Reduction of ultrasound transmissivity through a bone-phantom plate resulting from shear wave excitation

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The effects of clinical trials have suggested that ultrasound irradiation can enhance the effectiveness of thrombolytic agents and increase the reclamation rate for acute ischemic stroke patients [1]. Because of the risk of cerebral hemorrhage [2], a transcranial ultrasonic treatment device that uses a frequency ~500 kHz is under development [3,4] to lower mechanical and thermal risks. Development of such a device requires knowledge of the transmissivity of ultrasound through the temporal bone, which is a thin part of the human skull (~a few millimeters thick). To clarify the transmissivity dependence on bone thickness, we experimentally measured transmissivity through a bone-phantom plate in water (which has acoustic properties similar to those of soft tissue). Although our experimental results were almost consistent with those of a model calculation based on normal incident plane waves, we discerned a definitive discrepancy between them. In this Short Note, we report on a process for pursuing the cause of this discrepancy.

In the case of normal incidence of a plane wave to a bone-phantom plate in water, the energy transmissivity $\tau$ is given by the formula

$$\tau = \frac{4Z_bZ_w}{(Z_w + Z_b)e^{i\kappa d} - (Z_w - Z_b)e^{-i\kappa d}}, \tag{1}$$

where $\kappa_b$ is the wave number in the bone-phantom plate, $d$ is the thickness of the bone-phantom plate, $Z_w$ is the acoustic impedance of water, and $Z_b$ is the acoustic impedance of the bone-phantom plate (for the derivation, see, for example, [5]). The effect of absorption by the bone-phantom plate was taken into account by the substitution $\kappa_b \rightarrow \kappa_b - i\alpha f$, where $\alpha$ is the absorption coefficient and $f$ is the frequency. Figure 1 shows the transmissivity calculated by using the above formula as a function of bone-phantom plate thickness. There is a peak around 3 mm, which corresponds to half the wavelength of a 500 kHz wave in the bone-phantom plate.

In our experiment, we used 20 bone-phantom plates (of sound speed = 2,884 m/s, density = 1,664 kg/m$^3$, and absorption coefficient = 46.3 Np/m/MHz), ranging from 0.6 to 4.4 mm by 0.2 mm steps. Almost all Japanese temporal bones are considered to be within this range. The plates were special order products, and the acoustic parameters were measured by the manufacture. The parameters of the plate were similar to those of human skull [6]. A transducer, whose surface is a disk of 24 mm in diameter, was located 12 mm from the bone-phantom plate in a water-filled tank. Ultrasound entered the bone-phantom plate almost normally. The transmitted wave was measured by a needle-type hydrophone (Onda Corporation, Sunnyvale, CA, USA) at a position 60 mm away from the transducer surface on the central axis of the disk. Figure 2(a) shows the experimental result. There is a peak around 3 mm, as expected. Equation (1) can explain the overall tendency. However, we found a dip at 3.2 mm, which is not expected by using Eq. (1).

We repeated the experiment several times, and the 3.2 mm dip appeared each time. We investigated the reason for this reproducible 3.2 mm dip. At first, we suspected the quality of the material; one 3.2 mm plate might differ from others. This possibility was excluded because, when we used 650 kHz instead of 500 kHz, the transmittance behavior changed smoothly around 3.2 mm (see Fig. 2(b)). Next, we suspected the effect of the near field, which ranges from the transducer surface to ~48 mm. Changing the distance between the transducer and the bone-phantom plate to 70 mm, we measured the transmissivity. The 3.2 mm dip appeared again even in this far-field region. Thus, the transducer was not the problem. When we used 650 kHz or 400 kHz waves instead of 500 kHz waves, the dip appeared at 2.4 mm for 650 kHz waves and at 4.0 mm for 400 kHz waves. Hence, the appearance of the dip seemed to depend inversely on frequency. Considering the above fact, we inferred that some sort of yet-unknown phenomenon arose in the 3.2 mm bone-phantom plate. Subsequently, we suspected that nonlinear absorption of the bone-phantom plate might explain the dip. We numerically solved a wave equation including a possible nonlinear absorbing term. However, the sharp dip at 3.2 mm could not be reproduced. We finally concluded that the dip was caused by excitation of shear waves. Although shear waves cannot be excited at normal incidence, at incident angles slightly different from 0°, they can significantly change the transmissivity.

We investigated ultrasound transmissivity through a plate including shear waves. Our analysis is an extension of that in [7]. We used the coordinate system depicted in Fig. 3. The potential for the injected wave is given as follows:

$$\phi_i = \phi_0 e^{i(\omega t - k x \sin \theta - k \cos \theta)}, \tag{2}$$

where $\phi_0$ is a constant, $\omega$ is the angular frequency, $t$ is time, $k$ is the wave number, and $\theta$ is the angle of incidence. The potentials for the reflected wave (r), the longitudinal wave in the plate (L), the shear wave in the plate (S), the reflected longitudinal wave (Lr), the reflected shear wave (Sr), and the transmitted wave (T) are given by

$$\phi_r = R\phi_0 e^{i(\omega t - k x \sin \theta + k \cos \theta)},$$

$$\phi_L = T_L\phi_0 e^{i(\omega t - k_b x - K_L z)}, \tag{3}$$

$$\phi_L = T_L\phi_0 e^{i(\omega t - k_b x - K_L z)}, \tag{4}$$

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\[ S = T S/C_0 e^{i(\omega t - k S x - k S z)}, \]
\[ \phi_L = R_L \phi_0 e^{i(\omega t - k L x + K_{Lz} z)}, \]
\[ \psi_S = R_S \phi_0 e^{i(\omega t - k S x + K_{Sz} z)}, \]
\[ \phi_T = T \phi_0 e^{i(\omega t - k T x - \alpha \theta)}, \]

where \( R \) and \( T \) are ratio of pressure reflection, and ratio of pressure transmission, respectively. In the above formulas, \( K_{Lz} \) and \( K_{Sz} \) can be determined from

\[ (k L \sin \theta_L)^2 + (K_{Lz})^2 = \frac{\omega^2}{c_L^2}, \]
\[ (k S \sin \theta_S)^2 + (K_{Sz})^2 = \frac{\omega^2}{c_S^2}, \]

where \( c_L \) and \( c_S \) are the speeds of the longitudinal wave and shear wave in the plate, respectively. When \( K_{Lz} \) or \( K_{Sz} \) is a complex number, the imaginary part should be negative. We take into account absorption by complexification. For example,

\[ K_{Lz} \rightarrow K_{Lz} - i \frac{|K_{Lz}|}{\alpha} f. \]

Using these potentials, we can derive the \( z \) components of the particle velocity and the stress tensor as follows:

\[ v_z = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \]
\[ \iota \omega T_{zz} = \mu \left( 2 \frac{\partial^2 \phi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial z} \right), \]
\[ \iota \omega T_{xz} = \lambda \nabla^2 \phi + 2 \mu \left( \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} \right), \]

where \( \lambda \) and \( \mu \) are Lamé coefficients. (For a liquid whose density is \( \rho \) and sound speed is \( c \), \( \lambda = \rho c^2 \) and \( \mu = 0 \). For a solid whose density is \( \rho_s \), \( \lambda = \rho_s (c_L^2 - 2c_S^2) \) and \( \mu = \rho_s c_S^2 \).)

We impose connection conditions on \( v_z, T_{xz}, \) and \( T_{zz} \) at the boundaries. By considering the \( x \) dependence of (2)–(8), we find that...
$k \sin \theta = k \sin \theta_L = k \sin \theta_T = k \sin \theta_S = k \sin \theta_R$.  \hspace{1cm} (15)

These relations describe Snell’s law. By imposing continuity at $z = 0$, we obtain
\begin{equation}
\cos \theta R + \frac{K_{Lz}}{k} T_L + \sin \theta T_S - \frac{K_{Lz}}{k} R_L + \sin \theta R_S = \cos \theta, \hspace{1cm} (16)
\end{equation}

\begin{align}
-2 \sin \theta \frac{K_{Lz}}{k} T_L + \left( \frac{K_{Sz}}{k} \right)^2 - \sin^2 \theta \right] T_S \\
+2 \sin \theta \frac{K_{Lz}}{k} R_L + \left( \frac{K_{Sz}}{k} \right)^2 - \sin^2 \theta \right] R_S = 0,
\end{align}

and
\begin{align}
R + \frac{\rho_2}{\rho_1} \left( 2 \frac{c_S^2}{c_L^2} - 2 \frac{c_S^2}{2} \left( \frac{K_{Lz}}{k} \right)^2 - 1 \right) T_L \\
-2 \frac{\rho_2}{\rho_1} \frac{c_S^2}{c_L^2} \sin \theta T_S + \frac{\rho_2}{\rho_1} \left( 2 \frac{c_S^2}{c_L^2} - 2 \frac{c_S^2}{2} \left( \frac{K_{Lz}}{k} \right)^2 - 1 \right) R_L \\
+2 \frac{\rho_2}{\rho_1} \frac{c_S^2}{c_L^2} \sin \theta R_S = -1. \hspace{1cm} (18)
\end{align}

By imposing continuity at $z = d$, we obtain
\begin{align}
\frac{K_{Lz}}{k} e^{-ik_{d,z}d} T_L + \sin \theta e^{-ik_{d,z}d} T_S - \frac{K_{Lz}}{k} e^{ik_{d,z}d} R_L \\
+ \sin \theta e^{ik_{d,z}d} R_S - \cos \theta e^{-ik \cos \theta d} T = 0, \hspace{1cm} (19)
\end{align}

\begin{align}
-2 \sin \theta \frac{K_{Lz}}{k} e^{-ik_{d,z}d} T_L + \left( \frac{K_{Sz}}{k} \right)^2 - \sin^2 \theta \right] e^{-ik_{d,z}d} T_S \\
+2 \sin \theta \frac{K_{Lz}}{k} e^{ik_{d,z}d} R_L + \left( \frac{K_{Sz}}{k} \right)^2 - \sin^2 \theta \right] e^{ik_{d,z}d} R_S = 0, \hspace{1cm} (20)
\end{align}

and
\begin{align}
\frac{\rho_2}{\rho_1} \left( 2 \frac{c_S^2}{c_L^2} - 2 \frac{c_S^2}{2} \left( \frac{K_{Lz}}{k} \right)^2 - 1 \right) e^{-ik_{d,z}d} T_L \\
-2 \frac{\rho_2}{\rho_1} \frac{c_S^2}{c_L^2} \sin \theta T_S + \frac{\rho_2}{\rho_1} \left( 2 \frac{c_S^2}{c_L^2} - 2 \frac{c_S^2}{2} \left( \frac{K_{Lz}}{k} \right)^2 - 1 \right) e^{ik_{d,z}d} R_L \\
+2 \frac{\rho_2}{\rho_1} \frac{c_S^2}{c_L^2} \sin \theta T_S + \frac{\rho_2}{\rho_1} \left( 2 \frac{c_S^2}{c_L^2} - 2 \frac{c_S^2}{2} \left( \frac{K_{Lz}}{k} \right)^2 - 1 \right) e^{-ik \cos \theta d} T = 0. \hspace{1cm} (21)
\end{align}

Equations (16)–(21) comprise six linear equations with six unknowns. By numerically solving these equations, for example, using Kramer’s method, $T$ can be evaluated.

Figure 4 shows the energy transmissivity $|T|^2$. Here, we set the unknown parameter $c_S$ to be 1,600 m/s. Because, if we use this value, the wavelength of the shear wave corresponds to the dip-arising thickness. Additionally, the ratio of $c_S$ to $c_L$ is reasonable value for solid. Even though the incident angle is small (~3 degrees), the dip at the thickness corresponding to the wavelength of the shear wave is clearly discernible. It is reasonable to conclude that the 3.2 mm dip results from shear wave excitation. In Fig. 4, we can see a small dip at 1.2 mm

for 650kHz, 1.6 mm for 500 kHz, and 2.0 mm for 400 kHz case, respectively, which correspond to half the wavelength of shear waves. These small dips can be consistently seen in the experimental result (Fig. 2).

In this Short Note, we showed that the transmissivity through a bone-phantom plate can be significantly reduced by shear wave excitation at a certain thickness even for small angles of incidence. Observation of a dip may provide a method for measuring a speed of shear wave.

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