Sound emission of solid surfaces through bubble layer

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Abstract. The analysis of the influence of a thin homogeneous bubble layer on sound emission from a solid surface is carried out. Sound pulses and monochromatic wave packets with a carrier frequency equal to the resonant frequency of the bubbles forming the bubble layer are considered. It is shown that the bubble layer transforms short sound pulses into wave sound packets and significantly reduces the amplitude of the emitted sound. The structure of a sinusoidal wave packet is transformed similarly. A long sound pulse is stored in the form of a pulse, its shape changes significantly. A homogeneous bubble layer near a solid radiating surface is an open resonator. The layer generates far-field radiation with spectral lines depending on the method of layer excitation and the internal properties of the bubble layer. The resonant frequency of the bubble is the limiting frequency in the spectrum, but it is not distinguished by a separate line.

1. Introduction
A large number of works are devoted to the study of the effect of bubbles on the acoustic properties of the marine environment. Particular attention in the works is paid to the attenuation and structure of sound fields in the near-surface layers, where bubble clouds are formed due to the breaking of waves [1, 2]. These tasks are characterized by large spatial scales.

Sound significantly changes its characteristics, transiting even through thin layers of bubbles in water [3, 4]. Thin bubble layers can appear on the surface of sonars and significantly change their acoustic characteristics.

The present work aims to determine changes in the spectral characteristics of sound emitted by a solid surface into clean water through a thin layer of bubbles, which is located in the immediate vicinity of the surface.

2. Formulation of the problem
A solid surface is the boundary of a half-space filled with water. Gas bubbles are uniformly distributed over a thin layer near a solid surface. A solid surface emits sound in the form of sound pulses and sinusoidal wave packets. The solid surface is absolutely reflective. We investigated the structures and spectra of sound signals in pure water outside the layer.

3. Nonlinear wave system of equations
To study this problem, the wave model [5] is used in a one-dimensional formulation. In these equations, the layer is modeled by a discrete set of bubbles uniformly distributed over the layer thickness. The system of equations has the form
\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} = -\frac{1}{c^2} \frac{\partial}{\partial t} \left( p \frac{\partial}{\partial t} \ln(1 - \alpha) \right), \quad (1)
\]
\[
R_x \frac{d^2 R_k}{dt^2} + \frac{3}{2} \left( \frac{dR_k}{dt} \right)^2 + \frac{4 \mu}{\rho_0 R_k} \frac{dR_k}{dt} + \frac{2 \sigma}{\rho_0 R_k} = \frac{1}{\rho_0} \left[ P_0 + \frac{2 \sigma}{R_k} \left( \frac{R_{k_0}}{R_k} \right)^3 - P_0 - p(\vec{r}, t) \right], \quad (2)
\]
\[
\alpha(\vec{r}, t) = \sum_k v_k(t) \cdot \delta(\vec{r} - \vec{r}_k(t)), \quad (3)
\]
\[
v_k(t) = \frac{4}{3} \pi R_k^3(t). \quad (4)
\]

Index \( k \) denotes the ordinal number of the bubble in the layer, the function \( \delta(x - x_k) \) determines the position of the \( k_{th} \) bubble in the layer, \( P(x, t) \) is the pressure in the wave, \( \rho_0 \) is the initial pressure in the water, \( c \) is the speed of sound in water, \( \alpha \) is the volume concentration of bubbles, \( R_k \) is the radius of the \( k_{th} \) bubble, \( v_k \) is the volume of the \( k_{th} \) bubble, \( \rho \) is the density liquid, \( \sigma \) is the surface tension of the liquid, \( t \) is the time, \( x \) is the spatial coordinate. The gas in the bubbles obeys the adiabatic law with the adiabatic exponent \( \gamma = 1.4 \), the pressure and density of water are related by the relation \( P(x, t) = \rho(x, t) c^2 \).

The right-hand side of equation (1) depends on the pressure in the wave and the volumetric gas content in the bubble layer. It describes the change in the volumetric gas content \( \alpha \) with time at each point of the region, occupied by bubbles under the impact of a wave. The right side of equation (1) is written in a compact form. To compile a numerical algorithm, the right-hand side is differentiated. The result is linear and non-linear terms, with first and second time derivatives of the volumetric gas content. Expression (4) is a dimensionless quantity, which is equal to the total volume of bubbles per unit volume of the medium around the point \( x \), divided by the unit volume of the bubbly medium. At each point of the bubbly medium, the value of the volumetric gas content and the bubble radius can have arbitrary values, which are determined by the physical conditions of the problem.

For numerical solutions, system (1) - (4) is reduced to dimensionless form using the relations

\[
\delta R_k = R_k / R_{k_0}, \quad \delta P = P / P_0,
\]

\( \delta \) denotes the dimensionless value of the quantity. To introduce dimensionless time, the Rayleigh time is used as the collapse time of an empty bubble in the field of the emitted sound [6]. The same time is used to introduce a dimensionless spatial coordinate. Dimensionless time and spatial coordinate are determined by the relations

\[
\delta t = t / (R_0 \sqrt{\gamma (\rho_0 / P_0)}), \quad \delta x = x / (c R_0 \sqrt{\gamma (\rho_0 / P_0)}),
\]

index \( _0 \) denotes the initial value of the parameter. The studies were carried out for the values of the amplitude \( P_0 \) of sound pulses of 60 Pa. Two pulse sound signals with durations of \( 30 \cdot 10^6 \) s and \( 400 \cdot 10^6 \) s were considered. The duration of the sound wave packet was \( 1500 \) \( 10^6 \) s. The initial pressure in the medium was \( P_0 = 0.1 \) MPa, the radius of the bubbles was \( R_0 = 0.25 \) \( 10^{-3} \) m, the volume concentration of bubbles was \( \alpha = 10^{-2} \), the density of water was \( \rho_0 = 1000 \) kg/m\(^3\), the speed of sound in the liquid was \( c = 1500 \) m/s.

The width of the bubble layer was \( h = 0.01 \) m. The width of the computational domain was \( 0.3 \) m. A description of the method for numerically solving the system of equations (1) - (4) and checking the solutions for compliance with experimental data is given in [5]. Reflectorless boundary conditions were set at the boundary of the computational domain located in the water.

4. Discussion of results

The wave system of equations (1) - (4) describes the nonlinear formation of resonant solitons in the homogeneous, inhomogeneous, and polydisperse media at large wave amplitudes in a liquid with bubbles [7, 8]. The initial pressure in the radiated wave, equal to 60 Pa, refers to the linear range of
solutions to the system of equations (1) - (4).

Before proceeding with the analysis of the solutions to the problem posed, let us evaluate the quantities characterizing the process of the transmission of sound emitted by a solid wall through the layer of bubbles. To do this, we estimate the speed of sound in the bubble layer, the spatial size of the pulses inside the layer for the given signal durations, and the frequency of free pulsations of the bubble. In addition, we will determine the coefficients of reflection and transmission of sound through the interface between the layer with bubbles and pure water at the normal incidence of the sound wave.

The speed of sound in a liquid with bubbles is expressed by the well-known Mallok [9] formula

\[ c_m = \gamma P_0 \sqrt{\rho_0}. \]

The frequency of free volumetric bubble pulsations, the Minaert [10] frequency, is expressed by the formula

\[ f_r = 1/(2\pi R_0) \sqrt{3\gamma P_0 / \rho_0}. \]

The coefficients of reflection and transmission of sound at the boundary of the bubble layer are determined by the Fresnel formulas [11]. For the bubble layer, the Fresnel formulas for the reflection coefficient \( V \) and the transmission coefficient \( W \) have the form

\[ V = (\rho_0 c - \rho_0 (1 - \alpha) c_m) / (\rho_0 c + \rho_0 (1 - \alpha) c_m), \quad W = 2\rho_0 c / (\rho_0 c + \rho_0 (1 - \alpha) c_m). \]

According to (5), (6), and (7) the speed of sound in the bubble layer \( c_m = 118 \) m/s, therefore the spatial size of the pulse \( l \) with a duration of \( 30 \times 10^{-6} \) s inside the layer is \( 3.54 \times 10^{-3} \) m, and for a pulse with a duration of \( 400 \times 10^{-6} \) s, \( l = 4.72 \times 10^{-2} \) m. A sinusoidal wave packet with a duration of \( 1500 \times 10^{-6} \) s has a spatial dimension \( l = 6 \times 10^{-2} \) m. The frequency of free pulsations of bubbles is \( f_r = 13053 \) Hz, the reflection coefficient from the bubble layer - water interface is \( V = 0.85 \), and the transmission coefficient is \( W = 1.85 \). It follows from the estimates that the spatial size of a short pulse is much smaller than the layer width, and that of a long pulse is much larger. The value of the reflection coefficient at the boundary of the layer, due to the significant difference in the acoustic impedances of the layer and the water, ensures the accumulation of the sound energy emitted by the wall inside the layer. Then this energy is radiated into clean water. According to the conditions of the problem, a solid surface is absolutely reflective. Therefore, the energy introduced by the wave into the layer is radiated only into the water. The above estimates are valid for the low-frequency region of sound waves and, in this case, give a qualitative description of the ongoing processes.

Consider the solutions of system (1) - (4) for a short pulse. Figure 1 shows the structure of a sound signal emitted by a solid wall into a pure liquid and a sound signal transmitted through a layer of bubbles for an initial signal with a duration of \( 30 \times 10^6 \) s. The dimensionless signal duration is \( 3.8 \).

**Figure 1.** The structure of the initial broadband pulse and the pulse that transited

**Figure 2.** The spectrum of the original broadband pulse in a pure liquid
through the layer of bubbles. The sound signal transiting through the layer is transformed into a wave packet. Taking into account that the width of the signal spectrum and its duration obey the relationship $\Delta t \Delta f \approx 1$, the width of the spectrum of a sound pulse with a duration of $30 \cdot 10^{-6}$ s $\Delta f \approx 33300$ Hz, and the resonant frequency of the bubbles $f_r = 13053$ Hz. Dimensionless resonance frequency $\delta f_r = 0.158$. The spectrum of a short sound pulse is shown in figure 2. It follows from this that a significant fraction of the energy of a short pulse lies in the region above the resonance, and therefore the transformation of a pulse into a wave packet with these parameters is dispersive in nature [5]. Figure 3 shows a graph of the pressure in a sound signal in a pure liquid as a function of time, which formed after a solid wall emits a short pulse signal into a pure liquid through the layer. The signal in a pure liquid can be divided into initial and quasi-stationary phases. The initial phase is characterized by a relatively rapid decrease in the energy introduced into the bubble layer.

The amplitude of the emitted sound for a time interval equal to 150 durations of the original sound pulse decreases 40 times. The structure of the sound signal spectrum is determined by this phase. Figure 4 shows the emission spectrum of the bubble layer excited by a pulsed sound signal. The spectrum has a set of lines characterizing the bubble layer as a resonator. The limiting frequency of the spectrum is the resonant frequency of the bubble pulsations. In this case, the line at the resonance frequency of the bubble is absent in the spectrum. In the second phase, which can be called quasi-stationary and which lasts much longer than the initial phase, the damping is much less. On an interval of 1000 durations of the initial wave, the amplitude decreases by a factor of 8. Figure 5 shows the spectrum of a fragment of the bubble layer radiation at long times. The limiting frequency is also equal to the resonance frequency of the bubbles, but the width of the spectrum has almost halved, and the radiation spectrum itself has shifted to the high-frequency region. The intensity of the lines has
decreased by 5 times. For signals with a duration of $400 \times 10^6$ s, the spectrum width is $\Delta f \approx 2500$ Hz. The signal spectrum is located in the region below the Malloch frequency and the emitted signal practically does not experience dispersion distortion. Figure 6 shows the wave profile in pure water, the profile of the wave that passed through the layer, and the time variation of the bubble radius at the center of the layer. The duration of the signal that propagates through the layer increased by almost 10 times compared to the duration of the original wave. The amplitude of the wave decreased by 5 times and oscillations appeared on its envelope. Figure 7 shows the spectra of the original signal in a pure liquid and a wave emitted into a pure liquid through a layer of bubbles. In the spectrum of the wave emitted through the bubble layer, one line appeared, $\delta f = 0.068$, which is more pronounced when the bubble layer is excited by a gentle pulse.

Figure 6. The structure of the initial pulse, the pulse transited through the bubble layer, and the change in the bubble radius at the center of the layer.

Figure 7. Spectra of the original signal and transmitted through the layer.

Figure 8 shows the results of calculations when a solid wall emits a sinusoidal wave packet with a frequency equal to the natural frequency of the bubble pulsation. It follows from the graph that with a given excitation of the bubble layer, there are no pronounced phases in the signal, as in the case of excitation by a short pulse. For a comparable time, the signal amplitude decreased by a factor of 7. The emission spectrum is shown in figure 9. The spectrum also consists of lines and lies in the frequency range from the lower limit of the spectrum $\delta f = 0.066$ to the resonant frequency of the bubbles $\delta f_r = 0.158$.

Figure 8. Time base of sound emission from a bubble layer excited by a sinusoidal wave packet.

Figure 9. The sound emission spectrum of a bubble layer, excited by a sinusoidal wave packet.

The appearance of complex radiation with the presence of beats is since the bubble layer is excited
by the emitted wave not instantaneously, but over a finite time interval equal to the transite time of sound through the layer, as well as the presence of sound reflection back into the layer when transiting through the bubble layer - pure liquid interface. As a result, a complex dynamic superposition of waves is formed in the layer, which results in the radiation observed in a pure liquid.

**Conclusions**
The studies carried out show that in the presence of a bubble layer near a solid emitting surface, one should speak of radiation from a solid wall-bubble layer system. A complex system of reflected waves from the solid surface and the bubble layer - water interface is formed inside the layer. Energy radiation from the layer occurs only into the liquid, and the layer is a resonator with a transparent boundary. The emission of sound energy and the emission time are determined by the difference in acoustic impedances at the boundary between the layer and the water.

**Acknowledgments**
The work was carried out within the framework of the State order 2021-2025 №121031800217-8.

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