SUPERBUBBLES IN THE MULTIPHASE ISM AND THE LOADING OF GALACTIC WINDS

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ABSTRACT

We use numerical simulations to analyze the evolution and properties of superbubbles (SBs), driven by multiple supernovae (SNe), that propagate into the two-phase (warm/cold), cloudy interstellar medium (ISM). We consider a range of mean background densities $n_{\text{avg}} = 0.1$–10 cm$^{-3}$ and intervals between SNe $\Delta t_{\text{SN}} = 0.01$–1 Myr, and follow each SB until the radius reaches $\sim (1$–2)$H$, where $H$ is the characteristic ISM disk thickness. Except for embedded dense clouds, each SB is hot until a time $t_{\text{sf,m}}$ when the shocked warm gas at the outer front cools and forms an overdense shell. Subsequently, diffuse gas in the SB interior remains at $T_b \sim 10^6$–$10^7$ K, with an expansion velocity $v_b \sim 10^2$–$10^3$ km s$^{-1}$ (both highest for low $\Delta t_{\text{SN}}$). At late times, the warm shell gas velocities are several tens to $\sim 100$ km s$^{-1}$. While shell velocities are too low to escape from a massive galaxy, they are high enough to remove substantial mass from dwarfs. Dense clouds are also accelerated, reaching a few to tens of km s$^{-1}$. We measure the mass in hot gas per SN, $M_h$, and the total radial momentum of the bubble per SN, $\dot{p}_h$. After $t_{\text{sf,m}}$, $M_h \sim 10$–100 $M_\odot$ (highest for low $n_{\text{avg}}$), while $\dot{p}_h \sim 0.7$–$3 \times 10^7$ $M_\odot$ km s$^{-1}$ (highest for high $\Delta t_{\text{SN}}$). If galactic winds in massive galaxies are loaded by the hot gas in SBs, we conclude that the mass-loss rates would generally be lower than star formation rates. Only if the SN cadence is much higher than usual in galactic disks, as may occur for nuclear starbursts, can SBs breakout while hot and expel up to 10 times the mass locked up in stars. The momentum injection values, $\dot{p}_b$, are consistent with requirements to control star formation rates within galaxies at observed levels.

Key words: ISM: kinematics and dynamics – ISM: supernova remnants – methods: numerical – supernovae: general

1. INTRODUCTION

Many forms of energy originating in stars contribute to heating the gaseous interstellar, circumgalactic, and intergalactic media (ISM, CGM, and IGM, respectively), but the inputs from supernovae (SNe) play a unique role because they are so concentrated in space and time. This localized deposition of energy leads, through very strong shocks, to the creation of a hot “third” phase of the ISM (Cox & Smith 1974; McKee & Ostriker 1977), initially in SN remnants (SNRs) that are highly overpressured relative to their environment. Expansion of SN-breakout hot gas communicates momentum to the surrounding ISM and is crucial for maintaining turbulence in the warm neutral medium (WNM) and cold neutral medium (CNM) phases (Mac, Low, & Klessen 2004), which would otherwise rapidly collapse to make stars; it is believed that SN momentum injection is the most important element in the feedback loop that controls galactic star formation rates (SFRs; Kim et al. 2011; Ostriker & Shetty 2011). The hot phase created by SNe is observed to fill a substantial fraction of the ISM volume within the scale height of the turbulent CNM/WNM (e.g., Ferrière 1998; Könyves et al. 2007), sometimes surrounding small clouds of cooler phases (as in the Local ISM; e.g., Frisch et al. 2011), while on large scales being itself surrounded by shells of cooler gas (as in the Orion-Eridanus Bubble; e.g., Brown et al. 1995). Because of its high entropy, hot gas tends to rise to create a disk corona enveloping the cooler ISM phases (Norman & Ikeuchi 1989). Depending on its density, coronal gas may cool and condense into clouds that fall back to the disk, or remain hot and accelerate as a galactic wind to join the CGM (Shapiro & Field 1976; Bregman 1978; Chevalier & Clegg 1985).

The space–time concentration of SN energy inputs is further enhanced by stellar clustering. Massive stars are primarily born in clusters, and while some are ejected to become runaway O stars, the majority of core-collapse SNe explode in close proximity to each other over a period of several tens of millions of years. The combined action of many SNe leads to the development of an expanding superbubble (SB) with a hot interior and surrounding swept-up shell of cooled post-shock ISM gas (McCray & Snow 1979; Tomisaka et al. 1981). Large SBs, which energetically require contributions from multiple SNe, are ubiquitous in our Galaxy and our neighbors (e.g., Heiles 1979, 1984; Tenorio-Tagle & Bodenheimer 1988; Pidopryhora et al. 2007; Ochsendorf et al. 2015). The evolution of SBs depends on the SN rate and properties of the surrounding ISM. In cases with sufficiently many SN events (or frequent SNe), SB evolution in the absence of hot-gas cooling is expected to be analogous to the solutions for wind-driven bubbles powered by continuous energy injection, either in the simplified case of a uniform ambient medium (e.g., Castor et al. 1975; Weaver et al. 1977; McCray & Kafatos 1987), or taking into account stratification in the background disk (e.g., Tomisaka & Ikeuchi 1986; Mac Low & McCray 1988; Koo & McKee 1992).

Based on results from direct numerical simulations, an increasingly detailed understanding of the overall three-phase ISM disk is developing (e.g., de Avillez & Breitschwerdt 2004; Joung & Mac Low 2006; Hill et al. 2012; Hennebelle & Iffrig 2014; Li et al. 2015; Walsh et al. 2015). In recent simulations (Gatto et al. 2015; Walsh et al. 2015) the correlation (or lack thereof) of SNe with high-density gas has been shown to strongly shape the resulting character of the three-phase ISM, but the effects of SN clustering have not been investigated in detail. Instead, the detailed evolution of SBs has
mostly been studied via focused numerical models, in which the background ISM is treated in a simplified manner. Continuous thermal energy injection to a central region has been adopted for most SB simulations (e.g., Mac Low & Ferrara 1999; Strickland & Stevens 2000; Cooper et al. 2008; Tanner et al. 2016), although recently simulations allowing for discrete SN events have been considered in both spherical symmetry (Sharma et al. 2014; Gentry et al. 2016) and for the fully three-dimensional case (Yadav et al. 2016).

The realistic ISM has very large density (and temperature) contrasts, due to multiphase thermal structure and/or supersonic turbulence. For single SN events, the effect of non-uniform background states on SNR evolution and outcomes has been addressed by several recent direct numerical simulations. To model SNR interactions with molecular clouds, Iffrig & Hennebelle (2015) took as their background state cold clouds that have been seeded and evolved with supersonic turbulence, while Walch & Naab (2015) and Martizzi et al. (2015) adopted background states with an imposed distribution of density. In Kim & Ostriker (2015a; hereafter K15), we adopted a background state of a cloudy two-phase ISM that develops from nonlinear saturation of thermal instability. One of the main conclusions of these recent studies is that the total radial momentum injected into the CNM and WNM by the SNR expansion from an individual-SN explosion is insensitive to the mean background density and largely independent of the details of the ambient density distribution. In K15, we also considered a few cases of multiple SNe and found that the momentum injection per SN is slightly reduced, but it is still a weak (even weaker) function of the background density. This mean momentum per SN, \( p_a \), is a key parameter for turbulence driving and in the theory of self-regulation of star formation. The level of \( p_a \) obtained in these recent simulations can explain observations of the turbulent pressure and surface density of SFR \( \Sigma_{SFR} \) in a wide range of galaxies (Ostriker et al. 2010; Kim et al. 2011, 2013; Ostriker & Shetty 2011; Shetty & Ostriker 2012; Kim & Ostriker 2015b). In this work, we evaluate the momentum injection per SN for situations with multiple SNe, using a similar numerical setup to that in K15.

An issue of much interest in both analytic and numerical models of SBs has been the conditions that enable an SB to breakout of the “ambient” ISM disk into the galactic halo while still remaining overpressured relative to the environment (e.g., Mac Low & McCray 1988; Mac Low et al. 1989; Koo & McKee 1992; Basu et al. 1999). The original motivation for this question is that overpressured breakout and Rayleigh–Taylor instability were considered necessary for releasing hot gas into the galactic corona, where it could potentially launch a wind. However, in the modern understanding of the three-phase, turbulent ISM, there are many pre-existing low-density channels through which hot gas can vent from the disk, even if an SB is not powerful enough to remain intact until it reaches the disk scale height. Thus, even if bubble expansion stalled and there were no immediate escape routes for hot gas, its high entropy would make it buoyant. Although in this paper we do not directly model disk stratification, we discuss various conditions for SB breakout.

An important parameter in analytic and semi-analytic models of SN-driven galactic winds (e.g., Chevalier & Clegg 1985; Wang 1995; Bustard et al. 2015; Thompson et al. 2016) is the mass of hot gas launched in the wind per SN. An alternative parameterization is in terms of the “mass loading factor” \( \beta _h \), the ratio between the mass of hot gas launched in the wind and the mass of gas that has (by assumption) collapsed to form stars, including progenitors of the SNe that drive the wind. Here we will evaluate the evolution of the mass of hot gas per SN in the interior of an SB. The value of this quantity at the time the SB radius is comparable to the disk scale height allows us to obtain an upper limit on the mass loading in a galactic wind arising from a region with certain ISM conditions and SN rate. Another quantity that is often used to parameterize SN-driven winds is the energy loading (per SN or per unit mass of stars formed). As this is primarily used in combination with the mass loading to compute the specific enthalpy, here we will instead measure the temperature of the hot medium within the SB. This would represent the typical temperature of the hot ISM phase, and as it is proportional to the specific enthalpy, it can be used to constrain the asymptotic wind velocity (assuming adiabatic expansion such that the Bernoulli parameter is conserved along streamlines).

In this paper, we extend the previous simulations of KO15 for a more extensive investigation of SB evolution driven by multiple (discrete) SN events in the two-phase warm/cold cloudy ISM. We will show that, similar to the situation for individual SNRs, a key stage in the evolution is when a blast wave propagating into volume-filling warm ISM first cools, leading to shell formation. The shell formation time depends on both the ambient medium density and SN interval (or mass of star cluster). We will show that the SN interval must be smaller than the shell formation time for the early SB evolution to agree with the “continuous energy injection” limit.

Although we carry out simulations in an unstratified medium, we will connect to loading of winds by quantifying the properties of SBs when their radii are comparable to the scale height of an ISM disk with the same midplane density as the mean ambient density in the model. We will measure three key quantities in each simulation at this stage of evolution: the momentum per SN, the mass of hot gas per SN, and the temperature of the hot gas. We also evaluate the distribution of SB mass with velocity at this time.

The outline of this paper is as follows: in Section 2 we review the theory of adiabatic SB expansion, and provide reference values for the expected shell formation time and related quantities. We also discuss the analytic theory of SB breakout. In Section 3 we summarize the numerical methods and models we use for our simulations. Section 4 presents the results of our numerical SB simulations and analyses, and Section 5 discusses the implications of these results for wind loading. We summarize our conclusions in Section 6. We also provide an appendix to show convergence (as a function of resolution) in SB properties, and to demonstrate that SB evolution is independent of the method for injecting SN energy.

2. ANALYTIC THEORY

In this section, we reformulate the classical solution for SB evolution driven by continuous energy injection (McCray & Kafatos 1987), in which the physical properties of the SB were written in terms of the mechanical luminosity (or power) or number of SNe. These solutions are based on the analogous solutions for wind-blown interstellar bubbles (e.g., Avedisova 1972; Castor et al. 1975; Weaver et al. 1977). Here we instead parameterize the power in terms of the mean time interval between SNe, \( \Delta t_{6N} \).

We consider an SB driven by SN explosions originating in a star cluster with total mass \( M_c \). For \( M_c \gtrsim 10^3 M_\odot \), such that
the IMF is fully sampled, the expected number of SNe is \(N_{SN} = M_d/M_\ast\), where \(M_\ast\) is the total mass of stars formed per SN. For a Kroupa IMF (Kroupa 2001), \(M_\ast \approx 100 M_\odot\). The SN rate is relatively constant from \(\sim 3\) Myr to \(t_{life} \sim 40\) Myr (e.g., Leitherer et al. 1999), so that

\[
\Delta t_{SN} = \frac{t_{life}}{N_{SN}} = 0.4 \text{ Myr} \frac{1}{M_{cl,4}},
\]

where \(M_{cl,4} \equiv M_d/10^4 M_\odot\). With an energy per SN explosion \(E_{SN} = 10^{51} E_{51} \text{ erg}\), the total energy that has been injected to the bubble at time \(t\) is

\[
E_{SB} = E_{SN} \frac{t}{\Delta t_{SN}},
\]

and the mean power delivered by multiple SNe is given by

\[
L_{SB} = \frac{E_{SB}}{\Delta t_{SN}} = 3.2 \times 10^{37} \text{ erg s}^{-1} E_{51} \Delta t_{SN},
\]

where \(\Delta t_{SN,6} \equiv \Delta t_{SN}/\text{Myr}\).

### 2.1. Early Adiabatic Expansion

Successive multiple SN events contribute to the total energy of the SB, while the total mass is dominated by the material swept up from its environment. Before radiative losses become significant, the evolution is analogous to the Sedov–Taylor solution for a single SN, except with a steady increase in the energy contained within the expanding blast wave.

From dimensional analysis, the expansion velocities within the SB, as well as the sound speeds in the interior, will scale with its outer radius \(r\) as \(\rho \propto r/r_t\), while the mass contained is \(M \propto r^3 \rho_{amb}\), where \(\rho_{amb}\) is the density of the surrounding medium (treated as uniform); the total energy contained therefore varies as \(E \propto r^2 \rho_{amb}^2 / r_t^2\). For constant input power, energy must increase as \(E = L_{SB} t = E_{SB} / \Delta t_{SN}\), which yields \(r \propto \left(\frac{E_{SB}}{\rho_{amb}}\right)^{1/3} / \rho_{amb}\). A self-similar solution for the internal structure of the bubble determines the coefficient \((\sim 0.88\) for \(\gamma = 5/3\); see Weaver et al. 1977). In terms of the ambient hydrogen number density \(n_{amb} = \rho_{amb} / (1.4 m_H)\), the radius of the outer shock of the SB can be written during the adiabatic expansion stage as

\[
r_{ad} = 60 \text{ pc} \left(\frac{E_{51}}{\Delta t_{SN,6} \rho_{amb,0}}\right)^{1/5} t_6^{1/5},
\]

where \(t_6 \equiv t / \text{Myr}\) and \(n_{amb,0} \equiv n_{amb} / (1 \text{ cm}^{-3})\).

The expansion velocity of the outer SB shock during the adiabatic stage is

\[
v_{ad} = \frac{dr_{ad}}{dt} = 35 \text{ km s}^{-1} \left(\frac{E_{51}}{\Delta t_{SN,6} \rho_{amb,0}}\right)^{1/5} t_6^{-2/5},
\]

the total SB mass during the adiabatic stage is

\[
M_{ad} = \frac{4\pi}{3} r_{ad}^3 \rho_{amb} = 3.2 \times 10^{41} M_\odot \left(\frac{E_{51} \rho_{amb,0}}{\Delta t_{SN,6}}\right)^{1/5} t_6^{9/5},
\]

and the total radial momentum of the SB (treating the mass as concentrated near the outer shock) is

\[
P_{ad} \equiv \frac{4\pi}{3} r_{ad} \rho_{amb} v_{ad} = 1.1 \times 10^6 M_\odot \text{ km s}^{-1} \times \left(\frac{E_{51} \rho_{amb,0}}{\Delta t_{SN,6}}\right)^{1/5} t_6^{7/5}.
\]

For this energy-conserving solution, the momentum per SN in the shell is

\[
\dot{P}_{ad} \equiv P_{ad} \Delta t_{SN} = 1.1 \times 10^6 M_\odot \text{ km s}^{-1} \times (E_{51} \Delta t_{SN,6} \rho_{amb,0})^{1/5} t_6^{2/5}.
\]

### 2.2. Shell Formation and Post-radiative Evolution

As the SB evolves, the outer regions where the density is highest start to cool radiatively, forming a thin, dense shell. The shell formation time for a single SN explosion in a homogeneous medium is (e.g., Equation (7) in KO15):

\[
t_{sf,m} = 4.4 \times 10^4 \text{ yr} E_{51}^{0.22} n_{amb,0}^{-0.55}.
\]

For an SB formed from multiple SN explosions, we can estimate the shell formation time using Equation (9), with the energy equal to \(E_{SN,1d} / \Delta t_{SN}\) (see also Mac Low & McCray 1988; Koo & McKee 1992). This yields

\[
t_{sf,m} = 1.8 \times 10^4 \text{ year} E_{51}^{0.28} n_{amb,0}^{-0.71} \Delta t_{SN,6}^{-0.28}.
\]

Note that in order to be self-consistent with the assumption of continuous energy injection, it is necessary to have had multiple SN events prior to shell formation, i.e., \(\Delta t_{SN} < t_{sf,m}\). Only cases with sufficiently short SN interval and/or low ambient density, \(\Delta t_{SN} \rho_{amb,0} < 0.044 E_{51}^{1/3}\), satisfy this requirement. For cases that do not meet this requirement, shell formation occurs at the time given in Equation (9) for a single SN, when the radius is \(r_{sf,m} = 22.6 \text{ pc} E_{51}^{-0.29} n_{amb,0}^{-0.42}\) (e.g., Equation (8) in KO15).

Inserting in Equations (4) and (8), the corresponding radius and the momentum injection per SN at the time of shell formation, for multiple SNe in the continuous energy input limit, are

\[
r_{sf,m} \equiv r_{ad}(t_{sf,m}) = 5.5 \text{ pc} E_{51}^{0.37} n_{amb,0}^{-0.62} \Delta t_{SN,6}^{-0.37}
\]

and

\[
\dot{P}_{sf,m} \equiv \dot{P}_{ad}(t_{sf,m}) = 2.3 \times 10^5 M_\odot \text{ km s}^{-1} E_{51}^{0.91} n_{amb,0}^{-0.82} \Delta t_{SN,6}^{-0.087}.
\]

This is quite similar to the momentum in the remnant from a single SN at shell formation, \(P_d = 2.2 \times 10^5 M_\odot \text{ km s}^{-1} E_{51}^{0.93} n_{amb,0}^{-0.13}\) (e.g., Equation (17) in KO15).

Another interesting quantity is the mass of hot gas in the SB per SN. Up to the time of shell formation, the mass of hot gas is just the total mass of the SB (Equation (6)); dividing by the

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1 The predicted shell formation time depends on the definition of cooling time, which was described in the text of KO15 as \(t_{cool} \equiv \epsilon / |d\epsilon / dt|\). However, in practice we used \(t_{cool} \equiv 0.6 \epsilon / |d\epsilon / dt|\) to obtain Equation (7) in KO15, because this was in better agreement with numerical simulations. In this paper, we continue to use the same definition (i.e. with a factor of 0.6).
number of SNe at shell formation \( = t_{\text{sf,m}}/\Delta t_{\text{SN}} \) yields
\[
\dot{M}_{\text{h,sf,m}} = 1.3 \times 10^3 M_\odot \cdot \dot{E}_{\text{SN},0.16}^{0.83} n_{\text{amb,0}}^{0.16} \Delta t_{\text{SN},0.6}^{0.17}
\]  
\( (13) \)

Note that similar to the situation for the mass at shell formation in a single SNR (e.g., Equation (11) in KO15), this is insensitive to the ambient density, and it is also insensitive to the SN interval.

After shell formation, the low-density interior of the SB remains hot and is overpressured relative to the ambient medium. The classical solution for post-radiative SB evolution (e.g., Weaver et al. 1977; McCray & Kafatos 1987; Koo & McKee 1992) is similar to the pressure-driven snowplow stage of the SNR for a single SN. The expansion of the outer SB shell in this stage is assumed to be described by the momentum equation,
\[
\frac{d}{dt} \left( M_{\text{shell}} \frac{dr}{dt} \right) = 4\pi r^2 P_{\text{hot}},
\]  
\( (14) \)

where \( M_{\text{shell}} \approx \rho_{\text{amb}} 4\pi r^3/3 \), the exterior pressure is treated as negligible, and \( P_{\text{hot}} = E_{\text{hot}} (\gamma - 1)/(4\pi r^3/3) \), treating the interior as uniform. Under the assumption that the interior energy is reduced by adiabatic expansion but suffers no radiative losses, the energy equation of the interior hot gas would be
\[
\frac{dE_{\text{hot}}}{dt} = L_{\text{SB}} - 4\pi r^2 P_{\text{hot}} \frac{dr}{dt}
\]  
\( (15) \)

With \( \gamma = 5/3 \), this again yields \( r \propto (L_{\text{SB}} r^2/\rho_{\text{amb}})^{1/5} \) as in Equation (4). In contrast to the expansion of a single SNR, where there are distinguishable changes in the exponents \( r \propto t^{2/3} \) for energy conserving and \( t^{2/7} \) for pressure-driven snowplow, the radius of the bubble in both the Sedov–Taylor and the pressure-driven snowplow phases have identical parameter dependence, with only slightly different coefficients (0.88 for the former and 0.76 for the latter). Thus the SB radius would follow
\[
r_{\text{pds}} = 52 \text{ pc} \left( \frac{E_{51}}{\Delta t_{\text{SN},0.6} n_{\text{amb,0}}} \right)^{1/5} t_6^{3/5},
\]  
\( (16) \)

for the pressure-driven snowplow solution (Weaver et al. 1977). The shell velocity would be a factor of 0.86 below that in Equation (5), and the shell momentum would be a factor of 0.56 below that in Equation (7).

In practice, the assumptions adopted in the classical pressure-driven SB evolution are not satisfied in the real ISM. For the continuous energy injection model, it is assumed that the hot interior of the bubble is separated from the cooled shell by a contact discontinuity with a continuous velocity. If, however, the shell expands at a lower velocity than the hot interior, the separation between the high-velocity hot interior and the low-velocity cooled shell is instead mediated by shocks and/or cooling condensation layers. The latter situation occurs after shell formation in the expansion of the remnant from a single SN (e.g., Cioffi et al. 1988, KO15). For SBs driven by small clusters with large \( \Delta t_{\text{SN}} \), the evolution may then resemble a succession of individual SNe more than the continuous limit.

More generally, the high degree of inhomogeneity of the real ISM, combined with the development of hydrodynamic instabilities (e.g., Vishniac 1983, 1994), breaks the spherical symmetry assumed in the classical solution, such that the interface between the cooled shell and the hot interior will not be a simple contact discontinuity. Conduction at interfaces, combined with turbulent mixing between the dense cooled shell gas and hot interior gas, enhances radiative losses so that the energy grows more slowly than would be predicted by Equation (15). For SB expansion in the two-phase ISM, energy losses in the hot interior of the SB are also enhanced by losses from conduction and evaporation of dense clouds left behind by the expansion of the outer shell in the low-density intercloud medium; these clouds are also ablated by Kelvin–Helmholtz unstable interactions with the surrounding high-velocity hot gas, and mixing into the hot bubble gas increases its radiative losses. Recognition of the importance of these effects has led to intensive numerical investigation, with dozens of studies focused on the shocked cloud problem alone (see, e.g., Scannapieco & Brüggen 2015 and other citations within).

Because we consider expansion of SBs in a cloudy ISM, evolution after the shell formation stage is far from the classical pressure-driven bubble solution. Equation (16) therefore does not describe the realistic post-radiative evolution of the SB radius. We thus only compare our results with the early energy-conserving solutions (Equations (4), (6), and (7)), and also compare the onset time of strong cooling to Equation (10).

### 2.3. Superbubble Breakout

Under the assumption that SBs expand as pressure-driven snowplows (sweping up the ambient medium into a cooled shell), with no radiative cooling in their interior, i.e., following the generalizations of Equations (14) and (15) that allow for an external stratified pressure and density in the ISM disk (the Kompaneets approximation), several authors have proposed criteria for SB “breakout” from a disk (e.g., Mac Low & McCray 1988; Koo & McKee 1992; Basu et al. 1999). Based on Equation (16), \( t_{\text{pds}}(H) = H^{5/3}(\rho_{\text{amb}}^{2/3}/\Delta t_{\text{SN}})^{1/3} \) is (up to order-unitary factors) the characteristic timescale for an SB to expand to reach the disk scale height \( H \), assuming radiative losses are negligible in the interior. The “breakout” criterion under this assumption amounts to the requirement that \( t_{\text{pds}}(H) \) is sufficiently short (by at least a factor \( \sim 3 \)) compared to the sound-crossing time over the disk thickness, \( \sim H/\langle P_{\text{amb}}/P_{\text{amb}} \rangle^{1/2} \). Physically this is also equivalent to the pressure within the bubble at the time when \( t_{\text{pds}} = H \) being sufficiently large compared to \( P_{\text{amb}} \), or the expansion velocity \( (dP_{\text{pds}}/dt) \) of the shell being sufficiently large compared to the ambient sound speed. For an idealized SB with adiabatic interior, if the breakout criterion is satisfied, the shell would accelerate and develop Rayleigh–Taylor instability as it expands beyond a scale height, whereas otherwise it would stall.

While numerical simulations support the conclusions based on the Kompaneets approximation analysis for the case of a uniform ambient medium (Mac Low et al. 1989), the assumption that the SB interior remains adiabatic until the radius reaches \( \sim H \) is not satisfied for the realistic cloudy ISM. As we will show, while early expansion is generally consistent with the adiabatic relation of Equation (4) up to time \( t_{\text{sf,m}} \), for \( t > t_{\text{sf,m}} \) the SB expands with the total shell momentum (rather than internal energy) increasing approximately linearly in time. Also, since realistically the ambient pressure in the ISM is generally dominated by the turbulent component rather than the thermal component, SB shells merge into the turbulent
background as their expansion rates drop, rather than having expansion stalled by external pressure.

If momentum of the shell grows as

$$\frac{d}{dt} \left( M_{\text{shell}} \frac{dr}{dt} \right) = \frac{p_*}{\Delta t_{\text{SN}}}$$

(17)

for mean momentum per SN $p_*$, then the SB radius will follow a “momentum-driven snowplow” relation

$$r_{\text{mds}} = \left( \frac{3p_*}{\Delta t_{\text{SN}} 2 \pi \rho_{\text{amb}}} \right)^{1/4} t^{1/2} = 34 \text{ pc} \left( \frac{p_{*,5}}{\Delta t_{\text{SN,0},6 \rho_{\text{amb},0}}} \right)^{1/4} r^{1/2}_b,$$

(18)

where $p_{*,5} \equiv p_*/(10^5 \text{ km s}^{-1} M_\odot)$. As a function of shell radius, the SB shell velocity in the momentum-driven limit is

$$v_{\text{mds}} = \left( \frac{3p_*}{\Delta t_{\text{SN}} 8 \pi \rho_{\text{amb}}} \right)^{1/2} r_2^{-1} = 5.8 \text{ km s}^{-1} \left( \frac{p_{*,5}}{\Delta t_{\text{SN,0},6 \rho_{\text{amb},0}}} \right)^{1/2} r_2^{-1},$$

(19)

where $r_2 \equiv r/10^2$ pc. Clear breakout of an SB would correspond to the situation in which the shell expansion velocity is large enough compared to the typical velocity dispersion in the disk, $\sigma_v$, at the time the shell reaches $\sim H$.

Using Equation (19) and setting $r = H$ yields

$$\frac{v_{\text{mds}}(H)}{\delta v} = \left( \frac{3p_*}{\Delta t_{\text{SN}} \rho_{\text{amb}} 8 \pi H^2} \right)^{1/2},$$

(20)

where we have substituted $\rho_{\text{amb}} \delta v^2 \rightarrow P_{\text{amb}}$. The largest component of $P_{\text{amb}}$, is typically the turbulent pressure, and if the ISM disk overall is consistent with self-regulated equilibrium with feedback mainly provided by SNe, $P_{\text{amb}} \approx p_* \Sigma_{\text{SFR}}/(4m_*)$, where $\Sigma_{\text{SFR}}$ is the mean SFR per unit area in the disk (Kim et al. 2011; Ostriker & Shetty 2011). Letting $(\pi H^2 \Sigma_{\text{SFR}}/m_*)^{-1} \equiv \Delta t_{\text{SN,H}}$ be the mean interval between SNe within the disk area $\pi H^2$, $v_{\text{mds}}/\delta v = 1.2 (\Delta t_{\text{SN,H}}/\Delta t_{\text{SN}})^{1/2}$. If $\Delta t_{\text{SN}}/\Delta t_{\text{SN,H}}$ is sufficiently small, the SB shell will remain coherent until breakout occurs. For lower-mass clusters with larger $\Delta t_{\text{SN}}$, the shell velocity will drop below $\sigma_v$ at an earlier stage, and the SB shell will merge with the background turbulent ISM structure (which is itself driven by expanding SNRs and SBs from other SNe). If multiple clusters within an area $\sim \pi H^2$ act coherently to create an SB, the criterion for visible blowout is simply that the local SFR is sufficiently elevated compared to its time-averaged value.

These considerations imply that the more massive clusters will create SBs that remain intact until they emerge through the disk “surface,” producing distinctive signatures. However, even SBs created by lower-mass clusters with shells that are destroyed within the disk may still release hot overpressured gas that escapes into the galactic halo, as we will discuss in Section 5. There we will also discuss the requirement needed for an SB to breakout of the disk prior to $t_{\text{sf,em}}$, i.e., before the onset of strong cooling.

### 3. NUMERICAL METHODS AND MODELS

We use the same methods as in KO15. The inviscid hydrodynamical equations with optically thin cooling and heating are solved using the Athena code (Stone et al. 2008; Stone & Gardiner 2009). The mass and momentum conservation equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

(21)

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + P) = 0,$$

(22)

and the energy equation, including a source term for net cooling, is

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P) \mathbf{v}) = -\rho \mathbf{C}.$$  

(23)

The symbols have their usual meanings; $\rho$ is the mass density, $\mathbf{v}$ is the velocity, $E \equiv P/(\gamma - 1) + \rho v^2/2$ is the total energy density, $P$ is the gas pressure, and $\gamma = 5/3$ is the ratio of specific heats. The gas temperature is $T = P/(1.1 n_{\text{H}} k_B)$, where the hydrogen number density is $n_{\text{H}} = \rho/(1.4 m_{\text{H}})$ for 10% Helium abundance.

The net cooling rate per unit volume is $\rho \mathbf{C} = n_{\text{H}} [n_{\text{H}} \Lambda(T - \Gamma)]$. We combine cooling functions from Koyama & Inutsuka (2002) and Sutherland & Dopita (1993) for low ($T < 10^{4.2}$ K) and high ($T > 10^{4.2}$ K) temperature gas, respectively. A constant heating rate per particle $\Gamma$ is only adopted at $T < 10^{4.2}$ K, representing photoelectric heating for the CNM and WNM; for hotter gas, $\Gamma = 0$. As we vary the mean density of the ambient medium from one model to another, we also vary the heating rate as $\Gamma/\Gamma_0 = (n_{\text{H}}/2 \text{ cm}^{-3})^{-1}$, where $\Gamma_0 = 2 \times 10^{-26}$ erg s$^{-1}$ is the solar neighborhood value (Koyama & Inutsuka 2002). This scaling for the heating rate follows the form expected in galactic disks with self-regulated star formation, in which the photoelectric heating is approximately proportional to the local SFR per unit area, as well as to the midplane pressure and density (Ostriker et al. 2010; Kim et al. 2011, 2013). Explicit thermal conduction is neglected in this study (see the discussion in KO15), although numerical diffusion at interfaces between hot and cooler phases can lead to “evaporation” from the surface of dense clouds and energy loss from the hot medium, similar to the effects of physical conduction. Our convergence studies are used to assess how these and other resolution-dependent processes may affect our results.

We study the evolution of SBs produced by multiple SNe in a two-phase medium. Each SB expands in a “background” two-phase medium, which is the result of nonlinear saturation of the thermal instability in the atomic ISM (Field 1965). This yields CNM clouds embedded in an intercloud WNM that fills most of volume ($\sim 90\%$). These phases are in pressure equilibrium, with density and temperature differing by two orders of magnitude (Wolfire et al. 1995).

For each simulation, we represent multiple SNe via successive explosions at the center of the domain, with fixed time intervals between events. We consider nine models with three different values for the mean density of the ambient medium $n_{\text{avg}} = 0.1, 1, \text{ and } 10 \text{ cm}^{-3}$, and three different time

\footnote{Note that the temperature for fully ionized gas should be $T = P/(2.3 n_{\text{H}} k_B)$. Since we simply fix the mean molecular weight to that of the neutral gas, however, the temperatures in our simulations are higher than they should be by a factor of $2.3/1.1$ for ionized gas ($T \gtrsim 10^5$ K). This treatment only causes a slight offset in the adopted cooling rate, which depends on the temperature, but does not affect the sound speed of the gas $c_s^2 \equiv P/\rho$.}
Table 1

| Model Parameters | Model | $\Delta_{SN}$ | $n_{avg}$ | $P_0$ | $\Delta_r$ | $H$ |
|------------------|-------|---------------|----------|-------|-----------|-----|
| (1)              |       | (2)           | (3)      | (4)   | (5)       | (6) |
| n0.1-t0.01       | 0.01  | ...          | ...      | ...   | ...       | ... |
| n0.1-t0.1        | 0.1   | 0.1          | 0.017    | 99    | 6         | 329 |
| n0.1-t1          | 1     | ...          | ...      | ...   | ...       | ... |
| n1-t0.01         | 0.01  | ...          | ...      | ...   | ...       | ... |
| n1-t0.1          | 0.1   | 1            | 0.14     | $1.1 \times 10^3$ | 3   | 104 |
| n1-t1            | 1     | ...          | ...      | ...   | ...       | ... |
| n10-t0.01        | 0.01  | ...          | ...      | ...   | ...       | ... |
| n10-t0.1         | 0.1   | 10           | 1.5      | $1.2 \times 10^4$ | 0.75 | 33  |
| n10-t1           | 1     | ...          | ...      | ...   | ...       | ... |
| n1-t0.1-low      | 0.1   | 1            | 0.14     | $1.1 \times 10^3$ | 6   | 104 |
| n1-t0.1-high     | 0.1   | 1            | 0.14     | $1.1 \times 10^3$ | 1.5 | 104 |

Note. Column (1): model name. Column (2): time interval between SNe, in Myr. Column (3): mean density of the ambient medium, in $cm^{-3}$. Column (4): mean density of the WNM, in $cm^{-3}$. Column (5): mean pressure of the ambient medium, in $k_g cm^{-3} K$. Column (6): resolution, in parsecs. Column (7): reference ISM scale height (see Equation (24)), in pc.

This is a rough estimate using vertical dynamical equilibrium, $H = \sigma_1 \Sigma / (\rho_{mid} + 4 P_0)^{1/2}$, where the total gas surface density $\Sigma = H^2 / 2 \pi \rho_{mid}$. If the midplane volume density of stars and dark matter, $\rho_{sd}$, scales with the midplane gas density $\rho_{mid}$ (or else if the gas density dominates), this yields $H \propto \rho_{mid}^{-1/2}$. For the normalization, we use the results from Kim et al. (2013), in which we obtained $H \approx 85$ pc for the midplane density $n_{avg} \approx 1.5 \ cm^{-3}$ from self-consistent modeling of galactic disks with feedback from star formation. In the present simulations, we do not in fact include any vertical gravity, so that our models are unstratified. However, it is useful to keep in mind an approximate value for the scale height, in order to define SB properties at the time the bubble radius reaches what the warm/cold ISM scale height would be, and starts to breakout into circumgalactic space.

4. NUMERICAL SIMULATION RESULTS

Before describing the model results, we establish definitions for separate components of the SB. First, we define the “bubble” component as all the gas that has been affected by the blast wave. This is composed of all zones with $T > 10^5 K$ or $v > 1 \ km \ s^{-1}$. The ambient medium is composed of the remainder of zones in the domain (note that ambient gas is initially stationary, but small velocities develop since pressure balance between the warm and cold phases is not perfect). We define the “hot” gas as all zones with $T > 10^5 K$. All the hot gas is part of the bubble, but the bubble also contains gas that has been shocked and then radiatively cooled below $10^5 K$.

We measure the equivalent spherical radius, mass, total energy, pressure, and temperature of the hot and bubble gas. The radius is $r_c \equiv (3 \varepsilon / 4 \pi)^{1/3}$ and $V_c \equiv \sum \Delta_r^3$, where the gas component “c” can be either “hot” or “bubble,” and $\sum \Delta_r$ is summation over the zones that satisfy the definition of each gas component. The mass and energy are defined by $M_c \equiv \sum \rho \Delta_r^3$ and $E_c \equiv \sum \rho P (\gamma - 1) + \rho v^2 / 2 \Delta_r^3$, respectively. The pressure and temperature are defined with volume and mass-weighted means, respectively, as $P_c \equiv \sum \rho \Delta_r / V_c$ and $T_c \equiv 1.27 m_H P_c / V_c / (M_c k_b)$. Finally, the total radial momentum of the bubble is calculated by $p_b \equiv \sum \rho v \cdot \Delta_r^2$.

In Table 2, we summarize properties of SB evolution for each model. The expected shell formation time $t_{fin}$ from Equation (10) is listed in Column (2), and the measured times when $r_b = H$ and $2H$, $t_{121}$ and $t_{221}$, are listed in Columns (3) and (4), respectively. We also list the reference scale height in Column (5) and the measured bubble radius at $t_{fin}$ in Column (6). The measured bubble mass, mean velocity $v_b \equiv p_b / M_b$, and the hot gas temperature in the simulation at $t_{121}$ and $t_{221}$ are listed in Columns (7)-(12). As noted previously, because we do not allow the mean molecular weight to vary in the simulation, the true temperature of the hot medium would be a factor of two lower.

To connect our results to loading of galactic winds, we measure the hot gas mass and thermal energy per SN event defined by $M_b \equiv M_b / N_{SN}$ and $E_b \equiv E_{bh} / N_{SN}$, respectively, with $N_{SN} = [1 / \Delta t_{SN} + 1]$, where $[x]$ is the floor function that maps a real number $x$ to the largest previous integer. To connect our results to the driving of turbulence within galactic disks, we measure the total radial momentum of the bubble per SN event as $\dot{p}_b \equiv p_b / N_{SN}$. In Table 3, we summarize the SB properties per SN measured at $t_{121}$ and $t_{221}$.

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3 We have confirmed that if, rather than injecting thermal energy, we introduce the same amount of kinetic energy as expanding ejecta, our results are essentially the same. See the Appendix.
From Table 2, the SB would expand to \( H \) within \( 10^5-10^6 \) year for the parameter range considered, but because the expansion slows over time, reaching \( 2H \) requires \( 10^8-10^9 \) year. Table 2 also shows that \( r_b(t_{\text{sf,m}}) < H \) by a large margin for all cases except models with \( \Delta t_{\text{SN}} = 0.01 \) Myr. As we will discuss in Section 5, this implies that unless \( \Delta t_{\text{SN}} \) is quite short, SBs cool before breaking out of the disk. In turn, this suggests that substantial hot gas mass-loss in SN-driven winds can only occur in localized regions within galaxies where there are fairly massive clusters, or where several clusters are in close enough proximity (e.g., in galactic center regions) such that \( \Delta t_{\text{SN}} \) from the combined system is short. Indeed, Table 3 shows that \( \tilde{M}_b(H) > 100 M_\odot \) in only two cases with \( \Delta t_{\text{SN}} = 0.01 \) Myr. This implies that in most cases (for the present range of parameters), the hot gas mass in an SB at breakout is less than the total mass in newly formed stars of the cluster that drove the SB. However, Table 2 also shows that in essentially all cases, the hot gas has a temperature \( > 10^6 \) K at the time the SB would breakout of the disk. Thus, while the amount of hot gas expelled per star formed may not always be large, the sound speed is generally high enough to drive a wind that can escape the galactic potential well (see Section 4.4).

For most cases, \( \tilde{E}_b(H)/10^{51} \) erg is only a few percent or less, implying that most of the input energy is lost to a combination of radiative cooling and kinetic energy in the warm/cold ISM before SB breakout. Indeed, \( \tilde{E}_b(H) \gg \tilde{E}_b(H) \) in all cases, except those where \( t_{\text{sf,m}} \sim t_1 \).

The mean velocity of the SB substantially exceeds 10 km s\(^{-1}\) at \( t_H \) for the models with \( \Delta t_{\text{SN}} = 0.01 \) and 0.1 Myr. Since this

### Table 2
Superbubble Evolution Properties

| Model          | \( t_{\text{sf,m}} \) (1) | \( t_1 \) (2) | \( t_2 \) (3) | \( H \) (4) | \( r_b(t_{\text{sf,m}}) \) (5) | \( M_b(H) \) (6) | \( M_b(2H) \) (7) | \( v_b(H) \) (8) | \( v_b(2H) \) (9) | \( T_1(H) \) (10) | \( T_1(2H) \) (11) |
|----------------|--------------------------|--------------|-------------|-----------|----------------------------|----------------|-----------|-------------|--------------|--------------|--------------|
| n0.1-t0.01     | 1.23                     | 1.23         | 1.32        | 3.18      | ...                       | ...           | ...       | 3.98        | ...          | ...          | ...          |
| n0.1-t0.1      | 0.65                     | 3.59         | 3.29        | 3.15      | ...                       | ...           | ...       | 16.9        | ...          | ...          | ...          |
| n0.1-t1        | 0.34                     | 8.32         | 10.72       | 2.98      | ...                       | ...           | ...       | 7.87        | ...          | ...          | ...          |
| n1-t0.01       | 0.28                     | 0.32         | 1.69        | 0.81      | 8.13                      | 46.5          | 3.32      | 7.75        | 10.1         | ...          | ...          |
| n1-t0.1        | 0.15                     | 1.01         | 4.12        | 0.79      | 6.93                      | 19.6          | 2.07      | 4.12        | 4.06         | ...          | ...          |
| n1-t1          | 0.076                    | 1.86         | 8.72        | 0.64      | 4.75                      | 8.00          | 0.83      | 1.59        | 2.13         | ...          | ...          |
| n1-t0.01-high  | 0.051                    | 0.14         | 0.58        | 0.26      | 2.26                      | 45.2          | 4.60      | 14.0        | 11.4         | ...          | ...          |
| n1-t0.1-low     | 0.027                    | 0.25         | 1.24        | 0.21      | 1.81                      | 21.7          | 1.93      | 5.43        | 4.41         | ...          | ...          |
| n1-t0.1-1-ej    | 0.014                    | 0.28         | 2.05        | 0.15      | 1.18                      | 12.4          | 1.56      | 0.70        | 1.98         | ...          | ...          |

### Table 3
Superbubble Properties per SN

| Model          | \( \tilde{M}_b(H) \) (2) | \( \tilde{M}_b(2H) \) (3) | \( \tilde{p}_b(H) \) (4) | \( \tilde{p}_b(2H) \) (5) | \( \tilde{E}_b(H) \) (6) | \( \tilde{E}_b(2H) \) (7) | \( \tilde{E}_b(H) \) (8) | \( \tilde{E}_b(2H) \) (9) |
|----------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| n0.1-t0.01     | 386                      | ...                      | 1.02                     | ...                      | 0.15                     | ...                      | ...                      | ...                      |
| n0.1-t0.1      | 169                      | ...                      | 1.55                     | ...                      | 0.034                    | ...                      | ...                      | ...                      |
| n0.1-t1        | 48                       | ...                      | 2.61                     | ...                      | 0.0084                   | ...                      | ...                      | ...                      |
| n1-t0.01       | 51                       | 39                       | 1.40                     | 1.30                     | 0.025                    | 0.019                    | 0.14                     | 0.078                    |
| n1-t1          | 4.8                      | 11                       | 2.57                     | 1.98                     | 0.0008                   | 0.0028                   | 0.064                    | 0.045                    |
| n10-t0.01      | 21                       | 16                       | 0.84                     | 0.74                     | 0.034                    | 0.022                    | 0.21                     | 0.10                     |
| n10-t0.1       | 3.9                       | 9.0                       | 1.49                     | 1.16                     | 0.0025                   | 0.0045                   | 0.087                    | 0.043                    |
| n1-t0.1-high   | 3.7                       | 0.71                      | 2.27                     | 1.84                     | 0.0001                   | 0.0002                   | 0.083                    | 0.054                    |
| n1-t0.1-low     | 58                       | 38                       | 1.49                     | 1.37                     | 0.025                    | 0.018                    | 0.15                     | 0.078                    |
| n1-t0.1-1-ej    | 344                      | 38                       | 1.26                     | 0.87                     | 0.020                    | 0.030                    | 0.51                     | 0.12                     |
| n1-t1-ej       | 4.9                       | 11                       | 2.58                     | 1.81                     | 0.0008                   | 0.031                    | 0.064                    | 0.041                    |

Note. Column (1): model name. Columns (2–3): mass of the hot gas per SN at \( H \) and \( 2H \), in \( M_\odot \). Columns (4–5): total radial momentum of the bubble per SN at \( H \) and \( 2H \), in \( 10^{51} \) erg. Columns (6–7): thermal energy of the hot gas per SN at \( H \) and \( 2H \), in \( \text{km s}^{-1} \). Columns (8–9): total bubble energy per SN at \( H \) and \( 2H \), in \( 10^{51} \) erg.
exceeds typical background turbulence levels in observed galaxies, it suggests that SBs would remain coherent in their appearance until breakout for SBs driven with a high SN cadence, as argued in Section 2.3. Cases with $\Delta SN = 1$ Myr have lower $v_{\text{esc}}(H)$, suggesting that for lower-mass clusters, the SB shell would instead merge with the background ISM turbulence before breaking out of the disk. In all cases, the mean value of $v_{\text{esc}}(H)$ is smaller than the escape speed of all but very low-mass halos, indicating that the shell would not escape as a whole from most galaxies. However, we will see in Section 4.3 that there is substantial gas mass with velocities above 50 km s$^{-1}$ for the cases $\Delta SN = 0.01$ and 0.1 Myr, which would be able to escape from dwarf galaxies.

4.1. Time Evolution of Overall Bubble Properties

Figures 1–3 plot time evolution of SB properties for $\Delta SN = 0.01$, 0.1, and 1 Myr, respectively. Each panel shows (a) bubble radius $r_{\text{sf}}$; (b) hot gas radius $r_{\text{h}}$; (c) bubble mass $M_{\text{sf}}$; (d) hot gas mass $M_{\text{h}}$; (e) bubble momentum $p_{\text{h}}$; (f) bubble energy $E_{\text{h}}$; (g) bubble pressure $P_{\text{b}}$; and (h) hot gas temperature $T_{\text{h}}$. Analytic predictions of SB radius (Equation (4)), momentum (Equation (7)), and total injected energy (Equation (2)) in the energy-conserving (adiabatic) phase are shown as dotted lines in (a) and (b), (e), and (f), respectively. Analytic predictions of SB radius (Equation (18)) and momentum ($p_{\text{h}}/\Delta SN$) in the momentum-driven snowplow phase are shown as dashed lines in (a) and (e), respectively. Note that for $n_{\text{amb}}$, in those equations, we use the volume-filling WNM density, $n_{0.1}$, instead of the mean density of the background medium, $n_{\text{sys}}$, and for $p_{\text{h}}$, we use $p_{\text{h}}(t_{\text{final}})/\Delta SN/t_{\text{final}}$, where $t_{\text{final}}$ is the final time of each simulation. Also note that although we do not show the analytic pressure-driven bubble solutions, these are very close to the analytic adiabatic solutions shown, with a radius just 14% smaller (see discussion in Section 2.2). We also overplot as dotted lines the predictions for the swept-up WNM mass ($M_{\text{sw}, \text{v}} = \rho_{\text{v}, 4}\pi r^3/3$) in (d), again using Equation (4) for $r = r_{\text{sf}}(t)$. The circles in panel (a) denote $t_{\text{f}}$ and $t_{\text{final}}$, the time when $r_{\text{h}} = H$ and $2H$, respectively, while the squares in panels (b), (c), and (d) stand for $t_{\text{sf}, \text{m}}$, the predicted shell formation time for an SB driven by multiple SNe (Equation (9)). The solid horizontal lines in (g) show the ambient medium pressure for reference.

The SBs in our simulations can be categorized by comparing two timescales, $t_{\text{sf}, \text{m}}$ and $t_{\text{sf}, \text{m}}$. The models with $\Delta SN < t_{\text{sf}, \text{m}}$ (n0.1-t0.01, n0.1-t0.1, n1-t0.01, n10-t0.01) are in the limit of continuous energy injection, which we call the “continuous limit,” while the models with $\Delta SN > t_{\text{sf}, \text{m}}$ (n0.1-tn1, n1-tn1, n10-tn1, n10-t1) are in the opposite limit in which each SN acts discretely, which we call the “individual-SN limit.” Model n1-t0.1 does not satisfy either limit, $\Delta SN \sim t_{\text{sf}, \text{m}}$.

For the cases in the continuous limit, the overall evolution roughly follows the analytic predictions derived in Section 2 up to $t \sim t_{\text{sf}, \text{m}}$ (see Figure 1). Although the analytic prediction assumes a uniform background medium (rather than a two-phase state), the use of the volume-filling WNM density as the reference ambient value ($n_{\text{amb}} \rightarrow n_{0.1}$ for Equations (4) to (9)) provides a good estimate for the early-time bubble radius in most cases. The exception is when the bubble is big enough to enclose many cold clouds at $t \sim t_{\text{sf}, \text{m}}$ (e.g., n0.1-t0.01), in which case a significant amount of energy has already been radiated away before radiative cooling of the shocked WNM becomes important. The bubble and hot gas masses at $t \lesssim t_{\text{sf}, \text{m}}$ are also in rough agreement with the predicted swept-up total mass and warm gas mass, respectively. This implies that the hot gas is mainly produced by shocks propagating into the WNM.

Due to the shock heating, the WNM is traversed and loses energy by doing work and also by mixing with dense gas (leading to radiative cooling). Thus energy-conserving solutions would only be strictly applicable when the bubble expands in a single-phase (warm) medium. At $t \sim t_{\text{sf}, \text{m}}$ for the models in the continuous limit, the shocked WNM begins to cool, and the hot gas mass starts to decrease. The analytic predictions for $t_{\text{sf}, \text{m}}$ lie close to the time when the hot gas mass peaks in Figure 1(d). After a short period of decline, the hot gas mass again starts to rise, and the interval between SNe is short enough for these models that the evolution remains continuous. In this limit, the bubble interior remains filled with hot gas (see Figure 1(h)) and remains at much higher pressure than the ambient medium (see Figure 1(g)). The radial momentum of the bubble continues to increase as the overpressured interior pushes the outer shell, although the momentum increase stays far below the estimate for non-radiative pressure-driven expansion (cf. Equations (14) and (15)).

In the opposite limit, the individual-SN cases (see Figure 3), the analytic energy-conserving continuous-injection predictions are far from the real evolution, even at an early time. Instead, the evolution due to each SN is distinct. The shocks propagating into both the WNM and CNM cool down, and the bubble evolution enters the momentum conserving stage, before the next SN explosion. Each successive SN heats up the bubble and adds momentum to the shell, but the injected energy is largely radiated away. For Model n0.1-t1 (see also n10-t0.1 in Figure 2), the remaining hot gas and the residual pressure are non-negligible, so that at later times the bubble interior remains overpressured with respect to the ambient medium. The bubble continues to expand and injects momentum more continuously. However, for the extreme case of Model n10-t1 with very short $t_{\text{sf}, \text{m}}$, where the bubble completely cools down before the next SNe, the pressure of the bubble is even smaller than the ambient medium, so that the bubble cannot expand further, reaching a maximum size of ~130 pc.

For the intermediate case, Model n1-t0.1, the later time evolution is similar to that of the continuous-limit models, although this model does not have a phase that is consistent...
Figure 1. Time evolution of the models with $\Delta t_{SN} = 0.01$ Myr. Panels show (a) radius of the bubble $r_b$, (b) radius of hot gas $r_h$, (c) mass of the bubble $M_b$, (d) mass of hot gas $M_h$, (e) total radial momentum $p_b$, (f) total energy of the bubble $E_b$, (g) pressure of the bubble $P_b/k_B$, and (h) temperature of hot gas $T_h$. The circles in panel (a) indicate the times when the corresponding radii reached $H$ and $2H$, $t_f$ and $2t_f$, respectively. The circles in panel (a) denote the corresponding times at $t = t_{bm}$. The dotted lines in (a) and (b), (e), and (f) denote analytic predictions for radius, momentum, and total injected energy in the energy-conserving continuous limit from Equations (4), (7), and (2), respectively, while the dotted lines in (d) indicate the warm swept-up masses using the radius predicted from Equation (4). The dashed lines in (a) and (e) denote analytic predictions for radius and momentum in the momentum-driven snowplow stage from Equation (18) and $p_b t/\Delta t_{SN}$, respectively. The solid lines in (g) show the ambient medium pressure for reference.
with the energy-conserving bubble. Rather, the early evolution is similar to that of the individual-SN limit.

The late-time evolution of the bubble radius and radial momentum is very well described by the momentum-driven snowplow prediction (see dashed lines in (a) and (e)). Although we force the coefficient to match the final momentum by using $p_b = p_b(t_{\text{final}})/N_{\text{SN}}$, the time dependences of $r_b$ and $p_b$ are very close to $t^{1/2}$ and $t$, respectively. The agreement with Equation (18) is excellent for the models in the continuous limit, but is still reasonably good in the opposite limit.

Figure 2. Same as Figure 1, but for models with $\Delta t_{\text{SN}} = 0.1$ Myr.
4.2. Detailed Structure of Bubbles

To provide a sense of the evolution in SB morphology in a cloudy ambient medium, we show slices through Models n1-t0.01 (Figure 4), n1-t0.1 (Figure 5), and n1-t1 (Figure 6). Each figure consists of three rows, showing number density, pressure, and temperature from top to bottom, and three columns, showing snapshots at $t = t_{sf,m}$, $t = t_H$, and $t = t_{SH}$, from left to right. We select models n1-t0.01 and n1-t1 as representative of SBs in the continuous and individual-SN limits, respectively, while Model n1-t0.1 represents an intermediate case between these limits.

Figure 3. Same as Figure 1, but for models with $\Delta t_{SN} = 1$ Myr.
Until $t_{sf,m}$, the interior pressure is high enough that the expansion is nearly spherical in all cases. Since shocked WNM starts to cool earlier when the SN rate is lower, the size of bubbles is different at $t_{sf,m}$.

Interesting differences in morphology can be seen in the snapshots at $t_{H}$ (middle columns of Figures 4–6), in which the bubbles have similar physical sizes but are at different evolutionary stages. Since $t_{sf,m} \sim t_{H} \sim 0.3$ Myr for Model n1-t0.01, the bubble expands up to $r_{b} = H$ without suffering catastrophic energy loss. From Table 3 for Model n1-t0.01, 44% and 15% of the energy that has been injected remains as total energy in the bubble and thermal energy in the hot medium, respectively, at this time. With $t \sim t_{sf,m}$, the SB has retained a spherical shape and hot, highly overpressured interior. In contrast to the case of a bubble expanding in a uniform medium, however, there is non-negligible radiative energy loss through shocked CNM clouds in the SB interior, which are still dense but warm ($T \sim 10^{4}$ K).

In contrast, for Model n1-t1, the shell formed at early time ($t_{d} = 0.13$ Myr), and there was only one more SN event before $t_{H} \sim 1.9$ Myr for this case. Although Figure 3(g) shows that the mean bubble pressure remains higher than in the ambient medium, the interior pressure is in fact lower than in the ambient medium, since the bubble pressure is dominated by the shell (see pressure at $t = t_{H}$ in Figure 6). Therefore, the shell expands in a nearly force-free fashion (the RHS in Equation (14) is negligible). Radiative thin-shell instabilities (Vishniac 1983, 1994) produce wiggles in the shell. Model n1-t0.1 also forms a shell ($t_{d} \sim t_{sf,m} \sim 0.15$ Myr) well before $t_{H} \sim 1$ Myr, but there were 10 more SN explosions prior to $t_{H} \sim 1$ Myr, so the bubble interior is still overpressured and hot.

The overall morphology of bubbles at $t_{2H}$ looks more or less similar in all models, since this epoch is much later than the shell formation time ($t_{2H} \gtrsim 5t_{sf,m}$ even for Model n1-t0.01). However, the detailed internal structure and mass, momentum, and energy budgets are substantially different. Most importantly, the bubbles still have overpressured interiors for Models n1-t0.01 and n1-t0.1, while Model n1-t1 has a completely exhausted interior and an overpressured shell.

To show the detailed structure and interaction between ambient medium and shell gas (cooled bubble gas) and between shell and hot gas, Figure 7 displays from top to bottom zoom-in images of number density, temperature, ram

Figure 4. XY-slices for Model n1-t0.01. From top to bottom, logarithmic color scales show number density, pressure, and temperature. From left to right, columns correspond to snapshots at $t = t_{sf,m}, t = t_{H}, t = t_{2H}$. The white rectangle in the top-right panel indicates the region for which zoomed images are shown in Figure 7.
pressure $P_{\text{ram}} \equiv \rho v^2$, thermal pressure, and velocity magnitude $v \equiv |\mathbf{v}|$ at $t_{2H}$ for the regions marked in Figures 4–6 (columns from left to right). We also overplot isotherm temperature contours of $T = 500$ K and $10^5$ K in cyan and red to show the separation of the cold, warm, and hot phases.

The boundary between the ambient medium and the bubble is clear from the transition in the velocity magnitude maps, while the red contours delimit the boundary between the cooled gas in the bubble envelope and hot interior gas. For Model n1-t0.01 (left), a strong forward shock is propagating into the ambient medium, and the interior remains hot and highly overpressured. The bubble is bounded by a very thin overdense shell of cooled gas. However, for Model n1-t1 (right), the thermal and ram pressures of the shocked and cooled ambient gas exceed those of the bubble interior, and the bubble envelope is a broad overpressured region, rather than a thin shell. Rather than a forward shock between the shell and ambient gas seen in Model n1-t0.01, there is a smooth pressure wave propagating into the ambient medium.

In Model n1-t0.01, there are embedded dense clouds that are completely surrounded by hot gas, and some dense clouds remain warm. In Model n1-t1, most dense clouds have cooled back to the cold temperature. Model n1-t0.1 (middle) is intermediate, showing characteristics of both Models n1-t0.01 and n1-t1. Differences in the envelope structure (thin versus broad shell) are also quite clear in the top rows of Figures 4–6.

We note that the evolution of dense (initially cold) clouds within SBs is not fully resolved in the present simulations. In our simulations, dense clouds are initially shock-heated and accelerated when they are overrun by the outer forward shock of the SB. In cases with high-cadence SNe, these dense clouds in the interior can remain warm due to frequent shocks from subsequent explosions and the high pressure of surrounding hot gas. In cases with low-cadence SNe, embedded clouds cool down. With extremely high resolution simulations focused on individual clouds, hydrodynamical instabilities caused by shock-cloud interactions can be followed in detail (e.g., Klein et al. 1994; Mac Low et al. 1994; Scannapieco & Brüggen 2010); over time, these ablate small clouds and mix their material into the bubble interior. Here the resolution is much more limited, and we also neglect the thermal conduction and magnetic fields that would affect development of instabilities that tend to destroy clouds. Thus, although it is uncertain exactly how limited resolution and physics affect the evolution of individual dense clouds in our simulations, we believe that
our main results for the overall evolution of SBs are not strongly sensitive to this uncertainty. In particular, we measure in the Appendix the hot gas mass, momentum, and energy produced per SN at varying numerical resolution, and find these quantities are very well converged.

4.3. Gas Distributions in Temperature, Velocity, and Density

We next investigate the distributions of gas in temperature, velocity, and density at $t = t_H$ (i.e., when $r_H = H$). The probability density functions (PDFs) provide a detailed picture of the gas that would be available to create high speed winds when the bubble breaks out of the ISM disk into circumgalactic space. Figures 8 and 9 display the mass (contours) and volume (colors) fractions of all the gas within $r < 1.1H$ in the log $T$–log $v$ and log $n_H$–log $v$ planes, respectively. In these figures, results for models that are in the continuous energy injection limit (high SN cadence, with $\Delta t_{\text{SN}} < t_{\text{sf,m}}$) have red borders (panels (a), (b), (d), and (g)), while results for models that are in the individual-SN limit have blue borders (panels (c), (f), (h), and (i)).

In Figure 8, the dotted lines in each panel indicate the demarcation between gas that is defined as “ambient” ($T < 10^5 \text{ K}$ and $v < 1 \text{ km s}^{-1}$) and “bubble.” Although a portion of the gas in the “ambient” regime actually consists of dense gas clouds that have been shocked and subsequently cooled and slowed down, this represents at most $\sim 10\%$ of the total bubble mass. Thus, while not perfect, our definition represents a good practical criterion for distinguishing ambient and bubble gas. In each panel, the black dashed line shows the locus where the velocity, $v$, equals the sound speed, $c_s \equiv (k_B T/(1.27 n_H))^{1/2}$. Green dashed lines show the loci where the specific kinetic energy, $v^2/2$, equals the specific enthalpy $h \equiv \gamma P/[(\gamma - 1) \rho] = 5c_s^2/2$. Gas above and to the left of the black line is supersonic, and gas below and to the right of the green line has the Bernoulli parameter dominated by the enthalpy term.

The temperature–velocity distributions further distinguish different components of the bubble gas: hot interior, shocked warm shell gas and shocked warm clouds (originally WNM and CNM, respectively), and accelerated cold gas (shocked and then cooled CNM clouds). The volume-filling interior hot gas is easily seen in Figure 8 at $T > 10^6 \text{ K}$ and $v \sim 10^3 \text{ km s}^{-1}$. Moving from the continuous limit (top-left panels) to the individual-SN limit (bottom-right panels), this component gets cooler and slower. The hot medium consists of gas that was...
originally WNM, and was shock-heated and expanded into the SB interior to create this very hot and diffuse phase.

In Figure 9, the shocked dense clouds (originally CNM) can be found in a vertical band at high density, also enclosed by contours. For models with short $\Delta t_{SN}$, in the continuous limit (red borders), the dense gas has velocities up to a few tens of km s$^{-1}$. Although the cooling time of the shocked CNM is short due to its high density, clouds within the bubble are repeatedly shocked and surrounded by high pressure interior hot gas, so that the cooling is compensated by additional shock and compression heating for models with short $\Delta t_{SN}$ (see also the left column of Figure 7). Thus these shock-accelerated dense clouds remain warm. For the continuous-limit models (red borders) of Figures 8 and 9 (see contours for mass-weighted PDFs), there is no accelerated gas ($v > 1$ km s$^{-1}$) that has returned to cold temperatures ($T \sim 10^2$ K). However, models in the individual-SN limit (blue borders) of Figures 8 and 9 show a clear distribution of cold medium with velocity $\sim 1-10$ km s$^{-1}$ within contours; this material is dense clouds that have been shocked and accelerated, but for which the shock and compressional heating are inadequate to offset cooling.

The broadband in Figure 8 connecting the highest-temperature gas to gas at $T \sim 10^4$ K shows the effect of radiative cooling in the shell. Shocks at the boundary of the SB accelerate WNM to $v \sim 100$ km s$^{-1}$ and heat it to a high temperature, but it cools back to $T \sim 10^3$. This creates the warm shell of high- and moderate-velocity gas at the edge of the SB (see Figure 7). Models in the continuous limit show, in Figure 8, a broad warm gas distribution with a velocity range of $1-100$ km s$^{-1}$, which is a combination of the shocked and accelerated WNM and CNM; in Figure 9, these components can be distinguished based on their density. In models in the individual-SN limit, the warm gas is at a somewhat lower velocity, because the hot interior is lower pressure and the expansion into the ambient medium creates weaker shocks.

Most of the bubble gas at warm and cold temperatures is moving supersonically, since after it was accelerated and heated in a shock, its sound speed dropped by radiative cooling (see Figure 8). However, Figure 8 shows that the hot interior gas is at most transonic in its velocities, and generally has a specific enthalpy larger than the specific kinetic energy.

In addition to the mean expansion velocity of the bubble, it is also interesting to consider the distribution of mass with
velocity. Figures 8 and 9 show that the velocity increases toward lower density and higher temperature, and that the mass is divided between the denser (and slower) former CNM and the lower density (and faster) former WNM. As is also evident in Figures 8 and 9, the velocity distribution depends more on $\Delta v_{SN}$ than on $n_{\text{avg}}$. Except for the cases with the longest $\Delta v_{SN}$, there is $\sim 10 M_\odot$ per SN with $v_1 > 100-200$ km s$^{-1}$. As SBs are dominated by the more slowly moving warm and cold gas, the mass rises at lower velocity. For the $\Delta v_{SN} = 0.1$ Myr models, there is $\sim 100 M_\odot$ per SN with $v_1 > 50-70$ km s$^{-1}$, and for the $\Delta v_{SN} = 0.01$ Myr models, there are $> 100 M_\odot$ and $> 500 M_\odot$ per SN at $v_1 > 100$ km s$^{-1}$ and $> 50$ km s$^{-1}$, respectively. The gas at $v_1 \sim 50-70$ km s$^{-1}$ would form a galactic fountain in a massive galaxy like the Milky Way. However, these results suggest that in dwarf galaxies with shallower potential wells, substantial mass could escape as warm outflows driven by SBs.

4.4. Hot Gas Mass, Energy, and Momentum Injection per SN

SBs created by young, massive star clusters are some of the most plausible drivers of galactic winds. Thus we have shown in Figure 8 (see also Figure 10), only hot gas has high enough velocity (higher than a few hundred km s$^{-1}$) that it would be able to escape from a galaxy similar to the Milky Way. Warm and cold gas with $v$ velocities of several tens to a few hundred km s$^{-1}$ could, however, create a galactic fountain, while lower velocity warm and cold gas would interact with the surrounding ISM to drive turbulence. In a low mass galaxy with a shallow potential, warm and cold gas at $v_1 \sim 50-100$ km s$^{-1}$ would be able to escape as a wind.

In the classical adiabatic wind model of Chevalier & Clegg (1985), gas is accelerated to transonic velocities within a source region of a galaxy, and further accelerated to escape speeds by pressure gradients as the gas expands into circumgalactic space. In Chevalier & Clegg (1985) and subsequent models of thermal-pressure-driven winds, while the combined effects of multiple SNe are assumed to be responsible for producing the hot gas that feeds the outflow, this is not treated directly but parameterized in terms of the mass and energy injection per star formed (or per SN). For adiabatic steady winds, the conserved quantities beyond the source region are the mass flux, Bernoulli parameter, and specific entropy. Wind acceleration is associated with the increase of specific kinetic energy at the expense of decreasing specific enthalpy, while the sum of these terms (plus the gravitational potential energy) is equal to a fixed Bernoulli parameter.

In Figure 11, we plot mass (a) and (b)) and thermal energy (c) and (d)) of the hot gas per SN event as functions of the normalized time $t_{/t_{\text{d,m}}}$ ((a) and (c)) and radius of bubble $r_b/H$ (b) and (d)). Since the evolution of bubble properties can be spiky, especially for models in the individual-SN limit (see Figure 3), we show as symbols only values at the moment immediately before each SN event, and connect these symbols with dotted lines. The dotted lines represent lower/upper limits of mass/energy loading. We show the full evolution between the first and second SNe with continuous solid lines.

As already seen in Section 4.1, the hot gas mass initially increases rapidly as shocks propagate into the WNM, sharply drops at $t \sim t_{/t_{\text{d,m}}}$ when this shocked gas cools and forms a shell around the SB, and subsequently resumes a slower increase as shocks heat the inner surface of the shell bounding the SB and
Since we anticipate one SN for every shell formation time. The peak values are direction perpendicular to a disk velocity. To indicate the average one-dimensional velocity (e.g., in the direction perpendicular to a disk), we use $v_1 \equiv v/\sqrt{3}$.

The evolution of hot gas clouds left behind in the SB interior. The evolution of hot gas mass per SN, $\dot{M}_h$, reflects this behavior. The peaks of $\dot{M}_h$ line up very well at $t/t_{sf,m} \sim 1$ in Figure 11(a), implying that Equation (10) provides reasonably good estimates for the SB shell formation time. The peak values are $\dot{M}_h \sim 500-2000 M_\odot$. This is consistent with the prediction of Equation (13) that $\dot{M}_{h, sf,m} \sim 1000 M_\odot$.

Following the sharp drop in $\dot{M}_h$ at $t/t_{sf,m} \sim 1$, the late stages of evolution show a slow decline in $\dot{M}_h$. Except in the extreme case of Model n10-t1, in which hot gas produced by each SN event completely cools down before the next SN, the late-stage values ($t = t_1 - t_{2H}$) of $\dot{M}_h$ remain between $10 M_\odot$ and $100 M_\odot$. Since we anticipate one SN for every $m_h = 100 M_\odot$ of new stars formed from the IMF (e.g., Kroupa 2001), these values correspond to a “dimensionless mass loading factor” (e.g., Chevalier & Clegg 1985; Thompson et al. 2016) $\beta_h \equiv M_{hot}/M_h = M_h/m_h = 0.1-1$. Peak hot gas mass loading values for our set of parameters are $\beta_h = 5-20$, but except for cases with $\Delta t_{SN} = 0.01 \text{Myr}$ and $n_{avg} = 0.1$ and, $1 \text{cm}^{-3}$, the time for the peak is well before $t_{1H}$.

SBs are expected to breakout of the ISM, venting their hot gas into circumgalactic space, when their size exceeds the scale height of the warm/cold ISM. Although the present simulations are for unstratified ISM disks, we can obtain useful estimates of conditions at breakout by measuring the hot gas properties at $r_b = H$ and $r_b = 2H$. These are listed in Table 3 and shown in Figures 11(b) and (d). If the time interval between SNe is sufficiently short (or the star cluster is sufficiently massive), the bubble radius can reach $H$ during the energy-conserving phase, i.e., $H \leq r_b(t_{sf,m})$. In our simulations, Model n0.1-t0.01 is the only case that satisfies this condition. For this model, $\beta_h \sim 4$ at $t_{1H}$, but $\beta_h$ drops to less than one before $t_{2H}$. Model n1-t0.01 also has $r_b(t_{sf,m})$ close to $H$, and has $\beta_h = 1.7$ at $t_{1H}$. However, all other models have begun cooling before $r_b$ reaches $H$, yielding $\beta_h \sim 0.1-1$ at $t_{1H}$. For any given $\Delta t_{SN}$, there is a secular decrease in $\dot{M}_h(H)$ with increasing density. Similarly, for any given $n_{avg}$, there is a secular decrease in $\dot{M}_h(H)$ with increasing $\Delta t_{SN}$.

The dimensionless energy loading factor is defined by $\alpha_h \equiv \dot{E}_h/E_{SN}$, which is equivalent to the definition used in Thompson et al. (2016). In a uniform medium, by definition the total SB energy per SN is equal to $E_{SN} = 10^{51} \text{erg}$ for an SB during the energy-conserving phase, but for a multiphase ISM, some of the energy can be radiated away even at $t < t_{sf,m}$ via interactions with the CNM clouds. Similarly, the thermal energy per SN in the hot component would be fixed for $t < t_{sf,m}$ in a uniform medium, but not in a multiphase medium. Figure 11(c) shows that $\dot{E}_h$ declines slowly before the shell

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**Figure 9.** Same as Figure 8, but in the log $n_H$–log $v$ plane.

**Figure 10.** Cumulative mass in the bubble per SN at $t = t_{1H}$, as a function of velocity. To indicate the average one-dimensional velocity (e.g., in the direction perpendicular to a disk), we use $v_1 \equiv v/\sqrt{3}$. 

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formation time due to the cooling of shocked dense CNM clouds, and then drops more abruptly as the shocked WNM gas begins to cool at $t_{sf,m}$. At $t_{sf,m}$, $\dot{M}_h/E_{SN} = \alpha_h \sim 0.1-0.5$. After the strong drop in $\dot{E}_h$ at $t \sim t_{sf,m}$, the subsequent decline is similar to the decline in $\dot{M}_h$. In fact, after each SN event, the mean temperature of the hot gas returns to nearly the same value (see Figure 12). With nearly constant $T_h$, $\dot{E}_h \propto \dot{M}_h$. At $t_H$, $\dot{E}_h/E_{SN}$ has a wide range of values below 0.2, decreasing for higher $n_{avg}$ and for larger $\Delta t_{SN}$. At $t_{2H}$, there is a narrower range of $\dot{E}_h (\alpha_h \sim 0.002-0.5$, except for n10-t1), which maintains the trend of lower $\dot{E}_h$ at higher $n_{avg}$ and $\Delta t_{SN}$.

In Figure 12, we plot the mass-weighted mean temperature of the hot gas, which is a key quantity for controlling large-scale wind acceleration and escape from the galactic potential well. For a steady flow, the Bernoulli parameter (or function) is
2 \times 10^6 \text{ to } 2 \times 10^7 \text{ K for } t = t_1 - t_2H. \text{ For any given } \Delta t_{SN}, \text{ the range of } T_b \text{ for } t = t_1 - t_2H \text{ is even smaller, and } T_b \text{ increases with decreasing } \Delta t_{SN}. \text{ This suggests that the enthalpy of the hot gas that loads winds would be insensitive to exactly when and how breakout occurs. Furthermore, } T_b \text{ during the breakout stage depends more on the mass of the cluster driving the outflow (i.e., on } \Delta t_{SN}) \text{ than on the conditions of the ambient ISM (} n_{avg} \text{). Note that this behavior is opposite to the hot gas mass loading, which depends more strongly on } n_{avg} \text{ than on } \Delta t_{SN} \text{ (compare Figure 12(b) with Figure 11(b)). However, Figure 10 shows that the overall distributions of mass with velocity are more sensitive to } \Delta t_{SN} \text{ than } n_{avg}. 

In addition to loading of winds, SBs are important for driving turbulence in the warm/cold ISM, which in turn regulates SFRs. For self-regulated disk star formation, the turbulent pressure is proportional to the mean momentum injection per unit stellar mass formed } p_b/m_\ast, \text{ while the SFR is inversely proportional to } p_b/m_\ast \text{ (Kim et al. 2011; Ostriker \\& Shetty 2011). Previously, KO15 measured the final radial momentum of late-stage SNRs from single SNe in two-phase ISM backgrounds with a large range of } n_{avg} = 0.1-100, \text{ as well as a few different cases with multiple SNe and } \Delta t_{SN} = 1 \text{ Myr. Here we quantify momentum injection in terms of the mean radial momentum per SN for all our models.}

Figure 13 shows } \dot{p}_b, \text{ the radial momentum of the SB per SN, as a function of (a) normalized time and (b) normalized radius. At } t_{f,m}, \text{ the values of } \dot{p}_b \text{ are comparable to the prediction of Equation (12). For all models, } \dot{p}_b \text{ declines slightly after } t_{f,m}, \text{ but generally evolves very weakly at late stages, and is quite insensitive to parameter values. For single SNe, KO15 showed that the final momentum is } \sim 3 \times 10^5 M_\odot \text{ km s}^{-1} \text{ for } n_{avg} = 1 \text{ cm}^{-3} \text{, and weakly decreasing } \propto (n_{avg}/1 \text{ cm}^{-3})^{0.17}. \text{ Here our models with } \Delta t_{SN} = 1 \text{ Myr have similar } \dot{p}_b \text{ to the single-SN results at } t_H, \text{ while } \dot{p}_b \text{ is lower at } \Delta t_{SN} = 0.1 \text{ Myr } (\sim 1.5 \times 10^5 M_\odot \text{ km s}^{-1}) \text{ and } \Delta t_{SN} = 0.01 \text{ Myr } (\sim 1 \times 10^5 M_\odot \text{ km s}^{-1}). \text{ There is also a slight (<50%) decrease in } \dot{p}_b \text{ from } t = t_1 \text{ to } t_2H. \text{ The dependence of } \dot{p}_b \text{ on } n_{avg} \text{ is even weaker than in the single-SN case. These results imply that } p_\ast \approx 10^5 M_\odot \text{ km s}^{-1} \text{ quite generally.}

5. IMPLICATIONS FOR LOADING OF GALACTIC WINDS

In Section 4.4, we provided results for the mass of hot gas per SN as a function of time and radius (Figures 11(a) and (b)). Table 3 shows that except for the models that have } H_{s f,m} \lesssim t_{f,m}, \text{ } M_b \text{ is relatively constant for } r_b \sim H - 2H \text{ for any individual SB, and lies in the range } 10-100 M_\odot \text{ for the parameter set considered, with the lower end corresponding to ISM disks with larger } n_{avg}. \text{ As discussed in Section 2.3, the expanding shells of SBs from sufficiently massive clusters with short } \Delta t_{SN} \text{ are likely to remain coherent until breaking out of the disk, whereas SBs driven by lower-mass clusters with long } \Delta t_{SN} \text{ will have shells that merge with the turbulent ISM prior to breaking out.}

Even if the outer shell of an SB does not maintain its integrity, the high-entropy hot gas in the interior will tend to rise and make its way out of the galaxy. Since not all of the hot gas created in an SB will ultimately be able to escape, an upper limit on the contribution from each SN to a hot wind is } M_b. \text{ Dividing by a typical mass of stars } m_\ast = 100 M_\odot \text{ formed per SN, this implies that the hot wind “mass loading” factor } \beta_b = M_b/m_\ast \text{ would be less than unity, unless the conditions of}
the ISM and clusters driving SBs combine to enable the SB radius to exceed $H$ before $t_{\text{sf,m}}$. With velocities of warm gas in the shell only up to $\sim 100$ km s$^{-1}$ (see Figures 8 and 9), this warm gas could not immediately escape as a wind from a massive galaxy, although in principle some of this material could be further accelerated by interaction with the faster hot gas or cosmic ray wind that is flowing out of a galaxy. As noted earlier, at $t_{\text{h}}$ the total mass of gas with $[\text{H}] \geq 50$ km s$^{-1}$ exceeds $100 M_\odot$ for the models with $\Delta S_{\text{SN}} = 0.1, 0.01$ Myr, implying that for dwarf galaxies more material (mostly at warm temperatures) could escape as an SB-driven outflow than is locked up in stars.

Given the low $\beta_0$ values for our models with $t_{\text{sd,m}} < t_{\text{h}}$, we suggest that a heavily mass-loaded hot wind (i.e., $\beta_0 > 1$ in the hot component) is only possible if conditions enable ISM breakout prior to shell formation. Furthermore, from Equation (13), since the maximum mass in the SB per SN at shell formation is $\sim 10^3 M_\odot$ and not all of this gas would escape, there is an upper limit $\beta_0 \lesssim 10$ for SN-driven hot winds. Setting $t_{\text{sd,m}} = H$ and solving for $\Delta S_{\text{SN}}$ (using Equation (11)), the maximum interval (in Myr) between SNe that is consistent with the “hot breakout” condition is

$$\Delta t_{\text{hbo,6}} = 0.019 E_{51} (f_{n_{\text{avg}}})^{1.7} H_2^{2.7}$$

(25)

Here $H_2 \equiv H/100$ pc and we use $n_{\text{avg}} = n_w \approx f_w n_{\text{avg}}$ if the volume fraction of the CNM is negligible, where $f_w$ is the mass fraction of the WNM and $f_{n_{\text{avg}}} \equiv f_w/0.1$.$^5$

In Section 3, we adopted Equation (24) for the typical ISM scale height, but this can be generalized under the assumption of vertical dynamical equilibrium in the ISM to $H = \sigma_\perp^2 G n_{\text{avg}} (1 + \chi)^{-1/2}$, where $\chi$ is the approximate ratio of midplane stellar and dark-matter density to mean midplane gas density under typical disk conditions (Ostriker & Shetty 2011; Kim & Ostriker 2015b); $\chi \sim 1$ in the solar neighborhood, but gas may dominate in starburst regions. In addition, the mean midplane density is related to the total midplane pressure by $\rho_{\text{avg}} = R_{\text{tot}} / \sigma_\perp^2$, giving $H = \sigma_\perp^2 [G \rho_{\text{tot}} (1 + \chi)]^{-1/2}$.

Over long timescales, analytic theory (Ostriker et al. 2010; Kim et al. 2011; Ostriker & Shetty 2011) predicts, and numerical simulations (Kim et al. 2011, 2013; Kim & Ostriker 2015b) have verified, that the ISM will evolve to an equilibrium state that is self-regulated by feedback from star formation, in which $R_{\text{tot}}$ is approximately linearly proportional to the SFR per unit area, $\Sigma_{\text{SFR}}$. Based on theory and simulations, the expected total feedback yield $\eta \equiv R_{\text{tot}} / \Sigma_{\text{SFR}} \approx 10^3$ km s$^{-1}$; we define $\eta_3 \equiv \eta/10^3$ km s$^{-1}$. The normalized density and scale height can then be written as

$$n_{\text{avg,0}} = 0.28 \eta_3^{1/2} \sigma_{\perp,3}^{1/2} \Sigma_{\text{SFR,3}}^{-1}$$

(26)

and

$$H_2 = 3.5 \eta_3^{-1/2} \sigma_{\perp,3}^{1/2} \Sigma_{\text{SFR,3}}^{-1}$$

(27)

where $\sigma_{\perp} \equiv \sigma_{\perp}/10$ km s$^{-1}$, $\Sigma_{\text{SFR,3}} = \Sigma_{\text{SFR}}/10^{-3} M_\odot$ kpc$^{-2}$ yr$^{-1}$, and we set $\chi = 1$ for convenience.

Assuming that the background ISM state is consistent with self-regulated equilibrium, the limiting SN interval that allows hot breakout can then be computed using Equation (25), and the corresponding minimum star cluster mass (using

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5 If the ISM is primarily molecular, rather than two-phase atomic, $n_{\text{amb}}/n_{\text{avg}}$ could instead be computed based on the variance in the density PDF, and would scale inversely with the Mach number of the turbulence.

Equation (11) would be

$$M_{c_{\text{hbo}}} = 6.6 \times 10^5 M_\odot \ E_{51}^{-1} H_{13}^2 \eta_3^{0.35} \sigma_{\perp,3}^{2.0} \Sigma_{\text{SFR,3}}^{-0.35}$$

(28)

with the corresponding SFR obtained by dividing by $t_{\text{life}} = 40$ Myr. For solar neighborhood conditions, where $\Sigma_{\text{SFR,3}} \sim 3$, a very massive cluster ($\sim 10^8 M_\odot$) would be required to enable hot breakout.

In fact, the SNe that drive an SB need not all originate in a single cluster. Several clusters that are born within $t_{\text{life}} \sim 40$ Myr of each other, at distances $\lesssim H$, effectively act like a single cluster from the point of view of driving an SB (e.g., Yadav et al. 2016). It is therefore useful to compare Equation (28) with the average total mass of recently formed local stars that would contribute to a single SB (under self-regulated equilibrium, and again taking $\chi \sim 1$),

$$\langle M_{\text{young,3}} \rangle \equiv t_{\text{life}} \Sigma_{\text{SFR}} = 1.5 \times 10^4 M_\odot \eta_3^{-1} \sigma_{\perp,3}$$

(29)

For fiducial $\sigma$ and $\eta$, the corresponding mean SN interval and SFR within $H^2$ are $\Delta S_{\text{SN,3}} = m_{\text{SN}} (\Sigma H^2 \Sigma_{\text{SFR}}) \sim 0.3$ Myr and $M_{\text{SFR,3}} = \Sigma H^2 \Sigma_{\text{SFR}} \sim 4 \times 10^{-4} M_\odot$ yr$^{-1}$, respectively. Note that these are independent of the local gas surface density. Since local SFRs in fact vary as the ISM within $H$ cycles through condensation and dispersal, the value of $\Delta S_{\text{SN,3}}$ during active star formation will be somewhat lower than its average value. Our models with $\Delta S_{\text{SN}} = 0.1$ Myr are therefore likely the most representative of the regime present in self-regulated galactic disks.

A large upward fluctuation in the local SFR would be needed to increase the local mass in young stars by a factor $\sim 40 \sigma_{\perp,3}^{1/2} \Sigma_{\text{SFR,3}}^{-0.35}$ from the typical value in Equation (29) to the level required for hot breakout by Equation (28). Although the required level of upward fluctuation is higher in regions of increased $\Sigma_{\text{SFR}}$, this may be partly compensated if $\sigma$ also increases under these conditions. Indeed, while in observed disk galaxies $\Sigma_{\text{SFR}}$ varies by several orders of magnitudes and $\sigma$ varies by only a factor of a few, the variations are observed to be correlated (e.g., Tamburro et al. 2009; Wilson et al. 2011; Stilp et al. 2013; Ianjamasimanana et al. 2015). Nevertheless, unless most of the star formation in galaxies occurs in bursts that are well above the time-averaged SFR, SBs will generally undergo shell formation before breakout, and the SN-driven hot winds they create will only have a mass loading factor $\beta_h \sim 0.1–1$.

Starburst galaxies have very high central concentrations of gas, and correspondingly quite high localized values of $\Sigma_{\text{SFR}}$. Although these conditions are much more extreme than typical regions in galactic disks, the relationship between ISM equilibrium pressure (or weight) and the mean value of $\Sigma_{\text{SFR}}$ still appears to be consistent with the prediction of self-regulation by SN feedback (Ostriker & Shetty 2011; Narayanan et al. 2012; Shetty & Ostriker 2012). Equation (28) would therefore still represent the minimum mass of young stars within $\sim \pi H^2$ that is needed for a burst to produce a hot breakout. For starburst regions with $\Sigma_{\text{SFR,3}} = 10^{2–3}$, this corresponds to $M_{c_{\text{hbo}}} \sim 3 \times 10^{0–4} \times 10^5 M_\odot$ or $M_{c_{\text{hbo}}} \sim 0.1–1 M_\odot$ yr$^{-1}$. While assessment of the observed scale height or velocity dispersion of the atomic/molecular ISM in galactic centers is challenging due to limited resolution (but see Leroy et al. 2015), observed galaxies with winds powered by central starbursts do have total $M_\odot \sim 0.1–10^5 M_\odot$ yr$^{-1}$ within the central few hundred pc (Heckman et al. 2015). Intriguingly, the observed
values of $\beta$ in these starburst-driven winds decrease with increasing SFR, perhaps reflecting the greater difficulty of achieving hot breakout under the higher-density conditions that yield higher $\Sigma_{\text{SFR}}$ (as evident in the increase of $M_{\text{d,hot}}$ with $\Sigma_{\text{SFR}}$ in Equation (28)).

We conclude that the equilibrium SFR, based on temporal and spatial averages, is in general too low to drive a heavily loaded hot wind. Nevertheless, a massive cluster or large-amplitude fluctuation in $\Sigma_{\text{SFR}}$ could in principle lead to a hot outburst with maximum $\beta_h \sim 10$, and this appears to occur in nuclear regions for starburst-driven outflows. More typically, we expect $\beta_h \sim 0.1$–1 for SN-driven hot winds on large scales in disk galaxies. For disk-launched winds, the mass-loss rate per unit area on each disk face would be $\beta_h \Sigma_{\text{SFR}}/2$, whereas for quasi-spherical nuclear winds, the total mass-loss rate would be $\beta_h \Sigma_q/2$.

Finally, we note that for SN-driven steady-state hot winds, the flow velocity at large distance is obtained from the Bernoulli parameter $B = (1/2) v^2 + (5/2) P/\rho + \Phi$, which is constant along streamlines for an adiabatic flow. For the hot gas within SBs, the enthalpy term dominates (see Figure 8). However, after breakout, as streamlines expand and $P/\rho$ decreases ($\propto (vr^2)^{-2/3}$ for a spherical flow), the flow will accelerate and the kinetic term will begin to dominate. Neglecting the potential term, at large distance the velocity would approach $v_{\text{asy}} = (2B)^{1/2}$, where $B$ is set by the enthalpy of hot gas in the SB interior prior to breakout. For the range of values of $T_0(H)$ and $T_0(2H)$ in Table 2, $v_{\text{asy}} = (5P/\rho)^{1/2} = (3.9 k_B T_0/m_H)^{1/2}$ is in the range 200–600 km s$^{-1}$. This implies that SB-driven hot winds can escape at high velocity from the immediate vicinity of all but the most massive galaxies.

For SBs at $t > t_{\text{sf,m}}$, the effective momentum per unit time that the successive SNe impart to their surroundings is equal to $p_{\text{b}}/m_* \Phi$ multiplied by the SFR. From the results for $p_{\text{b}}$ in Table 3, and using $m_* = 100 M_\odot$, this is ($1$–$2$) $\times 10^{3} \text{ km s}^{-1}$, multiplied by the SFR. If this momentum is equally shared with all of the surrounding gas within the disk scale height $h$, the mean velocity at breakout will be comparable to the turbulent velocity dispersion in the disk—at most several tens of $\text{ km s}^{-1}$ (see Equation (20) and following, and the values for $v_\text{v}$ in Table 2). However, the initial breakout of an SB can be clear enough of the surrounding ISM. The time required for initial breakout, using the results of Section 2.3, is $(H/\sigma)/(\Delta_{\text{SN}}/6\Delta_{\text{SN,H}})^{1/2}$. For regions where the dynamical time $H/\sigma$ is shorter than $t_{\text{sf,m}}$, energy and momentum input from SNe will continue, but the momentum flux in the vertical direction will be shared with much less material. In this situation, a low value of $n_{\text{amb,0}}$ in Equation (19) can lead to a very fast outflow.

6. SUMMARY

The energy released by SNe is vital to the ISM and to the surrounding CGM and IGM on larger scales, and understanding the interaction of clustered SNe (the typical case) with their environment is essential to theories of both the ISM and galaxy formation. In this paper, we have used numerical simulations to study the evolution of SBs driven by multiple SNe as they expand into the two-phase (warm/cold) ISM, which in our simulations has a realistic complex cloudy structure that results from saturation of thermal instability. We consider models with a range of mean background density $n_{\text{avg}} = 0.1–10 \text{ cm}^{-3}$, and intervals between SNe $\Delta_{\text{SN}} = 0.01–1 \text{ Myr}$. The former corresponds to a typical range of gas surface density $\Sigma_{\text{gas}} \sim 5–50 M_\odot \text{ pc}^{-2}$ and SFR surface density $\Sigma_{\text{SFR}} \sim 4 \times 10^{-4}–4 \times 10^{-2} M_\odot \text{ kpc}^{-2} \text{ yr}^{-1}$. The latter corresponds to a range of star cluster mass (or total local mass in young stars) of $M_{\odot} = 4 \times 10^{3}–4 \times 10^{7} M_\odot$. Our simulations are idealized in that we do not include background stratification of the mean density and pressure. However, we can use expected relationships between mean midplane density and ISM scale height $H$ to define the times $t_H$ and $t_{2H}$ when the SB radius reaches $H$ or $2H$, such that if stratification were included, the SB would breakout of the warm/cold disk into the hot corona. We measure key SB properties—total radial momentum of the bubble $p_{\text{b}}$, hot gas mass $M_{\text{h}}$, and hot gas temperature $T_0$—at times up to $t_{2H}$. Taking ratios with the total number of SN events that have occurred, we compute $p_{\text{b}}/\Sigma_{\text{SFR}}$, the momentum and mass of hot gas injected per SN; we tabulate these at $t_H$ and $t_{2H}$ as $\beta_{p_{\text{b}}}(H)$, $\beta_{p_{\text{b}}}(2H)$, and so on (see Table 3).

Our main conclusions are as follows:

1. Evolution. As in the case of an SNR from a single SN, a blast driven by multiple SNe initially evolves similarly to analytic predictions for adiabatic expansion. Equation (10) provides a prediction for the time $t_{\text{sf,m}}$ when a cooled shell will form at the leading edge of the blast wave; this assumes continuous energy ejection, with $\Delta_{\text{SN}} < t_{\text{sf,m}}$. Figures 1–3 show that the mass in hot gas peaks at $t \sim t_{\text{sf,m}}$ for models with short $\Delta_{\text{SN}}$. After shell formation, SB radii expand more slowly than the classical prediction for an adiabatic pressure-driven snowplow. This is because energy is lost from the hot interior through cooling (due to mixing with material ablated from embedded dense clouds, and at the irregular interface with the cooled shell). For models with $\Delta_{\text{SN}} = 1 \text{ Myr}$, evolution behaves like a succession of individual events (with strong cooling after each one), whereas the evolution is continuous in models with $\Delta_{\text{SN}} = 0.01 \text{ Myr}$. For our set of parameters, the SB radius expands to $H$ within $\sim 1–10 \text{ Myr}$ (see Table 2 for the values of $t_H$ and $t_{2H}$). Equation (18), based on a constant rate of momentum injection (discussed later), describes the radial expansion after $t_{\text{sf,m}}$ quite well (see Figures 1–3(a)).

2. Morphology. Because of the highly inhomogeneous structure of the “background” warm/cold ISM into which they propagate, SBs have complex morphology (Figures 4–7). Fingers and islands of hot, warm, and cold gas phases interpenetrate, with irregular interfaces. Nevertheless, the SBs in our simulations retain the traditional elements of a very hot, very low-density interior contained within a shell consisting of shocked, cooled, and compressed ambient gas. Except at the earliest stages, the expansion velocity of the hot medium exceeds that of the surrounding shell. In models with $\Delta_{\text{SN}} = 0.01, 0.1 \text{ Myr}$, the bubble remains overpressured relative to the ambient ISM, whereas in models with $\Delta_{\text{SN}} = 1 \text{ Myr}$, the pressure can drop below ambient values at late time. Pressures in the hot interior can also be either higher or lower than those in the warm shell. SB interiors include dense clouds that were shock-heated and accelerated but left behind by the more rapid advance of the outer front; these clouds may remain warm if $\Delta_{\text{SN}}$ is
sufficiently small, or they may cool back down if $\Delta t_{SN}$ is large.

3. **Energetics of gas phases.** For all of our models, the mean temperature $T_b$ of the hot bubble interior remains $>10^4$ K throughout the simulation. Figures 1–3 show that $T_b$ remains close to $10^5$ K for models with $\Delta t_{SN} = 0.01$ Myr, evolving continuously when $n_{\text{avg}}$ is low. Models with higher $\Delta t_{SN}$ and $n_{\text{avg}}$ show spikes in $T_b$ after each event. PDFs in the temperature–velocity plane (Figure 8) at $t_1$ show differences for models in the “continuous” ($\Delta t_{SN} < t_{d,m}$) versus “discrete” ($\Delta t_{SN} > t_{d,m}$) limit. For the former, shocked dense clouds that are originally CNM are maintained at $T \sim 10^4$ K by continuous heating; they are also accelerated up to a few tens of km s$^{-1}$ (Figure 9). For the latter, dense CNM clouds are shocked and accelerated up to ~10 km s$^{-1}$, but they cool back to ~100 K. For all models, the SB shell is mostly composed of gas that was originally WNMM before being shocked and swept up; it remains at $T \sim 10^4$ K, with supersonic velocities of several tend to >100 km s$^{-1}$. Most of the mass of warm gas has a velocity below 100 km s$^{-1}$, so it would not be able to escape from the gravitational potential of a massive galaxy. However, substantial mass-loss in warm gas would be expected for dwarf galaxies (see Figure 10). For all cases except model n10–11, most of interior volume of the SB is filled by gas at $T \sim 10^2$–$10^4$ K. Mass-weighted mean values at $t_{i-1}$ are $T_b = 10^4$–10$^5$ K. Although the hot medium velocities exceed ~100 km s$^{-1}$ for all but models n10–11 and n11–12 (where $v_{\text{hot}}$ is several tens of km s$^{-1}$), the hot gas generally has enthalpy exceeding its kinetic energy and is at most transonic. Winds initiated with hot gas from SBs would accelerate as streamlines diverge after breakout, and have asymptotic velocities up to 200–600 km s$^{-1}$.

4. **Momentum.** Figure 13 shows that for all models, $\dot{p}_b$ remains relatively constant after $t_{d,m}$, in the range 0.7–3 $\times$ 10$^5 M_\odot$ km s$^{-1}$. That is, the SB evolves with nearly constant increase of momentum for each SN (or linear increase of momentum in time), quite different from the classical pressure-driven snowplow solution with constant increase of energy for each SN (linear increase of energy in time). Figures 1–3(e) show good agreement with $\dot{p}_b = \dot{p}_b(t)/\Delta t_{SN}$. The value of $\dot{p}_b$ is very insensitive to the ambient density, and increases slightly at higher $\Delta t_{SN}$. The values we obtain for $\dot{p}_b$ are similar to the final momentum obtained in recent simulations of SNR expansion following a single SN explosion in an inhomogeneous medium (Iffrig & Hennebelle 2015; Martizzi et al. 2015; Walch & Naab 2015, KO15), as well as for the homogeneous medium case with a single SN (Cioffi et al. 1988; Blondin et al. 1998; Thornton et al. 1998, KO15). Our results suggest that $p_a \approx 10^5 M_\odot$ km s$^{-1}$ may be expected to hold quite generally.

Recently, Gentry et al. (2016) have argued, based on spherically symmetric simulations of multiple SNe in a uniform background medium conducted with a semi-Lagrangian code, that the mean momentum injection per SN to the ISM, $p_a$, may be higher for an SB than for an individual SNR. Indeed, Equation (8) for the evolution prior to shell formation, or the same expression multiplied by 0.56 for the classical adiabatic pressure-driven snowplow, shows that if energy losses are small, the momentum per SN can exceed $10^6 M_\odot$ km s$^{-1}$ at late times. However, there are two difficulties in applying the results of Gentry et al. (2016) to the real ISM. First, high values of the momentum/SN are achieved only at quite late times, beyond the point that the SB radius would have exceeded $H$. Second, the extremely inhomogeneous conditions of the real ISM mean that a simple contact discontinuity between the hot interior and cooled shell cannot be maintained. Instabilities initiated at interfaces (both with the shell and with embedded dense clouds) develop into turbulence, and the subsequent mixing between the hot medium and denser phases enhances cooling. Spherically symmetric models cannot capture the energy losses that are inherent to evolution in a cloudy ISM. While simulations at higher resolution than the present ones would be valuable to investigate the mixing and cooling at interfaces in greater detail, we find (see the Appendix) that our results are converged. This suggests that the high values of $p_a$ proposed by Gentry et al. (2016) would not apply in the real ISM. Indeed, within the context of models in which SFRs are predominantly regulated by the momentum injection from SNe (Ostriker et al. 2010; Kim et al. 2011; Ostriker & Shetty 2011), a much larger value of $p_a$ would be inconsistent with observations of $\Sigma_{\text{SFR}}$ in both normal galaxies and starbursts.

5. **Hot gas mass and wind loading.** Figure 11 shows that the hot gas mass per SN peaks at a value $\dot{M}_b \sim 400$–2000 $M_\odot$ at $t \sim t_{d,m}$ and then drops. For most models, $\dot{M}_b \sim 10$–100 $M_\odot$ for $t \sim t_i - t_{d,m}$. The value of $\dot{M}_b$ decreases for increasing background ISM density. The late-time value of $\dot{M}_b$ does not depend strongly on $\Delta f_{SN}$, but because $\Delta f_{SN}$ determines the time $t_i$ when an SB would begin to breakout of the disk, the SN interval would affect the mass loading of winds by SBs. Taking the wind hot gas mass loading $\dot{p}_b = \dot{M}_b(t)/100 M_\odot$ for $t \sim t_i - t_{d,m}$, only our model n0.01–t0.01 has $\dot{p}_b > 1$, and this is only for the first part of the “breakout” period. We conclude that the potential for SBs to drive heavily mass-loaded hot winds depends strongly on $\Delta f_{SN}$, or equivalently the mass of the star cluster driving the bubble.

The time $t_i$ depends on the background ISM density and scale height, and Equation (25) provides an expression for the maximum SN interval ($\Delta f_{SN} < \Delta f_{\text{hot}}$) that would allow “hot breakout,” with the SB radius reaching $H$ prior to the onset of strong cooling ($t_i < t_{d,m}$). The value $\Delta t_{\text{hot}}$ can be converted to a minimum cluster mass (or local mass of young stars) that enables hot breakout; Equation (28) gives this mass $M_{\text{cl,hot}}$ as a function of local properties in the disk. Under typical galactic disk conditions, the condition for hot breakout would not be met. This implies that $\beta_h < 1$ would be expected for a hot wind driven by SBs for most regions in a galaxy. However, starbursts in the centers of galaxies have very high local concentrations of young stars, often exceeding $M_{\text{cl,hot}}$. These are indeed exactly the systems where strong wind signatures are observed (e.g., Heckman et al. 2015).

For dwarf galaxies with shallow potential wells, gas velocities need not reach hundreds of km s$^{-1}$ to escape as an outflow. Except for our models with the $\Delta f_{SN} \approx 1$ Myr (which exceeds the expected mean local SN interval $\Delta f_{SN,H} \sim 0.3$ Myr), at $t_i$ there is more than 100 $M_\odot$ in
mostly warm gas per SN that has $|v| > 50$ km s$^{-1}$ (see Figure 10). This suggests that SBs could effectively clear the baryons from low mass halos, as is required to reconcile observed statistics of dwarfs with $\Lambda$CDM cosmology (e.g., Somerville & Davé 2015).

Finally, we note that there are a number of physical effects that we have not included in the present simulations, which potentially could lead to substantial quantitative difference in some results. In particular, we have not incorporated thermal conduction, magnetic fields, turbulence in background state, or a pre-existing hot phase, all of which could alter the overall evolution and detailed density and thermal structure of SBs. Additionally, higher resolution would aid in investigating the details of turbulent mixing at the interfaces between phases. Many of these additional physical effects are best addressed in fully self-consistent simulations of three-phase ISM galactic disks with star formation and SNe, which we are currently pursuing (C.-G. Kim & E.C. Ostriker 2016, in preparation). Self-consistent star-forming ISM disk simulations are also helpful in directly measuring mass-loss rates in winds, without having to make an assumption that SB properties when $r_b \sim H$ determine mass-loss rates. (In fact, our galactic disk ISM simulations show $\beta_h \sim 0.1$–1 in hot gas, confirming the present results.) However, the isolation of individual elements is extremely helpful for building a deeper understanding of the ISM, and we believe it will continue be fruitful to conduct focused simulations and analyses of SBs, with enhanced physics and numerical resolution.

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APPENDIX

NUMERICAL CONVERGENCE

In KO15, we showed that the evolution of a radiative SNR is numerically converged, provided that the initial size of the feedback region is sufficiently small compared to the shell formation radius, $r_{\text{min}}/r_s < 1/3$, and the resolution is high enough to resolve the shell formation, $\Delta r/r_s < 1/3$. Physically these criteria can be understood considering that all of the hot gas, and most of the radial momentum, is produced via propagation of very strong shocks during energy conserving stages of evolution. In the post-shell formation stage for an

![Figure 14](image_url)

Figure 14. Slices at $t = 4$ Myr for low (left), standard (middle), and high (right) resolution simulations of Model n1-t0.1.
individual SNR, some additional momentum is acquired as the overpressured hot gas in the interior of the SNR pushes the surrounding shell outward, but this effect is less significant than originally thought (e.g., McKee & Ostriker 1977; Ostriker & McKee 1988). Therefore both momentum acquisition and hot gas creation can be numerically converged if one resolves the energy conserving phase.

The evolution of an SB is different from that of a single SNR. It is still important to resolve the onset of cooling in the shocked ambient medium, with a physical scale described by the shell formation radius. In principle, if $\Delta t_{SN}$ is sufficiently small, energy from subsequent SNe extends the energy-conserving stage to $t_{f,m} > t_f$ and produces a larger shell formation radius (see Equations (9) and (10)). This can in principle relax the resolution requirement for convergence, although in practice we still use the “single SN” criterion to set the feedback region size for each individual feedback event (see Section 3).

While early evolution of a single SNR and SB are similar, evolution after shell formation, and in particular the build-up of momentum and hot gas, is different for an SB from either the energy-conserving or pressure-driven snowplow phase of a single SNR. First, consider the case of a uniform ambient medium, and neglect development of instabilities in the shell that would lead to non-spherical morphology. After shell formation in a spherical SB, if the SB has sufficiently low internal density, ejecta from subsequent SNe would freely expand until reaching the dense shell. In this case, as the ejecta hit the dense shell, a shock would run into the dense medium and quickly cool down. At the same time, a reverse shock would propagate backward and heat up the interior. If the density in the interior of the SB is high enough for the ejecta to be slowed down before reaching the shell, then a Sedov-like solution could develop from forward and backward shock propagation, maintaining a hot and overpressured condition in the SB interior. If the SN interval is short enough, and thermalization of energy occurs in such a way that the interior and shell are separated by a contact discontinuity (i.e., without propagation of a shock into the shell, which would then radiatively cool), evolution would follow the limit of classical SB evolution driven by continuous energy injection (e.g., Weaver et al. 1977). Recent simulations have followed SB evolution with cooling for a uniform ambient medium, under the assumption that energy is fully thermalized at small scales. Gentry et al. (2016) impose spherical symmetry and use a semi-Lagrangian code to aid in resolving the interface between the SB interior and dense shell, while Yadav et al. (2016) conduct fully three-dimensional simulations resolving down to $\sim 1$ pc, showing evolution that agrees with corresponding spherical models.

Unlike the idealized 1D spherical theory (or simulations) for a uniform ambient medium, even in the limit of short $\Delta t_{SN}$ that approaches continuous energy injection, the evolution of an SB in the real ISM will be more complex. Multi-dimensionality allows instabilities to develop at the interface with the shocked cooled outer shell and internal overdense clumps that are inherent aspects of the warm/cold ISM. These instabilities result in hydrodynamic mixing between phases and enhance cooling. If thermal conduction is considered, the mass and energy exchanges between hot interior and cooled shell will also be enhanced. Especially considering the role of turbulence (driven by instabilities) in creating structure and mixing material at fine scales, the numerical requirements needed to capture the impact of multiple SN explosions in a cloudy ISM are not obvious—and indeed the numerical requirements may differ, depending on what issue is in question. Numerical simulations with grid resolution of order of parsec cannot resolve the realistic field length (Begelman & McKee 1990), so that the total cooling is dominated by unresolved interfaces. In spherical symmetry, one might expect the total cooling rate to vary as $\propto r_{x}^{2} \Delta x$, so that for a given shell size cooling would be overestimated at lower resolution. Also, with a clumpy medium, the usual realization of SN feedback with purely thermal energy is in question.

In order to address these concerns, we perform two numerical convergence tests. First, we conduct a resolution test by re-running Model n1-t0.1 with a factor of two higher and lower resolutions, n1-t0.1-high and n1-t0.1-low, respectively. In order to keep the background state for different resolutions, we adopt the same initial condition from the saturated state of thermal instability simulations with standard 3 pc resolution and then refine/degrade for different resolutions. Figure 14 illustrates the difference in structure at $t = 4$ Myr for different resolutions. In Figure 15, we plot all key quantities as a function of normalized size of bubble $r_b/H$: (a) hot gas mass per SN $\dot{M}_b$, (b) hot gas thermal energy per SN $\dot{E}_b$, (c) mass-weighted mean temperature of the hot gas $\bar{T}_b$, and (d) bubble radial momentum per SN event $\dot{p}_b$. The detailed evolution is slightly shifted toward the left for higher resolution
simulation. This means that the evolution is slightly faster at higher resolution. However, the results for mass, energy, and momentum loading, and for the mean interior temperature of the SB, are in agreement at all resolutions, indicating that these integrated quantities are converged.

Second, we conduct a test with a different realization of SN feedback. Instead of using pure thermal energy (“thermal” feedback), we dump ejecta mass 10 \( M_\odot \) and pure kinetic energy within a region that encloses ambient medium mass not exceeding 10% of the ejecta mass (“ejecta” feedback). Figure 16 plots the same key quantities as in Figure 15. We plot results using “ejecta” feedback as solid lines and results using the standard “thermal” feedback as dotted lines for Models n1-t1 (blue), n1-t0.1 (green), and n1-t0.01 (red). Again, there are small detailed differences, but the final results are generally in agreement for the two feedback treatments. In (b) and (c), the hot gas energy and temperature are slightly lower in n1-t0.01-e\( \beta \) than in n1-t0.01, since thermalization of the ejecta is not perfect when \( \Delta M \) is short. However, the hot gas mass (in (a)) is consistent for the two feedback treatments, implying that the main contributor to new hot gas is not the ejecta but shock-heated existing gas in the SB interior. From examining the detailed evolution of both models, we clearly observe the development of a shock that propagates through the hot interior and hits the CNM and WNM in the shell and fingers, generating new hot gas. As a consequence, the ejecta masse we use here also do not affect the results (unless it is too large). The injected momentum is slightly decreased (less than 10%) in larger SN interval models with ejecta feedback compared to thermal feedback.

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