On Matrix Description of IIA Center of Mass

Noriaki Ano

Department of Physics, Tokyo Metropolitan University,
Minamioosawa, Hachiooji, Japan

Abstract

The correspondence between the ground states of the BFSS matrix model and the type IIA string is investigated through the 11th direction. We derive the type IIA string from 11D supermembrane wrapped around the 11th direction of periodicity without a procedure of the double dimensional reduction, but by taking the limit $R_{11} \to \infty$. It is shown that the center of mass of this string as a function on the 11th coordinate $X^{11}$ has the same matrix description as the BFSS matrix model in the flat direction. The fact shows that the matrix model at the strong string coupling in the flat direction of large distance regime is directly connected with the 11D supermembrane theory in the transverse light-cone frame.

Feb. 1999

† E-mail: ano@phys.metro-u.ac.jp
1 Introduction

The BFSS matrix model \[1\] is regarded as a suitable candidate for realizing the eleven dimensional \( M \) theory. The most solid ground for this conjecture is the fact that the model contains the excitations of both the basic 11-dimensional supergravity multiplet with 256 degrees of freedom, and large classical supermembrane in the light-cone frame. The contents of 256 degrees of freedom are 44 gravitons, 84 components of a 3-form field and 128 gravitinos, i.e., the supergraviton. The supergraviton degrees of freedom are equivalent to the lowest states of type IIA string which can be obtain by the di- mensionally reduced supermembrane wrapped around the eleventh direction from the 11D viewpoint\[5][6]. Moreover, we know that by compactifying the ninth transverse direction (i.e., the 11-th direction of \( M \) theory with an 11-9 flip) on a circle of small radius \( R_9 \), the matrix model of weak string coupling can be reduced to a multiple of the light-front type IIA string on the dual torus, on the assumption that all the matrices are defined all over the ninth direction \[2\], based on the works by Motl \[3\] and by Sethi and Susskind \[4\]. The matrix model in this case is modified as noncommuting membrane theory extended in the ninth direction. We can see that the perturbatively interacting type II string spectrum which the matrix model may originally contains, become visible through this compactification procedure.

Clearly, these features indicate that the matrix model is intrinsically related to the IIA string. Originally the matrix model conjecture is motivated by this relation. In fact, the model is founded upon the \( N \) D0-branes mechanics with nonperturbative R-R charge of the type IIA string. On the other hand, the correspondence between the matrix model and the eleven dimensional \( M \) theory is not very strict. That is because the matrix model formulation is based only on the weakly coupled IIA string theory with small radius \( R_{11} \) of compactified 11th direction. In case of the limit \( R_{11} \to \infty \), i.e., the strong string coupling regime, it seems not sure that this conjecture could be correct for describing the nature of the 11 dimensional \( M \) theory.

In this paper, we will show that the matrix model in the flat direction has a relation with the 11 dimensional supermembrane wrapped around the 11th direction in \( R_{11} \to \infty \).
limit, or in string theory words, the strong coupling regime. In the ordinary Kaluza-Klein (KK) mechanism, the radius \( R \) of some periodic coordinate is set to small to obtain physics in the transverse space-time. Thus, it is found that the heavy KK nonzero-modes turn out to decouple from the massless sector. We instead discuss the large \( R \) limit. Taking this limit leads us to the massless KK nonzero-modes. All the KK modes then correspond to the massless degrees of freedom. We can see that the massless nonzero-modes in the large \( R \) limit could not be neglected. Henceforce, we will find that the 11D supermembrane in this limit has a matrix description.

The paper is organized as follows. In the next section 2, the 11D supermembrane wrapped around the 11th direction is considered. It is shown that by taking the limit \( R_{11} \to \infty \), the type IIA string is obtained without using the double dimensional reduction. It is found that this IIA string is on the periodic coordinate \( X^{11} \). In section 3, we study the center of mass of the \( X^{11} \)-dependent type IIA string obtained in section 2, which corresponds to the lowest states of the type IIA string. In order to deal with the physical degrees of freedom, we take the light-cone gauge. Furthermore, we next make the reduction of one of the unwrapped coordinates on the membrane world-volume to obtain the action of the center of mass of the \( X^{11} \)-dependent type IIA string. We are then lead to the action describing the center of mass on loop. In section 4, the matrix description of the \( X^{11} \)-dependent action obtained in the previous section is presented through the Fourier expansion with respect to the periodic \( X^{11} \) direction. In the final section 5, some discussions thought to be useful for justifying the results are given.

2 \( X^{11} \)-Dependent Type IIA String

In the KK-compactification of certain space direction with its small radius \( R \), the KK nonzero-modes become very heavy and decouple from the massless zero-mode in the result. Thus, it is allowed to pick up only the massless KK zero-mode in low energy theory. This procedure means the dimensional reduction in general. We know that the type IIA strings can be derived from 11D supermembrane wrapped around the 11th direction by using the double dimensional reduction along this direction \[5\]. In this
section, if we are concerned with only the elimination of the mass terms of the KK nonzero-modes, we will see that there could be another way to eliminate all the mass terms without decoupling the KK nonzero-modes. We must remind that the procedure of the double dimensional reduction amounts to pick up the type IIA string as the KK zero-mode which decouples from the KK nonzero-modes in the small $R_{11}$ limit.

2-1. IIA String from Supermembrane

Let us first recall the derivation of the type IIA string from the supermembrane wrapped around the compactified eleventh direction via double dimensional reduction of the world volume theory in $11D$ space-time to the world sheet theory in $10D$ space-time [5]. Let us suppose that the world volume coordinates are $\tau, \sigma^1$ and $\sigma^2$. The coordinate $\sigma^2$ is set identical to the compactified eleventh space direction as a gauge choice; $\sigma^2 = X^{11}$. The $11D$ supermembrane action $S_{SM}$ without a 3-form gauge field coupling can be presented as follows;

$$S_{SM} = T_{SM} \int d\tau d\sigma^1 dX^{11} \sqrt{| \det g_{\hat{a}\hat{b}}^{SM} |}, (2.1)$$

where $g_{\hat{a}\hat{b}}^{SM}$ is an induced metric on the world-volume of the supermembrane(SM), and the factor $T_{SM}$ denotes the membrane tension; $T_{SM} = 1/(2\pi)^2 l_p^3 = 1/(2\pi)^2 l_s^3 g_s$. The letters $\hat{a}, \hat{b}$ denote the world-volume coordinates $(\tau, \sigma^1, X^{11})$. The induced metric $g_{\hat{a}\hat{b}}^{SM}$ can be described by using super-space coordinates of supermembrane $Z^\hat{M} = (X^m, X^{11}, \theta^\mu)$ $(m = 0, 1, \cdots, 9$ and $\mu = 1, \cdots, 32)$ and the $11D$ supergravity background metric $G_{\hat{M}\hat{N}}$:

$$g_{\hat{a}\hat{b}}^{SM} = \partial_{\hat{a}} Z^\hat{M} \partial_{\hat{b}} Z^\hat{N} G_{\hat{M}\hat{N}}. (2.2)$$

The KK massive modes on the $\tau-\sigma^1$ world sheet are eliminated by the dimensional reduction of $X^{11}$, leaving only the KK massless zero mode on the world sheet. This procedure can be yielded by putting the following condition;

$$\frac{\partial}{\partial X^{11}} (\text{ ALL FIELDS } ) = 0. (2.3)$$

The condition (2.3) which corresponds also to setting all fields independent of $X^{11}$, leads us to the reduction of $g_{\hat{a}\hat{b}}^{SM}$ to the metric $g_{ab}^{GS}$ on the $\tau-\sigma^1$ world sheet. We can read the
reduced $g_{ab}^{GS}$ by using the super-coordinates $Z^M(\tau, \sigma^1) = (X^m, \theta^\mu)$ as follows;

$$g_{ab}^{GS} = \partial_a Z^M \partial_b Z^N G_{MN}, \quad (2.4)$$

where $G_{MN}$ is a reduced background metric from the 11D supergravity. We therefore reach at the GS-string (GS) action of type IIA;

$$S^{GS} = 2\pi R_{11} T_{SM} \int d\tau d\sigma^1 \sqrt{| \det g_{ab}^{GS} |}, \quad (2.5)$$

where $R_{11}$ is a radius of the compactified $X^{11}$ coordinate, and the following condition on the string tension $T_s$ must be fulfilled;

$$T_s = 2\pi R_{11} T_{SM}. \quad (2.6)$$

Consequently, the type IIA theory coupled with the 10D supergravity background is obtained by this procedure of the double dimensional reduction.

2-2. IIA String without Double Dimensional Reduction

We have now arrived at a subtle, but rather essential point. We would like to focus on eliminating the mass terms of KK nonzero-modes. We can see that there is another way to vanish the mass of all the KK-modes beside setting the condition (2.3), i.e., without vanishing the $X^{11}$ dependence. As will be seen in the following context, we will find that in this case, the KK nonzero-modes could not decouple from the massless sector.

The periodic $X^{11}$ coordinate is an $S^1$ coordinate with its radius $R_{11}$, and we naturally introduce a dimensionless angle parameter $\tilde{\tau}$, which satisfies $X^{11} = R_{11} \tilde{\tau}$. The differentiation associated with $X^{11}$ is written by using $R_{11}$ as follows;

$$\frac{\partial}{\partial X^{11}} = \frac{1}{R_{11}} \frac{\partial}{\partial \tilde{\tau}}. \quad (2.7)$$

It is well-known that the mass of the KK-modes is yielded from this factor $1/R_{11}$, and that in the limit $R_{11} \to \infty$ the mass of all the KK-modes vanishes. That is, all the KK-modes turn out to be the massless modes. The very important point we would like to emphasize is that in this limit, all the terms operated by $\partial/\partial X^{11}$ vanish. On the other hand, the terms which do not contain them survive with no change. Thus, we obtain
the massless KK nonzero-modes without putting the condition (2.3). In this procedure, we see that the $X^{11}$ dependence of all the fields is preserved in contrast to the result by making the ordinary dimensional reduction, that is, all the fields can be regarded as functions defined on $X^{11}$. Therefore, starting with the supermembrane action (2.1), we then obtain the following action by taking the limit $R_{11} \to \infty$:

$$\lim_{R_{11} \to \infty} S^{SM} = \int d\bar{\tau} S^{GS}(\bar{\tau}) \quad (2.8)$$

$$S^{GS}(\bar{\tau}) = R_{11} T_{SM} \int d\tau d\sigma^1 \sqrt{|\det g^{GS}(\bar{\tau})|}. \quad (2.9)$$

$S^{GS}(\bar{\tau})$ describes the $X^{11}$-dependent type IIA GS-string. In this case, the mass of all the modes turns out to be very small. We therefore see that the KK nonzero-modes could not decouple from the massless sector.

3 IIA Center of Mass on Loop

In order to consider the lowest states of the $X^{11}$-dependent type IIA string (2.9), we next deal with the string center of mass. Thus in the following procedure, we need to take the light-cone gauge and the dimensional reduction along the $\sigma^1$ coordinate on the world-sheet. In case of the type IIA string, the center of mass corresponds to the lowest state, and this fact is in contrast to the case of the type IIB string. Before going ahead with our discussion, let us now consider the lowest states of the GS-strings to specify a type of the GS-strings, which has supermultiplet of the string center of mass with 256 degrees of freedom.

3-1. Lowest States of Center of Mass

We first recall that the contents of the GS-string degrees of freedom corresponds to the $D=10$ super Yang-Mills multiplet, i.e., eight Bose states $|i\rangle$ of the vector representation $8_v$ of $spin(8)$ labeled by the letters $i, j$, and eight Fermi states $|a\rangle$ (or $|\dot{a}\rangle$) in opposite chirality) of the spinor representation $8_s$ (or $8_c$) of $spin(8)$ labeled by $a, b$ (or $\dot{a}, \dot{b}$). The $spin(8)$ contents of type IIA massless multiplet is given by the tensor product of two supermultiplets of opposite chirality which is described by 256 states; 128 Bose states
| \langle i \rangle | \langle j \rangle \oplus | \dot{a} \rangle | a \rangle \oplus | \dot{a} \rangle | j \rangle \rangle. \text{ In case of type IIB massless multiplet, the tensor product of two supermultiplets is given by Fermi states of the same chirality: 128 Bose states | \langle i \rangle | \langle j \rangle \oplus | a \rangle | b \rangle \rangle and 128 Fermi states | \langle i \rangle | a \rangle \oplus | b \rangle | j \rangle \rangle. On the other hand, the type IIA and IIB equations of motion of supermultiplets of the center of mass in the light cone frame are as follows; }

\begin{align}
\text{type IIA:} & \quad \frac{\partial^2}{\partial \tau^2} X^i(\tau) = 0, \quad \frac{\partial}{\partial \tau} S^{1a}(\tau) = 0, \quad \frac{\partial}{\partial \tau} S^{2a}(\tau) = 0, \\
\text{type IIB:} & \quad \frac{\partial^2}{\partial \tau^2} X^i(\tau) = 0, \quad \frac{\partial}{\partial \tau} S^{1(2)a}(\tau) = 0,
\end{align}

where $S^1$ and $S^2$ belong to different spinor representations of $\mathbf{8}_s$ and $\mathbf{8}_c$. These supermultiplets correspond to zero modes of the GS-strings in each case. Thus from Eq.(3.1), we see that the IIA field contents of supermultiplet of the center of mass is composed of fields of three different representations of spin(8), i.e., $\mathbf{8}_v$, $\mathbf{8}_s$ and $\mathbf{8}_c$, while in case of type IIB, supermultiplet of the center of mass contains two different representations of spin(8), i.e., $\mathbf{8}_v$ and $\mathbf{8}_s$ (or, $\mathbf{8}_c$). Then we find that the supersymmetric lowest states composed of the above supermultiplets of the center of mass are as follows; in case of type IIA, 256 states which are identical to type IIA massless multiplet (that is, 128 Bose states $| \langle i \rangle | \langle j \rangle \oplus | \dot{a} \rangle | a \rangle \rangle$ and 128 Fermi states $| \langle i \rangle | a \rangle \oplus | \dot{a} \rangle | j \rangle \rangle$), and in case of type IIB, 16 states which are identical with type I massless multiplet (that is, 8 Bose states $| \langle i \rangle \rangle$ and 8 Fermi states $| a \rangle$ (or $| \dot{a} \rangle$)). As a consequence, we find that only in case of type IIA, the supermultiplet of the center of mass has 256 degrees of freedom which is the same contents as the massless multiplet of the 11D supergravity.

### 3.2. Action of $X^{11}$-Dependent IIA Center of Mass

Let us here take the light-cone gauge and the dimensional reduction on the $X^{11}$-dependent IIA string (2.9), in order to study its lowest states which correspond to the string center of mass. We first take the light-cone gauge. In this gauge, the world-sheet parameter $\tau$ is usually described as

$$\tau \sim X^+/l_s^2 p^+,$$

3-2. Action of $X^{11}$-Dependent IIA Center of Mass

Let us here take the light-cone gauge and the dimensional reduction on the $X^{11}$-dependent IIA string (2.9), in order to study its lowest states which correspond to the string center of mass. We first take the light-cone gauge. In this gauge, the world-sheet parameter $\tau$ is usually described as

$$\tau \sim X^+/l_s^2 p^+,$$
where $X^+$ (and $X^-$) denotes the light-cone coordinate

$$X^\pm = \frac{X^0 \pm X^9}{\sqrt{2}},$$  \hspace{1cm} (3.3)$$

and $l_s$ is the string length. Here, we are free to set both $X^+$ and $p^+$ independent of $X^{11}$ together with taking the light-cone gauge, although all fields can be thought dependent upon $X^{11}$ as discussed above. It is natural to take this choice because $\tau$ is independent of $X^{11}$ in the above equation (3.2). The other light-cone coordinate $X^-$ can also be eliminated due to the Virasoro constraint equations as usual. Thus, we can obtain the reduced type IIA string action in the light-cone gauge from the 11D supermembrane action.

Let us next perform the dimensional reduction to obtain the center of mass. This procedure corresponds to reduce the IIA string action (2.8) along the $\sigma^1$ direction to one dimensional action. By taking this procedure, it is meant vanishing of $\sigma^1$-dependence of all the fields. The procedure of the reduction can be done by putting the condition; $\partial/\partial \sigma^1 = 0$. Because the world sheet parameter $\sigma^1$ is dimensionless, the integration with respect to $\sigma^1$ turns out to yield $2\pi$ alone.

Finally, we obtain the following $\tilde{\tau}$-dependent light-cone action of type IIA center of mass in the flat background coming from the 11D supergravity;

$$S(\tilde{\tau}) = \lambda \int dt d\tilde{\tau} \left( \left( \frac{\partial X^i(t, \tilde{\tau})}{\partial t} \right)^2 + i \left( S^{1a}(t, \tilde{\tau}) \frac{\partial S^{1a}(t, \tilde{\tau})}{\partial t} + S^{2a}(t, \tilde{\tau}) \frac{\partial S^{2a}(t, \tilde{\tau})}{\partial t} \right) \right),$$

(3.4)

where $S^1$ and $S^2$ are the 8 component spinors labeled by the letter $a$, and redefined as $S \rightarrow l_s \sqrt{p^+} S$. The letter $i$ runs from 1 to 8. The coefficient $\lambda$ is read as follows;

$$\lambda = \frac{2\pi l_s^2 p^+ R_{11} T_{SM}}{l_s g_s} = \frac{p^+ R_{11}}{l_s g_s}. \hspace{1cm} (3.5)$$

8
Matrix Description

The BFSS matrix model is a description of a system in the infinite momentum frame along the longitudinal direction. The situation corresponds to taking the small $R_{11}$ limit. This model contains $N$ D0-branes as well as very short open strings attached to D0-branes. Specifically under the large distance scale, the string sector becomes very massive and decouple largely from the parton sector. In this distance scale, all the matrices are commutative. We will show that the BFSS matrix model in this large distance regime can be derived from 11D supermembrane wrapped around the 11th direction in the large $R_{11}$ limit. The discussions on the consistency of resulting matrix description with the BFSS model in the large $R_{11}$ limit are given in the next section 5.

4.1. IIA Center of Mass as Diagonal Matrices

Due to the periodicity of $\tilde{\tau}$, we have the Fourier mode expansions of $X^i(t, \tilde{\tau}), S^1(t, \tilde{\tau})$, and $S^2(t, \tilde{\tau})$. Here we must set the reality conditions on $X^i, S^1, S^2$;

$$X^i_{-n} = X^i_n, \quad S^1_{-n} = S^{1*}_n, \quad S^2_{-n} = S^{2*}_n, \quad (n > 0).$$ (4.1)

We are free to set the additional reality conditions on the modes $X^i_n, S^1_n, S^2_n$ as follows;

$$X^i_n = X^{i*}_n, \quad S^1_n = S^{1*}_n, \quad S^2_n = S^{2*}_n, \quad (n > 0).$$ (4.2)

Therefore, we have the following mode expansions;

$$X^i(t, \tilde{\tau}) = \frac{1}{\sqrt{2\pi}} \sum_{n \geq 0} X^i_n(t) \cos n\tilde{\tau}$$

$$S^1(t, \tilde{\tau}) = \frac{1}{\sqrt{2\pi}} \sum_{n \geq 0} S^1_n(t) \cos n\tilde{\tau}$$

$$S^2(t, \tilde{\tau}) = \frac{1}{\sqrt{2\pi}} \sum_{n \geq 0} S^2_n(t) \cos n\tilde{\tau}$$ (4.3)

In ordinary treatment of Fourier expansion, the conditions which we need here are the reality conditions (4.1) on the original functions $X^i(\tilde{\tau})$. By setting the additional reality conditions (4.2), it is meant that these functions can be regarded as the space coordinates in the scheme of the matrix model.
Let us present matrix description of the resulting $\tilde{\tau}$-dependent action of the previous section. We first substitute Eq. (4.3) into Eq. (3.4), and integrate the action with respect to $\tilde{\tau}$. We then obtain the action:

$$S = \lim_{N \to \infty} \sum_{n=0}^{N} \left[ \lambda \int dt \left( \dot{X}_n^i \dot{X}_n^i + i (S_n^{1a} \dot{S}_n^{1a} + S_n^{2a} \dot{S}_n^{2a}) \right) \right]$$

$$= \lim_{N \to \infty} \lambda \int dt \text{Tr} \{(\dot{X}_i \dot{X}^i) + i \theta^T \dot{\theta} \}. \quad (4.4)$$

$X^i$ denotes $N \times N$ real matrix where the index $i$ runs from 1 to 8, and $\theta$ is a 16-component spinor which is a direct product of two 8-component spinors $S^1, S^2$;

$$\theta = (S^1, S^2)^T. \quad (4.5)$$

4-2. Correspondence with Matrix Model

Clearly, all these matrices $X^i$, $\theta$ do not have nonzero off-diagonal elements. Henceforce, we can see that the direction described by these matrices is flat, and that the action (4.4) is the same form as the well-known BFSS model in the flat direction of large distance regime, except for matching of the number of components of $X^i$. That is, the BFSS model has nine $X^i$'s, while the action of (4.4) contains eight $X^i$'s. It seems that the difference is superficial one, because both cases have the same degrees of freedom, 256. In the matrix model, this number comes from a representation of the algebra of the 16 fermionic field $\theta$'s with $2^8$ components.

To study this difference between the two matrix descriptions, let us remember the original theory upon which the BFSS model is founded, that is to say, the $D = 10$ super Yang-Mills theory. All of what we need here is instead the conserved energy-momentum tenser $T^{\mu \nu}$ of $D = 10$ Yang-Mills theory with no supersymmetry.

$$T^{\mu \nu} \sim - \text{Tr} \ F^{\mu \alpha} F^{\nu}_{\alpha} + \frac{1}{4} \text{Tr} \ g^{\mu \nu} F^{\alpha \beta} F_{\alpha \beta}. \quad (4.6)$$

We note here that the matrix model in the flat direction of long distance regime could be obtained by dimensional reduction of $D = 10$ U(1)$^N$ super Yang-Mills theory to no space direction. After dimensional reduction of $T^{\mu \nu}$ (4.6) associated with the broken U(1)$^N$ gauge symmetry, all the components of $T^{\mu \nu}$ are reduced to the following nontrivial components; $T^{00}$ and $T^{ij}$ alone. The letters “$i$, $j$” and “0” label the nine space directions.
and the time direction, respectively. Furthermore, only one non-trivial conservation equation of \( T^{\mu \nu} \) can survive:

\[
\partial_0 T^{00} = 0, \tag{4.7}
\]

where \( T^{00} \sim \sum_{i=1}^{9} \text{Tr} \left( \dot{A}^i \right)^2 \). We then find that \( \sum_{i=1}^{9} \text{Tr} \left( \dot{A}^i \right)^2 \) must be a constant number. This amounts to the dimensionally reduced \( \sum_{\mu = 1}^{9} T^{\mu \mu} \). Here we must recall that the trace \( T^{\mu \mu} \) can be a non-zero value in contrast to the case of four dimensions. Thus, the ninth component \( \text{Tr} \left( \dot{A}^9 \right)^2 \) can be obtained in terms of eight other components, so that only the transverse eight components are left as independent components:

\[
\text{Tr} \left( \dot{A}^9 \right)^2 = \text{const.} - \sum_{i=1}^{8} (\dot{A}^i)^2. \tag{4.8}
\]

In the matrix model, \( A^i \) is regarded as the position coordinate \( X^i \) of \( N \) bounded D0-particles. Thus, it is then possible to say that the bosonic matrix coordinate \( X^i \) has actually eight components rather than nine.

Moreover, we suppose that the action of the matrix model describing a system of low energy \( N \) D0-particles in large distance regime at weak string coupling could be preserved further in the large \( R_{11} \) limit, we can see that the action (4.4) actually corresponds to the matrix model in the flat direction of the large distance regime at strong string coupling. Hence force, we can also identify the coupling \( \lambda \) (3.5) with the D0-particle mass \( 1/l_s g_s \);

\[
\frac{p^+ R_{11}}{l_s g_s} \sim \frac{1}{l_s g_s}, \tag{4.9}
\]

and we have the relation with the string tension \( T_s \) on the assumption that the wrapped supermembrane after taking the limit \( R_{11} \to \infty \) must be identified with the type IIA string as in case of \( \mathbb{5} \): \( T_s = 2\pi R_{11} T_{SM} \) (2.6). We therefore obtain the following relation;

\[
p^+ \sim \frac{1}{l_s g_s}. \tag{4.10}
\]

Thus, the light-cone momentum \( p^+ \) becomes zero in the large \( R_{11} \) limit. Therefore, the momentum of the only 8 space directions can be thought to have non-zero value. The theory turns out to be free to set the 9-11 flip in the space directions in this limit as expected.
5 Discussions

In this paper, it is shown that the matrix model in the flat direction of large distance regime can be directly connected with the $11D$ theory of 256 degrees of freedom in the transverse light-cone frame by taking the limit $R_{11} \to \infty$. First, let us consider the meaning of this large $R_{11}$ limit in the matrix model. By taking account of the relations $R_{11} \sim g_s^{2/3} l_p \sim l_s^{-2/3}$, it seems that the large $R_{11}$ limit leads us to both the strong string coupling and short string scale region. The constant $l_p$ denotes the 11 dimensional Planck length. This means that the string sector is restricted to some local regions in the space-time. We need not take account of any effects of the strings in this limit. Thus, it is possible to say that the string sector substantially decouple from the parton sector. The situation seems to be the same one as in the flat direction condition of the large distance regime. Therefore, the fact shows that the large $R_{11}$ limit yields the flat direction in the matrix model. On the other hand, we have shown that the center of mass of the type IIA string obtained from the $11D$ supermembrane in the large $R_{11}$ limit has the same matrix description as the BFSS model. Taking these matters into consideration, we can see that the two matrix descriptions are consistent with each other under the limit $R_{11} \to \infty$.

We would next like to comment on the large $N$ limit with the large $R_{11}$ limit. In this limit $R_{11} \to \infty$, the energy of the whole system of the matrix model has infinite energy. In order to obtain the finite energy of the whole system at strong string coupling, it is necessary to take the limit $N \to \infty$, so that the energy of each state vanishes like $1/N$. Taking this large $N$ limit is equivalent to treating the whole Fourier modes. Thus, the energy of the matrix description derived from $11D$ supermembrane in the previous section naturally remain finite in spite of the strong string coupling.

Finally, let us make a remark about the 11th direction. This direction is a crucial one to bring the longitudinal momentum $p^{11}$ to the $10D$ theory as a nonperturbative R-R charge. In the scheme of the so-called matrix string theory, the role of the 11th direction is thought to be different from other direction which leads us to get a type IIA string, e.g., $X^9$. We can see that in this paper, the different roles of these directions...
belong to only one of all directions in the strong string coupling, i.e., the 11th direction $X^{11}$. In the weak coupling region, the situation is different from the case in the strong string coupling? Our inference is that the situation have no change. But we need to have an appropriate interpretation about the KK nonzero-modes which are very massive in turn, while the zero-mode corresponds to a single IIA string [3]. It seems undeniable that the KK nonzero-modes correspond to the matrix model in the small $R_{11}$ limit. In fact, we can see that the correspondence with the zero-mode is not mentioned in the matrix theory.

Acknowledgment: The author would like to thank Professor S.Saito for helpful discussions and careful reading of the manuscript.

References

[1] T. Banks, W. Fishler, S. H. Shenker and L. Susskind, “M Theory as a Matrix Model: A Conjecture,” Phys. Rev. D55 (1997) 5112, hep-th/9610043.

[2] R. Dijkgraaf, E. Verlinde and H. Verlinde, “Matrix String Theory,” Nucl. Phys. B500 (1997) 43, hep-th/9703030.

T. Banks, lecture at the Jerusalem Winter School on strings and Duality, Jan.5, 1997.

T. Banks and N. Seiberg, “String from Matrices,” Nucl. Phys. B497 (1997) 41, hep-th/97021187.

[3] L. Motl, “Proposals on Nonperturbative Superstrings Interactions,” hep-th/9701025.

[4] S. Sethi and L. Susskind, “Rotational Invariance in the M(atrix) Formulation of Type IIB Theory,” Phys. Lett. B400 (1997) 265, hep-th/9702101.

[5] M. J. Duff, P. S. Howe, T. Inami and K. S. Stelle, “Superstrings in D=10 from Supermembrane in D=11,” Phys. Lett. B191 (1987) 70.
[6] P. K. Townsend, “The Eleven-Dimensional Supermembrane Revisited,” Phys. Lett. B\textbf{350} (1995) 184, \texttt{hep-th/9501068}.