BIANCHI TYPE I MAGNETOFLUID COSMOLOGICAL MODELS WITH VARIOUS COSMOLOGICAL CONSTANT REVISITED

ANIRUDH PRADHAN
Department of Mathematics, Hindu Post-graduate College, Zamania, Ghazipur 232 331, India
acpradhan@yahoo.com, pradhan@iucaa.ernet.in

SANJAY KUMAR SINGH
Department of Computer Science, Post-graduate College, Ghazipur 233 001, India
sanjaysingh10@rediffmail.com

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The behaviour of magnetic field in anisotropic Bianchi type I cosmological model for bulk viscous distribution is investigated. The distribution consists of an electrically neutral viscous fluid with an infinite electrical conductivity. It is assumed that the component $\frac{1}{2}$ of shear tensor $\frac{1}{2}$ is proportional to expansion $\frac{1}{2}$ and the coefficient of bulk viscosity is assumed to be a power function of mass density. Some physical and geometrical aspects of the models are also discussed in presence and also in absence of the magnetic field.

Keywords: Cosmology, Bianchi type I universe, magnetofluid models

1. Introduction

A resurgence of interest in anisotropic, general-relativistic cosmological models of the universe $^{1,2}$ has been stimulated by the discovery of the cosmic microwave radiation, the study of its isotropy, the problem of the abundance of primordial helium, and the possibility of large-scale primordial magnetic fields. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model. $^3$ The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. $^4$ Also Harrison $^5$ has suggested that magnetic field could have a cosmological origin. As a natural consequence we should include magnetic fields in the energy-momentum tensor of the early universe. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors. $^6$ $^{15}$ Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic fields give rise
to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure was isotropic.\textsuperscript{16,17} Such fields can be generated at the end of an inflationary epoch.\textsuperscript{18,22} Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali\textsuperscript{23} obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Several authors\textsuperscript{24,28} have investigated Bianchi type I cosmological models with a magnetic field in different context.

Most cosmological models assume that the matter in the universe can be described by ‘dust’ (a pressure-less distribution) or at best a perfect fluid. Nevertheless, there is good reason to believe that - at least at the early stages of the universe - viscous effects do play a role.\textsuperscript{27,29} For example, the existence of the bulk viscosity is equivalent to slow process of restoring equilibrium states.\textsuperscript{30} The observed physical phenomena such as the large entropy per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn\textsuperscript{31} for a review on cosmological models with bulk viscosity). The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity.\textsuperscript{32,46}

In recent years, models with relic cosmological constant have drawn considerable attention among researchers for various aspects such as the age problem, classical tests, observational constraints on , structure formation and gravitational lenses have been discussed in the literature. It is remarkable here that in the absence of any interaction with matter or radiation this would force the cosmological constant to be constant, but, in the presence of the interaction with matter or radiation, a solution of Einstein’s field equation and assumed equation of covariant conservation of energy with a time varying can be found. For these solutions, conservation of energy requires that any decrease in the energy density of the vacuum component be compensated for by a corresponding increase in the energy density of matter or radiation. Some of the recent discussions on the cosmological constant “problem” and consequence on cosmology with a time-varying cosmological constant are contained in Bertolami,\textsuperscript{47} Ratra and Peebles,\textsuperscript{48} Dolgov,\textsuperscript{49} Sahni and Starobinsky,\textsuperscript{52} Padmanabhan,\textsuperscript{53} Vishwakarma.\textsuperscript{54} Recent observations by Perlmutter et al.\textsuperscript{55} and Riess et al.\textsuperscript{56} strongly favour a significant and positive . Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range 0:10 < z < 0:83 and suggest Friedmann models with negative pressure matter such as a cosmological constant, domain walls or cosmic strings (Vilenkin,\textsuperscript{57} Garnavich et al.\textsuperscript{58}) Recently, Carmeli and Kuzmenko\textsuperscript{59} have shown that the cosmological relativity theory (Behar and Carmeli\textsuperscript{60}) predicts the value \( \Lambda = 1.934 \times 10^{-35} \) s\(^{-2}\) for the cosmological constant. This value of \( \Lambda \) is in excellent agreement with the measurements recently obtained by the High-Z Supernova
Team and Supernova Cosmological Project (Garnavich et al.; Perlmutter et al.; Riess et al.; Schmidt et al.) The main conclusion of these works is that the expansion of the universe is accelerating.

Recently, Bali and Jain have obtained a generalized expanding and shearing anisotropic Bianchi type I magnetofluid cosmological model for perfect fluid distribution in general relativity. Motivated by the situations discussed above in regard with bulk viscous cosmologies, we extend their work by including an electrically neutral bulk viscous fluid as the source of matter in the energy-momentum tensor. The paper is organized as follows. In Sec. 2, we review the solutions and the main results of Bali and Jain. Here we have shown that all the solutions obtained by Bali and Jain and Bali can be derived from our solutions. In Sec. 3, we obtain the solutions for a universe filled with a bulk viscosity in presence of magnetic field. Sec. 4 includes the bulk viscous cosmological solutions in the absence of magnetic field. In Sec. 5, we discuss our main results and summarize our conclusions.

2. A magnetic fluid universe revisited

In this section, we review the solutions obtained by Bali and Jain. We consider an anisotropic homogeneous Bianchi type I metric in the form given by Marder

\[ ds^2 = A^2 (dx^2 + dt^2) + B^2 dy^2 + C^2 dz^2; \]

where the metric potentials are functions of \( t \) only. The energy momentum tensor is taken into the form

\[ T^i_j = (\rho + p)v^i v^j + \rho g^i_j + E^i_j; \]

where \( E^i_j \) is the electro-magnetic field given by Lichnerowicz as

\[ E^i_j = \frac{1}{2} g^i_j v^3 + \frac{1}{2} g^3_i h^3_j; \]

and

\[ p = p v^i_i; \]

Here, \( \rho, p, \rho_0, \) and are the energy density, isotropic pressure, effective pressure, bulk viscous coefficient respectively and \( v^i \) is the flow vector satisfying the relation

\[ g_{ij} v^i v^j = 1; \]

is the magnetic permeability and \( h_i \) the magnetic flux vector defined by

\[ h_i = \frac{1}{2} F_{ij} v^j; \]

where \( F_{ij} \) is the dual electro-magnetic field tensor defined by Synge to be

\[ F_{ij} = \frac{\rho - g}{2} \epsilon_{ijk} F^{jk}; \]
$F_{ij}$ is the electro-magnetic field tensor and $i_{jk1}$ is the Levi-Civita tensor density. Here, the comoving coordinates are taken to be $v^1 = 0 = v^2 = v^3$ and $v^4 = \frac{1}{A}$. We take the incident magnetic field to be in the direction of $x$-axis so that $h_1 = 0 = h_2 = h_3 = h_4$. This leads to $F_{12} = 0 = F_{13}$ by virtue of (6). Also due to assumption of infinite conductivity of the fluid, we get $F_{14} = 0 = F_{24} = F_{34}$. The only non-vanishing component of $F_{ij}$ is $F_{23}$. The first set of Maxwell’s equation

$$F_{ij;kl} + F_{jk;il} + F_{ki;lj} = 0;$$  \hspace{1cm} (8)

leads to

$$F_{23} = \text{I (const.);}$$  \hspace{1cm} (9)

where the semicolon represents a covariant differentiation. Hence

$$h_1 = \frac{A \text{I}}{B \text{C}};$$  \hspace{1cm} (10)

The Einstein’s field equations with time-dependent cosmological constant

$$R_{ij} + \frac{1}{2} R g_{ij} + g_{ij} = 8 \pi T_{ij}; \quad (c = 1, G = 1 \text{ in gravitational unit})$$  \hspace{1cm} (11)

for the line element (1) has been set up as

$$8 A^2 \quad p \quad \frac{I^2}{2 B^2 C^2} = \frac{B_{44}}{B} + \frac{C_{44}}{C} \quad \text{A}^2;$$  \hspace{1cm} (12)

$$8 A^2 \quad p + \frac{I^2}{2 B^2 C^2} = \frac{A_{44}}{A} + \frac{C_{44}}{C} \quad \text{A}^2;$$  \hspace{1cm} (13)

$$8 A^2 \quad p + \frac{I^2}{2 B^2 C^2} = \frac{A_{44}}{A} + \frac{B_{44}}{B} \quad \text{A}^2;$$  \hspace{1cm} (14)

$$8 A^2 \quad p + \frac{I^2}{2 B^2 C^2} = \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} \quad \text{A}^2;$$  \hspace{1cm} (15)

The suffix 4 by the symbols $A$, $B$ and $C$ denote differentiation with respect to $t$. Equations (12)-(15) are four equations in seven unknowns, $A$, $B$, $C$, $p$, and $\text{I}$. Referring to Thorn,\textsuperscript{66} current observations of the velocity-redshift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today to within 30 per cent.\textsuperscript{67,68} Put more precisely, redshift studies place the limit

$$\frac{\text{H}}{0.3}$$

on the ratio of shear, $\text{p}$, to Hubble constant, $\text{H}$, in the neighbourhood of our Galaxy today.

Following Bali and Jain,\textsuperscript{62} we assume that the expansion in the model is proportional to the eigen value $\frac{1}{1}$ of the shear tensor $\frac{1}{1}$. This condition leads to

$$A = \left( A B C \right)^0;$$  \hspace{1cm} (16)
where \( n \) is the proportionality constant and

\[
\frac{1}{1} = \frac{1}{3A} \frac{2A_4}{A} \frac{B_4}{B} \frac{C_4}{C}
\]

\[
= \frac{1}{A} \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}
\]

for the metric (1). Solving these fields equations, we obtain

\[
A^2 = \frac{\sin^2 (at)}{K_1};
\]

\[
B^2 = \frac{N}{K_1 \frac{n}{(n+1)}} \sin^{\frac{n}{2}} (at) \tan \frac{at}{2};
\]

\[
C^2 = \frac{1}{N K_1 \frac{n}{(n+1)}} \sin^{\frac{n}{2}} (at) \tan \frac{at}{2};
\]

where \( M, N \) and \( L \) are constants of integration and

\[
a^2 = 16 \frac{r^2}{1};
\]

\[
K_1 = \frac{a^2}{n (2n-1)M};
\]

Hence the metric (1) reduces to the form

\[
ds^2 = \frac{\sin^2 (at)}{K_1} dx^2 - \frac{\frac{1}{2}}{n^2 M} a^{\frac{1}{n}} \sin^{\frac{2(n-1)}{n}} (at) dt^2
\]

\[
+ \frac{N}{K_1 \frac{n}{(n+1)}} \sin^{\frac{n}{2}} (at) \tan \frac{at}{2} dy^2
\]

\[
+ \frac{1}{N K_1 \frac{n}{(n+1)}} \sin^{\frac{n}{2}} (at) \tan \frac{at}{2} dz^2;
\]

where

\[
K_2 = n (2n-1)M .
\]

Using the transformations

\[
\bar{\varphi} = \bar{\chi} ;
\]

\[
N \bar{y} = \bar{Y} ;
\]

\[
\frac{\bar{z}}{N} = \bar{Z};
\]
If we consider \( p = \text{constant} \) and coefficient of bulk viscosity \( \eta = 0 \), we obtain the solutions obtained by Bali and Jain.\(^{62}\) If we set \( \beta = \text{constant} \), \( \eta = 0 \) and \( n = 1 \), we get the solutions obtained by Bali.\(^{71}\)
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The equations with bulk viscosity can be obtained from the general relativistic field equations as follows. To obtain these equations, we replace $p$ by the effective pressure $p$ given by (4). The expansion scalar $\frac{\dot{a}}{a}$ is given by

$$\frac{\dot{a}}{a} = \frac{p}{M} (n + 1) K_1^{\frac{n+1}{n}} \frac{\cos (aT)}{\sin^{\frac{n+1}{n}} (aT)}$$  \hspace{1cm} (27)

Eq. (22) can be rewritten as

$$8 (p) = \frac{K_1^{\frac{n+1}{n}} K_3 L^2 + (2 K_2 M) \sin^2 (aT) - a^2 K_1^{\frac{1}{2}}}{4 \sin^{\frac{n+1}{n}} (aT)} \frac{a^2 K_1^{\frac{1}{2}}}{4 \sin^2 (aT)} ; \hspace{1cm} (28)$$

For the specification of $\dot{a}$, now we assume that the fluid obeys an equation of state of the form

$$p = \frac{\rho}{3}$$  \hspace{1cm} (29)

where $\rho$ and $m$ are constants. If $m = 1$, Eq. (30) may correspond to a radiative fluid. However, more realistic models are based on $m$ lying in the regime $0 < m < \frac{1}{2}$.

### 3.1. Model I: ($m = 0$)

When $m = 0$, Eq. (30) reduces to $\dot{a} = 0$ and hence Eqs. (28), with the use of Eq. (22), (27) and (29) lead to

$$8 (1 + ) = \frac{8 (n + 1) \frac{p}{M} K_1^{\frac{n+1}{n}} \cos (aT)}{\sin^{\frac{n+1}{n}} (aT)} + \frac{K_1^{\frac{n+1}{n}} K_3 L^2 + (2 K_2 M) \sin^2 (aT) - a^2 K_1^{\frac{1}{2}}}{4 \sin^{\frac{n+1}{n}} (aT)} \frac{a^2 K_1^{\frac{1}{2}}}{2 \sin^2 (aT)} ; \hspace{1cm} (31)$$

Eliminating $\dot{a}$ between Eqs. (22) and (31), we obtain

$$\frac{L^2}{8 (n + 1) \frac{p}{M} K_1^{\frac{n+1}{n}} \cos (aT)} \frac{K_1^{\frac{n+1}{n}} K_3 \cos^2 (aT)}{\sin^{\frac{n+1}{n}} (aT)} + \frac{K_1^{\frac{n+1}{n}} K_3 \cos^2 (aT)}{4 \sin^{\frac{n+1}{n}} (aT)} \frac{L^2}{8 (n + 1) \frac{p}{M} K_1^{\frac{n+1}{n}} \cos (aT)} + \frac{a^2 K_1^{\frac{1}{2}}}{2 \sin^2 (aT)} ; \hspace{1cm} (32)$$

where $K_4 = 2n^2$. 3n 1.

### 3.2. Model II: ($m = 0$)
When \( m = 1 \), Eq. (30) reduces to \( \lambda = 0 \) and hence Eqs. (28), with the use of Eq. (23), (27) and (29) lead to

\[
8 = \frac{1}{\left(1 + \frac{p}{M K^{\frac{n}{n+1}} \cos(aT)}\right)}
\]

\[
2 \left(\frac{K^\frac{n+1}{n} 2 (K^l L^2) + 2K^2 M \sin^2(aT)}{4 \sin^{\frac{2(n+1)}{n}}(aT)}\right) \frac{a^2 K^\frac{l}{2}}{2 \sin^\frac{a}{2}(aT)} \tag{33}
\]

Eliminating \( t \) between Eqs. (23) and (33), we obtain

\[
8 = \frac{1}{\left(1 + \frac{p}{M K^{\frac{n}{n+1}} \cos(aT)}\right)}
\]

\[
2 \left(\frac{K^\frac{n+1}{n} 2 (K^l L^2) + 2K^2 M \sin^2(aT)}{4 \sin^{\frac{2(n+1)}{n}}(aT)}\right) \frac{a^2 K^\frac{l}{2}}{2 \sin^\frac{a}{2}(aT)} \tag{34}
\]

### Some Physical and Geometrical Features of the Models

We shall now give the expressions for kinematical quantities and the components of conformal curvature tensor. The scalar of expansion calculated for the flow vector \( v^i \) is already given by (27).

The rotation \( \theta \) is identically zero and the shear in the model, is given by

\[
2 = K^\frac{n+1}{n} \left[(2n-1)^2 M \cos^2(aT) + 3L^2\right] \frac{12 \sin^{\frac{2(n+1)}{n}}(aT)}{12 \sin^{\frac{2(n+1)}{n}}(aT)} \tag{35}
\]

The non-vanishing components of conformal curvature tensor are obtained as

\[
C^{12}_{12} = K^\frac{n+1}{n} \left[\frac{L^2 M + 2nM + 6nL \cos(aT) + K_2 M \sin^2(aT)}{12 \sin^{\frac{2(n+1)}{n}}(aT)}\right] \tag{36}
\]

\[
C^{13}_{13} = K^\frac{n+1}{n} \left[\frac{L^2 M + 2nM + 6nL \cos(aT) + K_2 M \sin^2(aT)}{12 \sin^{\frac{2(n+1)}{n}}(aT)}\right] \tag{37}
\]

\[
C^{14}_{14} = K^\frac{n+1}{n} \left[\frac{L^2 M + 2nM + K_2 M \sin^2(aT)}{12 \sin^{\frac{2(n+1)}{n}}(aT)}\right] \tag{38}
\]

The model represents an expanding, shearing and non-rotating universe in general. The Eq. (21) requires that \( n (2n - 1) \neq 0 \). If we choose \( n > 0 \), then \( p \) becomes infinite.
at \( T = 0 \) and \( T = 0 \). Eq. (27) shows that \( g = 1 \) at \( T = 0 \), \( T = 0 \) at \( T = 0 \) and \( T = 0 \). Thus the model expands with a big bang at \( T = 0 \), the expansion stops at \( T = \frac{69}{2} \) and collapses at \( T = \frac{a}{a} \) much like the closed FRW model. The magnetic field is responsible for this.

It is observed that the metric exhibits singularities at \( T = 0 \) and \( T = 0 \). At \( T = 0 \) and \( T = 0 \), the model (21) has singularity of Point type, Barrel type and Cigar type if \( T < 0 \) and \( T > 0 \) respectively as given by MacCallum and Islam. At the singularity \( T = 0 \), \( g_{22} = 1 \), \( g_{33} = 1 \) according as \( \frac{1}{n} \) \( T \) \( 0 \) and \( \frac{1}{n} \) \( T \) \( 0 \). Also at \( T = a \), \( g_{22} = 1 \), \( g_{33} = 1 \) according as \( \frac{1}{n} \) \( T \) \( 0 \) and \( \frac{1}{n} \) \( T \) \( 0 \). Thus inflationary scenario exists near the initial singularities \( T = 0 \) and \( T = 0 \) in the model.

The cosmic time \( t \) is given by

\[
Z = k_1 \sin^m (aT) dt ;
\]

where \( k_1 \) is constant. Near \( T = 0 \), \( \sin^m (aT) \) \( a = T \) \( T \) i.e., \( t = k_2 T \), where \( k_2 \) is constant. Thus \( T = 0 \) implies \( t = 0 \). When \( T = a \) then \( t = t \) constant. Hence the model has a finite time span in cosmic time scale. The space time is Petrov type D when \( L = 0 \) and non-degenerate Petrov type I otherwise.

The expressions for \( E = \frac{m \text{ magnetic energy}}{m \text{ material energy}} \) are given as follows:

\[
E = \frac{a^2 \sin^2 (aT)}{K_1 \left( \frac{3 \cos^2 (aT)}{L^2} \right) \left[ a^2 \frac{\sin^m (aT)}{n} \right] \sin^m (aT) ;}
\]

\[
= \frac{1}{2 \cos (aT)} \left( \frac{(2n + 1) \sin^m (aT) + \frac{3L^2}{M} \sin^m (aT)}{3(n + 1)^2} \right) .
\]

It is observed that when \( T = 0 \), then \( E = 0 \), which shows that the material energy is more dominant than magnetic near the initial singularity.

4. Bulk Viscous Solutions in the Absence of Magnetic Field

In the absence of the magnetic field, the pressure and density for model (24) are given by Eqs. (25) and (26) respectively. Eq. (25) can be rewritten as

\[
8 \left( \frac{p}{M} \right) = \frac{K_3 L^2 + (2K_2 M) T^2}{4 \left( K_2 T^2 \right)^{\frac{n}{n}} \left( K_2 T^2 \right)^{\frac{1}{n}}} \cdot \frac{1}{4 \left( K_2 T^2 \right)^{\frac{1}{n}}} ;
\]

where the expansion scalar \( \bigg( \bigg) \) is given by

\[
= \frac{L \left( n + 1 \right)}{\left( K_2 T^2 \right)^{\frac{n}{n}} ;}
\]

Using Eqs. (29), (30) and (43) in (42), we obtain

\[
8 \left( \frac{p}{M} \right) = \frac{K_3 L^2 + (2K_2 M) T^2}{4 \left( K_2 T^2 \right)^{\frac{n}{n}}} ;
\]
\[
\frac{1}{4(K^2T^2)^\frac{1}{n}}
\]  

(44)

4.1. Model I: \( m = 0 \)

When \( m = 0 \), Eq. (30) reduces to \( m = 0 \) and hence Eq. (44) with the use of Eq. (26) leads to

\[
8(1 + \frac{1}{2}) = \frac{8}{(K^2T^2)^\frac{1}{n}} + \frac{2(K_3L^2) + (2K_2M)T^2}{4(K^2T^2)^\frac{1}{n}} - \frac{1}{2(K^2T^2)^\frac{1}{n}}
\]  

(45)

Eliminating \( t \) between Eqs. (26) and (45), we obtain

\[
(1 + \frac{1}{2}) = \frac{8}{(K^2T^2)^\frac{1}{n}} + \frac{K_3L^2 + (2K_2M)T^2}{4(K^2T^2)^\frac{1}{n}} - \frac{1}{4(K^2T^2)^\frac{1}{n}}
\]  

(46)

4.2. Model II: \( m = 0 \)

When \( m = 1 \), Eq. (30) reduces to \( m = 0 \) and hence Eq. (44) with the help of Eq. (26) lead to

\[
8 = \frac{1}{1 + \frac{8}{(K^2T^2)^\frac{1}{n}}}
\]

\[
\frac{1}{4(K^2T^2)^\frac{1}{n}} + \frac{K_3L^2 + (2K_2M)T^2}{4(K^2T^2)^\frac{1}{n}} - \frac{1}{2(K^2T^2)^\frac{1}{n}}
\]  

(47)

Eliminating \( t \) between Eqs. (26) and (47), we obtain

\[
= \frac{1}{1 + \frac{8}{(K^2T^2)^\frac{1}{n}}}
\]

\[
\frac{1}{4(K^2T^2)^\frac{1}{n}} + \frac{K_3L^2 + (2K_2M)T^2}{4(K^2T^2)^\frac{1}{n}} - \frac{1}{2(K^2T^2)^\frac{1}{n}}
\]  

(48)

From Eqs. (46) and (48), we observe that the cosmological constant in both models is a decreasing function of time and it approaches a small value as time progresses (i.e., the present epoch). The values of cosmological “constant” for both models are found to be small and positive which are supported by the results from recent supernovae observations (Garnavich et al.; Perlmutter et al.; Riess et al.; Schmidt et al. )
Some Physical and Geometric Aspects of Models

In absence of the magnetic field, the expansion is already given by (43). The expansion in the model stops for large values of $T$. The shear in the model is given by

$$\gamma = \left(2n + 1\right)^2 \left(\frac{1}{M} + \frac{3L^2}{12K^{1/3}} \left(\frac{aT}{aT} + \frac{1}{n+1}\right)\right)$$

(49)

The components of conformal curvature tensor are given by

$$C_{11}^{11} = \frac{L^2 M + 2nM + 6nL^2 P}{12 \left(\frac{K T^2}{2n} \right)^{2n+1}}$$

(50)

$$C_{22}^{22} = \frac{L^2 M + 2nM + 6nL^2 P}{12 \left(\frac{K T^2}{2n} \right)^{2n+1}}$$

(51)

$$C_{14}^{14} = \frac{L^2 M + 2nM}{6 \left(\frac{K T^2}{2n} \right)^{2n+1}}$$

(52)

Thus, in the absence of magnetic field, the space-time is Petrov type $D$ if $L = 0$ and non-degenerate Petrov type I otherwise. For large values of $T$, the space-time is conformally flat. For $n > 0$, when $T$ is large, the shear dies out.

5. Conclusions

We have obtained a new class of Bianchi type I anisotropic magnetofluid cosmological models with a bulk viscous fluid as the source of matter. Generally, the models are expanding, shearing and non-rotating. In all these models, we observe that they do not approach isotropy for large values of time in either the presence or in the absence of magnetic field. The cosmological constant in all models given in Sec. 4 are decreasing function of time and they all approach a small value as time increases (i.e., the present epoch). The values of cosmological “constant” for these models are found to be small and positive which are supported by the results from recent supernovae observations. If we consider as constant and coefficient of bulk viscosity as zero, we obtain the solutions obtained by Bali and Jain. If we set $= constant = 0$ and $n = 1$, we get the solutions obtained by Bali. Thus, with our approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where adhoc laws were used to arrive at a mathematical expressions for the decaying vacuum energy. Thus our models are more general than those studied earlier.

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