Turbulent pipe flow: Statistics, $Re$-dependence, structures and similarities with channel and boundary layer flows

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Abstract. Direct numerical simulation data of fully developed turbulent pipe flow are extensively compared with those of turbulent channel flow and zero-pressure-gradient boundary layer flow for $Re_\tau$ up to 1000. In the near-wall region, a high degree of similarity is observed in the three flow cases in terms of one-point statistics, probability density functions of the wall-shear stress and pressure, spectra, Reynolds stress budgets and advection velocity of the turbulent structures. This supports the notion that the near-wall region is universal for pipe and channel flow. Probability density functions of the wall shear stress, streamwise turbulence intensities, one-dimensional spanwise/azimuthal spectra of the streamwise velocity and Reynolds-stress budgets are very similar near the wall in the three flow cases, suggesting that the three canonical wall-bounded flows share many features. In the wake region, the mean streamwise velocity and Reynolds stress budgets show some expected differences.

1. Introduction

The study of the flow of a viscous fluid along the surface of a solid body and its interaction with the wall, is among the most fundamental problems in fluid mechanics. In wall-bounded flows, there are three simple geometrical configurations which are referred to as canonical flows: the spatially evolving boundary layer, the channel and the pipe. The difference between these cases comes not only from the presence of curvature at the wall in pipe flow, but also from the spatial development of boundary layer flow and the mutual interaction between the internal viscous region and the external inviscid flow leading to intermittency effects in the latter case. Moreover, in the outer region the large turbulence scales are strongly confined in pipe flow as opposed to channel and boundary layer flow.

Some of the earliest research in the field of turbulent wall-bounded flows was purely theoretical, and a key point was the pioneering work of Prandtl who introduced concept of boundary layers in 1904. Another major breakthrough was the formulation of the law of the wall by von Kármán in the 1930s. According to the law of the wall, the wall region is divided into a viscous sublayer attaining a linear profile, a logarithmic-law region with a buffer layer in between, and finally an outer layer. Ever since the formulation of the preceding theories, there has been a debate about, on the one hand, the universality of turbulence and, on the other hand, the differences between the canonical wall-bounded flows. At the early stages, some
of the controversies were about the effect of curvature in pipe flow on the near-wall region. Millikan (1938) derived the logarithmic law of the wall for pipe flow under the assumptions that the velocities close to the wall and in the core region are independent of the radius of curvature and viscosity, respectively. One of the first direct experimental comparisons between pipe and channel flows dates to Patel and Head (1969) who reported discrepancies in the mean velocity profile between the two cases and attributed them to the difference in geometrical configuration. Several years later, Huffman and Bradshaw (1972) examined the von Kármán constant by analysing data for pipe and channel flow, and stated that the transverse curvature in pipe flow may affect the viscous sublayer but not the rest of the inner layer.

It is hardly surprising that a clear understanding of the physical processes in wall-bounded flows could not be converted into a universal model for canonical flows for more than half a century. A point of dispute, which appeared from research on wall-bounded canonical flows, is the maximum of the streamwise/axial turbulence intensities. Channel flow and boundary layers show an increase in magnitude with increasing $Re$ (Marusic et al., 2010). This is attributed to the rising influence of the outer layer as higher Reynolds numbers are approached. However, the “Superpipe” experimental investigations by Zagarola and Smits (1997) indicated that this peak holds a constant value with increasing $Re$. There remain many other controversies in this context as, for instance, the scaling laws and the presence of a logarithmic law or a power law for pipe, channel and boundary layer flows; see e.g. Zagarola and Smits (1998), McKeon et al. (2004) and George (2007). Kim (2012) reviewed the progress in pipe and channel-flow turbulence research over a fifty years period, and highlighted many of the still remaining questions. Other recent reviews of wall-bounded flows are given by Marusic et al. (2010); Smits et al. (2011); Jiménez (2012).

The first comparison between pipe and channel flows from a direct numerical simulation (DNS) study, came to light with the work of Eggels et al. (1994) who reported on low- and high-order statistics. At a friction Reynolds number of 180, Eggels et al. (1994) stated that the mean velocity profile in pipes does not follow the law of the wall, in contrast to channels. In addition, the skewness factor of the wall-normal velocity showed differences while other statistical quantities, including turbulence intensities and energy budgets, showed insignificant differences. Jiménez et al. (2010) conducted comparisons between turbulent boundary layers and channels at moderate Reynolds numbers to study the behaviour of the velocity and pressure fluctuations in the outer layer. On the experimental side, Monty et al. (2007) investigated the presence of large-scale structures in turbulent pipe and channel flows, and compared it with boundary layers. In this work, the spanwise two-point correlations showed that the large-scale structures are significantly wider in pipe and channel flows than in boundary layer flow. Another profound difference, reported in the same work, was about the rate of growth of these structures with increasing wall distance. It was noted that they grow at a slower rate with wall distance in boundary layer flow than in pipe and channel flows. In a subsequent attempt to clarify more in detail the extent of similarity between the three canonical flow cases, Monty et al. (2009) conducted streamwise velocity measurements on pipe, channel and boundary layer flows at the same Reynolds number and with the same measurement resolution. The key point in their findings was that the mean statistics, in terms of mean velocity profile and streamwise/axial turbulence intensity, are similar in the near-wall region in the three cases, and the one-dimensional premultiplied spectra yielded identical results between pipe and channel flows throughout the flow, but was still different from that in boundary layers even close to the wall. Sillero et al. (2013) compared one-point statistics of boundary layer DNSs at friction Reynolds numbers up to 2 000 with experimental and DNS data of pipe and channel flow. They found that the three normal Reynolds stresses are higher in boundary layers than in pipe and channel flow in the outer layer. These differences are independent of the Reynolds number, and were also observed by Lee and Sung (2013).
The present study aims, through highly accurate well-resolved DNS data, to concretely prove for the first time that pipe, channel and turbulent boundary layer flows are practically indistinguishable in the near-wall region. In particular, we focus our attention on the near wall region up to a wall-normal position of 100 wall units, highlighting the similarities and differences, if any, between these canonical wall-bounded flows using various statistical tools, including probability density functions, one- and two-dimensional spectral analysis and energy budgets, as well as advection velocities of the turbulent structures.

2. Data sets and numerical methods
For the pipe flow simulations in this study, we consider a straight circular pipe of radius $R = 1$ and length $25R$, with the pipe axis taken along the axial $z$ direction. The incompressible Navier–Stokes equations are solved using the well-documented DNS solver nek5000 (Fischer et al., 2008). Nek5000 is a spectral element code in which the Navier–Stokes equations are solved using a Legendre polynomial based spectral element method (SEM), and are advanced in time with a semi-implicit scheme. The radial, azimuthal and axial grid spacing, measured in wall units, is set such that $\Delta r_{\text{max}}^+ \leq 5$ and $\Delta R\theta_{\text{max}}^+ \leq 5$ and $\Delta z_{\text{max}}^+ \leq 10$, respectively. A more detailed description, on the computational meshes and the usage of nek5000 in the current pipe DNSs, can be found in El Khoury et al. (2013).

The other data sets that we consider are from well-resolved DNS of channel flow (del Álamo et al., 2004; Lenaers et al., 2012) and zero-pressure gradient boundary layer flow (Schlatter and Örlü, 2010). All these simulations were carried out with spectral codes employing Chebyshev polynomials in the wall-normal direction.

The friction Reynolds number for pipe flow is defined as $Re_{\tau} = u_{\tau} R / \nu$, where $u_{\tau}$ is the friction velocity and $\nu$ is the kinematic viscosity, whereas for channel and boundary flows layer $Re_{\tau}$ is based on the channel half-width and the 99% boundary layer thickness, respectively. From now on, $r$ and $y$ refer to the wall-normal direction, $\theta$ to the azimuthal or spanwise direction and $z$ to the axial or streamwise direction.

3. Results
3.1. Axial/streamwise velocity statistics and visualisations
Figure 1 shows visualisations of the instantaneous axial or streamwise velocity field in a cross-stream plane of the pipe, channel and boundary layer DNSs at $Re_{\tau} \approx 1000$. We can observe the small-scale near-wall structures corresponding to the streamwise low- and high-speed streaks (Kline et al., 1967) as well as the large-scale turbulent structures in the outer regions. In particular, the visualisations illustrate the geometrical confinement of these large scales in the centre region of a pipe, while in channels and boundary layers there is no geometrical confinement in the spanwise direction. However, one should note that the periodic boundary conditions puts an upper limit on the size of the structures.

Figure 2(a) presents the mean axial or streamwise velocity profile obtained from the pipe, channel and boundary layer DNSs at $Re_{\tau} \approx 1000$. The mean velocity profiles in terms of wall units are very similar in the near-wall region and logarithmic-layer region but differ in the wake region. Here the boundary layer has the highest and the channel flow the lowest mean velocity. Monty et al. (2009) observed the same differences in the wake region in experiments of all three flow cases at a higher Reynolds number. The difference between pipe/channel flow and boundary layers is likely related to the intermittency caused by entrainment of irrotational fluid in the latter case (Jiménez et al., 2010), but there is no clear explanation for the difference between pipe and channel flow in the wake region.

Figure 2(b) shows the maximum of the root-mean-squared (rms) streamwise velocity fluctuations normalized by $u_{\tau}, u_{\tau,\text{rms}}$, found in the buffer layer for pipe, channel and boundary layer flow. The estimated confidence intervals for our pipe flow DNS results and several other
Figure 1. Visualisations of the instantaneous streamwise velocity field in a cross-stream plane in pipe (top figure), channel (middle figure) and boundary layer (bottom figure) at $Re_\tau \approx 1000$. The visualised spanwise extent of the channel flow plane is $3.85h$, where $h$ is the channel half-width. The spanwise and wall-normal (vertical) extent of the boundary layer plane are $3.85\delta$ and $2\delta$, respectively, where $\delta$ is the $99\%$ boundary layer thickness. A lighter colour corresponds to a higher velocity.
Figure 2. (a) Axial/streamwise mean velocity profiles in wall units in pipe (-----), channel (- - - - -) and boundary layer flow (-----) at $Re_\tau = 1000$. Dashed lines indicate the linear and logarithm behaviours. (b) Maximum of $u_{z,rms}^+$ as function of $Re_\tau$. Pipe flow: ⋆, El Khoury et al. (2013); ⋆, Chin et al. (2010), Klewicki et al. (2012); ⋆, Boersma (2011); ⋆, Wu and Moin (2008), Wu et al. (2012); ⋆, Wagner et al. (2001); ⋆, Eggels et al. (1994). Channel flow: ■, Lenaers et al. (2012); ■, del Álamo et al. (2004). Boundary layer flow: —, Schlatter and Örlü (2010). The error bars indicate the statistical uncertainty in our pipe flow DNS results.

DNS data of pipe and channel flow are included in the figure as well. We can observe a similar increase of the maximum of $u_{z,rms}^+$ with $Re_\tau$ in all three cases in our DNS data, in agreement with the observations by Sillero et al. (2013). We also note that there is no statistically significant difference in the maximum $u_{z,rms}^+$ in the three flow cases at $Re_\tau = 550$ and 1000 according to our DNS. However, the pipe flow DNS results of Chin et al. (2010); Klewicki et al. (2012); Wu and Moin (2008); Wu et al. (2012) appear to have a bit lower $u_{z,rms}^+$-maximum than channel flow and the boundary layer. The different pipe-flow DNS data show a significant scatter at low $Re_\tau$, which could be caused by insufficient resolutions or statistical errors related to short sampling times in some of the simulations.

3.2. Wall region

We compare the canonical wall-bounded flows by looking at the wall region in more detail. The probability density function of the wall-shear stress PDF($\tau_{z}^+$) is shown in figure 3 for pipe, channel and boundary-layer flows for a wide range of Reynolds numbers. Such functions highlight the occurrences associated with instantaneous fields, and the importance of looking at the wall-shear stress stems from its direct connection to the drag of an immersed body. The characteristic behaviour of PDF($\tau_{z}^+$) is observed to be the same between the canonical flows. At approximately the same Reynolds number, the probability density functions of $\tau_{z}^+$ collapse well, irrespective of the geometrical configuration, and indicate the presence of negative wall-shear stresses. This negative wall-shear stress is associated with negative axial/streamwise velocities, and is observed to occur more frequently and to increase in magnitude with increasing Reynolds number, as evidenced by the widening of the tail of PDF($\tau_{z}^+$). In this context, Lenaers et al. (2012) investigated in detail the presence of negative streamwise velocities in channel flow up to $Re_\tau = 1000$, and found that the widening of the negative tail of PDF($\tau_{z}^+$) with $Re$ was solely due to the rise in the number of regions of backflow which descended from a strong oblique vortex outside the viscous sublayer. Very large positive values of $\tau_{z}^+$ are also seen. These are likely caused by structures coming from outside the buffer layer that reach down into the viscous sublayer, see also figure 3 in Lenaers et al. (2012).
Figure 3. Probability density function (PDF) of wall-shear stress in wall units. Pipe by El Khoury et al. (2013); channel by Lenaers et al. (2012); boundary layer by Schlatter and Örlü (2010). (a) PDFs of pipe flows at \(Re_\tau = 180\), 550, 1000 and of boundary layers at \(Re_\tau = 400, 880, 1200\). The arrow indicates increasing \(Re_\tau\). (b) PDFs of pipe: \(Re_\tau = 1000\); channel: \(Re_\tau = 1000\); boundary layer: \(Re_\tau = 1200\).

Figure 4. Pipe and channel flows at \(Re_\tau = 1000\). (a) Joint probability density function of wall-shear stresses in the axial/streamwise (\(\tau^+_z\)) and azimuthal/spanwise (\(\tau^+_\theta\)) directions. (b) 2D-premultiplied spectra of wall-shear stress \(k_{(\tau^+_\theta)}\Phi_{(\tau^+_z)}(u^+_2/\nu)^2\). Pipe: (El Khoury et al., 2013); channel: (Lenaers et al., 2012).

The axial/streamwise wall-shear stress (\(\tau^+_z\)) is generated by means of azimuthal vorticity impinging on the wall. There exist another wall-shear stress component linked with axial vorticity and that is the azimuthal wall-shear stress (\(\tau^+_\theta\)). The joint PDFs of these two wall-shear stresses, shown in figure 4(a) for pipe and channel flows at \(Re_\tau = 1000\), are practically indistinguishable as well as the two-dimensional premultiplied spectra of \(\tau^+_z\) for these two cases, see figure 4(b). Note that the imprint of large-scale turbulent structures on the wall shear stress in pipe and channel flow is visible in these spectra. A similar large-scale imprint near the wall was also observed in two-dimensional spectra of the streamwise velocity and enstrophy in channel flow at \(Re_\tau = 2000\) by Hoyas and Jiménez (2008). The PDF of the pressure at the wall for pipe flow at \(Re_\tau = 180\), 550 and 1000 and channel flow at \(Re_\tau = 1000\) are shown in figure 5. The intermittency of the pressure at the wall in pipe flow becomes stronger with increasing \(Re_\tau\), similar to the wall shear stress. Also in this case, the PDFs of pipe and channel flow are extremely similar.
El Khoury et al. (2013) considered the development of the wall pressure fluctuations and wall-shear stresses with the Reynolds number for pipe, channel and boundary layer flows. In their results, there was a clear Reynolds-number dependence of $p_{w,rms}^+$, $\tau_{z,rms}^+$ and $\tau_{\theta,rms}^+$ for the three flow cases, being similar for pipe and channel flows, but lower than those in boundary layers. While the discrepancy in pressure fluctuations remained the same for boundary layer and pipe/channel flows as the Reynolds number increased, it was observed that the discrepancies in wall-shear stresses for the canonical flows tend to diminish with increasing $Re$. Therefore, it can be expected $\tau_{z,rms}^+$ and $\tau_{\theta,rms}^+$ would be the same for pipe, channel and boundary layer flows at sufficiently high $Re$. The higher pressure fluctuations in boundary layer flow, on the other hand, are linked to the intermittency of the potential and rotational flow regions for this configuration (Jiménez et al., 2010), and the difference to channels is thought not to disappear with Reynolds number.

3.3. Buffer layer

The buffer layer separates the viscous sublayer from the logarithmic-law region and sustains the presence of streaks responsible for the peak in axial/streamwise turbulence intensity and turbulence production. This layer extends between wall normal positions $y^+ = 5$ and $y^+ = 30$. In figure 6, the two-dimensional premultiplied spectra of the axial velocity and the co-spectra of Reynolds shear stress are shown for pipe and channel flows at $(1-r)^+ = 15$. There are no significant differences in the premultiplied spectra between the two flow cases. An inner peak is present in the two panels at axial/streamwise and azimuthal/spanwise wavelength of $\lambda_z^+ = 1000$ and $\lambda_\theta^+ = 100$, respectively, which corresponds to the spacing of the above mentioned streaks (Kline et al., 1967). Meanwhile, an outer peak is observed at axial/streamwise and spanwise/azimuthal wavelengths of $10000^+$ and $1000^+$, respectively. This peak is the sign of the large-scale motions embedded in the logarithmic layer and is also visible in the two-dimensional premultiplied spectra of wall-shear stress in figure 4. El Khoury et al. (2013) reported that the inner peak of pressure fluctuations shows negligible differences between pipe and channel flows but is clearly lower than for the boundary layer. The outer peak is noticeably absent in the two-dimensional co-spectrum of the Reynolds shear stress (figure 6(b)) implying that the large-scale structures observed in figure 6(a) contribute little to momentum transfer and are in that sense inactive (Hoyas and Jiménez, 2008).
3.4. Lower edge of the logarithmic-law region

In a similar fashion as in figure 6, the two-dimensional premultiplied spectra are presented in figure 7 at a wall-normal position of \((1 - r)^+ = 100\). Here, it can be readily observed that the axial/streamwise location of the inner peak is not affected by the wall-normal displacement, whereas the spanwise wavelength has tripled to \(\lambda_{r\theta}^+ = 300\). This indicates that the width of the structures increases while moving away from the wall, and that the rate of increase is similar between pipe and channel flows. Another interesting feature, that is observed in the axial/streamwise velocity field is the presence of large-scale structures in both flows at the same axial and azimuthal wavelengths. Measured in wall units, the secondary peak is observed at \(\lambda_z^+ = 10000\) and \(\lambda_{r\theta}^+ = 1000\). This corresponds to a structure that is approximately 10 radii or half channel widths long in the streamwise direction and one radius/channel half-width wide in the spanwise one.

3.5. Overview of wall regions

The one-dimensional premultiplied spectrum of the axial velocity component is shown in figure 8(a) for pipe, channel and boundary layer flows as a function of azimuthal/spanwise wavelength. An interesting observation in this case is that the canonical wall-bounded flows agree
Figure 8. One-dimensional premultiplied spectra in pipe, channel and boundary layer flows. (a, b) As a function of azimuthal/spanwise wavelength $\lambda_{(r\theta)}^+$; (c, d) As a function of axial/streamwise wavelength $\lambda_z^+$. (a) Axial/streamwise velocity, $k_{(r\theta)}\Phi_{(u_z u_z)}/u_\tau^2$. (b) Co-spectra, $k_{(r\theta)}\Phi_{(u_z u_r)}/u_\tau^2$. (c) Axial/streamwise velocity, $k_z\Phi_{(u_z u_z)}/u_\tau^2$. (d) Co-spectra, $k_z\Phi_{(u_z u_r)}/u_\tau^2$. Pipe: (thick line), El Khoury et al. (2013), $Re_\tau = 1000$; channel: red, del Álamo et al. (2004), $Re_\tau = 950$; boundary layer: blue (grey), Schlatter and Örlü (2010), $Re_\tau = 1000$.

very well up to $(1 - r)^+ = 100$ for azimuthal wavelengths that are less than 300 viscous units. Just above the logarithmic layer and at a wall position of $200^+$, a pronounced secondary peak, caused by large-scale structures, is observed for the three flow cases at an azimuthal/spanwise spacing of about one boundary-layer thickness or one pipe radius. For $Re_\tau = 1000$, this corresponds to a wavelength of 1000 wall units. In contrast to channel and boundary layer flows, where the spanwise length is constant, the circumferential length of a pipe decreases as the pipe centre is approached. This circumferential length is $5R$ at the position of the secondary peak, $(1 - r)^+ = 200$, indicating the presence of five large-scale structures. Meanwhile, the co-spectrum for pipe and channel flows in figure 8(b) shows negligible differences for the two flow cases. While the first two panels display the energy as a function of azimuthal/spanwise wavelength, figures 8(c, d) show the energy content as a function of the axial/streamwise lengths. This helps us to look at the large-scale structures and quantify the differences associated in this direction. The axial/streamwise component give no clear discrepancies between pipe and channel flows, whereas the co-spectrum of the Reynolds shear stress in the outer layer indicates more momentum transfer at longer wavelengths for channel flow when compared to pipe flow.
3.6. Advection velocity

Several methods of different complexity have previously been used by researchers in order to calculate the advection velocity of the turbulent structures in channel and boundary layer flows. For the present pipe flow simulations, we calculate the advection velocity of the large-scale motions using the method introduced by Del Álamo and Jiménez (2009). In their work, Del Álamo and Jiménez (2009) defined the average velocity of each mode for the streamwise velocity component as

\[ c_u(k_x, k_z, y) = -\frac{\langle \hat{u}^* \partial_t \hat{u} \rangle}{k_x \langle \hat{u} \hat{u}^* \rangle} = -\frac{\text{Im} \langle \hat{u}^* \partial_t \hat{u} \rangle}{k_x \langle \hat{u} \hat{u}^* \rangle}, \tag{1} \]

where \( \hat{u}(k_x, k_z, y, t) = |\hat{u}(k_x, k_z, y, t)| \exp[i\psi_u(k_x, k_z, y, t)] \) with \( k_x, k_z \) being the streamwise and spanwise wavenumbers, respectively. Since the contribution to the streamwise velocity component, in a turbulent flow field, can be a combination of a range of wavenumbers, Del Álamo and Jiménez (2009) proposed a more natural definition for the advection velocity by taking into consideration a collection of modes \( \Omega \), and the advection velocity associated with \( \Omega \) becomes

\[ C_u(y) = \frac{\int_{\Omega} c_u(k_x, k_z, y) |\hat{u}(k_x, k_z, y)|^2 k_x^2 dk_x dk_z}{\int_{\Omega} |\hat{u}(k_x, k_z, y)|^2 k_x^2 dk_x dk_z}. \tag{2} \]

The same approach can be used for pipe flows where the streamwise and spanwise wavenumbers are replaced by the axial and azimuthal ones, respectively.

In figure 9(a) and (b) we show the axial advection velocity of the turbulent structures including all Fourier modes and only including the larger scales, respectively, and the mean velocity profile in pipe flow at \( Re_r = 1000 \) together with the streamwise advection velocity of the turbulent structures in channel flow at \( Re_r = 950 \) computed by Del Álamo and Jiménez (2009). In both cases the advection velocities are computed using equation (1) and (2). The advection velocity is more uniform, i.e. higher near the wall and a bit lower in the wake region, when the small turbulent scales are excluded. Figure 9 also shows that the advection velocities are very similar in pipe and channel flow in the near-wall region, while in the wake region they are somewhat higher in pipe flow. This difference can be explained by the higher mean velocity in the wake region in the latter case.

![Figure 9. Advection velocity in pipe and channel flows at \( Re_r = 1000 \). (a) All Fourier modes. (b) Fourier modes with \( \lambda_z > 2R \) (axial/streamwise) and \( \lambda_{\theta} > 0.4R \) (azimuthal/spanwise). Pipe: mean velocity profile \( U^+_z \), advection velocity \( U^+_c \), (El Khoury et al., 2013). Channel: , advection velocity \( U^+_c \), (Del Álamo and Jiménez, 2009). \( U^+_z = (1 - r)^+ \) and \( U^+_c = \kappa^{-1} \ln(1 - r)^+ + B \) with \( \kappa = 0.41 \) and \( B = 5.2 \) are given as dashed lines.](image-url)
3.7. Reynolds-stress budget

The transport equation for the Reynolds stress tensor is given by

\[
\frac{D}{Dt} \langle u_i u_j \rangle = P_{ij} + \varepsilon_{ij} + \Pi_{ij} + D_{ij} + T_{ij},
\]

where the production, dissipation, pressure strain, divergence of pressure-velocity correlation, molecular diffusion and turbulent diffusion are defined as, respectively,

\[
P_{ij} = -\langle u_i u_k \rangle \frac{\partial U_j}{\partial x_k} - \langle u_j u_k \rangle \frac{\partial U_i}{\partial x_k}
\]

(4a)

\[
\varepsilon_{ij} = -2\nu \left( \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right)
\]

(4b)

\[
\Pi_{ij} = \frac{1}{\rho} \left( p u_i \frac{\partial u_j}{\partial x_j} + p u_j \frac{\partial u_i}{\partial x_i} \right)
\]

(4c)

\[
G_{ij} = -\frac{1}{\rho} \left( \frac{\partial}{\partial x_i} \langle p u_j \rangle + \frac{\partial}{\partial x_j} \langle p u_i \rangle \right)
\]

(4d)

\[
D_{ij} = \nu \frac{\partial^2}{\partial x_k^2} \langle u_i u_j \rangle
\]

(4e)

\[
T_{ij} = -\frac{\partial}{\partial x_k} \langle u_i u_j u_k \rangle.
\]

(4f)

The budget and premultiplied budget terms for the axial/streamwise stress and the Reynolds shear stress are shown in figures 10 and 11 for pipe, channel and boundary layer flows at \(Re_\tau = 1000\). Reynolds stress budgets for channel flow at \(Re_\tau = 180\) and 2000 were presented by Mansour et al. (1988) and Hoyas and Jiménez (2008), respectively. It can be readily observed that the various budget terms (non-premultiplied) are almost indistinguishable for pipe, channel and boundary layer flows apart from the magnitude of the production and viscous dissipation of the Reynolds shear stress, which are slightly larger in the boundary layer than in pipe and channel flow in the buffer layer. The budget for the axial/streamwise stress is dominated by production and viscous dissipation with the expected large peak of positive production in the buffer layer. The negative diffusion terms in this region imply an extraction and transport of energy away from the buffer region, whereas the negative pressure-strain serves to distribute energy from the normal stress to the wall-normal and azimuthal/spanwise stresses. The most important terms in the Reynolds shear stress budget are production and pressure-strain (figure 11a). The production term, in this case, is given by \(-\langle uu_r \rangle dU_z/dr\), \(-\langle vv \rangle dU/\partial y\) and \(-\langle uu \rangle \partial V/\partial x - \langle vv \rangle \partial U/\partial y\) for pipe, channel and boundary layer flows, respectively. The fact that the wall-normal Reynolds stress is identical between pipe and channel flow up to \((1-r)^+ = 200\) leads to the same production in these two flow cases, whereas the higher wall-normal stress \(\langle vv \rangle\) in boundary layer flow is responsible for the higher production observed here. Note that \(\langle uu \rangle \partial V/\partial x\) is negligible compared to \(\langle vv \rangle \partial U/\partial y\) in a boundary layer.

The terms of the Reynolds-stress budget in the outer layer can be highlighted through premultiplication and the usage of logarithmic abscissa. The apparent area below the curves between two wall-normal positions directly corresponds to the integral of the various budget terms when plotted in this way. The premultiplication for channels and boundary layer flows only involves the inner-scale wall distance \(y^+=\) whereas for pipe flow, the premultiplication contains two terms which are related to the integration along a logarithmic abscissa, \((1-r)^+\), and to the circular geometry, \(r\). The effect of premultiplication can be seen in the right panels of figures 10 and 11. In the wake region and for the streamwise/axial stress, the premultiplied production, viscous dissipation and pressure strain have the largest and smallest magnitude in
boundary layer and pipe flow, respectively. The pressure strain term in this case has a larger magnitude than the viscous dissipation. For the Reynolds shear stress, on the other hand, the most dominant terms are still production and pressure-strain, which tend to balance each other in the wake region and approach zero towards the centre (or towards the boundary-layer edge).

4. Concluding remarks

Single-point statistics, PDFs of the wall shear stress and pressure, spectra, advection velocities of the turbulent structures and Reynolds stress budgets obtained from our DNS data of turbulent pipe and channel flow up to $Re_\tau = 1000$ reveal a strong similarity up to at least $y^+ = 100$. The results strongly support the idea that the flow and dynamics, structure and even intermittency of
the turbulence have a high degree of universality in the near wall region in these two flow cases. Results on the turbulence intensity and spanwise spectrum of the streamwise velocity, the PDF of the wall shear stress and the Reynolds stress budgets obtained from DNS of zero-pressure-gradient turbulent boundary layer at $Re_\tau = 1000$ are also very similar to the corresponding results for pipe and channel flow, showing that many features of the three canonical wall-bounded flows are alike. The observed similarity appears consistent with Monty et al. (2009) who reported for experiments a similar correspondence in the near-wall statistics of the three flow cases. Only the largest structures in pipe and channel flow differ from the ones in boundary layer flow in their experiments. Further DNS and experimental studies should indicate if the universality of the near-wall region also pertains to higher Reynolds numbers, but there is no obvious reason why this should not be the case.

In the wake region, our DNS data reveal differences in the mean streamwise velocity and the Reynolds stress budgets between the three flow cases. A comprehensive comparison of one-statistics of pipe, channel and boundary layer data by Sillero et al. (2013) showed that also the Reynolds stresses are different in the outer region. Some of the differences are related to the intermittency and entrainment of irrotational fluid in the outer part of a boundary layer and the absence of this intermittency in pipe and channel flow. However, further studies are needed to find the reasons for the differences observed in pipe and channel-flow statistics, i.e. the flows that are not affected by intermittency.

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