Spin-orbit induced spin-density wave in a quantum wire

Jianmin Sun, Suhas Gangadharaiah, and Oleg A. Starykh

Department of Physics, University of Utah, Salt Lake City, UT 84112
(Dated: September 29, 2018)

We present analysis of the interacting quantum wire problem in the presence of magnetic field and spin-orbit interaction. We show that an interesting interplay of Zeeman and spin-orbit terms, facilitated by the electron-electron interaction, results in the spin-density wave (SDW) state when the magnetic field and spin-orbit axes are orthogonal. This strongly affects charge transport through the wire: with SDW stabilized, single-particle backscattering off an nonmagnetic impurity becomes irrelevant. Sensitivity of the effect to the direction of the magnetic field can be used for experimental verification of this proposal.

PACS numbers: 71.10.Pm,73.63.Nm,71.70.Ej

Introduction. Active current interest in devising schemes to manipulate electron spin has led to several interesting developments [1, 2, 3]. Most approaches rely on spin-orbit interaction, which couples particle’s momentum and spin, in order to achieve this goal. Significant progress has been made in clarifying the role of spin-orbit (SO) interaction, mostly of the Rashba type [4], on the electric and spin transport by noninteracting electrons during the past few years [3], our understanding of the combined effect of SO and electron-electron interactions is still limited [4, 5, 6, 7].

Here we study the combined effect of (Zeeman) magnetic field and spin-orbit interaction in a single-channel interacting quantum wire. This set-up allows for the well-controlled theoretical analysis of the interplay between broken time reversal $T$ (by applied magnetic field) and inversion $P$ (by spin-orbit interaction) symmetries and electron-electron interactions. The problem is formulated as follows. We consider a single-channel ballistic quantum wire, corresponding to the two-terminal conductance $G_0 = 2e^2/h$. Applied magnetic field creates two spin-split subbands, the wave functions of which are given by the standard orthogonal pair, $|\uparrow\rangle = |1,0\rangle$ and $|\downarrow\rangle = |0,1\rangle$ (the orbital effect of the field is neglected). It reduces spin-rotational symmetry to $U(1)$, rotations about $\hat{z}$ axis. Next we add weak spin-orbit interaction $H_R^{(1d)} = \alpha R p y \hat{\sigma}_x$, which is obtained by electrostatic gating of two-dimensional electron gas with Rashba SO interaction [3]. (Corrections to this form, due to the omitted “transverse” piece, $\alpha R p y$, and virtual transitions to the higher, unoccupied, subbands, can be taken into account [3] but are irrelevant for our purposes here.) Observe that $H_R^{(1d)}$ breaks spatial inversion ($y \to -y$) and $U(1)$ spin symmetry, $[\hat{\sigma}_z, H_R^{(1d)}] \neq 0$. The major consequence of this is the opening of a new, inter-subband Cooper scattering channel [10, 11]. In this process, a pair of electrons with opposite momenta in one subband is scattered (“tunnels”) into a similar pair in the other subband, see Figure 1. Note that this process requires spin non-conservation (i.e. $\alpha R \neq 0$), mentioned above, as it scatters two electrons with (almost) “up”-spins into a pair with (almost) “down”-spins (and vice versa). This simple observation is the key to our analysis: its derivation and consequences are presented below.

Technical formulation. The single-particle Hamiltonian, describing the scenario outlined above, reads

$$H_0 = \frac{p^2}{2m} - \mu - \frac{1}{2} g \mu_B B \sigma_z + \alpha_R \sigma_x,$$

(1)

where momentum along the wire ($y$-axis) is now denoted as $p$. The eigenstates $\psi_{\nu}(p) = e^{ipy} \chi_{\nu}(p) \ (\nu = \mp)$ are easily expressed in terms of the momentum-dependent spinors [2, 12]

$$\chi_{-}(p) = \begin{pmatrix} \cos[\gamma_p/2] \\
-\sin[\gamma_p/2] \end{pmatrix}, \chi_{+}(p) = \begin{pmatrix} \sin[\gamma_p/2] \\
\cos[\gamma_p/2] \end{pmatrix},$$

(2)

which describe momentum-dependent orientation of electron’s spin in $\hat{z} - \hat{x}$ plane. The rotation is specified by the angle $\gamma_p = \arctan(2\alpha_R p / g \mu_B B)$, Note that left- (right-) moving particle experiences clockwise (counter-clockwise) rotation of its spin away from “up”-spin ($\nu = -$) and “down”-spin ($\nu = +$) orientations at the subband’s center $p = 0$, see Figure 1. The corresponding eigenvalues

$$E_{\pm} = \frac{g}{2m} \mu \mp \sqrt{(\alpha_R p)^2 + (g \mu_B B / 2)^2}$$

describe two non-intersecting branches. The gap between them is again momentum-dependent and is minimal at $p = 0$, where it reduces to the Zeeman energy $g \mu_B B$.

![FIG. 1: Occupied subbands $E_{\pm}$ of Eq. 1. Arrows illustrate spin polarization in different subbands. Dashed (dotted) lines indicate exchange (direct) Cooper scattering processes.](image-url)
We consider the situation when the Fermi energy $E_F = v_F p_F$, where $p_F$ ($v_F$) are Fermi-momentum (velocity), crosses both branches, as shown in Fig. 1 resulting in four Fermi points, $\pm p_\pm$ in the wire. To describe low-energy excitations of the interacting wire we project single-particle spin-$s$ state $\Psi_s$ ($s = \uparrow, \downarrow$) onto the two-dimensional space spanned by $\psi_\pm$ eigenstates:

$$\Psi_s(y) = \sum_{\nu = \mp} (\chi_\nu(p_\nu)|s\rangle e^{ip_\nu y} R_\nu + (\chi_\nu(-p_\nu)|s\rangle e^{-ip_\nu y} L_\nu.$$ (3)

Operators $R_\nu$ ($L_\nu$) represent slow degrees of freedom: right- (left-) movers in the vicinity of $+p_\nu$ ($-p_\nu$) Fermi-points of the $\nu$-th subband, respectively. In this representation, the interaction term $H_{\text{int}} = \frac{1}{2} \sum_{s, s'} \int dy dy' U(y - y') \Psi_s^\dagger(y') \Psi_s(y)$ reduces to the sum of intra- and inter-subband scattering processes [11]. Keeping only low-energy momentum-conserving ones, the inter-subband terms include, in notations of Ref. [11] forward, exchange-backscattering and Cooper processes. The Cooper scattering represents two-particle (pair) tunneling between $-$ and $+$ subbands. It reads

$$H_C = \int dy \{ U(p_- - p_+) \sin^2[(\gamma_- - \gamma_+)/2] - U(p_- + p_+) \sin^2[(\gamma_- + \gamma_+)/2](R_\nu L_\nu^* R_\nu L_\nu + \text{h.c.}) \}.$$ (4)

Here $U(q) = \int dr U(r) e^{iqr}$ is the $q$-th Fourier component of electron interaction. The terms inside the figure brackets in [11] represent matrix elements for two different Cooper scatterings - direct and exchange, see Fig. 1. $U(p_- - p_+)$ describes direct scattering in which right-mover $R_\nu$ in the $\nu$-th subband scatters into right-moving $R_{\nu'}$ in the opposite, $-\nu$, subband, $R_\nu \rightarrow R_{\nu'}$, while its left-moving companion $L_\nu$ scatters into $L_{\nu'}$. The other possibility, exchange Cooper scattering, involving $U(p_- + p_+)$, describes right and left members of the pair scattering across: $R_\nu \rightarrow L_{-\nu}$ and $L_\nu \rightarrow R_{-\nu}$. It is crucial to observe here that in addition to involving two different Fourier components of the interaction potential, these two processes include squares of single-particle overlap integrals, $\sin^2[(\gamma_- \mp \gamma_+)/2]$. Relative magnitude of these is easy to understand in the limit of strong magnetic field and weak spin-orbit splitting, $\alpha_R [p_F/(g_\mu B)] \ll 1$, on which we concentrate now. As discussed in the introduction, in this limit eigen-spinors $\chi_\pm$ almost coincide with spin $|s = \uparrow, \downarrow\rangle$ eigenstates of the Zeeman Hamiltonian. Weak SO term, which can be thought of as momentum-dependent magnetic field, acting along the orthogonal, $\sigma_z$, direction, causes spins at $p_-$ and $p_+$ Fermi-points to tilt by only slightly different amount, resulting in a small overlap of single-particle wave functions, proportional to the difference $\delta_F = p_- - p_+ = g_\mu B/v_F$. At the same time, spins at, say, right $p_-$ and left $-p_+$ Fermi-points, tilt in opposite directions, resulting in relatively large angle (and bigger overlap) between them, proportional to the sum $p_- + p_+ = 2p_F$. This allows us to estimate the ratio of the two amplitudes as $(U(\delta_F)/(U(2p_F)))(g_\mu B/B_F)^2 \ll 1$ and neglect the contribution of the direct Cooper process in the following.

Bosonization. We now bosonize the problem [13] with the help of two conjugated fields, $\varphi_\nu$ and $\theta_\nu$, obeying commutation relation $[\varphi_\nu(x), \theta_\nu(y)] = (i/2)\delta_{\nu,}\delta(y - x)$. Fermions are represented as $R_\nu = \eta_\nu \exp[i\sqrt{\pi}(\varphi_\nu - \theta_\nu)]/\sqrt{2\pi a}$ and $L_\nu = \eta_\nu \exp[-i\sqrt{\pi}(\varphi_\nu + \theta_\nu)]/\sqrt{2\pi a}$. Klein factors $\eta_\nu$, satisfying $\{\eta_\nu, \eta_\nu\} = 2\delta_{\nu,\nu'}$, insure anticommutation of fermions from different subbands, and $\alpha \sim p_F^{-1}$ is a short-distance cutoff. We then transform to convenient symmetric, $\varphi_\nu = (\varphi_- + \varphi_+)/\sqrt{2}$, and antisymmetric, $\varphi_\nu = (\varphi_- - \varphi_+)/\sqrt{2}$, combinations (and similarly for $\theta_\nu$), in terms of which the Hamiltonian of the problem decouples into two commuting ones. As indicated by notations, symmetric (antisymmetric) combinations in fact coincide with the standard charge (spin) ones. This is not a generic property of the problem but rather a convenient feature of the limit $\alpha_R p_F \ll g\mu B < B_F$ which is used in the rest of the paper. Symmetric (charge) part $H_\rho$ is purely harmonic

$$H_\rho = \frac{1}{2} \int_y \frac{v_\rho}{K_\rho}(\partial_y \varphi_\rho)^2 + v_F(\partial_y \theta_\rho)^2,$$}

with stiffness $K_\rho^{-1} = \sqrt{1 + (U(0) - U(2p_F))/\pi v_F}$. The antisymmetric (spin) one includes nonlinear cosine term, representing Cooper process [11]

$$H_\sigma = \frac{1}{2} \int_y \frac{v_\sigma}{K_\sigma}(\partial_y \varphi_\sigma)^2 + v_F(\partial_y \theta_\sigma)^2 + \frac{g_\sigma}{(\pi a)^2} \cos[\sqrt{\pi} \theta_\sigma]$$

$$K_\sigma^{-1} = \sqrt{1 - U(2p_F)/\pi v_F},$$

$g_\sigma = U(2p_F)(2\alpha_R p_F)/(g_\mu B)$.

Renormalized velocities of these excitations follow from $v_{\rho,\sigma} = v_F/K_{\rho,\sigma}$. Equations (5,6) include $H_0$ as well as momentum-conserving intra-subband (forward and backscattering) and inter-subband forward ($\propto U(0)$) interactions, which are encoded in the stiffnesses $K_{\rho,\sigma}$. Inter-subband exchange backscattering, although momentum-conserving, is neglected because it is strictly marginal and small, of the order $\alpha_R^2$. We have also omitted marginal correction, small in $g_\mu B/B_F \ll 1$ factor, associated with weak dependence of subband velocities $v_\pm$ on magnetic field [14] - this is the main reason for the equivalence of symmetric (antisymmetric) modes with charge (spin) ones, mentioned above. Yet another simplification consists in replacing $U(2p_\pm)$ by $U(2p_F)$ in expressions for $K_{\rho,\sigma}$ - this is a valid approximation for any physical $U(r)$. Finally, we must keep Cooper term in (6), which, inspite of having small amplitude $g_\sigma$, is strictly relevant in the renormalization group (RG) sense. Its scaling dimension is $2/K_\sigma < 2$ for repulsive interactions [12].

Full argument in favor of Cooper term’s relevancy is a bit more delicate. It has to do with irrelevant inter-subband direct backscattering term $a g_{\text{bs}} \cos[\sqrt{8\pi} \varphi_\sigma - \pi/2]$.


2\delta_{PF} y]$, omitted from (6). Note that $K_\sigma = 1 + g_{bs}/2$. Backscattering decays as $g_{bs}(\ell) = g_{bs}(0)/(1 + g_{bs}(0)\ell)$ until the rescaled cutoff reaches $a(\ell) = a e^{-1/\deltaPF}$, see [15]. At that scale $\ell^* = \ln(p_F/\deltaPF) = \ln(E_B/g_{bs}B)$ and strongly oscillating spin backscattering cosine disappears from the problem (“averages out”) [12]. Spin stiffness $K^*_\sigma = 1 + g_{bs}(\ell^*)/2$ stops at the value above one [15], which implies the relevance of the Cooper term, as already mentioned above. In more detail, the Cooper coupling constant, evolution of which is described by the simple $\partial_t g_c = (2 - 2/K_\sigma)g_c$ changes little from its initial value by the time scale $\ell^*$ is reached: $g_c(\ell^*) = g_c(0)[1 + g_{bs}(0)\ell^*]$. From this point on, one is allowed to neglect $g_{bs}$ completely, and treat the Cooper scattering term Eq. [6] as the only relevant interaction. Both $g_c$ and $K_\sigma$ grow under RG as $\ell$ is increased past $\ell^*$, and reach strong coupling limit when $g_c(\ell) \sim v_F$ while $K_\sigma \to 2$ [16].

Consequences of (6). The flow to strong coupling implies the change in the ground state (spin sector) from gapless to gapped. The resulting spin gap can be estimated from the analysis of spin correlations. Choosing the simple coupling constant, evolution of which is described by $\partial_t \theta = - \cos[2\pi\theta]/\pi a$ and strongly oscillating spin backscattering cosine disappears from the problem (“averages out”) [12]. Spin stiffness $K^*_\sigma = 1 + g_{bs}(\ell^*)/2$ stops at the value above one [15], which implies the relevance of the Cooper term, as already mentioned above. In more detail, the Cooper coupling constant, evolution of which is described by the simple $\partial_t g_c = (2 - 2/K_\sigma)g_c$ changes little from its initial value by the time scale $\ell^*$ is reached: $g_c(\ell^*) = g_c(0)[1 + g_{bs}(0)\ell^*]$. From this point on, one is allowed to neglect $g_{bs}$ completely, and treat the Cooper scattering term Eq. [6] as the only relevant interaction. Both $g_c$ and $K_\sigma$ grow under RG as $\ell$ is increased past $\ell^*$, and reach strong coupling limit when $g_c(\ell) \sim v_F$ while $K_\sigma \to 2$ [16].

Consequences of (6). The flow to strong coupling implies the change in the ground state (spin sector) from gapless to gapped. The resulting spin gap can be estimated from the analysis of spin correlations. Choosing the simple coupling constant, evolution of which is described by $\partial_t \theta = - \cos[2\pi\theta]/\pi a$ and strongly oscillating spin backscattering cosine disappears from the problem (“averages out”) [12]. Spin stiffness $K^*_\sigma = 1 + g_{bs}(\ell^*)/2$ stops at the value above one [15], which implies the relevance of the Cooper term, as already mentioned above. In more detail, the Cooper coupling constant, evolution of which is described by the simple $\partial_t g_c = (2 - 2/K_\sigma)g_c$ changes little from its initial value by the time scale $\ell^*$ is reached: $g_c(\ell^*) = g_c(0)[1 + g_{bs}(0)\ell^*]$. From this point on, one is allowed to neglect $g_{bs}$ completely, and treat the Cooper scattering term Eq. [6] as the only relevant interaction. Both $g_c$ and $K_\sigma$ grow under RG as $\ell$ is increased past $\ell^*$, and reach strong coupling limit when $g_c(\ell) \sim v_F$ while $K_\sigma \to 2$ [16].

Consequences of (6). The flow to strong coupling implies the change in the ground state (spin sector) from gapless to gapped. The resulting spin gap can be estimated from the analysis of spin correlations. Choosing the simple coupling constant, evolution of which is described by $\partial_t \theta = - \cos[2\pi\theta]/\pi a$ and strongly oscillating spin backscattering cosine disappears from the problem (“averages out”) [12]. Spin stiffness $K^*_\sigma = 1 + g_{bs}(\ell^*)/2$ stops at the value above one [15], which implies the relevance of the Cooper term, as already mentioned above. In more detail, the Cooper coupling constant, evolution of which is described by the simple $\partial_t g_c = (2 - 2/K_\sigma)g_c$ changes little from its initial value by the time scale $\ell^*$ is reached: $g_c(\ell^*) = g_c(0)[1 + g_{bs}(0)\ell^*]$. From this point on, one is allowed to neglect $g_{bs}$ completely, and treat the Cooper scattering term Eq. [6] as the only relevant interaction. Both $g_c$ and $K_\sigma$ grow under RG as $\ell$ is increased past $\ell^*$, and reach strong coupling limit when $g_c(\ell) \sim v_F$ while $K_\sigma \to 2$ [16].

The first term is standard, and represents intra-subband contribution, while the second, involving $\theta$, is due to the subleading inter-subband contribution, which couples $\pm$ bands. Observe that both contributions disappear in the SDW phase (\(\theta \rightarrow \theta_0\)). Since $2p_F$-component of the charge density describes backscattering ($p \rightarrow p$) of electrons by potential impurity, Eq. 8 implies irrelevancy of the impurity in the SDW state. The reason for this is somewhat similar to that of backscattering suppression in the spin Hall effect [18]: in SDW phase right- and left-movers within a given subband have opposite (orthogonal) $S^z$-components, as can be seen from [17] and Fig. 11 which forbids intra-subband backscattering. (In the spin Hall case right- and left-movers form Kramers pair and backscattering is forbidden by the $T$ symmetry [18], which is broken here.) Figure 11 also suggests that backscattering between right-movers of the + subband and left-movers of the − one is possible: their $S^z$ components are parallel. Nonetheless such backscattering is still suppressed because of the destructive interference of the two scattering paths. Namely, the inter-subband part of $2p_F$ density oscillation, bosonic form of which is given by the second term in (8), reads $\left(R_1^L L_+ - R_1^R L_- + h.c.\right)$ in terms of original fermions. The crucial relative minus sign between the two backscattering processes can be traced to Eq. (23) and represents the noted destructive interference. It is useful to understand this result perturbatively: the intra-subband piece of (8) arises from fusing $\varphi_\sigma$ from the localized impurity potential (first term in (8)) with that in $H^{(1d)}_R$. This explains its magnitude \((\propto a_{PF}/\deltaPF)\) and oddness under inversion (about the impurity site) $P$. Thus, a potentially more relevant, but even under $P$, backscattering process $\left(R_1^L L_+ + R_1^R L_- + h.c.\right) \propto \cos[2\pi\varphi_\rho]\sin[2\pi\varphi_\sigma]$ (note the relative plus sign) cannot be generated.

Although the single-particle backscattering is suppressed, the two-particle in general is not [11, 19, 20]. By considering fluctuations $\partial_t \theta$ one indeed generates two-particle backscattering term $\propto (V^2/\Delta) \cos[2\pi\varphi_\rho]$. This spin-insensitive impurity affects finite-temperature linear conductance as $G = 2e^2/h \propto -(V^2/\Delta)^2K_\rho^{-2}$ [11]. The correction is seen to become strong (relevant) for strongly interacting wire with $K_{\rho} < 1/2$, when the impurity cuts off charge transport completely [21]. This leaves us with the finite window, $1/2 < K_\rho < 1$, where the impurity is irrelevant. This is an interesting, and, to the best of our knowledge, new, addition to the Kane-Fisher result of always relevant impurity in a single-channel repulsive Luttinger liquid [21]. Note however that our discussion assumes fully developed SDW phase, and thus implies...
weak disorder potential $V \ll \Delta$. Complete solution requires simultaneous RG analysis of Cooper and impurity terms [17]. The correlated state can also be probed via tunneling density of states (DOS) measurements. Skipping details, which are rather similar to the calculation of DOS in [11], we quote the result for the local DOS in SDW state: $\nu(\omega) \propto \Theta(\omega - \Delta)(\omega - \Delta)^3$, where $b = (K_p - 1)^2/(4K_p)$ and $\Theta$ denotes the step function. Naturally, DOS is zero for energies below the SDW gap, and is found to rise smoothly ($b > 0$) just above it.

Angular stability of the SDW state can be analyzed via angular dependence of subband dispersions $E_\pm$ in Fig. 1. Indeed, suppose that the two axes, $\tilde{B}$ and SO, are not orthogonal and denote the angle between them as $\pi/2 - \beta$. This will modify the SO term in (1) to $\alpha R(\sigma_z \cos \beta + \sigma_y \sin \beta)$. The eigenvalues of the modified Hamiltonian (1) now read $E_\pm = \frac{p^2}{2m} - \mu \mp \sqrt{(\alpha R \cos \beta)^2 + (g\mu_B B - \alpha R \sin \beta)^2}$, and describe two subbands ($\mp$) shifted in opposite directions along the momentum axis. For small $\beta$ the dispersion can be approximated as $E_\mp = \frac{(p \mp p_0)^2}{2m} - \mu \mp \sqrt{(\alpha R)^2 + (g\mu_B B/2)^2}$. Thus the lower ($-$) subband shifts left and is centered around $-p_0$, while the upper ($+$) one shifts toward positive momenta and centers around $+p_0$, where $p_0 \approx m\alpha R\beta$. This simple observation implies that opposite-Fermi-momenta pairs in $\pm$ subbands acquire opposite ($\pm p_0$) center-of-mass momenta. This can be pictured by shifting the bands in Fig. 1 horizontally in opposite directions. Thus two-particle Cooper tunneling processes illustrated in Fig. 1 become momentum-non-conserving ones. As a result, this important scattering channel will disappear above some critical misalignment angle $\beta^*$ which can be estimated as follows. Cooper order is destroyed once the misalignment cost $\propto 2R_Fp_0$ becomes comparable to the Cooper gap $\Delta$. Estimating the latter at $K_p = 2$ we find: $\beta^* \approx \alpha R\rho_0 U(2R_F)/(g\mu_B B)^2 \ll 1$. This estimate shows that the found SDW state has narrow but finite region of angular stability, and agrees fully with results of more detailed RG-based calculations in [17]. SDW state can also be destroyed by reducing magnetic field strength below the critical, $g\mu_B B_c \sim \alpha R F$, even while maintaining the orthogonal orientation (angle $\beta = 0$). This happens due to the decrease of the spin stiffness $K_x$ below 1 (so that the scaling dimension of Cooper term in (1) exceeds 2) once the Zeeman energy becomes smaller than the spin-orbit one [17]. This weak-field region, which includes the $B = 0$ limit of (1), has been studied previously [10, 11] and contains no relevant Cooper processes.

The sensitivity of the described SDW phase to the mutual orientation and magnitude of the magnetic and SO terms can be exploited in experimental searches of the novel field-induced SDW phase of the quantum wire with spin-orbit interaction. It appears that lateral quantum wells at vicinal surface of gold, which possess spin-orbit-split and highly one-dimensional subbands, [22] can serve as a nice experimental starting point.

We would like to thank L. Balents, G. Fiete, C. Kane, D. Mattis, E. Mishchenko, L. Levitov, J. Orenstein, M. Raikh, and Y.-S. Wu for useful discussions and suggestions. O.S. and S.G. are supported by ACS PRF 43219-AC10.

[1] S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).
[2] P. Streda and P. Seba, Phys. Rev. Lett. 90, 256601 (2003).
[3] L.S. Levitov and E.I. Rashba, Phys. Rev. B 67, 115324 (2003).
[4] Yu.A. Bychkov and E.I. Rashba, J. Phys. C 17, 6039 (1984).
[5] H.A. Engel, E.I. Rashba, and B.I. Halperin, cond-mat/0603306.
[6] A.V. Moroz, K.V. Samokhin and C.H.W. Barnes, Phys. Rev. B 62, 16900 (2000).
[7] V. Gritsev, G. Japaridze, M. Pletyukhov, and D. Baeriswyl, Phys. Rev. Lett. 94, 137207 (2005).
[8] A. De Martino, R. Egger, A.M. Tsvelik, Phys. Rev. Lett. 97, 076402 (2006).
[9] A.K. Farid and E.G. Mishchenko, Phys. Rev. Lett. 97, 096604 (2006).
[10] H.-H. Lin, L. Balents, and M.P.A. Fisher, Phys. Rev. B 56, 6569 (1997).
[11] O.A. Starykh, D.L. Maslov, W. Häusler, and L.I. Glazman, Lecture Notes in Physics 544, 37 (2000).
[12] R.G. Pereira and E. Miranda, Phys. Rev. B 71, 085318 (2005).
[13] T. Giamarchi. Quantum physics in one dimension, Oxford University Press, 2004.
[14] T. Kimura, K. Kuroki, and H. Aoki, Phys. Rev. B 53, 9572 (1996).
[15] I. Affleck and M. Oshikawa, Phys. Rev. B 60, 1038 (1999).
[16] A.O. Gogolin, A.A. Nersesyan, and A.M. Tsvelik, Bosonization and strongly correlated systems, Cambridge University Press, 1999.
[17] S. Gangadharaiah, J. Sun, and O.A. Starykh, unpublished.
[18] C.L. Kane and E.J. Mele, Phys. Rev. Lett. 95, 226801 (2005); C. Xu and J.E. Moore, Phys. Rev. B 73, 045322 (2006); C. Wu, B.A. Bernevig, and S.-C. Zhang, Phys. Rev. Lett. 96, 106401 (2006).
[19] E. Orignac and T. Giamarchi, Phys. Rev. B 56, 7167 (1997).
[20] R. Egger and A.O. Gogolin, Eur. Phys. J. B 3, 281 (1998).
[21] C.L. Kane and M.P.A. Fisher, Phys. Rev. B 46, 15233 (1992).
[22] A. Mugarza, A. Mascaraque, V. Repain, S. Rousset, K.N. Altman, F.J. Himpsel, Yu.M. Koroteev, E.V. Chulkov, F.J. García de Abajo, and J.E. Ortega, Phys. Rev. B 66, 245419 (2002).