Research on resolution of underwater acoustic imaging

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Abstract—In order to study the influence of underwater material on the resolution of acoustic imaging, the Rayleigh resolution limit is proposed for the acoustic imaging system. Based on the realization form of Bessel function, the theoretical calculation formula of sound field scattered by a rigid sphere is obtained. Based on the boundary element theory, the scattering sound field of the small ball is simulated by COMSOL software, and the feasibility of the simulation is verified. Finally, the applicability of Rayleigh resolution limit in acoustic imaging system is verified by underwater simulation experiment with sonar. The research results provide theoretical support for improving the resolution of underwater acoustic imaging, and have important guiding significance for the research and development of underwater detection equipment.

1. Introduction

Acoustic imaging system plays a very important role in the development of marine resources and the detection of underwater targets[1]. Compared with optical imaging in the underwater propagation distance is limited and the imaging effect is very poor[2], the advantage of acoustic imaging propagation distance is obvious. Due to the influence of Underwater Suspended Matter and the complexity of underwater environment, the resolution of acoustic imaging will be greatly affected. How to improve the resolution of acoustic imaging becomes very meaningful.

In this paper, based on the Rayleigh resolution limit theory of optics, Twersky[3] used Bessel Legendre series to express the scattering sound field of small balls with smaller equivalent frequency, and Marnevskaya[4][5] studied the problem that the distance between small balls is much larger than the wavelength on the basis of Bessel Legendre series. Bessel function analysis method is to express the scattered sound field of a rigid sphere by the superposition of spherical functions, which makes full use of the orthogonality of Legendre polynomials. According to the scattering sound field model based on the boundary element principle, the scattering sound field of a single rigid sphere is simulated by using COMSOL software, which has the advantages of good convergence, fast calculation speed and simple multi physical field coupling[6], and the directivity distribution of sound pressure is obtained. The scattering sound field of two rigid spheres is simulated, and the simulation experiment is carried out in combination with underwater. The influence of the distance between two spherical surfaces and wavelength on the resolution is analyzed. The function and applicability of Rayleigh resolution limit
are studied, and the resolution mechanism of high acoustic imaging is explored.

2. Theoretical analysis and calculation of underwater acoustic imaging resolution

After the point light source passes through the small circular hole of the optical instrument, it is affected by the diffraction and forms a circular light spot, which is called Airy spot. When two point light sources are close to each other, the formed light spots are easy to overlap, so the two point light sources cannot be distinguished well, so there is a resolution limit, which is just the resolution distance.

As shown in Figure 1, the Rayleigh criterion is used to determine the distance that can be distinguished. The formula is as follows

\[ \Delta x_{\text{min}} = \frac{0.61 \lambda}{n \sin \theta} = \frac{\lambda}{2} \]  

When Rayleigh criterion is used to determine the resolution limit in optics, the following conditions should be satisfied:[7]

1) The light source is incoherent point light source;
2) The intensity of two light sources is equal;
3) It is suitable for theoretical calculation.

The above conditions are also consistent with the acoustic imaging. We apply it to the study of acoustic imaging resolution. The Rayleigh resolution limit is expressed as the limit resolution distance is half of the wave length.

\[ \lambda = \frac{c}{f} \]  

\[ \Delta x_{\text{min}} = \frac{\lambda}{2} = \frac{750}{f} \]  

3. Calculation of sound field scattered by a rigid sphere

\[ \text{Fig 2} \quad \text{scattering diagram of small ball} \]
As shown in the figure, the plane sound wave is emitted by the transducer, and the sound wave acts on a rigid ball with radius \( r_0 \). The scattering sound field produced by the rigid sphere is independent of the azimuth\(^8\). First, we give the representation of plane acoustic wave

\[ P_i = P_0 \exp(jkz) = P_0 \exp(jkr \cos \theta) \]  \( \text{(4)} \)

It is assumed that the scattered sound wave of a rigid sphere is \( P_r \), so the total sound field is expressed as \( P = P_i + P_r \). In this paper, the plane wave is expressed in the form of superposition of spherical functions. Firstly, we expand the plane wave into a spherical function by Legendre polynomials

\[ P_i = P_0 \sum_{l=0}^{\infty} C_l P_l(\cos \theta) \]  \( \text{(5)} \)

By making full use of the orthogonality of Legendre polynomials, the coefficients of polynomials are expanded

\[ C_l = (l + \frac{1}{2}) \int_0^1 P_l(\cos \theta) \exp(jkr \cos \theta) \sin \theta \, d \theta \]
\[ = (l + \frac{1}{2}) \int_0^1 P_l(\cos \theta) \{1 + (jkr \cos \theta)^2/2 + \ldots\} \sin \theta \, d \theta \]
\[ = (2l + 1) j_l(kr) \]  \( \text{(6)} \)

The sound pressure of plane acoustic wave can be obtained by taking polynomial coefficients into the equation

\[ P_i = P_0 \sum_{l=0}^{\infty} (2l + 1) j_l P_l(\cos \theta) \]  \( \text{(7)} \)

We assume that the scattered wave and plane wave are completely symmetrical about the polar axis, so we give the expression of the scattered sound wave

\[ P_r = R_0 \sum_{l=0}^{\infty} A_l j_l^{(1)}(kr) j_l P_l(\cos \theta) \]  \( \text{(8)} \)

For a rigid sphere, the sum of the normal velocities of the incident and scattered waves of the surface particle is zero, so \( A_l \) can be obtained

\[ A_l = -\frac{(2l + 1) j_l^{(1)}(k_0)}{h_l(k_0)} \]  \( \text{(9)} \)

The expression of \( A_l \) is introduced into the expression of scattered sound wave to get the sound pressure of scattered sound wave

\[ P_r = -R_0 \sum_{l=0}^{\infty} (2l + 1) j_l^{(1)}(k_0) h_l^{(1)}(kr) P_l(\cos \theta) \]  \( \text{(10)} \)

Through the above steps, the sound pressure of plane sound wave and scattered sound wave can be obtained, and the total sound pressure at any position in space can be obtained by superposition of the two

\[ P = R_0 \sum_{l=0}^{\infty} (2l + 1) j_l^{(1)}(k_0) h_l^{(1)}(kr) P_l(\cos \theta) j_l(kr) \]  \( \text{(11)} \)

4. Simulation and analysis of scattering sound field of rigid sphere

4.1. Calculation principle of boundary element method
The calculation principle of BEM is obtained from the Helmholtz integral equation\(^9\). The next problem to be dealt with is the scattering of sound waves by a rigid sphere, which belongs to the
external problem of the direct BEM, so it satisfies the Helmholtz equation\[10].
\[
\nabla^2 p + k^2 p = 0
\]
\[
\nabla^2 G + k^2 G = 0
\]

(12)

\( G \) is the Green's function in free space
\[
G(X,Y) = \frac{e^{ikr}}{4\pi r}
\]

(13)

\[ \int_{S + \sigma + \Sigma} \left( p(Y) \frac{\partial G(X,Y)}{\partial n} - G(X,Y) \frac{\partial p(Y)}{\partial n} \right) dS(Y) = 0 \]

(14)

As shown in the figure, the surface of the steel ball is \( S \), and the X point is the singular point of Green's function in the area with volume \( V \) and surface \( \Sigma \). The points in other areas are nonsingular, so they are nonsingular in the space with surface \( \Sigma + \sigma + \Sigma \), so there are on the surface

\[ \int_{S + \sigma + \Sigma} \left( p(Y) \frac{\partial G(X,Y)}{\partial n} - G(X,Y) \frac{\partial p(Y)}{\partial n} \right) dS(Y) = 0 \]

(14)

The integral on the left side of the above formula is \(-p(X)\) on \( \sigma \) and zero on \( \Sigma \), so the integral on the surface \( S \) of a rigid sphere is \( p(X) \). Using the same method, the integral of point \( x \) on the surface \( S \) of a rigid sphere is \( 0.5p(X) \). Then the rigid sphere surface is discretized into quadrilateral elements, and the derivatives of \( p_i \) and normal and \( p_{ij} \) and normal are obtained

\[
p_i(\xi, \eta) = \sum_{j=1}^{4} N_j(\xi, \eta)p_j
\]

\[
\frac{\partial p_i(\xi, \eta)}{\partial n} = \sum_{j=1}^{4} N_j(\xi, \eta)\frac{\partial p_j}{\partial n}
\]

(15)

When the quadrilateral element is four nodes, the element shape function is

\[
N_j(\xi, \eta) = \frac{1}{4}(1+\xi)(1+\eta)
\]

(16)

\[
[A] = [B] \left( \frac{\partial p_i}{\partial n} \right)
\]

(17)

This expression represents the relationship between the sound pressure on the surface of a rigid sphere and its normal derivative. The sound pressure distribution on the surface of a rigid sphere can be obtained by solving the differential equations.

4.2. COMSOL acoustic simulation verification

Then, the solution obtained by COMSOL multiphysics software is compared with the previous solution of scattering sound field of rigid small sphere obtained from the equation of sound field
calculation.

The advantage of COMSOL software is that it has physical field model based on boundary element principle. After the mesh of the rigid sphere is divided, the boundary conditions and initial values are defined, and finally the solution is carried out.

For the convenience of calculation, we define the equivalent frequency $K_a$, where $k$ is the plane wave number. As shown in the figure, the blue line refers to the function calculation results, and the red line refers to the boundary element principle calculation results.

![Fig 4](image1.png)  
**Fig 4** Comparison of sound pressure directivity between function solution and boundary element principle at different equivalent frequencies

It can be seen from the figure that the directivity of sound pressure under different equivalent frequencies of function calculation and boundary element method is basically the same. Therefore, we can think that it is correct to use COMSOL software to study the scattering problem of rigid sphere based on boundary element method.

4.3. **Simulation analysis of sound field scattered by two small balls**

According to the previous research on Rayleigh resolution limit, Rayleigh resolution limit can be used for acoustic imaging. Next, we simulate and analyze the superposition of two identical spheres. The sound frequency is 12000hz, the wavelength is 0.125m, the radius of the ball is 0.2m, and the distances between the ball surfaces are 0.3m, 0.09m and 0.02m.

![image2.png](image2.png)  

D=0.3m
According to the figure, when the distance between the surfaces of the two spheres is greater than half of the wavelength, the sound field in the region of the sphere is greater than that between the two spheres, which can be well resolved; when the distance between the surfaces of the two spheres is about half of the wavelength, the sound field in the region where the spheres are located is slightly larger than that between the two spheres; when the distance between the surfaces of two spheres is less than half of the wavelength, the sound field in the region where the spheres are located is approximately equal to the sound field between the two spheres. Through the above analysis, it is completely possible to apply Rayleigh resolution limit theory to the resolution research of acoustic imaging system.

5. Experimental research on resolution limit
Using COMSOL software to simulate the sound field of two small balls, we can apply Rayleigh limit theory to acoustic imaging. Next, we use sonar to simulate two small balls with the same diameter underwater, and then analyze whether the images formed by two small balls at different surface distances can be distinguished.

The frequency of the sonar used is 900 kHz, so the wave length is 0.0017 m, the diameter of the simulated ball is 10 mm, and the resolution limit is 0.9 mm calculated by Rayleigh limit criterion. Control the distance between the two small balls to be 6 mm, 3 mm, 1 mm, and get the image of the small ball.
When the distance between the two balls is 6 mm, there is a gap between the balls, which can be well resolved. When the distance between the two spheres is 3 mm, the images of the two spheres almost coincide, but they can be distinguished. When the distance between the two balls is 1 mm, the images formed by the two balls are completely overlapped and cannot be distinguished.

From the above analysis results, we can know that the resolution limit of 900 kHz sonar is between 1 mm and 3 mm, which is slightly larger than our theoretical value. Due to the large number of underwater suspended particles, the sound wave will have a certain degree of attenuation in the process of transmission, which makes the sonar image error. Therefore, it is very effective to apply Rayleigh resolution limit to acoustic imaging.

6. Conclusions
In this paper, we use COMSOL software to simulate the scattering sound field of a single rigid sphere. The directivity of sound pressure under different equivalent frequencies is similar to that of function calculation, which verifies the feasibility of using COMSOL software to study the scattering problem of a sphere. Using COMSOL software to simulate the superimposed sound field of two rigid spheres and using sonar to analyze the underwater resolution limit experiment, the relationship between the surface distance of two rigid spheres and the wavelength is obtained, which verify that the Rayleigh polar resolution theory is very effective in the research of acoustic imaging resolution. The resolution limit is about half of the wavelength. The above analysis provides theoretical guidance for improving the resolution of underwater low visibility acoustic imaging. In the future research, we also need to consider the changes of the environment, and use the function to more accurately describe the relationship between distance, size and wavelength.

References
[1] Tong H W. Research on the influence of marine environment on sonar system [J]. Scientific and technological innovation and application, 2018250 (30): 62-63.
[2] Wang B. Research on sonar image processing and target recognition technology [D]. Northwest Normal University, 2005.
[3] Twersky, V. 1964. Rayleigh scattering. Appl. Opt. 3 (10), 1150–1162.
[4] Marnevskaya, L.A. 1969. Diffraction of a plane scalar wave by two spheres. Sov. Phys. Acoust. 14 (3), 356–360.
[5] Marnevskaya, L.A. 1970. Plane wave scattering by two acoustically rigid spheres. Sov. Phys. Acoust. 15 (4), 499–502.
[6] Zhou Y, Wen W. Application of COMSOL finite element software in acoustic simulation of large underwater targets [J]. Computer applications and software, 2020,37 (08): 74-78 + 84.
[7] Lin H. Study on Mie scattering and Brillouin scattering of marine suspended particles [D]. Huazhong University of science and technology, 2007.
[8] Zhou Y L, Fan J, Wang B. Phase characteristics of scattering sound field by underwater spherical target [J]. Journal of Harbin Engineering University, 2020,41 (07): 945-950.
[9] Yang D Q, Zhao Z S. Boundary element theory and application [M]. 2002.
[10] Peddie W. Helmholtz's Treatise on Physiological Optics[M]. 1924.