Complications in the interpretation of the charge asymmetry dependent π flow for the chiral magnetic wave

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The charge asymmetry ($A_{ch}$) dependence of the $\pi^-$ and $\pi^+$ elliptic flow difference, $\Delta v_2(A_{ch})$, has been regarded as a sensitive observable for the possible chiral magnetic wave (CMW) in relativistic heavy ion collisions. In this work, we first demonstrate that, due to non-flow backgrounds, the flow measurements by the Q-cumulant method using all charged particles as reference introduce a trivial linear term to $\Delta v_2(A_{ch})$. The trivial slope can be negative in the triangle flow difference $\Delta v_2(A_{ch})$ if the non-flow is dominated by back-to-back pairs. After eliminating the trivial term, we find that the non-flow between like-sign pairs gives rise to an additional positive slope to $\Delta v_2(A_{ch})$ because of the larger dilution effect to $\pi^+$ ($\pi^-$) at positive (negative) $A_{ch}$. We further find that the competition between different $\pi$ sources can introduce another non-trivial linear-$A_{ch}$ term due to their different multiplicity fluctuations and anisotropic flows. We then study the effect of neutral cluster (resonance) decays as a mechanism for local charge conservation on the slope parameter of $\Delta v_2(A_{ch})$. We find that the slope parameter is sensitive to the kinematics of those neutral clusters. Light resonances give positive slopes while heavy resonances give negative slopes. Local charge conservation from continuum cluster mass distribution can give a positive slope parameter comparable to experimental data. Our studies indicate that many non-CMW physics mechanisms can give rise to a $A_{ch}$-dependent $\Delta v_2(A_{ch})$ and the interpretation of $\Delta v_2(A_{ch})$ in terms of the CMW is delicate.

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I. INTRODUCTION

One of the most important phenomena observed in high-energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) is the strong collective flow of final state charged hadrons [1–5]. This is developed, presumably, by fast expansion of the quark gluon plasma (QGP) created in those collisions [6, 7]. The flow harmonics, such as elliptic flow ($v_2$) and triangular flow ($v_3$), are widely used to study and extract the transport properties of the QGP [8, 12]. One complication in those studies is the non-flow contaminations in the $v_2$ measurements [13–19]. These non-flow correlations can be caused by resonance decays, jet emission, etc. At low transverse momentum ($p_T$), however, non-flow contaminations in flow measurements are generally small.

Recently, pion elliptic flows have been proposed as a tool to search for a phenomenon called the chiral magnetic wave (CMW) [20, 21], which is related to the chiral anomaly and strong magnetic field in heavy ion collisions [20, 32]. The CMW is a gapless collective excitation of the QGP stemming from the interplay of the chiral magnetic and chiral separation effects [20, 24, 26]. The CMW could introduce an electric quadrupole moment, giving opposite contributions to the $v_2$’s of positive and negative pions [21]. These contributions would depend on

$$A_{ch} = \frac{N_+ - N_-}{N_+ + N_-},$$

(1)

where $N^+$ ($N^-$) is the multiplicity of positive (negative) charged hadrons in a given event. Namely, the CMW would cause the pion flows to linearly dependent on $A_{ch}$,

$$v_2^{\pi^{\pm}} = v_2^{\text{base}} + \frac{r(\pi^{\pm})}{2} A_{ch},$$

(2)

with opposite sign slope parameters for $\pi^+$ and $\pi^-$. The CMW-sensitive slope parameter ($r$) can be extracted from the $A_{ch}$ dependent pion flow difference

$$\Delta v_2(A_{ch}) = v_2^{\pi^-} - v_2^{\pi^+}.$$  

(3)

Experimental measurements by the STAR, ALICE and CMS collaborations qualitatively agree with the expectation from the CMW [23, 35]. Firm conclusions, however, have not been reached because it is generally perceived that non-CMW mechanisms can also generate $A_{ch}$-dependent $\pi$ flows, e.g., the Local Charge Conservation (LCC) [36] and the effect of isospin chemical potential [37]. Some of them are of non-flow nature, such as the LCC, but the effect of LCC on $\Delta v_2(A_{ch})$ is not coming from non-flow. The key reason for the LCC effect on $\Delta v_2(A_{ch})$ is that the charge asymmetry is more effectively affected by the decay products of neutral clusters with lower $p_T$, resulting in a smaller $\pi^+(\pi^-) v_2$ at positive (negative) $A_{ch}$ [36].
Non-flow contributions to $\Delta v_2(A_{ch})$ have not been thoroughly discussed in literature. Non-flow contributions may be small in the $\pi$ anisotropic flow measurements themselves, but could be important in the small difference between $\pi^+$ and $\pi^-$ flows. We will demonstrate in this paper that the non-flow correlations can give both trivial and non-trivial contributions to the slope parameters of $\Delta v_2(A_{ch})$ in Sec. [III]. We will then discuss the effect of multiple pion sources with different flows and contributions to the charge asymmetry in Sec. [IV].

We will revisit non-flow correlations in Sec. [V] and show that non-flow correlations can give further, non-trivial contributions to the slope parameters. We will further show in Sec. [VI] that the effect of LCC may not be as simple as it appears. It can give rise to both positive and negative slopes, depending on the details of the LCC physics.

We will illustrate that continuum mass distributions of charge conserving clusters may play an important role in $\Delta v_2(A_{ch})$.

II. TRIVIAL LINEAR TERM IN $v_n(A_{ch})$

The Q-cumulant method was widely used to calculate anisotropic flow observables [33–41]. The method is based on the $Q_n$-vector defined as

$$Q_n = \sum_{i=1}^{M} e^{in\varphi_i},$$

where $M$ is the multiplicity, $\varphi_i$ is the azimuthal angle of the emitted particle $i$, and $n$ is the harmonic order. Consider the case where there is no overlap between the particle of interest (POI) and the reference particle (REF), as in the case of our study, the two-particle cumulant can be calculated by

$$d_n\{2\} = \left\langle \frac{q_n}{mM} \right\rangle,$$

where $(m, q_n)$ and $(M, Q_n)$ are the (multiplicity, Q-vector) of POI and REF, respectively. Here and hereafter, $\left\langle \right\rangle$ denotes average over events. To reduce short range non-flow correlations in experimental studies, the sub-event method is often used with a minimum pseudorapidity gap $\Delta \eta$ applied between the POI and the REF. Following this common practice, we apply a gap of $\Delta \eta = 0.6$ in our phenomenological study here, although non-flow is known in our study and a $\Delta \eta$ gap may not be necessary to remove it. With all charged hadrons as REF, the anisotropic flow of $\pi^\pm$ can be written as

$$v_n^{\pi\pm}\{2\} = \frac{d_n\{2; \pi^\pm\-REF\}}{\sqrt{c_n\{2\}}}.$$  

Here we use $d_n\{2; A-B\}$ to represent the two-particle cumulant $d_n\{2\}$ between A (e.g. POI) and B (e.g. REF). $c_n\{2\} = d_n\{2; REF-REF\}$ is that of the REF pairs, and $\sqrt{c_n\{2\}}$ is the reference flow.

We now demonstrate that, if including all charged particles (denoted as $h$) as REF, the above method introduces a trivial term linear in $A_{ch}$. Using $M = N_+ + N_-$ and $Q_n = Q_n^+ + Q_n^-$, where $N_+$ and $Q_n^+$ are the multiplicities and Q-vectors of positive/negative particles, the two-particle cumulant $d_n\{2; \pi^\pm h\}$ [40] can be rewritten as

$$d_n\{2; \pi^\pm h\} = \left\langle \frac{q_n^{\pi^\pm} Q_n^*}{mM} \right\rangle$$

$$= \left\langle \frac{N_+ q_n^{\pi^\pm} Q_n^*}{mN_+} \right\rangle + \left\langle \frac{N_- q_n^{\pi^\pm} Q_n^-}{mN_-} \right\rangle$$

$$= \frac{d_n\{2; \pi^\pm h^+\} + d_n\{2; \pi^\pm h^-\}}{2}$$

$$+ \frac{d_n\{2; \pi^\pm h^+\} - d_n\{2; \pi^\pm h^-\}}{2} A_{ch},$$

where we have used the fact that the event average ($\left\langle \right\rangle$) is achieved in a given $A_{ch}$ interval. A trivial term appears, which is proportional to $A_{ch}$ (the second term on the r.h.s of Eq. (7)). If the correlations are from flow only, then $d_n\{2; \pi^\pm h^+\} = d_n\{2; \pi^\pm h^-\}$, and the trivial term vanishes. However, non-flow is present in experimental data and differs between like-sign and unlike-sign pairs, so the trivial term is finite. It is opposite in sign for $\pi^+$ and $\pi^-$. The trivial term arises when all charged particles are included in REF and the non-flow differs between like-sign and unlike-sign pairs. The slope of the trivial term, dubbed the trivial slope $r_{triv}$, reads,

$$r_{triv}(\pi^\pm) = \frac{d_n\{2; \pi^\pm h^+\} - d_n\{2; \pi^\pm h^-\}}{2\sqrt{c_n\{2\}}}.$$  

The slope parameter without removing the trivial term is denoted as $r_0$ in this paper.

We use a toy Monte Carlo (MC) model to illustrate the trivial term. We generate $\pi^+$ and $\pi^-$ with Poisson multiplicity fluctuations in each event. The $p_T$ of $\pi$ mesons correspond to the measured data in the 30-40% centrality of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [42, 43]. The $\eta$ spectra is parameterized as [44]

$$\frac{dN_{ch}}{d\eta} \propto \sqrt{1 - 1/\left(\alpha \cosh(\eta)\right)} + \frac{1}{1 + \exp((|\eta| - \beta)/a)},$$

with $\alpha = 2.74$, $\beta = 3.92$ and $a = 0.88$ for the centrality of interest. The mean multiplicity of charged hadrons is 380 in $|\eta| < 1$ with $p_T > 0.15$ GeV/c. To introduce a non-flow correlation difference between like-sign and unlike-sign pairs, we force, on average, 20% of the multiplicity of multiple pion sources with different flows and contributions to the charge asymmetry in Sec. III. We will revisit the case where there is no overlap between the particle of interest and the reference particle in Sec. V and show that non-flow correlations can give further, non-trivial contributions to the slope parameters. We will further show in Sec. VI that the effect of LCC may not be as simple as it appears. It can give rise to both positive and negative slopes, depending on the details of the LCC physics. We will illustrate that continuum mass distributions of charge conserving clusters may play an important role in $\Delta v_2(A_{ch})$. 

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$$d_n\{2; \pi^\pm h\} = \left\langle \frac{q_n^{\pi^\pm} Q_n^*}{mM} \right\rangle$$

$$= \left\langle \frac{N_+ q_n^{\pi^\pm} Q_n^+}{mN_+} \right\rangle + \left\langle \frac{N_- q_n^{\pi^\pm} Q_n^-}{mN_-} \right\rangle$$

$$= \frac{d_n\{2; \pi^\pm h^+\} + d_n\{2; \pi^\pm h^-\}}{2}$$

$$+ \frac{d_n\{2; \pi^\pm h^+\} - d_n\{2; \pi^\pm h^-\}}{2} A_{ch},$$

where we have used the fact that the event average ($\left\langle \right\rangle$) is achieved in a given $A_{ch}$ interval. A trivial term appears, which is proportional to $A_{ch}$ (the second term on the r.h.s of Eq. (7)). If the correlations are from flow only, then $d_n\{2; \pi^\pm h^+\} = d_n\{2; \pi^\pm h^-\}$, and the trivial term vanishes. However, non-flow is present in experimental data and differs between like-sign and unlike-sign pairs, so the trivial term is finite. It is opposite in sign for $\pi^+$ and $\pi^-$. The trivial term arises when all charged particles are included in REF and the non-flow differs between like-sign and unlike-sign pairs. The slope of the trivial term, dubbed the trivial slope $r_{triv}$, reads,

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Figure [1.a] shows the two-particle cumulant of $\pi^+$ from like-sign ($h^+$ as REF), unlike-sign ($h^-$ as REF) and all charged hadrons (all charge as REF) correlations.
The like-sign $d_2\{2; \pi^+ h^+\}$ is simply $v_2^2$. The unlike-sign $d_2\{2; \pi^+ h^-\}$ is larger because of the input non-flow of the $\pi^+\pi^-$ pairs. The elliptic flow of $\pi^\pm$ obtained from the $d_2$ with all charge REF (black points in Fig. 1(a) for $\pi^+$ POI) are shown as function of $A_{ch}$ in Fig. 1(b). The magnitudes are larger than the input value of 4% because of the input non-flow correlations. These non-flow backgrounds contribute trivial linear-$A_{ch}$ terms to $v_2^{\pi^+}(A_{ch})$ and $v_2^{\pi^-}(A_{ch})$, with opposite-sign slopes as shown in Fig. 1(b). With the input non-flow, we obtain a positive $r_{\text{triv}} = 2.28\%$ by Eq. (8). This is reflected in the fitted CMW-sensitive slope parameter $r_0$, shown in Fig. 1(c).

Non-flow differences are present between like-sign and unlike-sign pairs in experiment, and not much can be done to eliminate this non-flow difference. In order to eliminate the trivial linear $A_{ch}$ term, one is left with only one option, i.e., to use hadrons of a single charge sign instead of all charged hadrons as REF. One can use positive and negative particles as REF separately to obtain $v_n^{\pi^+}\{2; h^+\}$ and $v_n^{\pi^-}\{2; h^-\}$. One can then take the average

$$\bar{v}_n^A = \frac{v_n^{\pi^+}\{2; h^+\} + v_n^{\pi^-}\{2; h^-\}}{2}. \quad (10)$$

The $\Delta v_2(A_{ch})$ obtained using this technique is shown in Fig. 1(c) by the red stars. Indeed, the slope is zero as expected because there is no other physics in our toy model that would introduce a non-zero slope.

The Eq. (7) algebra holds for all orders of azimuthal harmonics. The trivial $A_{ch}$ term also exists in higher order flow harmonics, e.g., the $A_{ch}$-dependent $\pi$ triangle flow $v_3$. To illustrate this, we use the same MC model to generate the azimuth angle using input $v_3 = 4\%$. The
results are shown in Fig. 1(d), and are consistent with those for \( v_2 \). The underlying reason here is identical for \( v_2 \) and \( v_3 \).

The STAR preliminary results indicate a negative slope for \( \Delta v_3(A_{ch}) \) in central and peripheral collisions \([15]\). The slope in Fig. 1(d) due to close pairs is positive. A negative trivial slope can easily arise from back-to-back pairs because their effect on \( v_3 \) is negative while that of close pairs is positive. We illustrate this using the same toy model but force the pairs to emit back-to-back. The results are shown in Fig. 2. There is no difference between back-to-back pairs and close pairs for \( v_2 \) because they are symmetric with respect to the second harmonic (see Fig. 2(a) vis-a-vis Fig. 1(c)). While for \( v_3 \), the back-to-back pairs contribute a negative trivial slope as shown in Fig. 2(b), in contrast to that from close pairs in Fig. 1(d).

The trivial term has been removed in \( r \), obtained by flow measurements via Eq. (10). It is interesting to note, however, that the \( r \) slope for \( \Delta v_3(A_{ch}) \) is non-zero for back-to-back non-flow pairs, as shown in Fig. 2(b) by the red stars. This is in contrast to the \( r \) slopes for \( \Delta v_2(A_{ch}) \) and that of \( \Delta v_3(A_{ch}) \) using close pairs. The non-zero slope is at first glance hard to understand. The reason is due to the competition between two sources of pions, the paired pions and unpaired pions, which have effectively different final \( v_3 \) values, although both are generated by the same input \( v_3 \). This is not the case for \( v_2 \) and \( v_3 \) with the close-pair non-flow. We will discuss this point in the next section.

To summarize this section, a trivial linear \( A_{ch} \) term arises if the following two conditions are both met: (1) there exists non-flow difference between like-sign and unlike-sign pairs (which is generally true); and (2) both charged-sign hadrons are used as REF to calculate flow harmonics, which automatically introduces a \( A_{ch} \) dependence. Based on our toy model study, we found that for close pair unlike-sign non-flow, the trivial term introduces a positive slope to \( \Delta v_3(A_{ch}) \) for both \( n = 2, 3 \). For back-to-back pair unlike-sign non-flow, the trivial slope is positive for \( \Delta v_2(A_{ch}) \) and negative for \( \Delta v_3(A_{ch}) \).

### III. LINEAR \( A_{ch} \) TERM DUE TO MULTIPLE PION SOURCES

The magnitude of the slope parameter extracted from the \( \Delta v_n(A_{ch}) \) depends on the \( A_{ch} \) distribution: a narrower \( A_{ch} \) distribution will give a larger slope parameter. In heavy ion collisions, particle sources with different physics mechanisms can have different event-by-event fluctuations and thus different \( A_{ch} \) distributions. For example, the trivial statistical distribution of net-charges is a Skellam distribution, while the net-charge distribution with exact charge conservation is a Delta distribution.

Using a two-component model, i.e. primordial pions (denoted by subscript ‘P’ in the following) and pions from resonance decays (denoted by ‘D’), we show that the multiple pion sources introduce a natural \( A_{ch} \) dependence in \( \Delta v_n(A_{ch}) \), if these pion sources are different in both the \( A_{ch} \) distributions and the pion \( v_2 \) magnitudes. From the two-component model, we have

\[
v_{n,\pm} = \frac{N_{P,\pm} v_{n,P,\pm} + N_{D,\pm} v_{n,D,\pm}}{N_{D,\pm} + N_{P,\pm}},
\]

\[
A_{ch} = \frac{A_P + \epsilon A_D}{1 + \epsilon},
\]

\[
\epsilon = \frac{N_D + N_{D,\pm}}{N_P + N_{P,\pm}},
\]

where \( N_{P,\pm} \) and \( v_{n,P,\pm} \) are the multiplicity and elliptic flow of primordial (decay) \( \pi^\pm \), \( A_P = (N_{P,\pm} - N_{P,D,\pm})/(N_{P,D,\pm} + N_{P,D,\mp}) \), and \( A_D = (N_{D,\pm} - N_{D,D,\mp})/(N_{D,D,\pm} + N_{D,D,\mp}) \).

We first assume \( v_{n,P/D} = v_{n,P/D,\pm} = v_{n,P/D,\mp} \) and are independent of charge asymmetry. Then we have

\[
\Delta v_n = \frac{2\epsilon}{(1 + \epsilon)^2(1 - A_{ch}^2)}(A_D - A_P)(v_{n,P} - v_{n,D})
\]

\[
\simeq \frac{2\epsilon}{(1 + \epsilon)^2(1 - A_{ch}^2)}(A_D - A_P)(v_{n,P} - v_{n,D}).
\]

For the sake of simplicity, we assume the event-by-event distributions of \( A_P \) and \( A_D \) are both normal distributions, i.e. \( N(\mu_P, \sigma_P^2) \) and \( N(\mu_D, \sigma_D^2) \). Then the event-by-event distribution of \( A_{ch} \) is also a normal distribution with \( \mu_A = (\mu_P + \mu_D)/2 \) and \( \sigma_A^2 = (\sigma_P^2 \epsilon^2 + \sigma_D^2 \epsilon^2)/4 \). For a charge-neutral system, we have \( \mu_A = \mu_P = \mu_D = 0 \). Then from Eq. (14) we arrive at

\[
\Delta v_n = \frac{2\epsilon(\sigma_P^2 - \sigma_D^2)(v_{n,P} - v_{n,D})}{(1 + \epsilon)(\epsilon^2 \sigma_P^2 + \sigma_D^2)}A_{ch} \equiv r^{2C}A_{ch}.
\]

The slope \( r^{2C} \) from the two-component (2C) model is clearly non-zero if \( \sigma_P^2 \neq \sigma_D^2 \) and \( v_{n,P} \neq v_{n,D} \). In more general cases, the linear-A_{ch} term is also present in \( v_{n,P,\pm} \) and \( v_{n,D,\pm} \), i.e.,

\[
v_{n,P/D,\pm} = v_{n,P/D,\pm} \pm \frac{r_{P/D}}{2} A_{P/D}.
\]

In this case, \( v_{n,P/D} = (v_{n,P/D,\pm} + v_{n,P/D,\mp})/2 \) and

\[
\Delta v_n \simeq \frac{c^2 \sigma_P^2 r_{P/D}}{c^2 \sigma_P^2 + \sigma_D^2} + \frac{c^2 \sigma_D^2 r_{P/D}}{c^2 \sigma_D^2 + \sigma_P^2} + r^{2C}A_{ch}.
\]

Therefore, besides the linear-A_{ch} term from \( v_{n,P,\pm} \) and \( v_{n,D,\pm} \) themselves, the competition between two pion sources introduces another linear-A_{ch} term. The root reason is that the relative fractions of pions from different sources depend on the event-by-event \( A_{ch} \) value (because they contribute to \( A_{ch} \) differently), therefore the average \( v_2 \) from multiple sources, which have different \( v_2 \)’s, will depend on \( A_{ch} \).

We have used two “flow” sources in the above derivation. However, this also applies to the competition between flow and non-flow contributions to the observed \( \Delta v_n(A_{ch}) \). This is the reason for the non-zero slope in
Fig. 2(b) obtained from back-to-back pairs, because the eliminated slope is still negative.

Fig. 2: (Color online) Similar to Fig. 1 but the non-flow is simulated by back-to-back (B2B) unlike-sign pairs. (a) The effect of π like-sign pairs instead of unlike-sign pairs. We force 20% rameter. We modify our non-flow toy model to generate non-flow correlations still introduce a nonzero slope parameter. However, as we will show now, the like-sign π asymmetry: when more π are counted resulting in a positive A_{ch}, the π^+π^+ non-flow correlation is more diluted while the π^−π^− non-flow is less diluted. This is different in the unlike-sign correlations discussed before, because in that case, the dilution depends on the number of π^+π^− pairs. The dilution effect for unlike-sign non-flow is almost identical for π^+ and π^−.

IV. NON-FLOW BACKGROUND BEYOND THE TRIVIAL CONTRIBUTION

After addressing the trivial term by using REF of single charge sign, we now focus on the trivial slope eliminated results. In the rest of the paper, we only discuss the physical slope parameter r.

As shown in Sec. III, the physical slope parameter r of Δv_2(A_{ch}) is not affected by unlike-sign non-flow correlations. However, as we will show now, the like-sign non-flow correlations still introduce a nonzero slope parameter. We modify our non-flow toy model to generate like-sign pairs instead of unlike-sign pairs. We force 20% of π^+ (and π^-) to be paired in π^+π^+ (and π^-π^-) with the same azimuth. The results are shown in Fig. 3. The Δv_2(A_{ch}) has a positive slope r = 1.63%. This is due to the dilution effect: when more π^+ are counted resulting in a positive A_{ch}, the π^+π^+ non-flow correlation is more diluted while the π^-π^- non-flow is less diluted. This is different in the unlike-sign correlations discussed before, because in that case, the dilution depends on the number of π^+π^- pairs. The dilution effect for unlike-sign non-flow is almost identical for π^+ and π^-.

V. EFFECT OF DECAY KINEMATICS

As we have mentioned in the introduction, local charge conservation can also yield a nonzero slope of Δv_n(A_{ch}). Effect of the LCC mechanism on Δv_n(A_{ch}) comes from the resulting pion correlations and the p_T-dependent pion flow. Specifically, the charge asymmetry is more affected by neutral clusters with lower p_T, and thus smaller v_2. A lower p_T parent resonance has a larger decay opening angle. The decay daughters have a larger probability to cross acceptance boundaries to cause a finite A_{ch}, and those decay daughters have smaller v_2. Moreover, if the parent v_2 decreases with increasing |η|, the effect would be even stronger.

In principle, the trivial linear-A_{ch} term discussed in
Figure 4: (Color online) (a) The \( \pi \) elliptic flow differences \( \Delta v_2(A_{ch}) \) from a single source of \( \rho^0 \), \( K_s \), or \( \omega \) decays with fixed \( p_T = \langle p_T \rangle \) of the parent; The azimuth angle correlation between the parents (b) \( \rho^0 \), (c) \( K_s \), (d) \( \omega \) and their decay daughter pions. The elliptic flows are calculated from like-sign correlations. The acceptance cut for pions is \( 0.15 < p_T < 0.5 \text{ GeV/c} \) and \( |\eta| < 1 \).

Sec. [11] can be eliminated by using any kind of single-sign reference, but the non-flow backgrounds could be different. The resonance decays only contribute the unlike-sign non-flow correlations. To simplify our discussion of LCC, in the rest of this paper, the elliptic flow is calculated from the like-sign correlations to avoid the non-flow contamination. Namely, the slope parameter \( r \) is then extracted from

\[
\Delta v_n(A_{ch}) \equiv v_n^-\{2; h^-\} - v_n^+\{2; h^+\}.
\]  

(18)

In this section, we use a MC model to study the effect of resonance decays on \( \Delta v_2(A_{ch}) \). We first study the effect of resonance decays with the parent resonances at fixed \( p_T = \langle p_T \rangle \) (see below the related \( p_T \) spectra) and a pseudorapidity independent resonance flow, so that the aforementioned \( p_T \)-dependent LCC effect is absent. The averaged multiplicity of total charges in the STAR acceptance \( (p_T > 0.15 \text{ GeV/c}, |\eta| < 1.0) \) is about 380 with the \( \eta \) spectra given in Eq. [9]. The \( v_2 \) is kept uniform in \( \eta \). The slope \( r \) of \( \Delta v_2(A_{ch}) \) from \( \rho^0 \rightarrow \pi^+\pi^- \), \( K_s \rightarrow \pi^+\pi^- \) [17], and \( \omega \rightarrow \pi^+\pi^0\pi^- \) are shown in Fig. 4(a) individually. The slopes are all negative. This is because of the following. For resonances within the acceptance, those decay products closer to the parent (smaller angular difference from the resonance) are more likely to stay within the acceptance. For resonances outside the acceptance, the decay daughters further away more likely cross into the acceptance. Because of the nonlinear mapping between the decay angle and the acceptance gauge of \( \eta \), the net effect is that the daughter particles that are accepted within a given \( \eta \) window have, on average, smaller angle from the parent than those outside the acceptance. This results in a less angular smearing of the daughter particle flow. This is shown in Fig. 4(b-d) by the azimuthal angle correlations between parent (\( \rho^0 \), \( K_s \), \( \omega \)) and its decay pions as a function of \( A_{ch} \). At positive \( A_{ch} \), more \( \pi^+ \) are within the acceptance and they have a smaller average opening angle from the parent. The excess pions due to
Figure 5: (Color online) The π elliptic flow differences $\Delta v_2(A_{ch})$ from (a) a single source of $\rho^0$, $K_s$, or $\omega$ decays, and (b) multiple pion sources, primordial pions and decay pions from $K^0, \rho^0, \rho^\pm, \omega, \eta$, with the primordial to decay pion multiplicity ratio within acceptance of 0.94. The elliptic flows are calculated from like-sign correlations. The acceptance cut for pions is $0.15 < p_T < 0.5$ GeV/c and $|\eta| < 1$.

$A_{ch}$ selection will contribute a larger elliptic flow, thus the negative slope for $\Delta v_2(A_{ch})$.

VI. EFFECT OF LOCAL CHARGE CONSERVATION

We now include the $p_T$ distributions to the above analysis to study the LCC effects in our MC simulation. The $p_T$ spectra, and $v_2(p_T)$ of $\rho^0$, $K_s$, and $\omega$ we used correspond to the measured data in 30-40% centrality of Au+Au collisions [42,43]. The average multiplicity of total charges in the STAR acceptance ($p_T > 0.15$ GeV/c, $|\eta| < 1.0$) is about 380 with the $\eta$ spectra given in Eq. 9. The $v_2$ is kept uniform in $\eta$. The results are shown in Fig. [5](a). For the $\rho^0$ meson, the LCC contribution is small, and the slope parameter $r$ is still negative. This is because the decay opening angle is large for the relevant $p_T$ range where the $\rho^0$ yields are appreciable. The feeding into and out of the acceptance of the daughter pions are rather insensitive to the $p_T$ of the $\rho^0$. For decays with a smaller mass, like $K_s$, the decay opening angle is relatively small, so the positive contribution from LCC is appreciable. This makes the final slope positive. Three-body decays usually have small decay opening angles, so the LCC effect is also appreciable. Indeed, positive slope parameter is observed from $\omega$ decays. In general, smaller mass resonance decay gives a large positive slope due to
the LCC effect.

We apply the above simulations to multiple sources of pions, namely, decays of $K_s$, $\rho^0$, $\rho^\pm$, $\omega$, and $\eta$, together with primordial pions. The parameters are taken from Ref. [34] and the primordial to decay pion multiplicity ratio within acceptance is taken to be 0.94. The mean multiplicity of total charged pions within STAR acceptance is 380 with the $\eta$ distribution given in Eq. [5]. Poisson multiplicity fluctuations are applied for all the particle species. The result is shown in Fig. 3(b). The slope parameter is about $-0.4\%$, where contribution from the competition among multiple pion sources discussed in Sec. III is also included.

The negative slope parameter from the multiple pion source simulation is different from the one observed by Ref. [36], where a positive slope was obtained from a simplified MC simulation and a 3+1-dimension hydrodynamic calculation. The reason is that, besides the pions from resonance decays, the primordial $\pi^+\pi^-$ are also considered as correlated pairs by the LCC mechanism in Ref. [33]. The $(p_T,\eta)$ correlations of these primordial $\pi^+\pi^-$ pairs are constructed while the azimuthal angles are sampled from the input $v_n$ modulation. In our case, the azimuthal angles of decay pions are given by the parent azimuth and decay kinematics, see Fig. 2(b-d). Furthermore, in the MC and hydrodynamic calculation in Ref. [36] the LCC is implemented as neutral cluster decays with the mass spectrum, $p_T$ spectrum, $v_2(p_T)$, and decay kinematics different from the known resonances discussed here. One marked difference is the continuum-like invariant mass spectrum of those neutral clusters. To illustrate this effect, we use an exponential mass spectrum,

$$\rho(m) = \exp(-m/T_{\text{eff}}), \quad (19)$$

with $T_{\text{eff}} = 300$ MeV to mimic the invariant mass spectrum obtained by sampling pairs from the single pion $p_T$ spectrum keeping the $\eta$ the same. The correlations are introduced by the two-body decay mechanism. For the sake of comparison, the $p_T$ spectrum, $v_2(p_T)$ of neutral clusters, and the mean multiplicity of the decayed $\pi$’s in the STAR acceptance are as same as those used in the above $\rho^0$ study. The result is shown in Fig. 4(a). A positive slope parameter $r = 1.11\%$ is obtained from this mass spectra. This is in-line with the above observation that smaller resonance mass gives a more positive slope because the average mass of the spectrum of Eq. 19 is small. We note that the effect of the convex $v_2(\eta)$ distribution discussed in Ref. [30] is not included in our study, which will further increase the slope parameter.

We apply the LCC correlated pions from neutral clusters of the continuum mass to replace the independent primordial pions in the above multiple source simulations. The results are shown in Fig. 4(b) with the slope $r \sim 0.85\%$. The result differs from the one shown in Fig. 5(b)). The reason are twofold: First, the slope parameter of $\Delta v_2(A_{ch})$ generated from correlated pions is not zero, as we have shown in Fig. 4(a), while the one from independent primordial pions is obviously zero. Second, the multiplicity distribution of net-charge from correlated pions is much narrower than the Skellam distribution from independent primordial pions. This will give a different contribution to the slope of $\Delta v_2(A_{ch})$ from the competition among multiple pion sources.

It is worthwhile to note that the propose of our study is not to give quantitative descriptions of the measured data but to illustrate the potential non-CMW mechanisms contributing to the CMW-sensitive slope parameter. More elaborate studies are needed to pin down the exact LCC contribution to the slope observable. Considering that the CMW-sensitive observable is also sensitive to kinematics and multiplicity fluctuations, more extensive background studies are called for.

VII. SUMMARY

The charge asymmetry ($A_{ch}$) dependent pion elliptic flow difference $\Delta v_2(A_{ch})$ was proposed as a sensitive observable to the chiral magnetic wave (CMW) search. In this paper, we first demonstrated that the flow measurements can automatically introduces a linear-$A_{ch}$ dependence if (1) there exists non-flow difference between like-sign and unlike-sign pairs and (2) both charged-sign hadrons are used as reference in two-particle cumulants to calculate the flow harmonics. Based on our toy model study, we found that for close pair unlike-sign non-flow, the trivial term introduces a positive trivial slope to $\Delta v_n(A_{ch})$ for both $n = 2, 3$. For back-to-back pair unlike-sign non-flow, the slope is positive for $\Delta v_2(A_{ch})$ and negative for $\Delta v_3(A_{ch})$.

After eliminating the trivial term, we further study other non-CMW mechanisms that can produce non-zero slopes in $\Delta v_2(A_{ch})$. We found:

- Non-flow between like-sign pairs gives a positive slope to $\Delta v_2(A_{ch})$ because of the larger dilution effect to $\pi^+(\pi^-)$ at positive (negative) $A_{ch}$.

- Competition among multiple $\pi$ sources can introduce a linear-$A_{ch}$ term. This effect is sensitive to the differences in multiplicity fluctuations and anisotropic flows of those sources, and arises from the $A_{ch}$-dependent relative contributions of pions from those sources. Such $A_{ch}$ dependence does not need any $A_{ch}$ dependence of the elliptic flow of each individual $\pi$ source.

- The slope parameter is sensitive to the kinematics of neutral clusters decaying into pions. Light resonances give positive slopes while heavy resonances give negative slopes. Local charge conservation (LCC) from continuum cluster mass spectra can give positive slope parameters.

Our studies indicate that many physics mechanisms can give rise to a $A_{ch}$ dependent $\Delta v_2(A_{ch})$. One could
produce a positive slope of a few present, as experimentally observed, from those non-CMW mechanisms. The interpretation of $\Delta v_2(A_{ch})$ in terms of CMW is therefore delicate. In order to identify the CMW, those (and potentially more) non-CMW mechanisms have to be assessed in detail and to a precision that would leave no doubt to an experimental signal. This would require accurate modeling of particle/resonance production and dynamics of heavy ion collisions.

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[46] Here we have omitted the dash between $K_s$ and $K_0$.
[47] Although $K_s$ is a stable particle, we will use the term “resonance” everywhere for convenience.