A Spectrum Sensing Technology Exploiting Multiple Large Eigenvalues and Stochastic Resonance

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Abstract - Cognitive radio (CR) is a hopeful technology to reduce the negative effect of spectrum scarcity caused by the enormous amount of wireless mobile device. Spectrum sensing could alleviate the interference and harm for primary user, and enhance the wireless access capability. This paper proposes a novel spectrum sensing method by using the summation of multiple large eigenvalues (SMLE) and stochastic resonance (SR). SR is used to enforce the detection signal of multiple antennas in low SNR condition. Then the sample covariance matrix of the enforcing detection signals and its multiple large eigenvalues are computed for constructing test statistic. The simulation results demonstrate that the proposed detector based on SMLE and SR is superior than the existing detector based on SMLE, and is robust in strong noise background.

1. Introduction

Wireless mobile network is applied in a variety of important communication domain. But the relevant frequency resources are occupied by a large amount of mobile terminal devices. Spectrum resources are facing serious shortage issue in most sub GHZ Frequency bands which have been located fixedly [1]. Cognitive radio is considered to be a promising technology to relieve this challenge. It could also reduce the interference for authorized frequency band [2]. Accurate and faster spectrum sensing is the significant technology to guarantee the implement of cognitive radio in a variety of spectrum sensing methods. Energy detection and matched filter detection need to estimate the noise power, and are easily affected by the noise uncertainty [3]. To estimate the noise power, the noise only samples technology is applied during a long observation period when there is no primary user [4].

The future research found that the covariance based detector could avoid the noise power estimating problem [5]. This is because multiple path and oversampling give rise to the autocorrelation among the multiple antennas. The eigenvalue based detector is often proposed in the framework of generalized likelihood ratio test (GLRT). Reference [6] proposed the maximum eigenvalue detector (MED), and found that the maximum eigenvalue asymptotically obeyed the Tracy Widom (TW) distribution. Meanwhile the closed form expression of false alarm probability and detection probability of MED is derived. Reference [7] proposed mean-to-square extreme eigenvalue (MSEE) detector, which only employs the maximum and the minimum eigenvalues, and achieved lower computational complexity. However, the eigenvalue based detectors above are proposed based on the assumption that the rank of covariance matrix of primary signals is one. So multiple primary user model emerged in which the rank is more than one. Some algorithm considered this model. The arithmetic to geometric mean (AGM) eigenvalue based detector and weighted AGM were proposed to handle spectrum sensing in sparse sample and multiple rank scenarios [8]. Reference [9] proposed
eigenvalue based detectors with higher order moments to enhance the detection performance but brought about high computational complexity.

The relative research further indicated that the covariance matrix of received signal usually performed more than one larger eigenvalue [10]. The eigenvalue number can be estimated by some method, such as akaike information criterion, minimum description length criterion, or observing the eigenvalue distribution of sample covariance matrices [11]. Reference [12] proposed a signal subspace eigenvalues (SSE) detector, and adopt multiple large eigenvalues. but it depend on the knowledge of noise power, which result in the degenerated performance when the noise power is estimated. Reference [13] propose a new detector which employs multiple large eigenvalues based on the rank of the sample covariance matrix, and derived the distribution of the sum and product of the multiple large dependent eigenvalues of sample covariance matrices, and analyzed the theoretical performance when the noise powers is estimated.

Despite eigenvalue based spectrum sensing algorithms can improve sensing quality under the signal to noise ratio (SNR) wall or noise uncertainty conditions [14], in the circumstance of low SNR, eigenvalue based detector could only increase the antenna number to compensate the deterioration of spectrum sensing performance. Therefore, the design cost and complexity of wireless mobile device will be augmented.

Stochastic resonance (SR) is an important technology extracting weak signal from intense noise [15]. SR is a nonlinear physical and dynamical behavior. The output of SR is determined by the dynamic characteristics, i.e., noise level, SR system and input signal. When the noise power is proper, the system will become a desired state and the output signal can be improved. In SR, noise is no longer a disturbance and harm for signal quality. The SNR and power of weak signal could be amplified in the act of nonlinear SR system, which results in the increasement of signal detection ability rationally. the SNR wall and signal sample length could be alleviate in the aid of SR.

SR has been extensively applied to spectrum sensing in weak signal condition. Energy detection based on fixed parameter SR and adaptive parameter SR is proposed and analyzed in [15]. Adding an appropriate noise or adjusting the parameters in SR based CR terminal, the weak primary signal can be easily detected even in fading channel. Additionally, other detectors are further proposed, e.g., detector based on particle swarm algorithm and tri-stable SR [16], detector based on optimal dynamic overdamped bistable SR [17], detector based on polarization and SR [18]. However, the classical SR theories point out that the input signal of SR system could only worked in low frequency and small parameters [15], which restrict the actual wireless communication application. So frequency shifting technologies of SR are often applied to convert high frequency to low frequency equivalently.

This paper designed a detector based on multiple larger eigenvalues (SMLE) and SR. Firstly, multiple antenna signals are processed via SR system, in which normalized scale transformation (NST) frequency shifting scheme [19] is exploited. Secondly, the proposed detector employs multiple larger eigenvalues according to the rank of sample covariance matrix. The threshold is obtained by the theoretical results of false alarm probability. The simulation results are provided to demonstrate the superior performance of the proposed SMLE-SR detector compared to SMLE detector.

The notations are explained as follows: Boldface lowercase letters denote vectors. Boldface uppercase letters denote matrices. \(N(a, b)\) denotes Gaussian distribution with mean \(a\) and variance \(b\). The superscripts \((\cdot)^T\) denotes the transposition. \((\cdot)^{-1}\) denotes the inverse of matrix. \(\text{Tr}(\cdot)\) denotes the trace of matrix. \(\text{det}(\cdot)\) denotes the determinant of matrix. \(\text{diag}(a_1, ..., a_M)\) denotes a diagonal matrix and the diagonal elements are \(a_1, ..., a_M\).

2. System model
The system of spectrum sensing is usually modeled as a binary hypothesis problem:

\[
f(r(n)) = \begin{cases} 
    f(w(n)) & \mathcal{H}_0 \\
    f(sm(n) + w(n)) & \mathcal{H}_1 
\end{cases}
\]
When the primary users are absent, we make a decision with $\mathcal{H}_0$; when the primary users are present, we make a decision with $\mathcal{H}_i$. $r(n)$ denotes the received signal vector of a secondary user sampled in discrete time and baseband. $r(n)$ can be obtained from multiple antennas at time instant $n$. Supposing that $M$ is the antenna number and $r(n) = [r_1(n), ..., r_M(n)]$. In which, $n = 0, 1, ..., N_s - 1$ and $N_s$ is the length of sensing duration. $s(n) = [s_1(n), ..., s_M(n)]$ denotes the nth Sample vector of the primary signals captured at the secondary user. $w(n)$ denotes the additive circularly symmetric white Gaussian noise with mean zero and covariance $\sigma_w^2 I_M$. The distribution is $w(n) \sim \mathcal{N}(0, \sigma_w^2 I_M)$. $s(n)$ denotes the integral multiple primary signal via channel response. Meanwhile $w(n)$ is independent with $s(n)$. The received primary signal vector $s(n)$ can be symbolized as [13]:

$$s(n) = H * s_p(n)$$

where $s_p(n) = [s_{p1}(n), ..., s_{pP}(n)]^T$ denotes the primary signals vector of $p$ primary users. The matrix $H = [h_1, ..., h_M]$ denotes the channel response between the $M$ receiving antennas and the $p$ primary users. The channel response matrix $H$ is supposed to be constant during this sensing time, but differ from other sensing time. $f(.)$ is a nonlinear function which will be introduced in next section. The SNR is described as [13]:

$$SNR = \frac{Tr(R_{sm})}{M\sigma_w^2}$$

In which, $R_{sm} = \frac{1}{N_s} \sum_{n=0}^{N_s-1} s(n)s^T(n)$ is the sample covariance matrix.

### 3. Proposed Method:

This Section will introduce the proposed spectrum sensing method SMLE-SR. Firstly, the received signal in each antenna $r_i$ $(i = 1, ..., M)$ will be processed via the SR system. In order to recover the original signal furthest from intensive noise. The output signal vector is defined as $x(n) = [x_1(n), x_2(n), ..., x_M(n)]^T = f(r(n))$, where $f(.)$ is a nonlinear function of SR and can be defined by Langevin equation [15]:

$$\frac{dx_i(n)}{dt} = -\frac{U(x_i)}{dx_i} + s_m(n) + w_i(n)$$

Where $U(x_i)$ denotes the potential function: $U(x_i) = -\frac{a}{2} x_i^2 + \frac{b}{4} x_i^4$, and $a > 0$ and $b > 0$; $i$ denotes the $i$th antenna; $A_m$ and $f_c$ denotes amplitude and carrier frequency of weak signal. Equation (4) indicates the classic SR model, which is driven by three basic elements: the bistable nonlinear system, the Gaussian white noise $w_i(n)$, and the periodic excitation $s_m(t)$. There is a pair of symmetrical minimum values in $U(x_i)$ with the coordinates of $(x_m = \pm \frac{a}{\sqrt{4b}}, -U_0 = a^2/(4b))$. Two minimum value points are nominated as potential wells. Since these two potential wells represent two stable states of the SR system. So equation (4) is also named as bistable SR. In the coordinates of potential wells, $U_0$ denotes the potential barrier height. Under the effect of external signal $s_m(t)$, the balance of $U(x_i)$ will be affected, and potential wells will elevate alternatively and periodically. The classical SR theory, like adiabatic approximation theory and nonlinear response theory, regulate that SR could only worked in small parameters [19], that means:

1) The system parameters $a$ and $b$ must lie in the interval of $(0, 2)$

2) The signal frequency, signal amplitude, and noise power must be small, i.e., the Carrier frequency $f_c \gg 1$ Hz.

These conditions are opposite with the high frequency wireless communication. Thus, we exploit the normalized scale transformation (NST) method to convert high frequency to low frequency [19]. Let us review the NST theory. If $s_m(t) = A_m \cos(2\pi f_c t)$ is treated as a periodical signals, and the normalization and variable substitutions are shown as follows [19]:

$$\frac{dz_i(t)}{dt} = z_i - z_i^3 + A_0 \cos(2\pi f_0 \tau) + w_0(\tau)$$
Where \( z = x\sqrt{a/b}; \tau = at \) is the re-sample time interval; \( A_0 = \sqrt{b/a^2}A_t \) is the normalized amplitude; \( w_0(\tau) \) is the normalized noise with expectation 0 and variance \( \sigma_0^2 = \frac{\sigma_z^2}{b/a^2} \). In which, \( \sigma_z^2 \) is the variance of SR output signal. Equation (5) is the standard normalized form of equation (4), and they have the same dynamic characteristics. However, the main significances and contributions lie in that equation (5) can satisfy the preconditions of the adiabatic approximation theory. Because when \( a >> 1 \), the normalized frequency \( f_0 \) is 1/\( a \) times the original frequency \( f_c \), which can ensure the high carrier frequency signal to turn into a low frequency one and satisfy the precondition of small parameter. Through the preset of \( f_0 \) and \( A_0 \), \( a \) and \( b \) could be obtained. In addition, adjusting \( A_0 \) based on input SNR will achieve the desired output state.

In general, equation (5) is an expression form of one order ordinary differential equation, thus the exact solutions can not be obtained. However, it can be approximately numerical solved by the fourth order Runge-Kutta (RK), which is a process of multi-stage iteration [15].

Next we calculate the sample covariance matrix Of SR output signal \( x(n) \) as \( R_x = \frac{1}{N_s} \sum_{n=0}^{N_s-1} x(n)x^H(n) \). Next calculating the eigenvalues of the sample covariance matrix \( R_x \) and order the eigenvalues as \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \). It is already shown that the sample covariance matrix in practical applications like DTV signals has multiple large eigenvalues [10]. So we assume that the rank of covariance matrix \( R_s \) of the primary signals is \( p(p < M) \). So we could only use the \( p \) larger eigenvalues for detection [13]:

\[
T_{SMLE} = \frac{\sum_{l=1}^{p} \lambda_l}{\sigma_0^2}
\]

4. The calculation of threshold.

When the sample number is large enough and the input noise power is suitable, the output noise will obey by Gaussian distribution. The decision threshold \( th \) could be approximately obtained by the false alarm probability \( P_f \) [13]:

\[
th \approx \mu_{x|H_0} + \sigma_{x|H_0} Q^{-1}(P_f)
\]

Where \( Q^{-1}(\cdot) \) denotes the inverse of Q function. \( \mu_{x|H_0} \) and \( \sigma_{x|H_0} \) is the mean and standard variance of output SR noise:

\[
\mu_{x|H_0} = \frac{M(1 - v^p)}{1 - v^M}, \quad \sigma_{x|H_0} = \frac{M\sqrt{(1 - v)(1 - v^2p)}}{(1 - v^M)^{\frac{1}{2}}}2N_s(1 + v)
\]

In which

\[
v = \exp\left(-2\frac{15}{2(M^2 + 2)}\left(1 - \frac{4M(M^2 + 2)}{5N_s(M^2 - 1)}\right)\right)
\]

5. Simulation Results

In this section, we evaluate the proposed SMLE–SR detector by comparing with SMLE detector. Figure 1 test the effect of SR in arbitrary antenna when the input signal is \( r \) and the output signal is \( x_1 \). The parameters set as follows: The input signal is periodical and the amplitude is \( A_m = 3 \); The carrier frequency is \( f_c = 10 \) Hz; The normalized amplitude is \( A_0 = 0.3 \); The normalized frequency is \( f_0 = 10^{-2} \) Hz; \( SNR = -16 \) dB; The sample frequency is \( f_s = 10 \) KHz; The sample number is \( N_s = 4096 \). It can be found that the weak signal buried in noise is recovered well. because the critical value of normalized BSR system is \( A_c = 0.344 \) [15] while \( A_0 = 0.3 < A_c \). That means the signal itself can not cross the potential barrier to generate SR effect and the suitable noise is needed.

Figure 2 compare the detection probability \( P_d \) versus \( SNR \). In which, the antenna number is \( M = 5 \); The rank of primary signal is \( p = 4 \); The frequency band of primary user is [10 20] Hz. The sample number is \( N_s = 1024 \); The force alarm ability is \( P_f = 0.01 \); the channel matrix \( H \) is
constructed to make the signal $s_m$ obeyed by the Gaussian distribution. The Monte Carlo simulation times is 1000. It is shown that SMLE-SR exhibit the superior performance comparing with SMLE.

Figure 3 compare the detection probability $P_d$ versus rank $p$. In which $M = 10$. It is shown that the performance of SMLE-SR is enhanced extremely. when $p \approx M/2$, the detection probability is optimal.

6. Conclusion
In this paper, we proposed a new detector employing SR and multiple large eigenvalues of the covariance matrix of SR output signals. SR is adopted to achieve a better output signal for the received signals from multiple antennae. Normalized scale transformation technology is utilized to transform the high frequency application to low frequency. A test statistic is constructed according to the distribution of multiple large eigenvalues of the covariance matrix. The simulation results verify the robust performance of the proposed SMLE-SR detector compared to the SMLE detector under low SNR circumstances.

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