Emergent Universe as an interaction in the dark sector

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Abstract

In this work a cosmological scenario where dark matter interacts with a variable vacuum energy for a spatially flat Friedmann-Robertson-Walker space-time is analysed. One of the aims is to show how a particular interaction in the dark sector can be used to get a model of an Emergent Universe. After that we analyse the viability of two particular models by taking into account recent observations. The updated observational Hubble data is used in order to constrain the cosmological parameters of the models and the amount of dark energy in the radiation era is estimated. It is shown that the two models fulfill the severe bounds of $\Omega_x(z \approx 1100) < 0.009$ at the $2\sigma$ level of Planck.

1 Introduction

Since 1998, there are strong evidences that the universe is flat and it is in an accelerated expansion phase. Some of these evidences comes from the cosmological and astrophysical data from type Supernovae Ia (SNIa) [1], [2], [3], the spectra of the Cosmic Microwave Background (CMB) [4] radiation anisotropies and Large Scale Structure (LSS) [5], [6]. One of the alternatives to explain this faster expansion phase has been attributed to a mysterious dark energy component with negative pressure. The simplest type model of dark energy corresponds to a positive cosmological constant $\Lambda$. Another important component of our Universe is dark matter, it shares the non luminous nature with the dark energy. It is gravitationally attractive and leads to the formation of large scale structures.

There are several models which attempt to explain the origin or the dynamics of the dark matter and the dark energy. Some of them propose that the origin

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could be come from a kind of dynamical scalar field, as the quintessence model \[7\]-\[9\]. Other models expect that the cosmological term $\Lambda$ should not be strictly constant, they appear as smooth functions of the Hubble rate $H(t)$ \[10\]-\[14\].

A considerable alternative to the $\Lambda$CDM model is the possibility of interaction in the dark sector. This non-gravitational interaction gives rise to a continuous transfer of energy between dark, energy and matter, i.e. we suppose that one component can feel the presence of the other through the gravitational expansion of the Universe \[15\]. As it is expected, a connection between the dark components changes the background evolution of the dark sector \[16\]-\[19\], giving rise to a rich cosmological dynamics compared with non interacting models. Current data are compatible with interaction models although the evidence is, so far, not fully conclusive \[20\], \[21\].

As is known, the big bang cosmology scenario has some problems both in the early and in late universe. Many of these problems emerge when we describing the early Universe, the horizon problem, the flatness problem, fine-tuning, etc \[22\], \[23\]. These unresolved issues could be explained by the physics of inflation and the introduction of a small cosmological constant for late acceleration, but they are not clearly understood. An alternative is the Emergent Universe scenario, in which an inflationary universe emerges from a small static state that has within it the origin of the development of the macroscopic universe. The universe has a finite initial size and since the initial stage is Einstein static, there is no time-like singularity. It is an ever-existing universe, so there is no horizon problem. The possibilities of an emergent universe have been studied in few papers. Del Campo et. al. \[24\] studied the emergent universe model in the context of a self-interacting Jordan-Brans-Dicke theory, Mukherjee et. al. \[25\] in the framework of general relativity, Paul and Ghose in Gauss-Bonnet gravity \[26\], in a Horava gravity was studied by Mukherjee and Chakraborty \[27\], etc.

The emergent universes proposed by Mukherjee \[25\] are late-time de Sitter with an equation of state of the form $p = A\rho - B\rho^{1/2}$, where $A$ and $B$ are constants. This is a special case of the Chaplygin gas \[28\]-\[30\]. Recently, the onset of the recent accelerating phase had been determined by the constrains of the parameters $A$, $B$ with the observational data \[31\], \[32\].

The aim of this letter is two-folded. On one hand it is shown that, by assuming the existence of an interacting dark sector with a barotropic equation of state in the context of General Relativity, an emergent universe dynamics such as the ones considered in \[25\], \[31\] may arise. It should be emphasized that none of the previous emergent scenarios was obtained by taking into account a barotropic equation of state in General Relativity. This fact is one novel feature of the present work. On the other hand, we are considering models which differ those studied in \[25\], \[31\] since our model is described by a source equation which is of second order \[15\] while those authors consider a conservation equation which is of first order. Certain explicit solutions of this new models are reported in the text. In addition the explicit form of these solutions and the updated Hubble data and the severe bounds reported by the Planck mission on early dark energy are used below in order to constraint the parameters of our model.
2 Interaction Model

In the Interaction Scenario an spatially flat isotropic and homogeneous universe described by Friedmann-Robertson-Walker (FRW) spacetime is usually considered. The universe is filled with three components, baryonic matter, and two fluids that interacts in the dark sector. The first is a decoupled component. The evolution of the FRW universe is governed by the Friedmann and conservation equations,

\[ 3H^2 = \rho_T = \rho_r + \rho_b + \rho_m + \rho_x, \]
\[ \dot{\rho}_b + 3H\gamma_b \rho_b = 0, \]
\[ \dot{\rho}_m + \dot{\rho}_x + 3H(\gamma_m \rho_m + \gamma_x \rho_x) = 0, \]

where \( H = \dot{a}/a \) is the Hubble expansion rate and \( a(t) \) is the scale factor. The equation of state for each species, with energy densities \( \rho_i \), and pressures \( p_i \), take a barotropic form \( p_i = (\gamma_i - 1)\rho_i \), and the constants \( \gamma_i \) indicate the barotropic index of each component being \( i = \{x, m, b\} \), so that \( \gamma_x = 0 \), \( \gamma_b = 1 \), whereas \( \gamma_m \) will be estimated later on. Then \( \rho_x \) plays the role of a decaying vacuum energy or variable cosmological constant, \( \rho_b \) represents a pressureless barionic matter, and \( \rho_m \) can be associated with dark matter.

By solving the linear differential system (3) along with \( \rho = \rho_m + \rho_x \) allow to express both dark densities as functions of \( \rho \) and \( \dot{\rho} \)

\[ \rho_m = \frac{\gamma_x \rho + \rho'}{\gamma_m - \gamma_x}, \quad \rho_x = \frac{\gamma_m \rho + \rho'}{\gamma_m - \gamma_x}, \]

where the variable \( \eta = \ln(a/a_0)^3 \) has been introduced, with \( a_0 \) the present value of the scale factor (\( a_0 = 1 \)). In the following it is assumed that there is no interaction between the baryons and the dark sector, so the energy density is conserved and the prime indicates differentiation with respect to the new time variable \( \dot{t} \equiv d/d\eta \). Under this situation, Eqs. (2) leads to the energy density for baryonic matter, \( \rho_b \sim a^{-3} \).

In order to continue the analysis of the interacting dark sector it is convenient to introduce an energy transfer equation between the two fluids

\[ \rho_m' + \gamma_m \rho_m = -Q, \quad \rho_x' + \gamma_x \rho_x = Q. \]

Here a coupling with a factorized \( H \) dependence in the form \( 3HQ \) was considered, where \( Q \) indicates the energy exchange between the dark components. From Eqs. (4) and (5), the following source equation for the energy density \( \rho \) of the dark sector is obtained

\[ \rho'' + (\gamma_m + \gamma_x)\rho' + \gamma_m \gamma_x \rho = Q(\gamma_m - \gamma_x). \]

In this work an interaction \( Q \) between both dark components that depends on the scale factor will be considered of the form

\[ Q = -2A\sqrt{Ba}^{-3r} - B a^{-6r}. \]
where $A$ and $B$ are the coupling constants that measures the strength of the interaction in the dark sector. In this case, we will analyse the models with $r = 1/2$ and $r = 1/3$. These kind of interactions are now studied under the view of the new observations and gives rise to a dark energy model that can be viewed as an emergent universe \([31], [32]\).

Replacing the specific form of $Q$ into the source equation (6) and the value $\gamma_x = 0$, the second order differential equation for the total energy density $\rho(a)$ of the dark sector can be solved. The relation between the energy density and the redshift $z$ may be found by considering the relation between the scalar factor and the redshift, $z + 1 = 1/a$,

$$\rho = C_1 + C_2(1 + z)^{3\gamma_m} + \frac{2A\sqrt{B}}{r} \frac{\gamma_m}{\gamma_m - r}(1 + z)^{3r}$$

$$+ \frac{B}{2r} \frac{\gamma_m}{\gamma_m - 2r}(1 + z)^{6r}, \quad (8)$$

where $C_1$ and $C_2$ are the integration constants. Taking the derivative of Eq. (8) and replacing in the equations of (4) gives the dark matter and the dark energy of these models

$$\rho_m = C_2(1 + z)^{3\gamma_m} + \frac{2A\sqrt{B}}{r} (1 + z)^{3r} + \frac{B}{\gamma_m - 2r}(1 + z)^{6r}, \quad (9)$$

$$\rho_x = C_1 + \frac{2A\sqrt{B}}{r}(1 + z)^{3r} + \frac{B}{2r}(1 + z)^{6r}. \quad (10)$$

For the choice $C_2 = 0$, the energy density of Eq. [8] may be written in the form $\rho(a) = (\beta + \alpha a^3\gamma)^2 / a^{6r}$, with $\beta$ and $\alpha$ simple constants. This form of the energy density is the one obtained in [25], these authors find it by using a polytropic equation of state of the form $p = A\rho - B\rho^{1/2}$. One purpose of this research is to show that in a spatially flat universe with a barotropic equation of state (instead of polytropic) and the interaction Eq. (7) may cast the energy density of the an Emergent Universe. The observational constraints on the model parameters for $C_2 = 0$ will not be found here since it was already considered in [31], [32]. But we will analyse the models with $C_2 \neq 0$ for $r = 1/2$ and $r = 1/3$ and find the constrains to determine the model parameters, respectively. For the best of our knowledge, this was not done already in previous work.

By taking into account the present-density parameters $\Omega_{i0} = \rho_{i0}/3H_0^2$ along with the flatness condition $1 = \Omega_{i0} + \Omega_{x0} + \Omega_{m0}$, the integration constants $C_1$ and $C_2$ may be expressed in terms of the observational density parameters

$$C_1 = 3H_0^2\Omega_{x0} - \frac{2A\sqrt{B}}{r} \frac{B}{2r}, \quad (11)$$

$$C_2 = 3H_0^2\Omega_{m0} - \frac{2A\sqrt{B}}{\gamma_m - r} \frac{B}{\gamma_m - 2r}. \quad (12)$$
In this case the Friedmann equation (1) is given in terms of the redshift and density parameters by

\[ 3H^2(z) = (1 - \Omega_x0 - \Omega_m0)(1 + z)^3 + C_1 + C_2(1 + z)^3\gamma_m + 2A\sqrt{B}r\gamma_m + 2r(1 + z)^3\gamma_m. \]  

(13)

The specific models with \( r = 1/2 \) and \( r = 1/3 \) have six independent parameters \( (H_0, \Omega_x0, \Omega_m0, A, B, \gamma_m) \) to be completely specified. The above function (13) will be used in the next section for analysis with observational results and to determine the model parameters. For both models, in the limit case \( z \to -1 \), the energy density goes to a constant value like the \( \Lambda \)CDM model, so the universe exhibits a de Sitter phase at late times. In the dark energy domains, the energy density Eq. (8) for the model with \( r = 1/3 \) corresponds to a cosmic fluid that behaves as a composition of cosmological constant, domain walls and cosmic strings [25].

3 Constrains on model parameters form observational Hubble data

For our analysis we will consider a set of measurements for Hubble parameter \( H(z) \) at different redshifts [33] [34] [36] [37] [38] [39] [40]. We find a qualitative estimation of the cosmological parameters for the models with \( r = 1/2 \) and \( r = 1/3 \) described above. The values of the function \( H(z) \) are directly obtained from the cosmological observations, so this function plays a fundamental role in understanding the properties of the dark sector. The bibliography [41], [42], [43] shows \( H_{obs} \) for different redshifts with the corresponding 1\( \sigma \) uncertainties. The probability distribution for the \( \theta \)-parameters, for each model, is \( P(\theta) = N \exp(-\chi^2(\theta)/2) \) [44], being \( N \) a normalization constant. In order to obtain the parameters of the models we first minimized a chi-square function \( \chi^2 \) defined as

\[ \chi^2(\theta) = \sum_{i=1}^{N=29} \frac{(H(\theta; z_i) - H_{obs}(z_i))^2}{\sigma^2(z_i)}, \]  

(14)

where \( H_{obs}(z_i) \) and \( H(\theta, z_i) \) are the observed and observational values of the Hubble parameter \( H(z) \) at different redshifts \( z_i \) and \( \sigma(z_i) \) is the corresponding 1\( \sigma \) error. The Hubble function \( H(\theta, z_i) \) is (13) evaluated at \( z_i \), for both models, with \( r = 1/2 \) and \( r = 1/3 \) respectively. The variable \( \chi^2 \) is a random variable that depends on \( N = 29 \), the number of the data, and its probability distribution is a \( \chi^2 \) distribution for \( N - n \) degrees of freedom, with \( n = 2 \), where \( n \) is the number of parameters. The \( \chi^2 \) function reaches its minimum value at the best fit value \( \theta_c \) and the fit is good when \( \chi^2_{min}(\theta_c)/(N - n) \leq 1 \) [44]. For a given pair \( (\theta_1, \theta_2) \) of independent parameters, fixing the other ones, the confidence levels (C.L.) 1\( \sigma \) (68.3\%) or 2\( \sigma \) (95.4\%) will satisfy \( \chi^2(\theta) - \chi^2_{min}(\theta_c) \leq 2.30 \) or \( \chi^2(\theta) - \chi^2_{min}(\theta_c) \leq 6.17 \) respectively.
In theoretical models it is demanded that the parameters should satisfy the inequalities (i) $A > 0$ and (ii) $B > 0$. The regions of $1\sigma$ and $2\sigma$ confidence levels (C.L) obtained with the standard $\chi^2$ function are shown in Fig. 1 on the right the model with $r = 1/2$ and on the left the model with $r = 1/3$. The respectively estimation for the model is briefly summarized in Tables 1 and 2. For example, some best-fitting values obtained for the parameters are, $A = 49.39^{+21.59}_{-21.59}$ and $B = 30.95^{+17.95}_{-17.95}$ with $\chi^2_{d.o.f} = 0.764$ for the model with $r = 1/2$ and $A = 75.81^{+24.07}_{-24.07}$, $B = 24.34^{+16.84}_{-16.84}$ with $\chi^2_{d.o.f} = 0.765$ for $r = 1/3$. In both cases is satisfied the goodness condition $\chi^2_{d.o.f} < 1$. We get the best fit at the independence parameters $(\Omega_\gamma, \Omega_m) = (0.714^{+0.026}_{-0.025}, 0.249^{+0.077}_{-0.084})$ with $\chi^2_{d.o.f} = 0.831$ for the case with $r = 1/2$ by using the priors $(H_0 = 68, A = 60, B = 25, \gamma_m = 1.08)$; therefore the present day values obtained of the dark energy and dark matter parameter are in agreement with the data released by the WMAP-9 project [12] or with the data coming from the Planck Mission [15]. A similar result is obtained for the model with $r = 1/3$ as is shown in Table 2.

### Table 1: We show the observational bounds for the 2-D C.L. obtained in Fig. (1) by varying two cosmological parameters.

| No | Priors | Best fits | $\chi^2_{d.o.f}$ |
|----|--------|-----------|-----------------|
| I  | $(H_0, \Omega_L, \Omega_m, \gamma_m) = (69.2, 0.721, 0.235, 1.08)$ | $(A, B) = (70.13^{+1.23}_{-1.23}, 59.71^{+20.31}_{-20.31})$ | 0.764 |
| II | $(\Omega_L, \Omega_m, B, \gamma_m) = (0.72, 0.235, 30, 1.07)$ | $(H_0, A) = (70.14^{+1.48}_{-1.48}, 29.08^{+18.37}_{-18.37})$ | 0.749 |
| III | $(\Omega_L, \Omega_m, A, \gamma_m) = (0.721, 0.235, 60, 1.07)$ | $(H_0, B) = (70.14^{+1.48}_{-1.48}, 29.08^{+18.37}_{-18.37})$ | 0.748 |
| IV | $(H_0, A, B, \gamma_m) = (68, 60, 25, 1.08)$ | $(\Omega_L, \Omega_m) = (0.714^{+0.026}_{-0.025}, 0.249^{+0.077}_{-0.084})$ | 0.831 |

### Table 2: We show the observational bounds for the 2-D C.L. obtained in Fig. (1) by varying two cosmological parameters.

| No | Priors | Best fits | $\chi^2_{d.o.f}$ |
|----|--------|-----------|-----------------|
| I  | $(H_0, \Omega_L, \Omega_m, \gamma_m) = (69.2, 0.721, 0.235, 1.08)$ | $(A, B) = (75.81^{+24.07}_{-24.07}, 24.34^{+16.84}_{-16.84})$ | 0.765 |
| II | $(\Omega_L, \Omega_m, B, \gamma_m) = (0.72, 0.235, 30, 1.07)$ | $(H_0, A) = (70.24^{+1.56}_{-1.56}, 80.74^{+26.96}_{-26.96})$ | 0.745 |
| III | $(\Omega_L, \Omega_m, A, \gamma_m) = (0.721, 0.235, 60, 1.07)$ | $(H_0, B) = (70.25^{+1.56}_{-1.56}, 41.60^{+26.14}_{-26.14})$ | 0.744 |
| IV | $(H_0, A, B, \gamma_m) = (68, 60, 25, 1.08)$ | $(\Omega_L, \Omega_m) = (0.707^{+0.024}_{-0.026}, 0.247^{+0.085}_{-0.086})$ | 0.831 |

### 4 Other relevant parameters

For the models with $r = 1/2$ and $r = 1/3$ the behaviour of the density parameters $\Omega_L$, $\Omega_m$, and $\Omega_b$ nearly close to $z = 0$ is described in Fig. 2. As we well know, the dark energy is in particular the main source responsible of the Universe acceleration; far away from $z = 1$ the Universe is dominated by the dark matter which it is responsible of the structure formation. Note that these models are asymptotically de Sitter when $z \to -1$ and the total energy density tends to a constant value.

Other cosmological relevant parameter is the deceleration parameter at the present time $q(z = 0) = q_0$. The Figure 2 shows the behaviour of the de-
Figure 1: Two-dimensional C.L. associated with 1σ, 2σ for different θ planes with the interaction \( Q = \alpha \rho' \).
Figure 2: Plot of $\Omega_b(z)$, $\Omega_x(z)$, $\Omega_m(z)$, $r(z)$, and $q(z)$, using the best-fit values obtained with the Hubble data for different $\theta$ planes, for the model with $r = 1/2$.

Table 3: We show the cosmological parameters derived from the best fits value of 2-D C.L. obtained in Tables (1) and (2) by varying two cosmological parameters.

| N | $q(z = 0)$ | $\Omega_x(z \approx 1100)$ | $\Omega_x(z \approx 10^{10})$ | $\Omega_x(z \approx 10^{10})$ |
|---|---|---|---|---|
| I | 0.56 | 0.0032 | 0.00012 |
| II | 0.56 | 0.0032 | 0.00012 |
| III | 0.56 | 0.0031 | 0.00012 |
| IV | 0.54 | 0.0020 | 0.00004 |

We also determined the variation of the dark energy parameter behind recombination or big-bang nucleosynthesis epochs [16], [17] and compared with the severe bound for each epoch. This can be considered as a complementary tool for testing our models. One of the last constraints on early dark energy (ede) come from the Planck+WP+high L data: $\Omega_{ede} < 0.009$ at 95% C.L [45]. We found that $\Omega_x(z \approx 10^{7})$ is over the interval $[0.0020, 0.0032]$ for the model with $r = 1/2$ and $[2.7 \times 10^{-9}, 5.4 \times 10^{-9}]$ for $r = 1/3$, so our estimations satisfied the bound reported by the Planck mission [see Tables 3]. In regard to the bound reported from the joint analysis based on Euclid+CMBPol data, $\Omega_{ede} < 0.00092$ [46], [47], the model with $r = 1/2$ doesn’t satisfied the severe bound, but the model with $r = 1/3$ fulfill the bound reported. Around $z = 10^{10}$, in the nucleosynthesis epoch, we have $\Omega_x$ between $[10^{-15}; 10^{-13}]$ at the 1$\sigma$ level, therefore the model with $r = 1/3$ is in concordance with the conventional BBN processes that occurred at a temperature of 1Mev [48].

5 Discussions

In the present letter a Universe that presents a particular interaction in the dark sector has been analysed. It was found that with an interaction which depends...
of the scale factor of the form \( Q = -\sqrt{B}a^{-3r}(2A + \sqrt{B}a^{-3r}) \), with a barotropic equation of state in the dark sector, \( p = (\gamma - 1)\rho \), can cast a flat Emergent Universe solution that was already presented in [25]. However, it should be emphasized that the this Emergent Universe solutions was obtained in these references by using a non-linear equation of state, which is a particular case of the Chaplygin gas, and not with the interactions described by us.

The comparison with observational data was carried out by considering the parameter values \( r = 1/2 \) and \( r = 1/3 \), and the remaining cosmic set of parameters has been constrained by using the updated Hubble data and the severe bounds for dark energy found at early times. We have shown that both models interpolates between a cold dark matter regime and a De Sitter phase in the asymptotic future.

On the observational side, the best-fit values at 2\( \sigma \) level for the parameters of the model are represented in the Fig. 1 and Tables 1 and 2. We can observe that the obtained constant values of the models are \( A = 49.39^{+21.69}_{-21.59} \) and \( B = 30.95^{+22.19}_{-17.95} \) with \( \chi^2_{d.o.f} = 0.764 \) for the model with \( r = 1/2 \) and \( A = 75.81^{+24.07}_{-26.84} \), \( B = 24.34^{+16.84}_{-12.94} \) with \( \chi^2_{d.o.f} = 0.765 \) for \( r = 1/3 \), where \( A > 0 \) and \( B > 0 \) for both cases. They satisfy the goodness condition \( \chi^2_{d.o.f} < 1 \). The best fit is obtained at the independence parameters \( (\Omega_x, \Omega_m) = (0.714^{+0.026}_{-0.025}, 0.249^{+0.077}_{-0.084}) \) with \( \chi^2_{d.o.f} = 0.831 \) by using the priors \( (H_0 = 68, A = 60, B = 25, \gamma_m = 1.08) \) for the case with \( r = 1/2 \). The values obtained of the dark energy and dark matter density parameter are in agreement with the data coming form the WMAP-9 project [42] or with the data realised by the Planck Mission [45], see Table 1. For the model with \( r = 1/3 \) as is shown in Table 2 we get a similar result. In addition, the amount of early dark energy has been estimated, i.e. the energy density parameter in the radiation era. We found that the two models fulfill the severe bounds of \( \Omega_x(z \approx 1100) < 0.009 \) at the 2\( \sigma \) level of Planck. But the model with \( r = 1/2 \) didn’t satisfy the severe bound reported by the joint analysis based on Euclid+CMBPol data, \( \Omega_{ede} < 0.00092 \) [46], [47], while the model with \( r = 1/3 \) does it.

The present work shows that both models admits dark densities close to that predicted by observations in the concordance model. It should be emphasized however that the analysis performed does not take into account the evolution of density perturbations and the consequent structure formation of the universe. This important matter will be considered in a future investigation.

Acknowledgments

The authors are supported by CONICET.

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