Are really golden theories in supersymmetry excluded by LHC?

Abhijit Samanta

Nuclear and Particle Physics Research Centre, Department of Physics, Jadavpur University, Kolkata 700 032, India

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Among the promising ideas that have emerged over the past decades, arguably the most beautiful and far reaching is supersymmetry. The exclusion of more and more energy ranges at LHC is quite natural, but, the exclusions of the simplest supersymmetric grand unified theories take the physicists in quandary. However, in these calculations in literature, the whole spectrum in electroweak sector is conventionally generated at a single renormalization group evolution scale, the geometric mean of stop masses, at which the electroweak symmetry breaking potential is minimized. We first find the spectrum inaccurate and it leads to a wrong conclusion due to the facts that i) in every renormalized theory each physical quantity must be evaluated at its own renormalization scale, not all quantities at a single scale, ii) the renormalization scales spreads over a few orders of magnitude of energy due to the wide range of spectrum, and iii) moreover, above chosen conventional scale in literature is few orders of magnitude higher than the highest scale in standard model spectrum. The simplest supersymmetric grand unified theories like constrained minimal supersymmetric standard model and non-universal gaugino mass model etc. are seized out from measured branching ratio of $B^0_s \rightarrow \mu^+\mu^-$ at LHC become accessible when it is calculated at its correct renormalization scale. On the other hand, from the study of $b \rightarrow s\gamma$ decay we show an evidence that if one calculates the branching ratio at the above conventional scale or at a scale much higher than its actual renormalization scale then the theory becomes invalid having same value of branching ratio for different sets of input parameters.

INTRODUCTION

The supersymmetry (SUSY) is the most fascinating and far reaching theory over the decades to solve the shortcomings of the standard model of particle physics. It represents a new type of symmetry that relates bosons and fermions, thus unifying forces (mediated by vector bosons) with matter (quarks and leptons), and which endows space-time with extra fermionic dimensions. At present, there is no evidence of SUSY at LHC, it excludes more and more energy ranges, which is quite natural. Again, this exclusion depends on some specific conditions. But, the serious disappointments appear when the simplest supersymmetric theories have been excluded from the recently measured neutral Higgs Boson mass at CMS and ATLAS and measured branching ratio of $B^0_s \rightarrow \mu^+\mu^-$ decay at CMS and LHCb and it seems SUSY is weird. The physicists are then in a real quandary.

The spectrum in electroweak (EW) sector in standard model (SM) of particle Physics spreads over a few orders of magnitude of energy e.g., mass of electron is 0.51 MeV, and mass of top quark is about 175 GeV. In literature the electroweak symmetry breaking (EWSB) potential in the supersymmetric grand unified theories (SUSYGUTs) like constrained minimal supersymmetric standard model (cMSSM), and non-universal gaugino mass models etc. are minimized by default at the geometric mean of stop masses and the whole spectrum in EW sector is generated at this scale, which is few orders of magnitude higher than the mass of the heaviest particle in SM. We first find that this spectrum is inaccurate and it leads to a totally wrong conclusion due to the facts that in every renormalized theory each physical quantity must be evaluated at its own renormalization scale, not all quantities at a single scale, while the renormalization scales spreads over a few orders of magnitude of energy due to the wide range of spectrum, and moreover, above chosen conventional scale in literature is few orders of magnitude higher than the highest scale in SM spectrum. Then, we have shown that the simplest SUSYGUTs seized out from measured branching ratio of $B^0_s \rightarrow \mu^+\mu^-$ at LHC become accessible for testing when the branching ratio is calculated at its correct renormalization scale. From study of $b \rightarrow s\gamma$ decay we have also shown an evidence that if one calculates the branching ratio at the scale of geometric mean of stop masses or at a scale much higher than its own renormalization scale, then the theory becomes invalid having same value of branching ratio for different sets of input parameters.

We further focused from the study of EWSB potential that EW symmetry is truly broken at a very high scale than the scale conventionally known in literature, it is the highest renormalization group evolution (RGE) scale below which minimization of Higgs potential provides a stabilized minima with deep well in Higgs field space and squared of higgsino mass parameter $\mu^2 > 0$. We identify this as the true EWSB scale and the scale at which the EWSB potential is minimized is only the judicial choice to minimize the effect of higher order corrections to the potential, it has otherwise no special physical importance.

ELECTROWEAK SYMMETRY BREAKING

The tree level scalar potential of the MSSM keeping only the dependence on neutral Higgs fields $H_1^0$ and $H_2^0$...
is:

\[ V_{\text{tree}} = \Lambda + m_1^2 |H_1^0|^2 + m_2^2 |H_2^0|^2 + m_3^2 (H_1^0 H_2^0 + h.c.) \]
\[ + \frac{g^2 + g'^2}{8} (|H_1^0|^2 - |H_2^0|^2)^2 \]

(1)

where, \( \Lambda \) is a field-independent vacuum energy (which will be ignored for our present studies); \( m_1^2 = m_1^2 H_1 + \mu^2 \), \( m_2^2 = m_2^2 H_2 + \mu^2 \) (we assume \( \mu \) to be real ignoring all possible CP violating phases); \( m_1^2, m_2^2 \) and \( m_3^2 \) are soft SUSY-breaking mass parameters; \( g \) and \( g' \) are SU(2)\(_L\) and U(1)\(_Y\) gauge couplings, respectively. At classical level \( V_{\text{tree}} \) must satisfy \( m_1^2 + m_2^2 \geq 2|m_3^2| \) to ensure that the potential is bounded from below; and \( m_2^2 m_3^2 \leq m_1^4 \) to destabilize the origin in field space for ensuring nonzero values of the VEVs \( \langle H_1^0 \rangle = v_1/\sqrt{2} \), and \( \langle H_2^0 \rangle = v_2/\sqrt{2} \). Here, \( m_3^2 \) is not made restrictive to real and positive, so that \( v_1^2 \) and \( v_2^2 \) are real and positive, and the neutral Higgs fields can be decomposed into their VEVs plus CP-even and CP-odd fluctuations: \( H_1^0 = (v_1 + S_1 + i P_1)/\sqrt{2} \) and \( H_2^0 = (v_2 + S_2 + i P_2)/\sqrt{2} \). The parameters of \( V_{\text{tree}} \) are running ones and \( V_{\text{tree}}^{\text{min}} \) changes very rapidly with RGE scale. To obtain reliable results it requires minimization of the effective Higgs potential \( V_{\text{eff}} \) instead of \( V_{\text{tree}} \), and it is defined as \( V_{\text{eff}} = V_{\text{tree}} + \Delta V \); where, \( \Delta V \) contains radiative corrections to \( V_{\text{tree}} \). Then, the minimization conditions are

\[ \frac{1}{v_1} \frac{\partial V_{\text{eff}}}{\partial S_1} \bigg|_{\text{min}} = 0, \]

\[ \frac{1}{v_2} \frac{\partial V_{\text{eff}}}{\partial S_2} \bigg|_{\text{min}} = 0, \]

(2)

\[ m_1^2 H_1 + \mu^2 + \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) + m_1^2 v_1^2 + \Sigma_1 = 0, \]

\[ m_2^2 H_2 + \mu^2 + \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) + m_1^2 v_2^2 + \Sigma_2 = 0, \]

(3)

where, the tadpoles \( \Sigma_i \)s are defined as \( \Sigma_i = \frac{1}{v_i} \frac{\partial \Delta V}{\partial v_i} \bigg|_{\text{min}} \).

Generally, \( \tan \beta = \frac{v_2}{v_1} \) is considered as free input parameter of a theory, and \( v_2^2 = v_1^2 + v_2^2 \) is fixed by \( M_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)/4 \). Then, \( \mu^2 \) and \( m_3^2 \) are fixed from Eqs. (2,3)

\[ \mu^2 = -\frac{M_Z^2}{2} + \frac{m_1^2 H_1 + \Sigma_1 -(m_2 H_2 + \Sigma_2) \tan^2 \beta}{\tan \beta - 1}, \]

(4)

\[ m_3^2 = \frac{1}{2} \sin 2\beta \left( m_1^2 H_1 + m_2^2 H_2 + 2\mu^2 + \Sigma_1 + \Sigma_2 \right). \]

(5)

Now, \( V_{\text{eff}} \) can approximately be written as

\[ V_{\text{eff}} = (m_1^2 + \Sigma_1)|H_1^0|^2 + (m_2^2 + \Sigma_2)|H_2^0|^2 \]

\[ + m_3^2 (H_1^0 H_2^0 + h.c.) + \frac{g^2 + g'^2}{8}(|H_1^0|^2 - |H_2^0|^2)^2. \]

We study \( V_{\text{eff}} \) and \( V_{\text{tree}} \) as a function of \( H_1 \) keeping \( H_2 \) fixed at \( v_2 \) and evaluating the parameters at a particular value of RGE scale, which we denote by \( Q_{\text{PotMin}} \) since we also find the minima of the potentials at this scale. The potential has a well with \( V_{\text{eff}} < 0 \) and the width of the well decreases (increases) as \( Q_{\text{RGE}} \) is made larger (smaller) and it vanishes above a certain value of \( Q_{\text{PotMin}} \) providing no minima in Higgs field space. For the case when one gets a stable minima with a well in the Higgs field space, \( \mu^2 > 0 \) and masses are generated; but, otherwise \( \mu^2 < 0 \). This means that for a given set of input parameters EW symmetry remains unbroken above a certain value of RGE scale \( Q_{\text{PotMin}}^{\text{max}} \) and it breaks below this scale providing \( \mu^2 > 0 \). The result is demonstrated in the first plot of Fig. 1 for a typical set of input parameters. It is now clear that EW symmetry breaks at \( Q_{\text{PotMin}}^{\text{max}} \), but not at the scale at which the potential is minimized; it is our judicial choice to minimize the effect of higher order corrections to \( \mu \) and \( m_3^2 \), and it has otherwise no special physical importance. This scale \( Q_{\text{PotMin}}^{\text{max}} \) is much above than the scale of geometric mean of stop masses and it is few orders of magnitude larger than the top mass or the conventional EWSB scale.

The variations of the minimum of \( V_{\text{eff}} \) and \( V_{\text{tree}} \) have been studied with \( Q_{\text{PotMin}} \) below the \( Q_{\text{PotMin}}^{\text{max}} \) for different sets of input parameters. It is found that \( V_{\text{eff}} \) is quite stable with \( Q_{\text{PotMin}} \); but, the tree level potential has more and more rapid sharp variation when \( \tan \beta \) is increased. The variation becomes more severe for lower values of universal scalar mass \( m_0 \) and universal gaugino mass \( m_{1/2} \) with larger values of \( \tan \beta \). This result is demonstrated in the second plot of Fig. 1 for some bench mark values of input parameters.

It is wellknown fact that \( \Delta V_{1 - \text{loop}} \) is normally the smallest at \( Q_{\text{PotMin}} = Q_{\text{MS}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}} \) and all publicly available SUSY spectra generator packages and all publicly available SUSY spectra generator packages \[ \] have chosen this scale as the default scale for minimization of Higgs potential and all parameters and physical quantities in EW sector are calculated at this scale. But, here, it is found from above study that there exists no special choice of \( Q_{\text{PotMin}} \) after addition of two loop leading order corrections to \( V_{\text{tree}} \). It remains flat for low values of \( \tan \beta \) and slightly decreases with decrease in \( Q_{\text{PotMin}} \) for higher values of \( \tan \beta \). The reason for this slight variation is due to lack of more higher order corrections. Here, the package SuSpect has been used as the spectra generator, which uses full one loop radiative corrections and two loop leading order corrections \( O(\alpha_t \alpha_s + \alpha_s^{-}) \) to \( V_{\text{tree}} \).

**MEASUREMENTS AND RENORMALIZATION**

We have seen in previous section that EW symmetry truly breaks at much higher scale than the masses generated by EWSB or the scale of geometric mean of stop masses at which EWSB potential is conventionally minimized. The spectrum in EW sector spreads over a few orders of magnitude of energy, which implies that all the measurable quantities cannot be defined at a single renormalization scale. To define a theory using quantum
fields, one must specify the renormalization conditions. In renormalized theory, the physical mass and coupling constants of a theory are defined by the renormalization conditions. Each physical quantity in the spectrum must be defined at its own renormalization scale fixed by its renormalization conditions. For example, in $\phi^4$ theory without mass term ($m = 0$) these conditions are defined as:

$$\gamma = 0 \hspace{1cm} \text{at} \hspace{1cm} p^2 = -M^2 \hspace{1cm} \text{Eq. (7)}$$

$$\frac{d}{dp^2} (\gamma) = 0 \hspace{1cm} \text{at} \hspace{1cm} p^2 = -M^2 \hspace{1cm} \text{Eq. (8)}$$

The renormalization scale $M$ is called the renormalization scale of the theory. The two- and four-point Green’s functions are defined in this way at this certain point of $M$ and, in this process, it removes all ultraviolet divergences \[^{14}\]. In case $m \neq 0$ Eqs. \(^7\) and \(^8\) are evaluated at $p^2 = m^2$, and Eq. \(^9\) is evaluated at $s = 4m^2$ and $t = u = 0$.

The renormalization scale $M$ which is the scale of the theory must be matched with the Lorentz invariant energy scale at which a quantity is measured in the experiment. For example, if a particle is created by collision of two particles with centre of mass energy $E_{cm}$ and it decays, then the theoretical cross section of production of the particle is to be calculated at RGE scale equals to $E_{cm}$, but the decay rate of the particle is to be calculated at the RGE scale equals to the rest mass of the particle.

The physical mass $m$ of each particle is to be calculated from its RGE scale dependent parameters obtained at $Q_{RGE} = m$, and when the parameters are running below $m$, this calculated value of the mass $m$ should be frozen. This leads to the fact that physical mass of any particle obtained from EWSB cannot be larger than $Q_{PotMin}^{max}$.

**RENORMALIZED $Br[B_s^0 \rightarrow \mu^+\mu^-]$ AND $Br[b \rightarrow s\gamma]$**

We have studied theoretical values of $Br[B_s^0 \rightarrow \mu^+\mu^-]$, and $Br[b \rightarrow s\gamma]$ in details and demonstrated in Fig. \[^2\] for some typical sets of input parameters. We find very strongly RGE scale dependent. Then, the evaluation at exact renormalization scale is very crucial for correct evaluation to match it with the ones in experiments, otherwise, it may leads to wrong conclusion. From previous section it clear that $B_s^0 \rightarrow \mu^+\mu^-$ is to be calculated at $m_{B_s}$ and $b \rightarrow s\gamma$ at $m_b$. The program packages SuSpect \[^4\] and SuperIso \[^5\] are used for the calculation.

The branching ratio of $B_s^0 \rightarrow \mu^+\mu^-$ was calculated at the geometric mean of stop masses for all SUSYGUTs in literature. But, now it is clear from Fig. \[^2\] that if $Br[B_s^0 \rightarrow \mu^+\mu^-]$ is evaluated at its correct renormalization scale (i.e., at $m_{B_s}$) instead of the scale at which the Higgs potential is minimized, then the theories due to its recently measured value is quite favored. We have checked in detail and then have shown in Fig. \[^2\] that at $m_{B_s}$ the $Br[B_s^0 \rightarrow \mu^+\mu^-]$ is within the uncertainty range of recently measured value ($3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$ \[^10\]) at LHC. The results have been demonstrated for cMSSM inputs, but it happens to be true for almost all SUSYGUTs. All the analytical calculations are based on MSSM parameters and hence these are valid for all SUSYGUTs since they have only the difference in the pattern of input parameters.

In the plot of $b \rightarrow s\gamma$, there are crossings of the lines in $Br[b \rightarrow s\gamma]$ at RGE scales $>> m_b$. It means that multiple solutions (multiple sets of input parameters) exist for a physical quantity at the crossing point, which is not expected and it indicates a serious question on the validity of the theory. But, it does not appear if exact renormalization scale is considered for its calculation.
results strongly leads to the conclusion that evaluation at proper renormalization scale is very crucial.

CONCLUSION

We identified that the EW symmetry is truly broken at a very high scale than the scale conventionally known in literature. It is the highest RGE scale below which minimization of Higgs potential provides a stabilized minima with a deep well in Higgs field space and $\mu^2 > 0$. In literature, the evaluation of the spectrum in EW sector in SUSYGUTs is inaccurate since it is calculated at the geometric mean of stop masses and it leads to a totally wrong conclusion due to the facts that in every renormalized theory each physical quantity must be evaluated at its own renormalization scale, not all quantities at a single scale, while the renormalization scales spreads over a few orders of magnitude of energy due to the wide range of spectrum. The most of the seized out simplest SUSY theories are not now excluded. The exclusion of more and more energy ranges at LHC is quite natural, which again depends on specific conditions. Finally, the golden theories in SUSY are not now excluded by LHC and there is nothing at this moment for disappointing.

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[1] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].