Comparative study on a straight and helical capillary tube for CO₂ transcritical system

P L Jadhav¹ and N Agrawal²
¹,²Department of Mechanical Engineering,
Dr. B. A. Technological University Lonere MS-402103, India
Email: ¹pravin19580@rediffmail.com

Abstract. A comparative study on an adiabatic straight and helical capillary tube employing homogenous flow conditions for a CO₂ refrigerant. The numerical model is based on the basic principles of conservation of mass, momentum, and energy. A result of the present model is validated with previously published test results. The properties of CO₂ refrigerants are obtained from property code CO2PROP. Compared to the straight capillary tube, the percentage reduction in mass flow rate for various internal diameter, tube length, and coil diameter are evaluated. It is observed that percentage reduction in mass flow rate increases as the coil diameter, tube diameter, and length of helical capillary tube increases. Again comparison between straight and helical tube is done by calculating tube length at same working conditions. Compared to straight capillary, for similar operating and geometric conditions, the percentage reduction in length for coil diameter 50mm is 16.4%, for coil diameter 100mm is 14.2% and for coil diameter 150mm is 13.5%. This comparative study is useful for designing of compact refrigeration system using CO₂ as a refrigerant.

1. Introduction
A CO₂ refrigerant is a naturally available refrigerant, having excellent environmental-friendly properties. This refrigerant having less global warming Potential and ozone Depleting Potential. Due to that, it gets reviewed after twentieth century [1]. The capillary tubes are widely used in the refrigeration systems up to 10 kW with the extended benefits; low cost, less starting torque, simple in operation. The capillary tube is widely used as the pressure reducing element in vapour compression system. This is long narrow tube having inner diameter ranging from 0.5 to 3m. The tube having length ranging from 1m to 6m. The capillary tube, further grouped as coiled and straight capillary tube. A helical capillary tube is extensively used in the compact refrigeration system. The majority of study has been presented on the capillary tubes with different refrigerants. Wang et al.[2] studied the numerical model for adiabatic helical capillary tube with CO₂ as a refrigerant. The indicated that the mass flow rates rise nearly by 7% as coil diameter increases from 40mm to 600mm. Jadhav and Agrawal [3] compared straight and spirally coiled capillary tube using CO₂ and R22 refrigerant. For nearly same operating situation, mass flow rate and length of the tube is significantly more in CO₂ refrigerant. The mass flow rate of CO₂ and R22 for the spiral capillary tube is about 22% and 15% lesser than straight capillary tube. Agrawal and Bhattacharya [4] investigated a numerical straight capillary tube model for CO₂ refrigerant.Influence of various geometric and operating factors on the performance of capillary tube was evaluated. Comparison between various friction factor correlations is carried out. Some of the researcher [5-7] carried out a numerical study for the adiabatic helical capillary tube. The mass flow rate increases as the helical coil diameter of the helical tube increases,
the coiling effect plays a vital role in the designing of the helical tube. Bansal and Wang [8] conducted a numerical study on an adiabatic straight capillary tube. The model developed for R22 refrigerant, a graphical simulation diagram is formed for choked flow conditions for R134a and R600a in adiabatic capillary tubes which was the useful design of the capillary tube Agrawal and Bhattachrayya[9] carried study on helical and straight capillary tube with CO₂ and R22 refrigerant. The data shows that the reduction in the mass flow rate with a helical capillary tube compared with a straight one is more pronounced in case of R744 in comparison to R22. Zhou and Zhang [10] conducted theoretical and experimental analysis on the helical capillary tube for R22 refrigerant. The result of helical capillary tube is compared with a straight capillary tube. As the coil diameter increases, the refrigerant mass flow rate increases significantly.

Relatively less work was found in the open literature on a comparative study of straight and coiled capillary tube using transcritical CO₂ system. The comparative study on helical and straight capillary tube are less discussed and need to discuss in detail. The objective of the present study is to compare straight and helical capillary tube, which may be used to design the compact refrigeration system.

2. Mathematical Model

The model moves around the fundamental principles of mass, momentum, and energy. The total capillary tube is discretized into small elements to capture the change in thermodynamic property and minimize the numerical error. In CO₂ transcritical system, the supercritical and transcritical phase is single phase while the subcritical phase is the two-phase region (as shown in Figure 1). The total capillary length of straight and helical is calculated by adding all discretized elemental lengths of various flow regions (as shown in figure 2 and 3). For simplicity, the mathematical model of a single-phase and two-phase is developed for both straight and helical capillary for the CO₂ refrigerant.

To make the model simple, certain less important phenomenon in a preview of the present study are being neglected with the following assumptions as:

- The cross-sectional area is constant
- Internal surface roughness is the same throughout the tube.
- The flow is one-dimensional flow and no heat transfer through the capillary tube.
- Flow is homogeneous and no metastable occurrence in the capillary tube flow.
- Entrance losses are negligible.

![Figure 1 Transcritical CO₂ cycle with various flow regions in the capillary tube](image)
Figure 2 Schematic diagram of an adiabatic straight capillary tube showing various flow regions and discretized two-phase flow region

Figure 3 Schematic diagram of an adiabatic coiled capillary tube showing various flow regions and discretized two-phase flow region

2.1 Single-phase flow region

Applying conservation of mass and energy for an element

\[ pAV = \text{Constant} \]  
\[ \frac{dh}{G^2} + \frac{dp}{2 \rho g^2} = 0 \]  

2.1.1. Straight Capillary tube. A momentum conservation equation for the elemental length is given as

\[ dL = \frac{2d}{f_{sp}} \left( \frac{v}{dv} - \frac{dp}{vG^2} \right) \]  

The Churchill [11] correlation is used to calculate the single-phase friction factor \( f_{sp} \) as
\[ f_{sp} = 8 \left( \frac{8}{Re} \right)^{12} + (A^{16} + B^{16})^{\frac{1}{12}} \]  

\[ (4) \]

Where \( A = 2.457 \ln \left( \frac{1}{2 \pi^2 + 0.274} \right) \), \( B = \frac{37530}{Re} \), \( Re = \frac{Gd}{\mu} \)

2.1.2 Helical Capillary tube. A momentum conservation equation for the elemental length is given as

\[ \frac{dL}{L} = \frac{2d}{f_{sp}} \left( \frac{v}{d} - \frac{dp}{vG^2} \right) \]  

\[ (5) \]

Model is developed using a single-phase and two-phase model with Mori and Nakamaya friction factor model [12].

\[ f_{sp} = C_1 \left( \frac{d}{D_c} \right)^{\frac{7}{12}} \left[ 1 + C_2 \left( \frac{Re}{D_c} \right)^{\frac{1}{12}} \right] \]  

\[ (6) \]

Where \( C_1 \) and \( C_2 \) are the constant coefficient in the equation which can be calculated as

\[ \ln C_1 = \frac{1}{n+1} \left\{ \frac{1}{4} \left[ -3 \ln(2n+1) + (16n-7) \ln(2n-1) - \frac{1}{(8n-5)}(\ln n + \ln(4n-1) + (6n-1)) + n \ln n \right] + 9 \ln 2 \right\} \]

\[ \ln C_2 = \frac{1}{n+1} \left\{ \frac{1}{4} \left[ 3 \ln(2n+1) - (15n+4) \ln n + (19n-4) \ln(2n-1) - (7n-4) \ln(4n-1) + n \ln(6n-1) - 9n \ln 2 \right] \right\} \]

Where the value of \( n \) is considered as 5 (for \( Re \geq 10^5 \)), and \( \alpha \) is the relative coefficient calculated using the general friction factor formula indicated as: \( f_{sp} = \alpha \frac{Re^{-\frac{n}{2}}}{\pi} \) where \( n \) is the exponent component in equation

2.2 Two-Phase Models

Similar to the single-phase, the fundamental principle of fluid mechanics and thermodynamics is applied for the two-phase region for the straight and helical capillary tube. The modeling of the two-phase region is analogous to the single-phase, except the computation of the quality of the refrigerant. The conservation of the momentum equation is written for straight capillary tube. The equation for the differential length is obtained as

\[ \frac{dL}{L} = \frac{2d}{f_{tpm}} \left( \frac{v_m}{d} - \frac{dp}{v_m G^2} \right) \]  

\[ (7) \]

Where, \( f_{tpm} \) and \( v_m \) are mean friction factor and mean specific volume of the two-phase flow region, respectively.
2.2.1. Straight capillary tube. Lin [13] friction factor and Churchill [12] are employed for a straight capillary tube to determine the two-phase friction factor:

$$f_{tpm} = \phi_{tp} f_{sp} \left( \frac{v_{sp}}{v_{tp}} \right)$$

(8)

Where

$$\phi_{tp} = \left( \frac{8}{Re_{tp}} \right)^{12} + \left( A_{tp}^{16} + B_{tp}^{16} \right)^{\frac{1}{12}} \left( 1 + x \left( \frac{\mu_g}{\mu_l} - 1 \right) \right)$$

$$A_{tp} = 2.457 \ln \left( \frac{1}{Re} \right)^{0.13} + \left( \frac{0.277 x}{D} \right) \quad B_{tp} = \frac{37530}{Re} \quad \text{and} \quad Re = \frac{GD}{\mu_{tp}}$$

2.2.2. Helical Capillary tube. The Mean Friction factor, $f_{tpm}$ may be determined by the Mori and Nakamaya friction factor model and two-phase viscosity have been employed from McAdams model correlation

$$\frac{1}{\mu_{tp}} = \frac{1-x}{\mu_l} + \frac{x}{\mu_g}$$

Where $\mu_g$, $\mu_l$, and $\mu_{tp}$ are the dynamic viscosities of saturated vapour, saturated liquid and two-phase, respectively. The secondary flow also noted as Dean Effect, control the transfer of heat, momentum, and mass in coiled capillary tubes. Dean had suggested a dimensionless number (Dean Number) as the ratio of the viscous force enacting on a fluid flowing in a curved pipe to the centrifugal force.

3. Solution Technique

The equations are coupled equations and solved by an iterative numerical method to calculate the capillary tube length. The inception of vaporization is captured by taking into account the temperature and enthalpy simultaneously, in the subcooled region. Thermodynamic and transport properties of CO$_2$ refrigerant are obtained from property code CO2PROP which is developed by employing an iterative procedure of derivatives of Helmholtz free energy function[14]. In designing of the capillary tube, the mass of the tube is considered and the total length of the capillary tube is calculated. By considering mass momentum and energy equation elemental length is calculated. Addition of these elemental lengths at particular phase region (single-phase or Two-phase) is the total length of the phase region. The total capillary length is the addition of the lengths at single-phase and phase regions. In simulation studies, the mass flow rate is guessed initially for given capillary tube length. Taking mass flow into consideration calculating the total length of the capillary tube. Compare the calculated length with the desired length. If it has the deviation then change the mass flow rate and repeated the same procedure until the desired length is equal to the calculated length. At that time we achieve the desired mass flow for a given length.

4. Result and Discussion

Flow-through straight and helical capillary tube are numerically simulated for transcritical CO$_2$ refrigeration cycles. For CO$_2$, the analysis is carried under unchoked flow conditions at gas cooler pressure 100bar, gas cooler temperature 313K and evaporative temperature 273K. The internal surface roughness of the capillary tube is taken as 0.0088mm. Capillary length is taken as 1.5m for determining the mass flow rate. The authenticity of the model is checked by comparing the test results available in the open literature. The present model of straight capillary is validated with experimental
results of Agrawal and Bhattacharyya. [8]. The agreement of the results is within the ±1% acceptable limit as shown in Figures 4.

As shown in figure 5, the present model of the helical capillary tube is validated with the test result of Wang et al.[9]. The agreement of the results is within an acceptable range of 0 to 2.5 %.

![Validation of the result of the present model for straight capillary tube with experimental results of Agrawal and Bhattacharyya](image)

**Figure 4** Validation of the result of the present model for straight capillary tube with experimental results of Agrawal and Bhattacharyya

![Validation of the helical capillary tube with test results of Wang et al.](image)

**Figure 5** Validation of the helical capillary tube with test results of Wang et al.

Figure 6 shows a change in mass flow rate with coil diameter for capillary tube. At $d=1.3\text{mm}$, $P_{gc}=100\text{bar}$, $T_{ev}=273\text{K}$, $L=1.5\text{m}$, and $\varepsilon=0.0088\times10^{-3}\text{mm}$ the mass flow rate of the straight capillary tube is constant value i.e. 0.0158kg/s, while the mass flow rate of helical capillary tube is varied from 0.0149 to 0.154kg/s as the coil diameter varied from 30mm to 150mm. As the coil diameter increases, the secondary flow through the tube decreases, which increases mass flow rate of...
the helical capillary tube. Compared to straight capillary tube, the percentage reduction in mass in the helical capillary tube is in the range of the 5.8% to 2.2% as the coil diameter increases from 30mm to 150mm.

Figure 6 Change in Mass flow rate with the coil diameter

Figure 7 shows a change in mass flow rate with internal tube diameter for capillary tube at \( P_{gc} = 100 \) bar, \( T_{ev} = 273 \text{K} \), \( L = 1.5 \text{m} \), \( \varepsilon = 0.0088 \times 10^{-3} \text{mm} \) for the straight and helical capillary tube with coil diameter, \( D_c = 50 \text{mm}, 100 \text{mm}, \text{and} 150 \text{mm} \).
As the tube diameter increases from the 1mm to 1.5mm, the mass flow rate of the straight capillary tube is increases from 0.00792 to 0.023 kg/s. Similarly mass flow rate of helical capillary tube increases with increase in tube diameter. As the tube diameter increases the restriction to flow reduces, which results in an increase in the mass flow rate. Compared to straight capillary tube, the percentage reduction in mass in the helical capillary tube with coil diameter 50mm,100mm, and 150mm are observed. The percentage reduction in mass flow rate for d= 1 to 1.5mm with Dc= 50mm is 2.65 to 4.96%, for Dc=100mm is 2.14% to 3.39% and for Dc=150mm is 1.5 to 2.5%. It is observed, as the coil diameter increases percentage reduction in the mass flow decreases due decreasing coiling effect.

Figure 8 shows change in mass flow rate with capillary tube length at Pgc= 100bar, Tev=273K, d=1.3mm and ε=0.0088×10⁻³mm for coil diameter, Dc= 50mm, 100mm, and 150mm. As the tube length increases from 1.3m to 1.8m, the mass flow rate of the straight capillary tube is decreases from 0.01689 to 0.01449 kg/s. Similarly mass flow rate of the helical capillary tube decreases with increasing length. Pressure in the tube decreases continuously as length increases due to increase in wall resistance, result in a decrease of the mass flow rate considerably. Compared to straight capillary tube, the percentage reduction in mass in the helical capillary tube with coil diameter 50mm,100mm, and 150mm are observed. The percentage reduction in mass flow rate for L= 1.3m to 1.8m with Dc= 50mm is 4.44% to 4.55%, for Dc=100mm is 2.84% to 2.96% and for Dc=150mm is 2.0% to 2.20%.

Figure 8 Change in mass flow rate with capillary tube length

Figure 9 shows a change in capillary tube length with the gas cooler temperature. Pgc= 100bar, Tev=273K, d=1.3mm and ε=0.0088×10⁻³mm for coil diameter, Dc= 50mm, 100mm, and 150mm. For similar pressure and temperature reduction in the capillary tube, the length of straight and helical capillary tube is calculated. The length required for straight capillary tube for above geometric and operating condition is 3.88m. Similarly, the length calculated for the helical capillary tube with coil diameter 50, 100, and 150mm is 3.24m, 3.33m, and 3.36m respectively. Compared to straight capillary, the percentage reduction in length for Dc= 50mm is 16.4%, for Dc=100mm is 14.2% and for Dc=150mm is 13.5%. As the coil diameter increases from 50mm to 150mm, the resistance to flow, due to secondary flow decreases thats why capillary tube length increases.
Conclusion

Numerical study on the adiabatic straight and helical capillary tube are carried out using homogenous two-phase model for CO2 refrigerant. The model is developed fundamental equations of mass, momentum, and energy equation. At same operating and geometric situation, straight and helical capillary tubes are compared, by calculating the mass flow rate. Compared to straight capillary tube, the percentage reduction in mass flow rate for \( d = 1 \) to 1.5mm with \( D_c = 50\)mm is 2.65 to 4.96\%, for \( D_c = 100\)mm is 2.14\% to 3.39\% and for \( D_c = 150\)mm is 1.5 to 2.5\%. Similarly the percentage reduction in mass flow rate for \( L = 1.3\)m to 1.8m with \( D_c = 50\)mm is 4.44\% to 4.55\%, for \( D_c = 100\)mm is 2.84\% to 2.96\% and for \( D_c = 150\)mm is 2.0\% to 2.20\%. Likewise, the percentage reduction in mass flow decreases as the coil diameter increases. As the coil diameter increases percentage reduction in the mass flow decreases due decreasing coiling effect. Again comparison between straight and helical tube is done by calculating tube length at same working conditions. Compared to straight capillary, the percentage reduction in length for \( D_c = 50\)mm is 16.4\%, for \( D_c = 100\)mm is 14.2\% and for \( D_c = 150\)mm is 13.5\%. This comparative study is useful for designing of compact refrigeration system using CO2 as a refrigerant.

Figure 9  change gas cooler temperature with Capillary tube length
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## Nomenclature

| Symbol | Description |
|--------|-------------|
| $A$ | Cross-sectional area of capillary tube, [m²] |
| $C_p$ | Specific heat, [kJ kg⁻¹ K⁻¹] |
| $d$ | Capillary tube diameter, [mm] |
| $D_c$ | Coil diameter, [mm] |
| $De$ | Dean Number, $De = Re\sqrt{d/\overline{D}}$ [-] |
| $f$ | Friction factor, [-] |
| $f_c$ | Friction factor coiled capillary tube, [-] |
| $f_s$ | Friction factor straight capillary tube, [-] |
| $G$ | Mass flux, [kgm⁻² s⁻¹] |
| $h$ | Specific enthalpy, [kJ kg⁻¹] |
| $L$ | Capillary tube length,[m] |
| $m$ | Mass flow rate, [kgs⁻¹] |
| $p$ | Pressure, [bar] |
| $P$ | Pitch, [mm] |
| $Re$ | Reynolds number, $Re = \rho V d/\mu$ [-] |
| $R_e$ | Radius of curvature, [mm] |
| $T$ | Temperature,[K] |
| $V$ | Velocity, [ms⁻¹] |
| $x$ | Dryness fraction [-] |

| Subscripts | Description |
|------------|-------------|
| $f_c$ | Friction factor coiled capillary tube, [-] |
| $f_s$ | Friction factor straight capillary tube, [-] |
| $C$ | Capillary |
| $e$ | Evaporator |
| $g$ | Saturated vapour |
| $g_c$ | Gas cooler |
| $h_{sp}$ | Helical tube with single phase |
| $htpm$ | Mean of the two-phase helical tube |
| $i$ | Element |
| $k$ | Condenser |
| $l$ | Saturated liquid |
| $t$ | Two-phase |

## Greek Symbol

| Symbol | Description |
|--------|-------------|
| $\Delta T_{sub}$ | Degree of subcooling, [K] |
| $\epsilon$ | Internal surface roughness, [mm] |
| $\mu$ | Dynamic viscosity, [Pa s] |