We investigate the properties of the one-electron Green’s function in an interacting two-dimensional electron system in a strong magnetic field, which describes an electron tunneling into such a system. From finite-size diagonalization, we find that its spectral weight is suppressed near zero energy, reaches a maximum at an energy of about $0.2e^2/\epsilon l_c$, and decays exponentially at higher energies. We propose a theoretical model to account for the low-energy behavior. For the case of Coulomb interactions between the electrons, at even-denominator filling factors such as $\nu = 1/2$, we predict that the spectral weight varies as $e^{-\omega_0/|\omega|}$, for $\omega \to 0$. 

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High mobility $e^+$ or $e^-$ doped GaAs-GaAlAs quantum wells constitute a remarkable, almost ideal, many-body system. In such a system, Coulomb interactions among the electrons play a vital role in determining its properties. This is particularly true when the system is immersed in a strong perpendicular magnetic field which quenches the kinetic energy of the electrons, the so-called fractional quantum Hall regime. In order to experimentally probe the properties of such systems, investigations have primarily utilized measurements of the magnetotransport coefficients [1]. However, within the last year, a variety of spectroscopic probes have begun to play an important role [2] [3] [4] [5].

A recent experiment [5] measured the low temperature high field bulk tunneling characteristics of a bilayer system with electron density $n \equiv \nu / 2\pi l_c^2$ in each layer, coupled together by a weak tunnel barrier, separated by a distance $d \approx 2n^{-1/2}$. The experimental I-V characteristics in the range $0.48 < \nu < 0.83$ exhibited a strong suppression of the tunneling current at low biases, similar to what was observed in Ref. [4], and a broad, pronounced peak in the neighborhood of $eV_{\text{max}} \sim 0.4e^2/\epsilon l_c$. Here $\epsilon$ is the dielectric constant, and $l_c$ is the magnetic length.

In this paper we construct a theory of this type of tunneling experiment. In particular, we present exact numerical results on the one-electron Green’s function for a small number of particles which we believe gives reliable information at energies comparable to $eV_{\text{max}}$. At lower energies we construct a model which directly relates the I-V curve to the low-lying density fluctuations in the system. A comparison of the theory with experiment is made.

Since the layer separation $d$ is large, we can, to the first approximation, ignore coupling between the layers. In this limit, it may be shown that for electrons confined to the lowest Landau level, the tunneling current obtained from the general Golden rule expression becomes:

$$I = e\lambda^2 \Omega l_c^{-2} \int_0^{eV} d\omega A_+(\omega)A_-(eV - \omega),$$  (1)

where $V$ is the voltage difference between the layers, $\lambda$ is the tunneling matrix element, $\Omega$ is the area of the system, and $A_{\pm}(\omega)$ are the spectral weights to add (subtract) an electron.
to (from) the right (left) layer. Written in terms of the operators $\hat{\psi}^\dagger$ and $\hat{\psi}$ which creates or annihilates an electron at the origin, these spectral weights are:

$$A_+ (\omega) = \sum_{\alpha} |\langle \alpha, N + 1 | \hat{\psi}^\dagger | 0, N \rangle|^2 \times \delta (\omega + \mu + E_{0,N} - E_{\alpha,N+1}),$$

(2)

$$A_- (\omega) = \sum_{\alpha} |\langle \alpha, N - 1 | \hat{\psi} | 0, N \rangle|^2 \times \delta (\omega - \mu + E_{0,N} - E_{\alpha,N-1}),$$

(3)

where $\mu \equiv (\mu_+ + \mu_-)/2$, $\pm \mu_{\pm} = E_{0,N+1} - E_{0,N}$, and $\mu_{\pm}$ are the chemical potentials for adding and subtracting an electron from an $N$-electron system respectively. Note that for an incompressible state, the chemical potential jump $\Delta \mu \equiv \mu_+ - \mu_- > 0$. Therefore the low-bias suppression of the tunneling current observed in the experiment implies that the low-energy spectral weight of the one-electron Green’s function must vanish strongly for $\omega \to 0$, in contrast to the behavior of a normal Fermi liquid. We shall see that this behavior can be understood using rather simple arguments.

The mean value $\bar{\omega}_\pm$ of the energy in $A_\pm (\omega)$ is determined by the expectation value of the Hamiltonian $\hat{H}$ in the “initial state”, $|\Phi_1 \rangle \equiv \hat{\psi}^\dagger | 0, N \rangle$. We may estimate this if we consider $|\Phi_1 \rangle$ to be an assemblage of quasiparticles, packed together as closely as possible to give one extra electron charge in a disc about the origin. Since the maximum electron density in any Landau level is $(2\pi l_c^2)^{-1}$, and the background charge is $\nu (2\pi l_c^2)^{-1}$, the minimum radius of the charged disc is $r_+ = l_c \sqrt{2/(1 - \nu)}$. We may estimate $\bar{\omega}_+$ as being equal to the Coulomb self energy of this disc, or roughly $\bar{\omega}_+ \approx 0.2e^2/l_c$ for $\nu \approx 1/2$. The mean energy $\bar{\omega}_-$ of $A_- (\omega)$ may be estimated similarly assuming the hole charge is spread out in a disc of radius $r_- = l_c \sqrt{2/\nu}$. If the spectral weights are concentrated near $\bar{\omega}_\pm$, then the tunneling current should have a maximum at $eV_{\text{max}} \approx \bar{\omega}_+ + \bar{\omega}_- \approx 0.4e^2/\epsilon l_c$, the coefficient being roughly independent of $\nu$ in the range of $0.2 < \nu < 0.8$.

To understand crudely the suppression of $A_\pm (\omega)$ at small values of $\omega$, consider a circular droplet of electron liquid with a uniform density corresponding to filling factor $\nu$. In the
symmetric gauge for the magnetic vector potential, and in the absence of disorder, the total angular momentum $M$ perpendicular to the layer is a conserved quantity. The value of $M$ of a uniform droplet containing $N$ electrons is $M_{0,N} \approx N^2/2\nu$, so that $M_{0,N+1} - M_{0,N} \approx N/\nu$. The initial state $|\Phi_1\rangle$ with an electron added to the origin of the $N$-electron droplet has angular momentum $M_{0,N}$, which is very different from $M_{0,N+1}$. Thus not only is the state $|\Phi_1\rangle$ orthogonal to the groundstate of the $(N+1)$-electron droplet, it should also have little overlap with any of the low-lying excited states. In order for the low-energy excitations to carry away the necessary angular momentum, they must be created far from the origin, which means that there is little overlap with the initial state that is only perturbed at the origin.

If the filling fraction $\nu$ corresponds to a quantized Hall state with denominator $2p+1$, then the elementary charged excitations are quasiparticles with charge $e/(2p+1)$. Ignoring boundary excitations, and assuming a repulsive interaction between the quasiparticles, we expect that the lowest states of a droplet with one extra electron charge contain $2p+1$ quasiparticles well separated from each other. In order to obtain the correct total angular momentum, there must be at least one neutral excitation, such as a magnetoroton \[7\] in addition. Thus, in the quantized Hall case there should be a true energy gap in $A_+^{(\omega)}$, with a threshold $\omega_+$ that is slightly larger than $(2p+1)\bar{\varepsilon}_+$, where $\bar{\varepsilon}_+$ is the energy to add a single quasiparticle. In Ref. \[8\], it was proposed that for the principal quantized Hall states at $\nu = p/(2p+1)$, the combination $(2p+1)\bar{\varepsilon}_+$ should be only weakly dependent on $p$, and the threshold $\omega_+$ obtained in this way is only slightly smaller than the mean energy $\bar{\omega}_+$ estimated above. For other quantized Hall states the value of $\omega_+$ should be much smaller than this, and for a compressible state the true threshold must occur at $\omega = 0$. Nevertheless, to obtain a low energy state with the correct total charge and angular momentum, we must produce excitations far from the origin, so that $A_+^{(\omega)}$ should be very small for $\omega \to 0$, as we find in our quantitative estimates below.

In order to study $A_\pm^{(\omega)}$ for energies around $\bar{\omega}_\pm$, we have performed exact calculations of systems of up to 10 electrons in the lowest Landau level in a spherical geometry. In the
spherical geometry, due to the rotational symmetry, the eigenstates of the system can be classified by their total angular momentum \( \hat{L}^2 \) and \( \hat{L}_z \). We consider only systems where the groundstate of the initial \( N \) particle system has a zero total angular momentum. Since an electron has an intrinsic orbital angular momentum \( l = S \equiv N\phi/2 \) when it is confined to the surface of a sphere with a magnetic monopole of total flux \( N\phi \) at the center, we see that when one more electron is added, the resultant \( (N + 1) \)-electron system has a total angular momentum \( S \). Therefore the spectral weight is nonzero only for states with angular momentum \( S \).

In Fig. 1, we show the histograms of the spectral weight \( A(\omega) \), defined as \( A_+ (\omega) \) for \( \omega > 0 \) and \( A_- (-\omega) \) for \( \omega < 0 \), for a compressible system of 8 electrons and at an \( N\phi = 17 \) close to \( \nu = 1/2 \). Apparently there are a few dominant peaks in \( A_\pm (\omega) \) at energies comparable to \( 0.2e^2/\epsilon l_c \). There is no weight at energies below these peaks due to finite size effect: there are simply no states with the correct angular momentum in this energy interval for such a small system. The high energy behavior is shown in the inset. We find \( A(\omega) \sim e^{-|\omega|/\Gamma} \) with \( \Gamma \approx 0.026e^2/\epsilon l_c \). We attribute the high energy tail to states where the added electron is accompanied by one or more excitations of a short wavelength density mode within the lowest Landau level. A calculation including higher Landau levels would presumably show additional high-energy side bands shifted up by multiples of the cyclotron frequency. In Fig. 2, we show \( A(\omega) \) for a system of 8 electrons at a filling factor \( \nu = 1/3 \). Note that in the case, the tunneling peak position is \( eV_{\text{max}} \approx 0.6e^2/\epsilon l_c \). We have also done calculations at \( \nu = 1/3 \) for systems with up to 10 electrons. We find by \( 1/N \) extrapolation that \( eV_{\text{max}} \approx 0.5e^2/\epsilon l_c \) for an infinite system at \( \nu = 1/3 \). Calculations at other filling factors in the range \( 1/3 \leq \nu \leq 2/3 \) give results qualitatively similar to Fig. 1 and Fig. 2, and suggest a tunneling peak energy \( eV_{\text{max}} \sim (0.4 \sim 0.6)e^2/\epsilon l_c \) in all cases.

In order to predict the low-energy behavior in a compressible state, we employ a number of approximations which reduce the physics of this problem to that of the X-ray edge problem. We treat the added electron as if it were an infinitely massive foreign particle inserted into the \( N \)-electron system. Physically this should be a good approximation because of
the large magnetic field which suppresses the effect of exchange on a short distance scale and practically eliminates recoil effects. If $\hat{H}_N$ is the Hamiltonian of the $N$-electron system before insertion, the Hamiltonian after the insertion has the form

$$\hat{H} = \hat{H}_N + E' + \sum_q V_q \hat{\rho}_q$$

where $V_q = 2\pi e^2 / q$ is the Fourier transform of the Coulomb interaction, and $\hat{\rho}_q$ is the density operator for the original $N$-electron system, projected onto the lowest Landau level. The constant $E'$ is chosen so that the groundstate energy of $\hat{H}$ coincides with the correct groundstate energy of the $(N + 1)$-electron system.

Following the method of Mahan [6], Langreth [9], and Shung and Langreth [10], treating the last term in Eq.(4) as a weak perturbation in a linked cluster expansion of the extra electron self-energy, we immediately arrive at an analytic expression for the imaginary time electron Green’s function $G_+(\tau) = e^{-C(\tau)}$, where

$$C(\tau) = \sum_q V_q^2 \int_0^\infty \frac{d\omega}{\pi} \frac{\text{Im} \chi_{\omega}^2}{\omega^2} (1 - e^{-\tau\omega}).$$

where $\chi(q, \omega)$ is the density response function of the electron system. Since $G_+(\tau)$ is the Laplace transform of $A_+(\omega)$, Eq.(5) is equivalent to replacing the electron system by a set of Harmonic oscillators with the same density response [10].

Since we are interested in the low-energy behavior of $A_+(\omega)$ (i.e. long time behavior of $G_+(\tau)$), we need some estimate of $\chi(q, \omega)$ at low energies and long wavelengths. Recently Halperin, Lee, and Read [8] proposed a Chern-Simons Fermi liquid theory of the $\nu = 1/2$ state, which implies that, for Coulomb interaction, in the absence of impurity scattering, $\chi(q, \omega)$ is dominated by a diffusive mode at small $q$ and $\omega$, i.e.

$$\chi(q, \omega) = \frac{1}{V_q^2} \frac{1}{1 - i\omega / \beta q^2},$$

where, according to the RPA calculation in Ref. [8], $\beta = e^2 l_c / 4 \epsilon$ for $\nu = 1/2$. For a general compressible state, we expect that Eq.(6) still holds but with some modified value of $\beta$. Substituting Eq.(6) into Eq.(5), we obtain, for large $\tau$,
\[ C(\tau) \approx \frac{e^2}{\pi \epsilon} \sqrt{\frac{\tau}{\beta}} \int_0^\infty dx \int_0^\infty dy \frac{1}{xy^2(1+x^2)(1-e^{-xy^2})} \equiv 2\sqrt{\omega_0 \tau}, \]  

(7)

where \( \omega_0 = \pi e^2/2\epsilon l_c \). Since \( A_+(\omega) \geq 0 \), the \( \tau \to \infty \) behavior of the Laplace transform \( G_+ \) determines the low frequency behavior of \( A_+ \), and we find, for small \( \omega \),

\[ A_+(\omega) \sim e^{-\omega_0/\omega}. \]  

(8)

The identical result is obtained for the hole contribution, \( A_-(\omega) \). Combining this with Eq.(1), we find at small bias \( V \), at \( \nu = 1/2 \), the tunneling current is

\[ I \sim \int_0^{eV} d\omega \exp \left[ -\omega_0 \left( \frac{1}{\omega} + \frac{1}{eV-\omega} \right) \right] \sim e^{-V_0/V}. \]  

(9)

where \( eV_0 = 4\omega_0 \). Obviously, the tunneling current \( I \) at small \( V \) vanishes faster than any power law in \( V \).

For general filling fraction close to \( \nu = 1/2 \), we expect that Eqs.(7)(8)(9) should apply for intermediate times or energies, and that eventually there will be a crossover to some other behavior. For a pure system at a high order quantized Hall state, we expect to find a true gap at sufficiently low energies; for a compressible state at any even-denominator fraction that can be described as a Chern-Simons Fermi liquid, the ultimate \( \omega \to 0 \) behavior will be similar to Eq.(8), but with a value of \( \omega_0 \) that depends on the filling fraction. When impurity scattering is taken into account, one finds that at long wave lengths, the relaxation rate \( \beta q^2 \), in the denominator of Eq.(3), must be replaced by the two-dimensional dielectric relaxation rate, \( 2\pi q\sigma_{xx}/\epsilon \), where \( \sigma_{xx} \) is the diagonal conductivity at \( \nu = 1/2 \). This substitution leads to a modification of the low-energy behavior to \( A(\omega) \sim \exp \left( -x[\ln (2\pi \sigma_{xx}/\epsilon l_c |\omega|)]^2 \right) \), where \( x \equiv e^2/(2\pi^2 \sigma_{xx}) \) is of the order ten in typical experiments.

The interaction between layers may also be taken into account, although the analysis is more complicated in this case. It appears that the most important effect is a downward shift of the peak voltage \( V_{\text{max}} \), by an amount \( e^2/\epsilon d \), which reflects a reduction of the mean energy of the initial tunneling state due to the attraction between the electron and the hole. In
order to relax towards a uniform groundstate at large value of imaginary time $\tau$, the electron and the hole must spread in space so that on average they are far apart. The long time relaxation must be controlled by the relaxation of the density difference in the two layers, which for a system without impurities, and wavevectors $q \ll 1/d$, has a rate $\sim q^3$, according to the theory in Ref. [8]. This in turn gives a tunneling current $I \sim \exp(-\text{const}/\sqrt{V})$ for sufficiently small $V$.

The theoretical predictions obtained above from our calculations are in good overall agreement with the experiments of Eisenstein et al. [5,11]. For example, the peak position obtained in our simple model is quite close to the observed value of $0.4 e^2/\epsilon l_c$. The experimental behavior on the low bias side of the tunneling peak has also been fit by a formula $I \sim e^{-V_0/V}$, similar to Eq.(8) above. However, the experimental value $eV_0 \approx 0.9 e^2/\epsilon l_c$ is much smaller than the value $2\pi e^2/\epsilon l_c$ predicted by our theory. For the range of biases where this behavior is observed, the wavelength of the density fluctuation contributing significantly to the relaxation is estimated to be $\lambda \sim 3 l_c$, which is small compared to the inter-layer separation and the typical scattering lengths in the system. This implies that inter-layer screening and impurity scattering should not have a significant effect in modifying the diffusive behavior of $\chi(q,\omega)$ at the wavelengths and frequencies involved. Thus it seems that some other factor must be responsible for the difference between the experimental and theoretical results. Among the possibilities are that the frequencies probed by the experiment are still too high for the asymptotic form in Eq.(8) to be valid; or that the experimental dependence arises from residual long wavelength fluctuations in the concentration of the two doping layers, which would tend to smear out any steep rise in the tunneling characteristic. However, is is also possible that our theoretical analysis, simply using the RPA value of $\beta$, gives a poor estimate of the coefficient $\omega_0$ in Eq.(8).

In summary, we have investigated the properties of the one-electron Green’s function in an interacting two-dimensional electron system in a strong magnetic field through finite-size diagonalization and a low-energy model. We find that the spectral weight is always suppressed at low energies, with the form of $A(\omega) \sim \exp(-\omega_0/|\omega|)$ for the compressible
state at $\nu = 1/2$. We have also discussed the implications of our results on the electron tunneling into such a system.

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FIGURES

FIG. 1. The histograms of the spectral weight $A(\omega)$ of the one-electron Green’s function at a compressible state obtained for an 8-electron system with flux number $N_\phi = 17$. $\log_{10} A(\omega)$ is shown in the inset. All energies are measured from the chemical potential.

FIG. 2. The histograms of the spectral weight $A(\omega)$ of the one-electron Green’s function at an incompressible state obtained for an 8-electron system with flux number $N_\phi = 21$ corresponding to $\nu = 1/3$. 