An operational method for calculating the frequency fluctuations of a radio signal in a randomly inhomogeneous ionosphere

D Ch Kim¹, E T Ageeva¹, N T Afanasiev², S O Chudaev²,³, I G Mahro¹ and O I Medvedeva¹

¹ Department of Physics, Bratsk State University, 40 Makarenko St., 665709 Bratsk, Russia
² Department of Physics, Irkutsk State University, 1 K. Marx St., 664003 Irkutsk, Russia
³ Institute of Solar Terrestrial Physics of Siberian Branch of Russian Academy of Sciences, 126a Lermontov St., 664033 Irkutsk, Russia

E-mail: ch45st@gmail.com

Abstract. An operational method for calculating fluctuations of the Doppler frequency shift of a radio signal reflected from a randomly inhomogeneous ionosphere is proposed. The method is based on a numerical and analytical solutions of stochastic ray equations. The integral expressions are obtained for the average and root-mean-square deviations of the frequency of the radio signal along the oblique sounding path in the approximation of the perturbation method. The motion of chaotic ionospheric irregularities is taken into account in the framework of the hypothesis of the frozen turbulence transfer. The integral expressions for statistical moments are reduced to a system of ordinary differential equations of the first order and are solved numerically together with the ray equations in the regular ionosphere. This can significantly reduce the computer time spent calculating the Doppler frequency shift of the radio signal in a randomly inhomogeneous ionosphere. The results of mathematical modeling of frequency fluctuations of a decameter radio signal on a single-hop track in various geophysical conditions are presented.

1. Introduction

As is known [1], fluctuations of the Doppler frequency shift are observed. During the propagation of radio waves through an unsteady randomly inhomogeneous ionosphere with fixed coordinates of the source and receiver, therefore, the operational assessment of the Doppler shift on operating ionospheric paths is very relevant for predicting the optimal conditions for the passage of a radio signal and improving its quality.

In general, the ionosphere route is a complex, multiply connected system [2,3]. The prediction of the characteristics of this system is a big problem due to the anisotropy of the ionosphere, the variety of types of ionospheric irregularities and the characteristics of radio wave propagation mechanisms. In particular, it is important to know the shape of the spectrum of ionospheric irregularities in estimating the frequency fluctuations of the radio signal. The phenomenon of birefringence occurs and the calculation of the characteristics of the radio signal for both magnetoion components is required due to the influence of the Earth’s magnetic field on ionospheric radio paths. Despite these difficulties an
estimate of the frequency fluctuations of the radiosignal is possible on the basis of a model of the
ionosphere with generalized (integral) properties. At present, the geophysical parameters of the thin
structure of the ionosphere are known with a large degree of uncertainty[4], therefore, to predict the
characteristics of the radio signal propagating in the channel, the radiophysical (effective)
inhomogeneity parameters are also used, which are preliminarily obtained by approximate solution of
the inverse problem from measurements of some characteristics of test signals on the reference tracks
[5]. The radiophysical parameters of the irregularities obtained in this way are also of independent
interest, since they contain integral information on the statistical variability of the ionosphere.
Important results were obtained in this direction due to the introduction of ideas about a Gaussian
correlation ellipsoid that effectively describes random ionospheric irregularities [6] and allows to
significantly simplify the analytical calculations of the statistical moments of the radio signal. In
the general case, the ionosphere is a multiscale medium and it is characterized by a powerlaw spectrum of
irregularities. Nevertheless, the Gaussian spectrum of irregularities with effective parameters can be
used when calculating the lowest moments of phase fluctuations of a radio signal. In particular,
researches [7] showed that, when calculating the phase dispersion of a decameter radio signal in a
multiscale randomly inhomogeneous ionosphere, a Gaussian model of the correlation ellipsoid can be
used if the external scale of ionospheric turbulence specified by a powerlaw spectrum is taken as the
spatial scale of the irregularities. It is connected to the fact that the highfrequency part of the spectrum
of irregularities has a greater effect on the amplitude of the signal and to a lesser effect on its phase
[8]. We consider the case of decameter radio signal propagation in the isotropic ionosphere in this
paper. However, the proposed method for estimating the frequency fluctuations can also be used to
calculate the Doppler frequency shift of individual magnetoionic components of the radio signal in the
anisotropic ionosphere, if we take into account different refractive indices for the ordinary and
extraordinary rays. The calculation method allows the introduction of an anisotropic correlation ellipsoid model of irregularities oriented relative to the radio path [9]. The parameters of this ellipsoid
can be determined from the characteristics of the test signals on the reference paths, taking into
account a priori information about the typical properties of irregularities (for example, their elongation
along the lines of force of the geomagnetic field).

The aim of this work is to create an operational method for calculating the statistical characteristics
of the Doppler frequency shift of a decameter radio signal with singlehop propagation in an unsteady
randomly inhomogeneous ionosphere.

2. Derivation of analytic relations
By definition [10], Doppler frequency shift $\Delta \omega$ of the received signal is the time $\tau$ derivative of its
phase $\psi$

$$
\Delta \omega = -\frac{d\psi}{d\tau}.
$$

(1)

Using the approximation of geometric optics [11]:

$$
\psi = \frac{\omega}{c} \int_0^S \sqrt{\varepsilon(S, \tau)} dS,
$$

(2)

where $\omega$ is frequency, $c$ is speed of light in vacuum, $\varepsilon$ is random function of dielectric constant of
plasma; integration is carried out along an arc $S$, connecting the receiving and emission points. Substituting (2) in (1) and following [11], we have

$$
\Delta \omega = -\frac{\omega}{c} \frac{\partial}{\partial \tau} \int_0^S \sqrt{\varepsilon(S, \tau)} dS = -\frac{\omega}{c} \int_0^S 2\sqrt{\varepsilon(S, \tau)} \frac{\partial \varepsilon(S, \tau)}{\partial \tau} dS.
$$

(3)

We obtain the following expression, taking into account the curvature of the Earth's surface (figure 1) and equation (3):
\[ \Delta \omega = -\frac{\omega}{c} \int_0^\phi \frac{1}{2\sqrt{e(r,\phi,\tau)}} \cdot r(\phi) \cdot \frac{\partial e(S,\tau)}{\partial \tau} \, d\phi, \]  
(4)

where \( r(\phi), \beta(\phi) \) are beam path and angle of refraction, respectively; \( \varphi_k \) is angular coordinate of the radio reception point.

**Figure 1.** Coordinate system

We represent the permittivity function \( \varepsilon \) in the form

\[ \varepsilon(r,\phi,\tau) = \varepsilon_0(r,\phi,\tau) + \varepsilon_1(r,\phi,\tau), \]  
(5)

where \( \varepsilon_0(r,\phi,\tau) \) is a regular function characterizing the average dielectric constant of the ionosphere \( \varepsilon_0 = \langle \varepsilon \rangle \), function \( \varepsilon_1 \) describes random ionospheric irregularities. We find the average Doppler shift \( \Delta \omega_0 \) and frequency fluctuation \( \Delta \omega_1 \) using the perturbation method [12]. We obtain the following expression assuming in (4) \( |\varepsilon_1| << \varepsilon_0 \) and performing asymptotic expansions:

\[ \Delta \omega_0 = -\frac{\omega}{c} \int_0^\phi \frac{1}{2\sqrt{\varepsilon_0}} \cdot r_0(\phi) \cdot \frac{\partial \varepsilon_0}{\partial \tau} \, d\phi, \]  
(6)

\[ \Delta \omega_1 = -\frac{\omega}{c} \int_0^\phi \frac{1}{2\sqrt{\varepsilon_0}} \cdot \frac{r_0(\phi)}{\sin \beta_0(\phi)} \cdot \frac{\partial \varepsilon_1}{\partial \tau} \, d\phi \]  
+ \[ \frac{\omega}{c} \int_0^\phi \frac{\varepsilon_1}{4\sqrt{\varepsilon_0^3}} \cdot \frac{r_0(\phi)}{\sin \beta_0(\phi)} \cdot \frac{\partial \varepsilon_0}{\partial \tau} \, d\phi \].  
(7)

Integration is carried out along the average trajectory with characteristics \( r_0(\phi), \beta_0(\phi) \) in equations (6), (7). These characteristics can be determined by solving the system of ray equations in a plasma given by the average permittivity model \( \varepsilon_0(r,\phi,\tau) \) [11]

\[ \frac{dr_0}{d\phi} = r_0 \cdot c \tan \beta_0, \]  
(8)

\[ \frac{d\beta_0}{d\phi} = \frac{1}{2\varepsilon_0} \left( c \tan \beta_0 \frac{\partial \varepsilon_0}{\partial \phi} - r \frac{\partial \varepsilon_0}{\partial \tau} \right) - 1. \]  
(9)

We consider the motion velocities of chaotic inhomogeneities to be much greater than the velocity of the change in the average dielectric constant of the ionosphere:

\[ \frac{\partial \varepsilon_0}{\partial \tau} \ll \frac{\partial \varepsilon_1}{\partial \tau}. \]  
(10)

We have from (7)
\[
\Delta \omega_i = -\frac{\omega_0^{\phi_i}}{c} \int_0^1 \frac{1}{2\sqrt{\epsilon_0}} \frac{\eta_1(\phi)}{\sin \beta_0(\phi)} \frac{\partial \epsilon_1}{\partial \tau} d\phi.
\]

We use (11) to compile the statistical moment for the dispersion of the Doppler frequency shift of the radio signal:

\[
\sigma_{\omega}^2 = \left\langle (\Delta \omega)^2 \right\rangle = \frac{\omega_0^{\phi_i}}{c} \int_0^1 \frac{1}{2\sqrt{\epsilon_0}} \frac{\eta_1(\phi)}{\sin \beta_0(\phi)} \frac{\partial \epsilon_1}{\partial \tau} d\phi \frac{\omega_0^{\phi_i}}{c} \int_0^1 \frac{1}{2\sqrt{\epsilon_0}} \frac{\eta_2(\phi_2)}{\sin \beta_0(\phi_2)} \frac{\partial \epsilon_1}{\partial \tau_2} d\phi_2
\]

\[
= \frac{\omega_0^{2\phi_i}}{4c^2} \int_0^1 \frac{1}{\sqrt{\epsilon_0(\eta_1, \phi_1)\epsilon_0(\eta_2, \phi_2)}} \frac{\eta_1(\phi_1)\eta_2(\phi_2)}{\sin \beta_0(\phi_1)\sin \beta_0(\phi_2)} \frac{\partial^2 \epsilon_1}{\partial \tau \partial \tau_2} d\phi_1 d\phi_2,
\]

where \( K = \left\langle \epsilon_1(\eta_1, \phi_1, \tau_1) \epsilon_1(\eta_2, \phi_2, \tau_2) \right\rangle \) is spatiotemporal correlation function of the dielectric constant of the ionosphere irregularities, symbol \( \langle \rangle \) means averaging over an ensemble of heterogeneities.

Next, we consider a random field of irregularities that is quasihomogeneous in time and space. In this case, the function \( K \) has the form

\[
K = K_1(\eta + r_2, \phi + \phi_2, \tau + \tau_2) K_0(\eta - r_2, \phi - \phi_2, \tau - \tau_2).
\]

where \( K_0, K_1 \) are the homogeneous and inhomogeneous parts of the correlation function. For simplicity, we consider the vertical motion of chaotic isotropic irregularities and take this motion into account under the hypothesis of the frozen turbulence transfer [8]. The homogeneous part of the correlation function is follow expression:

\[
K_0 = K_0(\eta - r_2 - \nu(\tau_1 - \tau_2), \phi - \phi_2),
\]

where \( \nu \) is radial velocity of a random field of irregularities.

Assuming that the inhomogeneous part of \( K_1 \) changes in time much more slowly than \( K_0 \), the function \( K_1 \) can be taken out of the sign of the second mixed derivative in the equation (12). It is necessary to specify the analytical form of the function \( K_0 \) for further calculation of the integral (12). To describe the fluctuations of the dielectric constant of a randomly inhomogeneous ionosphere, correlation functions are usually used; the inverse Fourier transform of these functions gives a powerlaw spectrum of irregularities with a certain spectral index and parameters of the internal and external turbulence scales [9]. Meanwhile, the estimation of fluctuations of the Doppler frequency shift of the radio signal is also possible using the model of a Gaussian correlation ellipsoid of irregularities with generalized properties. Previously, such a model was effectively used in [6,13] to calculate the statistical moments of the angular characteristics of the radio signal reflected from the ionosphere. Studies [7,14] showed that when calculating the phase dispersion of a signal in a multiscale randomly inhomogeneous ionosphere, a Gaussian model of the correlation ellipsoid can be used, provided that the external scale of the plasma turbulence specified by the powerlaw spectrum is taken as the spatial scale of the irregularities. This possibility is explained by the fact that the highfrequency part of the irregularities spectrum has a greater effect on the amplitude of the radio signal and a lesser effect on its phase characteristics [8,9].

We define the homogeneous part of the correlation function \( K_0 \) in the form of a Gaussian function with the spatial correlation radius \( \rho \), equal to the external turbulence scale of a random field of inhomogeneities. In this case, we obtain following expression by performing the sum-difference integration [8] in equation (12)

\[
\sigma_{\omega}^2 = \frac{\omega_0^{2\phi_i}}{2c^2} \sqrt{\pi} \frac{\nu^2}{R} \frac{K_1(1 + \frac{z_0}{R})^2 \sin \beta_0}{\epsilon_0} dx,
\]

(15)
where $R$ is Earth radius, $z_0 = r_0 - R$, $dx = Rd\phi$; integration is carried out along the earth’s surface.

The following expression is obtained by assuming that the upper limit in the integral (15) is variable and differentiating the integral with respect to this limit

$$
\sigma^2 \omega = \frac{\omega^2 \sqrt{\pi} \sin^2(\varphi_0) \cdot K_1 \cdot (1 + \frac{z_0}{R})^2 \cdot \sin \beta_0}{2c^2 \cdot \rho \cdot \varepsilon_0} \cdot dx.
$$

We obtain a system of differential equations for simultaneously calculating the average trajectory, as well as the average and variance of the Doppler shift of the frequency of the radio signal, using equation (16) together with the system of deterministic ray equations (8), (9) and equation (6), differentiated by a variable upper limit. This system has the following form

$$
\begin{align*}
\frac{dz_0}{dx} &= (1 + \frac{z_0}{R}) \cdot \text{ctg} \beta_0, \\
\frac{d\beta_0}{dx} &= \frac{1}{2\varepsilon_0} \cdot \text{ctg} \beta_0 \cdot \frac{\partial \varepsilon_0}{\partial x} - (1 + \frac{z_0}{R}) \cdot \frac{\partial \varepsilon_0}{\partial x} - \frac{1}{R}, \\
\frac{d\sigma^2}{dx} &= \frac{\omega^2 \sqrt{\pi} \sin^2(\varphi_0) \cdot K_1 \cdot (1 + \frac{z_0}{R})^2 \cdot \sin \beta_0}{2c^2 \cdot \rho \cdot \varepsilon_0}, \\
\frac{d\Delta \alpha_0}{dx} &= \frac{\omega}{2c} \cdot \sqrt{\varepsilon_0} \cdot \sin \beta_0 \cdot \frac{1}{\frac{1}{R}} \cdot \frac{\partial \varepsilon_0}{\partial \alpha_0}. 
\end{align*}
$$

3. Mathematical modelling and discussion of calculation results

Based on system (17), mathematical modeling of the influence of the random plasma irregularities drift on the width of the spectral line during oblique sounding of the ionosphere was carried out. The model of a single-layer deterministic ionosphere was considered:

$$
\varepsilon_0(r, \tau) = 1 - \left(\frac{f_{sp}(\tau)}{f}\right)^2 \cdot \exp \left(-\left(\frac{z - z_m}{y_m}\right)^2\right),
$$

where $f_{sp}(\tau)$ is critical frequency of layer, $z_m, y_m$ are respectively, the height of maximum ionization and half-thickness of this layer, $f = \frac{\omega}{2\pi}$. The nonstationary ionosphere was given by the dependence:

$$
f_{sp}(\tau) = f_{sp0} - \eta \cdot \tau^2,
$$

where $f_{sp0}$ is critical frequency at the initial time $\tau = 0$, $\eta$ is dimensional coefficient. The inhomogeneous part of the correlation function was specified as:

$$
K_1 = \gamma^2 (1 - \varepsilon_0)^2,
$$

where $\gamma^2 = \left(\Delta n_e / n_e\right)^2$ is fluctuation intensity of electron concentration.

With typical parameters of model (18) the propagation of a radio signal between the receiving and transmission sounding points is possible in two ways: the lower and upper modes. The calculation results of the mean $\langle \Delta f \rangle$, and root-mean-square $\sigma = \sqrt{\langle \Delta f \rangle^2} = \frac{1}{2\pi} \sqrt{\langle \Delta n_e \rangle^2}$, Doppler frequency shifts of these modes are shown in figure 2, when the distance between the points of reception and transmission was $x = 1500$ km. The parameters of the determined ionosphere were: $z_m = 300$ km, $y_m = 100$ km, $f_{sp0} = 6.5$ MHz, $f = 13$ MHz, external turbulence scale $\rho = 10$ km, radial velocity irregularities motion $\nu = 100$ m/s, relative perturbation of electron concentration $\Delta n_e / n_e = 1.25\%$. The average values of the Doppler frequency shift $\langle \Delta f \rangle$ of both modes strongly
depend on the change rate of the ionosphere critical frequency, this follows from figure 2. Meanwhile, the main influence on the behavior of $\langle \Delta f \rangle$ is exerted by a change in the length of the trajectories of the lower and upper modes due to temporal variations of ionospheric parameters. A decrease in the critical frequency of the ionosphere leads to a decrease in the path length of the upper mode and an increase in the path of the lower mode. As a result, the average values $\langle \Delta f \rangle$ of both modes can be opposite in sign, which is an important property in their identification. The standard deviations of the Doppler frequency shift of the lower and upper modes increase with increasing $\Delta n_e / n_e$ and speed of chaotic irregularities. The root-mean-square deviations of the Doppler frequency shift of the lower and upper modes increase with increasing speed of the movement of chaotic inhomogeneities (see equations (15), (20)). Over time (decrease), the value $\sigma_f$ decreases for the upper rays and increases for the lower ones over time (with decreasing $f_{sp}$). Such changes of $\sigma_f$ are associated with an increase in signal frequency fluctuations with increasing ray paths in a randomly inhomogeneous ionosphere. The size of random irregularities affects the dispersion of the Doppler frequency shift, so that the value of $\sigma_f^2$ is inversely proportional to the external scale of turbulence $\rho$. Therefore, a decrease in this scale leads to an increase in $\sigma_f^2$.

Figure 2. Statistical characteristics of the Doppler frequency shift of a radio signal in a single-layer ionosphere

The case of propagation of a decameter radio signal in a horizontally inhomogeneous ionosphere specified by a two-layer dielectric permittivity model was further considered. Layering violation is introduced using the model of horizontal large-scale heterogeneity. These changes were made to the dependence in the expression (18). As a result, a more general regular model of the ionosphere has the form:

$$
e_0 = 1 - \frac{f_{sp}^2}{f^2} \exp \left( - \frac{\left( \frac{z - z_{mE}}{y_{mE}} \right)^2}{2} \right) - \frac{f_{sp}^2}{f^2} \exp \left( - \frac{\left( \frac{z - z_m}{y_m} \right)^2}{2} \right) \left[ 1 + \kappa \exp \left( - \left( \frac{x - x_L}{L} \right)^2 \right) \right], \quad (21)$$
where \( z_{mE}, y_{mE}, f_{xkE} \) is ionization maximum height, half thickness and critical frequency of the lower layer; parameters \( L, x_L, \kappa \) are respectively, the horizontal scale, the center coordinates and intensity of large-scale inhomogeneity. Figure 3 shows the of calculations results of the mean and standard deviation of the Doppler frequency shift of the radio signal in a horizontally inhomogeneous ionosphere (21). The function \( \gamma = -\chi \exp \left( -\frac{(z - z_m)^2}{y_m} - \frac{(x - x_L)^2}{L} \right) \) was given by the dependence:

\[
\gamma = -\chi \exp \left( -\frac{(z - z_m)^2}{y_m} - \frac{(x - x_L)^2}{L} \right),
\]

where \( \chi \) is parameter characterizing the maximum fluctuations of the electron concentration in the center of large-scale irregularity.

The usage of models (21) and (22) allows us to consider the different structure of a randomly inhomogeneous ionosphere. In particular, with \( \kappa = 0 \), but \( \chi \neq 0 \) we have the case of asymmetry of the random component of the ionosphere, if \( \kappa \neq 0, \chi \neq 0 \), then the case of asymmetry of both the middle ionosphere and the volume, where random inhomogeneities are concentrated, is realized. Calculations were performed for typical parameter values: \( z_m = 300 \text{ km}, z_{mE} = 125 \text{ km}, y_m = 100 \text{ km}, y_{mE} = 25 \text{ km}, L = 500 \text{ km} \). It follows from figure 3 that in the case of a spherical symmetric deterministic ionosphere (model (21) with \( \kappa = 0 \)) over time (decreasing the critical frequency of the upper layer) the average value of \( \langle \Delta f \rangle \) and the standard deviation or the upper and lower modes change nonlinearly, and the values \( \langle \Delta f \rangle \) and \( \sigma_f \) for the upper rays are significantly larger than for the lower rays. The standard deviation \( \sigma_f \) increases with increasing fluctuations in the electron concentration \( \chi \). The absolute value of \( \sigma_f \) depends on the length of the propagation path of the radio signal modes. The value of \( \sigma_f \) increases with increasing path length in the ionosphere. Calculations showed that in the presence of spherical asymmetry of both the deterministic and random components of the ionosphere (\( \kappa \neq 0, \chi \neq 0 \)), there is a significant increase in values \( \langle \Delta f \rangle \) and \( \sigma_f \) with increase \( f_{xk} \) and \( x_k \). For example, the value \( \sigma_f = 0,15 \text{ Hz} \) under the following conditions \( x_k = 3000 \text{ km}, \tau = 0 \) and \( \kappa = 0 \) for the upper rays, (figure 3 A), but with \( \kappa = 1 \) the value \( \sigma_f = 0,24 \text{ Hz} \) (figure 3 B). The values of \( \sigma_f \) and \( \langle \Delta f \rangle \) are rising with increasing in \( x_k, \kappa \) and \( \chi \).
Figure 3. Statistical characteristics of the Doppler frequency shift of a radiosignal in a horizontally inhomogeneous two-layer ionosphere

4. Conclusion
A mathematical apparatus has been developed for calculating the statistical characteristics of the operational Doppler decametre radio signal during propagation in the single-hop non-stationary random inhomogeneous ionosphere. The basis of the calculation method is a system of ordinary differential equations obtained in the approximation of geometric optics and the perturbation method. The proposed apparatus makes it possible to calculate the statistical characteristics of the Doppler frequency shift of a radio signal with oblique propagation and to evaluate the role of horizontal variability of the ionospheric plasma. The mean value and standard deviation of the Doppler frequency shift of the radio signal in a layered and horizontally inhomogeneous ionosphere with random irregularities are calculated. Numerical calculations showed that the mean value $\langle \Delta f \rangle$ Doppler frequency shift standard deviation $\sigma_f$ of the lower and upper modes of the radio signal depend on different conditions such as: the type of ray paths, the critical frequency $f_{cr}$, the distance of the radio path $x_k$, the intensities of deterministic $\kappa$ and random $\chi$ irregularities. The values of $\sigma_f$ and $\langle \Delta f \rangle$ are rising with increasing in $x_k$, $\kappa$ and $\chi$. The characteristic values of the Doppler frequency shift of the lower and upper modes obtained for different conditions can be used to predict the reliability of radio communications, for precision measurement tasks in the decameter range, to calculate and predict the propagation characteristics of decameter radio signals through the ionosphere, and also to increase the resolution of the methods Doppler filtering of ionospheric radio signals.

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