$B_s$ mixing phase and lepton flavor violation in supersymmetric SU(5)

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Abstract. The connection between $B_s$ mixing phase and lepton flavor violation is studied in SU(5) GUT. The $\mathcal{O}(1)$ phase, preferring a non-vanishing squark mixing, generically implies $\tau \rightarrow (e+\mu)\gamma$ and $\mu \rightarrow e\gamma$. In addition to the facts already well-known, stresses are put on the role of gaugino to scalar mass ratio at the GUT scale and the possible modifications due to Planck-suppressed non-renormalizable operators.

The $B_s$ mixing phase, denoted by $\phi_s$, is a theoretically clean observable, and one can make a close connection between its data and a theory possibly involving new physics. We choose the notation of $\phi_s$ used by the DØ collaboration. In the SM, one has $\phi_s \simeq -2\eta\lambda^2 \simeq -0.04$. On the experimental side, the latest world average of the $\phi_s$ deviation from the SM [3], appears to favor a negative $\phi_s$ value.

Another point to note is that $\phi_s \simeq -0.043$ (at $95\%$ C.L.) from the top Yukawa coupling to the scalar mass terms [4].

When one uses a low energy hadronic process to constrain the $\delta$ parameters at $M_{\text{GUT}}$, the running effect of soft mass terms results in an interesting behavior, illustrated in Fig. 1. Suppose that the cutoff scale of the GUT is two orders of magnitude higher than $M_{\text{GUT}}$, one can nevertheless constrain $\delta_{23}^{AB}$ using the combined mode, $\tau \rightarrow (e+\mu)\gamma$, exploiting the fact that the breakdown of $b\rightarrow \tau$ alignment is suppressed by $\cos\beta$ [4].

If one has a perfect alignment between the mass eigenstates of quarks and leptons, $(\delta_{23}^{d})_{RR}$ implies the transition of $l_j \rightarrow l_i$. However, this straightforward correspondence may be broken by the inclusion of non-renormalizable terms into the superpotential as a solution to the wrong quark–lepton mass relations of the lighter two families. With the assumption that the cutoff scale of the GUT is two orders of magnitude higher than $M_{\text{GUT}}$, one can nevertheless constrain $(\delta_{23}^{d})_{RR}$ using the combined mode, $\tau \rightarrow (e+\mu)\gamma$, exploiting the fact that the breakdown of $b\rightarrow \tau$ alignment is suppressed by $\cos\beta$ [4].

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FIGURE 2. Constraints on \((\delta_{LL}^d)_{\text{LL}}\). For \(\tau \to \mu \gamma\), the thin circle is an upper bound from the prospective branching ratio limit, \(10^{-8}\). For \(\mu \to e \gamma\), the thin circle shows the projected bound on the branching ratio, \(10^{-13}\). A yellow region is allowed by \(\Delta M_t\), given 30% uncertainty in the \(\Delta B = 2\) matrix element, and a cyan region is further consistent with \(\phi_s\). Of the two sides of the \(\delta_{\text{CP}}^K\) curve, the excluded one is indicated by thin short lines.

We restrict ourselves to \(LL\) and \(RR\) mixings of down-type squarks. We consider three scenarios: the \(LL\) scenario, the \(RR\) scenario, and the \(LL = RR\) scenario. It should be noted that we set an \(LL\) insertion, unless it is a scanning variable, to a value generated by RG running from the supersymmetry breaking mediation scale \(M_s\) down to \(M_{\text{GUT}}\), where \(M_s\) is taken to be the reduced Planck scale. These boundary conditions are given at \(M_{\text{GUT}}\) with which we solve one-loop RG equations down to the weak scale. The constraint from each observable is depicted on the complex plane of a GUT scale mass insertion. As for \(\phi_s\), we use the 95% probability region [3],

\[
\phi_s \in [-1.10, -0.36] \cup [-2.77, -2.07]. \tag{1}
\]

For concreteness, we assume that there is an exact quark-lepton flavor alignment. Regarding \(\tau \to \mu \gamma\), it is straightforward to translate their bounds presented below to a case with quark–lepton misalignment discussed above—interpret \(B(\tau \to \mu \gamma)\) as \(B(\tau \to (e + \mu) \gamma)\). This prescription is applicable to all the three scenarios considered here. As for \(\mu \to e \gamma\), barring accidental cancellations, a contour does not need a modification in the \(RR\) and \(LL = RR\) scenarios, while we do not have a systematic way to account for a misalignment in the \(LL\) scenario. We fix \(M_{1/2} = 180\) GeV, which makes the gluino mass be 500 GeV at the weak scale, and set \(m_0 = 600\) GeV, which optimizes the sensitivity of neutral meson mixing to \(\delta_s\) at the GUT scale. We use tan\(\beta = 5\). Other choices of parameters are considered in Refs. [4], which also explain other details.

First, the \(LL\) mixing scenario is shown in Fig. 2. One can find cyan regions that lead to \(\phi_s\) within its 95% CL intervals. They involve an \(\mathcal{O}(1)\) size of \((\delta_{LL}^d)_{\text{LL}}\). However, the supersymmetric disturbance is great also in \(B \to X_s \gamma\), which excludes the bulk of a cyan zone. The disturbance in this decay mode grows with \(\tan \beta\), as does that in \(S_{\text{CP}}\). Although not shown here, \(S_{\text{CP}}\) conflicts with \(\phi_s\) for tan\(\beta = 10\) [4] [7]. In this scenario, discovery of LFV seems to be difficult at a super \(B\) factory or MEG.

The \(RR\) scenario is shown in Fig. 3. Comparing this figure with Fig. 2, one notices that an \(RR\) insertion gives more effect on \(B_s\) mixing than an \(LL\) insertion. This is because an \(LL\) insertion is induced by RG running from \(M_s\) down to the weak scale even in the \(RR\) scenario. The presence of \((\delta_{LL}^d)_{\text{LL}}\) enhances the effect of \((\delta_{LL}^d)_{\text{RR}}\) on \(B_s\) mixing, and \(\phi_s\) can be easily pushed to its 95% probability region. However, an \(RR\) insertion is strictly limited by \(d_n\), the neutron EDM. A region allowed by \(d_n\) and \(\Delta M_s\) around the origin, is separated from the \(\phi_s\) region. The band obeying \(d_n\) can be rotated to overlap the cyan...
region by altering $(\delta_{23}^{d})_{LL}$ at $M_{\text{GUT}}$, since $d_{s}$ is influenced through the combination of $\text{Im}[\langle (\delta_{23}^{d})_{LL} (\delta_{23}^{d})^{*}_{RR} \rangle]$. The presented plots are valid for the phase of $(\delta_{23}^{d})_{LL}$ equal to $\arg(-V_{ts}^{*}V_{tb})$. Note that $B \rightarrow X_{\gamma} \gamma$ is not very tight. This is because the supersymmetric amplitude does not interfere with the SM one. LFV and $d_{a}$ are enhanced for high $\tan \beta$. Therefore, lowering $\tan \beta$ helps satisfy LFV and $d_{a}$ as well as $\phi_{t}$. One can find that the region preferred by $\phi_{t}$ involves the $\tau \rightarrow \mu \gamma$ rate in the vicinity of the current upper limit. For example, fitting the central value of $\phi_{t}$ causes $B(\tau \rightarrow \mu \gamma)$ to be around $10^{-7}$ which is already ruled out by the Belle data. The area still surviving could be explored by current and future experiments. The magnitude of mass insertion accessible with the sensitivity of $10^{-8}$, attainable at a super $B$ factory, is depicted by a thin circle inside the current upper bound. The cyan region is also expected to bring about $\mu \rightarrow e \gamma$ at a rate that can be probed by MEG.

The above restrictions on the $RR$ insertion, with slight modifications, can be applied to a popular scenario where the soft terms are flavor-blind at $M_{s}$ and large neutrino Yukawa couplings are the only source of flavor violation apart from the CKM mixing. In this case, the slepton mass matrix receives additional contribution running below $M_{\text{GUT}}$. Because of this, given the same $\delta$ at $M_{\text{GUT}}$, LFV rates are higher than in Fig. 3 and therefore one obtains tighter LFV bounds.

The last scenario where $(\delta_{23}^{d})_{LL} = (\delta_{23}^{d})_{RR}$ at $M_{\text{GUT}}$, is shown in Fig. 4. Comparison of Fig. 4 and Fig. 3 shows that the conflict between LFV and $\phi_{t}$ has been reduced here. Simultaneous presence of $LL$ and $RR$ mixings reinforces contribution to the $B_{s}$ mixing even with a smaller size of each insertion about 0.1, while the LFV bounds remain almost the same. Fig. 4 shows regions well inside the LFV bounds which lead to $\phi_{t}$ in perfect agreement with the latest world average. Part of those regions can satisfy $S_{\mu \gamma}^{X}$ and $d_{a}$ as well. An area preferred by $\phi_{t}$ gives rise to $B(\tau \rightarrow \mu \gamma)$ around $10^{-8}$ or larger. The rate of $\mu \rightarrow e \gamma$ expected from the same area is around the sensitivity of MEG. In this scenario, a higher $\tan \beta = 10$ is viable as well.

We summarize. We have examined three patterns of $(\delta_{23}^{d})_{LL}$ and $(\delta_{23}^{d})_{RR}$: $LL$, $RR$, and $LL = RR$. For reconciling $\phi_{t}$ with LFV, it greatly helps to choose the optimal value of the GUT scale gaugino to scalar mass ratio, in all these three scenarios. It appears that the most adequate to fit the current value of $\phi_{t}$ is $LL = RR$ among the three scenarios. The main difficulties for this purpose are $B \rightarrow X_{\gamma} \gamma$ and $S_{\mu \gamma}^{X}$ in the $LL$ scenario, and LFV and the neutron EDM in the $RR$ scenario. Inclusion of Planck-suppressed non-renormalizable terms for fixing the quark–lepton mass relations, in general, affects a LFV bound. This alteration can be estimated by weakening a $\tau \rightarrow \mu \gamma$ bound to that from $\tau \rightarrow (e + \mu) \gamma$. In the two scenarios involving an $RR$ mixing, this reduces the tension between LFV and $\phi_{t}$. In all cases, low $\tan \beta$ loosens $B \rightarrow X_{\gamma} \gamma$, $S_{\mu \gamma}^{X}$, and $d_{a}$ as well as LFV, providing for more room to accommodate $\phi_{t}$.

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