CP asymmetries in scalar bottom quark decays

A. Bartl\textsuperscript{a}, E. Christova\textsuperscript{b}, K. Hohenwarter-Sodek\textsuperscript{a}, T. Kernreiter\textsuperscript{a}

\textsuperscript{a}Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria
\textsuperscript{b}Institute for Nuclear Research and Nuclear Energy, Sofia 1784, Bulgaria

Abstract

We propose CP asymmetries based on triple product correlations in the decays $\bar{b}_m \to t\tilde{\chi}_j^-$ with subsequent decays of $t$ and $\tilde{\chi}_j^-$. For the subsequent $\tilde{\chi}_j^-$ decay into a leptonic final state $\ell^-\tilde{\nu}\tilde{\chi}_1^0$, we consider the three possible decay chains $\tilde{\chi}_j^- \to \ell^-\tilde{\nu} \to \ell^-\tilde{\nu}\tilde{\chi}_1^0$, $\tilde{\chi}_j^- \to \ell^-\tilde{\nu} \to \ell^-\tilde{\nu}\tilde{\chi}_1^0$ and $\tilde{\chi}_j^- \to 3\chi_1^0 \to \ell^-\tilde{\nu}\tilde{\chi}_1^0$. We consider two classes of CP asymmetries. In the first class it must be possible to distinguish between different leptonic $\tilde{\chi}_j^-$ decay chains, whereas in the second class this is not necessary. We consider also the 2-body decay $\tilde{\chi}_j^- \to W^-\tilde{\chi}_1^0$, and we assume that the momentum of the $W$ boson can be measured. Our framework is the minimal supersymmetric standard model with complex parameters. The proposed CP asymmetries are non-vanishing due to non-zero phases for the parameters $\mu$ and/or $A_b$. We present numerical results and estimate the observability of these CP asymmetries.
1 Introduction

In the Minimal Supersymmetric Standard Model (MSSM) [1, 2] the higgsino mass parameter $\mu$ and several of the Supersymmetry (SUSY) breaking parameters are complex in general. Among the SUSY breaking parameters the trilinear scalar couplings $A_f$ and two of the gaugino mass parameters $M_1$ and $M_3$ ($M_2$ is usually chosen to be real by redefining the fields) can be complex.

Current experimental upper bounds on the electric dipole moments (EDM) impose restrictions on the SUSY parameters that appear in supersymmetric models, in particular on their phases. To which extent the size of the phases have to be restricted, however, strongly depends on the underlying model. For instance, while only relatively small values of the phase of $\mu$, $|\phi_\mu| \lesssim 0.1$, are allowed in several versions of the MSSM with selectron masses of the order 100 GeV [3], this restriction may disappear if lepton flavour violating terms are included [4] or if the masses of the first and second generation scalar fermions are large (above the TeV scale) while the masses of the third generation scalar fermions are small (below 1 TeV) [5]. Recently it has been pointed out that for large trilinear scalar couplings $|A|$ one can simultaneously fulfill the EDM constraints of electron, neutron, and that of the atoms $^{199}$Hg and $^{205}$Tl, where at the same time, $\phi_\mu \sim O(1)$ [6]. The restrictions on the size of the phases of the trilinear couplings of the 3rd generation scalar fermions are far less important as their contributions to the EDMs appear only at two-loop level [7].

The various CP phases can have a big influence on the production and decay of supersymmetric particles. In particular the influence of the phases $\phi_{A_{c,t,b}}$ of the trilinear scalar coupling parameters on various observables (e.g. scalar fermion masses, cross sections, decay widths) can be important [8, 9]. However, a measurement of solely CP-even observables cannot be sufficient to unambiguously determine the SUSY parameters. Moreover, in order to clearly demonstrate that CP is violated, CP-odd observables have to be measured. Rate asymmetries have been proposed where the influence of the SUSY CP phases arise due to loop corrections (see for instance [10]). Another important class of CP-odd observables are based on triple product correlations (for an introduction see [11]). They arise already at tree-level and allow to define various CP asymmetries which are sensitive to the different CP phases. Such CP asymmetries have been proposed and analyzed for various SUSY processes (see for instance [12, 13]).

Recently, it has been shown [13] that triple product correlations among the decay products of the scalar top decay $\tilde{t} \rightarrow t\tilde{\chi}^0$ followed by the decays of $t$ and $\tilde{\chi}^0$, allow us to obtain information on CP violation in the scalar top system. Along the same line of the study performed in [13], in the present paper we analyze triple product correlations that arise in the decays of the scalar bottoms $\tilde{b}_m$. We focus on the influence of CP violation in the scalar bottom system, in particular on the influence
of the phase of the trilinear scalar coupling parameter $A_b, \phi_{A_b}$.

We study the decay

$$\tilde{b}_m \rightarrow t \tilde{\chi}_j^-,$$

followed by the subsequent decays of the top quark $t$ and the chargino $\tilde{\chi}_j^-$. We work in the approximation where $t$ and $\tilde{\chi}_j^-$ are both produced on mass-shell. As the top quark does not form a bound state this implies that both $t$ and $\tilde{\chi}_j^-$ decay with definite momenta and polarizations. Their polarizations can be retrieved from the distributions of their decay products. We consider the decays of the top quark

$$t \rightarrow b \ W^+ \quad \text{and} \quad t \rightarrow b \ l^+ \nu_l \ (b \ c \ \bar{s}),$$

and the following three possible decay chains for $\tilde{\chi}_j^-$:

$$\tilde{\chi}_j^- \rightarrow \ell_1^- \ \bar{\nu} \rightarrow \ell_1^- \bar{\nu} \tilde{\chi}_1^0,$$

$$\tilde{\chi}_j^- \rightarrow \ell_2^- \bar{\nu} \rightarrow \ell_2^- \bar{\nu} \tilde{\chi}_1^0,$$

$$\tilde{\chi}_j^- \rightarrow W^- \tilde{\chi}_1^0 \rightarrow \ell_3^- \bar{\nu} \tilde{\chi}_1^0,$$

which lead to the final states

$$\tilde{\chi}_j^- \rightarrow \ell^- \bar{\nu} \tilde{\chi}_1^0, \quad \ell = e, \mu, \tau.$$

We shall consider each of the decays (3),(4),(5) separately. The subscript of the leptons, $\ell_1, \ell_2, \ell_3$, is used in order to distinguish them in the different decay chains. For simplicity we shall work in the narrow width approximation for the intermediate particles in (3)-(5), i.e. we assume that these particles are produced on-mass-shell.

We consider also the 2-body decay of $\tilde{\chi}_j^-:

$$\tilde{\chi}_j^- \rightarrow W^- \tilde{\chi}_1^0,$$

assuming the momentum of the final $W$ boson can be reconstructed, which is possible for hadronic decays.

We consider the triple products

$$\mathcal{O} = q_1 \cdot (q_2 \times q_3) \equiv (q_1 q_2 q_3),$$

(8)
where \( \mathbf{q}_i \) are any 3-vectors of the particles in the considered process. With the help of the triple products \( \mathcal{O} \), Eq. (8), we define the T-odd observables (up-down asymmetries):

\[
A_T \equiv \int \frac{d\Omega \, \text{sgn}(\mathcal{O}) \, d\Gamma/d\Omega}{d\Omega \, d\Gamma/d\Omega} = \frac{N[\mathcal{O} > 0] - N[\mathcal{O} < 0]}{N[\mathcal{O} > 0] + N[\mathcal{O} < 0]},
\]

(9)

where \( d\Gamma \) stands for the differential decay width and \( d\Omega \) involves the angles of integration. The left hand side of Eq. (9) shows how the asymmetries are calculated, whereas the right hand side exemplifies how they are measured in experiment: \( N[\mathcal{O} > (\leq) 0] \) is the number of events for which \( \mathcal{O} > (\leq) 0 \).

The paper is organized as follows. In section 2 we present the results of our calculations in compact form using the formalism of [14]. We propose several T-odd asymmetries in section 3 and point out how the corresponding CP asymmetries can be obtained. In section 4 we perform a numerical analysis of the CP asymmetries proposed and estimate their observability. Finally, we summarize in section 5.

2 Formalism

In order to obtain analytic expressions for the sequential processes (1)–(7) we shall use the formalism of Kawasaki, Shirafuji and Tsai [14]. According to that formalism the differential decay rates of (1)–(7), when spin-spin correlations are taken into account, can be written as

\[
d\Gamma = d\Gamma(\tilde{b}_m \to t\tilde{\chi}_j^-) \frac{E_t}{m_t \Gamma_t} d\Gamma(t \to ...) \frac{E_{\chi_j}}{m_{\chi_j} \Gamma_{\chi_j}} d\Gamma(\tilde{\chi}_j^- \to ...) ,
\]

(10)

where \( d\Gamma(t \to ...) \) and \( d\Gamma(\tilde{\chi}_j^- \to ...) \) are the differential decay rates of the unpolarized top and unpolarized chargino. The factors \( E_{\chi_j}/(m_{\chi_j} \Gamma_{\chi_j}) \) and \( E_t/(m_t \Gamma_t) \) stem from the narrow width approximation used for \( t \) and \( \tilde{\chi}_j^- \), \( \Gamma_t \) and \( \Gamma_{\chi_j} \) are the total widths of \( t \) and \( \tilde{\chi}_j^- \), and \( (E_t, m_t) \) and \( (E_{\chi_j}, m_{\chi_j}) \) are their energies and masses, respectively.

\( d\Gamma(\tilde{b}_m \to t\tilde{\chi}_j^-) \) is the differential decay rate of the scalar bottom \( \tilde{b}_m \) into a top quark with the polarization 4-vector \( \xi_t^\alpha \) and a chargino with the polarization 4-vector \( \xi_{\chi_j}^\alpha \).

In the scalar bottom rest frame, we have:

\[
d\Gamma(\tilde{b}_m \to t \tilde{\chi}_j^-) = \frac{2}{m_{\tilde{b}_m}} |A|^2 d\Phi_{\tilde{b}_m} ,
\]

(11)
where $m_{\tilde{b}_m}$ is the mass of the decaying scalar bottom and the phase space element $\Phi_{\tilde{b}}$ is given in Eq. (76) in Appendix C. For the matrix element $A$ we have

$$A = g \bar{u} (p_t) (k_{\tilde{b}_m}^L P_L + l_{\tilde{b}_m}^R P_R) v(p_{\chi_j}) ,$$

where $P_{L,R} = \frac{1}{2} (1 - \gamma_5)$, $g$ is the SU(2) gauge coupling constant and the couplings are given in Eq. (70) in Appendix B. For the evaluation of $|A|^2$ we use the spin density matrices of $t$ and $\tilde{\chi}_j$:

$$\rho(p_t) = \Lambda(p_t) \frac{1 + \gamma_5 \xi_t}{2} , \quad \rho(-p_{\chi_j}) = -\Lambda(-p_{\chi_j}) \frac{1 + \gamma_5 \xi_{\chi_j}}{2} ,$$

with

$$\Lambda(p_t) = p_t + m_t , \quad \Lambda(p_{\chi_k}) = p_{\chi_j} + m_{\chi_j} .$$

The matrix element squared is then given by

$$|A|^2 = \frac{g^2}{2} \left\{ \left[ |l_{\tilde{b}_m}^L|^2 + |k_{\tilde{b}_m}^L|^2 \right] \left[ (p_{\chi_j} p_t) + m_{\chi_j} m_t (\xi_{\chi_j} \xi_t) \right] 
- \left[ |l_{\tilde{b}_m}^R|^2 - |k_{\tilde{b}_m}^R|^2 \right] \left[ m_t (p_{\chi_j} \xi_t) (p_{\chi_j} p_t) \right] 
- 2 \text{Re} (l_{\tilde{b}_m}^L k_{\tilde{b}_m}^R) \left[ m_{\chi_j} m_t - (p_{\chi_j} \xi_t) (p_{\chi_j} p_t) + (p_{\chi_j} p_t) (\xi_{\chi_j} \xi_t) \right] 
+ 2 \text{Im} (l_{\tilde{b}_m}^L k_{\tilde{b}_m}^R) \varepsilon^{\alpha \beta \gamma \delta} p_{\chi_j}^\alpha \xi_{\chi_j}^\beta \xi_t^\gamma p_t^\delta \right\}$$

where we use the convention $\varepsilon^{0123} = 1$. The polarization 4-vector $\xi_t$ is determined through the top quark decays (2) and the polarization 4-vector $\xi_{\chi_j}$ is determined through the $\tilde{\chi}_j$ decays (3)–(7). In the following we calculate the polarization 4-vectors $\xi_t$ and $\xi_{\chi_j}$, as well as the differential decay rates of $t$ and $\tilde{\chi}_j$ for their various decays (2) and (3)–(7). Some of the calculations are quite analogous to those carried out in [13] and in these cases we present the results only.

### 2.1 Decay rates for $\tilde{\chi}_j \rightarrow \ell^- \tilde{\nu} \rightarrow \ell^- \bar{\nu} \tilde{\chi}_1^0$

The polarization vector of the top quark was obtained in [13] and here we present the results for completeness. The polarization 4-vector of the top quark determined through the decay $t \rightarrow b \ W^+$, that we shall denote by $\xi_b$, equals

$$\xi_b^\alpha = \alpha_b \frac{m_t}{(p_t p_b)} \left( p_b^\alpha - \frac{(p_t p_b)}{m_t^2} p_t^\alpha \right) , \quad \alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} .$$
For the polarization vector of the top quark determined in $t \to b W^+ \to b l^+ \nu$ (and equivalently for $t \to b W^+ \to b c \bar{s}$, where we substitute the the $c$ quark for the lepton), that we denote by $\xi_t$, we have

$$\xi_t^\alpha = \alpha_l \frac{m_t}{(p_t p_l)} \left( p_l^\alpha - \frac{(p_t p_l)}{m_t^2} p_t^\alpha \right), \quad \alpha_l = -1 \ . \quad (17)$$

The polarization vector of $\tilde{\chi}_j^-$ is determined solely through the decay $\tilde{\chi}_j^- \to \ell^- \bar{\nu}$, as the subsequent decay of $\bar{\nu}$, being a scalar particle, does not contribute. We obtain:

$$\xi_{\chi_j}^\alpha = \alpha_{\tilde{\nu}} \frac{m_{\chi_j}}{(p_{\chi_j} p_{\ell_1})} \left( p_{\ell_1}^\alpha - \frac{(p_{\chi_j} p_{\ell_1})}{m_{\chi_j}^2} p_{\chi_j}^\alpha \right), \quad \alpha_{\tilde{\nu}} = \frac{|l_{\bar{\nu}}|^2 - |k_{\bar{\nu}}|^2}{|l_{\bar{\nu}}|^2 + |k_{\bar{\nu}}|^2} \ . \quad (18)$$

Further, according to Eq. (10), we need the differential decay rates of $t$ and $\tilde{\chi}_j^-$. The distribution of the leptons in the sequential decay (3), in the narrow width approximation for $\bar{\nu}$, is given by

$$d\Gamma_{X_j}^1 (\tilde{\chi}_j^- \to \ell^- \bar{\nu}) = d\Gamma (\tilde{\chi}_j^- \to \ell^1 \bar{\nu}) \cdot BR(\bar{\nu} \to \bar{\nu}X_1^0) \ , \quad (19)$$

where $BR(\bar{\nu} \to \bar{\nu}X_1^0)$ is the branching ratio of the decay $\bar{\nu} \to \bar{\nu}X_1^0$ and

$$d\Gamma (\tilde{\chi}_j^- \to \ell^- \bar{\nu}) = \frac{g^2 (|l_{\bar{\nu}}|^2 + |k_{\bar{\nu}}|^2) (p_{\chi_j} p_{\ell_1})}{2E_{\chi_j} m_{\chi_j}^2} d\Phi_{\chi_j}^1 \ , \quad (20)$$

where the couplings are given in Eq. (71) in Appendix B and the phase space element $d\Phi_{\chi_j}^1$ is given in Eq. (80) in Appendix C. The differential decay rates of the top quark are (see for instance [13]):

$$d\Gamma (t \to bW^+) = \frac{g^2 (m_t^2 - m_W^2) (2m_W^2 + m_t^2)}{8E_t m_W^2} d\Phi_t^b \ , \quad (21)$$

$$d\Gamma (t \to bl^+ \nu) = \frac{g^4 \pi (p_t p_l) (m_t^2 - 2(p_t p_l))}{2E_t m_W \Gamma_W} d\Phi_t^l \ , \quad (22)$$

with $d\Phi_t^{b,l}$ given in Eqs. (77) and (78) in Appendix C.

The angular distributions of the decay products of $t$ and $\tilde{\chi}_j^-$ decay mode (3) are obtained by inserting the differential decay rate of the scalar bottom, Eq. (11), the differential decay rates of the top quark, Eqs. (21) and (22), and the differential
decay rate of the chargino, Eq. (19), into Eq. (10), where we use the appropriate polarization vectors as given in Eqs. (16)–(18). The differential decay rates of $\tilde{b}_m$ then read

\[
d\Gamma^I_f = N_f \frac{g^6 BR(\tilde{\nu} \rightarrow \nu_\chi^0) (p_{\chi_j}p_{\ell_1}) \left( |l^\nu_j|^2 + |k^\nu_j|^2 \right)}{8 \, m_b \, m_t \, m_{\chi_j} \, m_{\chi_i} \, m_{\chi} \, \Gamma_{\chi i}} \times \left\{ (|l^\nu_j|^2 + |k^\nu_j|^2) (p_{\chi_j}p_{\nu}) - 2 \, \Re e (l^\nu_j k^\nu_j^*) m_{\chi_j} m_t + \cdots \right. \\
+ 2 \, \Im m (l^\nu_j k^\nu_j) \alpha_f \, \alpha_{\nu} \, m_t \, \left( p_{\ell_1} p_f \right) \left( p_{\chi_j} p_{\nu} \right) \left( p_{\chi_j} p_{\nu} \right) \right\} \, d\Phi^I_f,
\]

(23)

where the subindex $f = b, l$ is to distinguish the two $t$ quark decays in (2). The prefactors in Eq. (23) are

\[
N_b = \frac{(m_t^2 - m_W^2)(2m_W^2 + m_t^2)}{2m_W^2}, \\
N_l = \frac{g^2 \, 2 \, \pi \, (p_{\ell_1}p_t)(m_t^2 - 2(p_{\ell_1}p_t))}{m_W \, \Gamma_W},
\]

(24)

and the phase space elements equal

\[
d\Phi^I_f = d\Phi^I_{b_m} \, d\Phi^I_{l_f} \, d\Phi^I_{\chi_j}.
\]

(25)

In Eq. (23) we have only included those terms which are needed for the calculation of the up-down asymmetries in Eq. (9). The omitted terms, represented by dots, are T-even and thus, cannot contribute to the numerator of Eq. (9). Further, as they depend on the polarizations of either the top quark or the chargino, they cannot contribute to the denominator of Eq. (9).

### 2.2 Decay rates for $\chi^-_j \rightarrow \tilde{\ell}_n \tilde{\nu} \rightarrow \ell_2 \tilde{\nu} \chi^0_1$

In order to obtain the angular correlations among the $t$ decay products and the lepton $\ell_2$ stemming from the $\chi^-_j$ decay (4), we need the polarization 4-vector of $\chi^-_j$ determined in the decay (4). As $\tilde{\ell}_n$ is a scalar particle, $\xi_{\chi_j}$ is determined solely in the decay $\chi^-_j \rightarrow \tilde{\ell}_n \tilde{\nu}$. We obtain:

\[
\xi_{\chi_j}^\alpha = \alpha_{\ell} \, \frac{m_{\chi_j}}{(p_{\chi_j}p_{\nu})} \left( p_\nu^{-\alpha} - \frac{(p_{\chi_j}p_{\nu})}{m_{\chi_j}^2} \, p_\chi^{-\alpha} \right), \quad \alpha_{\ell} = -1.
\]

(26)
The differential decay rate of the decay chain (4), in the narrow width approximation for $\tilde{\ell}_n^-$, reads

$$d\Gamma_{\chi_j}(\tilde{\chi}_j^- \rightarrow \tilde{\ell}_n^- \bar{\nu}) = d\Gamma(\tilde{\chi}_j^- \rightarrow \tilde{\ell}_n^- \bar{\nu}) \frac{E_{\tilde{\ell}}}{m_{\tilde{\ell}}} d\Gamma(\tilde{\ell}_n^- \rightarrow \tilde{\chi}_1^0 \ell_2^-) ,$$

(27)

with the differential decay rates for $\tilde{\chi}_j^- \rightarrow \tilde{\ell}_n^- \bar{\nu}$ and $\tilde{\ell}_n^- \rightarrow \tilde{\chi}_1^0 \ell_2^-$ given by

$$d\Gamma(\tilde{\chi}_j^- \rightarrow \tilde{\ell}_n^- \bar{\nu}) = \frac{g^2}{2 E_{\chi_j}} |l_{nj}^\ell|^2 (p_{\chi_j} p_\nu) d\Phi_{\chi_j}^2 ,$$

(28)

and

$$d\Gamma(\tilde{\ell}_n^- \rightarrow \tilde{\chi}_1^0 \ell_2^-) = \frac{g^2}{E_{\tilde{\ell}}} (|a_{nk}|^2 + |b_{nk}|^2) (p_\ell p_{\ell_2}) d\Phi_{\tilde{\ell}} ,$$

(29)

where the couplings are given in Appendix C in Eqs. (72) and (74). The phase space elements $d\Phi_{\chi_j}^2$ and $d\Phi_{\tilde{\ell}}$ are given in Appendix B in Eqs. (81) and (82), respectively.

The angular distributions of the decay products of $t$ are the same as in the previous case. On the other hand, the angular distribution of the decay products of the $\tilde{\chi}_j^-$ decay mode (4) is given by Eq. (27) which we insert into Eq. (10) in order to obtain the differential decay rates of the combined process (1), (2) and (4). The polarization vector of the chargino is determined through the decay (4) and is given in Eq. (26). Then the differential decay rates of $\tilde{b}_m$ read

$$d\Gamma_f^\Pi = N_f \frac{g^8}{8 m_{\tilde{g}} m_{\tilde{t}} m_{\chi_j} \Gamma_{\chi_j} m_{\tilde{\ell}} \Gamma_{\tilde{\ell}}} \times \left\{ (|l_{mj}|^2 + |k_{mj}|^2) (p_{\chi_j} p_t) - 2 \Re(l_{mj}^* k_{mj}) m_{\chi_j} m_t + \cdots \
+ 2 \Im(l_{mj}^* k_{mj}) m_{\chi_j} m_{\tilde{g}} (p_{\ell_2} p_{\ell_1}) \right\} d\Phi_f^\Pi ,$$

(30)

where the phase space elements equal

$$d\Phi_f^\Pi = d\Phi_{\tilde{b}_m} d\Phi_{\tilde{\ell}} d\Phi_{\chi_j} d\Phi_{\tilde{\ell}} .$$

(31)

As in the previous case, we have omitted those terms in Eq. (30) (denoted by dots) which are unessential for the calculation of the up-down asymmetries, Eq. (9).
2.3 Decay rates for $\tilde{\chi}_j^- \rightarrow W^- \tilde{\chi}_1^0 \rightarrow \ell_3^- \bar{\nu} \tilde{\chi}_1^0$

When the decay of $\tilde{\chi}_j^-$ proceeds via the $W^-$ boson exchange, (5), the polarization 4-vector $\xi_{\chi_j}$ is parameterized by two components that are in the $\tilde{\chi}_j^-$ decay plane and a component normal to it. It can be written completely general as

$$\xi^\alpha_{\chi_j} = P_\ell Q_\ell^\alpha + P_\nu Q_\nu^\alpha + D^{CP} \epsilon^{\alpha\beta\gamma\delta} p_{\ell\beta} p_{\nu\gamma} p_{\chi_j\delta}$$

(32)

where the 4-vectors $Q_\ell^\alpha$ and $Q_\nu^\alpha$ are in the decay plane of $\tilde{\chi}_j^-$:

$$Q_\ell^\alpha = p_\ell - \frac{(p_\ell p_{\chi_j})}{m_{\chi_j}^2} p_{\chi_j}^\alpha,$$

$$Q_\nu^\alpha = p_\nu - \frac{(p_\nu p_{\chi_j})}{m_{\chi_j}^2} p_{\chi_j}^\alpha,$$

(33)

and $\epsilon^{\alpha\beta\gamma\delta} p_{\ell\beta} p_{\nu\gamma} p_{\chi_j\delta}$ is orthogonal to it. For the components in the decay plane we obtain

$$P_\ell = \frac{m_{\chi_j} |O_{ij}^L|^2 (2(p_\ell p_{\chi_j}) - m_W^2) - 2m_{\chi_j}^2 (p_\ell p_{\chi_j}) \Re (O_{ij}^L \chi_{ij})}{|C|^2},$$

$$P_\nu = \frac{-m_{\chi_j} |O_{ij}^R|^2 (2(p_\ell p_{\chi_j}) - m_W^2) + 2m_{\chi_j}^2 (p_\ell p_{\chi_j}) \Re (O_{ij}^R \chi_{ij})}{|C|^2},$$

(34)

with

$$|C|^2 = -m_W^2 \left[ |O_{ij}^L|^2 (p_\ell p_{\chi_j}) + |O_{ij}^R|^2 (p_\ell p_{\chi_j}) + m_{\chi_j}^2 (p_\ell p_{\chi_j}) \Re (O_{ij}^L \chi_{ij}) \right] + 2(p_\ell p_{\chi_j}) (|O_{ij}^L|^2 + |O_{ij}^R|^2),$$

(35)

where the couplings are given in Appendix B in Eq. (75). The component normal to the decay plane reads

$$D^{CP} = \frac{2m_{\chi_j} \Im (O_{ij}^L \chi_{ij})}{|C|^2}. $$

(36)

The component $D^{CP}$ is sensitive to CP violation in the $\tilde{\chi}_j^- \tilde{\chi}_1^0 W^+$ couplings, i.e. to the phases $\phi_\mu$ and $\phi_{M1}$. The decay rate distribution of $\tilde{\chi}_j^- \rightarrow W^- \tilde{\chi}_1^0 \rightarrow \ell_3^- \bar{\nu} \tilde{\chi}_1^0$ is given by

$$d\Gamma^{\text{III}}_{\chi_j} (\tilde{\chi}_j^- \rightarrow \ell_3^- \bar{\nu} \tilde{\chi}_1^0) = \sum_{\pm} \frac{g^4 \pi}{m_W \Gamma_W E_{\chi_j}} |C|^2 d\Phi^{\text{III}}_{\chi_j},$$

(37)
where \( d\Phi_{\chi_j}^{\text{III}} = \frac{1}{2\pi}(d\Phi_{\chi_j}^{3})^\pm \) with \((d\Phi_{\chi_j}^{3})^\pm\) being the phase space element for the decay \( \tilde{\chi}_j \to W^- \tilde{\chi}_1^0 \), Eq. (83) in Appendix C, and \( d\Phi_{W}^{3} \) is the phase space element for the decay \( W^- \to \ell^-_3 \tilde{\nu} \), Eq. (86).

The angular distributions of the decay products of \( \tilde{b}_m \), where the chargino decays according to (5), can now be obtained in the same manner as in the previous two cases. Again we only quote the terms that are essential for the calculation of the up-down asymmetries in Eq. (9):

\[
d\Gamma_{f}^{\text{III}} = \sum_{\pm} N_f \frac{g^8 \pi |C|^2}{4m_{\tilde{b}}m_{\ell} \Gamma_{\ell} \Gamma_{\chi_j} \Gamma_{W} \Gamma_{W}^*} \times \left\{ |l_{m_j}^{\tilde{b}}|^2 + |k_{m_j}^{\tilde{b}}|^2 \right\} (p_{\ell} p_{\tilde{\nu}}) - 2 \Re(l_{m_j}^{\tilde{b}} k_{m_j}^{\tilde{b}}) m_{\chi_j} m_{\ell} + \cdots \\
+ 2 \alpha_f \Re(l_{m_j}^{\tilde{b}} k_{m_j}^{\tilde{b}}) \frac{m_{\ell}}{(p_{\ell} p_{f})} (P_{\ell} - P_{f}) m_{\tilde{b}} (p_{\ell} p_{f} p_{\tilde{\nu}}) \right\} \ d\Phi_{f}^{\text{III}}, \tag{38}
\]

with

\[
d\Phi_{f}^{\text{III}} = d\Phi_{\tilde{b}} \ d\Phi_{\ell} \ d\Phi_{\chi_j}^{\text{III}}, \tag{39}
\]

where the sum in Eqs. (37) and (38) corresponds to the two kinematical solutions for \( E_{\ell_3} \) (for details see Appendix C).

In principle, the normal component of the chargino polarization vector in Eq. (32) will also give rise to triple products proportional to \( \Re(O_{kj} L_{kj}^{R}) \). However, in order to be sensitive to these triple products, the reconstruction of the decay plane of the chargino would be necessary. In practice, this cannot be accomplished, because the neutrino as well as the neutralino escape detection in experiment.

### 2.4 Decay rates for \( \tilde{\chi}_j^- \to W^- \tilde{\chi}_1^0 \)

Finally we consider the two-body decay mode of \( \tilde{\chi}_j^- \) (7). The polarization 4-vector of \( \tilde{\chi}_j^- \) in this case is given as

\[
\xi_{\chi_j}^\alpha = \alpha_W \frac{m_{\chi_j}}{(p_{\chi_j} p_{W})} \left( p_{W}^{\alpha} - \frac{(p_{W} p_{\chi_j})}{m_{\chi_j}^2} p_{\chi_j}^\alpha \right), \tag{40}
\]

with

\[
\alpha_W = 2 \left( \frac{|O_{L_{\chi_j}^{\alpha}}|^2 - |O_{R_{\chi_j}^{\alpha}}|^2}{|C_W|^2} \right) \left( \frac{m_{\chi_j}^2 - 2m_W^2 - m_{\chi_1}^0}{m_W^2} \right) (p_{\chi_j} p_{W}), \tag{41}
\]

\[\text{10}\]
where

\[ |C_W|^2 = (|O_{ij}^L|^2 + |O_{ij}^R|^2) \left[ \frac{(m_{\chi_i}^2 + m_{\chi_j}^2)m_W^2 + (m_{\chi_i}^2 - m_{\chi_j}^2)^2 - 2m_W^4}{m_W^2} \right] \]

\[ - 12 \Re (O_{ij}^L*O_{ij}^R) m_{\chi_i}^0 m_{\chi_j}. \] (42)

The decay rate distribution of \( \tilde{\chi}^{-}_j \rightarrow W^- \tilde{\chi}^0_1 \) is given by

\[ d\Gamma_W(\tilde{\chi}^{-}_j \rightarrow W^- \tilde{\chi}^0_1) = \sum_\pm \frac{g^2}{4E_{\chi_j}} |C_W|^2 (d\Phi_W^4)^\pm. \] (43)

The angular distribution of the decay products of \( \tilde{b}_m \), with the chargino two-body decay (7), is given by

\[ d\Gamma^W_f(\tilde{b}^-_m \rightarrow W^- f) = \sum_\pm \frac{g^6}{16 m_{\tilde{b}} m_t \Gamma_{\tilde{b}} m_{\chi_j} \Gamma_{\chi_j}} \]

\[ \times \left\{ (|l^\tilde{b}_{m_j}|^2 + |k^\tilde{b}_{m_j}|^2) \left( p_{\chi_j} p_t \right) - 2 \Re (l^\tilde{b}_{m_j} k^\tilde{b}_{m_j}) m_{\chi_j} m_t + \cdots \right. \]

\[ + 2 \Im (l^\tilde{b}_{m_j} k^\tilde{b}_{m_j}) \alpha_f \alpha_W \frac{m_t}{(p_{\tilde{b}} p_f)} \frac{m_{\chi_j}}{(p_{\chi_j} p_\ell)} m_{\tilde{b}} \right\} d\Phi_W^f, \] (44)

with

\[ d\Phi_W^f = d\Phi_{\tilde{b}} d\Phi_W (d\Phi_{\chi_j})^\pm. \] (45)

where again we have quoted only the terms that contribute to the up-down asymmetries in Eq. (9). The sum in Eq. (44) is due to the two kinematical solutions for \( |p_W| \) (for details see Appendix C).

## 3 T-odd asymmetries

A general definition of the T-odd observables which we study in this paper has been given in Eq. (9). For the following it is convenient to introduce a shorthand notation for the various T-odd asymmetries to be studied below:

\[ A_{ijk} = \frac{N [(p_i p_j p_k) > 0] - N [(p_i p_j p_k) < 0]}{N [(p_i p_k p_k) > 0] + N [(p_i p_j p_k) < 0]}, \] (46)
where \( N [(p_i p_j p_k) > 0] \) (\( N [(p_i p_j p_k) < 0] \)) are the number of events with \( (p_i p_j p_k) > 0 \) \(( (p_i p_j p_k) < 0 \). The indices \( i, j, k \) specify the observed particles appearing in the considered decay mode of \( b_m \). We choose the convention that \( p_i \) denotes the momentum of a particle originating from the \( \tilde{\chi}_j^- \) decay, \( p_j \) denotes the momentum of a fermion from the top quark decay and \( p_k \) either denotes the momentum of the top quark itself or of another particle stemming from the decay of the top quark. According to the different decay channels we group the considered triple products as follows:

(I) If the 3-body decay \( \tilde{\chi}_j^- \rightarrow \ell_i^- \bar{\nu} \chi_j^0 \) is considered, the only detectable particles are the final charged leptons \( \ell_1^-, \ell_2^-, \ell_3^- \). We shall distinguish two classes of asymmetries depending on whether the leptons \( \ell_1^-, \ell_2^-, \ell_3^- \), originating from the different decay chains (3)–(5), are distinguishable or not.

1. First we define the T-odd asymmetries where the leptons from the \( \tilde{\chi}_j^- \) decays (3)–(5) can be distinguished. The triple products on which the T-odd asymmetries are based in this case, are given by

\[
(p_{\ell_i^-} p_0 p_t) \quad \text{and} \quad (p_{\ell_i^-} p_0 p_W^+) \quad \text{when} \quad t \rightarrow bW^+ \rightarrow bq\bar{q}', \quad (47)
\]

\[
(p_{\ell_i^-} p_t p_0), \quad \text{when} \quad t \rightarrow bW^+ \rightarrow bl\nu, \quad (48)
\]

\[
(p_{\ell_i^-} p_0 p_t), \quad (p_{\ell_i^-} p_0 p_0) \quad \text{and} \quad (p_{\ell_i^-} p_0 p_s), \quad \text{when} \quad t \rightarrow bW^+ \rightarrow bc\bar{s}, \quad (49)
\]

where for the triple products in (49) it is necessary to identify the \( c \) quark which is expected to be possible with reasonable efficiency and purity [15–17]. With the triple products in (47)–(49) the associated T-odd asymmetries can be defined according to Eq. (46), where in the following we use the notation \( A_{\ell_i^- b} \) and \( A_{\ell_i^- bW^+} \) for the T-odd asymmetries based on the triple products in (47) etc. Note that \( A_{\ell_i^- b} \) and \( A_{\ell_i^- bW^+} \) have the same value due to momentum conservation.

2. We define a second class of T-odd asymmetries where it is not necessary to distinguish the different leptonic \( \tilde{\chi}_j^- \) decay chains, Eqs. (3)–(5). This class of T-odd asymmetries is based on the triple products as given in (47)–(49) where \( \ell_i^- \) is replaced by \( \ell^- \). Then \( N [(p_{\ell^-} p_0 p_t) > 0] \) in Eq. (46) means

\[
N [(p_{\ell^-} p_0 p_t) > 0] = N [(p_{\ell_1^-} p_0 p_t) > 0] + N [(p_{\ell_2^-} p_0 p_t) > 0] + N [(p_{\ell_3^-} p_0 p_t) > 0].
\]

For this class of T-odd asymmetries we will use the notation \( A_{\ell^- b} \) etc. The following formula relates \( A_{\ell^- jk} \) to the asymmetries \( A_{\ell_i^- jk} \) and the branching ratios \( BR_{\ell_i^-} \equiv \)

\[\text{In principle, the leptons from the decays (3)–(5) can be distinguished through their different angular or energy distributions.}\]
$BR(\tilde{\chi}_j^- \to \ell_i^- \tilde{\nu}_i^0)$ of the decay chains (3)–(5):\

$$A_{\ell^- jk} = \frac{BR_{\ell^-} A_{\ell^- jk}}{BR_{\ell^-}} + \frac{BR_{\ell^-} A_{\ell^- jk}}{BR_{\ell^-}} + \frac{BR_{\ell^-} A_{\ell^- jk}}{BR_{\ell^-}}.$$ (50)\

where we have introduced the shorthand notation $BR_{\ell^-}$:

$$BR_{\ell^-} = BR_{\ell^-} + BR_{\ell^-} + BR_{\ell^-}. \quad (51)$$

This formula allows us to calculate the contribution of $A_{\ell^- jk}$ to the asymmetry $A_{\ell^- jk}$, depending on the branching ratios of the different decay modes of $\tilde{\chi}_j^-$.\

(II) If we consider the 2-body decay mode $\tilde{\chi}_j^- \to W^- \tilde{\chi}_1^0$, where the $W$ boson decays hadronically so that its momentum vector can be reconstructed, we can define analogous triple products as in (47)–(49) with $\ell_i^-$ replaced by $W^-$. For the corresponding T-odd asymmetries again we use the notation $A_{W^- bt}$ etc.

At the end of this section, we discuss how CP-odd asymmetries can be obtained from the T-odd asymmetries defined above. It is well known that a non-zero value of the considered T-odd asymmetries does not necessarily imply that the CP symmetry is violated since final state interactions give rise (although only at loop level) to the same asymmetries. In order to identify a genuine signal of CP violation one needs to consider also the C-conjugate decay. T-odd asymmetries that are based on triple products analogous to the one given in (47)–(49) can be defined for the charge conjugate decay $\bar{b}_m \to \tilde{\chi}_j^+ \bar{t}$ as well, and we denote them by $\bar{A}_{ijk}$. One finds that the term of the matrix element squared for the C-conjugate decay $\bar{b}_m \to \tilde{\chi}_j^+ \bar{t}$ that comprises the triple product has the same sign as the corresponding term for the decay $\bar{b}_m \to \tilde{\chi}_j^- t$. Thus, true CP violating asymmetries are obtained when summing the T-odd asymmetries that arise in the decays $\bar{b}_m \to \tilde{\chi}_j^- t$ and $\bar{b}_m \to \tilde{\chi}_j^+ \bar{t}$:

$$A_{ij}^{CP} = \frac{A_{ijk} + \bar{A}_{ijk}}{2}. \quad (52)$$

4 Numerical results

Now we study numerically the CP asymmetries defined in the previous section, where we focus on their dependence on the CP phases, in particular on $\phi_{A_b}$. All CP asymmetries defined in the previous section are proportional to $\Im m(b_{m\bar{j}}^* b_{m\bar{j}})$, see Eq. (15). Hence they measure combinations of CP phases in the MSSM. In order to see more easily the dependence of the CP asymmetries on the parameters, it is
useful to expand:

\[ \Im m(i_{m j}^b k_{m j}^b) = -Y_t \left[ c_m \Im m(V^*_{j2} U_{j1}) - \frac{1}{2} Y_b \sin 2\theta_b \, d_m \, \Im m(V^*_{j2} U_{j1} e^{-i\phi_b}) \right], \]  

(53)

with \( Y_t \) and \( Y_b \) the top quark and bottom quark Yukawa couplings, \( c_1 = \cos^2 \theta_b \), \( c_2 = \sin^2 \theta_b \), \( d_1 = 1 \), \( d_2 = -1 \), and \( \theta_b \) and \( \phi_b \) the mixing angle and the CP phase of the scalar bottom system given in Appendix A. In general the quantity in Eq. (53) can be large due to the large \( t \) - and \( b \)-quark Yukawa couplings. The relevant phases are \( \phi_\mu \) and \( \phi_{A_b} \). For \( \phi_\mu = 0 \), we have \( \Im m(i_{m j}^b k_{m j}^b) \propto \sin 2\theta_b \sin \phi_b \) and from the explicit expressions given in Appendix A, we obtain \( \sin 2\theta_b \sin \phi_b \propto \sin \phi_{A_b} \). As we will see below also the asymmetries show such a \( \sin \phi \) behavior and thus, their largest values are attained at \( \phi_{A_b} = \pi/2, 3\pi/2 \). As \( \phi_b \) is sensitive to \( \phi_{A_b} \) if \( |A_b| \gtrsim |\mu| \tan \beta \), we need a small value for \( \tan \beta \) and a large value for \( |A_b| \) compatible with the constraint due to the tree-level vacuum stability condition [18]. Note that in the case where \( |\mu| \tan \beta \gg |A_b| \) we have \( \sin \phi_b \approx 0 \) if \( \phi_\mu = 0, \pi \).

For our numerical studies we adopt the two scenarios given in Table 1. In the two scenarios we have assumed the gaugino mass relation \( |M_1| = 5/3 \tan^2 \theta_W M_2 \), with \( \phi_{M_1} = 0 \), and we have fixed the scalar bottom masses assuming \( M_{\tilde{Q}} > M_{\tilde{D}} \). In scenario A the scalar bottom masses are heavy enough to allow for all considered decays of \( \tilde{\chi}_j^- \), Eq. (3)–(5), whereas the scalar bottom masses of scenario B are relatively light and the decay \( \tilde{\chi}_j^- \to W^- \tilde{\chi}_1^0 \) is not allowed. For the matter of simplicity, our numerical investigation is done for the first and second generation leptons where an influence of their Yukawa couplings can be safely neglected.

In Fig. 1 we show the CP asymmetries that are based on the triple products, (47)–(49), in the decays \( \tilde{b}_1 \to t\tilde{\chi}_1^- \), \( t \to bl^+\nu \) (bc\( \bar{s} \)) and \( \tilde{\chi}_1^- \to \ell_i^- \nu \tilde{\chi}_1^0 \) as a function of \( \phi_{A_b} \). Fig. 1(a) shows the three CP asymmetries \( A_{\ell_i^-bt} \) that are based on the triple products in Eq. (47) associated with the three different decay chains \( \tilde{\chi}_1^- \to \ell_1^- \nu \to \ell_1^- \tilde{\chi}_1^0 \) (dashed line), \( \tilde{\chi}_1^- \to \ell_2^- \nu \to \ell_2^- \tilde{\chi}_1^0 \) (dotted line) and \( \tilde{\chi}_1^- \to W^- \tilde{\chi}_1^0 \to \ell_3^- \tilde{\chi}_1^0 \) (dashdotdotted line). Fig. 1(a) also shows the CP asymmetry \( A_{\ell_i^-bt} \) (solid line), Eq. (50), where it is not necessary to distinguish the leptons from the different decay chains of the chargino. The asymmetry \( A_{\ell_i^-bt} \) is the largest one with a maximum value of about 11%. The CP asymmetries \( A_{\ell_i^-bt} \) and \( A_{\ell_i^+bt} \) have an additional phase space factor and are therefore suppressed compared to \( A_{\ell_i^-bt} \).

We now estimate the number of scalar bottoms \( \tilde{b}_1 \) necessary to observe the CP asymmetries for a given number of standard deviations \( N_\sigma \) by

\[ N_{\tilde{b}_1} = \frac{N_\sigma^2}{A_{\ell_i^-bt}^2 BR(\tilde{b}_1 \to t\tilde{\chi}_1^-)(\sum_{\ell=e,\mu} BR(\tilde{\chi}_1^- \to \ell^- \tilde{\chi}_1^0))(\sum_f BR(W^+ \to f))}, \]  

(54)

where \( f \) denotes the final state of the \( W^+ \) decay considered, i.e. \( f = ud, cs, \) or \( l^+\nu_l \),
\[ m_{\tilde{\nu}_\ell} = \sqrt{m_{\tilde{\ell}_L}^2 + m_\tau^2 \cos 2\beta \cos^2 \theta_W} \]  

(55)

This means that the partial decay widths \( \Gamma(\tilde{\chi}_1^- \rightarrow \ell^- \tilde{\nu}_\ell) \) are equal for \( \ell = e, \mu, \tau \). The same holds for the partial decay widths \( \Gamma(\tilde{\chi}_1^- \rightarrow \tilde{\ell}_R^- \tilde{\nu}_\ell) \) and \( \Gamma(\tilde{\ell}_R^- \rightarrow \tilde{\chi}_1^- \tilde{\nu}_1^-) \). Then we obtain \( \sum_{\ell=e,\mu} BR(\tilde{\chi}_1^- \rightarrow \ell_i^- \tilde{\nu}_1^-) = (31.3\%, 30.7\%, 1.5\%) \) corresponding to the three different decay chains of \( \tilde{\chi}_1^- \), \( (3)-(5) \). The values of the branching ratios of the W boson are given by \( BR(W^+ \rightarrow \sum_\ell \ell^+ \nu_\ell) = 21.4\% \) \( (l = e, \mu) \), \( BR(W^+ \rightarrow q\bar{q}') = 68\% \) and \( BR(W^+ \rightarrow c\bar{s} = 32\%) \) \[19\]. For an observation of the CP asymmetry \( A_{\ell^-\ell^+} \) at the 3-\( \sigma \) level, at least \( 7.1 \cdot 10^4 \) scalar bottoms have to be produced. The required number of scalar bottoms in order to measure the asymmetry \( A_{\ell^-\ell^+} = 6.4\% \) \( (\phi_{A_b} = 0.5\pi) \) at the 3-\( \sigma \) level is \( 1 \cdot 10^5 \).

In Fig. 1(b) we plot the CP asymmetries that are based on the triple products \( (p_{\ell^+} \cdot p_{\ell^-} \cdot \bar{p}_b) \), Eq. (48), as a function of \( \phi_{A_b} \). For the same reason as above the largest

| Scenario | A  | B  |
|----------|----|----|
| \( M_2 \) | 350| 250|
| \( |\mu| \) | 310| 140|
| \( \phi_\mu = \phi_{M_1} \) | 0  | 0  |
| \( \tan \beta \) | 3  | 3  |
| \( |A_b| \) | 1200| 1000|
| \( m_{\tilde{b}_1} \) | 480| 320|
| \( m_{\tilde{b}_2} \) | 600| 420|
| \( m_{\tilde{\ell}_R} \) | 200| 100|
| \( m_{\tilde{\ell}_L} \) | 220| 120|
| \( m_{\tilde{\nu}} \) | 208.1| 96.4|
| \( m_{\tilde{\ell}^0} \) | 164.3| 80.3|
| \( m_{\tilde{\ell}^-} \) | 257.3| 107.7|

Table 1: Input parameters \( M_2, |\mu|, \phi_\mu, \tan \beta, |A_b|, \phi_{A_b}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{\ell}_R} \) and \( m_{\tilde{\ell}_L} \) for scenarios A and B. All mass dimension parameters are given in GeV.
Figure 1: CP asymmetries $A_{ijk}$ which are based on the triple products (a) $(\mathbf{p}_t - \mathbf{p}_b \mathbf{p}_l)$, (b) $(\mathbf{p}_\ell^- \mathbf{p}_\ell^+ \mathbf{p}_b)$, (c) $(\mathbf{p}_\ell^- \mathbf{p}_\ell \mathbf{p}_t)$ and (d) $(\mathbf{p}_\ell^- \mathbf{p}_l \mathbf{p}_s)$ for the decays $\tilde{b}_1 \to t \tilde{\chi}^-_1$, $t \to b \ell^+ \nu$ (bcś) and $\tilde{\chi}^-_1 \to \ell^-_i \tilde{\nu} \tilde{\chi}^0_1$, as a function of $\phi_{Ab}$. The lepton $\ell^+_1$ ($\ell^+_2$, $\ell^+_3$) stems from the decay $\tilde{\chi}^-_1 \to \ell^+_1 \tilde{\nu} \to \ell^+_1\tilde{\nu} \tilde{\chi}^0_1$, $\tilde{\chi}^-_1 \to \ell^+_2 \tilde{\nu} \tilde{\chi}^0_1$, $\tilde{\chi}^-_1 \to \ell^+_3 \tilde{\nu} \tilde{\chi}^0_1$). The corresponding asymmetries are shown as dashed lines (dotted lines, dashdotted lines). The solid lines represent the combined asymmetries in Eq. (50). The MSSM parameters are for scenario A of Table 1.

asymmetry is due to the chargino decay chain $\tilde{\chi}^-_1 \to \ell^-_1 \tilde{\nu} \to \ell^-_1 \tilde{\nu} \tilde{\chi}^0_1$, (3). Its maximal value of about 13% is reached at $\phi_{Ab} = 0.5\pi$ and the number of scalar bottoms necessary to measure $A_{\ell^-_1 t^+ b}$ at the 3-$\sigma$ level is about $1.5 \cdot 10^5$. Fig. 1(c) shows the CP asymmetries that are based on $(\mathbf{p}_\ell^- \mathbf{p}_\ell \mathbf{p}_t)$ as a function of $\phi_{Ab}$. The asymmetries shown in Fig. 1(c) are more than twice as large as the asymmetries shown in Fig. 1. Their relative magnitudes can be attributed (i) to the different sensitivity factors of the top quark polarization which is $\alpha_t = 1$ for the asymmetries in Fig. 1(b), (c), (d) and $\alpha_b \simeq 0.4$ for the asymmetries in Fig. 1(a), and (ii) to the different 3-vectors involved in the triple products: for Figs. 1(a) and 1(c) it is $\mathbf{p}_t$, while for Figs. 1(b) and 1(d) it is the 3-vector of any of the decay products of the $t$-quark, which is always smaller or at most equal in magnitude than $|\mathbf{p}_t|$. For $\phi_{Ab} = 0.5\pi$ the CP asymmetry $A_{\ell^-_1 ct}$ is about 27%, which means that $2.5 \cdot 10^4$ scalar bottoms are necessary for its
A given in Table 1. The momentum vector \( \mathbf{p} \) is least 1, \( W \) that of the appropriate number of scalar bottoms to probe it at the 3-\( \sigma \) measurement at 3-\( \sigma \). The combined asymmetry in Eq. (50) can be as large as about 16% and the appropriate number of scalar bottoms to probe it at the 3-\( \sigma \) level is \( 3.6 \cdot 10^4 \). In Fig. 1(d) we plot the CP asymmetries which are based on the triple products \( (\mathbf{p}_W - \mathbf{p}_t \mathbf{p}_t) \) as a function of \( \phi_{A_b} \) for scenario A given in Table 1. The momentum vector \( \mathbf{p}_W \) involved in the triple products is that of the \( W \) boson stemming from the decay \( \tilde{\chi}_1^0 \to W^+ \tilde{\chi}_1^- \). The largest asymmetry \( A_{W_{-ct}} \) attains its maximum value of about 6% at \( \phi_{A_b} = 0.5\pi \). For the theoretical estimate of the number of scalar bottoms necessary to observe this asymmetry we replace \( \sum_{l=e,\mu} BR(\tilde{\chi}_1^0 \to l^- \overline{\nu} l_1^- \overline{\nu} l_{1}^0) \) by \( BR(\tilde{\chi}_1^0 \to W^- \tilde{\chi}_1^0) \cdot \sum_q BR(W^- \to q\overline{q}') = 4.8\% \) in Eq. (54). We then obtain that \( 1.1 \cdot 10^6 \) scalar bottoms are required for a 3-\( \sigma \) evidence.

In Fig. 3 the CP asymmetries for scenario B of Table 1 are displayed. In this case the decay \( \tilde{\chi}_1^- \to W^- \tilde{\chi}_1^0 \) is kinematically not accessible. We plot the CP asymmetries for the decay chains \( \tilde{\chi}_1^0 \to \ell_1^- \overline{\nu} \to \ell_1^- \overline{\nu} l_1^- \overline{\nu} l_{1}^0 \) and \( \tilde{\chi}_1^- \to \ell_2^+ \overline{\nu} \to \ell_2^+ \overline{\nu} l_2^+ \overline{\nu} l_{2}^+ \overline{\nu} l_{2}^0 \) as a function of \( \phi_{A_b} \). For the branching ratios we obtain \( BR(b_1 \to t\tilde{\chi}_1^-) = 7.2\% \), \( \sum_{l=e,\mu} BR(\tilde{\chi}_1^0 \to \ell_1^- \overline{\nu} l_{1}^0) = 54.3\% \) and \( \sum_{l=e,\mu} BR(\tilde{\chi}_1^- \to \ell_2^+ \overline{\nu} l_{2}^0) = 12.4\% \) in scenario B. Fig. 3(a) shows the CP asymmetries which are based on the triple products given in Eq. (47). The largest asymmetry results from the triple product \( (\mathbf{p}_{\tilde{\chi}_1^-} \mathbf{p}_b \mathbf{p}_t) \) where...
Figure 3: CP asymmetries $A_{ijk}$ that are based on the triple products (a) $(\mathbf{p}_l^- \mathbf{p}_b \mathbf{p}_t)$, (b) $(\mathbf{p}_l^- \mathbf{p}_l^- \mathbf{p}_b)$, (c) $(\mathbf{p}_l^- \mathbf{p}_l^+ \mathbf{p}_t)$ and (d) $(\mathbf{p}_l^- \mathbf{p}_c \mathbf{p}_t)$ for the decays $\tilde{b}_1 \rightarrow t \tilde{\chi}_1^-$, $t \rightarrow b l^+ \nu$ (bc$\bar{s}$) and $\tilde{\chi}_1^- \rightarrow \ell_l^- \nu\tilde{\chi}_1^0$, as a function of $\phi_{A_b}$. The lepton $\ell_l^-$ ($\ell_l^+$) stems from the decay $\tilde{\chi}_1^- \rightarrow \ell_l^- \nu \rightarrow \ell_l^- \bar{\nu} \tilde{\chi}_1^0$ ($\tilde{\chi}_1^- \rightarrow \ell_l^+ \nu \rightarrow \ell_l^+ \nu \tilde{\chi}_1^0$). The corresponding asymmetries are shown as dashed lines (dotted lines). The solid lines represent the combined asymmetries in Eq. (50). The MSSM parameters are for scenario B of Table 1.

the lepton $\ell_1^-$ originates from the first step of the decay chain $\tilde{\chi}_1^- \rightarrow \ell_1^- \bar{\nu} \rightarrow \ell_1^- \bar{\nu} \tilde{\chi}_1^0$, and its maximum value is of about 15%. For its measurement (at 3-$\sigma$) $16 \cdot 10^4$ scalar bottoms are required. The CP asymmetry that is based on $(\mathbf{p}_l^- \mathbf{p}_c \mathbf{p}_t)$, where the lepton $\ell_2^-$ comes from the decay chain $\tilde{\chi}_1^- \rightarrow \ell_2^- \bar{\nu} \rightarrow \ell_2^- \bar{\nu} \tilde{\chi}_1^0$ is phase space suppressed. Due to the large branching ratio of $\tilde{\chi}_1^- \rightarrow \ell_1^- \bar{\nu} \tilde{\chi}_1^0$ the combined asymmetry, Eq. (50), is about 12%, therefore $1.9 \cdot 10^4$ scalar bottoms would be necessary for a measurement at the 3-$\sigma$ level. In Fig. 3(b) we plot the CP asymmetries that are based on the triple products defined in Eq. (48). The largest asymmetry $A_{\ell_1^- t^+ b}$ reaches its maximum value of about 13% at $\phi_{A_b} = 1.5\pi$. Fig. 3(c) shows the CP asymmetry formed with the triple products $(\mathbf{p}_l^- \mathbf{p}_c \mathbf{p}_t)$. As expected, the asymmetry $A_{\ell_1^- c t}$ is the largest and its maximum value is of about 36%. In this case $5.5 \cdot 10^3$ scalar bottoms are necessary for a measurement of $A_{\ell_1^- c t}$ at the 3-$\sigma$ level. The CP asymmetry, where it is not necessary to distinguish from which $\tilde{\chi}_1^-$ decay chain the lepton originates,
reaches a maximum of about 30%. In this case the production of $6.7 \cdot 10^3$ scalar bottoms is necessary to probe the asymmetry $A_{\ell-ct}$ at 3-$\sigma$. In Fig. 3(d) the CP asymmetries that are based on the triple products $(p_{\ell}-p_{\bar{p}})$ are displayed. The maximum of $A_{\ell-ct}$ is about 9%, which means that $1.1 \cdot 10^5$ scalar bottoms are necessary to determine (at 3-$\sigma$) that the asymmetry is non-zero.

Fig. 4 shows the contours of the combined asymmetry $A_{\ell-ct}$, Eq. (50), in the $\phi_\mu$-$\phi_{Ab}$ plane. The slepton masses are $m_{\tilde{\ell}_L} = 140$ GeV and $m_{\tilde{\ell}_R} = 110$ GeV, $\tan \beta = 10$ and the other parameters are as given as in scenario B of Table 1. For the scenario chosen, the first term of Eq. (53) is small compared to the second term because $\cos \theta_{\tilde{b}} \ll \sin \theta_{\tilde{b}}$. Hence, the behavior of the asymmetry is given by the second term of Eq. (53), which is small in the $\phi_\mu$-$\phi_{Ab}$ plane where $\phi_\mu + \phi_{Ab} \approx 0, \pi$ because there $\phi_\mu - \arg[U_{12}^* V_{12}^*] \approx 0, \pi$ resulting in a cancellation of the two terms in Eq. (53). For CP phases of $\phi_\mu \approx 0.8\pi$ and $\phi_{Ab} \approx 0.6\pi$ the asymmetry reaches its maximum of about 11%.

5 Summary

We have proposed various T-odd asymmetries in the decay $\tilde{b}_m \rightarrow t\tilde{\chi}^-_j$, which are based on triple product correlations that involve the polarization vectors of $t$ and $\tilde{\chi}^-_j$. The distributions of their decay products depend on the polarizations of $t$ and $\tilde{\chi}^-_j$. For the $\tilde{\chi}^-_j$ decay into a leptonic final state $\ell^-\tilde{\nu}$ we have considered the three possible decay chains $\tilde{\chi}^-_j \rightarrow \ell^-\tilde{\nu} \rightarrow \ell^-\tilde{\nu}\tilde{\chi}^0_1$, $\tilde{\chi}^-_j \rightarrow \ell^-\tilde{\nu} \rightarrow \ell^-\tilde{\nu}\tilde{\chi}^0_1$ and
\(\tilde{\chi}_j \to W^- \tilde{\chi}_1^0 \to \ell^- \bar{\nu}_\ell \tilde{\chi}_1^0\). We have also considered the 2-body decay \(\tilde{\chi}_j^- \to W^- \chi_1^0\), where the \(W\) boson decays hadronically. The proposed T-odd asymmetries are proportional to the product of left- and right-couplings \(t \tilde{b}_m \bar{\chi}_k\) and are non-vanishing due to non-zero phases \(\phi_\mu\) and/or \(\phi_{Ab}\). Since scalar bottom mixing can be large these asymmetries will allow us to determine the CP violating phase \(\phi_{Ab}\), which is not easily accessible otherwise. We have also pointed out that true CP violating asymmetries can be obtained by summing the T-odd asymmetries that arise in the decays \(\tilde{b}_m \to \tilde{\chi}_j^+ t\) and \(\tilde{b}_m \to \tilde{\chi}_j^+ \bar{t}\). In this case an identification of the charges of the involved particles is not necessary.

In a numerical study we have presented results of these asymmetries for the decay \(\tilde{b}_1 \to t\tilde{\chi}_1^+\). The asymmetry \(A_{\ell^-ct}/\ell^+\), which is based on the triple product \((p_\ell^+ p_\ell^- p_t)\), is the largest one and its magnitude can be of the order 40%. We have also defined the asymmetry \(A_{\ell^-ct}/\ell^+\), Eq. (50), which is based on \((p_\ell^- p_\ell^+ p_t)\), and where it is not necessary to distinguish between the different leptonic \(\tilde{\chi}_1^+\) decay chains. We have found that this asymmetry can go up to 30%. By making a theoretical estimate of the number of \(\tilde{b}_1\) necessary to observe the T-odd asymmetries we have found that a \(\tilde{b}_1\) production rate of \(O(10^3)\) will be necessary to observe some of the proposed asymmetries, which should be possible at the LHC or at a future linear collider.

**Acknowledgements**

This work is supported by the 'Fonds zur Förderung der wissenschaftlichen Forschung' (FWF) of Austria, project No. P18959-N16.

**Appendix**

**A Scalar bottom masses and mixing**

The left-right mixing of the scalar bottoms is described by a hermitian \(2 \times 2\) mass matrix which in the basis \((\tilde{b}_L, \tilde{b}_R)\) reads

\[
\hat{\tilde{b}}_M^2 = -(\tilde{b}_L^\dagger, \tilde{b}_R^\dagger) \begin{pmatrix} M_{b_{LL}}^2 & e^{-i\phi_b} |M_{b_{LR}}^2| \\ e^{i\phi_b} |M_{b_{LR}}^2| & M_{b_{RR}}^2 \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix},
\]

\(\hat{\tilde{b}}_M^2\)

\[
M_{b_{LL}}^2 = M_{Q}^2 + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \Theta_W\right) \cos 2\beta \ m_Z^2 + m_b^2,
\]

where
\[ M_{b_{RR}}^2 = M_D^2 - \frac{1}{3} \sin^2 \Theta_W \cos 2\beta \, m_Z^2 + m_b^2, \]  
(58) 
\[ M_{b_{RL}}^2 = (M_{b_{LR}}^2)^* = m_b(A_b - \mu^* \tan \beta), \]  
(59) 
\[ \phi_b = \arg[A_b - \mu^* \tan \beta], \]  
(60)

where \( \tan \beta = v_2/v_1 \) with \( v_1(v_2) \) being the vacuum expectation value of the Higgs field \( H_1^0(H_2^0) \), \( m_b \) is the mass of the bottom quark and \( \Theta_W \) is the weak mixing angle, \( \mu \) is the Higgs–higgsino mass parameter and \( M_Q, M_D, A_t \) are the soft SUSY–breaking parameters of the scalar bottom system. The mass eigenstates \( \tilde{b}_i \) are \( (\tilde{b}_1, \tilde{b}_2) = (\tilde{b}_L, \tilde{b}_R)\bar{R}^{\tilde{b}^T} \) with

\[
\bar{R}^b = \begin{pmatrix}
e^{i\phi_b} \cos \theta_b & \sin \theta_b \\
-\sin \theta_b & e^{-i\phi_b} \cos \theta_b
\end{pmatrix},
\]  
(61)

with

\[
\cos \theta_b = \frac{-|M_{b_{LR}}^2|}{\sqrt{|M_{b_{LR}}^2|^2 + (m_{b_1}^2 - M_{b_{LL}}^2)^2}}, \quad \sin \theta_b = \frac{M_{b_{LL}}^2 - m_{b_1}^2}{\sqrt{|M_{b_{LR}}^2|^2 + (m_{b_1}^2 - M_{b_{LL}}^2)^2}}
\]  
(62)

The mass eigenvalues are

\[
m_{b_{1,2}}^2 = \frac{1}{2} \left( (M_{b_{LL}}^2 + M_{b_{RR}}^2) \mp \sqrt{(M_{b_{LL}}^2 - M_{b_{RR}}^2)^2 + 4|M_{b_{LR}}^2|^2} \right).
\]  
(63)

## B Lagrangian and couplings

The parts of the Lagrangian, necessary to calculate the decay rates of \( \tilde{b}_m \to \tilde{\chi}_j^- t \) with the subsequent decays \( \tilde{\chi}_j^- \to \ell^- \bar{\nu} \chi_i^0 \) are

\[
\mathcal{L}_{\tilde{b} \chi^+} = g \tilde{\ell} (l_{mij}^b P_R + k_{mij}^b P_L) \tilde{\chi}_j^+ \tilde{b}_m + \text{h.c.},
\]  
(64) 
\[
\mathcal{L}_{\tilde{b} \tilde{X}^+} = g \tilde{\ell} (k_{jL}^b P_L + t_{jL}^b P_R) \tilde{\chi}_j^+ \tilde{\nu}_\ell + \text{h.c.},
\]  
(65) 
\[
\mathcal{L}_{\tilde{\nu} \tilde{\chi}^+} = g \tilde{\nu} \ell_{mj} \tilde{\nu}_\ell P_R \tilde{\chi}_j^+ \tilde{\ell}_n + \text{h.c.},
\]  
(66) 
\[
\mathcal{L}_{W^{-} \tilde{\chi}_j^0} = g W^- \tilde{\chi}_j^0 \gamma \mu (\bar{O}^b_{kj} P_L + \bar{O}^b_{jk} P_R) \tilde{\chi}_j^+ + \text{h.c.},
\]  
(67) 
\[
\mathcal{L}_{\tilde{\ell} \tilde{\chi}_j^0} = g \tilde{\ell} (a_{nk}^b P_R + b_{nk}^b P_L) \tilde{\chi}_k^0 \tilde{\ell}_n + \text{h.c.},
\]  
(68) 
\[
\mathcal{L}_{\tilde{\nu} \tilde{\chi}_j^0} = g \tilde{\nu} P^\dagger_{R \tilde{\chi}_k} \tilde{\nu}_\ell + \text{h.c.},
\]  
(69)

where the couplings are defined as

\[ l_{mj} = -\tilde{R}_{m1} U_{j1} + Y_t \tilde{R}_{m2} U_{j2}, \quad \tilde{\nu}_{mj} = R_{m1} Y^{*}_t V_{j2}, \quad \tilde{\nu}_{mj} = R_{m1} Y^{*}_t V_{j2}, \quad (70) \]

\[ l_{j} = -V_{j1}, \quad k_{j} = Y_t U_{j2}, \quad (71) \]

\[ a_{nk} = R_{n1} f^{L}_L + R_{n2} h^{R}_R, \quad b_{nk} = R_{n1} h^{L}_L + R_{n2} f^{R}_R, \quad (72) \]

\[ f^{L}_L = \frac{1}{\sqrt{2}} (N_{k2} + \tan \theta_W N_{k1}), \]
\[ f^{R}_R = -\sqrt{2} \tan \theta_W N'^{*}_{k1}, \]
\[ h^{L}_R = (h^{L}_L)^{*} = -Y_t N_{k3}, \]
\[ f^{R}_L = \frac{1}{\sqrt{2}} (\tan \theta_W N_{k1} - N_{k2}), \quad (73) \]

\[ l_{nj} = -\tilde{R}_{n1} U_{j1} + Y_t \tilde{R}_{n2} U_{j2}, \quad (74) \]
\[ O^{L}_{kj} = -\frac{1}{\sqrt{2}} N_{k4} V^{*}_{j2} + N_{k2} V^{*}_{j1}, \quad O^{R}_{kj} = \frac{1}{\sqrt{2}} N^{*}_{k3} U_{j2} + N_{k2} U_{j1}, \quad (75) \]

where in the above equations \( U \) and \( V \) are the unitary \( 2 \times 2 \) mixing matrices that diagonalize the chargino mass matrix \( M_C, U^{*} M_C V^{-1} = \text{diag}(m_{\chi_1}, m_{\chi_2}) \), \( N_{ij} \) is the complex unitary \( 4 \times 4 \) matrix which diagonalizes the neutral gaugino-higgsino mass matrix \( Y_{\alpha \beta}, N'_{i\alpha} Y_{\alpha \beta} N'_{k\beta} = m_{\chi_{i \delta}} \delta_{ik} \) in the basis \((\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0) \) [2], \( R^{L}_L \) is the mixing matrix in the slepton sector (see for instance [8]) and the Yukawa couplings are given by \( Y_t = m_t / (\sqrt{2} m_W \sin \beta), Y_b = m_b / (\sqrt{2} m_W \cos \beta) \) and \( Y_{\ell} = m_{\ell} / (\sqrt{2} m_W \cos \beta) \), with \( m_W \) being the mass of the \( W \) boson.

\[\text{C Phase space and kinematics}\]

We will work in the rest frame of \( \tilde{b}_m \) and we fix the coordinate system so that the chargino momentum \( p_{\chi_j} \) points along the \( Z \)-axis.

\[ \text{Phase space element of the decay } \tilde{b}_m \rightarrow \tilde{\chi}_j^t t: \]

\[ d\Phi_{\tilde{b}_m} = \frac{|p_t|}{4\pi m_{\tilde{b}_m}}, \quad |p_t| = \frac{\lambda^2 (m_{\tilde{b}_m}^2, m_t^2, m_{\chi_j}^2)}{2m_{\tilde{b}_m}}, \quad (76) \]
where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \).

**Phase space elements of the top decays (2):**

The phase space element of the top decay \( t \to bW^+ \) is given as

\[
d\Phi^b_t = \frac{E_b^2}{2(m_t^2 - m_W^2)}\frac{d\Omega_b}{(2\pi)^2}, \quad E_b = \frac{m_t^2 - m_W^2}{2(E_t + |p_t| c_b)},
\]

(77)

The phase space element of the top decay \( t \to bl^+\nu \) reads

\[
d\Phi^l_t = \frac{1}{2\pi} d\Phi^b_t d\Phi_W,
\]

(78)

where we used the narrow width approximation for the \( W \) boson propagator. \( d\Phi_W \) is the phase space element for \( W^+ \to l^+\nu_l \):

\[
d\Phi_W = \frac{E_l^2}{2m_W^2}\frac{d\Omega_l}{(2\pi)^2}, \quad E_l = \frac{m_W^2}{2[E_l + |p_l| c_l - E_b(1 - c_bl)]},
\]

(79)

where \( c_b = \cos \theta_b, c_l = \cos \theta_l \) and \( c_bl = \cos \theta_{bl} \), with \( \theta_{bl} \) being the angle between \( p_b \) and \( p_l \), and \( d\Omega_b = \sin \theta_b d\theta_b d\phi_b \) etc.

**Phase space element for \( \tilde{\chi}^-_j \) decay via \( \tilde{\nu} \) exchange (3):**

The phase space element of the decay \( \tilde{\chi}^-_j \to \ell^-_1 \tilde{\nu} \) reads

\[
d\Phi^1_{\chi_j} = \frac{E_{\ell_1}^2}{2(m_{\chi_j}^2 - m_{\tilde{\nu}}^2)}\frac{d\Omega_{\ell_1}}{(2\pi)^2}, \quad E_{\ell_1} = \frac{m_{\chi_j}^2 - m_{\tilde{\nu}}^2}{2(E_{\chi_j} - |p_{\chi_j}| c_{\ell_1})},
\]

(80)

where \( c_{\ell_1} = \cos \theta_{\ell_1} \).

**Phase space elements for \( \tilde{\chi}^-_j \) decay via \( \tilde{\ell} \) exchange (4):**

The phase space element of the decay \( \tilde{\chi}^-_j \to \tilde{\ell}_n^{-} \tilde{\nu} \) is given by

\[
d\Phi^2_{\chi_j} = \frac{E_\nu^2}{2(m_{\chi_j}^2 - m_{\tilde{\ell}}^2)}\frac{d\Omega_\nu}{(2\pi)^2}, \quad E_\nu = \frac{m_{\chi_j}^2 - m_{\tilde{\ell}}^2}{2(E_{\chi_j} - |p_{\chi_j}| c_\nu)},
\]

(81)

where \( c_\nu = \cos \theta_\nu \). For the subsequent decay \( \tilde{\ell}_n^{-} \to \chi_1^{0} \ell_2^- \) the phase space element reads

\[
d\Phi_{\ell^-_2} = \frac{E_{\ell_2}^2}{2(m_{\tilde{\ell}}^2 - m_{\chi_1}^{02})}\frac{d\Omega_{\ell_2}}{(2\pi)^2}, \quad E_{\ell_2} = \frac{m_{\tilde{\ell}}^2 - m_{\chi_1}^{02}}{2(E_{\tilde{\ell}} - |p_{\tilde{\ell}}| c_{\ell_2})},
\]

(82)
where \( c_{\ell\ell_2} = \cos \theta_{\ell\ell_2} \) being the angle between \( p_\ell \) and \( p_{\ell_2} \).

**Phase space elements for \( \tilde{\chi}_j^- \) decay via \( W \) boson exchange (5):**

The phase space element of the decay \( \tilde{\chi}_j^- \rightarrow W^- \tilde{\chi}_0^1 \) is given by

\[
(d\Phi_{\chi_j}^3)^\pm = \frac{|p_W^\pm|^2}{4|E_W^\pm| |p_{\chi_j}| \cos \theta_W} \frac{d\Omega_W}{(2\pi)^2},
\]

with

\[
|p_W^\pm| = \left[ \left( m_{\chi_j}^2 + m_W^2 - m_{\chi_0}^2 \right) |p_{\chi_j}| \cos \theta_W \right]^{\pm} \sqrt{\lambda(m_{\chi_j}^2, m_W^2, m_{\chi_0}^2) - 4|p_{\chi_j}|^2 m_W^2 (1 - \cos^2 \theta_W) + 2|p_{\chi_j}|^2 (1 - \cos^2 \theta_W) + 2m_W^2} \right]^{-1}.
\]

(84)

There are two solutions \(|p_W^\pm|\) in the case \(|p_{\chi_j}^0| < |p_{\chi_j}|\), where \(|p_{\chi_j}^0| = \sqrt{\lambda(m_{\chi_j}^2, m_W^2, m_{\chi_0}^2)} / 2m_W\) is the chargino momentum if the \( W \) boson is produced at rest. The \( W \) decay angle \( \theta_W \) is constrained in that case and the maximal angle \( \theta_W^{\text{max}} \) is given as

\[
\sin \theta_W^{\text{max}} = \frac{|p_{\chi_j}^0|}{|p_{\chi_j}|} = \frac{m_{\tilde{\chi}_0} \sqrt{\lambda(m_{\chi_j}^2, m_W^2, m_{\chi_0}^2)}}{m_{\chi_j} \sqrt{\lambda(m_{\tilde{\chi}_0}^2, m_{\chi_j}^2, m_t^2)}} \leq 1.
\]

(85)

If \(|p_{\chi_j}^0| > |p_{\chi_j}|\), the decay angle \( \theta_W \) is not constrained and there is only the physical solution \(|p_W^+|\).

For the subsequent decay of the \( W \) boson, \( W^- \rightarrow \ell^- \nu \), the phase space element is analogous to the one given in (79) and reads

\[
d\Phi_{\ell_3}^3 = \frac{E_{\ell_3}^2 d\Omega_{\ell_3}}{2m_W^2 (2\pi)^2}, \quad E_{\ell_3} = \frac{m_W^2}{2(E_W^2 - |p_W^+|^2 c_{\ell_3 W})},
\]

(86)

where \( c_{\ell_3 W} = \cos \theta_{\ell_3 W} \) being the angle between \( p_{\ell_3} \) and \( p_W \).

**References**

[1] H. P. Nilles, Phys. Rep. 110, 1 (1984);
H. E. Haber and G. L. Kane, Phys. Rep. 117, 75 (1985);
R. Barbieri, Riv. Nuovo Cim. 11, 1 (1988).
[2] J. F. Gunion and H. E. Haber, Nucl. Phys. B 272, 1 (1986) [Erratum-ibid. B 402, 567 (1993)]; Nucl. Phys. B 278, 449 (1986).

[3] T. Ibrahim and P. Nath, Phys. Lett. B 418, 98 (1998) [arXiv:hep-ph/9707409]; Phys. Rev. D 57, 478 (1998) [Erratum-ibid. D 58, 019901 (1998), D 60, 079903 (1999), D 60, 119901 (1999)]; Phys. Rev. D 58, 111301 (1998) [Erratum-ibid. D 60, 099902 (1999)]; Phys. Rev. D 61, 093004 (2000) [arXiv:hep-ph/9910553]; M. Brhlik, G. J. Good and G. L. Kane, Phys. Rev. D 59, 115004 (1999) [arXiv:hep-ph/9810457]; M. Brhlik, L. L. Everett, G. L. Kane and J. Lykken, Phys. Rev. Lett. 83, 2124 (1999) [arXiv:hep-ph/9905215]; Phys. Rev. D 62, 035005 (2000) [arXiv:hep-ph/9908326]; A. Bartl, T. Gajdosik, W. Porod, P. Stockinger and H. Stremnitzer, Phys. Rev. D 60, 073003 (1999) [arXiv:hep-ph/9903402]; V. D. Barger, T. Falk, T. Han, J. Jiang, T. Li and T. Plehn, Phys. Rev. D 64, 056007 (2001) [arXiv:hep-ph/0101106]; A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer and O. Vives, Phys. Rev. D 64, 076009 (2001) [arXiv:hep-ph/0103324]; S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B 606, 151 (2001) [arXiv:hep-ph/0103320].

[4] A. Bartl, W. Majerotto, W. Porod and D. Wyler, Phys. Rev. D 68, 053005 (2003) [arXiv:hep-ph/0306050].

[5] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B 388, 588 (1996) [arXiv:hep-ph/9607394]; A. G. Akeroyd, Y. Y. Keum and S. Recksiegel, Phys. Lett. B 507, 252 (2001) [arXiv:hep-ph/0103008] and references therein.

[6] S. Yaser Ayazi and Y. Farzan, arXiv:hep-ph/0605272.

[7] D. Chang, W. Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999) [Erratum-ibid. 83, 3972 (1999)] [arXiv:hep-ph/9811202]; A. Pilaftsis, Phys. Lett. B 471, 174 (1999) [arXiv:hep-ph/9909485]; D. Chang, W. F. Chang and W. Y. Keung, Phys. Lett. B 478, 239 (2000) [arXiv:hep-ph/9910465]; A. Pilaftsis, Nucl. Phys. B 644, 263 (2002) [arXiv:hep-ph/0207277].

[8] A. Bartl, K. Hidaka, T. Kernreiter and W. Porod, Phys. Lett. B 538, 137 (2002) [arXiv:hep-ph/0204071]; Phys. Rev. D 66, 115009 (2002) [arXiv:hep-ph/0207186].

[9] A. Bartl, S. Hesselbach, K. Hidaka, T. Kernreiter and W. Porod, Phys. Lett. B 573, 153 (2003) [arXiv:hep-ph/0307317]; Phys. Rev. D 70, 035003 (2004) [arXiv:hep-ph/0311338].
[10] W. Bernreuther and P. Overmann, Z. Phys. C 72, 461 (1996) [arXiv:hep-ph/9511256];
B. Grzadkowski, Phys. Lett. B 305, 384 (1993) [arXiv:hep-ph/9303204];
E. Christova and M. Fabbrichesi, Phys. Lett. B 320, 299 (1994) [arXiv:hep-ph/9307298];
B. Grzadkowski and W. Y. Keung, Phys. Lett. B 316, 137 (1993); Phys. Lett. B 319, 526 (1993);
A. Bartl, E. Christova and W. Majerotto, Nucl. Phys. B 460, 235 (1996) [Erratum-ibid. B 465, 365 (1996)] [arXiv:hep-ph/9507445];
A. Bartl, E. Christova, T. Gajdosik and W. Majerotto, Phys. Rev. D 58, 074007 (1998) [arXiv:hep-ph/9802352];
S. Bar-Shalom, D. Atwood and A. Soni, Phys. Rev. D 57, 1495 (1998) [arXiv:hep-ph/9708357];
D. Atwood, S. Bar-Shalom, G. Eilam and A. Soni, Phys. Rept. 347, 1 (2001) [arXiv:hep-ph/0006032];
A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer and O. Vives, Phys. Rev. D 64, 076009 (2001) [arXiv:hep-ph/0103324];
E. Christova, S. Fichtinger, S. Kraml and W. Majerotto, Phys. Rev. D 65, 094002 (2002) [arXiv:hep-ph/0108076];
F. Browning, D. Chang and W. Y. Keung, Phys. Rev. D 64, 015010 (2001) [arXiv:hep-ph/0012258];
E. Christova, H. Eberl, W. Majerotto and S. Kraml, Nucl. Phys. B 639, 263 (2002) [Erratum-ibid. B 647, 359 (2002)] [arXiv:hep-ph/0205227];
E. Christova, H. Eberl, W. Majerotto and S. Kraml, JHEP 0212, 021 (2002) [arXiv:hep-ph/0211063];
E. Christova, E. Ginina and M. Stoilov, JHEP 0311, 027 (2003) [arXiv:hep-ph/0307319];
J. R. Ellis, J. S. Lee and A. Pilaftsis, Phys. Rev. D 70, 075010 (2004) [arXiv:hep-ph/0404167];
H. Eberl, T. Gajdosik, W. Majerotto and B. Schrausser, Phys. Lett. B 618, 171 (2005) [arXiv:hep-ph/0502112].

[11] G. Valencia, arXiv:hep-ph/9411441, and references therein; G.C. Branco, L. Lavoura, and J.P. Silva, CP violation, Oxford University Press, New York, 1999.

[12] S. Y. Choi, H. S. Song and W. Y. Song, Phys. Rev. D 61, 075004 (2000) [arXiv:hep-ph/9907474];
A. Bartl, T. Kernreiter and W. Porod, Phys. Lett. B 538, 59 (2002) [arXiv:hep-ph/0202198];
A. Bartl, H. Fraas, T. Kernreiter and O. Kittel, Eur. Phys. J. C 33, 433 (2004) [arXiv:hep-ph/0306304];
A. Bartl, T. Kernreiter and O. Kittel, Phys. Lett. B 578, 341 (2004) [arXiv:hep-ph/0309340];
A. Bartl, H. Fraas, O. Kittel and W. Majerotto, Phys. Rev. D 69, 035007 (2004) [arXiv:hep-ph/0308141]; Eur. Phys. J. C 36, 233 (2004) [arXiv:hep-ph/0402016]; Phys. Lett. B 598, 76 (2004) [arXiv:hep-ph/0406309]; Phys. Rev. D 70, 115005 (2004) [arXiv:hep-ph/0410054];
S. Y. Choi, M. Drees, B. Gaisser and J. Song, Phys. Rev. D 69, 035008 (2004) [arXiv:hep-ph/0310284];
S. Y. Choi, M. Drees and B. Gaisser, Phys. Rev. D 70, 014010 (2004) [arXiv:hep-ph/0403054];
J. A. Aguilar-Saavedra, Phys. Lett. B 596, 247 (2004) [arXiv:hep-ph/0403243];
Nucl. Phys. B 697, 207 (2004) [arXiv:hep-ph/0404104];
A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek and G. A. Moortgat-Pick, JHEP 0408, 038 (2004) [arXiv:hep-ph/0406190];
A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek, T. Kernreiter and G. Moortgat-Pick, JHEP 0601, 170 (2006) [arXiv:hep-ph/0510029];
K. Kiers, A. Szynkman and D. London, Phys. Rev. D 74, 035004 (2006) [arXiv:hep-ph/0605123];
A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek, T. Kernreiter and G. Moortgat-Pick, arXiv:hep-ph/0608065.

[13] A. Bartl, E. Christova, K. Hohenwarter-Sodek and T. Kernreiter, Phys. Rev. D 70, 095007 (2004) [arXiv:hep-ph/0409060].

[14] S. Kawasaki, T. Shirafuji, and S. Y. Tsai, Prog. Theor. Phys. 49, 1656 (1973).

[15] C. J. S. Damerell, private communication.

[16] C. J. Damerell and D. J. Jackson, Prepared for 1996 DPF / DPB Summer Study on New Directions for High-Energy Physics (Snowmass 96), Snowmass, Colorado, 25 Jun - 12 Jul 1996;
K. Abe et al. [SLD Collaboration], Phys. Rev. Lett. 88, 151801 (2002) [arXiv:hep-ex/0111035];
J. Abdallah [DELPHI Collaboration], arXiv:hep-ex/0311003.

[17] G. Alexander et al., TESLA Technical Design Report, Part IV, ‘A Detector for Tesla,’ http://tesla.desy.de/new_pages/ TDRCD/PartIV/detect.html

[18] J. P. Derendinger and C. A. Savoy, Nucl. Phys. B 237, 307 (1984);
J. A. Casas and S. Dimopoulos, Phys. Lett. B 387, 107 (1996) [arXiv:hep-ph/9606237].

[19] S. Eidelman et al. [Particle Data Group Collaboration], Phys. Lett. B 592, 1 (2004).