Cascade Global Symmetry and Quark Mass Hierarchy
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Abstract
Cascade global symmetry and multi-vacuum expectation values are combined to produce an initial texture of the quark mass matrices. Required corrections to the initial texture zeros (ITZ) are all at the order of $10^{-3} m_t$ or less. The possibility of radiative corrections as the source of the complete mass matrices is briefly discussed.

The discovery of the top quark makes the fermion spectrum in the standard model complete. We are now puzzled by the pattern of the mass spectrum in the standard model (SM) (Fig.1), as we were thirty years ago by the spectrum of hadrons. However, history does not repeat itself without some varieties. Obviously, the masses of quarks and leptons are regulated by mass ratios (hierarchy), instead of mass differences in the case of hadrons.

Dr. Tanaka has just reviewed some previous works in resolving this puzzle. I here would like to present an alternative approach which relates this puzzle to the structure of the Higgs sector, a sector yet completely in the dark. So far, no electroweak scalar (Higgs) has ever been found. However, the best limits are only set for the minimal standard model Higgs boson. For a more complicated spectrum of scalar particles (why are there not many scalars, while there are so many fermions ?), only one mass eigenstate, which is in the direction of the physical vacuum, has dimension three couplings with the weak gauge bosons; all the other Higgs particles, if they exist, are orthogonal to this single state and possess only dimension four couplings with gauge bosons, and therefore are more difficult to discover. The main feature of this approach is to use a natural global symmetry of the standard gauge-fermion interaction sector to protect the small matrix elements in the mass matrices of the quarks.

The basic ideas behind this approach[1,2] are the following:
1. Approximate global symmetry which is non-abelian in order to be more restrictive.
2. Multi-Higgs doublets with different vacuum expectation values (VEVs) contribute to dif-
ferent mass terms.

3. The naturalness principle which is engaged to control the sizes of the coupling constants whose ratios are at the order of $10^0$.

4. Rich scalar spectrum with a bunch of pseudo-Nambu-Goldstone bosons (PNGB).

Approximate global symmetries have been found in strong interactions. I-spin, chiral $SU(3) \times SU(3)$ and G-parity, to name just a few. These global symmetries are inexact. In the example of $SU(3) \times SU(3)$ chiral symmetry, it is a common belief that symmetry breaking comes from both the existence of a non-trivial QCD vacuum and the existence of the current quark masses. The former is called spontaneous symmetry breaking (SSB); and the latter, explicit symmetry breaking (ESB). A pattern was emerged from the relevant studies: While gauge symmetries are regarded as exact and dynamical symmetries, global symmetries are regarded as inexact and accidental. ESB of global symmetries are assumed ad hoc in the effective Lagrangian, although the ESB itself may be the effect of another SSB at a higher scale, as in the case of current masses of quarks in the chiral theory of light hadrons.

The specific global symmetry we use is $G = SU(3) \times U(1)$, where 3 corresponds to three generations and the extra $U(1)$ is an overall phase transition. Left-handed quarks are in a triplet (3), in addition to being in $SU(2)_L$ doublets. Right-handed quarks are in two anti-triplets ($3^*$). The global $U(1)$ quantum number is also assigned differently for the left and right handed fields. A Higgs triplet and a Higgs sextet, which are also weak doublets, are introduced to provide two different vacuum expectation values. These VEVs spontaneously break the $G$ symmetry down to a $U(1)$, because in the chosen parameter space the two VEVs are perpendicular to each other.

It should be emphasized that the Yukawa sector does not respect exact global symmetry. There are terms with smaller symmetries. These terms partially break the global symmetry. Since the ESB terms introduced in the Yukawa sector are dimension-4 operators, induced symmetry breaking terms in the Higgs potential are divergent (Fig. 2). Therefore corresponding ESB must also be introduced in the Higgs sector in order to provide counter terms to these divergences.

Typically, the mass matrices produced directly by the Yukawa sector through SSB are only the main texture. For example, the mass matrix for the up type quarks is

$$M^U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_1 v' \\ 0 & -G_1 v' & G_0 v \end{pmatrix}.$$ (1)
That for down type quarks is

$$M^D = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & G_3v' \\
0 & -G_3v' & G_2v
\end{pmatrix}. \quad (2)$$

Here $v$ is the VEV of the sextet Higgs and $v'$ is the VEV of the triplet Higgs. We assume $v : v' \sim 1 : 0.4$. $G_\alpha$ with $\alpha = 0, 1, 2, 3$ are the Yukawa coupling constants which share sequentially smaller symmetries. Approximately, the ratios of these couplings are

$$G_0 : G_1 \sim 5, \quad G_1 : G_2 \sim 7, \quad G_2 : G_3 \sim 2.5. \quad (3)$$

Both the mass matrices in Eqs. (1) and (2) are rank-2 matrices and it is easy to see that $m_t = |G_0v|$, $m_b = |G_1v|$ and the masses $m_c$ and $m_s$ are respectively decided by a “see-saw” like mechanism, which gives $m_c = |G_1v'|^2/m_t$, $m_s = |G_3v'|^2/m_b$. We also immediately obtain $V_{cb} = x - x'$, with $x = G_3v'/G_2v$, $x' = G_1v'/G_0v$. This $V_{cb}$ is within a factor of two close to the experimental data in magnitude. Considering the preliminary property of this calculation, this result is encouraging. Impressive progress has been made along other lines of thinking (to name just a few, see Refs.[3-7]). However, new features of this approach will immediately become apparent.

A few other interesting features of these two mass matrices should be noticed. First, there are only four different entries of sequentially smaller magnitudes. The sequence is caused by both smaller VEV and smaller Yukawa coupling constants. Second, each mass matrix has six initial texture zeros (ITZs). The nonzero matrix elements (2,3) or (3,2) in each matrix are about 10 times smaller than the corresponding (3,3) elements. The required corrections to all the ITZs are at the order of $10^{-3}m_t$ or less. Third, the ITZs are protected by the symmetrical property of the Yukawa sector. All lowest loop corrections to them are either vanishing or finite. Because the scalars which mediate flavor changed neutral (or charged) currents carry $SU(3) \times U(1)$ charges (called global charges), the loop will not be able to be closed, unless a mixing of scalars with different global charges is introduced. Because of these features, this approach may be called the ITZ.

This approach is different from other approaches in that it differentiates ITZs from non ITZs, protects them, and has a potential to correct the ITZs by radiative corrections[8]. It also predicts many new scalar mediated processes, such as $t \rightarrow cc\bar{u}$, and corrects the rates of many processes of the standard model. In addition, this approach, if successful, may change our concept on the desert between $10^4$ to $10^{15}$ GeV.
The following issues with respect to this approach are under consideration.

**A.** It is possible that the features of the initial mass matrices in Eqs.(1) and (2) allow one to produce (2,2), (1,2) and (2,1) elements and others, if necessary, by radiative corrections, in particular because the coupling constant $G_0$ is at order 1. Therefore the correct masses of the first generation and the whole mixing matrix (the CKM matrix) are calculable in some sense. This calculation may provide a method to restrict the parameters of the ESB terms, at least those which play significant roles in the calculations. With luck one may even over constrain these parameters. One is then in good shape to calculate corrections to e.g. $B_s - \bar{B}_s$ mixing, $b \to s + \gamma$ and other measurable processes. (See Fig. 3 for typical scalar mediated processes, where crosses on the dotted lines represent mixing vertices.) It is worth noting that because of the restriction of the $SU(2) \times U(1)$ gauge symmetry, it is impossible to introduce the needed mixing for loop diagrams which correct ITZs in the original Lagrangian. Therefore this mixing must be a combined effect of ESB and SSB.

**B.** The Higgs potential sector. In addition to the symmetric $SU(3) \times U(1)$ part, there must be partially symmetric and completely asymmetric terms in the Higgs potential. As discussed previously, the complete symmetry will be broken by the VEVs of the triplet and sextet Higgs down to $U(1)$. This SSB will cause 8+3 massless Nambu-Goldstone bosons. Three of these massless bosons which are related to gauge symmetry breaking are absorbed by the $W$ and $Z$ bosons. In order to transmute the other 8 massless bosons, which are related to the global symmetry breakdown, into massive ones, one must introduce ESB terms in the Higgs potential. ESB terms are necessary also to keep the theory consistent and renormalizable (Fig. 2). A third purpose of introducing ESB terms is to allow global charge breaking processes to occur, as discussed before. For these reasons, the Higgs sector could be complicated. In particular, there are many different ways to introduce ESB terms. A clever guideline for introducing ESB terms may drastically simplify the solution. As emphasized in **A**, much physics can be discussed before completely resolving the Higgs sector.

**C.** If the ESB terms in the Higgs sector are small, compared with the symmetric terms, then the masses of the formerly Nambu- Goldstone bosons will be small, compared with the masses in the original Higgs sector before SSB. Therefore there will be 8 psuedo-Nambu- Goldstone bosons (PNGB). Because these particles are relatively light, low energy physics is related to them. The properties of PNGBs are therefore worth studying. For example, the global quantum numbers of the PNGBs should be identified and the mass spectrum should be given in accordance with the pattern of the ESB.
In any case, the separation of the non-zero elements in Eqs.(1) and (2) from other small
elements will shed new light to produce a mass and mixing pattern which is sufficiently
hierarchical.

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