Constructing a cosmological model-independent Hubble diagram of type Ia supernovae with cosmic chronometers

Zhengxiang Li\textsuperscript{a}, J. E. Gonzalez\textsuperscript{b}, Hongwei Yu\textsuperscript{c}, Zong-Hong Zhu\textsuperscript{a}, J. S. Alcaniz\textsuperscript{b}

\textsuperscript{a}Department of Astronomy, Beijing Normal University, Beijing 100875, China
\textsuperscript{b}Observatório Nacional, 20921-400, Rio de Janeiro, RJ, Brazil
\textsuperscript{c}Center of Nonlinear Science and Department of Physics, Ningbo University, Ningbo, Zhejiang 315211, China

Abstract
We apply two methods to reconstruct the Hubble parameter $H(z)$ as a function of redshift from 15 measurements of the expansion rate obtained from age estimates of passively evolving galaxies. These reconstructions enable us to derive the luminosity distance to a certain redshift $z$, calibrate the light-curve fitting parameters accounting for the (unknown) intrinsic magnitude of type Ia supernova (SNe Ia) and construct cosmological model-independent Hubble diagrams of SNe Ia. In order to test the compatibility between the reconstructed functions of $H(z)$, we perform a statistical analysis considering the latest SNe Ia sample, the so-called JLA compilation. We find that, while one of the reconstructed functions leads to a value of the local Hubble parameter $H_0$ in excellent agreement with the one reported by the Planck collaboration, the other requires a higher value of $H_0$, which is consistent with recent measurements of this quantity from Cepheids and other local distance indicators.

1. Introduction
The inherent relation between the peak luminosity of type Ia supernovae (SNe Ia) and the speed of luminosity evolution after maximum light (known as the Phillips relationship) \cite{Phillips}, makes possible to standardize these events as a distance indicator to measure the geometry and dynamics of the universe. As well known, several years after the discovery of the Phillips relationship, observations of some dozens of distant SNe Ia led to the discovery of the cosmic acceleration \cite{Riess, Perlmutter}. In Einstein’s general relativity, such behavior implies either the existence of a new field, the so-called dark energy (see Refs. \cite{dark_energy} for recent reviews), or that the matter content of the universe is subject to dissipative processes \cite{dissipative}. The mysterious cause of the current cosmic acceleration can also be attributed to a modification of the standard theory of gravity itself on cosmologically relevant physical scales \cite{modified_gravity}.

In the past decade, several groups have put a lot of effort into enlarging the sample size of well-measured SNe Ia events. At the same time, improvements in precision and assessments of systematic errors have also been accomplished. Recently, several SNe Ia samples containing a large number of SNe Ia events with high quality have been released, such as the Sloan Digital Sky Survey-II Supernova Survey (SDSS-II SN Survey) \cite{SNLS3, SCP}, the Union2 and Union2.1 SN Ia released by the Supernova Cosmological Project (SCP) \cite{SCP, JLA}, the first three years of Supernova Legacy Survey (SNLS3) \cite{SNLS3, SCP}, and the joint light-curve analysis (JLA) of SDSS-II and SNLS3 \cite{JLA, SCP}.

The distance estimation from SNe Ia data is based on the empirical observation that these events form a homogeneous class whose remaining variability is reasonably well captured by two parameters. One of them describes the time stretching of the light-curve ($x_1$), and the other describes the SNe Ia color at maximum brightness ($c$). In the latest JLA SNe Ia sample \cite{JLA}, which uses the SALT2 model to reconstruct light-curve parameters ($x_1$, $c$, and the observed peak magnitude in rest frame $B$ band $m_B^*$), the distance estimator assumes that SNe Ia with identical color, shape, and galactic environment have on average the same intrinsic luminosity for all redshifts. This assumption can be quantified by a linear expression, yielding a standardized distance modulus $\mu = 5 \log \left[ \frac{d_L}{10^\text{10} \text{Mpc}} \right] + 25$:

\begin{equation}
\mu = m_B^* - M + \alpha \times x_1 - \beta \times c,
\end{equation}

where $\alpha$ and $\beta$ are nuisance parameters which characterize the stretch-luminosity and color-luminosity relationships, reflecting the well-known broader-brighter and bluer-brighter relationships, respectively. The value of $M$ is another nuisance parameter representing the absolute magnitude of a fiducial SNe. In general, light-curve fitting parameters, $\alpha$ and $\beta$, are left as free parameters being determined in the global fit to the Hubble diagram. This treatment results in the dependence of distance estimation on the cosmological model used in the analysis. Thus, implications derived from SNe Ia observations with the light-curve fitting parameters determined in the global fit...
to the Hubble diagram are somewhat cosmological-model-dependent. Therefore, \( \alpha \) and \( \beta \) should be explicitly fitted along with the other cosmological parameters to get rid of the model-biased results when nonstandard cosmologies are investigated.

Our goal in this paper is to construct a completely cosmological model-independent Hubble diagram of SNe Ia using observational data of the so-called cosmic chronometers [21], where the cosmic expansion rates \( H(z) \) are measured from age estimates of red galaxies without any assumption of cosmology. In order to obtain the Hubble expansion as a function of redshift from which the light-curve fitting parameters are determined we use two reconstruction methods for \( H(z) \) data, namely, Gaussian Processes (GP) and a non-parametric smoothing (NPS) procedure. Using the latest JLA SN Ia [20], cosmological implications of the model-independent Hubble diagrams as well as the compatibility between the reconstructed methods considered are also investigated.

2. Methodology

The expansion rate, \( H = \dot{a}/a \) where \( a = 1/(1 + z) \), at redshifts \( z \neq 0 \) can be obtained by calculating the derivative of redshift with respect to cosmic time, i.e., \( H(z) \approx \frac{1}{a(z) (1 + z)} \). The difficult task here is to estimate the change in the age of the Universe as a function of redshift \( \Delta t \). This can be done by calculating the age difference between two luminous red galaxies at different redshifts, as proposed in Ref. [21]. This method is usually referred to as differential age and the passively evolving galaxies from which \( \Delta t \) is estimated are called cosmic chronometers. Currently, 21 measurements of \( H(z) \) based on this method lying in the redshift range \( 0.97 \leq z \leq 1.965 \) have been obtained [22, 23, 24, 25, 26]. Although cosmological model-independent, these estimates rely on stellar population synthesis models whose influence on \( \Delta t \), according to Ref. [27], becomes important at \( z > 1.2 \). In our analysis, we follow Ref. [22] and consider only 15 \( H(z) \) measurements up to \( z < 1.2 \) which, in practice, given the redshift distribution of the \( H(z) \) data, means \( z \leq 1.037 \). We also increase slightly (20%) the error bar of the highest-\( z \) point to account for the uncertainties of the stellar population synthesis models. This ensures that the evolution of the Hubble parameter as a function of redshift reconstructed in this paper is neither dependent on the cosmology nor on the stellar population model.

Recently, \( H(z) \) measurements were proposed to estimate distances by solving numerically the comoving distance integral for non-uniformly spaced data with a simple trapezoidal rule [22]. Naturally, the precision of this simple trapezoidal rule is sensitive to the uniformity of the spaced data and the number of data points in a certain spaced range. As indicated in Ref. [30], the relative errors of this method decrease remarkably when the number of intervals averagely spaced in \( z = 0 - 1 \) increases.

Here, we use two methods to reconstruct the evolution of the expansion rate with redshift from cosmic chronometers \( H(z) \) measurements. This procedure enable us to achieve model-independent distance estimations by integrating the inverse of the reconstructed function using the approach of Ref. [29] with a very small and uniform step of \( \Delta z = z_{i+1} - z_i \) (see Sec. 3).

2.1. Gaussian processes

As a powerful non-linear interpolating tool, the Gaussian processes allow us to reconstruct a function from data without assuming a model or parameterization for it. This method has been applied to several purposes, for instance, to reconstruct the equation of state of dark energy [31], to perform null test of the concordance model [32, 33], to infer \( H_0 \) from cosmic chronometer data [34], and to reconstruct the distance-duality relation [35].

The reconstruction is based on a mean function with Gaussian error bands, where the function value at \( z \) is not independent of the function value at some other point \( \bar{z} \) (especially when \( z \) and \( \bar{z} \) are close to each other) and they are related through a covariance function \( k(z, \bar{z}) \). This covariance function depends on a set of hyperparameters and there is a wide range of possible candidates for it. As the function of Hubble parameter versus redshift is expected to be infinitely differentiable, we consider the squared exponential covariance function:

\[
k(z, \bar{z}) = \sigma_f^2 \exp \left\{ -\frac{(z - \bar{z})^2}{2l^2} \right\},
\]

where the two hyperparameters \( \sigma_f \) and \( l \) are, respectively, related to typical changes in the function value and the length scale one needs to move in input space to get significant change in the function value. In order to obtain the value of the function, the hyperparameters should be trained by maximizing the marginal likelihood which only depends on the locations of the observations.

In this work, we reconstruct the Hubble parameter as a function of the redshift from 15 \( H(z) \) measurements of cosmic chronometers by using the GaPP (Gaussian Processes in Python) [31]. The result is displayed in Fig. (1).

2.2. Non-parametric smoothing method

In order to verify the influence of the reconstructing methods on the results, we also use the non-parametric procedure of Ref. [36, 37] to reconstruct the \( H(z) \) function from cosmic chronometers data. The smoothing function taking into account the data errors is given by

\[
H^*(z, \Delta) = H^g(z) + N(z) \sum_i \frac{[H(z_i) - H^g(z_i)]}{\sigma^2_{H(z_i)}} \times K(z, z_i)
\]

\[\text{http://www.aege.uct.ac.za/~seikel/GAPP/index.html}\]
where $H^s(z, \Delta)$, $H^b(z_i)$, $H(z_i)$, $\sigma_H(z_i)$ and $\Delta$ correspond, respectively, to the smoothed data, the initial guess model, the observed data, the error associated with the $H(z_i)$ data and the smoothing scale. The function $N(z)$ is the normalization factor given by: \begin{equation} N(z)^{-1} = \sum_i \frac{K(z, z_i)}{\sigma_h(z_i)^2}. \end{equation}

Given the arbitrariness in the choice of the kernel $K(z, z_i)$, we tested a Gaussian function and the lognormal kernel of Ref. [36]. We found a small difference between the reconstructions above into distance following the analytical method. At the redshift range considered, $\sigma_{H^s(z)}$ is the $1\sigma$ error of the reconstruction, $v_i$ is the smoothing factor ($v_i = N(z)K(z, z_i)/\sigma_h^2(z_i)$) and $\hat{\sigma}^2$ is the estimate of the error variance given by \begin{equation} \hat{\sigma}^2 = \frac{\sum_i (H(z_i) - H^s(z_i))^2}{\sum_j (1 - v_j(z_j))}. \end{equation}

In order to test the validity of this extrapolation, we simulate $H(z)$ data using the LCDM scenario as fiducial model with different values of $\sigma$ (\(\sigma_{sim}\)) and compare them with those (\(\sigma_{rec}\)) calculated from Eq. (4). We find that $\sigma_{rec}$ is $\approx 30\%$ smaller than $\sigma_{sim}$. We, therefore, add $30\%$ to the $\sigma_{H^s(z)}$ obtained from Eq. (6). Figure 1 shows both the GP and NPS reconstructions of the expansion history of the Universe from the cosmic chronometers data using the best estimate for the hyperparameters (GP) and $\Delta$ (NPS).

### 3. Results

In order to calibrate the light-curve fitting parameters and construct cosmological model-independent Hubble diagrams for the JLA sample, we first transform the $H(z)$ reconstructions above into distance following the approach proposed in Ref. [29]. Assuming a spatially flat universe, we solve numerically the distance integral and the corresponding uncertainty with a uniform step, $\Delta z = z_{i+1} - z_i = 0.005$. In the case of the standard LCDM model, we check the difference between the results from this numerical treatment and the one from the analytical method. At the redshift range considered, $z < 1.2$, the difference in distance modulus is $< 0.003$ mag, which
is negligible when compared to the uncertainties of current SNe Ia observations.

We now discuss comparatively the SNe Ia results from both reconstructing methods. For this, we perform a statistical analysis assuming a spatially flat $\Lambda$CDM model whose expansion rate is given by $H(z) = H_0[\Omega_m(1 + z)^3 + (1 - \Omega_m)]^{1/2}$, where $H_0$ and $\Omega_m$ are the current values of the Hubble and the matter density parameters, respectively. The results are shown in Fig. 2a. Clearly, the constraints on the light-curve fitting parameters depend upon the method applied to reconstruct the Hubble expansion. We note that, while the NPS reconstruction provides values of $\alpha$ and $\beta$ very close to the standard results from a global fit to the Hubble diagram, the GP prediction favours smaller values of these parameters. It is worth mentioning that the constraining power of the results shown in Fig. 2a is practically of the same order of magnitude as the one achieved in the global fit of Ref. [24]. For completeness, we also show in Figure 2b, the residuals between distances estimated by using the light-curve fitting parameters calibrated from the GP and NPS methods. In general, at low-$z$, distances derived from the Gaussian processes reconstruction are larger than the ones obtained from the smoothing method.

We also investigate some cosmological implications of the model-independent Hubble diagrams of JLA SN Ia. As the degeneracy between the Hubble constant and the absolute magnitude of a fiducial SNe Ia existing in the global fit has been broken when the model-independent calibration for light-curve fitting parameters is performed, constraints on cosmological parameters can be obtained in the framework of a given cosmology. Assuming a spatially flat $\Lambda$CDM scenario, the resulting $\Omega_m - H_0$ space is shown in Figs. (3). It is indicated that cosmological implications derived with different reconstruction methods are significantly discrepant. For the Gaussian processes, we obtain $\Omega_m = 0.284 \pm 0.019$ and $H_0 = 68.44 \pm 0.32$, at the 68.3% confidence level. These are fairly compatible with the constraints from the latest $Planck + WMAP9 + BICEP2$ CMB measurements reported in Ref. [39]. For the smoothing method, we obtain $\Omega_m = 0.266 \pm 0.019$ and $H_0 = 71.54 \pm 0.32$ which is consistent with the recent measurement of the local Hubble parameter obtained by considering recession velocity of objects around us ($H_0 = 73.80 \pm 2.40$) [40], at the 68.3% confidence level.

4. Conclusions

As is well known, implications derived from current SNe Ia analyses, where the light-curve fitting parameters are determined in the global fit from Eq. (1), are cosmological-model-dependent. In this paper, we have applied two methods to reconstruct the Hubble expansion using 15 $H(z)$ measurements ($z \leq 1.2$) from cosmic chronometers and transformed the reconstructed functions into distances by carrying out a numerical integration. The choice of this reduced $H(z)$ sample is based on the arguments of Refs. [27, 28], which ensures that the evolution of the Hubble parameter reconstructed in our analysis is neither dependent on the cosmology nor on the stellar population model.

By using the derived model-independent distances we have calibrated the light-curve fitting parameters and

![Figure 2](image-url)

**Figure 2:** Left: Contours of 68.3% and 95.4% confidence level for cosmological model-independent bounds on the light-curve fitting parameters, $\alpha$ and $\beta$, using JLA SNe Ia sample. Right: The estimated distance difference obtained from the light-curve fitting parameters calibrated with the GP and NPS methods.
constructed a completely cosmological model-independent Hubble diagram for the JLA sample. The results suggest that the uncertainties on the light-curve fitting parameters obtained from the $H(z)$ reconstructions are almost of the same order of magnitude as the ones determined in the global fit for the ΛCDM model. Therefore, we expect the constraining power of any analysis derived from these $H(z)$-calibrated Hubble diagrams to be nearly identical to the one obtained when the global fit to a given model is performed. It should be emphasized, however, that cosmological implications of the Hubble diagrams constructed from $H(z)$ data do not suffer with cosmological model-dependence.

However, we have shown that these diagrams and their implications are heavily dependent on the method used to reconstruct the Hubble evolution. Assuming the spatially flat ΛCDM model, we have derived constraints on the matter density parameter $\Omega_m$ and Hubble constant $H_0$ from the JLA sample. For the analysis that uses Gaussian processes, it is shown that both model parameters are quite compatible with what is obtained from the latest Planck+WMAP9+BICEP2 CMB observations at 68.3% confidence level. When the smoothing method is applied, the bounds favor a lower value of $\Omega_m$ and a higher value of $H_0$, which is consistent with the recent local measurement of the expansion rate from Cepheids observations.

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