Quasi Goldstone Fermion As a Sterile Neutrino

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Abstract

The existence of sterile neutrino is hinted by simultaneous explanation of diverse neutrino anomalies. We suggest that the quasi Goldstone fermions (QGF) arising in supersymmetric theory as a result of spontaneous breaking of global symmetry like the Peccei-Quinn symmetry or the lepton number symmetry can play a role of the sterile neutrino. The smallness of mass of QGF ($m_S \sim 10^{-3} - 10$ eV) can be related to the specific choice of superpotential or Kähler potential (e.g., no-scale kinetic terms for certain superfields). Mixing of QGF with neutrinos implies the $R$-parity violation. It can proceed via the coupling of QGF with the Higgs supermultiplets or directly with the lepton doublet. A model which accounts for the solar and atmospheric anomalies and the dark matter is presented.
1 Introduction

All the experimentally known fermions transform non-trivially under the gauge group $SU(3) \times SU(2) \times U(1)$ of the standard model (SM). However there are experimental hints in the neutrino sector which suggest the existence of $SU(3) \times SU(2) \times U(1)$ - singlet fermions mixing appreciably with the known neutrinos. These hints come from (a) the deficits in the solar and atmospheric neutrino fluxes (b) possible need of significant hot component in the dark matter of the universe and (c) some indication of $\bar{\nu}_e - \bar{\nu}_\mu$ oscillations in the laboratory. These hints can be reconciled with each other if there exists a fourth very light ($< O(eV)$) neutrino mixed with some of the known neutrinos preferably with the electron one. The fourth neutrino is required to be sterile in view of the strong bounds on number of neutrino flavours coming both from the LEP experiment as well as from the primordial nucleosynthesis.

The existence of very light sterile neutrino demands theoretical justification since unlike the active neutrinos, the mass of the sterile state is not protected by the gauge symmetry of the SM and hence could be very large. Usually the sterile neutrino is considered on the same footing as the active neutrinos and some ad hoc symmetry is introduced to keep this neutrino light. Recently there are several attempts to construct models for sterile neutrinos which have the origin beyond the usual lepton structure. In particular in Ref. we suggested a possibility that supersymmetry (SUSY) may be responsible for both the existence and the lightness of the sterile fermions.

One could consider three different ways in which supersymmetry can keep sterile states very light.

1. Combination of supersymmetry and the (continuous) $R$ symmetry present in many supersymmetric models may not allow a mass term for the light sterile state.

2. Spontaneous breakdown of some other global symmetry in supersymmetric theory can lead to massless fermions which form the superpartners of the Goldstone bosons.

3. The spontaneous breakdown of the global supersymmetry itself would give rise to a massless fermion, the goldstino.

The mechanism (1) and its phenomenological consequences were discussed in Ref. Mechanism (3) though appealing is not favoured phenomenologically in view of the difficulties in building realistic models based on the spontaneously broken global SUSY. We discuss in this paper implications of the mechanism (2) concentrating for definiteness on the simplest
case of a global $U(1)_G$.

The spontaneously broken global symmetries are required for reasons unrelated to the existence of light sterile states. The most interesting examples being spontaneously broken lepton number symmetry $G_L$ and the Peccei-Quinn (PQ) symmetry imposed to solve the strong CP problem. The PQ symmetry arise naturally in many supersymmetric models. Apart from solving the strong CP problem, this symmetry can also explain the smallness of the $\mu$-parameter. Phenomenologically consistent breaking of these symmetries generally needs Higgs fields which are singlets of $SU(3) \times SU(2) \times U(1)$. In the supersymmetric context this automatically generates massless sterile fermion. While the existence of these quasi Goldstone fermions (QGF) is logically independent of neutrino physics, there are good reasons to expect that these fermions will couple to neutrinos. Indeed, in the case of lepton number symmetry the superfield which is mainly responsible for the breakdown of $U(1)_L$ carries nontrivial $U(1)_L$-charge and therefore it can directly couple to leptons if the charge is appropriate. In the case of the PQ symmetry, $U(1)_{PQ}$, this superfield could couple to the Higgs supermultiplet. If theory contains small violation of $R$ parity then this mixing with Higgs gets communicated to the neutrino sector. Thus the occurrence of the QGF can have implications for neutrino physics. We wish to discuss in this paper prospects for building realistic models based on this mechanism.

In the following section we elaborate upon the expected properties of the QGF, especially their masses when SUSY is broken. Section 3 discusses various mechanisms of mixing of these fermions with the active neutrinos. Explicit model based on the scenario presented in section 2 and 3 is given in section 4 and the last section presents our conclusions.

2 Quasi Goldstone fermions and their masses

In this section and subsequently, we will consider the following general superpotential

$$W = W_{MSSM} + W_S + W_{mixing},$$

where $W$ is assumed to be invariant under some global symmetry $U(1)_G$. As we outlined in the introduction, this symmetry may be identified with the PQ symmetry, lepton number symmetry or combination thereof. The first term in Eq. (1) refers to the superpotential of the minimal supersymmetric standard model (MSSM). The second term contains $SU(3) \times SU(2) \times$
$U(1)$ singlet superfields which are responsible for the breakdown of $U(1)_G$. The minimal choice for $W_S$ is

$$W_S = \lambda (\sigma \sigma' - f_G^2) y ,$$

where $\sigma, \sigma'$ carry non trivial $G$-charges and $f_G$ sets the scale of $U(1)_G$ breaking. The last term of Eq. (1) describes mixing of the singlet fields with the superfields of the MSSM.

In the supersymmetric limit the fermionic component of the Goldstone boson is massless. In the case (2) this Goldstone fermion is contained in

$$S = \frac{1}{\sqrt{2}} (\sigma - \sigma') .$$

However, SUSY breakdown results in generation of mass of the Goldstone fermion. In general, this mass can be as big as SUSY breaking scale, $m_{SUSY}$. Broken supersymmetry itself cannot automatically protect the masses of QGF in Eq. (3) much below $m_{SUSY}$. In fact, the mass of QGF depends on the manner in which SUSY is broken and on the way how this breaking is communicated to the singlet $S$. It also depends on the structure of superpotential and the scale $f_G$. In the below we identify theories which can allow for very light QGF ($m_S < 1$ eV). As the case of special interest we will consider the mass of QGF and its mixing with the electron neutrino:

$$m_S \simeq (2-3) \cdot 10^{-3} \text{ eV}$$

$$\sin \theta_{es} \simeq \tan \theta_{es} \simeq (2-6) \cdot 10^{-2} .$$

These values of parameters allow one to solve the solar neutrino problem through the resonance conversion $\nu_e \to S$.

One could consider different mechanisms for the QGF mass generation.

Let us note that in models with spontaneously broken global SUSY the QGF generically acquire a mass of $O\left(\frac{m^2_{SUSY}}{f_G}\right)$ [15]. But it can remain massless in spite of SUSY breaking (a) if SUSY is broken by a D-term of the gauge field or (b) if the F-terms that break SUSY do not carry any G-charges. The latter is exemplified by a simple generalization of Eq. (2):

$$W_S = \lambda_1 (\sigma \sigma' - f_1^2) y_1 + \lambda_2 (\sigma \sigma' - f_2^2) y_2 .$$

SUSY is broken in this example if $f_1^2 \neq f_2^2$. For a minimum with the F-terms: $F_\sigma = F_{\sigma'} = 0$, the Goldstone fermion in Eq. (3) remains massless at the tree level in spite of the SUSY
breakdown. As we noticed before this version has phenomenological problems and further on we will concentrate on possibilities related to supergravity.

The mass of the QGF in supergravity theory is typically of the order of gravitino mass $m_{3/2} (= m_{\text{SUSY}})$ \[16, 17, 18\]. For instance, the superpotential in Eq. (4) leads to $m_S \sim m_{3/2}$ when generic soft terms of SUSY breakdown are allowed \[16\]. However, the mass $m_S$ can be much smaller for specific choices of 1) the superpotential and/or 2) soft SUSY breaking terms. Let us consider these possibilities in order.

1). The superpotential

$$\lambda (\sigma \sigma' - X^2)y + \lambda' (X - f_G)^3$$

is shown \[17\] to generate the tree level mass

$$m_S \sim \frac{m_{3/2}^2}{f_G}$$

(5)
as in the global case if the minimal kinetic terms of the fields are assumed. For commonly accepted value of the PQ symmetry breaking scale, $f_G = f_{PQ} = 10^{10} - 10^{12}$ GeV, one gets from Eq. (5) $m_S \sim (10 - 10^3)$ eV. On the other hand, the value of $m_S$ in Eq. (4) desired for explanation of the solar neutrino deficit requires $f_G \sim 10^{16}$ GeV which can be related to the grand unification scale. To identify $f_G$ with $f_{PQ}$, one should overcome the cosmological bound $f_{PQ} < 10^{12}$ GeV. The bound can be removed by axion mixing with some other Goldstone boson in their kinetic terms \[19\] or by dilaton field driven to small values in inflationary period \[20\]. In this case however, the axion cannot play the role of cold dark matter.

2). Another possibility to get very light $S$ is based on the idea of no-scale supergravity \[21\]. The Kähler potential and the superpotential can be arranged in such a way that supersymmetry breaking is communicated to the singlet $S$ via a set of interactions. As the result, the mass of $S$ appears in one, two or even three loops.

Let us consider the following Kähler potential:

$$K = -3 \ln(T + T^* - Z_a Z_a^*) + C_i C_i^*,$$

(6)

where $T$ is the moduli field appearing in the underlying superstring theory, $Z_a$ and $C_i$ are the matter superfields which have the no-scale kinetic term ($Z$–sector) and the minimal kinetic term ($C$–sector) respectively. The corresponding scalar potential at the Planck scale reads,

$$V = |W_i|^2 + \{m_0 C_i W_i + \text{h.c.}\} + m_0^2 |C_i|^2 + |W_a|^2,$$

(7)
where \( m_0 = \mathcal{O}(m_{3/2}) \). The tree-level masses of the fermionic components of the fields \( Z_a \) are determined by the global supersymmetric results. Therefore, if the singlet fields triggering \( U(1)_G \) breaking are in the \( Z \)-sector, the QGF will be massless at tree level [18]. The QGF will acquire the mass through the interactions with fields \( C_i \) having minimal kinetic terms, and consequently, usual soft SUSY breaking terms. Moreover, \( S \) (or \( \sigma, \sigma' \)) may not couple to \( C_i \) directly. It can interact with \( C_i \) via couplings with some other fields \( Z_a \) having no-scale kinetic terms. In this case \( S \) will get the mass in two or larger number of loops.

Let us consider realizations of this idea in the context of the seesaw mechanism, when \( \sigma, \sigma' \) couple with right handed (RH) neutrinos \( N \). Let us introduce the following terms in the superpotential:

\[
W = \frac{m_D}{v_2} L N H^2 + \frac{M}{f_G} N N \sigma ,
\]

where we have omitted the generation indices. The first term in Eq. (8) produces the Dirac masses of neutrinos, whereas the second one gives the Majorana masses of RH neutrino components. The scale \( f_G \sim 10^{10} - 10^{12} \) GeV generates \( M \sim 10^{10} - 10^{11} \) GeV required by the HDM and atmospheric neutrinos.

(i) Suppose that only \( \sigma, \sigma', y \) superfields belong to the \( Z \)-sector, whereas all other superfields have minimal kinetic terms: \( N, H_2, L \in C \). Then SUSY breaking induces the soft term

\[
A_N \frac{M}{f_G} \tilde{N} \tilde{N} \sigma \]

which generates the mass of QGF in one loop (Fig. 1):

\[
m_S \simeq \frac{1}{16\pi^2} \left( \frac{M}{f_G} \right)^2 A_N .
\]

This mechanism is similar to that of the axino mass generation by coupling of \( S \) with heavy quarks [18, 22]. For \( A_N \sim \mathcal{O}(m_{3/2}) \) and \( (M/f_G) \sim 10^{-3} \), \( m_S \) is in the keV range.

(ii) Let us suppose that not only \( \sigma, \sigma', y \) but also \( N \) have the no-scale kinetic terms. In this case \( A_N = 0 \) at tree level, but non-zero \( A_N \) will be generated in one loop (see Fig. 2) by the soft breaking term related to usual Yukawa interaction \( L N H^2 \): \( A_D m_D^D \tilde{L} \tilde{N} H_2 \), and by the quartic coupling \( \sigma \tilde{N} \tilde{L}^* H_2^* \) which follows from \( |W_N|^2 \) term of the supersymmetric scalar potential. As the result one has

\[
A_N \sim \frac{1}{16\pi^2} \left( \frac{m_D}{v_2} \right)^2 A_D .
\]
Correspondingly, \( m_S \) appears in two loops (Fig. 2). Combining Eqs. (10) and (11) we get the estimation of \( m_S \):

\[
m_S \simeq \frac{1}{(16\pi^2)^2} \frac{A_D M^3}{v_2^2 f_G^2} m_\nu .
\]

Here \( m_\nu = (m^D)^2/M \). For the HDM mass scale \( m_\nu \simeq 3 \text{ eV}, A_D \simeq v_2 \simeq 100 \text{ GeV} \) and \( f_G \simeq 10^{12} \text{ GeV} \) it follows from Eq. (12) that \( m_S \simeq 3 \cdot 10^{-3} \text{ eV} \) can be achieved if the mass of RH component is \( M \simeq 10^9 \text{ GeV} \).

In this version of model the left and right neutrino components have different kinetic terms which may look unnatural.

(iii) Finally we consider the case where all chiral superfields belong to the Z-sector. This so-called strict no-scale model [23, 24] has only one seed of SUSY breakdown (i.e. gaugino mass). In this case \( A_D = 0 \) at tree level and non-zero \( A_D \) is generated in one loop by gaugino exchange. Correspondingly, \( m_S \) appears in three loops (Fig. 3) and its estimation can be written as

\[
m_S \simeq \frac{\alpha_2}{(4\pi)^3} \frac{m_{1/2} M^3}{v_2^2 f_G^2} m_\nu .
\]

Here \( \alpha_2 \) and \( m_{1/2} \) are the \( SU(2) \) fine structure constant and gaugino mass respectively. For \( m_\nu \simeq 3 \text{ eV}, m_{1/2} \simeq v_2 \simeq 100 \text{ GeV} \), and \( f_G \simeq 10^{12} \text{ GeV} \), one gets from Eq. (13) \( m_S \simeq 3 \cdot 10^{-3} \text{ eV} \) with a value of \( M \simeq 10^{10} \text{ GeV} \).

A contribution to the mass of the QGF can follow also from interactions, \( W_{\text{mixing}} \), which mix \( S \) with usual neutrinos (section 3).

### 3 Neutrino-QGF mixing

We now discuss possible ways which lead to mixing of the QGF with neutrinos. Such a mixing can occur only in the presence of either explicit or spontaneous violation of the \( R \) parity conventionally imposed in the MSSM [25]. Indeed, the Higgs field which breaks \( U(1)_G \) may belong either to \( R \) even or odd superfield depending upon the nature of the \( U(1)_G \). If it belongs to \( R \) even (i.e. Higgs like) superfield then the corresponding QGF is \( R \) odd and its mixing with neutrinos implies the \( R \)-violation. In contrast, if the QGF is \( R \) even, e.g. similar to the right-handed neutrino, then its scalar partner is \( R \) odd and the \( R \) symmetry gets broken together with the \( U(1)_G \) symmetry. The first alternative is realized when the \( U(1)_G \) is identified with the PQ symmetry. On the other hand, the lepton number symmetry containing right-handed
neutrino like superfield would provide an example of the second alternative. We discuss both these cases in turn.

1. *PQ symmetry.* The supersymmetric theories with Peccei-Quinn symmetry may contain a term

\[ \lambda H_1 H_2 \sigma, \]  

(14)

with \( \sigma \) being a superfield transforming non-trivially under the PQ symmetry. If the axionic superfield, \( S \), predominantly consists of the field \( \sigma \), the vacuum expectation value (VEV) \( \langle \sigma \rangle \sim f_{PQ} \) would be large \( \sim 10^{10} - 10^{12} \text{GeV} \). Since this VEV generates the parameter \( \mu = \lambda \langle \sigma \rangle \) of the MSSM through the interaction (14), one would need to fine tune \( \lambda \) in order to understand the smallness of \( \mu \). The coupling of axionic supermultiplet \( S \) to Higgs superfield is then given by

\[ W_{HS} = \frac{\mu}{f_{PQ}} H_1 H_2 S. \]  

(15)

The smallness of \( \mu \) can be understood if \( \sigma \) couples to Higgs through non-renormalizable term

\[ \lambda H_1 H_2 \frac{\sigma^2}{M_P}, \]  

(16)

where \( M_P \) is the Planck scale mass. In this case, \( \mu = \lambda \frac{\langle \sigma \rangle^2}{M_P} \) is naturally about the weak scale. Since \( f_{PQ} \approx \langle \sigma \rangle \), the axionic coupling following from Eq. (16) can be written as

\[ W_{HS} = 2 \frac{\mu}{f_{PQ}} H_1 H_2 S. \]  

(17)

Alternatively, the \( \sigma \) may acquire a small VEV \( \sim m_{3/2} \) and the scale of the PQ symmetry may be set by some other field which would predominantly contain the axionic multiplet. The \( \mu \)-parameter is naturally of the order \( m_{3/2} \) in this case. As long as the field \( \sigma \) transforms non-trivially under PQ symmetry, it will contain a small admixture \( \sim \langle \sigma \rangle / f_{PQ} \) of the axionic field \( S \). The interaction in Eq. (14) results in the following coupling

\[ W_{HS} \sim c_\mu \frac{\mu}{f_{PQ}} H_1 H_2 S, \]  

(18)

c\( \mu \) being \( \mathcal{O}(1) \).

It follows from Eqs. (15,17,18) that the axionic coupling to the Higgs superfield is insensitive to mechanism of implementation of the PQ symmetry. We can therefore consider the following
generic mixing term

\[ W_{\text{mixing}} = c_\mu \frac{\mu}{f_{PQ}} H_1 H_2 S + \mu H_1 H_2 + \epsilon L H_2. \]  

(19)

Here we also have included the explicit \( R \) violating coupling \( L H_2 \). The superpotential \((19)\) leads to the following mass matrix in the basis \((\nu, S, h_1, h_2)\):

\[
\begin{pmatrix}
0 & 0 & \epsilon \\
0 & m_S^0 & c\mu v \sin \beta/f_{PQ} & c\mu v \cos \beta/f_{PQ} \\
0 & c\mu v \sin \beta/f_{PQ} & 0 & \mu \\
\epsilon & c\mu v \cos \beta/f_{PQ} & \mu & 0 \\
\end{pmatrix},
\]  

(20)

where \( v \equiv \sqrt{v_1^2 + v_2^2} \) is the weak scale, \( \tan \beta \equiv v_2/v_1 \) and \( v_{1,2} \) are the VEV’s of \( H_{1,2} \). In matrix \((20)\) we have included also the direct axino mass \( m_S^0 \) that can be generated by the mechanisms of section 2. We have neglected the contribution from the interactions with the gauginos in Eq. \((19)\). In general gauginos mix with Higgsino through \( v_{1,2} \). This mixing will not change the qualitative results which follow from Eq. \((20)\). Moreover, the mixing can be small if the gaugino mass is chosen much larger than the \( \mu \)-parameter. Gauginos will also mix with neutrinos through the VEV of sneutrino field which may arise due to the presence of the \( \epsilon \) coupling in Eq. \((19)\) and soft SUSY breaking terms. This mixing generates \[26\] neutrino mass of order \( g^2 \langle \tilde{\nu} \rangle^2/m_{1/2} \) (\( g \) is the \( SU(2) \) coupling constant). For \( m_{1/2} > 100 \text{ GeV} \) and \( \langle \tilde{\nu} \rangle < 10 \) keV, this contribution is much smaller than \( m_S^0 \sim 10^{-3} \text{ eV} \) which can result from the radiative corrections.

Block diagonalization of the matrix \((20)\) leads to the following effective mass matrix for the neutrino and the axino, \((\nu, S)\):

\[
\begin{pmatrix}
0 & -c\nu \sin \beta/f_{PQ} \\
-c\nu \sin \beta/f_{PQ} & m_S^0 - c^2 \mu v^2 \sin 2\beta/f_{PQ}^2 \\
\end{pmatrix}.
\]  

(21)

If \( m_S^0 = 0 \) in Eq. \((21)\), the QGF mass, \( m_S = (2-3) \cdot 10^{-3} \text{ eV} \) can be obtained for the marginally allowed value of the PQ scale:

\[ f_{PQ} \approx v \sqrt{\frac{\mu \sin 2\beta}{m_S}} \lesssim 4 \cdot 10^9 \text{ GeV}. \]  

(22)

In this case, however, axions cannot provide the cold dark matter of the Universe. Note that the lightest supersymmetric particles cannot be cold dark matter either because of their instability due to the \( R \)-parity violation or due to their decay into the lighter axino. For
$f_{PQ} > 10^{10}$ GeV the QGF mass generated via $\mu$-term is too small for the MSW solution. For $f_{PQ} \sim 10^{11}$ GeV, $m_S \approx 10^{-5}$ eV is in the region of “just-so” solution of the solar neutrino problem. The axion can however serve as cold dark matter provided $f_{PQ} \sim 10^{12}$ GeV. In this case, the seesaw contribution to $m_S$ is very small and one needs a non-vanishing mass $m_0^S$.

If $m_0^S$ is the dominant contribution to the mass of $S$, $m_S \simeq m_0^S$, one obtains from Eq. (21) for the $\nu - S$ mixing

$$\tan \theta_{\nu s} \approx \frac{c_\mu \epsilon v \sin \beta}{m_0^S f_{PQ}} .$$

(23)

Then the desired value, $\tan \theta_{\nu s} \sim (2 - 6) \cdot 10^{-2}$ eV, can be obtained if the $R$ parity breaking parameter $\epsilon$ equals

$$\epsilon = \frac{m_0^S f_{PQ} \tan \theta_{\nu s}}{c_\mu v \sin \beta} \approx (2 - 6) \cdot 10^{-16} f_{PQ} / \sin \beta .$$

(24)

For $f_{PQ} \sim 10^{12}$ GeV one has $\epsilon \sim 0.1$ MeV. In general, the appropriate range of $\epsilon$ is $(10^{-3} - 10)$ MeV. It can be generated as a radiative correction: $\epsilon \sim h^2 m_{3/2}/16\pi^2$. Alternatively, $\epsilon$ may arise through the coupling of the product $LH_2$ to some fields carrying non zero lepton number. In this case the required smallness of $\epsilon$ may be understood in analogy with that of $\mu$-parameter.

2. Lepton number symmetry. Let us identify $U(1)_G$ with the lepton number symmetry. Unlike in the previous case, it is possible now to couple the QGF directly to neutrino through the term

$$hLH_2\sigma .$$

(25)

This is analogous to Eq. (14) but now the scalar component of $\sigma$ is $R$ odd and its VEV breaks $R$ parity. Electroweak symmetry breaking $v_2 \neq 0$ leads through the term (25) to the direct coupling between QGF and neutrino. Note that $\sigma$ is similar to the RH neutrino components. Just as the interaction in Eq. (14) generates the $\mu$, the interaction (25) generates the parameter $\epsilon$. Thus it is possible to correlate the origin of $\epsilon$ to the breaking of lepton number symmetry. The smallness of $\epsilon$ may be due to (i) fine tuning of $h$ or (ii) smallness of the VEV of $\sigma$ or due to (iii) occurrence of the non-renormalizable coupling analogous to that in Eq. (14). All these possibilities lead to the following effective coupling of $\nu$ to QGF:

$$W_{\text{mixing}} = c_L \frac{\epsilon}{f_L} LH_2 S + \epsilon LH_2 ,$$

(26)
where \( f_L \) denotes the scale associated with the spontaneous breaking of the lepton number symmetry and \( c_\epsilon \) is a parameter of order unity. The mass matrix generated by Eq. (26) is

\[
\begin{pmatrix}
0 & c_\epsilon \epsilon v \sin \beta / f_L \\
(1, -1, -3)
\end{pmatrix}
\begin{pmatrix}
c_\epsilon \epsilon v \sin \beta / f_L \\
(1, -1, -3)
\end{pmatrix}
\begin{pmatrix}
m_S^0 \\
0
\end{pmatrix}.
\]

(27)

and the desired \( \nu_e - S \) mixing can be obtained for \( \epsilon \simeq 0.1 \text{ MeV} \) and \( f_L \sim 10^{12} \text{ GeV} \).

Let us give an example of models which leads to the mixing term of Eq. (26). Consider the \( U(1)_L \) charge assignments \((1, -1, -3)\) for the fields \((\sigma, \sigma', L)\) respectively. All other fields are taken neutral. The relevant part for the \( U(1)_G \) invariant superpotential is given as follows:

\[
W = \lambda (\sigma \sigma' - f_L^2) y + \frac{\delta_\epsilon}{M_P^2} L H_2 \sigma^3,
\]

(28)

where the first term breaks the lepton symmetry and generates majoron supermultiplet of Eq. (3). The second term in Eq. (28) generates the effective interaction displayed in Eq. (26) with \( c_\epsilon = \frac{3}{\sqrt{2}} \) and \( \epsilon \sim \frac{\delta_\epsilon}{M_P^2} f_L^3 \). Thus specific choice for the lepton charges allows one to correlate \( \epsilon \) to the scale \( f_L \). In particular, for \( \delta_\epsilon \sim 0.1 \) and \( f_L \sim 10^{12} \text{ GeV} \), one has \( \epsilon \sim 1 \text{ MeV} \).

3. \textit{PQ as the lepton number symmetry.} If both Higgs and leptons transform non-trivially under the \( U(1)_G \) symmetry then the latter can play a dual role of the PQ symmetry and the lepton number symmetry as in Ref. [27]. In this case one can correlate the origin of \( \epsilon \) and \( \mu \) to the same symmetry breaking scale \( f_{PQ} \). The neutrino coupling to QGF is given by the combination of Eqs. (19) and (26):

\[
W_{\text{mixing}} = \mu H_1 H_2 + \epsilon L H_2 + c_\mu \frac{\mu}{f_{PQ}} H_1 H_2 S + c_\epsilon \frac{\epsilon}{f_{PQ}} L H_2 S.
\]

(29)

This \( W_{\text{mixing}} \) generates the following effective mass matrix for \( \nu \) and \( S \) which is the combination of Eq. (21) and Eq. (27):

\[
\begin{pmatrix}
0 & (c_\epsilon - c_\mu) \epsilon v \sin \beta / f_{PQ} \\
(c_\epsilon - c_\mu) \epsilon v \sin \beta / f_{PQ} & m_S^0 - c_\mu^2 \mu v^2 \sin 2\beta / f_{PQ}^2
\end{pmatrix}.
\]

(30)

According to Eq. (30) the \( \nu - S \) mixing angle \( \theta_{\nu S} \) is determined by

\[
\tan \theta_{\nu S} \sim \frac{(c_\mu - c_\epsilon) \epsilon v \sin \beta}{m_S^0 f_{PQ} - c_\mu^2 \mu v^2 \sin 2\beta / f_{PQ}^2}.
\]

(31)

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The $G$-charge prescription $(-1, -1, 1, -1, -2)$ for $(H_1, H_2, \sigma, \sigma', L)$ permits the following $U(1)_G$ invariant superpotential:

$$W = \lambda (\sigma \sigma' - f_{PQ}^2) y + \frac{\delta_\mu}{M_P} H_1 H_2 \sigma^2 + \frac{\delta_\epsilon}{M_P^2} L H_2 \sigma^3. \quad (32)$$

It gives the terms displayed in Eq. (29) with $c_\epsilon = \frac{3}{\sqrt 2}, c_\mu = \sqrt 2$.

4 Model

Let us put together the basic ingredients discussed in section 2 and 3 into a model which simultaneously explains the solar, atmospheric and the dark matter problems. In principle the sterile state, like axino, could mix with any of the neutrinos but the possibility of the $\nu_e - S$ mixing which solves the solar neutrino problem seems most preferred phenomenologically. The required range of the $\nu_e - S$ mixing and $S$ mass is given in Eq. (4). The alternative possibility of $\nu_\mu - S$ mixing accounting for the atmospheric neutrino deficit conflicts with the cosmological bound coming from the nucleosynthesis.

Let us consider the model with $U(1)_G = U(1)_{PQ}$ broken at $f_{PQ} \sim 10^{12}$ GeV in which the mass of QGF is generated in two or three loops via the interaction with the RH neutrino components (8) and the mixing is induced by the $L_e$-coupling described by the superpotential (32). To suppress the mixing of $S$ with $\nu_\mu, \tau$ and to get pseudo-Dirac structure for $\nu_\mu - \nu_\tau$ system (needed to explain simultaneously the HDM and the atmospheric neutrino problem), we suggest that $U(1)_G$ is generation dependent. Consider, for example, the following prescription of $U(1)_G$ charges:

$$H_1 \quad H_2 \quad \sigma \quad \sigma' \quad L_e \quad L_\mu \quad L_\tau \quad N_e \quad N_\mu \quad N_\tau,$$

$$-1 \quad -1 \quad 1 \quad -1 \quad -2 \quad -1/2 \quad 3/2 \quad 0 \quad 3/2 \quad -1/2. \quad (33)$$

This choice gives rise to the desired phenomenological results. Specifically,

- The mixing angle (31) following from the superpotential (32) can fall in the required range (4) if $\epsilon \sim 1$ MeV and $f_{PQ} \sim 10^{12}$ GeV.

- The above assignments lead to the following superpotential in the $\mu - \tau$ sector:

$$W = \sum_{\alpha = \mu, \tau} m^\alpha N_\alpha H_2 + \frac{M_{\tau}}{f_{PQ}} N_\tau N_\tau \sigma + \frac{M_{\mu \tau}}{f_{PQ}} N_\mu N_\tau \sigma'. \quad (33)$$

These couplings generate the axino mass $m^0_S$ in the MSW range as discussed in section 2.

\footnote{One can introduce for this an additional horizontal symmetry, suggesting that $U(1)_G$ is generation blind.}
The superpotential (33) leads to the mass matrix in $(\nu_\mu, \nu_\tau, N_\mu, N_\tau)$ basis:

$$
\mathcal{M} = \begin{pmatrix}
0 & 0 & m_D^\mu & 0 \\
0 & 0 & 0 & m_D^\tau \\
m_D^\mu & 0 & 0 & M_{\mu\tau} \\
0 & m_D^\tau & M_{\mu\tau} & M_\tau
\end{pmatrix}.
$$

The above mass matrix gives rise to pseudo-Dirac neutrino with a common mass

$$
m_{DM} \sim \frac{m_D^\mu m_D^\tau}{M_{\mu\tau}}.
$$

This mass can be in the eV range as required for the solution of the dark matter problem by taking the values $m_D^\mu \sim 0.1$ GeV, $m_D^\tau \sim 50$ GeV and $M_{\mu\tau} \sim 10^9$ GeV. The mass splitting is given by

$$
\frac{\Delta m^2}{m_{DM}^2} \approx 2 \left( \frac{m_D^\mu}{m_D^\tau} \right) \left( \frac{M_\tau}{M_{\mu\tau}} \right).
$$

Taking $\left( \frac{M}{m_{\mu\tau}} \right) \sim 1$, one reproduces both mixing and $\Delta m^2$ required to explain the atmospheric anomaly.

The charge prescription, $G(N_e) = 0$, permits the bare mass term $M N_e N_e$ or the non-renormalizable term $h N_e N_e \sigma^3/M_P$ which will produce $M_e \sim 10^6 - 10^{18}$ GeV. The Dirac mass term is generated by high-order non-renormalizable term: $h L_e N_e H_2^2 \sigma^3/M_P^3$, and therefore, $m_e^D \sim m_e (f_{PQ}/M_P)^3$ is negligibly small.

One can get more symmetric or regular charge prescription introducing more singlet fields or a horizontal symmetry in addition to $U(1)_G$.

The model presented above does not contain any mixing between $\nu_e$ and $\nu_{\mu,\tau}$. Such mixing can be induced, for example, by adding new Higgs field which could generate a Dirac mass term $m_{e\tau} \nu_e N_\tau$. This give rise to the $\nu_e - \nu_\mu$ mixing angle $\theta_{e\mu} \sim \frac{m_{e\tau}}{m_\mu}$ being in the range of sensitivity of KARMEN and LSND [4] for $m_{e\tau} \sim 30$ MeV, $m_\mu \sim$ GeV [6].

5 Conclusions

Simultaneous explanation of different neutrino anomalies hints to the existence of sterile neutrino. We have considered a possibility that the sterile neutrino is the quasi Goldstone fermion, which appears as the result of spontaneous breaking of a global $U(1)_G$ symmetry in supersymmetry theory. This global $U(1)_G$ symmetry can be identified with the PQ symmetry, the lepton number symmetry or the horizontal symmetry.
The mass of QGF generated by SUSY breaking can be as small as $10^{-3}$ eV so that $\nu_e \rightarrow S$ resonance conversion solves the solar neutrino problem. In the supergravity theories such a smallness of $m_S$ is related to special forms of superpotential and the scale of $U(1)_G$ breaking $f_G \sim 10^{16}$ GeV or to no-scale kinetic terms for certain superfields. In the last case, $m_S$ is generated in two or three loops.

The mixing of QGF with the neutrinos implies spontaneous or explicit violation of the $R$ parity. QGF can mix with neutrino via interaction with Higgs multiplets (in the case of PQ symmetry) or directly via coupling with the combination $LH_2$ (in the case of lepton number symmetry).

The $U(1)_G$-symmetry being generation dependent can simultaneously explain the dominance of QGF coupling with electron neutrino and pseudo-Dirac structure of $\nu_\mu - \nu_\tau$ system needed to explain the atmospheric neutrino problem and HDM.

The PQ breaking scale $f_{PQ} \sim 10^{10} - 10^{12}$ GeV determines several features of the model presented here. It provides simultaneous explanation of the parameters $\epsilon$ and $\mu$ and thus leads to small $R$-parity violation required in order to solve the solar neutrino problem in our approach. It also provides the intermediate scale for the RH neutrino masses which is required in order to solve the dark matter and the atmospheric neutrino problem. Finally, it controls the magnitude of the radiatively generated mass of the QGF and allows it to be in the range needed for the MSW solution of the solar neutrino problem. Thus the basic scenario presented here is able to correlate variety of phenomena.

If future solar neutrino experiments establish that the $\nu_e - S$ conversion is the cause of the solar neutrino deficit then one might be seeing indirect evidence for the PQ like symmetry or for that matter of SUSY itself.

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Fig. 1: One-loop diagram for the QGF mass. The solid lines are fermions and the dotted lines are bosons. $A_N$ is the soft parameter of $NN\sigma$. 
Fig. 2: Two-loop diagram for the QGF mass. $A_D$ is the soft parameter of $LNH_2$. 
Fig. 3: Three-loop diagram for the QGF mass. The cross with $m_{1/2}$ denotes gaugino mass insertion.