GENERALIZED HUBBLE LAW,
VIOLATION OF THE COSMOLOGICAL PRINCIPLE AND
SUPERNOVAE

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Abstract. The acceleration of the cosmic expansion has been discovered as a consequence of redshift Supernovae data. In the usual way, this cosmic acceleration is explained by the presence of a positive cosmological constant or quantum vacuum energy, in the background of standard Friedmann models. Recently, looking for an alternative explanation, I have considered an inhomogeneous barotropic spherically symmetric spacetime. Obviously, in this inhomogeneous model the philosophical cosmological principle is not verified. Within this framework, the kinematical acceleration of the cosmic fluid or, equivalently, the inhomogeneity of matter, is just the responsible of the SNe Ia measured cosmic acceleration. Moreover, this model gives rise to a generalized Hubble law with two anisotropic terms (dipole acceleration and quadrupole shear), besides the expansion one. The dipole term of this generalized Hubble law could explain, in a cosmological setting, the observed large scale flow of matter, without to have recourse to peculiar velocity-type newtonian models which assume a Doppler dipole.

1. Introduction

Recently [1], I have considered a specific inhomogeneous cosmological model, which could explain, in an alternative way, by the presence of a kinematic acceleration generated by a negative gradient of pressure (or mass-energy), the present negative deceleration parameter, which appears to be a result of high redshift Supernovae data [2]. In the current way, these data are explained by the presence of a positive cosmological constant or vacuum
energy or quintessence, in the background of standard Friedmann (FLRW) models with perfect fluid matter.

In this specific inhomogeneous model, which I will also consider in this work, the cosmic matter is described by a barotropic (B) perfect fluid and geometrically has spherical (S) symmetry (hereafter BS). The comoving perfect fluid matter congruence, in this vorticity-free BS inhomogeneous model, has expansion, kinematic acceleration and shear, at difference with the standard FLRW models, where it has expansion only.

2. The BS model can explain cosmic acceleration

In the BS model, the isotropy group is 1-dim, whereas the isometry group is a 3-dim $G_3$, which is acting multiply transitively on spacelike 2-dim surfaces orthogonal to a preferred direction (hereafter $e_1$). These 2-dim surfaces orbits, orthogonal both to the 4-velocity $u^a$ of the congruence and to $e_1$, are spheres.

Also, in the BS model the vorticity is zero. This implies both the existence of a global cosmic time and of 3-dim global spacelike hypersurfaces orthogonal to the fluid congruence.

Thus, the metric in comoving coordinates has the expression

$$ds^2 = -N^2(r, t) dt^2 + B^2(r, t) dr^2 + R^2(r, t) d\Omega^2.$$  \hspace{1cm} (1)

where $d\Omega^2$ is the spherical line element and the coefficient $N(r, t)$ is a lapse function which relates global cosmic and local proper times. In the BS spacetime exists a preferred central worldline, i.e. the spacelike hypersurfaces have a centre at $r = 0$, where the isotropy group is 3-dim, and a preferred radial direction $e_1$ at each point, associated with the direction of the only non-null component, $A$, of the kinematic acceleration of the matter fluid elements. This kinematic acceleration satisfies the spatially contracted equations of motion

$$p' + (\mu + p) A = 0,$$  \hspace{1cm} (2)

where a prime denotes the derivative along the preferred radial direction with unit vector field $e_1$. We see from the last formula that the kinematic acceleration, which opposes to the gravitational attraction towards the centre, is generated by a negative gradient of pressure or, equivalently, due to the supposed barotropic equation of state $p(r, t) = p(\mu(r, t))$, by a negative gradient of mass-energy density $\mu(r, t)$.

Note (see [1]), that the presence in the metric of the coefficient $N(r, t)$, gives rise to a non null kinematic acceleration and also to a new term in the expression of the deceleration parameter $q$,

$$q_0 = \frac{1}{2} \Omega_0 - \mathbb{I}_0,$$  \hspace{1cm} (3)
where $\Omega_0$ is the present matter density in units of the critical density and $I I_0$ was called in [1] the inhomogeneity parameter.

This additional inhomogeneity term $I I_0$ is positive and hence the deceleration parameter $q_0$ can be negative at present cosmic time, so one has an alternative explanation for the cosmic acceleration implied by the Supernovae data, in the realm of the BS model.

3. Cosmological principle

As the Cosmological Principle (CP) is not verified in the BS model, we would like to remind that nowadays there are not observational proofs of this philosophical assumption. The observational almost isotropy of the cosmic background radiation (CBR) temperature is insufficient to force exact isotropy into the spacetime geometry and hence exact spatial homogeneity of the 3-dim cosmic hypersurfaces, i.e., to force the verification of the Cosmological Principle, because neither the "exact" Ehlers-Geren-Sachs (EGS) theorem nor the "almost EGS theorem" [3] are necessarily verified.

Thus the measured almost isotropy of the CBR temperature is in principle compatible with large shear [4] and (or) nonzero kinematic acceleration, as happens in the BS model.

4. Linear Hubble law for the BS model

Now, we specialize the general linear Hubble law [5], valid for any inhomogeneous universe, for the specific case of the BS model considered. The final expression that adopts for off-center observers $P_0$, is [6]:

$$z = \left( \frac{\dot{R}}{R} - A \cos \Psi + \sqrt{3} \sigma \cos^2 \Psi \right)_0 D$$

By spherical symmetry, just the telescopic angle $\Psi$ is needed to describe off-centre observations in the rest space of the observer. $\Psi$ is the angle between the direction of observation of a light ray and the preferred vector $\varepsilon_1$, $\sigma$ is the scalar shear and $D$ is any cosmological distance. The generalized Hubble function has three terms which contribute to the cosmological redshift $z$.

The first is similar to the usual one of FLRW models due to the volume expansion, but in this model is due to the azimuthal expansion and depends not only on cosmic time but also on a radial comoving distance or position of the observer with respect to the centre. The second term in the generalized Hubble function is a dipolar one, due to the acceleration, and the third is a quadrupolar one, due to the shear.

The consequences of the generalized linear Hubble law are striking. For instance, when we are observing in the same sense that the acceleration
away from the centre, that is $\Psi = 0$, then the dipole term gives a maximum violetshift contribution, $-AD$. Of course, observing in the opposite sense to the acceleration, i.e., towards the centre, it gives an maximum dipole redshift, $+AD$. In both cases, if the emitters are at the same distance, the additional expansion and shear quadrupole terms have the same positive value. Hence, using the new Hubble law (4), the difference of redshifts of these kind of observations, is a pure dipole violetshift. Therefore, we consider this specific direction away from the centre, as the global direction of the matter dipole.

5. Final comments and conclusions

The usual dipole, manifested as a deviation of the isotropic Hubble law of Friedmann models, is a constant Doppler effect. This Doppler effect arises in the standard model from peculiar velocities and it can be eliminated going to the correct zero peculiar velocity frame. Whereas, in the BS model, the cosmological acceleration dipole, measured by off centre observers, is emitter’s distance dependent and cannot be eliminated. The same happens for the quadrupole shear term.

However, we do not claim that all the observed matter dipole is cosmological. In our model, a peculiar velocity dipole induced by a local inhomogeneity, must be calculated as a local perturbation of the inhomogeneous background spacetime, i.e. as a local deviation of the new Hubble law (4).

Finally note, that the new Hubble function of this model is not only cosmic time dependent, as in FLRW models, but observer’s position and angular dependent too. Therefore, this may justify the difference between its inferred values from observations performed with different telescopic angles.

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