Tight Guarantees for Static Threshold Policies in the Prophet Secretary Problem

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In the prophet secretary problem, \( n \) values are drawn independently from known distributions, and presented in a uniformly random order. A decision-maker must accept or reject each value when it is presented, and may accept at most \( k \) values in total. The objective is to maximize the expected sum of accepted values.

We analyze the performance of static threshold policies, which accept the first \( k \) values exceeding a fixed threshold (or all such values, if fewer than \( k \) exist). We show that using an appropriately-defined threshold guarantees a \( \gamma_k = 1 - e^{-k}k^k/k! \) fraction of the offline optimal solution, which equals \( 1 - 1/e \) when \( k = 1 \) and grows asymptotically as \( 1 - \Theta(1/\sqrt{k}) \). This represents the best-known guarantee for the prophet secretary problem for all \( k > 1 \), and we also show that it is tight for all \( k \) if one is restricted to the class of static threshold policies.

We also two provide distinct methods for setting the threshold, which make them implementable under different restrictions on what is known about the distributions. Our first method requires computing a threshold such that the expected number of values exceeding the threshold is equal to \( k \), and offers the guarantee of \( \gamma_k \) if \( k \geq 5 \) (but interestingly, not if \( k < 5 \)). Our second method requires computing a threshold such that \( k \cdot \gamma_k \) values are accepted in expectation, and offers the guarantee of \( \gamma_k \) for all \( k \). To establish these guarantees, we prove a new result in Bernoulli optimization about sums of independent Bernoulli random variables, which extends a classical result of Hoeffding (1956) and could be of general interest.

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CCS Concepts: · Theory of computation → Online algorithms.

Additional Key Words and Phrases: prophet inequalities, prophet secretary problem

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