Production of complex particles in low energy spallation and in fragmentation reactions by in-medium random clusterization.

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Abstract

Rules for in-medium complex particle production in nuclear reactions are proposed. These rules have been implemented in two models to simulate nucleon-nucleus and nucleus-nucleus reactions around the Fermi energy [1, 2]. Our work emphasizes the effect of randomness in cluster formation, the importance of the nucleonic Fermi motion as well as the role of conservation laws. The concepts of total available phase-space and explored phase-space under constraint imposed by the reaction are clarified. The compatibility of experimental observations with a random clusterization is illustrated in a schematic scenario of a proton-nucleus collision. The role of randomness under constraint is also illustrated in the nucleus-nucleus case.

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I. INTRODUCTION

Nuclear reactions around the Fermi energy have revealed that nuclei can break into several pieces of various sizes: the so-called multifragmentation process. A striking feature of experimental observation is the large number of charge and energy partitions that can be accessed. In order to understand the statistical aspects of the explored phase-space, several physical origins have been proposed. Among them, the nuclear liquid-gas phase transition appears as one of the best candidate. However, due to the complexity of nuclear reactions including impact parameter mixing, pre-equilibrium emission and thermal decay, it is hard to trace-back the process of cluster formation. Nowadays, the complexity of experimental analyses increases constantly. Conjointly, more and more elaborated models have been proposed to simulate reactions. However, the issue concerning cluster formation remains a highly debated question. In this work, we would like to contribute to the discussion on particle emission during the pre-equilibrium stage. We have tested a large number of hypothesis for the formation of clusters in the nuclear medium in order to provide event generators for the study of nuclear reactions. Guided by the experimental observation, surprising conclusions concerning the way cluster are formed may be assessed. Simple rules have been found for the formation and the emission of complex particles. The hypothesis retained are not only fully compatible with experiments on multifragmentation, but seems also to be adequate for nucleon-nucleus reactions in the same energy range.

The paper is organized as follows: first the rules for cluster formation and emission are introduced and illustrated in a schematic scenario for experiments. In a second part, additional effects that should be accounted to compare quantitatively with measurements are discussed. Finally, the compatibility of the rules with data are illustrated. Conclusions and perspectives are drawn at the end of the paper.

II. RULES FOR THE FORMATION AND THE EMISSION OF CLUSTERS

In this section, rules for the cluster formation and the production of fragmentation partitions are defined. In order to illustrate these rules we consider a proton colliding a heavy
FIG. 1: Schematic representation of a three step nucleon-induced reaction. A nucleon with beam energy close to the Fermi energy is first absorbed by a nucleus. Then two steps are identified for pre-equilibrium emission: the formation of the cluster and its emission.

target with an energy close to the Fermi energy \(^1\). Then a simplified three steps scenario is considered (see figure 1). First the incident nucleon is absorbed. The second step corresponds to the in-medium formation of the cluster while the last step is the emission in the continuum.

FIG. 2: Correlation between the position and the kinetic energy per nucleon for the \(\alpha\) particles using a random sampling assumption for the nucleons forming the \(\alpha\) particle. This two-dimensional plot corresponds to the total ”accessible” phase-space for the considered particle.

The hypothesis retained to describe the last two stages can be summarized as follow:

- **Cluster formation:** Considering a cluster of mass \(A_c\) and charge \(Z_c\) formed in the

\(^1\) The energy is chosen small enough to avoid strong influence of direct two-body nucleon-nucleon collisions.
medium, we assume that the cluster is composed of nucleons chosen randomly in the
target. Thus, the kinematics of the cluster is directly linked to the kinematics of the
nucleons. This defines the "total accessible phase-space" for the cluster \((A_c, Z_c)\) in the
medium. Figure 2 displays the correlation between the position \(r_c\) and the kinetic
energy \(\varepsilon_k\) of an \(\alpha\) particle produced in a Pb target obtained with the random sampling
assumption. We would like to stress that the random assumption is a generalization
of the pioneering work of Goldhaber \[16\].

- **Cluster emission:** we dissociate here the total accessible phase-space from the ex-
explored phase-space because the latter must take into account the constraints of the
reaction. Indeed, while the first rule described above leads to a large set of configu-
rations, all configurations will not necessarily lead to the emission of a cluster. Two
constraints can be identified. The first one, which is independent of the entrance
channel, is due to the mutual interaction between the cluster and the heavy emitter
\(^3\). Figure 3 shows an example of such an interaction in the case of an \(\alpha\) particle and
a \(\text{Hg}\) nucleus. In a classical picture, the cluster cannot escape from the heavy nucleus
if its energy is below the emission barrier. Let \(V_{A+A_c}(r_c)\) be the interaction potential
and \(V_B\) the associated barrier. We have the "local" condition

\[
\varepsilon_k(r_c) \geq V_B - V_{A+A_c}(r_c)
\]

leading to a lower limit on the cluster kinetic energy.

The second constraint is directly dependent on the reaction type and is due to the
energy balance. Indeed, the accessible configuration is further reduced due to the total
energy available in the reaction. In the simplified scenario presented here (accounting

\[2\] In the energy range considered here, a Thomas-Fermi distribution corresponding to the ground state of
the target is assumed. This means that in-medium nucleon-nucleon collisions are neglected in this first
approach. Such a sudden approximation is partially relaxed in a more realistic situation.

\[3\] Since, we are considering here rather low beam energy leading to small available energy, we do not expect
that two clusters are emitted at the same time, thus the outgoing channels are essentially binary. In
addition, the use of a heavy target is very helpful since in that case, due to the small available energy
in entrance channel, no particle can be emitted in the secondary decay stage. Therefore, in experimental
data, detected clusters are issued from the pre-equilibrium stage only.

\[4\] Due to the large mass assymetry, it is assumed for simplicity that the heavy target is at rest in the
laboratory frame.
for the fact that the initial nucleon is absorbed), we have the following inequality

\[ E_B - Q - V_{A+A_c}(r_e) \geq \varepsilon_k(r_e) \] (2)

which gives an upper limit. Here \( E_B \) denotes the incident energy while \( Q \) is the Q-value of the reaction. It is worth to notice that the second condition depends not only on the beam energy but also on the configuration itself. Therefore, only a fraction of the total phase-space accessible for the cluster will indeed lead to emission in the continuum. This fraction corresponds to the "explored phase-space" which takes into account the energy constraints induced by the reaction.

These two constraints are shown in Fig. 3 (top left) for a proton-induced reaction at \( E_B = 39 \) MeV. There, an \( \alpha \) particle can only be emitted in a small interval of kinetic energy (called "escape window" in the following) leading to a significant restriction in phase-space. The fraction of the phase-space available for the cluster emission is shown in Figure 3 (top-right). According to the energy constraint, all configurations between the two lines lead to the emission of an \( \alpha \) particle.

### A. Direct application of the rules

The prescription described above are compatible with experimental data as shown in the case of a proton-induced reaction at \( E_B = 39 \) MeV. Assuming that the proton is absorbed by the target, a Monte-Carlo sampling (using the cluster creation rules) of the \( \alpha \) particle is obtained in order to obtain the initial configurations in the medium. Then, using the emission rules, only those configurations allowed by the energy constraint are conserved. Last, each conserved configuration is propagated in the target potential. Thus, the \( \alpha \) kinetic energy distribution is obtained and successfully compared with the experimental data (from [18]). This is shown in Figure 3 (bottom part) where the calculated spectrum (open square) is compared to experimental data (filled circles). A similar agreement is found for the emission of protons, deuterons and tritons (see Fig. 4). However in this case, direct reactions are also present in the experimental data leading to an additional contribution at high energy.
III. TOWARDS NUCLEAR REACTIONS

Direct application of the rules to the simplified three steps scenario described above allows only qualitative comparisons with data. In order to provide quantitative comparisons, additional effects must be considered. Two phenomenological models (called n-IPSE$^5$ [2] and

\[ n \text{-IPSE: nucleon-Ion Phase-Space Exploration} \]
FIG. 4: From top to bottom, calculated kinetic energy distributions (open squares) obtained for proton, deuteron, triton and alpha particles. Distributions are compared to experimental data (open circles).

HIPSE\textsuperscript{6} \cite{1} based on the very same assumptions have been developed and confronted with the experimental data. We only give here the physical effects that have been added on top of the rules:

- **In medium nucleon-nucleon collisions:** At energies below the Fermi energy, the effect of in medium two-body collisions is small. However, as the beam energy increases, such collisions must be taken into account. Accordingly, the initial Thomas-Fermi distributions are distorted by two-body effects.

- **Influence of the impact parameter:** In nucleus-nucleus collisions, geometrical aspects associated with the impact parameter are accounted for by using a participant-spectator picture. In nucleon-induced reactions, this picture is replaced by the ”influence area” (equivalent to the participant region) notion which defines the number of

\textsuperscript{6} HIPSE: Heavy-Ion Phase-Space Exploration
nucleons of the target affected by the projectile.

- **One-body dissipation and nucleon absorption**: Depending on the incident energy, particles are exchanged by the two partners of the reaction: This is treated by means of a phenomenological parameter (see [1]). In nucleon induced collisions, this process is replaced by a probability that the incoming nucleon be absorbed by the target.

- **Application of cluster formation rules and ”nucleosynthesis” in the medium**: We have described previously a rule to obtain cluster properties when a single cluster is formed in the medium. However, the number of clusters is not a priori fixed. In order to solve this difficulty in both models, a coalescence algorithm is used to form clusters starting from the nucleons in the participant region. In the nucleon-induced reaction case, it was possible to show explicitly that this coalescence is equivalent to a random sampling assumption.

- **Application of cluster emission rules and Final-State interaction (FSI)**: After the coalescence stage, many configurations are accessible. However due to the energy-balance generalized to the many cluster case, only part of the accessible phase-space is really explored. In addition, if the relative energy between two clusters is lower than the barrier associated with their mutual interaction they will not separate during the expansion. In a realistic model, the recombination of fragments is allowed. In HIPSE, possible ”re-fusion” of fragments is accounted for before the freeze-out configuration is reached. This process can lead to important FSI’s and may relax completely the participant-spectator picture. For instance, the quasi-target and the quasi-projectile can fuse.

- **Freeze-out and the after-burner stage**: When the available energy is large, fragments are excited and once the chemical and thermal freeze-out are reached, the possible in-flight de-excitation of each cluster must be taken into account. This induces a complex mixing of pre- and post-equilibrium emission.

More details on technical aspects can be found in ref. [1, 2]. The important point we would like to stress is that rather different experimental data can be described using the same hypothesis on the production and the emission of clusters.
IV. CONTACT WITH EXPERIMENTAL DATA: TWO ILLUSTRATIVE EXAMPLES

A detailed comparison of the models with the experimental data can be found in [1, 2]. Here, we first concentrate on nucleon-induced reactions. Figure 5 shows a comparison between the kinetic energy differential cross-section of light clusters calculated with n-IPSE and data [19]. Note that there is no normalization between the data and the calculation. As a reference, we also show (right part of the figure) the calculated spectra obtained with GNASH [20]. A good agreement between the results of n-IPSE and the experimental data is obtained. This is true for a wide range of beam energy from 37 MeV to 135 MeV in both proton and neutron induced reactions on medium and heavy nuclei.

FIG. 5: Left: Kinetic energy differential cross-section of proton, deuteron, triton and alpha particles, obtained with the n-IPSE model calculation (solid line) for neutron induced reaction on $^{208}$Pb at beam $E_B = 96$ MeV (from [19]). Right: distributions obtained using the GNASH model [20].

Concerning nucleus nucleus reactions, a systematic comparison with the INDRA data [17] have demonstrated that the HIPSE model is able to reproduce not only the average properties [2] but also the fluctuations of the experimental observables [21]. It appears that besides mean properties and fluctuations, “internal” correlations inside each event are also correctly reproduced as shown in Figure 6 [22] where the distribution of the relative velocity (top) and the relative angle (bottom) between the three largest fragments taken two-by-two are presented for the reaction Xe+Sn at three different beam energies ($E_B = 25, 50, 80$ MeV/A). In each case, the calculated spectra are compared with the INDRA data [17]. The very good
FIG. 6: Distributions of the relative velocity (top) and the relative angle (bottom) between the three largest fragments taken two-by-two for the Xe + Sn system at three different beam energies. From left to right, the beam energies $E_B = 25$, 50 and 80 MeV/A are considered. In each case, the calculated spectra are compared with the INDRA data (filled circle). Events considered here correspond to well detected events (80 % of the total charge and impulsion). The same selection is used for the "filtered" HIPSE events.

agreement between HIPSE and the INDRA data gives additional proof of the compatibility between the rules for the production and the emission of complex particles and the experimental observables. Note that a similar agreement has been is found in different symmetric systems\(^7\).

\(^7\) Please note that, in both models, a few free parameters are used: they are related to the description of the participant region, the nucleon-nucleon collision rate, the exchange and absorption nucleon processes. Such parameters depend only on the beam energy. This means, that a single set of parameters is used to reproduce simultaneously nucleon reactions on Fe, Pb and U. Similarly, the parameters adjusted on Xe+Sn reactions have been used to the Ni+Ni and the Au+Au cases giving reasonable agreement with the data.
V. CONCLUSION

We have described simple rules that may be used to describe the pre-equilibrium emission of clusters in the course of nuclear reactions. These rules are based on a random sampling of the nucleons taking into account the Fermi motion and a proper account of nuclear effects as well as the conservation laws. Using these rules leads to a good agreement with data obtained from nucleus-nucleus reactions around the Fermi energy and surprisingly also for nucleon-nucleus reactions.

We would like to mention, that even if the complete randomness hypothesis appears compatible with the experimental data, this do not give any indication on the physical origin of randomness and several effects can be invoked: phase-transition, turbulence, self-organized criticality, quantum decoherence ...

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see also the INDRA Web site: [http://infodan.in2p3.fr](http://infodan.in2p3.fr)

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