A novel power-driven fractional accumulated grey model and its application in forecasting wind energy consumption of China

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Abstract

Wind energy is one of the most important renewable resources and plays a vital role in reducing carbon emission and solving global warming problem. Every country has made a corresponding energy policy to stimulate wind energy industry development based on wind energy production, consumption, and distribution. In this paper, we focus on forecasting wind energy consumption from a macro perspective. A novel power-driven fractional accumulated grey model (PFAGM) is proposed to solve the wind energy consumption prediction problem with historic annual consumption of the past ten years. PFAGM model optimizes the grey input of the classic fractional grey model with an exponential term of time. For boosting prediction performance, a heuristic intelligent algorithm WOA is used to search the optimal order of PFAGM model. Its linear parameters are estimated by using the least-square method. Then validation experiments on real-life data sets have been conducted to verify the superior prediction accuracy of PFAGM model compared with other three well-known grey models. Finally, the PFAGM model is applied to predict China’s wind energy consumption in the next three years.

Introduction

Wind energy is one of the important vital resources of renewable energy, which is widely distributed with large reserves. Wind energy and other renewables will play a vitally important role in solving global warming issues and reducing carbon emission in future decades. International energy agency (IEA) has optimistically estimated that renewables will account for 39% share of total electricity generation by 2050. In light of Global Wind Report 2018 of the GWEC, it noticed that new wind installed capacity had overtaken new fossil fuel capacity for the first time in many developing or mature markets. The global total cumulative wind installed capacity has brought up to 591GW at the end of 2018 with new installations 51.3GW. In China, the total wind installed capacity has reached 211GW in 2018, which indicates that the wind energy target of Five-Year-Plan 2016-2020 has been achieved two years ahead of
Along with the rapid development of the energy industry, it also brings a great challenge to make energy policy upgrade energy structure. According to the definition in the literature [1], the energy policy of an entity (especially a government) is used to solve the problems of energy production, consumption, and distribution [2]. Most scholars mainly focused on forecasting the state of wind energy, including wind speed forecasting and wind power forecasting, which is expected to provide a reference for formulating energy production planning from micro-perspective. These forecasting approaches mainly include deterministic forecasting and uncertainty analysis [3]. For these approaches, forecasting wind energy is considered as a prediction process of stochastic time series. Usually, the future values are predicted by these data-driven algorithms based on the historical wind speed or wind energy sequence or other related data. Many scholars applied Artificial Neural Network (ANN) to forecast wind energy. Numerous variants of ANN, such as Back Propagation Neural Network (BPNN) [4, 5], Radial Basis Function Neural Network (RBFNN) [6], Generalized Regression Neural Network (GRNN) [7] and Wavelet Neural Network (WNN) [8, 9], were also proposed to predict the future wind energy. Besides, some scholars utilized Support Vector Machine (SVM) [10] or its variants [11, 12] to forecast wind energy. Deep learning approaches, such as Autoencoder (AE), Deep Boltzmann Machine (DBM) [13], Convolutional Neural Network (CNN) [14], Recurrent Neural Networks (RNNs) [15] and so on, were also adopted to forecast the future wind energy. Though these approaches can be used to forecasting wind energy, they usually need a larger dataset, including historical data and some exogenous data, to train the predictors for better prediction performance. And they were often used to predict hours or minutes ahead of wind energy or speed for making the production plan of a company or energy farm in a local region [16, 17]. Besides, many scholars utilized ARMA, ARIMA, and their successors to predict wind energy or wind speed [18, 19]. However, the study of wind energy consumption prediction from the macro view is very few at present. Previously, many scholars mainly concentrated in study of forecasting electricity [20–29], natural gas [30, 31], oil [32, 33], nuclear [34, 35], renewable [36, 37] energy consumption and so on. Forecasting wind energy consumption is also an important task for making energy policy and plan for the government. Grey prediction model is an effective prediction approach, which is one of the best choices for solving the wind energy consumption prediction problem with small samples.

Recently, scholars have conducted numerous studies to improve prediction performance and enlarge the application range of grey models. Lots of novel or improved grey models have been put forward to solve prediction problems with partially know or unknown information effectively. On the one hand, many scholars devoted their efforts to optimize the initial GM (1,1) model. Wang et al. proposed a rolling grey model with PSO and data cleaning techniques to predict Beijing’s tertiary industry effectively [38]. Xia et al. [39] proposed an improved grey model with grey input of time power based on the new information priority. Ding et al. overcame the fixed structure and poor adaptability of GM(1,1) model and put forward an optimized grey model called NOGM(1,1) to forecast China’s electricity consumption [20]. Xu et al. proposed IRGM(1,1) model with optimizing the initial condition of time response function [21]. Wang et al. proposed SGM(1,1) model to forecast the seasonal time series [23]. Besides, other improved grey models such as WBGM(1,1) [40], GARGM(1,1) [41], Nash NBGM [42], SIGM [43], GRA-IGSA [44] and so on were proposed and obtained satisfactory prediction accuracy. But these models have the same insufficiency of traditional GM(1,1) model because they are essentially grey models with one order and one variable. For the multi-variable time series, Wang et al [45] proposed an improved GM(1,n) model which considered the effects of the dependent variables. Zeng et al. improved the GM(1,n) model with a dynamic background parameter [46]. Soon afterwards Zeng et al. [47] presented a new multi-variable...
grey model to enhance the structure compatibility of grey model. Ma et al. [48] proposed a parameter optimization method for CGM(1,n) model to promote its prediction accuracy. These efforts not only enriched the theory of grey system but also improved the prediction performance of grey prediction model.

On the other hand, fractional order accumulating operator was imported to decrease the stochasticity and uncertainty of raw sequence to boost the performance of grey model. Wu et al. initially improved traditional GM(1,1) with fractional order accumulated operator called FAGM(1,1), which guarantees the priority of new information under small fractional order and boosts the prediction accuracy compared with traditional GM(1,1) model [49]. Yang et al. generalized the traditional GM(1,1) models with fractional calculus of which fractional-order derivative is profoundly determined the accuracy of prediction and can be optimized by intelligent algorithms [27]. Mao et al. proposed a fractional grey model based on fractional order derivative called FGM(q,1) of which the whitening equation is a fractional order differential equation, and forecasted the gross national income per capita accurately [50]. Some researchers optimized the grey action quantity of FAGM(1,1) and achieved better prediction accuracy. Ma et al. optimized the grey input of the original FAGM(1,1) model with a fractional time delay term and applied it to predict the gas consumption and coal consumption [51]. Wu et al. proposed a fractional FAGM(1,1,k) model with linear grey input of time instead of constant grey input in initial FAGM(1,1) model and optimized it with optimal linear parameters and optimal order [34]. Then Wu et al. proposed a fractional accumulated Bernoulli grey model and adopted an intelligent optimization algorithm to seek optimal fractional order of this model [37]. Optimization of fractional order accumulation is another improvement for the traditional FAGM(1,1) model. Ma et al. firstly proposed a CFGM(1,1) model in which the computational complexity of accumulation is lower than that of traditional fractional accumulated operator [52]. Zeng proposed a self-adaptive intelligent fractional grey model called as SAIGMFO model and predicted the electricity consumption of China [53]. Besides, the fractional multivariate grey model is another kind of grey model to deal with multivariate time series. Ma et al. proposed a discrete multivariate grey prediction model (FDGM) and mathematically proved that it is an unbiased grey model that was applied in four real-life applications and achieved better accuracy compared with other well-known grey models [54]. Though these fractional grey models have gained better performance, boosting prediction accuracy of fractional grey models is still worth studying.

In this paper, we focus on forecasting wind energy consumption by using grey models to provide reference information for formulating and adjusting energy policy. In order to boost the prediction accuracy, we improved the classic fractional grey model with an exponential grey input term. There are two aspects of the contribution. On the one hand, we proposed a novel power-driven fractional accumulated grey model called PFAGM of which optimal order is sought out by WOA algorithm. Meanwhile, PFAGM model can be easily reduced into the classical grey model and fractional grey model. To some extends, PFAGM model has better adaptability than classical grey models. On the other hand, PFAGM model is applied to forecast China’s wind energy consumption in the next three years.

The rest of this paper is structured as follows. In section 2, we will introduce the fractional order accumulation and the classic fractional accumulated grey model in detail. In section 3, a power-driven fractional accumulated grey model (PFAGM) with optimization of grey action quantity is proposed. In section 4, we validate the prediction accuracy of PFAGM model on several real-life datasets compared with those of three well-known grey models. In section 5, PFAGM model is utilized to forecast the total wind energy consumption of China. In the last section, several conclusions are drawn.
**Fractional accumulated operation and fractional accumulated grey model**

The fractional accumulating operation plays a very important role in grey prediction applications. It can be used to decrease the randomness of raw sequence data and boost the performance of grey models. The detail of fractional accumulate operation and its inverse operation are introduced as follows.

**Fractional accumulated operation**

For an original data sequence \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)) \), the \( r \)-order accumulated operation sequence can be defined as follow:

**Definition 1** The \( r \)-order fractional accumulated operation sequence of raw data is defined as:

\[
X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n)).
\]

where \( x^{(r)}(k) = \sum_{i=1}^{k} \frac{(k-1) \cdots (k-r+1)}{(r-1)!} x^{(0)}(i), k = 1, 2, \ldots, n. \)

**Definition 2** The inverse \( r \)-order fractional accumulated operation sequence is defined as:

\[
X^{(-r)} = (x^{(-r)}(1), x^{(-r)}(2), \cdots, x^{(-r)}(n)).
\]

where \( x^{(-r)}(k) = \sum_{i=1}^{k} \frac{(k-1) \cdots (k-r+1)}{(r-1)!} x^{(0)}(i), k = 1, 2, \ldots, n. \)

**Methodology of fractional order accumulated grey model**

The fractional order accumulated grey model abbreviated as FAGM(1,1) was firstly proposed by Wu et al. in 2013 [49]. The whitening differential equation of FAGM(1,1) model is defined as:

\[
\frac{dx^{(r)}}{dt} + ax^{(r)}(t) = b.
\]

where \( a \) is the development coefficient, \( b \) is the grey input, and \( r \) is the fractional order of grey model. Integrating both side of Eq (3) within the interval \([k-1, k]\), the discrete form of FAGM model is obtained as follow:

\[
x^{(r)}(k) - x^{(r)}(k-1) + az^{(0)}(k) = b.
\]

where \( z^{(r)}(k) = 0.5(x^{(r)}(k) + x^{(r)}(k)). \)

In order to estimate the parameters of FAGM(1,1) model, the least-square method is used to solve the problem with the objective of minimizing the errors of simulation under the assumption that the order is given. So, the parameters \( a \) and \( b \) can be calculated as follow:

\[
[a, b]^T = (A^TA)^{-1}A^TY.
\]

where

\[
Y_u = \begin{bmatrix}
x^{(r)}(2) - x^{(r)}(1) \\
x^{(r)}(3) - x^{(r)}(2) \\
\vdots \\
x^{(r)}(u) - x^{(r)}(u)
\end{bmatrix},
A_u = \begin{bmatrix}
z^{(0)}(2) & 1 \\
z^{(0)}(3) & 1 \\
\vdots & \vdots \\
z^{(0)}(u) & 1
\end{bmatrix}.
\]
in which $n$ is the number of in-samples used to build model. Then we solve the differential Eq (3) and have

$$x^{(r)}(t) = Ce^{-at} + \frac{b}{a}. \tag{7}$$

Substituting the initial condition into Eq (7) and setting $t = k$, the time response function of FAGM model is obtained as follow:

$$x^{(r)}(k) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a}. \tag{8}$$

where $k = 2, 3, \ldots, n$. However, the time response sequence is only an intermediate result and needs to be restored to restored values by using inverse $r$-order fractional accumulated operation. The restored value can be represented as:

$$x^{(0)}(k) = \sum_{i=1}^{k} \frac{(k-i-r-1)!}{(-r-1)!(k-i)!} x^{(r)}(i). \tag{9}$$

where $k = 1, 2, \ldots, n$.

**The proposed fractional accumulated grey model**

In this section, we prove that the grey action quantity of fractional grey model built by various subsequences with the same length changes with time in a homogeneous exponential sequence. Then a novel power-driven fractional accumulated grey model is proposed, which optimizes the classical FAGM model with an exponential grey action quantity. It makes the grey action quantity change from a constant term to an exponential term of time.

**The basis of grey action quantity optimization**

The optimization of grey input is a remarkable approach to improve classical grey models and promote their prediction accuracy. The basis of grey action quantity optimization is that the grey input of the grey model changes with time in the real-world grey system. Xu et al. [55] presented a theory to illustrate that the grey action quantity of the classical grey model would change with time. The theory is represented as follow:

**Theorem 1** Assuming that the time series $X^{(0)}(k) = Ae^{\lambda(k-1)}$, $k = 1, 2, \ldots$ is raw sequence, there are two subsequence $X_1^{(0)}(k) = Ae^{\lambda(k-1)}$, $k = 1, 2, \ldots, n$ and $X_2^{(0)}(k) = Ae^{\lambda(k+1-1)}$, $k = 1, 2, \ldots, n$ with the same number of samples. The parameters $a_1$ and $b_1$ are respectively the development coefficient and grey input of the GM(1,1) model built by $X_1^{(0)}$. The parameters $a_2$ and $b_2$ are respectively the development coefficient and grey input of the GM(1,1) model built by $X_2^{(0)}$. Then, $a_1 = a_2$ and $b_2 = b_1 e^{\lambda t}$.

Theorem 1 indicates that the grey action quantity of GM(1,1) changes with time. If the first order term and constant term of Maclaurin’s series of $e^{\lambda t}$ are only remained, the SAIGM(1,1) model is obtained with the grey input term $bt + c$. If the first order term of Maclaurin’s series of $e^{\lambda t}$ is only remained, the NGM(1,1) model can be obtained with the grey input term $bt$. Obviously, the grey inputs of SAIGM and NGM are linear time functions. In fact, the grey input of the grey system is nonlinear. Notably, the EOGM(1,1) model has been presented when the term $b e^{\lambda t}$ is directly used as its grey action quantity.

For the fractional accumulation grey model, its grey action quantity should vary with time according to the above-mentioned optimization of the classical GM(1,1) model. This proposition can also be illustrated with the theorem as follow:
Theorem 2 Assuming that the sequence \( Y^{(0)} = \{ y^{(0)}(k) = Ae^{\lambda(k-1)} | k = 1, 2, \ldots \} \) is raw data, there are two subsequences \( Y^{(0)}_1 = \{ y^{(0)}_1(k) = Ae^{\lambda(k-1)} | k = 1, 2, \ldots , n \} \) and \( Y^{(0)}_2 = \{ y^{(0)}_2(k) = Ae^{\lambda(k-1)} | k = 1, 2, \ldots , n \} \) with the same number of samples. The parameters \( a_1 \) and \( b_1 \) are respectively the development coefficient and grey input of the FAGM(1,1) model built by \( Y^{(0)}_1 \). The parameters \( a_2 \) and \( b_2 \) are respectively the development coefficient and grey input of the FAGM(1,1) model built by \( Y^{(0)}_2 \). Then, \( a_1 = a_2 \) and \( b_1 = b_2 = b_1 e^{\lambda n} \).

Proof 1 According to the modeling procedure of FAGM(1,1) model, the fractional order accumulation generated sequence of \( Y^{(0)}_1 \) is represented as
\[
Y^{(0)}_1 = \left\{ y^{(0)}_1(k) = \sum_{i=1}^{\infty} \frac{[k-i-1]!}{[k-i]!} y^{(0)}_1(i) | k = 1, 2, \ldots , n \right\}.
\]
The fractional order accumulation generated sequence of \( Y^{(0)}_2 \) is represented as
\[
Y^{(0)}_2 = \left\{ y^{(0)}_2(k) = \sum_{i=1}^{\infty} \frac{[k-i-1]!}{[k-i]!} y^{(0)}_2(i) | k = 1, 2, \ldots , n \right\}.
\]
The grey differential equation of FAGM(1,1) can be built by using \( Y^{(0)}_1 \) as follow:
\[
y^{(0)}_1(k) - y^{(0)}_1(k-1) + a_1 z^{(0)}_1(k) = b_1,
\]
where \( z^{(0)}_1(k) = 0.5(y^{(0)}_1(k) + y^{(0)}_1(k-1)), k = 2, 3, \ldots , n \). The grey differential equation of FAGM(1,1) can be built by using \( Y^{(0)}_2 \) as follow:
\[
y^{(0)}_2(k) - y^{(0)}_2(k-1) + a_2 z^{(0)}_2(k) = b_2,
\]
where \( z^{(0)}_2(k) = 0.5(y^{(0)}_2(k) + y^{(0)}_2(k-1)), k = 2, 3, \ldots , n \). According to the Eqs (5) and (6), the grey parameters \( a_1 \) and \( b_1 \) can be obtained as follow:
\[
a_1 = - \frac{(n-1) \sum_{i=2}^{n} z^{(0)}_1(i) (y^{(0)}_1(i) - y^{(0)}_1(i-1))}{(n-1) \sum_{i=2}^{n} (z^{(0)}_1(i))^2 - \left( \sum_{i=2}^{n} z^{(0)}_1(i) \right)^2} \tag{12}
\]
\[
b_1 = - \frac{\sum_{i=2}^{n} z^{(0)}_1(i) \sum_{i=2}^{n} z^{(0)}_1(i) (y^{(0)}_1(i) - y^{(0)}_1(i-1))}{(n-1) \sum_{i=2}^{n} (z^{(0)}_1(i))^2 - \left( \sum_{i=2}^{n} z^{(0)}_1(i) \right)^2} \tag{13}
\]
\[\left\{\begin{aligned}
\sum_{i=2}^{n} \sum_{i=2}^{n} z^{(0)}_1(i) (y^{(0)}_1(i) - y^{(0)}_1(i-1)) \\
(n-1) \sum_{i=2}^{n} (z^{(0)}_1(i))^2 - \left( \sum_{i=2}^{n} z^{(0)}_1(i) \right)^2
\end{aligned}\right.\]
In a similar way, the grey parameters $a_2$ and $b_2$ can be obtained as:

$$a_2 = -\frac{(n - 1) \sum_{i=2}^{n} z_2(i) (y_2(i) - y_2(i - 1))}{(n - 1) \sum_{i=2}^{n} (z_2(i))^2 - \left( \sum_{i=2}^{n} z_2(i) \right)^2}$$

\[(14)\]

$$b_2 = -\frac{\sum_{i=2}^{n} z_2(i) \sum_{i=2}^{n} z_2(i) (y_2(i) - y_2(i - 1))}{(n - 1) \sum_{i=2}^{n} (z_2(i))^2 - \left( \sum_{i=2}^{n} z_2(i) \right)^2}$$

\[(15)\]

Based on the relation between the sequence $Y_1(0)$ and $Y_2(0)$, the equation between the accumulated sequence $Y_1(\tau)$ and $Y_2(\tau)$ are obtained as follow:

$$Y_2(\tau) = \sum_{i=1}^{\tau} \frac{(k - i - 1)!}{(-r - 1)! (k - i)!} Y_1(0)(i) e^{\lambda t} = e^{\lambda t} Y_1(\tau)$$

\[(16)\]

The equation between the sequence $z_2(\tau)(k)$ and $z_2(\tau)(k)$ are obtained as follow:

$$z_2(\tau)(k) = 0.5 (y_1(\tau)(k) e^{\lambda t} + y_1(\tau)(k - 1) e^{\lambda t}) = e^{\lambda t} z_2(\tau)(k)$$

\[(17)\]

Substituting Eqs (16) and (17) into Eqs (14) and (15), the equations can be obtained as follow:

$$a_2 = -\frac{(n - 1) \sum_{i=2}^{n} z_2(i) (y_2(i) - y_2(i - 1) e^{\lambda t}))}{(n - 1) \sum_{i=2}^{n} (z_2(i) e^{\lambda t})^2 - \left( \sum_{i=2}^{n} z_2(i) e^{\lambda t} \right)^2}$$

\[(18)\]
Therefore, the term $b e^{rt} + c$ is considered as the grey action quantity of the proposed fractional accumulated grey model. The definition of the proposed model is represented as follows.

**Definition 3** The differential equation

$$\frac{dx^{(i)}(t)}{dt} + ax^{(i)}(t) = be^{rt} + c.$$  \hspace{1cm} (20)

is the whitening equation of power-driven fractional order accumulated grey model abbreviated as PFAGM, in which $a$ is defined the same in FAGM model, $be^{rt} + c$ is the power-driven grey action quantity, and $r$ is the fractional order of this grey model.

Integrating the whitening Eq (20) within $[k - 1, k]$, we have

$$\int_{k-1}^{k} \frac{dx^{(i)}(t)}{dt} dt + a \int_{k-1}^{k} x^{(i)}(t)dt = \int_{k-1}^{k} (be^{rt} + c)dt.$$  \hspace{1cm} (21)

Then we have

$$x^{(i)}(k) - x^{(i)}(k - 1) + a \int_{k-1}^{k} x^{(i)}(t)dt = br^{-1}(e^r - 1)e^{r(k-1)} + c.$$  \hspace{1cm} (22)

Substituting the background value $z^{(i)}(k) = \int_{k-1}^{k} x^{(i)}(t)dt$ into Eq (22), the discrete form of PFAGM model can be obtained as follow.
**Definition 4** The discrete differential equation of PFAGM model is defined as:

\[
x^{(r)}(k) - x^{(r)}(k-1) + az^{(r)}(k) = br^{-1}(e^{r} - 1)e^{r(k-1)} + c.
\]

(23)

where \(z^{(r)}\) is the background value, and \(z^{(r)}(k) = 0.5x^{(r)}(k) + 0.5x^{(r)}(k-1)\).

When the parameter \(b\) of PFAGM model is set to 0, PFAGM model can be reduced into FAGM(1,1) model. When the parameter \(b\) is set to 0 and the fractional order is set to 1, PFAGM model can be degenerated into GM(1,1) model. Because \(e^{rt} \approx 1 + rt\), PFAGM model can be degenerated into FAGM(1,1,k) model if the term \(1 + rt\) replaces the term \(e^{rt}\) of the grey input in PFAGM model.

**Parameter estimation of power-driven factional grey model**

To realize the prediction of the fractional grey model, one of the most important problems is parameter estimation for building a model. In fact, it is effective to boost the prediction performance of grey model by using suitable parameters. For PFAGM model, it needs to determine three linear parameters and search out the optimal value of its fractional order. The optimal linear parameters can be gained by using the least-square method under the condition of the given fractional order of PFAGM model. In this subsection, we mainly introduce the principle of linear parameters’ estimation while the methodology of seeking out the optimal fractional order is presented in subsection 3.4.

Once the order of PFAGM model is given, the linear parameters of the PFAGM model can mathematically be calculated as:

\[
(a, b, c)^T = (B_u^TB_u)^{-1}B_u^TY_u.
\]

(24)

where

\[
Y_u = \begin{bmatrix}
  x^{(r)}(2) - x^{(r)}(1) \\
  x^{(r)}(3) - x^{(r)}(2) \\
  \vdots \\
  x^{(r)}(u) - x^{(r)}(u-1)
\end{bmatrix},
B_u = \begin{bmatrix}
  -z^{(r)}(2) & r^{-1}(e^{r} - 1)e^{r} & 1 \\
  -z^{(r)}(3) & r^{-1}(e^{r} - 1)e^{r} & 1 \\
  \vdots & \vdots & \vdots \\
  -z^{(r)}(u) & r^{-1}(e^{r} - 1)e^{r(u-1)} & 1
\end{bmatrix}.
\]

(25)

in which \(u\) is the number of samples for fitting. The proof process of parameter estimation is omitted here because it is like that of traditional FAGM model.

**The time response function and restored values**

After linear parameters of PFAGM model are calculated, the time response function and the restored values of PFAGM can be got by solving Eq (20). Firstly, we consider the homogeneous differential equation:

\[
\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = 0.
\]

(26)

which is corresponding to the whitening equation of PFAGM model and solve Eq (26). Then the solution of Eq (26) can be obtained as:

\[
x^{(r)}(t) = Ae^{-at}.
\]

(27)

where \(A\) is a constant which is determined by initial condition. Let \(x^{(r)}(t) = A(t)e^{-at}\) be the
solution of Eq (20). And substituting it into Eq (20), we have
\[
\frac{dA(t)}{dt} e^{-at} = be^{rt} + c.
\]  
(28)
Then the solution of Eq (28) can be easily obtained as:
\[
A(t) = \frac{b}{a + r} e^{(a+r)t} + \frac{c}{a} e^{at} + C.
\]  
(29)
So, the solution of the whitening equation of PFAGM model can be represented as:
\[
x^{(r)}(t) = Ce^{-at} + \frac{b}{a + r} e^{(a+r)t} + \frac{c}{a}.
\]  
(30)
Substituting the initial condition \(x^{(r)}(1) = x^{(0)}(1)\) into Eq (30), the arbitrary constant can be easily determined. And then the time response function of PFAGM model can be derived and represented as:
\[
x^{(r)}(k) = \left(x^{(0)}(1) - \frac{b}{a + r} e' - \frac{c}{a}\right) e^{-a(k-1)} + \frac{b}{a + r} e^{r} + \frac{c}{a}.
\]  
(31)
By using inverse \(r\)-order accumulated operation, the restored value \(x^{(0)}(k)\) can be obtained as:
\[
x^{(0)}(k) = (x^{(r)}(k))^{(-r)}.
\]  
(32)
where \(k = 1, 2, \ldots, n\).

**Searching the optimal order of power-driven fractional accumulated grey model**

In subsection 3.2, the linear parameters are estimated by the least-square method under the assumption that the order of fractional power-driven grey model has been given. In fact, the order of PFAGM model needs to be sought out and plays a vital role in boosting its prediction performance effectively. In order to search for an optimal value of the order, we establish a constrained optimization problem of which the objective is minimizing the mean absolute percentage errors for building the grey model. The constraints of PFAGM model have been derived in the above-mentioned modeling process. In summary, the optimization problem can be represented as follow:

\[
\min \text{MAPE}(r) = \frac{1}{u-1} \sum_{i=2}^{u} \left| \frac{x^{(0)}(i) - x^{(0)}(i)}{x^{(0)}(i)} \right| \times 100\%.
\]

\[
\begin{cases}
(a, b, c)^T = (B^T B_u)^{-1} B^T Y_u, \\
\text{s.t.}
\end{cases}
\]

\[
x^{(r)}(k) = \left(x^{(0)}(1) - \frac{b}{a + r} e' - \frac{c}{a}\right) e^{-a(k-1)} + \frac{b}{a + r} e^{r} + \frac{c}{a},
\]

\[
x^{(0)}(k) = (x^{(r)}(k))^{(-r)}, k = 2, 3, \ldots, u.
\]

where \(B_u\) and \(Y_u\) are defined as Eq (25). It can be clearly noticed that the optimization problem Eq (33) is a complex nonlinear programming problem with nonlinear objective function and a few nonlinear constraints. Obviously, it is difficult to derive the exact solution for optimal
fractional order. However, the optimal value of fractional order plays a very important role in boosting the prediction accuracy of existing grey models dramatically. For example, Ma et al. established a similar optimization problem to optimize the fractional order of FTDGM model and applied it to forecast energy consumption accurately [51]. Wu et al. also build a similar optimization problem to obtain the optimal fractional order and power index of FANGBM and forecasted the renewable energy consumption successfully [37]. These facts have shown that the optimal fractional order can be used to boost the prediction performance of grey models.

In order to obtain the optimal order of PFAGM, we adopted a nature-based heuristic intelligent method called Whale Optimization Algorithm (WOA) to solve the nonlinear programming problem Eq (33). WOA algorithm was firstly proposed by Mirjalili et al. in 2016 [56]. The inspiration of WOA is from the social behaviors of whales. It mimics the bubble-net feeding strategy including shrinking encircling, spiral updating position and randomly hunting behavior. Due to the simple rules and local optimization performance of WOA, it is widely used in many domains such as feature selection, clustering, classification, image processing and so on [57]. In this paper, it is assumed by WOA that there is a population with 30 humpback whales as search agents in search space. \( P(k) \) indicates the position vector of search agents at iteration \( k \) in search space. And set maximum iterations \( T_m \) to be 300. The value of each agent’s position indicates a possible fractional order of PFAGM model.

**Step 1:** Initialize the initial position \( P(k) \) of search agent randomly in search space at \( k = 1 \). According to Eq 33, the fitness function is defined as:

\[
\text{Fitness} = \frac{1}{u - 1} \sum_{i=2}^{u} \left| \frac{x_r^{(0)}(i) - x_0^{(0)}(i)}{x_0^{(0)}(i)} \right| \times 100\%. \tag{34}\]

The position of search agent with minimum fitness is considered as the candidate solution at first iteration.

**Step 2:** Compute the fitness of each agent and obtain the candidate solution \( P^*(k) \) at \( k = 1 \). According to Eq 33, the fitness function is defined as:

\[
\text{Fitness} = \frac{1}{u - 1} \sum_{i=2}^{u} \left| \frac{x_r^{(0)}(i) - x_0^{(0)}(i)}{x_0^{(0)}(i)} \right| \times 100\%. \tag{34}\]

The position of search agent with minimum fitness is considered as the candidate solution at first iteration.

**Step 3:** Update the position vectors of humpback whales. When humpback whales forage the prey, they usually move around the prey by shrinking encircling and spiral updating position simultaneously. In order to imitate this simultaneous behavior, we assume that there is a 50% probability to shrink encircling or update position spirally in the iteration of optimization. In each iteration, it generates a random number in \([0, 1]\) and set it to the parameter \( p \). If \( p \leq 0.5 \), humpback whales select to shrink encircling. Mathematically, the behavior of shrinking encircling can be represented as follows:

\[
\bar{A} = 2g(k) \cdot \bar{r} - g(k). \tag{35}\]

\[
\bar{B} = \left| 2 \cdot \bar{r} \cdot \bar{P}^*(k) - \bar{P}(k) \right|. \tag{36}\]

\[
\bar{P}(k + 1) = \bar{P}^*(k) - \bar{A} \cdot \bar{B}. \tag{37}\]

\[
g(k) = 2 - 2k/T_m. \tag{38}\]
where $\vec{r}$ is a random vector in interval $[0, 1]$. If $p > 0.5$, the humpback whales will select spiral updating position to imitate the helix-shaped moving behavior in process of optimization. The behavior is shown mathematically as follow:

$$\vec{P}(k + 1) = \vec{P}^* \cdot e^{\omega l} \cdot \cos(2\pi l) + \vec{P}(k).$$

(39)

where $l$ denotes a random value in $[-1, 1]$, $\omega$ is a constant. In fact, the humpback whales search for the prey randomly. If $|\vec{A}| > 1$, WOA algorithm imitates the behavior and performs a global search. Mathematically, the model is formulated as follows:

$$\vec{C} = |2 \cdot r \cdot \vec{P}_{\text{rand}}(k) - \vec{P}(k)|.$$  

(40)

$$\vec{P}(k + 1) = \vec{P}_{\text{rand}}(k) - \vec{A} \cdot \vec{C}.$$  

(41)

where $\vec{P}_{\text{rand}}(k)$ is the position of a random search agent.

**Step 4:** Calculate the fitness of each humpback whales. If there is a better candidate with minimum fitness value, it is needed to update the optimal solution $\vec{P}^*$. 

**Step 5:** If the ending condition is satisfied, the optimal fractional order is obtained, otherwise $k = k + 1$.

Based on the process of building PFAGM model and optimization of the fractional order, the detailed modeling procedure of power-driven fractional accumulated grey model can be shown in Fig 1. For enhancing the prediction performance of PFAGM model, the key is to obtain the optimal values of linear parameters and fractional order. The optimal value of fractional order can be sought out through solving the established optimization problem by WOA algorithm. Moreover, the linear parameters are estimated by using the least-square method. Then the established PFAGM model is used to predict future values in different applications.

### Applicability analysis of power-driven fractional grey model

Though the classical grey models are effective approaches to predict future values with small samples, each of these models has a certain scope of application. Similarly, the proposed power-driven fractional grey model also has its scope of application. Therefore, we construct a series of sequences, including two kinds of non-homogeneous exponential sequences and some other sequences with special shapes, to study the applicability of the proposed grey model. There are three cases studied and compared with GM(1,1), FAGM(1,1), and FAGM(1,1,k) as follows.

Firstly, we construct a series of non-homogeneous exponential sequences with the larger values of the development coefficient. These sequences are represented as $x^{(0)}(k) = Ae^{\lambda k} + B$ ($k = 1, 2, \ldots, 9$) in which $A$ is set to 1, $B$ is set to 10, and $a$ is set to 0.7, 1.0, 1.3, 1.5, 1.7 or 1.8. The first six digits of each sequence are used to train the grey models. The other three digits of each sequence are employed to examine the prediction performance of the proposed grey model. The MAPEs of different grey models for fitting and prediction are filled in Table 1. It can be noticed that PFAGM model obtains a significant advantage of prediction accuracy over the other three classical grey models. Meanwhile, PFAGM model has significant advantages in fitting. Secondly, we construct the other kind of non-homogeneous exponential sequences represented as $x^{(0)}(k) = Ae^{\lambda k} + B(k - 1)$ ($k = 1, 2, \ldots, 9$) in which $A$ is set to 1, $B$ is set to 5, and $a$ is set to 0.7, 1.0, 1.3, 1.5, 1.7 or 1.8. Similarly, the first six digits of each sequence are used to train the grey models. The other three digits of each sequence are employed to examine the
Fig 1. The flowchart of PFAGM model.

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| λ   | MAPE of Fitting | MAPE of Prediction |
|-----|-----------------|--------------------|
|     | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
| 0.7 | 8.0214 | 1.4179 | 2.0788 | **0.3766** | 43.4858 | 5.1797 | 26.5822 | **3.9745** |
| 1   | 69.4463 | 4.6622 | 5.2342 | **3.38E-07** | 88.7597 | 19.0313 | 27.3230 | **1.61E-06** |
| 1.3 | 188.766 | 10.4601 | 11.3918 | **1.0518** | 180.0536 | 46.3293 | 45.6655 | **0.9277** |
| 1.5 | 237.1642 | 15.7192 | 17.0864 | **1.9898** | 193.0147 | 59.6113 | 59.0422 | **2.3112** |
| 1.7 | 249.5205 | 22.4229 | 24.1564 | **3.0308** | 171.0404 | 71.1620 | 70.8141 | **3.9040** |
| 1.8 | 246.3506 | 26.4090 | 28.2787 | **3.5722** | 156.4693 | 76.0759 | 75.8225 | **4.7317** |

https://doi.org/10.1371/journal.pone.0225362.t001

Table 1. Comparison of different grey models for non-homogeneous exponential sequences with various development coefficients (X^0(k) = Ae^(λ(k-1)) + B, k = 1, 2, ..., n).
prediction performance of the proposed grey model. Table 2 shows the MAPEs of different grey models for fitting and prediction. The results indicate that PFAGM model has a significant advantage of fitting ability over the other three classical grey models. PFAGM model also has a significant superiority of prediction accuracy, especially when the development coefficient is large. From the above analysis results, it indicates that PFAGM model has better adaptability and applicability for the two kinds of non-homogeneous exponential sequences with the larger values of the development coefficient.

Finally, we generate a series of non-homogeneous exponential sequences with different characteristics randomly. The raw data of each sequence is tabulated in Table 3. Fig 2 exhibits the characteristics of these generated sequences. These sequences are divided into two groups. The first group, including the first eight digits of each sequence, is used to build grey models. The second group, including the other four digits of each sequence, is utilized to validate the prediction performance of the proposed grey model. The MAPEs of the grey models for fitting and prediction are filled in Table 4. It can be noticed that the prediction accuracy of the proposed grey model is better than those of the other three grey models. The fitting errors of the proposed grey model are the lowest or very approximate to the lowest.

From the above validations and analysis, it can be drawn that PFAGM model has a significant superiority to deal with the non-homogeneous exponential sequences (formulated as $X^{(0)}(k) = Ae^{\lambda(k-1)} + B(k-1)$ or $X^{(0)}(k) = Ae^{\lambda(k-1)} + B(k-1)$) compared with the classical grey models.

### Table 2. Comparison of different grey models for non-homogeneous exponential sequences with various development coefficients ($X^{(0)}(k) = Ae^{\lambda(k-1)} + B(k-1)$, $k = 1, 2, \ldots, n$).

| $\lambda$ | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|-----------|---------|-----------|-------------|-------|---------|-----------|-------------|-------|
| 0.7       | 5.4555  | 2.7347    | 2.4818      | 0.8435 | 14.2598 | 22.5767   | 14.4327     | 4.6936 |
| 1         | 26.9863 | 13.4895   | 9.2376      | 2.4296 | 54.5762 | 44.7105   | 45.4235     | 14.2598 |
| 1.3       | 111.0393| 10.5668   | 9.9856      | 3.6362 | 119.3470| 29.8071   | 36.2111     | 15.3936 |
| 1.5       | 163.5235| 9.7733    | 9.6019      | 3.9296 | 144.4912| 45.4235   | 45.4235     | 9.4585  |
| 1.7       | 190.9126| 13.5743   | 13.7557     | 3.8576 | 142.1207| 58.5609   | 58.0279     | 4.6953  |
| 1.8       | 195.8534| 16.2803   | 16.0250     | 3.7240 | 135.6079| 72.4797   | 72.4032     | 3.0292  |

https://doi.org/10.1371/journal.pone.0225362.t002

### Table 3. Raw data of special non-homogeneous exponential sequences.

| Index | S1   | S2   | S3   | S4   | S5   | S6   | S7   | S8   | S9   |
|-------|------|------|------|------|------|------|------|------|------|
| 1     | 10   | 10   | 10   | 10   | 10   | 10   | 10   | 1    | 1    |
| 2     | 13.6138 | 13.7664 | 16.7975 | 8.9671 | 7.8421 | 5.9371 | 1.32435 | 1.6487 | 6.6487 |
| 3     | 16.4143 | 13.4857 | 19.1673 | 8.4053 | 6.9623 | 4.7812 | 1.85915 | 2.7183 | 12.7183 |
| 4     | 18.7775 | 12.6666 | 19.7109 | 8.0436 | 6.5948 | 4.359  | 2.74085 | 4.4817 | 19.4817 |
| 5     | 20.8251 | 12.0434 | 19.4351 | 7.8582 | 6.5571 | 4.2515 | 4.19455 | 7.3891 | 27.3891 |
| 6     | 22.6213 | 11.7834 | 18.7967 | 7.8306 | 6.7771 | 4.3335 | 6.39125 | 12.1825 | 37.1825 |
| 7     | 24.2093 | 11.9347 | 18.0058 | 7.9513 | 7.2224 | 4.5737 | 10.54275 | 20.0855 | 50.0855 |
| 8     | 25.6219 | 12.5487 | 17.153  | 8.2224 | 7.8808 | 4.9824 | 17.05775 | 33.1155 | 68.1155 |
| 9     | 26.8853 | 13.7188 | 16.2686 | 8.658  | 8.7532 | 5.5959 | 27.7991 | 54.5982 | 94.5982 |
| 10    | 28.0212 | 15.5993 | 15.3514 | 9.2834 | 9.85  | 6.4732 | 45.50855 | 90.0171 | 135.0171 |
| 11    | 29.0477 | 18.4248 | 14.3833 | 10.1355| 11.19 | 7.6988 | 74.7066 | 148.4132 | 198.4132 |
| 12    | 29.9801 | 22.5364 | 13.3348 | 11.2646| 12.7997| 9.3895 | 122.846 | 244.6919 | 299.6919 |

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GM(1,1), FAGM(1,1) and FAGM(1,1,k) model. Meanwhile, PFAGM model has a certain advantage to handle the above-mentioned special sequences. Therefore, PFAGM model has better adaptability and applicability than the other classical grey models for non-homogeneous exponential sequences with larger development coefficients or special shape characteristics.

Validation of the power-driven grey model

Performance metrics and comparative grey models

In order to evaluate the accuracy of grey prediction models, four performance metrics are adopted in numerical validation and case studies, including residual percentage error (RPE), absolute percentage error (APE), mean absolute percentage error (MAPE) and Correlation coefficient (R) [58, 59]. The metrics RPE and APE are used to validate the prediction accuracy.
of a grey model for a single data point. Mathematically, they are represented as follows:

\[
RPE(k) = \frac{x^{(0)}_p(k) - x^{(0)}(k)}{x^{(0)}(k)} \times 100%.
\] (42)

\[
APE(k) = |RPE(k)|.
\] (43)

where \(x^{(0)}(k)\) is raw data, \(x^{(0)}_p(k)\) is the value produced by a grey model. Meanwhile, MAPE is used as a general metric to evaluate the accuracy of a prediction model. The lower value of MAPE indicates that the model has a better performance. Mathematically, MAPE is represented as follow:

\[
MAPE = \frac{1}{n} \sum_{k=1}^{n} |APE(k)|.
\] (44)

where \(n\) is the number of samples. For a raw sequence with \(n\) samples, the metric FMAPE is defined as the simulation performance metric while PMAPE is defined as a prediction performance metric. Mathematically, they can be formulated as:

\[
FMAPE = \frac{1}{v} \sum_{k=1}^{v} |APE(k)|.
\] (45)

\[
PMAPE = \frac{1}{n-v} \sum_{k=v+1}^{n} |APE(k)|.
\] (46)

where \(v\) is the number of samples used to build a model while the rest of the raw sequence is used to examine the prediction accuracy of the model. The total MAPE is used to evaluate the whole performance of a model and abbreviated as TMAPE. The correlation coefficient \((R)\) is used to describe the relationship between raw sequence \(X^{(0)}\) and the sequence \(X^{(0)}_p\) produced by grey models. The mathematical definition of \(R\) is represented as:

\[
R = \frac{Cov(X^{(0)}, X^{(0)}_p)}{\sqrt{Var(X^{(0)}) Var(X^{(0)}_p)}}.
\] (47)

where \(Cov(s, t)\) denotes the covariance of sequence \(s\) and \(t\), \(Var(s)\) denotes the variances of sequence \(s\).

For investigating its superiority of performance, PFAGM model compares with three existing grey models, including traditional integral-order grey model (GM(1, 1)), fractional accumulated grey model (FAGM(1,1)) and the improved fractional accumulated grey model (FAGM(1,1,k)). The whitening equation of integral-order GM(1,1) model [38] is represented as:

\[
\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b.
\] (48)

The whitening equation of FAGM(1,1,k) [34] is written as:

\[
\frac{dx^{(0)}(t)}{dt} + ax^{(0)}(t) = bt + c.
\] (49)

The details of FAGM(1,1) model are introduced in section 2. All grey prediction models, including the new proposed model and the three contrast grey models, are implemented in
MATLAB and performed on the platform MATLAB 2018. In particular, it needs to highlight that the orders of the two comparative fractional grey models are also sought out by the optimization algorithm WOA which also is applied in our proposed PFAGM model. In the following subsections, the validation experiments on some real-world data sets are conducted to illustrate the advantages of PFAGM compared with the other three existing grey models.

**Example A: Forecasting China’s nuclear energy consumption**

In this numerical validation, the raw sequence containing Chinese nuclear energy consumption (NEC) from 2006 to 2017 is obtained from section 8 of reference [34]. The dataset is tabulated in Table 5. The consumption from 2006 to 2012 is used to build a model respectively for PFAGM model and the other three contrast grey models, while the remainder of the dataset is used to validate the prediction performance of grey models. Firstly, we obtain the optimal orders of the three fractional grey models by using WOA algorithm. Then the linear parameters of all grey models are estimated by using the least-square method. The optimal values of linear and nonlinear parameters for each model are filled in Table 6. From Fig 3, it can be noticed that WOA algorithm converges rapidly into a stable status after a few iterations. And the optimal order of PFAGM model is obtained and equal to 0.24794. The results produced by these established grey models are also filled in Table 7. FMAPE, PMAPE, and TMAPE of PFAGM model are respectively 1.2024, 2.8591, and 1.8927. As can be noticed from Fig 4 and Table 7, all evaluation metrics’ values of PFAGM model are lowest compared with those of the other three contrast grey models. Fig 5 shows the detailed analysis between the raw data and the values produced by the four grey models. It can be clearly found that the linear regression line obtained from PFAGM model almost coincides with the equal line of which the point denotes that raw data is equal to the value produced by the grey model. It indicates that PFAGM model has a better prediction performance than the other three contrast models.

**Example B: Forecasting cumulative oil field production**

In this subsection, we consider forecasting the cumulative oil field production of which the raw data is collected from the paper [60] as a validation example. The raw data contains the cumulative oil field production from 1999 to 2012 of the RQ block of Huabei oil field company in China. The raw data is tabulated in Table 8. Then the dataset is divided into two subsets.

### Table 5. Raw data in Example A.

| Year | NEC | Year | NEC | Year | NEC |
|------|-----|------|-----|------|-----|
| 2006 | 12.4| 2010 | 16.7| 2014 | 30  |
| 2007 | 14.1| 2011 | 19.5| 2015 | 38.6|
| 2008 | 15.5| 2012 | 22  | 2016 | 48.3|
| 2009 | 15.9| 2013 | 25.3| 2017 | 56.1|

### Table 6. The optimal parameters of different grey models in Example A.

| Parameters | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|------------|---------|-----------|-------------|-------|
| $a$        | -0.08907| -0.50681  | 0.07111     | 0.80372|
| $b$        | 11.94888| -6.15082  | 0.67748     | 3.86999|
| $c$        | -       | -         | 1.99915     | 11.19314|
| $\gamma$   | 1       | -0.11349  | -0.2949     | 0.24794|

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The first subset, including the first 11 samples, is used to build a model for each grey model. The other subset, including the last 3 samples, is utilized to examine the accuracy of grey models. In the stage of building model, the optimal orders of all fractional grey models are obtained by using WOA algorithm. From Fig. 6, it can be obviously noticed that the optimal order of PFAGM is sought out after a few iterations of WOA. And then the linear parameters of these models are also obtained by using the least-square method after their orders are determined. The parameters of the integral order grey model can be directly estimated by using the least-square method. These optimal parameters are filled in Table 9. Then we utilize these different established grey models to calculate the fitted production from 1999 to 2009 and the predicted

Table 7. The results produced by proposed model and other comparative grey models in Example A.

| Year | Raw data | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|------|----------|---------|-----------|-------------|-------|
|      | Value    | RPE     | Value     | RPE         | Value | RPE | Value | RPE |
| 2006 | 12.4     | 12.4    | 0         | 12.4        | 12.4  | 0   | 12.4  | 0   |
| 2007 | 14.1     | 13.652  | 3.1752    | 13.9812     | 14.2607 | -1.1397 | 14.1  | 0   |
| 2008 | 15.5     | 14.9241 | 3.7156    | 15.0733     | 15.0798 | 2.7112 | 15.0037 | 3.2022 |
| 2009 | 15.9     | 16.3143 | -2.6059   | 16.1486     | 15.9525 | -0.3303 | 15.9486 | -0.3059 |
| 2010 | 16.7     | 17.8314 | -6.7911   | 17.4546     | 17.2798 | -3.4716 | 17.3112 | -3.6598 |
| 2011 | 19.5     | 19.4955 | 0.0233    | 19.2689     | 19.259  | 1.2359 | 19.2864 | 1.0955 |
| 2012 | 22       | 21.3116 | 3.1293    | 22          | 22     | 0    | 22.0337 | -0.1531 |
| 2013 | 25.3     | 23.2969 | 7.9176    | 26.3009     | 25.5662 | -1.0521 | 25.736  | -1.7235 |
| 2014 | 30       | 25.4671 | 15.1097   | 33.2414     | 29.9935 | 0.0215 | 30.6283 | -2.0943 |
| 2015 | 38.6     | 27.8395 | 27.877    | 44.5882     | 35.3004 | 8.5482 | 37.0175 | 4.0998 |
| 2016 | 48.3     | 30.4329 | 36.992    | 63.269      | 41.4932 | 14.0928 | 45.3032 | 6.2045 |
| 2017 | 56.1     | 33.2679 | 40.699    | 94.1392     | 48.5701 | 13.4223 | 56.0027 | 0.1734 |
| FMAPE | 2.7772 | 1.5517 | 1.2698 | 1.2024 |
| PMAPE | 25.7191 | 25.8144 | 7.4274 | 2.8591 |
| TMAPE | 12.3363 | 11.6612 | 3.8355 | 1.8927 |
| R    | 0.9642  | 0.9761  | 0.9959  | 0.998     |

The parameters of Table 7 are filled in Table 9. Then we utilize these different established grey models to calculate the fitted production from 1999 to 2009 and the predicted
Fig 4. Performance comparison of the proposed model and other comparative grey models in Example A.

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Fig 5. Analysis of detailed results obtained by using the proposed model and other comparative grey models in Example A.

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production from 2010 to 2012, which are filled in Table 10. As can be noticed from Table 10 and Fig 7, the proposed PFAGM model exhibits the most excellent prediction performance compared with those of the other contrast grey models. Though FMAPE of PFAGM model is not best, its PMAPE and TMAPE are superior to the other three contrast models. R of FMAPE is also best among those of these grey models. Meanwhile, it is shown that results produced by PFAGM almost approximate to the real value, and the regression line is almost coincident with the equal line for raw sequence and the produced sequence by the model in Fig 8. Above all, PFAGM model is slightly superior to the other three grey models in the aspect of prediction accuracy.

Table 8. Raw data of cumulative oil field production in example B.

| Year | Oil production | Year | Oil production | Year | Oil production |
|------|----------------|------|----------------|------|----------------|
| 1999 | 73.8217        | 2004 | 342.6394       | 2009 | 519.8508       |
| 2000 | 136.8817       | 2005 | 382.4312       | 2010 | 552.6569       |
| 2001 | 195.059        | 2006 | 420.0399       | 2011 | 581.6092       |
| 2002 | 247.8547       | 2007 | 454.043        | 2012 | 608.1863       |
| 2003 | 297.0902       | 2008 | 485.1171       |      |                |

Fig 6. Searching optimal fractional order of PFAGM by using WOA for Example B.

Table 9. The optimal parameters of different grey models in Example B.

| Parameters | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|------------|---------|-----------|-------------|-------|
| a          | -0.1142 | 0.0689    | 0.0513      | 0.3618|
| b          | 180.4262| 73.4323   | 81.0772     | -172.2099|
| c          | -       | -         | 31.5459     | 246.4802|
| γ          | 1       | 0.04046   | 1.12273     | -0.08235|

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Table 10. The results produced by proposed model and other comparative grey models in Example B.

| Year | Raw data | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|------|----------|---------|-----------|-------------|-------|
|      | Value    | RPE     | Value     | RPE         | Value  |
| 1999 | 73.8217  | 0       | 73.8217   | 0           | 73.8217|
| 2000 | 136.8817 | 46.155  | 136.8817  | 0           | 136.8817|
| 2001 | 195.059  | 14.9692 | 194.4272  | -0.3239     | 195.7859|
| 2002 | 247.8547 | 1.4236  | 247.2619  | -0.2392     | 248.754|
| 2003 | 297.0902 | -5.1502 | 295.9235  | -0.3927     | 297.0427|
| 2004 | 342.6394 | -7.8117 | 340.8246  | -0.5296     | 341.3715|
| 2005 | 382.4312 | -7.4134 | 382.305   | -0.033      | 382.2394|
| 2006 | 420.0399 | -5.5071 | 420.656   | 0.1467      | 420.025 |
| 2007 | 454.043  | -2.0101 | 456.1335  | 0.4604      | 455.0314|
| 2008 | 485.1171 | 2.8064  | 488.9656  | 0.7933      | 487.5107|
| 2009 | 519.8508 | 7.5416  | 519.3578  | -0.0948     | 517.6779|
| 2010 | 552.6569 | 13.3935 | 547.4965  | -0.9337     | 545.7198|
| 2011 | 581.6092 | 20.7816 | 573.5516  | -1.3854     | 571.8011|
| 2012 | 608.1863 | 29.4744 | 597.6785  | -1.7277     | 596.0686|

FMAPE 9.1626 0.274 0.2095 0.2412
PMAPE 21.2165 1.349 1.6447 1.2505
TMAPE 11.7456 0.5043 0.517 0.4575
R 0.9657 0.9998 0.9998 0.9999

Example C: Forecasting foundation settlement

In this validation, the raw data of foundation settlement during engineering construction is obtained from the paper [61], which is tabulated in Table 11. According to the previous practices, we divide the raw into two groups. The first one consists of the first eight digits of foundation settlement, which are used to build grey models for the proposed model and the
contrast grey models. The second one consists of the last two digits used to validate the prediction accuracy of each grey model. Firstly, the optimal orders of the fractional accumulated grey models are found out by using WOA algorithm. Then their linear parameters are estimated by using the least-square method under their determined fractional order. The parameters of GM (1,1) model are obtained by the least-square method directly. These optimal parameters are listed in Table 12. In Fig 9, it is shown that the convergence curve of WOA declines rapidly and then stays a constant stably after about 20 iterations. The results produced by these grey models are also tabulated in Table 13. From Table 13 and Fig 10, it can be apparently found that PFAGM model has the lowest PMAPE and TMAPE compared with the other three contrast grey models and has the second-lowest FMAPE which is almost approximate to the lowest FMAPE of FAGM(1,1,k) model. Meanwhile, it can be clearly seen from Fig 11 that the regression line between raw data and calculated values is almost coincident with their equal line. In brief, it can be concluded that PFAGM model has better prediction performance than the other three grey models in this example.

![Fig 8. Analysis of detailed results obtained by using the proposed model and other comparative grey models in Example B.](https://doi.org/10.1371/journal.pone.0225362.g008)

Table 11. Raw data of foundation settlement in Example C.

| Index | Observed value | Index | Observed value | Index | Observed value |
|-------|----------------|-------|----------------|-------|----------------|
| 1     | 43.19          | 6     | 99.73          | 9     | 112.19         |
| 2     | 58.73          | 7     | 105.08         | 10    | 113.45         |
| 3     | 70.87          | 8     | 109.73         |       |                |
| 4     | 83.71          | 5     | 92.91          |       |                |

https://doi.org/10.1371/journal.pone.0225362.t011
Table 12. The optimal parameters of different grey models in Example C.

| Parameters | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|------------|---------|-----------|-------------|-------|
| a          | -0.0920 | 0.1274    | 0.0421      | 0.0126|
| b          | 59.2681 | 30.1302   | -1.3201     | 42.9745|
| c          | -       | -         | 15.6763     | -25.9805|
| γ          | 0.95251 | 0.99867   | 0.99978     | 0.99980|

Fig 9. Searching optimal fractional order of PFAGM by using WOA for Example C.

Table 13. The results produced by proposed model and other comparative grey models in Example C.

| Index | Raw data | GM(1,1) | RPE | FAGM(1,1) | Value | RPE | FAGM(1,1,k) | Value | RPE | PFAGM | Value | RPE |
|-------|----------|---------|-----|-----------|-------|-----|-------------|-------|-----|-------|-------|-----|
| 1     | 43.19    | 43.19   | 0   | 43.19     | 43.19 | 0   | 43.19       | 43.19 | 0   | 43.19 | 43.19 | 0   |
| 2     | 58.73    | 66.243  | 12.7925 | 58.8407 | 0.1884 | 58.73 | 0 | 58.73 | 0 | 58.6616 | -0.1164 |
| 3     | 70.87    | 72.6275 | 2.4798 | 72.1069 | 1.7453 | 71.7718 | 1.2724 | 71.8027 | 1.3161 |
| 4     | 83.71    | 79.6272 | -4.8773 | 82.9702 | -0.8838 | 82.8814 | -0.9899 | 82.9569 | -0.8997 |
| 5     | 92.91    | 87.3016 | -6.0364 | 91.8531 | -1.1376 | 92.1827 | -0.7828 | 92.2435 | -0.7173 |
| 6     | 99.73    | 95.7156 | -4.0253 | 99.1141 | -0.6176 | 99.7299 | -0.0001 | 99.7491 | 0.0191 |
| 7     | 105.08   | 104.9405 | -0.1327 | 105.0387 | -0.0393 | 105.5627 | 0.4593 | 105.5524 | 0.4496 |
| 8     | 109.73   | 115.0546 | 4.8524 | 109.8553 | 0.1141 | 109.7181 | -0.0108 | 109.73 | 0 |
| 9     | 112.19   | 126.143 | 12.4373 | 113.749 | 1.3896 | 112.2341 | 0.0393 | 112.3567 | 0.1486 |
| 10    | 113.45   | 138.3009 | 21.9048 | 116.8717 | 3.0161 | 113.1498 | -0.2646 | 113.5059 | 0.0493 |

FMAPE 4.3996 | 0.5908 | 0.4394 | 0.4398
PMAPE 17.171 | 2.2029 | 0.152 | 0.0989
TMAPE 6.9539 | 0.9132 | 0.3819 | 0.3716
R 0.9525 | 0.9987 | 0.9998 | 0.9999
Fig 10. Performance comparison of the proposed model and other comparative grey models in Example C.

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Fig 11. Analysis of detailed results obtained by using the proposed model and other comparative grey models in Example C.

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Analysis and discussion

According to the results of the above three validation experiments, we can conclude that PFAGM model has a better prediction accuracy than the other three contrast grey models. From an overall perspective, the prediction performances of fractional grey models are superior to the classical GM(1,1) model. In Example A, the MAPE of PFAGM model for fitting is only slightly lower than the other grey models. However, the MAPE of PFAGM for prediction reaches 1.8927%, which is significantly lower than those of the other models. In Example B, the MAPE of PFAGM model for fitting is very closed to the lowest value though it is not the lowest. Moreover, the prediction performance of PFAGM model is slightly better than FAGM (1,1) and FAGM(1,1,k) model, and significantly better than GM(1,1) model. In Example C, PFAGM model obtains the secondary lowest MAPE of fitting which is also very closed to the lowest one among the four models. PFAGM model also achieves the lowest MAPE of prediction. Meanwhile, the overall MAPEs and correlation coefficients of PFAGM model in the three validation are lowest among all grey models. Such indicates that PFAGM model has better stability and adaptability. Fig 12 shows the rankings of various grey models for fitting and prediction performance. It can be clearly noticed that the prediction performance and overall performance of PFAGM is the best in the three validations. Moreover, the nuclear energy consumption of China is used to validate the performance of PFAGM model which obtains better accuracy in Example A. It indicates that PFAGM model can be applied in forecasting renewable energy consumption. In the next section, we apply the novel PFAGM model to predict Chinese wind energy consumption to provide a reference for corresponding decision departments.

Application

Wind energy is safe, renewable, green, and economical. It is worth developing in all countries all over the world. The 13th Five-Year Plan (2016-2020) of China pointed out that the target of total wind installed capacity will be over 210GW at the end of 2020. However, the cumulative
wind installed capacity of China has reached 211GW at the end of 2018, according to Global Wind Report 2018 of the GWEC. This means that the target of wind energy was achieved two years ahead of schedule. There is little room for wind energy growth in 2019 and 2020. To provide reference data for government, accurately predicting the wind energy is significant and necessary for formulating or adjusting corresponding wind energy policy. In this section, we focus on forecasting the annual wind energy consumption of China from macro perspectives. Because of small samples and some uncertainty, the grey prediction model is selected to apply in this application. According to the validation of the previous section, PFAGM model has a competitive edge compared with the other three grey models and can be used to forecast wind energy consumption. In this application, we collect the raw wind energy consumption of China from the BP Statistical Review of World Energy 2019. The wind energy consumption from 2009 to 2018, which are tabulated in Table 14, is used to forecast the consumption of the next three years. From Table 14, it can be apparently noticed that wind energy consumption increases rapidly. In 2018, the total wind energy consumption reached 82.8 million tonnes oil equivalent, which is 13.4 times as much as ten years ago. Meanwhile, wind energy is one of the cheapest forms of energy in many countries and is a kind of renewable energy used widely. Therefore, accurately forecasting wind energy consumption plays a vital role in reducing energy expenditure and carbon emissions. Firstly, we partition the raw sequence into two groups to build a model and test the model. The first group, including the consumption from 2009 to 2017, is used to build models for the four grey models separately. The second group, including wind energy consumption in 2018, is used to verify the prediction accuracy of these grey models. The linear parameters of GM(1,1) model can be estimated by using the least-square method. They are -0.22512 and 11.19559. The time response function (TRF) of GM(1,1) model can be represented as:

$$ x^{(1)}(k) = 55.9317 e^{-0.2251(k-1)} - 49.7317 $$

(50)

For fractional grey models, their fractional orders should be determined to obtain optimal values. The optimal orders of FAGM(1,1), FAGM(1,1,k), and PFAGM model are searched by using WOA algorithm, which are filled in Table 15. Fig 13 shows that the optimal order of PFAGM model is obtained after a few iterations of WOA algorithm. Then the linear parameters of each fractional grey model can be calculated by using the least-square method, which also are listed in Table 15. The time response function of the three fractional grey models can be respectively represented as follows. The TRF of FAGM(1,1) model is

$$ x^{(0.36971)}(k) = 37.55924 e^{0.17072(k-1)} - 31.35924. $$

(51)

| Year | Wind energy consumption | Year | Wind energy consumption |
|------|--------------------------|------|--------------------------|
| 2009 | 6.2                      | 2014 | 35.3                     |
| 2010 | 10.1                     | 2015 | 42                       |
| 2011 | 15.9                     | 2016 | 53.6                     |
| 2012 | 21.7                     | 2017 | 66.8                     |
| 2013 | 31.9                     | 2018 | 82.8                     |

Table 14. Raw data of China’s wind energy consumption (million tonnes oil equivalent).

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The TRF of FAGM(1,1,k) model is
\[ x^{(1.1336)}(k) = 283.1282e^{0.1385(k-1)} - 30.4471k - 246.4811. \] (52)

The TRF of PFAGM model is
\[ x^{(0.1787)}(k) = 19.6961e^{0.1787k} - 14.4727e^{-0.0812(k-1)} - 2.8781. \] (53)

By using inverse fractional or integral order accumulated operation, the restored results of the four grey models can be easily obtained and tabulated in Table 16. From Table 16 and Fig 14, FMAPE, PMAPE and TMAEP of PFAGM model are 3.1100, 2.1456 and 3.0136 respectively while those of GM(1,1) model are 8.3600, 3.2696 and 7.8510, those of FAGM(1,1) model are 3.1559, 3.3757 and 3.1779, and those of FAGM(1,1,k) model are 3.1901, 6.0252 and 3.4736. It can be clearly noticed that the total, fitting and validation MAPE of PFAGM are lowest. Meanwhile, Fig 15 shows that the regression line almost coincides with the equal line. The fitted points of PFAGM model almost are on the regression line, while more fitted points of other grey models deviate from the regression line. R of PFAGM model is also best among the four grey models. Above all, all the above-mentioned evidences suggest that the prediction performance of PFAGM model is better than the other three grey models, and PFAGM model can be used to forecast the wind energy consumption of the future. Then we utilize these established grey models to forecast the wind energy consumption in the next three years. The predicted wind energy consumptions from 2019 to 2021 are tabulated in Table 17.

Table 15. The optimal parameters of different grey models for forecasting Chinese wind energy consumption.

| Parameters | GM(1,1) | FAGM(1,1) | FAGM(1,1,k) | PFAGM |
|------------|---------|-----------|-------------|-------|
| \( a \)    | -0.22512| -0.17072  | -0.13851    | 0.08124|
| \( b \)    | 11.19559| 5.35365   | 4.21723     | 5.11976|
| \( c \)    | -       | -         | 3.69298     | -0.23365|
| \( \gamma \)| 1       | 0.36871   | 1.13366     | 0.17874|

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Fig 13. Searching optimal order of PFAGM by using WOA for forecasting Chinese wind energy consumption.

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results of PFAGM model are respectively 98.0472, 118.2918, and 142.4003 in the next three years.

### Conclusion

In this paper, a novel fractional grey model called PFAGM is put forward based on the grey action quantity optimization of the classic fractional grey model with an exponential term. For fractional order accumulated grey models, it is the key to seek out their optimal orders to...
obtain the best prediction accuracy. In PFAGM model, WOA algorithm is adopted to search its optimal order. Moreover, its linear parameters are estimated by using the least-square method on the basis of the optimal order. Meanwhile, there are some various structures of the grey model behind PFAGM model, which can be reduced into GM(1,1) and FAGM(1,1) easily. It maybe indicates that PFAGM model has more predictive power than traditional grey models. The results of validation experiments on real-life datasets show that PFAGM model has better prediction performance than the other three grey models. So PFAGM model is used to predict Chinese wind energy consumption. The predicted values of GM(1,1) in the next 3 years are 107.0951, 134.1331, 167.9974 respectively while those of FAGM(1,1) are 96.6314, 116.2954, 139.5689 and those of FAGM(1,1,k) are 92.3688, 109.0540, 128.1861 respectively. It indicates that the results of GM(1,1), FAGM(1,1,k) maybe have more some deviation from the truth. Results of PFAGM model are in the middle of the other grey models’ results. So, its

![Graphs showing GM(1,1), FAGM(1,1), FAGM(1,1,k), and PFAGM models for wind energy consumption prediction.](https://doi.org/10.1371/journal.pone.0225362.g015)

**Fig 15.** Analysis of detailed results obtained by using the proposed model and other comparative grey models for forecasting Chinese wind energy consumption.

![Table showing China's wind energy consumption predictions for 2019-2021.](https://doi.org/10.1371/journal.pone.0225362.t017)

**Table 17.** China's wind energy consumptions from 2019 to 2021 predicted by different grey models.

| Year | GM(1,1)  | FAGM(1,1) | FAGM(1,1,k) | PFAGM  |
|------|---------|-----------|-------------|--------|
| 2019 | 107.0951| 96.6314   | 92.3688     | 98.0472|
| 2020 | 134.1331| 116.2954  | 109.0540    | 118.2918|
| 2021 | 167.9974| 139.5689  | 128.1861    | 142.4003|

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predicted results can be as reference data of adjusting wind energy policy. Above all, it can be
drawn that PFAGM model is efficient to realize short term prediction for time series, especially
the sequence with small samples or uncertainty. In the future, the novel PFAGM model can be
applied in more applications such as carbon emission forecasting, management of the petro-
leum reservoirs and so on.

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