Solitons and Their Arrays: from Quasi One-Dimensional Conductors to Stripes.

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Abstract

We suggest a short review of literature on various solitonic lattices and individual solitons in quasi one-dimensional conductors. This information seems to be quite relevant to topics of stripes and their melted phases correspondingly. We shall quote also the latest experiments, which access solitons as elementary excitations in organic conductors and in charge density waves. We shall outline a theory for ordered phases, where solitons should acquire forms of combined topological configurations (kink-roton complexes). The extension of this picture to cuprates allows interpretation the latest STM observations on local rod-like structures.

\textbf{keywords}: Soliton, Confinement, Spinon, Holon, Super-lattice, Dislocation, Topological Defect.

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I. Solitonic lattices versus stripes.

Inhomogeneous states of electronic systems can be viewed as a phase separation stabilized by long range forces [1, 2, 3]. The most common form is a domain structure, alternating between different equivalent ground states [4, 5, 6]. This is what makes the stripes to be generically related to superstructures like solitonic lattices, commonly existing or being expected in quasi-1D conductors. The notion of a single soliton, as a stable elementary particle having a finite energy, appears in the 1D limit; here the soliton is a domain wall separating segments with different, while equivalent, ground states.

Since more than 3 decades of its development, the science of solitons has accumulated a vast amount of theoretical and experimental results. Only a slice of this notion has been touched, when theories of stripes have emerged [7, 8, 9, 10], and there was not much counterflow as well (as a cross-fields example, quote [11]). Recall the chain of conferences on ”Electronic Crystals” (see [12] for the latest event) which aims to relate all topics of electronic aggregation in solids. The goal of this short review is to give a reference guide to theory, with excursions to latest experiments, of solitonic lattices and individual solitons, and to outline extensions to general strongly correlated electronic systems.

I.1. Regular superstructures.

Flexibility of spontaneously formed states with a broken symmetry allows for their local modifications when electrons are added via doping, tunneling, photoemission, optics, or field effect. Lattices of solitons usually appear because of incommensurability, which itself can be derived from external or internal transfer of charge or of spin. The oldest, phenomenological version is a case of weak interaction between the underlying lattice and the superstructure. Its natural form is a sin-Gordon theory, firstly suggested [13] for helicoidal antiferromagnets (AFM) - spin density waves (SDW). A similar description is allowed for interesting cases of a mutual commensurability of coexisting superlattices, like the lock-in of two Charge Density Waves (CDW) observed in NbSe$_3$ [14].

Strength and stability of solitons are particularly high for the case of one electron per unit cell, i.e. a nearly half-filling of the bare zone. The interest to this case was boosted by studies of a doped polymer, polyacetylene (see reviews [15, 16]), which ground state shows a spontaneous bond dimerization - the Peierls effect. Exact solutions for periodic lattices of solitons (see the review [17]) were allowed for all basic physically realistic models.
They include: two principle limits of electronic concentration (small and nearly half band fillings); arbitrary filling with effects of breaking charge conjugation symmetry; Zeeman spin splitting; combined effect of doping and spin splitting; interchain hybridization; effects of lattice discreteness; case of superconductivity. Applications included: conducting polymers, spin-Peierls chains, CDWs and SDWs, FFLO phase in superconductors. The common spectral property: emergence of intragap bands partially filled by electrons, is the most important feature and the origin of energetic stability of solitonic lattices.

Typically, the solitons were found to be domain walls with exactly $\pi$ shifted (acquiring half a period) profile and anomalous quantum numbers: they are either charged or spin polarized. Both of these properties are consequences of the charge conjugation symmetry: its lifting due to momentum dispersion of interactions makes the phase shift arbitrary and brings to life a coexisting charge and spin density. The charge and the phase shift are not ultimately bound, as it is shown by the richest case of doubly periodic solitonic lattice originated by a combined effect of charge doping and spin polarization. Here, the secondary solitonic superlattice appearing above a threshold magnetic field, evolves drastically as a function of doping: from the lattice of spin carrying polarons embedded to the lattice of charged kinks to the overlapping superstructure of amplitude solitons. The electric charge associated to each spin varies from nearly $e$ to nearly 0, but the phase shift of spin-soliton superlattice stays exactly at $\pi$.

Taking into account the interchain hybridization of electrons, i.e. the curvature of their Fermi surfaces, is an important step toward higher dimensional systems, particularly the CuO$_2$ planes. In quasi-1D case, a strong band curvature can provoke formation of a neutral solitonic lattice, even with no doping or spin polarization: split-off intragap bands provide the necessary energy gain. Recently, this mechanism was invoked to explain the SDW-metal transition in organic conductors under pressure. Finally, recall the solitonic structures in quasi-1D superconductors, which appear as a 1D version of the well-known FFLO inhomogeneous state near the pair-breaking limit. Being very weak in 3D, this effect becomes quite pronounced in systems with nested Fermi surfaces which is the case of the 1D limit.

I.2. Inhomogeneity and melting.
The above classification implied that solitonic lattices are plane structures - the regular stripes. More complicated, and attracting much modern interest, is the case of an inhomogeneous doping, realized e.g. in the FET geometry, when the solitons’ chemical potential $\mu$ contains a variable electric potential $\mu \Rightarrow 2e\Phi(\vec{r}) + \text{const}$, to be determined self-consistently. Then, the solitonic lattice will change its on-chain $x$-period in, say $y$ direction, while the field is penetrating inside the host crystal $y > 0$ or between regions of varying dopant concentration. This regime results in a branching structure of dislocations of the solitonic lattice $^{27, 28}$.

In 1D, the solitonic lattice cannot possess a long range order: it becomes a liquid preserving the local periodicity in its correlation function. But what happens for a weak interchain interaction, competing with quantum or thermal fluctuations, which question is most relevant to stripes’ melting? Details are known at least for the gas of kink-solitons in systems with a double degeneracy (nearly half filling) $^{29}$. As a function of temperature, the system experiences two consecutive phase transitions at $T_1 > T_2$ (in D=3, while in D=2 the lower $T_2$ becomes a crossover). At highest $T > T_1$, the kinks are decoupled, serving as quasi-particles. Below the upper ”confinement transition”, at $T_1 > T > T_2$, the kinks and anti-kinks are bound into neutral pairs, which may be viewed as nucleolus droplets in magnetic semiconductors $^{1}$. Below the second transition of ”aggregation”, at $T < T_2$, a fraction of pairs is broken again, and the unpaired kinks assemble into domain walls crossing the sample. In case of charged solitons, this picture may be essentially affected by long range Coulomb forces $^{30}$, which e.g. lead to instability of plane solitonic lattices even within the coherent low $T$ phase. It may not be an accident, in this respect, that with a high experimental precision profiles of solitons have been obtained (by the NMR $^{31}$) only for spin chains, thanks to their high 3D coherence.

II. Solitons as quasi-particles.

Macroscopic instability of electronic systems to formation of solitonic lattices can manifest itself already at the single-particle level. The very character of elementary excitations can be modified; they acquire forms of topological configurations - solitons exploring the possibility to travel among different allowed ground states. These excitations can be also viewed as nucleuses of the melted stripe phase, which is typically observed under higher doping of oxides, or of the FFLO phase in spin polarized superconductors and CDWs.
II.1. New routes to topological excitations

New interest to solitons in electronic processes emerges from a discovery of the ferroelectric charge ordering in organic conductors (Monceau et al., Brown et al. (2001), see [32] - reviews [33, 34]) and from nano-scale experiments on internal tunneling in CDW materials (see [35, 36] and short reviews [37]). The charge ordering allows us to observe several types of solitons in conductivity, and solitons’ bound pairs in optics. The observed internal tunneling of electrons in CDWs goes through the channel of amplitude solitons ([35], see below), which correspond to the long sought quasi-particle, the spinon. Moreover, the resolved tunneling in the normally forbidden subgap region [36, 37] recovers collective quantum processes like coherent phase slips, 2\pi instantons. The same experiment gives an access to the reversible reconstruction of the junction via spontaneous creation of a special lattice of embedded 2\pi solitons, a grid of dislocations [36, 37].

On this solid basis we can extend the theory of solitons in quasi 1D systems to arrive at a picture of combined topological excitations in general strongly correlated systems: from nearly antiferromagnetic oxides to high gap superconductors. To extend physics of solitons to the higher-D world, the most important problem is the effect of confinement: as topological objects connecting different degenerate vacuums, the solitons at D=1 acquire an infinite energy unless they reduce or compensate their topological charges. The problem is generic to all solitons, but it becomes particularly interesting at the single electronic level, where the spin-charge reconfinement appears as the result of topological constraints. Especially interesting is the important case of coexisting discrete and continuous symmetries. As a result of their interference, the topological charge of solitons originated by the discrete symmetry can be compensated by gapless degrees of freedom originated by the continuous one. This is the scenario we shall discuss through the rest of the article. Details and references can be found in [38, 39, 40].

II.2. Amplitude solitons with phase wings in quasi-1D spin-gap systems: CDWs and superconductors.

Difference of states with even and odd numbers of particles is a common issue for correlated electrons and mesoscopics. Thanks to solitons, in an incommensurate CDW (ICDWs, an arbitrary CDW wave number Q) it also shows up in a spectacular way (S.B. 1978-80, see [17, 41]).
1. Additional pair of electrons or holes is accommodated to the extended ground state, for which the overall phase difference becomes $\pm 2\pi$. Phase increments are produced by phase slips, which provide the spectral flow from the upper $+\Delta_0$ to the lower $-\Delta_0$ rims of the single particle gap $2\Delta_0$. The phase slip requires for the CDW amplitude $A(x,t)$ to pass through zero, at which moment the complex order parameter has a shape of the amplitude soliton (AS) - the kink, $A(x = -\infty) \leftrightarrow -A(x = +\infty)$.

2. This instantaneous configuration of the AS (Fig.1) becomes the stationary state for the case when only one electron is added to the system, or when the total spin polarization is controlled to be nonzero. The AS carries the singly occupied mid-gap state, thus having a spin $1/2$, but its charge is compensated to zero by local dilatation of singlet vacuum states [17, 41] - the AS is a realization of the "spinon".

As a nontrivial topological object ($O_{\text{cdw}} = A(x) \cos[Qx + \varphi]$ does not map onto itself), the pure AS is prohibited in $D > 1$ environment. Nevertheless, the AS becomes allowed if it acquires phase tails with the total increment $\delta \varphi = \pi$. The length of these tails $\xi_{\varphi} \gg \xi_0 = \hbar v_F/\Delta_0$ is determined by the weak interchain coupling. As in 1D, the sign of $A(x)$ changes within the scale $\xi_0$ but further on, at the scale $\xi_{\varphi}$, the factor $\cos[Qx + \varphi]$ also changes the sign, thus leaving the product in $O_{\text{cdw}}$ to be invariant. As a result, the 3D allowed particle is formed with the AS core $\xi_0$ carrying the spin, and the two $\pi/2$-twisting wings stretched over $\xi_{\varphi}$, each carrying the charge $e/2$. This picture can be directly reformulated for quasi-1D superconductors by redefining the meaning of the phase. The phase tails form now the elementary $\pi$-junction [38, 42].

II.3. A hole in the AFM environment.

Consider the quasi-1D system with repulsion at a nearly half filled band, which is the SDW rout to a general doped antiferromagnetic (AFM) Mott-Hubbard insulator (see [40] for more details). The 1D bosonized Hamiltonian can be written schematically as

$$H \sim \{C_c(\partial \varphi)^2 - U \cos(2\varphi)\} + (C_s \partial \theta)^2$$

Here $\varphi$ is the analog of the CDW displacement phase, $\theta$ is the spin rotation angle and $U$ is the Umklapp scattering amplitude. In 1D, the excitations are the (anti)holon as the $\pm \pi$ soliton in $\varphi$, and the spin sound in $\theta$, which are decoupled. The $\pi-$ solitons have been clearly identified experimentally both in conductivity and optics (see [33] for a review and
interpretations). At $D \gtrsim 1$, below the SDW ordering transition, the order parameter (the staggered magnetization $< S_x + iS_y >$) is $O_{sdw} \sim \cos \varphi \exp(i\theta)$. To survive in $D > 1$, the $\pi-$ soliton in $\varphi$ ($\cos \varphi \rightarrow -\cos \varphi$) should enforce the $\pi$ rotation in $\theta$, then the sign changes in both of the two factors composing $O_{sdw}$, cancel each other and the configuration becomes allowed.

This construction can be generalized beyond quasi-1D systems by considering a vortex configuration bound to an unpaired electron. Extending to AFMs like the CuO$_2$ planes, the SDW becomes a staggered magnetization; the soliton becomes a hole, which motion leaves the string of reversed AFM sublattices; the $\pi$- wings become the magnetic semi-vortices. The resulting configuration is a half-integer vortex ring of the staggered magnetization (a semi roton) with the holon confined in its center (see Fig.2 - in 2D, the vortex ring is reduced to the pair of vortices.) Such combined semi-vortices might be significant in sliding incommensurate SDWs.

II.4. Nucleus stripes as elementary excitations.

The picture of combined topological excitations is well established theoretically, as well as confirmed experimentally, for weakly interacting chains. Since the results are symmetrically and topologically characterized, they can be extrapolated qualitatively to isotropic systems with strong coupling, where a clear microscopic derivation is not available. Here the hypothesis is that, instead of normal carriers excited or injected above the gap, the lowest states are the symmetry broken configurations described above as a semiroton-holon complex (spinon instead of holon for spin-gap cases).

Strong interactions are necessary for this extrapolation from quasi-1D. Stability of complex excitations is ultimately related to local robustness of a stripe phase, which must sustain a fragmentation due to quantum or thermal melting. Then any termination point of one stripe (the dislocation within the regular pattern) will be accompanied by the semiroton in accordance with the quasi-1D picture: the combined soliton becomes a minimal element (the nucleolus) of the melted stripe.

This picture allows us to answer the long staying question: are there stripes present when we do cannot visualize them, e.g. in cuprates away from the magic concentration $x = 1/8$. The answer is that they are still around - dispersed as an ensemble of their nucleuses. These particles determine most of observable properties, usually ascribed to conventional
electrons. Thus, the associated mid-gap states will give rise to observed growing arcs of Fermi surfaces. Individual solitons may be better accessed when they are trapped by dopants; that might be responsible for observation of rod-like local structures in recent STM experiments in cuprates [43], and more generally, for the whole issue of the strong inhomogeneity [44].

Details and illustrations can be found at the web site

www.lptms.u-psud.fr/membres/brazov/seminars.html

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[1] E.L. Nagaev, et al., "Physics of Magnetic Semiconductors" (Mir, Moscow, 1983); "Phase separation in high-Tc superconductors and related magnetic systems", PHYS-USP, 38, 497 (1995); "Cooperative electric phenomena in degenerate magnetic semiconductors with spontaneous phase separation", PHYS-USP, 39, 637 (1996); "Colossal magnetoresistance and phase separation in magnetic semiconductors", (Imperial College Press, 2001);

[2] A.I. Larkind and D.E. Khmelnitskii, Sov. Phys JETP 28, 1245 (1968).

[3] C. Castellani, C. Di Castro, and M. Grilli, Phys. Rev. Lett. 75, 4650 (1995).

[4] J.M. Tranquada, et al., Nature 375, 561 (1995).

[5] A. Bianconi, et al., Phys. Rev. Lett. 76, 3412 (1996).

[6] H.A. Mook, et al, Nature 395, 580 (1998).

[7] K. Machida, Physica 158C, 192 (1989).

[8] H.L. Schultz, J. Physique 50, 2833 (1989).

[9] J. Zaanen and O. Gunnarsson, Phys. Rev. B 40, 7391 (1989).

[10] D. Poilblanc and T.M. Rice, Phys. Rev. B 39, 9749 (1989).

[11] S.I. Matveenko and S.I. Mukhin, Phys. Rev. Lett. 84, 6066 (2000).

[12] Proceedings of ECRYS-2005, S. Brazovskii, N. Kirova, and P. Monceau eds., J. Physique IV 131 (2005).

[13] I.E. Dzyaloshinskii, Sov. Phys. JETP 20, 665 (1964).

[14] A. Ayari, et al., Phys. Rev. Lett. 93, 106404 (2004).

[15] A.J. Heeger, et al, Rev. Mod. Phys. 60, 781 (1988).

[16] Yu Lu, Solitons and Polarons in Conducting Polymers (World Scientific Publ. Co., 1988).
[17] S. Brazovskii and N. Kirova, ”Electron Self-localization and superstructures in quasi one-dimensional dielectrics”, Sov. Sci. Reviews, I.M. Khalatnikov ed., (Harwood Ac. Publ., 1984), v. A5, p. 99.
[18] S. Brazovskii, S. Gordyunin, and N. Kirova, JETP Letters, 31, 456 (1980).
[19] S. Brazovskii and S. Matveenko, Sov. Phys. JETP 60, 804 (1984).
[20] S. Brazovskii, I. Dzyaloshinskii, and N. Kirova, Sov. Phys. JETP 54, 1209 (1981).
[21] S. Brazovskii, L. Gor’kov, and J.R. Schrieffer, Physica Scripta 25, 423 (1982).
[22] S. Brazovskii, L. Gor’kov, and A. Lebed, Sov. Phys. JETP 56, 683 (1982).
[23] S. Brazovskii, I. Dzyaloshinskii and I. Krichever, Sov. Phys. JETP 56, 212 (1982); Phys. Lett. A 91, 40 (1982); S.I. Matveenko, Sov. Phys. JETP 60, 1026 (1984).
[24] S. Brazovskii and I. Dzyaloshinskii, J. Stat. Phys. 38, 115 (1985).
[25] A.I. Buzdin and V.V. Tugushev, Sov. Phys. JETP 58, 428 (1983); K. Machida and H. Nakanishi, Phys. Rev. B 30, 122 (1984).
[26] L.P. Gor’kov and P.D. Grigoriev, Europhys. Lett. 71, 425 (2005).
[27] S. Brazovskii and S.I. Matveenko, J. Physique I 2, 409 (1992); Synth. Met. 57 2696 (1993).
[28] N. Kirova and S. Brazovskii, J. Physique IV 131, 147 (2005).
[29] T. Bohr and S. Brazovskii, J. Phys. C 16, 1189 (1983).
[30] S. Teber, et al., J. Phys. C 13, 4015 (2001); ibid, 14, 7811 (2002).
[31] M. Horvatic, et al., Phys. Rev. Lett. 83, 420 (1999).
[32] Physics of Organic Conductors, A.G. Lebed ed. (Springer Series in Materials Sciences 2007) to be publ.
[33] S. Brazovskii, in [32]; (cond-mat/0606009).
[34] P. Monceau and F. Nad, J. Phys. Soc. of Japan 75, 051005 (2006); P. Monceau, et al, in [32].
[35] Yu.I. Latyshev, et al., Phys. Rev. Lett. 95, 266402 (2005).
[36] Yu.I. Latyshev, et al., Phys. Rev. Lett. 96, 116402 (2006).
[37] S. Brazovskii; Yu.I. Latyshev; S.I. Matveenko in [12].
[38] S. Brazovskii, in ”Electronic Correlations: From meso- to nano-physics”, T. Martin and G. Montambaux eds., EDP Sciences (2001), p. 315; cond-mat/0204147.
[39] N. Kirova and S. Brazovskii J. Physique IV 10, 183, (2000); cond-mat/0004313.
[40] S. Brazovskii, J. Physique IV, 10, 169 (2000); cond-mat/0006355.
[41] S. Brazovskii, in ”Modern Problems in Condensed Matter Science” (Elsevier Sci. Publ., 1990),
v. 25, p. 425.

[42] H.-J. Kwon and V. M. Yakovenko, Phys. Rev. Lett. 89, 017002 (2002).
[43] J.C. Seamus Davis, in this volume.
[44] A. Bianconi, in this volume; in [12], p.49.

FIG. 1: Profiles for the amplitude soliton in ICDW.

FIG. 2: Motion of the kink-roton complex. For SDW or AFM, the string of the amplitude reversal of the order parameter created by the holon is cured by the semi-vortex pair (the loop in 3D) of the staggered magnetization circulation. For ICDW or the superconductor, the amplitude kink is provided by the spinon. For the ICDW the curls are displacements contours for the half integer dislocation pair. For the superconductor, the curls are lines of electric currents circulating through the normal core carrying the unpaired spin.