A New Design Paradigm for Secure Full-Duplex Multiuser Systems

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Abstract—We consider a full-duplex (FD) multiuser system where an FD base station (BS) is designed to simultaneously serve both downlink (DL) and uplink (UL) users in the presence of half-duplex eavesdroppers (Eves). The problem is to maximize the minimum (max-min) secrecy rate (SR) among all legitimate users, where the information signals at the FD-BS are accompanied with artificial noise to debilitate the Eves’ channels. To enhance the max-min SR, a major part of the power budget should be allocated to serve the users with poor channel qualities, such as those far from the FD-BS, undermining the SR for other users, and thus compromising the SR per-user. In addition, the main obstacle in designing an FD system is due to the self-interference (SI) and co-channel interference (CCI) among users. We therefore propose an alternative solution, where the FD-BS uses a fraction of the time block to serve near DL users and far UL users, and the remaining fractional time to serve other users. The proposed scheme mitigates the harmful effects of SI, CCI and multiuser interference, and provides system robustness. The SR optimization problem has a highly nonconcave and nonsmooth objective, subject to nonconvex constraints. For the case of perfect channel state information (CSI), we develop a low-complexity path-following algorithm, which involves only a simple convex program of moderate dimensions at each iteration. We show that our path-following algorithm guarantees convergence at least to a local optimum. Then, we extend the path-following algorithm to the cases of partially known Eves’ CSI, where only statistics of CSI for the Eves are known, and worst-case scenario in which Eves can employ a more advanced linear decoder. The merit of our proposed approach is further demonstrated by the additional harvested energy requirements for Eves and by extensive numerical results.

Index Terms—Artificial noise, full-duplex radios, full-duplex self-interference, fractional time allocation, nonconvex programming, transmit beamforming, physical-layer security.

I. INTRODUCTION

The explosive demand for new services and data traffic is constantly on the rise, pushing the new development of signal processing and communication technologies that significantly upgrade an unprecedented performance of wireless communication systems, such as high reliability, massive connectivity and high throughput [1]. It is widely believed that multiple-antenna techniques can offer extra degrees-of-freedom (DoF) to efficiently allocate resources, which helps reduce the bandwidth and/or power while still maintaining the same quality-of-service (QoS) requirements [2]. However, the half-duplex (HD) radio, where downlink (DL) and uplink (UL) transmissions occur orthogonally in either time or frequency, leads to under-utilization of radio resources, and may no longer provide substantial improvements in system performance even if multiple antennas are employed.

By enabling simultaneous transmission and reception on the same channel, full-duplex (FD) radio, which offers considerable potential of doubling the spectral efficiency compared to its HD counterpart, has arisen as a promising technology for the fifth generation of mobile communications (5G) [3]. The major challenge in designing an FD radio is to suppress the self-interference (SI) caused by signal leakage from the DL transmission to the UL reception on the same device to a potentially suitable level, such as a few dB above background noise. Fortunately, recent advances in hardware design have allowed the FD radio to be implemented at a reasonable cost while canceling out most of the SI [4]–[7]. Since then, applying FD radio to a base station (BS) in small cell-based systems or to an access point in Wi-Fi, in which the transmit power is relatively low, has been widely considered. FD for multiple-input multiple-output (MIMO) in single-cell systems has been investigated to achieve a higher spectral efficiency [8]–[10], and an extension to multi-cell scenarios has also been considered in [11]–[13]. Another downside of FD radio in a typical cellular network is that co-channel interference (CCI) caused by the UL transmission of UL users severely impairs the DL reception of DL users. Therefore, it is challenging to fully capitalize on the benefits that FD radios may bring to 5G wireless networks. Wireless networks have a very wide range of applications, and an unprecedented amount of personal and sensitive information is transmitted over wireless channels. Consequently, wireless network security is a crucial issue due to the unalterable open nature of the wireless medium. Physical-layer (PHY)-layer security can potentially provide information privacy at the PHY-layer by taking advantage of the characteristics of the wireless medium [14]–[25]. An effective means to delivering PHY-layer security is to adopt artificial noise (AN) to degrade the decoding capability of the eavesdropper (Eve) [15]–[17], such that the confidential messages are useless for Eve. Notably, with FD radio, we can exploit AN even more effectively [14].

A. Related Work

Zheng et al. [18] proposed a self-protection scheme by exploiting FD radio at the desired user to simultaneously receive information and transmit AN in point-to-point transmission. The secrecy rate (SR) optimization was studied in both cases of known and unknown channel state information (CSI) of the Eve. The work in [19] considered a MIMO Gaussian wiretap channel with an FD jamming receiver to secure the DoF. The authors in [20] extended the prior work to a multiuser scenario, where the transmitter is equipped with multiple antennas to
guarantee the individual SR for multiple users. However, these approaches require high hardware complexity of the receivers, which may be difficult to achieve in practice because the end devices should be of very low complexity.

By shifting the computational hardware complexity from the receivers to the BS in cellular networks, FD-BS secure communications are of great interest thanks to providing communication secrecy to both UL and DL transmissions. In [21], joint information and AN beamforming at the FD-BS was investigated to guarantee PHY-layer security of a single-antenna UL and DL users. However, this work assumed that there is no SI and CCI, which is highly idealistic. Therefore, an extension was proposed in [22] by considering both SI and CCI. The work in [23] analyzed a trade-off between the DL and UL transmit power in FD systems to secure multiple DL and UL users. The common technique used in these works is the semi-definite relaxation (SDR) that relaxes nonconvex constraints to arrive at a semi-definite program (SDP). Though in-depth results were presented, the beamforming vectors that are recovered from the covariance matrices of SDR may perform poorly, and the dimension of covariance matrices is relatively large [26].

Another means of using AN is to deliver energy to users (potential eavesdroppers). This implies that the AN can play a dual role: one to jam Eves to guarantee secure communications and another to provide the radio-frequency (RF) energy for the users (may include Eves) to harvest [24], [25]. More recently, the beamformer/precoder design at the FD-BS has been used to focus the information signal and/or RF energy at the desired users [27], [28]. To the best of our knowledge, secrecy rate maximization under harvested energy requirements in FD systems is still an open problem.

B. Motivation and Contributions

Even with recent advances in hardware design for SI cancellation techniques, the harmful effect of SI cannot be neglected if it is not properly controlled, and is proportional to the DL transmission power. The CCI may become strong and uncontrolled whenever a UL user is located near DL users. These shortcomings limit the performance of FD systems [8]–[13], [23]. In addition, a major part of the FD-BS power budget allocated to the DL users with poor channel conditions to improve their QoS will significantly reduce the QoS for other users due to an increase in both the SI and multiuser interference (MUI). Recently, non-orthogonal multiple access (NOMA) [29], [30] has been introduced to improve the far users’ (i.e., the cell-edge users) throughput by allowing near users (i.e., the cell-center users) to access and decode their intended signals. In other words, far users must sacrifice their own information privacy in NOMA [29]–[33]. In all aforementioned work in FD security, the number of transmit antennas is usually required to be larger than the number of users to efficiently manage the network interference. Otherwise, the AN and MUI will impair the channel quality of the desired users, leading to a significant loss in system performance.

In this paper, we propose a new transmission design to further resolve the practical restrictions given above. Specifically, the near DL users and far UL users are served in a fraction of the time block, and then FD-BS uses the remaining fractional time to serve far DL users and near UL users. It is worth noting that the effects of SI, CCI and MUI are clearly reduced while the information privacy for far DL users is preserved (all DL users are allowed to access and decode their own information only). On the other hand, FD-BS can effectively perform transmit beamforming even if the number of DL users exceeds the number of transmit antennas because the number of users that are served at the same time is effectively reduced. There are multiple-antenna Eves that overhear the information signals from both DL and UL channels. We are concerned with the problem of jointly optimizing linear precoders/beamformers at the FD-BS, UL transmit power allocation and fractional time (FT) to maximize the minimum SR among all legitimate users subject to power constraints. In general, such a design problem involves optimization of nonconcave and nonsmooth objective functions subject to nonconvex constraints, for which the optimal solution is computationally difficult to find. Note that SDR cannot be directly applied to such a challenging problem since the problem resulting from the SDR is still highly nonconvex. The main contributions of this paper are summarized as follows:

1) We propose a new transmission model for FD security to optimize simultaneous DL and UL information privacy by exploiting user grouping-based FT model, which helps manage the network interference more effectively than aiming to focus the interference at Eves.

2) We first assume perfect CSI to realize the potential benefits of our new model, for which a path-following computational procedure is developed. The core idea behind our approach is to develop a new inner approximation of the nonconvex problem, which guarantees convergence at least to local optima. The convex program solved at each iteration is of moderate size since it does not require rank-constrained optimization, and thus, is very computationally efficient.

3) When only the statistics of CSI (SCSI) for Eves are known, we reformulate the optimization problem by replacing a nonconvex probabilistic constraint with a tractable nonconvex constraint which can be further shaped to have a set of convex constraints.

4) We determine the optimal solution for a worst-case scenario (WCS) of secure communications where Eves adopt a more advanced linear decoder to cancel all multiuser interference. Further, a scenario extension, where Eves are also energy receivers, is considered to provide certain wireless power transfers to Eves.

5) Extensive numerical results show that the proposed algorithms converge quite quickly in a few iterations and greatly improve the SR performance over existing schemes, i.e., HD, conventional FD and FD-NOMA. It also confirms the robustness of the proposed approach against the significant effects of SI and DoF bottleneck.

C. Paper Organization and Notation

The rest of the paper is organized as follows. The system model and problem statement are given in Section II.
Path-following algorithms based on a convex approximation for the SR maximization (SRM) problem with known CSI and statistical CSI of Eves are developed in Section III and Section IV, respectively. Section V is devoted to the computation for the SRM-WCS problem, and the extension to the SRM with energy receivers of Eves is considered in Section VI. Numerical results are illustrated in Section VII and Section VIII concludes the paper.

**Notation:** Lower-case letters, bold lower-case letters and bold upper-case letters represent scalars, vectors and matrices, respectively. $X^H$, $X^T$ and $X^\ast$ are the Hermitian transpose, normal transpose and conjugate of a matrix $X$, respectively. The trace of a matrix $X$ is denoted by $\text{Tr}(X)$. $\| \cdot \|$ and $| \cdot |$ denote a matrix’s Frobenius norm, a vector’s Euclidean norm and absolute value of a complex scalar, respectively. $\mathbb{I}_N$ represents an $N \times N$ identity matrix. $x \sim \mathcal{CN}(\eta, Z)$ means that $x$ is a random vector following a complex circularly symmetric Gaussian distribution with mean $\eta$ and covariance matrix $Z$. $\mathbb{E}\{\cdot\}$ denotes the statistical expectation. The notation $X \succeq 0$ ($X \preceq 0$) means that matrix $X$ is positive (negative) semi-definite (definite). $\mathbb{R}\{\cdot\}$ represents the real part of the argument. $\nabla_x f(x)$ is the gradient of $f(x)$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Signal Processing Model

Consider a multiuser communication system illustrated in Fig. 1 where the FD-BS is equipped with $N_t$ transmit antennas and $N_r$ receive antennas to simultaneously serve $2K$ DL users and $2L$ UL users, respectively, over the same RF band. Each legitimate user is equipped with a single antenna and operates in the HD mode to ensure low hardware complexity. Both DL and UL transmissions are overheard by $M$ non-colluding Eves, where Eve $m$ has $N_{e,m}$ antennas. Herein, we use the most natural and efficient divisions of the coverage area [30], [31]. In particular, users are randomly placed into two zones, such that there are $K$ DL users and $L$ UL users located in a zone nearer the FD-BS (referred to as zone-1 of near users), and $K$ DL users and $L$ UL users are located in a zone farther from the FD-BS (called zone-2 of far users) [30].

![Fig. 1. A multiuser system model with an FD-BS serving $2K$ DL users and $2L$ UL users in the presence of $M$ Eves.](image)

Note that our proposed algorithms can be further adjusted to the case of different numbers of users located in each zone.

In this paper, we split each communication time block, denoted by $T$, into two sub-time blocks orthogonally, as shown in Fig. 2. As previously mentioned, in order to mitigate the harmful effects of SI and CCI, far DL users (near DL users) and far UL users (near UL users) should be scheduled in a different time slot. The FD-BS serves the DL users (UL users) with similar channel conditions, which helps reduce the MUI. As a consequence, $K$ near DL users and $L$ far UL users are grouped into group-1 and are served in the first duration $\tau T$ ($0 < \tau < 1$), while $K$ far DL users and $L$ near UL users are grouped into group-2 and are served in the remaining duration $(1 - \tau)T$. Although each group still operates in the FD mode, the inter-group interference, i.e., interference across groups 1 and 2, is perfectly eliminated thanks to the FT allocation. This is in contrast with the conventional FD systems which simultaneously serve all UL and DL users. Without loss of generality, the communication time block $T$ is normalized to 1. Upon denoting $\mathcal{K} \triangleq \{1, 2, \cdots, K\}$ and $\mathcal{L} \triangleq \{1, 2, \cdots, L\}$, the sets of DL and UL users are $\mathcal{D} \triangleq \mathcal{I} \times \mathcal{K}$ and $\mathcal{U} \triangleq \mathcal{I} \times \mathcal{L}$ for $\mathcal{I} \triangleq \{1, 2\}$, respectively. Thus, the $k$-th DL user and the $\ell$-th UL user in the $i$-th group are referred to as DL user $(i,k) \in \mathcal{D}$ and UL user $(i,\ell) \in \mathcal{U}$, respectively.

1) Received Signal Model at the FD-BS and DL Users: We consider that the FD-BS deploys a transmit beamformer $w_{i,k} \in \mathbb{C}^{N_{t} \times 1}$ to transfer the information bearing signal $x_{i,k}$ with $\mathbb{E}\{|x_{i,k}|^2\} = 1$ to DL user $(i,k)$. Since all UL users have only a single antenna, they are unable to generate AN for jamming. To guarantee secure communication in both DL and UL channels, FD-BS also injects AN signals to interfere with the reception of the Eves. Hence, the DL transmit signals at the FD-BS intended for $K$ DL users in zone-$i$ can be expressed as

$$x_{i} = \sum_{k=1}^{K} w_{i,k} x_{i,k} + v_{i}, \forall i \in \mathcal{I} \tag{1}$$

where $v_{i} \in \mathbb{C}^{N_{t} \times 1}$ is the AN vector whose elements are zero-mean complex Gaussian random variables, i.e., $v_{i} \sim \mathcal{CN}(0, V_{i}^{H})$ with $V_{i} \in \mathbb{C}^{N_{t} \times N_{t}}$. All channels are assumed to follow frequency-flat fading, which accounts for the effects of both large-scale path loss and small-scale fading. The received signal at DL user $(i,k)$ can be expressed as

$$y_{i,k} = h_{i,k}^{H} w_{i,k} x_{i,k} + \sum_{j=1,j \neq k}^{K} h_{i,k}^{H} w_{i,j} x_{i,j} + h_{i,k}^{H} v_{i} + \sum_{\ell=1}^{L} f_{i,k,\ell} \rho_{i,\ell} \tilde{x}_{i,\ell} + n_{i,k} \tag{2}$$

Fig. 2. Time slot structure to serve $K$ near DL users and $L$ far UL users within the duration $\tau T$, as well as $K$ far DL users and $L$ near UL users in the remaining duration $(1 - \tau)T$. 

\[ \text{FD-BS} \rightarrow (1) \text{K DL users in Zone-1} \]

\[ \text{FD-BS} \rightarrow (2) \text{K DL users in Zone-2} \]

\[ \text{DL users in Zone-2} \rightarrow (1) \text{FD-BS} \]

\[ \text{UL users in Zone-1} \rightarrow (2) \text{FD-BS} \]
where $h_{i,k} \in \mathbb{C}^{N_i \times 1}$ is the transmit channel vector from the FD-BS to DL user $(i,k)$. In (3), the term $\sum_{\ell=1}^{L} f_{i,k,\ell} \rho_{i,\ell} \bar{x}_{i,\ell}$ represents the CCI from $L$ UL users to DL user $(i,k)$, where $f_{i,k,\ell} \in \mathbb{C}$, $\rho_{i,\ell}$ and $\bar{x}_{i,\ell}$ with $\mathbb{E}\{||\bar{x}_{i,\ell}||^2\} = 1$ are the complex channel coefficient from UL user $(i,\ell)$ to DL user $(i,k)$, transmit power and message of UL user $(i,\ell)$, respectively. $n_{i,\ell} \sim \mathcal{C}\mathcal{N}(0,\sigma^2)$ denotes the additive white Gaussian noise (AWGN) at DL user $(i,k)$. By defining $\tau_1 := \tau$ and $\tau_2 := 1 - \tau$, the information rate decoded by DL user $(i,k)$ in nat/sec/Hz is given by (3):

$$C_{i,k}^0(x_i,\tau_i) = \tau_i \ln\left(1 + \frac{||h_{i,k}^H w_{i,k}||^2}{\sigma^2}ight)$$

where $x_i = \{w_{i,k},\rho_{i,k}\}$, with $w_{i,k} \in \mathbb{C}^{N_i}$ and $\rho_{i,k} \in \mathbb{C}$, are the matrix encompassing the beamformers/precoders, $L$, are the matrix encompassing the beamformers/precoders in the UL and DL transmit power allocation of all users in the UL in the $i$-th group, and

$$\varphi_{i,k}(X_i) \triangleq \sum_{j=1}^{K} ||h_{i,k}^H w_{i,j}||^2 + ||h_{i,k}^H v_i||^2 + \sum_{\ell=1}^{L} \rho_{i,\ell}^2 ||f_{i,k,\ell}||^2 + \sigma^2.$$ 

The received signal at the FD-BS for reception of $L$ UL users in the $i$-th group can be expressed as

$$y_{i,bs} = \sum_{\ell=1}^{L} \rho_{i,\ell} \bar{g}_{i,\ell} \bar{x}_{i,\ell} + \sigma \sqrt{\sigma^2 \sum_{k=1}^{K} G_{i,1}^H w_{i,k} \bar{x}_{i,k}} + \sqrt{\sigma^2} \sigma G_{i,1}^H v_i + n_{i,bs}$$

where $g_{i,\ell} \in \mathbb{C}^{N_i \times 1}$ is the receive channel vector from UL user $(i,\ell)$ to the FD-BS. The term $\sqrt{\sigma^2} \sum_{k=1}^{K} G_{i,1}^H w_{i,k} \bar{x}_{i,k}$ $\in (4)$ represents the residual SI after all real-time cancellation in analog and digital domains [7]: $G_{i,1} \in \mathbb{C}^{N_i \times N_i}$ denotes a fading loop channel which impairs the UL signal detection at the FD-BS due to the concurrent DL transmission and $0 \leq \sigma^2 < 1$ is used to model the degree of residual SI propagation [4]. $n_{i,bs} \sim \mathcal{C}\mathcal{N}(0,\sigma^2 I_{N_i})$ denotes the AWGN at the FD-BS. To maximize the information rates of UL users, we adopt the minimum mean square error and successive interference cancellation (MMSE-SIC) decoder at the FD-BS [4]. For $L$ UL users in each group, we assume that the decoding order follows the UL users’ index, i.e., $\ell = 1, 2, \cdots, L$, with the Foschini ordering. In other words, the strongest signal is decoded first, while weakest signal is decoded last to support the most vulnerable UL users. Hence, the information rate in decoding UL user $(i,\ell)$’s message is given by (27):

$$C_{i,\ell}^0(x_i,\tau_i) = \tau_i \ln\left(1 + \rho_{i,\ell}^2 G_{i,\ell}^H \Phi_{i,\ell}(X_i)^{-1} g_{i,\ell}\right)$$

where

$$\Phi_{i,\ell}(X_i) \triangleq \sum_{j \neq \ell}^{K} \rho_{i,j} g_{i,j} g_{i,\ell} + \sigma \sqrt{\sigma^2 \sum_{k=1}^{K} G_{i,1}^H w_{i,k} w_{i,k}^H G_{i,1}} + \sigma G_{i,1}^H v_i v_i^H G_{i,1} + \sigma^2 I_{N_i}.$$ 

2) Received Signal Model at Eves: After performing hand-shaking with the FD-BS, we assume that the Eves are also aware of the FT $\tau_i$. The information signals of group-$i$ leaked out to the $m$-th Eve during the FT $\tau_i$ can be expressed as:

$$y_{i,m} = H_{i,m}^H \left(\sum_{k=1}^{K} w_{i,k} x_{i,k} + v_i\right) + \sum_{\ell=1}^{L} \rho_{i,\ell} g_{m,i,\ell}^H \bar{x}_{i,\ell} + n_{e,m}$$

where $H_{i,m} \in \mathbb{C}^{N_i \times N_{e,m}}$ and $g_{m,i,\ell} \in \mathbb{C}^{1 \times N_{e,m}}$ are the wiretap channel matrix and vector from the FD-BS and UL user $(i,\ell)$ to the $m$-th Eve, respectively. $n_{e,m} \sim \mathcal{C}\mathcal{N}(0,\sigma^2 I_{N_{e,m}})$ denotes the AWGN at Eve $m$. The worst-case information (WCI) rates at the $m$-th Eve, corresponding to the signals targeted for DL user $(i,k)$ and UL user $(i,\ell)$, are given by

$$C_{i,m,k}^{\text{RD}}(x_i,\tau_i) = \tau_i \ln\left(1 + \frac{||H_{i,m,k}^H w_{i,k}||^2}{\psi_{m,i,k}(X_i)}\right),$$

$$C_{m,i,\ell}^{\text{RD}}(X_i,\tau_i) = \tau_i \ln\left(1 + \frac{\rho_{i,\ell}^2 ||g_{m,i,\ell}^H||^2}{{\chi}_{m,i,\ell}(X_i)}\right)$$

respectively, where

$$\psi_{m,i,k}(X_i) \triangleq \sum_{j=1, \neq m}^{K} ||H_{i,m,k}^H w_{i,j}||^2 + ||H_{m}^H V_i||^2_F$$

$$\chi_{m,i,\ell}(X_i) \triangleq \sum_{k=1}^{K} ||H_{i,m,\ell}^H w_{i,k}||^2 + ||H_{m}^H V_i||^2_F$$

The channel of each legitimate user together with $M$ Eves form a compound wiretap channel for which the SR expressions of DL user $(i,k)$ and UL user $(i,\ell)$ can be expressed as (23), (35):

$$R_{i,k}^0(x_i,\tau_i) = \frac{C_{i,k}^0(x_i,\tau_i) - \max_{m \in M} C_{i,m,k}^{\text{RD}}(x_i,\tau_i)}{\max \{0,\tau_i\}},$$

$$R_{i,\ell}^0(x_i,\tau_i) = \frac{C_{i,\ell}^0(x_i,\tau_i) - \max_{m \in M} C_{m,i,\ell}^{\text{RD}}(x_i,\tau_i)}{\max \{0,\tau_i\}}$$

respectively, where $M = \{1,2,\cdots, M\}$ and $[x]^+ \triangleq \max\{0,x\}$.

We aim to jointly optimize the transmit information vectors, AN matrices $\{X_i\}_{i \in T}$ and the FT $\{\tau_i\}_{i \in T}$ to maximize the minimum (max-min) SR among all legitimate users. The SRM problem with Eves’ WCI rate, referred to as SRM-EWCI for short, can be mathematically formulated as

$$\text{maximize } \min_{\sum_{(i,\ell) \in D}} \{R_{i,k}^0(x_i,\tau_i), R_{i,\ell}^0(x_i,\tau_i)\}$$

$$\text{s.t. } \sum_{i=1}^{K} \tau_i \left(\sum_{k=1}^{K} ||w_{i,k}||^2 + ||V_i||^2_F\right) \leq P^\text{max}_{bs}$$

$$\tau_1 \rho_{1,\ell}^2 \leq P^\text{max}_{1,\ell}, \forall \ell \in L,$$

$$\tau_2 \rho_{2,\ell}^2 \leq P^\text{max}_{2,\ell}, \forall \ell \in L,$$

$$\rho_{i,\ell} \geq 0, \rho_{i,\ell} \geq 0, \forall \ell \in L,$$

$$\tau_1 > 0, \tau_2 > 0, \tau_1 + \tau_2 \leq 1,$$

where $X = \{X_1, X_2\}$ and $\tau = \{\tau_1, \tau_2\}$. Constraint 98 means that the total transmit power at the FD-BS, which

Note that if the Eves are not aware of the FT $\tau_i$, the received signals at Eve $m$ in (4) will include the inter-group interference, which leads to a lower bound on the information rate of the Eves. Such a design would be unfair to the Eves, and therefore, we do not pursue this here.
is allocated across different time fractions, does not exceed the power budget, $P_{bs}^{\text{max}}$, while constraints (9c) and (9d) are individual transmit power at UL user $(i, \ell)$ in its service time, with $P_{i,\ell}^{\text{max}}$ being the power budget (see e.g., [10], [27], [36] for these realistic power constraints). Finding an optimal solution to the SRM-EWC problem (8) is challenging because the objective (9a) is nonconcave and constraints (9b)-(9d) are also nonconvex due to coupling between $X$ and $\tau$.

Remark 1: In a practical scenario, the DL and UL traffic demands in current generation wireless networks are typically asymmetric. Another optimization problem of interest is to maximize the minimum SR of DL users subject to the SR constraints of UL users as follows:

\[
\begin{align*}
\text{maximize} & \quad \min_{(i,\ell) \in D} \left\{ R_i^{(U)}(X_i, \tau_i) \right\} \\
\text{s.t.} & \quad (9c), (9d), (10b), (10c)
\end{align*}
\]

where the QoS constraints in (10c) set a minimum SR requirement $R_i^{(U)}$ at UL user $(i, \ell)$. It should be emphasized that the systematic approach in this paper is expected to be applicable for such a problem (this will be elaborated in Section VII).

III. PROPOSED METHOD WITH KNOWN CSI

In this section, the CSI of the users (including Eves) is assumed to be perfectly known at the transmitters. Channel reciprocity of UL and DL channels in time duplex division (TDD) mode can be adopted for small cell systems as those considered in this paper. The channels for all users (with a low degree of mobility) can be acquired at the FD-BS by requesting them to send pilot signals to the FD-BS, and thus these estimated channels can be assumed to be perfectly available [8], [27]. Likewise, the CSI between a UL user and the receivers (DL users and Eves) can be estimated through TDD since any transmitted signal includes short-training and long-training sequences (e.g., a part of preamble). This way, any UL user can overhear and estimate the channels from DL users and Eves, and then these estimated channels can be acquired at the FD-BS by polling each UL user. After CSI acquisition, we assume that only 2K DL users and 2L UL users are scheduled to be simultaneously served as in IEEE 802.11ac. Herein, $M$ unscheduled users are not necessarily malicious, but are not trusted users. Thus, $M$ unscheduled users are treated as eavesdroppers but with perfectly known CSI.

A. Equivalent Transformations for (8)

To solve the max-min SR problem in (8), we present a path-following algorithm under which each iteration invokes only a simple convex program of low computational complexity. Toward a tractable form, several proper transformations need to be invoked. Let us start by introducing an additional variable $\eta$ to equivalently express (8) as

\[
\begin{align*}
\text{maximize} & \quad \eta \\
\text{s.t.} & \quad (8c), (8d), (8e), (8f), (9a) \\
& \quad R_i^{(U)}(X_i, \tau_i) \geq \eta, \forall (i, k) \in D, \\
& \quad R_i^{(D)}(X_i, \tau_i) \geq \eta, \forall (i, \ell) \in U.
\end{align*}
\]

Note that the equivalence between (9) and (11) can be readily verified by checking that constraints (11c)-(11d) hold with equality at optimum. We now provide a sketch of the proof to verify (11c), and other constraints follow immediately. Suppose that $R_i^{(U)}(X_i, \tau_i) > \eta$ for some $(i, k)$. Then, there may exist a positive constant $\Delta \eta > 0$ to satisfy $R_i^{(U)}(X_i, \tau_i) = \eta + \Delta \eta$. As a consequence, $\eta + \Delta \eta$ is also feasible for (11) but yielding a strictly larger objective, and thus, this is a contradiction with the optimality assumption. By observing that the objective function (11a) is monotonic in its argument, we further rewrite (11) as follows:

\[
\begin{align*}
\text{maximize} & \quad \eta \\
\text{s.t.} & \quad (9c), (9d), (10b), (10c), (12f) \\
& \quad C_i^{(D)}(X_i, \tau_i) - \Gamma_i^{(D)} \geq \eta, \forall (i, k) \in D, \\
& \quad C_i^{(U)}(X_i, \tau_i) \geq \eta, \forall (i, \ell) \in U, \\
& \quad C_i^{(m)}(X_i, \tau_i) \geq \eta, \forall m \in M, (i, k) \in D, \\
& \quad C_i^{(m)}(X_i, \tau_i) \geq \eta, \forall m \in M, (i, \ell) \in U.
\end{align*}
\]

where $\Gamma \triangleq \{ \Gamma_i^{(D)}, \Gamma_i^{(U)} \}_{i \in I, k \in K_i, \ell \in F}$ are newly introduced variables to tackle the maximum allowable rate of the Eves [24]. However, problem (12) still remains intractable.

To this end, we make the variable change:

\[
\tau_1 = \frac{1}{\alpha_1} \quad \text{and} \quad \tau_2 = \frac{1}{\alpha_2}
\]

(13) to equivalently rewrite (9) by the following convex constraint

\[
\frac{1}{\alpha_1} + \frac{1}{\alpha_2} \leq 1, \forall \alpha_i > 1, i \in I
\]

(14) where $\alpha \triangleq \{ \alpha_1, \alpha_2 \}$ are new variables. Using (13), constraints (12c) and (12d) become

\[
\begin{align*}
C_i^{(D)}(X_i, \alpha_i) & \triangleq \ln \left( \frac{1 + ||g_{i,\ell}^{(U)}\Phi_i^{(U)}(X_{i,\ell})^{-1}g_{i,\ell}||^2}{\alpha_i} \right) \geq \eta + \Gamma_i^{(D)} \geq \eta + \Gamma_i^{(D)} \geq \eta + \Gamma_i^{(D)}, \forall i, \ell.
\end{align*}
\]

(15a)

(15b)

Analogously, constraints (12d) and (12e) become

\[
\begin{align*}
C_i^{(m)}(X_i, \alpha_i) & \triangleq \ln \left( \frac{1 + ||h_{m,i}^{(m)}||^2}{\alpha_i} \right) \leq \Gamma_i^{(m)}, \forall m, i, k.
\end{align*}
\]

(16a)

\[
\begin{align*}
C_i^{(m)}(X_i, \alpha_i) & \triangleq \ln \left( \frac{1 + ||h_{m,i}^{(m)}||^2}{\alpha_i} \right) \leq \Gamma_i^{(m)}, \forall m, i, \ell.
\end{align*}
\]

(16b)

From (15) and (16), and substituting (13) and (14) into (9b-9d), the optimization problem (12) is re-expressed as

\[
\begin{align*}
\text{maximize} & \quad \eta \\
\text{s.t.} & \quad (12c), (12e), (12f) \\
& \quad \left( 1 - \frac{1}{\alpha_1} \right) \left( \sum_{k=1}^K ||w_{1,k}||^2 + ||V_1||^2 \right) \\
& \quad \leq \frac{1}{\alpha_2} \sum_{k=1}^K ||w_{2,k}||^2 + ||V_2||^2 \leq P_{bs}^{\text{max}}, \\
& \quad \left( 1 - \frac{1}{\alpha_1} \right) \rho_i^{(D)} \leq P_{bs}^{\text{max}}, \forall \ell \in L, \\
& \quad \frac{1}{\alpha_2} \rho_i^{(D)} \leq P_{bs}^{\text{max}}, \forall \ell \in L.
\end{align*}
\]

(17a)

(17b)

(17c)

(17d)

(17e)
Notice that $1/\alpha_1 + 1/\alpha_2 = 1$ hold at optimum, which means that the optimization problem (17) is equivalent to (12), and the optimal solution for $\tau$ is recovered by (13).

B. Proposed Convex Approximation-Based Path-Following Method

We are now in a position to approximate the equivalent formulation in (17). Note that except for (23), (14) and (17), the rest of the constraints are nonconvex. The proposed algorithm is mainly based on an inner approximation framework [37] under which the nonconvex parts are completely exposed.

Approximation of Constraints [15]. To develop a convex approximation, we first introduce the following approximation of function $\zeta(\gamma, t) \triangleq \ln(1+\gamma/t)$ at a feasible point $(\gamma^{(\kappa)}, t^{(\kappa)})$:

$$
\zeta(\gamma, t) \geq \mathcal{A}^{(\kappa)} - B^{(\kappa)} - C^{(\kappa)} t, \\
\forall \gamma > 0, \gamma^{(\kappa)}, t > 0, t^{(\kappa)} > 0
$$

where

$$
\mathcal{A}^{(\kappa)} \triangleq 2\zeta(\gamma^{(\kappa)}, t^{(\kappa)}) + \frac{\gamma^{(\kappa)}}{t^{(\kappa)}(\gamma^{(\kappa)} + 1)}, \\
B^{(\kappa)} \triangleq \frac{(\gamma^{(\kappa)})^2}{\gamma^{(\kappa)} + 1}, \\
C^{(\kappa)} \triangleq \frac{\zeta(\gamma^{(\kappa)}, t^{(\kappa)})}{t^{(\kappa)} + 1}.
$$

The proof of (18) is given in Appendix A. In the spirit of [38], for $\bar{w}_{i,k} = -e^{-i\pi}h_{i,k}w_{i,k}$ with $j = \sqrt{-1}$, it follows that $|h_{i,k}^H w_{i,k}| = h_{i,k}^H \bar{w}_{i,k} = \Re\{h_{i,k}^H w_{i,k}\}$ and $|h_{i,k}^H w_{i,k}| = |h_{i,k}^H \bar{w}_{i,k}|$ for all $(i, k') \neq (i, k)$. Thus, $\gamma^{(\kappa)}(X_{i,k}) \triangleq |h_{i,k}^H w_{i,k}|/\varphi_{i,k}(X_{i,k})$ can be equivalently replaced by

$$
\gamma^{(\kappa)}(X_{i,k}) = \left(\Re\{h_{i,k}^H w_{i,k}\}\right)^2/\varphi_{i,k}(X_{i,k}), \quad \forall (i, k) \in \mathcal{D}
$$

with the condition

$$
\Re\{h_{i,k}^H w_{i,k}\} \geq 0, \quad \forall (i, k) \in \mathcal{D}.
$$

By using (18), at a feasible point $(X_{i,k}^{(\kappa)}, \alpha_{i,k}^{(\kappa)})$ found at the $(\kappa-1)$-th iteration $C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k})$ is lower bounded by

$$
\ln(1 + \gamma^{(\kappa)}_{i,k}(X_{i,k})) \geq A_{i,k}^{(\kappa)} - B_{i,k}^{(\kappa)} - C_{i,k}^{(\kappa)} \varphi_{i,k}(X_{i,k}) - C_{i,k}^{(\kappa)} \alpha_{i,k}
$$

where

$$
A_{i,k}^{(\kappa)} \triangleq 2\ln(1 + \gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)})) + \frac{\gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)})}{\alpha_{i,k}^{(\kappa)}(\gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)}) + 1)}, \\
B_{i,k}^{(\kappa)} \triangleq \frac{(\gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)}))^2}{\alpha_{i,k}^{(\kappa)}(\gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)}) + 1)}, \\
C_{i,k}^{(\kappa)} \triangleq \frac{\ln(1 + \gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)}))}{\alpha_{i,k}^{(\kappa)}(\gamma^{(\kappa)}_{i,k}(X_{i,k}^{(\kappa)}) + 1)}.
$$

We make use of the following inequality

$$
\|x\|^2 \geq 2\Re\{\alpha^{(\kappa)}Hx\} - \|x^{(\kappa)}\|^2
$$

with $\forall x \in \mathbb{C}^N, x^{(\kappa)} \in \mathbb{C}^N$. Due to the convexity of the function $|x|^2$ to further expose the hidden convexity of the right-hand side (RHS) of (23) as

$$
\ln(1 + \gamma^{(\kappa)}_{i,k}(X_{i,k})) \geq A_{i,k}^{(\kappa)} - B_{i,k}^{(\kappa)} - C_{i,k}^{(\kappa)} \varphi_{i,k}(X_{i,k}) - C_{i,k}^{(\kappa)} \alpha_{i,k}
$$

over the trust region

$$
2\Re\{h_{i,k}^H w_{i,k}\} - \Re\{h_{i,k}^H w_{i,k}\} > 0, \quad \forall (i, k) \in \mathcal{D}
$$

where

$$
\Psi_{i,k}^{(\kappa)}(w_{i,k}) \triangleq \Re\{h_{i,k}^H w_{i,k}\} \left(2\Re\{h_{i,k}^H w_{i,k}\} - \Re\{h_{i,k}^H w_{i,k}\}\right).
$$

Note that $C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k})$ is a lower bounding concave function of $C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k})$, which also satisfies

$$
C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k}) = \frac{1}{\alpha_{i,k}}\ln(1 + \gamma_{i,k}^{(\kappa)}(X_{i,k}))
$$

As a result, (16a) can be iteratively replaced by the following inequality:

$$
C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k}) \geq \eta + \Psi_{i,k}^{(\kappa)}(w_{i,k}), \quad \forall (i, k) \in \mathcal{D}.
$$

By defining $\gamma_{i,k}^{(\kappa)}(X_{i,k}) \triangleq \rho_{i,k}^2 g_{i,k}^H \Phi_{i,k}(X_{i,k})^{-1} g_{i,k}$, the left-hand side (LHS) of (16b) is lower bounded at the feasible point $(X_{i,k}, \alpha_{i,k})$ as

$$
\ln(1 + \gamma_{i,k}^{(\kappa)}(X_{i,k})) \geq \hat{A}_{i,k}^{(\kappa)} + \hat{B}_{i,k}^{(\kappa)} \rho_{i,k} - \frac{\phi_{i,k}^{(\kappa)}(X_{i,k})}{\alpha_{i,k}} - \hat{C}_{i,k}^{(\kappa)} \alpha_{i,k}
$$

where

$$
\hat{A}_{i,k}^{(\kappa)} \triangleq 2\ln(1 + \gamma_{i,k}^{(\kappa)}(X_{i,k})) - \gamma_{i,k}^{(\kappa)}(X_{i,k}) \frac{\phi_{i,k}^{(\kappa)}(X_{i,k})}{\alpha_{i,k}}, \\
\hat{B}_{i,k}^{(\kappa)} \triangleq \frac{2\gamma_{i,k}^{(\kappa)}(X_{i,k})}{\rho_{i,k} \alpha_{i,k}}, \\
\hat{C}_{i,k}^{(\kappa)} \triangleq \frac{\ln(1 + \gamma_{i,k}^{(\kappa)}(X_{i,k}))}{\alpha_{i,k}^2}.
$$

It follows from (29) that $C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k})$ is a concave function, which agrees with $C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k})$ as

$$
C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k}) = \frac{1}{\alpha_{i,k}}\ln(1 + \gamma_{i,k}^{(\kappa)}(X_{i,k})).
$$

Thus, constraint (16b) can be iteratively replaced by

$$
C_{i,k}^{(\kappa)}(X_{i,k}, \alpha_{i,k}) \geq \eta + \Psi_{i,k}^{(\kappa)}(w_{i,k}), \quad \forall (i, k) \in \mathcal{U}.
$$

Approximation of Constraints [16]: Notice that the LHSs of (16) are neither convex nor concave with respect to (w.r.t.) $(X_{i,k}, \alpha_{i,k})$. For a given feasible point $x^{(\kappa)}$, the following inequality holds true:

$$
\ln(1 + x) \leq a(x^{(\kappa)}) + b(x^{(\kappa)})x, \quad \forall x^{(\kappa)} \geq 0, x \geq 0
$$

where

$$
a(x^{(\kappa)}) \triangleq \ln(1 + x^{(\kappa)}) - x^{(\kappa)}, \quad b(x^{(\kappa)}) \triangleq \frac{1}{1 + x^{(\kappa)}},
$$

which is a result of the concavity of the function $\ln(1 + x)$. In the sequel, we develop an upper bound of the LHSs of (16). Let us consider (16a) first. For $\gamma_{i,k}^{(\kappa)} \triangleq $
Moreover, at the feasible point \( X^{(c)} \) and using (30), its LHS is upper bounded by

\[
\ln\left(1 + \frac{\|H_m^T w_i\|^2}{\gamma_{m, i, k}(X_i)}\right) \leq \alpha_i \left(\gamma_{m, i, k}(X_i) + \frac{\|H_m^T w_i\|^2}{\gamma_{m, i, k}(X_i)}\right).
\]

(31)

We then introduce new variables \( \mu_{m, i, k} > 0, \forall m, i, k \), to decompose (16a) into a set of the following constraints:

\[
P_{m, i, k}(X_i, \alpha_i, \mu_{m, i, k}) \leq \Gamma_0 \forall m, i, k,
\]

(32a)

\[
\mu_{m, i, k} \leq \alpha_i \psi_{m, i, k}(X_i), \forall m, i, k
\]

(32b)

where (32a) is a convex constraint, and

\[
P_{m, i, k}(X_i, \alpha_i, \mu_{m, i, k}) := \frac{a(\gamma_{m, i, k})}{\alpha_i} + b(\gamma_{m, i, k}) \frac{\|H_m^T w_i\|^2}{\mu_{m, i, k}}.
\]

(33)

For the nonconvex constraint (32b), we can express it in more tractable form as

\[
F_{m, i, k}(X_i, \alpha_i, \mu_{m, i, k}) \leq N_{c, m} \sigma^2
\]

(33)

where

\[
F_{m, i, k}(X_i, \alpha_i, \mu_{m, i, k}) \triangleq \frac{\mu_{m, i, k}}{\alpha_i} - \left[\psi_{m, i, k}(X_i) - N_{c, m} \sigma^2\right].
\]

Let us define \( F_{m, i, k}^{(c)}(X_i, \alpha_i, \mu_{m, i, k}) \) to be the convex approximation of \( F_{m, i, k}(X_i, \alpha_i, \mu_{m, i, k}) \) at \( X_i^{(c)}, \alpha_i^{(c)}, \mu_{m, i, k}^{(c)} \), of which the derivation is given in Appendix B. As a consequence, the convex approximation of (33) reads as

\[
F_{m, i, k}^{(c)}(X_i, \alpha_i, \mu_{m, i, k}) \leq N_{c, m} \sigma^2, \forall m, i, k.
\]

(34)

Moreover, at the feasible point \( (X_i^{(c)}, \alpha_i^{(c)}, \mu_{m, i, k}^{(c)}) \), it is also true that

\[
C_{m, i, k}^{(c)}(X_i^{(c)}, \alpha_i^{(c)}), \mu_{m, i, k}^{(c)}) = \ln\left(1 + \frac{\gamma_{m, i, k}(X_i)}{\alpha_i}\right)
\]

(35)

since the inequality (32b) must hold with equality at optimum.

Similarly, constraint (16b) is iteratively approximated by

\[
P_{m, i, \ell}^{(c)}(X_i, \alpha_i, \mu_{m, i, \ell}) \leq 1_{\ell, t}, \forall m, i, \ell
\]

(36)

with the additional convex constraint

\[
P_{m, i, \ell}^{(c)}(X_i, \alpha_i, \mu_{m, i, \ell}) \geq 0, \forall m, i, \ell
\]

where \( \mu_{m, i, \ell} > 0, \forall m, i, \ell \), are new variables, and

\[
P_{m, i, \ell}^{(c)}(X_i, \alpha_i, \mu_{m, i, \ell}) := \frac{\alpha_i}{\gamma_{m, i, \ell}(X_i)} + b(\gamma_{m, i, \ell}) \frac{\|H_m^T w_i\|^2}{\mu_{m, i, \ell}}.
\]

(37)

for

\[
\gamma_{m, i, \ell}(X_i) \triangleq \left(\rho_{i, \ell}\right)^2 \|H_m^T w_i\|^2 / \chi_{m, i, \ell}(X_i).
\]

(38)

The function

\[
P_{m, i, \ell}^{(c)}(X_i, \alpha_i, \mu_{m, i, \ell}) \triangleq \frac{\mu_{m, i, \ell}}{\alpha_i} - \left[\chi_{m, i, \ell}(X_i) - N_{c, m} \sigma^2\right],
\]

(39)

of which the derivation is given in Appendix B.

**Inner Approximation of Power Constraints (17a) and (17b):**

Owing to the convexity of functions \( \|x\|^2 / t \), we have

\[
\frac{\|x\|^2}{t} \geq 2R\left(\frac{(x)^H X}{t}\right) - \frac{\|x\|^2}{t}.
\]

(38)

**Algorithm 1: Proposed path-following algorithm to solve SRM-WCD (9)**

**Initialization:** Set \( \kappa := 0 \) and solve (41) to generate an initial feasible point \( (X^{(0)}, \alpha^{(0)}, \mu^{(0)}) \).

1. repeat
2. Solve (40) to obtain the optimal solution \( (X^*, \alpha^*, \mu^*) \).
3. Update \( X^{(k+1)} := X^*, \alpha^{(k+1)} := \alpha^*, \mu^{(k+1)} := \mu^* \).
4. Set \( \kappa := \kappa + 1 \).
5. until Convergence

with \( \forall x \in C^n, x^{(c)} \in C^n, t > 0, t^{(c)} > 0 \). We make use of the inequality (38) to obtain inner convex approximations for nonconvex constraints (17a) and (17b) as

\[
\sum_{k=1}^{K} \|w_{1,k}\|^2 + \|V_1\|_F^2 + \frac{1}{\alpha_2} \left(\sum_{k=1}^{K} \|w_{2,k}\|^2 + \|V_2\|_F^2\right) + \frac{\|V_1\|_F^2}{\alpha_2} - \frac{\sum_{k=1}^{K} \|w_{1,k}\|^2}{\alpha_2} \leq P_{bs}^{\max},
\]

(39a)

\[
\rho_{1, \ell}^2 - 2\rho_{1, \ell} \rho_{1, \ell}^0 \rho_{1, \ell} + \frac{\left(\rho_{1, \ell}\right)^2}{\alpha_2} \leq P_{1, \ell}^{\max}, \forall \ell \in L.
\]

(39b)

Bearing all the above in mind, the following convex program is solved at the \( \kappa \)-th iteration:

\[
\max_{X, \eta, \Gamma, \alpha} \eta
\]

s.t. (9c), (14), (17a), (20), (24), (26), (29), (32a), (32b), (36), (37), (39)

(40a)

\[
\mu_{m, i, k} > 0, \tilde{\mu}_{m, i, k} > 0, \forall m, i, k, \ell
\]

(40c)

where \( \mu \triangleq \{\mu_{m, i, k}, \tilde{\mu}_{m, i, k}\}_{m \in M, i \in T, k \in K, \ell \in L} \) represents the collection of the auxiliary variables. A pseudo code-based path-following procedure used to solve (9) is given in Algorithm 1. After solving (40), we update the involved variables until convergence, i.e., until the difference between two successive values of the objective is less than a given threshold \( \epsilon \).

**Practical Implementations:**

The first step of Algorithm 1 requires a feasible point \( (X^{(0)}, \alpha^{(0)}, \mu^{(0)}) \) of (17) to successfully initialize the computational procedure, which is difficult to find in general. To circumvent this problem, initialized by any feasible \( (X^{(0)}, \alpha^{(0)}) \) to the convex constraints \{9c\}, (14), (17a), (20), (24), (29), (39\}, the following convex program

\[
\max_{X, \eta, \Gamma, \alpha} \eta - \eta_{\min}
\]

s.t. (9c), (14), (17a), (20), (24), (29), (39)

(41a)

without imposing Evs’ constraints, is successively solved until reaching: \( \eta - \eta_{\min} \geq 0 \). Herein, \( \eta_{\min} > 0 \) is a given value to further improve the convergence speed of solving (17). The initial feasible \( \mu^{(0)} \) is then found by calculating \( \mu^{(0)}_{m, i, k} \) and \( \mu^{(0)}_{m, i, k} \) of (17). We have numerically observed that it usually takes about 2 iterations to generate an initial feasible point of (17).
C. Convergence and Complexity Analysis

For the sake of notational convenience, let us define the set of optimization variables $\psi^{(k)}$, the feasible set $\mathcal{V}^{(k)}$, and the objective $\eta^{(k)}$ of the problem (40) at the $k$-th iteration by

$$
\psi^{(k)} = \{ X, \mathbf{\eta}, \mathbf{\Gamma}, \mathbf{\alpha}, \mathbf{\mu} \},
\mathcal{V}^{(k)} = \left\{ \psi^{(k)} \mid \text{s.t. (40b) and (40c)} \right\},
\eta^{(k)} = \left\{ \max \eta \mid \text{s.t.} \psi^{(k)} \in \mathcal{V}^{(k)} \right\},
$$

respectively.

Proposition 1: The feasible set $\mathcal{V}^{(k)}$ of the optimization problem (40) is also feasible for (17) (and hence the original problem (9)).

Proof: See Appendix C.

From Proposition 1, it is ready to show the convergence of Algorithm 1 which is stated as the following proposition.

Proposition 2: Algorithm 1 yields a non-decreasing sequence of the objective, i.e. $\eta^{(k+1)} \geq \eta^{(k)}$. After finitely many iterations, Algorithm 1 converges to the Karush-Kuhn-Tucker (KKT) point of (17) (and hence (9)).

Proof: See Appendix D.

Complexity Analysis: The computational complexity of solving (40) in each iteration is only polynomial in the number of constraints and optimization variables. To see this, the convex problem (40) involves $(4M + 6)(K + L) + 2$ linear and quadratic constraints, and $y \triangleq 2N^2 + 2(N_0 + M + 1)K + 2(M + 2)L + 3$ scalar real variables. The complexity required to solve (40) is thus $O\left( (4M + 6)(K + L) + 2 y^2 ((4M + 6)(K + L) + 2) \right)$.

IV. SECURE TRANSMISSION DESIGN WITH SCSI ON EVES

The perfect instantaneous CSI of the Eves may be difficult to obtain in some cases. For instance, Eves can move to another location to wiretap confidential messages more effectively by protecting its visibility from transmitters without exposing its CSI or they are passive users. As a result, the CSI of Eves may change, and thus, FD-BS and UL users can only estimate Eves’ channel statistics based on their last known CSI. In this section, we consider the case in which the FD-BS and UL users have perfect CSI of all legitimate users as explained in Section III but only the statistics of CSI for Eves (i.e., the first-order and second-order statistics [15, 20]) as follows:

$$
\mathbf{H}_m = \mathbb{E}\{ \mathbf{H}_m \mathbf{H}_m^H \}, \forall m \in \mathcal{M},
\mathbf{g}_{m,i,\ell} = \mathbb{E}\{ \mathbf{g}_{m,i,\ell} \mathbf{H}_m \mathbf{g}_{m,i,\ell}^H \}, \forall m \in \mathcal{M}, (i, \ell) \in \mathcal{U}.
$$

Upon the above settings and similar to (17), the SRM problem (9) with SCSI for Eves, called SRM-SCSI, can be re-expressed as

$$
\text{maximize} \quad \eta^{(k)}
\text{s.t.} \quad \text{(9a), (14), (15), (16d), (17d), (17e), (17f)},
\max_{m \in \mathcal{M}} C_{m,i,k}^{(k)}(X_i, \alpha_i) \leq \Gamma^0_{i,k}, \forall (i, k) \in \mathcal{D},
\max_{m \in \mathcal{M}} C_{m,i,\ell}^{(k)}(X_i, \alpha_i) \leq \Gamma^0_{i,\ell}, \forall (i, \ell) \in \mathcal{U}.
$$

We note that problem (43) is also nonconvex and more computationally difficult than solving (17). Fortunately, it is clear that the convex approximations presented in Section III is useful to obtain a solution to (43). To be specific, the nonconvex constraints in (15) were convexified by (26) and (29), while (17c) and (17d) were linearly approximated by convex constraints (39a) and (39b), respectively. The main obstacle to solving (43) is to handle (43c) and (43d) since Eves’ CSI is random. In the spirit of (17) and (25), we consider the replacement of constraints (43c) and (43d) by their minimum outage requirement as

$$
\max_{\eta, \mathbf{\alpha}, \mathbf{\Gamma}} \eta
\text{s.t.} \quad \text{(9a), (14), (15), (17c), (17d), (17e), (17f)},
\text{Prob} \left( \max_{m \in \mathcal{M}} C_{m,i,k}^{ED}(X_i, \alpha_i) \leq \Gamma^0_{i,k} \right) \geq \epsilon_{i,k}, \forall (i, k) \in \mathcal{D},
\text{Prob} \left( \max_{m \in \mathcal{M}} C_{m,i,\ell}^{SCSI}(X_i, \alpha_i) \leq \Gamma^0_{i,\ell} \right) \geq \epsilon_{i,\ell}, \forall (i, \ell) \in \mathcal{U}.
$$

It is stated that the probabilities of Eves’ maximum achievable rates targeted for DL user $(i, k)$ and UL user $(i, \ell)$ should be greater than or equal to certain constants $\epsilon_{i,k}$ and $\epsilon_{i,\ell}$, respectively. In fact, $\epsilon_{i,k}$ and $\epsilon_{i,\ell}$ are required to be close to 1 to provide secure communications. To evaluate (44c) and (44d), we first introduce the following lemma.

Lemma 1: Assuming all Eves have the same channel properties and they are independent, (44c) and (44d) are converted to the following constraints:

$$
\frac{w_{i,k}^H \mathbf{H}_m \mathbf{w}_{i,k}}{e_{\alpha,1}^m_{i,k} - 1} \leq \bar{\psi}_{m,i,k}(X_i) + (1 - \frac{1}{\epsilon_{i,k}}) N_{e,m} \sigma^2,
\forall m \in \mathcal{M}, (i, k) \in \mathcal{D},
$$
and

$$
\frac{\rho_{i,\ell}^2 \mathbf{g}_{m,i,\ell}^H \mathbf{g}_{m,i,\ell}}{e_{\alpha,1}^m_{i,\ell} - 1} \leq \bar{\chi}_{m,i,\ell}(X_i) + (1 - \frac{1}{\epsilon_{i,\ell}}) N_{e,m} \sigma^2,
\forall m \in \mathcal{M}, (i, \ell) \in \mathcal{U}.
$$

where

$$
\bar{\psi}_{m,i,k}(X_i) \triangleq \bar{\theta}_{m,i,k}(X_i) - w_{i,k}^H \mathbf{H}_m \mathbf{w}_{i,k},
\bar{\chi}_{m,i,\ell}(X_i) \triangleq \bar{\theta}_{m,i,\ell}(X_i) - \rho_{i,\ell}^2 \mathbf{g}_{m,i,\ell}^H \mathbf{g}_{m,i,\ell},
\bar{\theta}_{m,i,k}(X_i) \triangleq \sum_{k'=1}^K w_{i,k'}^H \mathbf{H}_m \mathbf{w}_{i,k'} + \text{Tr}(V_i^H \mathbf{H}_m V_i)
+ \sum_{\ell'=1}^L \rho_{i,\ell'}^2 \mathbf{g}_{m,i,\ell'}^H \mathbf{g}_{m,i,\ell'}.
$$

Proof: See Appendix E.

We note that constraint (45) is still nonconvex but can be further shaped to take the equivalent form:

$$
\frac{w_{i,k}^H \mathbf{H}_m \mathbf{w}_{i,k}}{\beta_{i,k}^D} \leq \bar{\psi}_{m,i,k}(X_i) + (1 - \frac{1}{\epsilon_{i,k}}) N_{e,m} \sigma^2,
$$

where

$$
\beta_{i,k}^D \leq e_{\alpha,1}^m_{i,k} - 1 \iff \frac{\ln (1 + \beta_{i,k}^D)}{\alpha_i} \leq \Gamma^0_{i,k}.
$$

where $\bar{\psi}_{m,i,k}(X_i)$ is the inner approximation of $\bar{\psi}_{m,i,k}(X_i)$.
using (22) as
\[
\Psi_{m,i,k}(x_i) = \Phi_{m,i}(x_i) - 2 \Re \left\{ (w_{i,k}^{(e)})^H H_m w_{i,k} \right\} + (w_{i,k}^{(e)})^H H_m w_{i,k},
\]
and \( \Phi_{m,i}(x_i) \) is obtained by taking the expectation operations on each individual random terms of \( \Phi_{m,i}(x_i) \) given in (32). By using (30), (47b) holds that
\[
Q \quad \text{and} \quad \mathcal{A}_{\kappa}(\alpha_i) \leq \mathcal{A}_{\kappa}(\alpha_i),
\]
and \( \mathcal{A}_{\kappa}(\alpha_i) \) is the inner approximation of \( \mathcal{A}(\alpha_i) \), is solved at the \( \kappa \)-th iteration. By following steps (47)-(51), we equivalently decompose (46) into a set of the following convex constraints:
\[
\begin{align*}
\max & \quad X \\
\text{s.t.} & \quad (\beta_{i,k}^{(e)})^2 + (\beta_{i,k}^{(f)})^2 \
& \quad \forall m, (i, k) \in D (52a)
\end{align*}
\]
where \( \beta_{i,k}^{(e)} > 0, \beta_{i,k}^{(f)} > 0, \forall (i, k) \in D, (i, \ell) \in U \) (53b) to generate the next feasible point \( (X^{(k+1)}, \alpha^{(k+1)}, \beta^{(k+1)}) \), where \( \beta \triangleq \{ \beta_{i,k}^{(e)}, \beta_{i,k}^{(f)} \} \in \mathcal{D}, (i, \ell) \in \mathcal{U}. \) The proposed Algorithm 2 outlines the steps to solve (44). An initial feasible point to solve (44) is easily found using the same method reported in Section III.

**Remark 2:** An important remark here is that the feasible point for (53) is also feasible for problem (44) but not vice versa because the results in (45) and (46) are obtained by using the Markov upper bound on the outage probabilities for DL and UL users (see Appendix E). Thus, the performance under the proposed method serves as a lower bound on the SR performance, which also coincides with the conclusion in [25].

**Algorithm 2:** Proposed path-following algorithm to solve SRM-SCSI (44)

**Initialization:** Set \( \kappa := 0 \) and generate an initial feasible point \( (X^{(0)}, \alpha^{(0)}, \beta^{(0)}) \).

1. **repeat**
   2. Solve (53) to obtain the optimal solution \( (X^*, \alpha^*, \beta^*) \).
   3. Update \( X^{(k+1)} := X^*, \alpha^{(k+1)} := \alpha^*, \beta^{(k+1)} := \beta^* \).
   4. Set \( \kappa := \kappa + 1 \).
   5. **until** Convergence

**V. Secure Transmission Design under Worst-case Scenario**

By introducing AN into a network, the signal strength distribution is essentially affected. The Eves can learn that additional interference is caused by AN, i.e., conduct a characterization of the average power of the received signals and classify their shape (whether a poor wiretap link quality is due to jamming) [39]. Therefore, each Eve can employ a more advanced linear decoder to combat such additional interference, i.e., the MMSE decoder for simplicity. In addition, we assume that targeted message at an Eve is decoded after canceling all multiuser interference caused by DL and UL users [23]. As a consequence, the information rates at the \( m \)-th Eve in (7) can be reformulated as
\[
\hat{C}_{m,i,k}(x_i, \tau_i) = \tau_i \ln (1 + w_{i,k}^H H_m \Xi^{-1} H_m w_{i,k}),
\]
where \( \Xi \) is the new variable, and for notational simplicity the AN signals contribute very much to neutralizing a higher potential for information leakage to the Eves.

The above assumptions lead to a worst-case scenario to provide secure communications of the SRM problem (9), called SRM-WCS, which can be mathematically formulated by
\[
\begin{align*}
\max & \quad \eta \\
\text{s.t.} & \quad (22), (44), (49), (50), (51), (52), (53), (54), (55),
\end{align*}
\]

**Corollary:** A secure transmission design problem (58) can be transformed to
\[
\begin{align*}
\max_{X,\eta,\Gamma,\alpha,\beta} & \quad \eta \\
\text{s.t.} & \quad (30), (42), (43), (44), (45), (46), (47), (48), (49), (50), (51), (52), (53), (54), (55),
\end{align*}
\]

where
\[
\begin{align*}
\hat{C}_{m,i,k}(x_i, \tau_i) & \leq \tilde{C}_{m,i,k}(x_i, \tau_i) - \max_{m \in M} \tilde{C}_{m,i,k}(x_i, \tau_i), \\
\hat{C}_{m,i,k}(x_i, \tau_i) & \geq \tilde{C}_{m,i,k}(x_i, \tau_i) - \min_{m \in M} \tilde{C}_{m,i,k}(x_i, \tau_i).
\end{align*}
\]
are more challenging than those of (16a) and (16b).

We first introduce new variables \( t_{m,i,k} > 0, \forall m, i, k \), to equivalently express (56c) as

\[
\frac{\ln(1 + t_{m,i,k})}{\alpha_i} \leq \Gamma^0_{i,k},
\]

\[
w_{i,k}^H H_m \Xi m^{-1}_m H_m^w_{i,k} \leq t_{m,i,k}.
\]

(57a)

(57b)

Constraint (57a) follows from (51) that

\[
a(t_{m,i,k}^{(\kappa)}) \leq \beta(t_{m,i,k}^{(\kappa)}) \text{Var}(t_{m,i,k}^{(\kappa)}, \alpha_i) \leq \Gamma^0_{i,k}, \forall m, i, k.
\]

(58)

Although (57b) is still a nonconvex constraint, its tractable form can be obtained by

\[
\left[ \begin{array}{c}
t_{m,i,k} \\
H_m^w w_{i,k} \\
\Xi m^{-1}_m
\end{array} \right] \geq 0, \forall m, i, k
\]

(59)

which is a result of applying Schur complement (40). Note that (59) is convex if and only if this set can be written as a linear matrix inequality (LMI). Let us define \( \Xi m_i, \Xi m_i + \sigma^2 I_{N_m} \), where \( \Sigma m_i = H_m^w V_i H_m^w \). The function \( \Sigma m_i \) can be linearized by the following linearization:

\[
\Sigma m_i^{(\kappa)} \triangleq H_m^w (V_i (V_i^{(\kappa)})^H) + V_i^H (V_i^{(\kappa)}) H_m^w.
\]

Thus, the LMI for (59) is given by

\[
\left[ \begin{array}{c}
t_{m,i,k} \\
H_m^w w_{i,k} \\
\Sigma m_i^{(\kappa)} + \sigma^2 I_{N_m}
\end{array} \right] \geq 0, \forall m, i, k
\]

(60)

over the trust region

\[
\Sigma m_i^{(\kappa)} \geq 0, \forall m \in M, i \in I.
\]

(61)

Analogously, applying similar steps from (57a) to (59a) for (56d) yields

\[
a(t_{m,i,k}^{(\kappa)}) \leq \beta(t_{m,i,k}^{(\kappa)}) \text{Var}(t_{m,i,k}^{(\kappa)}, \alpha_i) \leq \Gamma^0_{i,k}, \forall m, i, k, \ell
\]

(62a)

\[
\left[ \begin{array}{c}
t_{m,i,k} \\
\hat{t}_{m,i,\ell} \\
\rho_{m,i,\ell} \Xi m_i
\end{array} \right] \geq 0, \forall m, i, k, \ell
\]

(62b)

where \( \hat{t}_{m,i,\ell} > 0 \) are new variables.

In Algorithm 3, we propose a path-following algorithm to solve the SRM-WCS problem (55). At the \( \kappa \)-th iteration, it solves the following convex program:

\[
\text{maximize } \eta
\]

(63a)

s.t. (29), (30), (28a), (28b), (29), (30), (58), (60), (61), (62b)

\[
t_{m,i,k} > 0, \hat{t}_{m,i,\ell} > 0, \forall m, i, k, \ell
\]

(63b)

where \( t \triangleq \{t_{m,i,k}, \hat{t}_{m,i,\ell}\} \in \mathcal{M}, i \in I, k, \ell \in \mathcal{L} \), to generate a sequence \( \{X^{(\kappa+1)}, \alpha^{(\kappa+1)}, t^{(\kappa+1)}\} \) (and hence \( \{X^{(\kappa+1)}, \tau^{(\kappa+1)}\} \)) of improved points of (55), which also converges to a KKT point.

VI. EXTENSION TO SRM UNDER HARVESTED ENERGY REQUIREMENT FOR EVES

In the previous sections, we assumed that Eves are untrusted users attempting to eavesdrop information signals only. However, the Eves may also require certain wireless power transfer stored for later use in different power constrained operations (24), (25), i.e., assisting UL data transmission to the FD-BS under the same system model and optimization problem presented in Section III, we further consider an extended scenario by incorporating additional energy harvesting (EH) constraints for Eves. From (6), the EH at Eve \( m \), by exploiting the benefits of all interference channels in both groups, can be calculated as (27)

\[
E_m(X, \tau) = \delta \sum_{i=1}^2 \tau_i \left( \sum_{k=1}^K \|H_m^w w_i \|^2 + \|H_m^w v_i \|^2 \right) + \sum_{\ell=1}^L \rho_{\ell,m} \|g_{\ell,m} \|^2
\]

(64)

where \( \delta \in [0, 1] \) denotes the energy conversion efficiency at the receivers that accounts for the loss in the energy transduction (24), (25), (27).

Under (9), the SRM problem with the EH requirements (64), called SRM-EH, can be formulated as

\[
\text{maximize } \min \{R_{i,k}^D(X, \tau), R_{i,k}^U(X, \tau)\}
\]

(65a)

s.t. (9) – (9).

(65b)

\[
E_m(X, \tau) \geq E_m, \forall m \in M
\]

(65c)

where \( E_m \) denotes a minimum EH threshold for Eve \( m \). The developments presented in Section III can be employed to handle (65), with the exception of (65c). We first use (13) to equivalently express \( E_m(X, \alpha) \) by \( E_m(X, \alpha) \) and then exploit (58) to linearize \( E_m(X, \alpha) \) as

\[
E_m^{(\kappa)}(X, \alpha) \triangleq \delta \sum_{i=1}^2 \frac{1}{\alpha_i} \left( \sum_{k=1}^K \|H_m^w w_i \|^2 + \|H_m^w v_i \|^2 \right) + \sum_{\ell=1}^L \rho_{\ell,m} \|g_{\ell,m} \|^2
\]

(66)

Based on such, we can approximate (65) at the \( \kappa \)-th iteration by the following convex program:

\[
\text{maximize } \eta
\]

(67a)

s.t. (29), (30), (28a), (28b), (29), (30), (58), (60), (61), (62b)

\[
t_{m,i,k} > 0, \hat{t}_{m,i,\ell} > 0, \forall m, i, k, \ell
\]

(63b)

\[
E_m^{(\kappa)}(X, \alpha) \geq E_m, \forall m \in M.
\]

(67c)

In Algorithm 4, we summarize the proposed path-following optimization algorithm to solve the SRM-EH problem (65). To find a feasible point for starting Algorithm 4 initialized by any feasible \( \{X^{(0)}, \alpha^{(0)}\} \) to the convex constraints (9c),
Algorithm 4: Proposed path-following algorithm to solve SRM-EH (65)

Initialization: Set $\kappa := 0$ and solve (68) to generate an initial feasible point $(X^{(0)}, \alpha^{(0)}, \mu^{(0)})$.

1: repeat
2: Solve (67) to obtain the optimal solution $(X^*, \eta^*, \Gamma^*, \alpha^*, \mu^*)$.
3: Update $X^{(\kappa+1)} := X^*$, $\alpha^{(\kappa+1)} := \alpha^*$, $\mu^{(\kappa+1)} := \mu^*$.
4: Set $\kappa := \kappa + 1$.
5: until Convergence

| Table I | Simulation Parameters |
|---------|-----------------------|
| Parameter | Value |
| Carrier center frequency/ System bandwidth | 2 GHz/ 10 MHz |
| Distance between the FD-BS and nearest user | $\geq 10$ m |
| Noise power spectral density at the receivers | -174 dBm/Hz |
| PL model for LOS communications, $PL_{\text{LOS}}$ | 103.8 + 20 log10($d$) dB |
| PL model for NLOS communications, $PL_{\text{NLOS}}$ | 145.4 + 37.5 log10($d$) dB |
| Power budget at the FD-BS, $P_{b,\text{FD}}^{\text{max}}$ | 26 dBm |
| Power budget at UL users, $P_{\text{UL}}^{\text{max}}$ | 23 dBm |
| Degree of residual SI, $\sigma^2$ | -75 dB |
| Number of antennas at the FD-BS, $N_e$ | 5 |
| Threshold of all UL users, $R_{\text{UL}}^{\text{min}}$ | $\leq 0$ |
| Number of antennas at FD-BS, $N_t$ | 2 bps/Hz |

The following convex program

$$\max_{X, \eta, \Gamma, \alpha} \min_{m \in M} \left\{ \eta - \bar{\eta}_{\min}, E_m^{(x)}(X, \alpha) - E_{\min} \right\}$$

is successively solved until reaching:

$$\min_{m \in M} \left\{ \eta - \bar{\eta}_{\min}, E_m^{(x)}(X, \alpha) - E_{\min} \right\} \geq 0.$$  

By following the same arguments as in Proposition 2, we can prove that Algorithm 4 converges to a KKT point of (65).

VII. NUMERICAL RESULTS

We now evaluate the performance of the proposed FD model using realistic parameters. We consider the system topology illustrated in Fig. 3. Unless stated otherwise, most important parameters following FD radio evaluation methodology (66, 67) agreed in the 3GPP (42) are specified in Table I for ease of cross-referencing. Here, the power budgets of all UL users are set to be equal, i.e., $P_{\text{UL}}^{\text{max}} = P_{\text{UL}}^{\text{max}}$. The entries of the fading loop channel $G_{\text{SI}}$ are generated as independent and identically distributed Rician random variables with Rician factor $K_{\text{R}} = 5$ dB. The channel from UL user $(i, k)$ to DL user $(i, k)$ (CCI) at a distance $d$ in km is assumed to undergo path loss (PL) model for non-line-of-sight (NLOS) communications as $f_{i, k, t} = 10^{-PL_{\text{loss}}/10} f_{i, k, t}$, where PL$_{\text{loss}}$ is the PL in dB and $f_{i, k, t}$ follows $\mathcal{CN}(0, 1)$ representing small-scale effects (8, 42). All other channels follow the PL model for line-of-sight (LOS) communications as $L = 10^{-PL_{\text{loss}}/10} L$, where $L$ follows $\mathcal{CN}(0, 1)$. Herein, we have favored the channel quality of Eves due to their capability to select a good location to avoid a high possibility of obstruction. The error tolerance between two consecutive iterations in the proposed Algorithms is set to $\epsilon = 10^{-3}$. The achieved SR results in nat/sec/Hz are divided by $\ln(2)$ to have at units of bps/channel-use. We use MOSEK as the convex solver with the toolbox YALMIP (43) in the MATLAB environment. Results are obtained by averaging over 1,000 runs.

For comparison, the following three existing schemes are considered:

- We consider a conventional FD system, under which all DL and UL users are simultaneously served in the whole communication time block (i.e., without considering fractional times and user grouping (8, 23)). We call this scheme “Conventional FD.”
- Under the same system model with “Conventional FD,” the DL transmission can adopt NOMA (30–32) to further improve its performance. Here, each DL user in zone-1 is paired with a DL user in zone-2 to create a virtual cluster by using the clustering algorithm in (32). In each virtual cluster, the message intended for the DL user in zone-2 is decoded by both users while the message intended for the DL user in zone-1 is successively decoded by itself after processing SIC to cancel the interference of the former (31). Since the FD-BS already adopted MMSE-SIC decoder for the UL reception of UL users, we call this scheme “FD-NOMA.” Note that the achievable rate of DL users can be improved by considering a larger cluster size (31) See VI-D1, but their information privacy is more exposed (33).
- Additionally, an HD system is considered. Here, the HD-BS uses all available antennas $N = N_e + N_v$ to serve all DL users in the DL and all UL users in the UL, albeit in two separate communication time blocks. In such a case, there are no SI and CCI, but the effective SR suffers from a reduction by half.
To conduct a fair comparison, the BS in all those schemes also injects an AN in DL transmission and adopts an MMSE-SIC decoder for the reception of the UL signals with the same decoding order as previously presented. In parallel to the max-min SR among all DL and UL users (e.g., (9)), we also plot the max-min SR for DL users only but subject to UL users’ SR requirements (i.e., \( R^U_{\ell} \equiv R^U, \forall (i, \ell) \in U \) listed in Table I) mentioned in Remark 1. For convenience, the SR obtained by the former is referred to as “max-min SR” while that of the latter (i.e., by (10)) is referred to as the “max-min SR of DL users.” It is obvious that the optimization problems corresponding to the above schemes can be solved using our proposed methods after some modifications.

A. Numerical Results for Known CSI (9)

Fig. 4(a) depicts the average max-min SR versus the FD-BS transmit power with known CSI for different resource allocation schemes. The observations from the figure are as follows. First, one can see that the SR for the FD systems is better than that of HD system at a high transmit power \( P_{bs}^{max} \), and the SR curve of HD is nearly unchanged. The reasons for these results are three-fold: 1) The effective SR of the HD per resource block is divided by two; 2) The DL transmission in HD dominates the UL one, as the UL transmission is free of AN; 3) In the FD systems, FD-BS can better protect both DL and UL transmissions by using AN. Second, the SR of FD-NOMA outperforms conventional FD, which is a result of canceling out intra-cluster interference. Third, the SR of the proposed FD is fully superior to the other schemes and an improvement of almost 1.51 bps/Hz (≈ 38.2%) over HD is achieved at the practical value of \( P_{bs}^{max} \approx 26 \) dBm, which is defined in 3GPP TS 36.814. We recall that the proposed FD is advantageous over other schemes in terms of handling interference.

In Fig. 4(b), we plot the average max-min SR as a function of \( \sigma_{SI} \). We can see that the proposed FD scheme offers a significant gain over traditional FD schemes in terms of achievable SR. When \( \sigma_{SI} \) becomes stringent, it is even more essential. Although a major part of the power budget must be allocated to serve far DL users in improving their SR leading to a significant effect of the SI, the near UL users’ SR requirement is easily met. In other words, the effect of SI in the proposed FD becomes less due to the effectiveness of user grouping method. As \( \sigma_{SI} \) increases, the SRs of conventional FD and FD-NOMA drop quickly and tend to be worse than those of HD when \( \sigma_{SI} \) is higher than a certain level. These results are probably attributed to the fact that the FD-BS in those schemes needs to scale down its transmit power to maintain the UL users’ QoS, which leads to a significant loss in the system performance. In addition, Fig. 4(b) further shows that the degree of residual SI needs to be suppressed by at least 72 dB for the FD-NOMA and at least 78 dB for the conventional FD to guarantee a better SR per-user compared to HD. Interestingly, the SR of the proposed FD outperforms the HD for a broad range of \( \sigma_{SI} \), which confirms its robustness against the significant effect of SI.

A trade-off of the average max-min SR between DL and UL users is illustrated in Fig. 5 for different numbers of antennas at the FD-BS by solving (10). The results of the performance for HD are not shown here due to the independence of two resource blocks. In Fig. 5(a) for \( N_r = N_t = 5 \), we observe that the SRs of all schemes constantly decrease with an increase in \( R^D \) since the feasible set gets more restricted. For a high demand on the UL transmission, FD-BS must scale down its transmit power and allocate a large portion of power budget to produce the AN in order to degrade the Eve’s reception, leading to a drastic reduction in the SR for the DL transmission. As expected, the proposed FD scheme achieves better performance in terms of the SR compared to the others, especially in the range of \( R^D \geq 4 \) bps/Hz. For \( N_r = N_t = 3 \), Fig. 5(b) demonstrates the advantage of the proposed FD scheme when the number of users is larger than the number of transmit/receive antennas. As can be seen, traditional FD schemes cannot deliver a good SR for the given setup, owing to a lack of the DoF to leverage multiuser diversity (recall \( N_r = 3 < 4 \) DL users and \( N_r = 3 < 4 \) UL users). The UL users’ QoS ability of the FD-NOMA is inferior to the conventional FD due to inefficiency of using SIC in this setting. In contrast, the proposed FD scheme still has sufficient DoF to transmit and receive the information signals.
in both directions (at a specific time, FD-BS concurrently serves only 2 DL and 2 UL users). Consequently, this results in a substantial improvement, of about 3.37 bps/Hz and 3.22 bps/Hz at $\bar{R}_U = 2$ bps/Hz, in the SR of the proposed FD scheme when compared with FD-NOMA and conventional FD, respectively.

### B. Numerical Results for SCSI of Eves

In this subsection, we show numerical results for the SRM-SCSI problem (44). Under the same simulation setup as in the previous subsection, we set $\epsilon_{i,k} = 0.99$, $\forall (i, k) \in \mathcal{D}$ and $\bar{\epsilon}_{i,\ell} = 0.99$, $\forall (i, \ell) \in \mathcal{U}$ to guarantee secure communications in both directions. In Fig. 6(a), we also plot a benchmark of the proposed FD scheme, assuming perfect CSI for the Eves. The results in Fig. 6(a) reveal that the SRs of all resource allocation schemes degrade compared to Fig. 4(a), which is due to a lack of CSI on Eves. In other words, the perfectly known CSI at the transmitters plays a vital role in designing effective beamforming. Otherwise, this will result in the cost of a reduced system performance. Notably, the SRs of the proposed FD and FD-NOMA schemes have fewer loss than the others due to efficient proposed design and SIC, respectively. The SR of the proposed FD scheme also approaches that of the benchmark when $P_{\text{max}}^{\text{bs}}$ increases. This is because the proposed FD scheme aims to manage the network interference to improve the SR rather than concentrating the interference at Eves.

At $P_{\text{max}}^{\text{bs}} = 26$ dBm, significant gains of up to 126.8%, 57.1% and 45.5% are offered by the proposed FD scheme compared to conventional FD, HD and FD-NOMA, respectively. These results confirm that the proposed FD scheme is more robust and reliable in the presence of imperfect CSI of Eves compared to the others.

The average max-min SR of the DL users versus the FD-BS transmit power is given in Fig. 6(b) for $\bar{R}_U = 2$ bps/Hz. As can be observed, the SRs of all schemes grow very rapidly when $P_{\text{max}}^{\text{bs}}$ increases. The reasons behind this behavior are as follows. 1) The UL users can easily tune the power in meeting their QoS requirements (a loose QoS requirement) to avoid strong CCI to the DL users; 2) The FD-BS now pays more attention to serve the DL users by transferring more power to
them with less attention to the SI. Again, the proposed FD scheme achieves much better SR compared to the traditional FD schemes.

C. Numerical Results for Worst-Case Scenario

For the SRM-WCS, we plot the average max-min SR versus the number of antennas at the FD-BS in Fig. 7(a) and versus the degree of residual SI in Fig. 7(b). As expected, the SRs of all resource allocation schemes shown in Fig. 7(a) degrade (i.e., compared to Fig. 4(a) at $N_t = N_r = 5$), and this is even more drastic for HD. It is easy to see that the UL-user-to-Eve links make the HD scheme more vulnerable to interception than the FD ones, since the UL transmission is free of both MUI and AN. In contrast, FD-BS with AN can better protect the information signals in both directions, which further confirms the importance of using AN. As seen, for $N_t = N_r < 4$, the SRs of FD-NOMA and conventional FD are inferior to HD; however, as $N_t$ and $N_r$ increase, the benefit of exploiting FD radio outweighs the lack of the DoF to leverage multiuser diversity. Another interesting observation is that the SR of FD-NOMA is slightly worse than that of the conventional FD. The reason for this is because the FD-BS must allocate a major part of the power budget to produce the AN leading to less power to convey the desired signals, which may cause error propagation in SIC. In Fig. 7(b), the residual SI needs to be canceled no less than $45$ dB (i.e., $\sigma_{SI} < -45$ dB) for traditional FD schemes to achieve a better SR compared to HD. From both figures, it can be seen that the proposed FD scheme can exploit the spatial DoF and utilize the transmission power more efficiently, and thus, it achieves remarkable gains compared to the others.

D. Numerical Results for Eves’ EH Requirement

To enable practical wireless power transfer, the maximum FD-BS transmit power of $P_{bs}^{\text{max}} = 46$ dBm is set for the $10$ MHz bandwidth, according to 3GPP TR 36.942 v.9.0.1. All Eves are randomly located in zone-1, i.e., they are on the average nearer to the FD-BS in order to better harvest energy. We set an energy conversion efficiency of $\delta = 0.5$.

In Fig. 8(a), we compare the SR-EH regions achieved by different resource allocation schemes for the SRM-EH. We set a minimum EH threshold of $E_{min}^m := E_m, \forall m \in M$. In Fig. 8(b), we also compare the SR-EH regions for the sum EH...
threshold by Eves, $E_{\text{sum}}$. In this case, constraint (65c) becomes
$
\sum_{m=1}^{M} E_m(X, \tau) \geq E_{\text{sum}},$
and its convex approximation in (67c) is
$
\sum_{m=1}^{M} E_m^{(\kappa)}(X, \alpha) \geq E_{\text{sum}}.
$
From Figs. 3a and 3b, it is seen that the proposed FD scheme with AN achieves the best SR-EH trade-offs. The low SR of the conventional FD is a result of a strong wiretap link since Eves in this setup are nearer to the FD-BS. Moreover, FD-NOMA performs much better than the conventional FD because the strong information signals for the far DL users should be aligned to zone-1 to process SIC. The system design with sum EH by Eves of course can offer extra EH compared to the EH per Eve.

In addition, the performance of the proposed FD scheme without AN is plotted in both figures to examine the importance of using AN. From both Figs. 3a and 3b, it is observed that AN is not crucial for very low or strict EH requirements. For very low ($E_{\text{min}}, E_{\text{sum}}$), the network interference already fulfills the needed EH requirements, and there is no need to allocate power on AN. For very strict ($E_{\text{min}}, E_{\text{sum}}$), FD-BS must concentrate the RF energy at the Eves to satisfy (65c), which results in very low power received at the desired users. On the other hand, AN becomes important for a moderate EH requirement, where it is reasonable to allocate a suitable portion of power budget to produce the AN since it also debilitates the Eves’ reception.

E. Algorithm Convergence

Fig. 9 plots the typical convergence results of Algorithms 1 to 4 for a randomly generated channel realization. We can see that Algorithms 1 to 3 converge very fast to the optimal solution within about 10 iterations, while Algorithm 4 requires more iterations (i.e., about 20 iterations) to converge due to the additional EH constraint (65c). The optimization variables are accounted for to find a better solution for the next iteration. Intuitively, the SRs of all algorithms increase quickly within the first iterations and stabilize after a few more iterations. This is because in the first iterations, the approximation errors are large. However, when the algorithms reach the optimal solution, the approximation errors become small due to updating the involved optimization variables after each iteration.

Fig. 9. Convergence of the algorithms (in Algorithm 4 we set $E_{\text{min}} = 0.02$ mW, $\forall m \in \mathcal{M}$).

VIII. Conclusion

We have addressed the problem of secure FD multiuser wireless communication. To manage the network interference more effectively, a simple and very efficient user grouping-based fractional time model has been proposed. Depending on how much the transmitters know about the Eves’ CSI, three difficult nonconvex optimization problems have been considered: (i) SRM with known CSI, (ii) SRM with only Eves’ SCSI and (iii) SRM with WCS. We have developed new path-following optimization algorithms to jointly design the fractional times and power resource allocation to maximize the SR per-user in both DL and UL directions. Specifically, we have first transformed the nonconvex optimization problem to a tractable form and then solved a convex program with polynomial computational complexity in each iteration. Numerical results with realistic parameters have confirmed fast convergence to at least local optima of the original nonconvex design problems. They have revealed that the proposed FD scheme not only provides substantial improvement in terms of SR when compared to the known solutions (i.e., HD, conventional FD and FD-NOMA), but is also robust against the significant effects of SI, imperfect CSI on Eves and DoF bottleneck. The extension to the case of providing a certain SR when compared to the known solutions (i.e., HD, conventional FD and FD-NOMA), but is also robust against the significant effects of SI, imperfect CSI on Eves and DoF bottleneck.

Appendix A

Proof of Inequality (18)

Considering the function $\zeta(z, t) \triangleq \ln(1 + 1/z)/t$, we have
$\zeta(z, t) = -\ln(1 - 1/(1 + z))/t$. It is clear that $-\ln(1 - 1/(1 + z))$ is convex function and increasing on the domain $z > 0$, while the function $1/t$ is convex on the domain $t > 0$. Therefore, the composite function $\zeta(z, t)$ is convex on the domain $z > 0, t > 0$. Thus, it is true that

$$
\zeta(z, t) \geq \zeta(\kappa(z), t^{(\kappa)}) - \left(\nabla_{z} \zeta(z, t) \big|_{(\kappa(z), t^{(\kappa)})}\right) \left(\gamma - \zeta(\kappa(z), t^{(\kappa)})\right) + \frac{1}{t^{(\kappa)}(\kappa(z)) + 1} \gamma^{2} - \frac{\zeta(\kappa(z), t^{(\kappa)})}{t^{(\kappa)}}
$$

where the notation $(\cdot)_{\kappa(z), t^{(\kappa)}}$ is used to represent the value of the function at $(\kappa(z), t^{(\kappa)})$. The inequality (18) is then obtained by substituting $\gamma = z^{-1}$ and $\kappa(\gamma) = (z^{(\gamma)})^{-1}$ into (A.1).

Appendix B

Derivations of $F^{(\kappa)}_{m,i,k}(X_{i}, \alpha_{i}, \mu_{m,i,k})$ and $P^{(\kappa)}_{m,i,k}(X_{i}, \alpha_{i}, \mu_{m,i,k})$

For the concave function $\sqrt{yz}$, its convex upper bound can be found as

$$
\sqrt{yz} \leq \frac{\sqrt{y(g^{(\kappa)})}}{2\sqrt{z^{(\kappa)}}} z + \frac{\sqrt{z^{(\kappa)}}}{2\sqrt{y^{(\kappa)}}} y
$$

with $\forall y > 0, g^{(\kappa)} > 0, z > 0, z^{(\kappa)} > 0$. The convex approximation of $F^{(\kappa)}_{m,i,k}(X_{i}, \alpha_{i}, \mu_{m,i,k})$ can be found as follows. The
The first term $\mu_{m,i,k}/\alpha_i$ is convexified by using (B.1) while the second term is a quadratic function, which can be linearized for inner approximation by using (22). As a result, we have

$$F_{m,i,k}(X_i, \alpha_i, \mu_{m,i,k}) \triangleq \frac{1}{2}(\frac{\mu_{m,i,k}}{\mu_{m,i,k}^2} + \frac{\mu_{m,i,k}}{\alpha_i}) - \left[Q_{m,i}(X_i) - 2R\left(\left(w_{i,k}^H H_{m_i}H_{w,k}\right) + \|H_{m_i}w_{i,k}\|^2\right)\right].$$  

where

$$Q_{m,i}(X_i) \triangleq \frac{1}{2}\sum_{k'=1}^K \sum_{\ell=1}^L \rho_{i,\ell'} \left(\|H_{m_i}w_{i,k'}\|^2 + \left(\rho_{i,\ell'}\|g_{m_i,\ell'}\|^2\right)\right),$$

and $\alpha_i^2$ is linearized as $\alpha_i(2\alpha_i - \alpha_i^2)$. Analogously, the function $P_{m,i,j}(X_i, \alpha_i, \mu_{m,i,\ell})$ is approximated by

$$P_{m,i,\ell}(X_i, \alpha_i, \mu_{m,i,\ell}) \triangleq \frac{1}{2}(\frac{\mu_{m,i,\ell}}{\mu_{m,i,\ell}^2} + \frac{\mu_{m,i,\ell}}{\alpha_i}) - \left[Q_{m,i}(X_i) - 2\rho_{i,\ell} \rho_{i,\ell} \|g_{m_i,\ell'}\|^2\right].$$

and also for (31) and (35):

$$C^{ED}_{m,i,k}(X_i, \alpha_i) \leq C^{ED}_{m,i,k}(X_i, \alpha_i, \mu_{m,i,k}) \leq \Gamma^0_{i,k}$$

From (15a), it follows that

$$\eta(\kappa+1) \triangleq \min_{(i,k) \in D} \left\{ C^{i,k}_{i,k}(X_i^{(\kappa+1)}, \alpha_i^{(\kappa+1)}) - \max_{m \in M} C^{ED}_{m,i,k}(X_i^{(\kappa+1)}, \alpha_i^{(\kappa+1)}) \right\} \geq \min_{(i,k) \in D} \left\{ C^{i,k}_{i,k}(X_i^{(\kappa+1)}, \alpha_i^{(\kappa+1)}) - \max_{m \in M} C^{ED}_{m,i,k}(X_i^{(\kappa+1)}, \alpha_i^{(\kappa+1)}, \mu_{m,i,k}) \right\} \geq \min_{(i,k) \in D} \left\{ C^{i,k}_{i,k}(X_i^{(\kappa+1)}, \alpha_i^{(\kappa+1)}) - \max_{m \in M} C^{ED}_{m,i,k}(X_i^{(\kappa+1)}, \alpha_i^{(\kappa+1)}, \mu_{m,i,k}) \right\} \geq \eta(\kappa),$$

where the equality is obtained using the equalities in (D.1) and (D.2). This result shows that the objective is non-decreasing with iteration number, i.e., $\eta(\kappa+1) \geq \eta(\kappa)$. Also, it is clear that $X^{(\kappa+1)}$, $\alpha^{(\kappa+1)}$ is a better point for (17) than $(X^{(\kappa)}, \alpha^{(\kappa)})$ whenever $(X^{(\kappa+1)}, \alpha^{(\kappa+1)}) \neq (X^{(\kappa)}, \alpha^{(\kappa)})$. For problem (17), we have $\eta(\kappa+1) \geq \eta(\kappa)$, which is an immediate consequence and $\eta(\kappa) \geq 1$ is non-decreasing sequence. According to the Cauchy’s theorem, the sequence $\{\eta(\kappa)\}$ is bounded, i.e., $\lim_{\kappa \to +\infty} \eta(\kappa) = \tilde{\eta}$ with a limit point $\tilde{\eta}$. Then, each accumulation point $\tilde{\eta}(\kappa)$, if $\tilde{\eta}(\kappa+1) = \tilde{\eta}(\kappa)$, is a KKT point for (40), which obviously is also a KKT point for (17) according to [37 Theorem 1], Proposition 2 is thus proved. 

APPENDIX D

PROOF OF PROPOSITION 2

For the first point of Proposition 2, we focus on (15a) and the same arguments will be applicable to all remaining constraints. From (23) and (25), we recall that

$$C^{0}_{i,k}(X_i, \alpha_i) \geq C^{0}_{i,k}(X_i, \alpha_i)$$

and

$$C^{0}_{i,k}(X_i^{(\kappa)}, \alpha_i^{(\kappa)}) = C^{0}_{i,k}(X_i^{(\kappa)}, \alpha_i^{(\kappa)}).$$

Under the assumption of the independence of Eves’ channels, constraint (46) can be computed as

$$\text{Prob} \left( \max_{m \in M} C^{ED}_{m,i,k}(X_i, \alpha_i) \leq \Gamma^0_{i,k} \right) \geq \epsilon_{i,k},$$

Note that the inequality (E.1) holds easier if Eves’ channels are dependent since its RHS yields a smaller value. We further rewrite (E.2) based on the basic property of probability as

$$\text{Prob} \left( \max_{m \in M} C^{ED}_{m,i,k}(X_i, \alpha_i) \leq \Gamma^0_{i,k} \right) \geq \epsilon_{i,k}.$$
\[ \text{Prob}(Y \geq y) \leq \mathbb{E}[Y] / y, \text{ to manipulate the LHS of (E.2) as} \]
\[ \text{Prob}\left(\ln \left(1 + \frac{||H^H_m w_{i,k}||^2}{\psi_{m,i,k}(X_i)}\right) \geq \alpha_i D_{m,i,k}\right) \geq \frac{\alpha_i D_{m,i,k}}{(\alpha_i D_{m,i,k} - 1)N_{c,m}\sigma^2} \] 
\[ = \text{Prob}\left(\frac{1}{w_{i,k}^H H_m H^H w_{i,k}} \geq \frac{1}{\psi_{m,i,k}(X_i)} - N_{c,m}\sigma^2, \text{ and } \psi_{m,i,k}(X_i) \right) \ text{is the expectation of } \psi_{m,i,k}(X_i) \text{ w.r.t. (E.6). By replacing the LHS of (E.2) with (E.6) and after some straightforward manipulations, we arrive at (E.5). It can be shown in a similar manner that (E.4) is converted to (E.6), and thus the proof is completed.} \]

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