The proton spin puzzle: where are we today?

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Abstract

The proton spin puzzle has challenged our understanding of QCD for the last 20 years. New measurements of polarized glue, valence and sea quark polarization, including strange quark polarization, are available. What is new and exciting in the data, and what might this tell us about the structure of the proton? The proton spin puzzle seems to be telling us about the interplay of valence quarks with the complex vacuum structure of QCD.
1 Introduction

Protons behave like spinning tops. Unlike classical tops, however, the spin of these particles is an intrinsic quantum mechanical phenomenon. This spin is responsible for many fundamental properties of matter, including the proton’s magnetic moment, the different phases of matter in low-temperature physics, the properties of neutron stars, and the stability of the known universe. How is the proton’s spin built up from its quark and gluon constituents?

It is 20 years since the European Muon Collaboration (EMC) published their polarized deep inelastic measurement of the proton’s $g_1$ spin dependent structure function and the flavour-singlet axial-charge $g_{A}(0)|_{\text{PDIS}}$ [1]. Their results suggested that the quarks’ intrinsic spin contributes little of the proton’s spin. The challenge to understand the spin structure of the proton [2, 3] has inspired a vast programme of theoretical activity and new experiments at CERN, DESY, JLab, RHIC and SLAC. Where are we today?

We start by recalling the $g_1$ spin sum-rules.

These are derived starting from the dispersion relation for polarized photon-nucleon scattering and, for deep inelastic scattering, the light-cone operator product expansion. One finds that the first moment of the $g_1$ structure function is related to the scale-invariant axial charges of the target nucleon by

\begin{equation}
\int_{0}^{1} dx \ g_{1}^{p}(x, Q^2) = \left( \frac{1}{12} g_{A}^{(3)} + \frac{1}{36} g_{A}^{(8)} \right) \left\{ 1 + \sum_{\ell \geq 1} c_{N\ell} \alpha_{s}(Q) \right\} \\
+ \frac{1}{9} g_{A}(0)|_{\text{inv}} \left\{ 1 + \sum_{\ell \geq 1} c_{S\ell} \alpha_{s}(Q) \right\} + O(\frac{1}{Q^2}) + \beta_{\infty}.
\end{equation}

Here $g_{A}^{(3)}$, $g_{A}^{(8)}$ and $g_{A}(0)|_{\text{inv}}$ are the isovector, SU(3) octet and scale-invariant flavour-singlet axial charges respectively. The flavour non-singlet $c_{N\ell}$ and singlet $c_{S\ell}$ Wilson coefficients are calculable in $\ell$-loop perturbative QCD [4]. The term $\beta_{\infty}$ represents a possible leading-twist subtraction constant from the circle at infinity when one closes the contour in the complex plane in the dispersion relation [2]. If finite, the subtraction constant affects just the first moment sum-rule and, thus, corresponds to Bjorken $x = 0$. The first moment of $g_1$ plus the subtraction constant, if finite, is equal to the axial-charge contribution. The subtraction constant corresponds to a real term in the spin-dependent part of the forward Compton amplitude.

In terms of the flavour dependent axial-charges

\begin{equation}
2M s_{\mu} \Delta q = \langle p, s | \gamma_{\mu} \gamma_{5} q | p, s \rangle
\end{equation}

the isovector, octet and singlet axial charges are:

$$ g_{A}^{(3)} = \Delta u - \Delta d $$
\[ g_A^{(8)} = \Delta u + \Delta d - 2\Delta s \]
\[ g_A^{(0)}|_{\text{inv}} / E(\alpha_s) \equiv g_A^{(0)} = \Delta u + \Delta d + \Delta s. \]  

(3)

Here

\[ E(\alpha_s) = \exp \int_0^{\alpha_s} d\tilde{\alpha}_s \gamma(\tilde{\alpha}_s) / \beta(\tilde{\alpha}_s) \]  

(4)

is a renormalization group factor which corrects for the (two loop) non-zero anomalous dimension \( \gamma(\alpha_s) \) of the singlet axial-vector current

\[ J_{\mu 5} = \bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d + \bar{s}\gamma_\mu \gamma_5 s \]  

(5)

which goes to one in the limit \( Q^2 \to \infty \); \( \beta(\alpha_s) \) is the QCD beta function. We are free to choose the QCD coupling \( \alpha_s(\mu) \) at either a hard or a soft scale \( \mu \). The singlet axial charge \( g_A^{(0)}|_{\text{inv}} \) is independent of the renormalization scale \( \mu \) and corresponds to \( g_A^{(0)}(Q^2) \) evaluated in the limit \( Q^2 \to \infty \). The perturbative QCD expansion of \( E(\alpha_s) \) remains close to one – even for large values of \( \alpha_s \). If we take \( \alpha_s \sim 0.6 \) as typical of the infra-red then \( E(\alpha_s) \simeq 1 - 0.13 - 0.03 + ... = 0.84 + ... \) where -0.13 and -0.03 are the \( \mathcal{O}(\alpha_s) \) and \( \mathcal{O}(\alpha_s^2) \) corrections respectively.

If one assumes no twist-two subtraction constant (\( \beta_\infty = O(1/Q^2) \)) then the axial charge contributions saturate the first moment at leading twist. The isovector axial-charge is measured independently in neutron beta-decays (\( g_A^{(3)} = 1.270 \pm 0.003 \) [5]) and the octet axial charge is commonly taken to be the value extracted from hyperon beta-decays assuming good SU(3) properties (\( g_A^{(8)} = 0.58 \pm 0.03 \) [6]). From the first moment of \( g_1 \), polarized deep inelastic scattering experiments have been interpreted to imply a small value for the flavour-singlet axial-charge. Inclusive \( g_1 \) data with \( Q^2 > 1 \) GeV\(^2\) give [7]

\[ g_A^{(0)}|_{\text{pDIS, } Q^2 \to \infty} = 0.33 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.}) \]  

(6)

– considerably less than the value of \( g_A^{(8)} \) quoted above. In the naive parton model \( g_A^{(0)}|_{\text{pDIS}} \) is interpreted as the fraction of the proton’s spin which is carried by the intrinsic spin of its quark and antiquark constituents. When combined with \( g_A^{(8)} = 0.58 \pm 0.03 \) this value corresponds to a negative strange-quark polarization

\[ \Delta s_{Q^2 \to \infty} = \frac{1}{3}(g_A^{(0)}|_{\text{pDIS, } Q^2 \to \infty} - g_A^{(8)}) = -0.08 \pm 0.01(\text{stat.}) \pm 0.02(\text{syst.}) \]  

(7)

– that is, polarized in the opposite direction to the spin of the proton. The corresponding up and down quark polarizations are likewise extracted to be

\[ \Delta u_{Q^2 \to \infty} = 0.84 \pm 0.01(\text{stat.}) \pm 0.02(\text{syst.}) \]
\[ \Delta d_{Q^2 \to \infty} = -0.43 \pm 0.01(\text{stat.}) \pm 0.02(\text{syst.}) \]  

(8)

Relativistic quark models generally predict values \( g_A^{(0)} \sim 0.6 \) with little polarized strangeness in the nucleon[8] in agreement with the value of \( g_A^{(8)} \) extracted from
SU(3). The Bjorken sum-rule for the isovector part of $g_1$, $\int_0^1 dx g_1^{p-n} = \frac{1}{6} g_A^{(3)} \left\{ 1 + \sum_{\ell\geq 1} c_{\text{NSF}} \alpha_s^{(\ell)}(Q) \right\}$, has been confirmed in polarized deep inelastic scattering experiments at the level of 10% [9].

The results from polarized deep inelastic scattering pose the following questions:

- How is the spin $\frac{1}{2}$ of the proton built up from the spin and orbital angular momentum of the quarks and gluons inside?
- Why is the quark spin content $g_A^{(0)}|_{\text{DIS}}$ so small?
- How about $g_A^{(0)} \neq g_A^{(8)}$? What separates the values of the octet and singlet axial-charges?
- Is the proton spin puzzle a valence quark or sea/glue effect?

We next discuss the experiments that have been performed to address these questions.

2 The shape of $g_1$

Deep inelastic measurements of $g_1$ have been performed in experiments at CERN, DESY, JLab and SLAC. There is a general consistency among all data sets. COMPASS are yielding precise new data at small $x$, down to $x \sim 0.004$, which is shown for $g_1^d$ in Fig.1. JLab are focussed on the large $x$ region. To test deep inelastic sum-rules it is necessary to have all data points at the same value of $Q^2$. In the experiments the different data points are measured at different values of $Q^2$, viz. $x_{\text{expt.}}(Q^2)$. Next-to-leading order (NLO) QCD-motivated fits taking into account the scaling violations associated with perturbative QCD are frequently used to evolve all the data points to the same $Q^2$. 

![Figure 1: $g_1^d$ data from COMPASS.](image-url)
The COMPASS measurements of the deuteron spin structure function $g_1^d$ show the remarkable feature that $g_1^d$ is consistent with zero in the small $x$ region between 0.004 and 0.02 [7]. In contrast, the isovector part of $g_1$ is observed to rise at small $x$ ($0.01 < x < 0.1$) as $\sim x^{-0.5}$ and is much bigger than the isoscalar part of $g_1$. This is in sharp contrast to the situation in the unpolarized structure function $F_2$ where the small $x$ region is dominated by isoscalar pomeron exchange. The evolution of the Bjorken integral $\int_{x_{\text{min}}}^{1} dx g_{1}^{p-n}$ as a function of $x_{\text{min}}$ as well as the isosinglet integral $\int_{x_{\text{min}}}^{1} dx g_{1}^{p+n}$ are shown in Fig. 2 (HERMES data[10]). About 50% of the Bjorken sum-rule $\int_{0}^{1} dx g_{1}^{p-n}$ comes from $x$ values below about 0.12. The $g_{1}^{p-n}$ data are consistent with quark model and perturbative QCD predictions in the valence region $x > 0.2$ [11]. The size of $g_A^{(3)}$ forces us to accept a large contribution from small $x$ and the observed rise in $g_{1}^{p-n}$ is required to fulfil this non-perturbative constraint, perhaps signifying a hard Regge exchange[12] like the hard pomeron in unpolarized deep inelastic scattering[13].

The “missing spin” is associated with a “collapse” in the isosinglet part of $g_1$ to something close to zero instead of a valence-like rise $\sim x^{-0.5}$ for $x$ less than about 0.03. The isosinglet integral appears to converge at $x_{\text{min}} \sim 0.1$. This isosinglet part is the sum of SU(3)-flavour singlet and octet contributions. If there were a large positive polarized gluon contribution to the proton’s spin, this would act to drive the small $x$ part of the singlet part of $g_1$ negative[14] – that is, acting in the opposite direction to any valence-like rise at small $x$. However, gluon polarization measurements at COMPASS, HERMES and RHIC constrain this spin contribution to be small in measured kinematics – see below – meaning that the sum of valence and sea quark contributions is suppressed at small $x$. (Soft Regge theory predicts that the singlet term should behave as $\sim N \ln x$ in the small $x$ limit, with the coefficient $N$ to be determined from experiment[15, 16].) Further data from HERMES and COMPASS involving semi-inclusive measurements of fast pions and kaons in the final state is being used to constrain the sea and valence quark spin contributions and reveals no evidence for polarized strangeness, anti-up or anti-down spin polarization in the proton (in apparent contrast to the extraction of negative strangeness polarization extracted from inclusive measurements of $g_1$). We next discuss this spin decomposition and the different measurements.

### 3 Spin and the singlet axial charge

There has been considerable theoretical effort to understand the flavour-singlet axial-charge in QCD. QCD theoretical analysis leads to the formula

$$g_A^{(0)} = \left( \sum \Delta q - 3 \alpha_s \Delta g \right)_{\text{partons}} + C_\infty.$$  \hspace{1cm} (9)
Figure 2: Convergence of the first moment integrals for the proton, neutron, deuteron and isovector (NS) \( g_1 \) combination in HERMES data\(^{10} \).

Here \( \Delta g_{\text{partons}} \) is the amount of spin carried by polarized gluon partons in the polarized proton (\( \alpha_s \Delta g \sim \text{constant} \) as \( Q^2 \rightarrow \infty \) \([17, 18]\)) and \( \Delta q_{\text{partons}} \) measures the spin carried by quarks and antiquarks carrying “soft” transverse momentum \( k_t^2 \sim P^2, m^2 \) where \( P \) is a typical gluon virtuality and \( m \) is the light quark mass \([17, 18, 19, 20]\). The polarized gluon term is associated with events in polarized deep inelastic scattering where the hard photon strikes a quark or antiquark generated from photon-gluon fusion and carrying \( k_t^2 \sim Q^2 \) \([19]\). \( C_\infty \) denotes a potential non-perturbative gluon topological contribution \([21]\) which is associated with the possible subtraction constant in the dispersion relation for \( g_1 \) \([2]\). If finite it would mean that \( \lim_{\epsilon \to 0} \int_0^1 dx g_1 \) will measure the difference of the singlet axial-charge and the subtraction constant contribution; that is, polarized deep inelastic scattering measures the combination \( g_A^{(0)}|_{\text{pDIS}} = g_A^{(0)} - C_\infty \).

Possible explanations for the small value of \( g_A^{(0)}|_{\text{pDIS}} \) extracted from the polarized deep inelastic experiments include screening from positive gluon polarization, negative strangeness polarization in the nucleon, a subtraction at infinity in the dispersion relation for \( g_1 \) associated with non-perturbative gluon topology and connections to axial U(1) dynamics \([22, 23, 24, 25]\).

One would like to understand the dynamics which appears to suppress the singlet axial-charge extracted from polarized deep inelastic scattering relative to the OZI prediction \( g_A^{(0)} = g_A^{(8)} \sim 0.6 \) and also the sum-rule for the longitudinal spin structure of the nucleon

\[
\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \Delta g + L_q + L_g
\]  \hspace{1cm} (10)
where $L_q$ and $L_g$ denote the orbital angular momentum contributions.

There is presently a vigorous programme to disentangle the different contributions. Key experiments involve semi-inclusive polarized deep inelastic scattering (COMPASS and HERMES) and polarized proton-proton collisions (PHENIX and STAR at RHIC).

### 3.1 NLO QCD motivated fits to $g_1$

The first attempts to extract information about gluon polarization in the polarized nucleon used next-to-leading order (NLO) QCD-motivated fits to inclusive $g_1$ data.

Similar to the analysis that is carried out on unpolarized data, global NLO perturbative QCD analyses have been performed on the polarized structure function data sets. The aim is to extract the polarized quark and gluon parton distributions. These QCD fits are performed within a given factorization scheme. New fits are now being produced taking into account all the available data including new data from polarized semi-inclusive deep inelastic scattering. The largest uncertainties in these fits are associated with the ansatz chosen for the shape of the spin-dependent quark and gluon distributions at a given input scale. Further, the SU(3) value of $g_A^{(8)} (= 0.58 \pm 0.03)$ is assumed in these fits. Fits to the most recent world data on $g_1$ give “small” values of $|\Delta g| \simeq 0.2 - 0.3$ for $Q^2 = 3$ GeV$^2$ [7, 26]. To go further more direct measurements involving glue sensitive observables are needed to really extract the magnitude of $\Delta g$ and the shape of $\Delta g(x, Q^2)$ including any possible nodes in the distribution function.

### 3.2 Gluon polarization

There is a vigorous and ambitious global programme to measure $\Delta g$. Interesting channels include gluon mediated processes in semi-inclusive polarized deep inelastic scattering (COMPASS) and hard QCD processes in high energy polarized proton-proton collisions at RHIC.

The first experimental attempt to look at gluon polarization was made by the FNAL E581/704 Collaboration which measured the double-spin asymmetry $A_{LL}$ for inclusive multi-$\gamma$ and $\pi^0\pi^0$ production with a 200 GeV polarized proton beam and a polarized proton target suggesting that $\Delta g/g$ is not so large in the region of $0.05 < x_g < 0.35$ [27].

COMPASS has been conceived to measure $\Delta g$ via the study of the photon-gluon fusion process. The cross-section for this process is directly related to the gluon density at the Born level. The experimental technique consists of the reconstruction of charmed mesons [28] or high $p_t$ particles in the final state [29] to access $\Delta g$. The high $p_t$ particles method leads to samples with larger statistics but these have larger background contributions from QCD Compton processes and fragmentation. High
\[ p_t \text{ charged particle production has been used in earlier attempts by HERMES [30] and SMC [31] to access gluon polarization. These measurements are listed in Table 1 and shown in Fig. 3 for } x_g \sim 0.1. \]

The hunt for \( \Delta g \) is one of the main physics drives for polarized RHIC. Experiments using the PHENIX and STAR detectors are investigating polarized glue in the proton. Measurements of \( \Delta g/g \) from RHIC are sensitive to gluon polarization in the range \( 0.02 < x_g < 0.3 \) (\( \sqrt{s} = 200 \text{ GeV} \)) and \( 0.06 < x_g < 0.4 \) (\( \sqrt{s} = 62.4 \text{ GeV} \)) for the neutral pion \( A_{LL} \) measured by PHENIX [33, 34] and inclusive jet production measured by STAR at 200 GeV centre of mass energy [35, 36].

The RHIC data for these asymmetries appear in Fig. 4, together with the expectations based on different NLO fits to inclusive \( g_1 \) data. In Fig. 4 the curves “GRSV-min” (or “\( \Delta g = 0 \)”), “GRSV-std”, “GRSV-max” (or “\( \Delta g = g \)”) and “\( \Delta g = -g \)” correspond to a first moment of \( \Delta g \sim 0.1, 0.4, 1.9 \) and \(-1.8\) respectively at \( Q^2 \sim 1 \)

**Table 1: Polarized gluon measurements from deep inelastic experiments.**

| Experiment | process          | \( \langle x_g \rangle \) | \( \langle p^2 \rangle \) (GeV\(^2\)) | \( \Delta g/g \)          |
|------------|------------------|--------------------------|--------------------------------------|---------------------------|
| HERMES     | hadron pairs     | 0.17                     | \( \sim 2 \)                          | 0.41 \( \pm 0.18 \pm 0.03 \) |
| HERMES     | inclusive hadrons| 0.22                     | 1.35                                 | 0.071 \( \pm 0.034^{+0.105}_{-0.127} \) |
| SMC        | hadron pairs     | 0.07                     |                                      | -0.20 \( \pm 0.28 \pm 0.10 \) |
| COMPASS    | hadron pairs, \( Q^2 < 1 \) | 0.085               | \( \sim 3 \)                          | 0.016 \( \pm 0.058 \pm 0.054 \) |
| COMPASS    | hadron pairs, \( Q^2 > 1 \) | 0.082               | \( \sim 3 \)                          | 0.08 \( \pm 0.10 \pm 0.05 \) |
| COMPASS    | open charm       | 0.11                     | 13                                    | -0.49 \( \pm 0.27 \pm 0.11 \) |
GeV$^2$ in the analysis of Ref.[37]. The data are consistent with small gluon polarization in the measured kinematics and the value extracted by PHENIX from their $\sqrt{s} = 200$ GeV data is [33]

$$\Delta g_{\text{GRSV}}^{[0.02,0.3]} = 0.2 \pm 0.1(\text{stat.}) \pm 0.1(\text{sys.})\pm 0.0(\text{shape}) \pm 0.1(\text{scale})$$ (11)

at $Q^2 = 4$ GeV$^2$.

These measurements suggest that polarized glue is, by itself, not sufficient to resolve the difference between the small value of $g_A^{(0)}|_{p_{\text{DIS}}}$ and the naive constituent quark model prediction, $\sim 0.6$ through the polarized glue term $-3\frac{\alpha_s}{2\pi}\Delta g$. Note however that a gluon polarization $\sim 0.2 - 0.3$ is would still make a significant contribution to the spin of the proton in Eq.(10).

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**Figure 4:** PHENIX results on $A_{LL}^{p^0}$ together with the predictions from various QCD fits at $s = 200$ GeV [33] and 62.4 GeV [34], where $x_T = 2p_T/\sqrt{s}$ (above). STAR data on the longitudinal double spin inclusive jet asymmetry $A_{LL}$ at $\sqrt{s} = 200$ GeV versus jet $p_T$ [36] (below).
3.3 Valence and Sea polarization

Semi-inclusive measurements of fast pions and kaons in the current fragmentation region with final state particle identification can be used to reconstruct the individual up, down and strange quark contributions to the proton’s spin. In contrast to inclusive polarized deep inelastic scattering where the $g_1$ structure function is deduced by detecting only the scattered lepton, the detected particles in the semi-inclusive experiments are high-energy (greater than 20% of the energy of the incident photon) charged pions and kaons in coincidence with the scattered lepton. For large energy fraction $z = E_h/E_\gamma \rightarrow 1$ the most probable occurrence is that the detected $\pi^\pm$ and $K^\pm$ contain the struck quark or antiquark in their valence Fock state. They therefore act as a tag of the flavour of the struck quark [38].

Figure 5 shows the HERMES results on flavour separation [39, 40]. The polarizations of the up and down quarks are positive and negative respectively, while the sea polarization data are consistent with zero. There is no evidence from this semi-inclusive data for a large negative strange quark polarization. For the region $0.02 < x < 0.6$ the extracted $\Delta s$ integrates to the value $+0.037 \pm 0.019 \pm 0.027$[40] which contrasts with the negative value for the polarized strangeness, Eq.(7), extracted from inclusive measurements of $g_1$. New COMPASS measurements[42] also show no evidence of strangeness polarization in the region $x > 0.006$.

For semi-inclusive hadron production experiments it is important to match the theory with the acceptance of the detector [43]. For example, the anomalous polar-

![Figure 5: Recent HERMES results for the quark and antiquark polarizations extracted from semi-inclusive DIS. Left: (a) the flavour separation reported in Ref.[39]. Right: (b) 2008 HERMES results from charged kaon asymmetries[40]. Here $\Delta Q(x) = \Delta u(x) + \Delta d(x)$.](image)
Table 2: First moments for valence quark polarization $\Delta u_v + \Delta d_v$ and sea polarization $\Delta \bar{u} + \Delta \bar{d}$ from SMC$^{44}$, HERMES$^{39}$, and COMPASS$^{41}$. Note that these data are dominated by the singlet cf. octet contributions because of the extra factor of 4 weighting in the flavour-decomposition of $g_1$.

| Experiment | x-range | $Q^2$ (GeV$^2$) | $\Delta u_v + \Delta d_v$ | $\Delta \bar{u} + \Delta \bar{d}$ |
|------------|---------|-----------------|--------------------------|--------------------------|
| SMC98      | 0.003–0.7 | 10              | 0.26 ± 0.21 ± 0.11       | 0.02 ± 0.08 ± 0.06       |
| HERMES05   | 0.023–0.6 | 2.5             | 0.43 ± 0.07 ± 0.06       | −0.06 ± 0.04 ± 0.03      |
| COMPASS    | 0.006–0.7 | 10              | 0.40 ± 0.07 ± 0.05       | 0.0 ± 0.04 ± 0.03        |

A direct and independent measurement of the strange quark axial-charge through neutrino-proton elastic scattering$^{[45]}$ would be valuable. The axial-charge measured in $\nu p$ elastic scattering is independent of any assumptions about the presence or absence of a subtraction at infinity in the dispersion relation for $g_1$ and the $x \sim 0$ behaviour of $g_1$. The W-boson production programme at RHIC$^{[46]}$ will provide additional flavour-separated measurements of polarized up and down quarks and antiquarks. Further measurements to push the small $x$ frontier would be possible with a polarized $ep$ collider$^{[47]}$.

### 3.4 SU(3) breaking and $g_A^{(8)}$

Given that the measured $\Delta s$ and $-3\frac{\alpha_s}{\pi}\Delta g$ contributions to $g_A^{(0)}$ are small, it is worthwhile to ask about the value of $g_A^{(8)}$. The value 0.58 is extracted from a 2 parameter fit to hyperon $\beta$-decays in terms of the SU(3) $F = 0.46$ and $D = 0.80$ parameters$^{[6]}$ – see Table 3. The fit is good to 20% accuracy$^{[48]}$. More sophisticated fits would also include chiral corrections. Calculations of non-singlet axial-charges

Table 3: $g_A/g_V$ from $\beta$-decays with $F = 0.46$ and $D = 0.80$.

| Process     | measurement | SU(3) combination | Fit value |
|-------------|-------------|--------------------|-----------|
| $n \to p$   | 1.270 ± 0.003 | $F + D$            | 1.26      |
| $\Lambda^0 \to p$ | 0.718 ± 0.015 | $F + \frac{1}{3}D$ | 0.73      |
| $\Sigma^- \to n$ | $-0.340 \pm 0.017$ | $F - D$            | -0.34     |
| $\Xi^- \to \Lambda^0$ | 0.25 ± 0.05 | $F - \frac{1}{3}D$ | 0.19      |
| $\Xi^0 \to \Sigma^+$ | 1.21 ± 0.05 | $F + D$            | 1.26      |
in relativistic constituent quark models are sensitive to the confinement potential, effective colour-hyperfine interaction[49, 38], pion cloud plus additional wavefunction corrections[50] chosen to reproduce the physical value of \( g_A^{(3)} \). These effects have the potential to reduce \( g_A^{(8)} \) from the SU(3) value \( 3F - D \) to \( \sim 0.5 \), within the 20% variation. This value of \( g_A^{(8)} \) would reduce \( \Delta s_Q \rightarrow \infty \) in Eq.(7) to \( \sim -0.05 \), still leaving the OZI violation \( g_A^{(0)}|_{pDIS} - g_A^{(8)} \sim -0.15 \) to be explained.

We have seen in Section 2 that \( g_1^2 \) is flat and consistent with zero throughout the measured region \( 0.004 < x < 0.02 \) where Regge extrapolation[15, 16] would expect to see some divergence if there is a big contribution to \( g_A^{(0)} \) from partons at small but finite \( x \). In seeking to understand the data we are guided by QCD anomaly theory and the special role of gluon topology.

\section{Gluon topology and the QCD axial anomaly}

In QCD one has to consider the effects of renormalization. The flavour singlet axial vector current \( J_{\mu 5} \) in Eq.(5) satisfies the anomalous divergence equation

\[ \partial^\mu J_{\mu 5} = 6\partial^\mu K_\mu + \sum_{i=1}^3 2im_i \bar{q}_i \gamma_5 q_i \]  

(12)

where

\[ K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu
u\rho\sigma} \left[ A_\nu^\rho A_\sigma^\rho - \frac{1}{3} g f_{abc} A_\nu^a A_\rho^b \right] \]  

(13)

is the gluonic Chern-Simons current. Here \( A_\nu^a \) is the gluon field and \( \partial^\mu K_\mu = \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \) is the topological charge density. Eq.(12) allows us to define a partially conserved current \( J_{\mu 5} = J_{\mu 5}^{\text{con}} + 6K_\mu \), viz. \( \partial^\mu J_{\mu 5}^{\text{con}} = \sum_{i=1}^3 2im_i \bar{q}_i \gamma_5 q_i \).

The anomaly is the physical manifestation of a clash of classical symmetries under renormalization. When one renormalizes the flavour-singlet axial-vector current operator the triangle diagram with one axial-vector current vertex and two vector current vertices is important. One can choose an ultraviolet regularization which preserves current conservation (gauge-invariance) at the gluon vector-current vertices or one can preserve the partially conserved axial-vector current relation at the \( \gamma_\mu \gamma_5 \) vertex but not both. Gauge invariance must win because it is dynamical and is required for renormalization leading to the anomaly on the right hand side of Eq.(12).

When we make a gauge transformation \( U \) the gluon field transforms as

\[ A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{g} (\partial_\mu U)U^{-1} \]  

(14)

and the operator \( K_\mu \) transforms as

\[ K_\mu \rightarrow K_\mu + \frac{ig}{8\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial^\nu \left( U^\dagger \partial^\rho U A^\sigma \right) \]

\[ + \frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[ (U^\dagger \partial^\rho U)(U^\dagger \partial^\sigma U)(U^\dagger \partial^\tau U) \right]. \]

(15)
(Partially) conserved currents are not renormalized. It follows that $J_{\mu 5}^{\text{con}}$ is renormalization scale invariant and the scale dependence of $J_{\mu 5}$ associated with the factor $E(\alpha_s)$ is carried by $K_\mu$. Gauge transformations shuffle a scale invariant operator quantity between the two operators $J_{\mu 5}^{\text{con}}$ and $K_\mu$ whilst keeping $J_{\mu 5}$ invariant.

If we wish to understand the first moment of $g_1$ in terms of the matrix elements of anomalous currents ($J_{\mu 5}^{\text{con}}$ and $K_\mu$), then we have to understand the forward matrix element of $K_+$ and its contribution to $g_A^{(0)}$.

Here we are fortunate in that the parton model is formulated in the light-cone gauge ($A_+ = 0$) where the forward matrix elements of $K_+$ are invariant. In the light-cone gauge the non-abelian three-gluon part of $K_+$ vanishes. The forward matrix elements of $K_+$ are then invariant under all residual gauge degrees of freedom. Furthermore, in this gauge, $K_+$ measures the gluonic “spin” content of the polarized target [51, 52]. One finds

$$g_A^{(0)(A_+ = 0)} = \sum_q \Delta q_{\text{con}} - 3 \alpha_s \Delta G$$

where $\Delta q_{\text{con}}$ is measured by the partially conserved current $J_{\mu 5}^{\text{con}}$ and $-\frac{\alpha_s}{2\pi} \Delta G$ is measured by $K_+$. Positive gluon polarization tends to reduce the value of $g_A^{(0)}$ and offers a possible source for OZI violation in $g_A^{(0)}|_{\text{inv}}$. In perturbative QCD $\Delta q_{\text{con}}$ is associated with low $k_t$ partons and is identified with $\Delta q_{\text{partons}}$ and $\Delta G$ is identified with $\Delta g_{\text{partons}}$ (with the struck quark or antiquark carrying $k_t^2 \sim Q^2$) – see Eq.(9).

If we were to work only in the light-cone gauge we might think that we have a complete parton model description of the first moment of $g_1$. However, one is free to work in any gauge including a covariant gauge where the forward matrix elements of $K_+$ are not necessarily invariant under the residual gauge degrees of freedom [53]. Understanding the interplay between spin and gauge invariance leads to rich and interesting physics possibilities.

For example, consider a covariant gauge.

One can show [53] that the forward matrix elements of $K_\mu$ are invariant under “small” gauge transformations (which are topologically deformable to the identity) but not invariant under “large” gauge transformations which change the topological winding number. Perturbative QCD involves only “small” gauge transformations; “large” gauge transformations involve strictly non-perturbative physics. The second term on the right hand side of Eq.(15) is a total derivative; its matrix elements vanish in the forward direction. The third term on the right hand side of Eq.(15) is associated with the gluon topology [54].

The topological winding number is determined by the gluonic boundary conditions at “infinity”, viz.

$$\int d\sigma_\mu K^\mu = n$$

where $n$ is an integer and $\sigma_\mu$ is a large surface with boundary which is spacelike with
respect to the positions $z_k$ of any operators or fields in the physical problem. It is insensitive to local deformations of the gluon field $A_\mu(z)$ or of the gauge transformation $U(z)$. When we take the Fourier transform to momentum space the topological structure induces a light-cone zero-mode which can contribute to $g_1$ only at $x = 0$. Hence, we are led to consider the possibility that there may be a term in $g_1$ which is proportional to $\delta(x)$ \cite{2, 21} – hence the $C_\infty$ term in Eq.(9).

Note that we are compelled to consider this possibility by the QCD axial anomaly and gauge invariance under large gauge transformations – strictly non-perturbative physics. One can show mathematically that this contribution, if finite, corresponds to a subtraction constant in the dispersion relation for the $g_1$ spin structure function \cite{2}. It is associated with the residue of the massless Kogut-Susskind pole that arises in discussion of the axial U(1) problem. The subtraction constant, if finite, is a non-perturbative effect and vanishes in perturbative QCD. It is sensitive to the mechanism of axial U(1) symmetry breaking and the realisation of axial U(1) symmetry breaking by instantons: spontaneous U(1) symmetry breaking by instantons naturally generates a subtraction constant whereas explicit symmetry breaking does not \cite{21}. The QCD vacuum is a Bloch superposition of states characterised by non-vanishing topological winding number and non-trivial chiral properties. When we put a valence quark into this vacuum it can act as a source which polarizes the QCD vacuum with net result that the spin “dissolves” and some fraction of the spin of the constituent quark is associated with non-local gluon topology with support only at Bjorken $x = 0$.

In this scenario the finite “$\Delta s(x)$” (defined as one third the difference between the singlet and octet polarized quark distributions) extracted from NLO fits to inclusive $g_1$ data corresponds, in part, to the area that is shifted to Bjorken $x = 0$ through non-perturbative processes involving gluon topology. (In the NLO fits to just inclusive $g_1$ data, this negative “$\Delta s(x)$” was found to turn on strongly at threshold\cite{7} in contrast to the direct measurements of strangeness polarization in semi-inclusive scattering.)

5 Towards possible understanding

Where are we in our understanding of the spin structure of the proton and the small value of $g^{(0)}_A|_{p\text{DIS}}$? Measurements of valence, gluon and sea polarization suggest that the polarized glue term $-3\alpha_s/2\pi \Delta g_{\text{partons}}$ and strange quark contribution $\Delta s_{\text{partons}}$ in Eq.(9) are unable to resolve the small value of $g^{(0)}_A|_{p\text{DIS}}$. The spin puzzle appears to be a property of the valence quarks. Given that SU(3) works well, within 20%, in $\beta$-decays and the corresponding axial-charges, then the difference between $g^{(0)}_A|_{p\text{DIS}}$ and $g^{(8)}_A$ suggests a finite subtraction in the $g_1$ spin dispersion relation. If there is a finite subtraction constant, polarized high-energy processes are not measuring the full singlet axial-charge: $g^{(0)}_A$ and the partonic contribution $g^{(0)}_A|_{p\text{DIS}} = g^{(0)}_A - C_\infty$
can be different. Since the topological subtraction constant term affects just the first moment of $g_1$ and not the higher moments it behaves like polarization at zero energy and zero momentum. The proton spin puzzle seems to be telling us about the interplay of valence quarks with the complex vacuum structure of QCD.

Acknowledgements

I thank C. Aidala, A. Bravar, A. Korzenev, F. Kunne, H. Santos and R. Windmolders for conversations about experimental data, K. Aoki for help with Fig. 4, and B.L. Ioffe and A. W. Thomas for discussions about theoretical issues. The research of SDB is supported by the Austrian Science Fund (grant P20436).

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