Forecasting the Quarterly Production of Rice and Corn in the Philippines: A Time Series Analysis

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Abstract. Rice and corn are crops that are produced on most of the agricultural areas in the Philippine geographic territory. Gathered from the Philippine Statistics Authority (PSA), the quarterly volume of total production of the mentioned crops will be predicted using Seasonal Autoregressive Integrated Moving Average (SARIMA) Modelling through a Box-Jerkins method of forecasting. Investigators aims to forecast rice production and Corn production actual observed in the next 5 years of data, particularly in years mentioned. Obtaining models through SARIMA method, for rice production is SARIMA (2 1 8) (1 1 0)4 and for Corn production would be SARIMA (3 1 8) (0 1 1)4. Results suggest that two models have satisfactorily fitted their specified series and diagnostic checking for models’ accuracy. The dynamically forecasted values of both series were following the trend pattern of the past values of their allotted observed values which were going on an upward direction indicating favourable outcome for the study. This may help people for better understanding the possible changes in values of the data series gathered and may even help the government for any reforms they can make for the upward mobility of production of crops, in agriculture and especially, economy of the country.

1. Introduction
The Philippines has been one of the world’s largest suppliers in the global market since it has two season which could sustain the needs of the plants, forestry and fisheries. On that note, rice production gives off the supply for the demand of the local people and also makes good trading internationally. Rice is one most produced of the crops because it is most needed by people for each day, and is mainly consumed as the main part of the Filipino meals and for almost three times a day, moreover, it sustains proteins and calories that are essential for the body. However, the harvested areas are still
very small compared to the consumption. This leads to more importation of rice products from other countries like Vietnam and Thailand making the Philippines the biggest importer in Asia. On the other hand, the Philippine production of rice increased for approximately 70% which came from the irrigated areas from 1995 to 2010. \[1\]

Corn also is one utmost massively produced crops in the Philippines and in this country is known as the substitute food whenever there is a shortage for the production of rice. Various classification of corn like yellow corn which is mostly being locally utilized for the livestock and poultry since it is most commonly primary source of food in the animal industry, white corn as the staple food for almost 20 percent of the population primarily in the southern regions of Visayas and Mindanao and makes a major ingredients for producing raw material and industrial products like starch oil and organic liquids, a mere corn having as most of use like the two mentioned and others. \[2\] From 1980, the Philippines produces for about 3 million metric tons of corn crops from almost 3.2 million hectares of land which surged up to approximately 5 million after the land tilled were increased for about 0.6 million hectares for ten years. In year 2000, the production reduced by 0.3 million metric tons of total corn crops harvested from the number of areas declined which was about 1.6% annually. \[3\] It was signposted in an article by Louise Maureen Simeon (2016) that the production of corn in the Philippines at present has been 7% less than the earlier forecast of 4.37 million metric tons of corn products, by the Philippine Statistics Authority in 2015. \[4\]

On that note, the researchers plan on forecasting the rice and corn production for the year 2016-2020. The aim of this study is to provide information for the government’s use, and to those that are authorized to make decisions regarding the topics discussed to ensure the betterment of however they can improve the trading statuses of the crops mentioned in the study. This may even help the common people to attain information about the process regarding the production, the system of trading, and how the possible financial aspect is being assessed through the processes involved.

### 1.1. Objective of the Study

The main purpose of this study is to forecast the production of rice and corn utilizing the Seasonal Autoregressive Integrated Moving Average (SARIMA) through the Box-Jenkins Approach. With the use of SARIMA we can determine rice and corn can be produced for the year 2016-2020. Forecasting the amount of rice and corn production can support the government for their further decisions in the near future.

### 1.2. Statement of the Problem

This research aim to encourage the government to engross in rice and corn production by answering the following questions:

1) **What are the behavior of the graphs for:**
   a) Rice Production
   b) Corn Production

2) **What are the models that will be used in forecasting the Rice and Corn production in the Philippines?**

3) **What will be the predicted values of each of the Rice production and Corn Production?**

4) **What will be the Forecasted Values for the next five years of each of the two mentioned series?**

5) **Is there a significant difference between the values of:**
   a) Predicted and the actual values of Rice production?
   b) Predicted and the actual values of Corn production?

### 1.3. Scope and Limitations

The researchers limited the crops as rice and corn with a quarterly volume of production from year 1987-2015. The data were gathered from the Philippine Statistics Authority (PSA). The study will forecast the quarterly volume of production for the next five years (2016-2020) with the same location
and unit using the chosen methods of forecasting: Seasonal Autoregressive Integrated Moving Average (SARIMA) Modelling through a Box-Jerkins Approach.

1.4. Significance of the Study
This approach was already used as the statistical basis of analysis the huge amount data by the investigators for predicting the future values in which might be a benefit for the observation of a particular series, especially in economics like financials, vast units of goods, prices changes, and many others that are recorded through time.

The study may help the decision makers to generate any agricultural reforms that could support the production of the crops mentioned in the study, and for so, can help the people for better understanding of trading of the goods and to be more mobilised after knowing the results, the future values in the next 5 years, if ever might become favourable after the systematic changes in the production.

1.5. Research Paradigm

2. Methodology
The quarterly data of import and export trades Rice and corn of the Philippines from 2016-2020 were gathered to produces mathematical models using Seasonal Autoregressive Integrated Moving Average (SARIMA) with the same location and unit using the chosen methods of forecasting: Seasonal Autoregressive Integrated Moving Average (SARIMA) Modelling through a Box-Jerkins Approach.14

2.1. Box-Jenkins
This approach comprises four steps in analyzing time based data. A step by step process introduced by the statisticians, George Box and Gwilym Jenkins, fitting well identified model through the Autoregressive Integrated Moving average, a time series analysis represented by ARIMA (p, d, q), (P, D, Q)s where p, d, and q indicates the non-seasonal part of the model, and P, D, and Q the are seasonal part of the model for s seasonality interval of the time series.15 The approach uses four stages of modeling:

The model identification contains certain assumptions to be well thought-out in a time series analysis. On the first part of the model identification, supposing that the series is normally distributed after testing the normal distribution of the data through the Jarque-Bera test for normality, making certain classification of the model is primarily considered, whether the model to be used in the series has seasonally based or not, by which the series may have been seasonally adjusted to moderately deduce the seasonal component, if present within the data. Seasonality, along with the trend & cycle, is one of the seasonal components that are assumed to be present in a time series. It is the recurrence of a pattern in the same length of period after it happened. Another assumption is the stationarity of the
data. Assumption if there is a presence of a unit root within the series. Unit root is a random walk component that makes the data series be less possible to predict. In other words, unit root is a feature or a characteristic of a time series data having non-standard and non-normal asymptotic distributions that is not predictable. There is an augmented version of the test used to determine the stationarity of the data that is developed by the statisticians, David Dickey and Wayne Fuller, called the Dickey-Fuller Test applied to the equation

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \ldots + \delta_p \Delta y_{t-p+1} + \epsilon_t$$\textsuperscript{[16]}

Where $\alpha$ is constant, $\beta$ is the coefficient on a time trend, for a lag order of $p$ in an autoregressive process by which it tests the assumption of the presence of a unit root, preferably used by researchers for much larger amount of data and a lot more complicated set.\textsuperscript{[17]}

The second part will be the model estimation which comprises the estimation of the model where a series will be choosing Autoregressive terms and Moving Average terms out of the transformed series, to be regressed against the actual observed data series in which the series may have been differenced non-seasonally, undergone seasonal adjustment and logarithmically transformed based on the identification results on the first part of the approach in generating the model. It is where the possible combinations of the AR terms and the MA terms, as well as the seasonal terms that may possibly be included as part of the model. The plotted autocorrelation and partial-autocorrelation of the transformed series may show significant spikes indicates the AR and MA terms in specific lags.

In the diagnostic checking of the series, among the possible models that will be listed, one with favourable diagnostics will be the model to be used in forecasting the actual series. Diagnostic checking comprises the generation of the expected values that will be compared with the actual observed values so that the difference can be check with the mean zero. It also includes the R-square value that is that basis of the accuracy of the model and the Durbin-Watson test statistics for the goodness-of-fit-test, and the AR and MA Processes where to test if the autoregressive terms are stationary and if the moving average terms are invertible or not. Last part of the approach is regressing the model chosen in the process to forecast the series. Forecasting the series will be held to represent the possible outcomes in the near future. The graphical results as well as the listed values will be generated.

3. Results and Discussions

3.1. Rice

3.1.1. Actual Data and Logarithmic Transformation

The data shows an obvious trend started between the year 1995 and 2000, and the data series patterns exhibits which is an obvious seasonal recurrence of a pattern. At every 4\textsuperscript{th} quarter, the series exceeds the preceding values on an instantaneous upsurge. This indicates that the total production in the last three months, specifically: October, November and December, always having the highest production.
each year. This may be possible because the farmers are more likely to work on less warmer or more humid seasons, because it tends to have an occurrence of longer dry season called El Niño on the earlier part of the year due to climate change.

Also, the correlogram displays possible occurrence of a unit root in the series since the autocorrelation displays significant spikes that are recurring at every fourth lags in the autocorrelation which slowly turns to zero. This indicates the presence of unit root and seasonality of the data. However, it is necessary to have the presence of seasonality further tested in statistical basis.

The results of seasonality test suggest that there is a presence of seasonality failing to reject the null hypotheses which propose that seasonal adjustment is necessary for the data series. However, the autocorrelation has significant spikes at every after 3rd lags that are having values which were slowly turning to zero. This indicates that there might be a presence of a unit root within the series and it indicates that the series is not stationary.

To further test the stationarity of the data series, the augmented version of the Dickey-Fuller Test is performed. The results show that there exists a unit root in the actual observations. Though, it also shows that if the series undergoes an order of non-seasonal differencing, it becomes stationary.

It can be seen at the graph of the logarithmic transformed Rice production data series that the values has been reduced that utterly follows the same pattern, a recurrence of an upsurge value at every 4th quarter. Also, the variance has been minimized, the linear variations has been compressed which makes a minimal fluctuations in the preceding years. The significance level has reduced giving of more significant spikes at specific lags in the Correlogram of the series. At the fourth lag, both the autocorrelation and partial autocorrelation displayed significant spikes which mean a seasonal component is still present. Furthermore, it still follows the same outline of significant spikes of the correlogram in the actual observed values where at every after 3rd lags there occurs significant spikes that are slowly turning to zero value. These simply suggest that there might still be a presence of unit root and it means an order of non-seasonal differencing is still necessary.

### 3.1.2. Differencing Seasonally and Non-seasonally

![Figure 4. Non-seasonally Differenced Rice Production Series](image1)

![Figure 5. Seasonally Differenced Rice Production Series](image2)

Differencing the series reduced negative values and made the linear fluctuation having the mean zero, reduced its trend as well as the vertical values where it have now its units smaller than the preceding graph of the transformed data series. The correlogram have more spikes became significant on the negative side of the autocorrelation. The correlogram also states that the spikes at the 4th lag of autocorrelation and partial autocorrelation are significant having a seasonal component present. Moreover, the data series has already rejected the null hypothesis in the augmented Dickey-Fuller test on the first difference. It means that the data series is statistically tested to be stationary on the first order of differencing. It suggests that it no longer need of differencing the series non-seasonally.

The linear graph of the seasonally differenced series of the Rice production after it was transformed logarithmically and differenced on the first order of non-seasonal differencing, the linear plot shows...
that the line wavers vigorously while following zero trend and it also indicates that the mean now is much closer to zero. It can now be inferred the number of terms that can be used to create a model for the actual data series.

Estimating the number of terms to be included in the model, the possible combinations of the terms were regressed against the actual data series and produced the R-square values, Durbin-Watson statistics of each of the combinations, as well as their AR processes whether they are stationary or not, and the MA processes if it can be inverted or not. Among the listed combinations, 6 combinations with the highest R-square values were included to the selection of the possible models, however, it seems that their AR processes were not stationary, and one of them has its MA process appears not to be invertible. Regressing it again to the actual values of the Rice production, by which a constant now is included so that the AR terms would minimize its effect without adding more moving average terms, some combinations appeared stationary. One with highest R-square value is chosen to model the actual data series of the Rice production.

The model obtained has a total of 2 AR terms, 1 seasonal AR, and 8 MA terms including the constant. It was having the highest r-square value, which is about 86% probability, among the rest of the selection with acceptable Durbin-Watson test Statistics, invertible Moving Average processes, and stationary Autoregressive processes. The model will be SARIMA (2 1 8) (1 1 0), by which will be used in forecasting the actual series.

### 3.1.3. Static and Dynamic Forecasting of the Rice Production Series

The plotted graph of the predicted values in Figure 6, generated through static forecasting followed the same trend when compared with the actual series. Replicating the same pattern of the actual values, where on every 4th quarter, it has sudden up surged value then makes a downfall for the next three quarters after it. This may indicate that there might be no significant difference between the expected values from the observed values of the Rice production.

To test this assumption, paired t-test was performed to statistically infer that the two compared series have not much of a difference with each other. Paired t-test gives a 0.89335022 probability value which indicates the rejection of the null hypothesis which signifies that there is no significant difference between the compared values. Thus, we can say that the model is actually fitted to the data series.

Hence, in Figure 7, the forecasted values gives off a plotted graph that follows the same trend component, an upward direction of the line, where the line wavers on the same seasonal component vigorously. It shows an upsurge value at every 4th quarter of the year. However, it seems that the line reduces its variation year after year. This indicates that the model could forecast effectively for the next few years which is fairly favourable for predicting the data series.
3.2. Corn Production

3.2.1. Actual Data and Logarithmically Transformed

On the graph of the actual values of the total corn production, which includes the corn, white corn and yellow corn, it can be seen that the values follow a pattern where it has bigger changes of values that somehow it seemed that the values are almost the same at two consecutive quarters and makes a sudden huge increase in the 3rd quarter and a slight drop on the 4th quarter in a year. This sort of pattern may also indicate a seasonal component which means it might have seasonality. However, bring about on the beginning of the data until the after 1995 where it has wavered in to some extent, different than the previous form and procured into a trend going upward. Dropping into lesser amount except from a particular quarter in 1998, and continues to increase values after 2002. The plotted correlogram of the actual series of values of corn production has its significant spikes on both autocorrelation and partial autocorrelation at 4th lag which means it has a recurring pattern at every 4th quarter of the actual data series of the corn production. It indicates seasonality within the series of the actual observed values of Corn Production. Then, to statistically infer the assumed presence of seasonal component, seasonality test is established where the results indicates less than hypothesized probability F-stats value of 0.001 which means it has failed to reject the null hypothesis that there is a seasonal component within the series. Hence, the data series of the actual observed values of corn production have a presence of seasonality. Also, the autocorrelation displays significant spike at every after 3rd lags slowly decreasing in length until it reaches zero on a particular lag. This indicates that there might be a presence of a unit root which means the data might not be stationary. Testing the stationarity of the data, augmented dickey-fuller test results suggests that the actual values has a unit root since the value greater than 0.01 hypothesized probability, it has failed to reject the null hypothesis even when intercept has included, both trend and intercept, or none. The test result shows that the data needs to be differenced at least once with non-seasonal differencing.

The graph of logarithmic transformed data series of corn production displays the same pattern which is having two consecutive quarters having nearly similar values and makes an unexpected hug increase on the 3rd quarter and a slight drop on the 4th quarter. Then after 1995, it starts to waver on a slightly different way from the preceding term. The logarithmic transformed values slightly differ from the actual observed values in terms of trend component. It seems that the trend of the logarithmic transformed data has continuous trend from the very beginning of the series. On the other hand, the actual values are having a few changes with its trend on a particular group of years. Also, the correlogram have significant spikes on both 4th lags of autocorrelation and partial autocorrelation. Thus, indicating the presence of seasonality. Moreover, on every after 3rd lags significant spikes appears on the plotted autocorrelation that are slowly turning to zero. It means that there might still be a presence of unit root within the series and it signifies that transformed data is not stationary.

3.2.2. Differencing Seasonally and Non-seasonally
Differencing the series, the values are distributed and having negative values acquire the same distribution with the positive values. It means that the different logarithmic series are nearly having zero mean value. And also, the correlogram of the data series has reduced its significant level making more significant spikes on the left side of the plotted autocorrelation. The correlogram also suggest that there still might be a presence of seasonality within the series given that both autocorrelation and partial autocorrelation have significant spikes on the 4\textsuperscript{th} lags.

The linear graph of the seasonal adjusted differenced logarithmically transformed data of corn production wavers on a vigorous manner despite the fact of having no trend at present, which means it has now zero mean value. On the plotted correlogram, autocorrelation and partial autocorrelation exhibits significant spikes at specific lags which can be concluded as autoregressive and moving average terms to be used to create a list of combinations. These combinations will be used to estimate the model, utilizing it to produce expected predicted values. Additionally, forecasted values for the next five years of the data series of corn production will be generated as an output of the study.

It can now be inferred in the correlogram, the number of terms indicated by the significant spikes on the specific lags in the autocorrelation and partial autocorrelation, to be include to create combinations of autoregressive terms and moving average terms to be regressed with the actual data series of the corn production and thus, to produce the diagnostic tests results – the R-square values of each combinations, as well as their Durbin Watson statistics, and the AR and MA Processes.

Checking the diagnostics, there were few of the combinations that are having stationary AR processes and invertible MA processes. Hence, choosing the model to be used from these combinations, the one with the highest r-square value of is having 3 AR terms, 8 MA terms and 1 seasonal MA term, has its AR processes to be stationary, MA processes were invertible and the Durbin-Watson stats was very satisfactory at 73% R-square Probability. The model will be SARIMA (3 1 8) (0 1 1), which will be used to forecast the actual data series of the corn production.

3.2.3. Static and Dynamic Forecasting of the Corn Production Series
The graph of the Predicted values generated using static forecasting, emulating similar trend when plotting it along with the actual series. The dropping values every after two consecutive quarters and upturns to a huge change of value on the 3rd quarter and smaller dropping value each year starting from earlier than 1990 until after year 1995. Then, it wavers on a different manner on a slow upward trend until the end of the actual observed series. The predicted series differs on the variance since the actual observed values emits bigger variations. But to test if there is a significant difference between the two series, paired t-test was perform. Thus, the test result indicates that there is no significant difference between the two compared series. Since the probability value 0.955, it suggests that the null hypothesis was rejected.

Moving on to forecasting in Figure 2.5, the actual data series of the corn production, the forecasted values also gives off a plotted graph that follows the same upward trend linear wavering slowly reducing in variance. Therefore, it means the model can forecast fairly enough for the next few years. The values increases slowly upon having the same fluctuating pattern from the preceding years, however, having less and less variance.

4. Conclusions and Recommendation
The study utterly submits the models of the Rice and Corn production to be used in forecasting the two mentioned data series. Coming up with these models through SARIMA method, the investigators obtained the first model, for the Rice production which is SARIMA (2 1 8) (1 1 0)4 and the second model, for the Corn production would be SARIMA (3 1 8) (0 1 10)4. The corn production predicted values, as well as the predicted values of the Rice production, were statistically inferred that there are no significant differences with their particular actual observed values. Results suggesting that the two models has satisfactorily fitted their specified series and satisfied the diagnostic checking for the models. The dynamically forecasted values of both series were following the trend pattern of the past values of their allotted observed values which were going on an upward direction indicating favorable outcome for the study.

Showing results that are favourable enough, such as having the expected future values on both observations to continue to rise up time after time, the investigators suggests to further include the actual observed values in the near future to be compared with the forecasted values of the Rice and corn production in the study. This may be even of use for the decision making that the government has to make in the future instances, to make the possible changes and reforms for any possible improvement of the production of both corn and Rice.

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