Semileptonic decay of $B_c$ meson into $c\bar{c}$ states in a QCD potential model

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Abstract

The slope and curvature of Isgur Wise function for $B_c$ meson is computed in a QCD potential model in two different approaches of choosing the perturbative term of the Cornell potential. Based on heavy quark effective theory the exclusive semileptonic decay rates of $B_c$ meson into the $c\bar{c}$ ($\eta_c, J/\psi$) states are exploited. Spin symmetry breaking effects are ignored up to a particular point and the form factors are connected with Isgur-Wise function for other kinematic points since the recoil momentum of $c\bar{c}$ from $B_c$ is small due to its heavy mass.

Keywords: Dalгarno method, Isgur-Wise function, Form factors, Decay width.

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1 Introduction

The $B_c$ meson is a particularly interesting hadron, since it is the lowest bound state of two heavy ($b, c$) quarks with different flavors. Because of the fact that the $B_c$ meson carries the flavor explicitly, there is no gluon or photon annihilation via strong interaction or electromagnetic interaction but decay only via weak interaction. Since both $b$ and $c$ quarks forming the $B_c$ meson are heavy, the $B_c$ meson can decay through the $b \to q (q = c, u)$ transition with $c$ quark being a spectator as well as through the $c \to q (q = s, d)$ transition with $b$ quark being a spectator. The former transitions correspond to the semileptonic decays to $\eta_c$ and D mesons, while the latter transitions correspond to the decays to $B_s$ and $B$ mesons. The CDF Collaboration reported the discovery of the $B_c$ ground state in $p\bar{p}$ collisions already more than fourteen years ago \cite{1}. More experimental data on masses and decays of the $B_c$ meson are expected to come in near future from the Tevatron at Fermilab and the Large Hadron Collider (LHCb) at CERN. The estimates of the $B_c$ decay rates indicate that the $c$ quark transitions give the dominant contribution while the $b$ quark transitions and weak annihilation contribute less. However, from the experimental point of view the $B_c$ decays to charmonium are easier to identify. Indeed, CDF and D0 observed the $B_c$ meson and measured its mass analyzing its semileptonic decays of $B_c \to J/\psi l\nu$. 

There are many theoretical approaches to study the exclusive semileptonic decay of $B_c$ meson. The paper by Bjorken in 1986, on the decays of long lived $B_c$ meson is considered to be the pioneering work for $B_c$ meson \cite{2}. A lot of efforts was then directed to study this specific meson on the basis of modern understanding of QCD dynamics of heavy flavours in the framework of different approaches.
Some of these approaches are: QCD sum rules [3, 4, 5], the relativistic quark model [6, 7, 8], the quasi-potential approach to the relativistic quark model [9, 10, 11], the non-relativistic approach of the Bethe-Salpeter (BS) equation [12], based on the BS equation, the relativistic quark model [13, 14], the QCD relativistic potential model [15], the relativistic quark-meson model [16], the nonrelativistic quark model [17], the covariant light-front quark model [18] and the constituent quark model [19, 20, 21, 22] using BSW (Bauer, Stech, and Wirbel) model [23] and ISGW (Isgur, Scora, Grinstein, and Wise) model [24].

The consequence of heavy quark spin symmetry is that the number of form factors which parametrize the matrix elements is reduced and simplifies the semileptonic transitions. However, spin symmetry does not fix the normalisation of the form factors at any point of the phase space. The normalisation of the form factors near the zero recoil point must be computed by some nonperturbative approach [25]. So far, Jenkins et al. in ref. [26] estimated the universal form factors of semileptonic decays of $B_c$ meson using non-relativistic meson wave-functions and in ref. [27] it is computed by employing the ISGW model at the zero-recoil point.

In this paper, we extend a QCD potential model and check its sensitivity in studying the universal form factor Isgur-Wise function with two different approaches: linear part of the Cornell potential as perturbation with Coulombic part as parent and Coulombic part as perturbation with linear part as parent.

The formalism of the paper is presented in section 2 and in section 3 we place our results and conclusions.

2 Formalism

2.1 The wavefunction in the model

The QCD potential, so called the Cornell potential used in this work is the coulomb + linear potential, that takes into account a coulombic behaviour at short distances and a linear confining term at long distances, representing the perturbative one-gluon exchange and the non-perturbative chromoelectric flux tube of confinement. The potential with a constant shift $c$, which corresponds to quark self energy [28] is written as

$$V(r) = -\frac{4}{3r}\alpha_s + br + c,$$  \hspace{1cm} (1)

where $\alpha_s$ is the strong coupling constant and $b$ is the QCD string tension which is also known as the slope of the potential.

With Cornell potential one obtains the advantage of choosing the Coulombic part as perturbation with linear part as parent as well as linear part as perturbation with Coulombic part as parent. It is expected that a critical role is played by scale $r_0$, where the potential $V(r) = 0$. Aitchison and Dudek in ref. [29] put an argument that if the size of a state (meson here) measured by $\langle r \rangle < r_0$, then the Coulomb part as the ”Parent” will perform better and if $\langle r \rangle > r_0$, the linear part as ”parent” will perform better. Aitchison’s work also showed that the results with Coulombic part as parent (VIPT), bottomonium spectra are well explained than charmonium where as charmonium states are well explained with linear part as parent. Moreover in ref. [30], we have analysed that the critical distance $r_0$ is not a constant and can be enhanced by reducing $b$ and $c$ or by increasing $\alpha_s$. Thus for a fixed value...
of $b$ and $c$, $\alpha_s$ plays an important role in choosing the perturbative term. However in this manuscript we allow the same range of $\alpha_s$ obtained from the theoretical bounds of slope and curvature of I-W function and check the applicability of the model wavefunctions in the two approaches for the semileptonic decay of $B_c$ meson into $c\bar{c}$ ($\eta_c, J/\psi$) states.

The wavefunction computed by Dalgarno method \[31, 32\] with Coulombic part $-\frac{4\alpha_s}{3r} + c$ of the potential as perturbation and linear part $br$ as parent has been reported in ref.\[33\] and the alternate approach of choosing the linear part $br + c$ as perturbation has been reported earlier \[34, 35, 36\]. For completeness we summarise the main equations of the two wave functions in the model.

### 2.2 Wavefunction with linear part as perturbation

The wavefunction with linear part as perturbation and Coulombic part as parent with its confinement effect is given by\[34, 35, 36\],

$$\psi_{\text{linear}}(r) = \frac{N'}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}} \left( C' - \frac{\mu b a_0 r^2}{2} \right) \left( \frac{r}{a_0} \right)^{-\epsilon} \tag{2}$$

where the subscript "linear" means linear part of the potential as perturbation and

$$N' = \frac{2^{\frac{3}{2}}}{\sqrt{2^2 \Gamma (3 - 2\epsilon) C'^2 - \frac{1}{4} \mu b a_0^3 \Gamma (5 - 2\epsilon) C' + \frac{1}{64} \mu^2 b^2 a_0^6 \Gamma (7 - 2\epsilon) \left( 1 + a_0^2 Q^2 \right)}} \tag{3}$$

$$C' = 1 + c A_0 \sqrt{\pi a_0^3} \tag{4}$$

$$\mu = \frac{m_i m_j}{m_i + m_j} \tag{5}$$

$$a_0 = \left( \frac{4}{3} \mu \alpha_s \right)^{-1} \tag{6}$$

and

$$\epsilon = 1 - \sqrt{1 - \left( \frac{4}{3} \alpha_s \right)^2}. \tag{7}$$

It is important to note that the condition of convergence of the model is being discussed in ref.\[34, 39, 30\], which demands that the linear part "$br + c$" of the potential can be considered as perturbation provided

$$\frac{(4 - \epsilon)(3 - \epsilon) \mu b a_0^3}{2(1 + a_0^2 Q^2)} << C'. \tag{8}$$

### 2.3 Wavefunction with Coulombic part as perturbation

The wavefunction with linear part as parent becomes an Airy function, which in fact makes the total wavefunction a complicated one since Airy function is a diverging function. Thus, the total wave function corrected up to first order and considering up to order $r^3$ are given by\[33, 37\]

$$\psi_{\text{coul}}(r) = \psi^{(0)}(r) + \psi^{(1)}(r) \tag{9}$$

$$= \frac{N_1}{2 \sqrt{\pi}} \left[ Ai((2\mu b)^{\frac{1}{3}} + \rho_0) - \frac{4\alpha_s}{3} \left( \frac{a_0}{r} + a_1 + a_2 r \right) \right] \tag{10}$$
where \( N_1 \) is the normalisation constant for the total wave function \( \psi_{\text{coul}}(r) \) with subscript ‘coul’ means Coulombic potential as perturbation.

Where \( \rho_{0n} \) are the zeros of the Airy function and is given by \([29, 38]\):

\[
\rho_{0n} = -\left[\frac{3\pi (4n - 1)}{8}\right]^{\frac{2}{3}}
\]

(11)

\[
a_0 = \frac{0.8808 (b\mu)^\frac{1}{3}}{(E - c)} - \frac{a_2}{\mu (E - c)} + \frac{4W^1 \times 0.21005}{3\alpha_s (E - c)}
\]

(12)

\[
a_1 = \frac{ba_0}{(E - c)} + \frac{4 \times W^1 \times 0.8808 \times (b\mu)^\frac{1}{3}}{3\alpha_s (E - c)} - \frac{0.6535 \times (b\mu)^\frac{2}{3}}{(E - c)}
\]

(13)

\[
a_2 = \frac{4\mu W^1 \times 0.1183}{3\alpha_s}
\]

(14)

\[
W^1 = \int_0^{+\infty} r^2 |\psi^{(0)}(r)|^2 dr
\]

(15)

where \( H' = -\frac{4\alpha_s}{3r} + c \) is the perturbed hamiltonian and

\[
E = -\left(\frac{b^2}{2\mu}\right)^\frac{1}{3} \rho_{0n}
\]

(16)

### 2.4 The strong coupling constant \( \alpha_s \) in the Model

The strong running coupling constant appeared in the potential \( V(r) \), is considered to be related to the quark mass parameter as\([39, 28, 40]\):

\[
\alpha_s(\mu_0^2) = \frac{4\pi}{(11 - 2n_f/3)\ln\left(\frac{\mu_0^2 + M_B^2}{\Lambda^2}\right)}
\]

(17)

where, \( n_f = 3 \) is the number of flavours, \( \mu_0 \) is the renormalisation scale related to the constituent quark masses as \( \mu_0 = 2\frac{m_i m_j}{m_i + m_j} \), \( \Lambda \) (or \( \Lambda_{\text{QCD}} \)) is the characteristic scale of QCD and \( M_B \) is the background mass. The background mass can be calculated in the frame work of lattice QCD. The appearance of mass \( M_B \) in Eq\([17]\) is similar to the case of QED where \( \alpha \) has the mass of \( e^+e^- \) pair under logarithm \([28]\). Here we relate the background mass to the confinement term of the potential as \( M_B = 2.24 \times b^{1/2} = 0.95GeV \)[40 39]. With \( M_B \), thus we are able to incorporate the confinement effect so that with \( \mu_0^2 = \Lambda^2 \), the strong coupling constant \( \alpha_s \) becomes finite and with zero confinement \((b=0)\), Eq\([17]\) becomes equivalent to that of the \( \overline{MS} \) scheme.

In studying leptonic decay \([39]\), we fixed \( \Lambda_{\text{QCD}} = 200MeV \) in the Eq\([17]\) to obtain the value of running background coupling \( \alpha_s \). In case of leptonic decay of charged mesons, the quark(q) and antiquark(\( \overline{q} \)) annihilate to produce a virtual \( W^\pm \) boson so that \( q^2 = M^2 \) and hence we get only one form factor which absorb all the strong interaction effects. This form factor is known as the decay constant \( f_p \). However, in case of semileptonic decay, the \( q^2 \) is different for event to event and hence more than one form factor appears. This decrease of \( q^2 \) in semileptonic decay leads us to consider a larger value of \( \Lambda_{\text{QCD}} \) than that of the leptonic decay which effectively increases the strong coupling
constant $\alpha_s$.

The physically plausible range of effective $\Lambda_{QCD}$ can be deduced from the allowed range of the slope and curvature of the I-W function. Considering the theoretical bounds on slope $3/4 \leq \rho^2 < 1.51$ \cite{41,42} and curvature $C \geq \frac{5q^2}{4}$ \cite{42} of the I-W function, we obtained an allowed range of $\Lambda_{QCD}$ in the model as $382\, MeV \leq \Lambda_{QCD} \leq 430\, MeV$ for $B$ meson \cite{43}. We extend this theoretical bounds for $B_c$ meson and compute the slope and curvature of the Isgur Wise function.

### 2.5 Form factors and decay rates of $B_c \rightarrow c\bar{c}(\ell^+\nu_\ell)$ transitions

In case of semileptonic decay, the matrix element is the product of leptonic and hadronic matrix element. The hadronic part is contributed by the vector ($V^\mu = \bar{c}\gamma^\mu b$) or axial vector ($A^\mu = \bar{c}\gamma^\mu\gamma_5 b$) current between $B_c$ and $c\bar{c}$ states. For transition between two pseudoscalar mesons ($B_c \rightarrow \eta_c$), axial current $A^\mu$ vanishes and vector current $V^\mu$ only contributes. This hadronic current, $V^\mu$ between the two $J^P = 0^-$ mesons is expressed in terms of two form factors $f_\pm(q^2)$ as \cite{41}

$$
\langle \eta_c(p')|V^\mu|B_c(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu
$$

(18)

Where $q$ is the four momentum transfer which varies within the range $m_c^2 \leq q^2 \leq (m_{B_c} - m_{\eta_c})^2 = q_{max}^2$ and $f_+(q^2)$ and $f_-(q^2)$ are two weak transition form factors.

For the transition of pseudoscalar to vector mesons ($B_c \rightarrow J/\psi(p', \epsilon)$) both the vector and axial vector current contributes and we get four independent form factors as,

$$
\langle J/\psi(p', \epsilon)|\bar{c}\gamma^\mu b|B_c(p)\rangle = 2i\epsilon_{\mu\nu\alpha\beta} \frac{\epsilon_\nu p'_\alpha p'_\beta}{M_{B_c} + M_{J/\psi}} V(q^2)
$$

(19)

$$
\langle J/\psi(p', \epsilon)|\bar{c}\gamma^\mu\gamma_5 b|B_c(p)\rangle = (M_{B_c} + M_{J/\psi}) \left[ \epsilon^\mu - \frac{\epsilon \cdot q q^\mu}{q^2} \right] A_1(q^2)
$$

$$
-\epsilon \cdot q \left[ \frac{(p+p')^\mu}{M_{B_c} + M_{J/\psi}} - \frac{(M_{B_c} - M_{J/\psi}) q^\mu}{q^2} \right] A_2(q^2)
$$

$$
2M_{J/\psi} \frac{\epsilon \cdot q q^\mu}{q^2} A_0(q^2)
$$

(20)

In the present study we treat $B_c$ system as a heavy-light one in analogy to $D$ system as the ratio of the constituent quark masses in the $B_c$ meson is very close to that of $D$ meson and extend HQET (heavy quark effective theory) for the study of $B_c$ meson also. On the basis of HQET, the most general form of the transition discussed by Eqs \cite{18} and \cite{19} can be expressed as \cite{41,37},

$$
\frac{1}{\sqrt{M_{B_c} M_{\eta_c}}} \langle \eta_c(v')|V^\mu|B_c(v)\rangle = (v + v')^\mu \xi(\omega)
$$

(21)

$$
\frac{1}{\sqrt{M_{B_c} M_{J/\psi}}} \langle J/\psi(v', \epsilon_3)|V^\mu|B_c(v)\rangle = i\epsilon^{\mu\nu\alpha\beta} \epsilon_\nu v'_\alpha v_\beta \xi(\omega)
$$

(22)

$$
\frac{1}{\sqrt{M_{B_c} M_{J/\psi}}} \langle J/\psi(v', \epsilon_3)|A^\mu|B_c(v)\rangle = [(1 + \omega)\epsilon^\mu - (\epsilon \cdot v) v'^\mu] \xi(\omega),
$$

(23)
where \( v \) and \( v' \) is the four velocity of \( B_c \) meson before and after the transition in the rest frame of the initial meson and \( \xi(\omega) \) is the universal form factor known as Isgur Wise function.

For small, nonzero recoil, Isgur-Wise function can be written by the formula \([15]\) :

\[
\xi(v,v') = \xi(Y) = 1 - \rho^2 (Y - 1) + C (Y - 1)^2 + ... \tag{24}
\]

where \( Y \) is given by,

\[
Y = v \cdot v' = \frac{m_{B_c}^2 + m_{c\bar{c}}^2 - q^2}{2m_{B_c}m_{c\bar{c}}} \tag{25}
\]

The first order derivative of I-W function at \( Y = 1 \) is known as the slope \( \rho^2 \) of the function i.e

\[
\rho^2 = \frac{\partial \xi}{\partial Y} \bigg|_{Y=1} \tag{26}
\]

and the second order derivative is the curvature of the I-W function:

\[
C = \frac{1}{2} \left( \frac{\partial^2 \xi}{\partial Y^2} \right) \bigg|_{Y=1} \tag{27}
\]

For heavy-light mesons, I-W function can also be expressed by another formula \([16, 35]\) :

\[
\xi(Y) = \int_0^{+\infty} 4\pi r^2 |\psi(r)|^2 \cos pr dr \tag{28}
\]

where

\[
p^2 = 2\mu^2 (Y - 1). \tag{29}
\]

In Eq.28 we employ the two wavefunctions(Eq.2 and Eq.9) to compute the slope and curvature of the Isgur-Wise function and collect the result in Table.4. The input parameters used in the numerical calculation are the same as is used in our previous works \([39, 43]\) which are \( n_f = 3, m_{u/d} = 0.336 \text{ GeV} \), \( m_c = 1.55 \text{ GeV} \), \( m_b = 4.97 \text{ GeV} \), \( b = 0.183 \text{GeV}^2 \) and \( cA_0 = 1\text{GeV}^{2/3} \) with \( c = -0.4 \text{ GeV} \). For the masses of \( B_c, \eta_c \) and \( J/\psi \), we use the experimental masses from PDG2012 \([47]\).

Table 1: The slope \( \rho^2 \) and curvature \( C \) of the I-W function with linear part as perturbation and coulombic part as perturbation.

| \( \Lambda \) | Linear part as perturbation | Coulombic part as perturbation |
|---------------|-----------------------------|-----------------------------|
| \( \rho^2 \)  | \( C \)                   | \( \rho^2 \)                  | \( C \)                  |
| 382 MeV       | 9.59                       | 117.783                     | 3.78                    | 0.057                     |
| 430 MeV       | 5.45                       | 31.39                       | 3.83                    | 0.051                     |

In ref.[25], the slope and curvature of the universal form factor for \( B_c \) meson is computed in the framework of QCD relativistic potential model and is shown in Table.2. The result of Table.4 is found to be closer to that of ref.[25] in one of the approach of our model with Coulombic part as perturbation. Interestingly, the scale \( \Lambda = 397\text{MeV} \)(used in ref.[25]) lies within our range of
Table 2: Parameters of the form factors for the channel of $B_c \rightarrow \eta_c(J/\psi)$ with $\Lambda = 397$ MeV (from ref. [25]).

| Channel          | $F(1)$ | $\rho^2$ | $C$ |
|------------------|--------|----------|-----|
| $B_c \rightarrow \eta_c(J/\psi)$ | $0.94$ | $2.9$    | $3$  |

Figure 1: Variation of I-W function with $Y$ for different scales of $\Lambda$ with linear part as perturbation.

382 MeV $\leq \Lambda_{QCD} \leq 430$ MeV. In Fig[1] and Fig[2] we show the variation of Isgur-Wise function with its four velocity transfer ($Y=v.v'$) in the two different approaches.

Applying HQET, the most general form of the transition discussed by Eqs. [18] and [19] can be expressed in terms of Isgur-Wise function as

$$f_{\pm}(q^2) = \xi(Y) \frac{m_{B_c} \pm m_{\eta_c}}{2 \sqrt{m_{B_c}m_{\eta_c}}}$$  \hspace{1cm} (30)

and

$$V(q^2) = A_2(q^2) = A_0(q^2) = \left[1 - \frac{q^2}{(M_{B_c} + M_{J/\psi})^2}\right]^{-1} A_1(q^2) = \frac{(M_{B_c} + M_{J/\psi})^2}{4M_{B_c}M_{J/\psi}} \xi(Y)$$  \hspace{1cm} (31)

Here we have applied the HQET to relate the form factors of the semileptonic transitions of $B_c \rightarrow c\bar{c}$ states with the Isgur-Wise function in Eq[30] and Eq[31]. These equations are based on the heavy flavour symmetry and is broken in the case of mesons containing two heavy quarks [26]. Spin symmetry breaking effects can occur when the $c$-quarks recoil momentum is larger than $m_c$. However, we expects that the equations are applicable to other kinematic point since the recoil momentum $c\bar{c}$ state is small ($y_{max} - 1 = 0.26$) due to its heavy mass [48]. In ref. [25], Pietro Colangelo and Fulvia De Fazio showed that the normalization of the form factor $\Delta$ describing the transition $B_c \rightarrow J/\psi \ell^+ \nu_\ell$ is close to 1 ($\approx 0.94$) at the zero-recoil point, as being the overlap of wave-functions, although it is not constrained by symmetry arguments.

The differential semileptonic decay rates can be expressed in terms of these form factors by

(a) $B_c \rightarrow P \nu \bar{\nu}$ decay ($P = \eta_c$)

$$\frac{d\Gamma}{dq^2}(B_c \rightarrow P \nu \bar{\nu}) = \frac{G_F^2 \Delta^3 |V_{qb}|^2}{24\pi^3} |f_+(q^2)|^2.$$  \hspace{1cm} (32)
Figure 2: Variation of I-W function with Y for different scales of Λ with Coulombic part as perturbation.

(b) $B_c \rightarrow Ve\nu$ decay ($V = J/\psi$) The decay rate in transversely(T) and longitudinally(L) polarized vector mesons are defined by\[49\]

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 \Delta |V_{cb}|^2}{96\pi^3} \frac{q^2}{M_B^2} |H_0(q^2)|^2,$$ \hspace{1cm} (33)

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2} = \frac{G_F^2 \Delta |V_{cb}|^2}{96\pi^3} \frac{q^2}{M_B^2} \left(|H_+(q^2)|^2 + |H_-(q^2)|^2\right). \hspace{1cm} (34)$$

where helicity amplitudes are given by the following expressions

$$H_\pm (q^2) = \frac{2M_{B_c}\Delta}{M_{B_c} + M_V} \left[V(q^2) \mp \frac{(M_{B_c} + M_V)^2}{2M_{B_c}\Delta} A_1(q^2)\right], \hspace{1cm} (35)$$

$$H_0(q^2) = \frac{1}{2M_V\sqrt{q^2}} \left[(M_{B_c} + M_V)(M_{B_c}^2 - M_V^2 - q^2)A_1(q^2) - \frac{4M_{B_c}^2\Delta^2}{M_{B_c} + M_V} A_2(q^2)\right]. \hspace{1cm} (36)$$

Thus the total semileptonic decay rate is given by

$$\frac{d\Gamma}{dq^2}(B_c \rightarrow V\ell\nu) = \frac{G_F^2 \Delta |V_{cb}|^2}{96\pi^3} \frac{q^2}{M_B^2} \left(|H_+(q^2)|^2 + |H_-(q^2)|^2 + |H_0(q^2)|^2\right), \hspace{1cm} (37)$$

where $G_F$ is the Fermi constant, $V_{cb}$ is CKM matrix element,

$$\Delta \equiv |\Delta| = \sqrt{\frac{(M_{B_c} + M_{P,V} - q^2)^2}{4M_{B_c}^2} - M_{P,V}^2}. \hspace{1cm}$$

Integrating over $q^2$ of these formulas (Eq.32 and Eq.37), we compute the total decay rate of the corresponding semileptonic decay and collect the result in Table 3. In Fig.3 and Fig.4, we plot the differential semileptonic decay rates $d\Gamma/dq^2$ for semileptonic decays $B_c \rightarrow \eta_c\ell\nu$ and $B_c \rightarrow J/\psi\ell\nu$ within the two approaches of our model.

The computed decay rates and branching ratios for the semileptonic decay of $B_c \rightarrow c\bar{c}(\ell^+\nu_\ell)$ shows that the results overshoots in case of linear part as perturbation and falls short with Coulombic part as perturbation. With Coulombic part as perturbation, the decay rate and branching ratio for
Figure 3: Differential decay rates $(1/|V_{cb}|^2)d\Gamma/dq^2$ of $B_c \rightarrow \eta_{c}\nu$ (in $GeV^{-1}$). The red and blue curves correspond to $\Lambda = 382$ $MeV$ and $430$ $MeV$ respectively. The upper two curves are with linear part as perturbation.

Figure 4: Differential decay rates $(1/|V_{cb}|^2)d\Gamma/dq^2$ of $B_c \rightarrow J/\psi\nu$ (in $GeV^{-1}$). The red and blue curves correspond to $\Lambda = 382$ $MeV$ and $430$ $MeV$ respectively. The upper two curves are with linear part as perturbation.

$B_c \rightarrow J/\psi(\ell^+\nu_\ell)$ semileptonic decay give comparable results with that of ref. [10] for both $\Lambda = 382 MeV$ and $\Lambda = 430 MeV$. However, with $\Lambda = 382 MeV$ the numerical result is more comparable to that of ref. [10] and we consider this small difference of decay rate for $\Lambda = 382 MeV$ and $\Lambda = 430 MeV$ significantly. This is because the smaller value of the QCD scale $\Lambda$ in Eq. 17 provides a smaller value in $\alpha_s$ and hence weakens the coulombic part of the potential to treat the later as perturbation. Thus the results with $\Lambda = 382 MeV$ for Coulombic part as perturbation is considered to be more comparable.

3 Results and Discussion

In this paper, we have explored the possibility of treating $B_c$ meson as a typical heavy-light meson like $B$ or $D$ within a QCD potential model, considering the Coulombic part as perturbation in one approach and linear part as perturbation in the other. We have taken the prescription of the strong coupling constant of ref. [28, 40], which contains a QCD cut off parameter $\Lambda_{QCD}$ constrained in the region $382$ $MeV \leq \Lambda_{QCD} \leq 430$ $MeV$ by the theoretical bounds on Isgur-Wise function. [41, 42]. We
Table 3: Decay width and Branching ratio for $B_c \to \bar{c}c(\ell^+\nu_\ell)$ decay. In the braces "linear" means the result with linear part as perturbation and "coul" means Coulombic part as perturbation.

| Channel                  | Decay width($\Gamma$) x 10^{-15} GeV | Others                  | Branching ratio x 10^{-2} | Others |
|--------------------------|---------------------------------------|--------------------------|---------------------------|--------|
| $B_c \to \eta_c(\ell^+\nu_\ell)$ | 415 (linear) 1.8 (coul)               | 10.7 [7] 5.9 [10] 14.2 [50] 11.1 [14] 11 ± 1 [15] | 28 (linear) 0.12 (coul) 0.11 (coul) | 0.81 [8] 0.42 [10] 0.76 [14] 0.15 [15] 0.51 [16] |
|                          | 424 (linear) 15 (coul)                | 28.2 [7] 17.7 [10] 34.4 [50] 30.2 [14] 28 ± 5 [15] | 29 (linear) 1.0 (coul) 0.98 (coul) | 2.07 [8] 1.23 [10] 2.01 [14] 1.47 [15] 1.44 [16] |

have taken the limiting values of the scales for our computation with the same parameters $b$ and $c$ from our previous work [39] (i.e. $b = 0.183 GeV^2$ and $cA_0 = 1 GeV^{3/2}$).

The values of slope and curvature of the I-W function seems to be acceptable with Coulombic part as perturbation where as with linear part as perturbation, the result overshoot the possible values. The former is also closer to the result of ref. [25] obtained in a QCD relativistic potential model [52, 53]. For a slight lower value of $\Lambda_{QCD} < 382 MeV$ ($\Lambda \approx 280 MeV$), one can obtain the values of slope $\rho^2$ at par with the ref. [25] ($\rho^2 = 2.9$).

When one chooses the perturbative term, there should be an underlying assumption that perturbative term should not have dominant impact, otherwise the result is not stable. To that end, we have calculated the I-W function for a different $w$ with contribution of linear parent and coulomb parent alone for $\Lambda = 382 MeV$ and $\Lambda = 430 MeV$. We see that both the process of perturbative is viable for a different range of $Y$. The linear parent dominates within the range $1 \leq Y \leq 1.22$ where as Coulombic parent dominates very near to zero recoil ($1 \leq Y \leq 1.06$, ref. Appendix). It indicates that the former one is perturbatively more stable than the later (which is marginally stable). For Coulombic part to be progressively stable, one would expect a large scale of $\Lambda > 430 MeV$ (or equivalently larger strong coupling constant) beyond the theoretical constrain discussed in this work. The reality condition of the model parameter $\epsilon$ [Eq.7], however permits to consider $\alpha_s(\mu) \leq 3/4$ and hence $\Lambda \leq 460 MeV$ is the allowed limit in the model.

In a sense, the present work is complimentary to the work on leptonic decays [39] where $\Lambda_{QCD} = 200 MeV$ was chosen with linear part as perturbation within the same prescription of running coupling constant [28, 40]. The apparent change of $\Lambda_{QCD}$ (and hence equivalent strong coupling constant) in the present case is attributed to the decrease of available momentum transfer in semileptonic decays compared to leptonic ones.

The necessity of two scales of $\Lambda_{QCD}$ (or equivalent two scales of strong coupling constant) for heavy-light mesons was noticed earlier within V-scheme [54, 55] in ref. [36] with linear term as perturbation. The present work conforms to the conclusion even in the scheme of ref. [28, 40] and hence appears to be a scheme invariant feature of the potential model under study.
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4 Appendix

In the following table we show the dominance of parent term over the perturbation by comparing the numerical values for I-W function for the total wave function and parent term only.

Table 4: Variation of I-W function with total wavefunction and parent wavefunction only for $\Lambda = 382 \ MeV$.

| $Y$  | Linear part as perturbation | Coulombic part as perturbation |
|------|-----------------------------|-------------------------------|
|      | $\xi_{\text{total}}(Y)$    | $\xi_{\text{parent}}(Y)$     | $\xi_{\text{total}}(Y)$    | $\xi_{\text{parent}}(Y)$ |
| 1.01 | 0.916                       | 0.509                         | 0.962                       | 0.959                     |
| 1.06 | 0.848                       | 0.772                         | 0.773                       | 0.757                     |
| 1.08 | 0.9865                      | 2.145                         | 0.697                       | 0.676                     |
| 1.20 | 3.79                        | 25.06                         | 0.246                       | 0.192                     |
| 1.24 | -                           | -                             | 0.096                       | 0.031                     |