dddot{\phi}_{,kki} = -16\pi G \tau^L_{i0}, \tag{58}
\]
where L signifies the longitudinal component, and
\[
w_0^2 = \frac{(2 - c_{14}) c_{123}}{(2 + 3c_2 + c_+) (1 - c_+) c_{14}}. \tag{59}
\]

All these equations can be solved formally via Greens function methods, and the resulting integrals expanded in a far field, slow motion approximation. The expressions can be further simplified using the conservation of $\tau^{ab}$. A result that holds within the approximation scheme is that for a field $\psi$ satisfying a wave equation with speed $w$ evaluated at field point $x^i \equiv |x| \hat{n}^i$ with only outgoing waves,
\[
w \psi_{,i} = -\dot{\psi} \hat{n}^i. \tag{60}
\]
Also, differentially transverse becomes equivalent to geometrically transverse to $\hat{n}^i$.

The results to lowest PN order and ignoring static contributions are as follows. For spin-2,
\[
\phi_{ij} = \frac{2G}{|x|} \ddot{Q}^{TT}_{ij}, \tag{61}
\]
where the right-hand side is evaluated at time \((t - |x|/w_2)\) and the quadrupole moment \(Q_{ij}\) is the trace-free part of the system’s second mass moment \(I_{ij}\):

\[
I_{ij} = \sum_A m_A x_A^i x_A^j. \tag{62}
\]

For spin-1 variables,

\[
\nu^i = \frac{-2G}{|x|} \frac{1}{2c_1 - c_+ c_-} \left( \frac{\hat{n}^i}{w_1} \left( \frac{c_+}{1 - c_+} \tilde{Q}_{ij} + \tilde{Q}_{ij} \right) - 2\Sigma^i \right)^T, \tag{63}
\]

\[
\gamma_i = -c_+ \nu^i, \tag{64}
\]

where the right-hand side of the first equation is evaluated at time \((t - |x|/w_1)\), \(Q_{ij}\) is the trace-free part of the rescaled mass moment \(I_{ij}\):

\[
I_{ij} = \sum_A s_A m_A x_A^i x_A^j, \tag{65}
\]

and

\[
\Sigma^i = -\sum_A s_A m_A v_A^i. \tag{66}
\]

For spin-0 variables,

\[
F = \frac{-2G}{|x|} \frac{c_{14}}{2 - c_{14}} \left[\left( \frac{2\alpha_2 - \alpha_1}{2(2c_+ - c_{14})} + 3 \right) \tilde{Q}_{ij} + \frac{2}{w_0^2 c_{14}} \tilde{Q}_{ij} \right] \hat{n}^i \hat{n}^j + \frac{2\alpha_2 - \alpha_1}{2(2c_+ - c_{14})} \tilde{J} + \frac{2}{3w_0^2 c_{14}} \tilde{J} - \frac{4}{w_0 c_{14}} \hat{n}^i \Sigma^i, \tag{67}
\]

\[
h_{00} = \frac{1}{c_{14}} F, \tag{68}
\]

\[
\dot{\phi}_i = -\frac{1 + c_2}{c_{123}} \tilde{J}_i, \tag{69}
\]

where the right-hand side of the first equation is evaluated at time \((t - |x|/w_0)\), and \(I = I_{ii}\), \(\mathcal{I} = \mathcal{I}_{ii}\).

At this point, the expected smallness of the sensitivities, mentioned in the introduction, can be explained. One should take the weak field limit \((s_A \to \text{“small”})\) of the above wave forms and compare them with the perfect-fluid theory wave forms determined in [11]. The only \(s_A\)-dependence that remains at potentially leading order is in \(\Sigma_i\). Comparing (66) with Eqn. (85) of [11] indicates that in the small \(s_A\) limit,

\[
s_A = (\alpha_1 - \frac{2}{3} \alpha_2) \frac{\Omega_A}{m_A} + O(\frac{G N m}{d})^2, \tag{70}
\]
where $\Omega_A$ is the binding energy of the body: $\Omega/m \sim (G_N m/d)$, where $d$ is the characteristic size of the body. The implication is that when $\alpha_1 = \alpha_2 = 0$, $s$ must scale as $(G_N m/d)^2$, times a $c_n$ dependent coefficient. This coefficient should scale at least linearly in $c_n$, in the $c_n \to 0$ limit, to ensure finiteness of the perturbations.

B. Damping rate expression

For the next step, the wave forms are inserted into an expression for the rate of change of energy $\dot{\mathcal{E}}$. This expression can be derived via the Noether charge method of Iyer and Wald [24, 25], using the ae-theory Noether charges derived in [26], with the result:

$$\dot{\mathcal{E}} = -\frac{1}{16\pi G} \int d\Omega R^2 \left( \frac{1}{2w_2} \phi_{ij} \phi_{ij} + \frac{(2c_1 - c_c c_-)(1 - c_+)}{w_1} \phi^i \psi^i + \frac{2 - c_{14}}{4w_0 c_{14}} \dot{F} \dot{F} \right) + \dot{O}, \quad (71)$$

where $\dot{O}$ is a total time-derivative that will be argued away in a moment.

Using the above results for the wave forms, performing the angular integral, and ignoring $\dot{O}$ gives

$$\dot{\mathcal{E}} = -G_N \left( \mathcal{A}_1 \ddot{Q}_{ij} \ddot{Q}_{ij} + \mathcal{A}_2 \ddot{Q}_{ij} \ddot{Q}_{ij} + \mathcal{A}_3 \ddot{Q}_{ij} \ddot{Q}_{ij} + \mathcal{B}_1 \ddot{I} \ddot{I} + \mathcal{B}_2 \ddot{I} \ddot{I} + \mathcal{B}_3 \ddot{I} \ddot{I} + C \Sigma^i \Sigma^i \right), \quad (72)$$

where

$$\mathcal{A}_1 = \left( 1 - \frac{c_{14}}{2} \right) \left( \frac{1}{w_2} + \frac{2c_{14} c_+}{(2c_1 - c_c c_-)^2} \frac{1}{w_1} + \frac{c_{14}}{6(2 - c_{14})} \left( 3 + \frac{2\alpha_2 - \alpha_1}{2(2c_+ - c_{14})} \right) \frac{1}{w_0} \right), \quad (73)$$

$$\mathcal{A}_2 = \left( \frac{2 - c_{14}}{2c_1 - c_c c_-} \frac{1}{w_1^3} + \frac{c_{14}}{6(2c_+ - c_{14})} \frac{1}{w_0^3} \right), \quad (74)$$

$$\mathcal{A}_3 = \frac{1}{c_{14}} \left( \frac{2 - c_{14}}{4} \frac{1}{w_1^3} - \frac{1}{3} \frac{1}{w_0^3} \right), \quad (75)$$

$$\mathcal{B}_1 = \frac{c_{14}}{72} \left( \frac{2\alpha_2 - \alpha_1}{2(2c_+ - c_{14})} \right)^2 \frac{1}{w_0}, \quad (76)$$

$$\mathcal{B}_2 = \frac{2\alpha_2 - \alpha_1}{12(2c_+ - c_{14})} \frac{1}{w_0^3}, \quad (77)$$

$$\mathcal{B}_3 = \frac{1}{6c_{14}} \frac{1}{w_0}, \quad (78)$$

$$\mathcal{C} = \frac{2}{3c_{14}} \left( \frac{2 - c_{14}}{w_1^3} + \frac{1}{w_0^3} \right). \quad (79)$$

The coefficients $\mathcal{A}_1$, $\mathcal{B}_1$, and $\mathcal{C}$ are respectively identical to $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ of [11]. Taking the weak field limit corresponds to retaining only the $\mathcal{A}_1$, $\mathcal{B}_1$, and $\mathcal{C}$ terms, and invoking
the relation (70) for \( s_A \) in the \( C \) term. In the case that \( \alpha_1 = \alpha_2 = 0 \), \( B_1 \) vanishes, as does \( s_A \) in the weak field limit. The weak field damping rate in this case then contains only a quadrupole contribution and is identical to the GR rate when \( A_1 = 1 \). This remaining curve of \( c_n \) values intersects the range of values allowed by collected constraints considered in [8], as illustrated in Figure 1. Thus, this curve gives a one-parameter family of viable ae-theories if the weak field results alone are sufficient.

![Figure 1](image-url)

**FIG. 1:** Class of allowed ae-theories, if strong field effects in binary pulsar systems can be ignored. The four-dimensional \( c_n \) space has been restricted to the \((c_+, c_-)\) plane by setting the PPN parameters \( \alpha_1 \) and \( \alpha_2 \) to zero via the conditions (27). The shaded region is the region allowed by primordial nucleosynthesis, Čerenkov radiation, linearized stability and energy positivity, and PPN constraints, demarcated in [8]. The dashed curve is the curve along which binary pulsar tests will be satisfied, assuming ae-theory weak field expressions. Specifically, it is the curve along which \( A_1 = 1 \) in the \( \alpha_1 = \alpha_2 = 0 \) case, so that the damping rate (72) is identical to the quadrupole formula of general relativity. Along both this curve and the boundary of the allowed region, \( c_- \to \infty \) as \( c_+ \to 1 \). The curve remains within the allowed region for all \( c_+ \) between 0 and 1. As explained in Sec. IV strong field effects may lead to system dependent corrections to the binary pulsar curve for large \( c_n \); however, all such curves will coincide with the weak field curve for \(|c_n| \lesssim 0.01 \) given current observational uncertainties.
To simplify the expression (72), it is crucial to note that the damping rate is calculated to lowest PN order using the Newtonian results for the motion of the system. Thus, the system’s motion can be decomposed into a uniform center-of-mass motion—recall the conservation of $\mathbf{P}$—and a fixed Keplerian orbit in the center-of-mass frame. Since the motion is steady-state, the damping rate must have no secular time dependence. This observation implies that secular terms in $\dot{\mathcal{E}}$ arising from $\mathcal{I}_{ij}$, see below, must cancel with secular terms in $\dot{\mathcal{O}}$. In addition, the non-secular portion of $\dot{\mathcal{O}}$ will average to zero when a time average of the damping rate over an orbital period is taken, since it is a total time derivative. Thus, $\dot{\mathcal{O}}$ can be discarded.

Hence restricting attention to a binary system, and taking a time average over an orbital period, the expression reduces as follows. First, define the quantities

$$m = m_1 + m_2, \quad \mu_A = m_A/m, \quad \mu = m_1 m_2/m,$$  \hspace{1cm} (80)

and the vectors

$$r^i = x_1^i - x_2^i, \quad v^i = \dot{r}^i, \quad \text{ (81)}$$

$$X^i = \mu_1 x_1^i + \mu_2 x_2^i, \quad V^i = \dot{X}^i. \quad \text{ (82)}$$

To Newtonian order, $\dot{v}^i = -\left(Gm/r^2\right)\hat{r}^i$, and $\dot{V}^i = 0$. $I_{ij}$ can be diagonalized

$$I_{ij} = \mu r^i r^j + mX^i X^j,$$  \hspace{1cm} (83)

so that

$$\ddot{I}_{ij} = \frac{2G\mu m}{r^2}(3\hat{r}^i\hat{r}^j\hat{r} - 4v^{(i}\hat{r}^{j)}) \quad \text{ (84)}$$

As for $\mathcal{I}_{ij}$,

$$\mathcal{I}_{ij} = \mu(s_1 \mu_2 + s_2 \mu_1)r^i r^j + m(s_1 \mu_1 + s_2 \mu_2)X^i X^j + 2\mu(s_1 - s_2)r^{(i}\dot{X}^{j)} \quad \text{ (85)}$$

and

$$\ddot{\mathcal{I}}_{ij} = S\ddot{I}_{ij} - 6V^{(i}\dot{\mathcal{S}}^{j)} + 2\mu(s_1 - s_2)\dot{\hat{r}}^{(i}\dot{X}^{j)}, \quad \text{ (86)}$$

where

$$S = s_1 \mu_2 + s_2 \mu_1, \quad \text{ (87)}$$

and

$$\dot{\mathcal{S}}_i = (s_1 - s_2)\frac{G\mu m}{r^3} r^i. \quad \text{ (88)}$$
Terms in $\mathcal{T}_{ij}$ with $X^i$ dependence are secular; following the discussion above, they can be discarded.

Substituting into Eqn. (72) and imposing the time average gives the final expression

$$
\dot{E} = -G_N \left\langle \left( \frac{G \mu m}{r^2} \right)^2 \right. 
\times \left[ \frac{8}{15} (A_1 + S A_2 + S^2 A_3)(12v^2 - 11\dot{r}^2) 
+ 4(B_1 + S B_2 + S^2 B_3)\dot{r}^2 
+ (s_1 - s_2)^2 \left( C + \frac{6}{5} (3A_3 V^2 + (A_3 + 30 B_3)(V^i \dot{r}^i)^2) \right) 
+ (s_1 - s_2) \left( \frac{8}{5} (A_2 + 2S A_3)(3v^i V^i - 2V^i \dot{r}^i v^j \dot{r}^j) + 12(B_2 + 2S B_3)V^i \dot{r}^i v^j \dot{r}^j \right) \right],
$$

(89)

where the angular brackets denote the time average.

V. OBSERVATIONAL CONSTRAINTS

A. Center-of-mass velocity dependence

While the aether frame center-of-mass velocity $V^i$ of a binary system is not directly measurable, dependence of a binary system's motion on $V^i$ should actually be beneficial for constraining the theory. This is because constraints arise from a failure to observe $V^i$ dependent effects. It may be possible to formulate such constraints without having to determine the physical frame, as in the manner of bounds on the PPN parameter $\alpha_2$. The presence of alignment between the sun’s spin axis and the ecliptic plane signals the absence of frame dependent effects, and leads to a strong bound of $|\alpha_2| < 4 \times 10^{-7}$ [21]. This argument does require the assumption that the component of the preferred frame in the sun’s rest frame is not conveniently aligned with the sun’s spin axis; such an assumption may generally be required for similar arguments. For example, $V^i$ dependence should cause a binary’s orbital plane to precess, but not if $V^i$ happens to be normal to the plane.

An assumption on the order of magnitude of the norm $V$ is necessary to justify the use of just the leading PN order expressions for the PK parameters when applied to observed binary systems. The validity of the 1PN expressions depends on whether corrections of relative order $v^2$ and $(V^4/v^2)$ are smaller than observational uncertainties. Terms of order $v^2$ are negligible for all observed systems, for now, although the “double pulsar” [27] is
pushing this limit. For all but the double pulsar, \(v^2 \sim 10^{-6}\), and uncertainties are at least a thousand times this \([19]\). The double pulsar PSR J0737-3039A/B is the so-far unique binary containing two pulsars. The orbital velocity is high, \(v^2 \sim 10^{-5}\), and the presence of two pulsars happens to make measurement of system parameters much easier and thus more precise—the smallest relative uncertainty is \(10^{-4}\) on the rate of periastron advance. The \(v^2\) corrections are therefore small enough for now, but it is expected that precision will increase to probe the next PN order within the next 10-20 years \([19]\).

The \(V^i\) dependent terms must feature \(c_n\) dependent factors, since it is known that there is no center-of-mass velocity dependence at next PN order in pure GR \([21]\). Ignoring those factors for the moment, validity of leading PN order for the double pulsar requires that \((V^4/v^2) \lesssim 10^{-4}\), giving \(V^2 \lesssim 10^{-4.5}\), or \((V^2/v^2) \lesssim 10^{0.5} \approx 3\). For other systems, given uncertainties ranging from \((10^{-1} \sim 10^{-3})\), the conditions are \((V^4/v^2) \lesssim (10^{-1} \sim 10^{-3})\), giving \(V^2 \lesssim (10^{-3.5} \sim 10^{-2.5})\), or \((V^2/v^2) \lesssim (10^{2.5} \sim 10^{1.5}) \approx (300 \sim 30)\). Presumably, the \(c_n\) dependent factor actually goes to zero as some positive power of \(c_n\), so \(V\) can be larger in the small \(c_n\) limit. A reasonable first guess for the aether frame is the rest frame of the cosmic microwave background. A typical velocity for compact objects in our galaxy in this frame is \(V^2 \sim 10^{-6}\), so the restriction on \(V\) is met.

### B. Constraints in the small coupling regime

A formula for the sensitivities for a given source should be obtainable by comparing the strong field results of this article with analogous results in the exact perfect fluid theory. Higher order terms in the exact theory must be calculated, though, since the leading order results of \([11]\) only give the \(O(G_N m/d)\) part of \(s\) expressed in \((70)\). The calculation can be done in the case of a single body that is static except for a constant aether frame velocity, by, for example, continuing the iterative procedure used to determine the PPN parameters \([8, 23]\). The process may be lengthy, but straightforward.

I have shown that the sensitivity of a body will scale with the body’s self-potential like \(\beta[c_n](G_N m/d)^2\), where \(\beta\) is some \(c_n\)-dependent coefficient that scales at least as fast as \(c_n\) in the small \(c_n\) limit. Even in the absence of a formula for the sensitivities and precise knowledge of center-of-mass velocities, two useful comments can be derived. First, a constraint can be roughly stated: \(|\beta| \lesssim (0.1 \sim 1)\). Second, there exists a one-parameter family of theories that
passes all current constraints, obtained by restricting to \( c_n \) with magnitude less than roughly 0.1 and imposing the two PPN conditions, the one weak field damping rate condition, and the collected non-binary conditions.

The condition that \(|\beta| \lesssim (0.1 \sim 1)\) follows from constraints \([19]\) on the magnitude of violations of the strong equivalence principle—that is, that a body’s acceleration is independent of its composition. A violation would lead to a polarization of the orbit of pulsar systems due to unequal acceleration of the binary bodies in the gravitational field of the galaxy. The observed lack of polarization in neutron star–white dwarf systems leads to a constraint that can be stated here as \( s < 0.01 \), where here \( s \) is the sensitivity of the neutron star in the considered pulsars. Assuming that \((G_N m/d) \approx (0.1 \sim 0.3)\) for the pulsar, as it is in GR, the constraint on the size of \( \beta \) arises. It is possible that when the weak field conditions are imposed, \( \beta \) will automatically satisfy the above inequality; certainly it will in the small \( c_n \) regime when \(|c_n| < 0.01\).

The statement, that current tests will be satisfied if the weak field conditions are imposed and the remaining degree of \( c_n \) freedom satisfies \(|c_n| \lesssim 0.01\), can be derived by considering the battery of binary pulsar tests. First, consider tests that probe only the quasi-static PK parameters—that is, all but the damping rate. The tightest quasi-static test comes from the double pulsar \([27]\). The relative size of the strong field corrections to the weak field expressions will be \( O(s_A) \), while the prediction of GR has been confirmed to within a relative observational uncertainty of 0.05%. Requiring \( s \lesssim 10^{-3} \) and assuming that \((G_N m/d) \approx (0.1 \sim 0.3)\) for the pulsars, the condition \(|c_n| \lesssim 0.01\) arises. Given this and the two conditions that set the PPN parameters \( \alpha_1 \) and \( \alpha_2 \) to zero, all current quasi-static tests will be passed.

Tests that incorporate the damping rate will also be satisfied by the small-\( c_n \) condition and the weak field conditions. I note first that for systems in which the damping rate is probed, uncertainty on its measurement dominates uncertainties on quasi-static parameters \([19, 21]\). Thus, it is conventional to use the measurements of the quasi-static parameters to solve for the mass values of the binary bodies. When \( \alpha_1 = \alpha_2 = 0 \), and \(|c_n| \lesssim 0.01\), so that the expressions for the quasi-static parameters are close to those of GR, the predicted mass values will also be close.

Now, the dipole contribution to \( \dot{\mathcal{E}} \) can be significant in asymmetric systems where the
sensitivity of one body is much larger than the other. The dipole contribution is

\[ \dot{E}_{\text{Dipole}} = -G_N \langle \left( \frac{G \mu m}{r^2} \right)^2 \rangle C (s_1 - s_2)^2, \]  

which is of order \((Cs^2/10v^2)\) compared to the quadrupole and monopole contribution, where \(s\) is the dominant sensitivity. An applicable system is a neutron star–white dwarf binary, since for a typical white dwarf, \((G_N m/d) \sim 10^{-3}\). Constraints have been derived \([19]\) on the magnitude of dipole radiation from neutron star–white dwarf binaries PSR B0655+64 and PSR J1012+5307 by requiring that the dipole radiation rate be no larger than the observed rate. The analysis applied here leads to the condition \(Cs^2 \lesssim 10^{-4}\), where \(s\) is the sensitivity of the neutron star. In the small \(c_n\) regime, this translates again to the condition \(|c_n| \lesssim 0.01\).

For double neutron star binaries, the dipole rate is further suppressed by the similarities of the sensitivities, and the quadrupole and monopole contributions become dominant. The tightest test involving radiation is associated with the Hulse–Taylor binary PSR1913+16, with a relative uncertainty of 0.2% \([19, 21]\). In the small \(c_n\) regime, the condition \(A_1 = 1\) matches the leading order damping rate to that of GR. The strong field corrections are of relative order \(s\); to be smaller than the uncertainty again requires \(|c_n| \lesssim 0.01\).

This upper limit on \(|c_n|\) will decrease as observational uncertainties decrease. The most promising candidate for lowering the limit is the double pulsar: 2PN-order and spin-dependent effects should be observable within the next ten or twenty years \([27]\). Another type of system, yet undetected, for which high levels of accuracy could be obtained is a neutron star–black hole binary, as the structureless black hole would decrease noise due to finite-size effects and mass transfer between the bodies.

For \(|c_n| > 0.01\), strong field contributions to the expressions for the PK parameters may be significant. Those contributions for a given source cannot yet be calculated, so the theory cannot be checked against observations. Thus, there is no conclusion yet on the viability of large \(c_n\) values. If it were possible to calculate precise predictions for a given binary system, then each observed system would imply an extension from small to large \(c_n\) of the curve of allowed values. The only physically viable values would be those for which the curves for all observed systems overlapped within error.
Acknowledgments

I wish to thank Alessandra Buonanno, Cole Miller, Ira Rothstein, Clifford Will, and especially Ted Jacobson for fruitful discussions. This research was supported in part by the NSF under grant PHY-0601800 at the University of Maryland.

[1] V. A. Kostelecky and S. Samuel, Phys. Lett. B207, 169 (1988).
[2] R. Gambini and J. Pullin, Phys. Rev. D59, 124021 (1999), gr-qc/9809038.
[3] J. L. Hewett, F. J. Petriello, and T. G. Rizzo, Phys. Rev. D64, 075012 (2001), hep-ph/0010354.
[4] D. Mattingly, Living Rev. Rel. 8, 5 (2005), gr-qc/0502097.
[5] R. Bluhm, Overview of the sme: Implications and phenomenology of Lorentz violation, talk given at 339th WE Heraeus Seminar, Potsdam, Germany, 13-18 Feb 2005, hep-ph/0506054.
[6] J. W. Elliott, G. D. Moore, and H. Stoica, JHEP 08, 066 (2005), hep-ph/0505211.
[7] S. M. Carroll and E. A. Lim, Phys. Rev. D70, 123525 (2004), hep-th/0407149.
[8] B. Z. Foster and T. Jacobson, Phys. Rev. D73, 064015 (2006), gr-qc/0509083.
[9] C. Eling and T. Jacobson, Phys. Rev. D69, 064005 (2004), gr-qc/0310044.
[10] M. L. Graesser, A. Jenkins, and M. B. Wise, Phys. Lett. B613, 5 (2005), hep-th/0501223.
[11] B. Z. Foster, Phys. Rev. D73, 104012 (2006); Erratum: Phys. Rev. D75, 129904 (E) (2007); Beware typos, see latest version, gr-qc/0602004.
[12] A. Einstein, L. Infeld, and B. Hoffmann, Annals Math. 39, 65 (1938).
[13] D. Eardley, Astrophys. J. 196, L59 (1975).
[14] C. Will and D. Eardley, Astrophys. J. 212, L91 (1977).
[15] T. Damour and G. Esposito-Farese, Phys. Rev. D53, 5541 (1996), gr-qc/9506063.
[16] W. D. Goldberger and I. Z. Rothstein, Phys. Rev. D73, 104029 (2006), hep-th/0409156.
[17] R. A. Porto and I. Z. Rothstein, Phys. Rev. Lett. 97, 021101 (2006), gr-qc/0604099.
[18] C. Eling and T. Jacobson, Class. Quant. Grav. 23, 5625 (2006), gr-qc/0603058.
[19] I. H. Stairs, Living Rev. Rel. 6, 5 (2003), astro-ph/0307536.
[20] R. M. Wald, General Relativity (Univ. Pr., Chicago, 1984).
[21] C. M. Will, Living Rev. Rel. 4, 4 (2001), gr-qc/0103036.
[22] C. Eling, Phys. Rev. D73, 084026 (2006), gr-qc/0507059.
[23] C. M. Will, *Theory and Experiment in Gravitational Physics* (Univ. Pr., Cambridge, UK, 1993).

[24] R. M. Wald, Phys. Rev. **D48**, R3427 (1993), gr-qc/9307038.

[25] V. Iyer and R. M. Wald, Phys. Rev. **D50**, 846 (1994), gr-qc/9403028.

[26] B. Z. Foster, Phys. Rev. **D73**, 024005 (2006), gr-qc/0509121.

[27] M. Kramer et al., Science **314**, 97 (2006), astro-ph/0609417.