New massive supergravity and auxiliary fields

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Received 2 May 2013, in final form 24 July 2013
Published 2 September 2013
Online at stacks.iop.org/CQG/30/195004

Abstract

We construct a supersymmetric formulation of three-dimensional linearized new massive gravity without introducing higher derivatives. Instead, we introduce supersymmetrically a set of bosonic and fermionic auxiliary fields which, upon elimination by their equations of motion, introduce fourth-order derivative terms for the metric and third-order derivative terms for the gravitino. Our construction requires an off-shell formulation of the three-dimensional supersymmetric massive Fierz–Pauli theory. We discuss the nonlinear extension of our results.

PACS numbers: 04.65.+e, 11.30.Pb, 04.50.–h, 04.60.Kz

1. Introduction

New massive gravity (NMG) is a higher-derivative extension of three-dimensional (3D) Einstein–Hilbert gravity with a particular set of terms quadratic in the 3D Ricci tensor and Ricci scalar [1]. The interest in the NMG model lies in the fact that, although the theory contains higher derivatives, it nevertheless describes, unitarily, two massive degrees of freedom of helicity $+2$ and $-2$. Furthermore, it has been shown that even at the nonlinear level ghosts are absent [2]. The 3D NMG model is an interesting laboratory to study the validity of the AdS/CFT correspondence in the presence of higher derivatives. Its extension to 4D remains an open issue and has only been established so far at the linearized level [3].

For many purposes, it is convenient to work with a formulation of the model without higher derivatives, see, e.g. [4]. This can be achieved by introducing an auxiliary symmetric tensor that couples to (the Einstein tensor of) the 3D metric tensor and has an explicit mass term [1]. A supersymmetric version of NMG was constructed in [5]. Besides the fourth-order-derivative...
terms of the metric tensor this model also contains third-order-derivative terms involving the gravitino.

The purpose of this work is to construct a reformulation of the supersymmetric NMG model (SNMG) without higher derivatives. This requires that besides an auxiliary symmetric tensor, we introduce further auxiliary fermionic fields that effectively lower the number of derivatives of the gravitino kinetic terms.

At the linearized level the NMG model decomposes into the sum of a massless spin-2 Einstein–Hilbert theory and a massive spin-2 Fierz–Pauli (FP) model [1]. In the supersymmetric case we therefore need a 3D massless and a 3D massive spin-2 supermultiplet. We only consider the case of simple $\mathcal{N} = 1$ supersymmetry. In this paper we will explicitly construct the linearized, massive, off-shell spin-2 supermultiplet, paying particular attention to the auxiliary field structure. We will obtain this massive spin-2 FP multiplet by starting from a 4D (linearized) massless spin-2 supermultiplet, performing a Kaluza–Klein (KK) reduction over a circle and projecting onto the first massive KK sector. The final form of the 3D off-shell massive spin-2 supermultiplet is then obtained after a truncation and gauge-fixing a few Stückelberg symmetries. Along with the construction of the massive, off-shell spin-2 multiplet, we will look in detail at its massless limit. As is well-known, already in the bosonic case, this limit is non-trivial and should be taken with care. Indeed, the massless limit of the massive spin-2 FP theory, coupled to a conserved energy–momentum tensor, does not lead to linearized General Relativity, a result known as the van Dam–Veltman–Zakharov (vDVZ) discontinuity [6, 7]. Starting from the massive FP supermultiplet obtained, we will explicitly illustrate how a supersymmetric version of this vDVZ discontinuity arises (see also [8] for an earlier discussion).

Having constructed the off-shell massive spin-2 supermultiplet, it is rather straightforward to construct a linearized version of SNMG without higher derivatives, by appropriately combining a massless and a massive spin-2 multiplet. This theory contains three vector-spinors, whereas the higher-derivative version contains only one gravitino field. This is due to the fact that the massive multiplet contains two gravitini, unlike the massless multiplet that only contains one. The reason for this is that a massive gravitino describes a single helicity 3/2 state whereas $\mathcal{N} = 1$ SNMG contains two fermionic massive degrees of freedom of helicity $+3/2$ and $-3/2$. We will show that two of the vector-spinors are actually auxiliary like the auxiliary symmetric tensor in the bosonic case. In particular, we will explicitly show how, by eliminating the different bosonic and fermionic auxiliary fields, we re-obtain the linearized approximation of the higher-derivative SNMG model given in [5]. At the linearized level, we will distinguish between two types of auxiliary fields: the ‘trivial’ and ‘non-trivial’ ones. The difference between them is that only the elimination of the non-trivial auxiliary fields leads to higher derivatives in the action. The trivial ones are only needed to obtain a supersymmetry algebra that closes off-shell.

The extension to the nonlinear case, in the presence of both the trivial and non-trivial auxiliary fields, is not obvious. One way to see this, is by noting that our construction of the massive spin-2 multiplet is based upon a KK truncation which can only be performed consistently at the linearized level. We consider the alternative option that first, at the linearized level, one eliminates only the trivial auxiliary fields of the massive spin-2 supermultiplet but keeps all the other ones. This implies that at the linearized level the supersymmetry algebra closes on-shell but that the action does not contain higher derivatives. We will show that in principle the extension to the nonlinear case in this situation is possible but that the answer is not illuminating. This is in contrast to the higher-derivative formulation of SNMG where the contributions to the bosonic terms of the single auxiliary scalar $S$ of the massless multiplet can be nicely interpreted as a torsion contribution to the spin-connection [5].
This work is organized as follows. In section 2 we show how the 3D supersymmetric Proca theory is obtained from the KK reduction of a 4D massless spin-1 Maxwell multiplet. This serves as an explanatory discussion for section 3, in which we extend this analysis to the spin-2 case and obtain the supersymmetric FP model. The corresponding supersymmetry algebra closes off-shell and contains 3 auxiliary scalars and one auxiliary vector. In section 4 we use these results to construct a linearized version of SNMG without higher derivatives. We explicitly show how, after eliminating all bosonic and fermionic auxiliary fields, the higher derivatives of the metric and gravitino are introduced. In section 5 we discuss our attempts to extend our results to the nonlinear case. Our conclusions are presented in section 6. There are two appendices. In appendix A we summarize some properties of the off-shell massless multiplets that occur in this work. In appendix B we show, as a spin-off of the main discussion, how the trick that can be used to boost up the derivatives in the FP model, see e.g. [9], can be extended to the fermionic case to boost up the number of derivatives in a massive gravitino model.

2. Supersymmetric Proca

In this section we show how to obtain the 3D supersymmetric Proca theory from the KK reduction of an off-shell 4D $\mathcal{N} = 1$ supersymmetric Maxwell theory and a subsequent truncation to the first massive KK sector. This is a warming-up exercise for the spin-2 case which will be discussed in the next section.

2.1. Kaluza–Klein reduction

Our starting point is the 4D $\mathcal{N} = 1$ supersymmetric Maxwell multiplet which consists of a vector $\hat{V}_\mu$, a 4-component Majorana spinor $\hat{\psi}$ and a real auxiliary scalar $\hat{F}$. We indicate fields depending on the 4D coordinates and 4D indices with a hat. We do not indicate spinor indices. The supersymmetry rules, with a constant 4-component Majorana spinor parameter $\epsilon$, and gauge transformation, with local parameter $\hat{\Lambda}$, of these fields are given by

\begin{align}
\delta \hat{V}_\mu &= -\bar{\epsilon} \Gamma_\mu \hat{\psi} + \partial_\mu \hat{\Lambda}, \\
\delta \hat{\psi} &= \frac{1}{4} \Gamma_\mu \hat{F}_{\mu \nu} \epsilon + \frac{1}{4} \sqrt{2} \Gamma_3 \hat{\psi}, \\
\delta \hat{F} &= i \bar{\epsilon} \Gamma_3 \hat{V}_\mu \partial_\mu \hat{\psi} \tag{2.1}
\end{align}

where $\hat{F}_{\mu \nu} = \partial_\mu \hat{V}_\nu - \partial_\nu \hat{V}_\mu$.

In the following, we will split the 4D coordinates as $x^\hat{\mu} = (x^\mu, x^3)$, where $x^3$ denotes the compactified circle coordinate. Since all fields are periodic in $x^3$, we can write them as a Fourier series. For example:

\begin{equation}
\hat{V}_\mu(x^\hat{\mu}) = \sum_n V_{\mu,n}(x^\mu) e^{i n x^3}, \quad n \in \mathbb{Z}, \tag{2.2}
\end{equation}

where $m \neq 0$ has mass dimensions and corresponds to the inverse circle radius. The Fourier coefficients $V_{\mu,n}(x^\mu)$ correspond to 3D (un-hatted) fields. We first consider the bosonic fields. The reality condition on the 4D vector and scalar implies that only the 3D $(n = 0)$ zero modes are real. All other modes are complex but only the positive ($n > 0$) modes are independent, since

\begin{equation}
V_{\mu,-n} = V_{\mu,n}^*, \quad F_{-n} = F_{n}^* \quad n \neq 0, \tag{2.3}
\end{equation}

In the following we will be mainly interested in the $n = 1$ modes whose real and imaginary parts we indicate by

\begin{equation}
V_{\mu}^{(1)}(x^\mu) = \frac{1}{2} (V_{\mu,1} + V_{\mu,1}^*), \quad V_{\mu}^{(2)} = \frac{1}{2i} (V_{\mu,1} - V_{\mu,1}^*),
\end{equation}
\[ \phi^{(1)} = \frac{1}{2} (V_{3,1} + V_{3,1}^*), \quad \phi^{(2)} = \frac{1}{2i} (V_{3,1} - V_{3,1}^*), \]
\[ F^{(1)} = \frac{1}{2} (F_l + F_l^*), \quad F^{(2)} = \frac{1}{2i} (F_l - F_l^*). \]  

(2.4)

Similarly, the Majorana condition of the 4D spinor \( \tilde{\psi} \) implies that the \( n = 0 \) mode is Majorana but that the independent positive \( (n \geq 1) \) modes are Dirac. This is equivalent to two (4-component, 3D reducible) Majorana spinors which we indicate by
\[ \psi^{(1)} = \frac{1}{2} (\psi_1 + B^{-1} \psi_1^*), \quad \psi^{(2)} = \frac{1}{2i} (\psi_1 - B^{-1} \psi_1^*). \]  

(2.5)

Here \( B \) is the \( 4 \times 4 \) matrix \( B = i \Gamma_0 \), where \( C \) is the \( 4 \times 4 \) charge conjugation matrix.

Substituting the harmonic expansion (2.2) of the fields and a similar expansion of the gauge parameter \( \Lambda \) into the transformation rules (2.1), we find the following transformation rules for the first \( (n = 1) \) KK modes:
\[ \delta \phi^{(1)} = -\bar{\epsilon} \Gamma_3 \psi^{(1)} - m \Lambda^{(2)} - m \bar{\epsilon} \phi^{(2)}, \]
\[ \delta \phi^{(2)} = -\bar{\epsilon} \Gamma_3 \psi^{(2)} + m \Lambda^{(1)} + m \bar{\epsilon} \phi^{(1)}, \]
\[ \delta V^{(1)}_{\mu} = -\bar{\epsilon} \Gamma_{\mu} \psi^{(1)} + \partial_{\mu} \Lambda^{(1)} - m \bar{\epsilon} V^{(2)}_{\mu}, \]
\[ \delta V^{(2)}_{\mu} = -\bar{\epsilon} \Gamma_{\mu} \psi^{(2)} + \partial_{\mu} \Lambda^{(2)} + m \bar{\epsilon} V^{(1)}_{\mu}, \]
\[ \delta F^{(1)} = i \bar{\epsilon} \Gamma_5 \Gamma_{\mu} \partial_{\mu} \psi^{(1)} - im \bar{\epsilon} \Gamma_5 \Gamma_3 \psi^{(2)} - m \bar{\epsilon} F^{(2)}, \]
\[ \delta F^{(2)} = i \bar{\epsilon} \Gamma_5 \Gamma_{\mu} \partial_{\mu} \psi^{(2)} + im \bar{\epsilon} \Gamma_5 \Gamma_3 \psi^{(1)} + m \bar{\epsilon} F^{(1)}, \]
\[ \delta \psi^{(1)} = \frac{1}{8} \Gamma^{\mu \nu} F_{\mu \nu}^{(1)} \epsilon + \frac{1}{4} \Gamma^{\mu} \Gamma_3 \partial_{\mu} \psi^{(1)} \epsilon + \frac{1}{4} \Gamma_3 F^{(1)} \epsilon + \frac{m}{4} \Gamma^{\mu} \Gamma_3 V^{(2)}_{\mu} \epsilon - m \bar{\epsilon} \psi^{(2)}, \]
\[ \delta \psi^{(2)} = \frac{1}{8} \Gamma^{\mu \nu} F_{\mu \nu}^{(2)} \epsilon + \frac{1}{4} \Gamma^{\mu} \Gamma_3 \partial_{\mu} \psi^{(2)} \epsilon + \frac{1}{4} \Gamma_3 F^{(2)} \epsilon - \frac{m}{4} \Gamma^{\mu} \Gamma_3 V^{(1)}_{\mu} \epsilon + m \bar{\epsilon} \psi^{(1)}, \]  

(2.6)

where we have defined
\[ \Lambda^{(1)} = \frac{1}{2} (\Lambda + \Lambda^*), \quad \Lambda^{(2)} = \frac{1}{2i} (\Lambda - \Lambda^*). \]  

(2.7)

Apart from global supersymmetry transformations with parameter \( \epsilon \) and gauge transformations with parameters \( \Lambda^{(1)}, \Lambda^{(2)} \), the transformations (2.6) also contain a global SO(2) transformation with parameter \( \xi \), that rotates the real and imaginary parts of the 3D fields. This SO(2) transformation corresponds to a central charge transformation and is a remnant of the translation in the compact circle direction\(^4\).

In order to write the 3D 4-component Majorana spinors in terms of two irreducible 2-component Majorana spinors it is convenient to choose the following representation of the \( \Gamma \)-matrices in terms of \( 2 \times 2 \) block matrices:
\[ \Gamma_{\mu} = \begin{pmatrix} \gamma_{\mu} & 0 \\ 0 & -\gamma_{\mu} \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma_5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \]  

(2.8)

The 3D \( 2 \times 2 \) matrices \( \gamma_{\mu} \) satisfy the standard relations \{\( \gamma_{\mu}, \gamma_{\nu} \)\} = \( 2 \eta_{\mu \nu} \) and can be chosen explicitly in terms of the Pauli matrices by
\[ \gamma_{\mu} = (\sigma_1, \sigma_2, \sigma_3). \]  

(2.9)

In this representation the 4D charge conjugation matrix \( C \) is given by
\[ C = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}. \]  

(2.10)

\(^4\) This is a conventional central charge transformation. 3D supergravity also allows for non-central charges from extensions by non-central \( R \)-symmetry generators \([10]\), recently discussed in \([11]\).
where
\[ \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \] (2.11)
is the 3D charge conjugation matrix.

Using the above representation the 4-component Majorana spinors decompose into two 3D irreducible Majorana spinors as follows:
\[
\psi^{(1)} = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \quad \psi^{(2)} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}. \tag{2.12}
\]

In terms of these 2-component spinors the transformation rules (2.6) read
\[
\begin{align*}
\delta \phi^{(1)} &= -\tilde{\varepsilon}_1 \chi_2 + \tilde{\varepsilon}_2 \chi_1 - m\Lambda^{(2)} - m\xi \phi^{(2)}, \\
\delta \phi^{(2)} &= -\varepsilon_1 \psi_2 + \varepsilon_2 \psi_1 + m\Lambda^{(1)} + m\xi \phi^{(1)}, \\
\delta \psi^{(1)} &= -\varepsilon_1 \gamma_\mu \chi_1 - \varepsilon_2 \gamma_\mu \chi_2 + \partial_\mu \Lambda^{(1)} - m\xi \psi^{(2)}, \\
\delta \psi^{(2)} &= -\varepsilon_1 \gamma_\mu \psi_1 - \varepsilon_2 \gamma_\mu \psi_2 + \partial_\mu \Lambda^{(2)} + m\xi \psi^{(1)}, \\
\delta F^{(1)} &= -\varepsilon_1 \gamma^{\mu} \partial_\mu \chi_1 + \varepsilon_2 \gamma^{\mu} \partial_\mu \chi_2 - m(\varepsilon_1 \psi_1 + \varepsilon_2 \psi_2) - m\xi F^{(2)}, \\
\delta F^{(2)} &= -\varepsilon_1 \gamma^{\mu} \partial_\mu \psi_2 + \varepsilon_2 \gamma^{\mu} \partial_\mu \psi_1 + m(\varepsilon_1 \chi_1 + \varepsilon_2 \chi_2) + m\xi F^{(1)}, \\
\delta \chi_1 &= \frac{1}{8} \gamma^{\mu \nu} F_{\mu \nu}^{(1)} \varepsilon_1 + \frac{1}{2} \left( \gamma^{\mu} \partial_\mu \phi^{(1)} + F^{(1)} + my^{\mu} V^{(2)}_{\mu} \right) \varepsilon_1 - m\xi \psi_1, \\
\delta \chi_2 &= \frac{1}{8} \gamma^{\mu \nu} F_{\mu \nu}^{(2)} \varepsilon_2 - \frac{1}{2} \left( \gamma^{\mu} \partial_\mu \phi^{(2)} + F^{(2)} + my^{\mu} V^{(1)}_{\mu} \right) \varepsilon_2 + m\xi \psi_2, \\
\delta \psi_1 &= \frac{1}{8} \gamma^{\mu \nu} F_{\mu \nu}^{(2)} \varepsilon_2 + \frac{1}{2} \left( \gamma^{\mu} \partial_\mu \phi^{(2)} + F^{(2)} - my^{\mu} V^{(1)}_{\mu} \right) \varepsilon_2 + m\xi \chi_1, \\
\delta \psi_2 &= \frac{1}{8} \gamma^{\mu \nu} F_{\mu \nu}^{(1)} \varepsilon_1 - \frac{1}{2} \left( \gamma^{\mu} \partial_\mu \phi^{(1)} + F^{(1)} - my^{\mu} V^{(2)}_{\mu} \right) \varepsilon_1 + m\xi \chi_2. \tag{2.13}
\end{align*}
\]

If we take \( m \to 0 \) in the above multiplet we obtain two decoupled multiplets, \((\phi^{(1)}, V^{(1)}_{\mu}, F^{(1)}, \chi_1, \chi_2)\) and \((\phi^{(2)}, V^{(2)}_{\mu}, F^{(2)}, \psi_1, \psi_2)\). Either one of them constitutes a massless \( \mathcal{N} = 2 \) vector multiplet. This massless limit has to be distinguished from the massless limits discussed in subsections 2.3 and 3.2, which refer to limits taken after truncating to \( \mathcal{N} = 1 \) supersymmetry.

2.2. Truncation

In the process of KK reduction, the number of supercharges stays the same. The 3D multiplet (2.13) we found in the previous subsection thus exhibits four supercharges and hence corresponds to an \( \mathcal{N} = 2 \) multiplet, containing two vectors and a central charge transformation. One can, however, truncate it to an \( \mathcal{N} = 1 \) multiplet, not subjected to a central charge transformation and containing only one vector. This truncated multiplet will be the starting point to obtain an \( \mathcal{N} = 1 \) supersymmetric version of the Proca theory. The \( \mathcal{N} = 1 \) truncation is given by:
\[
\phi^{(2)} = V^{(1)}_{\mu} = F^{(2)} = \chi_2 = \psi_1 = 0, \tag{2.14}
\]
provided that at the same time we truncate the following symmetries:
\[
\varepsilon_1 = \Lambda^{(1)} = \xi = 0. \tag{2.15}
\]
Substituting this truncation into the transformation rules (2.13), we find the following \( \mathcal{N} = 1 \) massive vector supermultiplet:
\[
\delta \phi^{(1)} = \tilde{\varepsilon}_1 \chi_1 - m\Lambda^{(2)}, \tag{5}
\]
Note that the field content given in (2.16) is that of massless \( \mathcal{N} = 2 \). In the massive case, however, the scalar field \( \phi \) will disappear after gauge-fixing the St"uckelberg symmetry.
\[ \delta V^{(2)} = -\bar{\epsilon} \gamma^\mu \psi_2 + \partial_\mu \Lambda^{(2)}, \]
\[ \delta \psi_2 = \frac{1}{3} \gamma^{\mu \nu} F^{(2)}_{\mu \nu} \bar{\epsilon}_2, \]
\[ \delta \chi_1 = \frac{1}{8} \left( \gamma^\mu \partial_\mu \phi^{(1)} + F^{(1)} + m \gamma^\mu V^{(2)}_\mu \right) \bar{\epsilon}_2, \]
\[ \delta F^{(1)} = \bar{\epsilon}_2 \gamma^\mu \partial_\mu \chi_1 - m \bar{\epsilon}_2 \psi_2. \] (2.16)

Redefining \( \bar{\epsilon}_2 \rightarrow \bar{\epsilon}, \Lambda^{(2)} \rightarrow \Lambda \) and
\[ \phi^{(1)} \rightarrow 4 \phi, \quad V^{(2)}_\mu \rightarrow V_\mu, \quad F^{(1)} \rightarrow -F, \quad \psi_2 \rightarrow \psi, \quad \chi_1 \rightarrow \chi \] and \( m \rightarrow 4m, \) (2.17)
we obtain
\[ \delta \phi = \frac{1}{3} \bar{\epsilon} \chi - m \Lambda, \]
\[ \delta V_\mu = -\bar{\epsilon} \gamma_\mu \psi + \partial_\mu \Lambda, \]
\[ \delta \psi = \frac{1}{8} \gamma^{\mu \nu} F_{\mu \nu} \bar{\epsilon}, \]
\[ \delta \chi = \gamma^\mu D_\mu \phi \bar{\epsilon} - \frac{1}{4} F \bar{\epsilon}, \]
\[ \delta F = -\bar{\epsilon} \gamma^\mu \partial_\mu \chi + 4m \bar{\epsilon} \psi. \] (2.18)

where the covariant derivative \( D_\mu \) is defined as
\[ D_\mu \phi = \partial_\mu \phi + m V_\mu. \] (2.19)

The transformation rules (2.18) leave the following action invariant:
\[ I_1 = \int d^3 x \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} \partial_\mu \phi D^\mu \phi - 2 \bar{\psi} \slashed{D} \psi - \frac{1}{8} \bar{\chi} \slashed{D} \chi + m \bar{\psi} \chi + \frac{1}{32} F^2 \right). \] (2.20)

The gauge transformation with parameter \( \Lambda \) is a Stückelberg symmetry, that can be fixed by imposing the gauge condition
\[ \phi = \text{const.} \] (2.21)
Taking the resulting compensating gauge transformation
\[ \Lambda = \frac{1}{4m} \bar{\epsilon} \chi \] (2.22)
into account, we obtain the final form of the supersymmetry transformation rules of the \( \mathcal{N} = 1 \) supersymmetric Proca theory:
\[ \delta V_\mu = -\bar{\epsilon} \gamma_\mu \psi + \frac{1}{4m} \bar{\epsilon} \partial_\mu \chi, \]
\[ \delta \psi = \frac{1}{8} \gamma^{\mu \nu} F_{\mu \nu} \bar{\epsilon}, \]
\[ \delta \chi = m \gamma^\mu \partial_\mu \psi - \frac{1}{4} F \bar{\epsilon}, \]
\[ \delta F = -\bar{\epsilon} \gamma^\mu \partial_\mu \chi + 4m \bar{\epsilon} \psi. \] (2.23)

The supersymmetric Proca action is then given by
\[ I_{\text{Proca}} = \int d^3 x \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} \partial_\mu V_\mu V^\mu - 2 \bar{\psi} \slashed{D} \psi - \frac{1}{8} \bar{\chi} \slashed{D} \chi + m \bar{\psi} \chi + \frac{1}{32} F^2 \right). \] (2.24)

The supersymmetric Proca theory describes 2 + 2 on-shell and 4 + 4 off-shell degrees of freedom.

This finishes our description of how to obtain the 3D off-shell massive \( \mathcal{N} = 1 \) vector multiplet from a KK reduction and subsequent truncation onto the first massive KK sector of the 4D off-shell massless \( \mathcal{N} = 1 \) vector multiplet.
2.3. Massless limit

We end this section with some comments on the massless limit \((m \to 0)\). Taking the massless limit in (2.18), we see that the Proca multiplet splits into a massless vector multiplet and a massless scalar multiplet. Note that a massless vector multiplet can be coupled to a current supermultiplet. This is a feature that we would like to incorporate, in view of the upcoming spin-2 discussion. We will do so by coupling the above supersymmetric Proca system to a conjugate multiplet \((J_\mu, J_\psi, J_\chi, J_F)\), where \(J_\mu\) is a vector, \(J_\psi\) and \(J_\chi\) are spinors and \(J_F\) is a scalar. Our starting point is then the action

\[
I = I_{\text{Proca}} + I_{\text{int}},
\]

(2.25)

where the interaction part \(I_{\text{int}}\) describes the coupling between the Proca multiplet and the conjugate multiplet:

\[
I_{\text{int}} = V^\mu J_\mu + \bar{\psi} J_\psi + \bar{\chi} J_\chi + F J_F.
\]

(2.26)

Requiring that \(I_{\text{int}}\) is separately invariant under supersymmetry, determines the transformation rules of the conjugate multiplet:

\[
\delta J_\mu = \frac{1}{4} \bar{\epsilon} \gamma^\mu \partial^\nu J_\psi + m \bar{\epsilon} \gamma^\mu J_\chi,
\]

\[
\delta J_\psi = -\gamma^\mu \epsilon J_\mu - 4m \epsilon J_F,
\]

\[
\delta J_\chi = \frac{1}{4m} \bar{\epsilon} \partial^\mu J_\mu + \gamma^\mu \epsilon \partial^\mu J_F,
\]

\[
\delta J_F = \frac{1}{4} \bar{\epsilon} J_\chi.
\]

(2.27)

Taking the massless limit in the action (2.25) and transformation rules (2.23), (2.27) is non-trivial, due to the factors of \(1/m\) that appear in the transformation rules. In order to be able to take the limit in a well-defined fashion, we will work in the formulation where the St"uckelberg symmetry is not yet fixed. Note that this formulation can be easily retrieved from the gauge fixed version, by making the following redefinition in the action (2.24) and transformation rules (2.23):

\[
V_\mu = \bar{V}_\mu + \frac{1}{m} \partial_\mu \phi.
\]

(2.28)

Applying this redefinition to (2.24) and (2.23) indeed brings one back to the action (2.20) and to the transformation rules (2.18), whose massless limit is well-defined. The massless limit of the interaction part \(I_{\text{int}}\) (after performing the above substitution) and of the transformation rules (2.27) of the conjugate multiplet, is however not well-defined. In order to remedy this, we will impose the constraint that \(J_\mu\) corresponds to a conserved current, i.e. that

\[
\partial^\mu J_\mu = 0.
\]

(2.29)

In order to preserve supersymmetry, we will also take \(J_\chi = 0\) and \(J_F = 0\). The conjugate multiplet then reduces to a spin-1 current supermultiplet.

The massless limit is now everywhere well-defined. The transformation rules (2.18) reduce to the transformation rules of a massless vector \((\bar{V}_\mu, \bar{\psi})\) and scalar \((\phi, \chi, F)\) multiplet, see

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6 Strictly speaking, preservation of the constraint \(\partial^\mu J_\mu = 0\) under supersymmetry leads to the constraint \(\partial J_\chi = 0\) and preservation of this new constraint leads to the constraint \(\Box J_F = 0\). We are however interested in the massless limit, in which the conserved currents \((J_\mu, J_\chi)\) and the fields \((J_\psi, J_F)\) form two separate multiplets, that couple to a massless vector and scalar multiplet respectively. Since we are mostly interested in the coupling of the supercurrent multiplet \((J_\mu, J_\psi)\) to the vector multiplet, we will simply set the fields \((J_\chi, J_F)\) equal to zero.
equations (A.11) and (A.6), respectively. Performing the above outlined procedure and taking the limit $m \to 0$ leads to the following action
\[
I = \int d^3x \left[ \left( -\frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - 2 \hat{\psi} \hat{D} \hat{\psi} + \hat{V}^\mu J_\mu + \hat{\psi} \hat{\mathcal{J}} \hat{\psi} \right) - \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \hat{\chi} \hat{\chi} - \frac{1}{16} F^2 \right) \right].
\]
(2.30)
which is the sum of the supersymmetric massless vector and scalar multiplet actions, see equations (A.12) and (A.7), respectively. The vector multiplet action is coupled to a spin-1 current multiplet. Note that there is no coupling left between the current multiplet and the scalar multiplet. This will be different in the spin-2 case, as we will see later.

3. Supersymmetric Fierz–Pauli

In this section we extend the discussion of the previous section to the spin-2 case, skipping some of the details we explained in the spin-1 case. We use the same notation.

3.1. Kaluza–Klein reduction and truncation

Our starting point is the off-shell 4D $\mathcal{N} = 1$ massless spin-2 multiplet which consists of a symmetric tensor $\hat{h}_{\mu\nu}$, a gravitino $\hat{\psi}_\mu$, an auxiliary vector $\hat{A}_\mu$ and two auxiliary scalars $\hat{M}$ and $\hat{N}$. This corresponds to the linearized version of the ‘old minimal supergravity’ multiplet. The supersymmetry rules, with constant spinor parameter $\epsilon$, and gauge transformations of these fields, with local vector parameter $\hat{A}_\mu$ and local spinor parameter $\hat{\eta}$, are given by [12, 13]:
\[
\begin{align*}
\delta \hat{h}_{\mu\nu} &= \bar{\epsilon} \Gamma_{(\mu} \hat{\psi}_{\nu)} + \partial_{(\mu} \hat{A}_{\nu)}, \\
\delta \hat{\psi}_\mu &= -\frac{1}{2} \Gamma^{\mu\nu} \partial_\nu \hat{h}_{\mu\nu} \epsilon - \frac{1}{12} \Gamma_{\mu} (\hat{M} + i\Gamma_3 \hat{N}) \epsilon + \frac{1}{4} i \hat{A}_{\mu} \Gamma_5 \epsilon + \partial_\mu \hat{\eta}, \\
\delta \hat{M} &= -\bar{\epsilon} \Gamma^\mu \partial_\mu \hat{\psi}, \\
\delta \hat{N} &= -i \bar{\epsilon} \Gamma_3 \Gamma^\mu \partial_\mu \hat{\psi}, \\
\delta \hat{A}_\mu &= \frac{1}{2} i \bar{\epsilon} \Gamma_5 \Gamma_\mu \partial_\mu \hat{\psi} - i \bar{\epsilon} \Gamma_3 \Gamma_\mu \Gamma^{\nu\rho} \partial_\nu \partial_\rho \hat{\psi}.
\end{align*}
\]
(3.1)

Like in the spin-1 case we first perform a harmonic expansion of all fields and local parameters and substitute these into the transformation rules (3.1). Projecting onto the lowest KK massive sector we then obtain all the transformation rules of the real and imaginary parts of the $n = 1$ modes, like in equation (2.6) for the spin-1 case. We indicate the real and imaginary parts of the bosonic modes by:
\[
\begin{align*}
\hat{h}^{(1)}_{\mu\nu} &= \frac{1}{2} (\hat{h}_{\mu\nu,1} + \hat{h}^*_{\mu\nu,1}), \\
\hat{h}^{(2)}_{\mu\nu} &= \frac{1}{2i} (\hat{h}_{\mu\nu,1} - \hat{h}^*_{\mu\nu,1}), \\
\hat{V}^{(1)}_\mu &= \frac{1}{2} (\hat{V}_{\mu,1} + \hat{V}^*_{\mu,1}), \\
\hat{V}^{(2)}_\mu &= \frac{1}{2i} (\hat{V}_{\mu,1} - \hat{V}^*_{\mu,1}), \\
\hat{\phi}^{(1)} &= \frac{1}{2} (\hat{\phi}_{3,1} + \hat{\phi}^*_{3,1}), \\
\hat{\phi}^{(2)} &= \frac{1}{2i} (\hat{\phi}_{3,1} - \hat{\phi}^*_{3,1}), \\
\hat{A}^{(1)}_\mu &= \frac{1}{2} (\hat{A}_{\mu,1} + \hat{A}^*_{\mu,1}), \\
\hat{A}^{(2)}_\mu &= \frac{1}{2i} (\hat{A}_{\mu,1} - \hat{A}^*_{\mu,1}), \\
\hat{P}^{(1)} &= \frac{1}{2} (\hat{P}_{3,1} + \hat{P}^*_{3,1}), \\
\hat{P}^{(2)} &= \frac{1}{2i} (\hat{P}_{3,1} - \hat{P}^*_{3,1}), \\
\hat{M}^{(1)} &= \frac{1}{2} (\hat{M}_1 + \hat{M}^*_1), \\
\hat{M}^{(2)} &= \frac{1}{2i} (\hat{M}_1 - \hat{M}^*_1), \\
\hat{N}^{(1)} &= \frac{1}{2} (\hat{N}_1 + \hat{N}^*_1), \\
\hat{N}^{(2)} &= \frac{1}{2i} (\hat{N}_1 - \hat{N}^*_1).
\end{align*}
\]
(3.2)
while the fermionic modes decompose into two Majorana modes:
\[
\psi^{(1)}_\mu = \frac{1}{2}(\psi^{\ast}_\mu + B^{-1}\psi^{\ast}_{\mu 1}), \quad \psi^{(2)}_\mu = \frac{1}{21}(\psi^{\ast}_{\mu 1} - B^{-1}\psi^{\ast}_{\mu 1}).
\]
\[
\psi^{(1)}_3 = \frac{1}{2}(\psi_3 + B^{-1}\psi_{3 1}^{\ast}), \quad \psi^{(2)}_3 = \frac{1}{21}(\psi_{3 1} - B^{-1}\psi_{3 1}^{\ast}).
\]  

(3.3)

We next use the representation (2.8) of the \( \Gamma \)-matrices and decompose the 4-component spinors into two 2-component spinors as follows:
\[
\psi^{(1)} = \begin{pmatrix} \psi^{(1)}_1 \\ \psi^{(1)}_2 \end{pmatrix}, \quad \psi^{(2)} = \begin{pmatrix} \psi^{(2)}_1 \\ \psi^{(2)}_2 \end{pmatrix},
\]
\[
\eta^{(1)} = \begin{pmatrix} \eta^{(1)}_1 \\ \eta^{(1)}_2 \end{pmatrix}, \quad \eta^{(2)} = \begin{pmatrix} \eta^{(2)}_1 \\ \eta^{(2)}_2 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}.
\]  

(3.4)

Furthermore, we perform the following consistent truncation of the fields
\[
\phi^{(2)} = V^{(2)}_\mu = h^{(2)}_{\mu \nu} = M^{(2)} = N^{(1)} = P^{(2)} = A^{(1)} \quad \chi_1 = \psi_1 = \psi_{\mu 1} = \chi_{\mu 2} = 0
\]

(3.5)

and of the parameters
\[
\Lambda^{(2)}_\mu = \Lambda^{(1)} = \epsilon_1 = \eta^{(1)}_1 = \eta^{(2)}_1 = \xi = 0.
\]  

(3.6)

For simplicity, from now on we drop all numerical upper indices, e.g. \( \phi^{(1)} = \phi \), and all numerical lower indices, e.g. \( \psi_{\mu 1} = \psi_{\mu} \) of the remaining non-zero fields (but not of the parameters). We find that the transformation rules of these fields under supersymmetry, with constant 2-component spinor parameter \( \epsilon \), and Stückelberg symmetries, with local scalar and vector parameters \( \Lambda_3, \Lambda_\mu \), and 2-component spinor parameters \( \eta_1 \) and \( \eta_2 \), are given by
\[
\delta h_{\mu \nu} = \bar{\epsilon} \gamma_{\mu} \psi_0 + \bar{\partial}_{\mu} \Lambda_\nu,
\]
\[
\delta V_\mu = \frac{1}{2} \bar{\epsilon} \gamma_\mu \psi - \frac{1}{2} \bar{\epsilon} \chi_\mu + \frac{1}{2} \partial_\mu \Lambda_3 + \frac{1}{2} m \Lambda_\mu,
\]
\[
\delta \phi = -\bar{\epsilon} \chi - m \Lambda_3,
\]
\[
\delta \psi_{\mu} = -\frac{1}{2} \gamma^{\nu \rho} \partial_\rho \partial_\nu \psi_{\mu} + \frac{1}{2} \gamma_\mu \Lambda_3 + \frac{1}{2} m \Lambda_{\mu},
\]
\[
\delta \chi_\mu = -\frac{1}{2} \gamma^{\rho \nu} \partial_\rho \partial_\nu \chi_\mu + \frac{1}{2} \gamma_\mu \Lambda_3 + \frac{1}{2} m \Lambda_{\mu},
\]
\[
\delta \psi_0 = -\frac{1}{2} \gamma^{\rho \nu} \partial_\rho \partial_\nu \psi_0 + \frac{1}{2} \gamma_0 \Lambda_3 + \frac{1}{2} m \Lambda_3.
\]

(3.7)

The action invariant under the transformations (3.7) is given by
\[
I_m = \int d^3x \left[ h^{\mu \nu} G^{in}_{\mu \nu} (h) - m^2 (h^{\mu \nu} h_{\mu \nu} - h^2) + 2h^{\mu \nu} \partial_\mu \phi - 2h \partial_\mu \partial_\nu \phi - F^{\mu \nu} F_{\mu \nu} + 4mh^{\mu \nu} \partial_\mu \partial_\nu + 4h \partial_\mu \partial_\nu + 4 \bar{\psi} \gamma^{\mu \nu} \partial_\mu \partial_\nu \psi - 4 \bar{\psi} \gamma^{\mu \nu} \partial_\mu \partial_\nu \psi - 4 \bar{\psi} \gamma^{\mu \nu} \partial_\mu \partial_\nu \psi + 8 \bar{\psi} \gamma^{\mu \nu} \partial_\mu \partial_\nu \psi + 8 \bar{\psi} \gamma^{\mu \nu} \partial_\mu \partial_\nu \psi + 8 \bar{\psi} \gamma^{\mu \nu} \partial_\mu \partial_\nu \psi - \frac{2}{3} M^2 - \frac{2}{3} N^2 + \frac{2}{3} P^2 + \frac{2}{3} A_m A^m \right].
\]  

(3.8)

7 If we take the massless limit before the mentioned truncation we find two copies of a \( \mathcal{N} = 2 \) massless spin-2 multiplet plus two copies of a \( \mathcal{N} = 2 \) massless spin-1 multiplet, see also text after (2.13).

8 The 4D analogue of this multiplet, in superfield language, can be found in [14].
where \( h = \eta^{\mu\nu} h_{\mu\nu} \) and \( G_{\mu\nu}^{\text{lin}}(h) \) is the linearized Einstein tensor. We observe that the action is non-diagonal in the bosonic fields \((h_{\mu\nu}, V_{\mu}, \phi)\) and the fermionic fields \((\psi_{\mu}, \chi)\) and \((\chi_{\mu}, \psi)\).

Finally, we fix all Stückelberg symmetries by imposing the gauge conditions
\[
\phi = \text{const}, \quad V_{\mu} = 0, \quad \psi = 0, \quad \chi = 0. \tag{3.9}
\]

Taking into account the compensating gauge transformations
\[
\Lambda_3 = 0, \\
\Lambda_{\mu} = -\frac{1}{m} \epsilon \chi_{\mu}, \\
\eta_1 = -\frac{1}{12m} (M - 2P) \epsilon, \\
\eta_2 = \frac{1}{12m} (N + \gamma^\rho A_{\rho}) \epsilon, \tag{3.10}
\]
we obtain the final form of the supersymmetry rules of the 3D \( \mathcal{N} = 1 \) off-shell massive spin-2 multiplet:
\[
\delta h_{\mu\nu} = \bar{\epsilon} \gamma(\mu \psi_{\nu}) + \frac{1}{m} \bar{\epsilon} \partial(\mu \chi_{\nu}), \\
\delta \psi_{\mu} = -\frac{1}{4} \gamma^{\rho\sigma} \partial_{\rho} h_{\mu\sigma} \epsilon + \frac{1}{12} \gamma_{\mu} (M + P) \epsilon + \frac{1}{12m} \partial_{\mu} (N + \gamma^\rho A_{\rho}) \epsilon, \\
\delta \chi_{\mu} = \frac{1}{4} m \gamma^\rho h_{\mu\rho} \epsilon + \frac{1}{4} A_{\mu} \epsilon - \frac{1}{12} \gamma_{\mu} (N + \gamma^\rho A_{\rho}) \epsilon - \frac{1}{12m} \partial_{\mu} (M - 2P) \epsilon, \\
\delta M = \bar{\epsilon} \gamma^{\rho\lambda} \partial_{\rho} \psi_{\lambda} - m \bar{\epsilon} \gamma^\rho \chi_{\rho}, \\
\delta N = -\bar{\epsilon} \gamma^{\rho\lambda} \partial_{\rho} \chi_{\lambda} + m \bar{\epsilon} \gamma^\rho \psi_{\rho}, \\
\delta P = \frac{3}{2} \bar{\epsilon} \gamma_{\mu} \gamma^{\mu\rho} \partial_{\rho} \psi_{\lambda} - \bar{\epsilon} \gamma_{\mu} \gamma^{\mu\rho} \partial_{\rho} \chi_{\lambda} - \frac{1}{2} m \bar{\epsilon} \gamma_{\mu} \gamma^{\mu\rho} \psi_{\rho} + m \bar{\epsilon} \psi_{\mu}. \tag{3.11}
\]

These transformation rules leave the following action invariant:
\[
I_{\text{off-shell}} = \int d^3x \left\{ \eta^{\mu\nu} G_{\mu\nu}^{\text{lin}}(h) - m^2 (h^{\mu\nu} h_{\mu\nu} - h^2) - 4 \bar{\psi}_{\mu} \gamma^{\mu\rho\nu} \partial_{\nu} \psi_{\rho} - 4 \bar{\chi}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu} \chi_{\rho} - 8m \bar{\psi}_{\mu} \gamma^{\mu\nu} \chi_{\nu} - \frac{2}{3} M^2 - \frac{2}{3} N^2 + \frac{2}{3} P^2 + \frac{2}{3} A_{\mu} A_{\mu} \right\}. \tag{3.12}
\]

This action describes 2 + 2 on-shell and 12 + 12 off-shell degrees of freedom. The first line is the standard FP action. The fermionic off-diagonal mass term can easily be diagonalized by going to a basis in terms of the sum and difference of the two vector–spinors.\(^9\)

The above action shows that the three scalars \( M, N, P \) and the vector \( A_{\mu} \) are auxiliary fields which are set to zero by their equations of motion. We thus obtain the on-shell massive spin-2 multiplet with the following supersymmetry transformations:
\[
\delta h_{\mu\nu} = \bar{\epsilon} \gamma(\mu \psi_{\nu}) + \frac{1}{m} \bar{\epsilon} \partial(\mu \chi_{\nu}), \\
\delta \psi_{\mu} = -\frac{1}{4} \gamma^{\rho\sigma} \partial_{\rho} h_{\mu\sigma} \epsilon, \\
\delta \chi_{\mu} = \frac{m}{4} \gamma^\rho h_{\mu\rho} \epsilon. \tag{3.13}
\]

\(^9\) The +3/2 and −3/2 helicity states are described by the sum and difference of the two vector–spinors. See also appendix B.
It is instructive to consider the closure of the supersymmetry algebra for the above supersymmetry rules given the fact that, unlike in the massless case, the symmetric tensor $h_{\mu\nu}$ does not transform under the gauge transformations $\delta h_{\mu\nu} = \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu$ and the only symmetries left to close the algebra are the global translations. We find that the commutator of two supersymmetries on $h_{\mu\nu}$ indeed gives a translation,

$$\left[ \delta_1, \delta_2 \right] h_{\mu\nu} = \xi^\rho \partial_\rho h_{\mu\nu},$$  \hspace{1cm} (3.14)

with parameter

$$\xi^\mu = \frac{1}{2} \bar{\epsilon}_2 \gamma^\mu \epsilon_1.$$  \hspace{1cm} (3.15)

To close the commutator on the two gravitini requires the use of the equations of motion for these fields. From the action (3.12) we obtain the following equations:

$$\gamma^{\nu\rho\sigma} \partial_\nu \chi_\rho = m \gamma^{\mu\nu} \psi_\nu,$$  \hspace{1cm} (3.16)

and a similar equation for $\psi_\mu$. These equations of motion imply the standard spin $-3/2$ FP equations

$$\mathcal{R}_{\nu}^{(1)} \equiv \partial_\nu \chi_\mu + m \psi_\mu = 0,$$

$$\partial_\mu \chi_\mu = 0, \quad \gamma^\mu \chi_\mu = 0,$$  \hspace{1cm} (3.17)

and similar equations for $\psi_\mu$. A useful alternative way of writing the equations of motion (3.16) is

$$\mathcal{R}_{\mu\nu}^{(2)} \equiv \partial_\mu \chi_\nu + m \gamma_\nu [\psi_\nu] = 0.$$  \hspace{1cm} (3.18)

Using these two ways of writing the equations of motion as well as the FP conditions that follow from them we find that the commutator on the two gravitini gives the same translations (3.15) up to equations of motion. More specifically, we find the following commutators

$$\left[ \delta_1, \delta_2 \right] \psi_\mu = \xi^\nu \partial_\nu \psi_\mu - \frac{1}{4m} \xi^\alpha \partial_\mu \mathcal{R}_\alpha^{(1)} - \frac{1}{8m} \xi^\alpha \gamma_\alpha \partial_\mu (\gamma^{\nu\sigma} \partial_\nu \chi_\sigma)$$

$$+ \frac{1}{4m} \xi^\alpha \partial_\mu (\gamma^{\nu\sigma} \partial_\nu \chi_\sigma) - \frac{1}{8} \xi^\gamma \gamma_\gamma \gamma_\sigma (\gamma^{\nu\sigma} \partial_\nu \psi_\sigma),$$

$$\left[ \delta_1, \delta_2 \right] \chi_\mu = \xi^\nu \partial_\nu \chi_\mu + \frac{1}{2} \xi^\nu \mathcal{R}_\nu^{(2)} - \frac{1}{8} \xi^\rho \gamma_\rho \mathcal{R}_\mu^{(1)} - \frac{1}{8} \xi^\mu \gamma_\mu \partial_\mu \chi_\nu + \frac{m}{8} \xi^\rho \gamma_\rho \gamma_\mu (\gamma^\nu \psi_\nu).$$  \hspace{1cm} (3.19)

Hence, the algebra closes on-shell.

3.2. Massless limit

Finally, we discuss the massless limit $m \rightarrow 0$ of the supersymmetric FP theory. This is particularly interesting in view of the fact that the massless limit of the ordinary spin-2 FP system, coupled to a conserved energy–momentum tensor does not lead to linearized Einstein gravity. Instead, it leads to linearized Einstein gravity plus an extra force, mediated by a scalar that couples to the trace of the energy–momentum tensor with gravitational strength. This phenomenon is known as the vDVZ discontinuity. In the following, we will pay particular attention to this discontinuity in the supersymmetric case.

In order to discuss the massless limit, it turns out to be advantageous to trade the scalar fields $M$ and $P$ for scalars $S$ and $F$, defined by

$$S = \frac{1}{6} (M + P), \quad F = \frac{4}{3} (M - 2P).$$  \hspace{1cm} (3.20)

This field redefinition will make the multiplet structure of the resulting massless theory more manifest. In order to discuss the vDVZ discontinuity, we will include a coupling to a conjugate
multiplet \((T_{\mu\nu}, J^{\psi}_{\mu}, J^{\chi}_{\mu}, T_{S}, T_{N}, T_{F}, T_{A}^{\lambda})\), as we did in the Proca case. Here \(T_{\alpha\nu}\) is a symmetric two-tensor, \(J^{\psi}_{\mu}, J^{\chi}_{\mu}\) are vector–spinors, \(T_{\mu}^{A}\) is a vector and \(T_{F}, T_{S}, T_{N}\) are scalars. We will thus start from the action
\[
I = I_{FP} + I_{int},
\]
where \(I_{FP}\) is the supersymmetric FP action \((3.12)\) and the interaction part \(I_{int}\) is given by
\[
I_{int} = h_{\mu\nu}T^{\mu\nu} + \tilde{\psi}_{\mu}J^{\psi}_{\mu} + \tilde{\chi}_{\mu}J^{\chi}_{\mu} + ST_{S} + FT_{F} + NT_{N} + A_{\mu}T_{A}^{\mu}.
\]
Requiring that \(I_{int}\) is separately invariant under supersymmetry determines the transformation rules of the conjugate multiplet:
\[
\delta T_{\mu\nu} = \frac{1}{4}\varepsilon\gamma_{(\mu}\partial^{\nu)}J^{\psi}_{\nu} + \frac{m}{4}\varepsilon\gamma_{(\mu}J^{\chi}_{\nu)};
\]
\[
\delta J^{\psi}_{\mu} = \gamma^{\alpha}\epsilon T_{\alpha\mu} + \frac{1}{4}\gamma_{\alpha\beta}\partial^{\alpha}T_{\beta\mu} + my_{\mu}\epsilon T_{N} + \frac{m}{2}\gamma_{\alpha\beta}\epsilon T_{A}^{\beta} - meT_{\mu}^{A},
\]
\[
\delta J^{\chi}_{\mu} = \frac{1}{m}\epsilon\gamma^{\alpha}\partial_{\alpha}T_{\mu}\chi - \gamma_{\mu}\epsilon\partial_{\alpha}T_{\alpha} - 4my_{\mu}\epsilon T_{F} - \frac{3}{2}\gamma_{\mu\rho}\epsilon\partial^{\rho}T_{A}^{\alpha} + \gamma_{\mu}\gamma_{\rho}\epsilon\partial^{\rho}T_{A}^{\alpha},
\]
\[
\delta T_{S} = \frac{1}{2}\varepsilon\gamma^{\mu}J^{\psi}_{\mu},
\]
\[
\delta T_{N} = \frac{1}{12m}\epsilon\partial^{\mu}J^{\psi}_{\mu} - \frac{1}{12}\varepsilon\gamma^{\mu}\partial^{\mu}J^{\chi}_{\mu},
\]
\[
\delta T_{F} = \frac{1}{16m}\varepsilon\partial^{\mu}J^{\psi}_{\mu},
\]
\[
\delta T_{A}^{\mu} = \frac{1}{4}\epsilon J^{\psi}_{\mu} + \frac{1}{12}\varepsilon\gamma_{\alpha\beta}\partial^{\mu}J^{\chi}_{\alpha\beta} - \frac{1}{12m}\varepsilon\gamma_{\mu}\partial^{\rho}J^{\psi}_{\rho}.
\]
As in the Proca case, one should go back to a formulation that is still invariant under the St"uckelberg symmetries, in order to take the massless limit in a well-defined way. This may be achieved by making the following field redefinitions in the final transformation rules \((3.11)\) and action \((3.12)\) thereby re-introducing the fields \((V_{\mu}, \phi^{\prime}, \chi^{\prime}, \psi)\) that were eliminated by the gauge-fixing conditions \((3.9)\):
\[
h_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{1}{m}\left(\partial_{\mu}V_{\nu} + \partial_{\nu}V_{\mu}\right) + \frac{m}{2}\partial_{\mu}\partial_{\nu}\phi^{\prime},
\]
\[
\psi_{\mu} = \tilde{\psi}_{\mu} - \frac{1}{m}\partial_{\mu}\psi, \quad \chi_{\mu} = \tilde{\chi}_{\mu} + \frac{1}{4m}\partial_{\mu}\chi^{\prime}.
\]
Applying this field redefinition in \((3.11)\) then leads to transformation rules\(^{10}\), whose massless limit is well-defined. In order to make the massless limit of the interaction part \(I_{int}\) and of the transformation rules \((3.23)\) well-defined, we impose that \(T_{\mu\nu}\) and \(J^{\psi}_{\mu}\) are conserved
\[
\partial^{\nu}T_{\mu\nu} = 0, \quad \partial^{\mu}J^{\psi}_{\mu} = 0,
\]
and we put \(J^{\chi}_{\mu}, T_{S}, T_{N}\) and \(T_{A}^{\lambda}\) to zero in order to preserve supersymmetry and to obtain an irreducible multiplet in the massless limit. The conjugate multiplet \((3.23)\) then reduces to a spin-2 supercurrent multiplet \((T_{\mu\nu}, J^{\psi}_{\mu}, T_{S})\) that contains the energy–momentum tensor \(T_{\mu\nu}\) and supersymmetry current \(J^{\psi}_{\mu}\).
As in the Proca case, the massless limit is now well-defined. Performing the above outlined steps on the action \((3.21)\) and taking the massless limit leads, however, to an action that is in off-diagonal form. This action can be diagonalized by making the following field redefinitions:
\[
\tilde{h}_{\mu\nu} = h_{\mu\nu}^{\prime} + \eta_{\mu\nu}\phi^{\prime}, \quad \tilde{\psi}_{\mu} = \psi_{\mu}^{\prime} + \frac{1}{2}\gamma_{\mu}\chi^{\prime}, \quad S = S^{\prime} - \frac{1}{8}F, \quad \tilde{\chi}_{\mu} = \chi_{\mu}^{\prime} - \gamma_{\mu}\psi.
\]
\(^{10}\)These resulting transformation rules are given by the transformation rules \((3.7)\), provided one makes the following substitution: \(h_{\mu\nu} \rightarrow \tilde{h}_{\mu\nu}, \psi_{\mu} \rightarrow \tilde{\psi}_{\mu}, \chi_{\mu} \rightarrow \tilde{\chi}_{\mu}, \phi \rightarrow -\psi^{\prime}\) and \(\chi \rightarrow \chi^{\prime}/4.\)
The resulting action is given by

\[
I = \int d^3 x \left\{ R^{\mu \nu} G_{\mu \nu}^{\text{lin}} (h') - 4 \bar{\psi}_\mu y^{\mu \nu \rho} \partial_\nu \psi_\rho - 8 S'^2 + h'_{\mu \nu} T^{\mu \nu} + \bar{\psi}^{\mu} \not{\partial}_\mu \psi + ST_S \right. \\
- F^{\mu \nu} F_{\mu \nu} - \frac{2}{3} N^2 + \frac{2}{3} A^\mu A_\mu - 4 \bar{\chi} \gamma^{\mu \nu \rho} \partial_\nu \chi_\rho - 8 \bar{\psi} \gamma^\mu \partial_\mu \psi \\
+ 2 \left[ - \partial_\mu \phi \gamma^\mu \phi' - \frac{1}{4} \bar{\chi} \gamma^\mu \partial_\mu \chi' + \frac{1}{16} F^2 \right] \\
+ \phi' y^{\mu \nu} T^{\mu \nu} - \frac{1}{4} \bar{\chi} \gamma^\mu \not{\partial}_\mu \psi - \frac{1}{8} F T_S \right\}. \tag{3.27}
\]

This is an action for three massless multiplets: a spin two multiplet \((h'_\mu, \psi'_\mu, S')\), a mixed gravitino-vector multiplet \((V_\mu, \chi'_\mu, \psi, N, A_\mu)\) and a scalar multiplet \((\phi', \chi', F)\). These multiplets and their transformation rules are collected in appendix A.12. The spin-2 multiplet couples to the supercurrent multiplet in the usual fashion. Unlike the Proca case however, the supercurrent multiplet does not only couple to the spin-2 multiplet, but there is also a coupling to the scalar multiplet, given in the last line of (3.27). Indeed, defining

\[
T_\phi = \eta^{\mu \nu} T_{\mu \nu}, \quad \not{J} = -\frac{1}{4} \gamma^\mu \not{\partial}_\mu \psi, \quad T_F = -\frac{1}{8} T_S, \tag{3.28}
\]

one finds that the fields \((T_\phi, \not{J}, T_F)\) form a conjugate scalar multiplet with transformation rules

\[
\delta T_\phi = -\bar{\epsilon} \gamma^\mu \partial_\mu \psi, \\
\delta \not{J} = -\frac{1}{2} \epsilon T_\phi + \gamma^\mu \epsilon \partial_\mu T_F, \\
\delta T_F = \frac{1}{4} \bar{\epsilon} \not{J}, \tag{3.29}
\]

such that the last line of (3.27) is invariant under supersymmetry.

We have thus obtained a 3D supersymmetric version of the 4D vDVZ discontinuity. The above discussion shows that the massless limit of the supersymmetric FP theory coupled to a supercurrent multiplet, leads to linearized \(\mathcal{N} = 1\) supergravity, plus an extra scalar multiplet that couples to a multiplet that includes the trace of the energy–momentum tensor and the gamma-trace of the supercurrent.

### 4. Linearized SNMG without higher derivatives

Using the results of the previous section we will now construct linearized new massive supergravity without higher derivatives but with auxiliary fields. Furthermore, we will show how, by eliminating the different ‘non-trivial’ bosonic and fermionic auxiliary fields, one re-obtains the higher-derivative kinetic terms for both the bosonic and fermionic fields. We remind that by a ‘non-trivial’ auxiliary field we mean an auxiliary field whose elimination leads to higher-derivative terms in the action.

Consider first the bosonic case. The linearized version of lower-derivative (‘lower’) NMG is described by the following action [1]:

\[
I_{\text{SNMG}}^{\text{lin}} (\text{lower}) = \int d^3 x \left\{ -h^{\mu \nu} G_{\mu \nu}^{\text{lin}} (h) + 2 q^{\mu \nu} G_{\mu \nu}^{\text{lin}} (h) - m^2 (q^{\mu \nu} q_{\mu \nu} - q^2) \right\}, \tag{4.1}
\]

11 An on-shell version of this multiplet was introduced in [15].

12 The transformation rules of the different multiplets can also be found by starting from the transformation rules of the massive FP multiplet and carefully following all redefinitions as outlined in the main text, provided one performs compensating gauge transformations.
where \( h_{\mu\nu} \) and \( q_{\mu\nu} \) are two symmetric tensors and \( q = \eta^{\mu\nu} q_{\mu\nu} \). The above action can be diagonalized by making the redefinitions
\[
 h_{\mu\nu} = A_{\mu\nu} + B_{\mu\nu}, \quad q_{\mu\nu} = B_{\mu\nu},
\]
after which we obtain
\[
 I_{\text{NMG}}^{\text{lin}}[A, B] = \int d^3 x \left\{ -A^{\mu\nu} G_{\mu\nu}^{\text{lin}}(A) + B^{\mu\nu} G_{\mu\nu}^{\text{lin}}(B) - m^2 (B^{\mu\nu} B_{\mu\nu} - B^2) \right\}.
\]
Using this diagonal basis it is clear that we can supersymmetrize the action in terms of a massless multiplet \((A_{\mu\nu}, \lambda_\mu, S)\) and a massive multiplet \((B_{\mu\nu}, \psi_\mu, \chi_\mu, M, N, P, A_\mu)\). Transfoming this result back in terms of \( h_{\mu\nu} \) and \( q_{\mu\nu} \) and making the redefinition
\[
 \lambda_\mu = \rho_\mu - \psi_\mu
\]
we find the following linearized lower-derivative supersymmetric NMG action
\[
 I_{\text{NMG}}^{\text{lin}}(\text{lower}) = \int d^3 x \left\{ -h^{\mu\nu} G_{\mu\nu}^{\text{lin}}(h) + 2q^{\mu\nu} G_{\mu\nu}^{\text{lin}}(h) - m^2 (q^{\mu\nu} q_{\mu\nu} - q^2) + 8S^2 \right.
\]
\[
 - \frac{2}{3} M^2 - \frac{2}{3} N^2 + \frac{2}{3} P^2 + \frac{2}{3} A_{\mu} A^{\mu} + 4 \rho_\mu \gamma^{\mu\nu} \partial_\nu \rho_\mu
\]
\[
 - 8 \bar{\psi}_\mu \gamma^{\mu\nu} \partial_\nu \rho_\mu - 4 \bar{\chi}_\mu \gamma^{\mu\nu} \partial_\nu \chi_\mu + 8 m \bar{\psi}_\mu \gamma^{\mu\nu} \bar{\chi}_\nu \right\}. \tag{4.5}
\]
This action describes 2 + 2 on-shell and 16 + 16 off-shell degrees of freedom. It is invariant under the following transformation rules
\[
 \delta h_{\mu\nu} = \bar{\epsilon} \gamma_\mu (\rho_\nu), \quad \delta \rho_\mu = -\frac{1}{4} \gamma^{\rho\sigma} (\partial_\rho h_{\sigma\nu}) \epsilon + \frac{1}{2} S \gamma_\mu (M + P) \epsilon, \quad \delta S = \frac{1}{4} \bar{\epsilon} \gamma^{\mu\nu} \rho_{\mu\nu} - \frac{1}{2} \bar{\epsilon} \gamma^{\mu\nu} \psi_{\mu\nu}, \tag{4.6}
\]
where
\[
 \rho_{\mu\nu} = \frac{1}{2} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu), \quad \psi_{\mu\nu} = \frac{1}{2} (\partial_\mu \psi_\nu - \partial_\nu \psi_\mu), \tag{4.7}
\]
plus the transformation rules for the massive multiplet \((q_{\mu\nu}, \psi_\mu, \chi_\mu, M, N, P, A_\mu)\) which can be found in equation (3.11), with \( h_{\mu\nu} \) replaced by \( q_{\mu\nu} \). We have deleted \( 1/m \) terms in the transformation of \( h_{\mu\nu} \) and \( \rho_\mu \) since they take the form of a gauge transformation. Note also that the auxiliary field \( S \) transforms to the gamma-trace of the equation of motion for \( \rho_\mu \).

The action (4.5) contains the trivial auxiliary fields \((S, M, N, P, A_\mu)\) and the non-trivial auxiliary fields \((q_{\mu\nu}, \psi_\mu, \chi_\mu)\). The elimination of the trivial auxiliary fields does not lead to anything new. These fields can simply be set equal to zero and disappear from the action. Instead, as we will show now, the elimination of the non-trivial auxiliary fields leads to higher derivative terms in the action. To start with, the equation of motion for \( q_{\mu\nu} \) can be used to solve for \( q_{\mu\nu} \) as follows:
\[
 q_{\mu\nu} = \frac{1}{m^2} G_{\mu\nu}^{\text{lin}}(h) - \frac{1}{2m} \eta_{\mu\nu} G_{\alpha}^{\text{lin}}(h), \tag{4.8}
\]
where \( G_{\mu}^{\text{lin}}(h) = \eta^{\mu\nu} G_{\nu}(h) \). One of the vector–spinors, \( \psi_\mu \), occurs as a Lagrange multiplier. Its equation of motion enables one to solve for \( \chi_\mu \):
\[
 \chi_\mu = -\frac{1}{2m} \gamma^{\rho\sigma} \gamma_\mu \rho_{\rho\sigma}. \tag{4.9}
\]
The equation of motion of the other vector–spinor, \( \chi_\mu \), can be used to solve for \( \psi_\mu \) in terms of \( \chi_\mu \):
\[
 \psi_\mu = -\frac{1}{2m} \gamma^{\rho\sigma} \gamma_\mu \chi_{\rho\sigma}. \tag{4.10}
\]
and hence, via equation (4.9), in terms of \( \rho_\mu \). One can show that the solution of \( \psi_\mu \) in terms of (two derivatives of) \( \rho_\mu \) is such that it solves the constraint
\[
\gamma^{\mu\nu} \psi_{\mu\nu} = 0. \tag{4.11}
\]
We now substitute the solutions (4.8) for \( q_{\mu\nu} \) and (4.9) for \( \chi_\mu \) back into the action and make use of the identity
\[
-4 \tilde{\rho}_\mu \gamma^{\mu\nu} \partial_\nu \chi_\rho = \frac{8}{m^2} \tilde{\rho}^{\mu\nu} \rho_{\mu\nu} - \frac{2}{m^2} \tilde{\rho}_\mu \gamma^{\mu\nu} \gamma_\rho \rho_\rho, \tag{4.12}
\]
where we ignore a total derivative term. One thus obtains the following linearized higher-derivative (‘higher’) supersymmetric action of NMG [5]:
\[
I_{\text{SNMG}}^{\text{lin}}(\text{higher}) = \int d^3 x \left\{ -h^{\mu\nu} G_{\mu\nu}^{\text{lin}} (h) + \frac{4}{m^2} \tilde{\rho}^{\mu\nu} \partial_\nu \rho_\mu + 8S^2 + \frac{4}{m^2} \tilde{\rho}_{ab} \gamma_{cd} \rho_{cd} - \frac{2}{m^2} \tilde{\rho}_{ab} \gamma_{\rho\rho} \rho_{\rho\omega} \right\}. \tag{4.13}
\]
The action (4.13) is invariant under the supersymmetry rules
\[
\delta h_{\mu\nu} = \bar{\epsilon} \gamma_{(\mu} \rho_{\nu)}, \\
\delta \rho_\mu = -\frac{1}{4} \gamma^{\rho\sigma} \partial_\rho h_{\mu\sigma} \epsilon + \frac{1}{2} S \gamma_\mu \epsilon, \\
\delta S = \frac{1}{4} \bar{\epsilon} \gamma^{\mu\nu} \rho_{\mu\nu}, \tag{4.14}
\]
where we made use of the constraint (4.11) to simplify the transformation rule of \( S \). Under supersymmetry the auxiliary field \( S \) transforms to the gamma-trace of the equation of motion for \( \rho_\mu \), since the higher derivative terms in this equation of motion are gamma-traceless and therefore drop out.

Alternatively, the higher-derivative kinetic terms for \( \rho_\mu \) can be obtained by boosting up the derivatives in the massive spin-3/2 FP equations in the same way as that has been done for the spin-2 FP equations in the construction of NMG [1], except for one subtlety, see appendix B.

This finishes our construction of linearized SNMG. In the next section we will discuss to which extent this result can be extended to the nonlinear case.

5. The nonlinear case

Supersymmetric NMG without ‘non-trivial’ auxiliary fields, i.e. with higher derivatives, has already been constructed some time ago [5]. This action only contains the auxiliary field \( S \) of the massless multiplet. A characteristic feature is that there is no kinetic term for \( S \) and in the bosonic terms \( S \) occurs as a torsion contribution to the spin-connection. However, due to its coupling to the fermions it cannot be eliminated from the action. Thus, in the nonlinear case we cannot anymore identify \( S \) as a ‘trivial’ auxiliary field.

We recall that, apart from the auxiliary field \( S \), in the linearized analysis of section 3 and 4 we distinguish between the trivial auxiliary fields \( (M, N, P, A_\mu) \) and the non-trivial ones \( (q_{\mu\nu}, \psi_\mu, \chi_\mu) \). Only the elimination of the latter ones leads to higher derivatives in the Lagrangian. In the formulation of [5] only the auxiliary field \( S \) occurs. One could now search either for a formulation in which all other auxiliary fields occur or for an alternative formulation in which only the non-trivial auxiliary fields \( (q_{\mu\nu}, \psi_\mu, \chi_\mu) \) are present. In this work we will not consider the inclusion of all auxiliary fields any further. It is not clear to us whether such a formulation exists. This is based on the fact that our construction of the linearized massive multiplet makes use of the existence of a consistent truncation to the first massive KK level. Such a truncation can only be made consistently at the linearized level.
Before discussing the inclusion of the non-trivial auxiliary fields \((q_{\mu \nu}, \psi_{\mu}, \chi_{\mu})\) it is instructive to first consider the linearized case and see how, starting from the (linearized) formulation of [5] these three non-trivial auxiliary fields can be included and a formulation with lower derivatives can be obtained. Our starting point is the higher-derivative action (4.13) and corresponding transformation rules (4.14). We first consider the bosonic part of the action (4.13), i.e.

\[
l_{\text{bos}}^{\text{lin}}(\text{higher}) = \int d^3x \left\{ -h^{\mu \nu} G_{\mu \nu}^{\text{lin}}(h) + 8S^2 + \frac{4}{m^2} \left( R^{\mu \nu} R_{\mu \nu} - \frac{3}{8} R^2 \right)^{\text{lin}} \right\}. \tag{5.1}
\]

We already know from the construction of the bosonic theory that the derivatives can be lowered by introducing a symmetric auxiliary field \(q_{\mu \nu}\) and writing the equivalent bosonic action

\[
l_{\text{bos}}^{\text{lin}}(\text{lower}) = \int d^3x \left\{ -h^{\mu \nu} G_{\mu \nu}^{\text{lin}}(h) + 8S^2 + 2q^{\mu \nu} G_{\mu \nu}^{\text{lin}}(h) - m^2 (q^{\mu \nu} q_{\mu \nu} - q^2) \right\}. \tag{5.2}
\]

The field equation of \(q_{\mu \nu}\) is given by equation (4.8) and substituting this solution back into the lower-derivative bosonic action (5.2) we re-obtain the higher-derivative bosonic action (5.1).

We next consider the fermionic part of the higher-derivative action (4.13), i.e.

\[
l_{\text{ferm}}^{\text{lin}}(\text{higher}) = \int d^3x \left\{ 4\bar{\rho}_a Y^{\mu \nu \rho} \partial_\rho \rho_a + \frac{8}{m^2} \bar{\rho}_{ab} \bar{\rho}_{ab} - \frac{2}{m^2} \bar{\rho}_{ab} \gamma^{ab} \bar{\psi} \gamma^{cd} \rho_{cd} \right\}. \tag{5.3}
\]

To lower the number of derivatives we first replace the terms that are quadratic in \(\rho_{ab}\) by the kinetic term of an auxiliary field \(\chi_{\mu}\), while adding another term with a Lagrange multiplier \(\psi_{\mu}\) to fix the relation between \(\rho_{ab}\) and \(\chi_{\mu}\):

\[
l_{\text{ferm}}^{\text{lin}}(\text{lower}) = \int d^3x \left\{ 4\bar{\rho}_a Y^{\mu \nu \rho} \partial_\rho \rho_a - 4\bar{\chi}_a Y^{\mu \nu \rho} \partial_\rho \chi_a - 8\bar{\psi}_a (\gamma^{\mu \nu \rho} \rho_{\rho \mu} - m \gamma^{\mu \nu} \chi_a) \right\}. \tag{5.4}
\]

The equation of motion for \(\psi_{\mu}\) enables us to express \(\chi_{\mu}\) in terms of \(\rho_{ab}\). The result is given in equation (4.9). Substituting this solution for \(\chi_{\mu}\) back into the action, the terms linear in the Lagrange multiplier \(\psi_{\mu}\) drop out and we re-obtain the higher-derivative fermionic action given in equation (5.3).

Adding up the lower-derivative bosonic action (5.2) and the lower-derivative fermionic action (5.4) we obtain the lower-derivative supersymmetric action (4.5), albeit without the bosonic auxiliary fields \((M, N, P, A_{\mu})\). We only consider a formulation in which these auxiliary fields are absent.

Having introduced the new auxiliary fields \((q_{\mu \nu}, \psi_{\mu}, \chi_{\mu})\) we should derive their supersymmetry rules. They can be derived by starting from the solutions (4.8)–(4.10) of these auxiliary fields in terms of \(h_{\mu \nu}\) and \(\rho_{a}\) and applying the supersymmetry rules of \(h_{\mu \nu}\) and \(\rho_{a}\) given in equation (4.14). This leads to supersymmetry rules that do not contain the auxiliary fields. These can be introduced by adding to the supersymmetry rules a number of (field-dependent) equation of motion symmetries. We thus find the intermediate result:

\[
\begin{align*}
\delta h_{\mu \nu} &= \bar{\epsilon} Y_{(\mu} \rho_{\nu)}, \\
\delta \rho_{a} &= -\frac{1}{4} Y^{\rho a} \partial_\rho h_{\mu \nu} \epsilon + \frac{1}{2} S Y_{a} \epsilon, \\
\delta S &= \frac{1}{4} \bar{\epsilon} Y^{\mu \nu} \rho_{\mu \nu}, \\
\delta q_{a b} &= \bar{\epsilon} Y_{(a} \psi_{b)}, \\
\delta \psi_{\mu} &= -\frac{1}{4} Y^{\rho a} \partial_\rho q_{a} \epsilon, \\
\delta \chi_{\mu} &= \frac{m}{4} \gamma^{\nu} q_{\mu \nu} \epsilon + \frac{1}{2m} \bar{\epsilon} \partial_\mu S. \tag{5.5}
\end{align*}
\]
These transformation rules are not yet quite the same as the ones given in equation (4.6). In particular, the transformation rules of $S$ and $\chi_{\mu}$ are different. The difference is yet another ‘on-shell symmetry’ of the action equation (4.5), with spinor parameter $\eta$, given by

$$\delta S = -\frac{1}{4} \hat{\eta} \gamma^\mu \gamma^\nu \phi_{\mu\nu},$$

$$\delta \chi_{\mu} = -\frac{1}{2m} \eta \partial_\mu S.$$  \hfill (5.6)

The transformation rules in equations (4.6) and (5.5) are therefore equivalent up to an on-shell symmetry with parameter $\eta = \epsilon$:

$$\delta_{\text{usy}} \text{ (equation (4.6))} = \delta_{\text{usy}} \text{ (equation (5.5))} + \delta_{\text{on-shell}} (\eta = \epsilon).$$  \hfill (5.7)

We now wish to discuss in which sense the previous analysis can be extended to the nonlinear case. For simplicity, we take the approximation in which one considers only the terms in the action that are independent of the fermions and the terms that are bilinear in the fermions. Furthermore, we ignore in the supersymmetry variation of the action terms that depend on the auxiliary scalar $S$. Since terms linear in $S$ only occur in terms bilinear in fermions this effectively implies that we may set $S = 0$ in the action. In this approximation the higher-derivative action of SNMG is given by [5]

$$I_{\text{SNMG}}^{\text{nonlin}} \text{(higher)} = \int \, d^3 x \left \{ -4R(\hat{\omega}) + \frac{1}{m^2} R_{\mu
u ab}(\hat{\omega}) R_{\mu
u ab}(\hat{\omega}) - \frac{1}{2m^2} R^2(\hat{\omega}) + 4 \hat{\rho}_\mu \gamma^\mu \gamma^\nu D_\nu (\hat{\omega}) \rho_\mu \\
+ \frac{8}{m^2} \hat{\rho}_{ab}(\hat{\omega}) \mathcal{D}(\hat{\omega}) \rho_{ab}(\hat{\omega}) - \frac{2}{m^2} \hat{\rho}_{\mu
u}(\hat{\omega}) \gamma^\mu \mathcal{D}(\hat{\omega}) \gamma^\nu \rho_{\mu\nu}(\hat{\omega}) \\
- \frac{2}{m^2} R_{\mu
u ab}(\hat{\omega}) \hat{\rho}_\nu \gamma^\mu \gamma^\rho \rho_{\mu\nu}(\hat{\omega}) - \frac{2}{m^2} \hat{R}(\hat{\omega}) \hat{\rho}_\mu \gamma^\nu \rho_{\mu\nu}(\hat{\omega}) \\
+ \text{higher-order fermions and } S\text{-dependent terms} \right \}. \hfill (5.8)$$

Note that we have replaced the symmetric tensor $h_{\mu\nu}$ by a Dreibein field $e_{\mu}^a$. Keeping the same approximation discussed above the action (5.8) is invariant under the supersymmetry rules

$$\delta e_{\mu}^a = \frac{1}{2} \bar{\psi} \gamma^a \mu, \quad \delta \rho_\mu = D_\mu (\hat{\omega}) \epsilon. \hfill (5.9)$$

We first consider the lowering of the number of derivatives in the bosonic part of the action. Since the Ricci tensor now depends on a torsion-full spin-connection we need a non-symmetric auxiliary tensor $q_{\mu,v}$. The action (5.8) can then be converted into the following equivalent action:

$$I_{\text{SNMG}}^{\text{nonlin}} \text{(higher)} = \int \, d^3 x \left \{ -4R(\hat{\omega}) - m^2 (q_{\mu,v}^a q_{\mu,v} - q^2) + 2q_{\mu,v}^a G_{\mu,v}(\hat{\omega}) + 4 \hat{\rho}_\mu \gamma^\mu \gamma^\nu D_\nu (\hat{\omega}) \rho_\mu \\
+ \frac{8}{m^2} \hat{\rho}_{ab}(\hat{\omega}) \mathcal{D}(\hat{\omega}) \rho_{ab}(\hat{\omega}) - \frac{2}{m^2} \hat{\rho}_{\mu
u}(\hat{\omega}) \gamma^\mu \mathcal{D}(\hat{\omega}) \gamma^\nu \rho_{\mu\nu}(\hat{\omega}) \\
- \frac{2}{m^2} R_{\mu
u ab}(\hat{\omega}) \hat{\rho}_\nu \gamma^\mu \gamma^\rho \rho_{\mu\nu}(\hat{\omega}) - \frac{2}{m^2} \hat{R}(\hat{\omega}) \hat{\rho}_\mu \gamma^\nu \rho_{\mu\nu}(\hat{\omega}) \\
+ \text{higher-order fermions and } S\text{-dependent terms} \right \}. \hfill (5.10)$$

The equivalence with the previous action can be seen by solving the equation of motion for $q_{\mu,v}$:

$$q_{\mu,v} = \frac{1}{m^2} G_{\mu,v}(\hat{\omega}) - \frac{1}{2m^2} g_{\mu v} G_{\mu}(\hat{\omega}) \hfill (5.11)$$
and substituting this solution back into the action. Note that the solution for $q_{\mu,\nu}$ is not super-covariant.

We next consider the lowering of the number of derivatives in the fermionic terms in the action. Following the linearized case we define an auxiliary vector–spinor $\chi_\mu$ as

$$
\chi_\mu = -\frac{1}{2m} \gamma^{\rho\sigma} \gamma_\rho \rho_{\rho\sigma} (\hat{\omega}),
$$

or equivalently

$$
\rho_{\mu\nu} (\hat{\omega}) = -m \gamma_{\mu} \chi_\nu.
$$

The first equation is the nonlinear generalization of equation (4.9). Using this definition one can show the following identity

$$
\frac{8}{m^2} e_{\mu} \tilde{\rho}_{ab} (\hat{\omega}) \chi (\hat{\omega}) - \frac{2}{m^2} \tilde{e}_{\mu} \rho_{\mu\nu} (\hat{\omega}) \gamma^{\mu\nu} \rho_{\rho\sigma} (\hat{\omega}) =
\frac{8}{m^2} e_{\mu} \tilde{\rho}_{ab} (\hat{\omega}) \chi (\hat{\omega}) - \frac{1}{m} \bar{e} R_{\mu\nu\ab} (\hat{\omega}) \tilde{\rho}_{\mu} \gamma^{\mu\nu} \gamma^{a} \gamma^{b} \chi
+ \text{higher-order fermions and total derivative terms},
$$

which is the nonlinear generalization of the identity (4.12). This identity can be used to replace the higher-derivative kinetic terms of the fermions by lower-derivative ones. At the same time we may use equation (5.13) to replace $\rho_{\mu\nu}$ by $\chi_\mu$. This can be done by introducing a Lagrange multiplier $\psi_\mu$, whose equation of motion allows us to use equation (5.12). This leads to the following action:

$$
I_{\text{SNMG}}^{\text{nonlin} (\text{lower})} = \int d^3 x \left\{ -4 \bar{R} (\hat{\omega}) + 2 q_{\mu,\nu} G_{\mu,\nu} (\hat{\omega}) - m^2 \left( q^{\mu,\nu} q_{\mu,\nu} - q^2 \right) + 4 \bar{\rho}_{\mu} \gamma^{\mu\nu} D_\nu (\hat{\omega}) \rho_{\nu} 
- 8 \bar{\psi}_\mu \gamma^{\mu\nu} D_\nu (\hat{\omega}) \chi_\nu 
- \frac{1}{m} \bar{e} R_{\mu\nu\ab} (\hat{\omega}) \tilde{\rho}_{\mu} \gamma^{\mu\nu} \gamma^{a} \gamma^{b} \chi
+ \text{higher-order fermions and S-dependent terms} \right\}.
$$

Our next task is to derive the supersymmetry rules of the auxiliary fields $q_{\mu,\nu}$, $\psi_\mu$ and $\chi_\mu$. Using the solutions of the auxiliary fields in terms of $e_{\mu}^a$, $\rho_{\mu}$ we derived these supersymmetry rules. In this way one obtains supersymmetry rules that do not contain any of the auxiliary fields and, consequently, do not reduce to the supersymmetry rules (4.6) upon linearization. To achieve this, we must add to these transformation rules a number of field-dependent equation of motion symmetries, like we did in the linearized case. Since the results we obtained are not illuminating we refrain from giving the explicit expressions here.

A disadvantage of the present approach is that, although in principle possible in the approximation we considered, one cannot maintain the interpretation of $S$ as a torsion contribution to the spin-connection. This makes the result rather cumbersome. It would be interesting to see whether a superspace approach could improve on this. Without further insight the lower-derivative formulation of SNMG, if it exists at all at the full nonlinear level, does not take the same elegant form as the higher-derivative formulation presented in [5].

6. Conclusions

In this work we considered the $\mathcal{N} = 1$ supersymmetrization of NMG in the presence of auxiliary fields. All auxiliary fields are needed to close the supersymmetry algebra off-shell.
At the linearized level, we distinguished between two types of auxiliary fields: the ‘non-trivial’ ones whose elimination leads to higher derivatives in the Lagrangian (these are the fields $q_{\mu\nu}, \psi_\mu$ and $\chi_\mu$) and the ‘trivial’ ones whose elimination (if possible at all at the full nonlinear level) does not lead to higher derivatives (these are the fields $S, M, N, P$ and $A_\mu$). We found that at the linearized level all auxiliary fields could be included leading to a linearized SNMG theory without higher derivatives. At the nonlinear level we gave a partial answer for the case that only the trivial auxiliary $S$ and the non-trivial auxiliaries $q_{\mu\nu}, \psi_\mu$ and $\chi_\mu$ were included. To obtain the full nonlinear answer one should perhaps make use of superspace techniques. The answer without the non-trivial auxiliaries and with higher derivatives can be found in [5].

We discussed a 3D supersymmetric analog of the 4D vDVZ discontinuity by taking the massless limit of the supersymmetric FP model coupled to a supercurrent multiplet. We showed that in the massless limit there is a non-trivial coupling of a scalar multiplet (containing the scalar mode $\phi$ of the metric) to a current multiplet (containing the trace of the energy–momentum tensor). This is the natural supersymmetric extension of what happens in the bosonic case and supports the analysis of [8].

As a by-product we found a way to ‘boost up’ the derivatives in the spin-$3/2$ FP equation, see appendix B. The trick is based upon the observation that, before boosting up the derivatives like in the construction of the NMG model, one should first combine the equations of motion describing the helicity $+3/2$ and $-3/2$ states into a single parity-even equation with one additional derivative.

It is natural to extend the results of this work to the case of extended, i.e. $\mathcal{N} > 1$, supersymmetry, or to ‘cosmological’ massive gravity theories. Higher-derivative, linearized versions of NMG with extended supersymmetry, or anti-de Sitter vacua, were given in [16, 17]. Of special interest is the case of maximal supersymmetry since this would correspond to the KK reduction of the $\mathcal{N} = 8$ massless maximal supergravity multiplet which only exists in a formulation without (trivial) auxiliary fields. We expect that having a formulation of this maximal SNMG theory without higher-derivatives will be useful in finding out whether this massive 3D supergravity model has the same miraculous ultraviolet properties as in the 4D massless case.

Acknowledgments

We thank Olaf Hohm and Paul Townsend for comments on the draft. YY would like to thank Rakibur Rahman for inspiring discussions. Two of us (EB and JR) would like to thank the Simons Center for Geometry and Physics for its hospitality and generous financial support. They wish to thank the organizers of the Summer Workshop 2012 for providing a stimulating scientific environment in which part of this work was completed. The work of JR was supported by the START project Y 435-N16 of the Austrian Science Fund (FWF). LP acknowledges support by the Consejo Nacional de Ciencia y Tecnología (CONACyT), the Universidad Nacional Autónoma de México via the project UNAM-PAPIIT IN109013 and an Ubbo Emmius sandwich scholarship from the University of Groningen. The work of MK and YY is supported by the Ubbo Emmius Programme administered by the Graduate School of Science, University of Groningen. TZ acknowledges support by a grant of the Dutch Academy of Sciences (KNAW).

Appendix A. Off-shell $\mathcal{N} = 1$ massless multiplets

In this appendix we collect the off-shell formulations of the different 3D massless multiplets with $\mathcal{N} = 1$ supersymmetry. A useful reference where more properties about 3D
This table indicates the field content and off-shell/on-shell degrees of freedom of the different massless multiplets. Only the first three massless multiplets \((s = 2, 1, 0)\) occur in the massless limit of the FP model.

| Multiplet         | Fields                        | Off-shell | On-shell |
|-------------------|-------------------------------|-----------|----------|
| \(s = 2\)         | \(h_{\mu\nu}, \psi_\mu, S\) | 4+4       | 0+0      |
| \(s = 1\)         | \(V_\mu, N, A_\mu, \chi_\mu, \psi\) | 6+6       | 1+1      |
| \(s = 0\)         | \(\phi, \chi, F\)            | 2+2       | 1+1      |
| Gravitino multiplet| \(\chi_\mu, A_\mu, D\)       | 4+4       | 0+0      |
| Vector multiplet   | \(V_\mu, \psi\)              | 2+2       | 1+1      |

Supersymmetry can be found is [18]. The field content of the different multiplets can be found in table A1. 

\(s = 2\). The off-shell version of the 3D massless spin-2 multiplet is well-known. The multiplet is extended with an auxiliary real scalar field \(S\). The off-shell supersymmetry rules are given by

\[
\begin{align*}
\delta h_{\mu\nu} &= \bar{\epsilon} Y_{(\mu} \psi_{\nu)} , \\
\delta \psi_\mu &= -\frac{1}{4} Y^{\rho\sigma} \partial_\rho h_{\mu\sigma} \epsilon + \frac{1}{2} SY_\mu \epsilon , \\
\delta S &= \frac{1}{4} \bar{\epsilon} Y^{\mu\nu} \psi_{\mu\nu} ,
\end{align*}
\]

(A.1)

where

\[
\psi_{\mu\nu} = \frac{1}{2} (\partial_\mu \psi_\nu - \partial_\nu \psi_\mu).
\]

(A.2)

These transformation rules leave the following action invariant:

\[
I_{s=2} = \int d^3x \left\{ \frac{1}{2} \epsilon \left[ G^{\mu\nu}_{\text{lin}} (h) - 4 \bar{\psi}_\mu (Y^{|\mu|\rho} \partial_\rho \psi_\mu - 8S) \right] \right\}.
\]

(A.3)

\(s = 1\). The off-shell ‘mixed gravitino-vector’ multiplet consists of a propagating vector \(V_\mu\), an auxiliary vector \(A_\mu\), an auxiliary scalar \(N\), a vector–spinor \(\chi_\mu\) and a spinor \(\psi\). An on-shell version of this multiplet, called ‘vector–spinor’ multiplet, has been considered in [15]. The off-shell supersymmetry rules are given by

\[
\begin{align*}
\delta V_\mu &= \bar{\epsilon} Y_{\mu} \psi - \frac{1}{2} \bar{\epsilon} \chi_\mu , \\
\delta \psi &= -\frac{1}{8} Y^{\rho\sigma} F_{\rho\sigma} \epsilon - \frac{1}{12} N \epsilon - \frac{1}{12} \bar{\chi} Y_\mu A_\mu \epsilon , \\
\delta \chi_\mu &= -\frac{1}{4} Y^{\rho\sigma} A_{\rho\sigma} \epsilon - \frac{1}{4} Y_{\mu} Y^{\rho\sigma} F_{\rho\sigma} \epsilon - \frac{1}{6} Y_{\mu} N \epsilon + \frac{1}{6} A_\mu \epsilon - \frac{1}{6} \bar{\chi} Y_\mu A_\mu \epsilon , \\
\delta N &= \bar{\epsilon} Y^{\mu\nu} \partial_\mu \psi - \bar{\epsilon} Y^{\mu\nu} \partial_\nu \chi_\mu , \\
\delta A_\mu &= \frac{3}{2} \bar{\epsilon} Y_{\mu} \partial_\nu \psi - \frac{3}{2} \bar{\epsilon} Y_{\nu} \partial_\mu \psi + \bar{\epsilon} Y^{\mu\nu} \partial_\mu \chi_\mu + \bar{\epsilon} Y^{\mu\nu} \partial_\nu \chi_\mu .
\end{align*}
\]

(A.4)

Note that this multiplet is irreducible. It cannot be written as the sum of a gravitino and vector multiplet. These multiplets are given below. The supersymmetric action for this multiplet is given by

\[
I_{s=1} = \int d^3x \left\{ -F_{\mu\nu} F^{\mu\nu} - \frac{2}{3} N^2 + \frac{2}{3} A_\mu A^\mu - 4 \bar{\chi}_\mu Y^{\mu\nu} \partial_\nu \chi_\mu - 8 \bar{\psi} \gamma^\mu \partial_\mu \psi \right\}.
\]

(A.5)

with \(F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu\).

\(s = 0\). The off-shell scalar multiplet consists of a scalar \(\phi\), a spinor \(\chi\) and an auxiliary scalar \(F\). The off-shell supersymmetry rules are given by

\[
\begin{align*}
\delta \phi &= \frac{1}{2} \bar{\epsilon} \chi , \\
\delta \chi &= \gamma^\mu \epsilon (\partial_\mu \phi) - \frac{1}{4} F \epsilon , \\
\delta F &= -\bar{\epsilon} \gamma^\mu \partial_\mu \chi .
\end{align*}
\]

(A.6)
The supersymmetric action for a scalar multiplet is given by
\[
I_{\phi=0} = \int d^3x \left\{ -\partial^\mu \phi \partial_\mu \phi - \frac{1}{4} \bar{\chi} \gamma^\mu \partial_\mu \chi + \frac{1}{16} F^2 \right\}. \tag{A.7}
\]

Besides the massless multiplets discussed so far there is a separate gravitino and vector multiplet. The vector multiplet arises in section 2 in the massless limit of the Proca theory. For completeness we give these two multiplets below.

**Gravitino multiplet.** The off-shell gravitino multiplet consists of a gravitino \( \chi_\mu \), an auxiliary vector \( A_\mu \) and an auxiliary scalar \( D \). The off-shell supersymmetry rules are given by
\[
\begin{align*}
\delta \chi_\mu &= \frac{1}{4} \gamma^\mu \gamma^\rho \epsilon \chi_\rho + \frac{1}{2} \gamma^\mu \epsilon D, \\
\delta A_\mu &= \frac{1}{2} \bar{\epsilon} \gamma^\rho \sigma \gamma_\mu \chi_\rho \chi_\sigma, \\
\delta D &= \frac{1}{4} \bar{\epsilon} \gamma^\rho \sigma \chi_\rho \chi_\sigma, \\
\end{align*}
\] (A.8)
where
\[
\chi_{\mu \nu} = \frac{1}{2} (\partial_\mu \chi_\nu - \partial_\nu \chi_\mu). \tag{A.9}
\]
These transformation rules leave the following action invariant:
\[
I_{\chi=3/2} = \int d^3x \left\{ -4 \bar{\chi}_\mu \gamma^{\mu \nu \rho} \partial_\nu \chi_\rho - \frac{1}{2} A_\mu A_\mu + 2 D^2 \right\}. \tag{A.10}
\]

**Vector multiplet.** The off-shell vector multiplet consists of a vector \( V_\mu \) and a spinor \( \psi \). The off-shell supersymmetry rules are given by
\[
\begin{align*}
\delta V_\mu &= -\bar{\epsilon} \gamma_\mu \psi, \\
\delta \psi &= \frac{1}{4} \gamma^{\mu \nu \epsilon} F_{\mu \nu},
\end{align*}
\] (A.11)
with \( F_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \). The supersymmetric action for a vector multiplet is given by
\[
I_{\psi=1} = \int d^3x \left\{ - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} - 2 \bar{\psi} \gamma^\mu \partial_\mu \psi \right\}. \tag{A.12}
\]

This finishes our discussion of the massless multiplets in three dimensions.

**Appendix B. Boosting up the derivatives in spin-3/2 FP**

In this appendix we show how the higher-derivative kinetic terms for the gravitino \( \rho_\mu \) can be obtained by boosting up the derivatives in the massive spin-3/2 FP equations in the same way as that has been done for the spin-2 FP equations in the construction of NMG [1] except for one subtlety.

Our starting point is the following fermionic action with two massive gravitini, \( \psi_\mu \) and \( \chi_\mu \), each of which carries only one physical degree of freedom in 3D,
\[
I[\psi, \chi] = \int d^3x \left\{ -4 \bar{\psi}_\mu \gamma^{\mu \nu \rho} \partial_\nu \psi_\rho - 4 \bar{\chi}_\mu \gamma^{\mu \nu \rho} \partial_\nu \chi_\rho + 8m \bar{\psi}_\mu \gamma^{\mu \nu} \chi_\nu \right\}. \tag{B.1}
\]
The equations of motion following from this action are given by
\[
\gamma^{\mu \nu \rho} \partial_\nu \psi_\rho - m \gamma^{\mu \nu} \chi_\nu = 0, \quad \gamma^{\mu \nu \rho} \partial_\nu \chi_\rho - m \gamma^{\mu \nu} \psi_\nu = 0. \tag{B.2}
\]
Note that each one of the equations (B.2) can be used to solve for one gravitino in terms of the other one. However, this solution does not solve the other equation. Therefore, one cannot substitute only one solution back into (B.1) because one would lose information about the differential constraint encoded in the other equation.
After diagonalization
\[ \zeta_1^\mu = \psi_\mu + \chi_\mu, \quad \zeta_2^\mu = \psi_\mu - \chi_\mu, \]  
we obtain the massive FP equations for a helicity +3/2 and −3/2 state:
\[ (\partial + m) \zeta_1^\mu = 0, \quad \gamma^\mu \zeta_1^\mu = 0, \quad \partial_\mu \zeta_1^\mu = 0, \]  
\[ (\partial - m) \zeta_2^\mu = 0, \quad \gamma^\mu \zeta_2^\mu = 0, \quad \partial_\mu \zeta_2^\mu = 0. \]  
(B.3)

(B.4)

(B.5)

To boost up the derivatives in these equations we may proceed in two ways. One option is to boost up the derivatives in each equation separately by solving the corresponding differential constraint. In a second step one should then combine the two higher-derivative equations by a single equation in terms of \( \rho_\mu \) by a so-called ‘soldering’ technique which has also been applied to construct NMG out of two different topologically massive gravities \[19\]. Alternatively, it is more convenient to first combine the two equations into the following equivalent second-order equation which is manifestly parity-invariant:
\[ (\Box - m^2) \zeta_\mu = (\partial \mp m) (\partial \pm m) \zeta_\mu = 0, \quad \gamma^\mu \zeta_\mu = 0, \quad \partial_\mu \zeta_\mu = 0. \]  
(B.6)

Note that the action corresponding to these equations of motion cannot be used in a supersymmetric action since the fermionic kinetic term would have the same number of derivatives as the standard bosonic kinetic term describing a spin-2 state.

We are now ready to perform the procedure of “boosting up the derivatives” in the same way as in the bosonic theory where it leads to the higher-derivative NMG theory. To be specific, we solve the divergenceless condition \( \partial_\mu \zeta_\mu = 0 \) in terms of a new vector–spinor \( \rho_\mu \) as follows:
\[ \zeta_\mu = \mathcal{R}_\mu (\rho) \equiv \epsilon_\mu^{\nu\rho} \partial_\nu \rho_\rho. \]  
(B.7)

Substituting this solution back into the other two equations in (B.6) leads to the higher-derivative equations
\[ (\Box - m^2) \mathcal{R}_\mu (\rho) = 0, \quad \gamma^\mu \mathcal{R}_\mu (\rho) = 0. \]  
(B.8)

These equations of motion are invariant under the gauge symmetry
\[ \delta \rho_\mu = \partial_\mu \eta. \]  
(B.9)

Furthermore, they can be integrated to the following action:
\[ I [\rho] = \int d^3 x \left\{ \bar{\rho} \mathcal{R}_\mu (\rho) - \frac{1}{2m^2} \bar{\rho} [\partial \mathcal{R}_\mu (\rho) + \epsilon_\mu^{\nu\tau} \partial_\nu \mathcal{R}_\tau (\rho)] \right\}. \]  
(B.10)

One can show that the equations of motion following from this action implies the algebraic constraint given in (B.8). The action (B.10) is precisely the fermionic part of the action (4.13) of linearized SNMG.

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