Axial Anomaly and Transition Form Factors

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Abstract

We investigate the properties of the amplitude induced by the anomaly. In a relatively high energy region those amplitudes are constructed by the vector meson poles and the anomaly terms, in which the anomaly terms can be essentially evaluated by the triangle quark graph. We pay our attention to the anomaly term and make intensive analysis of the existing experimental data, i.e., the electromagnetic $\pi^0$ and $\omega$ transition form factors. Our result shows that it is essential to use the constituent quark mass instead of the current quark mass in evaluating the anomaly term from the triangle graph.

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Much interests have been paid to the non-abelian anomaly because of its fundamental property of quantum gauge theories. These kinds of anomaly are known to be caused by the quantum effects of triangle fermion loops shown in Fig 1. Their amplitudes in the low energy limit is known to be independent either of the higher order corrections or of the internal fermion masses. On the other hand the asymptotic freedom of QCD facilitates us to predict the hadronic amplitudes at extremely high energy region, where again the triangle diagram becomes dominant. Indeed this triangle amplitude correctly reproduces the high energy behavior obtained from the operator product expansion (OPE) technique[1, 2], up to the ambiguity of the coefficient factor.

The important observation here is that the amplitudes induced by the anomaly are constrained not only by the low energy theorem[3, 4, 5] but also by the high energy behavior because of the asymptotically freedom, and therefore we may expect that the amplitudes induced by the anomaly are essentially controlled by the triangle quark graph. The most well-known example of the processes induced by the axial anomaly is $\pi^0 \rightarrow \gamma\gamma$. The triangle quark amplitude reproduces the experimental data for this process excellently.

If we want to apply the anomaly induced amplitudes to the relatively higher energy region, the non-anomalous contributions come in the process in addition to the anomaly terms. Then the QCD corrections to this triangle graph becomes important. Especially we have to take account of the following nonperturbative QCD effects: 1) chiral symmetry breaking (generation of constituent quark mass); 2) confinement effects; 3) hadron contributions. Our interest here is investigating the processes induced by the axial anomaly in the high energy region.

In the separate paper[6], taking account of the above effects, we have proposed an interpolating formula for the amplitude induced by the axial anomaly. There the amplitudes are constructed by the “vector meson pole terms” and the “anomaly terms”, the latter being essentially evaluated by the quark triangle graph of Fig. 1.

In this paper we pay our attention to the anomaly term and make intensive comparison with the existing experimental data. The candidates for investigating the structure of this triangle anomaly amplitudes are the $\pi^0$ transition form factors.

The result shows that, in addition to the vector meson poles, it is essential to use the constituent quark mass instead of the current quark mass for the internal quark lines of the triangle graph.
Let us start with the following three-point function:

\[
T^{AB,\mu\nu}(p, q) = -i \int d^4 x d^4 y e^{-ik_5y+ipx} \langle 0 | T j^{A\nu}(x) j^{B\rho}(0) j_5^\mu(y) | 0 \rangle,
\]

where \( q \equiv k-p, j_5^\mu \) is the axial-vector current which generally couples to Nambu-Goldstone (NG) boson \( P \) : \( j_5^\mu = \bar{\psi} T^\mu \gamma_5 \gamma_5 \psi, \) \( \langle 0 | j_5^\mu(0) | P(k) \rangle = ik_\mu f_P, \) and \( j^{A\nu}(x) \) and \( j^{B\rho}(0) \) are the relevant currents, \( j^{A\nu} = \bar{\psi} Q^A_\gamma \gamma^\nu \psi \) with \( Q^A \) being the charge matrix of the quark field \( \psi \).

Denote the amplitude the NG-boson pole removed as \( \hat{T}^{AB,\mu\nu}(p, q) \):

\[
T^{AB,\mu\nu}(p, q) = \hat{T}^{AB,\mu\nu}(p, q) + \frac{f_P k^\mu}{m_P^2 - k^2} T^{AB,\nu\rho}(p, q),
\]

where

\[
T^{AB,\nu\rho}(p, q) = -i \int d^4 x e^{-ipx} \langle 0 | T j^{A\nu}(x) j^{B\rho}(0) | P(k) \rangle.
\]

If one fixes the current \( j^{A\nu} = j^{\nu}_{\text{em.}} \), the general form of this \( T^{AB,\nu\rho}(p, q) \) is given by\(^\text{[9, 10]}\):

\[
T^{AB,\nu\rho}(p, q) = N_c e^2 4\pi^2 f_P \text{tr} \left\{ T^P \{ Q^A, Q^B \} \right\} \times \left[ \begin{array}{c} (p \cdot q) g^{\nu\rho} - q^\nu p^\rho \rangle F_P^p(5)(p^2, q^2) + (p^2 g^{\nu\rho} - p^\nu p^\rho \rangle G_P^p(5)(p^2, q^2) \\
+ (p^2 q^\nu q^\rho - (p \cdot q) q^\nu p^\rho \rangle H_P^p(5)(p^2, q^2) + \varepsilon^{\alpha\beta\nu\rho} p_\alpha q_\beta F_P(p^2, q^2) \end{array} \right],
\]

where \( N_c (= 3) \) is the number of colors and we use the electromagnetic current conservation: \( p_\nu T^{AB,\nu\rho}(p, q) = 0 \). We have normalized in such a way that \( F_P(p^2 = 0, q^2 = 0) = 1 \) (see Eq.\(^\text{[10]}\)). If we further set \( p^2 = 0 \) (real photon), we have the electromagnetic transition amplitude \( \langle P(k) | j^{B\rho}(0) | \gamma(p) \rangle \), which is expressed in terms of the vector and axial form factors \( F_P(p^2, q^2) \) and \( F_P^p(5)(p^2, q^2) \). In the case where the current \( j^{B\rho}(0) \) couples to the photon or \( Z \) boson, the axial vector form factor vanishes, and only the term proportional to \( \varepsilon^{\alpha\beta\nu\rho} \) remains. Thus the only anomaly term \( F_P(p^2, q^2) \) contributes to such processes. In this paper, we consider this form factor \( F_P(p^2, q^2) \), which is directly related to the anomaly part of this three-point amplitude.

To see how this form factor is incorporated to the anomaly term, we study the anomalous Ward-Takahashi identity expressed as\(^\text{[9, 10]}\):

\[
\partial_\mu j_5^\mu = 2m_0 j_5 + \frac{e^2}{16\pi^2} \varepsilon \vec{F} \vec{F},
\]

where \( j_5 \) is the corresponding pseudoscalar density: \( 2m_0 j_5 \equiv 2\bar{\psi} \mathcal{M} T^P i\gamma_5 \psi (\mathcal{M} \text{: the mass matrix of the quarks}) \). This leads us to

\[
k_\mu T^{AB,\mu\nu}(p, q) = M^{AB,\mu\nu}(p, q) + \frac{N_c e^2}{4\pi^2} \text{tr} \left\{ T \left\{ Q^A, Q^B \right\} \right\} \varepsilon^{\alpha\beta\nu\rho} p_\alpha q_\beta.
\]
where
\[ M_{AB,\nu\rho}(p, q) = \int d^4x d^4y e^{-ipx + ik} \langle 0 | T j^{A\nu}(x) j^{B\rho}(0) 2m_0 j_5(y) | 0 \rangle. \] (7)

This amplitude \( M_{AB,\nu\rho}(p, q) \), as well as the three-point function \( T_{AB,\mu\nu\rho}(p, q) \), contains the NG-boson pole contributions. We extract the NG-boson pole contributions in both sides:

\[ \left[ k_\mu \tilde{T}_{AB,\mu\nu\rho}(p, q) + \frac{f_P k^2}{m_P^2 - k^2} T_{AB,\nu\rho}(p, q) \right] \]

\[ = \left[ \tilde{M}_{AB,\nu\rho}(p, q) + \frac{f_P m_P^2}{m_P^2 - k^2} T_{AB,\nu\rho}(p, q) \right] + \frac{N_c e^2}{4\pi^2} \text{tr} \langle T \{ Q^A, Q^B \} \rangle \varepsilon^{\alpha\beta\nu\rho} p_\alpha q_\beta. \tag{8} \]

where use has been made of \( \langle 0 | 2m_0 j_5(0) | P \rangle = m_P^2 f_P \). Thus the transition amplitude for NG-boson is obtained:

\[ T_{AB,\nu\rho}(p, q) = \frac{1}{f_P} \left[ k_\mu \tilde{T}_{AB,\mu\nu\rho}(p, q) - \tilde{M}_{AB,\nu\rho}(p, q) \right] - \frac{N_c e^2}{4\pi^2} f_P \text{tr} \langle T \{ Q^A, Q^B \} \rangle \varepsilon^{\alpha\beta\nu\rho} p_\alpha q_\beta. \tag{9} \]

which is rewritten in terms of the invariant amplitude

\[ T_{AB,\nu\rho}(p, q) = \frac{N_c e^2}{4\pi^2 f_P} \text{tr} \langle T \{ Q^A, Q^B \} \rangle \varepsilon^{\alpha\beta\nu\rho} p_\alpha q_\beta \left[ 1 - T(p^2, q^2, k^2) \right] \]

\[ - \frac{N_c e^2}{4\pi^2 f_P} \text{tr} \langle T \{ Q^A, Q^B \} \rangle \varepsilon^{\alpha\beta\nu\rho} p_\alpha q_\beta. \tag{10} \]

From this expression we easily see that \( T(p^2, q^2, k^2) \) corresponds to \( F_P(p^2, q^2) \) of Eq.(4). It is well known that the triangle diagram containing fermion loops couples to vector or axial vector currents leads to anomalies, and in the standard renormalization procedure the gauge invariance guarantees that any higher order corrections do not modify the structure of this anomaly.

So let us first calculate the quark triangle graph shown in Fig. which is expected to give the dominant contribution to the function \( T(p^2, q^2, k^2) \) in Eq.(10). The result is expressed as\(^\text{‡}\)

\[ T(p^2, q^2, k^2) \bigg|_{\text{triangle}} = \frac{2 \sum_i T_i Q_i^A Q_i^B}{\text{tr} \langle T \{ Q^A, Q^B \} \rangle} \int [dz] \frac{m_i^2}{m_i^2 - z_2 z_3 k^2 - z_3 z_1 p^2 - z_1 z_2 q^2}, \tag{11} \]

with the Feynman parameter integral defined by \( \int [dz] = 2 \int_0^1 dz_1 dz_2 dz_3 (1 - z_1 - z_2 - z_3) \).

It is easy to see that in the low energy limit in which \( k^2 = p^2 = q^2 = 0 \), this reduces to

\[ T(p^2 = 0, q^2 = 0, k^2 = 0) \bigg|_{\text{triangle}} = 1, \tag{12} \]

\(^\text{‡}\)Here we restrict ourselves to neutral currents.
Figure 1: The quark triangle anomaly graph

independently of $m_i$. This is the expression that the anomaly term is exactly reproduced by the lowest order triangle graph.

At this point, we should stress that the quark mass $m_i$ appearing in $T(p^2, q^2, k^2)$ is taken to be the constituent quark masses instead of the original current quark masses. This reflects the most prominent feature of QCD: since the chiral symmetry has been spontaneously broken and there appear NG-bosons, the quarks necessarily acquire dynamical masses. This replacement never changes the value of the low energy limit, because $T(p^2 = 0, q^2 = 0, k^2 = 0) = 1$ independently of the quark mass (see Eq.(13)).

In the following we restrict ourselves to the $\pi^0$ case, where only the $u$ and $d$ quarks come into play. Then $j_5^\mu = \sum_i \bar{q}_i T_i \gamma^\mu \gamma_5 q_i$, $q_i = (u, d)$, $T_i = (1/2, -1/2)$, and $2m_0j_5 \equiv \sum_i 2m_0\bar{q}_i T_i i\gamma_5 q_i$. The triangle graph contribution to $T(p^2, q^2, k^2 = 0)$ is obtained as

$$I(p^2, q^2; m) \equiv T(p^2, q^2, k^2 = 0)\bigg|_{\text{triangle}} = \int [dz] \frac{m^2}{m^2 - z_3 z_1 p^2 - z_1 z_2 q^2},$$

where $m (=m_u \simeq m_d)$ is the constituent quark mass (of the $u$ and $d$ quarks).

If one photon is on its mass shell (e.g., $p^2 = 0$), then the function $I(p^2, q^2; m)$ reduces to

$$J(q^2; m) \equiv I(p^2 = 0, q^2; m) = -\frac{m^2}{q^2} \left[ \ln \frac{\sqrt{4m^2 - q^2} + \sqrt{-q^2}}{\sqrt{4m^2 - q^2} - \sqrt{-q^2}} \right]^2,$$

which gives the $\pi^0\gamma$ transition form factor. The high energy behavior of this function is
given by

\begin{equation}
J(q^2; m) \Rightarrow \frac{m^2}{q^2} \left[ \pi^2 - \left( \ln \frac{m^2}{q^2} \right)^2 - 2i\pi \ln \frac{m^2}{q^2} \right].
\end{equation}

(15)

It is well known that, in the low energy limit, the higher order corrections do not change the value of the three-point function induced by the anomaly in Eq.(1), or \(T(p^2 = q^2 = k^2 = 0)\) itself. However, QCD corrections do contribute appreciably to its higher energy behavior. Especially nonperturbative QCD effects are alleged to take place: 1) chiral symmetry breaking (generation of constituent quark mass); 2) confinement effects; 3) hadron contributions.

As we have mentioned below Eq.(12), we have already taken account of the effect 1) by replacing the current quark masses by the constituent masses of the quarks. The next important QCD effects are represented as a rich hadron spectrum. As an average, most part of those resonances with broad widths may be already involved by the above replacement in the triangle contributions of Fig. 1.

However, one should further modify the functions \(I(p^2, q^2; m)\) and \(J(q^2; m)\) due to the effect 2). The graph in Fig. 1 shows the contribution of the \(q\bar{q}\) threshold at \(p^2 = 4m^2\) \((q^2 = 4m^2)\) from which an imaginary part emerges as shown in Fig. 2 (the dotted lines). However, since the quarks are confined, the intermediate states are not multi-quark states but actually multi-hadron states \((2\pi, 4\pi, \ldots \text{etc.})\). The above effect may be properly taken into account by smearing out the original \(J(q^2; m)\). Here we adopt the following function:

\begin{equation}
\tilde{J}(q^2; m) \equiv -\frac{(\ln(1+x))^2}{x} + \ln(1+x) + \alpha x \exp(-\beta x) + 2\pi i \frac{\ln(1+x)}{x} \exp\left(-\frac{\lambda}{x}\right), \quad x = \frac{q^2}{m^2}.
\end{equation}

(16)

Both \(J\) and \(\tilde{J}\) are shown in Fig. 2 where we take the parameter choice \(\alpha = 1.4, \ \beta = 0.35\) and \(\lambda = 2.5\). From this figure, we see that the modified amplitudes \(\tilde{J}\) are properly smeared out. This form has been chosen so as to coincide with the original \(J\) (the dotted lines) both in the high and low energy limits.

Finally, we must take account of the dominant hadron poles as the effect 3). Among hadron spectrum, NG-bosons, which have been put in as elementary fields, are of course most important. The next important poles are lowest energy bound states, i.e., the vector mesons. This can be done by introducing the vector meson poles to the amplitude, where we should be careful to include them to avoid the double counting and to satisfy the low energy theorem Eq.(12).
Figure 2: The QCD corrected-improved functions $\tilde{J}(q^2; m)$ (solid lines) comparing with the original functions $J(q^2; m)$ (dotted lines), where we take the parameter choice $\alpha = 1.4$, $\beta = 0.35$ and $\lambda = 2.5$. 
In the high energy region, on the other hand, in addition to the energy properties as the parameter \( \tilde{\rho} \), we have terms for the coefficient factor of the framework of HLS. Then of course, this form factor again satisfies the low energy theorem:

\[
\tilde{\rho} \sim O(\omega),
\]

where \( \tilde{\rho} \) and \( \tilde{\omega} \) have been chosen to be consistent with the flavor symmetry. Since the above form factor should satisfy the low energy theorem, the normalization condition Eq. (12) gives

\[
\kappa_1 + 4\kappa_2 + \kappa_3 = 1 \text{ (note that } \tilde{\rho}(p^2 = 0) = \tilde{\omega}(p^2 = 0) = 1 \text{).}
\]

This expression is most elegantly derived in the framework of hidden local symmetry (HLS) (see, for a review, Ref. [11]). In the following we use the parameters \( c_3 \) and \( c_4 \): \( c_3 = 2\kappa_3; \ c_4 = 1 - \kappa_1 \), where \( c_3 \) and \( c_4 \) correspond to the terms appearing in the Lagrangian constructed in the framework of HLS. Then of course, this form factor again satisfies the low energy theorem:

\[
F_{\pi^0\gamma\gamma^*}(p^2, q^2) = \kappa_1 \tilde{I}(p^2, q^2; m) + \kappa_2 \left[ \left\{ D_\rho(p^2) + D_\omega(p^2) \right\} \tilde{J}(q^2; m) + \left\{ D_\rho(q^2) + D_\omega(q^2) \right\} \tilde{J}(p^2; m) \right] + \kappa_3 \left[ D_\rho(p^2)D_\omega(q^2) + D_\rho(q^2)D_\omega(p^2) \right],
\]

(17)

where \( D_\rho(p^2) \) and \( D_\omega(p^2) \) are the Breit-Wigner type propagators for \( \rho \) and \( \omega \) mesons:

\[
D_\rho(p^2) = \frac{m_\rho^2}{m_\rho^2 - p^2 - i\sqrt{p^4\Gamma_\rho}}, \quad D_\omega(p^2) = \frac{m_\omega^2}{m_\omega^2 - p^2 - i\sqrt{p^4\Gamma_\omega}},
\]

with \( \Gamma_\rho \) and \( \Gamma_\omega \) being the widths of \( \rho \) and \( \omega \) mesons, respectively. The weight of \( D_\rho \) and \( D_\omega \) has been chosen to be consistent with the flavor symmetry. Since the above form factor should satisfy the low energy theorem, the normalization condition Eq. (12) gives

\[
\kappa_1 + 4\kappa_2 + \kappa_3 = 1 \text{ (note that } \tilde{\rho}(p^2 = 0) = \tilde{\omega}(p^2 = 0) = 1 \text{).}
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\[
F_{\pi^0\gamma\gamma^*}(p^2, q^2 = 0) = 1, \text{ independently of the parameters } c_3 \text{ and } c_4.
\]

This together with the coefficient factor \( g_{\gamma\gamma\pi} = -e^2/(4\pi^2f_\pi) \), excellently reproduces the experimental value of the \( \pi^0 \to \gamma\gamma \) process: this is nothing but the expression of the low energy theorem.

In the high energy region, on the other hand, in addition to the \( \tilde{J}(q^2; m) \sim (m^2/q^2) \) contribution, we have terms \( D_\rho(q^2) \sim m_\rho^2/q^2 \) and \( D_\omega(q^2) \sim m_\omega^2/q^2 \). If one notices that \( m_\rho \sim m_\omega \sim O(m) \), it is easy to see that \( \rho \) and \( \omega \) pole contributions have the same high energy properties as \( \tilde{I} \) and \( \tilde{J} \).

Now, we compare the above proposed form factor with the experimental data and examine the \( q^2 \) dependence of the anomaly terms \( \tilde{I} \) and \( \tilde{J} \). To do this, we first determine the parameter \( c_3 \) and \( c_4 \) from the experiment.

By setting \( p^2 = 0 \) we have the \( \pi^0 \) electromagnetic transition form factor:

\[
F_{\pi^0\gamma}(q^2) = \left( 1 - \frac{c_3 + c_4}{2} \right) \tilde{J}(q^2; m) + \frac{c_3 + c_4}{4} \left\{ D_\rho(q^2) + D_\omega(q^2) \right\},
\]

(19)

\( ^\dagger \) The function \( \tilde{I} \) is obtained by the same procedure as \( \tilde{J} \).
in which \((c_3 + c_4)/2\) is to be determined. The comparison of our formula with the experimental data\(^\text{[12]}\) indicates

\[
\frac{c_3 + c_4}{2} = 1.0. \tag{20}
\]

This value implies that the vector meson dominance (VMD) is almost realized in this process. The result Eq.\((20)\) implies that the \(q^2\) dependence of \(J\) is not so important for the \(\pi^0\gamma\) transition form factor, so far as we analyze the existing data (see Ref.\(\text{[1]})\). So we can not get the information of the function \(J(q^2; m)\) from this process alone.

On the other hand, the vector meson form factors can also be obtained from our formula. The expression for the \(\omega\pi\) transition form factor is given by extracting the \(\omega\)-pole contributions from Eq.\((17)\) and by making proper normalization of the amplitude we find

\[
\begin{align*}
\text{(case 1)} & \quad F_\omega(q^2) = -\tilde{c}J(q^2; m) + (1 + \tilde{c})D_\rho(q^2), \\
& \quad \tilde{c} \equiv \frac{c_4 - c_1}{c_3 + c_4}, \tag{21}
\end{align*}
\]

where \(\tilde{c}\) is to be determined. We take here \(m = m_\rho/2\) (case 1). For reference, we examine the case where we adopt the current quark mass \(m = m_0 \approx 5\text{MeV}\) in \(J(q^2; m_0)\) (case 2), and the case in which the triangle anomaly term is taken to be a constant (case 3):

\[
\begin{align*}
\text{(case 2)} & \quad F_\omega^{(m_0)}(q^2) = -\tilde{c}J(q^2; m_0) + (1 + \tilde{c})D_\rho(q^2), \\
\text{(case 3)} & \quad F_\omega^C(q^2) = -\tilde{c} + (1 + \tilde{c})D_\rho(q^2). \tag{22}\tag{23}
\end{align*}
\]

The form factor of case 3 was often used in the extensive analysis of the form factors in relatively lower \(q^2\) region (see, for example Ref.\([13]\)).

Let us determine the value of \(\tilde{c}\) for each case using the experimental data \(\Gamma(\omega \to \pi^0\mu^+\mu^-)\). It is convenient to use the following expression:

\[
\frac{\Gamma(\omega \to \pi^0\mu^+\mu^-)}{\Gamma(\omega \to \pi^0\gamma)} = \int_{4m_\mu^2}^{(m_\omega-m_\mu)^2} dq^2 \frac{\alpha}{3\pi q^2} \left(1 + \frac{2m_\mu^2}{q^2}\right) \sqrt{\frac{q^2 - 4m_\mu^2}{q^2}} \times \left[\left(1 + \frac{q^2}{m_\omega^2 - m_\pi^2}\right)^2 - \frac{4m_\omega^2q^2}{(m_\omega^2 - m_\pi^2)^2}\right]^{3/2} |F_\omega(q^2)|^2, \tag{24}
\]

where \(q^2\) is the intermediate photon momentum (or invariant mass of final muons). The experimental data\([14]\) shows

\[
\frac{\Gamma(\omega \to \pi^0\mu^+\mu^-)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{(9.6\pm 2.3) \times 10^{-5}}{(8.5 \pm 0.5) \times 10^{-2}} \approx (1.1 \pm 0.3) \times 10^{-3}, \tag{25}
\]

\(^\text{4}\)We could also obtain the \(\rho^0\pi^0\) transition form factor. However, the branching ratio \(B(\rho^0 \to \pi^0\gamma)\) \((= (7.9 \pm 2.0) \times 10^{-4})\) is far smaller than \(B(\omega \to \pi^0\gamma)\) \((= (8.5 \pm 0.5) \times 10^{-2})\). Moreover, no experimental data for \(\rho^0 \to \pi^0\gamma^*\) has been reported.
which gives\[\]

case 1 \quad \tilde{c} = 0.49^{+0.12}_{-0.13}, \quad \text{for} \quad F_\omega(q^2),

case 2 \quad \tilde{c} = 0.11^{+0.16}_{-0.18}, \quad \text{for} \quad F^{(m_0)}_\omega(q^2),

case 3 \quad \tilde{c} = 0.88^{+0.16}_{-0.18}, \quad \text{for} \quad F^C_\omega(q^2). \quad (26)

The average values for all the above cases show that the complete \(\rho\) meson dominance is incapable of describing the \(\omega\pi^0\) form factor\[15\] (VMD corresponds to \(\tilde{c} = 0\), see Eq.(21)).

Now that we have found that the \(\omega\pi^0\) transition form factor receives appreciable contributions from the anomaly term, we use the \(\omega\pi^0\gamma^*\) process to get the information on the \(q^2\) dependence of the anomaly term from the experimental data. The available experimental data for \(\omega\pi^0\gamma^*\) lies in a relatively wide energy range (up to 1.4GeV). We use the value of \(\tilde{c}\) for each case obtained above, and compare the above three cases. The predicted curves are shown in Fig. 3 (a) (b). In those figures the three data points around \(q^2 = 0.4\text{GeV}^2\) are located far from the theoretical curves. We should remark that the deviation of the above data points should not be taken seriously: because the above region corresponds to the kinematical boundary, the event rate is very rare. As a result, the behavior of the form factor strongly depends on the normalization uncertainty\[13\]. From Fig. 3 (a) (b) we find that the form factor \(F^C_\omega(q^2)\) (case 3) can not reproduce the high \(q^2\) data, the theoretical values are much larger than the data points. For the case 2 \((F^{(m_0)}_\omega(q^2))\) the suppression of the form factor is too strong and the theoretical values are too small to fit the data points in the high \(q^2\) region. On the other hand, aside from the three points mentioned above, the improved form factor \(F_\omega(q^2)\) (case 1) is found to be in good agreement with the data in the whole energy region. This successful result is owing to the triangle anomaly term described by the modified function \(\tilde{J}(q^2; m)\) in addition to the vector meson pole effect. In particular, in the above analysis we have taken the constituent quark mass \(m = m_\rho/2\).

One might wonder whether the above results depend on the smearing procedure of the function \(J\). To check this we show the curve obtained from the form factor with the unsmeared function \(J\) in Fig. 3. From this figure, we see that the smearing procedure does not affect the value of the form factor so far as in the region where the experimental data exists, although it does around the threshold \((q^2 \simeq 0.6\text{GeV}^2)\).

In conclusion, the experimental data in a relatively wide energy range is found to be reproduced very well by using the QCD corrected triangle amplitude as the “anomaly

\[\]
\[\]\[\]The another solutions for \(\tilde{c}\), for example, \(\tilde{c} = -2.5 \pm 0.1\) (case 1), are excluded by experiments.
term”. It must be stressed that the constituent quark mass plays an essential role in the triangle anomaly term.

It is a common understanding that the low-energy effective Lagrangian for quarks and gluons is described by the chiral Lagrangian in terms of the NG-bosons. Their anomaly term has been introduced by the Wess-Zumino action\cite{16, 17} in the effective Lagrangian. Another way to incorporate the anomaly term is to introduce the quarks (to be more exact, we should say “constituent quarks”) and gluon fields in place of the hadrons other than NG-bosons in the chiral Lagrangian\cite{18}.

At extremely low energy limit, the above two effective Lagrangians work very well for reproducing the phenomena and they are completely equivalent because the latter induces the anomaly term via loop effects. This is the result of low energy theorem. If one wants to apply the Lagrangian to nonzero but relatively lower energy scale, the vector mesons should be properly included in the Lagrangian. So far as we examine the processes in which the complete VMD is realized, the whole anomaly terms are replaced by the vector meson pole terms. Indeed we have seen that the anomaly term disappears from the $\pi^0\gamma$ transition form factor and we cannot check the $q^2$ dependence of the anomaly term from this process. On the other hand, we have found that $\omega\pi^0$ transition form factor, in which the complete VMD hypothesis is not valid, recieves appreciable contributions from the anomaly term. For such processes the above two approaches give different predictions. Our result indicates that the WZ action alone cannot reproduce the experimental data for the $\omega\pi^0$ transition form factor in the relatively high energy region, because the WZ action gives the constant anomaly term. This is in remarkable contrast to the anomaly term calculated from the QCD corrected quark triangle graph. In this sense our treatment here may have somewhat similar spirit as the latter approach.

Also the high energy behavior of the anomaly term, with the usage of the constituent quarks in the internal loop, seems to suggest the possibility that this anomaly term can be applicable even to the phenomena in the extremely high energy region, say for examples, $Z \rightarrow P\gamma$ ($q^2 = m_Z^2$) or $W$ decay processes\cite{2, 4, 5, 7, 8, 9, 10}. The processes induced by the axial anomaly may yet provide us with the information on how to construct the effective theory applicable to higher energy regions.

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Figure 3 (a) (b): The form factors in the energy range below 1.4 GeV: (a: case 1) the improved form factor $F_\omega(q^2)$; (b: case 2) $F_\omega^{(m_0)}(q^2)$. The low energy data (Exp.1) is given in Ref.[15], and the high energy data (Exp.2) is translated from the cross section data[19]. The values of $\tilde{c}$ are determined using $\Gamma(\omega \rightarrow \pi^0\mu^+\mu^-)$ (see Eq.(26)).
Figure 3 (c): The form factors in the energy range below 1.4 GeV: (c: case 3) $F^C_\omega(q^2)$.

Figure 4: The form factor in the energy range below 1.4 GeV using the unsmeared function $J(q^2; m)$. 

\[ |F^C_\omega(q^2)|^2 \]
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