Little-Parks oscillations in superconducting ring with Josephson junctions

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Abstract. Nb nano-rings connected serially by Nb wires exhibit, at low bias currents, the typical parabolic Little-Parks magnetoresistance oscillations. As the bias current increases, these oscillations become sinusoidal. This result is ascribed to the generation of Josephson junctions caused by the combined effect of current-induced phase slips and the non-uniformity of the order parameter along each ring due to the Nb wires attached to it. This interpretation is validated by further increasing the bias current, which results in magnetoresistance oscillations typical of a SQUID.

1. Introduction

The Little-Parks magnetoresistance oscillations in superconducting rings have been extensively studied both theoretically and experimentally, see e.g. [1-13]. However, quite frequently the observed oscillations’ waveform deviates from the predicted parabolic oscillations, exhibiting sinusoidal-like oscillations, see e.g. [2-4]. Such deviations can be related to a distribution of the ratio \( \xi/R \) in a wide ring [2] (\( \xi \) is the coherence length and R is the radius of the ring), or to a size distribution of rings in a network [3]. In this article we propose an alternative explanation associated with the existence of Josephson junctions (JJ) in a ring. The existence of such junctions is highly probable in superconducting nano-rings with superconducting leads (‘arms’) attached to them. Such superconducting structures comprising a ring with two arms are common in nano-fabrication in which the arms serve as leads to the ring. As shown by de-Gennes [14] and Alexander [15] the arms cause a non-uniform order parameter along the ring with two minima at equal distances from the connection points of the arms to the ring. In the presence of large enough bias-current, enhanced phase slips at these minima can generate Josephson junctions in the ring [16]. In this paper we show that in such a ring with Josephson junctions, the Little-Parks (LP) oscillations should become sinusoidal. We demonstrate this effect in Nb nano-rings by showing that the parabolic Little-Parks oscillations at low bias currents are switched into sinusoidal oscillations by increasing the bias current.

2. Experimental

E-beam lithography was used to fabricate Nb square loops (340x340 nm²) connected serially by 66 nm wide Nb wires, see right panel of Figure 1. The ring rim (~57 nm) is of the order of the zero
temperature coherence length in Nb, $\xi_0 = 40$ nm. For details of the fabrication process see Ref. [16]. Magnetoresistance measurements were performed using a commercial system (PPMS, Quantum-Design).

3. Results

Current-induced switching of the classical LP parabolic oscillations into sinusoidal ones is demonstrated in the left panel of Figure 1 which shows typical magnetoresistance oscillations measured at $T = 7.1$ K. At low currents (1 µA and below) parabolic LP oscillations are obtained [1], exhibiting upward cusps at odd multiples of $\Phi_0 / 2$, and a field-period of $\sim 170$ Oe, corresponding to the area of a single ring ($\sim 1.2 \cdot 10^{-9}$ cm$^2$). For higher currents, (2 - 3 µA), the cusps disappear and the oscillations become sinusoidal. As we argue below, this change results from generation of Josephson junctions in the rings. A clear manifestation of the existence of these junctions is obtained by further increasing the current to 4 µA, yielding oscillations with downward cusps at multiples of $\Phi_0$, typical of the magnetoresistance response of a SQUID biased with a current that is equal to its maximum critical current [17].

![Figure 1](image.png)

**Figure 1.** Left panel: Magnetoresistance of a single Nb ring measured at $T = 7.1$ K with currents between 1 and 4 µA. The magnetic flux, $\Phi$, is normalized to the quantum flux, $\Phi_0$, and calculated for a ring area of $1.2 \cdot 10^{-9}$ cm$^2$. The solid curves through the data points are guide to the eye. Right panel: A scanning electron microscope image of the Nb rings.

4. Discussion

We begin by considering the effect of a single arm on a ring. As the arm is not affected by the magnetic flux, the order parameter along the ring has a maximum at the connection point and a minimum at the antipodal point. This minimum drops to zero when the flux becomes equal to a half flux quantum, $\Phi = \Phi_0 / 2$ [18, 19]. When two symmetrical arms are connected to a ring, the order parameter is maximum at the connection points and minimum at equal distances from these points [18, 19]. Current-induced phase slips at these minima further reduce the order parameter down to a level required for the creation of effective Josephson junctions.
We recall that screening current in a ring without Josephson junctions is linear with the magnetic flux, with discontinuities at odd multiplications of \( \Phi_0 / 2 \):

\[
(2) \quad I_s^{LP-JJ} = I_c \sin \left( \pi \Phi_0 / \Phi_0 \right) \cos (\pi n); \quad \left( n - \frac{1}{2} \right) \Phi_0 \leq \Phi \leq \left( n + \frac{1}{2} \right) \Phi_0, \quad n = 0, \pm 1, \pm 2, \ldots
\]

as shown by the dashed line in Figure 2. Consequently, the magnetoresistance, which is proportional to the kinetic energy \( I_s^2 \) [20-22] is parabolic (see the dashed line in Figure 3). However, for a ring with Josephson junctions of critical current \( I_c \) the dependence of the screening current \( I_s^{(LP-JJ)} \) on \( \Phi / \Phi_0 \) is [23]:

\[
(2) \quad I_s^{LP-JJ} = I_c \sin \left( \pi \Phi_0 / \Phi_0 \right) \cos (\pi n); \quad \left( n - \frac{1}{2} \right) \Phi_0 \leq \Phi \leq \left( n + \frac{1}{2} \right) \Phi_0, \quad n = 0, \pm 1, \pm 2, \ldots
\]

as described by the solid line in Figure 2. As a result, the magnetoresistance is sinusoidal, as shown by the solid line in Figure 3. The switching of the magnetoresistance waveform from parabolic to sinusoidal in our data can, therefore, be ascribed to the generation of Josephson junctions in the ring due to the combined effects of current induced phase slips and non-uniform order parameter along the ring caused by the superconducting arms. The existence of the Josephson junctions in the ring is clearly manifested by the SQUID-like magnetoresistance oscillations, with cusps down, obtained when the current is increased to 4 µA (see Fig. 1). At this current the magnetoresistance oscillations result from the flux dependence of the critical current of the SQUID rather than by oscillation of the critical temperature due to oscillations of the screening current.

Note that in a conventional SQUID it is assumed that the rim width is larger than the superconducting penetration depth, \( \lambda \), and, therefore, the Little-Parks effect is unobservable. However, in our case, as is usually the case in most nano-rings, the rim width is smaller than \( \lambda \). Thus, magnetoresistance oscillations due to Little-Parks effect, Eq. (2), are expected in such SQUIDs near \( T_c \).

![Figure 2](image)

Figure 2. Flux dependence of the screening current in homogeneous ring and in a ring with Josephson junctions (dashed and solid lines, respectively).
Figure 3. Flux dependence of the superconducting kinetic energy in homogeneous ring and in a ring with Josephson junctions (dashed and solid lines, respectively).

5. Summary and Conclusion
Little-Parks oscillations can transform from parabolic to sinusoidal when a Josephson junction is generated in the superconducting loop. We demonstrated that such an effect can be induced by external current in Nb nano-rings with two arms. Moreover, we demonstrated that such a ring exhibits SQUID-like magnetoresistance oscillations when large enough bias current is applied.

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