Localized nonlinear electrostatic structures in a multispecies space plasma

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Abstract. In this research paper, authors have studied ion-acoustic solitons in a warm multicomponent plasma consisting of warm positive and negative ions alongwith nonthermal electrons and an electron beam. Reductive perturbation method is used to derive KdV equation and dispersion relation, later a polynomial of six degree in phase velocity which corresponds to six ion-acoustic modes. Compressive as well as rarefactive solitons were found to exist corresponding to four linear modes, while other two wave modes support either compressive or rarefactive solitons in the given parameter regime. Study of KdV equation shows that nonthermality and electron beam parameters play crucial role in the characterization of solitons.

1. Introduction
Nonlinear wave structures are beautiful and amazing manifestation of nature, arising out of competition between properties like nonlinearity, dispersion and dissipation. Such structures have received considerable attention both for space and laboratory plasmas and been extensively studied in the last several years. Further, they have been paying rich dividends to the scientists as they offer deep physical insight underlying the nonlinear phenomena. Space environment constitutes a magnificent laboratory to capture the physics of nonlinear plasma phenomena. Different nonlinear wave structures such as solitons, shock waves, double layers, vortices etc. have been studied by a number of researchers. The earlier experimental observations confirmed the existence of various solitons in laboratory [1-4] and are reported in an excellent review [5].

In models used to study coherent nonlinear wave structures, some form of perturbation method usually is adopted. In small amplitude approximation, we derive some form of nonlinear partial differential equations, which are integrable and possess special solutions. To quote a few, we use reductive perturbation technique to obtain Korteweg-de-Vries (KdV), modified Korteweg-de-Vries
(m-KdV), Kadomtsev-Petviashvili (KP) equation or Nonlinear Schrödinger equation (NLSE) etc. All these are the members of the large class of nonlinear evolution equations [6].

The solitons in electron-ion plasma are compressive type and therefore have a positive potential in a plasma. However, the inclusion of extra negative ions in such plasma modifies the usual characteristics of solitons and this motivated the investigations of solitons in multicomponent plasmas. Both theoretically [5, 7-14] and experimentally [15-19], the study of KdV and m-KdV ion-acoustic solitons in multispecies plasmas consisting of positive ion, electrons and negative ions confirmed that there exists a critical concentration of negative ions below which compressive solitons exist and above which rarefactive solitons exist. Functional relationship of relative density and idea of critical density have been introduced theoretically [20] and confirmed experimentally.

Various on board satellite observations have confirmed that space plasmas are invariably observed to be multicomponent type, of particular interest here is case when an electron beam is present in such plasma. Such a situation is typically encountered in upper layers of the magnetosphere where the co-existence of two different electron populations have been reported by Satellite missions [21-27]. Further, such beam plasma systems have been created in laboratory [28-30]. Specifically, stationary nonlinear ion-acoustic waves may be excited when an electron beam is injected into a plasma [31-32]. In an actual situation, electron beam component is frequently observed in the region of space where ion-acoustic waves exist. The observations of solitary waves in auroral zone suggest that there are two classes of solitary waves : the first kind associated with electron beam and other is associated with the ion beam [21]. From a nonlinear plasma-theoretical point of view, the existence of an electron beam has been shown to significantly modify the properties and conditions for the existence of nonlinear excitation, both from the Sagdeev pseudopotential formalism [33] or small amplitude solitary waves [34]. The investigation of electron beam plasma system has also considerable importance in the areas of magnetosphere and solar physics. Further, it is known that high speed electrons have considerable influence on the excitation of the various kinds of nonlinear waves in interplanetary space and Earth’s magnetosphere [35]. In the recent past, this motivated the researchers to study multicomponent plasma with electrons in variety of systems [36-38]. Yadav et al [39] showed that above a critical beam velocity four ion-acoustic branches appear in electron beam plasma system. However, the present investigation shows the existence of six modes above a critical electron beam velocity.

Most recent investigations show that particles may not follow Maxwellian distribution due to the formation of space holes. Accordingly, in most space plasmas, particles follow non-Maxwellian distribution [14, 26,43]. Further, it is worth noting that the electron trapping is observed not only in space plasma but also in laboratory experiments [44, 45]. Further, it had been found that electrons and ion distributions play crucial role in characterizing the physics of nonlinear investigations. Particle distribution offer a considerable increase in richness and variety of wave motion that can exist in plasma and further significantly influence the conditions required for the formation of these waves. Moreover, it is also known that electron and ion distribution can be significantly modified in the presence of large amplitude waves [46]. Recently, energetic electron distributions have been observed in different regions of magnetosphere. Cairn et al [40] used nonthermal electron distribution of electrons to study ion-acoustic solitary structures observed by Freja satellite. This motivated the researchers to undertake such particle distributions in investigations and several articles have been reported with such particle distributions. The (H⁺,O₂⁻) and (H⁺,H⁻) plasma composition occur in D-region of ionosphere, where negative ions are found [41]. As an approximation, plasma can be treated collisionless in the upper part of the D-region of Earth's ionosphere where collision frequency is expected to be small and neglected [42]. We have chosen (H⁺,H⁻) plasma for our numerical computation.

The major aim of the present investigation is to study both the electron beam and energetic population parameter β in a plasma with positive and negative ions having different temperatures.
Our choice for nonthermal electron distribution is prompted as a matter of convenience rather than appropriately explaining some observational data. The manuscript is organized as follows: Sections 2 and 3 are devoted to formulation of basic equations and derivation of KdV equation. Soliton solution is found in section 3. In section 4, detailed discussion of the results is presented and finally conclusion is given.

2. Basic equations

We consider a collisionless unmagnetized plasma consisting of nonthermal electron distribution, warm positive and negative ions species with a propagating electron beam having temperatures $T_1$, $T_2$, and $T_b$, respectively. The nonthermal distribution for electrons was used by a number of authors in various investigations. The number density of the electron fluid associated with nonthermal electrons is given by [14]:

$$n_e = (1 - \beta \phi + \beta \phi^2)e^\gamma$$  \hspace{1cm} (1)

Where

$$\beta = \frac{4\gamma}{1 + 3\gamma}$$

and we have chosen the appropriate normalization. The real parameter $\gamma$ is an arbitrary parameter which defines the shape of distribution function and expresses the deviation from the Maxwellian state. Here $\gamma$ determines the number of nonthermal electrons present in our nonthermal plasma model [14,40]. This is non-Maxwellian distribution function which contains high energy electron component. Such distributions are very common in auroral zone of ionosphere. As an example, when $\gamma=0$, we get Maxwellian distribution while for $\gamma \to 1$, two counter streaming beams with cold core distribution is obtained [52].

The nonlinear behaviour of ion-acoustic waves in multispecies plasma is governed by the following set of normalized fluid equations:

Fluid equations for the positive ions:

$$\frac{\partial n_1}{\partial t} + \frac{\partial (n_1 v_1)}{\partial x} = 0$$

$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x} = -\frac{1}{\sigma_1} \frac{\partial \phi}{\partial x} - \frac{1}{\delta_1} \frac{\partial n_1}{\partial x}$$  \hspace{1cm} (2)

Fluid equations for the negative ions:

$$\frac{\partial n_2}{\partial t} + \frac{\partial (n_2 v_2)}{\partial x} = 0$$

$$\frac{\partial v_2}{\partial t} + v_2 \frac{\partial v_2}{\partial x} = \frac{\varepsilon_2}{\eta_2} \frac{\partial \phi}{\partial x} - \frac{\sigma_2}{\eta_2} \frac{\partial n_2}{\partial x}$$  \hspace{1cm} (3)

Fluid equations for the electron beam:

$$\frac{\partial n_b}{\partial t} + \frac{\partial (n_b v_b)}{\partial x} = 0$$

$$\frac{\partial v_b}{\partial t} + v_b \frac{\partial v_b}{\partial x} = -\frac{1}{\sigma_b} \frac{\partial \phi}{\partial x} - \frac{1}{\delta_b} \frac{\partial n_b}{\partial x}$$  \hspace{1cm} (4)
Poisson's equation
\[ \frac{\partial^2 \phi}{\partial x^2} = n_e + \alpha_e n_b - \frac{n_1}{\rho} + \frac{\alpha e}{\rho} n_2 \]  
(5)

Where
\[ \rho = 1 - \alpha_e - \alpha_z \]
and
\[ \alpha_b = \frac{n_b^{(0)}}{n_e^{(0)}}, \alpha_e = \frac{n_e^{(0)}}{n_1^{(0)}}, \alpha_i = \frac{n_i^{(0)}}{n_1^{(0)}}, \alpha_z = \frac{\alpha_b}{Z_1}, \]
\[ \varepsilon_e = \frac{Z_2}{Z_1}, \eta = \frac{m_e}{m_1}, \eta_i = \frac{m_i}{m_1}, \sigma_i = \frac{T_i}{T_e}, \]
\[ \sigma_b = \frac{T_b}{T_e}, \delta = \frac{\eta + \alpha e^2}{\eta(1 - \alpha e z)} \]  
(6)

It must be mentioned that we have adopted the fluid description for our model equations to study the ion-acoustic solitons. The temperatures \( T_1, T_2, \) and \( T_b \) enter in above equations through pressure terms. In another approach [49, 50], the coherent structures encountered in space plasmas are interpreted in terms of phase space holes, ion phase holes or electron holes. This is the viewpoint of someone who watches the phase density. But the phase space is not directly accessible from observation data. Therefore, the same objects are also called solitary waves, electrostatic shocks or double layers. However, for low electron beam velocity, kinetic approach used to study phase space holes is more rigorous and valid. Here \( (n_1, v_1), (n_2, v_2), \) and \( (n_b, v_b) \), are the densities and fluid velocities of positive and negative ion species and electron beam respectively. \( n_1^{(0)}, n_2^{(0)}, n_b^{(0)} \) are the equilibrium densities of two ion components and beam respectively. Further, \( \phi \) is the electrostatic potential, \( \eta \) is the mass ratio of the negative ion species to the positive ion species, \( \eta_i \) is the mass ratio of the beam electrons to the positive ion species, \( \alpha \) is the equilibrium density ratio of electron beam to positive ion species, \( \alpha_b \) is the equilibrium density ratio ofelectron beam to positive ion species, \( \sigma \) is the charge multiplicity ratio of the negative ion to positive ion species. Here \( Z_1 \) and \( Z_2 \) are the charge on positive and negative ions respectively. For \( \text{H}^+\text{H}^- \) plasma \( Z_i = Z_e = 1 \). \( m_1 \) is the mass of positive ion, \( m_2 \) is the mass of negative ion and \( m_e \) is the mass of electron in electron beam respectively. In equations (2) to (5), velocities \( (v_1, v_2, v_b) \) potential \( (\phi) \), time \( (t) \) and space coordinate \( (x) \) have been normalized with respect to the ion-acoustic speed in the mixture, \( C_s = (T_s, \delta Z_1/m_1)^{1/2} \), thermal potential \( T_e/e \), inverse of ion plasma frequency in the mixture \( \omega_{pe}^{-1} = (m_e/4\pi n_e \sigma Z_1 Z_e)^{1/2} \) and Debye length \( \lambda_D = (T_e/4\pi n_e e^2)^{1/2} \) respectively. Ion densities \( (n_1, n_2, n_b) \) are normalized with their corresponding equilibrium values, whereas electron densities are normalized by \( n_{e0} \).

### 3. Dispersion Relation and KdV Equation

To study small but finite amplitude ion-acoustic solitary waves in our multispecies plasma model, we construct here a weakly nonlinear theory of the ion-acoustic waves which leads to scaling of the independent variables through the stretched co-ordinates \( \xi \) and \( \tau \):

\[ \xi = \varepsilon^{1/2} (x - \lambda t) \]
\[ \tau = \varepsilon^{3/2} t \]  
(7)

where \( \varepsilon \) is a small parameter measuring the weakness of the dispersion and \( \lambda \) is the phase velocity of the wave to be determined later. Now to strike a balance between nonlinear and dispersive terms, we...
use reductive perturbation technique where we expand all dependent quantities in equations (2) to (5) around the equilibrium values in powers of $\varepsilon$ in the following form:

$$
\begin{pmatrix}
  n_1 \\
  n_2 \\
  n_b \\
  v_1 \\
  v_2 \\
  \phi
\end{pmatrix} =
\begin{pmatrix}
  1 \\
  1 \\
  0 \\
  0 \\
  0
\end{pmatrix} + \sum_{n=1}^{\infty} \varepsilon^n 
\begin{pmatrix}
  n_1^{(n)} \\
  n_2^{(n)} \\
  n_b^{(n)} \\
  v_1^{(n)} \\
  v_2^{(n)} \\
  \phi^{(n)}
\end{pmatrix}
$$

(8)

Here, $v_0$ is the initial electron beam velocity. Using equations (7) and (8) into Poisson's equation (5), to the lowest order of $\varepsilon$, we get the following dispersion relation:

$$
\frac{Z_1(1+\alpha_b)}{1-\alpha_b} \left[ \frac{1}{(\partial Z, \lambda^2 - \sigma_i)^2} + \frac{\alpha \varepsilon_i^2}{(\partial \eta Z, \lambda^2 - \sigma_i)^2} \right] + \frac{\alpha \varepsilon_i^2}{(v_0 - \lambda)^2 \delta \eta Z_1 - \sigma_i} = 1 - \beta = F(\lambda)
$$

(9)

It may be noted that the dispersion relation (9) is a six degree polynomial in $\lambda$, thereby giving six modes propagating with different phase velocities. However, in the earlier investigation [14], where analysis was done without the electron beam only four roots were obtained. Thus inclusion of electron beam gives two extra modes. Further, these modes are significantly modified by the presence of nonthermality parameter $\beta$. Polynomial equation (9) is not amenable to analytical solution and we, therefore, either use graphical or numerical methods for its solution.

We use (8) in equation (2) to (5) and compare various physical quantities in same order. The resulting equations are further used to eliminate other dependent parameters in terms $\phi^{(1)}$. Then after a long algebraic but straightforward manipulations, we arrive at the following KdV equation:

$$
\frac{\partial \phi^{(1)}}{\partial \tau} + Q \frac{\partial \phi^{(1)}}{\partial \xi} + P \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0
$$

(10)

Where $Q=Q'/P'$ and $P=1/P'$. Here $P'$ and $Q'$ are given by

$$
P' = \frac{2\delta \lambda Z_1^2(1+\alpha_b)}{1-\alpha_b} \left[ \frac{1}{(\partial Z, \lambda^2 - \sigma_i)^2} + \frac{\alpha \varepsilon_i^2}{(\partial \eta Z, \lambda^2 - \sigma_i)^2} \right] - \frac{2Z_1 \eta_i \delta \alpha_b (v_0 - \lambda)}{[(v_0 - \lambda)^2 \delta \eta Z_1 - \sigma_b]^3} = -\frac{dF}{d\lambda}
$$

(11)

$$
Q = \frac{Z_1^2(1+\alpha_b)}{1-\alpha_b} \left[ \frac{3(\partial Z, \lambda^2 - \sigma_i)}{(\partial Z, \lambda^2 - \sigma_i)^3} + \alpha \varepsilon_i (\partial \eta Z, \lambda^2 - \sigma_i)^3 (\delta \eta Z, \lambda^2 - \sigma_i)^3 - \delta \eta Z_1 - \sigma_b \right] - \frac{3Z_1 \eta_i \delta (v_0 - \lambda)^2 \delta \eta Z_1 - \sigma_b}{[(v_0 - \lambda)^2 \delta \eta Z_1 - \sigma_b]^3}
$$

(12)

Coefficient $Q$ being a function of a number of parameters, is a measure of nonlinearity, while coefficient $P$ is the measure of dispersion. It is observed that in the absence of electron beam (i.e, $\alpha_b=0$), the KdV equation (10) becomes same as that of reference [14]. However, for $\alpha = \beta = 0$, the KdV equation is reduced to that obtained by Yadav et al [39] by taking $\gamma=1$. 


4. Soliton Solution

The steady state solution of KdV equation (10) is obtained by transforming the independent variables $\xi$ and $\tau$ as

$$\xi = \xi - u \tau$$

(13)

where $u$ is a normalized constant velocity. Using equation (13) into (10) and integrating w.r.t. $\zeta$, we get

$$\frac{1}{2} \frac{d\phi}{d\zeta}^2 + V(\phi) = 0$$

(14)

where $\phi^{(1)}$ is replaced by $\phi$ for the sake of convenience and $V(\phi)$ is the Sagdeev potential given by

$$V(\phi) = \frac{1}{6} \left( 2Q \phi^3 - 3P u \phi^2 \right)$$

(15)

For deriving equation (15), we have used the following boundary conditions:

$$\zeta \to \pm \infty, \left( \phi, \frac{d\phi}{d\zeta}, \frac{d^2\phi}{d\zeta^2} \right) \to 0$$

(16)

However, for the soliton solution, the Sagdeev potential $V(\phi)$ should satisfy the following boundary conditions:

$$V(\phi) = 0 \text{ at } \phi = 0 \text{ and } \phi = \phi_0$$

$$V'(\phi) = 0 \text{ at } \phi = 0$$

$$V(\phi) > 0 \text{ at } \phi = \phi_0 \text{ for compressive solitons.}$$

$$V(\phi) < 0 \text{ at } \phi = \phi_0 \text{ for rarefactive solitons.}$$

(17)

From the boundary condition (17), we get

$$u P' > 0 \quad \text{or} \quad u \frac{dF}{d\lambda} < 0$$

(18)

The soliton solution for equation (10) is given by

$$\phi = \phi_0 \sec h^2 \left( \frac{\xi}{d} \right)$$

(19)

where the amplitude $\phi_0$ and width $d$ are given by

$$\phi_0 = \frac{3P u}{Q}$$

$$d = \left( \frac{4}{P u} \right)^{1/2}$$

(20)

5. Discussion

The critical beam velocity is found numerically from equations (9) and (11) with the condition that $dF/d\lambda = 0$ for $F(\lambda) = 1$ [39]. It is found that critical beam velocity depends on electron beam concentration $\alpha_b$, negative ion concentration $\alpha$, electron beam temperature $\sigma_b$, relative ion temperatures $\sigma_1$, $\sigma_2$, $\eta$ and $\eta_1$. Using the above condition, the analytical expression for $\nu_c$ (with $\sigma_1 = 0$, $\sigma_2 = 0$ and $\sigma_0 = 0$) is given by
The variation of critical beam velocity as a function of $\alpha_b$ is shown in figure 1 for four different sets of parameters (see figure caption). The other parameters for (H$^+$H) plasma are chosen as follows:

$$\alpha = 0.1, \sigma_1 = 0.1, \sigma_2 = 0.01, \eta_1 = 1/1836, \eta = 1, \varepsilon = Z_1 = Z_2 = 1$$

![Figure 1. Variation of critical velocity $v_{cr}$ with $\alpha_b$ with the chosen set of parameters as $\alpha = 0.1, \eta = 1, \eta_1 = 1/1836, \sigma_1 = 0.1, \sigma_2 = 0.01, \varepsilon = Z_1 = Z_2 = 1$. Curve 'a' corresponds to $(\beta = 0, \sigma_0 = 0.01)$, curve 'b' to $(\beta = 0.1, \sigma_0 = 0.01)$, curve 'c' to $(\beta = 0.1, \sigma_0 = 0.001)$ and curve 'd' to $(\beta = 0, \sigma_0 = 0.02)$ respectively.](image)

Apparently, $v_{cr}$ increases with $\alpha_b$. This behaviour is similar to one observed by Yadav et al [39]. From curves a, b and c, it is observed that critical velocity increases with $\beta$. Further, from curves b and d, critical beam velocity is observed to increase with electron beam temperature $\sigma_b$. Thus, the values of $\alpha_b$, $\beta$ and beam temperature $\sigma_b$ influence the value of critical beam velocity. It was further observed that beam velocity $v_0$ play important role in the characterization of solitons. There are four real roots and apparently other two corresponding to complex conjugate pair appear in case of $v_0 < v_{cr}$. However, when beam velocity exceeds the critical velocity (i.e. $v_0 > v_{cr}$), the number of real roots becomes six [not shown]. This behaviour is also in accordance with that observed by Yadav et al [39], where four real roots are possible for $v_0 > v_{cr}$. So, the introduction of negative ions leads to the formation of two extra modes. These six roots correspond to six modes moving with different phase velocities.

In our case, if we take $\alpha_0 = 0.01$ and $\sigma_0 = 0.01$ with other parameters mentioned above, the beam critical velocity $v_{cr} \approx 14.877$. Six modes are obtained corresponding to $v_0 > v_{cr}$ [not shown]. From the numerical computation, it is further observed that five ion acoustic modes having phase velocities less than the beam velocity, are slow ion acoustic modes. There is only one mode which has phase velocity greater than beam velocity and hence corresponds to fast mode. The phase velocities of first five modes range from $10^7 - 10^6$ cm/s. Further, it may be mentioned that the sixth mode, which we call the fast mode possesses very large phase velocity. From our numerical computation, for the chosen set of parameters, the five modes having phase velocities lower than the beam velocity are slow ion acoustic modes. However, fast mode has phase velocity $= 1.9 \times 10^7$ cm/s. Since the range of phase velocity is of the order of electron acoustic wave [51], the fast mode here is
electron acoustic mode type which exists in a plasma with multiple populations of electrons with
different temperatures.

Corresponding to six modes of given plasma system studied here, we have six nonlinear wave
structures, five of which are slow correspond to ion-acoustic solitons and sixth fast mode
corresponds to electron acoustic solitons. The electrostatic solitary structures are observed in
different regions of the space plasma e.g. auroral zone of ionosphere as well as magnetospheric
plasma. In the following, we shall give the detailed study of these models as well as the
corresponding solitary structures. Particularly, we have numerically calculated the phase velocity,
electrostatic field and width of the solitons. It is found that the results are consistent with the direct
observations of localized parallel electric fields in a space plasma [53]. Though we have only
calculated these physical quantities associated with the sixth mode, however, the numerical analysis
can be extended to the other modes as well. We discuss all of these modes one by one. It may be
mentioned here that for $\alpha = 0$ and $\eta = Z_1 = 1$, the relation for critical velocity i.e. equation (21) reduces
to the one obtained by Yadav et al [39].

5.1. Mode - $\lambda_1$
For the soliton propagating in negative direction $\lambda_1$, the nonlinearity coefficient $Q$ of the KdV
equation (10) may become positive, negative or zero depending upon the different plasma
parameters. Therefore, compressive ($Q > 0$) as well as rarefactive ($Q < 0$) solitons are possible for
this mode. For mode - $\lambda_1$, the curve $Q = 0$, is plotted in the parameter space ($\sigma_b, \alpha_b$) for three different
sets of parameters and results are shown in figure 2(a). The other parameters for (H’ H’) plasma are
same as above. It is observed that for the above set of parameters, there exist critical values of $\sigma_b$
($\sigma_{c1}$) and $\alpha_b$ ($\alpha_{c1}$) above (below) which we always get rarefactive (compressive) solitons. Also
increase in critical velocity $v_0$ increases the region of existence of compressive solitons. It is further
observed that with increase in nonthermal parameter $\beta$, the region of existence of compressive
solitons decreases. This point is also obvious from the figure 2(b), where the variation of soliton
solution ($\phi$) with parameter $\zeta$ for seven sets of parameters is displayed. The subscript '1' here
corresponds to mode- $\lambda_1$ and similarly for other modes are taken.

![Figure 2(a). Contour plot of $Q = 0$, for mode $\lambda_1$ in the parameter space ($\sigma_b, \alpha_b$) for three different
set of parameters. The other parameters are same as in figure 1.](image-url)
It is observed from curves $a_1$ and $b_1$ that with the increase in nonthermality (i.e. $\beta$), the peak amplitude of compressive solitons increases. However, with further increase in $\beta$ (see curve $c_1$), the transition from compressive to rarefactive ($C \rightarrow R$) solitons occurs. It may be mentioned that for given values of $\alpha$, $\sigma_1$, $\sigma_2$, $\sigma_b$, and $v_0$, the transition from $C \rightarrow R$ occurs at the lower value of $\alpha_b$ [not shown]. Peak amplitude is also observed to decrease with beam velocity $v_0$ [see curves $b_1$ and $d_1$]. Thus increase in beam velocity $v_0$ decreases/ increases the amplitude of compressive/ rarefactive solitons. This behaviour is in agreement with Yadav et al [39]. From curves $b_1$ and $e_1$, it is seen that increase in electron beam temperature $\sigma_b$, leads to transition from $C \rightarrow R$ solitons. In other words, we can say that the region $Q < 0$ is encountered for these parameters. Also $\varphi_{01}$ increases with increase in relative electron beam density $a_0$ [curves $f_1$ and $g_1$] i.e. in region $Q > 0$ while in region $Q < 0$, amplitude decreases with increase in $a_0$. This behaviour of variation of $\varphi_{01}$ with $a_0$ is also understandable from figure 2(a). It is observed that for given values of $\alpha$, $\beta$, $\eta$, $\eta_1$, $\sigma_1$, $\sigma_2$ and $v_0$, $Q$ is positive for small values of $\alpha_b$ and compressive solitons are observed. With the increase in $\alpha_b$, one approaches towards the curve $Q = 0$ (i.e. $Q$ decreases) and hence amplitude (width) of the compressive solitons increases (decreases). $Q$ becomes negative with further increase in $a_0$ and correspondingly rarefactive solitons are observed. Now as one moves away from the curve $Q = 0$ (i.e. $Q$ increases) the amplitude (width) will decrease (increase).

![Figure 2(b)](image)

**Figure 2(b).** Plot of soliton solution ($\varphi$) of mode- $\lambda_1$ as a function of $\zeta$ for seven different sets of parameters ($\alpha_0$, $\beta$, $\sigma_0$, $v_0$). Here curves ‘$a_1$’ correspond to (0.005, 0, 0.05, 18), ‘$b_1$’ to (0.005, 0.1, 0.05, 18), ‘$c_1$’ to (0.005, 0.3, 0.05, 18), ‘$d_1$’ to (0.005, 0.1, 0.05, 20), ‘$e_1$’ to (0.005, 0.1, 0.1, 18), ‘$f_1$’ to (0.005, 0.1, 0.05, 18) and ‘$g_1$’ to (0.008, 0.1, 0.05, 18) respectively. The other parameters are same as in figure 1 with soliton velocity $u$ as -0.01.
A 3D-profile of the peak amplitude $\phi_{01}$ as a function of $\alpha_b$ and $v_0$ is shown in figure 2(c), from which it is seen that transition from $R \rightarrow C$ solitons occurs with $v_0$. However, the transition comes earlier for low values of $\alpha_b$.

5.2. Mode $\lambda_2$

For the soliton branch $\lambda_2$, contour plot of curve $Q = 0$ is shown in Fig. 3(a) for two different values of $v_0$ with other parameters of figure 1. Also there is critical value of electron beam temperature below (above) which $Q < 0$ ($Q > 0$) and correspondingly rarefactive (compressive) solitons exists.

![Figure 2(c). 3D plot of peak amplitude $\phi_{01}$ as a function of relative beam density $\alpha_b$ and beam velocity $v_0$ for mode $\lambda_1$ with $\beta = 0.1$ $\sigma_b = 0.05$ and $u = -0.01$ and other parameters of figure 1.](image)

![Figure 3(a). Contour plot of $Q=0$, for mode $\lambda_2$ in the parameter space $(\alpha_b, \sigma_b)$ for two different values of $v_0$. The other parameters are same as in figure 1.](image)
From figure 3(b), it is observed that for given values of \( \alpha, \sigma_1, \sigma_2 \) and \( u \), with the increase in the nonthermality parameter \( \beta \), beam velocity \( v_0 \) and \( a_0 \), the amplitude of rarefactive solitons increases [see curves \( a_2, b_2, c_2 \) and \( f_2 \)]. However, increase in electron beam temperature decreases the amplitude [curve \( d_2 \)] and further increase in \( \sigma_0 \) leads to transition from \( R \rightarrow C \) solitons as obvious from curve \( e_2 \). Thus, beam temperature is an important parameter, is also characterizing the behaviour of solitons. Further increase in \( \alpha_b \) leads to transition from \( C \rightarrow R \) solitons as obvious from curve \( e_2 \). Thus, beam temperature is an important parameter, is also characterizing the behaviour of solitons. From the three dimensional view of peak amplitude of mode- \( \lambda_2 \) \( \phi_{02} \) as a function of \( \alpha_b \) and \( v_0 \) for mode \( \lambda_2 \) with \( \beta = 0.1, \sigma_b = 0.2 \) and \( u = -0.01 \) and other parameters of figure 1.

**Figure 3(b).** Plot of soliton solution \( (\varphi)_2 \) of mode- \( \lambda_2 \) as a function of \( \zeta \) for six different sets of parameters \( (a_0, \beta, \sigma_0, v_0) \). Here curves \( 'a_2' \) correspond to \((0.1,0.1,0.1,18)\), \( 'b_2' \) to \((0.1,0.7,0.1,18)\), \( 'c_2' \) to \((0.1,0.1,0.1,20)\), \( 'd_2' \) to \((0.1,0.1,0.2,18)\), \( 'e_2' \) to \((0.1,0.1,0.24,18)\), and \( 'f_2' \) to \((0.3,0.1,0.1,18)\) respectively. The other parameters are same as in figure 1 with soliton velocity \( u \) as -0.01.

**Figure 3(c).** 3D plot of peak amplitude \( \phi_{02} \) as a function of relative beam density \( \alpha_0 \) and beam velocity \( v_0 \) for mode \( \lambda_2 \) with \( \beta = 0.1, \sigma_b = 0.2 \) and \( u = -0.01 \) and other parameters of figure 1.

5.3. Mode - \( \lambda_3 \)

For slow mode soliton branch \( \lambda_3 \), the curve \( Q = 0 \) in the parameter space \( (\sigma_0, a_0) \) is shown in figure 4(a) for three different sets of parameters. Again, it is observed that compressive as well as
rarefactive solitons are possible corresponding to \( Q > 0 \) and \( Q < 0 \) [Figure 4(a)].

The region of the existence of compressive solitons \((i.e. Q > 0)\) increases with the increase in \( v_0 \) and \( \beta \). There are two values of critical electron beam densities \((\alpha_{l3} \text{ and } \alpha_{h3})\) and electron beam temperature \((\sigma_{l3} \text{ and } \sigma_{h3})\). For region I and III, for a given value of electron beam temperature, rarefactive solitons are observed as shown in Fig. 4(b), for which peak amplitude \( \phi_{03} \) increases with increase in \( \beta \) and \( \alpha_b \) [see curves \( a_3, b_3, \) and \( e_3 \)] and is independent of \( v_0 \).

![Figure 4(a)](image)

**Figure 4(a).** Contour plot of \( Q = 0 \), for mode \( \lambda_3 \) in the parameter space \((\alpha_b, \sigma_b)\) for three different set of parameters. The other parameters are same as in figure 1.

![Figure 4(b)](image)

**Figure 4(b).** Plot of soliton solution \((\phi)_3\) of mode \( \lambda_3 \) as a function of \( \zeta \) for six different sets of parameters \((\alpha_b, \beta, \sigma_b, v_0)\). Here curves ‘\( a_3 \)’ correspond to \((0.1,0.1,0.1,17.3)\), ‘\( b_3 \)’ to \((0.1,0.7,0.1,17.3)\), ‘\( c_3 \)’ to \((0.1,0.1,0.18,17.3)\), ‘\( d_3 \)’ to \((0.1,0.1,0.22,17.3)\), ‘\( e_3 \)’ to \((0.1,0.1,0.24,17.3)\), and ‘\( f_3 \)’ to \((0.1,0.3,0.24,17.3)\) and ‘\( g_3 \)’ to \((0.3,0.1,0.1,17.3)\) respectively. The other parameters are same as in figure 1.
For a given value of electron beam density, with increase in beam temperature $\sigma_b$, $\phi_{03}$ first decreases and then starts increasing [curve $c_3$ and $d_3$]. However, transition occurs with further increase in $\sigma_b$ [not shown] and region III is encountered [curve $e_3$], that too corresponds to rarefactive solitons. With further increase in $\sigma_b$, the amplitude of rarefactive solitons decreases [not shown] and increases with $\alpha_b$ as clear from curves $f_3$ and $g_3$.

Figure 4(c). Plot of peak amplitude of mode $\lambda_3$ i.e. $|\phi_{03}|$ as a function of $\alpha_b$ for four different sets of parameters ($\beta$, $\sigma_b$, $v_0$, $u$). Here curve $m_3$ corresponds to (0, 0.15, 17.3, 0.01), $n_3$ to (0, 0.15, 17.4, 0.01), $p_3$ to (0, 0.15, 17.3, 0.012) and $q_3$ to (0.5, 0.15, 17.3, 0.01) respectively. The other parameters are same as in figure 1.

For region I ($Q < 0$), for a given set of parameters and given value of $\sigma_b < \sigma_{03}$, rarefactive solitons are obtained for which $|\phi_{03}|$ increases with increase in $\alpha_b$, reaches a maximum value and then starts decreasing [see Figure 4(c)]. This is similar one as observed by Yadav et al [39]. Gell and Nakach [48] described this as "resonance like structure". It is further observed from Figure 4(c) that increase in electron beam velocity shifts the peak of resonance to higher value of $\alpha_b$. However if the beam is kept cold, then the resonance like behaviour disappears [not shown]. The reason for this resonance like behaviour is that for $\sigma_b > \sigma_{03}$, increase in electron beam concentration $\alpha_b$ leads towards the curve $Q = 0$ and hence the value of $\alpha_b$ decreases. As amplitude of solitons is inversely related to $Q$, so increase in $\alpha_b$.

Figure 4(d). Plot of soliton solution ($\phi$) of mode- $\lambda_3$ as a function of $\zeta$ for four different sets of parameters ($\alpha_b$, $\beta$, $\sigma_b$, $v_0$). Here curves 'h_3' correspond to (0.1, 0.1, 0.21, 17.3), 'i_3' to (0.1, 0.1, 0.21, 17.35), 'j_3' to (0.1, 0.5, 0.21, 17.3), 'k_3' to (0.15, 0.1, 0.21, 17.3) respectively. The other parameters are same as in figure 1.
leads to increase in amplitude of solitons. It attains a maximum value and then starts decreasing. Nonthermality also plays an important role in characterization of the behaviour of solitons. It may be mentioned here that the width also follows the resonance like structure [not shown]. However, increase in soliton velocity increases the amplitude of soliton. For region II ($Q > 0$), for $\sigma_{b1} < \sigma_b < \sigma_{b2}$ and fixing other parameters, we see that compressive solitons are observed as shown in Figure 4(d) where a plot of $\phi_j$ as a function of $\zeta$ is shown. We see that for given value of $\sigma_b$, peak amplitude decreases with increase in $\beta$ and $v_0$ [see curves $h_3, i_3, j_3$]. Curve $k_3$ shows that $\phi_{0h}$ increases with $\alpha_b$.

5.4. Mode – $\lambda_4$

For this mode, the curve $Q = 0$ is plotted in figure 5(a). The behaviour is of this mode is identical to that of mode – $\lambda_1$. Increase in beam velocity increases the region of existence of compressive solitons while increase in nonthermality decreases it. For a given value of $\alpha_b$, there exists a critical value of beam temperature above/below which rarefactive/compressive solitons exist. In the region $\sigma_b < \sigma_{c4}$, for a given value of $\alpha_b$ and other parameters fixed, the variation of peak amplitude $|\phi_{04}|$ with $\alpha_b$ is shown in figure 5(b).

From curves $a_4 (a_4')$ and $b_4 (b_4')$ [figure 5(b)], we see that introduction of $\beta$ decreases (increases) the amplitude (width) of solitons. Curves $a_4 (a_4')$ and $e_4 (e_4')$ show that amplitude (width) increases (decreases) with increase in electron beam velocity $v_0$. Curves $a_4 (a_4')$ and $e_4 (e_4')$ show that finite beam temperature reduces the amplitude of solitons. Further, with increase in $u$, increases the amplitude.

![Figure 5(a). Contour plot of $Q = 0$, for mode $\lambda_4$ in the parameter space ($\alpha_b, \sigma_b$) for three different set of parameters. The other parameters are same as in figure 1.](image)

For $\sigma_b > \sigma_{c4}$ and for given parameters, only compressive solitons are observed for which the absolute value of peak amplitude decreases with $\beta$ and $\alpha_b$ [not shown]. It is further observed that amplitude increases with $v_0$ and soliton velocity $u$. correspondingly, the width decreases with $\alpha_b$ and $\beta$ and increases with $v_0$ and $u$.  

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5.5. Mode – $\lambda_5$

It is observed that only compressive solitons are obtained for this slow mode for which peak amplitude decreases with nonthermality and relative density of electron beam as is obvious from curves $a_5$, $b_5$ and $c_5$ in figure 6(a). Also the peak amplitude increases with increase in beam velocity $v_0$ and beam temperature $\sigma_b$ [see curves $d_5$ and $e_5$]. From Fig. 6(b), where a 3D profile of peak amplitude $\varphi_{05}$ as a function of $v_0$ and $a_b$ is given, effect of parameters $v_0$ and $a_b$ again becomes clear.

Figure 5(b). Plot of peak amplitude of mode $\lambda_4$, i.e. $|\varphi_{0i}|$ as a function of $a_b$ for five different sets of parameters ($\beta$, $\sigma_b$, $v_0$, $u$). Here curves $a_4$ ($a_4'$) corresponds to (0, 0.05, 21.5, 0.01), $b_4$ ($b_4'$) to (0.1, 0.05, 21.5, 0.01), $c_4$ ($c_4'$) to (0, 0.05, 22, 0.01), $d_4$ ($d_4'$) to (0, 0.05, 21.5, 0.015), and $e_4$ ($e_4'$) to (0, 0.0, 21.5, 0.01) respectively. The other parameters are same as in figure 1.

Figure 6(a). Plot of soliton solution $(\varphi)_5$ of mode- $\lambda_5$ as a function of $\zeta$ for five different sets of parameters ($a_b$, $\beta$, $\sigma_b$, $v_0$, $u$). Here curves ’$a_5$’ correspond to (0.05,0.1,0.3, 20,0.01), ’$b_5$’ to (0.05, 0.3, 0.3, 20, 0.01), ’$c_5$’ to (0.1, 0.1, 0.3, 20, 0.01), ’$d_5$’ to (0.05, 0.1, 0.32, 20, 0.01), and ’$e_5$’ to (0.05, 0.1, 0.3, 19.5, 0.01) respectively. The other parameters are same as in figure 1.
5.6. Mode – $\lambda_6$

For this fast mode, only rarefactive solitons are observed. This is consistent with the observations made by the Freja Satellite [40], where observed rarefactive solitons were explained in form of nonthermal particle distributions. The variation of soliton solution $(\phi)_6$ as a function of $\zeta$ is shown in figure 7 for five different sets of parameters corresponding to the curves $a_6$ and $b_6$. It is observed that with increase in $\beta$, peak amplitude as well as width of soliton increases. From curves $a_6$ and $c_6$, it is observed that with increase in beam temperature, amplitude as well as width of soliton decreases. Also curves $a_6$ and $d_6$ represent that peak amplitude and width increases with increase in the relative density of electron beam. Further, the peak amplitude/ width of soliton increases/ decreases with soliton velocity $u$ [curve $e_6$]. It is further observed that the finite ion temperatures and electron beam velocity have negligible effect on the amplitude and width of solitons. This behaviour is similar to the one observed for the fast mode by Yadav et al [39].

![Figure 6(b). 3D plot of peak amplitude $\phi_{05}$ as a function of relative beam density $a_b$ and beam velocity $v_0$ for mode $\lambda_5$ with $\beta = 0.1$, $\sigma_0 = 0.3$, and $u = -0.01$ and other parameters of figure 1.](image)

![Figure 7. Plot of soliton solution $(\phi)_6$ of mode-$\lambda_6$ as a function of $\zeta$ for five different sets of parameters $(\alpha_b, \beta, \sigma_b, v_0, u)$. Here curves ‘$a_6$’ correspond to $(0.05, 0.1, 0.3, 18, 0.01)$, ‘$b_6$’ to $(0.05, 0.3, 0.3, 18, 0.01)$, ‘$c_6$’ to $(0.05, 0.1, 0.5, 18, 0.01)$, ‘$d_6$’ to $(0.1, 0.1, 0.3, 18, 0.01)$, and ‘$e_6$’ to $(0.05, 0.1, 0.3, 18, 0.02)$ respectively. The other parameters are same as in figure 1.](image)

The numerical computation has been carried out for $T_e = 1\text{eV}$, $n=10^4 \text{ cm}^{-3}$, $\sigma_1=0.1$, $\sigma_2=0.01$, $\sigma_0=0.3$, and $u=0.01-0.1$, $v_0=18$, $\beta=0.1$, $\alpha=0.1$. For these parameters, the phase velocity of the electron
acoustic mode is ~ $1.9 \times 10^7$ cm/s. Further the debye length hand width of the solitary structures are respectively 8.2 cm and 80.1 cm. These observations qualitatively agree with the FAST Satellite [54,55].

6. Conclusions
In the present investigation, we have studied a plasma system consisting of both electron and energetic electron population described by nonthermal electron distribution in a plasma with positive and negative ions. We have derived dispersion relation of this plasma system to study the mode structures existing in such plasma. It is found that for beam velocity greater than some critical value, the six linear modes are obtained, five of which are slow and one is fast. Corresponding to the linear modes $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, compressive as well as rarefactive solitons are obtained for the chosen set of parameters. However, for mode $\lambda_5$ only compressive solitons are observed while for mode $\lambda_6$, only rarefactive solitons are obtained. The relative negative ion density $\alpha$, ion temperatures $\sigma_1$ and $\sigma_2$, charge multiplicity ratio $\varepsilon$, nonthermal parameter $\beta$, beam velocity $v_0$, beam temperature $\sigma_b$ and relative beam density $\alpha_b$ affect the behaviour of the solitons. The effect of all these parameters is discussed in detail.

Numerical calculations are done for the fast acoustic mode for characterizing its nature. It is found that this mode is electron-acoustic type, reminiscent of multi electron population plasma system. Further, numerical analysis show that the localized structures for the chosen set of parameters has electric field ~ $207 \text{mV/m}$, width is ~ 80.1 cm and phase velocity is ~ $1.9 \times 10^7$ cm/s. They qualitatively compare well with the direct observations of localized parallel electric field in the space plasma [53]. However, some limitations of the present model are listed as follows:

(i) Collisions are neglected here.
(ii) The $KdV$ model shown here is limited to study small amplitudes. However, Sagdeev Pseudopotential can be used to study large amplitudes [54].
(iii) Model used here is applicable for study of electrostatic localized structures and cannot be used for electromagnetic structures.
(iv) Magnetic field, which usually permeates the space plasma, has not been considered in the present investigation.

However, this model may be used for studying the electric field structures in space plasmas.

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