SCALING FOR THE COALESCENCE OF MICROFRAC TURES BEFORE BREAKDOWN

S. ZAPPERS1, P. RAY2, H.E. STANLEY1 AND A. VESPIGNANI3
1 Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA
2 The Institute of Mathematical Sciences, CIT Campus, Madras - 600 113, India
3 Instituut-Lorentz, University of Leiden, P.O. Box 9506 The Netherlands.

ABSTRACT

We study the behavior of fracture in disordered systems close to the breakdown point. We simulate numerically both scalar (resistor network) and vectorial (spring network) models with threshold disorder, driven at constant current and stress rate respectively. We analyze the scaling of the susceptibility and the cluster size close to the breakdown. We observe avalanche behavior and clustering of the cracks. We find that the scaling exponents are consistent with those found close to a mean-field spinodal and present analogies between the coalescence of microfractures and the coalescence of droplets in a metastable magnetic system. Finally, we discuss different experimental conditions and some possible theoretical interpretations of the results.

INTRODUCTION

The breakdown of solids under external forces is a longstanding problem, that has practical and theoretical relevance. The way a material breaks, under the effect of an external electric field or under mechanical stress are closely related problems, due to the formal similarities in the underlying laws governing those phenomena. The first theoretical approach to fracture mechanics dates back to the twenties with the work of Griffith [1], who formulated a theory of crack formation, which is similar to the classical theory of nucleation in first-order phase transitions. Cracks grow or heal, depending on whether the external stress prevail over the resistance at surface of the crack. Similarly in bubble nucleation [2], a critical droplet will form when the change in free energy due to the bulk exceeds that of the surface. Griffith theory assume the presence a single microcrack of a particular shape surrounded by an homogeneous medium, and therefore is not appropriate in disordered systems, where cracks can start from different positions and coalescence may take place.

Spinodal nucleation [3], contrary to classical nucleation, is characterized by scaling properties and fractal droplets. The spinodal point in fact has some characteristics of a critical point in second order phase transitions. The similarity between a solid driven to the threshold of mechanical instability and spinodal nucleation has been discussed in the past. Rundle and Klein [4], using a Landau-Ginzburg analysis of a single crack, showed that the crack growth obeyed scaling laws expected for spinodal nucleation. Selinger et al. [5] have shown by numerical simulations and mean-field theory of thermally activated fracture that the breakdown has the characteristics of a spinodal point.

In this paper we concentrate on disordered media and we disregard the effect thermal fluctuations. The system is driven by an increasing external load to the point of global failure. It has been experimentally observed that the response, detected by acoustic emission (AE) measurements, to an increasing external stress takes place in bursts or avalanches distributed over a wide range of scale. Examples of this are found in foam glasses [6], fiber matrix composites [7], concretes [8], hydrogen precipitation [9] and volcanic rocks [10]. We
observe a similar behavior for two dimensional discrete models. We show that the scaling behavior close to the breakdown is in quantitative agreement with the mean-field theory and it is suggestive of a first-order transition. The values of the scaling exponents are consistent with those found close to a spinodal point in thermally driven homogeneous systems.

THE MODELS

We study here two models, the random fuse model [11] for electric breakdown and a spring network model [12] for fracture. In the random fuse model [11] each bond of a two-dimensional lattice is occupied by a fuse of conductivity $\sigma = 1$, which burns when the current flowing in it exceeds a quenched random threshold. We consider a rotated square lattice with periodic boundary conditions in one direction. We impose a constant external current on the two other edges of the lattice. The currents in each bond are computed solving the kirchhoff equations. This step corresponds to the minimization of the total energy dissipated in the lattice

$$E(\{\sigma\}) = \frac{1}{2} \sum_i \sigma_i (\Delta V)_i^2, \quad (1)$$

where $(\Delta V)_i$ is the voltage drop in the bond $i$. We employ a multigrid relaxation algorithm with precision $\epsilon = 10^{-10}$. When all the currents are below the threshold we increase the current until the next bond reaches the threshold. The process is continued until a path of broken bonds spans the lattice and no current flows anymore. We chose a uniform distribution of thresholds, $D \in [0, 2]$.

The second model is an elastic network [12] which has central and rotationally invariant bond-bending forces. The potential energy is

$$E = \frac{a}{2} \sum_i (\delta r_i)^2 \sigma_i + \frac{b}{2} \sum_{<i,j>} (\delta \theta_{ij})^2 \sigma_i \sigma_j \quad (2)$$

where $\delta r_i$ is the change in the length of the bond $i$ and $\delta \theta_{ij}$ is the change in the angle between neighboring bonds $i$ and $j$. The constant $\sigma_i$ is equal to one if the bond is present and it is zero otherwise. A slowly increasing external stress is applied on all the edges and the lattice dynamics is obtained by numerically solving the equations of motion for each spring. Bonds break when stretched beyond a randomly chosen threshold.

SIMULATION RESULTS

The response of the model to the increase of the external force takes place in widely distributed avalanches. The average size of the avalanches (i.e. the number of broken bonds) increases when the global failure is approached. We were able to show [13] using mean-field theory that the average avalanche size $\langle m \rangle$ diverges at the breakdown as

$$\langle m \rangle \sim (f_c - f)^{-\gamma} \quad \gamma = 1/2. \quad (3)$$

where $f$ is the stress or the current per unit length imposed on the lattice. We note that the same scaling law is expected close to a spinodal point, in the case of thermally driven first-order transitions. The macroscopic quantities of the system (i.e. elasticity) have a finite jump at the breakdown, indicative of a first-order transition.
Figure 1: a) The average avalanche size $\langle m \rangle$ is plotted as a function of $f = I/L$, the fit is done using the mean-field value $\gamma = 1/2$. b) The “susceptibility” of the spring network with the mean field fit ($\gamma = 1/2$). The parameter $\phi$ is the fraction of bond that are not broken. The average avalanche size $\langle m \rangle$ is proportional to $d\phi/df$.

We confirm the validity of mean-field scaling by computer simulations of two dimensional models. For both models mean-field theory is obeyed remarkably well (see Fig. 1a and Fig. 1b).

The reason for the observed mean-field behavior is probably due to the long-range nature of elastic interactions. The formation of cracks in those models takes place by the coalescence of several microcracks. This is confirmed by the behavior of the average crack size which does not diverge at the breakdown (see Fig 2).

DISCUSSION AND CONCLUSIONS

The breakdown of driven disordered media is described by scaling law which are reminiscent of those found close to a spinodal point. It appears that the behavior of a driven disordered system is similar to that of a thermally driven homogeneous system. This analogy is not too strict since the concept of metastability and spinodal are not well defined in the first case.

Despite several experimental investigations of avalanche dynamics in fractures [6, 7, 3]...
Figure 2: The average crack size for the fuse model as a function of the current for different systems sizes. The crack size does not seem to diverge at the breakdown.

There is not a clear theoretical interpretation of the results. We believe that different experimental conditions can all give rise to similar scaling behavior, but the underlying physical mechanisms could be quite different. We can distinguish the following experimental setups:

1. A solid driven by a constant stress rate can be described in the framework discussed in this paper. The system responds to the increase of the external load by AE bursts of increasing size, diverging at the point of global failure. It would be interesting to check if the scaling exponents agree with the mean-field theory.

2. A solid subject to a constant load breaks because of thermal fluctuations. The AE is due to the formation of “droplets” and should be power law distributed close to the limit of stability (spinodal). Scaling exponents consistent with those of spinodal nucleation were observed in a recent experiment on cellular glass. To confirm this interpretation it would be necessary to study the scaling for different values of the applied load.

3. A solid in a perfectly plastic state could respond to the increase of the external strain by a stationary AE signal. In this case one can interpret the results as a manifestation of self-organized criticality. Such a behavior was shown in numerical models, but to our knowledge it has not yet been observed in experiments.

We believe that extensive and systematic experiments along these lines can resolve these longstanding problems.

ACKNOWLEDGMENTS

We wish to thank P. Cizeau, W. Klein, F. Sciortino and H. Strauven for useful discussions and remarks. The Center for Polymer studies is supported by NSF.

References
[1] A. A. Griffith, Phil. Trans. Roy. Soc. London A 221, 163 (1920).

[2] J. D. Gunton, M. San Miguel and P. S. Sahini, in Phase Transitions and Critical Phenomena, Vol. 8, edited by C. Domb and J. L. Lebowitz (Academic, London, 1983).

[3] C. Unger and W. Klein, Phys. Rev. B 29, 2698 (1984); ibid. 31, 6127 (1985); for a review, see L. Monette, Int. J. of Mod. Phys B 8, 1417 (1994).

[4] J. B. Rundle and W. Klein, Phys. Rev. Lett. 63, 171 (1989).

[5] R. L. B. Selinger, Z.-G. Wang, W. M. Gelbart and A. Ben-Saul, Phys. Rev. A 43, 4396 (1991).

[6] H. Strauven, G. Claes, G. Heylen, G. Crevecoer and C. Maes, in 22nd European Conference on Acoustic Emission Testing Proceedings (Aberdeen, 1996).

[7] J.-C. Anifrani, C. Le Floch, D. Sornette and B. Souillard, J. de Phys. I 5, 631 (1995).

[8] A. Petri, G. Paparo, A. Vespignani, A. Alippi and M. Costantini, Phys. Rev. Lett. 73, 3423 (1994).

[9] G. Cannelli, R. Cantelli and F. Cordero, Phys. Rev. Lett. 70, 3923 (1993).

[10] P. Diodati, F. Marchesoni and S. Piazza, Phys. Rev. Lett. 67, 2239 (1991).

[11] L. de Arcangelis, S. Redner and H. J. Herrmann, J. Phys. Lett. (Paris) 46, L585 (1985); P. Duxbury, P. D. Beale and P. L. Leath, Phys. Rev. Lett. 57, 1052 (1986).

[12] P. Ray and G. Date, Physica A 229, 26 (1996).

[13] S. Zapperi, P. Ray, H. E. Stanley and A. Vespignani, unpublished.

[14] S. Zapperi, A. Vespignani and H. E. Stanley in Fracture-Instability Dynamics, Scaling and Ductile/Brittle Behavior, edited by R. B. Selinger, J. Mecholsky, E. R. Fuller, Jr. and A. Carlsson (Mat. Res. Soc. Proc. 409, Pittsburgh, 1996) pp. 355-358.

[15] H. J. Tillemans and H. J. Herrmann, Physica A 217, 261 (1995).