The hardware function of the URAGAN muon hodoscope

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Abstract. The problem of the hardware response function calculation for the URAGAN hodoscope (MEPhI) is analyzed. The simplified hodoscope model is developed for which the response function is found out analytically. It is shown that the presence of non-detecting intervals (gaps between detectors) leads to the response function that includes azimuthal dependence. A numerical procedure of the response function calculation is developed for the URAGAN hodoscope. The result is in correlation with the averaged matrix data from the hodoscope. The application of response function analyses to detecting of reduced muon flux areas is demonstrated. Further refinement of response function calculation is discussed.

1. Introduction

Muon hodoscope is a device which is able to measure both individual muon tracks and, under continuous operation, angular distribution of muon flux in wide solid angle range. The design of hodoscope involves a number of particle detectors located at two or more parallel planes.

Muons are generated in upper atmosphere by primary cosmic rays as a product of secondary pion decay. Thus the observation of muon flux intensity and its angular distribution \cite{1} provides important information about the processes in heliosphere, particularly, may serve the basis for magnetic storms forecast. Muon flux is also imprinted by atmospheric processes and muon flux observations may be used for weather forecasts \cite{2}. Recently muon hodoscope was successively used for detection of previously unknown cavity (probably chamber or gallery) in Khufu’s Pyramid \cite{3}.

An important aspect of hodoscope data interpretation is a correct account for the distortions introduced by the device itself, so called hardware response function. It may be defined as angular distribution which would be measured in the case of purely isotropic muon flux. The direct calibration of hodoscope seems impossible since we don’t possess any standard muon source which generates isotropic muon flux and is strong enough to accumulate sufficient statistical data. The natural muon flux is approximately isotropic over azimuth but has strong dependence on zenith angle, moreover it is subject to diurnal variations and the influence of atmosphere. Thus theoretical estimations of response function are of great importance.
the present paper we consider theoretical estimation of response function for URAGAN muon hodoscope.

2. URAGAN hodoscope design
The detecting unit of URAGAN muon hodoscope [4, 5] is gas-discharge chamber of square cross-section (8.95x8.95 mm inner and 9.95x9.95 outer) and 3.5 m length. The inner surface of the chamber is covered with graphite, anod wire is spanned along the axis. The chambers are located in 8 parallel planes, 320 chambers in each plane, the coordinates of chamber’s axes (in mm) equal to
\[ x_{ij} = 9.95 \cdot i + 3.15 \cdot [i/8] + 4.47 \cdot [i/16], \quad i = 0, \ldots, 319, \]
\[ z_{ij} = 65.5 \cdot j, \quad j = 0, \ldots, 7. \]
([...] denotes integer part, z axis is vertical, y axis points along the chamber length.) Thus there are non-detecting gaps after each 8-chambers set, the widths of odd gaps are 3.15 mm, the widths of even ones are 7.62 mm.

To prevent anod wire from sagging the so-called holders are used: 6 plastic plugs of 5 mm wide distributed homogeneously along the chamber length, coordinates of holder’s centers (in mm) equal to
\[ y_k = 525 + 490 \cdot k, \quad k = 0, \ldots, 5. \]
The set of all holders in a given plane form 6 additional non-detecting gaps perpendicular to the gaps between chambers.

The chambers operate in streamer mode. The signal is inducted on so-called strips: aluminium bands located over and below each chambers plane. The strips parallel to chambers are called X-strips, their number is equal to chamber’s number (320), their width is equal to chamber’s width and they are located exactly over the chambers. The strips perpendicular to chambers are called Y-strips, their number equals 288, width equals 12 mm and their center’s coordinates (in mm) equal to
\[ y_m = 12 \cdot m + 3.05 \cdot [m/16], \quad m = 0, \ldots, 287. \]
The location of Y-strips is not correlated with location of holders. Several X- and Y-strips in the same plane may be activated under oblique muon incidence.

The design of URAGAN hodoscope has two main features: discrete detectors of rectangular shape and irregular detector location with the presence of non-detecting gaps.

The raw hodoscope data is the set of activated strips. However the main purpose is to measure muon flux angular distribution so the raw data is converted first to muon track (the straight line that best fits the activated strips) and then to muon matrix: the 76 rows by 90 columns table with the number of muon’s tracks in given 1° zenith and 4° azimuth angle intervals in each cell.

3. Averaged muon matrix features
Muon matrices averaged over extended time interval (hour and longer) exhibit, apart from zenith dependence, also weak azimuth variations. On figure 1 90° and 180° periodical azimuthal variations as well as rib-like ornament are clearly visible. In what follows we investigate the influence of two main hodoscope design features, the discreteness and rectagular shape of detectors and the presence of non-detecting gaps on measured hodoscope data. For the purposes of such investigation the exact number of planes and vertical thickness of detectors as well as total horizontal dimensions of the device are in our opinion of secondary importance. Thus we consider two simple models in both detectors being located in two infinite parallel planes and have zero vertical thickness. In these simplified models, contrary to original design, both modelling of activated strips and muon track reconstruction are very simple since muon is able
to activate only one X- and one Y-strip in each plane. We assume detectors are rectangular with $p$ and $q$ dimensions, let the distance between the planes equals $l$. The first model assumes that detectors densely fill the plane, the second one assumes that periodical non-detecting gaps both in $x$- and $y$-directions occur. We find that neither rectangular detector’s shape (in the case of high angular resolution, $l \gg p,q$) no rectangular shape of the device itself lead to azimuthal dependence. It is found that azimuthal dependence of hodoscope response function is due to presence of non-detecting gaps. We also consider numerical simulation of URAGAN response function completely taking into account all its design features. The result of this simulation is in accordance with measured hodoscope data. We discuss the possibilities of further refinement of response function evaluation.

4. The first simple model: two planes densely filled with rectangular detectors

Let two infinite parallel horizontal planes, one at distance $l$ from another, be densely filled with detectors of $p \times q$ size. Let muon flux be isotropic in upper half-space, in other words, let $\cos \theta \sin \phi \, d\phi \, d\theta$ muons with tracks in zenith angle interval from $\theta$ to $\theta + d\theta$ and azimuth angle interval from $\phi$ to $\phi + d\phi$ pass the square $dx \, dy$ in one second (factor $\cos \theta$ accounts for oblique incidence, $\sin \theta \, d\phi \, d\theta$ is infinitesimal solid angle). Detectors in each plane may be enumerated with pairs of integer numbers $(m, n)$. The event of muon registration is then described by quartet $(i, j, m, n)$, in which first two numbers point to detector in one plane and next two in the other. Because of homogeneity we may restrict ourselves to events $(0,0,m,n)$. The calculation of response function is divided into two steps. On the first step we calculate probabilities of events $(0,0,m,n)$ with different $m$, $n$, on the second we reconstruct muon track. The second step is trivial: it is naturally to assume that the track passes through the centers of activated detectors, spherical angles $(\theta, \phi)$ of the track satisfying $pm = l \tan \theta \cos \phi$, $qn = l \tan \theta \sin \phi$. The calculation of probability requires integration of muon flux distribution over the squares of detectors in one and another planes, the integral takes the form

$$P(0,0,m,n) = \frac{1}{pq} \int_0^p dx \int_0^q dy \int_0^p dx' \int_0^q dy' \frac{l^2}{[l^2 + (pm + x - x')^2 + (qn + y - y')^2]^2}.$$ 

Though the integration can be carried out in elementary functions it is much more illustrative to expand the result in powers of $(p,q)/l$. The reconstructed muon tracks distribution, $N(\theta, \phi) = P(0,0, (l/p) \tan \theta \cos \phi, (l/q) \tan \theta \sin \phi)$, equals

$$N(\theta, \phi) = \frac{pq \cos^4 \theta}{l^2} + \frac{pq \cos^6 \theta}{3l^4} [6 \sin^2 \theta (p^2 \cos^2 \phi + q^2 \sin^2 \phi) - (p^2 + q^2)] + \ldots$$  (1)
Figure 2. Response function for simplified two-plane model with non-detecting intervals. $A = 8, a = 0.8, B = 50, b = 0.5, l = 50$ cm.

The leading term does not depend on azimuth. As for altered zenith dependence it just comes from the change of variables $(p, q) \rightarrow (\theta, \phi)$ with Jacobian equal to $D(p, q)/D(\theta, \phi) = (l^2/pq)\sin \theta/\cos^3 \theta$. The next-to-leading term, dependent on azimuth, is of order of magnitude $10^{-3}$ in comparison with leading term (for high angular resolution, $p, q \sim 1$ cm, $l \sim 50$ cm) and is far beyond hodoscope measuring accuracy.

The dependence on azimuth also comes from account for finite dimensions of device. Muon registered in the upper plane is not necessarily registered in the lower one in the case of oblique incidence. If hodoscope has rectangular shape with dimensions $P$ and $Q$ additional factor $\frac{(P - p|m|)(Q - q|n|)}{PQ}(P - l\tan \theta \cos \phi)(Q - l\tan \theta \sin \phi)$ appears in formula for $N(\theta, \phi)$. However this factor is also unable to explain measured azimuthal dependence. We have to look for some other source.

5. The second simple model: periodical non-detecting gaps

Bearing in mind the experience of the first model consideration we do not take into account the discreteness of detectors and finite dimensions of the device while considering the model with non-detecting gaps. We just take the leading term from (1). We assume that non-detecting gaps are located periodically with period $A$ along $x$ axis and period $B$ along $y$ axis, the widths of gaps being $a$ and $b$ correspondingly. Then the presence of the gaps leads to additional factor equal to the ratio of the non-shadowed by gaps square to the full square of $A \times B$ cell. Calculating shadowed square one should keep in mind that the bars of one plane are shifted relatively to the bars of the other by the amounts of $l\tan \theta \cos \phi$ and $l\tan \theta \sin \phi$ along $x$ and $y$ axes correspondingly.

The result of calculation depends on if the bars overlap or not. Let’s introduce periodical function (assuming $c > 2$)

$$g(x, c) = \begin{cases} 
  x, & 0 < x \leq 1, \\
  1, & 1 < x \leq c - 1, \\
  c - x, & c - 1 < x \leq c.
\end{cases}$$
Then response function can be written in the form

\[ N(\theta, \phi) = \frac{pq \cos^4 \theta}{l^2} \left( 1 - \frac{a}{A} - \frac{a}{A^2} g \left( \frac{l}{a} \tan \theta \cos \phi, \frac{A}{a} \right) \right) \left( 1 - \frac{b}{B} - \frac{b}{B^2} g \left( \frac{l}{b} \tan \theta \sin \phi, \frac{B}{b} \right) \right). \]

Figure 2 shows that this function reproduces the features of measured muon matrix. Thus we come to the conclusion that exactly the non-detecting gaps are responsible for azimuthal dependence of muon matrices.

6. Numerical simulation of URAGAN response function

Due to complexity of URAGAN design it is impossible to calculate its response function analytically, however it may be successively simulated numerically [6]. The result of simulation is shown on figure 3. The simulation was carried out as follows.

At the first step random muon track was generated. The track is a line segment with the ends in two planes above and below all detectors. Standard uniform random numbers on \([0, 1]\) generator was used to generate four random numbers. The first number was multiplied by 2\(\pi\) and represented \(\phi\) azimuth angle. The second random number \(F\) was used to calculate zenith angle \(\theta = \arccos F^{1/(1+\alpha)}\). This angle is distributed with density \(\cos \alpha \theta\), \(\alpha = 0\) corresponds to uniform distribution and is used to simulate response function, \(\alpha \approx 2.4\) is used to account for zenith dependence of incident muon flux in order to compare directly with hodoscope data. Two more numbers were multiplied by dimensions of the device and represented \(x\) and \(y\) coordinates of muon incident point in upper plane.

At the second step the segments of track inside detectors were determined. Under the detector we assumed the part of chamber between two neighbouring holders. The set of planes bounding the detectors were introduced: 8 pairs perpendicular to \(z\) axis, 320 pairs perpendicular to \(x\) axis and 7 pairs perpendicular to \(y\) axis. The determination was carried out sequentially: the segments of track between planes perpendicular to \(z\) axis were determined, then their subsegments between planes perpendicular to \(x\) and finally subsubsegments between planes perpendicular to \(y\).

At the third step we determine activated strips. We assume that the strip is activated if it overlaps one of the track segments inside detectors in the corresponding plane. The set of activated strips is a simulation of the hodoscope raw data.

The subsequent track reconstruction is performed in accordance with real data processing. The raw sets containing strips in less then 5 planes are omitted. If several \(X\)- (and/or \(Y\)-) strips in the same plane are activated we assume one \(X\)- (and/or \(Y\)-) strip is activated with coordinate equal to mean activated strips coordinates value. Least squares method is applied to reconstruct track direction separately in \(xz\) and \(yz\) planes. Namely, the track projection angle \(\theta_x\) in \(xz\) plane is found by minimization of \(\sum_n (x_n \cos \theta_x - z_n \sin \theta_x - a_x)^2\) with respect to fitting parameters \(\theta_x, a_x, x_n\) and \(z_n\) being coordinates of activated \(X\)-strips, the same for \(yz\) plane. Track projection angles \(\theta_x\) and \(\theta_y\) are transformed into standard spherical angles \(\theta\) and \(\phi\) according to \(\tan \theta_x = \tan \theta \cos \phi, \tan \theta_y = \tan \theta \sin \phi\). The tracks that don’t cross all 8 detectors planes are omitted.

The result of simulation is in qualitative agreement with observed data though quantitative agreement is not that good. Our simulation was purely geometrical and we attribute the deviation to simplified assumption which strips are activated. This step of modelling requires more accurate account for physical processes in the gas-discharge chamber itself as well as the processes of strips signals induction. We hope that further development of these issues will improve quantitative agreement between simulations and observed data.

Figures 4 and 5 demonstrate how the response function can be used to detect the areas with reduced/enlarged muon flux. Typically the variation is very small, 5% and lower, and is shadowed by poor statistics. One can see that data to response function ratio has much better.
Figure 4. Artifi-cally simulated data with reduced by 5% muon flux in the range 20 < θ < 40, 240 < ϕ < 300. The reduced flux area is shadowed by poor statistics.

Figure 5. Data to response function ra-tio. The reduced flux area is clearly seen.

signal to noise ratio then the data itself. Further noise reduction may be achieved by filtering based on local rolling approximation models [7, 8].

7. Conclusion
URAGAN muon hodoscope response function theoretical estimation is considered. Two simplified hodoscope models are proposed for which the response function can be found analytically. These analytical solution demonstrate, on one hand, that azimuthal dependences of hodoscope data can not be attributed to the discreteness and rectangular shape of detectors as well as rectangular shape of the device itself, and, on the other hand, that the presence of non-detecting gaps leads to azimuthal dependence similar to that of observed data. The response function of URAGAN hodoscope is simulated numerically. The result is in agreement with muon matrices averaged over extended time interval. Further re-fi nement of response function simulation requires more accurate account for physical processes in gas-discharge chambers and strips.

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