Fractionalized Fermi liquids

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We will also discuss the quantum transition over a finite range of parameters. For \( n_e = 1 \) the FL* is determined by the density of conduction electrons alone. A number of earlier works [8,14] have considered a Fermi surface of ordinary \( S = 1/2 \), charge \(-e\), sharp quasiparticles, enclosing a volume

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as the exchange couplings are varied: this transition is preempted by a superconducting state.

We note that the FL* state does not contradict the non-perturbative computation by Oshikawa [8] of $\mathcal{V}_{FL}$; on the contrary, this argument helps establish the intimate connection between FL and topological order. Oshikawa placed the system on a torus, and considered the adiabatic evolution of the ground state upon threading a magnetic flux of $\hbar c/e$ felt by the electrons with spin up (in some basis) through one of the holes of the torus. For insulating antiferromagnets with a fractionalized spin liquid ground state (a resonating valence bond (RVB) state), this procedure connects two of the topologically distinct states which become degenerate in the thermodynamic limit in a toroidal geometry [4,5,6]. i.e. it connects states with and without a vison threading the hole of the torus. The FL* state of the Kondo lattice models we are discussing here has a similar topological order, and the toroidal system has global vison excitations which are degenerate with the ground state in the thermodynamic limit. Oshikawa did not consider such excitations, and only included the electron-like Fermi surface quasiparticles. Consequently, his argument does not directly apply to the FL* state, and a modification accounting for vison excitations shows that the volume $\mathcal{V}_{FL}$ is allowed. In other words, the Fermi volume is still quantized, but different from that in a Fermi liquid.

The volume $\mathcal{V}_{FL}$ is observed in many compounds, and in particular in those with weak direct exchange $J_H$ between different local moments. Doniach [17] pointed out that increasing $J_H$ would lead to magnetically ordered states. However, the effective exchange interactions between the local moments are strongly frustrated in many common lattices, so that the magnetic order may be very fragile or entirely absent: it is these frustrated systems which are favorable candidates for displaying a non-magnetic FL* state. The generic appearance of superconductivity in the crossover between the $Z_2$ FL* and FL states is experimentally significant: this may be regarded as a proposed ‘mechanism’ for superconductivity in heavy fermion systems, which bears some similarity to the RVB theory [8]. The critical temperature ($T_c$) for the onset of superconductivity, $T_c$, can be small.

The $T > 0$ behavior of the $Z_2$ FL* state depends on $d$, as discussed for other fractionalized states in Ref. [1]. In $d = 3$ there is a finite temperature phase transition associated with the onset of topological order. This is absent in $d = 2$ where the topological order is present only at $T = 0$. In layered quasi-two dimensional materials, both types of behavior (corresponding to two distinct $T = 0$ phases) are possible.

To understand the origin of our results in the context of [1], consider first the limiting case $J_K = 0$, when the $c_\sigma$ fermions and the $\mathcal{S}_j$ spins are decoupled. While the $c_\sigma$ fermions will occupy states inside a Fermi surface enclos-
charge $e$ and are spin singlet: Condensation of these bosons implies a non-zero amplitude that a local moment has formed a Kondo singlet with the conduction electrons. This condensation indicates that the $Z_2$ gauge theory enters a Higgs phase which can also be identified with a phase in which $Z_2$ charges are confined [24]. Moreover, as the spinon pairing amplitude $\langle \varepsilon_{\sigma'} f_{\sigma'} f_{\sigma} \rangle$ is generically non-zero in the small $J_K$ fractionalized phase [11], the condensation of $B_1$ implies condensation of $B_2$ (and vice-versa), and there is only a single $Z_2$ confinement transition. More importantly, the pairing of the spinons and the condensation of $B_{1,2}$ implies that the resulting phase also has pairing of the conduction electrons, and is a superconductor at $T = 0$.

Consider now the behavior when $J_K, t \gg J_H$. In the limit $J_H = 0$, the usual FL state is expected (at least at generic incommensurate conduction electron density). Turning on a weak non-zero $J_H$ potentially introduces a weak instability toward superconductivity, as will be the case in our mean-field theory below. However, the FL state may still be stabilized by a weak nearest neighbor repulsive interaction between the conduction electrons.

The general considerations above can be illustrated by a simple mean-field computation of the phase diagram of $H$. We applied the large $N$ method associated with a generalization of $H$ to $Sp(N)$ symmetry [25] on the triangular lattice. It is important to note that both the symmetry group and the lattice have been carefully chosen to allow for a mean field state with $Z_2$ topological order, stable under gauge fluctuations [12]; in particular, there are topologically distinct mean-field ground states in a toroidal geometry, differing in the $Z_2$ flux through the holes of the torus. Other choices [8] for the lattice or the symmetry group lead to mean-field solutions which are generically disrupted by $U(1)$ or $SU(2)$ gauge fluctuations in $d = 2$. We used self-conjugate, fully antisymmetric (fermionic) representations for the spin states, and the computations were then similar to earlier work on the $t$-$J$ model [25]. For $J_K = 0$ and nearest neighbor $J_H$, these representations yields globally stable solutions in which the $S_j$ spins are paired in fully dimerized states which break lattice symmetries. As we are not interested in such states here, we restricted our analysis to saddle points which preserve all lattice symmetries. Such RVB saddle points can be stabilized by additional couplings between the local moments; they are also stable for nearest-neighbor $J_H$ for bosonic spin representations [20,25], but these, unfortunately, do not allow a simple description of the FL state at large $J_K$. It is possible that the spinons undergo a change from bosonic to fermionic statistics with increasing $J_K$ within the FL* state, but this will not be captured by our present mean field theory which has only fermionic spinons.

The phase diagram is shown in Fig 1 as a function of $J_K$ and $T$ for fixed $J_H$, $t$, and $\rho_c$. 

\[ \begin{array}{|c|c|c|}
\hline
J_K & T & \text{Phase} \\
\hline
0 & 0 & \text{FL} \\
0.1 & 0 & \text{Superconductor} \\
0.2 & 0.1 & \text{decoupled} \\
0.3 & 0.2 & \text{FL*} \\
\hline
\end{array} \]

FIG. 1. Mean field phase diagram of $H$ on the triangular lattice. We used fermionic representations of $Sp(N)$ for the spins, and restricted attention, by hand, to saddle points which preserve all lattice symmetries. We had nearest-neighbor $t = 1$, $J_H = 0.4$, and $\rho_c = 0.7$. The superconducting $T_c$ is exponentially small, but finite, for large $J_K$, while it is strictly zero for small $J_K$. Thin (thick) lines are second (first) order transitions. The transitions surrounding the superconductor will survive beyond mean field theory, while the others become crossovers.

In addition to the $Z_2$ FL* and FL states, and an intermediate superconducting state, whose character we have already discussed, there is also a high temperature “decoupled” state. Here, in the mean field saddle point, the spins are mutually decoupled from each other, and from the conduction electrons. This decoupling is, of course, an artifact of the saddle point, and it points to a regime where all excitations are incoherent but strongly interacting with each other. For the case where the superconducting phase is present only at very low temperatures (as may well be the case beyond mean field theory), this incoherent regime represents the quantum-critical region of the $Z_2$ FL*-FL transition. A separate description of this incoherent quantum critical dynamics was provided by the large-dimensional saddle point studied by Burdin et al. [11], where it was related to the gapless spin liquid state of Ref. [27].

An interesting $T = 0$ quantum phase transition appearing in Fig 1 is that between the FL* and superconducting states. As we discussed earlier, this transition is associated with the condensation of the charge $c$ bosons $B_{1,2}$. A critical theory of the transition can be written down in terms of $B_{1,2}$ and the conduction electrons: the methods and resulting field theory are identical to those discussed in Ref. [28]. The renormalization group analysis shows that the $T = 0$ transition can be either first or second order, depending upon the values of microscopic parameters. The gapped vison excitations in the FL* state may be detected through the flux trapping experiments discussed in Ref. [29]. Furthermore, provided the transition is not too strongly first order, the presence of a critical charge $c$ bosonic mode implies that the su-
perconducting state in the vicinity of this transition is a candidate for displaying stable $he/e$ vortices [3][4].

Interesting physics obtains in the presence of an external uniform Zeeman magnetic field in the FL* state. As the local moment and conduction electron systems are essentially decoupled in this phase, they both respond independently to the magnetic field. If the spinons are gapped in the fractionalized phase, then there would be a critical field $B_c$ associated with the onset of magnetization in the local moment system. Experimentally, this would be seen as a "metamagnetic" transition in the response of the system to an applied field. Interestingly, this onset transition could clearly be generically (i.e. without any fine tuning) second order. Metamagnetic quantum criticality in strongly correlated systems has been the subject of some recent experimental [5][8] and theoretical studies [6], although accidental fine tuning has been invoked to obtain a second order transition.

This paper has established that metals with local moments in dimensions $d \geq 2$ can have non-magnetic ground states (FL*) which are distinct from the familiar heavy Fermi liquid state (FL). The latter state has a Fermi surface enclosing a volume $V_{FL}$ determined by the density, $\rho_0$, of both the conduction electrons and local moments; our topologically ordered FL* state has sharp electron-like excitations on a Fermi surface enclosing a volume $V_{FL^*}$ determined by $(\rho_0 - 1)$ (for $n_\ell = 1$ this is the density of conduction electrons alone), along with additional ‘fractionalized’ excitations. In between these FL and FL* states, a plethora of additional states associated with magnetic, superconducting, and charge order appear possible, along with non-trivial quantum critical points between them. We believe this rich phenomenology should find experimental realizations in the heavy fermion compounds.

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