Production scheduling for continuous manufacturing systems with quality constraints

Jalel Ben Hmida*, Jim Leea, Xinchun Wanga and Fathi Boukadib

aSystems Engineering, University of Louisiana at Lafayette, P.O. Box 44170, Rougeou, Hall Room # 244, Lafayette, LA 70504, USA; bDepartment of Petroleum Engineering, University of Louisiana at Lafayette, P.O. Box 44408, Madison Hall, Room # 134, Lafayette, LA 70504, USA

(Received 18 November 2013; accepted 6 February 2014)

This research is motivated by a real world production scheduling problem in a continuous manufacturing system involving multiple objectives, multiple products and multiple processing lines with various inventory, production and quality constraints. Because of the conflicting objectives, a global optimization approach is considered as not feasible by the plant management. Given a customer demand forecast, two practical heuristic or sequential optimization algorithms are developed to generate daily production schedules for two primary objectives: minimize shipment delays (pull-backward procedure) and minimize average inventory levels (push-forward procedure). A third heuristic algorithm (reduce switch-over procedure) which is based on the current management practice is also developed to serve as a benchmark. A factorial experiment was performed to evaluate the performance of the heuristic procedures and to identify factors that might affect the performance differences among heuristics. Since each heuristic is designed to give priority to one of the three conflicting objectives, none of them is absolutely superior to the other algorithms in all aspects. However, the first two heuristic procedures performed better than the current management practice in shipment delays and average daily inventory. The production schedules generated by the two procedures also satisfy the quality constraints. The experimental results also showed that the performance of the algorithms is significantly affected by product mix, inventory levels, and demand pattern.

**Keywords:** production scheduling; continuous manufacturing system; product quality; multiple objectives; heuristic algorithms

1. Introduction

Scheduling is concerned with efforts to see that the resources of a manufacturing system are well utilized so that the products are produced within reasonable accord with the customer demand. Numerous planning and scheduling methods have been developed for discrete part manufacturing systems, and published research has shown that the approach for scheduling production depends on the type of manufacturing system involved. Reviews of the characteristics of continuous manufacturing systems in Vollmann, Berry, and Whybark (1992), Askin and Standridge (1993) suggest that the continuous manufacturing systems require different modeling approaches.

*Corresponding author. Email: jalel.benhmida@louisiana.edu

© 2014 The Author(s). Published by Taylor & Francis.
This is an open-access article distributed under the terms of the Creative Commons Attribution License http://creativecommons.org/licenses/by/3.0/, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The moral rights of the named author(s) have been asserted.
This research is motivated by an actual production scheduling problem in a continuous manufacturing system of a large producer of chemical products. On average, the chemical plant receives more than 200 customer orders in a month, totaling about four million pounds of chemical products. The production system is capable of producing a large number of different types of products through multiple processing lines. A large number of process constraints exist due to varied capabilities of the production lines and the processing requirements. Most of the products can be produced on more than one line and some of the processes require the sharing of special tools. Some products have precedence constraints, and some products have similar production conditions that should be scheduled for production consecutively.

Currently, scheduling daily production in the system is subjectively based on the management’s experience. With an increasing emphasis on the multiple objectives of on-time shipment, low inventory, and production quality; the management of the plant needs a scheduling tool to improve the production scheduling for better system performance. The three objectives identified by the plant management are described as below:

1. **On-time shipments.** The current overdue shipment rate (OSR) is unacceptable by the top management. The lack of on-time delivery may prevent the plant from being considered as a major supply candidate by the downstream industries. A measure of on-time shipment is commonly considered as the most important performance indicator of a continuous manufacturing system.

2. **Low inventory.** To reduce overdue shipments and if processing lines are available, finished products are normally produced before the due dates. This results in high level of inventory. While “zero inventory” may not be practically feasible, efforts must be made to reduce the overdue shipments and the inventory level at the same time.

3. **Product quality.** Production run length is defined as the number of days a processing line is scheduled to produce the same product type. Longer runs can produce finished products with consistent quality. Frequent product switchovers in the processing line can result in quality problems. However, a run length larger than necessary can increase the inventory level.

The management of continuous manufacturing systems of this type faces the dilemma of meeting customer delivery dates while operating the system efficiently. This involves conflicting objectives. The conflict arises because improvement in one objective can be made to the detriment of one or more of the other objectives. Moreover, various production and quality constraints need to be satisfied. All of these concerns are addressed in this research. Currently the approach used by the plant management gives priority to product quality where the product switchover is minimized. A scheduling tool, which can generate several daily production schedules based on algorithms with priorities given to different objectives, would be helpful to the plant management.

2. **Literature review**

Production scheduling, perhaps because of its considerable quantifiable structure and richness of virtually endless variations, continues to attract many researchers for scholarly work. Patterson, Slowinski, Talbit, and Weglarz (1989) stated that most of the realistic scheduling problems could not be solved in a computationally efficient manner. Linear programming (LP) is commonly used to solve production planning and
scheduling problems optimally, but the problem formulation is normally over-simplified and the solution is used only at the strategic level. Heuristic rules have been developed to deal with practical scheduling problems on the shop floor.

Deepark and Grossmann (1990) proposed a multi-period linear programming model for the simultaneous production planning and scheduling of multi-product batch manufacturing systems that may consist of several non-identical parallel lines. The model involves a simplistic representation of the scheduling problem where many possible constraints are not considered. Gupta, Hector, and Gupta (1990) provided an approach to determine the optimal sequence of products with due date in a single machine system. Linear programming models were developed for three different due date assignment methods. Minton, Johnston, Phillips, and Laird (1992) described a simple heuristic approach to solve large-scale constraint satisfaction and scheduling problems. Anwar and Nagi (1997) formulated an integrated scheduling and lot-sizing problem. An efficient heuristic is developed that schedules operations by exploiting the critical path of a network and iteratively groups in order to determine the lot-size that minimizes the make span as well as set-up and holding costs. Certain aspects of scheduling problems such as operational pre-conditions (e.g. product sequencing and cleaning requirements) are important to avoid compromising final product quality, safety and feasibility of operations. (Amaro & Barbosa-Pôvoa, 1999) developed a Mixed Integer Linear Programming (MILP) formulation so as to account for operational conditions. The formulation is solved using a Branch and Bound (B&B) procedure. In some production systems, early shipments are forbidden. Scudder, Hoffmann, and Rohleder (1993) explored various policies on shipments using a simulation model and also came up with best policies.

Many scheduling problems are multi-objective that can be efficiently solved by a heuristic approach. Daniels (1994) described a multi-objective scheduling situation where information concerning the relative importance of the criteria is available through interaction with a given decision maker. Yim and Lee (1996) used Petri nets and heuristic search for multiple objectives scheduling in a flexible manufacturing systems (FMSs). Tung, Lin, and Nagi (1999) presented a hierarchical approach to scheduling in a FMS that pursues multiple performance objectives. Viana and de Sousa (2000) used heuristics to solve multi-objective resource constrained project scheduling problem. Lou, Liu, Zhou, Wang, and Sun (2012) present a method for developing a pro-active scheduling model that anticipates known uncertainties and a reactive model that accounts for and handles unknown uncertainties.

One approach to avoid frequent product switchover in production is to assign a long setup time between product switchovers. He and Kusiak (1992) considered a mixed-integer formulation of the single-machine scheduling program with sequence-dependent setup times and precedence constraints. Seki and Kogure (1996) developed a stochastic-demand model to deal with lot scheduling problem for multi-item continuous demand utilizing a single machine with setup times. A recent study by Wagner and Smits (2004) used local search heuristic for stochastic economic lot scheduling problem that considered multiple product in a single facility with limited capacity and significant setup times and costs.

Recent research in scheduling has also incorporated various management concepts. Modarress, Ansari, and Willis (2000) presented a technique using Just-In-Time (JIT) and Statistical Process Control (SPC) methods to minimize the short run production in an environment with demand uncertainties. Bose and Pekny (2000) presented Model Predictive Control (MPC) concept for planning and scheduling problem. A forecasting
A model was developed to calculate target inventory. The importance of Supply Chain (SC) production planning was studied in Caramanis, Anli, and Paschalidis (2003). Their model produces weekly production schedule that minimizes inventory and backlog costs subjected to non-linear constraints on production imposed by weekly varying dynamic lead times and inventory hedging policies.

Giannelos and Georgiadis (2003) investigated scheduling in a continuous manufacturing process. They developed a model for scheduling fast consumer good manufacturing process using MILP techniques combined with continuous representation of time so as to alleviate some of the computational problems incurred by discrete time-index formulations. Kogan, Leu, and Perkins (2002) suggested polynomial-time algorithm for the continuous-time scheduling problem in a manufacturing system where multiple product types are produced on a set of parallel machines. The main objective of the study was to schedule the production so that inventory, production, and planning horizon costs are minimized.

Review of scheduling literature indicates that most of the realistic scheduling problems are solved by efficient heuristic methods. These methods are developed primarily for discrete manufacturing systems and do not consider the effects of scheduling on product quality. In this paper, mathematical models which formulate the problem in continuous manufacturing systems with multiple objectives are presented first. Several heuristic algorithms are then proposed to solve the scheduling problem with priority given to different objectives. The management can determine an acceptable production schedule based on the proposed decision support procedures. The performance of the heuristic procedures is evaluated based on a simulation experiment.

3. Problem description

The scheduling problem addressed in this research can be stated as follows: A continuous manufacturing system can produce $M$ types of products. There are $N$ processing lines, each being capable of producing a subset of $M$ products. If finished products are produced earlier than the due date, they are stocked in the warehouse where each product has a limited storage area. The production rate varies by products on a line, and only one product can be produced by one line at any time.

Table 1 shows a sample of the data of maximum inventory level allowed and production rate of each processing line by product. For the purpose of modeling, the

| Product | Maximum inventory (×1000 lb) | Production rate by line (×1000 lb/day) |
|---------|-----------------------------|-------------------------------------|
|         |                             | $A$ | $B$ | $C$ | $D$ | $E$ |
| 1SB     | 150                         | 25  | 25  | 0   | 0   | 0   |
| 1SC     | 262                         | 25  | 25  | 0   | 0   | 0   |
| 2SB     | 80                          | 24  | 0   | 24  | 0   | 0   |
| 2SC     | 126                         | 24  | 0   | 24  | 0   | 0   |
| 3SB     | 150                         | 23  | 0   | 0   | 23  | 0   |
| 3SC     | 159                         | 23  | 0   | 0   | 23  | 0   |
| 4B      | 150                         | 19  | 0   | 0   | 0   | 19  |
| 4C      | 112                         | 19  | 0   | 0   | 0   | 19  |
| 5       | 180                         | 34  | 0   | 0   | 0   | 0   |
| 6B      | 120                         | 0   | 29  | 0   | 0   | 0   |
| 6C      | 180                         | 0   | 29  | 0   | 0   | 0   |
setup time is included in our estimation of production rate by processing line. For example, if line $A$ is used to produce product 1SB and the minimum run length is 4 days. The total production in this run of 4 days is estimated to be 100 (thousand pounds), and the daily production is considered to be $100/4 = 25$ (thousand pounds). The real production time of the last day in a run is normally shorter than the other days, as the line must be cleaned and adjusted (i.e. setup time) for another run with a different product. As a result, the total estimation is normally slightly lower than the actual production to cover the loss of setup time. When the run length is long, it is possible that the production run can be completed 20–30 min early. The effect of the difference does not significantly affect the estimation on the production rate.

For example, if line $A$ is used to process product 1SB for one day, then 25 thousand pounds of 1SB can be produced. While most of the products can be produced on more than one line, some of the products can be produced only on one line (e.g. product 5 can be produced on Line $A$ only). A sample of customer orders by days is shown in Table 2. Based on the customer orders, the scheduling problem is to determine the daily production schedule for each processing line with production, inventory and quality constraints.

To formulate the problem we require the following notation:

- $T$: planning horizon
- $t$: day in planning horizon ($t = 1, \ldots, T$)
- $M$: number of product types
- $N$: number of processing lines
- $i, j$: product index ($i, j = 1, \ldots, M$)
- $k$: processing line index ($k = 1, \ldots, N$)
- $L_i$: waiting period for product $i$ in days. Waiting period is the number of days that a product needs to stay in warehouse before shipment for quality assurance
- $U_i$: actual due date of product $i$
- $E_i$: adjusted due date of product $i$ based on waiting period
- $HL_i$: minimum run length (in days) of product $i$
- $IH_i$: maximum inventory allowed for product $i$
- $I_{it}$: inventory of product $i$ in day $t$
- $D_{it}$: demand of product $i$ in day $t$
- $Q_{ikt}$: production quantity of product $i$ on line $k$ in day $t$
- $Q_{Oi}$: quantity of overdue product $i$
- $W_{kt}$: times of product switchovers on line $k$ in day $t$
- $C_{ik}$: daily production rate for product $i$ on line $k$
- $P$: set of jobs $[i, j]$, where product $i$ precedes product $j$
- $G_g$: set of product group which should be produced sequentially
- $g$: represents the type of groups
- $R_r$: set of products that require the same special tool (resource) $r$ for processing
- $r$: represents the type of resource
- $RA_r$: number of units of resources available for products in set $R_r$
- $H_{ik}$: production run length (in days) of product $i$ on line $k$
- $B_{ik}$: production starting day of product $i$ on line $k$
- $O_{it}$: demand of product $i$ that cannot be produced on time in day $t$
- $X_{ikt}$: a zero-one variable. $X_{ikt}$ is 1 if product $i$ is scheduled on line $k$ in day $t$
The three objectives to be considered in the production scheduling are: minimizing overdue shipment, minimizing inventory, and minimizing product switchovers. They are discussed below:

1) OSR measures the amount of customer demands that cannot be met by a production schedule. In this research, it is expressed as ratio of overdue shipment amount to the total demands in the planning horizon. The overdue shipment of production day \( t \) (\( O_{it} \)) is demand (\( D_{it} \)) minus the inventory \( I_{i, t} \) and minus the production of product \( i \) on all lines (\( Q_{ikt} \)).

\[
\text{OSR} = \frac{\sum_i QO_i}{\sum_i \sum_t D_{it}}
\]

where \( QO_i = \sum_t O_{it} \) and \( O_{it} = D_{it} - I_{i, t-1} - \sum_k Q_{ikt} \)

If \( O_{it} < 0 \), let \( O_{it} = 0 \)

2) Average daily inventory (ADI) reflects the average daily amount of finished products produced before the due date. The inventory of product \( i \) on day \( t \) (\( I_{it} \)) is the initial inventory plus production on day \( t \) and minus the demand on day \( t \). The ADI is determined by the total inventory divided by the planning horizon \( T \). It is shown as follow:

\[
\text{ADI} = \frac{\sum_t \sum_i I_{it}}{T}
\]

where \( I_{it'} = I_{i0} + \sum_{t=1}^t \sum_k Q_{ikt} - \sum_{t=1}^t D_{it} \)

If \( I_{it'} < 0 \), let \( I_{it'} = 0 \)

3) Total Switch Times (TST) of the product is used to check the quality of a product as frequent product switchovers could lead to poor product quality. For everyday \( t \) and every line \( k \), product switchover count \( W_{kt} \) is 1 if the product scheduled on day \( t \) is different from \( t-1 \). The TST can be defined as:

| Due day | 1SB | 1SC | 2SB | 2SC | 3SB | 3SC | 4B | 4C | 5 | 6B | 6C | 7 | 8B |
|---------|-----|-----|-----|-----|-----|-----|----|----|---|----|----|---|----|
| 1       | 0   | 15  | 20  | 0   | 0   | 0   | 0  | 0  | 35| 0  | 0  | 0  | 0  |
| 2       | 0   | 0   | 0   | 0   | 0   | 0   | 0  | 10| 0  | 0  | 35| 0  | 0  | 0  |
| 3       | 20  | 10  | 0   | 20  | 0   | 30  | 0  | 0  | 0  | 20| 0  | 0  | 0  | 0  |
| 4       | 0   | 30  | 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 5       | 5   | 0   | 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 6       | 10  | 0   | 15  | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 7       | 0   | 0   | 0   | 0   | 0   | 0   | 0  | 0  | 5  | 0  | 0  | 0  | 0  | 0  |
| 8       | 0   | 30  | 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 9       | 20  | 0   | 0   | 0   | 0   | 0   | 0  | 0  | 0  | 10| 35| 0  | 40 | 0  |
| 10      | 0   | 0   | 30  | 0   | 0   | 0   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
TST = \sum_{k} \sum_{t} W_{kt} \tag{3}

where \( W_{kt} = 1 \) if the product scheduled on day \( t \) is different from \( t - 1 \).

Several types of production, inventory and quality constraints are considered in scheduling the daily production.

(1) \textit{Production capacity and capability constraints.} The quantity of product \( i \) that can be produced on line \( k \) in day \( t \), \( Q_{ikt} \), should be less than the capacity \( C_{ik} \) of line \( k \). The capacity of line \( k \) is a function of product type \( i \). For every \( i \) and every \( k \),

\[ Q_{ikt} \leq C_{ik} \tag{4} \]

where \( C_{ik} = 0 \) if line \( k \) is not capable of producing product \( i \).

(2) \textit{Inventory constraints.} Each product is allocated limited storage space in the warehouse. A maximum inventory level \( I_{i} \) is imposed on each product \( i \).

\[ I_{io} + \sum_{t} \sum_{k} Q_{ikt} - \sum_{t} D_{it} \leq I_{i} \tag{5} \]

(3) \textit{Minimum run length constraints.} To achieve the desired quality level, some products have minimum run length requirements as defined by \( H_{Li} \). For every \( i \), \( H_{ik} \) represents the actual run length in days of product \( i \) on line \( k \):

\[ H_{ik} \geq H_{Li} \tag{6} \]

(4) \textit{Precedence constraints.} If product \( i \) must be produced before product \( j \), then the production starting date of product \( i \) on line \( k \) (\( B_{ik} \)) should be completed before the line starts to process product \( j \).

\[ B_{ik} + H_{ik} \leq B_{jk} \tag{7} \]

In our example, product 2SB must be produced prior to product 4B and 4C.

(5) \textit{Group constraints.} Products with similar production conditions should be produced sequentially to ensure product quality. If product \( i \) and \( j \) are in the same group, it is desired to produce these products back to back on line \( k \).

\[ B_{ik} + H_{ik} = B_{jk}, \text{ or } B_{jk} + H_{jk} = B_{ik} \tag{8} \]

For example, Product 17B, 18B, 20 require similar production conditions. They should be produced sequentially in the production schedule.

(6) \textit{Resource sharing constraints.} The processing operations of some products require the use of special equipment with capacity constraints. Let \( RA_{r} \) represent the number of tools available and \( R_{r} \) is a set of products required in the production. At any time, no more than \( RA_{r} \) products in \( R_{r} \) can be produced.

\[ \sum_{i \in R_{r}} \sum_{k} X_{ikt} \leq RA_{r} \tag{9} \]

where \( X_{ikt} = 1 \) if \( i \) is scheduled on \( k \).

For example, no more than two S type products can be produced at one time on all lines as the processing of a S type product requires the use of a special tool 1, and \( RA_{1} = 2 \).

(7) \textit{Preferable product-line assignment.} Since most of the products can be produced on more than one processing line, priorities on the product-line assignment can be established by the management. The assignment may affect the production efficiency and
the quality of the product in some systems. For example, products 2SB and 3SB are preferably produced on line B though they can also be produced on line A.

The number of constraints for this multi-line, multi-product facility is more than one hundred, and the very challenging task is how to satisfy these constraints in the scheduling process while considering the conflicting objectives. This research proposes the development of practical heuristic scheduling algorithms to generate various production schedules based on the demand data and the constraints. With the scheduling tool, the management will be able to generate several feasible production schedules with estimated overdue shipment, daily inventory levels and the number of product switchovers. This information will allow the management to determine the actual daily production schedule that balances the three objectives.

4. Heuristic scheduling algorithms

The research problem is to construct production schedules that minimize (1) OSR, (2) ADI and (3) TST while satisfying the various production, inventory and quality constraints. With three conflicting objective functions, it is infeasible to develop a global optimization model which optimizes all three functions. Therefore, three heuristic or sequential optimization algorithms are developed in this research to optimize each of the three objectives. The Pull-Backward Heuristic Procedure (PB) first produces an initial production schedule to minimize the OSR. The initial schedule is then adjusted to reduce ADI without affecting the OSR. This procedure is basically a sequential optimization method which optimizes the OSR and ADI in two steps. The Push-Forward Heuristic Procedure (PF) first produces an initial schedule which minimizes the ADI. The initial schedule is then adjusted to reduce OSR without affecting the ADI. Similar to the PB schedule, the PF is a sequential optimization method which optimizes the ADI first then OSR in two steps. Finally, the Reduce Switchover Heuristic Procedure (RS) is used to produce an optimal solution on TST first. Then the OSR is considered next, followed by ADI.

The methodology underlying the three procedures is similar and follows the three basic steps: problem decomposition, product sequencing, and demand shifting. First, the planning horizon is broken into several smaller periods. If the planning horizon is one month, the length of a period can be about one week. All orders of the same product in a period are lumped together to form one product demand. The production for each of the periods is scheduled according to the lumped demand. After all the period schedules are obtained, they are merged together to form the schedule for the whole planning horizon.

In creating a period schedule, two sequencing rules are used to create priorities. The Earliest Due Date (EDD) rule is chosen as the primary rule, where the choice is made based on the product that has the earliest due date. The Shortest Processing Time (SPT) rule, where the choice is made based on the processing operation with the shortest duration, is used as a secondary rule. For example, if 1SB, 1SC and 2SB are produced in line A (see Table 2) in the first 7-day period, 1SB will be produced last as it has an order of 20 due on day 3 compared to the due date of 1 for 1SC (with order quantity 15) and 2SB (with order quantity 20). Since 1SC and 2SB have the same due date, they are sequenced based on SPT rule. The processing priority is given to 2SB because the demand of 35 thousand pounds of 2SB can be produced with a shorter processing time than the time required to produce 55 thousand pounds of 1SC.

When the production cannot be scheduled for on-time delivery due to capacity constraints, the demand can be shifted to another processing line according to line
capability. If other lines are also occupied, the demand can be shifted to the next period. To satisfy precedence constraints, group constraints, and resource constraints; the product demand can be shifted to other dates based on the three objectives of the scheduling. Different approaches of shifting are employed for different heuristic procedures. However, the demand of a product is always shifted forward to satisfy the maximum inventory constraints. The procedures are summarized below.

4.1. Pull-Backward Heuristic Procedure (PB)

The PB procedure is basically a sequential optimization method which optimizes the OSR and ADI in two steps. If the demand is greater than the production capacity in a given period, then not all production runs can be scheduled before the due dates. In this case, an attempt is made to schedule the production in the previous period to avoid shipment delay. However, the ADI may go up.

Step 0. Initialization

Adjust the due date of customer order $U$ according to waiting period $L$ to form an adjusted demand date $E$ in planning horizon. Given a product $i$, let $E_i = U_i - L_i$ for all products with a non-zero waiting period. Break planning horizon $T$ into PN scheduling periods; let $v$ be the length of the period and $n$ be the $n$th period ($n = 1, \ldots, PN$) in the planning horizon, Then $PN = \frac{T}{v}$. If PN is non-integer, it is rounded down to the nearest integer.

Step 1. Product-processing line assignment

Assign products to processing lines based on preferable product-line assignment, subject to capability constraints. One product is assigned to one line.

Step 2. Period schedule generation

Construct a trial production schedule based on demand for period $n$ subject to production capacity constraints and minimum production run length.

(2.1.) Compute the total demand due in period $n$ by product and by line. Let $DD_{ikn}$ represents this demand for product $i$ assigned to processing line $k$. For each $k$, sort due day $E_i$ for all products in ascending order, resulting in $k$ sorted lists. If a product has several orders in period $n$, $E_i$ is computed based on the order with the earliest due date. Products with smaller $E_i$ values have higher priorities to be scheduled for production. If two or more products have the same due day, the product with the smallest processing time (SPT) has a higher priority to be scheduled than other products.

(2.2.) For each processing line $k$, sequence products for the line based on the priority established in the sorted list from step 2.1.

(2.3.) Determine production run length $H$ for each of product scheduled in the period. Run length of a product in a period is determined according to the total quantity of demand, capability and minimum run length constraints:

$$H_{ik} = \begin{cases} \frac{DD_{ikn}/C_{ik}}{HL_i} & \text{for } H_{ik} > HL_i \\ H_{ik} & \text{for } H_{ik} \leq HL_i \end{cases}$$

where $DD_{ikn}$ is the demand of product $i$ on line $k$ in period $n$. $H_{ik}$ is rounded up to the next integer if $DD_{ikn}/C_{ik}$ is not an integer. Repeat this step for all $k$. (For example, product 1SC in the first 5-day period in Table 2 has $DD_{ik1} = 55,000$ and $C_{ik} = 25,000$. $DD_{ik1}/C_{ik}$ is 55,000/25,000, or 2.2. Therefore, $H_{ik}$ would be rounded up to 3.)
(2.4.) Shift the unfulfilled demand of a product to another processing line in accordance with capability constraints. If \( H_{ik} \) cannot be totally assigned to line \( k \) in the period, shift the unscheduled demand to another line which is not fully utilized. If product \( i \) has an unfulfilled demand in period \( n \) and cannot be assigned to any other processing line, shift the unfulfilled demand to period \( n + 1 \).

Step 3. Period schedule revision

(3.1.) Modify the schedule to meet the precedence constraints. If product \( j \) has been scheduled at time \( t_1 \) before product \( i \) at time \( t_2 \) where \( [i, j] \) has precedence constraint \( P \), then re-schedule \( i \) at \( t \), where \( t < t_1 \). Adjust production schedule on the processing line accordingly. Repeat this step for all processing lines.

(3.2.) Modify the schedule to meet the group constraints. If production of a set of products \([i, j]\) in \( G_g \) is separated in the schedule and product \( i \) is in the earliest position, shift all other products in the same group next to and immediately following the product \( i \). Shift the production of products without group constraints to a later day in the schedule if needed.

Step 4. Period iteration

Repeat step 1 to step 3 until PN schedules \( S_1, S_2, \ldots, S_{PN} \) are obtained for each of the PN periods.

Step 5. Schedule Combination

Merge period schedules and check constraints.

(5.1.) Merge subsets of schedules \( S_1, S_2, \ldots, S_{PN} \) to form a complete schedule \( S \).

(5.2.) Exchange the position of products scheduled at the same line if precedence constraints are violated.

(5.3.) Check resource constraints for the production schedule across all processing lines. If a constraint is violated in day \( t \), shift the demand of the product with lower \( DD_{ikn} \) to an earlier day if possible. If the product cannot be produced on an earlier day, shift the demand to day \( t + 1 \). Adjust the production schedule on the same processing line correspondingly.

(5.4.) Satisfy the maximum inventory constraints. Assume the production on product \( i \) in line \( k \) can be shipped in the same day to meet the demand. Determine the inventory level for all products. If the inventory of product is greater than the maximum level allowed, shift that demand to day \( t + 1 \) and then check the resource constraints. The production schedule will be shifted to the future if the new schedule violates the resource sharing constraints.

4.2. Push-forward heuristic procedure (PF)

The PF is a sequential optimization method which optimizes the ADI first then OSR in two steps. If the demand is greater than the production capacity in a given period, however, the demand is shifted to the next period to avoid inventory. Based on the previous description, only two steps need to be replaced. They are described below.

(3.1.) Modify the schedule to meet the precedence constraints. If product \( j \) has been scheduled at time \( t_1 \) before product \( i \) at time \( t_2 \) where \([i, j]\) has precedence constraint...
Then re-schedule \( j \) at \( t \), where \( t > t_2 \). Adjust production schedule on the processing line accordingly. Repeat this step for all processing lines.

(3.2.) Modify the schedule to meet the group constraints. If production of a set of products \([i, j]\) in \( G_g \) is separated in the schedule and product \( i \) is in the earliest position, shift all other products in the same group to be produced immediately before the product \( i \). Shift the production of products without group constraints to a later day in the schedule if needed.

(5.3.) Check resource constraints for the production schedule across all processing lines. If a constraint is violated in day \( t \), shift the demand of the product with lower \( DD_{ikn} \) to day \( t + 1 \). Adjust the production schedule on the same processing line correspondingly.

### 4.3. Reduce Switchover Heuristic Procedure (RS)

The RS procedure focuses on minimizing product switchovers on processing lines which is similar to the current practice of the plant management. It is used to produce an optimal solution on TST first. Then the OSR is considered next, followed by ADI. Based on the current practice, once a processing line is assigned, the production run is continued until the total demand for the planning horizon (normally one month) is satisfied. The advantage of the current practice is that the method is easy to implement. The disadvantage is that both the OSR and ADI are not acceptable.

To implement the RS algorithm, the number of days in a period used in the previous procedures must be increased to allow a longer run. A test on various period lengths shows that the length of the period should be doubled or the number of periods (PN) should be reduced by about 50% in this procedure to observe a statistically significant different number of product switchovers. With RS, all the steps remain unchanged as procedure PB. Only the PN is changed. Other steps remain unchanged as procedure PB.

From the above discussions, it is noted that different production schedules can be obtained by using different heuristic procedures. A computer program written in C language is then implemented to include the three heuristic procedures. Many variables, such as the initial assignment of products to the processing lines and the number of periods in the planning horizon can be changed in the program. Consequently, alternative schedules can be produced for the management to make scheduling decisions.

### 5. Simulation experiments

Simulation experiments were carried out to evaluate the performance of the three heuristic procedures. The purpose of the experiment was to compare the performance of the procedures by answering the following questions:

1. What is the magnitude of the relative performance of one procedure over another?
2. If the performances of the two procedures are statistically significant different, then what are the factors related to the problem attributes affecting their differences?

To answer these questions, four factors that may have influence on the performance measures are identified. They are as follows:

1. **Group ratio** (G). The group ratio is expressed as a ratio of the number of
products with group constraints to the total number of products produced in the system. For example, if the total number of products is 50, and five of them are involved in group constraints, then the group ratio is 5/50 or 10%. Three levels are tested: 10, 25, and 50%.

(2) **Precedence strictness ratio (P).** The precedence constraints specify the products that must be produced before others. The precedence strictness ratio is defined as the number of precedence requirements among the products over the number of feasible pairs of products. For example, in a system producing five products, the maximum number of precedence constraints is 5!/2!3! or 10. If only one constraint exists for a pair of products, then the precedence strictness ratio is 1/10 or 0.1. Two levels are tested in the experiment: 0.1 and 0.2. These two ratios are relatively low because the type of products in the continuous manufacturing system we studied does not have too many precedence constraints.

(3) **Maximum inventory (I).** The maximum finished-goods inventory allowed in the system is basically limited by the space in the warehouse. To reduce the inventory cost, it is desired to reduce the maximum inventory level. Two levels of maximum inventory are tested. A level of 100% represents the actual maximum inventory allowed in a chemical plant, which is about 4500 thousand pounds of products in total. The level of 67% means that the maximum inventory allowed is reduced to 4,500 × 67% or 3000 thousand pounds in total.

(4) **Demand (D).** Three sets of real demand data were collected from a chemical plant. They are referred to as D1, D2, and D3. Each data set contains the daily demand for over 60 product types over a month period. The average total daily demands for D1, D2 and D3 are 96.6, 108.1 and 109.4 thousand pounds respectively. The standard deviations of the total daily demand of the three demand patterns are 93.90 for D1, 74.63 for D2, and 75.24 for D3.

Since the production scheduling issues are unique, a well-known data is not available in the literature. Using the three data sets from industry, a factorial experiment was designed and performed to evaluate the performance of the three heuristic procedures and to identify the factors affecting the three objectives scheduling.

Table 3 summarizes the factors and associated levels tested in the experiment. A full factorial experimental design is used, so the number of basic experiments designed for each heuristic procedure is $3 \times 2 \times 2 \times 3 = 36$. In order to account for variations in the experiments, five replications are created for each of the basic experiments. Considering three heuristic procedures and five replications per experiment, the total number of experiments would be $36 \times 5 \times 3 = 540$.

The method to replicate the experiment is based on a 10% random perturbation on demands. For every customer order in the planning horizon, a uniformly distributed random number between (0, 1] is generated to determine the direction of the perturbation. If the random number is between (0, 0.25], then the value is decreased by 10%. If the random number is between (0.25, 0.5], the order is decreased by 5%. If the random number is between (0.5, 0.75], the corresponding value will be increased by 5%. Finally, the value is increased by 10% if the random number is between (0.75, 1). This allows the demand data of replications to be varied within a 20% range. For example, if the original demand for a product is 25, it may be changed to 22.5 (i.e. $25 \times 0.90$), 23.75 (i.e. $25 \times 0.95$), 26.25 (i.e. $25 \times 1.05$) or 27.5 (i.e. $25 \times 1.10$) in the replications.

Three performance measures corresponding to each of the three scheduling objectives are used in the experiments: (1) OSR, (2) ADI and (3) Total switch times (TST).
The three performance measures have been defined in Equations (1)–(3). Based on OSR, ADI and TST, a total cost index (TCI) can be estimated to represent an overall economical performance for a schedule where:

\[
TCI = \frac{(400 \times QO + ADI + 2500 \times TST)}{100}
\] (11)

The quantity QO is the total amount (in thousand pounds) of overdue shipment products, ADI is the ADI in thousand pounds, and TST is the number of times the switch-over occurs in the production schedule. Note that the coefficients of the three parameters are estimated to present a certain relation between parameters, and the values vary in accordance with different production systems.

### 6. Analysis of experimental results

Based on the experimental design described in the previous section, a total of 540 experiments were performed. The performance measures of the 180 experiments for each of the heuristic procedures were averaged and presented in Table 4.

Among the three objectives of the scheduling, OSR and ADI can be considered as the primary or key performance measures for comparing different production schedules. As the product quality is critical to the success of the business, every production schedule developed by management must satisfy the run length constraints to insure that the quality of products is acceptable. Therefore, TST is simply used as a benchmark for schedule comparison purpose. In the real world a smaller TST is easier to implement and control, so it can be also used by management to evaluate the feasibility of schedule implementation. With the consideration of the minimum run length constraints in our scheduling problem, all three algorithms proposed can schedule production with acceptable product quality.

Table 4 shows that each heuristic procedure performs best for one of the three objectives as expected. Procedure RS results in an average of 12% OSR for the test problems. If the reduction of OSR is critical, then procedure PB can reduce the rate from 12% to 9%, and procedure PF is also slightly better than RS by 1%. For the ADI, PB is better than RS by 1%, and PF can reduce the inventory by more than 5%.

| Performance measure | Heuristic procedure | % Difference |
|---------------------|---------------------|--------------|
|                     | PB                  | PF           | RS           | PB-RS | PF-RS |
| OSR                 | 0.09\(^a\)          | 0.11         | 0.12         | 3     | 1     |
| ADI                 | 73,164              | 69,883\(^a\) | 73,852       | 1     | 5     |
| TST                 | 42.40               | 40.33        | 36.51\(^a\)  | −14   | −9    |
| TCI                 | 3032\(^a\)          | 3233         | 3211         | 6     | 1     |

\(^a\)Indicates the best performer by performance measure.
However, the product switchover (TST) for PB and PF will go up by 14 and 9%. Based on the TCI with the coefficients defined in equation (11), procedure PB is the most economical procedure among the three. However, this superiority can be diminished if the coefficients in equation (11) are changed.

The summary results shown in Table 4 reveal that the performance measures of the three heuristic procedures are different. To determine whether the differences are significantly away from 0, $t$ tests were performed. Under the null hypothesis, there should be no difference between the performance measures obtained from each procedure. If the null hypothesis is rejected at a selected level of significance, it is concluded that the difference exists. The assumption of normality on the data was first tested and verified. The test was set at 0.01 level of significance.

Four types of variables were used to measure the relative difference of the heuristic procedures. Variables DOSR($i$, $j$) measures the difference in OSR between two heuristic procedures $i$ and $j$:

$$DOSR(i,j) = OSR(i) - OSR(j)\quad (12)$$

Variables for measures of ADI, TST and TCI are expressed as percentage difference between two procedures $i$ and $j$:

$$PADI(i,j) = \frac{ADI(i) - ADI(j)}{ADI(j)}\quad (13)$$

$$PTST(i,j) = \frac{TST(i) - TST(j)}{TST(j)}\quad (14)$$

$$PTCI(i,j) = \frac{TCI(i) - TCI(j)}{TCI(j)}\quad (15)$$

The reason of using only difference in OSR is that the OSR is already a percentage value. Moreover, the value for measure of percentage difference between two procedures sometimes might be infinite since OSR($j$) could be near zero.

The difference of the four statistics (OSR, ADI, TST and TCI) and the results of the $t$ tests appear in Table 5. A ‘*’ is marked in the “Significance” column if the hypothesis that the difference is 0 cannot be rejected at the 0.01 level of significance.

| Variable      | No. Obs | $M$ (%) | SD  | Significance |
|---------------|---------|---------|-----|--------------|
| DOSR(PB,RS)   | 180     | −2.24   | 0.036 | 0.0001       |
| DOSR(PB,PF)   | 180     | −1.96   | 0.052 | 0.0001       |
| DOSR(PF,RS)   | 180     | −0.28   | 0.064 | 0.554*       |
| PADI(PF,RS)   | 180     | −5.43   | 0.017 | 0.0001       |
| PADI(PF,PB)   | 180     | −4.77   | 0.019 | 0.0001       |
| PADI(PB,RS)   | 180     | −0.94   | 0.015 | 0.0001       |
| PTST(RS,PB)   | 180     | −14.80  | 0.081 | 0.0001       |
| PTST(RS,PF)   | 180     | −9.71   | 0.032 | 0.0001       |
| PTST(PF,PB)   | 180     | −7.69   | 0.192 | 0.0001       |
| PTCI(PB,RS)   | 180     | −5.17   | 0.214 | 0.0002       |
| PTCI(PB,PF)   | 180     | −4.70   | 0.214 | 0.0037       |
| PTCI(RS,PF)   | 180     | −4.43   | 0.428 | 0.071*       |

*Indicates that the difference is insignificant at level 0.01.
The $t$ tests show that procedure PB significantly outperforms PF and RS, but the difference between PF and RS is not significant. The OSR generated by procedure PB is about 2% less than that of procedure PF and is 2.24% less than that of procedure RS. Procedure PF can generate the lowest level of ADI, and procedure PB ranks second in ADI. The $t$ tests indicate that all the differences are significant, although the percent deviations are not very large. Procedure RS is the best on TST as the approach gives priority to reduce the number of product switchovers. The differences among the three procedures are statistically significant different. The statistics of the TCI show that procedure PB is significantly superior to the other two procedures for a system with cost coefficients similar to Equation (11). The difference between PF and RS is not significant. The results show that no procedure is absolutely superior to other procedures in every aspect. On the other hand, the experiment shows that the three procedures perform as desired since each of them is given a different priority.

The preceding observation suggests that further statistic analyses can be done to evaluate the effects of various system factors and their interaction effects on the difference between the procedures. To identify important factors affecting the performance of the heuristic procedures, an analysis of variance (ANOVA) was done using the SAS software system to evaluate the effects of the four factors and all their possible interactions. The ANOVA results on the four performance measures are summarized in Tables 6.

The variable column identifies the data set tested in the ANOVA model. For example, DOSR(PB, PF) represents the set of experimental data showing DOSR between procedures PB and PF. The level of significance is set at 0.05. All significant effects are listed in the tables.

The observations from ANOVA results are summarized as below:

(1) Group ratios (G) and demand patterns (D) significantly affect the performance differences in most of the cases.

(2) The interaction effects of G*D, I*D and G*I*D can also significantly affect the difference in performance measures.

(3) Three of the four factors (G, I, D) affect the performance. It seems that the performance of the heuristic strongly depends on the operating conditions. Only the precedence constraints (P) do not significantly affect the difference in performance measures.
7. Conclusion

This research presents a multiple objective formulation of production scheduling problem in continuous manufacturing systems and the development of three heuristic procedures to schedule daily production. Each heuristic procedure gives priority to one of three objectives. A computer program was developed to implement the heuristics. Many variables such as assignment of product to processing lines, coefficients of the cost function and the number of periods in the planning horizon can be changed in the program. Consequently, these heuristic procedures can be considered as “decision support” rules because the management will still need to make decisions themselves based on the various schedules produced.

A factorial experiment was designed and performed to evaluate the performance of the three heuristic procedures and to identify the factors that affect the performance differences. The analyses of the results show that no procedure is absolutely superior to the other procedures in all aspects. This is not a surprise because each of the three algorithms produces a local optimal solution. However, both the PB and PF algorithms can generate schedules with better OSR and ADI than the current management approach modeled by the RS algorithm. To provide a composite performance measure, a TCI was suggested. It is noted that the rank of the heuristics can be changed as a function of the cost coefficients in TCI.

Based on the ANOVA results, the differences between procedures are significantly affected by group ratio, maximum inventory and demand pattern. This information is of important value because it suggests that if one of the above factor changes in the manufacturing system, the method to schedule production needs to be re-evaluated. Further research is currently underway to study the sensitivity of the resource availability to the performance measures, and the development of a decision support system based on the heuristics proposed in this research.

References

Amaro, A. C. S., & Barbosa-Póvoa, A. P. F. D. (1999). Scheduling of industrial distribution manifolds with pre-conditions. European Journal of Operational Research, 119, 461–478.

Anwar, M. F., & Nagi, R. (1997). Integrated lot-sizing and scheduling for just-in-time production of complex assemblies with finite set-ups. International Journal of Production Research, 35, 1447–1470.

Askin, R. G., & Standridge, C. R. (1993). Modeling and analysis of manufacturing systems. New York, NY: John Wiley.

Bose, S., & Pekny, J. F. (2000). A model predictive framework for planning and scheduling problems: A case study of consumer goods supply chain. Computers & Chemical Engineering, 24, 329–335.

Caramanis, M. C., Anli, O. M., & Paschalidis, I. C. (2003). Supply chain (SC) production planning with dynamic lead time and quality of service constraints. Proceedings of the IEEE Conference on Decision and Control, 5, 5478–5485.

Daniels, R. (1994). Incorporating preference information into multi-objective scheduling. European Journal of Operational Research, 77, 272–286.

Deepark, B. B., & Grossmann, I. E. (1990). Simultaneous production planning and scheduling in multiproduct batch plants. Industrial & Engineering Chemistry Research, 29, 570–580.

Giannelos, N., & Georgiadis, M. (2003). Efficient scheduling of consumer goods manufacturing processes in the continuous time domain. Computers & Operations Research, 30, 1367–1381.

Gupta, Y. P., Bector, C. R., & Gupta, M. C. (1990). Optimal schedule on a single machine using various due date determination methods. Computers in Industry, 15, 245–254.

He, W., & Kusiak, A. (1992). Scheduling manufacturing systems. Computers in Industry, 20, 163–175.
Kogan, K., Leu, Y., & Perkins, J. (2002). Parallel-machine, multiple-product-type, continuous-time scheduling: Decomposable cases. *IIE Transactions*, *34*, 11–22.

Lou, P., Liu, Q., Zhou, Z., Wang, H., & Sun, S. X. (2012). Multi-agent-based proactive–reactive scheduling for a job shop. *The International Journal of Advanced Manufacturing Technology, 59*, 311–324.

Minton, S., Johnston, M. D., Phillips, A. B., & Laird, P. (1992). Minimizing constraint satisfaction and scheduling problems. *Artificial Intelligence, 58*, 61–205.

Modarress, B., Ansari, A., & Willis, G. (2000). Controlled production planning for just-in-time short-run suppliers. *International Journal of Production Research, 38*, 1163–1182.

Patterson, J. H., Slowinski, R., Talbit, F. B., & Weglarz, J. (1989). An algorithm for a general class of research and resource constrained scheduling problems. *Advances in Project Scheduling*. Amsterdam: Elsevier Science.

Scudder, G. D., Hoffmann, T. R., & Rohleder, T. R. (1993). Scheduling with forbidden early shipments: Alternative performance criteria and conditions. *International Journal of Production Research, 31*, 2287–2305.

Seki, Y., & Kogure, K. (1996). Lot scheduling problem for continuous demand. *International Journal of Production Economics, 44*, 7–15.

Tung, L., Lin, L., & Nagi, R. (1999). Multiple-objective scheduling for the hierarchical control of flexible manufacturing systems. *International Journal of Flexible Manufacturing Systems, 11*, 379–409.

Viana, A., & de Sousa, J. P. (2000). Using metaheuristics in multiobjective resource constrained project scheduling. *European Journal of Operational Research, 120*, 359–374.

Vollmann, T. E., Berry, W. L., & Whybark, D. C. (1992). *Manufacturing planning and control systems*. Boston, MA: Irwin.

Wagner, M., & Smits, S. (2004). A local search algorithm for the optimization of the stochastic economic lot scheduling problem. *International Journal of Production Economics, 90*, 391–402.

Yim, S., & Lee, D. Y. (1996). Multiple objective scheduling for flexible manufacturing systems using Petri nets and heuristic search. *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, 4*, 2984–2989.