Impedance Modeling and Analysis for DFIG-Based Wind Farm in SSO Studies

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\section*{ABSTRACT} Several models were developed in the literatures for studying the system impedance characteristics during sub-synchronous oscillations (SSO). However, the doubly-fed induction generator (DFIG) impedance presents a $2 \times 2$ matrix with the frequency coupling component at the off-diagonal position, which increases the difficulty for analyzing the impedance characteristics of DFIG. The coupling effect in this case is caused by the asymmetric impedance characteristics of controllers, which are composed of voltage source converters (VSCs). Accordingly, simplifications are often conducted to decouple the impedance matrix into single sequences. Outer loop controller and phase-locked loop (PLL), which are often neglected in simplifications, are the main causes of the asymmetry of impedance matrix. Such simplifications could also hamper proper system stability analyses. In this paper, a comprehensive impedance model is proposed in which the outer loop controller, PLL, and grid-side controller (GSC) are all considered. The proposed comprehensive impedance model can provide exact stability data, in which stable boundaries for compensation level and transmission distances are presented. Furthermore, the proposed model is converted to simplified models in which the impacts of outer loop controller, PLL, GSC and coupling components on boundary stability of SSO are analyzed. The results are validated by time domain simulations and eigenvalue analyses, and deviations of the simplified models are analyzed for different operating parameters and conditions leading to simplifications.

\section*{INDEX TERMS} Doubly-fed induction generator, impedance model, sub-synchronous oscillation, stable boundary.

\section*{I. INTRODUCTION} One of the critical subjects in the application of doubly-fed induction generator (DFIG) is the sub-synchronous oscillation (SSO) caused by the interaction of DFIG with series compensation capacitors in power systems [1]–[4]. Impedance analysis [5]–[17] is widely applied in the SSO analyses, but due to the DFIG’s asymmetric impedance characteristic, DFIG is presented by a multi-in-and-multi-out (MIMO) system with a $2 \times 2$ impedance matrix [5]. Accordingly, coupling effects caused by non-zero off-diagonal components of impedance matrix aggravate the difficulties for analyzing the system impedance characteristic. The asymmetric characteristic of impedance matrix can be due to phase-locked loop (PLL), outer loop controller, and asymmetric inner loop controller parameters [6]. In other words, the coupling components could be caused by asymmetric characteristics of DFIG controllers which are composed of voltage source converters (VSCs). Hence, the key points and difficulties of modeling for DFIG are traced back to the VSC modeling.

However, simplifications have been conducted in order to decouple the impedance matrix and convert the complex MIMO system to a single-in-and-single-out (SISO) system. In [7], outer loop controller and PLL are neglected, a DFIG and series-compensated network impedance model is derived, and the corresponding equivalent circuit is presented. PLL is considered in [8]–[11], but the coupling components of PLL are neglected for decoupling. Although neglecting coupling components can simplify the impedance model
into single dimension impedance and obtain the equivalent circuit to study the SSO mechanism explicitly, the simplifications could cause inaccurate results in SSO stability analyses [12]–[14]. Coupling effects are considered in [15]–[17], where impedance matrix is converted to single dimension impedance in [15] and [16], and a stability criterion based on the frequency characteristic of the system impedance matrix is proposed in [17]. Although it is pointed out that simplifications could cause inaccurate results, but valid conditions for simplifications are not discussed in SSO’s frequency range.

The existing impedance models for DFIG with different considerations are not unified. It is difficult to obtain a comprehensive form of impedance model with coupling components, as impedance matrix will be very complex with the consideration of outer loop controller and PLL. Besides, grid side controller (GSC) is often neglected in impedance models, and the consideration of GSC will make the problem more difficult. It is necessary to build a comprehensive impedance model to bridge the gap between different models.

In this paper, we propose a new form of impedance model for DFIG with complete controllers and GSC, which could be converted to the existing simplified impedance models. The deviations in the proposed impedance model and eigenvalue analyses are analyzed to verify the correctness of the proposed model. The number of DFIGs, wind speed, compensation level, and transmission distance have significant impact on SSO. Hence, , , , and are currents in stator, rotor, GSC and transmission line, respectively. The deviations in the proposed impedance model and eigenvalue analyses are analyzed to verify the correctness of the proposed model. The number of DFIGs, wind speed, compensation level, and transmission distance have significant impact on SSO. Hence, , , , and are currents in stator, rotor, GSC and transmission line, respectively.

The control scheme for the DFIG controller employed in our work with the reference direction in Fig. 1 is shown in Fig. 2. The outer loop controller of rotor side controller (RSC) aims to regulate the stator active power and reactive power, while the inner loop controller regulates the rotor current and . The GSC function is to maintain a constant DC bus voltage and regulate the GSC current and . The reference value is denoted by subscript ref, and are dq components of rotor voltage, while and are dq components of GSC voltages. is transfer function of PI controller where is the Laplace transform. The parameters for the controllers are shown in Table 1.

This paper is organized as follows. A comprehensive form of impedance model for DFIG is proposed in Section II. Impedance analysis is applied to power system studies in Section III. The stable boundaries of parameters obtained by eigenvalue and impedance analyses are compared in Section IV. In order to analysis the impacts of PLL, GSC, outer loop controller and coupling components on SSO analyses, we propose respective simplified impedance models and analyze the stable boundaries with different operation parameters in Section V, where proper conditions for simplifications are discussed. The corresponding test results are validated by time domain simulation (TDS) based on Simulink. The conclusion of this paper is presented in Section VI.

II. IMPEDANCE MODEL IN DFIG-BASED POWER SYSTEM

A. DFIG-BASED POWER SYSTEM DESCRIPTION

A sample system shown in Fig. 1 is used in our study which is derived from the IEEE first benchmark model [18]. A DFIG-based wind farm represented by an aggregated DFIG model [19] is connected to the infinite bus via a 220 kV transmission line and a 500 kV series compensated transmission line. The system parameters and wind speed data are provided in [20]. In Fig. 1, and represent impedance of transformers, and represent impedance of transmission lines, is the reactance of filter inductor and is the reactance of series compensation capacitors; are currents in stator, rotor, GSC and transmission line, respectively.

The synchronous reference frame PLL (SRF-PLL) is adopted in this paper [5], [21]. The relationship between PLL

| Table 1: Controller parameters. |
|---|
| Symbols | Value (p.u.) | Symbols | Value (p.u.) |
| \( K_{p1} \) | 0.5 | \( K_{p2} \) | 0.3 |
| \( K_{i1} \) | 25 | \( K_{i2} \) | 15 |
| \( K_{p3} \) | 10 | \( K_{i4} \) | 1.2 |
| \( K_{i3} \) | 500 | \( K_{i4} \) | 20 |
| \( K_{pPLL} \) | 50 | \( K_{pPLL} \) | 600 |
| \( K_{i} \) | 0.025 |

B. IMPEDANCE MODEL FOR DFIG

The synchronous reference frame PLL (SRF-PLL) is adopted in this paper [5], [21]. The relationship between PLL...
dq frame, system dq frame, and stationary αβ frame are shown in Fig. 3, in which the system dq frame spins anti-clockwise at the nominal speed $\omega_0$, while the PLL dq frame spins at a speed $\omega$. The speed and angel deviations between the two dq frames are caused by perturbations.

FIGURE 3. Reference frames in the system.

Complex equivalent method in [5] is conducted in this paper by which variables are converted into a complex form. The complex variables in dq frame are distinguished from complex variables in αβ frame by subscript dq in this paper. The controller variables in PLL dq frame are labeled with superscript PLL, and the variables are converted from the PLL dq frame to the system dq frame as shown in (1) where $f$ denotes controller variables, $u_{dqd}$ is stator voltage in complex form, $H_{pll}(s)$ is the transfer function of PLL. The coefficients of $H_{pll}(s)$ are shown in Table 1. $\Delta$ and $*$ are deviation and complex conjugate operation, respectively.

\[
\begin{bmatrix}
\Delta f_{dld}^\text{PLL} \\
\Delta f_{dq}^\text{PLL}
\end{bmatrix} = \begin{bmatrix}
\Delta f_{dq} \\
\Delta f_{dq}^*
\end{bmatrix}
+ \frac{H_{pll}(s)}{2}\begin{bmatrix}
-f_{dq} & f_{dq} \\
f_{dq} & -f_{dq}
\end{bmatrix} \begin{bmatrix}
\Delta u_{dqd} \\
\Delta u_{dq}^*
\end{bmatrix}
\]

(1)

The DFIG model is stated in stationary αβ frame in this paper. Given the impedance matrix $Z_{dq}$ in dq frame,

\[
\begin{bmatrix}
\Delta u_{dq} \\
\Delta u_{dq}^*
\end{bmatrix} = \begin{bmatrix}
Z_{11}^d(s-j\omega_0) & Z_{12}^d(s-j\omega_0) \\
Z_{22}^d(s-j\omega_0) & Z_{22}^d(s-j\omega_0)
\end{bmatrix} \begin{bmatrix}
\Delta i_{dq} \\
\Delta i_{dq}^*
\end{bmatrix}
\]

$Z_{dq}$ is converted to the stationary αβ frame as [5],

\[
\begin{bmatrix}
\Delta u \\
e^{j2\theta}\Delta u^*
\end{bmatrix} = \begin{bmatrix}
Z_{11}^d(s-j\omega_0) & Z_{12}^d(s-j\omega_0) \\
Z_{22}^d(s-j\omega_0) & Z_{22}^d(s-j\omega_0)
\end{bmatrix} \begin{bmatrix}
\Delta i \\
e^{j2\theta}\Delta i^*
\end{bmatrix}
\]

(2)

where $\theta = \omega_0 t$, $Z_{12}^d(s-j\omega_0)$ and $Z_{21}^d(s-j\omega_0)$ are coupling impedances. The impedance matrix will be symmetric and decoupled when coupling impedances are zero. Here, a given perturbation voltage with angular frequency $2\omega$ will correspond to currents with an angular frequency $\omega$ and a coupling angular frequency $2\omega_0-\omega$.

Considering (1), (2) and the control scheme shown in Fig. 2, we linearize RSC and GSC equations in the αβ frame as shown in (3) and (4), respectively. In (3) and (4), there are three parts corresponding to outer loop controller, inner loop controller and PLL. In (3), $M_{RSC}^{outer-U}$ and $M_{RSC}^{outer-I}$ are coefficient matrices of outer loop controller, $M_{RSC}^{inner}$ is the coefficient matrix of inner loop controller, $M_{PLL}^{RSC}$ is the PLL coefficient matrix generated by the conversion of PLL dq frame to the system dq frame. In (4), $u_g$ and $i_g$ are the GSC voltage and current, $U$ and $I$ are the corresponding steady-state voltage and current, and $C_{dc}$ is DC capacitor. $M_{GSC}^{outer-PGI}$, $M_{GSC}^{outer-PGU}$, $M_{GSC}^{outer-PRU}$ and $M_{GSC}^{outer-PRI}$ are coefficient matrices of outer loop controller; $M_{GSC}^{inner}$ is the coefficient matrix of inner loop controller; $M_{PLL}^{GSC}$ is the PLL coefficient matrix. All of the coefficient matrices are presented in Appendix.

\[
\begin{bmatrix}
\Delta u_r \\
e^{j2\theta}\Delta u_r^*
\end{bmatrix} = M_{inner}^{RSC} \begin{bmatrix}
\Delta i_r \\
e^{j2\theta}\Delta i_r^*
\end{bmatrix} + M_{PLL}^{RSC} \begin{bmatrix}
\Delta u_s \\
e^{j2\theta}\Delta u_s^*
\end{bmatrix} + M_{outer-PL} \begin{bmatrix}
\Delta u_t \\
e^{j2\theta}\Delta u_t^*
\end{bmatrix}
\]

inner loop controller

PLL

(3)

\[
\begin{bmatrix}
\Delta u_g \\
e^{j2\theta}\Delta u_g^*
\end{bmatrix} = M_{inner}^{GSC} \begin{bmatrix}
\Delta i_g \\
e^{j2\theta}\Delta i_g^*
\end{bmatrix} + M_{PLL}^{GSC} \begin{bmatrix}
\Delta u_s \\
e^{j2\theta}\Delta u_s^*
\end{bmatrix} - M_{outer-PG} \begin{bmatrix}
\Delta i_g \\
e^{j2\theta}\Delta i_g^*
\end{bmatrix} + M_{outer-PU} \begin{bmatrix}
\Delta i_s \\
e^{j2\theta}\Delta i_s^*
\end{bmatrix}
\]

inner loop controller

PLL

outer loop controller

GSC

(4)

Assembling (3) and (4) with induction generator model and filter inductor model, we can obtain a comprehensive DFIG impedance model with two parallel branches presented in (5) and (6). The complete derivation process can be found in Appendix. The comprehensive form of $Z_{B1}$ and $Z_{B2}$ are presented in (7) and (8), as shown at the bottom of the next page. In (8), $M_u$ and $M_{ur}$ are the coefficient matrices which are stated in Appendix. In (7) and (8), the non-zero off-diagonal elements of the impedance matrix are caused by asymmetry outer loop controller and PLL matrices shown in (3) and (4). Furtherly, the impedance matrix of DFIG can be calculated as (9).

\[
\begin{bmatrix}
\Delta u_s \\
e^{j2\theta}\Delta u_s^*
\end{bmatrix} = M_{B1} \begin{bmatrix}
Z_{B1,11}(s) & Z_{B1,12}(s) \\
Z_{B1,21}(s) & Z_{B1,22}(s)
\end{bmatrix} \begin{bmatrix}
\Delta i_s \\
e^{j2\theta}\Delta i_s^*
\end{bmatrix}
\]

(5)
In order to decouple \( Z_{DFIG} \) into single sequence and analyze the characteristic of SSO, two types of simplifications are conducted. One is to neglect outer loop controller and PLL directly, and the other is to neglect outer loop controller and the coupling impedances in PLL coefficient matrix. Neglecting outer loop controller and PLL will set the corresponding matrix in (5) and (6) to zero, in which case the impedance characteristic of DFIG will be decoupled. Further neglecting GSC, we can have (7) as

\[
Z_{B1} = M_{SS} - M_{SM} \left( M_{RR} - \frac{M_{inner}^{RSC}}{inner \ loop \ controller} \right)^{-1} M_{RM}
\]

(10)

Accordingly, the positive sequence is formed as,

\[
Z_{B1,11} = (R_s + sL_s) + \frac{[Z_{RSC} / \text{slip}(s) + R_r / \text{slip}(s) + sL_r]}{Z_{RSC} / \text{slip}(s) + R_r / \text{slip}(s) + sL_r + sL_m}
\]

(11)

where \( Z_{RSC} = [H_2 (s - j\omega_0) - jK_{sr}] \), \( \text{slip}(s) = (s - j\omega_r) / s \).

Here, (11) has the same form as that in [7]. While considering PLL on the base of (11) by setting off-diagonal components of PLL matrix to zero, (7) will be converted to (12), as shown at the bottom of the page, and the positive impedance is shown in (13), as shown at the bottom of the page, where

\[
K_{sr} = \frac{1}{2} U_r H_{pol} (s - j\omega_0) \left\{ [H_2 (s - j\omega_0) - jK_{sr}] \frac{I_r}{U_r} + 1 \right\}.
\]

If we neglect \( L_m \) in (13), it will correspond to the DFIG impedance model given in [11].

C. IMPEDANCE MODEL FOR RLC TRANSMISSION LINE

The transmission system can be equivalent to a RLC series circuit with a decoupled impedance matrix presented in (14),

\[
\begin{bmatrix}
\Delta u_s \\
e^{j2\theta} \Delta u_s^* 
\end{bmatrix} = \begin{bmatrix}
Z_{line} & 0 \\
0 & Z_{line}^* 
\end{bmatrix} \begin{bmatrix}
\Delta i_e \\
e^{j2\theta} \Delta i_e^* 
\end{bmatrix}
\]

(14)

where \( Z_{line} = R + sL + 1 / sC, Z_{line}^* = R + (s - 2j\omega_0) L + 1 / (s - 2j\omega_0) C \). and \( R, L \) and \( C \) are the equivalent resistance, induction and capacitance. Using (9) and (14), we calculate the impedance model shown in (15) for the studied power system.

\[
Z_{SYS} = Z_{DFIG} + Z_L
\]

(15)
III. FREQUENCY SCAN RESULT FOR THE STUDIED SYSTEM

We set compensation level $K_C = 0.7$, wind speed $V_w = 8$ m/s, the number of DFIGs $N$ is 600, the time of transmission distance $T_{L2}$ is 1. The frequency scan is applied to the determinant of the system impedance matrix $Z_{sys}$ shown in (15). The determinant of $Z_{sys}$ can be calculated as,

$$
\text{det} \left( Z_{sys} \right) = Z_{sys} \left( 1, 1 \right) \cdot Z_{sys} \left( 2, 2 \right) - Z_{sys} \left( 1, 2 \right) \cdot Z_{sys} \left( 2, 1 \right)
$$

(16)

In (16), $Z_{sys}(1,1)$ and $Z_{sys}(2,2)$ are diagonal components of $Z_{sys}$, while $Z_{sys}(1,2)$ and $Z_{sys}(2,1)$ are non-diagonal components, we furtherly define

$$
\begin{align*}
\text{det} \left( Z_{sys} \right) &= R_d + jX_d \\
Z_{self} &= R_{self} + jX_{self} = Z_{sys} \left( 1, 1 \right) \cdot Z_{sys} \left( 2, 2 \right) \\
Z_{coup} &= R_{coup} + jX_{coup} = -Z_{sys} \left( 1, 2 \right) \cdot Z_{sys} \left( 2, 1 \right)
\end{align*}
$$

(17)

The impacts of coupling components on impedance matrix determinant can be denoted by $Z_{coup}$. The impedance curves of $\text{det}(Z_{sys})$, $Z_{self}$ and $Z_{coup}$ are presented in Fig. 4 where the coupling impedance curves for $R_{coup}$ and $X_{coup}$ are more prominent in the neighborhood of normal frequency (50 Hz), and the determinant curve is distorted severely by $Z_{coup}$ curves around 50 Hz. The zero-crossing points of $X_d$ in Fig. 4 are shown in Table 2, where the stability information of the points can be obtained from the corresponding real parts $R_d$ and the slope of $X_d$. Here, the mode will be stable when $R_d$ and slope have the same signs [17]. In Table 2, the first and the seventh zero-crossing points, which correspond to SSO frequencies of 9.15 Hz and 90.85 Hz, are unstable. We focus on the first zero-crossing point in the remaining parts of this paper.

| TABLE 2. Zero-crossing points of imaginary part and stability information. |
|-----------------|----------|----------|----------|
| $f$(Hz) | $R_d$ | $X_d$ | Stability |
| 9.15 | -0.02 | -2.34 | Negative |
| 35.71 | 5.85 | 13.77 | Positive |
| 42.37 | 50.00 | Unstable |

A. IMPEDANCE AND EIGENVALUE ANALYSIS

Eigenvalue and impedance analyses are conducted for the designated system with increasing $K_C$. The corresponding results are shown in Fig. 5 which are marked as IA (Impedance Analysis) and EA (Eigenvalue Analysis), respectively. The SSO indices obtained by eigenvalue analysis are damping ratio $\xi$ [2] and SSO frequency. In Fig. 5, SSO frequencies obtained from eigenvalue and impedance analyses are nearly the same when $R_d$ and $\xi$ are nearly zero. Also, the deviations increase with increasing absolute values of $R_d$ and $\xi$, because the SSO frequency is obtained by scanning the frequencies which makes the imaginary part of the system impedance determinant to be zero as $\left| Z_{abc}^{sys} (\alpha + jo) \right| = 0$. The SSO frequencies obtained from the two methods will be the same with $\alpha = 0$, and the frequency deviations will be enlarged by the increasing of absolute value of $\alpha$, the smaller the absolute value of $\alpha$ is, the accuracy of SSO frequencies obtained by impedance analysis will be higher.

B. STABLE BOUNDARIES FOR IMPEDANCE AND EIGENVALUE ANALYSIS

In Fig. 5, although the SSO frequencies obtained from the impedance analysis differ from those of the eigenvalue analysis, they can be applied to stability analyses as the corresponding curves for $R_d$ and $\xi$ behave similarly. The stable boundaries of the operating parameters obtained by impedance analysis are consistent with the results obtained by eigenvalue analysis, as demonstrated in Fig. 6. In Fig. 6, stable boundaries of $K_C$ and $T_{L2}$ are calculated by impedance and eigenvalue analysis respectively for $N$ DFIG units.

In order to study the impact of transmission line distance on SSO analysis, we increase the transmission distance from 0.5 times ($T_{L2} = 0.5$) to 3 times ($T_{L2} = 3$) of the original length. When the system is operated at above the boundaries, the system will present to be unstable. For example, when $K_C = 0.6$, $T_{L2} = 2$ and $N = 600$, the $R_d$ value obtained from

![FIGURE 4. Frequency scans for system impedances ($N = 600$, $V_w = 8$ m/s, $K_C = 0.7$ and $T_{L2} = 1$).](image-url)
impedance analysis is -0.090 and ζ obtained from eigenvalue analysis is -1.52%. Accordingly, the system will present to be unstable, while the system will be stable when \( K_C = 0.2, T_{L2} = 1 \) and \( N = 600 \), with \( R_d = 0.112 \) and \( ζ = 1.51% \). We can also conclude from Fig. 6 that the stable domain of operating the variables will shrink by increasing the number of DFIGs (N).

V. STABLE BOUNDARIES CONSIDERING DIFFERENT IMPEDANCE MODELS

Simplifications are often conducted by neglecting PLL or outer loop controller to decouple the impedance matrix; GSC is also neglected to simplify the problem. In order to study the impacts of PLL, outer loop controller, GSC and off-diagonal coupling elements in coefficient matrixes on SSO, we neglect the corresponding parts and build impedance models respectively to analyze SSO and compare the results. The validity of simplifications for different operating conditions is also discussed. The corresponding acronyms for the models with different considerations are shown in Table 3.

| Table 3. Impedance models with different consideration. |
|---|---|
| Symbols | Meanings |
| \( M_C \) | Comprehensive system model |
| \( M_{NP} \) | System model without considering PLL |
| \( M_{NO} \) | System model without considering outer loop controller |
| \( M_{NG} \) | System model without considering GSC |
| \( M_{NC} \) | System model without considering coupling elements |

A. STABLE BOUNDARIES WITH DIFFERENT MODELS

The stable boundaries for impedance models are shown in Fig. 7, and the stable threshold values of \( K_C \) versus different \( T_{L2} \) are shown in Table 4. In Fig. 7, stable boundaries of \( M_{NP} \) and \( M_{NG} \) differ from those of \( M_C \), deviations between \( M_{NO}, M_{NC} \) and \( M_C \) are more prominent. By increasing \( T_{L2} \) and \( K_C \), the deviations will be reduced. It can be concluded that neglecting outer loop controller, PLL and GSC will cause inaccurate results and ignore the risk of SSO. Also, neglecting the outer loop controller will cause large deviations in SSO analysis.

Time domain simulation (TDS) is conducted at specified points shown in Fig. 7 to verify the accuracy of the results. The corresponding phase A currents in 500 kV transmission lines are shown in Fig. 8, where compensation capacitor is added to the 500 kV transmission line at 2 s. In Figs. 8(a) and 8(b), SSO occurs at 2 s, when participating currents are unstable. In Figs. 8(c) and 8(d), SSO occurs, although currents will get attenuated. Fast Fourier Transformation (FFT) is applied to the currents, and the corresponding SSO frequencies are shown in Table 5. Compared with
the results obtained by impedance and eigenvalue analyses, it can be concluded that TDS results are closer to those of eigenvalue analysis which verifies the validity of simulations. We can also find that deviations between IA and EA are smallest when $K_C = 0.58$ which is located close to the stable boundary. As the real part of the eigenvalue will be zero at the boundary, this result is consistent with the conclusion presented in Section IV.

B. STABLE BOUNDARIES FOR DIFFERENT NUMBERS OF DFIGS

We increase $N$ from 400 to 1200, and the corresponding stable boundaries are shown in Fig. 9. In Figs. 7 and 9, we can find that with the increasing of $N$, deviations of $M_{NO}$ and $M_{NC}$ between $M_{C}$ will decrease, and deviations between $M_{NP}$ and $M_C$ will be minute. Neglecting GSC will ignore SSO risk with smaller $N$, and will judge system state to be SSO with larger $N$ inaccurately. The proposed simplification which neglects PLL can be feasible for larger $N$. However, the deviations caused by outer loop control and coupling components are still very prominent.

C. STABLE BOUNDARIES WITH DIFFERENT WIND SPEEDS

We obtain stable boundaries with different models shown in Fig. 10 when $V_w$ is set to 4 m/s, 5 m/s, 7 m/s and 9 m/s, respectively. Stable domains for the models will be enlarged by the increasing of wind speed, and deviations vary among different models. The deviation caused by neglecting outer loop controller keeps significant with the varying of wind speed. The result obtained by simplified model $M_{NP}$ is very close with the result obtained by the comprehensive model with lower wind speed, and the deviation will increase with the increasing wind speed. The results obtained by $M_{NG}$ and $M_{NC}$ present to be conservative with lower wind speed, and present to be aggressive with higher wind speed. In other words, the system will be wrong judged to unstable state with lower wind speed, and be judged to stable state with higher wind speed accurately.
VI. CONCLUSION

In this paper, a comprehensive model of DFIG impedance is proposed which can utilize existing simplified models under different considerations conveniently. The corresponding deviations in SSO results are studied and analyzed by impedance and eigenvalue analyses. It is proved that the impedance analysis can present the same stable boundaries with eigenvalue analysis.

Using the proposed impedance model, stable boundaries of operating parameters obtained by different impedance models with simplifications are compared. The impacts of simplifications on SSO analysis are studied, and the validity of simplifications is examined with different operating conditions. The verifications of the obtained results are conducted by time domain simulations based on Simulink, and the conclusions of this paper are presented as follows:

1) Simplified impedance model will cause misleading judgement in SSO analysis. Neglecting the outer loop controller will cause more deviations than neglecting GSC, PLL and the coupling elements (off-diagonal elements) in coefficient matrices. Furthermore, the deviations will decrease with increasing \( K_C \) and \( T_{ll2} \).

2) With the increasing of \( N \), PLL can be neglected. Deviations caused by outer loop control and coupling coefficients will be decreased, although they are still significant. Neglecting GSC will ignore SSO risk with smaller \( N \), and system will be wrong judged to be SSO state with larger \( N \).

3) The derivation caused by neglecting outer loop controller is significant with different wind speed. Stable boundaries obtained by neglecting PLL are close to the result obtained by comprehensive impedance model with low wind speeds. When the wind speed increases, the derivations caused by neglecting PLL and outer loop controller will be enlarged. Neglecting GSC and coupling coefficient will judge system to be unstable with lower wind speed inaccurately, and ignore SSO risk with higher wind speed.

APPENDIX

The induction generator model for the DFIG can be denoted as (A1) and (A2). (A1) denotes the stator side model, while (A2) denotes the rotor side model.

\[
\begin{align*}
\Delta u_s &= M_{SS} \Delta i_s + M_{SM} \Delta i_r \\
\Delta u_r &= M_{RR} \Delta i_r + M_{RM} \Delta i_s
\end{align*}
\]  

(A1)

(A2)

In the above (A1) and (A2), \( M_{SS}, M_{SM}, M_{RR} \) and \( M_{RM} \) are the coefficient matrices of stator and rotor. \( L_{ss}, L_{rr} \) and \( L_m \) are stator winding inductance, stator winding inductance and magnetizing inductance, respectively. \( \omega_2 = \omega_0 - \omega_r \) is rotor speed.

The impedance models for RSC and GSC can be presented as (A3) and (A4). In order to distinguish impacts of outer loop controller, inner loop controller and PLL on the impedance models, the impedance models are labeled with subscripts for different parts in (A3) and (A4).

\[
\begin{align*}
\Delta u_s &= M_{inner}^{RSC} \Delta i_r + M_{PLL}^{RSC} \Delta u_s^* + M_{outer}^{RSC} \Delta u_s^* \\
\Delta u_r &= M_{inner}^{GSC} \Delta i_g + M_{PLL}^{GSC} \Delta u_s^* + M_{outer}^{GSC} \Delta u_s^*
\end{align*}
\]  

(A3)

\[
\begin{align*}
\Delta u_g &= M_{inner}^{GSC} \Delta i_g + M_{PLL}^{GSC} \Delta u_s^* + M_{outer}^{GSC} \Delta u_s^* \\
\Delta u_r &= M_{inner}^{GSC} \Delta i_r + M_{PLL}^{GSC} \Delta u_s^* + M_{outer}^{GSC} \Delta u_s^*
\end{align*}
\]  

(A4)

\( M_{outer}^{U} \) and \( M_{outer}^{I} \) are the coefficient matrices for the RSC outer loop controller, \( M_{inner}^{RSC} \) is the coefficient matrix for RSC inner loop controller, \( M_{outer}^{PGI} \), \( M_{outer}^{PGU} \), \( M_{inner}^{GSC} \) and \( M_{outer}^{GSC} \) are the coefficient matrices for the GSC outer loop controller, \( M_{inner}^{GSC} \) is the coefficient matrix for GSC inner loop controller, \( M_{PLL}^{RSC} \) and \( M_{PLL}^{GSC} \) are the PLL coefficient matrices. All of the coefficient matrices in (A3) and (A4) are presented as follows, as shown at the bottom of the next page.

The function of filter induction is shown in (A5).

\[
\begin{align*}
\Delta u_g &= M_{TG} \Delta i_g^* + \Delta u_s^* - M_{TG} \Delta i_s^*
\end{align*}
\]  

(A5)

where \( M_{TG} = \begin{bmatrix} sL_{tg} & 0 \\ 0 & (s - 2j\omega_0) L_g \end{bmatrix} \).

Assembling (A1)-(A5), the comprehensive impedance model for DFIG can be derived. The detailed process is presented as follows.

Firstly, we derive the impedance relationship between \( \Delta u_s \) and \( \Delta i_r \). Using (A3) and (A2) to eliminate \( \Delta u_r \), we can have,

\[
\begin{align*}
\Delta i_r &= M_{Ir}^{Us} \Delta u_s^* + M_{Ir}^{Us} \Delta i_r^* \\
\Delta i_s &= M_{Ir}^{Us} \Delta u_s^* + M_{Ir}^{Us} \Delta i_r^*
\end{align*}
\]  

(A6)

where

\[
M_{Ir}^{Us} = \left( M_{RR} - M_{inner}^{RSC} \right)^{-1} \left( M_{outer}^{U} - M_{RM} \right)
\]

\[
M_{Ir}^{Us} = \left( M_{RR} - M_{inner}^{GSC} \right)^{-1} \left( M_{outer}^{I} - M_{RM} \right)
\]
Substituting (A6) into (A1) to eliminate \( \Delta i_r \), the relationship between \( \Delta u_s \) and \( \Delta i_s \) can be presented as,

\[
\begin{bmatrix}
\Delta u_s \\
e^{j2\theta} \Delta u_s^*
\end{bmatrix} = Z_{B1} \begin{bmatrix}
\Delta i_s \\
e^{j2\theta} \Delta i_s^*
\end{bmatrix}
\]  
(A7)

where \( Z_{B1} = [I - M_{SM} M_{Ir}^U]^{-1} \left[ M_{SS} + M_{SM} M_{Ir}^L \right] \).

And then, we derive the impedance relationship between \( \Delta u_s \) and \( \Delta i_e \). Substituting (A5) into (A4) to eliminate \( \Delta i_g \), we can have,

\[
\begin{bmatrix}
\Delta u_s \\
e^{j2\theta} \Delta u_s^*
\end{bmatrix} - M_{TG} \begin{bmatrix}
\Delta i_g \\
e^{j2\theta} \Delta i_g^*
\end{bmatrix}
\]

Substituting \( \Delta u_r \) and \( \Delta i_r \) with \( \Delta u_s \) and \( \Delta i_g \) in (A8), we can get the relationship between \( \Delta u_s \) and \( \Delta i_g \). Using (A6) to eliminate \( \Delta i_r \) in (A2), we can replace \( \Delta u_r \) with \( \Delta u_s \).

| \( M_{SS} \) | \( \begin{bmatrix}
R_s + sL_{ss} & 0 \\
0 & R_s + (s - 2j\omega_0) L_{ss}
\end{bmatrix} \) |
| \( M_{SM} \) | \( \begin{bmatrix}
(sL_m) & 0 \\
0 & (s - 2j\omega_0) L_m
\end{bmatrix} \) |
| \( M_{RR} \) | \( \begin{bmatrix}
R_r + (s - j\omega_0 + j\omega_2) L_{rr} & 0 \\
0 & R_r + (s - j\omega_0 - j\omega_2) L_{rr}
\end{bmatrix} \) |
| \( M_{RM} \) | \( \begin{bmatrix}
(s - j\omega_0 + j\omega_2) L_m & 0 \\
0 & (s - j\omega_0 - j\omega_2) L_m
\end{bmatrix} \) |

\[
\begin{align*}
M_{outer-PGI}^{RSC} &= H_1 (s - j\omega_0) H_2 (s - j\omega_0) \begin{bmatrix}
U_s^* & 0 \\
0 & U_s
\end{bmatrix} \\
M_{outer-PGU}^{GSC} &= \frac{H_3 (s - j\omega_0) H_4 (s - j\omega_0)}{2sC_{dc} U_{dc}} \begin{bmatrix}
I_g^* & I_g \\
U_g^* & U_g
\end{bmatrix} \\
M_{outer-PRI}^{GSC} &= \frac{-H_3 (s - j\omega_0) H_4 (s - j\omega_0)}{2sC_{dc} U_{dc}} \begin{bmatrix}
I_r^* & I_r \\
I_r^* & I_r
\end{bmatrix} \\
M_{outer-PRU}^{GSC} &= \frac{-H_3 (s - j\omega_0) H_4 (s - j\omega_0)}{2sC_{dc} U_{dc}} \begin{bmatrix}
U_r^* & U_r \\
U_r^* & U_r
\end{bmatrix} \\
\end{align*}
\]

\[
\begin{align*}
M_{inner-PRU}^{GSC} &= \begin{bmatrix}
H_4 (s - j\omega_0) & 0 \\
0 & H_4 (s - j\omega_0)
\end{bmatrix} \\
M_{PLL}^{GSC} &= \frac{H_{pll} (s - j\omega_0)}{2} \begin{bmatrix}
K_{GSC} & -K_{GSC} \\
-K_{GSC} & K_{GSC}
\end{bmatrix} \\
K_{GSC} &= -H_4 (s - j\omega_0) I_g + U_g
\end{align*}
\]
\[ \begin{bmatrix} \Delta u_s \\ e^{j2\theta} \Delta u_s^* \end{bmatrix} = \left[ I - M_{SM} \left( M_{RR} - M_{inner RSC}^{outer loop controller} \right)^{-1} M_{outer-l RSC} - M_{SM} \left( M_{RR} - M_{inner RSC}^{outer loop controller} \right)^{-1} M_{PLL RSC} \right]^{-1} \cdot \left[ M_{SM} \left( M_{RR} - M_{inner RSC}^{outer loop controller} \right)^{-1} M_{outer-U RSC} + M_{SM} - M_{SM} \left( M_{RR} - M_{inner RSC}^{inner loop controller} \right)^{-1} M_{RM} \right] \left[ \Delta i_s \\ e^{j2\theta} \Delta i_s^* \right] \] (A13)

\[ \begin{bmatrix} \Delta u_s \\ e^{j2\theta} \Delta u_s^* \end{bmatrix} = \left[ I - \left( M_{outer-GSC}^{outer loop controller} - M_{outer-PG I GSC} - M_{outer-PRI M_{ur}} G_{IR} - M_{outer-PRI M_{ir}} G_{IR} \right) + M_{PLL GSC} \right]^{-1} \cdot \left[ M_{outer-GSC}^{outer loop controller} - M_{outer-PG I GSC} + M_{TG} + M_{inner RSC}^{inner loop controller} \right] \left[ \Delta i_s \\ e^{j2\theta} \Delta i_s^* \right] \] (A14)

\[ M_{ur} = M_{RR} \left( M_{RR} - M_{inner RSC}^{outer loop controller} \right)^{-1} \left( M_{outer-l RSC} + M_{outer-U RSC} G_{RB1} \right) + M_{RR} \left( M_{RR} - M_{inner RSC}^{inner loop controller} \right)^{-1} M_{PLL RSC} \]

\[ M_{ir} = \left( M_{RR} - M_{inner RSC}^{outer loop controller} \right)^{-1} \left( M_{outer-l RSC} + M_{outer-U RSC} G_{RB1} \right) + \left( M_{RR} - M_{inner RSC}^{inner loop controller} \right)^{-1} M_{PLL RSC} - \left( M_{RR} - M_{inner RSC} \right)^{-1} M_{RM G_{RB1}} \]

and \( \Delta i_s \) as,

\[ \begin{bmatrix} \Delta i_r \\ e^{j2\theta} \Delta i_r^* \end{bmatrix} = M_{RR} M_{Ir}^{Us} \left[ \Delta u_s \\ e^{j2\theta} \Delta u_s^* \right] + \left( M_{RR} M_{Ir}^{Us} + M_{RM} \right) \left[ \Delta i_s \\ e^{j2\theta} \Delta i_s^* \right] \] (A9)

Substituting \( \Delta i_s \) in (A9) by (A7), we can have the relationship between \( \Delta u_r \) and \( \Delta u_s \) as,

\[ \begin{bmatrix} \Delta u_r \\ e^{j2\theta} \Delta u_r^* \end{bmatrix} = M_{ur} \left[ \Delta u_s \\ e^{j2\theta} \Delta u_s^* \right] \] (A10)

where \( M_{ur} = \left[ M_{RR} M_{Ir}^{Us} + \left( M_{RR} M_{Ir}^{Us} + M_{RM} \right) G_{RB1} \right] \).

Substituting \( \Delta i_s \) in (A6) by (A7), we can have the relationship between \( \Delta i_r \) and \( \Delta u_s \) as,

\[ \begin{bmatrix} \Delta i_r \\ e^{j2\theta} \Delta i_r^* \end{bmatrix} = M_{ir} \left[ \Delta u_s \\ e^{j2\theta} \Delta u_s^* \right] \] (A11)

where \( M_{ir} = \left( M_{Ir}^{Us} + M_{Ir}^{IR} G_{RB1} \right) \) and \( G_{RB1} = Z_{RB1}^{-1} \).

Finally, we replace \( \Delta u_r \) and \( \Delta i_s \) in (A8) by (A10) and (A11), the relationship of \( \Delta u_s \) between \( \Delta i_s \) can be obtained and shown in (A12).

\[ \begin{bmatrix} \Delta u_s \\ e^{j2\theta} \Delta u_s^* \end{bmatrix} = Z_{B2} \left[ \Delta i_s \\ e^{j2\theta} \Delta i_s^* \right] \] (A12)

The comprehensive form of (A7), (A12), \( M_{ur} \) and \( M_{ir} \) can be found at the top of this page. (A13) and (A14) are two parallel impedance branches of DFIG, and they are shown as (7) and (8) in Section II.

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