Berry curvature unravelled by the Nernst effect

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The discovery of topological quantum materials represents a striking innovation in modern condensed matter physics with remarkable fundamental and technological implications. Their classification has been recently extended to topological Weyl semimetals, i.e., solid state systems which exhibit the elusive Weyl fermions as low-energy excitations. Here we show that the Nernst effect can be exploited as a sensitive probe for determining key parameters of the Weyl physics, applying it to the non-collinear antiferromagnet Mn3Ge. This compound exhibits anomalous thermoelectric transport due to enhanced Berry curvature from Weyl points located extremely close to the Fermi level. We establish from our data a direct measure of the Berry curvature at the Fermi level and, using a minimal model of a Weyl semimetal, extract for the first time the Weyl point energy and their distance in momentum space.

Weyl semimetals[1–4] are certainly one of the most stunning representatives of topological material classes. Their electronic band structure is predicted to host Weyl points (WP), i.e., three-dimensional linear band crossings that represent massless Weyl fermions of defined chirality. Two Weyl points always form a pair of opposite chirality which is separated in momentum space due to spin-orbit coupling and breaking of the time-reversal symmetry or inversion symmetry. WPs act as a source/sink of Berry curvature, a vector field in momentum space which represents the topological properties in Weyl semimetals. Understanding, probing and controlling this quantity is of enormous importance to emergent fields of basic and applied research. For example, in spintronics[5] the Berry curvature is causing a spin-orbit torque that drives spin dynamics in transition-metal bilayers[6]. A further example is quantum computing, where the Berry curvature provides a superior robustness to noise in photonic networks of solid-state qubits[7].

The Berry curvature can be seen as an effective magnetic field in the reciprocal lattice, determining an additional component to the electron velocity v(k), the so-called anomalous velocity, which is always perpendicular to the force driving the electron motion[8]. As a natural consequence, anomalous transverse transport properties, namely the anomalous Hall effect (AHE) and its thermoelectric counterpart, the anomalous Nernst effect (ANE), are expected to arise[8–10]. A clear advantage of the Nernst effect, being determined by a temperature gradient, is given by its direct relation to the entropy transport. Since in metals, semimetals and semiconductors, the entropy accumulates at the interface between occupied and unoccupied states due to the effect of a finite temperature on the Fermi-Dirac distribution, the ANE is a measurement of the Berry curvature at the Fermi surface and not a property of the whole Fermi sea like the AHE[11]. Therefore, compared to other transport properties, the sensitivity of the ANE to the Berry curvature is exceptionally high[12]. Furthermore this implies that the appearance of an ANE is expected only if the Weyl points are close enough to the Fermi level.

Recently, the isostructural non-collinear antiferromagnets Mn3Ge and Mn3Sn attracted a tremendous interest[5] due to giant anomalous transport coefficients[3, 11, 13–16]. Mn3Sn has been extensively studied in Hall- and Nernst-measurements[11, 14], but it lacks magnetic order below $T = 50$ K and develops a glassy ferromagnetic ground state[17, 18], where both ANE and AHE vanish[11]. Here we report the first comprehensive study of the ANE[19] in Mn3Ge, in which magnetic order and anomalous transport persist down to lowest temperatures. Mn3Ge is characterized by a hexagonal crystal structure (space group $P6_3/mmc$), where Mn atoms form a kagome lattice of mixed triangles and hexagons with Ge atoms being situated at the center of the hexagons. In the noncollinear antiferromagnetic ground state of Mn3Ge the Mn moments are oriented at 120° with respect to their neighbors (see Figure 1a)[20]. Only a very small net moment appears in-plane due to a slight tilting of the Mn-moments[21, 22]. Thanks to this peculiar lattice symmetry, multiple Weyl points have been predicted to exist in the band structure of Mn3Ge[20].

We start with a clear demonstration that the anomalous transport (which is driven by Berry curvature) dominates the Nernst effect in Mn3Ge. The different measurement configurations are labeled as $S_{ij}$, with the Nernst signal measured along the $i$-direction with an applied temperature gradient $\nabla T_j$ and a magnetic field $B_k$ (compare Figure 1b). Figure 1c shows the Nernst coefficient $S_{xz}$ which exhibits a totally anomalous behaviour (no magnetic field dependence) without any visible normal
(linear $B$-dependence) contribution as a function of field for all the investigated temperatures, exhibiting a step-like feature at very low fields and reaching a saturation in a flat plateau for $B < 0.02$ T. $S_{xz}$ reveals a very similar behaviour as presented in Figure 1d. Both configurations show the peculiar saturating behaviour up to room temperature with a large Nernst signal of around 0.4 - 1.5 $\mu$V/K, depending on the temperature. On the other hand, a different phenomenology characterizes $S_{xy}$, as reported in Figure 1e. In this configuration the Nernst coefficient is much smaller, with the step-like behaviour just slightly visible and shows a much weaker temperature dependence.

The experimental observation in Fig. 1 allows us to draw an important conclusion about the Berry curvature in Mn$_3$Ge. The Nernst signal $S_{xz}$ of the transverse transport is generally determined by the thermoelectric tensor $\alpha_{ij}$ and the charge conductivity tensor $\sigma_{ij}$ as follows,

$$S_{xz} = \frac{\alpha_{xz}\sigma_{xx} - \alpha_{xx}\sigma_{xz}}{\sigma_{xx}^2 + \sigma_{xz}^2}.$$  

Since we observe a purely anomalous Nernst signal for $S_{xz}$ we conclude, according to the well established Boltzmann transport theory, that the transverse components $\alpha_{xz}$ and $\sigma_{xz}$ dominate the Nernst signal in Eq. 1. These components are directly related to the $y$ component of the momentum integrated Berry curvature via the expressions

$$\sigma_{xx} = \frac{e^2}{h} \int \frac{d^3k}{(2\pi)^3} \Omega^y(k)f_k,$$

$$\alpha_{xx} = \frac{k_B e}{h} \int \frac{d^3k}{(2\pi)^3} \Omega^y(k)s_k,$$

where $f_k$ is the Fermi distribution function and $s_k$ is the entropy density. Thus, if a large anomalous Nernst signal $S_{xz}$ is observed the $y$ component of the integrated Berry curvature must be large. An analog argument can be found for $S_{yz}$ and the corresponding component $\Omega^y$. These considerations allow us to conclude that the $x$- and $y$-components of the Berry curvature are large compared to its $z$ component. This finding is consistent with symmetry considerations of the band structure[20, 23].

The saturation values of $S_{xz}$, $S_{yz}$, and $S_{xy}$ are plotted in Figure 1f as a function of temperature $T$ for $B = 14$ T: a broadened maximum of about 1.5 $\mu$V/K is visible for $S_{xz}$ and $S_{yz}$ at around 100 K. This remarkable temperature dependence is leading us to a second qualitative fundamental conclusion. As is explained in Ref. [12], the peak position in the temperature dependence of the ANE represents a coarse correspondence with the lowest Weyl
point energy $\mu$ with respect to the Fermi level. This is because for $k_B T \ll |\mu|$ essentially states with energy $|\epsilon| < |\mu|$ probe the Berry curvature, giving rise to an increasing ANE upon the thermal energy approaching $|\mu|$ from below. On the other hand, if $k_B T$ exceeds $|\mu|$, the then additionally contributing higher energy states provide an opposite contribution to the ANE. Thus, we estimate $|\mu| \sim 10$ meV.

After having established a qualitative understanding of the ANE, we now move on to extract material specific parameters of the Weyl system. In order to provide a quantitative evaluation we derive (see Supplemental Material) the transverse Peltier coefficient

$$\alpha_{xz} = \frac{\rho_{zz} S_{xz} - \rho_{zz} S_{zz}}{\rho_{xx} \rho_{zz} + \rho_{xz}^2}$$

(4)

using experimental values for the required transport coefficients (see Supplemental Material). The temperature dependence of $\alpha_{xz}$ in zero field is shown in Fig. 2. It resembles the behaviour of $S_{xx}$ but exhibits a narrower maximum which is shifted to lower temperatures ($\approx 75$ K). We elaborated a model to explain the temperature dependence of $\alpha_{xz}$, extending the theory proposed in Ref. [12]. $\alpha_{xz}$ is dominated by the anomalous Nernst contribution in Eq. 4 and thus is truly anomalous as well (see Comparison of $\alpha_{xz}$-contributions in Supplemental Material). We assume that this anomalous contribution is predominantly determined by one particular pair of Weyl nodes which are placed extremely close to the Fermi level[20]. Using a simplified model of two linearized bands which touch each other in such a pair of Weyl nodes (compare Fig. 2 b) we have developed an analytical approach to the temperature dependence of $\alpha_{xz}$, obtaining the following formula (see Supplemental Material for a detailed description of our theoretical approach and derivation),

$$\alpha_{xz} = C_\alpha k_B T \left[ 1 - \frac{E_s}{\mu} - \frac{E_s k_B T}{\mu^2} \right]$$

$$+ F(-E_s - \mu) \left( 1 + \frac{E_s}{\mu} - 2 \frac{E_s k_B T}{\mu^2} \right)$$

$$+ F(E_s - \mu) \left( -1 + \frac{E_s}{\mu} - 2 \frac{E_s k_B T}{\mu^2} \right)$$

$$+ 2 \frac{E_s k_B T}{\mu^2} \left( F^2(E_s - \mu) - F^2(-E_s - \mu) \right)$$

(5)

where $F(E) = 1/(1 + e^{E/(k_B T)})$ is the Fermi distribution function and $\mu = \mu(T) = \sqrt{\mu_0^2 + (0.162 \cdot N_W k_B T)^2}$ is the temperature-dependent level of the chemical potential relative to the Weyl-point energy (compare Fig. 2 b). The saddle point energy $E_s = v_F \Delta k$ is related to the separation of the Weyl points of opposite chirality in momentum space, $\Delta k$, and $\mu_0$ describes the energy difference between the Weyl point and the Fermi level at $T = 0$. Note that $\mu_0$ is usually considered in band structure calculations as the Weyl point energy and is therefore of particular interest. Within our model the values $E_s$ and $\mu_0$ are assigned to the particular low-energy Weyl point which arises from the crossing of our two linearized bands. The dimensionless number $N_W$ is a measure for degrees of freedom of the electronic system at the Fermi level. In the case of trivial bands being absent it roughly corresponds to the number of high-energy Weyl points in the material (see Supplemental Material). The remaining parameter $C_\alpha = (e^2 \hbar \Omega_0 \ln 2)/2$ is a constant which represents the order of magnitude of $\alpha_{xz}$; $\rho_0$ describes the amplitude of the density of states and $\Omega$ represents...
the experimentally relevant strength of the Berry curvature of the considered Weyl system near the Fermi level (see Supplemental Material).

In order to extract the important geometric properties of the underlying system of Weyl fermions close to the Fermi level, we fit the experimental data (red line in Fig. 2a) using the formula 5. As can be seen clearly, the fit works well in a wide temperature range. The deviation at high temperature above 250 K can be explained due to interplay between the Berry curvature strength at the Fermi level which is, to good approximation, determined by the band structure of Mn3Sn[20].

FIG. 3. a) Zoom-in of $S_{xz}(B)$ with a clearly visible hysteresis of the Nernst signal at different selected temperatures. b) Comparison of the hysteresis curves of $S_{xz}(B)$ and the magnetization $M$ in y-direction at $T=100$ K. $B$ denotes the external magnetic field. c) $S_{xz}$ vs. $M$. There is no obvious scaling of the anomalous Nernst signal on the magnetization of the sample.

would like to draw attention to the low-field region of the Nernst data. As highlighted in Figure 3, both the $S_{xz}$ vs $B$ and the $S_{yz}$ vs $B$ curves (the latter are not shown) exhibit a hysteresis cycle which remains almost unaltered from 5 K up to room temperature. This cycle is rectangular-shaped and closes at around 20 mT, in agreement with the previous report on Mn3Sn[14]. Remarkably, the cycle exhibits a total extension of around $\Delta S_{xz} = 2 \mu V/K$ at 100 K (even overcoming the peak value of $\Delta S_{xz} = 0.7 \mu V/K$ observed in Mn3Sn[14]) with negligible net magnetization. This exceptionally large value underpins that non-collinear antiferromagnets such as Mn3Ge and Mn3Sn may constitute a new material paradigm in the field of spintronic[3, 5, 25–27] and thermoelectric technologies[14, 28]. Another crucial outcome is the absence of a proportionality between the ANE and the magnetization $M$ in contrast to traditional ferromagnetic compounds where the ANE is linear in $M$. A comparison of $S_{xz}$ and $M$ vs magnetic field is shown in Figure 3b. Even if $M$ somehow reproduces the overall hysteresis shape in the low field region it closes the cycle at much higher fields with respect to $S_{xz}$. Furthermore, while $S_{xz}$ is anomalous and as such stays constant with increasing $B$, $M$ undergoes an almost linear drift. We also verified the absence of a scaling of the two quantities by plotting $S_{xz}$ vs $M$ in Figure 3c.

In summary, we measured for the first time the anomalous Nernst effect in Mn3Ge and developed a theoretical model to obtain quantitative information on the Weyl nodes in this material. Our analysis reveals an access to fundamental properties of Weyl systems through anomalous transverse transport. On the one hand, we find geometrical properties such as position of the low energy Weyl points in energy and momentum space. Remarkably, the energy and the k-space distance of Weyl nodes closest to the Fermi level are in perfect agreement with the theoretical predictions of band structure calculations. On the other hand, and most importantly, our analysis yields for the first time a measure of the Berry curvature strength at the Fermi level which is, to
the best of our knowledge, not accessible through other experimental probes. In this way, our study promotes the anomalous Nernst effect as an exceptional bulk probe to detect and study Weyl physics in solid state materials.

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SUPPLEMENTAL MATERIAL

Sample Synthesis

The Mn$_2$Ge single crystals were grown using the Bridgman-Stockbarger technique. First, the high purity metals were premelted in an alumina crucible using induction melting. Then the crushed powder was filled in a custom-designed sharp-edged alumina tube and sealed inside a tantalum tube. The crystal growth temperature was controlled using a thermometer at the bottom of the ampule. The sample was heated to 1000$^\circ$C, held there for 12 h to ensure homogeneous mixing of the melt, and then slowly cooled to 750$^\circ$C. Finally, the sample was quenched to room temperature to retain its high temperature hexagonal phase. The single crystallinity was checked by white-beam backscattering Laue x-ray diffraction at room temperature. The crystal structures were analyzed with a Bruker D8 VENTURE x-ray diffractometer using Mo-K radiation.

Magnetization

The DC Magnetization was measured as a function of the temperature and magnetic field by means of a quantum interference device magnetometer (SQUID-VSM) by Quantum Design. For the purpose of a direct comparison with the Nernst effect measurements, the magnetization curves where obtained by applying an external magnetic field in the direction(s) [2-1-10] (and [01-10]). In order to probe the field dependent magnetization, M(B) (Fig. 4) was measured upon sweeping the magnetic field between -7 T to 7 T at constant temperatures 1.8, 50, 100, 150, 200, 250, 300 K. Given the large magnitude of the measured magnetic signal, the small background correction due to the diamagnetic contribution of the glue GE-Varnish used to fix the sample has been neglected.

![Graph of Magnetic moment of Mn$_2$Ge](image)

**FIG. 4.** Magnetic moment of Mn$_2$Ge in a) x-direction and b) y-direction. The magnetization has been corrected for geometry effects. However, demagnetization corrections are small and have not been applied. B denotes the external magnetic field.

Hall effect and resistivity

Electrical transport measurements were done in different configurations to calculate the Peltier coefficient $\alpha_{xz}$. The temperature dependent resistivity for in-plane and out-of-plane configurations is shown in Fig. 5. Fig. 6 shows the $xz$-component of the Hall conductivity. Both quantities are in good agreement with previous measurements[13].

Thermoelectric measurements

The thermoelectric measurements were done in a home-built probe in a Helium cryostat with a magnetic field of up to 15 T. The thermal gradient is generated with a chip resistor on one end of the sample, the other end is glued to a cold bath with an Al$_2$O$_3$-plate in between to establish electrical current free conditions. The
gradient along the sample is measured with a magnetic field calibrated AuFe/Chromel-P thermocouple. The Nernst voltage is measured perpendicular to the thermal gradient and the applied field.

The temperature dependent Nernst effect measurements were done at positive and negative magnetic fields. The data was antisymmetrized to get rid of any field dependent symmetric effect. This was not possible for the magnetic field dependent measurements, due to the peculiar hysteretic behaviour at small fields, which leads to different $S_{ij}(B)$ curves depending of the field history. Therefore, the Nernst signal was measured in full field cycles, from $B = -15$ T to $+15$ T and back to $-15$ T. By subtraction of the symmetric contribution (which shows no field dependence), the curves were centered around $S_{ij}(B) = 0$ to allow a comparison of different temperatures.

Seebeck effect

The Seebeck effect was measured using the Nernst setup with one additional electrical contact. The Seebeck coefficient $S_{ii}$ along the $y$- and $z$-axis has been studied. The data is shown in Fig. 7.

Peltier coefficient

Since Hall- and Nernst effect show an anomalous behavour, $\alpha_{xz}$ can be calculated using zero field values of all involved transport coefficients. The Peltier tensor $\bar{\alpha}$ can be written as

$$\bar{\alpha} = \bar{\sigma} \bar{S} \quad (6)$$

with the conductivity tensor $\bar{\sigma}$ and the thermoelectric tensor $\bar{S}$, the former describing longitudinal and Hall conductivities and the latter Seebeck- and Nernst coefficients. Focussing on the $xz$-plane, the conductivity tensor can be expressed as the inverse of the resistivity tensor

$$\bar{\sigma} = \frac{1}{\rho} = \frac{1}{\text{det} \rho} \left( \begin{array}{ccc} \rho_{zz} & -\rho_{xx} \\ -\rho_{xz} & \rho_{xx} \end{array} \right), \quad (7)$$

leading to the following form of $\bar{\alpha}$:

$$\bar{\alpha} = \left( \begin{array}{cc} \alpha_{xx} & \alpha_{xz} \\ \alpha_{xz} & \alpha_{zz} \end{array} \right) = \frac{1}{\rho_{xx} \rho_{zz} - \rho_{xz}^2} \times \left( \begin{array}{ccc} \rho_{zz} S_{xx} - \rho_{xx} S_{xz} & \rho_{zz} S_{xx} - \rho_{xx} S_{zz} \\ \rho_{xx} S_{zz} - \rho_{zz} S_{xz} & \rho_{xx} S_{zz} - \rho_{zz} S_{zz} \end{array} \right), \quad (8)$$

With this, one can easily express $\alpha_{xz}$ as

$$\alpha_{xz} = \frac{\rho_{zz} S_{xz} - \rho_{xx} S_{zz}}{\rho_{xx} \rho_{zz} - \rho_{xz}^2}. \quad (9)$$
or, using the relation $\rho_{xx} = -\rho_{zz}$, as

$$\alpha_{xz} = \frac{\rho_{xx} S_{xz} - \rho_{xz} S_{zz}}{\rho_{xx}\rho_{zz} + \rho_{zz}^2},$$

(10)

**Fitting of Mn$_3$Ge data**

Since the Peltier coefficient $\alpha_{xz}$ in Fig. 2 is extracted from the Nernst effect measurements in zero-field we assume that according to the Boltzmann transport theory the measured $\alpha_{xz}$ is dominated by an anomalous contribution proportional to the $y$-component of the integrated Berry curvature,

$$\alpha_{xz} = \frac{e^2}{\hbar} \sum_n \int \frac{d^3k}{(2\pi)^3} \Omega_{n,k}^y s_{n,k},$$

(11)

where $s_{n,k} = -f_{n,k} \ln f_{n,k} - [(1 - f_{n,k}) \ln(1 - f_{n,k})]$ is the entropy density ($f_{n,k} = f(E_{n,k})$: Fermi distribution function) for the dispersion $E_{n,k}$ of the conduction electron band $n$. The vector $\Omega_{n,k}$ is the Berry curvature with respect to the band 'n'. We consider two bands which are separated in energy and which touch each other in a pair of Weyl nodes (compare Fig. 2b). They are indexed by $n = 0$ (high energy band with $E_{0,k} > 0$) and $n = 1$ (low energy band with $E_{1,k} < 0$). For the dispersion we apply the following minimal model\[29\] of linearized Weyl fermions,

$$E_{n,k} = \pm v_F \sqrt{k_x^2 + (k_y \pm \Delta k)^2 + k_z^2}.$$  

(12)

It describes a pair of Weyl points which are placed at energy $E = 0$ and are separated in momentum space by $2\Delta k$.

We now present an analytical approach to the temperature dependence of $\alpha_{xz}$ based on the simplified model 12 of a pair of Weyl cones. The resulting formula is then used to fit the experimental data in Fig. 2a with the aim of extracting important geometric properties of the underlying system of Weyl fermions. To derive such a fitting formula we replace at first the momentum integral in Eq. 11 with an energy integral,

$$\alpha_{xz} = \frac{e^2}{\hbar} \int_{-\infty}^{0} dE \rho_0(E) \Omega_{n,k}^y(E) s(E) + \frac{e^2}{\hbar} \int_{0}^{\infty} dE \rho_0(E) \Omega_{n,k}^y(E) s(E),$$

(13)

where $\rho_0(E)$ is the density of states with respect to the band 'n' according to the notation of one high-energy band $n = 0$ for $E > 0$ and one low-energy band $n = 1$ for $E < 0$ as illustrated in Fig. 2b. For the linearized band structure 12 the density of states is given by,

$$\rho_0(E) = \begin{cases} 0 & : \quad E < 0 \\ \rho_0 E^2 & : \quad 0 \leq E \leq E_s \\ \frac{\rho_0}{2} E(E_s + E) & : \quad E > E_s \end{cases},$$

(14)

$$\rho_1(E) = \begin{cases} 0 & : \quad E > 0 \\ \rho_0 E^2 & : \quad -E_s \leq E \leq 0 \\ \frac{\rho_0}{2} E(-E_s + E) & : \quad E < -E_s \end{cases},$$

(15)

where $\rho_0$ is a constant in energy and $E_s = v_F \Delta k$ is the energy difference between the Weyl point and the saddle point at $k = 0$ (compare Fig. 2b). Note that the total density of states $\rho = \rho_0 + \rho_1$ is symmetrical around the Weyl point, i.e. $\rho(-E) = \rho(E)$, and it vanishes at the Weyl points $E = 0$. The explicit formula for the energy dependence of the velocity matrix elements and as-

$$\Omega_{n,k} = i\hbar^2 \sum_{m \neq n} \frac{\langle \psi_{n,k} | \nabla \psi_{m,k} \times | \psi_{n,k} \rangle}{E_{n,k} - E_{m,k}}$$

(17)

where $| \psi_{n,k} \rangle$ are the Bloch states and $\nabla$ is the velocity operator. Within our linearized model we neglect the energy dependence of the velocity matrix elements and assume that the Berry curvature is equal for the two bands. Thus, according to Eq. 17, the energy dependence of one component of $\Omega_{n}$ is determined by the quadratic energy denominator in the Kubo formula 17. Hence, we set for the $y$-component

$$\Omega_{n,k}^y(E) = \Omega_{n,k}^y(0) = \frac{\hbar^2 \tilde{\Omega}}{4E^2},$$

(18)

with an open parameter $\tilde{\Omega}$ representing the off-diagonal velocity matrix elements in Eq. 17 in a field which breaks the time-reversal symmetry of the system. Note that the Berry curvature diverges at the Weyl-point energy $E = 0$.

Inserting the expressions 14, 15, and 18 in the formula 13 for $\alpha_{xz}$ we obtain

$$\alpha_{xz} = \frac{e^2 \hbar \tilde{\Omega} \rho_0}{4} \left[ \int_{-E_s}^{E_s} s(E) dE + \frac{1}{2} \int_{-\infty}^{-E_s} \frac{-E_s + E}{E} s(E) dE + \frac{1}{2} \int_{E_s}^{\infty} \frac{E_s + E}{E} s(E) dE \right],$$

(19)

where for $s(E)$ the expression 16 (considering $\mu$ and $\beta$ as constant in energy) has to be used. To simplify the solution of the integrals with the aim of obtaining a fitting formula for the temperature dependence we substitute at first the variable $E$ with the Fermi distribution function
\[ F(E) = \frac{1}{1+e^{\beta E}}. \]

Exploiting the property that \( F(E) \) can only take values between 0 and 1 we then perform a Taylor expansion in terms of \( F \) around the value \( 1/2 \) up to the first order in \( (F - 1/2) \). Due to the exponential functions in \( s(E) \) the power series of \( F \) converges quickly if the temperature is not too large. After this expansion we integrate over \( F \) and obtain the following result

\[
\alpha_{xz} = C_\alpha k_B T \left[ 1 - \frac{E_s}{\mu} - \frac{E_s k_B T}{\mu^2} \right] + F(-E_s - \mu) \left( 1 + \frac{E_s}{\mu} - \frac{2 E_s k_B T}{\mu^2} \right) + F(E_s - \mu) \left( -1 + \frac{E_s}{\mu} - \frac{2 E_s k_B T}{\mu^2} \right) + 2 \frac{E_s k_B T}{\mu^2} \left( F^2(E_s - \mu) - F^2(-E_s - \mu) \right),
\]

with \( F(E) = 1/(1+e^{\beta E}) \) and \( C_\alpha = (e^2 h \tilde{\Omega} \rho_0 \ln 2)/2 \). Note that the chemical potential \( \mu \), which is derived below, is in general temperature-dependent, i.e. \( \mu = \mu(T) \). After inserting this function \( \mu(T) \) into Eq. 20 we obtain the desired fitting formula for the temperature dependence of the Peltier coefficient \( \alpha_{xz} \).

As can be seen in Eq. 20, the parameter \( C_\alpha \) plays a specific role which is used in our analysis. \( C_\alpha = (e^2 h \tilde{\Omega} \rho_0 \ln 2)/2 \), where \( \tilde{\Omega} \) is according to Eqs. 17 and 18 a parameter which characterizes the off-diagonal velocity matrix elements of the Berry curvature. \( \rho_0 \) is defined in Eqs. 14 and 15 and represents the amplitude of the density of states which is mainly determined by the band dispersion.

Let us finally derive an approximate expression for \( \mu(T) \). Generally, this function can be found from the relation between the total particle number and the chemical potential which is given by an integral over the Fermi distribution as follows,

\[
N = \int_{-\infty}^{\infty} F(E - \mu) \rho(E) dE,
\]

where \( \rho(E) = \rho_0(E) + \rho_1(E) \) is the total density of states with the two parts \( \rho_0 \) and \( \rho_1 \) as given by the Eqs. 14 and 15. Evaluating the energy integral in Eq. 21 and then solving the resulting expression for \( \mu \) gives rise to the function \( \mu = \mu(N, T) \). Unfortunately, the exact solution can only be found numerically. To find an approximate analytical formula for \( \mu(T) \) we simplify the integration in Eq. 21 by replacing the exponential behavior of the Fermi distribution function \( F(E - \mu) \) around \( E = \mu \) with a linear function in \( E \). More specifically, we model the function \( F(E - \mu) \) to linearly drop to zero in the energy range \( k_B T \) around \( E = \mu \). For \( E \) values below and above this range we set \( F \) equal to 1 and 0, respectively. Such an approximation is valid if the temperature is not too large \( (k_B T < 2E_s) \). We obtain from Eq. 21

\[
N \approx \int_{-\infty}^{-E_s} \frac{\rho_0}{2} E(-E_s + E) dE + \int_{E_s}^{\infty} \rho_0 E^2 dE + \rho_0 \mu^2 \frac{k_B T}{2}.
\]

Forming the derivative with respect to \( (k_B T) \) on both sides of Eq. 22 leads to the following approximate differential equation for the chemical potential,

\[
0 \approx \mu^2 \mu' + \frac{k_B T}{2} \left( \frac{k_B T}{2} \right)^2 \left( \mu' - \frac{1}{2} \right). \tag{23}
\]

At zero temperature \( T = 0 \) we immediately find \( \mu' = 0 \). In the high-temperature limit \( \mu \ll k_B T \), Eq. 23 suggests a linear behavior of \( \mu \) with temperature, i.e. \( \mu \propto k_B T \). Therefore, we assume the following approximate temperature behavior,

\[
\mu(T) \approx \sqrt{\mu_0^2 + (A k_B T)^2}, \tag{24}
\]

where \( \mu_0 \) is the chemical potential at zero-temperature, i.e. \( \mu_0 = \mu(T = 0) \). This ansatz fulfills the above properties as one can easily verify by considering the corresponding limiting cases. According to Fig. 2b, \( \mu_0 \) defines the energy of the Weyl point relative to the Fermi level. The dimensionless constant \( A \) can be obtained by inserting the ansatz 24 into the differential equation 23 in the limit \( k_B T \ll \mu \). We find the following equation for \( A \),

\[
A^2 + \frac{3}{4} A - \frac{1}{8} \approx 0,
\]

which has the solution \( A \approx 0.162 \). Note that this constant determines the change of the chemical potential with temperature in the high-energy regime. Therefore, in the real multiband material where several Weyl points are present we expect for \( A \) a value larger than 0.162.

In summary, the expression 20 together with Eq. 24 provides a fitting formula for the temperature dependence of the Peltier coefficient \( \alpha_{xz} \). The fitting parameters are \( A, C_\alpha, \mu_0 \), and \( E_S \). The energy \( E_S = v_F \Delta k \) is related to the separation of the Weyl points of opposite chirality in momentum space and \( \mu_0 \) describes its distance to the Fermi level. These energy values are related to the Weyl point with the lowest possible energy. The dimensionless number \( A > 0.162 \) is a measure for degrees of freedom of the electronic system at the Fermi level. In the case of trivial bands being absent it roughly corresponds to the number of high-energy Weyl points in the material times the number 0.162. Therefore we can slightly modify Eq. 24, using the quantity of \( N_W \) instead of \( A \):

\[
\mu(T) \approx \sqrt{\mu_0^2 + (0.162 \cdot N_W \cdot k_B T)^2}. \tag{26}
\]

Using the same arguments as described in the previous paragraph we have derived a similar fitting formula for the anomalous Hall coefficient \( \sigma_{xz} \), based on the usual expression

\[
\sigma_{xz} = \frac{e^2}{\hbar} \int \frac{d^3k}{(2\pi)^3} \Omega^y(k) F(\epsilon_k) \tag{27}
\]
as obtained from the Boltzmann transport theory. The evaluation of the momentum integration results in the following formula,

\[ \sigma_{xz} = C_\sigma k_B T \left[ 2 \ln \left( 1 + e^{(E_S/2-\mu)/k_B T} \right) - \ln \left( 1 + e^{(-E_S/2-\mu)/k_B T} \right) \right], \] (28)

where \( C_\sigma \) is a specific fitting parameter of \( \sigma_{xz} \). This formula can be used, together with the parameters obtained by fitting the temperature dependence of \( \sigma_{xz} \) (\( \mu_0 = 6.6 \pm 0.7 \) meV, \( E_S = 90 \pm 25 \) meV, and \( N_W = 17.8 \pm 2.2 \)), to fit the Hall conductivity \( \sigma_{xz} \). The parameters \( \mu_0, E_S \) and \( N_W \) were allowed to vary inside the corresponding errorbar. The result is displayed in Fig. 8, the remaining parameter \( C_\sigma = -3.6 \pm 0.3 \) (\( \Omega \text{ cm meV} \)) \(^{-1}\). Note that due to the presence of the Fermi function the momentum integration in Eq. 28 is not restricted to states close to the Fermi level. Therefore, an additional approximation needs to be included to perform the momentum integration analytically. This is the reason why the fitting of \( \sigma_{xz} \) based on Eq. 28 to the actual experimental data is less perfect than the fitting of \( \alpha_{xz} \) based on Eq. 20.

**Comparison of \( \alpha_{xz} \)-contributions**

The Peltier coefficient can be written as

\[ \alpha_{xz} = \frac{\rho_{xx} S_{xz} - \rho_{xz} S_{zz}}{\rho_{xx} \rho_{zz} + \rho_{zz}^2}. \] (29)

To evaluate and compare the influence of the anomalous Nernst and Hall signal on the Peltier coefficient, the two contributions \( \rho_{xx} S_{xz}/(\rho_{xx} \rho_{zz} + \rho_{zz}^2) \) and \( \rho_{zz} S_{xz}/(\rho_{xx} \rho_{zz} + \rho_{zz}^2) \) are displayed in Fig. 9. The first term (including the Nernst signal \( S_{xz} \)) is clearly dominating the Peltier coefficient.

**Fitting of Mn\textsubscript{3}Sn data**

We compare our findings with available results on the similar compound Mn\textsubscript{3}Sn, where a strong anomalous Nernst coefficient exists at room temperature\[11, 14\]. At \( T = 300 \) K, \( S_{xx} \) and \( S_{yz} \) exhibit exactly the same step-like behavior than our data. Remarkably, the saturation value is also the same as in Mn\textsubscript{3}Ge, settled around 0.5 \( \mu \text{V/K} \). Interestingly, though the two compounds share the same kagome-type lattice structure with emergent chiral antiferromagnetism, the predicted positions and numbers of Weyl points at the Fermi energy are different. In particular, according to Ref.[20], Mn\textsubscript{3}Ge displays many more Weyl points than Mn\textsubscript{3}Sn, the lowest in energy lying much closer to the Fermi energy. To make a similar analysis for Mn\textsubscript{3}Sn, we extracted the data points of \( \alpha_{xz} \) from Ref. [14]. Since the triangular spin structure in Mn\textsubscript{3}Sn is destroyed below \( T = 200 \) K[11], Ikhlas et al. used a slight stoichiometric offset between Mn and Sn in the Mn\textsubscript{3}Sn crystals (Mn\textsubscript{3.06}Sn\textsubscript{0.94} and Mn\textsubscript{3.99}Sn\textsubscript{0.01}) to shift the transition to lower temperatures. Fitting the temperature dependence of \( \alpha_{xz} \) with our formula (see Fig. 10), we obtained a Weyl point energy in the range of \( \mu_0 \approx 40 - 100 \) meV. The lower accuracy of the estimation in this case is due to the very limited number of available data points. However, the magnitude corresponds well to the value of \( \mu = 86 \) meV for Mn\textsubscript{3}Sn given by band structure calculations in Ref. [20].

The amplitude of \( \alpha_{xz} \) and thus \( C_\alpha \) are significantly smaller (by approximately one order of magnitude) than in Mn\textsubscript{3}Ge (compare Figs. 10 and 2a). Since the DOS \( \rho_0 \)
at the Fermi level of both compounds is approximately the same, we conclude that the Berry curvature at the Fermi level $\Omega$ is significantly reduced in Mn$_3$Sn as compared to Mn$_3$Ge.

FIG. 10. $\alpha_{ij}$ in a) Mn$_{3.06}$Sn$_{0.94}$ and b) Mn$_{3.00}$Sn$_{0.94}$ from Ref. [14]. Applying our fitting formula we can extract values of a) $\mu_0 = 108$ meV, $C_\alpha = 0.010$ and b) $\mu_0 = 45$ meV, $C_\alpha = 0.004$.

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Upon finalizing this manuscript we became aware of the research of Xu et al. (arXiv:1812.04339). The Nernst data is in well agreement to our measurement, however, only one configuration of $S_{ij}$ was measured and the focus is lying on the relations between different transverse transport coefficients such as the Wiedemann-Franz law.

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