Variability of Binary Stochastic Neurons Employing Low Energy Barrier Nanomagnets with In-plane Anisotropy

Rahnuma Rahman and Supriyo Bandyopadhyay  
Department of Electrical and Computer Engineering  
Virginia Commonwealth University, Richmond, VA 23284, USA

Received xxxxxx  
Accepted for publication xxxxxx  
Published xxxxxx

Abstract

Binary stochastic neurons (BSNs) are excellent hardware accelerators for machine learning. An ideal platform for implementing them are low- or zero-energy-barrier nanomagnets (LBMs) possessing in-plane anisotropy (e.g. circular or slightly elliptical disks) whose fluctuating magnetization encodes a probabilistic bit. Here, we show that such a BSN’s activation function, the pinning current (which pins the output to a particular binary state), and the correlation time associated with the decay of the auto-correlation of the fluctuation (which determines the speed of the BSN) – all exhibit strong sensitivity to very slight geometric variations in the LBM. For example, a mere 1% change in the diameter of a circular LBM in any arbitrary direction can change the correlation time by a factor of 3 - 4 at room temperature and a 10% variation can change the pinning current by a factor of ~2. All this poses a design challenge. It appears that slightly elliptical LBMs may be preferable over perfectly circular LBMs. We also show that spin inertia, which gives rise to nutation during the initial few fs or ps of the magnetization dynamics and can sometimes have long-term outcomes, has no significant effect on the BSN characteristics.

Keywords: Binary stochastic neurons, low barrier nanomagnets, pinning current, correlation time

1. Introduction

Artificial intelligence (AI) platforms typically employ deep neural networks (DNNs). DNNs are also dominant in statistical machine learning which has advanced computer vision (image detection, recognition and segmentation [1]), natural language processing [2], etc. There is now a strong desire to improve the energy and speed performances of these systems, leading to the interest in spiking neural networks (SNN) activated by binary stochastic impulses [2, 3]. They are remarkably energy-efficient and fast. The primitive element of such SNNs is a binary stochastic neuron (BSN) that has two distinct output states (-1 and +1). The BSN will output either a -1 or a +1 (in the bipolar representation) with a probability determined by a specific function of the input that is provided to it. It is also an excellent representation of an Ising spin used to solve combinatorial optimization problems.

Low (or zero) energy barrier nanomagnets (LBMs) possessing in-plane magnetic anisotropy are a natural (and popular) choice for BSNs and have been shown to be capable of acting as efficient hardware accelerators for machine learning [4-6]. The magnetization vectors of these LBMs, which are circular or nearly circular disks, fluctuate at room temperature owing to thermal perturbations and the fluctuating (stochastic) magnetization can produce, with suitable design, either a -1 or a +1 state [4, 5] whose probabilities are engineered by passing a spin polarized current through the LBM [4, 5]. The current, which is the input to the BSN, biases the output towards either -1 or +1 and the degree of bias can be tuned with the current.

1 Invited paper
There are two critical parameters for a BSN. The first is the “activation function” which represents the probability of outputting a given state (−1 or +1) as a function of the input. In the case of LBMs implementing BSNs, the input is a spin polarized current of magnitude \( I_0 \) and the activation function is the \( \langle \hat{s} \cdot \hat{m} \rangle \) “versus \( I_0 \)” relation where \( \hat{s} \) and \( \hat{m} \) are the unit vectors in the direction of the spin polarization of the current and the magnetization of the LBM, respectively. The angular bracket denotes time averaging and hence \( \langle \hat{s} \cdot \hat{m} \rangle \) can be viewed as the (time-averaged) probability of outputting +1 for one direction of spin polarization of the current and -1 for the opposite direction. The second critical parameter, which determines the response speed of BSNs with LBMs, is the “correlation time” \( \tau_c \) which is the full-width-at-half maximum (FWHM) of the decay characteristic of the auto-correlation function of the magnetization fluctuations [5]. LBMs with in-plane anisotropy typically have a value of \( \tau_c \) that is about two orders of magnitude smaller than LBMs with perpendicular magnetic anisotropy [5] and are therefore favored because the BSN speed (i.e. the speed of hardware accelerators) will be higher if they are employed.

In this paper, we present three important results pertaining to BSNs implemented with LBMs possessing in-plane anisotropy. First, we show that the activation function \( \langle \hat{s} \cdot \hat{m} \rangle \) is quite sensitive to small changes in the shape of a circular nanomagnet. Second, we show that the auto-correlation function of the magnetization fluctuation and its FWHM (i.e. \( \tau_c \)) can change significantly depending on whether the LBM is perfectly circular or very slightly elliptical. We show this with the example of a perfectly circular LBM of diameter 100 nm and a very slightly elliptical LBM with major axis of 100 nm and minor axis of 99 nm. Their \( \tau_c \)'s differ by a factor of 3-4, even though their lateral dimensions in just one direction differ by a mere 1%. The slightly elliptical LBM has a longer (shorter) correlation time than the circular LBM for the magnetization component along the major (minor) axis. Third, we show that “spin-inertia” [7], which causes short-lived nutation in magneto-dynamics and can sometimes have long-term effects [8], has no significant influence on the auto-correlation decay characteristics of BSNs implemented with LBMs and hence has no serious effect on \( \tau_c \). It also has no perceptible effect on the activation function.

2. Theory

To study the dynamical characteristics of BSNs implemented with LBMs, such as a circular or slightly elliptical disk shown in Fig. 1, we first simulate their magneto-dynamics in the presence of thermal noise by solving the stochastic Landau-Lifshitz-Gilbert (LLG) equation (sometimes referred to as the Landau-Lifshitz-Gilbert-Langevin equation) with the effect of spin-inertia included as a second order time derivative of the magnetization with a characteristic pre-factor \( \tau \) which is the time scale over which the angular momentum of magnetization relaxes [7]. It is also the time scale over which nutation takes place in the magneto-dynamics [7]. Additionally, we model the effect of a spin polarized dc current flowing normal to the cross-section of the nanomagnet with the spin polarized along either the x-axis or the y-axis (minor and major axes of an elliptical nanomagnet) shown in Fig. 1. This allows us to evaluate the activation function and the pinning current, which will “pin” the magnetization along the direction of spin polarization of the injected current. Pinning it to the minor axis (hard axis) with spin-polarized current will be harder than pinning it to the major axis (easy axis) in an elliptical LBM since the shape anisotropy energy in the LBM is minimum when the magnetization is aligned along the easy axis and maximum when the magnetization is aligned along the hard axis.

The stochastic LLG equation (including the effect of spin inertia) is

\[
\frac{d\hat{m}(t)}{dt} = -\frac{\alpha}{\mu_0 M_s \Omega} \left[ \left( H(t) - \gamma \hat{m}(t) \times \frac{d\hat{m}(t)}{dt} + \tau \frac{d^2\hat{m}(t)}{dt^2} \right) \right] \\
+ a\hat{m}(t) \times \left[ \frac{\xi_2}{q M_s^2} \hat{m}(t) \right] + b\frac{\xi_1}{q M_s^2} \times \hat{m}(t)
\]

where \( H(t) = H_{\text{thermal}}(t) + H_{\text{dam}}(t) \)

\[
H_{\text{thermal}}(t) = \frac{2a kT}{\sqrt{1 + (\alpha^2) \mu_0 M_s \Omega \Delta t}} \times \left[ G_{01}(t) \hat{\xi} + G_{02}(t) \hat{\eta} + G_{03}(t) \hat{\zeta} \right]
\]

\[
H_{\text{dam}}(t) = -M_s N_{d,xx} m_{xx}(t) \hat{\xi} - M_s N_{d,yy} m_{yy}(t) \hat{\eta} - M_s N_{d,zz} m_{zz}(t) \hat{\zeta}
\]

\[
\tilde{I} = I_0 \hat{\tilde{x}} \quad \text{(for spin polarized in the x-direction)}
\]

\[
\tilde{I} = I_0 \hat{\tilde{y}} \quad \text{(for spin polarized in the y-direction)}
\]

\[
N_{d,xx} = \frac{\pi}{4} \left( \frac{d}{l} \right) \left[ \frac{1}{4} \left( \frac{l - w}{l} \right) + \frac{3}{16} \left( \frac{l - w}{l} \right)^2 \right]
\]

\[
N_{d,yy} = \frac{\pi}{4} \left( \frac{d}{l} \right) \left[ \frac{1}{4} \left( \frac{l - w}{l} \right) - \frac{3}{16} \left( \frac{l - w}{l} \right)^2 \right]
\]

\[
N_{d,zz} = 1 - N_{d,xx} - N_{d,yy}
\]
Fig. 1: A nanomagnet shaped like an elliptical disk with the major axis along the y-axis and minor axis along the x-axis. The nanomagnet has in-plane magnetic anisotropy.

Here \( \vec{m}(t) \) is the magnetization vector normalized to the saturation magnetization of the magnetic material \( M_s \), and \( m_i(t) \) is its \( i \)-th component. The quantity \( \vec{H}(t) \) is the effective magnetic field acting on the magnetization, and it has two components due to demagnetization and noise.

The quantity \( \gamma \) is the gyromagnetic precession constant \( (2.21 \times 10^4 \text{ rad-m/A-s}) \), \( \alpha \) is the Gilbert damping factor in the nanomagnet associated with damping of the magnetization’s precession, and \( \tau \) is the relaxation time of the angular momentum associated with spin inertia \( [7] \). The magnitude of the charge current injected into the nanomagnet is \( I_0 \) and the degree of spin polarization of the current is \( \alpha \), with the spin polarized along the intended pinning direction, which is either the major axis of the nanomagnet \( (y \text{-direction}) \) or the minor axis \( (x \text{-direction}) \) in our case.

The last two terms in the first equation above represent the Slonczewski and field-like torques, respectively, with \( \mu_B \) being the Bohr magneton. The relative strengths of these two torques are given by the quantities \( a \) and \( b \), and following ref. \([9]\), we assume \( a = 1.0 \) and \( b = 0.3 \). The value of \( \tau \) can range from a few fs to \( \sim 100 \text{ ps} \) in ferromagnets \([10]\).

The demagnetizing field \( H_{\text{demag}}(t) \) depends on the demagnetization coefficients \( N_{dxz}, N_{dyz} \) and \( N_{dzx} \), which, in turn, depend on the nanomagnet dimensions \( l \) (major axis), \( w \) (minor axis) and \( d \) (thickness) as shown in Fig. 1 \([11]\). In the expression for the thermal noise field term \( H_{\text{thermal}}(t) \), \( \Omega \) is the volume of the nanomagnet \( \Omega = (\pi/4)l \times w \times d \), \( \Delta t \) is the time step used in the simulation, \( \mu_0 \) is the permeability of free space and the quantities \( G_{i,j}^{(l)} (i = x, y, z) \) are three statistically independent Gaussians with zero mean and unit standard deviation \([12]\). All simulations assume room temperature \( (T = 300 \text{ K}) \). We also assume the following material parameters: \( M_s = 8 \times 10^5 \text{ A/m} \), \( \alpha = 0.1 \).

The vector equation (1) is decomposed into three coupled scalar equations. When the spin in the injected current is polarized along the \( x \)-direction, the coupled equations are:

\[
\frac{dm_x(t)}{dt} = -\gamma \left[ m_y(t) H_x(t) - m_z(t) H_z(t) \right] + \alpha \left[ m_x(t) \frac{dm_x(t)}{dt} - m_y(t) \frac{dm_y(t)}{dt} \right] + \alpha \left[ m_z(t) \frac{dm_z(t)}{dt} - m_y(t) \frac{dm_y(t)}{dt} \right] - b \left( \frac{\xi_f \mu_B}{q M_s \Omega} \right) m_y(t) - \left( \frac{\xi_f \mu_B}{q M_s \Omega} \right) m_z(t)m_y(t)
\]

\[
\frac{dm_y(t)}{dt} = -\gamma \left[ m_y(t) H_y(t) - m_x(t) H_x(t) \right] + \alpha \left[ m_y(t) \frac{dm_y(t)}{dt} - m_x(t) \frac{dm_x(t)}{dt} \right] + \alpha \left[ m_z(t) \frac{dm_z(t)}{dt} - m_x(t) \frac{dm_x(t)}{dt} \right] + b \left( \frac{\xi_f \mu_B}{q M_s \Omega} \right) m_x(t) - \left( \frac{\xi_f \mu_B}{q M_s \Omega} \right) m_z(t)m_x(t)
\]

where \( H_i(t) (i = x, y, z) \) is the \( i \)-th component of the effective magnetic field at time \( t \).

When the spin in the injected current is polarized in the \( y \)-direction, the coupled equations are:
\[ \frac{dm_x(t)}{dt} = -H \left[ m_y(t) H_y(t) - m_z(t) H_z(t) \right] + \alpha \left[ m_y(t) \frac{dm_y(t)}{dt} - m_z(t) \frac{dm_z(t)}{dt} \right] \\
+ \alpha \left[ m_z(t) \frac{dm_z(t)}{dt} - m_y(t) \frac{dm_y(t)}{dt} \right] + \gamma m_y(t) - \alpha \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_y(t) m_z(t) \\
+ a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) \left[ m_z^2(t) + m_y^2(t) \right] \\
+ \frac{\xi J_{dbu}}{qM_{\Omega}} \left[ m_z(t) - a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_y(t) m_z(t) \right] \\
- \frac{\xi J_{dbu}}{qM_{\Omega}} \left[ m_z(t) - a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_y(t) m_z(t) \right] \\
+ \beta \left[ \frac{\xi J_{dbu}}{qM_{\Omega}} \right] m_y(t) - a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_y(t) m_z(t) \\
(2b) \\
\frac{dm_y(t)}{dt} = -H \left[ m_x(t) H_x(t) - m_z(t) H_z(t) \right] \\
+ \alpha \left[ m_z(t) \frac{dm_z(t)}{dt} - m_x(t) \frac{dm_x(t)}{dt} \right] \\
- \alpha \left[ m_x(t) \frac{dm_x(t)}{dt} - m_z(t) \frac{dm_z(t)}{dt} \right] + \gamma m_z(t) \\
+ \gamma m_z(t) - \alpha \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_z(t) m_x(t) \\
+ a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) \left[ m_x^2(t) + m_z^2(t) \right] \\
+ a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) \left[ m_x(t) - a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_z(t) m_x(t) \right] \\
(2c) \\
\frac{dm_z(t)}{dt} = -H \left[ m_x(t) H_x(t) - m_y(t) H_y(t) \right] \\
+ \alpha \left[ m_y(t) \frac{dm_y(t)}{dt} - m_x(t) \frac{dm_x(t)}{dt} \right] \\
- \alpha \left[ m_x(t) \frac{dm_x(t)}{dt} - m_y(t) \frac{dm_y(t)}{dt} \right] + \gamma m_y(t) \\
+ \gamma m_y(t) - \alpha \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_y(t) m_z(t) \\
+ a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) \left[ m_y^2(t) + m_z^2(t) \right] \\
+ a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) \left[ m_y(t) - a \left( \frac{\xi J_{dbu}}{qM_{\Omega}} \right) m_z(t) m_y(t) \right] \\
\] 

\[ \mathbf{3.1 \ \text{Pinning current and the activation function of the BSN}} \]

Equation (2) has an implicit random term associated with thermal noise field (see Equation (1)) and hence the magnetization versus time trajectories \[ \left[ m_i(t) \right] vs \ t \] calculated from it will differ slightly from run to run. Consequently, all results reported here are ensemble averaged over 1000 runs.

In order to determine the activation function and pinning current of the BSN, we first calculate the quantity \( \langle \hat{s} \cdot \hat{m} \rangle \) as a function of the current amplitude \( I_0 \) after steady state is reached. Here \( \hat{s} \) is the unit vector along the direction of the spin polarization in the injected current. When the spin-polarization is along the \( y \)-direction, \( \langle \hat{s} \cdot \hat{m} \rangle = \langle m_y \rangle \) and when the spin-polarization is along the \( x \)-direction, \( \langle \hat{s} \cdot \hat{m} \rangle = \langle m_z \rangle \).

In Figs. 2(a) and 2(b), we show a plots of \( m_x(t) \) vs \( t \) and \( m_y(t) \) vs \( t \) for an elliptical nanomagnet (major axis = 100 nm, minor axis = 90 nm and thickness = 2 nm) for \( I_0 = 2 \) mA. The spin polarization in the injected current is in the +\( x \) direction and +\( y \) direction, respectively, for the two cases, where the \( x \)-axis is along the minor (hard) axis and the \( y \)-axis is along the major (easy) axis. The initial orientation of the magnetization is assumed to lie in the plane of the LBM, orthogonal to the direction of the current’s spin polarization. Note that the time to achieve pinning in the two cases are different with the latter being shorter. This is because the minor axis (\( x \)-axis) is the hard axis and the major axis (\( y \)-axis) is the easy axis. Since it is harder to pin the magnetization along the hard axis and easier to pin it along the easy axis, the time to pin is always longer in the former case for any given value of the current.

![Fig. 2: Plots of (a) \( m_x(t) \) vs \( t \) and (b) \( m_y(t) \) vs \( t \) for an elliptical nanomagnet (major axis = 100 nm, minor axis = 90 nm and thickness = 2 nm) for \( I_0 = 2.0 \) mA. The spin polarization in the injected current is in the +\( x \) direction and +\( y \) direction, respectively, for the two cases. The time to pin is the time taken to pin the magnetization in the orientation of the spin polarization. The nutation time \( \tau \) was taken to be 10 ps.](image-url)
The component \( m_x(t) \) or \( m_y(t) \) takes different amounts of time to reach steady-state (and thereafter get pinned) for different values of the current \( I_0 \) and never reaches steady-state for \( I_0 = 0 \).

For the non-zero values of \( I_0 \) that we considered, both \( m_x(t) \) and \( m_y(t) \) always reach steady-state and get pinned within 25 ns of turning on the current. Hence, we take the value of \( m_x(t) \) or \( m_y(t) \) at the end of 25 ns for every value of \( I_0 \) that we consider and then average that over 1000 runs to calculate \( \langle \mathbf{\hat{s}} \cdot \mathbf{\hat{m}} \rangle = \left\langle m_x \right\rangle \) and \( \langle \mathbf{\hat{s}} \cdot \mathbf{\hat{m}} \rangle = \left\langle m_y \right\rangle \), when the spin is polarized along the \( x \)-direction and along the \( y \)-direction, respectively. At steady-state, the system is ergodic and hence the time-average is equal to the ensemble-average.

In Fig. 3, we show the results of \( \langle \mathbf{\hat{s}} \cdot \mathbf{\hat{m}} \rangle \) versus \( I_0 \) (i.e. the activation function) for three different values of the nutation duration \( \tau \) associated with spin inertia, and find no significant \( \tau \)-dependence. We consider three cases: (a) elliptical nanomagnet with the injected current spin-polarized along the major axis (\( y \)-axis), (b) same nanomagnet with the injected current spin-polarized along the minor axis (\( x \)-axis), and (c) perfectly circular nanomagnet. In all cases, the magnetization initially points orthogonal to the axis of spin polarization of the injected current (i.e. orthogonal to the intended direction of pinning) while lying in the plane of the LBM. The positive and negative values of the current represent opposite spin polarizations. The curve has the expected \( \tanh(I_0/\tau) \) \( [I_0 = \text{constant}] \) behavior \([4, 5]\). The values of \( I_0 \) where \( \langle \mathbf{\hat{s}} \cdot \mathbf{\hat{m}} \rangle \) reaches the value \( \pm 1 \) are the pinning currents for pinning the magnetization along the direction of spin polarization. These results reveal that spin inertia does not affect the activation function or the pinning current perceptibly. However, there is clearly some dependence on whether the nanomagnet is circular or elliptical and whether the spin polarization of the injected current is along the major axis or minor axis in the case of the elliptical nanomagnet. In fact, a 10\% variation in a principal axis dimension (minor axis varying from 100 nm to 90 nm, i.e. circular to slightly elliptical) decreases the pinning current from 1.0 mA to 0.5 mA when the current is spin-polarized along the major axis (easy axis) and increases the pinning current from 1.0 mA to 2.0 mA when the current is spin-polarized along the minor (hard) axis. This happens because it is easier to pin the magnetization along the easy axis and harder to pin it along the hard axis, whereas the circular nanomagnet has no distinction between easy and hard axes. Therefore, we expect that the pinning current will be least in an elliptical nanomagnet with spin polarization along the major axis and most in the same nanomagnet with the spin polarization along the minor axis, with the circular nanomagnet being in between. That is what we observe.

![Fig. 3: Plot of the steady-state value of \( \langle \mathbf{\hat{s}} \cdot \mathbf{\hat{m}} \rangle \) as a function of current for different values of the parameter \( \tau \) representing the duration of nutation caused by spin inertia.](image)

- (a) Elliptical nanomagnet (major axis = 100 nm, minor axis = 90 nm and thickness = 2 nm) with an in-plane shape anisotropy energy barrier of 4.6 kT at room temperature and the current is spin-polarized along the \( y \)-axis or major axis (easy axis) and
- (b) same nanomagnet when the current is spin-polarized along the \( x \)-axis or minor axis (hard axis), and
- (c) circular nanomagnet of diameter 100 nm and thickness 2 nm with no in-plane shape anisotropy energy barrier.
Correlation time of the BSN

Next, we calculate the auto-correlation function of the magnetization component along a particular direction (say, the $x$-axis) defined as $C_x(t') = \int_0^\infty m_x(t)m_x(t+t')dt$ in the absence of any spin-polarized current, starting with two initial conditions: first, the magnetization initially pointing approximately along the $+x$-axis, and second, approximately along the $+y$-axis. These two initial conditions are, respectively, $m_x(0) = 0.995; m_y(0) = 0.095; m_z(0) = 0.031$ and $m_x(0) = 0.095; m_y(0) = 0.995; m_z(0) = 0.031$. Starting with either initial condition, we follow the time trajectory of the magnetization component $m_x(t)$ [obtained by solving Equation (2)] to obtain the auto-correlation function $C_x(t')$ versus $t'$ as $m_x(t)$ fluctuates between -1 and +1 owing to thermal noise. Because the fluctuation history is slightly different from one run to another owing to the randomness of thermal noise, we average $C_x(t')$ versus $t'$ over 1000 runs to obtain the final $C_x(t')$ versus $t'$ plots. The purpose of this calculation is to find the FWHM of the auto-correlation function, which is the correlation time $\tau_c$ that determines the response time of BSNs. Our purpose was to study if slight changes in the nanomagnet shape (from perfectly circular to slightly elliptical) can have a significant effect on the correlation time $\tau_c$.

We also obtain the auto-correlation function of the magnetization component along the $y$-axis which would be the major axis of the elliptical nanomagnet. This is defined as $C_y(t') = \int_0^\infty m_y(t)m_y(t+t')dt$. We calculate this for the same two initial conditions, namely, $m_x(0) = 0.995; m_y(0) = 0.095; m_z(0) = 0.031$ and $m_x(0) = 0.095; m_y(0) = 0.995; m_z(0) = 0.031$.

In Fig. 4, we plot the calculated auto-correlation functions as a function of the delay [i.e. $C_x(t')$ versus $t'$ and $C_y(t')$ versus $t'$] for a slightly elliptical nanomagnet (major axis = 100 nm, minor axis = 99 nm) and a perfectly circular nanomagnet (diameter = 100 nm), both with the same thickness of 6 nm. The first has an in-plane shape anisotropy barrier of $4kT$ at room temperature and the second has no such barrier. In the case of the slightly elliptical nanomagnet, we plot the auto-correlation functions for two cases: initial orientation of the magnetization is along the minor axis ($x$-axis) and initial orientation is along the major axis ($y$-axis).

**Fig. 4:** Auto-correlation function of the fluctuations in the magnetization component along the $x$-axis (minor axis of the elliptical nanomagnet) versus delay for (a) a slightly elliptical (100 nm, 99 nm) nanomagnet with initial orientation of the magnetization along the major ($y$-) axis, (b) the same slightly elliptical nanomagnet with initial orientation along the minor ($x$-) axis, and (c) a perfectly circular nanomagnet (diameter = 100 nm). Auto-correlation function of the fluctuations in the magnetization component along the $y$-axis (major axis of the elliptical nanomagnet) versus delay with (d) initial orientation along the major ($y$-) axis and (e) initial orientation along the minor ($x$-) axis.
Table I: The correlation time $\tau_c$ in circular and slightly elliptical LBMs for three different spin inertia relaxation times $\tau$.

| $\tau$ (ps) | $\tau_c$ in circular LBM | $\tau_c$ associated with $m_x(t)$ in slightly elliptical nanomagnet. Initial orientation along major axis | $\tau_c$ associated with $m_x(t)$ in slightly elliptical nanomagnet. Initial orientation along minor axis | $\tau_c$ associated with $m_y(t)$ in slightly elliptical nanomagnet. Initial orientation along major axis | $\tau_c$ associated with $m_y(t)$ in slightly elliptical nanomagnet. Initial orientation along minor axis |
|-------------|--------------------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| 0           | 5.8 ns                   | 1.5 ns                                                                                           | 1.6 ns                                                                                           | 20.25 ns                                                                                         | 20.79 ns                                                                                         |
| 10          | 5.6 ns                   | 1.5 ns                                                                                           | 1.6 ns                                                                                           | 21.07 ns                                                                                         | 20.42 ns                                                                                         |
| 100         | 5.7 ns                   | 1.7 ns                                                                                           | 1.7 ns                                                                                           | 20.24 ns                                                                                         | 20.79 ns                                                                                         |

In Table I, we list the calculated full-width-at-half-maximum (FWHM) of the auto-correlation functions (or the correlation time $\tau_c$) for the five cases.

From Table I, we find that spin inertia (i.e., the value of $\tau$) does not cause any significant change in the correlation time, but even a slight change in geometry (1% change in the diameter of the circular nanomagnet in any arbitrary direction to make it slightly elliptical) can decrease the correlation time associated with fluctuation of the magnetization component along the minor axis by a factor of nearly 4 and increase the correlation time associated with fluctuation of the magnetization component along the major axis by a factor of nearly 4 as well! The opposite behaviors of the two magnetization components have to do with the fact that the major axis is the easy axis and the minor axis is the hard axis. The magnetization prefers to loiter around the easy axis and shun the hard axis. That is why the auto-correlation function of the fluctuation of the magnetization component along the easy axis decays much more slowly than that along the hard axis. What is surprising is that a very slight ellipticity (major axis = 100 nm and minor axis = 99 nm) can make so much difference. When probabilistic (p-) bits are encoded in the randomly fluctuating magnetization component of a LBM [4-6], it makes a very significant difference as to whether the component along the major axis or the minor axis is chosen to encode the p-bit, since they have very different correlation times that can differ by more than an order of magnitude.

We also notice that the initial magnetization orientation does not make any significant difference in the case of the slightly elliptical nanomagnet. One might have expected to see some dependence on the initial orientation because the major axis ($y$-axis) is the easy axis and hence corresponds to a stable direction while the minor axis ($x$-axis) is the hard axis and corresponds to an unstable direction. Therefore, if the magnetization is initially along the easy axis, it will resist straying from it while if it is initially along the hard axis, it will quickly leave that state. We do not see this effect since the decay time of the auto-correlation function exceeds 1 ns and over that long duration, the magnetization loses any memory of the initial orientation. That is why we see no effect of the initial state on the auto-correlation function decay characteristic or the correlation time $\tau_c$. Because the decay is relatively slow and occurs over a time scale exceeding 1 ns, which is much longer than the nutation time $\tau$, we also do not see any effect of spin inertia. Sometimes spin inertia can have a very significant effect in slow magneto-dynamics that lasts over time scales orders of magnitude longer than $\tau$ [8], but those cases are unusual. This is not one such case.

The most interesting point to note is that the correlation time in the circular nanomagnet differs by a factor of ~4 from that in the slightly elliptical nanomagnet (either $4 \times$ larger or $4 \times$ smaller depending on which magnetization component is chosen to encode the p-bit). The difference between the two nanomagnets is that the circular nanomagnet has no in-plane shape anisotropy energy barrier and the in-plane component of the demagnetizing field is isotropic in the plane of the nanomagnet, whereas in the slightly elliptical nanomagnet, the in-plane shape anisotropy energy barrier is small (4kT) but non-zero, and the in-plane component of the demagnetizing field is slightly anisotropic in the nanomagnet’s plane. Surprisingly, these very small differences have a very strong effect on the correlation time and cause the factor of ~4 difference.
4. Conclusion

We have shown that the temporal characteristics (decay of the auto-correlation function) of binary stochastic neurons implemented with low- or zero-energy barrier nanomagnets (LBMs) with in-plane anisotropy can exhibit significant variability and sensitivity to small differences in geometry. In the past, we showed that extended defects introduce large variability in the auto-correlation functions of BSNs implemented with LBMs possessing in-plane anisotropy [13], and here we show that slight variation in the geometry can also have a similar deleterious effect. It may be advisable to avoid perfectly circular nanomagnets, such as those used to fashion p-bits in refs. [14, 15], because a slight change in shape can cause a large change in the correlation time, and use slightly elliptical nanomagnets instead to ameliorate this effect.

APPENDIX

Ref. [5] derived approximate analytical expressions for the correlation time and pinning currents. These expressions are

\[
\tau_c = \frac{\sqrt{8\ln(2)} \frac{1}{\gamma} \sqrt{\frac{M \Omega}{4\pi H_{\text{demag}}^2}}}{kT} \quad (A1)
\]

\[
I_p = \frac{2q}{h} \sqrt{\frac{\sigma}{\pi}} \sqrt{\frac{4\pi H_{\text{demag}}^2}{M \Omega kT}} \quad (A2)
\]

where \( q \) is the electronic charge, \( h \) is the reduced Planck constant and \( H_{\text{demag}} \) is the normal-to-plane demagnetizing field given by \( H_{\text{demag}} = \frac{1}{M} N_{d-\pi} \). The other quantities were previously defined. These expressions were probably derived for the perfectly circular nanomagnet since no account is made of any shape anisotropy. It is informative to compare the analytical results with the numerically calculated results.

In our case, the quantity \( H_{\text{demag}} \) is \( 7.25 \times 10^5 \) A/m for the circular nanomagnet. The volume of the perfectly circular nanomagnet that we considered is \( 47,124 \) nm\(^3\). Using these results in Equation (A1), we find that the correlation time \( \tau_c \) is 333 ns in the perfectly circular nanomagnet, which is about two orders of magnitude larger than what we found and report in Table I.

The value of the pinning current calculated from Equation (A2) is about 2.9 A, which is three orders of magnitude larger than what we find in the circular nanomagnet.

These observations show that the analytical expressions are at odds with the more accurate numerical simulations. This is not surprising since stochastic dynamics is rarely amenable to precise analytical treatment.

Acknowledgement: The authors acknowledge support from the US National Science Foundation under grants CCF-2001255 and CCF-2006843.

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