Type I Vacua from Diagonal $Z_3$-Orbifolds

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Abstract

We discuss the open descendants of diagonal irrational $Z_3$ orbifolds, starting from the $c = 2$ case and analyzing six-dimensional and four-dimensional models. As recently argued, their consistency is linked to the presence of geometric discrete moduli. The different classes of open descendants, related to different resolutions of the fixed-point ambiguities, are distinguished by the number of geometric fixed points surviving the unoriented projection.

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1 Introduction

Open superstrings models descend from oriented closed superstrings [1, 2, 3, 4], and display the same level of consistency of their heterotic counterparts. However, for several years they have been considered less attractive, since it was believed that weakly-coupled heterotic superstrings compactified on Calabi-Yau threefolds could provide a natural bridge between a unified theory of all interactions and the Standard Model [5]. Even the unusual low-energy properties of open-string models [3], a first clear indication of their potential, were not immediately appreciated. This viewpoint has dramatically changed since weak/strong coupling dualities have effective led to the unification of all five known superstrings (heterotic, Type I and Type II), that together with the eleven-dimensional supergravity are now regarded as different asymptotic expansions of M-theory [6]. Moreover, the perturbative oscillations of D-branes, BPS solitons carrying Ramond-Ramond charges [7] that play a central role in the web of string dualities, are described by open superstrings. These findings have completely changed our picture of how the Standard Model should be embedded within string theory or M-theory. To wit, while in a heterotic susy-GUT scenario the string scale and the Planck scale are essentially identified, in Type I vacua both the string scale and the dimensions felt by gauge interactions could be as low as a few TeV. Moreover, in Type I vacua the gauge fields generally invade only some of the dimensions, while the remaining ones, felt only via gravitational interactions, could be far larger, and even of millimeter size [8]. In this new Kaluza-Klein scenario, often called “Brane World” [9], several fundamental issues like the gauge hierarchy problem and the nature of supersymmetry breaking have to be reconsidered. At any rate, Type I vacua have a central role in this setting, but only a limited number of them has been explored so far.

In this paper we construct a class of six and four dimensional Type I vacua from “nongeometric” $Z_3$ orbifolds of the parent Type IIB theory. The more familiar “geometric” Type I vacua are open descendants of Type IIB models obtained modding out the closed spectrum with the world-sheet parity operator $\Omega$, that interchanges left and right moving sectors, and adding suitable twisted states, unoriented open superstrings. From a Conformal Field Theory point of view, their construction translates into a set of rules based on sewing constraints that, for a given left-right symmetric model, allow in general several possible descendants [10, 11, 12, 13, 14]. One has indeed the freedom of changing the unoriented truncation of the closed sector (the Klein-bottle amplitude) compatibly with the “crosscap constraint” [12, 13, 14, 15] and of adding suitable boundary-states, that may or may not [16] respect all the symmetries of the bulk. The final ingredient is the solution of tadpole conditions that fix (partly) the Chan-Paton gauge groups. This brings about a number of surprises. For instance, one can have Type I models without open strings [17, 18], while unconventional Klein-bottle projections can even require that supersymmetry be broken to lowest order in the open sector [19, 20] as a result of the simultaneous presence of branes and anti-branes. These last models are the first in
which supersymmetry breaking is a consistency condition rather than an option.

There is another possibility, that consists in “dressing” Ω with the action of involutions I of order two. Again, from a Conformal Field Theory point of view, this is equivalent to applying the construction not to the “geometric” Type IIB, but to a different parent theory based on a different, non geometric, GSO projection. Rational models of this type were studied very early in [3], but a closer look at the relation between the different approaches can be very useful, and in particular shed some light on the generalization of the intuitive concepts of D-branes and O-planes to non geometric settings [16, 21].

To be less generic, let us consider the open descendants of the Type IIB superstring compactified on a $Z_3$ orbifold to $D = 6$. The “geometric” torus partition function corresponds to the so-called “charge conjugation” modular invariant. The open descendants of this model have been constructed by several authors both at rational [17] and at generic points of moduli space [18, 22]. Recently, another class of open descendants was constructed in [23, 24] combining Ω with a conjugation of the complex internal coordinates (and with a corresponding action on the fermions). The resulting models, already known as open descendants of Gepner models [17], i.e. at special points of moduli space, exhibit some interesting properties. First, the spectrum includes twisted open strings, and this feature, common to other non-geometric Type I vacua [25], can be interpreted in terms of D7-branes at angles. Second, their consistency rests on the presence of quantized “geometric” moduli (the off-diagonal components of the target-space metric). These are also responsible for the rank reduction of the Chan-Paton groups [24], much in the same way as the $B$-field (now a continuous deformation) is in the conventional case [4, 26, 27]. In this paper we explore further the world sheet structure of these models, and show how they may be regarded as open descendants of theories whose GSO projections correspond to “diagonal” modular invariants. We also extend the construction to four dimensional models that have similar properties.

The plan of the paper is the following. In Section 2 we begin by discussing the open descendants of the bosonic $c = 2$ diagonal $Z_3$-orbifold. This is useful to address the behavior of the irrational deformations. In Section 3 we recover the six-dimensional models of [23, 24] and extend the procedure to investigate additional models in four dimensions. Section 4 is devoted to our conclusions and to some conjectures. Notation and conventions are illustrated in Appendix A.

2 The $c = 2$ diagonal $Z_3$-orbifold and its open descendants

A pair of bosonic fields compactified on a $Z_3$-orbifold of a two-torus can be described by a single complex field $Z$ modded out by the action of the orbifold group that, in the $k$-th twisted sector, is

$$Z(\sigma + 2\pi, \tau) = \omega^k Z(\sigma, \tau)$$

(1)
where $\omega = e^{2i\pi/3}$. The partition function of the resulting theory can be written in terms of a sesquilinear combination of chiral blocks (see Appendix A). In particular, the geometric orbifold corresponds to the so called “charge conjugation” modular invariant

$$T_{cc} = \frac{1}{3} \left[ \Phi_{00} \Phi_{00} \Lambda + \Phi_{01} \Phi_{01} + \Phi_{02} \Phi_{02} + 3 (\Phi_{10} \bar{\Phi}_{20} + \Phi_{11} \bar{\Phi}_{11} + \Phi_{12} \bar{\Phi}_{12}) 
+ 3 (\Phi_{20} \bar{\Phi}_{10} + \Phi_{21} \bar{\Phi}_{12} + \Phi_{22} \bar{\Phi}_{11}) \right],$$

where $\Lambda$ is the Narain lattice sum and the factors of three are connected to the three points fixed under the $Z_3$ action. Denoting by $m_i$ and $n_i$ respectively momenta and winding modes, and letting

$$p = \frac{1}{\sqrt{2X_2 Y_2}} \left[ X m_1 - m_2 - Y ( n_1 + X n_2 ) \right],$$

$$\bar{p} = \frac{1}{\sqrt{2X_2 Y_2}} \left[ X m_1 - m_2 - Y ( n_1 + X n_2 ) \right],$$

where $X = X_1 + iX_2$ and $Y = Y_1 + iY_2$ are the two complex moduli of the target two-torus, connected with metric and antisymmetric tensor by the relations

$$g = \frac{\alpha^\prime Y_2}{X_2} \left( \frac{1}{X_1} \frac{X_1}{|X|^2} \right), \quad B = \alpha^\prime Y_1 \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right),$$

the lattice sum is

$$\Lambda = \sum_{(p,\bar{p}) \in \Gamma_{2,2}} q^{\frac{|p|^2}{2}} q^{\frac{|\bar{p}|^2}{2}}.$$

We set for simplicity $B = 0$ (i.e. $Y_1 = 0$), and once the lattice has been normalized to have basis vectors of length 2 in $R^2$ units, it is easy to recognize that a possible choice for a sensible $Z_3$ action is $X_1 = -1/2$, $X_2 = \sqrt{3}/2$ and $Y_2 = \sqrt{3}R^2/\alpha^\prime$. The open descendants of this geometric orbifold have been analyzed in several contexts, and result in a class of theories with only Neumann strings (i.e. excitations of D25-branes if one refers to the bosonic critical string theory, or of D9-branes in Type IIB) in the open sector and with (bulk) twisted sectors flowing in the tree channel, including “massless” ones.

In order to produce a diagonal GSO projection, one has to combine the orbifold action (1) with an involution that conjugates the eigenvalues of the right-moving coordinates. As observed in [23], this diagonal action is natural on the complex field obtained T-dualizing one of the real components of the field $Z$, rather than on the field $Z$ itself. The resulting open descendants, when dressed with Type II critical superstring coordinates, are sometimes referred to as Type I' models, and may be regarded as orientifolds of the Type IIA superstring [28]. Coming back to the bosonic model, the torus partition function displays neatly the diagonal combination

$$T_d = \frac{1}{3} \left[ \Phi_{00} \Phi_{00} \Lambda + \Phi_{01} \Phi_{01} + \Phi_{02} \Phi_{02} + 3 (\Phi_{10} \bar{\Phi}_{10} + \Phi_{11} \bar{\Phi}_{11} + \Phi_{12} \bar{\Phi}_{12}) \right] + 3 (\Phi_{20} \bar{\Phi}_{20} + \Phi_{21} \bar{\Phi}_{21} + \Phi_{22} \bar{\Phi}_{22}),$$

where

$$\Phi_{00} \Phi_{00} \Lambda + \Phi_{01} \Phi_{01} + \Phi_{02} \Phi_{02} + 3 (\Phi_{10} \bar{\Phi}_{20} + \Phi_{11} \bar{\Phi}_{11} + \Phi_{12} \bar{\Phi}_{12}) \right].$$
as also apparent when the same expression is written in terms of “characters” (see Appendix A)

\[
\mathcal{T}_d = \frac{1}{3} \left[ \Phi_{00} \Phi_{00} A' \right] + \chi_{00} + \chi_{01} \Phi_{00} + \chi_{02} \Phi_{00} + 3 \left( \chi_{10} \Phi_{10} + \chi_{11} \Phi_{11} + \chi_{12} \Phi_{12} \right) + 3 \left( \chi_{20} \Phi_{20} + \chi_{21} \Phi_{21} + \chi_{22} \Phi_{22} \right),
\]

(7)

where the prime denotes a lattice sum without the zero-mode contribution. The crucial point is now to understand how T-duality [29], that is a symmetry of the toroidal partition function, can be combined with \( \Omega \) on the lattice. Typically, this can be done using a reflection with respect to some symmetry axis of the orbifold [23], and in the \( Z_3 \) case one has two choices. The first corresponds to a reflection \( I_1 \) with respect to the diagonal of the unit cell (or, equivalently, with respect to the vertical axis) and leaves invariant all the three fixed points. The second, \( I_2 \), is a reflection with respect to the horizontal axis, and leaves invariant only the origin. The two choices are connected to different resolutions of “fixed-point ambiguities” [30] and give rise to different modular invariants, and thus to different Klein-bottle projections. In ref. [24] the reflection \( I_2 \) was related to \( I_1 \) on a lattice rotated by an angle \( \pi/6 \). If the T-duality is associated to \( I_1 \), the \( \Omega \) projection fixes states that satisfy the \( p = \bar{p} \) condition, equivalent to a slice of the lattice with \( n_2 = 0, m_1 = 0 \), with conventions chosen in such a way that the complex coordinate \( z \) corresponds to \( i(x_1 + ix_2) \). The closed twist produces the amplitudes \( (\Phi_{g,h} - h' \delta_{g,g'}) \) from the bulk term \( (\Phi_{g,h} \bar{\Phi}_{g',h'}) \), and the resulting Klein-bottle amplitude is

\[
\mathcal{K} = \frac{1}{2} \left[ \sum_{m,n} \left( e^{-2\pi t} \frac{X_2 Y_2 [m^2 + Y_2^2 n^2]}{\eta^2 (2it)} + 3 \Phi_{10} + 3 \Phi_{20} \right) \right].
\]

(8)

Notice that eq. (8) exactly symmetrizes the \( \mathcal{T}_d \) amplitude, as neatly evidenced by the corresponding expression in terms of characters

\[
\mathcal{K} = \frac{1}{2} \left[ \sum_{m,n} \left( e^{-2\pi t} \frac{X_2 Y_2 [m^2 + Y_2^2 n^2]}{\eta^2 (2it)} + \chi_{00} + \chi_{01} + \chi_{02} + 3 \left( \chi_{10} + \chi_{11} + \chi_{12} \right) + 3 \left( \chi_{20} + \chi_{21} + \chi_{22} \right) \right) \right],
\]

(9)

where again the primed sum means that we are extracting the term corresponding to \( m = n = 0 \). An S modular transformation exposes the tree channel

\[
\mathcal{K} = \frac{2^{D/2}}{2} \left[ 2X_2 \sum_{m,n} \left( e^{-2\pi t} \frac{X_2 Y_2 [m^2 + Y_2^2 n^2]}{\eta^2 (i\ell)} + \sqrt{3} \Phi_{01} + \sqrt{3} \Phi_{02} \right) \right],
\]

(10)

where, as usual, the powers of two account for additional dimensions and come from the (omitted) modular measure. Because of T-duality, the lattice sum in (8) contains both momenta and windings. Consequently, the transverse channel depends on a ratio of volumes, and is independent of the radius \( R \). The consistency of (10) is precisely
linked to the value of $X_2$. It generalizes to the irrational case the property, familiar from Rational Conformal Field Theory, that only the identity appears in the transverse channel of diagonal invariants. The “massless contribution” is indeed

$$\tilde{K}_0 = \frac{2^{D/2}}{2} - 3\sqrt{3} \chi_{00},$$

as expected from the complete projector in the tree channel.

To construct the annulus amplitude, two observations are in order. First, the transverse annulus must contain only the states that can be reflected from a boundary or, equivalently, that are paired with their conjugates in the bulk GSO. This amounts to selecting terms with $p = -\bar{p}$. From (3) and the form of $I_2$, it is easy to see [24] that only $m_1$ and $n_2$ survive the projection, but with the additional condition that both $2X_1m_1$ and $2X_1n_2$ must be even. Second, the Chan Paton matrices reflect the structure of the orbifold group [2], and in the $Z_3$ case one has

$$Tr[A_k] = N + \omega^k M + \bar{\omega}^k \bar{M}.$$  

In the diagonal annulus amplitude, however, we expect both untwisted and twisted chiral blocks, without projections. In other words, the traces of the breaking matrices $A_k$ with $k \neq 0$ should vanish, allowing only a single Chan Paton charge. Introducing a suitable projector, the transverse channel annulus becomes

$$\tilde{A} = \frac{2^{-D/2}}{2} \frac{N^2}{2} \left[ \frac{X_2}{2} \sum_{\epsilon_1, \epsilon_2 = 0, 1} \sum_{m,n} \left( e^{-2\pi \ell} \right) \frac{X_2}{4} \left[ m^2 + Y_2^2 n^2 \right] e^{2\pi i X_1 (m\epsilon_1 + n\epsilon_2)} \eta^2(\ell) \right] + \sqrt{3} \Phi_{01} + \sqrt{3} \Phi_{02}$$

where the completeness of the projector is due to the zero mode contributions that supply exactly the necessary factor of 4 in front of the $\eta^{-2}$ term. The “massless contribution” is then

$$\tilde{A}_0 = \frac{2^{-D/2}}{2} \frac{N^2}{2} 3\sqrt{3} \chi_{00}.$$  

As usual, the direct channel annulus amplitude exhibits windings and momenta shifted by the (quantized) value of $X_1$

$$A = \frac{N^2}{2} \left[ \sum_{\epsilon_1, \epsilon_2 = 0, 1} \sum_{m,n} \left( e^{-2\pi \ell} \right) \frac{X_2}{\eta^2(\ell)} \left[ (m+X_1\epsilon_1)^2 + Y_2^2 (n+X_1\epsilon_2)^2 \right] + 3 \Phi_{10} + 3 \Phi_{20} \right],$$

and in terms of characters is

$$A = \frac{N^2}{2} \left[ \chi_{00} + \chi_{01} + \chi_{02} + \sum_{\epsilon_1, \epsilon_2 = 0, 1} \sum_{m,n} \left( e^{-2\pi \ell} \right) \frac{X_2}{\eta^2(\ell)} \left[ (m+X_1\epsilon_1)^2 + Y_2^2 (n+X_1\epsilon_2)^2 \right] \right]$$

$$+ 3 \left( \chi_{10} + \chi_{11} + \chi_{12} \right) + 3 \left( \chi_{20} + \chi_{21} + \chi_{22} \right).$$

(16)
Sector by sector, the transverse Möbius amplitude is the geometric mean of the transverse annulus and Klein bottle amplitudes, with signs needed to normalize correctly the projector \[4, 26, 27, 24\]

\[\tilde{M} = 2 N^2 e^{i \pi} \sum_{\epsilon, \epsilon' = 0,1} \sum_{m,n} \gamma_{\epsilon, \epsilon'} \left( e^{-2 \pi \ell} \right)^{X_2[m^2 + Y_2^2 n^2]} e^{2 \pi i (m \epsilon + n \epsilon')} \eta^2 (i \ell + \frac{1}{2}) + \sigma_{01} \sqrt{3} \Phi_{01} + \sigma_{02} \sqrt{3} \Phi_{02} \right] . \tag{17}\]

Notice that, barring the overall phase that appear in the definition of hatted quantities (see Appendix A), a sensible projector requires \(\sigma_{00} = \sigma_{01} = \sigma_{02}\) and the condition

\[\sum_{\epsilon, \epsilon' = 0,1} \gamma_{\epsilon, \epsilon'} = 2 \quad , \tag{18}\]

in such a way that the “massless” term includes only the “hatted” identity:

\[\tilde{M}_0 = \frac{2N}{2} \sigma_{00} 3 \sqrt{3} \hat{\chi}_{00} \quad . \tag{19}\]

It should be appreciated that the transverse “massless” terms of \(\tilde{K}, \tilde{A}\) and \(\tilde{M}\) combine to give a perfect square. It is also nice that the three amplitudes must contribute the same coefficient, so that the size of the Chan-Paton group in a critical model would depend solely on the additional coordinates, with a rank reduction by a factor of two, as already observed in [24]. As noticed before, the reflection coefficients in front of boundaries and crosscaps are independent of the radius \(R\). In Type I vacua the sign \(\sigma_{00}\) would be fixed by the cancellation of the identity tadpole, as we shall see in the next Section. The matrix \(P^{-1}\) allows one to display the direct channel Möbius amplitude

\[M = \frac{N}{2} e^{i \pi} \sigma_{00} \left[ \sum_{\epsilon, \epsilon' = 0,1} \sum_{m,n} \gamma_{\epsilon, \epsilon'} \left( e^{-2 \pi \ell} \right)^{X_2[m^2 + Y_2^2 n^2]} e^{2 \pi i (m \epsilon + n \epsilon')} \eta^2 (i \ell + \frac{1}{2}) + 3 e^{-i \pi \frac{\ell}{2}} (\Phi_{11} + \Phi_{22}) \right] \quad , \tag{20}\]

an expression that, in terms of hatted characters,

\[M = \frac{N}{2} \sigma_{00} \left[ \sum_{\epsilon, \epsilon' = 0,1} \sum_{m,n} \gamma_{\epsilon, \epsilon'} \left( e^{-2 \pi \ell} \right)^{X_2[m^2 + Y_2^2 n^2]} \eta^2 (i \ell + \frac{1}{2}) + \gamma_{00} \left( \hat{\chi}_{00} + \hat{\chi}_{01} + \hat{\chi}_{02} \right) + 3 (\hat{\chi}_{10} - \hat{\chi}_{11} - \hat{\chi}_{12}) + 3 (\hat{\chi}_{20} - \hat{\chi}_{21} - \hat{\chi}_{22}) \right] \tag{21}\]

is manifestly compatible with the annulus amplitude of eq. (16) and allows both orthogonal and symplectic global Chan-Paton groups.

It is also interesting to look at \(I_2\), the reflection with respect to the horizontal axis. In this case, states satisfying the condition \(p = -\tilde{p}\) flow in the direct Klein bottle amplitude, and only one of the chiral twist fields survives the unoriented projection,
reflecting a different resolution of the fixed-point ambiguity in the twisted sectors. In fact,

\[
K = \frac{1}{2} \left[ \frac{1}{4} \sum_{\epsilon_1, \epsilon_2 = 0, 1} \sum_{m, n} \left( e^{-2\pi t} \frac{X^2}{2\pi} [m^2 + Y z^2 n^2] X (m \epsilon_1 + n \epsilon_2) \right) \frac{e^{2\pi i X (m \epsilon_1 + n \epsilon_2)}}{\eta^2 (2it)} + \Phi_{01} + \Phi_{02} \right],
\]

that in terms of characters becomes

\[
K = \frac{1}{2} \left[ \frac{1}{4} \sum_{\epsilon_1, \epsilon_2 = 0, 1} \sum_{m, n} \left( e^{-2\pi t} \frac{X^2}{2\pi} [m^2 + Y z^2 n^2] X (m \epsilon_1 + n \epsilon_2) \right) \frac{e^{2\pi i X (m \epsilon_1 + n \epsilon_2)}}{\eta^2 (2it)} + \chi_{00} + \chi_{01} + \chi_{02} 
\right. \\
+ \left. \left( \chi_{10} + \chi_{11} + \chi_{12} \right) + \left( \chi_{20} + \chi_{21} + \chi_{22} \right) \right],
\]

and it is evident, comparing with (7), that one of the three degenerate twist fields is diagonal while the remaining two are off-diagonal in the would-be resolved torus amplitude. In (22), the projector is at work in the direct channel. Consequently, even if in the transverse channel windings and momenta are now shifted

\[
\tilde{K} = \frac{2D/2}{2} \left[ \frac{1}{4X^2} \sum_{\epsilon_1, \epsilon_2 = 0, 1} \sum_{m, n} \left( e^{-2\pi t} \frac{X^2}{2\pi} [(m + X \epsilon_1) X (m + X \epsilon_1) \epsilon_{12}^2 + Y z^2 (n + X \epsilon_2)^2] \right) \frac{e^{2\pi i X (m \epsilon_1 + n \epsilon_2)}}{\eta^2 (i\ell)} \\
+ \frac{1}{\sqrt{3}} \Phi_{01} + \frac{1}{\sqrt{3}} \Phi_{20} \right],
\]

the “massless contribution” involves again only the identity

\[
\tilde{K}_0 = \frac{2D/2}{2} \frac{3}{\sqrt{3}} \chi_{00} ,
\]

with a different reflection coefficient for the crosscap. The transverse annulus amplitude now receives contributions from states with \( p = \tilde{p} \), without the need of any projections

\[
\tilde{A} = \frac{2^{-D/2}}{2} N^2 \left[ \frac{1}{2X^2} \sum_{m, n} \left( e^{-2\pi \ell} \frac{X^2}{2\pi} \frac{1}{X^2} [m^2 + Y z^2 n^2] \right) \frac{e^{2\pi i X (m \epsilon_1 + n \epsilon_2)}}{\eta^2 (i\ell)} \\
+ \frac{1}{\sqrt{3}} \Phi_{01} + \frac{1}{\sqrt{3}} \Phi_{20} \right].
\]

At “zero mass”, eq. (26) gives

\[
\tilde{A}_0 = 2^{-D/2} \frac{N^2}{2} \frac{3}{\sqrt{3}} \chi_{00} ,
\]

while an \( S \) modular transformation yields the direct channel amplitude

\[
A = \frac{N^2}{2} \left[ \sum_{m, n} \left( e^{-2\pi \ell} \frac{X^2}{2\pi} \frac{1}{X^2} [m^2 + Y z^2 n^2] \right) \frac{e^{2\pi i X (m \epsilon_1 + n \epsilon_2)}}{\eta^2 (i\ell)} + \Phi_{10} + \Phi_{20} \right]
\]

or, in terms of characters,

\[
A = \frac{N^2}{2} \left[ \chi_{00} + \chi_{01} + \chi_{02} + \sum_{m, n} \left( e^{-2\pi \ell} \frac{X^2}{2\pi} \frac{1}{X^2} [m^2 + Y z^2 n^2] \right) \frac{e^{2\pi i X (m \epsilon_1 + n \epsilon_2)}}{\eta^2 (i\ell)} \\
+ \left( \chi_{10} + \chi_{11} + \chi_{12} \right) + \left( \chi_{20} + \chi_{21} + \chi_{22} \right) \right].
\]
Notice that, due to the quantization of $X_1$, the transverse Möbius amplitude involves shifts as well. The result is

$$
\tilde{M} = 2 \frac{N}{2} e^{\frac{2\pi i}{3}} \left[ \frac{1}{2X_2} \sum_{\epsilon_1,\epsilon_2=0,1} \sum_{m,n} \gamma_{\epsilon_1,\epsilon_2} \left( e^{-2\pi t} \right) \frac{1}{2i\eta^2(i\ell + \frac{1}{2})} \left[ (m + X_1\epsilon_1)^2 + Y_2^2(n + X_1\epsilon_2)^2 \right] \right],
$$

and consistency requires $\gamma_{00} = \sigma_{01} = \sigma_{02}$ to produce the correct “massless” contribution

$$
\tilde{M}_0 = 2 \frac{N}{2} \frac{3}{\sqrt{3}} \hat{\chi}_{00}. \tag{31}
$$

In the direct channel, the $\epsilon$-dependence is again crucial. In fact,

$$
\mathcal{M} = \frac{N}{2} e^{\frac{2\pi i}{3}} \left[ \frac{1}{2} \sum_{\epsilon_1,\epsilon_2=0,1} \sum_{m,n} \gamma_{\epsilon_1,\epsilon_2} \left( e^{-2\pi t} \right) \frac{X_2^{1/2}}{2^{1/2} \eta^2(i\ell + \frac{1}{2})} \left[ m^2 + Y_2^2 n^2 \right] \right],
$$

and the correct normalization of the projector demands that

$$
\sum_{\epsilon_1,\epsilon_2=0,1} \gamma_{\epsilon_1,\epsilon_2} = 2 \delta \tag{33}
$$

be satisfied, with $\delta$ a sign, in such a way that, in terms of hatted characters,

$$
\mathcal{M} = \frac{N}{2} \left[ \delta (\hat{\chi}_{00} + \hat{\chi}_{01} + \hat{\chi}_{02}) + \sum_{\epsilon_1,\epsilon_2=0,1} \sum_{m,n} \left( e^{-2\pi t} \right) \frac{X_2^{1/2}}{2^{1/2} \eta^2(i\ell + \frac{1}{2})} \right]. \tag{34}
$$

Notice that the sign $\gamma_{00}$ is fixed in Type I vacua by the cancellation of the $\chi_{00}$ tadpole while $\delta$ determines the type of the Chan-Paton group, whose rank is reduced by a factor of 2 as a result of the equality of the reflection coefficients for boundaries and crosscaps.

## 3 Type I vacua in six and four dimensions

We now apply the results of the previous Section to the construction of Type I vacua in six and four dimensions, starting from the corresponding diagonal $Z_3$ orbifold of the Type IIB theory. In $D = 10 - d$ ($D = 4$ or 6), one can resolve the fixed-point ambiguity in several ways, allowing a number $n = \prod_{i=1}^{d/2} k_i$ (with $k_i$ equal to 1 or 3) of twist fields associated to as many fixed points in the direct Klein bottle amplitude. We thus find three models with $n$ equal to 1, 3 and 9 in $D = 6$, and four models with $n$ equal to 1, 3, 9 and 27 in $D = 4$. Using supersymmetric chiral blocks (see Appendix A), the Type
IIB diagonal modular invariant is

\[
\mathcal{T}_d = \frac{1}{3} \left[ \rho_{00} \rho_{00} \prod_{i=1}^{d/2} \Lambda_i + \rho_{01} \rho_{01} + \rho_{02} \rho_{02} + 3^{d/2} \left( \rho_{10} \tilde{\rho}_{10} + \rho_{11} \tilde{\rho}_{11} + \rho_{12} \tilde{\rho}_{12} \right) \right.
\]

\[
+ 3^{d/2} \left( \rho_{20} \tilde{\rho}_{20} + \rho_{21} \tilde{\rho}_{21} + \rho_{22} \tilde{\rho}_{22} \right) \right],
\]

that in terms of supersymmetric characters becomes

\[
\mathcal{T}_d = \frac{1}{3} \left[ \rho_{00} \rho_{00} \left( \prod_{i=1}^{d/2} \Lambda_i \right)' + \chi_{00} \chi_{00} + \chi_{01} \chi_{01} + \chi_{02} \chi_{02} \right.
\]

\[
+ 3^{d/2} \left( \chi_{10} \tilde{\chi}_{10} + \chi_{11} \tilde{\chi}_{11} + \chi_{12} \tilde{\chi}_{12} \right) + 3^{d/2} \left( \chi_{20} \tilde{\chi}_{20} + \chi_{21} \tilde{\chi}_{21} + \chi_{22} \tilde{\chi}_{22} \right) \right].
\]

From now on, we do not specify the explicit form of the lattice sums, that should be clear from the previous Section. In complete analogy with the \( c = 2 \) case

\[
\mathcal{K} = \frac{1}{2} \left[ \rho_{00} \prod_{i=1}^{d/2} \Gamma_i^K + n \rho_{10} + n \rho_{20} \right],
\]

and extracting the zero modes from the lattice sums, it can be written in terms of \( Z_3 \)-characters as

\[
\mathcal{K} = \frac{1}{2} \left[ \rho_{00} \left( \prod_{i=1}^{d/2} \Gamma_i^K \right)' + \chi_{00} + \chi_{01} + \chi_{02} +
\]

\[
+ n \left( \chi_{10} + \chi_{11} + \chi_{12} \right) + n \left( \chi_{20} + \chi_{21} + \chi_{22} \right) \right].
\]

It is clear from (38) that only the twist fields associated to \( n \) of the \( 3^{d/2} \) fixed points survive the unoriented projection. Exposing the transverse channel by an \( S \) transformation gives

\[
\tilde{\mathcal{K}} = \frac{2^{D/2}}{2} \left[ n \left( \rho_{00} \prod_{i=1}^{d/2} \tilde{\Gamma}_i^K \right)' + \rho_{01} + \rho_{02} \right]
\]

\[
= \frac{2^{D/2}}{2} \left[ n \left( \frac{1}{3} \rho_{00} \prod_{i=1}^{d/2} \tilde{\Gamma}_i^K \right)' \right],
\]

and it is worthwhile to observe that, as in the \( c = 2 \) model, the complete projector is restored in virtue of the right values of the \( X_2 \) moduli in the compactified directions.

A single type of Chan-Paton charge is allowed in the annulus amplitude, that can be associated to Dirichlet \( (9 - d/2) \)-branes at angles [23], the natural objects in the theory after \( d/2 \) T-duality transformations. Thus

\[
\mathcal{A} = \frac{N^2}{2} \left[ \rho_{00} \prod_{i=1}^{d/2} \Gamma_i^A + n \rho_{10} + n \rho_{20} \right],
\]

9
and, as should be clear at this point, the transverse channel gives back the right contributions

\[
\tilde{A} = \frac{2^{-D/2} N^2}{2} \left[ n 3^{-d/2} (\rho_{00} \prod_{i=1}^{d/2} \tilde{\Gamma}_i^A + \rho_{01} + \rho_{02}) \right]
\]

\[
= \frac{2^{-D/2} N^2}{2} n 3^{(1-d/2)} \left[ \chi_{00} + \frac{1}{3} \rho_{00} \left( \prod_{i=1}^{d/2} \tilde{\Gamma}_i^A \right)' \right].
\] (41)

The Möbius amplitude completes the one-loop Type I partition function. In the open-string loop channel,

\[
\mathcal{M} = -(-1)^{d/2} \frac{N}{2} \left[ \rho_{00} \prod_{i=1}^{d/2} \tilde{\Gamma}_i^M + n \rho_{11} + n \rho_{22} \right],
\] (42)

while in the transverse channel

\[
\tilde{\mathcal{M}} = -2 \frac{N}{2} n 3^{-d/2} \left[ \rho_{00} \prod_{i=1}^{d/2} \tilde{\Gamma}_i^M + \rho_{01} + \rho_{02} \right]
\]

\[
= -2 \frac{N}{2} n 3^{(1-d/2)} \left[ \tilde{\chi}_{00} + \frac{1}{3} \rho_{00} \left( \prod_{i=1}^{d/2} \tilde{\Gamma}_i^M \right)' \right].
\] (43)

Notice that, in this case, the sign is fixed by the tadpole of \(\chi_{00}\). From \(\tilde{K}, \tilde{A}\) and \(\tilde{\mathcal{M}}\) we get \(N = 2^{D/2}\), with symmetrization of the Chan-Paton charges for the gauge vectors in four dimensions and antisymmetrization in six dimensions. As a consequence, all the \(D = 6\) models have an \(SO(8)\) gfe group, while all the \(D = 4\) models have an \(Sp(4)\) gauge group. Their massless spectra are summarized in Table 1 and in Table 2.

The closed oriented massless spectrum of Type IIB on the \(Z_3\) orbifold in \(D = 6\) results in \(N = (2, 0)\) supergravity coupled to 21 tensor multiplets. The unoriented truncation produces \(N = (1, 0)\) models with \(12 + n\) hypermultiplets and \(9 - n\) tensor multiplets. The open sector adds the gauge multiplet and \(n + 1\) hypermultiplets in the adjoint representation of \(SO(8)\). These models were already described in [24], and as open descendants of Gepner models in [17]. The model with \(n = 3\) also corresponds to a recently described open descendant of a \(Z_{3L} \times Z_{3R}\) asymmetric orbifold [25]. It is

| \(n\) | \(T\) | \(H\) | \(CP\) group | \(H\) |
|---|---|---|---|---|
| 1 | 8 | 13 | \(SO(8)\) | 2 (28) |
| 3 | 6 | 15 | \(SO(8)\) | 4 (28) |
| 9 | 0 | 21 | \(SO(8)\) | 10 (28) |

Table 1: Massless spectra of \(D = 6\) models.
amusing to stress how all these descriptions are related to different resolutions of the fixed-point ambiguities in the diagonal modular invariant. It is also easy to check that the gravitational anomaly cancellation condition

$$H - V = 273 - 29T$$

is satisfied for every integer value of $n$ between 1 and 9. As usual, the models with several tensor multiplets are anomaly-free due to a GSS mechanism [31, 32].

The four dimensional models are related to a compactification of the Type IIB on a Calabi-Yau threefold with Hodge numbers $h_{1,1} = 0$ and $h_{1,2} = 36$. After the unoriented truncation, the massless closed spectrum contains the $N = 1$ supergravity multiplet coupled to $V = (27 - n)/2$ vector multiplets and to $10 + n + V$ chiral multiplets. A gauge vector together with $n + 3$ chiral multiplets in the adjoint of the $Sp(4)$ Chan-Paton groups results from the open and unoriented massless spectrum. These models, differently from the geometric open descendants [33], are not chiral and are clearly anomaly-free. Compared to the model with $n = 27$ (“the true diagonal”), the presence of additional vector multiplets in the unoriented closed spectrum reduces the number of marginal deformations.

### 4 Conclusions and discussion

Starting from a pair of free bosonic fields propagating on a $Z_3$ orbifold of a two-torus, we have discussed the open descendants of the diagonal model. Its consistency, as stressed in ref. [24], is deeply connected to discrete values of some geometric moduli. In fact, the diagonal GSO projection results from the combination of the geometric orbifold action and a T-duality. For the $Z_3$ group, on a two-dimensional lattice there are two possible choices, corresponding to two different resolutions of the fixed-point ambiguities, that give rise to different classes of open descendants. We have then discussed the application to (already known) six dimensional and to (new) four dimensional open descendants of Type IIB $Z_3$ diagonal orbifolds. Their spectra are parameterized by the number $n$ of fixed points surviving the closed unoriented projection. In $D = 6$, $n$ can take the
values 1, 3 and 9, giving rise to Type I models with an $SO(8)$ Chan-Paton gauge group and $13 + 2n$ hypermultiplets and $9 - n$ tensor multiplets. In $D = 4$, $n$ can be 1, 3, 9 and 27, yielding non-chiral Type I vacua with an $Sp(4)$ Chan-Paton gauge group, $V = (27 - n)/2$ additional vector multiplets and $13 + 2n + V$ chiral multiplets. An inspection of the complete one-loop partition function and of the Rational Conformal Field Theory examples suggests that all these models should be consistent for every integer value of $n$ between 1 and 9 in six dimensions and for every odd-integer value of $n$ between 1 and 27 in four dimensions. In order to implement this conjecture, one should be able to find a suitable involution that represents the combined action of $\Omega$ and the T-duality on the lattice and leaves exactly $n$ fixed points invariant. This is not allowed for orbifolds that are products of two-tori, but more complicated slices of four dimensional or six-dimensional lattices could exist that realize this settings.

It would be interesting to extend this construction to open descendants of $Z_N$ orbifolds, and to investigate the possibility of introducing anti-branes in this context. Notice that we do not find, for the $Z_3$ case, unsolvable tadpole conditions as in recently proposed open descendants of the Type IIA in four dimensions [34]. Finally, it should be noticed that all these models are defined perturbatively and are tightly constrained by Conformal Field Theory consistency conditions on surfaces with boundaries and/or crosscaps. This is true, in particular, for the six dimensional model with zero tensor multiplets. One is not allowed to append to a given closed unoriented spectrum an open sector that does not respect the aforementioned constraints. Some recently proposed “non-perturbative” orientifolds are built violating the anomaly-cancellation conditions and calling for non-perturbative states that supply the missing multiplets [35]. We have no way to check the consistency of these models, but certainly we are able to define perturbative orientifolds compatibly with the closed unoriented spectra of some of the models of ref. [35].

5 Acknowledgments

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6 Note added

For the sake of comparison with [36], where models slightly different from these are presented, we would like to add the following comments. Actually, there are more solutions, both in six and in four dimensions, connected to the presence of discrete open-string Wilson-lines [3, 4, 27]. In fact, the Möbius-strip amplitude in (42) should
be written as
\[
\mathcal{M} = -\frac{N}{2} \left[ \gamma (\hat{x}_{00} + \hat{x}_{01} + \hat{x}_{02}) + \rho_{00} \prod_{i=1}^{d/2} \Gamma_i^M \right] + (-1)^{d/2} n \rho_{11} + (-1)^{d/2} n \rho_{22} \right] , \tag{45}
\]
or, in terms of characters,
\[
\mathcal{M} = -\frac{N}{2} \left[ \gamma (\hat{x}_{00} + \hat{x}_{01} + \hat{x}_{02}) + \rho_{00} \prod_{i=1}^{d/2} \Gamma_i^M \right] + (-1)^{d/2} n \left( \hat{x}_{10} - \hat{x}_{11} - \hat{x}_{12} \right) + (-1)^{d/2} n \left( \hat{x}_{20} - \hat{x}_{21} - \hat{x}_{22} \right) \right] , \tag{46}
\]
where \(\gamma\) is a sign hidden in the lattice sum, as pointed out in Section 2. Comparing with the annulus amplitude of eq. (40), one can see that \(\gamma = +1\) leads to orthogonal Chan-Paton groups, while \(\gamma = -1\) leads to symplectic ones. The six-dimensional models in Table 1 correspond to \(\gamma = +1\), while the four-dimensional models in Table 2 correspond to \(\gamma = -1\). Two other series of models, whose massless spectra are reported in Tables 3 and 4, are thus available. Notice that the closed spectra are equal to the previous ones, but the open sectors are changed. In particular, in \(D = 6\) one has the gauge multiplet and one hypermultiplet in the adjoint representation of \(Sp(8)\), with \(n\) hypermultiplets in the antisymmetric representation. Eq. (44) is again satisfied, because the Wilson line affects both the vector and a corresponding untwisted hypermultiplet. The gauge anomalies in the models with several tensor multiplets are again absent, due to the GGS mechanism \[31, 32\].

| \(n\) | \(T\) | \(H\) | \(CP\) group | \(H\) | \(H\) |
|------|------|------|--------------|------|------|
| 1    | 8    | 13   | \(Sp(8)\)    | 1 (36) | 1 (28) |
| 3    | 6    | 15   | \(Sp(8)\)    | 1 (36) | 3 (28) |
| 9    | 0    | 21   | \(Sp(8)\)    | 1 (36) | 9 (28) |

Table 3: Massless spectra of \(D = 6\) models with \(\gamma = -1\).

| \(n\) | \(C\) | \(V\) | \(CP\) group | \(C\) | \(C\) |
|------|------|------|--------------|------|------|
| 1    | 24   | 13   | \(SO(4)\)   | 3 (6) | 1 (10) |
| 3    | 25   | 12   | \(SO(4)\)   | 3 (6) | 3 (10) |
| 9    | 28   | 9    | \(SO(4)\)   | 3 (6) | 9 (10) |
| 27   | 37   | 0    | \(SO(4)\)   | 3 (6) | 27 (10) |

Table 4: Massless spectra of \(D = 4\) models with \(\gamma = +1\).
In $D = 4$, one has the gauge vector multiplet and three chiral multiplets in the adjoint representation of $SO(4)$, together with $n$ chiral multiplets in the symmetric representation. All these models are not chiral and thus anomaly-free, and coincide with the $Z_3$-models in ref. [36].

### 7 Appendix A: notations and conventions

Let us start by defining the chiral blocks for the $c = 2$ model describing a pair of free bosons on the $Z_3$ orbifold. The chiral traces in the untwisted sector are

$$\Phi_{0,h} = \left[ q^{1/12} \prod_{n=1}^{\infty} (1 - \omega^h q^n) (1 - \overline{\omega}^h q^n) \right]^{-1} , \quad (47)$$

while in the twisted sectors

$$\Phi_{1,h} = \Phi_{2,-h} = \left[ q^{-1/36} \prod_{n=1}^{\infty} (1 - \omega^h q^{n-2/3}) (1 - \overline{\omega}^h q^{n-1/3}) \right]^{-1} , \quad (48)$$

with $\omega = e^{2\pi i/3}$ and $h = (0, 1, 2) \mod 3$. Notice that $\Phi_{0,0} = \eta^{-2}$.

The chiral supertraces entering the Type II and Type I superstring orbifold models are defined by

$$\rho_{g,h} \equiv \text{Tr}_{NS,g} \frac{1}{2} (1 - (-F) h q^{L_0 - \frac{c}{24}}) - \text{Tr}_{R,g} \frac{1}{2} (1 + (-F) h q^{L_0 - \frac{c}{24}}) , \quad (49)$$

where $g, h \in Z_3$, the trace runs over the $g$-twisted sector with a plus sign for NS states and a minus sign for R states and we are omitting the measure and the contribution of non-compact coordinates. For the $h$ projection in a given $g$-twisted sector, one thus obtains

$$\rho_{00} \equiv \frac{1}{2} \sum_{\alpha,\beta = 0,1/2} (-)^{2\alpha + 2\beta + 4\alpha\beta} \vartheta_{[\alpha \beta]}^4 \frac{\vartheta_{[\alpha \beta]}^{4-d/2}}{\eta^{d/2}} \prod_{i=1}^{d/2} \left( 2 \sin \pi h_i \right) \vartheta_{\left[ \frac{\alpha + h_i}{2} \right]} \vartheta_{\left[ \frac{\beta + h_i}{2} \right]} \quad h \neq 0$$

$$\rho_{0h} \equiv \frac{1}{2} \sum_{\alpha,\beta = 0,1/2} (-)^{2\alpha + 2\beta + 4\alpha\beta} \left( \frac{\vartheta_{[\alpha \beta]}^{4-d/2}}{\eta} \right) \prod_{i=1}^{d/2} \left( 2 \sin \pi h_i \right) \frac{\vartheta_{\left[ \frac{\alpha + h_i}{2} \right]}^{4}}{\vartheta_{\left[ \frac{\beta + h_i}{2} \right]}^{4}} \quad g, h \neq 0$$

$$\rho_{gh} \equiv -(i)^{\frac{d}{2}} \frac{1}{2} \sum_{\alpha,\beta = 0,1/2} (-)^{2\alpha + 2\beta + 4\alpha\beta} \left( \frac{\vartheta_{[\alpha \beta]}^{4-d/2}}{\eta} \right) \prod_{i=1}^{d/2} \left( 2 \sin \pi h_i \right) \frac{\vartheta_{\left[ \frac{\alpha + g_i}{2} \right]}^{4}}{\vartheta_{\left[ \frac{\beta + h_i}{2} \right]}^{4}} \quad g, h \neq 0$$

where $\vartheta_{[\alpha \beta]}$ are the standard Jacobi theta functions with characteristics and $\sum_{i=1}^{d/2} g_i = \sum_{i=1}^{d/2} h_i = 0 \mod 1$.

Under S-modular transformations ($\tau \to -1/\tau$)

$$\Phi_{00} \to (-i\tau)^{-1} \Phi_{00}$$

$$\Phi_{0h} \to (\sqrt{3}) \Phi_{0h} \quad h \neq 0$$
$$\Phi_{h0} \rightarrow (1/\sqrt{3}) \Phi_{0,-h} \quad h \neq 0$$
$$\Phi_{gg} \rightarrow (e^{i\pi/18}) \Phi_{g,-g} \quad g \neq 0$$
$$\Phi_{g,-g} \rightarrow (e^{-i\pi/18}) \Phi_{-g,-g} \quad g \neq 0 \quad ,$$

(51)

and

$$\rho_{00} \rightarrow (-i\tau)^{-d/2} \rho_{00}$$
$$\rho_{0h} \rightarrow (2 \sin \pi h)^{d/2} \rho_{0h} \quad h \neq 0$$
$$\rho_{h0} \rightarrow (2 \sin \pi h)^{-d/2} \rho_{0,-h} \quad h \neq 0$$
$$\rho_{gg} \rightarrow (i)^{d/2} \rho_{g,-g} \quad g \neq 0$$
$$\rho_{g,-g} \rightarrow (-i)^{d/2} \rho_{-g,-g} \quad g \neq 0 \quad ,$$

(52)

while under T-modular transformations ($\tau \rightarrow \tau + 1$)

$$\Phi_{0,h} \rightarrow (e^{-i\pi/6}) \Phi_{0,h}$$
$$\Phi_{g,h} \rightarrow (e^{i\pi/18}) \Phi_{g,g+h} \quad g \neq 0$$

(53)

and

$$\eta^{D/2} \rho_{gh} \rightarrow \eta^{D/2} \rho_{g,g+h} \quad .$$

(54)

The characters are combinations of chiral blocks that respect the $Z_3$ group structure. In the $c = 2$ case they are

$$\chi_{\alpha,\beta} = \frac{1}{3} \left[ \Phi_{\alpha,0} + \omega^{\beta} \Phi_{\alpha,1} + \bar{\omega}^{\beta} \Phi_{\alpha,2} \right] \quad ,$$

(55)

while in the superstring models they are

$$\chi_{\alpha,\beta} = \frac{1}{3} \left[ \rho_{\alpha,0} + \omega^{\beta} \rho_{\alpha,1} + \bar{\omega}^{\beta} \rho_{\alpha,2} \right] \quad ,$$

(56)

The modular transformation $P = ST^2ST$ relates chiral traces in the transverse and direct Möbius-strip amplitudes and corresponds to $\hat{P} = T^{1/2}ST^2ST^{1/2}$ on “hatted” real characters [3], defined by

$$\hat{\chi}_{\alpha,\beta} = e^{-i\pi (h_{\alpha,\beta}-c/24)} \chi_{\alpha,\beta}$$

(57)

where $h_{\alpha,\beta}$ is the conformal weight of the character $\chi_{\alpha,\beta}$.

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