Charting standard concrete based on the theory of degradation

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Abstract. The article describes the simple dependence of the behavior of concrete based on the theory of degradation of the object in time. The revealed dependence is most closely meet the requirements of the design standards. Based on two simple dependences analyzed the dependences of the energy and elastic characteristics of concretes offered by modern design standards. Received characteristics power for case of rectangular and trapezoidal distributions of energies. Each of the characteristics analyzed separately. Obtained the theoretical values for the initial elastic zone and elastic modulus. The article describes the general equation describing diagrams behavior of concrete. Putting into the notion the greatest theoretical deformations. The greatest value of the theoretical deformations were calculated. For the first time proposed the General theory of the destruction of the object in time.

1. Introduction
At the present stage of the deformation diagram of the material play a significant role in the assessment, in the calculations and analysis of the behavior of materials [1, 2]. Questions of the material behavior during exposure to aggressive environments and temperatures continue to be the center of attention [3]. The need in the construction of modern high-strength materials also is one of the first places [4]. Reinforced concrete structures continue to occupy leading positions in construction. Therefore, all mentioned above, is directly related to them [5]. It should be noted that the regulation and the description of the concrete in different countries differs slightly among themselves. All constructs are based on the description of experimental data the most statistically close convenient for operation form. Still not revealed the true of the diagram of concrete. Same can be said about almost all materials used in construction. The theory of degradation to answer the deep questions of the material behavior in time.

2. Features of the model theory of degradation
The difference between the theory of degradation from existing theories in that is not considered here, the internal structure of the object. It is assumed that in all objects there is some finite set of energies, which determine the properties of the object. For each type of energy is adopting its own form of distribution in time. the simplest of shapes – rectangular, to the more sophisticated triangular and further to more complex forms described, for example, a power series. Energy of an object consists of some of the more simple forms of the energy distribution. In this case, we can decompose the well-known diagrams of the behavior of the object in a more simple and understandable form from the point of view of the behavior of energy. As the analysis shows, in this case, it becomes clear the term total destruction – as a
complete destruction of the internal connections of the individual components of the object. The same preliminary analysis showed that the basis for the analysis of the distribution of energy can be the simplest rectangular form distribution. This form of energy gives the average shape of the potential of all forms of energy distribution as deviations from it. Thus, the behavior of almost all generalized objects can be analyzed on the basis of this simplest form of distribution of the energy characteristics.

Should add, that the energy approach is often used when analysis the graphs the behavior of concrete [6,7].

3. The approach to building models

Discussed the concrete in which internal energy exchange has not happening. If we consider the standard concrete at 28 days age, for them, the theory will be true, but built on the basis of its model in another age will differ from the standard. For hardening concrete occurs internal and external exchange of energy. Will have to build two models, and then unification them.

Previously performed analysis [8] revealed a complex pattern of energy distribution in the concrete specimen. Traced three zones of distribution of energy. The initial zone with drastic changes in the values of acceleration energy (the area of unstable operation, which accounts for about 0.2...0.3 of the breaking load). In this area, the slightest change in the estimated parameters lead to significant changes in the definition of the energy characteristics of concrete. We can assume that the deformation within this zone can be restored. The analysis shows that in this zone there is a value of the modulus E, wherein the initial area is rectified and included in the five percent interval error (error during the graphical processing of the data). On the other hand, this area shows the arbitrariness of the adoption of the initial modulus of the elastic material or the insufficient accuracy of measurements in the initial stage of the work material. All this confirms the complex nature of deformation in such a heterogeneous material such as concrete. The second zone - the zone of stability, within the interval of error of 5% was in the range from 0.3 to 0.75 on a scale of relative voltage. And the third zone is a smooth increase the value of acceleration energy. It is possible to assume that in the third zone to happen of the influence of crack growth (on a diagram of concrete). The third area can be described inclined straight or slightly curved line. In moment the destruction the third zone a diagram of concrete is bent larger. Zones are correlated with the parametric points of the concrete [9,10].

To assess the applicability of the theory of degradation for the making up diagram concrete - propose be using the a two most simple models of the theory of degradation [11]:

The model with rectangular shape of power distribution

\[ \sigma = B_1 (L \ln (\varepsilon/\alpha_1) - \varepsilon + \alpha_1), \]  
(1)

and model summation of rectangular and triangular forms of power distribution

\[ \sigma = L \left( B + \frac{C}{2} \right) \ln \varepsilon - \frac{C}{4L} \varepsilon^2 - B\varepsilon + \left[ \frac{ca^2}{4L} + B\alpha - L \left( B + \frac{C}{2} \right) \ln \alpha \right]. \]  
(2)

In the proposed models \( L \) - strain of concrete corresponding to its prism strength; \( B \) - characteristic, for rectangular shaped distribution of the power, \( C \) - characteristic, for triangular shape of the distribution of power \( a \) - the value of the initial elastic zone concrete.

This dependence can be rewritten, revealing the influence of rectangular and triangular parts of the power distribution to the graph of concrete:

\[ \sigma = \left\{ \left[ B L \ln \varepsilon - B\varepsilon + (B\alpha - B L \ln \alpha) \right] + \left[ + \frac{C L}{2} \ln \varepsilon - \frac{C}{4L} \varepsilon^2 + \frac{ca^2}{4L} - \frac{C L}{2} \ln \alpha \right] \right\}. \]  
(3)

It should be noted that in the following dependencies (1) - (3) the values \( B \) and \( C \) do not depend on time \( (L \) and \( a) \). In other words, the energy characteristics independent of the temporal characteristics. The dependence between \( B, C \) and \( L \) to result to via voltage.

4. The results build the model

As a reference take a diagram work of concrete, given in the sources [12,13]. The chart on the dependencies (1) and (2) is built on two points of the reference diagram corresponding the levels of loading for 0.2 and 1. In Fig.1 shows eleven diagrams of deformation of concrete B40, diagrams numbered 1, 2,
4, 5, 8-11, built on dependence (2) when the values of $\alpha = 12, 11, 10, 7.5, 2.5, 1, 0.5 \times 10^{-5}$ respectively. Graph 3 is constructed by the formula (1) for the value $\alpha = 10^{-5}$.

$$L(\sigma_0, V/C) = \frac{\sigma_0^2 V}{C}$$

![Figure 1. Comparison of diagrams of deformation of concrete.](image)

As can be seen the theoretical values obtained for a rectangular distribution of the acceleration energy, lie above the experimental plots. The increase in the elastic zone is more than $\alpha = 10 - 10^{-5}$ leads to significant deviations from normative and experimental diagrams for a trapezoidal distribution of energy and for a rectangular distribution. The area reduction of the elastic work for a trapezoidal distribution has led to the almost complete coincidence with the schedule given in the rulebook of the Russian Federation SR 63.13330.2012 "Concrete and reinforced Concrete structures. Basic provisions" (with $\alpha = 7$ standard deviation was 2.1%, with $\alpha = 6$ is 1.8%, with $\alpha = 5$ and $\alpha = 8 - 2.5\%$) and with our experimental graph (with $\alpha = 5$ the standard deviation is 1.3% for $\alpha = 6 - 2.6\%$). Here we must note that our experimental schedule was obtained for the concrete age of 180 days. For rectangular distribution, the reduction of the elastic zone of concrete with little impact on the change in the shape of the graph. Curves obtained by the formula (2) for the trapezoidal distribution of the acceleration energy when the values of $\alpha$ smaller 5 practically merge into a single line. Get the energy characteristics of behavior charts for known classes of concrete. Characteristics calculated from the original diagrams SR 63 along two points (when the voltage levels of 0.2 and 1) for the equation (1) and three points (at the stress level of 0.2; 0.5 and 1) for the equation (2). For ascending lines of the original diagrams. Obtained values are given in table 1.

| Feature | B5 | B10 | B100 | B20 | B30 | B40 | B50 | B60 | B70 | B80 | B90 | B100 | B100 |
|---------|----|-----|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $R$ (MPa) | 3.5 | 7.5 | 15   | 22  | 29  | 36  | 43  | 50  | 57  | 64  | 71  | 71  |      |
| $E_{10^5}$ (MPa) | 13 | 19 | 20 | 27.5 | 32.5 | 36 | 38 | 39.5 | 41 | 42.0 | 42.5 | 43 | 43 |
| $L(\sigma_0, \cdot 10^{-5})$ | 174 | 196 | 170 | 198 | 203 | 211 | 224 | 237 | 249 | 268 | 278 | 293 | 280 |
| $B$ (MPa) | 633 | 1352 | --- | 3101 | 4879 | 6666 | 8271 | 9812 | 11312 | 12818 | 14066 | 15200 | --- |
| $a_1 \cdot 10^{-5}$ | 2.7 | 4.3 | --- | 6.5 | 8.4 | 10.4 | 12.5 | 14.6 | 16.6 | 19.2 | 21.5 | 23.7 | --- |
| $E_1 \cdot 10^{-5}$ (MPa) | 40 | 60 | --- | 91 | 113 | 129 | 140 | 150 | 158 | 166 | 168 | 173 | --- |
| $\sigma$ (MPa) | -152 | +1452 | +2494 | +12698 | +25945 | +37701 | +50372 | +62230 | +77319 | +80586 | +92979 | +101937 | +11541 |
| $B$ (MPa) | +724 | +485 | +120 | -4583 | -11159 | -16875 | -23566 | -29850 | -38315 | -39332 | -46436 | -51266 | -59341 |
| $\alpha \cdot 10^{-5}$ | 2.8 | 4.1 | 3.9 | 5.7 | 7.1 | 6.8 | 6.5 | 6.2 | 3 | 6.8 | 2.3 | --- | --- |
| $E_{20} \cdot 10^{-5}$ (MPa) | 41 | 54 | --- | 68 | 86 | 100 | 112 | 120 | 128 | 158 | 166 | 168 | 173 |
| $B/C$ | -4.8 | +0.25 | 0.05 | -0.24 | -0.30 | -0.36 | -0.44 | -0.47 | -0.49 | -0.49 | -0.5 | -0.51 | -0.51 |
| $2B/C$ | -9.5 | +0.5 | 0.1 | 0.48 | 0.60 | 0.72 | 0.88 | 0.94 | 0.98 | 0.98 | -1 | -1.028 | 1.028 |
| $\sigma_m$ (MPa) | -1.12 | -3.36 | -3.7 | -6.34 | -7.99 | -9.12 | -8.30 | -7.29 | -2.10 | -7.58 | -1.19 | +0.41 | +11.42 |
| $L_{10\sigma}$ (MPa) | >1000 | 690 | --- | 485 | 450 | 460 | 470 | 490 | 502 | 545 | 555 | 580 | --- |
Calculation of the characteristics with use the other base points gives some other values than those values listed in the dependency. Overall, however, the direction of change of these characteristics remains unchanged.

The table also includes the values of the ultimate strain of concrete $L_U$ (the end of the descending line charts – the line crosses the graph with the axis of deformation) obtained according to (2). The value $a$ is determined based on values of the free members of the equations, but if they have a positive value of variable is not defined. Therefore, in the following table to show and the values of free members of the equations. The moduli of elasticity $E_1$ and $E_2$ calculated in accordance with equations (1) and (2) as the tangents at the points $x = a$. In the table and in subsequent equations are substituted strain is increased to $10^5$, voltages are obtained in MPa.

Calculated in accordance with the table 1, dependencies for concretes of different classes get the following appearance:

$$
\sigma_{B5} = + 1.128 \ln \varepsilon + 0.0000022 \varepsilon^2 - 0.00724 \varepsilon - 1.12
$$

$$
\sigma_{B10} = + 2.370 \ln \varepsilon - 0.0000185 \varepsilon^2 - 0.00485 \varepsilon - 3.36
$$

$$
\sigma_{B20} = + 3.500 \ln \varepsilon - 0.0000160 \varepsilon^2 + 0.04583 \varepsilon - 6.34
$$

$$
\sigma_{B30} = + 3.681 \ln \varepsilon - 0.0000320 \varepsilon^2 + 0.11159 \varepsilon - 7.99
$$

$$
\sigma_{B40} = + 4.160 \ln \varepsilon - 0.0000447 \varepsilon^2 + 0.16875 \varepsilon - 9.12
$$

$$
\sigma_{B50} = + 3.620 \ln \varepsilon - 0.0005620 \varepsilon^2 + 0.23566 \varepsilon - 8.30
$$

$$
\sigma_{B60} = + 3.000 \ln \varepsilon - 0.0005600 \varepsilon^2 + 0.29850 \varepsilon - 7.29
$$

$$
\sigma_{B70} = + 0.858 \ln \varepsilon - 0.000763 \varepsilon^2 + 0.38315 \varepsilon - 2.10
$$

$$
\sigma_{B80} = + 2.576 \ln \varepsilon - 0.000751 \varepsilon^2 + 0.39332 \varepsilon - 7.58
$$

$$
\sigma_{B90} = + 0.149 \ln \varepsilon - 0.0008361 \varepsilon^2 + 0.46436 \varepsilon - 1.19
$$

$$
\sigma_{B100} = - 0.872 \ln \varepsilon - 0.0008698 \varepsilon^2 + 0.51266 \varepsilon + 0.41
$$

5. Consideration

1. With increasing concrete strength has been increasingly influenced by the second and third members of the equation. For concrete class B100 is impeding the logarithmic members of the equation in the description of the chart. Conversely, concrete B5 and B10 dominated by the influence of the logarithmic members of equation.

2. According to the data obtained using equation (1) with the increasing of class of concrete will occur proportional to the increase in the total energy performance of concrete $B^*$ and the elastic zone to work of concrete $a^*$. Other features, obtained according to equation (1), is not revealed.

3. The calculation of the energy characteristics according to (2) leads to the separation of the classes concrete along these energy characteristics. The first area - concrete class below B20. In this area there is a change of signs of the characteristics $B$ and $C$ and the change in the characteristics of $B/C$. The Second area is a normal concrete. And the third area of the concrete class above B90. In this zone, the magnitude characteristics $2B/C$ becomes greater than one and the value of the elastic zone to work of concrete $a$ is not determined.

4. The change in the characteristics $B$ and $C$ is almost linear. With the failure occurs after B70. Similar data are obtained when processing the data in accordance with European standards.

5. Value $B/C$ characterizes the deformation in the diagram in which the values $B$ and $C$ become equal (compaction and decompaction are aligned). The value of $2B/C$ characterizes the deformation at which "density" of concrete overcomes the "density" of the initial structure of concrete.

6. The value of $a$, $B$ and $C$ characterize the parametric point of concrete. As noted in [8] the curvature of the graph depends not only on strength characteristics of concrete, but also from its structure. Therefore, the parametric point for concretes of different composition will slightly differ from those obtained in this work. The model parameters will also depend on the structure of the material.

As an example, we present the structure of the components of the formula for concrete B40 in accordance with formulas (1), (2) and (3) which can be written, for is identifying the effect of rectangular and triangular parts of the acceleration energy

$$
\sigma_{B40} = + 14.065 \ln \varepsilon - 0.06666 \varepsilon - 32.24
$$

$$
\sigma_{B40} = + 4.160 \ln \varepsilon - 0.0004470 \varepsilon^2 + 0.16875 \varepsilon - 9.12
$$

$$
\sigma_{B40} = \begin{cases} 
- 35.606 \ln \varepsilon + 0.16875 \varepsilon + 67.11 \\
+ 39.766 \ln \varepsilon - 0.0004470 \varepsilon^2 - 76.23 
\end{cases}
$$
Equation (4) from of solved by equation (1), equation (5) by equation (2), equation (6) by equation (3). It is seen that in independent parts of the equations (4) and (6) big influence of the free term of the equation and logarithmic functions. In the combined equation (5) the impact of these members compensate.

Graphs of functions in comparison with the curvilinear charts, constructed according to sets of rules SR63.13330.212 shown in figure 2. Analysis diagrams for the energy parameters depends on the accuracy of their construction. From of the exact build can be identified and the impact of higher degrees of distribution of acceleration energy on the material behavior.

For comparison we present the results of our tests of concretes with polymeric binders (fillers - crushed stone and sand are taken as concrete with cement binder): strength 65 MPa at deformation $14\cdot10^{-3}$, values $C=2370$ MPa, $B=357$ MPa, $\alpha=3810-5$. Accordingly, the first parametric points (characteristic) of the ultrasound examinations by did not detect.

![Figure 2 Diagrams for concrete of class B20, B70... (solid lines according to SR 63) and theoretical (dotted)](image)

On the drop-down branches of the diagram marked out the level $0.85 R_b$ (figure 2 leased line). For concretes with high and low strength grade of divergence of the theoretical and normative charts at this level is smaller than for concretes of medium (B40, B50) classes strength.

6. Theoretical determination of the maximum deformations of concrete

The greatest theoretical deformation $L_U$ (the point of intersection of the pull-down branches with the axis of deformation) obtained for concrete B20 to compiles $500\cdot10^{-5}$ (for formula SR 63 more than $1000\cdot10^{-5}$), for concrete B70: - $534\cdot10^{-5}$. Table 2 below lists the values of these deformations obtained along different points of the chart. Usually these values are impossible to obtain through standard samples tests, and they are not considered when creating charts. However, as will be shown later, these values are important in determining the creep deformation of concrete. This is the deformation at which concrete ceases completely to resist external influences.

| B20  | B30  | B40  | B50  | B60  | B70  |
|------|------|------|------|------|------|
| 485...502 | 450...502 | 460...505 | 470...512 | 490...519 | 502...534 |

The following is the equation for a concrete grade B40 with a maximum relative deformation $L(\varepsilon_R)$ corresponding to its class strength equal to $300\cdot10^{-5}$, $340\cdot10^{-5}$ and $400\cdot10^{-5}$. We can conclude the existence of a maximum deformation of $L(\varepsilon_R)$ under which the logarithmic and the free terms of the
equation becomes zero. With the growth of the maximum deformation the influence members of the equations, having degree, reduced

\[
\begin{align*}
\sigma_{B40-211} &= + 4.160 \ln \varepsilon - 0.00045 \varepsilon^2 + 0.1688 \varepsilon - 9.12 \\
\sigma_{B40-300} &= 0.4785 \ln \varepsilon - 0.00029 \varepsilon^2 + 0.1726 \varepsilon - 2.1 \\
\sigma_{B40-340} &= - 0.0085 \ln \varepsilon - 0.00024 \varepsilon^2 + 0.1622 \varepsilon - 1.46 \\
\sigma_{B40-400} &= - 0.962 \ln \varepsilon - 0.00019 \varepsilon^2 + 0.1535 \varepsilon + 0.7
\end{align*}
\]

We use dependences to determine the ultimate strain of concrete. For concrete grade B40, the smallest module (capacity) in accordance with the standard chart will be \(\tan \alpha = \sigma / \varepsilon = 29/211 \cdot 10^{-5} = 0.13744\) MPa. On a curve of the potential for concrete B40 (\(B_n = 6.88/\varepsilon - 0.00066\varepsilon + 0.10639\)), we obtain the corresponding deformation of \(81.3 \cdot 10^{-5}\) and the stress \(\sigma = 21.1\) MPa. In fact, long-term strength of concrete will be \(29 \times 0.9 = 26.1\) MPa, which corresponds to the potential 7262 MPa and ultimate deformation of concrete \(\varepsilon_{\text{cr}}\); for voltages of 26.1 MPa equal to 26.1/7262 = 359 \cdot 10^{-5} for the voltages 29 MPa is equal to 29/7262 = 399 \cdot 10^{-5}. The obtained values are in good agreement with known values of the ultimate strain. Therefore, in the standard diagram is already laid possibility to determine the long-term strength and deformation of concrete. The strength of concrete on the basis of external influences is determined in the desired time period. The corresponding deformations of the concrete are determined along by the curves of deformation of concrete and of potential curves of concrete.

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