Thermoelectrical detection of Majorana states

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We discuss the thermoelectrical properties of nanowires hosting Majorana edge states. For a Majorana nanowire directly coupled to two normal reservoirs the thermopower always vanishes regardless of the value of the Majorana hybridization. This situation changes drastically if we insert a quantum dot. Then, the dot Majorana side coupled system exhibits a different behavior for the thermopower depending on the Majorana hybridization parameter $\varepsilon_M$. Thermopower reverses its sign when the half fermionic state is fully developed, i.e., when $\varepsilon_M = 0$. As long as $\varepsilon_M$ becomes finite the Seebeck coefficient behaves similarly to a resonant level system. The sign change of the thermopower when Majorana physics takes place and the fact that both, the electrical and thermal conductances reach, their half fermionic value could serve as a proof of the existence of Majorana edge states in nanowires. Finally, we perform some predictions about the gate dependence of the Seebeck coefficient when Kondo correlations are present in the dot.

I. INTRODUCTION

Nowadays there is a lot of interest in the interplay between heat and charge flows in nanostructures. Thermovoltages generated in response to a temperature gradient have been shown to be much bigger at the nanoscale due to the peculiar properties of quantum systems. For example, delta like density of states occurring in confined nanostructures like quantum wells alter dramatically their thermoelectrical properties. The main utility of thermoelectrical devices is the heat-to-electricity conversion processes. However, from a more fundamental point of view, both thermal and electrical transport reveal information on the intrinsic nature of a quantum system. An instance is the departure of the Wiedemann Franz law attributed to the non Fermi liquid behavior. In addition, thermoelectric transport measurements are able to distinguish between distinct types of carriers, like electrons and holes in Andreev systems and molecular junctions.

Our motivation is to address to what extent Majorana physics can be reflected in the thermoelectrical transport properties of a system. The unambiguous detection of Majorana fermions in solid state devices is still a discussion issue. Majorana physics, in the low energy domain, was predicted to occur as quasiparticle excitations. The first proposals suggested their observation in quantum Hall states, the Moore Read state at filling factor $\nu = 5/2$. Then, other suggestions considered some exotic superconductors like Sr$_2$RuO$_4$ or $p$-wave superconductors. Later on, the pioneering work by Fu and Kane demonstrated that such quasiparticles could be created in a topological insulator brought in close proximity with a superconductivity source. However, the Majorana search has been very prolific in the realm of quasi one dimensional semiconductor nanowires, and in particularly in large $g$ factor materials like InAs and InSb. Most of the experiments

![FIG. 1: (a) Majorana nanowire tunnel coupled to two normal contacts by tunneling barriers of probability $\Gamma$. Here, $\eta_1$, and $\eta_2$ denote the two Majorana ends states at the semiconductor nanowire. Left(right) metallic contact is electric and thermal biased with $V_L(R)$, and $\theta_L(R)$. (b) A quantum dot is inserted and symmetrically coupled to the metallic reservoirs with tunneling rate $\gamma$. The dot is side coupled to the Majorana nanowire, such coupling is characterized by the parameter $\zeta$.](image-url)
designed to detect these elusive quasiparticles have been performed via electrical transport measurements\(25,26\) by tunnel spectroscopy. A voltage shift, \(\delta V\), is applied to the nanowire edges that generates an electrical current \(I\). The Majorana signature appears as a zero bias anomaly in the nonlinear conductance \(dI/dV\).\(30–33\) In semiconductor nanowires, Majorana quasiparticles arise when superconductivity (source of electrons and holes), strong spin orbit interaction, and magnetic field work together. Then, under certain conditions the nanowire enters in the named topological phase and shows up spinless, chargeless zero energy states, very elusive quasiparticle excitations. We refer to this as Majorana nanowire. However, the presence of a zero bias anomaly in the nonlinear conductance does not warrant the presence of Majorana quasiparticles. Kondo physics can be observed in normal superconductor nanowires as well.\(24\)\(25\) Furthermore, nearly zero energy Andreev states\(36,37\) or weak antilocalization\(28\) effects are possible sources of zero bias anomaly in normal superconductor nanowires. There are other suggestions to detect Majorana zero energy states in Josephson junctions and rings.\(39–46\) The Josephson current displays an anomalous periodicity of \(4\pi\) if Majorana physics takes place. However, so far the experimental verification is not yet definitive.\(47\)

Our goal consists in utilizing the thermoelectrical properties as a tool to detect the presence of Majorana edge states formed in normal superconductor nanowires. The only attempt to study similar issues has done in \(p\)-wave superconductors.\(48\) Here, we propose a way of detecting Majorana edge states in semiconductor nanowires when a temperature gradient \((\delta \theta = \theta_L - \theta_R)\) is applied and an induced electrical shift \((\delta V = V_L - V_R)\) is generated. We analyze a two terminal device as depicted in Fig. [IIa] and determine both the electrical and energy currents. Here, the Majorana nanowire is contacted to two normal reservoirs. In general, the linear response electric \(I\) and energy \(J\) currents can be expressed as

\[
\left( \begin{array}{c} I \\ J \end{array} \right) = \left( \begin{array}{cc} G & L \\ M & K \end{array} \right) \left( \begin{array}{c} \delta V \\ \delta \theta \end{array} \right).
\]

The \(2 \times 2\) matrix is the Onsager matrix that includes diagonal elements, the electric \(G\) and thermal \(K\) conductances, and non diagonal coefficients, the thermoelectric \(L\) and electrothermal \(M\) conductances. The latter are related due to microreversibility condition.\(49,50\) More specifically, we are interested in the determination of the Seebeck coefficient or thermopower that measures how efficient is the conversion of heat into electricity in a thermoelectrical machine. The larger the Seebeck coefficient, the more efficient this conversion is. Seebeck coefficient is easily determined from the relation: \(S = -\delta V/\delta \theta = L/G\).

Our results for a two terminal Majorana nanowire [see Fig. [IIa]] show that both, the electrical and heat conductances reach their maximum value only when Majorana edge states do not overlap. On the contrary, the thermoelectrical(electrothermal) response always vanishes irrespectively of the Majorana hybridization. As a result, the Seebeck coefficient vanishes owing to the intrinsic particle hole symmetry of the system under consideration. However, this physical scenario can be dramatically altered by inserting a quantum dot in between the two normal contacts and side coupled to the Majorana nanowire.\(21,53,54\) Figure. [IIb] illustrates the sample configuration. In this arrangement, the Seebeck coefficient can be tuned by gating the dot i.e., \(S = S(\varepsilon_d)\) with \(\varepsilon_d\) the dot level position.

II. GENERAL FORMALISM

We present our theory for the thermoelectrical transport by employing the nonequilibrium Keldysh Green function framework. We consider a semiconductor nanowire with strong Rashba spin orbit interaction with proximity induced \(s\)-wave superconductivity, and an applied magnetic field \(B\). We assume a sufficiently long wire to neglect charging effects. The magnetic field is such that the wire is in the topological phase, \(\Delta_Z > \sqrt{\Delta^2 + \mu^2}\), with \(\Delta_Z = g\mu_B B/2\), and \(\mu\) the wire chemical potential. Then, isolated Majorana zero energy states \(\eta_1 = f + f\dagger\), and \(\eta_2 = i(f\dagger - f)\) (in terms of \(f\) Dirac fermions) are formed at the nanowire ends points. We consider that two normal contacts are tunnel coupled to the wire ends as shown in Fig. [IIa]. The Hamiltonian describing this system is given by these three contributions: \(\mathcal{H} = \mathcal{H}_C + \mathcal{H}_M + \mathcal{H}_T\), where

\[
\mathcal{H}_C = \sum_{\alpha,k} \varepsilon_{\alpha k} c_{\alpha k}^\dagger c_{\alpha k},
\]

\[
\mathcal{H}_M = \frac{i}{2} \varepsilon_M \eta_1 \eta_2,
\]

\[
\mathcal{H}_T = \mathcal{H}_{TL} + \mathcal{H}_{TR} = \sum_{\alpha, k; \beta} \left[ V_{\alpha k, \beta} c_{\alpha k}^\dagger \eta_\beta + V_{\alpha k, \beta} \eta_\beta c_{\alpha k} \right].
\]

Here, \(\mathcal{H}_C\) describes the two normal leads, with \(c_{\alpha k}^\dagger(c_{\alpha k})\) being the creation (annihilation) operator for an electron with wavevector \(k\) in the lead \(\alpha\). Note that the spin degree of freedom is omitted. This can be understood considering that we need to apply a large magnetic field to observe the edge Majoranas, so that only one kind of spin is effectively involved. \(\mathcal{H}_M\) characterizes the coupling between the two end Majorana states where \(\varepsilon_M \sim f(B, \Delta)e^{-L/\xi_0}\) with \(L\) the length of the wire and \(\xi_0\) the superconducting coherence length. \(f(B, \Delta)\) is a complicated function of \(B\) and \(\Delta\) that determines \(\varepsilon_M\). For our purpose we assume that \(\varepsilon_M\) is a parameter. The last contribution, \(\mathcal{H}_T\) corresponds to the tunnel Hamiltonian between normal leads and the Majorana end states. Below, the tunnel amplitude \(V_{\alpha k, \beta}\) is taken as \(V_0\) for \(\alpha = \beta\) and zero for \(\alpha \neq \beta\). This defines \(\Gamma = \pi V_0^2 \rho_0\) with \(\rho_0\) the contact density of states.

The charge and energy currents have the Landauer and...
Büttiker form

$$I = \frac{e}{h} \int d\omega T(\omega) [f_L(\omega) - f_R(\omega)],$$

and

$$J = \frac{1}{h} \int d\omega \omega T(\omega) [f_L(\omega) - f_R(\omega)],$$

with a transmission coefficient given by

$$T(\omega) = \frac{4\Gamma^2 (\omega^2 + 4\Gamma^2 + \varepsilon_M^2)}{(\omega^2 + 4\Gamma^2)^2 + \varepsilon_M^2 [\varepsilon_M^2 - 2 (\omega^2 - 4\Gamma^2)]}.$$  (5)

Here $f_L = 1/[1 + \exp(\omega - (\mu + V_L)/k_B T + 1]$ ($k_B$ Boltzmann constant) and $f_R = 1/[1 + \exp(\omega - (\mu + V_R)/k_B T + 1]$ are the Fermi Dirac distribution function for the left and right contacts respectively with $V_{L,R} = \pm \delta V/2$, and $\delta T = T_L - T_R = \pm \delta \theta/2$. The linear conductances are (with $\mu = 0$)

$$G = \frac{e^2}{h} \int d\omega T(\omega) \left[ -\frac{\partial f_{eq}}{\partial \omega} \right],$$

$$L = \frac{e}{h T_b} \int d\omega \omega T(\omega) \left[ -\frac{\partial f_{eq}}{\partial \varepsilon} \right],$$

$$M = \frac{e}{h} \int d\omega \omega^2 T(\omega) \left[ -\frac{\partial f_{eq}}{\partial \omega} \right],$$

$$K = \frac{1}{h T_b} \int d\omega \omega^2 T(\omega) \left[ -\frac{\partial f_{eq}}{\partial \omega} \right],$$

where $f_{eq}$ is the equilibrium Fermi Dirac distribution function when $\delta T = 0$ and $\delta V = 0$. In a Sommerfeld expansion, at sufficiently low temperatures, the linear response conductances $G$, and $K$ have the same behavior with the transmission coefficient up to a proportionality factor: $G_0$, and $K_0$. Thus,

$$G(K) = \lim_{\delta V \to 0} \left( \frac{dI}{dV} \right) \left( \frac{dJ}{d\theta} \right) = \frac{4\Gamma^2}{\varepsilon_M^2 + 4\Gamma^2}.$$  (10)

with $G_0 = e^2/h$ (quantum electrical conductance), and $K_0 = \pi^2 k_B^2 T_b/3h$ (quantum thermal conductance). They take their maximum value $G_0$, and $K_0$, respectively when $\varepsilon_M = 0$, otherwise, they vanish as $\varepsilon_M$ grows. Importantly, the off diagonal conductances are always zero, $L = L_0 = \delta T(\omega)/\delta \omega = 0$ with $L_0 = e\pi^2 k_B^2 T_b/3h$ (and $M = M_0 = L_0 = T_b/4\pi$). The vanishing value of the $L(M)$ has profound consequences in the thermopower or Seebeck coefficient (we recall that $S = L/G$). The Seebeck coefficient vanishes regardless of the value of $\varepsilon_M$. The reason for this result lies in the inherent particle hole symmetry of our system, there is no electrical response to a thermal gradient.

Asymmetry in the particle and hole subspaces can happen if we insert a quantum dot between the two normal contacts. Here the dot is side coupled to the Majorana as illustrated in Fig. 1(b). The thermoelectrical transport through the dot Majorana system shows a non zero value for the off diagonal Onsager conductances when the dot is off resonance, i.e., a nonzero Seebeck coefficient. Importantly, we can tune the Seebeck coefficient from zero when the dot is on resonance to large values when is off resonance. Besides, the behavior of the Seebeck coefficient with the dot level is quite different depending on the value of the Majorana hybridization parameter, $\varepsilon_M$. Thus, Seebeck coefficient might allow us to detect truly zero energy Majorana states for which $\varepsilon_M$ is negligible.

### III. SIDE TUNNEL COUPLED DOT MAJORANA SYSTEM

In order to include the quantum dot we need to reformulate the Hamiltonian as follows. First, we consider the dot Hamiltonian

$$\mathcal{H}_d = \sum_{\alpha k} \varepsilon_d d^\dagger \alpha_k d + h.c.$$

where $d(d^\dagger)$ operator annihilates(creates) an electron on the dot site. We consider a single dot level with energy $\varepsilon_d$. The dot is connected to the left and right normal contacts by tunnel barriers

$$\mathcal{H}_{T_d} = \sum_{\alpha k} (W_c e_{\alpha k}^\dagger d + h.c).$$

We consider symmetrically dot coupling to the normal contacts with a common tunneling rate: $\gamma = \pi W^2 \rho_0$, with $W = W_L = W_R$. The dot is side coupled to the Majorana nanowire as

$$\mathcal{H}_{T_M} = \sum_{\beta} \zeta (d^\dagger \eta_\beta + \eta_\beta d),$$

with $\beta = 1, 2$. Here, we assume that only the closest Majorana state to the dot is coupled, say $\eta_1$. The total Hamiltonian is the sum of all these contributions, and the contact and Majorana Hamiltonians $[\mathcal{H}_c$, and $\mathcal{H}_M$, see Eq. (2)]: $\mathcal{H} = \mathcal{H}_C + \mathcal{H}_d + \mathcal{H}_M + \mathcal{H}_{T_d} + \mathcal{H}_{T_M}$. Now, the charge and energy flows can be expressed in terms of the dot transmission (see Ref. [53] for details)

$$T_d(\omega) = \frac{1}{2\pi} \text{Im} G_d^R(\omega),$$

where $G_d^R$ is the retarded dot Green function

$$G_d^R(\omega) = \frac{1}{\omega - \varepsilon_d + i\pi/2 - B(\omega) [1 + \tilde{B}(\omega)]},$$

with

$$\tilde{B}(\omega) = \frac{B(\omega)}{\omega + \varepsilon_d + i\pi/2 - B(\omega)}.$$  (16)

The parameter $\zeta$ in Eq. (14) characterizes the dot Majorana coupling where $B(\omega) = |\zeta|^2/\omega - \varepsilon_M/\omega$ being the dot Majorana selfenergy coupling.
FIG. 2: Dot transmission \( T_0(\omega) \) for (a) various \( \zeta \) values as indicated and \( \gamma = 0.25 \); (b) for different \( \gamma \) values and \( \zeta = 0.05 \). Parameters: \( \varepsilon_d = 0.0 \), \( \varepsilon_M = 0 \).

IV. DISCUSSION

Before starting the discussion of the thermoelectrical properties in the dot Majorana system it is worth to re-visit the behavior of the dot transmission with the system parameters, \( \varepsilon_M, \varepsilon_d, \zeta \) and \( \gamma \). Hereafter, we employ \( D = 50 \) for the contact bandwidth that determines our energy unit. The dependence of \( T_0(\omega) \) with \( \zeta \) and \( \gamma \) is illustrated in Fig. 2 when the dot is on resonance and no Majorana overlap occurs (\( \varepsilon_d = 0 \), and \( \varepsilon_M = 0 \)). For the uncoupled Majorana situation the transmission corresponds to the resonant level model with unitary transmission. As \( \zeta \) is turn on two peaks at \( \omega = \pm \zeta \) appear due to the dot Majorana finite coupling. Now, keeping fixed \( \zeta \) and tuning \( \gamma \) the dot transmission shows a three peak structure when \( \gamma \approx \zeta \) in which the zero energy peak is the signature of the presence of Majorana edge states [see Fig. 2(b)]. In all cases, when \( \zeta \neq 0 \), the dot transmission is always half fermionic.\(^{33,54}\)

When \( \varepsilon_M \) acquires a finite value, \( T_0 \) becomes unitary, as shown in Fig. 3(a). For large \( \varepsilon_M \), \( T_0 \) corresponds to the one for a resonant level mode, with resonances at \( \omega \pm \varepsilon_M \) due to the coupling of the dot state with the \( f \) Dirac fermions in the wire (resulting from the large Majorana hybridization).

Thermoelectrical effects appears when the transmission becomes asymmetric. In order to observe such asymmetric transmission for \( \omega < 0 \), and \( \omega > 0 \) the dot level must be positioned off resonance, i.e., \( \varepsilon_d \neq 0 \). This situation is presented in Fig. 3(b) for several values of the Majorana hybridization parameter when \( \varepsilon_d = 0.12 \). Note that, the transmission is asymmetric even for \( \varepsilon_M = 0 \) although is still half fermionic. For a nonzero Majorana overlap, the transmission depends strongly on the dot gate value leading to a non unitary electrical(thermal) conductance.

The dot gate dependence of \( T_0(\omega) \) for an ideal Majorana nanowire (\( \varepsilon_M = 0 \)) is depicted in Fig. 4(a) and its energy derivative in Fig. 4(b). These curves show that the transmission at zero energy is always half fermionic as should be for \( \varepsilon_M = 0 \), regardless of the dot gate value. However, it is interesting to observe that the energy derivative of the transmission at zero energy acquires some dot gate dependence reflecting the asymmetry between the particle and hole sectors. This result is important for the thermoelectrical conductance \( L \), we recall that \( L = L_0 \partial T_0(\omega)/\partial \omega|_{\omega=0} \) implying that \( L \) becomes gate dependent. Whereas the diagonal conductances are not sensitive to the particle hole asymmetry introduced by \( \varepsilon_d \neq 0 \), the off diagonal conductances show a dot gate dependence with important consequences in the thermoelectrical transport.

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Our previous analysis for the dot transmission explains the curves for the conductances illustrated in Fig. 4. Both, the electrical and thermal conductances, \( G \), and \( K \) depend strongly on \( \varepsilon_d \) whenever the two end Majorana states overlap. Otherwise, in the ideal situation where \( \varepsilon_M = 0 \), \( G \), and \( K \) take its maximum value and they becomes half fermionic.\(^{33,54}\) This important result it serves to us to detect the presence of Majorana edge states in side coupled dot nanowires systems. However, the previous results are applicable only for purely electrical or thermal transport measurements. Here, we are interested more in the thermoelectrical signatures of the Majorana edge states. For that purpose, we analyze how the off diagonal conductances behave with the dot gate values. We find, that when Majorana edge states have negligible overlap (i.e., \( \varepsilon_M = 0 \)) the off diagonal conductance \( L(M) \) reverses it sign in comparison with a situation with finite overlap, i.e., \( \varepsilon_M \neq 0 \). Our results show that for zero Majorana overlap \( \varepsilon_M = 0 \), the thermoelectrical conductance \( L \) depends linearly with \( \varepsilon_d \) with a negative slope \(-1/2\varepsilon_d^2\) that depends inversely on the dot Majorana strength. However, for a finite Majorana overlap, when \( \varepsilon_M \neq 0 \) the thermoelectrical conductance \( L|L_0 = \varepsilon_d/[4\varepsilon_d^2 + \gamma_0^2][2\gamma_0^2(\varepsilon_M^2 + \gamma_0^2)/\varepsilon_M^2] \), displays two extrema at \( \varepsilon_d = \pm \gamma/2 \). In this case, \( L \) behaves similarly to the resonant level model. Importantly, the different behavior found for the gate dependence of the thermoelectrical conductance \( L \) could be utilized as an smooking
gun for the Majorana detection in thermoelectrical transport measurements.

Using the previous results, we discuss the gate dependence of the thermopower $S = L/G = -\delta V/\delta \theta$, where $S = (\pi^2 k_B^2 T_b/3e)\ln T(\omega)/d\omega|_{\omega=0}$ is the Mott formula. We define $S_0 = \pi^2 k_B^2 T_b/3e$. For the dot Majorana uncoupled case, $\zeta = 0$, the thermopower $S/S_0 = -\varepsilon_d/(4\varepsilon_d^2 + \gamma^2)$ vanishes when $\varepsilon_d = 0$ and follows the resonant level model as expected. For the coupled system, when $\zeta \neq 0$, the thermopower $S$ versus the dot gate position is plotted in Fig. 4. Remarkably, the thermopower is linear with $\varepsilon_d$ for zero Majorana overlap: $S/S_0 = -\varepsilon_d/\zeta^2$ when $\varepsilon_M = 0$ and $\zeta \neq 0$. The dot gate dependence of $S$ is due to the particle hole asymmetry introduced when $\varepsilon_d$ is tuned from the on to the off resonance situation. The way to understand this result is by the addition of two effects. First, the Majorana state contributes to the thermopower in a rigid way with a constant term $-1/\zeta^2$. Second, the particle hole asymmetry grows as $\varepsilon_d$ does and this explains why the thermopower grows with $\varepsilon_d$. Then, both features add up and produce a linear dependence of the Seebeck coefficient with the dot gate with a negative slope that depends on the inverse of the dot Majorana coupling $\zeta$.

Figure 4 displays our results for the thermopower for various values of $\varepsilon_M$. For $\varepsilon_M = 0$, Fig. 4 shows that the thermopower is positive(negative) for negative(positive) $\varepsilon_d$ having $\delta V < 0$ by heating up(cooling down) the left contact. The thermopower sign dependence with $\varepsilon_d$ is inverted when the Majorana overlap is finite. Here, for $\varepsilon_M$ finite the thermopower is: $S/S_0 = [\varepsilon_d/(4\gamma^2 + \varepsilon_d^2)]([\gamma^2 + \zeta^2]/\varepsilon_M^2)]$, This means that when $\varepsilon_d < 0(\varepsilon_d > 0)$ the heating(cooling) of the left contact induced a positive(negative) voltage difference. Here, the Seebeck coefficient follows the behavior for a resonant model with two extrema at $\varepsilon_d = \pm \gamma/2$. All these differences for $S(\varepsilon_d)$ depending on $\varepsilon_M$ it allows us to distinguish situations where nanowires can host truly Majorana edge states or not.

Some of the previous results allow us to predict the dot gate dependence of the Seebeck coefficient, when Coulomb interactions take place. A quantum dot with a free local moment is able to form a Kondo singlet with the delocalized electrons in the normal reservoirs when is strongly tunnel coupled to them. Then, at temperatures much lower than the Kondo scale $T_K$ the dot physics can be explained within the Fermi Liquid theory. In this scenario, both the dot gate position $\varepsilon_d \rightarrow \varepsilon_d + \lambda$, and the lead dot tunneling rate $\Gamma \rightarrow \tilde{\Gamma}$ are renormalized by Kondo correlations as $\lambda = -\varepsilon_d$, and $\tilde{\Gamma} = T_K$. Under this situation, the Seebeck coefficient, in the Kondo regime is zero (with $T_K$ larger that the dot Majorana coupling selfenergy, i.e., in the Kondo dominant regime). In the pure Kondo regime spin fluctuations carry the charge and energy transport in a particle and hole symmetric situation, then, it quite reasonable to expect a vanishing Seebeck coefficient no matter the Majorana overlap is.

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FIG. 6: Thermopower $S$ versus $\varepsilon_d$ for various $\varepsilon_M$ values. The curve corresponding to $\varepsilon_M = 0$ has been enlarged by a factor 20 for comparison purposes. Parameters: $\gamma = 0.25$, $\zeta = 0.15$, and $T_b = 0.025$.

V. CONCLUSION

We have investigated the linear response conductances to a thermal and electrical voltage shift in a two-terminal geometry with normal superconductor nanowires showing Majorana physics. Firstly, we have considered a nanowire directly coupled to two normal reservoirs. Due to the intrinsic particle hole symmetry this system exhibits a null thermopower, no voltage is generated in response to a thermal gradient. Then, we insert a quantum dot between the two normal contacts which is side coupled to the Majorana nanowire. With this arrangement the detection of the Majorana edge states can be performed by looking at the sign of the thermoelectrical conductance or the thermopower $S$. Besides, we show that both, the electrical and thermal conductances take their half fermionic values whenever a true Majorana fermion state is formed, when $\varepsilon_M = 0$. Finally, we make some predictions for the gate dependence of the Seebeck coefficient for interacting dots in the Kondo regime. We believe that our results could serve as an unambiguous tool for the detection of Majorana edge states in semiconductor nanowires.

Note added—During the completion of this paper we become aware of a related work dealing with thermoelectric transport in normal-dot-Majorana nanowires systems. The difference is that we consider thermal and electrical bias applied to the normal contacts, in Ref. [58] the thermoelectrical forces are applied to the normal and Majorana parts.

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